Nishant Jindal

Geometrical Interpretation of C.N

1) We Can Show C.N on Argand Plane.

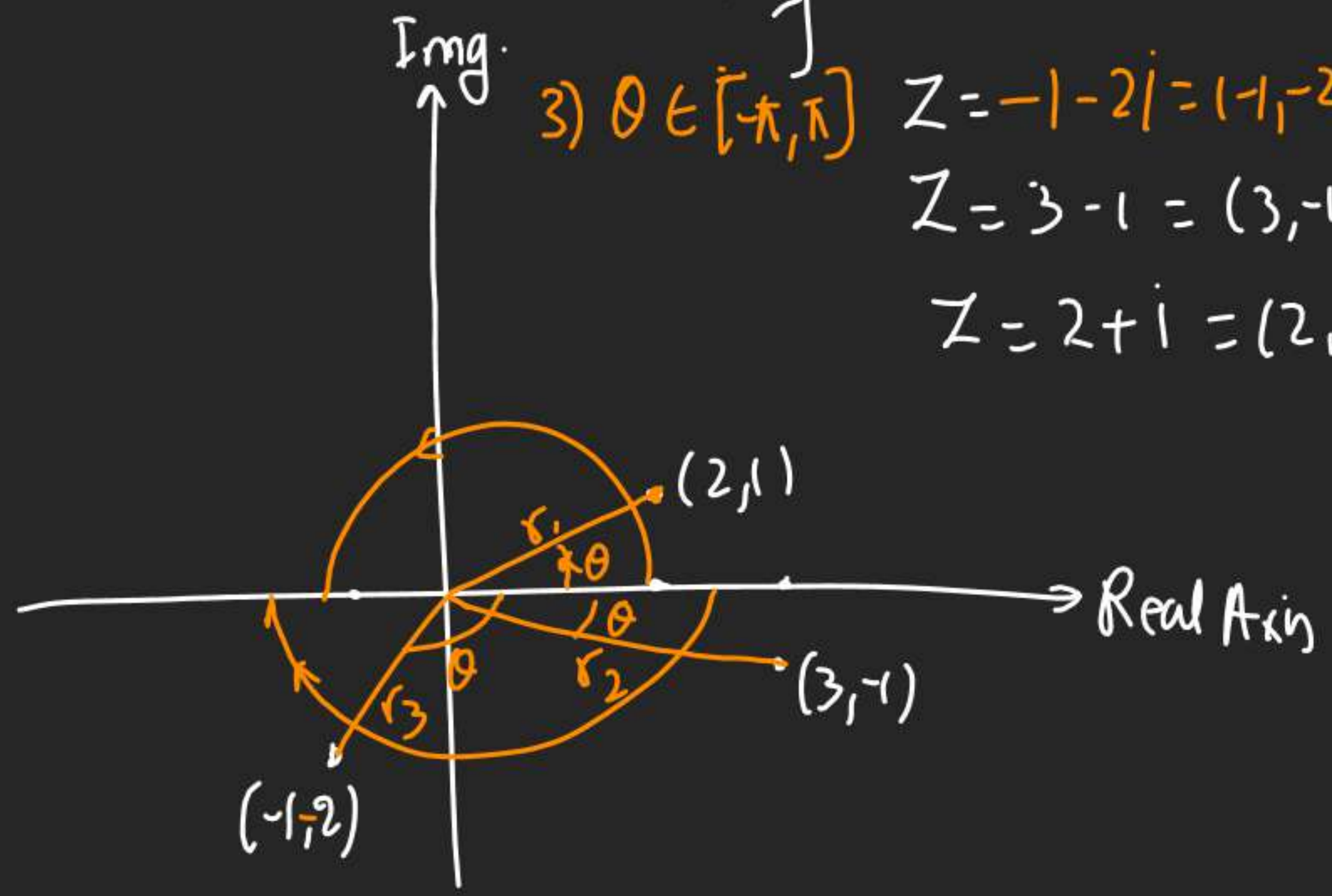
2) It has 2 Axes → Real Axis
   → Imaginary Axis

3) $\theta \in [-\pi, \pi]$

$Z = -1-2i = (-1,-2)$

$Z = 3-i = (3,-1)$

$Z = 2+i = (2,1)$



(3) If $Z = (x+iy)$ then in Argand Plane it is Rep. by (x,y); $x, y \in R$.

(4) A Complex No. behaves Like a Position Vector.

(5) No line can Cross 180°

Beyond this we take angle from $[-\pi, 0)$

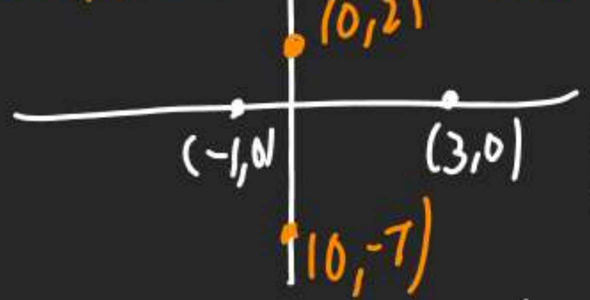
(6) $0+0i$ Rep. origin $(0,0)$ here

(7) A Purely Real No $Z = a = a+0i = (a,0)$

Rep. themselve at Real Axis

$Z = 2i = 0+2i = (0,2)$

$Z = -7i = 0-7i = (0,-7)$



$Z = 3 = 3+0i = (3,0)$

$Z = -1 = -1+0i = (-1,0)$

(8) A Purely Imaginary No $Z = bi = (0,b)$ Placed on Img. Axis

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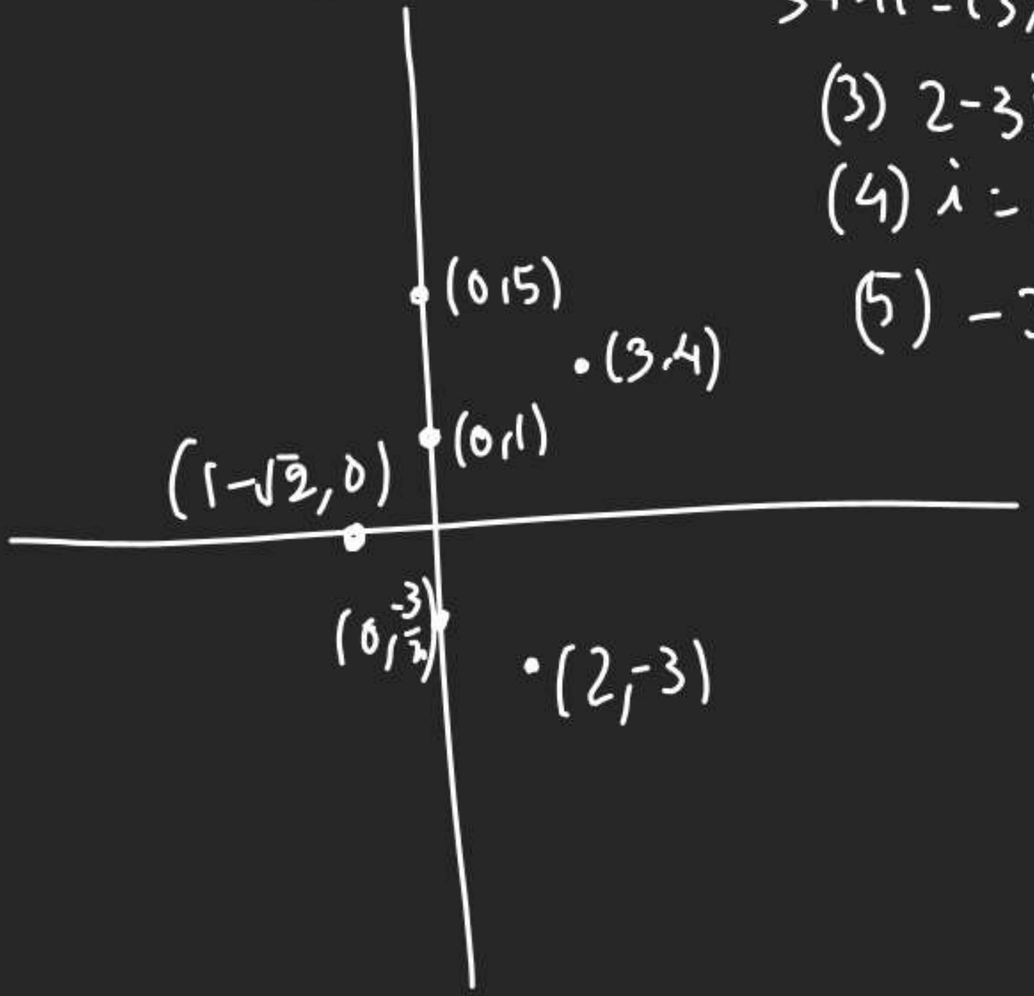
Q Put following C.N. on Argand Plane.

1) $3+4i$ (2) $5i$ (3) $2-3i$ (4) i

(5) $-\frac{3i}{2}$ (6) $1-\sqrt{2}$



$3+4i=(3,4)$ (2) $5i=0+5i=(0,5)$

(3) $2-3i=(2,-3)$

(4) $i=0+1\cdot i=(0,1)$

(5) $-\frac{3i}{2}=0-\frac{3}{2}i=\left(0,-\frac{3}{2}\right)$

(6) $1-\sqrt{2}=1-\sqrt{2}+0\cdot i$

$=(1-1.414,0)$

$=(-.414,0)$

Q Which of the following is not a C.N.? Ex1,2

A) (i^5, i^4)

B) $(\sqrt{-2}, i^8)$

C) $(\tan\frac{\pi}{2}, \log_e 2)$ ⊗

D) $(\sqrt{e}, i^{12})$

all given option are in the form of (x,y) & $x,y \in$ Real No. Whosoever is not $\in$ Real No. is not a C.N.

(A) $(x,y)=(i^5,i^4)=(i,1)$ ⊗ $\uparrow \notin R$

(B) $(\sqrt{-2}, i^8)=(\sqrt{2}i, 1)$ ⊗ $\notin R$

(C) $(\tan\frac{\pi}{2}, \ln 2)=(\infty, \ln 2)$ $\notin R$, R

(D) $(\sqrt{e}, 1)$ $\in R$ $\in R$

Yes it is a CN

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Properties of C.N.

(A) Equality of 2 C.N.

A) If $Z_1 = x_1 + iy_1$ & $Z_2 = x_2 + iy_2$

& $Z_1 = Z_2$

$x_1 + iy_1 = x_2 + iy_2$

$\rightarrow x_1 = x_2$ & $y_1 = y_2$

(B) 2 C.N. are Said to be Eql when Real Part of both & Img Part of Both is equal.

(C) If $Z_1 = Z_2$ then

$Re(z_1) = Re(z_2)$ &

$Im(z_1) = Im(z_2)$

Q If $\underline{2}x + (x-y)i = \underline{4} + 2i$ find (x, y)?

$2x = 4$ & $(x-y) = 2$

$x = 2$ & $2 - y = 2$

$y = 0$

$(x, y) = (2, 0)$

Rk :- Im C.N.

(1) No C.N can be gr. than or less than to another.

(2) $x_1 + iy_1 > x_2 + iy_2$

It is Absurd.

$3i > 2i$ (He He He)

(3) If Somebody forcefully Present

$a + bi > c + di$

then we take b & $d = 0$

then $a > c$ is allowed

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Q $4x+i(3x-y)=3-6i$

find $(x,y)=?$

$4x=3$ & $3x-y=-6$

$x=\frac{3}{4}$ $\frac{9}{4}-y=-6$

$y=\frac{9}{4}+6$

$=\frac{33}{4}$

$(x,y)=\left(\frac{3}{4},\frac{33}{4}\right)$

(2) Multiplication of Constant

$K(a+ib)=Ka+iKb$

$2\cdot(3-i)=6-2i$ ✓

(3) Sum of 2 C.N.

$Z=Z_1+Z_2=(x_1+iy_1)+(x_2+iy_2)$

$x+iy=(x_1+x_2)+i(y_1+y_2)$

$$\boxed{\begin{array}{l} x=x_1+x_2 \Rightarrow Re(z)=Re(z_1)+Re(z_2) \\ y=y_1+y_2 \Rightarrow Im(z)=Im(z_1)+Im(z_2) \end{array}}$$

Q $Z_1=2+3i$, $Z_2=3-2i$

find $Re(Z_1+Z_2)$ & $Im(Z_1+Z_2)$?

$Z_1+Z_2=(2+3i)+(3-2i)$

$=(2+3)+i(3-2)$

$=\underline{5+i}$

$Re(Z_1+Z_2)=5$

$Im(Z_1+Z_2)=1$

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(4) Subtraction of C.N.

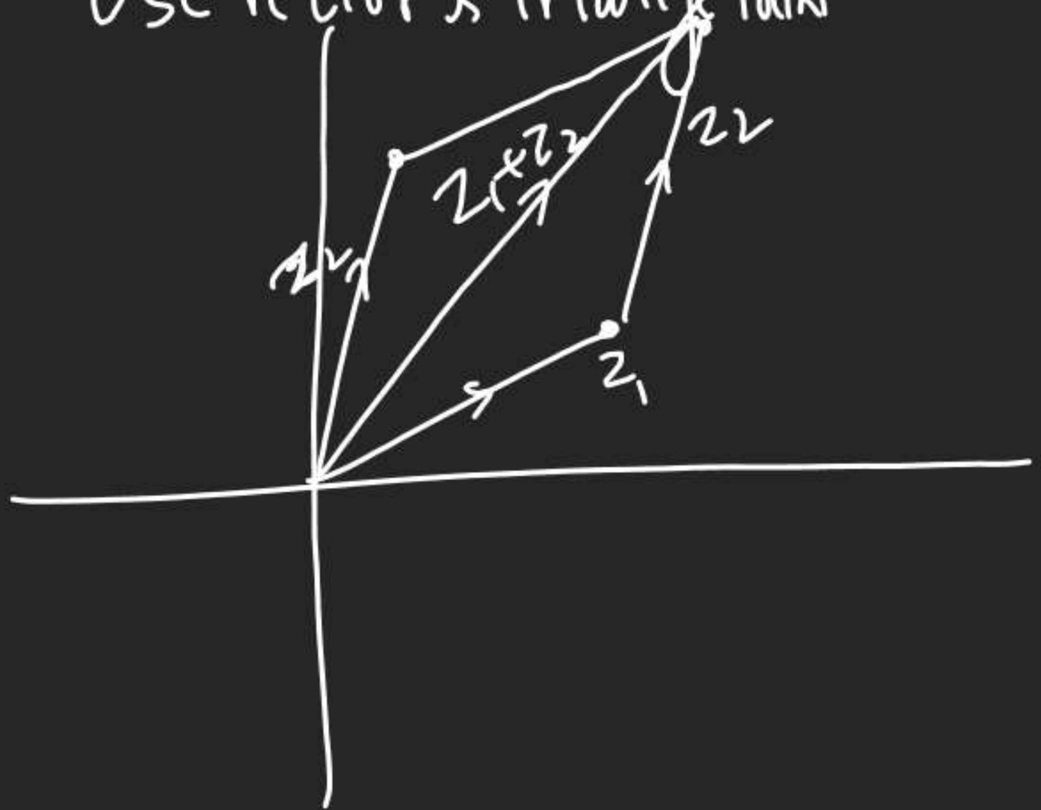
1) $z_1 - z_2 = (x_1 - x_2) + i(y_1 - y_2)$

2) $Re(z_1 - z_2) = x_1 - x_2 = Re(z_1) - Re(z_2)$

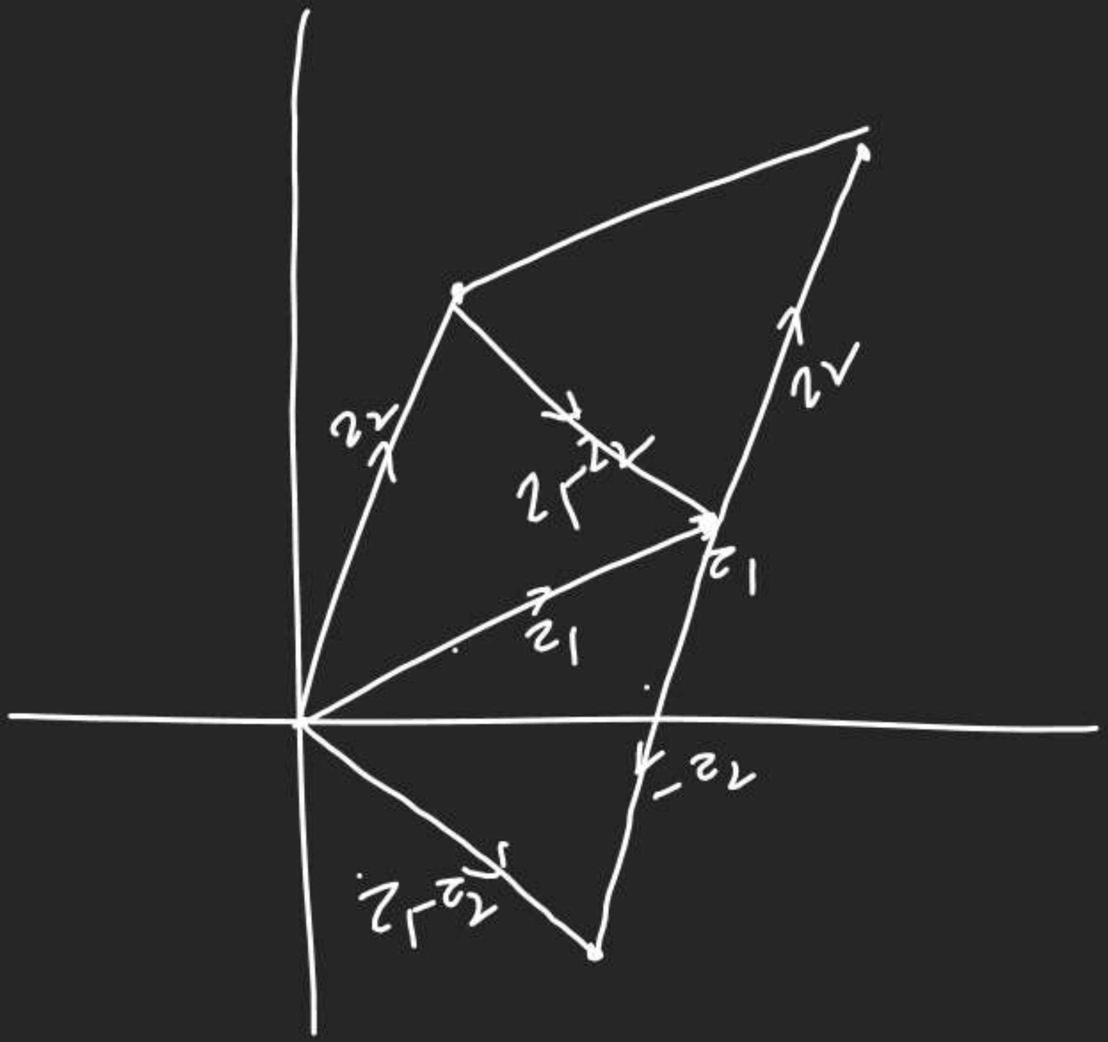
$Im(z_1 - z_2) = y_1 - y_2 = Im(z_1) - Im(z_2)$

(5) Argand Representation of Addition & Subtraction

Use Vector's triangle law



Subtraction



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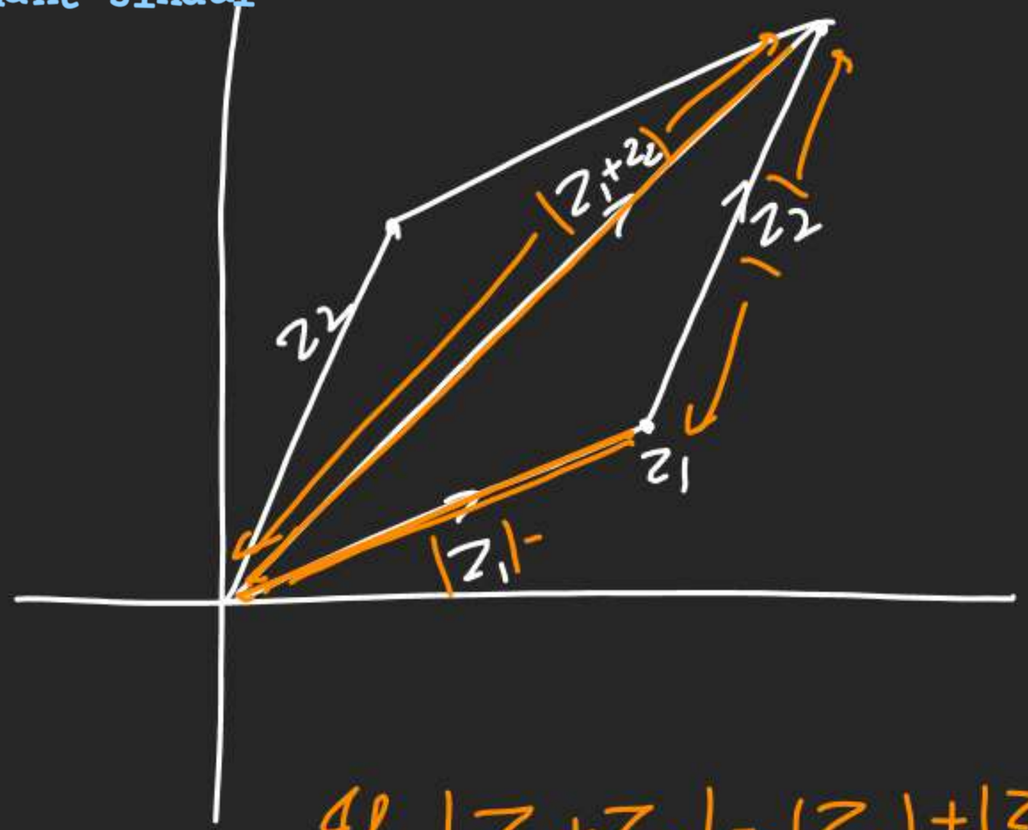
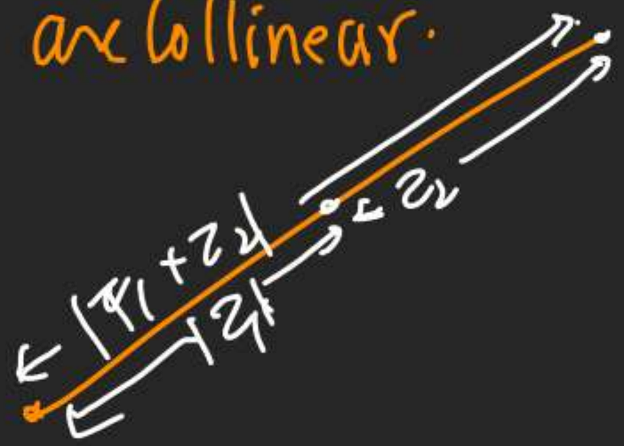


If $|z_1+z_2| = |z_1|+|z_2|$

is PSBL when z_1, z_2 are collinear.



(6) Multiplication of 2 CN.

1) $(a+ib)\cdot(c+id) = ?$

$a\cdot c + iad + ibc + i^2 bd$

$ac + i(ad+bc) - bd$

$(ac-bd) + i(ad+bc)$

(2) If $z_1 = x_1+iy_1$ & $z_2 = x_2+iy_2$

$z_1\cdot z_2 = (x_1+iy_1)(x_2+iy_2)$

$z_1\cdot z_2 = (x_1x_2 - y_1y_1) + i(x_1y_2 + x_2y_1)$

(3) $\boxed{\begin{array}{l} Re(z_1z_2) = x_1x_2 - y_1y_2 \\ Re(z_1z_2) = Re(z_1)Re(z_2) - Im(z_1)Im(z_2) \end{array}}$

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Basic Algebra

$(z_1+z_2)^2 = z_1^2+z_1^2+2z_1z_2$

$(z_1-z_2)^2 = z_1^2+z_2^2-2z_1z_2$

$(z_1^2-z_2^2) = (z_1+z_2)(z_1-z_2)$

$(z_1+z_2)^3 = z_1^3+3z_1^2z_2+3z_1z_2^2+z_2^3$

Q $(2+3i)(2-3i)=?$

$2^2-(3i)^2$

$4-(-9)$

13

Q $(-2+3i)(1-i)=?$

$-2+2i+3i-3i^2$

$-2+5i+3$

$1+5i$

Q $(a+ib)^2=?$

$(a)^2+(ib)^2+2a(ib)$

$\underbrace{a^2-b^2}+2iab$

Q $Re(a+ib)^2=?$

$Re(a+ib)^2 = a^2-b^2$

Q Simplify $\frac{1+i}{1-i}=?$

$\frac{1+i}{1-i}\times\frac{1+i}{(1+i)}$

$=\frac{(1+i)^2}{1^2-(i)^2} = \frac{1+i^2+2i}{2}$

$=\frac{\cancel{1}+\cancel{-1}+2i}{2} = i$

Try to Remember

1) $\frac{1+i}{1-i} = i$

2) $\frac{1-i}{1+i} = -i$

3) $(1+i)^2 = 2i$

4) $(1-i)^2 = -2i$

(5) $\frac{1}{i} = -i$

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Q Find Multiplicative Inverse of $\frac{2+3i}{1+5i} = ?$

1) A No. whose product with given No gives 1.

2) Multi Inverse $= \frac{1+5i}{2+3i} \times \frac{2-3i}{2-3i}$

$$= \frac{2-3i+10i-15i^2}{(2)^2-(3i)^2}$$

$$= \frac{17+7i}{13}$$

Q If $a+ib = \frac{(1+2i)(1-i)^2}{2+3i}$ find $\frac{a+5b}{4} = ?$

$$= \frac{(1+2i)(-2i)}{2+3i}$$

$$= \frac{-4i^2-2i}{2+3i}$$

$$= \frac{4-2i}{2+3i} \times \frac{2-3i}{2-3i}$$

$$= \frac{8-12i-4i+6i^2}{13}$$

$$= \frac{2-16i}{13}$$

$a = \frac{2}{13}, b = -\frac{16}{13}$

$$\frac{a+5b}{4} = \frac{\frac{2-80}{13}}{4} = \frac{-\cancel{78}^{39}}{13\times\cancel{4}2} = -\frac{3}{2}$$

Q If $\frac{(1+i)^2}{(1-i)^2} + \frac{1}{x+iy} = 1+i$ find $|x-2y| = ?$

$$(i)^2 + \frac{1}{x+iy} = 1+i$$

$$\frac{1}{x+iy} = 2+i$$

$$\Rightarrow x+iy = \frac{1}{2+i} \times \frac{2-i}{2-i}$$

$$= \frac{2-i}{2^2-(i)^2}$$

$$= \frac{2-i}{5}$$

$x = \frac{2}{5}$, $y = -\frac{1}{5}$

$$|x-2y| = \left|\frac{2}{5}+\frac{2}{5}\right| = \frac{4}{5}$$

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Q If $\frac{3+2i\sin\theta}{1-2i\sin\theta}$ is Purely Imag. then find value of θ?

$$Z = \frac{3+2i\sin\theta}{1-2i\sin\theta} \times \frac{1+2i\sin\theta}{1+2i\sin\theta}$$

$$= \frac{3+6i\sin\theta+2i\sin\theta+4\sin^2\theta\, i^2}{(1)^2-(2i\sin\theta)^2}$$

$$Z = \frac{(3-4\sin^2\theta)+8i\sin\theta}{1+4\sin^2\theta}$$

Z is Purely Img $\Rightarrow$ $Re(z)=0$

$$Re(Z) = \frac{3-4\sin^2\theta}{1+4\sin^2\theta} = 0$$

$$3-4\sin^2\theta = 0$$

$$\sin^2\theta = \frac{3}{4} = \left(\frac{\sqrt{3}}{2}\right)^2$$

$$\sin^2\theta = \sin^2\frac{\pi}{3}$$

$$\boxed{\theta = 2n\pi \pm \frac{\pi}{3}}$$

Q If $(x+iy)^{\frac{1}{3}} = a+ib$ & $\frac{x}{a}+\frac{y}{b} = K(a^2-b^2)$ then $K=?$

$$x+iy = (a+ib)^3$$

$$= a^3+3a^2(ib)+3a(ib)^2+(ib)^3$$

$$= a^3+3ia^2b-3ab^2-ib^3$$

$$x+iy = (a^3-3ab^2)+i(3a^2b-b^3)$$

$$\Rightarrow x = a(a^2-3b^2), \quad y = b(3a^2-b^2)$$

$$\Rightarrow \frac{x}{a} = a^2-3b^2 \qquad \frac{y}{b} = 3a^2-b^2$$

Acc to QS

$$\frac{x}{a}+\frac{y}{b} = 4a^2-4b^2$$

$$= 4(a^2-b^2)$$

Comparison

$$K=4$$

Conjugate of a C.N.

A) Conjugate of z is $\bar{z}$

(B) If $z = a+ib$ then

$\bar{z} = a - ib$

$z = 2-3i \qquad \bar{z} = 2+3i$

$z = 3+7i \qquad \bar{z} = 3-7i$

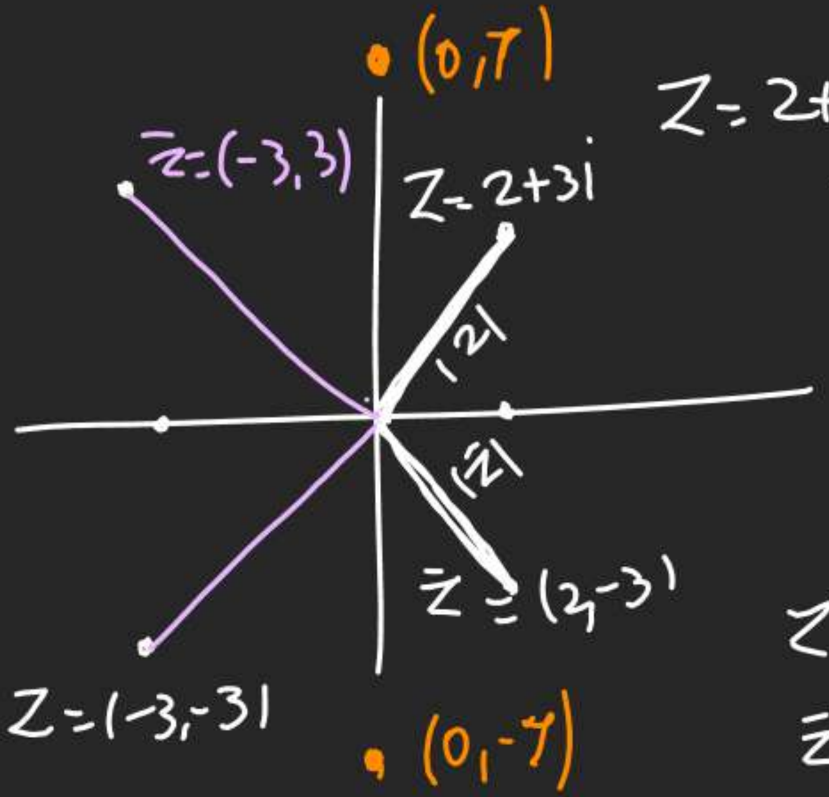
$z = -5i \qquad \bar{z} = +5i$

$z = 7 \qquad \bar{z} = 7$

i का Sign change.

(C) Geometrical Meaning of $\bar{z}$

1) $\bar{z}$ Represent Image of z in Real Axis.



$z = 2+3i = (2,3)$

$\bar{z} = 2-3i$

$= (2,-3)$

$z = 7i = (0,7)$

$\bar{z} = -7i = (0,-7)$

$z = -3-3i = (-3,-3)$

$\bar{z} = -3+3i = (-3,3)$

(2) Distance of Any C.N from Origin is Represented by $|z|$

(3) $|z| = |\bar{z}| = |-z|$

$= |-\bar{z}|$

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Ex. If $z = 2+3i = (2,3)$

then $\underline{\bar{z} = 2-3i} = (2,-3)$

$\underline{-z = -2-3i} = (-2,-3)$

$\underline{-\bar{z} = -2+3i} = (-2,3)$