Nishant Jindal

$$40 = \int_1^3 x^3 \underset{\text{II}}{F''(x)}\,dx = x^3 F'(x)\Big|_1^3 - 3\int_1^3 x^2 F'(x)\,dx$$

$$-4 \qquad 0$$

$$40 = 9\left(f'(3) - \cancel{F(3)}\right) - \left(f'(1) - \cancel{F(1)}\right) + 36$$

$$\frac{F'(x)}{G'(x)} = \lim_{x\to 1} \frac{f(x)}{x\,|f(f(x))|} \Rightarrow \frac{1}{2\,|f(\underbrace{f(1)}_{1/2})|} = \frac{1}{14}$$

$$f(x) = xF(x)$$

$$f'(x) = \underline{xF'(x)} + F(x)$$

$$3F'(3) = f'(3) - F(3)$$

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$$f(x) = \int_0^x \frac{t^2}{1+t^4}\,dt - 2x + 1$$

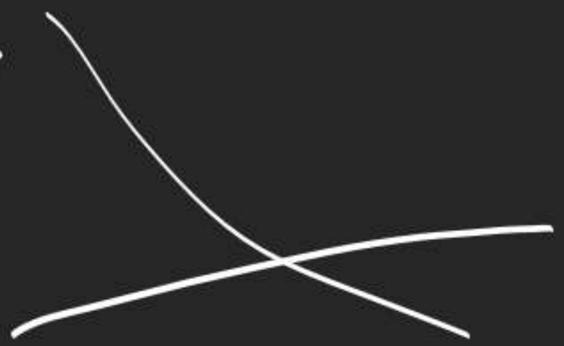
$$f'(x) = \frac{x^2}{1+x^4} - 2 \le \frac{1}{2} - 2$$

$$f \downarrow$$

$$f(0) = 1 \checkmark$$

$$f(1) = \int_0^1 \frac{t^2\,dt}{1+t^4} - 1 \le \frac{1}{2} - 1 = -\frac{1}{2}$$

$$\int_0^1 \frac{1}{\frac{1}{t^2}+t^2}\,dt \le \int_0^1 \frac{1}{2}\,dt = \frac{1}{2}$$



0

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$$g(x) = x - \int_0^{\frac{\pi}{2}-x} f(t)\cos t\, dt$$

$$g(0) = -\underbrace{\int_0^{\pi/2} \overbrace{f(t)\cos t}^{>0}\, dt}_{>0} < 0$$

$0 < \quad < 1$

$$g(1) = 1 - \underbrace{\int_0^{\frac{\pi}{2}-1} \underbrace{f(t)\cos t}_{<1}\, dt}_{<\left(\frac{\pi}{2}-1\right)} > 0$$

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$(0,1)$

$$\cos x = 1 - \frac{x^2}{2!} + \underbrace{\frac{x^4}{4!} - \frac{x^6}{6!} + \cdots}_{} > 1 - \frac{x^2}{2!}$$

$$I > \int_0^1 x\left(1 - \frac{x^2}{2!}\right)dx = \frac{3}{8}$$

$$g(x) = \int_0^x f(t)\,dt - \sin 3x$$

$$\sin x > \frac{x}{1!} - \frac{x^3}{3!}$$

$g(0) = 0$

$g\left(\frac{\pi}{3}\right) = 0$

Note → Let $f(x)$ be continuous in $[a,b]$

and $\int_a^b f(x)\,dx = 0$, then

$g(x) = \int_a^x f(t)\,dt$ $g(a)=0$, $g(b)=0$

$\exists\, c \in (a,b)$ s.t. $f(c) = 0$ $\exists\, c \in (a,b)$, $g'(c) = 0$