HOMEWORK

Link to View Video Solution: Click Here

1. The domain of the function $f(x)=\frac{\sqrt{-\log_{0.3}(x-1)}}{\sqrt{x^2+2x+8}}$ is

(A) (1,4) (B) (−2,4) (C) (2,4) (D) [2, ∞)

Ans. **(D)**

Sol. Given,

$f(x)=\frac{\sqrt{-\log_{0.3}(x-1)}}{\sqrt{-x^2+2x+8}}$

For given function to be defined,

$-\log_{0.3}(x-1)\geq 0 \Rightarrow \log_{103}(x-1)\geq 0 \Rightarrow x-1\geq 1 \Rightarrow x\geq 2$

and, $-x^2+2x+8>0 \Rightarrow x^2-2x-8<0 \Rightarrow (x+2)(x-4)<0 \Rightarrow -2<x<4$

Combining above two we get required domain [2,4)

2. The domain of the function $f(x)=\log_{1/2}\left(-\log_2\left(1+\frac{1}{\sqrt[4]{x}}\right)-1\right)$ is

(A) $0<x<1$ (B) $0<x\leq 1$ (C) $x\geq 1$ (D) null set

Sol. f(x) is defined if $-\log_{1/2}\left(1+\frac{1}{x^{\frac{1}{4}}}\right)-1>0$

$\Rightarrow \log_{1/2}\left(1+\frac{1}{x^{\frac{1}{4}}}\right)<-1$ $\Rightarrow 1+\frac{1}{x^{\frac{1}{4}}}>\left(\frac{1}{2}\right)^{-1}$

$\Rightarrow \frac{1}{x^{\frac{1}{4}}}>1$ $\Rightarrow 0<x<1$

3. If $q^2-4pr=0, p>0$, then the domain of the function,

$f(x)=\log(px^3+(p+q)x^2+(q+r)x+r)$ is

(A) $R-\left\{-\frac{q}{2p}\right\}$ (B) $R-\left[(-\infty,-1]\cup\left\{-\frac{q}{2p}\right\}\right]$

(C) $R-\left[(-\infty,-1]\cap\left\{-\frac{q}{2p}\right\}\right]$ (D) none of these

Ans. **(B)**

Sol. If $q2-4pr=0, p>0$, then the domain of the function, $f(x)=\cdots$

$f(x)=\log(px^3+(p+q)x^2+(q+r)x+r)$

$px^3+(p+q)x^2+(q+r)x+r>0$

$x(px^2+qx+r)+(px^2+qx+r)>0$

Since $q^2-4pr=0$

$x\left(x+\frac{q}{2p}\right)^2+\left(x+\frac{q}{2p}\right)^2>0$ $\Rightarrow \left(x+\frac{q}{2p}\right)^2(x+1)>0$

This inequality is true for

$x>-1\,\&\, x\neq -\frac{q}{2p}$ $\Rightarrow x=R-(-\infty,-1]\cup\left\{-\frac{q}{2p}\right\}$

4. The domain of the function $\sqrt{\log_{1/3}\log_4([x]^2-5)}$ is (where [x] denotes greatest integer function)

(A) $[-3,-2)\cup[3,4)$ (B) $[-3,-2)\cup(2,3]$

(C) $R-[-2,3)$ (D) $R-[-3,3]$

Link to View Video Solution: Click Here

Ans. **(A)**

Sol. $\sqrt{\log_{1/3}\log_4([x]^2-5)}$

$\Rightarrow \log_{1/3}\log_4([x]^2-5) \geq 0$

$\Rightarrow 0 < \log_4([x]^2-5) \leq 1$

$\Rightarrow 1 < [x]^2 - 5 \leq 4 \Rightarrow 6 < [x]^2 \leq 9$

[x] always gives integer value so square of GTF will also give Integer value. In between 6 and 9 are only perfect square value possible.

$[x]^2 = 9 \Rightarrow$ $[x] = 3$ $[x] = -3$

$3 \leq x < 4$ $-3 \leq x < -2$

$\therefore x \in [-3,-2) \cup [3,4)$

5. If $f(x) = 2\sin^2\theta + 4\cos(x+\theta)\sin x \cdot \sin\theta + \cos(2x+2\theta)$ then value of $f^2(x) + f^2\left(\frac{\pi}{4} - x\right)$ is

(A) 0 (B) 1 (C) -1 (D) x^2

Ans. (B)

Sol. $f(x) = 2\sin^2\theta + 4\cos(x+\theta)\sin x\sin\theta + \cos(2x+2\theta)$

$= 2\sin^2\theta + 2\cos(x+\theta)[2\sin x\sin\theta] + \cos2(x+\theta)$

$\cos(A-B) - \cos(A+B) = 2\sin A\sin B$

$\therefore f(x) = 2\sin^2\theta + 2\cos(x+\theta)[\cos(x-\theta) - \cos(x+\theta)] + \cos^2(x+\theta) - \sin^2(x+\theta)$

$f(x) = 2\sin^2\theta + 2\cos(x+\theta)\cos(x-\theta) - 2\cos^2(x+\theta) + \cos^2(x+\theta) - \sin^2(x+\theta)$

$f(x) = 2\sin^2\theta + 2\cos(x+\theta)\cos(x-\theta) - [\cos^2(x+\theta) + \sin^2(x+\theta)]$

$\cos(A+B) + \cos(A-B) = 2\cos A\cos B$

$f(x) = 2\sin^2\theta + \cos(x+\theta+x-\theta) + \cos(x+\theta-x+\theta) - 1$

$f(x) = 2\sin^2\theta + \cos2x + \cos2\theta - 1$

$f(x) = 2\sin^2\theta + \cos2x - (1-\cos2\theta)$

$f(x) = 2\sin^2\theta + \cos2x - 2\sin^2\theta$

$f(x) = \cos2x$

$f^2(x) + f^2\left(\frac{\pi}{4} - x\right) = \cos^2 2x + \cos^2 2\left(\frac{\pi}{4} - x\right)$

$= \cos^2 2x + \left[\cos\left(\frac{\pi}{2} - 2x\right)\right]^2$

$= \cos^2 2x + \sin^2 2x = 1$

6. Let $P(x,y)$ be a moving point in the $x-y$ plane such that $[x].[y] = 2$, where [.] denotes the greatest integer function, then area of the region containing the points $P(x,y)$ is equal to :

(A) 1 sq. units (B) 2 sq. units

(C) 4 sq. units (D) None of these

Ans. **(C)**

Sol. $[x][y] = 2$

Hence the area $= 4 \times 1 = 4$

7. Total number of solution of $2^{\cos x} = |\sin x|$ in $[-2\pi, 5\pi]$ is equal to :

(A) 12 (B) 14 (C) 16 (D) 15

Link to View Video Solution: Click Here

Ans. **(B)**

Sol. According to question.

$\frac{1}{2} \leq 2^{\cos x} \leq 2$

we know, max&min value of the cosx = 1

so, when, $\cos x = 1, \Rightarrow x = 0, 2\pi, 4\pi$

$\cos x = 0 \Rightarrow x = \frac{\pi}{2}, \frac{3\pi}{2}$

and, $\cos x = -1 \Rightarrow x = \pi, 3\pi$

we know points up to 5π.

$0 - \pi \Rightarrow 2$

$2\pi - 3\pi \Rightarrow 2$

$4\pi - 5\pi = 2$

$[-2\pi, 5\pi]$

$-\pi - 0 \Rightarrow 2$

$\pi - 2\pi \Rightarrow 2$

$3\pi - 4\pi \Rightarrow 2$

$\Rightarrow \pi - 2\pi \Rightarrow 2$

$\Rightarrow 3\pi - 4\pi \Rightarrow 2$

$\Rightarrow$ Now, we find out from $[-2\pi, 5\pi]$:

$\Rightarrow -2\pi - (-\pi) = 2$

$\Rightarrow 0 - \pi \Rightarrow 2$

$\Rightarrow 2\pi - 3\pi \Rightarrow 2$

$\Rightarrow 4\pi - 5\pi = 2$

So, the total no. of solution is 14, and the correct option is B.

8. The sum $\left[\frac{1}{2}\right] + \left[\frac{1}{2} + \frac{1}{2000}\right] + \left[\frac{1}{2} + \frac{2}{2000}\right] + \left[\frac{1}{2} + \frac{3}{2000}\right] + \cdots .. + \left[\frac{1}{2} + \frac{1999}{2000}\right]$ is equal to (where $[*]$ denotes the greatest integer function)

(A) 1000 (B) 999 (C) 1001 (D) None of these

Ans. (A)

Sol. $\left[\frac{1}{2}\right] + \left[\frac{1}{2} + \frac{1}{2000}\right] + \cdots, \left\{\left[\frac{1}{2} + \frac{1000}{2000}\right] + \left[\frac{1}{2} + \frac{1001}{2000}\right] + \cdots - \left[\frac{1999}{2000}\right]\right.$

$(1 + 999)$ boxes

will give value $= 0$

$\because$ it is less than 1

these 1000 boxen will give value 1 each and add upto give 1000

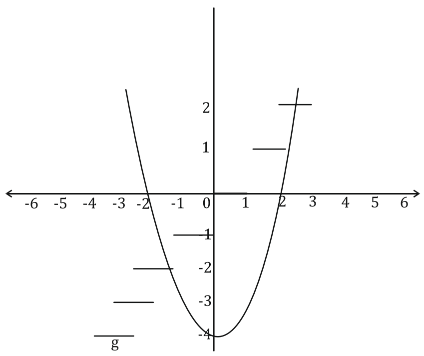
9. Total number of solutions of the equation $x^2 - 4 - [x] = 0$ are (where [.] denotes the greatest integer function):

(A) 1 (B) 2 (C) 3 (D) 4

Ans. **(B)**

Sol. Given: $x^2 - 4 - [x] = 0 \Rightarrow x^2 - 4 = [x] = y(\text{ say })$

From the graph, it is seen that both the



Link to View Video Solution: Click Here

graphs intersect at 2 points.

Hence, there will be 2 solutions of the equation.

10. $y = 2[x] + 3$ & $y = 3[x - 2] + 5$ then $[x + y] = ?$

(A) 0 (B) 15 (C) 30 (D) 45

Ans. (B)

Sol. $2[x] + 3 = 3[x - 2] + 5 \Rightarrow 2[x] + 3 = 3[x] - 6 + 5$

$\Rightarrow [x] = 4 \Rightarrow 4 \le x < 5$

Let f be the fractional part of x $\therefore x = 4 + f$

$y = 2[x] + 3 = 11$ $\Rightarrow$ Hence, $[x + y] = [4 + f + 11] = [15 + f] = 15$

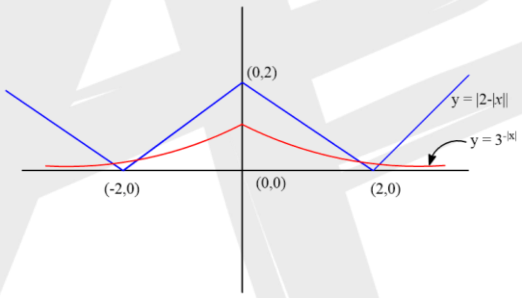
11. How many roots the following equation posses $3^{|x|}(2 - |x|) = 1$.

(A) 1 (B) 2 (C) 3 (D) 4

Ans. (D)

Sol. Here, $3^{|x|}\{|2 - |x||\} = 1 \Rightarrow |2 - |x|| = 3^{-|x|}$

In order to determine the number of roots, it is sufficient to find the points of intersection of the curves $y = |2 - |x||$ and $y = 3^{-|x|}$, shown in the graph; We observe the two curves intersect at four points. $\therefore$ Four real solutions exist.



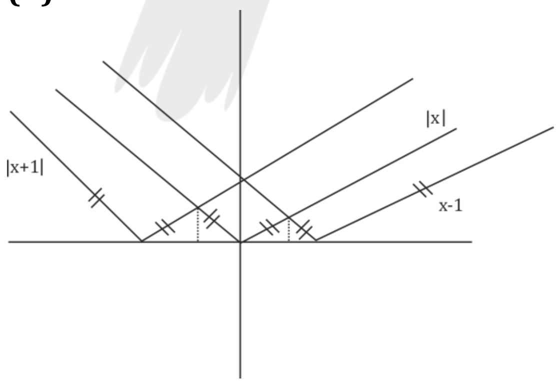
12. If $f(x) = \min\{|x - 1|, |x|, |x + 1|\}$, then:

(A) f is odd (B) f is even

(C) f is periodic (D) None of these

Ans. (B)

Sol.



Clearly graph is symmetric about y-axis. So, f is even function.

13. Let $g(x) = 1 + x - [x]$ and $f(x) = \begin{cases} -1 & \text{if} \quad x < 0 \\ 0 & \text{if} \quad x = 0 \\ 1 & \text{if} \quad x > 0 \end{cases}$, then $\forall x$, fog(x) equals (where [*] represents greatest integer function).

(A) x (B) 1 (C) f(x) (D) g(x)

Link to View Video Solution: Click Here

Ans. **(B)**

Sol. Determine the value of f(g(x))

We know that:

$x = [x] + \{x\}$ $\Rightarrow x - [x] = \{x\},\ \{x\} \in [0,1)$

Here $\{x\}$ is the fractional part of x. Now we have:

$f(g(x)) = f(1 + x - [x]) = f(1 + \{x\})$ $\Rightarrow$ Here, $(1 + \{x\}) > 0$. So, for $x > 0, f(x) = 1$.

Therefore, $f(g(x)) = 1$

14. Domain of the function

$f(x) = \log_e \left\{\log_{|\sin x|}(x^2 - 8x + 23) - \frac{3}{\log_2|\sin x|}\right\}$ is given by :

(A) (3,5)
(B) $(3,\pi) \cup (\pi,5)$
(C) $(3,\pi) \cup (3\pi/2,5)$
(D) None of these

Ans. **(D)**

Sol. f(x) is defined if $\left(\log_{|\sin x|}(x^2 - 8x + 23) - \frac{3}{\log_{|\sin x|}|\sin x|}\right) > 0$

$\Rightarrow \log_{|\sin x|}\left(\frac{x^2 - 8x + 23)}{8} > 0\right)\left(\text{as } \frac{3}{\log_2|\sin x|} = \frac{\log_2 8}{\log_2}|\sin x|\right) = \log_{|\sin x|}8$

The is true, if

$|\sin x| \equiv 0,1$ and $\frac{x^2 - 8x + 23}{8} < 1$

$\left(\text{as } |\sin x| < 1 \Rightarrow \log_{|\sin x|}a > 0 \Rightarrow a < 1\right)$

Now, $\frac{x^2-8x+23}{8} < 1 \Rightarrow x^2 - 8x + 23 < 0$ $\Rightarrow x \in (3,5) - \left\{\pi, \frac{3\pi}{4}\right\}$

Hence, domain $= (3,\pi) \cup \left(\pi, \frac{3\pi}{2}\right) \cup \left(\frac{3\pi}{2}, 5\right)$

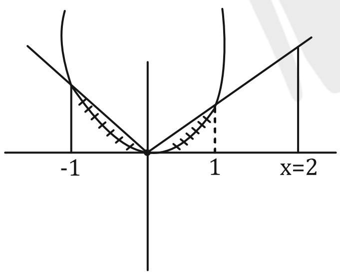
15. Which of the following has range above $y = 2$

(A) $f(x) = \mathrm{Sgn}(1 - |x|)$
(B) $f(x) = \mathrm{Sgn}([x^2 - x])$
(C) $f(x) = \mathrm{Min}(|x|, x^2, 2)$
(D) $f(x) = \mathrm{Max}\{|\tan x|, \cos|x|\}xt[-\pi,\pi]$

Ans. **(D)**

Sol. We know signum function has range {1,0,-1}.

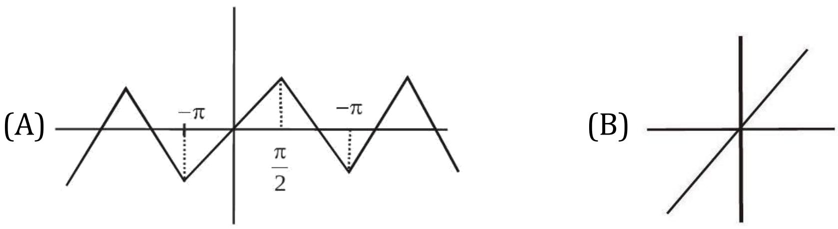
Graph ?



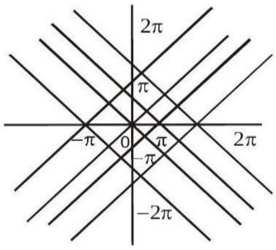
Clearly, min $(|x|, x^2, 2)$ does not have range above $y = 2$.

So,(D) is correct.

16. sin y = sin x has graph



Link to View Video Solution: Click Here

(C)  (D) Not

Ans. **(C)**

Sol. Clearly, option (c) will graph of the given function.

17. Find the domain of each of the following functions

(i) $f(x) = \frac{x^3-5x+3}{x^2-1}$

Here, f(x) is not defined if $x^2 - 1 \neq 0$

$(x-1)(x+1) \neq 0$

$x \neq 1, x \neq -1$

Hence, the domain of $f = R - \{-1,1\}$ a

(ii) $f(x) = \frac{1}{\sqrt{x+|x|}}$

We have, $f(x) = \frac{1}{\sqrt{x+|x|}}$

Now, $|x| = \{x$, when $x \geq 0$

Now, $|x| = \{-x$, when $x < 0$

$\Rightarrow x + |x| = \begin{cases} x + x, \text{ when } x \geq 0 \\ x - x, \text{ when } x < 0 \end{cases}$ $\Rightarrow x + |x| = \begin{cases} 2x, \text{ when } x \geq 0 \\ 0, \text{ when } x < 0 \end{cases}$

$\Rightarrow x + |x| > 0$, when $x > 0$

$\Rightarrow f(x) = \frac{1}{\sqrt{x+|x|}}$ assumes real values only when $x + |x| > 0$ and this happens only when $x > 0$

$\therefore \text{dom}(f) = (0, \infty)$.

(iii) $f(x) = e^{x+\sin x}$

Let $g(x) = e^x$, and $h(x) = x + \sin x$. The given function $f(x) = g(h(x))$.

The function $g(x) = e^x$ is defined $\forall x \in R$.

$h(x) = x + \sin x$ is defined $\forall x \in R$.

Polynomial functions and the sine function are continuous on R. So the function $f(x) = e^{x+\sin x}$ has the domain $x \in (-\infty, \infty)$. f(x) is continuous on $x \in R$ as well.

(iv) $f(x) = \frac{1}{\log_{10}(1-x)} + \sqrt{x+2}$

$$f(x) = \frac{1}{\log_{10}(1-x)} + \sqrt{(x+2)}$$

For this function to be defined, $(1-x) > 0, \equiv 1$ and $(x+2) \geq 0$

$\Rightarrow x < 1, \equiv 0$ and $x \geq -2$

Thus domain of f(x) is $x \in [-2,0) \cup (0,1)$

(v) $\log_x \log_2 \left(\frac{1}{x-1/2}\right)$

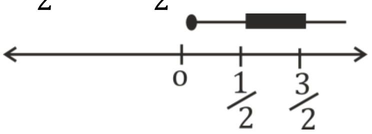
given: $f(x) = \log_x \log_2 \left(\frac{1}{x-1/2}\right)$

$\Rightarrow \log_2 \left(\frac{1}{x-1/2}\right) > 0$ and $0 < x < 1$ or $x > 1$ and $x \neq 1$

ie. $\frac{1}{x-\frac{1}{2}} > 1$ so, $0 < x - \frac{1}{2} < 1$

Link to View Video Solution: Click Here

$= \frac{1}{2} < x < \frac{3}{2}$



$\therefore x \in \left(\frac{1}{2}, 1\right) \cup \left(1, \frac{3}{2}\right)$

(vi) $f(x) = \sqrt{3 - 2^x - 2^{1-x}}$

$3 - 2^x - 2^{1-x} \geq 0$

$\Rightarrow 3.2^x - (2^x)^2 - 2 \geq 0$

$\Rightarrow (2^x)^2 - 3.2^x + 2 \leq 0$

$\Rightarrow (2^x - 1)(2^x - 2) \leq 0$

$\Rightarrow 1 \leq 2^x \leq 2$

$\Rightarrow 2^0 \leq 2^x \leq 2^1$

$\Rightarrow 0 \leq x \leq 1$

Thus, $D_f = [0,1]$

(vii) $f(x) = \sqrt{1 - \sqrt{1 - x^2}}$

$1 - \sqrt{1 - x^2} \geq 0$

$\Rightarrow 1 - x^2 \geq 0$

$X \in [-1,1]$

$\Rightarrow$ for $x \in [-1,1]$, $1 - x^2 < 1$

$\therefore$ f is defined for all $x \in [-1,1]$

(viii) $f(x) = (x^2 + x + 1)^{-3/2}$

$f(x) = (x^2 + x + 1)^{-3/2} = \dfrac{1}{(x + x + 1)\sqrt{x^2 + x + 1}}$

$x^2 + x + 1 > 0$ always (as $D < 0$)

So, domain is R

(ix) $f(x) = \sqrt{\frac{x-2}{x+2}} + \sqrt{\frac{1-x}{1+x}}$

$$f(x) = \sqrt{\frac{x-2}{x+2}} + \sqrt{\frac{1-x}{1+x}}$$

For f(x) to be defined,

$x + 2 \neq 0$

$\Rightarrow x \neq -2.$ ....(1)

And $1 + x \neq 0$

$\Rightarrow x \neq -1$ ....(2)

Also, $\frac{x-2}{x+2} \geq 0$

$\Rightarrow \frac{(x-2)(x+2)}{(x+2)^2} \geq 0$

$\Rightarrow (x - 2)(x + 2) \geq 0$

$\Rightarrow x \in (-\infty, -2) \cup [2, \infty]$.........(3)

And, $\frac{1-x}{1+x} \geq 0$

$$\Rightarrow \frac{(1 - x)(1 + x)}{(1 + x)^2} \geq 0$$

$\Rightarrow (1 - x)(1 + x) \geq 0$

$\Rightarrow x \in [-1, 1]$ ... ... (4)

From (1), (2), (3) and (4), we get, $x \in \phi$

Thus, $\text{dom}(f(x)) = \phi$

Link to View Video Solution: Click Here

(x) $f(x) = \sqrt{\tan x - \tan^2 x}$

$\tan x - \tan^2 x \geqslant 0$

$\tan x(\tan x - 1) \leq 0$

$\tan x = 0$ $\Rightarrow \tan x = 1$

$x = n\pi$ $\Rightarrow x = n\pi + \frac{\pi}{4}$

Domain is $\cup_{n\in I}\left[n\pi, n\pi + \frac{\pi}{4}\right]$

(xi) $f(x) = \frac{1}{\sqrt{1-\cos x}}$

We know the value of cosx lies between $-1,1$,

$-1 \leq \cos x \leq 1$

Multiplying by negative sign, we get or $1 \geq -\cos x \geq -1$ Adding 1, we get

$2 \geq 1 - \cos x \geq 0$

Now, $f(x) = \frac{1}{\sqrt{1-\cos x}}$

$1 - \cos x \neq 0$

$\Rightarrow \cos x \neq 1$

Or, $x \neq 2n\pi \;\forall\; n \in Z$

Therefore, the domain of $f = R - \{2n\pi;$

$; n \in Z\}$

(xii) $f(x) = \sqrt{\log_{1/4}\left(\frac{5x-x^2}{4}\right)}$

Therefore, for real value of f(x),

$0 < \left(\frac{5x-x^2}{4}\right) < 1$ $\Rightarrow 0 < 5x - x^2 < 4$

Now, $5x - x^2 > 0$

or $x(5-x) > 0$

$x > 0$ and $x < 5$

$\Rightarrow x \in (0,5)$ ... (i)

And $5x - x^2 < 4$ implies

$\Rightarrow x^2 - 5x + 4 > 0$

$(x-4)(x-1) > 0$

$\Rightarrow x > 4$ and $x < 1$

$\Rightarrow x \in (-\infty, -1) \cup (4, \infty)$

Hence, $x \in (0,1) \cup (4,5)$

(xiii) $f(x) = \log_{10}\left(1 - \log_{10}(x^2 - 5x + 16)\right)$ f is defined for

$\left(1 - \log_{10}(x^2 - 5x + 16)\right) > 0$

$\Rightarrow \log_{10}(x^2 - 5x + 16) < 1$ $\Rightarrow (x^2 - 5x + 16) < 10$

$\Rightarrow (x^2 - 5x + 6) < 0$ $\Rightarrow (x-2)(x-3) < 0$

$\Rightarrow 2 < x < 3$

Thus, $D_f = (2,3)$

Link to View Video Solution: Click Here

ANSWER KEY

1.	D	**2.**	D	**3.**	B	**4.**	A	**5.**	B	**6.**	C	**7.**	B
8.	A	**9.**	B	**10.**	B	**11.**	B	**12.**	B	**13.**	B	**1/4.**	D
15.	D	**16.**	C										

17.

(i)	$R-\{-1,1\}$	**(ii)**	$(0,\infty)$	**(iii)**	R
(iv)	$[-2,0)\cup(0,1)$	**(v)**	$\left(\frac{1}{2},1\right)\cup\left(1,\frac{3}{2}\right)$	**(vi)**	$[0,1]$
(vii)	$[-1,1]$	**(viii)**	R	**(ix)**	ϕ
(x)	$\bigcup_{n\in 1}\left[n\pi, n\pi+\frac{\pi}{4}\right]$	**(xi)**	$R-\{2n\pi\}, n\in I$	**(xii)**	$(0,1]\cup[4,5)$
(xiii)	$(2,3)$				