Nishant Jindal

PROBABILITY

Q. 4 Cards are selected out of 52 cards. Find Probability

(A) all 4 cards are of same suit

(B) card of every suit available

(C) all 4 are of same denomination

(D) all 4 are of same colour

(I) 2 Red & 2 Black cards

(F) all 4 Face cards

(E) 2 King 2 Queen

(H) atleast one Queen

M2

$$\frac{{}^4C_4 \times {}^{48}C_0 + {}^4C_3 \times {}^{48}C_1 + {}^4C_2 \times {}^{48}C_2 + {}^4C_1 \times {}^{48}C_3}{{}^{52}C_3}$$

(A) $$\frac{{}^{13}C_4 + {}^{13}C_4 + {}^{13}C_4 + {}^{13}C_4}{{}^{52}C_4}$$

(B) $$\frac{{}^{13}C_1 \times {}^{13}C_1 \times {}^{13}C_1 \times {}^{13}C_1}{{}^{52}C_4}$$

(C) $$\frac{{}^{13}C_1 \times {}^4C_4}{{}^{52}C_4}$$

4 Red ← $${}^{26}C_4$$ → 4 Black

(I) $$\frac{{}^{26}C_4 + {}^{26}C_4}{{}^{52}C_4}$$

(E) $$\frac{{}^{26}C_2 \times {}^{26}C_2}{{}^{52}C_4}$$

(F) $$\frac{{}^{12}C_4}{{}^{52}C_4}$$

(E) $$\frac{{}^4C_2 \times {}^4C_2}{{}^{52}C_4}$$

(F) At least one Queen

$$= 1 - \text{No Queen}$$

$$= 1 - \frac{{}^{48}C_4}{{}^{52}C_4}$$

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PROBABILITY

Q. Ram has 3 shares of a lottery. (3 tickets) In which these are 3 prizes & 6 blanks, what is Probability that Ram gets atleast one Prize.

$$= \frac{{}^3C_3 \times {}^6C_0 + {}^3C_2 \times {}^6C_1 + {}^3C_1 \times {}^6C_2}{{}^9C_3}$$

Teeno Prize	6 Blank	3 Prize & 6 Blank	3 Prize & 6 Blank
		2	1

$$1 - (\text{No Prize}) = 1 - \frac{{}^6C_3}{{}^9C_3}$$

PROBABILITY

Q. $S = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ a set is given.

(1) A number of 9 digits is to be made by using digit of sets (RNA). What is Probability that 1^{st}, 5^{th}, 9^{th} digit of number is odd

(2) What is Probability that number is greater than 3×10^8

(3) What is Probability that sum of digit at first 2 places from left is equal to digit used in last place.

Total odd No = 1, 3, 5, 7, 9 (5 odd)

$\frac{0}{1} \_ \_ \_ \frac{0}{5} \_ \_ \_ \frac{0}{9}$

5 में से 3 चुनना → 3 odd को 3 place पर रखना
→ बचे 6 No को 6 place पर लगाना।

(1) $P(A) = \frac{{}^5C_3 \times \lfloor 3 \times \lfloor 6}{\lfloor 9}$

(2) 300000000 (2)

(3) $\frac{\_ \_ \_ \_ \_ \_ \_ \_ \_}{12}$ 3 $4, 5, 6, 7, 8, 9$

$\frac{7_{C_1} \times \lfloor 8}{\lfloor 9}$

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$$\frac{32C_1 \times 1 \times 6}{9}$$

2,3,4,5,8,9

1,5	6
5,1	6
2,4	6
4,2	6

(last digit)

Sum	Case	
3	(1,2) (2,1)	2 Case. 2 Cases
4	(1,3) (3,1)	
5	(2,3) (3,2) (4,1) (1,4)	4
6	(1,5) (5,1) (2,4) (4,2)	4
7	(1,6) (6,1) (2,5) (5,2) (3,4) (4,3)	6
8	(1,7) (7,1) (2,6) (6,2) (3,5) (5,3)	6
9	(1,8) (8,1) (2,7) (7,2) (3,6) (6,3) (4,5) (5,4)	8

32 (case में कोई 1 selection की This will fill 1st 2 Blocks & last Block will be filled automatically. So 3 Blocks will be filled

Now take care for rest 6 Blocks.

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PROBABILITY

Q. 2 Natural number p & q are randomly selected from set of Whole number.

Find Probability that $3^p + 7^q$ is div. by 5 (cyclicity.

$3^1 \to 3$
$3^2 \to 9$
$3^3 \to 27$
$3^4 \to 81$
$3^5 \to 243$
$3^6 \to 729$
$3^7 \to 2187$
$3^8 \to 6561$

3^p \ 7^q	7	9	3	1
3	0	x	x	x
9	x	x	x	0
7	x	x	0	x
1	x	0	x	x

$$P\left(\frac{3^p + 7^q}{5}\right) = \frac{4}{16} = \frac{1}{4}$$

$3^1 \to 3$
$3^2 \to 9$
$3^3 \to 7$
$3^4 \to 1$
$3^5 \to 3$
$3^6 \to 9$
$3^7 \to 7$
$3^8 \to 1$

$7^1 \to 7$
$7^2 \to 9$
$7^3 \to 3$
$7^4 \to 1$
$7^5 \to 7$
$7^6 \to 9$
$7^7 \to 3$
$7^8 \to 1$

PROBABILITY

Q. 2 Natural number p & q are randomly selected from set of Whole number.

Find Probability that $3^p + 7^q$ is div. by 5

$a = 5n_1 + r_1 \rightarrow r_1 \in [0,4]$

Q 2 No a & b are selected from set of Natural No. then Prob. that a^2+b^2 is div. by 5, is?

0^2+0^2
1^2+1^2
2^2+2^2

r_1, r_2

Same	different
0 0 ✓	0 1 2 3 4
1 1 ✗	$r_1=1$: 0 2 3 4 5
2 2 ✗	$r_1=2$: 0 1 3 4 ✓
3 3 ✗	$r_1=3$: 0 1 2 4 ✓
4 4 ✗	$r_1=4$: 0 1 2 3 ✓

Let $a = 5n_1 + r_1$; $0 \le r_1 \le 4$

$b = 5n_2 + r_2$; $0 \le r_2 \le 4$

$a^2+b^2 = (5n_1+r_1)^2 + (5n_2+r_2)^2 = \frac{25(n_1^2+n_2^2)}{5} + \frac{10(n_1r_1+n_2r_2)}{5} + \frac{r_1^2+r_2^2}{5}$

$P(A) = \frac{1}{25} + \frac{8}{25} = \frac{9}{25}$

div. by 3

$a = 3n_1 + r_1$

$0 \le r_1 \le 2$

$a = 0, 1$
$a = 0+1 = 1$
$a = 0+2 = 2$
$a = 0+3 = 3$
$a = 0+4 = 4$
$a = 5+0 = 5$
$5+1 = 6$
$5+2 = 7$
$5+3 = 8$
$5+4 = 9$

$n_1=0, r_1=0$
$n_1=0, r_1=1$
$= 2$

$n_1=1$, $r_1=0$
$r_1=1$
$r_1=2$
$r_1=3$
$r_1=4$

$n_1=2$ $r_1=0$
$r_1=1$
$r_1=2$
$r_1=3$
$r_1=4$

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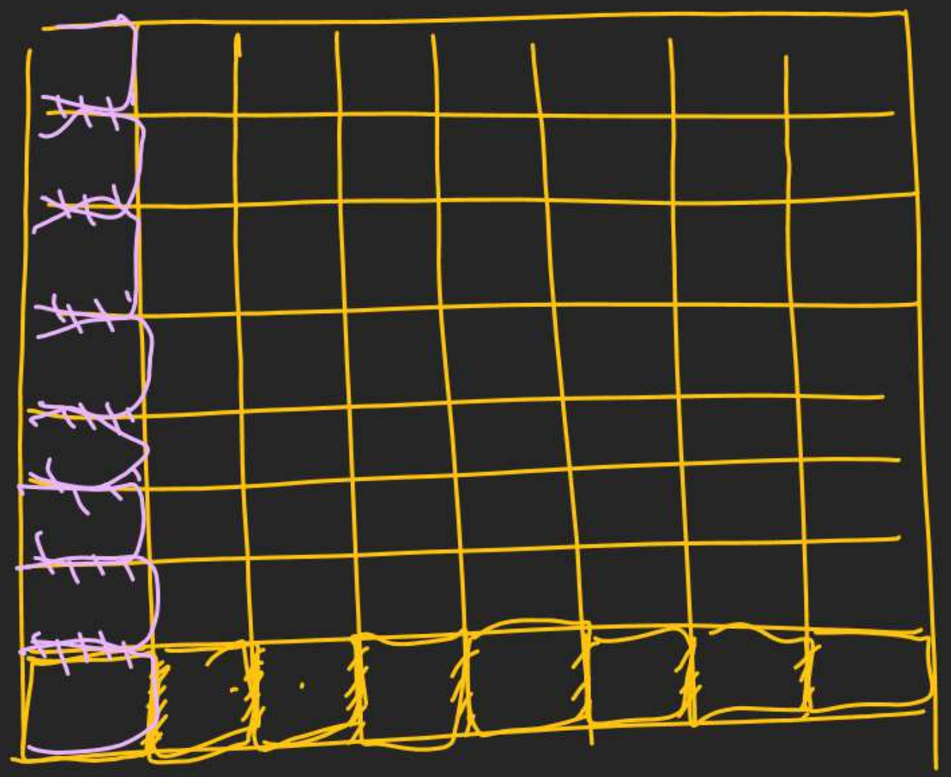
PROBABILITY

Q. If two of the 64 squares are chosen at random on a chess board, the probability that they have a side in common is



$$P(A) = \frac{7\times8 + 7\times8}{{}^{64}C_2} = \frac{112\times2}{64\times63}$$

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PROBABILITY

Q. $x + \frac{100}{x} > 50$. **Find Probability of x**

$x \in N$

$x \leq 100$

$1, 2, 48, 49, 50, 51, \ldots, 100$

$1 + \frac{100}{1} > 50$

$2 + \frac{100}{2} > 50$

$3 + \frac{100}{3} > 50$ ×

$4 + \frac{100}{4} > 50$

$47 + \frac{100}{47}$ | $48 + \frac{100}{48} > 50$

$49 + \frac{100}{49} > 50$ ✓

$50 + \frac{100}{50}$

$51 + \frac{100}{51}$

$60 + \frac{100}{60}$

$100 + \frac{100}{100} > 50$

$P(A) = \frac{55}{100}$

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PROBABILITY

Q. If p & q are selected randomly with replacement from first 10 Natural number

Find Probability that roots of equation $x^2 + px + q = 0$ are real.

$D = p^2 - 4q \geq 0$

$p, q \in \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

$$P(A) = \frac{62}{{}^{10}C_1 \times {}^{10}C_1}$$

$$= \frac{62}{100} \text{ Ans}$$

p	q	$D \geq 0$
1	x	0
2	1	1
3	1, 2	2
4	1, 2, 3, 4	4
5	1, 2, 3, 4, 5, 6	6
6	1, 2, 3, 4, 5, 6, 7, 8, 9	9
7	1, 2, - - - 10	10
8	1, 2 - — 10	10
9		10
10		10

$2^2 - 4 \times 1 = 0$

$9 - 4 \times 1$
4×2

$16 - 4 \times 1$
4×2
4×3
4×4

$25 - 4 \times 1$
4×2
4×3
4×4
4×5
4×6

$30 - 4 \times 1$
4×2
4×3
4×4
$\vdots$
4×9

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Q. Probability of Birth date same for 3 friends

$$Prob = \frac{{}^{365}C_1 \times 1 \times 1 \times 1}{365_1 \times 365_1 \times 365_1}$$

PROBABILITY

Q. Probability of Birth month same for 3 friends

$$P(A) = \frac{{}^{12}C_1 \times 1 \times 1 \times 1}{{}^{12}C_1 \times {}^{12}C_1 \times {}^{12}C_1}$$

PROBABILITY

Q. Probability of Birth date is any 2 months for 6 friends

6 दोस्तों का Birth month कोई दो month पर आने के chance.

$$P(A) = \frac{{}^{12}C_2 \times ({}^{2}C_1 \times {}^{2}C_1 \times {}^{2}C_1 \times {}^{2}C_1 \times {}^{2}C_1 \times {}^{2}C_1 - 1 - 1)}{(12)^6}$$

$$= \frac{{}^{12}C_2 (2^6 - 2)}{12^6}$$