7. Z-Test for Quality Control

When to Use

- Large samples ($n \geq 30$) or known population variance (σ^2)
- Testing **means** or **proportions** against a standard

Formulas

Test Type	**Formula**
One-sample mean	$z = \frac{\bar{X}-\mu_0}{\sigma/\sqrt{n}}$
Two-sample means	$z = \frac{(\bar{X}_1-\bar{X}_2)}{\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}}}$
One-sample proportion	$z = \frac{\hat{p}-p_0}{\sqrt{p_0(1-p_0)/n}}$
Two-sample proportions	$z = \frac{\hat{p}_1-\hat{p}_2}{\sqrt{\hat{p}(1-\hat{p})(\frac{1}{n_1}+\frac{1}{n_2})}}$

Where $\hat{p} = \frac{x_1+x_2}{n_1+n_2}$ (pooled proportion).

Critical Values

- 95% CI: $z_{\alpha/2} = 1.96$
- 99% CI: $z_{\alpha/2} = 2.576$

8. Fundamental Statistical Laws

Law of Large Numbers

$$\lim_{n\to\infty} P(|\bar{X}_n - \mu| \geq \epsilon) = 0$$

Interpretation: As sample size grows, sample mean converges to population mean.

Central Limit Theorem (CLT)

For any distribution with mean μ and variance σ^2:

$$\bar{X}_n \sim N\left(\mu, \frac{\sigma^2}{n}\right) as n \to \infty$$

Quality application: Justifies normality assumption in control charts.

Empirical Rule (68-95-99.7)