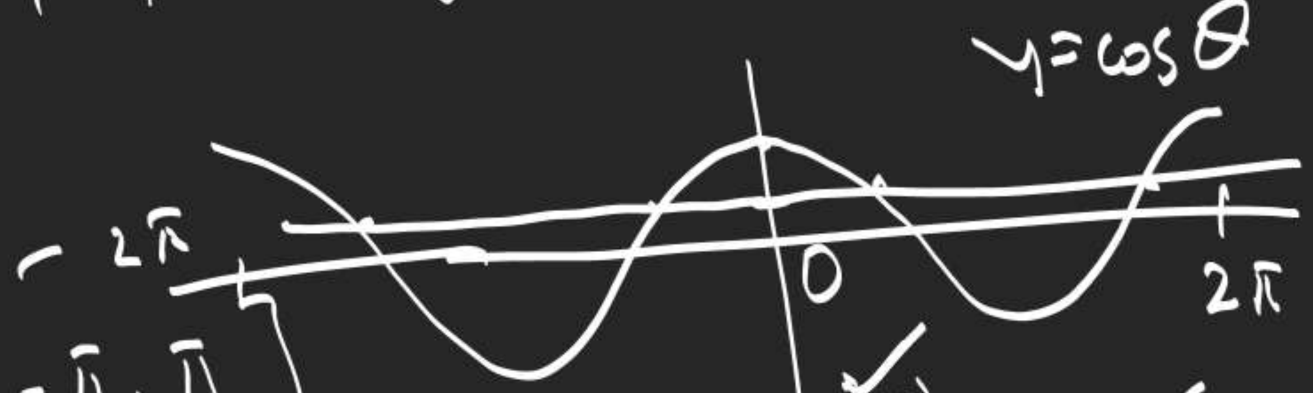
Nishant Jindal

$$2\sin x + 1 = 2\cos^2 x = 2 - 2\sin^2 x$$



$$\alpha \in [-\pi, \pi]$$

$$-\beta \in [-\pi, \pi]$$

$$\alpha - \beta \in [-2\pi, 2\pi]$$

$(-\pi, \pi)$

$\beta = \alpha + 2\pi$

$\beta = 0 + \alpha$

$\beta = \alpha - 2\pi$

$$\alpha - \beta = -2\pi, 0, 2\pi$$

$$(\alpha, \beta) = (\alpha, \alpha)$$

$$\cos(\alpha + \beta) = \cos 2\alpha = \frac{1}{e}$$

$$\boxed{2\alpha \in [-2\pi, 2\pi]}$$

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5.

$$(y+z)\cos 3\theta = xyz\sin 3\theta$$

$$xyz\sin 3\theta = 2z\cos 3\theta + 2y\sin 3\theta$$

$$xyz\sin 3\theta = (y+2z)\cos 3\theta + y\sin 3\theta$$

$$(y-z)\cos 3\theta - 2y\sin 3\theta = 0$$

$$0 = -y\cos 3\theta + y\sin 3\theta$$

$$\boxed{\tan 3\theta = 1}$$

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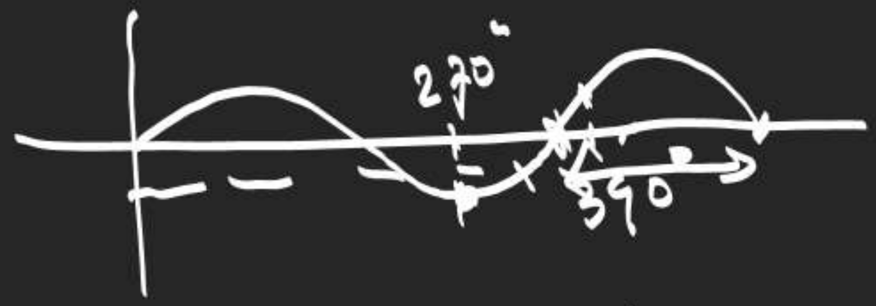
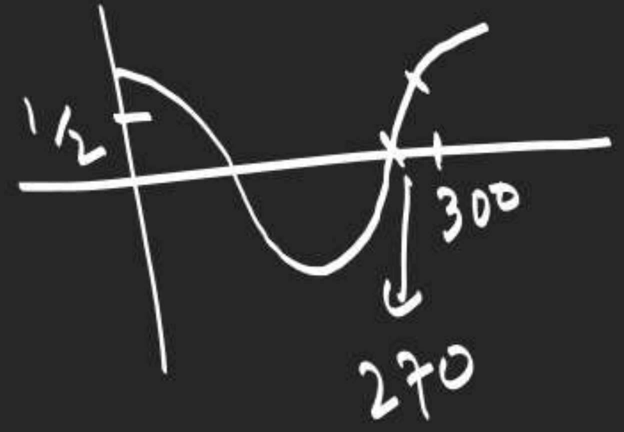
$$2\cos\theta - 2\cos\theta\sin\phi = 2\sin\theta\cos\phi - 1$$

$$(1,2) \Rightarrow 2\underbrace{\cos\theta}_{(0,\frac{1}{2})} + 1 = 2\sin(\theta+\phi)$$

$$\boxed{\sin(\theta+\phi) \in \left(\tfrac{1}{2}, 1\right)}\ \checkmark$$



$$\theta \in (270^\circ, 300^\circ)$$



$$\theta+\phi \in (270^\circ, 390^\circ)$$

$90, 240$ $\quad \boxed{360, 540}$ $\quad \phi \in 0, 90^\circ$

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$$\frac{1}{\sqrt{2}}(3-\sin2\theta)(\sin\theta-\cos\theta)-\frac{1}{\sqrt{2}}(\sin3\theta+\cos3\theta)$$

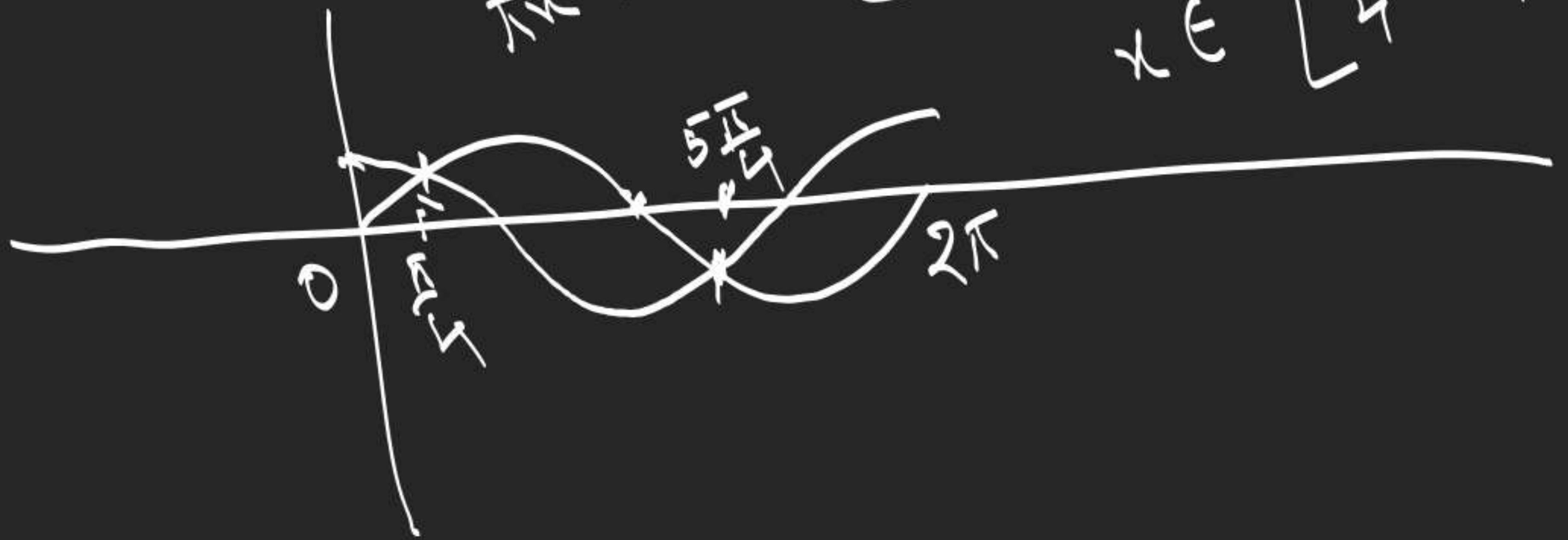
$$3(s-c)-4(s^3-c^3)$$

$$\frac{1}{\sqrt{2}}(s\theta-c\theta)\left[(3-s2\theta)-\left(3-4(1+s\theta c\theta)\right)\right]$$

$$\frac{1}{\sqrt{2}}(s\theta-c\theta)\left(\underbrace{4+\sin2\theta}_{>0}\right)\geq 0$$

$$\pi x=\theta\in\left[\frac{\pi}{4},\frac{5\pi}{4}\right]$$

$$x\in\left[\frac{1}{4},\frac{5}{4}\right]$$



1: Solve for x satisfying

$$\begin{vmatrix} u+a^2x & l+abx & m+acx \\ l+abx & v+b^2x & n+bcx \\ m+acx & n+bcx & w+c^2x \end{vmatrix} = 0$$ in terms of determinant.

$$C_2 \to aC_2 - bC_1, \quad C_3 \to aC_3 - cC_1$$

$$\frac{1}{a^2}\begin{vmatrix} u+a^2x & al-bu & am-cu \\ l+abx & av-bl & an-cl \\ m+acx & an-bm & aw-cm \end{vmatrix} = \frac{1}{a^2}\left[\begin{vmatrix} u & al-bu & am-cu \\ l & av-bl & an-cl \\ m & an-bm & aw-cm \end{vmatrix} + x\begin{vmatrix} a^2 & al-bu & am-cu \\ ab & av-bl & an-cl \\ ac & an-bm & aw-cm \end{vmatrix}\right] = 0$$

$=$

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Q. Simplify
$$\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix}$$

$\downarrow C_1 \to C_1 - bC_3$; $C_2 \to C_2 + aC_3$

$$= (1+a^2+b^2)^3$$

$$(1+a^2+b^2)^2 \begin{vmatrix} 1 & 0 & -2b \\ 0 & 1 & 2a \\ b & -a & 1-a^2-b^2 \end{vmatrix}$$

$R_3 \to R_3 - bR_1$

$$(1+a^2+b^2)^2 \begin{vmatrix} 1 & 0 & -2b \\ 0 & 1 & 2a \\ 0 & -a & 1-a^2+b^2 \end{vmatrix}$$

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Q: Let 3 digit numbers $\underline{A28}$, $3B9$, $62C$ where A, B, C are integer between 0 and 9 be divisible by integer K, then P.T.

$$\begin{vmatrix} A & 3 & 6 \\ \underset{\lambda_1 K}{\underline{A28}} & \underset{\lambda_2 K}{\underline{3B9}} & \underset{\lambda_3 K}{\underline{62C}} \\ 2 & B & 2 \end{vmatrix} = \begin{vmatrix} A & 3 & 6 \\ 8 & 9 & C \\ 2 & B & 2 \end{vmatrix}$$ is also divisible by K.

$R_2 \to R_2 + 10R_3 + 100R_1$

$$= K \begin{vmatrix} A & 3 & 6 \\ \lambda_1 & \lambda_2 & \lambda_3 \\ 2 & B & 2 \end{vmatrix} = \lambda K$$

$\lambda \in I$

$a_n a_{n-1} a_{n-2} \cdots a_1 a_0$

$= a_0 10^0 + a_1 10^1 + a_2 10^2 + \cdots a_n 10^n$

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4. Simplify $\begin{vmatrix} a & b-c & b+c \\ a+c & b & c-a \\ a-b & a+b & c \end{vmatrix}$

$C_1 \to aC_1 + bC_2 + cC_3$

$$\frac{1}{abc}\begin{vmatrix} a^2 & ab-ac & ab+ac \\ ba+bc & b^2 & bc-ba \\ ca-cb & ca+cb & c^2 \end{vmatrix}$$

$R_1 \to R_1 + R_2 + R_3$

$(a+b+c)$

$$\frac{1}{abc}\begin{vmatrix} a^2 & b^2-bc & bc+c^2 \\ a^2+ac & b^2 & c^2-ac \\ a^2-ab & ab+b^2 & c^2 \end{vmatrix} \xrightarrow{C_1 \to C_1 + C_2 + C_3} \frac{a^2+b^2+c^2}{abc}\begin{vmatrix} 1 & b^2-bc & bc+c^2 \\ 1 & b^2 & c^2-ac \\ 1 & ab+b^2 & c^2 \end{vmatrix}$$

$$\frac{(a^2+b^2+c^2)}{a}\begin{vmatrix} 1 & b-c & b+c \\ 1 & b & c-a \\ 1 & a+b & c \end{vmatrix}$$

$R_2 \to R_2 - R_1$

$R_3 \to R_3 - R_1$

$$\frac{(a^2+b^2+c^2)}{a}\begin{vmatrix} 1 & b-c & b+c \\ 0 & c & -a-b \\ 0 & a+c & -b \end{vmatrix}$$

$$\| $$

$$(a^2+b^2+c^2)(a+b+c)$$

5.

$$\begin{vmatrix} a^2+x^2 & ab & ca \\ ab & b^2+x^2 & bc \\ ca & bc & c^2+x^2 \end{vmatrix} = \frac{1}{abc}\begin{vmatrix} a(a^2+x^2) & a^2b & ca^2 \\ ab^2 & b(b^2+x^2) & b^2c \\ c^2a & bc^2 & c(c^2+x^2) \end{vmatrix}$$

$$= \begin{vmatrix} a^2+x^2 & a^2 & a^2 \\ b^2 & b^2+x^2 & b^2 \\ c^2 & c^2 & c^2+x^2 \end{vmatrix}$$

$$\xleftarrow{R_1 \to R_1+R_2+R_3}$$

$$(a^2+b^2+c^2+x^2)\begin{vmatrix} 1 & 1 & 1 \\ b^2 & b^2+x^2 & b^2 \\ c^2 & c^2 & c^2+x^2 \end{vmatrix}$$

$$\downarrow \quad C_2 \to C_2 - C_1, \quad C_3 \to C_3 - C_1$$

$$= (a^2+b^2+c^2+x^2)x^4$$

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6. $\begin{vmatrix} ax-by-cz & ay+bx & cx+az \\ ay+bx & by-cz-ax & bz+cy \\ cx+az & bz+cy & cz-ax-by \end{vmatrix}$

$C_1 \to xC_1 + yC_2 + zC_3$

$\frac{(x^2+y^2+z^2)}{x}\begin{vmatrix} a & ay+bx & cx+az \\ b & by-cz-ax & bz+cy \\ c & bz+cy & cz-ax-by \end{vmatrix} \xrightarrow{R_1 \to aR_1 + bR_2 + cR_3}$

$\frac{(a^2+b^2+c^2)(x^2+y^2+z^2)}{ax}\begin{vmatrix} 1 & y & z \\ b & by-cz-ax & bz+cy \\ c & bz+cy & cz-ax-by \end{vmatrix}$

$\xleftarrow[R_3 \to R_3 - cR_1]{R_2 \to R_2 - bR_1}$

$\left(\sum a^2\right)\left(\sum x^2\right)\left(\sum ax\right)$