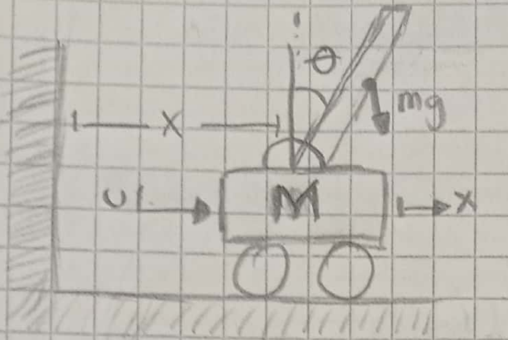
Sistema de péndulo invertido

Encontrar representación en espacio de estados



Las coordenadas del centro de masa son:

en x: $x + l\,\text{sen}\,\theta$ en y: $l\cos\theta$

Por lo tanto: $U = M\dfrac{d^2x}{dt^2} + m\dfrac{d^2}{dt^2}(x + l\,\text{sen}\,\theta)$

$$U = M\frac{d^2x}{dt^2} + m\frac{d^2x}{dt^2} + m\frac{d^2}{dt^2}l\,\text{sen}\,\theta = \frac{d^2x}{dt^2}(M+m) + ml\frac{d}{dt}\cos\theta\,\dot{\theta}$$

$$U = \frac{d^2x}{dt^2}(M+m) - ml\,\text{sen}\,\theta\,\dot{\theta}^2 + ml\cos\theta\,\ddot{\theta}$$

$$U = \ddot{x}(M+m) - ml\,\text{sen}\,\theta\,\dot{\theta}^2 + ml\cos\theta\,\ddot{\theta}$$

Para el péndulo, aplicando la segunda ley de Newton

$$mgl\,\text{sen}\,\theta = m\frac{d^2}{dt^2}(x + l\,\text{sen}\,\theta)\,l\cos\theta - m\frac{d^2}{dt^2}(l\cos\theta)\,l\,\text{sen}\,\theta$$

$$mgl\,\text{sen}\,\theta = \left[m\frac{d^2x}{dt^2} - ml\,\text{sen}\,\theta\,\dot{\theta}^2 + ml\cos\theta\,\ddot{\theta}\right]l\cos\theta + \left[ml\cos\theta\,\dot{\theta}^2 + ml\,\text{sen}\,\theta\,\ddot{\theta}\right]l\,\text{sen}\,\theta$$

$$mg\,\text{sen}\,\theta = m\cos\theta\,\ddot{x} - ml\,\text{sen}\,\theta\cos\theta\,\dot{\theta}^2 + ml\cos^2\theta\,\ddot{\theta} + ml\,\text{sen}\,\theta\cos\theta\,\dot{\theta}^2 + ml\,\text{sen}^2\theta\,\ddot{\theta}$$

$$mg\,\text{sen}\,\theta = m\cos\theta\,\ddot{x} + ml\,\ddot{\theta}$$

Las ecuaciones del sistemas son:

$$U = (M+m)\ddot{x} - ml\,\text{sen}\,\theta\,\dot{\theta}^2 + ml\cos\theta\,\ddot{\theta}$$

$$mg\,\text{sen}\,\theta = m\cos\theta\,\ddot{x} + ml\,\ddot{\theta}$$

$\Rightarrow$ Si suponemos que θ y $\dot{\theta}$ son pequeños (debido a que se desea mantener el péndulo vertical)

$\text{sen}\,\theta \simeq \theta$ $\cos\theta \simeq 1$

$\dot{\theta}^2 \simeq 0$

$$U = (M+m)\ddot{x} + ml\,\ddot{\theta}$$

$$mg\theta = m\ddot{x} + ml\,\ddot{\theta}$$

$$\Rightarrow U - mg\theta = M\ddot{x}$$

$$\ddot{x} = \frac{U}{M} - \frac{mg}{M}\theta$$

Definiendo estados:

$x_1 = x \qquad x_3 = \theta$

$x_2 = \dot{x}_1 \qquad x_4 = \dot{x}_3$

$$\dot{x}_2 = \frac{U}{M} - \frac{mg}{M} x_3$$

$$g\theta = \ddot{x} + l\ddot{\theta}$$

$$g x_3 = \dot{x}_2 + l\dot{x}_4 \Rightarrow g x_3 = \frac{U}{M} - \frac{mg}{M} x_3 + l\dot{x}_4$$

$$\dot{x}_4 = \left(\frac{g}{l} + \frac{mg}{Ml}\right) x_3 - \frac{U}{ML} = \frac{Mg + mg}{Ml} x_3 - \frac{U}{ML}$$

$$\dot{x}_4 = \frac{(M+m)g}{Ml} x_3 - \frac{U}{Ml}$$

$$\dot{\vec{x}} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -\frac{mg}{M} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & \frac{(M+m)g}{Ml} & 0 \end{bmatrix} \vec{x} + \begin{bmatrix} 0 \\ \frac{1}{M} \\ 0 \\ -\frac{1}{Ml} \end{bmatrix} U$$

$$\begin{bmatrix} x_1 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \vec{x} + \vec{0}\, U$$