# Homework 3

Simulation and Performance Evaluation – University of Trento

**DEADLINE: May 14, 2024**

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You can solve the following assignments using any programming language. You are allowed to use any utility functions you wish.

## Exercise 1

Consider an M/M/1 queue-server system. Remember that this means that arrivals follow a Poisson process of rate $\lambda$, and services (or departures) follow a Poisson process of rate $\mu$, where you should have $\mu > \lambda$, e.g., $\lambda = 1$ and $\mu = 2$. Moreover, there is a single server and the queue is managed according to a FIFO policy.

Implement a discrete-event simulator to evaluate the performance of the M/M/1 system. The simulator should manage at least the below events:

- Start of the simulation
- End of the simulation
- Arrival of a packet
- Departure of a packet

To do this, create first an ordered queue/list of events where:

- Every event always links to the one that immediately follows it in time;
- When you insert an event in the queue, you always insert it in order of increasing time; (i.e., say that the queue contains three events: event 1 taking place at time $t_1$ and linking to event 2, which takes place at time $t_2$ and which links to event 3 at time $t_3$; if another event 4 taking place at time $t_4$ is inserted in the queue, and $t_2 < t_4 < t_3$, then you have to make event 2 link to event 4, and event 4 link to event 3.

Finally implement the system behavior as seen in class, namely:

- When a packet arrives: if the server is free, seize the server and schedule the departure of the packet; if the server is busy, increase the number of packets in queue;
- When a departure event is triggered: if the queue is empty, release the server; otherwise keep the server busy and schedule the next departure event.

Use your simulator to do the following:

1. Show how the number of packets in the system (those in queue plus those currently in service) varies over time. Compare your results with the theoretical average number of packets in the system in stationary conditions, $\rho/(1-\rho)$, where $\rho = \lambda/\mu$.
2. Play with $\lambda$ and $\mu$, and discuss how their values affect the convergence of the system to the theoretical value.

(*Hint*: you will need to run your simulator several times to do the above. Remember the contents of the class on output analysis.)

## Exercise 2

In the same setting of Exercise 1, consider now a different service model where the service time obeys a (long-tailed) Pareto distribution

$$f(x) = \begin{cases} \left(\frac{x_m}{x}\right)^\alpha & x \geq x_m \ , \\ 0 & x < x_m \ . \end{cases} \tag{1}$$

E.g., use the values $x_m = 0.5$ and $\alpha = 1.5$. Consider that the average service time is $\alpha x_m/(\alpha - 1)$.

Check the evolution of the number of packets in the system for different values of the arrival rate $\lambda$ and discuss how the results change with respect to those of Exercise 1.