

## LIMIT

$$\lim_{n \rightarrow \infty}$$

Always try to take max term.

(common & cancel them, giving  $\infty$ )

$$\lim_{n \rightarrow \infty} \frac{\underbrace{n+2}_{(n+2)} + \underbrace{n+1}_{(n+1)}}{\underbrace{n+2}_{(n+2)} - \underbrace{n+1}_{(n+1)}}$$

$$\frac{\cancel{n+2} \left\{ 1 + \frac{\cancel{n+1}}{\cancel{n+2}} \right\}}{\cancel{n+2} \left\{ 1 - \frac{\cancel{n+1}}{\cancel{n+2}} \right\}}$$

$$\lim_{n \rightarrow \infty} \frac{1 + \cancel{\frac{1}{n+2}}}{1 - \cancel{\frac{1}{n+2}}} = \frac{1+0}{1-0} = 1$$

$$Q \lim_{n \rightarrow \infty} \frac{\cancel{3(n+1)}}{(n+1)^3} \cancel{3n}^{\sqrt{3n+3}}$$

$$\lim_{n \rightarrow \infty} \frac{(3n+3)(3n+2)(3n+1)}{(n+1)^3} \cancel{3n}^{\sqrt{3n+3}}$$

$$\lim_{n \rightarrow \infty} \frac{\cancel{3}^{\sqrt{3}} \left( 3 + \cancel{\frac{3}{n}} \right) \left( 3 + \cancel{\frac{2}{n}} \right) \left( 3 + \cancel{\frac{1}{n}} \right)}{\cancel{3}^{\sqrt{3}} \left( 1 + \cancel{\frac{1}{n}} \right)^3}$$

$$\frac{3+0}{3} \left( 3+0 \right) \left( 3+0 \right) = \frac{27}{3} = 9$$

## LIMIT

$$\lim_{x \rightarrow \infty} \frac{2x+7}{3x+4}$$

$$\lim_{x \rightarrow \infty} \frac{x(2+\frac{7}{x})}{x(3+\frac{4}{x})} = \frac{2}{3}$$

$$\lim_{x \rightarrow \infty} \frac{x^2(2-\frac{3}{x}+\frac{5}{x^2})}{x^2(4+\frac{7}{x}+\frac{10}{x^2})}$$

$$\frac{2}{4} = \frac{1}{2}$$

$$\lim_{x \rightarrow \infty} \frac{x(3+\frac{5}{x})}{x(4+\frac{7}{x}+\frac{10}{x^2})} = \frac{\infty}{\infty}$$

Poly type

$$\lim_{x \rightarrow \infty} \frac{x^2(2-\frac{3}{x}+\frac{5}{x^2})}{x(7+\frac{10}{x})} > \infty$$

$$\lim_{x \rightarrow \infty} \frac{\sqrt{x} \rightarrow \frac{1}{2}}{\sqrt{x+\sqrt{x}} + \sqrt{x} \rightarrow \frac{1}{2}}$$

$$\frac{\frac{1}{2}}{1+1} = \frac{1}{2}$$

$$(1) \frac{e}{\bar{e}} = \frac{(off)}{(off)}$$

(2) Smaller Bigger  $\rightarrow 0$

(3) Bigger smaller  $\rightarrow \infty$

$$\lim_{x \rightarrow \infty} \frac{\sqrt{2x^2-7} - \sqrt{3x^2+5}}{x^2-1}$$

Rationalise

$$\frac{\sqrt{2}-\sqrt{3}}{1} = \sqrt{2}-\sqrt{3}$$

## LIMIT

$$\text{Q} \lim_{x \rightarrow \infty} \frac{(2x-7)(3x+1)}{(4x+2)(7x-9)} = ?$$

2

$$\frac{2 \times 3}{4 \times 7} = \frac{3}{14}$$

$2x+2$

$$\text{Q} \lim_{x \rightarrow \infty} \left( \frac{x^2 - 3x + 1}{5x^2 + 7} \right) \rightarrow \frac{\infty}{\infty}$$

$$= \left( \frac{1}{5} \right)^\infty = (1^{\frac{1}{5}})^{\infty}$$

$$= 0$$

$$\frac{2-3x}{4x+1}$$

$$\text{Q} \lim_{x \rightarrow \infty} \left( \frac{2x^2 + 13}{5x^2 + x} \right) \rightarrow \frac{\infty}{\infty}$$

$$\left( \frac{2}{5} \right)^{-\frac{3}{4}} \cdot \left( \frac{5}{2} \right)^{3/4}$$

$$\text{Q} \lim_{x \rightarrow \infty} \frac{8x^3 + 7x^2 + \sqrt{6x^5 + 2}}{\sqrt{7x^6 + 2x^3} + 2x - 1} = \frac{\infty}{\infty}$$

3      2      3      2.5  
3      1.5      1

$$2\frac{8}{\sqrt{7}}$$

## LIMIT

$$\text{Q} \lim_{x \rightarrow \infty} \frac{2\sqrt{x} + 3x^{1/3} + 4x^{1/4} + \dots + nx^{1/n}}{(2x-3)^{1/2} + (2x-3)^{1/3} + \dots + (2x-3)^{1/n}}$$

$$\frac{2}{\sqrt{2}} = \sqrt{2}$$

$$\text{Q} \lim_{x \rightarrow \infty} \frac{\sqrt{x^2+1} + 3\sqrt[3]{x^2+1}}{5\sqrt[5]{2x^5+3} - 5\sqrt[5]{9x^4+27}} = \frac{1}{(2)^{1/5}}$$

$$\text{Q} \lim_{n \rightarrow \infty} \frac{\frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \dots + \frac{n}{n+1}}{25n^2 - 7n + 100}$$

$$\lim_{n \rightarrow \infty} \frac{\frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \dots + \frac{n}{n+1}}{25n^2 - 7n + 100}$$

$$\frac{1}{2} \lim_{n \rightarrow \infty} \frac{1+2+3+\dots+n}{25n^2 - 7n + 100}$$

$$\frac{1}{2} \lim_{n \rightarrow \infty} \frac{(n)(n+1)}{2(25n^2 - 7n + 100)} \stackrel{n \rightarrow \infty}{\rightarrow} \frac{1}{2} \times \frac{1}{50} = \frac{1}{100}$$

## LIMIT

$$\text{Q} \lim_{n \rightarrow \infty} \frac{1+2+3+\dots+n^1}{n^2} = ?$$

*Ans: 1/2*

$$\lim_{n \rightarrow \infty} \frac{(n)(n+1)}{2 \cdot n^2} = \frac{\infty}{\infty}$$

$$= \frac{1}{2}$$


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$$\text{Q} \lim_{n \rightarrow \infty} \frac{1^3+2^3+\dots+n^3}{n^5}$$

$$\frac{(n)^2(n+1)^2}{4n^5} \xrightarrow[n \rightarrow \infty]{} \frac{1}{4}$$

*Ans: 1/4*

$$= 0$$

$$\text{Q} \lim_{n \rightarrow \infty} \frac{1^2+2^2+3^2+\dots+n^2}{n^3} = ?$$

*Ans: 1/3*

$$\lim_{n \rightarrow \infty} \frac{(n)(n+1)(2n+1)}{6n^3} = \frac{1 \times 1 \times 2}{6} = \frac{1}{3}$$

$$\text{Q} \lim_{n \rightarrow \infty} \frac{1^3+2^3+3^3+\dots+n^3}{n^4} = ?$$

$$= \frac{1}{4}$$
  

$$\text{Q} \lim_{n \rightarrow \infty} \frac{1^p+2^p+3^p+\dots+n^p}{n^{p+1}} = ?$$

*(Definite)*

$$= \frac{1}{(p+1)}$$

$$\text{Q} \lim_{n \rightarrow \infty} \frac{1^3+2^3+\dots+n^3}{n^5} = ?$$

*Ans: 0*

## LIMIT

$$Q \lim_{n \rightarrow \infty} \frac{(1^4 + 2^4 + \dots + n^4) - (1^3 + 2^3 + \dots + n^3)}{n^5}$$

$$\left| \begin{array}{c} \lim_{n \rightarrow \infty} \frac{1^4 + 2^4 + \dots + n^4}{n^5} \xrightarrow{\text{diff=1}} \lim_{n \rightarrow \infty} \frac{1^3 + 2^3 + \dots + n^3}{n^5} \xrightarrow{\text{diff=2}} \\ \frac{1}{5} - 0 = \frac{1}{5} \end{array} \right.$$

$$Q \lim_{n \rightarrow \infty} \left[ \frac{1}{2 \cdot 5} + \frac{1}{3 \cdot 6} + \frac{1}{4 \cdot 7} + \dots + \frac{1}{(n)(n+3)} \right] = \frac{1}{\text{diff} \times 1^{\text{st}} \text{ term}}$$

$$\left| \begin{array}{c} Q \lim_{n \rightarrow \infty} \left[ \frac{1}{1 \cdot 2 \cdot 3} + \frac{1}{2 \cdot 3 \cdot 4} + \frac{1}{3 \cdot 4 \cdot 5} + \frac{1}{4 \cdot 5 \cdot 6} + \dots + \frac{1}{(n)(n+1)(n+2)} \right] \\ \frac{1}{\text{diff} \times 1^{\text{st}} \text{ term}} \\ \frac{1}{2} \left[ \frac{1}{1 \cdot 2} - \frac{1}{2 \cdot 3} \right] + \frac{1}{2} \left[ \frac{1}{2 \cdot 3} - \frac{1}{3 \cdot 4} \right] + \frac{1}{2} \left[ \frac{1}{3 \cdot 4} - \frac{1}{4 \cdot 5} \right] + \dots + \frac{1}{2} \left[ \frac{1}{(n)(n+1)} - \frac{1}{(n+1)(n+2)} \right] \end{array} \right. = \frac{1}{4}$$

$$\frac{1}{2 \cdot 5} = \frac{1}{\text{diff}} \left[ \frac{1}{\text{Chhotu}} - \frac{1}{\text{Bda}} \right]$$

diff=3

$$\frac{1}{2 \cdot 5} = \frac{1}{3} \left[ \frac{1}{2} - \frac{1}{5} \right]$$

$$\frac{1}{1 \cdot 2 \cdot 3} = \frac{1}{2} \left[ \frac{1}{1^2 + (\text{oub})} - \frac{1}{2^{n+1}(\text{oub})} \right]$$

diff=2

$$= \frac{1}{2} \left[ \frac{1}{1 \cdot 2} - \frac{1}{2 \cdot 3} \right]$$

$$\lim_{n \rightarrow \infty} \frac{\frac{1}{2} \left[ \frac{1}{1 \cdot 2} - \frac{1}{(n+1)(n+2)} \right]}{\frac{1}{2} \left[ \frac{1}{1 \cdot 2} - \frac{1}{(n+1)(n+2)} \right]} = \frac{1}{(1 \cdot 2) \times 2}$$

## LIMIT

$$\text{Q} \lim_{n \rightarrow \infty} \frac{1}{\underbrace{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot \dots}_{3 \times [1 \cdot 2 \cdot 3]}} =$$

$$\text{Q} \lim_{n \rightarrow \infty} \frac{-3n + (-1)^n}{4n - (-1)^n}$$

$$\lim_{n \rightarrow \infty} \frac{n[-3 + \frac{(-1)^n}{n}]^0}{n[4 - \frac{(-1)^n}{n}]^0} = -3$$

$$\text{Q} \lim_{n \rightarrow \infty} (3^n + 4^n)^{\frac{1}{n}} \xrightarrow{\text{com.}} (3^\infty + 4^\infty)^{\frac{1}{\infty}} = (\infty)^0$$

$$4 \left( \frac{3^n}{4^n} + 1 \right)^{\frac{1}{n}}$$

$$\lim_{n \rightarrow \infty} 4 \left( \left( \frac{3}{4} \right)^n + 1 \right)^{\frac{1}{n}}$$

$$4(0+1)^0$$

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Bigger is the Answer

$$\text{Q} \lim_{n \rightarrow \infty} (3^n - 4^n + 5^n - 6^n)^{\frac{1}{n}}$$

$$\lim_{n \rightarrow \infty} 6 \left( \left( \frac{3}{6} \right)^n - \left( \frac{4}{6} \right)^n + \left( \frac{5}{6} \right)^n - 1 \right)$$

$$6(0-0+0-1)^0$$

$$6 \times 1 = 6$$

## LIMIT

$$Q \lim_{x \rightarrow 0} \left[ (1) \frac{1}{\sin^2 x} + (2) \frac{1}{\sin x} + (3) \frac{1}{\sin^2 x} + \dots + (n) \frac{1}{\sin^2 x} \right]^{\frac{1}{\sin^2 x}} \quad \begin{array}{l} x \rightarrow 0 \\ \sin x \rightarrow 0 \\ \sin^2 x \rightarrow 0 \\ \frac{1}{\sin^2 x} \rightarrow \infty \end{array}$$

$$(1^\infty + 2^\infty + 3^\infty + \dots + n^\infty)^0$$

$$= n \quad \begin{array}{c} (1^n + 2^n)^{\frac{1}{n}} = 2 \\ (2^n + 3^n)^{\frac{1}{n}} = 3 \end{array}$$

$$Q \lim_{\substack{n \rightarrow \infty \\ m \rightarrow \infty}} \frac{1 + \sqrt[n]{1^n + 2^n} + \sqrt[n]{2^n + 3^n} + \sqrt[n]{3^n + 4^n} + \dots + \sqrt[m]{(m-1)^n + m^n}}{m^2}$$

$$\lim_{m \rightarrow \infty} \frac{1 + 2 + 3 + 4 + \dots + m}{m^2} = \frac{1}{2}$$

## LIMIT

$$\frac{0}{0} = 0$$

Q  $\lim_{x \rightarrow \infty} \left( \frac{x^2+1}{x+1} - ax - b \right) = 0$  find  $a, b$ ?

$$\lim_{x \rightarrow \infty} \left( \frac{x^2+1 - ax(x+1) - b(x+1)}{x+1} \right) = 0$$

$$\lim_{x \rightarrow \infty} \left( \frac{x^2+1 - ax^2 - ax - bx - b}{x+1} \right) = 0$$

$$\lim_{x \rightarrow \infty} \left[ \frac{x^2(1-a)}{x+1} + \frac{(-a-b)x}{x+1} + \frac{(1-b)}{x+1} \right] = 0$$

$\underbrace{\hspace{1cm}}_{B/S}$        $\underbrace{\hspace{1cm}}_{E/R}$

$1-a=0$	$-a-b=0$	$(a, b) = (1, -1)$
$a=1$	$b=-a$	
	$b=-1$	

Q  $\lim_{x \rightarrow \infty} \frac{ax^2+bx+c}{x} = 0$  then  $a, b, c$ ?

$$\lim_{x \rightarrow \infty} \left[ \frac{ax^2}{x} + \frac{bx}{x} + \frac{c}{x} \right] = 0$$

$\underbrace{\hspace{1cm}}_A$        $\underbrace{\hspace{1cm}}_B$        $\underbrace{\hspace{1cm}}_C$

$a=0$	$b=0$	$c=0$
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(G.R.)

Q  $\lim_{x \rightarrow 0} \frac{ax^2+bx+c}{x} \neq 0$  then  $a, b, c$ ?

$$\lim_{x \rightarrow 0} \left[ \frac{ax^2}{x} + \frac{bx}{x} + \frac{c}{x} \right] \neq 0$$

$\underbrace{\hspace{1cm}}_A$        $\underbrace{\hspace{1cm}}_B$        $\underbrace{\hspace{1cm}}_C$

$a \neq 0$	$b=0$	$c=0$
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## LIMIT

$$\text{Q} \lim_{x \rightarrow \infty} \frac{a(2x^3 - x^2) + b(x^3 + 5x^2 - 1) - c(3x^3 + x^2)}{a(5x^4 - x) - bx^4 + c(4x^4 + 1) + 2x^2 + 5c} = 1 \text{ then } a, b, c?$$

$$\lim_{x \rightarrow \infty} \frac{x^3(2a+b-3c) + \cancel{x^2}(-a+5b-c) + 0 \cdot x - b}{x^4(5a-b+4c) + 0 \cdot x^3 + \cancel{x^2}(-a+5c) + c} = \boxed{1} \rightarrow \text{Ansatz bei } \begin{cases} 0 \\ \infty \end{cases} \quad \begin{cases} 0 \\ \infty \end{cases}$$

$\frac{5a-b+4c=0}{2a+b-3c=0} \quad \left| \begin{array}{l} 2a+b-3c=0 \\ -a+5b-c=0 \end{array} \right. \quad \frac{-a+5b-c}{2}=1$

$$\left. \begin{array}{l} 5a-b+4c=0 \\ 2a+b-3c=0 \\ -a+5b-c=0 \end{array} \right\} \quad \left. \begin{array}{l} a=-\frac{2}{10g} \\ b=\frac{46}{10g} \\ c=\frac{14}{10g} \end{array} \right| \quad \text{Check yourself}$$

## LIMIT

Q. If  $a_n + b_n + c_n = 2n+1$ ;  $a_n b_n + b_n c_n + c_n a_n = 2n-1$ ;  $a_n \cdot b_n \cdot c_n = -1$  | Dekha Dekha sa lgta.  
 Ans  $a_n < b_n < c_n$  Then.  $\lim_{n \rightarrow \infty} n \cdot a_n = ?$   
 → In teeno se Lubric

$$\lambda^3 - (a_n + b_n + c_n)x^2 + (a_n b_n + b_n c_n + c_n a_n)x - a_n b_n c_n = 0$$

$$\lambda^3 - (2n+1)x^2 + (2n-1)x + 1 = 0$$

$$\lambda^3 - 2n(\lambda^2 - \lambda^2 + 2n)\lambda - \lambda + 1 = 0$$

$$\lambda^2(\lambda - 1) - 2n\lambda(\lambda - 1) - 1(\lambda - 1) = 0$$

$$[(\lambda - 1)(\lambda^2 - 2n\lambda + 1)] = 0$$

$$\therefore \frac{n + \sqrt{n^2 - 1}}{c_n}, \frac{1}{b_n}, \frac{n - \sqrt{n^2 - 1}}{a_n}$$

$$\begin{cases} \lim_{n \rightarrow \infty} n \cdot a_n \\ \lim_{n \rightarrow \infty} n(n - \sqrt{n^2 - 1}) \end{cases}$$

$$\lim_{n \rightarrow \infty} \frac{n(n^2 - (n^2 - 1))}{n + \sqrt{n^2 - 1}} = \frac{n^2}{n + \sqrt{n^2 - 1}} = n \pm \sqrt{n^2 - 1}$$

$$= \frac{1}{1+1} = \frac{1}{2}$$

$\left. \begin{array}{l} \alpha + \beta + \gamma = \\ \alpha \cdot \beta + \beta \gamma + \gamma \alpha = \\ \alpha \cdot \beta \cdot \gamma = \end{array} \right\}$  Kha  
 $\left. \begin{array}{l} \alpha + \beta + \gamma = \\ \alpha \cdot \beta + \beta \gamma + \gamma \alpha = \\ \alpha \cdot \beta \cdot \gamma = \end{array} \right\}$  Dekha  
 (akeegyan)

$$\lambda^3 - (\alpha + \beta + \gamma)x^2 + (\alpha\beta + \beta\gamma + \gamma\alpha)x - \alpha\beta\gamma = 0$$

$$\lambda = \frac{2n \pm \sqrt{4n^2 - 4}}{2}$$

## LIMIT

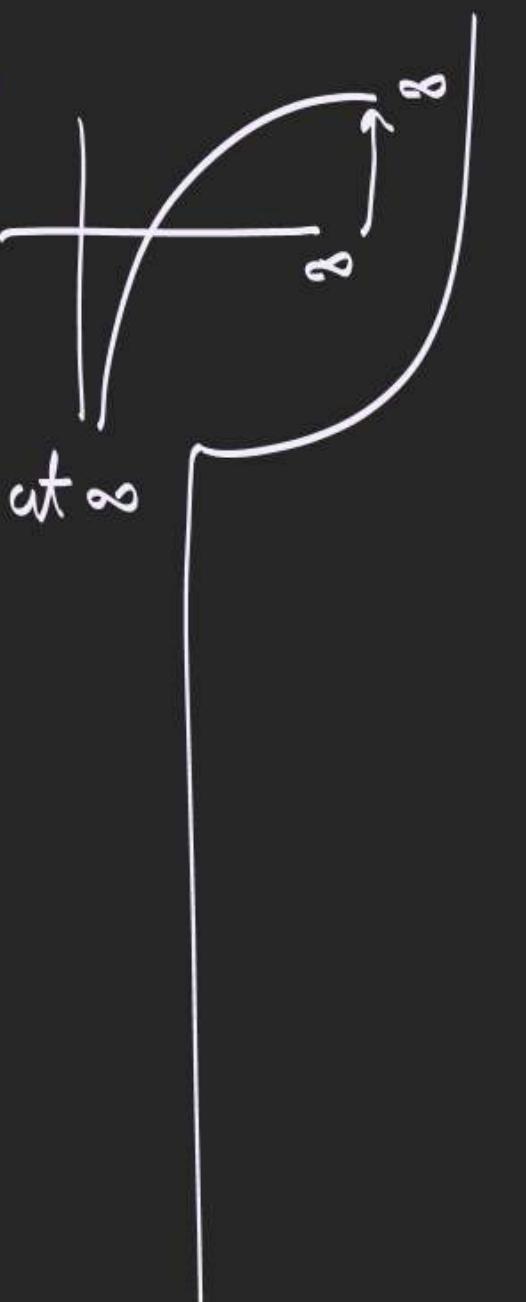
$$\text{Q} \lim_{x \rightarrow \infty} \frac{\log x^n - \lceil x \rceil}{\lfloor x \rfloor} \quad \lim_{x \rightarrow \infty} \lceil x \rceil = x$$

$$\lim_{n \rightarrow \infty} \frac{n \log x - x}{x}$$

$$n \lim_{x \rightarrow \infty} \frac{\log x}{x} \stackrel{x \rightarrow \infty}{=} 1 \quad \boxed{DL}$$

$$n \lim_{x \rightarrow \infty} \frac{\frac{1}{x}}{1} = 1$$

$$n \times 0 = -1$$



$$\text{Q} 1-15 \quad \underline{\underline{Ex 1}}$$

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$\underline{\underline{Ex 2}}$  1, 2, 5, 6, 7, 10  
13, 14

## LIMIT

DPP2Q8

$$f(x) = \begin{cases} \sin x + \text{cosec } x & x \in (-\infty, -1] \cup [1, \infty) \\ 1 & -1 \leq x \leq 1 \end{cases}$$

$$\lim_{x \rightarrow \frac{\pi}{2}} f(x) = 1 \quad \lim_{x \rightarrow \frac{3\pi}{2}} f(x) = 1 \quad \boxed{B}$$

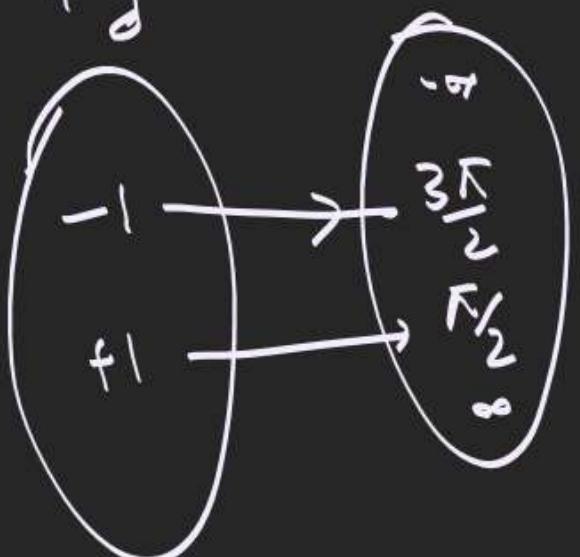


$$f(-1) = \left( \sin(-1) \right) + \text{cosec } \frac{1}{1} \\ = -1 + \infty = \boxed{B}$$

$$f(1) = \left( \sin 1 \right) + \text{cosec } \frac{1}{1} \\ = 1 + 0 = \boxed{C}$$

2 Ans 

$$x \in \{-1, 1\} \quad \text{Inf} \rightarrow 1-2-1 \quad \text{A} \quad \checkmark$$



$$1-2-1 \\ \text{Ans} \quad \text{B} \quad \times$$

## LIMIT

$$(P) f(x) = \lim_{n \rightarrow \infty} \left( \frac{x}{1+|x|} \right)$$

$$\lim_{x \rightarrow \infty} \frac{x}{1+x} \in \frac{0}{\infty} = 0 \\ = 1$$

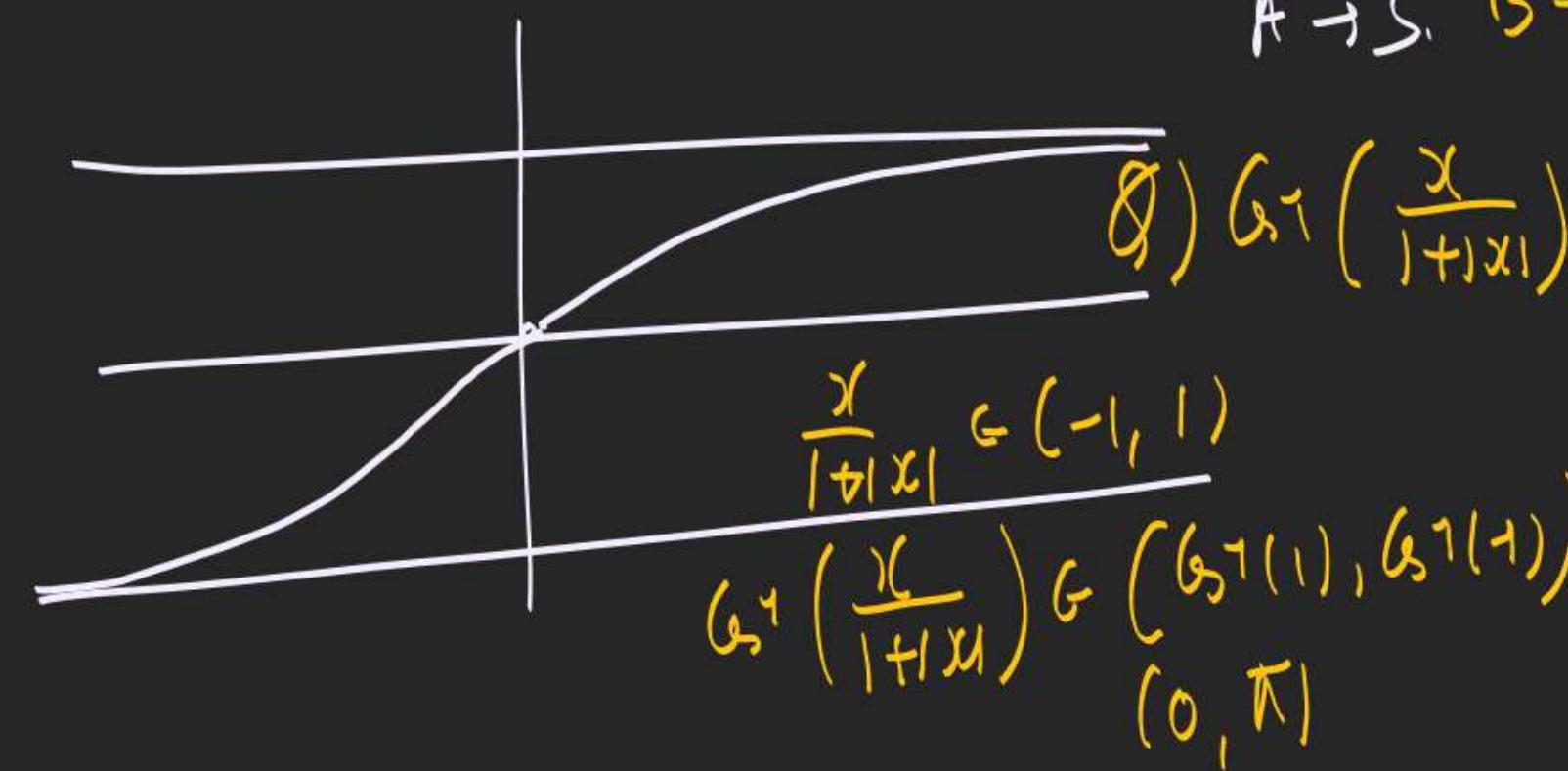
$$\frac{x}{1+|x|} \in (-1, 1)$$

$$\lim_{n \rightarrow \infty} \left( \frac{x}{1+|x|} \right) \in \left( \lim_{n \rightarrow \infty} (-1), \lim_{n \rightarrow \infty} (1) \right) \\ \left( -\frac{\pi}{2}, \frac{\pi}{2} \right)$$

$$y = \frac{x}{1+|x|}$$

$\frac{x}{1+|x|} < 1 \quad x > 0$

$\frac{x}{1+|x|} > 1 \quad x < 0$



$$\frac{x(+)}{x(+)} = 1 \quad (C) \lim_{x \rightarrow 0} \left( \frac{x}{1+|x|} \right)$$

$$\lim_{x \rightarrow \infty} \frac{x}{1+|x|} = 1 \quad \frac{x}{1+|x|} \in (-1, 1)$$

$$\frac{0}{0+1} = 0 \quad \text{in } (-\frac{\pi}{2}, \frac{\pi}{2})$$

A  $\rightarrow$  S. B  $\rightarrow$  P ( $\rightarrow$  R)