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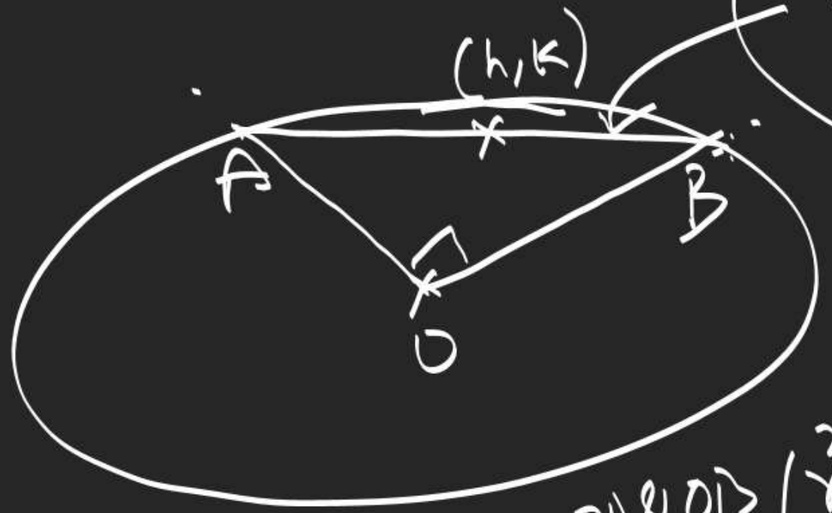

$$\frac{xh}{a^2}+\frac{yk}{b^2}=\frac{h^2}{a^2}+\frac{k^2}{b^2}$$

OA & OB $\Rightarrow \left(\frac{x^2}{a^2}+\frac{y^2}{b^2}\right)\left(\frac{h^2}{a^2}+\frac{k^2}{b^2}\right)-\left(\frac{xh}{a^2}+\frac{yk}{b^2}\right)^2=0$

$$\left(\frac{h^2}{a^2}+\frac{k^2}{b^2}\right)^2\left(\frac{1}{a^2}+\frac{1}{b^2}\right)-\frac{h^2}{a^4}-\frac{k^2}{b^4}=0$$

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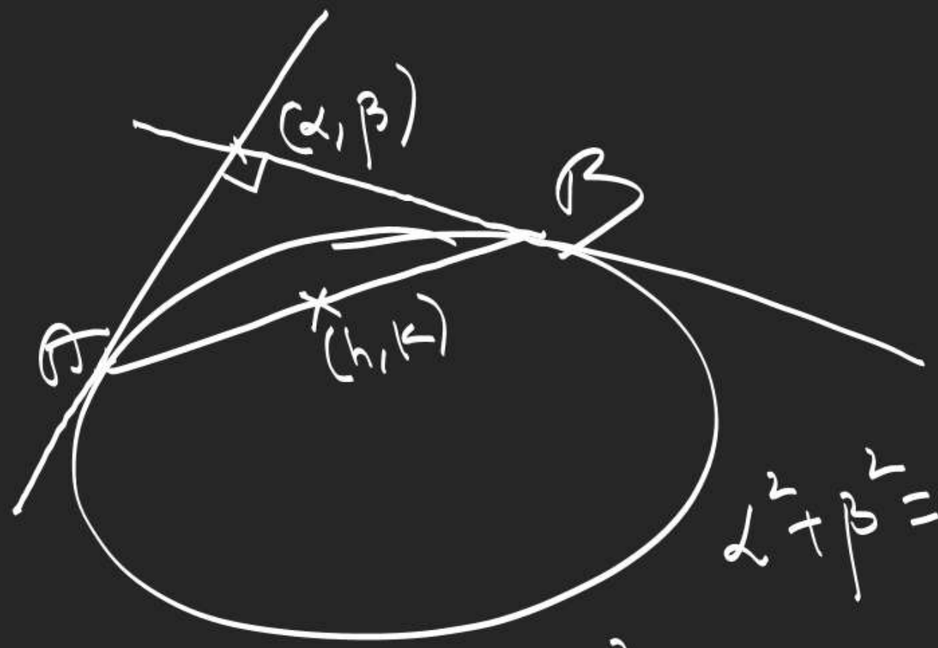

$$\alpha^2+\beta^2=a^2+b^2$$

$$\frac{x\alpha}{a^2}+\frac{y\beta}{b^2}=1$$

$$\frac{xh}{a^2}+\frac{yk}{b^2}=\frac{h^2}{a^2}+\frac{k^2}{b^2}$$

$$\frac{\alpha}{h}=\frac{\beta}{k}=\frac{1}{\frac{h^2}{a^2}+\frac{k^2}{b^2}}$$

$$\frac{h^2+k^2}{\left(\frac{h^2}{a^2}+\frac{k^2}{b^2}\right)^2}=a^2+b^2$$

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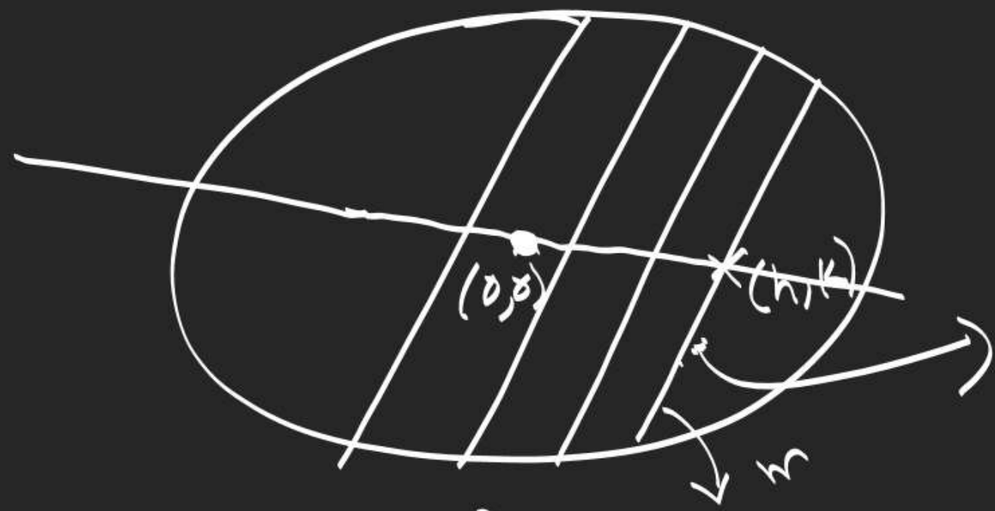
Diameters.



$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

$$\frac{xh}{a^2}+\frac{yk}{b^2}=\frac{h^2}{a^2}+\frac{k^2}{b^2}$$

$$-\frac{h}{k}\frac{b^2}{a^2}=m$$

$$y=-\frac{b^2}{a^2 m}x.$$

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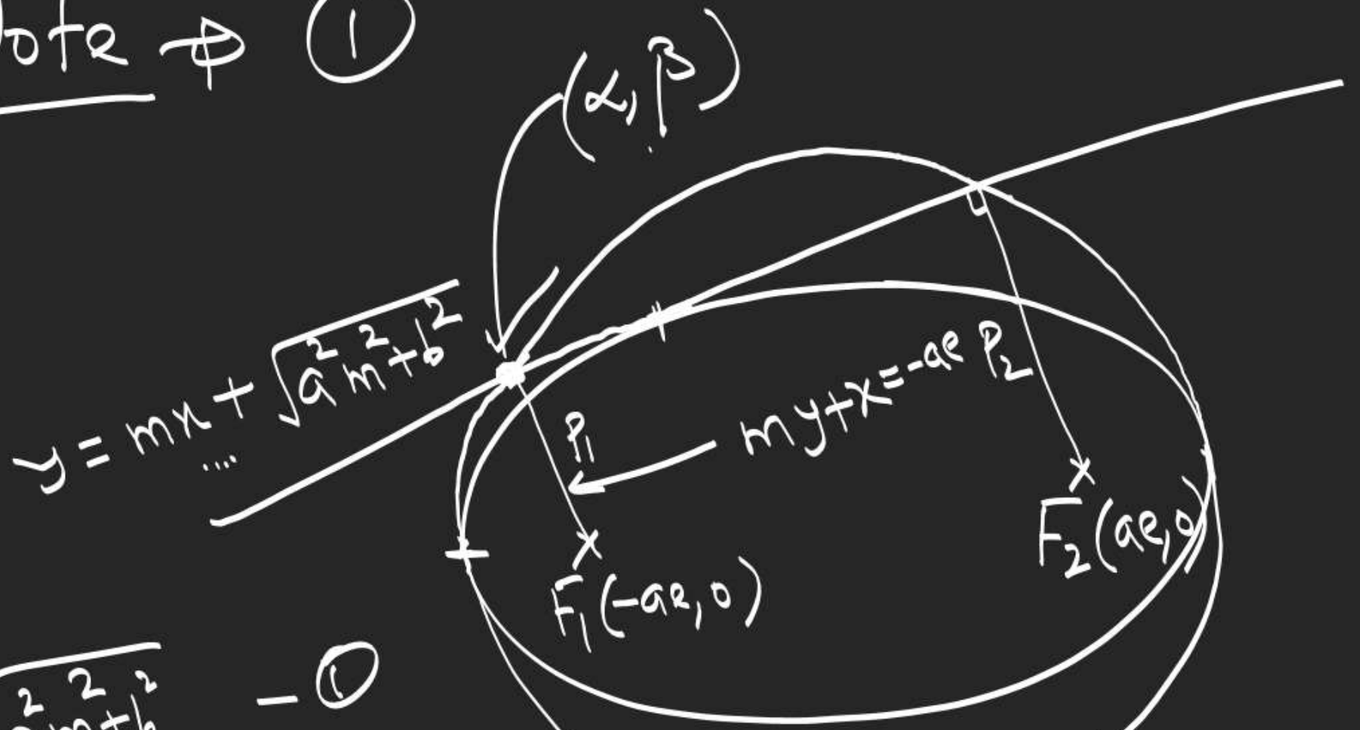
Note ⇒ (1)



$\beta - m\alpha = \sqrt{a^2m^2 + b^2}$ — (1)

$\alpha + m\beta = -ae$ — (2)

(1)² + (2)²

$(\alpha^2 + \beta^2)(1 + m^2) = a^2m^2 + b^2 + a^2 - b^2$

$\alpha^2 + \beta^2 = a^2$

$P_1P_2 = \dfrac{\left|-mae + \sqrt{a^2m^2 + b^2}\right|}{\sqrt{1+m^2}} \cdot \dfrac{\left|mae + \sqrt{a^2m^2 + b^2}\right|}{\sqrt{1+m^2}}$

$= \dfrac{\left|a^2m^2 + b^2 - m^2(a^2 - b^2)\right|}{1 + m^2} = b^2$

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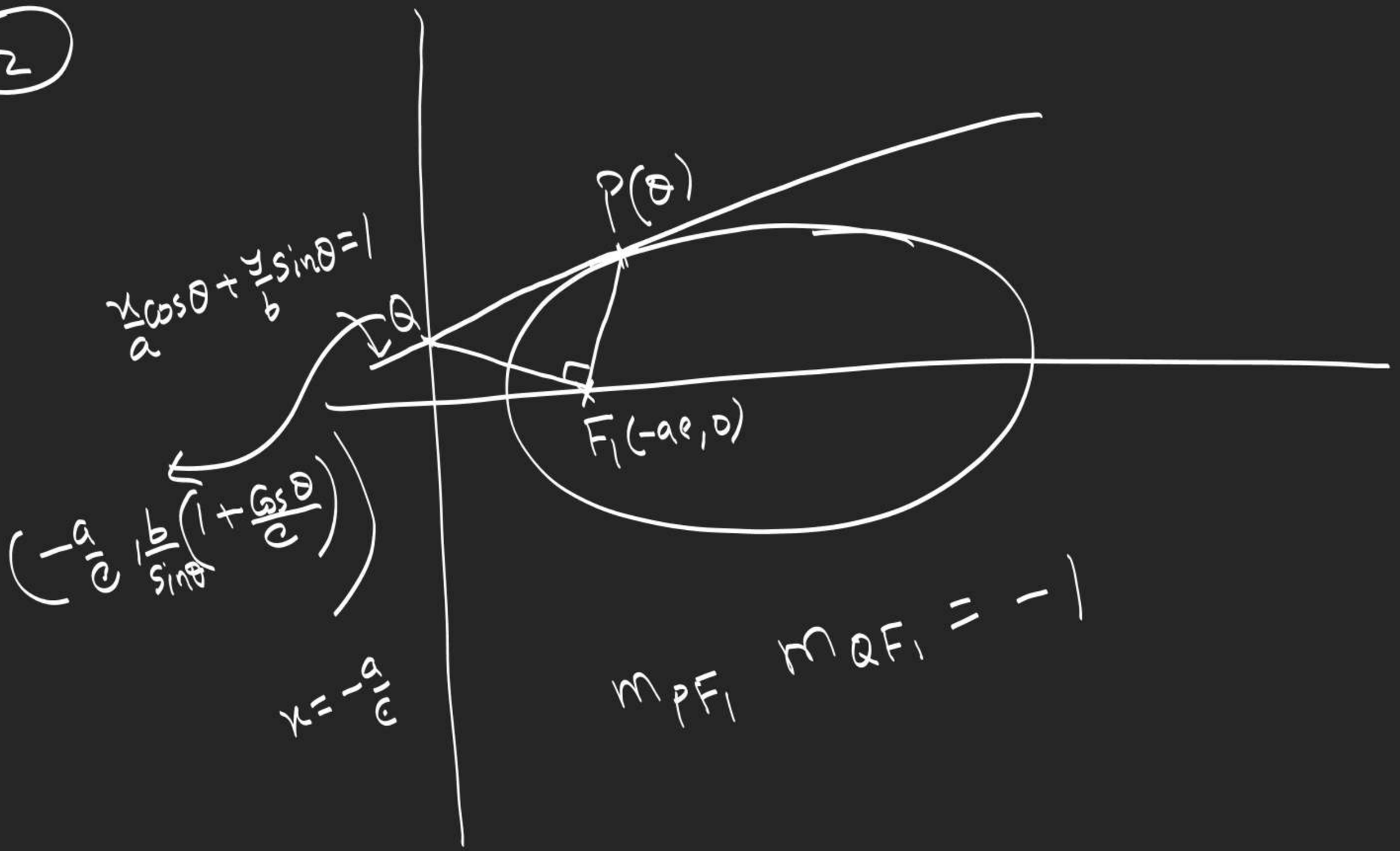
(2)



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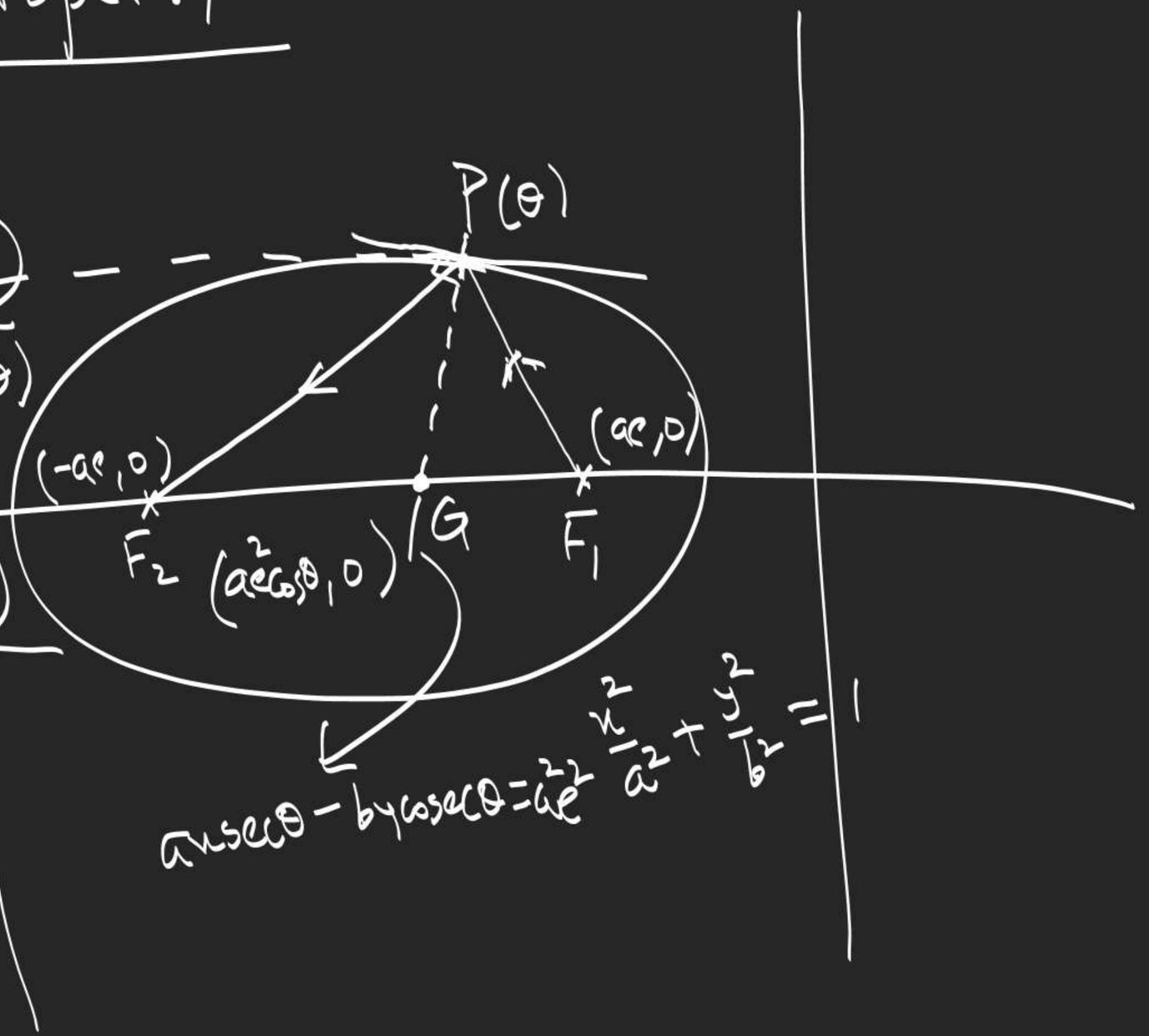
(3) Reflection Property



$$\frac{F_2G}{GF_1} = \frac{ae(e\cos\theta + 1)}{ae(1 - e\cos\theta)} \quad - - -$$

$$\frac{PF_2}{PF_1} = \frac{e\left(a\cos\theta + \frac{a}{e}\right)}{e\left(\frac{a}{e} - a\cos\theta\right)}$$

$$= \frac{1 + e\cos\theta}{1 - e\cos\theta}$$

$$a\sec\theta - b\cot\theta\csc\theta = a^2e^2 \quad \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

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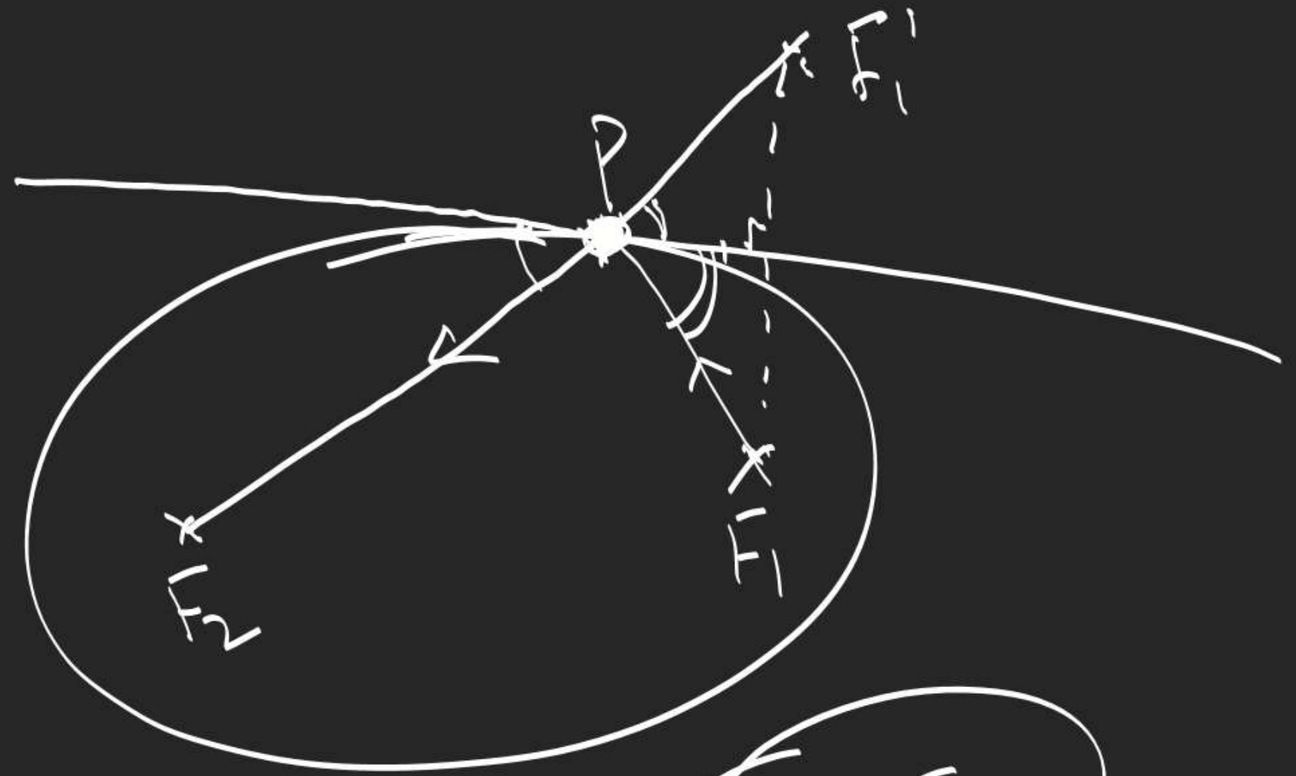

P = ?

$P\vec{F}_1 + P\vec{F}_2$

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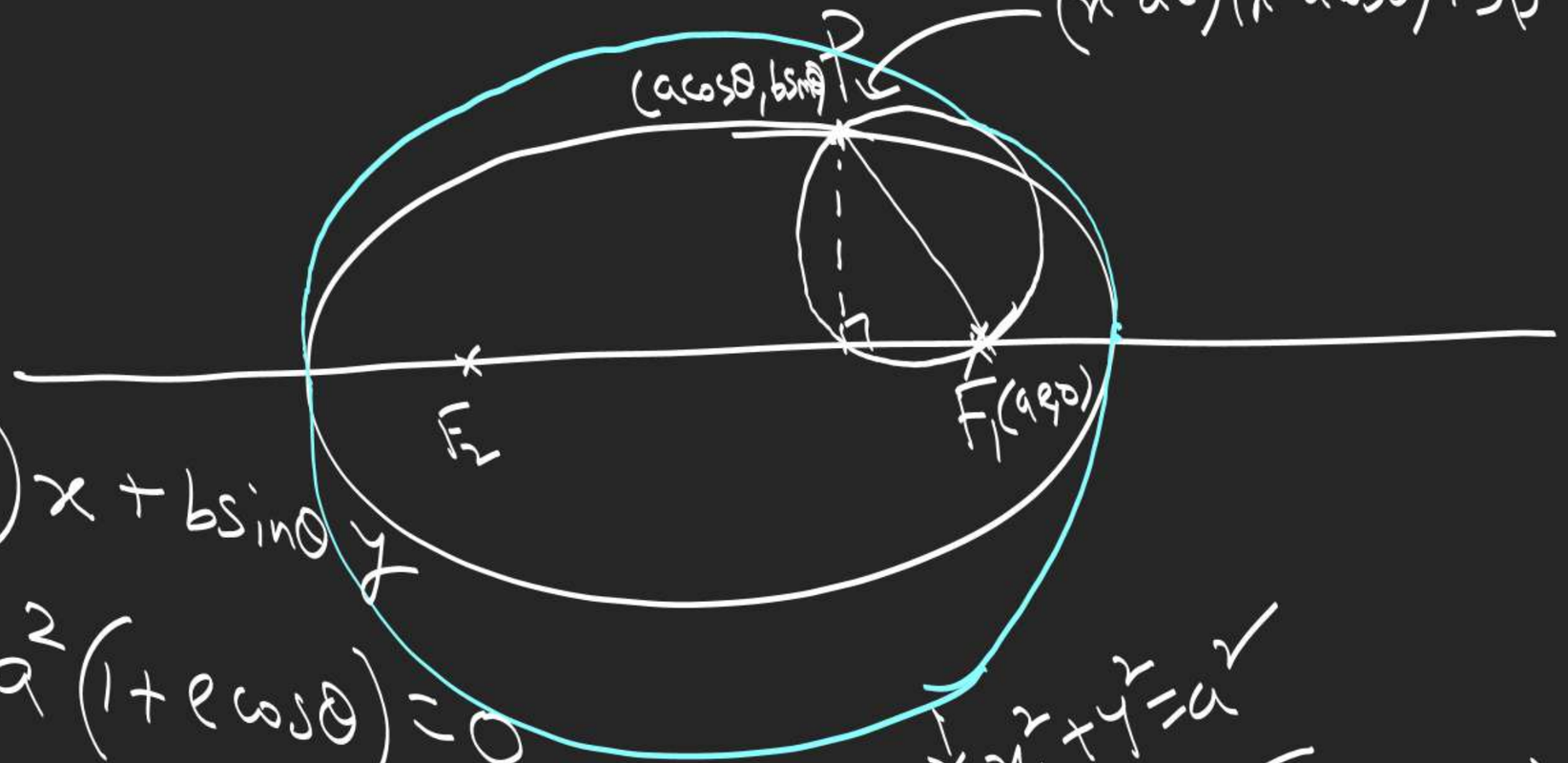
4.



$(x-ae)(x-a\cos\theta)+y(y-b\sin\theta)=0$

RA

$(ae+a\cos\theta)x+b\sin\theta\, y - a^2(1+e\cos\theta)=0$

$\downarrow x^2+y^2=a^2$

$$p=\frac{|a^2(1+e\cos\theta)|}{\sqrt{a^2(e^2+\cos^2\theta+2e\cos\theta)+a^2(1-e^2)\sin^2\theta}}=\frac{a^2(1+e\cos\theta)}{a\sqrt{e^2\cos^2\theta+1+2(\cos\theta)e}}=a$$

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Significance of eccentricity

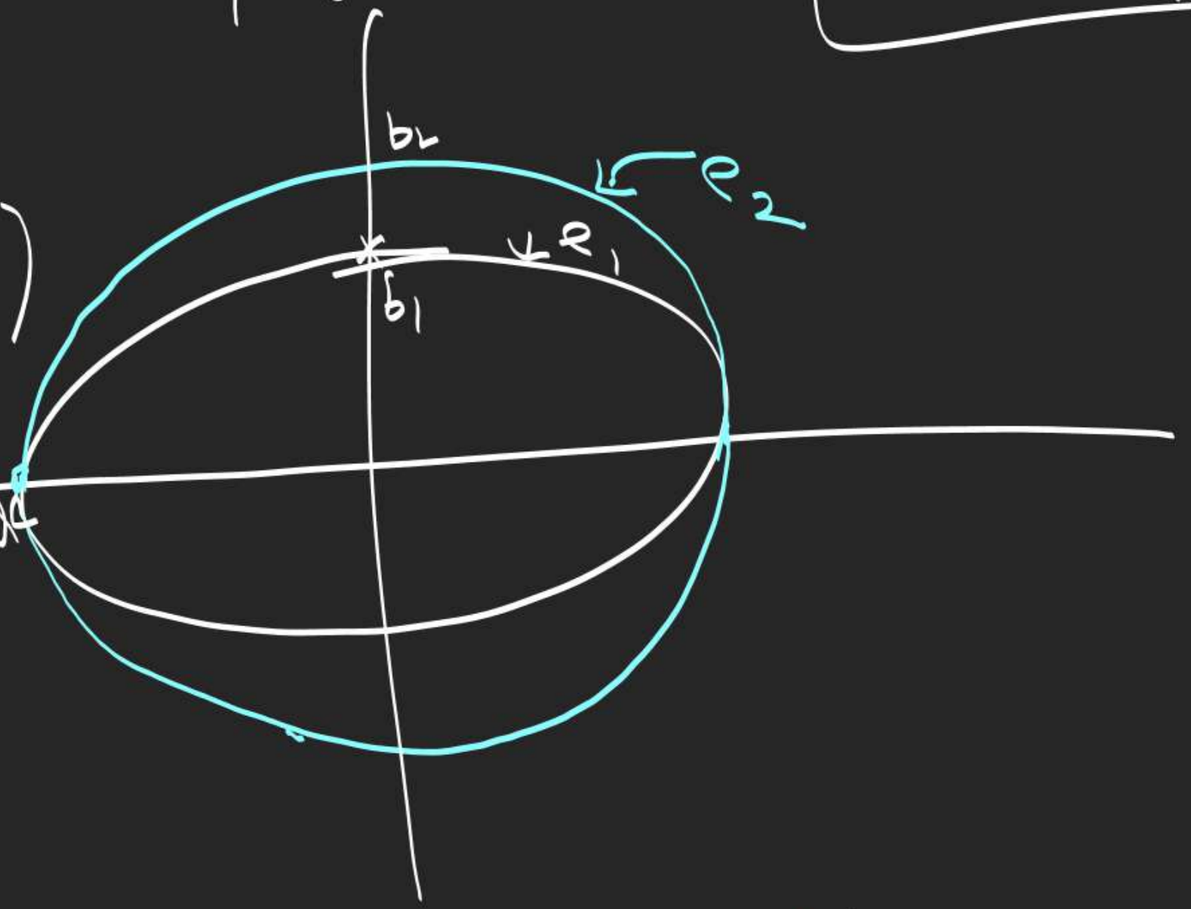
↓

Degree of flatness

remaining Ex-3

$b_2 > b_1$

$a^2(1-e_2^2) > a^2(1-e_1^2)$

$e_2 < e_1$

$e \to 0$ ⇒ Ellipse → Circle