FUNCTIONS

15.

$$\tan\frac{\pi}{24} = \left(\sqrt{3}-\sqrt{2}\right)\left(\sqrt{2}-1\right)$$

$$\downarrow$$

$$\frac{1-\cos\frac{\pi}{12}}{\sin\frac{\pi}{12}} = \frac{1-\frac{\sqrt{3}+1}{2\sqrt{2}}}{\frac{\sqrt{3}-1}{2\sqrt{2}}}$$

16. 24

$$\downarrow$$

$$\frac{1}{4}\sin 75^\circ$$

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FUNCTIONS

2.

$$f(x) = \frac{\sqrt{\underbrace{\cos x - \tfrac{1}{2}}_{\geq 0}}}{\sqrt{\underbrace{6 + 35x - 6x^2}_{> 0}}}$$

$$D_f = \left(-\frac{1}{6}, \frac{\pi}{3}\right] \cup \left[\frac{5\pi}{3}, 6\right)$$

$$6x^2 - 35x - 6 < 0$$

$$(6x+1)(x-6) < 0$$

$$x \in \left(-\frac{1}{6}, 6\right) \checkmark$$

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Find domain

3. $f(x) = \sqrt{\log_{\frac{1}{2}} |3-x|}$

$|x|^2 = x^2$

$\log_{\frac{1}{2}} |3-x| \geq 0 = \log_{\frac{1}{2}} 1$

$(x-3)^2 \leq 1$

$(x-2)(x-4) \leq 0$

$0 < |3-x| \leq 1$, $\Rightarrow$ $x-3 \in [-1,0) \cup (0,1]$

$x \neq 3$

$D_f \Rightarrow \boxed{x \in [2,3) \cup (3,4]}$

4. $f(x) = \sqrt{x} + \sqrt[3]{\frac{1}{x-2}} - \log_{10}(2x-3)$

$\boxed{x \geq 0}$ $\boxed{x \neq 2}$

$2x-3 > 0$

$\boxed{x > 3/2}$

$\boxed{D_f = \left(\frac{3}{2}, 2\right) \cup (2, \infty)}$

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$$\boxed{D_f = \left[\log_3 \frac{9}{10},\ 2\right)}$$

5.

$$f(x) = \sqrt{1 - \frac{(x-1)}{\log_3(9-3^x) - 3}}$$

$$\frac{x-1}{\underbrace{\log_3(\underbrace{9-3^x}_{<9}) - 3}_{<2}} \le 1 \Rightarrow x-1 \geq \log_3(9-3^x) - 3$$

$$\Rightarrow \log_3 3^{x+2} = x+2 \geq \log_3(9-3^x)$$

$$3^{\log_3 \frac{9}{10}}$$

$$3^x < 3^2$$

$$0 < 9 - 3^x \le 3^{x+2}$$

$$\boxed{x < 2}$$

$$10 \cdot 3^x \geq 9 \Rightarrow 3^x \geq \frac{9}{10}$$

$$\boxed{x \geq \log_3 9/10}$$

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$[x] > 3$, $x = ?$

$[\cdot] = G.I.F$

$[x] \in \{4, 5, 6, \ldots\}$

$x \in [4, \infty)$

$[x] \leq -2$, $x = ?$

$x \in (-\infty, -1)$

$[-1.98] = -2$

$[-1.01] = -2$

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FUNCTIONS

$[\cdot] = G.I.F$

$$-27.6 < \left[x + \underbrace{\left[x + \left[x + [x+3]\right]\right]}_{\in I}\right] \leq 35.8$$

$x = ?$

$$[x] + \left[x + \underbrace{\left[x + [x+3]\right]}_{\in I}\right]$$

$$\Rightarrow -27.6 < 4[x] + 3 \leq 35.8$$

$$[x] + [x] + \left[x + [x+3]\right]$$

$$\frac{-30.6}{4} < [x] \leq \frac{32.8}{4}$$

$$[x] + [x] + [x] + [x+3]$$

$$[x] \in \{-7, -6, \ldots 7, 8\}$$

$$\boxed{x \in [-7, 9)}$$

Ans

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FUNCTIONS

Find domain

6. $f(x) = \sqrt{\left[\ln \frac{x}{[x]}\right]}$.

$[\cdot] = G.I.F.$

$\left[\ln \frac{x}{[x]}\right] \geq 0 \Rightarrow \ln \frac{x}{[x]} \geq 0 \Rightarrow \frac{x}{[x]} \geq 1 \Rightarrow \frac{x}{[x]} - 1 \geq 0$

$\Rightarrow \frac{\{x\}}{[x]} \geq 0$

$[x] > 0 \Rightarrow \boxed{x \in [1, \infty) \cup \{n\}}$

$n \in I^-$

$D_f = \{\ldots, -2, -1\} \cup [1, \infty)$

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FUNCTIONS

Find range of

1. $$f(x) = \ln\left(\overbrace{5x^2-8x+4}^{\left[\frac{4}{5},\infty\right)}\right) \in \left[\ln\frac{4}{5}, \infty\right)$$

$$\boxed{R_f = \left[\ln\frac{4}{5}, \infty\right)}$$

$$5x^2-8x+4 = 5\left(x^2-\frac{8}{5}x\right)+4$$

$$= 5\left(x-\frac{4}{5}\right)^2+4-\frac{16}{5}$$

$$\left(\frac{4}{5}, \frac{4}{5}\right) \quad = 5\left(x-\frac{4}{5}\right)^2+\frac{4}{5} \geq \frac{4}{5}$$

FUNCTIONS

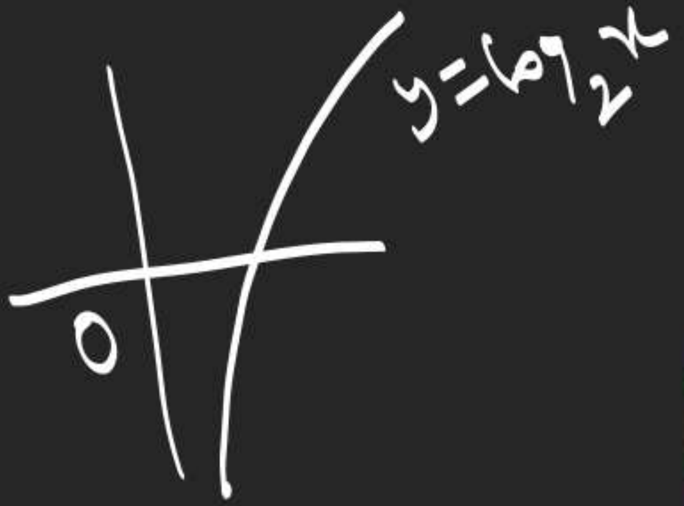
2. $f(x) = \log_2\Big(\underbrace{2 - \underbrace{\log_{\sqrt{2}}(\overbrace{16\sin^2x+1}^{[1,\,17]})}_{\in\,[0,\ \log_{\sqrt{2}}17]}}_{>0}\Big)$

$$\boxed{0 < 2 - \log_{\sqrt{2}}(16\sin^2x+1) \le 2}$$

$$-\infty < f(x) \le 1$$

$$0 \le \log_{\sqrt{2}}(16\sin^2x+1) \le \log_{\sqrt{2}}17$$

$$\boxed{R_f = (-\infty, 1]}$$



$$-\log_{\sqrt{2}}17 \le -\log_{\sqrt{2}}(16\sin^2x+1) \le 0$$

$$\underbrace{2-\log_{\sqrt{2}}17}_{<0} \le 2-\log_{\sqrt{2}}(16\sin^2x+1) \le 2$$

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FUNCTIONS

3. $f(x) = \dfrac{5-4x}{2x+7} = \dfrac{-2(2x+7)+19}{2x+7}$

$y = \dfrac{ax+b}{cx+d}$

$\boxed{R_f = R - \left\{\frac{a}{c}\right\}}$

$f(x) = -2 + \dfrac{19}{2x+7} \neq -2$

$y+2 = \dfrac{19}{2(x+7/2)}$

$\boxed{y = \dfrac{19}{2x}}$

$\boxed{R_f = R - \{-2\}}$

$\dfrac{19}{2x+7} = 5$

$\dfrac{1}{2}\left(\dfrac{19}{5} - 7\right) = x$

FUNCTIONS

4. $f(x) = \dfrac{x^2 - x + 1}{x^2 + x + 1}$

$$\boxed{R_f = \left[\tfrac{1}{3}, 3\right]}$$

$$y = \frac{x^2 - x + 1}{x^2 + x + 1} \Rightarrow (y-1)x^2 + (y+1)x + (y-1) = 0$$

$y - 1 = 0$

$\Rightarrow x^2 + x + 1 = x^2 - x + 1$

$\Rightarrow \boxed{x = 0}$

OR

$\boxed{y - 1 \neq 0}$

$D \geq 0 \Rightarrow (y+1)^2 - 4(y-1)^2 \geq 0$

$(y + 1 - 2y + 2)(y + 1 + 2y - 2) \geq 0$

$(y - 3)(3y - 1) \leq 0$

$\left[\frac{1}{3}, 3\right] - \{1\}$

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$$y = \frac{x^2 - x + 1}{x^2 + x + 1} = 1 - \frac{2x}{x^2 + x + 1}$$

① $D_f = \mathbb{R}$

② $$y' = -2\left(\frac{(x^2+x+1) - x(2x+1)}{(x^2+x+1)^2}\right) = \frac{2(x^2-1)}{(x^2+x+1)^2}$$

$x \in (-\infty, -1) \cup (1, \infty)$ $\uparrow$

$x \in (-1, 1)$ $\downarrow$

$x \to -\infty$, $y = \dfrac{1 - \frac{1}{x} + \frac{1}{x^2}}{1 + \frac{1}{x} + \frac{1}{x^2}}$, $y \to 1$

$x \to \infty$, $y \to 1$

$x = -1$, $y = 3$

$x = 1$, $y = \frac{1}{3}$

$R_f = \left[\frac{1}{3}, 3\right]$

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<u>5.</u> $y = \dfrac{x^2+2x-11}{2(x-3)}$

$$x^2+(2-2y)x+6y-11=0$$

$$D \geq 0$$

$$(y-1)^2-(6y-11) \geq 0$$

$$y^2-8y+12 \geq 0$$

$$R_f = (-\infty, 2] \cup [6, \infty)$$

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$$f(x) = \frac{x^2+2x-11}{2(x-3)} = \frac{(x-3)(x+5)+4}{2(x-3)} = \left(x+5+\frac{4}{x-3}\right)\frac{1}{2}$$

$$y = x\left(\frac{\left(1+\frac{2}{x}-\frac{11}{x^2}\right)}{2\left(1-\frac{3}{x}\right)}\right)$$

① $D_f = R - \{3\}$

$x \to 3.0\ldots$

② $f'(x) = \frac{1}{2}\left(1-\frac{4}{(x-3)^2}\right) = \frac{(x-5)(x-1)}{2(x-3)^2}$

$x \to -\infty, y \to -\infty$

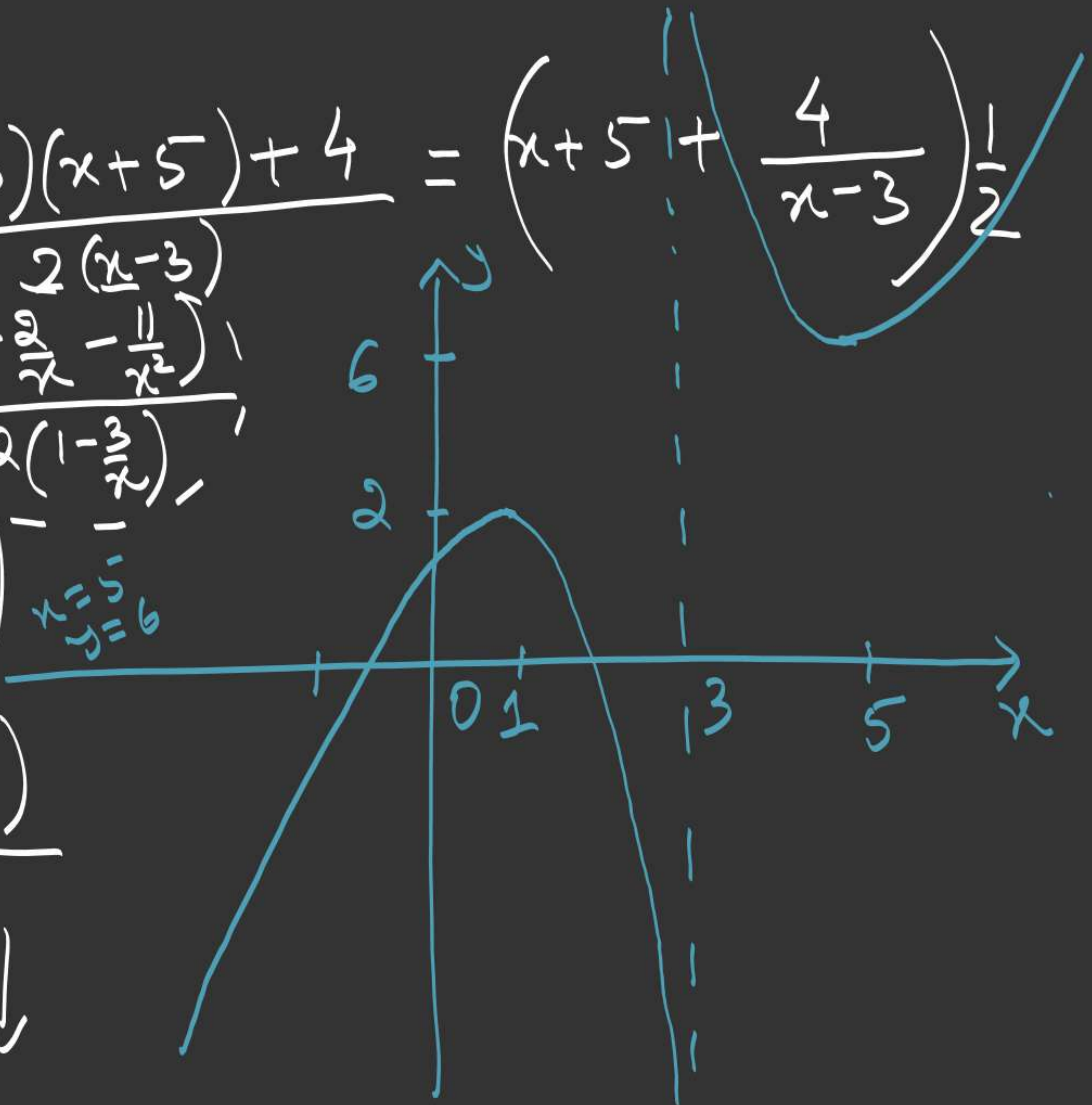
$x \to \infty, y \to \infty$

$x \to 3^-, y \to -\infty$

$2.99\ldots$

$(1,3) \cup (3,5) \downarrow$

$x=5$
$y=6$



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FUNCTIONS

Proficiency Test

1., 2, 3, 4, 5

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Equal or Identical Functions

f & g are identical if

(1) $D_f = D_g$ and

(2) $\forall x \in D_f$, $f(x) = g(x)$

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FUNCTIONS

$f(x) = \text{sgn}(\underbrace{x^2+1}_{>0}) \rightarrow D_f = R$

$= 1$

$g(x) = \sin^2 x + \cos^2 x \quad D_g = R$

$= 1$

$f(x)$ & $g(x)$ are identical

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FUNCTIONS

$$f(x) = \frac{x}{1+x}$$

$$D_f = R - \{-1\}.$$

not identical

$$g(x) = \frac{1}{1+\frac{1}{x}}$$

$$D_g = R - \{0, -1\}$$

$$\frac{x}{1+x}$$