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Exercise 1

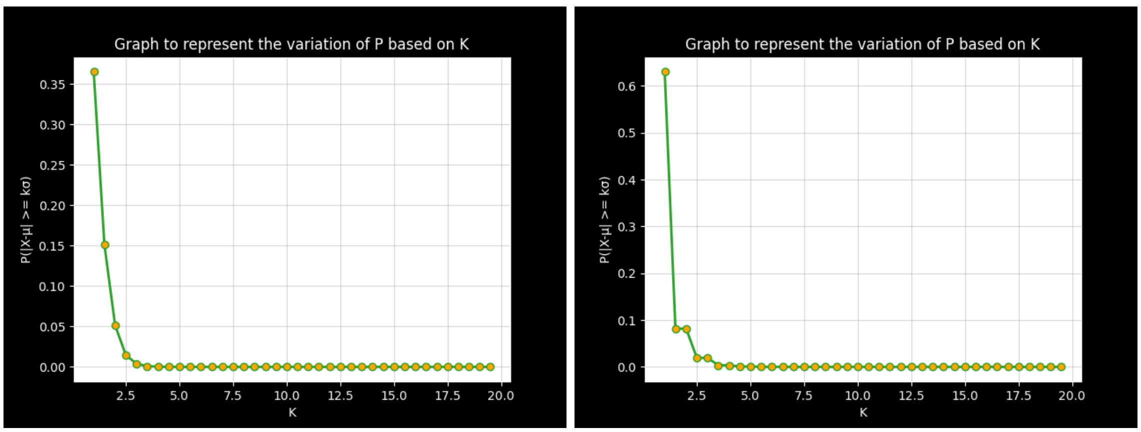
Chebyshev's inequality states that for any random variable X with finite expected value μ and finite standard deviation σ, the probability that X will be a certain number of standard deviations from the mean is given by the following inequality:

$$P(|X - \mu| >= k\sigma) <= 1/k^2$$

In other words, the probability that a random variable will be more than k standard deviations from the mean is less than or equal to $1/k^2$. This inequality holds for any random variable, regardless of its distribution.

We calculate the veritable of this inequality for a Poisson random variable and deepen the study by changing the value of k increasing it by 0.5 for each step and plotting it. The results it give us verify the inequality for all the possible values that k assume.

The plot in Figure 1 give us the value of the probability calculated as k increase: we can see that the more bigger the k is the less the probability is.

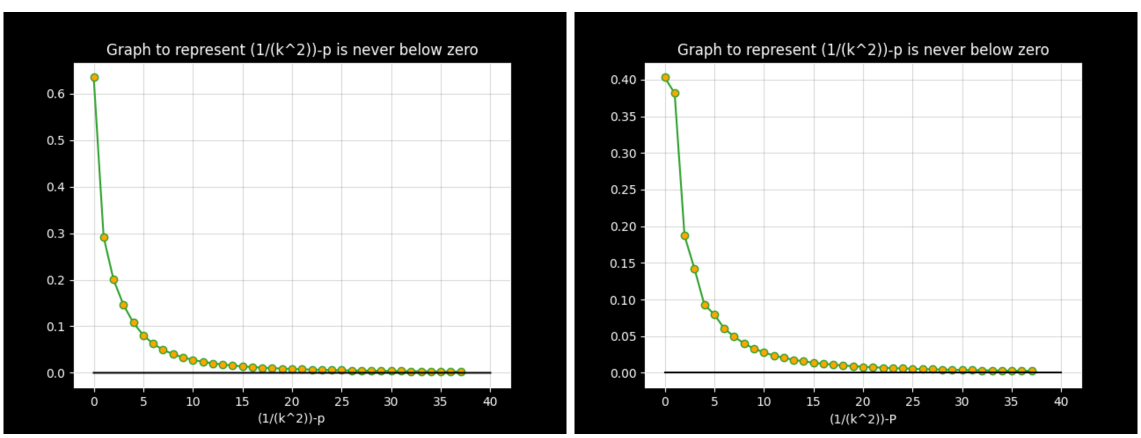


(a) lambda=15.

(b) lambda=1

Figure 1: Graph to represent the variation of P based on K

The plot in Figure 2 give us the difference between $1/k^2$ and P: we can see that the difference is never negative so $1/k^2$ will always be greater than P and his lowest value will be 0.

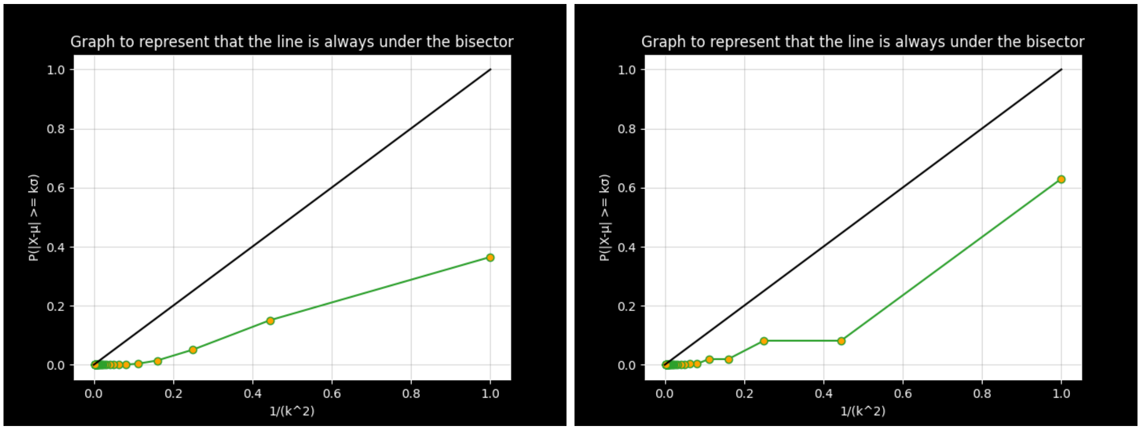


(a) lambda=15

(b) lambda=1

Figure 2: Graph of $(1/k^2)$-P is never below zero.

The plot in Figure 3 is the curve with coordinate ($1/k^2$, P): we can see that it never surpass the bisector and as a consequence $1/k^2$ is always grater than P. The more bigger is lambda the smaller will be the coefficient of the curve's declivity.



(a) lambda=15 (b) lambda=1

Figure 3: Graph is always under the bisector

The seem apparent duplication of the values two by two present in the graph with lambda=1 that we can see more clearly in Figure 1(b) is due the step used to choose the value k; the step used is of 0.5 change from the previous one and because sigma is the $\sqrt{(}var)$ in which var = lambda and in this case is equal to 1 the $\sqrt{(}lambda)$ become an irrelevant calculus that not change the final value which will always be an integer, so the k's variation of 0.5 do not change from the previous result if k not grow of a whole number.

Exercise 2

The probability that a random variable uniformly distributed in the interval [3,5] is greater than a random variable uniformly distributed in the interval [2, 4] using a simulation resulted to be between 87% and 88%. The simulation was based on random variables uniformly distributed created from a chosen seed to permit the repeatability of the execution. The variable included in the interval [4,5] of the interval [3,5] and the variables in [2,3] of the interval [2,4] can be momentarily excluded from the calculus because these interval do not influence the result seeing as for all value in X[4,5] the condition of X>Y is always satisfied and the same is for Y[2,3]

The interested part of the interval is the interval [3,4] because it is overlapped by the two distribution. I calculate all the possible combinations for the couple of numbers (X[3,4], Y[3,4]) and counted the case in which the condition of X>Y is the opposite, i.e. Y>X to facilitate the following operations.

The result of this operation will be all the cases in which the event of X>Y do not occur. So the simple operation of all possible event minus the event we don't want to occur give us the events in which the condition X>Y happens. This number divided by all possible events give us the probability of the condition X>Y to happen.

$$P\{X[3,5] > Y[2,4]\} = \frac{number\ of\ time\ X > Y\ happens}{all\ possible\ events} = 0.875$$

In conclusion the probability obtained is 87,5% or 0.875 if expressed as a decimal.

The opposite, X<Y is the entire sample space minus the probability obtained before .

$$P\{X[3,5] < Y[2,4]\} = \frac{all\ possible\ events - number\ of\ time\ X > Y\ happens}{all\ possible\ events} = 0.124$$

This gives us a probability of 12,4% or 0.124 if expressed as a decimal.

Theoretical we can divide the X[3,5] in X[3,4] and X[4,5] and Y[2,4] in Y[2,3] and Y[3,4]. Now we try all possible combinations which is 2^2 so 4 combinations. We obtain :

$$\begin{aligned} P\{X[3,5] < Y[2,4]\} = \frac{P\{X[3,4] > Y[2,3]\} + P\{X[3,4] > Y[3,4]\} + P\{X[4,5] > Y[2,3]\} + P\{X[4,5] > Y[3,4]\}}{P\{\Omega\} * number\ of\ combinations} \\ = \frac{3,5}{4} = 0,875 \end{aligned} \quad (1)$$