Nishant Jindal

Q19 43, 44, 45, 46, 47, 48, 49

Q20, $a_1, a_2 \dots a_{49}$, $a_9 + a_{43} = 66$

$$a + 8d + a + 42d = 66.$$

$$a + 25d = 33 \rightarrow (1)$$

2) $\sum_{k=0}^{12} a_{4k+1} = 416$

$$13a + d(4 + 8 + \dots 48) = 416$$

$$13a + 4d(1 + 2 + \dots 12)$$

$$13a + 4d \times \frac{12 \times 13}{2} = 416$$

$$13(a + 24d) = 416 \rightarrow (2)$$

$d = 1$, $a = 8$

Add

21 Sum of 1st n terms of AP $= cn^2$

Jee 2009

then sum of sq^r of these n these n terms = ?

1) $S_n = cn^2 \Rightarrow S_{n-1} = c(n-1)^2$

$$T_n = S_n - S_{n-1} = cn^2 - c(n-1)^2$$

$$= c\{n^2 - (n-1)^2\} = (2n-1)c$$

2) $\sum T_n^2 = c^2 \sum (2n-1)^2$

$$c^2 \sum 4n^2 - 4n + 1$$

$$c^2 \{4\sum n^2 - 4\sum n + \sum 1\}$$

Future

$\sum 1 = n$

$\sum n = \frac{(n)(n+1)}{2}$

$\sum n^2 = \frac{(n)(n+1)(2n+1)}{6}$

Nishant Jindal

Adv

$2a_2 < 27 \Rightarrow a_2 < \frac{27}{2}\ 13.5$

Q22 Let $a_1, a_2, \ldots, a_{11}$ be Real No. Satisfying. $a+c=2b$

Adv 2010

$a_1 = 15$, $\boxed{27 - 2a_2 > 0}$ & $\boxed{a_k = 2a_{k-1} - a_{k-2}}$

for $k = 3, 4, \ldots, 11$. If $\frac{a_1^2 + a_2^2 + \ldots + a_{11}^2}{11} = 90$

then value of $\frac{a_1 + a_2 + \ldots + a_{11}}{11} = ?$

$$\frac{a^2 + (a+d)^2 + (a+2d)^2 + \ldots + (a+10d)^2}{11} = 90$$

$$\frac{11a^2 + d^2(1^2 + 2^2 + \ldots + 10^2) + 2ad(1+2+3+\ldots+10)}{11} = 90$$

$11 \times 225 + 385d^2 + 15 \times 110 d = 990$

$385d^2 + 1650d + 1485 = 0$

$35d^2 + 150d + 135 = 0 \Rightarrow 7d^2 + 30d + 27 = 0 \Rightarrow (7d+9)(d+3) = 0$

$1+2+3+\ldots+10 = \frac{(10)(11)}{2}$

$1^2 + 2^2 + \ldots + 10^2 = \frac{(10)(11)(21)}{6}$

$d = -3, \ -\frac{9}{7}$ ✗

Demand $= \frac{a_1 + a_2 + \ldots + a_{11}}{11}$

$= \frac{\frac{11}{2}[30 + 10 \times -3]}{11} = 0$

Nishant Jindal

HW 2

Q5 $\boxed{a}, ar, ar^2$

$a = ar + ar^2$

$\Rightarrow r^2 + r = 1$

$\Rightarrow r^2 + r - 1 = 0$

$\Rightarrow r = \frac{-1 \pm \sqrt{1+4}}{2}$

$r = \frac{-1 \pm \sqrt{5}}{2}$ → $\frac{\sqrt{5}-1}{2}$ ⊕ $= \left(\frac{\sqrt{5}-1}{4}\right) \times 2 = 2 \sin 18^\circ$ ✓

→ $\frac{-\sqrt{5}-1}{2}$ ⊖ ✗

(4) $x^2 - 3x + a = 0$ → α, β ; $\alpha + \beta = 3$

$x^2 - 12x + b = 0$ → γ, δ ; $\gamma + \delta = 12$

$\alpha, \beta, \gamma, \delta \to$ G.P.

a, ar, ar^2, ar^3

① $a + ar = 3$

$a(1+r) = 3$

② $ar^2 + ar^3 = 12$

$ar^2(1+r) = 12$

$\frac{\cancel{a} r^2 \cancel{(1+r)}}{\cancel{a} \cancel{(1+r)}} = \frac{\cancel{12}\,4}{\cancel{3}}$

$\Rightarrow r = 2$ or -2

$a(1+2) = 3$

$a = 1$

$\alpha\ \beta\ \gamma\ \delta$

(2) $1, 2, 4, 8$

$a = \alpha \cdot \beta = 2$

$b = \gamma \cdot \delta = 4 \times 8 = 32$

Nishant Jindal

Q3 $S =$ Sum of ∞ GP.

$\leftarrow \infty \rightarrow$

$a, ar, ar^2, ar^3 \dots ar^{n-1} \dots \dots \infty$

$\leftarrow$ n term $\rightarrow$

$S_n = \boxed{\dfrac{a(1-r^n)}{(1-r)}}$

$S_n = S\left(1-\left(1-\frac{a}{S}\right)^n\right)$ $\boxed{B}$

$S = \dfrac{a}{(1-r)} \Rightarrow 1-r = \dfrac{a}{S}$

2) $1-\dfrac{a}{S} = r$

23) $a, \dots ar^{n-1}$

$ar^{n-1} = b$

$b = ar^{n-1}$

$\dfrac{b}{a} = r^{n-1}$

$\left(\dfrac{b}{a}\right)^{\frac{1}{n-1}} = (r)$

Q24 $(a^2p^2 - 2abP + b^2) + (b^2P^2 - 2bcP + c^2) + (c^2P^2 - 2cdP + a^2) = 0$

$(aP-b)^2 + (bP-c)^2 + (cP-d)^2 = 0$

$aP - b = 0$ & $bP - c = 0$, $cP - d = 0$

$P = \dfrac{b}{a} = \dfrac{c}{b} = \dfrac{d}{c} \Rightarrow a, b, c, d$ GP

(2) $P = a \cdot ar \cdot ar^2 \dots ar^{n-1}$

$= (a \cdot ar^{n-1}) \cdot (ar \cdot ar^{n-2})$

$(ar^2 \cdot ar^{n-3}) \dots$

$= \underbrace{(a \cdot b)(ab) \dots (ab)}_{n/2}$

$P = (ab)^{\frac{n}{2}} \Rightarrow P^2 = (ab)^n$

Nishant Jindal

Selection of terms in GP-

	AP (Sum is given)	(Prod is given) GP
3 terms.	$a-d, a, a+d$	$\frac{a}{r}, a, ar$
4 terms.	$a-3d, a-d, a+d, a+3d$	$\frac{a}{r^3}, \frac{a}{r}, ar, ar^3$
5 terms	$a-2d, a-d, a, a+d, a+2d$	$\frac{a}{r^2}, \frac{a}{r}, a, ar, ar^2$

Q If Sum of 3 Consecutive terms of a GP = 19 & Product = 216 find S_n & S_∞?

let terms are $\frac{a}{r}, a, ar$

(1) $\frac{a}{r} + a + ar = 19$

$a\left(\frac{1}{r} + 1 + r\right) = 19$

(2) Prod $= \frac{a}{r} \times a \times ar = a^3 = 216$

$\boxed{a = 6}$

$\frac{r^2 + r + 1}{r} = \frac{19}{6}$

$\Rightarrow$

$6r^2 + 6r + 6 = 19r$

$6r^2 - 13r + 6 = 0$

$6r^2 - 9r - 4r + 6 = 0$

$3r(2r-3) - 2(2r-3) = 0$

$(3r-2)(2r-3) = 0$

$\Rightarrow r = \frac{3}{2}, \frac{2}{3}$

(A) $S_n = \frac{a(r^n - 1)}{r-1} = \frac{6\left(\left(\frac{3}{2}\right)^n - 1\right)}{\left(\frac{3}{2} - 1\right)}$

$= 12\left(\left(\frac{3}{2}\right)^n - 1\right)$

(B) $S_\infty = \frac{6}{1 - \frac{2}{3}} = 12$

Nishant Jindal

Q Find Sum of given Series

A) $1+2+3+4+5+8+7+16+\dots 40 \text{ term}$.

$(1+3+5+7 \dots 20 \text{ term}) + (2+4+8+16+32 \dots 20 \text{ te})$

$\leftarrow$ 20 odd terms $\rightarrow$ $\qquad$ $\leftarrow$ GP $\rightarrow$

$$= 20^2 + \frac{2(2^{20}-1)}{2-1} = 400 + 2\cdot 2^{20} - 2 = 398 + 2^{21}$$

(B) $\frac{1}{7} + \frac{2}{7^2} + \frac{1}{7^3} + \frac{2}{7^4} + \frac{1}{7^5} + \dots \infty$

$$\left(\frac{1}{7} + \frac{1}{7^3} + \frac{1}{7^5} \dots \infty\right) + 2\left(\frac{1}{7^2} + \frac{1}{7^4} + \frac{1}{7^6} \dots \infty\right)$$

$$\frac{\frac{1}{7}}{\left(1-\frac{1}{7^2}\right)} = \frac{1}{7} \times \frac{\cancel{49}^{7}}{48} + 2\left(\frac{\frac{1}{7^2}}{1-\frac{1}{7^2}}\right) = \frac{7}{48} + \frac{2}{\cancel{7^2}} \times \frac{\cancel{7^2}}{48} = \frac{9}{48}$$

Nishant Jindal

Q $2^{\frac{1}{2}} \cdot 2^{\frac{1}{4}} \cdot 2^{\frac{1}{8}} \cdot 2^{\frac{1}{16}} \cdots$

$(2)^{\frac{1}{2}+\frac{1}{2^2}+\frac{1}{2^3}+\frac{1}{2^4} \cdots \infty}$

$(2)^{\frac{\frac{1}{2}}{1-\frac{1}{2}}} = (2)^1 = 2$

Q $\sqrt{2}+\sqrt{6}+\sqrt{18}+\sqrt{54} \cdots 10$ term.

$\sqrt{2}\left(1+\sqrt{3}+\sqrt{9}+\sqrt{27}+\cdots\right)$

$\sqrt{2}\left(1+\sqrt{3}+(\sqrt{3})^2+(\sqrt{3})^3+\cdots\right)$

← GP of 10 term

$\sqrt{2}\left(\frac{1\cdot((\sqrt{3})^{10}-1)}{(\sqrt{3}-1)}\right)$

Q

Q $6+66+666+6666+\dots$ n term. **Yad**

$$6\{1+11+111+1111+\dots\}$$

$$\frac{6}{9}\{9+99+999+9999+\dots\}$$

$$\frac{6}{9}\{(10-1)+(10^2-1)+(10^3-1)+(10^4-1)\dots\}$$

$$\frac{6}{9}\{(\underbrace{10+10^2+10^3+\dots}_{\leftarrow n\text{ times}\rightarrow})-(\underbrace{1+1+\dots}_{\leftarrow n \text{ बार} \rightarrow})\}$$

$$\frac{6}{9}\left\{\frac{10\cdot(10^n-1)}{(10-1)}-n\right\}$$

Q $\frac{3}{19}+\frac{33}{19^2}+\frac{333}{19^3}+\dots$ n term.

$$3\left\{\frac{1}{19}+\frac{11}{19^2}+\frac{111}{19^3}+\dots n\text{tu}\right\}$$

$$\frac{3}{9}\left\{\frac{9}{19}+\frac{99}{19^2}+\frac{999}{19^3}+\dots\right\}$$

$$\frac{3}{9}\left\{\frac{(10-1)}{19}+\frac{(10^2-1)}{19^2}+\frac{(10^3-1)}{19^3}+\frac{(10^4-1)}{19^4}+\dots\right\}$$

$$\frac{3}{9}\left\{\left(\frac{10}{19}+\frac{10^2}{19^2}+\frac{10^3}{19^3}+\frac{10^4}{19^4}+\dots\right)-\left(\frac{1}{19}+\frac{1}{19^2}+\frac{1}{19^3}+\dots\right)\right\}$$

$$\frac{3}{9}\left\{\frac{\frac{10}{19}\left(1-\left(\frac{10}{19}\right)^n\right)}{\left(1-\frac{10}{19}\right)}-\frac{\left(\frac{1}{19}\right)\left(1-\left(\frac{1}{19}\right)^n\right)}{\left(1-\frac{1}{19}\right)}\right\}$$

Nishant Jindal

Q $\boxed{x = \sum_{n=0}^{\infty} a^n}$, $y = \sum_{n=0}^{\infty} b^n$, $z = \sum_{n=0}^{\infty} (ab)^n$

$xz + yz = xy + z$ (T/F)

$$x = \sum_{n=0}^{\infty} a^n$$

$= a^0 + a^1 + a^2 + a^3 + \dots \infty$

$= 1 + a + a^2 + a^3 \dots \infty$

$x = \frac{1}{1-a}$

$1 - a = \frac{1}{x}$

$1 - \frac{1}{x} = a$

$\frac{x-1}{x} = a$

$$y = \sum_{n=0}^{\infty} b^n$$

$= 1 + b + b^2 + b^3 \dots \infty$

$\Rightarrow y = \frac{1}{1-b}$

$\Rightarrow 1 - b = \frac{1}{y}$

$\Rightarrow 1 - \frac{1}{y} = b$

$\Rightarrow b = \frac{y-1}{y}$

$z = 1 + ab + (ab)^2 + (ab)^3 \dots$

$z = \frac{1}{1-ab}$

$ab = \frac{z-1}{z}$

$\left(1 - \frac{1}{x}\right)\left(1 - \frac{1}{y}\right) = 1 - \frac{1}{z}$

$x + \frac{1}{x} + \frac{1}{y} + \frac{1}{xy} = x + \frac{1}{z}$

$\frac{y + x + 1}{xy} = \frac{1}{z}$

$yz + xz + z = xy$