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1. $$\int_0^{\frac{\pi}{2}^-} \frac{\sec^2 x\,dx}{(\tan x+\sec x)^n}, \quad n>1$$

$$\tan x+\sec x=t$$
$$\sec x-\tan x=\frac{1}{t}$$
$$\tan x=\frac{1}{2}\left(t-\frac{1}{t}\right)$$

$$\frac{1}{2}\int_1^\infty\left(1+\frac{1}{t^2}\right)\frac{1}{t^n}\,dt=\frac{1}{2}\left\{\left.\frac{1}{(-n+1)t^{n-1}}\right|_1^\infty+\left.\frac{1}{-(n+1)t^{n-1}}\right|_1^\infty\right\}$$

$$=\frac{1}{2}\left[\frac{1}{n-1}+\frac{1}{n+1}\right]=\frac{n}{n^2-1}$$

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2.

$$\int_a^b \frac{x^{n-1}\left((n-2)x^2 + (n-1)(a+b)x + nab\right) dx}{(x+a)^2(x+b)^2}$$

$$\int_a^b \frac{nx^{n-1}(x+a)(x+b) - x^n\left(2x+(a+b)\right)}{\left((x+a)(x+b)\right)^2}\, dx$$

$$\int_a^b \frac{d}{dx}\left(\frac{x^n}{(x+a)(x+b)}\right) dx = \left.\frac{x^n}{(x+a)(x+b)}\right|_a^b$$

$$= \frac{b^{n-1} - a^{n-1}}{2(a+b)}$$

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3: Let $I = \int_0^{\frac{\pi}{2}} \frac{\cos x\, dx}{(a\cos x + b\sin x)}$, $J = \int_0^{\frac{\pi}{2}} \frac{\sin x\, dx}{(a\cos x + b\sin x)}$

where $a>0,\ b>0$

find I, J.

$$aI + bJ = \frac{\pi}{2} \quad \text{—— (1)}$$

$$bI - aJ = \int_0^{\pi/2} \frac{(b\cos x - a\sin x)\,dx}{a\cos x + b\sin x} = \ln|a\cos x + b\sin x|\Big]_0^{\pi/2}$$

$$bI - aJ = \ln\left(\frac{b}{a}\right) \quad \text{—— (2)}$$

4. $$\int_0^1 e^{\sqrt{e^x}}\,dx + 2\int_e^{e^{\sqrt{e}}} \ln(\ln x)\,dx$$

$$f(x) = e^{\sqrt{e^x}} \qquad = \int_0^1 f(x)\,dx + \int_e^{e^{\sqrt{e}}} f^{-1}(x)\,dx \quad ✓$$

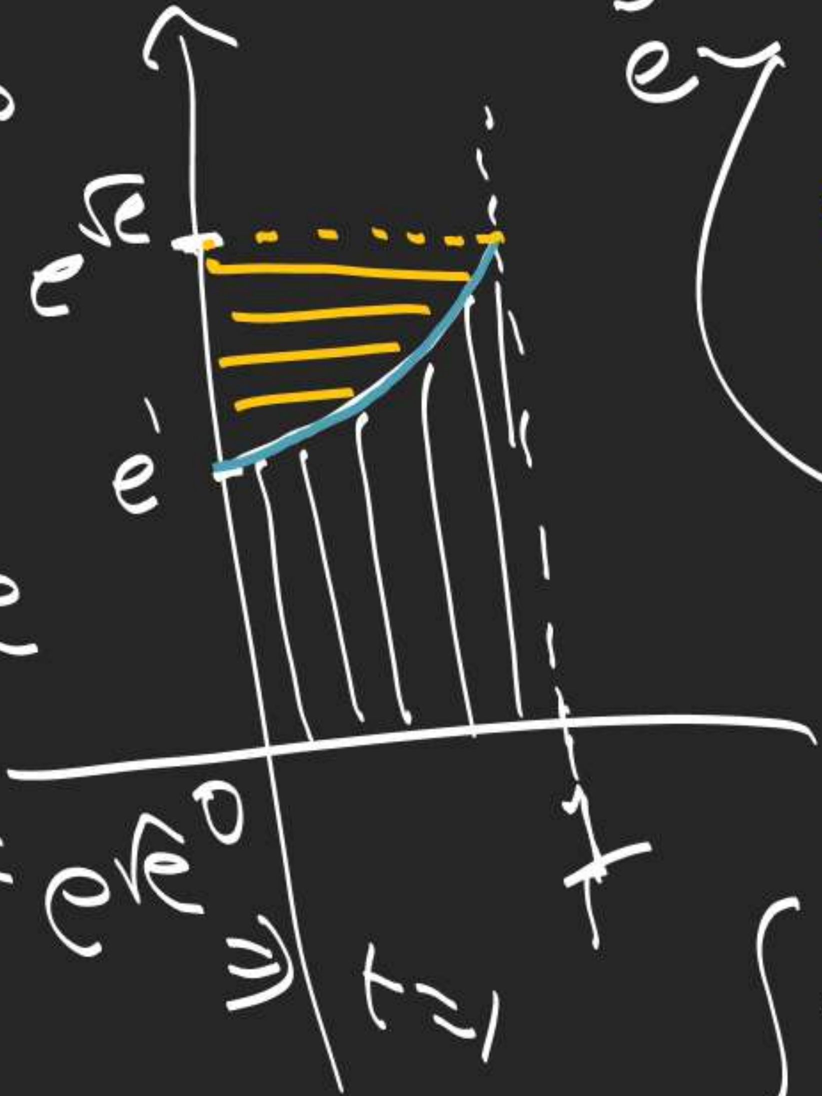
$$e^{\sqrt{e^{f^{-1}(x)}}} = x \qquad \int_0^1 \left(f(x)+x f'(x)\right)dx$$

$$\sqrt{e^{f^{-1}(x)}} = \ln x \qquad x f(x)\Big|_0^1 = 1 f(1) = e^{\sqrt{e}}$$

$$e^{f^{-1}(x)} = \ln^2 x$$

$$f^{-1}(x) = \ln\left(\ln^2 x\right)$$

$$f(t) = e \qquad f(t) = e^{\sqrt{e^0}} \Rightarrow t = 1$$



$$= 1 \times e^{\sqrt{e}}$$

$$f^{-1}(x) = t$$

$$x = f(t)$$

$$dx = f'(t)\,dt$$

$$\int_0^1 t\, f'(t)\,dt = \int_0^1 x f'(x)\,dx$$

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5. If $\lim_{n\to\infty} \int_{-\sqrt[3]{a}}^{\sqrt[3]{a}} \left(1-\frac{t^3}{n}\right)^n t^2\,dt = \frac{2\sqrt{2}}{3}$, $n \in N$, find 'a'.

$$-\frac{n}{3}\int_{-\sqrt[3]{a}}^{\sqrt[3]{a}} \left(1-\frac{t^3}{n}\right)^n \left(-\frac{3t^2}{n}\right) dt = -\frac{n}{3(n+1)}\left(1-\frac{t^3}{n}\right)^{n+1}\Bigg|_{-\sqrt[3]{a}}^{\sqrt[3]{a}}$$

$$e^{-\frac{a(n+1)}{n}}$$

$$\lim_{n\to\infty} -\frac{n}{3(n+1)}\left[\left(1-\frac{a}{n}\right)^{n+1} - \left(1+\frac{a}{n}\right)^{n+1}\right]$$

$$-\frac{1}{3}\left(e^{-a} - e^{a}\right) = \frac{2\sqrt{2}}{3}$$

$a = ?$

$\boxed{\text{Monotonocity Ex-5 (Remaining)}}$

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$$\lim_{n\to\infty}\int_{-\sqrt[3]{a}}^{\sqrt[3]{a}}\left(1-\frac{t^3}{n}\right)^n t^2\,dt = \frac{2\sqrt{2}}{3}$$