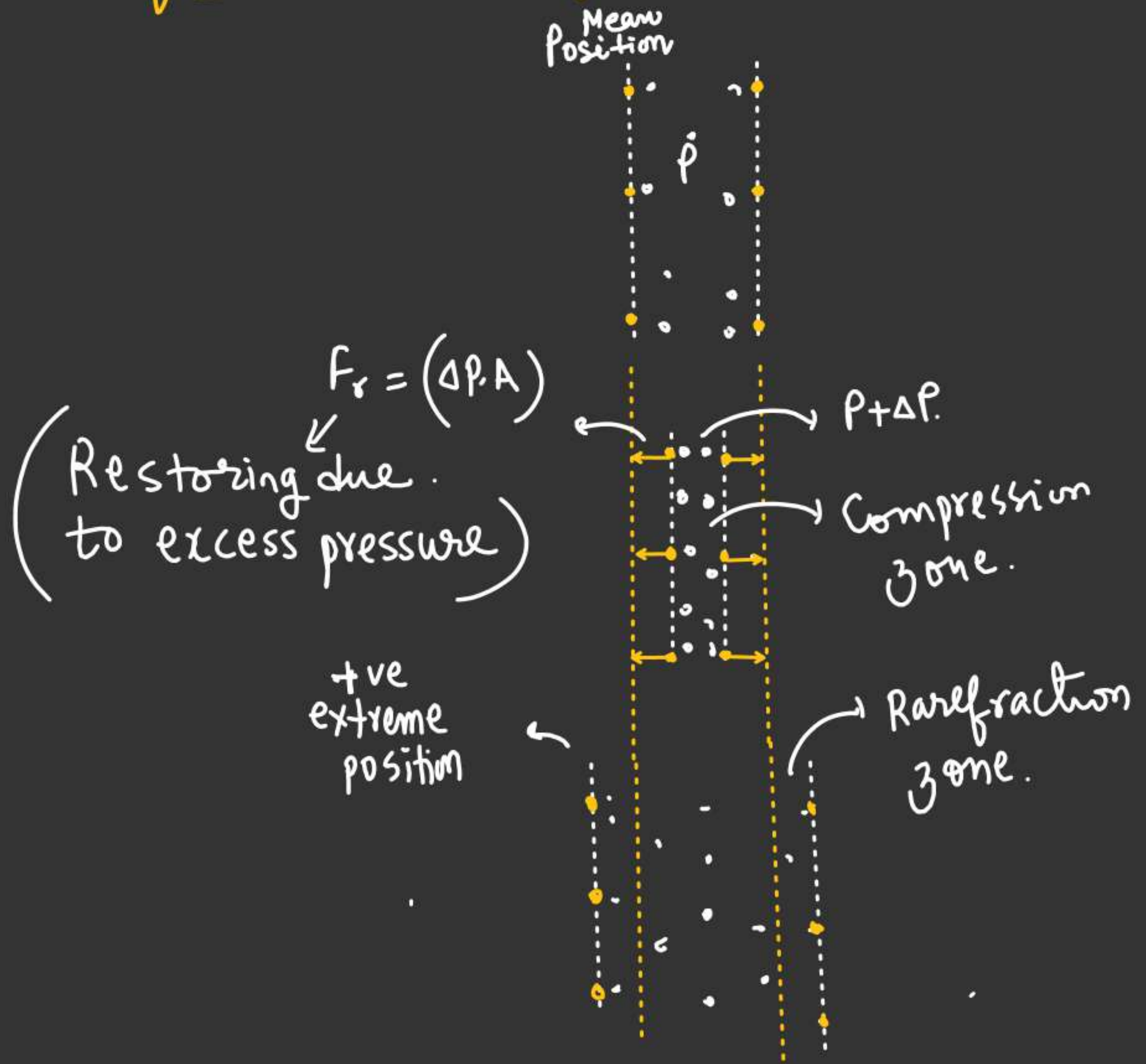
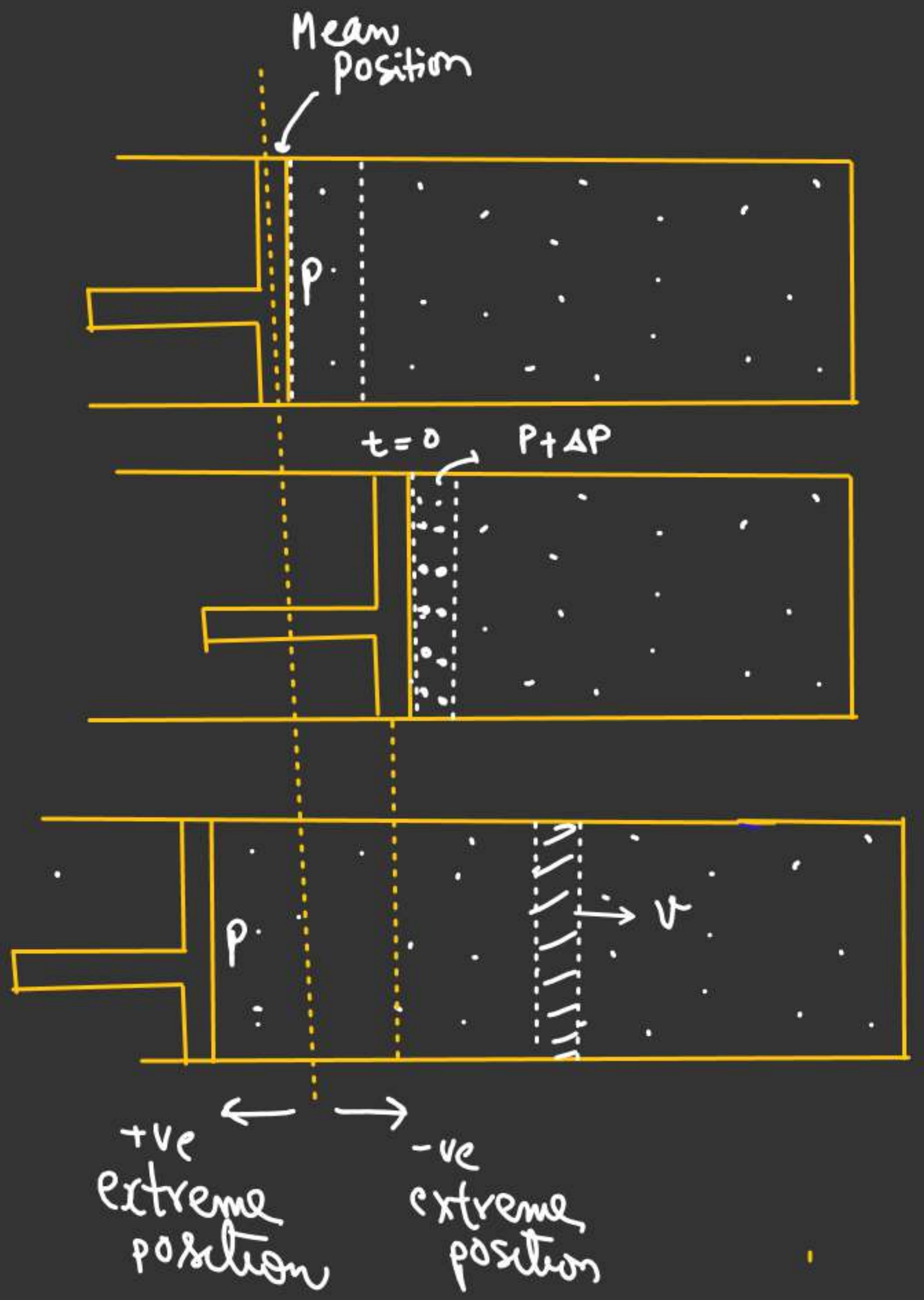
Nishant Jindal

WAVE

# Equation of longitudinal wave



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$$\boxed{S = S_0 \sin \omega\left(t - \frac{x}{v}\right)}$$

$S$ → Displacement of particle

$S_0$ → Maximum displacement

$$\boxed{S = S_0 \sin(\omega t - Kx)}$$

$$\boxed{P = P_0 \cos(\omega t - Kx)}$$

Longitudinal wave in terms of excess pressure.

Longitudinal wave in terms of displacement.

$$\boxed{\Delta P = \Delta P_0 \sin\left[\omega\left(t - \frac{x}{v}\right) + \frac{\pi}{2}\right]}$$

$\Delta P$ → Excess pressure

$\Delta P_0$ → Excess pressure Amplitude

In general $\Delta P \rightarrow P$

$\Delta P_0 \rightarrow P_0$

$$\boxed{P_0 = BKS_0}$$ ** → Relation b/w excess pressure amplitude & displacement amplitude

$B$ → Bulk Modulus of the Medium

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## Wave Speed of Longitudinal wave

$$\boxed{v = \sqrt{\frac{B}{\rho}}}$$

Speed of Longitudinal wave in gas or liquid

$B$ = Bulk Modulus of Medium

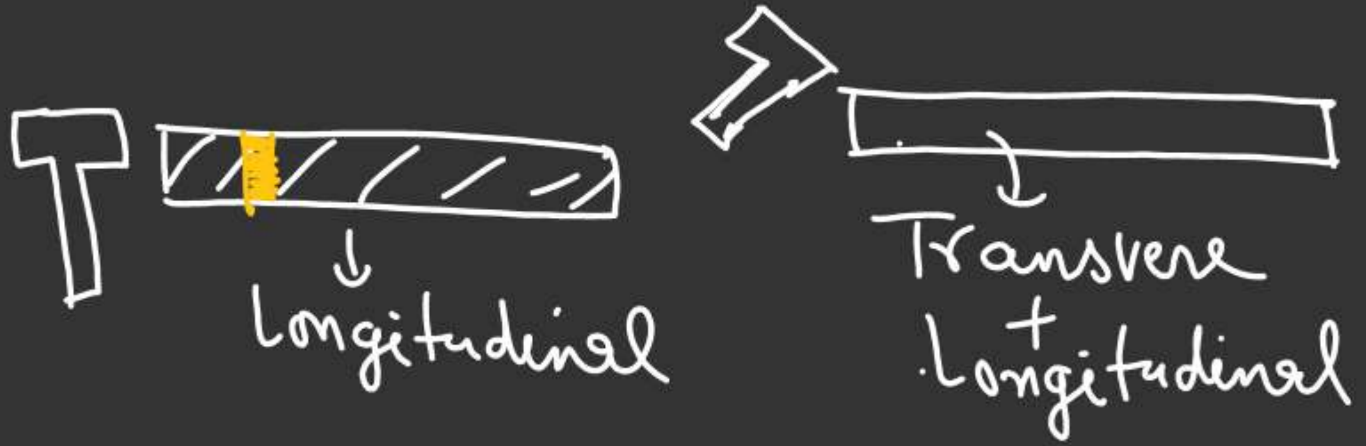
$\rho$ = density of Medium

$$\boxed{v = \sqrt{\frac{Y}{\rho}}}$$

Speed of Longitudinal wave in solid.

$Y$ = Young's Modulus of solid.

$\rho$ = density of solid body.



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## Speed of Sound wave

### According to Newton

↳ Compression & rarefraction phenomena is an isothermal process. (ie Temperature Constant)

$$PV = nRT \rightarrow \text{constant}$$

$$B = \left(\frac{-dP}{\frac{dV}{V}}\right)$$

$$v = \sqrt{\frac{B}{\rho}}$$

$$\boxed{v = \sqrt{\frac{P}{\rho}}} \times$$

$\downarrow$

Not Matched with experimental value.

Differentiating both side w.r.t Volume

$$P \cdot \frac{d(V)}{dV} + V\frac{dP}{dV} = 0$$

$$P = -V\frac{dP}{dV}$$

$$P = \left(-\frac{dP}{\left(\frac{dV}{V}\right)}\right) \rightarrow B_{isothermal}$$

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## LAPLACE CORRECTION

According to Laplace, Compression & rarefraction is an adiabatic process i.e no heat exchange b/w System & Surrounding.

$$PV^{\gamma} = C$$

Differentiating both side w.r.t volume

$$P\frac{d}{dV}(V^{\gamma}) + V^{\gamma}\cdot\frac{dP}{dV} = 0$$

$$P\gamma V^{\gamma-1} = -V^{\gamma}\left(\frac{dP}{dV}\right)$$

$$\gamma P = -\frac{V^{\gamma}}{V^{\gamma-1}}\left(\frac{dP}{dV}\right)$$

$$\gamma P = -V\left(\frac{dP}{dV}\right)$$

$$\gamma P = -\frac{dP}{\left(\frac{dV}{V}\right)} \rightarrow B_{adiabatic}$$

$$\boxed{v = \sqrt{\frac{\gamma P}{\rho}}} \rightarrow \text{Speed of Sound in air}$$

$$\gamma_{air} = 1.4$$

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## Characteristics of Sound

$$\text{Intensity} = \frac{\text{Energy}}{\text{Area} \times \text{time}} = \frac{\text{Power} \to W}{\text{Area} \to m^2}$$

Pitch → By pitch we can differentiate b/w different voices. i.e wether the voice is of Male or female

Quality — Higher the pitch quality of sound is good.

Loudness → It tells us about intensity of sound.
Intensity of sound measured by unit (decible → dB)

$$\boxed{\beta = 10 \log \left(\frac{I}{I_0}\right)}$$

(Sound level)

$I_0$ = Refrance intensity of sound

$$\boxed{I_0 = 10^{-12}\ W/m^2}$$

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\# If intensity is increased by a factor of 20 then how many decible sound level increased ??

$$\beta = 10 \log\left(\frac{I}{I_0}\right)$$

$$I_0 = 10^{-12}\ W/m^2$$

$$I_1 = 20I$$

$$\beta_1 = ??$$

$$\beta_1 = 10 \log\left(\frac{20I}{I_0}\right)$$

$$\beta_1 - \beta = 10\left[\log\left(\frac{20I}{I_0}\right) - \log\left(\frac{I}{I_0}\right)\right]$$

$$\beta_1 - \beta = 10\left[\log 20\right]$$

$$= 10\left[\log 2 + \log 10\right]$$

$$\log 10 \downarrow 1$$

$$= 10\left[0.3 + 1\right]$$

$$\beta_1 - \beta = \underline{13}$$

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## Superposition principle

$$y_1 = f_1\left(t - \frac{x}{v}\right)$$

$$y_2 = f_2\left(t - \frac{x}{v}\right)$$

If these two wave pulse interfere at a point then according to Superposition principle.

$$y_R = y_1 + y_2$$

$$= f_1\left(t - \frac{x}{v}\right) + f_2\left(t - \frac{x}{v}\right)$$

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## Phasor Method

$$y_1 = \underline{A_1} \sin(\omega t - kx)$$

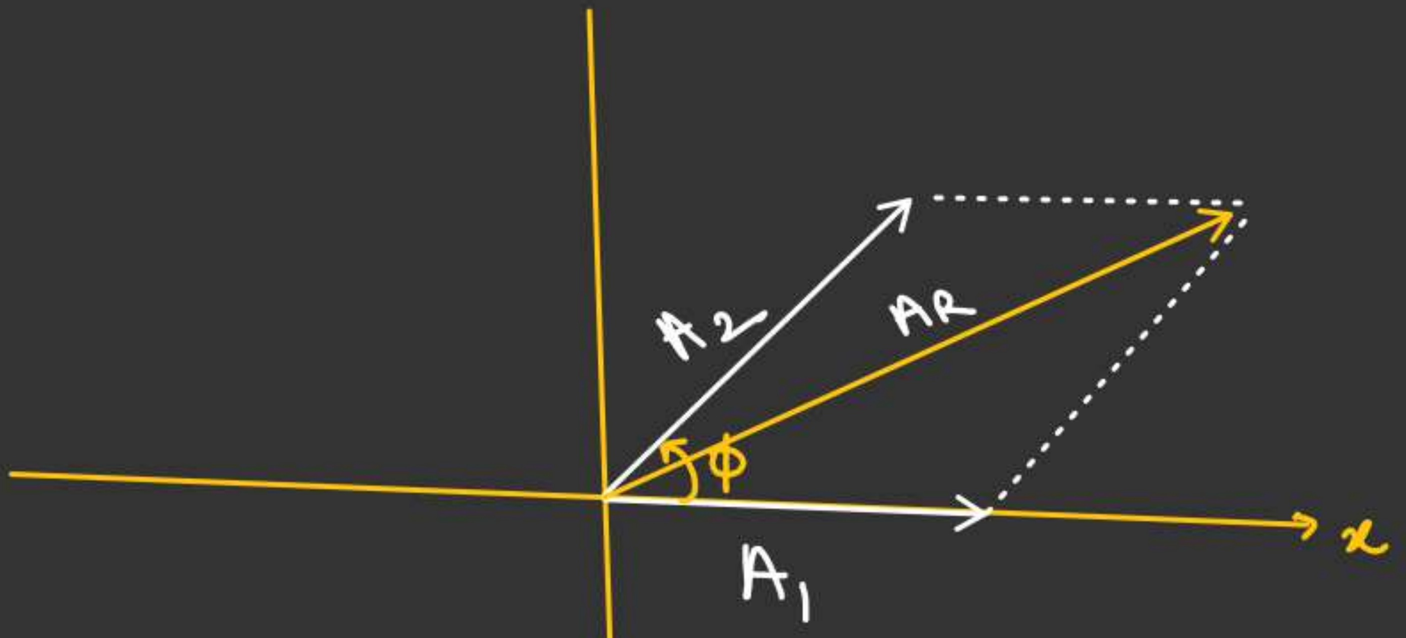
$$y_2 = \underline{A_2} \sin(\omega t - kx + \phi)$$

$$\Delta\phi = \phi - 0 = \phi$$

Imagine two vectors of magnitude equal to their amplitudes.

$+\phi \rightarrow$ Leading

$-\phi \rightarrow$ Lagging




$$\boxed{A_R = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\phi}}$$

$$\boxed{I \propto A^2}$$

$$A_R^2 = A_1^2 + A_2^2 + 2A_1A_2\cos\phi$$

$\Downarrow$

$$\boxed{I_R = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi}$$

$I_1 \propto A_1^2$

$I_2 \propto A_2^2$

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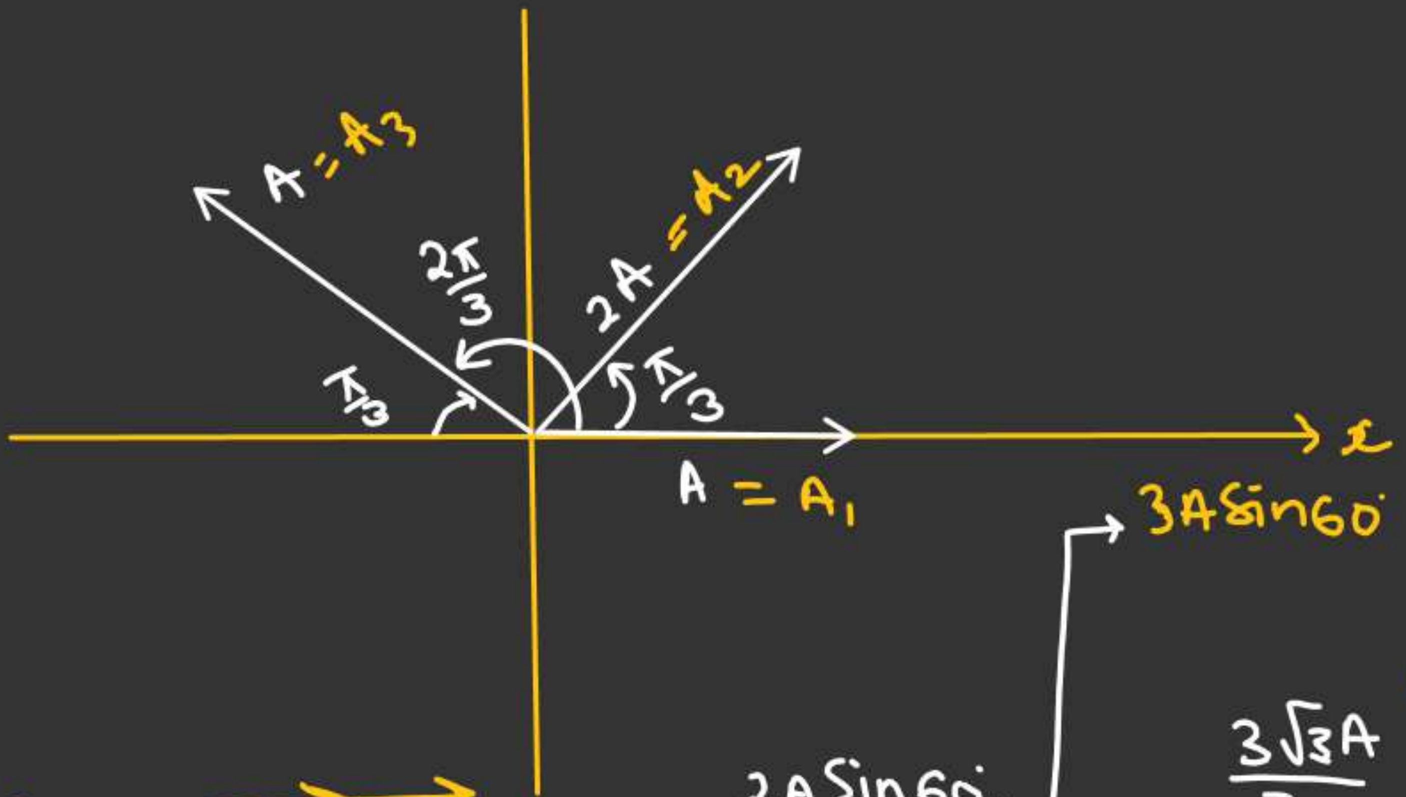
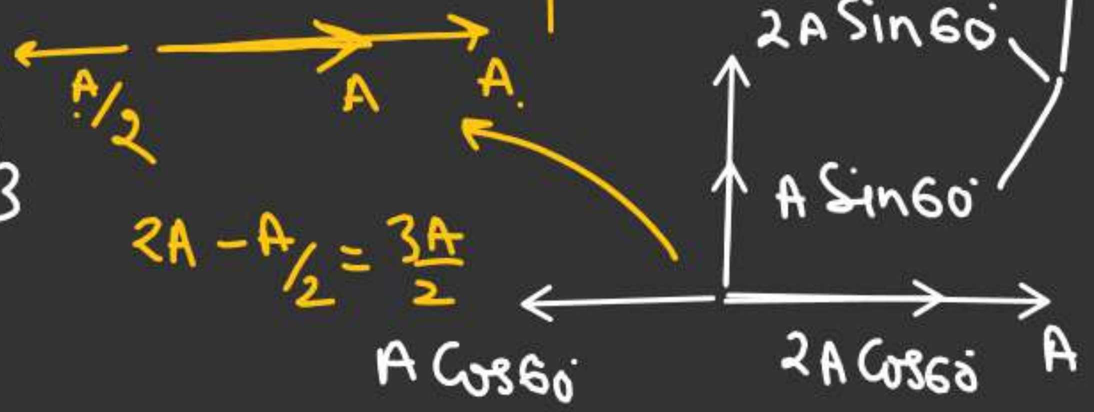
$$\left.\begin{aligned} y_1 &= \overset{A_1}{A}\sin\omega t \\ y_2 &= \overset{A_2}{2A}\sin(\omega t + \pi/3) \\ y_3 &= \overset{A_3}{A}\sin\left(\omega t + \frac{2\pi}{3}\right) \end{aligned}\right.$$

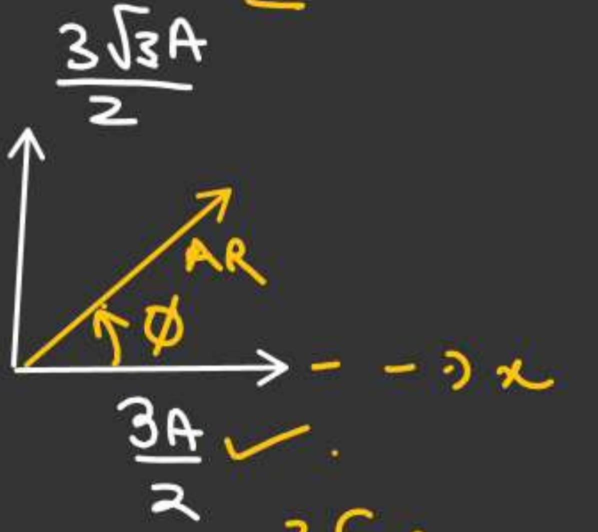
$$y_R = (y_1 + y_2 + y_3)$$




$\pi - \pi/3$




$$3A\sin 60^\circ = 3A \times \frac{\sqrt{3}}{2}$$




$$A_R = \sqrt{\left(\frac{3\sqrt{3}A}{2}\right)^2 + \left(\frac{3A}{2}\right)^2}$$

$$A_R = \sqrt{\frac{27A^2}{4} + \frac{9A^2}{4}} = \sqrt{9A^2} = 3A$$

$$y_R = A_R\sin(\omega t + \phi)$$

$$\tan\phi = \frac{\frac{3\sqrt{3}A}{2}}{3A/2} = \sqrt{3}$$

$$\phi = 60^\circ$$

$$\boxed{y_R = 3A\sin\left(\omega t + \frac{\pi}{3}\right)}$$

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## INTERFERANCE OF TWO WAVE

$$S_1 = S_{01} \sin(\omega t - Kx)$$

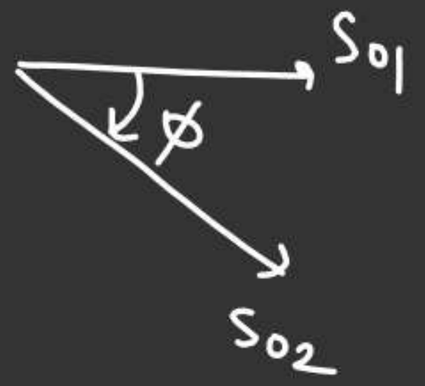
$$S_2 = S_{02} \sin(\omega t - K(x + \Delta x))$$

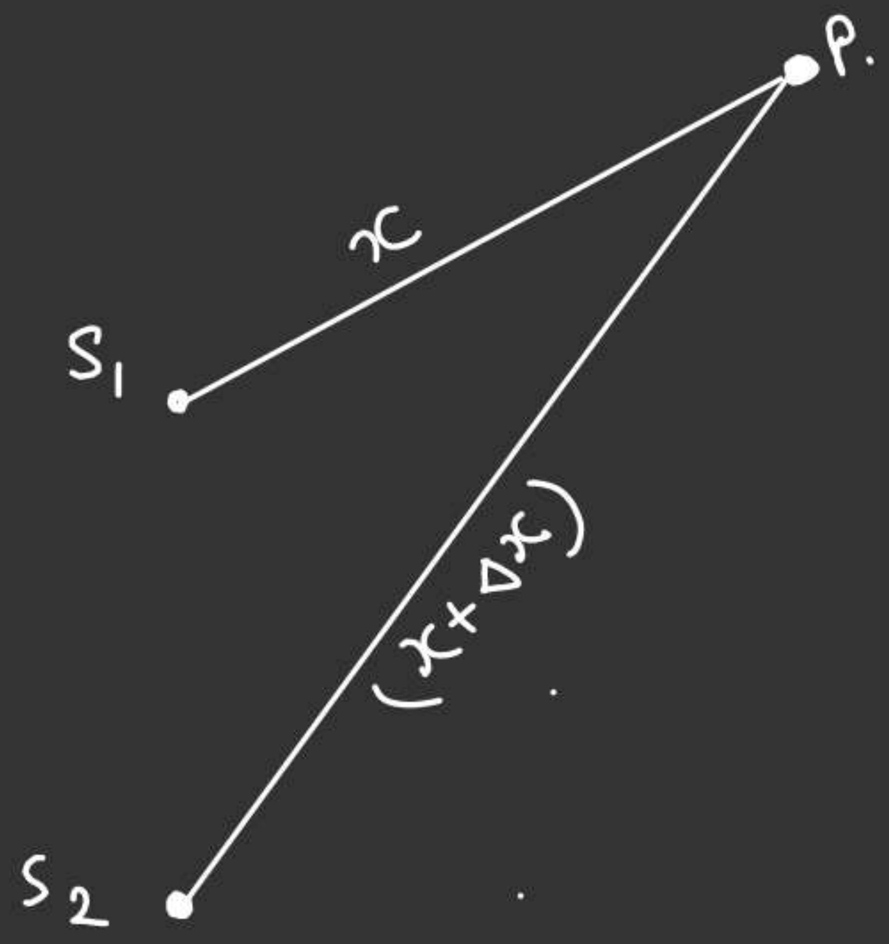
$$S_2 = S_{02} \sin(\omega t - Kx - \underbrace{K\Delta x}_{\phi})$$

$$\Delta\phi = \phi = K\Delta x$$

$$\boxed{\Delta\phi = \frac{2\pi}{\lambda} . \Delta x}$$

(Phase difference) (Path difference)



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$$S_R = \sqrt{S_{01}^2 + S_{02}^2 + 2 S_{01} S_{02} \cos\phi}$$

$I_1 \propto S_{01}^2$

$I_2 \propto S_{02}^2$

$$\downarrow$$

$$I_R = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$$

For $(I_R)_{max}$, $\cos\phi = +1$ (Constructive Interference)

$$\boxed{\phi = 2n\pi}$$

$n \in I^+$

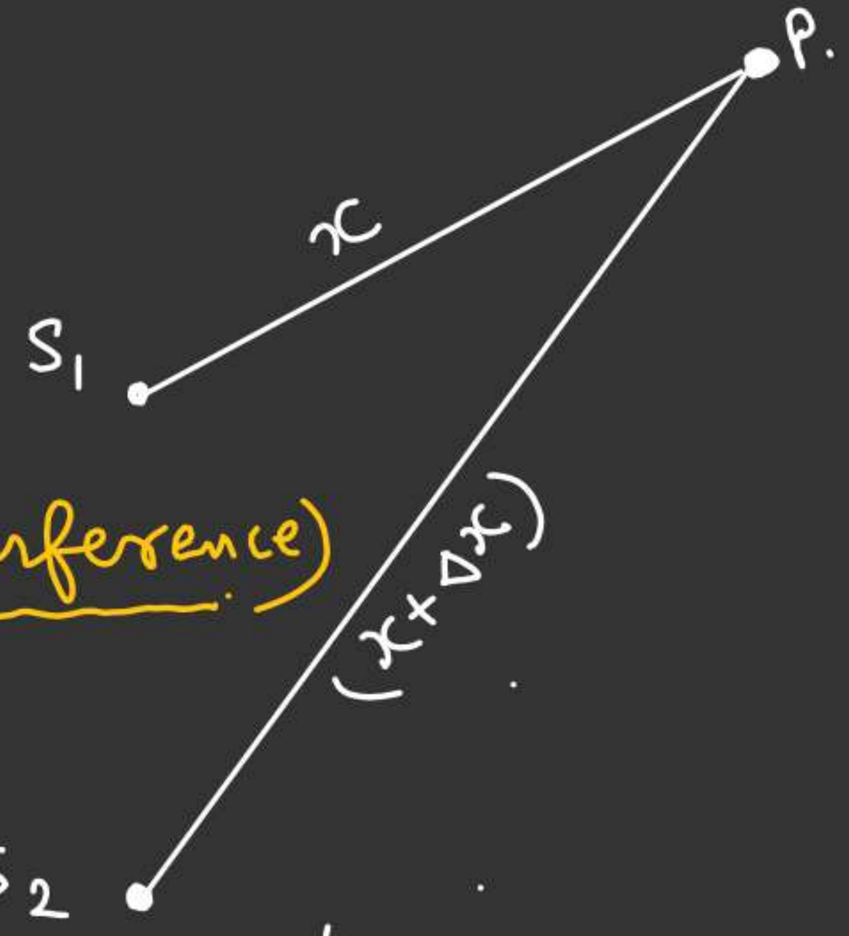
$n = 0, 1, 2, \ldots$

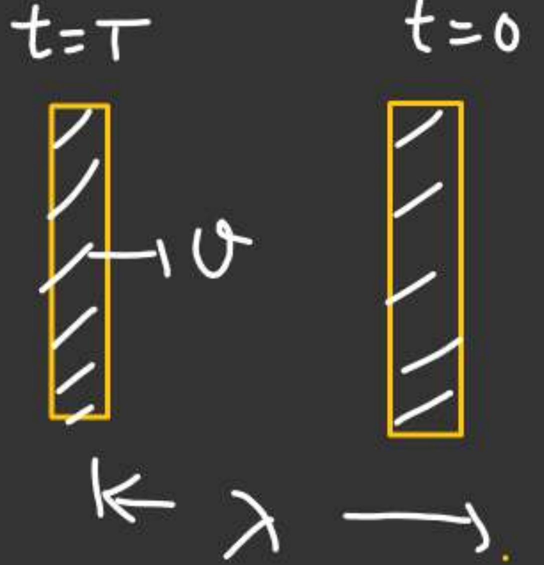
$$\downarrow$$

$$\frac{2\pi}{\lambda}\cdot\Delta x = 2n\pi$$

$$\boxed{\Delta x = n\lambda}$$

$n = 0, 1, 2, \ldots$




$$v = \frac{\lambda}{T}$$

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**Destructive interference** (Amplitude, or Intensity minimum)

$I_R$ or $S_R$ is minimum for this

$$\cos\phi = -1$$

$$\boxed{\phi = (2n-1)\pi \quad \text{or} \quad (2n+1)\pi}$$

↳ $n = 1, 2, 3 \ldots$ ↳ $n = 0, 1, 2, 3, \ldots$

$$\frac{2\pi}{\lambda} \times \Delta x = (2n-1)\pi \qquad \frac{2\pi}{\lambda} \cdot \Delta x = (2n+1)\pi$$

$$\boxed{\Delta x = (2n-1)\frac{\lambda}{2}} \quad \text{or} \quad \boxed{\Delta x = (2n+1)\frac{\lambda}{2}}$$

↳ $n = 1, 2, 3 \ldots$ ↳ $n = 0, 1, 2, 3 \ldots$

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S = Sound Source

D = Detector

Find $x_{min}$ so that detector detects maxima of sound

Frequency of sound source = 180 Hz

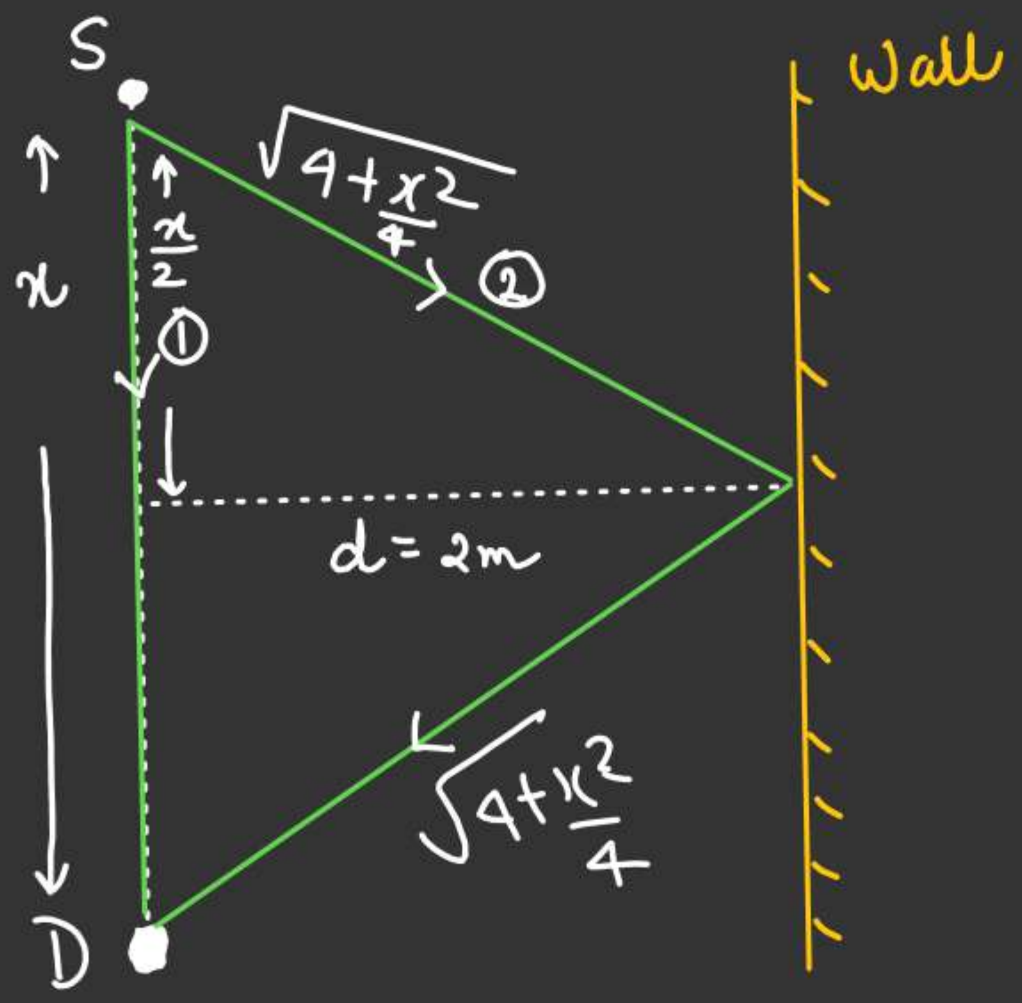
Speed of sound in air = 360 m/s

$\frac{1}{T} = f$

$v = \frac{\lambda}{T} = \lambda \cdot f$

$\lambda = \frac{v}{f} = \frac{360}{180}$

$\lambda = 2m$




Sol$^n$ Interference of two sound wave one directly reaching to D and other which is reaching to D by reflecting with wall

For constructive intereference

$\Delta x = n\lambda$

$2\sqrt{4+\frac{x^2}{4}} - x = n\lambda$

$2\sqrt{4+\frac{x^2}{4}} - x = \lambda$

$\Downarrow$

2

For $x_{min}$, $n = 1$

$2\sqrt{4+\frac{x^2}{4}} = (2+x)$

$4\left(4+\frac{x^2}{4}\right) = 4+x^2+4x$

$x_{min} = 3$

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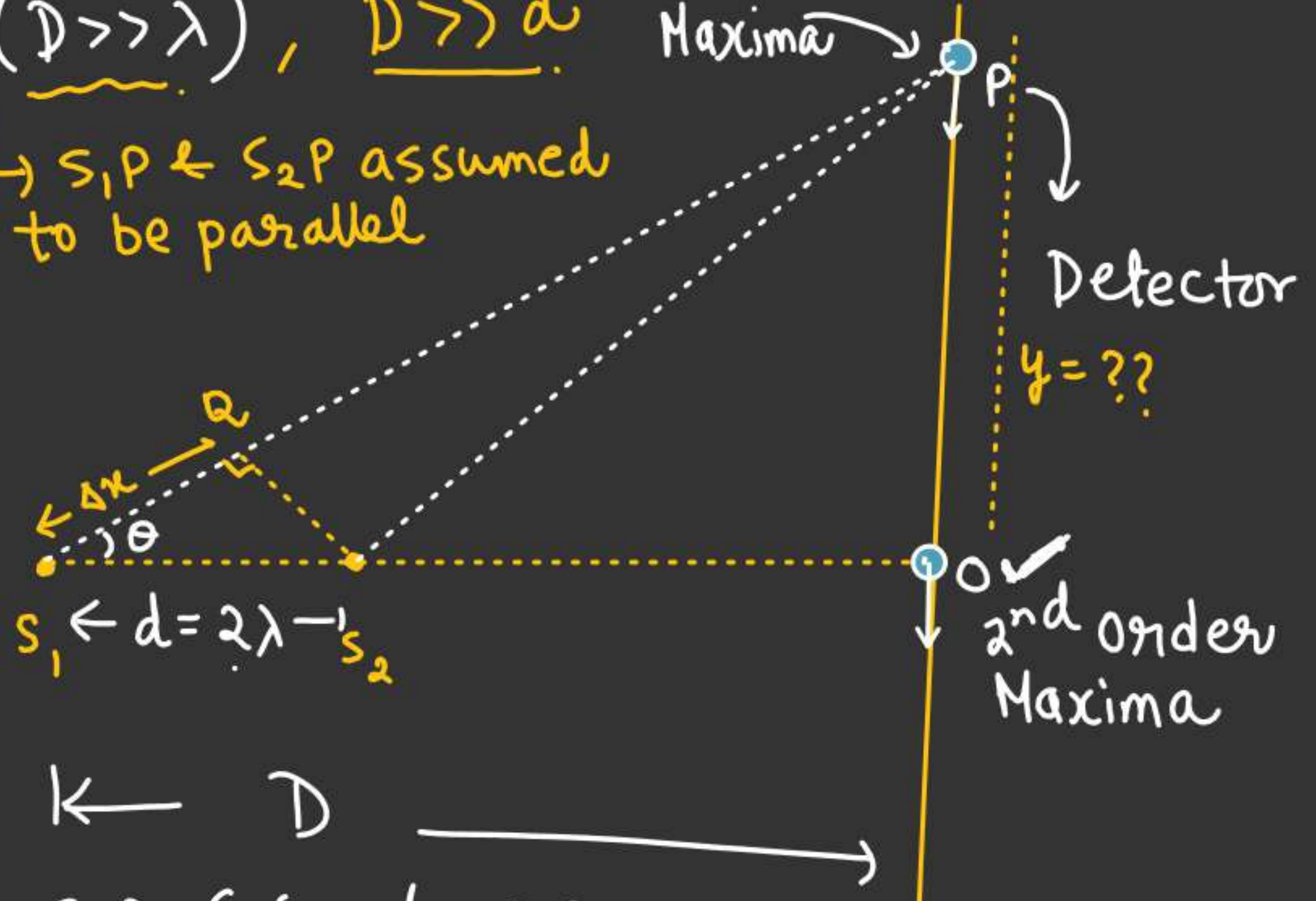
A detect moving perpendicular to line joining $S_1$ & $S_2$. Find distance b/w P & O so that detector detects same intensity when it is a P & O.

$S_1$ & $S_2$ are Coherent Source

↳ Sources vibrating in Same phase or have Constant phase difference

$(D >> \lambda)$, $D >> d$

↳ $S_1P$ & $S_2P$ assumed to be parallel




$\Delta x_0 = S_1O - S_2O = d = 2\lambda$

$$\Delta x = d\cos\theta$$

At O, $\theta \to 0$, $\Delta x = d = 2\lambda$

↳ 2nd order Maxima

$\Delta x = n\lambda$

For maxima

$\Delta x = n\lambda$

$d\cos\theta = n\lambda$

$$\boxed{n = \frac{d}{\lambda}\cos\theta}$$

$n_{max}$ when $\theta \to 0$

$n_{max} = \left(\frac{d}{\lambda}\right)$

↓

At O

# WAVE

At P $1^{st}$ order maxima.

$$d\cos\theta = \lambda \quad (n=1)$$

$$d = 2\lambda$$

$$\cos\theta = \frac{\lambda}{2\lambda} = \frac{1}{2}$$

$$\theta = 60^\circ$$

In $\Delta S_1OP$

$$\tan 60^\circ = \frac{y}{D}$$

$$y = \sqrt{3}D \checkmark$$