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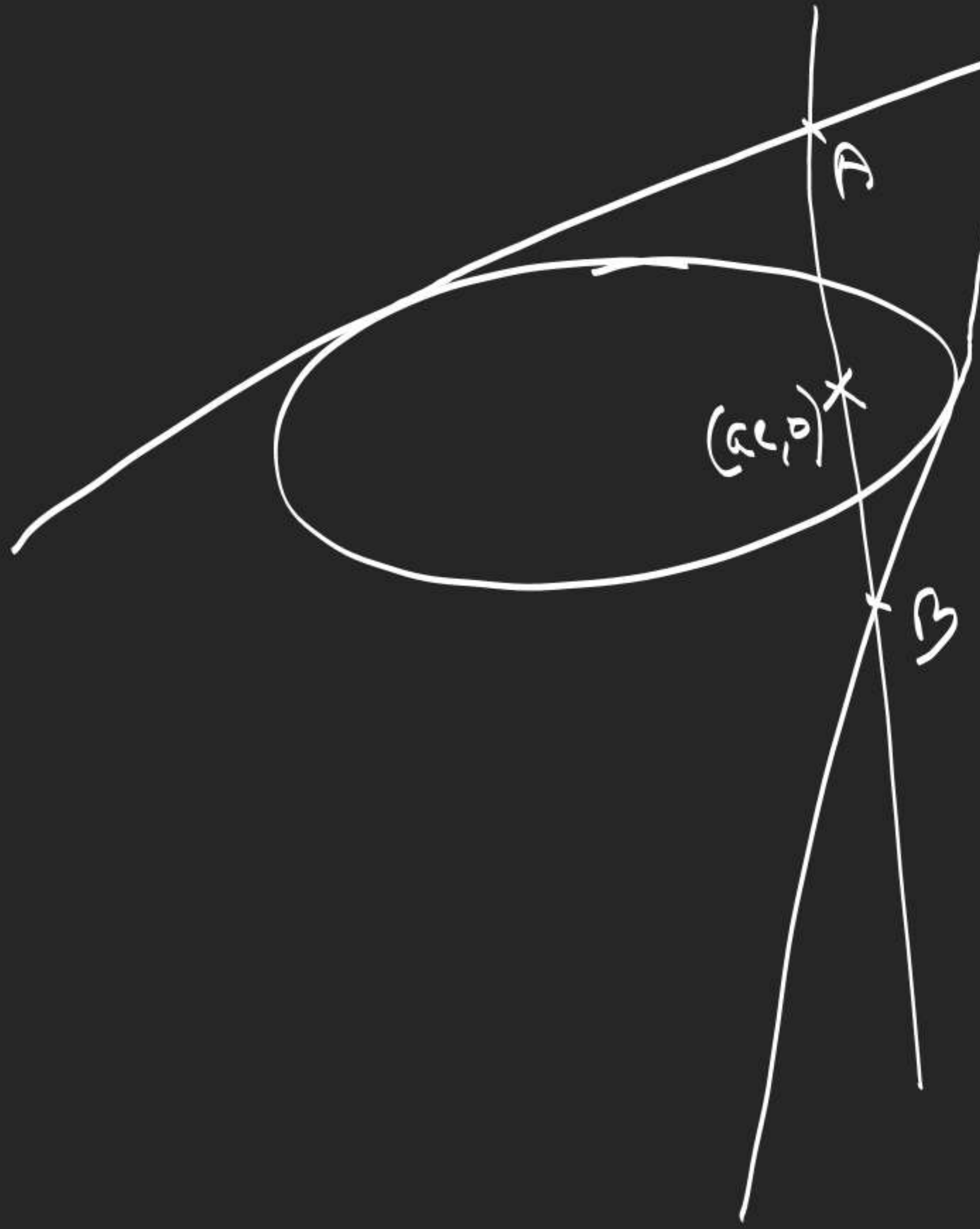

$\left(\frac{a}{e}, \sqrt{a^2+b^2}\right)$

$$\left(\frac{x}{ae} + \frac{y\sqrt{a^2+b^2}}{b^2} - 1\right)^2 = \left(\frac{x^2}{a^2} + \frac{y^2}{b^2} - 1\right)\left(\frac{1}{e^2} + \frac{a^2+b^2}{b^2} - 1\right)$$

$x = ae$

$$y^2 = a^2$$

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Complex

$z = a + ib \qquad a, b \in R. \quad , \; i = \sqrt{-1}$

$a = Re(z) \quad , \quad b = Im(z)$

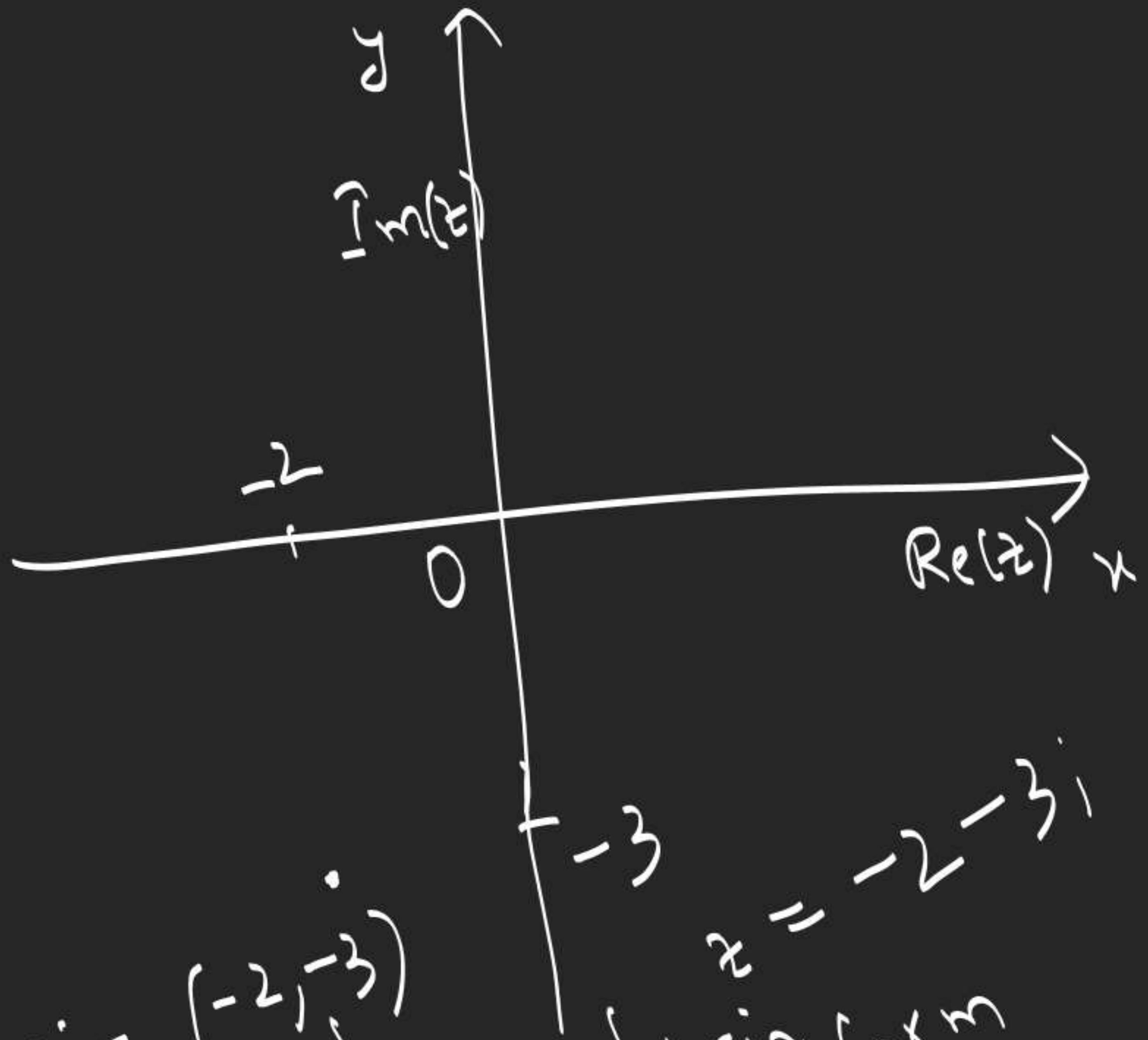
- If $b = 0$, z is purely real.
- If $a = 0$, z is purely imaginary
- If $b \neq 0$, z is imaginary

$\boxed{z = 0 \text{ purely real or purely imag.}}$

$2 + 3i \rightarrow$ imag.

$3i$

Argand Plane (Complex plane)



$z = -2 - 3i$

$-2-3i = (-2,-3)$

Ordered pair form

$z = (0,2) = \boxed{2i}$

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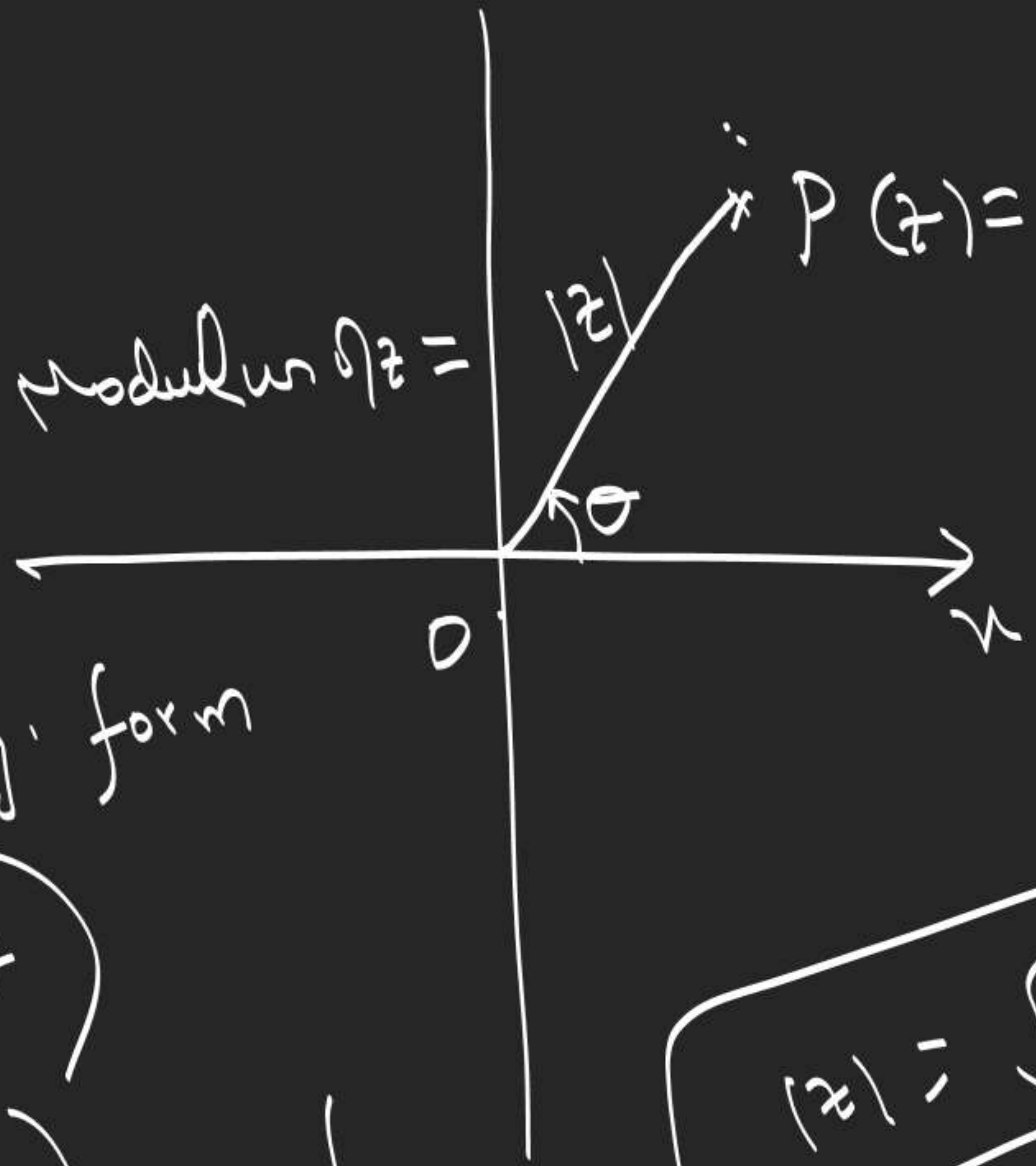


$P(z) = a+ib$

Modulus of $z = |z|$

$$\boxed{z = |z|e^{i\theta} = |z|\,\mathrm{cis}\,\theta}$$

Principle argument

$\theta \in (-\pi, \pi]$

Trig. form

$(|z|, \theta)$

$(2, -\frac{\pi}{3})$

$$\boxed{|z| = \sqrt{a^2+b^2} \qquad |z| \text{ is non negative real}}$$

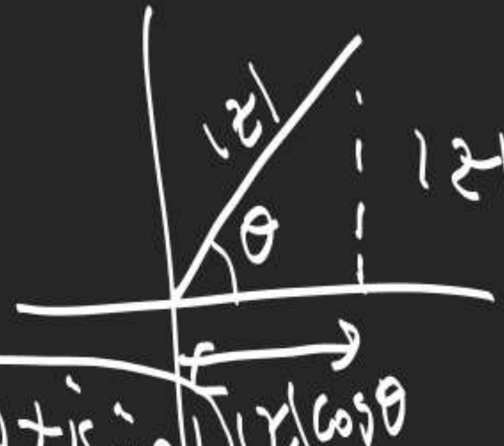
θ = argument of $z = \arg(z)$

$\tan\theta = \frac{b}{a}$

$$\boxed{z = 1-\sqrt{3}\,i}$$

Euler's rule

$$\boxed{z = |z|(\cos\theta + i\sin\theta)}$$



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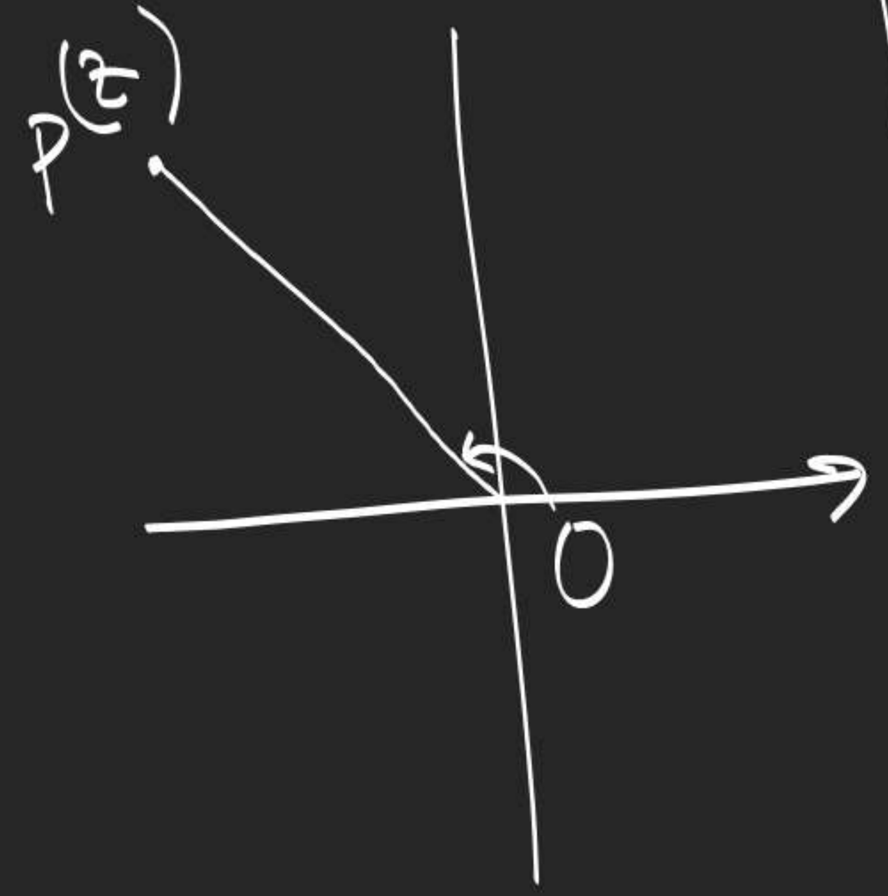


$\boxed{z = \overrightarrow{OP}}$

$z = 0$ has mag. $= 0$ and arg. not defd.

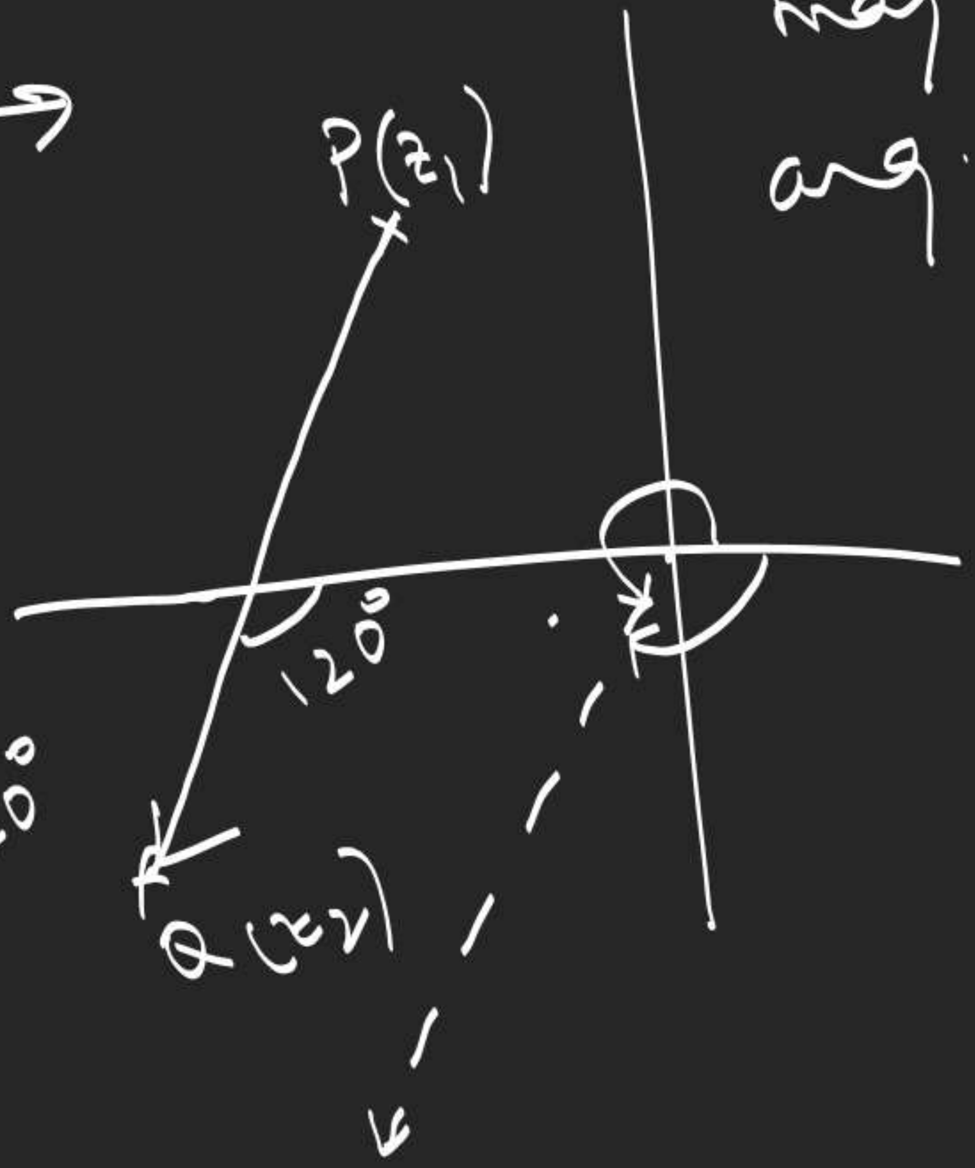
principle arg. of $-1-i$

$= -\frac{3\pi}{4}$

$z_2 - z_1 = \overrightarrow{PQ}$

$|z_2 - z_1| = PQ$

$\theta = -120^\circ$



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Conjugate of z

$z = a + ib$

$\bar{z} = a - ib$

$z_1 + z_2 = x_1 + iy_1 + x_2 + iy_2$

$= (x_1 + x_2) + i(y_1 + y_2)$

$z_1 z_2 = (x_1 + iy_1)(x_2 + iy_2)$

$= (x_1 x_2 - y_1 y_2) + i(x_2 y_1 + x_1 y_2)$

$\frac{z_1}{z_2} = \frac{x_1 + iy_1}{x_2 + iy_2} = \frac{(x_1 + iy_1)(x_2 - iy_2)}{(x_2^2 + y_2^2)}$

$= \left(\frac{x_1 x_2 + y_1 y_2}{x_2^2 + y_2^2}\right) + i\left(\frac{y_1 x_2 - y_2 x_1}{x_2^2 + y_2^2}\right)$

Inequality

$1 + 2i \not> 1 + i$

Equality of Complex no.s

$z_1 = z_2$

$\Rightarrow Re(z_1) = Re(z_2) \;\&\; Im(z_1) = Im(z_2)$

or

$|z_1| = |z_2| \;\&\; arg(z_1) = arg(z_2)$

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$$\boxed{\begin{aligned} \frac{z+\bar{z}}{2} &= Re(z) \\ \frac{z-\bar{z}}{2i} &= Im(z) \\ z\bar{z} &= |z|^2 \end{aligned}}$$

$$\overline{z_1 \pm z_2} = \bar{z}_1 \pm \bar{z}_2$$

$$\overline{z_1 z_2} = \bar{z}_1 \bar{z}_2$$

$$\overline{\left(\frac{z_1}{z_2}\right)} = \frac{\bar{z}_1}{\bar{z}_2}$$

$$\overline{\left(\frac{z_1 z_2^2 z_3 + z_4 z_5}{z_6 + z_7^3 z_8}\right)} = \frac{\bar{z}_1 \bar{z}_2^{\,2} \bar{z}_3 + \bar{z}_4 \bar{z}_5}{\bar{z}_6 + \bar{z}_7^{\,3} \bar{z}_8}$$

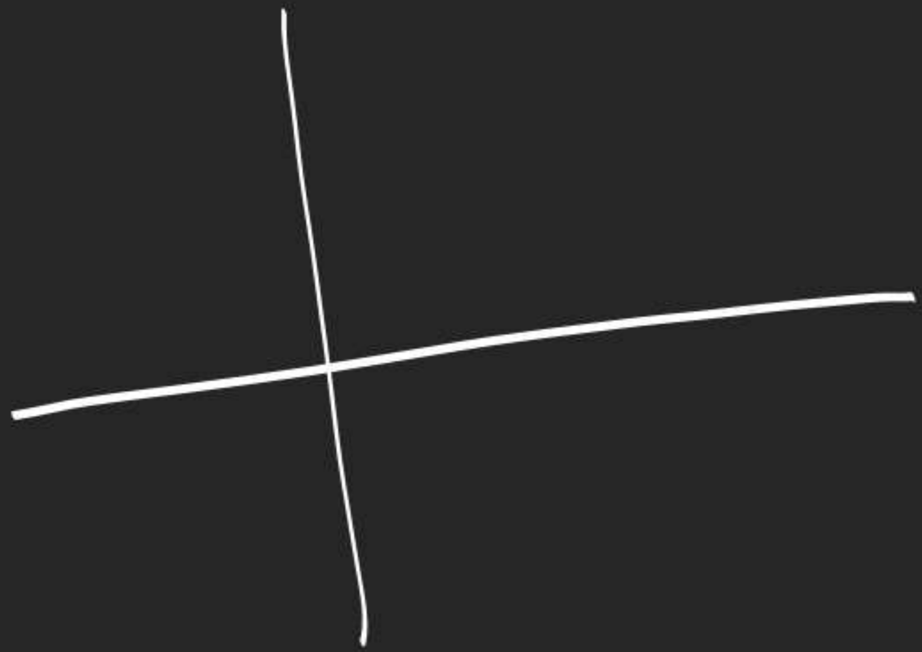
Prop of $|z|$.

$$|z_1 z_2| = |z_1||z_2|$$

$$\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}$$

$$\left|\frac{z_1}{z_2}\right|^2 = \left(\frac{z_1}{z_2}\right)\left(\frac{\bar{z}_1}{\bar{z}_2}\right) = \frac{|z_1|^2}{|z_2|^2}$$

$$\big||z_1| - |z_2|\big| \le |z_1 + z_2| \le |z_1| + |z_2| \rightarrow \text{Triangular Inequality.}$$



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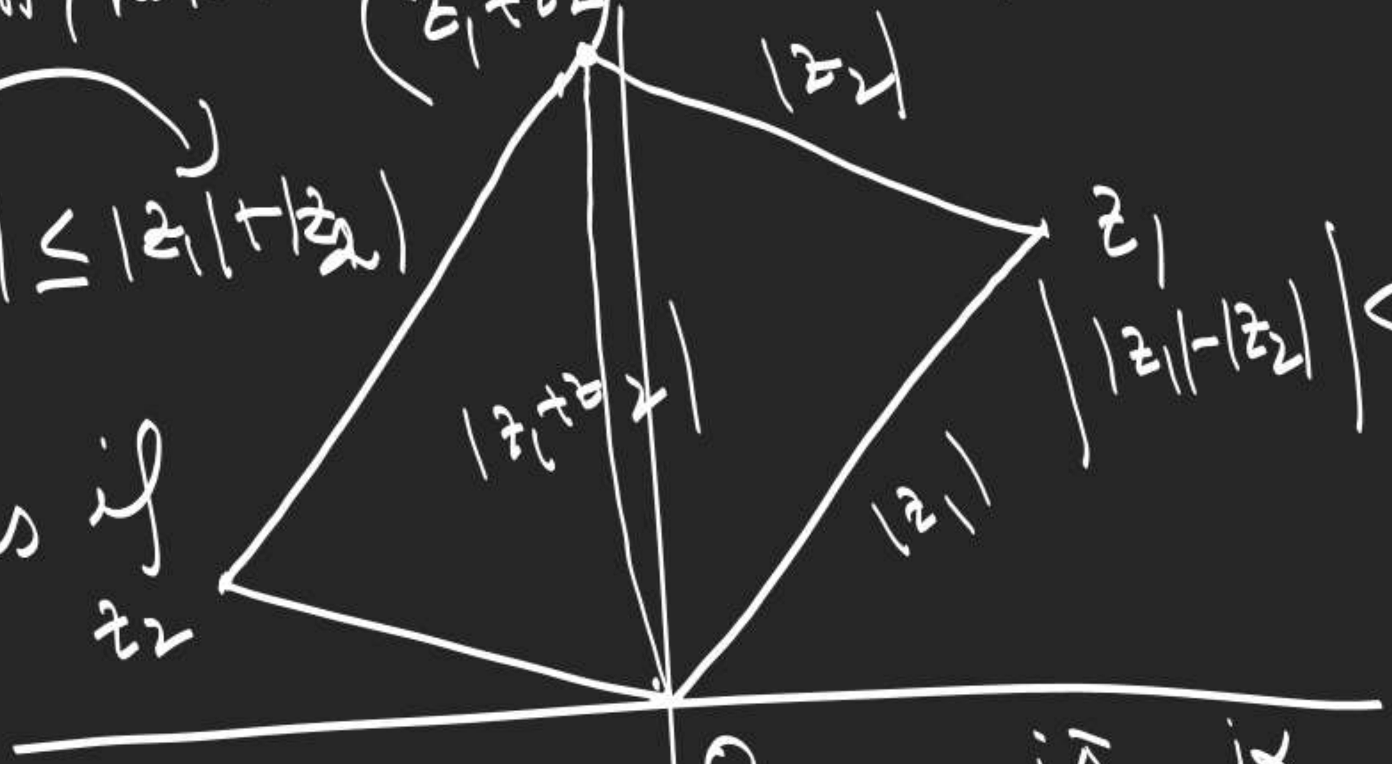
Equality holds if $\arg(z_1)=\arg(z_2)$

$$||z_1|-|z_2|| \le |z_1+z_2| \le |z_1|+|z_2|$$

Equality holds if

$$\arg z_1 - \arg z_2 = \pi$$



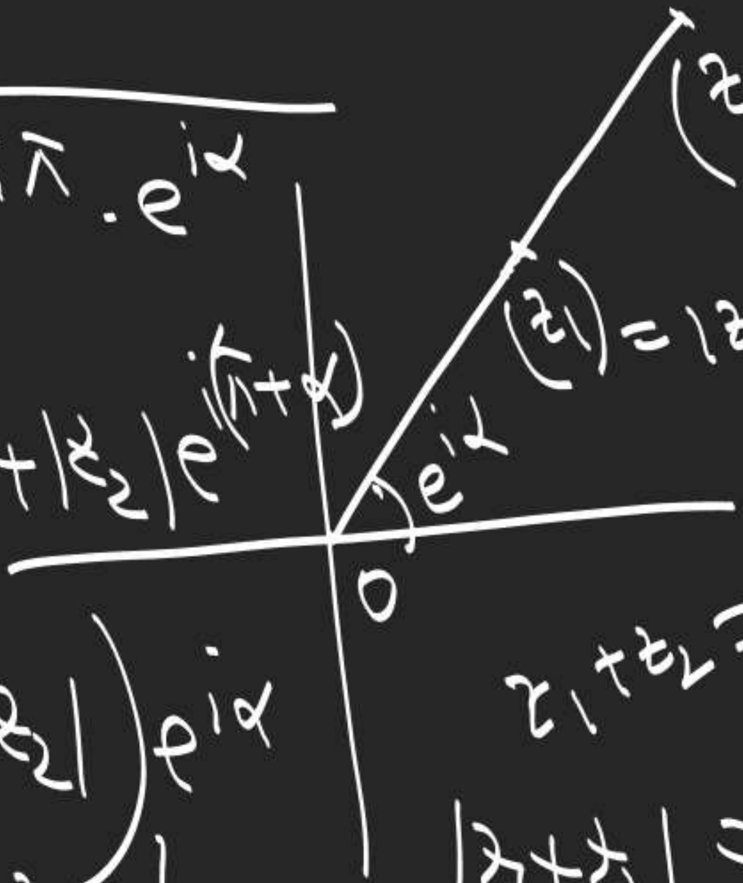
$$||z_1|-|z_2|| < |z_1+z_2| < |z_1|+|z_2|$$

$$e^{i\pi}\cdot e^{i\alpha}$$

$$z_1+z_2 = |z_1|e^{i\alpha}+|z_2|e^{i(\pi+\alpha)}$$

$$= (|z_1|-|z_2|)e^{i\alpha}$$

$$|z_1+z_2| = ||z_1|-|z_2||$$



$$(z_2) = |z_2|e^{i\alpha}$$

$$(z_1) = |z_1|e^{i\alpha}$$

$$z_1+z_2 = (|z_1|+|z_2|)e^{i\alpha}$$

$$|z_1+z_2| = |z_1|+|z_2|$$

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$$z_1 + z_2 = |z_1| e^{i\theta_1} + |z_2| e^{i\theta_2}$$

$$= \left(|z_1|\cos\theta_1 + |z_2|\cos\theta_2\right) + i\left(|z_1|\sin\theta_1 + |z_2|\sin\theta_2\right)$$

$$|z_1+z_2|^2 = \left(|z_1|c\theta_1 + |z_2|c\theta_2\right)^2 + \left(|z_1|s\theta + |z_2|s\theta\right)^2 = |z_1|^2 + |z_2|^2 + 2|z_1||z_2|\cos(\theta_1-\theta_2)$$

$$|z_1|^2 + |z_2|^2 - 2|z_1||z_2| \le |z_1+z_2|^2 \le |z_1|^2 + |z_2|^2 + 2|z_1||z_2|$$

P-2

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Principle $\arg(z_1 z_2 z_3 \cdots z_n)$

$$= (\arg z_1 + \arg z_2 + \cdots + \arg z_n) + 2k\pi \qquad k \in I.$$

$$\text{prin.} \arg\left(\frac{z_1 z_2}{z_3 z_4 z_5}\right) = (\theta_1 + \theta_2 - \theta_3 - \theta_4 - \theta_5) + 2k\pi, \ k \in I.$$