

Q find  $z$  of  $4z^2 + 2z + 1 = 0$

$$(2z)^2 + 2z + 1 = 0$$

$$x^2 + x + 1 = 0 \begin{matrix} \nearrow \omega \\ \searrow \omega^2 \end{matrix}$$

$$2z = \omega \text{ or } 2z = \omega^2$$

$$z = \frac{\omega}{2} \text{ or } z = \frac{\omega^2}{2}$$

Q find  $z$  of  $z^4 + z^3 + 2z^2 + z + 1 = 0$

$$\text{find } |z|$$

$$(z^4 + z^3 + z^2) + (z^2 + z + 1) = 0$$

$$z^2(z^2 + z + 1) + (z^2 + z + 1) = 0$$

$$(z^2 + z + 1)(z^2 + 1) = 0$$

$$z^2 + z + 1 = 0 \text{ or } z^2 = -1$$

$$\downarrow$$

$$\omega, \omega^2$$

$$|z| = |\omega| = 1$$

$$|\omega^2| = 1$$

$$z = i, -i$$

$$|z| = |i| = |-i|$$

$$= 1$$

$$|z| = 1$$

Q  $x^2 + x + 1 = 0$  has Roots

$$\alpha \text{ \& } \beta \text{ find } \alpha^{19} + \beta^{19} = ?$$

$$x^2 + x + 1 = 0 \begin{matrix} \nearrow \omega \\ \searrow \omega^2 \end{matrix}$$

$$\alpha = \omega, \beta = \omega^2$$

$$\alpha^{19} + \beta^{19} = \omega^{19} + (\omega^2)^{19}$$

$$= \omega^{19} + \omega^{38}$$

$$= (\omega^3)^6 \cdot \omega + (\omega^3)^{12} \cdot \omega^2$$

$$= \omega + \omega^2$$

$$= -1$$

$$\textcircled{Q} \frac{a+b\omega+c\omega^2}{a\omega+b\omega^2+c} + \frac{a\omega+b\omega^2+c}{a+b\omega+c\omega^2}$$

$$\frac{1}{\omega} + \omega = \omega + \omega^2 = -1$$

$$-1 = G\pi + i8m\pi$$

$$-1 = e^{i\pi}$$

$$Q(x-1)^3 + 3 = 0$$

## Find Roots 2

$$(-1)^3 = -8 \quad (-1)^{1/3}$$

$$x(-1 - (-8))^{1/3}$$

$$= \left\{ |8|^{1/3} e^{i \frac{(\pi + 2K\pi)}{3}} \right\}_{K=0,1,2}$$

$$x-1 = 2 e^{i\frac{\pi}{3}} \cdot e^{i\frac{\pi}{2}} \cdot e^{i\frac{5\pi}{3}}$$

$$x = 1 + 2 \cdot e^{i\frac{\pi}{3}} = \underbrace{1 + 2 \cdot e^{in}}_{=-1} \quad 1 + 2 \cdot e^{i\frac{5\pi}{3}}$$

Q If  $\alpha_1, \alpha_2, \dots, \alpha_{n-1}$  are  $n^{\text{th}}$  Roots of unity

(other than 1) find

1)  $(1-\alpha_1)(1-\alpha_2) \dots (1-\alpha_{n-1})$

2)  $(\omega-\alpha_1)(\omega-\alpha_2) \dots (\omega-\alpha_{n-1})$   
 $1, \alpha, \alpha^2, \alpha^3, \dots, \alpha^{n-1}$

$\chi = 1^{1/n} \rightarrow 1, \alpha_1, \alpha_2, \alpha_3, \dots, \alpha_{n-1}$

$\chi^n = 1 \Rightarrow \chi^n - 1 = 0$

$\chi^n - 1 = (\chi - 1)(\chi - \alpha_1)(\chi - \alpha_2) \dots (\chi - \alpha_{n-1})$

$\frac{\chi^n - 1}{\chi - 1} = (\chi - \alpha_1)(\chi - \alpha_2) \dots (\chi - \alpha_{n-1})$

$\Rightarrow 1 + \chi + \chi^2 + \dots + \chi^{n-1} = (\chi - \alpha_1)(\chi - \alpha_2) \dots (\chi - \alpha_{n-1})$   
 Put  $\chi = 1$

①  $n = (1-\alpha_1)(1-\alpha_2) \dots (1-\alpha_{n-1})$

② Put  $\chi = \omega$

$1 + \omega + \omega^2 + \omega^3 + \dots + \omega^{n-1} = (\omega - \alpha_1)(\omega - \alpha_2) \dots (\omega - \alpha_{n-1})$   
 Terms of HP  $\rightarrow \frac{\omega^n - 1}{\omega - 1} = \begin{cases} \frac{1-1}{\omega-1} = 0 & n = 3K \\ \frac{\omega-1}{\omega-1} = 1 & n = 3K+1 \\ \frac{\omega^2-1}{\omega-1} = \omega+1 = -\omega^2 & n = 3K+2 \end{cases}$   
 $\omega^{3K+1} = \omega$   
 $\omega^{3K+2} = \omega^2$

$$1) Z = e^{j\frac{2\pi}{2n+1}} + i\sin\frac{2\pi}{2n+1} \quad n = +ve \text{ int}$$

find Eq whose roots are.

$$\alpha = Z + Z^3 + \dots + Z^{2n+1} \quad \beta = Z^2 + Z^4 + \dots + Z^{2n}$$

← n terms →

DMT use already

$$1) Z = \left( e^{j\frac{2\pi}{2n+1}} + i\sin\frac{2\pi}{2n+1} \right)^{\frac{1}{2n+1}} \text{ DMT}$$

$$\Rightarrow Z^{2n+1} = 1 \Rightarrow Z^{2n} = \frac{1}{Z}$$

$$(2) \alpha = \frac{Z(Z^{2n}-1)}{(Z^2-1)} = \frac{Z(\frac{1}{Z}-1)}{(Z^2-1)} = \frac{Z(Z-1)}{Z(Z^2-1)}$$

$$\beta = \frac{Z^2((Z^2)^n-1)}{(Z^2-1)} = \frac{Z^2(\frac{1}{Z}-1)}{(Z^2-1)} = -\frac{Z}{Z+1}$$

$$(3) \alpha + \beta = -\frac{1}{Z+1} - \frac{Z}{Z+1} = -\frac{(Z+1)}{(Z+1)} = -1$$

Answer → Eq in Z with  $6^{20}h$  —

$$\alpha \cdot \beta = -\frac{1}{(Z+1)} \times -\frac{Z}{(Z+1)}$$

$$\frac{2000s}{100 + 150} = \frac{Z}{Z^2 + 2Z + 1}$$

$$\frac{2000}{30 + 40} = \frac{1}{Z + \frac{1}{Z} + 2}$$

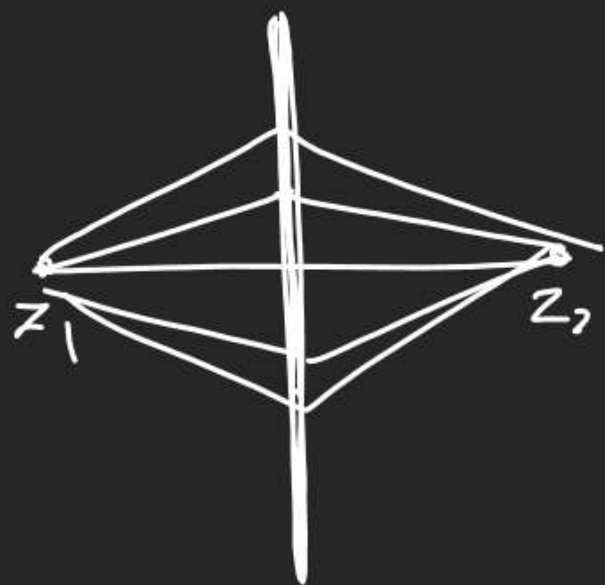
$$= \frac{Z}{(Z+1)^2} = \frac{Z}{Z^2 + 2Z + 1} = \frac{1}{Z + \frac{1}{Z} + 2}$$

$$\alpha \cdot \beta = \frac{1}{2(1+6\theta)} = \frac{1}{4(6^{20}h)}$$

$$Z^2 + Z + \frac{1}{4(6^{20}h)} = 0$$

$$\frac{Z - 6\theta + i\sin\theta}{\frac{1}{Z} - 6\theta - i\sin\theta} = \frac{Z - 6\theta + i\sin\theta}{Z + \frac{1}{Z} = 2(6\theta)}$$

①  $|z - z_1| = |z - z_2|$



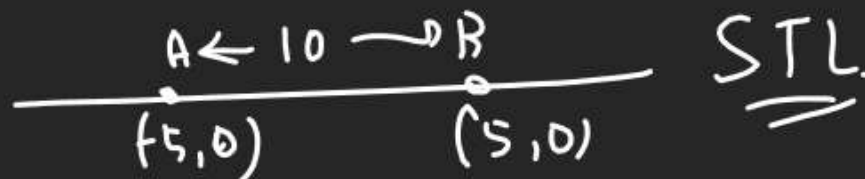
$|PA - PB| < AB$

hyperbola  $||z - z_1| - |z - z_2|| < |z_1 - z_2|$

$|z - z_1|^2 + |z - z_2|^2 = |z_1 - z_2|^2 \rightarrow \text{Circle} \rightarrow \text{Pythagorean}$

Q  $|z + 5| + |z - 5| = 10$  Rep?

$PA + PB = AB$



Q  $|z + 5| + |z - 5| = 8$

$PA + PB = 10$  &  $AB = 8$

$PA + PB > AB \rightarrow \text{Ellipse}$

Q  $|3z - 2| + |3z + 2| = 4$  How?

$|z - \frac{2}{3}| + |z + \frac{2}{3}| = \frac{4}{3} \Rightarrow PA + PB = AB$

(2) A)  $PA + PB = AB$



St.L (AB)

$|z - z_1| + |z - z_2| = |z_1 - z_2|$

B)  $PA - PB = AB$



STL inside  $\Rightarrow |z - z_1| - |z - z_2| = |z_1 - z_2|$

C)  $PB - PA = AB$



STL  $\Rightarrow |z - z_2| - |z - z_1| = |z_1 - z_2|$

(D)  $PA + PB > AB$

$PA + PB < AB$



No Locus



Ellipse  $|z - z_1| + |z - z_2| > |z_1 - z_2|$