

$$\begin{aligned}
 & \text{Given } \sin^{-1} \frac{2}{3} + \sin^{-1} \frac{6}{7} = \sin^{-1} \sin(\theta_1 + \theta_2) \text{ where } \theta_1, \theta_2 \in (0, \frac{\pi}{2}) \\
 & \text{(a) } \sin^{-1} \left( \frac{2\sqrt{13} + 6\sqrt{5}}{21} \right) \quad \text{(b) } \pi - \sin^{-1} \left( \frac{2\sqrt{13} + 6\sqrt{5}}{21} \right) \\
 & \text{(c) } \cos^{-1} \left( \frac{\sqrt{65} - 12}{21} \right) \quad \text{(d) } 2\pi - \cos^{-1} \left( \frac{\sqrt{65} - 12}{21} \right)
 \end{aligned}$$

$$\begin{aligned}
 \theta_1 + \theta_2 &= \cos^{-1} (\cos(\theta_1 + \theta_2)) = \cos^{-1} \left( \frac{\sqrt{5}}{3} \frac{\sqrt{13}}{7} - \frac{2}{3} \frac{6}{7} \right) \\
 \sin(\theta_1 + \theta_2) &\leq \cos(\theta_1 + \theta_2) = \frac{\sqrt{65} - 12}{21} < 0
 \end{aligned}$$

$$\begin{aligned}
 & \theta_1 \in (0, \pi/2) \quad \theta_2 \in (0, \frac{\pi}{2}) \quad \cos^{-1} \cos(\theta_1 - \theta_2) = \cos^{-1} \left( \frac{3 + \sqrt{56}}{12} \right) \\
 & \text{Given: } \theta_1 - \theta_2 \in (-\frac{\pi}{2}, \frac{\pi}{2}) \quad \cos^{-1} \frac{3}{4} = -\cos^{-1} \frac{1}{3} \quad \theta_1 - \theta_2 \in (-\frac{\pi}{2}, 0) \\
 & \text{(a) } \cos^{-1} \left( \frac{3 + \sqrt{56}}{12} \right) = -(0, -\theta_2) \\
 & \quad \downarrow y = -x \\
 & \quad \text{Graph: A Cartesian coordinate system with x-axis and y-axis. The origin is labeled 'O'. The x-axis has tick marks at -\pi, -\frac{\pi}{2}, 0, \frac{\pi}{2}, \pi. A line passes through the second and fourth quadrants, starting from the negative x-axis and ending at the positive x-axis. It intersects the x-axis at (\frac{3}{4}, 0) and the negative y-axis at (0, -\frac{1}{3}). The angle between the positive x-axis and this line is labeled } \theta_1 - \theta_2. \\
 & \quad \sin^{-1} \left( \frac{\sqrt{7} - 3\sqrt{8}}{12} \right) \\
 & \quad \cancel{(\text{c})} \\
 & \quad \cancel{\text{Graph: A Cartesian coordinate system with x-axis and y-axis. The origin is labeled 'O'. The x-axis has tick marks at -\pi, -\frac{\pi}{2}, 0, \frac{\pi}{2}, \pi. A line passes through the second and fourth quadrants, starting from the negative x-axis and ending at the positive x-axis. It intersects the x-axis at (\frac{3}{4}, 0) and the negative y-axis at (0, -\frac{1}{3}). The angle between the positive x-axis and this line is labeled } \theta_1 - \theta_2.} \\
 & \quad \text{(d) } \pi - \sin^{-1} \left( \frac{\sqrt{7} - 3\sqrt{8}}{12} \right) \\
 & \quad \sin^{-1} \sin(\theta_1 - \theta_2) = \sin^{-1} \left( \frac{\sqrt{7}}{4} \frac{1}{3} - \frac{\sqrt{8}}{3} \frac{3}{4} \right) \\
 & \quad = (\theta_1 - \theta_2)
 \end{aligned}$$

$$\sin^{-1}x + \sin^{-1}y = \begin{cases} \sin^{-1}(x\sqrt{1-y^2} + y\sqrt{1-x^2}) & 0 < x < 1, 0 < y < 1, \\ & x^2 + y^2 < 1 \\ \pi - \sin^{-1}(x\sqrt{1-y^2} + y\sqrt{1-x^2}) & 0 < x < 1, 0 < y < 1, \\ & x^2 + y^2 > 1 \end{cases}$$

$\theta_1 \in (0, \frac{\pi}{2}) \quad \theta_2 \in (0, \frac{\pi}{2})$

$$\sin^{-1}(x\sqrt{1-y^2} - y\sqrt{1-x^2}) + \cos^{-1}x + \cos^{-1}y$$

$0 < x < 1, 0 < y < 1$

$$\cos^{-1}x - \cos^{-1}y$$


$0 < x < 1, 0 < y < 1$

$$\sin(\theta_1 + \theta_2) = \sin(x\sqrt{1-y^2} + y\sqrt{1-x^2})$$

$x^2 + y^2 < 1 \iff (1-x^2)(1-y^2) > x^2y^2$

$\theta_1 + \theta_2 \in (0, \frac{\pi}{2})$

$$\cos^{-1}x + \cos^{-1}y = \cos^{-1}\left(xy - \sqrt{1-x^2}\sqrt{1-y^2}\right) \quad 0 < x < 1, 0 < y < 1$$

$$\cos^{-1}x - \cos^{-1}y = \begin{cases} \cos^{-1}\left(xy + \sqrt{1-x^2}\sqrt{1-y^2}\right) & 0 < x < 1, 0 < y < 1, \\ & x \leq y \\ -\cos^{-1}\left(xy + \sqrt{1-x^2}\sqrt{1-y^2}\right) & 0 < x < 1, 0 < y < 1, \\ & x > y \end{cases}$$

1.  $f : \mathbb{R} \rightarrow \mathbb{R}$ ,  $f(x) = \frac{x(x^4+1)(x+1)+x^4+2}{x^2+x+1}$

*continuous*  $\leftarrow$  *cont.  $x > 0$*

*as  $x \rightarrow \pm\infty, y \rightarrow \infty$*   $\frac{x^6(1+\frac{1}{x^4}+\frac{1}{x^2})}{x^2(1+\frac{1}{x}+\frac{1}{x^2})}$

2. Find range of  $f(x) = \frac{(1+x+x^2)(1+x^4)}{x^3}$  for  $x > 0$

$y=6$  at  $x=1$

$$\left(\frac{1}{x} + x + 1\right) \left(\frac{1}{x^2} + x^2\right) \geq 6$$

$$\geq 3 \quad \geq 2$$

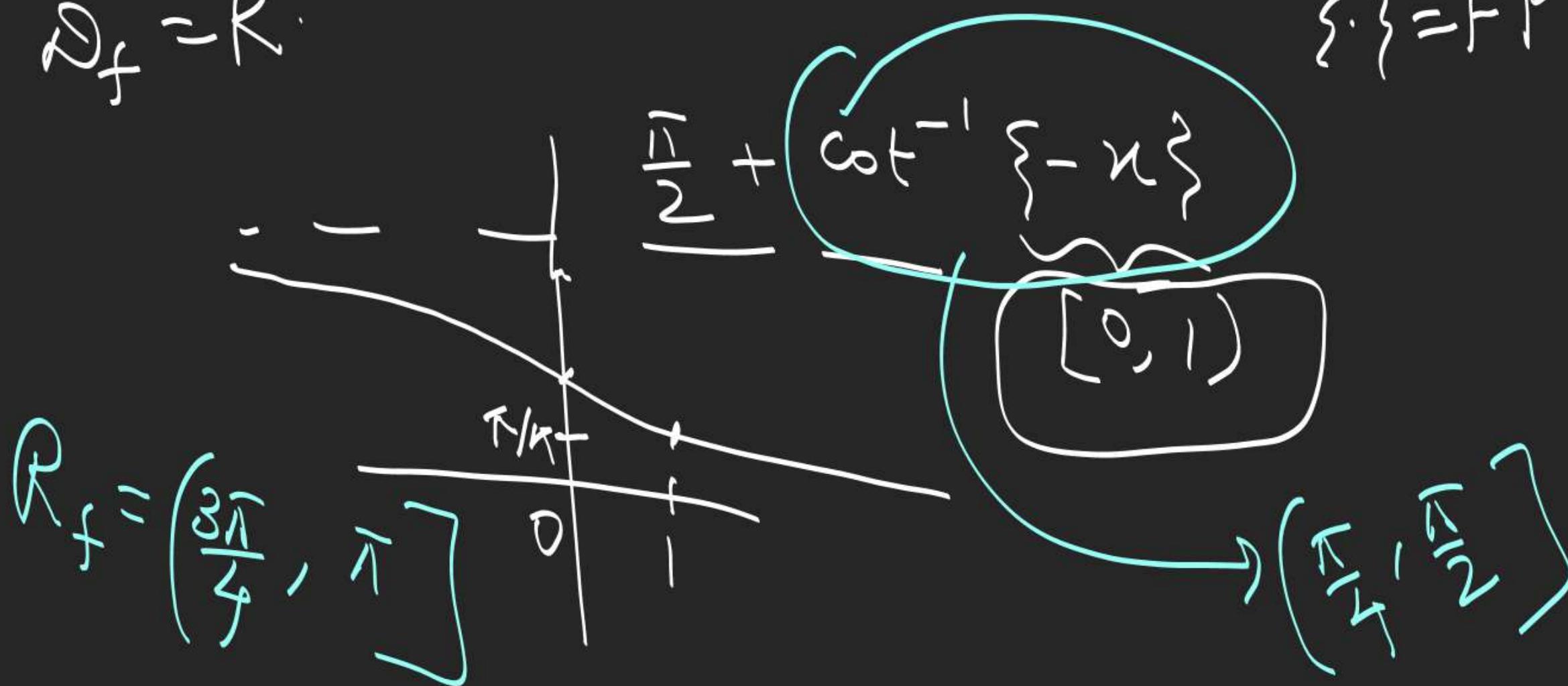
$M-1$ , into  
 $R_f = [6, \infty)$

3. Find the range of

$$f(x) = \cot^{-1}\{-x\} + \sin^{-1}\{x\} + \cos^{-1}\{x\}$$

$$D_f = \mathbb{R}$$

$$\{\cdot\} = \overline{F}PF$$

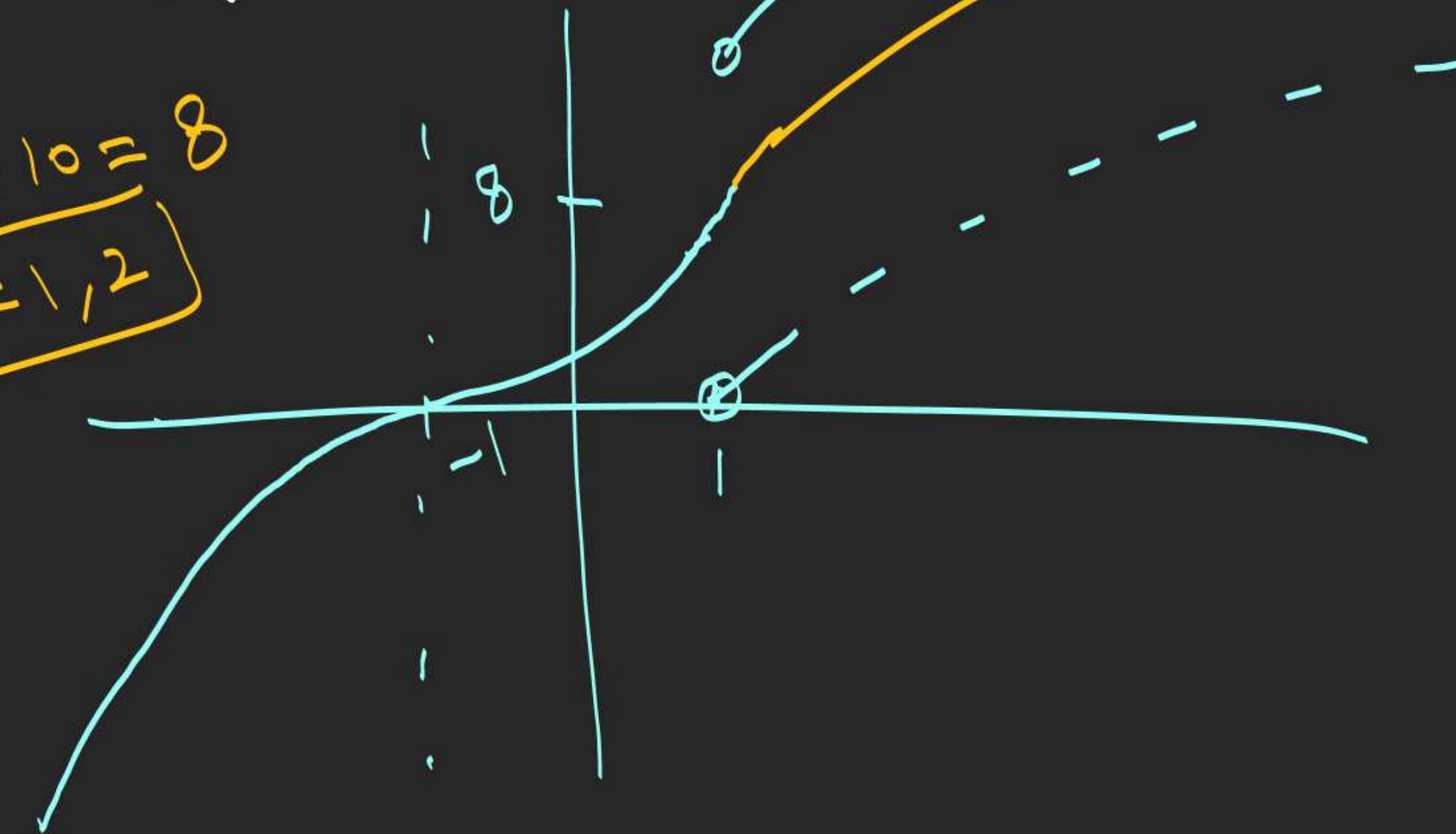


4. Let  $f : \mathbb{R} \rightarrow \mathbb{R}$ ,  $f(x) = \begin{cases} (x+1)^3, & x \leq 1 \\ \ln x + (b^2 - 3b + 10), & x > 1 \end{cases}$

is invertible, find  $b$ .

$$\boxed{b^2 - 3b + 10 = 8}$$

$$\boxed{b = 1, 2}$$



5.  $f: (-\infty, 2] \rightarrow (-\infty, 4]$ ,  $f(x) = x(4-x)$ , find  $f^{-1}(x)$

$$f^{-1}: (-\infty, 4] \rightarrow \underbrace{(-\infty, 2]}$$

$$f^{-1}(x)(4-f^{-1}(x)) = x$$

$$t^2 - 4t + x = 0 \Rightarrow f^{-1}(x) = 2 \pm \sqrt{4-x}$$

$$\boxed{f^{-1}(x) = 2 - \sqrt{4-x}}$$

6.  $f: \mathbb{R} \rightarrow \left[0, \frac{\pi}{2}\right]$ ,  $f(x) = \tan^{-1}(3x^2 + bx + c)$  is surjective, then

(a)  $b^2 = 3c$     (b)  $b^2 = 4c$     (c)  $b^2 = 8c$     (d)  ~~$b^2 = 12c$~~

