

ENERGY CONSERVATION

① No non-Conservative force.

② Defined initial & final state

$$U_i + K \cdot E_i = U_f + K \cdot E_f$$

③ Choose as reference potential as zero potential line.

If above zero potential line $U = +mgh$

If below zero potential line $U = -mgh$

Block is released from rest, Find maximum Compression in the Spring.

$$U_i + K \cdot E_i = U_f + K \cdot E_f$$

$$mgh + 0 = [U_{\text{block}} + U_{\text{spring}}] + 0$$

$$mgh = 0 + \frac{1}{2} k x_{\text{max}}^2$$

$$x_{\text{max}} = \sqrt{\frac{2mgh}{k}}$$

$$W_{mg} + W_N + W_{\text{spring force}} = \Delta K \cdot E$$

$$W_{mg} = -\Delta U$$

$$= U_i - U_f$$

$$= mgh - 0$$

$$W_N = 0$$

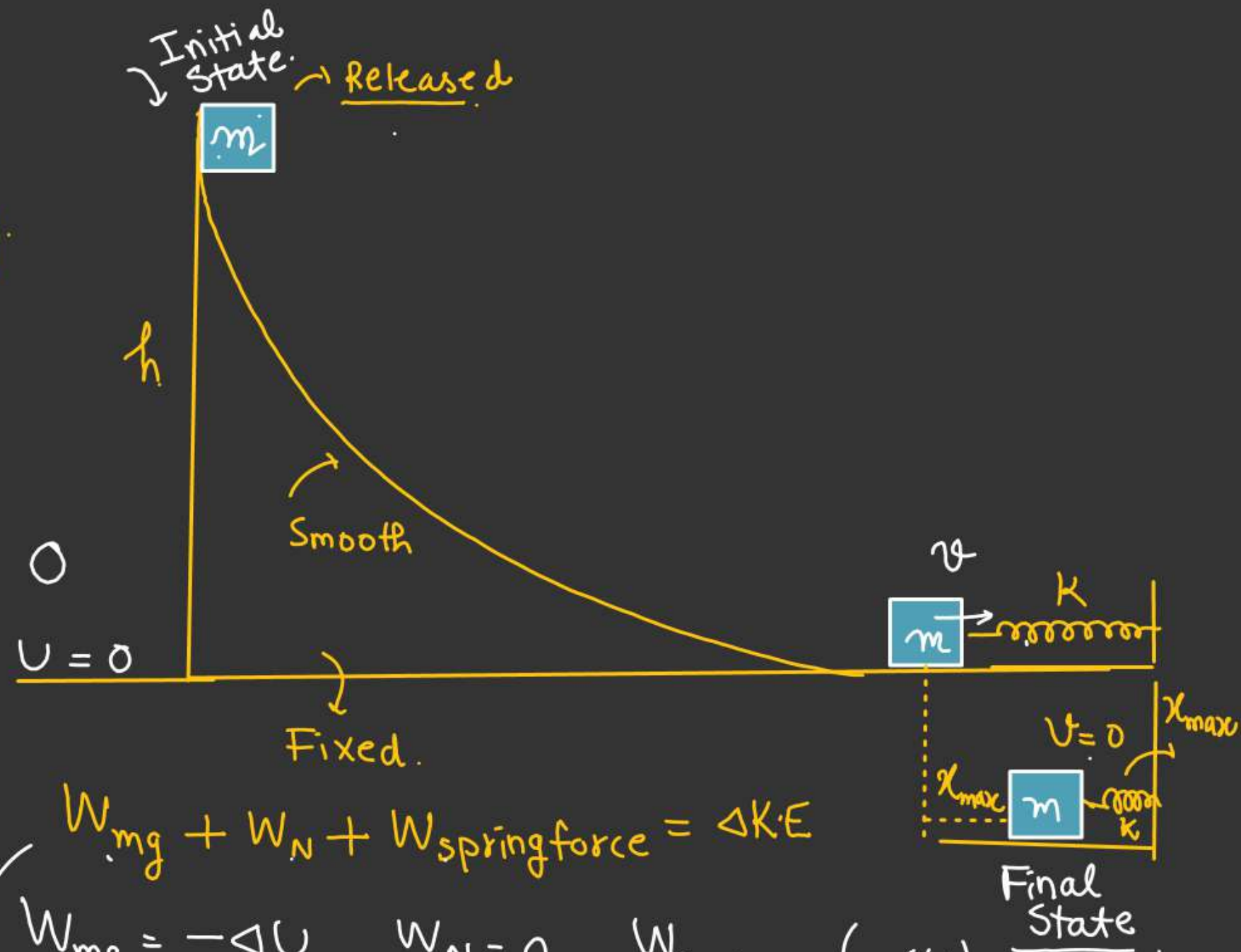
$$W_{\text{spring force}} = (-\Delta U)$$

$$= U_i - U_f$$

$$= 0 - \frac{1}{2} k x_{\text{max}}^2$$

$$= -\frac{1}{2} k x_{\text{max}}^2$$

$$mgh + 0 - \frac{1}{2} k x_{\text{max}}^2 = 0$$



(b) Find velocity of block if compression in the Spring is half of its maximum compression

$$U_i + K \cdot E_i = \underline{U_f} + K \cdot E_f$$

$$\downarrow$$

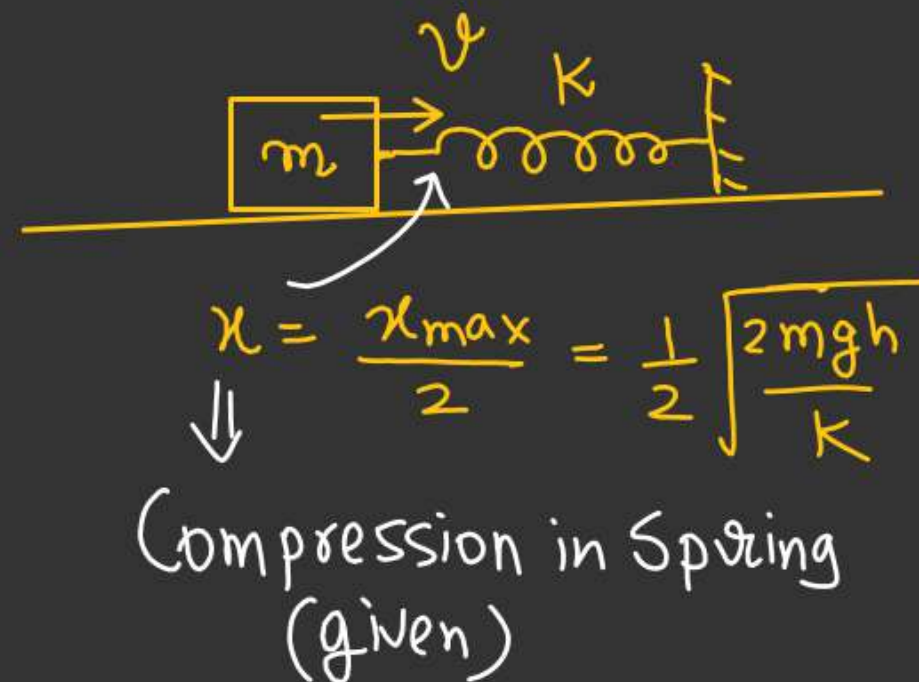
$$mgh + 0 = \frac{1}{2}kx^2 + \frac{1}{2}mv^2$$

$$mgh = \frac{1}{2} \cancel{k} \times \frac{1}{4} \times \left(\frac{2mgh}{\cancel{k}} \right) + \frac{1}{2}mv^2$$

$$mgh - \frac{mgh}{4} = \frac{1}{2}mv^2$$

$$\frac{3}{4} \cancel{mgh} = \frac{1}{2} \cancel{mv^2}$$

$$\underline{v = \sqrt{\frac{3}{2}gh}}$$



#

Block is released from rest
Find the value of h for
maximum range.

Energy Conservation from A to B.

$$U_i + K \cdot E_i = U_f + K \cdot E_f$$

$$mgH + 0 = mgh + \frac{1}{2}mv^2$$

$$mg(H-h) = \frac{1}{2}mv^2$$

$$v = \sqrt{2g(H-h)}$$

$$t_{Bc} = \sqrt{\frac{2h}{g}}$$

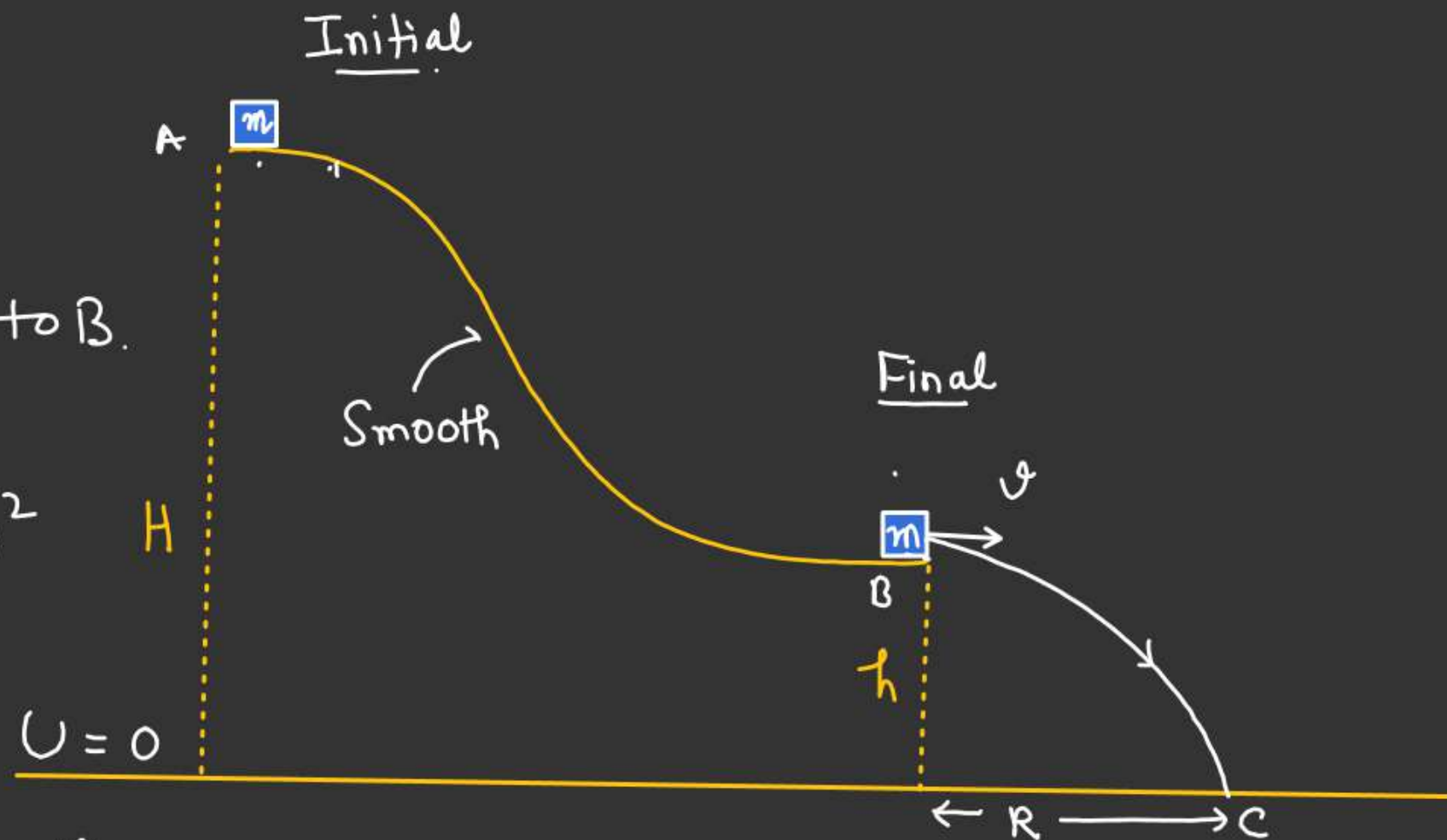
↓

$$h = \frac{1}{2}gt_{Bc}^2$$

$$R = t_{Bc} \cdot v$$

$$R = \sqrt{\frac{2h}{g}} \sqrt{2g(H-h)}$$

$$R = 2\sqrt{h(H-h)}$$



$$R = 2\sqrt{h(H-h)}$$

For R to be maximum, or minimum.

$$\frac{dR}{dh} = 0$$

$$\frac{H-2h}{2\sqrt{Hh-h^2}} = 0$$

$$2 \frac{d}{dh}(\sqrt{Hh-h^2}) = 0$$

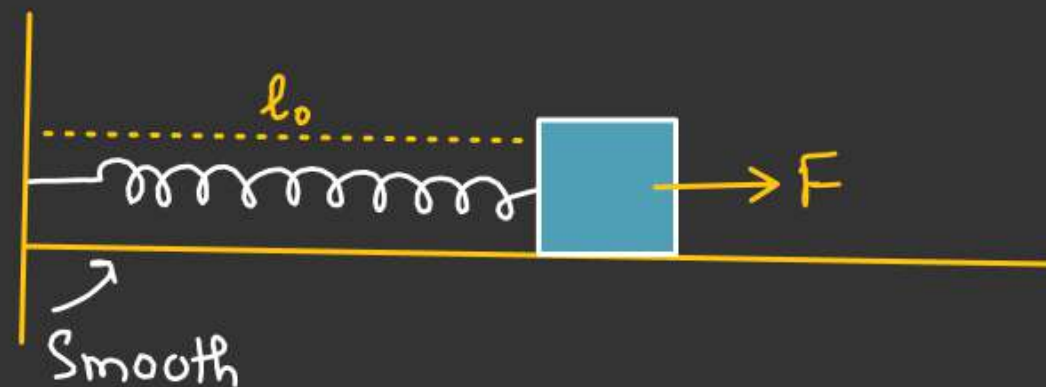
$$\underline{h = \frac{H}{2} \text{ Ans.}}$$

$$\text{put } \underline{Hh-h^2} = t.$$

$$\begin{aligned} \frac{d\sqrt{t}}{dh} &= \frac{d\sqrt{t}}{dt} \times \frac{dt}{dh} \\ &= \frac{1}{2\sqrt{t}} (H-2h) \\ &= \left(\frac{H-2h}{2\sqrt{Hh-h^2}} \right) \end{aligned}$$

QA l_0 = Natural length of the Spring k = Spring Constant.

Block pulled by a Constant force F . When Spring at its natural length.



- ① Find x_{\max} i.e. Maximum elongation in the Spring.
- ② Find v_{\max} of the block.

M-1

By Force Concept
At Equilibrium.

$$F = Kx$$

$$x = \left(\frac{F}{K} \right)$$

$$x_{\max} = 2x = \left(\frac{2F}{K} \right)$$

Work-Energy theorem.

$$\underline{W_F} + W_{\text{spring force}} + \cancel{W_N} + \cancel{W_{mg}} = \Delta K.E$$

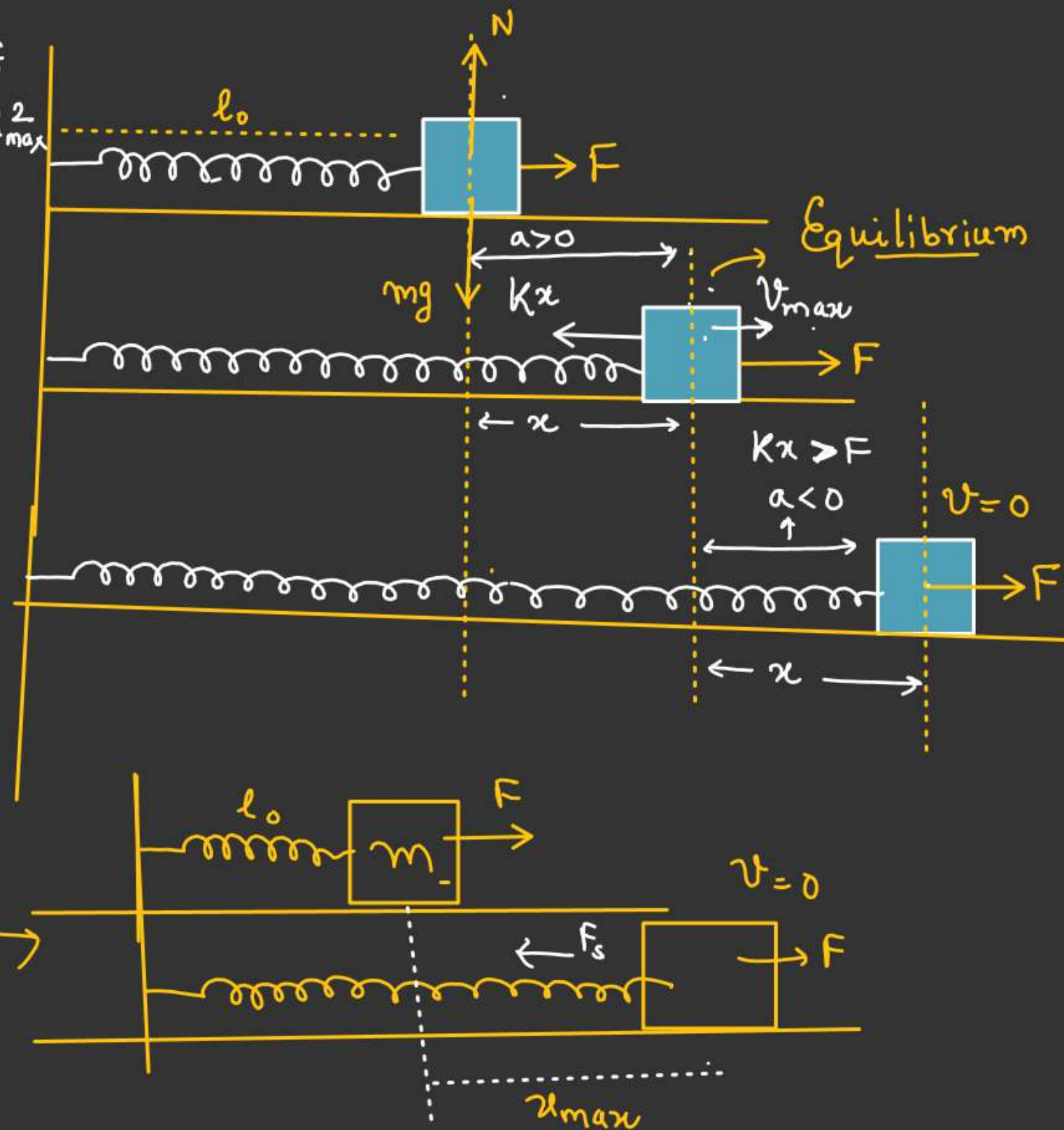
For Maximum elongation block must be at rest at that moment

$$\Delta K.E = K.E_f - K.E_i = 0$$

$$Fx_{\max} - \frac{1}{2}Kx_{\max}^2 = 0$$

$$x_{\max} = \left(\frac{2F}{K} \right)$$

$$\begin{aligned} W_{\text{spring}} &= -\Delta U \\ &= U_i - U_f \\ &= 0 - \frac{1}{2}Kx_{\max}^2 \end{aligned}$$



For $v_{\max} = ??$

v_{\max} always at Equilibrium position.

At $x = \left(\frac{F}{K}\right)$ Equilibrium.

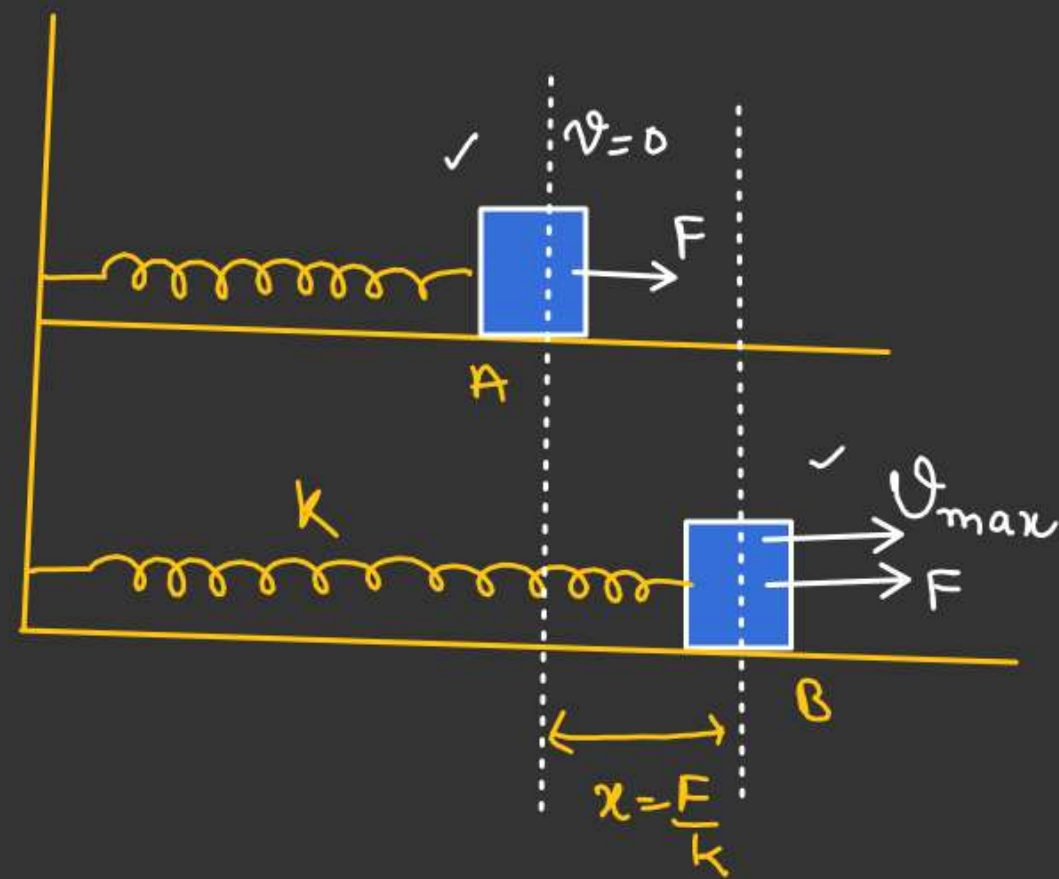
Work-Energy theorem from A to B.

$$W_F + W_{\text{spring}} = \Delta K.E$$

$$F \cdot \left(\frac{F}{K}\right) - \frac{1}{2} K \left(\frac{F}{K}\right)^2 = \frac{1}{2} m v_{\max}^2 - 0$$

$$\frac{F^2}{K} - \frac{F^2}{2K} = \frac{1}{2} m v_{\max}^2$$

$$\frac{1}{2} m v_{\max}^2 = \frac{F^2}{2K} \Rightarrow \left[v_{\max} = \frac{F}{\sqrt{mK}} \right] \checkmark$$



★★

Block is pulled by constant force F when spring at its natural length.

Find a) $x_{\max} = ? \Rightarrow$ Maximum elongation in the spring.
b) $v_{\max} = ?$

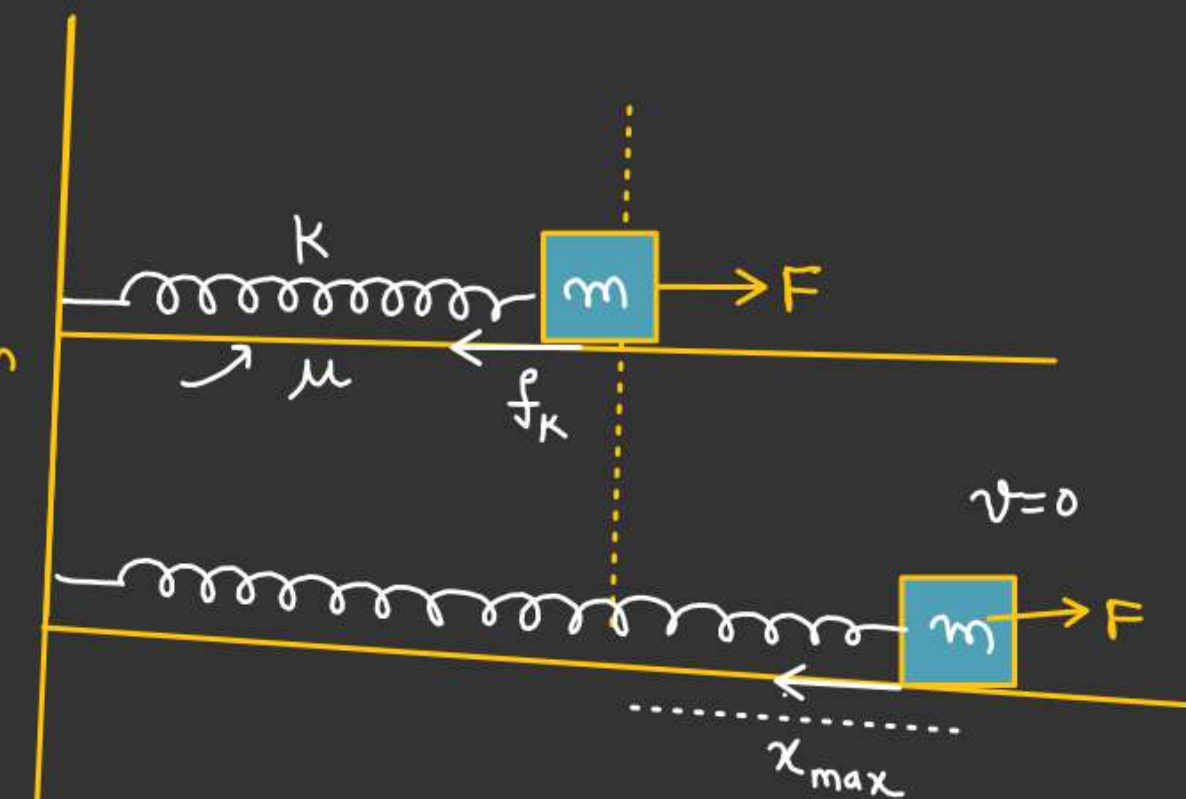
By work-Energy theorem

$$W_F + W_{f_k} + W_{\text{spring}} = \Delta K.E$$

$$F \cdot x_{\max} - \mu mg \cdot x_{\max} - \frac{1}{2} K x_{\max}^2 = 0$$

$$F - \mu mg = \frac{1}{2} K x_{\max}$$

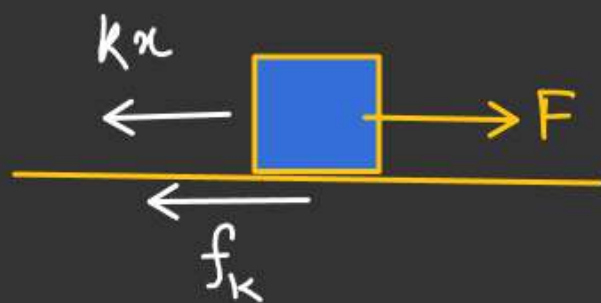
$$x_{\max} = \left[\frac{2(F - \mu mg)}{K} \right] \underline{\text{Ans.}}$$



$$\begin{aligned} W_{\text{spring}} &= -\Delta U \\ &= U_i - U_f \\ &= 0 - \frac{1}{2} K x_{\max}^2 \end{aligned}$$

For v_{\max}

At Equilibrium



$$F = kx + f_k$$

$$F = kx + \mu mg$$

$$\frac{F - \mu mg}{k} = x$$

↓
elongation at
Equilibrium.

Work-Energy theorem

$$W_F + W_{\text{Spring}} + W_{f_k} = \Delta K.E$$

$$F \cdot x - \frac{1}{2} kx^2 - \mu mgx = \frac{1}{2} m v_{\max}^2 - 0$$

$$F \left(\frac{F - \mu mg}{k} \right) - \frac{1}{2} \cancel{k} \frac{(F - \mu mg)^2}{\cancel{k^2}} - \mu mg \left(\frac{F - \mu mg}{k} \right)$$

$$\left(\frac{F - \mu mg}{k} \right) \left[F - \left(\frac{F - \mu mg}{2} \right) - \mu mg \right] = \frac{1}{2} m v_{\max}^2$$

$$\frac{(F - \mu mg)^2}{\cancel{2k}} = \frac{1}{2} m v_{\max}^2 \Rightarrow v_{\max} = \sqrt{\frac{(F - \mu mg)^2}{mk}}$$

H.W. · H.C. Verma ·

Q. No (1 to 21)

Q. No (31 to 37)

Q- No (40 to 43)

Q. No (47, 48, 51)