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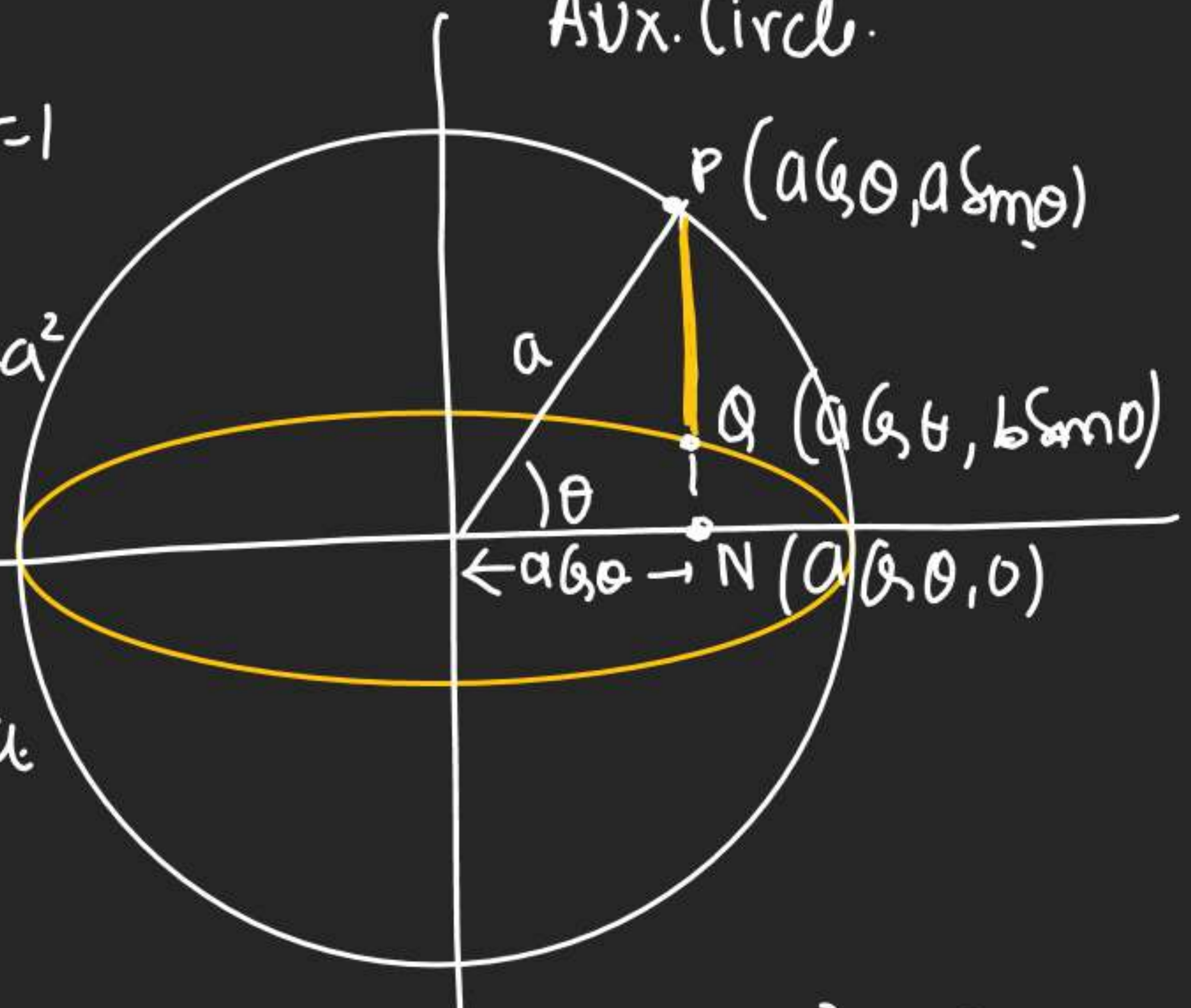
Eccentric Angle & Eccentric Circle
|| Aux. Circle

E: $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

Cir:→ $x^2+y^2=a^2$

dia of Circle = Maj Axis = 2a

$\boxed{r=a}$



(2) $x=a\cos\theta$ put $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

$\frac{a^2\cos^2\theta}{a^2}+\frac{y^2}{b^2}=1 \Rightarrow y=b\sin\theta$

(3) here Q is Par. Coord of $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

$Q=(a\cos\theta, b\sin\theta)$

(4) θ = eccentric angle to Eccentric Circle.

$0\le\theta<2\pi$

(5) P, Q, N are known as corresponding Pt

Q P.T. $\frac{PN}{QN}$ = Constant?

$PN=a\sin\theta$, $QN=b\sin\theta$

$\frac{PN}{QN}=\frac{a\sin\theta}{b\sin\theta}=\frac{a}{b}$

Q P.T. $\frac{PN}{PQ}$ = Const.?

$PN=a\sin\theta$

$PQ=a\sin\theta-b\sin\theta$

$\frac{PN}{PQ}=\frac{a}{a-b}$

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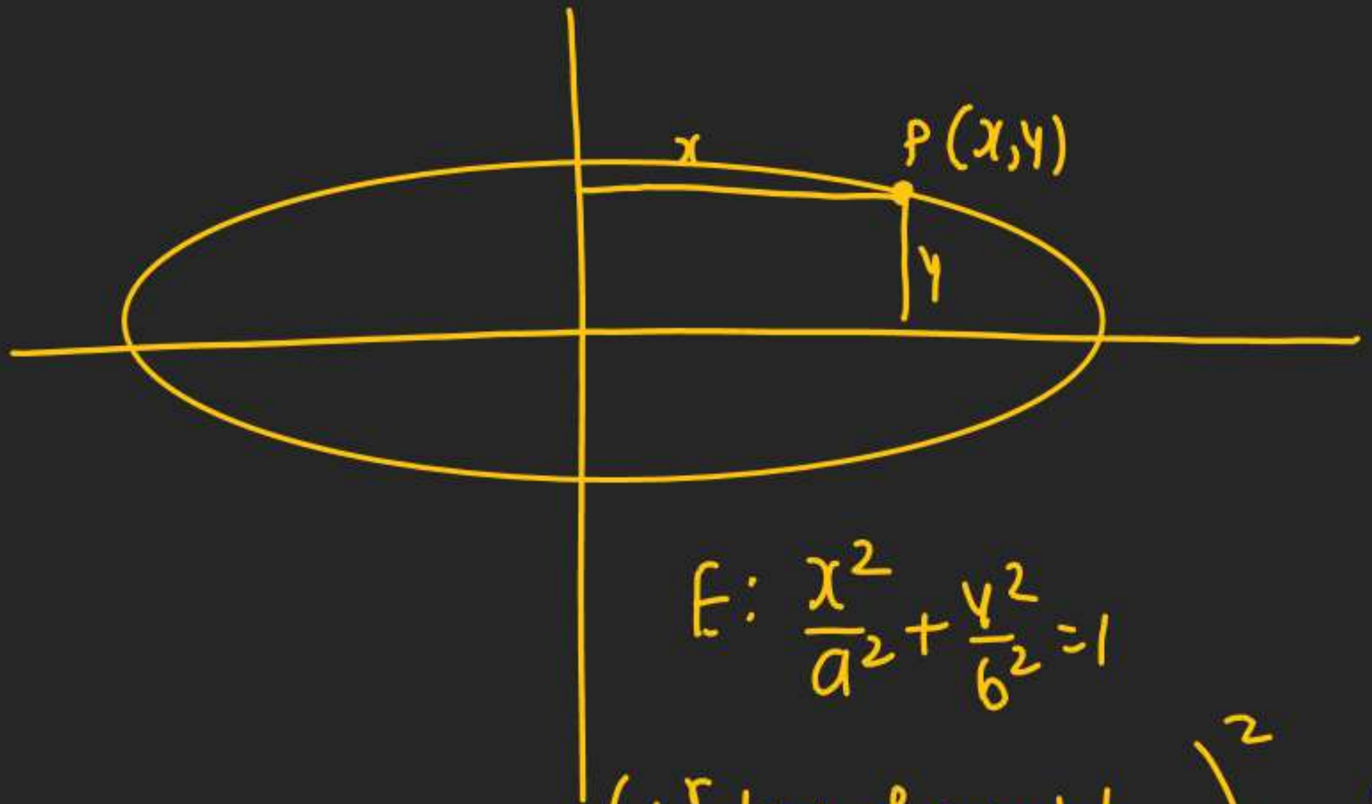
Teda Ellipse



E: $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

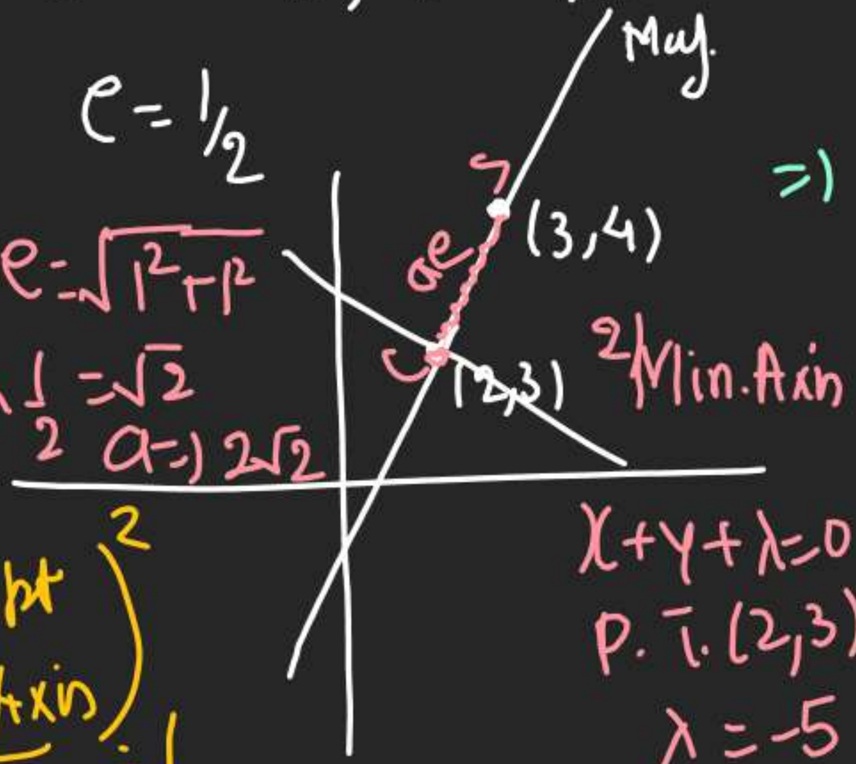
$$\frac{\left(\perp^r \text{ dist. of any pt from Minor Axis}\right)^2}{a^2}+\frac{\left(\perp^r \text{ Dist of any pt from Major Axis}\right)^2}{b^2}=1$$

$$\boxed{\frac{(\text{Min. Axis})^2}{a^2}+\frac{(\text{Maj. Axis})^2}{b^2}=1}$$

Q Find Eqn of Ellipse Whose Focus (3,4) & Centre (2,3) with $e=\frac{1}{2}$

(3) $ae=\sqrt{1^2+1^2}$

$a\times\frac{1}{2}=\sqrt{2}$ $a\Rightarrow 2\sqrt{2}$



$x+y+\lambda=0$

P. T. (2,3)

$\lambda=-5$

(1) Maj. Axis

$(y-3)=\frac{4-3}{3-2}(x-2)$

$x-y+1=0$

Min Axis $x+y-5=0$

(4) $1-e^2=\frac{b^2}{a^2}$

$1-\frac{1}{4}=\frac{b^2}{8}$

$b^2=6$

$$\frac{\left(\frac{x+y-5}{\sqrt{1^2+1^2}}\right)^2}{8}+\frac{\left(\frac{x-y+1}{\sqrt{1^2+1^2}}\right)^2}{6}=1$$

$$\Rightarrow \frac{(x+y-5)^2}{16}+\frac{(x-y+1)^2}{12}=1$$

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Q Find Eq^n of Ellipse whose axes are of lengths 6 & $2\sqrt{6}$ & their Eq^n are $x-3y+3=0$ & $3x+y-1=0$

(1) $2a=$ Major Axis $=6$ | Minor Axis $=2b=2\sqrt{6}$

$a=3$ | $b=\sqrt{6}$

(3) Maj. Axis $\Rightarrow x-3y+3=0$

Minor Axis $\Rightarrow 3x+y-1=0$

$$E: \frac{(\text{Min})^2}{a^2}+\frac{(\text{Maj})^2}{b^2}=1$$

$$E: \frac{\left(\frac{3x+y-1}{\sqrt{3^2+1^2}}\right)^2}{9}+\frac{\left(\frac{x-3y+3}{\sqrt{1^2+3^2}}\right)^2}{6}=1$$

$$\frac{(3x+y-1)^2}{90}+\frac{(x-3y+1)^2}{60}=1$$

Q If distance of any pt of Ellipse $\frac{x^2}{12}+\frac{y^2}{4}=1$ from Centre is $2\sqrt{3}$ then Ecc. angle?

$(a\cos\theta, b\sin\theta)$

$(2\sqrt{3}\cos\theta, 2\sin\theta)$

$$\sqrt{12\cos^2\theta+4\sin^2\theta}=2\sqrt{3}$$

$12\cos^2\theta+4\sin^2\theta=12$

$3\cos^2\theta+\sin^2\theta=3 \quad 0\le\theta<2\pi$

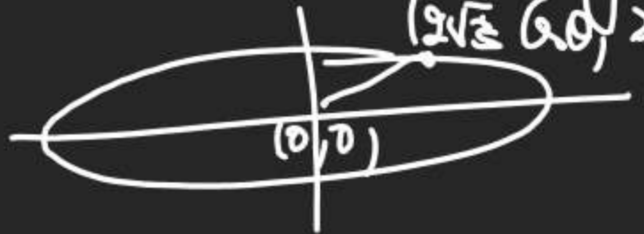
$2\cos^2\theta=2$

$\cos^2\theta=1,-1$

$\theta=0,\pi$

Q If distance of any pt. on Ellipse $\frac{x^2}{12}+\frac{y^2}{4}=1$ from Centre is $2\sqrt{2}$ then Ecc. angle?

$(2\sqrt{3}\cos\theta, 2\sin\theta)$



$12\cos^2\theta+4\sin^2\theta=8$

$3\cos^2\theta+\sin^2\theta=2$

$2\cos^2\theta=1 \Rightarrow \cos^2\theta=\frac{1}{2}$

$\cos\theta=\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}$

$\theta=\frac{\pi}{4}, 2\pi-\frac{\pi}{4}$ | $\pi-\frac{\pi}{4}, \pi+\frac{\pi}{4}$

$=\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

Line & Ellipse

$$y = mx + c$$

$$E: \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Combine Eqn

$$\frac{x^2}{a^2} + \frac{(mx+c)^2}{b^2} = 1$$

$$b^2x^2 + a^2m^2x^2 + a^2c^2 + 2a^2mcx = a^2b^2$$

$$\Rightarrow x^2(a^2m^2+b^2) + 2a^2mcx + c^2a^2 - a^2b^2 = 0$$

$$D = 4a^4m^2c^2 - 4(a^2m^2+b^2)(c^2a^2 - a^2b^2)$$

$$= \cancel{4a^4m^2c^2} - \cancel{4a^4m^2c^2} + 4a^4m^2b^2 - 4a^2b^2c^2 + 4a^2b^4$$

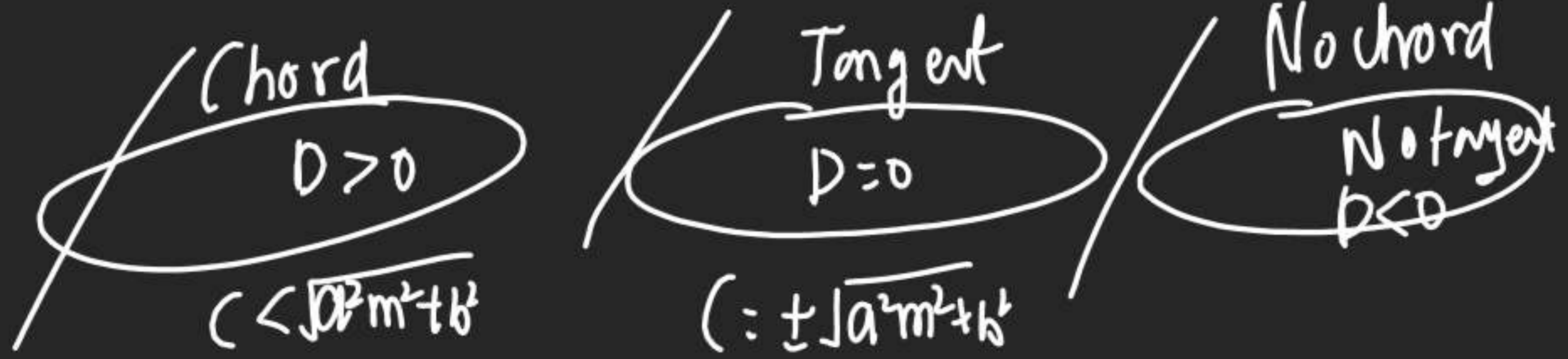
for tangent $\Rightarrow$ Cond^n of tangency

$$D = 0$$

$$4a^2b^2(a^2m^2 - c^2 + b^2) = 0$$

$$\Rightarrow c^2 = a^2m^2 + b^2$$

$$\boxed{c = \pm\sqrt{a^2m^2+b^2}}$$



Tangent

1) $y = mx \pm \sqrt{a^2m^2+b^2}$ is tangent (Slope form)

(2) $k = mh \pm \sqrt{a^2m^2+b^2}$

$$\Rightarrow k - mh = \pm\sqrt{a^2m^2+b^2}$$

$$k^2 + m^2h^2 - 2mkh = a^2m^2 + b^2$$

$$m_1, m_2 \leftarrow m^2(h^2-a^2) - 2mkh + k^2 - b^2 = 0$$

$$\boxed{m_1 + m_2 = \frac{2kh}{h^2-a^2} \qquad m_1 m_2 = \frac{k^2-b^2}{h^2-a^2}}$$

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Director Circle:

1) Locus.

2) Locus of $\perp^r$ tangent.

3) $m_1 m_2 = -1$

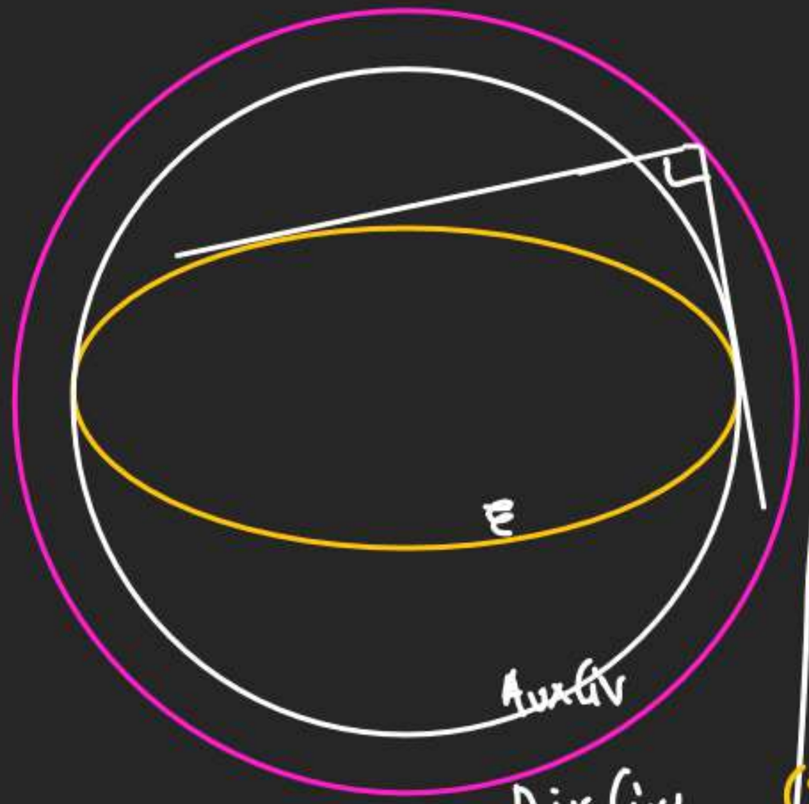
$$\frac{k^2-b^2}{h^2-a^2} = -1$$

$$\Rightarrow k^2-b^2 = a^2-h^2$$

$$\Rightarrow h^2+k^2 = a^2+b^2$$

$$\Rightarrow \boxed{x^2+y^2 = a^2+b^2}$$

$\text{Rad} = \sqrt{a^2+b^2}$



Q D.C. for $\frac{x^2}{12}+\frac{y^2}{4}=1$ is?

D.C: $x^2+y^2 = 12+4$

$x^2+y^2=16$

Q D.C. for $\frac{(x+1)^2}{3}+\frac{(y-1)^2}{7} = 1$ is

$(x+1)^2+(y-1)^2 = 3+7$

$(x+1)^2+(y-1)^2 = 10$

Q Find E.O.T. to $9x^2+16y^2=144$ from $(2,3)$.

Position of $(2,3)$

1) $E: \frac{9x^2}{144}+\frac{16y^2}{144}=1$

$\frac{x^2}{16}+\frac{y^2}{9}=1$

$E(2,3)$ $\quad \frac{4}{16}+\frac{9}{9}-1>0$ (outside)

(2) $y = mx \pm \sqrt{16m^2+9}$ P.T. $(2,3)$

$3 = 2m \pm \sqrt{16m^2+9}$

$\Rightarrow (3-2m) = \pm\sqrt{16m^2+9}$

$\Rightarrow \cancel{9}+4m^2-12m = 16m^2+\cancel{9}$

$12m^2+12m=0 \Rightarrow m=0,-1$

1) $(y-3)=0(x-2)$

$\Rightarrow y=3$

(2) $y-3=-1(x-2)$

$x+y=5$

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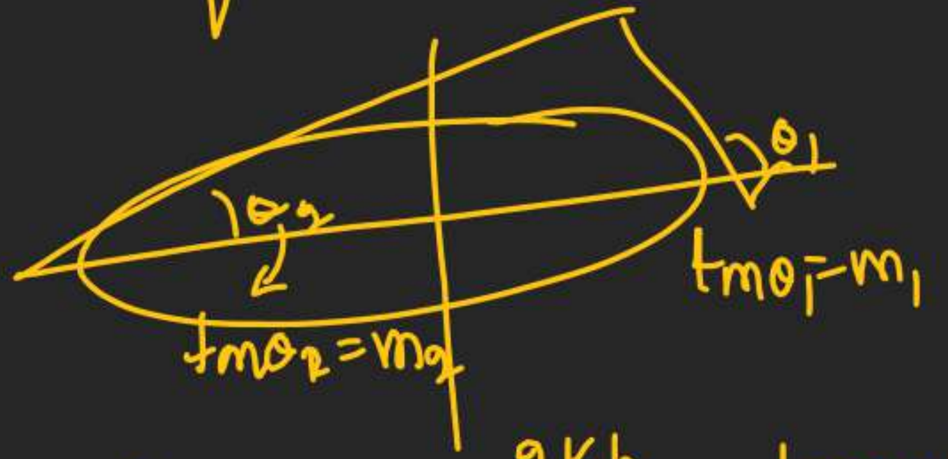
Q Tangent to Ellipse makes angles θ_1 & θ_2 with major Axis. Find Locus of their Point of Int. if $\cot\theta_1 + \cot\theta_2 = \lambda^2$



$$m_1 + m_2 = \frac{2kh}{h^2-a^2} = \tan\theta_1 + \tan\theta_2$$

$$m_1 m_2 = \frac{k^2-b^2}{h^2-a^2} = \tan\theta_1 \cdot \tan\theta$$

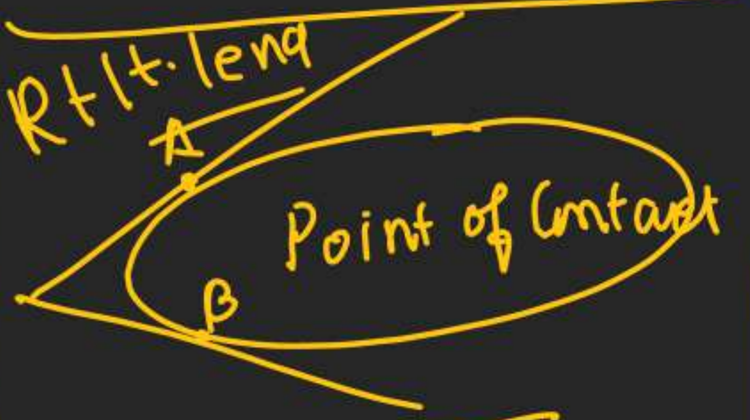
given $\Rightarrow \left(\frac{1}{\tan\theta_1} + \frac{1}{\tan\theta_2}\right) = \lambda^2$

$$\frac{\tan\theta_1 + \tan\theta_2}{\tan\theta_1 \cdot \tan\theta_2} = \lambda^2$$

$$\frac{\frac{2kh}{\cancel{h^2-a^2}}}{\frac{k^2-b^2}{\cancel{h^2-a^2}}} = \lambda^2$$

$\Rightarrow 2xy = \lambda^2(y^2-b^2)$ Locus

Rt lt. leng



$$y = mx \pm \sqrt{a^2m^2+b^2}$$

$$\left\{\mp \frac{a^2m}{\sqrt{a^2m^2+b^2}}, \pm \frac{b^2}{\sqrt{a^2m^2+b^2}}\right\}$$

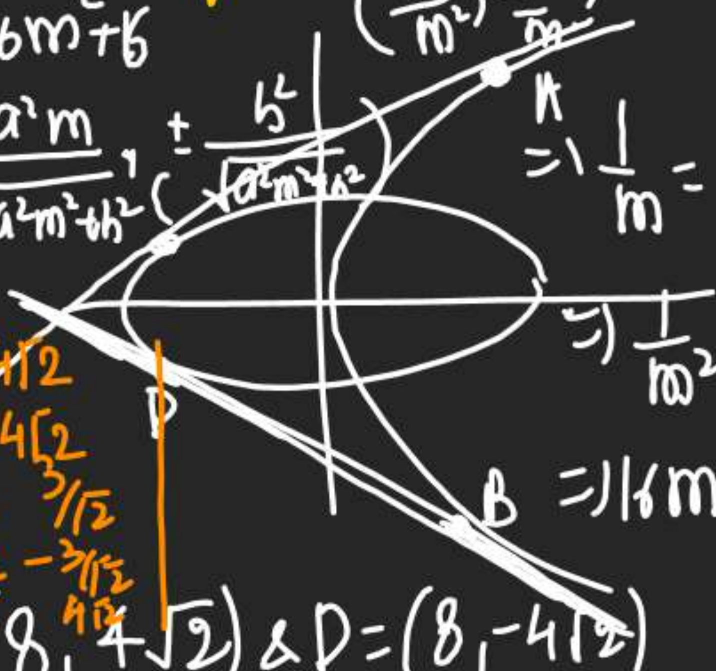
Q Com. tangents are drawn to $y^2 = 4x$ ($a=1$) & $3x^2 + 8y^2 = 48$ touching Parabola at A & B, Ellipse at C & D Find Area of ABCD Quad.

$$\frac{x^2}{16} + \frac{y^2}{6} = 1$$

$$y = mx \pm \sqrt{16m^2+6}$$

$$\left(\mp \frac{a^2m}{\sqrt{a^2m^2+b^2}}, \pm \frac{b^2}{\sqrt{a^2m^2+b^2}}\right)$$

$$\left(\frac{a}{m^2}, \frac{2a}{m}\right)$$



$$y = mx + \frac{1}{m}$$

$\Rightarrow \frac{1}{m} = \pm\sqrt{16m^2+6}$

$\Rightarrow \frac{1}{m^2} = 16m^2+6$

$\Rightarrow 16m^4 + 6m^2 - 1 = 0$

$m = \pm \frac{1}{2\sqrt2}$

$$\Delta = \frac{1}{2}\begin{vmatrix} 8 & 4\sqrt2 \\ 8 & -4\sqrt2 \\ -2 & \frac{3}{\sqrt2} \\ -2 & -\frac{3}{\sqrt2} \\ 8 & 4\sqrt2 \end{vmatrix}$$

$C = (8, 4\sqrt2)$ & $D = (8, -4\sqrt2)$

$$A \equiv \left(-\frac{16 \times \frac{1}{2\sqrt2}}{\sqrt{\frac{16}{8}+6}}, +\frac{6}{\sqrt{\frac{16}{8}+6}}\right) \equiv \left(-2, \frac{3}{\sqrt2}\right)$$

$$B \equiv \left(+\frac{16 \times \frac{-1}{2\sqrt2}}{\sqrt8}, -\frac{6}{\sqrt8}\right) \equiv \left(-2, -\frac{3}{\sqrt2}\right)$$