Nishant Jindal

Trigonometry

A.P. in Sine & Cosine Series

1) $\sin(A)+\sin(A+d)+\sin(A+2d)+\dots+\sin(a+(n-1)d) = \dfrac{\sin\left(\frac{n\times C.D.}{2}\right)}{\sin\left(\frac{C.D}{2}\right)} \times \sin\left(\dfrac{1^{st}\text{term}+\text{Last term}}{2}\right)$

2) $\cos(A)+\cos(A+d)+\cos(A+2d)+\dots+\cos(A+(n-1)d) = \dfrac{\sin\left(\frac{n\times C.D}{2}\right)}{\sin\left(\frac{C.D}{2}\right)} \times \cos\left(\dfrac{1^{st}\text{ term}+\text{Last term}}{2}\right)$

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Trigonometry

Q $\cos\frac{\pi}{11}+\cos\frac{3\pi}{11}+\cos\frac{5\pi}{11}+\cos\left(\frac{7\pi}{11}\right)+\cos\left(\frac{9\pi}{11}\right)$

$\frac{2\pi}{11}$

1) Some + + +
2) AP Series → C.D $=\frac{2\pi}{11}$
3) $n=5$ (5 cos add)
4) $2\sin\theta\cos\theta=\sin 2\theta$

(5) $\sin(\pi-\theta)=\sin\theta$

$$\frac{\sin\left(\frac{5\times 2\pi}{2\cdot 11}\right)}{\sin\left(\frac{2\pi}{11\times 2}\right)}\times\cos\left(\frac{\frac{\pi}{11}+\frac{9\pi}{11}}{2}\right)$$

$$\left\{\frac{2\sin\left(\frac{5\pi}{11}\right)}{2\sin\left(\frac{\pi}{11}\right)}\times\cos\left(\frac{5\pi}{11}\right)\right\}=\frac{\sin\left(\frac{10\pi}{11}\right)}{2\sin\left(\frac{\pi}{11}\right)}=\frac{\sin\left(\frac{11\pi-\pi}{11}\right)}{2\sin\left(\frac{\pi}{11}\right)}=\frac{\sin\left(\pi-\frac{\pi}{11}\right)}{2\sin\left(\frac{\pi}{11}\right)}=\frac{\cancel{\sin\left(\frac{\pi}{11}\right)}}{2\cancel{\sin\left(\frac{\pi}{11}\right)}}$$

$$=\frac{1}{2}$$

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Trigonometry

Q If $\sin\frac{\pi}{n}+\sin\frac{2\pi}{n}+\sin\frac{3\pi}{n}+\dots+n$ terms $=2+\sqrt{3}$ then $n=?$

$\frac{\pi}{n}$ AP

$$\frac{\sin\left(\frac{\cancel{n}\cdot\pi}{\cancel{n}\times 2}\right)}{\sin\left(\frac{\pi}{2n}\right)}\cdot\sin\left(\frac{\frac{\pi}{n}+\frac{n\pi}{n}}{2}\right)=2+\sqrt{3}$$

$$\frac{1}{\sin\left(\frac{\pi}{2n}\right)}\times\sin\left(\frac{(n+1)\pi}{2n}\right)=\cot 15^\circ$$

$$\frac{\sin\left(\frac{\pi}{2}\left(\frac{n+1}{n}\right)\right)}{\sin\left(\frac{\pi}{2n}\right)}=\cot 15^\circ \Rightarrow \frac{\sin\left(\frac{\pi}{2}\left(1+\frac{1}{n}\right)\right)}{\sin\left(\frac{\pi}{2n}\right)}=\cot 15^\circ$$

$\boxed{2}$

$$\frac{\sin\left(\frac{\pi}{2}+\frac{\pi}{2n}\right)}{\sin\left(\frac{\pi}{2n}\right)}=\cot 15^\circ$$

$$\frac{+\cos\left(\frac{\pi}{2n}\right)}{\sin\left(\frac{\pi}{2n}\right)}=\frac{\cos(15^\circ)}{\sin 15^\circ}$$

$$\frac{\pi}{2n}=\frac{\pi}{12}$$

$2n=12$

$\boxed{n=6}$

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Trigonometry

Q Jab AP $\sin^2$ me ho to????

$$S = \underline{\sin^2\theta} + \sin^2(2\theta) + \underline{\sin^2(3\theta)} + \cdots + \sin^2(n\theta)$$

AP

$$S = \frac{1-\cos 2\theta}{2} + \frac{1-\cos 4\theta}{2} + \frac{1-\cos 6\theta}{2} + \quad + \frac{1-\cos 2n\theta}{2}$$

$$= \left(\frac{1}{2} - \frac{\cos 2\theta}{2}\right) + \left(\frac{1}{2} - \frac{\cos 4\theta}{2}\right) + \left(\frac{1}{2} - \frac{\cos 6\theta}{2}\right) + \cdots + \left(\frac{1}{2} - \frac{\cos 2n\theta}{2}\right)$$

Cos मे AP AA GYI

← n लोग

$$= \frac{n}{2} - \frac{1}{2}\left(\cos 2\theta + \cos 4\theta + \cos 6\theta + \cos 8\theta \cdots + \cos 2n\theta\right)$$

$$= \frac{n}{2} - \frac{1}{2}\left\{\frac{\sin\left(\frac{n \times 2\theta}{2}\right)}{\sin\left(\frac{2\theta}{2}\right)} \times \cos\left(\frac{2\theta + 2n\theta}{2}\right)\right\}$$

$$= \frac{n}{2} - \frac{1}{2}\left\{\frac{\sin n\theta \times \cos((n+1)\theta)}{\sin\theta}\right\}$$ Ans

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Trigonometry

Q $S = \cos^2\theta + \cos^2 2\theta + \cos^2(3\theta) + \cos^2(4\theta) + \dots + \cos^2(n\theta)$

1) AP – $\cos^2$ me Dikkat!!!

2) $\cos^2\theta = \frac{1+\cos 2\theta}{2}$ Use

$$S = \left(\frac{1+\cos 2\theta}{2}\right) + \left(\frac{1+\cos 4\theta}{2}\right) + \left(\frac{1+\cos 6\theta}{2}\right) + \dots + \left(\frac{1+\cos 2n\theta}{2}\right)$$

$$= \left(\frac{1}{2} + \frac{1}{2} + \frac{1}{2} \dots n \text{ बार}\right) + \frac{1}{2}\left(\cos 2\theta + \cos 4\theta + \cos 6\theta + \dots + \cos 2n\theta\right)$$

2θ

$$= \frac{n}{2} + \frac{1}{2}\left\{\frac{\sin\left(\frac{n \times 2\theta}{2}\right)}{\sin\left(\frac{2\theta}{2}\right)} \times \cos\left(\frac{2\theta + 2n\theta}{2}\right)\right.$$

$$\frac{n}{2} + \frac{1}{2}\left\{\frac{\sin n\theta}{\sin\theta} \times \cos(\theta(n+1))\right\}$$

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Trigonometry

gandaa.

Q $S = \sin\theta - \sin 2\theta + \sin 3\theta - \sin 4\theta \cdots n \text{ terms}.$

AP आ जाएगा

Yad

$\sin(\pi-\theta) + \sin(2\pi - 2\theta) + \sin(3\pi - 3\theta) + \sin(4\pi - 4\theta)$

$+ \cdots + \sin(n\pi - n\theta)$

$\pi-\theta \quad \pi-\theta \quad \pi-\theta$

$$\frac{\sin\left(\frac{n\cdot(\pi-\theta)}{2}\right)}{\sin\left(\frac{\pi-\theta}{2}\right)} \times \sin\left(\frac{\pi-\theta+n\pi-n\theta}{2}\right)$$

$$\frac{\sin\left(\frac{n\pi}{2}-\frac{n\theta}{2}\right)}{\sin\left(\frac{\pi}{2}-\theta/2\right)} \times \sin\left((n+1)\left(\frac{\pi}{2}-\frac{\theta}{2}\right)\right)$$

(1) Sir AP तो है Butttt

$+ - + - + -$ आ रहे हैं

(2) Sabko + + + + Bdl Denge

Kese ???

(3) Use $\sin\theta = \sin(\pi-\theta)$

$-\sin 2\theta = \sin(2\pi - 2\theta)$

$\sin(2\pi - 2\theta) = -\sin 2\theta$

2π $2\pi-2\theta$ (4)

(4) Me Bataaunga Na to Pta Chl Jayega

Trigonometry

$\phi+\phi+\phi+\ldots+\phi=n\phi$

← Sare Ext. An

Q If ϕ is exterior angle of a regular Polygon of n side

& θ is any Constant then P.T.

$\sin\theta+\sin(\theta+\phi)+\sin(\theta+2\phi)+\ldots$ n term $=0$

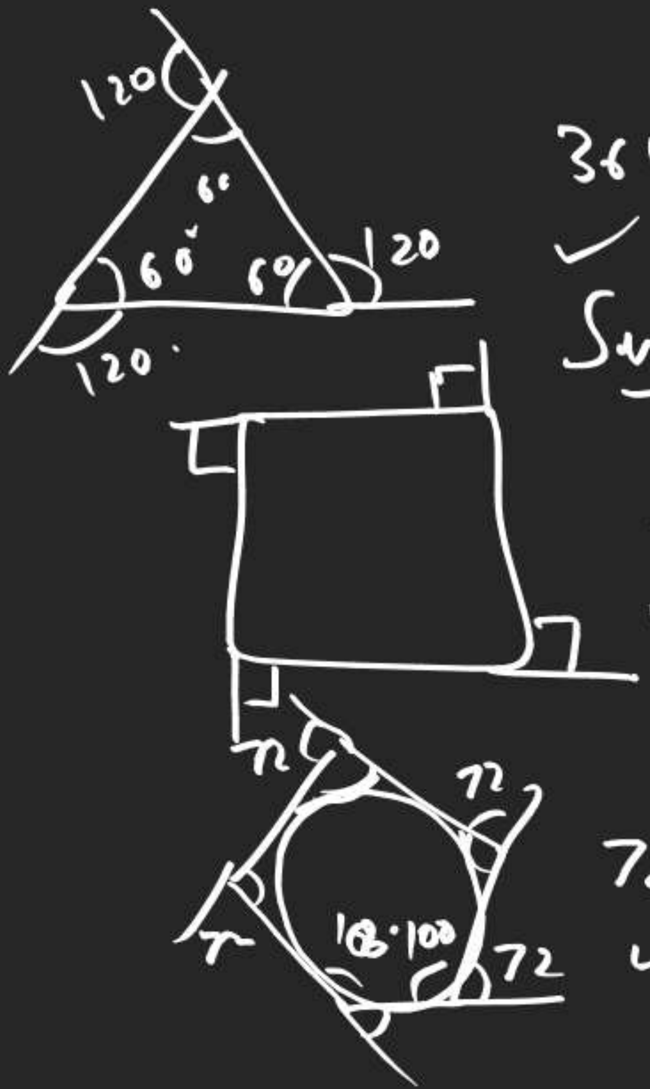
1) $\boxed{n\phi=2\pi}$

AP ③

2) $\sin\theta+\sin(\theta+\phi)+\sin(\boxed{\theta+2\phi})+\ldots$

$$\frac{\sin\left(\frac{n\times\phi}{2}\right)}{\sin\left(\frac{\phi}{2}\right)}\times\sin\left(\frac{\theta+\theta+(n-1)\phi}{2}\right)$$

$$=\frac{\sin\left(\frac{2\pi}{2}\right)}{\sin\left(\frac{\phi}{2}\right)}\times\sin\left(\theta+\frac{(n-1)\phi}{2}\right)=0$$



360 ✓

Sum of Exterior $=2\pi$

360 ✓

$72\times5=360$ ✓

n Sided Polygon → Interior Angles

$=(n-2)\pi$

$=(5-2)\pi=540°$

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$\sin 178° = \sin(180-2°) = \sin(\pi - 2°) = \sin 2°$

Q P.T. average of Numbers $n \sin n°$; $n = 2, 4, 6, 8 \ldots 180$ is $\cot 1°$?

$$\text{Average} = \frac{2\sin 2° + 4\sin 4° + 6\sin 6° + \quad 176\sin 176° + 178\sin 178° + \cancel{180\sin 180°}}{90}$$

$90 \cdot \sin 90°$

$$= \frac{2\sin 2° + 4\sin 4° + 6\sin 6° \quad 174\sin 6° + 176\sin 4° + 178\sin 2°}{90}$$

$$= \frac{\sin 2°(2+178) + \sin 4°(4+176) + \sin 6°(6+174) + \ldots}{90}$$

$$= \frac{180(\sin 2° + \sin 4° + \sin 6° + \sin 8° \ldots \sin 88°) + 90}{90}$$

$$= 2 \times \frac{\sin\left(\frac{44 \cdot 2°}{2}\right)}{\sin\left(\frac{2°}{2}\right)} \cdot \sin\left(\frac{2°+88°}{2}\right) + 1$$

$$= \frac{2\sin 44° \cdot \sin 45°}{\sin 1°} + 1 \Rightarrow \frac{\cos(1°) - \cos(89°)}{\sin 1°} + 1 \Rightarrow \frac{\cos 1°}{\sin 1°} - \frac{\sin 1°}{\sin 1°} + 1 \Rightarrow \cot 1° - 1 + 1 = \cot 1°$$

पहला No → $2\sin 2°$

दूसरा No → $4\sin 4°$

तीसरा No → $6\sin 6°$

last No → $180 \cdot \sin 180°$

90 लोग

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Trigonometry

(A+B+C) angles

① $\sin(A+\boxed{B+C}) = \sin(A+D)$, where $D = B+C$

$= \sin A \cos D + \cos A \sin D$

$= \sin A \cdot \cos(B+C) + \cos A \cdot \sin(B+C)$

$\Rightarrow \sin A \cdot (\cos B \cos C - \sin B \cdot \sin C) + \cos A (\sin B \cos C + \cos B \sin C)$

$\sin(A+B+C) = \sin A \cos B \cos C - \sin A \cdot \sin B \cdot \sin C + \cos A \cdot \sin B \cos C + \cos A \cdot \cos B \cdot \sin C$

(2) $\cos(A+\boxed{B+C}) = \cos A \cdot \cos D - \sin A \cdot \sin D$

$= \cos A \cdot \cos(B+C) - \sin A \cdot \sin(B+C)$

$= \cos A (\cos B \cos C - \sin B \cdot \sin C) - \sin A \cdot (\sin B \cos C + \cos B \sin C)$

$= \cos A \cdot \cos B \cos C - \cos A \sin B \sin C - \sin A \cdot \sin B \cos C - \sin A \cdot \cos B \sin C$

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3) $\tan(A+B+C) = \tan(A+D) = \dfrac{\tan A + \tan D}{1 - \tan A \cdot \tan D} = \dfrac{\tan A + \tan(B+C)}{1 - \tan A \cdot \tan(B+C)}$

A – B
D – C

$$= \dfrac{\tan A + \dfrac{\tan B + \tan C}{1 - \tan B \tan C}}{1 - \tan A \cdot \left(\dfrac{\tan B + \tan C}{1 - \tan B \tan C}\right)} = \dfrac{\tan A - \tan A \tan B \tan C + \tan B + \tan C}{1 - \tan B \tan C - \tan A \cdot \tan B - \tan A \cdot \tan C}$$

(3) $$\boxed{\tan(A+B+C) = \dfrac{(\tan A + \tan B + \tan C) - (\tan A \cdot \tan B \cdot \tan C)}{1 - (\tan A \cdot \tan B + \tan B \cdot \tan C + \tan C \tan A)}} = \dfrac{S_1 - S_3}{1 - S_2}$$

(SOPOT 3 AAT)

(4) $$\tan(A+B+C+D) = \dfrac{\underbrace{(\tan A + \tan B + \tan C + \tan D)}_{\text{Sum of all tangent}} - (\tan A \cdot \tan B \cdot \tan C + \tan B \tan C \tan D + \tan C \tan D \tan A + \tan D \tan A \tan B)}{1 - \underbrace{(\tan A \tan B + \tan B \tan C + \tan C \tan D + \tan D \tan A + \tan A \cdot \tan C + \tan B \tan D)}_{(\text{SOPOT 2 AAT})} + \underbrace{\tan A \cdot \tan B \cdot \tan C \cdot \tan D}_{\text{Prod of all 4}}}$$

$$\tan(A+B+C+D) = \dfrac{S_1 - S_3}{1 - S_2 + S_4}$$

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$$\tan(A+B+C+D+E) = \frac{S_1 - S_3 + S_5}{1 - S_2 + S_4}$$

$$\boxed{\tan 3x = \frac{3\tan x - \tan^3 x}{1 - 3\tan^2 x}}$$

$$\boxed{\begin{aligned} \tan(A+B+C) &= \frac{S_1 - S_3}{1 - S_2} \\ \tan(A+B+C+D) &= \frac{S_1 - S_3}{1 - S_2 + S_4} \end{aligned}}$$

$\boxed{Ex 1}$

$$\tan(A+B+C) = \frac{\tan A + \tan B + \tan C - \tan A \cdot \tan B \cdot \tan C}{1 - (\tan A \cdot \tan B + \tan B \tan C + \tan C \tan A)}$$

$\boxed{A = B = C = x}$

$$\tan(x+x+x) = \frac{\tan x + \tan x + \tan x - \tan x \cdot \tan x \cdot \tan x}{1 - (\tan x \cdot \tan x + \tan x \tan x + \tan x \cdot \tan x)} = \frac{3\tan x - \tan^3 x}{1 - 3\tan^2 x}$$