Nishant Jindal

- $$[\vec{a}\ \vec{b}\ \vec{c}] = [\vec{b}\ \vec{c}\ \vec{a}] = [\vec{c}\ \vec{a}\ \vec{b}]$$
$$= -[\vec{b}\ \vec{a}\ \vec{c}]$$

- $$(\vec{a} \times \vec{b}) \cdot \vec{c} = \vec{a} \cdot (\vec{b} \times \vec{c})$$
$$(\vec{a} \times \vec{b}) \cdot \vec{c} = \vec{c} \cdot (\vec{a} \times \vec{b}) = \vec{a} \cdot (\vec{b} \times \vec{c})$$

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- Right handed System

$\vec{a}, \vec{b}, \vec{c}$ non coplanar are said to form a right handed system if $[\vec{a}\ \vec{b}\ \vec{c}] > 0$.

$\vec{a}, \vec{b}, \vec{c}$ non coplanar are said to form a left handed system if $[\vec{a}\ \vec{b}\ \vec{c}] < 0$.

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1. P.T. $\left[\vec{l}\ \vec{m}\ \vec{n}\right]\left[\vec{a}\ \vec{b}\ \vec{c}\right] = \begin{vmatrix} \vec{l}\cdot\vec{a} & \vec{l}\cdot\vec{b} & \vec{l}\cdot\vec{c} \\ \vec{m}\cdot\vec{a} & \vec{m}\cdot\vec{b} & \vec{m}\cdot\vec{c} \\ \vec{n}\cdot\vec{a} & \vec{n}\cdot\vec{b} & \vec{n}\cdot\vec{c} \end{vmatrix}$

$$\begin{vmatrix} l_1 & l_2 & l_3 \\ m_1 & m_2 & m_3 \\ n_1 & n_2 & n_3 \end{vmatrix} \cdot \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

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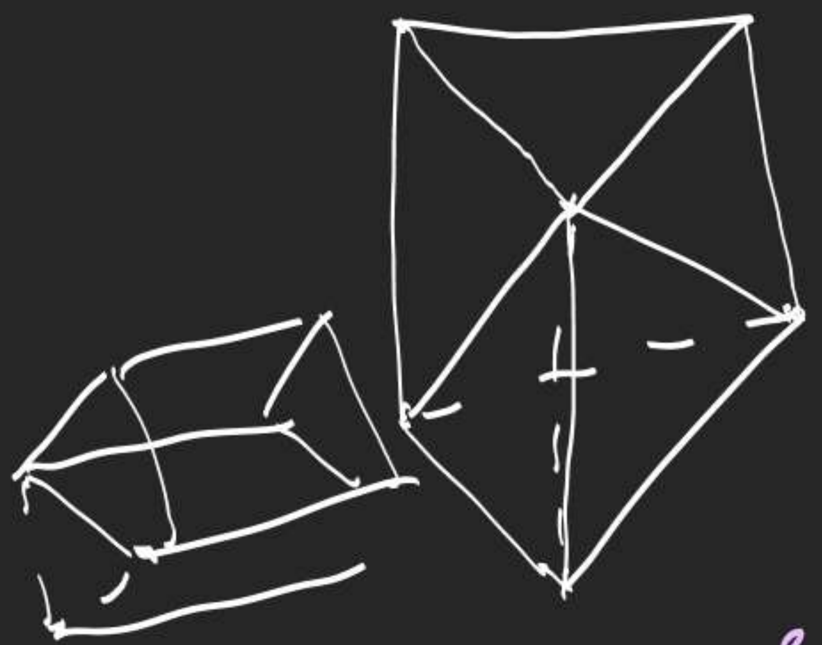
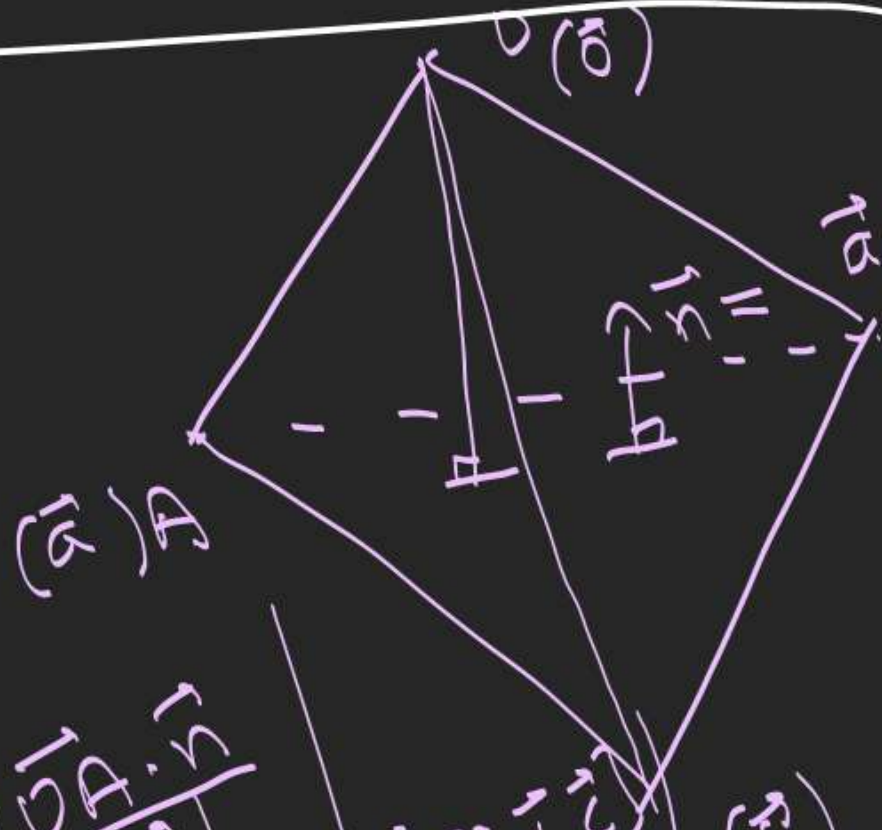
2: P.T. volume of tetrahedron OABC is

$$\boxed{V = \frac{1}{6}\left|[\vec{OA}\ \vec{OB}\ \vec{OC}]\right|}$$





$\vec{n} = \vec{a}\times\vec{b} + \vec{b}\times\vec{c} + \vec{c}\times\vec{a}$

$$h = \left|\frac{\vec{OA}\cdot\vec{n}}{|\vec{n}|}\right| = \left|\frac{\vec{a}\cdot\vec{n}}{|\vec{n}|}\right| = \left|\frac{[\vec{a}\ \vec{b}\ \vec{c}]}{|\vec{n}|}\right|$$

$$V = \frac{1}{3}\times\frac{1}{2}\cancel{|\vec{n}|}\times\left|\frac{[\vec{a}\ \vec{b}\ \vec{c}]}{\cancel{|\vec{n}|}}\right|$$

$$= \frac{1}{6}\left|[\vec{a}\ \vec{b}\ \vec{c}]\right|$$

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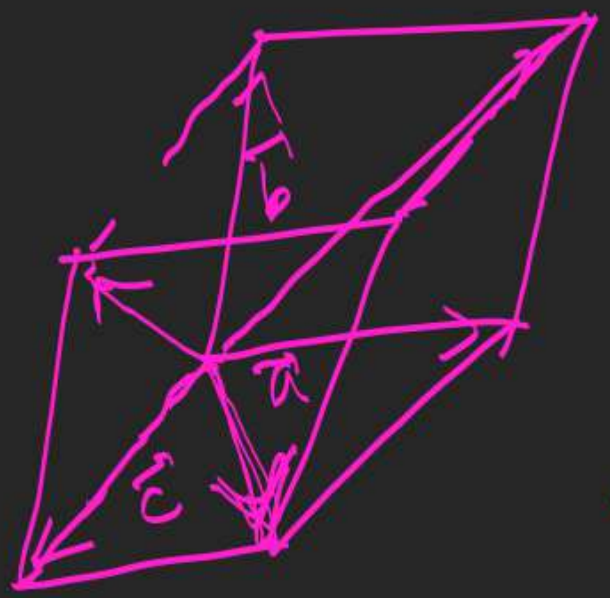
3. P.T. $\boxed{[\vec{a}+\vec{b} \quad \vec{b}+\vec{c} \quad \vec{c}+\vec{a}] = 2[\vec{a} \quad \vec{b} \quad \vec{c}]}$

Also give its geometric interpretation.



$$\left((\vec{a}+\vec{b})\times(\vec{b}+\vec{c})\right)\cdot(\vec{c}+\vec{a})$$

$$= \left(\vec{a}\times\vec{b}+\vec{a}\times\vec{c}+\vec{b}\times\vec{c}\right)\cdot(\vec{c}+\vec{a}) = 2[\vec{a} \quad \vec{b} \quad \vec{c}]$$

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Vector Triple Product

$\boxed{(\vec{a} \times \vec{b}) \times \vec{c} \text{ is coplanar with } \vec{a}, \vec{b} \text{ and } \perp \text{ to } \vec{c}}$

(c_1, c_2, c_3)

(a_1, a_2, a_3) (b_1, b_2, b_3)

$\vec{v} = (\vec{a} \times \vec{b}) \times \vec{c}$

$\vec{v} = x\vec{a} + y\vec{b}$

$\vec{v} \cdot \vec{c} = 0 = x(\vec{a} \cdot \vec{c}) + y(\vec{b} \cdot \vec{c})$

$$\frac{x}{\vec{b} \cdot \vec{c}} = \frac{-y}{\vec{a} \cdot \vec{c}} = \lambda$$

$\vec{a} = \hat{i}, \ \vec{b} = \hat{j}, \ \vec{c} = \hat{i}$

$LHS = \hat{k} \times \hat{i} = \hat{j}$

$RHS = \lambda(-\hat{j})$

$\vec{v} = \lambda\left((\vec{b} \cdot \vec{c})\vec{a} - (\vec{a} \cdot \vec{c})\vec{b}\right)$

$\lambda = -1$

$$\boxed{(\vec{a} \times \vec{b}) \times \vec{c} = (\vec{a} \cdot \vec{c})\vec{b} - (\vec{b} \cdot \vec{c})\vec{a}}$$

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1. P.T. $\vec{V}_1 = \vec{a} \times (\vec{b} \times \vec{c})$, $\vec{V}_2 = \vec{b} \times (\vec{c} \times \vec{a})$ & $\vec{V}_3 = \vec{c} \times (\vec{a} \times \vec{b})$ are coplanar.

$$\vec{V}_1 = (\vec{a}\cdot\vec{c})\vec{b} - (\vec{a}\cdot\vec{b})\vec{c}$$
$$\vec{V}_2 = (\vec{b}\cdot\vec{a})\vec{c} - (\vec{b}\cdot\vec{c})\vec{a}$$
$$\vec{V}_3 = (\vec{c}\cdot\vec{b})\vec{a} - (\vec{c}\cdot\vec{a})\vec{b}$$

$$\vec{V}_1 + \vec{V}_2 + \vec{V}_3 = \vec{0}$$

2. If $\hat{a} \times (\hat{b} \times \hat{c}) = \frac{1}{2}\hat{b}$, where $\hat{b}, \hat{c}$ are non collinear

Find the angle between $\hat{a}$ & $\hat{b}$ and b/n $\hat{a}$ & $\hat{c}$.

$\pi/2$ $\qquad$ $\frac{\pi}{3}$

$$(\hat{a}\cdot\hat{c})\hat{b} - (\hat{a}\cdot\hat{b})\hat{c} = \frac{1}{2}\hat{b}$$

$$\hat{a}\cdot\hat{c} = \frac{1}{2}$$
$$\hat{a}\cdot\hat{b} = 0$$

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1. $$\underbrace{(\vec{a}\times\vec{b})}_{\vec{u}}\cdot(\vec{c}\times\vec{d}) = ((\vec{a}\times\vec{b})\times\vec{c})\cdot\vec{d}$$

$$= \left((\vec{a}\cdot\vec{c})\vec{b} - (\vec{b}\cdot\vec{c})\vec{a}\right)\cdot\vec{d}$$

$$= (\vec{a}\cdot\vec{c})(\vec{b}\cdot\vec{d}) - (\vec{b}\cdot\vec{c})(\vec{a}\cdot\vec{d})$$

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2. $(\vec{a}\times\vec{b})\times\underbrace{(\vec{c}\times\vec{d})}_{\vec{u}} = [\vec{a}\ \vec{b}\ \vec{d}]\vec{c} - [\vec{a}\ \vec{b}\ \vec{c}]\vec{d}$

$$= [\vec{a}\ \vec{c}\ \vec{d}]\vec{b} - [\vec{b}\ \vec{c}\ \vec{d}]\vec{a}$$

3. If $\vec{a}, \vec{b}, \vec{c}, \vec{d}$ are 4 vectors s.t. no 3 of them are coplanar, then any vector among them can be expressed as linear combination of remaining vectors.

$$\vec{d} = \frac{[\vec{b}\ \vec{c}\ \vec{d}]}{[\vec{a}\ \vec{b}\ \vec{c}]}\vec{a} - \frac{[\vec{a}\ \vec{c}\ \vec{d}]}{[\vec{a}\ \vec{b}\ \vec{c}]}\vec{b} + \frac{[\vec{a}\ \vec{b}\ \vec{d}]}{[\vec{a}\ \vec{b}\ \vec{c}]}\vec{c}$$

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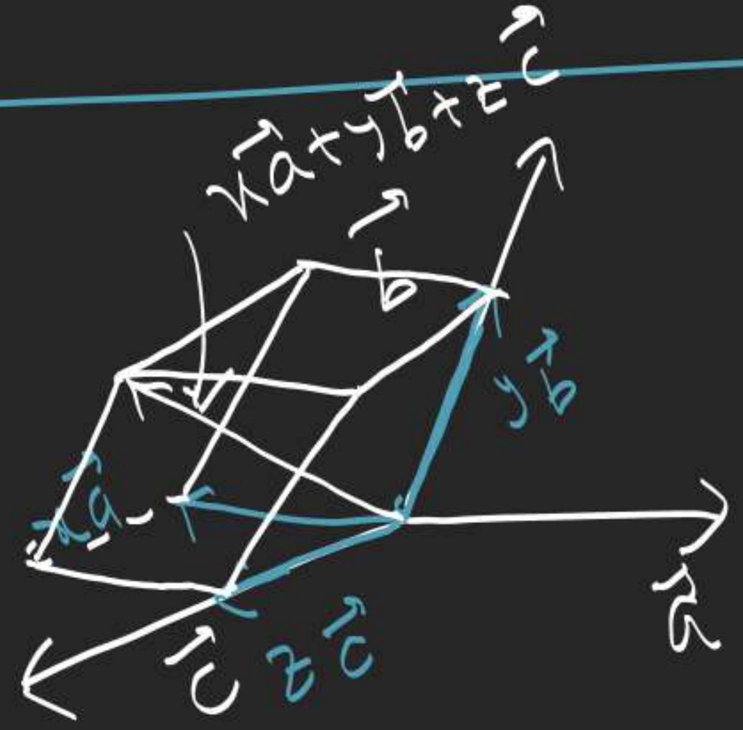
Theorem for Space

If $\vec{a}, \vec{b}, \vec{c}$ are non coplanar vectors, then any vector in 3D space can be expressed as their linear combination.

$$\vec{v} = x\vec{a} + y\vec{b} + z\vec{c} \quad , x, y, z \in R.$$



$$\vec{v} = x\hat{i} + y\hat{j} + z\hat{k}$$

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Note $\rightarrow$ 4 or more vectors are always linearly dependent.

Case I Let $\vec{a}, \vec{b}, \vec{c}, \vec{d}$ s.t. atleast one set of 3 vectors among them is coplanar

$$\vec{c} = x\vec{a} + y\vec{b}$$

$$(0)\vec{d} + (1)\vec{c} + (-x)\vec{a} + (-y)\vec{b} = \vec{0}$$

Case II no 3 of them are coplanar.

$$\vec{d} = x\vec{a} + y\vec{b} + z\vec{c} \Rightarrow (1)\vec{d} + (-x)\vec{a} + (-y)\vec{b} + (-z)\vec{c} = \vec{0}$$

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1. P.T. $\underbrace{(\vec{a}\times\vec{b})}_{\vec{u}}\times(\vec{c}\times\vec{d})+(\vec{a}\times\vec{c})\times\underbrace{(\vec{d}\times\vec{b})}_{\vec{u}}$

$+(\vec{a}\times\vec{d})\times\underbrace{(\vec{b}\times\vec{c})}_{\vec{u}}$ is parallel to $\vec{a}$

$$\cancel{[\vec{a}\ \vec{b}\ \vec{d}]\vec{c}}-[\vec{a}\ \vec{b}\ \vec{c}]\vec{d}+\cancel{[\vec{a}\ \vec{d}\ \vec{b}]\vec{c}}-[\vec{c}\ \vec{d}\ \vec{b}]\vec{a}$$

$$+[\vec{a}\ \vec{b}\ \vec{c}]\vec{d}-[\vec{d}\ \vec{b}\ \vec{c}]\vec{a}$$

$$=-2[\vec{c}\ \vec{d}\ \vec{b}]\vec{a}$$

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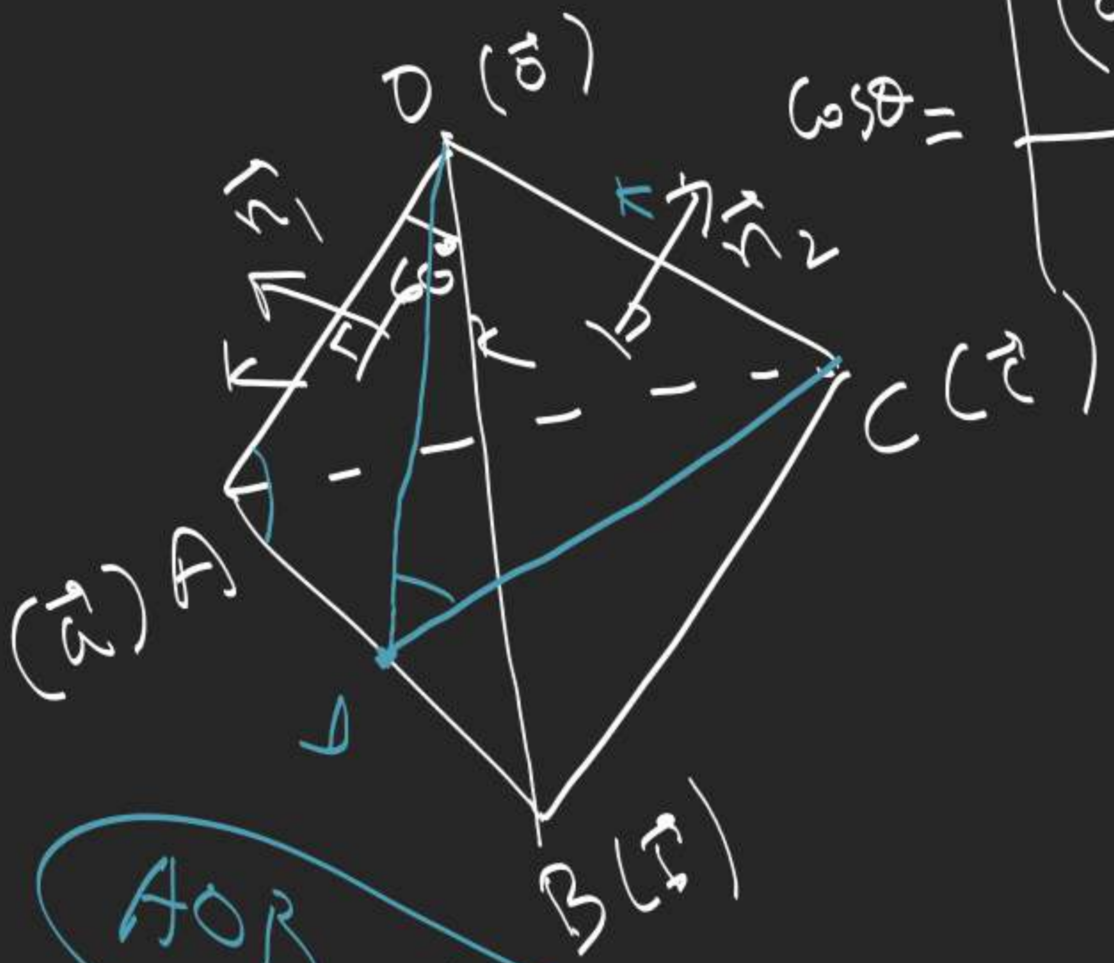
Q. Find the acute angle b/n two plane faces of a regular tetrahedron

$(\vec{a}\times\vec{b})\times\vec{b}\cdot\vec{c}$



AOB _ ABC

$$\cos\theta = \left|\frac{(\vec{a}\times\vec{b})\cdot(\vec{b}\times\vec{c})}{|\vec{a}\times\vec{b}|\,|\vec{b}\times\vec{c}|}\right| = \left|\frac{(\vec{a}\cdot\vec{b})(\vec{b}\cdot\vec{c})-(\vec{b}\cdot\vec{b})(\vec{a}\cdot\vec{c})}{|\vec{a}\times\vec{b}|\,|\vec{b}\times\vec{c}|}\right|$$

$$= \left|\frac{\frac{k^2}{2}\,\frac{k^2}{2} - k^2\,\frac{k^2}{2}}{k^2\frac{\sqrt{3}}{2}\;k^2\frac{\sqrt{3}}{2}}\right|$$

$\boxed{1-15\ (Ex-2)}$

$$\cos\theta = \frac{1}{3}$$

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3. If a, b, c, d are four lines in space and (ad) represents any plane parallel to a and d and so on.
If $(ad) \perp (bc)$ and $(bd) \perp (ca)$, then P.T.
$(cd) \perp (ab)$.