Nishant Jindal

$$2+\tan x-\tan^2 x+1=2-2\tan x$$

$$\operatorname{arc}(\tan 3)=\tan^{-1}3$$

$$(\sin x-1)(\sqrt{3}+\tan x)=0$$

$$\tan^3 x+\tan^2 x-3\tan x-3=0$$

$$\cos 2x=\sin 3x$$

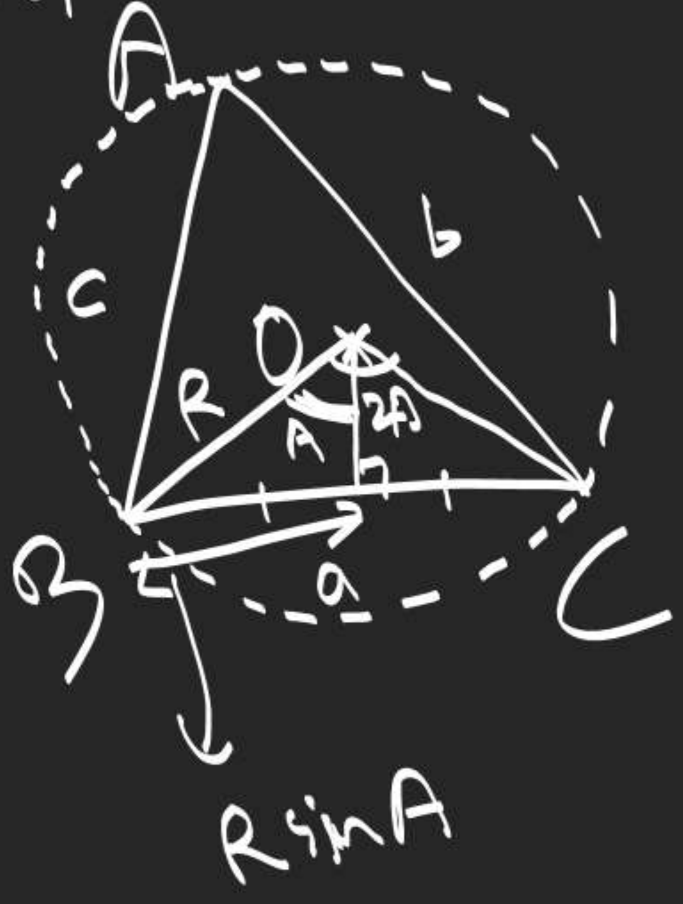
$$2x=2n\pi\pm\left(\frac{\pi}{2}-3x\right)$$

Nishant Jindal

Sine Rule

R = circumradius of ΔABC

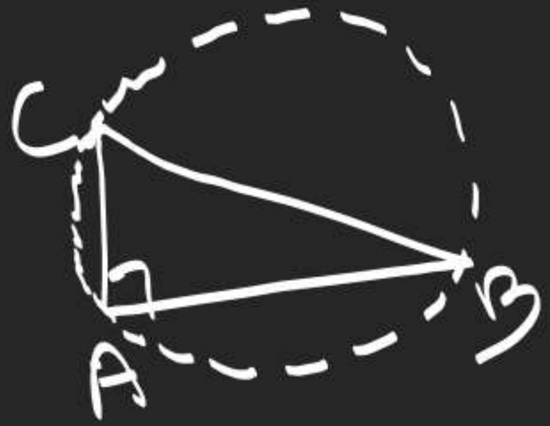
$$\boxed{\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R}$$

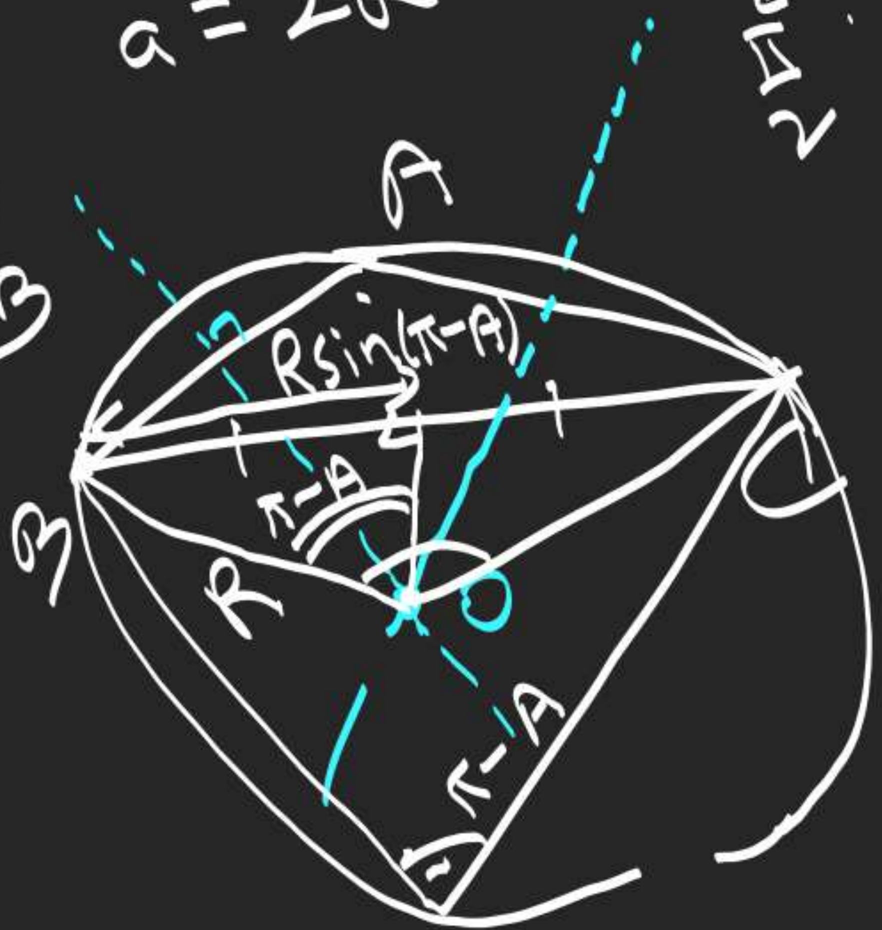


Elements of triangle

sides & angles.

$a = 2R = 2R\sin\cancel{A}$ $\frac{\pi}{2}$





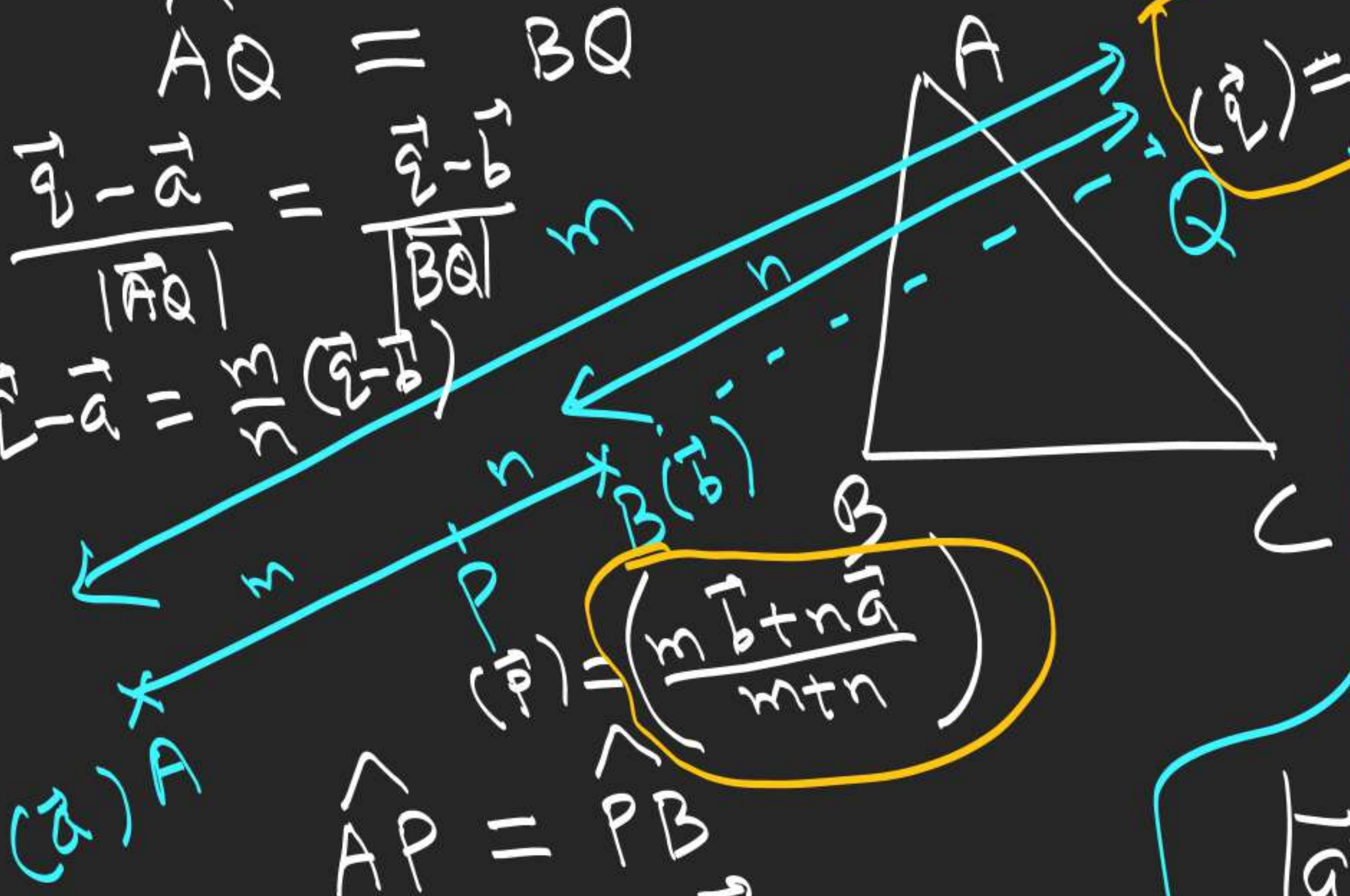
$a = 2R\sin A$

$$\widehat{AQ} = \widehat{BQ}$$

$$\frac{\vec{q} - \vec{a}}{|\vec{AQ}|} = \frac{\vec{q} - \vec{b}}{|\vec{BQ}|}$$

$$\vec{q} - \vec{a} = \frac{m}{n}(\vec{q} - \vec{b})$$

$$(\vec{q}) = \frac{m\vec{b} - n\vec{a}}{m-n}$$



$$|\vec{a}|^2 = \vec{a}\cdot\vec{a}$$

$$|\vec{a}+\vec{b}|^2 = (\vec{a}+\vec{b})\cdot(\vec{a}+\vec{b})$$

$$|\vec{a}+\vec{b}|^2 = |\vec{a}|^2 + 2\vec{a}\cdot\vec{b} + |\vec{b}|^2$$

$$|\vec{a}-\vec{b}|^2 = |\vec{a}|^2 - 2\vec{a}\cdot\vec{b} + |\vec{b}|^2$$

$$|\vec{a}+\vec{b}+\vec{c}|^2 = |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a})$$

$$(\vec{p}) = \left(\frac{m\vec{b}+n\vec{a}}{m+n}\right)$$

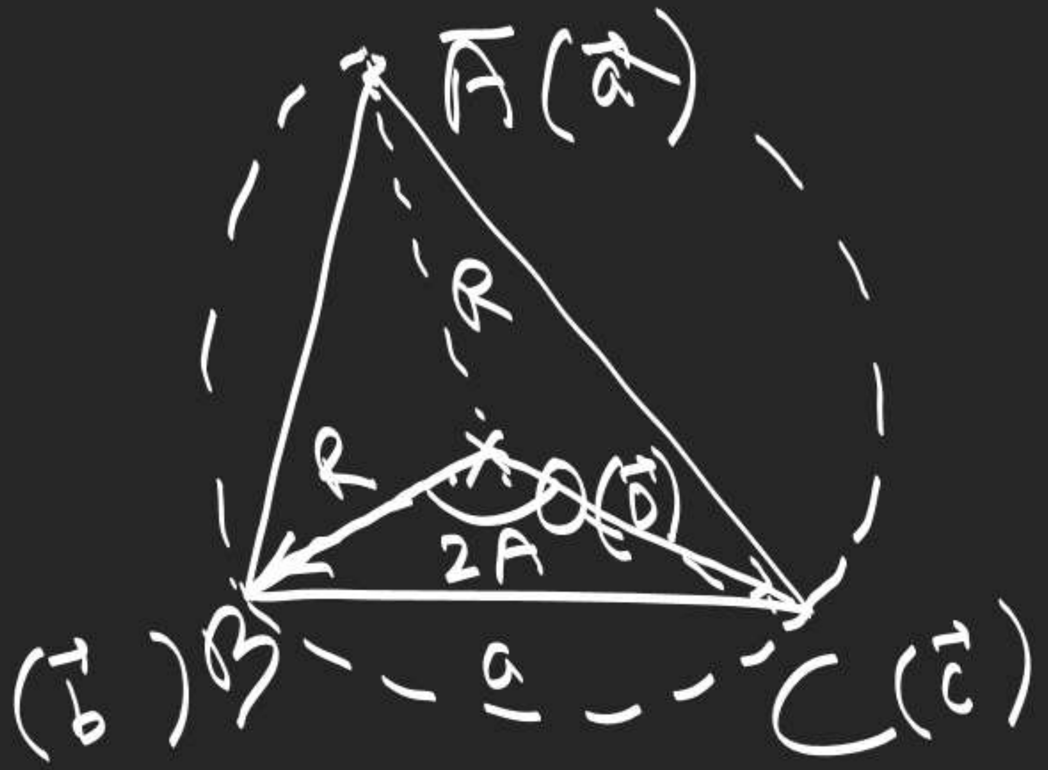
$$\widehat{AP} = \widehat{PB}$$

$$\frac{\vec{p} - \vec{a}}{|\vec{AP}|} = \frac{\vec{b} - \vec{p}}{|\vec{PB}|}$$

$$\vec{p} - \vec{a} = \frac{m}{n}(\vec{b} - \vec{p})$$

$$\vec{p} = \frac{m\vec{b}+n\vec{a}}{m+n}$$

Nishant Jindal



$$a = |\vec{BC}| = |\vec{c} - \vec{b}|$$

$$a^2 = |\vec{c} - \vec{b}|^2$$

$$= |\vec{c}|^2 + |\vec{b}|^2 - 2\,\vec{b}\cdot\vec{c}.$$

$$= R^2 + R^2 - 2(R)(R)\cos 2A$$

$$= 2R^2(1 - \cos 2A)$$

$$= 4R^2 \sin^2 A.$$

$$\boxed{a = 2R\sin A}$$

Nishant Jindal

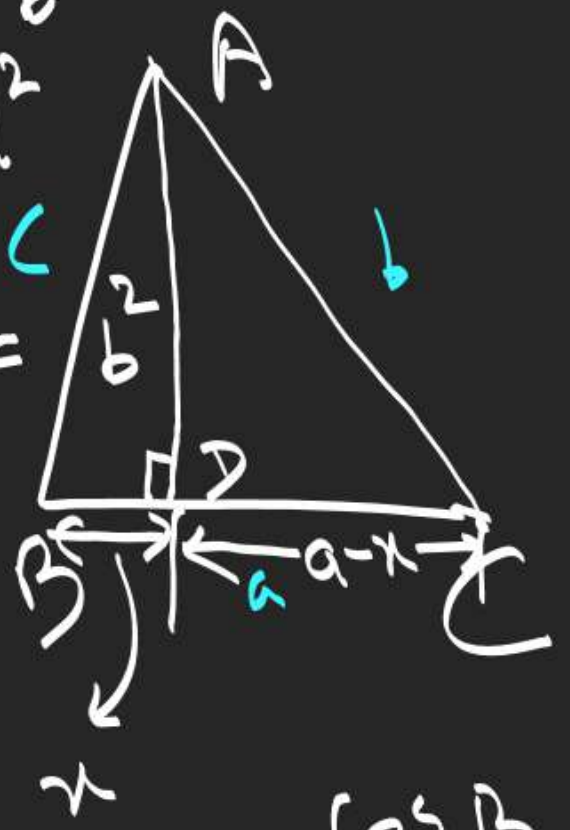
Cosine Rule

$$\boxed{\begin{aligned}\cos A &= \frac{b^2+c^2-a^2}{2bc}\\ \cos B &= \frac{c^2+a^2-b^2}{2ca}\\ \cos C &= \frac{a^2+b^2-c^2}{2ab}\end{aligned}}$$

$\vec{c} - \vec{a} = \vec{b}$

$|\vec{c}-\vec{a}|^2 = |\vec{b}|^2$

$c^2 + a^2 - 2ca\cos B =$

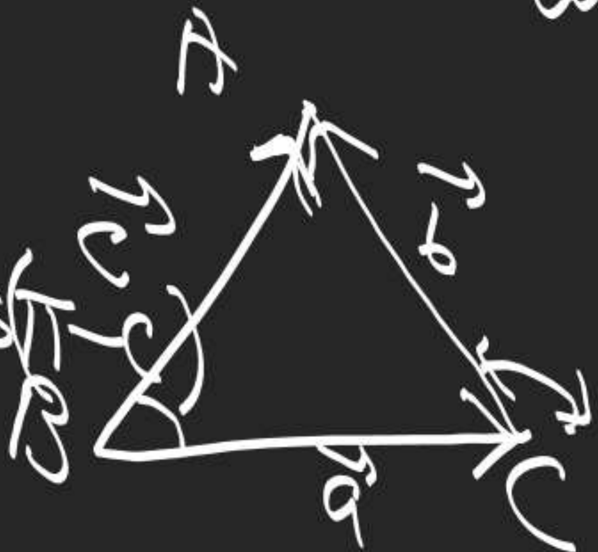


$AD^2 = c^2 - x^2 = b^2 - (a-x)^2$

$c^2 - \cancel{x^2} = b^2 - a^2 + 2ax - \cancel{x^2}$

$$x = \frac{c^2 - b^2 + a^2}{2a}$$

$$\cos B = \frac{x}{c} = \frac{a^2 + c^2 - b^2}{2ac}$$



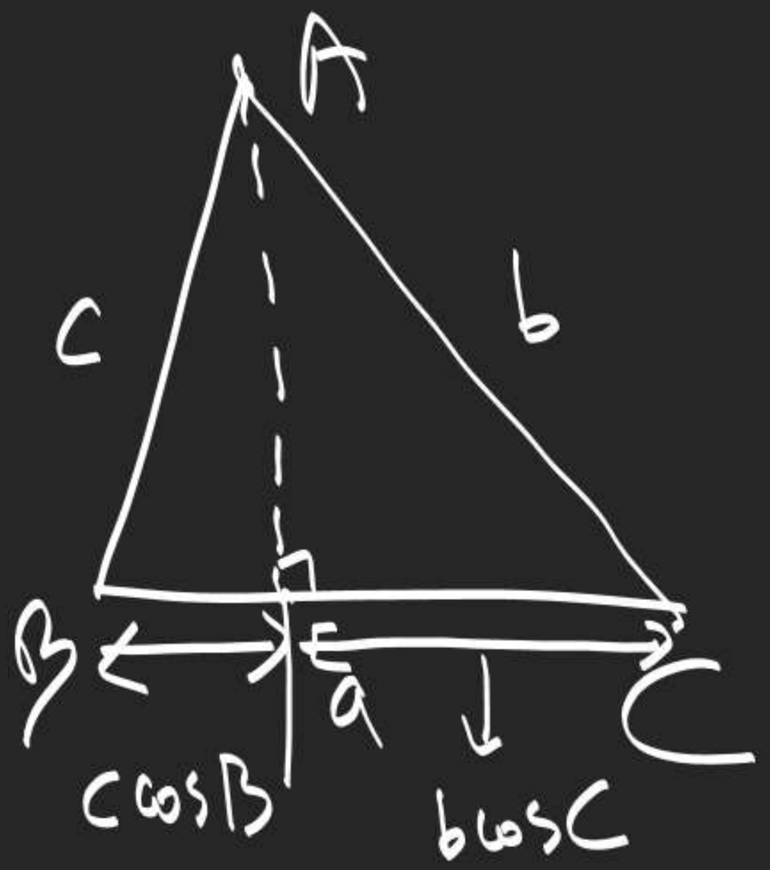
$$\boxed{\vec{c} = \vec{a} + \vec{b}}$$

$c^2 = a^2 + b^2 + 2ab\cos(\pi - C)$

$|\vec{c}|^2 = |\vec{a}+\vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 + 2\vec{a}\cdot\vec{b}$

Projection Rule

$$\boxed{\begin{aligned} a\cos B + b\cos A &= c \\ a\cos C + c\cos A &= b \\ b\cos C + c\cos B &= a \end{aligned}}$$



$$\vec{c} = \vec{a} + \vec{b}$$

$$\vec{c}\cdot\vec{c} = \vec{a}\cdot\vec{c} + \vec{b}\cdot\vec{c}$$

$$c^2 = ac\cos B + bc\cos A$$

Nishant Jindal

Napier's Analogy

$$\boxed{\begin{aligned}\tan\left(\frac{B-C}{2}\right) &= \frac{b-c}{b+c}\cot\frac{A}{2}\\ \tan\left(\frac{C-A}{2}\right) &= \frac{c-a}{c+a}\cot\frac{B}{2}\\ \tan\left(\frac{A-B}{2}\right) &= \frac{a-b}{a+b}\cot\frac{C}{2}\end{aligned}}$$

$$\frac{\pi}{2}-\frac{A}{2}$$

$$\frac{b-c}{b+c} = \frac{2R\sin B - 2R\sin C}{2R\sin B + 2R\sin C}$$

$$= \frac{2\sin\frac{B-C}{2}\cos\frac{B+C}{2}}{2\sin\frac{B+C}{2}\cos\frac{B-C}{2}}$$

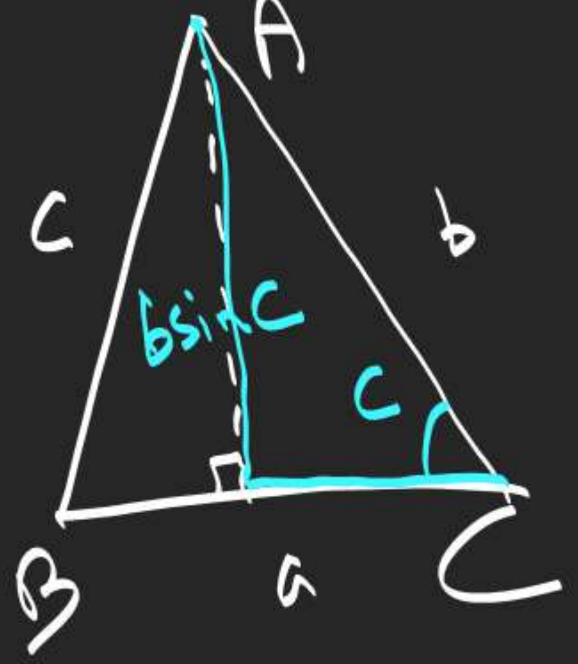
$$= \frac{\sin\frac{B-C}{2}\sin\frac{A}{2}}{\cos\frac{B-C}{2}\cos\frac{A}{2}}$$

$$= \tan\frac{B-C}{2}\tan\frac{A}{2}.$$

Nishant Jindal

Area of triangle

$$\text{Area, } \boxed{\begin{aligned}\Delta &= \tfrac{1}{2} bc\sin A \\ &= \tfrac{1}{2} ca\sin B \\ &= \tfrac{1}{2} ab\sin C\end{aligned}}$$



$$\Delta = \frac{1}{2}\left|\vec{CB} \times \vec{CA}\right|$$

$$\frac{1}{2} a(b\sin C)$$

Nishant Jindal

Half Angle Formulae

$$\sin\frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{bc}}$$

$$\sin\frac{B}{2} = \sqrt{\frac{(s-c)(s-a)}{ca}}$$

$$\sin\frac{C}{2} = \sqrt{\frac{(s-a)(s-b)}{ab}}$$

$$s = \frac{a+b+c}{2}$$

$$\sin\frac{A}{2} = \sqrt{\frac{1-\cos A}{2}}$$

$$= \sqrt{\frac{1-\frac{b^2+c^2-a^2}{2bc}}{2}}$$

$$= \sqrt{\frac{a^2-(b-c)^2}{4bc}} = \sqrt{\frac{(a-b+c)(a+b-c)}{4bc}} = \sqrt{\frac{(2s-2b)(2s-2c)}{4bc}}$$

Nishant Jindal

$$\boxed{\begin{aligned}\cos\frac{A}{2} &= \sqrt{\frac{s(s-a)}{bc}}\\ \cos\frac{B}{2} &= \sqrt{\frac{s(s-b)}{ca}}\\ \cos\frac{C}{2} &= \sqrt{\frac{s(s-c)}{ab}}\end{aligned}}$$

$$\cos\frac{A}{2} = \sqrt{\frac{1+\cos A}{2}} = \sqrt{\frac{1+\frac{b^2+c^2-a^2}{2bc}}{2}}$$

$$= \sqrt{\frac{(b+c)^2-a^2}{4bc}}$$

$$= \sqrt{\frac{(b+c-a)(b+c+a)}{4bc}}$$

$$= \sqrt{\frac{(2s-2a)(2s)}{4bc}}$$

$$\sqrt{s(s-a)(s-b)(s-c)} = \Delta = \frac{1}{2}bc\sin A$$

$$bc\sqrt{\frac{(s-b)(s-c)}{bc}}\sqrt{\frac{s(s-a)}{bc}} = bc\sin\frac{A}{2}\cos\frac{A}{2}$$

Nishant Jindal

$$\tan\frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}} = \frac{\Delta}{s(s-a)}$$

$$\tan\frac{B}{2} = \sqrt{\frac{(s-c)(s-a)}{s(s-b)}} = \frac{\Delta}{s(s-b)}$$

$$\tan\frac{C}{2} = \sqrt{\frac{(s-a)(s-b)}{s(s-c)}}$$

$$= \frac{\Delta}{s(s-c)}$$

$$\tan\frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}} = \sqrt{\frac{s(s-a)(s-b)(s-c)}{s^2(s-a)^2}}$$

$$= \frac{\Delta}{s(s-a)}$$

PT-3