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1. $f(x) = \cos(\cos x) \rightarrow T = \pi$

$$f(x+T) = f(x) \Rightarrow \cos(\cos(x+T)) = \cos(\cos x) \quad \forall x \in R.$$

$$\cos(x+T) = 2n\pi \pm \cos x. \quad , n \in I.$$

2π

$$\boxed{T = \pi} \qquad \cos(x+T) = \cos x \text{ or } -\cos x$$

π

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2.

$$f(x) = \underbrace{\cos(\sin x)}_{T=\pi} + \underbrace{\cos(\cos x)}_{T=\pi}$$

$$T = \frac{\pi}{2}$$

$$f(x) = \sin^2 x + \cos^2 x$$

T not defined.

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3. P.T. $f(x) = \cos(x^2)$ is aperiodic

$\cos(x+T)^2 = \cos x^2 \quad \forall x \in R$

$\left[0, \sqrt{2\pi}\right)$ ✓

$(x+T)^2 = 2n\pi \pm x^2$

$\left[\sqrt{2\pi}, \sqrt{4\pi}\right)$ ✓

$T^2 + 2xT - 2n\pi = 0$ or

$T^2 + 2xT + 2x^2 - 2n\pi = 0$

$T \neq$ const. $\quad T = g(x)$

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4. $f(x) = x\sin x$ is aperiodic.

$$(x+T)\sin(x+T) = x\sin x$$

$$x\left(\sin(x+T) - \sin x\right) = -T\sin(x+T)$$

$$x\, 2\sin\frac{T}{2}\cos\left(x+\frac{T}{2}\right) = -T\sin(x+T) \quad \forall x \in R$$

$$\frac{2\sin\frac{T}{2}}{-T} = \frac{\sin(x+T)}{x\cos\left(x+\frac{T}{2}\right)} \quad \forall x \in R$$

const. varies.

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5. P.T. $f(x) = \sin x + \cos(ax)$ is periodic if 'a' is rational

$\sin x \to 2\pi$

$\cos(ax) \to \frac{2\pi}{|a|}$

$$T = 2\pi n_1 = \frac{2\pi}{|a|} n_2$$

$$\frac{n_1}{n_2} = \frac{1}{|a|}$$

$n_1, n_2 \in N$

$$\boxed{a \in Q}$$

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Q. If $f(x)=\dfrac{\sin(nx)}{\sin\left(\frac{x}{n}\right)}$ has period 4π, find integral values of 'n'.

$f(x+4\pi)=f(x) \quad \forall x \in D_f$.

$n \in I$, $\dfrac{\sin(nx+4\pi n)}{\sin\left(\frac{x}{n}+\frac{4\pi}{n}\right)} = \dfrac{\sin(nx)}{\sin\left(\frac{x}{n}\right)} \quad \forall x \in D_f$.

$\sin\left(\frac{x}{n}+\frac{4\pi}{n}\right) = \sin\frac{x}{n} \quad \forall x \in D_f$

$\underline{n=1}$

$f(x)=1,\ x \in R-\{n\pi\}$

$0 \quad \pi \quad 2\pi$

$\boxed{n, K \in I}$

$\frac{4\pi}{n} = 2\pi K \Rightarrow \boxed{n=\frac{2}{K}}$ $\boxed{n=\pm1, \pm2}$ ✓

$K=\frac{2}{n}$

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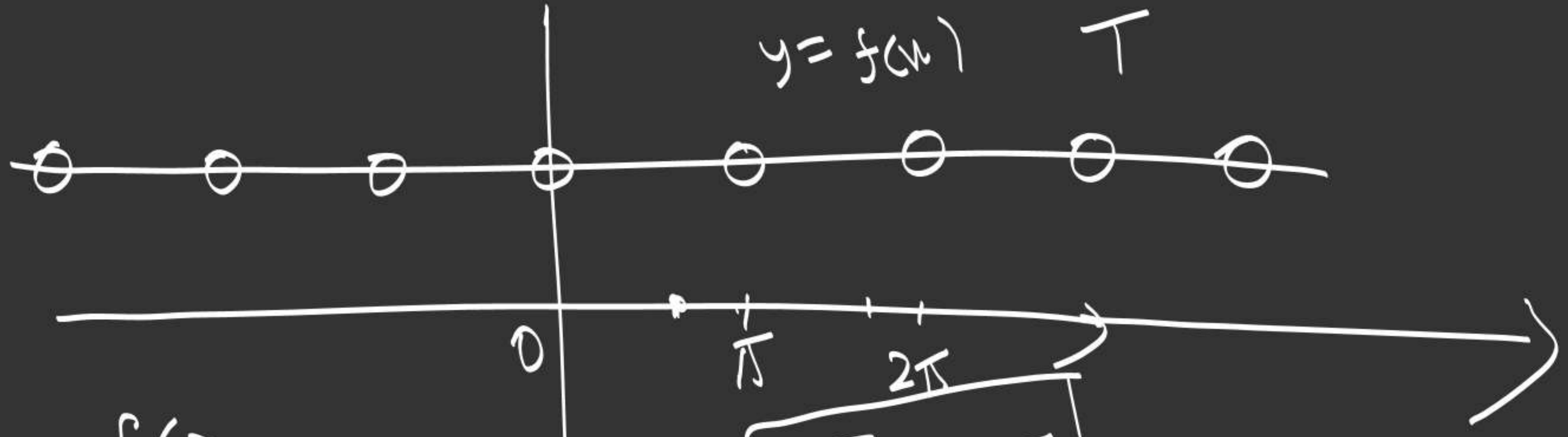
$$f(x) = \frac{\sin x}{\sin x}$$

$$f(x) = 1 \quad \forall x \in R - \{n\pi\}, \; n \in I.$$



$f\left(\frac{\pi}{2}\right)$ $\quad f(\pi)$

$$\boxed{T = \pi}$$

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7. P.T. f is periodic if

$$f(x+1)+f(x-1)=f(x) \quad -(1) \quad \forall x \in R.$$

$x \to x+1$ $\quad f(x+2)+f(x)=f(x+1) \quad -(2)$

(1) + (2) $\quad f(x+2)=-f(x-1)$

$x \to x+1$ $\quad f(x+3)=-f(x)$

$x \to x+3$ $\quad f(x+6)=-f(x+3)=-(-f(x))=f(x)$

$$\boxed{f(x+6)=f(x)}$$

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Q. If $f(x+1) + f(x-1) = \sqrt{3}\, f(x) \quad \forall x \in R$.

P.T. $f(x)$ is periodic.

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Algebraic Function

$$f(x) = \frac{x^2 - 3x + 6}{x^{2/3} + 7 - 5x}$$

algebraic

$\pm$

$\times$

$/$

$(\ )^n$

$$f(x) = \sqrt{x^3 - 3x + 7}$$

algebraic

Transcedental Function $\rightarrow$ not algebraic

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$$f(x) = \frac{\sqrt{x^2-2x} + x}{\sqrt[3]{x-24} - x^2+3}$$

algebraic

$$f(x) = \underline{\sin x} + x^2 - 7x$$

Transcedental

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Note → Let $f(x)$ be a polynomial satisfying

$$f(x)f\left(\frac{1}{x}\right) = f(x) + f\left(\frac{1}{x}\right) \quad \forall x \in R - \{0\},$$

then $f(x) = 1 \pm x^n$

$$g(x) = x^2 + x^3$$
$$g\left(\frac{1}{x}\right) = \frac{1}{x^2} + \frac{1}{x^3}$$

$$f(x)f\left(\frac{1}{x}\right) - f(x) - f\left(\frac{1}{x}\right) = 0$$

$$\left(f\left(\frac{1}{x}\right) - 1\right)\left(\underbrace{f(x) - 1}_{g(x)}\right) = 1 = g\left(\frac{1}{x}\right) g(x)$$

polynomial

$$g\left(\frac{1}{x}\right) = \pm\frac{1}{x^n} \Leftarrow g(x) = \pm x^n$$

$$\alpha, \quad \beta \quad , \alpha\beta$$

$$\alpha + \underbrace{2\beta - \alpha\beta}_{} = 0$$

$$(\beta - 1)(2 - \alpha) = -2$$

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Explicit & Implicit Expressions

$y = f(x)$ → Explicit function

$y^2 = x^3$ → Implicit eqn

$y = x^{3/2}$

$y = -x^{3/2}$

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$$x = 2y - y^2 \quad \begin{cases} y = 1 + \sqrt{1-x} \\ y = 1 - \sqrt{1-x} \end{cases}$$

$$1 - (y-1)^2 = x$$

$$y = 1 \pm \sqrt{1-x}$$

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1. Find the domain of the explicit form of the function represented implicitly by the equation $(1+x)\cos y = x^2$

$$\cos y = \frac{x^2}{1+x}$$

$$-1 \le \frac{x^2}{1+x} \le 1 \Rightarrow$$

$$\frac{\overbrace{x^2+x+1}^{>0}}{1+x} \ge 0 \;\&\; \frac{x^2-x-1}{x+1} \le 0$$

$$x \in (-1, \infty) \qquad (-\infty, -1) \cup \left[\frac{1-\sqrt{5}}{2}, \frac{1+\sqrt{5}}{2}\right]$$

$-$ $\quad -1 \quad$ $+$ $\quad \frac{1-\sqrt{5}}{2} \quad$ $-$ $\quad \frac{1+\sqrt{5}}{2} \quad$ $+$

$$\boxed{D_f = \left[\frac{1-\sqrt{5}}{2}, \frac{1+\sqrt{5}}{2}\right]}$$

Homogeneous Expression

$f(x, y)$ is homogeneous if

Homogenous $\quad f(tx, ty) = t^n f(x, y)$

$$f(y, x) = \cos\frac{y}{x} + 2\left(\frac{x}{y}\right) + \frac{x^3 + y^3}{x^2y - 3xy^2}$$

$$t^0 f(x, y) = f(ty, tx) = \cos\left(\frac{y}{x}\right) + 2\left(\frac{x}{y}\right) + \frac{\cancel{t^3}(x^3 + y^3)}{\cancel{t^3}(x^2y - 3xy^2)}$$

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Bounded Function

If $|f(x)| \le M \quad \forall\, x \in D_f$

where M is a finite real number.

$\boxed{\text{Ex - I (remaining)} \\ \text{PT - 3}}$

$f(x) = \sin x$ is bounded

$|\sin x| \le 1$

$-\{x\}^{3}\sqrt{\tan}$

Ex-I " (i) {