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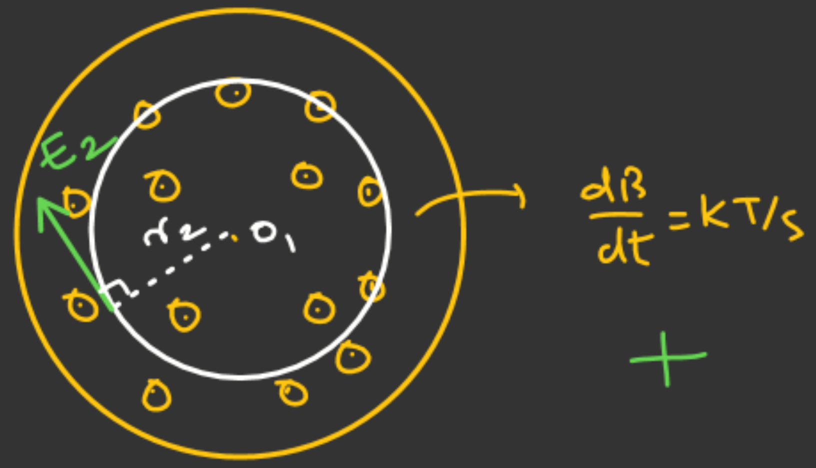
Induced Electric field inside a Spherical Cavity :-

$\frac{dB}{dt} = K\,(T/s)$. Find E_{ind} at any point inside the Cavity.



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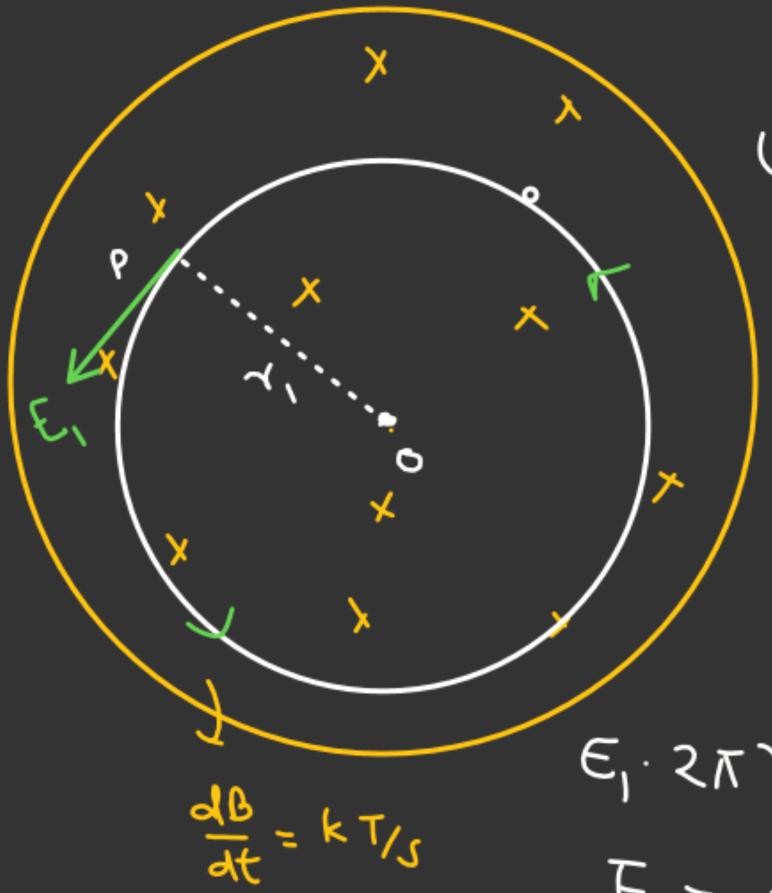

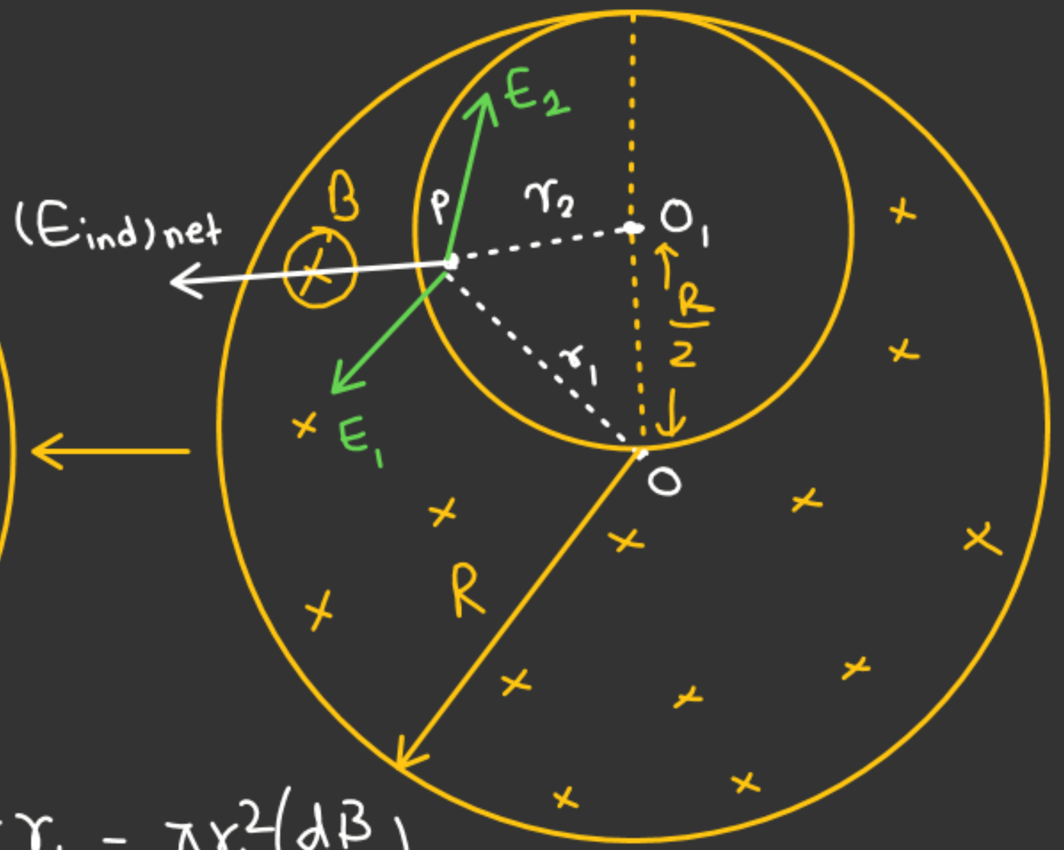

$$E_2 = \left(\frac{k r_2}{2}\right)$$

$$E_1 \cdot 2\pi r_1 = \pi r_1^2 \left(\frac{dB}{dt}\right)$$

$$E_1 = \frac{r_1}{2}\left(\frac{dB}{dt}\right) = \frac{k r_1}{2}$$

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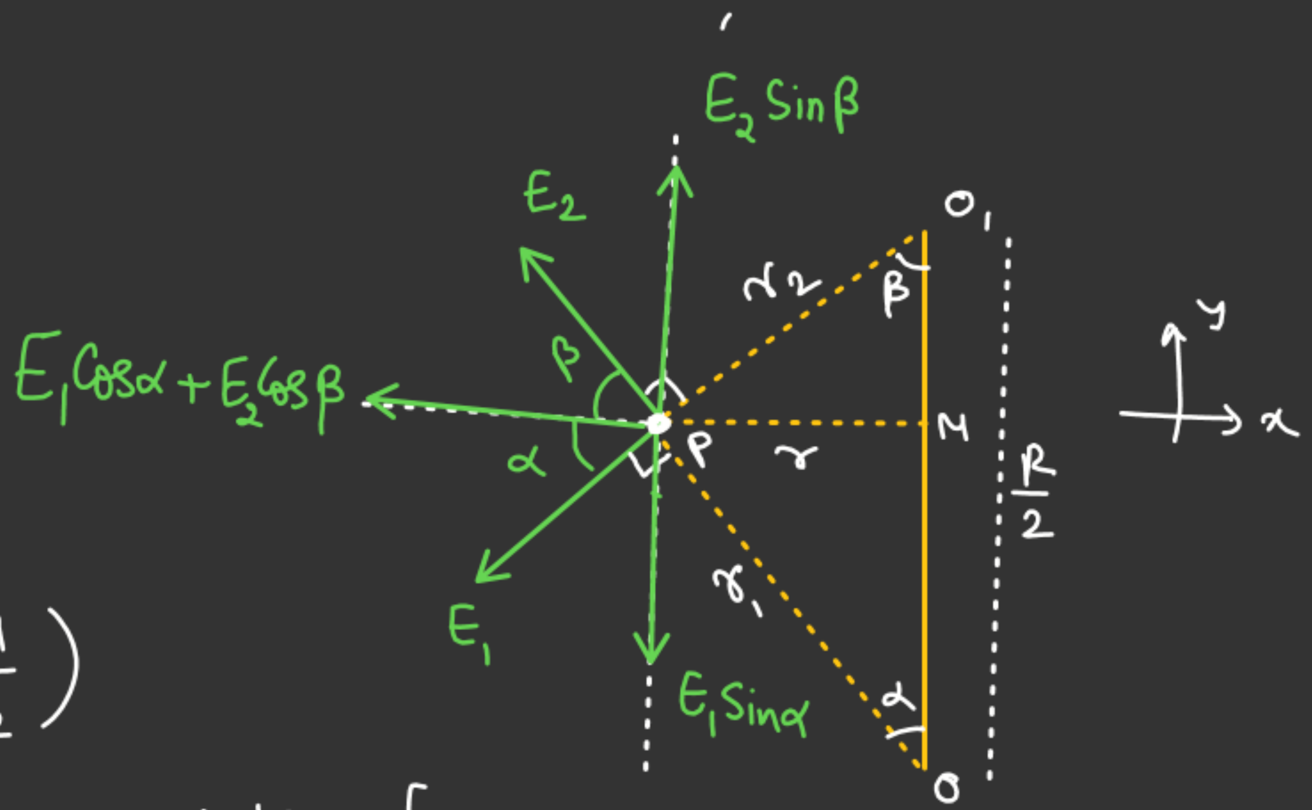
$$(E_{ind})_y = E_2\sin\beta - E_1\sin\alpha$$
$$= \frac{k r_2}{2}\left(\frac{r}{r_2}\right) - \frac{k r_1}{2}\cdot\left(\frac{r}{r_1}\right)$$
$$= 0$$



$$(E_{ind})_x = (E_1\cos\alpha + E_2\cos\beta)$$
$$= \frac{k r_1}{2}\left(\frac{OM}{r_1}\right) + \frac{k r_2}{2}\left(\frac{O_1M}{r_2}\right)$$
$$= \frac{k}{2}(OM + O_1M)$$
$$= \frac{k}{2}\left(\frac{R}{2}\right) = \left(\frac{kR}{4}\right)$$

$$\underline{\vec{E}_{net}} = \left(-\frac{kR}{4}\hat{i}\right)$$

Note:- [Resultant induced electric field is perpendicular to the line joining O & O_1

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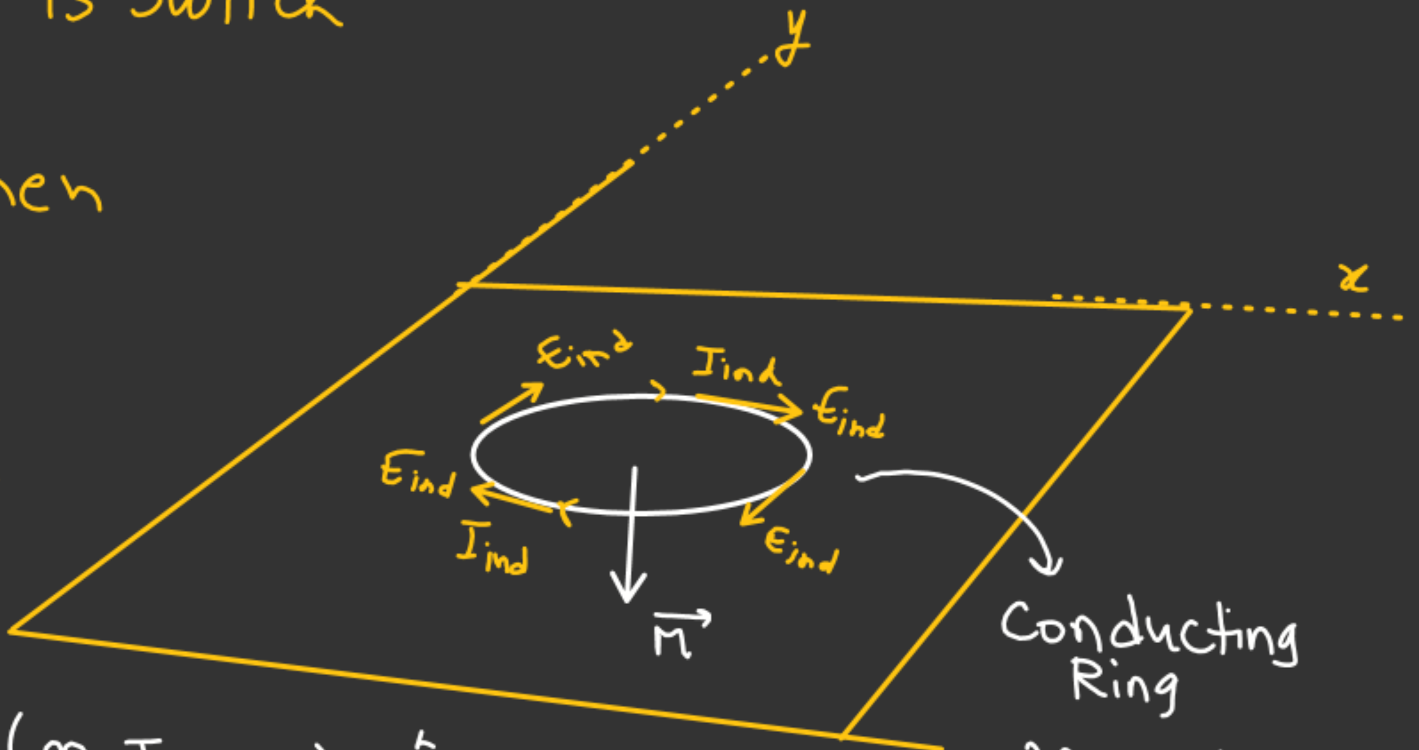
~~Q.~~ A Magnetic field $\vec{B} = (2\hat{j} + t^2\hat{k})$ is Switch on.

Find a) Induced Electric field when ring starts toppling.

b) Total heat dissipiated from $t=0$ to the time when ring starts toppling.



$m = \pi$ Kg

$R = \frac{1}{2}$ m

r = resistance of ring $= \pi\,\Omega$ ⇓ Ohm

Sol^n: →

$\phi = \vec{B}\cdot\vec{A}$

$\phi = (2\hat{j} + t^2\hat{k})\cdot(\pi R^2)(-\hat{k})$

$\phi = -(\pi R^2 t^2)$

$\mathcal{E}_{ind} = -\frac{d\phi}{dt} = (\pi R^2 2t) = \underline{(2\pi R^2)t}$

$I_{ind} = \frac{\mathcal{E}_{ind}}{r} = \frac{2\pi R^2}{\pi}t = \underline{(2R^2)t}$

$\vec{M} = (\underset{\substack{\Downarrow \\ 1}}{n} I_{ind} A)\, -\hat{k}$

$= (2R^2 t)(\pi R^2)(-\hat{k})$

$\vec{M} = (2\pi R^4)t\,(-\hat{k})$

$\vec{\tau}_B = \vec{M} \times \vec{B}$

$= (2\pi R^4 t)(-\hat{k}) \times (2\hat{j} + t^2\hat{k})$

$\vec{\tau}_B = (4\pi R^4 t) + \hat{i}$

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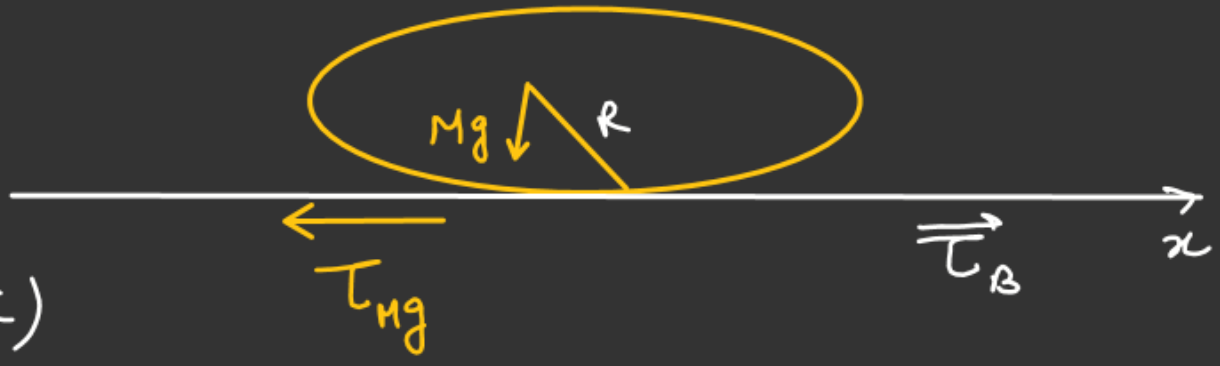
Ring will start to topple when $\tau_B = \tau_{mg}$.

$$4\pi R^4 t = MgR$$

$$t = \left(\frac{mg}{4\pi R^3}\right)$$

$$t = \frac{\cancel{\pi}\times 10}{4\times\cancel{\pi}\times\left(\frac{1}{2}\right)^3} = (20\,\text{sec})$$



$$E_{ind}\cdot 2\pi R = \mathcal{E}_{ind}$$

$$E_{ind}\cdot 2\pi R = (2\pi R^2)\,t$$

$$E_{ind} = (Rt)$$

$$(E_{ind})_{t=20\,sec} = \frac{1}{2}\times 20 = 10\ V/m$$

$$P = i_{ind}^2\cdot \underline{r}$$

$$P = 4R^4 t^2 \pi = 4\pi R^4 t$$

$$\frac{dH}{dt} = 4\pi R^4 t^2$$

$$\int_0^H dH = 4\pi R^4 \int_0^{20} t^2\,dt$$

$$H = (4\pi R^4)\frac{t^3}{3} = \underline{\frac{4\pi R^4}{3}\cdot t^3}$$

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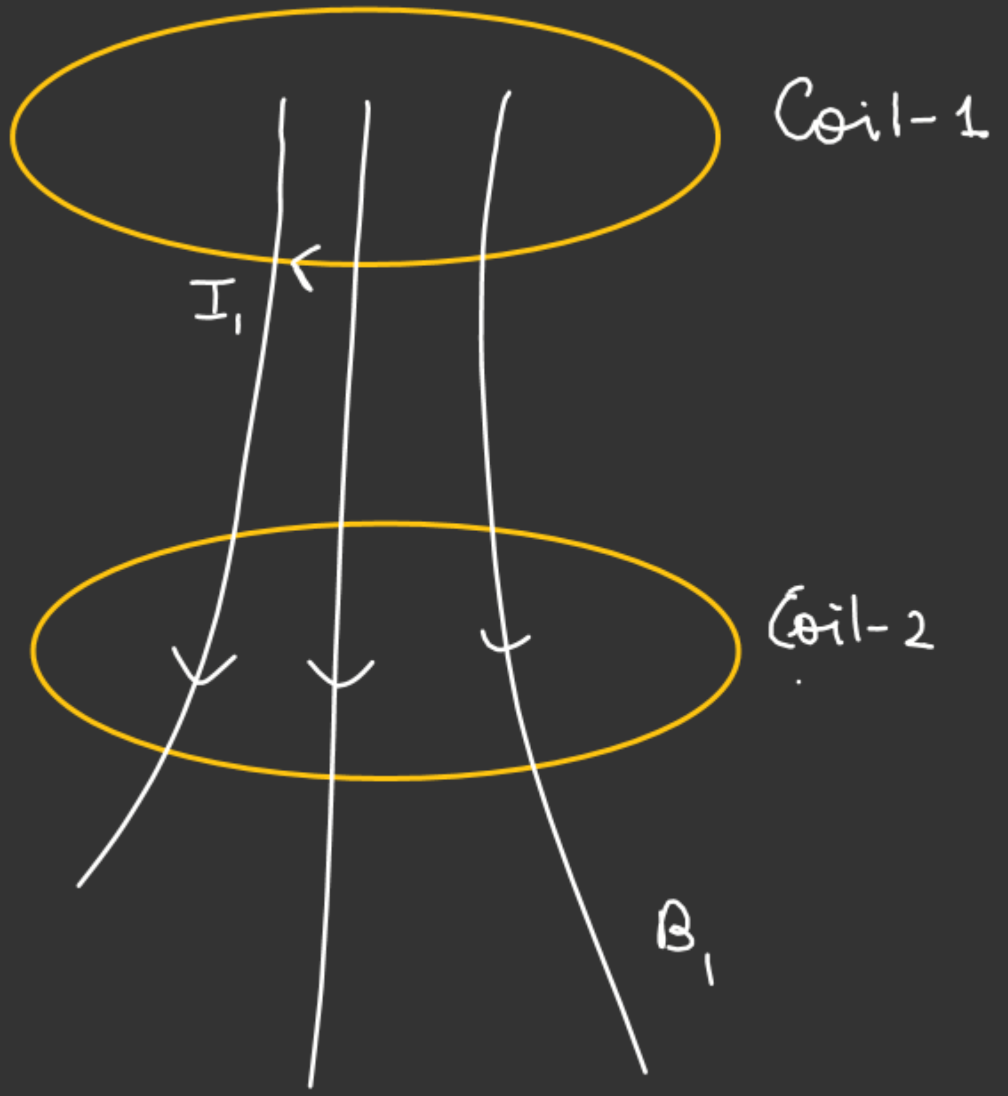
Concept of Mutual Induction

$\phi_{2-1} \propto I_1$

$\phi_{2-1} = M_{2-1} I_1$

M_{2-1} = Mutual Inductance of Coil 2 due to 1.



If I_2 be the Current in Coil 2

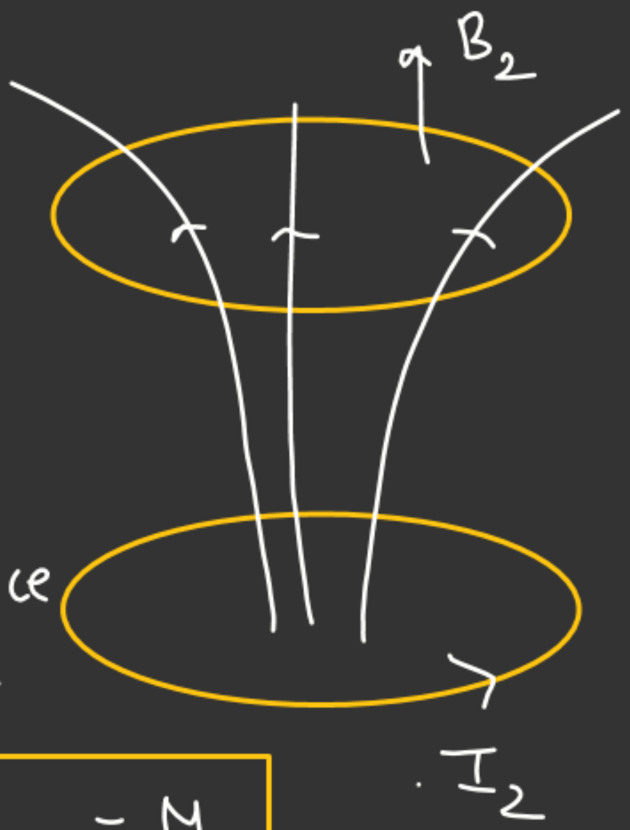
$\phi_{1-2} \propto I_2$

$\phi_{1-2} = M_{1-2} I_2$

$\Downarrow$

Mutual inductance of 1 due to 2

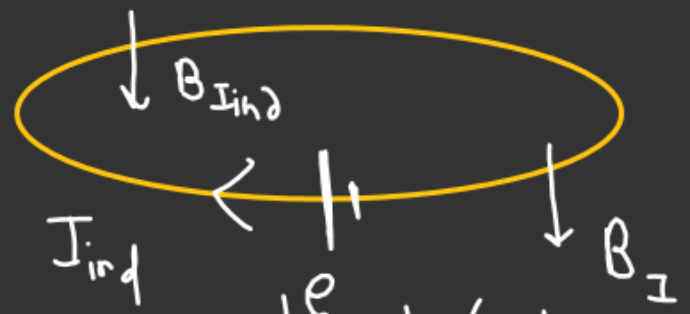
$$\boxed{M_{1-2} = M_{2-1} = M}$$



$$\phi = MI$$

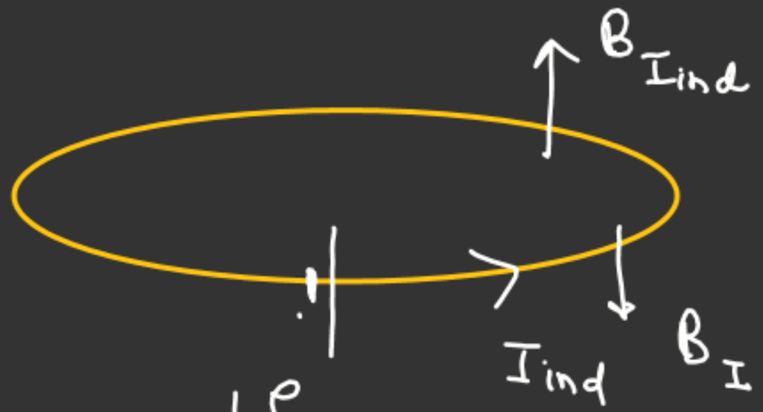
$$\mathcal{E}_{ind} = \frac{-d\phi}{dt} = -M\left(\frac{dI}{dt}\right)$$

$I \rightarrow$ decreasing w.r.t time.



$$|\mathcal{E}_{ind}| = \left(M\frac{dI}{dt}\right)$$

If I increasing



$$|\mathcal{E}_{ind}| = M\left(\frac{dI}{dt}\right)$$

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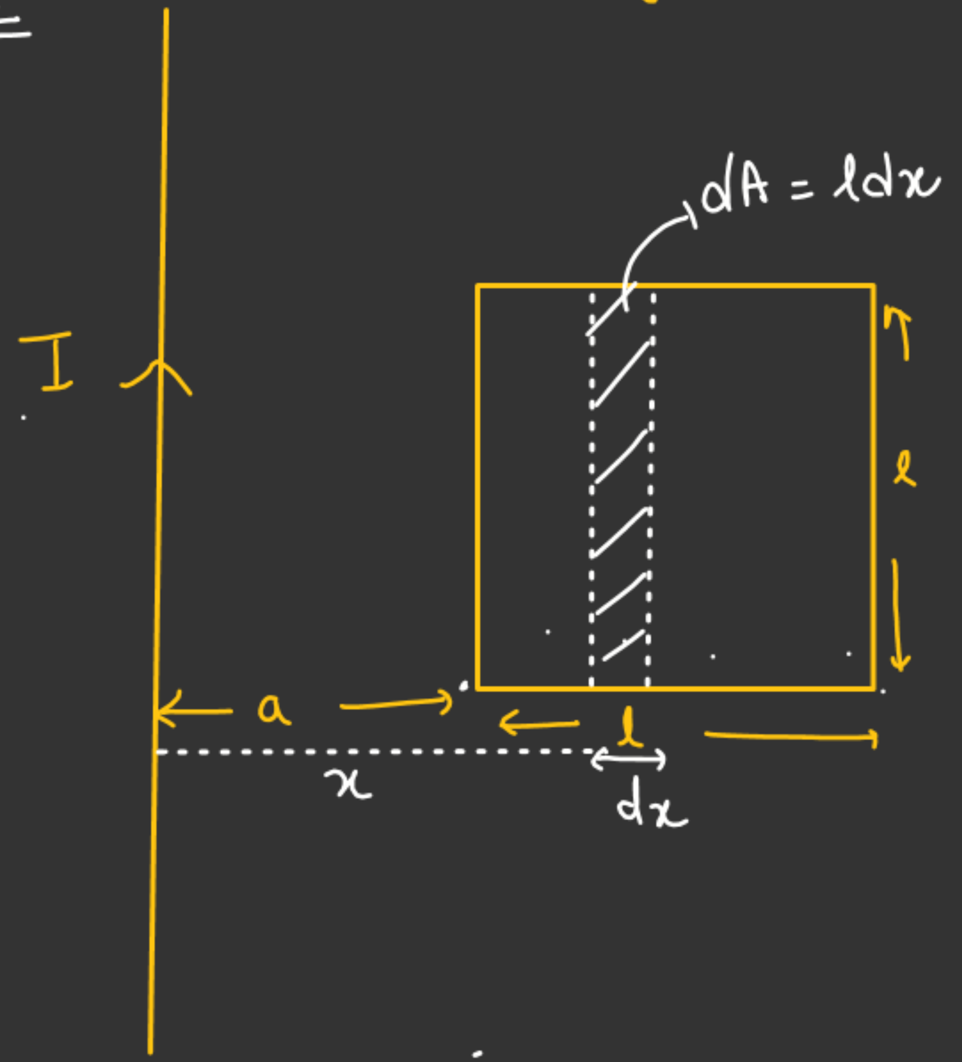
How to find $M = ??$

- Assume any current in one of the loop or body
- Find flux enclosed on the other body.
- Compare the result with $[\phi = MI]$ & find M.

#



$$d\phi = \left(\frac{\mu_0 I}{2\pi x}\right) l\,dx$$

$$\int_0^{\phi} d\phi = \frac{\mu_0 I l}{2\pi}\int_a^{a+l}\frac{dx}{x}$$

$$\phi = \frac{\mu_0 I l}{2\pi}\ln\left(\frac{a+l}{a}\right)$$

$$\phi = \left[\frac{\mu_0 l}{2\pi}\ln\left(\frac{a+l}{a}\right)\right] I$$

$$\phi = M\,I$$

$$M = \frac{\mu_0 l}{2\pi}\ln\left(\frac{a+l}{a}\right)$$

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Find M if $b << a$

$\Downarrow$

Magnetic field at c due to bigger loop is Same as in the Smallar loop.

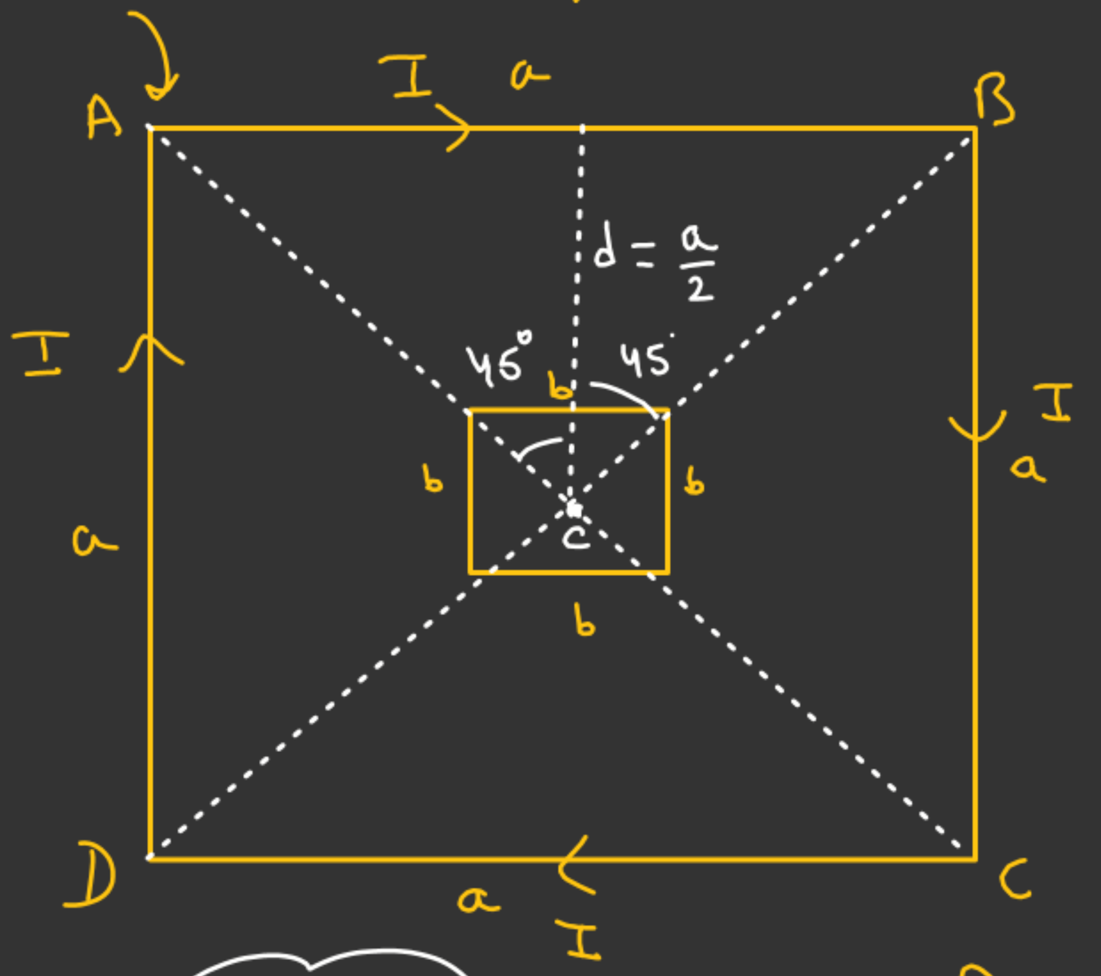
$$\phi = \underbrace{\frac{\mu_0 I}{4\pi(\frac{a}{2})} \times 2\sin 45^\circ \times 4}_{B} \times b^2$$

$$\phi = \left(\frac{4\mu_0 b^2}{\sqrt{2}\pi a}\right) I.$$

$$\phi = \left(\frac{2\sqrt{2}\,\mu_0 b^2}{\pi a}\right) I$$

$$\phi = M I$$

Conducting Square frame.



$$M = \frac{2\sqrt{2}\,\mu_0 b^2}{\pi a}$$ ✓

$$B = \frac{\mu_0 I}{4\pi d}\left[\sin\theta_1 + \sin\theta_2\right]$$

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Both the ring are in the same plane

(I) Find $M = ??$

(II) Find force b/w two rings.