Nishant Jindal

$\underline{\Delta ABC}$

1. $\sin A + \sin B + \sin C \le \dfrac{3\sqrt{3}}{2}$

$\sum \sin A = \dfrac{3\sqrt{3}}{2}$

if $A = B = C$

$\Rightarrow \dfrac{\sum \sin A}{3} \le \sin\left(\dfrac{A+B+C}{3}\right) = \dfrac{\sqrt{3}}{2}$

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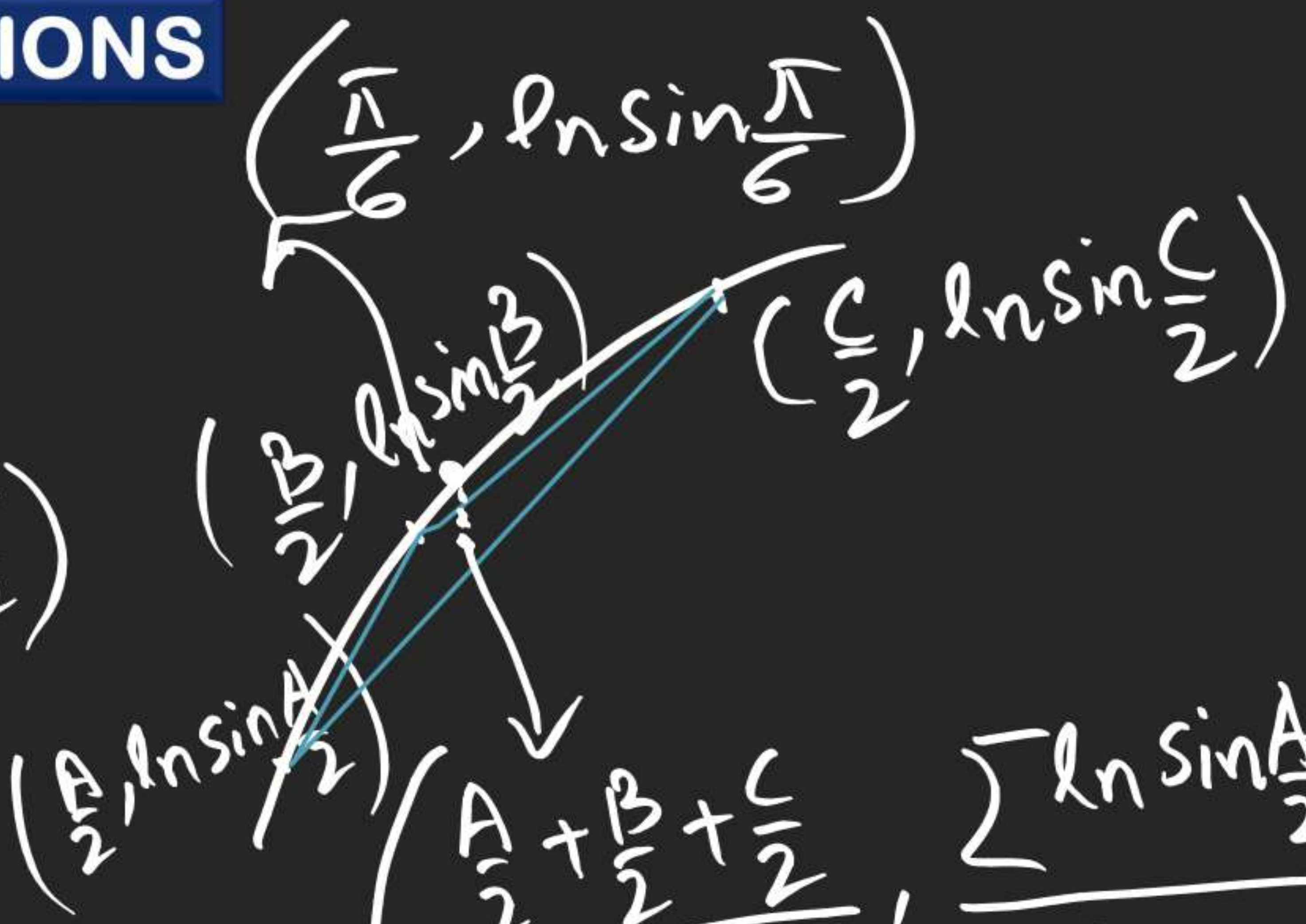
FUNCTIONS

(2) $\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$

$f(x) = \ln\sin x \quad , x \in \left(0, \frac{\pi}{2}\right)$

$f'(x) = \cot x$

$f''(x) = -\operatorname{cosec}^2 x < 0$



$$\left(\frac{\frac{A}{2}+\frac{B}{2}+\frac{C}{2}}{3}, \frac{\sum \ln\sin\frac{A}{2}}{3}\right)$$

$$= \frac{\pi}{6}$$

$$\Rightarrow \frac{1}{3}\ln\left(\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}\right) \le \ln\sin\frac{\pi}{6}$$

$$\boxed{\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2} \le \frac{1}{8}}$$

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FUNCTIONS

2: B

3: B

5: A

4: $2\left(\cos^4\frac{\pi}{8}+\cos^4\frac{3\pi}{8}\right)$ $\rightarrow \frac{\pi}{2}-\frac{\pi}{8}$

$=2\left(\cos^4\frac{\pi}{8}+\sin^4\frac{\pi}{8}\right)$

$=2\left(1-\frac{1}{2}\sin^2\frac{\pi}{4}\right)$

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FUNCTIONS

5. $\tan\frac{x}{2} = \frac{1-\cos x}{\sin x}$

$\tan x = \frac{3}{4}$

$\pi < x < \frac{3\pi}{2}$

$= \dfrac{1-\left(-\frac{4}{5}\right)}{-\frac{3}{5}}$

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FUNCTIONS

8. (a) -1

(b) $\sqrt{3}$

(c) $\frac{5}{4}$

(d) $\sqrt{3}$

7. $1+\underbrace{\cos^2\alpha-\sin^2\beta}+\cos^2\gamma$

$$= 1+\cos(\alpha-\beta)\cos(\underbrace{\alpha+\beta}_{=\gamma})+\cos^2\gamma$$

$$= 1+\cos\gamma\Big(\cos(\alpha-\beta)+\cos(\alpha+\beta)\Big)$$

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8. (a) $\dfrac{4\cos 20^\circ \sin 20^\circ - \sqrt{3}\cos 20^\circ}{\sin 20^\circ}$

$60^\circ - 20^\circ$

$= \dfrac{2\sin 40^\circ - \sqrt{3}\cos 20^\circ}{\sin 20^\circ}$

$= \dfrac{2\left(\frac{\sqrt{3}}{2}\cos 20^\circ - \frac{1}{2}\sin 20^\circ\right) - \sqrt{3}\cos 20^\circ}{\sin 20^\circ}$

FUNCTIONS

(d) $\tan 10^\circ - \tan(60^\circ - 10^\circ) + \tan(60^\circ + 10^\circ)$

$$= t - \frac{\sqrt{3} - t}{1 + \sqrt{3}t} + \frac{\sqrt{3} + t}{1 - \sqrt{3}t}$$

$$= \frac{3(3t - t^3)}{1 - 3t^2} = 3\tan 30^\circ$$

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Greatest Integer Function / Step Up Function

$[x]$ $\qquad$ $[\cdot]$ denote G.I.F

$[x]$ is the greatest integer $\leq x$

$[-13.0078] = -14$ $\qquad$ $[-2] = -2$

$[13.876] = 13$

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x

$$[x] = \begin{cases} \vdots & \\ -1 & x \in [-1, 0) \\ 0 & x \in [0, 1) \\ 1 & x \in [1, 2) \\ 2 & x \in [2, 3) \\ \vdots & \end{cases}$$

$$0 \le x - [x] < 1$$

$$[x] \le x < [x] + 1$$

$$x - 1 < [x] \le x$$

$[-1] = -1$

$[-0.979] = -1$

$[-0.00006] = -1$

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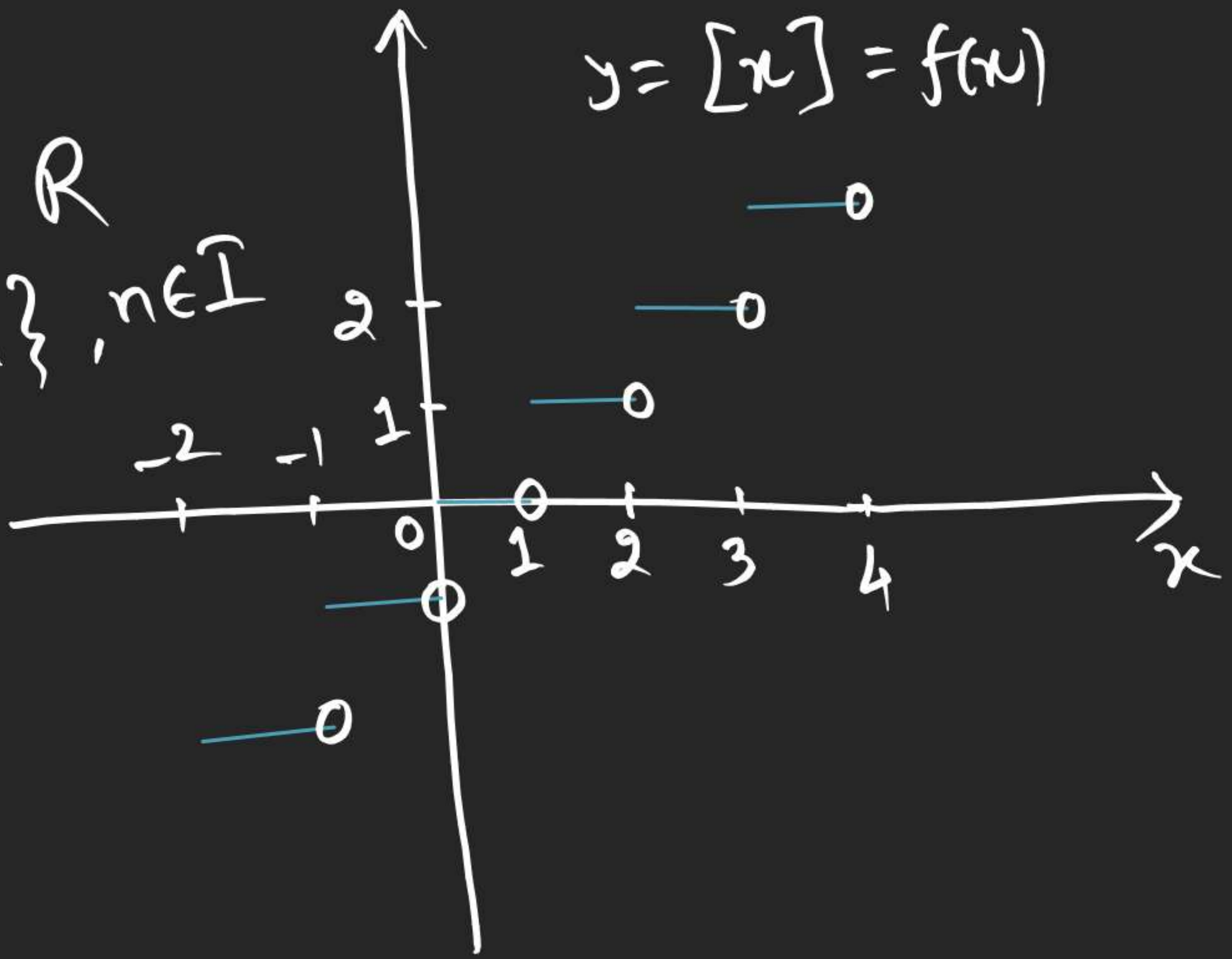
$y = [x] = f(x)$

$D_f = R$

$R_f = \{n\}, n \in I$



FUNCTIONS

Properties

① $[x] \le x < [x]+1$

② $x-1 < [x] \le x$

③ $[x+n] = n+[x]$, $n \in I$

④ $[x]+[-x] = \begin{cases} -1 & \text{if } x \notin I \\ 0 & \text{if } x \in I \end{cases}$

⑤ $\left[\frac{x}{n}\right]+\left[\frac{x+1}{n}\right]+\left[\frac{x+2}{n}\right]+\dots+\left[\frac{x+(n-1)}{n}\right] = [x]$, $n \in N$

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FUNCTIONS

$[x+n]$, $n \in I$

$[x] \leq x < [x]+1$

$[x]+n \leq x+n < [x]+n+1$

$\Rightarrow [x+n] = [x]+n$

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FUNCTIONS

$[x]+[-x]$

$[x] \le x < [x]+1$

$-[x]-1 < -x \le -[x]$

$(3,4]$

If $x \notin I$

$[x] < x < [x]+1$

$-[x]-1 < -x < -[x]$

$\Rightarrow [-x] = -[x]-1$

If $x \in I$

$[-x]+[x] = -x+x = 0$

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P.T. $\sum_{r=0}^{n-1} \left[\frac{x+r}{n}\right] = [x] \qquad x = nQ+R \qquad Q \in I, \; 0 \le R < n$

T.P.T. $\left[\frac{nQ+R}{n}\right] + \left[\frac{nQ+R+1}{n}\right] + \dots + \left[\frac{nQ+R+(n-1)}{n}\right] = [nQ+R]$

$Q+\frac{R}{n} \qquad Q+\frac{R+1}{n} \qquad Q+\frac{R+(n-1)}{n}$

$Q+\left[\frac{R}{n}\right] + Q + \left[\frac{R+1}{n}\right] + \dots Q + \left[\frac{R+(n-1)}{n}\right] = nQ + [R]$

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FUNCTIONS

T.P.T. $\left[\frac{R}{n}\right]+\left[\frac{R+1}{n}\right]+\left[\frac{R+2}{n}\right]+\cdots+\left[\frac{R+(n-1)}{n}\right]=[R]$ $\quad R\in[0,n)$

$k\le R<k+1$, $k\in\{0,1,2,\ldots,n-1\}$

$RHS = k$

$LHS = \left[\frac{R}{n}\right]+\left[\frac{R+1}{n}\right]+\cdots+\underbrace{\left[\frac{R+n-k}{n}\right]}_{=1}+\left[\frac{R+n-k+1}{n}\right]+\cdots+\left[\frac{R+n-2}{n}\right]+\left[\frac{R+(n-1)}{n}\right]$

$= k$

$0\le\frac{R+r}{n}<\frac{n+n-1}{n}<2$

$n\le R+(n-k)<n+1$

$1\le\frac{R+(n-k)}{n}<\frac{n+1}{n}$

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FUNCTIONS

$$\left[\frac{x}{3}\right]+\left[\frac{x+1}{3}\right]+\left[\frac{x+2}{3}\right]=[x]$$

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FUNCTIONS

Fraction Part Function

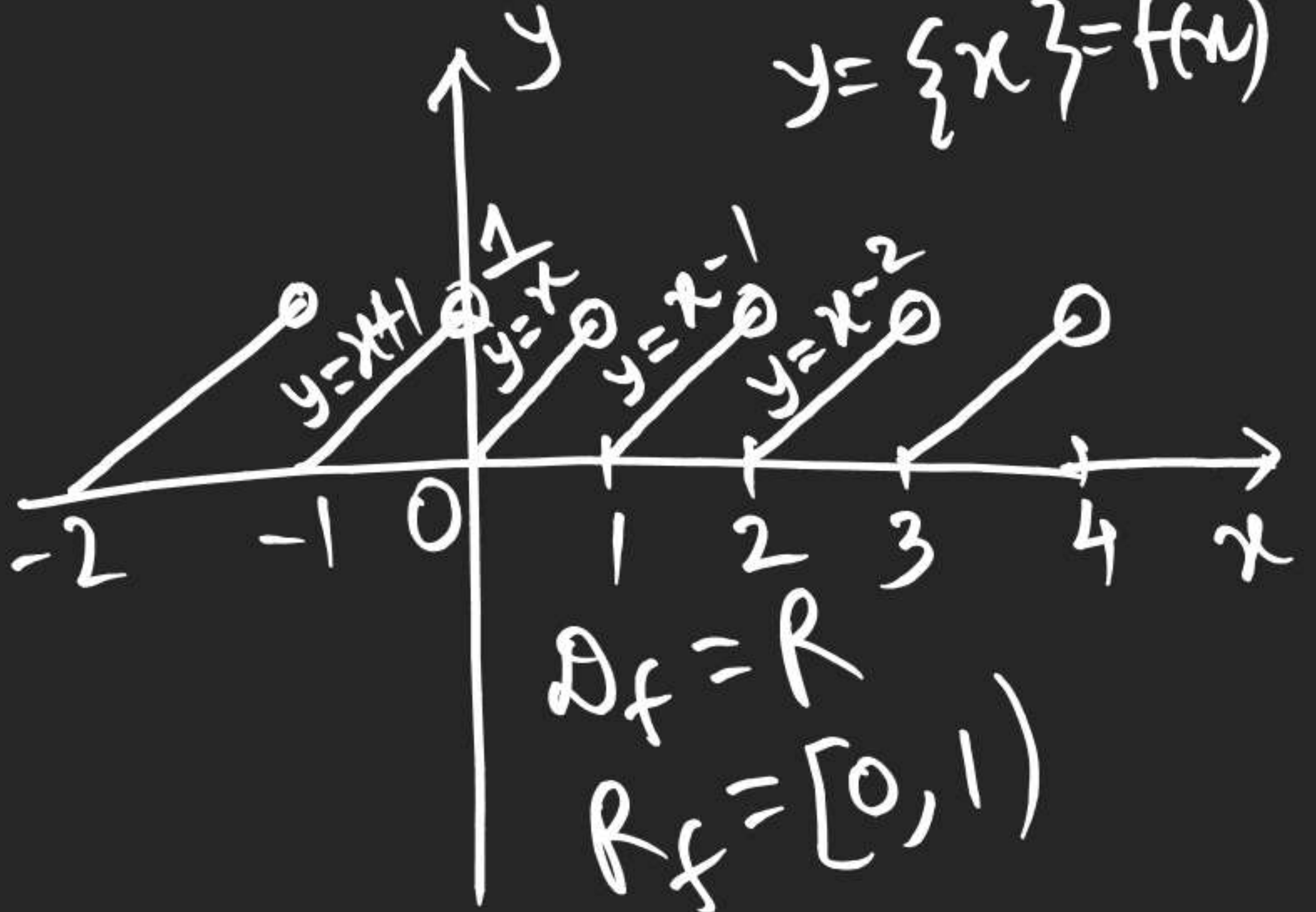
$\{\cdot\} = \text{FPF}$

$$f(x) = \{x\}$$

$$= x - [x]$$

$$= \begin{cases} \vdots & \\ x+1 & x \in [-1, 0) \\ x-0 & x \in [0, 1) \\ x-1 & x \in [1, 2) \\ x-2 & x \in [2, 3) \\ \vdots & \end{cases}$$

$y = \{x\} = f(x)$



$D_f = R$

$R_f = [0, 1)$

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FUNCTIONS

Properties

① $\{x+n\} = \{x\}$, $n \in I$

② $\{x\} + \{-x\} = \begin{cases} 1 & x \notin I \\ 0 & x \in I \end{cases}$

③ $[\{x\}] = 0$

④ $\{[x]\} = 0$

② $x - [x] + (-x) - [-x]$
$= -([x] + [-x])$

$\{f(x)\} = f(x) - [f(x)]$

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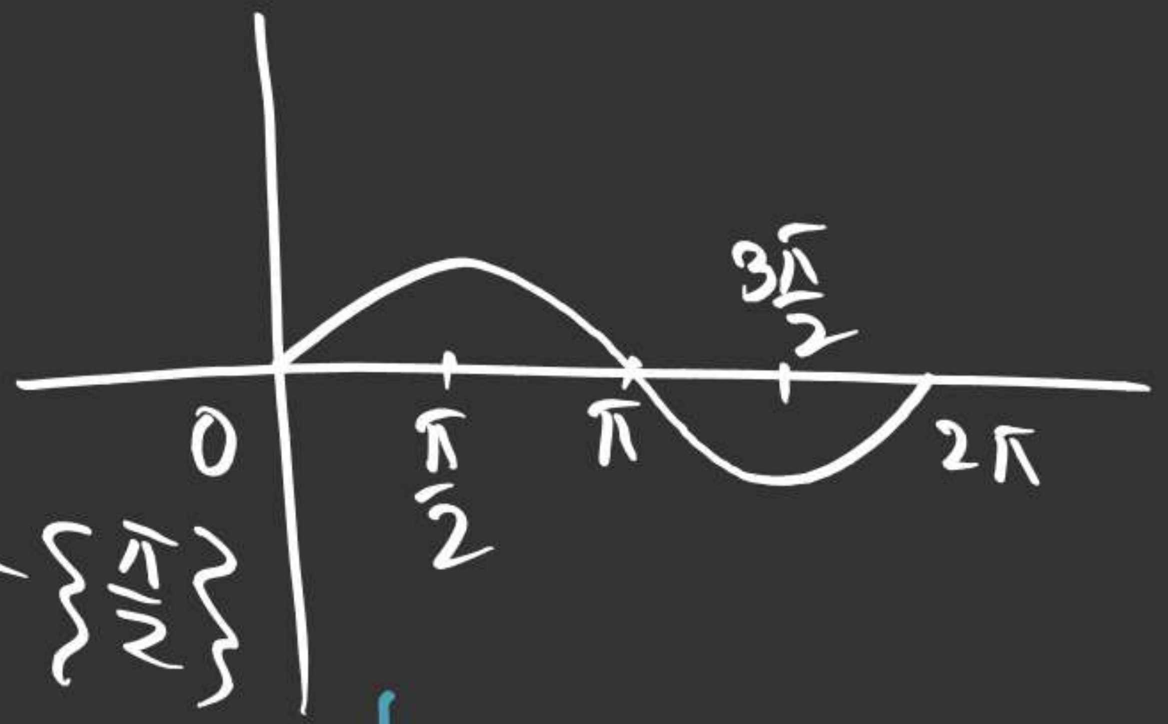
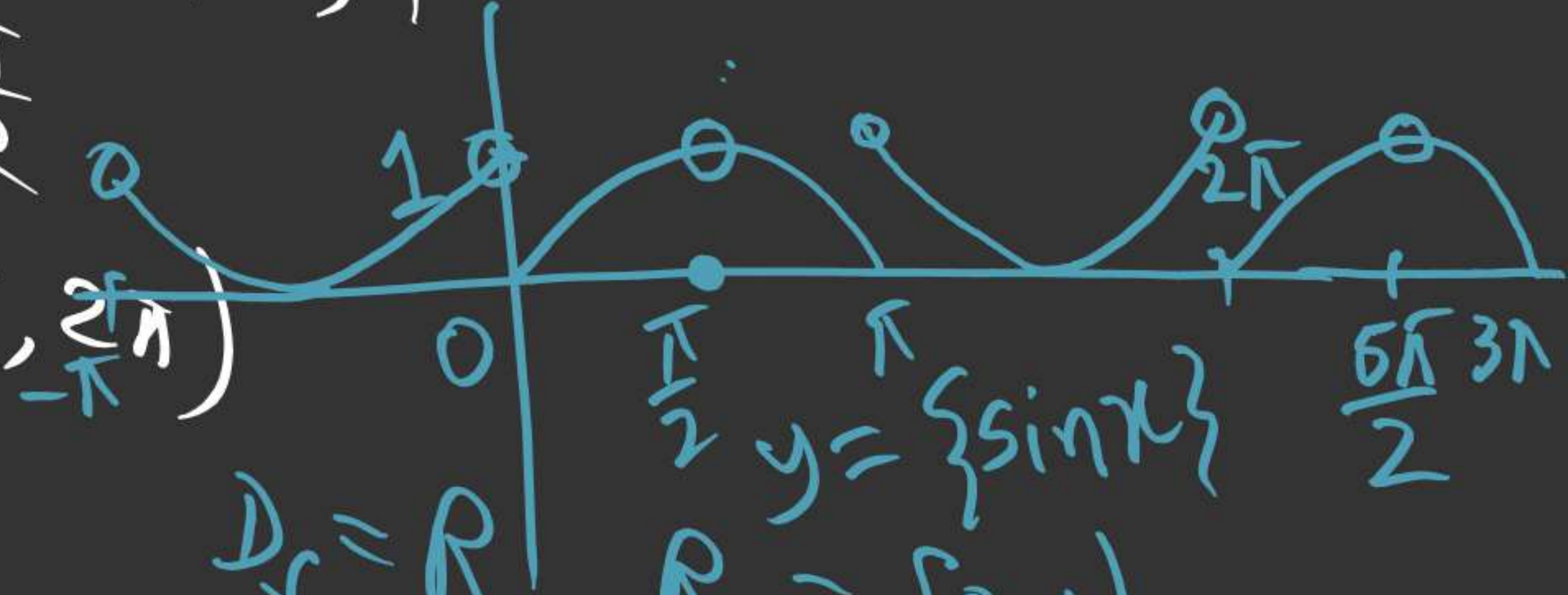
$y = \{\sin x\}$ $\{\cdot\} = \text{FPF}$

$= \sin x - [\sin x]$

$$= \begin{cases} \sin x - 0 & x \in [0, \pi] - \{\frac{\pi}{2}\} \\ 0 & x = \frac{\pi}{2} \\ \sin x - (-1) & x \in (\pi, 2\pi) \end{cases}$$

$= (\sin x) + 1$





$D_f = R$, $R_f = [0, 1)$

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① $y = \sin\{x\}$ $\qquad \{\cdot\} = \text{FPF}$

Draw the graph, find domain & range.

② $y = \text{sgn}(\{x\})$ $\qquad \{\cdot\} = \text{FPF}$

③ $y = \text{sgn}(x^3 - x)$

Signum Function

$$f(x) = \operatorname{sgn}(x) = \begin{cases} 1 & x > 0 \\ -1 & x < 0 \\ 0 & x = 0 \end{cases}$$

$$= \begin{cases} \dfrac{x}{|x|} & x \neq 0 \\ 0 & x = 0 \end{cases}$$

$D_f = \mathbb{R}$

$R_f = \{-1, 0, 1\}$