ELECTROSTATICS

Type of Equilibrium $[F_{net} = 0]$

Stable ↓   Unstable ↓   Neutral ↓

Stable

↳ There must be a restoring force which always restore the position of the body towards the equilibrium position (Mean position)

Unstable

↳ [No restoring force]
If body is displaced for equibrium position it never come back towards equilibrium position

Neutral

↳ [Body always at rest]

ELECTROSTATICS

mg

Mean
position
or
Equilibrium
position

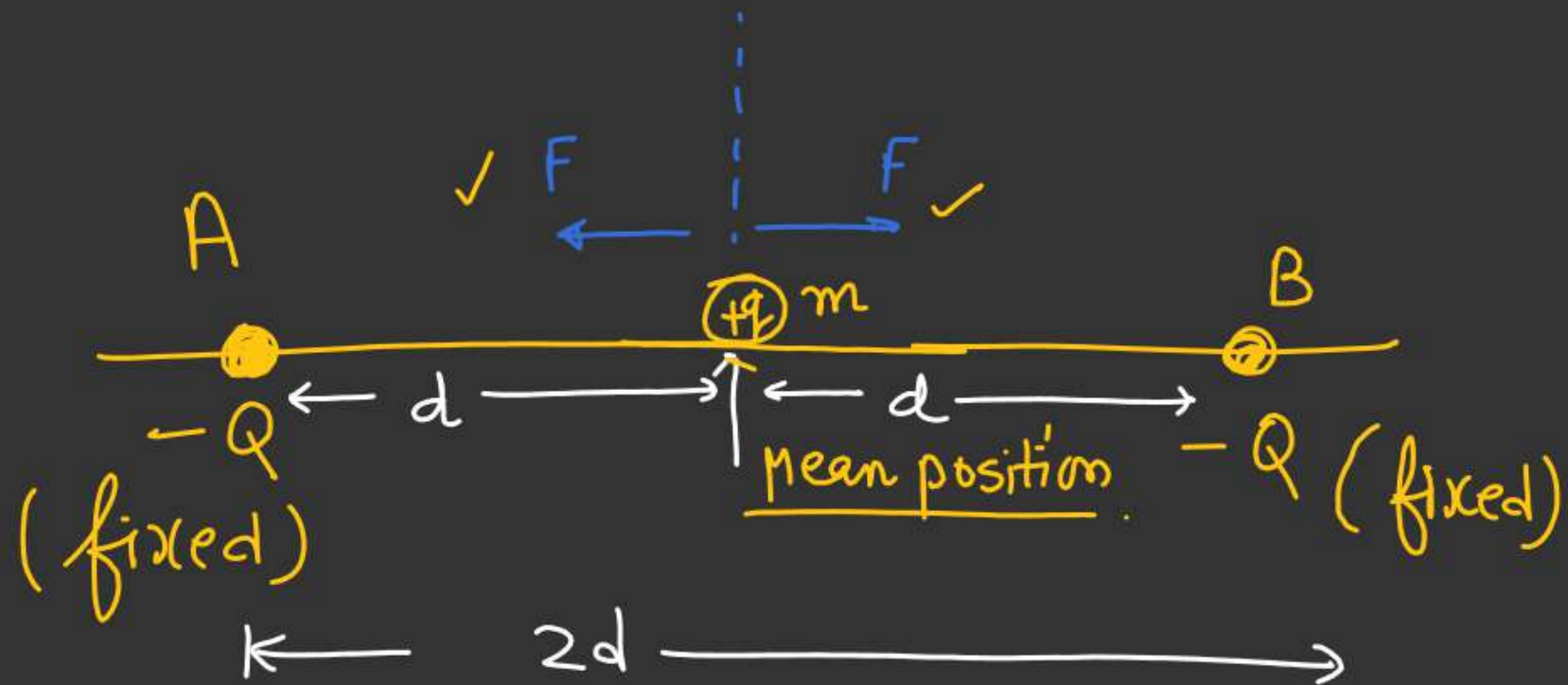$F_{net} = 0$

Stable
Equilibrium

$mg\sin\theta$

F_{net}

$mg\sin\theta$

(Unstable Equilibrium)

$F_{net} = 0$

Neutral
Equilibrium.
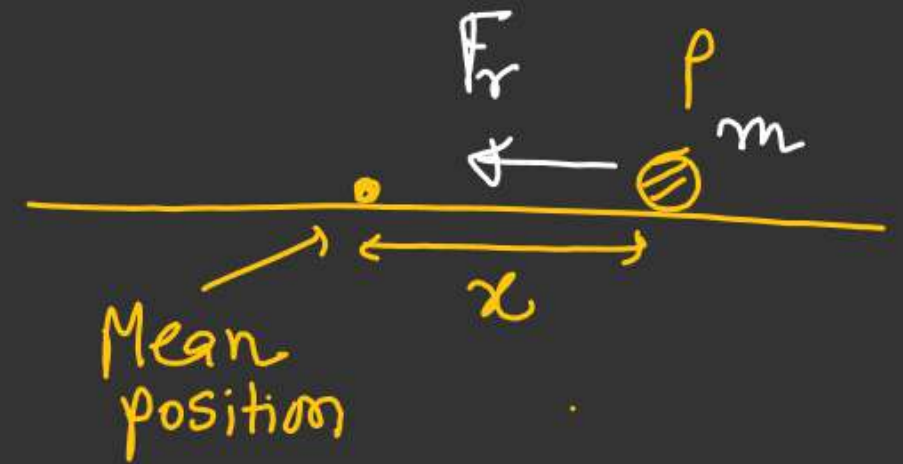
Discuss the motion of $-q$ charge when
a) displaced along the line joinig
b) displaced perpendicular to line joing.

a)

A ✓ F $\longleftarrow$ | $\longrightarrow$ F ✓ B

$+q$ m

$-Q$ (fixed) $\longleftarrow d \longrightarrow$ | $\longleftarrow d \longrightarrow$ $-Q$ (fixed)

mean position

$\longleftarrow 2d \longrightarrow$

Recape.

S.H.M

F_r ← P m

Mean position

x

$F_r \propto x$

$$F_r = \ominus Kx$$

always

restoring in nature
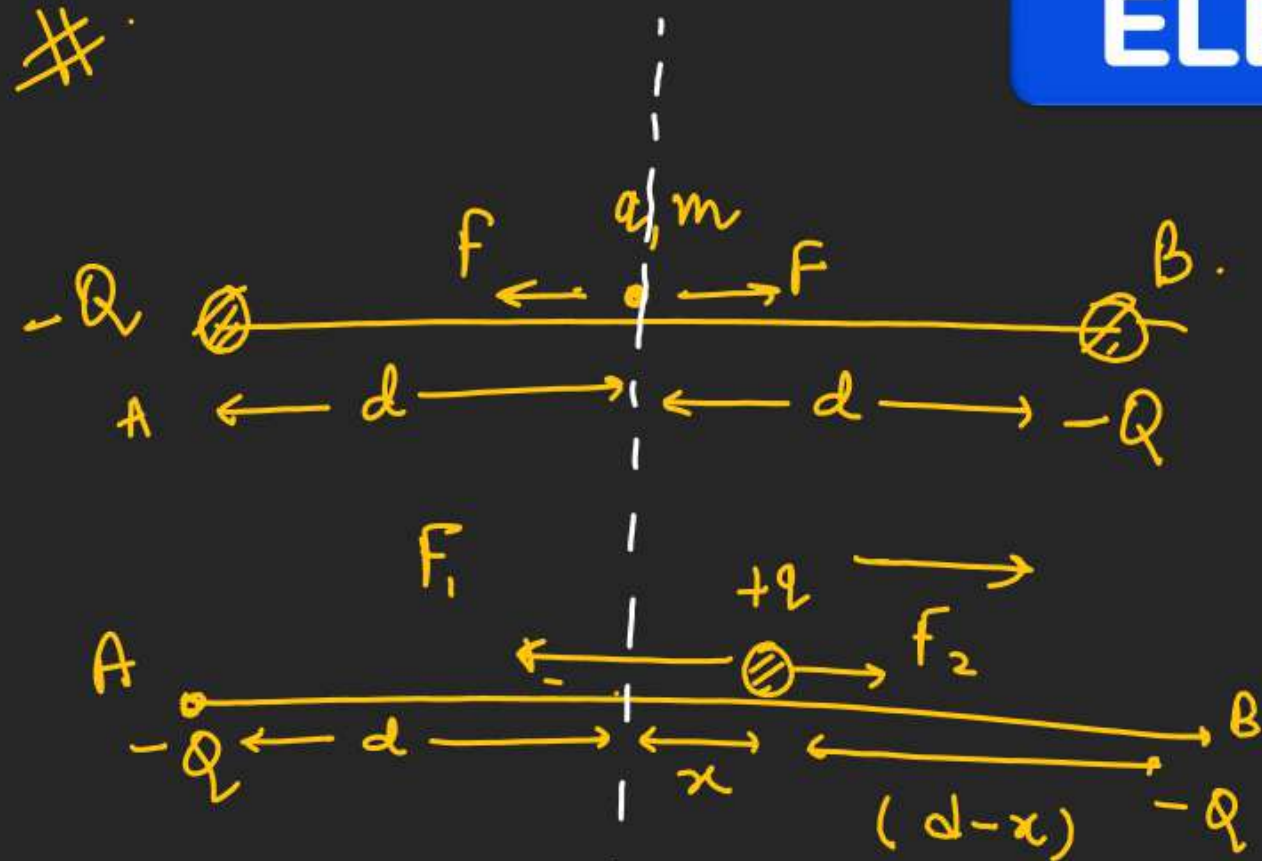
$$a = \frac{F_r}{m} = \frac{-Kx}{m}$$

$$a = -\omega^2 x$$

$\omega^2 = \frac{K}{m}$

$\omega = \sqrt{\frac{K}{m}}$

$$T = \frac{2\pi}{\omega}$$

ELECTROSTATICS

$-Q$ $\xleftarrow{F}$ q,m $\xrightarrow{F}$ B.

$A \xleftarrow{\quad d \quad} \xleftarrow{\quad d \quad} -Q$

F_1 $+q$ $\xrightarrow{F_2}$

A $-Q \xleftarrow{\quad d \quad} \xleftarrow{x} \xleftarrow{(d-x)} -Q$ B

$$F_1 = \frac{KQq}{(d+x)^2}$$

$$F_2 = \frac{KQq}{(d-x)^2}$$

$F_2 > F_1$

$\Rightarrow \boxed{\text{Unstable Equilibrium}}$

(b)

$fSin\theta$ C $+q$ $fSin\theta$

$\sqrt{d^2+y^2}$ F θ θ F

A $2F\cos\theta$ y D B

$-Q \xleftarrow{\quad d \quad} \xleftarrow{\quad d \quad} -Q$

$\sqrt{d^2+y^2}$ θ y C

A d D

$$F_r = -2F\cos\theta$$

$$F_r = \frac{-2KQq}{(\sqrt{d^2+y^2})^2} \times \frac{y}{(\sqrt{d^2+y^2})}$$

$$F_r = -\frac{2KQq}{(d^2+y^2)^{3/2}} y$$

ELECTROSTATICS

$$\Rightarrow \quad F_\gamma = -\frac{2KQq}{(d^2+y^2)^{3/2}} \cdot y$$

$$\boxed{F_\gamma = -Kx}$$

$$\text{if} \quad \boxed{y << d}$$

$$F_\gamma = -(2KQq) \frac{y}{d^3\left(1+\dfrac{y^2}{d^2}\right)^{3/2}}$$

$$y << d \qquad \frac{y}{d} \to 0$$

$$F_\gamma = -\left(\frac{2KQq}{d^3}\right) y$$

$$\underbrace{\qquad\qquad}_{K}$$
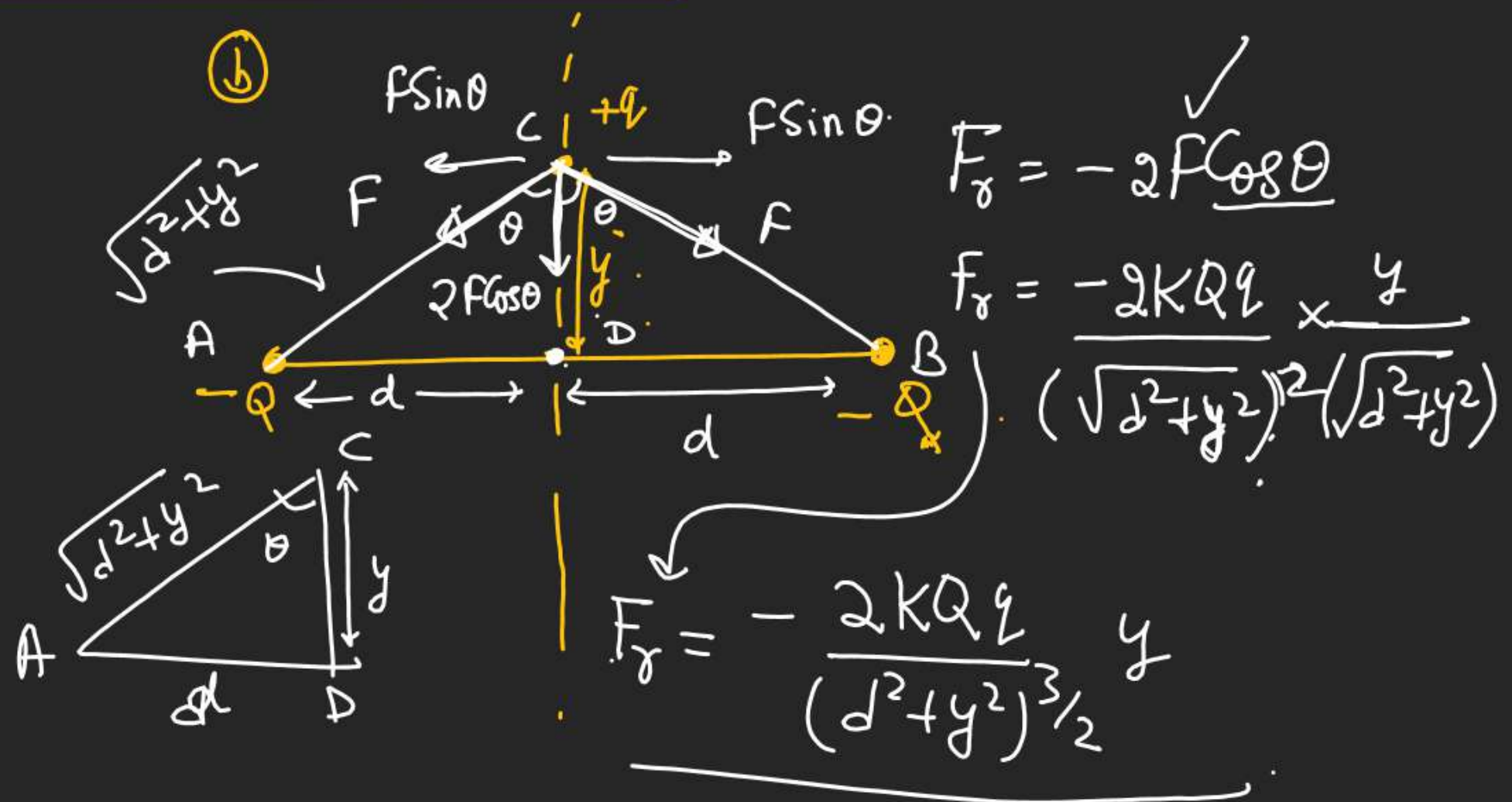
$$\circledast \quad (1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2$$

$$\text{if} \quad x << 1$$

$$\boxed{(1+x)^n = 1+nx}$$

$$\circledast$$

$$a = -\left(\frac{2KQq}{md^3}\right) y$$

$$a = - \quad \omega^2 y$$

$$\omega = \sqrt{\frac{2KQq}{md^3}} , \quad T = \frac{2\pi}{\omega}$$

ELECTROSTATICS

$$\omega = \sqrt{\frac{2KQq}{md^3}} \quad \#$$
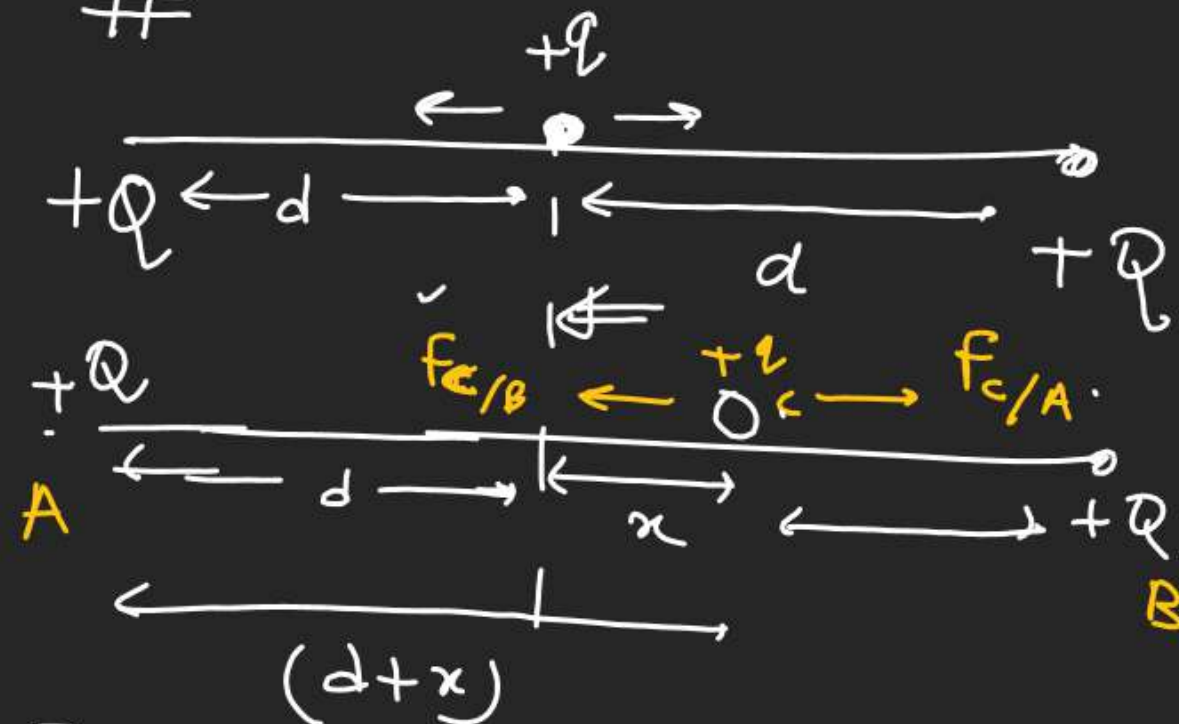
$$\#$$

$$T = \frac{2\pi}{\omega}$$

$$T = 2\pi \sqrt{\frac{md^3}{2KQq}}$$

$$K = \frac{1}{4\pi\varepsilon_0}$$

$$T = 2\pi \sqrt{\frac{md^3}{2Qq \times \frac{1}{4\pi\varepsilon_0}}}$$

$$\boxed{T = 2\pi \sqrt{\frac{2\pi\varepsilon_0 md^3}{Qq}}}$$

$$F_\gamma = -\left[F_{c/B} - F_{c/A} \right]$$

$$F_\gamma = -\left[\frac{KQq}{(d-x)^2} - \frac{KQq}{(d+x)^2} \right]$$

ELECTROSTATICS

$$x << d$$

$$F_r = -\left[\frac{KQq}{(d-x)^2} - \frac{KPq}{(d+x)^2} \right]$$

$$F_r = -KQq \left[\frac{1}{(d-x)^2} - \frac{1}{(d+x)^2} \right]$$

$$F_r = -KQq \left[\frac{(d+x)^2 - (d-x)^2}{(d^2-x^2)^2} \right]$$

$$F_r = -KQq \left[\frac{\cancel{d^2}+\cancel{x^2}+2dx - \cancel{d^2}-\cancel{x^2}+2dx}{(d^2-x^2)^2} \right]$$

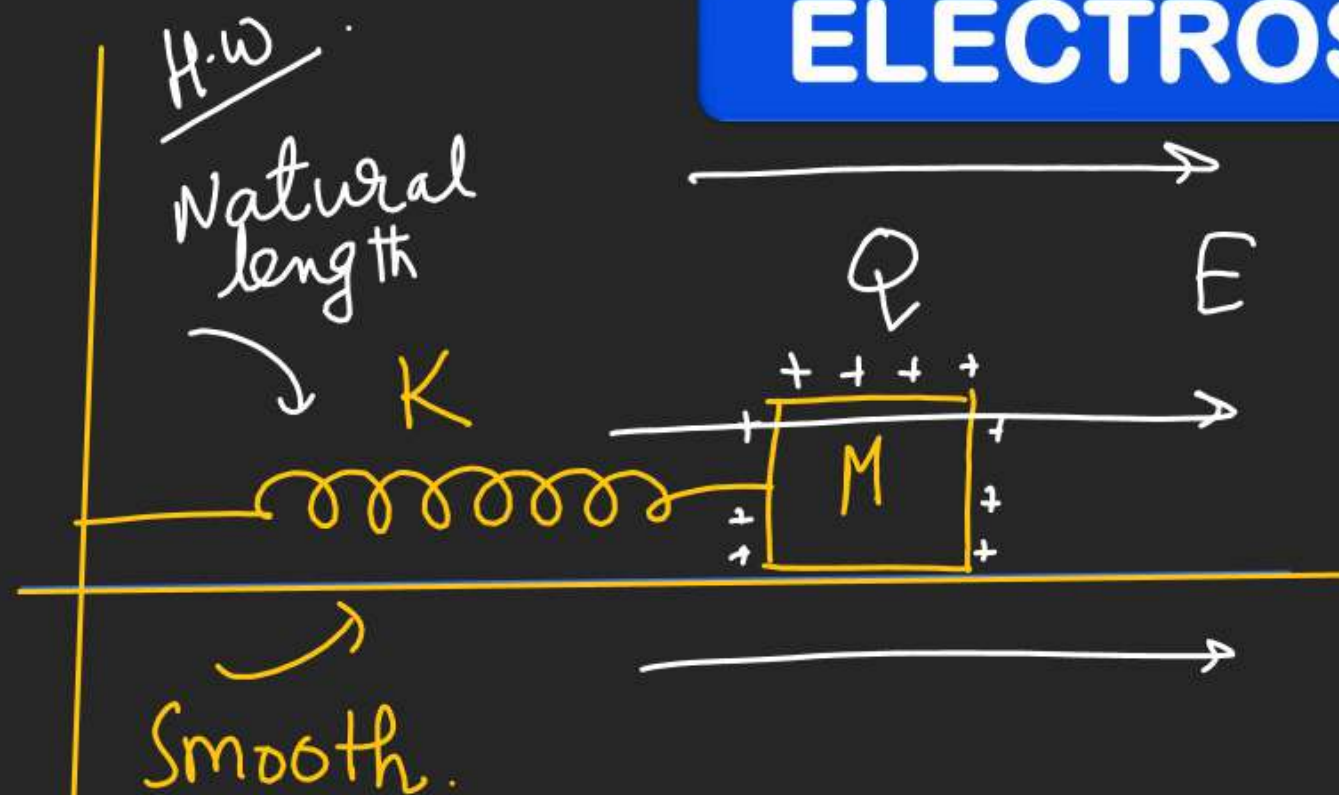$$F_r = \frac{-KQq \, 4d \, x}{(d^2-x^2)^2}$$

$$F_r = -\frac{KQq \, 4d}{\cancel{d^4}\left(1 - \frac{x^2}{d^2}\right)^2} x$$

$$F_r = \frac{-1}{4\pi\varepsilon_o} \frac{Qq \, 4}{d^3} x \downarrow_{0}$$

$$F_r = -\left(\frac{Qq}{\pi\varepsilon_o d^3} \right) x.$$

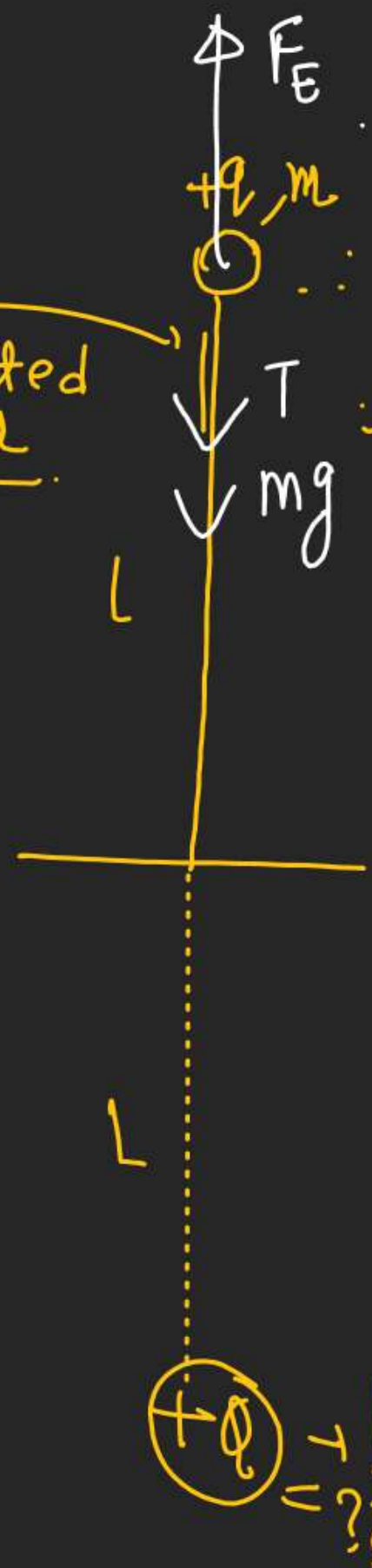**ELECTROSTATICS**

H.W :

Natural length

K

Q   E

M

Smooth.

An Uniform Electric field Switched on. prove that Motion is S.H.M. find its time period.

ELECTROSTATICS

F_E

$+q, m$

Insulated thread

T

mg

L

L

$\boxed{F_E > mg}$ ✓

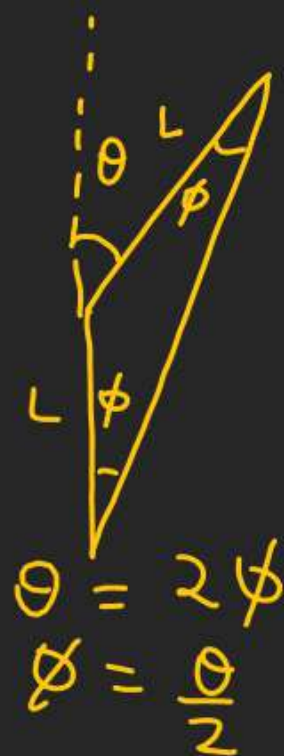$\dfrac{KQq}{(2L)^2} > mg$

$Q > \left(\dfrac{4mgL^2}{Kq}\right)$ ①

$+Q \rightarrow$ fixed
$=??$

Find the value of $+Q$ so the $+q$ charge is in stable equilibrium for a very small horizontal displacement
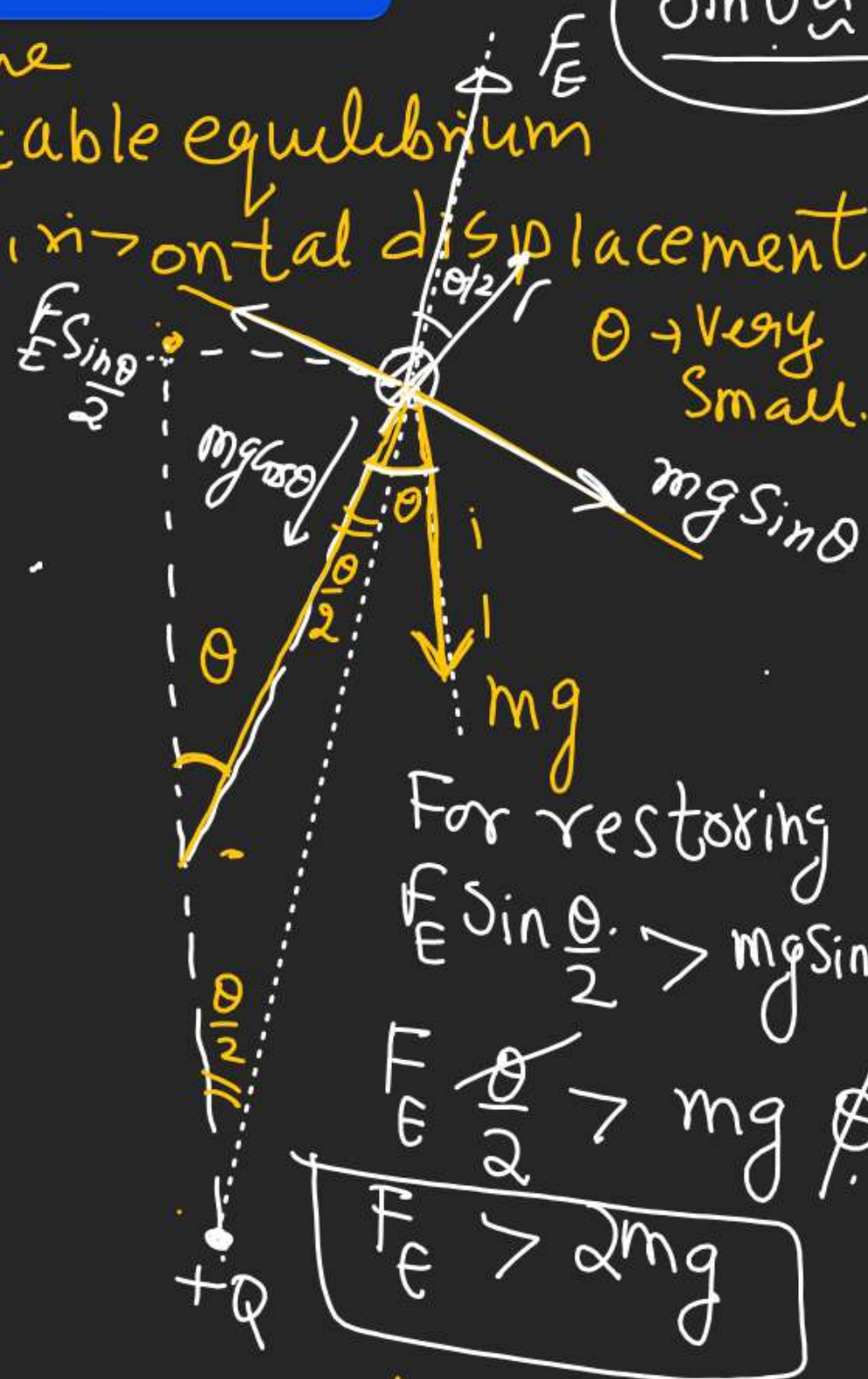
$F_E > mg$

$\boxed{F_E > mg}$

θ L
ϕ

L ϕ

$\theta = 2\phi$

$\phi = \dfrac{\theta}{2}$

$\boxed{\sin\theta \approx \theta}$

F_E

$\dfrac{\theta}{2}$

$\theta \rightarrow$ very small.

$\dfrac{F_E \sin\theta}{2}$

$mg\cos\theta$

θ

$\dfrac{\theta}{2}$

$mg\sin\theta$

mg

For restoring

$F_E \sin\dfrac{\theta}{2} > mg\sin\theta$

$F_E \dfrac{\theta}{2} > mg\,\theta$

$\boxed{F_E > 2mg}$

$+Q$

ELECTROSTATICS

$$F_E > 2mg$$

$$\frac{1}{4\pi\varepsilon_0} \frac{Q \cdot q}{(2L)^2} > 2mg$$

$$Q > \left(\frac{32\pi\varepsilon_0 L^2 mg}{q}\right)$$

$F_E \sin\frac{\theta}{2}$ ✓

$+q$

$mg\sin\theta$

$+q$

$2L$

θ

$\frac{\theta}{2}$

$\frac{\theta}{2}$
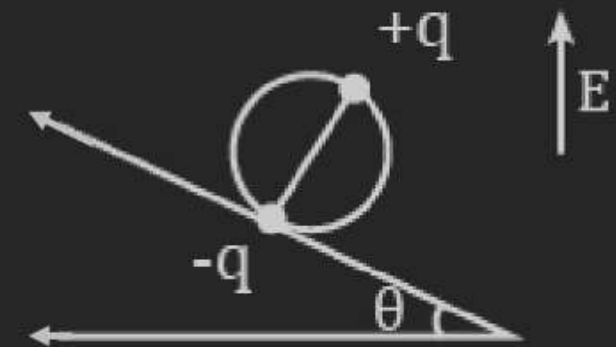
$+q$

$+q$

F_E

mg

$2L$
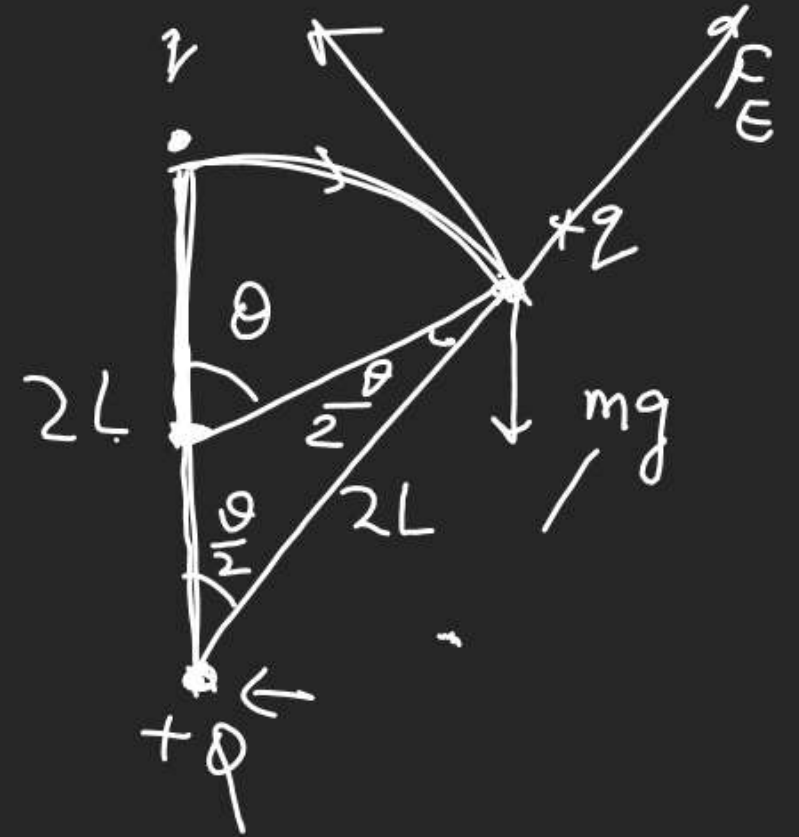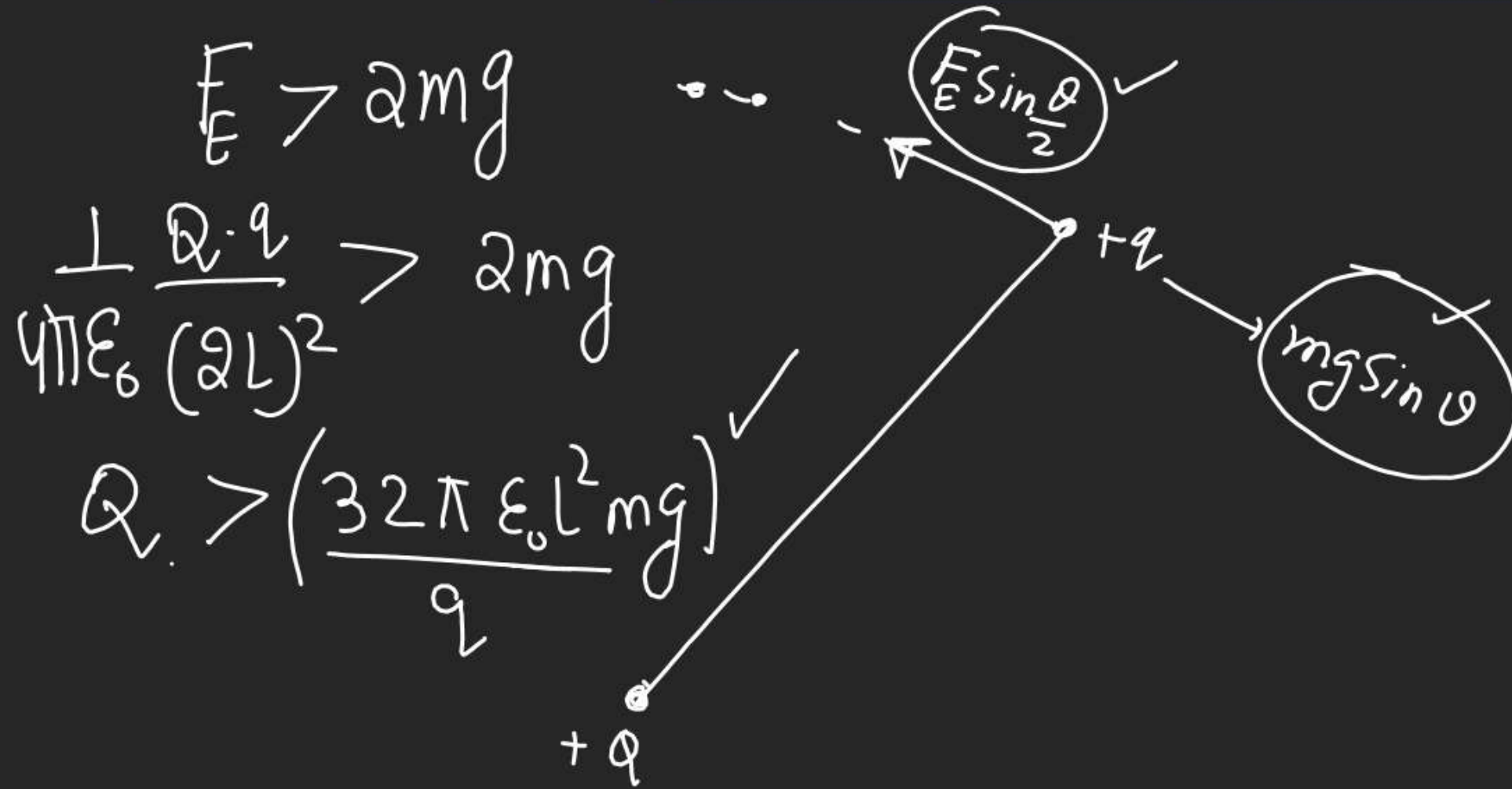
COULOMB'S LAW

Q. A wheel having mass m has charges $+q$ and $-q$ on diametrically opposite points. It remains in equilibrium on a rough inclined plane in the presence of uniform vertical electric field $E =$:

(A) $\dfrac{mg}{q}$

(B) $\dfrac{mg}{2q}$

(C) $\dfrac{mg\tan\theta}{2q}$

(D) none

COULOMB'S LAW

Q. A certain charge Q is divided into two parts q and $(Q-q)$. How should the charges Q and q be divided so that q and $(Q-q)$ placed at a certain distance apart experience maximum electrostatic repulsion?   **[JEE Mains - 2021]**

(A) $Q = 4q$

(B) $Q = \dfrac{q}{2}$

(C) $Q = 3q$

(D) $Q = 2q$

COULOMB'S LAW

Q. Two identical conducting spheres with negligible volume have 2.1nC and -0.1nC charges, respectively. They are brought into contact and then separated by a distance of 0.5 m. The electrostatic force acting between the spheres is _____ $\times 10^{-9}$ N.

[Given : $4\pi\varepsilon_0 = \dfrac{1}{9 \times 10^9}$ SI unit]

COULOMB'S LAW

Q. A particle of mass 1mg and charge q is lying at the mid-point of two stationary particles kept at a distance ' 2 m ' when each is carrying same charge ' q '. If the free charged particle is displaced from its equilibrium position through distance ' x ' $(x < 1\text{ m})$. The particle executes SHM. Its angular frequency of oscillation will be_____ $\times 10^5\,\text{rad/s}$ if $q^2 = 10C^2$

[JEE Mains - 2021]

COULOMB'S LAW

Q. Point charges $5C$ and $-2C$ are located at $(2, 0, 4)$ and $(-3, 0, 5)$ respectively

(A) Force experienced by $1C$ point charge at $(1, -3, 7)$ is

$$\left[\frac{5}{4\pi\varepsilon_0} \frac{(-\hat{i} - 3\hat{j} + 3\hat{k})}{(19)^{\frac{3}{2}}} - \frac{2}{4\pi\varepsilon_0} \frac{(4\hat{i} - 3\hat{j} + 2\hat{k})}{(29)^{\frac{3}{2}}} \right]$$

(B) Force experienced by $1C$ point charge at $(1, -3, 7)$ is

$$\left[\frac{5}{4\pi\varepsilon_0} \frac{(\hat{i} + 3\hat{j} - 3\hat{k})}{(19)^{\frac{3}{2}}} + \frac{2}{4\pi\varepsilon_0} (4\hat{i} - 3\hat{j} + 2\hat{k}) \right]$$

(C) Force on $-2C$ charge is $\dfrac{10}{4\pi\varepsilon_0} \left(\dfrac{+5\hat{i} - \hat{k}}{(26)^{\frac{3}{2}}} \right)$

(D) Force on $5C$ charge is $\dfrac{10}{4\pi\varepsilon_0} \left(\dfrac{+5\hat{i} - \hat{k}}{(26)^{\frac{3}{2}}} \right)$

COULOMB'S LAW

Q. A tiny spherical oil drop carrying a net charge q is balanced in still air with a vertical uniform electric field of strength $\frac{81\pi}{7} \times 10^5 \text{Vm}^{-1}$. When the field is switched off, the drop is observed to fall with terminal velocity $2 \times 10^{-3} \text{ ms}^{-1}$. Given $g = 9.8 \text{ ms}^{-2}$, viscosity of the air $= 1.8 \times 10^{-5} \text{Nsm}^{-2}$ and the density of oil $= 900 \text{ kg m}^{-3}$, the magnitude of q is [JEE (Adv.) - 2019]

(A) $1.6 \times 10^{-19} \text{C}$

(B) $3.2 \times 10^{-19} \text{C}$

(C) $4.8 \times 10^{-19} \text{C}$

(D) $8.0 \times 10^{-19} \text{C}$