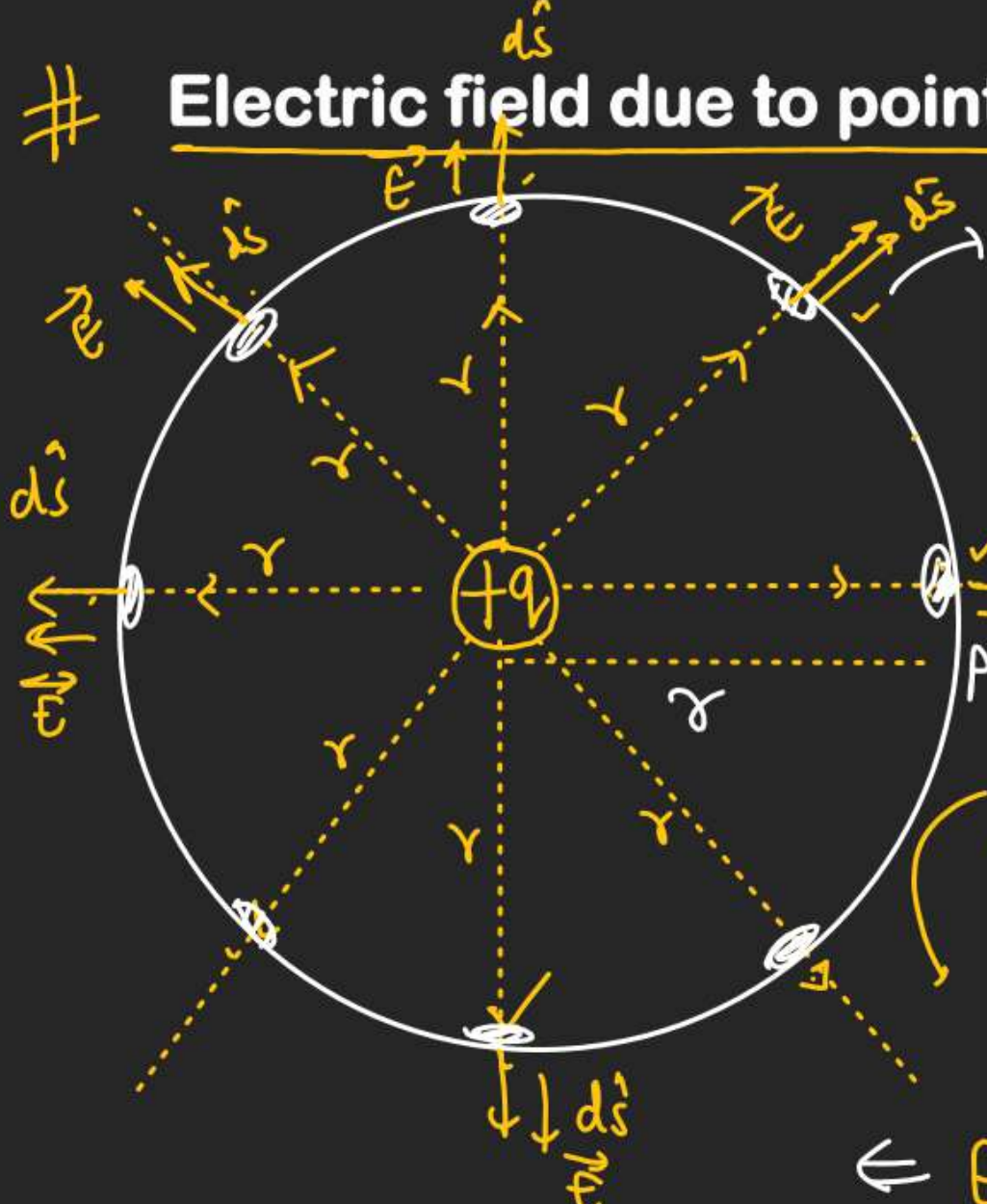


APPLICATION OF GAUSS'S LAW

Electric field due to point charge.



Spherical Gaussian Surface.

$$\vec{dS} = (ds)\hat{dS}$$

$$\oint \vec{E} \cdot \vec{dS} = \frac{q}{\epsilon_0}$$

$\vec{E} \parallel \vec{dS}$

$$\oint E ds = \frac{q}{\epsilon_0}$$

$$E \oint ds = \frac{q}{\epsilon_0}$$

$$E \cdot 4\pi r^2 = \frac{q}{\epsilon_0}$$

$$E = \frac{q}{4\pi\epsilon_0 r^2}$$

Surface Integral
of Sphere \rightarrow Gaussian Surface.

$$\phi = \frac{q_{enc}}{\epsilon_0}$$

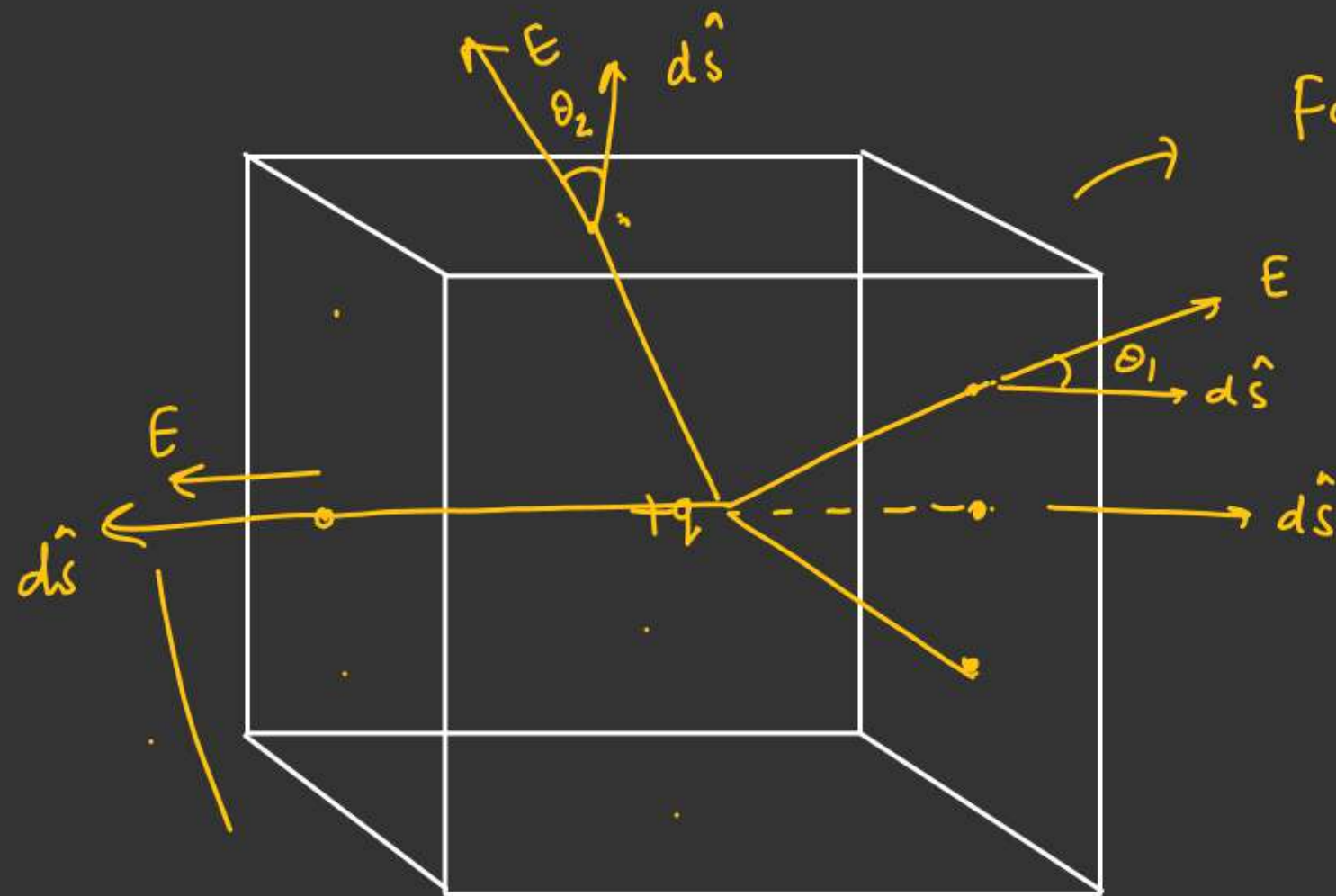
$$\oint \vec{E} \cdot \vec{dS} = \frac{q_{enc}}{\epsilon_0}$$

$$\oint (E ds \cos\theta) = \frac{q_{enc}}{\epsilon_0}$$

For Symmetrical Charge distribution
if we can choose a Gaussian Surface
Where $\vec{E} \perp \vec{dS}$ or $\vec{E} \parallel \vec{dS}$ then

$$\vec{E} \cdot \vec{dS} = 0 \quad \vec{E} \perp \vec{dS}$$

$$\vec{E} \cdot \vec{dS} = E ds \quad \vec{E} \parallel \vec{dS}$$



For point Charge
Gaussian Surface as a Cube
not possible for calculating
 E .

APPLICATION OF GAUSS'S LAW

Electric field due to infinite line charge

λ = linear charge density

l = length of the cylindrical Gaussian Surface.
 r = radius of the Gaussian Surface.

$$\oint \vec{E} \cdot d\vec{s} = \frac{q_{enc}}{\epsilon_0}$$

$$q_{enc} = \lambda L$$

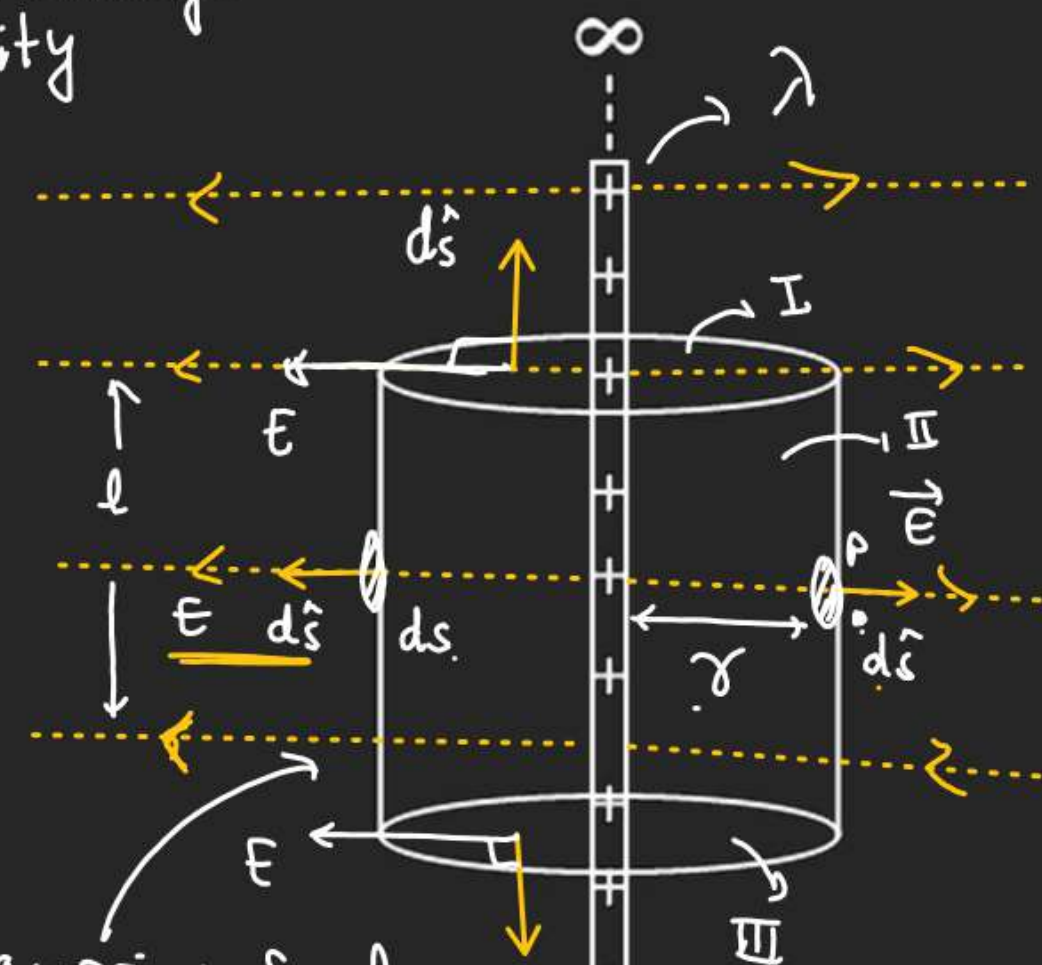
$$\int_I \vec{E} \cdot d\vec{s} + \int_{II} \vec{E} \cdot d\vec{s} + \int_{III} \vec{E} \cdot d\vec{s} = \frac{\lambda L}{\epsilon_0}$$

$$\int_I \vec{E} \cdot d\vec{s} = \int_{III} \vec{E} \cdot d\vec{s} = 0 \quad [\vec{E} \perp d\vec{s}]$$

$$\int_{II} \vec{E} \cdot d\vec{s} = \int_{II} E ds \quad [\vec{E} \parallel d\vec{s}]$$

$$E \int_{II} ds = \frac{\lambda L}{\epsilon_0} \Rightarrow E \cdot 2\pi r L = \frac{\lambda L}{\epsilon_0} \Rightarrow$$

$$E = \frac{\lambda}{2\pi\epsilon_0 r}$$



Gaussian Surface $d\vec{s}$ for infinite line Charge \rightarrow cylindrical ∞

APPLICATION OF GAUSS'S LAW

Electric field due to a long uniformly charged Conducting Cylinder

① $r < R$ (Inside the cylinder)

$$\oint \vec{E} \cdot d\vec{s} = \frac{q_{\text{enc}}}{\epsilon_0}$$

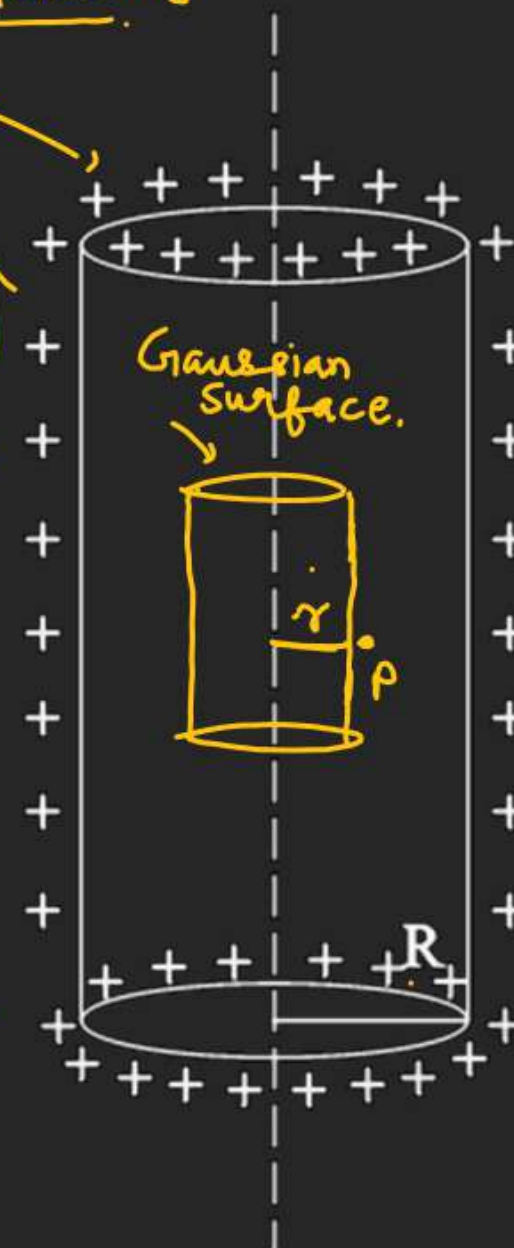
Here, $q_{\text{enc}} = 0$

$$\oint \vec{E} \cdot d\vec{s} = 0$$

$$\boxed{\vec{E} = 0}$$

\Rightarrow Always Charge resides at the surface of the cylinder
 Since cylinder is very long so its field lines is perpendicular to the cylinder so, we choose cylindrical Gaussian surface

Conducting



σ = Surface Charge density.

For Curve part of Cylinder.

$$\vec{E} \parallel d\vec{s}$$

$$E \oint ds = \frac{q_{enc}}{\epsilon_0}$$

Curve part of Cylinder

$$E_{surce} = \frac{\sigma}{\epsilon_0}$$

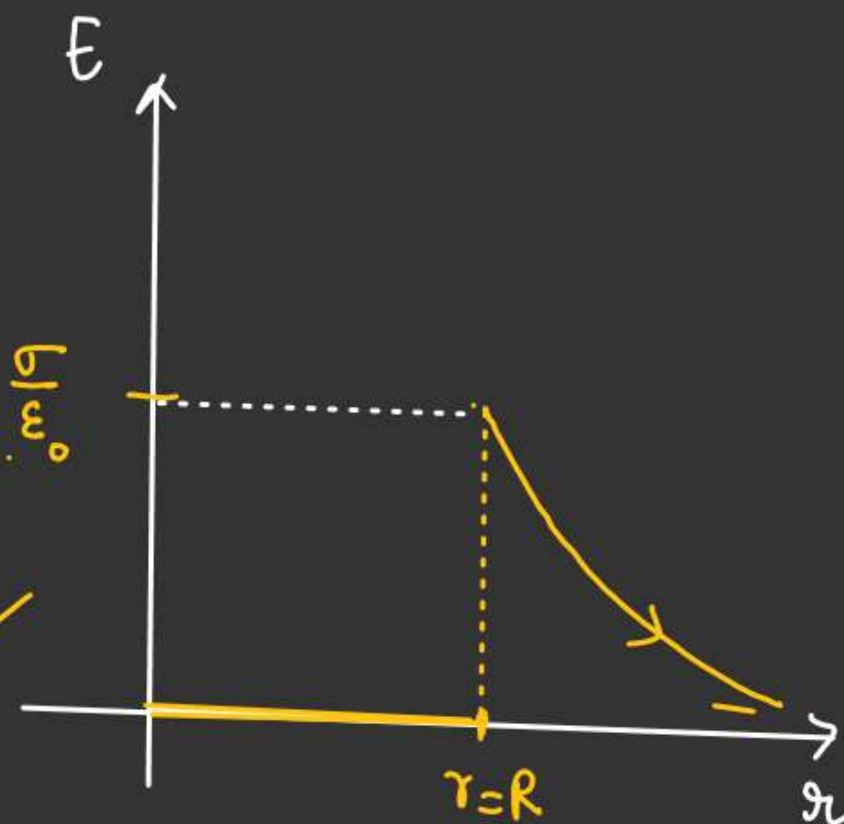
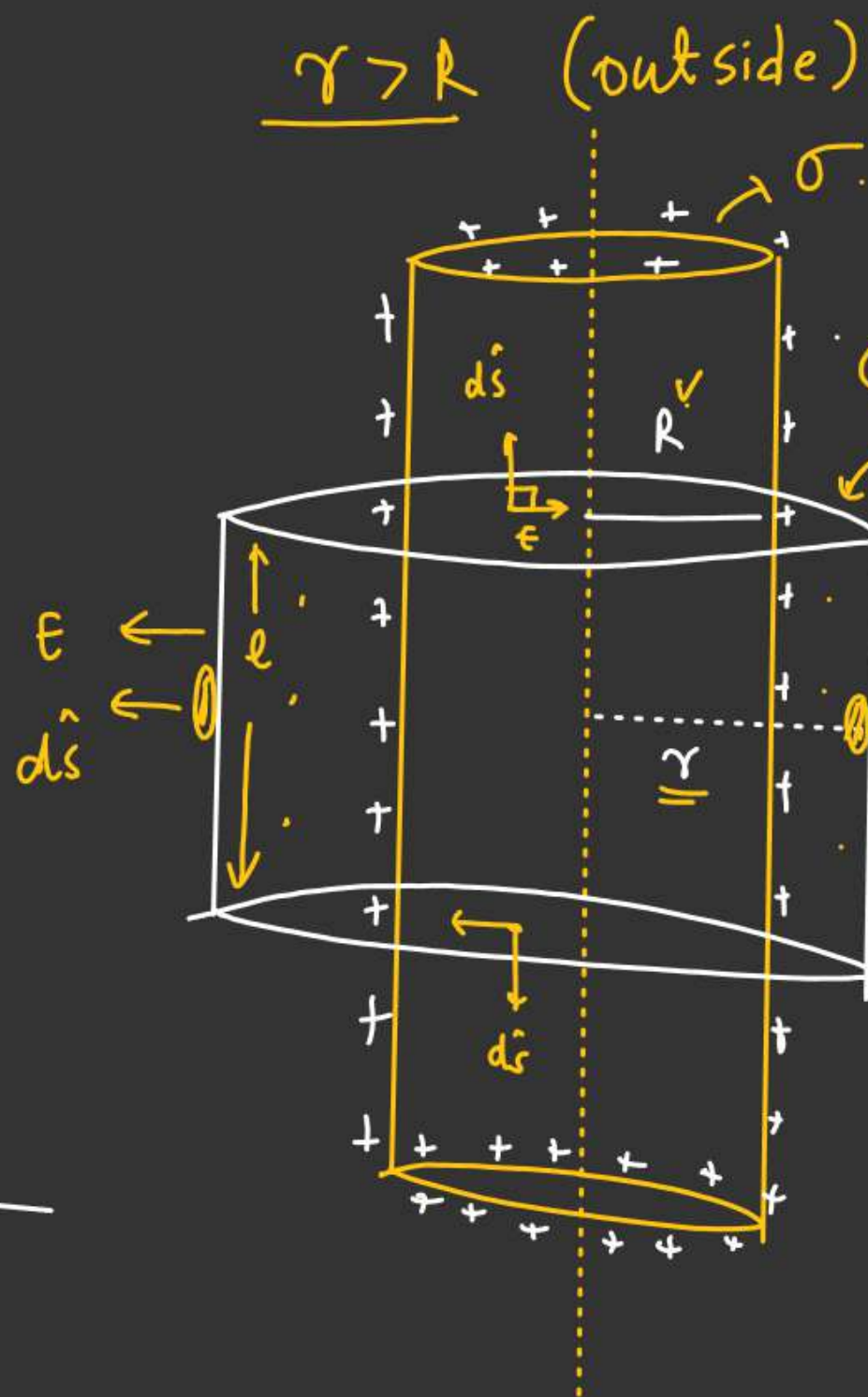
$$E \cdot (2\pi r l) = \frac{\sigma 2\pi R l}{\epsilon_0}$$

$$E = \left(\frac{\sigma R}{\epsilon_0 r} \right)$$

outside

$$E \propto \frac{1}{r}$$

$$q_{enc} = \sigma \cdot 2\pi R L$$



APPLICATION OF GAUSS'S LAW

Electric field due to a long uniformly charged non-conducting cylinder.

"For very long cylinder Electric field perpendicular to axis of cylinder."

ρ = Volume Charge density.

$\rho = \text{Constant} \Rightarrow$ uniformly charged.

$$\frac{Q}{\pi R^2 h} = \rho$$

For $r > R$

$$E \cdot 2\pi r l = \frac{\rho \cdot \pi R^2 l}{\epsilon_0}$$

$$E_{\text{out side}} = \frac{\rho R^2}{2\epsilon_0 r}$$

1) $r < R$

$\vec{E} \parallel d\vec{s}$ (For Curve part)

$$\oint \vec{E} \cdot d\vec{s} = \frac{q_{\text{enc}}}{\epsilon_0}$$

$$E \cdot 2\pi r l = \frac{(\rho \pi r^2 l)}{\epsilon_0}$$

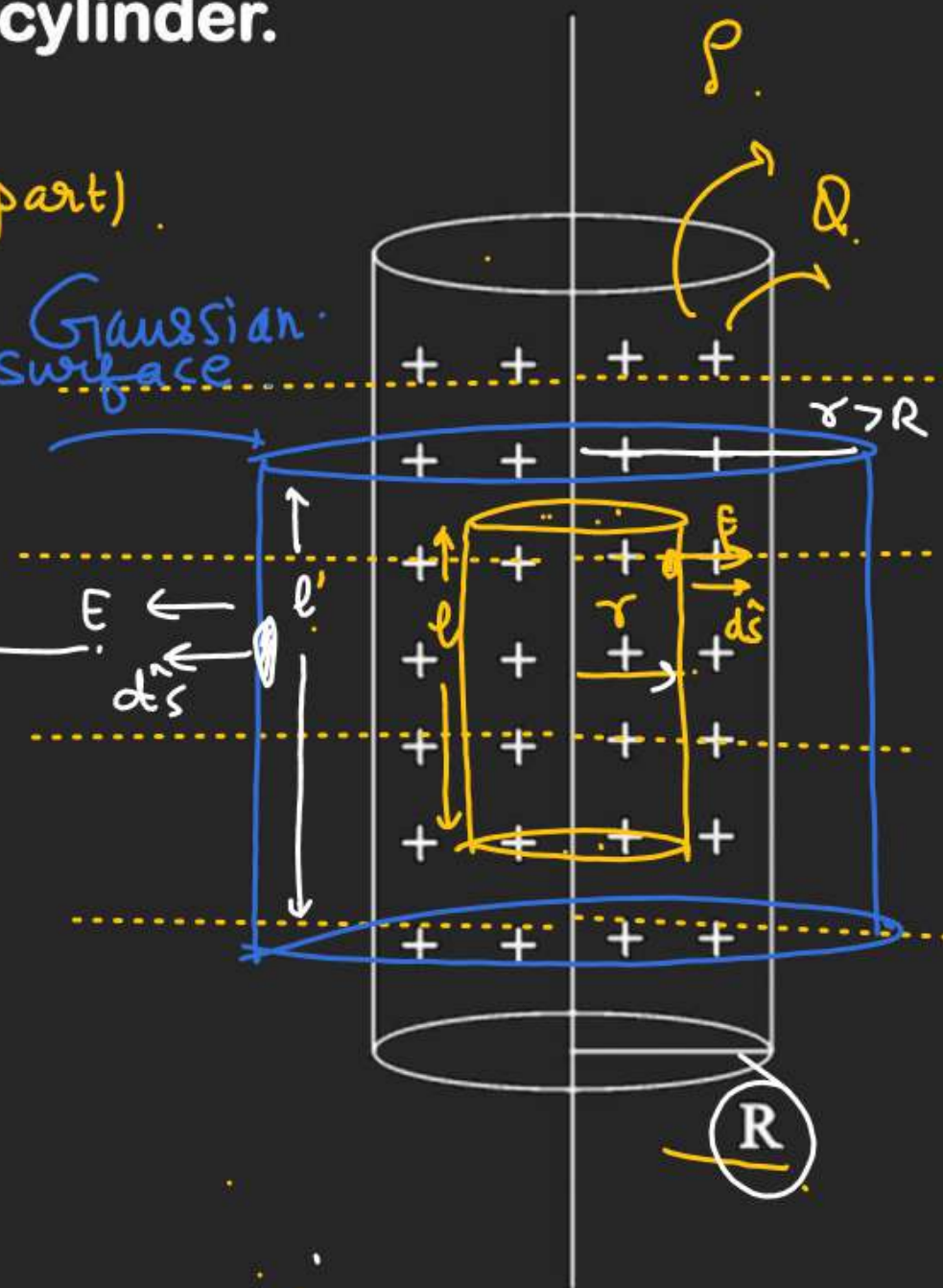
$$E_{\text{inside}} = \frac{\rho r}{2\epsilon_0}$$

$$\vec{E}_{\text{inside}} = \frac{\rho \vec{r}}{2\epsilon_0}$$

$$E_{\text{surface}} = \frac{\rho R}{2\epsilon_0}$$

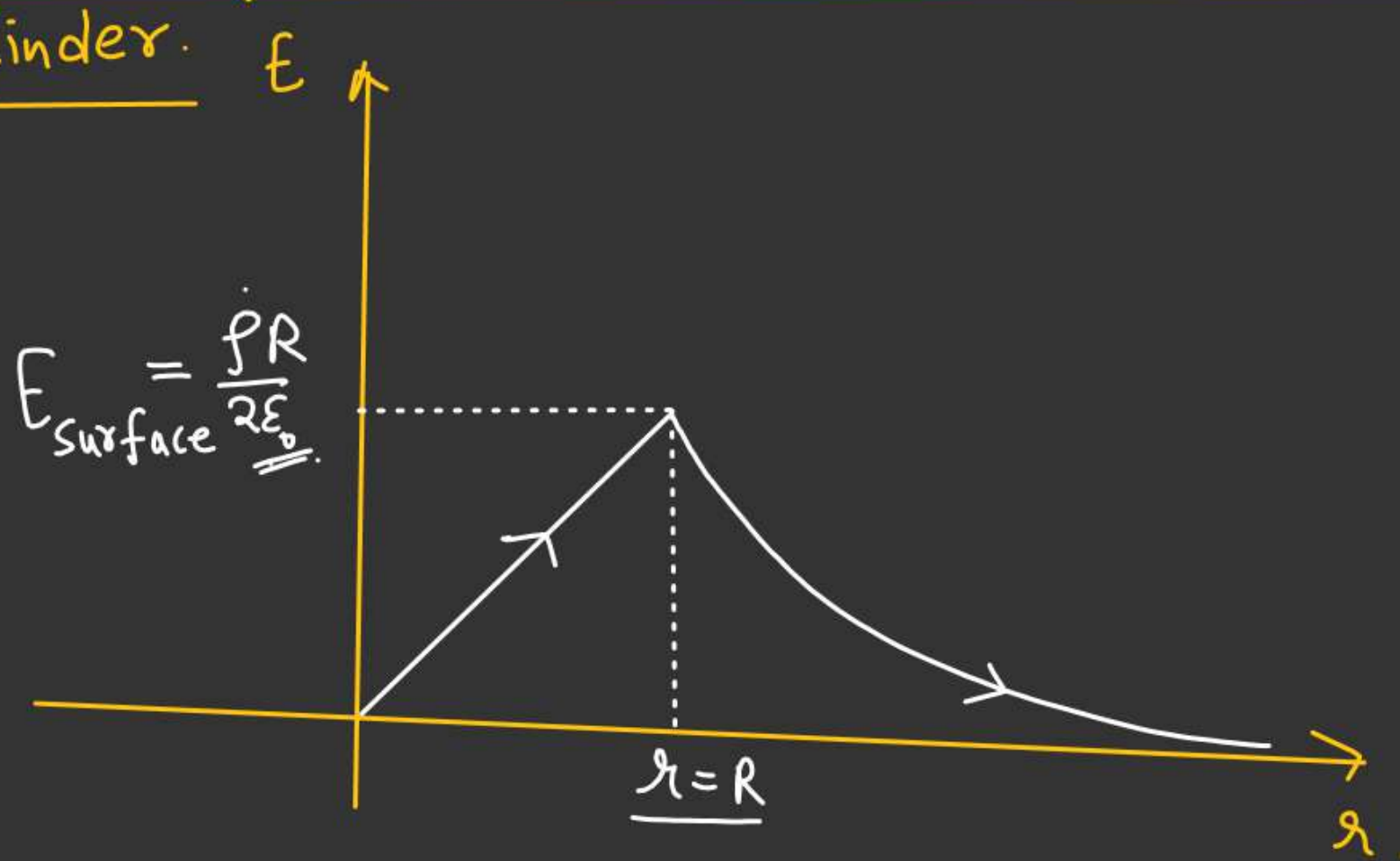
$$r = R$$

Gaussian surface



E Vs r graph for uniformly charge non-Conducting long.

Cylinder.



$$E_{\text{inside}} = \frac{\rho r}{2\epsilon_0} \checkmark$$

$$E_{\text{outside}} = \frac{\rho R^2}{2\epsilon_0 r} \checkmark$$