## DPP 02

## SOLUTION

**1.** $T_1 = 2\pi\sqrt{\frac{M}{k}}$ ..... (i)

When a mass m is placed on mass M, Mass of new system $= (M + m)$ attached to the spring. New time period of oscillation

$$T_2 = 2\pi\sqrt{\frac{(m+M)}{k}} \text{ ... (ii)}$$

Say $v_1$ is the velocity of mass M passing through mean position and $v_2$ velocity of mass $(m + M)$ passing through mean position.

By conservation of linear momentum

$Mv_1 = (m + M)v_2$

$M(A_1\omega_1) = (m + M)(A_2\omega_2)(\because v_1 = A_1\omega_1 \text{ and } v_2 = A_2\omega_2)$

or $\frac{A_1}{A_2} = \frac{(m+M)}{M}\frac{\omega_2}{\omega_1} = \left(\frac{m+M}{M}\right) \times \frac{T_1}{T_2}$

$\left(\because \omega_1 = \frac{2\pi}{T_1} \text{ and } \omega_2 = \frac{2\pi}{T_2}\right)$

$\frac{A_1}{A_2} = \sqrt{\frac{m+M}{M}}$ (Using (i) and (ii))

**2.** $v_1 = \frac{d}{dt}(y_1) = (0.1 \times 100\pi)\cos\left(100\pi t + \frac{\pi}{3}\right)$

$v_2 = \frac{d}{dt}(y_2) = (-0.1 \times \pi)\sin \pi t$

$= (0.1 \times \pi)\cos\left(\pi t + \frac{\pi}{2}\right) \quad \therefore \Delta\phi = \frac{\pi}{3} - \frac{\pi}{2} = -\frac{\pi}{6}$

**3.** Standard of SHM is

$\frac{d^2x}{dt^2} + \omega^2 x = 0$

Given equation is $\frac{d^2x}{dt^2} + \alpha x = 0$

$\therefore \omega^2 = \alpha \text{ or } \omega = \sqrt{\alpha} \therefore T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{\alpha}}$

**4.** When springs are in series, $k = \frac{k_1 k_2}{k_1 + k_2}$

For first spring, $t_1 = 2\pi\sqrt{\frac{m}{k_1}}$

For second spring $t_2 = 2\pi\sqrt{\frac{m}{k_2}}$

$\therefore t_1^2 + t_2^2 = \frac{4\pi^2 m}{k_1} + \frac{4\pi^2 m}{k_2} = 4\pi^2 m\left(\frac{k_1 + k_2}{k_1 k_2}\right)$

or $t_1^2 + t_2^2 = \left[2\pi\sqrt{\frac{m(k_1 + k_2)}{k_1 k_2}}\right]^2$ or $t_1^2 + t_2^2 = T^2$

**5.** Maximum velocity under simple harmonic motion

$v_m = a\omega$

$\therefore v_m = \frac{2\pi a}{T} = (2\pi a)\left(\frac{1}{T}\right) = (2\pi a)\left(\frac{1}{2\pi}\sqrt{\frac{k}{m}}\right)$

or $v_m = a\sqrt{\frac{k}{m}}$

$\because (v_m)_A = (v_m)_B$

$\therefore a_1\sqrt{\frac{k_1}{m}} = a_2\sqrt{\frac{k_2}{m}} \Rightarrow \frac{a_1}{a_2} = \sqrt{\frac{k_2}{k_1}}$

**6.** Initially, $T = 2\pi\sqrt{M/k}$

Finally, $\frac{5T}{3} = 2\pi\sqrt{\frac{M+m}{k}}$

$\therefore \frac{5}{3} \times 2\pi\sqrt{\frac{M}{k}} = 2\pi\sqrt{\frac{M+m}{k}}$ or $\frac{25}{9}\frac{M}{k} = \frac{M+m}{k}$

or $9m + 9M = 25M$ or $\frac{m}{M} = \frac{16}{9}$

**7.** $x = 4(\cos \pi t + \sin \pi t)$

$= 4 \times \sqrt{2}\left[\frac{1}{\sqrt{2}}\cos \pi t + \frac{1}{\sqrt{2}}\sin \pi t\right]$

or $x = 4\sqrt{2}\left[\sin\frac{\pi}{4}\cos \pi t + \cos\frac{\pi}{4}\sin \pi t\right]$

$= 4\sqrt{2}\sin\left(\pi t + \frac{\pi}{4}\right)$

Hence amplitude $= 4\sqrt{2}$

**8.** For simple harmonic motion,

Total energy$= \frac{1}{2}ma^2\omega^2$

It means total energy is independent of x.

**9.** $y = \sin^2 \omega t = \frac{1-\cos 2\omega t}{2} = \frac{1}{2} - \frac{\cos 2\omega t}{2}$

It is a periodic motion but it is not SHM.

$\therefore$ Angular speed $= 2\omega$

$\therefore$ Period $T = \frac{2\pi}{\text{angular speed}} = \frac{2\pi}{2\omega} = \frac{\pi}{\omega}$

**10.** In a simple harmonic oscillator, kinetic energy is maximum and potential energy is minimum at mean position.