Nishant Jindal

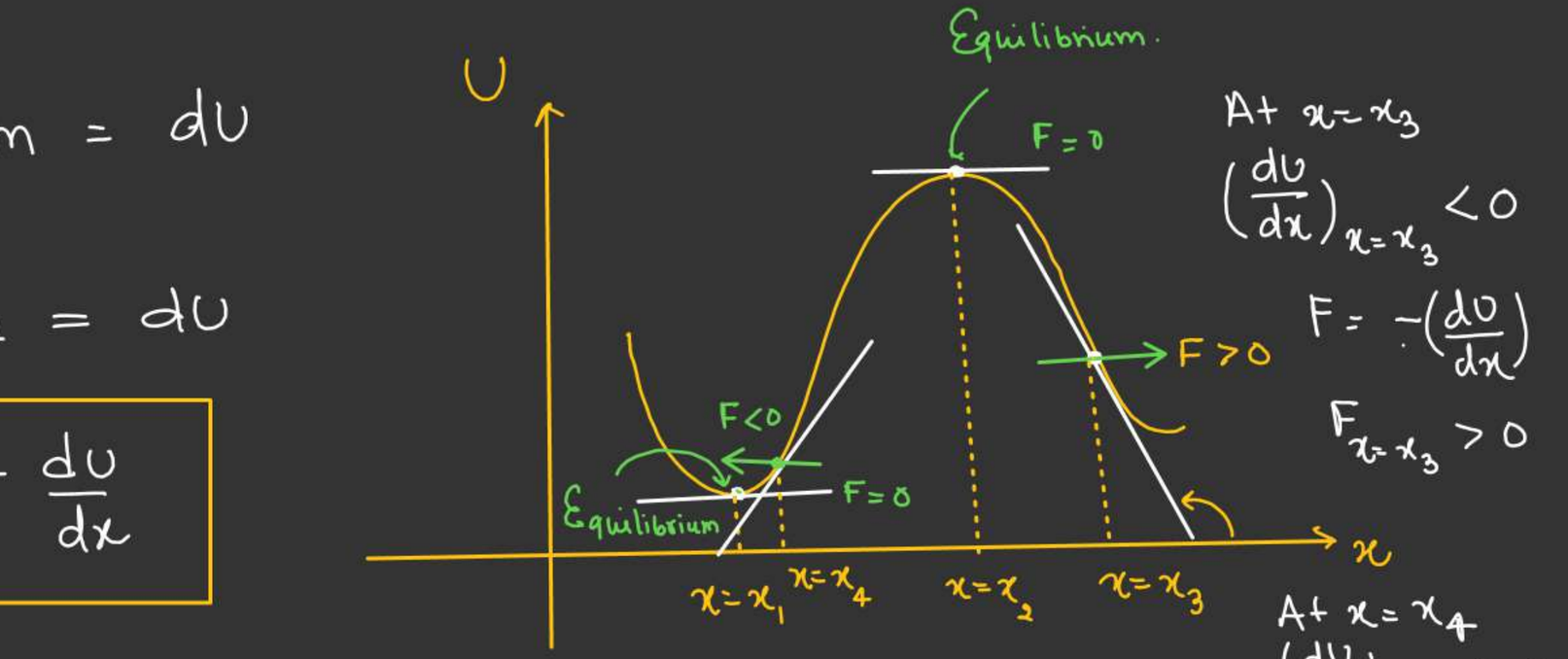
Relation b/w Conservative force and potential Energy

$$-W_{system} = dU$$

$$-F \cdot dx = dU$$

$$\boxed{F = -\frac{dU}{dx}}$$

(Conservative force)



At $x = x_3$

$\left(\frac{dU}{dx}\right)_{x=x_3} < 0$

$F = -\left(\frac{dU}{dx}\right)$

$F_{x=x_3} > 0$

At $x = x_4$

$\left(\frac{dU}{dx}\right)_{x=x_4} > 0$

$F_{x=x_4} < 0$

$F = \left(\frac{dU}{dx}\right)_{x=x_1} = 0$

$F = \left(\frac{dU}{dx}\right)_{x=x_2} = 0$

Equilibrium at $x = x_1$ & x_2.

Nishant Jindal

Q4 $U = (x^2 - 2x + 1)$

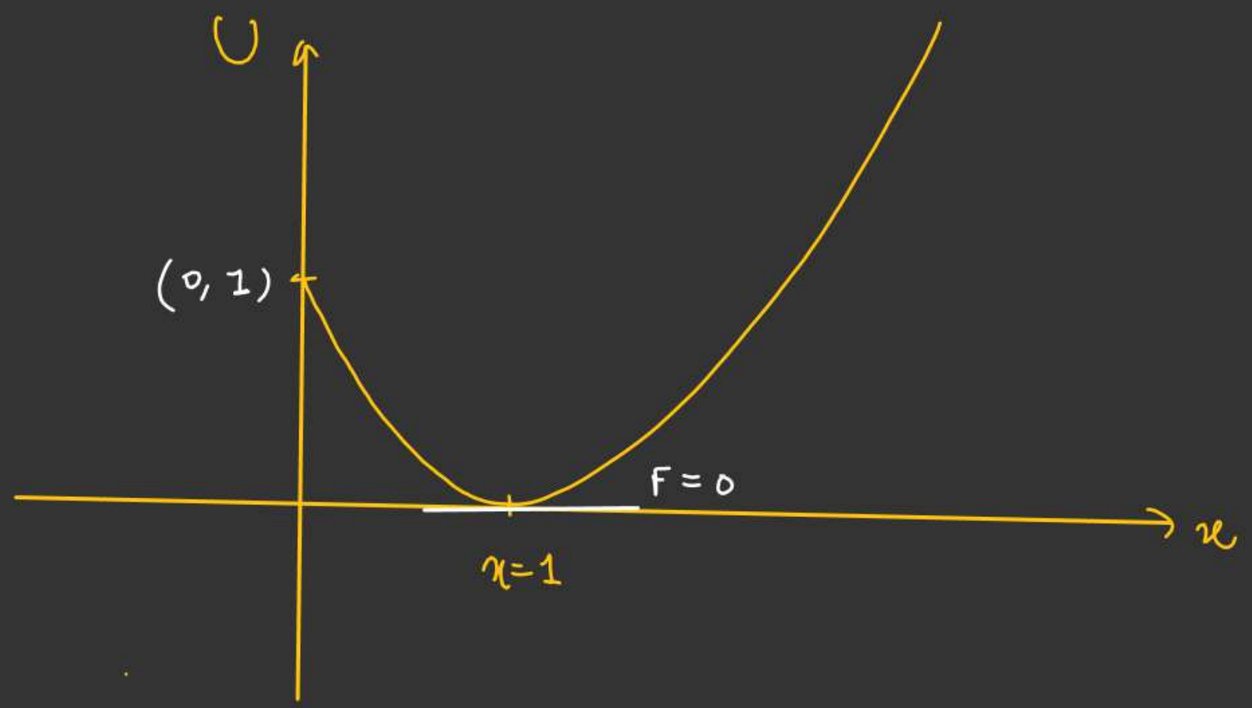
a) Find F at $x = 2$

$$F = -\frac{dU}{dx} = -[2x - 2]$$

$$F = -2(x-1)$$

At $x = 2$

$$F = -2(2-1) = -2\,N$$



b) Find Equilibrium point

For $\frac{dU}{dx} = 0$ ie $F = 0$

$$(x-1) = 0$$

$$x = 1$$

Nishant Jindal

$\Delta\Delta$ $U = f(x,y,z)$.

$$F = -\left[\frac{\partial U}{\partial x}\hat{i} + \frac{\partial U}{\partial y}\hat{j} + \frac{\partial U}{\partial z}\hat{k}\right]$$

$\frac{\partial U}{\partial x} \Rightarrow$ y & z assumed to be Constant.

$\frac{\partial U}{\partial y} \Rightarrow$ Assuming x & z as Constant

$\frac{\partial U}{\partial z} \Rightarrow$ Assuming x & y as constant.

$\#$ $U = (x^2y + yz)$

Find Conservative force for this potential Energy.

$$\frac{\partial U}{\partial x} = \frac{\partial}{\partial x}(x^2y + yz) = \frac{\partial}{\partial x}(x^2y) + \frac{\partial}{\partial x}(yz)$$
$$= y\frac{\partial}{\partial x}(x^2) + 0$$
$$= (2xy)$$

$$\frac{\partial U}{\partial y} = x^2\frac{\partial}{\partial y}(y) + z\cdot\frac{\partial}{\partial y}(y)$$
$$= (x^2+z)$$

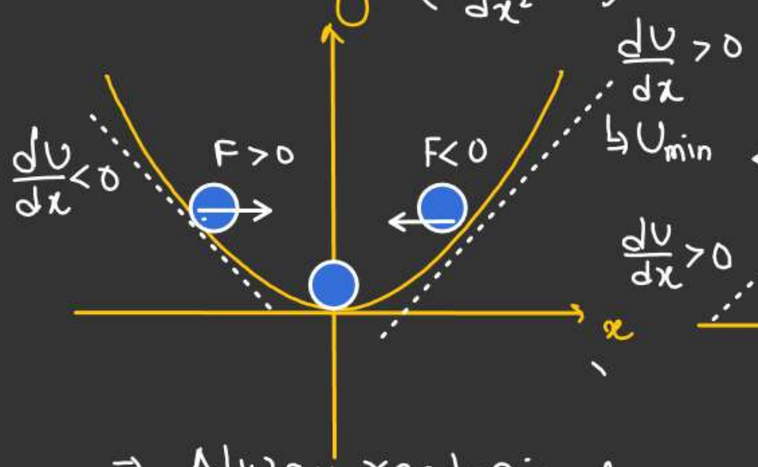
$$\frac{\partial U}{\partial z} = \frac{\partial}{\partial z}(x^2y) + y\frac{\partial}{\partial z}(z) = y.$$
($x^2y \to C$, $\frac{\partial}{\partial z}(C) \to 0$; $\frac{\partial}{\partial z}(z) \to 1$)

$$F = -\left[(2xy)\hat{i} + (x^2+z)\hat{j} + y\hat{k}\right]$$

Nishant Jindal

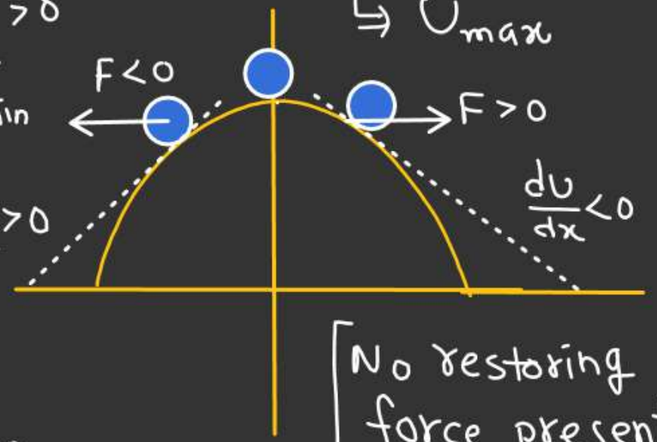
Type of Equilibrium $\Rightarrow \left(\frac{dU}{dx}=0\right)$

Stable Equilibrium $\left(\frac{d^2U}{dx^2}>0\right)$



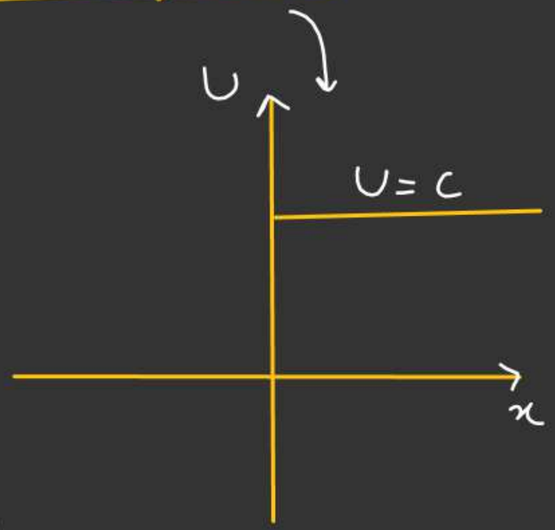
$\Rightarrow$ Alway restoring force which restore the position of the body. for stable Equilibrium.

Unstable Equilibrium $\left(\frac{d^2U}{dx^2}<0\right)$



[No restoring force present for unstable Equilibrium.

Neutral Equilibrium.



Nishant Jindal

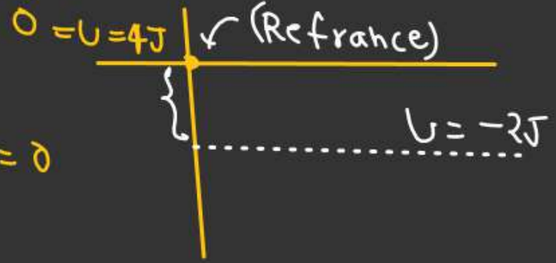
For Conservative force field

$$(E_T) = (P.E) + (K.E)$$

- E_T ⇓ Total Mechanical Energy
- P.E ⇓
- K.E ⇓ Kinetic Energy

⇒ $K.E = \frac{1}{2}mv^2 > 0$ ⇒ Always +ve

⇒ P.E may be +ve or −ve.
(depends on zero potential)



⇒ E_T also be +ve or −ve

⇒ For $(K.E)_{max}$

$(K.E)_{max} = E_T - P.E$

E_T always constant.

For $(K.E)_{max}$, P.E Should be minimum.

Potential Energy

WORK POWER ENERGY

Q.1 Potential energy of a particle is related to x coordinate by equation $x^2 - 2x$.

Particle will be in stable equilibrium at :

(A) $x = 0.5$

(B) $x = 1$ ✓

(C) $x = 2$

(D) $x = 4$

$U = x^2 - 2x$

For Equilibrium

$\frac{dU}{dx} = 0$

$(2x - 2) = 0$

$x = 1$ ⇒ Point of Equilibrium

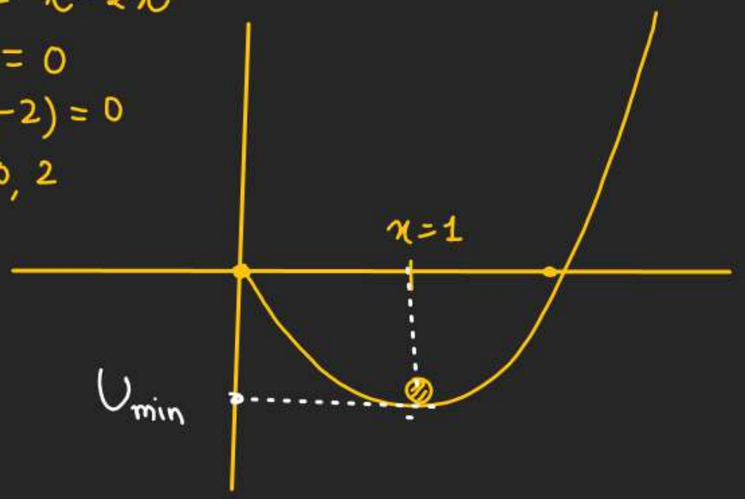
$\frac{d^2U}{dx} = (2) > 0$ ⇒ Point of Minimia or Stable Equilibrium

$U = x^2 - 2x$

$U = 0$

$x(x-2) = 0$

$x = 0, 2$



Potential Energy

WORK POWER ENERGY

Q.2 A particle is released from rest at origin. It moves under influence of potential field $U = x^2 - 3x$, kinetic energy at $x = 2$ is:

(A) 2 J ✓✓

(B) 1 J

(C) 1.5 J

(D) 0 J

$U_{x=0} = 0$

$(K.E)_{x=0} = 0$

$(E_T) = U_{x=0} + (K.E)_{x=0}$

$(E_T) = 0$

$U_{x=2} = (4 - 3\times 2)$

$= -2J$

At $x = 2$

$E_T = U_{x=2} + (K.E)_{x=2}$

$\Downarrow$

$0 = U_{x=2} + (K.E)_{x=2}$

$(K.E)_{x=2} = -(U_{x=2})$

$= -(-2J)$

$= +2J$ ✓

WORK POWER ENERGY

Q.6 A particle of mass m is moving in a horizontal circle of radius r under a centripetal force equal to $\left(-\frac{k}{r^2}\right)$, where k is a positive constant. Then if kinetic energy, potential energy and mechanical energy of the particle are KE, PE and ME respectively. Which one is correct?

(A) $KE = \left(\frac{k}{2r}\right), PE = -\left(\frac{k}{r}\right), ME = -\left(\frac{k}{2r}\right)$

(B) $KE = \left(\frac{k}{2r}\right), PE = -\left(\frac{k}{2r}\right), ME =$ zero

(C) $KE =$ zero, $PE =$ zero, $ME =$ zero

(D) $KE = \left(\frac{k}{r}\right), PE = -\left(\frac{k}{2r}\right), ME = \left(\frac{k}{2r}\right)$

$F_c = \frac{k}{r^2}$

$\frac{mv^2}{r} = \frac{k}{r^2}$

$mv^2 = \frac{k}{r}$

$K.E = \frac{mv^2}{2} = \left(\frac{k}{2r}\right)$

$E_T = K.E + P.E$

$= \left(\frac{k}{2r} - \frac{k}{r}\right) = \left(-\frac{k}{2r}\right)$

$F = -\frac{dU}{dr}$

$\int_0^U dU = -\int_\infty^r F \cdot dr$

$U = k\int_\infty^r \frac{dr}{r^2} = \left(-\frac{k}{r}\right)$

WORK POWER ENERGY

Q.9 The force between two atoms in a diatomic molecule can be represented approximately by the potential energy function

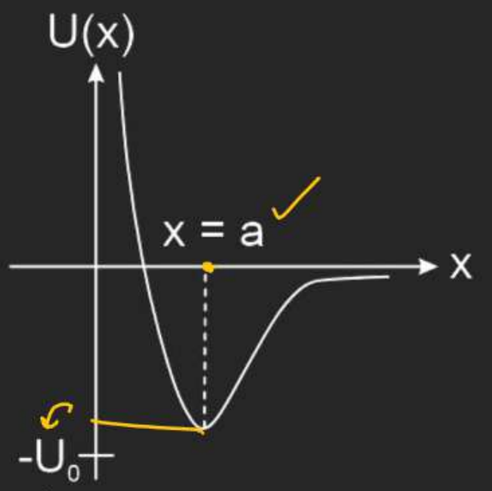
$$U = U_0\left[\left(\frac{a}{x}\right)^{12} - 2\left(\frac{a}{x}\right)^{6}\right]$$

where U_0 and a are constants.

(a) At what value of x is the potential energy zero?

(b) Find the force F_x.

(c) At what value of x is the potential energy a minimum?



x = a

a) $U = 0$

$U_0\left(\frac{a}{x}\right)^6\left[\left(\frac{a}{x}\right)^6 - 2\right] = 0$

$\left(\frac{a}{x}\right)^6 = 2$

$\frac{a}{x} = (2)^{1/6} \Rightarrow x = \frac{a}{(2)^{1/6}}$

↳ At $x = a$ P.E minimum.

$U_{x=a} = ??$

Nishant Jindal

$$U = U_0\left[\left(\frac{a}{x}\right)^{12} - 2\left(\frac{a}{x}\right)^{6}\right]$$

$$U = U_0\left[a^{12}(x^{-12}) - 2a^6 \cdot x^{-6}\right]$$

$$F = -\frac{dU}{dx} = -U_0\left[a^{12}\frac{d}{dx}(x^{-12}) - 2a^6\frac{d}{dx}(x^{-6})\right]$$

$$F = -U_0\left[a^{12}(-12)x^{-13} - 2a^6(-6)x^{-7}\right]$$

$$F = -U_0\left[-\frac{12a^{12}}{x^{13}} + \frac{12a^6}{x^7}\right]$$

$$F = -U_0 12a^6\left[\frac{1}{x^7} - \frac{a^6}{x^{13}}\right]$$

$$|E_T| = (K\cdot E) = \frac{|P\cdot E|}{2}$$

$$\Downarrow$$

$$F \propto \frac{1}{r^2} \Rightarrow$$ only true for F follow inverse square law.