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Differential Equations

$$f\left(x, y, \frac{dy}{dx}, \frac{d^2y}{dx^2}, \ldots, \frac{d^ny}{dx^n}\right) = 0 \longrightarrow \text{DE}$$

$$y^3 \frac{d^2y}{dx^2} - e^x y + x = 0 \longrightarrow \text{DE}$$

$$\frac{dy}{dx} = 1 \longrightarrow$$

$$y = x + C$$

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Order of DE

$$x\frac{d^3y}{dx^3} - \ln y\,\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} - e^x = 0$$

Order = 3.

Degree of DE

$$x\left(\frac{dy}{dx}\right)^7 - y^2\left(\frac{d^3y}{dx^3}\right)^2 + e^x + 5\left(\frac{d^2y}{dx^2}\right)^3 = \ln y.$$

Order = 3

Degree = 2.

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$$\left(\frac{dy}{dx}+7\right)^{\frac{1}{5}} = \left(\frac{d^2y}{dx^2}-7x\right)^{\frac{1}{7}}$$

$$\left(\frac{dy}{dx}+7\right)^{7} = \left(\frac{d^2y}{dx^2}-7x\right)^{5}$$

Order = 2

Degree = 5

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Solution of DE

Given: DE, $f\left(x, y, \frac{dy}{dx}, \frac{d^2y}{dx^2}, \ldots, \frac{d^ny}{dx^n}\right) = 0$

→ General Soln.

→ Particular Soln

$$y = \frac{x^3}{3} + 2x^2 - x + 5$$

Particular soln.

Integrate to get soln.

$$\frac{d^2y}{dx^2} = 2x + 4$$

$$\frac{dy}{dx} = x^2 + 4x + c_1$$

$$y = \frac{x^3}{3} + 2x^2 + c_1 x + c_2$$

General Soln.

General Soln → $\boxed{g(x, y, c_1, c_2, \ldots, c_n) = 0}$, where $c_i \to$ arbitrary constants

no. of arbitrary constants = Order of DE.

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Formation of DE

<u>Given</u> : a family of curves $f(x, y, c_1, c_2, \ldots, c_n) = 0$

c_i = arbitrary constants.

Differentiate and eliminate arbitrary constants.

Order of DE = no. of arbitrary constants.

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1: Find order and degree of the D E for curves $y^2 = 2c(x+\sqrt{c})$ — (1)

$c \rightarrow$ arbitrary constants.

$$\boxed{\begin{array}{l}\text{Order} = 1 \\ \text{Degree} = 3\end{array}}$$

$2yy' = 2c$ — (2)

$$\frac{y}{2y'} = x + \sqrt{c} = x + \sqrt{yy'}$$

$$\left(\frac{y}{2y'} - x\right)^2 = yy'$$

$$(y - 2xy')^2 = 4y(y')^3$$

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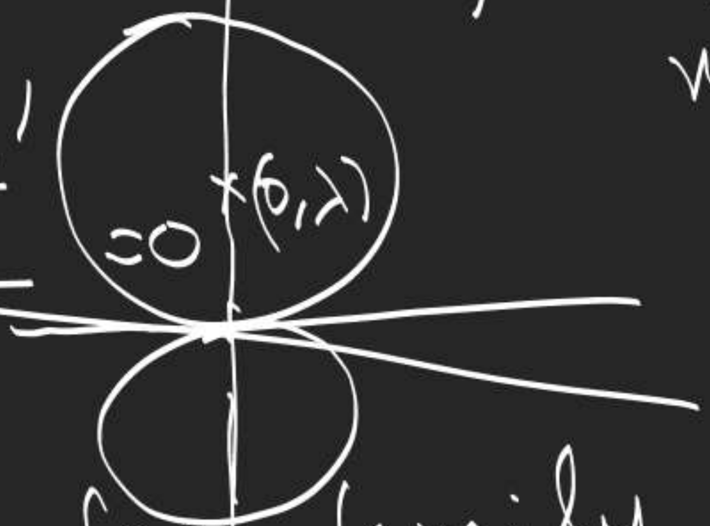
2: Obtain the DE for all circles touching x-axis at origin and having centre on y-axis



$$x^2+(y-\lambda)^2=|\lambda|^2$$

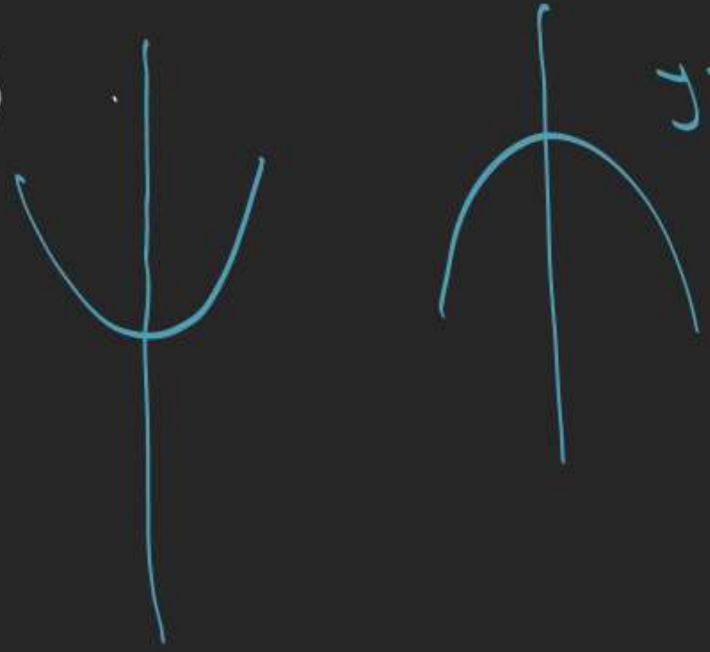
$$x^2+y^2-2\lambda y=0$$

$$\frac{x^2+y^2}{y}=2\lambda$$

$$\frac{y(2x+2yy')-(x^2+y^2)y'}{y^2}=0$$

3: Obtain the DE for family of parabolas having axis of symmetry parallel to y-axis

$$(-y^2+x^2)y'=2xy$$



$$y=ax^2+bx+c$$

$$y'=2ax+b$$

$$y''=2a$$

$$\boxed{y'''=0}$$

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4.

$$y = (c_1 + c_2)\cos(x + c_3) - c_4 e^x e^{c_5}$$

$c_1, c_2, c_3, c_4, c_5 \to$ arbitrary constant.

$$\boxed{y''' - y'' + y' - y = 0}$$

Order = ? 3

Degree = 1

Obtain DE.

$$y = \alpha\cos(x + c_3) + \beta e^x$$

$$y' = -\alpha\sin(x + c_3) + \beta e^x$$

$$y'' = -\alpha\cos(x + c_3) + \beta e^x$$

$$e^{-x}(y''' + y' - y'' - y) = 0 \Leftarrow e^{-x}(y'' + y) = 2\beta$$

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$$e^{\frac{dy}{dx}+7} = \ln\left(\frac{d^3y}{dx^3} - 7x + 2\right)$$

Order = 3

Degree = not defined

$$\ln \frac{d^2y}{dx^2} = y e^{x} \Rightarrow \frac{d^2y}{dx^2} = e^{ye^{x}}$$

Order = 2

Degree = 1

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Method.

Variable Separable

$$f(x)\,dx + g(y)\,dy = 0$$

$$\int f(x)\,dx + \int g(y)\,dy = C.$$

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DE of form

$$\boxed{\frac{dy}{dx} = f(ax+by+c)}$$

a, b, c → given constants.

$$\boxed{\text{Put } ax+by+c = t}$$

$$\frac{dt}{dx} = a + b\frac{dy}{dx}$$

$$\Rightarrow \frac{1}{b}\left(\frac{dt}{dx} - a\right) = f(t)$$

$$\frac{dt}{dx} = a + bf(t)$$

$$\int \frac{dt}{a+bf(t)} = \int dx$$

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DE of form

$$\boxed{\begin{array}{l} \dfrac{dy}{dx} = \dfrac{a_1x+b_1y+c_1}{a_2x+b_2y+c_2}, \quad b_1+a_2=0 \\ \text{Change into differential.} \end{array}}$$

$$a_2x\,dy + b_2y\,dy + c_2\,dy - a_1x\,dx - \underline{b_1y\,dx} - c_1\,dx = 0$$

$$\int a_2\underbrace{(x\,dy + y\,dx)} + b_2y\,dy + c_2\,dy - a_1x\,dx - c_1\,dx = 0$$

$$a_2(xy) + b_2\frac{y^2}{2} + c_2y - a_1\frac{x^2}{2} - c_1x = C$$

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Polar Substitution

$x = r\cos\theta$, $y = r\sin\theta$

$x\,dx + y\,dy = r\,dr$

$x\,dy - y\,dx = r^2 d\theta$

$x^2 + y^2 = r^2$

$\frac{y}{x} = \tan\theta$

$\frac{x\,dy - y\,dx}{x^2} = \sec^2\theta\, d\theta$

$x = r\sec\theta$, $y = r\tan\theta$

$x\,dx - y\,dy = r\,dr$

$x\,dy - y\,dx = r^2 \sec\theta\, d\theta$

$x\,dy + y\,dx = d(xy)$

$x\,dy - y\,dx$

$x^2 - y^2 = r^2$

$\frac{y}{x} = \sin\theta$

$\frac{x\,dy - y\,dx}{x^2} = \cos\theta\, d\theta$

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1. Find a particular solution of the DE

$(1+e^x)y\frac{dy}{dx} = e^x$, satisfying the initial condition

$y(0) = 1$

$$\int y\,dy = \int \frac{e^x\,dx}{1+e^x}$$

$$\frac{y^2}{2} = \ln(1+e^x) + C$$

$x=0$ $\quad \frac{1}{2} = \ln(2) + C$

$$\frac{y^2}{2} = \ln(1+e^x) + \frac{1}{2} - \ln 2$$

$$y^2 - 1 = 2\ln\left(\frac{1+e^x}{2}\right)$$

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2. $$\frac{dy}{dx} = \frac{2x+3y-1}{4x+6y-5}$$

$$2x+3y-1 = t$$

$$2+3\frac{dy}{dx} = \frac{dt}{dx}$$

$$\left(\frac{dt}{dx} - 2\right) = \frac{3t}{2t-3}$$

$$\frac{dt}{dx} = \frac{7t-6}{2t-3} \Rightarrow \int \frac{(2t-3)}{7t-6}\,dt = \int dx$$

3. $$\frac{dy}{dx} = \frac{x-2y+5}{2x+3y-1} \Rightarrow 2x\,dy + 3y\,dy - dy - x\,dx + 2y\,dx - 5dx = 0$$

$$\boxed{2xy + \frac{3y^2}{2} - y - \frac{x^2}{2} - 5x = C}$$

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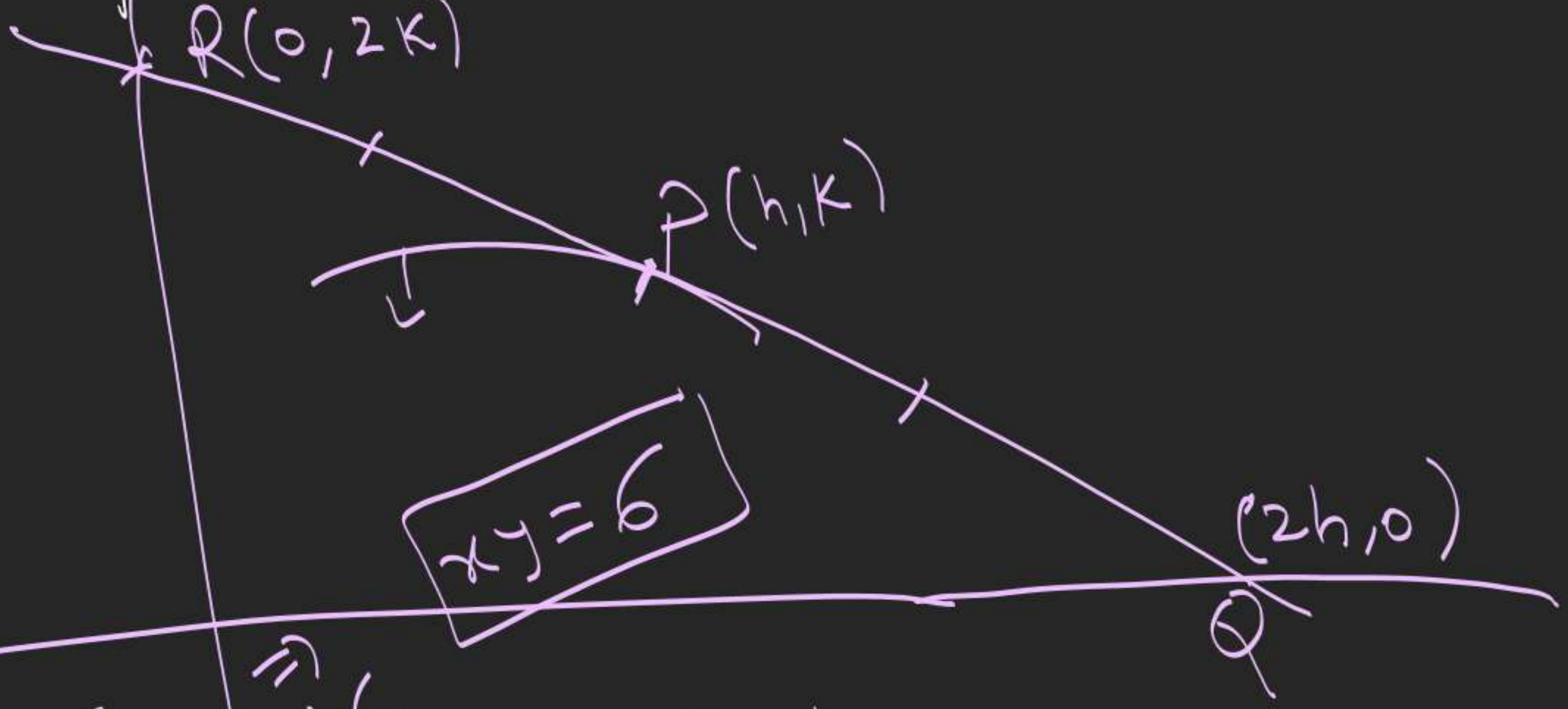
4. Find the curve for which the segment of the tangent contained between coordinate axes is bisected by the point.

Also given that curve passes thru $(2,3)$.



$$\left(\frac{dy}{dx}\right)_{(h,k)} = -\frac{k}{h}$$

$$\frac{dy}{dx} = -\frac{y}{x}$$

$$\int \frac{dy}{y} = \int -\frac{dx}{x}$$

$$\ln y = -\ln x + C \qquad (2,3)$$

$$\ln 3 = -\ln 2 + C \Rightarrow C = \ln 6$$

$$\boxed{xy = 6}$$

$$\frac{dy}{dx} = \frac{2x^2 - 3y}{x+y}$$

$$\left.\frac{dy}{dx}\right|_{(h,k)} = \frac{2h^2 - 3k}{h+k}$$

$$\frac{2x^2-3y}{x+y}$$