Complete Quality Statistics Reference

Hypothesis Testing, ANOVA, Six Sigma (DMAIC), and SPC

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1. Hypothesis Testing Framework

Key Components

- **Null Hypothesis (H):** Default assumption (e.g., "no difference").
- **Alternative Hypothesis (H):** Claim to test (e.g., "process improved").
- **Significance Level ():** Typically 0.05 (5% risk of Type I error).
- **p-value:** Probability of observing data if H is true.

Test Selection Table

Scenario	**Test**	**Formula**
Compare mean to target	One-sample t-test	$t = \frac{\bar{X} - \mu_0}{s/\sqrt{n}}$
Compare two means	Two-sample t-test	$t = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{s_p^2(\frac{1}{n_1} + \frac{1}{n_2})}}$
Proportions test	Z-test	$z = \frac{\hat{p} - p_0}{\sqrt{p_0(1-p_0)/n}}$
Non-normal data	Mann-Whitney U	$U = R_1 - \frac{n_1(n_1+1)}{2}$

2. ANOVA (Analysis of Variance)

One-Way ANOVA Table

Source	SS	df	MS	F
Between	SS_B	$k-1$	$MS_B = \frac{SS_B}{k-1}$	$\frac{MS_B}{MS_W}$
Within	SS_W	$N-k$	$MS_W = \frac{SS_W}{N-k}$	
Total	SS_T	$N-1$		

Where:

$$SS_B = \sum n_i(\bar{X}_i - \bar{X}_{\text{total}})^2$$
$$SS_W = \sum (X_{ij} - \bar{X}_i)^2$$

3. Quality Control Metrics

Six Sigma (DMAIC)

- **Defects per Million (DPMO):**

$$\text{DPMO} = \frac{\text{Defects} \times 10^6}{\text{Units} \times \text{Opportunities}}$$

- **Sigma Level:** Use Z-table to convert DPMO to (e.g., 3.4 DPMO = 6).

Statistical Process Control (SPC)

- **Control Limits (X-R Chart):**

$$\text{UCL} = \bar{\bar{X}} + A_2\bar{R}, \quad \text{LCL} = \bar{\bar{X}} - A_2\bar{R}$$

- **Process Capability (Cp, Cpk):**

$$C_p = \frac{USL - LSL}{6\sigma}, \quad C_{pk} = \min\left(\frac{USL - \mu}{3\sigma}, \frac{\mu - LSL}{3\sigma}\right)$$

4. Common Distributions in Quality

Distribution	Use Case
Normal	Continuous data (e.g., dimensions)
Binomial	Pass/fail outcomes
Poisson	Defect counts
Weibull	Lifetime/failure analysis

5. Hypothesis Testing in Quality

Examples

- **t-test:** "Does new supplier material reduce thickness variation?"
- **Chi-square:** "Is defect type independent of production shift?"
- **ANOVA:** "Do any of 5 machines produce different diameters?"

6. Practical Workflow

1. Define problem (DMAIC "Define" phase)
2. Collect data (ensure representative sampling)
3. Test normality (Shapiro-Wilk or Anderson-Darling)
4. Choose appropriate test (refer to Section 1 table)
5. Calculate p-value and effect size
6. Implement control charts if process is stable

Crafted by Ouzidane Reda for quality engineering professionals.