EXAMPLES(XVII)

SL LONEY

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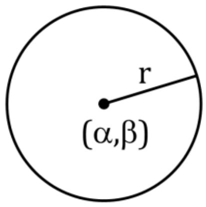
Find the equation to the circle:

1. Whose radius is 3 and whose centre is $(-1,2)$.

Ans. $x^2 + y^2 + 2x - 4y = 4$

Sol. Standard Equation $\Rightarrow (x-\alpha)^2 + (y-\beta)^2 = r^2$



$[x-(-1)]^2 + (y-2)^2 = 3^2$

$(x+1)^2 + (y-2)^2 = 9$

$x^2 + y^2 + 2x - 4y - 4 = 0$

2. Whose radius is 10 and whose centre is $(-5,-6)$.

Ans. $x^2 + y^2 + 10x + 12y = 39$

Sol. $(x+5)^2 + (y+6)^2 = 10^2$
$x^2 + y^2 + 10x + 12y - 39 = 0$

3. Whose radius is $a+b$ and whose centre is $(a,-b)$.

Ans. $x^2 + y^2 - 2ax + 2by = 2ab$

Sol. $(x-a)^2 + (y+b)^2 = (a+b)^2$

$x^2 - 2ax + a^2 + y^2 + 2by + b^2 = a^2 + 2ab + b^2$
$x^2 + y^2 - 2ax + 2by - 2ab = 0$

4. Whose radius is $\sqrt{a^2 - b^2}$ and whose centre is $(-a,-b)$.

Ans. $x^2 + y^2 + 2ax + 2by + 2b^2 = 0$

Sol. $(x+a)^2 + (y+b)^2 = a^2 - b^2$

$x^2 + 2ax + a^2 + y^2 + 2by + b^2 = a^2 - b^2$
$x^2 + y^2 + 2ax + 2by + 2b^2 = 0$

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Find the coordinates of the centres and the radii of the circles whose equations are:

5. $x^2 + y^2 - 4x - 8y = 41$

Ans. $(2,4)\ ;\sqrt{61}$

Sol. $x^2 + y^2 - 4x - 8y - 41 = 0$

$$\text{Centre } \equiv \left(-\frac{(-4)}{2}, -\frac{(-8)}{2}\right) = (2,4)$$

$$r = \sqrt{4 + 16 - (-41)} = \sqrt{61}$$

6. $3x^2 + 3y^2 - 5x - 6y + 4 = 0$

Ans. $\left(\frac{5}{6}, 1\right) : \frac{1}{6}\sqrt{13}$

Sol. $x^2 + y^2 - \frac{5}{3}x - 2y + \frac{4}{3} = 0$

$$\text{Centre } \equiv \left(\frac{5}{6}, 1\right)$$

$$r = \sqrt{\frac{25}{36} + 1 - \frac{4}{3}} = \sqrt{\frac{25}{36} - \frac{1}{3}} = \sqrt{\frac{13}{36}} = \frac{1}{6}\sqrt{13}$$

7. $x^2 + y^2 = k(x + k)$

Ans. $\left(\frac{k}{2}, 0\right) : \frac{\sqrt{5}}{2}k$

Sol. $x^2 + y^2 - kx - k^2 = 0$

$$\text{Centre } \equiv \left(\frac{k}{2}, 0\right)$$

$$r = \sqrt{\frac{k^2}{4} + k^2} = \sqrt{\frac{5k^2}{4}} = \frac{\sqrt{5}k}{2}$$

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8. $x^2 + y^2 = 2gx - 2fy$

Ans. $(g, -f); \sqrt{f^2 + g^2}$

Sol. $x^2 + y^2 - 2gx + 2fy = 0$

Centre $\equiv (g, -f)$
$r = \sqrt{g^2 + f^2}$

9. $\sqrt{1+m^2}(x^2 + y^2) - 2cx - 2mcy = 0$

Ans. $\left(\frac{c}{\sqrt{1+m^2}}, \frac{mc}{\sqrt{1+m^2}}\right); c$

Sol. $\div \sqrt{1+m^2}$

$$x^2 + y^2 - \frac{2c}{\sqrt{1+m^2}}x - \frac{2mc}{\sqrt{1+m^2}}y = 0$$

$$\text{Centre} \equiv \left(\frac{c}{\sqrt{1+m^2}}, \frac{mc}{\sqrt{1+m^2}}\right)$$

$$r = \sqrt{\frac{c^2}{1+m^2} + \frac{m^2c^2}{1+m^2}}$$

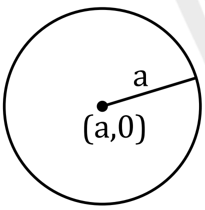
$$r = \sqrt{c^2\left(\frac{1+m^2}{1+m^2}\right)} = c$$

Draw the circles whose equations are:

10. $x^2 + y^2 = 2ay$

Sol. $x^2 + y^2 - 2ay = 0$

Centre $\equiv (a, 0)$
$r = a$



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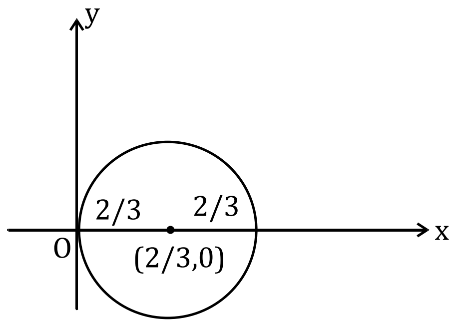
11. $3x^2 + 3y^2 = 4x$

Sol. $(\div 3)$



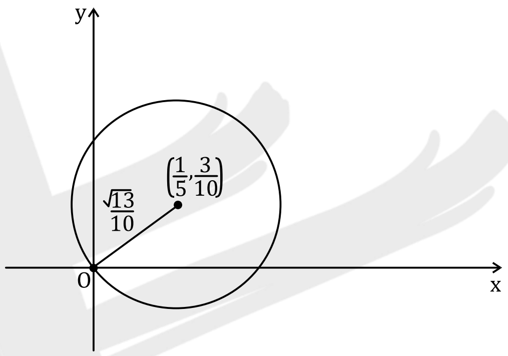
$$x^2 + y^2 - \frac{4x}{3} = 0$$

$$\text{Centre } = \left(\frac{2}{3}, 0\right)$$

$$r = \frac{2}{3}$$

12. $5x^2 + 5y^2 = 2x + 3y$

Sol. $(\div 5)$



$$x^2 + y^2 - \frac{2x}{5} - \frac{3y}{5} = 0$$

$$\text{Centre } = \left(\frac{1}{5}, \frac{3}{10}\right)$$

$$r = \sqrt{\frac{1}{25} + \frac{9}{100}} = \frac{\sqrt{13}}{10}$$

13. Find the equation to the circle which passes through the points $(1, -2)$ and $(4, -3)$ and which has its centre on the straight line $3x + 4y = 7$.

Ans. $15x^2 + 15y^2 - 94x + 18y + 55 = 0$

Sol. let Equation of Circle be $x^2 + y^2 + 2gx + 2fy + c = 0$ with centre $\equiv (-g, -f)$

$\because$ Circle passes through $(1, -2)$

$\therefore 1 + 4 + 2g - 4f + c = 0$

$2g - 4f + c = -5 \quad \cdots\cdots\cdots(1)$

$\because$ Circle passes through $(4, -3)$

$16 + 9 + 8g - 6f + c = 0$

$8g - 6f + c = -25 \quad \cdots\cdots\cdots(2)$

$\because$ centre of circle lies on $3x + 4y = 7$

$\therefore -3g - 4f = 7$

$3g + 4f = -7 \quad \cdots\cdots\cdots(3)$

From (3) $-$(2)

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$8g - 6f + c = -25$

$2g - 4f + c = -5$

$-\quad+\quad-\quad+$

$6g - 2f = -20$

or $12g - 4f = -40 \quad \cdots\cdots\cdots(4)$

Now ,from (3) & (4)

$15g = -47$

$g = -\frac{47}{15}$

$f = \frac{3}{5}$

$C = \frac{11}{3}$

Equation of Circle $\Rightarrow x^2 + y^2 - \frac{94x}{15} + \frac{6y}{5} + \frac{11}{3} = 0$ or $15x^2 + 15y^2 - 94x + 18y + 55 = 0$

14. Find the equation to the circle passing through the points $(0, a)$ and (b, h), and having its centre on the axis of x.

Ans. $b(x^2 + y^2 - a^2) = x(b^2 + h^2 - a^2)$

Sol. let Equation of Circle is $x^2 + y^2 + 2gx + c = 0$............(1)

Circle (1) passes through $(0, a) \Rightarrow a^2 + c = 0 \Rightarrow c = -a^2$ {Put in (1)}

Circle (1) passes through $(b, h) \Rightarrow b^2 + h^2 + 2gb - a^2 = 0$

$2g = \frac{(a^2-b^2-h^2)}{b}$ {Put in (2)}

$x^2 + y^2 + \frac{(a^2-b^2-h^2)x}{b} - a^2 = 0$

$b(x^2 + y^2) + (a^2 - b^2 - h^2)x - a^2b = 0$

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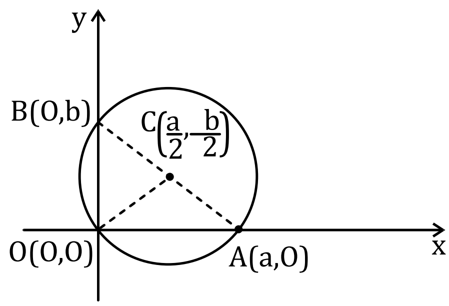
Find the equations to the circles which pass through the points;

15. $(0,0), (a,0)$ and $(0,b)$

Ans. $x^2 + y^2 - ax - by = 0$

Sol. $r = \sqrt{\frac{a^2}{4} + \frac{b^2}{4}} = \frac{\sqrt{a^2+b^2}}{2}$



Equation of Circle $\Rightarrow \left(x - \frac{a}{2}\right)^2 + \left(y - \frac{b}{2}\right)^2 = \frac{a^2 + b^2}{4}$

$x^2 - ax + \frac{a^2}{4} + y^2 - by + \frac{b^2}{4} = \frac{a^2}{4} + \frac{b^2}{4}$

$x^2 + y^2 - ax - by = 0$

16. $(1,2), (3,-4)$ and $(5,-6)$

Ans. $x^2 + y^2 - 22x - 4y + 25 = 0$

Sol. Let Equation of Circle be $x^2 + y^2 + 2gx + 2fy + c = 0$...............(1)

Circle (1)passes through $(1,2) \Rightarrow 1 + 4 + 2g + 4f + c = 0$
$2g + 4f + c = -5.$ ....(2)

Circle (1)passes through $(3,-4) \Rightarrow g + 16 + 6g - 8f + c = 0$
$6g - 8f + c = -25$ ....(3)

Circle (1)passes through $(5,-6) \Rightarrow 25 + 36 + log - 12f + c = 0$
$log - 12f + c = -61$ ....(4)

Solving (2),(3),(4) we get $g = -11, f = -2, c = 25$ put all these values in (1)

$\therefore x^2 + y^2 - 22x - 4y + 25 = 0$

17. $(1,1), (2,-1)$ and $(3,2)$

Ans. $x^2 + y^2 - 5x - y + 4 = 0$

Sol. Let Equation of Circle be $x^2 + y^2 + 2gx + 2fy + c = 0$...............(1)

Circle (1)passes through $(1,1) \Rightarrow 1 + 1 + 2g + 2f + c = 0$
$2g + 2f + c = -2$ ....(2)

Circle (1)passes through $(2,-1) \Rightarrow 4 + 1 + 4g - 2f + c = 0$
$4g - 2f + C = -5$ ....(3)

Circle (1)passes through $(3,2) \Rightarrow 9 + 4 + 6g + 4f + c = 0$
$6g + 4f + c = -13$ ....(4)

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Solving (2),(3),(4) we get $g = -\frac{5}{2}, f = -\frac{1}{2}, c = 4$

$\therefore x^2 + y^2 - 5x - y + 4 = 0$

18. (5,7), (8,1) and (1,3)

Ans. $3x^2 + 3y^2 - 29x - 19y + 56 = 0$

Sol. Let Equation of Circle be $x^2 + y^2 + 2gx + 2fy + c = 0 \dots\dots\dots\dots\dots (1)$

Circle (1)passes through (5,7) $\Rightarrow$ $10g + 14f + c = -74$..............(2)

Circle (1)passes through (8,1) $\Rightarrow$ $16g + 2f + c = -65$................(3)

Circle (1)passes through (1,3) $\Rightarrow$ $2g + 6f + c = -10$..................(4)

Solving (2),(3),(4) we get $g = \frac{-29}{6}, f = \frac{-19}{6}, c = \frac{56}{3}$

$\therefore 3x^2 + 3y^2 - 29x - 19y + 56 = 0$

19. $(a, b), (a, -b)$ and $(a + b, a - b)$

Ans. $b(x^2 + y^2) - (a^2 + b^2)x + (a - b)(a^2 + b^2) = 0$

Sol. Let Equation of Circle be $x^2 + y^2 + 2gx + 2fy + c = 0 \dots\dots\dots\dots\dots (1)$

Circle (1)passes through $(a, b) \Rightarrow 2ag + 2af + c = -(a^2 + b^2)$ ....(2)

Circle (1)passes through $(a - b) \Rightarrow 2ag - 2af + c = -(a^2 + b^2)$ ....(3)

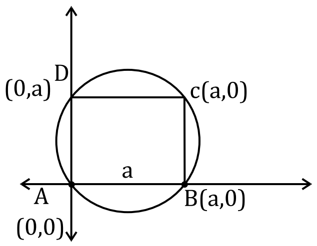
Circle (1)passes through $(a + b, a - b) \Rightarrow 2(a + b)g + 2(a - b) + +c = -2(a^2 + b^2)$ ....(4)

Solving (2), (3), (4)we get $g = -\frac{(a^2 + b^2)}{2b}, f = 0,$ $c = (a - b)\frac{(a^2 + b^2)}{b}$

$\therefore b(x^2 + y^2) - (a^2 + b^2)x + (a - b)(a^2 + b^2) = 0$

20. ABCD is a square whose side is a; taking AB and AD as axes, prove that the equation to the circle circumscribing the square is, $x^2 + y^2 = a(x + y)$.

Sol. $\therefore$ Equation of Circle $x^2 + y^2 - ax - ay = 0$



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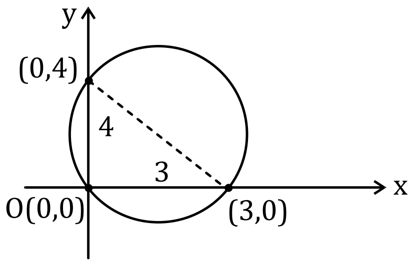
21. Find the equation to the circle which passes through the origin and cuts off intercepts equal to 3 and 4 from the axes.



Ans. $x^2 + y^2 - 3x - 4y = 0$

Sol. $x^2 + y^2 - 3x - 4y = 0$

22. Find the equation to the circle passing through the origin and the points (a, b) and (b, a). Find the lengths of the chords that it cuts off from the axes.

Ans. $x^2 + y^2 - \frac{a^2+b^2}{a+b}(x + y) = 0; \frac{a^2+b^2}{a+b}$

Sol. Let Equation of Circle be $x^2 + y^2 + 2gx + 2fy + c = 0 \ldots \ldots \ldots \ldots \ldots (1)$

Circle (1)passes through (a,b) $\Rightarrow$ $a^2 + b^2 + 2ag + 2bf = 0$

$ag + bf = -\left(\frac{a^2+b^2}{2}\right)$...........(2)

Circle (1)passes through (b,a) $\Rightarrow a^2 + b^2 + 2bg + 2af = 0$

$bg + af = -\left(\frac{a^2+b^2}{2}\right)$ ....(3)

Solving (2) & (3) we get ,

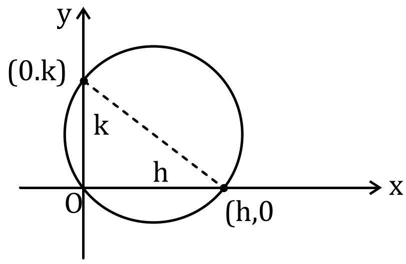
$g = \frac{-(a^2+b^2)}{2(a+b)}$

$f = \frac{-(a^2+b^2)}{2(a+b)}$ Put in (1)

$\therefore x^2 + y^2 - \left(\frac{a^2+b^2}{a+b}\right)(x + y) = 0$

x/y Intercepts $= \frac{a^2+b^2}{a+b}$

23. Find the equation to the circle which goes through the origin and cuts off intercepts equal to h and k from the positive parts of the axes.



Ans. $x^2 + y^2 - hx - ky = 0$

Sol. $x^2 + y^2 - hx - ky = 0$

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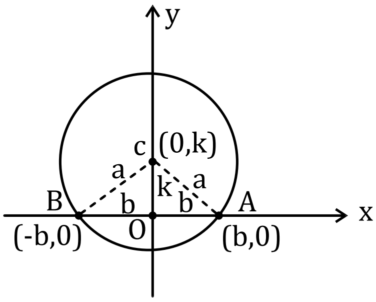
24. Find the equation to the circle, of radius a, which passes through the two points on the axis of x which are at a distance b from the origin.

Ans. $x^2 + y^2 \pm 2y\sqrt{a^2 - b^2} = b^2$

Sol. $AC = a$



$\sqrt{b^2 + k^2} = a$

$b^2 + k^2 = a^2$

$k^2 = a^2 - b^2$

$k = \pm\sqrt{a^2 - b^2}$

Centre $= \left(0, \pm\sqrt{a^2 - b^2}\right)$

r $= a$

Equation of $(x - 0)^2 + \left(y \pm \sqrt{a^2 - b^2}\right)^2 = a^2$

$x^2 + y^2 \pm 2\sqrt{a^2 - b^2}y + a^2 - b^2 = a^2$

$x^2 + y^2 \pm 2y\sqrt{a^2 - b^2} - b^2 = 0$

Find the equation to the circle which:

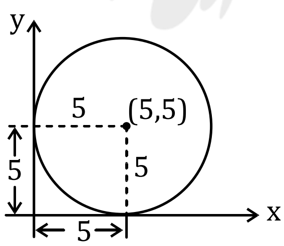
25. touches each axis at a distance 5 from the origin.

Ans. $x^2 + y^2 - 10x - 10y + 25 = 0$

Sol. Centre $\equiv (5,5)$, $r = 5$

$Eq^n \Rightarrow (x - 5)^2 + (y - 5)^2 = 5^2$

$x^2 + y^2 - 10x - 10y + 25 = 0$



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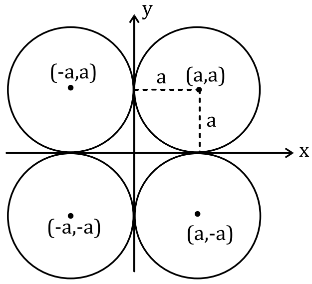
26. touches each axis and is of radius a.

Ans. $x^2 + y^2 \pm 2ax \pm 2ay + a^2 = 0$

Sol. $Eq^n \Rightarrow (x \pm a)^2 + (y \pm a)^2 = a^2$

$x^2 + y^2 \pm 2ax \pm 2ay + a^2 = 0$



27. touches both axes and passes through the point $(-2, -3)$.

Ans. $x^2 + y^2 + 2(5 \pm \sqrt{12})(x + y) + 37 \pm 10\sqrt{12} = 0$

Sol. $Eq^n \Rightarrow (x + a)^2 + (y + a)^2 = a^2$.............(1)

Passes through $(-2, -3)$

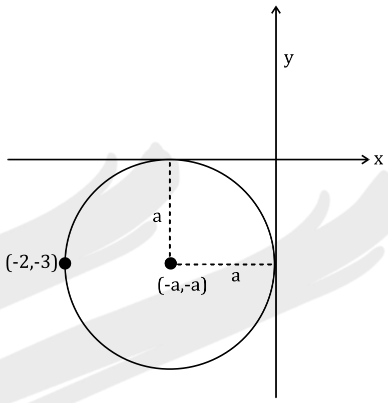
$(-2 + a)^2 + (-3 + a)^2 = a^2$

$a^2 - 4a + 4 + y^2 - 6a + 9 = a^2$

$a^2 - 10a + 13 = 0$

$a = \frac{10 \pm \sqrt{100 - 52}}{2}$

$a = \frac{10 \pm 2\sqrt{12}}{2} = 5 \pm \sqrt{12}$ Put in (1)



28. touches the axis of x and passes through the two points $(1, -2)$ and $(3, -4)$

Ans. $x^2 + y^2 - 6x + 4y + 9 = 0$, or $x^2 + y^2 + 10x + 20y + 25 = 0$

Sol. Let Equation of Circle be $(x - h)^2 + (y - k)^2 = k^2$...........(1)

Passes through $(1, -2) \Rightarrow (1 - h)^2 + (-2 - k)^2 = k^2$

$h^2 - 2h + 1 + h^2 + 4k + 4 = h^2$

$h^2 - 2h + 4k + 5 = 0$....................(2)

Passes through $(3, -4) \Rightarrow (3 - h)^2 + (-4 - r)^2 = h^2$

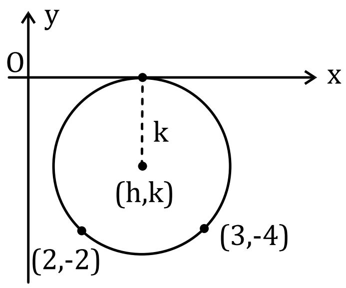
$h^2 - 6h + 8k + 25 = 0$ ....(3)

$(3) - (2)$

$-4h + 4k + 20 = 0$

$-h + k + 5 = 0$

$k = h - 5$ put in (2)



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$h^2 - 2h + 4(h-5) + 5 = 0$

$h^2 + 2h - 15 = 0$

$(h+5)(h-3) = 0$

When $h = -5 \Rightarrow k = -10$

Centre $(-5, -10)$ & $r = 10$

Equation of circle is $(x+5)^2 + (y+10)^2 = 100$

or $x^2 + y^2 + 10y + 20y + 25 = 0$

& **When $h = 3 \Rightarrow k = -2$**

Centre $(3, -2)$ & $r = 2$

Equation of Circle is $(x-3)^2 + (y+2)^2 = 4$

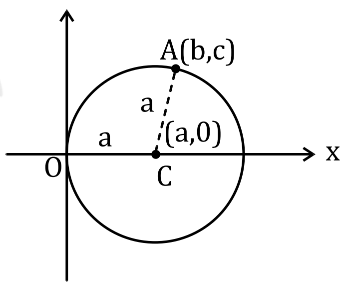
or $x^2 + y^2 - 6x + 4y + 9 = 0$

29. touches the axis of y at the origin and passes through the point (b, c).

Ans. $b(x^2 + y^2) = x(b^2 + c^2)$

Sol. Let Equation of Circle be $(x-a)^2 + y^2 = a^2$..............(1)
$x^2 + y^2 - 2ax = 0$

$AC = a$



$\sqrt{(b-a)^2 + c^2} = a$

$b^2 + a^2 - 2ab + c^2 = a^2$

$b^2 + c^2 = 2ab \Rightarrow 2a = \frac{b^2+c^2}{b}$ put in (1)

$\therefore x^2 + y^2 - \left(\frac{b^2+c^2}{b}\right)x = 0 (xb)$

$b(x^2 + y^2) = (b^2 + c^2)x$

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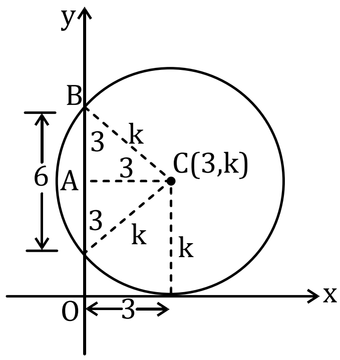
30. touches the axis of x at a distance 3 from the origin and intercepts a distance 6 on the axis of y.

Ans. $x^2+y^2\pm 6\sqrt{2}y-6x+9=0$

Sol. In $\triangle ABC$



$BC^2 = AB^2+AC^2$

$k^2 = 3^2+3^2$

$k = \pm 3\sqrt{2} = r$

$\therefore$ centre $= (3,\pm 3\sqrt{2})$

$r = k = 3\sqrt{2}$

$\therefore$ Equation of Circle $\Rightarrow (x-3)^2+(y\pm 3\sqrt{2})^2=(3\sqrt{2})^2$

$x^2+y^2-6x\pm 6\sqrt{2}y+9=0$

31. points (1,0) and (2,0) are taken on the axis of x, the axes being rectangular. On the line joining these points an equilateral triangle is described, its vertex being in the positive quadrant. Find the equations to the circles described on its sides as diameters.

Ans. $x^2+y^2-3x+2=0;\ 2x^2+2y^2-5x-\sqrt{3}y+3=0$

$2x^2+2y^2-7x-\sqrt{3}y+6=0$

Sol. $AC=1$

$(h-1)^2+k^2=1$..................(1)

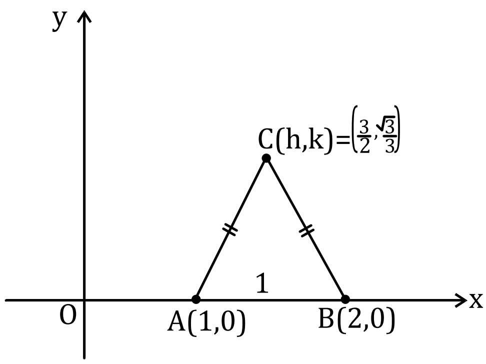
$BC=1$

$(h-2)^2+k^2=1$..................(2)

$(2)-(1)\Rightarrow (h-2)^2-(h-1)^2=0 \Rightarrow (h-2+h-1)(h-2-h+1)=0$

$h=3/2 \Rightarrow k=\sqrt{3}/2 \Rightarrow C\left(\dfrac{3}{2},\dfrac{\sqrt{3}}{2}\right)$

Equation of circle whose diameter, is



$AB \Rightarrow (x-1)(x-2)+(y-0)(y-0)=0$

$BC \Rightarrow (x-2)(x-3/2)+(y-0)\left(y-\dfrac{\sqrt{3}}{2}\right)=0$

$AC \Rightarrow (x-1)(x-3/2)+(y-0)\left(y-\frac{\sqrt{3}}{2}\right)=0$

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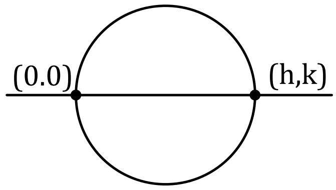
32. If $y = mx$ be the equation of a chord of a circle whose radius is ^{4} the origin of coordinates being one extremity of the chord and the axis of ? being a diameter of the circle, prove that the equation of a circle of which this chord is the diameter is,

$(1 + m^2)(x^2 + y^2) - 2a(x + my) = 0.$

Sol. First circle

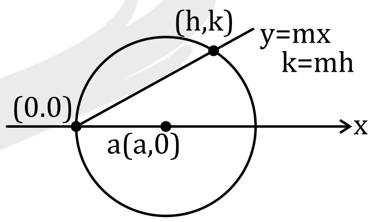


$r = a$
Chord $\Rightarrow y = mx$
Diameter $\Rightarrow$ x-axis
$\therefore$ Equation of of First circle $(x - a)^2 + y^2 = a^2$

Passes through (h, k) $\therefore (h - a)^2 + h^2 = a^2$ or $h^2 - 2ah + a^2 + m^2h^2 = a^2$

$h - 2a + m^2h = 0 \Rightarrow h = \frac{2a}{1+m^2} \Rightarrow k = \frac{2am}{1+m^2} (\because y = mh)$

Equation of other circle having this chord as diameter



$(x - 0)(x - h) + (y - 0)(y - k)$

$x^2 + y^2 - hx - ky = 0$

$x^2 + y^2 - \frac{2ax}{1+m^2} - \frac{2amy}{1+m^2} = 0$

33. Find the equation to the circle passing through the points (12,43), (18,39), and (42,3) and prove that it also passes through the points $(-54, -69)$ and $(-81, -38)$.

Ans. $(x + 21)^2 + (y + 13)^2 = 65^2$

Sol. let equation of circle be $x^2 + y^2 + 2gx + 2fy + c = 0$..........(1)

Circle (1) passes through (12,43) $\Rightarrow 12^2 + 43^2 + 24g + 86f + c = 0$ … ....(2)

Circle (1) passes through (18,39) $\Rightarrow 18^2 + 3g^2 + 36g + 78f + c = 0$ ....(3)

Circle (1) passes through (42,3) $\Rightarrow 42^2 + 3^2 + 84g + 6f + c = 0$ … … .. (4)

Solve (2),(3) ,(4) and get values of g , f , c & put them in (1)

$\therefore (x + 21)^2 + (y + 13)^2 = 65^2$

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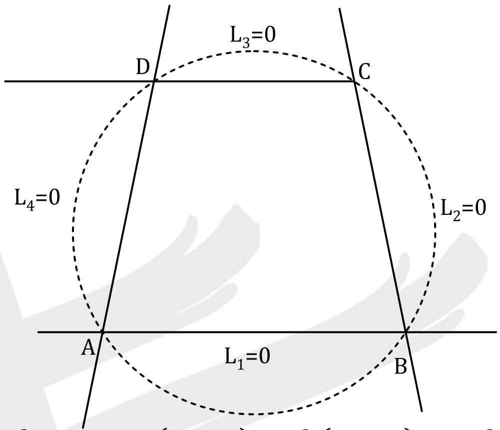
34. Find the equation to the circle circumscribing the quadrilateral formed by the straight lines, $2x+3y=2, 3x-2y=4, x+2y=3$ and $2x-y=3$

Ans. $8x^2+8y^2-25x-3y+18=0$

Sol. Equation of circle circumscribing the quadrilateral having equation of sides as $L_1=0$, $L_2=0, L_3=0, L_4=0$ is $L_1L_3+\mu L_2L_4=0$

$\therefore\ (2x+3y-2)(x+2y-3)+\mu(3x-2y-4)(2x-y-3)=0$ ....(1)

Now, as this representer a circle



$\therefore$ coefficient of x^2 = coefficient of y^2

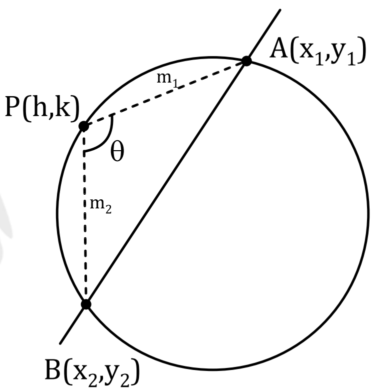
$2+6\mu=6+2\mu \Rightarrow \mu=1$ Put in (1)

$8x^2+8y^2-25x-3y+18=0$

35. Prove that the equation to the circle of which the points (x_1, y_1) and (x_2, y_2) are the ends of a chord of a segment containing an angle θ is,

$(x-x_1)(x-x_2)+(y-y_1)(y-y_2)\pm\cot\theta[(x-x_1)(y-y_2)-(x-x_2)(y-y_1)]=0.$

Sol. $m_{AP}=m_1=\frac{k-y_1}{h-x_1}$



$\& \ m_{BP}=m_2=\dfrac{k-y_2}{h-x_2}$

$\therefore \tan\theta=\left|\dfrac{m_1-m_2}{1+m_1m_2}\right|$

$$\tan\theta=\pm\left(\frac{\frac{k-y_1}{h-x_1}-\frac{k-y_2}{h-x_2}}{1+\left(\frac{k-y_1}{h-x_1}\right)\left(\frac{k-y_2}{h-x_2}\right)}\right)$$

$$\frac{1}{\cot\theta}=\pm\frac{(k-y_1)(h-x_2)-(k-y_2)(h-x_1)/(h-x_1)(h-x_2)}{(h-x_1)(h-x_2)+(k-y_1)(k-y_2)/(h-x_1)(h-x_2)}$$

Replace $h\to x$ & $k\to y$

$(x-x_1)(x-x_2)+(y-y_1)(y-y_2)\pm\cot\theta\{(y-y_1)(x-x_2)-(y-y_2)(x-x_1)\}$

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36. Find the equations to the circles in which the line joining the points (a, b) and $(b, -a)$ is a chord subtending an angle of $45°$ at any point on its circumference.

Ans. $x^2 + y^2 = a^2 + b^2; x^2 + y^2 - 2(a + b)x + 2(a - b)y + a^2 + b^2 = 0$

Sol. Put $\theta = 45°$ in expression proved in previous problem

Equation $\Rightarrow$ $(x - a)(x - b) + (y - b)(y + a) \pm \cot 45°\{(x - a)(y + b) - (x - b)(y - b)\}$

Taking + Sign

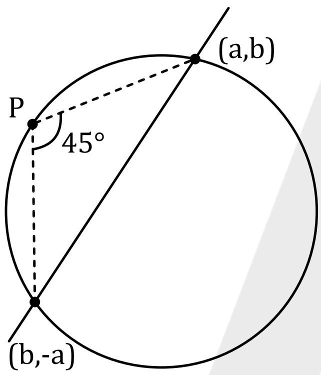
$x^2 - (a + b)x + ab + y^2 + (a - b)y - ab + xy + ax - ay - a^2 - xy + bx + by - b^2 = 0$

$\therefore x^2 + y^2 = a^2 + b^2$

Taking − Sign

$x^2 - (a + b)x + ab + y^2 + (a - b)y - ab - xy - ax + ay + a^2 + xy - bx - by + b^2$

$\therefore$ $x^2 + y^2 - 2(a + b)x + 2(a - b)y + a^2 + b^2 = 0$



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ANSWER KEY

1. $x^2+y^2+2x-4y=4$

2. $x^2+y^2+10x+12y=39$

3. $x^2+y^2-2ax+2by=2ab$

4. $x^2+y^2+2ax+2by+2b^2=0$

5. $(2,4);\sqrt{61}$

6. $\left(\frac{5}{6},1\right):\frac{1}{6}\sqrt{13}$

7. $\left(\frac{k}{2},0\right):\frac{\sqrt{5}}{2}k$

8. $(g,-f);\sqrt{f^2+g^2}$

9. $\left(\frac{c}{\sqrt{1+m^2}},\frac{mc}{\sqrt{1+m^2}}\right);c$

13. $15x^2+15y^2-94x+18y+55=0$

14. $b(x^2+y^2-a^2)=x(b^2+h^2-a^2)$

15. $x^2+y^2-ax-by=0$

16. $x^2+y^2-22x-4y+25=0$

17. $x^2+y^2-5x-y+4=0$

18. $3x^2+3y^2-29x-19y+56=0$

19. $b(x^2+y^2)-(a^2+b^2)x+(a-b)(a^2+b^2)=0$

21. $x^2+y^2-3x-4y=0$

22. $x^2+y^2-\frac{a^2+b^2}{a+b}(x+y)=0;\frac{a^2+b^2}{a+b}$

23. $x^2+y^2-hx-ky=0$

24. $x^2+y^2\pm 2y\sqrt{a^2-b^2}=b^2$

25. $x^2+y^2-10x-10y+25=0$

26. $x^2+y^2\pm 2ax\pm 2ay+a^2=0$

27. $x^2+y^2+2(5\pm\sqrt{12})(x+y)+37\pm 10\sqrt{12}=0$

28. $x^2+y^2-6x+4y+9=0$, or $x^2+y^2+10x+20y+25=0$

29. $b(x^2+y^2)=x(b^2+c^2)$

30. $x^2+y^2\pm 6\sqrt{2}y-6x+9=0$

31. $x^2+y^2-3x+2=0;2x^2+2y^2-5x-\sqrt{3}y+3=0$

$2x^2+2y^2-7x-\sqrt{3}y+6=0$

33. $(x+21)^2+(y+13)^2=65^2$

34. $8x^2+8y^2-25x-3y+18=0$

36. $x^2+y^2=a^2+b^2;x^2+y^2-2(a+b)x+2(a-b)y+a^2+b^2=0$