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11.

$$LHL = \lim_{x\to 0^-}\left(\frac{1-a^x+a^x\ln(1+a^x-1)}{a^x\, x^2}\right)$$

$a^x - 1 = t$

$a^x = 1+t$

$$\lim_{t\to 0} \ln^2 a\left(\frac{(1+t)\ln(1+t) - t}{t^2(1+t)\,\frac{\ln^2(1+t)}{t^2}}\right)$$

$$\frac{1}{2}\ln^2 a = \lim_{t\to 0}\ln^2 a\left(\frac{\ln(1+t)}{t} + \frac{\ln(1+t)-t}{t^2}\right)\frac{1}{(1+t)\,\frac{\ln^2(1+t)}{t^2}}$$

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$$\frac{\cos^{-1}(1-\{x\}^2)}{\{x\}}\cdot\frac{\sin^{-1}(1-\{x\})}{\sqrt{2}\,(1-\{x\}^2)}$$

Note → We discuss cont. of $f(x)$ in domain only

$\cos\theta = 1-\{x\}^2$

$\{x\} = \sqrt{1-\cos\theta}$

$$\lim_{\theta\to 0^+}\frac{\theta = |\theta|}{\sqrt{1-\cos\theta}}\cdot\frac{\sin^{-1}(1}{\sqrt{2}\,(\quad)}$$

$$= \sqrt{2}\,\frac{\pi}{2\sqrt{2}}$$

$$\sqrt{\frac{1-\cos\theta}{\theta^2}}$$

$$\lim_{x\to 0^+}\left|g(\underbrace{f(x)}_{=0})\right| = \left|0-\frac{\pi}{2}\right| = \frac{\pi}{2}\quad h(0)=\frac{\pi}{2}$$

$[0,$

$$h(0^-) = \lim_{x\to 0^-} h(x)$$

$$g(f(x))$$

$$\frac{[x]\{x\}}{1+[x]}$$

$(0-h,0)$

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$$f(a^+) = \lim_{x \to a^+} \sin\left(\frac{a-x}{2}\right) \tan\underbrace{\left[\frac{\pi x}{2a}\right]}_{1} = 0$$

$$0 \tan 1$$

$$\lim_{x \to 0^+} \frac{\sin x}{x} = 1 \quad 1^-$$

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Let f is differentiable at $x = a$.

$$\lim_{x \to a^+} f'(x) \neq \text{RHD}$$

$$\lim_{x \to a^+} f'(x) = \text{RHD}$$

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1. $f(x) = \begin{cases} \dfrac{\sin(x^2)}{x} & x \neq 0 \\ 0 & x = 0 \end{cases}$. Find eqn. of tangent and normal at $x = 0$, if exist.

$$f'(0) = \lim_{x\to 0} \frac{\frac{\sin x^2}{x} - 0}{x - 0} = 1$$

$(0,0)$

$y = x \rightarrow$ Tangent

$y = -x \rightarrow$ Normal

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2. Discuss diff. of $f(x) = \begin{cases} |1-4x^2| & 0\le x<1 \\ [x^2-2x] & 1\le x\le 2 \end{cases}$ in $[0,2]$

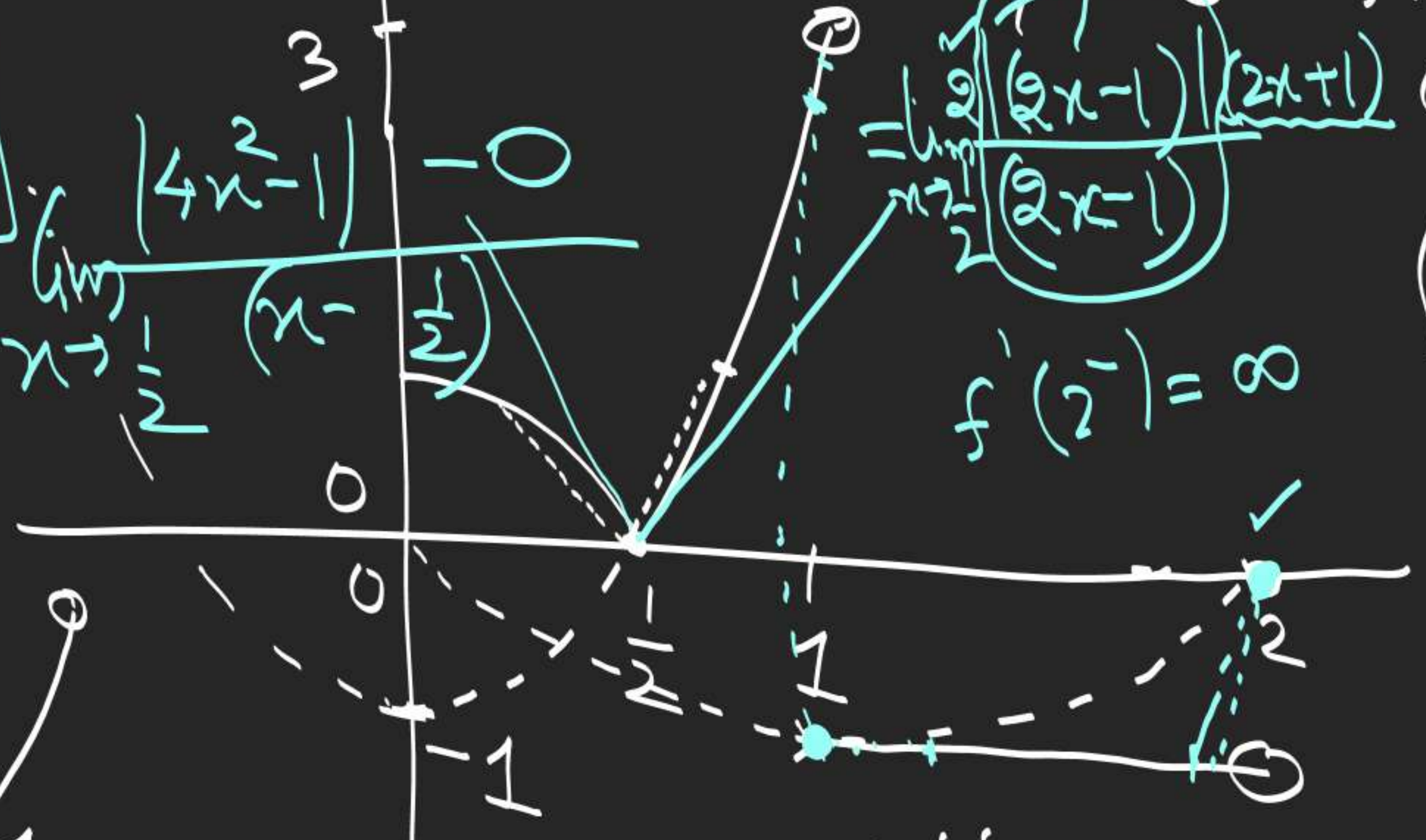
$[\cdot] =$ G.I.F.

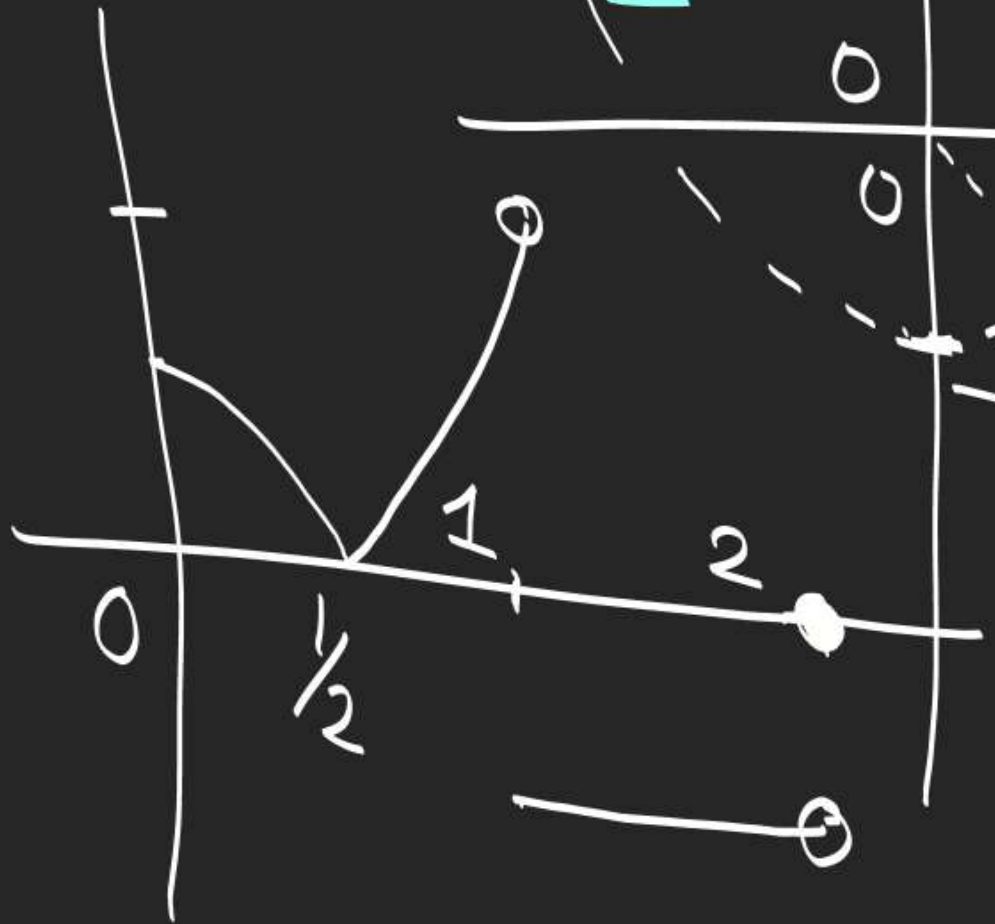
$\boxed{x=\frac{1}{2}}$

$\lim_{x\to\frac{1}{2}} \frac{|4x^2-1| - 0}{\left(x-\frac{1}{2}\right)}$

$= \lim_{x\to\frac{1}{2}} \frac{2|(2x-1)||(2x+1)|}{(2x-1)}$

LHD $= -4$, RHD $= 4$





$f'(2^-) = \infty$

$f'(1^-) = -\infty$

$f'(1^+) = 0$

Non diff.

$x = \frac{1}{2}, 1, 2$

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3. If $f(x) = \begin{cases} ax+b & x \le -1 \\ ax^3+x+2b & x > -1 \end{cases}$ is differentiable $\forall x \in \mathbb{R}$, find a, b.

Diff. at $x=-1$

$3a+1=a \Rightarrow \boxed{a=-\frac{1}{2}}$

$$f'(-1^-) = \lim_{h\to 0} \frac{a(-1-h)+b-(-a+b)}{-h} = a$$

$$f'(-1^+) = \lim_{h\to 0} \frac{\left(a(-1+h)^3+(-1+h)+2b\right)-(-a+b)}{h}$$

$$\text{LHD} = \lim_{h\to 0} \frac{f(-1-h)-f(-1)}{-h}$$

$$\text{RHD} = \lim_{h\to 0} \frac{f(-1+h)-f(-1)}{h} = \lim_{h\to 0} \frac{ah^3-3ah^2+(3a+1)h+(b-1)}{h}$$

$\boxed{b-1=0}$ $= 3a+1$

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$$f(x) = \begin{cases} ax+b & x \le -1 \\ ax^3+x+2b & x > -1 \end{cases}$$

Cont at $x=-1$

$$-\cancel{a}+b = -\cancel{a}-1+2b$$

$$\boxed{b=1}$$

Cont. of $f'(x)$

$$a = 3a(-1)^2+1$$

$$a = -\frac{1}{2}$$

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4. If $f(x) = \begin{cases} x^m \sin\frac{1}{x} & , x > 0 \\ 0 & , x = 0 \end{cases}$ is <u>continuous</u> but not diff. at $x=0$, find m.

$$\boxed{m \in (0, 1]}$$

$$\lim_{x \to 0^+} x^m \sin\frac{1}{x} = 0 = f(0), \quad \boxed{m > 0}$$

$$f'(0) = \lim_{x \to 0^+} \left(\frac{x^m \sin\frac{1}{x} - 0}{x} \right)$$

$m < 0$

$$\boxed{m - 1 \le 0} \Longleftarrow \lim_{x \to 0^+} x^{m-1} \sin\frac{1}{x} = \text{not exist}$$

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5:

Discuss cont. & diff. at $x=0$

$$f(x) = \begin{cases} \left(\ln\left(e\left|[x]+[-x]\right|\right)\right)^{x}\left(\dfrac{2e^{\frac{\{x\}+\{-x\}}{|x|}}-5}{3+e^{\frac{1}{|x|}}}\right), & x<0 \\ 0, & x=0 \\ x\left(\dfrac{1-e^{|x|+\{x\}}}{|x|+\{x\}}\right)^{x-[x]}, & x>0 \end{cases}$$

$\boxed{\text{Non diff; but continuous}}$

$$f'(0^-) = \lim_{x\to 0^-} \frac{\cancel{x}\left(\dfrac{2e^{\frac{1}{|x|}}-5}{3+e^{\frac{1}{|x|}}}\right)-0}{\cancel{x}-0} = \lim_{x\to 0}\left(\frac{2-\frac{5}{e^{1/|x|}}}{1+\frac{3}{e^{\frac{1}{|x|}}}}\right)$$

$$= 2$$

$$f'(0^+) = \lim_{x\to 0^+} \frac{\cancel{x}\left(\dfrac{1-e^{2x}}{2x}\right)-0}{\cancel{x}} = -1$$

$[\cdot]=$ G.I.F, $\{\cdot\}=$ FPF

Q.

$\boxed{\text{Ex - IV}}$

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6. $f(x) = \sin x, \quad g(x) = \cos x$

$$h(x) = \begin{cases} \max\left(f(t) \mid 0 \le t \le x\right), & x \in \left[0, \frac{\pi}{2}\right] \\ \min\left(f(t) \mid x \le t \le \pi\right), & x \in \left(\frac{\pi}{2}, \pi\right] \\ \min\left(g(t) \mid \pi \le t \le 2\pi\right), & x \in \left(\pi, 2\pi\right] \end{cases}$$

Discuss cont. & diff. of $h(x)$ in $[0, 2\pi]$.

$$h\left(\frac{\pi}{6}\right) = \max\left(\underline{f(t)} \mid 0 \le t \le \frac{\pi}{6}\right) = \sin\frac{\pi}{6}$$