$$\text{Ex-II} \ (11, 12, 13)$$

$$\text{Ex-I (remaining)}$$

1. Find the greatest angle of triangle having sides equal to $a, b, \underbrace{\sqrt{a^2+ab+b^2}}_{=c}$

$$\cos C = \frac{a^2+b^2-(a^2+ab+b^2)}{2ab} = -\frac{1}{2}$$

$$\boxed{C = \frac{2\pi}{3}}$$

$$rms\left(x_1, x_2, \ldots x_n\right) = \boxed{\frac{x_1^2+x_2^2+\cdots+x_n^2}{n}}$$

2. In $\triangle ABC$, if $\underline{c^4} - 2\underline{(a^2+b^2)}\underline{c^2} + a^4 + b^4 + a^2b^2 = 0$, find $\angle C$.

$$+a^2b^2 \quad +a^2b^2$$

$$\left(c^2 - (a^2+b^2)\right)^2 = a^2b^2$$

$$\left(\frac{a^2+b^2-c^2}{2ab}\right)^2 = \frac{1}{4}$$

$$\cos^2 C = \frac{1}{4}$$

$$\boxed{\frac{\pi}{3} \;/\; \frac{2\pi}{3}}$$

3. In $\triangle ABC$, if $\dfrac{2\cos A}{a} + \dfrac{\cos B}{b} + \dfrac{2\cos C}{c} = \dfrac{a}{bc} + \dfrac{b}{ca}$
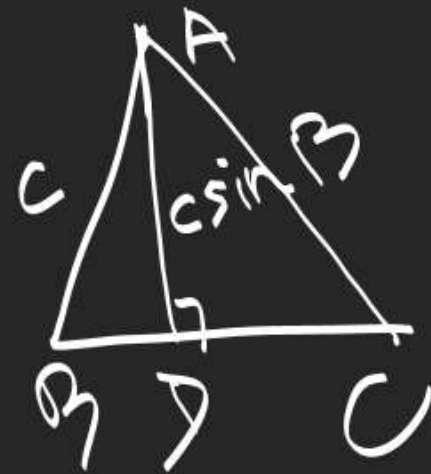
find $\angle A$.

$\boxed{\angle A = \dfrac{\pi}{2}}$

$\dfrac{(b^2+c^2-a^2)}{abc} + \dfrac{a^2+c^2-b^2}{2abc} + \dfrac{a^2+b^2-c^2}{abc} = \dfrac{a^2+b^2}{abc}$

$b^2+c^2=a^2 \Leftarrow 2(b^2+c^2-a^2) + a^2+c^2-b^2 + 2(a^2+b^2-c^2) = 2(a^2+b^2)$

4. In $\triangle ABC$, AD is altitude from A. Given $b > c$,

$\angle C = 23° \ \& \ AD = \dfrac{abc}{b^2-c^2}$, find $\angle B$.

$\dfrac{\sin A}{\sin(B-C)\sin(B+C)} = \ \angle \sin B = \dfrac{abc}{b^2-c^2}$

$\sin B = \dfrac{\sin A \ \sin B}{\sin^2 B - \sin^2 C}$

$\sin(B-C) = 1$

$B - C = 90°$

$\boxed{B = 113°}$

5. P.T. (i) $\quad (a+b+c)\left(\tan\dfrac{A}{2}+\tan\dfrac{B}{2}\right) = 2c\cot\dfrac{C}{2}$.

(ii) $\quad \dfrac{\cot\dfrac{A}{2}+\cot\dfrac{B}{2}+\cot\dfrac{C}{2}}{\cot A+\cot B+\cot C} = \dfrac{(a+b+c)^2}{(a^2+b^2+c^2)}$ $\checkmark$

$$\dfrac{\dfrac{s(s-a)}{\Delta}+\dfrac{s(s-b)}{\Delta}+\dfrac{s(s-c)}{\Delta}}{\dfrac{b^2+c^2-a^2}{2bc\sin A}+\dfrac{c^2+a^2-b^2}{2ca\sin B}+\dfrac{a^2+b^2-c^2}{2ab\sin C}} = \dfrac{\dfrac{s}{\Delta}(s)}{2\Delta\left(\dfrac{a^2+b^2+c^2}{4\Delta}\right)} = \dfrac{(2s)^2}{(a^2+b^2+c^2)}$$

$$2s\left(\dfrac{c^2}{\Delta}{\!}\dfrac{}{s(s-a)}+\dfrac{}{\Delta}{\!}\dfrac{}{s(s-b)}\right)$$

$$= 2c\,\dfrac{\sqrt{s(s-a)(s-b)(s-c)}}{(s-a)(s-b)} = 2c\sqrt{\dfrac{s(s-c)}{(s-a)(s-b)}}$$

$$= 2c\cot\dfrac{C}{2}$$

6. 2) a, b, c are in A.P., then P.T.

(i) $\cot \dfrac{A}{2}$, $\cot \dfrac{B}{2}$, $\cot \dfrac{C}{2}$ are in A.P.

(ii) $\overset{\frac{S(S-a)}{\Delta}}{\cos A \cot \dfrac{A}{2}}$, $\cos B \cot \dfrac{B}{2}$, $\cos C \cot \dfrac{C}{2}$ are in A.P.

$\left(1 - 2\sin^2 \dfrac{A}{2}\right) \cot \dfrac{A}{2} = \cot \dfrac{A}{2} - \sin A$, $\cot \dfrac{B}{2} - \sin B$, $\cot \dfrac{C}{2} - \sin C$

$-a, -b, -c \qquad \rightarrow A.P.$

$s-a, s-b, s-c \qquad \rightarrow A.P.$

$\dfrac{S(s-a)}{\Delta}, \dfrac{S(s-b)}{\Delta}, \dfrac{S(s-c)}{\Delta} \rightarrow A.P$

$\cot \dfrac{A}{2}, \cot \dfrac{B}{2}, \cot \dfrac{C}{2} \rightarrow A.P.$

$\sin A, \sin B, \sin C \rightarrow A.P.$

$\cot \dfrac{A}{2} - \sin A, \cot \dfrac{B}{2} - \sin B, \cot \dfrac{C}{2} - \sin C \rightarrow A.P.$

$T_1, T_2, T_3, \cdots \cdots, T_n \rightarrow A.P$

$T_1', T_2', T_3', \cdots \cdots, T_n' \rightarrow A.P.$

$\boxed{A.P} \leftarrow T_1 + T_1', T_2 + T_2', T_3 + T_3', \cdots ; T_n + T_n'$

7. If the altitude, angle bisector and median drawn from vertex A of $\triangle ABC$ divide the angle A in 4 equal parts. Find $\angle A$.

$$4\theta = \frac{\pi}{2}$$

$$\angle A = \frac{\pi}{2}$$
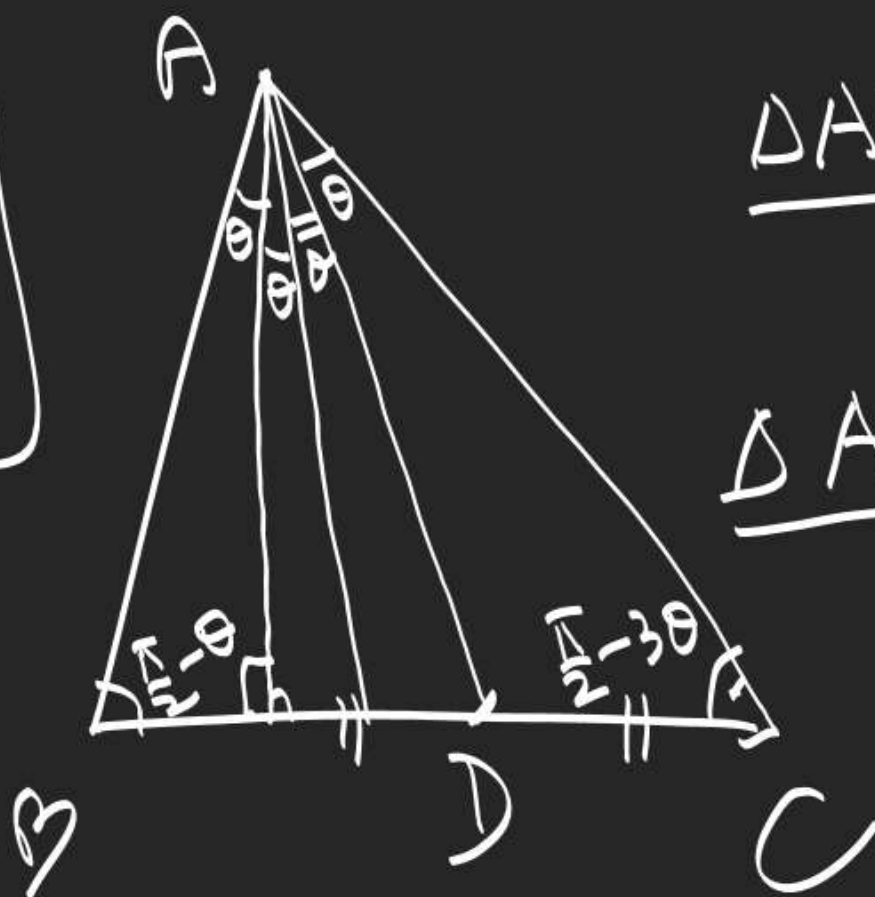
$\triangle ABD \to \dfrac{BD}{\sin 3\theta} = \dfrac{AD}{\sin\left(\frac{\pi}{2}-\theta\right)}$ ——①

$\triangle ADC \to \dfrac{CD}{\sin\theta} = \dfrac{AD}{\sin\left(\frac{\pi}{2}-3\theta\right)}$ ——②

$\dfrac{①}{②} \quad \dfrac{\sin\theta}{\sin 3\theta} = \dfrac{\cos 3\theta}{\cos\theta}$

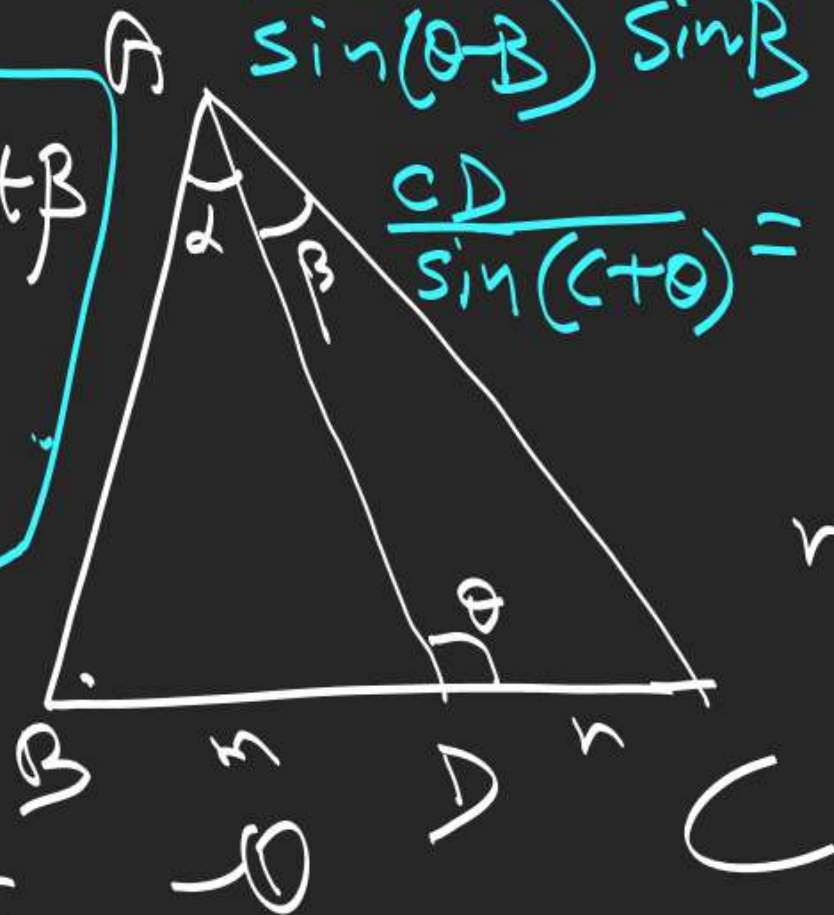$2\sin 2\theta \cos 4\theta = 0 \Longleftarrow \sin 6\theta = \sin 2\theta$

m-n theorem

$$(m+n)\cot\theta = m\cot\alpha - n\cot\beta$$

$$(m+n)\cot\theta = n\cot B - m\cot C$$

$$\frac{BD}{\sin(\theta-\beta)} = \frac{AD}{\sin\beta}$$

$$\frac{CD}{\sin(C+\theta)} = \frac{AD}{\sin C}$$

$$\frac{BD}{DC} = \frac{m}{n}$$

$$n(\cot\theta + \cot\beta) = m(\cot\alpha - \cot\theta)$$

ABD

$$\frac{BD}{\sin\alpha} = \frac{AD}{\sin(\theta-\alpha)}$$

ACD

$$\frac{CD}{\sin\beta} = \frac{AD}{\sin(\pi-(\theta+\beta))}$$ —②

①

②

$$\frac{m\sin\beta}{n\sin\alpha} = \frac{\sin(\theta+\beta)}{\sin(\theta-\alpha)}$$

$$n\frac{\sin(\theta+\beta)}{\sin\theta\sin\beta} = \frac{m\sin(\theta-\alpha)}{\sin\theta\sin\alpha}$$

1. If the median from vertex C on the opposite side is $\perp$ar to AC. Then

P.T. $2\tan A + \tan C = 0$

S.L. Loney $\rightarrow$ Ex-27 (1-15)