FINDING INTERVALS OF MONOTONOCITY

1. The interval in which the function x^3 increases less rapidly than $6x^2 + 15x + 5$ is
(A) $(-\infty, -1)$ (B) $(-5,1)$ (C) $(-1,5)$ (D) $(5, \infty)$

2. The function $\frac{|x-1|}{x^2}$ is monotonically decreasing in
(A) $(2, \infty)$ (B) $(0,1)$ (C) $(0,1)$ and $(2, \infty)$ (D) $(-\infty, \infty)$

3. The true set of real values of x for which the function, $f(x) = x\ln x - x + 1$ is positive is
(A) $(1, \infty)$ (B) $(1/e, \infty)$ (C) $[e, \infty)$ (D) $(0,1)$ and $(1, \infty)$

4. The set of all x for which $\ln(1 + x) \leq x$ is equal to
(A) $x > 0$ (B) $x > -1$ (C) $-1 < x < 0$ (D) null set

5. If $f(x) = \frac{x^2}{2-2\cos x}$; $g(x) = \frac{x^2}{6x-6\sin x}$ where $0 < x < 1$, then
(A) both ' f ' and ' g ' are increasing functions
(B) ' f ' is decreasing & ' g ' is increasing function
(C) ' f ' is increasing & ' g ' is decreasing function
(D) both 'f' & ' g ' are decreasing function

6. $f(x) = x^2 - x\sin x$ is
(A) increasing for $0 \leq x \leq \pi/2$ (B) decreasing for $0 \leq x \leq \pi/2$
(C) decreasing for $[\pi/4, \pi/2]$ (D) non increasing for $0 < x < \pi/2$

7. $x^3 - 3x^2 - 9x + 20$ is
(A) - ve for $x < 4$ (B) + ve for $x > 4$
(C) - ve for $x \in (0,1)$ (D) $-$ve for $x \in (-1,0)$

8. The function $f(x) = \tan^{-1}(\sin x + \cos x)$ is an increasing function in
(A) $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$ (B) $\left(-\frac{\pi}{2}, \frac{\pi}{4}\right)$ (C) $\left(0, \frac{\pi}{2}\right)$ (D) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

9. Function $f(x) = \log\sin x$ is monotonic increasing when
(A) $x \in (\pi/2, \pi)$ (B) $x \in (-\pi/2, 0)$ (C) $x \in (0, \pi)$ (D) $x \in (0, \pi/2)$

10. Which of the following statements is/are correct
(A) $x + \sin x$ is increasing function
(B) $\sec x$ is neither increasing nor decreasing function
(C) $x + \sin x$ is decreasing function
(D) $\sec x$ is an increasing function

11. Let $f(x) = x^{m/n}$ for $x \in R$ where m and n are integers, m even and n odd and $0 < m < n$. Then
(A) f(x) decreases on $(-\infty, 0]$ (B) f(x) increases on $[0, \infty)$
(C) f(x) increases on $(-\infty, 0]$ (D) f(x) decreases on $[0, \infty)$

12. The function $y = 2x^2 - \ln|x|$ is monotonically increasing in the interval I_1 and monotonically decreasing in the interval $I_2, x(\neq 0)$, then
(A) $I_1 = \left(-\frac{1}{2}, 0\right) \cup \left(\frac{1}{2}, \infty\right)$ (B) $I_2 = \left(-\infty, -\frac{1}{2}\right) \cup \left(0, \frac{1}{2}\right)$
(C) $I_1 = \left(-\infty, -\frac{1}{2}\right) \cup \left(0, \frac{1}{2}\right)$ (D) $I_2 = \left(-\frac{1}{2}, 0\right) \cup \left(\frac{1}{2}, \infty\right)$

13. If $\phi(x) = f(x) + f(2a - x)$ and $f'(x) > a, a > 0, 0 \leq x \leq 2a$, then
(A) $\phi(x)$ increases in $(a, 2a)$ (B) $\phi(x)$ increases in $(0, a)$
(C) f(x) decreases in $(0, a)$ (D) $\phi(x)$ decreases in $(1, 2a)$

14. If $f(x) = 2x + \cot^{-1} x + \log\left(\sqrt{1+x^2} - x\right)$, then f(x)
(A) increases in $[0, \infty)$
(B) decreases in $[0, \infty)$
(C) neither increases nor decreases in $[0, \infty)$
(D) increases in $(-\infty, \infty)$

15. Let $g(x) = 2f(x/2) + f(1-x)$ and $f'(x) < 0$ in $0 \le x \le 1$ then g(x)
(A) decreases in $\left[0, \frac{2}{3}\right]$
(B) decreases $\left[\frac{2}{3}, 1\right]$
(C) increases in $\left[0, \frac{2}{3}\right]$
(D) increases in $\left[\frac{2}{3}, 1\right]$

16. Let $\phi(x) = f(x)^3 - 3(f(x))^2 + 4f(x) + 5x + 3\sin x + 4\cos x \ \forall x \in R$, then
(A) ϕ is increasing whenever f is increasing
(B) ϕ is increasing whenever f is decreasing
(C) ϕ is decreasing whenever f is decreasing
(D) ϕ is decreasing if $f'(x) = -11$

17. The function $f(x) = 3x^4 + 4x^3 - 12x^2 - 7$ is
(A) $\uparrow$ in $[-2,0]\&[1,\infty)$
(B) $\downarrow$ in $(-\infty,-2]\&[0,1]$
(C) $\downarrow$ in $[-2,0]\&[1,\infty)$
(D) $\downarrow$ in $(-\infty,-2]\&[0,1]$

18. The function $f(x) = x^2/(x-1), x \neq 1$ is
(A) $\uparrow [0,1) \cup (1,2]$
(B) $\downarrow$ ln $(-\infty, 0] \cup [2, \infty)$
(C) $\downarrow [0,1) \cup (1,2]$
(D) $\uparrow$ ln $(-\infty, 0] \cup [2, \infty)$

19. The function $f(x) = 2\ln(x-2) - x^2 + 4x + 1$ increases in the intervals
(A) (1,2)
(B) (2,3)
(C) $\left[\frac{5}{2}, 3\right]$
(D) (2,4)

20. Let the function $f(x) = \sin x + \cos x$, be defined in $[0, 2\pi]$, then f(x)
(A) increases in $(\pi/4, \pi/2)$
(B) decreases in $[\pi/4, 5\pi/4]$
(C) increases in $[0, \pi/4] \cup [5\pi/4, 2\pi]$
(D) decreases in $[0, \pi/4) \cup (\pi/2, 2\pi]$

21. (i) Show that $f(x) = \tan^{-1}(\sin x + \cos x)$ is a decreasing function for $x \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$.
(ii) Show that $f(x) = \frac{x}{\sqrt{1+x}} - \ln(1+x)$ is an increasing function for $x > -1$.

22. Find the intervals of monotonocity for the following functions.
(i) $\frac{x^4}{4} + \frac{x^3}{3} - 3x^2 + 5$
(ii) $\sin \frac{\pi}{x}$
(ii) M.I. in $\left[\frac{2}{4n+3}, \frac{2}{4n+1}\right], n \in Z$
M.D. in $\left[\frac{2}{4n+1}, \frac{2}{4n-1}\right], n \in Z$
(iii) $\log_3^2 x + \log_3 x$

23. Find the intervals of monotonocity for the following functions & represent your solution set on the number line.
Also plot the graph in each case.
(a) $f(x) = 2 \cdot e^{x^2-4x}$
(b) $f(x) = e^x/x$
(c) $f(x) = x^2 e^{-x}$
(d) $f(x) = 2x^2 - \ln|x|$

24. **Column – I** | **Column - II**

Column – I	Column - II
(A) The function $f(x)=\frac{x}{(1+x^2)}$ decreases in the interval	(P) $(-\infty,-1)$
(B) The function $f(x)=\tan^{-1} x-x$ decreases in the interval	(Q) $(-\infty,0)$
(C) The function $f(x)=x-e^x+\tan\left(\frac{2\pi}{7}\right)$ increases in the interval	(R) $(0,\infty)$
	(S) $(1,\infty)$
	(T) $(-\infty,\infty)$

25. The function $f(x)=\tan^{-1}(\sin x+\cos x)$ is an increasing function in - **[AIEEE 2007]**
(A) $(\pi/4,\pi/2)$ (B) $(-\pi/2,\pi/4)$ (C) $(0,\pi/2)$ (D) $(-\pi/2,\pi/2)$

FINDING VALUE OF VARIABLE GIVEN MONOTONIC BEHAVIOUR

26. If $y=(a+2)x^3-3ax^2+9ax-1$ decreases monotonically $\forall x\in R$ then ' a ' lies in the interval
(A) $(-\infty,-3]$ (B) $(-\infty,-2)\cup(-2,3)$
(C) $(-3,\infty)$ (D) $(0,\infty)$

27. For what values of a does the curve $f(x)=x(a^2-2a-2)+\cos x$ is always strictly monotonic $\forall x\in R$.
(A) $a\in R$ (B) $|a|<\sqrt{2}$
(C) $1-\sqrt{2}\le a\le 1+\sqrt{2}$ (D) $|a|<\sqrt{2}-1$

28. The values of p for which the function $f(x)=\left(\frac{\sqrt{p+4}}{1-p}-1\right)x^5-3x+\ln 5$ decreases for all real x is
(A) $(-\infty,\infty)$ (B) $\left[-4,\frac{3-\sqrt{21}}{2}\right]\cup(1,\infty)$
(C) $\left[-3,\frac{5-\sqrt{27}}{2}\right]\cup(2,\infty)$ (D) $(1,\infty)$

29. The set of values of the parameter ' a ' for which the function;
$f(x)=8ax-a\sin 6x-7x-\sin 5x$ increases & has no critical points for all $x\in R$, is
(A) $[-1,1]$ (B) $(-\infty,-6)$ (C) $(6,+\infty)$ (D) $[6,+\infty)$

30. For what values of x, the function $f(x)=x+\frac{4}{x^2}$ is monotonically decreasing
(A) $x<0$ (B) $x>2$ (C) $x<2$ (D) $0<x<2$

31. Let $f(x)=\begin{cases}x^2 & x\ge 0\\ ax & x<0\end{cases}$. Find real values of a such that f(x) is strictly monotonically increasing at $x=0$.

32. If $f(x)=x^3+(a-1)x^2+2x+1$ is strictly monotonically increasing for every $x\in R$ then find the range of values of ' a '

33. Find the values of ' a ' for which the function $f(x)=(a+2)x^3-3ax^2+9ax-1$ decreases for all real values of x.

CHECKING MONOTONOCITY AT POINT OR IN AN INTERVAL

34. In the interval $(0,1)$, $f(x)=x^2-x+1$ is -
(A) monotonic (B) not monotonic
(C) decreasing (D) increasing

35. Function $f(x) = \sin x - \cos x$ is monotonic increasing when -

(A) $x \in (0, \pi/2)$ (B) $x \in (-\pi/4, 3\pi/4)$

(C) $x \in (\pi/4, 3\pi/4)$ (D) No where

36. Function $f(x) = (x-1)^2(x-2)$ is monotonically decreasing when -

(A) $x \in (1,2)$ (B) $x \in (1, 5/3)$

(C) $x \in R - (1, 5/3)$ (D) No where

37. For $0 \le x \le 1$, the function $f(x) = |x| + |x-1|$ is

(A) monotonically increasing (B) monotonically decreasing

(C) constant function (D) identity function

38. $f(x) = 2x^2 - \log |x| (x \neq 0)$ is monotonic increasing in the interval -

(A) $(1/2, \infty)$ (B) $(-\infty, -1/2)(1/2, \infty)$

(C) $(-\infty, -1/2) \cup (0, 1/2)$ (D) $(-1/2, 0) \cup (1/2, \infty)$

39. Let $f(x)$ and $g(x)$ be two continuous and differentiable functions from $R \to R$ such that $f(x_1) > f(x_2)$ & $g(x_1) < g(x_2) \forall x_1 > x_2$ then possible values of x satisfying $f\left(g(2x^2 - 8x)\right) > f(g(x-4))$ is/are

(A) 0 (B) 1 (C) 2 (D) 3

40. Let $f(x) = x^3 - x^2 + x + 1$ and $g(x) = \begin{cases} \max\{f(t): 0 \le t \le x\} & , 0 \le x \le 1 \\ 3 - x & , 1 < x \le 2 \end{cases}$ Discuss the continuity & differentiability of $g(x)$ is in the interval $(0,2)$

MAXIMUM & MINIMUM VALUE IN CLOSED INTERVAL BY MONOTONOCITY

41. If $f(x) = x^2 + kx + 1$ is increasing function in the interval $[1,2]$, then least value of k is -

(A) 2 (B) 4 (C) -2 (D) -4

42. The function $f(x) = |px - q| + r|x|, x \in (-\infty, \infty)$, where $p > 0, q > 0, r > 0$ assumes its minimum value only at one point if

(A) $p \neq q$ (B) $r \neq q$ (C) $r \neq p$ (D) $p = q = r$

43. Find the greatest & least value of $f(x) = \sin^{-1}\frac{x}{\sqrt{x^2+1}} - \ln x$ in $\left[\frac{1}{\sqrt{3}}, \sqrt{3}\right]$.

44. Using monotonocity find range of the function $f(x) = \sqrt{x-1} + \sqrt{6-x}$.

45. Let $f: R \to R$ be defined by **[JEE 2021]**

$$f(x) = \frac{x^2 - 3x - 6}{x^2 + 2x + 4}$$

Then which of the following statements is (are) TRUE?

(A) f is decreasing in the interval $(-2, -1)$

(B) f is increasing in the interval $(1,2)$

(C) f is onto

(D) Range of f is $\left[-\frac{3}{2}, 2\right]$

PROVING INEQUATION BY MONOTONOCITY

46. If $\frac{x}{(1+x)} < \log(1+x) < x$ then

(A) $x > 0$ (B) $x < 0$ (C) $x = 0$ (D) none

47. If $f(x) = 2x\sec x + x$ and $g(x) = 3\tan x$ then in interval $x \in (0, \pi/2)$ is

(A) $f(x) > g(x)$ (B) $f(x) < g(x)$ (C) $f(x) = g(x)$ (D) None of these

48. If $f(x) = \sin x \tan x$ and $g(x) = x^2$ then in interval $x \in (0, \pi/2)$ is

(A) $f(x) > g(x)$ (B) $f(x) < g(x)$ (C) $f(x) = g(x)$ (D) None of these

49. Which of the following inequalities are valid

(A) $|\tan^{-1} x - \tan^{-1} y| \leq |x - y| \forall x, y \in R$

(B) $|\tan^{-1} x - \tan^{-1} y| \geq |x - y|$

(C) $|\sin x - \sin y| \leq |x - y|$

(D) $|\sin x - \sin y| \geq |x - y|$

50. Using monotonicity prove that

(i) $x < -\ln(1 - x) < x(1 - x)^{-1}$ for $0 < x < 1$

(ii) $\frac{x}{1-x^2} < \tan^{-1} x < x$ for every $x \geq 0$

51. Prove that $\tan^2 x + 6 \ln\sec x + 2\cos x + 4 > 6\sec x$ for $x \in \left(\frac{3\pi}{2}, 2\pi\right)$.

52. If $g(x)$ is monotonically increasing and $f(x)$ is monotonically decreasing for $x \in R$ and if $(gof)(x)$ is defined for $x \in R$, then prove that $(gof)(x)$ will be monotonically decreasing function. Hence prove that $(gof)(x + 1) < (gof)(x - 1)$

53. For $x \in \left(0, \frac{\pi}{2}\right)$ identify which is greater $(2\sin x + \tan x)$ or $(3x)$. Hence find $\lim_{x \to 0}\left[\frac{3x}{2\sin x + \tan x}\right]$ where $[*]$ denote the greatest integer function.

54. For the function $f(x) = x\cos\frac{1}{x}, x \geq 1$, **[JEE 2009]**

(A) for at least one x in the interval $[1, \infty)$, $f(x + 2) - f(x) < 2$

(B) $\lim_{x\to\infty} f'(x) = 1$

(C) for all x in the interval $[1, \infty)$, $f(x + 2) - f(x) > 2$

(D) $f'(x)$ is strictly decreasing in the interval $[1, \infty)$

BASED ON ROLLE'S THEOREM

55. The function $f(x) = x^3 - 6x^2 + ax + b$ satisfy the conditions of Rolle's theorem in [1, 3]. The value of a and b are

(A) $11, -6$ (B) $-6, 11$ (C) $-11, 6$ (D) $6, -11$

56. If $f(x)$ and $g(x)$ are differentiable in [0,1] such that $f(0) = 2, g(0) = 0, f(1) = 6, g(1) = 2$, then Rolle's theorem is applicable for which of the following

(A) $f(x) - g(x)$ (B) $f(x) - 2g(x)$ (C) $f(x) + 3g(x)$ (D) None of these

57. A function f is defined by $f(x) = 2 + (x - 1)^{2/3}$ in [0,2], Which of the following is not correct?

(A) f is not derivable in (0,2) (B) f is continuous in [0,2]

(C) $f(0) = f(2)$ (D) Rolle's theorem is true in [0,2]

58. If $a + b + c = 0$, then the equation $3ax^2 + 2bx + c = 0$ has, in the interval (0,1)

(A) atleast one root (B) atmost one root (C) no root (D) None of these

59. If $27a + 9b + 3c + d = 0$, then the equation $4ax^3 + 3bx^2 + 2cx + d = 0$, has atleast one real root lying between -

(A) 0 and 1 (B) 1 and 3 (C) 0 and 3 (D) None of these

60. The function $f(x) = x(x + 3)e^{-x/2}$ satisfies all the conditions of Rolle's theorem in $[-3, 0]$. The value of c which verifies Rolle's theorem, is

(A) 0 (B) -1 (C) -2 (D) 3

61. If $\frac{C_0}{1}+\frac{C_1}{2}+\frac{C_2}{3}=0$, where $C_0C_1C_2$ are all real, then the quadratic equation $C_2x^2+C_1x+C_0=0$ has
(A) at least one root in $(0,1)$
(B) one root in $(1,2)$ and the other in $(3,4)$
(C) one root in $(-1,1)$ and the other in $(-5,-2)$
(D) both roots imaginary

62. If $\frac{a_0}{n+1}+\frac{a_1}{n}+\frac{a_2}{n-1}+\cdots+\frac{a_{n-1}}{2}+a_n=0$, then the equation $a_0x^n+a_1x^{n-1}+\cdots+a_{n-1}x+a_n=0$ has, in the interval $(0,1)$,
(A) exactly one root (B) atleast one root
(C) atmost one root (D) No root.

63. If the polynomial equation $a_nx^n+a_{n-1}x^{n-1}+\cdots+a_2x^2+a_1x+a_0=0$; n positive integer, has two different real roots α and β, then between α and β, the equation $na_nx^{n-1}+(n-1)a_{n-1}x^{n-2}+\cdots.+a_1=0$ has
(A) exactly one root (B) atmost one root
(C) atleast one root (D) No root

65. If the function $f(x)=x^3-6x^2+ax+b$ defined on $[1,3]$, satisfies the Rolle's theorem for $c=\frac{2\sqrt{3}+1}{\sqrt{3}}$, then-
(A) $a=11, b=6$ (B) $a=-11, b=6$ (C) $a=11, b\in R$ (D) None of these

66. If $f(x)=\begin{vmatrix}\sin x & \sin a & \sin b\\ \cos x & \cos a & \cos b\\ \tan x & \tan a & \tan b\end{vmatrix}$, where $0<a<b<\frac{\pi}{2}$, then the equation $f(x)=0$ has, in the interval (a,b) -
(A) atleast one root (B) atmost one root (C) no root (D) None of these

67. Which of the following functions do not satisfy conditions of Rolle's Theorem?
(A) $e^x\sin x, x\in\left[0,\frac{\pi}{2}\right]$ (B) $(x+1)^2(2x-3)^5, x\in\left[-1,\frac{3}{2}\right]$
(C) $\sin|x|, x\in[\pi,2\pi]$ (D) $\sin\frac{1}{x}, x\in\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$

68. Verify Rolle's theorem for the function $f(x)=\log_e\left(\frac{x^2+ab}{x(a+b)}\right)+p$, for $[a,b]$ where $0<a<b$.

69. Using Rolle's theorem prove that the equation $3x^2+px-1=0$ has at least one real root in the interval $(-1,1)$.

70. $f(x)$ and $g(x)$ are differentiable functions for $0\le x\le 2$ such that $f(0)=5, g(0)=0, f(2)=8, g(2)=1$. Show that there exists a number c satisfying $0<c<2$ and $f'(c)=3g'(c)$.

COMPREHENSION

Suppose a, b, c, d be non-zero real numbers and $ab>0$,
and
$$\int_0^1\left(1+e^{x^2}\right)(ax^3+bx^2+cx+d)dx=\int_0^2\left(1+e^{x^2}\right)(ax^3+bx^2+cx+d)dx=0$$

71. The equation $ax^3+bx^2+cx+d=0$ has
(A) three positive roots (B) three negative roots
(C) two positive and one negative roots (D) one positive & two negative roots

72. Rolle's theorem can be applied for $ax^3+bx^2+cx+d=0$ the interval
(A) $[0,1]$ (B) $[0,2]$ (C) $[1,2]$ (D) None of these

73. If $F(x) = \int f(x)dx = \int_0^1 f(x)dx = \int_0^2 f(x)dx = 0$ then in which of the following can Rolle's theorem can be applied for F(x), where $f(x) = \left(1 + e^{x^2}\right)(ax^3 + bx^2 + cx + d)$

(A) [0,1] (B) [0,2] (C) [1,2] (D) all of these

74. Let $f(x) = 2 + \cos x$ for all real x. **[JEE 2007]**

Statement-1 : For each real t, there exists a point ' c ' in $[t, t + \pi]$ such that $f'(c) = 0$.

because

Statement-2 : $f(t) = f(t + 2\pi)$ for each real t.

(A) Statement-1 is true, statement-2 is true; statement-2 is correct explanation for statement-1.

(B) Statement- 1 is true, statement-2 is true; statement-2 is NOT a correct explanation for statement-1.

(C) Statement-1 is true, statement- 2 is false.

(D) Statement- 1 is false, statement- 2 is true.

BASED ON LMVT

75. Function for which LMVT is applicable but Rolle's theorem is not

(A) $f(x) = x^3 - x, x \in [0,1]$

(B) $f(x) = \begin{cases} x^2 & ,0 \le x < 1 \\ x, & 1 < x \le 2 \end{cases}$

(C) $f(x) = e^x, x \in [-3,3]$

(D) $f(x) = 1 - \sqrt[3]{x^2}, x \in [-1,1]$

76. A value of C for which the conclusion of Mean Value Theorem holds for the function $f(x) = \log_e x$ on the interval [1,3] is

(A) $2\log_3 e$ (B) $\frac{1}{2}\log_e$ (C) $\log_3 e$ (D) $\log_e 3$

77. The number of values of 'c' of Lagrange's mean value theorem for the function, $f(x) = (x-1)(x-2)(x-3), x \in (0,4)$ is

(A) 1 (B) 2 (C) 3 (D) None of these

78. LMVT is not applicable for which of the following?

(A) $f(x) = x^2, x \in [3,4]$

(B) $f(x) = \ln x, x \in [1,3]$

(C) $f(x) = 4x^2 - 5x^2 + x - 2, x \in [0,1]$

(D) $f(x) = \{x^4(x-1)\}^{1/5}, x \in \left[-\frac{1}{2}, \frac{1}{2}\right]$

79. Equation $3x^2 + 4ax + b = 0$ has at least one root in (0,1) if

(A) $4a + b + 3 = 0$

(B) $2a + b + 1 = 0$

(C) $b = 0, a = -\frac{4}{3}$

(D) None of these

80. Let $f(x) = (x-4)(x-5)(x-6)(x-7)$ then,

(A) $f'(x) = 0$ has four roots

(B) Three roots of $f'(x) = 0$ lie in $(4,5) \cup (5,6) \cup (6,7)$

(C) The equation $f'(x) = 0$ has only one real root

(D) Three roots of $f'(x) = 0$ lie in $(3,4) \cup (4,5) \cup (5,6)$

81. If the function $f(x) = x^3 - 6ax^2 + 5x$ satisfies the conditions of Lagrange's mean theorem for the interval [1,2] and the tangent to the curve $y = f(x)$ at $x = 7/4$ is parallel to the chord joining the points of intersection of the curve with the ordinates $x = 1$ and $x = 2$. Then the value of a is

(A) 35/16 (B) 35/48 (C) 7/16 (D) 5/16

82. $f: [0,4] \to R$ is a differentiable function then for some $a, b \in (0,4)$, $f^2(4) - f^2(0)$ equals

(A) $8f'(a) \cdot f(b)$ (B) $4f'(a)f(b)$ (C) $2f'(a)f(b)$ (D) $f'(a)f(b)$

83. Let $f'(x) = e^{x^2}$ and $f(0) = 10$ and $A < f(1) < B$ can be concluded from the Mean Value Theorem then the largest value of $(A - B)$ is less than

(A) 0 (B) 1 (C) 2 (D) e

84. Explain the failure of Lagrange's mean value theorem in the interval $[-1,1]$ for the function $f(x) = \frac{1}{x}$

85. If a, b are two real numbers with $a < b$ show that a real number ' c ' can be found between a and b such that $3c^2 = b^2 + ab + a^2$.

86. Using LMVT prove that :

(a) $\tan x > x$ in $\left(0, \frac{\pi}{2}\right)$,

(b) $\sin x < x$ for $x > 0$

87. A value of C for which the conclusion of Mean Value Theoremholds for the function $f(x) = \log_e x$ on the interval $[1,3]$ is - **[AIEEE 2007]**

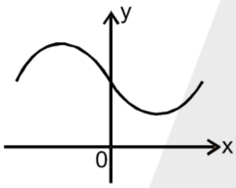
(A) $2\log_3 e$ (B) $\frac{1}{2}\log_e 3$ (C) $\log_3 e$ (D) $\log_e 3$

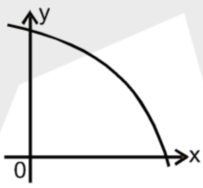
88. If f and g are differentiable functions in $[0,1]$ satisfying $f(0) = 2 = g(1), g(0) = 0$ and $f(1) = 6$, then for some $c \in]0,1[$: **[AIEEE 2014]**

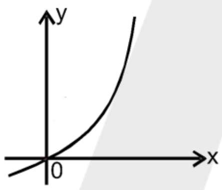
(A) $2f'(c) = g'(c)$ (B) $2f'(c) = 3g'(c)$ (C) $f'(c) = g'(c)$ (D) $f'(c) = 2g'(c)$

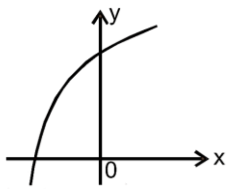
CURVE SKETCHING, QUESTION ON FINDING NUMBER OF SOLUTION

89. The curve $y = f(x)$ which satisfies the condition $f'(x) > 0$ and $f''(x) < 0$ for all real x, is

(A) 

(B) 

(C) 

(D) 

90. f: R → R be a differentiable function $\forall x \in R$. If tangent drawn to the curve at any point $x \in (a, b)$ always lie below the curve, then

(A) $f'(x) > 0 f''(x) < 0 \forall x \in (a, b)$ (B) $f'(x) < 0 f''(x) < 0 \forall x \in (a, b)$

(C) $f'(x) > 0 f''(x) > 0 \forall x \in (a, b)$ (D) None

91. If $f(x) = a^{\{a^{|x|}\operatorname{sgn} x\}}; g(x) = a^{[a^{|x|}\operatorname{sgn} x]}$ for $a > 1, a \neq 1$ and $x \in R$, where $\{*\}\&[*]$ denote the fractional part and integral part functions respectively, then which of the following statements holds good for the function h(x), where $(\ln a)h(x) = (\ln f(x) + \ln g(x))$.

(A) ' h ' is even and increasing (B) ' h ' is odd and decreasing

(C) ' h ' is even and decreasing (D) ' h ' is odd and increasing

92. Given that f is a real valued differentiable function such that $f(x)f'(x) < 0$ for all real x, it follows that

(A) f(x) is an increasing function (B) f(x) is a decreasing function

(C) |f(x)| is an increasing function (D) |f(x)| is a decreasing function

93. For which values of ' a ' will the function $f(x)=x^4+ax^3+\frac{3x^2}{2}+1$ will be concave upward along the entire real line

(A) $a\in[0,\infty)$ (B) $a\in(-2,2)$ (C) $a\in[-2,2]$ (D) $a\in(0,\infty)$

94. If $f(x)=1+x^m(x-1)^n, m,n\in N$, then $f'(x)=0$ has atleast one root in the interval

(A) $(0,1)$ (B) $(2,3)$ (C) $(-1,0)$ (D) None of these

95. Let $f(x)=ax^4+bx^3+x^2+x-1$. If $9b^2<24a$, then number of real roots of $f(x)=0$ are

(A) 4 (B) >2 (C) 0 (D) can't say

96. The equation $xe^x=2$ has

(A) one root of $x<0$ (B) two roots for $x>1$

(C) no root in $(0,1)$ (D) one root in $(0,1)$

97. Construct the graph of the function $f(x)=-\left|\frac{x^2-9}{x+3}-x+\frac{2}{x-1}\right|$ and comment upon the following

(a) Range of the function,

(b) Intervals of monotonocity,

(c) Point(s) where f is continuous but not differentiable,

(d) Point(s) where f fails to be continuous and nature of discontinuity.

(e) Gradient of the curve where f crosses the axis of y.

98. Show that exactly two real values of x satisfy the equation $x^2=x\sin x+\cos x$.

99. **Match the column.**

In the following [x] denotes the greatest integer less than or equal to x. **[JEE 2007]**

Column-I	**Column-II**
(A) $x\|x\|$	(P) continuous in $(-1,1)$
(B) $\sqrt{\|x\|}$	(Q) differentiable in $(-1,1)$
(C) $x+[x]$	(R) strictly increasing in $(-1,1)$
(D) $\|x-1\|+\|x+1\|$	(S) non differentiable at least at one point in $(-1,1)$

Paragraph for Question 100 and 101

Let $f:[0,1]\to R$ (the set of all real numbers) be a function. Suppose the function f is twice differentiable, $f(0)=f(1)=0$ and satisfies
$f''(x)-2f'(x)+f(x)\geq e^x, x\in[0,1]$. **[JEE 2013]**

100. Which of the following is true for $0<x<1$?

(A) $0<f(x)<\infty$ (B) $-\frac{1}{2}<f(x)<\frac{1}{2}$

(C) $-\frac{1}{4}<f(x)<1$ (D) $-\infty<f(x)<0$

101. If the function $e^{-x}f(x)$ assumes its minimum in the interval $[0,1]$ at $x=\frac{1}{4}$, which of the following is true?

(A) $f'(x)<f(x), \frac{1}{4}<x<\frac{3}{4}$ (B) $f'(x)>f(x), 0<x<\frac{1}{4}$

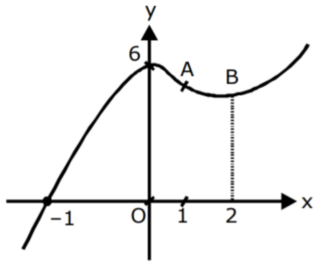
(C) $f'(x)<f(x), 0<x<\frac{1}{4}$ (D) $f'(x)<f(x), \frac{3}{4}<x<1$

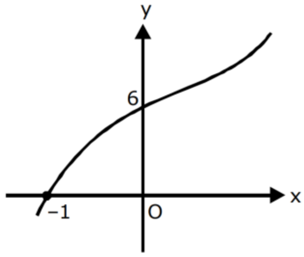
102. The number of points in $(-\infty,\infty)$, for which $x^2-x\sin x-\cos x=0$, is **[JEE 2013]**

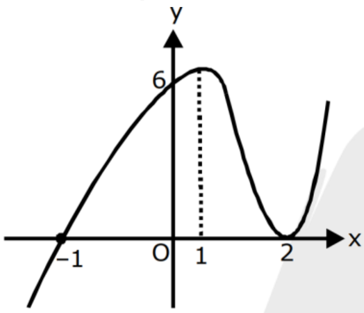
(A) 6 (B) 4 (C) 2 (D) 0

MIXTED PROBLEMS

103. Sketch the graph of $y = \frac{x^3}{3} - \frac{3x^2}{2} + 2x + 6$

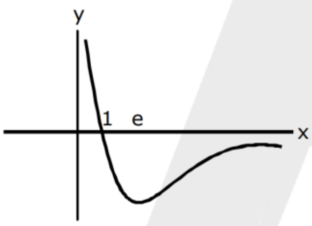
(A)


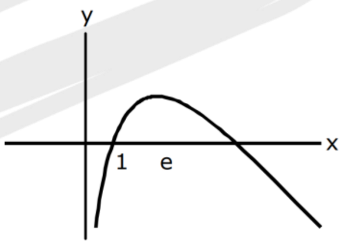
(B)


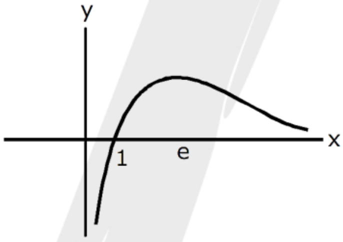
(C)


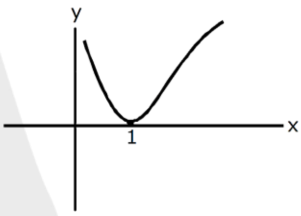
(D) None of these

104. Sketch graph of $y = \frac{\ln x}{x}$

(A)


(B)


(C)


(D)


105. A function $y = f(x)$ has a second order derivative $f'' = 6(x-1)$. If its graph passes through the point $(2,1)$ and at that point the tangent of the graph is $y = 3x - 5$, then the function is

(A) $(x-1)^2$ (B) $(x-1)^3$ (C) $(x+1)^3$ (D) $(x+1)^2$

106. If $0 < a < b < \frac{\pi}{2}$ and $f(a,b) = \frac{\tan b - \tan a}{b-a}$, then

(A) $f(a,b) \geq 2$ (B) $f(a,b) \geq 1$ (C) $f(a,b) \leq 1$ (D) None of these

107. If p, q, r be real then the intervals in which, $f(x)=\begin{vmatrix} x+p^2 & pq & pr \\ pq & x+q^2 & qr \\ pr & qr & x+r^2 \end{vmatrix}$

(A) increases is $x<-\frac{2}{3}(p^2+q^2+r^2), x>0$

(B) decrease is $\left(-\frac{2}{3}(p^2+q^2+r^2), 0\right)$

(C) decrease is $x<-\frac{2}{3}(p^2+q^2+r^2), x>0$

(D) increase is $\left(-\frac{2}{3}(p^2+q^2+r^2), 0\right)$

108. Let f and g be two functions defined on an interval I such that $f(x)\geq 0$ and $g(x)\leq 0$ for all $x\in I$ and f is strictly decreasing on I while g is strictly increasing on I then

(A) the product function fg is strictly increasing on I

(B) the product function fg is strictly decreasing on I

(C) $fog(x)$ is monotonically increasing on I

(D) $fog(x)$ is monotonically decreasing on I

109. If $f(x)=\tan^{-1}x-(1/2)\ln x$ then

(A) the greatest value of $f(x)$ on $[1/\sqrt{3}, \sqrt{3}]$ is $\pi/6+(1/4)\ln 3$

(B) the least value of $f(x)$ on $[1/\sqrt{3}, \sqrt{3}]$ is $\pi/3-(1/4)\ln 3$

(C) $f(x)$ decreases on $(0, \infty)$

(D) $f(x)$ increases on $(-\infty, 0)$

110. For the function $f(x)=x^4(12\ln x-7)$

(A) the point $(1,-7)$ is the point of inflection

(B) $x=e^{1/3}$ is the point of minima

(C) the graph is concave downwards in $(0,1)$

(D) the graph is concave upwards in $(1, \infty)$

111. If $f(x)=\log(x-2)-1/x$, then

(A) $f(x)$ is M.I. for $x\in(2,\infty)$ (B) $f(x)$ is M.I. for $x\in[-1,2]$

(C) $f(x)$ is always concave downwards (D) $f^{-1}(x)$ is M.I. wherever defined

112. Let $f: R\to R$ be a positive increasing function with $\lim_{x\to\infty}\frac{f(3x)}{f(x)}=1$. Then $\lim_{x\to\infty}\frac{f(2x)}{f(x)}=$

(A) 1 (B) $\frac{2}{3}$ (C) $\frac{3}{2}$ (D) 3 **[AIEEE 2010]**

113. Paragraph **[JEE 2007]**

If a continuous function f defined on the real line R, assumes positive and negative values in R then the equation $f(x)=0$ has a root in R. For example, if it is known that a continuous function f on R is positive at some point and its minimum value is negative then the equation $f(x)=0$ has a root in R.

Consider $f(x)=ke^x-x$ for all real x where k is a real constant.

(i) The line $y=x$ meets $y=ke^x$ for $k\leq 0$ at

(A) no point (B) one point (C) two points (D) more than two points

(ii) The positive value of k for which $ke^x-x=0$ has only one root is

(A) $1/e$ (B) 1 (C) e (D) $\log_e 2$

(iii) For $k>0$, the set of all values of k for which $ke^x-x=0$ has two distinct roots is

(A) $(0,1/e)$ (B) $(1/e,1)$ (C) $(1/e,\infty)$ (D) $(0,1)$

114. **(a)** Let the function $g: (-\infty, \infty) \to \left(-\frac{\pi}{2} \cdot \frac{\pi}{2}\right)$ be given by $g(u) = 2\tan^{-1}(e^u) - \frac{\pi}{2}$. Then, g is
(A) even and is strictly increasing in $(0, \infty)$ **[JEE 2008, 3 + 4]**
(B) odd and is strictly decreasing in $(-\infty, \infty)$
(C) odd and is strictly increasing in $(-\infty, \infty)$
(D) neither even nor odd, but is strictly increasing in $(-\infty, \infty)$
(b) Let f(x) be a non-constant twice differentiable function defined on $(-\infty, \infty)$ such that $f(x) = f(1-x)$ and $f'(1/4) = 0$. Then
(A) $f''(x)$ vanishes at least twice on [0,1]
(B) $f'(1/2) = 0$
(C) $\int_{-1/2}^{1/2} f\left(x + \frac{1}{2}\right) \sin x dx = 0$
(D) $\int_0^{1/2} f(t) e^{\sin \pi t} dt = \int_{1/2}^{1} f(1-t) e^{\sin \pi t} dt$

115. Let f be a real valued function defined on the interval $(0, \infty)$ by $f(x) = \ell nx + \int_0^x \sqrt{1 + \sin t}\, dt$.
Then which of the following statement(s) is/are true ? **[JEE 2010]**
(A) $f''(x)$ exists for all $x \in (0, \infty)$
(B) $f'(x)$ exists for all $x \in (0, \infty)$ and f' is continuous on $(0, \infty)$ but not differ entiable on $(0, \infty)$.
(C) there exists $\alpha > 1$ such that $|f'(x)| < |f(x)|$ for all $x \in (\alpha, \infty)$
(D) there exists $\beta > 0$ such that $|f(x)| + |f'(x)| \le \beta$ for all $x \in (0, \infty)$

Paragraph for Question 116 and 117

Let $f(x) = (1-x)^2 \sin^2 x + x^2$ for all $x \in IR$, and let $g(x) = \int_1^x \left(\frac{2(t-1)}{t+1} - \ln t\right) f(t) dt$ for all $x \in (1, \infty)$

116. Which of the following is true ? **[JEE 2012]**
(A) g is increasing on $(1, \infty)$
(B) g is decreasing on $(1, \infty)$
(C) g is increasing on (1,2) and decreasing on $(2, \infty)$
(D) g is decreasing on (1,2) and increasing on $(2, \infty)$

117. Consider the statements :
P : There exists some $x \in IR$ such that $f(x) + 2x = 2(1 + x^2)$
Q:There exists some $x \in IR$ such that $2f(x) + 1 = 2x(1 + x)$
Then
(A) both **P** and **Q** are true
(B) **P** is true and **Q** is false
(C) **P** is false and **Q** is true
(D) both **P** and **Q** are false

118. Let $f: (0, \infty) \to R$ be given by
$f(x) \int_{\frac{1}{x}}^{x} e^{-\left(t + \frac{1}{t}\right)} \frac{dt}{t}$. **[JEE 2014]**
Then
(A) f(x) is monotonically increasing on $[1, \infty)$
(B) f(x) is monotonically decreasing on (0,1)
(C) $f(x) + f\left(\frac{1}{x}\right) = 0$, for all $x \in (0, \infty)$
(D) $f(2^x)$ is an odd function of x on R

119. Let b be a nonzero real number. Suppose $f: \mathbb{R} \to \mathbb{R}$ is a differentiable function such that $f(0) = 1$ If the derivative f' of f satisfies the equation **[JEE 2020]**

$f'(x) = \frac{f(x)}{b^2+x^2}$ for all $x \in \mathbb{R}$, then which of the following statements is/are TRUE?

(A) If $b > 0$, then f is an increasing function

(B) If $b < 0$, then f is a decreasing function

(C) $f(x)f(-x) = 1$ for all $x \in \mathbb{R}$

(D) $f(x) - f(-x) = 0$ for all $x \in \mathbb{R}$

ANSWER KEY

1. (C) **2.** (C) **3.** (D) **4.** (B) **5.** (C) **6.** (A) **7.** (B)

8. (B) **9.** (D) **10.** (AB) **11.** (AB) **12.** (AB) **13.** (AC) **14.** (AD)

15. (BC) **16.** (AD) **17.** (AB) **18.** (CD) **19.** (BC) **20.** (BC) **21.**

22. **(i)** M.D. in $(-\infty,-3]\cup[0,2]$

M.I. in $(-3,0]\cup[2,\infty)$

(ii) M.I. in $\left[\frac{2}{4n+3},\frac{2}{4n+1}\right], n\in Z$ M.D. in $\left[\frac{2}{4n+1},\frac{2}{4n-1}\right], n\in Z$

(iii) M.D. in $\left(0,\frac{1}{\sqrt{3}}\right]$

M.I. in $\left[\frac{1}{\sqrt{3}},\infty\right)$

23. **(a)** I in $(2,\infty)$&D in $(-\infty,2)$ **(b)** I in $(1,\infty)$&D in $(-\infty,0)\cup(0,1)$

(c) I in $(0,2)$&D in $(-\infty,2)\cup(2,\infty)$ **(d)** I for $x>\frac{1}{2}$ or $-\frac{1}{2}<x<0$&D for $x<-\frac{1}{2}$ or $0<x<\frac{1}{2}$

24. A-PS, B-P,Q,R,S,T, C-P,Q

25. (B) **26.** (A) **27.** (C) **28.** (B) **29.** (C) **30.** (D) 31. $a\in R^+$

32. $[1-\sqrt{6},1+\sqrt{6}]$ **33.** $(-\infty,-3]$ **34.** (B) **35.** (B) **36.** (B)

37. (C) **38.** (D) **39.** (BCD) **40.** continuous but not diff. at $x=1$

41. (C) **42.** (C) **43.** $(\pi/6)+(1/2)\cdot\ln\ 3,(\pi/3)-(1/2)\ln\ 3$

44. $[\sqrt{5},\sqrt{10}]$ **45.** (AB) **46.** (A) **47.** (A) **48.** (A) **49.** (AC)

53. $2\sin\ x+\tan\ x>3x$, limit $=0$ **54.** (BCD) **55.** (A) **56.** (B) **57.** (D)

58. (A) **59.** (C) **60.** (C) **61.** (A) **62.** (B) 63. (C) **64.** (B)

65. (C) **66.** (A) **67.** (AD) **71.** (C) **72.** (B) **73.** (D) **74.** (B)

75. (C) **76.** (A) **77.** (B) **78.** (D) **79.** (B) **80.** (B) **81.** (B)

82. (A) **83.** (ABCD) **87.** (A) **88.** (D) **89.** (D) 90. (C)

91. (D) **92.** (D) **93.** (C) **94.** (A) **95.** (B) **96.** (D)

97. **(a)** $(-\infty,0]$ **(b)** $\uparrow$ in $\left(1,\frac{5}{3}\right)$ and $\downarrow$ in $(-\infty,1)\cup\left(\frac{5}{3},\infty\right)-\{-3\}$ **(c)** $x=\frac{5}{3}$

(d) removable discont. at $x=-3$ (missing point) and non removable discont. at $x=1$ (infinite type)

(e) -2

99. (A)-P, Q, R, (B)-P, S, (C)-R, S (D)-P, Q

100. (D) **101.** (C) **102.** (C) **103.** (D) **104.** (C) **105.** (B) **106.** (B)

107. (AB) **108.** (AD) **109.** (ABC) **110.** (ABCD) **111.** (ACD) **112.** (A)

113. (i)B (ii)A (iii)A **114.** (a)C (b)ABCD **115.** (BC) **116.** (B) **117.** (C)

118. (ACD) **119.** (AC)