INTEGRATION BY PARTS

1. $\int \frac{\tan^{-1}x-\cot^{-1}x}{\tan^{-1}x+\cot^{-1}x}dx$ is equal to

(A) $\frac{4}{\pi}x\tan^{-1}x+\frac{2}{\pi}\ln(1+x^2)-x+c$ (B) $\frac{4}{\pi}x\tan^{-1}x-\frac{2}{\pi}\ln(1+x^2)+x+c$

(C) $\frac{4}{\pi}x\tan^{-1}x+\frac{2}{\pi}\ln(1+x^2)+x+c$ (D) $\frac{4}{\pi}x\tan^{-1}x-\frac{2}{\pi}\ln(1+x^2)-x+c$

2. $\int \frac{(\log x)}{x^2}dx =$

(A) $\frac{1}{2}(\log x+1)+c$ (B) $-\frac{1}{x}(\log x+1)+c$

(C) $\frac{1}{x}(\log x-1)+c$ (D) $\log(x+1)+c$

3. $\int \left(xe^{\ell n\sin x}-\cos x\right)dx$ is equal to:

(A) $x\cos x+c$ (B) $\sin x-x\cos x+c$

(C) $-e^{\ln x}\cos x+c$ (D) $\sin x+x\cos x+c$

4. If $\int \frac{x\tan^{-1}x}{\sqrt{1+x^2}}dx=\sqrt{1+x^2}f(x)+A\ln\left(x+\sqrt{x^2+1}\right)+C$, then

(A) $f(x)=\tan^{-1}x, A=-1$ (B) $f(x)=\tan^{-1}x, A=1$

(C) $f(x)=2\tan^{-1}x, A=-1$ (D) $f(x)=2\tan^{-1}x, A=1$

5. $\int (x-1)e^{-x}dx$ is equal to

(A) $-xe^x+C$ (B) xe^x+C (C) $-xe^{-x}+C$ (D) $xe^{-x}+C$

6. $\int e^{\tan^{-1}x}\left(\frac{1+x+x^2}{1+x^2}\right)dx$ is equal to

(A) $xe^{\tan^{-1}x}+c$ (B) $x^2e^{\tan^{-1}x}+c$ (C) $\frac{1}{x}e^{\tan^{-1}x}+c$ (D) None of these

7. $\int [f(x)g''(x)-f''(x)g(x)]dx$ is equal to

(A) $\frac{f(x)}{g'(x)}$ (B) $f'(x)g(x)-f(x)g'(x)$

(C) $f(x)g'(x)-f'(x)g(x)$ (D) $f(x)g'(x)+f'(x)g'(x)$

8. $\int \frac{e^{\sqrt{x}}}{\sqrt{x}}(x+\sqrt{x})dx$ is equal to

(A) $2e^{\sqrt{x}}[\sqrt{x}-x+1]+c$ (B) $2e^{\sqrt{x}}[x-2\sqrt{x}+1]+c$

(C) $2e^{\sqrt{x}}[x-\sqrt{x}+1]+c$ (D) $2e^{\sqrt{x}}[x+\sqrt{x}+1]+c$

9. $\int e^{\tan\theta}(\sec\theta-\sin\theta)d\theta$ is equal to

(A) $-e^{\tan\theta}\sin\theta+c$ (B) $e^{\tan\theta}\sin\theta+c$ (C) $e^{\tan\theta}\sec\theta+c$ (D) $e^{\tan\theta}\cos\theta+c$

10. $\int \left\{\frac{(\log x-1)}{1+(\log x)^2}\right\}^2 dx$ is equal to

(A) $\frac{x}{(\log x)^2+1}+c$ (B) $\frac{xe^x}{1+x^2}+c$ (C) $\frac{x}{x^2+1}+c$ (D) $\frac{\log x}{(\log x)^2+1}+c$

11. $\int \left\{\ln(1+\sin x)+x\tan\left(\frac{\pi}{4}-\frac{x}{2}\right)\right\}dx$ is equal to

(A) $x\ln(1+\sin x)+c$ (B) $\ln(1+\sin x)+c$

(C) $-x\ln(1+\sin x)+c$ (D) $\ell n(1-\sin x)+c$

INTEGRATION BY PARTS FRACTION

12. If $\int \frac{4e^x+6e^{-x}}{9e^x-4e^{-x}}dx=Ax+B\ln(9e^{2x}-4)+C$, then

(A) $A=-\frac{3}{2}, B=\frac{35}{36}, c=0$ (B) $A=\frac{35}{36}, B=-\frac{3}{2}, c\in R$

(C) $A=-\frac{3}{2}, B=\frac{35}{36}, c\in R$ (D) None of these

13. $\int \frac{x^3-1}{x^3+x}dx$ is equal to

(A) $x - \ell nx + \ell n(x^2+1) - \tan^{-1}x + c$ (B) $x - \ln x + \frac{1}{2}\ln(x^2+1) - \tan^{-1}x + c$

(C) $x + \ln x + \frac{1}{2}\ln(x^2+1) + \tan^{-1}x + c$ (D) None of these

14. If $\int \frac{dx}{x^4+x^3} = \frac{A}{x^2} + \frac{B}{x} + \ell n\left|\frac{x}{x+1}\right| + C$, then

(A) $A = \frac{1}{2}, B = 1$ (B) $A = 1, B = -\frac{1}{2}$

(C) $A = -\frac{1}{2}, B = 1$ (D) None of these

15. $\int \frac{1}{x(x^n+1)}dx$ is equal to

(A) $\frac{1}{n}\ln\left(\frac{x^n}{x^n+1}\right) + c$ (B) $\frac{1}{n}\ln\left(\frac{x^n+1}{x^n}\right) + c$

(C) $\ln\left(\frac{x^n}{x^n+1}\right) + c$ (D) None of these

16. $\int \frac{1-x^7}{x(1+x^7)}dx$ is equal to

(A) $\ell nx + \frac{2}{7}\ell n(1+x^7) + c$ (B) $\ell nx - \frac{2}{7}\ell n(1-x^7) + c$

(C) $\ell nx - \frac{2}{7}\ell n(1+x^7) + c$ (D) $\ell nx + \frac{2}{7}\ell n(1-x^7) + c$

MISCELLANEOUS

17. Antiderivative of $\frac{\sin^2 x}{1+\sin^2 x}$ w.r.t. x is :

(A) $x - \frac{\sqrt{2}}{2}\arctan(\sqrt{2}\tan x) + c$ (B) $x + \frac{1}{\sqrt{2}}\arctan\left(\frac{\tan x}{\sqrt{2}}\right) + c$

(C) $x - \sqrt{2}\arctan(\sqrt{2}\tan x) + c$ (D) $x - \sqrt{2}\arctan\left(\frac{\tan x}{\sqrt{2}}\right) + c$

18. If $y = \int \frac{dx}{(1+x^2)^{3/2}}$ and $y = 0$ when $x = 0$, then value of y when $x = 1$ is

(A) $\sqrt{\frac{2}{3}}$ (B) $\sqrt{2}$ (C) $3\sqrt{2}$ (D) $\frac{1}{\sqrt{2}}$

19. If $\int \frac{x^4+1}{x(x^2+1)^2}dx = A\ell n|x| + \frac{B}{1+x^2} + c$ where c is the constant of integration then

(A) $A = 1; B = -1$ (B) $A = -1; B = 1$ (C) $A = 1; B = 1$ (D) $A = -1, B = -1$

20. $\int \frac{dx}{\cos x - \sin x}$ is equal to-

(A) $\frac{1}{\sqrt{2}}\log\left|\tan\left(\frac{x}{2} - \frac{\pi}{8}\right)\right| + C$ (B) $\frac{1}{\sqrt{2}}\log\left|\cot\left(\frac{x}{2}\right)\right| + C$

(C) $\frac{1}{\sqrt{2}}\log\left|\tan\left(\frac{x}{2} - \frac{3\pi}{8}\right)\right| + C$ (D) $\frac{1}{\sqrt{2}}\log\left|\tan\left(\frac{x}{2} + \frac{3\pi}{8}\right)\right| + C$

21. If $f(x) = \int \frac{2\sin x - \sin 2x}{x^3}dx$ where $x \neq 0$ then Limit $_{x\to 0} f'(x)$ has the value

(A) 0 (B) 1 (C) 2 (D) Not defined

22. $\int \sqrt{\frac{x-1}{x+1}} \cdot \frac{1}{x^2}\, dx$ is equal to

(A) $\sin^{-1}\frac{1}{x} + \frac{\sqrt{x^2-1}}{x}$ (B) $\frac{\sqrt{x^2-1}}{x} + \cos^{-1}\frac{1}{x} + c$

(C) $\sec^{-1}x - \frac{\sqrt{x^2-1}}{x} + c$ (D) $\tan^{-1}\sqrt{x^2-1} - \frac{\sqrt{x^2-1}}{x} + c$

23. $\int \tan(x-\alpha)\tan(x+\alpha)\tan 2x dx$ is equal to

(A) $\ln\left|\frac{\sqrt{\sec 2x}\cdot\sec(x+\alpha)}{\sec(x-\alpha)}\right| + C$ (B) $\ln\left|\frac{\sqrt{\sec 2x}}{\sec(x-\alpha)\sec(x+\alpha)}\right| + C$

(C) $\ln\left|\frac{\sqrt{\sec 2x}\cdot\sec(x+\alpha)}{\sec(x+\alpha)}\right| + C$ (D) None of these

24. $\int x \cdot \frac{\ln(x+\sqrt{1+x^2})}{\sqrt{1+x^2}}dx$ is equal to

(A) $\sqrt{1+x^2}\cdot\ln(x+\sqrt{1+x^2})-x+c$ (B) $\frac{x}{2}\cdot\ln^2(x+\sqrt{1+x^2})-\frac{x}{\sqrt{1+x^2}}+c$

(C) $\frac{x}{2}\cdot\ln^2(x+\sqrt{1+x^2})+\frac{x}{\sqrt{1+x^2}}+c$ (D) $\sqrt{1+x^2}\ln(x+\sqrt{1+x^2})+x+c$

25. $2\int \sin x\cdot \operatorname{cosec}4x\,dx$ is equal to

(A) $\frac{1}{2\sqrt{2}}\ln\frac{1+\sqrt{2}\sin x}{1-\sqrt{2}\sin x}-\frac{1}{4}\ln\frac{1+\sin x}{1-\sin x}+C$ (B) $\frac{1}{2\sqrt{2}}\ln\frac{1+\sqrt{2}\sin x}{1-\sqrt{2}\sin x}+\frac{1}{4}\ln\frac{1+\sin x}{1-\sin x}+C$

(C) $\frac{1}{2\sqrt{2}}\ln\frac{1-\sqrt{2}\sin x}{1+\sqrt{2}\sin x}-\frac{1}{4}\ln\frac{1+\sin x}{1-\sin x}+C$ (D) None of these

26. $\int \frac{1}{x^2(x^4+1)^{3/4}}dx$ is equal to

(A) $\left(1+\frac{1}{x^4}\right)^{1/4}+c$ (B) $(x^4+1)^{1/4}+c$

(C) $\left(1-\frac{1}{x^4}\right)^{1/4}+c$ (D) $-\left(1+\frac{1}{x^4}\right)^{1/4}+c$

27. If $\int \frac{1}{x+x^5}dx = f(x)+C$, then the value of $\int \frac{x^4}{x+x^5}dx$ is equal to

(A) $\log x - f(x)+C$ (B) $f(x)+\log x+C$

(C) $f(x)-\log x+C$ (D) None of these

28. $\int \frac{1+x^4}{(1-x^4)^{3/2}}dx$ is equal to

(A) $\frac{1}{\sqrt{x^2-\frac{1}{x^2}}}+c$ (B) $\sqrt{x^2-\frac{1}{x^2}}+c$ (C) $\frac{1}{\sqrt{\frac{1}{x^2}+x^2}}+c$ (D) None of these

29. $\int \frac{\cos^3 x}{\sin^2 x+\sin x}dx$ is equal to

(A) $\ln|\sin x|+\sin x+c$ (B) $\ln|\sin x|-\sin x+c$

(C) $-\ln|\sin x|-\sin x+c$ (D) $-\ln|\sin x|+\sin x+c$

30. Primitive of $\frac{3x^4-1}{(x^4+x+1)^2}$ w.r.t. x is

(A) $\frac{x}{x^4+x+1}+c$ (B) $-\frac{x}{x^4+x+1}+c$ (C) $\frac{x+1}{x^4+x+1}+c$ (D) $-\frac{x+1}{x^4+x+1}+c$

SUBJECTIVE QUESTION

31. $\int \sin^2 x\cos^2 x\,dx$

32. $\int \frac{1}{\sin(x-a)\cos(x-b)}dx$

33. $\int \frac{x+\sqrt{x+1}}{x+2}dx$

34. $\int \frac{(x-1)^2}{x^4+x^2+1}dx$

35. $\int \frac{2\sin 2\phi-\cos\phi}{6-\cos^2\phi-4\sin\phi}d\phi$

36. $\int \frac{1}{1-\sin^4 x}dx$

37. $\int \frac{1+x\cos x}{x(1-x^2e^{2\sin x})}dx$

38. $\int \frac{x\cos\alpha+1}{(x^2+2x\cos\alpha+1)^{3/2}}dx = \frac{f(x)}{\sqrt{g(x)}}+c$ then find f(x) and g(x)

39. $\int \left[\left(\frac{x}{e}\right)^x+\left(\frac{e}{x}\right)^x\right]\ln x\,dx$

40. $\int \sqrt{\frac{\sin(x-a)}{\sin(x+a)}}dx$

41. $\int \left[\frac{\sqrt{x^2+1}[\ln(x^2+1)-2\ln x]}{x^4}\right]dx$

42. $\int \frac{(ax^2-b)dx}{x\sqrt{c^2x^2-(ax^2+b)^2}}$

43. For any natural number m, evaluate $\int (x^{3m}+x^{2m}+x^m)(2x^{2m}+3x^m+6)^{\frac{1}{m}}dx$, where $x>0$

44. $\int e^x\frac{x^3-x+2}{(x^2+1)^2}dx$

45. $\int \frac{x\ln x}{(x^2-1)^{3/2}}dx$

46. $\int \cos 2x \ln(1+\tan x)dx$

47. $\int \tan^{-1}x \cdot \ln(1+x^2)dx$

48. $\int \frac{e^{\cos x}(x\sin^3 x+\cos x)}{\sin^2 x}dx$

49. $\int \frac{e^x(2-x^2)}{(1-x)\sqrt{1-x^2}}dx$

50. $\int \frac{x+1}{x(1+xe^x)^2}dx$

51. $\int \sqrt{\frac{\operatorname{cosec}x-\cot x}{\operatorname{cosec}x+\cot x}} \cdot \frac{\sec x}{\sqrt{1+2\sec x}}dx$

52. $\int \frac{\cos x-\sin x}{7-9\sin 2x}dx$

53. $\int \frac{3+4\sin x+2\cos x}{3+2\sin x+\cos x}dx$

54. $\int \frac{dx}{\cos^3 x-\sin^3 x}$

55. $\int \frac{(\sqrt{x}+1)dx}{\sqrt{x}(\sqrt[3]{x}+1)}$

56. $\int \frac{5x^4+4x^5}{(x^5+x+1)^2}dx$

57. $\int \frac{dx}{\sin x+\sec x}$

58. $\int \frac{4x^5-7x^4+8x^3-2x^2+4x-7}{x^2(x^2+1)^2}dx$

59. Evaluate $\int \sin^{-1}\left(\frac{2x+2}{\sqrt{4x^2+8x+13}}\right)dx$

60. $\int \frac{\sqrt{4+x^2}}{x^6}dx$

61. $\int \frac{1+\cos\alpha\cos x}{\cos\alpha+\cos x}dx$

62. $\int \frac{(\cos 2x-3)}{\cos^4 x\sqrt{4-\cot^2 x}}dx$

63. $\int \frac{\tan 2\theta}{\sqrt{\cos^6\theta+\sin^6\theta}}d\theta$

64. $\int \frac{x^2+x}{(e^x+x+1)^2}dx$

65. $\int \sqrt{\frac{(1-\sin x)(2-\sin x)}{(1+\sin x)(2+\sin x)}}dx$

66. $\int \frac{\sqrt{2-x-x^2}}{x^2}dx$

67. $\int \frac{\cos^2 x}{1+\tan x}dx$

JEE MAIN

68. $\int \frac{dx}{\cos x+\sqrt{3}\sin x}$ equals **[AIEEE 2007]**

(A) $\frac{1}{2}\log\tan\left(\frac{x}{2}+\frac{\pi}{12}\right)+C$ (B) $\frac{1}{2}\log\tan\left(\frac{x}{2}-\frac{\pi}{12}\right)+C$

(C) $\log\tan\left(\frac{x}{2}+\frac{\pi}{12}\right)+C$ (D) $\log\tan\left(\frac{x}{2}-\frac{\pi}{12}\right)+C$

69. The value of $\sqrt{2}\int \frac{\sin x dx}{\sin\left(x-\frac{\pi}{4}\right)}$ is **[AIEEE 2008]**

(A) $x-\log\left|\sin\left(x-\frac{\pi}{4}\right)\right|+c$ (B) $x+\log\left|\sin\left(x-\frac{\pi}{4}\right)\right|+c$

(C) $x-\log\left|\cos\left(x-\frac{\pi}{4}\right)\right|+c$ (D) $x+\log\left|\cos\left(x-\frac{\pi}{4}\right)\right|+c$

70. If the integral $\int \frac{5\tan x}{\tan x-2}dx = x+a\ln|\sin x-2\cos x|+k$ then a is equal to : **[AIEEE 2012]**

(A) 1 (B) 2 (C) -1 (D) -2

71. If $\int f(x)dx = \Psi(x)$, then $\int x^5 f(x^3)dx$ is equal to: **[JEE-MAIN 2013]**

(A) $\frac{1}{3}x^3\Psi(x^3)-\int x^2\Psi(x^3)dx+C$ (B) $\frac{1}{3}\left[x^3\Psi(x^3)-\int x^3\Psi(x^3)dx\right]+C$

(C) $\frac{1}{3}\left[x^3\Psi(x^3)-\int x^2\Psi(x^3)dx\right]+C$ (D) $\frac{1}{3}x^3\Psi(x^3)-3\int x^3\Psi(x^3)dx+C$

72. The integral $\int \left(1+x-\frac{1}{x}\right)e^{x+\frac{1}{x}}dx$ is equal to: **[JEE-MAIN 2014]**

(A) $(x-1)e^{x+\frac{1}{x}}+c$ (B) $x_e^{x+\frac{1}{x}}+c$ (C) $(x+1)e^{x+\frac{1}{x}}+c$ (D) $-xe^{x+\frac{1}{x}}+c$

73. The integral $\int \frac{dx}{x^2(x^4+1)^{3/4}}$ equals : **[JEE-MAIN 2015]**

(A) $-(x^4+1)^{1/4}+c$ (B) $-\left(\frac{x^4+1}{x^4}\right)^{1/4}+c$

(C) $\left(\frac{x^4+1}{x^4}\right)^{1/4}+c$ (D) $\left(\frac{x^4+1}{x^4}\right)^{1/4}+c$

74. The integral $\int \frac{2x^{12}+5x^9}{(x^5+x^3+1)^3}dx$ is equal to: **[JEE-MAIN 2016]**

(A) $\frac{x^{10}}{2(x^5+x^3+1)^2}+C$ (B) $\frac{x^5}{2(x^5+x^3+1)^2}+C$ (C) $\frac{-x^{10}}{2(x^5+x^3+1)^2}+C$ (D) $\frac{-x^5}{2(x^5+x^3+1)^2}+C$

where C is an arbitrary constant.

75. Let $I_n=\int \tan^n x dx, (n>1) m\ I_4+I_6=a\tan^5 x+bx^5+C$, where C is a constant of integration ,then the ordered pair (a, b) is equal to : **[JEE-MAIN 2017]**

(A) $\left(-\frac{1}{5},1\right)$ (B) $\left(\frac{1}{5},0\right)$ (C) $\left(\frac{1}{5},-1\right)$ (D) $\left(-\frac{1}{5},0\right)$

76. The integral $\int \frac{\sin^2 x\cos^2 x}{(\sin^5 x+\cos^3 x\sin^2 x+\sin^3 x\cos^2 x+\cos^5 x)^2}dx$ is equal to : **[JEE-MAIN 2018]**

(A) $\frac{-1}{1+\cot^3 x}+C$ (B) $\frac{1}{3(1+\tan^3 x)}+C$ (C) $\frac{-1}{3(1+\tan^3 x)}+C$ (D) $\frac{1}{1+\cot^3 x}+C$

(where C is a constant of integration)

77. If $\int e^{\sec x}(\sec x\tan x f(x)+(\sec x\tan x+\sec^2 x))dx=e^{\sec x}f(x)+C$, then a possible choice of f(x) is: **[JEE-MAIN 2019]**

(A) $\sec x+\tan x+\frac{1}{2}$ (B) $\sec x-\tan x-\frac{1}{2}$

(C) $\sec x+x\tan x-\frac{1}{2}$ (D) $x\sec x+\tan x+\frac{1}{2}$

78. Find integration $\int \frac{dx}{(x-3)^{6/7}\cdot(x+4)^{8/7}}$ **[JEE-MAIN 2020]**

(A) $\left(\frac{x-3}{x+4}\right)^{\frac{1}{7}}+c$ (B) $7\left(\frac{x-3}{x+4}\right)^{\frac{1}{7}}+c$

(C) $7\left(\frac{x-3}{x+4}\right)^{\frac{6}{7}}+c$ (D) $7\left(\frac{x+4}{x-3}\right)^{\frac{6}{7}}+c$

79. If $\int \frac{\cos x-\sin x}{\sqrt{8-\sin 2x}}dx=a\sin^{-1}\left(\frac{\sin x+\cos x}{b}\right)+c$, where c is a constant of integration, then the ordered pair (a, b) is equal to: **[JEE-MAIN 2021]**

(A) $(1,-3)$ (B) $(1,3)$ (C) $(-1,3)$ (D) $(3,1)$

80. For $I(x)=\int \frac{\sec^2 x-2022}{\sin^{2022} x}dx$, if $I\left(\frac{\pi}{4}\right)=2^{1011}$, then **(JEE-MAIN 2022)**

(A) $3^{1010}I\left(\frac{\pi}{3}\right)-I\left(\frac{\pi}{6}\right)=0$ (B) $3^{1010}I\left(\frac{\pi}{6}\right)-I\left(\frac{\pi}{3}\right)=0$

(C) $3^{1011}I\left(\frac{\pi}{3}\right)-I\left(\frac{\pi}{6}\right)=0$ (D) $3^{1011}I\left(\frac{\pi}{6}\right)-I\left(\frac{\pi}{3}\right)=0$

81. If $\int \sqrt{\sec 2x-1}dx=\alpha\log_e\left|\cos 2x+\beta+\sqrt{\cos 2x\left(1+\cos\frac{1}{\beta}x\right)}\right|+$ constant, then $\beta-\alpha$ is equal to _____. **(JEE MAIN 2023)**

JEE ADVANCED

82. $\int \frac{x^2-1}{x^3\sqrt{2x^4-2x^2+1}}dx$ is equal to **[JEE 2006]**

(A) $\frac{\sqrt{2x^4-2x^2+1}}{x^2}+C$ (B) $\frac{\sqrt{2x^4-2x^2+1}}{x^3}+C$ (C) $\frac{\sqrt{2x^4-2x^2+1}}{x}+C$ (D) $\frac{\sqrt{2x^4-2x^2+1}}{2x^2}+C$

83. **(a)** Let $f(x)=\frac{x}{(1+x^n)^{1/n}}$ for $n\geq 2$ and [JEE 2007] $g(x)=\underbrace{(f\circ f\circ....\circ f)}_{f\text{ occurs n times}}(x)$. Then $\int x^{n-2}g(x)dx$ Equals

(A) $\frac{1}{n(n-1)}(1+nx^n)^{1-\frac{1}{n}}+K$ (B) $\frac{1}{(n-1)}(1+nx^n)^{1-\frac{1}{n}}+K$

(C) $\frac{1}{n(n+1)}(1+nx^n)^{1+\frac{1}{n}}+K$ (D) $\frac{1}{(n+1)}(1+nx^n)^{1+\frac{1}{n}}+K$

(b) Let F(x) be an indefinite integral of $\sin^2 x$.

Statement-1 : The function F(x) satisfies $F(x+\pi)=F(x)$ for all real x. because

Statement-2 : $\sin^2(x+\pi)=\sin^2 x$ for all real x.

(A) Statement (1) is true and statement (2) is true and statement (2) is correct explanation for (1)
(B) Statement (1) is true and statement (2) is true and statement (2) is NOT correct explanation for (1)
(C) Statement (1) is true but (2) is false
(D) Statement (1) is false but (2) is true **[JEE 2007]**

84. Let $I = \int \frac{e^x}{e^{4x}+e^{2x}+1}dx, J = \int \frac{e^{-x}}{e^{-4x}+e^{-2x}+1}dx$ Then, for an arbitrary constant C, the value of $J - I$ equals **[JEE 2008]**

(A) $\frac{1}{2}\ln\left(\frac{e^{4x}-e^{2x}+1}{e^{4x}+e^{2x}+1}\right)+C$ (B) $\frac{1}{2}\ln\left(\frac{e^{2x}+e^{x}+1}{e^{2x}-e^{x}+1}\right)+C$

(C) $\frac{1}{2}\ln\left(\frac{e^{2x}-e^{x}+1}{e^{2x}+e^{x}+1}\right)+C$ (D) $\frac{1}{2}\ln\left(\frac{e^{4x}+e^{2x}+1}{e^{4x}-e^{2x}+1}\right)+C$

85. The integral $\int \frac{\sec^2 x}{(\sec x+\tan x)^{9/2}}dx$ equals (for some arbitrary constant K) **[JEE 2012]**

(A) $-\frac{1}{(\sec x+\tan x)^{11/2}}\left\{\frac{1}{11}-\frac{1}{7}(\sec x+\tan x)^2\right\}+K$

(B) $\frac{1}{(\sec x+\tan x)^{\frac{11}{2}}}\left\{\frac{1}{11}-\frac{1}{7}(\sec x+\tan x)^2\right\}+K$

(C) $-\frac{1}{(\sec x+\tan x)^{\frac{11}{2}}}\left\{\frac{1}{11}+\frac{1}{7}(\sec x+\tan x)^2\right\}+K$

(D) $\frac{1}{(\sec x+\tan x)^{11/2}}\left\{\frac{1}{11}+\frac{1}{7}(\sec x+\tan x)^2\right\}+K$

86. Let $f: R \to R$ and $g: R \to R$ be two nonconstant differentiable functions. If $f'(x) = \left(e^{(f(x)-g(x))}\right)g'(x)$ for all $x \in R$, and $f(1) = g(2) = 1$, then which of the following statement(s) is (are) TRUE ? **[JEE Adv. 2018]**

(A) $f(2) < 1 - \log_e 2$ (B) $f(2) > 1 - \log_e 2$

(C) $g(1) > 1 - \log_e 2$ (D) $g(1) < 1 - \log_e 2$

ANSWER KEY

1.	(D)	**2.**	(B)	**3.**	(C)	**4.**	(A)	**5.**	(C)	**6.**	(A)	**7.**	(C)
8.	(C)	**9.**	(D)	**10.**	(A)	**11.**	(A)	**12.**	(C)	**13.**	(B)	**14.**	(C)
15.	(A)	**16.**	(C)	**17.**	(A)	**18.**	(D)	**19.**	(C)	**20.**	(D)	**21.**	(B)
22.	(C)	**23.**	(B)	**24.**	(A)	**25.**	(A)	**26.**	(D)	**27.**	(A)	**28.**	(B)
29.	(B)	**30.**	(B)	**31.**	$\frac{1}{8}\left[x-\frac{\sin 4x}{4}\right]+C$	**32.**	$\frac{1}{\cos(a-b)}\ln\left\lvert\frac{\sin(x-a)}{\cos(x-b)}\right\rvert+c$						

33. $(x+1)+2\sqrt{x+1}-2\ln|x+2|-2\tan^{-1}\sqrt{x+1}+c$

34. $\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{x^2-1}{x\sqrt{3}}\right)-\frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right)+c$

35. $2\ln|\sin^2\phi-4\sin\phi+5|+7\tan^{-1}(\sin\phi-2)+c$

36. $\frac{1}{2\sqrt{2}}\tan^{-1}(\sqrt{2}\tan x)+\frac{1}{2}\tan x+c$

37. $\ln\left(xe^{\sin x}\right)-\frac{1}{2}\ln\left(1-x^2e^{2\sin x}\right)+c$

38. $x/\sqrt{x^2+2\cos\alpha+1}$

39. $\left(\frac{x}{e}\right)^x-\left(\frac{e}{x}\right)^x+C$

40. $\cos a\cdot\arccos\left(\frac{\cos x}{\cos a}\right)-\sin a\cdot\ell n\left(\sin x+\sqrt{\sin^2x-\sin^2a}\right)+c$

41. $\frac{(x^2+1)\sqrt{x^2+1}}{9x^3}\left[2-3\ln\left(1+\frac{1}{x^2}\right)\right]$

42. $\sin^{-1}\left(\frac{ax^2+b}{cx}\right)+k$

43. $\frac{1}{6(m+1)}(2x^{3m}+3x^{2m}+6x^m)^{m+1/m}+C$

44. $e^x\left(\frac{x+1}{x^2+1}\right)+c$

45. $\operatorname{arcsec}x-\frac{\ln x}{\sqrt{x^2-1}}+c$

46. $\frac{1}{2}[\sin 2x\cdot\ln(1+\tan x)-x+\ln(\sin x+\cos x)]+c$

47. $x\tan^{-1}x\cdot\ln(1+x^2)+(\tan^{-1}x)^2-2x\tan^{-1}x+\ln(1+x^2)-\left(\ln\sqrt{1+x^2}\right)^2+c$

48. $C-e^{\cos x}(x+\operatorname{cosec}x)$

49. $e^x\sqrt{\frac{1+x}{1-x}}+c$

50. $\ln\left(\frac{xe^x}{1+xe^x}\right)+\frac{1}{1+xe^x}+C$

51. $\sin^{-1}\left(\frac{1}{2}\sec^2\frac{x}{2}\right)+c$

52. $\frac{1}{24}\ln\frac{(4+3\sin x+3\cos x)}{(4-3\sin x-3\cos x)}+c$

53. $2x-3\arctan\left(\tan\frac{x}{2}+1\right)+c$

54. $\frac{2}{3}\tan^{-1}(\sin x+\cos x)+\frac{1}{2\sqrt{3}}\ln\left\lvert\frac{\sqrt{2}+\sin x+\cos x}{\sqrt{2}-\sin x-\cos x}\right\rvert+C$

55. $6\left[\frac{t^4}{4}-\frac{t^2}{2}+t+\frac{1}{2}\ln(1+t^2)-\tan^{-1}t\right]+C$ जहाँ $t=x^{1/6}$

56. $-\frac{x+1}{x^5+x+1}+C$ or $C-\frac{x^5}{x^5+x+1}$

57. $\frac{1}{2\sqrt{3}}\ln\frac{\sqrt{3}+\sin x-\cos x}{\sqrt{3}-\sin x+\cos x}+\arctan(\sin x+\cos x)+c$

58. $4\ln x+\frac{7}{x}+6\tan^{-1}(x)+\frac{6x}{1+x^2}+C$

59. $(x+1)\tan^{-1}\frac{2(x+1)}{3}-\frac{3}{4}\ln(4x^2+8x+13)+c$

60. $\frac{(4+x^2)^{3/2}\cdot(x^2-6)}{120x^5}+C$

61. $x\cos\alpha+\sin\alpha\ell n\left\{\frac{\cos\frac{1}{2}(\alpha-x)}{\cos\frac{1}{2}(\alpha+x)}\right\}+c$

62. $-\frac{1}{3}\tan x\cdot(2+\tan^2 x)\cdot\sqrt{4-\cot^2 x}$

63. $\ln\left(\frac{1+3\cos^2 2\theta}{\cos 2\theta}\right)+C$

64. $C-\ln(1+(x+1)e^{-x})-\frac{1}{1+(x+1)e^{-x}}$

65. $\sqrt{3}\ln\frac{t-\sqrt{3}}{1+\sqrt{3}}+2\tan^{-1}(t)+C$

66. $\frac{\sqrt{2-x-x^2}}{x}+\frac{\sqrt{2}}{4}\ln\left(\frac{4-x+2\sqrt{2}\sqrt{2-x-x^2}}{x}\right)-\sin^{-1}\left(\frac{2x+1}{3}\right)+c$

67. $\frac{1}{4}\ln(\cos x+\sin x)+\frac{x}{2}+\frac{1}{8}(\sin 2x+\cos 2x)+c$

68.	(A)	**69.**	(B)	**70.**	(B)	**71.**	(A)	**72.**	(B)	**73.**	(B)	**74.**	(A)		
75.	(B)	**76.**	(C)	**77.**	(A)	**78.**	(A)	**79.**	(A)	**80.**	(A)	**81.**	1		
82.	(D)	**83.**	**(a)** (A) ,**(b)** (D)			**84.**	(C)	**85.**	(C)	**86.**	(B,C)				