

$$\frac{1}{5} + i \frac{2}{5} - 4 - i \frac{5}{2}$$

$$\left(\frac{1}{5} - 4\right) + i\left(\frac{2}{5} - \frac{5}{2}\right) \quad \textcircled{20} \quad \left(\frac{1+i}{1-i}\right)^m \cdot (i)^m = 1$$

$$-\frac{19}{5} + i\left(-\frac{21}{10}\right)$$

$$9 \quad (a+b)^3 = - -$$

$$(1) \quad \text{~~~~~} \quad (12) \quad \left(1^2 + (-1)^{25}\right)$$

$$(-1 - 1')$$

$$(-1 - i)$$

$$(6) \quad x=0 \quad z=i y \rightarrow \text{Punely Imag. N.} \\ y \neq 0$$

Q If  $|z_1 + z_2|^2 = |z_1|^2 + |z_2|^2$  then  $\frac{z_1}{z_2}$  is

Punely Imag. = ?

$$|z_1 + z_2|^2 = |z_1|^2 + |z_2|^2$$

$$(z_1 + z_2) \cdot (\bar{z}_1 + \bar{z}_2) = |z_1|^2 + |z_2|^2$$

$$(z_1 + z_2)(\bar{z}_1 + \bar{z}_2) =$$

$$z_1 \bar{z}_1 + z_1 \bar{z}_2 + z_2 \bar{z}_1 + z_2 \bar{z}_2 = |z_1|^2 + |z_2|^2$$

$$|z_1|^2 + z_1 \bar{z}_2 + z_2 \bar{z}_1 + |z_2|^2 = |z_1|^2 + |z_2|^2$$

$$z_1 \bar{z}_2 = - \cancel{z_2 \bar{z}_1}$$

$$\frac{z_1}{z_2} = - \left( \frac{z_1}{z_2} \right) \rightarrow \frac{z_1}{z_2} \text{ is Punely Imag.}$$

Q If  $z$  is a N.S. such that

$\frac{z-1}{z+1}$  is purely Imag. then  $|z|=?$

$$z = -\bar{z}$$

$$\frac{z-1}{z+1} = -\left(\frac{\bar{z}-1}{\bar{z}+1}\right)$$

$$\frac{z-1}{z+1} = -\left(\frac{\bar{z}-1}{\bar{z}+1}\right)$$

$$z \cdot \bar{z} - \bar{z} + z - 1 = -(z \bar{z} - z + \bar{z} - 1)$$

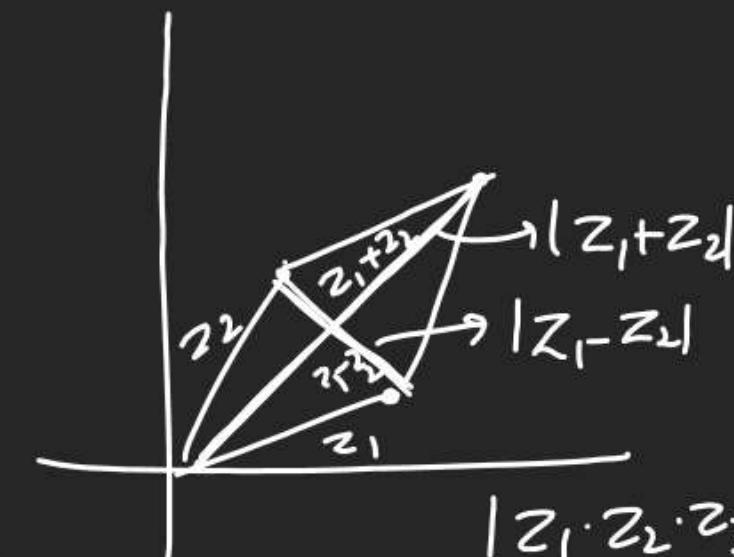
$$|z|^2 - 1 + 1 - 1 = -|z|^2 + 1 - |z| + 1$$

$$|z|^2 = 1$$

$$|z| = 1$$

Q What is  $|z_1 + z_2|$

&  $|z_1 - z_2|?$



$$Q z = (1+i)(1+2i)(1+3i)$$

then  $|z|=?$

$$|z| = \sqrt{(1+i)(1+2i)(1+3i)}$$

$$= \sqrt{(1+i)(1+2i)(1+3i)}$$

$$= \sqrt{2} \sqrt{5} \sqrt{10} = 10$$

Q  $z = \frac{(1+i)(1+2i)}{(1+3i)}$  then  $|z|=?$

$$|z| = \sqrt{\frac{(1+i)(1+2i)}{(1+3i)}}$$

$$= \frac{\sqrt{(1+i)(1+2i)}}{\sqrt{1+3i}}$$

$$= \frac{\sqrt{2} \sqrt{5}}{\sqrt{10}} = 1$$

Q If  $z = \frac{(1-i)^2}{(1+2i)}$  then  $|z|=?$

$$|z| = \frac{(1-i)^2}{(1+2i)} = \left| \frac{(-2i)}{1+2i} \right|$$

$$= \frac{|-2i|}{|1+2i|} = \frac{2}{\sqrt{5}}$$

Q  $z = 1 + 6s_2\theta + i8m_2\theta$

then  $|z|=?$ ;  $\theta \in \left(\pi, \frac{3\pi}{2}\right)$

$$|z| = \sqrt{(1+6s_2\theta)^2 + (8m_2\theta)^2}$$

$$= \sqrt{1+26s_2\theta + 6s_2^2\theta^2 + 64m_2^2\theta^2}$$

$$= \sqrt{2+26s_2\theta} = \sqrt{2} \sqrt{1+6s_2\theta}$$

$$\begin{aligned} &= \sqrt{2} \sqrt{26\theta^2} \quad \theta \in (180^\circ, 270^\circ) \\ &= 2 \sqrt{6\theta} \\ &= -2\sqrt{6\theta} \end{aligned}$$

Q If  $\left(\frac{3}{2} + i\frac{\sqrt{3}}{2}\right)^{50} = 3^{24}(x+iy)$

$$\text{Then } x^2 + y^2 \rightarrow |x+iy| = \sqrt{x^2+y^2}$$

$$\left| \left(\frac{3}{2} + i\frac{\sqrt{3}}{2}\right)^{50} \right| = \left| 3^{24}(x+iy) \right|$$

$$\left| \frac{3}{2} + i\frac{\sqrt{3}}{2} \right|^{50} = 3^{24} |x+iy|$$

$$\left( \sqrt{\frac{9}{4} + \frac{3}{4}} \right)^{50} = 3^{24} \sqrt{x^2+y^2}$$

$$3^{3^{25}} = 3^{24} \sqrt{x^2+y^2}$$

$$g = x^2+y^2$$

Q Prove that

$$|z_1+z_2|^2 = |z_1|^2 + |z_2|^2 + 2\operatorname{Re}(z_1\bar{z}_2)$$

LHS  $|z_1+z_2|^2$

$$= (z_1+z_2)(\bar{z}_1+\bar{z}_2)$$

$$= (z_1+z_2)(\bar{z}_1+\bar{z}_2)$$

$$= z_1\bar{z}_1 + z_1\bar{z}_2 + z_2\bar{z}_1 + z_2\bar{z}_2$$

$$= |z_1|^2 + |z_2|^2 + (z_1\bar{z}_2 + z_2\bar{z}_1)$$

$$= |z_1|^2 + |z_2|^2 + (z_1\bar{z}_2 + \bar{z}_1\bar{z}_2)$$

$$= |z_1|^2 + |z_2|^2 + 2\operatorname{Re}(z_1\bar{z}_2)$$

RHS

Q Find  $|z_1 - z_2|^2 = ?$

$$|z_1 - z_2|^2 = (z_1 - z_2)(\bar{z}_1 - \bar{z}_2)$$

$$= (z_1 - z_2)(\bar{z}_1 - \bar{z}_2)$$

$$= z_1 z_1 - z_1 \bar{z}_2 - z_2 \bar{z}_1 + z_2 \bar{z}_2$$

$$= |z_1|^2 + |z_2|^2 - (z_1 \bar{z}_2 + z_2 \bar{z}_1)$$

$$= |z_1|^2 + |z_2|^2 - (z_1 \bar{z}_2 + \bar{z}_1 z_2)$$

$$= |z_1|^2 + |z_2|^2 - 2R(z_1 z_2)$$

Q  $|z_1 + z_2|^2 + |z_1 - z_2|^2 = ?$

$$|z_1 + z_2|^2 = |z_1|^2 + |z_2|^2 + 2R\phi(z_1 \bar{z}_2)$$

$$\frac{|z_1 + z_2|^2 = |z_1|^2 + |z_2|^2 - 2R\phi(z_1 \bar{z}_2)}{|z_1 + z_2|^2 + |z_1 - z_2|^2 = 2(|z_1|^2 + |z_2|^2)}$$

Q Find  $(az_1 - bz_2)^2 + (bz_1 + az_2)^2 = ?$

Hence

$$(a^2 + b^2)(|z_1|^2 + |z_2|^2)$$

Q If  $|z - 2 + 3i| = |z - 1 + 2i|$  then locus of  $z$ ? Imp Note:-

$z = x + iy$  Put

$$|x + iy - 2 + 3i| = |x + iy - 1 + 2i|$$

$$|(x-2) + i(y+3)| = |(x-1) + i(y+2)|$$

$$\sqrt{(x-2)^2 + (y+3)^2} = \sqrt{(x-1)^2 + (y+2)^2}$$

$$(-2)^2 + (4+3)^2 = (-1)^2 + (4+2)^2$$

$$x^2 + y^2 - 4x + 6y + 13$$

$$= x^2 + y^2 - 2x + 4y + 5$$

$$2x - 2y = 8$$

$x - y = 4 \rightarrow \text{St. Line}$

Locus of given Express

$\alpha$  st. Line

$$Q. |z - (2-3i)| = |z - (1-2i)|$$

$\perp$  Bisector

Line  $\rightarrow x - y = 4$

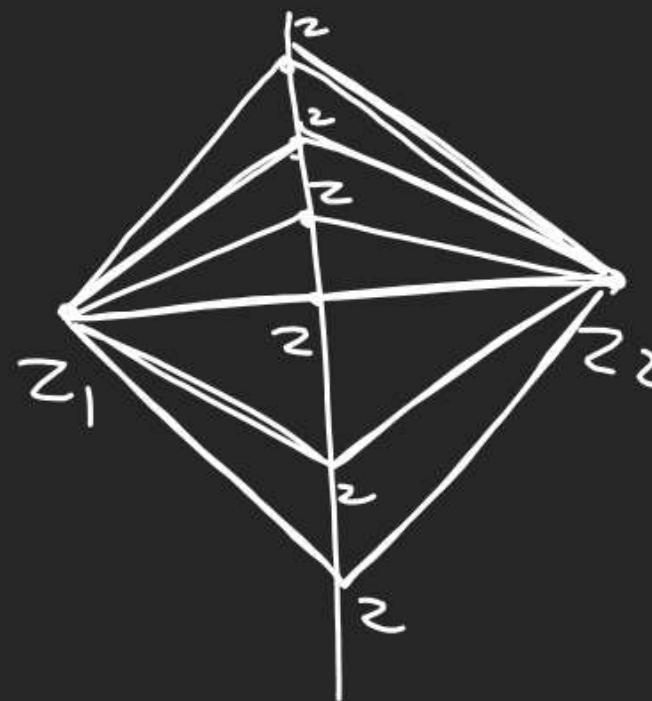
$z_1 (2, -3) \quad (3, -5) \quad (1, -2) \quad z_2$

(2)  $|z - z_1| = |z - z_2|$  given them.

It Rep. Locus of Line  $\perp$  to Bisector

Line joining  $z_1$  &  $z_2$

(3)



(4)  $|z - z_1|$  Rep. Dist. of  $z$  to  $z_1$

Q  $|z - 2 + 3i| = 5$  Then locus of  $z$ .

(1) Meaning of  $|z - 2 + 3i|$

$$\text{II } |z - (2 - 3i)|$$

= Dist. of  $z$  to Pt  $(2 - 3)$

2) Meaning of this Q.S.

$$|z - (2 - 3i)| = 5$$

3) dist of  $(2 - 3)$  to  $z = 5$   
fix pt var pt

Q  $|z - 2 + 3i| = 5$  ifnd  
Locus of  $z$ ?

dist of  $(2, -3)$  from  $z = \underline{5}$

In this is Pmly Meaningless.

No Locus.

$$(4) |x + iy - 2 + 3i| = 5$$

$$|(x-2) + i(y+3)| = 5$$

$$\sqrt{(x-2)^2 + (y+3)^2} = 5$$

$$(x-2)^2 + (y+3)^2 = 5^2$$

[i.e.  $\sqrt{(2, -3)}$  Rad = 5]

Q find  $z$  if

$$|z+1| = z+2(1+i)$$

$$|(x+i)y+1| = x+iy+2+2i$$

$$|(x+1)+iy| = (x+2) + i(y+2)$$

$$\frac{\sqrt{(x+1)^2+y^2}}{\text{Real}} = \frac{(x+2)}{\text{Real}} + \frac{i(y+2)}{\text{Imag.}}$$

$$\Rightarrow \sqrt{(x+1)^2+y^2} = (x+2) \quad |y+2=0$$

$$\sqrt{(x+1)^2+(-2)^2} = (x+2) \quad |y=-2$$

$$x^2+2x+1+4 = x^2+4x+4$$

$$2x=1$$

$$x=\frac{1}{2}$$

$$\therefore z = x+iy = \frac{1}{2} - 2i$$

$$Q \left| \left( \frac{\sqrt{3}+1}{2\sqrt{2}} \right) - i \left( \frac{\sqrt{3}-1}{2\sqrt{2}} \right) \right| = ?$$

$$\left| 6^{15^\circ} - i \sin 15^\circ \right|$$

$$\sqrt{6^2 15^\circ + \sin^2 15^\circ} = 1$$

$$Q \text{ If } |z_1|=1, |z_2|=2, |z_3|=3$$

$$|z_1+z_2+z_3|=1$$

$$\text{then } |z_1z_2z_3 + 4z_1z_3 + 9z_2z_3|=?$$

$$\text{Demand: } |z_1z_2z_3 \left( \frac{1}{z_1} + \frac{4}{z_2} + \frac{9}{z_3} \right)|$$

$$= |z_1||z_2||z_3| \left| \frac{1}{z_1} + \frac{4}{z_2} + \frac{9}{z_3} \right|$$

$$= (x_1x_2x_3) \left| \frac{\bar{z}_1}{|z_1|^2} + \frac{4\bar{z}_2}{|z_2|^2} + \frac{9\bar{z}_3}{|z_3|^2} \right|$$

$$\left[ \frac{1}{z} = \frac{\bar{z}}{|z|^2} \right], |\bar{z}|=|z|$$

$$6 \left| \bar{z}_1 + 4\bar{z}_2 + 9\bar{z}_3 \right| = 6 \left| \bar{z}_1 + \bar{z}_2 + \bar{z}_3 \right|$$

$$= 6 \left| \overrightarrow{z_1+z_2+z_3} \right| = 6|z_1+z_2+z_3|$$

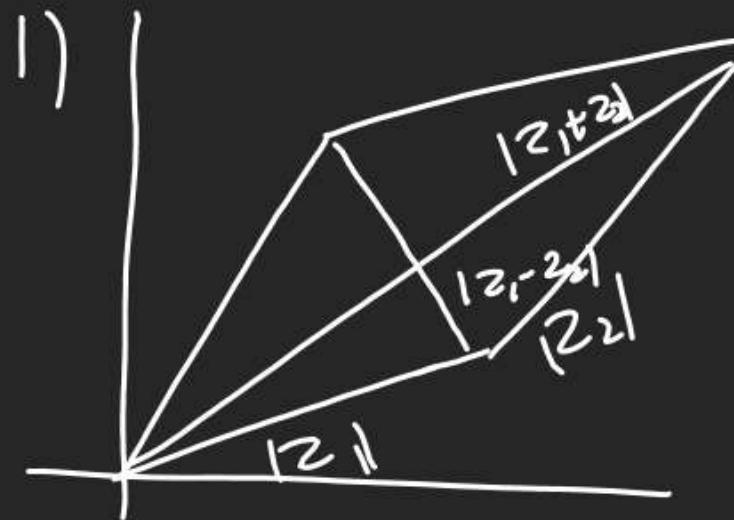
$$= 6 \times 1 = 6$$

$$Q \text{ for } z=a+ib \text{ (check } \frac{1}{z} = \frac{\bar{z}}{|z|^2})$$

$$\text{LHS} \Rightarrow \frac{1}{z} = \frac{1}{a+ib} \times \frac{a-ib}{a-ib} = \frac{a-ib}{a^2+b^2}$$

$$\therefore \frac{\bar{z}}{|z|^2} = \text{RHS}$$

## Triangular Inequality



$$|z_1| + |z_2| \geq |z_1 + z_2| \geq |z_1 - z_2| \geq |z_1| - |z_2|$$

2)  $-\sqrt{x^2+y^2} \leq z \leq \sqrt{x^2+y^2}$

3)  $\operatorname{Re}(z) \leq |z|$

(4)  $|z_1 + z_2|^2 = |z_1|^2 + |z_2|^2 + 2\operatorname{Re}(z_1 \bar{z}_2) \leq |z_1|^2 + |z_2|^2 + 2|z_1||z_2|$

$\operatorname{Re}(z) \leq |z|$

$|\bar{z}| = |z| = |-z|$

$|z_1 + z_2|^2 \leq |z_1|^2 + |z_2|^2 + 2|z_1||z_2|$

$|z_1 + z_2|^2 \leq (|z_1| + |z_2|)^2$

$|z_1 + z_2| \leq |z_1| + |z_2|$

$\leq |z_1| + |z_2|$

$|z_1 - z_2| \geq ||z_1| - |z_2||$

$||z_1| - |z_2|| \leq |z_1 - z_2| \leq |z_1 + z_2| \leq |z_1| + |z_2|$

(5)

Min. of  $|z_1 + z_2| = (|z_1| - |z_2|)$

Max. of  $|z_1 + z_2| = |z_1| + |z_2|$

Q If  $z$  is C.N. S.T.

$$|x| \leq 9 \Rightarrow -9 \leq x \leq 9$$

$|z-2+i| \leq 2$  then gr. & Least value of  $|z|$

$$|z-(2-i)| \leq 2$$

$$||z|-|2-i|| \leq |z-(2-i)| \leq 2$$

$$||z|-\sqrt{5}| \leq 2$$

$$-2 \leq |z| - \sqrt{5} \leq 2$$

$$-2 + \sqrt{5} \leq |z| \leq 2 + \sqrt{5}$$

Least

Q  $z$  is a C.N. S.T.

$$|z+4| \leq 3 \text{ find gr. value of } |z+1|$$

$$|z+4| \leq 3$$

$$||z+1|-3| \leq |(z+1)+(3)| \leq 3$$

$$|z+1|-3 \leq 3$$

$$-3 \leq |z+1|-3 \leq 3$$

$$0 \leq |z+1| \leq \frac{6}{|z|}$$

Max.

Q If  $|z - \frac{6}{z}| = 4$  find

gr. value of  $|z|$ ?

$$|z - \frac{6}{z}| = 4 \text{ (given)}$$

$$|z - \frac{6}{z}| = \left| z - \frac{6}{z} \right|$$

$$4 - \left| \frac{6}{z} \right| \leq |z|$$

$$-|z| \leq 4 - \frac{6}{|z|} \leq |z|$$

$$\textcircled{1} \quad 4 - \frac{6}{|z|} \leq |z|$$

$$|z| + \frac{6}{|z|} \geq 4$$

$$|z|^2 - 4|z| + 6 \geq 0$$

Solve for  $|z|$