

Q Find coord. of Point
of Intersection of line
 $2x - y + 1 = 0$ & Circle
 $x^2 + y^2 = 2$?

Solving Line with Circle.

POI
Needed?
Then solve
Line with
Circle

$$x^2 + (2x+1)^2 = 2$$

$$5x^2 + 4x + 1 = 2$$

$$5x^2 + 4x - 1 = 0$$

$$5x^2 + 5x - x - 1 = 0$$

$$5x(x+1) - 1(x+1) = 0$$

$$x = -1 \quad x = \frac{1}{5}$$

$$-2 - y + 1 = 0 \quad \frac{2}{5} - y + 1 = 0$$

$$y = -1 \quad y = \frac{7}{5}$$

$$(-1, -1) \quad (\frac{1}{5}, \frac{7}{5})$$



Q Line $4x + 3y + k = 0$
is tangent to $x^2 + y^2 = 4$
find $k = ?$ $\rightarrow a = 2$

Concept \rightarrow Line $y = mx + c$ is
tangent to $x^2 + y^2 = a^2$ then
 $c = \pm a\sqrt{1+m^2}$

$$3y = -4x - k \quad | \quad m = -\frac{4}{3}$$

$$y = -\frac{4}{3}x - \frac{k}{3}$$

$$\text{then } -\frac{k}{3} = \pm 2\sqrt{1 + \frac{16}{9}}$$

$$-\frac{k}{3} = \pm 2 \times \frac{5}{3}$$

$$-k = \pm 10$$

$$k = -10, +10$$

Q Find value of c for which Line $y = 2x + c$
is tangent to $x^2 + y^2 = 5 \rightarrow a = \sqrt{5} \quad m = 2$
 $c = \pm \sqrt{5}\sqrt{1+2^2} = \pm \sqrt{5} \times \sqrt{5} \Rightarrow c = \pm 5$

Q E.O.T. to Circle $x^2 + y^2 = 4$ which is
inclined to 60° at x Axis? $\rightarrow a = 2$

$$m = \sqrt{3}$$

$$\text{E.O.T.} \Rightarrow y = mx \pm a\sqrt{1+m^2}$$

$$y = \sqrt{3}x \pm 2\sqrt{1+(\sqrt{3})^2} \Rightarrow y = \sqrt{3}x \pm 4$$

Q Find E.O.T. to Circle $x^2 + y^2 = 9$ ||^r to
Line $2x + y - 3 = 0$. $\rightarrow a = 3$
Tangent's Slope = Line's Slope
 $m = -2$

$$y = -2x \pm 3\sqrt{1+(-2)^2} \Rightarrow y = -2x \pm 3\sqrt{5}$$

$$2x + y \mp 3\sqrt{5} = 0$$

Q Find EOT to circle.

6 $4x^2 + 4y^2 - 25 = 0$ i.e. to

Line $12x - 5y + 3 = 0$

$m = \frac{12}{5}$

$x^2 + y^2 = \frac{25}{4}$ Tangent's Slope $= -\frac{5}{12} = m$
 $a = \frac{5}{2}$

$y = -\frac{5}{12}x \pm \frac{5}{2}\sqrt{1 + \left(\frac{5}{12}\right)^2}$

$y = -\frac{5}{12}x \pm \frac{5}{2} \times \frac{13}{12}$

$12y + 5x = \pm 65/2$

Q Find EOT to circle $x^2 + y^2 = 2$
 7 at (1,1)?

(1,1) lying on circle
 So using Cart. form.

$x \cdot x_1 + y \cdot y_1 = 2$

$x + y - 2 = 0$

Q EOT to circle $x^2 + y^2 = 25$ at (-3, -4)

8

$xx_1 - 3 + yy_1 - 4 = 25$

$3x + 4y + 25 = 0$

$x^2 \rightarrow xx_1, y^2 \rightarrow yy_1, 2x \rightarrow x+x_1$
 $2y \rightarrow y+y_1$

Q EOT to circle $x^2 + y^2 - 30x + 6y + 109 = 0$

9 at (4, -1)

$16 + (-1)^2 - 120 - 6 + 109 = 0$ Satisfying

Cart form $T=0$

$x \cdot 4 + y \cdot (-1) - 15(x+4) + 3(y-1) + 109 = 0$

$-11x + 2y - 60 - 3 + 109 = 0$

$-11x + 2y + 46 = 0$ EOT.

Q Find EOT to circle $x^2 + y^2 - 26x - 2y + 45 = 0$

10

at (2, 3)?

$4 + 9 - 52 - 2 + 45 = 0$ EOT

$T=0$

$2x + 3y - 13(x+2) - (y+3) + 45 = 0$

$-11x + 2y - 26 - 3 + 45 = 0$

$-11x + 2y + 16 = 0$

Q Find EOT to circle $x^2 + y^2 - 2ax = 0$

11 at $(a(1+b), a(1+b))$?

$a^2(1+b)^2 + a^2(1+b)^2 - 2a^2(1+b) = 0$

$a^2 + a^2 + 2a^2b - 2a^2 - 2a^2b = 0$

Satisfy

$T=0$

EOT $\rightarrow x \cdot a(1+b) + y \cdot a(1+b) - a(x + a(1+b)) = 0$

$\Rightarrow ax(1+b) + ay(1+b) - ax - a^2(1+b) = 0$

$\Rightarrow axb + ayb - a^2(1+b) = 0$

$\Rightarrow xby + yb - a(1+b) = 0$

Q 12 S.T. $x^2 + y^2 - 4x + 6y + 8 = 0$
 & $x^2 + y^2 - 10x - 6y + 14 = 0$
 touch at $(3, -1)$?

Concept If 2 Circle touches each other then they must have common tangent



1) $9 + 1 - 12 - 6 + 8 = 0$ ✓
 $9 + 1 - 30 + 6 + 14 = 0$ ✓

Both Circles have Same com. tangent

2) $3x - y - 2(x+3) + 3(y+1) + 8 = 0$ at $(3, -1)$
 $x + 2y - 6 - 3 + 8 = 0$
 $x + 2y - 1 = 0$ ✓
 \Rightarrow touching each other.

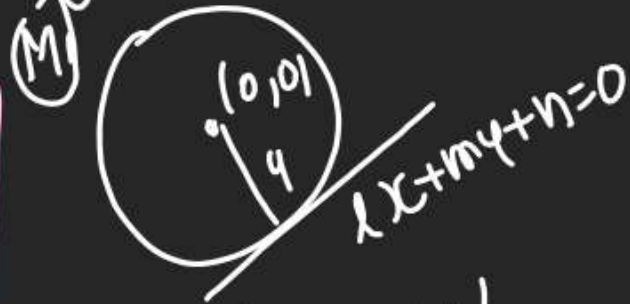
(EOT), $x + 2y - 1 = 0$ ✓
 $3x - y - 5(x+3) - 3(y+1) + 14 = 0$
 $-2x - 4y - 15 + 3 + 14 = 0$
 $x + 2y - 1 = 0$ ✓

Q P.T. tangent to Circle.

$x^2 + y^2 = 16$ at $(5, 12)$ & $(12, -5)$
 are \perp to each other.

$\perp = 0$ $5x + 12y - 16 = 0 \rightarrow m = -5/12$
 $\perp = 0$ $12x - 5y - 16 = 0 \rightarrow m' = 12/5$

Q. Find condⁿ that line $lx + my + n = 0$
 is tangent to $x^2 + y^2 = a^2$?



$p = \frac{|0 + 0 + n|}{\sqrt{l^2 + m^2}} = a$

$n^2 = a^2(l^2 + m^2)$
 is Required condⁿ

$\perp = 0$ $lx + n = -my \Rightarrow y = \frac{lx + n}{-m}$
 $x^2 + y^2 = a^2$

$x^2 + \left(\frac{lx + n}{-m}\right)^2 = a^2$

$m^2 x^2 + l^2 x^2 + 2n lx + n^2 - a^2 m^2 = 0$

$x^2(m^2 + l^2) + 2n lx + n^2 - a^2 m^2 = 0$

Touch $\rightarrow D = 0$

$4n^2 l^2 = 4(l^2 + m^2)(n^2 - a^2 m^2)$

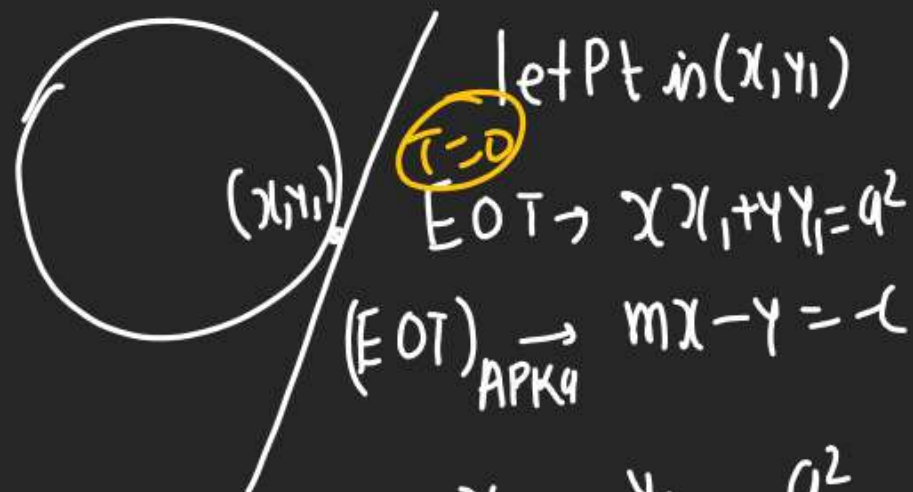
$n^2 l^2 = l^2 n^2 - a^2 m^2 l^2 + m^2 n^2 - a^2 m^4$

$= 1 \quad m^2 n^2 = a^2 m^2 (l^2 + m^2)$

$\Rightarrow \boxed{n^2 = a^2(l^2 + m^2)}$

Q Find Coord of Pt of Contact

15 if $y = mx + c$ touches $x^2 + y^2 = a^2$



Let Pt in (x_1, y_1)
 $EOT \rightarrow x x_1 + y y_1 = a^2$
 $(EOT)_{APK9} \rightarrow mx - y = -c$

$$\frac{x_1}{m} = \frac{y_1}{-1} = \frac{a^2}{-c}$$

$$x_1 = \frac{a^2 m}{-c} \quad y_1 = \frac{a^2}{c}$$

$$\therefore \text{Pt of Contact} = \left(-\frac{a^2 m}{\pm a \sqrt{1+m^2}}, \frac{a^2}{\pm a \sqrt{1+m^2}} \right)$$

$$= \left(\mp \frac{am}{\sqrt{1+m^2}}, \pm \frac{a}{\sqrt{1+m^2}} \right)$$

Q $(\because x^2 + y^2 = 16$ has tangent
 $y = 2x + 4\sqrt{5}$ find Pt of
 Contact. $m = 2 \rightarrow m_{\perp} = -\frac{1}{2}$

Let Pt. of Cont = (x_1, y_1)

$$EOT \rightarrow x x_1 + y y_1 = 16$$

$$2x - y = -4\sqrt{5}$$

$$\frac{x_1}{2} = \frac{y_1}{-1} = \frac{16}{-4\sqrt{5}}$$

$$x_1 = -\frac{8}{\sqrt{5}}, y_1 = +\frac{4}{\sqrt{5}}$$

$$\left(-\frac{8}{\sqrt{5}}, \frac{4}{\sqrt{5}} \right)$$



$$y - 0 = -\frac{1}{2}(x - 0)$$

$$2y + x = 0 \quad \times 2$$

$$4y + 2x = 0$$

$$-4y + 2x = -4\sqrt{5}$$

$$5y = +4\sqrt{5} \Rightarrow y = +\frac{4}{\sqrt{5}}$$

Q $(\because x^2 + y^2 - 2x - 4y + 1 = 0$ & External
 Pt (7,4)

Q Find EOT from (7,4)



Whenever pt is from
 outside of circle we use
 $y = mx \pm a\sqrt{1+m^2}$ (Slope form)

$$r = \sqrt{1^2 + 2^2} = 2$$

Q Changing circle in central form.

$$(x-1)^2 + (y-2)^2 = 2^2$$

EOT.
 (2) $y - 2 = m(x - 1) \pm 2\sqrt{1+m^2}$ in P.T. (7,4)

$$12 = m \times 6 \pm 2\sqrt{1+m^2} \Rightarrow 1 - 3m = \pm \sqrt{1+m^2}$$

$$\Rightarrow 9m^2 + 1 - 6m = 1 + m^2 \Rightarrow 8m^2 - 6m = 0$$

$$m = 0 \quad m = 3/4$$

(3) EOT $(y - 4) = 0(x - 7) \rightarrow y = 4$
 $(y - 4) = \frac{3}{4}(x - 1) \Rightarrow 4y - 16 = 3x - 3 \Rightarrow 3x - 4y = 11$

Q₁₈ If tangent is drawn from external
 Pt (7, 4) to circle $x^2 + y^2 - 2x - 4y + 1 = 0$
 find Pt of contact?

① From Prev. Qs - EOT at (7, 4) are

$$y = 4, \quad 3x - 4y = 5 \quad m = \frac{3}{4}$$

②



EON

$$(y - 2) = -\frac{1}{3}(x - 1)$$

$$3y - 6 = -x + 1$$

$$4x + 3y = 10 \quad \times 4$$

$$9x - 12y = 15$$

$$16x + 12y = 40$$

$$25x = 55 \Rightarrow x = \frac{11}{5}$$

$$\frac{33}{5} - 4y = 5$$

$$4y = \frac{8}{5}$$

$$\left(\frac{11}{5}, \frac{2}{5}\right)$$

$y = 4$ is \perp

Line P.T. (1, 2) is

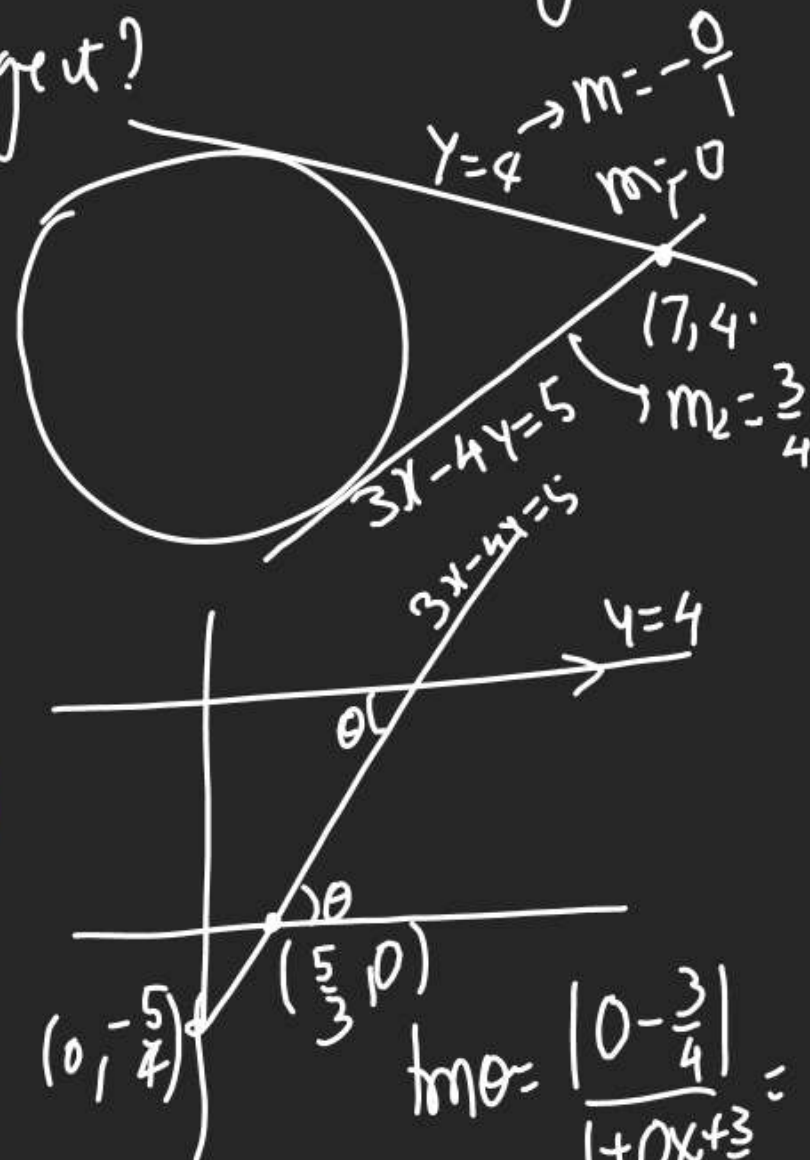
$$x = 1$$

\therefore Another Pt of contact is (1, 4)

Q If tangents are drawn from

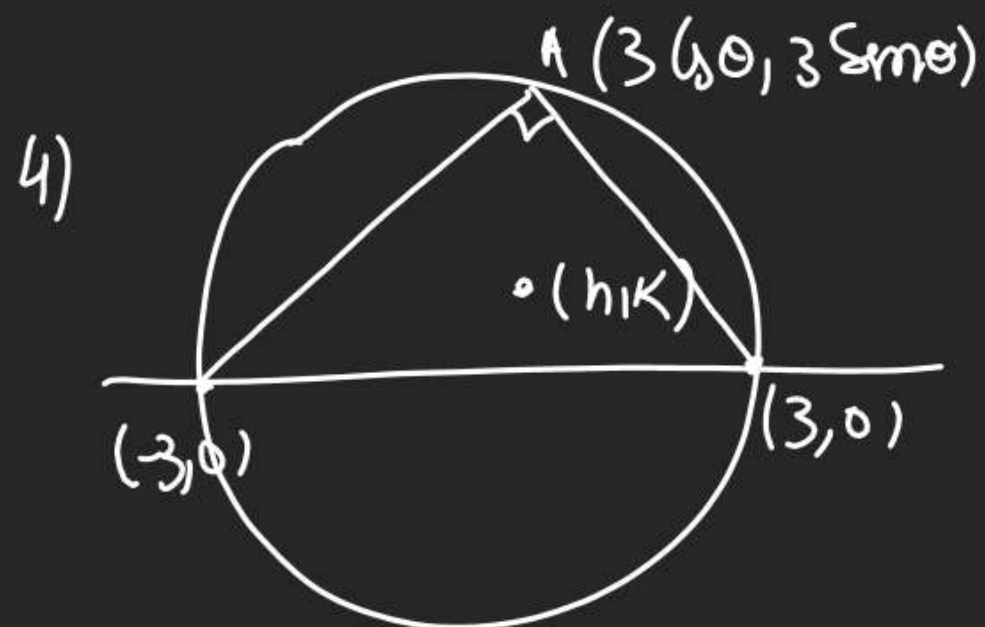
External Pt (7, 4) to circle

$x^2 + y^2 - 2x - 4y + 1 = 0$ find Angle
 betⁿ tangents?



$$\tan \theta = \frac{\left|0 - \frac{3}{4}\right|}{1 + 0 \times \frac{3}{4}} = \frac{3}{4}$$

$$\theta = \tan^{-1} \left| \frac{3}{4} \right|$$



$$(x+3)(x-3) + (y-0)(y-0) = 0$$

$$x^2 + y^2 = 9$$

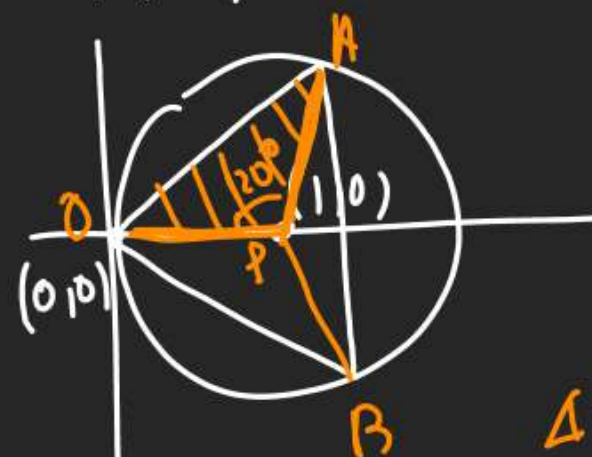
$$h = \frac{3(-3) + 3 \cos \theta}{3} \quad k = \frac{0 + 0 + 3 \sin \theta}{3}$$

$$h^2 + k^2 = 1$$

$$x^2 + y^2 = 1$$

(5) $x^2 + y^2 - 2x = 0$

$$(1, 0), R = \sqrt{1^2 + 0^2} = 1$$



$$\Delta = \frac{1}{2} ab \sin \theta$$

$$OAB = 3 \Delta OAP$$

$$= 3 \times \frac{1}{2} \times 1 \times 1 \cdot \sin 2\theta$$

$$= \frac{3}{2} \times \frac{\sqrt{3}}{2} = \frac{3\sqrt{3}}{4}$$