Nishant Jindal

$$\frac{x^{\frac{1}{13}}-1}{x-1}$$

$$8\times\frac{1}{64}\times\frac{1}{4} = 8x\left(\frac{1-\cos\frac{x^2}{2}}{4x\left(\frac{x^2}{2}\right)^2}\right)\left(\frac{1-\cos\frac{x^2}{4}}{16x\left(\frac{x^2}{4}\right)^2}\right)$$

5. (a) $\lim_{x\to 0} \tan^{-1}\frac{a}{x^2} = \begin{cases} \frac{\pi}{2} & a>0 \\ -\frac{\pi}{2} & a<0 \\ 0 & a=0 \end{cases}$

$\lim_{x\to 0} \tan^{-1}\frac{0}{x^2}$

8. $\left(\frac{\cos 2x-\cos 4x}{\cos x-\cos 3x}\right) \frac{\cos x \cos 3x}{\cos 2x \cos 4x} = \frac{3}{2}$

$$\frac{3\,\frac{\sin x}{x}\,\frac{\sin 3x}{3x}}{2\,\frac{\sin x}{x}\,\frac{\sin 2x}{2x}}$$

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$$\sqrt{2}\left(\frac{1-\cos\left(\theta-\frac{\pi}{4}\right)}{16\left(\theta-\frac{\pi}{4}\right)^2}\right)$$

$$-3x\sin\frac{1}{x}$$

$$= \frac{\sqrt{2}}{32}$$

$$\lim_{x\to-\infty}\frac{2\cdot\cancel{|x|^3}\left[\left(3|x|+\frac{2}{|x|}\right)\sin\frac{1}{x}+1+\frac{5}{|x|^3}\right]}{\cancel{|x|^3}\left(1+\frac{1}{|x|}+\frac{1}{|x|^2}+\frac{1}{|x|^3}\right)}$$

$$= -2$$

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1.
$$\sum_{r=2}^{n}\left((r+1)\sin\frac{\pi}{r+1} - r\sin\frac{\pi}{r}\right) = \lim_{n\to\infty} \pi\left(\frac{(n+1)}{\pi}\sin\frac{\pi}{n+1}\right) - 2\sin\frac{\pi}{2}$$

$\downarrow$ $r=n$ $\qquad$ $\downarrow$ $r=2$

$$\{\pi - 2\} = \pi - 2 - 1$$

$$= \pi - 3 .$$

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7. $\lim_{n\to\infty}\left(\cos\left(\ln\left(\frac{n-1}{n+1}\right)\right)\right)^{(n+1)^2}$ ✓

$\frac{1}{2}(n^2-2n+1)$

$$= e^{\lim_{n\to\infty}\frac{\cos\left(\ln\left(\frac{n-1}{n+1}\right)\right)-1}{\ln^2\left(\frac{n-1}{n+1}\right)}\left(\frac{\ln\left(1+\frac{-2}{n+1}\right)}{\left(\frac{-2}{n+1}\right)}\right)^2\times 4}$$

$$\frac{\frac{1-x+\ln(1+(x-1))}{(x-1)^2}}{\pi^2\frac{1-\cos(\pi-\pi x)}{(\pi(1-x))^2}} = -\frac{1}{\pi^2}$$

$$e^{-\frac{1}{2}\times 4} = e^{-2}$$

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$$\lim_{x\to 1}(ax^2+bx+c) = \lim_{x\to 1}\left(\frac{ax^2+bx+c}{(x-1)^2}(x-1)^2\right) = 0$$

$$\boxed{a+b+c=0}$$

$$\boxed{2a+b=0}$$

$$\lim_{x\to 1}\frac{ax^2+bx-a-b}{(x-1)^2}$$

$$2 = \lim_{x\to 1}\frac{(a(x+1)+b)}{x-1}$$

$$\lim_{x\to 1}(a(x+1)+b) = \lim_{x\to 1}\left(\frac{a(x+1)+b}{(x-1)}(x-1)\right) = 0$$

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$$\lim_{y\to 0}\lim_{x\to\infty}\frac{e^{ay\frac{\ln\left(1+\frac{ay}{x}\right)}{\left(\frac{ay}{x}\right)}}-e^{by\frac{\ln\left(1+\frac{by}{x}\right)}{\frac{by}{x}}}}{y}$$

$$=\lim_{y\to 0}\frac{e^{ay}-e^{by}}{y}$$

$$\lim_{y\to 0}\left(\lim_{x\to\infty}\right.$$

$$\lim_{y\to 0}(a-b)e^{by}\frac{\left(e^{(a-b)y}-1\right)}{(a-b)y}$$

$$=a-b$$

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1.
$$\lim_{x\to 0}\left(\frac{(1+x)^{\frac{1}{x}}-e}{x}\right)=\lim_{x\to 0}\frac{e^{\frac{1}{x}\ln(1+x)}-e}{x}$$

$$e\lim_{x\to 0}\frac{e^{\frac{1}{x}\ln(1+x)-1}-1}{\frac{\ln(1+x)-x}{x}}\quad\frac{\ln(1+x)-x}{x^2}$$

$$=-\frac{e}{2}$$

$$\cancel{e^{-\frac{e}{x}}}$$

$$\lim_{x\to 0^-}\left(\frac{\cot^{-1}\frac{1}{x}}{x}\right)\to-\infty.$$

$$\lim_{x\to 0^+}\frac{\cot^{-1}\frac{1}{x}}{x}=\lim_{x\to 0^+}\frac{\tan^{-1}x}{x}$$

$$=1$$

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Q. $\lim_{x\to 0}\left(\dfrac{\sin x - x^2 - \{x\}\{-x\}}{x\cos x - x^2 - \{x\}\{-x\}}\right)$ $\qquad \{\cdot\} = FPF$

$= \dfrac{1}{3}$

$\{-h\} = -h - [-h] = -h+1$

$\{h\} = h - [h] = h$

$LHL = \lim_{h\to 0}\dfrac{-\sin h - h^2 - \{-h\}\{h\}}{-h\cos h - h^2 - \{-h\}\{h\}} = \lim_{h\to 0}\dfrac{-\sin h + h^2 - (1-h)h}{-h\cos h - h^2 - (1-h)h}$

$= \lim_{h\to 0}\dfrac{-\sin h - h}{-h\cos h - h} = \lim_{h\to 0}\dfrac{\frac{\sin h}{h}+1}{\cos h + 1} = 1$

$RHL = \lim_{h\to 0}\left(\dfrac{\sin h - h^2 - h(1-h)}{h\cos h - h^2 - h(1-h)}\right) = \lim_{h\to 0}\dfrac{\sin h - h}{h(\cos h - 1)} = \lim_{h\to 0}\dfrac{\left(\frac{\sin h - h}{h^3}\right)}{\left(\frac{\cos h - 1}{h^2}\right)}$

Nishant Jindal

3. If $\lim_{x\to 0}\left(\frac{A\cos x + Bx\sin x - 5}{x^4}\right)$ exists and finite, find A, B and the limit.

$\lim_{x\to 0}(A\cos x + Bx\sin x - 5) = 0$

$= \frac{A\cos x + Bx\sin x - 5}{x^4} \, x^4$

$\boxed{A - 5 = 0}$

$\lim_{x\to 0} \frac{\frac{5(\cos x - 1)}{x^2} + \frac{Bx\sin x}{x^2}}{x^2}$

$\boxed{B - \frac{5}{2} = 0}$

$= \lim_{x\to 0}\left(\frac{5(\cos x - 1) + Bx\sin x}{x^4}\right) = \left(-\frac{5}{24}\right) = 5\left(-\frac{1}{3}\right)\frac{1}{8}$

$\frac{5(\cos x - 1) + \frac{5}{2}x\sin x}{x^4}$

$= \frac{-10\sin^2\frac{x}{2} + 10\,\frac{x}{2}\sin\frac{x}{2}\cos\frac{x}{2}}{x^4}$

$= \frac{10\sin\frac{x}{2}\cos\frac{x}{2}\left(\frac{x}{2} - \tan\frac{x}{2}\right)}{x^4} = \frac{\left(\frac{x}{2}\right)^3}{8}$

Nishant Jindal

4. If $\lim_{x\to 0}\left(\dfrac{4+\sin 2x + A\sin x + B\cos x}{x^2}\right)$ exists & finite, find A, B and the limit.

$\boxed{4+B=0}$

$$\lim_{x\to 0}\frac{4(1-\cos x)+\sin x\,(2\cos x + A)}{x^2} = \lim_{x\to 0}\frac{\frac{4(1-\cos x)}{x^2}x + \frac{\sin x}{x}(2\cos x + A)}{x}$$

$\boxed{2+A=0}$

$$\lim_{x\to 0}\frac{4+\underline{\sin 2x}-2\sin x-4\cos x}{x^2} = \lim_{x\to 0}\frac{(1-\cos x)(4-2\sin x)}{x^2}$$

$$2=\frac{1}{2}\times 4$$

Nishant Jindal

$$\lim_{x\to 0} \frac{A\cos x + Bx\sin x - 5}{x^4}$$

$$= \lim_{x\to 0} \frac{A\left(1-\frac{x^2}{2!}+\frac{x^4}{4!}-\frac{x^6}{6!}+\cdots\right)+Bx\left(\frac{x}{1!}-\frac{x^3}{3!}+\frac{x^5}{5!}-\frac{x^7}{7!}+\cdots\right)-5}{x^4}$$

$A-5=0$, $-\frac{A}{2}+B=0$, $\boxed{l=\frac{A}{24}-\frac{B}{6}}$

$$\lim_{x\to 0} \frac{a_3x^3+a_4x^4+\cdots}{x^4}$$

$$\frac{a_3+a_4x+\cdots}{x}$$

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5: If $f(x) = \lim_{n\to\infty} \dfrac{\tan(\pi x^2) + (x+1)^n \sin x}{x^2 + (x+1)^n}$, find $\lim_{x\to 0} f(x)$

$\boxed{\begin{array}{l} \text{Ex-I (Complete)} \\ \text{Ex-II (1-5)} \end{array}}$

$$\lim_{x\to 1} \lim_{n\to\infty} \left(\frac{\tan(\pi x^2) + (x+1)^n \sin x}{x^2 + (x+1)^n} \right)$$

$a>0$

$a>0$

$$\lim_{n\to\infty} a^n = \begin{cases} 0 & 0<a<1 \\ \infty & a>1 \\ 1 & a=1 \end{cases}$$

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$$\lim_{x \to \frac{\pi}{4}^+} \left(\tan\left(\frac{\pi}{8} + x \right) \right)^{\tan 2x} = 0$$

$$\left(\to \sqrt{2}+1 \right)^{-\infty} = 0$$

$\sqrt{2}+1$

$\frac{3\pi}{8}$