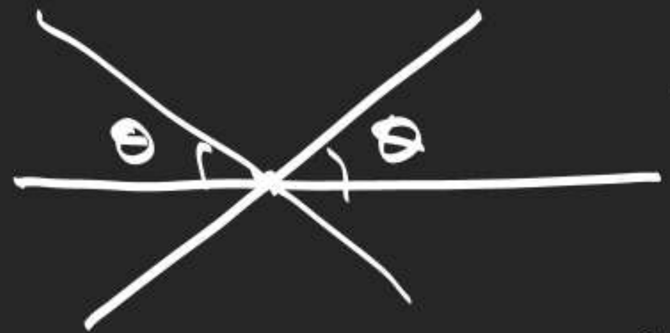
Nishant Jindal

Angle b/n lines $\boxed{ax^2+2hxy+by^2=0}$



2 distinct real lines
D > 0, $h^2 > ab$

$h^2 = ab$, equal real lines

$h^2 < ab$, imaginary lines

$$m = \frac{y-0}{x-0}$$

$$bm^2 + 2hm + a = 0 \to m_1, m_2$$

$$m_1 m_2 = \frac{a}{b}, \quad m_1 + m_2 = \frac{-2h}{b}$$

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right| = \frac{\sqrt{(m_1+m_2)^2 - 4m_1m_2}}{|1+m_1m_2|} = \frac{\sqrt{\frac{4h^2}{b^2} - \frac{4a}{b}}}{\left|1 + \frac{a}{b}\right|}$$

non parallel Lines equally inclined with x-axis ⇒ h = 0

Lines are ⊥ar ⇒ a + b = 0

$$\boxed{\tan\theta = \frac{2\sqrt{h^2 - ab}}{|a+b|}}$$

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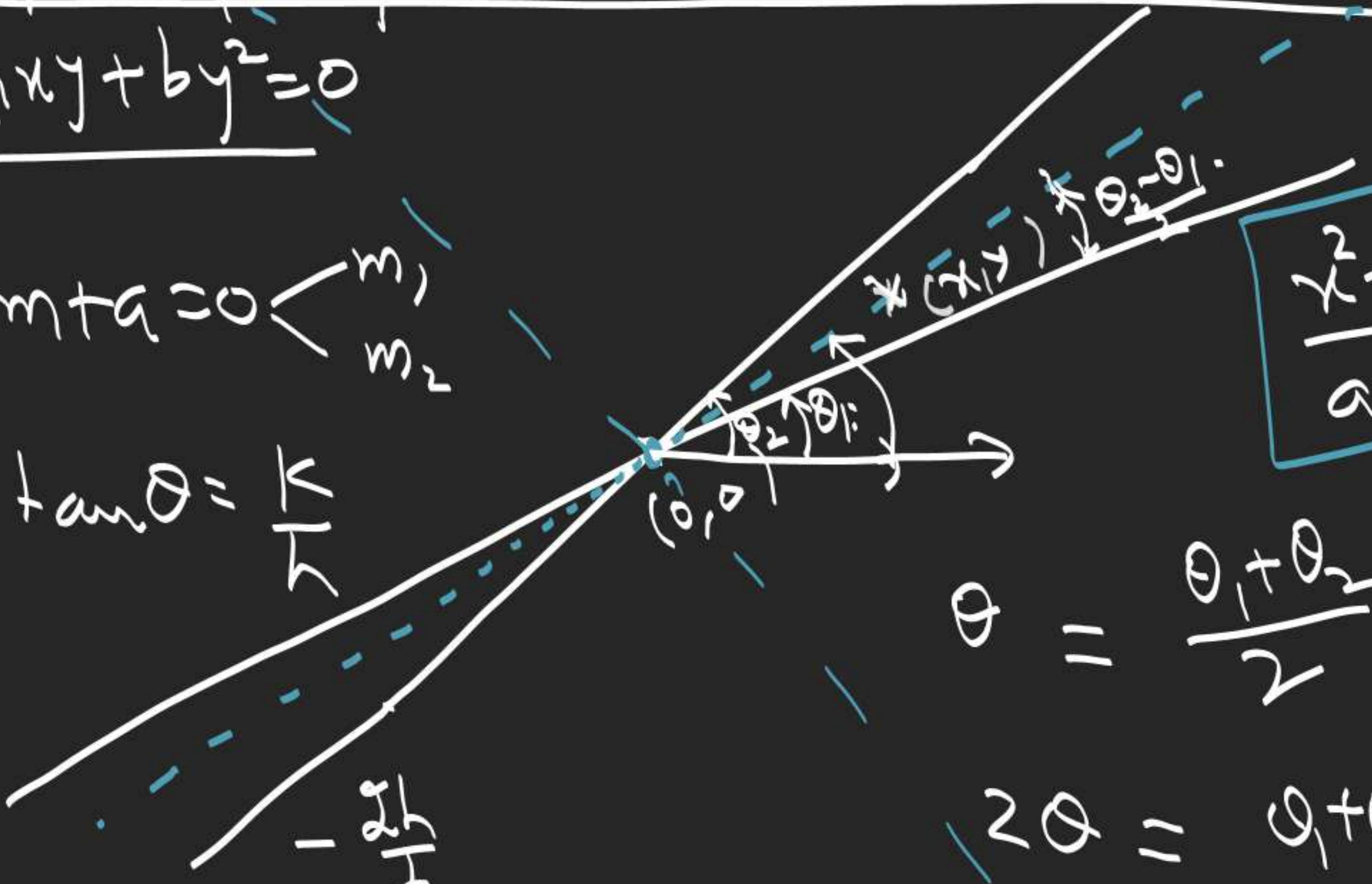
Eqn. of pair of angle bisectors to the pair of lines

$$ax^2+2hxy+by^2=0$$

$$bm^2+2hm+a=0 \;<\; \begin{matrix} m_1 \\ m_2 \end{matrix}$$

$$\tan\theta = \frac{k}{h}$$



$$\boxed{\frac{x^2-y^2}{a-b} = \frac{xy}{h}}$$

$$\theta = \frac{\theta_1+\theta_2}{2},\ \frac{\theta_1+\theta_2}{2}+\frac{\pi}{2}$$

$$2\theta = \theta_1+\theta_2,\ \theta_1+\theta_2+\pi$$

$$\tan 2\theta = \tan(\theta_1+\theta_2)$$

$$\frac{2\tan\theta}{1-\tan^2\theta} = \frac{m_1+m_2}{1-m_1m_2}$$

$$\frac{\frac{2y}{x}}{1-\frac{y^2}{x^2}} = \frac{-\frac{2h}{b}}{1-\frac{a}{b}}$$

$$\frac{xy}{x^2-y^2} = \frac{h}{a-b}$$

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$ax^2+2hxy+by^2+2gx+2fy+c=0$ represent pair of lines

if $\boxed{abc+2fgh-af^2-bg^2-ch^2=0}$

$$\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = 0$$

$$ax^2+2hxy+by^2+2gx+2fy+c=(l_1x+m_1y+n_1)(l_2x+m_2y+n_2)$$

$$\frac{1}{8}\begin{vmatrix} 2l_1l_2 & l_1m_2+m_1l_2 & l_1n_2+n_1l_2 \\ l_1m_2+m_1l_2 & 2m_1m_2 & \\ & & \end{vmatrix} = \begin{vmatrix} l_1 & l_2 & 0 \\ m_1 & m_2 & 0 \\ n_1 & n_2 & 0 \end{vmatrix}\begin{vmatrix} l_2 & m_2 & n_2 \\ l_1 & m_1 & n_1 \\ 0 & 0 & 0 \end{vmatrix}$$

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Angle b/w lines represented by $ax^2+2hxy+by^2+2gx+2fy+c=0$

$$ax^2+2hxy+by^2+2gx+2fy+c = b\,(y-m_1x-c_1)(y-m_2x-c_2)$$

coeff of x^2 $\quad a = bm_1m_2$ $\qquad m_1m_2 = \frac{a}{b}$

Coeff of xy $\quad 2h = -b(m_1+m_2)$ $\qquad m_1+m_2 = -\frac{2h}{b}$.

$$\boxed{\tan\theta = \frac{2\sqrt{h^2-ab}}{|a+b|}}$$

Lines are parallel

$\Rightarrow h^2 = ab$

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1. Find the condition for which $ax^3+bx^2y+cxy^2+dy^3=0$ represent three lines, two of which are at right angles.

$\frac{y}{x}=m$

$$dm^3+cm^2+bm+a=0 \quad m_1, m_2, m_3$$

$$m_1m_2=-1$$

$$m_1m_2m_3=-m_3=-\frac{a}{d}$$

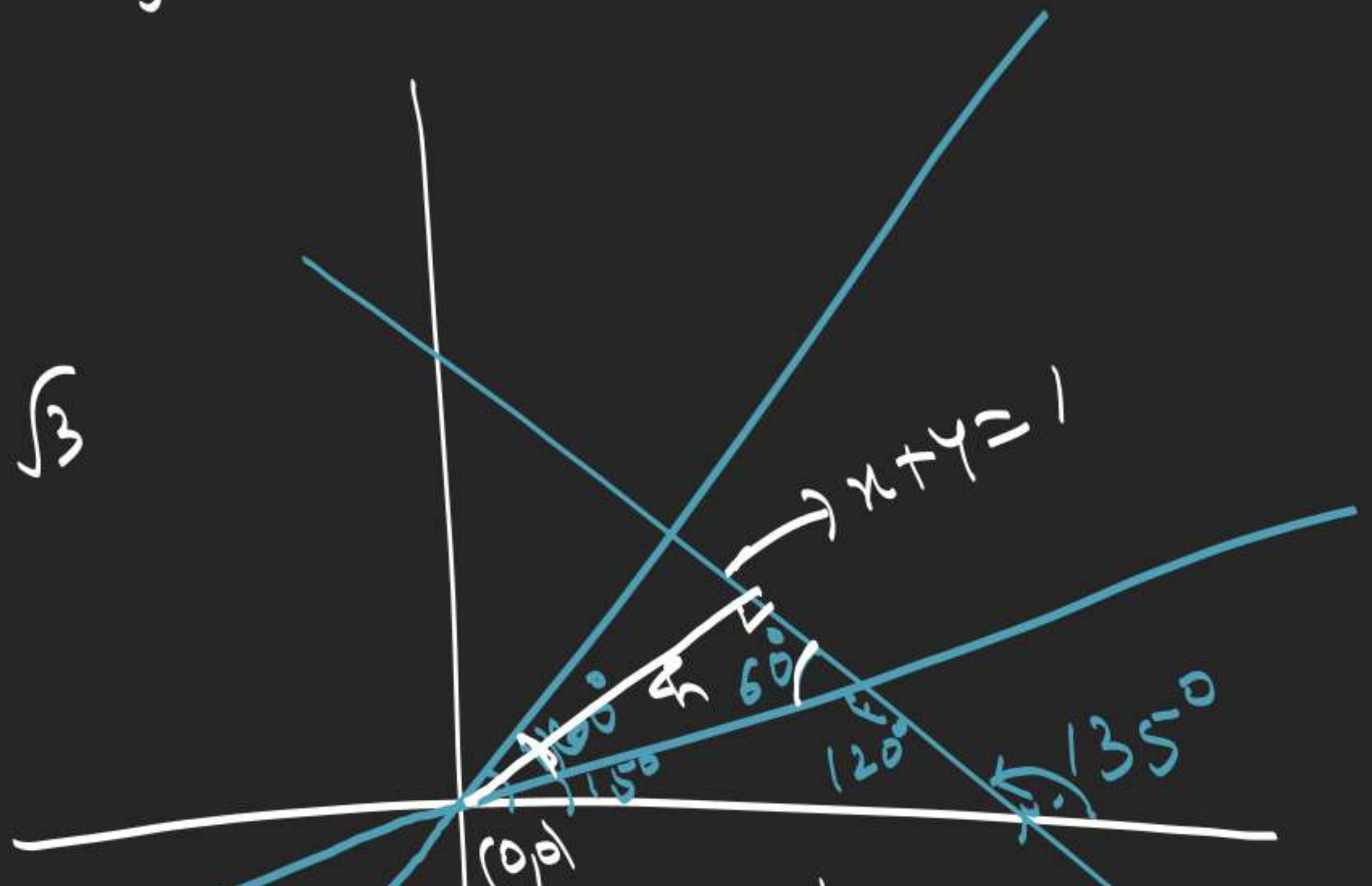
$$m_3=\frac{a}{d}$$

$$d\frac{a^3}{d^3}+c\frac{a^2}{d^2}+b\frac{a}{d}+a=0$$

Q. P.T. lines $x^2-4xy+y^2=0$ and $x+y=1$ enclose an equilateral triangle. Also find its area.

$$m^2-4m+1=0$$

$$m=\frac{4\pm\sqrt{12}}{2}=2\pm\sqrt{3}$$



$$\frac{a\sqrt{3}}{2}=h$$

$$\Delta=\frac{\sqrt{3}}{4}\left(\frac{2h}{\sqrt{3}}\right)^2=\frac{h^2}{\sqrt{3}}.$$

$$h=\frac{1}{\sqrt{2}}$$

$$\boxed{\Delta=\frac{1}{2\sqrt{3}}}$$

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Homogenisation

$$a\alpha^2+2h\alpha\beta+b\beta^2+(2g\alpha+2f\beta)\left(\frac{l\alpha+m\beta}{-n}\right)+c\left(\frac{l\alpha+m\beta}{-n}\right)^2$$

$$0 = a\alpha^2+2h\alpha\beta+b\beta^2+2g\alpha+2f\beta+c=$$

Given: a two degree curve $C: ax^2+by^2+2hxy+2gx+2fy+c=0$ and a line $L: lx+my+n=0$ such that 'L' intersects C at 2 points A, B.

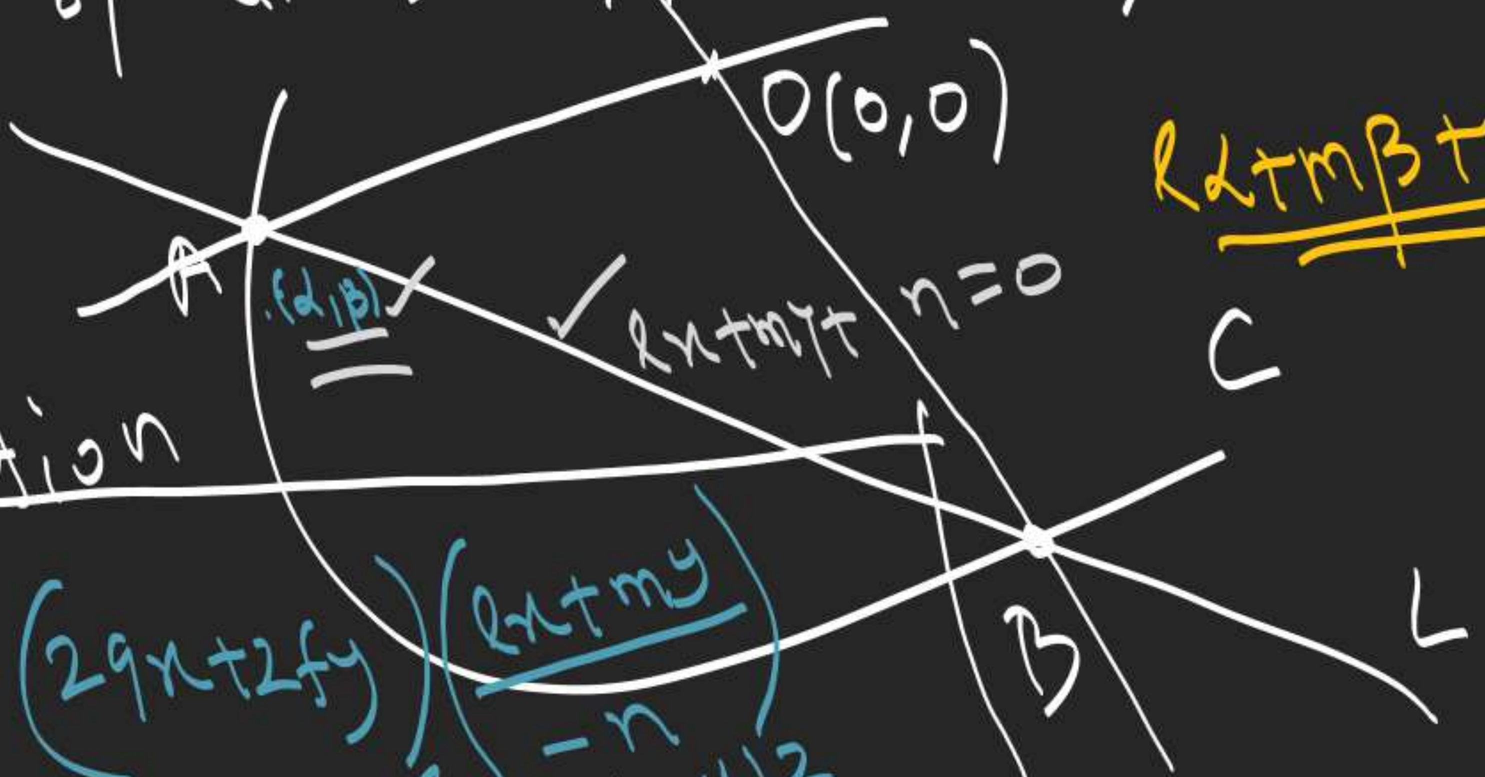
$$\frac{lx+my}{-n}=1$$

$$\frac{lx+n}{-my}=1$$

To find: Equation of pair of lines OA & OB, where 'O' is origin.

$$l\alpha+m\beta+n=0$$

$\boxed{11-25-\{22\}}$



Method: Use homogenisation

OA&OB: $$ax^2+2hxy+by^2+(2gx+2fy)\left(\frac{lx+my}{-n}\right)+c\left(\frac{lx+my}{-n}\right)^2=0$$