

Q Eqⁿ of Ellipse

In whose Major Axis = 8 & ecc = $\frac{1}{2}$

$$\left. \begin{array}{l} 2a = 8 \\ a = 4 \end{array} \right| e = \frac{1}{2}$$

$$b^2 = a^2(1 - e^2) \\ = 16(1 - \frac{1}{4})$$

$$b^2 = 12$$

$$\frac{x^2}{16} + \frac{y^2}{12} = 1$$

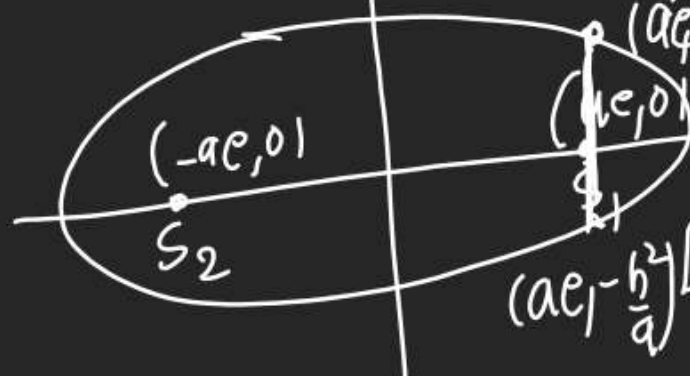
$$\frac{x^2}{4^2} + \frac{y^2}{2\sqrt{3}} = 1$$

Q Ecc of Ellipse is $\frac{5}{8}$
& distance betⁿ foci = 10
find L.R.
L.R.?

$$e = \frac{5}{8} \quad \left| \quad \begin{array}{l} 2ae = 10 \\ 2a \times \frac{5}{8} = 10 \end{array} \right.$$

$$a = 8$$

$$b^2 = a^2(1 - e^2) = 64(1 - \frac{25}{64}) = 39$$



$$L.R. = \frac{2b^2}{a} = \frac{2 \times 39}{8} = \frac{39}{4}$$

Q An ellipse is such that
its L.R. = Sum of length
of its Principle Axis
Then Ellipse becomes
Circle.

2 Pr. Axis \rightarrow Maj./Min

Sum of Pr. Axis = $a + b$

$$\frac{2b^2}{a} = a + b$$

$$2b^2 = a^2 + ab \quad \div b^2$$

$$\Rightarrow \left(\frac{a}{b}\right)^2 + \left(\frac{a}{b}\right) - 2 = 0$$

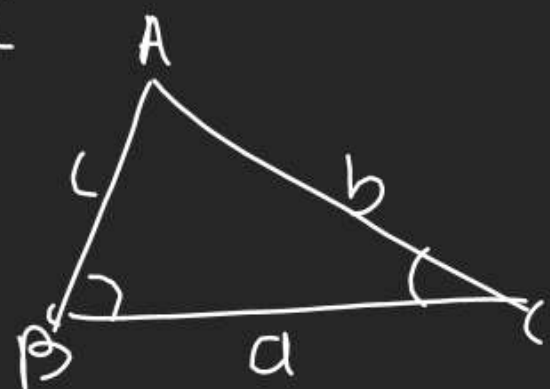
$$\Rightarrow \left(\frac{a}{b} + 2\right)\left(\frac{a}{b} - 1\right) = 0$$

$$\frac{a}{b} - 1 = 0 \Rightarrow a = b \text{ (circle)}$$

Q In $\triangle ABC$ with fixed base BC

Vertex A moves such that

$$\sin B + \sin C = 4 \sin^2 \left(\frac{A}{2} \right)$$



If a, b, c denotes length of sides $\angle A + \angle B + \angle C = \pi$

of \triangle opp to A, B, C then

$$\frac{B+C}{2} = \frac{\pi - A}{2}$$

① $b+c=4a$ ~~$b+c=2a$~~

$$\sin \left(\frac{B+C}{2} \right) = \sin \left(\frac{\pi - A}{2} \right) = \sin \frac{A}{2}$$

(3) Locus of A is Ellipse (2) A in Pair of

Locus of
St. line

$$\sin B + \sin C = 4 \sin^2 \frac{A}{2}$$

$$2 \sin \left(\frac{B+C}{2} \right) \cos \left(\frac{B-C}{2} \right) = 4 \sin^2 \frac{A}{2}$$

$$\sin \frac{A}{2} \cos \left(\frac{B-C}{2} \right) = 2 \sin^2 \frac{A}{2}$$



$$\cos \left(\frac{B-C}{2} \right) = 2 \sin \left(\frac{A}{2} \right)$$

$$\cos \frac{A}{2} \cdot \cos \left(\frac{B-C}{2} \right) = 2 \sin \frac{A}{2} \cos \frac{A}{2}$$

$$2 \cos \left(\frac{\pi - (B+C)}{2} \right) \cos \left(\frac{B-C}{2} \right) = 2 \sin A$$

$$2 \sin \left(\frac{B+C}{2} \right) \cos \left(\frac{B-C}{2} \right) = 2 \sin A$$

$$\sin B + \sin C = 2 \sin A$$

$$\frac{\sin B + \sin C}{\sin A} = 2$$

$$\frac{b+c}{a} = 2 \Rightarrow \underline{\underline{b+c=2a}}$$

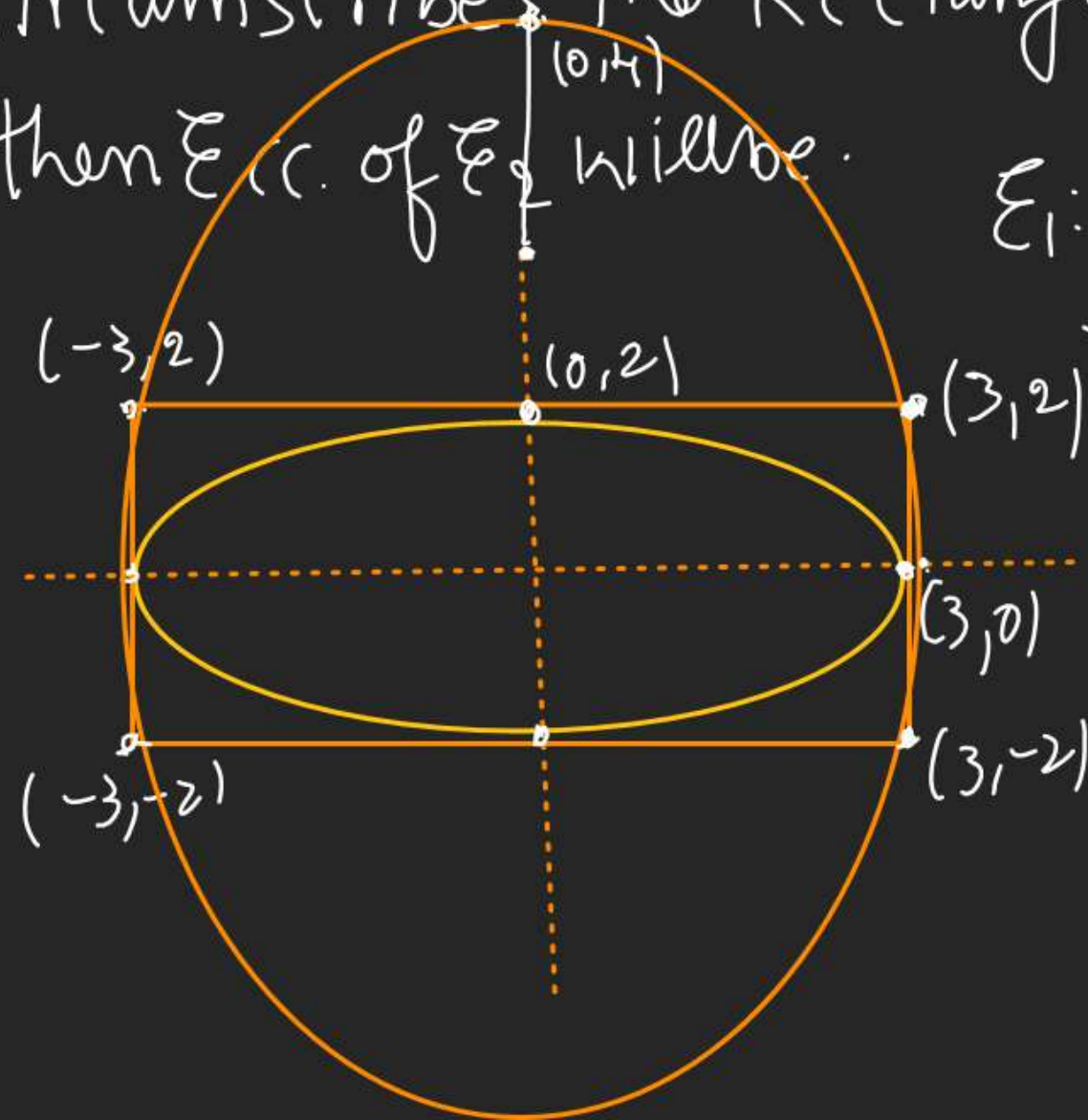
Q $E_1: \frac{x^2}{9} + \frac{y^2}{4} = 1$ is Inscribed in a Rect.

Whose sides are \parallel^e to (o-axis).

Another Ellipse E_2 P.T. (0,4)

(Circumscribes the Rectangle

then Ecc. of E_2 will be.



$$E_1: \frac{x^2}{9} + \frac{y^2}{4} = 1$$

New Ellipse

$$E_2: \frac{x^2}{a^2} + \frac{y^2}{16} = 1 \text{ P.T. } (3, 2)$$

$$\frac{9}{a^2} + \frac{4}{16} = 1$$

$$\frac{36}{a^2} = \frac{12}{4}$$

$$a^2 = 12$$

Normally

$$b^2 = a^2(1 - e^2)$$

$$\text{Vert. } a^2 = b^2(1 - e^2)$$

$$12 = 16(1 - e^2)$$

$$\Rightarrow 1 - e^2 = \frac{3}{4} \Rightarrow e^2 = \frac{1}{4}$$

$$e = \frac{1}{2}$$

Q $P(x, y)$, $F_1 = (3, 0)$

$F_2 = (-3, 0)$

$16x^2 + 25y^2 = 400$ then $PF_1 + PF_2 = ?$

$$\frac{x^2}{25} + \frac{y^2}{16} = 1$$

$a = 5, b = 4$

$2a = 10$

$$\begin{array}{r} 2x + 24y + 28 = 0 \\ -2x - y + 3 = 0 \\ \hline 25y = -25 \end{array}$$

$y = -1$
 $x = -2$ } $(-2, -1)$
Centre

Q find Range of a

for Ellipse

$$\frac{x^2}{6-a} + \frac{y^2}{a-2} = 1$$

\oplus $6-a > 0$ $a-2 > 0$
 $a < 6$ $a > 2$

$a \in (2, 6) - \{4\}$

$(a \neq 4)$

Q Centre of $x^2 + 24xy - 6y^2 + 28x + 36y + 10 = 0$ is

Non-Hom Eqn then Centre

$$\frac{\partial f}{\partial x} = 0 \quad | \quad \frac{\partial f}{\partial y} = 0$$

E_{q1} E_{q2}

$y(\text{const.})$ $x(\text{const.})$

$$\begin{array}{r} 2x + 24y - 0 + 28 = 0 \\ x + 12y + 14 = 0 \rightarrow (1) \end{array}$$

$$\begin{array}{r} 0 + 24x - 12y + 36 = 0 \\ 2x - y + 3 = 0 \rightarrow (2) \end{array}$$

Q Find Area of figure Bounded betⁿ.

Foci of E_1 : $\frac{x^2}{25} + \frac{y^2}{16} = 1$ $\rightarrow a > b$

E_2 : $\frac{x^2}{24} + \frac{y^2}{49} = 1$ $\rightarrow a < b$

$a = 5, b = 4$



$b^2 = a^2(1 - e^2)$

$16 = 25(1 - e^2)$

$\frac{16}{25} = 1 - e^2$

$e = \frac{3}{5}$

Foci = $(3, 0)$
 $(-3, 0)$

$a = \sqrt{24}, b = 7$

$(0, be) = (0, 5)$

$(0, -be) = (0, -5)$

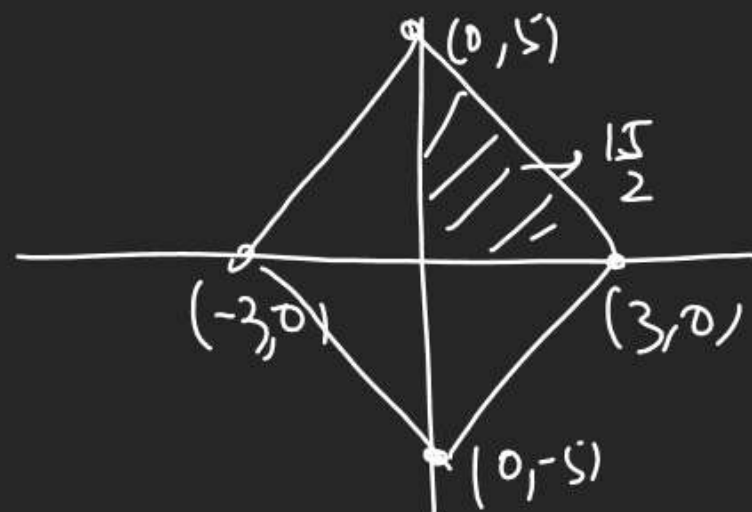
$a^2 = b^2(1 - e^2)$

$24 = 49(1 - e^2)$

$\frac{24}{49} = 1 - e^2$

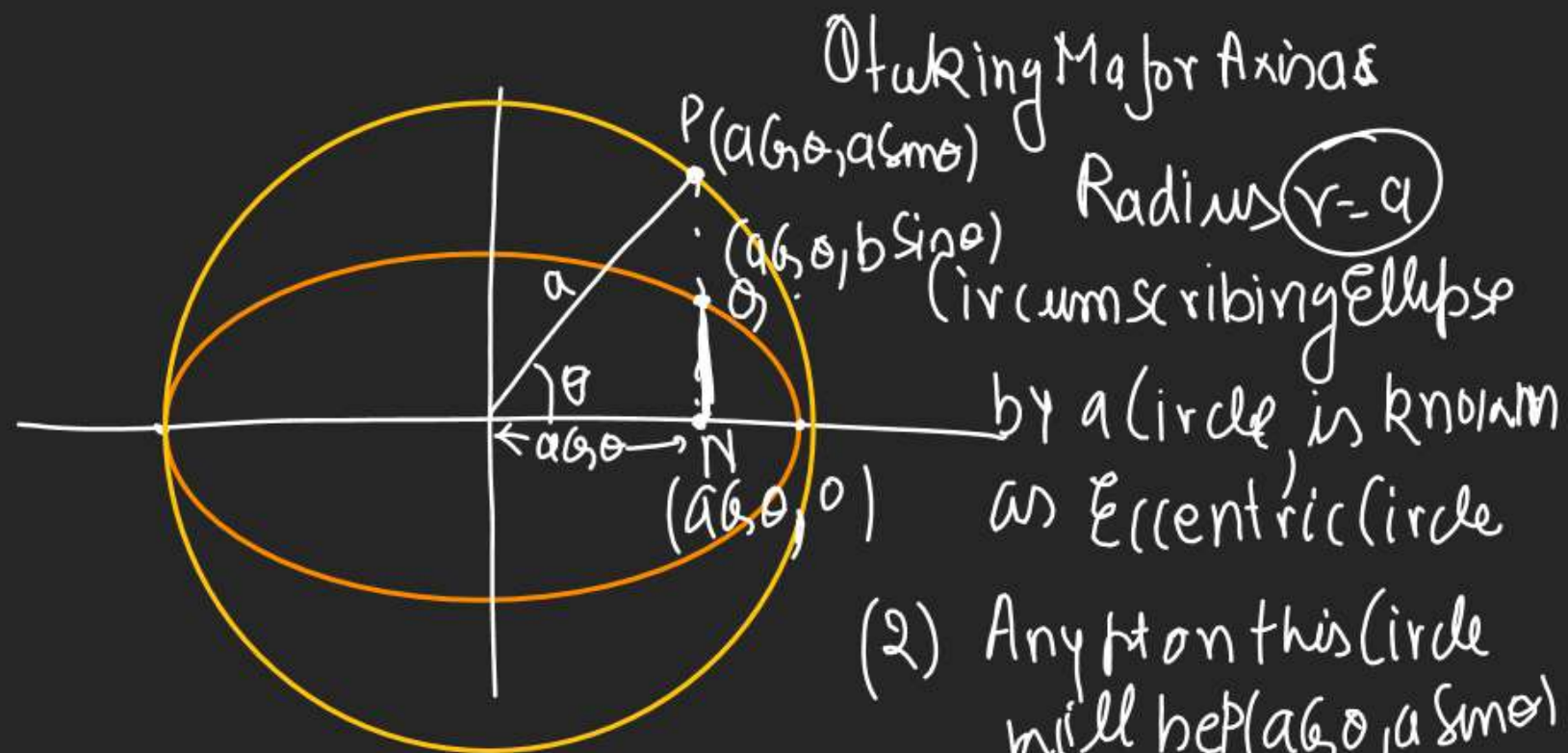
$e = \frac{5}{7}$

$be = 5$



Area = $4 \times \frac{15}{2}$
 $= 30$

Eccentric Angle & Eccentric Circle:



(2) Any pt on this circle will be $(a \cos \theta, a \sin \theta)$

(3) No Dropping a \perp from P at x-axis (coming at N $(a \cos \theta, 0)$)

(4) This \perp Intersecting Ellipse at Q
 $x = a \cos \theta$ in $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

(6) P & Q are known as corresponding pts

Q P.T. $\boxed{\frac{PN}{PQ} = \text{Const}}$

$$\frac{PN}{PQ} = \frac{a \sin \theta}{a \sin \theta - b \sin \theta} = \frac{a}{a-b}$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\frac{y^2}{b^2} = \sin^2 \theta \Rightarrow y = b \sin \theta$$

So we are getting Q $(a \cos \theta, b \sin \theta)$ on Ellipse

& This Par. Coord of Ellipse.

(5) Par. Coord of Ellipse

$$\begin{cases} x = a \cos \theta \\ y = b \sin \theta \end{cases}$$

Where θ - Ecc. Angle
 $0 \leq \theta < 2\pi$

Q find coord. of any pt on Ellipse

Whose foci are $(-1, 0)$ & $(7, 0)$

& Ecc = $\frac{1}{2}$?

$2ae$

$$2ae = 8$$

$$ae = 4$$

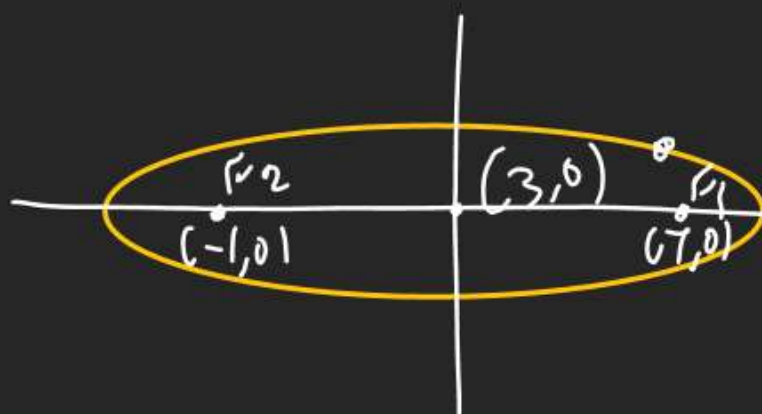
$$ax \frac{1}{2} = 4$$

$$a = 8$$

$$b^2 = a^2(1 - e^2)$$

$$b^2 = 64(1 - \frac{1}{4})$$

$$= 48$$



$$\frac{(x-3)^2}{64} + \frac{(y-0)^2}{48} = 0$$

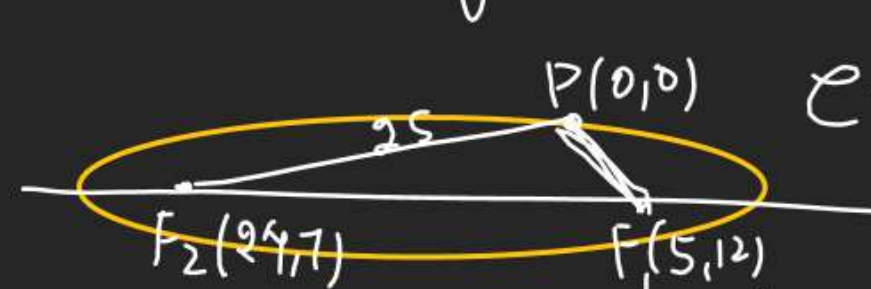
$$x - 3 = 8 \cos \theta$$

$$y - 0 = 4\sqrt{3} \sin \theta$$

$$\text{coord} = (3 + 8 \cos \theta, 4\sqrt{3} \sin \theta)$$

$$R_K: \begin{cases} e = \frac{2ae}{2a} \\ e = \frac{F_1 F_2}{PF_1 + PF_2} \end{cases}$$

Q If (5, 12) & (24, 7) are foci of Ellipse P.T. Origin. then find Ecc. of Ellipse



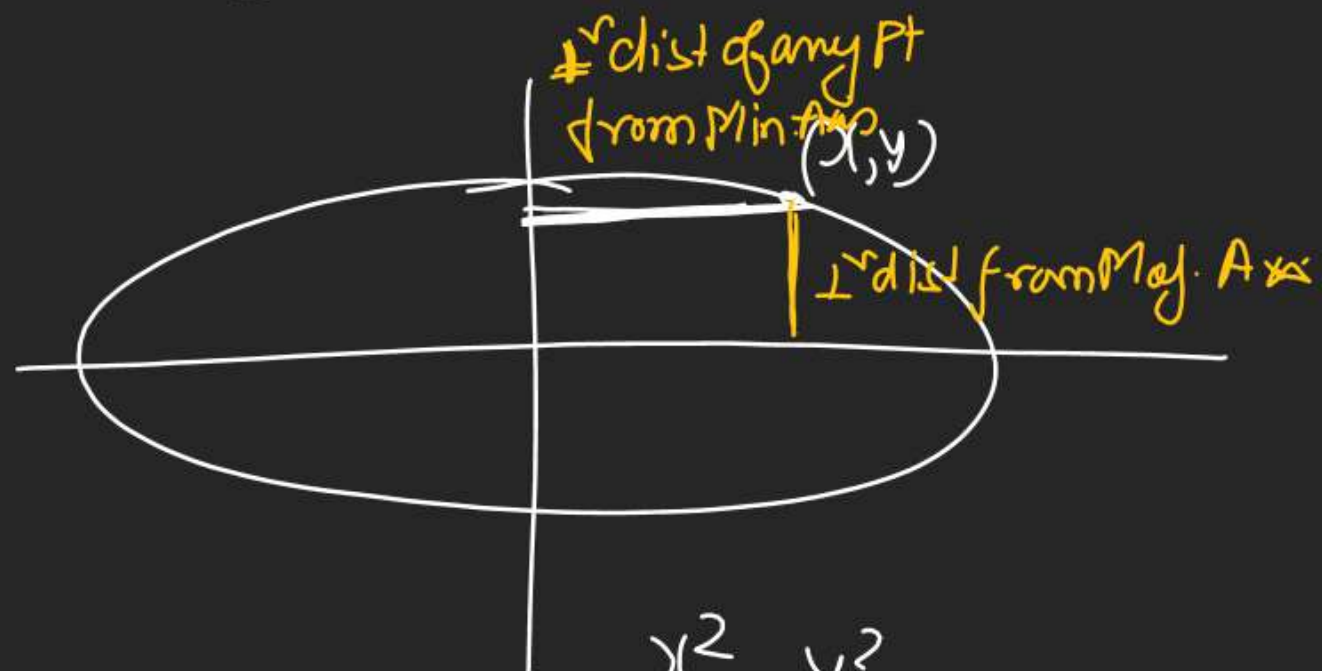
$$F_1 F_2 = \sqrt{(24-5)^2 + (7-12)^2}$$

$$= \sqrt{361 + 25}$$

$$= \sqrt{386}$$

$$e = \frac{\sqrt{386}}{13+25} = \frac{\sqrt{386}}{38}$$

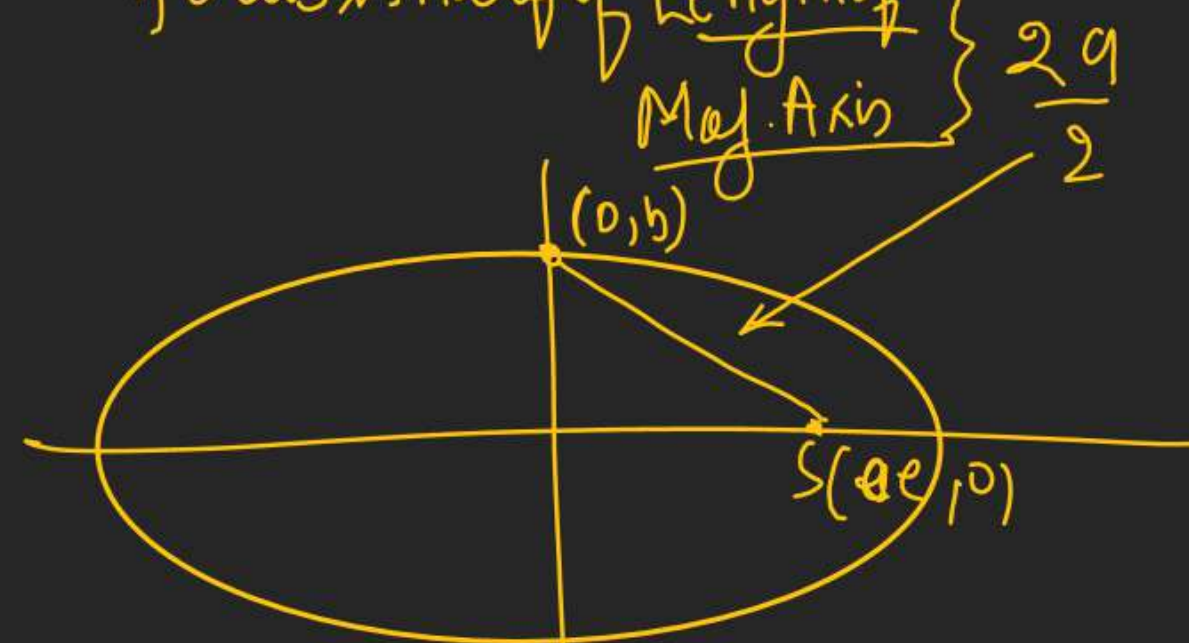
Teda Ellipse hai Per Mera Ellipse hai



$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\frac{(\text{Min Axis})^2}{a^2} + \frac{(\text{Maj Axis})^2}{b^2} = 1$$

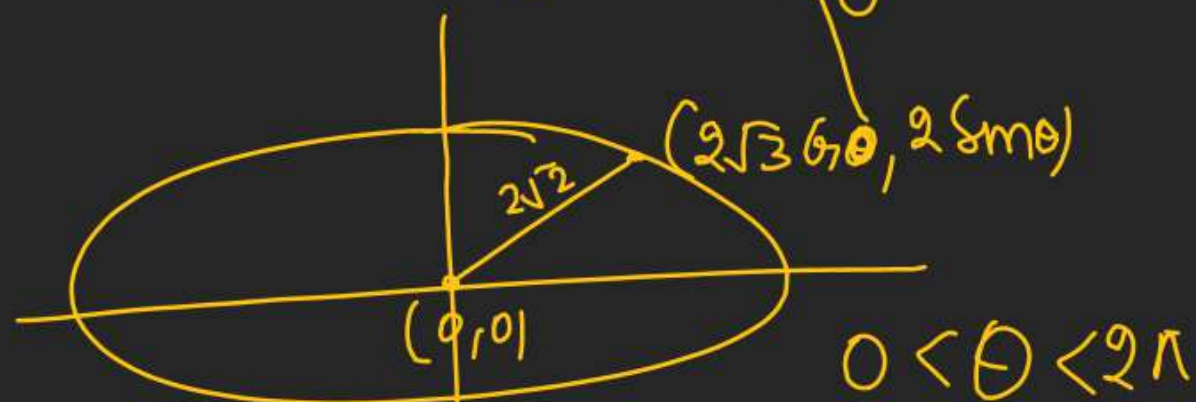
Q. P.T. distance of Extremity of Minor Axis from focus is half of Length of Maj. Axis



$$\begin{aligned} \text{dis} &= \sqrt{a^2e^2 + b^2} \\ &= \sqrt{a^2e^2 + a^2(1-e^2)} \\ &= \sqrt{a^2} = a = \frac{2a}{2} \end{aligned}$$

Q If distance of any pt on $\frac{x^2}{12} + \frac{y^2}{4} = 1$

from centre is $2\sqrt{2}$ find θ (c. angle)?



$$(2\sqrt{3}\cos\theta)^2 + (2\sin\theta)^2 = 2\sqrt{2}$$

$$12\cos^2\theta + 4\sin^2\theta = 8$$

$$4 + 8\cos^2\theta = 8$$

$$8\cos^2\theta = 4 \Rightarrow \cos^2\theta = \frac{1}{2}$$

$$\cos\theta = \pm \frac{1}{\sqrt{2}} \quad \theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$$

Q Eqn of ellipse whose focus = (3,4)

(centre (2,3)) $e = \frac{1}{2}$
 $(3) ae = \sqrt{-1^2 + -1^2} = \sqrt{2}$

$$a \times \frac{1}{2} = \sqrt{2} \Rightarrow a = 2\sqrt{2}$$

$$(4) b^2 = 8(1 - \frac{1}{4}) = 6$$

1) Major Axis

$$(y-3) = \frac{4-3}{3-2}(x-2)$$

$$\text{M. } x - y + 1 = 0$$

2) Minor Axis

$$x + y + k = 0$$

$$2 + 3 + k = 0$$

$$k = -5$$

$$\text{Min } x + y - 5 = 0$$



P.T. (2,3)