

ANOVA Example for Quality Control

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One-Way ANOVA Example: Machine Performance Comparison

Scenario

A factory has 4 machines producing the same gear. Quality engineers measure gear diameters (mm) from 5 random samples per machine to test if all machines produce identical sizes ($\alpha = 0.05$).

Table 1: Gear Diameter Measurements (mm)

Machine A	Machine B	Machine C	Machine D
10.2	10.5	10.3	10.6
10.1	10.7	10.4	10.5
10.3	10.6	10.5	10.7
10.0	10.4	10.2	10.6
10.2	10.3	10.3	10.4

Step 1: Calculate Group Means and Overall Mean

$$\begin{aligned}\bar{X}_A &= \frac{10.2 + 10.1 + 10.3 + 10.0 + 10.2}{5} = 10.16 \\ \bar{X}_B &= 10.50, \quad \bar{X}_C = 10.34, \quad \bar{X}_D = 10.56 \\ \bar{X}_{\text{total}} &= \frac{10.16 + 10.50 + 10.34 + 10.56}{4} = 10.39\end{aligned}$$

Step 2: Sum of Squares Calculations

$$\begin{aligned}SS_{\text{between}} &= 5[(10.16 - 10.39)^2 + (10.50 - 10.39)^2 \\&\quad + (10.34 - 10.39)^2 + (10.56 - 10.39)^2] = 0.492 \\SS_{\text{within}} &= \sum (X_{ij} - \bar{X}_i)^2 \\&= (10.2 - 10.16)^2 + \cdots + (10.4 - 10.56)^2 = 0.192 \\SS_{\text{total}} &= SS_{\text{between}} + SS_{\text{within}} = 0.684\end{aligned}$$

Step 3: Degrees of Freedom

$$\begin{aligned}df_{\text{between}} &= k - 1 = 4 - 1 = 3 \\df_{\text{within}} &= N - k = 20 - 4 = 16 \\df_{\text{total}} &= N - 1 = 19\end{aligned}$$

Step 4: Mean Squares and F-Statistic

$$\begin{aligned}MS_{\text{between}} &= \frac{0.492}{3} = 0.164 \\MS_{\text{within}} &= \frac{0.192}{16} = 0.012 \\F &= \frac{0.164}{0.012} = 13.67\end{aligned}$$

Step 5: Critical Value and Conclusion

- Critical $F_{3,16}$ (from F-table at $\alpha = 0.05$) = 3.24
- Since $13.67 > 3.24$, **reject H**

Interpretation

At least one machine produces significantly different gear diameters ($p < 0.05$). Post-hoc tests (e.g., Tukey's HSD) are needed to identify which machine(s) differ.

Table 2: ANOVA Summary Table

Source	SS	df	MS	F	p-value
Between	0.492	3	0.164	13.67	< 0.001
Within	0.192	16	0.012		
Total	0.684	19			