Mean Squared Error (MSE) and Its Mathematical Indicators

REDA OUZIDANE

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1 Introduction

In regression tasks, evaluating the accuracy of a model's predictions is essential. One of the most common metrics used to measure prediction accuracy is the **Mean Squared Error (MSE)**. MSE is the average of the squares of the differences between the predicted values and the actual values. It is a widely used metric due to its simplicity and ease of computation.

2 Formula for Mean Squared Error

The formula for calculating MSE is:

$$\text{MSE} = \frac{1}{n} \sum_{i=1}^{n} (y_i - \hat{y}_i)^2$$

Where:

- n is the number of data points.
- y_i is the actual value of the i-th data point.
- $\hat{y}_i$ is the predicted value of the i-th data point.

3 Mathematical Indicators Involved in MSE

3.1 Error (Residual)

The error, also called the residual, for a single data point is the difference between the actual value and the predicted value:

$$e_i = y_i - \hat{y}_i$$

3.2 Squared Error

The squared error is the square of the residual:

$$(e_i)^2 = (y_i - \hat{y}_i)^2$$

Squaring the error ensures that all values are positive and that larger errors are penalized more than smaller ones.

3.3 Mean of Squared Errors

The MSE is the average of the squared errors over all data points:

$$\text{MSE} = \frac{1}{n}\sum_{i=1}^{n}(y_i - \hat{y}_i)^2$$

4 Interpretation of MSE

- A lower MSE value indicates that the model's predictions are closer to the actual values, meaning the model performs better.
- A higher MSE value suggests that the model's predictions deviate significantly from the actual values, indicating poor model performance.

5 Properties of MSE

- **Non-Negativity**: MSE is always non-negative, as it involves squaring the errors. The smallest value for MSE is 0, which occurs when all predictions are perfect.
- **Sensitivity to Outliers**: MSE is sensitive to outliers because larger errors are squared. This means that MSE can be disproportionately influenced by data points with large errors.

6 Comparison with Other Metrics

6.1 Root Mean Squared Error (RMSE)

RMSE is the square root of MSE and returns the error in the same units as the original data:

$$\text{RMSE} = \sqrt{\text{MSE}}$$

6.2 Mean Absolute Error (MAE)

Unlike MSE, MAE does not square the errors and is less sensitive to outliers. The formula for MAE is:

$$\text{MAE} = \frac{1}{n}\sum_{i=1}^{n}|y_i - \hat{y}_i|$$

7 Conclusion

MSE is an essential metric for evaluating regression models, particularly when you want to penalize larger errors. While it is widely used, it is sensitive to outliers, which can sometimes lead to misleading results in the presence of extreme values. In practice, you might consider using other metrics like RMSE or MAE depending on the specific requirements of your model evaluation.

By REDA OUZIDANE