

DPP 03

Solution

1. Effective force constant in Case (3) and (4) is

$$k_{\text{eff}} = 2k + 2k = 4k$$

$$\text{Therefore, } T_1 = T_2 = 2\pi\sqrt{\frac{m}{k}}$$

$$\text{and } T_3 = T_4 = 2\pi\sqrt{\frac{m}{4k}} = \pi\sqrt{\frac{m}{k}}$$

2. Since, $\frac{1}{k_s} = \frac{1}{k} + \frac{1}{2k} + \frac{1}{4k} + \frac{1}{8k} + \dots$

$$\Rightarrow \frac{1}{k_s} = \frac{1}{k} \left(1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots \right)$$

$$\Rightarrow \frac{1}{k_s} = \frac{1}{k} \left(\frac{1}{1 - 1/2} \right) = \frac{2}{k} \Rightarrow k_s = \frac{k}{2}$$

$$\Rightarrow T = 2\pi\sqrt{\frac{m}{k_s}} = 2\pi\sqrt{\frac{2m}{k}}$$

3. $K_{\text{eff}} = \frac{(2K)(2K)}{2K+2K} + K + K$

$$\Rightarrow K_{\text{eff}} = 3K$$

$$\Rightarrow f = \frac{1}{2\pi} \sqrt{\frac{3K}{M}}$$

4. Since, $T = 2\pi\sqrt{\frac{m}{k_p}} = 2\pi\sqrt{\frac{m}{2k}}$

$$\text{and } T' = 2\pi\sqrt{\frac{m}{k}} = \frac{T}{\sqrt{2}}$$

5. Phase difference between the two SHMs is 90° . Then, resultant amplitude of both SHM is

$$A_R = \sqrt{2}A$$

$$\Rightarrow E = \frac{1}{2}m\omega^2 A_R^2 = \frac{1}{2}m\omega^2 (\sqrt{2}A)^2 = m\omega^2 A^2$$

6. Since, $\omega = \sqrt{\frac{k}{m}}$

General equation of motion is

$$x = 2\ell \cos(\omega t)$$

$$\Rightarrow \ell = 2\ell \cos(\omega t)$$

$$\Rightarrow \omega t = \frac{\pi}{3}$$

$$\Rightarrow \sqrt{\frac{k}{m}} t = \frac{\pi}{3}$$

$$\Rightarrow t = \frac{\pi}{3} \sqrt{\frac{m}{k}}$$

$$\Rightarrow T = 2t = \frac{2\pi}{3} \sqrt{\frac{m}{k}}$$

7. Since, $k_{\text{eff}} = 2k$, so $T = 2\pi \sqrt{\frac{M}{2k}}$

Period (or k_{eff}) is independent of θ .

8. Time period of pendulum

$$T = 2\pi \sqrt{\frac{l}{g}} = 2\pi \sqrt{\frac{1}{\pi^2}} = 2 \text{ s}$$

On the right side, it completes half an oscillation, whereas on the left side, it only goes from mean to half the amplitude and comes back. So, time of oscillation is

$$T' = \frac{T}{2} + 2\left(\frac{T}{12}\right) = \frac{2}{3}T = \frac{4}{3} \text{ s}$$

9. Block of mass m_2 shorts off carrying some kinetic energy away from the system.

By Law of Conservation of Mechanical Energy

$$\left(\begin{array}{c} \text{Potential Energy} \\ \text{of Spring} \end{array} \right) = \left(\begin{array}{c} \text{Maximum Kinetic} \\ \text{Energy of Blocks} \end{array} \right)$$

$$\Rightarrow \frac{kd^2}{2} = (m_1 + m_2) \frac{v^2}{2}$$

$$\Rightarrow v^2 = \frac{kd^2}{m_1 + m_2}$$

With m_1 alone on the spring, we have

$$\left(\begin{array}{c} \text{Maximum Potential} \\ \text{Energy} \end{array} \right) = \left(\begin{array}{c} \text{Maximum Kinetic} \\ \text{Energy of } m_1 \end{array} \right)$$

$$\Rightarrow \frac{1}{2} kA^2 = \frac{1}{2} m_1 v^2$$

$$\Rightarrow kA^2 = \frac{km_1 d^2}{m_1 + m_2}$$

$$\Rightarrow A = d \sqrt{\frac{m_1}{m_1 + m_2}}$$

10. Beyond point P, length of pendulum becomes $\frac{\ell}{4}$.

Since $T \propto \sqrt{\ell}$, so beyond P time period will become $T = \frac{T}{2}$. Hence required time is

$$t = \frac{T}{2} + \frac{T'}{2} = \frac{T}{2} + \frac{T}{4} = \frac{3T}{4}$$

11. Time period of linear oscillations of a spring mass system is independent of any constant force acting on the block.

12. Since, $v^2 = \omega^2(a^2 - x^2)$

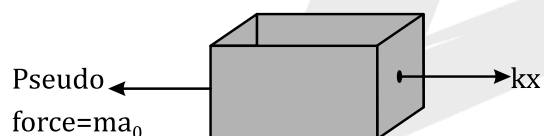
$$\Rightarrow v_1^2 = \omega^2(a^2 - y_1^2) \quad \dots (i)$$

$$\Rightarrow v_2^2 = \omega^2(a^2 - y_2^2) \quad \dots (ii)$$

From (1) and (2), we get

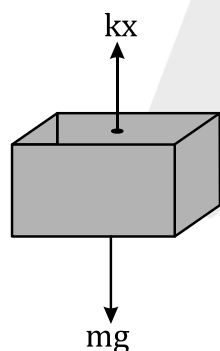
$$T = 2\pi \sqrt{\frac{y_1^2 - y_2^2}{v_2^2 - v_1^2}}$$

13. Free body diagram of the truck from non-inertial frame of reference.



This is similar to a situation when a block is suspended from a vertical spring.

So, the block will execute simple harmonically with time period $T = 2\pi \sqrt{\frac{m}{k}}$



Amplitude will be given by

$$A = x = \frac{ma_0}{k} \quad \{\because ma_0 = kx\}$$

Energy of oscillation will be

$$E = \frac{1}{2}kA^2 = \frac{1}{2}k\left(\frac{ma_0}{k}\right)^2 = \frac{m^2a_0^2}{2k}$$

14. Since the particle starts from the extreme position, so

$$x = a \cos \omega t$$

$$\Rightarrow \frac{a}{2} = a \cos \omega t$$

$$\Rightarrow \cos \omega t = \frac{1}{2} = \cos \frac{\pi}{3}$$

$$\Rightarrow t = \frac{T}{6} = \frac{0.60}{6} = 0.10 \text{ s}$$

15. $\langle v \rangle = \frac{a/2}{T/6} = \frac{3a}{T}$

$$\Rightarrow \langle v \rangle = \frac{3 \times 10 \times 10^{-2}}{0.60} \text{ ms}^{-1} = 0.50 \text{ ms}^{-1}$$

16. Since the particle starts from the mean position, so

$$x = a \sin \omega t$$

$$\Rightarrow \frac{a}{2} = a \sin \omega t$$

$$\Rightarrow \frac{1}{2} = \sin \omega t \sin \frac{\pi}{6} = \sin \omega t$$

$$\Rightarrow \frac{\pi}{6} = \frac{2\pi}{T} t$$

$$\Rightarrow t = \frac{T}{12} = \frac{0.60}{12} \text{ s} = 0.05 \text{ s}$$

17. $\langle v \rangle = \frac{a/2}{T/12} = \frac{6a}{T}$

$$\Rightarrow \langle v \rangle = \frac{6 \times 10 \times 10^{-2}}{0.60} \text{ ms}^{-1} = 1 \text{ ms}^{-1}$$

18. The given equation, $y = A \sin(\omega t) + A \sin \left(\omega t + \frac{2\pi}{3} \right)$ can also be written as.

$$y = 2A \sin \left(\omega t + \frac{\pi}{3} \right) \cdot \cos \left(\frac{\pi}{3} \right)$$

$$y = A \sin \left(\omega t + \frac{\pi}{3} \right)$$

Now, we can see that this is SHM with amplitude A and initial phase $\frac{\pi}{3}$.

$$v = A\omega \cos \left(\omega t + \frac{\pi}{3} \right)$$

$$\Rightarrow v_{\max} = A\omega$$