Nishant Jindal

$$4\sqrt{3} = \frac{1}{2}\, 2\times 5\, \frac{\sqrt{3}}{2} + \frac{1}{2} xy \frac{\sqrt{3}}{2}$$

$$xy = 7$$

$$2^2 + 5^2 - 2(2)(5)\cos 60^\circ = x^2 + y^2 - 2xy\cos 120^\circ$$

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1. Simplify $\begin{vmatrix} 23 & 66 & 11 \\ 36 & 55 & 26 \\ 63 & 143 & 37 \end{vmatrix} = 11 \begin{vmatrix} 23 & 6 & 11 \\ 36 & 5 & 26 \\ 63 & 13 & 37 \end{vmatrix}$

$\xrightarrow{\substack{C_1 \to C_1 - 4C_2 \\ C_3 \to C_3 - 2C_2}} 11 \begin{vmatrix} -1 & 6 & -1 \\ 16 & 5 & 16 \\ 11 & 13 & 11 \end{vmatrix}$

$0 =$

2. Show that $\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$

$= abc \begin{vmatrix} 1+\frac{1}{a} & \frac{1}{b} & \frac{1}{c} \\ \frac{1}{a} & 1+\frac{1}{b} & \frac{1}{c} \\ \frac{1}{a} & \frac{1}{b} & 1+\frac{1}{c} \end{vmatrix} \xrightarrow{C_1 \to C_1 + C_2 + C_3} \left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right) abc \begin{vmatrix} 1 & \frac{1}{b} & \frac{1}{c} \\ 1 & 1+\frac{1}{b} & \frac{1}{c} \\ 1 & \frac{1}{b} & 1+\frac{1}{c} \end{vmatrix}$

$R_2 \to R_2 - R_1$

$R_3 \to R_3 - R_1$

$abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right) = abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right) \begin{vmatrix} 1 & \frac{1}{b} & \frac{1}{c} \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$

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3.

Simplify
$$\begin{vmatrix} 1 & \sin 3\theta & \sin^3\theta \\ 2\cos\theta & \sin 6\theta & \sin^3 2\theta \\ \underbrace{4\cos^2\theta - 1}_{3-4\sin^2\theta} & \sin 9\theta & \sin^3 3\theta \end{vmatrix} = 0$$

$$C_2 \to C_2 - 3\sin\theta\, C_1 + 4C_3$$

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4. Let $\underline{A28}$, $3B9$ and $62C$ are 3 digit numbers divisible by integer K, then P.T. the determinant

$$\begin{vmatrix} A & 3 & 6 \\ 8 & 9 & C \\ 2 & B & 2 \end{vmatrix}$$ is also divisible by K.

$$\Big\downarrow \; R_2 \to R_2 + 10R_3 + 100R_1$$

$$\begin{vmatrix} A & 3 & 6 \\ \underline{A28} & \underbrace{3B9} & \underset{\sim}{62C} \\ 2 & B & 2 \end{vmatrix} = \underline{K} \begin{vmatrix} A & 3 & 6 \\ \alpha & \beta & \gamma \\ 2 & B & 2 \end{vmatrix}$$

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5. Let $D_r = \begin{vmatrix} r & x & \frac{n(n+1)}{2} \\ 2r-1 & y & n^2 \\ 3r-2 & z & \frac{n(3n-1)}{2} \end{vmatrix}$, find $\sum_{r=1}^{n} D_r$

$$\sum_{r=1}^{n} D_r = D_1 + D_2 + \cdots D_n = \begin{vmatrix} \sum_{r=1}^{n} r & x & \frac{n(n+1)}{2} \\ \sum_{r=1}^{n}(2r-1) & y & n^2 \\ \sum_{r=1}^{n}(3r-2) & z & \frac{n(3n-1)}{2} \end{vmatrix} = 0$$

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6. If $\begin{vmatrix} p & b & c \\ a & q & c \\ a & b & r \end{vmatrix} = 0$, find the value of

$$\boxed{\frac{p}{p-a} + \frac{q}{q-b} + \frac{r}{r-c} = 2}$$

$R_2 \to R_2 - R_1$
$R_3 \to R_3 - R_1$

$$\begin{vmatrix} p & b & c \\ a-p & q-b & 0 \\ a-p & 0 & r-c \end{vmatrix} = p(q-b)(r-c) + b(p-a)(r-c) + c(p-a)(q-b) = 0$$

$$0 = \frac{p}{p-a} + \frac{b-q+q}{q-b} + \frac{c-r+r}{r-c} \Leftarrow \frac{p}{p-a} + \frac{b}{q-b} + \frac{c}{r-c} = 0$$

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7. Simplify $\begin{vmatrix} (b+c)^2 & a^2 & bc \\ (c+a)^2 & b^2 & ca \\ (a+b)^2 & c^2 & ab \end{vmatrix}$

$C_1 \to C_1 + C_2 - 2C_3$

$$= (a^2+b^2+c^2)\begin{vmatrix} 1 & a^2 & bc \\ 1 & b^2 & ca \\ 1 & c^2 & ab \end{vmatrix}$$

$$= \frac{(a^2+b^2+c^2)}{abc}\begin{vmatrix} a & a^3 & abc \\ b & b^3 & abc \\ c & c^3 & abc \end{vmatrix}$$

$$= (a^2+b^2+c^2)\begin{vmatrix} a & a^3 & 1 \\ b & b^3 & 1 \\ c & c^3 & 1 \end{vmatrix}$$

$$= (a^2+b^2+c^2)\begin{vmatrix} 1 & a & a^3 \\ 1 & b & b^3 \\ 1 & c & c^3 \end{vmatrix}$$

$$\boxed{(a^2+b^2+c^2)(a-b)(b-c)(c-a)(a+b+c)}$$

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8. P.T.

$$\begin{vmatrix} a & b-c & b+c \\ a+c & b & c-a \\ a-b & a+b & c \end{vmatrix} = (a^2+b^2+c^2)(a+b+c)$$

$$\frac{1}{abc}\begin{vmatrix} a^2 & ab-ac & ab+ac \\ ba+bc & b^2 & bc-ba \\ ca-cb & ca+cb & c^2 \end{vmatrix}$$

$$R_1 \to R_1+R_2+R_3$$

$$\frac{(a+b+c)}{a}\begin{vmatrix} a & b & c \\ a+c & b & c-a \\ a-b & a+b & c \end{vmatrix} = \frac{(a+b+c)}{abc}\begin{vmatrix} a & b & c \\ ab+bc & b^2 & bc-ba \\ ca-cb & ca+cb & c^2 \end{vmatrix}$$

$$\frac{(a+b+c)}{a^2bc}\begin{vmatrix} a^2 & b^2 & c^2 \\ a^2+ac & b^2 & c^2-ac \\ a^2-ab & ba+b^2 & c^2 \end{vmatrix}$$

$$C_1 \to C_1+C_2+C_3$$

$$\frac{\Sigma a(\Sigma a^2)}{a^2bc}\begin{vmatrix} 1 & b^2 & c^2 \\ 1 & b^2 & c^2-ac \\ 1 & ba+b^2 & c^2 \end{vmatrix}$$

$$\frac{\Sigma a\,\Sigma a^2}{a^2}\begin{vmatrix} 1 & b & c \\ 1 & b & c-a \\ 1 & a+b & c \end{vmatrix}$$

$$\frac{\Sigma a\,\Sigma a^2}{a^2}\begin{vmatrix} 1 & b & c \\ 0 & 0 & -a \\ 0 & a & 0 \end{vmatrix}$$

$$\begin{vmatrix} a & b-c & b+c \\ a+c & b & c-a \\ a-b & a+b & c \end{vmatrix} \xrightarrow{} \quad C_1 \to aC_1 + bC_2 + cC_3$$

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Ex-6

$$\frac{1}{a}\begin{vmatrix} a^2+b^2+c^2 & b-c & b+c \\ a^2+b^2+c^2 & b & c-a \\ a^2+b^2+c^2 & a+b & c \end{vmatrix} = \frac{(a^2+b^2+c^2)}{a}\begin{vmatrix} 1 & b-c & b+c \\ 1 & b & c-a \\ 1 & a+b & c \end{vmatrix}$$

$R_2 \to R_2 - R_1$

$R_3 \to R_3 - R_1$

$$\frac{(a^2+b^2+c^2)}{a}\begin{vmatrix} 1 & b-c & b+c \\ 0 & c & -a-b \\ 0 & a+c & -b \end{vmatrix}$$