

Distribution of alike objects

Aim → To find 'n' over 'p' persons
 $(x_1, x_2, x_3) = (1, 2, 3), (1, 3, 2), \dots$
 $n+p-1$

Method →
$$\frac{(n + (p-1))!}{n! (p-1)!} = {}^{n+p-1}C_{p-1}$$

$6+2 {}^2C_2$

alike
6 objects over 3 persons

$x_1 + x_2 + x_3 = 6$

$x_1, x_2, x_3 \in \text{Whole no.s}$

—, (0, 0, 6), (0, 6, 0), (6, 0, 0)

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To distribute n alike objects over p persons
 ($n > p$) so that every person gets atleast one
 object.

\downarrow X
 Distribute 1 object
 to each person

$$(n-p) + p-1 \quad \binom{n-p}{p-1}$$

$$x+y+z=6, x,y,z \in \mathbb{N}$$

$$\downarrow \quad \downarrow \quad \downarrow$$

$$p_1 \quad p_2 \quad p_3$$

$$n-1 \quad \binom{n-1}{p-1}$$

$$6-3+2 \quad \binom{6-3+2}{2}$$

$$0x \ 0x \ 0x \dots x \ 0x \dots x \ 0$$

$$Q_1 \quad Q_2 \quad Q_3 \quad Q_8$$

$$3$$

$$30 - 2(8) + 7$$

$$C_7$$

$$= \boxed{{}^{21}C_7}$$

Find no. of solns of $\xleftarrow{Q_1+Q_2} x_1 + x_2 + x_3 + \dots + x_8 = 30$
 s.t. $x_i \geq 2, x_i \in \mathbb{I}$.
 mark $\cap Q_1$

2. Find no. of natural number soln.
of eqn. $x + y + z = 102$

$$\begin{matrix} \geq 1 & \geq 1 & \geq 1 \\ x & + & y & + & z & = & 102 \end{matrix}$$

$$102 - 3 + 2 = \binom{101}{2}$$

DPP-9 (remaining)
DPP-10 (1-7)

$$x + y + z + t = 0, 1, 2, 3, \dots, 30$$

$$\binom{101}{2} = \binom{4}{3} + \binom{5}{3} + \binom{6}{3} + \dots + \binom{33}{3}$$

3. Find no. of non negative integral solution
of (i) inequality $x + y + z + t \leq 30$

$$= \boxed{{}^{34}C_4}$$

(ii) in equality

$$23 < x + y + z + t \leq 30$$

$$x + y + z + t + u = 30$$

$$\boxed{{}^{34}C_4 - {}^{27}C_4}$$

$$24 \leq \leq 30 \quad u = 0, 1$$

$$23 < x+y+z+t \leq 30$$

$$x+y+z+t = 24, 25, 26, 27, 28, 29, 30$$

$$-2^7 \binom{27}{4} + \left(2^7 \binom{27}{4} + 2^7 \binom{27}{3} + 2^8 \binom{28}{4} + 2^9 \binom{29}{4} + 2^{10} \binom{30}{3} + 2^{11} \binom{31}{3} + 2^{12} \binom{32}{3} + 2^{13} \binom{33}{3} \right)$$

$$= -2^7 \binom{27}{4} + 2^{14} \binom{34}{4}$$