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$$X(s) = \frac{2s^3+8s^2+4s+8}{s(s+1)(s^2+4s+8)} = \frac{2s^3+8s^2+4s+8}{s(s+1)(s+2-2j)(s+2+2j)}$$

$$\frac{2s^3+8s^2+4s+8}{s(s+1)(s+2-2j)(s+2+2j)} = \frac{A}{s} + \frac{B}{s+1} + \frac{C}{s+2-2j} + \frac{C^*}{s+2+2j}$$

$$2s^3+8s^2+4s+8 = A(s+1)(s+2-2j)(s+2+2j) + Bs(s+2-2j)(s+2+2j) + Cs(s+1)(s+2+2j)$$
$$+ C^* s(s+1)(s+2-2j)$$

si $s=0$

$8 = A8 \Rightarrow A=1$

si $s=-1$

$-2+8-4+8 = -B(1-2j)(1+2j)$

$10 = -B5 \Rightarrow B = -2$

si $s=-2+2j$

$32-24j = C(-2+2j)(-1+2j)(4j)$

$32-24j = C(24-8j)$

$4-3j = C(3-j)$

$$\frac{4-3j}{3-j} = C = \frac{(4-3j)(3+j)}{10} = \frac{15-5j}{10} = 1.5-0.5j$$

$C^* = 1.5+0.5j$

$$X(s) = \frac{1}{s} - \frac{2}{s+1} + \frac{1.5-0.5j}{s+2-2j} + \frac{1.5+0.5j}{s+2+2j}$$

problema de diseño:

Diseñar un $O_S = 9.5\%$ y $t_s = 0.74\ s$ $G(s) = \frac{20(s+5)}{s(s+1)(s+4)}$

$$O_S = e^{\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}} = 9.5\% \Rightarrow \ln(0.095) = \frac{-\zeta\pi}{\sqrt{1-\zeta^2}}$$

$\zeta = \pm 0{,}5996$

$5{,}54 - \zeta^2 5.54 = \zeta^2\pi^2 \Rightarrow \zeta^2(\pi^2+5.54) = 5.54$

$S = \sigma + j\omega d$ $\quad t_s = \frac{4}{\sigma} = 0.74$ $\quad \sigma = 5.41 = \zeta \omega_n$

$$\omega_n = \frac{\sigma}{\frac{3}{4}} = 9,0227$$

$$\omega d = \omega_n \sqrt{1-\zeta^2} = 7,2209$$

$$S = -5,41 \pm j7,22$$

$$G(s) = \frac{20(s+5)}{s^3+5s^2+4s} = \frac{20(s+5)X(s)}{U(s)} = \frac{Y(s)}{U(s)}$$

$$U(s) = s^3 X(s) + 5s^2 X(s) + 4s X(s)$$

$$Y(s) = 20s X(s) + 100 X(s)$$

$$U(t) = \dddot{x} + 5\ddot{x} + 4\dot{x} \Rightarrow y(t) = 20\dot{x} + 100x$$

Definiendo estados:

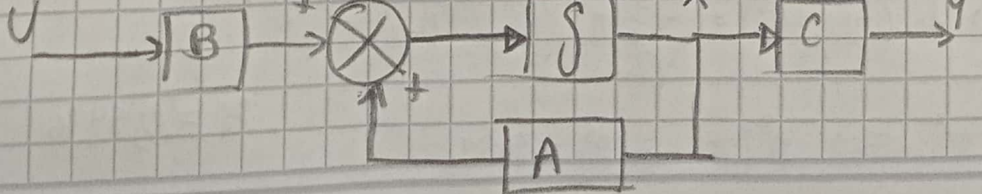
$X_1 = x \quad X_2 = \dot{x}_1 \quad X_3 = \dot{x}_2$

$U = \dot{x}_3 + 5x_3 + 4x_2 \qquad y = 20x_2 + 100x_1$

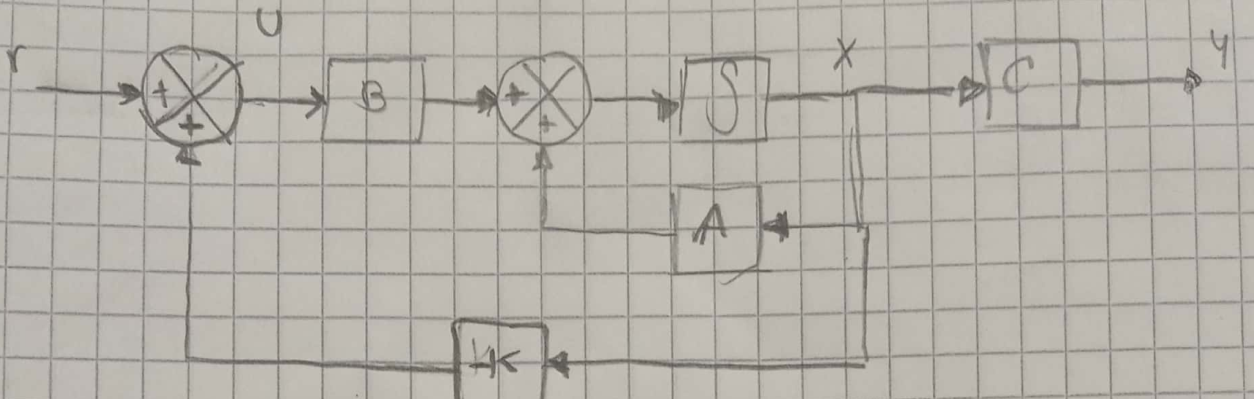
$\dot{x}_3 = U - 4x_2 - 5x_3$

$$\dot{\vec{x}} = \underbrace{\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -4 & -5 \end{bmatrix}}_{A} \vec{x} + \underbrace{\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}}_{B} U \qquad y = \underbrace{\begin{bmatrix} 100 & 20 & 0 \end{bmatrix}}_{C} \vec{x} + \underbrace{\vec{0}}_{D} U$$

Representación en bloques



Ya que se desea modificar $O_s = 9.5\%$ y $t_s = 0.74\,s$ se agrega un bloque de realimentación al sistema



Donde $\dot{x} = Ax + BU$ $\quad U = r - Kx$

$$\dot{x} = Ax + B(r - Kx) = (A - BK)x + Br$$

$$\dot{\vec{x}} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -k_1 & -(4+k_2) & -(5+k_3) \end{bmatrix} \vec{x} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} r$$

La Ecuación Característica es

$$\det[sI - (A - BK)] = s^3 + (5+k_3)s^2 + (4+k_2)s + k_1 = 0$$

Recordemos que el polo dominante está dado por:

$$s = -5,41 + j7,22$$

$$(s + 5,41 - j7,22)(s + 5,4 + j7,22)(s + 5.1) = s^3 + 15,9s^2 + 136,08\,s + 413.1 = 0$$

Comparando con la ecuación característica:

$k_1 = 413,1$ $\quad k_2 = 132,08$ $\quad k_3 = 10.9$