Nishant Jindal

$(1-i)^n((1+i)^n = (1+1)^n = 2^n$

22 → |z| निकालो

$$\frac{z_1 z_2}{\bar{z}_1} = \frac{(2-i)(-2+i)}{(2+i)}$$

$$= \frac{-4-(i)^2}{2+i} = \frac{-3}{2+i} \times \frac{2-i}{2-i}$$

$$= \frac{-3(2-i)}{4-(-1)} = -\frac{6+3i}{5}$$

26 Copy

27) $\left(\frac{1+i}{1-i}\right)^3 - \left(\frac{1-i}{1+i}\right)^3 = (i)^3 - (-i)^3$

$= -i + (-i) = -2i$

29) $\left(\frac{1-i}{1+i}\right)^{100} = (-i)^{100} = i^{100} = 1$

(30) $a = \cos\theta + i\sin\theta$, $\frac{1+a}{1-a} = \frac{(1+\cos\theta)+i\sin\theta}{1-\cos\theta-i\sin\theta}$

$$= \frac{2\cos^2\theta/2 + i\,2\sin\theta/2\cos\theta/2}{2\sin^2\theta/2 - i\,2\sin\theta/2\cdot\cos\theta/2}$$

$$= \frac{2\cos\theta/2\,(\cos\theta/2 + i\sin\theta/2)}{2\sin\theta/2\,(\sin\theta/2 - i\cos\theta/2)} \times \frac{(\sin\theta/2 + i\cos\theta/2)}{(\sin\theta/2 + i\cos\theta/2)}$$

$$= \cot\theta/2\,\frac{(\sin\theta/2\cos\theta/2 + 2i \quad - \sin\theta/2\cos\theta/2)}{(\sin^2\theta/2) - i^2(\cos^2\theta/2)}$$

$= 2i\cot\theta/2$

Nishant Jindal

$\overline{a+ib} = a-ib$

(34) $\frac{z-1}{z+1} = -\left(\overline{\frac{z-1}{z+1}}\right)$

$\frac{z-1}{z+1} = -\left(\frac{\bar{z}-1}{\bar{z}+1}\right)$

C.M & Soln.

35) $\mathrm{Re}(\quad) = 0 \; 5 < i$

(39) $\sin x - i\cos 2x = \cos x - i\sin 2x$

If $\left|z-\frac{6}{z}\right| = 4$ then gr. value of $|z| = ?$

$|z| = \left|\left(z-\frac{6}{z}\right)+\left(\frac{6}{z}\right)\right| \le \left|z-\frac{6}{z}\right| + \left|\frac{6}{z}\right|$

$|z| \le 4 + \frac{6}{|z|}$

$|z|^2 - 4|z| - 6 \le 0$

$(|z| - (2+\sqrt{10}))(|z| - (2-\sqrt{10})) \le 0$

$|z| = \frac{4 \pm \sqrt{16+24}}{2}$

$|z| = \frac{4 \pm 2\sqrt{10}}{2}$

$|z| = 2 \pm \sqrt{10}$

$2-\sqrt{10} \le |z| \le \underbrace{2+\sqrt{10}}_{\text{gr. value}}$

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Q If $|z-1|<1,\ |z_2-2|<2,\ |z-3|<3$

then P.T. $|z_1+z_2+z_3|<12$

Demand $= |z_1+z_2+z_3|$

$= |(z_1-1)+(z_2-2)+(z_3-3)+(6)| \le |z_1-1|+|z_1-2|+|z_2-3|+6$

$\Rightarrow |z_1+z_2+z_3| < 1+2+3+6$

$|z_1+z_2+z_3|<12$

Q If $z_1 z_2 z_3 \ldots z_n$ are n C.N. Such that

$|z_1|=|z_2|=|z_3| \ldots = |z_n|=1$

then P.T. $|z_1+z_2+z_3 \ldots z_n| = \left|\frac{1}{z_1}+\frac{1}{z_2}+\frac{1}{z_3} \ldots \frac{1}{z_n}\right|$

$|z_1|=1$

$|z_1|^2=1$

$z_1\bar{z}_1=1$

$\frac{1}{z_1}=\bar{z}_1$

$|z|=|\bar{z}|$

RHS $= \left|\frac{1}{z_1}+\frac{1}{z_2}+\frac{1}{z_3} \ldots \frac{1}{z_n}\right|$

$|\bar{z}_1+\bar{z}_2+\bar{z}_3+\ldots\bar{z}_n|$

$|\overline{z_1+z_2+z_3 \ldots z_n}|$

$= |z_1+z_2+z_3 \ldots z_n|$

RHS

Q For C.N. z_1, z_2 P.T.

$$\left|\frac{z_1}{|z_1|}+\frac{z_2}{|z_2|}\right|(|z_1|+|z_2|) \le 2(|z_1|+|z_2|)$$

Carefully see Qs.

demand

$$||x|| = |x|$$

$$\left|\frac{z_1}{|z_1|}+\frac{z_2}{|z_2|}\right| \le 2 \quad (\text{To prove})$$

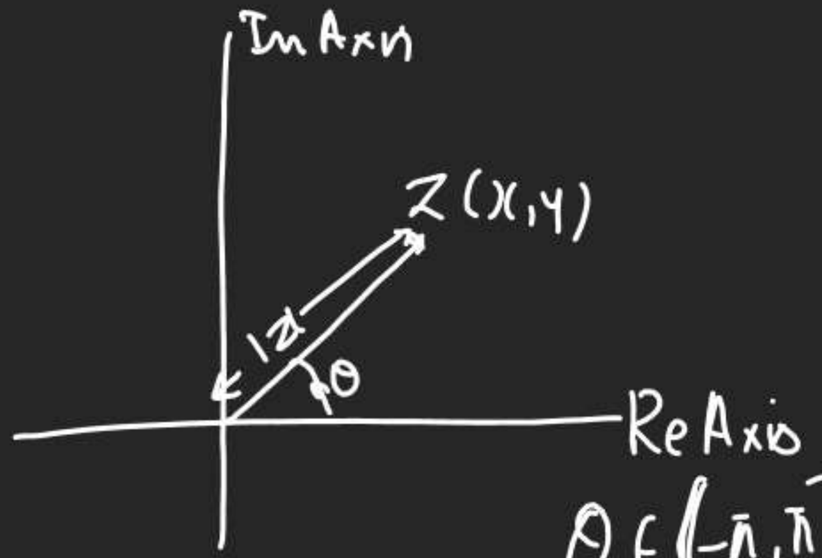
LHS $\rightarrow \left|\frac{z_1}{|z_1|}+\frac{z_2}{|z_2|}\right| \le \left|\frac{z_1}{|z_1|}\right| + \left|\frac{z_2}{|z_2|}\right|$

$$\le \frac{|z_1|}{|z_1|}+\frac{|z_2|}{|z_2|}$$

$$\le 1+1$$

$$\left|\frac{z_1}{|z_1|}+\frac{z_2}{|z_2|}\right| \le 2$$

Argument of a C.N.



$\theta \in (-\pi, \pi]$

1) θ = Argument z

$= Arg(z) = Amp(z)$

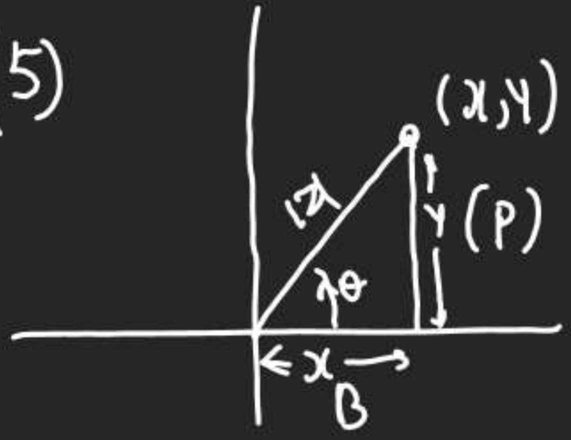
$Arg(z) = \theta + 2n\pi$

(2) $-\pi < \theta \le \pi$, without $2n\pi$ is known as Principle Argument

(3) $\theta + 2n\pi$ is General Argument.

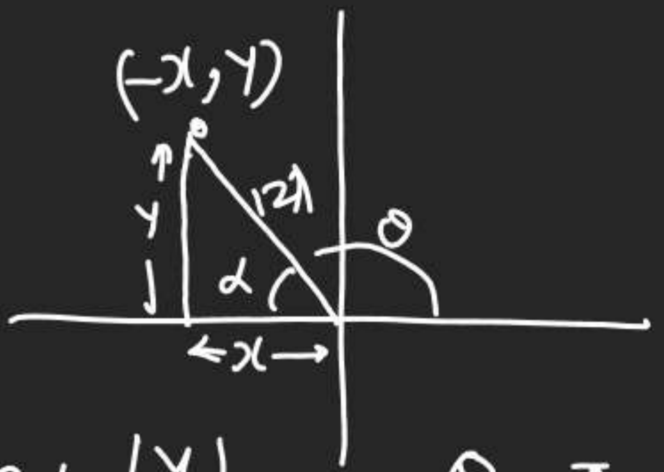
(4) $z = 0+0i$ only C.N. whose $Arg(z)$ is not defined.

(5)



$\tan\theta = \frac{y}{x} \Rightarrow \theta = \tan^{-1}\frac{y}{x}$

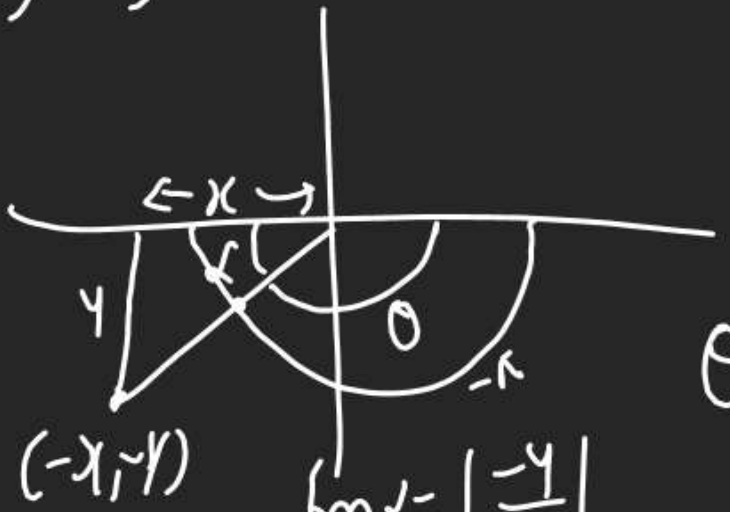
(6) 2^{nd} Quad



$\tan\alpha = \left|\frac{y}{-x}\right|$ $\qquad \theta = \pi - \alpha$.

$\alpha = \tan^{-1}\left|\frac{y}{-x}\right|$ $\quad \theta = \pi - \tan^{-1}\left|\frac{y}{-x}\right|$

(7) 3^{rd}

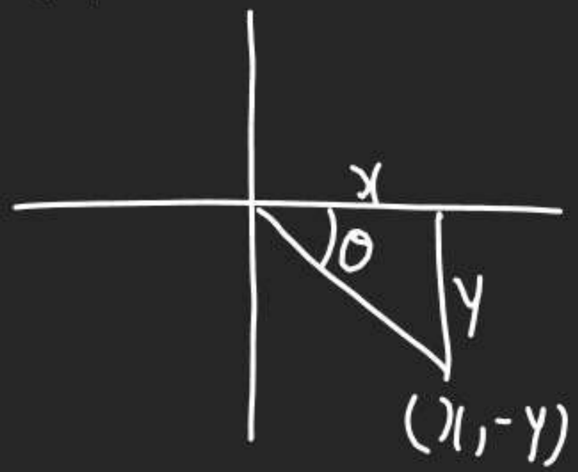


$\theta = -\pi + \alpha = -\pi + \tan^{-1}\left|\frac{y}{x}\right|$

$\tan\alpha = \left|\frac{-y}{-x}\right|$

$\alpha = \tan^{-1}\left|\frac{-y}{-x}\right|$

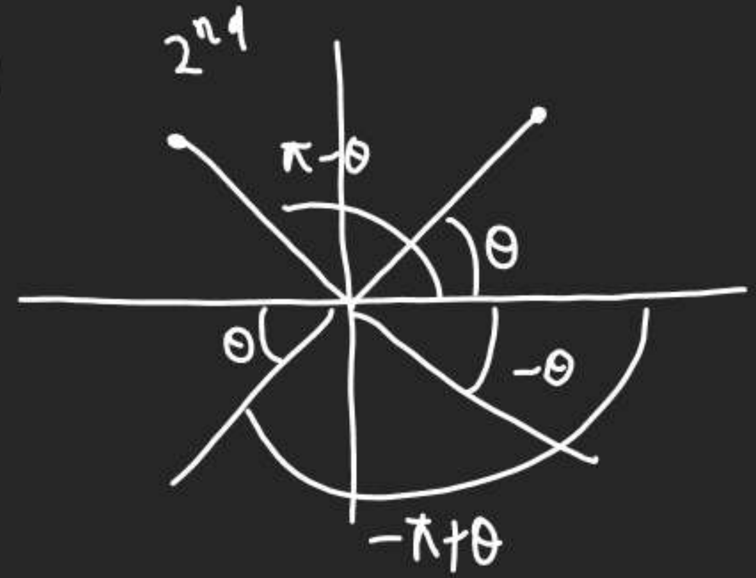
(8) 4^{th} Q



$\tan\theta = \left|\frac{-y}{x}\right|$

$\mathrm{Arg}(z) = -\theta = -\tan^{-1}\left|\frac{-y}{x}\right|$

(9)



(x, y)	$\theta = \tan^{-1}\left\lvert\frac{y}{x}\right\rvert$	1
$(x, -y)$	$-\tan\left\lvert\frac{y}{x}\right\rvert$	4
$(-x, -y)$	$-\pi + \tan^{-1}\left\lvert\frac{y}{x}\right\rvert$	3
$(-x, y)$	$\pi - \tan^{-1}\left\lvert\frac{y}{x}\right\rvert$	2

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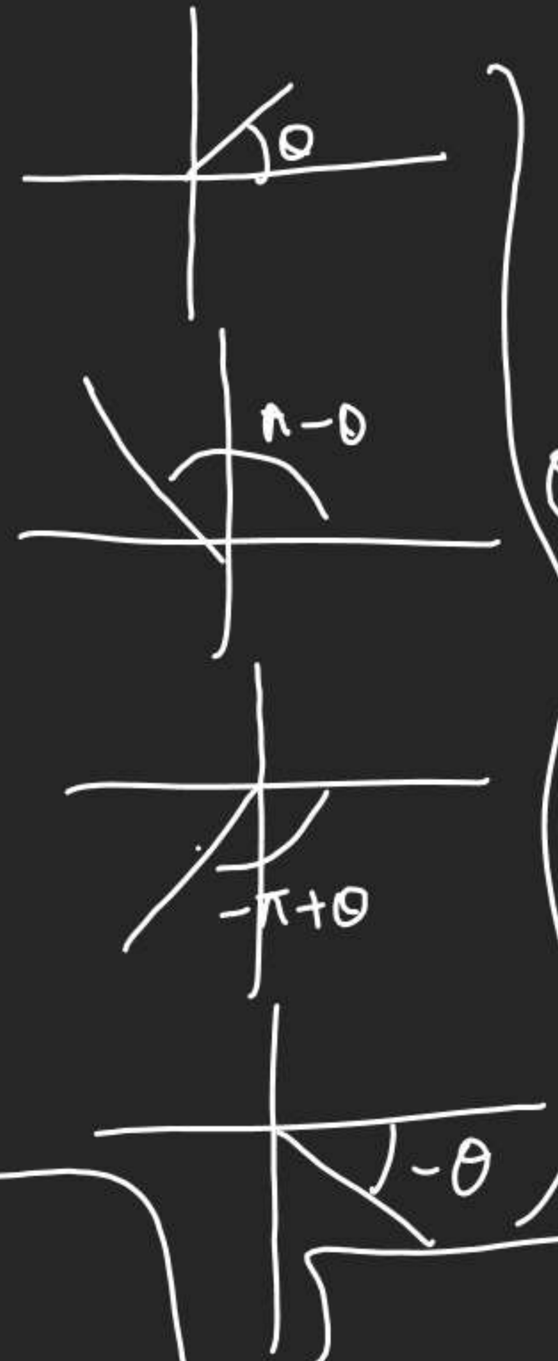
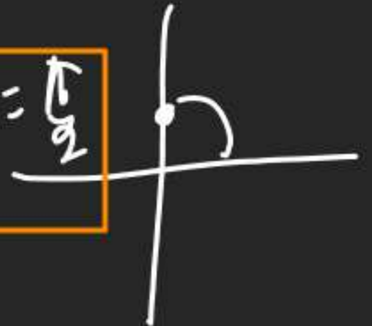
	(x,y)	Q	Arg z
① $Arg(1+i)$ =	$(1,1)$	1	$\theta = \tan^{-1}\left\lvert\frac{1}{1}\right\rvert = \tan^{-1}(1) = \frac{\pi}{4}$
2) $Arg(1-i)$ =	$(1,-1)$	4	$-\theta = -\tan^{-1}\left\lvert\frac{-1}{1}\right\rvert = -\tan^{-1}1 = -\frac{\pi}{4}$
3) $Arg(-1+i)$ =	$(-1,1)$	2	$\pi-\theta = \pi - \tan^{-1}\left\lvert\frac{1}{-1}\right\rvert = \pi - \tan^{-1}1 = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$
4) $Arg(-1-i)$ =	$(-1,-1)$	3	$-\pi+\theta = -\pi + \tan^{-1}\left\lvert\frac{-1}{-1}\right\rvert = -\pi + \tan^{-1}1 = -\pi + \frac{\pi}{4} = -\frac{3\pi}{4}$
(5) $Arg(-1-\sqrt{3}i)$ =	$(-1,-\sqrt{3})$	3^{rd}	$-\pi+\theta = -\pi + \tan^{-1}\left\lvert\frac{-\sqrt{3}}{-1}\right\rvert = -\pi + \tan^{-1}(\sqrt{3}) = -\pi + \frac{\pi}{3} = -\frac{2\pi}{3}$
(6) $Arg(3-4i)$	$(3,-4)$	4^{th}	$-\theta = -\tan^{-1}\left\lvert\frac{-4}{3}\right\rvert = -\tan^{-1}\left(\frac{4}{3}\right)$



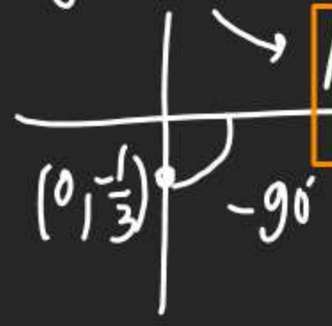
$\theta = \tan^{-1}\left\lvert\frac{y}{x}\right\rvert$

(7) $Arg(i) = (0,1) = \frac{\pi}{2}$

$\boxed{Arg(+ve\ img\ No) = \frac{\pi}{2}}$



(8) $Arg\left(-\frac{i}{3}\right) = \left(0,-\frac{1}{3}\right) = -\frac{\pi}{2}$



$\boxed{Arg(-ve\ img\ No) = -\frac{\pi}{2}}$

(9) $Arg(5) = (5,0) = 0$

$\boxed{Arg(+ve\ Real\ No) = 0}$

Q $Arg(-1) = (-1,0) = \pi$

$\boxed{Arg(-ve\ Real\ No) = \pi}$

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Q $Arg(3-\sqrt{3}i)$

$= (3,-\sqrt{3}) = 4^{th}$

$Arg z = -\tan^{-1}\left|\frac{-\sqrt{3}}{3}\right|$

$= -\tan^{-1}\left|\frac{1}{\sqrt{3}}\right|$

$= -\frac{\pi}{6}$

Q $Arg(3-\sqrt{3})$

No i is Real No.

$3-1.72 = +ve$

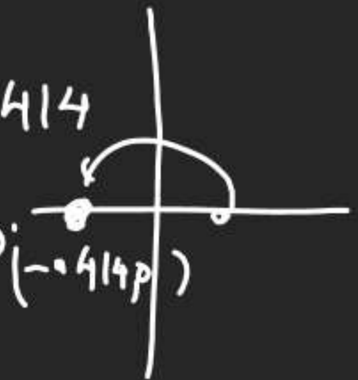
+ve Real No

$Arg(z) = 0$

Q $Arg(1-\sqrt{2})$

$1-1.414 = -.414$

-ve Real No.



$Arg z = \pi$

Q $Arg(5i) =$

$\frac{\pi}{2}$

$Arg(-3i) =$

$= -\frac{\pi}{2}$

Q $Arg\left(\frac{1+i\sqrt{3}+i-\sqrt{3}}{2\sqrt{2}}\right)$

$= Arg\left(\frac{1-\sqrt{3}}{2\sqrt{2}} + i\left(\frac{1+\sqrt{3}}{2\sqrt{2}}\right)\right) = \left(\frac{-.72}{2\sqrt{2}}, \frac{2.72}{2\sqrt{2}}\right)$

-ve ⊕

2^{nd}

$= \pi - \tan^{-1}\left|\frac{\frac{1+\sqrt{3}}{2\sqrt{2}}}{\frac{1-\sqrt{3}}{2\sqrt{2}}}\right| = \pi - \tan^{-1}\left|\frac{\sqrt{3}+1}{\sqrt{3}-1}\right|$

$= \pi - \tan^{-1}\left|\frac{1+\frac{1}{\sqrt{3}}}{1-\frac{1}{\sqrt{3}}}\right| = \pi - \tan^{-1}\left|\frac{\tan\frac{\pi}{4}+\tan\frac{\pi}{6}}{1-\tan\frac{\pi}{4}\times\tan\frac{\pi}{6}}\right|$

$= \pi - \tan^{-1}\left|\tan\left(\frac{\pi}{4}+\frac{\pi}{6}\right)\right|$

$= \pi - 75^\circ = 105^\circ = \frac{7\pi}{12}$

Q $\text{Arg}\left(1+\cos\frac{6\pi}{5}+i\sin\frac{6\pi}{5}\right) = \left(1+\cos\frac{6\pi}{5}, \sin\frac{6\pi}{5}\right)$

$\sin\left(\pi+\frac{\pi}{5}\right) = -\sin\frac{\pi}{5}$

$= \left(1+\cos\frac{6\pi}{5}, -\sin\frac{\pi}{5}\right)$

$= (+ve, -ve)$

4th.

$\boxed{\frac{\pi}{5} = 36^\circ}$

$\Rightarrow -\tan^{-1}\left|\frac{-\sin\frac{\pi}{5}}{1-\cos\frac{\pi}{5}}\right|$

$= -\tan^{-1}\left|\frac{-\sin\theta}{1-\cos\theta}\right|$

$= -\tan^{-1}\left|\frac{2\sin\theta/2 \cdot \cos\theta/2}{2\sin^2\theta/2}\right|$

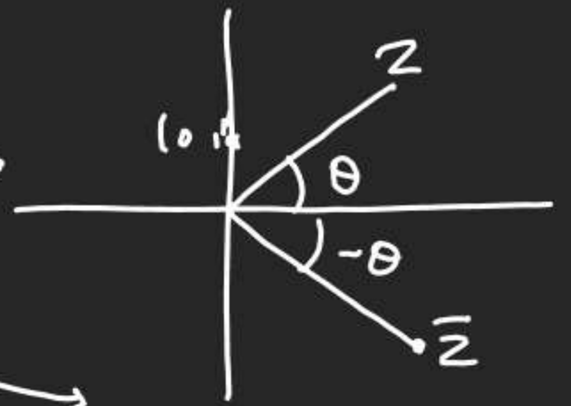
$= -\tan^{-1}\left|\cot\frac{\theta}{2}\right| = -\tan^{-1}(\cot 18^\circ)$

$= -\tan^{-1}(\tan(90-18^\circ))$

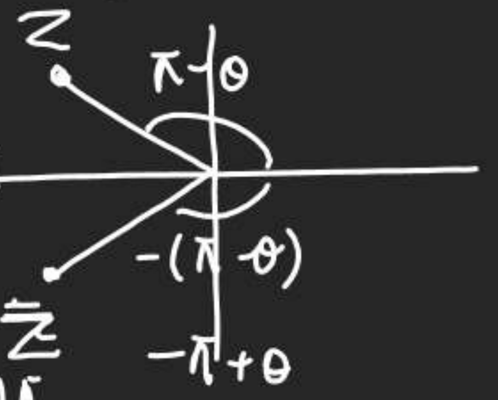
$= \underline{\underline{-72^\circ}}$

Properties of Argument:

(1) $\text{Arg}(\bar{z}) = -\text{Arg}\, z$



(2) $\text{Arg}(-\bar{z}) = \pi - \text{Arg}\, z$



(3) $\text{Arg}(z_1 \cdot z_2) = \text{Arg}\, z_1 + \text{Arg}\, z_2 + 2n\pi$

(4) $\text{Arg}\left(\frac{z_1}{z_2}\right) = \text{Arg}(z_1) - \text{Arg}(z_2) + 2n\pi$

(5) $\text{Arg}(z^n) = n\,\text{Arg}\, z$

(6) $\text{Arg}(iz) = \text{Arg}\, i + \text{Arg}\, z = \frac{\pi}{2} + \text{Arg}\, z$

(7) $\text{Arg}(\omega z) = \text{Arg}\,\omega + \text{Arg}\, z = \frac{2\pi}{3} + \text{Arg}\, z$

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Q $Arg(z_1)=\theta_1$, $Arg(z_2)=\theta_2$

$Arg(z_3)=\theta_3$ find $Arg\left(\frac{z_1\bar{z}_2}{z_3}\right)=?$

$$Arg\left(\frac{z_1\cdot\bar{z}_2}{z_3}\right)=Arg\,z_1+Arg\,\bar{z}_2-Arg\,z_3$$

$$=Arg\,z_1-Arg\,z_2-Arg\,z_3$$

$$=\theta_1-\theta_2-\theta_3 \text{ Ans}$$

Q Value of $Arg\{(1+i)(1+i\sqrt{3})(\cos\theta+i\sin\theta)\}$

$$Arg(1+i)+Arg(1+i\sqrt{3})+Arg(\cos\theta+i\sin\theta)$$

$(1,1)$ $\quad(1,\sqrt{3})\quad$ $(\cos\theta,\sin\theta)$

$$\tan^{-1}\left|\frac{1}{1}\right|+\tan^{-1}\left|\frac{\sqrt{3}}{1}\right|+\tan^{-1}\left|\frac{\sin\theta}{\cos\theta}\right|$$

$$\frac{\pi}{4}+\frac{\pi}{3}+\tan^{-1}(\tan\theta)=\frac{7\pi}{12}+\theta$$

Q $Arg\left|\frac{1+i}{1-i}\right|$

$\sqrt{a^2+b^2}$

+ve Real No. होगा

$=0$

Q $Arg(z_1)=170^\circ$, $Arg(z_2)=70^\circ$

find $Arg(z_1\cdot z_2)$

$$Arg(z_1\cdot z_2)=Arg\,z_1+Arg\,z_2$$

$$=170^\circ+70^\circ=240^\circ$$

$$Arg(z)=-\pi+60^\circ=-\frac{2\pi}{3}$$

Q If z_1,z_2 & z_3,z_4 are 2 pairs of Conjugate C.N. then value of $Arg\left(\frac{z_1}{z_4}\right)+Arg\left(\frac{z_2}{z_3}\right)$

$z_1=\bar{z}_2$ & $z_3=\bar{z}_4$

$$Arg\left(\frac{z_1}{z_4}\right)+Arg\left(\frac{z_2}{z_3}\right)$$

$$=Arg\left(\frac{z_1\cdot z_2}{z_3\,z_4}\right)$$

$$=Arg\left(\frac{\bar{z}_2\cdot z_2}{\bar{z}_4\cdot z_4}\right)=Arg\left(\frac{|z_2|^2}{|z_4|^2}\right)$$

+ve Real

$=0$

7 तक HW