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AMPERE'S LAW

Line integral of $\vec{B}\cdot\vec{dl}$ around a Closed loop (Amperian loop) is equal to μ_0 times the Current enclosed within the Amperian loop.

$$\boxed{\oint \vec{B}\cdot\vec{dl} = \mu_0 i_{enc}}$$

$\Rightarrow$ Prefer for Calculating field where $(\vec{B}\cdot\vec{dl} = 0)$ or $(\vec{B}\cdot\vec{dl} = B\,dl)$

$$\frac{\mu_0}{4\pi} = 10^{-7} \Rightarrow \underline{\mu_0 = 4\pi\times10^{-7}}$$

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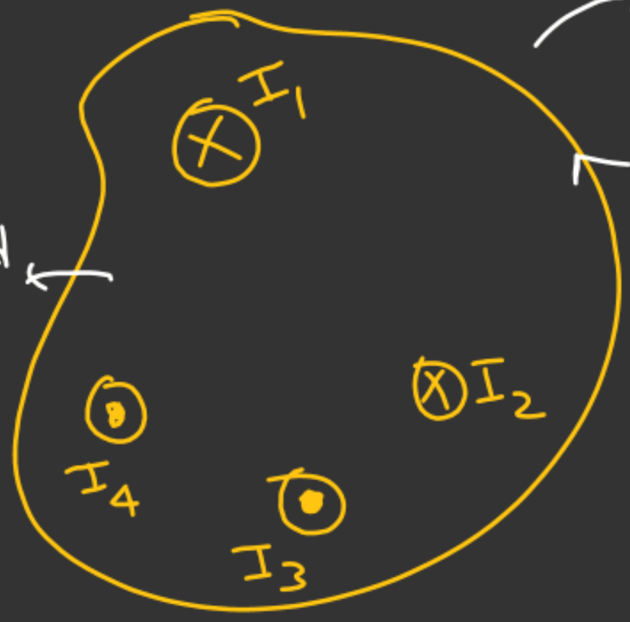
** How to decide +ve Side of Amperian loop :-

Amperian loop

+ve Side → outward



I (assumed)

$$I_{enc} = (I_4 + I_3 - I_1 - I_2)$$

For I_{enc}

↳ Enclosed Current : along the +ve Side of Amperian loop taken as +ve & opposite to +ve Side taken as -ve

↳ Imagine any arbitary direction of Current in Amperian loop i.e either Clockwise or anticlockwise.

If assumed Current in anticlockwise the outward Normal is the +ve Side of Amperian loop.

&

If current in clockwise inward Normal is the +ve Side of Amperian loop.

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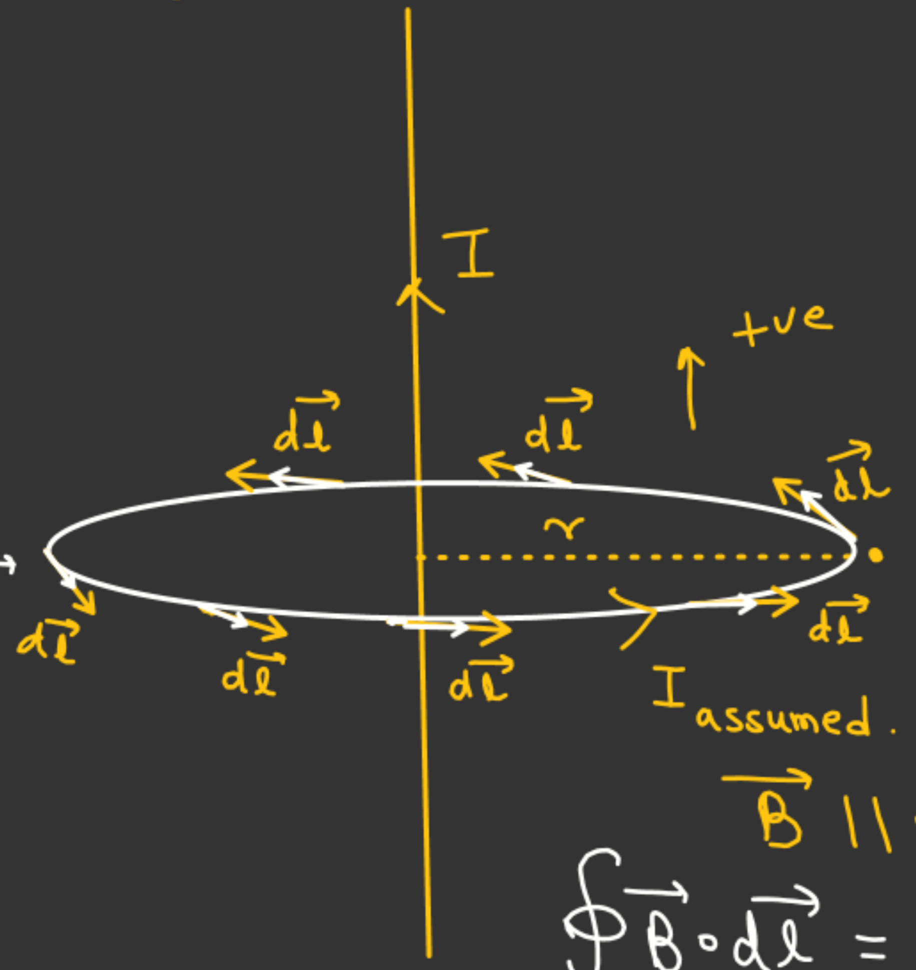
$\oint \vec{B} \cdot \vec{dl}$ (Always taken along the direction of current flow)

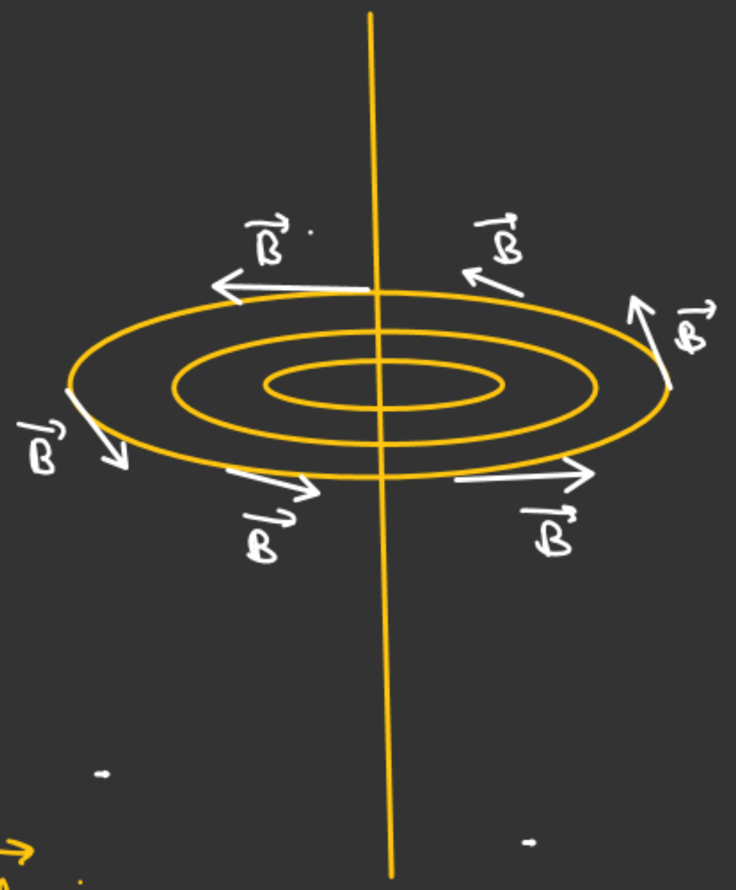
↳ Line integral of Amperian loop.

(*) B due to infinitly long wire

(*) In general Amperian loop in the shape of Magnetic field lines.

(Circular Amperian loop)





$\vec{B} \parallel \vec{dl}$

$$\oint \vec{B} \cdot \vec{dl} = \mu_0 i_{enc} \quad (i_{enc} = +I)$$

$$\oint B\, dl = \mu_0 i_{enc}$$

$$B \oint dl = \mu_0 I$$

$$B \cdot 2\pi r = \mu_0 I$$

$$\boxed{B = \frac{\mu_0 I}{2\pi r}}$$

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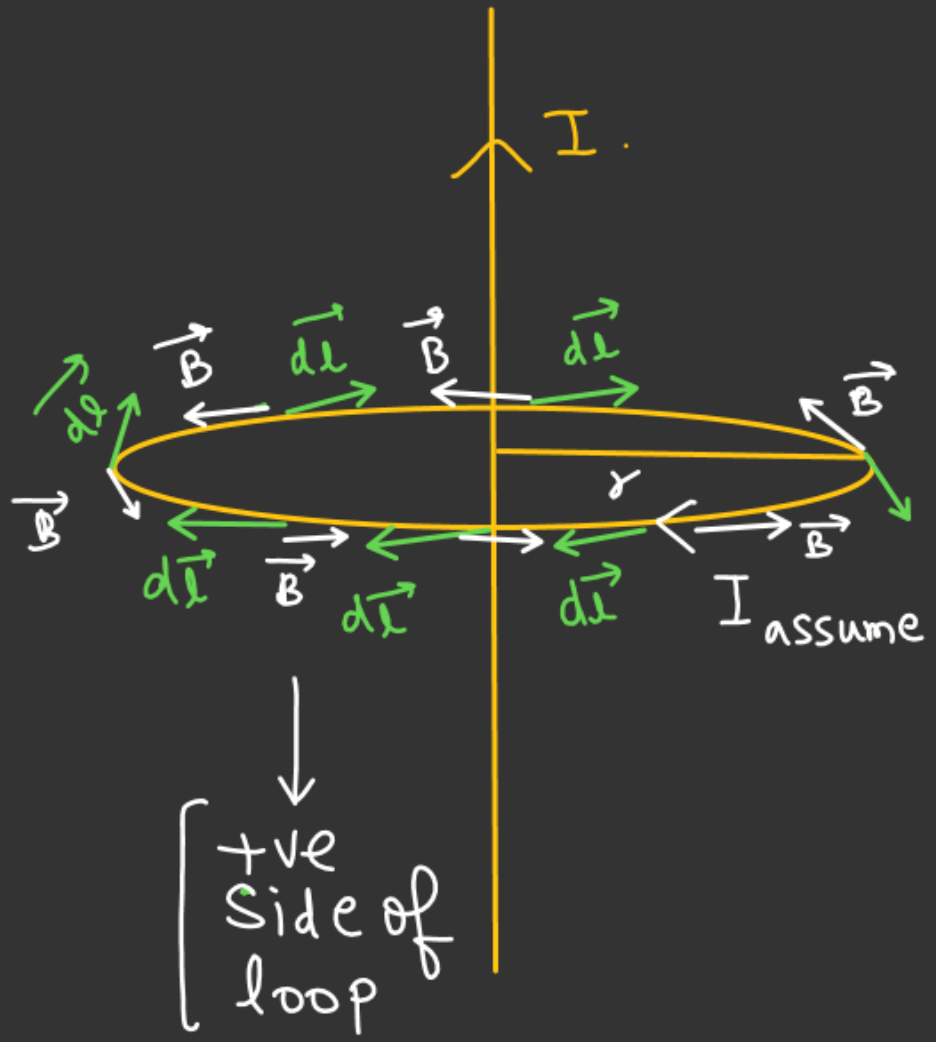

$i_{enc} = (-I)$

$\vec{B}$ & $\vec{dl}$ antiparallel.

$\theta = \pi$.

$$\oint \vec{B} \cdot \vec{dl} = \mu_0 i_{enc}$$

$$-B \oint \vec{dl} = \mu_0 (-I)$$

$$B \cdot 2\pi r = \mu_0 I.$$

$$\boxed{B = \frac{\mu_0 I}{2\pi r}}$$

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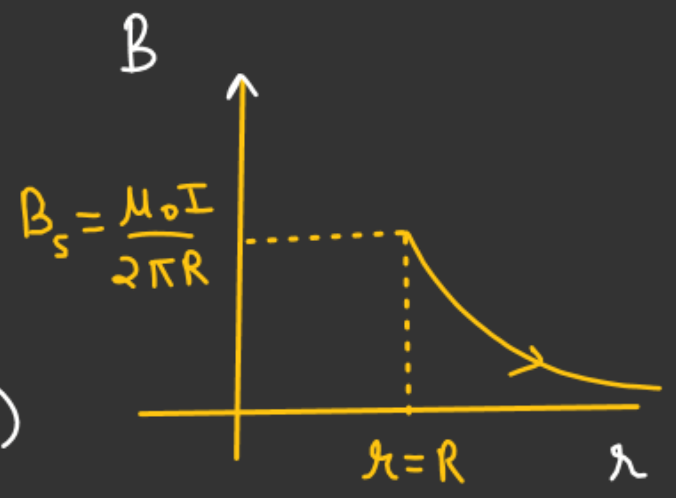
Magnetic field due to a (Very long) Cylindrical hollow Conductor :→

For very long Cylinder magnetic field lines are Concentric Circles.

Inside $(r < R)$

$I_{enc} = 0$.

$B = 0$.

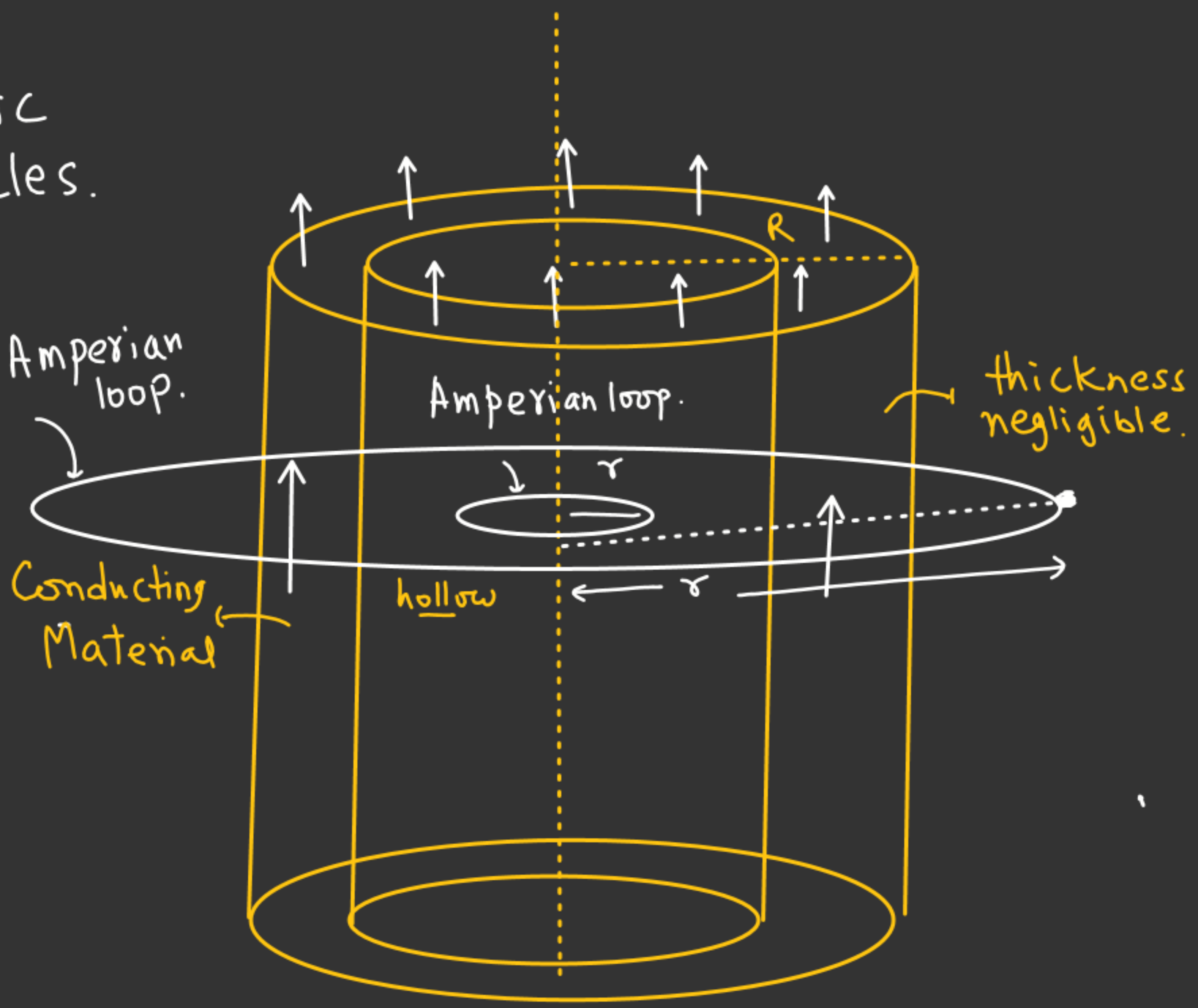


Outside $(r > R)$

$$B \oint dl = \mu_0 I$$

$$B \cdot 2\pi r = \mu_0 I$$

$$\boxed{B = \frac{\mu_0 I}{2\pi r}}$$



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(*) Magnetic field due to a Solid Conducting very long Cylinder: →

r < R (Inside)

$J = \left(\frac{I}{\pi R^2}\right)$ → Total Current.

⇓

Current density.

$$B \oint dl = \mu_0 i_{enc}$$

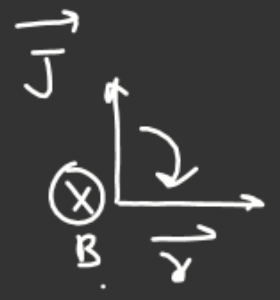
$$B(2\pi r) = \mu_0 \left(\frac{I}{\pi R^2}\right)(\pi r^2)$$

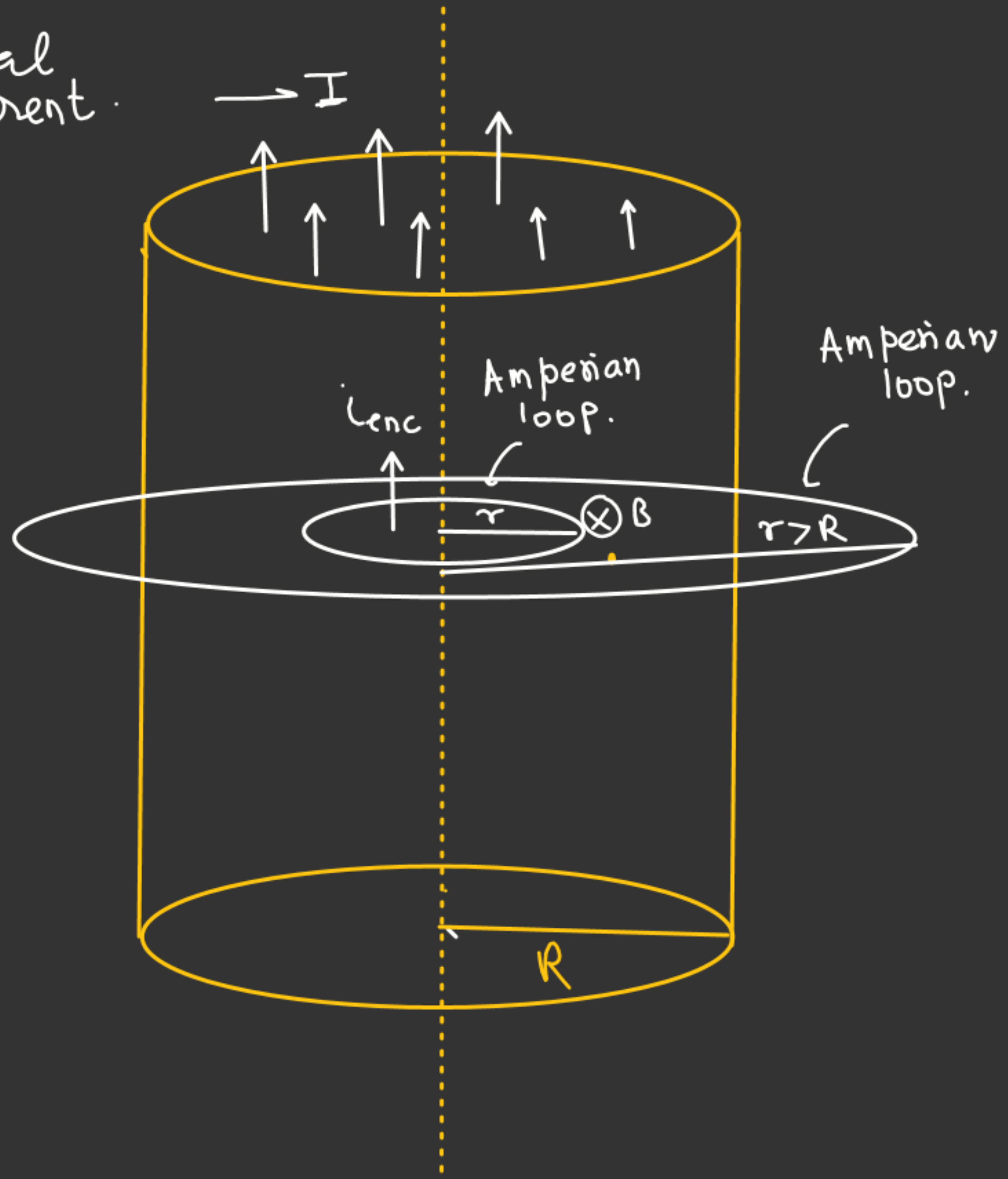
$$\boxed{B = \left(\frac{\mu_0 I}{2\pi R^2}\right) r}$$

or

$$B = \frac{\mu_0}{2\pi R^2} \times (J \cdot \pi R^2) r$$

$$\boxed{B = \frac{\mu_0 J r}{2}} \Rightarrow \boxed{\vec{B} = \frac{\mu_0}{2}(\vec{J} \times \vec{r})}$$





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$\underline{r > R}$ (outside)

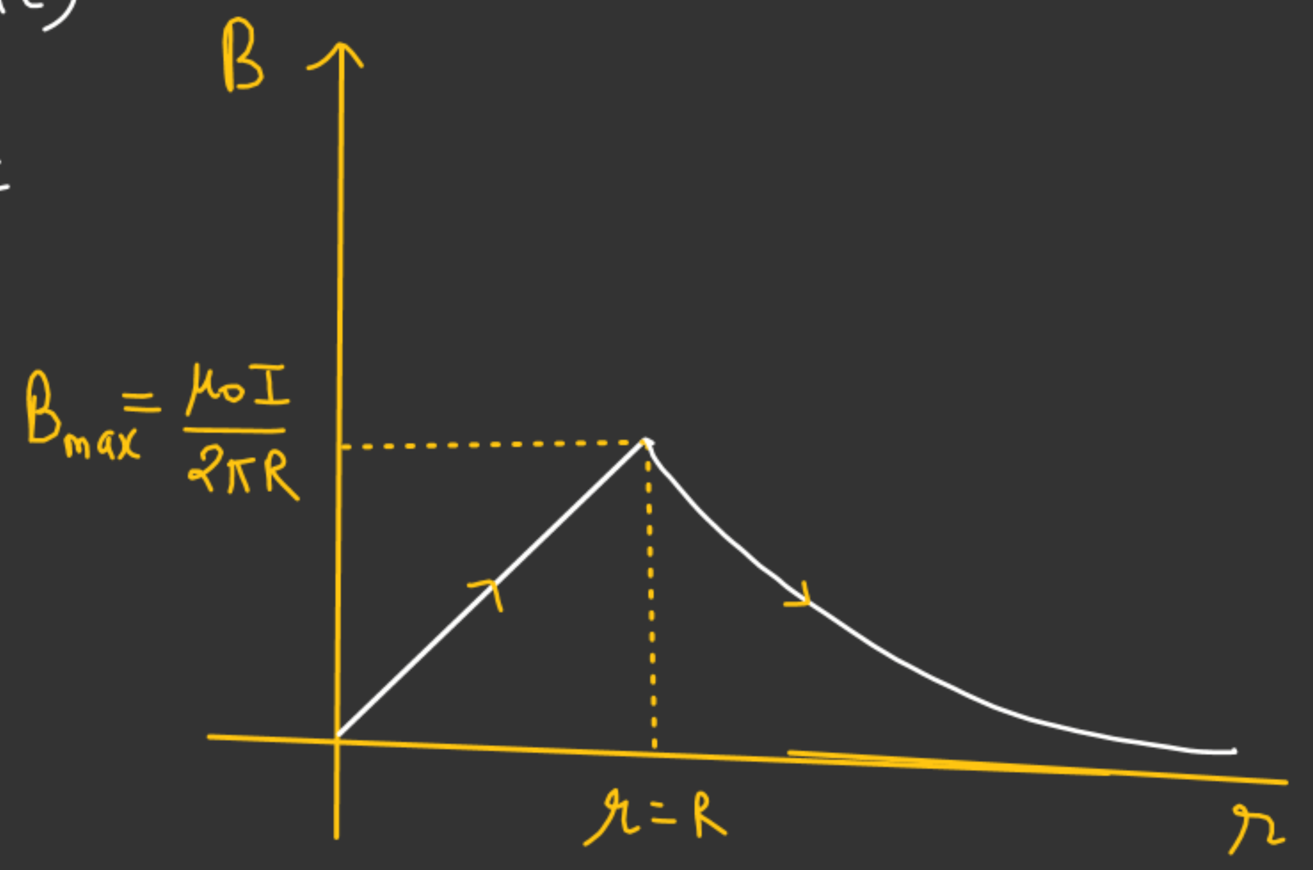
$$B \oint dl = \mu_0 i_{enc}$$

$$i_{enc} = I.$$

$$B \cdot 2\pi r = \mu_0 I.$$

$$\boxed{B = \frac{\mu_0 I}{2\pi r}}$$

Same as infinitly long wire.



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Q. Magnetic field due to a Conducting very long Cylinder.

a) $J = J_0 r^2$, b) $J = J_0\left(1 - \frac{r}{R}\right)$

J_0 is a constant.

$$J_x = (J_0 x^2)$$

$di_{enc} \rightarrow i_{enc}$ is dx thickness

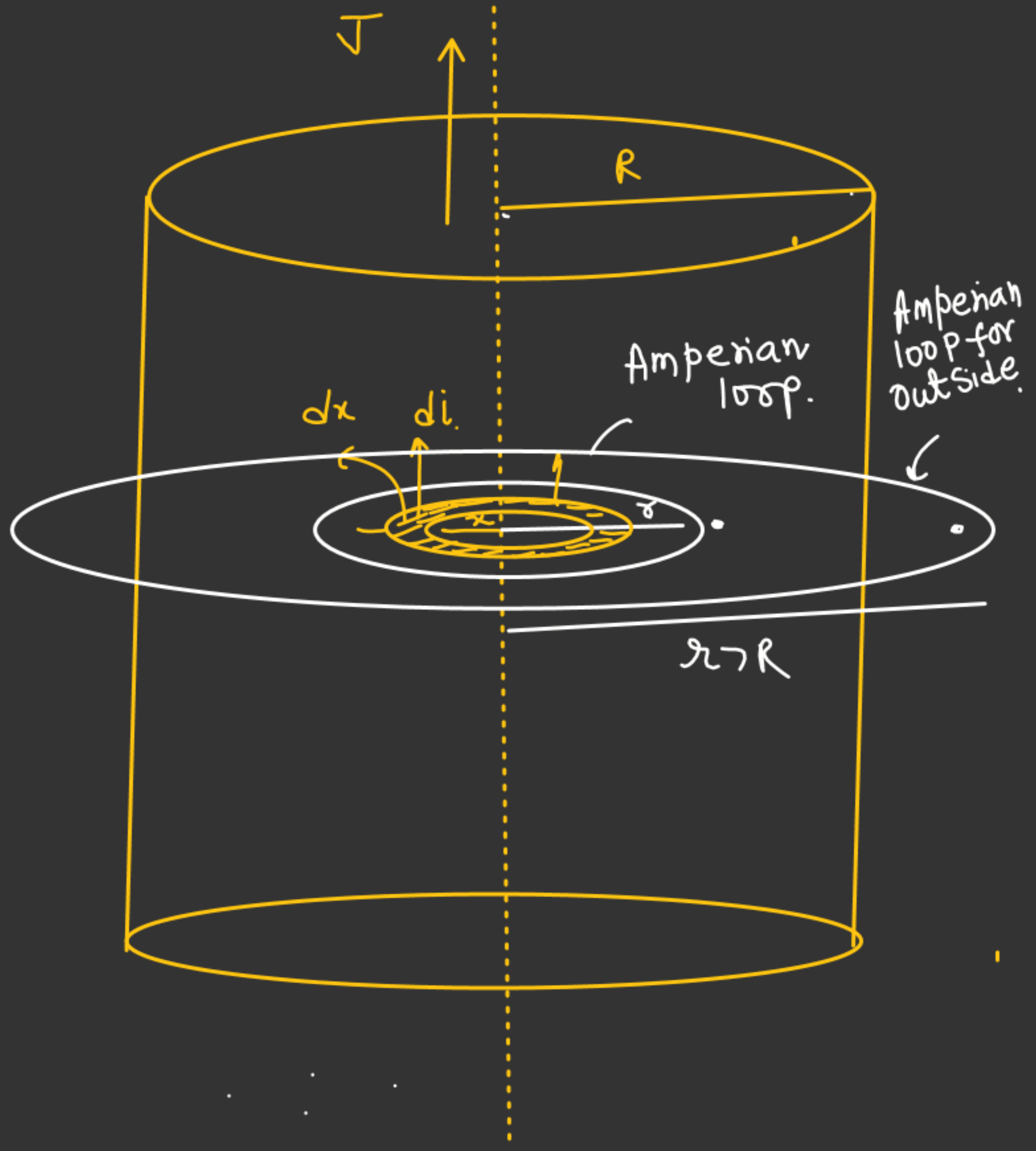
$$I_{enc} \quad di_{enc} = J_x \cdot dA$$

$dA \rightarrow$ differential area of of ring

$$\int_0 di_{enc} = \int_0^r J_0 x^2 (2\pi x)\, dx$$

$$I_{enc} = J_0 2\pi \int_0^r x^3\, dx = \frac{J_0 2\pi r^4}{4}$$

$$I_{enc} = \left(\frac{J_0 \pi r^4}{2}\right)$$



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$\underline{r<R}$

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 i_{enc}$$

$$B \cdot 2\pi r = \mu_0 \frac{J_0 \pi r^4}{2}$$

$$B = \left(\frac{\mu_0 J r^3}{4}\right) \text{Ans.}$$

$\underline{r>R}$ (outside)

$$i_{enc} = \left(\frac{\mu_0 J \pi R^4}{2}\right)$$

$$B \cdot 2\cancel{\pi} r = \frac{\mu_0 J \cancel{\pi} R^4}{2}$$

$$B = \frac{\mu_0 J R^4}{4r}.$$ ✓