Nishant Jindal

2.

$$\frac{\sin^5\theta - \sin^3\theta + \cos^5\theta - \cos^3\theta}{} = \frac{-\sin^3\theta\cos^2\theta - \cos^3\theta\sin^2\theta}{\sin^5\theta}$$

$$\frac{-\sin^3\theta\cos^2\theta - \cos^3\theta\sin^2\theta}{-\sin^5\theta\cos^2\theta - \cos^5\theta\sin^2\theta}$$

$$= \frac{\sin\theta + \cos\theta}{\sin^3\theta + \cos^3\theta} = \frac{T_1}{T_3}$$

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5.
$$\frac{\tan\frac{x+y}{2}}{\tan\frac{x-y}{2}} = \frac{5}{3} \Rightarrow \frac{\tan\frac{x+y}{2} - \tan\frac{x-y}{2}}{\tan\frac{x+y}{2} + \tan\frac{x-y}{2}} = \frac{5-3}{5+3}$$

$$\frac{\sin y}{\sin x} = \frac{1}{4}$$

Comp. & Dividendo

Ratio & Proportion

$$\boxed{\frac{a}{b} = \frac{c}{d} = \frac{a+c}{b+d} = \frac{a-c}{b-d}}$$

$$\frac{a}{b} = \frac{c}{d} = K$$

$$\frac{a+c}{b+d} = \frac{bK+dK}{b+d} = K$$

$$\frac{a}{b} = \frac{c}{d}$$

$$\frac{a-b}{a+b} = \frac{c-d}{c+d}$$

$$\frac{a}{b} = \frac{c}{d} = K, \quad \frac{a-b}{a+b} = \frac{bK-b}{bK+b} = \frac{K-1}{K+1}$$

$$\frac{c-d}{c+d} = \frac{dK-d}{dK+d} = \frac{K-1}{K+1}$$

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3.

$$\frac{9\left(2\tan\frac{x}{2}\right)}{1+\tan^2\frac{x}{2}} + \frac{2\left(1-\tan^2\frac{x}{2}\right)}{\left(1+\tan^2\frac{x}{2}\right)} = 6$$

$$9t + 1 - t^2 = 3 + 3t^2$$

$$4t^2 - 9t + 2 = 0$$

$$-8t - t$$

$$(4t-1)(t-2) = 0$$

$$\tan\frac{x}{2} = \frac{1}{4}, 2.$$

$$\cot x = \frac{1-\frac{1}{16}}{2\times\frac{1}{4}}$$

$$\text{or } \frac{1-4}{2\times 2}$$

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$$-4 \mp 4$$

$$16 = 16$$

$$\cot\beta = \frac{7}{6} \text{ or } -\frac{7}{6}$$

$$\cot\alpha = \frac{9}{2}$$

$$x + \alpha = \beta$$

$$\cot x = \cot(\beta - \alpha) = \frac{1 + \frac{9}{2}\left(\frac{7}{6}\right)}{\frac{9}{2} - \frac{7}{6}} \quad \text{or} \quad \frac{1 + \frac{9}{2}\left(-\frac{7}{6}\right)}{\frac{9}{2} + \frac{7}{6}}$$

$$9\sin x + 2\cos x = 6$$

$$\sqrt{85}\left(\underbrace{\frac{9}{\sqrt{85}}}_{\cos\alpha}\sin x + \underbrace{\frac{2}{\sqrt{85}}}_{\sin\alpha}\cos x\right) = 6$$

$$\sin(x+\alpha) = \frac{6}{\sqrt{85}} = \sin\beta$$

6.

$1+\cos\frac{\pi}{6}$

$$\left(2\cos^2\frac{\pi}{18}\right)\left(2\cos^2\frac{3\pi}{18}\right)\left(2\cos^2\frac{5\pi}{18}\right)\left(2\cos^2\frac{7\pi}{18}\right)$$

$$16\left(\cos 10^\circ \cos 50^\circ \cos 70^\circ\right)^2 \quad \frac{3}{4}$$

$x\in(0,\pi)$

$$9\underline{x\sin x}+\frac{4}{x\sin x}=9t+\frac{4}{t}$$

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11. $(2\sin x+1)(2\cos y+1)=0$

$ab=0$

$\sin x=-\frac{1}{2}$ or $\cos y=-\frac{1}{2}$

$x, y \in [0, 2\pi]$

$(x+y)_{max} = \boxed{2\pi-\frac{\pi}{6}+2\pi}$

$\alpha, \beta, \alpha\beta$

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13. $\left(\tan^2\frac{\pi}{16}+\cot^2\frac{\pi}{16}\right)+\left(\tan^2\frac{3\pi}{16}+\cot^2\frac{3\pi}{16}\right)$

$=\left(\tan\frac{\pi}{16}-\cot\frac{\pi}{16}\right)^2+2+\left(\tan\frac{3\pi}{16}-\cot\frac{3\pi}{16}\right)^2+2$

$=\left(-2\cot\frac{\pi}{8}\right)^2+\left(-2\cot\frac{3\pi}{8}\right)^2+4$

$\boxed{\tan\theta-\cot\theta}=\frac{\tan^2\theta-1}{\tan\theta}=-2\left(\frac{1-\tan^2\theta}{2\tan\theta}\right)=\boxed{-2\cot 2\theta}$

$=4\left(\cot^2\frac{\pi}{8}+\tan^2\frac{\pi}{8}\right)+4$

$=4\left(\left(\tan\frac{\pi}{8}-\cot\frac{\pi}{8}\right)^2+2\right)+4$

$=12+4\left(-2\cot\frac{\pi}{4}\right)^2$

$=\boxed{28}$

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$$\left(\tan^2\frac{\pi}{16}+\cot^2\frac{\pi}{16}\right)+\left(\tan^2\frac{3\pi}{16}+\cot^2\frac{3\pi}{16}\right)$$

$$\frac{1-\frac{1}{2}\sin^2\frac{\pi}{8}}{\frac{1}{4}\sin^2\frac{\pi}{8}}+\frac{1-\frac{1}{2}\cos^2\frac{\pi}{8}}{\frac{1}{4}\cos^2\frac{\pi}{8}}-4$$

$$\left(\tan\frac{\pi}{16}+\cot\frac{\pi}{16}\right)^2+\left(\tan\frac{3\pi}{16}+\cot\frac{3\pi}{16}\right)^2-4$$

$$=\left(2\left(\frac{1+\tan^2\frac{\pi}{16}}{2\tan\frac{\pi}{16}}\right)\right)^2+$$

$$=\frac{4}{\sin^2\frac{\pi}{8}}+\frac{4}{\sin^2\frac{3\pi}{8}}-4=\frac{4}{\sin^2\frac{\pi}{8}\cos^2\frac{\pi}{8}}$$

$$\cos^2\frac{\pi}{8}$$

$$=\frac{16}{\left(2\sin\frac{\pi}{8}\cos\frac{\pi}{8}\right)^2}-4$$

$$\frac{16}{\sin^2\frac{\pi}{4}}-4$$

$$=16\times 2-4$$

$$=28$$

$$-4$$

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8. $\tan\alpha + 2\tan 2\alpha + 4\tan 4\alpha + 8\tan 8\alpha + \cdots + 2^{n-1}\tan 2^{n-1}\alpha$

$$\cot\alpha + (-\cot\alpha + \tan\alpha) + 2\tan 2\alpha + 2^2\tan 2^2\alpha + 2^3\tan 2^3\alpha + \cdots + 2^{n-1}\tan 2^{n-1}\alpha$$

$$\cot\alpha + (-2\cot 2\alpha + 2\tan 2\alpha) + 2^2\tan 2^2\alpha + \cdots + 2^{n-1}\tan 2^{n-1}\alpha$$

$$\cot\alpha - 2^2\cot 2^2\alpha + 2^2\tan 2^2\alpha + 2^3\tan 2^3\alpha + \cdots + 2^{n-1}\tan 2^{n-1}\alpha$$

$$\boxed{\tan\theta - \cot\theta = -2\cot 2\theta}$$

$$\cot\alpha - 2^3\cot 2^3\alpha + 2^3\tan 2^3\alpha + \cdots + 2^{n-1}\tan 2^{n-1}\alpha$$

$$= \cot\alpha - 2^n\cot 2^n\alpha$$

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$$\cos\theta \cos 2\theta \cos 2^2\theta \cdots \cos 2^{n-1}\theta$$

$$= \frac{\sin 2^n\theta}{2^n \sin\theta}$$

Differentiation

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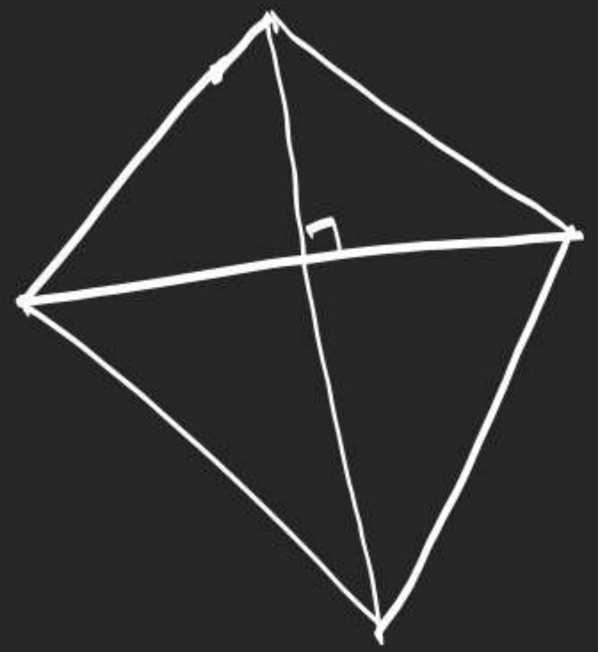
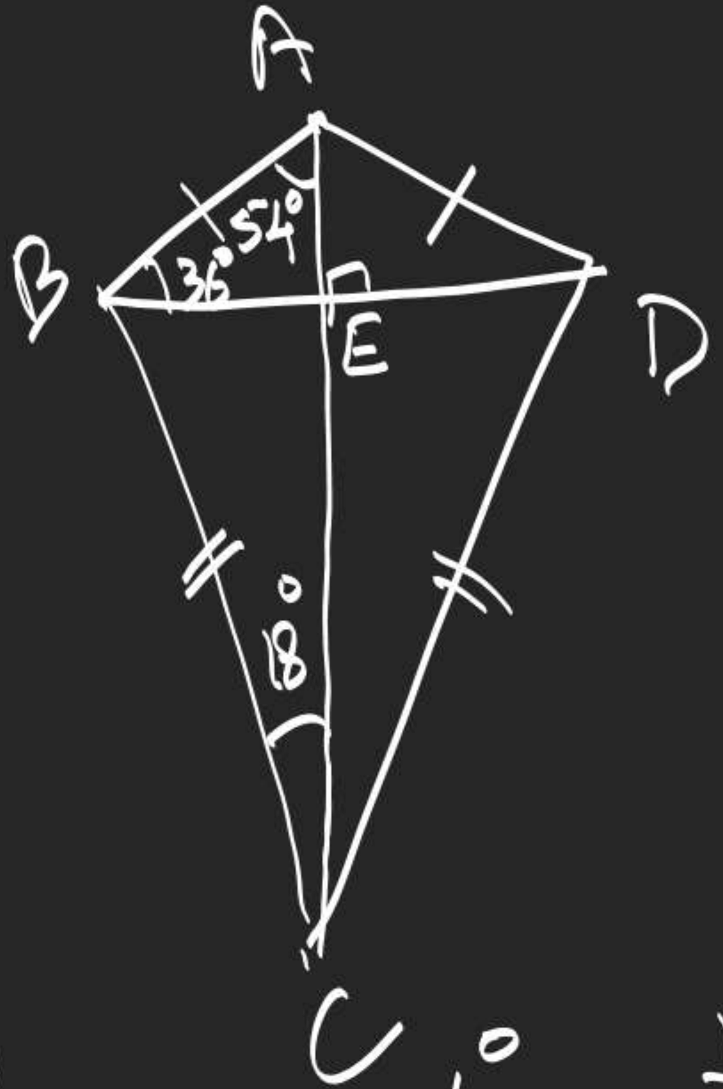
20.





$$\frac{ABD}{CBD} = \frac{\frac{1}{2}(AE)\,2(BE)}{\frac{1}{2}(CE)(2BE)}$$

$$\frac{\frac{AE}{BE}}{\frac{CE}{BE}} = \left(\frac{AE}{BE}\right)\left(\frac{BE}{CE}\right) = \frac{\tan 36^\circ}{\cot 18^\circ} = \frac{\tan 36^\circ}{\tan 72^\circ} = \frac{\cancel{\tan 36}\,(1-\tan^2 36^\circ)}{2\cancel{\tan 36}^\circ}$$

Nishant Jindal

1. Find the range of function

$$f(x) = \frac{\sin 3x}{\sin x}$$

$$\boxed{R_f = [-1, 3)} \Rightarrow \text{Ans.}$$

$\sin x \neq 0$.

$$f(x) = \frac{3\sin x - 4\sin^3 x}{\sin x} = 3 - 4\sin^2 x$$

$\sin x \in [-1, 0) \cup (0, 1]$

$0 < \sin^2 x \leq 1$

$\sin^2 x \in (0, 1]$

$-4 \leq -4\sin^2 x < 0$

$-4\sin^2 x \in [-4, 0)$

$-1 \leq 3 - 4\sin^2 x < 3$

$\Rightarrow 3 - 4\sin^2 x \in [-1, 3)$

Nishant Jindal

2: Find the minimum value of $f(x)$, $x \in \left(0, \frac{\pi}{2}\right)$

$$f(x) = \frac{1+\cos 2x + 8\sin^2 x}{\sin 2x},$$

$$= \frac{2\cos^2 x + 8\sin^2 x}{2\sin x\cos x}$$

$$= \cot x + 4\tan x$$

$$= \left(2\sqrt{\tan x} - \sqrt{\cot x}\right)^2 + 4 \geq 4$$

$$f(x)_{min} = 4$$

$f(x) = 4$ if

$$2\sqrt{\tan x} = \sqrt{\cot x}$$

$$\boxed{\tan x = \frac{1}{2}}$$

Nishant Jindal

3: If $x^2+y^2=4$ and $a^2+b^2=8$, find the maximum and minimum values of $ax+by$

min $= -4\sqrt{2}$, max $= 4\sqrt{2}$.

$$x^2+y^2=4$$

$$x = 2\cos\theta$$
$$y = 2\sin\theta.$$

$$a = 2\sqrt{2}\cos\phi$$
$$b = 2\sqrt{2}\sin\phi$$

$$ax+by = 4\sqrt{2}\cos\theta\cos\phi + 4\sqrt{2}\sin\theta\sin\phi$$
$$= 4\sqrt{2}\cos(\theta-\phi) \in [-4\sqrt{2}, 4\sqrt{2}]$$

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$ax + by =$ $\boxed{\{x - 4\}}$

$(ax+by)^2 + (ay-bx)^2 = a^2(x^2+y^2) + b^2(x^2+y^2)$

$= (a^2+b^2)(x^2+y^2)$

$a^2+b^2 = 8$

$x^2+y^2 = 4$

$\boxed{(ax+by)^2 \le 32 \Rightarrow ax+by \in [-4\sqrt{2}, 4\sqrt{2}]}$

$8 \times 4 = 32$

$x^2 \le 4$

$x \in [-2, 2]$

$\underbrace{(ax+by)^2}_{\le 32} + (ay-bx)^2 = 32$

$(ax+by)^2 = 32 - \underbrace{(ay-bx)^2}_{\ge 0} \le 32$