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14. $\sqrt{\underline{\log_2 3}(2+\log_2 3)(4+\log_2 3)\underline{(6+\log_2 3)}+16} - (\log_2 3+2)(\log_2 3+4)+10$

$= \sqrt{(t^2+6t)(t^2+6t+8)+16} - (t^2+6t+8)+10$

$= \sqrt{(t^2+6t)^2+8(t^2+6t)+16} - (t^2+6t+8)+10$

$= t^2+6t+4 - (t^2+6t+8)+10$

$\log\left(\frac{x+4}{x}\right) = \log\left(\frac{3-x}{x-1}\right)$ or $\log\left(\frac{x+4}{x}\right) = \log\left(\frac{x-1}{3-x}\right)$

$x + y = 4$

$\boxed{\frac{1}{x} - \frac{1}{4-x} = 1}$

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20.

$$\frac{\log_{10} 45}{\log_{10} 3x} = \frac{\log_{10} 40\sqrt{3}}{\log_{10} 4x} = \frac{\log_{10}\left(\frac{3\sqrt{3}}{8}\right)}{\log_{10}\left(\frac{3}{4}\right)} = \log\left(\frac{3\sqrt{3}}{8}\right) = \left(\frac{\sqrt{3}}{2}\right)^3$$

$$\left(\frac{\sqrt{3}}{2}\right)^2 = \frac{3}{4}$$

$$= \frac{3}{2}$$

$$\log_{10}(3x)^{3/2} = \log_{10} 45 \qquad \log_7 x^3 = \log_7 \boxed{75}$$

$$(3x)^{3/2} = 45$$

$$x^3 = \frac{45^2}{3^3} = \frac{(9\times5)^2}{} \qquad 3\times25 = 75$$

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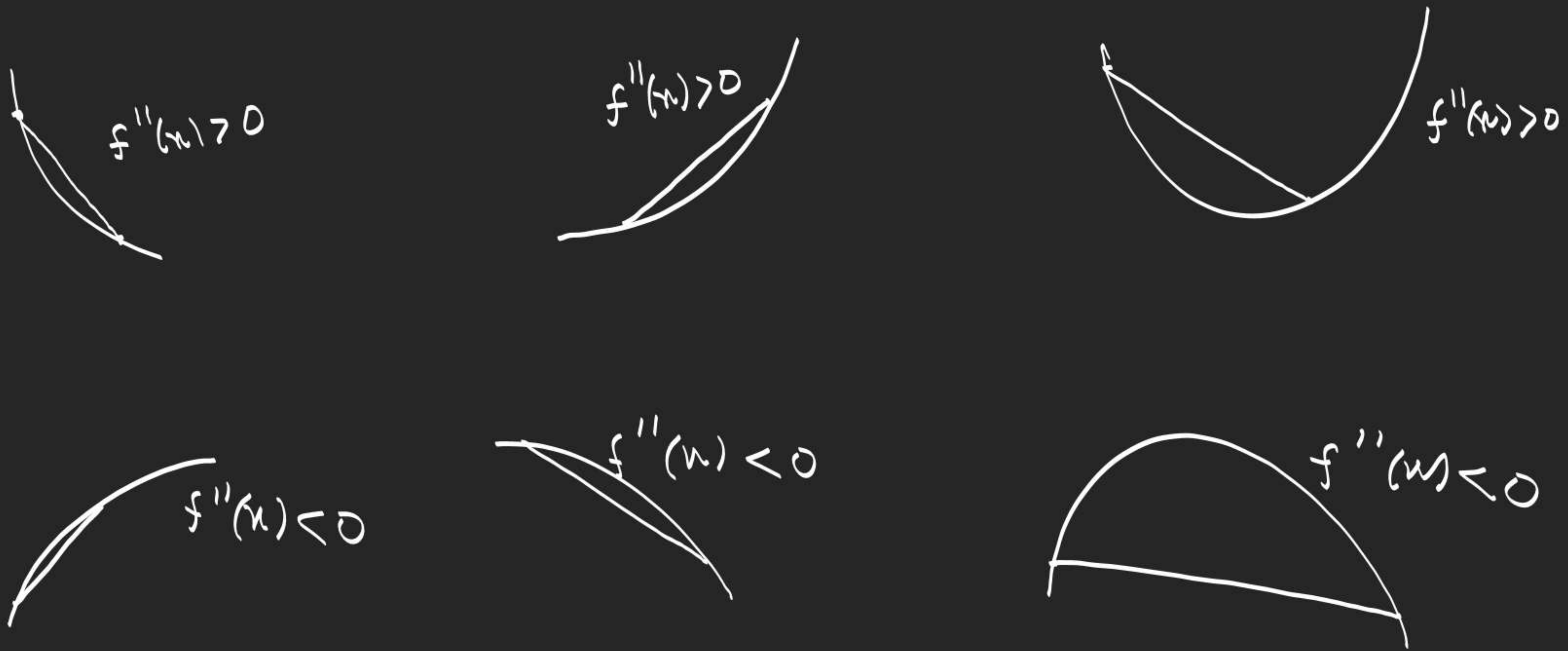

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Jensen's Inequality

If $f''(x) > 0$ in interval $[a, b]$ and $x_1, x_2, \ldots, x_n \in (a, b)$

then $$\frac{\lambda_1 f(x_1) + \lambda_2 f(x_2) + \cdots + \lambda_n f(x_n)}{\lambda_1 + \lambda_2 + \cdots + \lambda_n} \geq f\left(\frac{\lambda_1 x_1 + \lambda_2 x_2 + \cdots + \lambda_n x_n}{\lambda_1 + \lambda_2 + \cdots + \lambda_n}\right)$$

where $\lambda_1, \lambda_2, \lambda_3, \ldots, \lambda_n > 0$. Equality holds if $x_1 = x_2 = \cdots = x_n$

and Inequality gets reversed if $f''(x) < 0$.

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$$\boxed{\frac{\sum_{i=1}^{n} \lambda_i f(x_i)}{\sum \lambda_i} \geq f\left(\frac{\sum \lambda_i x_i}{\sum \lambda_i}\right)}$$

$$\left(\frac{\sum_{i=1}^{n} \lambda_i x_i}{\sum_{i=1}^{n} \lambda_i}, \frac{\sum_{i=1}^{n} \lambda_i f(x_i)}{\sum_{i=1}^{n} \lambda_i}\right)$$

$$\left(\frac{\lambda_1 x_1+\lambda_2 x_2}{\lambda_1+\lambda_2}, \frac{\lambda_1 f(x_1)+\lambda_2 f(x_2)}{\lambda_1+\lambda_2}\right)$$

$(x_1, f(x_1))$

λ_2 λ_1 λ_3

$$\left(\frac{\lambda_1 x_1+\lambda_2 x_2+\lambda_3 x_3}{\lambda_1+\lambda_2+\lambda_3}, \frac{\lambda_1 f(x_1)+\lambda_2 f(x_2)+\lambda_3 f(x_3)}{\lambda_1+\lambda_2+\lambda_3}\right)$$

$(2,4)$

$(2,3)$

y, x

$(x_2, f(x_2))$

$3 > 2$, $3 \geq 2$

$x \geq y$

$x > y$ or $x = y$

$\lambda_1+\lambda_2$ λ_4

$\lambda_1+\lambda_2+\lambda_3$

$y = f(x)$

$$\frac{\lambda_1 f(x_1)+\lambda_2 f(x_2)+\lambda_3 f(x_3)+\lambda_4 f(x_4)}{\lambda_1+\lambda_2+\lambda_3+\lambda_4}$$

$(x_3, f(x_3))$ $(x_4, f(x_4))$

$$\left(\frac{\sum_{i=1}^{4} \lambda_i x_i}{\sum \lambda_i}, \frac{\sum_{i=1}^{4} \lambda_i f(x_i)}{\sum_{i=1}^{4} \lambda_i}\right)$$

$$\left(\frac{\sum_{i=1}^{n} \lambda_i x_i}{\sum_{i=1}^{n} \lambda_i}, f\left(\frac{\sum_{i=1}^{n} \lambda_i x_i}{\sum_{i=1}^{n} \lambda_i}\right)\right)$$

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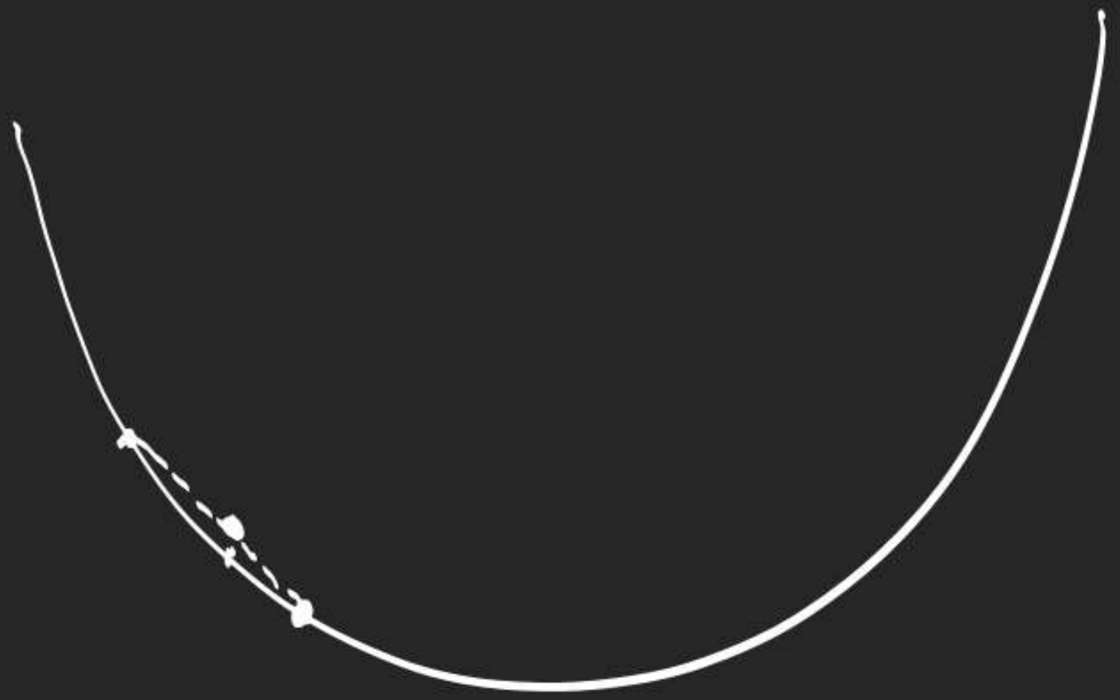

Equality holds if

$$x_1 = x_2 = x_3 = \dots = x_n$$

$$f\left(\frac{\lambda_1 x_1+\lambda_2 x_2+\dots+\lambda_n x_n}{\lambda_1+\lambda_2+\dots\lambda_n}\right) = \frac{\lambda_1 f(x_1)+\lambda_2 f(x_2)+\dots+\lambda_n f(x_n)}{\lambda_1+\lambda_2+\dots+\lambda_n}$$

$x_1=x_2$
$=x_3$

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Graph of function

$y = f(x)$

- Domain of function
- Intervals of increase/decrease of function

  $f'(x) > 0 \Rightarrow f$ is $\uparrow$

  $f'(x) < 0 \Rightarrow f$ is $\downarrow$

- Concavity
- Sketch the graph

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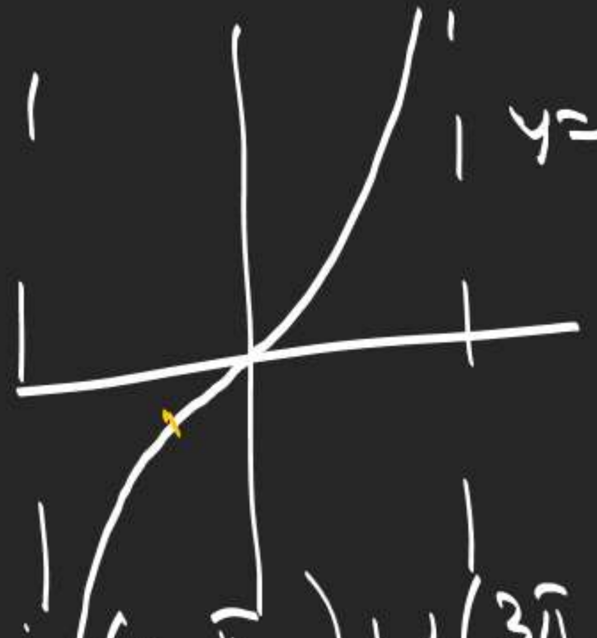
$f(x) = \sin x$

① $D_f = \mathbb{R}$



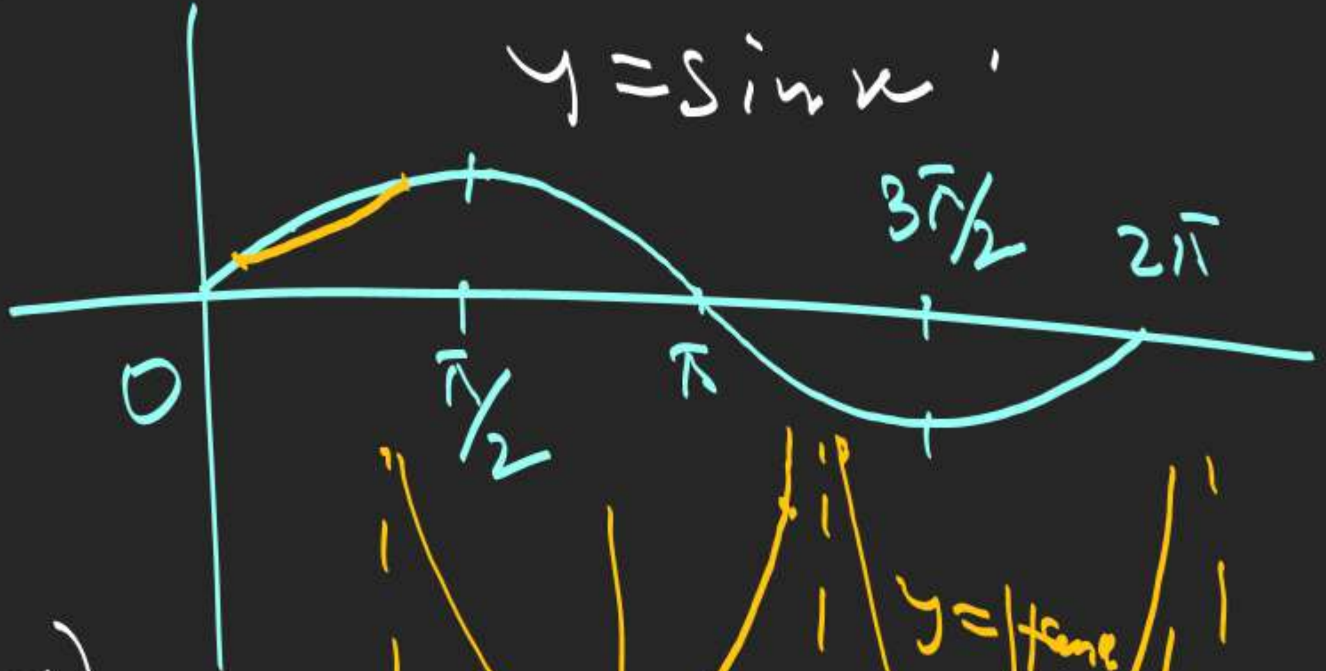
$f'(x) = \sec^2 x$

$f''(x) = 2\sec x(\sec x \tan x)$

$= 2\sec^2 x \tan x$

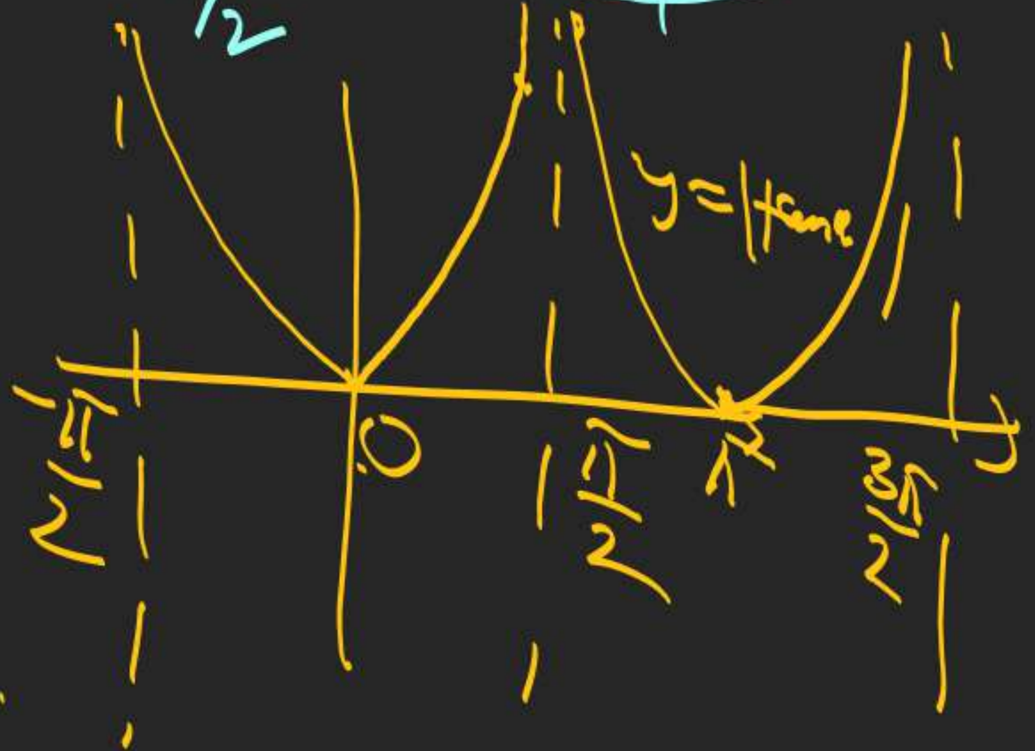
② $f'(x) = \cos x \;\; > 0 \quad x \in \left(0, \frac{\pi}{2}\right) \cup \left(\frac{3\pi}{2}, 2\pi\right)$

$< 0 \quad x \in \left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$



③ $f''(x) = -\sin x \;\; < 0 \quad x \in (0, \pi)$

$> 0 \quad x \in (\pi, 2\pi)$



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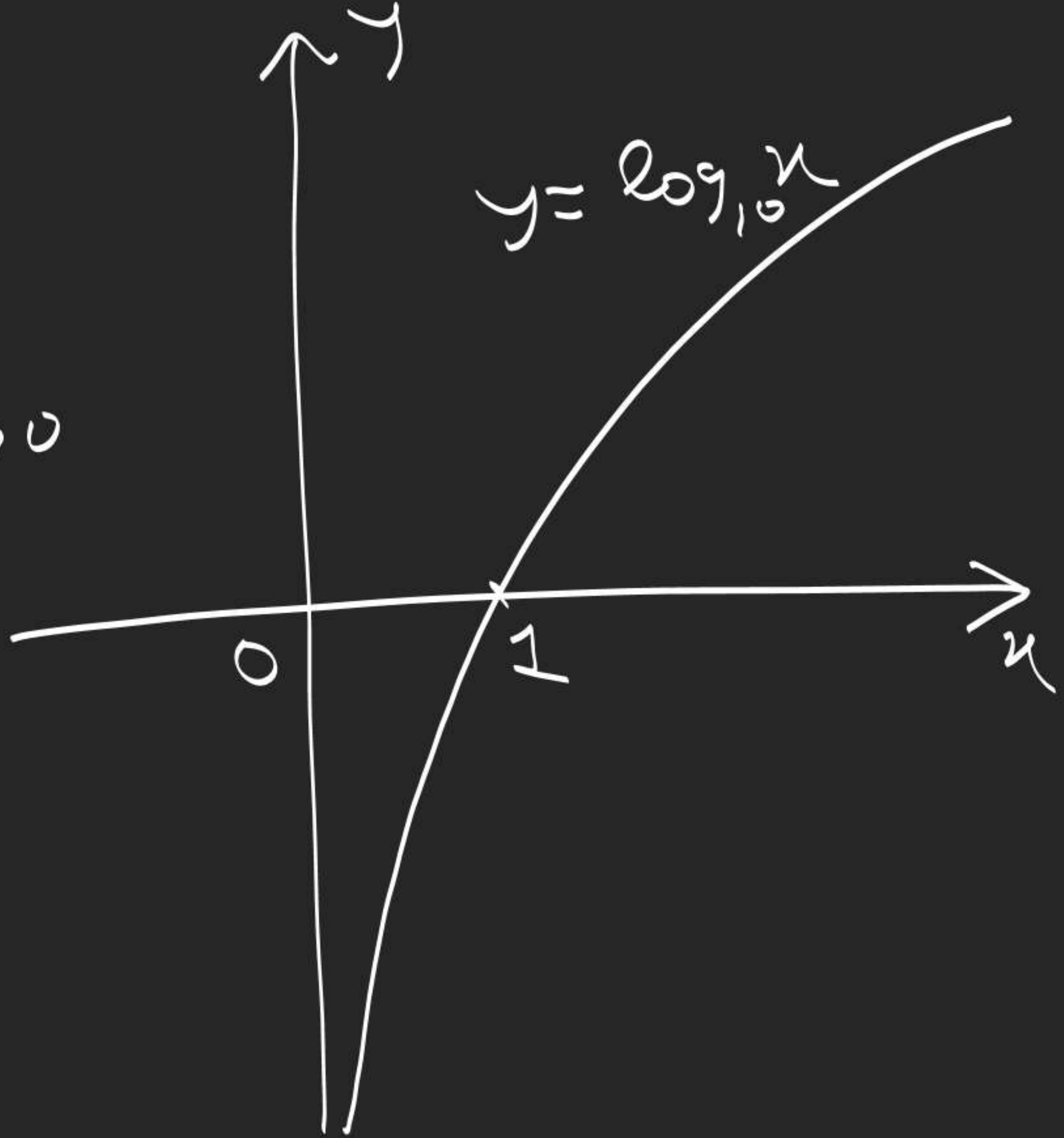
$$f(x) = \log_{10} x = \frac{\ln x}{\ln 10}$$

① $D_f = (0, \infty)$

② $f'(x) = \frac{1}{x \ln 10} > 0 \quad \forall x > 0$

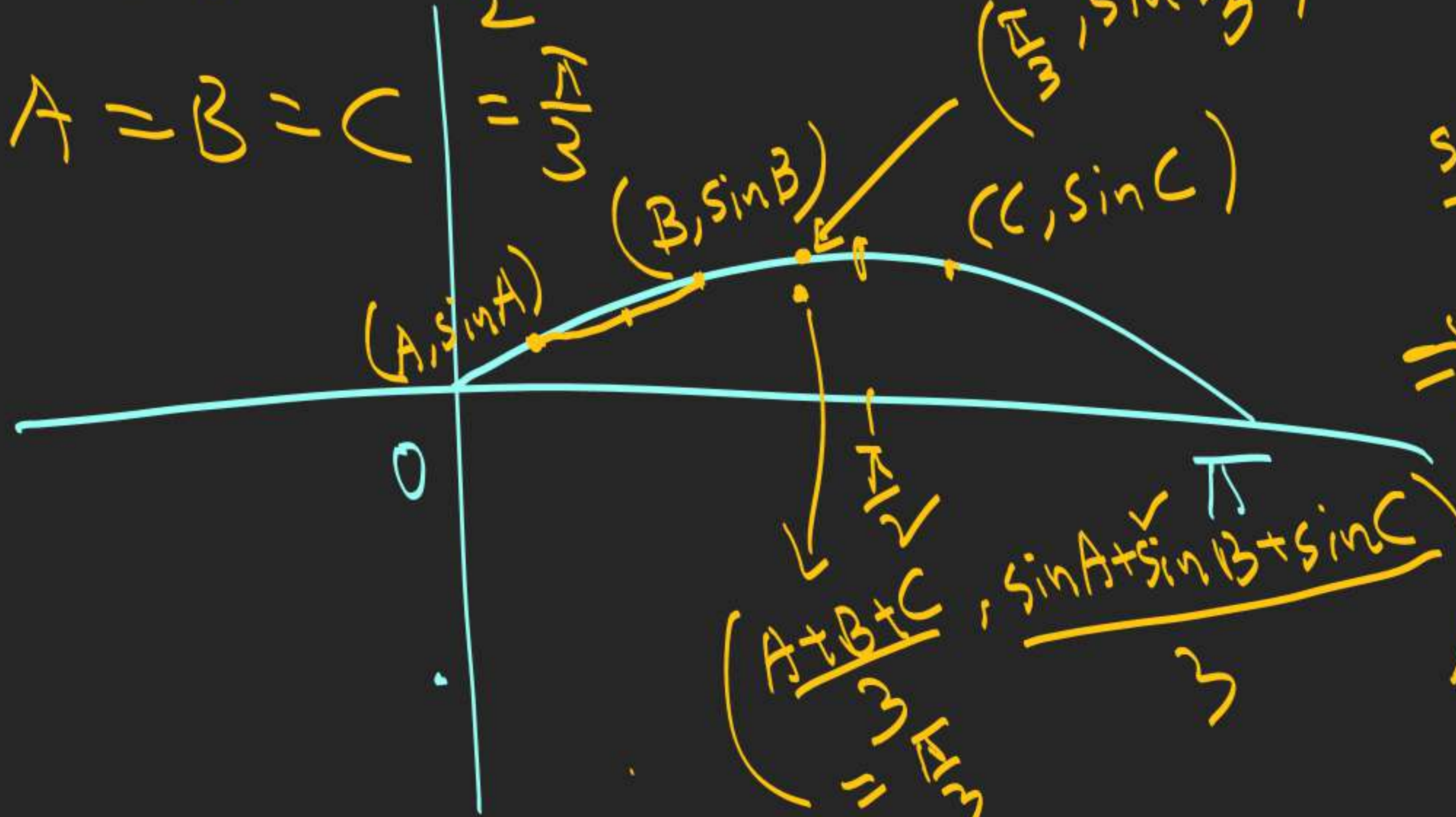
③ $f''(x) = -\frac{1}{x^2 \ln 10} < 0$



① P.T. for any triangle ABC, the maximum value of $\sin A + \sin B + \sin C$ is $\frac{3\sqrt{3}}{2}$.

$\sum \sin A = \frac{3\sqrt{3}}{2}$

if $A = B = C = \frac{\pi}{3}$



$\frac{\sin A + \sin B + \sin C}{3} \le \sin\frac{\pi}{3}$

$\Rightarrow \sin A + \sin B + \sin C \le \frac{3\sqrt{3}}{2}$

$(\sin A + \sin B + \sin C)_{max} = \frac{3\sqrt{3}}{2}$

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For any ΔABC, $\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2} \le \frac{1}{8}$.

$$\frac{\ln\sin\frac{A}{2} + \ln\sin\frac{B}{2} + \ln\sin\frac{C}{2}}{3} \le \ln\sin\frac{\pi}{6}$$

$$\frac{\log\sin\frac{A}{2} + \log\sin\frac{B}{2} + \log\sin\frac{C}{2}}{3}$$

$\boxed{\text{Ex-4}}$

$$\ln\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2} \le \ln\left(\sin\frac{\pi}{6}\right)^3 = \ln\frac{1}{8}$$

$$\boxed{\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2} \le \frac{1}{8}}$$

$f(x) = \ln(\sin x)$, $x \in \left(0, \frac{\pi}{2}\right)$

$$f'(x) = \frac{\cos x}{\sin x} = \cot x$$

$$f''(x) = -\operatorname{cosec}^2 x < 0$$