Nishant Jindal

Q Mark

23 (A) $1+0i$, (B) $-1+0i$, (C) $3+4i$

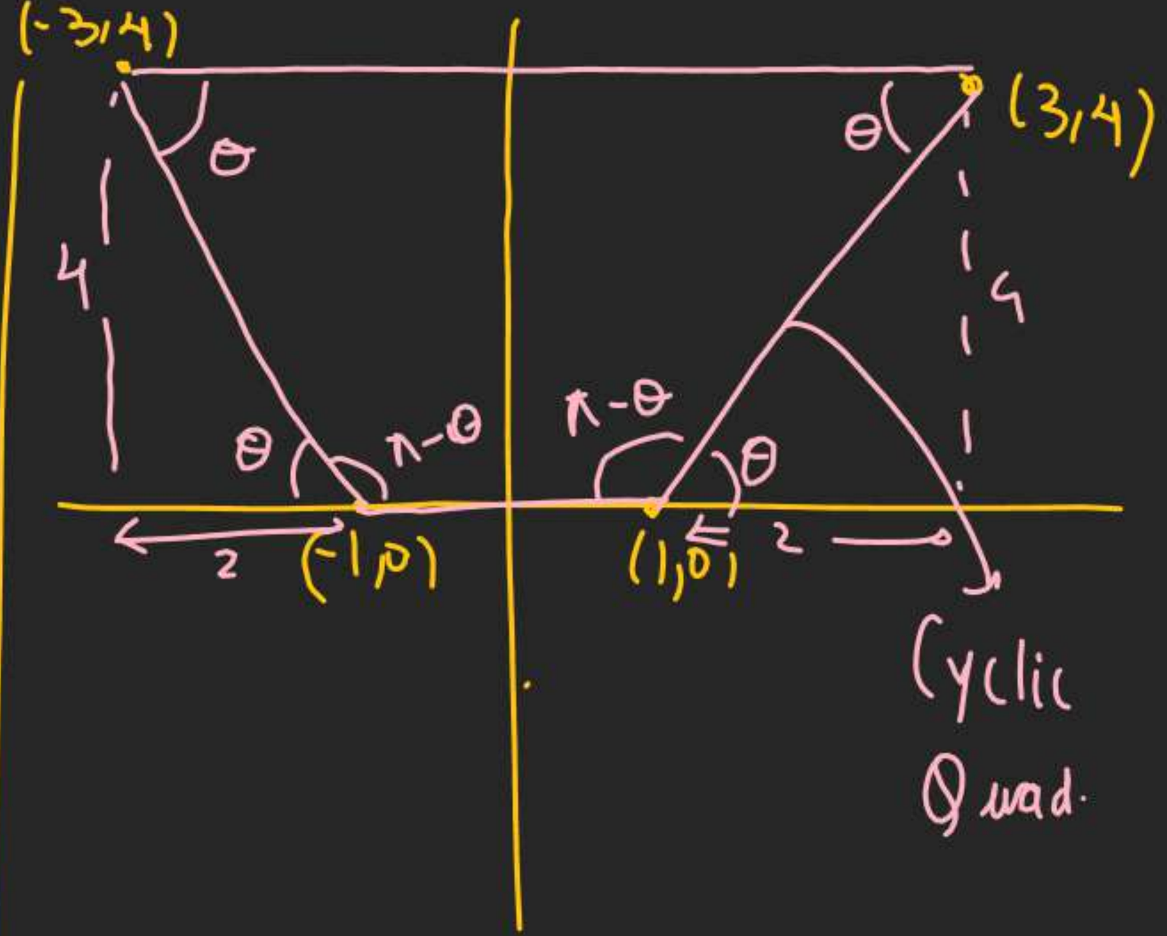
& $\dfrac{25}{-3-4i}$ at Argand Plane

$$\frac{25}{-3-4i}\times\frac{-3+4i}{-3+4i}$$

$$=\frac{25(-3+4i)}{(-3)^2-(4i)^2}$$

$$=\frac{\cancel{25}(-3+4i)}{\cancel{9+16}}=-3+4i$$

(D)



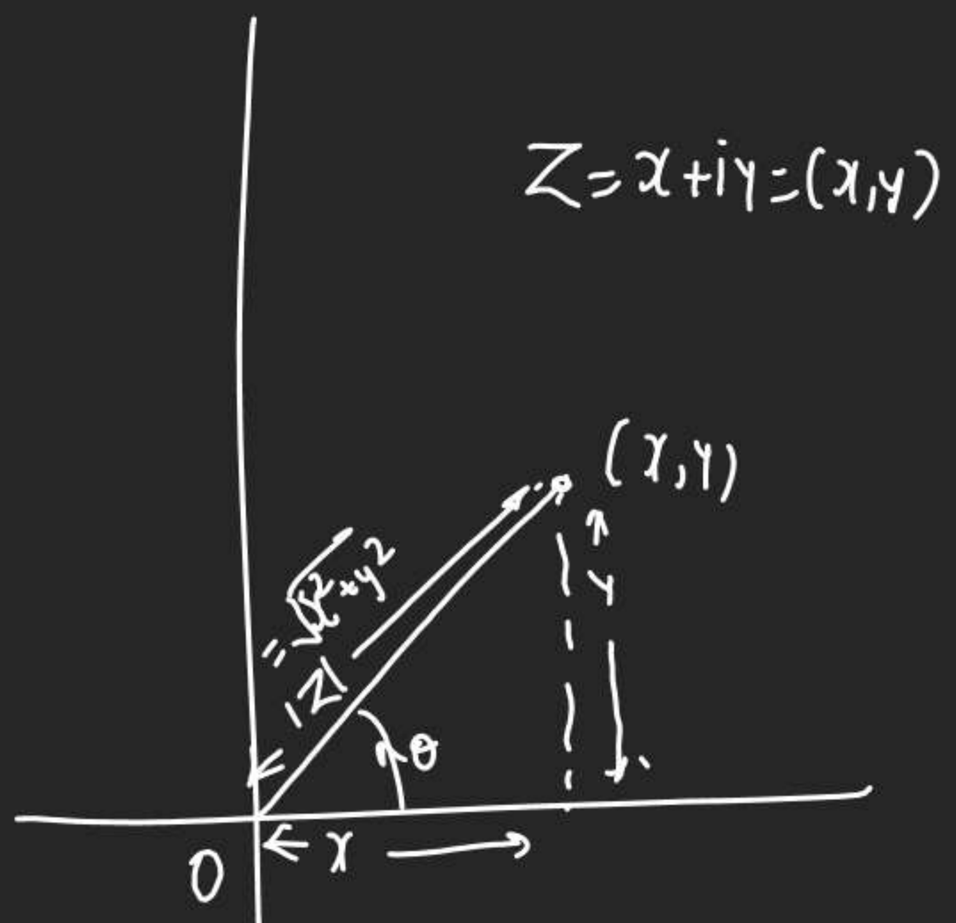
$1+0i=(1,0)$

$-1+0i=(-1,0)$

$3+4i=(3,4)$

$-3+4i=(-3,4)$ (D)

$Z=x+iy=(x,y)$



Position of any C.N. can be shown by its distance from origin (|Z|) & angle made by line joining Z to origin from Real Axis

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Modulus z = Modulus of C.N.

A) Rep by $|z|$.

B) $z = a+ib$.

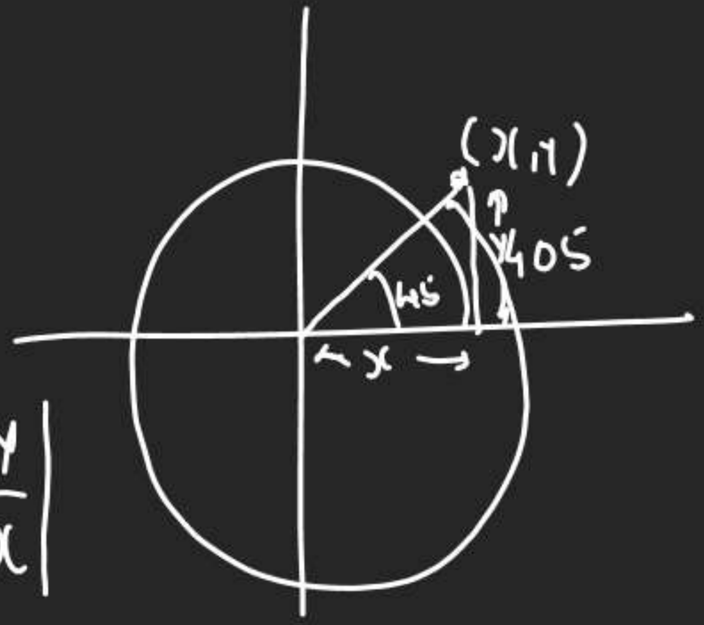
$$\boxed{\begin{gathered}|z| = \sqrt{a^2+b^2} \\ |z| = \sqrt{(\mathrm{Re}\,z)^2+(\mathrm{Im}\,z)^2}\end{gathered}}$$

(C) $|z|$ Rep. dist. of any C.N. z from origin.

θ = Argument of z.

① Arg(z) / Amp(z)

(2)



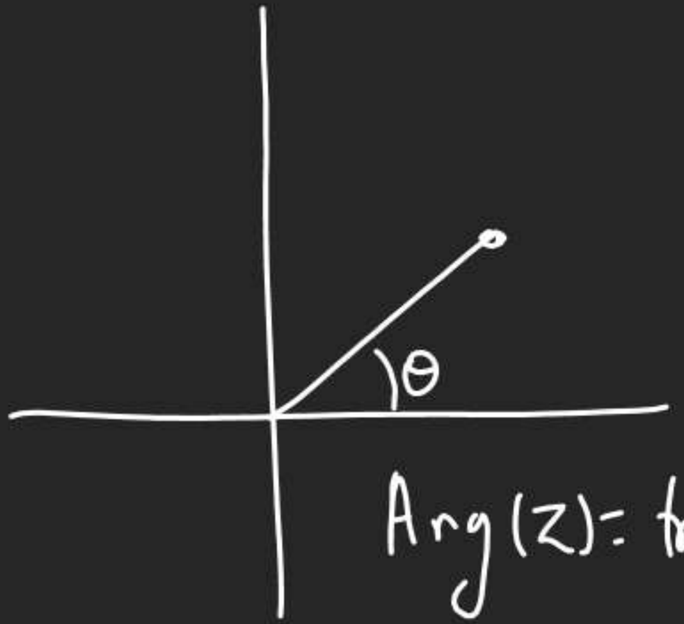
(3) $\mathrm{Arg}(z) = \theta = \tan^{-1}\left|\frac{y}{x}\right|$

(4) $\mathrm{Arg}(z) = \theta$

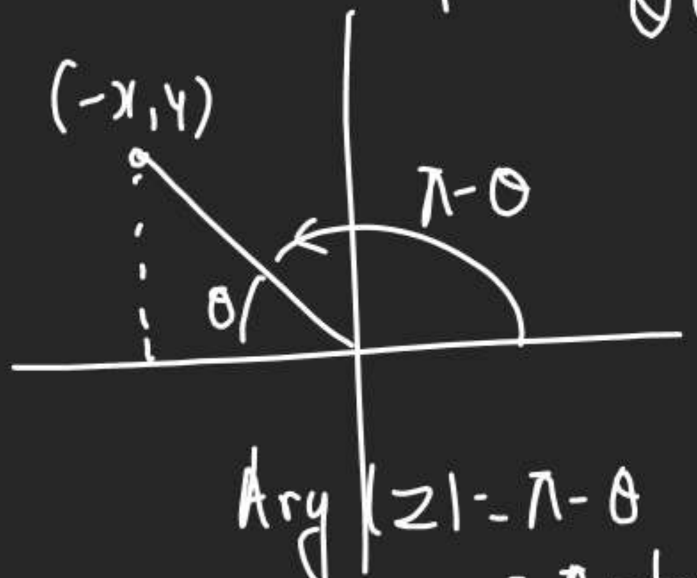
Gen. value of Arg(z)	Principle Value of Arg(z)	LPA Least +ve Arg(z)
$= \theta + 2k\pi$, $k \in I$	$= \mathrm{Amp}(z)$, $-\pi < \theta \le \pi$	$0 < \theta \le 2\pi$

(5) We Predict Arg depends on Quadrants.

$\theta \in (-\pi, \pi]$



$Arg(z) = \tan^{-1}\left|\frac{y}{x}\right|$

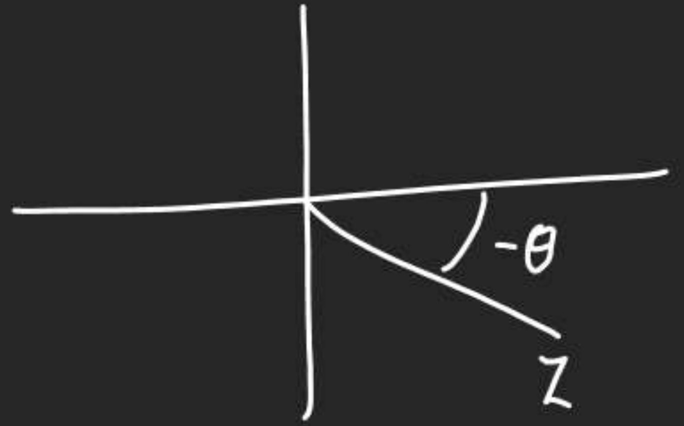


$Arg(z) = \pi - \theta$

$= \pi - \tan^{-1}\left|\frac{y}{x}\right|$

$Arg(z) = -\pi + \tan^{-1}\left|\frac{y}{x}\right|$

4^{th} Q



$Arg(z) = -\theta = -\tan^{-1}\left|\frac{y}{x}\right|$

Q $Arg(1+\sqrt{3}i) = (1, \sqrt{3}) = 1^{st}$ Q.

$Arg(z) = \tan^{-1}\left|\frac{\sqrt{3}}{1}\right| = \frac{\pi}{3}$

B) $Arg(1-\sqrt{3}i) = (1, -\sqrt{3}) \rightarrow 4^{th}$ Q $\rightarrow -\theta$

$Arg(z) = -\tan^{-1}\left|\frac{-\sqrt{3}}{1}\right| = -\frac{\pi}{3}$

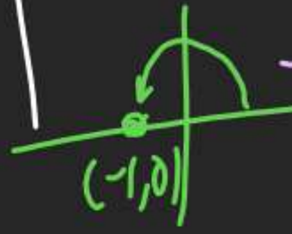
C) $Arg(-1+\sqrt{3}i) \rightarrow (-1, \sqrt{3}) \rightarrow 2^{nd} \rightarrow \pi - \theta$

$Arg(z) = \pi - \tan^{-1}\left|\frac{\sqrt{3}}{-1}\right| = \pi - \frac{\pi}{3} = \frac{2\pi}{3}.$

(D) $Arg(-1-\sqrt{3}i) \rightarrow (-1, -\sqrt{3}) \rightarrow 3^{rd} \rightarrow -\pi + \theta$

$Arg(z) = -\pi + \tan^{-1}\left|\frac{-\sqrt{3}}{-1}\right| = -\pi + \frac{\pi}{3} = -\frac{2\pi}{3}$

$Arg(1) \rightarrow (1, 0)$ (1,0) +ve Real Axis. $Arg(z) = 0$



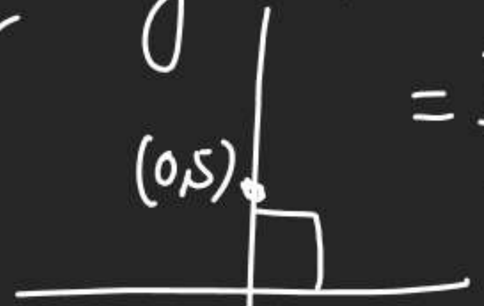
$Arg(-1) = \pi$ (-1,0) -ve Real Axis

Q $\mathrm{Arg}(1+\sqrt{-2})$

$\mathrm{Arg}(1+\sqrt{2}i) \rightarrow (1,\sqrt{2}) \rightarrow$ ①

$\mathrm{Arg}(z) = \tan^{-1}\left|\frac{\sqrt{2}}{1}\right| = \tan^{-1}\sqrt{2}$

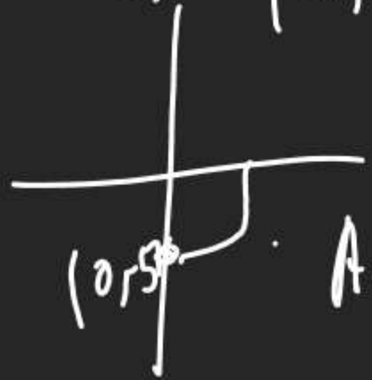
Q $\mathrm{Arg}(5i) \rightarrow (0,5)$



$= \frac{\pi}{2}$

Purely Img ⊕ तो $\mathrm{Arg} = \frac{\pi}{2}$

Q $\mathrm{Arg}(-5i) = (0,-5)$



$\mathrm{Arg}(z) = -\frac{\pi}{2}$

$\mathrm{Arg}(+\text{Real No}) = 0$

$\mathrm{Arg}(-\text{ve Real No}) = \pi$

$\mathrm{Arg}(+\text{ve Img No}) = \frac{\pi}{2}$

$\mathrm{Arg}(-\text{ve Img No}) = -\frac{\pi}{2}$

$\mathrm{Arg}(\text{Img No}) = \pm\frac{\pi}{2}$

Q $\frac{z_1}{z_2}$ is Purely Img then $\mathrm{Arg}\left(\frac{z_1}{z_2}\right) = ?$

$\mathrm{Arg}\left(\frac{z_1}{z_2}\right) = \pm\frac{\pi}{2}$

Algebra of C.N.

① Equality of C.N.

$z_1 = x_1 + iy_1$

$z_2 = x_2 + iy_2$

If $z_1 = z_2$

$x_1 + iy_1 = x_2 + iy_2$

$\boxed{x_1 = x_2}$ & $\boxed{y_1 = y_2}$

If $z_1 = z_2$ then

$\mathrm{Re}(z_1) = \mathrm{Re}(z_2)$

$\mathrm{Img}(z_1) = \mathrm{Img}(z_2)$

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(2) Inequality of 2 C N.

$$z_1 > z_2$$

$x_1 + iy_1 > x_2 + iy_2$ ← Meaningless.

But forcefully given then $y_1 = y_2 = 0$

(3) Sum of 2 C.N.

$$z_1 + z_2 = (x_1 + iy_1) + (x_2 + iy_2)$$

$$= (x_1 + x_2) + i(y_1 + y_2)$$

$$\downarrow$$

$$Re(z_1 + z_2) = Re(z_1) + Re(z_2)$$

(4) difference of 2 C N.

(5) Multiplication of 2 C N.

$$z_1 \cdot z_2 = (x_1 + iy_1)(x_2 + iy_2)$$

$$= x_1x_2 + ix_1y_2 + iy_1x_2 + i^2 y_1y_2$$

$$z_1 z_2 = (x_1x_2 - y_1y_2) + i(x_1y_2 + x_2y_1)$$

Q $z = \dfrac{(3+4i)(1-2i)}{(5i)}$ then $|Re z| + |Im z| = ?$

$$z = \frac{(3 - 6i + 4i - 8i^2)}{5}(-i)$$

$$z = \frac{(11 - 2i)}{5}(-i)$$

$$z = \frac{-11i + 2i^2}{5} = \frac{-2 - 11i}{5}$$

$$|Re z| + |Im z|$$

$$= \left|-\frac{2}{5}\right| + \left|\frac{-11}{5}\right|$$

$$= \frac{2}{5} + \frac{11}{5} = \frac{13}{5} \text{ Ans}$$

$$\boxed{\begin{array}{l} \text{RK: If } z_1^2 + z_2^2 = 0 \\ \text{then } z_1 = z_2 = 0 \\ \text{only } z_1, z_2 \text{ Real} \end{array}}$$

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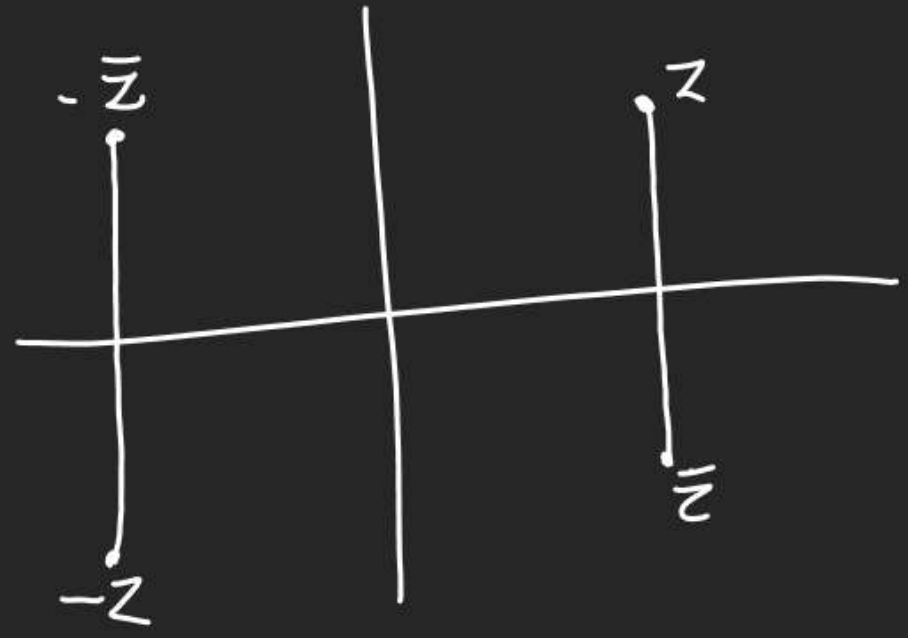
Conjugate of a C.N.

(1) $Z = x+iy$ then it Conjugate is Rep. by $\bar{Z}$

(2) $\bar{z} = x - iy$ (Change Sign of i)

(3) $\bar{z}$ is Image of z in Real Axis

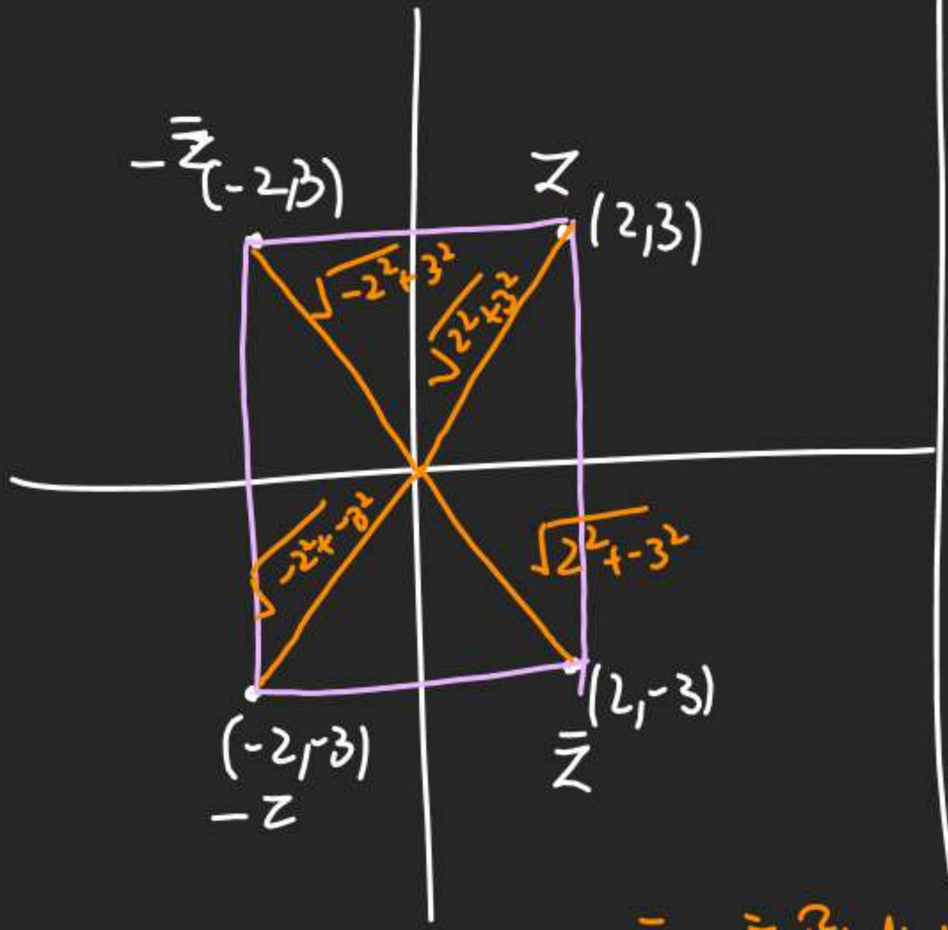


$Z = 2+3i = (2,3)$

$\bar{z} = 2-3i = (2,-3)$

$-Z = -2-3i = (-2,-3)$

$-\bar{z} = -2+3i = (-2,3)$



origin से $z, -z, \bar{z}, -\bar{z}$ की distance eq.

$|z| = |-z| = |\bar{z}| = |-\bar{z}|$

(4) $z + \bar{z} = x + i\cancel{y} + x - i\cancel{y}$

$= 2x$

$$\boxed{z + \bar{z} = 2\,Re(z)}$$

(5) $z - \bar{z} = x + iy - (x - iy)$

$= 2iy$

$= 2i\,(y)$

$$\boxed{z - \bar{z} = 2i\,(Im\,z)}$$

(6) $\boxed{x = \dfrac{z+\bar{z}}{2} \quad y = \dfrac{z-\bar{z}}{2i}}$

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Q Convert $x+2y=3$ in Complex form?

$$x+2y=3$$

$$\frac{1}{i}=-i$$

$$\frac{z+\bar{z}}{2}+\cancel{2}\cdot\frac{z-\bar{z}}{2i}=3$$

$$\left(\frac{z+\bar{z}}{2}\right)-i(z-\bar{z})=3$$

$$z+\bar{z}-2iz+2i\bar{z}=6$$

C. form $\boxed{z(1-2i)+\bar{z}(1+2i)=6}$

RK: A) If in Qs. z is given Real then $z=\bar{z}$

B) If in Qs z is given Img. then take $z=-\bar{z}$

Properties of $\bar{z}$

(1) $(\bar{\bar{z}})=z$

(2) $z+\bar{z}=0 \Rightarrow 2x=0$

$x=0$

z is Purely Img.

(3) $z-\bar{z}=0 \Rightarrow 2iy=0$

$y=0$

z is Purely Real C.N.

(4) $x=\frac{z+\bar{z}}{2}, y=\frac{z-\bar{z}}{2i}$

(5) $(\overline{z_1+z_2})=\bar{z}_1+\bar{z}_2$

$(\overline{z_1+z_2+z_3})=\bar{z}_1+\bar{z}_2+\bar{z}_3$

(6) $(\overline{z_1-z_2})=\bar{z}_1-\bar{z}_2$

(7) $(\overline{z_1 z_2})=\bar{z}_1\cdot\bar{z}_2$

(8) $\left(\overline{\frac{z_1}{z_2}}\right)=\frac{\bar{z}_1}{\bar{z}_2}$

(9) $w=f(x+iy)$ then

$\bar{w}=f(x-iy)$

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Properties of |z|

(1) $z = x + iy \Rightarrow |z| = \sqrt{x^2 + y^2}$

(2) $|z| = |\bar{z}| = |-z| = |-\bar{z}|$

(3) $z \cdot \bar{z} = |z|^2$

$(x+iy)(x-iy) = \left(\sqrt{x^2+y^2}\right)^2$

$(x)^2 - (iy)^2 = x^2 + y^2$

$x^2 + y^2 = x^2 + y^2$

(4) Reciprocal of CN $= \dfrac{1}{z}$

$\dfrac{1}{z} = \dfrac{1}{z} \times \dfrac{\bar{z}}{\bar{z}} = \dfrac{\bar{z}}{|z|^2}$

$\boxed{\dfrac{1}{z} = \dfrac{\bar{z}}{|z|^2}} \Rightarrow \boxed{\dfrac{1}{z} = \lambda \bar{z}}$

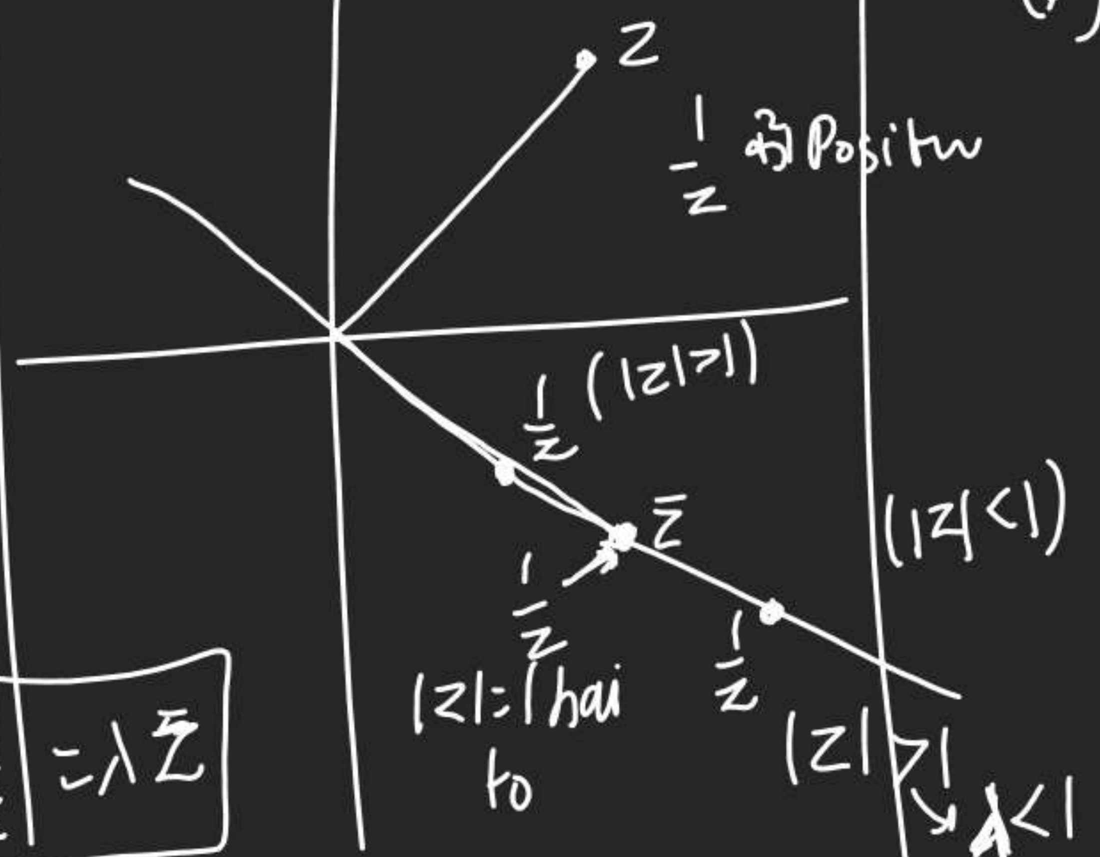
(5) z is Unimodulus

$|z| = 1$

$\Rightarrow |z|^2 = 1$

$\Rightarrow z \cdot \bar{z} = 1$

$\Rightarrow \boxed{\bar{z} = \dfrac{1}{z}}$ of $|z| = 1$



$\boxed{\dfrac{1}{z}, \text{ origin } \& \ \bar{z} \text{ are collinear.}}$

(6) $|z_1 \cdot z_2| = |z_1||z_2|$

$|z_1 z_2 z_3| = |z_1||z_2||z_3|$

(7) $\left|\dfrac{z_1}{z_2}\right| = \dfrac{|z_1|}{|z_2|}$

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① $(a+b)^2 = a^2+b^2+2ab$

$(a-b)^2 = a^2+b^2-2ab$

$(a+b)^3 = a^3+3a^2b+3ab^2+b^3$

$(a-b)^3 = a^3-3a^2b+3ab^2-b^3.$

Q If $\dfrac{(1+i)^2}{(1-i)^2} + \dfrac{1}{(x+iy)} = 1+i$

find $(x,y) = ?$

$\Rightarrow \left(\dfrac{1+i}{1-i}\right)^2 + \left(\dfrac{1}{x+iy}\right) = 1+i$

$\Rightarrow (i)^2 + \dfrac{1}{x+iy} = 1+i$

$\dfrac{1}{x+iy} = 2+i$

$x+iy = \dfrac{1}{2+i} \times \dfrac{2-i}{2-i} = \dfrac{2-i}{5} \Rightarrow x = \dfrac{2}{5}, y = -\dfrac{1}{5}$

Q $\dfrac{3+2i\sin\theta}{1-2i\sin\theta}$ is Purely Img. find θ.

$\mathrm{Re}(\quad) = 0$

$z = \dfrac{3+2i\sin\theta}{1-2i\sin\theta} \times \dfrac{1+2i\sin\theta}{1+2i\sin\theta}$

$= \dfrac{3+6i\sin\theta+2i\sin\theta-4\sin^2\theta}{1^2-(2i\sin\theta)^2}$

$z = \dfrac{3-4\sin^2\theta}{1+4\sin^2\theta} + i(\quad)$

$\Rightarrow \dfrac{3-4\sin^2\theta}{1+4\sin^2\theta} = 0$

$\sin^2\theta = \dfrac{3}{4} = \left(\dfrac{\sqrt{3}}{2}\right)^2$

$\sin^2\theta = \sin^2\dfrac{\pi}{3}$

$\theta = n\pi \pm \pi/3$

Q If $(x+iy)^{1/3} = a+ib$

& $\boxed{\dfrac{x}{a}+\dfrac{y}{b} = k(a^2-b^2)}$ find $k = ?$

① $(x+iy)^{1/3} = a+ib$

$x+iy = (a+ib)^3$

$= a^3+3a^2(ib)+3a(ib)^2+(ib)^3$

$x+iy = a^3-3ab^2+i(3a^2b-b^3)$

2) $x = a^3-3ab^2$ | $y = 3a^2b-b^3$

$\dfrac{x}{a} = a^2-3b^2$ | $\dfrac{y}{b} = 3a^2-b^2$

(3) $\dfrac{x}{a}+\dfrac{y}{b} = (a^2-3b^2)+(3a^2-b^2)$

$= 4(a^2-b^2)$

$k = 4$

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Q $Re\left(\frac{1}{z}\right) < 2$ find locus of z.

$$\frac{1}{z} = \frac{1}{x+iy} \times \frac{x-iy}{x-iy}$$

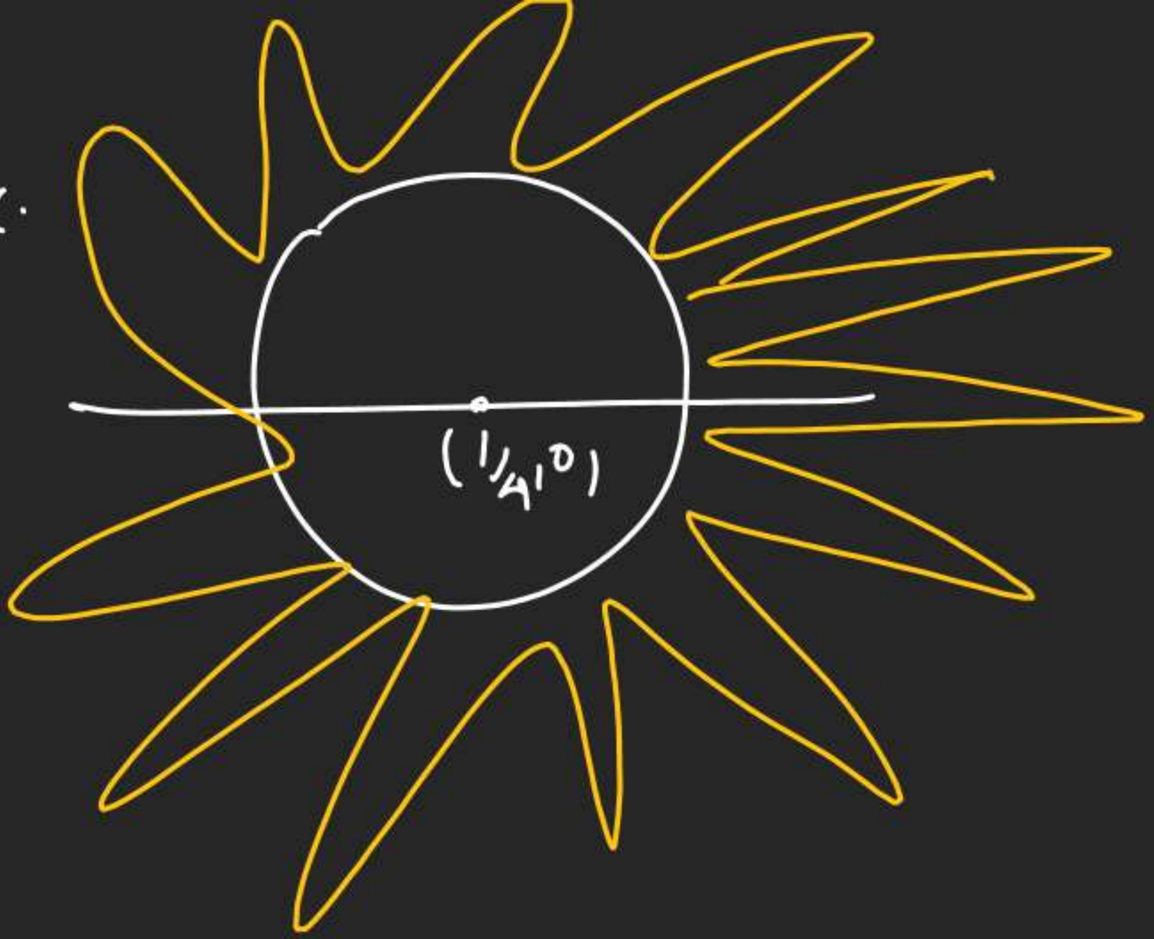
$$\frac{1}{z} = \frac{x-iy}{x^2+y^2}$$

$$Re\left(\frac{1}{z}\right) = \left(\frac{x}{x^2+y^2}\right) < 2$$

$$2(x^2+y^2) > x$$

$$\underbrace{x^2+y^2-\frac{x}{2}}_{} > 0$$

Circle → Centre $= \left(\frac{1}{4}, 0\right)$, $R = \frac{1}{4}$



Locus Rep. all pt. outside Circle.

Q Z is C.N.S.T. $\boxed{\frac{z-1}{z+1} \text{ is Purely Img}}$ then $|z| = ?$

$$\frac{z-1}{z+1} = -\left(\overline{\frac{z-1}{z+1}}\right)$$

$$\frac{z-1}{z+1} = -\frac{(\bar{z}-1)}{(\bar{z}+1)}$$

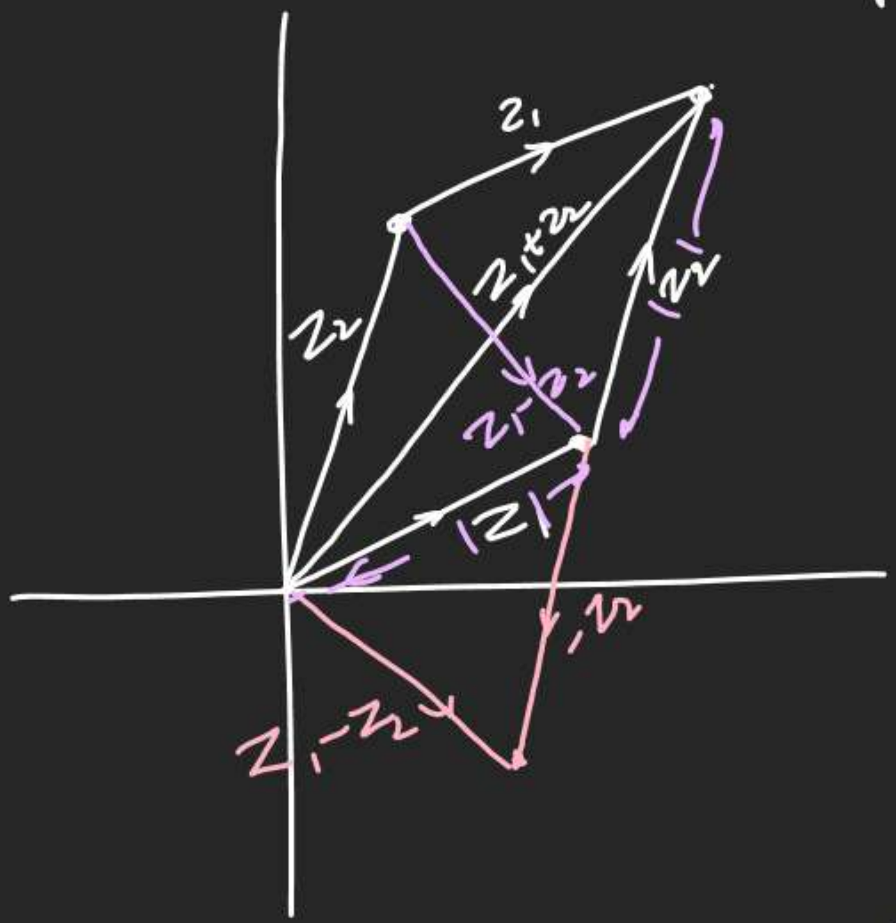
$$\Rightarrow z\bar{z} - \cancel{\bar{z}} + \cancel{z} - 1 = -z\bar{z} + \cancel{z} - \cancel{\bar{z}} + 1$$

$$\Rightarrow |z|^2 - 1 = -|z|^2 + 1$$

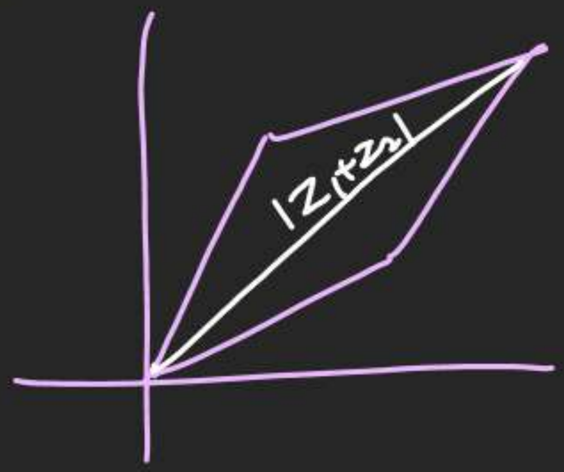
$$2|z|^2 = 2 \Rightarrow |z|^2 = 1$$

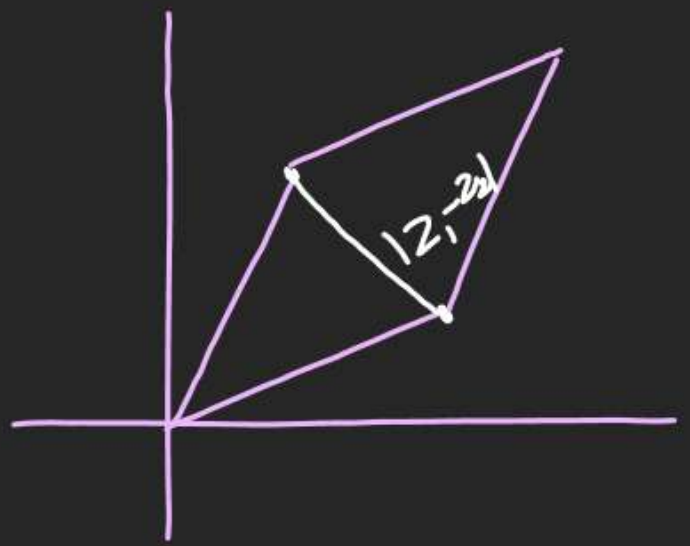
$$|z| = 1$$

Q Rep. z_1+z_2 & z_1-z_2 at Complex Plane?



Q What is $|z_1+z_2|$ & $|z_1-z_2|$





$$|z_1+z_2| = |x_1+iy_1+x_2+iy_2|$$
$$= |(x_1+x_2)+i(y_1+y_2)|$$
$$= \sqrt{(x_1+x_2)^2+(y_1+y_2)^2}$$

Q $z=(1+i)(1+2i)(1+3i)$ find $|z|=?$

$$|z| = |(1+i)(1+2i)(1+3i)|$$
$$= |1+i|\,|1+2i|\,|1+3i|$$
$$= \sqrt{1^2+1^2} \times \sqrt{1^2+2^2}\sqrt{1^2+3^2} = \sqrt{2}\sqrt{5}\sqrt{10}$$
$$= 10$$

Q $z = \dfrac{(1+i)(2+i)}{(3+i)}$ then $|z|=?$

$$\left|\frac{(1+i)(2+i)}{(3+i)}\right| = \frac{|1+i|\,|2+i|}{|3+i|}$$
$$= \frac{\sqrt{1^2+1^2}\sqrt{2^2+1^2}}{\sqrt{3^2+1^2}} = \frac{\sqrt{2}\times\sqrt{5}}{\sqrt{10}}$$
$$= 1$$

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Q If $\left(\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)^{50}=3^{24}(x+iy)$

then $x^2+y^2=?$

hint of $|z|$

Rem $\boxed{\begin{array}{c}(\overline{z^n})=(\bar{z})^n \\ |z^n|=|z|^n\end{array}}$

take Mod.

$\left|\left(\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)^{50}\right|=\left|3^{24}(x+iy)\right|$

$\left|\frac{3}{2}+i\frac{\sqrt{3}}{2}\right|^{50}=3^{24}|x+iy|$

$\left(\sqrt{\frac{9}{4}+\frac{3}{4}}\right)^{50}=3^{24}\sqrt{x^2+y^2}$

$3\ \cancel{3^{25}}=\cancel{3^{24}}\sqrt{x^2+y^2}$

$x^2+y^2=9$

Q If $(a+ib)^5=P+iq$ then

S.T. $(b+ia)^5=q+ip$

proof it q without i

Fix style

conjugat

$(a+ib)^5=P+iq$

$\overline{(a+ib)^5}=\overline{(P+iq)}$

$(a-ib)^5=P-iq$

$(-i)^5\left(b+\frac{a}{-i}\right)^5=-i\left(q+\frac{P}{-i}\right)$

$-i(b+ai)^5=-i(q+ip)$

$(b+ai)^5=q+ip$ H.P

Q find $|z_1+z_2|^2$ & $|z_1-z_2|^2$

Result

& P.T. $|z_1+z_2|^2+|z_1-z_2|^2=2(|z_1|^2+|z_2|^2)$

$|z|^2=z\cdot\bar{z}$

① $|z_1+z_2|^2=(z_1+z_2)\cdot(\overline{z_1+z_2})$

$=(z_1+z_2)(\bar{z}_1+\bar{z}_2)$

$=z_1\bar{z}_1+z_1\bar{z}_2+z_2\bar{z}_1+z_2\bar{z}_2$

$=|z_1|^2+|z_2|^2+z_1\bar{z}_2+z_2\bar{z}_1$

$=|z_1|^2+|z_2|^2+(z_1\bar{z}_2+\overline{z_1\bar{z}_2})$

$z+\bar{z}=2\,\mathrm{Re}(z)$

$\boxed{|z_1+z_2|^2=|z_1|^2+|z_2|^2+2\,\mathrm{Re}(z_1\bar{z}_2)}$

Sly $\boxed{|z_1-z_2|^2=|z_1|^2+|z_2|^2-2\,\mathrm{Re}(z_1\bar{z}_2)}$

Add $|z_1+z_2|^2+|z_1-z_2|^2=2(|z_1|^2+|z_2|^2)$

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Q If $|z-2+3i| = |z-1+2i|$

find locus of z.

$$|x+iy-2+3i| = |x+iy-1+2i|$$

$$|(x-2)+i(y+3)| = |(x-1)+i(y+2)|$$

$$\sqrt{(x-2)^2+(y+3)^2} = \sqrt{(x-1)^2+(y+2)^2}$$

$$\cancel{x^2}+\cancel{y^2}-4x+6y+13 = \cancel{x^2}+\cancel{y^2}-2x+4y+5$$

$$2x-2y = 8$$

Analysis

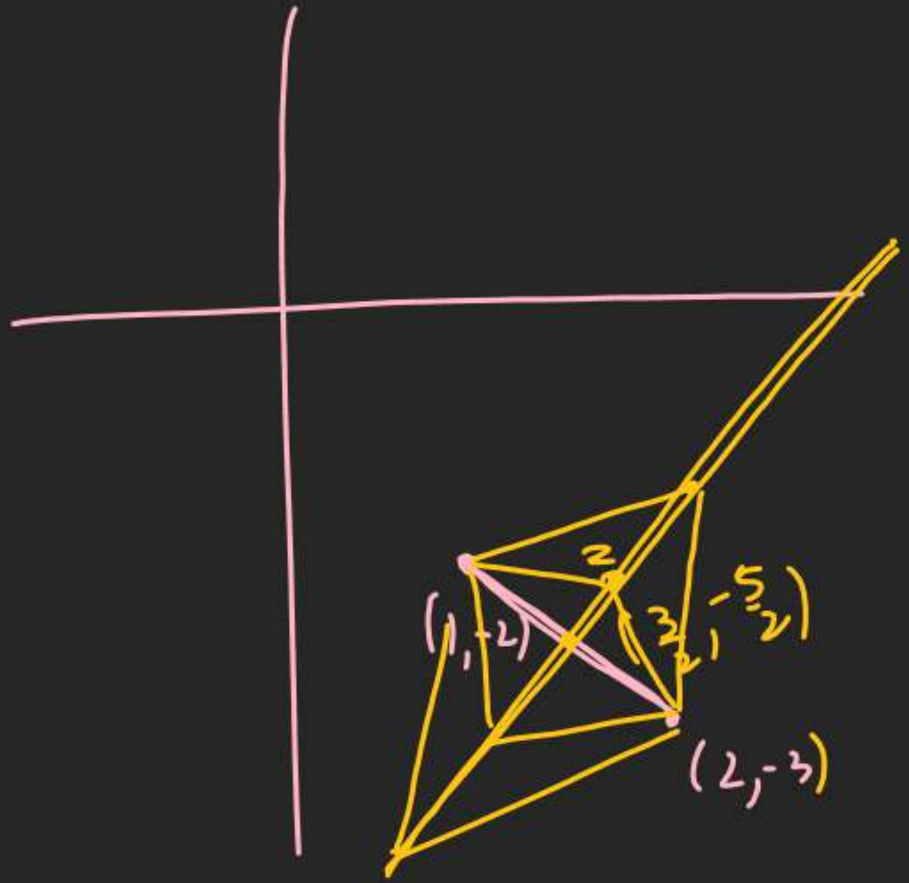
1) $x-y=4$ → Locus is a St. Line

2) This is also $\perp^r$ Bisector Line of $(2,-3)$ & $(1,-2)$

3) $|z-2+3i| = |z-1+2i|$

$|z-(2-3i)| = |z-(1-2i)|$ → $|z-z_1| = |z-z_2|$ Likha hua hai

$(2,-3)$ $(1,-2)$



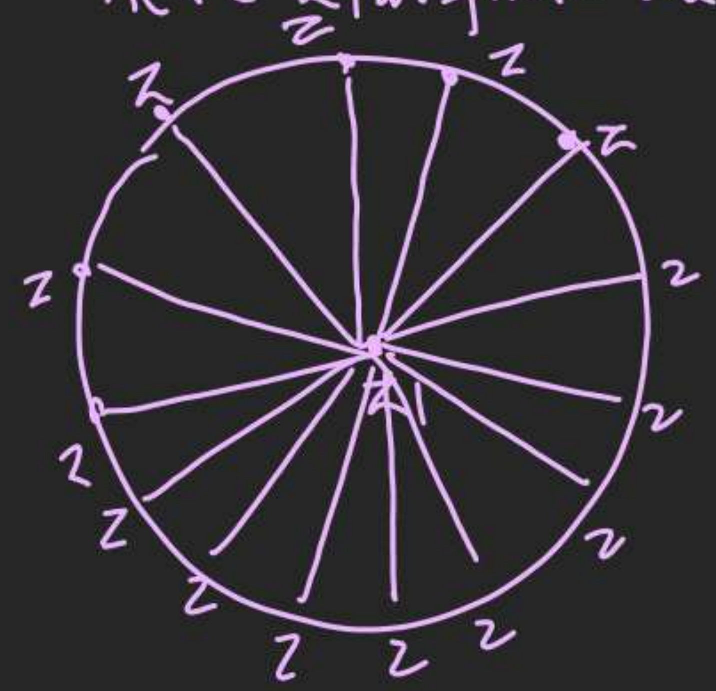
RK: A) $|z-z_1| =$ z का z_1 से dist.

B) $|z_1-z_2| =$ dist. betn z_1 & z_2

C) $|z-z_1| = 6$

dist betn z & $z_1 = 6$

here z_1 is fix pt & z is a var. pt



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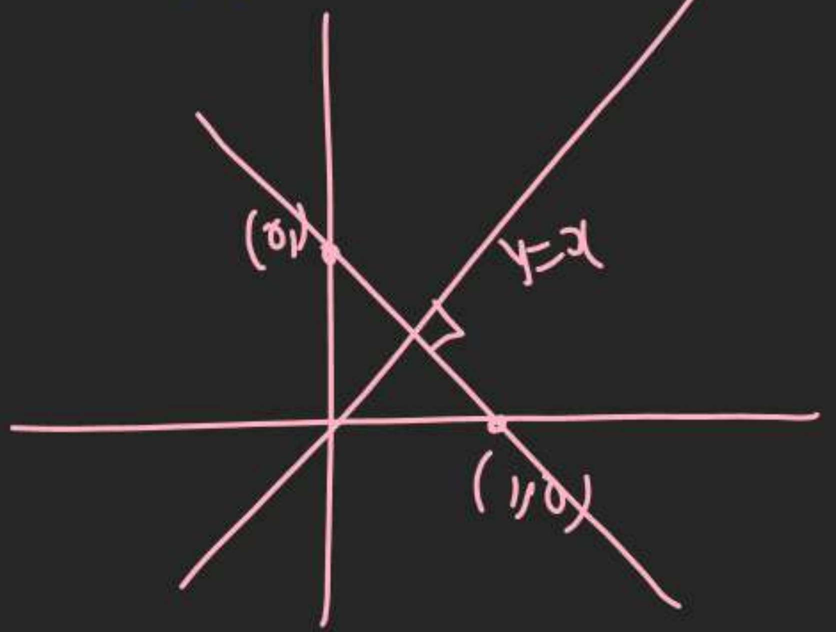
Q $|z-i| = |z-1|$ the Locus.

$z_1 = (0,1)$ $z_2 = (1,0)$



1) $PA = PB \Rightarrow$ Locus of P is ⊥r Bis.

2) $PA = 2PB \Rightarrow$ Locus of P is Circle

Sqr Root of a C.N.

Q find $\sqrt{8-15i}$

① $\sqrt{C.N.} = C.N.$

Let $\sqrt{8-15i} = a+ib$

1) $8-15i = a^2-b^2+2iab$

$a^2-b^2=8 \;|\; 2ab=-15$

(2) $(a^2+b^2)^2 = (a^2-b^2)^2+4a^2b^2$

$= 64+225$

$a^2+b^2 = 17^2$

$a^2+b^2=17$

$a^2-b^2=8$

$a = \pm\frac{5}{\sqrt{2}}, \; b = \mp\frac{3}{\sqrt{2}}$

$\sqrt{8-15i} = \pm\left(\frac{5}{\sqrt{2}} - \frac{3i}{\sqrt{2}}\right)$

We Use Short Cut

$$\sqrt{x \pm iy} = \pm\left\{\sqrt{\frac{|z|+x}{2}} \pm i\sqrt{\frac{|z|-x}{2}}\right\}$$

$$\sqrt{7+24i} = \pm\left\{\sqrt{\frac{25+7}{2}} + i\sqrt{\frac{25-7}{2}}\right\} = \pm(4+3i)$$

$$\sqrt{-7+24i} = \pm\left\{\sqrt{\frac{25-7}{2}} + i\sqrt{\frac{25+7}{2}}\right\} = \pm(3+4i)$$

$$\sqrt{7-24i} = \pm\left\{\sqrt{\frac{25+7}{2}} - i\sqrt{\frac{25-7}{2}}\right\} = \pm(4-3i)$$