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Q R_f of $y = \cos^{-1}(2x - x^2)$

$2x - x^2 = -(x^2 - 2x + 1) + 1$

$= 1 - (x-1)^2$

$\infty > (x-1)^2 \geq 0$

$-\infty < -(x-1)^2 \leq 0$

$-\infty < 1 - (x-1)^2 \leq 1$

$\cos^{-1}(-1) > \cos^{-1}(2x - x^2) \geq \cos^{-1} 1$

$\pi > y \geq 0 \therefore y \in [0, \pi)$

Q R_f of $y = \cos^{-1}\left(\frac{x^4 + x^2 + 1}{x^2 + x + 1}\right)$

$\frac{x^4 + x^2 + 1}{x^2 + x + 1} = x^2 - x + 1$

$y \in [0, \cos^{-1}(\frac{3}{4})]$

$= (x - 1/2)^2 - (1/2)^2 + 1$

$= (x - 1/2)^2 + \frac{3}{4} \geq \frac{3}{4}$

$\frac{3}{4} \leq \frac{x^4 + x^2 + 1}{x^2 + x + 1} < \infty$

$\cos^{-1}(\frac{3}{4}) \geq \cos^{-1}\left(\frac{x^4 + x^2 + 1}{x^2 + x + 1}\right) > \cos^{-1}(1)$

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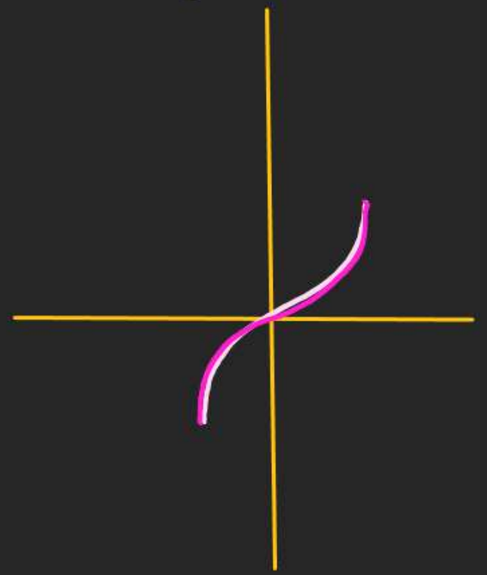
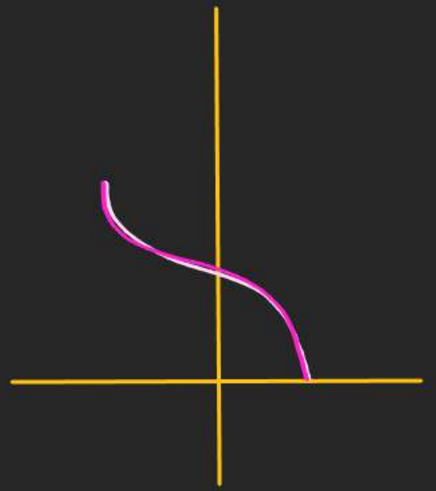
Graph of all ITF फ ल

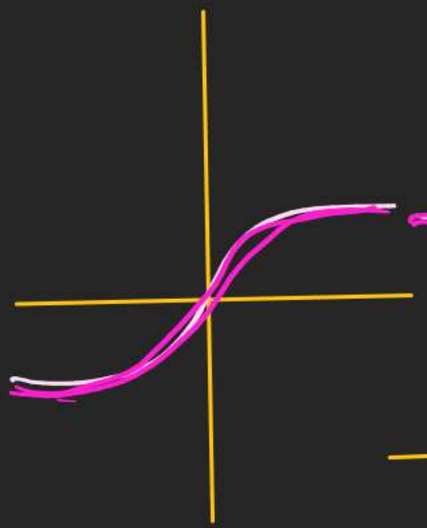
① $y = \sin^{-1} x$



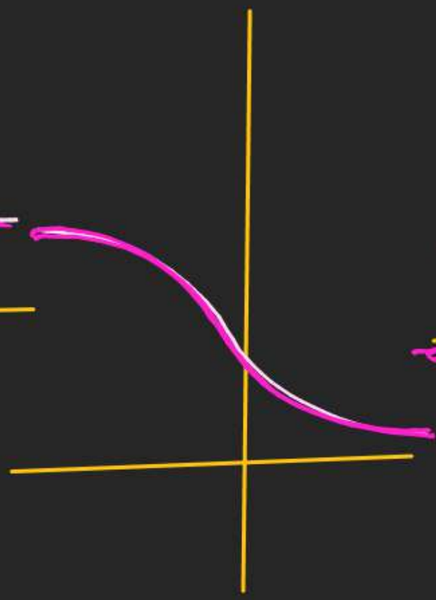
② $y = \cos^{-1} x$



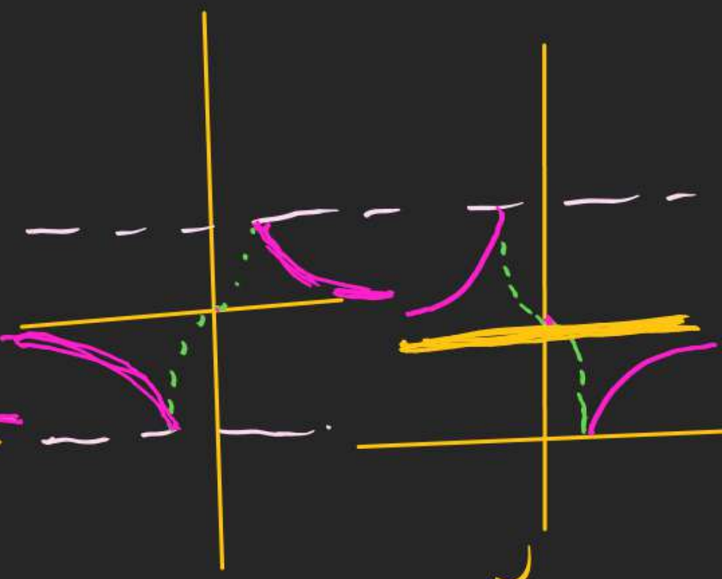
③ $y = \tan^{-1} x$



④ $y = \cot^{-1} x$



⑤ $y = \operatorname{cosec}^{-1} x$



⑥ $y = \sec^{-1} a$

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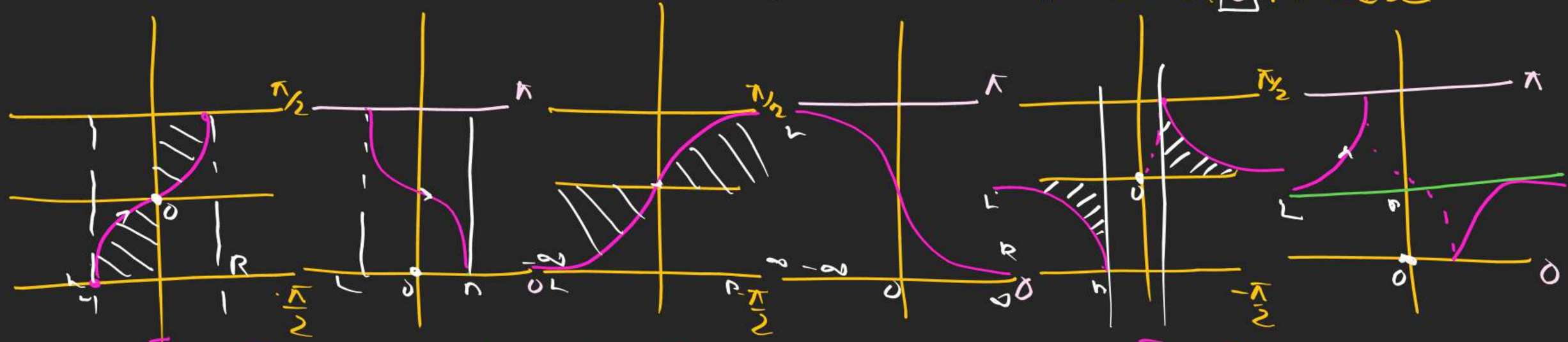
1. $y = \sin^{-1} x$ 2. $y = \cos^{-1} x$ 3. $y = \tan^{-1} x$ 4. $y = \cot^{-1} x$ 5. $y = \sec^{-1} x$ 6. $y = \operatorname{cosec}^{-1} x$



$y \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ | $y \in [0, \pi]$ | $y \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ | $y \in (0, \pi)$ | $y \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$ | $y \in [0, \pi] - \left\{\frac{\pi}{2}\right\}$

$-1 \le x \le 1$ | $-1 \le x \le 1$ | $x \in \mathbb{R}$ | $x \in \mathbb{R}$ | $x \le -1 \cup x \ge 1$ | $x \le -1 \cup x \ge 1$

odd | NENO | odd | NEN | odd | NONO

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Range of $\sin^{-1}x \rightarrow -\frac{\pi}{2} \le \sin^{-1}x \le \frac{\pi}{2}$

Range of $\cos^{-1}x \rightarrow 0 \le \cos^{-1}x \le \pi$

Qs Base on Maxm / Minm value of $\sin^{-1}x / \cos^{-1}x$

Q If $\sin^{-1}x + \sin^{-1}y + \sin^{-1}z = \frac{3\pi}{2}$ then $xy+yz+zx$ = ?

3 $\sin^{-1}$ Ka Sum = $\frac{3\pi}{2}$ given $\Rightarrow$ all $\sin^{-1}$ are giving $\frac{\pi}{2}$

$\Rightarrow \sin^{-1}x = \sin^{-1}y = \sin^{-1}z = \frac{\pi}{2}$

$x = \sin\frac{\pi}{2} = 1$ | $y = \sin\frac{\pi}{2} = 1$, $z = 1$

Demand $= xy+yz+zx = 1\times1+1\times1+1\times1 = 3$

Q If $\sum_{i=1}^{20} \sin^{-1}x_i = 10\pi$

then find $\sum_{i=5}^{10} x_i = ?$

$\sin^{-1}x_1 + \sin^{-1}x_2 + \sin^{-1}x_3 - - - + \sin^{-1}x_{20} = \frac{20\pi}{2}$

20 $\sin^{-1}$ ka Sum $= 20\frac{\pi}{2}$

$\sin^{-1}x_1 = \sin^{-1}x_2 \ldots \sin^{-1}x_{20} = \frac{\pi}{2}$

$x_1 = x_2 = x_3 - - x_{20} = 1$

Demand $= \sum_{i=5}^{10} x_i = x_5 + x_6 + x_7 + x_8 + x_9 + x_{10}$

$1+1+1+1+1+1 = 6$

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Q If $(\sin^{-1}x)^3+(\sin^{-1}y)^3+(\sin^{-1}z)^3=\frac{3\pi^3}{8}$

then find $2x-3y+4z=?$

$(\sin^{-1}x)^3+(\sin^{-1}y)^3+(\sin^{-1}z)^3=\left(\frac{\pi}{2}\right)^3+\left(\frac{\pi}{2}\right)^3+\left(\frac{\pi}{2}\right)^3$

$\sin^{-1}x=\sin^{-1}y=\sin^{-1}z=\frac{\pi}{2}$

$x=y=z=1$

Demand $=2x-3y+4z$

$=2\times1-3\times1+4\times1$

$=3$

Q If $\cos^{-1}x+\cos^{-1}y+\cos^{-1}z=0$

then $\frac{x^{2001}+y^{2023}}{z^{2019}}=?$

$(\cos^{-1}x)_{Min}=0$ $\qquad 0\le\cos^{-1}x\le\pi$

$\cos^{-1}x+\cos^{-1}y+\cos^{-1}z=0$

$0+0+0=0$

$\cos^{-1}x=0=\cos^{-1}y=\cos^{-1}z$

$x=1=y=z$

$\frac{x^{2001}+y^{2023}}{z^{2019}}=\frac{1+1}{1}=2$

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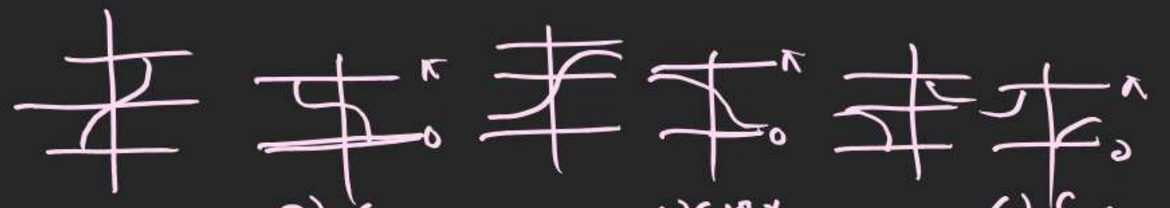

2) $\cos^{-1}x$ — Neno $\oplus$ — $\cos^{-1}x \geq 0$

4) $\cot^{-1}x$ — neno — $\cot^{-1}x > 0$ $\oplus$

6) $\sec^{-1}x$ — neno — $\sec^{-1}x \geq 0$

Int → Q $[\cos^{-1}x] + [\cot^{-1}x] = 0$ $\oplus$ $\oplus$

$-\sin\theta = \sin(-\theta)$

$-\cos\theta = \cos(\pi-\theta)$

$-\tan\theta = \tan(-\theta)$

$-\cot\theta = \cot(\pi-\theta)$

$\Rightarrow [\cos^{-1}x] = [\cot^{-1}x] = 0$

$0 \leq \cos^{-1}x < 1$ & $0 < \cot^{-1}x < 1$

B) $\theta = \cos^{-1}(-x)$

$\cos\theta = -x$

$x = -\cos\theta$

$x = \cos(\pi-\theta)$

$\cos^{-1}x = \pi-\theta$

$\theta = \pi - \cos^{-1}x$

$\cos^{-1}(-x) = \pi - \cos^{-1}x$

Properties.

1) P-1 $T^{-1}(-x)$

Odd	Neno
A) $\sin^{-1}(-x) = -\sin^{-1}x$	$\cos^{-1}(-x) = \pi - \cos^{-1}x$
$\tan^{-1}(-x) = -\tan^{-1}x$	$\cot^{-1}(-x) = \pi - \cot^{-1}x$
$\operatorname{cosec}^{-1}(-x) = -\operatorname{cosec}^{-1}x$	$\sec^{-1}(-x) = \pi - \sec^{-1}x$

A) $\theta = \sin^{-1}(-x)$

$\sin\theta = -x$

$x = -\sin\theta$

$x = \sin(-\theta)$

$\sin^{-1}x = -\theta$

$\theta = -\sin^{-1}x$

$\sin^{-1}(-x) = -\sin^{-1}x$

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Q $\sin^{-1}(+1)$

$-\sin^{-1}(1)$

$-\frac{\pi}{2}$

Q $\cos^{-1}(-1/2)$ } $\cos^{-1}(-x) = \pi - \cos^{-1}x$

$\pi - \cos^{-1}\left(\frac{1}{2}\right)$

$\pi - \frac{\pi}{3} = \frac{2\pi}{3}$

Q $\tan^{-1}(-\sqrt{3})$

$-\tan^{-1}(\sqrt{3})$

$-\frac{\pi}{3}$

Q $\sec^{-1}(-1)$

$\pi - \sec^{-1}(1)$

$\pi - \frac{\pi}{4} = \frac{3\pi}{4}$

Q $\cot^{-1}(-\sqrt{3})$

2) $\sin\left(\frac{\pi}{2} - \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right)$

1) $2\left(\pi - \cot^{-1}(\sqrt{3})\right)$

$2\left(\pi - \frac{\pi}{6}\right) = 2 \times \frac{5\pi}{6} = \frac{5\pi}{3}$

2) $\sin\left(\frac{\pi}{2} + \sin^{-1}\left(\frac{\sqrt{3}}{2}\right)\right)$

$\sin\left(\frac{\pi}{2} + \frac{\pi}{3}\right)$

$\sin(90° + 60°) = \sin 150° = \frac{1}{2}$

Nishant Jindal

P(2) Constant Prop.

A) $\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$; $-1 \le x \le 1$

B) $\tan^{-1}x + \cot^{-1}x = \frac{\pi}{2}$ $\quad x \in R$

C) $\operatorname{cosec}^{-1}x + \sec^{-1}x = \frac{\pi}{2}$ $\quad x \le -1 \cup x \ge 1$

$\sin^{-1}x = \theta$

$x = \sin\theta$

$x = \cos\left(\frac{\pi}{2} - \theta\right)$

$\cos^{-1}x = \frac{\pi}{2} - \theta$

$\cos^{-1}x = \frac{\pi}{2} - \sin^{-1}x$

$\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$ [HP]

Q Range of $y = \sin^{-1}x + \cos^{-1}x + \tan^{-1}x$

$y = \sin^{-1}x + \cos^{-1}x + \tan^{-1}x$

$-1 \le x \le 1 \cap -1 \le x \le 1 \cap x \in R$

Dom $\quad x \in [-1, 1]$

Range

$y = \sin^{-1}x + \cos^{-1}x + \tan^{-1}x$

$y = \frac{\pi}{2} + \tan^{-1}x$

$x \in [-1, 1]$

$\tan^{-1}x \in [\tan^{-1}(-1), \tan^{-1}(1)]$

$\tan^{-1}x \in \left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$

$y \in \frac{\pi}{2} + \tan^{-1}x \in \left[\frac{\pi}{4}, \frac{3\pi}{4}\right]$

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Q $f(x)=\left([\{x\}],\tan^{-1}\left(\frac{x^2-3x-1}{x^2-3x+5}\right)+(3-x^7)\right)^{\frac{1}{7}}$

Value of $f^{-1}(50)-f(50)+f(f(100))=?$

$f(x)=\left(\underbrace{[\{x\}]}_{=0}\cdot\tan^{-1}\left(\frac{x^2-3x-1}{x^2-3x+5}\right)+(3-x^7)\right)^{1/7}$

$[\{2.4\}]=[.4]=0$

$[\{-2.4\}]=[1-.4]$

$=[.6]=0$

$[\{6\}]=[0]=0$

$f(x)=(a-x^7)^{1/n}$

$f(f(x))=x$

$f(x)=(3-x^7)^{1/7}$

$f(f(x))=x$

$f(x)=f^{-1}(x)$

$f(50)=f^{-1}(50)$

Demand

$=f^{-1}(50)-f(50)+f(f(100))$

$=100$

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Q let $f(x) = \frac{1}{\pi}(\sin^{-1}x + \cos^{-1}x + \tan^{-1}x) + \frac{x+1}{x^2+2x+10}$

If absolute max^m value of $f(x)$ is M then $5\,2M = ?$

$-1 \le x \le 1$

$f(x) = \frac{1}{\pi}\left(\boxed{\sin^{-1}x + \cos^{-1}x + \tan^{-1}x}\right) + \frac{x+1}{(x^2+2x+1)+9}$

$= \frac{1}{\pi}\left(\frac{\pi}{2} + \tan^{-1}x\right) + \frac{x+1}{(x+1)^2+9}$

HW

$f(x) = \frac{1}{\pi}\left(\frac{\pi}{2} + \tan^{-1}x\right) + \left(\frac{x+1}{(x+1)^2+9}\right)$

$\ge 0 + 9$

≥ 9

Max Tabaayega

When Dr is Min^n

$\frac{\pi}{2} + \tan^{-1}x \in \left[\frac{\pi}{4}, \frac{3\pi}{4}\right]$

$\frac{1}{\pi}\left(\frac{\pi}{2} + \tan^{-1}x\right) \in \left[\frac{1}{4}, \frac{3}{4}\right]$

Nishant Jindal

RELATION FUNCTION

Q. For the function $f(x) = \frac{e^x+1}{e^x-1}$, if $n(d)$ denotes the number of integers which are not in its domain and $n(r)$ denotes the number of integers which are not in its range, then $n(d) + n(r)$ is equal to

(A) 2

(B) 3

(C) 4

(D) Infinite

$y = \frac{e^x+1}{e^x-1}$

$e^x \cdot y - y = e^x + 1$

$e^x(y-1) = 1+y$

$e^x = \left(\frac{y+1}{y-1}\right) > 0$

$\{-1, 0, 1\}$ $n(r) = 3$ $\pm$ $n(d) = 1$

4

$y = \frac{e^x+1}{e^x-1}$

$e^x - 1 \neq 0$

$e^x \neq 1$

$e^x \neq e^0$

$x \neq 0$

Nishant Jindal

RELATION FUNCTION

Q. Number of integral values of x in the domain of function

$$f(x) = \sqrt{\ln|\ln|x||} + \sqrt{7|x| - |x|^2 - 10}$$ **is equal to**

(A) 4

(B) 5

(C) 6

(D) 7

$|x|\ln|x| \geq 0$

$|\ln|x|| \geq 1$

~~$\ln|x| \leq -1 \cup \ln|x| \geq 1$~~

$|x| \leq e^{-1} \cup |x| \geq e$

$|x| \leq \frac{1}{e}$

$7|x| - |x|^2 - 10 \geq 0$

$|x|^2 - 7|x| + 10 \leq 0$

$(|x| - 2)(|x| - 5) \leq 0$

$2 \leq |x| \leq 5$

$x \in [-5, -e] \cup [e, 5]$

Nishant Jindal

RELATION FUNCTION

Q. If a polynomial function ' f ' satisfies the relation

$$\log_2(f(x)) = \log_2\left(2+\frac{2}{3}+\frac{2}{9}+\cdots.\infty\right)\cdot\log_3\left(1+\frac{f(x)}{f\left(\frac{1}{x}\right)}\right) \text{ and } f(10) = 1001$$

then the value of $f(20)$ is

(A) 2002

(B) 7999

(C) 8001

(D) 16001

$f(x) = 1 + x^3$

$n = 3$

$f(x) = 1 + x^n$

$f(10) = 1 + 10^n = 1001$

$\log_2 2\left(1+\frac{1}{3}+\frac{1}{3^2}-\cdots\right) \times \log_3\left(\frac{f(x)+f(1/x)}{f(1/x)}\right)$

$\left(\frac{1}{1-1/3}\right)$

$f(x)\cdot f\left(\frac{1}{x}\right) = f(x)+f(1/x)$

$\log_2 2\left(\frac{3}{2}\right)$

$\log_2 f(x) = \frac{\log 3}{\log 2} \times \log_3\left(\frac{f(x)+f(1/x)}{F(1/x)}\right) \Rightarrow \log_2 f(x) = \log_2\left(\frac{f(x)+f(1/x)}{F(1/x)}\right)$

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RELATION FUNCTION

Q. If the range of function $f(x)=\frac{x^2+x+c}{x^2+2x+c}, x \in R$ is $\left[\frac{5}{6},\frac{3}{2}\right]$ then c is equal to

(A) -4

(B) 3

(C) ~~4~~

(D) 5

$$y=\frac{x^2+x+c}{x^2+2x+c}$$

$$x^2(y-1)+x(2y-1)+(y-c)=0$$

$$D \geq 0$$

$$(2y-1)^2-4(y-1)((y-1) \geq 0$$

$$4y^2-4y+1-4c(y^2-2y+1) \geq 0$$

$$4y^2(1-c)-4y(1-2c)+(1-4c) \geq 0$$

$$4y^2(c-1)-4y(2(c-1)+(4c-1) \leq 0$$

$$12y^2-28y+15 \leq 0$$

$$\frac{5}{6} \leq y \leq \frac{3}{2}$$

$$(6y-5)(2y-3) \leq 0$$

$$12y^2-28y+15 \leq 0$$

$$\frac{4(c-1)}{12}=\frac{4(2(c-1)}{28}=\frac{4c-1}{15}$$

$$c=4$$

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RELATION FUNCTION

Q. If $x = \frac{4l}{1+l^2}$ and $y = \frac{2-2l^2}{1+l^2}$ where ' l ' is a parameter and range of $f(x, y) = x^2 - xy + y^2$ is $[a, b]$ then $(a + b)$ is equal to

(A) 4

(B) 6

(C) 8

(D) 12

$l = \tan\theta$

$x = 2 \times \frac{2l}{1+l^2}$ $\quad y = 2\left(\frac{1-l^2}{1+l^2}\right)$

$x = 2\sin 2\theta$ $\quad y = 2\cos 2\theta$

$= 4\sin^2 2\theta - 4\sin 2\theta \cos 2\theta + 4\cos^2 2\theta$

$= 4 - 2\sin 4\theta$

$-1 \le \sin 4\theta \le 1$

$2 \ge -2\sin 4\theta \ge -2$

$6 \ge 4 - 2\sin 4\theta \ge 2$

$a = 2, b = 6$

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RELATION FUNCTION

Q. If minimum and maximum values of $f(x) = 2|x-1| + |x+3| - 3|x-4|$ are m and M respectively then $(m+M)$ equals

draw graph.

(A) 0

(B) 1

(C) 2

(D) 3

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RELATION FUNCTION

Q. If the domain of $g(x)$ is $[3,4]$, then the domain of $g\left(\log_2\left(x^2+3x-2\right)\right)$ is

(A) $[-4,-1]\cup[2,7]$

(B) $[-3,2]$

(C) $[-6,-5]\cup[2,3]$

(D) $\left[\frac{3}{2},5\right]$

$3 \le x \le 4$

$3 \le \log_2(x^2+3x-2) \le 4$

$8 \le x^2+3x-2 \le 16$

And

$x^2+3x-2 \ge 8$

$x^2+3x-10 \ge 0$

$(x+5)(x-2) \ge 0$

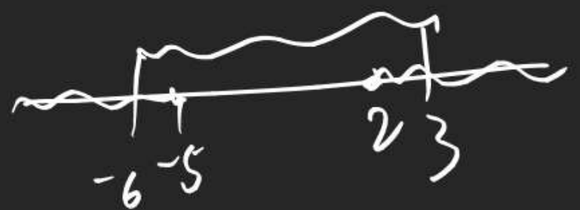
$x \le -5 \cup x \ge 2$

$x^2+3x-2 \le 16$

$x^2+3x-18 \le 0$

$(x+6)(x-3) \le 0$

$-6 \le x \le 3$



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RELATION FUNCTION

level

Q. Consider the function $f(x) = x + \sqrt{1 - x^2}$, then which of the following is/are ORRECT?

(A) Range of $f(x)$ is $[-1, \sqrt{2}]$.

$x^2 - 1 \le 0$

$-1 \le x \le 1$

$Y = x + \sqrt{1-x^2}$

$= -1 + \sqrt{x-(x)^2} = -1$

$Y = -0 + \sqrt{1-0}$

$= 1$

$Y = 1 + \sqrt{1-1^2}$

$= 1$

(B) f is many one.

(C) f is either even or odd.

(D) Range of $f(x)$ is identical to range of $g(x) = \sqrt{2}\cos\left(x - \frac{\pi}{4}\right)$.

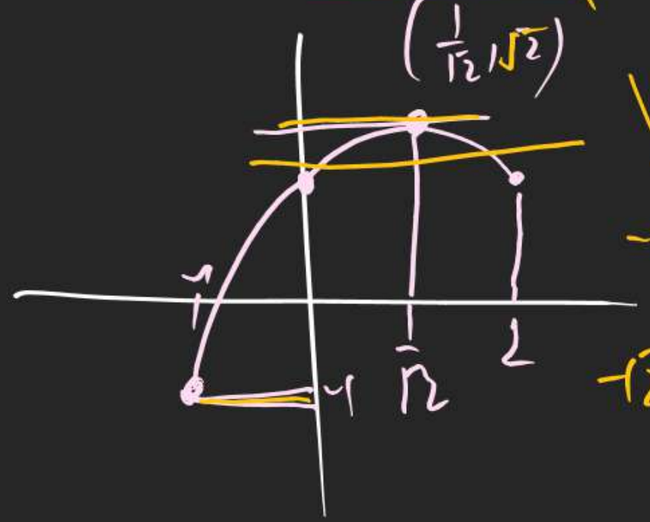
$\frac{dy}{dx} = 1 - \frac{2x}{2\sqrt{1-x^2}} = 0$

$\frac{x}{\sqrt{1-x^2}} = 1$

$x = \sqrt{1-x^2}$

$x^2 = 1 - x^2$

$2x^2 = 1 \Rightarrow x = \frac{1}{\sqrt{2}}$



$Y = \frac{1}{\sqrt{2}} + \sqrt{1 - \frac{1}{2}} = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}$

$= \sqrt{2}$

$-1 \le \sqrt{2}\cos\left(x - \frac{\pi}{4}\right) \le 1$

$-\sqrt{2} \le \sqrt{2}\cos(\quad) \le \sqrt{2}$

Nishant Jindal

$\{x+1\} = \{x\}$

RELATION FUNCTION

Q. Consider, $f(x) = \{x + [\log_2(2+x)]\} + \{x + [\log_2(2+x^2)]\} + \cdots \ldots + \{x + [\log_2(2+x^{10})]\}$

Identify the correct statement(s)

(A) $[f(e)] = 7$

(B) $f(\pi) = 20\pi - 60.$

(C) the number of solutions of the equation $f(x) = x$ is 9.

(D) the number of solutions of the equation $f(x) = x$ is 10.

[Note : $\{y\}$ and $[y]$ denotes the fractional part function and greatest integer function respectively.]

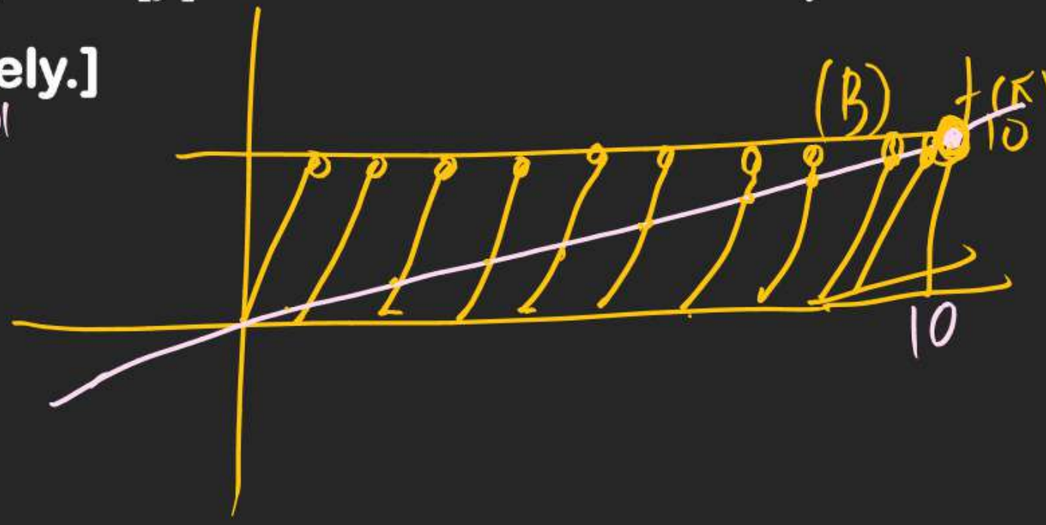
$= \{x\} + \{x\} \ldots\ldots \{x\}$

$f(x) = 10\{x\} \Rightarrow [f(e)] = 10\{e\} = 10\{2.718\}$

$= 10 \times .718$

$= [7.18] = 7$

$= 20 \times 3.14 - 60 = 62.8 - 60 = 2.8$



(B) $f(\pi) = 10\{\pi\} = 10\{3.14\} = 10 \times .14$

$= 1.4$

(D) $Y = 10\{x\}$

$0 \le \{x\} < 1$

$\le 10\{x\} < 10$