Nishant Jindal

Trigonometry

$\sin(A+B) \leftarrow B \leftrightarrow -B$

(1) $\sin(A+B) = \sin A\cos B + \cos A\sin B$

(2) $\sin(A-B) = \sin A\cos B - \cos A\sin B$

(3) $\cos(A+B) = \cos A\cos B - \sin A\sin B$

(4) $\cos(A-B) = \cos A\cos B + \sin A\sin B$

(5)* $\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A\tan B}$

(6) $\tan(A-B) = \dfrac{\tan A - \tan B}{1 + \tan A\tan B}$

(7) $\cot(A+B) = \dfrac{\cot A\cot B - 1}{\cot A + \cot B}$

(8) $\cot(A-B) = \dfrac{\cot A\cot B + 1}{\cot B - \cot A}$

$\sin(A-B) = \sin A\cos(-B) + \cos A\sin(-B)$

$= \sin A\cos B - \cos A\sin B$

(9)* $\sin(A+B)\cdot\sin(A-B) = \sin^2 A - \sin^2 B$

(10)* $\cos(A+B)\cdot\cos(A-B) = \cos^2 A - \sin^2 B$ ✓

$= \cos^2 B - \sin^2 A$ ✓

Trigonometry

Q $\boxed{\dfrac{\tan A}{\tan B} = \dfrac{x}{y}}$ then $\dfrac{\sin(A+B)}{\sin(A-B)} = ?$

Given

$\Rightarrow \dfrac{\tan A}{\tan B} = \dfrac{x}{y} \Rightarrow \dfrac{\frac{\sin A}{\cos A}}{\frac{\sin B}{\cos B}} = \dfrac{x}{y}$

$\Rightarrow \dfrac{\sin A \cos B}{\cos A \sin B} = \dfrac{x}{y}$ $\boxed{C \,\&\, D}$

$\dfrac{(\sin A \cos B + \cos A \sin B)}{\sin A \cos B - \cos A \sin B} = \dfrac{x+y}{x-y}$

$\boxed{\dfrac{\sin(A+B)}{\sin(A-B)} = \dfrac{x+y}{x-y}}$ Ans

Pichhle Qs Ko Result ki trh Use kia

Q $m \tan(\theta - 30^\circ) = n \cdot \tan(\theta + 120^\circ)$ find $\cos 2\theta = ?$

$\dfrac{\tan A}{\tan B} \leftarrow \dfrac{\tan(\theta - 30^\circ)}{\tan(\theta + 120^\circ)} = \dfrac{n}{m}$

$\dfrac{\sin(\theta - 30^\circ + \theta + 120^\circ)}{\sin(\cancel{\theta} - 30^\circ - \cancel{\theta} - 120^\circ)} = \dfrac{n+m}{n-m}$

$\dfrac{\sin(90^\circ + 2\theta)}{\sin(-150^\circ)} = \dfrac{n+m}{n-m}$

$\dfrac{+\cos(2\theta)}{-\frac{1}{2}} = \dfrac{n+m}{n-m}$

$\cos 2\theta = \dfrac{1}{2}\left(\dfrac{n+m}{m-n}\right)$

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Trigonometry

Q $\dfrac{\cos 8^\circ - \sin 8^\circ}{\boxed{\cos 8^\circ} + \sin 8^\circ} = ?$

$\div \cos 8^\circ$

1 Bnane Ki Socho

$$\frac{\frac{\cancel{\cos 8^\circ}}{\cancel{\cos 8^\circ}} - \frac{\sin 8^\circ}{\cos 8^\circ}}{\frac{\cancel{\cos 8^\circ}}{\cancel{\cos 8^\circ}} + \frac{\sin 8^\circ}{\cos 8^\circ}} = \frac{1 - \tan 8^\circ}{1 + \tan 8^\circ \times 1}$$

$$= \frac{\tan 45^\circ - \tan 8^\circ}{1 + \tan 8^\circ \times \tan 45^\circ} = \tan(45^\circ - 8^\circ)$$

$$= \tan(37^\circ)$$

$$\frac{\tan A - \tan B}{1 + \tan A \tan B} = \tan(A - B)$$

Q $\dfrac{\cos 9^\circ + \sin 9^\circ}{\boxed{\cos 9^\circ} - \sin 9^\circ}$

★ $\dfrac{\cos A + \sin B}{\cos A - \sin B}$ Jesi Shkl

$$\frac{\frac{\cancel{\cos 9^\circ}}{\cancel{\cos 9^\circ}} + \frac{\sin 9^\circ}{\cos 9^\circ}}{\frac{\cos 9^\circ}{\cos 9^\circ} - \frac{\sin 9^\circ}{\cos 9^\circ}} = \frac{1 + \tan 9^\circ}{1 - \tan 9^\circ} = \frac{\tan 45^\circ + \tan 9^\circ}{1 - \tan 9^\circ \times \tan 45^\circ}$$

$$= \tan(45^\circ + 9^\circ)$$

$$= \tan(54^\circ)$$

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Trigonometry

Direct Formula Bnale Kya?

$$\frac{\cos A+\sin A}{\cos A-\sin A}=\frac{\frac{\cos A}{\cos A}+\frac{\sin A}{\cos A}}{\frac{\cos A}{\cos A}-\frac{\sin A}{\cos A}}$$

$$=\frac{1+\tan A}{1-\tan A}$$

$$\frac{\tan\frac{\pi}{4}+\tan A}{1-\tan A\times\tan\frac{\pi}{4}}$$

$$=\tan\left(\frac{\pi}{4}+A\right)$$

$$\boxed{\begin{aligned}\tan\left(\frac{\pi}{4}+A\right)&=\frac{1+\tan A}{1-\tan A}\\ \tan\left(\frac{\pi}{4}-A\right)&=\frac{1-\tan A}{1+\tan A}\end{aligned}}$$

Q $\dfrac{\tan\left(\frac{\pi}{4}+x\right)}{\tan\left(\frac{\pi}{4}-x\right)}=\dfrac{(1+\tan x)^2}{(1-\tan x)^2}$ [T/F] ?

$$\Rightarrow\frac{\frac{1+\tan A}{1-\tan A}}{\frac{1-\tan A}{1+\tan A}}=\frac{1+\tan A}{1-\tan A}\times\frac{1+\tan A}{1-\tan A}=\frac{(1+\tan A)^2}{(1-\tan A)^2}$$

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Trigonometry = KhuB Practice

Q $\tan 70^\circ = 2\tan 50^\circ + \tan 20^\circ$ [T/F]

Practice

LHL $\tan 70^\circ = \tan(50^\circ + 20^\circ)$

$$\tan 70^\circ = \frac{\tan 50^\circ + \tan 20^\circ}{1 - \tan 50^\circ \cdot \tan 20^\circ}$$

CrossMultiply

$\Rightarrow \tan 70^\circ - \boxed{\tan 70^\circ} \tan 50^\circ \cdot \tan 20^\circ = \tan 50^\circ + \tan 20^\circ$

$\tan 70^\circ - \cot \cancel{20^\circ} \tan 50^\circ \cdot \cancel{\tan 20^\circ} = \tan 50^\circ + \tan 20^\circ$

$\tan 70^\circ = 2\tan 50^\circ + \tan 20^\circ$

Q $\tan 80^\circ = 2\tan 70^\circ + \tan 10^\circ$ (P.T.)

LHS

$\tan 80^\circ = \tan(70^\circ + 10^\circ)$

$$\tan 80^\circ = \frac{\tan 70 + \tan 10}{1 - \tan 70 \cdot \tan 10^\circ}$$

$\Rightarrow \tan 80^\circ - \boxed{\tan 80^\circ} \tan 70^\circ \cdot \tan 10^\circ$

$= \tan 70^\circ + \tan 10^\circ$

$\Rightarrow \tan 80^\circ - \cot \cancel{10^\circ} \tan 70^\circ \cdot \cancel{\tan 10^\circ}$

$= \tan 70^\circ + \tan 10^\circ$

$\Rightarrow \tan 80^\circ = 2\tan 70^\circ + \tan 10^\circ$

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Trigonometry

Q $\tan 13\theta - \tan 9\theta - \tan 4\theta = \tan 13\theta \cdot \tan 9\theta \cdot \tan 4\theta$ $\boxed{P.T}$

Start Urself.

$\tan 13\theta = \tan(9\theta + 4\theta)$

$$\tan 13\theta = \frac{\tan 9\theta + \tan 4\theta}{1 - \tan 9\theta \cdot \tan 4\theta}$$

$\tan 13\theta - \tan 13\theta \cdot \tan 9\theta \cdot \tan 4\theta = \tan 9\theta + \tan 4\theta$

$\tan 13\theta - \tan 9\theta - \tan 4\theta = \tan 13\theta \cdot \tan 9\theta \cdot \tan 4\theta$

$\boxed{H.P}$

P.T.

Q $\boxed{\tan 8\theta} - \tan 6\theta - \tan 2\theta = \tan 8\theta \cdot \tan 6\theta \cdot \tan 2\theta$

$\tan 8\theta = \tan(6\theta + 2\theta)$

$$\tan 8\theta = \frac{\tan 6\theta + \tan 2\theta}{1 - \tan 6\theta \cdot \tan 2\theta}$$

$\tan 8\theta - \tan 8\theta \cdot \tan 6\theta \cdot \tan 2\theta = \tan 6\theta + \tan 2\theta$

$\tan 8\theta - \tan 6\theta - \tan 2\theta = \tan 8\theta \cdot \tan 6\theta \cdot \tan 2\theta$

Q $\tan 7A \cdot \tan 4A \cdot \tan 3A = ?$

$= \tan 7A - \tan 4A - \tan 3A$

Q $\tan 3A \cdot \tan 2A \cdot \tan A = ?$

Ans $\tan 3A - \tan 2A - \tan A$

Nishant Jindal

Trigonometry

Q $\dfrac{\cos 17^\circ + \sin 17^\circ}{\cos 17^\circ - \sin 17^\circ} = ?$

$\tan\left(\frac{\pi}{4} + 17^\circ\right)$

$= \tan(45^\circ + 17^\circ)$

$= \tan 62^\circ$

Q $\dfrac{\cos 9^\circ + \sin 9^\circ}{\cos 9^\circ - \sin 9^\circ} = ?$

A: $\tan(45^\circ + 9^\circ)$

$= \tan 54^\circ$

Q If $\tan A = \frac{5}{6}$ & $\tan B = \frac{1}{11}$

$A + B = ?$

$\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A \cdot \tan B}$

$= \dfrac{\frac{5}{6} + \frac{1}{11}}{1 - \frac{5}{6} \times \frac{1}{11}}$

$\tan(A+B) = \dfrac{55+6}{66-5} = \dfrac{\cancel{61}}{\cancel{61}} = 1$

$\tan(A+B) = \tan\frac{\pi}{4} \Rightarrow A+B = \frac{\pi}{4}$

Q If $\tan A = \frac{1}{2}$ & $\tan B = \frac{1}{3}$

$A + B = ?$

Q. $\tan A = \dfrac{n}{n+1}$, $\tan B = \dfrac{1}{2n+1}$

$A + B = ?$

$\tan(A+B) = \dfrac{\frac{n}{n+1} + \frac{1}{2n+1}}{1 - \frac{n}{n+1} \times \frac{1}{2n+1}}$

$= \dfrac{2n^2+n+n+1}{2n^2+3n+1-n}$

$= \dfrac{\cancel{2n^2+2n+1}}{\cancel{2n^2+2n+1}} = 1$

$A+B = \frac{\pi}{4}$

Nishant Jindal

Trigonometry

Q $\sin\left(\frac{\pi}{4}+x\right)-\sin\left(\frac{\pi}{4}-x\right)=?$

$$\left(\sin\frac{\pi}{4}\cdot\cancel{\cos x}+\cos\frac{\pi}{4}\cdot\sin x\right)-\left(\cancel{\sin\frac{\pi}{4}\cos x}-\cos\frac{\pi}{4}\cdot\sin x\right)$$

$$2\times\cos\frac{\pi}{4}\times\sin x=2\times\frac{1}{\sqrt{2}}\sin x=\sqrt{2}\sin x$$

Q $\cos(30^\circ-A)-\cos(30^\circ+A)=?$

$$\left(\cos 30\cdot\cancel{\cos A}+\sin 30\cdot\sin A\right)-\left(\cancel{\cos 30\cos A}-\sin 30\cdot\sin A\right)$$

$$2\cdot\sin 30^\circ\cdot\sin A$$

$$\cancel{2}\times\frac{1}{\cancel{2}}\cdot\sin A=\sin A$$

Q $\frac{\cos(\alpha-\beta)}{\cos\alpha\cdot\sin\beta}=\tan\alpha\cdot\cot\beta$ [T/F]

LHS $\frac{\cos\alpha\cdot\cos\beta+\sin\alpha\cdot\sin\beta}{\cos\alpha\cdot\sin\beta}$

$$\frac{\cancel{\cos\alpha}\cdot\cos\beta}{\cancel{\cos\alpha}\cdot\sin\beta}+\frac{\sin\alpha\cdot\cancel{\sin\beta}}{\cos\alpha\cdot\cancel{\sin\beta}}$$

$$\cot\beta+\tan\alpha \quad [\text{False}]$$

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Trigonometry

$$\sin 120^\circ = \sin\left(\frac{\pi}{2}+30^\circ\right) \quad \boxed{2}$$

$$= +\cos 30^\circ = \frac{\sqrt{3}}{2}$$

$$\cos 120^\circ = \cos\left(\frac{\pi}{2}+30^\circ\right) \to \boxed{2}$$

$$= -\sin 30^\circ = -1/2$$

$$\sin 2\frac{\pi}{3} = \sin 120^\circ$$

$$\sin 4\frac{\pi}{3} = \sin 240^\circ = \sin(180^\circ+60^\circ)$$

$$= \sin(\pi+60^\circ)$$

$$= -\sin 60^\circ = -\frac{\sqrt{3}}{2}$$

$$\cos 240^\circ = \cos(\pi+60) = -\cos 60^\circ = -\frac{1}{2}$$

Q $\sin\theta + \sin\left(\frac{2\pi}{3}+\theta\right) + \sin\left(\frac{4\pi}{3}+\theta\right) = ?$

$$\sin\theta + (\sin 120^\circ \cdot \cos\theta + \cos 120 \cdot \sin\theta) + (\sin 240 \cdot \cos\theta + \cos 240 \cdot \sin\theta)$$

$$\sin\theta + \left(\frac{\sqrt{3}}{2}\cancel{\cos\theta} - \frac{1}{2}\sin\theta\right) + -\frac{\sqrt{3}}{2}\cancel{\cos\theta} - \frac{1}{2}\sin\theta$$

$$\cancel{\sin\theta} - \cancel{\sin}\theta = 0$$

Nishant Jindal

Trigonometry

Q $\dfrac{1}{\tan 3\theta - \tan\theta} - \dfrac{\tan 3\theta \cdot \tan\theta}{\tan\theta - \tan 3\theta} = ?$

$\dfrac{1}{\tan 3\theta - \tan\theta} + \dfrac{\tan 3\theta \cdot \tan\theta}{\tan 3\theta - \tan\theta}$

$\dfrac{1 + \tan 3\theta \cdot \tan\theta}{\tan 3\theta - \tan\theta} = \dfrac{1}{\left(\dfrac{\tan 3\theta - \tan\theta}{1 + \tan 3\theta \cdot \tan\theta}\right)}$

$= \dfrac{1}{\tan(3\theta - \theta)} = \cot 2\theta$

Q If $\sin(x+y) = \frac{5}{13}$ & $\cos(x-y) = \frac{4}{5}$

$(x+y)$ & $(x-y)$ are Acute Angle

then $\tan 2y = ?$

$\tan 2y = \tan\{(x+y) - (x-y)\}$ → $\tan(A-B)$ Ki trh treat

$= \dfrac{\tan(x+y) - \tan(x-y)}{1 + \tan(x+y)\cdot\tan(x-y)}$

$= \dfrac{\frac{5}{12} - \frac{3}{4}}{1 + \frac{5}{12} \times \frac{3}{4}} = \dfrac{20-36}{48+15} = \dfrac{-16}{63}$

$\cos(x-y) = \frac{4}{5}$

$\sin(x-y) = \frac{3}{5}$

$\tan(x-y) = \frac{3}{4}$

$\sin(x+y) = \frac{5}{13}$

$\cos(x+y) = \frac{12}{13} \Rightarrow \tan(x+y) = \frac{5}{12}$

Nishant Jindal

Trigonometry

Q $\sin^2\left(\frac{\pi}{8}+\frac{\theta}{2}\right)-\sin^2\left(\frac{\pi}{8}-\frac{\theta}{2}\right)=?$

$\sin^2 A-\sin^2 B=\sin(A+B)\cdot\sin(A-B)$
feel

$\sin\left\{\left(\frac{\pi}{8}+\frac{\theta}{2}\right)+\left(\frac{\pi}{8}-\frac{\theta}{2}\right)\right\}\cdot\sin\left\{\left(\frac{\pi}{8}+\frac{\theta}{2}\right)-\left(\frac{\pi}{8}-\frac{\theta}{2}\right)\right\}$

$\sin\left(\frac{\pi}{4}\right)\cdot\sin\theta=\frac{\sin\theta}{\sqrt{2}}$

Q $\sin^2 75^\circ-\sin^2 15^\circ\rightarrow\sin^2 A-\sin^2 B$ feel

$\sin(75^\circ+15^\circ)\cdot\sin(75^\circ-15^\circ)$

$\sin 90^\circ\cdot\sin 60^\circ=1\times\frac{\sqrt{3}}{2}=\frac{\sqrt{3}}{2}$

$\boxed{\text{Note}}$ HB sir

$\tan\frac{3\pi}{4}=\tan 135^\circ=\tan\left(\frac{\pi}{2}+45^\circ\right)$ $\boxed{2}$

Q $\tan\left(\frac{3\pi}{4}+\theta\right)\cdot\tan\left(\frac{\pi}{4}+\theta\right)=?$ | $=-\cot 45^\circ=-1$

$\frac{\tan\frac{3\pi}{4}+\tan\theta}{1-\tan\frac{3\pi}{4}\cdot\tan\theta}\times\frac{\tan\frac{\pi}{4}+\tan\theta}{1-\tan\frac{\pi}{4}\cdot\tan\theta}$

$\frac{-1+\tan\theta}{\cancel{1-(-1)\tan\theta}}\times\frac{\cancel{1+\tan\theta}}{1-\tan\theta}$

$-\frac{\cancel{(1-\tan\theta)}}{\cancel{(1-\tan\theta)}}=-1$

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Trigonometry

Q. $\cos^2(A-B)+\cos^2B-2\cos(A-B)\cdot\cos A\cos B=?$

$\cos(A+B)\cdot\cos(A-B)=\cos^2B-\sin^2A$

$\cos^2B+\cos(A-B)\{\cos(A-B)-2\cos A\cos B\}$

$+\cos(A-B)\{\cos A\cos B+\sin A\sin B-2\cos A\cos B\}$

$\cos^2B\quad+\cos(A-B)\cdot\{\sin A\cdot\sin B-\cos A\cos B\}$ ← Ulta Likha.

$\cos^2B-\cos(A-B)\{\cos A\cos B-\sin A\cdot\sin B\}$

$\cos^2B-\cos(A-B)\cos(A+B)$

$\cancel{\cos^2B}-(\cancel{\cos^2B}-\sin^2A)=\sin^2$