Q Angle betⁿ $x^3+y^3+x+2y=0$ & $xy+2x=y$ at origin.

(0,0) $3x^2+3y^2\frac{dy}{dx}+1+2\frac{dy}{dx}=0 \Rightarrow \frac{dy}{dx}=\frac{-1}{2}$

(0,0) $x\frac{dy}{dx}+y+2=\frac{dy}{dx} \Rightarrow \frac{dy}{dx}=2$

$$\tan\theta=\left|\frac{-\frac{1}{2}-2}{1+-\frac{1}{2}\times 2}\right| \Rightarrow \theta=\frac{\pi}{2}$$

Q Find angle at which curve

$x^4-2xy^2+y^2+3x-3y=0$

cuts x Axis at (0,0)

(0,0) $\cancel{4x^3}-\cancel{4x}y\frac{dy}{dx}-\cancel{2y^2}+\cancel{2y\frac{dy}{dx}}+3-3\frac{dy}{dx}=0$

$\frac{dy}{dx}=1 \Rightarrow \theta=\frac{\pi}{4}$

Q3 If curve $a_1x^2+b_1y^2=1$ & $a_2x^2+b_2y^2=1$ are orthogonal find condⁿ?

Imp

1) $a_1x^2+b_1y^2=1$

$a_2x^2+b_2y^2=1$

$(a_1-a_2)x^2+(b_1-b_2)y^2=0$

$(a_1-a_2)x^2=-(b_1-b_2)y^2$

$\frac{y^2}{x^2}=-\frac{(a_1-a_2)}{(b_1-b_2)}$

(2) $2a_1x+2b_1y\cdot\frac{dy}{dx}=0$

$m_1=\frac{dy}{dx}=-\frac{a_1x}{b_1y}$

$2a_2x+2b_2y\cdot\frac{dy}{dx}=0$

$m_2=\frac{dy}{dx}=-\frac{a_2x}{b_2y}$

(3) $m_1m_2=-1$

$\frac{a_1a_2x^2}{b_1b_2y^2}=-1 \Rightarrow -\frac{a_1a_2}{b_1b_2}=\frac{y^2}{x^2}$

(4) $-\frac{a_1a_2}{b_1b_2}=-\frac{(a_1-a_2)}{(b_1-b_2)}$

$\frac{b_1-b_2}{b_1b_2}=\frac{a_1-a_2}{a_1a_2}$

याद

condⁿ of orthogonality $\boxed{\left(\frac{1}{b_1}-\frac{1}{b_2}\right)=\left(\frac{1}{a_1}-\frac{1}{a_2}\right)}$

for $a_1x^2+b_1y^2=1$ & $a_2x^2+b_2y^2=1$

Nishant Jindal

Q If Curve $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ & $\frac{x^2}{l^2}-\frac{y^2}{m^2}=1$

are intersecting Ortogonal.

Nishant Jindal

Q If Curve $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ & $\frac{x^2}{l^2}-\frac{y^2}{m^2}=1$

are Intersecting orthogonally

find Cond^n?

$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ | $\frac{x^2}{l^2}-\frac{y^2}{m^2}=1$

$a_1x^2+b_1y^2=1$ | $a_2x^2+b_2y^2=1$

Cond^n $\frac{1}{a_1}-\frac{1}{a_2}=\frac{1}{b_1}-\frac{1}{b_2}$

$\frac{1}{1/a^2}-\frac{1}{1/l^2}=\frac{1}{1/b^2}-\frac{1}{-1/m^2}$

$a^2-l^2=b^2+m^2$

$a^2-b^2=l^2+m^2$

Q S.T.

$\frac{x^2}{a^2+K_1}+\frac{y^2}{b^2+K_1}=1$ & $\frac{x^2}{a^2+K_2}+\frac{y^2}{b^2+K_2}=1$

are Intersecting orthogonally?

$a_1x^2+b_1y^2=1$ $\qquad$ $a_2x^2+b_2y^2=1$

If given Curve are Orthogonal then they must satisfy Cond^n

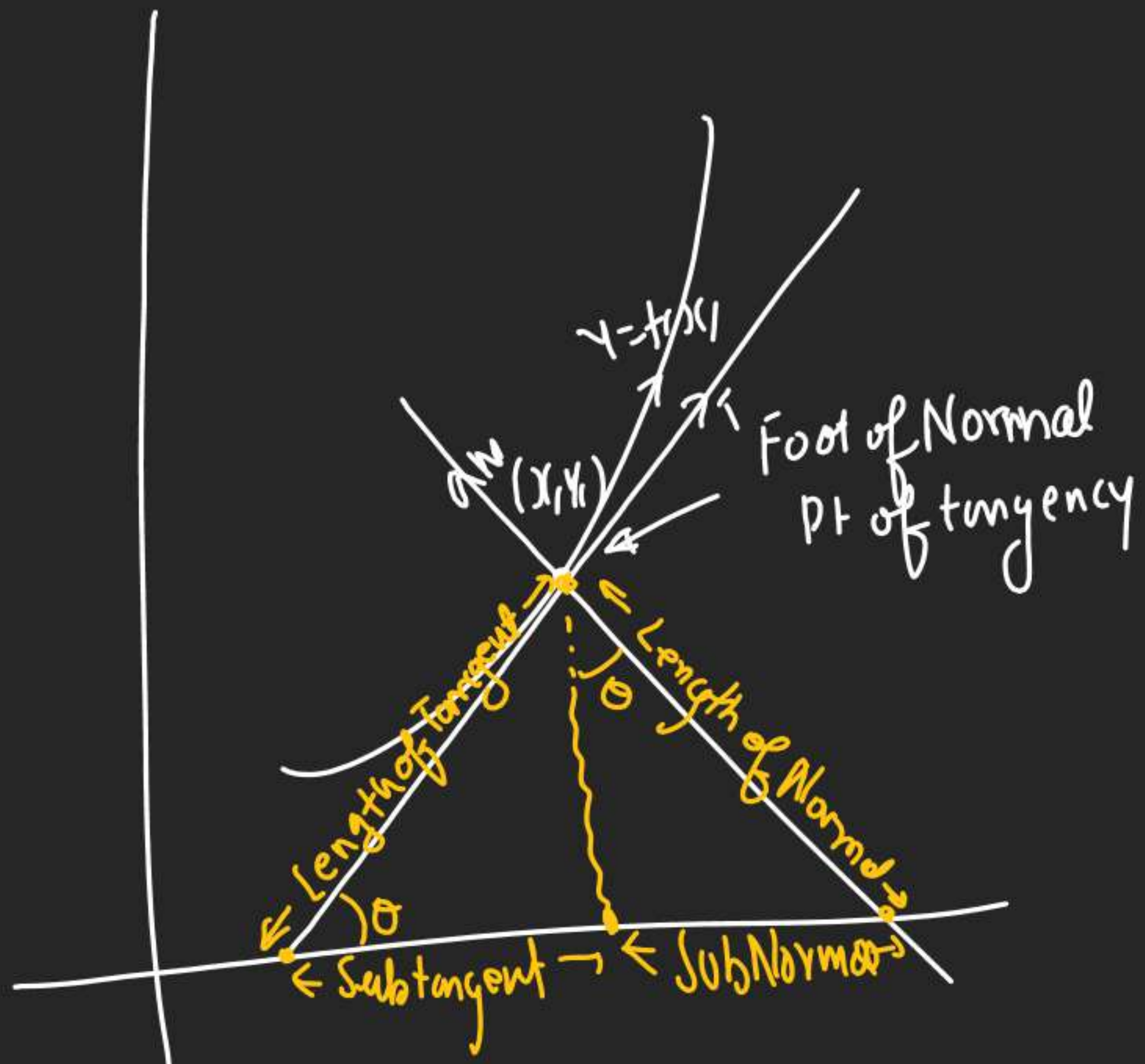
$\frac{1}{\frac{1}{a^2+K_1}}-\frac{1}{\frac{1}{a^2+K_2}}=\frac{1}{\frac{1}{b^2+K_1}}-\frac{1}{\frac{1}{b^2+K_2}}$

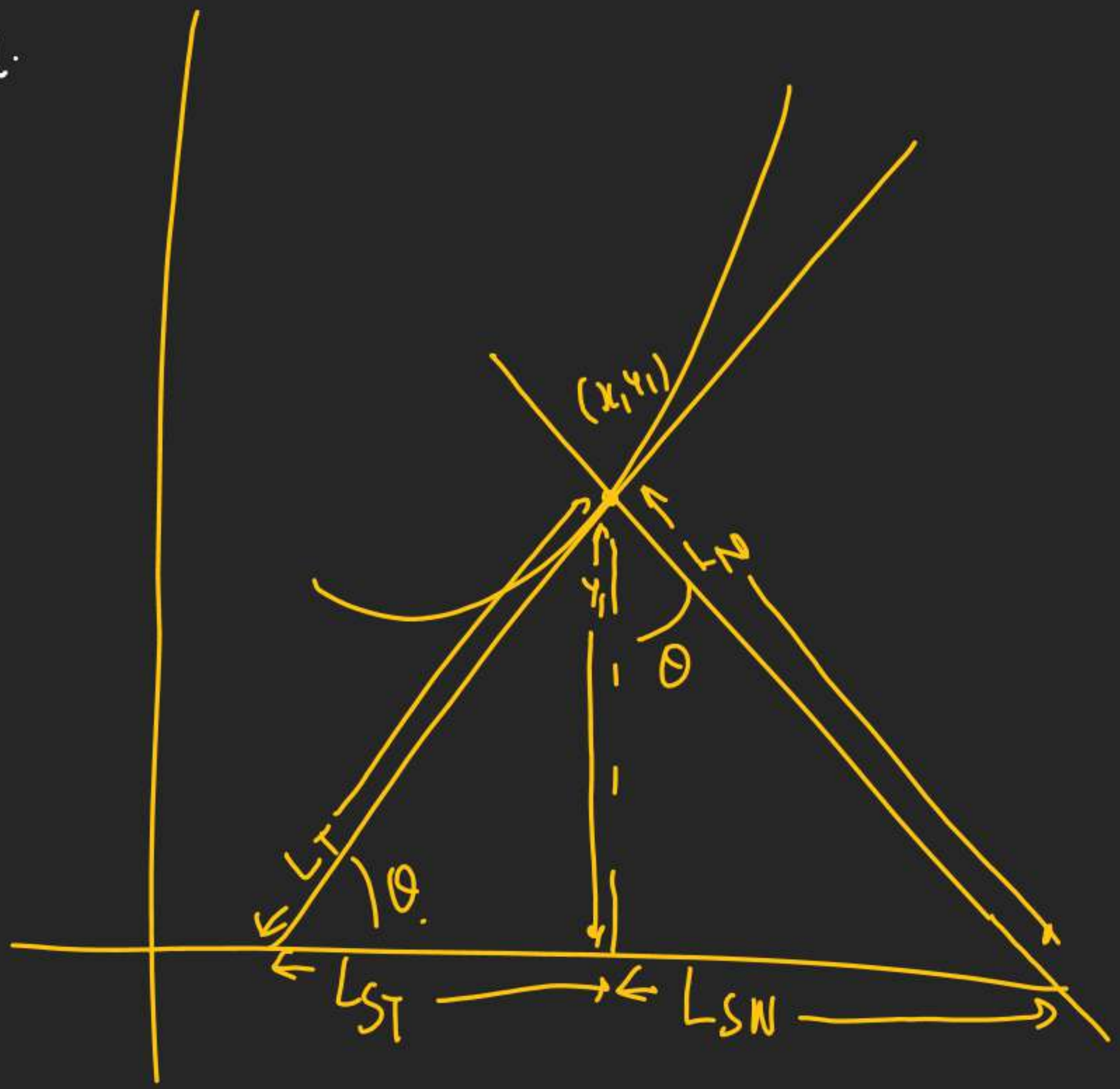
$K_1-K_2=K_1-K_2$

Satisfying

Nishant Jindal

Length of Tangent, Normal, Subtangent, Subnormal.





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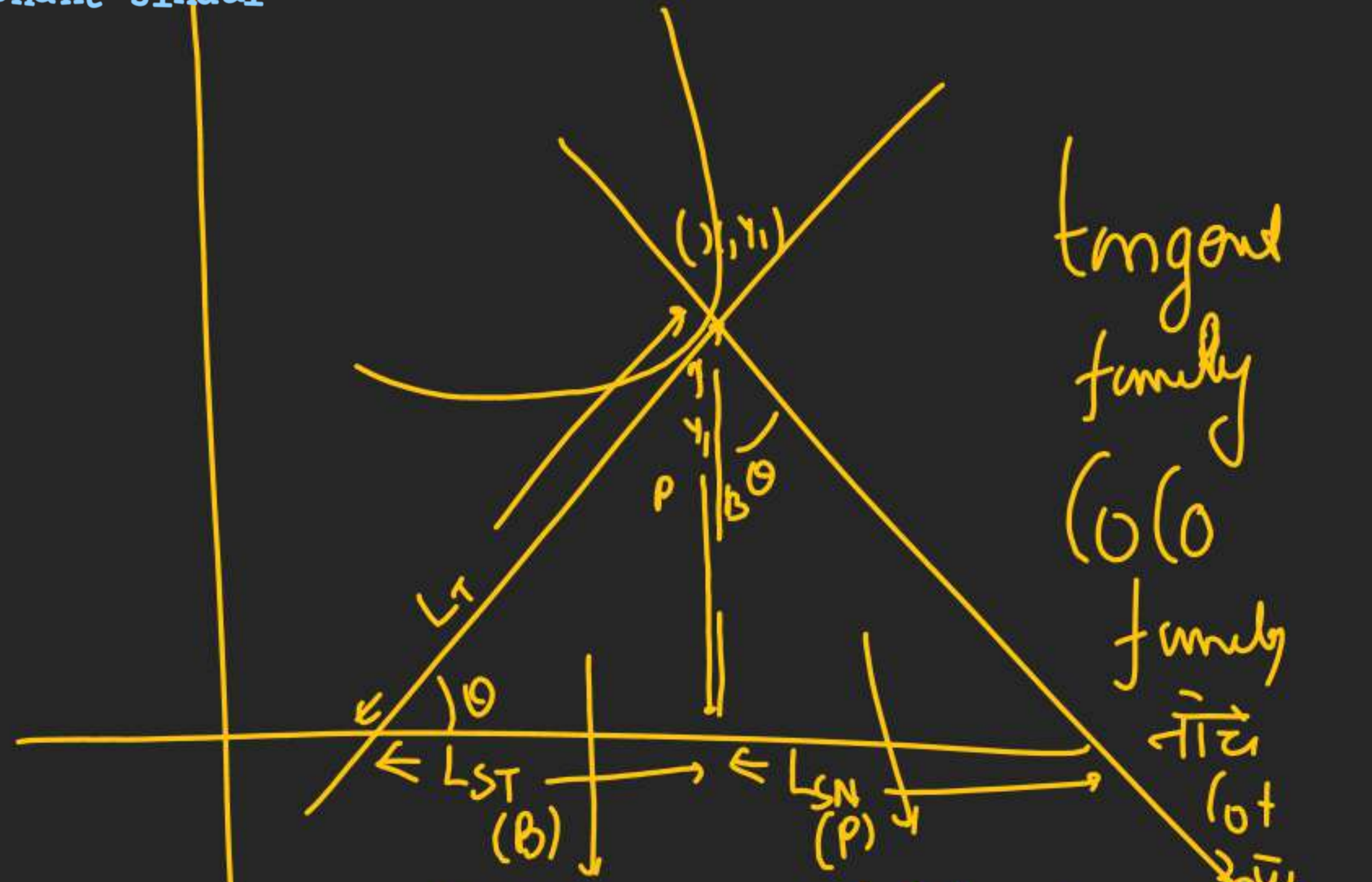
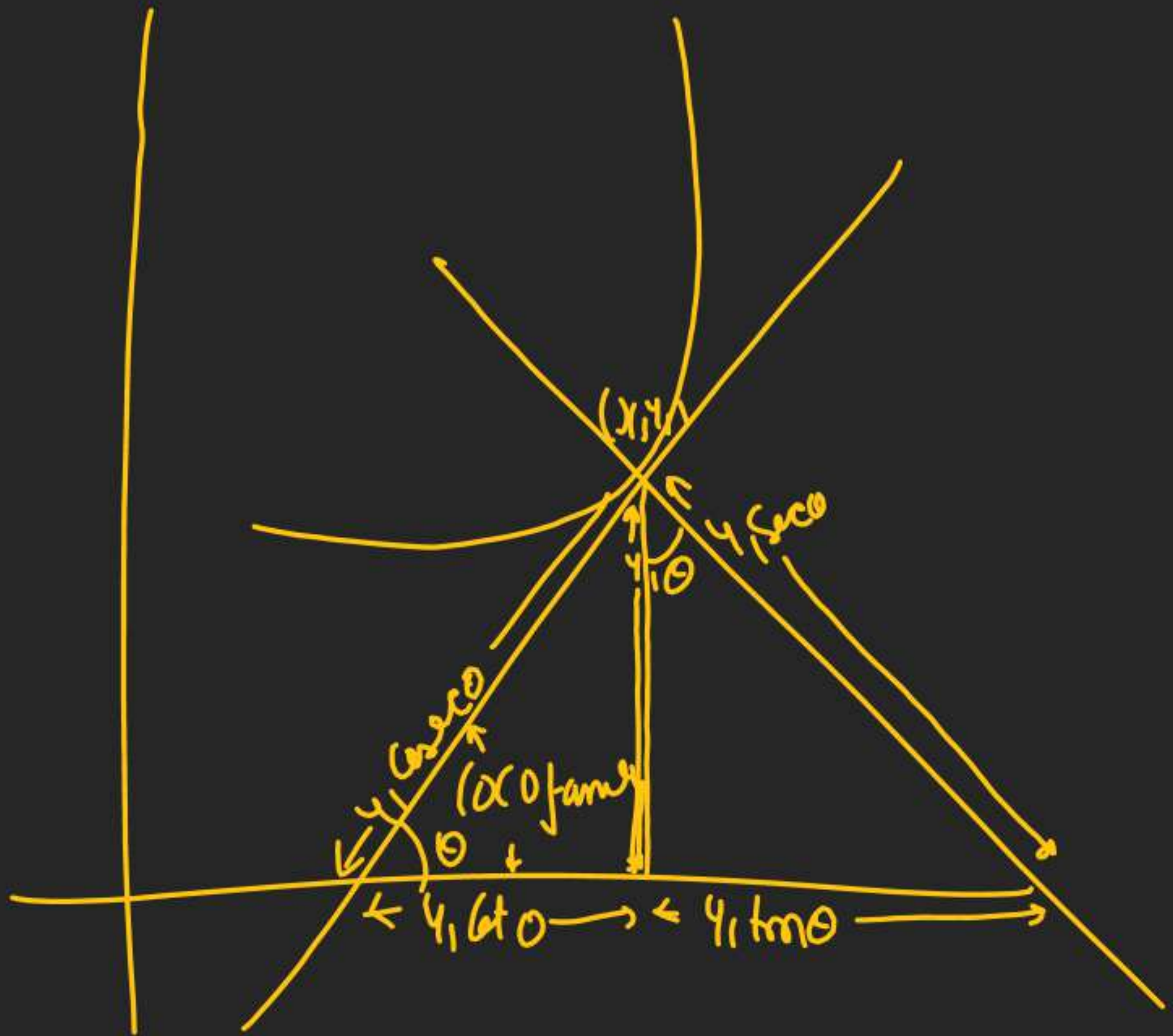


$$\frac{y_1}{L_{ST}} = \tan\theta$$

$$L_{ST} = \frac{y_1}{\tan\theta}$$

$$\frac{L_{SN}}{y_1} = \tan\theta$$

$$L_{SN} = y_1 \tan\theta$$



Nishant Jindal

Q Find length of SN for $y^2 = 4ax$ at any pt on curve.

let Pt. (x, y) | $2y\frac{dy}{dx} = 4a$

$L_{SN} = y\tan\theta = y \cdot \frac{dy}{dx}$

$= \frac{4a}{2}$

$= 2a$

Result

$\boxed{L_{SN} \text{ for } y^2 = 4ax \text{ is half of its L.R.}}$

Q 7 L_{ST} for $\sqrt{x} + \sqrt{y} = 3$ at $(4, 1)$ x_1, y_1

$L_{ST} = y_1 \cot\theta = \frac{y_1}{\frac{dy}{dx}}$

diff

$\frac{1}{2\sqrt{x}} + \frac{1}{2\sqrt{y}} \cdot \frac{dy}{dx} = 0$

$\left.\frac{dy}{dx}\right|_{(4,1)} = -\frac{\sqrt{y}}{\sqrt{x}} = -\frac{\sqrt{1}}{\sqrt{4}} = -\frac{1}{2}$

$L_{ST} = \frac{1}{-1/2} = |-2| = 2$

Q 8 find length of tangent for $y = x^3 + 3x^2 + 4x - 1$ at $x = 0$

1) Pt $\to$ $x = 0, y = -1 \Rightarrow (0, -1)$ x_1, y_1

2) $L_T = y_1 \operatorname{cosec}\theta = y_1\sqrt{1 + \frac{1}{\tan^2\theta}}$

$= y_1\sqrt{1 + \frac{1}{\left(\frac{dy}{dx}\right)^2}} = -1\sqrt{1 + \frac{1}{(4)^2}}$

$= \left|-1\sqrt{17/16}\right|$

$= \frac{\sqrt{17}}{4}$ A.

$\left.\frac{dy}{dx}\right|_{x=0} = 3x^2 + 6x + 4 = 4$

Nishant Jindal

Q9 Find Length of Normal to $y = a(t+\sin t)$, $x = a(1-\cos t)$

1) $\frac{dy}{dx} = \frac{\cancel{a}(1+\cos t)}{\cancel{a}(\sin t)} = \frac{\cancel{2}\cos^2 t/2}{\cancel{2}\sin t/2 \cdot \cancel{\cos t/2}}$

$= \cot t/2$

(2) $L_N = y_1 \sec\theta$

$= y_1\sqrt{1+\tan^2\theta} = y_1\sqrt{1+\left(\frac{dy}{dx}\right)^2}$

$= \left|a(t+\sin t)\sqrt{1+\cot^2 t/2}\right|$

Q10 L_{ST} at any pt on curve $x^m y^n = a^{m+n}$ $\qquad \left|\frac{n \cdot x}{m}\right|$

1) $m\ln x + n\ln y = m+n\ln a$

$\frac{m}{x} + \frac{n}{y}\cdot\frac{dy}{dx} = 0$

$\frac{dy}{dx} = -\frac{m}{x}\cdot\frac{y}{n}$

(2) $L_{ST} = \frac{y_1}{\frac{dy}{dx}} = \frac{y}{-\frac{my}{nx}}$

$= \left|\frac{nx}{m}\right|$

Q11 L_{ST} at $x=a$ for $ay^2 = (a+x)^2(3a-x)$, $a>0$

① Pt. $ay^2 = (a+a)^2(3a-a) \Rightarrow \cancel{a}y^2 = 4a^2 \times 2\cancel{a}$

$y = 2\sqrt{2}a$

$(a, 2\sqrt{2}a)$

(2) $2ay\cdot\frac{dy}{dx} = 2(a+x)(3a-x)+(a+x)^2(-1)$

$x=a$, $y=2\sqrt{2}a$

$4\sqrt{2}a^2\frac{dy}{dx} = 2\times 2a\times 2a - 4a^2 = 4a^2$

$\frac{dy}{dx} = \frac{1}{\sqrt{2}}$

(3) $L_{ST} = \frac{y_1}{\frac{dy}{dx}} = \frac{2\sqrt{2}a}{\frac{1}{\sqrt{2}}} = 4a$

Nishant Jindal

Q_{12} Ordinate of $y = \frac{a}{2}\left(e^{\frac{x}{a}}\right.$

Nishant Jindal

Q12 Ordinate of $y=\frac{a}{2}\left(e^{\frac{x}{a}}+e^{-\frac{x}{a}}\right)$ is Geometric Mean of L_N & Quantity A then find A.

1) y is G.M of L_N & A

$$y=\sqrt{L_N\cdot A} \Rightarrow y^2 = L_N\cdot A$$

2) $$L_N = y\sec\theta = y\sqrt{1+\left(\frac{dy}{dx}\right)^2}$$

$$= y\sqrt{1+\frac{e^{\frac{2x}{a}}+e^{-\frac{2x}{a}}-2}{4}} = \frac{y}{2}\sqrt{\left(e^{x/a}+e^{-x/a}\right)^2}$$

3) $$\frac{dy}{dx} = \frac{\cancel{a}}{2}\left(e^{\frac{x}{a}}\cdot\frac{1}{\cancel{a}} + e^{-\frac{x}{a}}\cdot -\frac{1}{\cancel{a}}\right) = \left(\frac{e^{\frac{x}{a}}-e^{-\frac{x}{a}}}{2}\right)$$

4) Using $y^2 = L_N\cdot A \Rightarrow$ $$\frac{a^{\cancel{2}}}{\cancel{4}}\cancel{\left(e^{\frac{x}{a}}+e^{-\frac{x}{a}}\right)^2} = \frac{\cancel{a}}{\cancel{4}}\cancel{\left(e^{\frac{x}{a}}+e^{-\frac{x}{a}}\right)^2}\cdot A$$

$$A = a$$

Q If $L_{SN} = L_{ST}$ at $(3,4)$ for $y=f(x)$ & tangent meeting Co axes at A, B & O is origin, then find max area of ΔOAB.

① $L_{SN} = L_{ST} \Rightarrow y\tan\theta = \frac{y}{\tan\theta}$

$$\tan^2\theta = 1 \Rightarrow \left(\frac{dy}{dx}\right)^2 = 1$$

$$\frac{dy}{dx} = \pm 1$$

② EOT at $(3,4)$

$(y-4) = 1(x-3)$

$x - y = -1$

$\frac{1}{2}\times 1\times 1 = \frac{1}{2}$

$y-4 = -(x-3)$

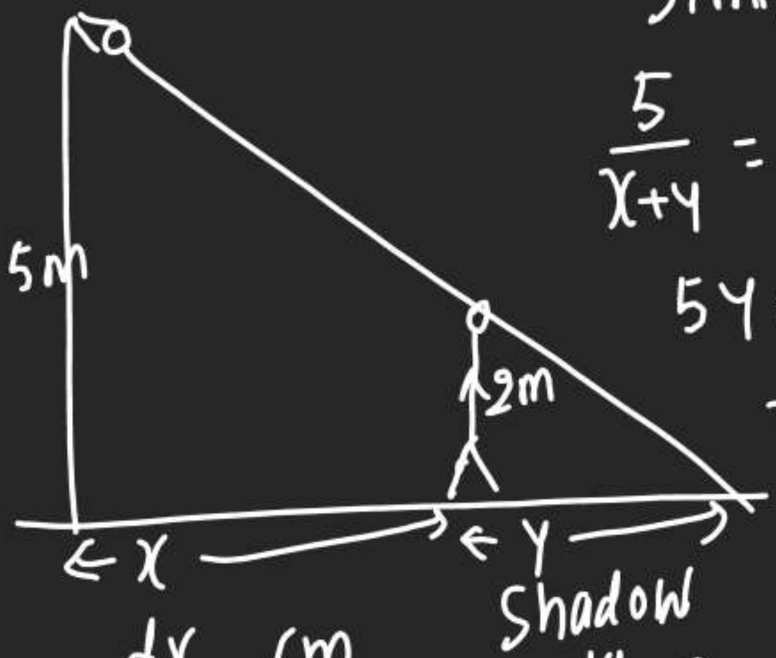
$x + y = 7$

Max $\Delta = \frac{49}{2}$

Nishant Jindal

Rate Measurer

Q A man 2m high is walking away
13. from lamp post of 5m ht. at 6 m/min speed. Find rate of increase of its shadow.



$\frac{dx}{dt} = \frac{6m}{min}$ $\qquad \frac{dy}{dt} = ?$

Similar $\triangle$

$\frac{5}{x+y} = \frac{2}{y}$

$5y = 2x + 2y$

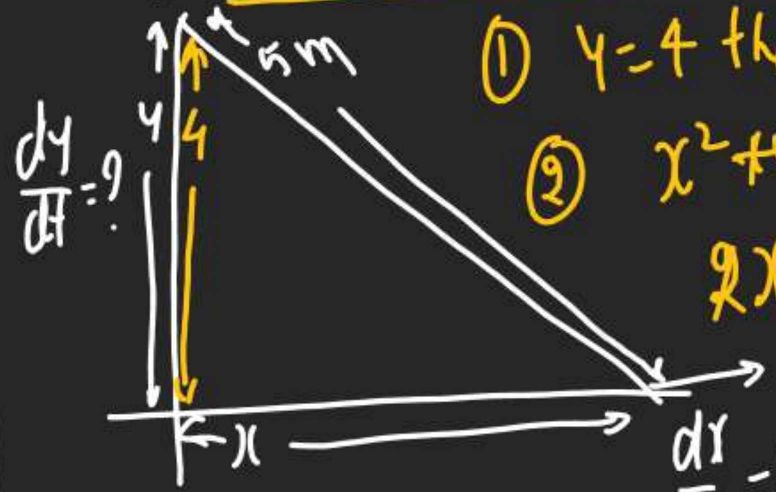
$3y = 2x$

$3 \cdot \frac{dy}{dt} = \frac{2dx}{dt}$

$\cancel{3}\frac{dy}{dt} = 2 \times \cancel{6}2$

$\frac{dy}{dt} = 4 m/min$

Q A ladder 5m long is leaning against wall. Bottom of the ladder is pulled away @ 2m/sec. How fast the ht of ladder on wall is decreasing when ladder's ht is 4m.



$\frac{dx}{dt} = \frac{2m}{sec}$

① $y = 4$ then $x = 3$

② $x^2 + y^2 = 25$

$\cancel{2}x \cdot \frac{dx}{dt} + \cancel{2}y \cdot \frac{dy}{dt} = 0$

$3 \cdot 2 + 4 \cdot \frac{dy}{dt} = 0$

$\frac{dy}{dt} = \frac{-6}{4} = -1.5$

Nishant Jindal

Q Water is dripping out from a
15 Conical vessel having semi vertical angle $\frac{\pi}{3}$ @ $\frac{4\,cm^3}{sec}$
When Slant ht of water is 4 cm find the Rate of change of Slant ht.

$l = 4$

$\frac{dV}{dt} = 4\,cm^3/sec$

$V = \frac{\pi}{3} r^2 h$

$h = l\cos 60^\circ = \frac{l}{2}$

$r = l\sin 60^\circ = \frac{\sqrt{3}\,l}{2}$

$V = \frac{\pi}{3} \cdot \frac{3}{4} l^2 \cdot \frac{l}{2} = \frac{3\pi l^3}{8}$

$\frac{dV}{dt} = \frac{3\pi}{8 \cdot 3}\, 3l^2 \cdot \frac{dl}{dt}$

$4 = \frac{3\pi}{\cancel{3}\,\cancel{8}_2} \times 3(4)^2 \cdot \frac{dl}{dt} \rightarrow \frac{dl}{dt} = \frac{2}{3\pi}$

AOD

T&N → 94 Qs

R.M + Appr...

84/80 Qs

40 Qs

Vector → DPP 1,2