HOMEWORK–2

1. DIFFERENTIATION OF IMPLICIT FUNCTION

1. If $y=\sqrt{\sin x+y}$, then $\frac{dy}{dx}=$

(A) $\frac{\sin x}{2y-1}$ (B) $\frac{\sin x}{1-2y}$ (C) $\frac{\cos x}{1-2y}$ (D) $\frac{\cos x}{2y-1}$

Ans. (D)

Sol. $y=\sqrt{\sin x+y}$

squaring both side

$y^2=\sin x+y$

differentiating w.r.t. to x

$2yy'=\cos x+y'$

$y'=\frac{\cos x}{2y-1}$

2. If $ax^2+2hxy+by^2=0$, then $\frac{dy}{dx}$ equals-

(A) $\frac{ax+hy}{hx+by}$ (B) $-\frac{ax+hy}{hx+by}$ (C) $\frac{hx+by}{ax+hy}$ (D) $-\frac{hx+by}{ax+hy}$

Ans. (B)

Sol. $ax^2+2hxy+by^2=0$

$$2ax+2hx\frac{dy}{dx}+2hy\cdot 1+2by\frac{dy}{dx}=0$$

$$\frac{dy}{dx}(hx+by)=-(ax+hy)$$

$$\frac{dy}{dx}=-\left(\frac{ax+hy}{hx+by}\right)$$

3. If $x\sqrt{y}+y\sqrt{x}=1$, then $\frac{dy}{dx}$ equals-

(A) $-\frac{y+2\sqrt{xy}}{x+2\sqrt{xy}}$ (B) $-\sqrt{\frac{x}{y}}\left(\frac{y+2\sqrt{xy}}{x+2\sqrt{xy}}\right)$ (C) $-\sqrt{\frac{y}{x}}\left(\frac{y+2\sqrt{xy}}{x+2\sqrt{xy}}\right)$ (D) $\frac{x}{y}$

Ans. (C)

Sol. $x\sqrt{y}+y\sqrt{x}=1$

$$x\cdot\frac{1}{2\sqrt{y}}\frac{dy}{dx}+\sqrt{y}\cdot 1+y\frac{1}{2\sqrt{x}}+\sqrt{x}\frac{dy}{dx}=0$$

$$\frac{dy}{dx}\left[\frac{x}{2\sqrt{y}}+\sqrt{x}\right]=-\left(\frac{y}{2\sqrt{x}}+\sqrt{y}\right)$$

$$\frac{dy}{dx}\left(\frac{x+2\sqrt{xy}}{2\sqrt{y}}\right)=-\left(\frac{y+2\sqrt{xy}}{2\sqrt{x}}\right)$$

$$\frac{dy}{dx}=-\frac{\sqrt{y}}{\sqrt{x}}\left(\frac{y+2\sqrt{xy}}{x+2\sqrt{xy}}\right)$$

4. If $e^x\sin y-e^y\cos x=1$, then $\frac{dy}{dx}$ equals-

(A) $\frac{e^x\sin y+e^y\sin x}{e^y\cos x-e^x\cos y}$ (B) $\frac{e^x\sin y+e^y\sin x}{e^y\cos x+e^x\cos y}$

(C) $\frac{e^x\sin y-e^y\sin x}{e^y\cos x-e^x\cos y}$ (D) $\frac{e^x}{e^y}$

Ans. **(A)**

Sol. $e^x\sin y-e^y\cos x=1$

$$e^x\cdot\cos y\frac{dy}{dx}+\sin ye^x-e^y(-\sin x)-\cos x.$$

$$e^y\cdot\frac{dy}{dx}=0$$

$$\frac{dy}{dx}(e^x\cos y-e^y\cos x)=-(e^x\sin y+e^y\sin x)$$

$$\frac{dy}{dx}=-\left(\frac{e^x\sin y+e^y\sin x}{e^x\cos y-e^y\cos x}\right)$$

$$=\frac{e^x\sin y+e^y\sin x}{e^y\cos x-e^x\cos y}$$

5. If $\sqrt{1-x^2}+\sqrt{1-y^2}=a(x-y)$, then the value of dy/dx is -

(A) $\frac{\sqrt{1-x^2}}{\sqrt{1-y^2}}$ (B) $\frac{\sqrt{1-y^2}}{\sqrt{1-x^2}}$ (C) $-\frac{\sqrt{1-x^2}}{\sqrt{1-y^2}}$ (D) $-\frac{\sqrt{1-y^2}}{\sqrt{1-x^2}}$

Ans. **(B)**

Sol. $\frac{\sqrt{1-y^2}}{\sqrt{1-x^2}}$

Substituting $x=\sin\theta$ and $y=\sin\phi$ in the given equation, we get

$\cos\theta+\cos\phi=a(\sin\theta-\sin\phi)$

$$\Rightarrow 2\cos\frac{\theta+\phi}{2}\cdot\cos\frac{\theta-\phi}{2}=2a\cos\frac{\theta+\phi}{2}\cdot\sin\frac{\theta-\phi}{2}$$

$$\Rightarrow \cot\frac{\theta-\phi}{2} = a \Rightarrow \theta-\phi = 2\cot^{-1}a$$

$$\Rightarrow \sin^{-1}x - \sin^{-1}y = 2\cot^{-1}a$$

Differentiating with respect to x, we get

$$\frac{1}{\sqrt{1-x^2}} - \frac{1}{\sqrt{1-y^2}}\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = \frac{\sqrt{1-y^2}}{\sqrt{1-x^2}}$$

6. If $x^3\cos(xy) + y^3\sin(xy) + 1 = 0$, then $\frac{dy}{dx}$ equals

(A) $\frac{x^3y\tan(xy)-(3x^2+y^4)}{xy^3+(3y^2-x^4)\tan xy}$ (B) $\frac{x^3y\tan(xy)+(3x^2+y^4)}{xy^3-(3y^2-x^4)\tan xy}$

(C) $\frac{x^3y-(3x^2+y^4)\tan(xy)}{xy^3\tan(xy)+(3y^2-x^4)}$ (D) $x^2y\tan(xy) + 3x^2$

Ans. **(A)**

Sol. $f(x) = x^3\cos xy + y^3\sin xy + 1$

$$\frac{\partial f}{\partial y} = x^3\cdot(-\sin xy)\cdot y + \cos xy\cdot 3x^2 + y^3\cdot\cos xy\cdot y$$

$$= -x^3y\sin xy + 3x^2\cos xy + y^4\cos xy$$

$$\&\frac{\partial f}{\partial y} = x^3(-\sin xy)\cdot x + y\cos xy\cdot x + \sin xy\cdot 3y^2$$

$$= -x^4\sin xy + y^3x\cos xy + 3y^2\sin xy$$

$$\therefore \frac{dy}{dx} = -\frac{\partial f/\partial x}{\partial f/\partial y}$$

$$= -\left(\frac{-x^3\sin xy + 3x^2\cos xy + y^4\cos xy}{-x^4\sin xy + y^3x\cos xy + 3y^2\sin y}\right)$$

$$= -\left(\frac{-x^3y\tan xy + 3x^2 + y^4}{-x^4\tan xy + y^3x + 3y^2\tan xy}\right)$$

$$= \frac{x^3y\tan xy - (3x^2+y^4)}{y^3x + (3y^2-x^4)\tan xy}$$

7. If $y = 2^{\log_2 x^{2x}} + \left(\tan\frac{\pi x}{4}\right)^{\frac{4}{\pi x}}$ then $\left.\frac{dy}{dx}\right]_{x=1}$ is

(A) 4 (B) 1 (C) 0 (D) Not defined

Ans. **(A)**

Sol. $y = x^{2x} + \left(\tan\left(\frac{\pi x}{4}\right)\right)^{4/\pi x}$

$y = u + v$

$$\Rightarrow \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$$

$$u = x^{2x}, v = \left(\tan\left(\frac{\pi x}{4}\right)\right)^{4/\pi x}$$

2. DIFFERENTIATION OF LOGARITHMIC FUNCTION

8. If $x^m \cdot y^n = (x+y)^{m+n}$, then $\frac{dy}{dx}$ is

(A) $\frac{x+y}{xy}$ (B) xy (C) $\frac{x}{y}$ (D) $\frac{y}{x}$

Ans. **(D)**

Sol. Given that, $x^m y^n (x+y)^{m+n}$
Taking log on both sides, we get
$m.\log x + n\log y = (m+n)\log(x+y)$
On differentiating w.r.t. x, we get

$$\frac{m}{x} + \frac{n}{y}\frac{dy}{dx} = \frac{(m+n)}{(x+y)}\left(1 + \frac{dy}{dx}\right)$$

$$\Rightarrow \frac{dy}{dx}\left(\frac{m+n}{x+y} - \frac{n}{y}\right) = \frac{m}{x} - \frac{m+n}{x+y}$$

$$\Rightarrow \frac{dy}{dx}\left(\frac{my+ny-nx-ny}{y(x+y)}\right)$$

$$= \frac{mx+my-mx-nx}{x(x+y)} \Rightarrow \frac{dy}{dx} = \frac{y}{x}$$

9. If $f(x) = |x|^{|\sin x|}$ then $f'(\pi/4)$ equals

(A) $\left(\frac{\pi}{4}\right)^{1/\sqrt{2}}\left(\frac{\sqrt{2}}{2}\ln\frac{4}{\pi} - \frac{2\sqrt{2}}{\pi}\right)$ (B) $\left(\frac{\pi}{4}\right)^{1/\sqrt{2}}\left(\frac{\sqrt{2}}{2}\ln\frac{4}{\pi} + \frac{2\sqrt{2}}{\pi}\right)$

(C) $\left(\frac{\pi}{4}\right)^{1/\sqrt{2}}\left(\frac{\sqrt{2}}{2}\ln\frac{\pi}{4} - \frac{2\sqrt{2}}{\pi}\right)$ (D) $\left(\frac{\pi}{4}\right)^{1/\sqrt{2}}\left(\frac{\sqrt{2}}{2}\ln\frac{\pi}{4} + \frac{2\sqrt{2}}{\pi}\right)$

Ans. **(D)**

Sol. $f(x) = |x|^{|\sin x|}$

$x > 0, f(x) = x^{\sin x} \Rightarrow y = x^{\sin x}$

$\ell ny = \sin x \ln x$

$$y' = x^{\sin x}\left\{\frac{\sin x}{x} + \cos x \ln x\right\}$$

$$y|_{x=\pi/4} = \left(\frac{\pi}{4}\right)^{\frac{1}{\sqrt{2}}}\left\{\frac{1}{\sqrt{2}} \times \frac{4}{\pi} + \frac{1}{\sqrt{2}}\ln\frac{\pi}{4}\right\}$$

$x < 0 \; y = -x^{-\sin x}$

$\ell ny = -\sin x \ell n(-x)$

10. If $y = (1+x)(1+x^2)(1+x^4) \ldots .(1+x^{2n})$, then $\frac{dy}{dx}$ at $x = 0$ is

(A) -1 (B) 1 (C) 0 (D) 2^n

Ans. **(B)**

Sol. $y = (1+x)(1+x^2) \ldots (1+x^{2n})$

$$y = \frac{(1-x^2)(1+x^2)(1+x^4) \ldots (1+x^{2n})}{(1-x)}$$

$$y = \frac{1-x^{4n}}{1-x}$$

$$\frac{dy}{dx} = \frac{(1-x)(-4nx^{4n-1}) + (1-x^{4n})}{(1-x)^2}$$

$$\frac{dy}{dx} = \frac{-4nx^{4n-1} + 4nx^{4n} + 1 - x^{4n}}{(1-x)^2}$$

$$\left.\frac{dy}{dx}\right|_{x=0} = \frac{-4n \times 0 + 0 + 1 - 0}{1} = 1$$

3. DIFFERENTIATION OF INFINITE SERIES

11. If $x = e^{y+e^{y+\cdots \text{upto } \infty}}$, $x > 0$, then $\frac{dy}{dx}$

(A) $\frac{x}{1+x}$ (B) $\frac{1}{x}$ (C) $\frac{1-x}{x}$ (D) $\frac{1+x}{x}$

Ans. **(C)**

Sol. $x = e^y + e^{y+e^{y+\cdots\infty}}$

$x = e^{y+x}$

$$x = e^{(y+x)}\left\{\frac{dy}{dx} + 1\right\}$$

$$\frac{dy}{dx} = \frac{1-e^{x+y}}{e^{x+y}} = \frac{1-x}{x}$$

12. If $y = \sqrt{x}^{\sqrt{x}^{\sqrt{x} \ldots\ldots \infty}}$, then the value of $\frac{dy}{dx}$ is

(A) $\frac{xy^2}{2-y\log x}$ (B) $\frac{x^2}{y(2-y\log x)}$ (C) $\frac{y^2}{x(2-y\log x)}$ (D) $\frac{y^2}{x(2+y\log x)}$

Ans. **(C)**

Sol. $y = \sqrt{x}^{\sqrt{x}^{\sqrt{x}^{\sqrt{x}\ldots\ldots\infty}}}$

$y = (\sqrt{x})^y$

$\log y = y\log\sqrt{x}$ ....(i)

$\frac{1}{y}\frac{dy}{dx} = y\cdot\frac{1}{\sqrt{x}}\times\frac{1}{2\sqrt{x}} + \log\sqrt{x}\cdot\frac{dy}{dx}$

$\left(\frac{1}{y} - \log\sqrt{x}\right)\frac{dy}{dx} = \frac{y}{2x}$

$\left(\frac{1}{y} - \frac{\log y}{y}\right)\frac{dy}{dx} = \frac{y}{2x}$ (from (1))

$\frac{dy}{dx} = \frac{y^2}{2x(1-\log y)}$

or $\frac{dy}{dx} = \frac{y^2}{2x(1-y\log x^{1/2})}$

$= \frac{y^2}{2x\left(1-\frac{1}{2}y\log x\right)} = \frac{y^2}{x(2-y\log x)}$

13. If $y = \sqrt{\log x + \sqrt{\log x + \sqrt{\log x + \ldots\ldots\ldots}}}$, then $\frac{dy}{dx}$ equals-

(A) $x/(2y+1)$ (B) $1/x(2y-1)$ (C) $(2y-1)/x$ (D) $x(2y-1)$

Ans. **(B)**

Sol. $y = \sqrt{\log x + \sqrt{\log x + \sqrt{\log x + \cdots\ldots\ldots}}}$

$y = \sqrt{\log x + y}$

$y^2 = \log x + y$

$2y\frac{dy}{dx} = \frac{1}{x} + \frac{dy}{dx}$

$\frac{dy}{dx}(2y-1) = \frac{1}{x}$

$\frac{dy}{dx} = \frac{1}{x(2y-1)}$

14. Let $f(x) = x + \frac{1}{2x+\frac{1}{2x+\frac{1}{2x+\ldots\ldots\infty}}}$. Compute the value of $f(100).f'(100)$

Ans. **100**

Sol. $2y = \frac{1}{2x+(y-x)} + y - x + 2x$

$$y = \frac{1}{y+x} + x$$

$$y^2 - x^2 = 0 \Rightarrow yy^1 - x = 0$$

$$f(100) \cdot f^1(100) = 100$$

15. If $y = x + \frac{1}{x+\frac{1}{x+\frac{1}{x+\ldots\ldots\ldots}}}$, prove that $\frac{dy}{dx} = \frac{1}{2-\frac{x}{x+\frac{1}{x+\frac{1}{x+\ldots\ldots\ldots}}}}$

Sol. $y = x + \frac{1}{x+\frac{1}{x+\frac{1}{x+\cdots}}}$

$$y = x + \frac{1}{y}$$

$$y^2 = xy + 1$$

$$2yy' = xy' + y \Rightarrow y' = \frac{y}{2y - x}$$

$$y' = \frac{1}{2 - \frac{x}{y}} \quad y' = \frac{1}{2 - \frac{x}{x + \frac{1}{x+\cdots}}}$$

16. If $y = \tan^{-1}\frac{1}{x^2+x+1} + \tan^{-1}\frac{1}{x^2+3x+3} + \tan^{-1}\frac{1}{x^2+5x+7} + \tan^{-1}\frac{1}{x^2+7x+13} + \ldots\ldots$ to n terms.

Find dy/dx, expressing your answer in 2 terms,

Ans. $\frac{1}{1+(x+n)^2} - \frac{1}{1+x^2}$

Sol. $y = \tan^{-1}(x+1) - \tan^{-1}x$

$$+\tan^{-1}(x+2) - \tan^{-1}(x+1)$$

$$+\tan^{-1}(x+n) - \tan^{-1}(x+(n-1))$$

$$y = \tan^{-1}(x+n) - \tan^{-1}x$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{1+(x+n)^2} - \frac{1}{1+x^2}$$

4. DIFFERENTIATION OF DETERMINANT

17. If $f(x), g(x), h(x)$ are polynomials in x of degree 2

and $F(x)=\begin{vmatrix} f & g & h \\ f' & g' & h' \\ f'' & g'' & h'' \end{vmatrix}$, then $F'(x)$ is equal to

(A) 1 (B) 0 (C) -1 (D) $f(x)\cdot g(x)\cdot h(x)$

Ans. **(B)**

Sol. $F'(x)=\begin{vmatrix} f' & g' & h' \\ f' & g' & h' \\ f'' & g'' & h'' \end{vmatrix}+\begin{vmatrix} f & g & h \\ f'' & g'' & h'' \\ f'' & g'' & h'' \end{vmatrix}+\begin{vmatrix} f & g & h \\ g' & g' & h' \\ f''' & g''' & h''' \end{vmatrix}=0$

18. If $y=\sin mx$ then the value of $\begin{vmatrix} y & y_1 & y_2 \\ y_3 & y_4 & y_5 \\ y_6 & y_7 & y_8 \end{vmatrix}$

(where subscripts of y shows the order of derivative) is

(A) independent of x but dependent on m (B) dependent of x but independent of m

(C) dependent on both m & x (D) independent of m & x

Ans. **(D)**

Sol. $\begin{vmatrix} \sin mx & m\cos mx & -m^2\sin mx \\ -m^3\cos mx & m^4\sin mx & m^5\cos mx \\ -m^6\sin mx & -m^7\cos mx & m^8\sin mx \end{vmatrix}$

by expanding $=0$

19. If $f(x)=\begin{vmatrix} \cos(x+x^2) & \sin(x+x^2) & -\cos(x+x^2) \\ \sin(x-x^2) & \cos(x-x^2) & \sin(x-x^2) \\ \sin 2x & 0 & \sin 2x^2 \end{vmatrix}$ then

(A) $f(-2)=0$ (B) $f'(-1/2)=0$

(C) $f'(-1)=-2$ (D) $f''(0)=4$

Ans. **(B, C, D)**

Sol. $f(x)=\begin{vmatrix} \cos(x+x^2) & \sin(x+x^2) & -\cos(x+x^2) \\ \sin(x-x^2) & \cos(x-x^2) & \sin(x-x^2) \\ \sin 2x & 0 & \sin 2x^2 \end{vmatrix}$

5. HIGHER ORDER DERIVATIVE

20. If $y=a\cos(\ln x)+b\sin(\ln x)$, then $x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}$

(A) 0 (B) y (C) $-y$ (D) xy

Ans. **(C)**

Sol. $y = a\cos\ell nx + b\sin\ell nx$

differentiating w.r.t. to x

$y' = -\frac{a}{x}\sin\ell nx + \frac{b}{x}\cos\ell nx$

$xy' = -a\sin\ell nx + b\cos\ell nx$

$xy'' + y' = -\frac{a\cos\ell nx}{x} - \frac{b\sin\ell nx}{x}$

$x^2y'' + xy' = -y$

21. $\frac{d^2x}{dy^2}$ equals :

(A) $\left(\frac{d^2y}{dx}\right)^{-1}$

(B) $-\left(\frac{d^2y}{dx^2}\right)^{-1}\left(\frac{dy}{dx}\right)^{-3}$

(C) $\left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right)^{-2}$

(D) $-\left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right)^{-3}$

Ans. **(D)**

Sol. Here, $\frac{dy}{dx} = \left(\frac{dy}{dx}\right)^{-1}$

Differentiating both sides w.r.t. y, we get

$\frac{d^2x}{dy^2} = \left(\frac{dy}{dx}\right)^{-2} \cdot \frac{d}{dy} \cdot \left(\frac{dy}{dx}\right)$

$= -\left(\frac{dy}{dx}\right)^{-2} \cdot \frac{d}{dy}\left(\frac{dy}{dx}\right) \cdot \frac{dx}{dx}$

$= -\left(\frac{dy}{dx}\right)^{-2} \cdot \frac{d}{dx} \cdot \left(\frac{dy}{dx}\right) \cdot \frac{dx}{dy}$

$= -\left(\frac{dy}{dx}\right)^{-2} \cdot \frac{d^2y}{dx^2} \cdot \left(\frac{dy}{dx}\right)^{-1}$

$= -\left(\frac{dy}{dx}\right)^{-3} \cdot \left(\frac{d^2y}{dx^2}\right)$

6.TRIGONOMETRIC SUBSTITUTIONS

22. The derivative of $\sec^{-1}\left(\frac{1}{2x^2-1}\right)$ w.r.t. $\sqrt{1-x^2}$ at $x = \frac{1}{2}$ is

(A) 4 (B) 1/4 (C) 1 (D) 7

Ans. **(A)**

Sol. Let $u = \sec^{-1}\left(\frac{1}{2x^2-1}\right)$

$= \cos^{-1}(2x^2 - 1)$ Let $x = \cos\theta$

$= \cos^{-1}(2\cos^2\theta - 1)$

$= \cos^{-1}(\cos 2\theta)$

$u = 2\theta \qquad \Rightarrow u = 2\cos^{-1}x$

$$\Rightarrow \frac{du}{dx} = 2 \times \frac{-1}{\sqrt{1-x^2}} = \frac{-2}{\sqrt{1-x^2}}$$

Let $v = \sqrt{1-x^2}$

$$\Rightarrow \frac{dv}{dx} = \frac{1}{2\sqrt{1-x^2}} x - 2x = \frac{-x}{\sqrt{1-x^2}}$$

$$\therefore \frac{du/dx}{dv/dx} = \frac{2/\sqrt{1-x^2}}{x/\sqrt{1-x^2}} = \frac{2}{x}$$

$$\Rightarrow \frac{du}{dv} = \frac{2}{x} \Rightarrow \left(\frac{du}{dv}\right) = \frac{2}{1/2} = 2 \times 2 = 4$$

$$\therefore \left(\frac{du}{dv}\right)_{x=1/2} = 4$$

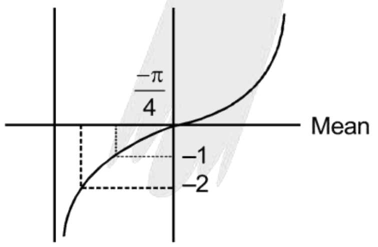
23. If $y = \sin^{-1}\frac{2x}{1+x^2}$ then $\left.\frac{dy}{dx}\right]_{x=-2}$ is

(A) $\frac{2}{5}$ (B) $\frac{2}{\sqrt{5}}$ (C) $-\frac{2}{5}$ (D) $\frac{\sqrt{5}}{2}$

Ans. **(C)**

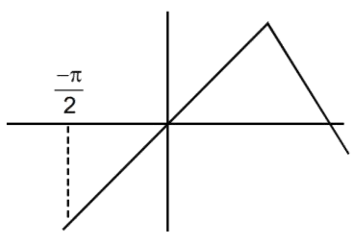
Sol. $y = \sin^{-1}\left(\frac{2x}{1+x^2}\right) \cdot \left.\frac{dy}{dx}\right|_{x=-2}$

$x = \tan\theta \;\Rightarrow\; y = \sin^{-1}(\sin 2\theta)$



$$\theta < -\frac{-\pi}{4}$$

$$2\theta < -\frac{-\pi}{2}$$



$y = \pi - 2\theta = \pi - 2\tan^{-1}x$

$\left.\frac{dy}{dx}\right|_{x=-2} = \frac{-2}{(1+x^2)} = \frac{-2}{5}$

24. $d/dx\left[\tan^{-1}\left(\frac{\sqrt{x^2+a^2}+x}{\sqrt{x^2+a^2}-x}\right)^{1/2}\right]$, is equal to -

(A) $\frac{a}{2(x^2+a^2)}$ (B) $\frac{a}{x^2+a^2}$

(C) ½ (D) None of these

Ans. **(A)**

Sol. $y = \tan^{-1}\left(\frac{\sqrt{a^2+x^2}+x}{\sqrt{a^2+x^2}-x}\right)^{1/2}$

put $x = a\tan\theta$

$$= \tan^{-1}\left(\frac{\sqrt{a^2 + a^2\tan^2\theta} + a\tan\theta}{\sqrt{a^2 + a^2\tan^2\theta} - a\tan\theta}\right)^{1/2}$$

$$= \tan^{-1}\left(\frac{\sec\theta + \tan\theta}{\sec\theta - \tan\theta}\right)^{1/2}$$

$$= \tan^{-1}\left(\frac{1 + \sin\theta}{1 - \sin\theta}\right)^{1/2}$$

$$= \tan^{-1}\left\{\frac{\cos\frac{\theta}{2} + \sin\frac{\theta}{2}}{\cos\frac{\theta}{2} - \sin\frac{\theta}{2}}\right\}^{-1/2}$$

$$= \tan^{-1}\left(\frac{1 + \tan\theta/2}{1 - \tan\theta/2}\right) = \tan^{-1}\tan(\pi/4 + \theta/2)$$

$$= \pi/4 + \theta/2$$

$$y = \frac{\pi}{4} + \frac{1}{2}\tan^{-1}\frac{x}{a}$$

$$\frac{dy}{dx} = 0 + \frac{1}{2} \times \frac{1}{1+\frac{x^2}{a^2}} \times \frac{1}{a} = \frac{fa^2}{2(x^2+a^2)}$$

25. $d/dx\left[\sin^2\cot^{-1}\frac{1}{\sqrt{\frac{1+x}{1-x}}}\right]$, is equal to -

(A) 0 (B) 1/2 (C) $-1/2$ (D) -1]

Ans. **(B)**

Sol. $y = \sin^2\cot^{-1}\frac{1}{\sqrt{\frac{1+x}{1-x}}}$

$$= \sin^2\tan^{-1}\sqrt{\frac{1+x}{1-x}} \quad \left\{\because \tan^{-1}x = \sin^{-1}\frac{x}{\sqrt{1+x}}\right\}$$

$$= \sin^2\sin^{-1}\frac{\sqrt{\frac{\sqrt{1+x}}{1-x}}}{\sqrt{1+x}} = \sin^2\sin^{-1}\frac{\sqrt{1+x}}{\sqrt{2}}$$

$$= \left(\sin^{-1}\sin^{-1}\frac{\sqrt{1+x}}{\sqrt{2}}\right)^2 = \left(\frac{\sqrt{1+x}}{\sqrt{2}}\right)^2$$

$$\frac{dy}{dx} = 0 + \frac{1}{2}$$

26. If $y = \tan^{-1}\frac{u}{\sqrt{1-u^2}}$ & $x = \sec^{-1}\frac{1}{2u^2-1}$

$u \in \left(0, \frac{1}{\sqrt{2}}\right) \cup \left(\frac{1}{\sqrt{2}}, 1\right)$ prove that $2\frac{dy}{dx} + 1 = 0$

Sol. $y = \tan^{-1}\frac{u}{\sqrt{1-u^2}}$ &$x = \sec^{-1}\frac{1}{\sqrt{2u^2-1}}$

$$u \in \left(0, \frac{1}{\sqrt{2}}\right) \cup \left(\frac{1}{\sqrt{2}}, 1\right)$$

$u = \sin\theta \qquad 0 < u < \frac{1}{\sqrt{2}}$ OR $\frac{1}{\sqrt{2}} < u < 1$

$\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$ OR $\left(0, \frac{\pi}{4}\right) \to 1^{st}\,Q^{nt}$

$$y = \tan^{-1}\left(\frac{\sin\theta}{\cos\theta}\right) = \theta$$

$$x = \sec^{-1} = \sec^{-1}\left(-\frac{1}{\cos 2\theta}\right)$$

$$\Rightarrow \sec^{-1}(-\sec 2\theta)$$

$$x = \pi - 2\theta$$

$$\frac{dy}{dq} = 1, \frac{dx}{dq} = -2$$

$$\frac{dy}{dx} = -\frac{1}{2} \Rightarrow 2\cdot\frac{dy}{dx} + 1 = 0$$

7. MIXED PROBLEMS

27. If $y = \sec(\tan^{-1}x)$, then $\frac{dy}{dx}$ at $x = 1$ is equal to: **[JEE Main 2013]**

(A) 1 (B) $\sqrt{2}$ (C) $\frac{1}{\sqrt{2}}$ (D) $\frac{1}{2}$

Ans. **(C)**

Sol. $y^{-1} = \sec(\tan^{-1}x)\tan(\tan^{-1}x) \times \frac{1}{1+x^2}$

at $x = 1$

$$y' = \sqrt{2} \times \frac{1}{2} = \frac{1}{\sqrt{2}}$$

28. For $x \in R, f(x) = |\log 2 - \sin x|$ and $g(x) = f(f(x))$, then: **[JEE MAIN 2016]**

(A) $g'(0) = \cos(\log 2)$

(B) $g'(0) = -\cos(\log 2)$

(C) g is differentiable at $x = 0$ and $g'(0) = -\sin(\log 2)$

(D) g is not differentiable at $x = 0$

Ans. **(A)**

Sol. We have, $f(x) = |\log 2 - \sin x|$
and $g(x) = f(f(x)), x \in R$
Note that, for $x \to 0, \log 2 > \sin x$

$f(x) = \log 2 - \sin x$

$\Rightarrow g(x) = \log 2 - \sin(f(x))$

$= \log 2 - \sin(\log 2 - \sin x)$

Clearly, $g(x)$ is differentiable at $x = 0$ as $\sin x$ is differentiable.

Now, $g'(x) = -\cos(\log 2 - \sin x)(-\cos x)$

$= \cos x \cdot \cos(\log 2 - \sin x)$

$\Rightarrow g'(0) = 1 \cdot \cos(\log 2)$

29. If for $x \in \left(0, \frac{1}{4}\right)$, then derivative of $\tan^{-1}\left(\frac{6x\sqrt{x}}{1-9x^3}\right)$ is $\sqrt{x} \cdot g(x)$, then $g(x)$ equals

(A) $\frac{9}{1+9x^3}$ (B) $\frac{3x\sqrt{x}}{1-9x^3}$ **[JEE MAIN 2017]**

(C) $\frac{3x}{1-9x^3}$ (D) $\frac{3x}{1+9x^3}$

Ans. **(A)**

Sol. Let $y = \tan^{-1}\left(\frac{6x\sqrt{x}}{1-9x^3}\right) = \tan^{-1}\left[\frac{2\cdot(3x^{3/2})}{1-(3x^{3/2})^2}\right]$

$$= 2\tan^{-1}\left(3x^{3/2}\right)\left[\because 2\tan^{-1}x = \tan^{-1}\frac{2x}{1-x^2}\right]$$

$$\frac{dy}{dx} = 2\cdot\frac{1}{1+\left(3x^{3/2}\right)^2}\cdot 3\times\frac{3}{2}(x)^{1/2}$$

$$= \frac{9}{1+9x^3}\cdot\sqrt{x}$$

$$\therefore\ g(x) = \frac{9}{1+9x^3}$$

30. Let $f(\theta) = \sin\left(\tan^{-1}\left(\frac{\sin\theta}{\sqrt{\cos 2\theta}}\right)\right)$ where $-\frac{\pi}{4} < \theta < \frac{\pi}{4}$, Then the value of $\frac{d}{d(\tan\theta)}(f(\theta))$ is

[JEE 2011]

Ans. 1

Sol. Given $f(\theta) = \sin\left(\tan^{-1}\left(\frac{\sin\theta}{\sqrt{\cos 2\theta}}\right)\right)$

Let $\tan^{-1}\left(\frac{\sin\theta}{\sqrt{\cos 2\theta}}\right) = y$

$$\Rightarrow \tan y = \frac{\sin\theta}{\sqrt{2\cos^2\theta - 1}}$$

Refer to the figure below,

$$\Rightarrow \sin y = \frac{\sin\theta}{\cos\theta}$$

$$f(\theta) = \sin y = \frac{\sin\theta}{\cos\theta} = \tan\theta$$

$$\Rightarrow \frac{d}{d(\tan\theta)}(f(\theta)) = 1$$

ANSWER KEY

1. DIFFERENTIATION OF IMPLICIT FUNCTION

1.	(D)	**2.**	(B)	**3.**	(C)	**4.**	(A)	**5.**	(B)	**6.**	(A)	**7.**	(A)

2. DIFFERENTIATION OF LOGARITHMIC FUNCTION

8.	(D)	**9.**	(D)	**10.**	(B)

3. DIFFERENTIATION OF INFINITE SERIES

11.	(C)	**12.**	(C)	**13.**	(B)	**14.**	100	**16.**	$\frac{1}{1+(x+n)^2}-\frac{1}{1+x^2}$

4. DIFFERENTIATION OF DETERMINANT

17.	(B)	**18.**	(D)	**19.**	(B,C,D)

5. HIGHER ORDER DERIVATIVE

20.	(C)	**21.**	(D)

6.TRIGONOMETRIC SUBSTITUTIONS

22.	(A)	**23.**	(C)	**24.**	(A)	**25.**	(B)

7. MIXED PROBLEMS

27.	(C)	**28.**	(A)	**29.**	(A)	**30.**	1