(MATHEMATICS)   **INDEFINITE INTEGRATION**

DPP-03

(STANDARD INTEGRATION)

1. $\int \left[\sin^2\left(\frac{9\pi}{8} + \frac{x}{4}\right) - \sin^2\left(\frac{7\pi}{8} + \frac{x}{4}\right)\right] dx$

2. $\int \frac{\cos 4x + 1}{\cot x - \tan x} dx$

3. A function g defined for all positive real numbers satisfies $g'(x^2) = x^3$ for all $x > 0$ ànd $g(1) = 1$. Compute $g(4)$.

4. $\int \left[\sin\alpha \sin(x - \alpha) + \sin^2\left(\frac{x}{2} - \alpha\right)\right] dx$

5. $\int \frac{\sin 2x + \sin 5x - \sin 3x}{\cos x + 1 - 2\sin^2 2x} dx$

6. $\int \left[\frac{\cot^2 2x - 1}{2\cot 2x} - \cos 8x \cot 4x\right] dx$

7. $\int \frac{\cos^4 x - \sin^4 x}{\sqrt{1 + \cos 4x}} dx \ (\cos 2x > 0)$

8. $\int \frac{2x^3 + 3x^2 + 4x + 5}{2x + 1} dx$

9. $\int \frac{(x^2 + \sin^2 x)\sec^2 x}{1 + x^2} dx$

10. $\int \frac{dx}{\sqrt{9 - 16x^2}}$

11. $\int \frac{dx}{25 + 4x^2}$

12. $\int \frac{2x + 3}{3x + 2} dx$

13. $\int \frac{\cos 8x - \cos 7x}{1 + 2\cos 5x} dx$

14. $\int \frac{2 + 3x^2}{x^2(1 + x^2)} dx$

15. $\int \frac{(\sin 2x) - (\sin 2k)}{\sin x - \sin k + \cos x - \cos k} dx$

16. $\int \frac{x^2 + 3}{x^6(x^2 + 1)} dx$

17. $\int \sin x \cos x \cos 2x \cos 4x \, dx$

18. $\int x^x \ln(ex) dx$

19. $\int \frac{dx}{x^2 + x + 1}$ is equal to

(A) $\frac{\sqrt{3}}{2}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right) + c$

(B) $\frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right) + c$

(C) $\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right) + c$

(D) $\sqrt{3}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right) + c$

20. If $\int \frac{1}{1 + \sin x} dx = \tan\left(\frac{x}{2} + a\right) + b$, then

(A) $a = -\frac{\pi}{4}, b \in R$

(B) $a = \frac{\pi}{4}, b \in R$

(C) $a = \frac{5\pi}{4}, b \in R$

(D) $a = \frac{\pi}{2}, b \in R$

21. $\int \{1 + 2\tan x(\tan x + \sec x)\}^{1/2} dx$ is equal to

(A) $\ln \sec x(\sec x - \tan x) + c$

(B) $\ln \csc x(\sec x + \tan x) + c$

(C) $\ln \sec x(\sec x + \tan x) + c$

(D) $\ln(\sec x + \tan x) + c$

22. $\int \frac{\cos 2x - 1}{\cos 2x + 1} dx =$

(A) $\tan x - x + c$

(B) $x + \tan x + c$

(C) $x - \tan x + c$

(D) $-x - \cot x + c$

23. If $\int \frac{\cos 4x+1}{\cot x-\tan x} dx = A\cos 4x + B$ where A & B are constants, then

(A) $A = -1/4$ & B may have any value

(B) $A = -\frac{1}{8}$ & B may have any value

(C) $A = -\frac{1}{2}$ & $B = -1/4$

(D) $A = \frac{1}{2}$ & $B \in R$

24. $\int 4\sin x \cos\frac{x}{2} \cos\frac{3x}{2} dx$ is equal to

(A) $\cos x - \frac{1}{2}\cos 2x + \frac{1}{3}\cos 3x + c$

(B) $\cos x - \frac{1}{2}\cos 2x - \frac{1}{3}\cos 3x + c$

(C) $\cos x + \frac{1}{2}\cos 2x + \frac{1}{3}\cos 3x + c$

(D) $\cos x + \frac{1}{2}\cos 2x - \frac{1}{3}\cos 3x + c$

25. $\int \sin x \cdot \cos x \cdot \cos 2x \cdot \cos 4x \cdot \cos 8x \cdot \cos 16x \, dx$ is

3 equal to

(A) $\frac{\sin 16x}{1024} + c$

(B) $-\frac{\cos 32x}{1024} + c$

(C) $\frac{\cos 32x}{1096} + c$

(D) $-\frac{\cos 32x}{1096} + c$

26. $\int \frac{\sin^8 x - \cos^8 x}{1 - 2\sin^2 x \cos^2 x} dx$ is equal to

(A) $\frac{1}{2}\sin 2x + c$

(B) $-\frac{1}{2}\sin 2x + c$

(C) $-\frac{1}{2}\sin x + c$

(D) $-\sin^2 x + c$

27. If $\int \frac{\sin x}{\sin(x-\alpha)} dx = Ax + B\log \sin(x-\alpha) + C$, then value of (A, B) is-

(A) $(\sin\alpha, \cos\alpha)$

(B) $(\cos\alpha, \sin\alpha)$

(C) $(-\sin\alpha, \cos\alpha)$

(D) $(-\cos\alpha, \sin\alpha)$

28. $\int (\sin 2x - \cos 2x) dx = \frac{1}{\sqrt{2}}\sin(2x-a) + b$, then

(A) $a = \frac{5\pi}{4}, b \in R$

(B) $a = -\frac{5\pi}{4}, b \in R$

(C) $a = \frac{\pi}{4}, b \in R$

(D) $a = -\frac{\pi}{4}, b \in R$

29. $\int [1 + \tan x \cdot \tan(x+\alpha)] dx$ is equal to

(A) $\cos\alpha \cdot \ln\left|\frac{\sin x}{\sin(x+\alpha)}\right| +$

(B) $\tan\alpha \cdot \ln\left|\frac{\sin x}{\sin(x+\alpha)}\right| + C$

(C) $\cot\alpha \cdot \ln\left|\frac{\sec(x+\alpha)}{\sec x}\right| + C$

(D) $\cot\alpha \cdot \ln\left|\frac{\cos(x+\alpha)}{\cos x}\right| + C$

30. $\int \left(\sqrt{\frac{a+x}{a-x}} - \sqrt{\frac{a-x}{a+x}}\right) dx$ is equal to

(A) $-2\sqrt{a^2 - x^2} + C$

(B) $\sqrt{a^2 - x^2} + C$

(C) $-\sqrt{x^2 - a^2} + C$

(D) None of these

31. $\int \frac{x^2+\cos^2 x}{1+x^2}\operatorname{cosec}^2 x\, dx$ is equal to :

    (A) $\cot x - \cot^{-1} x + c$ (B) $c - \cot x + \cot^{-1} x$

    (C) $-\tan^{-1} x - \frac{\operatorname{cosec} x}{\sec x} + c$ (D) $-e^{\ell \ln \tan^{-1} x} - \cot x + c$

32. $\int \frac{4x^5-7x^4+8x^3-2x^2+4x-7}{x^2(x^2+1)^2}\, dx$ **33.** $\int \frac{x\, dx}{x^4+1}.$

34. $\int \frac{x\, dx}{\sqrt{a^2-x^4}}.$ **35.** $\int \frac{x^2\, dx}{x^6+4}$

36. $\int \frac{x^3\, dx}{\sqrt{1-x^8}}$ **37.** $\int \frac{e^x\, dx}{e^{2x}+4}.$

38. $\int \frac{\cos \alpha\, d\alpha}{a^2+\sin^2 \alpha}.$ **39.** $\int (e^x + 1)^3\, dx.$

40. $\int \frac{1+x}{\sqrt{1-x^2}}\, dx.$ **41.** $\int \frac{3x-1}{x^2+9}\, dx.$

42. $\int \sqrt{\frac{1-x}{1+x}}\, dx.$ **43.** $\int \frac{dx}{1+\sin x}.$

44. $\int \frac{1-\cos x}{1+\cos x}\, dx.$ **45.** $\int \frac{1+\sin x}{1-\sin x}\, dx.$

46. $\int \cos^3 x\, dx$ **47.** $\int \tan^4 x\, dx$

48. $\int \sin^4 x\, dx$ **49.** $\int \tan^3 x\, dx.$

ANSWER KEY

1. $-\sqrt{2}\cos\frac{x}{2} + C$ 2. $-\frac{\cos 4x}{8} + C$

3. $\frac{67}{5}$ 4. $\frac{1}{2}(x - \sin x) + C$

5. $-2\cos x + C$ 6. $-\frac{\cos 8x}{8} + C$

7. $\frac{x}{\sqrt{2}} + C$ 8. $\frac{x^3}{3} + \frac{x^2}{2} + \frac{3x}{2} + \frac{7}{4}\ln(2x + 1)$

9. $\tan x - \tan^{-1} x + C$ 10. $\frac{1}{4}\sin^{-1}\frac{4}{3}x + C$

11. $\frac{1}{10}\tan^{-1}\frac{2x}{5} + C$ 12. $\frac{2}{3}x + \frac{5}{9}\ln(3x + 2) + C$

13. $\frac{\sin 3x}{3} - \frac{\sin 2x}{2} + C$ 14. $-\frac{2}{x} + \tan^{-1} x + C$

15. $(\sin x - \cos x) + (\sin k + \cos k)x + C$ 16. $C - \frac{2}{x} + \frac{2}{3}\frac{1}{x^3} - \frac{3}{5}\frac{1}{x^5} - 2\tan^{-1}x$

17. $-\frac{1}{64}\cos 8x + C$ 18. $x^x + C$

19. (B) **20.** (A) **21.** (C)   22. (C) **23.** (B) **24.** (B) **25.** (B)

26. (B) **27.** (B) **28.** (B)   29. (C) **30.** (A) **31.** (B,C,D)

32. $4\ln x + \frac{7}{x} + 6\tan^{-1}(x) + \frac{6x}{1+x^2} + C$ 33. $\frac{1}{2}\arctan x^2 + C.$

34. $\frac{1}{2}\arcsin\frac{x^2}{a} + C.$ 35. $\frac{1}{6}\arctan\frac{x^3}{2} + C.$

36. $\frac{1}{4}\arcsin x^4 + C.$ 37. $\frac{1}{2}\arctan\frac{e^x}{2} + C.$

38. $\frac{1}{a}\arctan\frac{\sin\alpha}{a} + C.$ 39. $\frac{1}{3}e^{3x} + \frac{3}{2}e^{2x} + 3e^x + x + C$

40. $\arcsin x - \sqrt{1 - x^2} + C.$ 41. $\frac{3}{2}\ln(x^2 + 9) - \frac{1}{3}\arctan\frac{x}{3} + C.$

42. $\arcsin x + \sqrt{1 - x^2 + C}.$ 43. $\tan\left(\frac{x}{2} - \frac{\pi}{4}\right) + C.$

44. $2\tan\frac{x}{2} - x + C$ 45. $2\tan\left(\frac{x}{2} + \frac{\pi}{4}\right) - x + C$

46. $\sin x - \frac{\sin^3 x}{3} + C$ 47. $\frac{1}{3}\tan^3 x - \tan x + x + C$

48. $\frac{3}{8}x - \frac{1}{4}\sin 2x + \frac{1}{32}\sin 4x + C$ 49. $\frac{1}{2}\tan^2 x + \ln|\cos x| + C$