Nishant Jindal

KINEMATICS

H.W.:

Q. (i) A particle is moving in three dimension. Its position vector is given by

$$\vec{r} = 6\hat{i} + (3 + 4t)\hat{j} - (3 + 2t - t^2)\hat{k}$$

Distance are in meters, and the time, t, in seconds.

(a) What is the velocity vector at $t = +3$?

(b) What is the speed (in m/sec) at $t = +3$?

(c) What is the acceleration vector and what is its magnitude (in m/sec^2) at $t = +3$?

(ii) Now the particle is moving only along the z-axis, and its position is given by, $(t^2 - 2t - 3)\hat{k}$ at what time does the particle stand still?

Nishant Jindal

KINEMATICS

H.W :

Q. A motor boat of mass ' m ' moves along a lake with velocity v_0. At the moment $t = 0$ the engine of the boat is shut down. Assuming the resistance of water to be proportional to the velocity of the boat $F = -kv$, find-

(a) how long the motorboat moved with the shut down engine.

(b) the velocity of the motor boat as a function of the distance covered with the shut-down engine, as well as the total distance covered till it stops completely.

(c) the mean velocity of the motor boat over the time interval (beginning with the moment $t = 0$), during which its velocity decreases to $(1/\eta)$ times.

KINEMATICS

Q. A particle moves along x-axis with an initial speed $\boxed{v_0 = 5\ \mathrm{m\,s^{-1}}}$. If its acceleration varies with time as shown in $a - t$ graph in Fig.,

a. Find the velocity of the particle at $t = 4$ s.

b. Find the time when the particle starts moving along $-x$ direction.

HW

$t = 0$

$-ve$

$\Delta V = (A_1 + A_2)$

$V_f - V_i = $

$V_f = V_i + (A_1 + A_2)$

$= 5 + \left(\frac{35}{6} - \frac{40}{3}\right)$

$= -5/2$ m/s

Sol^n :-

$a = \frac{dv}{dt}$

$\int_{v_i}^{v_f} dv = \int_0^4 a\,dt$

$V_f - V_i = $

$\tan\theta = \left(\frac{5}{x} = \frac{10}{1-x}\right)$

$1 - x = 2x$

$1 = 3x$

$\left(x = \frac{1}{3}\right)$

$A_1 = \frac{1}{2}\left\{\left[1 + \frac{1}{3}\right] + 1\right\} \times 5$ $(35/6)$

$A_2 = \frac{1}{2}\left(2 + \left(1 - \frac{1}{3}\right)\right) \times 10$

$= -5\left(2 + \frac{2}{3}\right) = -40/3$

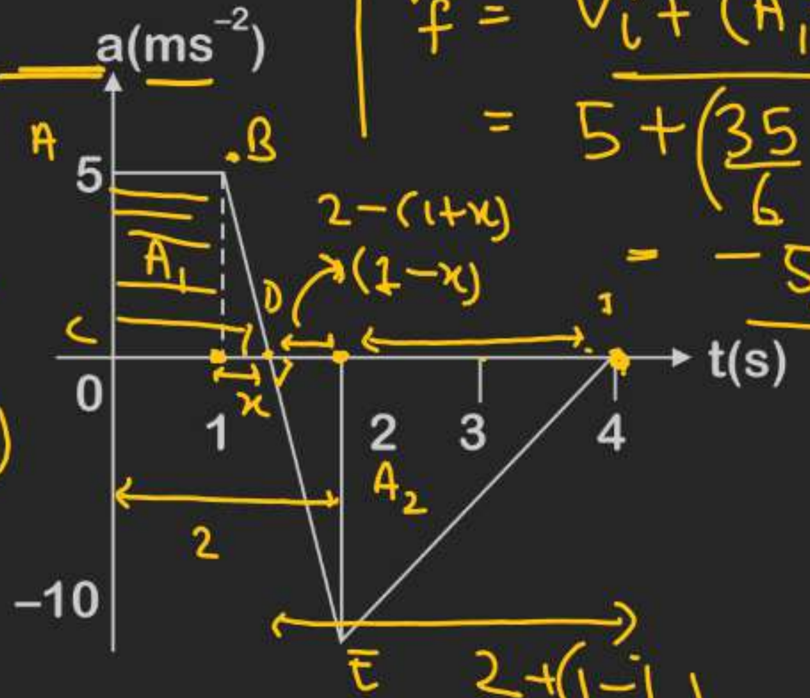


Fig. 4.90

KINEMATICS

Q. The velocity of a particle varies with time as shown below. The distance travelled by the particle during $t = 2s$ and $t = 6\ s$ is :

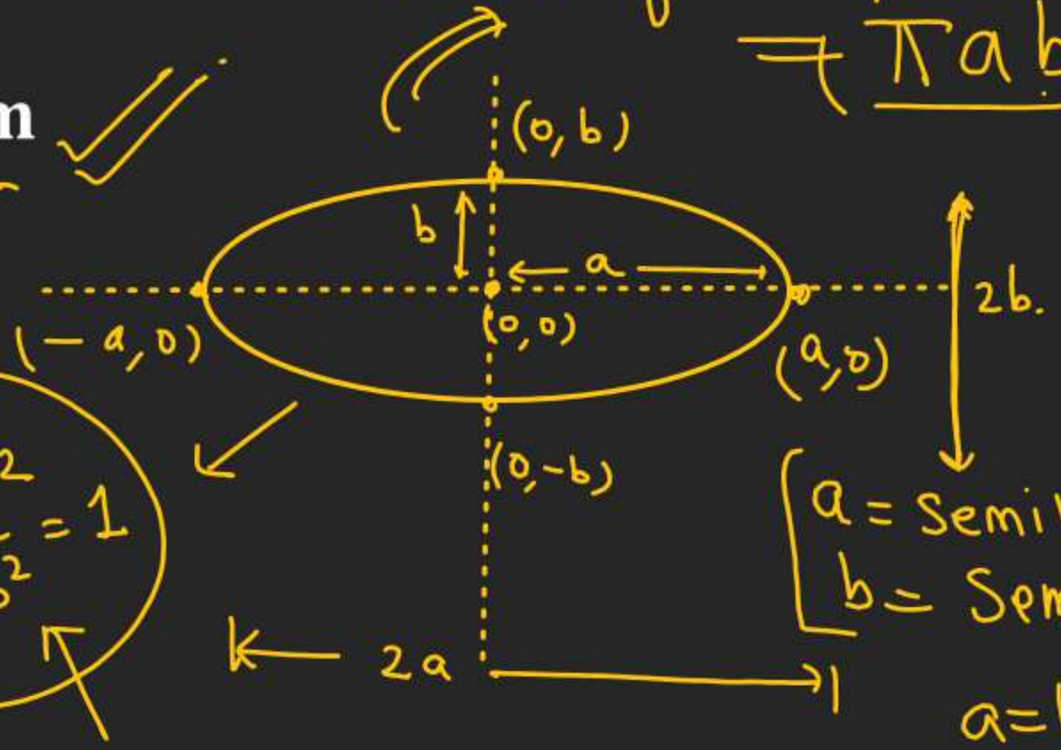
(A) $2\pi m$

(B) $(2\pi + 40)m$

(C) $4\pi m$

(D) $40\ m$

Area of ellipse $= (\pi a b)$

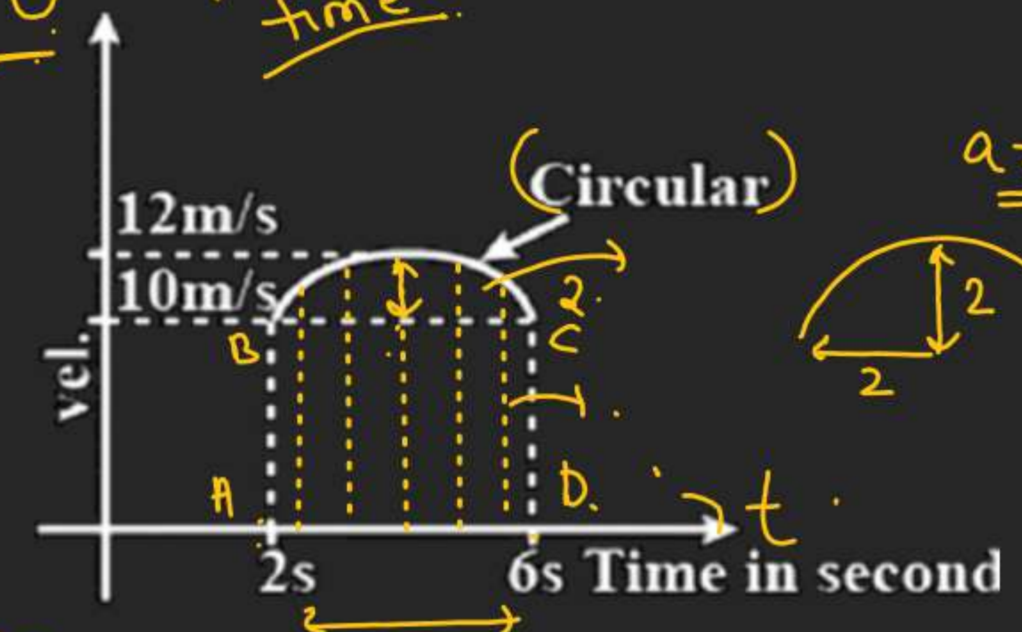


$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

a = Semi major axis
b = Semi minor axis

$a = b \Rightarrow$ Circle

$v \rightarrow$ Speed vs. time



$a = b \Rightarrow$ Circle

Distance = (Area of Semi circle) + (Area of rectangle)

$= \left(\frac{\pi 4}{2}\right) + (10 \times 4)$

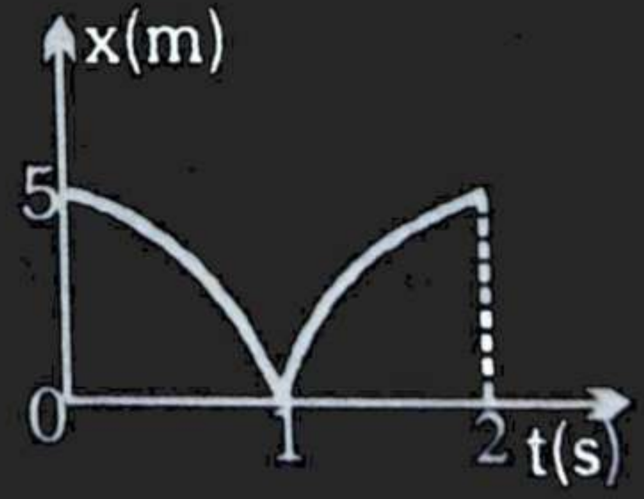
$= (2\pi + 40)$

$= 2(\pi + 20)$

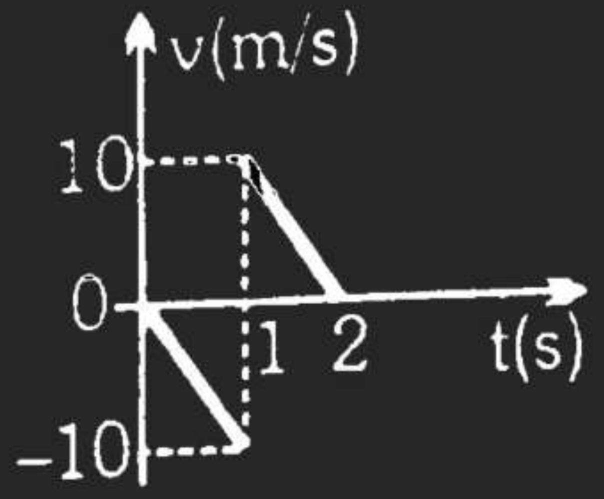
H.W.

KINEMATICS

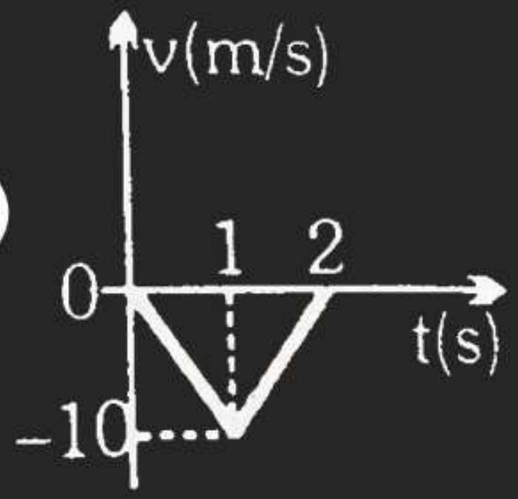
Q. The Position-time graph of a moving particle with constant acceleration is shown in the figure. The velocity-time graph is given by;



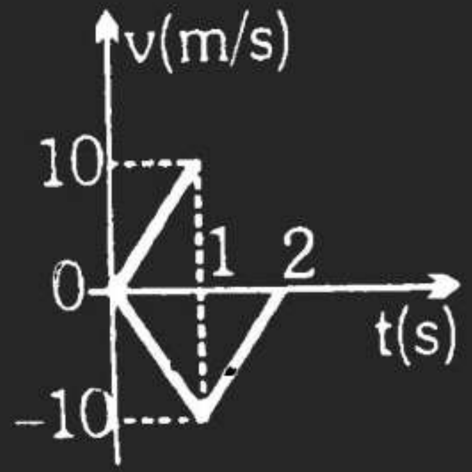
(A)



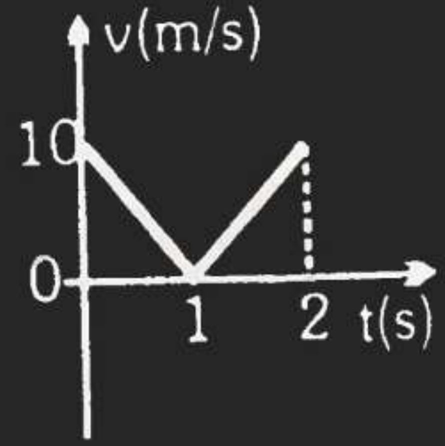
(B)



(C)

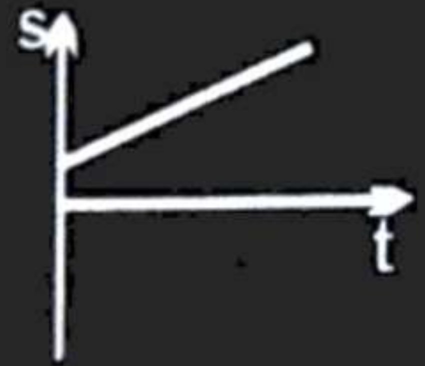


(D)

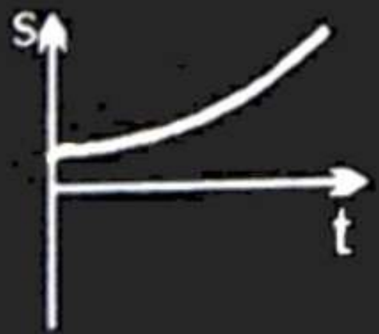


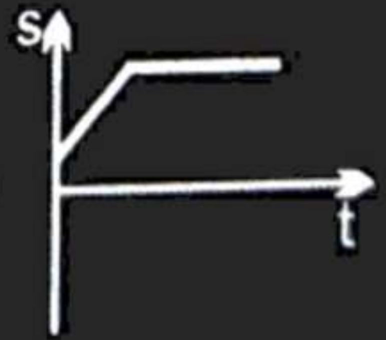
H.W

Q. A particle moving along a straight line has a velocity $v = \mu s^2$ where 's' is its displacement. If initially $s = s_0$, then which of the following graphs best represents 's' versus 't' ? $\{\text{for } t < (t/\mu s_0)\}$

(A) 

(B) 

(C) 

(D) 

KINEMATICS

$V_i = 0$ → at rest

Q. A particle of mass ' m ', initially arrest, is acted upon by a variable force F for a brief interval of time T. It begins to move with a velocity ' u ' after the force stops acting. Variation of force with time is shown in the graph. The curve is a semicircle. Then ' u ' is given by;

u → v_f

semicircle → ellipse

(A) $u = \frac{\pi F_0^2}{2\ m}$

(B) $u = \frac{\pi T^2}{8\ m}$

(C) $u = \frac{\pi F_0\ T}{4\ m}$ ✓

(D) $u = \frac{F_0\ T}{2\ m}$??

Area under a vs t graph $= \Delta V$

$F = ma$

$a = \frac{F}{m}$

↓ semi

(Area of ellipse) $= v_f - \cancel{v_i}^{\,0}$

$= u$

$\frac{\pi ab}{2} = u$

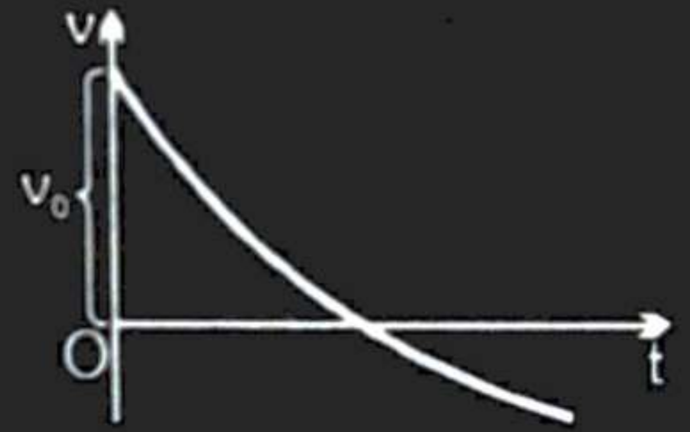
$u = \frac{\pi}{2} \times \frac{T}{2} \times \frac{F_0}{m} = \frac{\pi T F_0}{4m}$

$a = T/2,\ b = \left(\frac{F_0}{m}\right)$

KINEMATICS

H.W.

Q. An object is thrown upward with an initial velocity v_0. The air drag on the object is assumed to be proportional to the velocity as shown in the figure. The intercept on time axis is; (λ is constant)



(A) $\ln\left(2+\frac{\lambda v_0}{g}\right)$

(B) $\frac{1}{\lambda}\ln\left(1+\frac{\lambda v_0}{g}\right)$

(C) none of these.

(D) can't be ascertained.

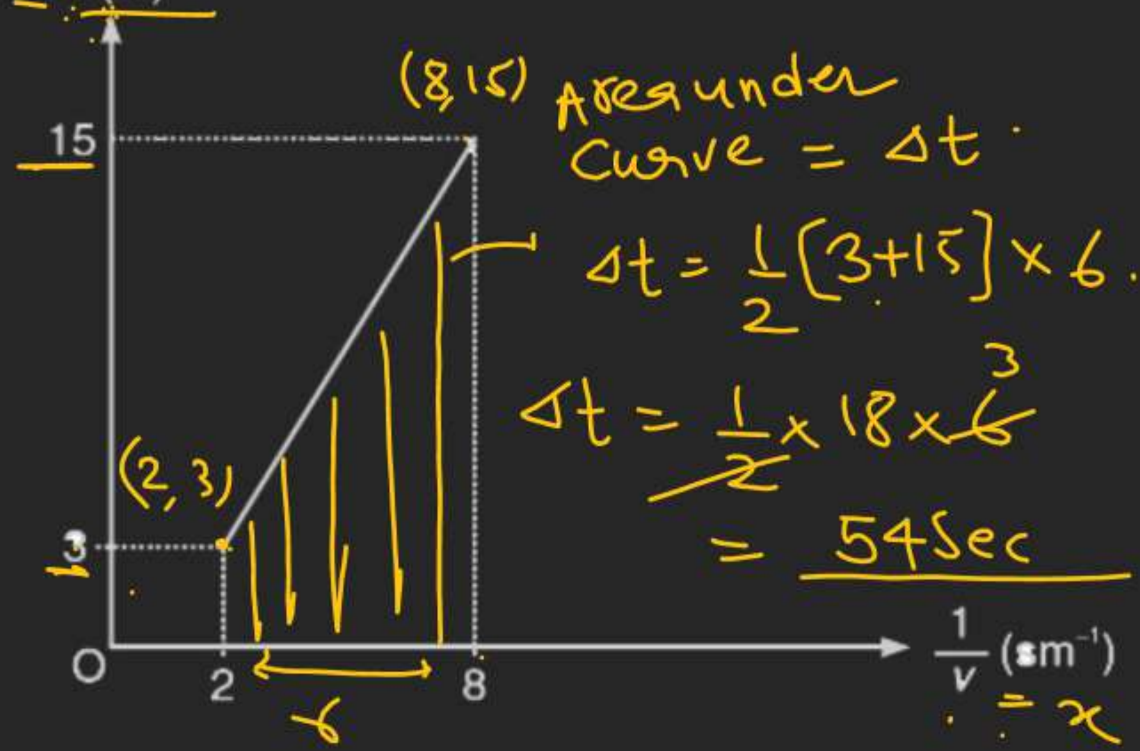
Nishant Jindal

KINEMATICS

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

(x_1, y_1) (x_2, y_2)

Q. Graph of position (x) vs inverse of velocity $\left(\frac{1}{v}\right)$ for a particle moving on a straight line is as shown. Find the time taken by the particle to move from $x = 3$ m to $x = 15$ m.



$y = x(m)$, $\frac{1}{v} = x$

Area under curve $= \Delta t$

$\Delta t = \frac{1}{2}[3+15] \times 6$

$\Delta t = \frac{1}{\cancel{2}} \times 18 \times \cancel{6}^{3}$

$= 54$ Sec

Trick :- To Check Slope

$= \left(\frac{y}{x}\right) = \frac{m}{s/m}$

$= (m^2/s) = ??$

Area $= (y \times x)$

$= \cancel{m} \times \frac{s}{\cancel{m}} = s \Rightarrow$ Time

H.W

2nd Method :-

$$\boxed{x - 3 = \frac{15-3}{8-2}\left(\frac{1}{v} - 2\right)}$$

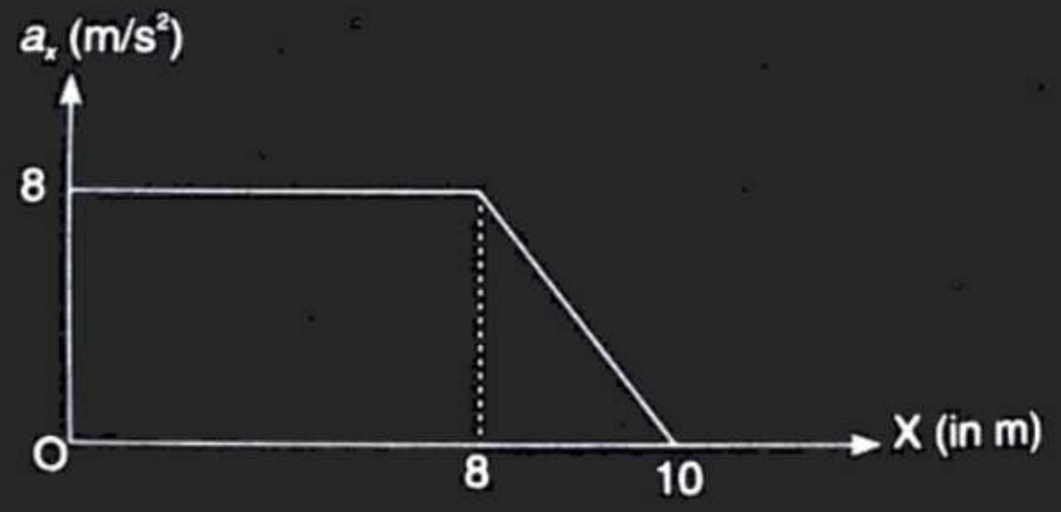
$v \Rightarrow f(x)$

$\frac{dx}{dt} =$

H.W.

KINEMATICS

Q. A particle starts from rest (at $x = 0$) when an acceleration is applied to it. The acceleration of the particle changes with its co-ordinate as shown in the fig. Find the speed of the particle at $x = 10m$



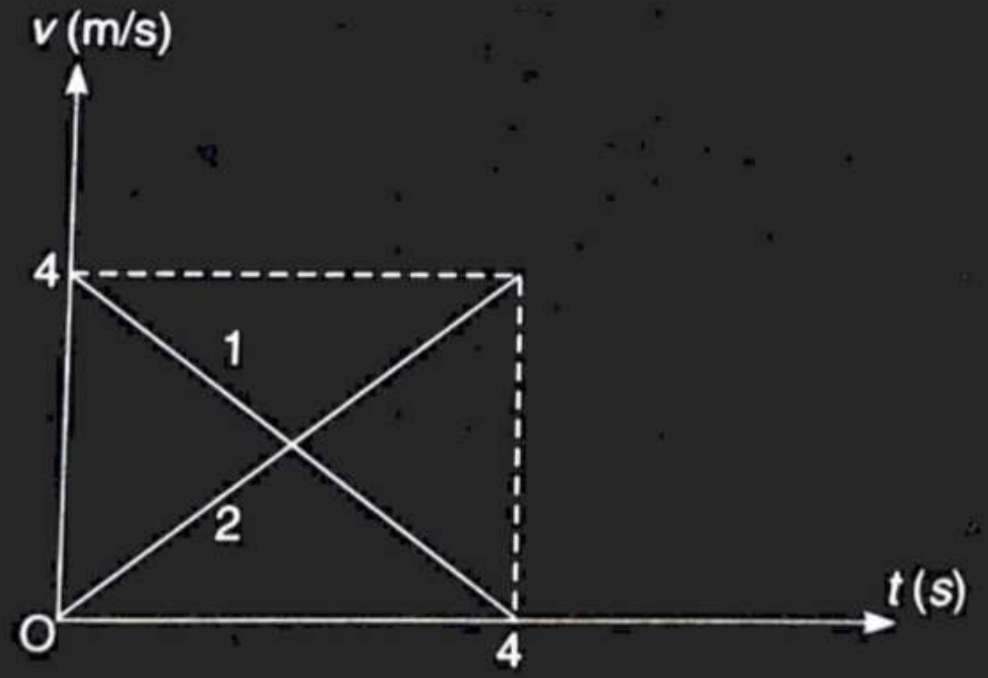
H.W.

KINEMATICS

Q. The velocity time graph for two particles (1 and 2) moving along X axis is shown in fig. At time $t = 0$, both were at origin.

(a) During first 4 second of motion what is maximum separation between the particles? At what time the separation is maximum?

(b) Draw position (x) vs time (t) graph for the particles for the given interval.



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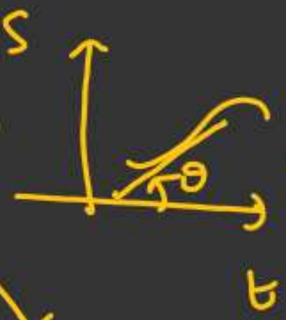
$\rightarrow$

$ds/dt = v \Rightarrow$



$\tan\theta = \frac{ds}{dt}$

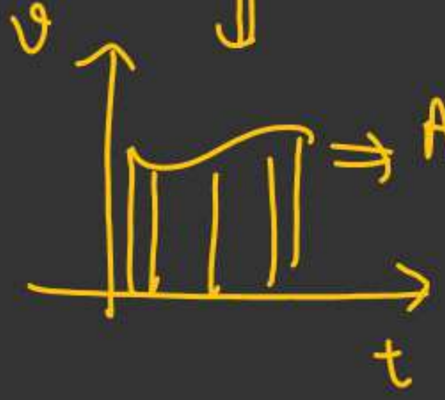
$\Downarrow$

slope of s-vs t graph gives velocity

$$S \rightarrow f(t) \xrightarrow{\;ds/dt = v\;} v \rightarrow f(t)$$

$$\int v.dt$$

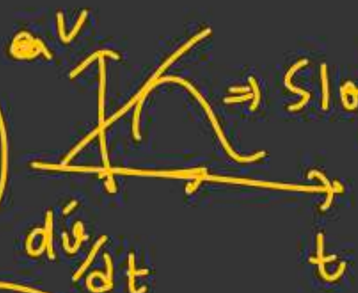
$\Downarrow$



$\Rightarrow$ Area under v vs t graph = displacement. Δs

$\left(a = \frac{dv}{dt}\right)$

$\downarrow$



dv/dt

$\Rightarrow$ Slope $\rightarrow$ velocity.

$$v \rightarrow f(t) \longrightarrow a \rightarrow f(t)$$

$$\int a.dt.$$

$\Rightarrow$ Area $= \Delta v$

$$\int a\,ds = \int_u^v v\,dv$$

$$v \rightarrow f(s) \xrightarrow{\;a = v\frac{dv}{ds}\;} a \rightarrow f(s)$$

$$\int a.ds$$

$A = \left(\frac{v^2 - u^2}{2}\right)$

$$\boxed{\frac{v^2 - u^2}{2} = \text{Area}}$$ **

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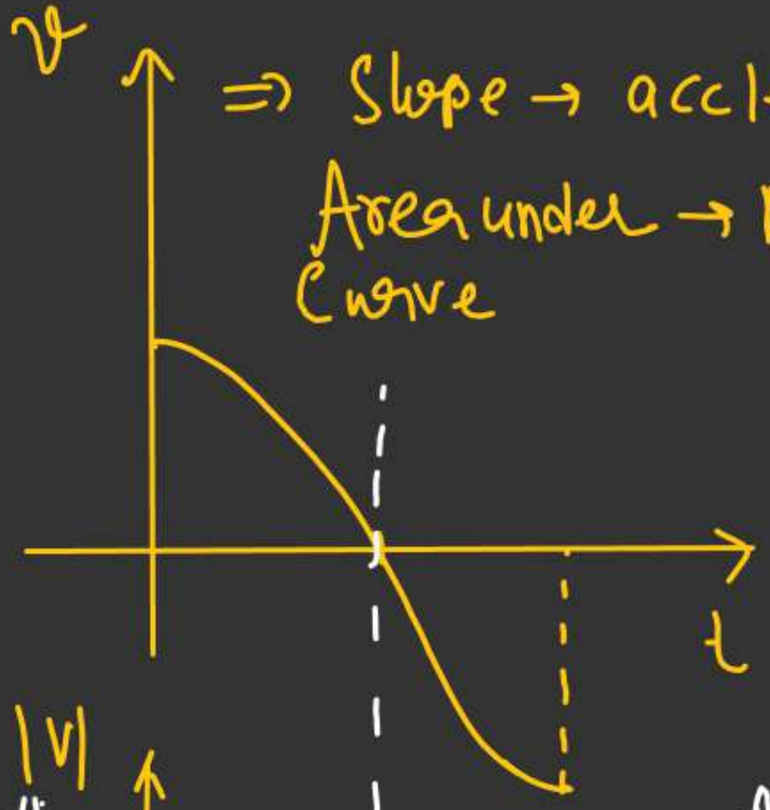
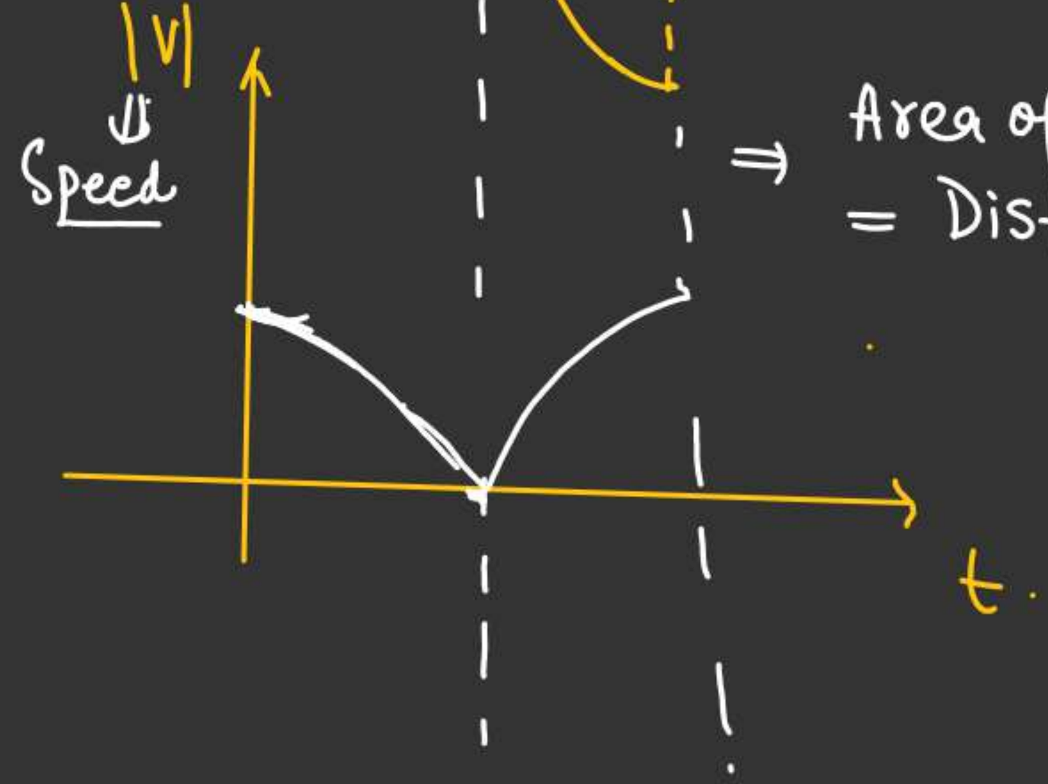
⇒ Concept of graph :-

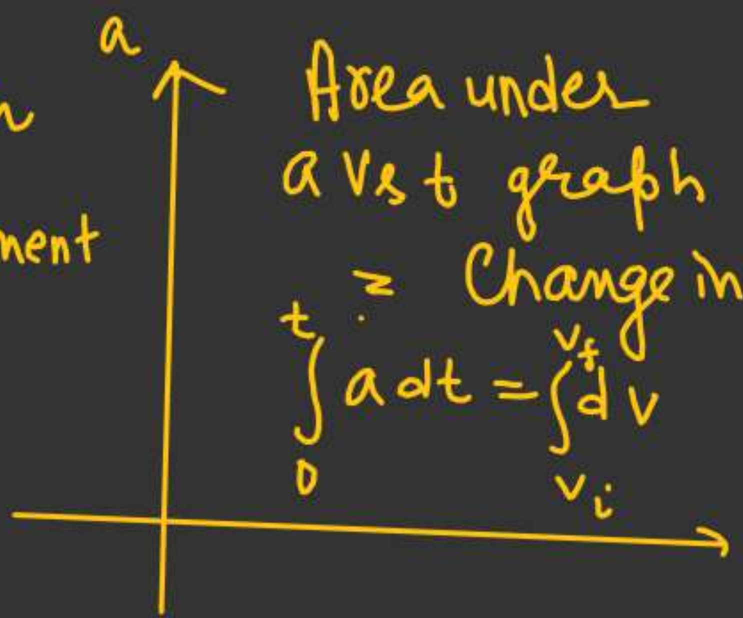


⇒ Slope → accleration

Area under Curve → Displacement

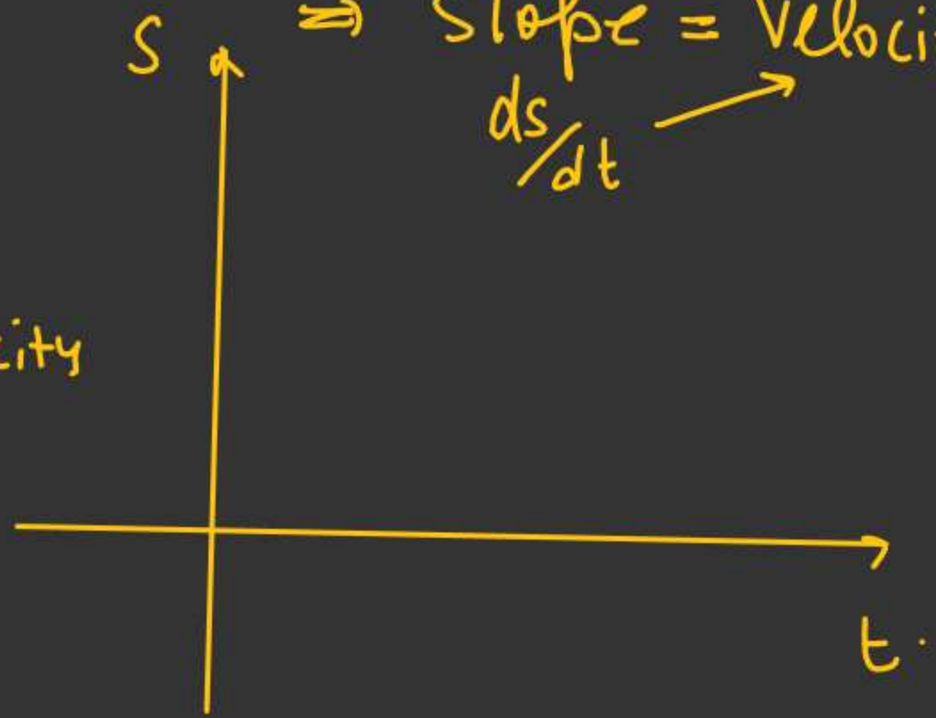


⇒ Area of |v| Vs t = Distance



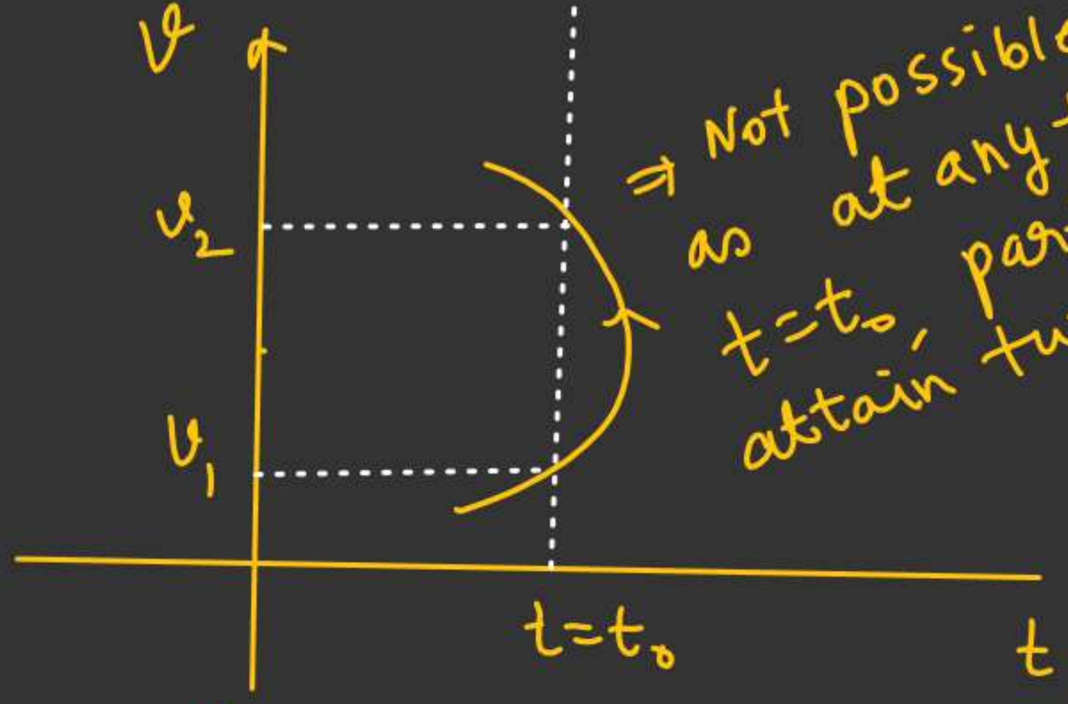
Area under a Vs t graph = Change in velocity

$$\int_0^t a\,dt = \int_{v_i}^{v_f} dv$$

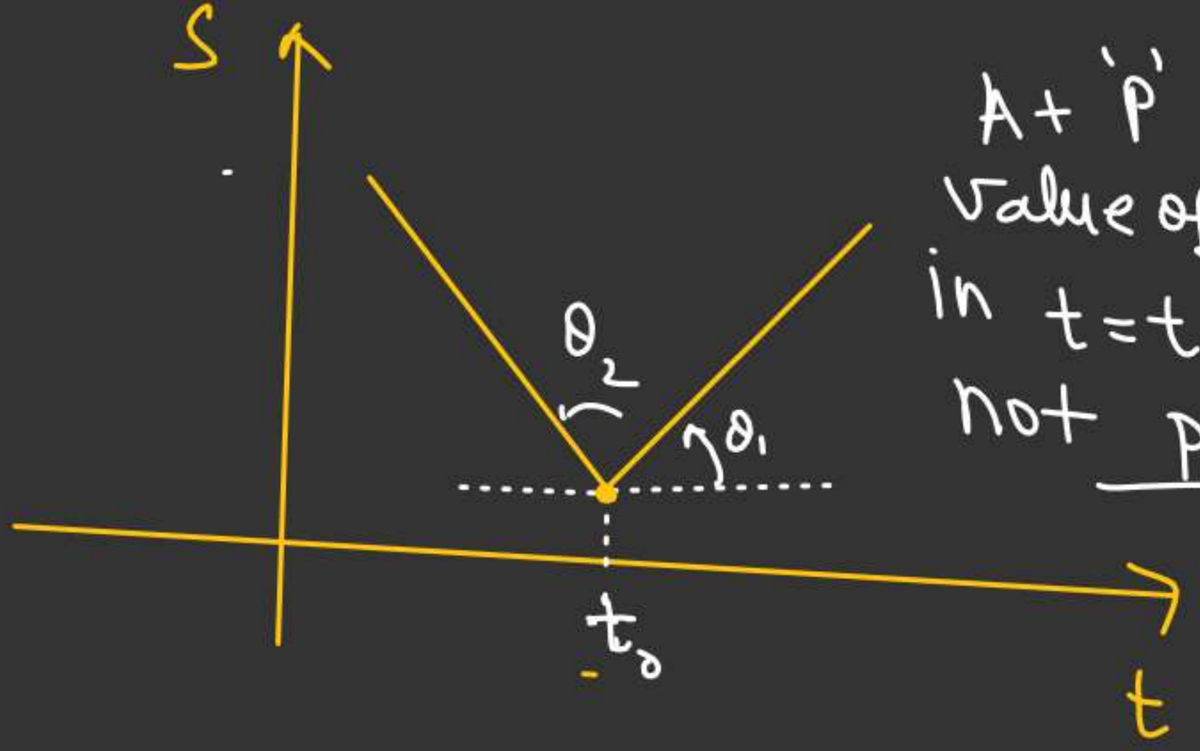


⇒ Slope = Velocity

ds/dt

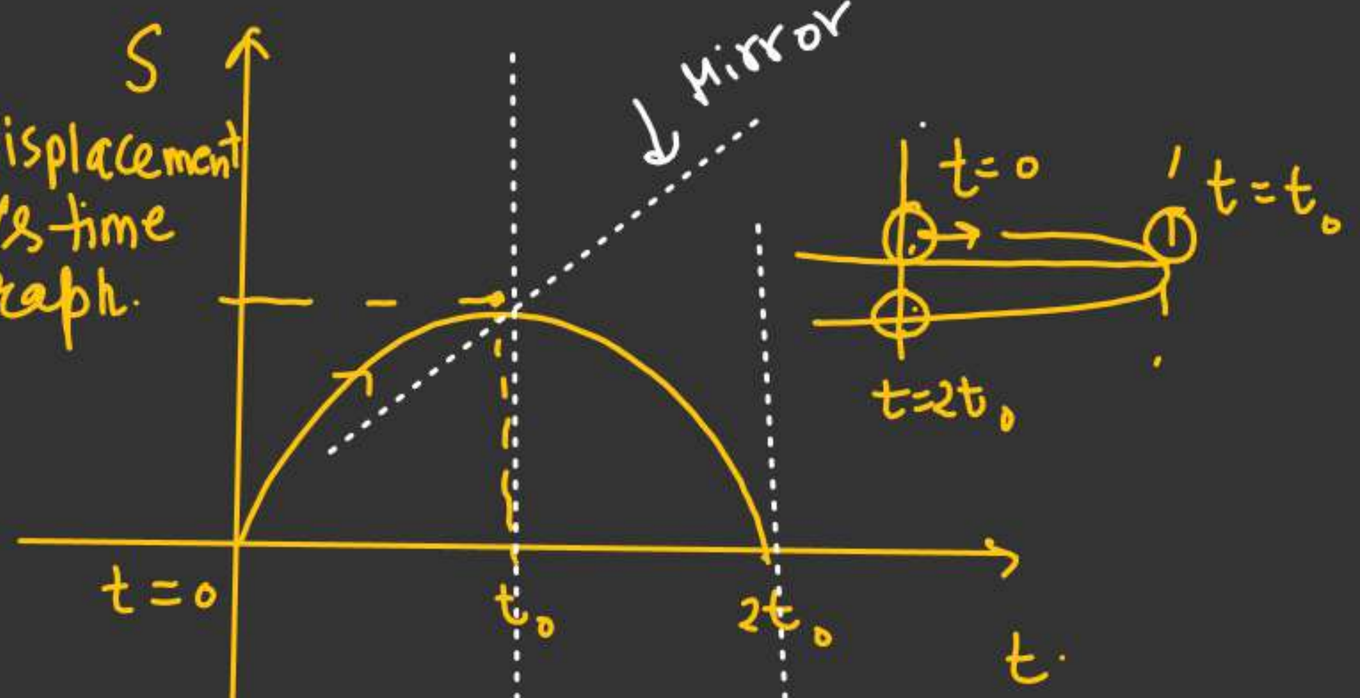


$\Rightarrow$ Not possible. as at any time $t=t_0$, particle attain two velocities.

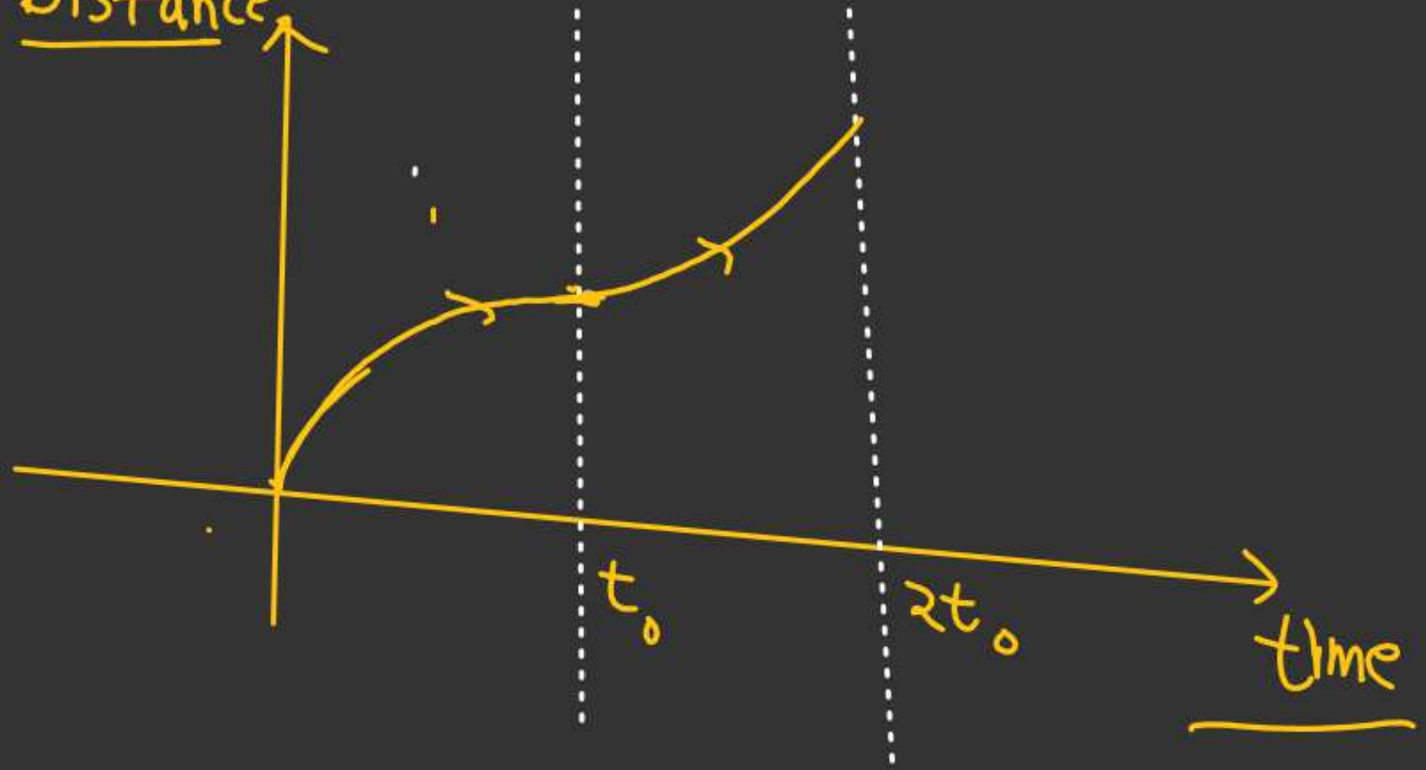


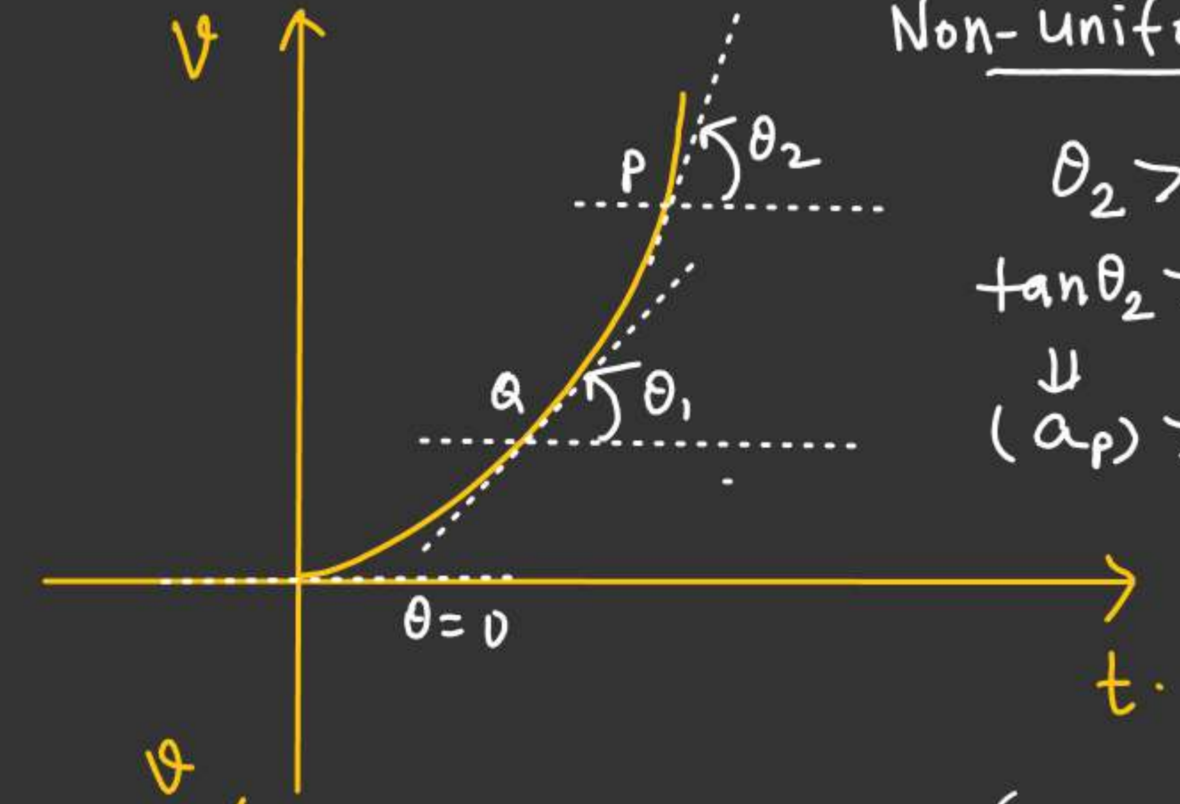
At 'P' point two value of velocity in $t=t_0$ which is not possible.

Displacement Vs time graph.



Distance





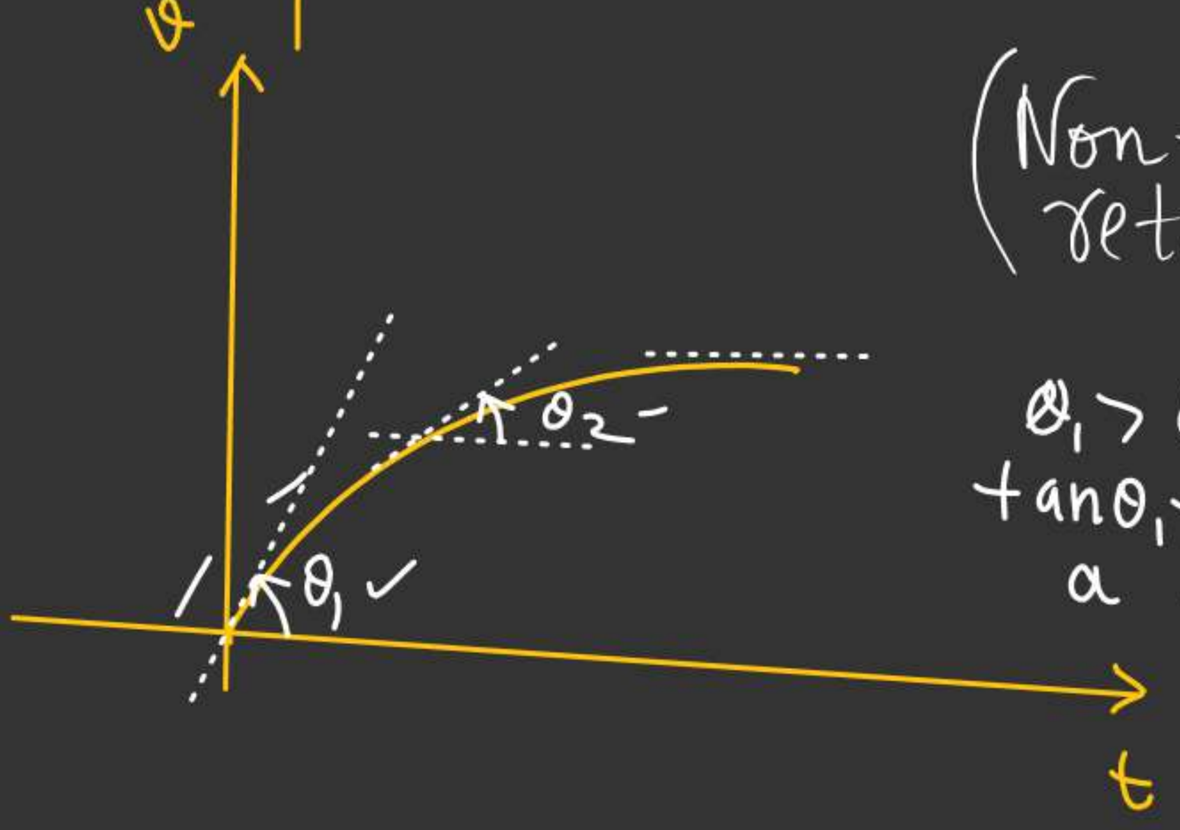
Non-uniform acclerated motion :-

$\theta_2 > \theta_1$ ↳ (accleration increasing)

$\tan\theta_2 > \tan\theta_1$

$\Downarrow$

$(a_P) > (a_Q)$

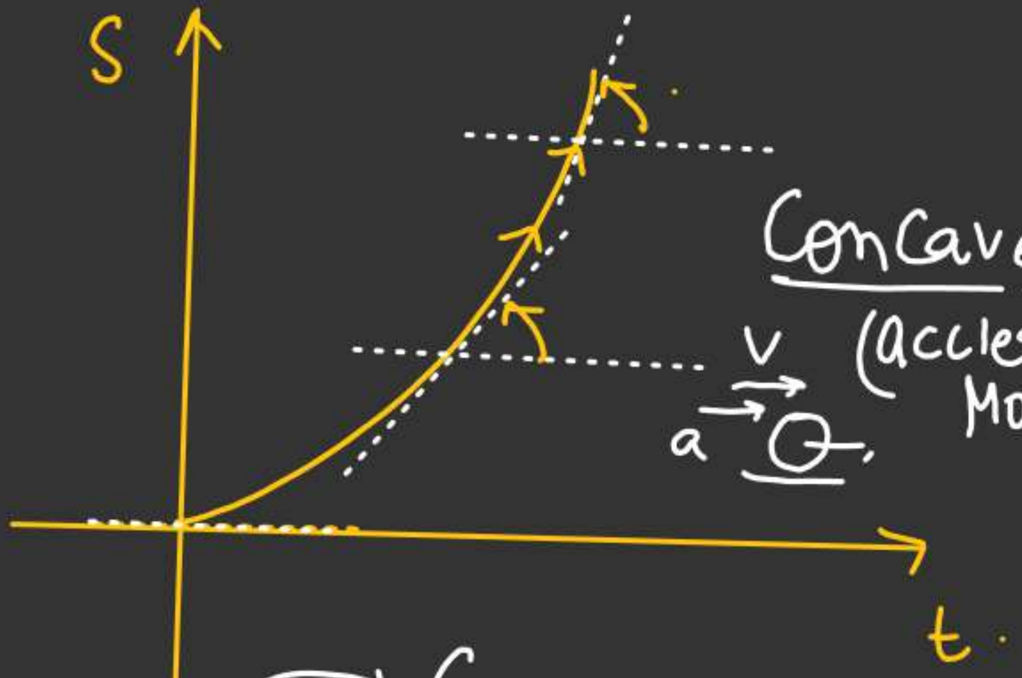


(Non-uniform retadation)

$\theta_1 > \theta_2$

$\tan\theta_1 > \tan\theta_2$

a



Concave (accleraated Motion)

Convex ⇒ Retardation