CIRCLE

Q. If $y = c$ is a tangent to the circle $x^2 + y^2 - 2x + 2y - 2 = 0$ at $(1, 1)$, then the value of c is

(A) 1

(B) 2

(C) -1

(D) -2

$y = 1 = c$

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CIRCLE

Q. Line $3x + 4y = 25$ touches the circle $x^2 + y^2 = 25$ at the point

(A) $(4, 3)$
(B) $(3, 4)$
(C) $(-3, -4)$
(D) none of these

(x_1, y_1)

$xx_1 + yy_1 - 25 = 0$

$3x + 4y - 25 = 0$

$\frac{x_1}{3} = \frac{y_1}{4} = \frac{-25}{-25}$

$\left.\begin{array}{l} x_1 = 3 \\ y_1 = 4 \end{array}\right\}$

CIRCLE

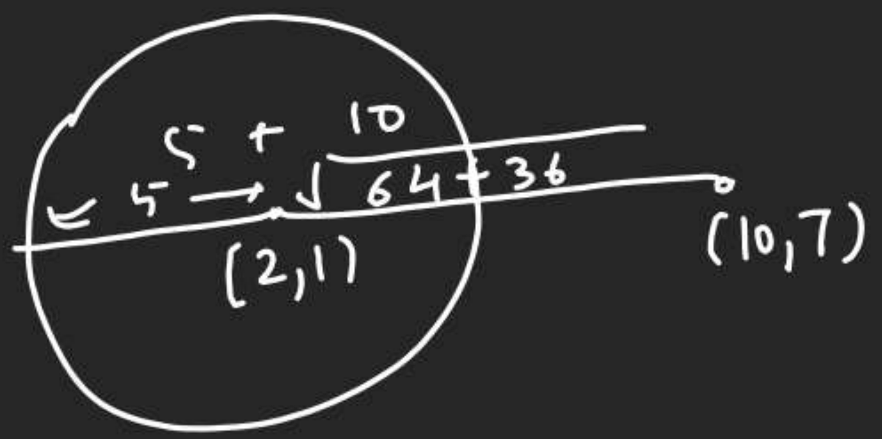
Q. The greatest distance of the point $\mathrm{P}(10, 7)$ from the circle $x^2 + y^2 - 4x - 2y - 20 = 0$ is

(A) 5

(B) 15

(C) 10

(D) none of these



$r = \sqrt{4+1+20} = 5$

CIRCLE

Q. Cartesian equations of a circle whose parametric equation are $x = -7 + 4\cos q, y = 3 + 4\sin q$ is -

(A) $(x+7)^2 + (y-3)^2 = 16$

(B) $(x-7)^2 + (y-3)^2 = 16$

(C) $(x-7)^2 + (y+3)^2 = 16$

(D) $(x+7)^2 + (y+3)^2 = 16$

$x = -7 + 4\cos\theta$

$y = 3 + 4\sin\theta$

$\frac{x+7}{4} = \cos\theta, \frac{y-3}{4} = \sin\theta$

$(x+7)^2 + (y-3)^2 = 4^2$

CIRCLE

Q. The equation of the normal to the circle $x^2 + y^2 = 9$ at the point $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ is

(A) $x - y = \frac{\sqrt{2}}{3}$

(B) $x + y = 0$

(C) $x - y = 0$

(D) none of these

$(0,0)$

$$(y-0) = \frac{\frac{1}{\sqrt{2}} - 0}{\frac{1}{\sqrt{2}} - 0}(x-0)$$

$$y = x$$

CIRCLE

Q. The length of the tangent drawn from the point $(2, 3)$ to the circles $2(x^2 + y^2) - 7x + 9y - 11 = 0$.

(A) 18

(B) 14

(C) $\sqrt{14}$

(D) $\sqrt{28}$

$S: x^2 + y^2 - \frac{7x}{2} + \frac{9y}{2} - \frac{11}{2} = 0$

$L_T = \sqrt{S_1} = \sqrt{2^2 + 3^2 - \frac{7 \times 2}{2} + \frac{9 \times 3}{2} - \frac{11}{2}}$

CIRCLE

Q. The angle between the two tangents from the origin to the circle $(x-7)^2+(y+1)^2=25$ equals

(A) $\frac{\pi}{2}$

(B) $\frac{\pi}{3}$

(C) $\frac{\pi}{4}$

(D) none

$2\tan^{-1}\frac{a}{\sqrt{S_1}} = 2\tan^{-1}\frac{5}{5}$

$= \frac{\pi}{2}$

$\sqrt{49+1-25}$

CIRCLE

Q. The point from which the tangents to the circles

$x^2 + y^2 - 8x + 40 = 0, 5x^2 + 5y^2 - 25x + 80 = 0$ $x^2 + y^2 - 8x + 16y + 160 = 0$

are equal in length is

(A) $\left(8, \frac{15}{2}\right)$

(B) $\left(-8, \frac{15}{2}\right)$

(C) $\left(8, -\frac{15}{2}\right)$ //

(D) none of these

Such Pt in R.C

$S_1 - S_3 = 0$

$\cancel{x^2} + \cancel{y^2} - \cancel{8x} + 40 = 0$

$\cancel{x^2} + \cancel{y^2} - \cancel{8x} + 16y + 160 = 0$

$- \quad - \quad + \quad - \quad -$

$-16y = 120$

$y = \frac{120}{-16} = \frac{30}{-4} = -\frac{15}{2}$

Nishant Jindal

PoI

$2x-3y=-1 \quad \times 2 \Rightarrow \quad 4x-6y=-2$

$3x-2y=1 \quad \times 3 \Rightarrow \quad 9x-6y=3$

$-5x=-5$

$x=1, y=1$

$(1,1)$

$(x^2+2x+\quad)+y^2-4y$

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CIRCLE

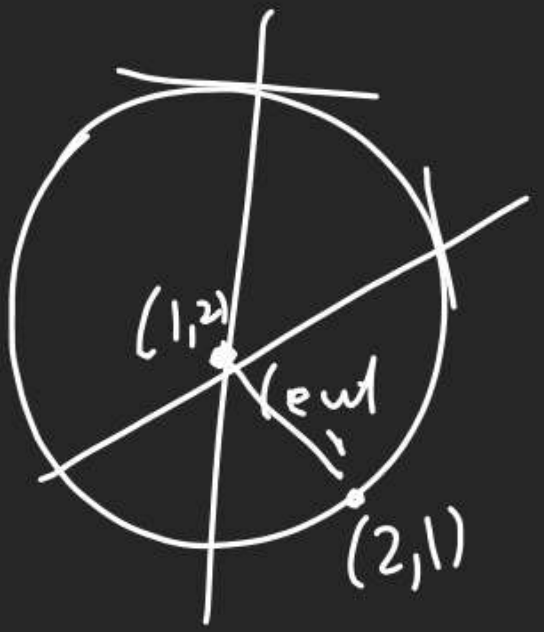
Q. The equation of the circle having the lines $y^2 - 2y + 4x - 2xy = 0$ as its normals & passing through the point $(2, 1)$ is

(A) $x^2 + y^2 - 2x - 4y + 3 = 0$

(B) $x^2 + y^2 - 2x + 4y - 5 = 0$

(C) $x^2 + y^2 + 2x + 4y - 13 = 0$

(D) $x^2 + y^2 - 2x - 8y = 0$



$y^2 - 2y + 4x - 2xy = 0$

$y(y-2) + 2x(2-y) = 0$

$(y-2x)(y-2) = 0$

$y = 2,\ y = 2x$

$x = 1$

Centre $\to (1,2)$

$(x-1)^2 + (y-2)^2 = r^2$

CIRCLE

Q. The equation of director circle to the circle $x^2 + y^2 = 8$ is-

(A) $x^2 + y^2 = 8$

(B) $x^2 + y^2 = 16$

(C) $x^2 + y^2 = 4$

(D) $x^2 + y^2 = 12$

D. C $\rightarrow x^2 + y^2 = 16$

CIRCLE

Q. Two perpendicular tangents to the circle $x^2 + y^2 = a^2$ meet at P. Then the locus of P has the equation-

(A) $x^2 + y^2 = 2a^2$

(B) $x^2 + y^2 = 3a^2$

(C) $x^2 = y^2 = 4a^2$

(D) None of these

D.C

$x^2+y^2=2a^2$

CIRCLE

Q. The locus of the mid-points of the chords of the circle $x^2 + y^2 - 2x - 4y - 11 = 0$ which subtend 60° at the centre is

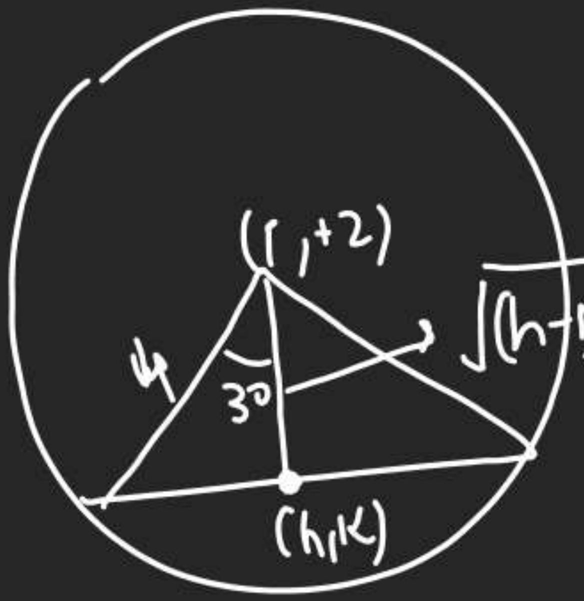
(A) $x^2 + y^2 - 4x - 2y - 7 = 0$

(B) $x^2 + y^2 + 4x + 2y - 7 = 0$

(C) $x^2 + y^2 - 2x - 4y - 7 = 0$

(D) $x^2 + y^2 + 2x + 4y + 7 = 0$

hanger



$\sqrt{(h-1)^2 + (k-2)^2} = 4\cos 30^\circ$

$(x^2 - 2x + 1) + (y^2 - 4y + 4) = 16$

$(x-1)^2 + (y-2)^2 = 16^2$

CIRCLE

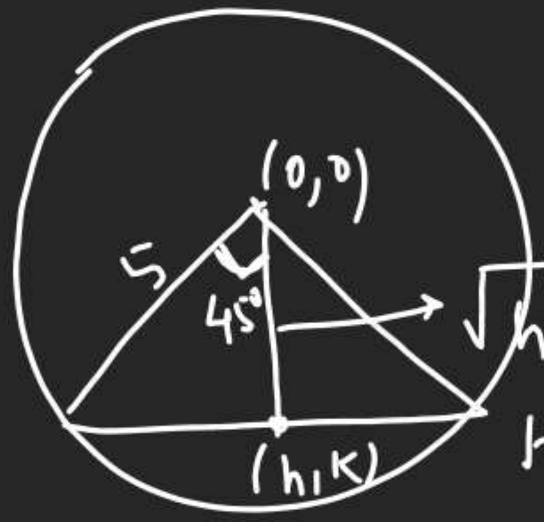
Q. Find the locus of mid point of chords of circle $x^2 + y^2 = 25$ which subtends right angle at origin-

(A) $x^2 + y^2 = 25/4$

(B) $x^2 + y^2 = 5$

(C) $x^2 + y^2 = 25/2$ ✓

(D) $x^2 + y^2 = 5/2$



$\sqrt{h^2+k^2} = 5\cos 45°$

$h^2+k^2 = \left(\frac{5}{\sqrt{2}}\right)^2$

$x^2+y^2 = \frac{25}{2}$

CIRCLE

Q. The equation to the chord of the circle $x^2 + y^2 = 16$ which is bisected at $(2, -1)$ is-

(A) $2x + y = 16$

(B) $2x - y = 16$

(C) $x + 2y = 5$

(D) $2x - y = 5$ ✓

$T = S_1$

$2x - y - \cancel{16} = 4 + 1 - \cancel{16}$

$2x - y = 5$

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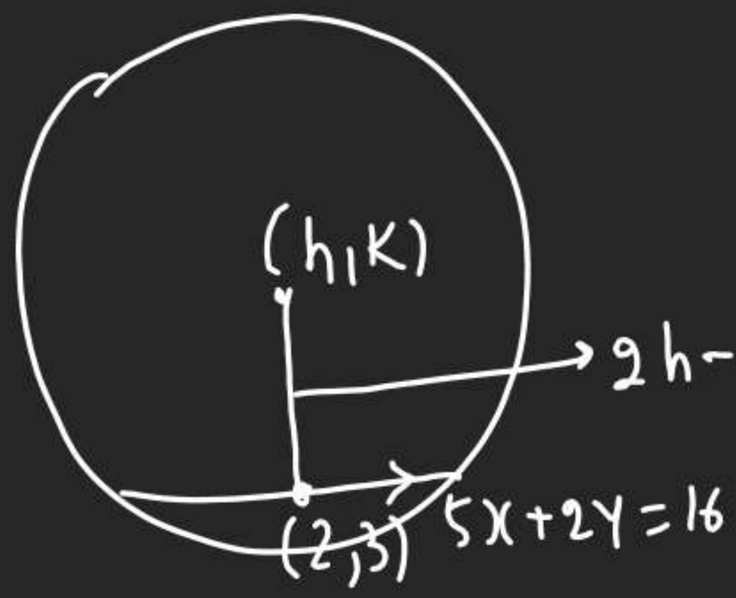
Q. The locus of the centres of the circles such that the point $(2, 3)$ is the mid point of the chord $5x + 2y = 16$ is

(A) $2x - 5y + 11 = 0$ //

(B) $2x + 5y - 11 = 0$

(C) $2x + 5y + 11 = 0$

(D) none



$2h - 5K = \lambda \rightarrow (2,3)$

$4 - 15 = \lambda$

$\lambda = -11$

$2x - 5y = -11$

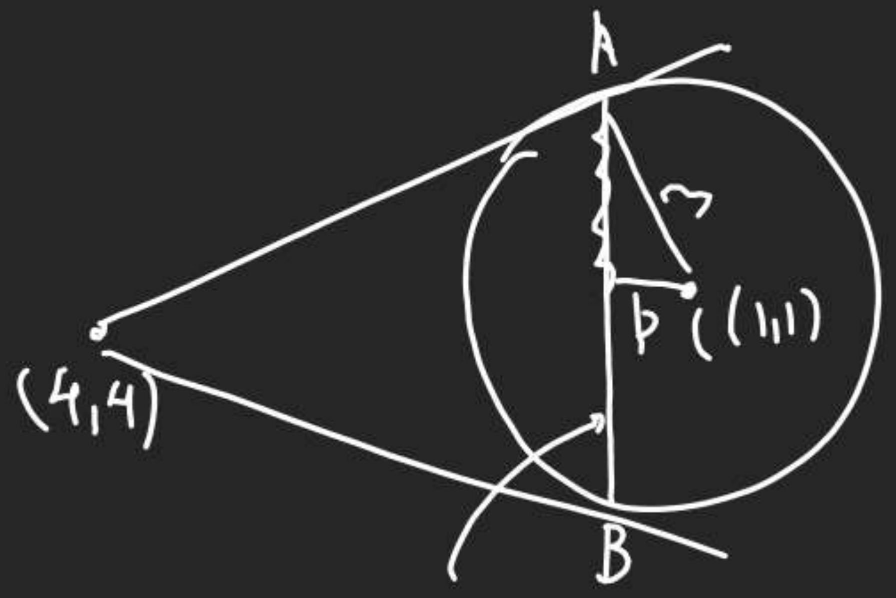
Q. Tangents are drawn from $(4, 4)$ to the circle $x^2 + y^2 - 2x - 2y - 7 = 0$ to meet the circle at A and B. The length of the chord AB is

(A) $2\sqrt{3}$

(B) $3\sqrt{2}$

(C) $2\sqrt{6}$

(D) $6\sqrt{2}$



$r = \sqrt{1+1+7} = 3$

find p & ⊥ Pythagoras

$T = 0$

$4x + 4y - (x+4) - (y+4) - 7 = 0$

$3x + 3y = 15$

$x + y = 5$

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CIRCLE

Q. Pair of tangents are drawn from every point on the line $3x + 4y = 12$ on the circle $x^2 + y^2 = 4$. Their variable chord of contact always passes through a fixed point whose co-ordinates are

Mains

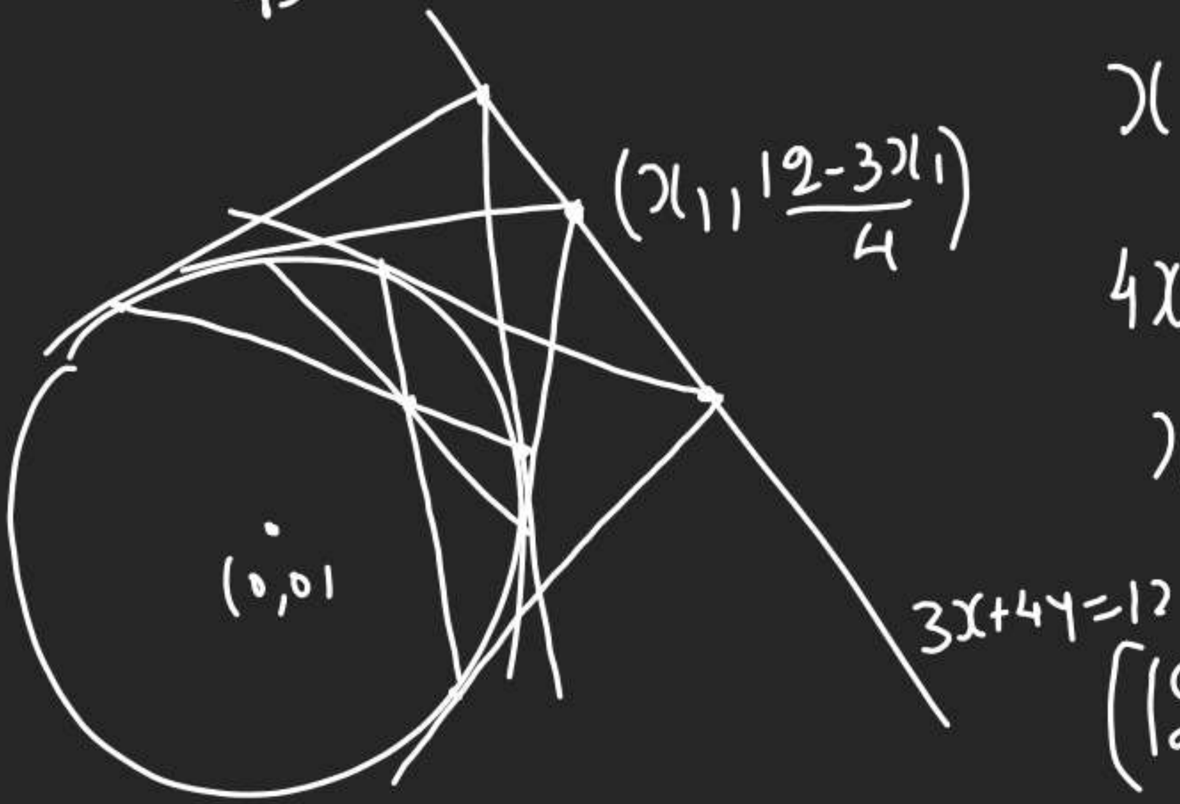
Imp

(A) $\left(\frac{4}{3}, \frac{3}{4}\right)$

(B) $\left(\frac{3}{4}, \frac{3}{4}\right)$

(C) $(1, 1)$

(D) $\left(1, \frac{4}{3}\right)$



$L_1 + \lambda L_2 = 0$

COC

$x x_1 + y\left(\frac{12-3x_1}{4}\right) = 4$

$4x x_1 + 12y - 3x_1 y = 16$

$x_1(4x - 3y) + (12y - 16) = 0$

$(12y - 16) + x_1(4x - 3y) = 0$

$L_1 + \lambda L_2 = 0$

Fix pt

L_1 $12y - 16 = 0 \Rightarrow y = \frac{4}{3}$

L_2: $4x - 3y = 0 \Rightarrow x = 1$

$\left(1, \frac{4}{3}\right)$

CIRCLE

Q. The equation of pair of tangents drawn from the point $(0, 1)$ to the circle $x^2 + y^2 - 2x + 4y = 0$ is-

(A) $4x^2 - 4y^2 + 6xy + 6x + 8y - 4 = 0$

(B) $4x^2 - 4y^2 + 6xy - 6x + 8y - 4 = 0$

(C) $x^2 - y^2 + 3xy - 3x + 2y - 1 = 0$

(D) $x^2 - y^2 + 6xy - 6x + 8y - 4 = 0$

$SS_1 = T^2$

Use

CIRCLE

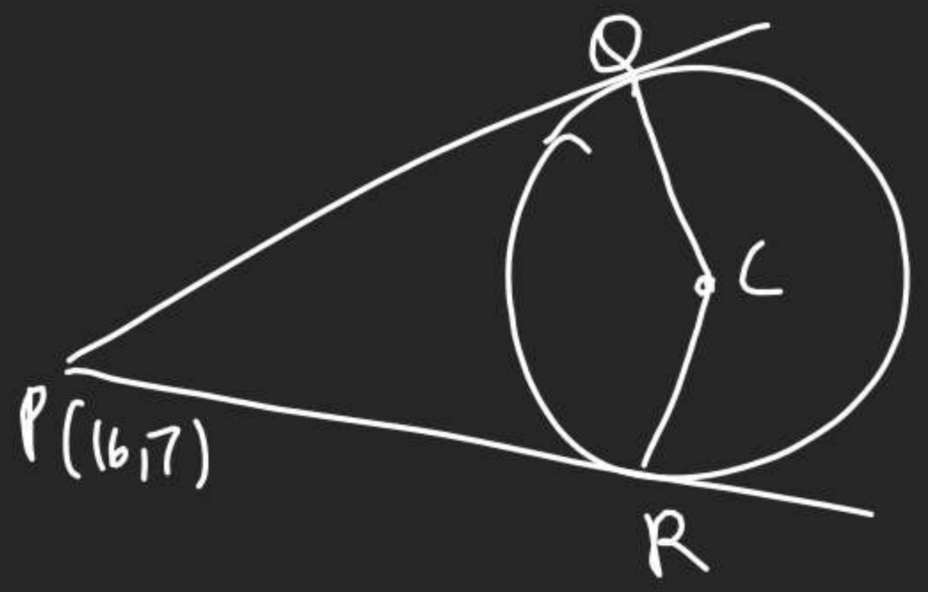
Q. From the point $P(16, 7)$ tangents PQ and PR are drawn to the circle $x^2 + y^2 - 2x - 4y - 20 = 0$. If C be the centre of the circle then area of the quadriteral PQCR is-

(A) 450 sq. units

(B) 15 sq. units

(C) 50 sq. units

(D) 75 sq. units3



$\square = a\sqrt{S_1}$

CIRCLE

Q. Equation of the circle touching the circle $x^2 + y^2 - 15x + 5y = 0$ at the point $(1, 2)$ and passing through the point $(0, 2)$ is

(A) $13x^2 + 13y^2 - 13x - 61y + 70 = 0$

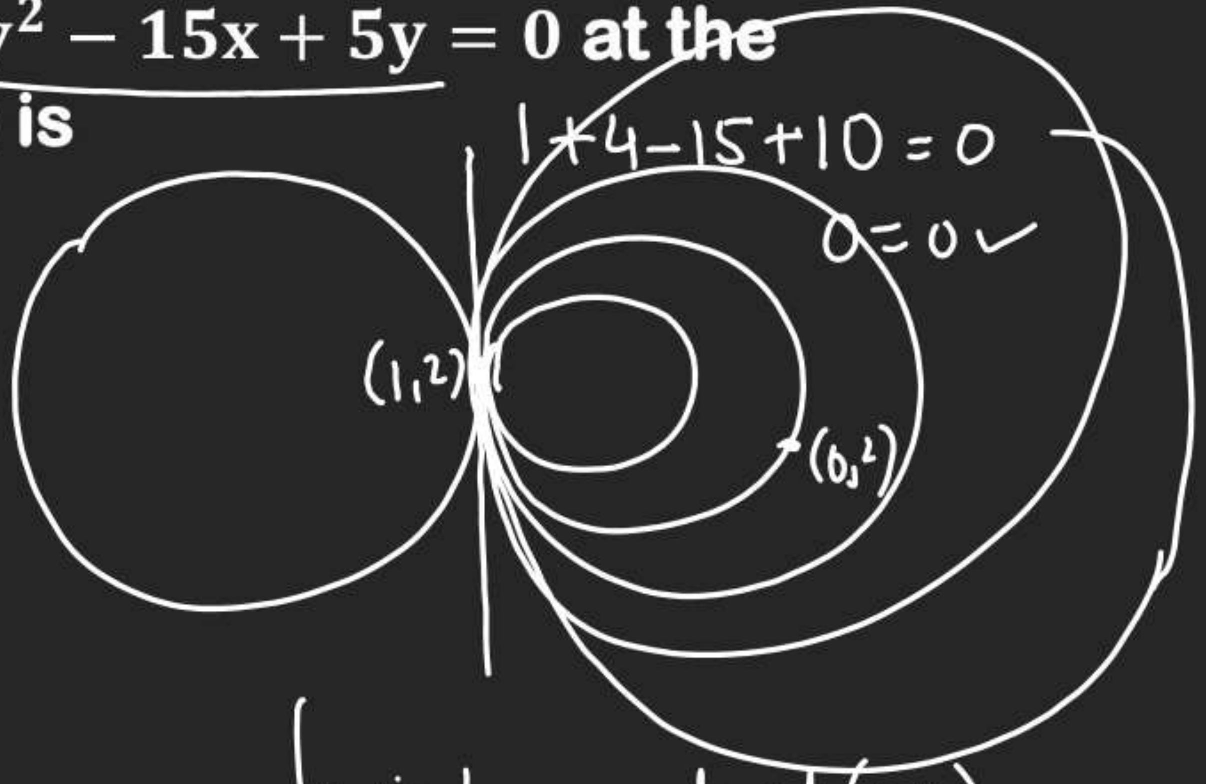
(B) $x^2 + y^2 + 2x = 0$

(C) $13x^2 + 13y^2 - 13x - 61y + 9 = 0$

(D) none of these

$1+4-15+10=0$

$0=0$ ✓



$S + \lambda L = 0$

$(x^2+y^2-15x+5y) + \lambda(13x-9y+5)=0$

$(0,2)$

$(+4-0+10)+\lambda(0-18+5)=0$

$-13\lambda = -14 \Rightarrow \lambda = \frac{14}{13}$

$(x^2+y^2-15x+5y) + \frac{14}{13}(13x-9y+5)=0$

$13x^2+13y^2-195x+65y+182x-126y+70=0$

$13x^2+13y^2-13x$

L is tangent at (1,2)

$x\cdot 1+y\cdot 2-\frac{15}{2}(x+1)+\frac{5}{2}(y+2)=0$

$2x+4y-15x+5y-5=0$

$-13x+9y-5=0$

$13x-9y+5=0$ ✓

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CIRCLE

Q. The number of common tangents of the circles $x^2 + y^2 - 2x - 1 = 0$ and $x^2 + y^2 - 2y - 7 = 0$

(A) 1

(B) 3

(C) 2

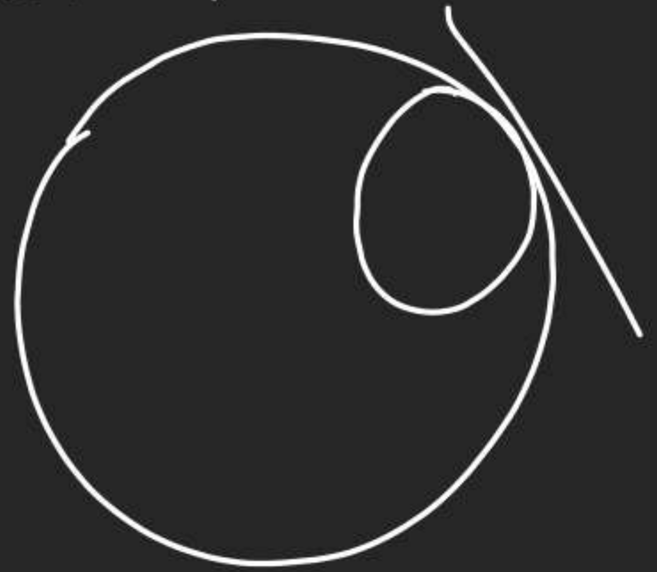
(D) 4

$C_1(1,0)$ | $r_1 = \sqrt{1+0+1} = \sqrt{2}$

$C_2(0,1)$ | $r_2 = \sqrt{0+1+7} = 2\sqrt{2}$

$\sqrt{2}$ | $r_1 - r_2 = |\sqrt{2} - 2\sqrt{2}|$

$\sqrt{2} = C_1C_2 = |r_1 - r_2| = \sqrt{2}$



CIRCLE

Q. If the circle $x^2 + y^2 = 9$ touches the circle $x^2 + y^2 + 6y + c = 0$, then c is equal to

(A) -27 //
(B) 36
(C) -36
(D) 27a

$C_1 = (0,0)$

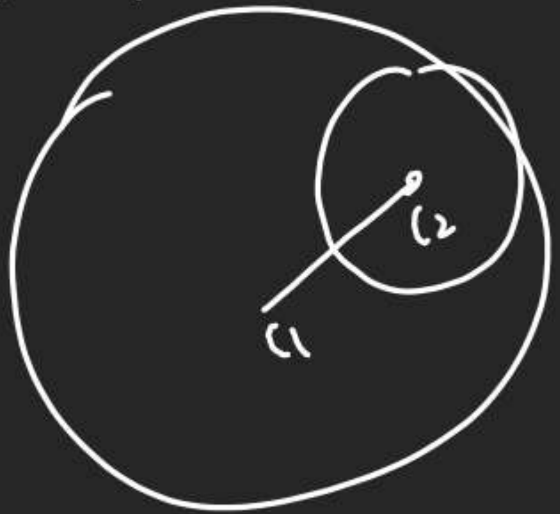
$C_2 = (0,-3)$

$d = 3$

$r_1 = 3$

$r_2 = \sqrt{0+9-c}$

Internally touch.



$C_1C_2 = |r_1 - r_2|$

$3 = |3 - \sqrt{9-c}|$

$\oplus$

$3 = 3 - \sqrt{9-c}$

$c = 9$

$\ominus$

$3 = -3 + \sqrt{9-c}$

$6 = \sqrt{9-c}$

$36 = 9 - c$

$c = -27$

CIRCLE

Q. The distance of the centre of the circle $x^2 + y^2 = 2x$ from the common chord of the circles $x^2 + y^2 + 5x - 8y + 1 = 0$ and $x^2 + y^2 - 3x + 7y - 25 = 0$ is

(A) 1

(B) 3

(C) 2 ✓

(D) $\frac{1}{3}$

$S_1 - S_2 = 0$

$x^2 + y^2 - 2x = 0$

$8x - 15y + 26 = 0$

$C = (1, 0)$

$d = \frac{|8 - \cancel{15(0)} + 26|}{\sqrt{64 + 225}} = \frac{34}{17} = 2$

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CIRCLE

Q. Two given circles $x^2 + y^2 + ax + by + c = 0$ and $x^2 + y^2 + dx + ey + f = 0$ will intersect each other orthogonally, only when-

Coby

(A) $ad + be = c + f$

(B) $a + b + c = d + e + f$

(C) $ad + be = 2c + 2f$

(D) $2ad + 2be = c + f$

CIRCLE

Q. If the circles of same radius a and centres at $(2, 3)$ and $(5, 6)$ cut orthogonally, then a is equal to-

Copy

(A) 6

(B) 4

(C) 3

(D) 10

Q. If $a^2 + b^2 = 1$, $m^2 + n^2 = 1$, then

(A) $|am + bn| \leq 1$

(B) $|am - bn| \geq 1$

(C) $|am + bn| \geq 1$

(D) none of these

$AM \geq GM$

CIRCLE

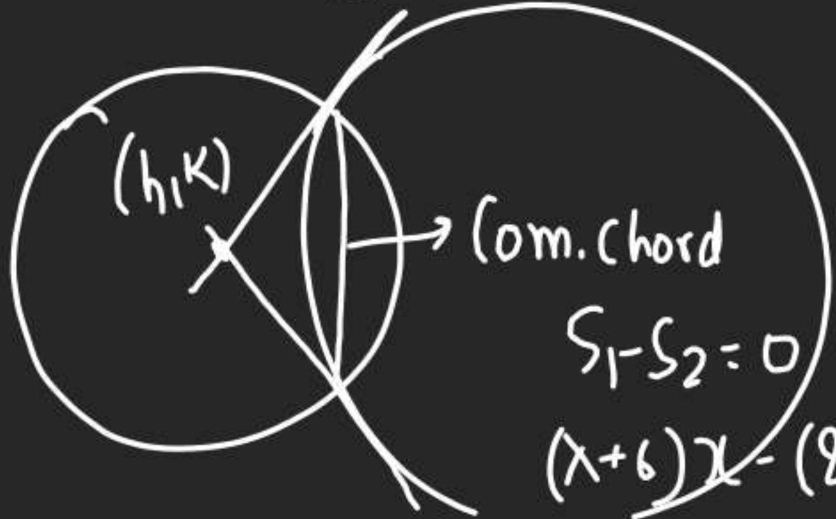
Q. Tangents are drawn to the circle $x^2 + y^2 = 1$ at the points where it is met by the circles. $x^2 + y^2 - (\lambda + 6)x + (8 - 2\lambda)y - 3 = 0, \lambda$ being the variable. The locus of the point of intersection of these tangents is

(A) $2x - y + 10 = 0$ //

(B) $x + 2y - 10 = 0$

(C) $x - 2y + 10 = 0$

(D) $2x + y - 10 = 0$ //



$S_1 - S_2 = 0$

$(\lambda+6)x - (8-2\lambda)y + 2 = 0$

$hx + Ky - 1 = 0$

$\frac{h}{\lambda+6} = \frac{K}{-(8-2\lambda)} = \frac{-1}{2}$

$h = \frac{-\lambda-6}{2}$ | $K = \frac{8-2\lambda}{2}$

$-\lambda - 6 = 2h$ $8 - 2\lambda = 2K$

$\lambda = -2h - 6$ $2\lambda = 8 - 2K$

$-4h - 12 = 8 - 2K$

$4x - 2y + 20 = 0$

$2x - y + 10 = 0$

CIRCLE

Comprehension

A circle C of radius 1 is inscribed in an equilateral triangle PQR. The points of contact of C witt the sides PQ, QR, RP and D, E, F respectively. The line PQ is given by the equation $\sqrt{3}x + y - 6 = 0$ and the point D is $\left(\frac{3\sqrt{3}}{2}, \frac{3}{2}\right)$. Further, it is given that the origin and the centre of C are on the same side of the line PQ

Q. (i) The equation of circle C is

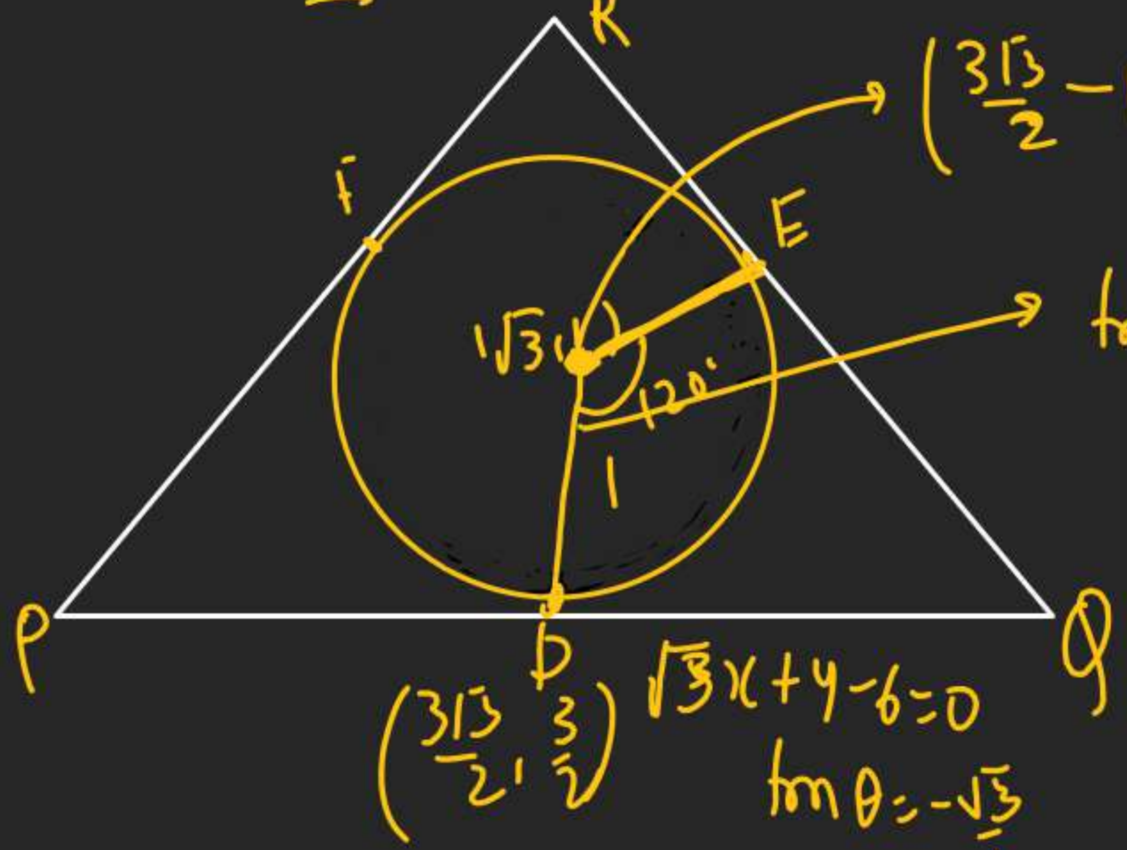
(A) $(x - 2\sqrt{3})^2 + (y - 1)^2 = 1$

(B) $(x - 2\sqrt{3})^2 + \left(y + \frac{1}{2}\right)^2 = 1$

(C) $(x - \sqrt{3})^2 + (y + 1)^2 = 1$

(D) $(x - \sqrt{3})^2 + (y - 1)^2 = 1$ ✓

$(x-\sqrt{3})^2+(y-1)^2=1$



$\tan\theta = -\frac{\sqrt{3}}{1}$

$\left(\frac{3\sqrt{3}}{2} - 1\cdot\cos 30°, \frac{3}{2} - 1\cdot\sin 30°\right)$

$= \left(\frac{3\sqrt{3}}{2} - \frac{\sqrt{3}}{2}, \frac{3}{2} - \frac{1}{2}\right)$

$= (\sqrt{3}, 1)$

$\tan\theta = \frac{1}{\sqrt{3}}$

$\theta = 30°$

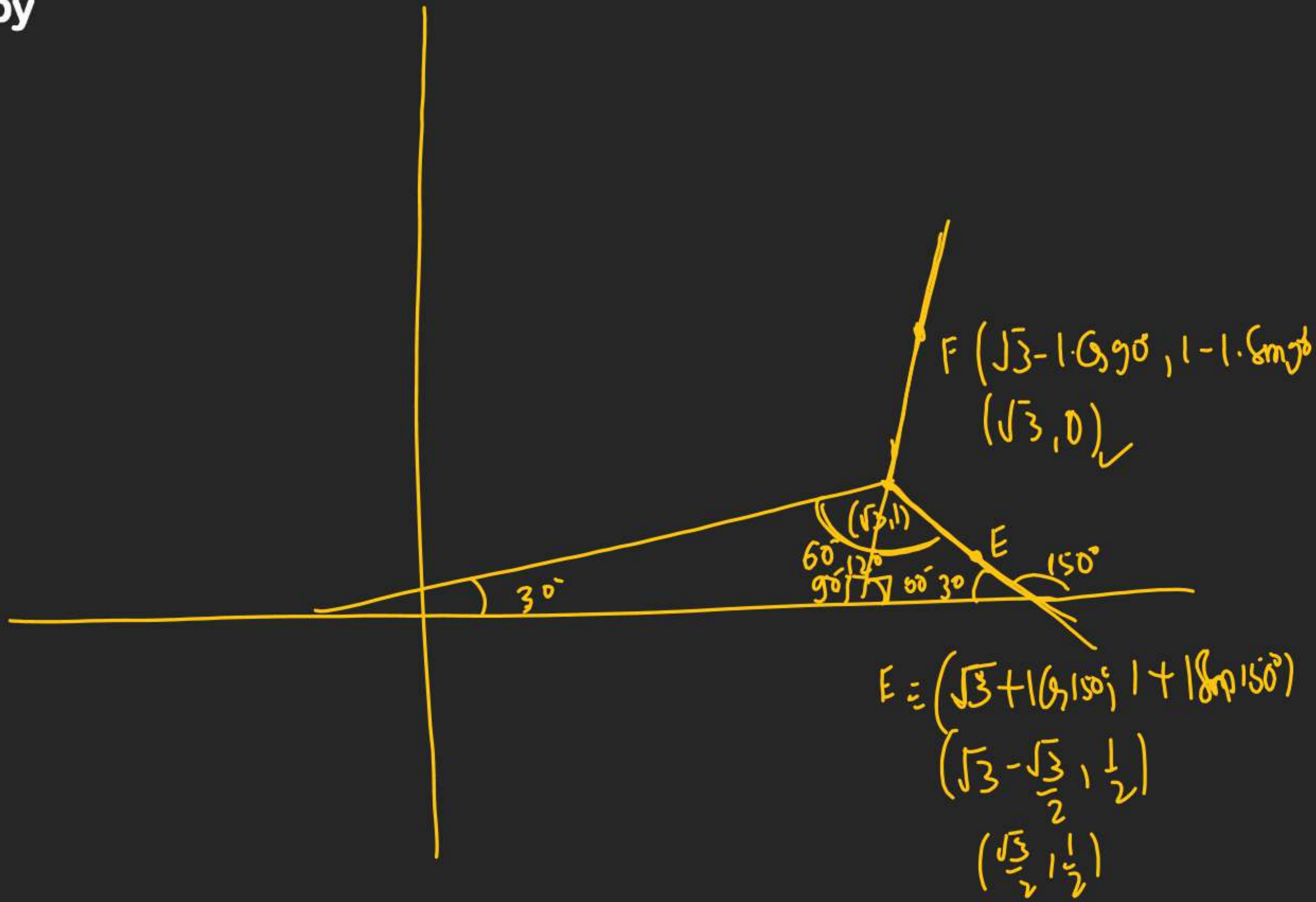
(ii) Points E and F are given by

(A) $\left(\frac{\sqrt{3}}{2}, \frac{3}{2}\right), (\sqrt{3}, 0)$

(B) $\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right), (\sqrt{3}, 0)$

(C) $\left(\frac{\sqrt{3}}{2}, \frac{3}{2}\right) \cdot \left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

(D) $\left(\frac{3}{2}, \frac{\sqrt{3}}{2}\right) \cdot \left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$



(iii) Equations of the sides RP, RQ are

(A) $y = \frac{2}{\sqrt{3}}x + 1, y = -\frac{2}{\sqrt{1}}x - 1$

(B) $y = \frac{1}{\sqrt{3}}x_4 y = 0$

(C) $y = \frac{\sqrt{3}}{z}x + 1, y = -\frac{\sqrt{3}}{z}x - 1$

(D) $y = \sqrt{3}x, y = 0$

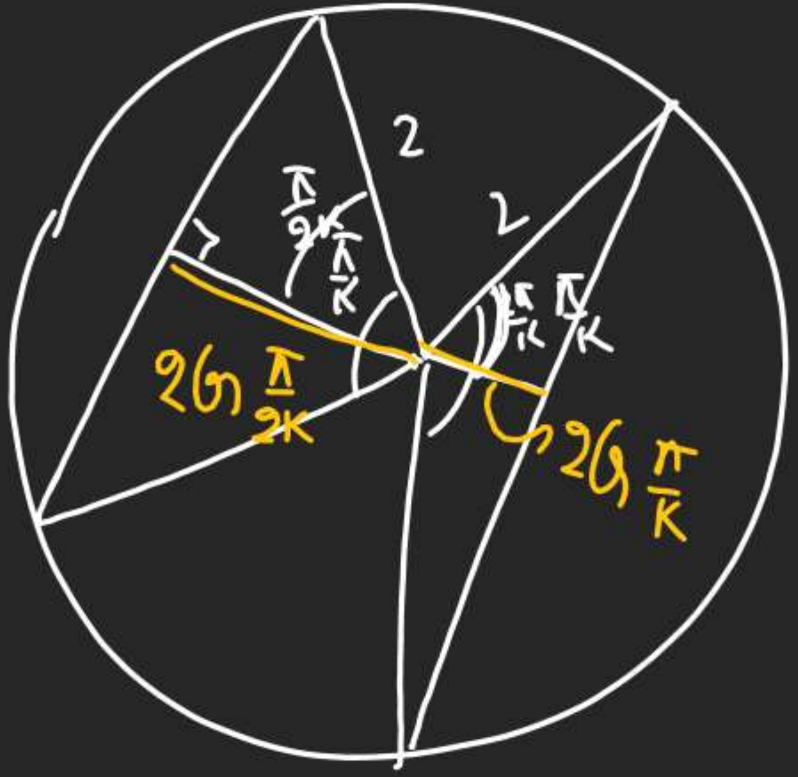
CIRCLE

Q. Two parallel chords of a circle of radius 2 are at a distance $\underline{\sqrt{3}+1}$ apart. If the chords subtend at the center, angles of $\frac{\pi}{k}$ and $\frac{2\pi}{k}$, where $k > 0$, then the value of $[k]$ is

[Note: $[\,k\,]$ denotes the largest integer less than or equal to k]

Sh. Dhracharya

$$t = \frac{-2 \pm \sqrt{4 + 4\times 4\times(3+\sqrt{3})}}{8}$$



$$2\cos\frac{\pi}{k} + 2\cos\frac{\pi}{2k} = \sqrt{3}+1$$

$$\cos\theta + \cos\frac{\theta}{2} = \frac{\sqrt{3}+1}{2}$$

$$2\cos^2\frac{\theta}{2} - 1 + \cos\frac{\theta}{2} = \frac{\sqrt{3}+1}{2}$$

$$2t^2 + t - 1 - \frac{\sqrt{3}+1}{2} = 0$$

$$4t^2 + 2t - 2 - \sqrt{3} - 1 = 0$$

$$4t^2 + 2t - 3 - \sqrt{3} = 0$$

CIRCLE

Q. Let T be the line passing the points $P(-2, 7)$ and $Q(2, -5)$. Let F be the set of all pairs of circles (S_1, S_2) such that T is tangent to S_1 at P and tangent to S_2 at Q, and also such that S_1 and S_2 touch each other at a point, say M. Let E_1^2 be the set representing the locus of M as the pair (S_1, S_2) varies in F_1, Let the set of all straight line segments foining a pair of distinct points of E_2 and passing through the point $R(1, 1,) \mathrm{be} F_2$. Let E_2 be the set of the mid-points of the line segments in the set F_2. Then, which of the following statement(s) is (are) TRUE?

(A) The point $(-2, 7)$ lies in E_1

(B) The point $\left(\frac{4}{5}, \frac{7}{5}\right)$ does NOT lie in E_2

(C) The point $\left(\frac{1}{2}, 1\right)$ lies in E_2

(D) The point $\left(0, \frac{3}{2}\right)$ does NOT lie in E_1