Nishant Jindal

Projectile Motion

(*)

Condition when projectile hit the inclined plane perpendicularly:—

Along the inclined plane.

$$v_x = (u\cos\theta) - (g\sin\alpha)t$$

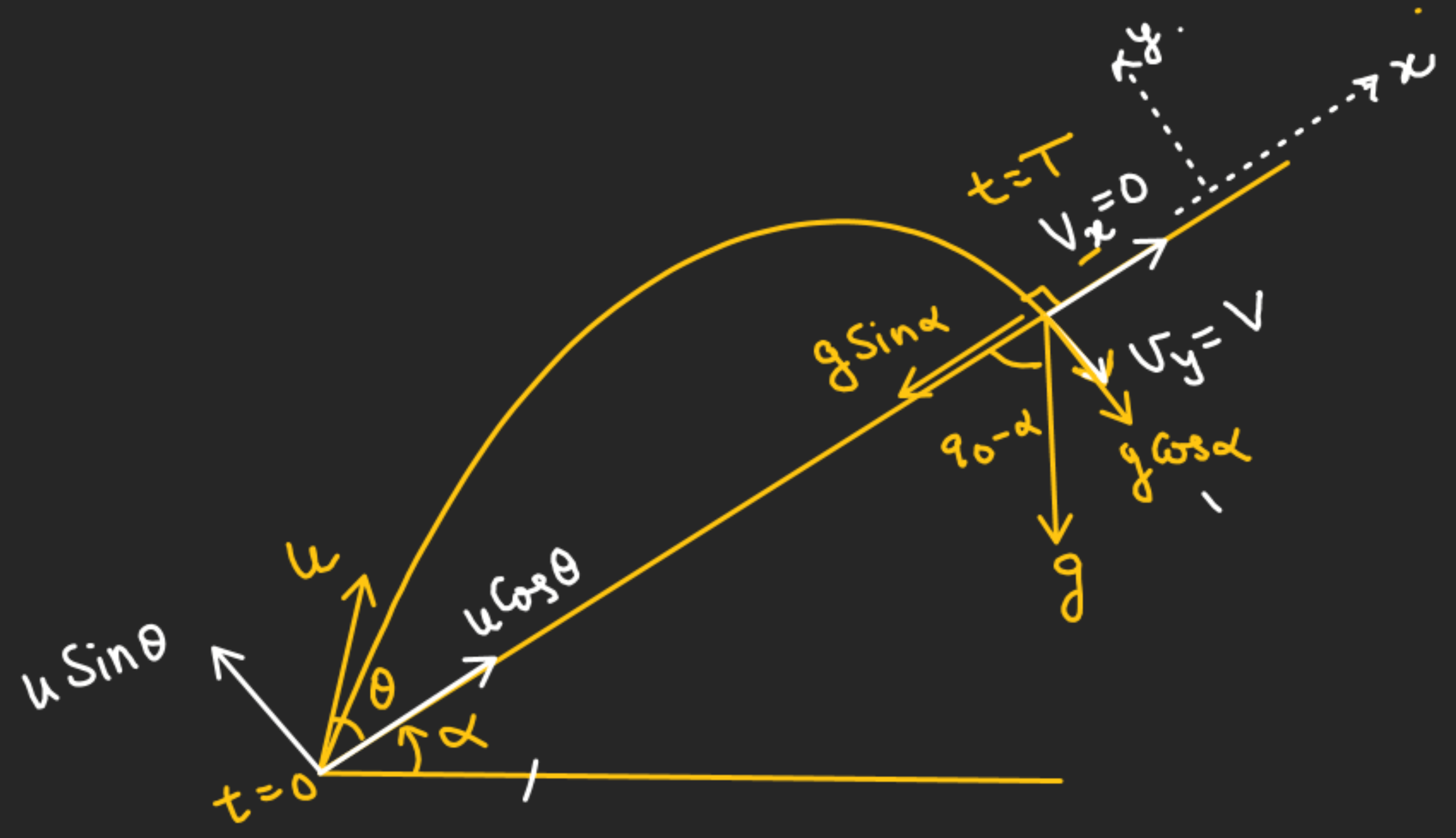
$t = T$, $v_x = 0$.

$$T = \left(\frac{2u\sin\theta}{g\cos\alpha}\right)$$

$$0 = (u\cos\theta) - (\cancel{g}\sin\alpha)\left(\frac{2u\sin\theta}{\cancel{g}\cos\alpha}\right)$$

$$2\cancel{u}\sin\theta(\tan\alpha) = \cancel{u}\cos\theta$$

$$\boxed{2\tan\alpha = \cot\theta}$$ ✓



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Projectile Motion

Condition when projectile hit the inclined plane horizontally :→ ✓

$$V_y = u_y - gt \quad \begin{bmatrix} At\ t = T \\ V_y = 0 \end{bmatrix} \quad T = \left(\frac{2u\sin\theta}{g\cos\alpha}\right)$$

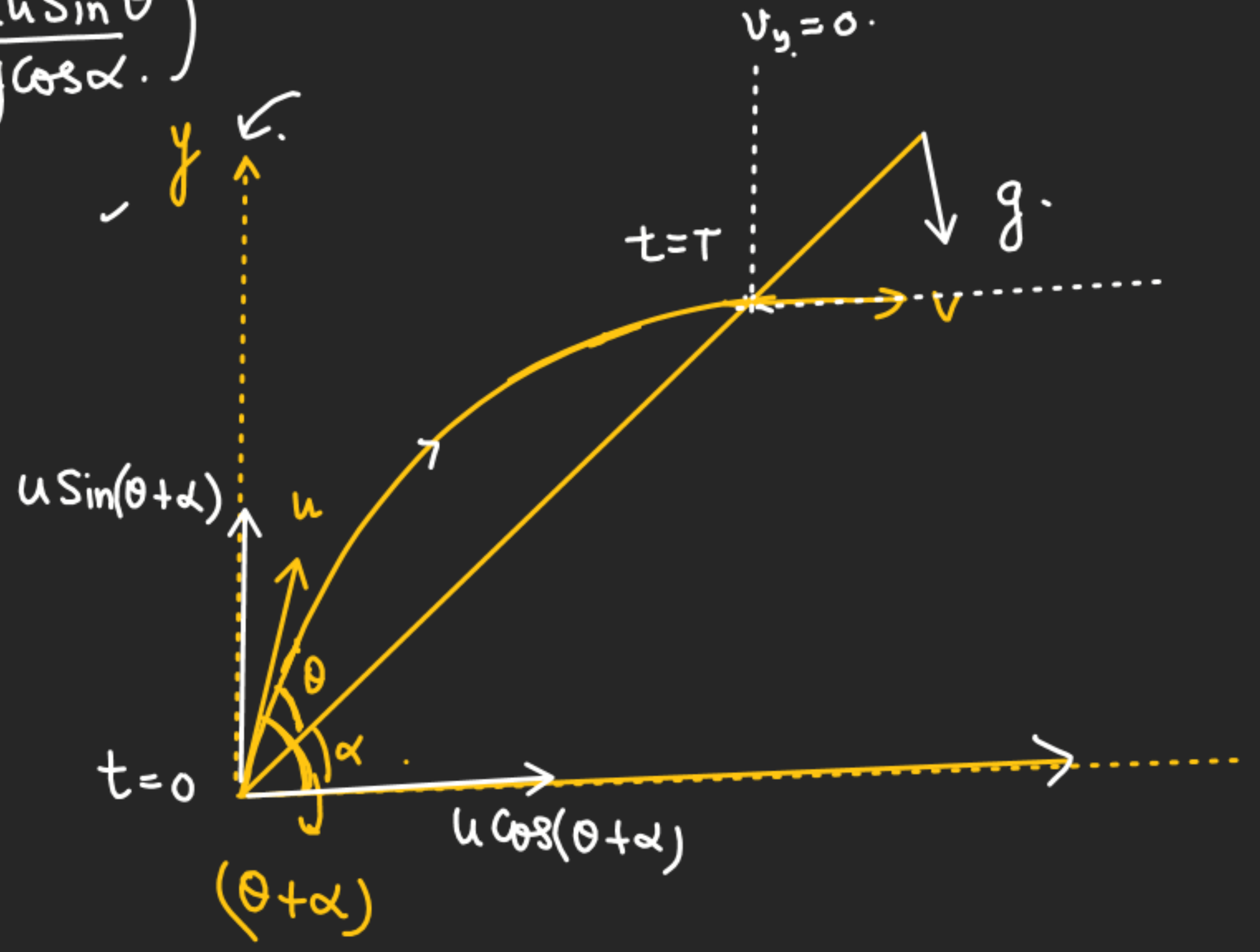
$$\Downarrow$$

$$0 = u\sin(\theta+\alpha) - g.T$$

$$0 = u\sin(\theta+\alpha) - g\left(\frac{2u\sin\theta}{g\cos\alpha}\right)$$

$$\frac{2\cancel{u}\sin\theta}{\cos\alpha} = \cancel{u}\sin(\theta+\alpha)$$

$$\left[\frac{2\sin\theta}{\sin(\theta+\alpha)} = \cos\alpha\right]$$



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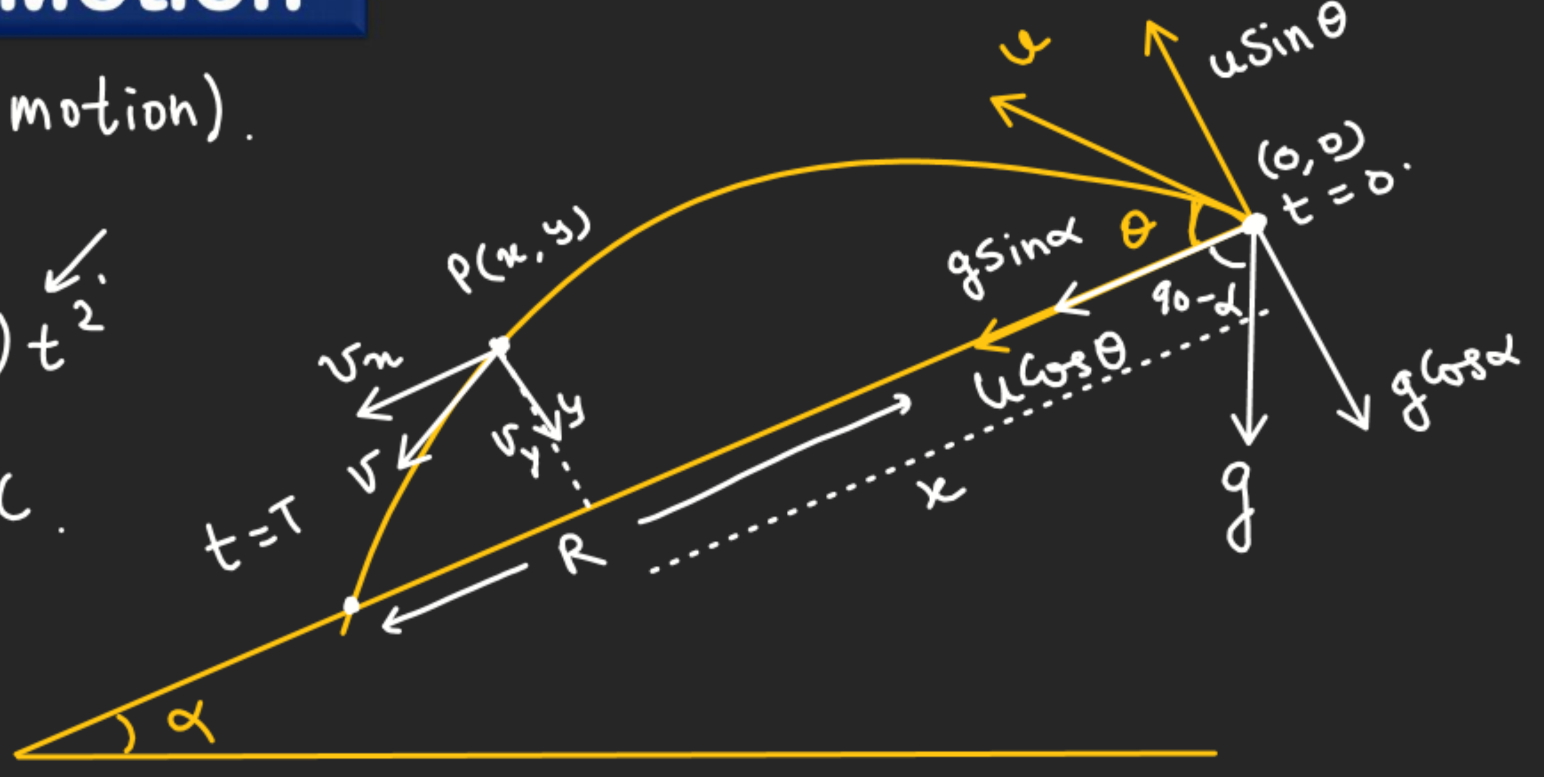
Projectile Motion

In x-direction (Acclerated motion).

$$V_x = u\cos\theta + (g\sin\alpha)t$$

$$x = (u\cos\theta)t + \frac{1}{2}(g\sin\alpha)t^2$$

$$V_x^2 = (u\cos\theta)^2 + 2(g\sin\alpha)x$$



In-y direction

$$V_y = (u\sin\theta) - (g\cos\alpha)t$$

$$y = (u\sin\theta)t - \frac{1}{2}(g\cos\alpha)t^2$$

$$V_y^2 = (u\sin\theta)^2 - 2(g\cos\alpha)y$$

$$T = \frac{2u\sin\theta}{g\cos\alpha}$$

$$H_{max} = \frac{u^2\sin^2\theta}{2(g\cos\alpha)}$$

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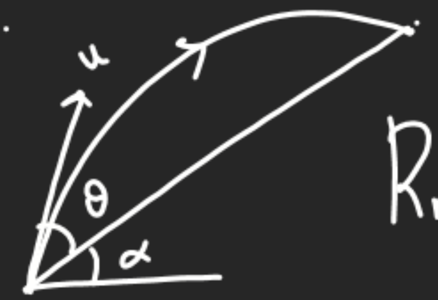
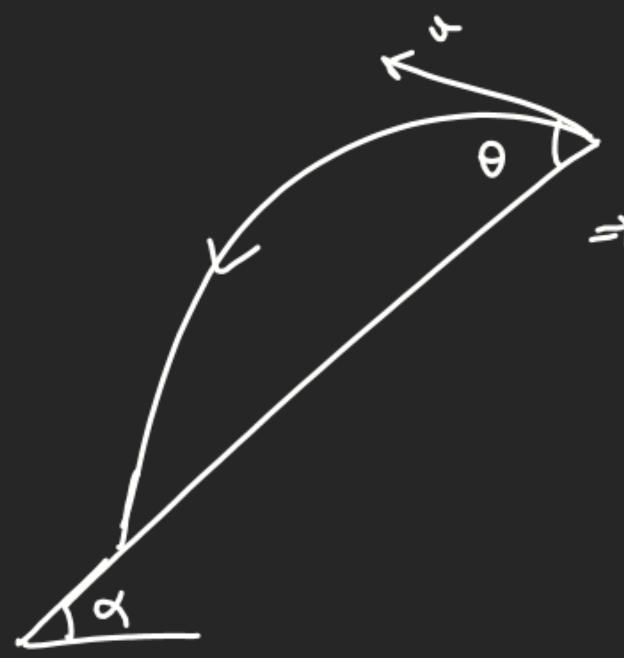
Projectile Motion



$$R_{max} = \frac{u^2}{g(1 \pm \sin\alpha)}, \quad \theta = \left(\frac{\pi}{4} - \frac{\alpha}{2}\right)$$



$$\Rightarrow \boxed{R_{max} = \frac{u^2}{g(1 - \sin\alpha)}}$$

$$\theta = \left(\frac{\pi}{4} + \frac{\alpha}{2}\right)$$

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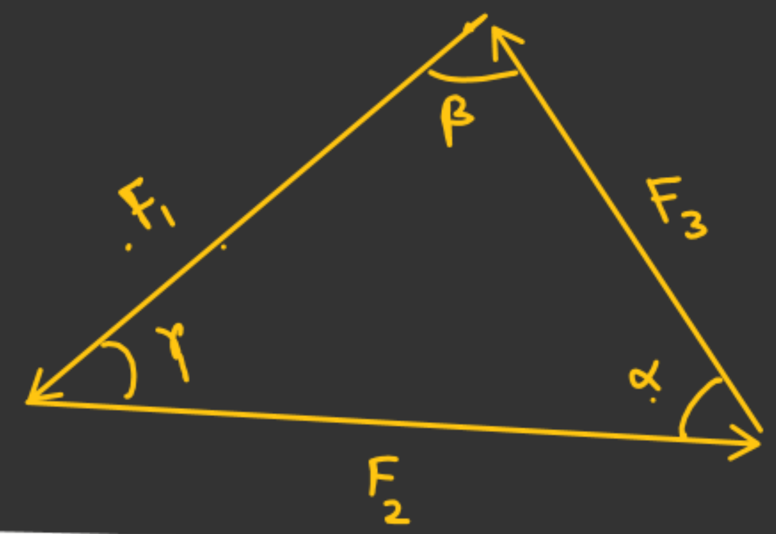
Sine rule :→



$$\boxed{\frac{F_1}{\sin\alpha} = \frac{F_2}{\sin\beta} = \frac{F_3}{\sin\gamma}}$$

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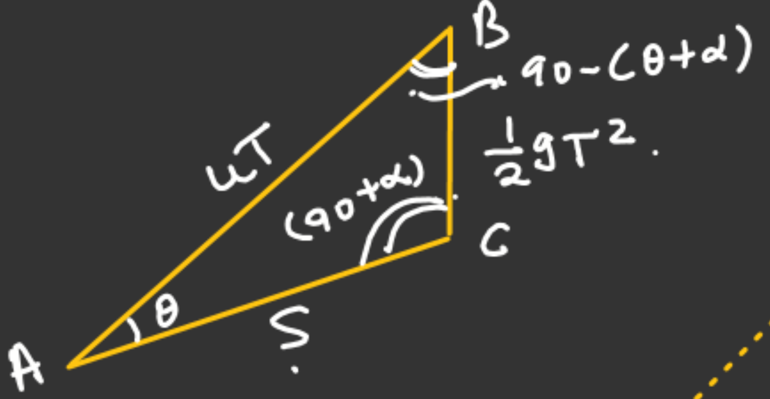
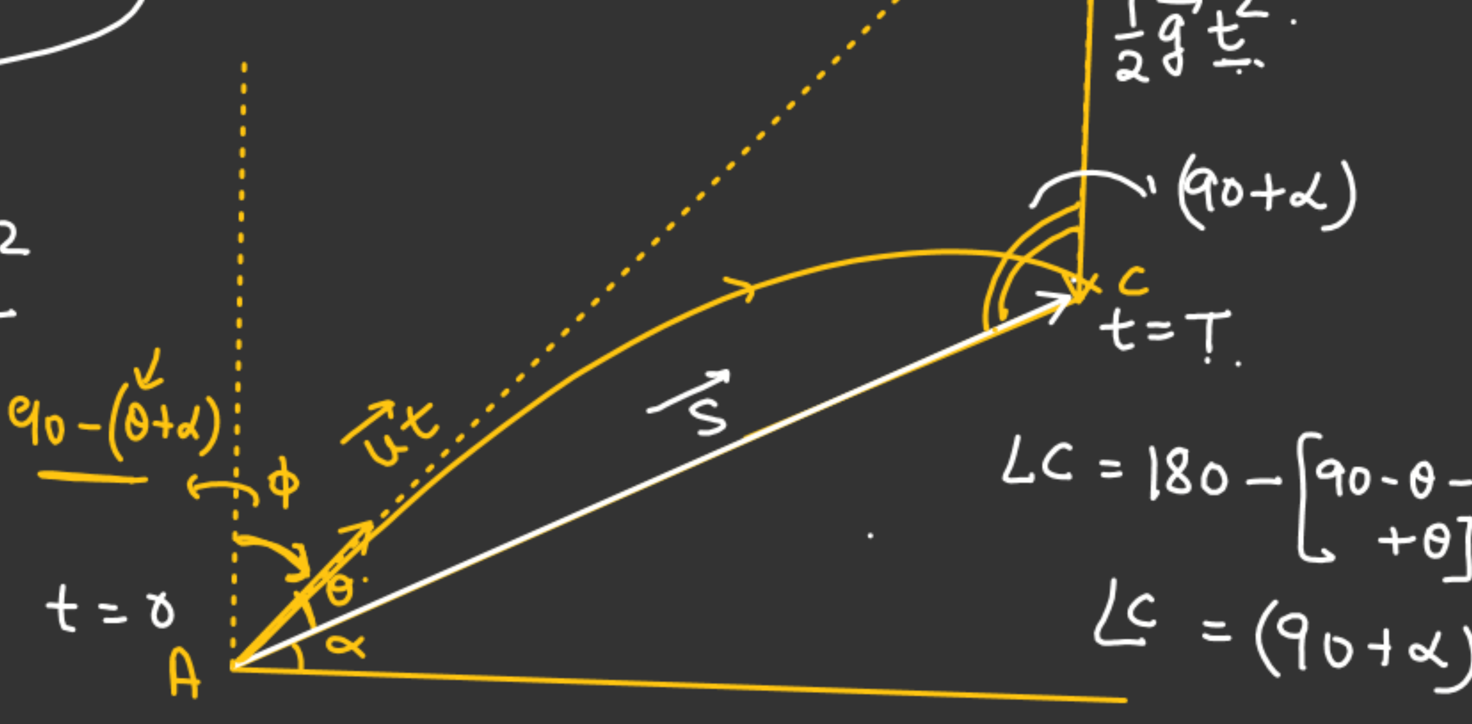
$$T = \left(\frac{2u\sin\theta}{g\cos\alpha}\right)$$

$$\left[\underbrace{\vec{S}}_{\vec{A}} = \underbrace{\vec{u}t}_{\vec{B}} + \underbrace{\tfrac{1}{2}\vec{g}t^2}_{\vec{C}}\right]$$

$$[\vec{A} = \vec{B} + \vec{C}]$$





$\angle C = 180 - [90 - \theta - \alpha + \theta]$

$\angle C = (90+\alpha)$

By Sine rule

$$\frac{S}{\sin[90-(\theta+\alpha)]} = \frac{uT}{\sin(90+\alpha)} = \frac{\frac{1}{2}gT^2}{\sin\theta}$$

$$\frac{S}{\cos(\theta+\alpha)} = \frac{uT}{\cos\alpha} = \frac{\frac{1}{2}gT^2}{\sin\theta}$$

$$S = \frac{u\cos(\theta+\alpha)}{\cos\alpha} \times \frac{2u\sin\theta}{g\cos\alpha}$$

Range

$$S = \frac{2u^2}{g}\left[\frac{\cos(\theta+\alpha)\cdot\sin\theta}{\cos^2\alpha}\right]$$

$$S = \frac{u^2}{g\cos^2\alpha}[2\sin\theta\cdot\cos(\theta+\alpha)]$$

For S to be maximum $\phi = 90-(\theta+\alpha) = \frac{\pi}{2} - \left[\left(\frac{\pi}{4}-\frac{\alpha}{2}\right)+\alpha\right]$

$$\theta = \left(\frac{\pi}{4} - \frac{\alpha}{2}\right)$$

$$\boxed{\phi = \left(\frac{\pi}{4} - \frac{\alpha}{2}\right) = \theta}$$

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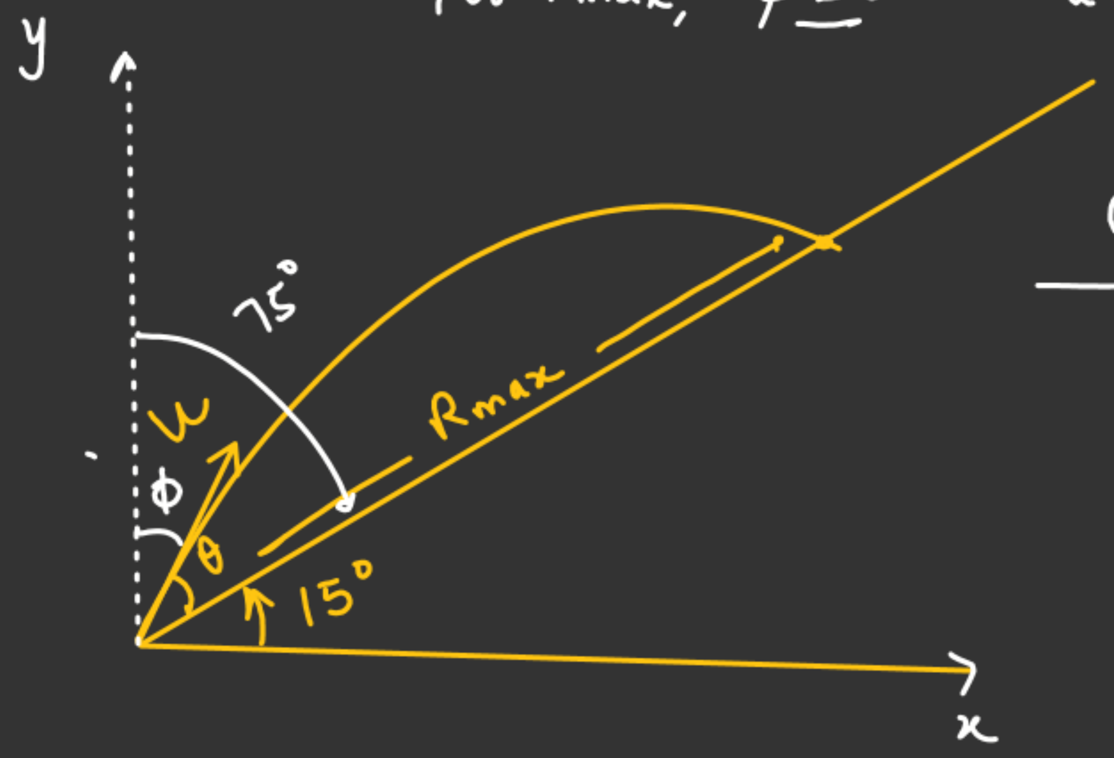
For R_{max}, $\theta = ??$

For R_{max}, $\phi = \theta$.



$2\theta = 75°$

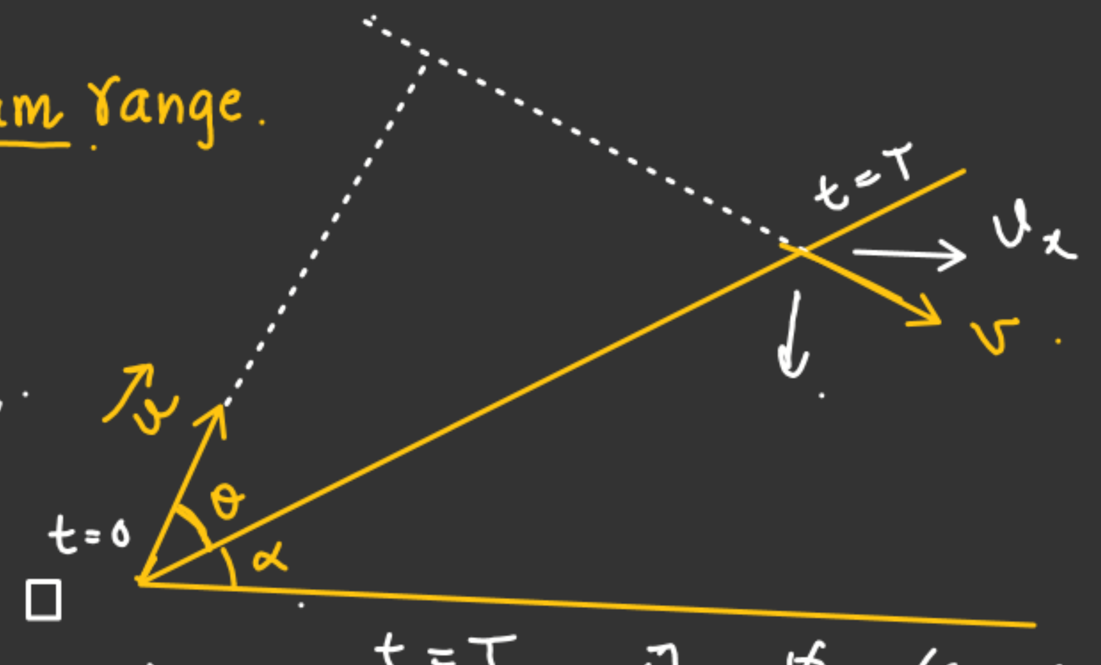
$\theta = \frac{75°}{2}$

$\theta = 37.5°$

For maximum range.

$$\boxed{\vec{v} \cdot \vec{u} = 0}$$

$$\vec{v} = \vec{u} + \vec{g}t$$

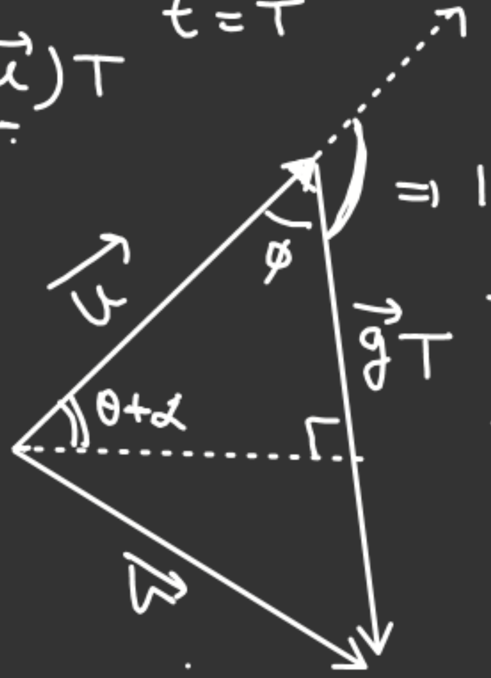


$$\vec{v} \cdot \vec{u} = \vec{u} \cdot \vec{u} + (\vec{g} \cdot \vec{u})T$$

$\Downarrow$

t=T

$\phi = (90 - \theta - \alpha)$



$\Rightarrow 180 - \phi$

$\Rightarrow 180 - (90 - \theta - \alpha)$

$\Rightarrow 90 + (\theta + \alpha)$