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1. (c), (e), (g) $\rightarrow f(x) = \dfrac{e^{x^2} + e^{-x^2} - 2e^{-x^2}}{e^{x^2} + e^{-x^2}} = 1 - \dfrac{2}{e^{2x^2}+1}$ ≥ 0, ≥ 2

$y = (x^2 + x + 5)(x^2 + x - 3) \rightarrow \boxed{\text{cont}}$ ✓

$-15 = y \quad x = 0, -1$

$\boxed{\text{M-1}}$

M-1

$x \to -\infty, y \to \infty$

$x \to \infty, y \to \infty$

$\boxed{\text{Into}}$

$2 \leq e^{2x^2} + 1 < \infty$

$0 < \dfrac{1}{e^{2x^2}+1} \leq \dfrac{1}{2}$

$\dfrac{-2}{e^{2x^2}+1} \in [-1, 0)$

$y \in [0, 1)$

$\boxed{\text{Into}}$

$\boxed{\text{M-1}}$

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FUNCTIONS

5:

$$f(f(x)) = \begin{cases} \sqrt{2} & \text{when } f(x) \text{ is rational} \\ 0 & f(x) \text{ is irr.} \end{cases}$$

$$f(f(x)) = \begin{cases} \sqrt{2} & x \notin Q \\ 0 & x \in Q \end{cases}$$

7:

$$g(3x+4) = 2x-1 = \qquad g(t) = 2\left(\frac{t-4}{3}\right) - 1$$

$$3x+4 = t \Rightarrow x = \frac{t-4}{3}$$

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9.

$$f(x) = \sin^2 x - \cos^2\left(x+\frac{\pi}{3}\right) + 1 + \frac{1}{2}\, 2\cos x \cos\left(x+\frac{\pi}{3}\right)$$

$$= -\cos\frac{\pi}{3}\cancel{\cos\left(2x+\frac{\pi}{3}\right)} + 1 + \frac{1}{2}\left(\cancel{\cos\left(2x+\frac{\pi}{3}\right)} + \cos\frac{\pi}{3}\right)$$

$$= \frac{5}{4}$$

$g(x) =$ one-one

$$g(f(x)) = g\left(\frac{5}{4}\right) = \text{const.}$$

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$$D_{gof} = [-2,-1) \cup (-1,1) \cup (1,3]$$

$$R_{gof} = [-2,-1) \cup (-1,1] \cup (2,10]$$

$$f(x) = \begin{cases} x^2 - 1 & -2 \le x \le 1 \\ x^2 + 3 & 1 < x \le 3 \end{cases}$$

$$g(x) = \begin{cases} -1 - x^2 & x \in [-1,0) \\ 1 - x^2 & x \in (0,1] \\ x - 2 & x \in (-\infty,-1) \cup (1,\infty) \end{cases}$$

$(1,3]$ $(0,1]$

$$g(f(x)) = \begin{cases} -1 - f^2(x) & f(x) \in [-1,0) \\ 1 - f^2(x) \in (0,1] & f(x) \in (0,1] \\ f(x) - 2 & f(x) \in (-\infty,-1) \cup (1,\infty) \end{cases}$$

$[0,1)$

$\in [-2,-1)$

$(2,10]$

$(-1,1]$

$$g(f(x)) = \begin{cases} -1 - (x^2-1)^2 & x \in (-1,1) \\ 1 - (x^2-1)^2 & x \in [-\sqrt{2},-1) \\ (x^2-1) - 2 & x \in [-2,-\sqrt{2}) \\ (x^2+3) - 2 & x \in (1,3] \end{cases}$$

$(5, 12]$

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Inverse of Function

Let $f : A \to B$ is bijective, then there always exists a unique function $g : B \to A$ such that

If $f(a) = b$, then $g(b) = a$ $\forall a \in A$. Then

f & g are called mutually inverse to each other; $f = g^{-1}$, $g = f^{-1}$.

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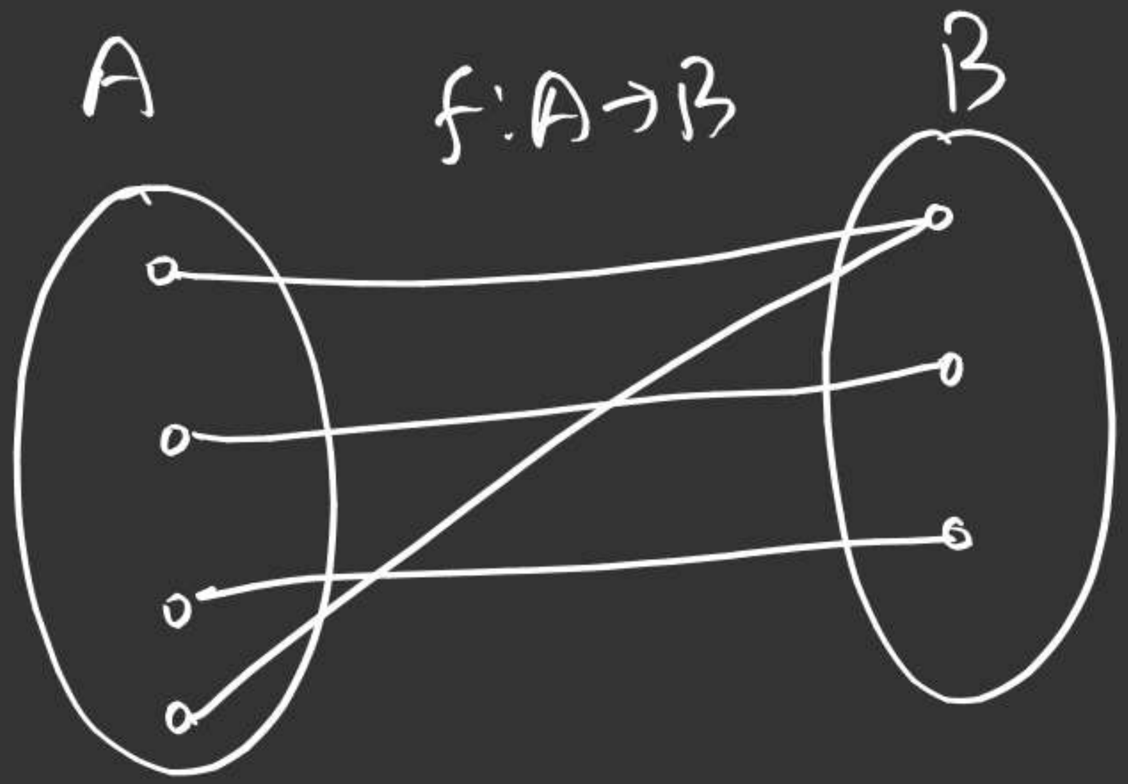

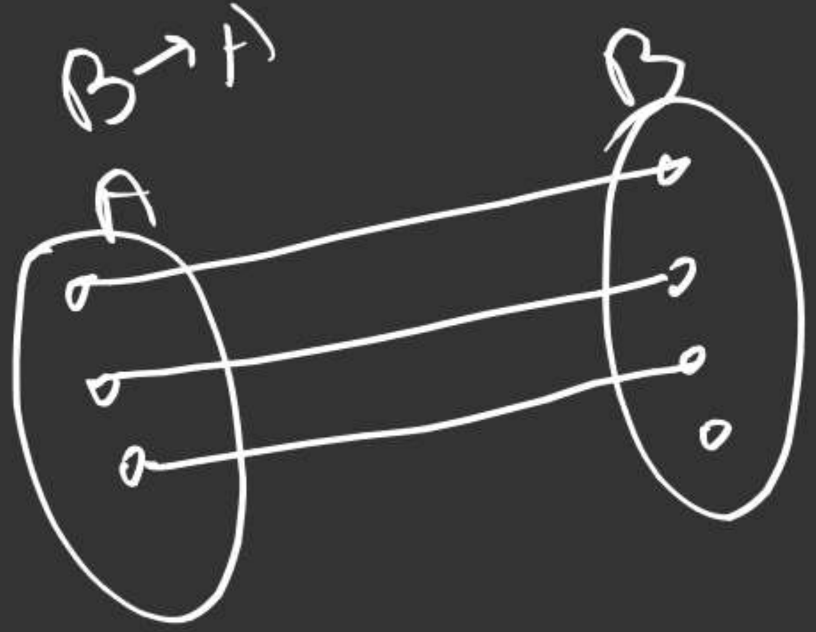

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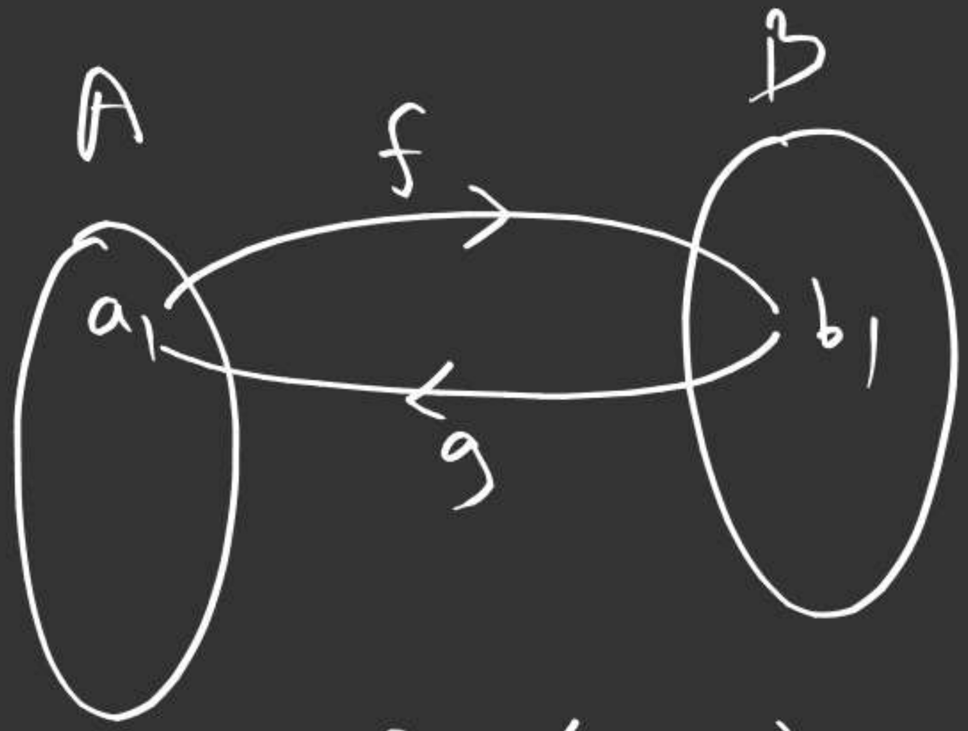
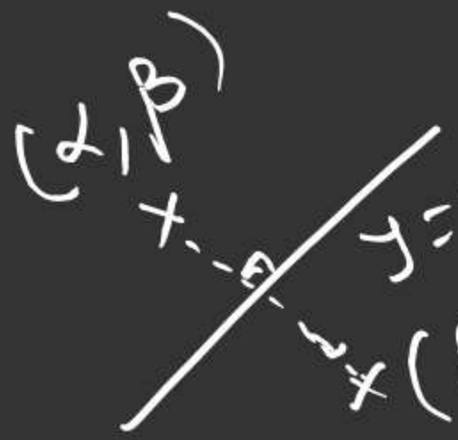
$f : A \to B$

$f(a) = b \quad , \; g(b) = a$



$f = (\alpha, \beta)$

$f^{-1} = g = (\beta, \alpha)$



$y = x \Rightarrow y - x = 0$

$(\beta, \alpha) = x, y$

$$\frac{x-\alpha}{-1} = \frac{y-\beta}{1} = -2\,\frac{\beta-\alpha}{1^2+1^2}$$

$$= \alpha - \beta$$

$x = \beta \quad , \; y = \alpha$

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① Graphs of $y=f(x)$ & $y=f^{-1}(x)$ are mirror image to each other about line $y=x$.

② $f: A \to B$, $g: B \to A$, $g = f^{-1}$

$g \circ f : A \to A$, $g \circ f(x) = x$

$f \circ g : B \to B$, $(f \circ g)(x) = x$.

③

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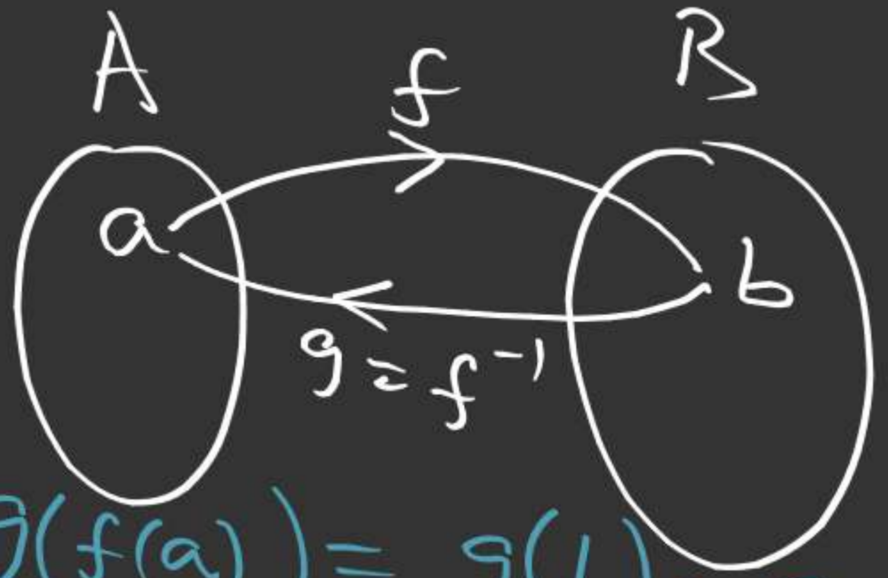
$f : A \to B \quad , \quad g : B \to A$



$f(a) = b$

$\Rightarrow \quad g(b) = a.$

$gof(a) = g(f(a)) = g(b) = a.$

$\boxed{gof(a) = a \quad \forall a \in A}$

$fog(b) = f(g(b)) = f(a) = b$

$fog(b) = b \quad \forall b \in B$

$\boxed{fog = I_B}$

$\boxed{gof = I_A}$

$\boxed{f(f^{-1}(x)) = x \quad x \in B}$

$\boxed{f^{-1}(f(x)) = x \quad \forall x \in A}$

FUNCTIONS

Identity Function

$f : A \to A$, $f(x) = x$

$f = I_A$

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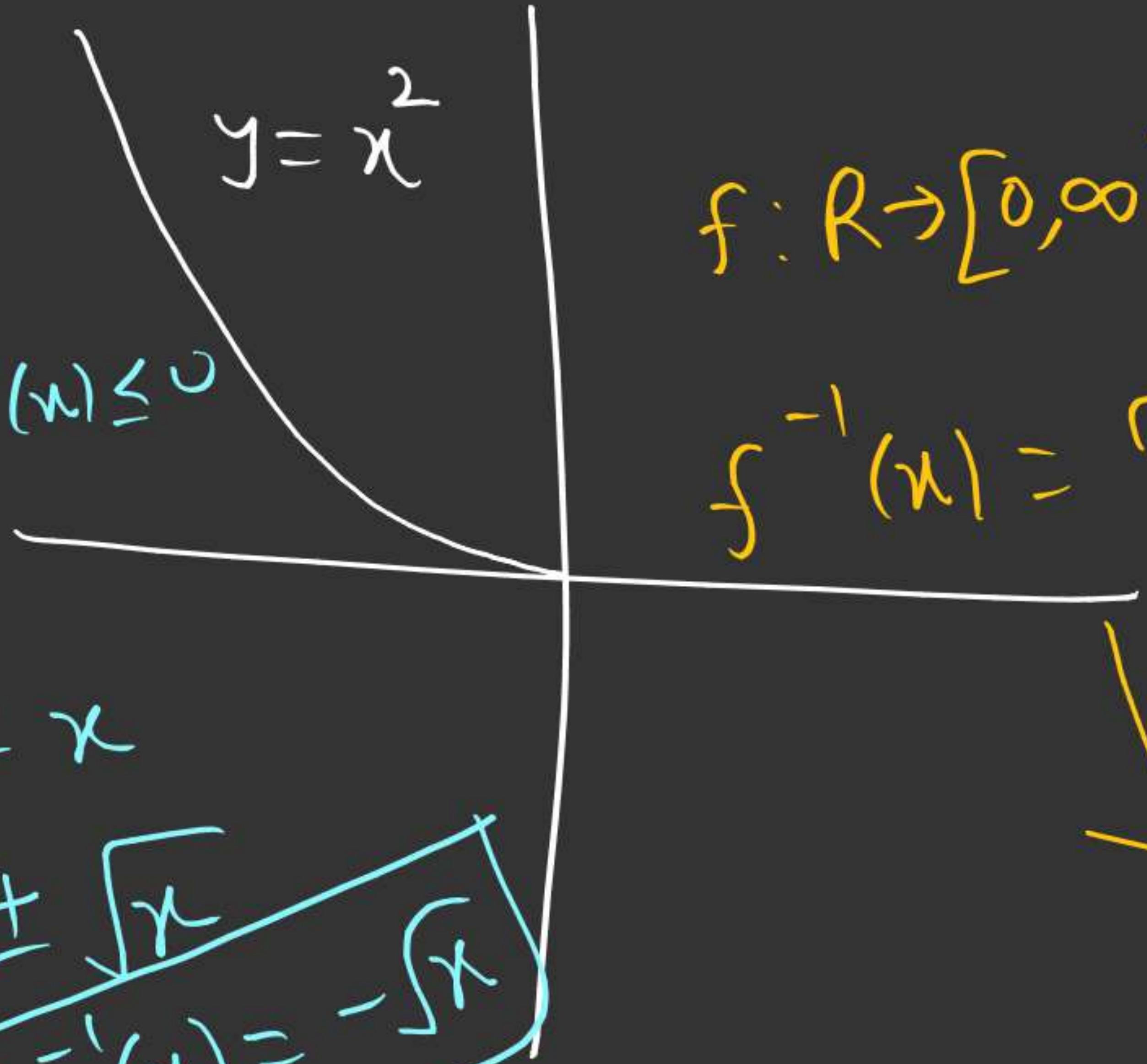
1. $f:(-\infty,0]\to[0,\infty)$, $f(x)=x^2$, $f^{-1}(x)=?$

$f^{-1}:[0,\infty)\to(-\infty,0]$

$f^{-1}(x)\le 0$



$f(f^{-1}(x))=x$

$f(f^{-1}(x))=(f^{-1}(x))^2=x$

$f^{-1}(x)=\pm\sqrt{x}$

$\boxed{f^{-1}(x)=-\sqrt{x}}$

$f:R\to[0,\infty)$, $f(x)=x^2$

$f^{-1}(x)=?$ not def

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2. $f:(-\infty,0]\to[1,\infty)$, $f(x)=\dfrac{e^{x}+e^{-x}}{2}$, $f^{-1}(x)=?$

$\underset{\geq 1}{x}+\underset{\geq 0}{\sqrt{x^2-1}}\geq 1$

$f'(x)=\frac{1}{2}\left(e^{x}-e^{-x}\right)=\dfrac{\overset{\leq 1}{e^{2x}}-1}{2e^{x}}<0$

$f\downarrow$ $\quad$ 1-1

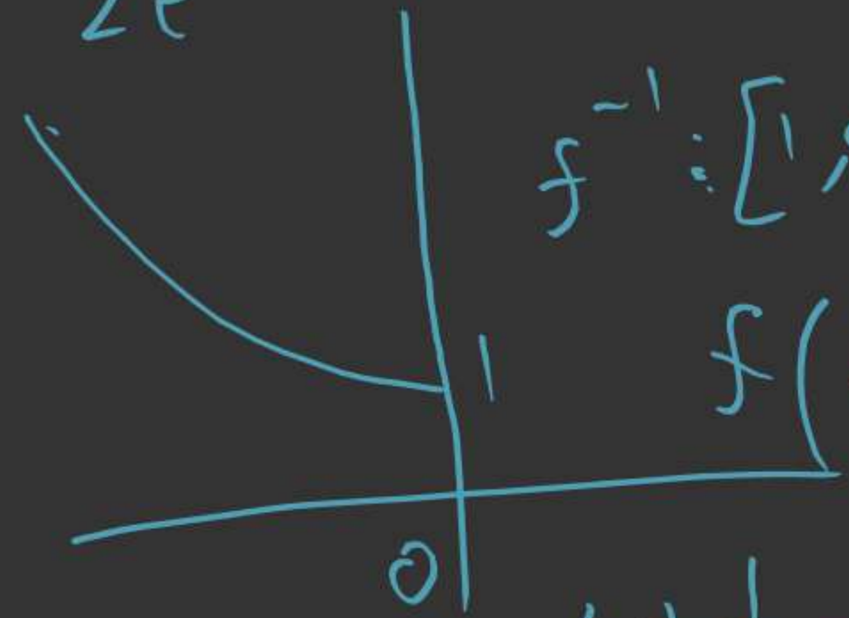
$$\boxed{f^{-1}(x)=\ln\left(x-\sqrt{x^2-1}\right)}\leq$$

$x\to-\infty,\ y\to\infty$

$x=0,\ y=1$

$R_f=[1,\infty)$

$x-\sqrt{x^2-1}=\dfrac{1}{x+\sqrt{x^2-1}}\leq 1$



$f^{-1}:[1,\infty)\to\boxed{(-\infty,0]}$

$f\left(f^{-1}(x)\right)=\dfrac{e^{f^{-1}(x)}+e^{-f^{-1}(x)}}{2}=x$

$t+\frac{1}{t}=2x\Rightarrow t^2-2xt+1=0$

$f^{-1}(x)=\ln\left(x\pm\sqrt{x^2-1}\right)\Leftarrow e^{f^{-1}(x)}=t=x\pm\sqrt{x^2-1}$

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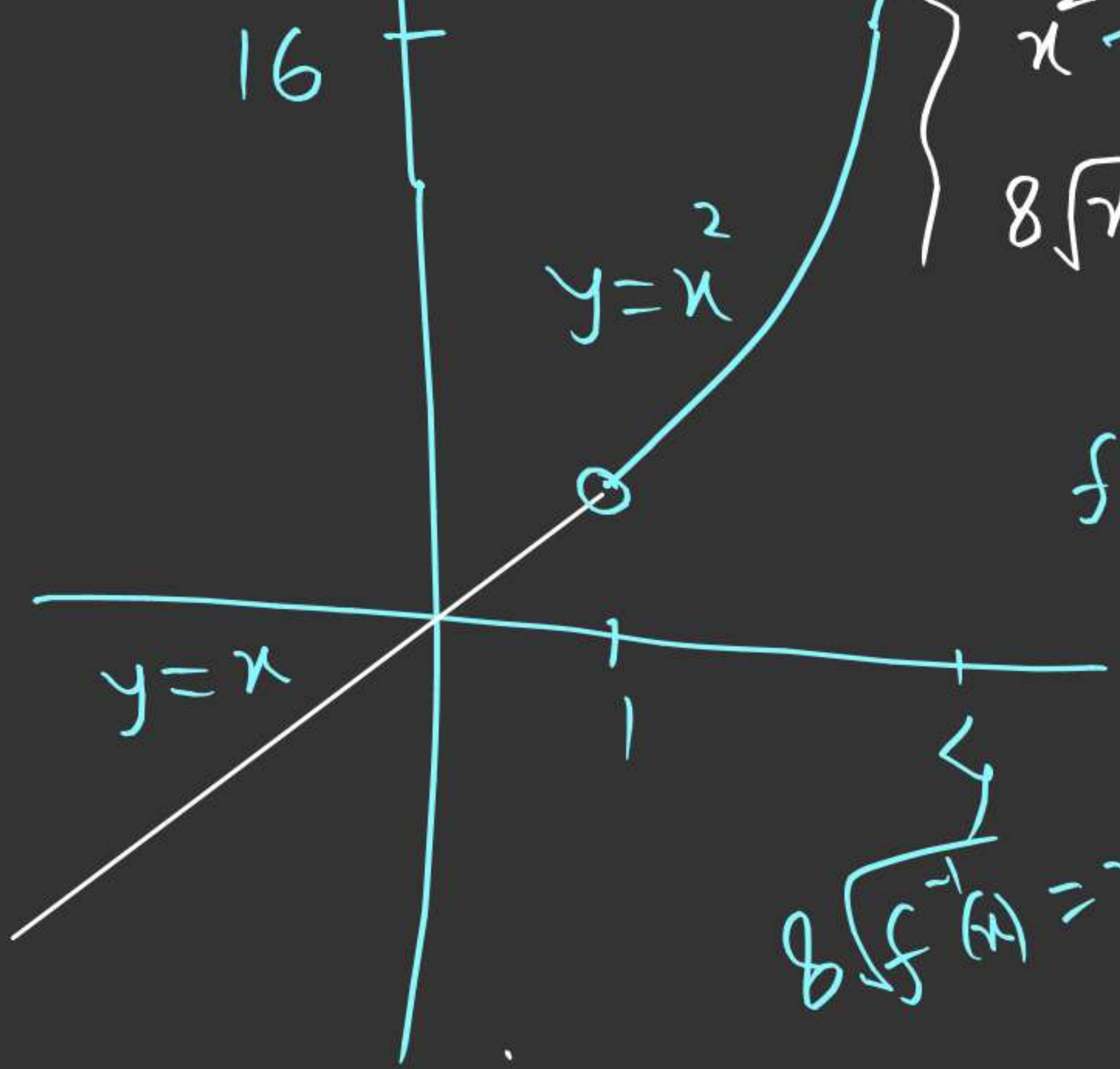
3. $f : R \to R$, $\quad f(x) = \begin{cases} x & x < 1 \\ x^2 & 1 \le x \le 4 \\ 8\sqrt{x} & x > 4 \end{cases}$, find $f^{-1}(x)$.

$(-\infty, 1)$

$[1, 16]$

$(16, \infty)$



$8\sqrt{f^{-1}(x)} = x$

$$f^{-1}(x) = \begin{cases} x & x \in (-\infty, 1) \\ \sqrt{x} & x \in [1, 16] \\ \dfrac{x^2}{64} & x \in (16, \infty) \end{cases}$$

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4. If $f: R \to R$, $f(x) = x^3 + (a+2)x^2 + 3ax + 5$ is an invertible mapping, find 'a'.

HW

PT-2 → 3, 4, 10

Ex-I → 1, 2, 3, 4, 5, 6, 7, 8