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$$\int_0^{\pi} 2x \sin\frac{x}{2}\, dx$$

$$8\int_0^{\pi/2} x \sin x\, dx$$

$8ax + 2bx$

$$\int_0^x \left(\int_0^u f(t)\,dt\right) du = u\int_0^u f(t)\,dt\Big]_0^x - \int_0^x u f(u)\,du$$

$$\int_0^x 1\cdot g(u)\,du$$

$$= x\int_0^x f(t)\,dt - \int_0^x t f(t)\,dt$$

$$= \int_0^x (x-t) f(t)\,dt.$$

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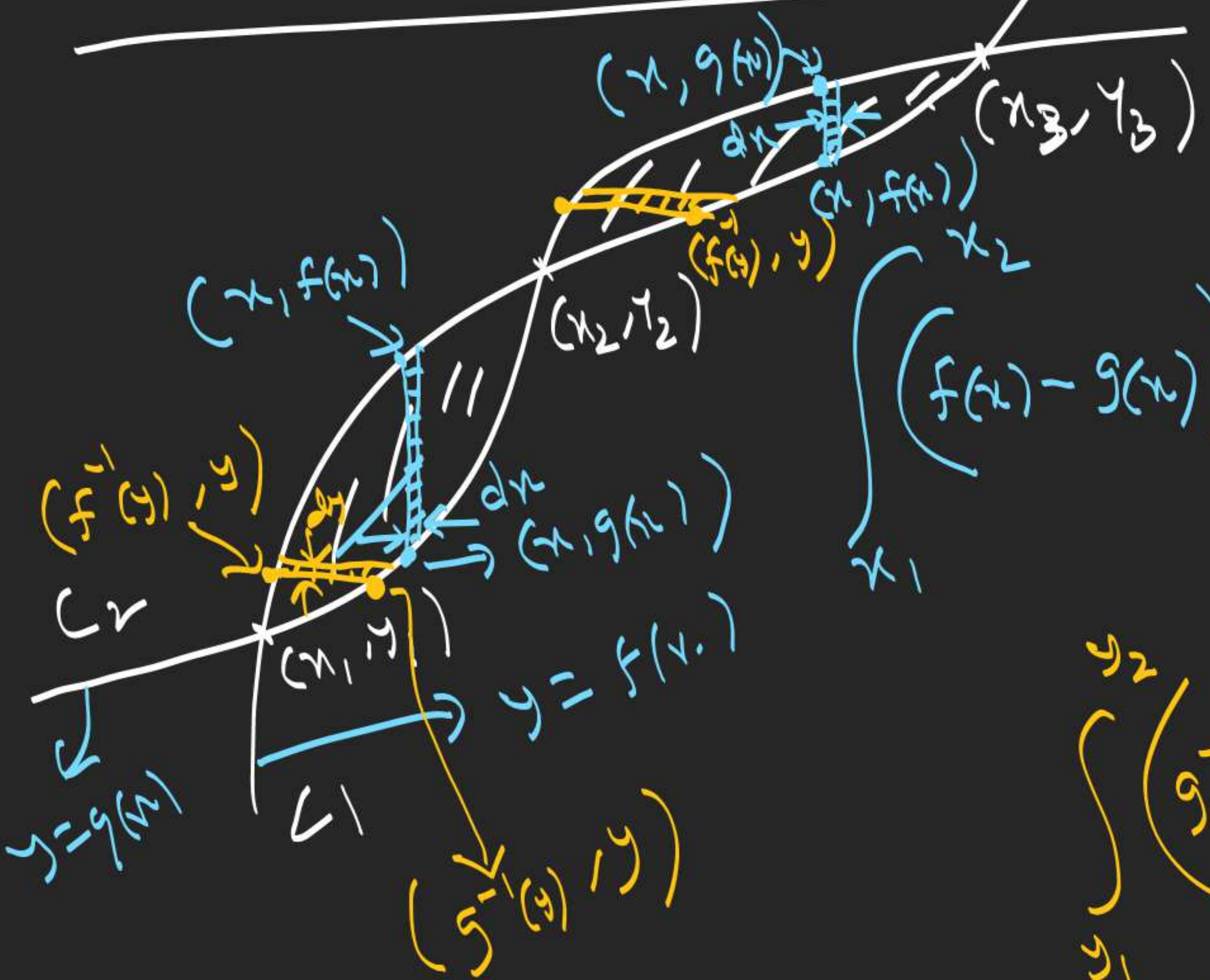
Area bounded by Curves



$$\int_{x_1}^{x_2} \big(f(x) - g(x)\big)\,dx + \int_{x_2}^{x_3} \big(g(x) - f(x)\big)\,dx$$

$$\int_{y_1}^{y_2} \big(g^{-1}(y) - f^{-1}(y)\big)\,dy + \int_{y_2}^{y_3} \big(f^{-1}(y) - g^{-1}(y)\big)\,dy$$

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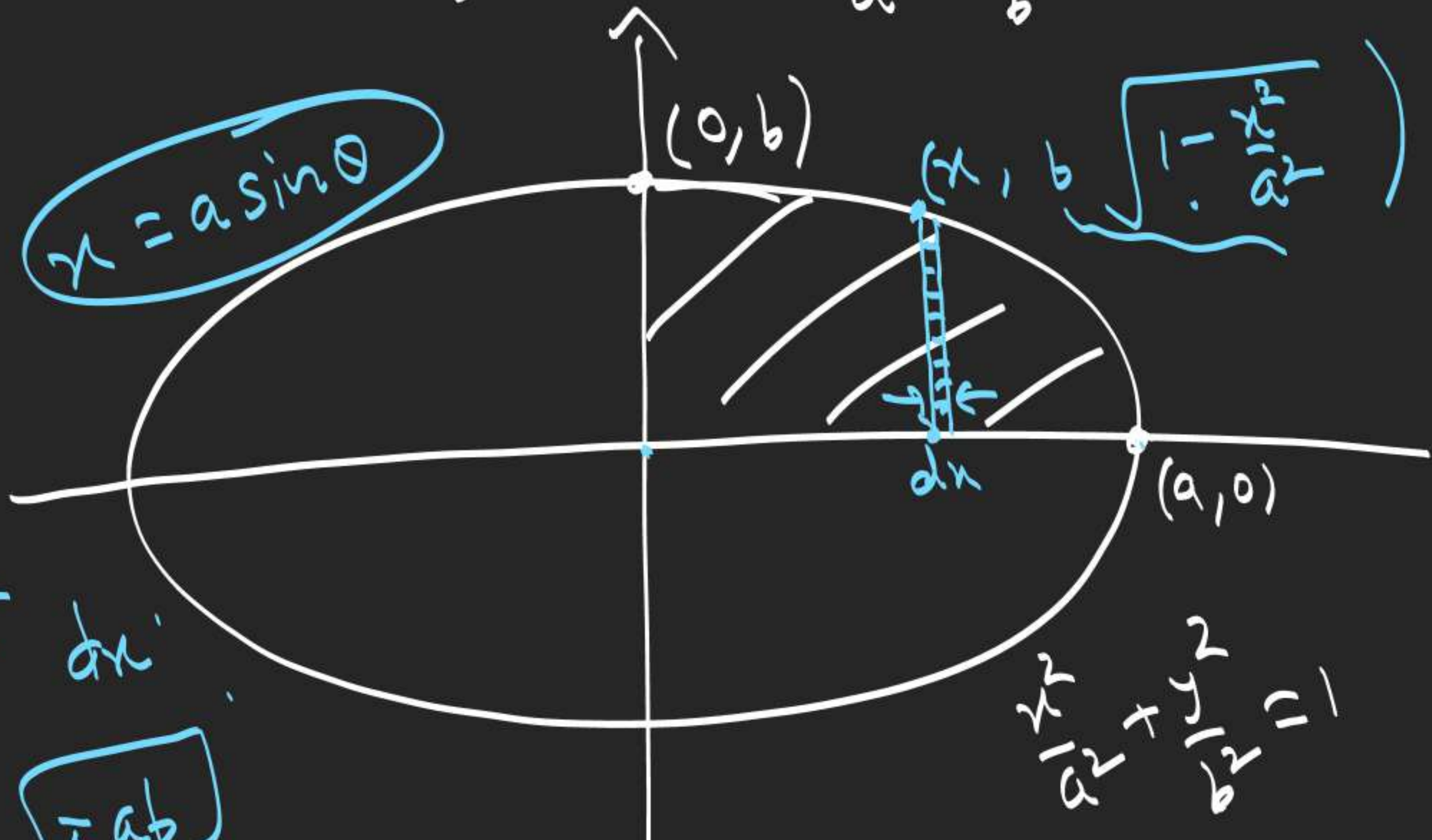
Area enclosed by ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$



$$4\int_0^a b\sqrt{1-\frac{x^2}{a^2}}\,dx$$

$x = a\sin\theta$

$$4ba\int_0^1 \sqrt{1-x'^2}\,dx'$$

$$= 4ab\,\frac{\pi}{4}(1)^2 = \boxed{\pi ab}$$

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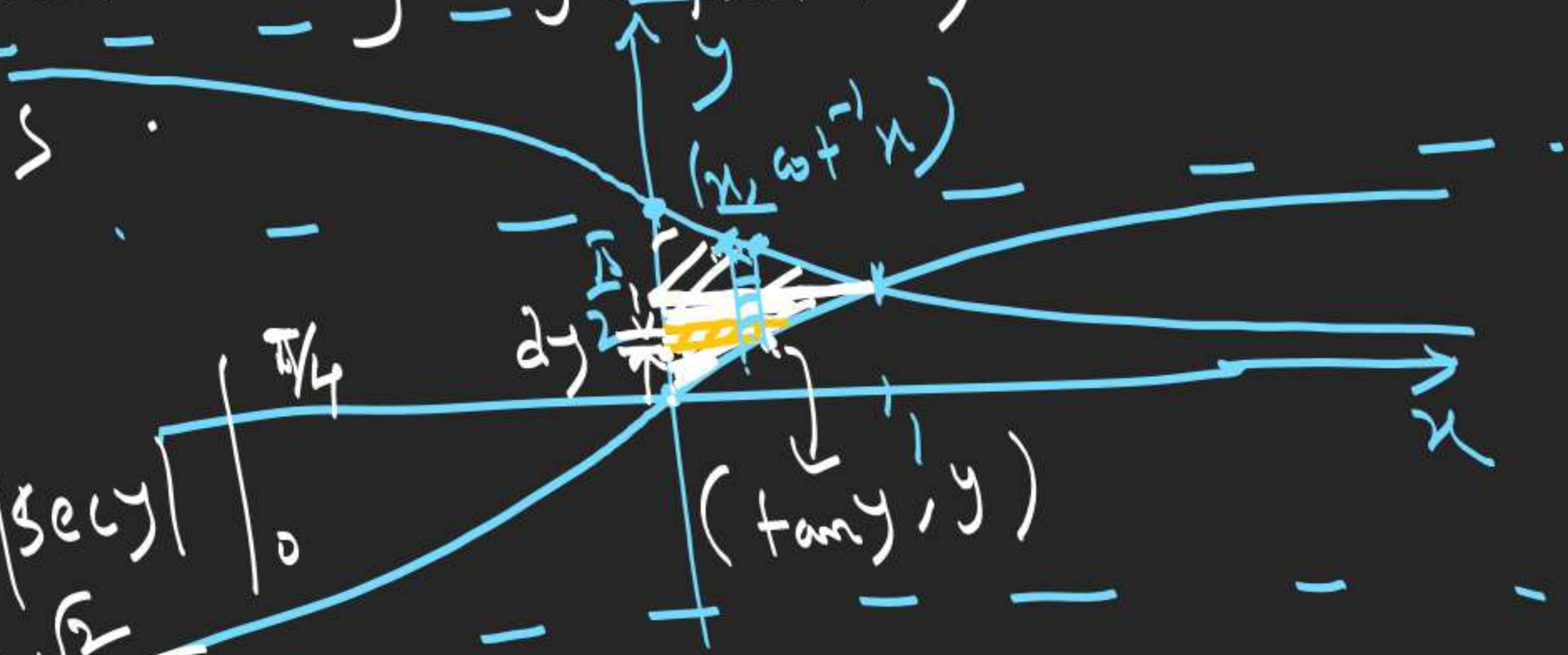
1. Find the area enclosed by $y=\tan^{-1}x$, $y=\cot^{-1}x$ and y-axis.



$$\int_0^1 \left(\cot^{-1}x-\tan^{-1}x\right)dx$$

$$\int_0^1 \left(\frac{\pi}{2}-2\tan^{-1}x\right)dx$$

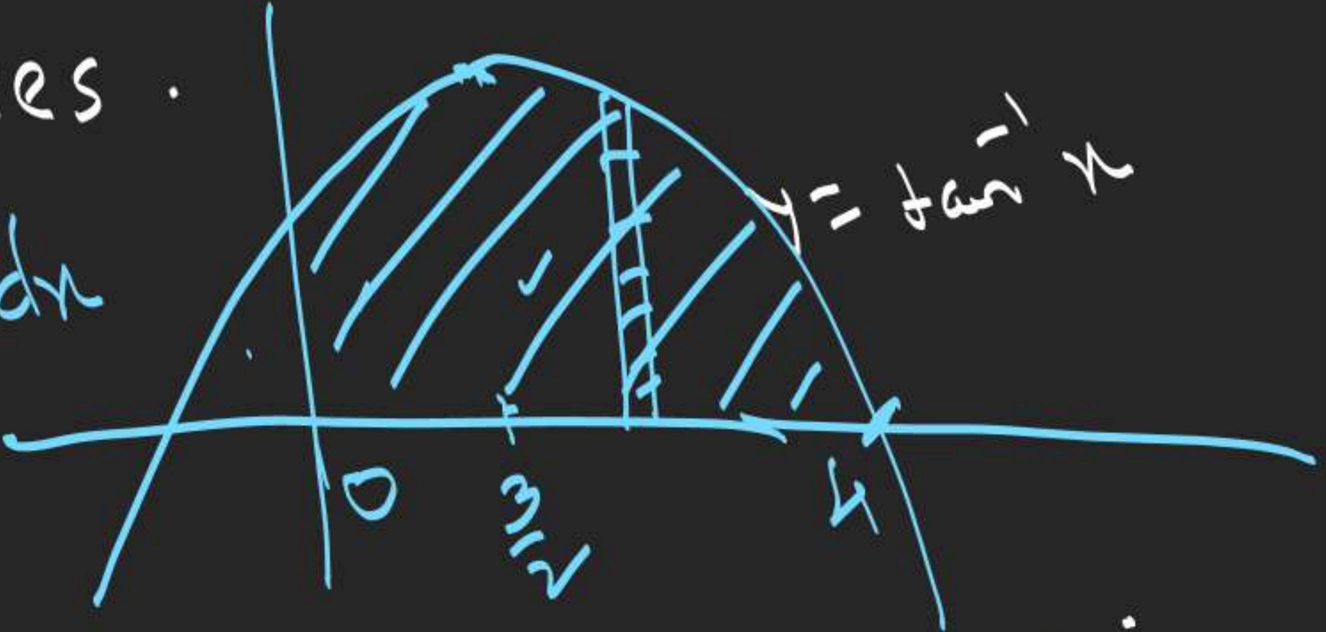
$$2\int_0^{\pi/4}\tan y\,dy$$

$$=2\ln|\sec y|\Big|_0^{\pi/4}$$

$$=2\ln\sqrt{2}$$

$$=\boxed{\ln 2}$$

2. Compute the larger area bounded by $y=4+3x-x^2$ and coordinate axes.



$$\int_0^4 \left(4+3x-x^2\right)dx$$

$$=\frac{56}{3}$$

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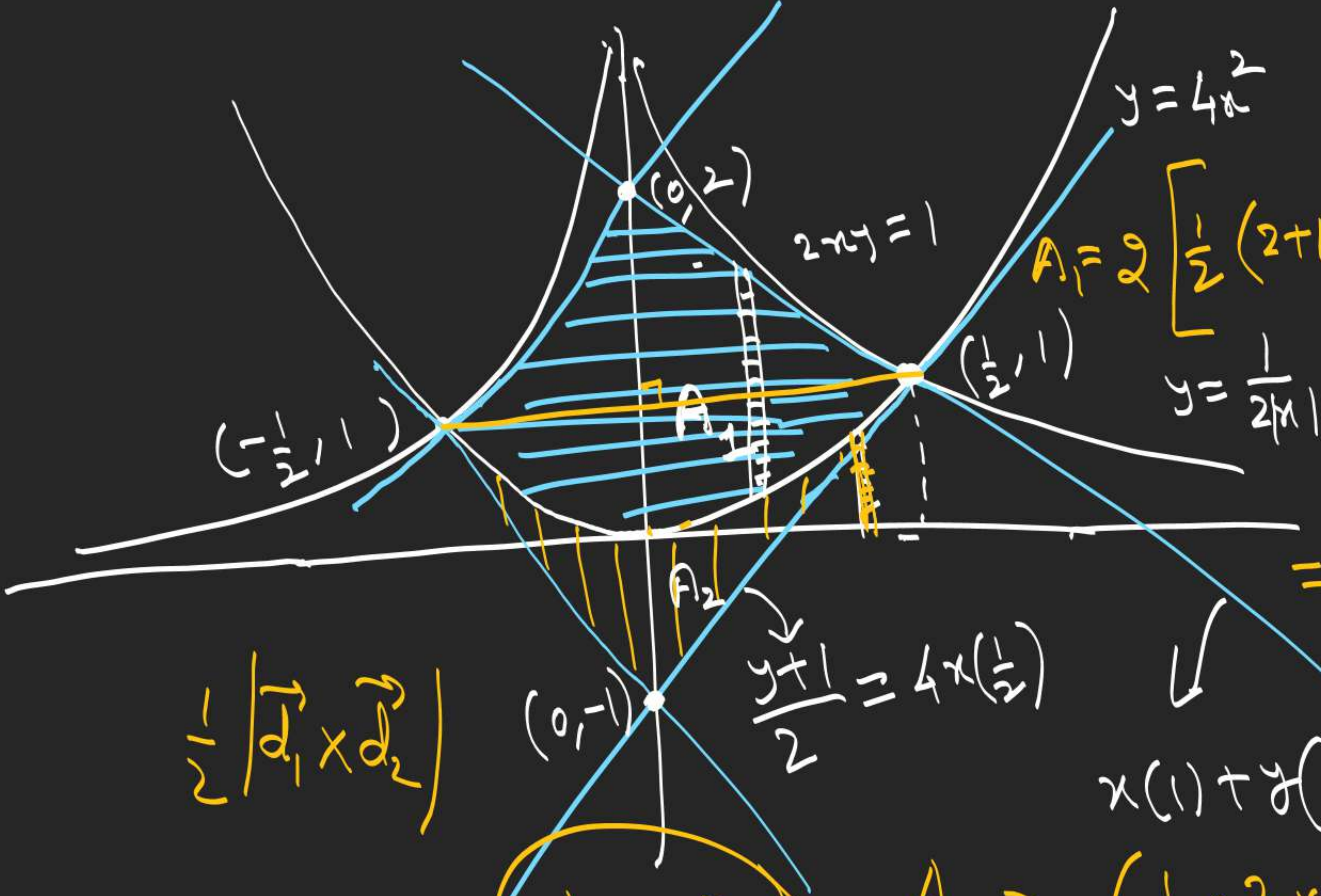
3. Find the ratio in which $y = 4x^2$ divides the region enclosed by tangents to $y = \frac{1}{2|x|}$ and $y = 4x^2$ drawn at their points of intersection.



$$A_1 = 2\left[\frac{1}{2}(2+1)\frac{1}{2} - \int_0^{\frac{1}{2}} 4x^2\,dx\right]$$

$$= 2\left[\frac{3}{4} - \frac{4}{3}\left(\frac{1}{8}\right)\right]$$

$$= \frac{3}{2} - \frac{1}{3} = \frac{7}{6}$$

$$\frac{y+1}{2} = 4x\left(\frac{1}{2}\right)$$

$$x(1) + y\left(\frac{1}{2}\right) = 1$$

$$\frac{1}{2}\left|\vec{d}_1 \times \vec{d}_2\right|$$

$$A_2 = \left(\frac{1}{2}\times 3\times 1 - \frac{7}{6}\right) = \frac{2}{6} = \frac{1}{3}$$

$$\frac{A_1}{A_2} = \frac{7}{2}$$

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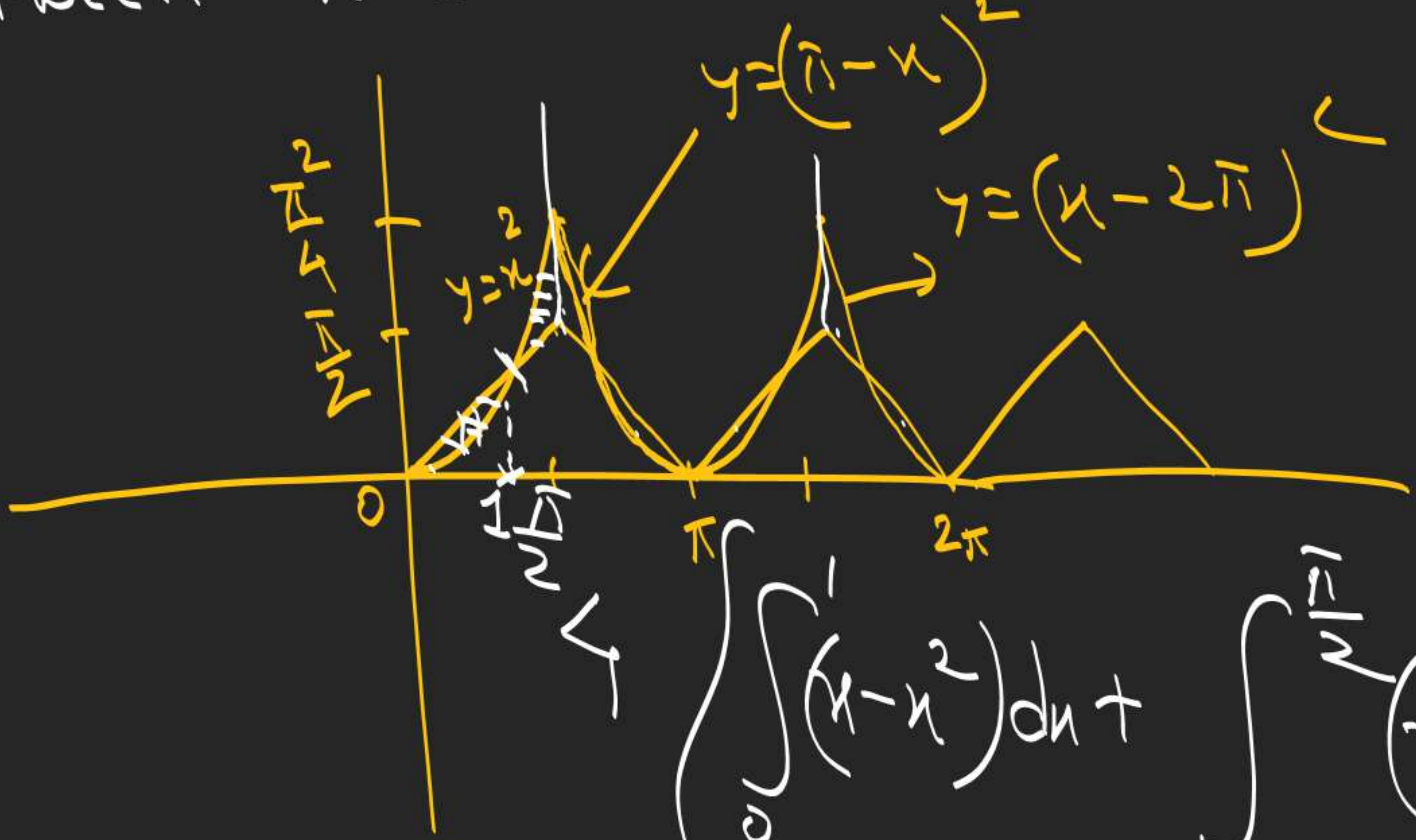
4. Find the area bounded by

$y = \sin^{-1}|\sin x|$ and $y = (\sin^{-1}\sin x)^2$

between $x = 0$ and $x = 2\pi$.



$$4\left[\int_0^1 (x-x^2)\,dx + \int_1^{\frac{\pi}{2}} (x^2-x)\,dx\right]$$

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5. Find area of region formed by points (x, y) satisfying $\log_x(\log_y x) > 0$, $\frac{1}{2} < x < 2$.

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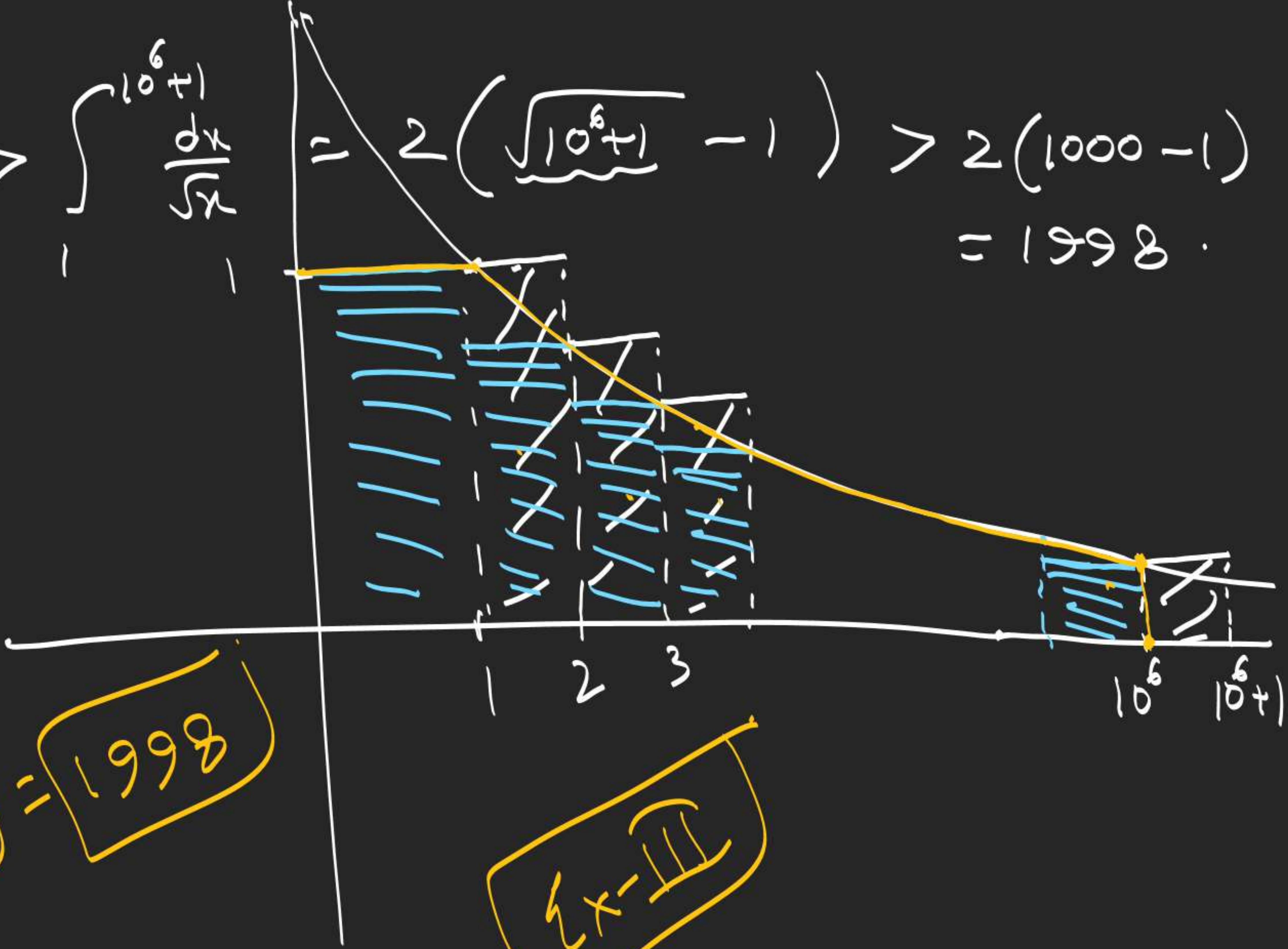
$$1+\int_{1}^{10^6}\frac{dx}{\sqrt{x}} > \sum_{r=1}^{10^6}\frac{1}{\sqrt{r}} > \int_{1}^{10^6+1}\frac{dx}{\sqrt{x}} = 2\left(\sqrt{10^6+1}-1\right) > 2(1000-1)$$

$$= 1998.$$

$$1+2\left(\sqrt{10^6}-1\right) = 1999$$



$$[\cdot] = \boxed{1998}$$

$\boxed{\text{Ex-III}}$