KEY CONCEPTS

1. DEFINITIONS :

A Vector may be described as a quantity having both magnitude & direction. A vector is generally represented by a directed line segment, say $\overrightarrow{AB}$. A is called the **initial point** & B is called the **terminal point**. The magnitude of vector $\overrightarrow{AB}$ is expressed by $\left|\overrightarrow{AB}\right|$.

Zero Vector a vector of zero magnitude i.e.which has the same initial & terminal point, is called a **Zero Vector**. It is denoted by O.

Unit Vector a vector of unit magnitude in direction of a vector $\vec{a}$ is called unit vector along $\vec{a}$ and is denoted by $\hat{a}$ symbolically $\hat{a}=\frac{\vec{a}}{|\vec{a}|}$.

Equal Vectors two vectors are said to be equal if they have the same magnitude, direction & represent the same physical quantity.

Collinear Vectors two vectors are said to be collinear if their directed line segments are parallel disregards to their direction. Collinear vectors are also called **Parallel Vectors**. If they have the same direction they are named as like vectors otherwise unlike vectors.

Simbolically, two non zero vectors $\vec{a}$ and $\vec{b}$ are collinear if and only if, $\vec{a}=K\vec{b}$, where K $\in$ R

Coplanar Vectors a given number of vectors are called coplanar if their line segments are all parallel to the same plane. Note that **"Two Vectors Are Always Coplanar"**.

Position Vector let O be a fixed origin, then the position vector of a point P is the vector $\overrightarrow{OP}$. If $\vec{a}$ & $\vec{b}$ & position vectors of two point A and B, then, $\overrightarrow{AB}$ = $\vec{b}-\vec{a}$ = pv of B – pv of A .

2. VECTOR ADDITION :

☞ If two vectors $\vec{a}$ & $\vec{b}$ are represented by $\overrightarrow{OA}$ & $\overrightarrow{OB}$, then their sum $\vec{a}+\vec{b}$ is a vector represented by $\overrightarrow{OC}$, where OC is the diagonal of the parallelogram OACB.

☞ $\vec{a}+\vec{b}=\vec{b}+\vec{a}$ (commutative)

☞ $(\vec{a}+\vec{b})+\vec{c}=\vec{a}+(\vec{b}+\vec{c})$ (associativity)

☞ $\vec{a}+\vec{0}=\vec{a}=\vec{0}+\vec{a}$

☞ $\vec{a}+(-\vec{a})=\vec{0}=(-\vec{a})+\vec{a}$

3. MULTIPLICATION OF VECTOR BY SCALARS :

If $\vec{a}$ is a vector & m is a scalar, then m$\vec{a}$ is a vector parallel to $\vec{a}$ whose modulus is $|m|$ times that of $\vec{a}$. This multiplication is called **Scalar Multiplication**. If $\vec{a}$ & $\vec{b}$ are vectors & m, n are scalars, then:

$$m(\vec{a})=(\vec{a})m=m\vec{a} \qquad m(n\vec{a})=n(m\vec{a})=(mn)\vec{a}$$

$$(m+n)\vec{a}=m\vec{a}+n\vec{a} \qquad m(\vec{a}+\vec{b})=m\vec{a}+m\vec{b}$$

4. SECTION FORMULA :

If $\vec{a}$ & $\vec{b}$ are the position vectors of two points A & B then the p.v. of a point which divides AB in the ratio m : n is given by : $\vec{r}=\frac{n\vec{a}+m\vec{b}}{m+n}$. Note p.v. of mid point of AB = $\frac{\vec{a}+\vec{b}}{2}$.

5. DIRECTION COSINES :

Let $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ the angles which this vector makes with the +ve directions OX,OY & OZ are called **Direction Angles** & their cosines are called the **Direction Cosines**.

$\cos\alpha = \dfrac{a_1}{|\vec{a}|}$, $\cos\beta = \dfrac{a_2}{|\vec{a}|}$, $\cos\Gamma = \dfrac{a_3}{|\vec{a}|}$. **Note that, $\cos^2\alpha + \cos^2\beta + \cos^2\Gamma = 1$**

6. VECTOR EQUATION OF A LINE :

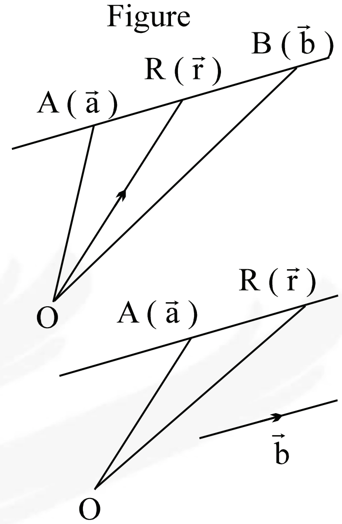
Parametric vector equation of a line passing through two point $A(\vec{a})$ & $B(\vec{b})$ is given by, $\vec{r} = \vec{a} + t(\vec{b} - \vec{a})$ where t is a parameter. If the line passes through the point $A(\vec{a})$ & is parallel to the vector $\vec{b}$ then its equation is,

$\vec{r} = \vec{a} + t\vec{b}$

Note that the equations of the bisectors of the angles between the lines $\vec{r} = \vec{a} + \lambda\vec{b}$ & $\vec{r} = \vec{a} + \mu\vec{c}$ is :

$\vec{r} = \vec{a} + t\left(\hat{b} + \hat{c}\right)$ & $\vec{r} = \vec{a} + p\left(\hat{c} - \hat{b}\right)$.

Figure



7. TEST OF COLLINEARITY :

Three points A,B,C with position vectors $\vec{a}, \vec{b}, \vec{c}$ respectively are collinear, if & only if there exist scalars x , y, z not all zero simultaneously such that ; $x\vec{a} + y\vec{b} + z\vec{c} = 0$, where x + y + z = 0.

8. SCALAR PRODUCT OF TWO VECTORS :

☞ $\vec{a}.\vec{b} = |\vec{a}||\vec{b}|\cos\theta (0 \le \theta \le \pi)$,

note that if θ is acute then $\vec{a}.\vec{b} > 0$ & if θ is obtuse then $\vec{a}.\vec{b} < 0$

☞ $\vec{a}.\vec{a} = |\vec{a}|^2 = \vec{a}^2, \vec{a}.\vec{b} = \vec{b}.\vec{a}$ (commutative) ☞ $\vec{a}.(\vec{b} + \vec{c}) = \vec{a}.\vec{b} + \vec{a}.\vec{c}$ (distributive)

☞ $\vec{a}.\vec{b} = 0 \Leftrightarrow \vec{a} \perp \vec{b}$ $(\vec{a} \ne 0 \;\; \vec{b} \ne 0)$

☞ $\hat{i}.\hat{i} = \hat{j}.\hat{j} = \hat{k}.\hat{k} = 1$; $\hat{i}.\hat{j} = \hat{j}.\hat{k} = \hat{k}.\hat{i} = 0$

☞ projection of $\vec{a}$ on $\vec{b} = \dfrac{\vec{a}.\vec{b}}{|\vec{b}|}$.

Note : That vector component of $\vec{a}$ along $\vec{b} = \left(\dfrac{\vec{a}\cdot\vec{b}}{\vec{b}^2}\right)\vec{b}$ and perpendicular to $\vec{b} = \vec{a} - \left(\dfrac{\vec{a}\cdot\vec{b}}{\vec{b}^2}\right)\vec{b}$.

☞ the angle ϕ between $\vec{a}$ & $\vec{b}$ is given by $\cos\phi = \dfrac{\vec{a}.\vec{b}}{|\vec{a}||\vec{b}|}$ $0 \le \phi \le \pi$

☞ if $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ & $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ then $\vec{a}.\vec{b} = a_1b_1 + a_2b_2 + a_3b_3$

$|\vec{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$, $|\vec{b}| = \sqrt{b_1^2 + b_2^2 + b_3^2}$

Note :

(i) Maximum value of $\vec{a}\cdot\vec{b} = |\vec{a}|\,|\vec{b}|$

(ii) Minimum values of $\vec{a}\cdot\vec{b} = \vec{a}\cdot\vec{b} = -|\vec{a}|\,|\vec{b}|$

(iii) Any vector $\vec{a}$ can be written as , $\vec{a} = (\vec{a}\cdot\hat{i})\,\hat{i} + (\vec{a}\cdot\hat{j})\,\hat{j} + (\vec{a}\cdot\hat{k})\,\hat{k}$.

(iv) A vector in the direction of the bisector of the angle between the two vectors $\vec{a}\,\&\,\vec{b}$ is $\frac{\vec{a}}{|\vec{a}|} + \frac{\vec{b}}{|\vec{b}|}$. Hence bisector of the angle between the two vectors $\vec{a}\,\&\,\vec{b}$ is $\lambda\left(\hat{a} + \hat{b}\right)$, where $\lambda \in R^+$. Bisector of the exterior angle between $\vec{a}\,\&\,\vec{b}$ is $\lambda\left(\hat{a} - \hat{b}\right)$, $\lambda \in R^+$.

9. VECTOR PRODUCT OF TWO VECTORS :

(i) ☞ If $\vec{a}\,\&\,\vec{b}$ are two vectors & θ is the angle between them then $\vec{a}\times\vec{b} = |\vec{a}||\vec{b}|\sin\theta\,\vec{n}$, where $\vec{n}$ is the unit vector perpendicular to both $\vec{a}\,\&\,\vec{b}$ such that $\vec{a}, \vec{b}\,\&\,\vec{n}$ forms a right handed screw system .

(ii) ☞ Lagranges Identity : for any two vectors $\vec{a}\,\&\,\vec{b}$; $(\vec{a}\times\vec{b})^2 = |\vec{a}|^2|\vec{b}|^2 - (\vec{a}\cdot\vec{b})^2 = \begin{vmatrix} \vec{a}\cdot\vec{a} & \vec{a}\cdot\vec{b} \\ \vec{a}\cdot\vec{b} & \vec{b}\cdot\vec{b} \end{vmatrix}$

(iii) ☞ Formulation of vector product in terms of scalar product:

The vector product $\vec{a}\times\vec{b}$ is the vector $\vec{c}$, such that

(i) $|\vec{c}| = \sqrt{\vec{a}^2\vec{b}^2 - (\vec{a}\cdot\vec{b})^2}$ (ii) $\vec{c}\cdot\vec{a} = 0$; $\vec{c}\cdot\vec{b} = 0$ and

(iii) $\vec{a}, \vec{b}, \vec{c}$ form a right handed system

(iv) ☞ $\vec{a}\times\vec{b} = 0 \Leftrightarrow \vec{a}\,\&\,\vec{b}$ are parallel (collinear) $(\vec{a} \neq 0, \vec{b} \neq 0)$ i.e. $\vec{a} = K\vec{b}$, where K is a scalar.

☞ $\vec{a}\times\vec{b} \neq \vec{b}\times\vec{a}$ (not commutative)

☞ $(m\vec{a})\times\vec{b} = \vec{a}\times(m\vec{b}) = m(\vec{a}\times\vec{b})$ where m is a scalar .

☞ $\vec{a}\times(\vec{b}+\vec{c}) = (\vec{a}\times\vec{b}) + (\vec{a}\times\vec{c})$ (distributive)

☞ $\hat{i}\times\hat{i} = \hat{j}\times\hat{j} = \hat{k}\times\hat{k} = 0$ ☞ $\hat{i}\times\hat{j} = \hat{k},\ \hat{j}\times\hat{k} = \hat{i},\ \hat{k}\times\hat{i} = \hat{j}$

(v) ☞ If $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ & $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ then $\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$

(vi) ☞ Geometrically $|\vec{a}\times\vec{b}|$ = area of the parallelogram whose two adjacent sides are represented by $\vec{a}\,\&\,\vec{b}$.

(vii) ☞ Unit vector perpendicular to the plane of $\vec{a}\,\&\,\vec{b}$ is $\hat{n} = \pm\frac{\vec{a}\times\vec{b}}{|\vec{a}\times\vec{b}|}$

☞ A vector of magnitude ‘r’ & perpendicular to the palne of $\vec{a}\,\&\,\vec{b}$ is $\pm\frac{r(\vec{a}\times\vec{b})}{|\vec{a}\times\vec{b}|}$

☞ If θ is the angle between $\vec{a}\,\&\,\vec{b}$ then $\sin\theta = \frac{|\vec{a}\times\vec{b}|}{|\vec{a}||\vec{b}|}$

(viii) Vector area

☞ If $\vec{a},\vec{b}\,\&\,\vec{c}$ are the pv's of 3 points A, B & C then the vector area of triangle ABC =

$\frac{1}{2}\left[\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}\right]$. The points A, B & C are collinear if $\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}=0$

☞ Area of any quadrilateral whose diagonal vectors are $\vec{d}_1\,\&\,\vec{d}_2$ is given by $\frac{1}{2}\left|\vec{d}_1\times\vec{d}_2\right|$

10. SHORTEST DISTANCE BETWEEN TWO LINES :

If two lines in space intersect at a point, then obviously the shortest distance between them is zero. Lines which do not intersect & are also not parallel are called **SKEW LINES**. For Skew lines the direction of the shortest distance would be perpendicular to both the lines. The magnitude of the shortest distance vector would be equal to that of the projection of $\overrightarrow{AB}$ along the direction of the line of shortest distance, $\overrightarrow{LM}$ is parallel to $\vec{p}\times\vec{q}$ i.e.

$$\overrightarrow{LM}=\left|\text{Projection of }\overrightarrow{AB}\text{ on }\overrightarrow{LM}\right| = \left|\text{Projection of }\overrightarrow{AB}\text{ on }\vec{p}\times\vec{q}\right| = \left|\frac{\overrightarrow{AB}\cdot(\vec{p}\times\vec{q})}{\vec{p}\times\vec{q}}\right| = \left|\frac{(\vec{b}-\vec{a})\cdot(\vec{p}\times\vec{q})}{|\vec{p}\times\vec{q}|}\right|$$

1. The two lines directed along $\vec{p}\,\&\,\vec{q}$ will intersect only if shortest distance = 0 i.e.

$(\vec{b}-\vec{a})\cdot(\vec{p}\times\vec{q})=0$ i.e. $\left(\vec{b}-\vec{a}\right)$ lies in the plane containing $\vec{p}\,\&\,\vec{q}$. $\Rightarrow \left[\left(\vec{b}-\vec{a}\right)\,\vec{p}\,\vec{q}\right]=0$.

2. If two lines are given by $\vec{r}_1=\vec{a}_1+K\vec{b}\,\&\,\vec{r}_2=\vec{a}_2+K\vec{b}$ i.e. they are parallel then , $d=\left|\frac{\vec{b}\times(\vec{a}_2-\vec{a}_1)}{|\vec{b}|}\right|$

11. SCALAR TRIPLE PRODUCT / BOX PRODUCT / MIXED PRODUCT :

☞ The scalar triple product of three vectors $\vec{a},\vec{b}\,\&\,\vec{c}$ is defined as :

$\vec{a}\times\vec{b}\cdot\vec{c}=|\vec{a}||\vec{b}||\vec{c}|\sin\theta\cos\phi$ where θ is the angle between $\vec{a}\,\&\,\vec{b}$ & ϕ is the angle between $\vec{a}\times\vec{b}\,\&\,\vec{c}$.

It is also defined as $[\vec{a}\;\vec{b}\;\vec{c}]$, spelled as box product .

☞ Scalar triple product geometrically represents the volume of the parallelopiped whose three couterminous edges are represented by $\vec{a},\vec{b}\,\&\,\vec{c}$ i.e. $V=[\vec{a}\;\vec{b}\;\vec{c}]$

☞ In a scalar triple product the position of dot & cross can be interchanged i.e.

$\vec{a}\cdot(\vec{b}\times\vec{c})=(\vec{a}\times\vec{b})\cdot\vec{c}$ OR $[\vec{a}\;\vec{b}\;\vec{c}]=[\vec{b}\;\vec{c}\;\vec{a}]=[\vec{c}\;\vec{a}\;\vec{b}]$

☞ $\vec{a}\cdot(\vec{b}\times\vec{c})=-\vec{a}\cdot(\vec{c}\times\vec{b})$ i.e. $[\vec{a}\;\vec{b}\;\vec{c}]=-[\vec{a}\;\vec{c}\;\vec{b}]$

☞ If $\vec{a}=a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$; $\vec{b}=b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$ & $\vec{c}=c_1\hat{i}+c_2\hat{j}+c_3\hat{k}$ then $[\vec{a}\,\vec{b}\,\vec{c}]=\begin{vmatrix}a_1 & a_2 & a_3\\ b_1 & b_2 & b_3\\ c_1 & c_2 & c_3\end{vmatrix}$.

In general , if $\vec{a}=a_1\vec{l}+a_2\vec{m}+a_3\vec{n}$; $\vec{b}=b_1\vec{l}+b_2\vec{m}+b_3\vec{n}$ & $\vec{c}=c_1\vec{l}+c_2\vec{m}+c_3\vec{n}$

then $\left[\vec{a}\,\vec{b}\,\vec{c}\right]=\begin{vmatrix}a_1 & a_2 & a_3\\ b_1 & b_2 & b_3\\ c_1 & c_2 & c_3\end{vmatrix}\left[\vec{l}\,\vec{m}\,\vec{n}\right]$; where $\vec{\ell},\vec{m}\,\&\,\vec{n}$ are non coplanar vectors .

☞ If $\vec{a}, \vec{b}, \vec{c}$ are coplanar $\Leftrightarrow [\vec{a}\ \vec{b}\ \vec{c}] = 0$.

☞ Scalar product of three vectors, two of which are equal or parallel is 0 i.e. $[\vec{a}\,\vec{b}\,\vec{c}] = 0$,

Note : If $\vec{a}, \vec{b}, \vec{c}$ are non – coplanar then $[\vec{a}\ \vec{b}\ \vec{c}] > 0$ for right handed system & $[\vec{a}\ \vec{b}\ \vec{c}] < 0$ for left handed system .

☞ [i j k] = 1 ☞ $[K\vec{a}\ \vec{b}\ \vec{c}] = K[\vec{a}\ \vec{b}\ \vec{c}]$ ☞ $[(\vec{a} + \vec{b})\ \vec{c}\ \vec{d}] = [\vec{a}\ \vec{c}\ \vec{d}] + [\vec{b}\ \vec{c}\ \vec{d}]$

☞ The volume of the tetrahedron OABC with O as origin & the pv's of A, B and C being $\vec{a}, \vec{b}$ & $\vec{c}$ respectively is given by $V = \frac{1}{6}[\vec{a}\ \vec{b}\ \vec{c}]$

☞ The positon vector of the centroid of a tetrahedron if the pv's of its angular vertices are $\vec{a}, \vec{b}, \vec{c}$ & $\vec{d}$ are given by $\frac{1}{4}[\vec{a} + \vec{b} + \vec{c} + \vec{d}]$.

Note that this is also the point of concurrency of the lines joining the vertices to the centroids of the opposite faces and is also called the centre of the tetrahedron. In case the tetrahedron is regular it is equidistant from the vertices and the four faces of the tetrahedron .

Remember that : $\left[\vec{a}-\vec{b}\quad \vec{b}-\vec{c}\quad \vec{c}-\vec{a}\right] = 0$ & $\left[\vec{a}+\vec{b}\quad \vec{b}+\vec{c}\quad \vec{c}+\vec{a}\right] = 2\left[\vec{a}\ \vec{b}\ \vec{c}\right]$.

*12. VECTOR TRIPLE PRODUCT :

Let $\vec{a}, \vec{b}, \vec{c}$ be any three vectors, then the expression $\vec{a} \times (\vec{b} \times \vec{c})$ is a vector & is called a vector triple product .

Geometrical Interpretation of $\vec{a} \times (\vec{b} \times \vec{c})$

Consider the expression $\vec{a} \times (\vec{b} \times \vec{c})$ which itself is a vector, since it is a cross product of two vectors $\vec{a}$ & $(\vec{b} \times \vec{c})$. Now $\vec{a} \times (\vec{b} \times \vec{c})$ is a vector perpendicular to the plane containing $\vec{a}$ & $(\vec{b} \times \vec{c})$ but $\vec{b} \times \vec{c}$ is a vector perpendicular to the plane $\vec{b}$ & $\vec{c}$, therefore $\vec{a} \times (\vec{b} \times \vec{c})$ is a vector lies in the plane of $\vec{b}$ & $\vec{c}$ and perpendicular to $\vec{a}$. **Hence we can express** $\vec{a} \times (\vec{b} \times \vec{c})$ **in terms of** $\vec{b}$ & $\vec{c}$ **i.e.** $\vec{a} \times (\vec{b} \times \vec{c})$ **=** $x\vec{b} + y\vec{c}$ where x & y are scalars .

☞ $\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c}$ ☞ $(\vec{a} \times \vec{b}) \times \vec{c} = (\vec{a} \cdot \vec{c})\vec{b} - (\vec{b} \cdot \vec{c})\vec{a}$

☞ $(\vec{a} \times \vec{b}) \times \vec{c} \neq \vec{a} \times (\vec{b} \times \vec{c})$

13. LINEAR COMBINATIONS / Linearly Independence and Dependence of Vectors :

Given a finite set of vectors $\vec{a}, \vec{b}, \vec{c}, ...$ then the vector $\vec{r} = x\vec{a} + y\vec{b} + z\vec{c} + ...$ is called a linear combination of $\vec{a}, \vec{b}, \vec{c}, ...$ for any x, y, z ...... $\in$ R. We have the following results :

(a) **FundamentalTheorem In Plane :** Let $\vec{a}, \vec{b}$ be non zero , non collinear vectors. Then any vector $\vec{r}$ coplanar with $\vec{a}, \vec{b}$ can be expressed uniquely as a linear combination of $\vec{a}, \vec{b}$ i.e. There exist some unique x,y $\in$ R such that $x\vec{a} + y\vec{b} = \vec{r}$.

(b) **Fundamental Theorem In Space :** Let $\vec{a}, \vec{b}, \vec{c}$ be non–zero, non–coplanar vectors in space. Then any vector $\vec{r}$, can be uniquily expressed as a linear combination of $\vec{a}, \vec{b}, \vec{c}$ i.e. There exist some unique x,y $\in$ R such that $x\vec{a} + y\vec{b} + z\vec{c} = \vec{r}$.

(c) If $\vec{x}_1, \vec{x}_2, \ldots\ldots \vec{x}_n$ are n non zero vectors, & $k_1, k_2, \ldots\ldots k_n$ are n scalars & if the linear combination $k_1\vec{x}_1 + k_2\vec{x}_2 + \ldots\ldots k_n\vec{x}_n = 0 \Rightarrow k_1 = 0, k_2 = 0 \ldots\ldots k_n = 0$ then we say that vectors $\vec{x}_1, \vec{x}_2, \ldots\ldots \vec{x}_n$ are **Linearly Independent Vectors** .

(d) If $\vec{x}_1, \vec{x}_2, \ldots\ldots \vec{x}_n$ are not **Linearly Independent** then they are said to be **Linearly Dependent** vectors . i.e. if $k_1\vec{x}_1 + k_2\vec{x}_2 + \ldots\ldots + k_n\vec{x}_n = 0$ & if there exists at least one $k_r \neq 0$ then $\vec{x}_1, \vec{x}_2, \ldots\ldots \vec{x}_n$ are said to be **Linearly Dependent** .

Note :

☞ If $\vec{a} = 3i + 2j + 5k$ then $\vec{a}$ is expressed as a **Linear Combination** of vectors $\hat{i}, \hat{j}, \hat{k}$. Also, $\vec{a}, \hat{i}, \hat{j}, \hat{k}$ form a linearly dependent set of vectors. In general , every set of four vectors is a linearly dependent system.

☞ $\hat{i}, \hat{j}, \hat{k}$ are **Linearly Independent** set of vectors. For

$$K_1\hat{i} + K_2\hat{j} + K_3\hat{k} = 0 \Rightarrow K_1 = 0 = K_2 = K_3.$$

☞ Two vectors $\vec{a}$ & $\vec{b}$ are linearly dependent $\Rightarrow \vec{a}$ is parallel to $\vec{b}$ i.e. $\vec{a} \times \vec{b} = 0 \Rightarrow$ linear dependence of $\vec{a}$ & $\vec{b}$. Conversely if $\vec{a} \times \vec{b} \neq 0$ then $\vec{a}$ & $\vec{b}$ are linearly independent .

☞ If three vectors $\vec{a}, \vec{b}, \vec{c}$ are linearly dependent, then they are coplanar i.e. $[\vec{a}, \vec{b}, \vec{c}] = 0$, conversely, if $[\vec{a}, \vec{b}, \vec{c}] \neq 0$, then the vectors are linearly independent.

14. COPLANARITY OF VECTORS :

Four points A, B, C, D with position vectors $\vec{a}, \vec{b}, \vec{c}, \vec{d}$ respectively are coplanar if and only if there exist scalars x, y, z, w not all zero simultaneously such that $x\vec{a} + y\vec{b} + z\vec{c} + w\vec{d} = 0$ where, x + y + z + w = 0.

15. RECIPROCAL SYSTEM OF VECTORS :

If $\vec{a}, \vec{b}, \vec{c}$ & $\vec{a}', \vec{b}', \vec{c}'$ are two sets of non coplanar vectors such that $\vec{a}.\vec{a}' = \vec{b}.\vec{b}' = \vec{c}.\vec{c}' = 1$ then the two systems are called Reciprocal System of vectors.

Note : $$a' = \frac{\vec{b} \times \vec{c}}{[\vec{a}\,\vec{b}\,\vec{c}]}\ ;\ b' = \frac{\vec{c} \times \vec{a}}{[\vec{a}\,\vec{b}\,\vec{c}]}\ ;\ c' = \frac{\vec{a} \times \vec{b}}{[\vec{a}\,\vec{b}\,\vec{c}]}$$

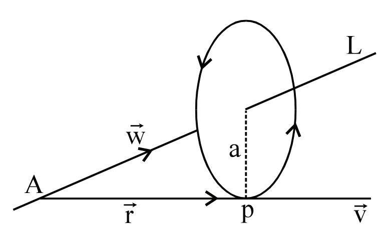
16. EQUATION OF A PLANE :

(a) The equation $(\vec{r} - \vec{r}_0).\vec{n} = 0$ represents a plane containing the point with p.v. $\vec{r}_0$ where $\vec{n}$ is a vector normal to the plane . $\vec{r}.\vec{n} = d$ is the general equation of a plane.

(b) Angle between the 2 planes is the angle between 2 normals drawn to the planes and the angle between a line and a plane is the compliment of the angle between the line and the normal to the plane.

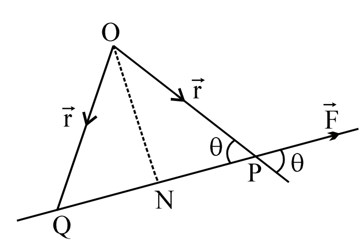
17. APPLICATION OF VECTORS :

(a) Work done against a constant force $\vec{F}$ over a displacement $\vec{s}$ is defined as $W = \vec{F}.\vec{s}$



(b) The tangential velocity $\vec{V}$ of a body moving in a circle is given by $\vec{V} = \vec{w} \times \vec{r}$ where $\vec{r}$ is the pv of the point P.

(c) The moment of $\vec{F}$ about 'O' is defined as $\vec{M} = \vec{r} \times \vec{F}$ where $\vec{r}$ is the pv of P wrt 'O'. The direction of $\vec{M}$ is along the normal to the plane OPN such that $\vec{r}, \vec{F}$ & $\vec{M}$ form a right handed system.



(d) Moment of the couple $= (\vec{r}_1 - \vec{r}_2) \times \vec{F}$ where $\vec{r}_1$ & $\vec{r}_2$ are pv's of the point of the application of the forces $\vec{F}$ & $-\vec{F}$.

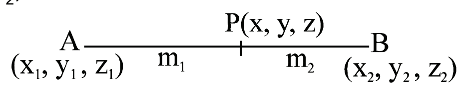
3 -D COORDINATE GEOMETRY

USEFUL RESULTS

A General :

(1) Distance (d) between two points (x_1, y_1, z_1) and (x_2, y_2, z_2)

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$



(2) **Section Fomula**

$$x = \frac{m_2 x_1 + m_1 x_2}{m_1 + m_2} \quad ; \quad y = \frac{m_2 y_1 + m_1 y_2}{m_1 + m_2} \quad ; \quad z = \frac{m_2 z_1 + m_1 z_2}{m_1 + m_2}$$

(For external division take –ve sign)

Direction Cosine and direction ratio's of a line

(3) Direction cosine of a line has the same meaning as d.c's of a vector.

(a) Any three numbers a, b, c proportional to the direction cosines are called the direction ratios i.e.

$$\frac{l}{a} = \frac{m}{b} = \frac{n}{c} = \pm \frac{1}{\sqrt{a^2 + b^2 + c^2}}$$

same sign either +ve or –ve should be taken through out.

note that d.r's of a line joining x_1, y_1, z_1 and x_2, y_2, z_2 are proportional to $x_2 - x_1$, $y_2 - y_1$ and $z_2 - z_1$

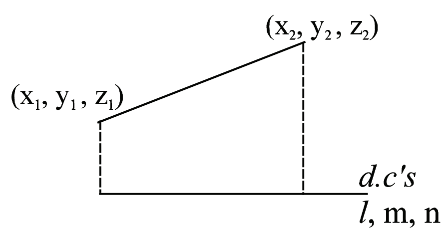
(b) If θ is the angle between the two lines whose d.c's are l_1, m_1, n_1 and l_2, m_2, n_2

$\cos\theta = l_1 l_2 + m_1 m_2 + n_1 n_2$

hence if lines are perpendicular then $l_1 l_2 + m_1 m_2 + n_1 n_2 = 0$

if lines are parallel then $\frac{l_1}{l_2} = \frac{m_1}{m_2} = \frac{n_1}{n_2}$

note that if three lines are coplanar then $\begin{vmatrix} l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ l_3 & m_3 & n_3 \end{vmatrix} = 0$



(4) Projection of the join of two points on a line with d.c's l, m, n are

$l(x_2 - x_1) + m(y_2 - y_1) + n(z_2 - z_1)$

B PLANE :

(i) General equation of degree one in x, y, z i.e. $ax + by + cz + d = 0$ represents a plane.

(ii) Equation of a plane passing through (x_1, y_1, z_1) is

$a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$

where a, b, c are the direction ratios of the normal to the plane.

(iii) Equation of a plane if its intercepts on the co-ordinate axes are x_1, y_1, z_1 is

$$\frac{x}{x_1} + \frac{y}{y_1} + \frac{z}{z_1} = 1.$$

(iv) Equation of a plane if the length of the perpendicular from the origin on the plane is p and d.c's of the perpendicular as l, m, , n is $l x + m y + n z = p$

(v) **Parallel and perpendicular planes** – Two planes

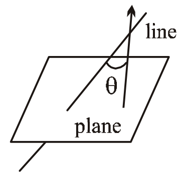
$a_1 x + b_1 y + c_1 z + d_1 = 0$ and $a_2 x + b_2 y + c_2 z + d_2 = 0$ are

perpendicular if $a_1 a_2 + b_1 b_2 + c_1 c_2 = 0$

parallel if $\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$ and

coincident if $\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}=\frac{d_1}{d_2}$

(vi) Angle between a plane and a line is the compliment of the angle between the normal to the plane and the line. If $\left.\begin{array}{l}\text{Line} \ : \vec{r}=\vec{a}+\lambda\vec{b} \\ \text{Plane} : \vec{r}.\vec{n}=d\end{array}\right]$ then $\cos(90-\theta)=\sin\theta=\frac{\vec{b}.\vec{n}}{|\vec{b}|.|\vec{n}|}$.



where θ is the angle between the line and normal to the plane.

(vii) Length of the perpendicular from a point (x_1, y_1, z_1) to a plane ax + by + cz + d = 0 is

$$p=\left|\frac{ax_1+by_1+cz_1+d}{\sqrt{a^2+b^2+c^2}}\right|$$

(viii) Distance between two parallel planes $ax+by+cz+d_1=0$ and $ax+by+cz+d_2=0$ is

$$\left|\frac{d_1-d_2}{\sqrt{a^2+b^2+c^2}}\right|$$

(ix) Planes bisecting the angle between two planes
$a_1x+b_1y+c_1z+d_1=0$ and $a_2+b_2y+c_2z+d_2=0$ is given by

$$\left|\frac{a_1x+b_1y+c_1z+d_1}{\sqrt{a_1^2+b_1^2+c_1^2}}\right|=\pm\left|\frac{a_2x+b_2y+c_2z+d_2}{\sqrt{a_2^2+b_2^2+c_2^2}}\right|$$

Of these two bisecting planes, one bisects the acute and the other obtuse angle between the given planes.

(x) Equation of a plane through the intersection of two planes P_1 and P_2 is given by $P_1+\lambda P_2=0$

C STRAIGHT LINE IN SPACE

(i) Equation of a line through A (x_1, y_1, z_1) and having direction cosines l, m, n are

$$\frac{x-x_1}{l}=\frac{y-y_1}{m}=\frac{z-z_1}{n}$$

and the lines through (x_1, y_1, z_1) and (x_2, y_2, z_2)

$$\frac{x-x_1}{x_2-x_1}=\frac{y-y_1}{y_2-y_1}=\frac{z-z_1}{z_2-z_1}$$

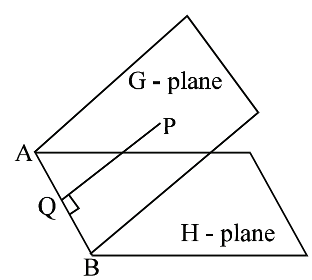
(ii) Intersection of two planes $a_1x+b_1y+c_1z+d_1=0$ and $a_2x+b_2y+c_2z+d_2=0$ together represent the unsymmetrical form of the straight line.

(iii) General equation of the plane containing the line $\frac{x-x_1}{l}=\frac{y-y_1}{m}=\frac{z-z_1}{n}$ is

$A(x-x_1)+B(y-y_1)+c(z-z_1)=0$ where $Al+bm+cn=0$.

LINE OF GREATEST SLOPE

AB is the line of intersection of G-plane and H is the horizontal plane. Line of greatest slope on a given plane, drawn through a given point on the plane, is the line through the point 'P' perpendicular to the line of intersetion of the given plane with any horizontal plane.



EXERCISE–I

1. If $\vec{a}\,\&\,\vec{b}$ are non collinear vectors such that, $\vec{p}=(x+4y)\vec{a}+(2x+y+1)\vec{b}$ & $\vec{q}=(y-2x+2)\vec{a}+(2x-3y-1)\vec{b}$, find x & y such that $3\vec{p}=2\vec{q}$.

2. (a) Show that the points $\vec{a}-2\vec{b}+3\vec{c};\,2\vec{a}+3\vec{b}-4\vec{c}\,\&-7\vec{b}+10\vec{c}$ are collinear.

(b) Prove that the points A = (1,2,3), B (3,4,7), C (–3,–2,–5) are collinear & find the ratio in which B divides AC.

3. Points X & Y are taken on the sides QR & RS, respectively of a parallelogram PQRS, so that $\vec{QX}=4\vec{XR}$ & $\vec{RY}=4\vec{YS}$. The line XY cuts the line PR at Z. Prove that $\vec{PZ}=\left(\frac{21}{25}\right)\vec{PR}$.

4. Find out whether the following pairs of lines are parallel, non-parallel & intersecting, or non-parallel & non-intersecting.

(i) $\vec{r}_1=\hat{i}+\hat{j}+2\hat{k}+\lambda\left(3\hat{i}-2\hat{j}+4\hat{k}\right)$
$\vec{r}_2=2\hat{i}+\hat{j}+3\hat{k}+\mu\left(-6\hat{i}+4\hat{j}-8\hat{k}\right)$

(ii) $\vec{r}_1=\hat{i}-\hat{j}+3\hat{k}+\lambda\left(\hat{i}-\hat{j}+\hat{k}\right)$
$\vec{r}_2=2\hat{i}+4\hat{j}+6\hat{k}+\mu\left(2\hat{i}+\hat{j}+3\hat{k}\right)$

(iii) $\vec{r}_1=\hat{i}+\hat{k}+\lambda\left(\hat{i}+3\hat{j}+4\hat{k}\right)$
$\vec{r}_2=2\hat{i}+3\hat{j}+\mu\left(4\hat{i}-\hat{j}+\hat{k}\right)$

5. 'O' is the origin of vectors and A is a fixed point on the circle of radius 'a' with centre O. The vector $\vec{OA}$ is denoted by $\vec{a}$. A variable point 'P' lies on the tangent at A & $\vec{OP}=\vec{r}$. Show that $\vec{a}.\vec{r}=|a|^2$. Hence if P ≡ (x,y) & A ≡ (x_1,y_1) deduce the equation of tangent at A to this circle.

6. Let $\vec{u}$ be a vector on rectangular coordinate system with sloping angle 60°. Suppose that $\left|\vec{u}-\hat{i}\right|$ is geometric mean of $|\vec{u}|$ and $\left|\vec{u}-2\hat{i}\right|$ where $\hat{i}$ is the unit vector along x-axis then $|\vec{u}|$ has the value equal to $\sqrt{a}-\sqrt{b}$ where a, b ∈ N, find the value $(a + b)^3 + (a – b)^3$.

7. The resultant of two vectors $\vec{a}\,\&\,\vec{b}$ is perpendicular to $\vec{a}$. If $\left|\vec{b}\right|=\sqrt{2}\left|\vec{a}\right|$ show that the resultant of $2\vec{a}\,\&\,\vec{b}$ is perpendicular to $\vec{b}$.

8. $\vec{a},\vec{b},\vec{c}$ and $\vec{d}$ are the position vectors of the points A ≡ (x, y, z) ; B ≡ (y, – 2z, 3x) ; C ≡ (2z, 3x, – y) and D≡(1,–1, 2) respectively. If $|\vec{a}|=2\sqrt{3}$; $\left(\vec{a}\,\hat{}\,\vec{b}\right)=\left(\vec{a}\,\hat{}\,\vec{c}\right)$; $\left(\vec{a}\,\hat{}\,\vec{d}\right)=\frac{\pi}{2}$ and $\left(\vec{a}\,\hat{}\,\hat{j}\right)$ is obtuse, then find x, y, z.

9. If $\vec{r}$ and $\vec{s}$ are non zero constant vectors and the scalar b is chosen such that $\left|\vec{r}+b\vec{s}\right|$ is minimum, then show that the value of $\left|b\vec{s}\right|^2+\left|\vec{r}+b\vec{s}\right|^2$ is equal to $|\vec{r}|^2$.

10. (a) Find a unit vector â which makes an angle (π/4) with axis of z & is such that â + i + j is a unit vector.

(b) Prove that $\left(\frac{\vec{a}}{\vec{a}^2}-\frac{\vec{b}}{\vec{b}^2}\right)^2=\left(\frac{\vec{a}-\vec{b}}{|\vec{a}|\,|\vec{b}|}\right)^2$

11. Given four non zero vectors $\vec{a}, \vec{b}, \vec{c}$ and $\vec{d}$. The vectors $\vec{a}, \vec{b}$ & $\vec{c}$ are coplanar but not collinear pair by pair and vector $\vec{d}$ is not coplanar with vectors $\vec{a}, \vec{b}$ & $\vec{c}$ and $(\widehat{\vec{a}\vec{b}}) = (\widehat{\vec{b}\vec{c}}) = \frac{\pi}{3}, (\widehat{\vec{d}\vec{a}}) = \alpha, (\widehat{\vec{d}\vec{b}}) = \beta$ then prove that $(\widehat{\vec{d}\ \vec{c}}) = \cos^{-1}(\cos\beta - \cos\alpha)$.

12. Given three points on the xy plane on O(0, 0), A(1, 0) and B(–1, 0). Point P is moving on the plane satisfying the condition $\left(\overrightarrow{PA}\cdot\overrightarrow{PB}\right) + 3\left(\overrightarrow{OA}\cdot\overrightarrow{OB}\right) = 0$

If the maximum and minimum values of $\left|\overrightarrow{PA}\right|\left|\overrightarrow{PB}\right|$ are M and m respectively then find the value of $M^2 + m^2$.

13. In the plane of a triangle ABC, squares ACXY, BCWZ are described , in the order given, externally to the triangle on AC & BC respectively. Given that $\overrightarrow{CX} = \vec{b}$, $\overrightarrow{CA} = \vec{a}$, $\overrightarrow{CW} = \vec{x}$, $\overrightarrow{CB} = \vec{y}$. Prove that $\vec{a}.\vec{y} + \vec{x}.\vec{b} = 0$. Deduce that $\overrightarrow{AW}\,.\,\overrightarrow{BX} = 0$.

14. Given that $\vec{u} = \hat{i} - 2\hat{j} + 3\hat{k}$; $\vec{v} = 2\hat{i} + \hat{j} + 4\hat{k}$; $\vec{w} = \hat{i} + 3\hat{j} + 3\hat{k}$ and $(\vec{u}\cdot\vec{R} - 10)\hat{i} + (\vec{v}\cdot\vec{R} - 20)\hat{j} + (\vec{w}\cdot\vec{R} - 20)\hat{k} = 0$. Find the unknown vector $\vec{R}$.

15. If O is origin of reference, point $A(\vec{a})$; $B(\vec{b})$; $C(\vec{c})$; $D(\vec{a}+\vec{b})$; $E(\vec{b}+\vec{c})$; $F(\vec{c}+\vec{a})$; $G(\vec{a}+\vec{b}+\vec{c})$ where $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$; $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ and $\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$ then prove that these points are vertices of a cube having length of its edge equal to unity provided the matrix.

$\begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix}$ is orghogonal. Also find the length XY such that X is the point of intersection of CM and GP; Y is the point of intersection of OQ and DN where P, Q, M, N are respectively the midpoint of sides CF, BD, GF and OB.

16. (a) If $\vec{a} + \vec{b} + \vec{c} = 0$, show that $\vec{a} \times \vec{b} = \vec{b} \times \vec{c} = \vec{c} \times \vec{a}$. Deduce the Sine rule for a ΔABC.

(b) Find the minimum area of the triangle whose vertices are A(–1, 1, 2); B(1, 2, 3) and C(t, 1, 1) where t is a real number.

17. (a) Determine vector of magnitude 9 which is perpendicular to both the vectors :

$$4\hat{i} - \hat{j} + 3\hat{k} \ \ \& \ -2\hat{i} + \hat{j} - 2\hat{k}$$

(b) A triangle has vertices (1, 1, 1) ; (2, 2, 2), (1, 1, y) and has the area equal to csc$(\pi/4)$ sq. units. Find the value of y.

18. Consider a parallelogram ABCD. Let M be the centre of line segment $\overrightarrow{BC}$ and S denote the point of intersection of the line segment $\overrightarrow{AM}$ and the diagonal $\overrightarrow{BD}$. Find the ratio of the area of the parallelogram to the area of the triangle BMS.

19. The length of the edge of the regular tetrahedron D – ABC is 'a'. Point E and F are taken on the edges AD and BD respectively such that E divides $\overrightarrow{DA}$ and F divides $\overrightarrow{BD}$ in the ratio 2:1 each . Then find the area of triangle CEF.

20. Let $\vec{a}=\sqrt{3}\,\hat{i}-\hat{j}$ and $\vec{b}=\frac{1}{2}\hat{i}+\frac{\sqrt{3}}{2}\hat{j}$ and $\vec{x}=\vec{a}+(q^2-3)\vec{b}$, $\vec{y}=-p\vec{a}+q\vec{b}$. If $\vec{x}\perp\vec{y}$, then express p as a function of q, say $p = f(q)$, ($p \neq 0$ & $q \neq 0$) and find the intervals of monotonicity of $f(q)$.

EXERCISE–II

1. The vector $\overrightarrow{OP}$ = $\hat{i}+2\hat{j}+2\hat{k}$ turns through a right angle, passing through the positive x-axis on the way. Find the vector in its new position.

2. The position vectors of the points A, B, C are respectively (1, 1, 1) ; (1, –1, 2) ; (0, 2, –1). Find a unit vector parallel to the plane determined by ABC & perpendicular to the vector (1, 0, 1).

3. Let $\begin{vmatrix}(a_1-a)^2 & (a_1-b)^2 & (a_1-c)^2\\ (b_1-a)^2 & (b_1-b)^2 & (b_1-c)^2\\ (c_1-a)^2 & (c_1-b)^2 & (c_1-c)^2\end{vmatrix}$ = 0 and if the vectors $\vec{\alpha}=\hat{i}+a\hat{j}+a^2\hat{k}$; $\vec{\beta}=\hat{i}+b\hat{j}+b^2\hat{k}$; $\vec{\gamma}=\hat{i}+c\hat{j}+c^2\hat{k}$ are non coplanar, show that the vectors $\vec{\alpha}_1=\hat{i}+a_1\hat{j}+a_1^2\hat{k}$;$\vec{\beta}_1=\hat{i}+b_1\hat{j}+b_1^2\hat{k}$ and $\vec{\gamma}_1=\hat{i}+c_1\hat{j}+c_1^2\hat{k}$ are coplaner.

4. Given non zero number x_1, x_2, x_3 ; y_1, y_2, y_3 and z_1, z_2 and z_3

(i) Can the given numbers satisfy

$$\begin{vmatrix}x_1 & x_2 & x_3\\ y_1 & y_2 & y_3\\ z_1 & z_2 & z_3\end{vmatrix} = 0 \text{ and } \begin{cases}x_1x_2+y_1y_2+z_1z_2=0\\ x_2x_3+y_2y_3+z_2z_3=0\\ x_3x_1+y_3y_1+z_3z_1=0\end{cases}$$

(ii) If $x_i > 0$ and $y_i < 0$ for all i = 1, 2, 3 and P = (x_1, x_2, x_3) ; Q (y_1, y_2, y_3) and O (0, 0, 0) can the triangle POQ be a right angled triangle?

5. The pv's of the four angular points of a tetrahedron are: $A\left(\hat{j}+2\hat{k}\right)$; $B\left(3\hat{i}+\hat{k}\right)$; $C\left(4\hat{i}+3\hat{j}+6\hat{k}\right)$ & $D\left(2\hat{i}+3\hat{j}+2\hat{k}\right)$. Find :

(i) the perpendicular distance from A to the line BC.
(ii) the volume of the tetrahedron ABCD.
(iii) the perpendicular distance from D to the plane ABC.
(iv) the shortest distance between the lines AB & CD.

6. The length of an edge of a cube $ABCDA_1B_1C_1D_1$ is equal to unity. A point E taken on the edge $\overrightarrow{AA_1}$ is such that $\left|\overrightarrow{AE}\right| = \frac{1}{3}$. A point F is taken on the edge $\overrightarrow{BC}$ such that $\left|\overrightarrow{BF}\right| = \frac{1}{4}$. If O_1 is the centre of the cube, find the shortest distance of the vertex B_1 from the plane of the $\Delta\, O_1EF$.

7. $A(\vec{a})$; $B(\vec{b})$; $C(\vec{c})$ are the vertices of the triangle ABC such that $\vec{a}=\frac{1}{2}(2\hat{i}-\vec{r}-7\hat{k})$; $\vec{b}=3\vec{r}+\hat{j}-4\hat{k}$; $\vec{c}=22\hat{i}-11\hat{j}-9\vec{r}$. A vector $\vec{p}=2\hat{j}-\hat{k}$ is such that $(\vec{r}+\vec{p})$ is parallel to $\hat{i}$ and $(\vec{r}-2\hat{i})$ is parallel to $\vec{p}$. Show that there exists a point $D(\vec{d})$ on the line AB with $\vec{d}=2t\,\hat{i}+(1-2t)\hat{j}+(t-4)\hat{k}$. Also find the shortest distance of C from AB.

8. Find the point R in which the line AB cuts the plane CDE where

$\vec{a} = \hat{i}+2\hat{j}+\hat{k}$, $\vec{b} = 2\hat{i}+\hat{j}+2\hat{k}$, $\vec{c} = -4\hat{j}+4\hat{k}$, $\vec{d} = 2\hat{i}-2\hat{j}+2\hat{k}$ & $\vec{e} = 4\hat{i}+\hat{j}+2\hat{k}$.

9. If $\vec{a} = a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$; $\vec{b} = b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$ and $\vec{c} = c_1\hat{i}+c_2\hat{j}+c_3\hat{k}$ then show that the value of the scalar triple product $[n\vec{a}+\vec{b} \quad n\vec{b}+\vec{c} \quad n\vec{c}+\vec{a}]$ is $(n^3+1)\begin{vmatrix} \vec{a}\cdot\hat{i} & \vec{a}\cdot\hat{j} & \vec{a}\cdot\hat{k} \\ \vec{b}\cdot\hat{i} & \vec{b}\cdot\hat{j} & \vec{b}\cdot\hat{k} \\ \vec{c}\cdot\hat{i} & \vec{c}\cdot\hat{j} & \vec{c}\cdot\hat{k} \end{vmatrix}$

10. (a) Prove that $|\vec{a} \times \vec{b}| = \sqrt{-\vec{b}\cdot[\vec{a} \times (\vec{a} \times \vec{b})]}$

(b) Given that $\vec{a},\vec{b},\vec{p},\vec{q}$ are four vectors such that $\vec{a}+\vec{b} = \mu\vec{p}, \vec{b}.\vec{q} = 0$ & $(\vec{b})^2 = 1$, where μ is a scalar then prove that $|(\vec{a}.\vec{q})\vec{p}-(\vec{p}.\vec{q})\vec{a}| = |\vec{p}.\vec{q}|$.

11. ABCD is a tetrahedron with pv's of its angular points as A(–5, 22, 5); B(1, 2, 3); C(4, 3, 2) and D(–1, 2, – 3). If the area of the triangle AEF where the quadrilaterals ABDE and ABCF are parallelograms is $\sqrt{S}$ then find the value of S.

12. If $\vec{A}, \vec{B}$ & $\vec{C}$ are vectors such that $|\vec{B}| = |\vec{C}|$, Prove that: $\left[\left(\vec{A}+\vec{B}\right)x\left(\vec{A}+\vec{C}\right)\right] x \left(\vec{B}x\vec{C}\right).\left(\vec{B}+\vec{C}\right) = 0$.

13. Find the scalars α & β if $\vec{a}\,x(\vec{b}\,x\,\vec{c}) + (\vec{a}.\vec{b})\vec{b} = (4-2\beta-\sin\alpha)\vec{b} + (\beta^2-1)\vec{c}$ & $(\vec{c}.\vec{c})\vec{a} = \vec{c}$ while $\vec{b}$ & $\vec{c}$ are non zero non collinear vectors.

14. Let $\vec{a} = \alpha\hat{i} + 2\hat{j} - 3\hat{k}$, $\vec{b} = \hat{i} + 2\alpha\hat{j} - 2\hat{k}$ and $\vec{c} = 2\hat{i} - \alpha\hat{j} + \hat{k}$. Find the value(s) of α, if any, such that $\left\{\left(\vec{a}\times\vec{b}\right)\times\left(\vec{b}\times\vec{c}\right)\right\}\times\left(\vec{c}\times\vec{a}\right) = 0$. Find the vector product when α = 0.

15. Find a vector $\vec{v}$ which is coplanar with the vectors $\hat{i}+\hat{j}-2\hat{k}$ & $\hat{i}-2\hat{j}+\hat{k}$ and is orthogonal to the vector $-2\hat{i}+\hat{j}+\hat{k}$. It is given that the projection of $\vec{v}$ along the vector $\hat{i}-\hat{j}+\hat{k}$ is equal to $6\sqrt{3}$.

16. Given four points P_1, P_2, P_3 and P_4 on the coordinate plane with origin O which satisfy the condition

$$\overrightarrow{OP}_{n-1} + \overrightarrow{OP}_{n+1} = \frac{3}{2}\overrightarrow{OP}_n, \quad n = 2, 3$$

(i) If P_1, P_2 lie on the curve xy = 1, then prove that P_3 does not lie on the curve.
(ii) If P_1, P_2, P_3 lie on the circle $x^2 + y^2 = 1$, then prove that P_4 lies on this circle.

17. Prove the result (Lagrange's identity) $(\vec{p}\times\vec{q})\cdot(\vec{r}\times\vec{s}) = \begin{vmatrix} \vec{p}.\vec{r} & \vec{p}.\vec{s} \\ \vec{q}.\vec{r} & \vec{q}.\vec{s} \end{vmatrix}$ & use it to prove the following. Let (ab)denote the plane formed by the lines a,b. If (ab) is perpendicular to (cd) and (ac) is perpendicular to (bd) prove that (ad) is perpendicular to (bc).

18. Consider the non zero vectors $\vec{a},\vec{b},\vec{c}$ & $\vec{d}$ such that no three of which are coplanar then prove that $\vec{a}\left[\vec{b}\,\vec{c}\,\vec{d}\right] + \vec{c}\left[\vec{a}\,\vec{b}\,\vec{d}\right] = \vec{b}\left[\vec{a}\,\vec{c}\,\vec{d}\right] + \vec{d}\left[\vec{a}\,\vec{b}\,\vec{c}\right]$. Hence prove that if $\vec{a},\vec{b},\vec{c}$ & $\vec{d}$ represent the position vectors of the vertices of a plane quadrilateral then $\dfrac{\left[\vec{b}\,\vec{c}\,\vec{d}\right]+\left[\vec{a}\,\vec{b}\,\vec{d}\right]}{\left[\vec{a}\,\vec{c}\,\vec{d}\right]+\left[\vec{a}\,\vec{b}\,\vec{c}\right]} = 1$.

19. The base vectors $\vec{a}_1, \vec{a}_2, \vec{a}_3$ are given in terms of base vectors $\vec{b}_1, \vec{b}_2, \vec{b}_3$ as, $\vec{a}_1 = 2\vec{b}_1 + 3\vec{b}_2 - \vec{b}_3$; $\vec{a}_2 = \vec{b}_1 - 2\vec{b}_2 + 2\vec{b}_3$ & $\vec{a}_3 = -2\vec{b}_1 + \vec{b}_2 - 2\vec{b}_3$. If $\vec{F} = 3\vec{b}_1 - \vec{b}_2 + 2\vec{b}_3$, then express $\vec{F}$ in terms of $\vec{a}_1, \vec{a}_2$ & $\vec{a}_3$.

20. Let $\vec{a} = \begin{bmatrix} 1 \\ 0 \\ -3 \end{bmatrix}$; $\vec{b} = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$; $\vec{c} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$. Find the numbers α, β, γ such that $\alpha\vec{a} + \beta\vec{b} + \gamma\vec{c} = \begin{bmatrix} -2 \\ -5 \\ 6 \end{bmatrix}$.

21. (a) If $p\vec{x} + (\vec{x} \times \vec{a}) = \vec{b}; (p \neq 0)$ prove that $\vec{x} = \dfrac{p^2\vec{b} + (\vec{b}.\vec{a})\vec{a} - p(\vec{b} x \vec{a})}{p(p^2 + \vec{a}^2)}$.

(b) Solve the following equation for the vector $\vec{p}$; $\vec{p} x \vec{a} + \left(\vec{p}.\vec{b}\right)\vec{c} = \vec{b} x \vec{c}$ where $\vec{a}, \vec{b}, \vec{c}$ are non zero non coplanar vectors and $\vec{a}$ is neither perpendicular to $\vec{b}$ nor to $\vec{c}$, hence show that $\left(\vec{p} \times \vec{a} + \dfrac{\left[\vec{a}\,\vec{b}\,\vec{c}\right]}{\vec{a} \cdot \vec{c}}\vec{c}\right)$ is perpendicular to $\vec{b} - \vec{c}$.

22. Solve the simultaneous vector equations for the vectors $\vec{x}$ and $\vec{y}$.

$\vec{x} + \vec{c} \times \vec{y} = \vec{a}$ and $\vec{y} + \vec{c} \times \vec{x} = \vec{b}$ where $\vec{c}$ is a non zero vector.

EXERCISE–III

1. Find the angle between the two straight lines whose direction cosines *l*, m, n are given by 2*l* + 2m – n = 0 and mn + n*l* + *l*m = 0.

2. P is any point on the plane *l*x + my + nz = p. A point Q taken on the line OP (where O is the origin) such that OP. OQ = p^2. Show that the locus of Q is p(*l*x + my + nz) = $x^2 + y^2 + z^2$.

3. Find the equation of the plane through the points (2, 2, 1), (1, –2, 3) and parallel to the x-axis.

4. Through a point P (f, g, h), a plane is drawn at right angles to OP where 'O' is the origin, to meet the coordinate axes in A, B, C. Prove that the area of the triangle ABC is $\dfrac{r^5}{2fgh}$ where OP = r.

5. The plane *l*x + my = 0 is rotated about its line of intersection with the plane z = 0 through an angle θ. Prove that the equation to the plane in new position is *l*x + my $\pm$ $z\sqrt{l^2 + m^2}\,\tan\theta$ = 0

6. Find the equations of the straight line passing through the point (1, 2, 3) to intersect the straight line x + 1 = 2 (y – 2) = z + 4 and parallel to the plane x + 5y + 4z = 0.

7. Find the equations of the two lines through the origin which intersect the line $\dfrac{x-3}{2} = \dfrac{y-3}{1} = \dfrac{z}{1}$ at an angle of $\dfrac{\pi}{3}$.

8. A variable plane is at a constant distance p from the origin and meets the coordinate axes in points A, B and C respectively. Through these points, planes are drawn parallel to the coordinates planes. Find the locus of their point of intersection.

9. Find the distance of the point P (– 2, 3, – 4) from the line $\frac{x+2}{3}=\frac{2y+3}{4}=\frac{3z+4}{5}$ measured parallel to the plane 4x + 12y – 3z + 1 = 0.

10. Find the equation to the line passing through the point (1, –2, –3) and parallel to the line
$2x + 3y - 3z + 2 = 0 = 3x - 4y + 2z - 4$.

11. Find the equation of the line passing through the point (4, –14, 4) and intersecting the line of intersection of the planes : 3x + 2y – z = 5 and x – 2y – 2z = –1 at right angles.

12. Let P = (1, 0, – 1) ; Q = (1, 1, 1) and R = (2, 1, 3) are three points.
(a) Find the area of the triangle having P, Q and R as its vertices.
(b) Give the equation of the plane through P, Q and R in the form ax + by + cz = 1.
(c) Where does the plane in part (b) intersect the y-axis.
(d) Give parametric equations for the line through R that is perpendicular to the plane in part (b).

13. Find the point where the line of intersection of the planes x – 2y + z = l and x + 2y – 2z = 5, intersects the plane 2x + 2y + z + 6 = 0.

14. Feet of the perpendicular drawn from the point P (2, 3, –5) on the axes of coordinates are A, B and C. Find the equation of the plane passing through their feet and the area of ΔABC.

15. Find the equations to the line which can be drawn from the point (2, –1, 3) perpendicular to the lines

$$\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4} \quad \text{and} \quad \frac{x-4}{4}=\frac{y}{5}=\frac{z+3}{3} \quad \text{at right angles.}$$

16. Find the equation of the plane containing the straight line $\frac{x-1}{2}=\frac{y+2}{-3}=\frac{z}{5}$ and perpendicular to the plane x – y + z + 2 = 0.

17. Find the value of p so that the lines $\frac{x+1}{-3}=\frac{y-p}{2}=\frac{z+2}{1}$ and $\frac{x}{1}=\frac{y-7}{-3}=\frac{z+7}{2}$ are in the same plane. For this value of p, find the coordinates of their point of intersection and the equation of the plane containing them.

18. Find the equations to the line of greatest slope through the point (7, 2 , –1) in the plane x – 2y + 3z = 0 assuming that the axes are so placed that the plane 2x + 3y – 4z = 0 is horizontal.

19. Let L be the line given by $\vec{r}=\begin{bmatrix}2\\-2\\-1\end{bmatrix}+\lambda\begin{bmatrix}-1\\0\\1\end{bmatrix}$ and let P be the point (2, – 1, 1). Also suppose that E be the plane containing three non collinear points A = (0, 1, 1); B(1, 2, 2) and C = (1, 0, 1)
Find
(a) Distance between the point P and the line L.
(b) Equation of the plane E.
(c) Equation the plane F containing the line L and the point P.
(d) Acute between the plane E and F.
(e) Volume of the parallelopiped by A, B, C and the point D(– 3, 0, 1).

20. The position vectors of the four angular points of a tetrahedron OABC are (0, 0, 0); (0, 0, 2); (0, 4, 0) and (6, 0, 0) respectively. A point P inside the tetrahedron is at the same distance 'r' from the four plane faces of the tetrahedron. Find the value of 'r'.

21. The line $\frac{x+6}{5}=\frac{y+10}{3}=\frac{z+14}{8}$ is the hypotenuse of an isosceles right angled triangle whose opposite vertex is (7, 2, 4). Find the equation of the remaining sides.

22. Find the foot and hence the length of the perpendicular from the point (5, 7, 3) to the line $\frac{x-15}{3}=\frac{y-29}{8}=\frac{5-z}{5}$. Also find the equation of the plane in which the perpendicular and the given straight line lie.

23. Find the equation of the line which is reflection of the line $\frac{x-1}{9}=\frac{y-2}{-1}=\frac{z+3}{-3}$ in the plane 3x – 3y + 10z = 26.

24. Find the equation of the plane containing the line $\frac{x-1}{2}=\frac{y}{3}=\frac{z}{2}$ and parallel to the line $\frac{x-3}{2}=\frac{y}{5}=\frac{z-2}{4}$.

Find also the S.D. between the two lines.

25. Consider the plane

E : $\vec{r} = \begin{bmatrix}-1\\1\\1\end{bmatrix} + \lambda\begin{bmatrix}1\\2\\0\end{bmatrix} + \mu\begin{bmatrix}1\\0\\1\end{bmatrix}$

Let F be the plane containing the point A (– 4, 2, 2) and parallel to E.

Suppose the point B is on the plane E such that B has a minimum distance from the point A. If C (– 3, 0, 4) lies in the plane F. Find the area of the angle ABC.

EXERCISE–IV

1. (a) Let $\vec{a}=2\hat{i}+\hat{j}-2\hat{k}$ & $\vec{b}=\hat{i}+\hat{j}$. If $\vec{c}$ is a vector such that $\vec{a}\cdot\vec{c}=|\vec{c}|$, $|\vec{c}-\vec{a}|=2\sqrt{2}$ and the angle between $(\vec{a}\times\vec{b})$ and $\vec{c}$ is 30º, then $|(\vec{a}\times\vec{b})\times\vec{c}|$ =

(A) 2/3 (B) 3/2 (C) 2 (D) 3

(b) Let $\vec{a}=2\hat{i}+\hat{j}+\hat{k}$, $\vec{b}=\hat{i}+2\hat{j}-\hat{k}$ and a unit vector $\vec{c}$ be coplanar. If $\vec{c}$ is perpendicular to $\vec{a}$, then $\vec{c}$ =

(A) $\frac{1}{\sqrt{2}}\left(-\hat{j}+\hat{k}\right)$ (B) $\frac{1}{\sqrt{3}}\left(-\hat{i}-\hat{j}-\hat{k}\right)$

(C) $\frac{1}{\sqrt{5}}\left(\hat{i}-2\hat{j}\right)$ (D) $\frac{1}{\sqrt{3}}\left(\hat{i}-\hat{j}-\hat{k}\right)$

(c) Let $\vec{a}$ & $\vec{b}$ be two non-collinear unit vectors . If $\vec{u}=\vec{a}-\left(\vec{a}\,.\,\vec{b}\right)\vec{b}$ & $\vec{v}=\vec{a}\text{ x }\vec{b}$, then $|\vec{v}|$ is :

(A) $|\vec{u}|$ (B) $|\vec{u}|+|\vec{u}\,.\,\vec{a}|$

(C) $|\vec{u}|+|\vec{u}\,.\,\vec{b}|$ (D) $\vec{u}+\vec{u}\,.\left(\vec{a}+\vec{b}\right)$

(d) Let $\vec{u}$ & $\vec{v}$ be unit vectors . If $\vec{w}$ is a vector such that $\vec{w}+\left(\vec{w}\text{ x }\vec{u}\right)=\vec{v}$, then prove that $\left|\left(\vec{u}\text{ x }\vec{v}\right).\,\vec{w}\right|\le\frac{1}{2}$ and the equality holds if and only if $\vec{u}$ is perpendicular to $\vec{v}$.

[JEE '99, 2 + 2 + 3 + 10, out of 200]

2. (a) An arc AC of a circle subtends a right angle at the centre O. The point B divides the arc in the ratio 1 : 2. If $\overrightarrow{OA}=\vec{a}$ & $\overrightarrow{OB}=\vec{b}$, then calculate $\overrightarrow{OC}$ in terms of $\vec{a}$ & $\vec{b}$.

(b) If $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar vectors and $\vec{d}$ is a unit vector, then find the value of,
$\left|(\vec{a}.\vec{d})(\vec{b} \times \vec{c}) + (\vec{b}.\vec{d})(\vec{c} \times \vec{a}) + (\vec{c}.\vec{d})(\vec{a} \times \vec{b})\right|$ independent of $\vec{d}$. **[REE '99, 6 + 6]**

3. (a) Select the correct alternative :

(i) If the vectors $\vec{a}, \vec{b}$ & $\vec{c}$ form the sides BC, CA & AB respectively of a triangle ABC, then

(A) $\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a} = 0$
(B) $\vec{a} \times \vec{b} = \vec{b} \times \vec{c} = \vec{c} \times \vec{a}$
(C) $\vec{a}.\vec{b} = \vec{b}.\vec{c} = \vec{c}.\vec{a}$
(D) $\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a} = 0$

(ii) Let the vectors $\vec{a}, \vec{b}, \vec{c}$ & $\vec{d}$ be such that $(\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d}) = \vec{0}$. Let P_1 & P_2 be planes determined by the pairs of vectors $\vec{a}, \vec{b}$ & $\vec{c}, \vec{d}$ respectively. Then the angle between P_1 and P_2 is :

(A) 0
(B) $\pi/4$
(C) $\pi/3$
(4) $\pi/2$

(iii) If $\vec{a}, \vec{b}$ & $\vec{c}$ are unit coplanar vectors, then the scalar triple product

$$\left[2\vec{a} - \vec{b} \quad 2\vec{b} - \vec{c} \quad 2\vec{c} - \vec{a}\right] =$$

(A) 0 (B) 1 (C) $-\sqrt{3}$ (D) $\sqrt{3}$

[JEE ,2000 (Screening) 1 + 1 + 1 out of 35]

(b) Let ABC and PQR be any two triangles in the same plane. Assume that the perpendiculars from the points A, B, C to the sides QR, RP, PQ respectively are concurrent. Using vector methods or otherwise, prove that the perpendiculars from P, Q, R to BC, CA, AB respectively are also concurrent.

[JEE '2000 (Mains) 10 out of 100]

4. (i) If $\vec{a} = \hat{i} + \hat{j} - \hat{k}$, $\vec{b} = -\hat{i} + 2\hat{j} + 2\hat{k}$ & $\vec{c} = -\hat{i} + 2\hat{j} - \hat{k}$, find a unit vector normal to the vectors $\vec{a} + \vec{b}$ and $\vec{b} - \vec{c}$.

(ii) Given that vectors $\vec{a}$ & $\vec{b}$ are perpendicular to each other, find vector $\vec{v}$ in terms of $\vec{a}$ & $\vec{b}$ satisfying the equations, $\vec{v} \cdot \vec{a} = 0$, $\vec{\upsilon} . \vec{b} = 1$ and $[\vec{v}\ \vec{a}\ \vec{b}] = 1$

(iii) $\vec{a}, \vec{b}$ & $\vec{c}$ are three unit vectors such that $\vec{a} \times (\vec{b} \times \vec{c}) = \frac{1}{2}(\vec{b} + \vec{c})$. Find angle between vectors $\vec{a}$ & $\vec{b}$ given that vectors $\vec{b}$ & $\vec{c}$ are non-parallel.

(iv) A particle is placed at a corner P of a cube of side 1 meter. Forces of magnitudes 2, 3 and 5 kg weight act on the particle along the diagonals of the faces passing through the point P. Find the moment of these forces about the corner opposite to P.

[REE '2000 (Mains) 3 + 3 + 3 + 3 out of 100]

5. (a) The diagonals of a parallelogram are given by vectors $2\hat{i} + 3\hat{j} - 6\hat{k}$ and $3\hat{i} - 4\hat{j} - \hat{k}$. Determine its sides and also the area.

(b) Find the value of λ such that a, b, c are all non-zero and
$(-4\hat{i} + 5\hat{j})a + (3\hat{i} - 3\hat{j} + \hat{k})b + (\hat{i} + \hat{j} + 3\hat{k})c = \lambda(a\hat{i} + b\hat{j} + c\hat{k})$ **[REE '2001 (Mains) 3 + 3]**

6. (a) Find the vector $\vec{r}$ which is perpendicular to $\vec{a} = \hat{i} - 2\hat{j} + 5\hat{k}$ and $\vec{b} = 2\hat{i} + 3\hat{j} - \hat{k}$ and $\vec{r} \cdot (2\hat{i} + \hat{j} + \hat{k}) + 8 = 0$.

(b) Two vertices of a triangle are at $-\hat{i} + 3\hat{j}$ and $2\hat{i} + 5\hat{j}$ and its orthocentre is at $\hat{i} + 2\hat{j}$. Find the position vector of third vertex. **[REE '2001 (Mains) 3 + 3]**

7. (a) If $\vec{a}, \vec{b}$ and $\vec{c}$ are unit vectors, then $|\vec{a}-\vec{b}|^2 + |\vec{b}-\vec{c}|^2 + |\vec{c}-\vec{a}|^2$ does NOT exceed

(A) 4 (B) 9 (C) 8 (D) 6

(b) Let $\vec{a} = \hat{i} - \hat{k}, \vec{b} = x\hat{i} + \hat{j} + (1-x)\hat{k}$ and $\vec{c} = y\hat{i} + x\hat{j} + (1+x-y)\hat{k}$. Then $[\vec{a}, \vec{b}, \vec{c}]$ depends on

(A) only x (B) only y
(C) NEITHER x NOR y (D) both x and y

[JEE '2001 (Screening) 1 + 1 out of 35]

8. Let $\vec{A}(t) = f_1(t)\hat{i} + f_2(t)\hat{j}$ and $\vec{B}(t) = g_1(t)\hat{i} + g_2(t)\hat{j}$, t ∈ [0, 1], where f_1, f_2, g_1, g_2 are continuous functions. If $\vec{A}(t)$ and $\vec{B}(t)$ are nonzero vectors for all t and $\vec{A}(0) = 2\hat{i} + 3\hat{j}$, $\vec{A}(1) = 6\hat{i} + 2\hat{j}$, $\vec{B}(0) = 3\hat{i} + 2\hat{j}$ and $\vec{B}(1) = 2\hat{i} + 6\hat{j}$, then show that $\vec{A}(t)$ and $\vec{B}(t)$ are parallel for some t.

[JEE '2001 (Mains) 5 out of 100]

9. (a) If $\vec{a}$ and $\vec{b}$ are two unit vectors such that $\vec{a} + 2\vec{b}$ and $5\vec{a} - 4\vec{b}$ are perpendicular to each other then the angle between $\vec{a}$ and $\vec{b}$ is

(A) 45^0 (B) 60^0
(C) $\cos^{-1}(1/3)$ (D) $\cos^{-1}(2/7)$

(b) Let $\vec{V} = 2\hat{i} + \hat{j} - \hat{k}$ and $\vec{W} = \hat{i} + 3\hat{k}$. If $\vec{U}$ is a unit vector, then the maximum value of the scalar triple product $[\vec{U}\ \vec{V}\ \vec{W}]$ is

(A) –1 (B) $\sqrt{10} + \sqrt{6}$
(C) $\sqrt{59}$ (D) $\sqrt{60}$ **[JEE 2002(Screening), 3 + 3]**

10. Let V be the volume of the parallelopiped formed by the vectors $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$, $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$, $\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$. If a_r, b_r, c_r, where r = 1, 2, 3, are non-negative real numbers and $\sum_{r=1}^{3}(a_r + b_r + c_r) = 3L$, show that $V \le L^3$. **[JEE 2002(Mains), 5]**

11. If $\vec{a} = \hat{i} + a\hat{j} + \hat{k}$, $\vec{b} = \hat{j} + a\hat{k}$, $\vec{c} = a\hat{i} + \hat{k}$, then find the value of 'a' for which volume of parallelopiped formed by three vectors as coterminous edges, is minimum, is

(A) $\frac{1}{\sqrt{3}}$ (B) $-\frac{1}{\sqrt{3}}$ (C) $\pm\frac{1}{\sqrt{3}}$ (D) none **[JEE 2003(Scr.), 3]**

12. (i) Find the equation of the plane passing through the points (2, 1, 0), (5, 0, 1) and (4, 1, 1).

(ii) If P is the point (2, 1, 6) then find the point Q such that PQ is perpendicular to the plane in (i) and the mid point of PQ lies on it. **[JEE 2003, 4 out of 60]**

13. If $\vec{u}, \vec{v}, \vec{w}$ are three non-coplanar unit vectors and α, β, γ are the angles between $\vec{u}$ and $\vec{v}$, $\vec{v}$ and $\vec{w}$, $\vec{w}$ and $\vec{u}$ respectively and $\vec{x}, \vec{y}, \vec{z}$ are unit vectors along the bisectors of the angles α, β, γ respectively. Prove that $[\vec{x}\times\vec{y}\ \ \vec{y}\times\vec{z}\ \ \vec{z}\times\vec{x}] = \frac{1}{16}[\vec{u}\ \vec{v}\ \vec{w}]^2 \sec^2\frac{\alpha}{2}\ \sec^2\frac{\beta}{2}\ \sec^2\frac{\gamma}{2}$.

[JEE 2003, 4 out of 60]

14. (a) If the lines $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4}$ and $\frac{x-3}{1} = \frac{y-k}{2} = \frac{z}{1}$ intersect, then k =

(A) 2/9 (B) 9/2 (C) 0 (D) – 1

(b) A unit vector in the plane of the vectors $2\hat{i}+\hat{j}+\hat{k}$, $\hat{i}-\hat{j}+\hat{k}$ and orthogonal to $5\hat{i}+2\hat{j}+6\hat{k}$

(A) $\dfrac{6\hat{i}-5\hat{k}}{\sqrt{61}}$ (B) $\dfrac{3\hat{j}-\hat{k}}{\sqrt{10}}$ (C) $\dfrac{2\hat{i}-5\hat{k}}{\sqrt{29}}$ (D) $\dfrac{2\hat{i}+\hat{j}-2\hat{k}}{3}$

(c) If $\vec{a}=\hat{i}+\hat{j}+\hat{k}$, $\vec{a}\cdot\vec{b}=1$ and $\vec{a}\times\vec{b}=\hat{j}-\hat{k}$, then $\vec{b}$ =

(A) $\hat{i}$ (B) $\hat{i}-\hat{j}+\hat{k}$ (C) $2\hat{j}-\hat{k}$ (D) $2\hat{i}$

[JEE 2004 (screening)]

15. (a) Let $\vec{a}, \vec{b}, \vec{c}, \vec{d}$ are four distinct vectors satisfying $\vec{a}\times\vec{b} = \vec{c}\times\vec{d}$ and $\vec{a}\times\vec{c}=\vec{b}\times\vec{d}$. Show that $\vec{a}\cdot\vec{b}+\vec{c}\cdot\vec{d}\neq\vec{a}\cdot\vec{c}+\vec{b}\cdot\vec{d}$.

(b) Let P be the plane passing through (1, 1, 1) and parallel to the lines L_1 and L_2 having direction ratios 1, 0, –1 and –1, 1, 0 respectively. If A, B and C are the points at which P intersects the coordinate axes, find the volume of the tetrahedron whose vertices are A, B, C and the origin.

[JEE 2004, 2 + 2out of 60]

16. (a) If $\vec{a}, \vec{b}, \vec{c}$ are three non-zero, non-coplanar vectors and $\vec{b}_1=\vec{b}-\dfrac{\vec{b}\cdot\vec{a}}{|\vec{a}|^2}\vec{a}$, $\vec{b}_2=\vec{b}+\dfrac{\vec{b}\cdot\vec{a}}{|\vec{a}|^2}\vec{a}$,

$\vec{c}_1=\vec{c}-\dfrac{\vec{c}\cdot\vec{a}}{|\vec{a}|^2}\vec{a}+\dfrac{\vec{b}\cdot\vec{c}}{|\vec{c}|^2}\vec{b}_1, \vec{c}_2=\vec{c}-\dfrac{\vec{c}\cdot\vec{a}}{|\vec{a}|^2}\vec{a}-\dfrac{\vec{b}_1\cdot\vec{c}}{|\vec{b}_1|^2}\vec{b}_1, \vec{c}_3=\vec{c}-\dfrac{\vec{c}\cdot\vec{a}}{|\vec{c}|^2}\vec{a}+\dfrac{\vec{b}\cdot\vec{c}}{|\vec{c}|^2}\vec{b}_1, \vec{c}_4=\vec{c}-\dfrac{\vec{c}\cdot\vec{a}}{|\vec{c}|^2}\vec{a}-\dfrac{\vec{b}\cdot\vec{c}}{|\vec{b}|^2}\vec{b}_1$

then the set of orthogonal vectors is

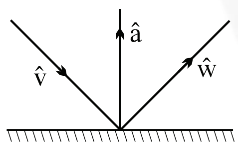
(A) $\left(\vec{a}, \vec{b}_1, \vec{c}_3\right)$ (B) $\left(\vec{a}, \vec{b}_1, \vec{c}_2\right)$ (C) $\left(\vec{a}, \vec{b}_1, \vec{c}_1\right)$ (D) $\left(\vec{a}, \vec{b}_2, \vec{c}_2\right)$

(b) A variable plane at a distance of 1 unit from the origin cuts the co-ordinate axes at A, B and C. If the centroid D (x, y, z) of triangle ABC satisfies the relation $\dfrac{1}{x^2}+\dfrac{1}{y^2}+\dfrac{1}{z^2}$ = k, then the value of k is

(A) 3 (B) 1 (C) 1/3 (D) 9

(c) Find the equation of the plane containing the line 2x – y + z – 3 = 0, 3x + y + z = 5 and at a distance of $1/\sqrt{6}$ from the point (2, 1, – 1).

(d) Incident ray is along the unit vector $\hat{v}$ and the reflected ray is along the unit vector $\hat{w}$. The normal is along unit vector $\hat{a}$ outwards. Express $\hat{w}$ in terms of $\hat{a}$ and $\hat{v}$. **[JEE 2005 (Screening), 3]**



[JEE 2005 (Mains), 2 + 4 out of 60]

17. (a) A plane passes through (1, –2, 1) and is perpendicular to two planes 2x – 2y + z = 0 and x – y + 2z = 4. The distance of the plane from the point (1, 2, 2) is

(A) 0 (B) 1 (C) $\sqrt{2}$ (D) $2\sqrt{2}$

(b) Let $\vec{a}=\hat{i}+2\hat{j}+\hat{k}$, $\vec{b}=\hat{i}-\hat{j}+\hat{k}$ and $\vec{c}=\hat{i}+\hat{j}-\hat{k}$. A vector in the plane of $\vec{a}$ and $\vec{b}$ whose projection on $\vec{c}$ has the magnitude equal to $\dfrac{1}{\sqrt{3}}$, is **[JEE 2006,3 marks each]**

(A) $4\hat{i}-\hat{j}+4\hat{k}$ (B) $3\hat{i}+\hat{j}-3\hat{k}$ (C) $2\hat{i}+\hat{j}-2\hat{k}$ (D) $4\hat{i}+\hat{j}-4\hat{k}$

(c) Let $\vec{A}$ be vector parallel to line of intersection of planes P_1 and P_2 through origin. P_1 is parallel to the vectors $2\hat{j} + 3\hat{k}$ and $4\hat{j} - 3\hat{k}$ and P_2 is parallel to $\hat{j} - \hat{k}$ and $3\hat{i} + 3\hat{j}$, then the angle between vector $\vec{A}$ and $2\hat{i} + \hat{j} - 2\hat{k}$ is

(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{6}$ (D) $\frac{\pi}{3}$

[JEE 2006, 5]

(d) **Match the following**

Column-I	**Column-II**
(i) Two rays in the first quadrant x + y = | a | and ax – y = 1 intersects each other in the interval $a \in (a_0, \infty)$, the value of a_0 is	(A) 2
(ii) Point (α, β, γ) lies on the plane x + y + z = 2. Let $\vec{a} = \alpha\hat{i} + \beta\hat{j} + \gamma\hat{k}$, $\hat{k} \times (\hat{k} \times \vec{a})$ = 0, then γ =	(B) 4/3
(iii) $\left\| \int_0^1 (1-y^2)\,dy \right\| + \left\| \int_1^0 (y^2-1)\,dy \right\|$	(C) $\left\| \int_0^1 \sqrt{1-x}\,dx \right\| + \left\| \int_{-1}^0 \sqrt{1+x}\,dx \right\|$
(iv) If sinA sinB sinC + cos A cosB = 1, then the value of sin C =	(D) 1

[JEE 2006, 6]

(e) **Match the following**

(i) $\sum_{i=1}^{\infty} \tan^{-1}\left(\frac{1}{2i^2}\right) = t$, then tan t =	(A) 0
(ii) Sides a, b, c of a triangle ABC are in A.P. and $\cos\theta_1 = \frac{a}{b+c}$, $\cos\theta_2 = \frac{b}{a+c}$, $\cos\theta_3 = \frac{c}{a+b}$, then $\tan^2\frac{\theta_1}{2} + \tan^2\frac{\theta_3}{2}$ =	(B) 1
(iii) A line is perpendicular to x + 2y + 2z = 0 and passes through (0, 1, 0). The perpendicular distance of this line from the origin is	(C) $\frac{\sqrt{5}}{3}$
	(D) 2/3

[JEE 2006, 6]

18. **(a)** The number of distinct real values of λ, for which the vectors $-\lambda^2\hat{i} + \hat{j} + \hat{k}$, $\hat{i} - \lambda^2\hat{j} + \hat{k}$ and $\hat{i} + \hat{j} - \lambda^2\hat{k}$ are coplanar, is

(A) zero (B) one (C) two (D) three

(b) Let $\vec{a}, \vec{b}, \vec{c}$ be unit vectors such that $\vec{a} + \vec{b} + \vec{c} = \vec{0}$. Which one of the following is correct?

(A) $\vec{a} \times \vec{b} = \vec{b} \times \vec{c} = \vec{c} \times \vec{a} = \vec{0}$

(B) $\vec{a} \times \vec{b} = \vec{b} \times \vec{c} = \vec{c} \times \vec{a} \neq \vec{0}$

(C) $\vec{a} \times \vec{b} = \vec{b} \times \vec{c} = \vec{a} \times \vec{c} \neq \vec{0}$

(D) $\vec{a} \times \vec{b}, \vec{b} \times \vec{c}, \vec{c} \times \vec{a}$ are mutually perpendicular.

(c) Let the vectors $\overrightarrow{PQ}$, $\overrightarrow{QR}$, $\overrightarrow{RS}$, $\overrightarrow{ST}$, $\overrightarrow{TU}$ and $\overrightarrow{UP}$ represent the sides of a regular hexagon.

Statement-1: $\overrightarrow{PQ} \times \left(\overrightarrow{RS}+\overrightarrow{ST}\right) \neq \vec{0}$

because

Statement-2: $\overrightarrow{PQ}\times\overrightarrow{RS} = \vec{0}$ and $\overrightarrow{PQ}\times\overrightarrow{ST} \neq \vec{0}$

(A) Statement-1 is true, statement-2 is true; statement-2 is a correct explanation for statement-1.
(B) Statement-1 is true, statement-2 is true; statement-2 is NOT a correct explanation for statement-1.
(C) Statement-1 is true, statement-2 is false.
(D) Statement-1 is false, statement-2 is true.

(d) Consider the planes 3x – 6y – 2z = 15 and 2x + y – 2z = 5.
Statement-1: The parametric equations of the line of intersection of the given planes are x = 3 + 14t, y = 1 + 2t, z = 15t.
because

Statement-2: The vector $14\hat{i}+2\hat{j}+15\hat{k}$ is parallel to the line of intersection of given planes.

(A) Statement-1 is true, statement-2 is true; statement-2 is a correct explanation for statement-1.
(B) Statement-1 is true, statement-2 is true; statement-2 is NOT a correct explanation for statement-1.
(C) Statement-1 is true, statement-2 is false.
(D) Statement-1 is false, statement-2 is true.

MATCH THE COLUMN :

(e) Consider the following linear equations

ax + by + cz = 0
bx + cy + az = 0
cx + ay + bz = 0

Match the conditions/ expressions in **Column I** with statements in **Column II**.

	Column I		Column II
(A)	a + b + c ≠ 0 and $a^2 + b^2 + c^2 = ab + bc + ca$	(P)	the equation represent planes meeting only at a single point.
(B)	a + b + c = 0 and $a^2 + b^2 + c^2 \neq ab + bc + ca$	(Q)	the equation represent the line x = y = z
(C)	a + b + c ≠ 0 and $a^2 + b^2 + c^2 \neq ab + bc + ca$	(R)	the equation represent identical planes
(D)	a + b + c = 0 and $a^2 + b^2 + c^2 = ab + bc + ca$	(S)	the equation represent the whole of the three dimensional space.

[JEE 2007, 3+3+3+3+6]

19. (a) The edges of a parallelopiped are of unit length and are parallel to non-coplanar unit vectors $\hat{a}, \hat{b}, \hat{c}$ such that $\hat{a}\cdot\hat{b} = \hat{b}\cdot\hat{c} = \hat{c}\cdot\hat{a} = \frac{1}{2}$. Then the volume of the parallelopiped is

(A) $\frac{1}{\sqrt{2}}$ (B) $\frac{1}{2\sqrt{2}}$ (C) $\frac{\sqrt{3}}{2}$ (D) $\frac{1}{\sqrt{3}}$

(b) Let two non-collinear unit vector $\hat{a}$ and $\hat{b}$ form an acute angle. A point P moves so that at any time t the position vector $\overrightarrow{OP}$ (where O is the origin) is given by $\hat{a}\cos t + \hat{b}\sin t$. When P is farthest from origin O, let M be the length of $\overrightarrow{OP}$ and $\hat{u}$ be the unit vector along $\overrightarrow{OP}$. Then,

(A) $\hat{u} = \dfrac{\hat{a}+\hat{b}}{|\hat{a}+\hat{b}|}$ and $M = (1+\hat{a}\cdot\hat{b})^{\frac{1}{2}}$
(B) $\hat{u} = \dfrac{\hat{a}-\hat{b}}{|\hat{a}-\hat{b}|}$ and $M = (1+\hat{a}\cdot\hat{b})^{\frac{1}{2}}$
(C) $\hat{u} = \dfrac{\hat{a}+\hat{b}}{|\hat{a}+\hat{b}|}$ and $M = (1+2\hat{a}\cdot\hat{b})^{\frac{1}{2}}$
(D) $\hat{u} = \dfrac{\hat{a}-\hat{b}}{|\hat{a}-\hat{b}|}$ and $M = (1+2\hat{a}\cdot\hat{b})^{\frac{1}{2}}$

(c) Consider three planes

$P_1 : x - y + z = 1$
$P_2 : x + y - z = -1$
$P_3 : x - 3y + 3z = 2$

Let L_1, L_2, L_3 be the lines of intersection of the planes P_2 and P_3, P_3 and P_1, and P_1 and P_2, respectively.

Statement-1 : At least two of the lines L_1, L_2 and L_3 are non-parallel.

and

Statement-2 : The three planes do not have a common point.

(A) Statement-1 is True, Statement-2 is True; statement-2 is a correct explanation for statement-1
(B) Statement-1 is True, Statement-2 is True; statement-2 is **NOT** a correct explanation for statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

Paragraph for Question Nos. (i) to (iii)

(d) Consider the lines **[JEE 2008, 3+3+3+4+4+4]**

$L_1 : \dfrac{x+1}{3} = \dfrac{y+2}{1} = \dfrac{z+1}{2}$; $L_2 : \dfrac{x-2}{1} = \dfrac{y+2}{2} = \dfrac{z-3}{3}$

(i) The unit vector perpendicular to both L_1 and L_2 is

(A) $\dfrac{-\hat{i}+7\hat{j}+7\hat{k}}{\sqrt{99}}$ (B) $\dfrac{-\hat{i}-7\hat{j}+5\hat{k}}{5\sqrt{3}}$ (C) $\dfrac{-\hat{i}+7\hat{j}+5\hat{k}}{5\sqrt{3}}$ (D) $\dfrac{7\hat{i}-7\hat{j}-\hat{k}}{\sqrt{99}}$

(ii) The shortest distance between L_1 and L_2 is

(A) 0 (B) $\dfrac{17}{\sqrt{3}}$ (C) $\dfrac{41}{5\sqrt{3}}$ (D) $\dfrac{17}{5\sqrt{3}}$

(iii) The distance of the point (1, 1, 1) from the plane passing through the point (–1, –2, –1) and whose normal is perpendicular to both the lines L_1 and L_2 is

(A) $\dfrac{2}{\sqrt{75}}$ (B) $\dfrac{7}{\sqrt{75}}$ (C) $\dfrac{13}{\sqrt{75}}$ (D) $\dfrac{23}{\sqrt{75}}$

20. If $\vec{a}, \vec{b}, \vec{c}$ and $\vec{d}$ are unit vectors such that $(\vec{a}\times\vec{b}).(\vec{c}\times\vec{d}) = 1$ and $\vec{a}\,.\,\vec{c} = \dfrac{1}{2}$, then

(A) $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar
(B) $\vec{b}, \vec{c}, \vec{d}$ are non-coplanar
(C) $\vec{b}, \vec{d}$ are non-parallel
(D) $\vec{a}, \vec{d}$ are parallel and $\vec{d}, \vec{c}$ are parallel **[JEE 2009]**

21. Let P(3, 2, 6) be a point in space and Q be a point on the line $\vec{r} = (\hat{i}-\hat{j}+2\hat{k}) + \mu(-3\hat{i}+\hat{j}+5\hat{k})$.

Then the value of μ for which the vector $\overrightarrow{PQ}$ is parallel to the plane x – 4y + 3z = 1 is

(A) $\dfrac{1}{4}$ (B) $-\dfrac{1}{4}$ (C) $\dfrac{1}{8}$ (D) $-\dfrac{1}{8}$ **[JEE 2009]**

22. A line with positive direction cosines passes through the points P(2, –1, 2) and makes equal angles with the coordinate axes. The line meets the plane 2x + y + z = 9 at point q. The length of the line segment PQ equals.

(A) 1 (B) $\sqrt{2}$ (C) $\sqrt{3}$ (D) 2 **[JEE 2009]**

23. Match the statements/expressions given in **Column I** with the values given in **Column II**.

	Column I		**Column II**
(A)	The number of solutions of the equation $xe^{\sin x} - \cos x = 0$ in the interval $\left(0, \frac{\pi}{2}\right)$	(P)	1
(B)	Value(s) of k for which the planes kx + 4y + z = 0, 4x + ky + 2z = 0 and 2x + 2y + z = 0 intersect in a straight line	(Q)	2
(C)	Value(s) of k for which $\|x-1\| + \|x-2\| + \|x+1\| + \|x+2\| = 4k$ has integer solution(s)	(R)	3
(D)	If y′ = y + 1 and y(0) = 1, then value(s) of y (ln 2)	(S)	4
		(T)	5

[JEE 2009]

24. Match the statements/expressions given in **Column I** with the values given in **Column II**.

	Column I		**Column II**
(A)	Root(s) of the equation $2\sin^2\theta + \sin^2 2\theta = 2$	(P)	$\frac{\pi}{6}$
(B)	Points of discontinuity of the function $f(x) = \left[\frac{6x}{\pi}\right]\cos\left[\frac{3x}{\pi}\right]$, where [y] denotes the largest integer less than or equal to y	(Q)	$\frac{\pi}{4}$
(C)	Volume of the parallelopiped with its edges represented by the vectors $\hat{i}+\hat{j}, \hat{i}+2\hat{j}$ and $\hat{i}+\hat{j}+\pi\hat{k}$	(R)	$\frac{\pi}{3}$
(D)	Angle between vectors $\vec{a}$ and $\vec{b}$ where $\vec{a}, \vec{b}$ and $\vec{c}$ are unit vectors satisfying $\vec{a}+\vec{b}+\sqrt{3}\,\vec{c}=\vec{0}$	(S)	$\frac{\pi}{2}$
		(T)	π

[JEE 2009]

25. Let P, Q, R and S be the points on the plane with position vectors $-2\hat{i}-\hat{j}, 4\hat{i}, 3\hat{i}+3\hat{j}$ and $-3\hat{i}+2\hat{j}$ respectively.

The quadrilateral PQRS must be a **[JEE 2010]**

(A) parallelogram, which is neither a rhombus nor a rectangle

(B) square

(C) rectangle, but not a square

(D) rhombus, but not a square

26. Equation of the plane containing the straight line $\frac{x}{2}=\frac{y}{3}=\frac{z}{4}$ and perpendicular to the plane containing the straight lines $\frac{x}{3}=\frac{y}{4}=\frac{z}{2}$ and $\frac{x}{4}=\frac{y}{2}=\frac{z}{3}$ is **[JEE 2010]**

(A) x + 2y – 2z = 0 (B) 3x + 2y – 2z = 0 (C) x – 2y + z = 0 (D) 5x + 2y – 4z = 0

27. If $\vec{a}$ and $\vec{b}$ are vectors in space given by $\vec{a}=\dfrac{\hat{i}-2\hat{j}}{\sqrt{5}}$ and $\vec{b}=\dfrac{2\hat{i}+\hat{j}+3\hat{k}}{\sqrt{14}}$, then the value of $(2\vec{a}+\vec{b}).[(\vec{a}\times\vec{b})\times(\vec{a}-2\vec{b})]$ is **[JEE 2010]**

28. If the distance between the plane Ax – 2y + z = d and the plane containing the lines $\dfrac{x-1}{2}=\dfrac{y-2}{3}=\dfrac{z-3}{4}$ and $\dfrac{x-2}{3}=\dfrac{y-3}{4}=\dfrac{z-4}{5}$ is $\sqrt{6}$, then |d| is **[JEE 2010]**

29. If the distance of the point P(1, –2, 1) from the plane x + 2y – 2z = α, where α > 0, is 5, then the foot of the perpendicular form P to the plane is **[JEE 2010]**

(A) $\left(\dfrac{8}{3},\dfrac{4}{3},-\dfrac{7}{3}\right)$ (B) $\left(\dfrac{4}{3},-\dfrac{4}{3},\dfrac{1}{3}\right)$ (C) $\left(\dfrac{1}{3},\dfrac{2}{3},\dfrac{10}{3}\right)$ (D) $\left(\dfrac{2}{3},-\dfrac{1}{3},\dfrac{5}{2}\right)$

30. Two adjacent sides of a parallelogram ABCD are given by $\overrightarrow{AB} = 2\hat{i}+10\hat{j}+11\hat{k}$ and $\overrightarrow{AD} = -\hat{i}+2\hat{j}+2\hat{k}$. The side AD is rotated by an acute angle α in the plane of the parallelogram so that AD becomes AD′, If AD′ makes a right angle with the side AB, then the cosine of the angle α is given by **[JEE 2010]**

(A) $\dfrac{8}{9}$ (B) $\dfrac{\sqrt{17}}{9}$ (C) $\dfrac{1}{9}$ (D) $\dfrac{4\sqrt{5}}{9}$

31.

Column-I	**Column-II**
(A) A line from the origin meets the lines $\dfrac{x-2}{1}=\dfrac{y-1}{-2}=\dfrac{z+1}{1}$ and $\dfrac{x-\frac{8}{3}}{2}=\dfrac{y+3}{-1}=\dfrac{z-1}{1}$ at P and Q respectively. If length PQ = d, then d^2 is	(P) –4
(B) The value of x satisfying $\tan^{-1}(x + 3) - \tan^{-1}(x - 3) = \sin^{-1}\left(\dfrac{3}{5}\right)$ are	(Q) 0
(C) Non-zero vectors $\vec{a},\vec{b}$ and $\vec{c}$ satisfy $\vec{a}.\vec{b}=0$, $(\vec{b}-\vec{a}).(\vec{b}+\vec{c})=0$ and $2\lvert\vec{b}+\vec{c}\rvert=\lvert\vec{b}-\vec{a}\rvert$. If $\vec{a}=\mu\vec{b}+4\vec{c}$, then the possible values of μ are	(R) 4
(D) Let f be the function on [$-\pi$, π] given by f(0) = 9 and f(x) = $\sin\left(\dfrac{9x}{2}\right)\Big/\sin\left(\dfrac{x}{2}\right)$ for $x \neq 0$. The value of $\dfrac{2}{\pi}\int_{-\pi}^{\pi} f(x)\,dx$ is	(S) 5
	(T) 6

[JEE 2010]

32. Let $\vec{a}=\hat{i}+\hat{j}+\hat{k}$, $\vec{b}=\hat{i}-\hat{j}+\hat{k}$ and $\vec{c}=\hat{i}-\hat{j}-\hat{k}$ be three vectors. A vector $\vec{v}$ in the plane of $\vec{a}$ and $\vec{b}$, whose projection on $\vec{c}$ is $\dfrac{1}{\sqrt{3}}$, is given by **[JEE 2011]**

(A) $\hat{i}-3\hat{j}+3\hat{k}$ (B) $-3\hat{i}-3\hat{j}+\hat{k}$ (C) $3\hat{i}-\hat{j}+3\hat{k}$ (D) $\hat{i}+3\hat{j}-3\hat{k}$

33. The vector(s) which is/are complanar with vectors $\hat{i}+\hat{j}+2\hat{k}$, and $\hat{i}+2\hat{j}+\hat{k}$, and perpendicular to the vector $\hat{i}+\hat{j}+\hat{k}$ is/are **[JEE 2011]**

(A) $\hat{j}-\hat{k}$ (B) $-\hat{i}+\hat{j}$ (C) $\hat{i}-\hat{j}$ (D) $-\hat{j}+\hat{k}$

34. Let $\vec{a}=-\hat{i}-\hat{k}, \vec{b}=-\hat{i}+\hat{j}$ and $\vec{c}=\hat{i}+2\hat{j}+3\hat{k}$ be three given vectors. If $\vec{r}$ is a vector such that $\vec{r}\times\vec{b}=\vec{c}\times\vec{b}$ and $\vec{r}.\vec{a}=0$, then the value of $\vec{r}.\vec{b}$ is **[JEE 2011]**

35. The point P is the intersection of the straight line joining the points Q(2,3,5) and R(1, –1,4) with the plane $5x-4y-z=1$. If S is the foot of the perpendicular drawn from the point T(2,1,4) to QR, then the length of the line segment PS is **[JEE 2012]**

(A) $\frac{1}{\sqrt{2}}$ (B) $\sqrt{2}$ (C) 2 (D) $2\sqrt{2}$

36. The equation of a plane passing through the line of intersection of the planes x + 2y + 3z = 2 and x – y + z = 3 and at a distance $\frac{2}{\sqrt{3}}$ from the point (3, 1, –1) is **[JEE 2012]**

(A) 5x – 11y + z = 17 (B) $\sqrt{2}x+y=3\sqrt{2}-1$ (C) x + y + z = $\sqrt{3}$ (D) $x-\sqrt{2}y=1-\sqrt{2}$

37. If $\vec{a}$ and $\vec{b}$ are vectors such that $|\vec{a}+\vec{b}|=\sqrt{29}$ and $\vec{a}\times(2\hat{i}+3\hat{j}+4\hat{k})=(2\hat{i}+3\hat{j}+4\hat{k})\times\vec{b}$, then a possible value of $(\vec{a}+\vec{b}).(-7\hat{i}+2\hat{j}+3\hat{k})$ is : **[JEE 2012]**

(A) 0 (B) 3 (C) 4 (D) 8

38. If the straight lines $\frac{x-1}{2}=\frac{y+1}{k}=\frac{z}{2}$ and $\frac{x+1}{5}=\frac{y+1}{2}=\frac{z}{k}$ are coplanar, then the plane(s) containing these two lines is(are) **[JEE 2012]**

(A) y + 2z = – 1 (B) y + z = – 1 (C) y – z = – 1 (D) y – 2z = – 1

39. If $\vec{a},\vec{b}$ and $\vec{c}$ are unit vectors satisfying $\left|\vec{a}-\vec{b}\right|^2+\left|\vec{b}-\vec{c}\right|^2+\left|\vec{c}-\vec{a}\right|^2=9$, then $\left|2\vec{a}+5\vec{b}+5\vec{c}\right|$ is **[JEE 2012]**

40. If the lines $\frac{x-2}{1}=\frac{y-3}{1}=\frac{z-4}{-k}$ and $\frac{x-1}{k}=\frac{y-4}{2}=\frac{z-5}{1}$ are coplanar, then k can have : **[IIT Mains - 2013]**

(A) exactly three values (B) any value
(C) exactly one value (D) exactly two values

41. If the vectors $\overrightarrow{AB}=3\hat{i}+4\hat{k}$ and $\overrightarrow{AC}=5\hat{i}-2\hat{j}+4\hat{k}$ are the sides of a triangle ABC, then the length of the median through A is : **[IIT Mains - 2013]**

(A) $\sqrt{45}$ (B) $\sqrt{18}$ (C) $\sqrt{72}$ (D) $\sqrt{33}$

42. Distance between two parallel planes 2x + y + 2z = 8 and 4x + 2y + 4z + 5 = 0 is : **[IIT Mains - 2013]**

(A) $\frac{9}{2}$ (B) $\frac{3}{2}$ (C) $\frac{5}{2}$ (D) $\frac{7}{2}$

43. Let $\overrightarrow{PR} = 3\hat{i}+\hat{j}-2\hat{k}$ and $\overrightarrow{SQ}=\hat{i}-3\hat{j}-4\hat{k}$ determine diagonals of a parallelogram PQRS and $\overrightarrow{PT}=\hat{i}+2\hat{j}+3\hat{k}$ be another vector. Then the volume of the parallelepiped determined by the vectors $\overrightarrow{PT},\overrightarrow{PQ}$ and $\overrightarrow{PS}$ is : **[IIT Advance - 2013]**

(A) 5 (B) 20 (C) 10 (D) 30

44. Perpendiculars are drawn from points on the line $\frac{x+2}{2}=\frac{y+1}{-1}=\frac{z}{3}$ to the plane x + y + z = 3. The feet of perpendiculars lie on the line **[IIT Advance - 2013]**

(A) $\frac{x}{5}=\frac{y-1}{8}=\frac{z-2}{-13}$ (B) $\frac{x}{2}=\frac{y-1}{3}=\frac{z-2}{-5}$ (C) $\frac{x}{4}=\frac{y-1}{3}=\frac{z-2}{-7}$ (D) $\frac{x}{2}=\frac{y-1}{-7}=\frac{z-2}{5}$

45. A line ℓ passing through the origin is perpendicular to the lines **[IIT Advance - 2013]**

$\ell_1 : (3+t)\hat{i} + (-1+2t)\hat{j} + (4+2t)\hat{k}, -\infty < t < \infty$

$\ell_2 : (3+2s)\hat{i} + (3+2s)\hat{j} + (2+s)\hat{k}, -\infty < s < \infty$

Then, the coordinate(s) of the point(s) on ℓ_2 at a distance of $\sqrt{17}$ from the point of intersection of ℓ and ℓ_1 is(are) :

(A) $\left(\frac{7}{3},\frac{7}{3},\frac{5}{3}\right)$ (B) (–1, –1, 0) (C) (1, 1, 1) (D) $\left(\frac{7}{9},\frac{7}{9},\frac{8}{9}\right)$

46. Two lines $L_1 : x = 5, \frac{y}{3-\alpha}=\frac{z}{-2}$ and $L_2 : x=\alpha, \frac{y}{-1}=\frac{z}{2-\alpha}$ are coplanar. Then α can take value(s)

(A) 1 (B) 2 (C) 3 (D) 4 **[IIT Advance - 2013]**

47.

List-I	**List-II**
(A) Volume of parallelepiped determined by vectors $\vec{a}, \vec{b}$ and $\vec{c}$ is 2. Then the volume of the parallelepiped determined by vectors $2(\vec{a}\times\vec{b}), 3(\vec{b}\times\vec{c})$ and $(\vec{c}\times\vec{a})$ is	(P) 100
(B) Volume of parallelepiped determined by vectors $\vec{a},\vec{b}$ and $\vec{c}$ is 5. Then the volume of the parallelepiped determined by vectors $3(\vec{a}+\vec{b}),(\vec{b}+\vec{c})$ and $2(\vec{c}+\vec{a})$ is	(Q) 30
(C) Area of a triangle with adjacent sides determined by vectors $\vec{a}$ and $\vec{b}$ is 20. Then the area of the triangle with adjacent sides determined by vectors $(2\vec{a}+3\vec{b})$ and $(\vec{a}-\vec{b})$ is	(R) 24
(D) Area of a parallelogram with adjacent sides determined by vectors $\vec{a}$ and $\vec{b}$ is 30. Then the area of the parallelogram with adjacent sides determined by vectors $(\vec{a}+\vec{b})$ and $\vec{a}$ is	(S) 60

[IIT Advance - 2013]

(A) $P\to4$; $Q\to2$; $R\to3$; $S\to1$ (B) $P\to2$; $Q\to3$; $R\to1$; $S\to4$
(C) $P\to3$; $Q\to4$; $R\to1$; $S\to2$ (D) $P\to1$; $Q\to4$; $R\to3$; $S\to2$

48. Consider the lines $L_1 : \frac{x-1}{2} = \frac{y}{-1} = \frac{z+3}{1}$, $L_2 : \frac{x-4}{1} = \frac{y+3}{1} = \frac{z+3}{2}$ and the planes $P_1 : 7x + y + 2z = 3$, $P_2 : 3x + 5y - 6z = 4$. Let $ax + by + cz = d$ be the equation of the plane passing through the point of intersection of lines L_1 and L_2, and perpendicular to planes P_1 and P_2. **[IIT Advance - 2013]**
Match List-I with List-II and select the correct answer using the code given below the lists :

	List-I		List-II
(P)	a =	(1)	13
(Q)	b =	(2)	–3
(R)	c =	(3)	1
(S)	d =	(4)	–2

(A) $P \to 3$; $Q \to 2$; $R \to 4$; $S \to 1$ (B) $P \to 1$; $Q \to 3$; $R \to 4$; $S \to 2$
(C) $P \to 3$; $Q \to 2$; $R \to 1$; $S \to 4$ (D) $P \to 2$; $Q \to 4$; $R \to 1$; $S \to 3$

49. Consider the set of eight vectors V = $\{a\hat{i} + b\hat{j} + c\hat{k} : a, b, c \in \{-1, 1\}\}$. Three non-coplanar vectors can be chosen from V in 2^P ways. Then p is : **[IIT Advance - 2013]**

50. The image of the line $\frac{x-1}{3} = \frac{y-3}{1} = \frac{z-4}{-5}$ in the plane $2x - y + z + 3 = 0$ is the line **[IIT Main - 2014]**
(A) $\frac{x-3}{-3} = \frac{y+5}{-1} = \frac{z-2}{5}$ (B) $\frac{x+3}{3} = \frac{y-5}{1} = \frac{z-2}{-5}$
(C) $\frac{x+3}{-3} = \frac{y-5}{-1} = \frac{z+2}{5}$ (D) $\frac{x-3}{3} = \frac{y+5}{1} = \frac{z-2}{-5}$

51. If $\left[\vec{a} \times \vec{b}\ \vec{b} \times \vec{c}\ \vec{c} \times \vec{a}\right] = \lambda \left[\vec{a}\ \vec{b}\ \vec{c}\right]^2$ then λ is equal to **[IIT Main - 2014]**
(A) 1 (B) 2 (C) 3 (D) 0

52. The angle between the lines whose direction cosines satisfy the equations $\ell + m + n = 0$ and $\ell^2 = m^2 + n^2$ is : **[IIT Main - 2014]**
(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{4}$ (D) $\frac{\pi}{6}$

53. Let $\vec{x}$, $\vec{y}$ and $\vec{z}$ be three vectors each of magnitude $\sqrt{2}$ and the angle between each pair of them is $\frac{\pi}{3}$. If $\vec{a}$ is a non-zero vector perpendicular to $\vec{x}$ and $\vec{y} \times \vec{z}$ and $\vec{b}$ is a non-zero vector perpendicular to $\vec{y}$ and $\vec{z} \times \vec{x}$, then **[IIT Advance - 2014]**
(A) $\vec{b} = (\vec{b}.\vec{z})(\vec{z} - \vec{x})$ (B) $\vec{a} = (\vec{a}.\vec{y})(\vec{y} - \vec{z})$ (C) $\vec{a}.\vec{b} = -(\vec{a}.\vec{y})(\vec{b}.\vec{z})$ (D) $\vec{a} = (\vec{a}.\vec{y})(\vec{z} - \vec{y})$

54. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three non-coplanar unit vectors such that the angle between every pair of them is $\frac{\pi}{3}$. If $\vec{a} \times \vec{b} + \vec{b} \times \vec{c} = p\vec{a} + q\vec{b} + r\vec{c}$, where p, q and r are scalars, then the value of $\frac{p^2 + 2q^2 + r^2}{q^2}$ is
[IIT Advance - 2014]

55. The distance of the point (1, 0, 2) from the point of intersection of the line $\frac{x-2}{3} = \frac{y+1}{4} = \frac{z-2}{12}$ and the plane $x - y + z = 16$, is : **[IIT Main - 2015]**
(A) 13 (B) $2\sqrt{14}$ (C) 8 (D) $3\sqrt{21}$

56. The equation of the plane containing the line $2x - 5y + z = 3$; $x + y + 4z = 5$, and parallel to the plane, $x + 3y + 6z = 1$, is : **[IIT Main - 2015]**

(A) $2x + 6y + 12z = -13$
(B) $2x + 6y + 12z = 13$
(C) $x + 3y + 6z = -7$
(D) $x + 3y + 6z = 7$

57. Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three non-zero vectors such that no two of them are collinear and $(\vec{a}\times\vec{b})\times\vec{c} = \frac{1}{3}|\vec{b}||\vec{c}|\vec{a}$. If θ is the angle between vectors $\vec{b}$ and $\vec{c}$, then a value of $\sin\theta$ is **[IIT Main - 2015]**

(A) $\frac{-2\sqrt{3}}{3}$
(B) $\frac{2\sqrt{2}}{3}$
(C) $\frac{-\sqrt{2}}{3}$
(D) $\frac{2}{3}$

58. In $\mathbb{R}^3$, let L be a straight line passing through the origin. Suppose that all the points on L are at a constant distance from the two planes $P_1 : x + 2y - z + 1 = 0$ and $P_2 : 2x - y + z - 1 = 0$. Let M be the locus of the feet of the perpendiculars drawn from the points on L to the plane P_1. Which of the following points lie(s) on M? **[IIT Advance - 2015]**

(A) $\left(0, -\frac{5}{6}, -\frac{2}{3}\right)$
(B) $\left(-\frac{1}{6}, -\frac{1}{3}, \frac{1}{6}\right)$
(C) $\left(-\frac{5}{6}, 0, \frac{1}{6}\right)$
(D) $\left(-\frac{1}{3}, 0, \frac{2}{3}\right)$

59. Let ΔPQR be a triangle. Let $\vec{a} = \overrightarrow{QR}$, $\vec{b} = \overrightarrow{RP}$ and $\vec{c} = \overrightarrow{PQ}$. If $|\vec{a}| = 12$, $|\vec{b}| = 4\sqrt{3}$ and $\vec{b}.\vec{c} = 24$, then which of the following is (are) true? **[IIT Advance - 2015]**

(A) $\frac{|\vec{c}|^2}{2} - |\vec{a}| = 12$
(B) $\frac{|\vec{c}|^2}{2} + |\vec{a}| = 30$
(C) $|\vec{a}\times\vec{b} + \vec{c}\times\vec{a}| = 48\sqrt{3}$
(D) $\vec{a}\bullet\vec{b} = -72$

60. In $\mathbb{R}^3$, consider the planes $P_1 : y = 0$ and $P_2 : x + z = 1$. Let P_3 be a plane, different from P_1 and P_2, which passes through the intersection of P_1 and P_2. If the distance of the point (0, 1, 0) from P_3 is 1 and the distance of a point (α, β, γ) from P_3 is 2, then which of the following relations is (are) true? **[IIT Advance - 2015]**

(A) $2\alpha + \beta + 2\gamma + 2 = 0$
(B) $2\alpha - \beta + 2\gamma + 4 = 0$
(C) $2\alpha + \beta - 2\gamma + 10 = 0$
(D) $2\alpha - \beta + 2\gamma - 8 = 0$

61. Suppose that $\vec{p}$, $\vec{q}$ and $\vec{r}$ are three non-coplanar vectors in $\mathbb{R}^3$. Let the components of a vector $\vec{s}$ along $\vec{p}$, $\vec{q}$ and $\vec{r}$ be 4, 3 and 5, respectively. If the components of this vector $\vec{s}$ along $\left(-\vec{p}+\vec{q}+\vec{r}\right)$, $\left(\vec{p}-\vec{q}+\vec{r}\right)$ and $\left(-\vec{p}-\vec{q}+\vec{r}\right)$ are x, y and z, respectively, then the value of $2x + y + z$ is **[IIT Advance - 2015]**

62. The distance of the point (1, –5, 9) from the plane $x - y + z = 5$ measured along the line $x = y = z$ is : **[IIT Main - 2016]**

(A) $3\sqrt{10}$
(B) $10\sqrt{3}$
(C) $\frac{10}{\sqrt{3}}$
(D) $\frac{20}{3}$

63. If the line, $\frac{x-3}{2} = \frac{y+2}{-1} = \frac{z+4}{3}$ lies in the plane, $\ell x + my - z = 9$, then $\ell^2 + m^2$ is equal to : **[IIT Main - 2016]**

(A) 26
(B) 18
(C) 5
(D) 2

64. Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three unit vectors such that $\vec{a} \times (\vec{b} \times \vec{c}) = \frac{\sqrt{3}}{2}(\vec{b} + \vec{c})$. If $\vec{b}$ is not parallel to $\vec{c}$, then the angle between $\vec{a}$ and $\vec{b}$ is : **[IIT Main - 2016]**

(A) $\frac{3\pi}{4}$ (B) $\frac{\pi}{2}$ (C) $\frac{2\pi}{3}$ (D) $\frac{5\pi}{6}$

65. Consider a pyramid OPQRS located in the first octant ($x \geq 0$, $y \geq 0$, $z \geq 0$) with O as origin, and OP and OR along the x-axis and the y-axis, respectively. The base OPQR of the pyramid is a square with OP = 3. The point S is directly above the mid-point T of diagonal OQ such that TS = 3. Then **[IIT Advance - 2016]**

(A) the acute angle between OQ and OS is $\frac{\pi}{3}$

(B) the equation of the plane containing the triangle OQS is $x - y = 0$

(C) the length of the perpendicular from P to the plane containing the triangle OQS is $\frac{3}{\sqrt{2}}$

(D) the perpendicular distance from O to the straight line containing RS is $\sqrt{\frac{15}{2}}$

66. Let P be the image of the point (3, 1, 7) with respect to the plane $x - y + z = 3$. Then the equation of the plane passing through P and containing the straight line $\frac{x}{1} = \frac{y}{2} = \frac{z}{1}$ is **[IIT Advance - 2016]**

(A) $x + y - 3z = 0$ (B) $3x + z = 0$ (C) $x - 4y + 7z = 0$ (D) $2x - y = 0$

67. Let $\hat{u} = u_1\hat{i} + u_2\hat{j} + u_3\hat{k}$ be a unit vector in $\mathbb{R}^3$ and $\hat{w} = \frac{1}{\sqrt{6}}(\hat{i} + \hat{j} + 2\hat{k})$. Given that there exists a vector $\vec{\upsilon}$ in $\mathbb{R}^3$ such that $|\hat{u} \times \vec{\upsilon}| = 1$ and $\hat{w} \cdot (\hat{u} \times \vec{\upsilon}) = 1$. Which of the following statement(s) is(are) correct?

[IIT Advance - 2016]

(A) There is exactly one choice for such $\vec{\upsilon}$
(B) There are infinitely many choices for such $\vec{\upsilon}$
(C) If $\hat{u}$ lies in the xy-plane then $|u_1| = |u_2|$
(D) If $\hat{u}$ lies in the xy-plane then $2|u_1| = |u_3|$

68. Let $\vec{a} = 2\hat{i} + \hat{j} - 2\hat{k}$ and $\vec{b} = \hat{i} + \hat{j}$. Let $\vec{c}$ be a vector such that $|\vec{c} - \vec{a}| = 3$, $|(\vec{a} \times \vec{b}) \times \vec{c}| = 3$ and the angle between $\vec{c}$ and $\vec{a} \times \vec{b}$ be 30º. Then $\vec{a} \cdot \vec{c}$ is equal to : **[JEE Mains-2017]**

(A) 5 (B) $\frac{1}{8}$ (C) $\frac{25}{8}$ (D) 2

69. If the image of the point P(1, – 2, 3) in the plane, $2x + 3y - 4z + 22 = 0$ measured parallel to the line, $\frac{x}{1} = \frac{y}{4} = \frac{z}{5}$ is Q, then PQ is equal to : **[JEE Mains-2017]**

(A) $\sqrt{42}$ (B) $6\sqrt{5}$ (C) $3\sqrt{5}$ (D) $2\sqrt{42}$

70. The distance of the point (1, 3, –7) from the plane passing through the point (1, –1, –1), having normal perpendicular to both the lines $\frac{x-1}{1}=\frac{y+2}{-2}=\frac{z-4}{3}$ and $\frac{x-2}{2}=\frac{y+1}{-1}=\frac{z+7}{-1}$, is :**[JEE Mains-2017]**

(A) $\frac{5}{\sqrt{83}}$ (B) $\frac{10}{\sqrt{74}}$ (C) $\frac{20}{\sqrt{74}}$ (D) $\frac{10}{\sqrt{83}}$

71. Let O be the origin and let PQR be an arbitary triangle. The point S is such that **[IIT Advance - 2017]**

$\overrightarrow{OP}.\overrightarrow{OQ}+\overrightarrow{OR}.\overrightarrow{OS}=\overrightarrow{OR}.\overrightarrow{OP}+\overrightarrow{OQ}.\overrightarrow{OS}=\overrightarrow{OQ}.\overrightarrow{OR}+\overrightarrow{OP}.\overrightarrow{OS}$

Then the triangle PQR has S as its

(A) incentre (B) orthocenter (C) circumcentre (D) centroid

72. The equation of the plane passing through the point (1, 1, 1) and perpendicular to the planes 2x + y – 2z = 5 and 3x – 6y – 2z = 7, is : **[IIT Advance - 2017]**

(A) 14x + 2y + 15z = 31 (B) 14x + 2y – 15z = 1

(C) – 14x + 2y + 15z = 3 (D) 14x – 2y + 15z = 27

PARAGRAPH (Q.73 TO Q.74) **[IIT Advance - 2017]**

Let O be the origin, and $\overrightarrow{OX},\overrightarrow{OY},\overrightarrow{OZ}$ be three unit vectors in the directions of the sides $\overline{QR},\overline{RP},\overline{PQ}$ respectively of a triange PQR.

73. $|\overrightarrow{OX}\times\overrightarrow{OY}|=$

(A) sin (Q + R) (B) sin (P + R) (C) sin 2R (D) sin(P + Q)

74. If the triangle PQR varies, then the minimum value of cos (P + Q) + cos (Q + R) + cos (R + P) is

(A) $\frac{3}{2}$ (B) $-\frac{3}{2}$ (C) $\frac{5}{3}$ (D) $-\frac{5}{3}$

75. Let $\vec{u}$ be a vector coplanar with the vectors $\vec{a}=2\hat{i}+3\hat{j}-\hat{k}$ and $\vec{b}=\hat{j}+\hat{k}$. If $\vec{u}$ is perpendicular to $\vec{a}$ and $\vec{u}\,.\,\vec{b}=24$, tjhen $|\vec{u}|^2$ is equal to : **[JEE Mains-2018]**

(A) 84 (B) 336 (C) 315 (D) 256

76. If L_1 is the line of intersection of the planes 2x – 2y + 3z – 2 = 0, x – y + z + 1 = 0 and L_2 is the line of intersection of the planes x + 2y – z – 3 = 0, 3x – y + 2z – 1 = 0, then the distance of the origin from the plane, containing the lines L_1 and L_2 is : **[JEE Mains-2018]**

(A) $\frac{1}{\sqrt{2}}$ (B) $\frac{1}{4\sqrt{2}}$ (C) $\frac{1}{3\sqrt{2}}$ (D) $\frac{1}{2\sqrt{2}}$

77. The length of the projection of the line segement joining the points (5, – 1, 4) and (4, – 1, 3) on the plane, x + y + z = 7 is **[JEE Mains-2018]**

(A) $\sqrt{\frac{2}{3}}$ (B) $\frac{2}{\sqrt{3}}$ (C) $\frac{2}{3}$ (D) $\frac{1}{3}$

78. Let $P_1 : 2x + y - z = 3$ and $P_2 : x + 2y + z = 2$ be two planes. Then, which of the following statement(s) is (are) **TRUE**? **[JEE Advanced-2018]**

(A) The line of intersection of P_1 and P_2 has direction ratios 1, 2, –1

(B) The line $\frac{3x-4}{9} = \frac{1-3y}{9} = \frac{z}{3}$ is perpendicular to the line of intersection of P_1 and P_2

(C) The acute angle between P_1 and P_2 is 60°

(D) If P_3 is the plane passing through the point (4, 2, –2) and perpendicular to the line of intersection of P_1 and P_2, then the distance of the point (2, 1, 1) from the plane P_3 is $\frac{2}{\sqrt{3}}$

79. Let $\vec{a}$ and $\vec{b}$ be two unit vectors such that $\vec{a}.\vec{b} = 0$. For some x, y $\in \mathbb{R}$, let $\vec{c} = x\vec{a} + y\vec{b} + (\vec{a} \times \vec{b})$. If $|\vec{c}| = 2$ and the vector $\vec{c}$ is inclined at the same angle α to $\vec{a}$ and $\vec{b}$, then the value of $8\cos^2\alpha$ is ______________.

[JEE Advanced-2018]

80. Let P be a point in the first octant, whose image Q in the plane x + y = 3 (that is, the line segment PQ is perpendicular to the plane x + y = 3 and the mid-point of PQ lies in the plane x + y = 3) lies on the z-axis. Let the distance of P from the x-axis be 5. If R is the image of P in the xy-plane, then the length of PR is ______.

[JEE Advanced-2018]

81. Consider the cube in the first octant with sides OP, OQ and OR of length 1, along the x-axis, y-axis and z-axis, respectively, where O(0, 0, 0) is the origin. Let $S\left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right)$ be the centre of the cube and T be the vertex of the cube opposite to the origin O such that S lies on the diagonal OT. If $\vec{p} = \overrightarrow{SP}$, $\vec{q} = \overrightarrow{SQ}$, $\vec{r} = \overrightarrow{SR}$ and $\vec{t} = \overrightarrow{ST}$, then the value of $\left|(\vec{p} \times \vec{q}) \times (\vec{r} \times \vec{t})\right|$ is __________. **[JEE Advanced-2018]**

82. Let L_1 and L_2 denote the lines

$\vec{r} = \hat{i} + \lambda(-\hat{i} + 2\hat{j} + 2\hat{k}), \lambda \in R$ and

$\vec{r} = \mu(2\hat{i} - \hat{j} + 2\hat{k}), \mu \in R$

respectively. If L_3 is a line which is perpendicular to both L_1 and L_2 and cuts both of them then which of the following options describe(s) L_3? **[IIT Advanced - 2019]**

(A) $\vec{r} = \frac{2}{9}(2\hat{i} - \hat{j} + 2\hat{k}) + t(2\hat{i} + 2\hat{j} - \hat{k}),\ t \in R$

(B) $\vec{r} = \frac{2}{9}(4\hat{i} + \hat{j} + \hat{k}) + t(2\hat{i} + 2\hat{j} - \hat{k}), t \in R$

(C) $\vec{r} = t(2\hat{i} + 2\hat{j} - \hat{k}), t \in R$

(D) $\vec{r} = \frac{1}{3}(2\hat{i} + \hat{k}) + t(2\hat{i} + 2\hat{j} - \hat{k}), t \in R$

83. Three lines are given by

$\vec{r} = \lambda\hat{i}, \lambda \in R$

$\vec{r} = \mu(\hat{i} + \hat{j}), \mu \in R$ and

$\vec{r} = \nu(\hat{i} + \hat{j} + \hat{k}), \nu \in R$

Let the lines cut the plane x + y + z = 1 at the points A, B and C respectively. If the area of the triangle ABC is Δ then the value of $(6\Delta)^2$ equals________ **[IIT Advanced - 2019]**

84. Three lines

$L_1: \vec{r} = \lambda\hat{i},\ \lambda \in R$

$L_2: \vec{r} = \hat{k} + \mu\hat{j},\ \mu \in R$ and

$L_3: \vec{r} = \hat{i} + \hat{j} + v\hat{k},\ v \in R$

are given. For which point(s) Q on L_2 can we find a point P on L_1 and a point R on L_3 so that P, Q and R are collinear? **[IIT Advanced - 2019]**

(A) $\hat{k} - \frac{1}{2}\hat{j}$ (B) $\hat{k}$ (C) $\hat{k} + \frac{1}{2}\hat{j}$ (D) $\hat{k} + \hat{j}$

85. Let $\vec{a} = 2\hat{i} + \hat{j} - \hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} + \hat{k}$ be two vectors. Consider a vector $\vec{c} = \alpha\vec{a} + \beta\vec{b},\ \alpha, \beta \in R$. If the projection of $\vec{c}$ on the vector $(\vec{a} + \vec{b})$ is $3\sqrt{2}$, then the minimum value of $\left(\vec{c} - (\vec{a} \times \vec{b})\right).\vec{c}$ equals___.

[IIT Advanced - 2019]

86. Let L_1 and L_2 be the following straight lines.

$$L_1: \frac{x-1}{1} = \frac{y}{-1} = \frac{z-1}{3} \text{ and } L_2: \frac{x-1}{-3} = \frac{y}{-1} = \frac{z-1}{1}$$

Suppose the straight line

$$L: \frac{x-a}{l} = \frac{y-1}{m} = \frac{z-\gamma}{-2}$$

lies in the plane containing L_1 and L_2, and passes through the point of intersection of L_1 and L_2. If the line L bisects the acute angle between the lines L_1 and L_2, then which of the following statements is/are TRUE ? **[IIT Advanced - 2020]**

(A) $\alpha - \gamma = 3$ (B) $l + m = 2$ (C) $\alpha - \gamma = 1$ (D) $l + m = 0$

87. Let $\alpha, \beta, \gamma, \delta$ be real numbers such that $\alpha^2 + \beta^2 + \gamma^2 \neq 0$ and $\alpha + \gamma = 1$. Suppose the point (3, 2, –1) is the mirror image of the point (1, 0, –1) with respect to the plane $\alpha x + \beta y + \gamma z = \delta$. Then which of the following statements is/are TRUE? **[IIT Advanced - 2020]**

(A) $\alpha + \beta = 2$ (B) $\delta - \gamma = 3$ (C) $\delta + \beta = 4$ (D) $\alpha + \beta + \gamma = \delta$

88. In a triangle PQR, let $\vec{a} = \overrightarrow{QR}, \vec{b} = \overrightarrow{RP}$ and $\vec{c} = \overrightarrow{PQ}$. If $|\vec{a}| = 3$, $|\vec{b}| = 4$ and $\frac{\vec{a}.(\vec{c} - \vec{b})}{\vec{c}.(\vec{a} - \vec{b})} = \frac{|\vec{a}|}{|\vec{a}| + |\vec{b}|}$, then the value of $|\vec{a} \times \vec{b}|^2$ is ____________. **[IIT Advanced - 2020]**

89. Let O be the origin and $\overrightarrow{OA} = 2\hat{i}+2\hat{j}+\hat{k}$, $\overrightarrow{OB} = \hat{i}-2\hat{j}+2\hat{k}$ and $\overrightarrow{OC} = \frac{1}{2}\left(\overrightarrow{OB}-\lambda\overrightarrow{OA}\right)$ for some $\lambda > 0$. If $|\overrightarrow{OB}\times\overrightarrow{OC}| = \frac{9}{2}$, then which of the following statements is (are) **True** ? **[IIT Advanced - 2021]**

(A) Projection of $\overrightarrow{OC}$ on $\overrightarrow{OA}$ is $-\frac{3}{2}$

(B) Area of the triangle OAB $\frac{9}{2}$

(C) Area of the triangle ABC is $\frac{9}{2}$

(D) The acute angle between the diagonals of the parallelogram with adjacent sides $\overrightarrow{OA}$ and $\overrightarrow{OC}$ is $\frac{\pi}{3}$

Question Stem (Q.90 to Q.91) **[IIT Advanced - 2021]**

Let α, β and γ be real numbers such that the system of linear equations

$$x + 2y + 3z = \alpha$$
$$4x + 5y + 6z = \beta$$
$$7x + 8y + 9z = \gamma - 1$$

is consistent. Let |M| represent the determinant of the matrix

$$M = \begin{bmatrix} \alpha & 2 & \gamma \\ \beta & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$$

Let P be the plane containing all those (α, β, γ) for which the above system of linear equations is consistent, and D be the **square** of the distance of the point (0, 1, 0) from the plane P.

90. The value of |M| is________.

91. The value of D is______.

92. Let $\vec{u}, \vec{v}$ and $\vec{w}$ be vectors in three-dimensional space, where $\vec{u}$ and $\vec{v}$ are unit vectors which are not perpendicular to each other and

$$\vec{u}\cdot\vec{w} = 1, \quad \vec{v}\cdot\vec{w} = 1, \quad \vec{w}\cdot\vec{w} = 4$$

If the volume of the parallelopiped, whose adjacent sides are represented by the vectors $\vec{u}$, $\vec{v}$ and $\vec{w}$, is $\sqrt{2}$, then the value of $|3\vec{u}+5\vec{v}|$ is_________. **[IIT Advanced - 2021]**

ANSWER KEY
EXERCISE–I

1. $x = 2, y = -1$ **2.** **(b)** externally in the ratio 1 : 3

4. **(i)** parallel **(ii)** the lines intersect at the point p.v. $-2\hat{i} + 2\hat{j}$ **(iii)** lines are skew

5. $xx_1 + yy_1 = a^2$ **6.** 28

8. $x = 2, y = -2, z = -2$ **10.** **(a)** $\frac{-1}{2}i - \frac{1}{2}j + \frac{1}{\sqrt{2}}k$ **12.** 34

14. $-\hat{i}+2\hat{j}+5\hat{k}$ **15.** $\frac{\sqrt{11}}{3}$ **16. (b)** $\frac{\sqrt{3}}{2}$

17. (a) $\pm 3(\hat{i}-2\hat{j}-2\hat{k})$, (b) $y = 3$ or $y = -1$

18. 12 **19.** $\frac{5a^2}{12\sqrt{3}}$ sq. units **20.** $p = \frac{q(q^2-3)}{4}$; decreasing in $q \in (-1, 1)$, $q \neq 0$

EXERCISE–II

1. $\frac{4}{\sqrt{2}}\hat{i} - \frac{1}{\sqrt{2}}\hat{j} - \frac{1}{\sqrt{2}}\hat{k}$ **2.** $\pm\frac{1}{3\sqrt{3}}(\hat{i}+5\hat{j}-\hat{k})$ **4.** NO, NO

5. **(i)** $\frac{6}{7}\sqrt{14}$ **(ii)** 6 **(iii)** $\frac{3}{5}\sqrt{10}$ **(iv)** $\sqrt{6}$ **6.** $\frac{11}{\sqrt{170}}$ **7** $2\sqrt{17}$

8. p.v. of $\vec{R} = r = 3i + 3k$ **11.** 110

13. $\alpha = n\pi + \frac{(-1)^n\pi}{2}, n \in I \,\&\, \beta = 1$ **14.** $\alpha = 2/3$; if $\alpha = 0$ then vector product is $-60\left(2\hat{i}+\hat{k}\right)$

15. $9\left(-\hat{j}+\hat{k}\right)$ **19.** $F = 2\vec{a}_1 + 5\vec{a}_2 + 3\vec{a}_3$ **20.** $\alpha = -1, \beta = -2, \gamma = 3$

21. **(b)** $\left\{\vec{p} = \frac{[\vec{a}\,\vec{b}\,\vec{c}]}{(\vec{a}.\vec{c})(\vec{a}.\vec{b})}\left(\vec{a}+\vec{c}\times\vec{b}\right) + \frac{(\vec{b}.\vec{c})\vec{b}}{(\vec{a}.\vec{b})} - \frac{(\vec{b}.\vec{b})\vec{c}}{(\vec{a}.\vec{b})}\right\}$

22. $\vec{x} = \frac{\vec{a}+(\vec{c}.\vec{a})\vec{c}+\vec{b}\times\vec{c}}{1+\vec{c}^2}$, $y = \frac{\vec{b}+(\vec{c}.\vec{b})\vec{c}+\vec{a}\times\vec{c}}{1+\vec{c}^2}$

EXERCISE–III

1. $\theta = 90^0$ **3.** $y + 2z = 4$ **6.** $\frac{x-1}{2} = \frac{y-2}{2} = \frac{z-3}{-3}$

7. $\frac{x}{1} = \frac{y}{2} = \frac{z}{-1}$ or $\frac{x}{-1} = \frac{y}{1} = \frac{z}{-2}$ **8.** $\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2} = \frac{1}{p^2}$ **9.** $\frac{17}{2}$

10. $\frac{x-1}{6} = \frac{y+2}{13} = \frac{z+3}{17}$ **11.** $\frac{x-4}{3} = \frac{y+14}{10} = \frac{z-4}{4}$

12. (a) $\frac{3}{2}$; (b) $\frac{2x}{3}+\frac{2y}{3}-\frac{z}{3} = 1$; (c) $\left(0, \frac{3}{2}, 0\right)$; (d) $x = 2t + 2$; $y = 2t + 1$ and $z = -t + 3$

13. (1, –2, – 4)

14. $\frac{x}{2}+\frac{y}{3}+\frac{z}{-5}=1$, Area = $\frac{19}{2}$ sq. units

15. $\frac{x-2}{11}=\frac{y+1}{-10}=\frac{z-3}{2}$

16. 2x + 3y + z + 4 = 0

17. p = 3, (2, 1, –3) ; x + y + z = 0

18. $\frac{x-7}{22}=\frac{y-2}{5}=\frac{z+1}{-4}$

19. (a) $\sqrt{3}$; (b) x + y – 2z + 1 = 0; (c) x – 2y + z = 5; (d) $\pi/3$; (e) 4

20. 2/3

21. $\frac{x-7}{3}=\frac{y-2}{6}=\frac{z-4}{2}$; $\frac{x-7}{2}=\frac{y-2}{-3}=\frac{z-4}{6}$

22. (9, 13, 15) ; 14 ; 9x – 4y – z = 14

23. $\frac{x-4}{9}=\frac{y+1}{-1}=\frac{z-7}{-3}$

24. x – 2y + 2z – 1 = 0; 2 units

25. 9/2

EXERCISE–IV

1. (a) B (b) A (c) A, C

(a) $\cancel{e} = \sqrt{3}\,\vec{a}+2\vec{b}$ (b) $\left|\left[\vec{a}\ \vec{b}\ \vec{c}\right]\right|$

3. (a) (i) B (ii) A (iii) A

4. (i) $\pm\ \hat{i}$; (ii) $\frac{\vec{b}}{\vec{b}^2}+\frac{\vec{a}\times\vec{b}}{(\vec{a}\times\vec{b})^2}$; (iii) $\frac{2\pi}{3}$;(iv) $|\vec{M}|=\sqrt{7}$

5. (a) $\frac{1}{2}(5\hat{i}-\hat{j}-7\hat{k}),\frac{1}{2}(-\hat{i}+7\hat{j}-5\hat{k});\frac{1}{2}\sqrt{1274}$ sq. units (b) $\lambda = 0$, $\lambda = -2 \pm \sqrt{29}$

6. (a) $\vec{r}=-13\hat{i}+11\hat{j}+7\hat{k}$; (b) $\frac{5}{7}\hat{i}+\frac{17}{7}\hat{j}$

7. (a) B (b) C

9. (a) B ; (b) C

11. D

12. (i) x + y – 2z = 3 ; (ii) (6, 5, –2)

14. (a) B, (b) B, (c) A

15. (b) 9/2 cubic units

16. (a) B, (b) D ; (c) 2x – y + z – 3 = 0 and 62x + 29y + 19z – 105 = 0, (d) $\hat{w} = \hat{v} - 2(\hat{a}\cdot\hat{v})\,\hat{a}$

17. **(a)** D; **(b)** A; **(c)** B ; **(d)** (i) D, (ii) A, (iii) B, C, (iv) D; **(e)** (i) B, (ii) D, (iii) C

18. **(a)** C; **(b)** B; **(c)** C; **(d)** D; **(e)** (A) R; (B) Q; (C) P; (D) S

19. **(a)** A; **(b)** A; **(c)** D; **(d)** (i) B; (ii) D; (iii) C

20. C **21.** A **22.** C

23. (A) $\to$ P ; (B) $\to$ Q, S ; (C) $\to$ Q, R, S, T ; (D) $\to$ R

24. (A) $\to$ Q, S, ; (B) $\to$ P, R, S, T ; (C) $\to$ T ; (D) $\to$ R

25. A **26.** C **27.** 5 **28.** 6 **29.** A **30.** B

31. A – T ; B – P, R ; C – Q ; D – R **32.** C **33.** AD **34.** 9

35. A **36.** A **37.** C **38.** B, C **39.** 3 **40.** D **41.** D

42. D **43.** C **44.** D **45.** B, D **46.** A, D **47.** C **48.** A

49. 5 **50.** B **51.** A **52.** B **53.** A, B, C **54.** 4 **55.** A

56. D **57.** B **58.** A, B **59.** A, C, D **60.** B, D **61.** BONUS

62. B **63.** D **64.** D **65.** B, C, D **66.** C **67.** B,C

68. D **69.** D **70.** D **71.** B **72.** A **73.** D **74.** B

75. B **76.** C **77.** A **78.** C,D **79.** 3 **80.** 8 **81.** 0.5

82. A,B,D **83.** 0.75 **84.** A,C **85.** 18.00 **86.** AB **87.** ABC **88.** 108.00

89. A,B,C **90.** 1.00 **91.** 1.50 **92.** 7.00