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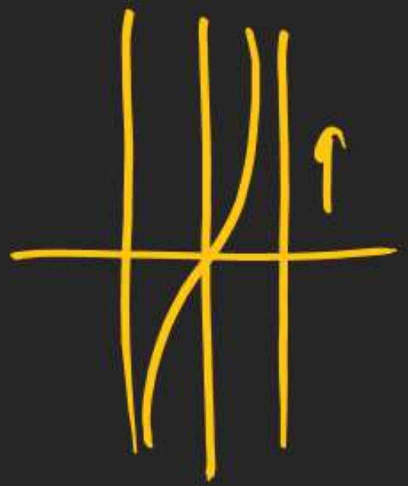
Q17 If $f(x)$ is defined for $\boxed{x \in [0, 1]}$

find D_f of $f(\tan x)$?



$f(\tan x)$ will be defined

$$0 \le \tan x \le 1$$

$$\tan 0 \le \tan x \le \tan \frac{\pi}{4}$$

$$0 \le x \le \frac{\pi}{4}$$

$$x \in \left[n\pi + 0,\ n\pi + \frac{\pi}{4}\right]$$

Q

{Dm}

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$x \in \bigcup_{n \in I} \left[n+\frac{1}{3}, n+\frac{1}{2}\right]$

RELATION FUNCTION

Q If $f(x)$ is defined for $x \in (0,1)$ then domain of $f(e^x) + f(\ln|x|) = ?$

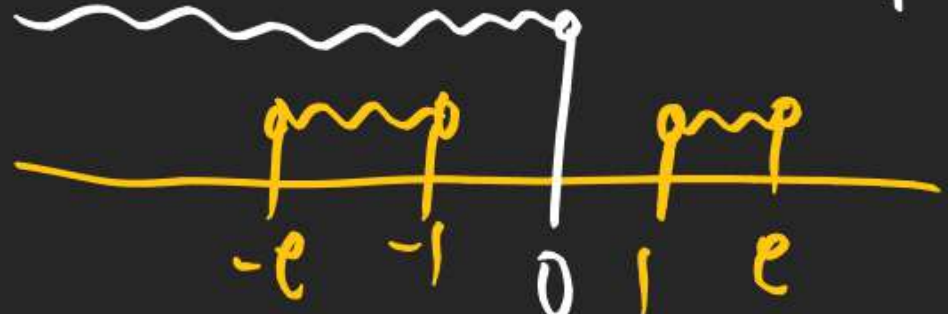
$x \in (-e, -1)$

log at 0

$0 < e^x < 1$

$\boxed{\log_e 0} < \log_e e^x < \log_e 1$

$-\infty < x < 0$



$0 < \ln|x| < 1$

$e^0 < |x| < e^1$

$1 < |x| < e$

$\{1+\frac{1}{3}\} = \frac{1}{3}$

$\{2+\frac{1}{3}\} = \frac{1}{3}$

$\{3+\frac{1}{3}\} = \frac{1}{3}$

GHODA

Q 2 If domain of $f(x)$ is $x \in (-\infty, 0]$ then dom of $f(6\{x\}^2 - 5\{x\} + 1) = ?$

$-\infty < \boxed{6\{x\}^2 - 5\{x\} + 1 \le 0}$

$(3\{x\} - 1)(2\{x\} - 1) \le 0$

BH

$\frac{1}{3} \le \{x\} \le \frac{1}{2}$

$n + \frac{1}{3} \le x \le n + \frac{1}{2}$

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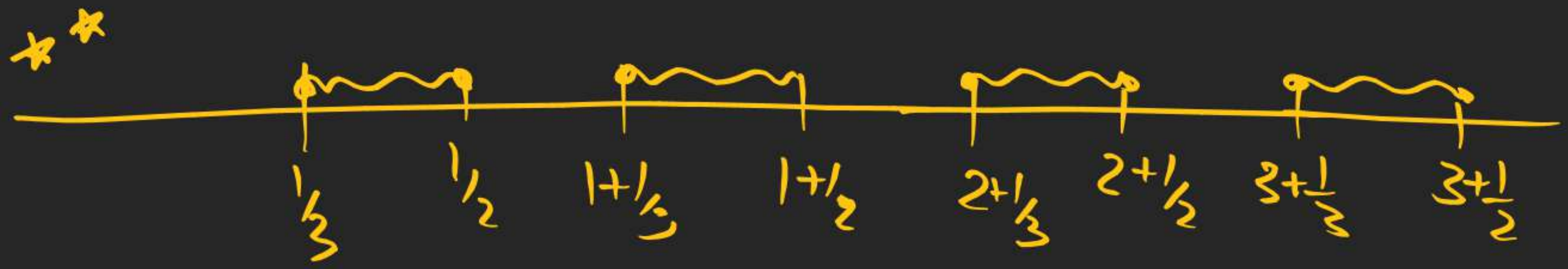
RELATION FUNCTION



RELATION FUNCTION

Range :

$\boxed{Y = \text{Range} = \text{Answer} = \text{ht.} = \text{Image}}$

A) Trigo fxn's Range

$$\boxed{\begin{aligned} -1 \le \sin x \le 1 \\ -1 \le \cos x \le 1 \end{aligned}}$$

$$\boxed{\begin{aligned} -1 \le \sin(ax+b) \le 1 \\ -1 \le \cos(ax+b) \le 1 \end{aligned}}$$

$$\boxed{\begin{aligned} -a \le a\sin x \le a \\ -b \le b\cos x \le b. \end{aligned}}$$

Q3 $Y = 2\cos x - 3$'s Range?

$-1 \le \cos x \le 1$

$-2 \le 2\cos x \le 2$

$-2-3 \le 2\cos x - 3 \le 2-3$

$\boxed{-5 \le y \le -1}$ $\quad y \in [-5, -1]$

RELATION FUNCTION

M2) $y = 2\cos x - 3$. Trick

$$\left[2(-1)-3,\ 2(1)-3\right] = [-5,-1]$$

Q4 $y = \dfrac{1}{9\sin x - 3}$, Range?

$$-1 \le \sin x \le 1$$

$$-9 \le 9\sin x \le 9$$

$$-12 \le 9\sin x - 3 \le 9-3$$

$$-\frac{1}{12} \ge \frac{1}{9\sin x - 3} \ge \frac{1}{6}$$

$y \in \left[-\frac{1}{12}, \frac{1}{6}\right)$

$$y \in \left[\frac{1}{9(1)-3}, \frac{1}{9(-1)-3}\right)$$

$$y \in \left[-\frac{1}{12}, \frac{1}{6}\right)$$

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RELATION FUNCTION

(B) Range of $A\sin x + B\cos x$

$$\boxed{-\sqrt{A^2+B^2} \le A\sin x + B\cos x \le \sqrt{A^2+B^2}}$$

Q Find Range of $y = \underbrace{3\sin x + 4\cos x}\,-7$

$$-\sqrt{3^2+4^2} \le 3\sin x + 4\cos x \le \sqrt{3^2+4^2}$$

$$-5 \le 3\sin x + 4\cos x \le 5$$

$$-5-7 \le 3\sin x + 4\cos x - 7 \le 5-7$$

$$-12 \le y \le -2 \rightarrow y \in [-12, -2]$$

$y \in [-4, 10]$

Q6 $y = 5\cos\theta + 3\underbrace{\cos\left(\theta + \frac{\pi}{3}\right)} + 3$ R_f?

$$y = 5\cos\theta + 3\left\{\cos\theta \cdot \frac{1}{2} - \sin\theta \cdot \frac{\sqrt{3}}{2}\right\} + 3$$

$$y = 5\cos\theta + \frac{3}{2}\cos\theta - \frac{3\sqrt{3}}{2}\sin\theta + 3$$

$$= \underbrace{\left(\frac{13}{2}\right)\cos\theta - \frac{3\sqrt{3}}{2}\sin\theta} + 3$$

$$A = \frac{13}{2},\ B = -\frac{3\sqrt{3}}{2} \qquad +3$$

$$R_f \in \left[-\sqrt{\frac{169}{4} + \frac{27}{4}},\ \sqrt{\frac{169}{4} + \frac{27}{4}}\right]$$

$$\in \left[-\frac{14}{2}, \frac{14}{2}\right] + 3$$

$$\in [-7, 7] + 3$$

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Q7

$$f(x) = \begin{vmatrix} \cos\frac{x}{2} & 1 & 1 \\ 1 & \cos\frac{x}{2} & -\cos\frac{x}{2} \\ -\cos\frac{x}{2} & 1 & 1 \end{vmatrix}$$

$$\Rightarrow \cos\frac{x}{2}\begin{vmatrix} \cos\frac{x}{2} & -\cos\frac{x}{2} \\ 1 & -1 \end{vmatrix} - 1\begin{vmatrix} 1 & -\cos\frac{x}{2} \\ -\cos\frac{x}{2} & -1 \end{vmatrix} + 1\begin{vmatrix} 1 & \cos\frac{x}{2} \\ -\cos\frac{x}{2} & 1 \end{vmatrix}$$

$$\Rightarrow \cos\frac{x}{2}\left(\cancel{-\cos\frac{x}{2}} + \cancel{\cos\frac{x}{2}}\right) - 1\left(-1 - \cos^2\frac{x}{2}\right) + 1\left(1 + \cos^2\frac{x}{2}\right)$$

$$= 1 + \cos^2\frac{x}{2} + 1 + \cos^2\left(\frac{x}{2}\right)$$

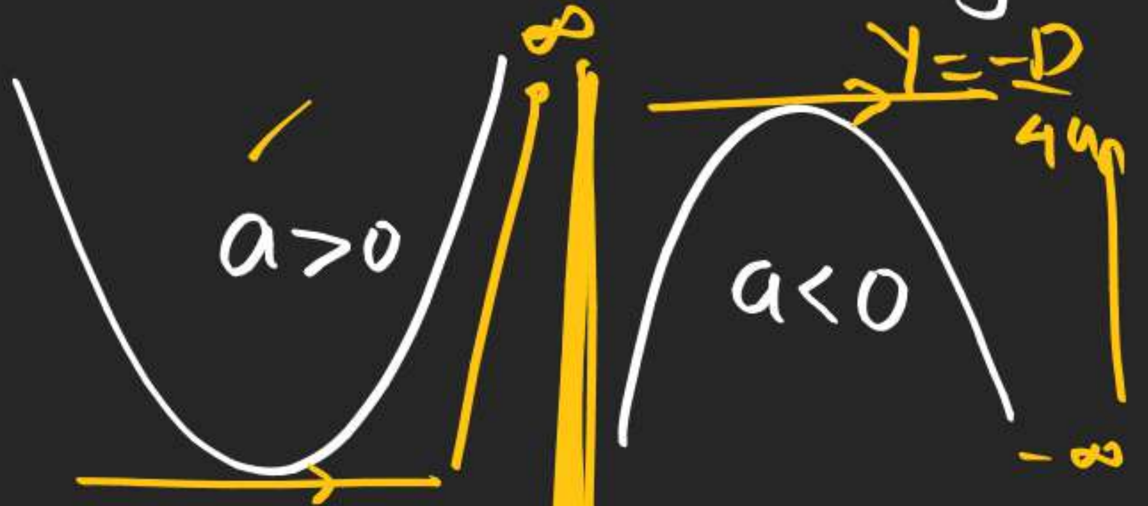
$$Y = 2\left(1 + \boxed{\cos^2\left(\frac{x}{2}\right)}\right)$$

$$0 \le \cos^2\left(\frac{x}{2}\right) \le 1 \qquad \therefore Y \in [2, 4]$$

$$1 \le 1 + \cos^2\frac{x}{2} \le 2$$

$$2 \le 2\left(1 + \cos^2\left(\frac{x}{2}\right)\right) \le 4$$

(C) $y = ax^2+bx+c$'s Range



$y \in \left[\frac{-D}{4a}, \infty\right)$ $\quad$ $y \in \left(-\infty, \frac{-D}{4a}\right]$

$\in \left[\frac{4ac-b^2}{4a}, \infty\right)$

Q_8 $y = x^2+x+1$ R_f?

$a=1$ $\oplus$, $b=1$, $c=1$

$y \in \left[\frac{4\times1\times1-1^2}{4\times1}, \infty\right)$

$\in \left[\frac{3}{4}, \infty\right)$

Q_9 $y = 3x^2+4x+5$'s R_f?

$a=3$ $\oplus$, $b=4$, $c=5$

$y \in \left[\frac{4\times3\times5-4^2}{4\times3}, \infty\right) \Rightarrow y \in \left[\frac{\cancel{44}^{11}}{\cancel{4}\times3}, \infty\right)$

$y \in \left[\frac{11}{3}, \infty\right)$

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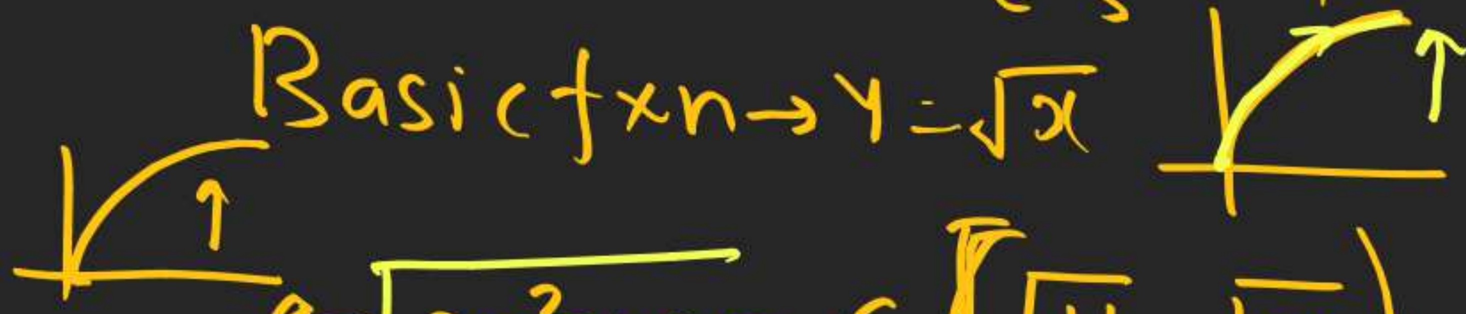
(D) $f(g(x))$'s R_f

Q9 $y = \sqrt{3x^2+4x+5}$'s R_f ?

$3x^2+4x+5 \in \left[\frac{11}{3}, \infty\right)$

Basic fxn $\to y = \sqrt{x}$



$\sqrt{3x^2+4x+5} \in \left[\sqrt{\frac{11}{3}}, \sqrt{\infty}\right)$

$\in \left[\sqrt{\frac{11}{3}}, \infty\right)$

Q10 $y = \log_e(3x^2+4x+5)$

$R+$

$3x^2+4x+5 \in \left[\frac{11}{3}, \infty\right)$

B.F $\to$ $f(x) = \log_e x$

$\log_e(3x^2+4x+5)$

$\in \left[\log_e \frac{11}{3}, \log_e \infty\right)$

$y \in \left[\log_e \frac{11}{3}, \infty\right)$

Q11 $y = e^{3x^2+4x+5}$

$3x^2+4x+5 \in \left[\frac{11}{3}, \infty\right)$

BF $\to y = e^x$

$e^{3x^2+4x+5} \in \left[e^{\frac{11}{3}}, e^{\infty}\right)$

$y \in \left[e^{\frac{11}{3}}, \infty\right)$

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RELATION FUNCTION

Q12 $y = \dfrac{1}{3x^2+4x+5}$, R_f

$3x^2+4x+5 \in \left[\frac{11}{3}, \infty\right)$

BF → $y = \frac{1}{x}$

$\dfrac{1}{3x^2+4x+5} \in \left(\frac{1}{\infty}, \frac{1}{\frac{11}{3}}\right]$

$\in \left(0, \frac{3}{11}\right]$

R_f

Q13 $y = e^{-(3x^2+4x+5)}$ R_f

$3x^2+4x+5 \in \left[\frac{11}{3}, \infty\right)$

$e^{-(3x^2+4x+5)} \in \left(e^{-\infty}, e^{-\frac{11}{3}}\right]$

$\in \left(\frac{1}{e^{\infty}}, e^{-\frac{11}{3}}\right]$

$\in \left(\frac{1}{\infty}, e^{-\frac{11}{3}}\right]$

$\in \left(0, e^{-\frac{11}{3}}\right]$

$\therefore \sin(3x^2+4x+5) \in [-1, 1]$

$3x^2+4x+5 \in \left[\frac{11}{3}, \infty\right)$

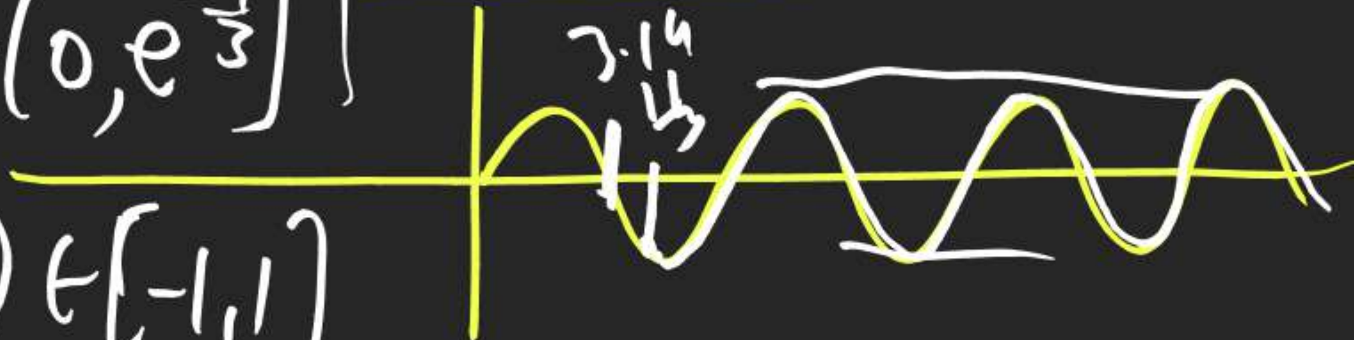
Q14 $y = \sin(3x^2+4x+5)$ 3.6

BF → $y = \boxed{\sin x}$

NIND

1) Andr Wale Ki R_f K Brabar Graph Dark Kr

2) Dark hue Graph Ki Uchhaal Dekho



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RELATION FUNCTION

Q_{15} $y = Gs(\ln\{x\})$'s Range?

JTHSJJH

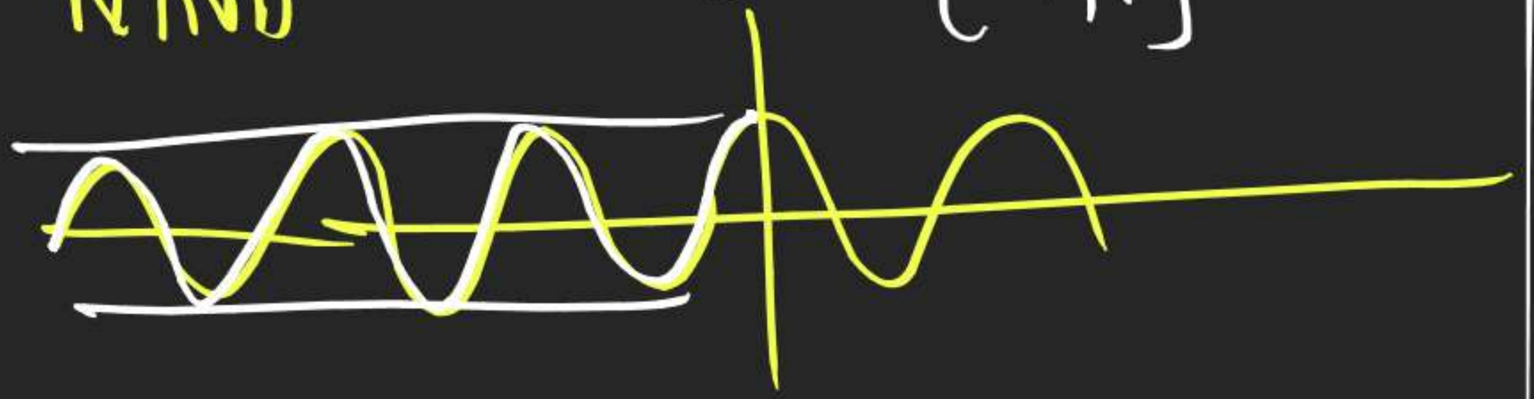
$\{x\} \in [0, 1)$

$\ln\{x\} \in [\ln 0, \ln 1)$

$\ln\{x\} \in (-\infty, 0)$

$G(\ln\{x\}) \in [-1, 1]$

NIND



Q_{16} $y = \sin(\ln(|x|))$ R_f?

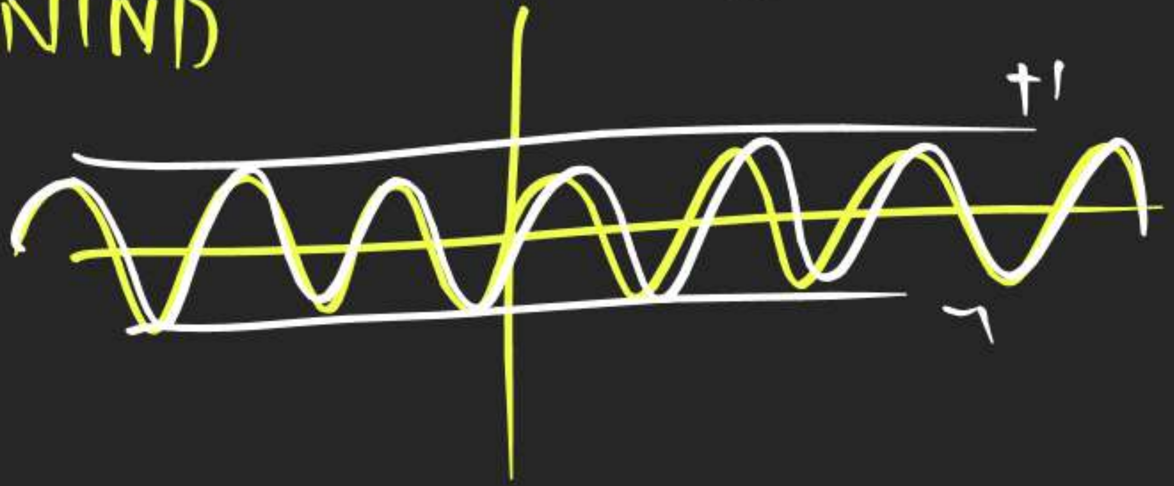
$|x| \in [0, \infty)$

$\ln|x| \in [\ln 0, \ln\infty)$

$\ln|x| \in (-\infty, \infty)$

$\sin\ln|x| \in [-1, 1]$

NIND



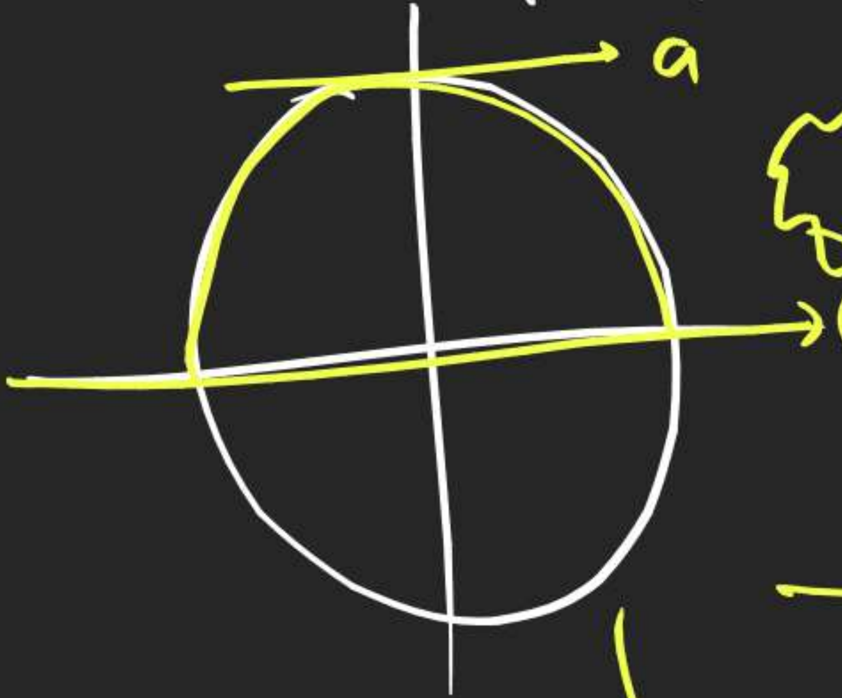
(E) Range of $y=\sqrt{a^2-x^2}$

$y=\sqrt{a^2-x^2}$ $\oplus$ → Above x Axis

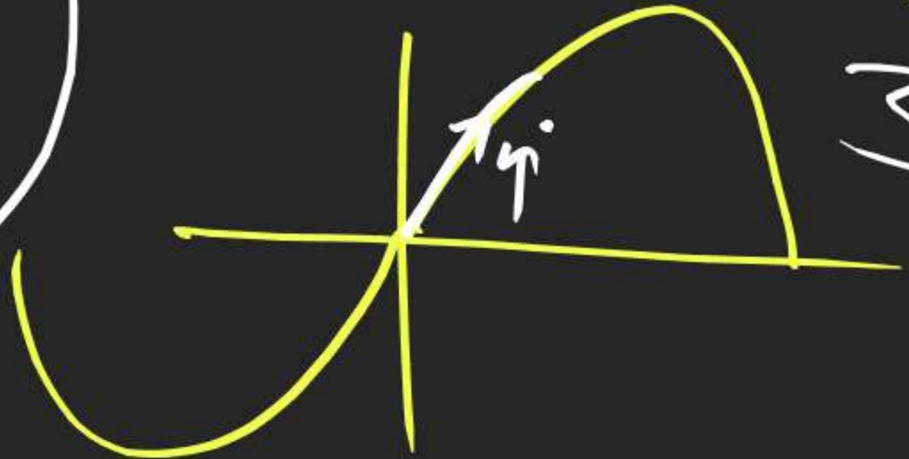
$y^2=a^2-x^2$

$x^2+y^2=a^2$ Circle

(0,0), a



$\boxed{\sqrt{a^2-x^2}\in[0,a]}$



Ex:- $y=\sqrt{a^2-x^2}\in[0,a]$

$y=\sqrt{16-x^2}\in[0,4]$

$y=\sqrt{9-x^2}\in[0,3]$

Q17 $y=3\sin\sqrt{\frac{\pi^2}{16}-x^2}$'s R_f ?

$\sqrt{\frac{\pi^2}{16}-x^2}\in\left[0,\frac{\pi}{4}\right]$

$3\sin\sqrt{\frac{\pi^2}{16}-x^2}\in\left[3\sin 0, 3\sin\frac{\pi}{4}\right]$

$\in\left[0,\frac{1}{\sqrt{2}}\right]\times 3$

$\in\left[0,\frac{3}{\sqrt{2}}\right]$

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Q_{18} $y = 2\tan\sqrt{\frac{\pi^2}{9} - x^2}$'s R_f?

$\sqrt{a^2 - x^2}$

$$\sqrt{\frac{\pi^2}{9} - x^2} \in \left[0, \frac{\pi}{3}\right]$$

$$\tan\sqrt{\frac{\pi^2}{9} - x^2} \in \left[\tan 0, \tan\frac{\pi}{3}\right]$$

$$\in \left[0, \sqrt{3}\right]$$

$$2\tan\sqrt{\frac{\pi^2}{9} - x^2} \in \left[0, 2\sqrt{3}\right]$$

Q_{19} R_f of $y = \log_{\sqrt{2}}\left(2 - \log_2\left(\underline{16\sin^2 x + 1}\right)\right)$

$$0 \le \sin^2 x \le 1$$

$$0 \le 16\sin^2 x \le 16$$

$$1 \le 16\sin^2 x + 1 \le 17$$

4 से छोटा है

$$\log_2 1 \le \log_2(16\sin^2 x + 1) \le \log_2 17$$

$$0 \le \log_2(16\sin^2 x + 1) \le \log_2 17$$

$$2 + 0 \ge 2 - \log_2(16\sin^2 x + 1) \ge \underbrace{2 - \log_2 17}_{-ve}$$

$$\log_{\sqrt{2}} 2 \ge \log_{\sqrt{2}}\left(2 - \log_2(16\sin^2 x + 1)\right) > \log_{\sqrt{2}} 0$$

$$2 \ge y > -\infty \rightarrow (-\infty, 2]$$

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RELATION FUNCTION

Q20 $y = G(\sin x)$

R_f:

$y = \sqrt{G(\sin x)}$

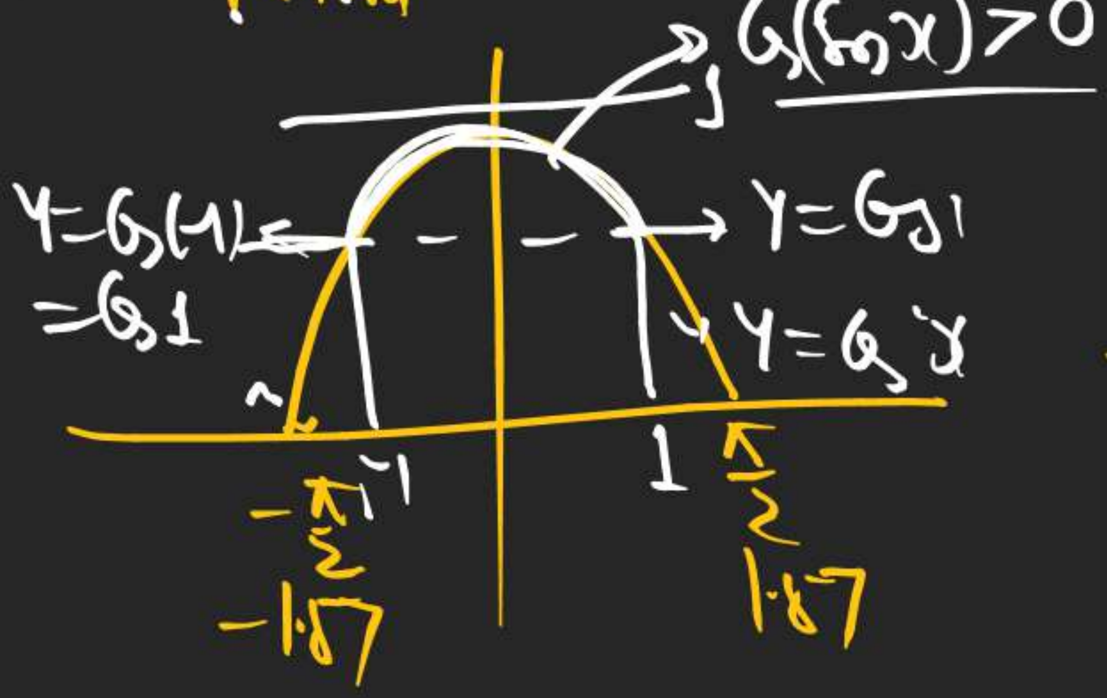
⊕ Always

$x \in R$

$\sin x \in [-1, 1]$

$G(\sin x) \in [G1, 1]$

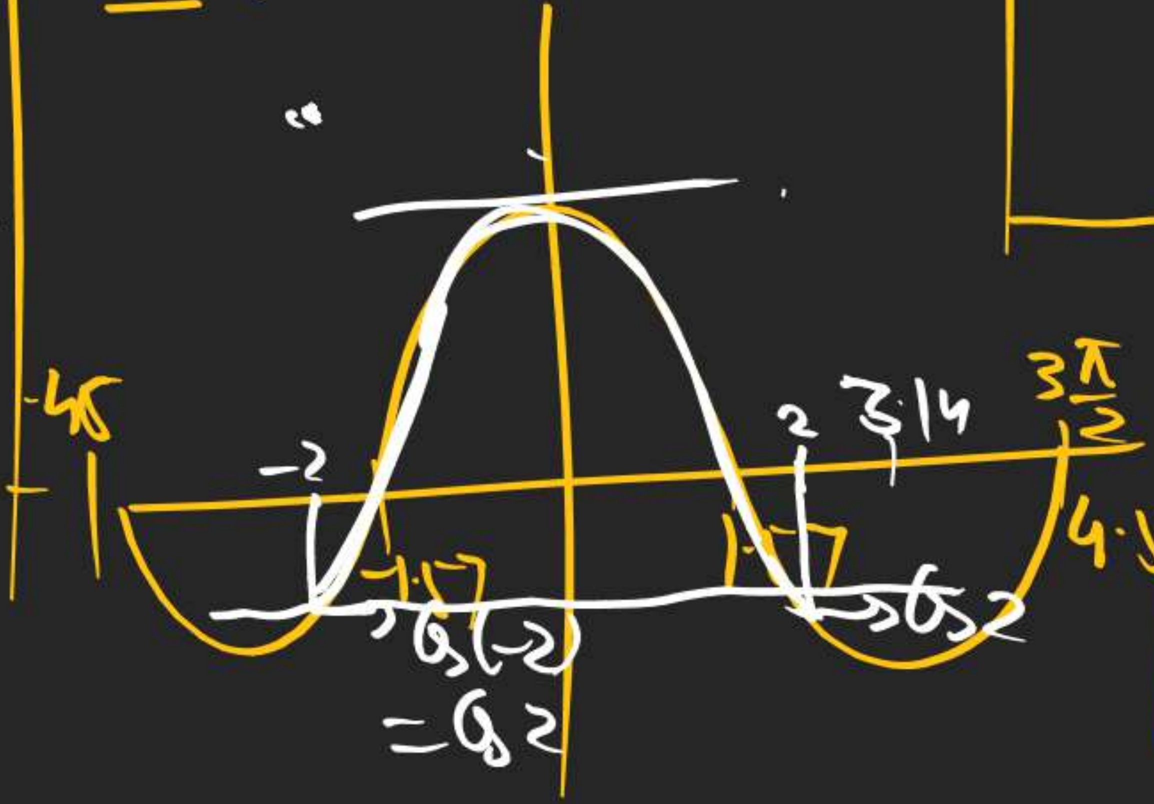
Mind



Q21 $y = G(2\sin x)$

$2\sin x \in [-2, 2]$

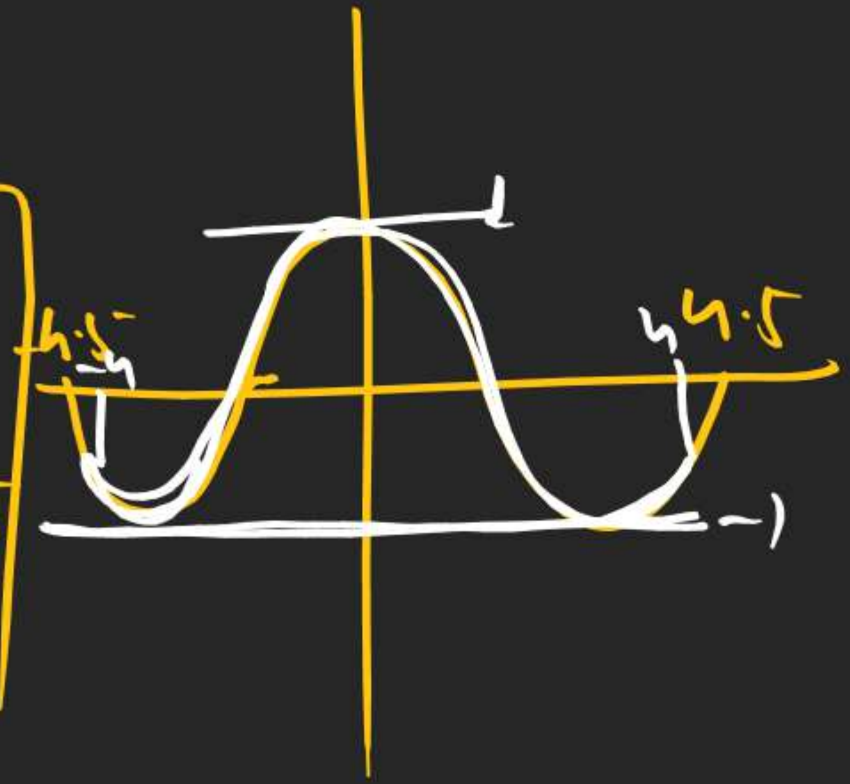
$G(2\sin x) \in [G2, 1]$



Q22 $y = G(4\sin x)$

$4\sin x \in [-4, 4]$

$G(4\sin x) \in [-1, 1]$



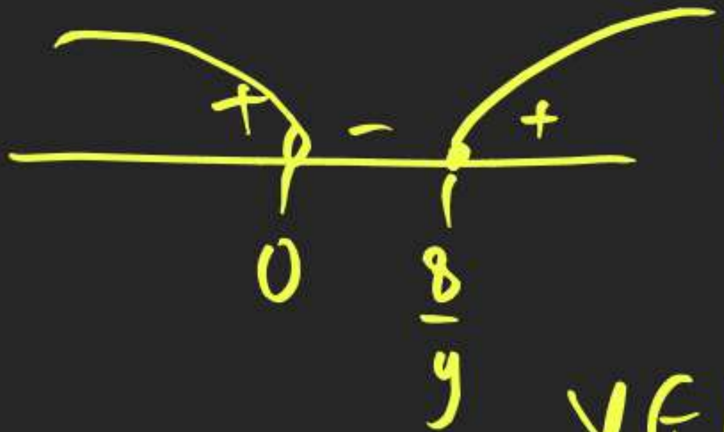
(G) Known fxn Ki Range me Bdl Kar.

Q23 R_f of $y = \dfrac{8}{9-x^2}$

$$9 - x^2 = \frac{8}{y}$$

$$9 - \frac{8}{y} = x^2 \geq 0$$

$$\frac{(9y-8)}{(y)} \geq 0$$



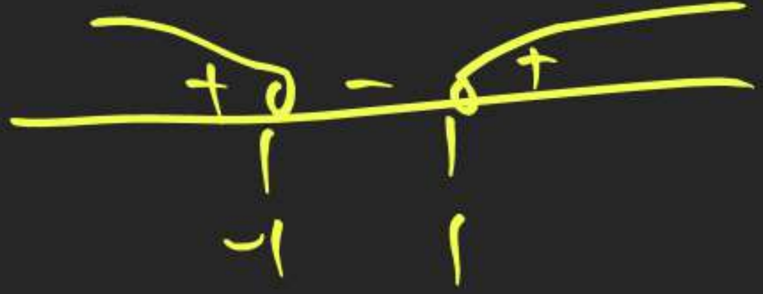
$$y \in (-\infty, 0) \cup \left[\frac{8}{9}, \infty\right)$$

Q25 $y = \dfrac{e^x+1}{e^x-1}$ find R_f.

$$e^x \cdot y - y = e^x + 1$$

$$e^x(y-1) = 1 + y$$

$$0 < e^x = \frac{1+y}{y-1} \Rightarrow \frac{y+1}{y-1} > 0$$



$$y \in (-\infty, -1) \cup (1, \infty)$$