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7. (iii) // (iv)

$$\tan(22.5^\circ) = \frac{1-\cos 45^\circ}{\sin 45^\circ} = \frac{1-\frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}} = \sqrt{2}-1$$

$$\boxed{\begin{aligned} &\tan\left(\frac{\pi}{8}\right) = \tan 22.5^\circ = \sqrt{2}-1 = \cot 67.5^\circ = \cot\frac{3\pi}{8} \\ &\cot\frac{\pi}{8} = \cot 22.5^\circ = \sqrt{2}+1 = \tan 67.5^\circ = \tan\frac{3\pi}{8} \end{aligned}}$$

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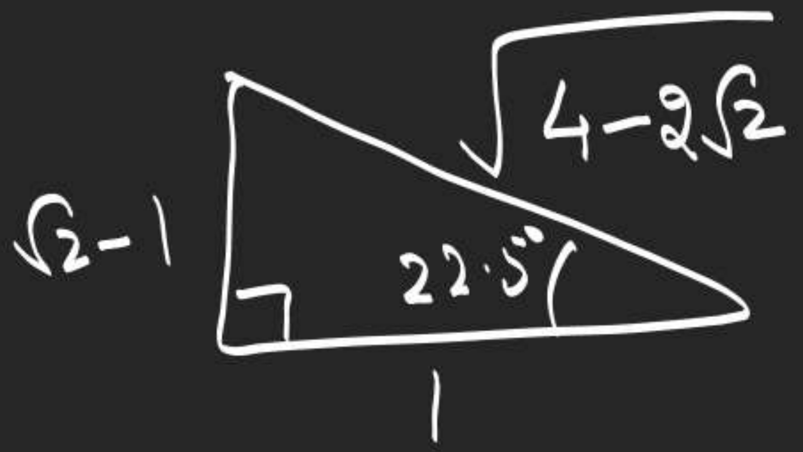
$$\tan(11.25^\circ) = \frac{1-\cos 22.5^\circ}{\sin 22.5^\circ}$$




$$= \frac{1-\dfrac{1}{\sqrt{4-2\sqrt{2}}}}{\dfrac{\sqrt{2}-1}{\sqrt{4-2\sqrt{2}}}} = \frac{\left(\sqrt{4-2\sqrt{2}}-1\right)}{\sqrt{2}-1}$$

$$= \left(\sqrt{4-2\sqrt{2}}-1\right)\left(\sqrt{2}+1\right)$$

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8:

$$2\sin\left(\frac{\theta+\phi}{2}\right)\cos\left(\frac{\theta-\phi}{2}\right) = a \quad — (1)$$

$$2\cos\left(\frac{\theta+\phi}{2}\right)\cos\left(\frac{\theta-\phi}{2}\right) = b \quad — (2)$$

$$\boxed{\tan\frac{\theta+\phi}{2} = \frac{a}{b}}$$

$$4\cos^2\left(\frac{\theta-\phi}{2}\right) = a^2+b^2$$

(2) $\sin(\theta+\phi) = ? \quad \dfrac{2\frac{a}{b}}{1+\left(\frac{a}{b}\right)^2}$

$$\sec^2\frac{\theta-\phi}{2} = \frac{4}{a^2+b^2} = 1+\tan^2\left(\frac{\theta-\phi}{2}\right)$$

(3) $\cos(\theta+\phi) = ?$

$$= \frac{1-\left(\frac{a}{b}\right)^2}{1+\left(\frac{a}{b}\right)^2}$$

(1) $\tan\frac{\theta-\phi}{2} = \pm\sqrt{\frac{4}{a^2+b^2}-1}$

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14. $$\frac{1}{\frac{1}{2}-\sin^2\theta} = \frac{2}{\cos 2\theta}$$

15. $$\tan\left(\frac{\pi}{4}+\frac{A}{2}\right) = \frac{1+\tan\frac{A}{2}}{1-\tan\frac{A}{2}} = \sqrt{\left(\frac{\cos\frac{A}{2}+\sin\frac{A}{2}}{\cos\frac{A}{2}-\sin\frac{A}{2}}\right)^2}$$

$$= \sqrt{\frac{1+\sin A}{1-\sin A}} = \sqrt{\frac{(1+\sin A)^2}{1-\sin^2 A}}$$

$$= \left|\frac{1+\sin A}{\cos A}\right| = \left|\sec A+\tan A\right|$$

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$$\sin^4\theta+\cos^4\theta = (\sin^2\theta+\cos^2\theta)^2-2\sin^2\theta\cos^2\theta$$

$$= 1-\frac{1}{2}(2\sin\theta\cos\theta)^2$$

$$= 1-\frac{1}{2}\sin^2 2\theta$$

$$(\sin^2\theta+\cos^2\theta)^3 - 3\sin^2\theta\cos^2\theta(\sin^2\theta+\cos^2\theta)$$

$$\boxed{\begin{aligned} \sin^4\theta+\cos^4\theta &= 1-\frac{1}{2}\sin^2 2\theta \\ \sin^6\theta+\cos^6\theta &= 1-\frac{3}{4}\sin^2 2\theta \end{aligned}}$$

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# Value of $\sin 18^\circ$, & $\cos 36^\circ$

$= \sin\frac{\pi}{10}$ $\qquad$ $= \cos\frac{\pi}{5}$

$ab = bc \not\Rightarrow a = c$

$$\boxed{b(a-c) = 0}$$

$\Rightarrow b = 0$ or $a = c$

$\theta = \frac{\pi}{10}$

$5\theta = \frac{\pi}{2}$

$2\theta + 3\theta = \frac{\pi}{2} \Rightarrow 2\theta = \frac{\pi}{2} - 3\theta$

$\sin 2\theta = \sin\left(\frac{\pi}{2} - 3\theta\right) = \cos 3\theta$

$2\sin\theta\cos\theta = 4\cos^3\theta - 3\cos\theta$

$2\sin\theta = 4 - 4\sin^2\theta - 3 \Leftarrow 2\sin\theta = 4\cos^2\theta - 3.$

$4\sin^2\theta + 2\sin\theta - 1 = 0$

$4x^2 + 2x - 1 = 0$

$\sin\theta = \frac{-2 \pm \sqrt{4+16}}{8} = \frac{-1 \pm \sqrt{5}}{4}$

$$\boxed{\sin 18^\circ = \frac{\sqrt{5}-1}{4}}$$

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$$\cos 36^\circ = 1 - 2\sin^2 18^\circ = 1 - 2\left(\frac{\sqrt{5}-1}{4}\right)^2$$

$$= 1 - \frac{6 - 2\sqrt{5}}{8}$$

$$= 1 - \frac{3-\sqrt{5}}{4} = \frac{1+\sqrt{5}}{4}$$

$$\boxed{\begin{array}{l} \sin 18^\circ = \sin\frac{\pi}{10} = \frac{\sqrt{5}-1}{4} = \cos\frac{2\pi}{5} = \cos 72^\circ \\ \cos 36^\circ = \cos\frac{\pi}{5} = \frac{\sqrt{5}+1}{4} = \sin\frac{3\pi}{10} = \sin 54^\circ \end{array}}$$

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1. Find the value of

$$\cos^2 48^\circ - \sin^2 12^\circ = \cos 36^\circ \cos 60^\circ$$

$$= \left(\frac{\sqrt{5}+1}{4}\right)\frac{1}{2} = \frac{\sqrt{5}+1}{8}$$

2. $$\sin 132^\circ \sin 12^\circ = \frac{1}{2}\left(\cos \underbrace{120^\circ}_{180-60^\circ} - \cos \underbrace{144^\circ}_{180-36}\right) = \frac{1}{2}\left(-\cos 60^\circ + \cos 36^\circ\right)$$

$= -\cos 60^\circ$ $\quad$ $-\cos 36^\circ$

$$\frac{\sqrt{5}-1}{8} = \frac{1}{2}\left(-\frac{1}{2} + \frac{\sqrt{5}+1}{4}\right)$$

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$\underline{3.}$ $\boxed{4\cos\frac{\pi}{10} - 3\sec\frac{\pi}{10}} - 2\tan\frac{\pi}{10}$ $\quad \frac{\sin}{\cos}$

$$\frac{\left(4\cos^2\frac{\pi}{10} - 3 - 2\sin\frac{\pi}{10}\cos\frac{\pi}{10}\right)}{\cos^2\frac{\pi}{10}}$$

$$\frac{\cos\frac{\pi}{10}\left(4\cos^2\frac{\pi}{10} - 3\right)}{\cos^2\frac{\pi}{10}} - 2\tan\frac{\pi}{10}$$

$$= \frac{\cos\frac{3\pi}{10}}{\cos^2\frac{\pi}{10}} - 2\tan\frac{\pi}{10}$$

$\sin\frac{2\pi}{10}$

$\frac{\pi}{2} - \frac{2\pi}{10}$

$$= \frac{\cos\frac{3\pi}{10} - \sin\frac{2\pi}{10}}{\cos^2\frac{\pi}{10}} = \frac{\sin\frac{2\pi}{10} - \sin\frac{2\pi}{10}}{\cos^2\frac{\pi}{10}}$$

$$= 0$$

$$= 2\tan\frac{\pi}{10} - 2\tan\frac{\pi}{10}$$

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4. $\sin\frac{\pi}{5}\sin\frac{2\pi}{5}\sin\frac{3\pi}{5}\sin\frac{4\pi}{5} = \left(\frac{2\sin\frac{\pi}{5}\sin\frac{2\pi}{5}}{2}\right)^2$

$\frac{3\pi}{5} \to \pi - \frac{2\pi}{5}$, $\frac{4\pi}{5} \to \pi - \frac{\pi}{5}$

$\frac{108^\circ}{2}$, $108^\circ = 90^\circ + 18^\circ$

$$= \left(\frac{1}{2}\left(\cos\frac{\pi}{5} - \cos\frac{3\pi}{5}\right)\right)^2 = \frac{1}{4}\left(\cos\frac{\pi}{5} + \sin\frac{\pi}{10}\right)^2$$

$$= \frac{1}{4}\left(\frac{\sqrt{5}+1}{4} + \frac{\sqrt{5}-1}{4}\right)^2 = \frac{1}{4}\left(\frac{\sqrt{5}}{2}\right)^2 = \frac{5}{16}.$$

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5.

$$\prod_{r=1}^{7} \sin(2r-1)\frac{\pi}{14} = \sin\frac{\pi}{14}\sin\frac{3\pi}{14}\sin\frac{5\pi}{14}\sin\frac{7\pi}{14}\sin\frac{9\pi}{14}\sin\frac{11\pi}{14}\sin\frac{13\pi}{14}$$

$\pi-\frac{5\pi}{14}$, $\pi-\frac{3\pi}{14}$, $\pi-\frac{\pi}{14}$

$$= \left(\sin\frac{\pi}{14}\sin\frac{3\pi}{14}\sin\frac{5\pi}{14}\right)^2$$

$\frac{\pi}{2}-\frac{4\pi}{14}$, $\frac{\pi}{2}-\frac{2\pi}{14}$

$$\cos\left(\frac{\pi}{2}+\frac{\pi}{14}\right) = -\sin\frac{\pi}{14}$$

$\pi+\frac{\pi}{7}$

$$= \left(-\cos\frac{8\pi}{14}\cos\frac{4\pi}{14}\cos\frac{2\pi}{14}\right)^2 = \left(\frac{\sin\frac{8\pi}{7}}{8\sin\frac{\pi}{7}}\right)^2 = \boxed{\frac{1}{64}}$$

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6. $\sin^4\frac{\pi}{16}+\sin^4\frac{3\pi}{16}+\sin^4\frac{5\pi}{16}+\sin^4\frac{7\pi}{16}$.

$\frac{5\pi}{16} \to \frac{\pi}{2}-\frac{3\pi}{16}$ $\qquad$ $\frac{7\pi}{16} = \frac{\pi}{2}-\frac{\pi}{16}$

$$=\left(\sin^4\frac{\pi}{16}+\cos^4\frac{\pi}{16}\right)+\left(\sin^4\frac{3\pi}{16}+\cos^4\frac{3\pi}{16}\right)$$

$$= 1-\frac{1}{2}\sin^2\frac{\pi}{8}+1-\frac{1}{2}\sin^2\frac{3\pi}{8}$$

$\frac{3\pi}{8} \to \frac{\pi}{2}-\frac{\pi}{8}$

$$= 2-\frac{1}{2}\left(\sin^2\frac{\pi}{8}+\cos^2\frac{\pi}{8}\right)=2-\frac{1}{2}=\boxed{\frac{3}{2}}$$

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## HW

Ex-18 → Q 16 to 22

23, 27; 26.

Ex-19 → Q.1 to 11