Introduction

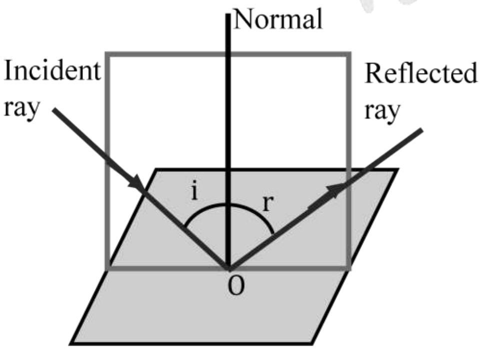
- The human eye's retina has been naturally equipped with the capability to perceive electromagnetic waves only in a limited portion of the electromagnetic spectrum.
- Light is the term used to describe electromagnetic radiation with wavelengths ranging from 400 nm to 750 nm, which primarily enables the sense of vision.
- Light travels in a straight line at an incredibly fast speed, with its velocity in a vacuum being the maximum achievable speed in the natural world.The speed of light in vacuum is

  $$c = 2.99792458 \times 10^{8}\,\mathrm{ms}^{-1}.$$

  $$\approx 3 \times 10^{8}\,\mathrm{ms}^{-1}$$

- The wavelength of light is very small compared to the size of ordinary objects that we encounter commonly (generally of the order of a few cm or larger). A light wave can be considered to travel from one point to another, along a straight line joining them. The path is called a ray of light, and a bundle of such rays constitutes a beam of light. The phenomena of reflection, refraction and dispersion of light are explained using the ray picture of light. We shall study image formation by plane and spherical reflection and refracting surfaces, using the basic laws of reflection and refraction. The construction and working of some important optical instruments, including the human eye are also explained.

  The size of ordinary objects commonly encountered is generally several centimeters or larger, making the wavelength of light appear minuscule in comparison. Light waves travel in a straight line between two points, which is known as a ray of light, and a group of rays form a beam of light. The behavior of light, such as reflection, refraction, and dispersion, can be explained using this ray model of light. The fundamental laws of reflection and refraction will be utilized to study the creation of images by planar and spherical reflective and refractive surfaces. Additionally, this will cover the construction and operation of important optical instruments, including the human eye.
- **Relection of Light :** When a light ray strikes the boundary of two media such as air and glass, a part of light is turned back into the same medium. This is called reflection of light.



In case of reflection at the point of incidence 'O', the angle between incident ray and normal to the reflecting surface is called the angle of incidence (i). The angle between reflected ray and normal to the reflecting surface is called angle of reflection (r).

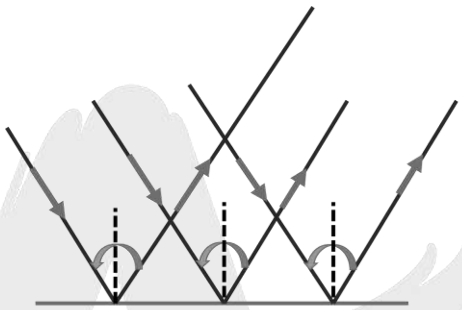
The plane containing incident ray and normal is called plane of incidence.

- **Laws of reflection :** The incident ray, the reflected ray and the normal to the reflecting surface at the point of incidence, all lie in the same plane.
- The angle of incidence is equal to the angle of reflection $\angle i = \angle r$

Types of reflections

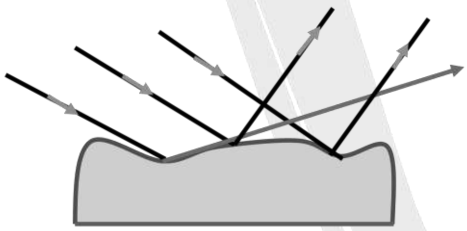
- **Regular reflection :** When the reflection takes place from a perfect smooth plane surface, then the reflection is called regular reflection (or) specular reflection.

  In this case, a parallel beam of light incident will remain parallel even after reflection as shown in the figure.



  In case of regular reflection, the reflected light ray has large intensity in one direction and negligibly small intensity in other direction. Regular reflection of light is useful in determining the property of mirror.

- **Diffused reflection :** If the reflecting surface is rough (or uneven), parallel beam of light is reflected in randome direction. This kind of reflection is called diffused reflection.

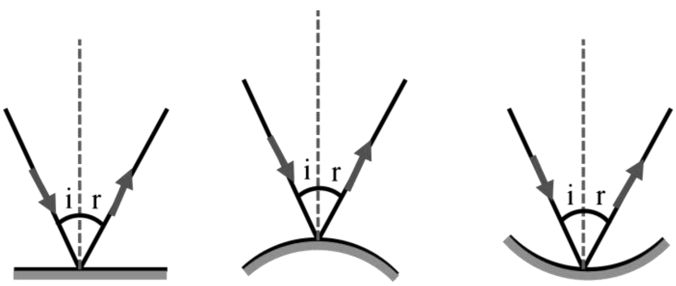


  As shown in the above figure if the reflecting surface is rough, the normal at different points will be in different directions, so the ray that are parallel before reflection will be reflected in random directions.

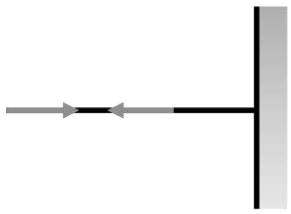
  We see non-luminous objects by diffused reflection.

Important points regarding reflection

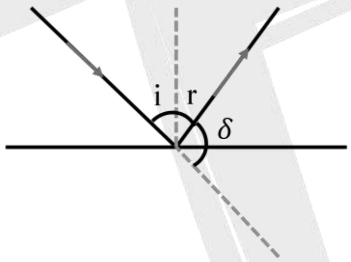
- Laws of reflection are valid for all reflecting surfaces either plane or curved.



- If a light ray is incident normally on a reflecting surface, after reflection it retraces its path i.e., if $\angle i = 0$ then $\angle r = 0$



- In case of reflection of light frequency, wavelength and speed does not change. But the intensity of light on reflection will decreases.
- If the reflection of light takes place from a denser medium, there is a phase change of π rad.
- If $\hat{I}$, $\hat{N}$ and $\hat{R}$ are vectors of any magnitude along incident ray, the normal and the reflected ray respectively then $\hat{R}\cdot\left(\hat{I}\times\hat{N}\right)=\hat{N}\cdot\left(\hat{I}\times\hat{R}\right)=\hat{I}\cdot\left(\hat{N}\times\hat{R}\right)=0$
- This is because incident ray, reflected ray and the normal at the point of incidence lie in the same plane.
- **Deviation of a ray due to reflection :** The angle between the direction of incident ray and reflected light ray is called the angle of deviation (δ).

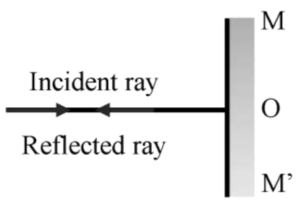


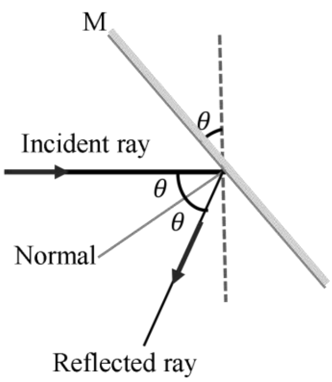
From the above figure $\delta = -(i + r)$

But i = r

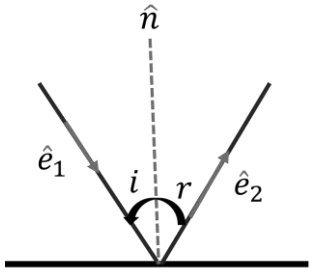
Hence angle of deviation in the case of reflection is $\delta = \pi - 2i$

By keeping the incident ray fixed, the mirror is rotated by an angle $'\theta'$, about an axis in the plane of mirror, the reflection ray in rotated through an angle $'2\theta'$.





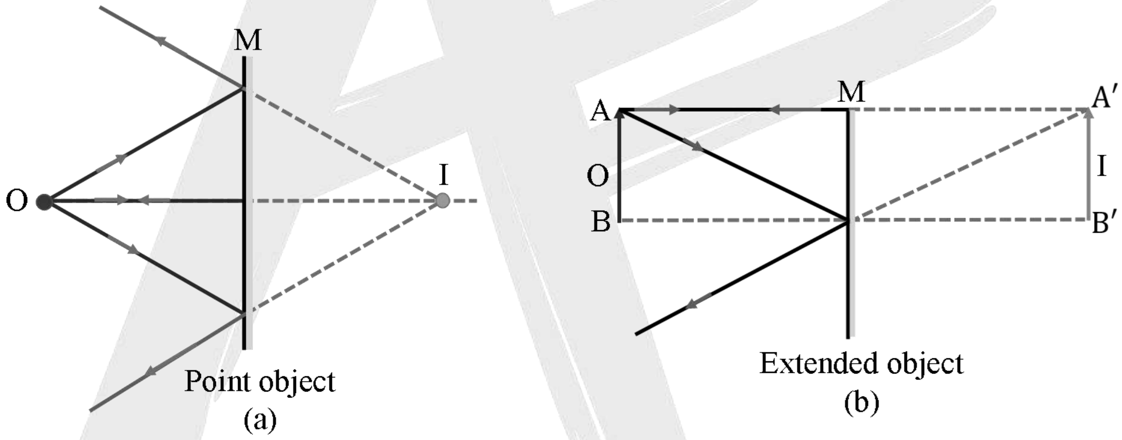
➢ **Vector from of law of reflection :**



If $\hat{e}_1$ is unit vector along the incident ray $\hat{e}_2$ is the unit vector along the reflected ray $\hat{n}$ is the unit vector along the normal then, $\hat{e}_2 = \hat{e}_1 - 2(\hat{e}_1 \cdot \hat{n})\hat{n}$

Reflection from Plane Surface

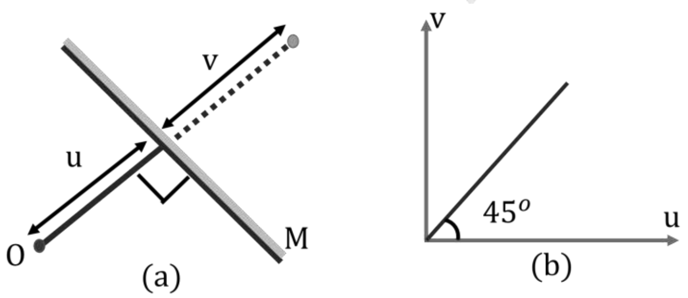
- When you look into a plane mirror, you see an image of yourself that has three properties.
- The image is up right.
- The image is the same size as you are
- The image is located as far behind the mirror as you are infront of it. This is shown in the figure (b).



Point object
(a)

Extended object
(b)

- A plane mirror always form virtual image to a real object and vice versa and the line joining object and image is perpendicular plane mirror as shown in figure (a).

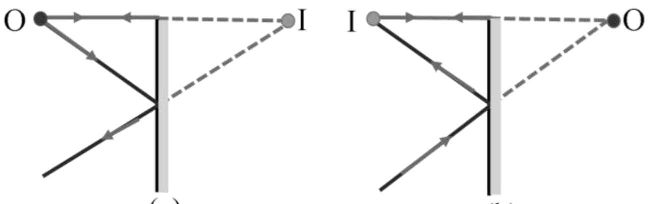


(a) (b)

The graph between image distance (v) and object distance (u) for a plane mirror is a straight line as shown in figure (b).

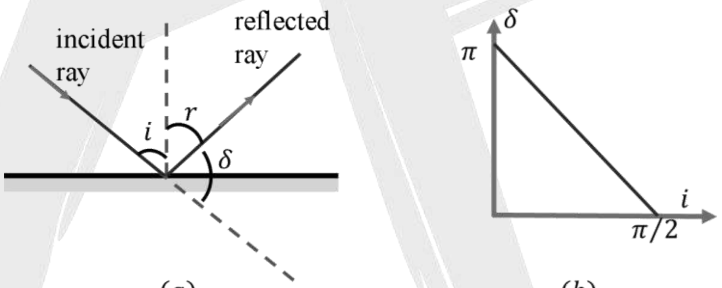
The ratio of image height to the object height is called lateral magnification (m). Thus in case of plane mirror 'm' is equal to one.

- The principle of reversibility states that rays retrace their path when their direction is reversed. In accordance with the principle of reversibility object and image positions are interchangable. The points corresponding to object and image are called conjugate points.
  This is illustrated in figure.



(a) (b)

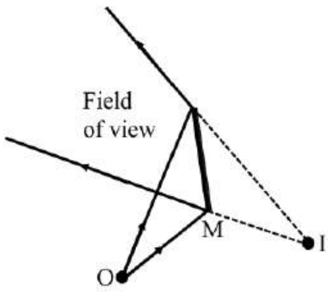
- Regardless of its size, a mirror produces a full image of the object positioned in front of it. A larger mirror provides a brighter image compared to a smaller one. However, it is observed that the reflector's size must be significantly larger than the wavelength of the incident light; otherwise, the light will scatter in multiple directions.
- The angle between directions of incident ray and reflected or refracted ray is called deviation (δ). A plane mirror deviates the incident light through angle $\delta = 180 - 2i$ where 'i' is the angle of incidence. The deviation is maximum for normal incidence, hence $\delta_{max} = 180°$.



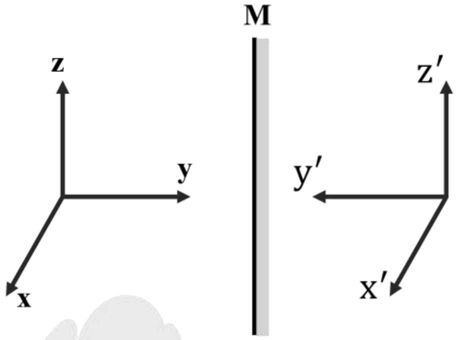
(a) (b)

It is noted that, generally anti-clock wise deviation is taken as positive and clock wise deviation as negative.

- For a given mirror, each object possesses a distinct field of view. The field of view is defined as the area between the outermost reflected rays and is determined by the object's placement in front of the mirror. Only when our eyes are within this field of view can we see the object's image; otherwise, we cannot. This concept is depicted in the accompanying figure.



- Instead of a left-right reversal, a plane mirror produces a front-back reversal. It is important to note that the mirror creates this reversal effect in a direction perpendicular to its plane. As shown in figure (a), the right-handed coordinate system is transformed into a left-handed coordinate system.



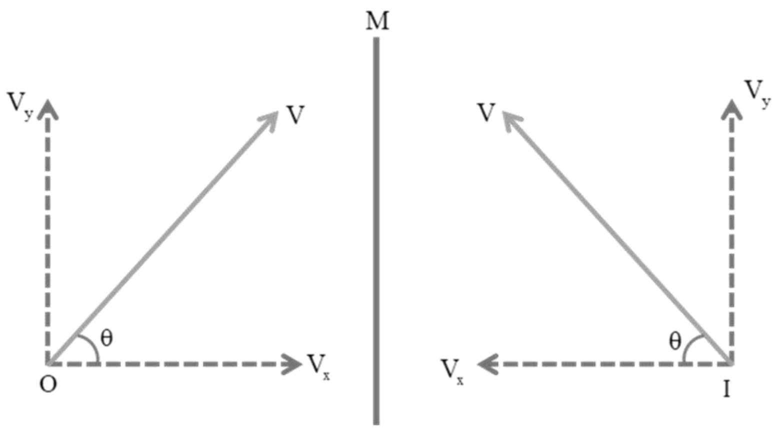
(a)

i.e., the image formed by a plane mirror left is turned into right and vice versa with respect to object as shown in figure (b).

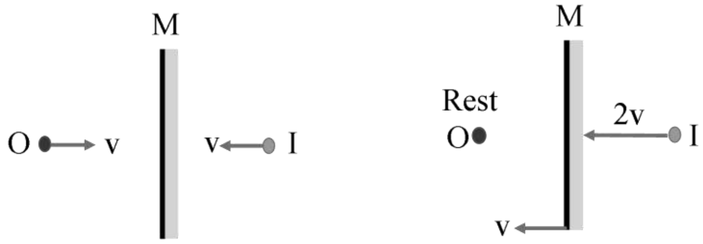
(b)

- As an object moves in front of a stationary mirror, the relative speed of the object and its image is zero along the mirror's plane and twice the speed of the object perpendicular to the mirror's plane.

$\left(V_{IO}\right)_y = 0$ and $\left(V_{IO}\right)_x = 2v_x$



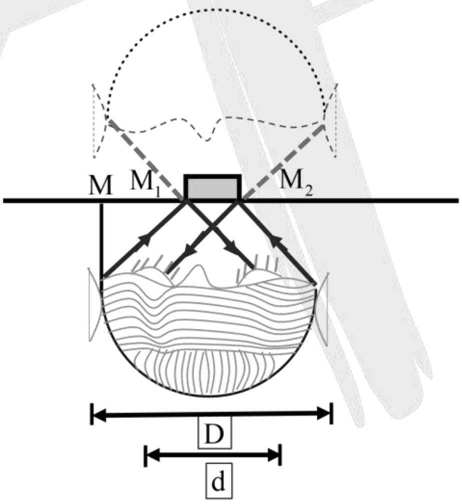
- When an object moves towards or away from a plane mirror at a velocity of v, the image also moves towards or away from the object at the same speed of v. The relative velocity of the image with respect to the object will be 2v, as illustrated in figure (a). In contrast, if the mirror moves towards or away from a stationary object at a velocity of v, the image also moves towards or away from the object at a speed of 2v, as demonstrated in figure (b).



- A person of height 'h' can see his full image in a mirror of minimum length $\ell = \frac{h}{2}$

  If a person stands at the center of a room and looks towards a plane mirror hanging on a wall, they can see the entire height of the wall behind them if the length of the mirror equals one-third of the height of the wall.

- The minimum width of a plane mirror required for a person to see the complete width of his face is $\frac{(D-d)}{2}$, where, D is the width of his face and d is the distance between his two eyes.



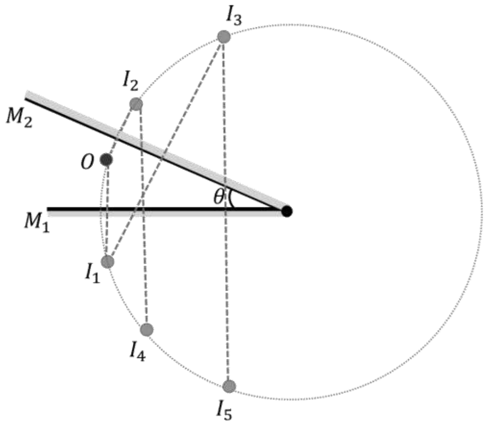
$$MM_1 = \frac{1}{2}\left[D - \frac{1}{2}(D-d)\right] \Rightarrow MM_1 = \frac{(D+d)}{4} \quad \text{.... (i)}$$

and $MM_2 = D - \frac{(D+d)}{4} \Rightarrow MM_2 = \frac{(3D-d)}{4}$ .... (ii)

$\therefore$ Width of the mirror = $M_1M_2 = MM_2 - MM_1 = \frac{2D-2d}{4}$ {From (i) and (ii)}

$$= \frac{2(D-d)}{4} = \frac{D-d}{2}$$

The number of images formed of a point object when two plane mirrors are inclined to each other at an angle is determined as follows:

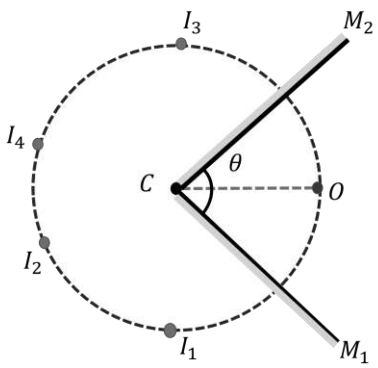


- If $\frac{360}{\theta}$ is even number (say m) Number of images formed $n=(m-1)$, for all positions of objects in between the mirrors.
- If $\frac{360}{\theta}$ is odd integer (say m) number of images formed n = m, if the object is not on the bisector of mirrors. $n=(m-1)$, if the object is one the bisector of mirrors.
- If $\frac{360}{\theta}$ is a fraction (say m). The number of images formed will be equal to its integer part i.e., $n=[m]$.

Ex : If m = 4.3, the total number of images $n=[4.3]=4$

$m=\frac{360}{\theta}$	Position of the object	Number of images (n)
Even	Anywhere	$m-1$
Odd	Symmetric	$m-1$
	Asymmetric	M
Fraction	Anywhere	[m]

- All the images lie on a circle whose radius is equal to the distance between the object 'O' and the point of intersection of mirrors C. If θ is less more number of images on circle with large radius.

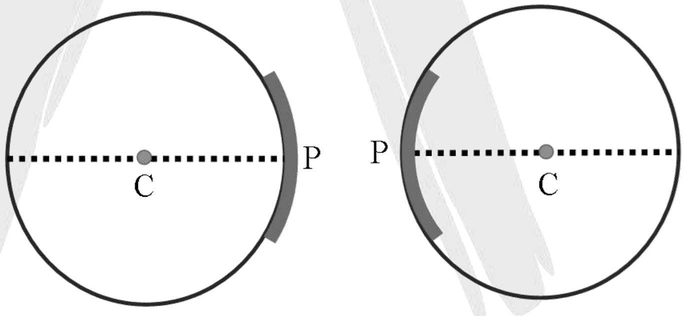


- If the objects is placed in between two parallel mirrors $\theta = 0°$, the number of images formed is infinite but of decreasing intensity in according with $I \alpha r^{-2}$.
- If 'θ' is given n is unique but if 'n' is given θ is not unique. Since same number of images can be formed for different θ.
- The number of images that an observer sees may differ from the number of images formed, and this depends on the relative positions of the object, mirror, and observer. When a light ray vector strikes a mirror, only the component vector that is parallel to the normal of the mirror changes its sign without any change in its magnitude upon reflection. It is important to note that a mirror can reflect the entire energy incident on it, resulting in the magnitude of the reflected vector being the same as that of the incident vector. The incident vector corresponds to an object, and the reflected vector corresponds to an image, which could be based on its position, velocity, or acceleration.

Example: If a plane mirror lies on x-z plane, a light vector $2\hat{i} + 3\hat{j} - 4\hat{k}$ on reflection becomes $2\hat{i} - 3\hat{j} - 4\hat{k}$.

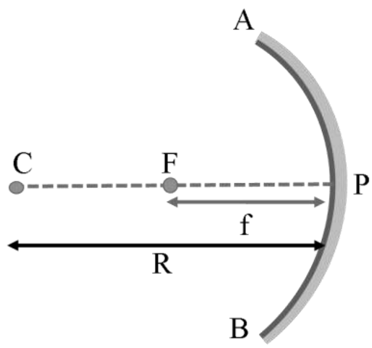
Reflection from Curved Surface

- A curved mirror is a smooth reflecting part (in any shape) of a symmetrical curved surface such as spherical, cylindrical or ellipsoidal. In this chapter we consider a piece of spherical surface only.

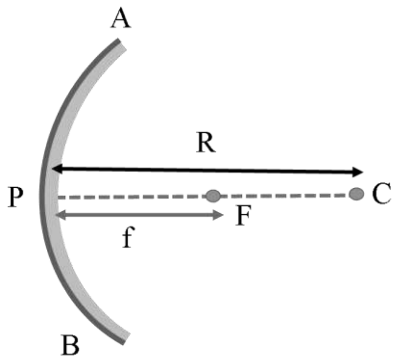


Concave mirror (a) Convex mirror (b)

The mirror is called concave if the reflection takes place from the inner surface and convex if it takes place from the outer surface, as shown in the figure. For a thin spherical mirror, the centre 'C' of the sphere of which the mirror part is a section, is called the centre of curvature of the mirror. P is the centre of the mirror surface and is called the pole. The line CP produced is the principal axis, AB represents the aperture or the effective diameter of the light-reflecting area of the mirror. The distance CP represents the radius of curvature (R). The point F is the focus and the distance between F and P is called the focal length (f), which is related to R as $f = R/2$.



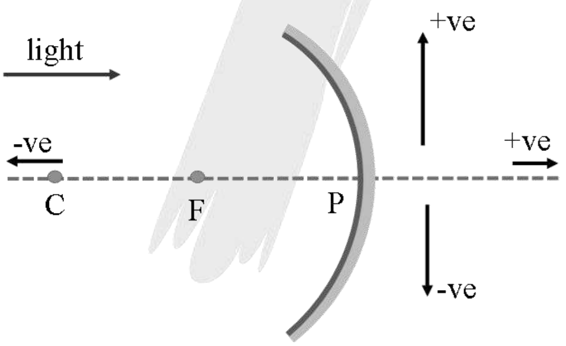
Convex mirror
(a)



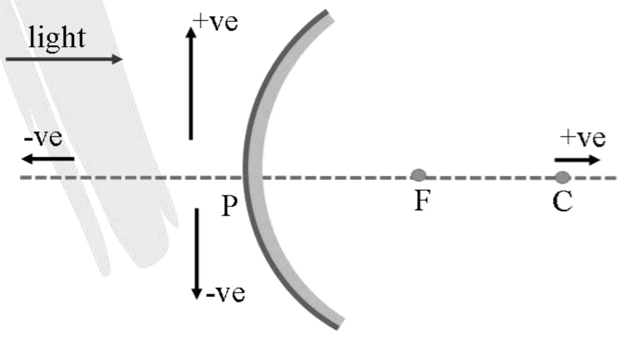
Convex mirror
(b)

Sign Convention :

In order to derive the appropriate formulas for reflection and refraction by spherical mirrors and surfaces, a sign convention must be established for measuring distances. The Cartesian sign convention will be used in this book, which states that all distances are measured from the pole of the mirror. Distances measured in the same direction as the incident light are considered positive, while those measured in the opposite direction are negative. Heights measured on one side of the principal axis of the mirror are taken as positive, while heights measured on the other side are negative. Acute angles measured from the normal (principal axis) in the counterclockwise direction are positive, while those measured in the clockwise direction are negative.



Concave mirror
(a)

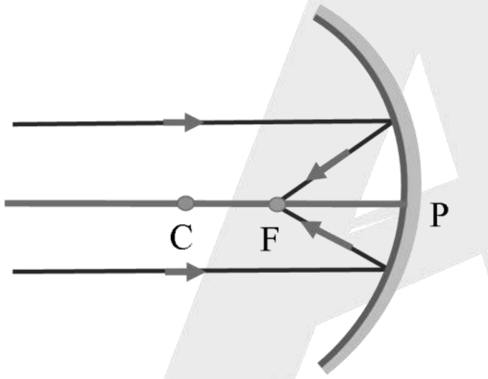


Convex mirror
(b)

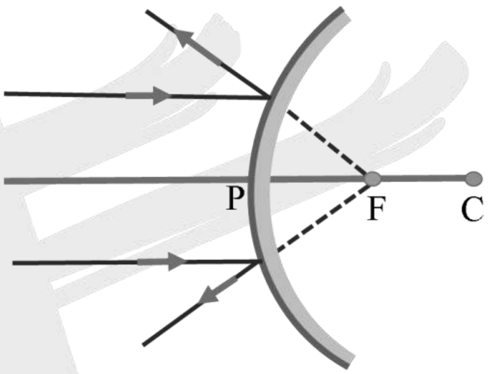
- **Paraxial Approximation :** Rays which are close to the principal axis or make small angle $\left(\theta < 10^\circ\right)$ with it i.e. they are nearly parallel to the axis, are called paraxial rays. Accordingly we set $\cos\theta \approx 1$, $\sin\theta \approx \theta$ and $\tan\theta \approx \theta$. This is known as paraxial approximation or first order theory or "Gaussian" optics. In spherical mirrors we restrict to mirror with small aperture and to paraxial rays.

Focal Length of Spherical Mirrors

- We make the assumption that light rays are paraxial, meaning they make small angles with the principal axis. When a beam of parallel paraxial rays is reflected from a concave mirror, they converge at a point F on the principal axis. This point is called the principal focus of the mirror and is a real focus. On the other hand, when a narrow beam of paraxial rays falls on a convex mirror, they are reflected to form a divergent beam that appears to come from a point 'F' behind the mirror. This means that a convex mirror has a virtual focus 'F'. The distance between the focus (F) and the pole (P) is called the focal length 'f'. Concave mirrors are also referred to as converging mirrors and are commonly used in car headlights, searchlights, and telescopes. Conversely, convex mirrors are called diverging mirrors and provide a wider field of view than a plane mirror or concave mirror. Due to this property, convex mirrors are frequently used as rear-view mirrors in vehicles.



Converging mirror
(a)



Diverging mirror
(b)

(b) Diverging mirror

When using the Cartesian sign convention, the focal length of a concave mirror with a real object is negative since the distance PF (measured from P to F) is in the opposite direction of the light. Conversely, the focal length of a convex mirror is positive for the same reason. This sign convention also applies to virtual objects, where imaginary light rays are treated in the same way.

Relation between F and R

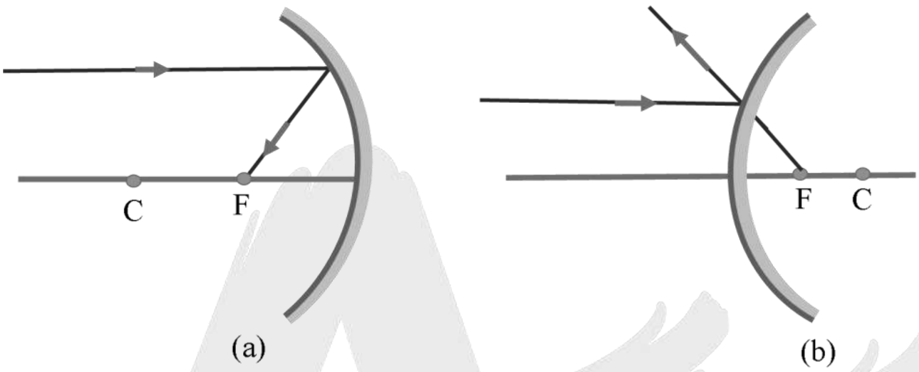
(a) Concave mirror

(b) Convex mirror

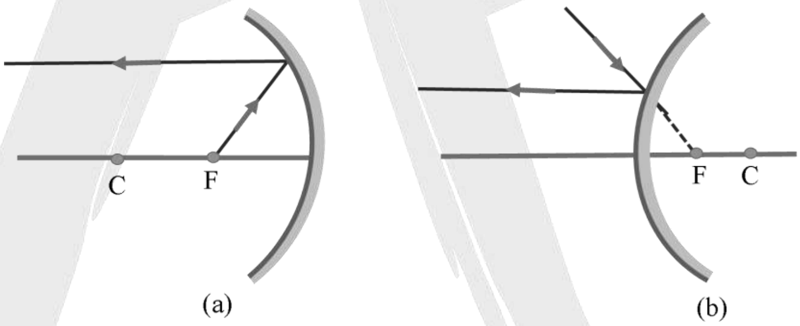
$$f = \frac{R}{2}$$

The focal length of mirror is independent on medium in which it placed and wavelength of incident light. To a plane mirror focal length 'f' is infinite $(\text{as } R = \infty)$

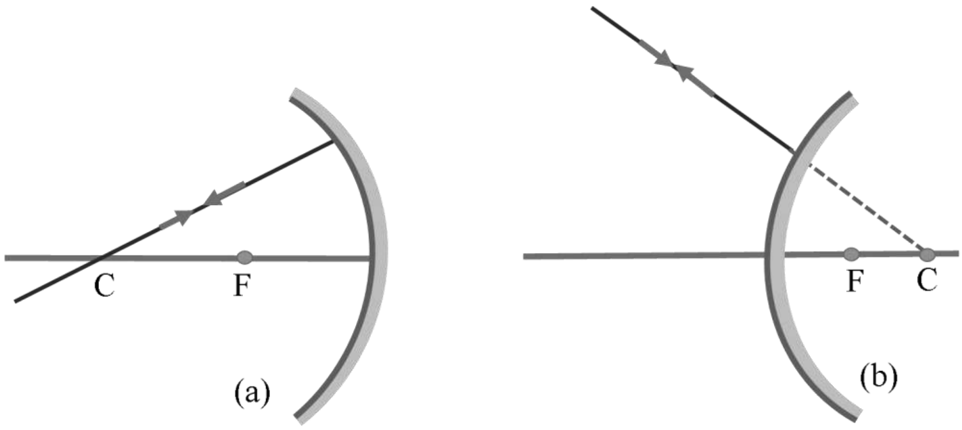
- The characteristics of an image, including its position, nature (whether it is real or virtual, erect or inverted, magnified or diminished), are generally dependent on the object's distance from the mirror. To determine the nature of the image, a ray diagram can be drawn. It is important to note that unless otherwise specified, the object is assumed to be real and may be either a point object or an extended object.
- After reflection from the mirror, a ray parallel to the principal axis either passes through or appears to pass through the focus F.



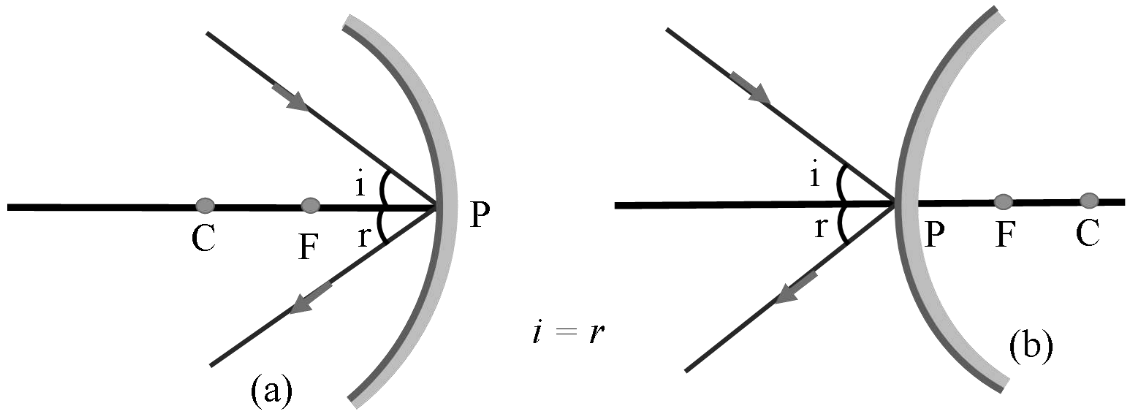
- The principle of reversibility states that a ray directed towards the focus of a mirror will reflect off the mirror and become parallel to the principal axis. Similarly, a parallel ray incident on the mirror will reflect in a way that appears to come from the focus.



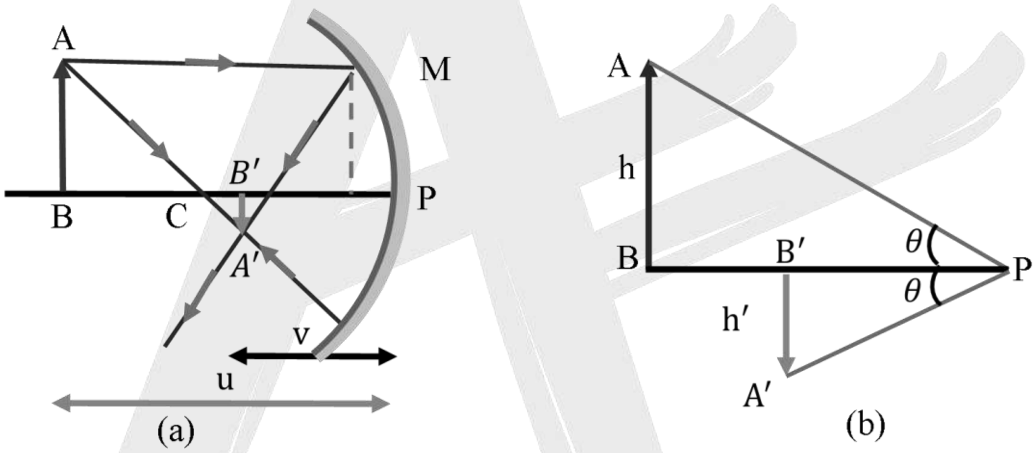
- After reflection from the mirror, a ray that passes through or is directed towards the centre of curvature C will follow the same path in the opposite direction. In other words, the ray retraces its path.



- When a ray of light strikes the pole P of a mirror, it reflects back in a symmetrical manner on the opposite side.



The Mirror Equation : Figure (a) shows the ray diagram considering two rays and the image A^1B^1 (in this case real image) of an object AB formed by a concave mirror.



$$\frac{1}{u}+\frac{1}{v}=\frac{1}{f}\left(\because \quad f=\frac{R}{2}\right)$$

Gauss's formula for a spherical mirror is a relationship that is applicable to all situations involving a spherical mirror, including convex mirrors. This formula involves substituting the known quantities with their proper signs in order to calculate the unknown quantities.

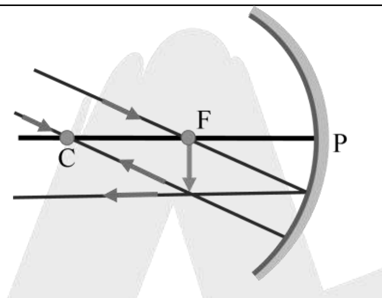
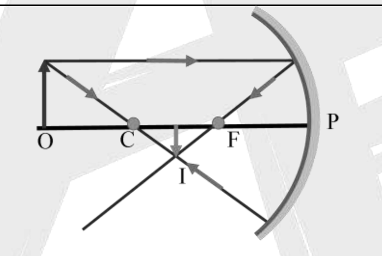
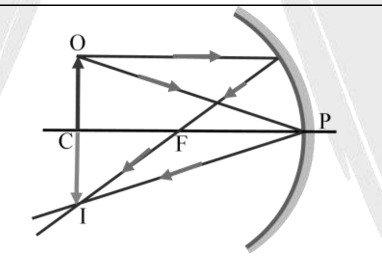
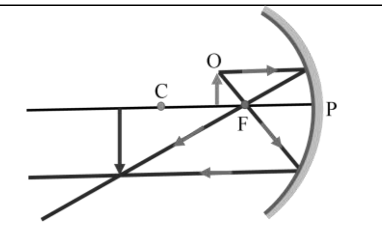
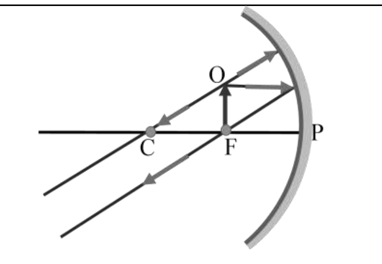
Image Formation by Spherical Mirrors

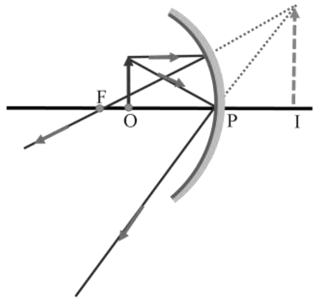
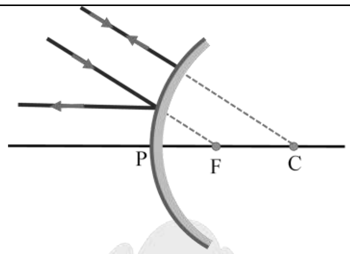
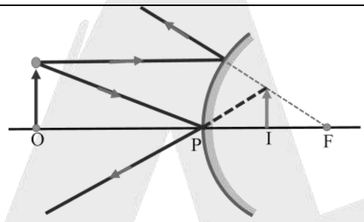
From the ray diagrams we understand that

- To a real object in case of concave mirror the image is erect, virtual and magnified when the object is placed between F and P. In all other positions of object the image is real and inverted.
- To a real object the image formed by convex mirror is always virtual, erect and diminished no matter where the object is.
- A concave mirror with virtual object behaviour is similar to convex mirror with real object and convex mirror with virtual object behaviour similar to concave mirror with real object.

- By principle of reversibility a convex mirror can form real and magnified image to a virtual object which is with in the focus and virtual images when virtual object beyond the focus. i.e., the convex mirror can form real and virtual images to virtual object. A concave mirror with virtual object always forms real images.
- If the given mirror breaks in to pieces, each piece of that mirror has own principal axis, but behaviour is similar to that of main mirror with less intensity of image.

(a) Concave mirror (b) Convex Mirror

Position of the object	Ray diagram	Image details
At Infinity		Real, inverted, very small, at F
Between ∞ and c		Real, inverted, diminished between F and C
At C		Real, inverted, equal, at C
Between F and C		Real, inverted, enlarged, beyond C
At F		Real, inverted, very large at infinity

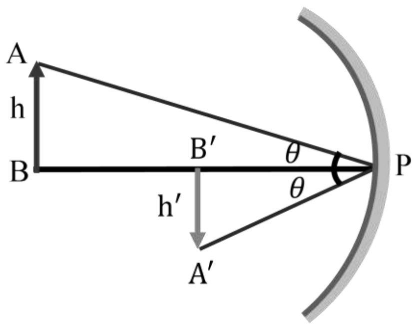
Between F and P		Virtual, erect, elarged behind the mirror
At Infinity		Virtual, erect, very small at F
Infront of mirror		Virtual, erect, diminished between P and F

Magnification :

In addition to position and nature of the image, the relative size of the image with respect to the object is also significant. For this purpose, magnification is defined. It is important to note that magnification does not necessarily imply that the image is magnified. The size of the image formed by the optical system can be greater than, smaller than, or equal to that of the object.

Lateral magnification :

The transverse or lateral or linear magnification (m) is defined as the ratio of the transverse dimension of the final image formed by an optical system to the corresponding dimension of the object. Hence it is the ratio of the height of image $\left(h^1\right)$ to the height of the object (h). From the figure.



Lateral magnification $m = \frac{A^1B^1}{AB} = \frac{h^1}{h}$

Here h and h^1 will be taken positive or negative in accordance with the accepted sign convention.

In triangles A^1B^1P and ABP, we have $\frac{B^1A^1}{BA}=\frac{B^1P}{BP}$, with sign convention this becomes

$\frac{-h^1}{h}=\left(\frac{-v}{-u}\right)$, so that, $m=\frac{h^1}{h}=-\frac{v}{u}$

Negative magnification indicates that the image is inverted relative to the object, while positive magnification indicates that the image is erect relative to the object. In other words, when the magnification (m) is negative, a real image is formed for a real object, and a virtual image is formed for a virtual object. On the other hand, when the magnification is positive, a virtual image is formed for a real object and a real image is formed for a virtual object.

Ex: If $m=-2$, means, if the object is real, image is real, inverted, magnified and mirror used is concave.

Longitudinal magnification :

However, if the one dimensional object is placed with its length along the principal axis. The ratio of length of image to length of object is called longitudinal magnification (m_L).

Longitudinal magnification can be expressed as $m_L=\frac{(v_2-v_1)}{(u_2-u_1)}$

Where v_1 and v_2 are image positions corresponding to u_1 and u_2 positions.

For small objects $m_L=-\frac{dv}{du}$

We have $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$

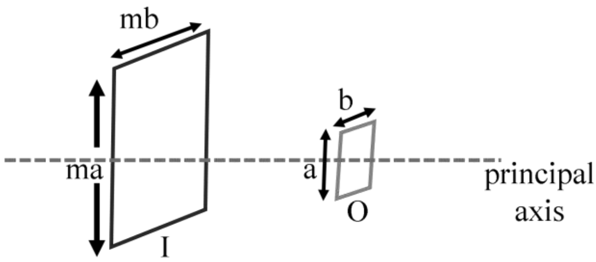
In case of small linear object $-\frac{dv}{v^2}-\frac{du}{u^2}=0$

$$\therefore\quad m_L=-\frac{dv}{du}=\left[\frac{v}{u}\right]^2=m^2$$

Areal magnification:

If a two dimensional object is placed with its plane perpendicular to principal axis, its magnification is called a real or superfical magnification. If m is the lateral magnification and m_A is the areal magnification.

$$m_A=\frac{\text{area of image}}{\text{area of object}}=\frac{(ma)(mb)}{ab}=m^2$$



Overall magnification :

When there are multiple optical components, the image formed by the first component becomes the object for the second component, and the image of the second component becomes the object for the third component, and so on. The overall magnification of the system is the product of all individual magnifications.

$$m_0 = \frac{I}{O} = \frac{I_1}{O_1} \times \frac{I_2}{O_2} \times - - \frac{I_n}{O_n} = m_1 \times m_2 \times - - \times m_n$$

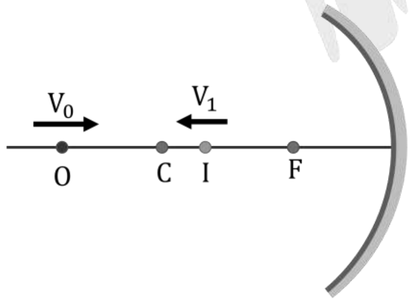
Newton's Formula : In case of spherical mirror if the object distance (x_1) and image distance (x_2) are measured from focus instead of the pole of the mirror. Then mirror formula reduces to a simple form called the Newton's formula.

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f} \text{ reduces to } \frac{1}{f + x_2} + \frac{1}{f + x_1} = \frac{1}{f}$$

Which on simplification gives $x_1 x_2 = f^2$ (Newton's Formula) $\left(f = \sqrt{x_1 x_2}\right)$

Motion of Object in front of Mirror Along the Principal Axis

- When there is a relative motion between the object and the mirror along the principal axis, the position of the image changes with respect to the mirror. In such cases, we use the mirror equation to determine the relationship between the speed of the object and the speed of the image.



$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

Differentiate with respect to time,

we get $-\frac{1}{v^2} \cdot \frac{dv}{dt} - \frac{1}{u^2} \cdot \frac{du}{dt} = 0$ (or) $\frac{dv}{dt} = -\left(\frac{v}{u}\right)^2 \cdot \frac{du}{dt}$ (or) $V_1 = -\left(\frac{v}{u}\right)^2 \cdot V_0$

Where v_1 velocity of image with respect to mirror and v_0 is the velocity of object with respect to mirror along the principal axisHere a negative sign in the mirror equation indicates that the object and image are moving in opposite directions. For a concave mirror, the image speed may be greater, lesser, or equal to the object speed depending on the position of the object.

(a) $R < u < \infty$ $\quad |m| < 1$ $\quad V_1 < V_0$

(b) $u = R$ $\quad |m| = 1$ $\quad V_1 = V_0$

(c) $f < u < R$ $\quad |m| > 1$ $\quad V_1 > V_0$

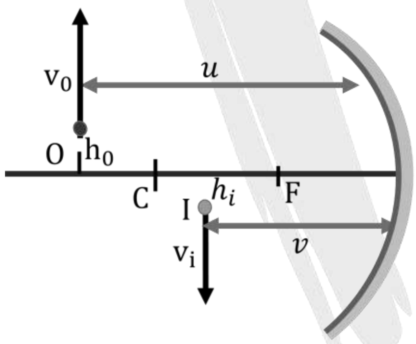
(d) $u < f$ $\quad |m| > 1$ $\quad V_1 > V_0$

(e) $u \approx 0$ $\quad |m| \approx 1$ $\quad V_1 \approx V_0$

The above equation that relates the velocity of the object and image is also applicable for convex mirrors. However, for convex mirrors, the speed of the image is always slower than the speed of the object, regardless of the position of the object. It should be noted that this equation is not valid for acceleration of the object and image.

Motion of the object Transverse to the Principal Axis

If the object moves transverse to principal axis then the image also moves transverse to principal axis.

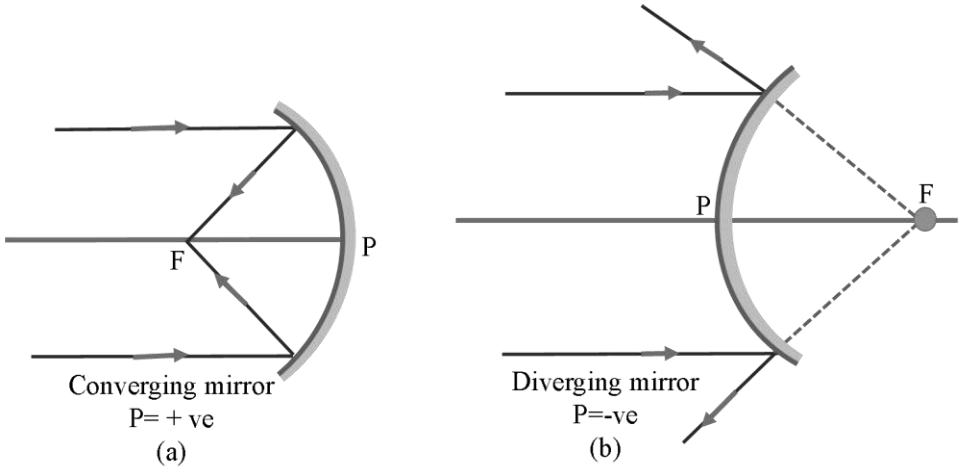


Consider the diagram. In a mirror $\frac{h_i}{h_0} = \frac{v}{u} = \text{constant} \quad (-m)$

$$\therefore \quad \frac{\frac{dh_i}{dt}}{\frac{dh_0}{dt}} = -m \ (\text{or}) \ V_1 = -mV_0$$

Power of Curved Mirror :

The power of an optical instrument refers to its ability to alter the path of incident light rays. If the instrument converges the rays parallel to the principal axis, its power is considered positive. Conversely, if the instrument diverges the rays, its power is regarded as negative.



For a mirror Power 'P'

$$P == -\frac{1}{f\left(\text{metre}\right)} \;(\text{or})\; P = -\frac{100}{f\left(\text{cm}\right)}$$

S.I unit of power is dioptre $\left(D\right) = m^{-1}$

For concave mirror (converging mirror) P is positive and for convex mirror (diverging mirror) power is negative.

- $\frac{1}{V} - \frac{1}{U}$ Graph to Mirrors : The graph between $\frac{1}{v}$ and $\frac{1}{u}$ to a concave mirror is shown in figure (a)

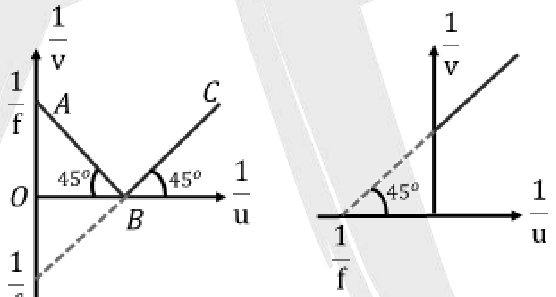


Fig (a)

Since $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

For all real image $-\frac{1}{v} - \frac{1}{u} = -\frac{1}{f}$ $\quad\therefore \frac{1}{v} = -\frac{1}{u} + \frac{1}{f}$

This is a straight line equation with slope –1.

This is represented by the line AB.

For virtual image, $\frac{1}{v} - \frac{1}{u} = -\frac{1}{f}$ $\quad\therefore \frac{1}{v} = \frac{1}{u} - \frac{1}{f}$

This is a straight line equation with slope +1.

This represents line BC.

- The graph between $\frac{1}{v}$ and $\frac{1}{u}$ to a convex mirror as shown in figure (b).

  Since convex mirror always form virtual image to a real object.

$$\frac{1}{v}+\frac{1}{-u}=\frac{1}{f} \quad \therefore \quad \frac{1}{v}=\frac{1}{u}+\frac{1}{f}$$

  This is a straight line equation with slope +1.

- U-V Graph in Curved Mirror : In case of concave mirror, the graph between u and v is hyperbola as shown in figure.

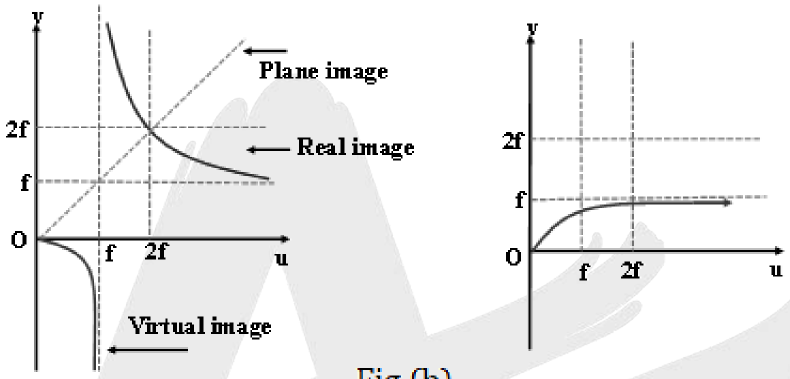


Fig (b)

For real image $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}\ (\text{or})\ \frac{1}{v}=\frac{u-f}{uf} \Rightarrow v=\frac{f}{1-\frac{f}{u}}$

For virtual image $\frac{1}{v}-\frac{1}{u}=-\frac{1}{f} \Rightarrow \frac{1}{v}=\frac{f-u}{fu}\ (\text{or})\ v=\frac{f}{\frac{f}{u}-1}$

- In the case of a convex mirror, the graph showing the relationship between object distance (u) and image distance (v) is a hyperbola as shown in Figure (b). This is because convex mirrors only form virtual images.

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}\ (\text{or})\ v=\frac{f}{1+\frac{f}{u}}$$

Graph in Spherical Mirror :

In a spherical mirror : $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$

$$\therefore\ 1+\frac{v}{u}=\frac{v}{f}\ (\text{or})\ \frac{v}{u}=\frac{v}{f}-1$$

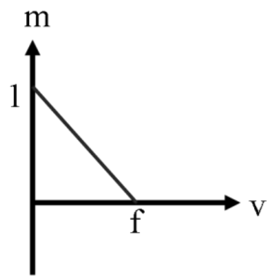


Fig. (a)

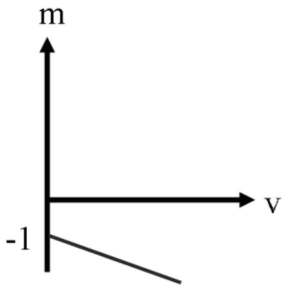


Fig. (b)

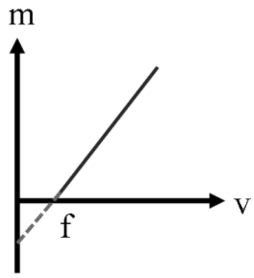


Fig. (c)

- Concave mirror : If the objects is real,
- For real image, $u = -ve, \ v = -ve, \ f = -ve$,

  $\therefore \ -m = \frac{v}{f} - 1 \ (or) \ m = -\frac{v}{f} + 1$
- Graph as shown in figure (a)

  For virtual image, $u = -ve, \ v = +ve, \ f = -ve$

  $\therefore \ \ m = -\frac{v}{f} - 1$, Graph as shown in figure (b)
- Convex mirror : Since convex mirror always form virtual image to a real object,

  $u = -ve, \ v = +ve, \ f = +ve$,

  $\therefore \ \ m = \frac{v}{f} - 1$, graph as shown in figure (c).

  From the above graph it is observed that for $v \approx 0, \ m = 1$. i.e., when an object is very near to pole of the mirror $(u \approx 0)$, then the curved mirror behaves like a plane mirror.
- **Refraction of light:**

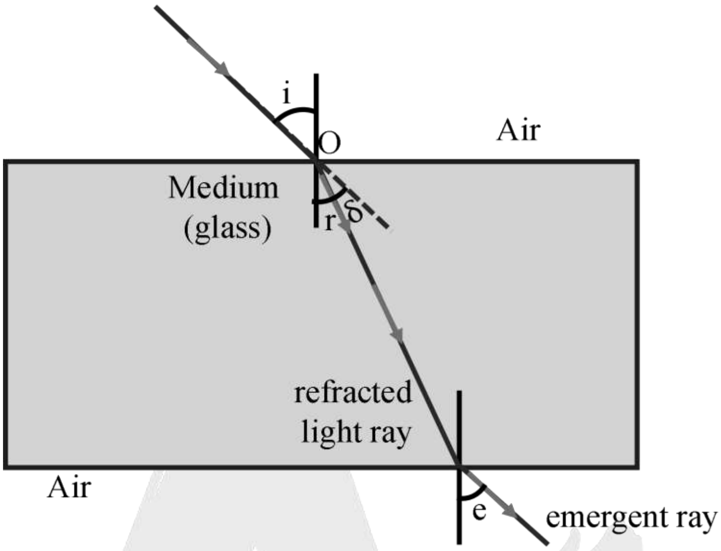
  When light passes from one medium to another, some of it is reflected back into the first medium at the interface of the two media while the rest continues to travel through the second medium in a different direction. The change in the direction of light occurs at the interface of the two media. The bending or deviation of light rays from their original path while passing from one medium to another is known as refraction.

  (OR)

  The bending of light from its original path when passing from one optical medium to another is referred to as refraction. This phenomenon occurs due to a change in the speed of light as it moves from one medium to another. Refraction does not affect the frequency (colour) or phase of light, but it does cause changes in wavelength and velocity.

Note: When light passes from one medium to another, its frequency (which determines the color) remains the same, but its wavelength and velocity change due to refraction.

- Refraction of light at plane surface:



- **Incident ray:** A light ray that travels towards another optical medium is referred to as an incident ray.
- **Point of incidence:** The point O at which an incident ray meets the interface of another optical medium is called the point of incidence.
- **Normal:** A line perpendicular to the surface separating two optical media at the point of incidence is referred to as the normal.
- **Angle of incidence (i):** The angle formed between the incident ray and the normal at the point of incidence is called the angle of incidence.
- **Refracted ray:** A ray of light that changes direction when it enters a different optical medium is known as a refracted ray.
- **Angle of refraction (r):** The angle between the refracted ray and the normal at the point of incidence is known as the angle of refraction.
- **Angle of deviation due to refraction** (δ) **:** It is the angle between the direction of incident light ray and the emergent ray, which is the ray that exits the second medium after refraction.
- **Emergent ray:** The ray of light that exits from an optical medium, as illustrated in the figure above, is known as the emergent ray.
- **Angle of emergence (e):** The angle formed between the emergent ray and the normal to the surface at the point of emergence is known as the angle of emergence.
- **Laws of Refraction:** Incident ray, refracted ray and normal always lie in the same plane
- The produce of refractive index and sine of angle of incidence at a point in a medium is constant,

$\mu \times \sin i$ = constant

$\mu_1 \sin i_1 = \mu_2 \sin i_2$

If $i_1 = i$ and $i_2 = r$ then

$\mu_1 \sin i = \mu_2 \sin r$;

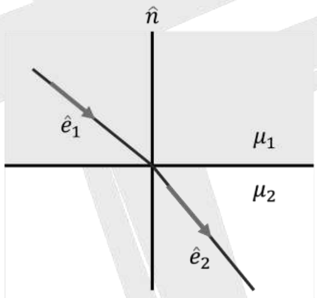
This law is called snell's law.

According to Snell's law,

$\frac{\sin i}{\sin r}$ = constant $\left(= \frac{\mu_2}{\mu_1}\right)$ for any pair of medium and for light of given wavelength

Note: The ratio of the sine of the angle of incidence to the sine of the angle of refraction is known as the refractive index of a material with respect to the medium in which the angle of incidence is located. When light ray travells from medium 1 to medium 2 then $\frac{\sin i}{\sin r} = \frac{\mu_2}{\mu_1} = {}_1\mu_2$ = refractive index of medium (2) with respect to medium (1)

Vector form of Snell's law: $\mu_1(\hat{e}_1 \times \hat{n}) = \mu_2(\hat{e}_2 \times \hat{n})$



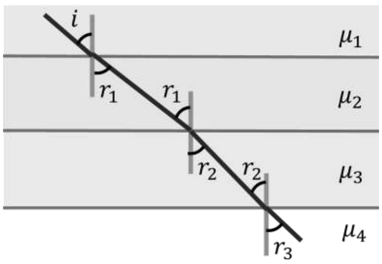
There $\hat{e}_1$ = unit vector along incident ray

$\hat{e}_2$ = unit vector along refracted ray

$\hat{n}$ = unit vector along normal incidence point

Note: Let us consider a ray of light travelling in situation as shown in figure

Applying Snell's law at each interface, we get



$\mu_1 \sin i = \mu_2 \sin r_1$; $\mu_2 \sin r_1 = \mu_3 \sin r_2$

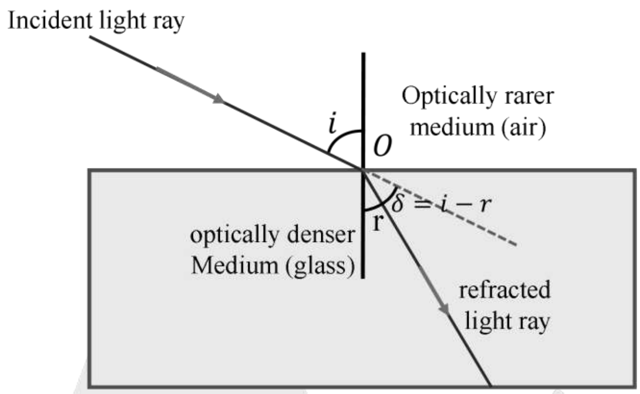
$\mu_3 \sin r_2 = \mu_4 \sin r_3$; it is clear that

$\mu_i \sin i = \mu_2 \sin r_1 = \mu_3 \sin r_2 = \mu_4 \sin r_3$ (or) $\mu \sin i$ = constant

Note: When light ray travels from medium of refractive index μ_1 to another medium of refractive index μ_2 to another medium of refractive index μ_2 then, $\mu_1 \sin i_1 = \mu_2 \sin i_2$

$$\frac{\sin i_1}{V_1} = \frac{\sin i_2}{V_2} = \frac{\sin i_1}{\lambda_1} = \frac{\sin i_2}{\lambda_2}$$

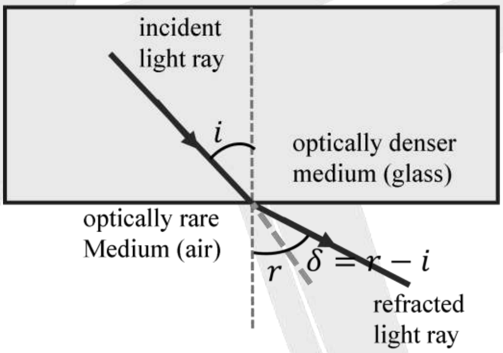
When a light travels from optically rarer medium to optically denser medium obliquely:



(a) it bends towards normal

(b) angle of incidence is greater than angle of refraction

When a ray of light travels from optically denser medium to optically rarer medium obliquely



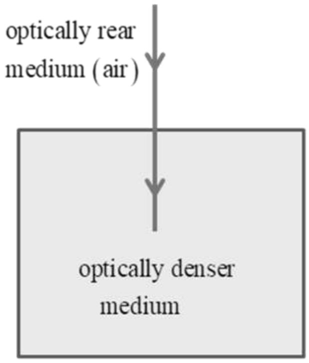
(a) it bends away from the normal at the point of incidence

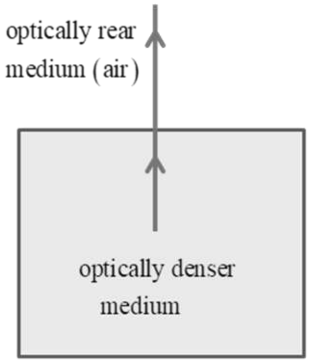
(b) angle of refraction is greater than angle of incidence

(c) angle of deviation $\delta = r - i$

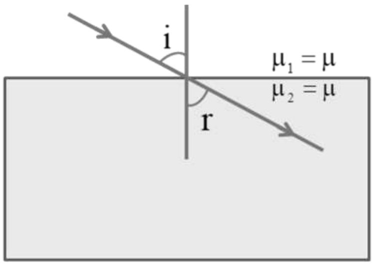
- **Condition for no refraction:**

When an incident ray strikes normally at the point of incidence, it does not deviates from its path i.e., it suffers no deviation.





- In this case angle of incidence (i) and angle of refraction (r) are equal and $\angle i = \angle r = 0$
  If the refraction indices of two media are equal



$\mu_1 = \mu_2 = \mu$

From Snell's law,

$\mu \sin i = \mu \sin r$, $\sin i = \sin r$

$\angle i = \angle r$

Hence, the ray passes without any deviation at the boundary.

Note: Due to the fact that when light travels from a medium to another with the same refractive index, it does not change its direction, a transparent solid immersed in a liquid with the same refractive index will become invisible.

Refractive index:

Absolute refractive index (μ)**:**

The absolute refractive index of a medium is the ratio of speed of light in free space (C) to speed of light in a given medium (V).

$$\mu = \frac{\text{velocity of light in free space (C)}}{\text{velocity of light in a given medium (V)}}$$

It is a scalar

It has no units and dimensions

From electron magnetic theory if ε_0 and μ_0 are the permittivity and permeability of free space, ε and μ are the permittivity and refractive index of the given medium

$$\mu = \frac{C}{V} = \frac{\frac{1}{\sqrt{\varepsilon_0 \mu_0}}}{\frac{1}{\sqrt{\varepsilon \mu}}} = \sqrt{\frac{\varepsilon \mu}{\varepsilon_0 \mu_0}} = \sqrt{\varepsilon_r \mu_r}$$

Where ε_r & μ_r are the relative permittivity and permeability of the given medium

- The speed of light of all wavelengths in vacuum or free space is same and equal to c.
  So, for all wavelengths the refractive index of free space is $\mu = \frac{C}{C} = 1$
- For a given medium the speed of light is different for different wavelengths of light, greater will be the speed and hence lesser will be refractive index $\lambda_R > \lambda_V$, so in medium $\mu_V > \mu_R$

Note: Actually, refractive index μ varies with λ according to the equation $\mu = A + \frac{B}{\lambda^2}$.

(where A & B are constants)

- For a given light, denser the medium lesser will be the speed of light and so greater will be the refractive index.

Example: Glass is denser medium when compared to water, so $\mu_{glass} > \mu_{water}$

The refractive index of water $\mu_w = \frac{4}{3}$

The refractive index of glass $\mu_g = \frac{3}{2}$

- For a given light and given medium, the refractive index is also equal to the ratio of wavelength of light in free space to that in the medium.

$$\mu = \frac{C}{V} = \left(\frac{f\,\lambda_{vauum}}{f\,\lambda_{medium}}\right) = \frac{\lambda_{vauum}}{\lambda_{medium}}$$

(when light travels from vacuum to a medium, frequency does not change)

Note: If C is velocity of light in free space λ_0 is wavelength of given light in free space then velocity of light in a medium of refractive index

$$(\mu) \text{ is } V_{medium} = \frac{C}{\mu}$$

Wavelength of given light in a medium of refractive index

$$(\mu) \text{ is } \lambda_{medium} = \frac{\lambda_0}{\mu}$$

- Relative Refractive index: When light passes. From one medium to the other, the refractive index of medium 2 relative to medium 1 is written as ${}_1\lambda_2$ and is given by

$${}_1\mu_2 = \frac{\mu_2}{\mu_1} = \frac{v_1}{v_2} = \frac{\lambda_1}{\lambda_2} \quad \text{... (1)}$$

Refractive index of medium 1 relative to medium 2 is ${}_2\mu_1$ and

$${}_2\mu_1 = \frac{\mu_1}{\mu_2} = \frac{v_2}{v_1} = \frac{\lambda_2}{\lambda_1} \quad \text{... (2)}$$

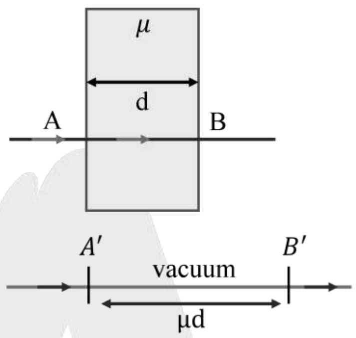
From equation (1) and (2), we get

$${}_1\mu_2 = \frac{1}{{}_2\lambda_1}$$

i.e., $({}_1\mu_2).({}_2\mu_1) = 1$

Optical Path (Δx):

Consider two points A and B in a medium as shown in figure. The distance between two points A and B in a medium is called the geometrical path, which is independent of the surrounding medium. When light travels from point A to point B, it moves at a velocity of c in vacuum and a lesser velocity v in any other medium. Thus, light takes more time to travel from A to B in a non-vacuum medium. The optical path of a given geometrical path in a medium is the distance that light travels in vacuum in the same time it takes to travel the given path length in that medium.



AB = real path or geometrical path

$A'B'$ = Optical path

If the light travels a path length 'd' in a medium at speed v, the time taken by it will be $\left(\frac{d}{v}\right)$

so, optical path length,

$$\Delta x = c \times t = c \times \left[\frac{d}{v}\right] = \mu d \left(\text{as } \mu = \frac{c}{v}\right)$$

Therefore, optical path is μ times the geometrical path. As for all media $\mu > 1$, optical path length is always greater than actual path length.

Note: If in a given time t, light has same optical path length in different media, and if light travels a distance d_1 in a medium of refractive index μ_2 in same time t, then $\mu_1 d_1 = \mu_2 d_2$.

Note: The difference in distance travelled by light in vacuum and in a medium in the same interval of time is called optical path difference due to that medium.

$\Delta x = A'B' - AB = \mu d - d$ $\qquad \Delta x = (\mu - 1)d$

Note: A slab of thickness d and refractive index μ is kept in a medium of refractive index $\mu'(<\mu)$. If the two rays parallel to each other passes through such a system with one ray passing through the slab, then path difference between two rays due to slab will be $\Delta x = \left(\frac{\mu}{\mu'} - 1\right)d$

Note: The optical phase change $\phi = \frac{2\pi}{\lambda}$ (Optical path difference)

Principal of Reversibility of Light

- According to principal of reversibility, if a ray of light travels from X to Z along a certain path, it will follow exactly the same path, while travelling from Z to X. In other words the path of light is reversible.

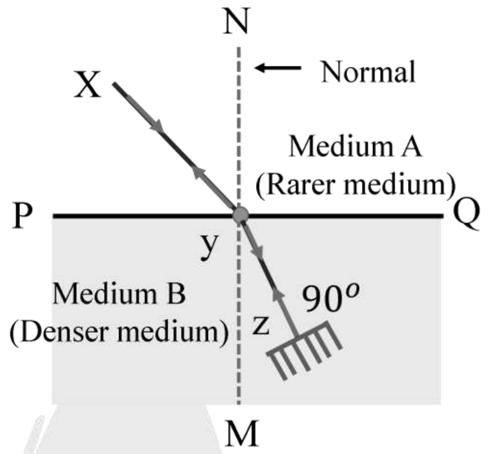


Figure shows a ray of light XY travelling through medium ‘A’, such that it travels along YZ, while travelling medium ‘B’. NM is the normal at point Y, such $\angle XYN$ is the angle of incidence and $\angle MYZ$ is the angle of refraction.

$$\therefore {}_a\mu_b = \frac{\sin\angle XYN}{\sin\angle MYZ} \quad \text{... (1)}$$

If a plane mirror is placed at right angles to the path of refracted ray ‘YZ’, it found that light retraces back its path. Now ray ZY acts as incident ray YX as refracted ray, such that $\angle MYZ$ is angle of incidence and $\angle XYN$ is the angle of refraction.

$$\therefore {}_b\mu_a = \frac{\sin\angle MYZ}{\sin\angle XYN} \qquad \therefore \frac{1}{{}_b\mu_a} = \frac{1}{\frac{\sin\angle MYZ}{\sin\angle XYN}} = \frac{\sin\angle XYN}{\sin\angle MYZ} \quad \text{... (2)}$$

Comparing (1) and (2) ${}_a\mu_b = \frac{1}{{}_b\mu_a}$

Thus, the refractive index of medium ‘b’ with respect to ‘a’ is equal to the reciprocal of refractive index of medium ‘a’ with respect to medium ‘b’.

Apparent Depth:

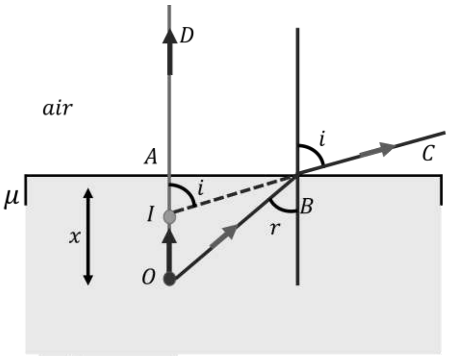
- **Case (1):** Object in denser medium and observer in rarer medium.

  When object ‘O’ is placed at a distance ‘x’ from A in denser medium of refraction index μ as shown in figure. Ray OA, which falls normally on the plane surface, passes undeviated as AD. Ray OB, which ‘r’ (with normal) on the plane surface, bends away from the normal and passes as BC in air. Rays AD and BC meet at ‘I’ after extending these two rays backwards. This ‘I’ is the virtual image of real object ‘O’ to an observer in rarer medium near to transmitted ray.

$\sin i \approx \tan i = \frac{AB}{AI}$ ...(i)

$\sin r \approx \tan r = \frac{AB}{AO}$ ...(ii)

Dividing equation (i) and (ii)



$$\frac{\sin i}{\sin r} = \frac{AO}{AI};$$

According to Snell's law $\mu = \frac{\sin i}{\sin r}$ $\therefore \mu = \frac{AO}{AI}$ $\therefore AI = \frac{AO}{\mu} = \frac{x}{\mu}$

The distance of image AI is called apparent depth or apparent distance. The apparent depth x_{app} is given by i.e., $x_{app} = \frac{x_{real}}{\mu}$

The apparent shift $(OI) = AO - AI = x - \frac{x}{\mu}$

Hence the apparent shift $(OI) = \left(1 - \frac{1}{\mu}\right)x$

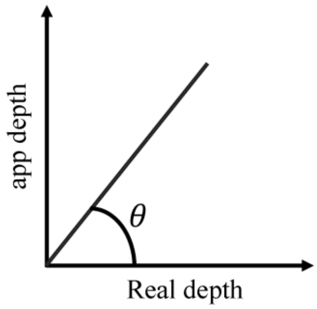
If the observer is in other than air medium of refractive index $\mu(<\mu)$

The apparent depth $= \frac{\text{real depth}}{\mu_{relative}} = \frac{\text{real depth}}{\left(\frac{\mu}{\mu'}\right)}$

$\therefore$ apparent depth $= \frac{\mu'}{\mu}$ (real depth)

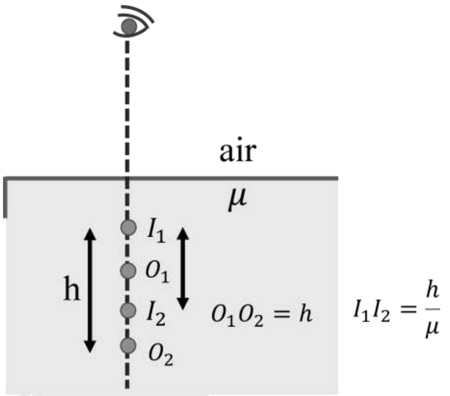
Apparent shift $= \left(1 - \frac{\mu'}{\mu}\right)x$

diagram shows variation of apparent depth with real depth of the object.

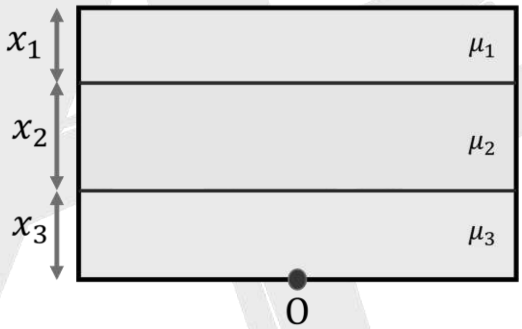


Slope $= \tan\theta = \frac{\mu'}{\mu}(<1)$

Note: If two objects O_1 and O_2 separated by 'h' on normal line to the boundary in a medium of refractive index μ. These objects are observed from air near to normal line of boundary. The distance between the images I_1 and I_2 of O_1 and O_2 is $\frac{h}{\mu}$.



Note: Apparent depth of object due to to composite slab is $x_a = \frac{x_1}{\mu_1} + \frac{x_2}{\mu_2} + \frac{x_3}{\mu_3}$



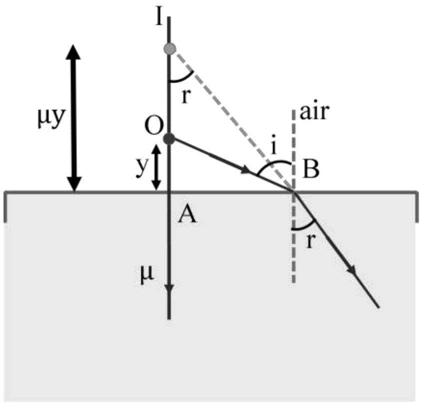
Note: If there are 'n' number of parallel slabs which are may be in contact or may not with different refractive indices are placed between the observer and the object, then the total apparent shift

$$s = \left(1 - \frac{1}{\mu_1}\right)x_1 + \left(1 - \frac{1}{\mu_2}\right)x_2 + \ldots\ldots + \left(1 - \frac{1}{\mu_n}\right)x_n$$

Where x_1, x_2 ..... x_n are the corresponding refractive indices.

- Object in rarer medium and observer in denser medium:

  When the object in rarer medium (air) at a distance 'y' from boundary and an observer near to normal in denser medium of refractive index 'μ'. By ray diagram in figure it is observed that the image is virtual, on same side to boundary and its distance from the boundary is μ times the object distance. Since $\mu > 1$ image distance is more than object distance.



$\sin i \approx \tan i = \frac{AB}{AO}, \ \sin r \approx \tan r = \frac{AB}{AI}$

According to Snell's law $1.\sin i = \mu \sin r$

$\frac{AB}{AO} = \mu \frac{AB}{AI}, \ AI = \mu.AO$

Therefore, apparent height of object (AI) $= \mu$ x real height of object (AO)

i.e., $y_{app} = \mu . y_{real}$ Apparent shift $= AI - AO$

apparent shift $= (\mu - 1) y$

if the object is in other than air medium of refractive index $\mu'(<\mu)$.

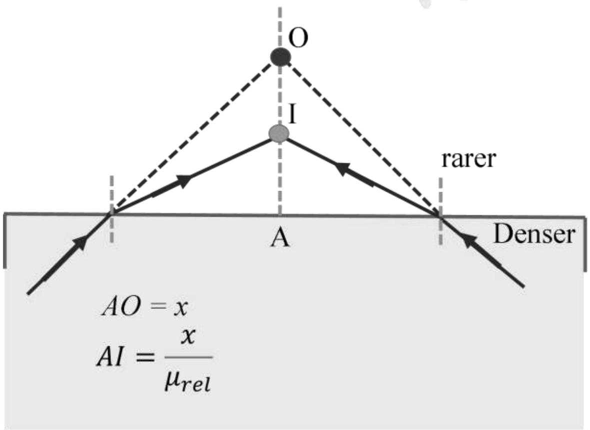
Then apparent height $= \mu_{rel}$ (real height); i.e., $y_a = \left(\frac{\mu}{\mu'}\right) y$

apparent shift $= \left(\frac{\mu}{\mu'} - 1\right) y$

Diagram shows variation of apparent height with real height of the object.

Slope $= \tan\phi = \frac{\mu}{\mu'} (>1)$

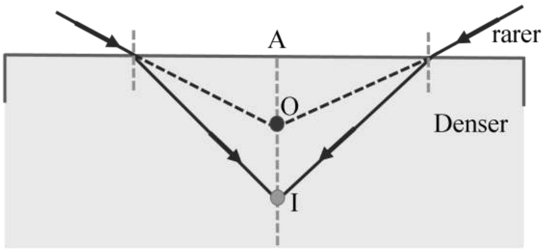
Note: When a convergent beam of rays passes from a denser medium to a rarer medium, a real image is formed in the rarer medium that is closer to the boundary than the virtual object.



Shift $= x\left(1 - \frac{1}{\mu_{real}}\right)$

Note: When a convergent beam of rays passes from a rarer medium to a denser medium, as shown in the figure, a real image is formed in the denser medium, which is farther from the boundary than the virtual object.

Shift $=(\mu_{real}-1)x$



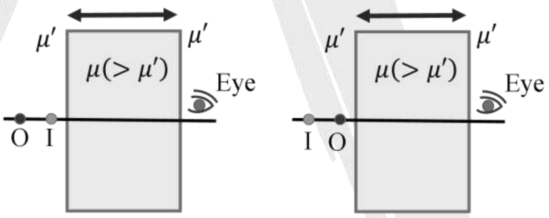
$AO = x$, $AI = \mu_{rel}.x$

- **Application**

Normal shift due to glass slab: When an object is placed on normal line to the boundary of slab whose thickness is 't' and refractive index 'μ'. When observing a real object through a transparent slab from the other side, the image of the object appears to shift from its original position along the normal line due to refraction. This shift is known as the normal shift. If the slab is denser than the surroundings, the shift is towards the slab, and if the slab is rarer than the surroundings, the shift is away from the slab. Then the normal shift

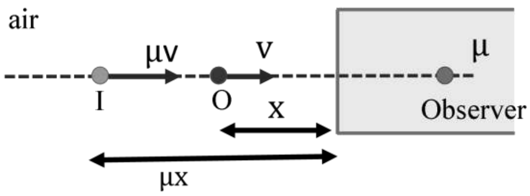
$$OI=\left(1-\frac{1}{\mu_{rel}}\right)t=\left(1-\frac{\mu'}{\mu}\right)t \text{ or } OI=\left(1-\frac{1}{\mu}\right)t$$

For $\mu'=1$, normal shift



Relation between the velocities of object and image:

The figure shows an object O moving towards the plane boundary of a denser medium.



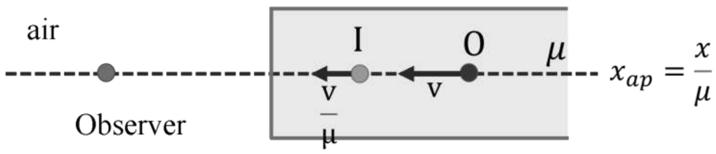
$x_{ap}=\mu x$

Differentiating the above equation with respective to time, we get

$V_{ap}=\mu V$

To an observer in the denser medium, the object appears to be more distant but moving faster. If the speed of the object is v, then the speed of the image will be μv

(b) Similarly to an observer in rarer medium and object in denser medium, the image appears to be closer but moving slowly.



Differentiating the above equation with respective to time, we get $V_{ap} = \frac{V}{\mu}$

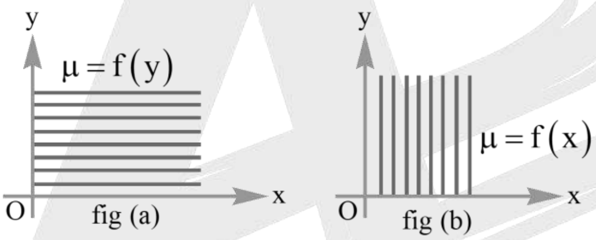
If the speed of the object is v, then the speed of the image will be $\frac{v}{\mu}$.

GEOMETRICAL OPTICS [RS AND PM]

- **Concept of Refraction in a medium of variable refractive index:**

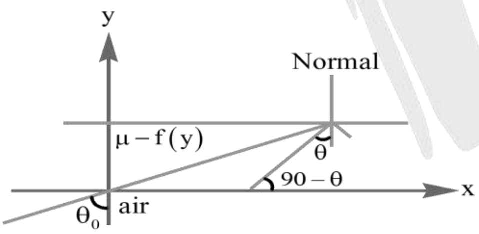
In the previous scenario, we assumed that the refractive index of the slab remains constant. However, this assumption is not valid for the atmosphere. As we move upward from the Earth's surface, the atmosphere becomes less dense, and thus the refractive index decreases. In reality, the refractive index of the atmosphere changes continuously in all directions. To simplify this complex situation, we can consider the medium as a collection of many thin layers with varying refractive indices.

Let refractive index be a function of y i.e., $\mu = f(y)$ then the medium can be considered as to be made up of large number of thin slabs placed parallel to x-axis and optical normal at any interface is parallel to y-axis. Similarly, if $\mu = f(x)$ then slabs are parallel to y-axis and optical normal at any interface is parallel to x-axis.



Examples of variable refractive index:

➢ $\mu = f(y)$. Now the medium can be divided into thin slices parallel to x axis and optical normal parallel to y axis. Let θ_0 be the angle of incident in the variable medium at point (0,0). And θ is the angle made by tangent with normal parallel to y axis at any point (x, y) on the trajectory.



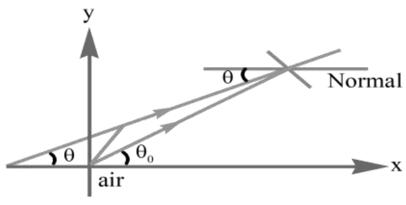
Trajectory of light ray when $\mu - f(y)$

From the snell's law: $1.\ \sin\theta_0 = \mu \sin\theta$ ...(1)

Geometrically, relate the slope of this tangent to the angle θ i.e., $\frac{dy}{dx} = \tan(90 - \theta)$ ...(2)

Substitute for θ from equation (1) and determine $\frac{dy}{dx}$ as a function of y. integrate and obtain an expression of y as a function of x.

➢ Let us consider a situation $\mu = f(x)$. Now the medium can be divided into thin slices parallel to y axis and optical normal parallel to x-axis.



Trajectory of light ray when $\mu = f(x)$

Trajectory of light ray when $\mu = f(x)$

From the Snell's law: $1.\sin\theta_0 = \mu\sin\theta$ (1)

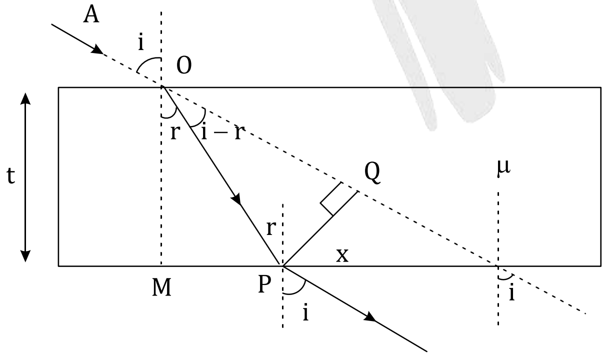
[Since $\mu = 1$ at $x = 0$]

Geometrically, relate the slope of this tangent to the angle of incidence θ i.e., $\frac{dy}{dx} = \tan\theta$ (2)

Substitute θ from equation (1) and determine $\frac{dy}{dx}$ as a function of y. Integrate and obtain an expression of y as a function of x.

Lateral Shift :

Consider a ray of light (AO) incident on a slab at an angle of incidence 'i' and passing through a thickness 't' of the slab, as shown in the figure. After two refractions at the boundary, the ray emerges parallel to the incident ray. The perpendicular distance between the direction of the incident ray and the direction of the emergent ray is known as the lateral shift or lateral displacement (x).



From the figure, the distance PQ is called lateral displacement (or) lateral shift

From the triangle PQO, $\sin(i-r) = \frac{PQ}{OP}$

$PQ(=x) = OP\sin(i-r)$

$x = OP\sin(i-r)$ ...(1)

But $\cos r = \frac{OM}{OP}$, $OP = \frac{OM}{\cos r} = \frac{t}{\cos r}$ ...(2)

From (1) and (2)

$\therefore\ x = t\left[\frac{\sin(i-r)}{\cos r}\right]$

For small angle of incidence $\sin i \simeq i$

$\sin r \simeq$ and $\cos r \simeq 1, \sin(i-r) \simeq (i-r)$ or

$x = t(i-r) = ti\left(1-\frac{r}{i}\right)$ Lateral shift $x = \left(1-\frac{1}{\mu}\right)ti$

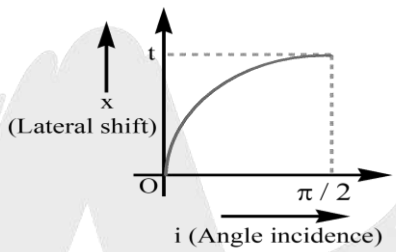


Figure plot of lateral shift versus angle of incidence

Note: When $i \to \frac{\pi}{2}$ (grazing incidence)

$x_{max} = \frac{t}{\cos r}\sin\left(\frac{\pi}{2}-c\right) = t\,(\because r = c)$

Note: The later shift $x = t\left[\frac{\sin(i-r)}{\cos r}\right]$ can also be expressed as follows.

$x = \frac{t}{\cos r}\sin(i-r) = \frac{t}{\cos r}(\sin i \cos r - \cos i \sin r)$

On simplification, $x = t\left(1-\frac{\cos i}{\sqrt{\mu^2-\sin^2 i}}\right)\sin i$

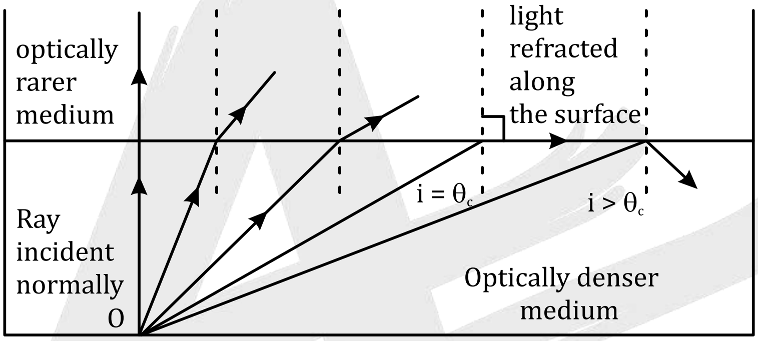
- **Examples of Refraction:** Visibility of two images of an object:
  When an object is placed in a glass container filled with a liquid and viewed from outside at a higher level than the liquid, two images are formed. One image is formed due to refraction through the liquid, and the other image is formed due to refraction through the glass.
- **Twinkling of stars:** The irregular fluctuations in the refractive index of the atmosphere can cause the refraction of light to vary, leading to some instances where the light reaches the observer's eye and other instances where it does not. This phenomenon is what causes stars to appear to twinkle.

Critical Angle and total Internal Reflection:

Consider a point object O placed in an optically denser medium as shown in the figure. Rays of light travel from O in all possible directions. As the light travels from the denser medium to the rarer medium, it is refracted at the surface and bends away from the normal.

Therefore, as the angle of incidence increases, the angle of refraction also increases till for a certain angle of incidence, the angle of refraction is $90°$ and light is refracted along the surface separating the two media. The corresponding angle of incidence is called the critical angle (θ_C).

When light is incident at a point beyond point P, where the angle of incidence is greater than the critical angle, the light is not refracted and instead, the entire incident light is reflected back into the same medium. This phenomenon is referred to as total internal reflection.



➢ **Expression for critical angle (θ_C):**

According to Snell's law, at critical angle of incidence

$$\mu_D.\sin\theta_C = \mu_R.\sin 90°, \quad \sin\theta_C = \frac{\mu_R}{\mu_D}$$

$$\sin\theta_C = \frac{\mu_R}{\mu_D} = \frac{V_D}{V_R} = \frac{\lambda_D}{\lambda_R}$$

$$\text{For } \mu_R = 1, \quad \mu_D = \frac{1}{\sin\theta_C}$$

➢ **Condition for total internal reflection:**

Total internal reflection can only occur when light propagates from a denser medium to a rarer medium.

Ex: Ray from water to air, glass to water.

Total internal reflection occurs when the angle of incidence of a light ray in a denser medium is greater than the critical angle for that particular boundary with a rarer medium.

$$\text{i.e., } i > \theta_C \text{ with } \theta_C = \sin^{-1}\left(\frac{\mu_R}{\mu_D}\right)$$

Note: Critical angle (θ_C) depends on nature of pair of media. Greater $\frac{\mu_R}{\mu_D}$ ratio greater will be the critical angle

For glass – air:

$$\mu_D = \frac{3}{2},\ \mu_R = 1,\ \theta_C = \sin^{-1}\left(\frac{2}{3}\right),\ \theta_C = 42^\circ$$

For water – air:

$$\mu_D = \frac{4}{3},\ \mu_R = 1,\ \theta_C = \sin^{-1}\left(\frac{3}{4}\right),\ \theta_C = 49^\circ$$

For glass – water:

$$\mu_D = \frac{3}{2},\ \mu_R = \frac{4}{3},\ \theta_C = \sin^{-1}\left(\frac{8}{9}\right),\ \theta_C = 63^\circ$$

For diamond – air:

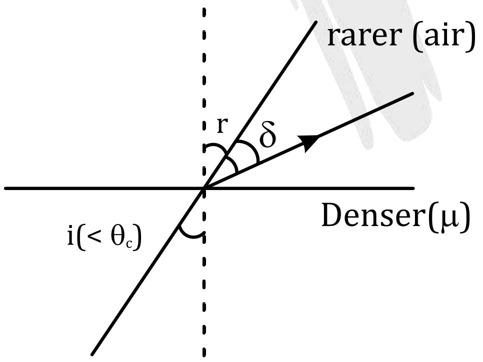
$$\mu_D = 2.5,\ \mu_R = 1,\ \theta_C = \sin^{-1}\left(\frac{1}{2.5}\right),\ \theta_C = 24^\circ$$

Note: In the case of total internal reflection, all (100%) incident light is reflected back into the same medium without any loss of intensity.This is why images formed by total internal reflection are much brighter than that formed by mirrors and lenses.

Note: Image due to total internal reflection is real, lateral and inverted with respect to object.

➢ **<u>Deviation of light under total internal reflection:</u>**

The figure shows a light ray travelling from denser to rarer medium at an angle i, less than the critical angle θ_C.

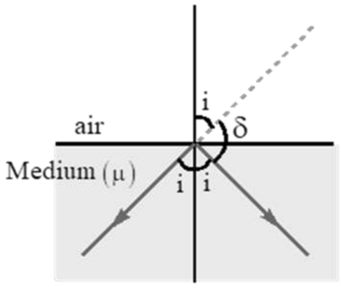


The deviation δ of the light ray is given by $\delta = r - i$

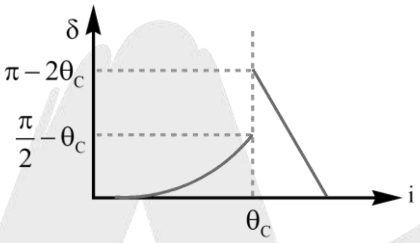
Since $\mu \sin i = \sin r$, therefore $\delta = \sin^{-1}(\mu \sin i) - i$

This is a non linear equation. The maximum value of 'δ' occurs when $i = \theta_C$, and is equal to $\frac{\pi}{2} - \theta_C$ i.e., $\delta_{max} = \frac{\pi}{2} - \theta_C$

- If the light incident at an angle $i > \theta_C$, as shown in the figure then the angle of deviation is given by $\delta = \pi - 2i$. The maximum value of δ occurs when $i = \theta_C$ and is equal to $\delta_{max} = \pi - 2\theta_C$

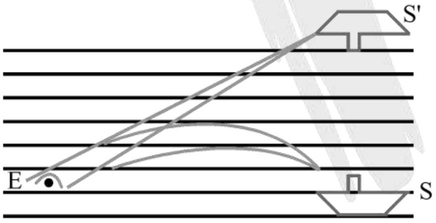


The variation of 'δ' with the angle of incidence 'i' is plotted in figure.



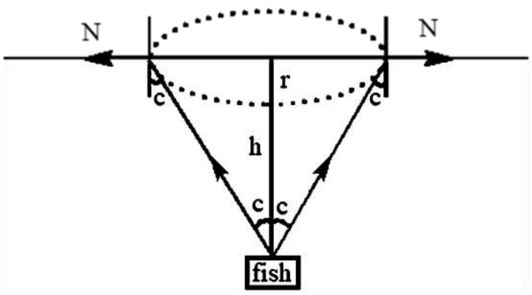
- **Looming:**

This phenomenon is known as superior mirage and it occurs when there is a strong temperature inversion where the air near the ground is cooler than the air above it. In this situation, light rays from an object such as a ship S are bent downwards, making the object appear higher than its actual position S'. When these rays reach the observer's eye, they appear to be coming from a higher position, creating the illusion that the ship is floating in the air. This effect is often observed in cold regions near the surface of a sea or lake.



- **Field of vision of fish:**

A fish at a depth 'h' from the surface of water of refractive index μ can see the outer world through an inverted cone with



- Vertex angle $= 2C$
- Radius of the circular base of the cone formed on surface of water is given by

$$\left[r = \tan C = \frac{h}{\sqrt{\mu^2 - 1}} \right]$$

$$\left[\because \sin C = \frac{1}{\mu},\ \tan C = \frac{1}{\sqrt{\mu^2 - 1}} \right]$$

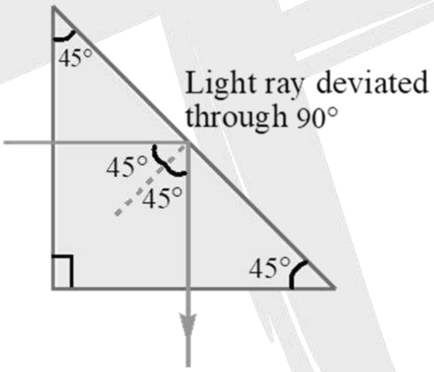
- For water $2C = 98°$ and $r = \frac{3h}{\sqrt{7}}$

(d) Area of the base: $A = \frac{\pi h^2}{\mu^2 - 1}$

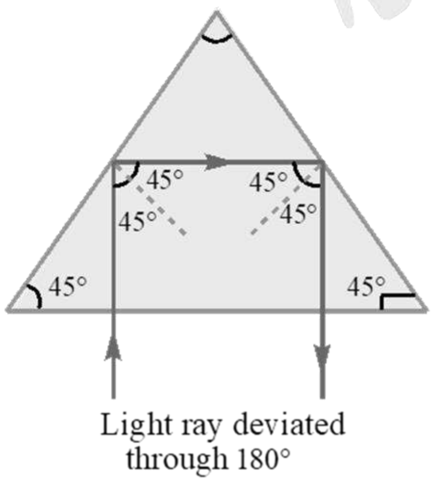
- **Total internal reflection in Prisms:**

The critical angle of ordinary glass is very nearly $42°$. If light is incident inside a prism at an angle greater than $42°$, then the light will be totally internally reflected. This is achieved by taking a right prism ($90°$ prism) so that the other angles of the prism are $45°$ each.

- **Deviation through** $90°$**:**

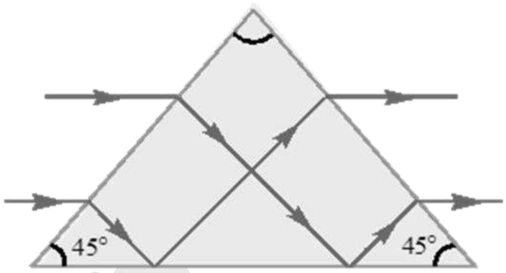


- **Deviation through** $180°$**:**



- **Erecting prism (No deviation Prism):**

  In this scenario, the incident rays of light are parallel to the base of the prism. When these rays are refracted and reach the hypotenuse face of the prism, they make an angle greater than the critical angle (42^0). As a result, total internal reflection occurs, and the rays emerge from the prism parallel to the base.

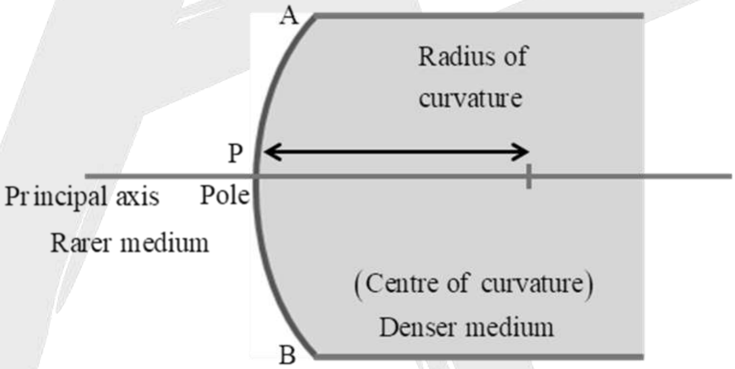


However, on emergence the rays are inverted. Therefore, this prism is used for making an inverted image erect.

Refraction at spherical surfaces and by Lenses

- A part of a sphere of refracting material is called a spherical refracting surface.

Some important terms related to spherical refracting surface are given below:



- The point 'P' in the figure is the pole (P)
- The spherical refracting surface is part of a sphere, and the center of this sphere is known as the center of curvature (C) of the refracting surface.
- The spherical refracting surface is part of a sphere, and the center of this sphere is known as the center of curvature (C) of the refracting surface.
- The aperture of a spherical refracting surface is the diameter of the surface. In the given figure, the line connecting points A and B represents the aperture of the spherical refracting surface.
- The principal axis is the line that passes through the pole of the spherical refracting surface and the center of curvature and extends on both sides of the surface.

  All distances related to the spherical refracting surface are measured from its pole. Distances measured in the direction of the incident light are considered positive.

Refraction at spherical surfaces:

When light undergoes refraction at a spherical interface between two transparent media, the normal at the point of incidence is perpendicular to the tangent plane to the spherical surface at that point. As a result, the normal passes through the center of curvature of the sphere.

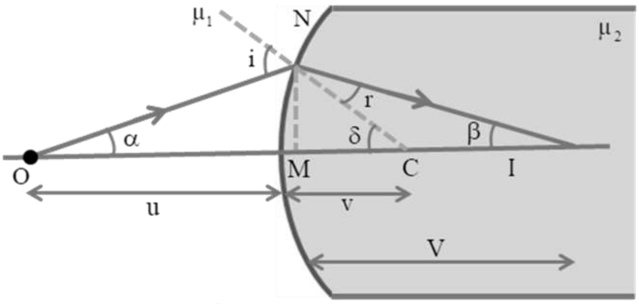


Figure shows the geometry of formation of image I of an object 'O' on the principal axis of spherical surface with centre of curvature C, and radius of curvature R. the rays are incident from a medium of refractive index μ_1 to another of refractive index μ_2. As before, we take like in curved mirrors the aperture of the surface to be small compared to other distances involved. Hence NM will be taken to be nearly equal to the length of the perpendicular from the point N on the principal axis.

$$\tan\alpha = \frac{MN}{OM},\ \tan\delta = \frac{MN}{MC},\quad \tan\beta = \frac{MN}{MI}$$

Now for ΔNOC, 'i' is the exterior angle.

Therefore, $i = \alpha + \delta$

Similarly $r = \angle NCM - \angle NIM = \delta - \beta$

Now by Snell's law $\mu_1.\sin i = \mu_2.\sin r$ or for small angles $\mu_1.r = \mu_2.r$ substituting i & r, we get

$$\mu_1(\alpha+\delta) = \mu_2(\delta-\beta)$$

$$\mu_1\alpha + \mu_2\beta = \delta(\mu_2 - \mu_1)$$

$$\frac{\mu_1}{OM} + \frac{\mu_2}{MI} = \frac{\mu_2-\mu_1}{MC}$$

Here OM, MI and MC represent magnitude of distances, applying sign conversions.

$$OM = -u,\quad MI = +v,\ MC = +R$$

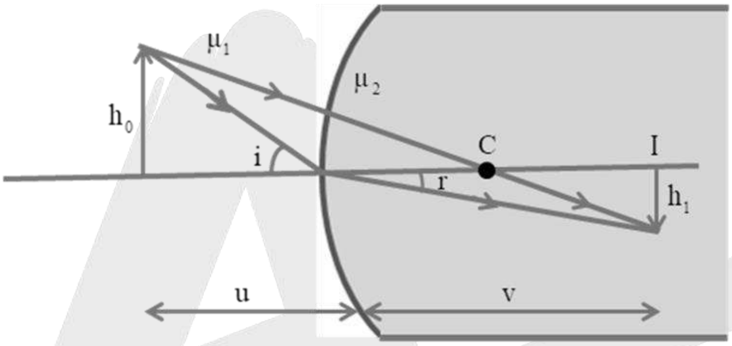
$$\therefore\ \frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2-\mu_1}{R}$$

This is the Gaussion's relation for a single spherical refracting surface. Though above relation is derived for a convex surface and for a real object and real image, it is equally valid for all other conditions.

- If we move in the direction of light, μ_1 is the refractive index of the medium which comes before the boundary and μ_2 is the refractive index of the medium which comes after the boundary
- When an object or image is present at a refracting surface, refraction at that surface is not taken into account.
- It is note that with respect to real object convex refracting surfaces can form real image (for distant object) as well as virtual image (for nearer object), where as concave refracting surface forms only virtual image.

Magnification:

- **Lateral magnification or transverse magnification:**



From figure, the lateral magnification is $m_t = \frac{h_i}{h_0}$

From Snell's law:

$\mu_1 i = \mu_2 r$ (for small angles)

Therefore, $\mu_1 \frac{h_0}{u} = \mu_2 \frac{h_i}{v}$

Thus, lateral magnification $m_t = \frac{h_i}{h_0} = \frac{\mu_1}{\mu_2} \cdot \left(\frac{v}{u}\right)$

- **Longitudinal magnification at refracting curved surface:**

If a small object of length 'du' is placed on the axis, produces an image of length 'dv' along the axis of the refracting surface, then longitudinal magnification

$m_L = \frac{dv}{du}$; since $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$

On differentiating, $\frac{dv}{du} = \frac{\mu_1}{\mu_2} \cdot \frac{v^2}{u^2}$ $\quad\quad \therefore\ m_L = \frac{\mu_1}{\mu_2} \frac{v^2}{u^2}$

Longitudinal magnification $m_L = m^2 \left(\frac{\mu_2}{\mu_1}\right)$

Where 'm_t'is traverses magnification.

Motion of Object:

- **Along the principal axis:**

Since $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$

As the object position changes with time, the image position also changes.

Hence $-\frac{\mu_2}{v^2}\frac{dv}{dt} + \frac{\mu_1}{u^2}\frac{du}{dt} = 0,\ \frac{\mu_2}{v^2}v_1 + \frac{\mu_1}{u^2}v_0 = 0 \qquad \therefore\ V_i = \left(\frac{\mu_1}{\mu_2}\right)\left(\frac{v}{u}\right)^2 v_0$

v_i = (longitudinal magnification) v_0

$v_i = m_L v_0$

On applying proper sign conversion, we get direction of motion of image.

- **Along perpendicular (Transverse) to the Principal Axis:**

If the object moves transverse to the principal axis with the speed V_0. If m is the magnification, then

$$m_t = \frac{\mu_2}{\mu_1}\left(\frac{v}{u}\right) = \frac{h_i}{h_0} = \frac{\frac{dh_i}{dt}}{\frac{dh_0}{dt}} \qquad \therefore \text{ velocity of image is } V_i = \frac{\mu_2}{\mu_1}\left(\frac{v}{u}\right).V_0 \text{ or } V_i = m_t V_0$$

- **Principal FOCI:**

In the equation $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$

- If the object at infinity i.e., $u = \infty$

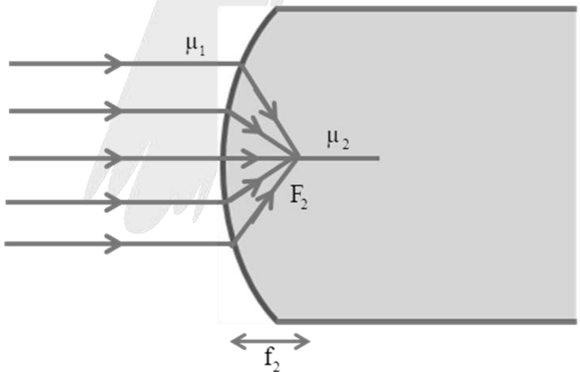
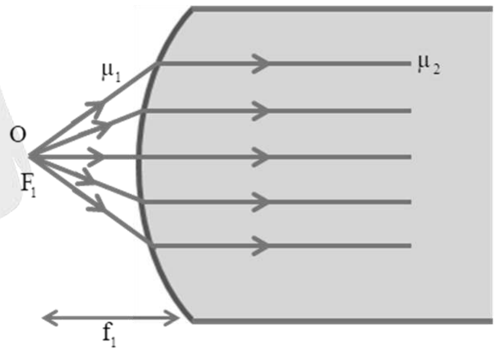


Figure (a)



Figure(b)

$$\frac{\mu_2}{v} - 0 = \frac{\mu_2 - \mu_1}{R}$$

From figure, it is clear that $v = f_2 \qquad \therefore f_2 = \frac{\mu_2 R}{\mu_2 - \mu_1}$

i.e., The position of image corresponding to the object at infinity, is called the second principal focus of the refracting surface. This is shown in figure (a)

- Similarly if $v=\infty$, i.e., the object is so placed that the refracting rays becomes parallel to the principal axis, then $\frac{\mu_2}{\infty}-\frac{\mu_1}{u}=\frac{\mu_2-\mu_1}{R}$

  From figure, it is clear that $u=f_1$ $\quad \therefore f_1=\frac{\mu_1 R}{\mu_2-\mu_1}$

  i.e., the position of the object, whose image is formed at infinity to known as the first principal focus of the refracting surface. This is shown in figure (b).

  hence $\frac{f_1}{f_2}=\frac{\mu_1}{\mu_2}$

  it is easy to see that first focal length f_1 for spherical refracting surface is not equal to the second focal length f_2.

  Further $\frac{\mu_2}{v}-\frac{\mu_1}{u}=\frac{\mu_2-\mu_1}{R}$ $\quad \therefore \frac{\mu_2 R}{v(\mu_2-\mu_1)}-\frac{\mu_1 R}{u(\mu_2-\mu_1)}=1$ $\quad \therefore \frac{f_2}{v}+\frac{f_1}{u}=1$

- **Power of Refracting spherical surface:**

  A distance that is divided by the refractive index of the medium in which it is measured is referred to as a reduced distance. The refracting power of a spherical surface is difined as the reciprocal of the reduced focal length. If f_1 and f_2 are first and second principal focal length of refracting surface.

  The ratios $\frac{f_1}{\mu_1}$ and $\frac{f_2}{\mu_2}$ are the reduced focal lengths.

  Then refracting power $P=\frac{\mu_1}{f_1}=-\frac{\mu_2}{f_2}$ (As $\frac{f_1}{\mu_1}=-\frac{f_2}{\mu_2}$)

  Power (P) $=\frac{\mu_2-\mu_1}{R}\left(\because f_1=-\frac{\mu_1 R}{\mu_2-\mu_1}\right) \Rightarrow \frac{\mu}{V_1}-\frac{1}{-x}=0(\because R=\infty) \Rightarrow V_1=-\mu x$

  Here, I_1 behaves like object for pole P_2 (Plane mirror): for this I_2 is the image from refraction through the plane mirror.

  Again for pole P_1, I_2 behaves as an object, then apply

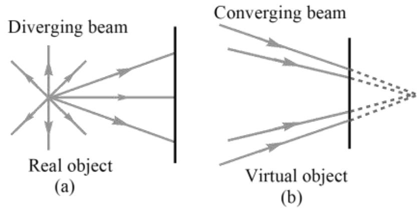
  $\frac{\mu_2}{V}-\frac{\mu_1}{u}=\frac{\mu_2-\mu_1}{R}$ (for pole P_1)

  $\Rightarrow \frac{1}{V_3}-\frac{\mu}{-(\mu x+2t)}=0\ (\because R=\infty)$

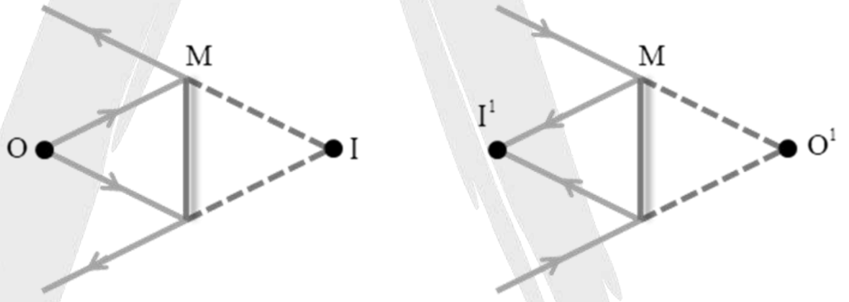
  $\Rightarrow V_3=-\frac{(\mu x+2t)}{\mu}$ (This is the final image formed by the combination)

- **Real and virtual object:**

  If a surfac is incident with a divergent beam, it means a real object is placed infront of the surface at the position where the rays are divergent as shown in figure (a)



- If a surface is incident with a converging beam, it means a virtual object is placed behind the surface at the position where the rays appear to converge as shown in figure (b)
- In case of image formation unless stated object is taken to be real, it may be point object denoted by dot $(\bullet)$ or extended and is denoted by an arrow $(\uparrow)$
- Real and virtual image: The optical image is a point, where the rays of light either interesect or appear to intersect.
- If the real rays after reflection or refraction actually converge at a point, the image is said to be real as shown in figure (a)
- When rays of light appear to converge but do not actually converge, the resulting image is called a virtual image.This is as shown in the figure (b).



$O \rightarrow$ Real object
$I \rightarrow$ Virtual object

$O \rightarrow$ Virutal object
$I \rightarrow$ Real object

Note: A virtual image can be seen by the eye, but it cannot be projected onto a screen or photographed. In contrast, a real image can be captured on a screen or photographed as it contains light energy.

Refraction by Lenses

- **Lenses theory:** A lens is an optical device made of transparent material with at least one curved refracting surface and a refractive index different from that of the surrounding medium. When the curved surfaces of a lens are spherical, it is referred to as a spherical lens, and if its thickness is small compared to the radii of curvature of the lens surfaces, it is called a thin lens.

  Here we shall limit ourselves to thin spherical lenses.

Different types of Spherical lens are shown in figure (a) and (b)

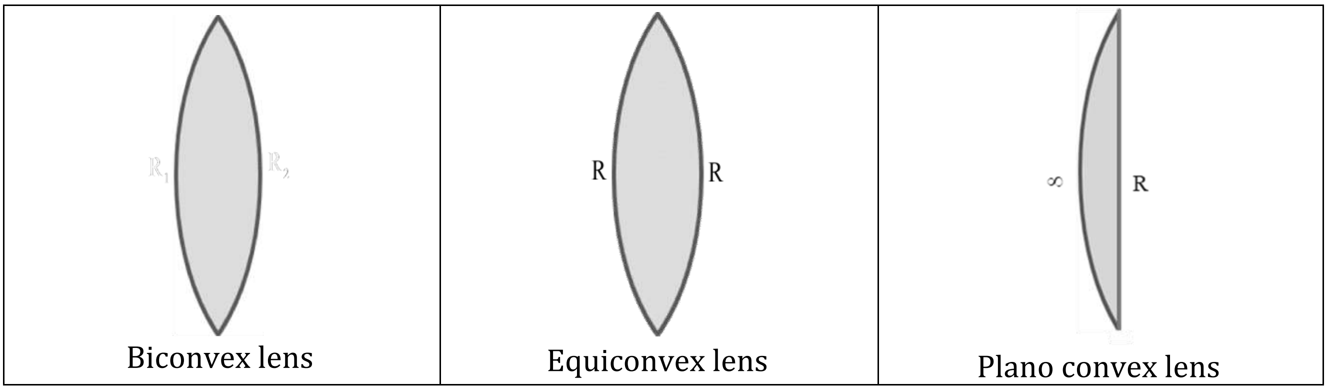


Fig. (a) Convex Lens

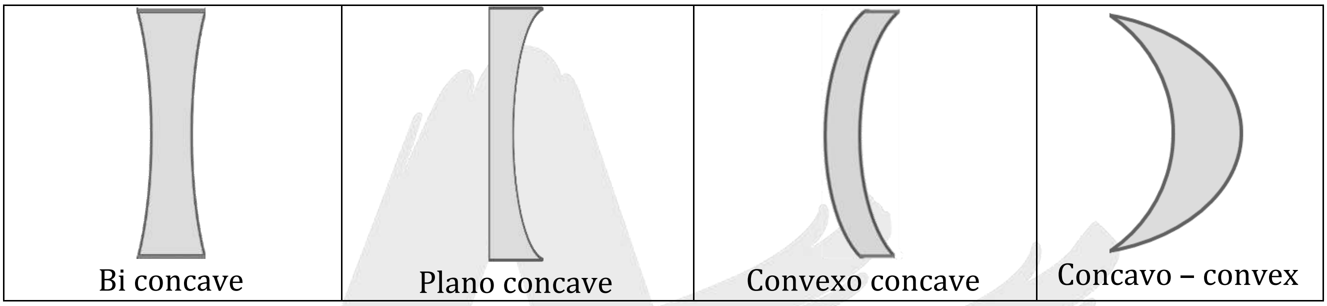


Fig. (b) Concave Lens

Fig. (c) Meniscus

Note: while calling the name of the lens we called first the shape of the surface which has more radius of curvature is to be considered.

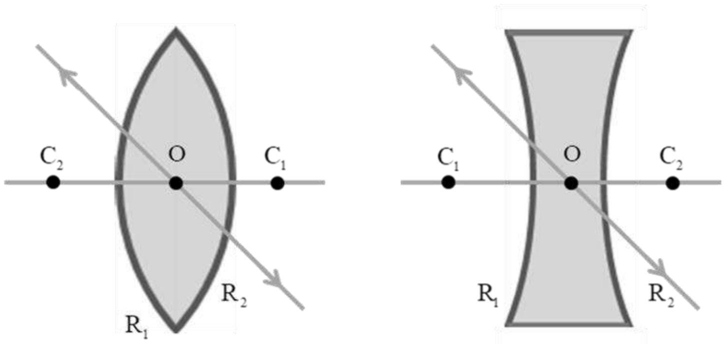
➢ A thin lens with a refractive index greater than that of its surroundings acts as a converging or convex lens when its central portion is thicker than the marginal portion. This means that it will converge parallel rays incident on it.

➢ If the central portion of a lens (with $\mu_L > \mu_m$) is thinner than marginal, it diverges parallel rays and behaves as divergent or concave lens

Note: A thin lens is characterized by having a thickness much smaller than the object distance, the image distance, or either of the two radii of curvature of the lens.

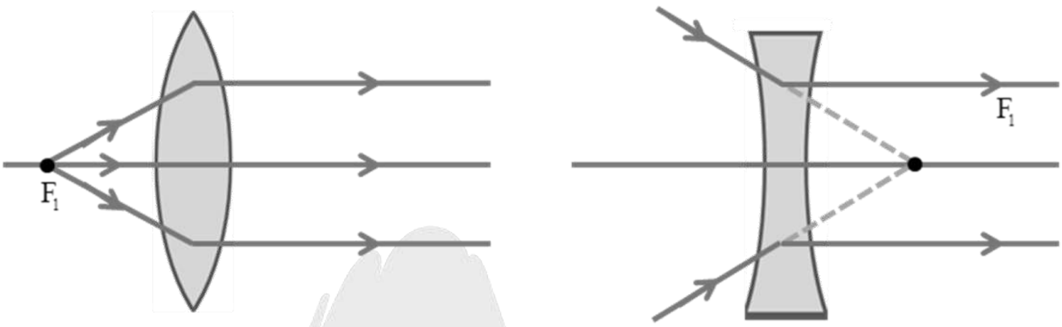
➢ **In case of thin spherical lenses:**

Optical centre (or) pole O is a point for a given lens through which any ray passes undeviated.



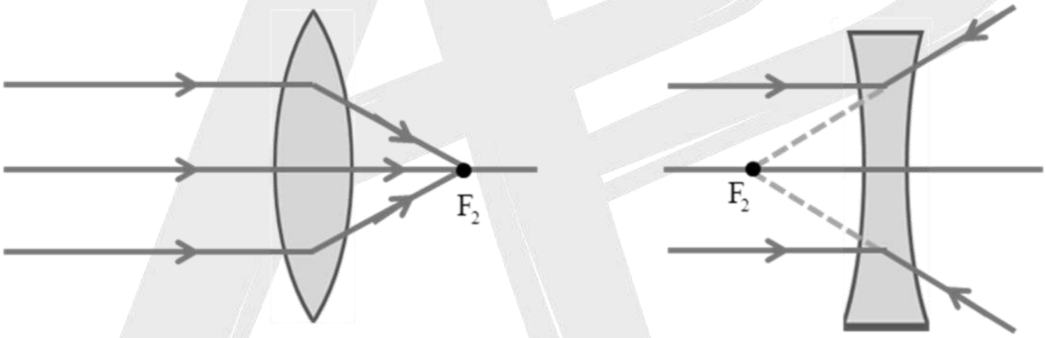
If the lens has two spherical surfaces, there are two centres of curvature C_1 and C_2 and correspondingly two radii of curvature R_1 and R_2.

- The principal axis (C_1C_2) of a lens is a line that passes through the optical center and the centers of curvature of the two refracting surfaces. It is perpendicular to the lens.
- A lens has two surfaces and thus has two focal points. The first focal point (F_1) is the object point on the principal axis where an image is formed at infinity, which is real in the case of a convex lens and virtual for a concave lens.



(a) (b)

Second focal point (F_2) is an image point on the principal axis for which object lie at infinity



(a) (b)

- The distance between optical centre of a lens and the point where the parallel beam of light converges or appears to converge. i.e., second principal focal point (F_2), is called focal length f.
- To a lens, if the media on the two sides is same, then first principal focal distance is equal to second principal focal distance. i.e., $|f_1| = |f_2|$

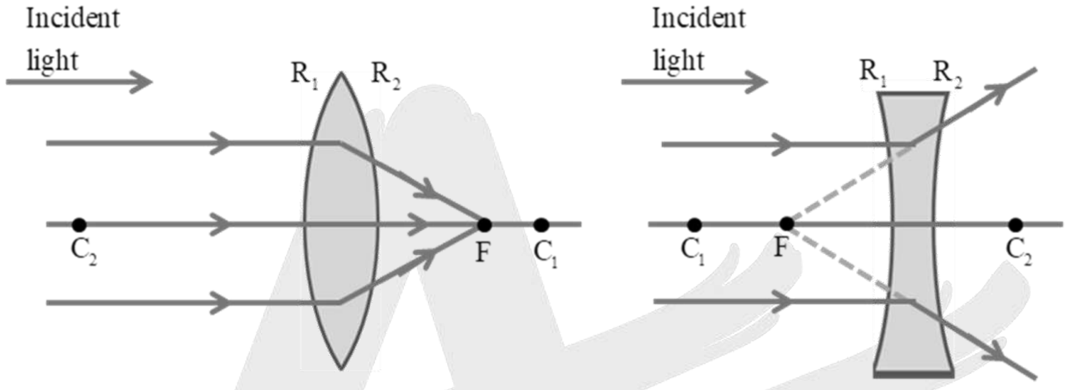
Note: We are mainly concerned with the second focus F_2 because wherever we write the focal length 'f' measures second principal focal length.

- **Focal plane:** The plane perpendicular to the principal axis and passing through the principal focus is called the focal plane.
- **Aperture:** To a lens, aperture refers to the effective diameter of the area that transmits light. The intensity of the image formed by a lens is dependent on the square of the aperture.i.e., $I \alpha (\text{apeture})^2$

➢ **Sign convention:**

Whenever it is feasible, light rays are assumed to travel from left to right. Positive values are assigned to transverse distances measured above the principal axis, and negative values are assigned to distances measured below it. Longitudinal distances are measured from the optical center and are taken as positive in the direction of light propagation and negative in the opposite direction.

Note: While using the sign convention it must be kept in mind that, to calculate an unknown quantity the known quantities are substituted with sign in a given formula.



(a) (b)

For convex lens as shown in figure (a).

$R_1(=OC_1)$ is +ve; $R_2(=OC_2)$ is -ve

$f=(OF)$ is +ve

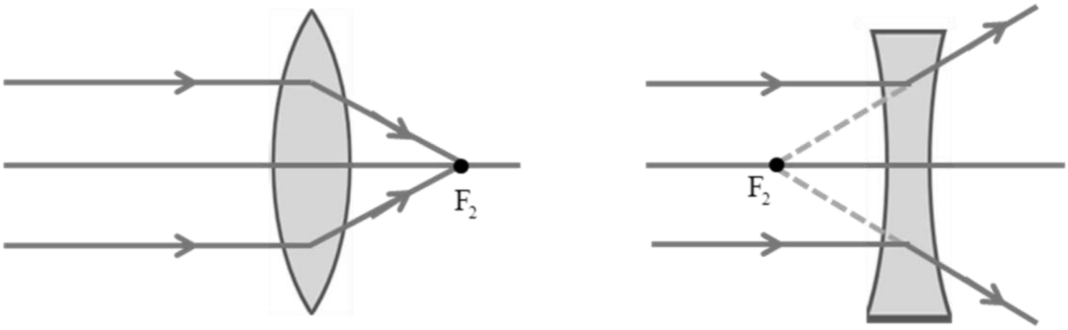
For concave lens as shown in figure (b)

$R_1(=OC_1)$ is -ve; $R_2(=OC_2)$ is +ve

$f=(OF)$ is -ve

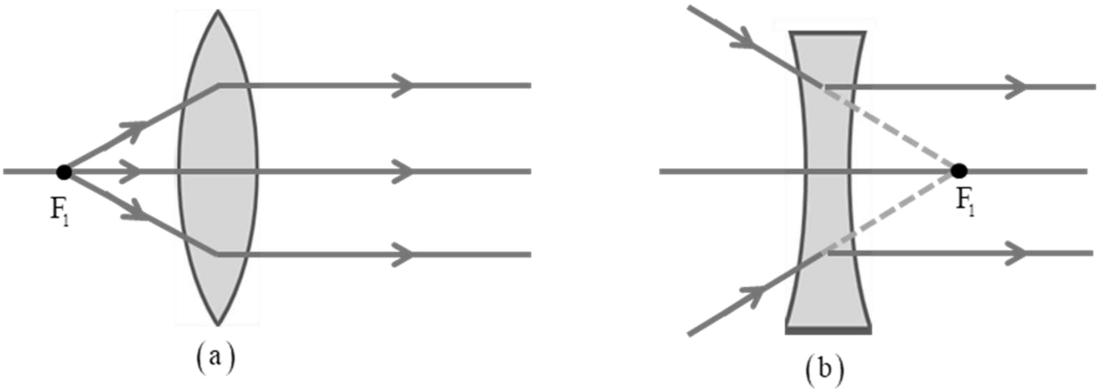
➢ **Rules for image formation:** To graphically determine the location and characteristics of an image formed by a lens, the following rules are applied.

After refraction, a ray that is parallel to the principal axis of a convex lens passes through the principal focus, while in the case of a concave lens, it appears to diverge from the focus.



(a) (b)

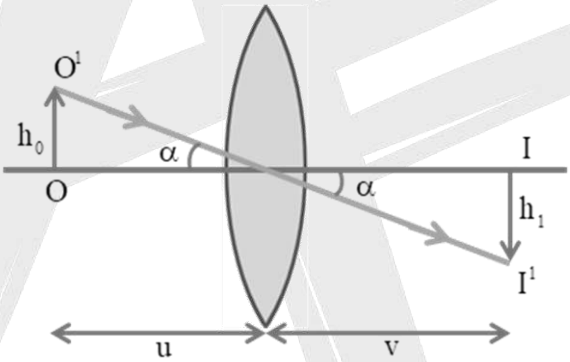
- A ray passing through the first focus F_1 becomes parallel to the principal axis after refraction.



Magnification:

- **Lateral magnification:** The magnification produced by a lens is determined by the ratio of the size of the image to the size of the object, with both sizes being measured perpendicular to the principal axis.

$$m_t = \frac{II'}{OO'} = \frac{h_1}{h_0} = \frac{v}{u}$$



When we apply the sign convention, for erect (and virtual) image formed by a convex or concave lens 'm' is positive, while for an inverted (and real) image, m is negative.

Note: Linear magnification for a lens can also be expressed as $m = \frac{I}{O} = \frac{v}{u} = \frac{f-v}{f} = \frac{f}{f+u}$

- **Longitudinal magnification:**

Longitudinal magnification is defined as the ratio of infinitesimal axial length (dv) in the region of the image to the corresponding length (du) in the region of the object.

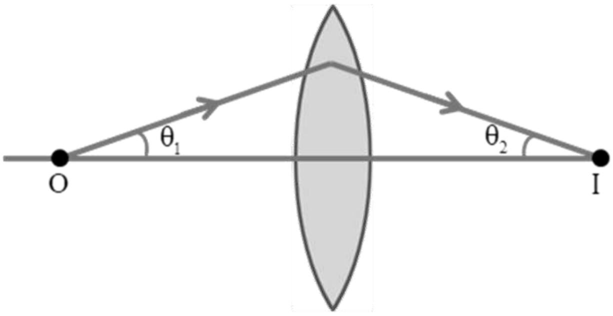
Longitudinal magnification $(m_L) = \frac{dv}{du}$

On differentiating equation $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ $-\frac{dv}{v^2} - \left(-\frac{du}{u^2}\right) = 0$ or $\frac{dv}{v^2} - \frac{du}{u^2} = 0$

Therefore, $m_L = \frac{dv}{du} = \frac{v^2}{u^2} = \left(\frac{v}{u}\right)^2 = m^2$

So, longitudinal magnification is proportional to the square of the lateral magnification.

➢ **Angular magnification of lens:** The ratio of the slopes of emergent ray and corresponding incident ray with principal axis is called the angular magnification.



Angular magnification $(\gamma) = \frac{\tan\theta_2}{\tan\theta_1}$

Note: when several lenses are used co-axially, the total magnification

$m = m_1 \times m_2 \times ......m_n$

From the ray diagrams it is clear that

➢ **Regarding convex lens:**

When a convex lens is used to form an image, a real image is formed for a real object when the object is placed beyond the focal point. If the object is placed within the focal point, a virtual image is formed for the real object. The real image formed by the convex lens is always inverted, while the virtual image is always erect.

➢ **Regarding concave lens:**

A concave lens always produces a virtual image for a real object. The virtual image formed by a concave lens is always upright and smaller in size. However, if the object is virtual, a concave lens can form both a real and virtual image.

➢ **Lens formula:**

Lens formula is a relation connecting focal length of the lens with the object distance and image distance.

The formula is $\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$

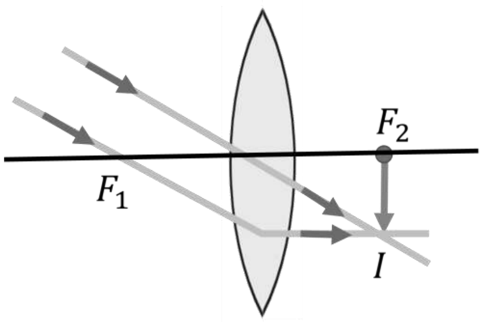
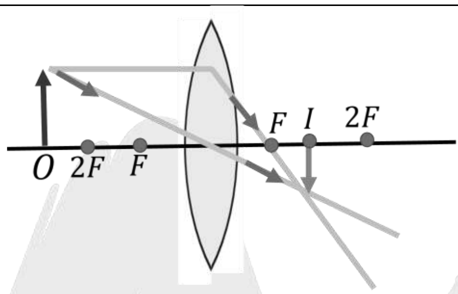
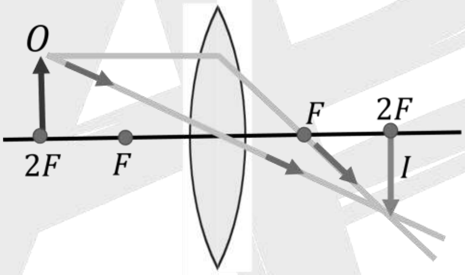
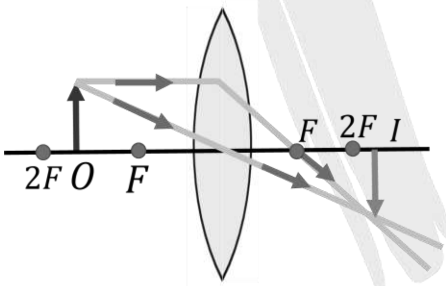
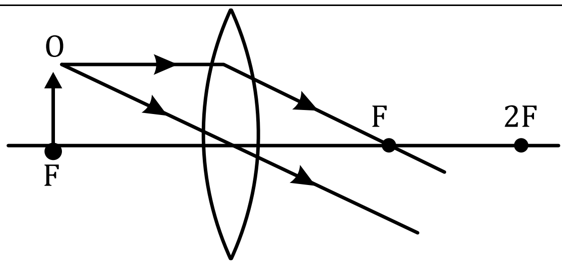
Note: The above formula is applicable to both convex and concave lenses and is independent of the image's nature (real or virtual).

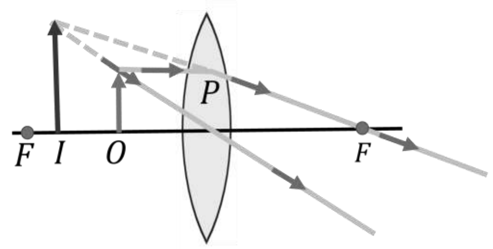
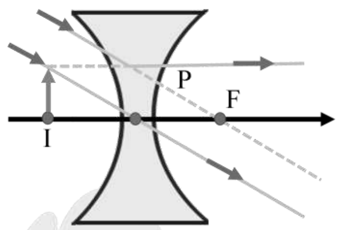
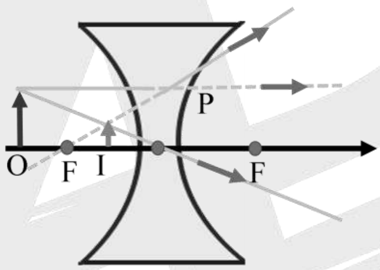
Note: To solve the problems, the above equation can also be expressed as follows

$$v = \frac{uf}{u+f},\ u = \frac{vf}{f-v},\ f = \frac{vu}{u-v}$$

(a) Convex lens &

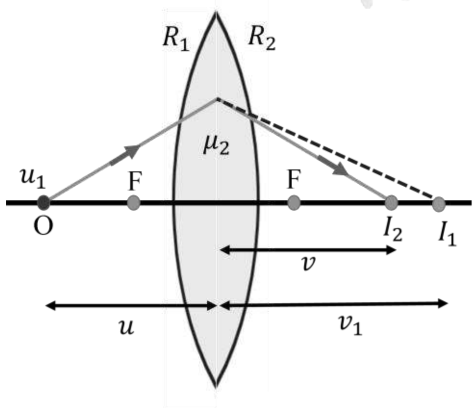
(b) Concave lens

Position of the object	Ray diagram	Image details
At infinity		Real inverted, diminished at F
Between ∞ and 2F		Real, inverted, diminished between F and 2F
At 2F		Real, inverted, equal, at 2F
Between 2F and F		Real, inverted, enlarged, between 2F and infinity
At F		Real, inverted, enlarged at infinity

Between F and P		Virtual, erect, enlarged and on the side of the object.
At infinity		Virtual, erect, diminished at F
Infront of mirror		Virtual, erect, diminished Between F and P

Lens maker's formula and lens formula:

When an image is formed by a lens, the incident ray is refracted at both the first and second surface of the lens. The image formed by the first surface of the lens acts as an object for the second surface to form the final image. Let us consider an object O placed at a distance u from a convex lens as shown in the figure. Let its image is I_1 after refraction through first surface. So from the formula for refraction at curved surface.



$$\frac{\mu_2}{v}-\frac{\mu_1}{u}=\frac{\mu_2-\mu_1}{R}$$

For first surface $\frac{\mu_2}{v_1}-\frac{\mu_1}{u}=\frac{\mu_2-\mu_1}{R_1}$,,,(1)

The image I_1 is acts as object to second surface, and form final image I_2

For second surface $\frac{\mu_1}{v}-\frac{\mu_2}{v_1}=\frac{\mu_1-\mu_2}{R_2}$ ...(2)

So adding (1) and (2) equation, we have

$$\mu_1\left[\frac{1}{v}-\frac{1}{u}\right]=(\mu_2-\mu_1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right) \text{ or } \left(\frac{1}{v}-\frac{1}{u}\right)=\left(\frac{\mu_2}{\mu_1}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

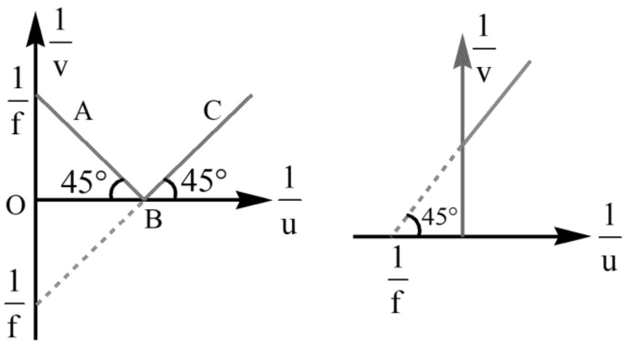
$$\frac{1}{v}-\frac{1}{u}=(\mu_r-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

With $\mu_r=\frac{\mu_2}{\mu_1}(\text{or})\ \frac{\mu_L}{\mu_M}$

- If object is at infinity, image will be formed at the focus i.e., for $u=-\infty,\ v=f$, so that above equation becomes $\frac{1}{f}=(\mu_r-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$ which is known as lens – maker's formula and
- For a lens it becomes $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$ which is known as the "lens-formula" or "Gauss's formula" for a lens.
- Though we derived it for a real image formed by a convex lens, the formula is valid for both convex as well as concave lens and for both real and virtual images.

Note: The lens maker's formula is applicable for thin lenses only and the value of R_1 and R_2 are to be put in accordance with the Cartesian sign convention.

- $\frac{1}{v}$ **and** $\frac{1}{u}$ **Graphs:**
- **Convex lens:** The graph between $\frac{1}{v}$ and $\frac{1}{u}$ in case convex lens is as shown in figure.



For real image:

$$\frac{1}{v}-\frac{1}{(-u)}=\frac{1}{f}; \quad \frac{1}{v}=-\frac{1}{u}+\frac{1}{f}$$

It is a straight line with slope – 1, for virtual image

$$\frac{1}{(-v)}-\frac{1}{(-u)}=\frac{1}{f}; \quad \frac{1}{v}=\frac{1}{u}-\frac{1}{f}$$

It is a straight line with slope +1. Hence AB line when the image is real. BC line when the image is virtual.

- **Concave lens:** The graph between $\frac{1}{v}$ and $\frac{1}{u}$ in case of concave lens as shown in figure.

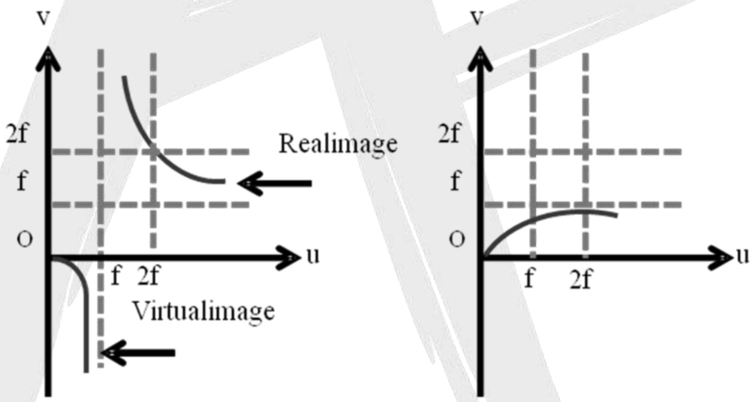
  Since concave lens only from virtual image,

$$\frac{1}{-v}-\frac{1}{-u}=-\frac{1}{f}; \quad \frac{1}{v}=\frac{1}{u}+\frac{1}{f}$$

  It is a straight line with slope +1.

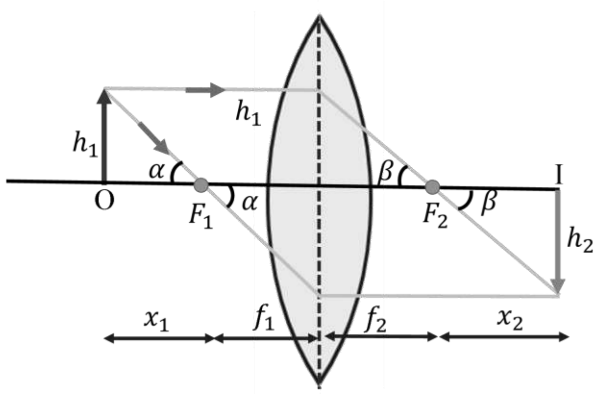
U and V graph:

- **Convex lens:** The graph between v and u is hyperbola to convex lens as shown in figure.



- **Concave lens:** The graph between v and u is hyperbola to concave lens as shown in figure.

  In case of thin convex lens if an object is placed at a distance x_1 from first focus and its image is formed at a distance x_2 from the second focus.



From properties of triangles, to the left of the lens

$\frac{h_1}{x_1} = \frac{h_2}{f_1}$ To the right of the lens

$\frac{h_1}{f_2} = \frac{h_2}{x_2}$

From above two equations $\frac{x_1}{f_1} = \frac{x_2}{f_2}$

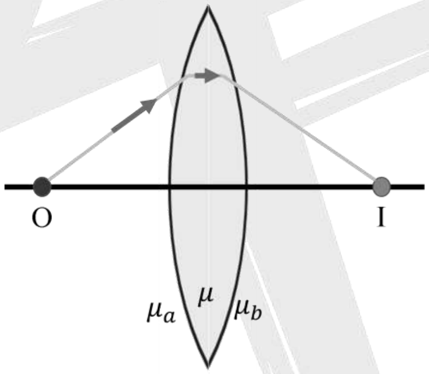
$\therefore x_1x_2 = f_1f_2$

For $f_1 = f_2$

$x_1x_2 = f^2$ is called Newton's formula or lens user formula. This relation can also prove by using lens formula.

- Lenses with different media on either side

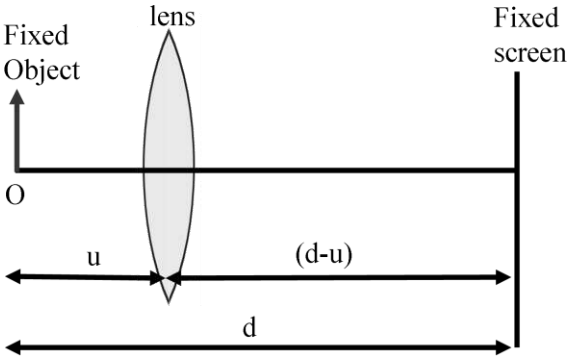
Consider a lens made of a material with refractive index μ with a liquid μ_a on the left and a liquid μ_b



The governing equation for this system is

$$\frac{\mu_b}{v} - \frac{\mu_a}{u} = \frac{\mu - \mu_a}{R_1} + \frac{\mu_b - \mu}{R_2}$$

- **<u>Determination of the focal length of a convex lens (or) size of the object by "LENS DISPLACEMENT METHOD".</u>**



The displacement method is a laboratory technique used to determine the focal length of a convex lens. It involves placing the object at a fixed distance from the lens and moving the screen to different positions until two distinct and equally sized images of the object are formed. If the distance between the object and screen is greater than four times the focal length of the lens, then there will be two possible positions of the lens that will produce the two images.

If the object is at a distance u from the lens, the distance of image from the lens $v=(d-u)$, so from lens formula $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$ or $\frac{1}{d-u}+\frac{1}{u}=\frac{1}{f}$ i.e., $u^2-du+df=0$

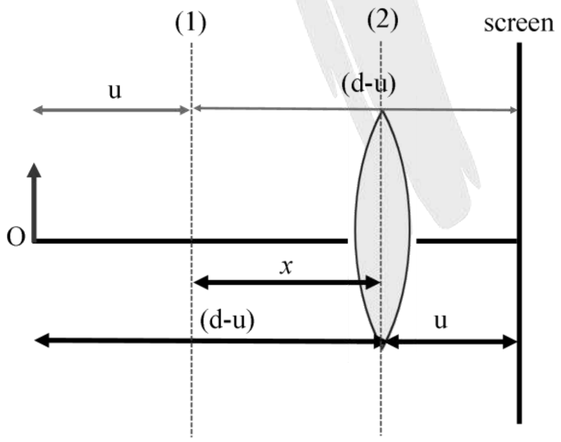
So that $u=\frac{d\pm\sqrt{d(d-4f)}}{2}$

Now there are three possibilities.

- If, $d<4f$, u will be imaginary, so physically no position of lens is possible.
- If $d=4f$, in this $u=\frac{d}{2}=2f$ so only one position is possible and in this $v=2f$. That is why the minimum separation between the real object and real image is 4f
- If $d>4f$, $u_1=\frac{d-\sqrt{d(d-4f)}}{2}$ and $u_2=\frac{d+\sqrt{d(d-4f)}}{2}$ for these two positions of the object real images are formed for $u=u_1$, $v=d-u_1=\frac{d+\sqrt{d(d-4f)}}{2}=u_2$

For $u=u_2$, $v_2=d-u_2=\frac{d-\sqrt{d(d-4f)}}{2}=u_1$

i.e., for two positions of the lens object and image distances are interchangeable as shown in the figure.



So the magnification for the both positions of the object are related as $m_1=\frac{1}{m_2}$ i.e., $m_1.m_2=1$

$$\therefore m_1m_2=\frac{I_1}{O}.\frac{I_2}{O}=\frac{I_1I_2}{O^2}=1$$

Therefore $O=\sqrt{I_1I_2}$ where I_1 & I_2 are the sizes of images for two positions of the object and O is size of the object.

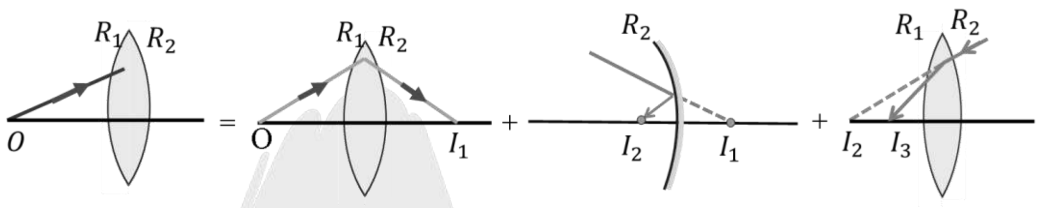
➢ It means that the size of object is equal to the geometric mean of the two images. This method of measuring the size of the object is useful when the object inaccessible.

➢ If 'x' is the distance between the two positions of the lens. Then $f = \frac{x}{m_1 - m_2}$

Lens with one silvered surface

➢ When the back surface of a convex lens is silvered

The rays are first refracted by lens, then refracted from the silvered surface and finally refracted by lens, so that we get two refractions and one reflection.



In the diagram if f_1 and f_m are respective the focal lengths of lens and mirror. Then

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_m} + \frac{1}{f_1} = \frac{2}{f_1} + \frac{1}{f_m}$$

In terms of focal powers of the lens and mirror

$$P = P_\ell + P_m + P_\ell = 2P_\ell + P_m \text{ with } P_\ell = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \text{ and } P_m = \frac{2}{R_2}$$

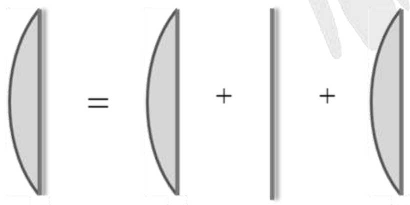
Here P_ℓ and P_m are substituted with sign.

➢ The system will behave as a concave mirror if 'P' is positive and

➢ The system will behave as a convex mirror if "P" is negative.

The replacement with the mirror is due to overall reflection of given rays.

➢ When the plane surface of plano convex lens is silvered.



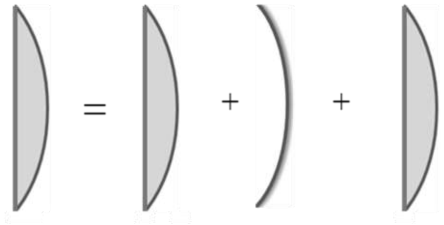
Then, the focal power of the given lens is $(P_m = 0)$

$$P = 2P_\ell + P_m$$

$$P = 2.\left(\frac{\mu - 1}{R}\right) + 0 = \frac{2(\mu - 1)}{R}$$

Since $\mu > 1$, 'P' is positive, the system behaves as a concave mirror with focal length $\frac{R}{2(\mu - 1)}$

- When curved surface of a plano convex lens is silvered.



Then, the focal power of the given lens is

$$P = 2P_{\ell} + P_m = \frac{2(\mu-1)}{R} + \frac{2}{R} = \frac{2\mu}{R}$$

Since 'P' is positive, the system behaves a concave mirror with focal length $\frac{R}{2\mu}$

- **<u>Lens maker's formula – special Cases</u>**

It relates the focal length of the lens to the refractive index of material of the lens and the radii of curvature of the two surfaces.

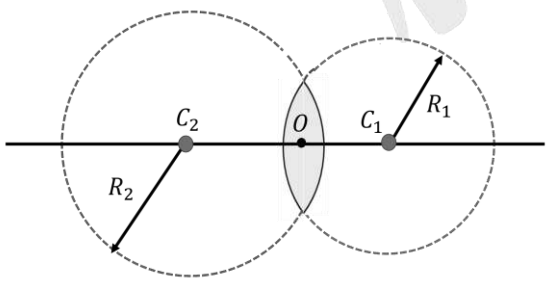
The formula is $\frac{1}{f} = \left(\frac{\mu_{lens}}{\mu_{medium}} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

Where μ_{lens} is the absolute refractive index of material of the lens, μ_{medium} is the absolute refractive index of the medium in which the lens is placed. R_1 and R_2 are the radii of curvature of two surfaces of the lens.

If the lens is placed in vacuum, then $\frac{1}{f} = (\mu_{lens} - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

The lens maker's formula is applicable for thin lenses only and the value of R_1 and R_2 are to be put in accordance with the Cartesian sign convention.

Note: For convex lens

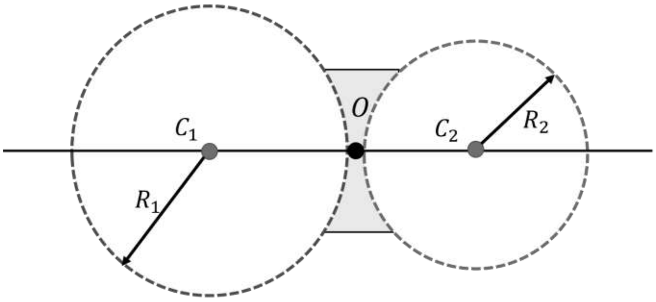


Convex lens

For convex lens R_1 is +ve and R_2 is -ve so the lens makers formula is $\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$

For equiconvex lens $\frac{1}{f} = (\mu - 1)\left(\frac{2}{R}\right)$

Note: For concave lens



For concave lens R_1 is -ve and R_2 is +ve so the lens makers formula is $\frac{1}{f} = (\mu - 1)\left(-\frac{1}{R_1} - \frac{1}{R_2}\right)$

For equiconcave lens $\frac{1}{f} = -(\mu - 1)\left(\frac{2}{R}\right)$

Note: For converging meniscus

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right), \text{ if } (R_1 < R_2)$$

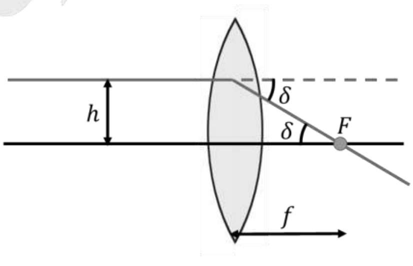
Note: For diverging meniscus

$$\frac{1}{f} = -(\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right), \text{ if } (R_1 < R_2)$$

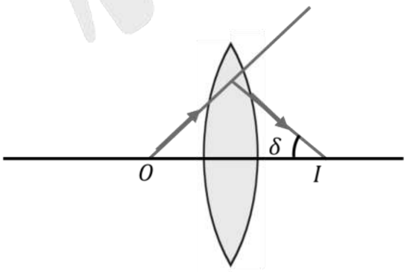
Note: For plano convex lens

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R}\right) \qquad \because R_2 = \infty$$

- **Power of A Lens :** The power of a lens is a measure of its ability to either converge or diverge a parallel beam of light. The power, denoted by P, is defined as the tangent of the angle at which the lens converges or diverges the beam of light that falls at a unit distance from the optical center.



(a)



(b)

$$\tan\delta = \frac{h}{f};\ \text{if } h = 1,\ \tan\delta = \frac{1}{f}$$

As per definition power $(P) = \tan\delta = \frac{1}{f}$

- If lens is placed in a medium other than air of refractive index μ. Then power $P = \frac{\mu}{f}$

  The S.I unit of power is diopter (D) and $1D = 1m^{-1}$

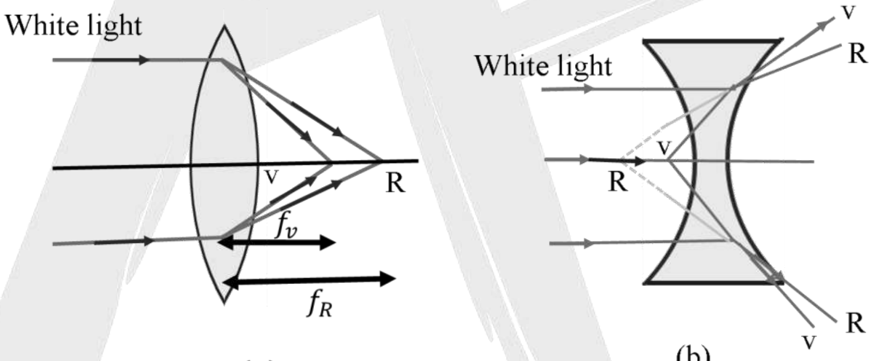
  i.e. $P = \frac{1}{f\,(\text{in m})} = \frac{100}{f\,(\text{in cm})} D$

- A convex lens has the ability to converge incident rays, which is why its power is assigned a positive value. In contrast, a concave lens has the ability to diverge incident rays, which is why its power is assigned a negative value.

Some important points regarding lens :

- Each part of a lens is capable of forming a complete image. When a portion of the lens is obstructed, the full image will still be formed, but its intensity will be reduced.

- The focal length of a lens depends on its refractive index i.e $\frac{1}{f} \propto (\mu - 1)$, so the focal length of a given lens is different for different wave lengths and maximum for red and minimum for violet whatever the nature of the lens as shown in figure.



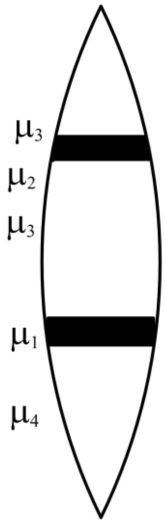
(a) (b)

Filling up of a lens :

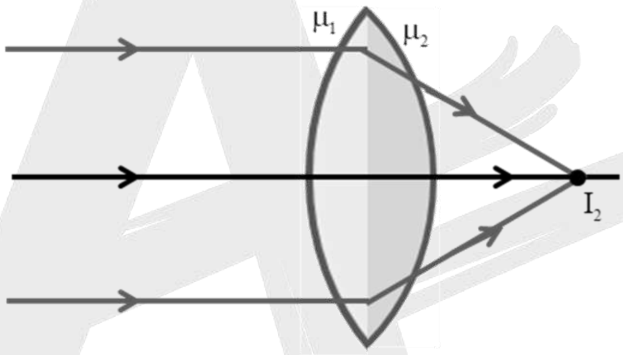
- If a lens made a number of layers of different refractive indices as shown in figure, for a given wavelength of light it will have many focal length as

$$\frac{1}{f} \propto (\mu - 1)$$

  Hence it will form many images as there are different μ's. According to given diagram number of images formed by lens is 4.



- If a lens is made of two or more materials and are placed side by side as shown in below, then there will be one focal length and hence one image



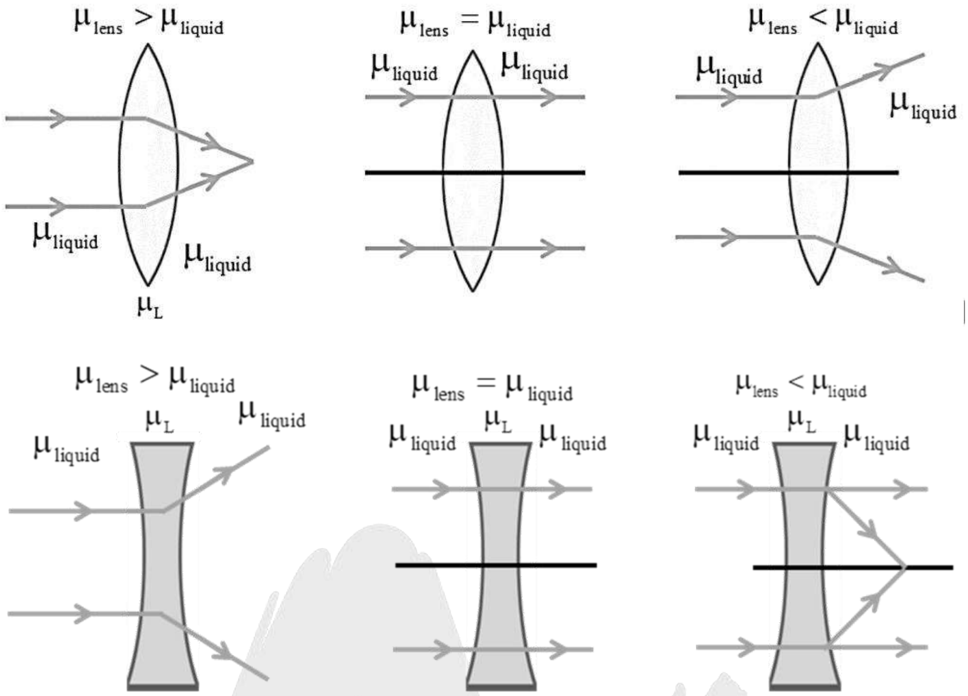
- **Lens immersed in a liquid :**

  If a lens made of material of refractive index μ_{lens} is immersed in a liquid of refractive index μ_{liquid}, if f_a is the focal length of a lens placed in air, then $\frac{1}{f_a} = (\mu_{lens} - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \to (1)$

  If f_1 is the focal length of lens immersed in a liquid then $\frac{1}{f_1} = \left(\frac{\mu_{lens}}{\mu_{liquid}} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \to (2)$

  $$\frac{(1)}{(2)} \Rightarrow \frac{f_1}{f_a} = \frac{(\mu_{lens} - 1)}{\left(\frac{\mu_{lens}}{\mu_{liquid}} - 1\right)}$$

- Depending upon the values of μ_{lens} and μ_{liquid}, we have three cases



(a) (b) (c)

Case (a) : If $\mu_{lens} > \mu_{liquid}$, then f_ℓ and f_a are of same sign and $f_\ell > f_a$ i.e. the nature of lens remains unchangged, but it's focal length increases and hence power of lens decreases.

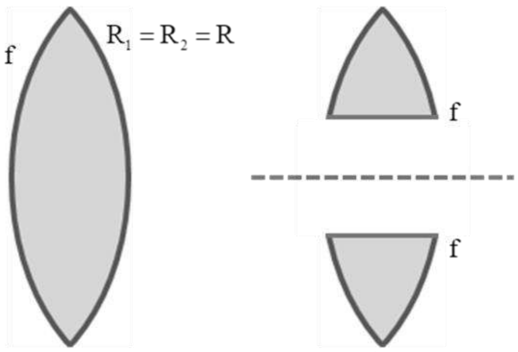
Case (b) : If $\mu_{lens} = \mu_{liquid}$ then $f_\ell = \infty$, i.e. the lens behaves as a plane glass plate and becomes invisible in the medium.

Case (c) : If $\mu_{lens} < \mu_{liquid}$, then f_ℓ and f_a have opposite sign and the nature of lens changes i.e. a convex lens diverges the light rays and concave lens converge the light rays.

If a thin lens has different radii of curvature for its two surfaces, the focal length remains the same regardless of which surface the light is incident upon.

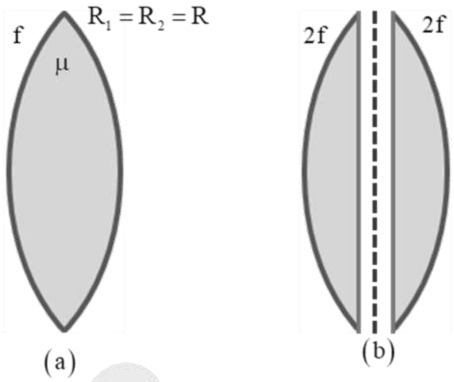
Cutting of a lens :

- If an equi convex lens of focal length 'f' is cut into two equal parts along its principal axis, as shown in figure, as none of μ, R_1 and R_2 will change, the focal length of each part will be equal that of initial lens, but intensity of image formed by each part will reduced to half.



(a) (b)

- When an equi-convex lens with a focal length of 'f' is cut into two equal parts transverse to the principal axis, as shown in the figure, the focal length of each part becomes twice the initial value. However, the intensity of the image remains the same.

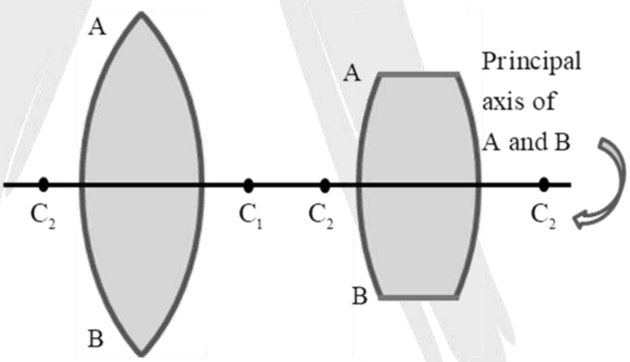


For original lens $\frac{1}{f} = \frac{2(\mu-1)}{R} \rightarrow (1)$

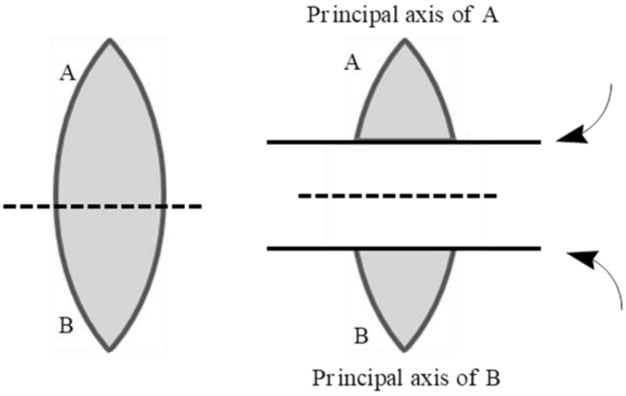
For each part of cut lens $\frac{1}{f'} = \frac{(\mu-1)}{R} \rightarrow (2)$

From (1) and (2) we get $f' = 2f$

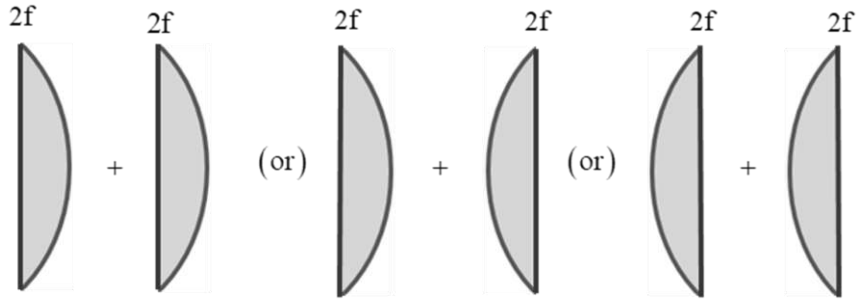
- On removing a part of lens with out disturbing remaining part, the principal axis position of the remaining part is same as earlier, but intensity of image is reduced



(b) If given lens is cut along the principal axis and the separation between them increase in a direction transverse to principal axis, each part has own principal axis.

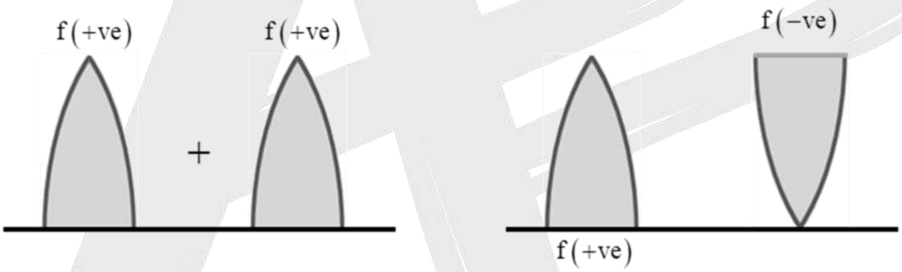


- If the equiconvex lens of focal length 'f' is divided into two equal parts transverse to the principal axis as shown in figure, the focal length of each part is 2f. If these two parts are put in contact in different combinations as shown in figure



$$\frac{1}{F_{net}}=\frac{1}{2f}+\frac{1}{2f},\ \frac{1}{F_{net}}=\frac{2}{2f} \text{ and } F_{net}=f,\ P_{net}=\frac{1}{f}$$

- If an equi convex lens of focal length 'f' is divided into two equal parts along its principal axis as shown in figure, the focal length of each part is f. If these two parts are put in contact in different combinations as shown in figure



For the first combined $\frac{1}{f_{net}}=\frac{1}{f}+\frac{1}{f}\Rightarrow f_{net}=\frac{f}{2} \quad \therefore \quad P_{net}=\frac{2}{f}$

For the second combination as shown in figure, fist part will behave as convergent lens of focal length f while the other divergent of same focal length (being thinner near the axis), so in this case, $\frac{1}{F_{net}}=\frac{1}{f}-\frac{1}{f'}$; $F_{net}=\infty,\ P_{net}=0$

To a lens $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$

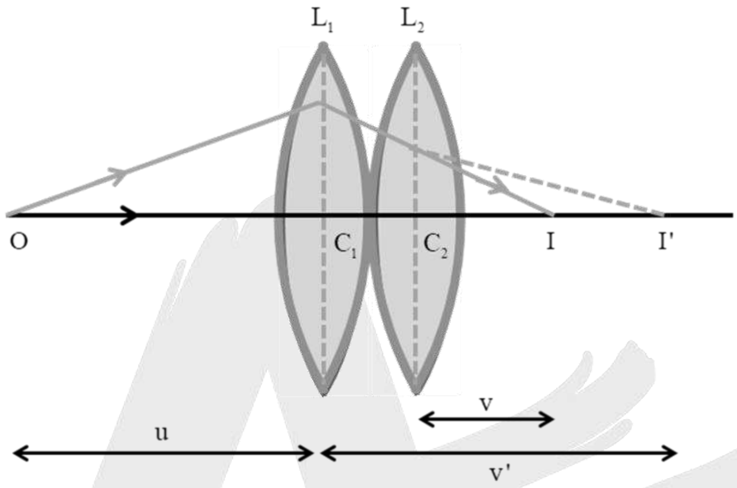
On differentiating above equation, we get $-\frac{1}{v^2}\cdot\frac{dv}{dt}+\frac{1}{u^2}\frac{du}{dt}=0$

(or) $V_i=\left(\frac{v}{u}\right)^2 V_0$ where V_i = velocity of image with respect to lens, V_0 = velocity of object with respect to lens. ; i.e. $V_i=m^2\cdot V_0=\left[\frac{f}{u+f}\right]^2\cdot V_0$

When an object moves at a constant speed towards a convex lens, from infinity to the focus, the image moves slowly at first and then faster away from the lens. On the other hand, if the object moves from the focus to the optical center, the image moves faster on the same side of the object, from infinity towards the lens.

Combination of Lenses :

Consider two thin lenses kept in contact as shown in figure. Let a point object 'O' is placed on the axis as shown in figure.



First lens of focal length f_1 from the image I_1 of the object 'O' at a distance v_1 from it.

$$\frac{1}{v_1}-\frac{1}{u}=\frac{1}{f_1} \quad \rightarrow (1)$$

Now the image I_1 will act as an object for second lens and the second lens forms image I at a distance 'v' from it, then

$$\frac{1}{v}-\frac{1}{v_1}=\frac{1}{f_2} \quad \rightarrow (2)$$

So adding (1) and (2) equations we have $\frac{1}{v}-\frac{1}{u}=\frac{1}{f_1}+\frac{1}{f_2}$ (or) $\frac{1}{v}-\frac{1}{u}=\frac{1}{F}$

So $\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2}$

i.e., the combination behaves as a single lens of equivalent focal length "F" given by

$$\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2}$$

This derivation is valid for any number of thin lenses in contact co-axially.

$$\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2}+\frac{1}{f_3}+\ldots\ldots\ldots\frac{1}{f_n}$$

In terms of power $P_{net}=P_1+P_2+P_3+\ldots\ldots.P_n$

Here focal length values are to be substituted with sign.

Note: If the two thin lens are separated by a distance 'd', then $\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2}-\frac{d}{f_1f_2}$ and

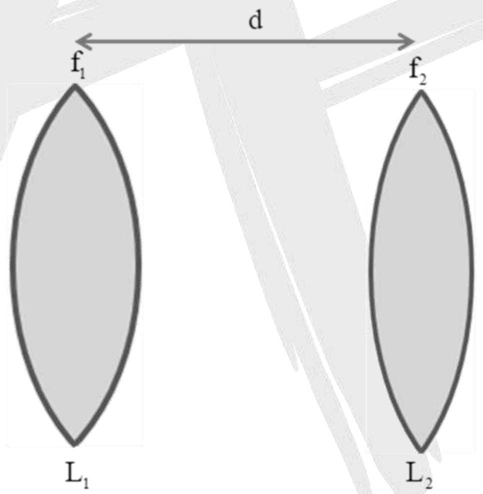
$$P_{net}=P_1+P_2-dP_1P_2$$

Note: If the medium on either side of the lens is air and the medium between the lens is one having refractive index μ, we can imagine that the rays emerging from the first lens are incident on the second lens as if they have traversed a thickness $\frac{d}{\mu}$ in air.

Hence $\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2}-\frac{(d/u)}{f_1f_2}$ $\qquad \therefore \quad P=p_1+p_2-\left(\frac{d}{\mu}\right)P_1P_2$

Note: If two thin lenses of equal focal length but of opposite nature are pair in contact, the resultant focal length of the combination will be $\frac{1}{F}=\frac{1}{f}+\left(-\frac{1}{f}\right)=0$ i.e. $F=\infty$ and $P=0$

Note: If f_1 and f_2 are focal lengths of two lenses $(L_1 \text{ and } L_2)$ are separated by a distance 'd' on common principal axis and 'F' is the equivalent focal length of the system.

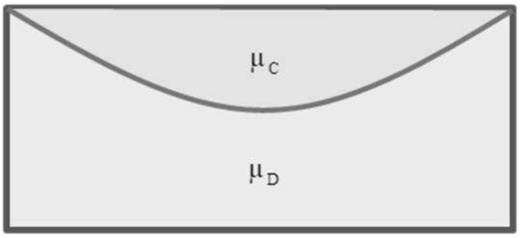


Then

(i) The distance of equivalent lens from second lens L_2 is $\frac{Fd}{f_1}$ towards the object if the value is positive and away from the object if the value is negative

(ii) The distance of equivalent lens from the first lens L_1 is $\frac{Fd}{f_2}$ away from the object, if the value is positive and towards the object if the value is negative. It is note that F, f_1 and f_2 are to be substituted according to sign convention.

Note: A plane glass plate is constructed by combining a plano-convex lens and a plano-concave lens of differentt materials as shown in figure. (μ_C is the refractive index of convergence lens, μ_D is the refractive index of divergent lens and R is the radius of curvature of common interface).



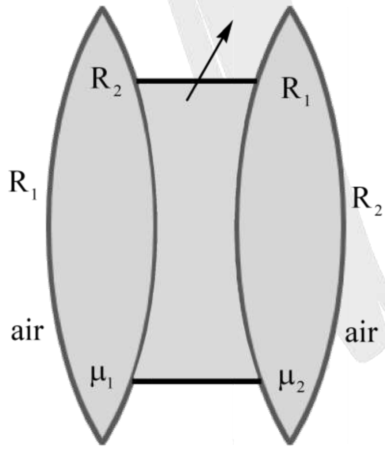
by lens maker's formula $\frac{1}{f_C} = (\mu_C - 1)\left[\frac{1}{\infty} - \frac{1}{-R}\right] = \frac{(\mu_C - 1)}{R} \rightarrow (1)$

and $\frac{1}{f_D} = (\mu_D - 1)\left[\frac{1}{-R} - \frac{1}{\infty}\right] = \frac{-(\mu_D - 1)}{R} \rightarrow (2)$

Now as the lenses are in contact, $\frac{1}{F} = \frac{1}{f_C} + \frac{1}{f_D} = \frac{(\mu_C - \mu_D)}{R}$; $F = \frac{R}{(\mu_C - \mu_D)}$

As $\mu_C \neq \mu_D$, the system will act as lens. The system will behave as convergent lens if $\mu_C > \mu_D$ (as its focal length will be positive) and as divergent lens if $\mu_C < \mu_D$ (as F will be negative)

Note: Two convex lenses made of materials of refractive indices μ_1 & μ_2, they are placed as shown in figure, the gap between them is filled with a liquid of refractive index μ_{liquid}. This combination is placed in air then



The system is equal to combination of three thin lenses in contact so $\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_{liquid}} + \frac{1}{f_2}$

Where $\frac{1}{f_1} = (\mu_1 - 1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$

$\frac{1}{f_{liquid}} = (\mu_{liquid} - 1)\left(\frac{1}{R_2} + \frac{1}{R'_1}\right) \Rightarrow \frac{1}{f_2} = (\mu_2 - 1)\left[\frac{1}{R'_1} + \frac{1}{R'_2}\right]$

If the effective focal length F of the combination is +ve then the combination behaves like converging lens, if F is –ve then the combination behaves like diverging lens.

Note: If two convex lenses made of materials of refractive indices μ_1 & μ_2 are kept in contact and the whole arrangement is placed in a liquid of refractive index μ_{liquid} then this is equivalent to combination of two lenses kept in contact in a medium.

In this case $\frac{1}{F}=\frac{1}{f_1(m)}+\frac{1}{f_2(m)}$

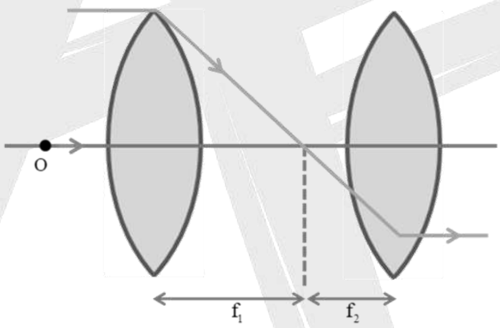
Where $\frac{1}{f_1(m)}=\left(\frac{\mu_1}{\mu_{liquid}}-1\right)\left(\frac{1}{R_1}+\frac{1}{R_2}\right)$

$\frac{1}{f_2(m)}=\left(\frac{\mu_1}{\mu_{liquid}}-1\right)\left(\frac{1}{R_1'}+\frac{1}{R_2'}\right)$

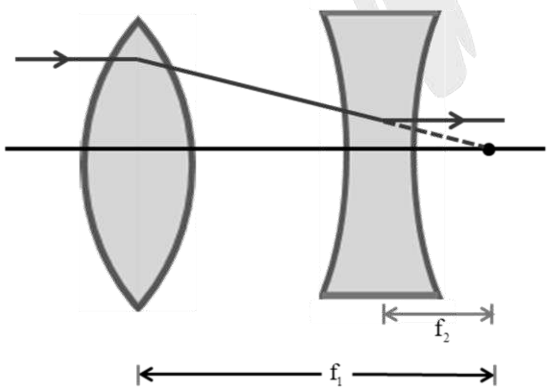
Note: If parallel incident ray on first lens emerges parallel from the second lens, then $f_e=\infty$

$\frac{1}{\infty}=\frac{1}{f_1}+\frac{1}{f_2}-\frac{d}{f_1f_2} \Rightarrow d=f_1+f_2$

(i) If both the lenses are convex, then $d=f_1+f_2$

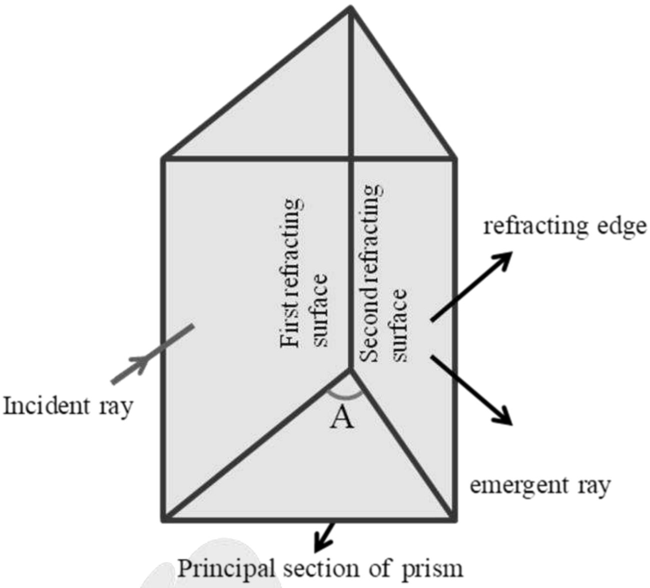


(ii) If second lens is concave, then $d=f_1+(-f_2)=f_1-f_2$

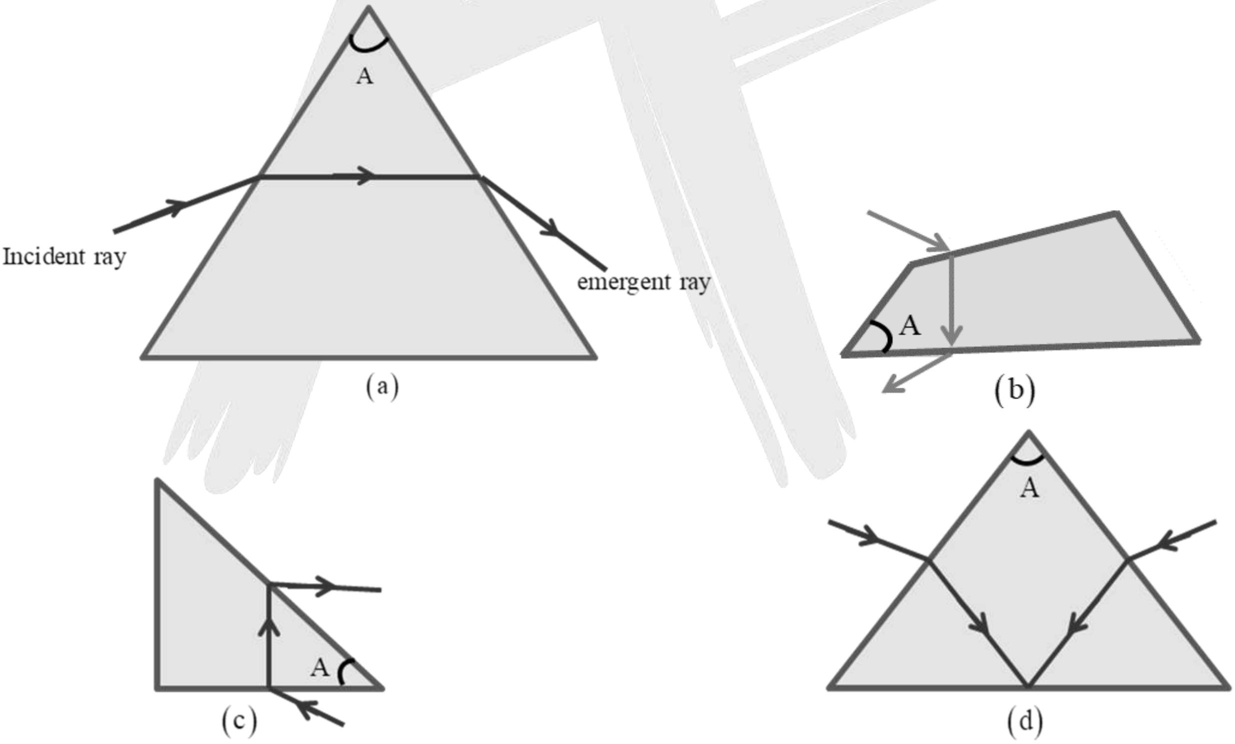


Refractive through Prism

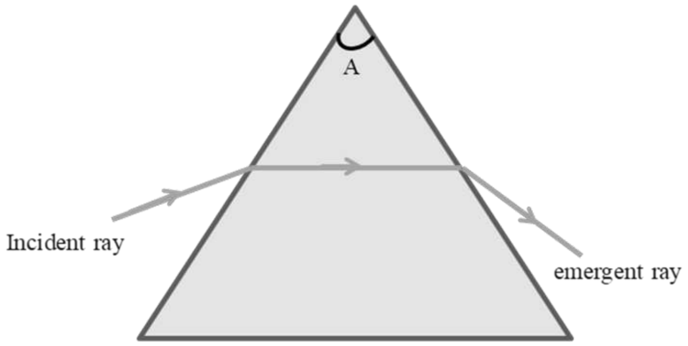
- Prism is a transparent medium bounded by any number of surfaces in such a way that the surface on which light is incident and the surface from which the light emerges are plane and non-parallel as shown in figure.



- The plane surface on which light is incident and emerges are called refracting faces.
- The angle between the faces on which light is incident and from which it emerges is called refracting angle or apex angle or angle of prism (A).
- The two refracting surfaces meet each other in a line called refracting edge.
- A section of the prism by a plane perpendicular to the refracting edge is called principal section



- The angle of deviation (δ) refers to the angle between the incident and emergent rays of light after passing through a prism. When measuring the angle of deviation, the counterclockwise direction is considered positive, while the clockwise direction is considered negative.

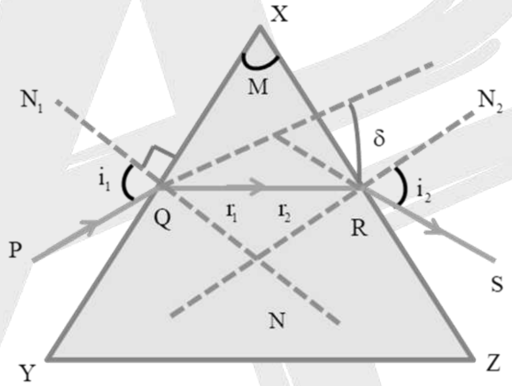


δ = deviation angle

Note: If the refractive index of the prism's material is equal to that of its surroundings, then there will be no refraction at the prism's surfaces, and light will pass through the prism undeviated. ie. $[\delta = 0]$.

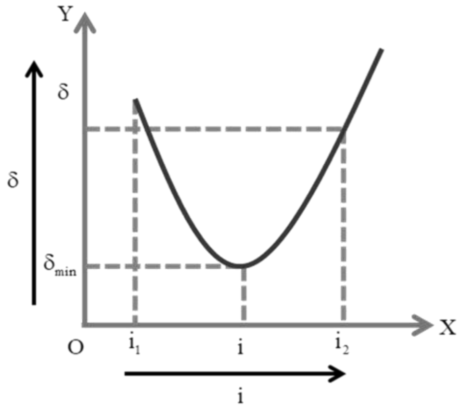
(vii) Generally we use equilateral or right angled or Isosceles prism.

Determination of Refractive index of material of the prism for minimum deviation



- **Minimum Deviation**

From the equation $\delta = (i_1 + i_2) - A$, the angle of deviation δ depends upon angle of incidence (i_1). If we determine experimentally, the angle of deviation corresponding to different angles of incidence and then plot a graph by taking angle of incidence (i) on x-axis, angle of deviation (δ) on y-axis, we get the curve as shown in figure.



By snell's law $\mu = \frac{\sin i}{\sin r} = \frac{\sin i_1}{\sin r_1} = \frac{\sin i_2}{\sin r_2} \Rightarrow \mu = \frac{\sin\left(\frac{A+\delta_{min}}{2}\right)}{\sin\frac{A}{2}} \Rightarrow \frac{\mu_p}{\mu_m} = \frac{\sin\left(\frac{A+\delta_{min}}{2}\right)}{\sin\frac{A}{2}}$

Note: Deviation produced by small angled prism for small angle, from equation above

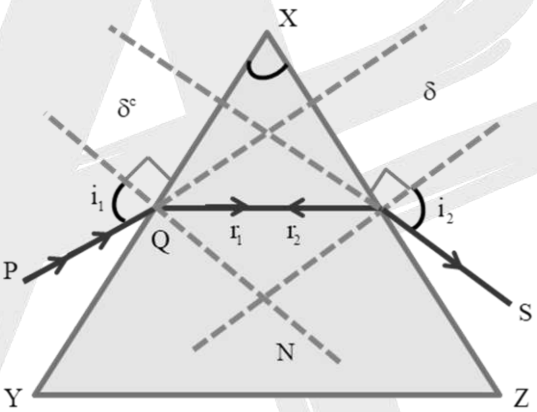
$$\mu = \frac{i_1}{r_1} = \frac{i_2}{r_2};\ i_1 = \mu r_1,\ i_2 = \mu r_2 \text{ but } \delta = (i_1 + i_2) - A$$

$$\delta = \mu r_1 + \mu r_2 - A;\ \ \delta = \mu(r_1 + r_2) - A \text{ but } r_1 + r_2 = A$$

For a prism immersed in a medium of refractive index μ_m

$$\delta = (\mu - 1)A \Rightarrow \delta = \left(\frac{\mu_p}{\mu_m} - 1\right)A$$

Note: There are two values of angle of incidence for same angle of deviation :



When a light ray is incident at an angle i_1 at the surface (xy), it emerges at an angle i_2 from the surface (zx) with a deviation angle δ. As the path of light is reversible, therefore if angle of incidence is i_2, at the face (xy), then the angle of emergence will be i_1, with the same angle of deviation (δ)

Note :

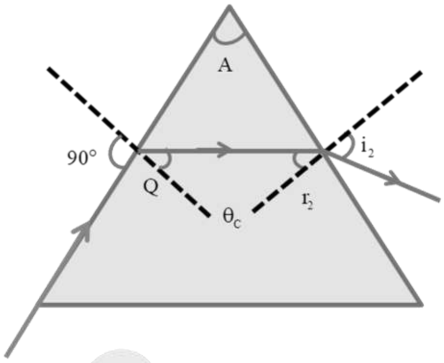
(i) For a given material of prism, wave length of light and angle of incidence. When the angle of prism increases angle of deviation also increases as $\delta \propto A$.

(ii) With increase in wavelength, deviation decreases ie. deviation for red is least while maximum for violet as $\delta \propto (\mu - 1)$ $\left\{\mu \alpha \frac{1}{\lambda}\right\}$

(iii) When a given prism is immersed in liquid, the angle of deviation changes as $\delta \propto (\mu - 1)$

- **Maximum deviation :**

  Deviation of ray will be maximum when the angle of incidence is maximum i.e $i = 90°$. Therefore the maximum deviation $\delta_{max} = 90 + i_2 - A$



To find the angle of emergence in this case let us apply Snell's law at second surface.

$$\frac{\mu_a}{\mu} = \frac{\sin r_2}{\sin i_2} = \frac{1}{\mu}$$

As $i_1 = 90°,\ r_1 = \theta_C$

Also $r_1 + r_2 = A,\ \theta_C + r_2 = A$

So, $r_2 = A - \theta_C$

$$\mu \sin(A - \theta_C) = 1 \sin i_2$$

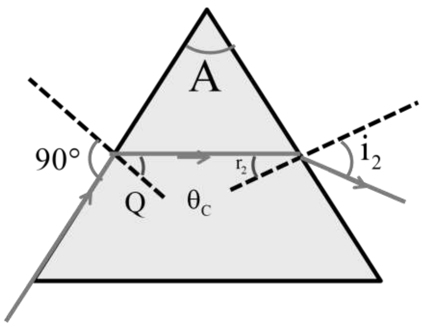
$$i_2 = \sin^{-1}\left[\mu \sin(A - \theta_C)\right]$$

Maximum deviation is $\delta_{max} = 90° + i_2 - A$

$$\delta_{max} = 90° + \sin^{-1}\left[\mu \sin(A - \theta_C)\right] - A$$

Note: Since the law of reversibility always true, then for an angle of incidence $i = \sin^{-1}\left[\mu \sin(A - \theta_C)\right]$, the ray grazes at the other surface.

- Condition of grazing emergence : If a ray can emerge out of a prism, the value of angle of incidence i_1 for which angle of emergence $i_2 = 90°$ is called condition of grazing emergence. In this situation as the ray emerges out of face XZ, i.e., TIR does not take place at it.



$r_2 < \theta_C \rightarrow (1)$

But as in a prism $r_1 + r_2 = A;\ r_1 = A - r_2$

So $r_1 = A - (<\theta_C)$ ie. $r_1 > A - \theta_C \rightarrow (2)$

Now from snell's law at face XY, we have $1\sin i_1 = \mu \sin r_1$

But inview of equation (2)

$$\sin r_1 > \sin(A-\theta_C);\quad \frac{\sin i_1}{\mu} > \sin(A-\theta_C)$$

$$\sin i_1 > \mu \sin(A-\theta_C)$$

$$\text{ie. } \sin i_1 > \mu[\sin A \cos\theta_C - \cos A \sin\theta_C]$$

$$\text{ie } \sin i_1 > \mu\left[(\sin A)\sqrt{(1-\sin^2\theta_C)} - \cos A \sin\theta_C\right]$$

$$\text{i.e } \sin i_1 > \left[\sqrt{(\mu^2-1)}\sin A - \cos A\right]$$

$$\left(\text{as } \sin\theta_C = \left(\frac{1}{\mu}\right)\right) \text{ or } i_1 > \sin^{-1}\left[\sqrt{(\mu^2-1)}\sin A - \cos A\right] \rightarrow (3)$$

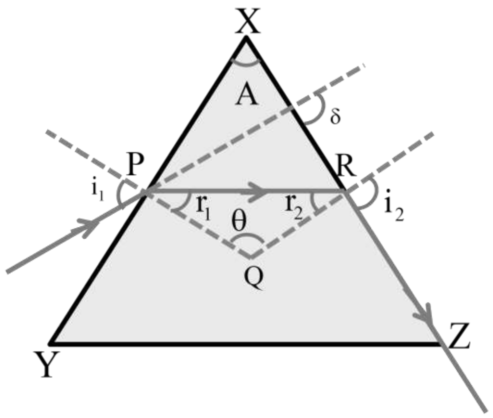
i.e light will emerge out of prism only if angle of incidence is greater than $(i_1)_{min}$ given by eq... (3).

In this situation deviation will be given by $\delta = (i_1 + 90° - A)$ with i_1 given by eq...(3)

- **Condition of no emergence :**

  The light will not emerge out of a prism for a values of angle of incidence if at face AB for $i_{1(max)} = 90°$ at face AC

  $r_2 > \theta_C \rightarrow (1)$



Now for Snell's law at face AB, we have $1\times\sin 90^\circ = \mu \sin r_1$

ie. $r_1 = \sin^{-1}\left(\frac{1}{\mu}\right)$; or $r_1 = \theta_C \quad \rightarrow \quad (2)$

From equation (1) and (2) ; $r_1 + r_2 > 2\theta_C \quad \rightarrow \quad (3)$

However in prism ; $r_1 + r_2 = A \quad \rightarrow \quad (4)$

So from equation (3) and (4) ; or $A > 2\theta_C \quad \rightarrow \quad (5)$

$$\frac{A}{2} > \theta_C \text{ or } \sin\left[\frac{A}{2}\right] > \sin\theta_C \Rightarrow \sin\left(\frac{A}{2}\right) > \frac{1}{\mu}$$

i.e. $\mu > \left[\operatorname{cosec}\left(\frac{A}{2}\right)\right] \quad \rightarrow \quad (6)$

i.e., A ray of light will not emerge out of a prism (what ever be the angle of incidence) if $A > 2\theta_C$

, i.e if $\mu > \operatorname{cosec}\left(\frac{A}{2}\right)$

(or) $\mu = \sqrt{\cot^2(A/2)+1}$

Note: Limiting Angle : In order to have an emergent ray, the maximum angle of the prism is $2\theta_C$, where θ_C is the critical angle of the prism w.r.t the surrounding medium $2\theta_C$ is called the limiting angle of the prism.

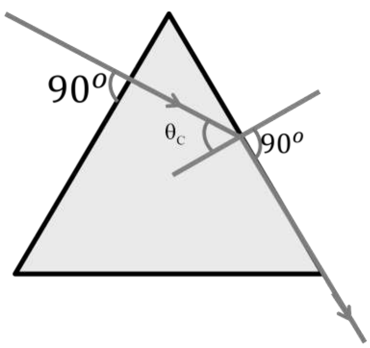
Note: If the angle of incidence at first surface i is such that

(a) If $i = \sin^{-1}\left[\mu \sin(A-\theta_C)\right]$, the ray grazes at the other surface.

(b) If $i > \sin^{-1}\left[\mu \sin(A-\theta_C)\right]$, then the ray emerges out of a prism from the other surface.

(c) If $i < \sin^{-1}\left[\mu \sin(A-\theta_C)\right]$, the ray undergo TIR at the other surface.

Note: Normal incidence – grazing emergence : If the incident ray falls normally on the prism and grazes from the second surface, then

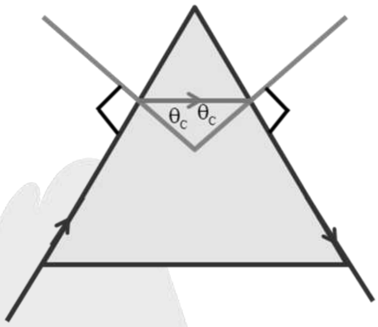


(a) $i_1 = r_1 = 0, i_2 = 90°$ and $r_2 = \theta_C = A$

(b) $A = \theta_C = \sin^{-1}\left(\frac{1}{\mu}\right)$

(c) Deviation $d = 90 - \theta_C$

Note: Grazing incidence – grazing emergence : If the incident ray falls on the prism with grazing incidence and grazes from the second surface, then



(i) $i_1 = i_2 = 90°$

(ii) $r_1 = r_2 = \theta_C$

(iii) Angle of prism $A = 2\theta_C$

(iv) Deviation $d = 180 - 2\theta_C = 180 - A$

Note : In this situation angle of emergence is equal to angle of incidence = $49°$ and deviation $\delta_m = (2i - A) = (2 \times 49 - 60) = 38°$

(b) For maximum deviation, $i_1 = 90°$ so that $r_1 = \theta_C = \sin^{-1}\left(\frac{2}{3}\right) = 42°$, but as in a prism $r_1 + r_2 = A$ so $r_2 = A - r_1 = 60° - 42° = 18°$

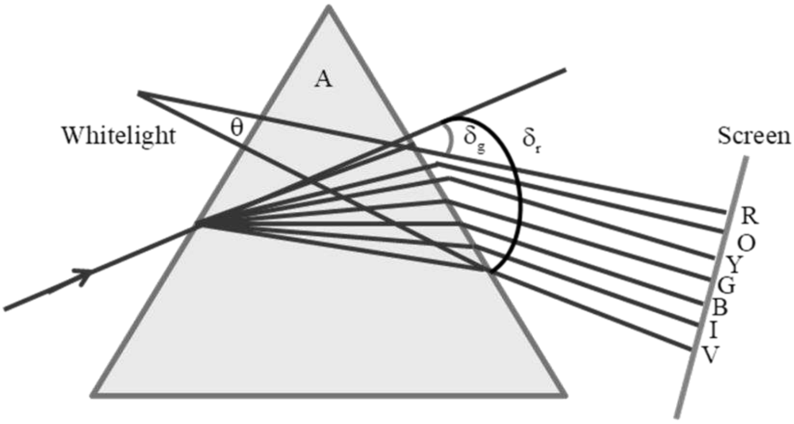
Now applying Snell's law at the second face,

$\mu \sin r_2 = \sin i_2$, i.e., $\frac{3}{2}\sin 18° = \sin i_2$

ie. $i_2 = \sin^{-1}[1.5 \times 0.31] = \sin^{-1}(0.465) \cong 28°$

- **Dispersion by A Prism :**

When white light passes through a prism, it separates into its constituent colors, which is known as dispersion. This happens because the refractive index of the prism is different for different wavelengths of light. So different wave lengths in passing through a prism are deviated through different angles and as $\delta \propto (\mu - 1)$, violet is deviated most while red is least devited giving rise to display of colours known as spectrum. The spectrum consists of visible and invisisble regions.



In the visible spectrum, the mean deviation and refractive index are determined using the yellow ray. When dispersion occurs in a medium and follows the order of "VIBGYOR," it is known as normal dispersion. On the other hand, if the dispersion does not follow the "VIBGYOR" rule, it is referred to as anomalous dispersion. A medium that causes dispersion is known as a dispersive medium. Dispersive prisms separate light based on wavelength and are widely used in spectrometers to distinguish closely adjacent spectral lines. Glass prisms are commonly used for dispersion in the visible region, while different materials are used for dispersion in the UV and IR regions.

- **Angular dispersion :**

The difference in the angles of deviations of any pair of colours is called angular dispersion (θ) for those two colours. If the refractive indices of violet, red and yellow are indicated by μ_v, μ_R and μ_y. The deviation δ_y corresponding to yellow colour is taken as mean deviation. The deviations δ_v, δ_R and δ_y can be written as

$$\delta_v = (\mu_v - 1)A, \ \delta_R = (\mu_R - 1)A \text{ and } \delta_y = (\mu_y - 1)A$$

Angular dispersion for violet and red $\theta = (\delta_v - \delta_R) = (\mu_v - \mu_R)A$

The angular dispersion of a prism depends on both the prism material and the prism angle. The angular dispersion refers to the extent to which the visible spectrum is separated into its component colors, specifically the angular separation between violet and red.

- **Dispersive Power :**

The ability of the material of a prism to disperse light rays is indicated by its dispersive power. It is calculated as the ratio of the angular dispersion of two extreme colors to their mean deviation.

$$\omega = \frac{\text{Angular dispersion}}{\text{Mean deviation}} \quad \text{or} \quad \omega = \frac{\delta_v - \delta_R}{\left(\frac{\delta_v + \delta_R}{2}\right)}$$

But the mean colour of red and violet colours is yellow colour,

so $\frac{\delta_v + \delta_R}{2} = \delta_y$

So, $\omega = \frac{\theta}{\delta_y} = \frac{\delta_v - \delta_R}{\delta_y}$

Where δ_y is the deviation for yellow light $\omega = \frac{\mu_v - \mu_R}{(\mu_r - 1)} = \frac{d\mu}{(\mu - 1)}$

It is seen that the dispersive power is independent of the angle of prism and angle incidence, but depends on material of prism.

The dispersive power more precisely expressed with reference to C, D and F Fraunhoffer's lines in the solar spectrum. The C, D and F lines lies in the red, yellow and blue regions of the spectrum and their wavelengths are $6563\overset{o}{A}$, $5893\overset{o}{A}$ and $4861\overset{o}{A}$ respectively. Then the dispersive power may be expressed as $\omega = \frac{\mu_F - \mu_C}{\mu_D - 1}$

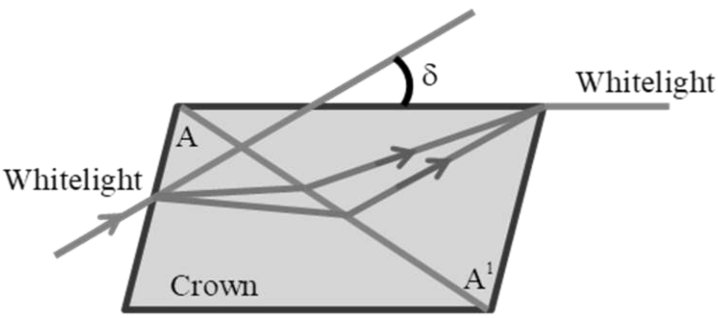
Where $\mu_D = \frac{\mu_F + \mu_C}{2}$

It is observed that a single prism exhibits both dispersion and deviation simultaneously. However, by combining two prisms made of different materials such as crown and flint glass, we can achieve either dispersion or deviation alone. The dispersive power of flint glass is greater than that of crown glass for the same refracting angle. This means that the angular separation of spectral colors in flint glass is higher than that in crown glass. If two prisms of prism angles A and A' and refractive indices μ and μ' respectively are placed together then the total deviation

$\delta = \delta_1 + \delta_2 = (\mu_y - 1)A + (\mu'_y - 1)A'$ and total dispersion

$\theta = \theta_1 + \theta_2 = (\mu_v - \mu_R)A + (\mu'_v - \mu'_R)A'$

Deviation without Dispersion or achromatic Prism :



An achromatic prism is made by combining two prisms of different materials and specified angles in such a way that it produces no colors. Flint glass has a higher dispersive power than crown glass, which makes it possible to create an achromatic prism. By carefully selecting the angles and materials of the two prisms, it is possible to ensure that a ray of white light can pass through the combination without any dispersion, although it may still experience deviation.Such a combination is called achromatic combination.

i.e $\delta \neq 0$ and $\theta = 0$

$$\therefore \ (\mu_v - \mu_R)A + (\mu'_v - \mu'_R)A' = 0 \text{ or } \frac{(\mu_v - \mu_R)A}{(\mu_y - 1)}(\mu_y - 1) + \frac{(\mu'_v - \mu'_R)A}{(\mu'_y - 1)}(\mu'_y - 1) = 0$$

i.e $\omega_C \delta_C + \omega_f \delta_f = 0$

In this case as the deviation produced by flint prism is opposite to crown prism. Therefore the net deviation $\delta = \delta_C - \delta_f$

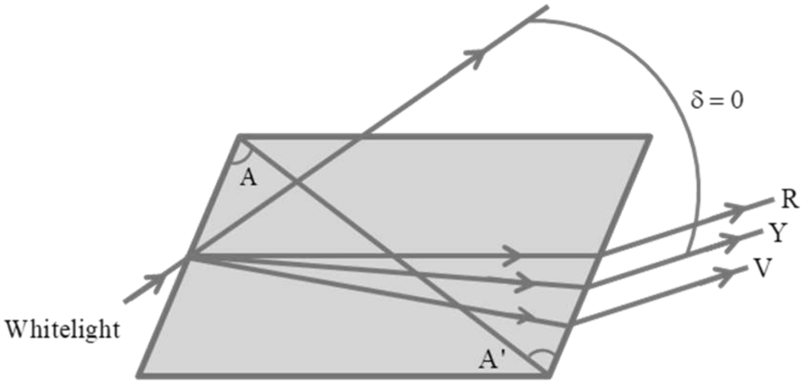
$$\delta = (\mu_y - 1)A - (\mu'_y - 1)A'$$

$$\delta = \frac{(\mu_y - 1)}{(\mu_V - \mu_R)}(\mu_V - \mu_R)A - \frac{(\mu'_y - 1)}{(\mu'_v - \mu'_R)}(\mu'_v - \mu'_R)A'$$

$$\delta = \frac{\theta_C}{\omega_C} - \frac{\theta_f}{\omega_f}$$

Dispersion without Deviation OR Direct Vision Prism :

A combination of two prisms made of crown and flint glass can be designed such that the angles are adjusted in such a way that the deviation produced by the first prism for the mean rays is equal and opposite to that produced by the second prism. When this happens, the final beam will be parallel to the incident beam. This combination of two prisms is used to produce dispersion of the incident beam without any deviation.



i.e $\delta = 0$ and $\theta \neq 0$

$$\therefore \quad (\mu_y - 1)A + (\mu'_y - 1)A' = 0$$

$$\frac{(\mu_y - 1)}{(\mu_v - \mu_R)}(\mu_v - \mu_R)A + \frac{(\mu'_y - 1)}{(\mu'_v - \mu'_R)}(\mu'_v - \mu'_R)A' = 0$$

ie. $\dfrac{\theta_C}{\omega_C} + \dfrac{\theta_f}{\omega_f} = 0$

In this case as the dispersion produced by flint glass prism is opposite to crown glass prism. Therefore the net angular dispersion $\theta = \theta_C - \theta_f$

$$\theta = (\mu_V - \mu_R)A - (\mu'_V - \mu'_R)A' \text{ (or) } \theta = \frac{(\mu_V - \mu_R)A}{(\mu_y - 1)}(\mu_y - 1) - \frac{(\mu'_V - \mu'_R)A'}{(\mu'_y - 1)}(\mu'_y - 1)$$

$$\theta = \omega_C\delta_C - \omega_f\delta_f$$

Optical Instruments : Optical instruments are used primarily to assist the eye in viewing the object. Optical instruments are classified into three groups, they are

(a) visual instruments

Ex : microscope, telescope

(b) photographing and projecting instruments

Ex: cameras

(c) analysing and measuring instruments

Ex : spectrometer

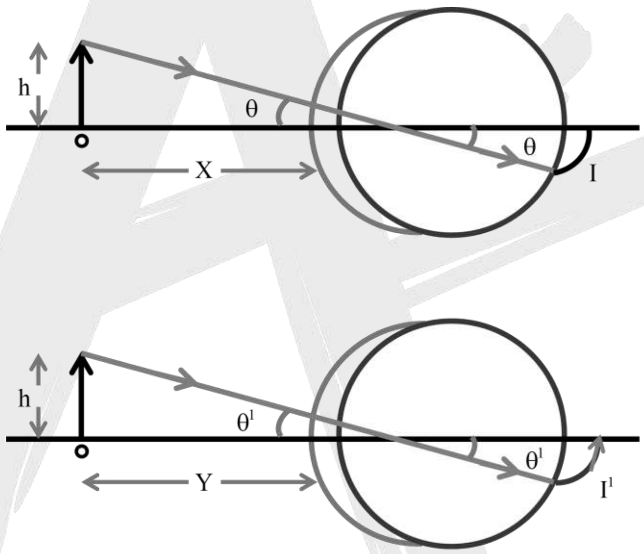
Optical instruments such as telescopes and microscopes typically consist of an objective lens and an eyepiece lens. The objective lens is positioned towards the object being viewed, while the eyepiece lens is positioned towards the observer's eye. However, a single lens can produce images with aberrations or defects. Placing the eye too close to the eyepiece lens results in reduced field of view and uneven intensity distribution across the field, with the central part being brighter than the marginal part.

To overcome these issues, combinations of lenses are used in the design of telescopes and microscopes for practical use. An eyepiece lens made up of multiple lenses is called an eyepiece. In any eyepiece, the lens closer to the eye is referred to as the eye lens, while the field lens increases the field of view. The eye lens magnifies the image. Two commonly used eyepieces are Ramsden's eyepiece and Huygen's eyepiece.

- **The Eye :**

The eye is a complex optical system through which light enters via a curved front surface called the cornea and passes through the central hole in the iris known as the pupil. The size of the pupil can change due to control muscles. The combination of the cornea, lens, and fluid inside the eye act as a single converging lens. This lens focuses the light onto the retina, which is a layer of nerve fibers. The retina contains rods and cones that detect the intensity and color of the light respectively, and transmit electrical signals to the brain through the optic nerve. The ciliary muscles can adjust the shape and focal length of the eye lens. The image formed by the eye lens is real, inverted, and diminished at the retina. The size of the image on the retina is proportional to the angle subtended by the object on the eye, which is known as the visual angle.

Therefore it is known as the angular size.



When the object is distant, its visual angle θ and hence image at retina is small and object looks smaller.

When the object is brought near to the eye its visual angle θ and hence size of image will increase and object looks larger as shown in figure (b)

Optical instruments are used to increase this visual angle artificially in order to improve the clarity.

Eg : Microscope, Telescope

When the eye is focussed on a distant object $(\theta \approx 0)$ the ciliary muscles are relaxed so that the focal length of the eye-lens has maximum value which is equal to its distance from the retina. When the eye is focussed on a closer object (θ increases) the ciliary muscles of the eye are strained and focal length of eye lens decreases.

The focal length of the eye lens can be adjusted by the ciliary muscles to form a clear image on the retina. This process is called accommodation. However, if an object is brought too close to the eye, the focal length cannot be adjusted enough to form a clear image on the retina. This sets a minimum distance for clear vision, known as the near point of the eye. The least distance of distinct vision is the distance between the eye and the near point, which is typically 25cm for a normal eye and is denoted by D. On the other hand, the farthest point from the eye at which an object can be distinctly seen is called the far point. For a normal eye, the far point is theoretically at infinity.

- **Deffects of Vision :**

  Our eyes are extraordinary organs that possess the ability to interpret incoming electromagnetic waves as images through a complex process. However, eyes can develop defects due to various reasons.

Some common optical defects of the eye are

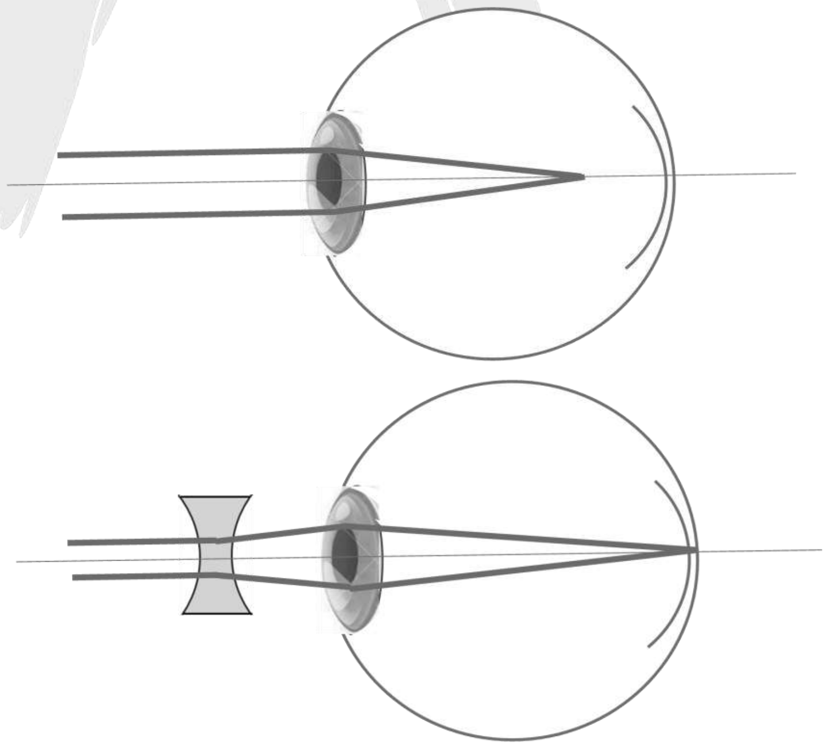
(a) myopia

(b) hypermetropia

(c) presbyopia

- **Myopia :**

  When light from a distant object enters the eye, it may converge in front of the retina, causing a vision defect known as my opia or shortsightedness. In this condition, the farthest point from the eye that can be seen clearly is at a distance less than infinity, making distant objects appear blurry.



This defect is rectified by using spectacles having divergent lens (concave lens) which forms the image of a distant object at the far point of defected eye. From lens formula

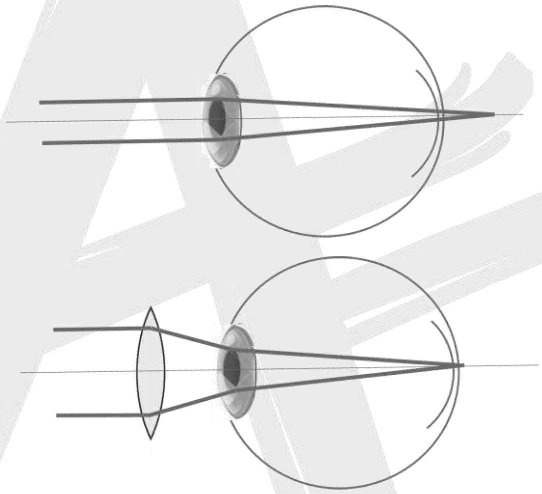
$$\frac{1}{\text{F.P}} - \frac{1}{-(\text{distance of object})} = \frac{1}{f} = P$$

Where F.P = Far point of the defective eye. If the object is at infinity

Power of lens $(p) = \frac{1}{f} = \frac{1}{\text{F.P}}$

- **Hypermetropia : (or) Long-sightendness.**

  The light from an object at the eye lens may be converged at a point behind the retina. This defect is called in this type of defect, near point is at a distance greater than 25 cm and near objects are not clearly visible.



  This defect is rectified by using spectacles having convergent lens (i.e convex lens) which forms the image of near objects at the near point of the defeated eye (which is more than 25cm)

$$\frac{1}{-\text{N.P.}} - \frac{1}{-(\text{distance of object})} = \frac{1}{f} = P$$

  N.P. = Near Point of defected eye.

  If the objective is placed at $D = 25\text{cm} = 0.25\text{m}$

$$P = \frac{1}{f} = \left(\frac{1}{0.25} - \frac{1}{\text{N.P.}}\right)$$

- **Presbyopia :** Presbyopia is a condition where the power of accommodation of the eye lens decreases due to the weakening of ciliary muscles. As a result, the far point of the eye is less than infinity and the near point is greater than 25 cm, making it difficult to see both near and far objects clearly. To correct this condition, bifocal lenses are commonly used.

- **Astigmatism :**
  Astigmatism is caused by an imperfectly shaped lens, resulting in different focal lengths in two orthogonal directions. This can cause difficulty in focusing on both horizontal and vertical lines simultaneously. To correct this defect, a cylindrical lens is used in the direction where the focal length is incorrect.
- **Simple Microscope :**
  To view an object with the naked eye, the object should be positioned between the least distance of distinct vision (D) and infinity. The maximum angle subtended by the object is achieved when it is located at D.
- **Magnifying power of simple microscope :**
  The magnifying power or angular magnification of a simple microscope can be defined as the ratio of the visual angle with the instrument to the maximum visual angle that an unaided eye can perceive when the object is placed at the least distance of distinct vision.

$$M=\frac{\text{visual angle with instrument}}{\text{maximum visual angle for unaided eye}}$$

$$M=\frac{\theta}{\theta_0}$$

Case (1): When the final images is formed at far point (or) When the final image is formed at infinity

In this case $u=f,\ V=\infty$; So $M_\infty=\frac{D}{f}$

As here u is maximum, magnifying power is minimum and as in this situation parallel beam of light enters the eye, eye is least strained and is said to be normal, relaxed and unstrained.

Case (2): When the final image is formed at near point (or) When the final image is formed at D

$v=-D$, u is –ve

$$\frac{1}{f}=-\frac{1}{D}-\frac{1}{-u};\ \frac{1}{u}=\frac{1}{f}+\frac{1}{D}$$

$$M_D=D\left(\frac{1}{f}+\frac{1}{D}\right);\ m_D=\left(1+\frac{D}{f}\right)$$

As the minimum value of v(=D) in this situation u is minimum and magnifying power is maximum and eye is under maximum strain.

Note : If lens is kept at a distance 'a' from the eye then D is replaced by $(D-a)$

$$M_D=1+\left(\frac{D-a}{f}\right);\ M_\infty=\frac{D-a}{f}$$

- **Some important points regarding microscope :**
- As $M_D = 1 + \frac{D}{f}$; $M_\infty = \frac{D}{f}$, so $M_D > M_\infty$
- As $M_D = 1 + \frac{D}{f}$; $M_\infty = \frac{D}{f}$, so smaller the focal length fo the lens greater the magnifying power of the simple microscope.

  When longer wavelengths of light are used in a microscope, the focal length will increase, causing the magnifying power to decrease. The maximum possible magnifying power of a simple microscope for an image without defects is about 4. Simple microscopes use a single lens to form an image, which can result in defects such as spherical aberration and astigmatism. As the magnification increases, the image becomes more distorted. Therefore, for higher magnifying power, a compound microscope is used. Simple magnifiers are essential components of most optical instruments, such as microscopes and telescopes, and are used as eyepieces.

Compound Microscope

A simple magnifying lens is not sufficient for producing high magnification. Instead, a highly magnified image is produced in two stages using a compound microscope.

Magnifying power:

$$M = \frac{\text{visual angle with instrument}}{\text{max.visual angle for unaided eye}} = \frac{\theta}{\theta_0}$$

$$M = -\left(\frac{v}{u}\right)\left(\frac{D}{u_e}\right)$$

Where u is the object distance for the objective lens, v is the image distance for the objective lens, u_e is the object distance for the eye piece.

i.e., $M = m_0 \times m_e$

the length of the tube $L = v + u_e$

case (i): If the final image is formed at infinity (far point):

In this case $u_e = f_e$

$$\therefore M_\infty = -\frac{v}{u}\left[\frac{D}{f_e}\right] \text{ with } L_\infty = v + f_e$$

A microscope is usually considered to operate in this mode unless stated otherwise. In this mode u_e is maximum and hence magnifying power is minimum.

Note: When the object is very close to the principal focus F_0 of the objective, the image due to the objective becomes very close to the eye piece. Then replace u with f_0 and ν_0 with L so the expression for magnifying power. $M_{\infty} \approx -\frac{L}{f_0}\left(\frac{D}{f_e}\right)$

Case – ii: If the final image is formed at D (Near point):

In this case, for eye piece $V_e = -D, u_e$ is –ve

$$-\frac{1}{D} - \frac{1}{-u_e} = \frac{1}{f_e}$$

i.e., $\frac{1}{u_e} = \frac{1}{D}\left[1+\frac{D}{f_e}\right]$; $m = m_0 m_e$ $\quad\therefore M_D = -\frac{\nu}{u}\left[1+\frac{D}{f_e}\right]$ with $L_D = \nu + \frac{f_e D}{f_e + D}$

In this situation as u_e is minimum magnifying power is maximum and eye is most strained. When the object is very close to the principal focus F_0 of the objective, the image due to the objective becomes very close to the eyepiece. Then replace u with f_0 and v with L so the expression for magnifying power.

$$M_D \approx -\frac{L}{f_0}\left(1+\frac{D}{f_e}\right)$$

- **Some important points regarding compound microscope:**
- As magnifying power of a compound microscope is negative, the image seen is always truly inverted.
  For a microscope magnifying power is minimum when final image is at ∞ and maximum when final image is at least distance of distinct vision D, i.e., and $M_{max} = -\frac{\nu}{u}\left(1+\frac{D}{f_e}\right)$
- For a given microscope magnifying power for normal setting remain practically unchanged if field and eye lens are interchanged as $M = \frac{LD}{f_0 f_e}$
- In an actual compound microscope each of the objective and eye piece consists of a combination of several lenses instead of a single lens to eliminate the aberrations and to increase the field of view.
- In low power microscopes, the magnifying power is about 20 to 40, while in high power microscopes, the magnifying power is about 500 to 2000.

- **Telescopes:**

A microscope is used to view the objects placed close to it. To look at distant objects such as star, a planet or a cliff etc, we use another optical instrument called telescope, which increases the visual angle of distant object.

The telescope that uses a lens as an objective is called refracting telescope. However, may telescopes use a curved mirror as an objective such telescopes are known as reflecting telescopes. There are three types of refracting telescopes in use.

(i) Astronomical telescope

(ii) Terrestrial telescope

(iii) Galilean telescope

- **Astronomical telescope:**

Magnifying power (M):

Magnifying power of a telescope is given by

$$M = \frac{\text{Visual angle with instrument}}{\text{Visual angle for unaided eye}} = \frac{\theta}{\theta_0}$$

From the above figure,

$$\theta_0 = \frac{h}{f_0} \text{ and } \theta = \frac{h}{-u_e}; M = \frac{\theta}{\theta_0} = \frac{-\left(\frac{h}{u_e}\right)}{\left(\frac{h}{f_0}\right)} = -\frac{f_0}{u_e}$$

The length of the tube $L = f_0 + u_e$

Case – i: If the final image is at infinity (far point):

In this case, for eyepiece $v_e = -\infty$,

$$u_e = -ve \quad \frac{1}{-\infty} - \frac{1}{-u_e} = \frac{1}{f_e}$$

Hence $u_e = f_e$

Hence $M_\infty = -\frac{f_0}{f_e}$ and $L_\infty = f_0 + f_e$

Usually a telescope is operated in this mode unless stated othewise. In this mode u_e is maximum, hence magnifying power is minimum, while length of tube is maximum.

This case is also called normal adjustment because in this case eye is least strained are relaxed.

Case – ii: If the final image is at D (Near point)

In this situation for eyepiece $v_e = -D$

$$\frac{1}{-D} - \frac{1}{u_e} = \frac{1}{f_e} \text{ i.e., } \frac{1}{u_e} = \frac{1}{f_e}\left[1+\frac{f_e}{D}\right] = M_D = \frac{-f_0}{f_e}\left[1+\frac{f_e}{D}\right]$$

In this case length of the tube $L_D = f_0 + \dfrac{f_e D}{f_e + D}$

In this situation u_e is minimum, hence magnifying power is maximum while the length of the tube is minimum and eye is most strained.

- **Some important points regarding astronomical telescope:**
- In case of telescope if object and final image are at infinity and total light entering the telescope leaves it, parallel to its axis.

  $\therefore$ magnifying power $= \dfrac{f_0}{f_e} = \dfrac{A_0}{A_e}$

  Where A_0 and A_e are the apertures of objectives and eyepiece.
- The magnifying power of an astronomical telescope is negative, resulting in a truly inverted image where left appears as right and upside down simultaneously. Although this may seem disadvantageous, it does not usually affect the observations of astronomical objects as most of them are symmetrical in nature.
- For given telescope, magnifying power is minimum when final image is at infinity (Far point) and maximum when it is at least distance of distinct vision (Near point)

  i.e., $M_{min} = -\left(\dfrac{f_0}{f_e}\right)$ and $M_{max} = -\dfrac{f_0}{f_e}\left(1+\dfrac{f_e}{D}\right)$
- In case of a telescope when the final image is at ∞, now if field and eye lenses are interchanged magnifying power will change from $\left(\dfrac{f_0}{f_e}\right)$ to $\left(\dfrac{f_e}{f_0}\right)$ i.e., it will change from m to $\left(\dfrac{1}{m}\right)$ i.e., will become $\left(\dfrac{1}{m^2}\right)$ times of its initial value.
- As magnifying power for normal setting as $\left(\dfrac{f_0}{f_e}\right)$ to have large magnifying power f_0 must be as large as practically possible and f_e is small. This is why in a telescope, objective is of large focal length while eyepiece of smaller focal length.

- A larger objective aperture helps in collecting more light from a distant object, thereby improving the brightness of the image. However, it can also increase aberrations, particularly spherical aberration. If a fly lands on the objective lens of a telescope and a photograph of a distant astronomical object is taken through it, the fly will not be visible, but the intensity of the image will be slightly reduced as the fly will act as an obstruction to the light and decrease the aperture of the objective.

  Telescopes produce angular magnification, while microscopes produce linear magnification. The image produced by a telescope appears closer to the eye, which increases the visual angle.

- **Terrestrial Telescope:**

  The magnifying power and length of telescope for relaxed eye will be

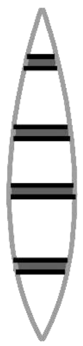
$$M_{\infty} = \frac{-f_0}{f_e}(-1) = \frac{f_0}{f_e}, \quad L_{\infty} = f_0 + f_e + 4f$$

  The magnifying power and the length of telescope for image at D will be

$$M_D = \frac{f_0}{f_e}\left(1 + \frac{f_e}{D}\right), \quad L_D = f_0 + 4f + \frac{Df_e}{D + f_e}$$

EXERCISE-I

1. A layered lens as shown in figure is made of two types of transparent materials indicated by different shades. A point object is placed on its axis. The object will form:



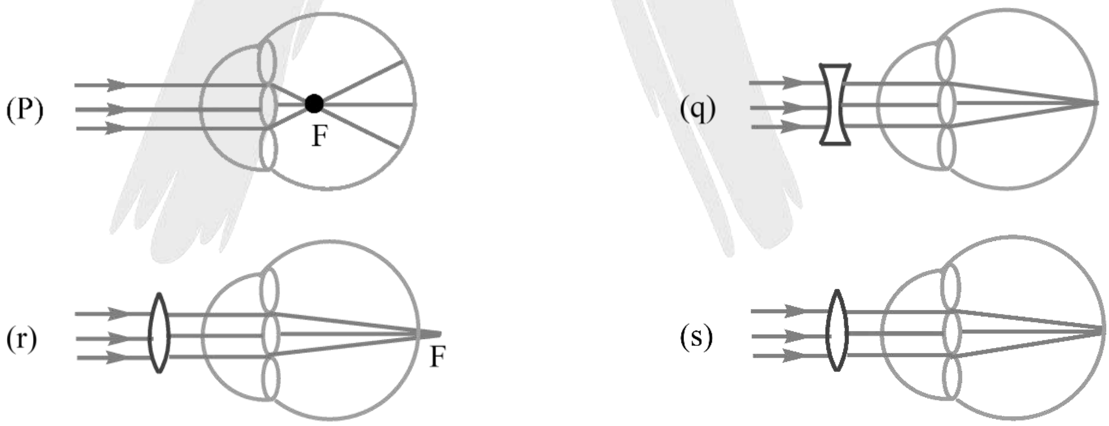
(A) 1 image (B) 2 images (C) 3 images (D) 9 images

2. In telescope, if the powers of an objective and eye lens are +1.25 D and +20 D respectively, then for relaxed vision, the length an magnification will be:

(A) 85 cm and 25 (B) 85 cm and 16 (C) 21.25 cm and 16 (D) 21.25 cm and 25

3. A microscope objective gathers light over a cone of semi-vertex $30°$ and used visible light of wavelength $5500\,\overset{o}{A}$. Its resolving limit is $n \times 10^{-6}\,\text{cm}$. Find 'n'?

4. Identify the wrong description of the given figures:



(A) p represents far-sightedness (B) q correction for short-sightedness

(C) r represents far-sightedness (D) s correction for far-sightedness

5. The angular magnification of a simple microscope can be increased by increasing:

(A) focal length of lens (B) size of object

(C) aperture of lens (D) power of lens

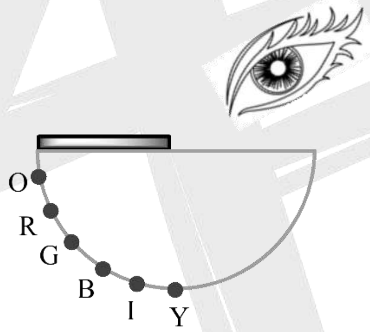
6. Modern telescope use mirrors, rather than lenses, to form images. One advantage of mirrors over lenses is that the images formed by mirrors are not affected by:

(A) interference
(B) spherical aberration
(C) atmospheric refraction
(D) chromatic aberration

7. A parallel beam of white light falls on a convex lens. Images of blue, red and green light are formed on other side of the lens at distances x, y and z respectively from the pole of the lens. Then:

(A) $x > y > z$
(B) $x > z > y$
(C) $y > z > x$
(D) none of these

8. A person is looking at the flat surface of transparent hemi sphere of refractive index $\mu = \sqrt{2}$. Half of the flat surface is coloured black and half of curved surface is marked at 6 points equally spaced as shown in the figure. If the person sees only through the flat surface, he can see:



(A) all the points
(B) Y and I only
(C) Y, I and B
(D) O, Y and G

9. Let spherical aberration and chromatic aberration be denoted by S and C respectively. In case of concave mirror:

(A) S and C both may be present
(B) Both S and C can not exist
(C) S may be present but C cannot exist
(D) C may be present but S cannot exist

10. Two thin prisms of flint glass, with refracting angles of 6° and 8° respectively, possess dispersive powers in the ratio:

(A) 4 : 3
(B) 3 : 4
(C) 1 : 1
(D) 9 : 16

11. A light ray travelling parallel to the principal axis of a concave mirror strikes the mirror at angle of incidence θ. If radius of curvature of the mirror is R, then after reflection, the ray meets the principal axis at distance d from the centre of curvature, then d is:

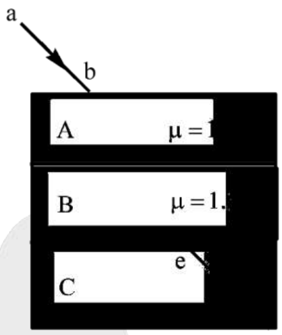
(A) $\frac{R}{2}$

(B) $R\left(1-\frac{1}{2\cos\theta}\right)$

(C) $\frac{R}{2\cos\theta}$

(D) $\frac{R}{2}(1+\cos\theta)$

12. In the given figure. The refractive index of slab C if ray ef is parallel to ray ab, is



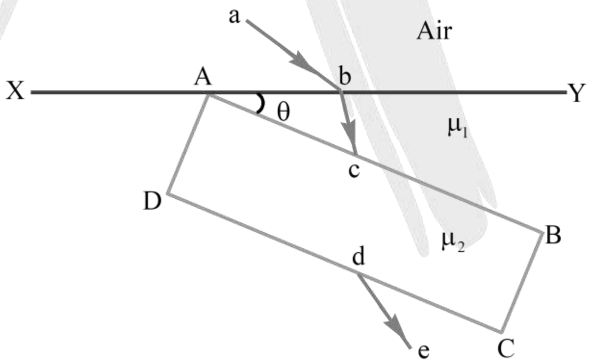
(A) 1.5

(B) 1.45

(C) 1.55

(D) 1

13. A ray of light ab passing through air, enters a liquid of refractive index μ_1, at the boundary XY. In the liquid, the ray is shown as bc. The angle between ab and bc is δ (angle of deviation). The ray then passes through a rectangular slab ABCD of refractive index $\mu_2\left(\mu_2>\mu_1\right)$, and emerges from the slab as ray de. The angle between XY and AB is θ. The angle between ab and de is:



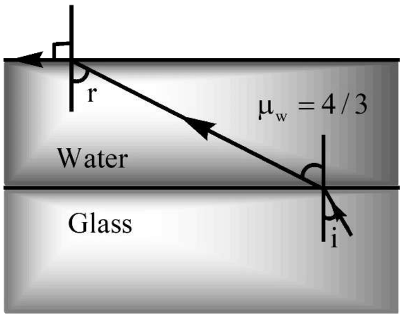
(A) δ

(B) $\delta+\theta$

(C) $\delta+\sin^{-1}\left(\frac{\mu_1}{\mu_2}\right)$

(D) $\delta+\theta-\sin^{-1}\left(\frac{\mu_1}{\mu_2}\right)$

14. A ray is light is incident at the glass-water interface at an angle i. It emerges finally parallel to the surface of water, then the value of μ_g would be:



(A) $\left(\frac{4}{3}\right)\sin i$ (B) $\frac{1}{\sin i}$ (C) $\frac{4}{3}$ (D) 1

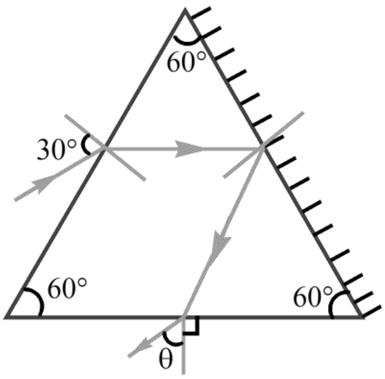
15. A ray of light is incident at a small angle i on a small angle prism $(A = 4°)$ and emerges normally from the opposite surface. If the refractive index of the material of the prism is $\mu = \frac{3}{2}$, the angle of incidence i (in degrees) is nearly equal to:

16. The minimum angle of deviation of a prism of refractive index μ is equal to its refracting angle. The refracting angle o prism is:

(A) $2\cot^{-1}\left(\frac{\mu}{2}\right)$ (B) $2\tan^{-1}\left(\frac{\mu}{2}\right)$

(C) $2\sin^{-1}\left(\frac{\mu}{2}\right)$ (D) $2\cos^{-1}\left(\frac{\mu}{2}\right)$

17. A ray diagram for a prism whose one surface is reflecting as shown. The value of θ will be:



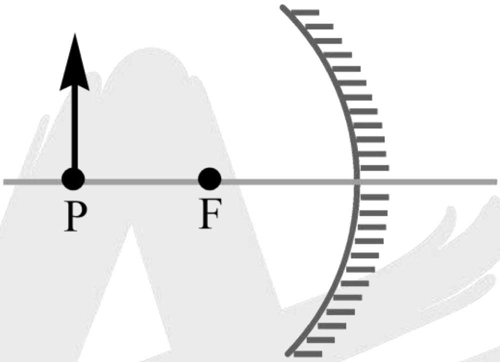
(A) 30° (B) 60° (C) 0° (D) 90°

18. f_v and f_r are the focal lengths of a convex lens for violet and red lights respectively and f_v and f_r are the focal lengths of a concave lens for violet and red lights respectively. Then:

(A) $f_v < f_r$ and $f_v > f_r$ (B) $f_v < f_r$ and $f_v < f_r$

(C) $f_v > f_r$ and $f_v > f_r$ (D) $f_v > f_r$ and $f_v < f_r$

19. The diagram below shows an object located at point P, 0.25 meter from a concave spherical mirror with principal focus F. The focal length of the mirror is 0.10 meter.



How does the image change as the object is moved from point P towards point F?

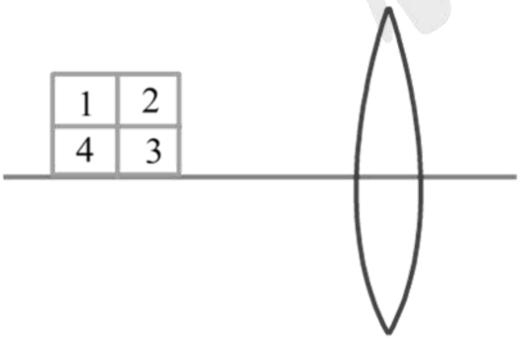
(A) Its distance from the mirror decreases

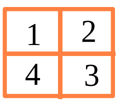
(B) The size of image decreases

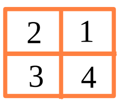
(C) Its distance from the mirror increases

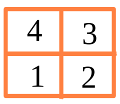
(D) The size of images increases

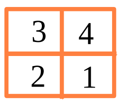
20. A convex lens is used to form a real image of the object as shown in the figure. Then the real inverted image is as shown in the following figure:



(A) 

(B) 

(C) 

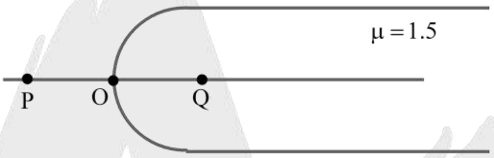
(D) 

EXERCISE-II

1. A convergent (biconvex) lens is placed inside a jar filled with a liquid. The lens has focal length 20cm, when in air and its material has a refractive index of 1.5. If the liquid has a refractive index of 1.6, the focal length of the lens while in the jar, is:

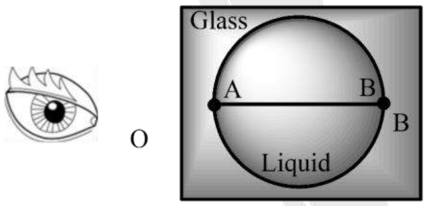
(A) -110 cm (B) -130 cm (C) -160 cm (D) -180 cm

2. One end of a glass rod of refractive index $n = 1.5$ is a spherical surface of radius of curvature R. The centre of the spherical surface lies inside the glass. A point object placed in air on the axis of the rod at the point P has its real image inside glass at the point Q (see figure). A line joining the points P and Q cuts the surface at O such that $OP = 2(OQ)$. The distance PO is:



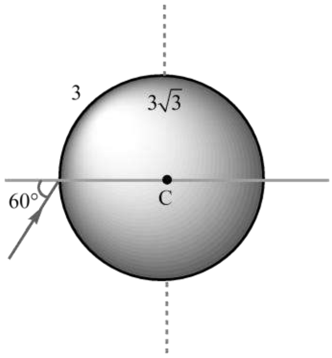
(A) 8R (B) 7R (C) 2R (D) None of these

3. The observer 'O' sees the distance AB as infinitely large. If refractive index of liquid is μ_1 and that of glass is μ_2, then $\frac{\mu_1}{\mu_2}$ is:



(A) 2 (B) $\frac{1}{2}$ (C) 4 (D) None of these

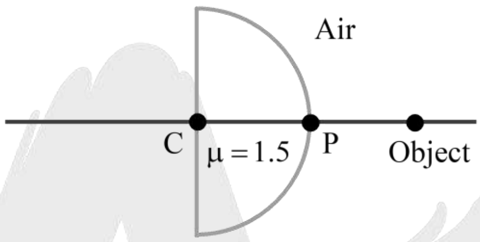
4. Left half of a glass sphere is surrounded with a medium having refractive index $\sqrt{3}$ as shown. A ray is incident at an angle of $60°$ as shown. Find the total deviation as the ray comes out of sphere.



(A) $60°$ CW (B) $60°$ ACW (C) $120°$ CW (D) $120°$ ACW

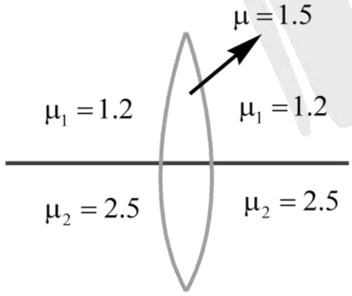
5. An object and a screen are fixed on the uprights of an optical bench. The distance between them is 100 cm. A convex lens is placed in between the object and the screen and the position of the lens is so adjusted that the image of the object is formed on the screen at two conjugate positions of the lens. The distance between these conjugate positions of the lens is 40 cm. what is the focal length (in cm) of the lens:

6. A point object is placed at a distance of 9 meter from a glass hemisphere of refractive index $\mu = \frac{3}{2}$ then position of final image from C is (radius of hemisphere is 3 meter):



(A) 15 m right of C
(B) 16 m left of C
(C) 22.5 m left of C
(D) 22.5 m right of C

7. A thin lens of material having refractive index $\mu = 1.5$ and focal length of 20 cm in air has two mediums of different refractive indices $\mu_1 = 1.2$ and $\mu_2 = 2.5$ cover upper and lower halves of the lens, respectively as shown in figure. If an object is placed on the principal axis, then its two images will form one after refraction from upper part and other after refraction from lower part. Consider the object to be at ∞, the separation (in cm) between two images formed would be:



8. Two identical thin planoconvex lenses of refractie index n are silvered, one on the plane side and the other on the convex side. The ratio of their focal lengths is:

(A) $\frac{n}{n-1}$
(B) $\frac{n-1}{n}$
(C) $\frac{n+1}{n}$
(D) n

9. The least distance of distinct vision for a far-sighted person is 1m. The optical power of the lens of his specs which effectively reduces his LDDV to 25 cm, is:

(A) +3D (B) +2D (C) -3D (D) None of these

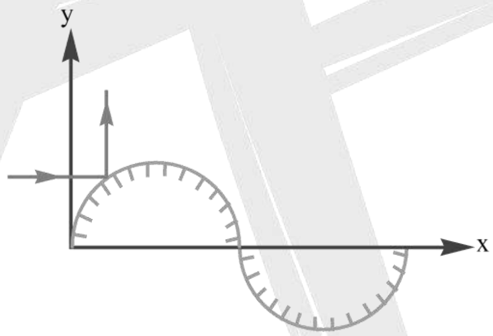
10. In telescope, if the powers of an objective and eye lens are +1.25 D and +20 D respectively, then for relaxed vision, the length and magnification will be:

(A) 85 cm and 25 (B) 85 cm and 16 (C) 21.25 cm and 16 (D) 21.25 cm and 25

11. The power of a lens having refractive index 1.25 is +3 diopters. When placed in a liquid its power is -2 diopters. The refractive index of the liquid is:

(A) 1.2 (B) 1.4 (C) 1.5 (D) 1.6

12. Equation of reflecting surface is defined as $y = \frac{2L}{\pi} \sin \frac{\pi x}{L}$. A ray of light incident on this surface in first quadrant parallel to X axis becomes parallel to Y axis just after reflection. Find X co-ordinate point of incidence:



(A) $\frac{L}{6}$ (B) $\frac{L}{3}$ (C) $\frac{L}{4}$ (D) $\frac{L}{8}$

13. A spherical mirror is polished on both sides. When the convex side is used as a mirror the image is erect with magnification $\frac{1}{4}$. What is the magnification when the concave side is used as a mirror, the object remaining the same distance from the mirror?

(A) $-\frac{1}{4}$ (B) $-\frac{1}{2}$ (C) $-\frac{1}{3}$ (D) $+\frac{1}{4}$

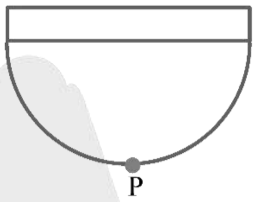
14. A pole 6 m hign stands amidst a lake having water to a depth of 2m. If the sun is at angle of inclination $37°$, what will be the length of the shadow in water?

(A) 4.5 m (B) 6.8 m (C) 5.3 m (D) 6 m

15. A beam of light in air of width t is incident on an air-water boundary at an angle of incidence 45°. The width of the beam in water is: (Refractive index of water = μ)

(A) $(\mu - 1)t$ (B) μt (C) $\frac{\sqrt{2\mu^2 - 1}}{\mu}.t$ (D) $\frac{\sqrt{\mu^2 - 1}}{\mu}.t$

16. A hemispherical bowl of radius 10 cm if filled with liquid of refractive index $\mu = \frac{4}{3}$. A glass plate of refractive index 1.5 is placed on the top of bowl. If for the observer above the plate the shift in position of a point P on the bottom is 3 cm. Find the thickness of glass plate.

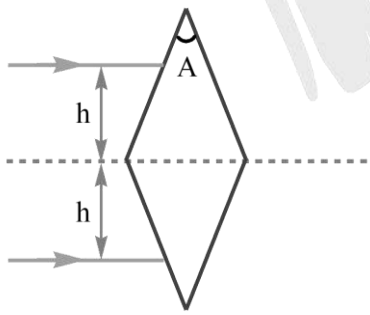


(A) 1.5 cm (B) 1 cm (C) 7 cm (D) 10 cm

17. It is necessary to illuminate the bottom of a well by reflected solar beam when the light is incident at an angle 40° to the vertical. At what angle to the horizontal should a plane mirror be placed?

(A) 65° (B) 70° (C) 75° (D) 80°

18. Two identical isosceles thin prisms of prism angle A and refractive index μ are placed with their bases touching each other. This system acts as a converging lens. What is the focal length of this system for parallel rays at a distance h from the base of the prism?



(A) $\frac{2h}{(\mu - 1)A}$ (B) $\frac{h}{(\mu - 1)A}$

(C) $\frac{h}{2(\mu - 1)A}$ (D) None of these

19. A concave spherical mirror has a radius of curvature of 50 cm. Find two positions of an object for which the image is four times as large as the object.

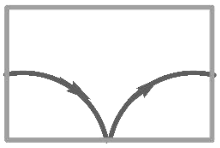
(A) $\frac{75}{4}$ cm from mirror

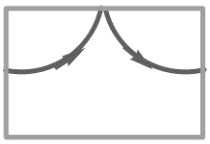
(B) 125 cm from mirror

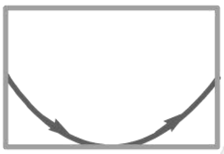
(C) $\frac{125}{4}$ cm from mirror

(D) 32 cm from mirror

20. A cubic container is filled with a liquid whose refractive index increases linearly from top to bottom. Which of the following figures may represent the path of a ray of light inside the liquid?

(A) 

(B) 

(C) 

(D) 

21. A number of thin prism of prism angle A and refractive index μ are arranged on periphery of circle such that any light ray entering from one prism moves along a regular polygon.

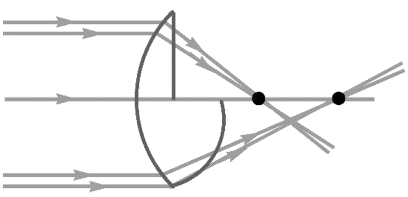
(A) Number of prism used will be $\frac{2\pi}{(\mu-1)A}$

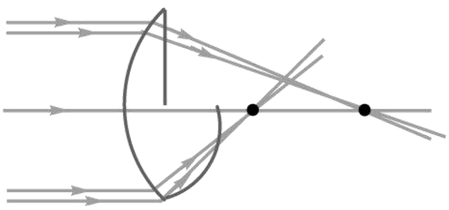
(B) If A is rational, μ must be rational

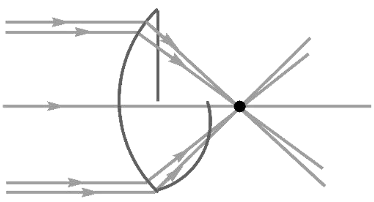
(C) If A is rational, μ must be irrational

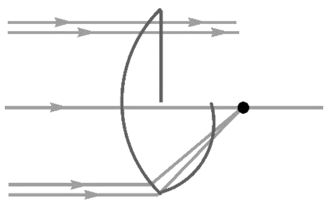
(D) If A is irrational, μ must be irrational

22. Choose the correct ray diagram of an equiconvex lens which is cut as shown :

(A) 

(B) 

(C) 

(D) 

23. A thin concavo – convex lens has two surfaces of radii of curvature R and 2R. The material of the lens has a refractive index μ. When kept in air, the focal length of the lens :

(A) will depend on the direction from which light is incident on it

(B) will be the same, irrespective of the direction from which light is incident on it

(C) will be equal to $\frac{R}{\mu-1}$

(D) will be equal to $\frac{2R}{\mu-1}$

EXERCISE-III

1. A watch glass has uniform thickness, and the average radius of curvature of its two surfaces is much larger than its thickness. It is placed in the path of a beam of parallel light in air. The beam will:

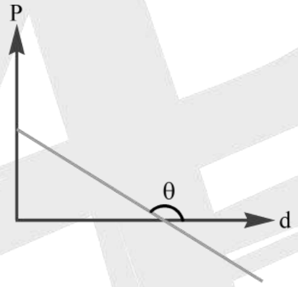
(A) converge slightly

(B) diverge slightly

(C) be completely unaffected

(D) converge or diverge slightly depending on whether the beam is incident from the concave or the convex side

2. Two coaxial convex lenses have an effective power of P_0 when in contact, figure shows the graph of power P versus d, the distance between their centre. The difference in their focal lengths is: (Given that $P = P_1 + P_2 - dP_1P_2$)



(A) $\sqrt{P_0^2 \cot^2\theta - 4\cot\theta}$

(B) $\sqrt{P_0^2 \cot^2\theta + 4\cot\theta}$

(C) $P_0^2 \cot^2\theta - 4\cot\theta$

(D) $P_0^2 \cot^2\theta + 4\cot\theta$

3. A convex spherical refracting surface with radius R separates a medium having refractive index $\frac{5}{2}$ from air. As an object (in air) approaches the surface from far away from the surface along the central axis, its image:

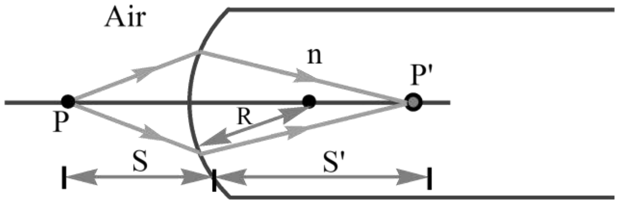
(A) changes from real to virtual when it is at a distance R from the surface

(B) changes from virtual to real when it is at a distance R from the surface

(C) changes from real to virtual when it is at a distance $\frac{2R}{3}$ from the surface

(D) changes from virtual to real when it is at a distance $\frac{2R}{3}$ from the surface

4. Which of these actions will move the real image point P' farther from the boundary?



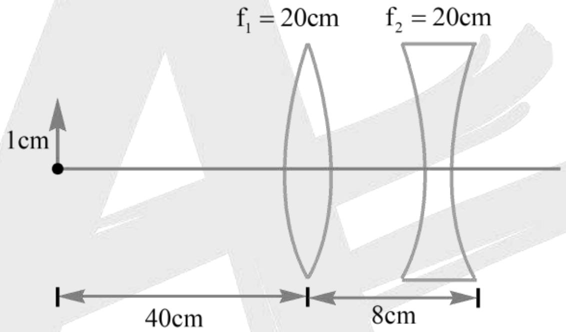
[P] Decrease the index of refraction n

[Q] Increase the distance S

[R] Decrease the radius of curvature R

(A) P, Q, R (B) R only (C) Q and R only (D) Q only

5. Consider a system of two thin lenses as shown in figure. An object of height 1 cm is placed at a 40 cm from convex lens. Mark the correct option related to final image formed by the two-lens-system:



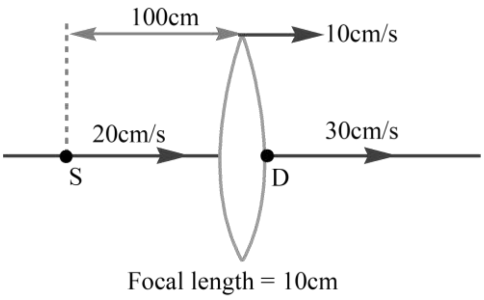
(A) Final image is formed at 32 cm on right of concave lens and is $\frac{1}{2.2}$cm in size

(B) Final image is formed at 32 cm on left side of concave lens and is 1 cm in size

(C) Final image is formed at 14.5 cm on the left on concave lens and is $\frac{1}{2.2}$cm in size

(D) None of the above

6. The figure shows the initial position of a point source of light S, a detector D and a lens L. Now at $t = 0$, all the three start moving towards right with different velocities as shown.

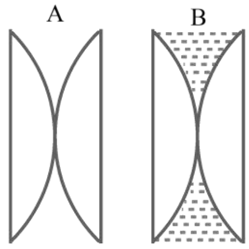


The time(s) at which the detector receives the maximum light is:

(A) 0.56 s and 8.94 s (B) 3.8 s

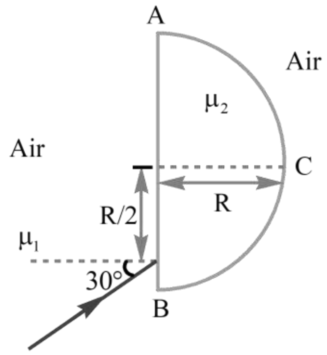
(C) 8.94 s and 19.62 s (D) 0.56 s

7. Figure A shows two identical plano-convex lenses in contact as shown. The combination has focal length 24 cm. Figure B shows the same with a liquid introduced between them. If refractive index of glass of the lenses is 1.50 and that of the liquid is 1.60, the focal length of the system in figure B will be:



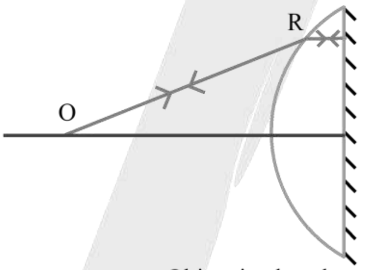
(A) -120 cm (B) 120 cm (C) -24 cm (D) 24 cm

8. In the adjacent figure, a ray of light travelling in air $(\mu_1 = 1)$ strikes the plane surface AB of a solid semi-cylinder with refractive index μ_2, at an angle 30° with the normal to the surface. The emergent ray comes out at C. The value of μ_2 is (Axis of cylinder is perpendicular to plane of the paper and incident ray lies in the plane of the paper):

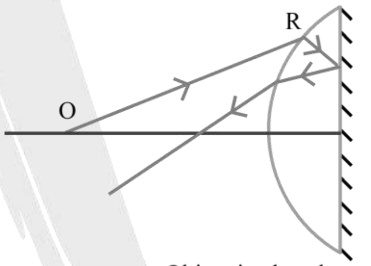


(A) $\frac{\sqrt{5}}{2}$ (B) $\frac{\sqrt{5}}{\sqrt{2}}$ (C) $\frac{2\sqrt{5}}{3}$ (D) $\frac{4}{3}$

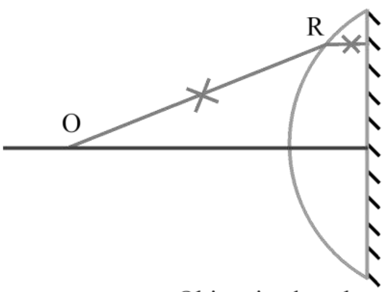
9. A thin plano-convex glass lens $(\mu = 1.5)$ has its plane surface reflecting and R is the radius of curvature of curved part, then which of the following ray diagram is true for an object placed at O?

(A)


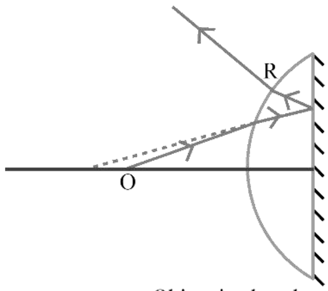
Object is placed at distance 2R from lens

(B)


Object is placed at distance 2R from lens

(C)


Object is placed at distance 2R from lens

(D)


Object is placed at distance 2R from lens

10. A plano-convex lens, when silvered at its plane surface is equivalent to a concave mirror of focal length 28 cm. when its curved surface is silvered and the plane surface not silvered, it is equivalent to a concave mirror of focal length 10 cm, then the refractive index of the material of the lens is:

(A) $\frac{9}{14}$ (B) $\frac{14}{9}$ (C) $\frac{17}{9}$ (D) None of these

11. A hollow biconcave lens is made of a very thin transparent material. Refractive index of this material is nearly equal to 1. It can be filled with water (refractive index μ_w) or either of two liquids L_1 or L_2 with refractive indices μ_1 and μ_2 respectively $(\mu_2 > \mu_w > \mu_1)$. The lens will not diverge a parallel beam of light incident on it, if it is filled with:

(A) L_2 and immersed in L_1

(B) L_2 and immersed in water

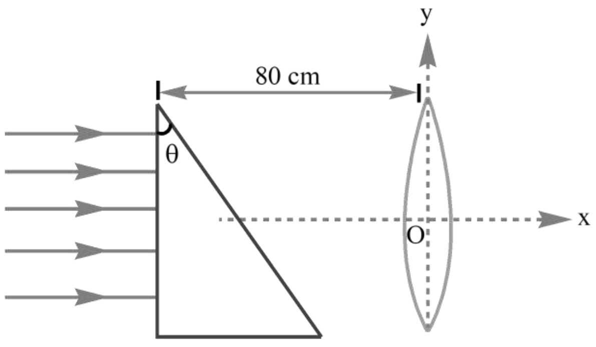
(C) water and immersed in L_1

(D) air and immersed in either water or L_1 or L_2

12. In the displacement method for determining the focal length of a convex lens, the lens is displaced by distance x to again obtain a sharp image on the screen. If the magnification in two positions are m_1 and m_2 respectively, the focal length of the lens is:

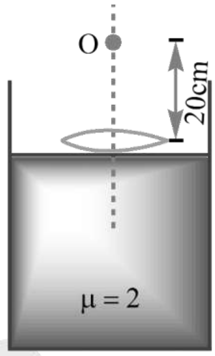
(A) $\frac{x}{|m_1+m_2|}$ (B) $\frac{x}{|m_1-m_2|}$ (C) $\frac{x}{|m_1-m_2|^2}$ (D) $\frac{x}{|m_1+m_2|^2}$

13. A parallel beam of light is incident on a thin prism of prism angle of $\frac{4}{\pi}$ degrees. The refractive index of the prism is 1.5. The focal length of the lens is 60 cm. The coordinates of converging point of the beam is:

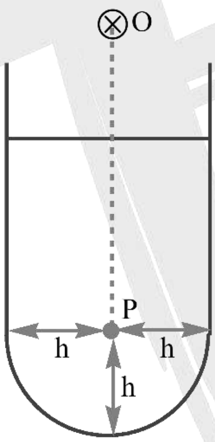


(A) $\left(60\,\text{cm}, \frac{2}{3}\,\text{cm}\right)$ (B) $\left(60\,\text{cm}, \frac{1}{3}\,\text{cm}\right)$ (C) $\left(60\,\text{cm}, -\frac{1}{3}\,\text{cm}\right)$ (D) $\left(60\,\text{cm}, -\frac{2}{3}\,\text{cm}\right)$

14. A point object O is placed at a distance of 20 cm in front of a equip-convex lens $\left({}_{a}\mu_{g}=1.5\right)$ of focal length 10 cm. The lens is placed on a liquid of refractive index 2 as shown. Image will be formed at a distance h from lens. The value of h (in cm) is:



15. A concave mirror of radius h is placed at the bottom of a tank containing a liquid of refractive index μ upto a depth d. An object P is placed at height h above the bottom of the mirror. Outside the liquid, an observer O views the object and its image in the mirror. The apparent distance between these two will be:



(A) 0

(B) $\frac{2h}{\mu}$

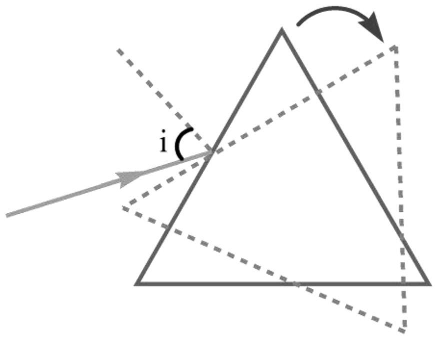
(C) $h\left(1-\frac{1}{\mu}\right)$

(D) $\frac{2h}{\mu-1}$

16. A simple telescope consisting of an objective of focal length 60 cm and a single eye lens of focal length 5cm is focused on a distant object in such a way that parallel rays emerge from the eye lens. If the object subtend an angle of 2° at the objective then the angular width of the image is:

(A) 10° (B) 24° (C) 50° (D) 36°

17. An equilateral prism produces a deviation of $30°$. When prism is rotated by an angle $30°$, keeping the incident ray fixed as shown, angle of deviation remains same:



(A) angle of incidence before rotation is $45°$

(B) angle of incidence before rotation is $30°$

(C) initially prism is in a state producing minimum deviation

(D) after rotation prism is in a state producing minimum deviation

18. A prism of angle $3°$ is made of glass having refractive index 1.64. Two thin prisms made of glass having refractive index 1.48 are intended to be coupled with the former prism to yield a combination without an average deviation. Which of the following angles cannot correspond to the two prisms?

(A) $2°$ and $2°$ (B) $1.5°$ and $2.5°$ (C) $6°$ and $2°$ (D) $5.5°$ and $3.5°$

19. A luminous point object is moving along the principal axis of a concave mirror of focal length 12 cm towards it. When its distance from the mirror is 20 cm and its velocity is 4 cm/s. The velocity of the image in cm/s at that instant is:

(A) 6, towards the mirror

(B) 6, away from the mirror

(C) 9, away from the mirror

(D) 9, towards the mirror

20. Ram is looking at his face in a mirror kept 10 cm away and he finds that his image is erect and magnified $(m = 1.8)$. If he holds the mirror 50 cm away:

(A) He cannot see the image because reflected rays falling on his eyes are converging

(B) He sees a magnified and erect image

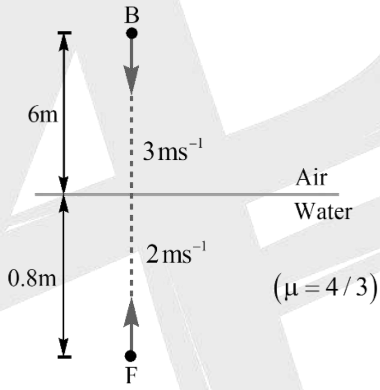
(C) He sees a diminished and inverted image

(D) He sees a magnified and inverted image.

21. A submarine is 300 m horizontal out from the shore and 100 m beneath the surface of the water. A laser beam is sent from the submarine, so that it strikes the surface of the water at a point 200 m from the shore. If the outgoing beam from the surface of the water just strikes the top of building standing vertically at shore, then the height of the building is $\left[\mu_w = \frac{4}{3}\right]$:

(A) 300 m (B)140 m (C) 240 m (D) 170.7 m

22. A fish, F in the pond, is at a depth of 0.8 m from water surface and is moving vertically upwards with velocity $2\,ms^{-1}$. At the same instant, a bird B is at a height of 6 m from water surface and is moving downwards with velocity $3ms^{-1}$. At this instant both are on the same vertical lines as shown in the figure. Which of the following statement(s) is (are) correct?



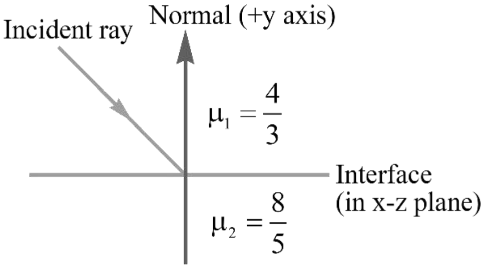
(A) Height of B, observed by F (from itself) is equal to 8.00 m

(B) Depth of F, observed by B (from itself) is equal to 6.60 m

(C) Velocity of B, observed by F (relative to itself) is equal to $5.00\,ms^{-1}$

(D) Velocity of F, observed by B (relative to itself) if equal to $4.50\,ms^{-1}$

23. A light ray incident in a medium of refractive index $\frac{4}{3}$ is shown in diagram. Interface seperating the two mediums lies in xz plane. Vector along incident ray is $3\hat{i} - 4\hat{j}$.



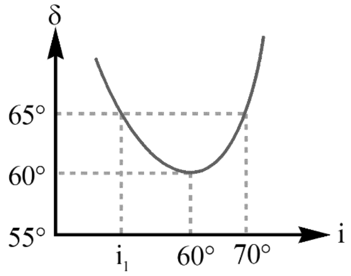
(A) Angle of incidence is $37°$

(B) The angle of refraction is $60°$

(C) The angle of refraction is $30°$

(D) If angle of incidence of the incident ray is greater than to the critical angle then total internal reflection takes place

24. The angle of deviation (δ) v/s angle of incidence (i) is plotted for a prism. Choose the correct statement(s).



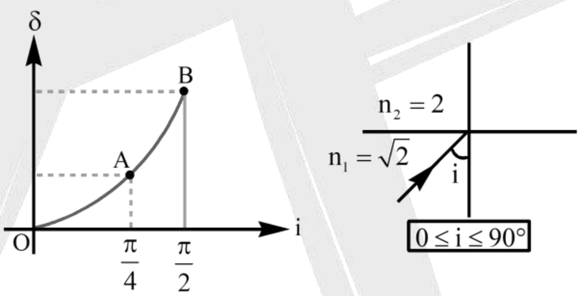
(A) The angle of prism is $60°$

(B) The refractive index of the prism is $n=\sqrt{3}$

(C) For deviation to be $65°$ the angle of incidence $i_1 = 55°$

(D) for minimum deviation, the angle of emergence $e = 60°$

25. A ray incidents on an interface as shown and graph between angle of deviation 'δ' and angle of incidence i, $(0 \le i \le 90°)$ is also shown in the figure. Choose the correct statement(s) :



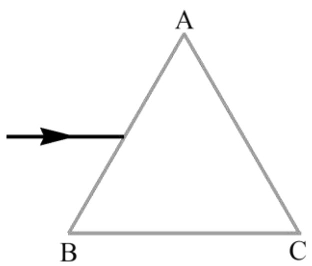
(A) Graph after point A is not possible

(B) At point A value of $\delta = \frac{\pi}{2} - \theta_C$

(C) Graph after point A is straight line

(D) At point A deviation δ is $15°$

26. A ray of light is incident on a equilateral triangular prism parallel to its base as shown in the figure. The ray just fails to emerge from the face AC. If μ be the refractive index of the prism then the incorrect relation(s) is/are :



(A) $2\sin^{-1}\left(\frac{1}{\mu}\right)=\frac{\pi}{3}$ (B) $\sin^{-1}\left(\frac{1}{\mu}\right)+\sin^{-1}\left(\frac{1}{2\mu}\right)=\frac{\pi}{6}$

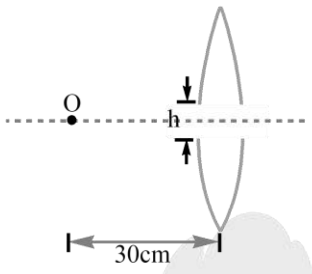
(C) $\sin^{-1}\left(\frac{1}{\mu}\right)+\sin^{-1}\left(\frac{1}{2\mu}\right)=\frac{\pi}{3}$ (D) $\sin^{-1}\left(\frac{\mu}{2}\right)+\sin^{-1}\left(\frac{\mu}{4}\right)=\frac{\pi}{3}$

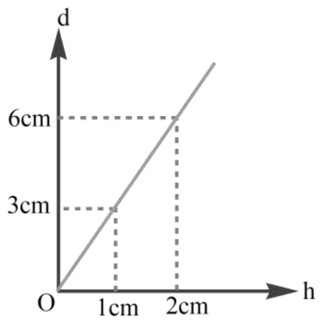
27. The object is at a distance of 45 cm from the screen. With the help of lens we obtain a small image of the object on the screen. By moving the lens, we receive a different image on the screen, whose size is 4 times greater than the first. What is the focal length of the lens?

(A) 5cm (B) 15cm (C) 10cm (D) 20cm

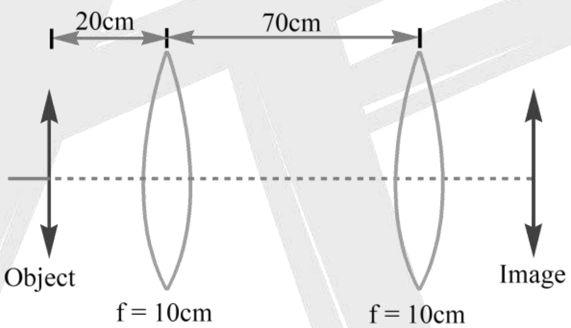
EXERCISE-IV

1. Figures shows a convex lens cut symmetrically into two equal halves and separated laterally by a distance h. A point object placed at a distance 30cm, from the lens halves, forms two real images separated by a distance d. A plot of d versus h is shown in figure. The focal length of the lens (in cm) is:



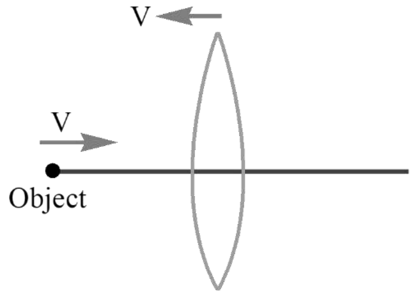


2. In the given optical instrument, in which manner third lens can be inserted in order to increase the collecting efficiency of light without changing the position of object and image. Calculate the position of that lens.



(A) 20 cm from left lens
(B) 30 cm from left lens
(C) 20 cm from right lens
(D) 45 cm from right lens

3. In the diagram shown, the lens is moving towards the object with a velocity V m/s and the object is also moving towards the lens with the same speed. What is speed of the image with respect to earth when the object is at a distance 2f from the lens? (f is the focal length):

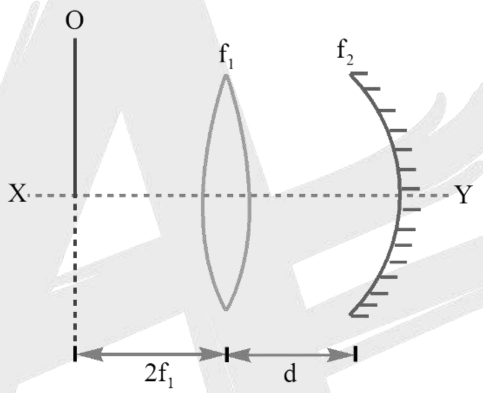


(A) 2V
(B) 4V
(C) 3V
(D) V

4. A student wants to set up an experiment with a thin convex lens of focal length f such that a thin real object produces a focused real image on a movable screen. At how many locations along the optical axis (principal axis) can the object be placed so that the distance between the object and the focused image on the screen is equal to 3f?

(A) There is no location
(B) There is exactly one location
(C) There are exactly two locations
(D) There are exactly four locations

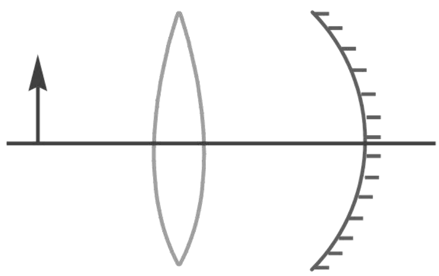
5. An erect object O is placed transverse to the axis of a converging lens at a distance equal to twice the focal length f_1 of the lens. One the other side of the lens, is placed a converging mirror of focal length f_2 at a distance d from the lens. The lens and the mirror have a common optic axis XY.



It is observed that the image of the object O seen after one refraction from lens and one reflection from mirror is erect, real and of the same size as the object O. Then the distance d is given by:

(A) $d = f_1 + f_2$
(B) $d = 2(f_1 + f_2)$
(C) $d = \frac{f_1 + f_2}{2}$
(D) $d = 2f_2$

6. Figure shows a thin converging lens for which the focal length is 5.00 cm. The lens is in front of a concave spherical mirror of radius $R = 30cm$. If the lens and mirror are 20.0 cm apart and an object is placed 15 cm to the left of the lens, determine the approximate distance of the final image from lens.

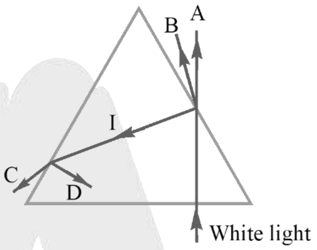


(A) 5.3 cm
(B) 4.6 cm
(C) 6.1 cm
(D) 12.7 cm

7. The refractive index of a medium for light of wavelength λ is given by $\mu = 1.45 + \frac{128\times10^4}{\lambda^2}$, where λ is in $\overset{o}{A}$. Assuming the white light to comprise of colours from violet $\left(\lambda = 4000\overset{o}{A}\right)$ to red $\left(\lambda = 8000\overset{o}{A}\right)$, then the wavelength whose μ will be the mean of the two extreme colours is:

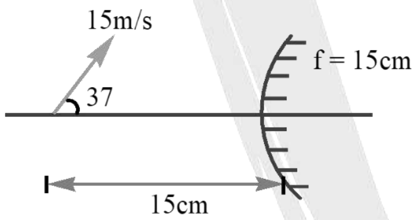
(A) $6000\overset{o}{A}$ (B) $6560\overset{o}{A}$ (C) $5333\overset{o}{A}$ (D) $5060\overset{o}{A}$

8. A white light ray is incident on a glass prism and it creates three refracted rays A, B and C and one reflected ray D. Select the correct alternative(s). (I and D are total internally reflected rays).



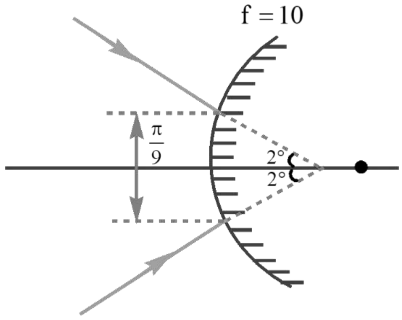
(A) Best possible colour for A is red, B is green, C is yellow and D is blue.
(B) Best possible colour for A is red, B is yellow, C is green and D is blue
(C) Best possible colour for A is blue, B is green, C is yellow and D is red
(D) Best possible colour for A is blue, B is yellow, C is green and D is red

9. In the figure shown, the speed of image with respect to mirror is:



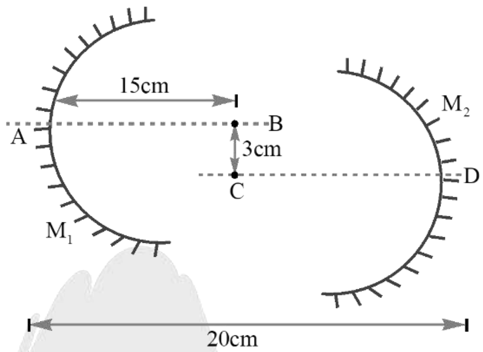
(A) 3 m/s (B) 4.5 m/s (C) 5.41 m/s (D) 29.25 m/s

10. A converging beam of light having angle of convergence 4° is incident upon a convex mirror as shown. Find the angle of convergence after reflection. focal length of mirror is 10 cm.



(A) 0.5° (B) 1° (C) 1.5° (D) 2°

11. M_1 and M_2 are two concave mirrors of the same focal length 10 cm. AB and CD are their principal axes respectively. An object is kept on the line AB at distance 15 cm from M_1. The distance between the mirrors is 20 cm. considering two successive reflections, first on M_1 and then on M_2. The distance of final image from the line AB is:

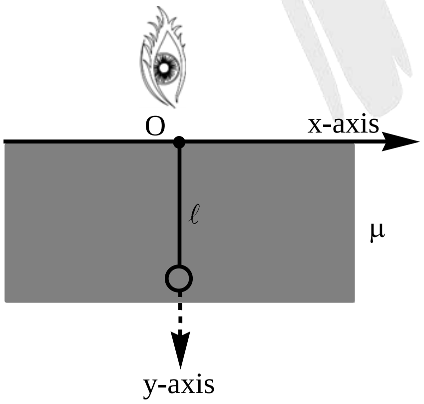


(A) 3 cm (B) 1.5 cm (C) 4.5 cm (D) 1 cm

12. A vessel of depth H is filled with a non-homogeneous liquid wose refractive index varies with the depth 'y' as: $\mu = \left(1 + \frac{y}{H}\right)$. The apparent depth as seen by an observer from above is:

(A) 0.693 H (B) 0.667 H (C) 0.75 H (D) 0.500 H

13. A pendulum of length ℓ is free to oscillate in vertical plane about point O in a medium of refractive index μ. An observer in air is viewing the bob of the pendulum directly from above. The pendulum is performing small oscillations about its equilibrium position. The equation of trajectory of bob as seen by observer is:



(A) $x^2 + y^2 = \ell^2$

(B) $\frac{x^2}{(\ell/\mu)^2} + \frac{y^2}{\ell^2} = 1$

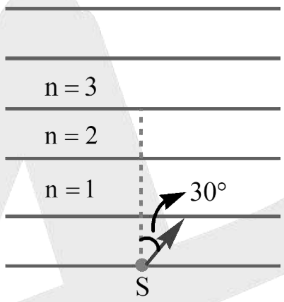
(C) $\frac{x^2}{\ell^2} + \frac{y^2}{(\ell/\mu)^2} = 1$

(D) $x^2 + y^2 = \left(\frac{\ell}{\mu}\right)^2$

14. A light ray is travelling from air to a liquid of refractive index $\sqrt{3}$. If the incident beam is rotated at a rate of 3 rad/sec then the angular speed of refracted beam when angle of incident is $60°$, is:

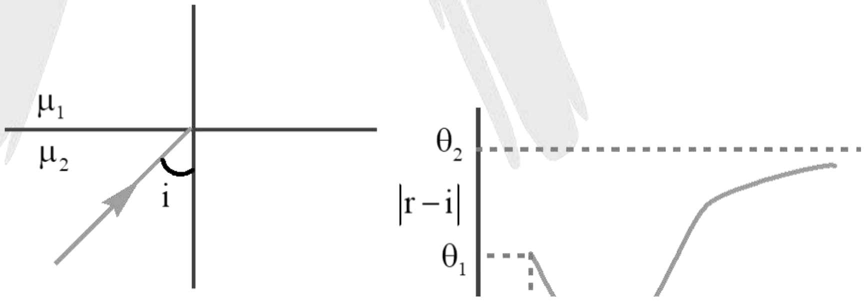
(A) 3 rad/sec (B) $\sqrt{3}$ rad/sec (C) 1 rad/sec (D) $3\sqrt{3}$ rad/sec

15. A point source S is placed at the bottom of different layers as shown in figure. The refractive index of bottom most layer is μ_0. The refractive index of any other upper layer is $\mu(n)=\mu_0-\frac{\mu_0}{4n-18}$ where $n=1,2,\ldots\ldots$. A ray of light starts from the source S as shown. Total internal reflection takes place at the upper surface of layer having n equal to:



(A) 3 (B) 5 (C) 4 (D) 6

16. The figure shows a ray incident at an angle $i=\frac{\pi}{3}$. If the plot drawn shows the variation of $|r-i|$ versus $\frac{\mu_1}{\mu_2}=k$, (r = angle of refraction) choose the wrong alternative.



(A) The valueof k_1 is $\frac{2}{\sqrt{3}}$

(B) The value of θ_1 is $\frac{\pi}{6}$

(C) The value of θ_2 is $\frac{\pi}{3}$

(D) The value of k_2 is 2

17. A particle moves towards a concave mirror of focal length 30 cm along its axis and with a constant speed of 4 cm/sec.

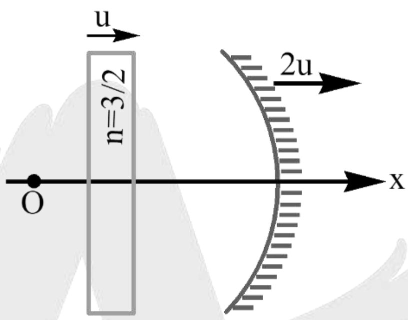
(A) At the instant the particle is 90 cm from the pole speed of image is 1 cm/sec

(B) At the instant the particle is 90 cm from the pole speed of image w.r.t. particle is cm/sec

(C) At the instant the particle is 90 cm from the pole particle and image move towards each other

(D) The particle approaches pole, velocity of image always increases

18. For the system shown in the figure, the image formed by the concave mirror :



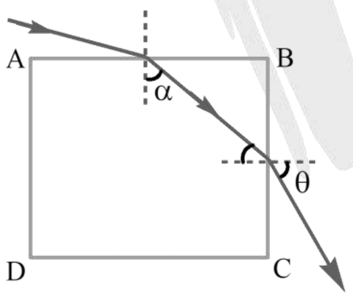
(A) may have speed greater than the speed of the object

(B) will have positive velocity

(C) may have speed greater than the speed of the mirror

(D) sign of velocity will depend upon thickness of slab

19. ABCD is plane glass cube. A nearly horizontal beam of light enters the face AB at grazing incidence. Then :



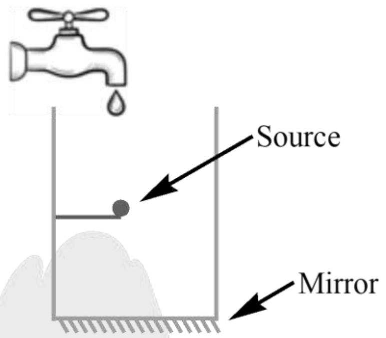
(A) $\sin\theta = \cot\alpha$ where α is critical angle

(B) Critical angle $\alpha \leq 45°$, for TIR at second surface

(C) Refracting index $\mu \leq \sqrt{2}$ for TIR at second surface

(D) None of the above

20. A small source of light is mounted inside a cylindrical container of height h. The bottom of the container is covered with a mirror. Initially, the container is empty. Then a clear liquid with the index of refraction n is slowly poured into the container. The level of liquid H rises steadily, reaching the top of the container in time T. Let h_1 be the distance of source of the light from the bottom of the container. Consider paraxial ray approximation.



Consider two cases

$[P] H \le h_1$ $\quad$ $[Q] h_1 \le H \le h$

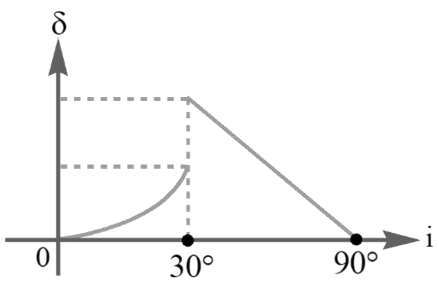
(A) The speed of the image of the source in case [P] during this process is $\frac{2h}{T}\left(1-\frac{1}{n}\right)$

(B) The speed of the image of the source in case [Q] during this process is $\frac{h}{T}\left(1-\frac{1}{n}\right)$

(C) The speed of the image of the source in case [P] during this process is $\frac{h}{2T}\left(1-\frac{1}{n}\right)$

(D) The speed of the image of the source in case [Q] during this process is $\frac{h}{2T}\left(\frac{1}{n}\right)$

21. Figure shows graph of angle of deviation v/s angle of incidence for a light ray. Incident ray goes from medium 1 (μ_1) to medium 2 (μ_2). Mark the correct option(s) :



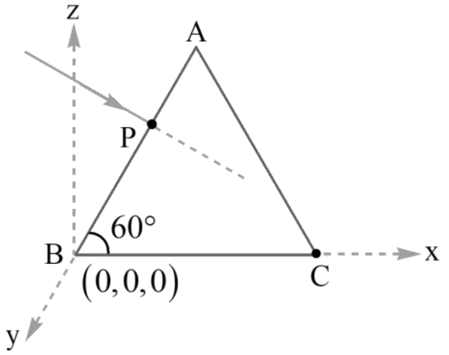
(A) $\frac{\mu_1}{\mu_2} = \frac{1}{2}$

(B) Critical angle is $30°$

(C) $\mu_1 > \mu_2$

(D) Maximum deviation is $120°$

22. An equilateral prism ABC is placed in air with its base side BC lying horizontally along x-axis as shown in the figure. A ray given by $\sqrt{3}z + x = 10$ is incident at a point P on face AB of prism :



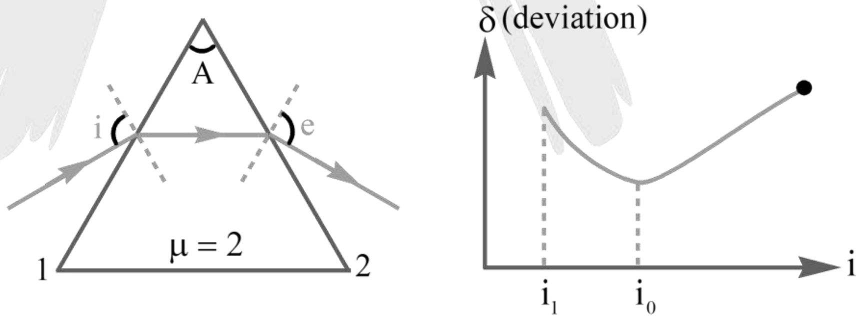
(A) For $\mu = \frac{2}{\sqrt{3}}$ the ray grazes the face AC

(B) For $\mu = \frac{3}{2}$ finally refracted ray is parallel to z-axis

(C) For $\mu = \frac{2}{\sqrt{3}}$ the ray emerges perpendicular to the face AC

(D) For $\mu = \frac{3}{2}$ finally refracted ray is parallel to x-axis

23. A monochromatic light is incident on a prism as shown. Deviation suffered by ray varies with angle of incidence as shown. Assuming refraction takes place at 1 and 2. Choose the correct alternative(s).



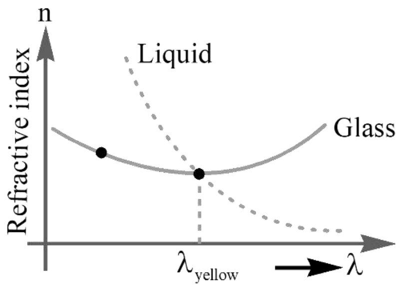
(A) i_1 can be equal to zero if $A = 30°$

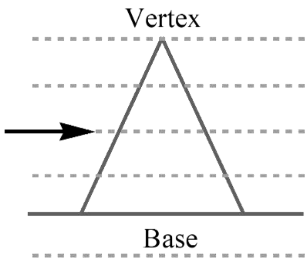
(B) If angle of incidence is increased by $2°$, then angle of emergence may decrease by less than $2°$

(C) If angle of incidence is increased by $2°$, then angle of emergence may decrease by more than $2°$

(D) If angle of incidence is increased by $2°$, then angle of emergence may increase by equal to $2°$

24. When a ray of white light is incident on the prism parallel to the base. The variation in refractive index v/s wavelength (λ) graph is given as shown.





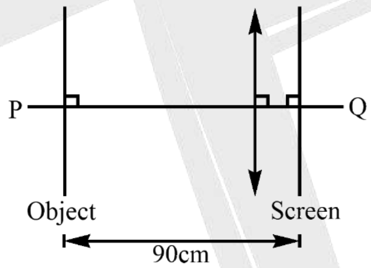
(A) Yellow ray travels without deviation

(B) Blue ray is deviated towards the vertex

(C) Red ray is deviated towards the base

(D) there is no dispersion

25. A convex lens forms an image of a fixed object on a fixed screen. The height of the image is 9 cm. The lens is now displaced along line PQ joining lens and screen until an image is again obtained on the screen. The height of this image is 4cm. The distance between the object and the screen is 90cm.



(A) The distance between the two position of the lens is 30cm

(B) The distance of the object from the lens in its first position is 36cm

(C) The height of the object is 6cm

(D) The focal length of the lens is 21.6 cm

26. A beam of light parallel to main optical axis falls on a converging lens of focal length $f_1 = 20$cm. Behind the lens, coaxially at some distance L, is diverging lens of focal length $f_2 = -20$cm. After passing through the diverging lens, the light focuses at a point 'A' 5 cm behind the lens. The position of lens is then interchanged :

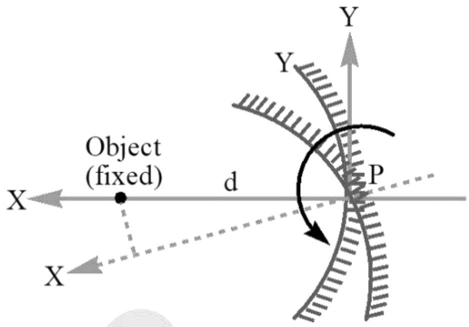
(A) The value of L is 16 cm

(B) The value of L is $\frac{80}{3}$ cm

(C) Point A shifts by 40 cm after the lenses are interchanged

(D) Point A shifts by $\frac{125}{3}$ cm after the lenses are interchanged

27. A particle is situated on the principal axis of a concave mirror at a distance d from the pole. Now a small angular displacement is given to the mirror in the anticlockwise direction about pole. Graph between the "y-coordinate of image" and the angular displacement of the mirror: (Considering new principal axis as x-axis.)



(A) Straight line of slope $\frac{fd^2}{(f+d)^2}$

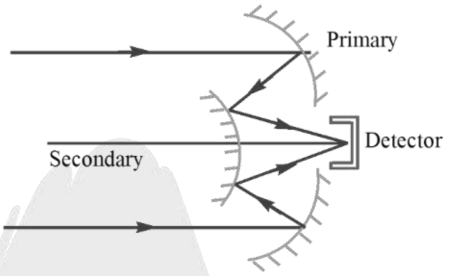
(B) Straight line of slope $\frac{f^2d}{(f-d)^2}$

(C) Straight line of slope $\frac{fd}{f-d}$

(D) Straight line of slope $\frac{fd}{(f+d)^2}$

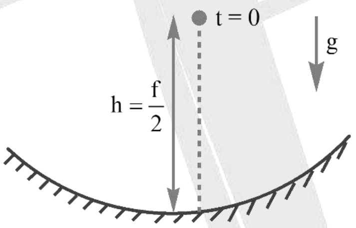
EXERCISE-V

1. A Cassegrain telescope is a reflecting telescope that uses two mirrors, the secondary mirror focusing the image through a hole in the primary mirror (similar to that shown in figure). You wish to focus the image of a distant galaxy onto the detector shown in the figure. If the primary mirror has a focal length of -2.5 m, the secondary mirror has a focal length of +1.5 m and the distance from the vertex of the primary mirror to the detector is 2m. What should be the distance between the vertices of the two mirrors?



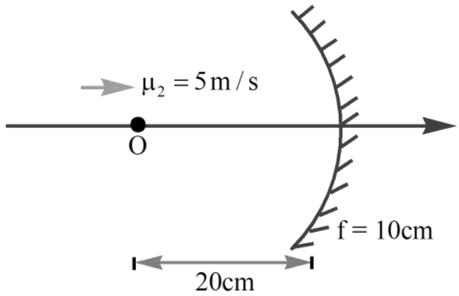
(A) 2.8 m (B) 3.50 m (C) 0.78 m (D) 1.3 m

2. A particle is dropped along the axis from a height $\frac{f}{2}$ on a concave mirror of focal length f as shown in figure. The maximum speed of image is:



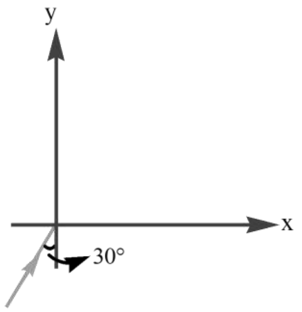
(A) ∞ (B) $\frac{3}{4}\sqrt{3fg}$ (C) $\frac{3}{2}\sqrt{3fg}$ (D) None of these

3. An object moves with a uniform velocity u_0 along the axis of a concave spherical mirror. Consider the instant shown in diagram, object is moving with $u_0 = 5\,m/s$ and $f = -10\,cm$. If object is at the centre of curvature at this instant then the magnitude of acceleration of image at this instant is $a\,m/s^2$. Find a.



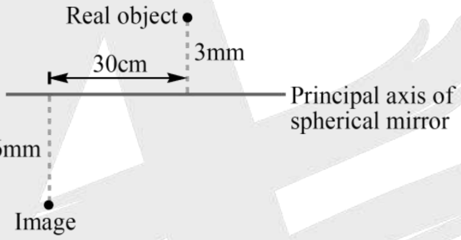
(A) 10 (B) 5 (C) 25 (D) None of these

4. Refractive indices of a medium in x-y coordinate system as shown are $\mu = \frac{2}{1+y^2}$ for $y > 0$ and $\mu = \frac{4}{3}$ for $y < 0$. A ray incidents on medium as shown then maximum y-coordinate of ray is:



(A) $y=\sqrt{2}$ (B) $y=0$ (C) $y=\sqrt{(\sqrt{3}-1)}$ (D) None of these

5. A spherical mirror forms image of a real object as shown, choose the correct option(s):



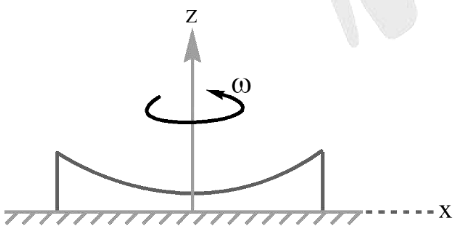
(A) Mirror is concave of radius 40 cm

(B) Mirror of convex of radius 40 cm

(C) Mirror is 30 cm from object along principal axis

(D) Mirror is 60 cm from the image along the principal axis

6. It was once suggested that the mirror for an astronomical telescope could be produced by rotating a flat disk of mercury at a prescribed anggular velocity ω about a vertical axis.



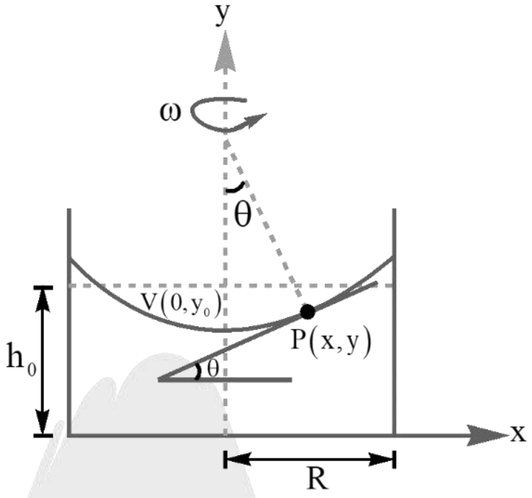
(A) The equation of the reflection (free) surface so obtained is $z=\omega^2\frac{x^2}{g}$

(B) The angular velocity of rotation of the disk to produce a mirror of focal length 10 cm $5\sqrt{2}$ rad/s

(C) The equation of the reflection (free) surface so obtained is $z=\omega^2\frac{x^2}{2g}$

(D) The angular velocity of rotation of the disk to produce a mirror of focal length 10 cm is 5 rad/s

7. Consider mercury (Hg) rotating about a vertical axis with uniform angular velocity ω filled in a cylindrical container. The liquid surface is curved. The figure shows a cross sectional view of the curved surface.



Ignore surface tension and viscosity. Mark the correct statement(s) :

[Hint : coordinates of focus for parabola $x^2 = 4ay$ is given by (0, a)]

(A) Steady state angle θ made by tangent to the surface at P (x, y) with the horizontal is given by

$\tan^{-1}\left(\frac{\omega^2 x}{g}\right)$

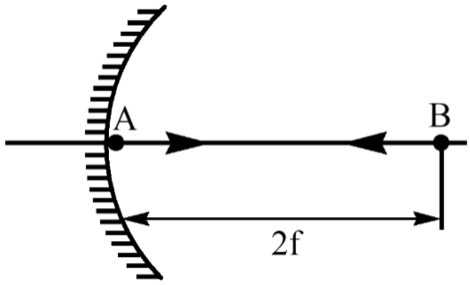
(B) Steady state angle θ made by tangent to the surface at P (x, y) with the horizontal is given by

$\sin^{-1}\left(\frac{\omega^2 x}{g}\right)$

(C) If the focal length of the mirror formed by shiny liquid surface is 20 cm then ω is 5 rad/s

(D) If the focal length of the mirror formed by shiny liquid surface is 20 cm then ω is 10 rad/s

8. At time t = 0, two point objects A and B respectively are at pole and centre of curvature of fixed concave mirror of focal length f ; the velocity vectors of A and B are always $\vec{V}_A = u\hat{i}$ and $\vec{V}_B = -u\hat{i}$ respectively, where $\hat{i}$ is unit vector along principal axis directed from pole towards focus and u is a positive constant :



(A) The distance between images of A and B will be 4f at time $t = \frac{f}{2u}$

(B) Magnitude of relative velocity of image of A and image of B at t = 0 is 2u

(C) Starting from t = 0 and before the particles come in contact, distance between image A and B increases

(D) Starting from t = 0 and before the particles come in contact, distance between A and B first increases and then decreases

9. A car is moving with at a constant speed of 20 m/s on a straight road. Looking at the rear view mirror, the driver finds that the car following him is at a distance of 50 m and is approaching with a speed of 10 m/s. In order to keep track of the car in the rear, the driver begins to glance alternatively at the rear and side mirror of his car after every 2 s till the other car overtakes. If the two cars were maintaining their speeds, which of the following statement(s) is/are correct?

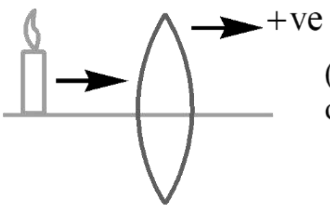
(A) The speed of the car in the rear is 30 m/s

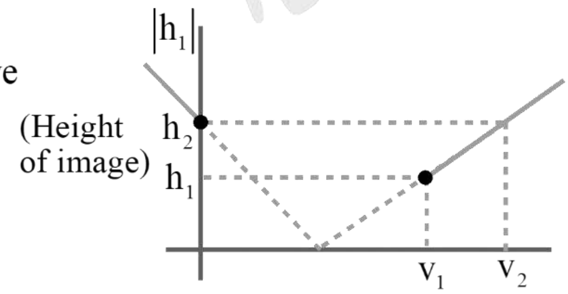
(B) In the side mirror the car in the rear would appear to approach with a speed of 10 m/s to the driver of the leading car

(C) In the rear view mirror the speed of the approaching car would appear to decrease as the distance between the cars decreases

(D) In the side mirror, the speed of the approaching car would appear to increase as the distance between the cars decreases

10. A candle of height h is being moved along the axis from $u = 3f$ towards a thin converging lens of focal length f. The adjacent graph represents the height of image as a function of the position of image w.r.t. lens as shown in figure.





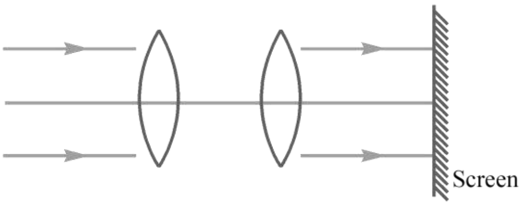
(A) The value of h_2 is h

(B) The value of h_2 is 2h

(C) The value of h_1 is $\frac{h}{2}$

(D) The ratio $\frac{v_1}{v_2}$ is $\frac{3}{4}$

11. Two converging lenses of same aperture size are placed with their principal axis coinciding. Their focal lengths are in the ratio K. When a light beam is incident parallel to their common principal axis, it is observed that final emergent beam is also parallel to their common principal axis.



It is observed that when beam is incident from left intensity recorded on screen is I_1, but when positions of lenses are interchanged, same point on screen records an intensity I_2. Then $\frac{I_1}{I_2}$ may be equal to :

(A) K^2 (B) K^4 (C) $\frac{1}{K^4}$ (D) $\frac{1}{K^2}$

12. A convex lens made of glass $\left(\mu_g = \frac{3}{2}\right)$ has focal length f in air. The image of an object placed in front of it is inverted, real and magnified. Now whole arrangement is immersed in water $\left(\mu_g = \frac{4}{3}\right)$ without changing distance between object and lens. The :

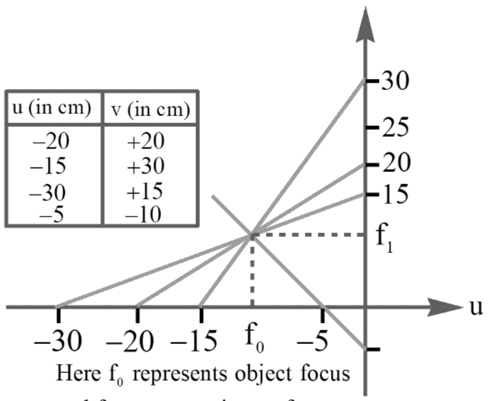
(A) new focal length will become 4f

(B) new focal length will become $\frac{f}{4}$

(C) new image will be virtual and magnified

(D) new image will be real, inverted and smaller in size

13. There is a simple and useful method for finding the focal length of a thin less or a spherical mirror. We make two perpendicular lines as we draw on a graph. Suppose the object position is u and image positions is v. We mark u and v on horizontal and vertical lines and join these points by a straight line. Similarly we draw straight lines for other pairs of (u, v). The common point of intersection of all these lines represents the focal length of the lens (or mirror) e.g. consider the table for (u, v) as shown below. This is a converging lens. The charge looks like this. For a concave mirror graph for (Use Cartesian sign convention) :



u (in cm)	v (in cm)
–20	+20
–15	+30
–30	+15
–5	–10

Here f_0 represents object focus and f_1 represents image focus

Here focal length comes out to be 10 cm.

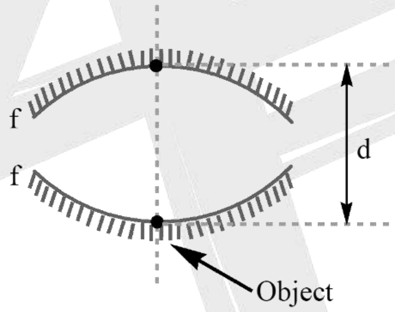
(A) Real object and virtual image pair is a straight line with negative slope and positive intercept on y-axis

(B) Real object and virtual image pair is a straight line with negative slope and negative intercept on y-axis

(C) Real object and virtual image pair is a straight line with positive slope and positive intercept on y-axis

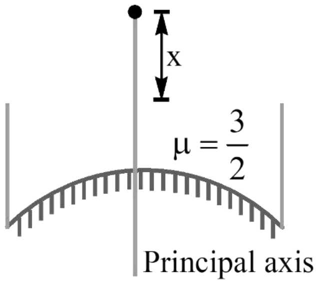
(D) Real object and virtual image pair is a straight line with positive slope and negative intercept on y-axis

14. Two concave mirrors with equal focal length f, is placed one above other at a seperation of d. Upper mirror has a small hole at its centre as given. A small object placed at centre of lower mirror. Take first reflection at above mirror and second at lower and answer the given questions for these two reflections only. For final image to be at the hole of the upper mirror. Value of d for given possibility :



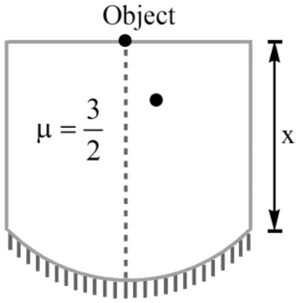
(A) f (B) 2f (C) 3f (D) $\frac{f}{3}$

15. If a ray incident normally on a mirror, the ray retraces its former path and final image of the object forms on the object itself. If amount of liuqid in curved part of the mirror is very small, focal length of mirror is 10 cm, the value of x for which final image forms on the object itself :



(A) 10 cm (B) 15 cm (C) 20 cm (D) None of the above

16. If a ray incident normally on a mirror, the ray retraces its former path and final image of the object forms on the object itself. Radius of curvature of spherical surface is 20 cm. If image of the object forms on the object itself. The value of x is :



(A) 10 cm (B) 15 cm (C) 20 cm (D) None of the above

17. The following objects are placed on after each other in given order onto a central axis with a separation of 40 cm each. A point source of light O, a diverging lens of focal length 40 cm, a converging lens of focal length 40 cm and converging mirror of focal length 80 cm. The aperture diameter of lenses and mirror is d = 20 cm. If a point source of light is placed at a perpendicular distance of x from central axis then : (You have to consider single optical event at any optical element). Mark the correct statement(s).

(A) Height of final image is $\frac{x}{2}$

(B) Height of final image is $\frac{x}{3}$

(C) Image formed by converging lens is real and inverted

(D) Image formed by converging mirror is real

18. The following objects are placed on after each other in given order onto a central axis with a separation of 40 cm each. A point source of light O, a diverging lens of focal length 40 cm, a converging lens of focal length 40 cm and converging mirror of focal length 80 cm. The aperture diameter of lenses and mirror is d = 20 cm. If a point source of light is placed at a perpendicular distance of x from central axis then : (You have to consider single optical event at any optical element). Mark the correct statement :

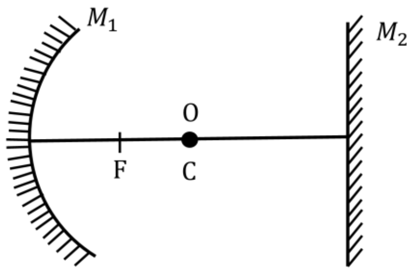
(A) Final image can be formed in the plane of converging lens

(B) For x > d final real image can be captured on a screen after one optical event (i.e., refraction or reflection) at every optical system

(C) Final image is real and inverted

(D) Final image is virtual and erect

19. An object is placed at centre of curvature C of the spherical mirror. Column – II corresponds to velocity of image (I_1) in mirror M_1 with respect to image (I_2) in mirror M_2. Column– I corresponds to motion of object and mirrors M_1 and M_2.



	Column – I		Column – II
(a)	O $\rightarrow$ V towards right $M_1 \rightarrow$ at rest, $M_2 \rightarrow$ at rest	(p)	Zero
(b)	$O \rightarrow \frac{V}{2}$ towards right $M_1 \rightarrow V$ towards right, $M_2 \rightarrow$ at rest	(q)	V
(c)	$O \rightarrow$ at rest $M_1 \rightarrow V$ towards right, $M_2 \rightarrow \frac{V}{2}$ towards left	(r)	2V
(d)	$O \rightarrow V$ towards right, $M_1 \rightarrow \frac{V}{2}$ towards right, $M_2 \rightarrow$ at rest	(s)	3V
		(t)	V/2

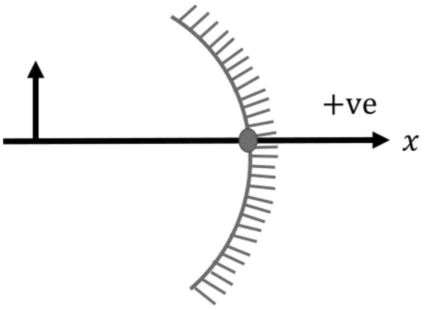
(A) a-p; b-r; c-s; d-q

(B) a-q; b-r; c-s; d-p

(C) a-p; b-s; c-r; d-q

(D) a-p; b-r; c-q; d-s

20. An extended object is moving in front of concave mirror as shown in figure. On L.H.S various velocity of object and position is given. On R.H.S some properties of image and its velocity is given. Consider velocity along x-axis only.



	Column-I (Object)		Column-II (Image)
(a)	+ve velocity and object is between focus and centre of curvature	(p)	+ve velocity
(b)	-ve velocity and object is between focus and pole	(q)	-ve velocity
(c)	ve velocity and object is beyond centre of curvature	(r)	size of image is increasing
(d)	-ve velocity and object is virtual	(s)	size of image is decreasing

(A) a-q,r; b-p,r; c-p,s; d-p,s

(B) a-p,r; b-p,s; c-p,r; d-p,s

(C) a-q,r; b-p,q; c-p,s; d-p,s

(D) a-p,s; b-p,s; c-q,r; d-q,r

21. The greatest thickness of a plano-convex lens when viewed normally through the plane surface appears to be 3 cm and when viewed normally through the curved surface it appears to be 3.6 cm. Actual thickness of lens is 4.5 cm.

	Column-I		Column – II
(a)	The refractive index of the material of the lens in the multiple of 10^{-1} is	(p)	9
(b)	The radius of curvature of lens is cm	(q)	3
(c)	The focal length if its plane surface is silvered in cm	(r)	16
(d)	The focal length if its curved surface is silvered in cm	(s)	15

(A) a-p; b-s; c-p; d-q

(B) a-s; b-p; c-p; d-q

(C) a-s; b-q; c-p; d-r

(D) a-q; b-q; c-s; d-s

PROFICIENCY TEST-I

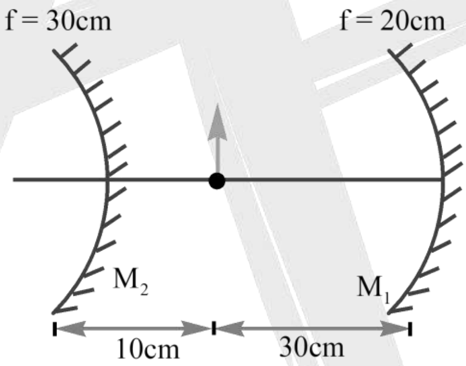
1. When a telescope is adjusted for parallel light, the distance of the objective from the eyepiece is 100cm for normal adjustment. The magnifying power of the telescope in this case is 9. If an old man cannot see beyond 90 cm and wishes to use the telescope then he will have to reduce the tube length by:

(A) 1.5 cm (B) 0.5 cm (C) 1 cm (D) 2 cm

2. An astronomical telescope has magnifying power 6 for distance objects. The separation between objective and the eye piece is 42 cm and the final image is formed at infinity. The focal length f_0 of objective and focal length f_e of eye-piece are (in cm):

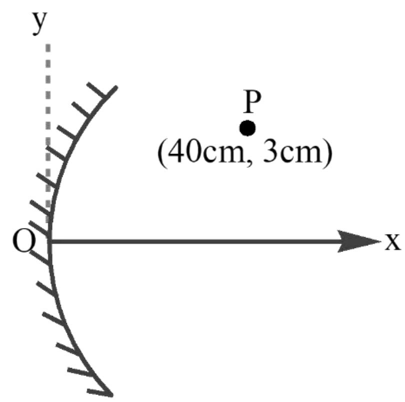
(A) $f_0 = 36, f_e = 6$ (B) $f_0 = 50, f_e = 8$ (C) $f_0 = 20, f_e = 10$ (D) $f_0 = 40, f_e = 8$

3. In the figure shown find the total magnification after two successive reflections first on M_1 and then on M_2. (Assume paraxial rays only)



(A) +6 (B) -6 (C) +3 (D) -3

4. The co-ordinates of the image of point object P formed by a concave mirror of radius of curvature 20 cm (consider paraxial rays only) as shown in the figure is:



(A) $(13.33\text{cm}, -1\text{cm})$ (B) $(13.33\text{cm}, +1\text{cm})$

(C) $(-13.33\text{cm}, +1\text{cm})$ (D) $(-13.33\text{cm}, -1\text{cm})$

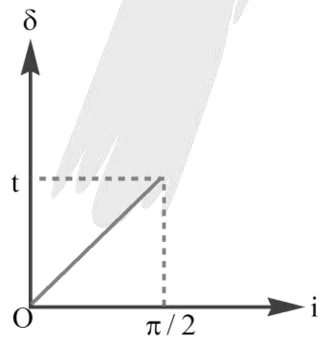
5. The x-z plane separates two media A and B of refractive indices $\mu_1 = 1.6$ and $\mu_2 = 1.8$. A ray of light travels from A to B. Its directions in the two media are given by unit vectors $\vec{\mu}_1 = a\hat{i} + b\hat{j}$ and $\vec{\mu}_2 = c\hat{i} + d\hat{j}$. Then:

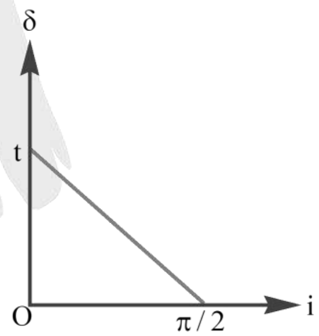
(A) $\frac{a}{c} = \frac{4}{3}$ (B) $\frac{a}{c} = \frac{3}{4}$ (C) $\frac{a}{c} = \frac{9}{8}$ (D) $\frac{a}{c} = \frac{8}{9}$

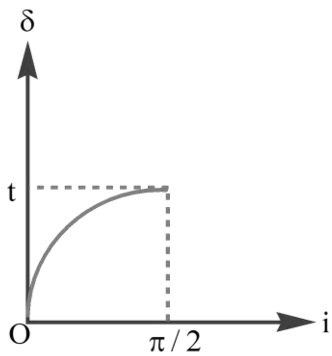
6. An opaque disc of radius 10.00 cm floats on the surface of a transparent homogeneous liquid. An isotropic light source is lit at the bottom of the beaker containing the liquid, vertically below the centre of the disc. A person views the source from a point vertically above the centre of the disc. The liquid is slowly allowed to drain out through a tap. When the height of the liquid becomes 13.33 cm, the source disappears. The refractive index of the liquid is:

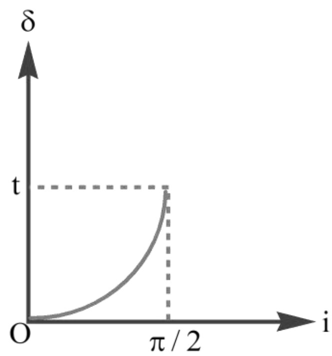
(A) $\frac{4}{3}$ (B) $\frac{5}{3}$ (C) $\frac{3}{2}$ (D) $\frac{5}{4}$

7. A ray of light passes through a rectangular slab of thickness t. The variation of lateral shift (δ) with angle of incidence (i) is:

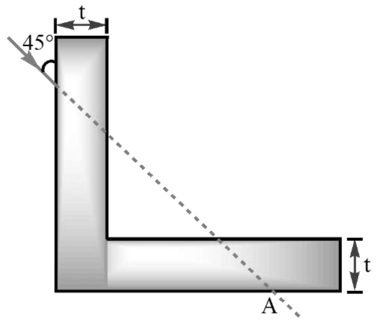
(A)


(B)


(C)


(D)


8. Two glass slabs of same thickness are joined to form a L as shown. A ray is incident of the combination as shown (in the plane of paper). The final emergent ray:



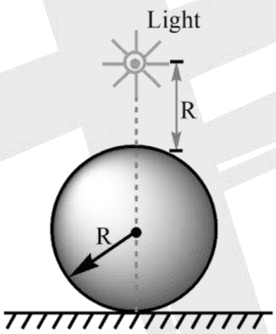
(A) comes out at point A in the same direction as that of the original ray

(B) comes out at point A but not in the same direction as that of the original ray

(C) does not come out at point A but is in the same direction as that of the original ray

(D) does not come out at point A and is not in the same direction as that of the original ray

9. An opaque sphere of radius R lies on a horizontal plane. A light source is placed above sphere as shown. Then:



[P] area of shadow on the plane is $2\pi R^2$

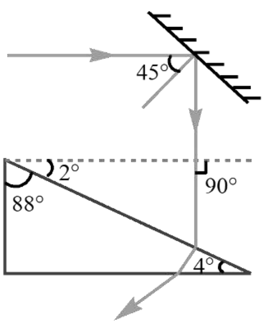
[Q] area of shadow on the plane is $3\pi R^2$

[R] if the sphere is just submerged in some liquid, area of the shadow on plane decreases

[S] if the sphere is just submerged in some liquid, area of the shadow on plane increases

(A) P, Q (B) Q, R (C) P, R (D) Q, S

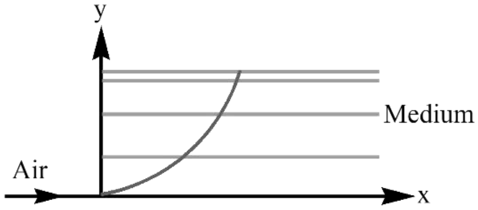
10. A ray of light strikes a plane mirror at an angle of incidence $45°$ as shown in the figure. After reflection, the ray passes through a prism of refractive index 1.5, whose apex angle is $4°$. The angle through which the mirror should be rotated if the total deviation of the ray is to be $90°$ is:



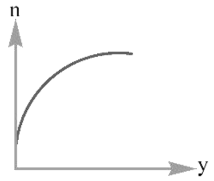
(A) 1° clockwise (B) 1° anticlockwise

(C) 2° clockwise (D) 2° anticlockwise

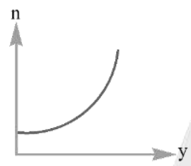
11. The refractive index of the medium within a certain region $x > 0,\ y > 0$, changes continuously with y. A thin light ray travelling in air in the x-direction strikes the medium at right angles and moves through the medium along a circular arc of radius R.



(A) Referactive index of medium varies with y as



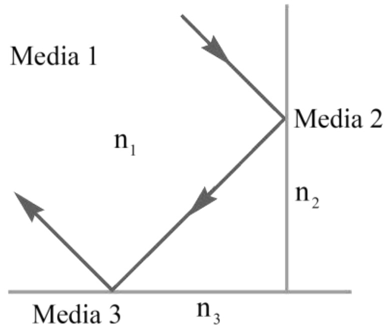
(B) Refractive index of medium varies with y as



(C) If refractive index of medium can increase upto a value n = 2.5, the maximum value of y is $\frac{3R}{5}$

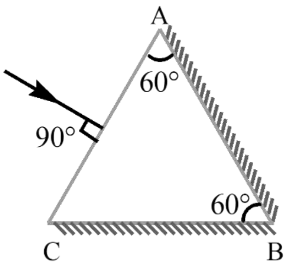
(D) If refractive index of medium can increase upto a value n = 2.5, the maximum value of y is 5R

12. In the diagram shown, light is inciddent on the interface between media 1 (refractive index n_1) and 2 (refractive index n_2) at angle slightly greater than the critical angle, and is totally reflected. The light is then also totally reflected at the interface between media 1 and 3 (refractive index n_3), after which it travels in a direction opposite to its initial direction. The media must have a refractive indices such that :



(A) $n_1 < n_2 < n_3$ (B) $n_1^2 - n_3^2 > n_2^2$ (C) $n_1^2 - n_2^2 < n_3^2$ (D) $n_1^2 + n_2^2 > n_3^2$

13. The two faces of a prism of side length a are silvered. A ray incidents normally on unsilvered face as shown. Then :



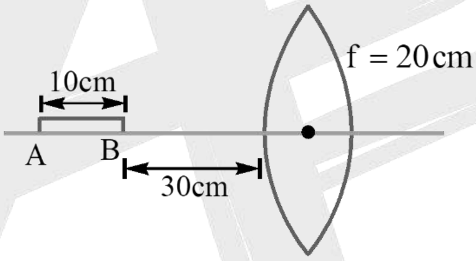
(A) net deviation of ray through prism is $90°$

(B) net deviation of ray through prism is $180°$

(C) length of path travelled by light ray inside the prism is $\sqrt{5}a$

(D) length of path travelled by light ray inside the prism is $\sqrt{3}a$

14. AB is a linear object placed along optical axis as shwon in figure. Tick the **incorrect** statement(s) :



(A) The length of image is smaller than the length of object

(B) The length of image is larger than the length of object

(C) The length of image is equal to the length of object

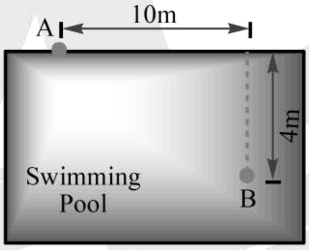
(D) If middle portion of the lens is painted then the length of image is smaller than length of object

PROFICIENCY TEST-II

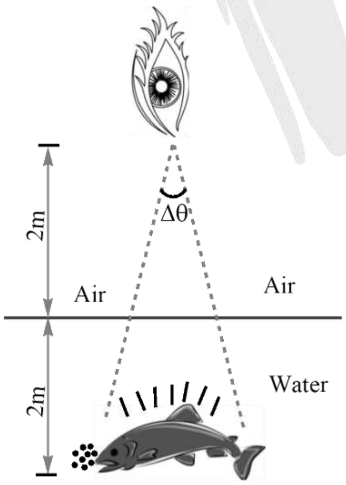
1. A large convex spherical mirror in an amusement park is facing a plane mirror 10 m away. A boy of height 1m standing midway between the two sees himself twice as tall as in plane mirror as in spherical one. In other words, the angle subtended at the observer by the image in plane mirror is twice the angle subtended by the image in the spherical mirror. What is the focal length of the convex mirror?

(A) 5 m (B) 2.5 m (C) 1.8 m (D) 3.6 m

2. On one boundayr of a swimming pool, there is a person at point A whose speed of running on ground (boundary) is $10\,ms^{-1}$, while that of swimming is $6\,ms^{-1}$. He has to reach a point B in the swimming pool. The distance (in m) covered on the boundary so that the time required to reach the point B in the pool is minimum, is:

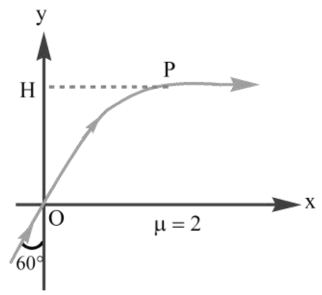


3. A man looks down on a fish of length 5 cm. His eye is 2m above the surface of the water $\left(\mu = \frac{4}{3}\right)$ and the fish is 2m below the surface as shown in the figure. The ratio of angular width $\Delta\theta$ of the fish as seen by the man in presence of water to be $\Delta\theta$ in the absence of water is:



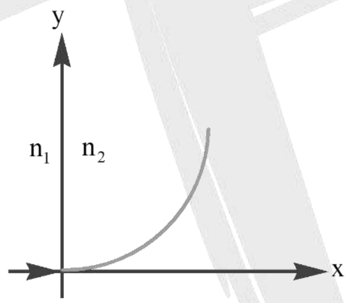
(A) $\frac{6}{5}$ (B) $\frac{5}{6}$ (C) $\frac{7}{8}$ (D) $\frac{8}{7}$

4. A system of coordinates is drawn in a medium whose refractive index varies as $\mu = \frac{2}{1+y^2}$, where $0 \le y \le 1$ and $\mu = 2$ for $y < 0$ as shown in figure. A ray of light is incident at origin at an angle $60°$ with y-axis as shown in the figure. At point P ray becomes parallel to x-axis. The value of H is



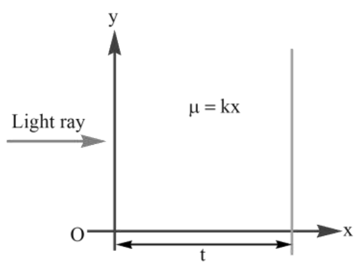
(A) $\left\{\left(\frac{2}{\sqrt{3}}\right)-1\right\}^{1/2}$ (B) $\left\{\frac{2}{\sqrt{3}}\right\}^{1/2}$ (C) $\left\{\left(\sqrt{3}\right)-1\right\}^{1/2}$ (D) $\left(\frac{4}{\sqrt{3}-1}\right)^{1/2}$

5. The refractive index of the medium within a certain region $x > 0$, $y > 0$, changes with y. A thin light ray travelling in the x-direction strikes the medium at nearly right angles and moves through the medium along a circular arc of radius R. It is given that greatest refractive index of any material is 2.5. What is the maximum possible y-coordinate of the circular arc?



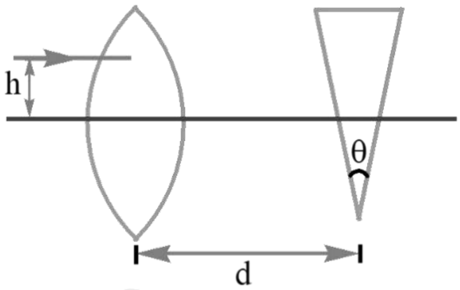
(A) $\frac{3}{5}R$ (B) $\frac{4}{5}R$ (C) $\frac{5}{3}R$ (D) can't be determined

6. Refractive index of a transparent slab varies as $\mu = kx$ where x is the distance from origin. Time taken by the light to travel the slab of thickness t (as shown in the figure):



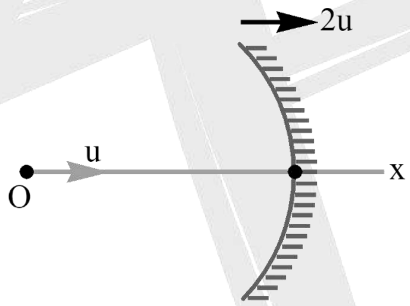
(A) $\frac{t^2k}{2c}$ (B) $\frac{t^2k}{c}$ (C) $\frac{tk}{c}$ (D) $\frac{2tk}{2c}$

7. A ray of light parallel to the axis of a converging lens (having focal length f) strikes it at a small distance 'h' from its optical centre. A thin prism having angle θ and refractive index μ is placed normal to the axis of lens at a distance 'd' from it. What should be the value of μ so that the ray emerges parallel to the lens axis?



(A) $\frac{h}{f\,\theta}$ (B) $\frac{h}{f\,\theta}+1$ (C) $\frac{h}{(d+f)\theta}$ (D) $\frac{h}{(d+f)\theta}+1$

8. An object and a concave mirror is moving with velocities $u\hat{i}$ and $2u\hat{i}$ respectively as shown in figure. The image formed by the concave mirror :



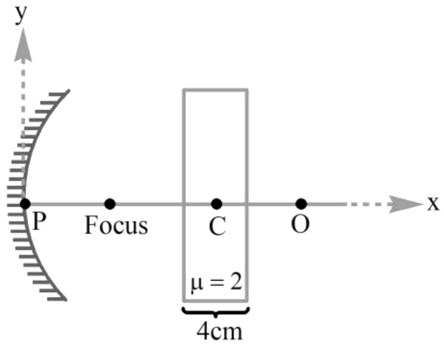
(A) will have speed greater than the speed of the object

(B) may have speed greater than the speed of the object

(C) will have speed greater than the speed of the mirror

(D) must move away from the mirror

9. A concave mirror of focal length 50 cm is placed at origin as shown in diagram. A point sized object is placed at O (x = 152 cm). A glass slab of thickness 4 cm, $\mu = 2$, is placed symmetrically around C. Light cannot reach from object to mirror without passing through slab. Choose the correct option(s) :



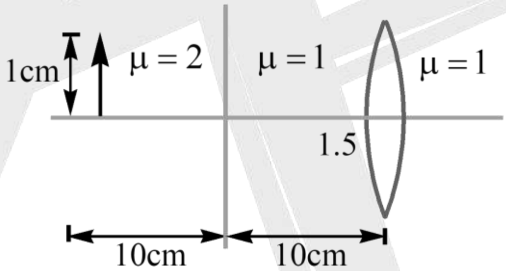
(A) Image made by concave mirror will be at x = 75 cm

(B) Final image made by entire optical system will be at x = 77 cm

(C) Final image of the system will be real

(D) Final image of the system will be virtual

10. An object of length 1 cm is placed on a principal axis of an equiconvex lens of radii 5cm. Distance between the lens and object is 20 cm. Space between the lens and object is filled with medium of two different refractive index 2 and 1, lens being placed in the medium of refractive index 1. Boundary of both medium is mid-way between the object and lens as shown in figure.



(A) The image will be formed at distance of 7.5cm

(B) The image will be formed at distance of $\frac{20}{3}$ cm from the optical centre

(C) The size of the image is 0.5cm

(D) The size of the image is 0.4cm

11. The principal axis of an optical device is along $y = -1$. If the image of a small body placed at $(-30, 3)$ is formed at a point $(60, -3)$, then the optical device may be :

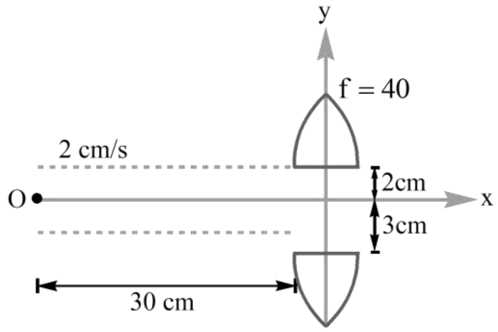
(A) A convex lens of focal length 20cm

(B) A concave mirror of focal length 60 cm

(C) A concave lens of focal length 20 cm

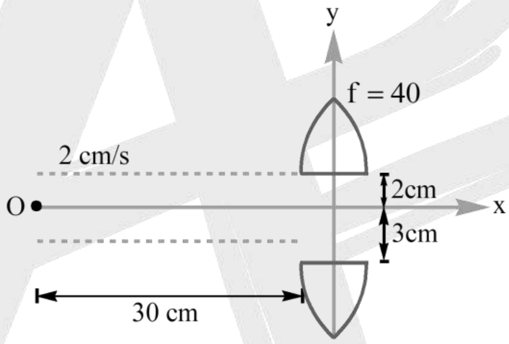
(D) A convex mirror of focal length 60 cm

12. An object is approaching two pieces of a lens halves are placed according to diagram. Then find x coordinate of images :



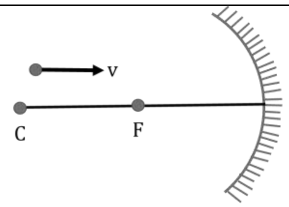
(A) -120 (B) -30 (C) -40 (D) $+120$

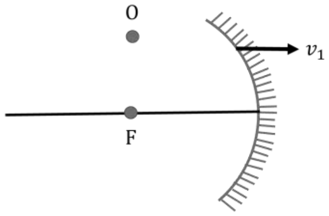
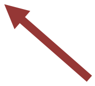
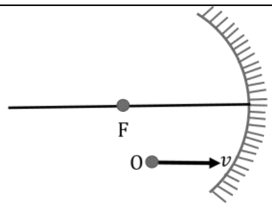
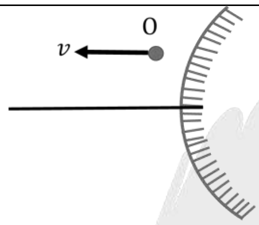
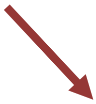
13. An object is approaching two pieces of a lens halves are placed according to diagram. Then find y coordinates of image formed by lower and upper half :



(A) -6cm, 9cm
(B) -8cm, -12cm
(C) 8cm, -12cm
(D) -8cm, $+12$cm

14. In Column-I possible instantaneous velocity vector of the image with respect to ground are shown. The corresponding velocity vectors of images are the situations shown in Column-II. Match Column-I with the Column-II.

	Column – I		Column – II
(a)		(p)	

(b)		(q)	
(c)		(r)	
(d)		(s)	
		(t)	None of these

(A) a-r; b-p; c-s; d-q

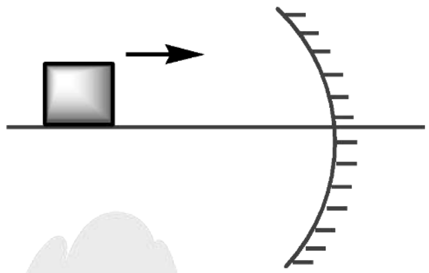
(B) a-p; b-r; c-s; d-q

(C) a-r; b-p; c-q; d-s

(D) a-r; b-p; c-q; d-s

PROFICIENCY TEST-III

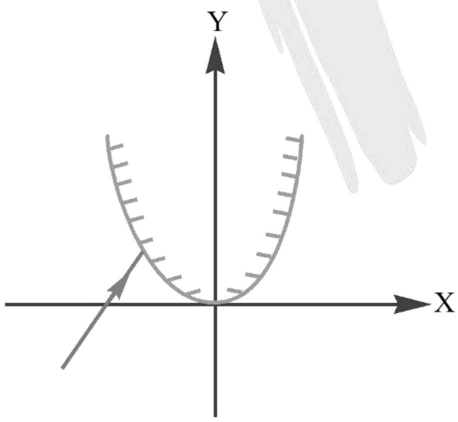
1. A block of mass 1 kg is moving on a rough horizontal surface along the principal axis of a convcave mirror as shown. At t = 0, it is 15 m away from the pole, moving with a velocity of 7 m/s. At t = 1 sec, it's image is at $\frac{57}{13}$ m away from the pole on left hand side of the mirror. Where will the image be at t = 3 sec?



(A) 5 m left of mirror (B) $\frac{123}{23}$ m to left of mirror

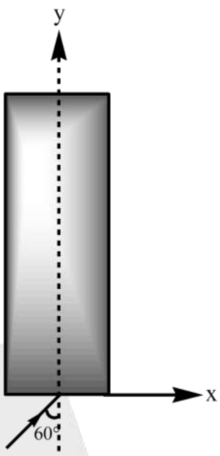
(C) $\frac{138}{23}$ m to left of mirror (D) 7.5 m left of mirror

2. A parabolic mirror is silvered at inner surface. The equation of the curve formed by its intersection with X-Y plane is given by $y=\frac{x^2}{4}$. A ray travelling in X-Y plane along line $y=x+3$ hits the mirror in second quadrant and gets reflected. The unit vector in the direction of reflected ray will be:



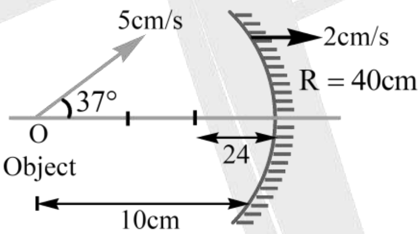
(A) $\frac{1}{\sqrt{2}}\left(-\hat{i}-\hat{j}\right)$ (B) $\frac{1}{\sqrt{2}}\left(\hat{i}+\hat{j}\right)$ (C) $\frac{-2\hat{i}-3\hat{j}}{\sqrt{13}}$ (D) $\frac{-\left(2\hat{i}+\hat{j}\right)}{\sqrt{5}}$

3. A ray of light enters into a thick glass slab from air as shown in figure. The refractive index varies as $\mu = (2\sqrt{3} - y)$. If width of slab is very large then maximum value of y-coordinate of the ray is:



(A) $\frac{\sqrt{3}}{2}$ (B) $\frac{3\sqrt{3}}{2}$ (C) ∞ (D) date insufficient

4. Velocity of object and concave mirror are shown in the diagram. At the given instant choose the correct statement(s) :



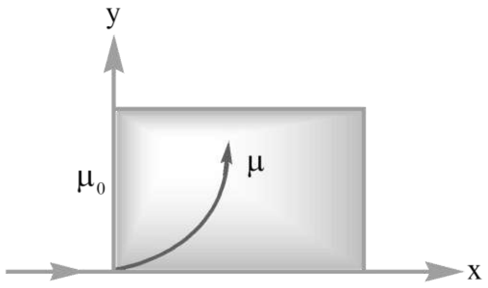
(A) Angle between velocity of image and velocity of object is $98°$

(B) Velocity of image with respect to object is $\sqrt{109}\,cm/s$

(C) Velocity of image with respect to object is $\sqrt{72}\,cm/s$

(D) Velocity of image is in the direction of velocity of plane mirror

5. The refractive index of the medium with a certain region, $x > 0,\ y > 0$, changes with y. A thin light ray travelling in the x-direction in medium having refractive index $\mu_0 = 1$ strikes another medium of refractive index μ at right angles and moves through the medium along a circular arc of radius R as shown in the figure. The material with the greatest known refractive index is diamond, but even the refractive index of this material does not reach the value $\mu_{max} = 2.5$. It is this limit that sets the maximum angular size of the arc the light ray can cover. Angular size of arc is the angle subtended by the arc at the centre.



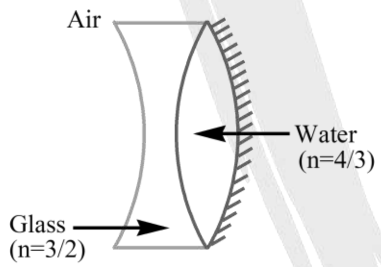
(A) The variation of refractive index μ with y is given as $\mu = \frac{R}{R-y}$

(B) The unit vector in the direction of refracted light at $y = \frac{R}{2}$ is $\frac{1}{2}\hat{i} + \frac{\sqrt{3}}{2}\hat{j}$

(C) If the maximum angular size of the arc of light is θ_{max} then $\sin\theta_{max} = \frac{2}{5}$

(D) If the maximum angular size of the arc of light is θ_{max} then $\cos\theta_{max} = \frac{2}{5}$

6. In the figure shown the radiuus of curvature of the left and right surface of the concave lens are 10cm and 15cm respectively. The radius of curvature of the mirror is 15cm.



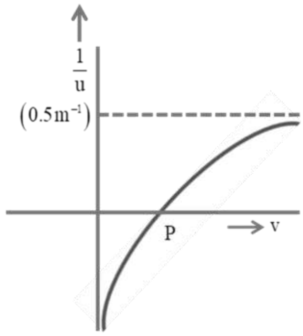
(A) modulus of equivalent focal length of the combination is 18cm

(B) modulus of equivalent focal length of the combination is 36cm

(C) the system behaves like a concave mirror

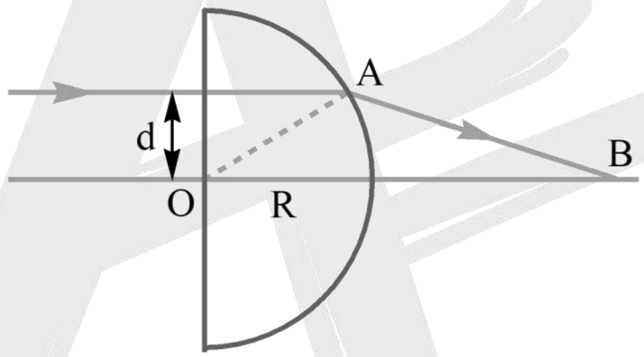
(D) the system behaves like a convex mirror

7. Sign convention is taken as +ve direction in the direction of light ray and the graph is drawn between $\frac{1}{u}$ and v for a spherical mirror. What is the focal length of the mirror ?



(A) -50cm (B) -200cm (C) $+200$cm (D) $+50$cm

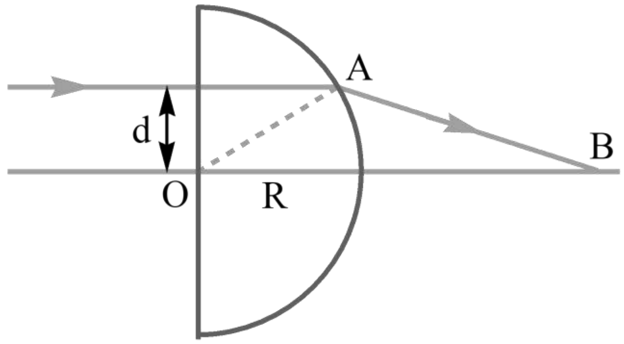
8. A semi-cylinder made of a transparent plastic has a refraction index of $n=\sqrt{2}$ and a radius of R. There is a narrow incident light ray perpendicular to the flat side of the semi cylinder at d distance from the axis of symmetry. What can the maximum value of d be so that the light ray can still leave the other side of the semi cylinder ?



(A) $d=\frac{R}{\sqrt{2}}$ (B) $d=\frac{R}{2\sqrt{2}}$

(C) $\frac{R}{2}$ (D) None of these

9. A semi-cylinder made of a transparent plastic has a refraction index of $n=\sqrt{2}$ and a radius of R. There is a narrow incident light ray perpendicular to the flat side of the semi cylinder at d distance from the axis of symmetry. When the value of d chosen is such that TIR just takes place then time for which light remains inside the cylinder is :



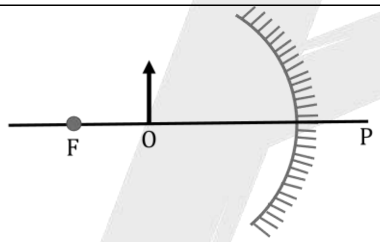
(A) $\frac{4R}{c}$

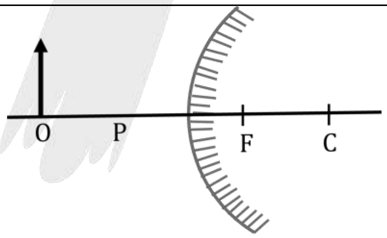
(B) $\frac{4\sqrt{2}R}{c}$

(C) $\frac{2\sqrt{2}R}{c}$

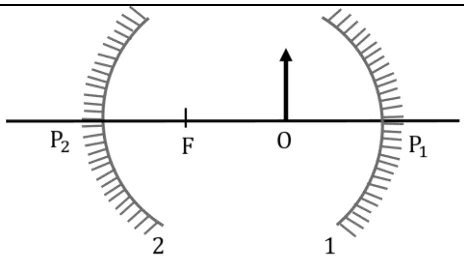
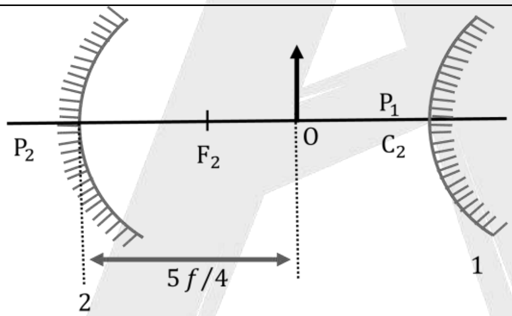
(D) $\frac{2R}{c}$

10. In Column-I, different situations are given and in Column-II, nature of image is given. Match the entries of Column-I with Column-II.

<table>
<tr><th></th><th>Column – I</th><th></th><th>Column – II</th></tr>
<tr><td>(a)</td><td>

Object lies between Pole and Focus of a concave mirror</td><td>(p)</td><td>Image is real</td></tr>
<tr><td>(b)</td><td>

Object is kept in front of convex mirror</td><td>(q)</td><td>Image is enlarged</td></tr>
</table>

(c)	 Object is kept between two similar concave mirrors. Object is kept between pole and focus of concave mirror-1 and concave mirror-2 is kept at center of curvature of concave mirror-1. Consider the image formed after two reflection, first reflection from concave mirror-2 and second from mirror-1.	(r)	Image is inverted
(d)	 Object is kept between convex mirror 1 and concave mirror-2 having same radius of curvature. f is the focal length of convex mirror. Consider the image after two reflection, first reflection from mirror-2 and second from mirror 1. C2 is the centre of curvature of mirror 2.	(s)	Image is diminished
		(t)	Image is virtual

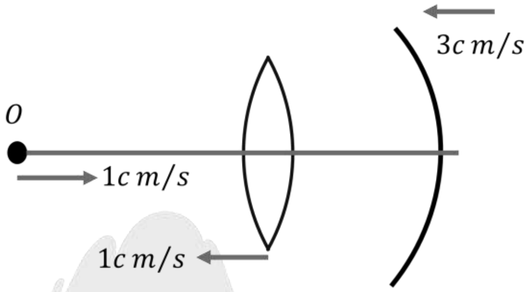
(A) a-s; b-q; c-p, d-q,t

(B) a-q, b-s; c-p; d-q,t

(C) a-q; b-s; c-q; d-p,t

(D) a-q; b-s; c-q,t; d-p

11. An equiconvex lens of refractive index $\frac{3}{2}$ and of radius of curvature 40 cm is placed co-axially at a distance of 40 cm in front of a concave mirror of radius of curvature 60cm. The medium is air. A point object O is placed on the common axis of the lens. At an instant (t=0), the object, the lens and the mirror start moving with a speed 1cm/s, 1cm/s and 3 cm/s respectively in the direction shown in the figure. Find the time after which the object and its real image coincide.



(A) 10 sec (B) 20 sec (C) 30 sec (D) 40 sec

EXERCISE-I KEY

1	2	3	4	5	6	7	8	9	10
B	B	67	A	D	D	C	C	C	C
11	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
C	D	A	B	6	D	A	A	CD	D

EXERCISE-II KEY

1	2	3	4	5	6	7	8	9	10
C	A	A	A	21	B	65	A	A	B
11	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
C	B	B	B	C	A	B	B	AC	AD
21	**22**	**23**							
AC	B	BD							

EXERCISE-III KEY

1	2	3	4	5	6	7	8	9	10
B	A	C	C	B	A	A	A	A	B
11	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
D	B	D	40	A	B	B	D	C	C
21	**22**	**23**	**24**	**25**	**26**	**27**			
D	BD	AC	ABCD	D	ABD	C			

EXERCISE-IV KEY

1	2	3	4	5	6	7	8	9	10
60	A	D	A	B	A	D	B	C	D
11	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
B	A	C	C	C	C	ABC	ABC	AB	AB
21	**22**	**23**	**24**	**25**	**26**	**27**			
BCD	AB	ABC	ABC	BCD	AC	C			

EXERCISE-V KEY

1	2	3	4	5	6	7	8	9	10

D	B	D	A	ACD	BC	AC	ABC	AD	ACD
11	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
BC	AC	C	AC	D	C	ACD	AC	A	A
21									
B									

PROFICIENCY TEST-I KEY

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
C	A	B	A	C	B	C	A	B	B
11	**12**	**13**	**14**						
BC	BD	BD	ACD						

PROFICIENCY TEST-II KEY

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
B	7	D	A	A	A	B	AC	ABD	AC
11	**12**	**13**	**14**						
AB	A	A	D						

PROFICIENCY TEST-III KEY

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
B	A	B	AB	ABD	AC	C	A	A	B
11									
D									