Homework 2

Simulation and Performance Evaluation – University of Trento

DEADLINE: April 25, 2023

You can solve the following assignments using any programming language. In doing so, try to implement the formulas explained in class. You can use built-in functions in your programming language to draw random variates, compute means and standard deviations, as well as for any utility purposes (e.g., for managing data, sorting, plotting, printing messages etc).

Exercise 1

In wireless communications, the Rayleigh pdf is typically used to model the small-scale statistical fluctuations that affect the amplitude gain of non-line-of-sight (NLoS) radio propagation paths:

$$f_{\text{Rayl}}(x) = \frac{x}{\sigma^2} \exp\left(-\frac{x^2}{2\sigma^2}\right) , \tag{1}$$

whereas the Rice distribution models the statistical fluctuations of line-of-sight (LoS) channels:

$$f_{\text{Rice}}(x) = \frac{x}{\sigma^2} \exp\left(-\frac{x^2+\nu^2}{2\sigma^2}\right) I_0\left(\frac{x\nu}{\sigma^2}\right) . \tag{2}$$

In (2), $I_0(x)$ is the modified Bessel function of the first kind and order 0. There exist utilities that implement $I_0(\cdot)$ in most programming languages, e.g., check `besseli` in Matlab/Octave or `scipy.i0` in Python, or `besselI` in R.

Use rejection sampling to draw from the Rayleigh distribution for $\sigma = 1$, and from the Rice distribution for $\sigma = 2$ and $\nu = 2$. (*Hint: use an exponential pdf as a bounding function.*)

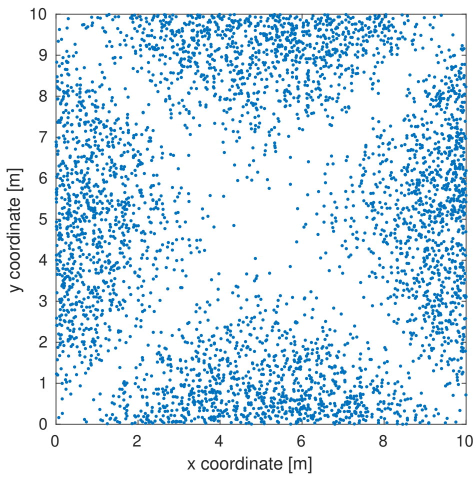
In both cases, compute the average number of trials required to draw a variate. For the Rice case, is it more convenient to just increase the saling factor or also to change the mean of the bounding exponential pdf?

Facultative: in the Rayleigh case, find the optimal scaling factor that you need to apply to the exponential pdf in order to minimize the number of attempts before a random variate draw is accepted.

Exercise 2

We are going to simulate a simplified case of wireless communications, both without and with interference. Follow the below steps.

1. Let us start from the scenario. Throw wireless terminals at random in a square of 10 m × 10 m, in such a way that the density of the terminals is minimum along any of the 2 diagonals, and progressively increasing towards the four sides. (*Hint: follow the indications on slides 66-67 of the module on RNGs and modify the method to suit your needs.*) The result should be like in the figure below.



2. Take pairs of 2 terminals at random, let one of them be the "transmitter" (TX) and the other be the "receiver" (RX). Compute the Signal-to-Noise Ratio (SNR) γ as follows:

$$\gamma = \frac{P_{\text{tx}} \cdot d_{\text{tx,rx}}^{-2}}{N} , \tag{3}$$

where $P_{\text{tx}} = 0.1$ Watt (or equivalently, 20 dBm), $N = 1.6 \cdot 10^{-4}$ Watt, and $d_{\text{tx,rx}}$ is the distance between the TX and the RX. Assuming that a transmission is correctly received whenever $\gamma > 8$ (or 6 dB), find the probability of success of a transmission and give a 95% confidence interval for the probability of success. You will probably need to repeat your simulation for several different draws of the scenario. However, you definitely can draw multiple pairs from the same scenario before changing to a new one.

3. Now consider a case with interference. After drawing the terminals, take them in 3-tuples, let the first of each tuple be the TX, the second be the RX, and the third be an interfering TX. The SNR now should take interference into consideration, so consider this modified formula for the signal-to-noise-and-interference ratio (SINR):

$$\gamma_I = \frac{P_{\text{tx}} \cdot d_{\text{tx-rx}}^{-2}}{N + P_{\text{tx}} \cdot d_{\text{txI,rx}}^{-2}} , \tag{4}$$

where $d_{\text{txI,rx}}$ is the distance between the interfering transmitter and the receiver. Compute the probability of success of a transmission in this case and give a 95% confidence interval. Check a few cases where the transmission is successful: what do you observe?

4. Repeat points 2 and 3 for a case with fading affecting each transmission. For the case of the SNR, the new formula to consider is:

$$\gamma_{\text{fad}} = \frac{P_{\text{tx}}\xi_{\text{tx}} \cdot d_{\text{tx,rx}}^{-2}}{N} , \tag{5}$$

where ξ_{tx} is an exponentially distributed random variable of mean $1/\lambda = 1$. For the case of the SINR, the new formula to consider is:

$$\gamma_I = \frac{P_{\text{tx}}\xi_{\text{tx}} \cdot d_{\text{tx-rx}}^{-2}}{N + P_{\text{tx}}\xi_{\text{I}} \cdot d_{\text{txI,rx}}^{-2}} , \tag{6}$$

where ξ_{tx} and ξ_{I} are *independent* exponentially distributed randoms variable of mean $1/\lambda = 1$, that should be drawn anew for each transmission. Do you observe any differences?