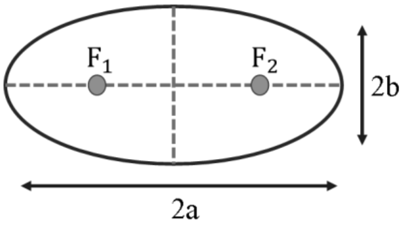
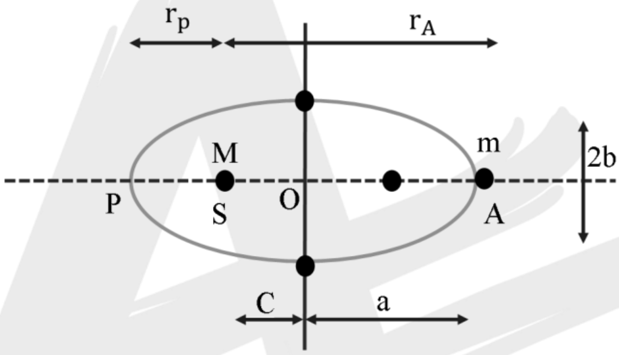
Kepler's Laws:

Kepler's first law or law of orbits: Each planet follows an elliptical orbit around the sun, with the sun positioned at one of the focal points of the ellipse.



As shown in the fig., sun may be at F_1 or F_2. Here a and b denote the lengths of semi major and semi minor axes.



- Eccentricity of the elliptical path $e = \frac{SO}{OA}$ $e = \frac{c}{a} \Rightarrow c = ea$
- From fig, $r_p = a - c = a - ea = a(1-e)$ Similarly $r_a = a + c = a + ea = a(1+e)$
- From conservation of angular momentum at A and P, we have $mV_p r_p = mV_A r_A$

$$\frac{V_P}{V_A} = \frac{r_A}{r_p} = \frac{1+e}{1-e}$$

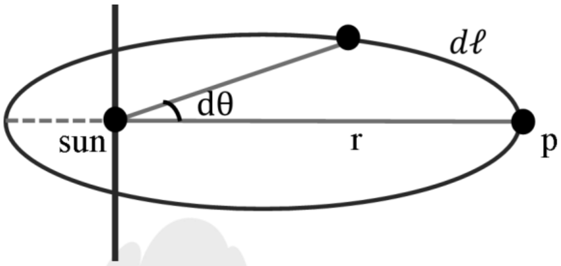
- From conservation of energy, we have

$$V_A = \sqrt{\frac{GM}{a}\left(\frac{1-e}{1+e}\right)} \text{ and } V_P = \sqrt{\frac{GM}{a}\left(\frac{1+e}{1-e}\right)}$$

- Ife > 1 and total energy (K.E +P. E) > 0, the path of the satellite is hyperbolic and it escapes from its orbit.
- If e < 1 and total energy is negative, it moves in an elliptical path.
- If e=0 and total energy is negative, it moves in circular path.
- Ife = 1 and total energy is zero, it will take parabolic path.
- The path of the projectile thrown to lower heights is parabolic and thrown to greater heights is elliptical.

- **Kepler's second law of Law of Areas:** The line connecting the planet to the sun covers the same area during equal time intervals.
- Areal Velocity of radius vector $\left(\frac{dA}{dt}\right)$ joining the planet to sun remains constant. Mathematically

  $\frac{dA}{dt}$ = constant.



But $A = \frac{1}{2}(d\ell)r = \frac{1}{2}(rd\theta)r = \frac{1}{2}r^2 d\theta$

$$\Rightarrow \frac{1}{2}\frac{mr^2\omega}{m} = \frac{I\omega}{2m} = \frac{L}{2m} = \text{constant}$$

L = constant

- **Kepler's third law or Law of periods:** The duration of a planet's orbit around the sun squared is directly proportional to the cube of the planet's average distance from the sun (represented by its semi-major axis in the elliptical orbit).

  $$r_{mean} = \frac{r_{max} + r_{min}}{2} = \frac{(1+e)a + (1-e)a}{2} = a$$

  Hence, $T^2 \propto a^3$

  where ' a' is length of semimajor axis of ellipse

Newton's Law of Gravitation:

The magnitude of gravitational force of attraction between two point masses is given by

$$F = G\frac{m_1 m_2}{r^2}$$

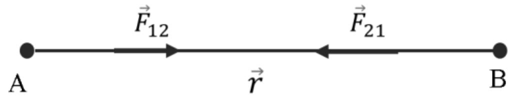
Where G is universal gravitational constant

- $G = 6.67 \times 10^{-11} Nm^2kg^{-2}$ (or) $6.67 \times 10^{-8} dyne\,cm^2 gm^{-2}$
- G is a scalar, Dimensional formula $\left\lfloor M^{-1}L^3T^{-2} \right\rfloor$
- In vector form $\vec{F} = \frac{-Gm_1m_2}{r^2}\hat{r} = \frac{-Gm_1m_2}{r^3}\vec{r}$ -

  Here $\hat{r}$ is the unit vector in the direction of $\vec{r}$ -and '-'sign indicates that the force is attractive.

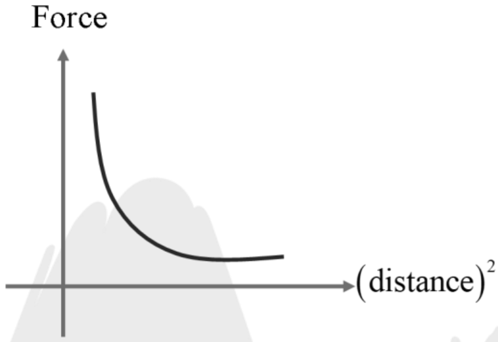
Properties of gravitational force:-

- Gravitational force form an action - reaction pair. So gravitational force obeys Newton IIIrd Law



$\Rightarrow \vec{F}_{12} = -\vec{F}_{21} \Rightarrow \vec{F}_{12} + \vec{F}_{21} = 0$

- $F - d^2$ graph is a rectangular hyperbola as shown.



Acceleration due to gravity (g):

- Relation between G and g is, $g = \frac{GM}{R^2}$
- 'g' is a vector quantity with $\left[LT^{-2}\right]$ as its dimensional formula. Its SI unit is ms^{-2}.

Variation of 'g':

Variation of g with altitude:

If g and g_h are acceleration due to gravities on the surface of the Earth and at height 'h' above the surface of the Earth of mass M and radius R then

$g = \frac{GM}{R^2}$ and $g_h = \frac{GM}{(R+h)^2}$

$$g_h = g\left(\frac{R^2}{(R+h)^2}\right)$$

Variation of with depth:

If "g" represents the gravitational acceleration experienced at the Earth's surface, while "g, " signifies the gravitational acceleration encountered at a depth "d" beneath the Earth's surface, then

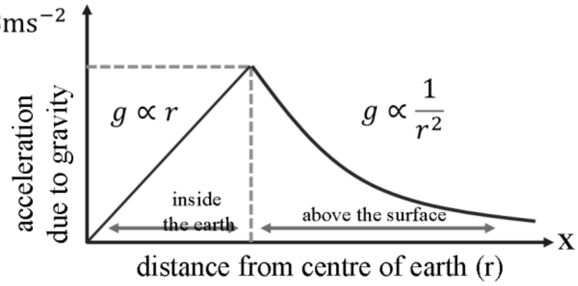
- on surface, $g = \frac{GM}{R^2} = \frac{4}{3}\pi G R \rho$
- at a depth, $g_d = \frac{4}{3}\pi G(R-d)\rho$
- $g_d = g\left(1 - \frac{d}{R}\right)$

Thus, as depth increases, the acceleration due to gravity decreases.

- **Graphical representation of variation of with height and depth:**

The variation of g with the distance r from the centre of the earth is shown below

(i) Above the earth: $g_h = \dfrac{gR^2}{(R+h)^2}$

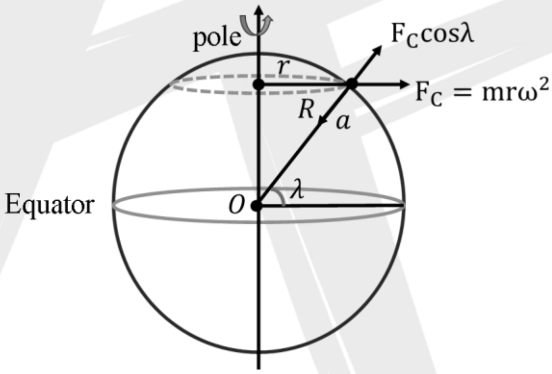


(ii) Inside the earth: $g_d = \dfrac{g}{R}(R-d)$

Variation of 'g' with latitude :

$g_\lambda = g - r\omega^2 \cos\lambda$

where $r\omega^2 \cos\lambda$ is the component of centrifugal acceleration along the radius of the Earth.



where r is the radius of the circle in which the object is revolving. Here $r = R\cos\lambda$

$\therefore g_\lambda = g - \omega^2 R\cos^2\lambda$

where ω is the angular velocity. R is radius of the earth and λ is latitude of the place.

Gravitational Field:

It designates the area or expanse surrounding a massive particle where its gravitational impact is detectable.

Gravitational field strength (or) Intensity of Gravitational Field:

- Gravitational field strength at a specific location within a gravitational field is characterized as the gravitational force encountered by a unit mass situated at that point.

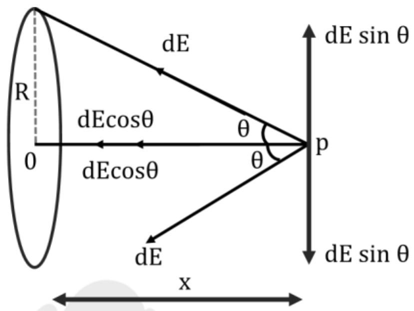
$\therefore$ Gravitational field strength, $\vec{E}_g = \dfrac{\vec{F}}{m_0}$

Units of gravitational field strength are Nkg^{-1} or ms^{-2} and dimensional formula is LT^{-2}

Field due to Circular Ring:

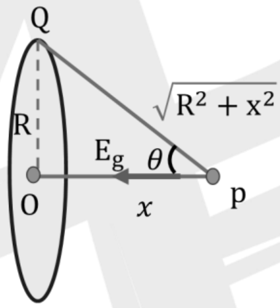
- Gravitational field intensity due to a uniform circular ring of mass M at any point at a distance 'x' (from the centre of the ring) on its axis is

$$E_g = \frac{GMx}{\left(x^2 + R^2\right)^{3/2}} \text{ along } \overrightarrow{PO}$$



Field due to Circular Disc:

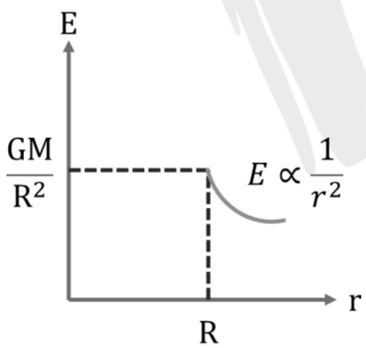
- Gravitational field intensity due to a circular disc of mass M at any point on the axial line



or $E_g = \frac{2GM}{R^2}(1 - \cos\theta)$ (in terms of 'θ')

Field due to Hollow Sphere (or) Spherical Shell (E or I):

- Gravitational field intensity due to a uniform spherical shell



At a point inside the spherical shell,

$\left(E_g\right)_{inside} = 0, \left(E_g\right)_{centre} = \text{zero}$

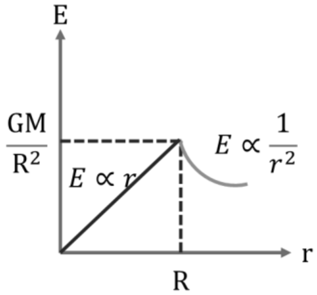
At a point outside the spherical shell,

$\left(E_g\right)_{surface} = \frac{GM}{R^2} (\text{here } r = R)$

$\left(E_g\right)_{outside} = \frac{GM}{r^2} (\text{here } r > R)$

Field due to Solid Sphere(uniform mass density):

Gravitational field intensity due to a solid sphere



Gravitational Potential:

The amount of work done in bringing a unit mass from infinity to a certain point in the gravitational field of another massive object is called as gravitational potential at that point due to massive object.

Let W is the work done m_0 is the test mass then $V = \frac{W}{m_0}$

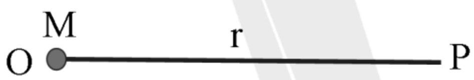
As this work done is negative, the gravitational potential is negative.

S.I unit: J/Kg

Dimensional formula: $\left[M^0L^2T^{-2}\right]$

Potential due to a Point Mass:

- The gravitational potential at a point P which is at a distance r from a point mass M is given by $V = -\frac{GM}{r}$

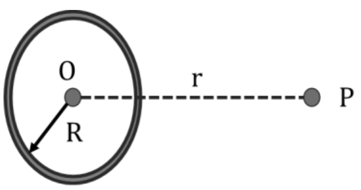


- If the system has a number of masses $m_1, m_2, m_3 .....m_n$ at distances $r_1, r_2, r_3 ....r_n$ from the point p, the resultant gravitational potential at a point p can be written as

$$V = V_1 + V_2 + V_3 + ...V_n \Rightarrow V = -G\left[\frac{m_1}{r_1} + \frac{m_2}{r_2} + \frac{m_3}{r_3} + ... + \frac{m_n}{r_n}\right] \Rightarrow V = -G\sum_{i=1}^{n}\frac{m_i}{r_i}$$

Potential due to Circular Ring:

- Gravitational potential due to a circular ring, at a distance r from the centre and on the axis of a ring of mass M and radius R is given by $V = \frac{-GM}{\sqrt{R^2 + r^2}}$



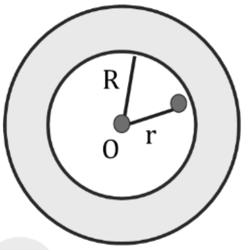
At $r = 0$, $V = -\frac{GM}{R}$, i.e., at the centre of the ring gravitational potential is $-\frac{GM}{R}$

Gravitational Potential due to a Spherical Shell:

Let M be the mass of spherical shell and R is its radius $V = \frac{-GM}{r}$

- At a point inside the spherical shell, (If r < R)

$V_{inside} = \frac{-GM}{R}$



- At a point on the surface of the spherical shell,

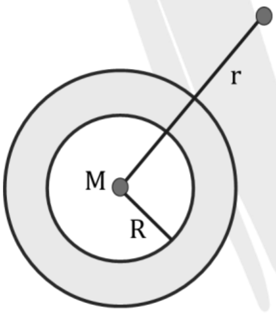
$V_{surface} = \frac{-GM}{R}$ (If r = R)

$V_{centre} = -\frac{GM}{R}$ (r = 0 at centre)

$V_{inside} = V_{surface} = V_{centre} = -\frac{GM}{R}$,

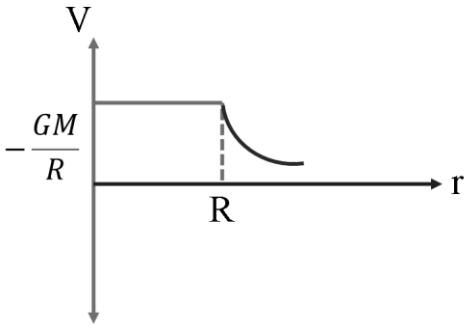
- At a point outside the spherical shell, $V_{outside} = \frac{-GM}{r}$ (If r > R)

- At infinity, $V_\infty = 0$



- At infinity, $V_\infty = 0$

- **The variation of magnitude of V with r is as shown (For a spherical shell)**

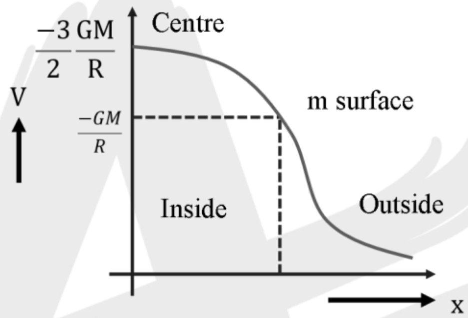


Gravitational Potential due to Solid Sphere:

- At a point inside the solid sphere, $V_{inside} = \frac{-GM}{2R^3}(3R^2 - x^2)$

$$V_{inside} = -GM\left(\frac{3}{2R} - \frac{x^2}{2R^3}\right) \quad (\text{if } x < R)$$

- At a point on the surface of the solid sphere, $V_{surface} = \frac{-GM}{R}$ (If x = R)
- At a point outside the solid sphere, $V_{outside} = \frac{-GM}{x}$ (If x > R)
- At the centre, $x = 0 \Rightarrow V_c = -\frac{3}{2}\frac{GM}{R} = \frac{3}{2}V_{surface}$.
- The variation of V with x is as shown:



- In case of solid sphere potential is maximum at centre.

Newton's Shell Theorem:

Gravitational potential at a point outside of a solid (or) hollow sphere of mass M is same as potential at that point due to a point mass of M separated by same distance.

Hence, the sphere can be replaced by a point mass.

Gravitational potential difference:

The energy needed to move a one-kilogram mass between two spots in a gravitational field is referred to as the gravitational potential difference between those spots.

$$\Delta V = V_b - V_a = -\left(\frac{W_b - W_a}{m_0}\right) \text{ or } W_{ab} = -m_o(V_b - V_a) = -Gmm_0\left(\frac{1}{r_b} - \frac{1}{r_a}\right)$$

Relation between gravitational field and potential:

- Gravitational field and the gravitational potential are related by $\vec{E} = -$ gradient V = - grad V

$$\vec{E} = -\left[\frac{\partial V}{\partial x}\hat{i} + \frac{\partial V}{\partial y}\hat{j} + \frac{\partial V}{\partial z}\hat{k}\right]$$

Here, $\frac{\partial V}{\partial x} =$ Partial derivative of potential function V with respect to x, i.e., differentiate V wrt *x* assuming y and z to be constant.

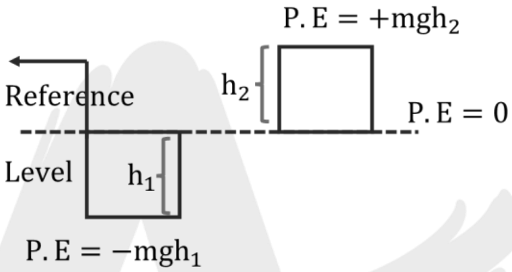
- The above equation can be written in the following forms.
- $E = \frac{-dV}{dx}$, If gravitational field is along x-direction only.
- $dV = -\vec{E}.\vec{dr}$, (where $\vec{dr} = dx\hat{i} + dy\hat{j} + dz\hat{k}$ and $\vec{E} = E_x\hat{i} + E_y\hat{j} + E_z\hat{k}$)

Gravitational potential energy:

The energy expended by the gravitational force when a body is brought from an infinite distance to a specific location within the gravitational field is referred to as the gravitational potential energy at that location.

$$\Rightarrow U = -\int_{\infty}^{r} \vec{F}.\vec{dr} = -W \left[\text{as} \int_{\infty}^{r} \vec{F}.\vec{dr} = W \right]$$

The potential energy of a body or system is the result of the conservative forces performing negative work to bring it from an infinite distance to its current position.



- By the definition of gravitational potential, $V = -\frac{W}{m} = \frac{U}{m} \Rightarrow U = mV$

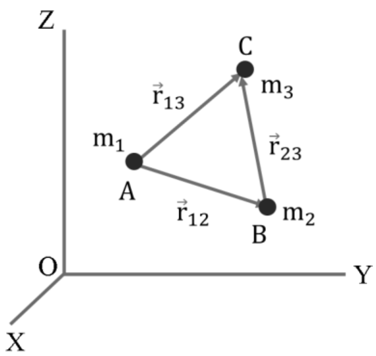
Gravitational Potential Energy of Two Particle System:

- The gravitational potential energy of two particles of masses m_1 and m_2 separated by a distancer is given by $U = -\frac{Gm_1m_2}{r}$

Gravitational Potential Energy of Three Particle System:

Consider a system consists of three particles of masses m_1, m_2 and m_3 located at A, B and C respectively. Total potential energy 'U' of the system is

$$U = -G\left[\frac{m_1m_2}{|\vec{r}_{12}|} + \frac{m_2m_3}{|\vec{r}_{23}|} + \frac{m_1m_3}{|\vec{r}_{13}|}\right]$$



If a body is moving only under the influence of gravitational force, from law of conservation of mechanical energy $U_1 + K_1 = U_2 + K_2$

Gravitational Potential Energy of a System of Particles:

The gravitational potential energy for a system of n particles is given by

$$U = \sum U_i = -\left[\frac{Gm_1m_2}{r_{12}} + \frac{Gm_2m_3}{r_{23}} + \dots\dots\dots\right]$$

- For n particle system there are $\frac{n(n-1)}{2}$ pairs and the potential energy is calculated for each pair and added to get the total potential energy of the system.

Gravitational Potential Energy of a body in Earth's Gravitational Field:

- If a point mass 'm' is at a distancer r from the centre of the earth, then, $U = -\frac{GMm}{r}$
- On the surface of earth, $U_{surface} = -\frac{GMm}{R} = -mgR\left(\because g = \frac{GM}{R^2}\right)$
- At a height 'h' above the surface of earth, $U_h = -\frac{GMm}{R+h}$
- The difference in potential energy of the body of mass m at a height h and on the surface of earth is $\Delta U = U_h - U_{surface}$

$$= -\frac{GMm}{R+h} - \left(-\frac{GMm}{R}\right) = GMm\left(\frac{1}{R} - \frac{1}{R+h}\right)$$

$$= -\frac{GMmh}{(R+h)R} - \frac{GMmh}{R^2\left(1+\frac{h}{R}\right)} \Rightarrow \Delta U = \frac{mgh}{1+\frac{h}{R}}$$. If $h << R,\ \Delta U \approx mgh$

- Gravitational potential energy at the centre of the earth is given by

$$U_c = mV_c = -\frac{3}{2}\frac{GMm}{R}$$

Here, $V_c = \frac{3}{2}V_s = \frac{-3GM}{2R}$ (It is minimum but not zero. However, 'g' at centre of earth is zero)

- **Self-potential energy of a uniform sphere of mass' and radius 'R':**

It is the amount of work done to bring identical massive particles to construct a sphere of mass M radius R and density ρ

$$= \frac{-16\pi^2 GR^5}{15}\left(\frac{M}{\frac{4}{3}\pi R^3}\right) = \frac{-3}{5}\frac{GM^{-2}}{R}$$

= Gravitational self-potential energy of a sphere.

- **Self-potential energy of a thin uniform shell of mass 'm' and radius 'R' is** $-\frac{Gm^2}{2R}$
- Change in the gravitational potential energy in lifting a body from the surface of the earth to a height equal to 'nR' from the surface of the earth

$$\Delta U = \frac{GMmh}{R(R+h)} = \frac{GMm(nR)}{R(R+nR)} = \frac{GMmn}{R(n+1)} = \frac{mgRn}{n+1}$$

Escape Velocity:

Escape velocity is the lowest velocity a body must attain to break free from a planet's gravitational influence.

If V_e is the escape velocity from the surface of the planet then $V_e = \sqrt{\frac{2GM}{R}}$

Also $V_e = \sqrt{2gR}$ and $V_e = \sqrt{2\left(\frac{4}{3}\pi R\rho G\right)R}$

(where ρ is the mean density of the planet)

Escape Velocity of a body From certain height above the surface of a planet:

- At a height 'h' above the surface of a planet $PE_{body} = \frac{-GMm}{R+h}$

$$\Rightarrow v_e = \sqrt{\frac{2GM}{R+h}} = \sqrt{2g_h(R+h)}$$

Here, g_h is acceleration due to gravity at height h.

Earth Satellites

Orbital speed of Satellites:.

$$v_o = \sqrt{\frac{GM}{r}} = \sqrt{\frac{GM}{(R+h)}} = \sqrt{\frac{gR^2}{(R+h)}}$$

Angular velocity $\omega = \sqrt{\frac{GM}{(R+h)^3}}$

Time period $T = 2\pi\sqrt{\frac{(R+h)^3}{GM}} = 2\pi\sqrt{\frac{(R+h)^3}{gR^2}}$

- For a satellite orbiting very close to earth.

$h << R$ then, $v_0 = \sqrt{\frac{GM}{R}} = \sqrt{gR}$

$$\omega^2 \propto \frac{1}{R^3} \Rightarrow T^2 \propto R^3$$

- For two satellites revolving around the earth in different circular orbits of radii r_1 and r_2 at vertical heights h_1 and h_2, $\frac{v_1}{v_2} = \sqrt{\frac{r_2}{r_1}} = \sqrt{\frac{R+h_2}{R+h_1}}$

Frequency of Revolution (n):

$$n=\frac{1}{T}=\frac{1}{2\pi}\sqrt{\frac{GM}{r^3}}=\frac{1}{2\pi}\sqrt{\frac{GM}{(R+h)^3}}=\frac{1}{2\pi}\sqrt{\frac{gR^2}{(R+h)^3}}\quad\left[\because g=\frac{GM}{R^2}\right]$$

- If the satellite revolves close to the earth surface,

$(h<<R)$ then $n=\frac{1}{2\pi}\sqrt{\frac{GM}{r^3}}=\frac{1}{2\pi}\sqrt{\frac{g}{R}}$

Angular Momentum: The angular momentum of the satellite is given by

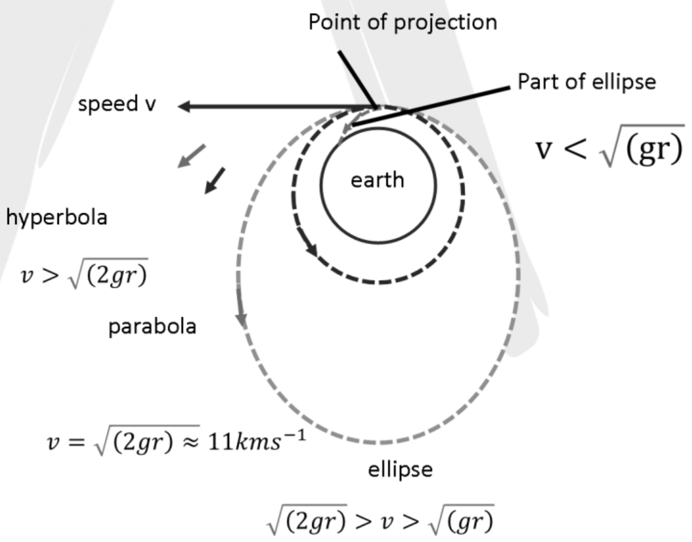
$$L=mv_0r=mr\sqrt{\frac{GM}{r}}=\sqrt{GMm^2r}$$

Energy of Orbiting Satellite:

- The potential energy of the system is $U=\frac{-GMm}{r}$
- The kinetic energy of the satellite is, $K=\frac{1}{2}mv_0^2=\frac{1}{2}m\left(\frac{GM}{r}\right)$ or $K=\frac{GMm}{2r}$
- The total energy is $E=K+U=-\frac{GMm}{2r}$

Trajectories of a body projected with different velocities:

An object orbits a planet exclusively when it is launched with enough velocity perpendicular to the planet's gravitational attraction.



If $v<\sqrt{gr}$ object falls on the surface of earth

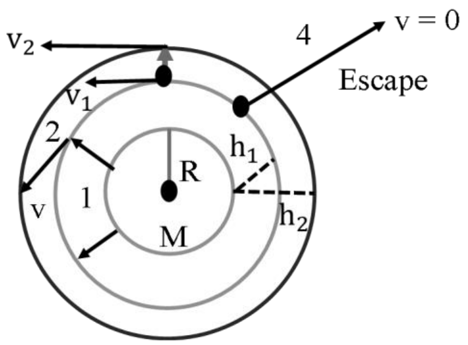
If $v=\sqrt{gr}$ object revolve in a circular orbit.

If $\sqrt{gr}<v<\sqrt{2gr}$ object revolves in an elliptical orbit.

If $v=\sqrt{2gr}$ object escapes from the field and follows parabolic path.

If $v>\sqrt{2gr}$ object escapes from the field and follows hyperbolic path.

Special cases:



Case I: Work done to lift an object at rest from the surface of a planet to a height h is

$$TE_i = TE_{surface} = \frac{GMm}{R} \quad 0 \quad \frac{GMm}{R}$$

$$TE_f = TE_{height} = \frac{GMm}{R \quad h} \quad 0 \quad \frac{GMm}{R \quad h}$$

$$\text{Work done } W = TE_f - TE_i = \frac{GMm}{R} \quad \frac{GMm}{R \quad h} \Rightarrow W \quad \frac{Gmmh}{R\,R\,h} \quad \frac{mgh}{1 \quad \frac{h}{R}}$$

Case II: Work done to shift an object at rest from the surface of planet into an orbit in which object revolves around the planet is

$$TE_i = TE_{surface} = \frac{GMm}{R} \quad 0 \quad \frac{GMm}{R}$$

$$TE_f = TE_{orbit} = \frac{GMm}{R \;\; h} \quad \frac{1}{2} mv_0^2 \quad \frac{GMm}{2 \;\; R \;\; h}$$

$$\text{Work done } W = TE_f - TE_i = \frac{GMm}{R} \; \frac{GMm}{2 \; R \; h}$$

$$W \quad GMm \frac{R \qquad 2h}{2R \qquad R \qquad h}$$

Case III: Work done to shift an object revolving around the planet from one orbit in to another orbit is

$$TE_i \left(TE\right) h_1 = \frac{GMm}{R \quad h_1} \frac{1}{2} mv_1^2 \quad \frac{GMm}{2R \quad h_1}$$

$$TE_f = TE_{h_2} = \frac{GMm}{R \quad h_2} \quad \frac{1}{2} mv_2^2 \quad \frac{GMm}{2 \;\; R \;\; h_2}$$

Work done

$$W = TE_f - TE_i = \frac{GMm}{2 \quad R \quad h_1} \quad \frac{GMm}{2 \;\; R \qquad h_2} \Rightarrow W \frac{GMm}{2} \quad \frac{h_2 \quad h_1}{R \;\; h_1 \;\; R \;\; h_2}$$

Case IV:

An object (satellite) revolving around the planet escapes when

1) It's KE is doubled (increases by 100%)

2) It's velocity is increased to $\sqrt{2}$ times of present value (increases by 41.4%)

Additional velocity imparted to the body $= v_e - v_0 = \sqrt{2}v_0 \quad v_0$

$\sqrt{2} \quad 1\, v_0 \qquad\qquad 3.2\text{km/s}$ (nearly)

Note: In the above case if the object initially revolves around the planet at a height h from the surface then it's TE $\frac{GMm}{2\ R\ \ h}$

Additional energy required to escape the object is $\frac{GMm}{2\ R\ \ h}$

Binding Energy:

The energy required to remove the satellite from its orbit to infinity is called binding energy of the system.

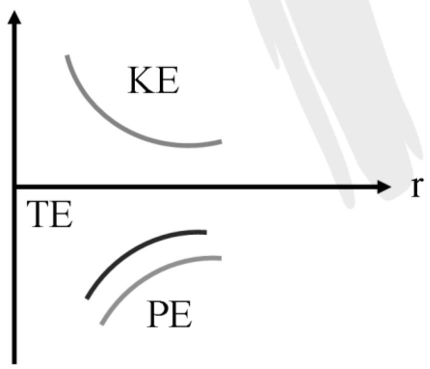
$$\frac{1}{2}mV_0^2 - \frac{GMm}{r} + B.E. = 0$$

Binding energy (BE) = $\frac{GMm}{2r}$

- For a satellite $\frac{PE}{KE} = -2, \frac{PE}{TE} = 2, \frac{KE}{TE} = -1$

PE: KE:TE=-2:1:-1

- Energy graph for a satellite is



Change in Orbit of a Satellite:

- Energy required to shift a satellite from an orbit radius r_1 into an orbit of radius r_2 is

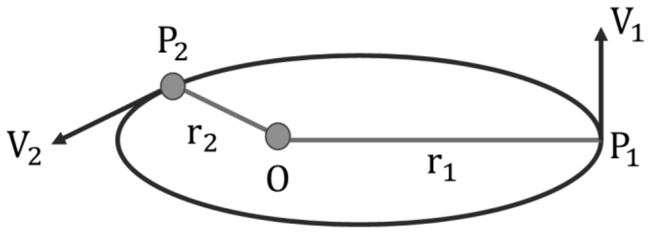
$$E = E_2 - E_1 = \frac{GMm}{2}\left(\frac{1}{r_1} - \frac{1}{r_2}\right) \text{ (or) } E = \frac{mgR^2}{2}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

- When the satellite is transferred to a higher orbit $(r_2 > r_1)$ then variation in different quantities are as shown in the following table.

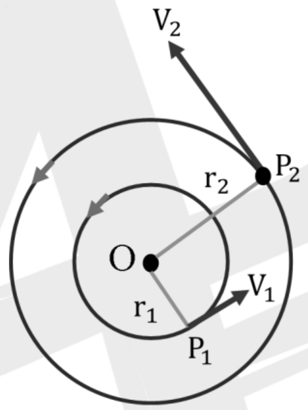
Important Features Regarding Satellite:

- **For a given satellite in an orbit,**

  $L = mvr = \text{constant} \Rightarrow v \propto \frac{1}{r}$.



- For satellites in different orbits $v \propto \frac{1}{\sqrt{r}}$



Weightlessness:

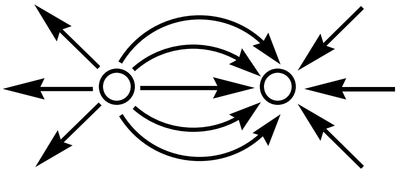
- Weightlessness is a phenomenon in which the object is in a state of free fall.
- $W_{app} = m(g-a)$ Here $a = g \Rightarrow W_{app} = 0$

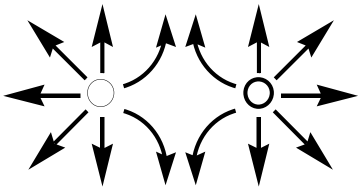
Condition for Weightlessness in a satellite:

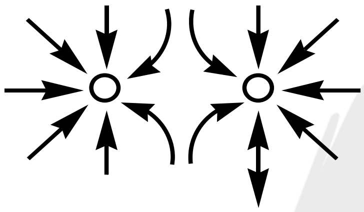
- The force acting on the astronaut of mass 'm' is $\frac{GMm}{r^2} - F_R = \frac{mv_0^2}{r}$
- The reactional force on the floor of the satellite is zero. Hence, there is the state of weightlessness in a satellite i.e., $\frac{GMm}{r^2} = \frac{mv_0^2}{r}$
- As the frame of reference attached to the satellite is an accelerated frame, whose acceleration towards the centre of the earth is $a = \frac{v_0^2}{r} = \frac{GM}{r^2} = g$

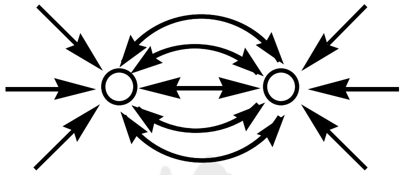
EXERCISE-I

1. Assume that gravitational lines of forces represent gravitational field just like electric lines of forces represent electric field. Which of the following diagram correctly represents the gravitational field lines for a pair of point masses shown in options below?

(A) 

(B) 

(C) 

(C) 

2. A simple pendulum is taken to 64 km above the earth's surface. It's time period will:

(A) increase by 1%

(B) decrease by 1%

(C) increase by 2%

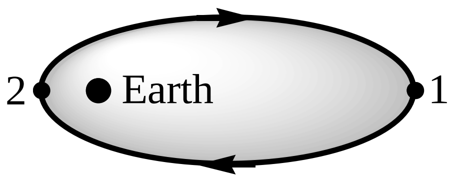
(D) decrease by 2%

3. A satellite is revolving around the Earth in a circular orbit with a constant speed. If its speed is made 2 times by supplying energy from an external source then what will be the path of satellite after this?

(A) Straight line (B) Circular (C) Parabola (D) Hyperbola

4. The gravitational potential difference between the surface of a planet and a point 20 m above the surface is 16 J/kg. The work done in moving a 4 kg body by 8 m on a slope of 60° from the horizontal is $\frac{2216}{n}$ J. Find n?

5. A small satellite is in elliptical orbit around Earth as shown. If L denotes the magnitude of its angular momentum and K denotes kinetic energy :



(A) $L_2 > L_1$ and $K_2 > K_1$

(B) $L_2 > L_1$ and $K_2 = K_1$

(C) $L_2 < L_1$ and $K_2 = K_1$

(D) $L_2 = L_1$ and $K_2 > K_1$

6. Three planets of same density and with radii R_1, R_2 and R_3 such that $R_1 = 2R_2 = 3R_3$ have gravitational fields on the surface g_1, g_2, g_3 and escape velocity v_1, v_2 and v_3 respectively. Then

(A) $\frac{g_1}{g_2} = \frac{1}{2}$ (B) $\frac{g_1}{g_3} = 3$ (C) $\frac{v_1}{v_2} = 2$ (D) $\frac{v_1}{v_3} = \frac{1}{3}$

7. A point mass of 0.5 kg moving with a constant speed of $5\,ms^{-1}$ on an elliptical track experiences an outward force of 10 N when at either endpoint of the major axis and a similar force of 1.25 N at each end of the minor axis. How long is the semi major axes of the ellipse?

(A) 5 m (B) 10 m (C) 15 m (D) 20 m

8. A particle of mass m_0 is projected vertically upward from surface of earth with a speed of $\sqrt{\frac{5GM_e}{4R}}$. Find the maximum height of the particle from the earth surface is [Mass of the earth M_e and R is radius of earth].

(A) $\frac{5R}{3}$ (B) $\frac{2R}{3}$ (C) $\frac{7R}{3}$ (D) $\frac{11R}{3}$

9. If the areal velocity of one planet is 2 times the areal velocity of other planet, then find the ratio of their radii of circular motion around the sun.

(A) 2 (B) 4 (C) 6 (D) 8

10. Six point masses of mass m each are at the vertices of a regular hexagon of side ℓ. The force on any one of the masses is given by $F = \frac{Gm^2}{\ell^2}\left(\frac{1}{x} + \frac{1}{\sqrt{y}} + \frac{1}{z}\right)$, then find $x + y - z$. (Take: $x < y < z$)

(A) 0 (B) 1 (C) 3 (D) 4

11. Two particles of mass 'm' and 3m are initially at rest at infinite distance apart. Both the particles start moving due to gravitational attracction. Find the their relative velocity of approach

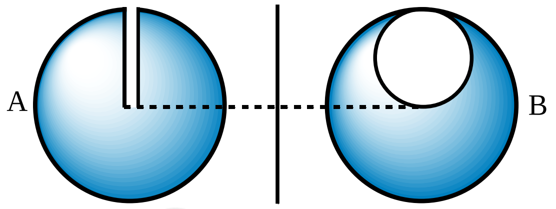
(A) $\sqrt{\frac{2GM}{d}}$ (B) $\sqrt{\frac{4GM}{d}}$

(C) $\sqrt{\frac{6GM}{d}}$ (D) $\sqrt{\frac{8GM}{d}}$

12. Two point particles of mass 3 mg and 1.8 mg each are kept at a distance r apart. Equal number of electrons are removed from both particles till they are in equilibrium under influence of mutual electrostatic repulsion and gravitational attraction. How many electrons were removed from one of the particles?(Take : $G = \frac{20}{3} \times 10^{-11} Nm^2/kg^2$)

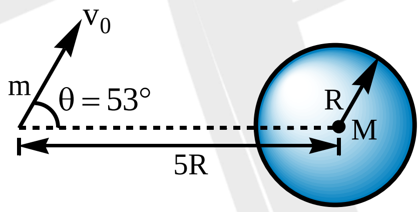
(A) 750 (B) 950 (C) 1050 (D) 1250

13. Now-a-days ISRO is working on a space research program. In this program they discover spherical Asteroid made up titanium, a precious metal. They dig a tunnel from surface to centre to find depth of titanium. Titanium was uniformly distributed in the sphere. Now they want to mine this precious matterial out of the asteroid due to which cavity is formed as shown in figure B. If a piece of metal falls in the tunnel it takes time t_1 and when it falls in cavity it takes time t_2 to reach at the bottom. Ratio of time $\frac{t_1}{t_2}$ is



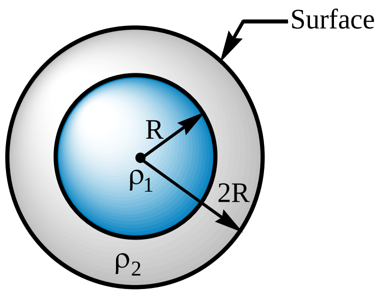
(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{4}$ (D) $\frac{\pi}{5}$

14. A spaceship is sent to investigae a planet of mass M and radius R. While hanging motionless in space at a distance 5R from the centre of the planet, the spaceship fires an instrument package with speed V_0 as shown in the figure. The package has mass m, which is much smaller than the mass of the spaceship. The package just grazes the surface of the planet and at that moment it's speed is V_0. Find V_0?



(A) $\sqrt{\frac{2GM}{75R}}$ (B) $\sqrt{\frac{4GM}{75R}}$ (C) $\sqrt{\frac{6GM}{75R}}$ (D) $\sqrt{\frac{8GM}{75R}}$

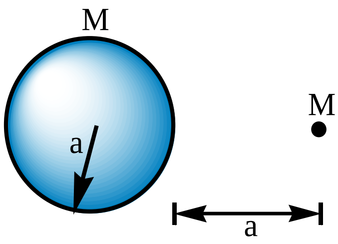
15. A planet is made of two materials of density ρ_1 and ρ_2 as shown in figure. The acceleration due to gravity at surface of planet is same as a depth 'R'. Find the ratio of $\frac{\rho_1}{\rho_2}$ is



(A) $\frac{7}{3}$ (B) $\frac{3}{7}$ (C) $\frac{5}{7}$ (D) $\frac{7}{5}$

EXERCISE-II

1. A particle of mass M is at a distance 'a' from surface of a thin spherical shell of uniform equal mass and having radius a.



(A) Gravitational field and potential both are zero at centre of the shell

(B) Gravitational field is zero not only inside the shell but at a point outside the shell also

(C) Inside the shell, gravitational field alone is zero

(D) Neither gravitational field nor gravitational potential is zero inside the shell

2. An object is placed at a distance of $\frac{R}{2}$ from the centre of earth. Knowing mass is distributed uniformly, acceleration of that object due to gravity at that point is $\frac{g}{n}$. Find n?

(g = acceleration due to gravity on the surface of earth and R is the radius of earth)

3. Two masses, 800 kg and 450 kg are at a distance 25 m apart. The magnitude of gravitational field intensity at a point 20 m distant from the 800 kg mass and 15 m distant from the 450 kg mass will be : (in N/kg) – (G : universal gravitational constant)

(A) 2G (B) $2\sqrt{2}G$ (C) 4G (D) $5\sqrt{2}G$

4. A small area is removed from a uniform spherical shell of mass M and radius R. Then the gravitational field intensity near the hollow portion is $\frac{GM}{\alpha R^{\beta}}$. Find $\alpha+\beta$?

5. A spaceship orbits the earth at a cosntant speed along a circular path. When an astronaut inside the spaceship release an object it does no move away from him. Which of the following is the most accurate reason for this?

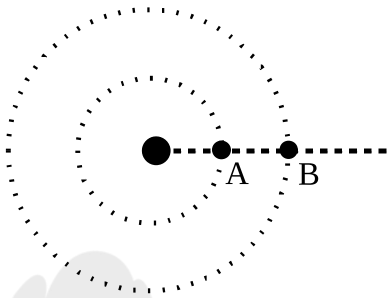
(A) the astronaut and the object move along the same circular path due to the earth's gravitational pull

(B) an object moving in a circular path round the earth experiences no gravitational pull

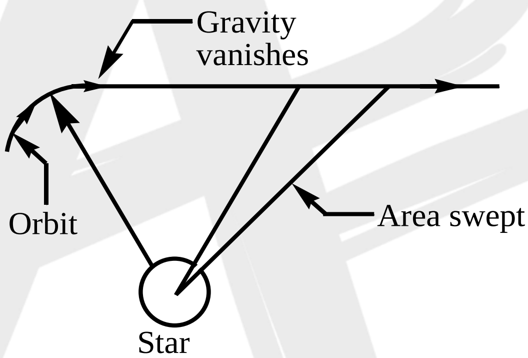
(C) the gravitational forces on the object due to the spaceship exactly balance the gravitational pull on it due to the earth

(D) the gravitational pull on the object due to the earth is very weak at a large distance from the earth

6. Two planets A and B travel counter clockwise in circular orbits around a fixed star. The radii of their orbits are in the ratio 1 : 4. At some time, they are aligned as shown in the figure, making a straight line with the star. After a certain time, planet A comes back to its initial position, completing one full circle about the star. In the same time, angular displacement (in degree) of the planet B is :



7. A planet is orbiting a star when for no apparent reason the star's gravity suddenly vanishes. After which planet moves in a straight line. Mark the correct statement(s)



(A) Newton's first law is obeyed on planet after gravity vanishes

(B) Kepler's law of areas is obeyed only till the planet is in gravity of star

(C) Kepler's law of areas is obeyed even after gravity vanishes

(D) Angular momentum of planet about centre of star is conserved through out its motion

8. Which of the following options are incorrect?

(A) Gravitational potential inside a uniform solid sphere is constant

(B) Gravitational field intensity inside a uniform solid sphere is zero

(C) Gravitational field intensity inside a uniform spherical shell is zero

(D) Gravitational potential inside a uniform spherical shell is constant

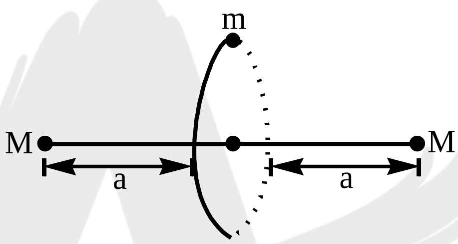
9. Satellite A is in a circular orbit of radius r and satellite B in another circular orbit of radius 4r, both revolving around the earth. The masses of the satellites A and B are in the ratio 3:1. If the symbols V, E, u and T represent the speed, total energy, escape velocity and period of revolution of satellite, then

(A) $\frac{V_A}{V_B} = 2$ (B) $\frac{E_A}{E_B} = 12$ (C) $\frac{u_A}{u_B} = 2$ (D) $\frac{T_A}{T_B} = \frac{1}{16}$

10. A spherical planet has no atmosphere and consists of pure gold. Find the minimum orbital period for a satellite circling the planet. Take density of gold as $5\pi \times 10^3 \text{kg/m}^3$ and $G = \frac{20}{3} \times 10^{-11} \text{Nm}^2/\text{kg}^2$.

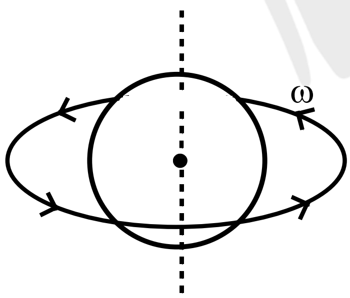
(A) 3×10^{3} sec
(B) 4×10^{4} sec
(C) 3×10^{-3} sec
(D) 4×10^{-4} sec

11. A particle of mass m is moving in circular motion of radius r with constant speed v in influence of two fixed particles of mass M. Plane of circular motion is perpendicular to line joining fixed particles as shown. Find the speed of particle is $\left(a = r\sqrt{3}\right)$



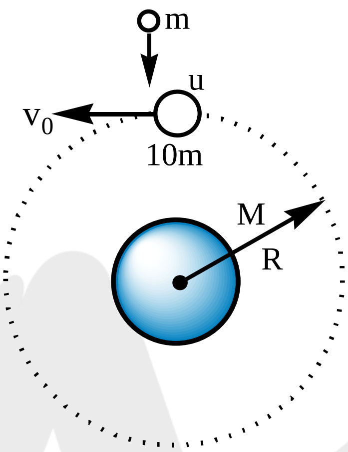
(A) $\sqrt{\frac{GM}{2r}}$
(B) $\sqrt{\frac{GM}{3r}}$
(C) $\sqrt{\frac{GM}{4r}}$
(D) $\sqrt{\frac{GM}{r}}$

12. A ring of mass m and radius 3R is rotating with constant angular speed ω around a planet of mass M and radius R. Centre of ring and planet coincide with each other. Find the tension in the ring is $\left(\omega = \sqrt{\frac{GM}{9R^3}}\right)$.



(A) $\frac{GMm}{3\pi R^2}$
(B) $\frac{GMm}{6\pi R^2}$
(C) $\frac{GMm}{9\pi R^2}$
(D) $\frac{GMm}{12\pi R^2}$

13. A meteorite of mass 'm' strikes the satellite of mass 10 m moving in circular path of radius 'R' around planet of mass 'M' (>>m). Meteorite strikes perpendicular to the orbital velocity of satellite. The combined satellite and meteorite has minimum distance $\frac{R}{2}$ from planet's centre during subsequent motion. Find the velocity u of meteorite just before the collision is



(A) $\sqrt{\frac{29GM}{R}}$ (B) $\sqrt{\frac{38GM}{R}}$ (C) $\sqrt{\frac{48GM}{R}}$ (D) $\sqrt{\frac{58GM}{R}}$

14. India plans to place geostationary satellites, in equatorial orbit at a height 36000 km from surface of earth. Each of these satellites are arranged equispaced in same orbit and should be able to see (exchange signals directly with each other) at any instant of time. If maximum number of such geostationary satellites is $2N+1$, then find the value of N. (Radius of Earth = 6400 km, $\sin^{-1}(0.15)$ = 0.15 radian)

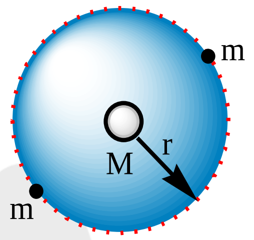
(A) 2 (B) 3 (C) 4 (D) 4

15. On a planet which has the same density as that of earth, we construct a frictionless inclined plane of inclination $53°$ with the horizontal. When a block is released on the inclined plane, it is seen to accelerate at $1\ m/s^2$. What is the radius of planet? (Take radius of Earth = 6400 km)

(A) 2×10^5 m (B) 4×10^5 m (C) 6×10^5 m (D) 8×10^5 m

EXERCISE-III

1. An isolated triple star system consists of two identical stars, each of mass m and a fixed star of mass M. They revolve around the central star in the same circular orbit of radius r. The two orbiting stars are always at opposite ends of a diameter of the orbit. The time period of revolution of each star around the fixed star is equal to $\frac{\alpha \pi r^{3/2}}{\sqrt{G(\beta M + m)}}$. Find $\alpha + \beta$?



2. A satellite is launched int the equatorial plane in such a way that it can transmit signals upto $60°$ latitude on the earth. Then the angular velocity of the satellite is $\sqrt{\frac{\alpha M}{\beta R^{\gamma}}}$. Find $\alpha + \beta + \gamma$?

3. A satellite is moved from one circular orbit around the earth, to another of lesser radius. Which of the following statement is true?

(A) The kinetic energy of satellite increases and the gravitational potential energy of satellite-earth system increases

(B) The kinetic energy of satellite increases and the gravitational potential energy of satellite-earth system decreases

(C) The kinetic energy of satellite decreases and the gravitational potential energy of satellite-earth system decreases

(D) The kinetic energy of satellite decreases and the gravitational potential energy of satellite-earth system increases

4. A satellite is in a circular orbit very close to the surface of a planet with speed V_0. At some point it is given an impulse along its direction of motion causing its velocity to increases η times (that is, its speed becomes ηV_0).

It now goes into an elliptical orbit. The maximum possible value of η for satellite to move in the elliptical orbit is :

(A) 2 (B) $\sqrt{2}$ (C) $\sqrt{2}+1$ (D) $\frac{1}{\sqrt{2}-1}$

5. A satellite with mass 2000 kg and angular momentum magnitude $2\times10^{12}\ kgm^2/s$ is moving in an elliptical orbit around a planet. The rate at which area is being swept out by the satellite around the planet, is equal to $\alpha\times10^8\ m^2/s$. Find α ?

6. A satellite is fired from the surface of the moon of mass M and radius R, with speed V_0 at $30°$ with the vertical. The satellite reaches a maximum distance of $\frac{5R}{2}$ from the centre of the moon. The value of V_0 is $\sqrt{\frac{\alpha GM}{\beta R}}$. Find $\alpha+\beta$?

7. A binary star system consists of two stars one of which has double the mass of the other. The stars rotate about their common centre of mass :

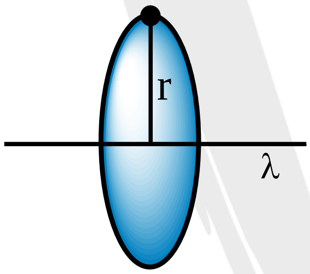
(A) both the stars have same angular momentum about centre of mass

(B) star having the smaller mass has larger angular momentum about the centre of mass

(C) the lighter star has smaller linear speed

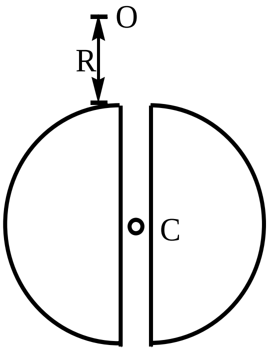
(D) the heavier star has higher kinetic energy

8. Consider a long heavy object of linear mass density λ in free space. A small particle of mass m is orbiting this long object at a distance r from its axis as shown. The time period of revolution and kinetic energy of the particle will be :



(A) $\pi r\sqrt{\frac{2}{\lambda G}}, \lambda Gm$ (B) $\pi r\sqrt{\frac{1}{\lambda G}}, \lambda Gm$ (C) $\frac{2\pi r}{\sqrt{\lambda G}}, \lambda G/m$ (D) $\pi r\sqrt{\frac{1}{2\lambda G}}, \lambda G/m$

9. A particle is dropped from a height equal to the radius of the earth above the tunnel dug through the earth as shown in the figure.



R : Radius of earth

M: Mass of earth

(A) Particle will oscillate through the earth to a height R on both sides

(B) Particle will excute simple harmonic motion

(C) Motion of the particle is periodic

(D) Particle passes the centre of earth with a speed $\sqrt{\frac{GM}{R}}$

10. If the law of gravity were inverse law instead of inverse square law, considering two satellites of same mass but in orbits of radii "r" and "2r" around the earth

(A) The ratio of speeds of two satellites would be 1:1

(B) The ratio of angular speeds of these two satellites would be 2:1

(C) The ratio of the potential energy of these two satellites would be 1:2

(D) The ratio of kinetic energies of these two satellites would be 2:1

11. At the equator on a spherical planet a body weighs one third of that at the pole. Radius of planet is R. the time period of revolution of the planet around its axis is equal to T = 60min.

(take $G = \frac{20}{3} \times 10^{-11} Nm^2 / kg^2$)

(A) The average density ρ of the planet is $\frac{\pi}{192} \times 10^6 kg / m^3$

(B) The height of geostationary satellite from the surface of planet is $\left(\sqrt[3]{\frac{3}{2}} - 1\right)R$

(C) At a latitude of $60°$, the weight of the body is half the weight at pole

(D) The geostationary satellite can be in a plane passing through the poles

12. Two smooth tunnels are dug from one side of the earth's surface to the other side, one along a diameter and the other along a chord. Now two particles are dropped from one end of each of the tunnels. Both the particle oscillate simple harmonically along the tunnels. Let T_1 and T_2 be the time period, v_1 and v_2 be the maximum speed of the particle along the diameter and along the chord respectively. Then

(A) $T_1 = T_2$

(B) $T_1 > T_2$

(C) $v_1 = v_2$

(D) $v_1 > v_2$

13. A near earth orbit satellite of earth experiences very small air resistance. It's period of revolution decreases by 1.08 sec/day.

(A) It's radius decreased by $\frac{160}{3}$ m in 1 day

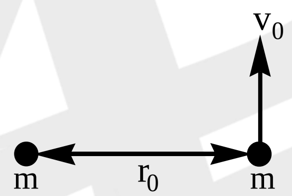
(B) It's radius decreses by $\frac{160}{3}$ m in 1 revolution

(C) It's angular momentum decreases

(D) It's mechanical energy increases

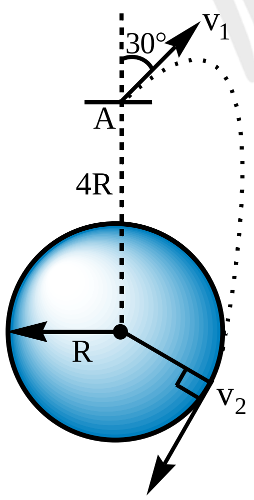
14. Two equal masses are situated at a separation r_0. One of them is imparted a velocity $v_0 = \sqrt{\frac{GM}{r_0}}$ perpendicular to the line joining them both are free to move. Treating motion only under mutual gravitational force find the ratio of maximum and minimum separation between them.

[**Hint :** Solve in CM frame]



(A) 2 (B) 3 (C) 4 (D) 5

15. A particle is projected from point A, that is at a distance 4R from the centre of the Earth, with speed V_1 in a direction making $30°$ with the line joining the centre of the Earth and point A, as shown. Find the speed V_1 if particle passes grazing the surface of the earth. Consider gravitational interaction only between these two. (Use $\frac{GM}{R} = 6.4 \times 10^7 \text{m}^2/\text{s}^2$)



(A) $\frac{8000}{\sqrt{2}}$ m / s (B) $\frac{6000}{\sqrt{2}}$ m / s (C) $\frac{4000}{\sqrt{2}}$ m / s (D) $\frac{2000}{\sqrt{2}}$ m / s

EXERCISE-IV

1. If T_1 = time period of a sample pendulum of infinite length, T_2 = time period of simple harmonic motion of a body dropped in a tunnel dug along diameter of earth and T_3 = time period of circular motion of a satellite revolving near the surface of the earth, then :

(A) $T_1 > T_2 = T_3$ (B) $T_1 = T_2 = T_3$ (C) $T_1 = T_2 > T_3$ (D) $T_1 < T_2 < T_3$

2. Maximum height reached by a rocket fired with a speed equal to 50% of the escape velocity from earth's surface is $\frac{\alpha R}{\beta}$. Find $\alpha + \beta$?

3. A planet revolves abut the sun in elliptical orbit of semimajor axis 2×10^{12} m. The areal velocity of the planet when it is nearest to the sun is 4.4×10^{16} m^2/s. The least distance between planet and the sun is 1.8×10^{12} m. Then the minimum speed of the planet in km/s is :

4. Three identical stars, each of mass M, form an equilateral triangle (stars are positioned at the corners) that rotates around the centre of the triangle. The system is isolated and edge length of the triangle is L. The amount of work done, that is required to dismantle the system, is $\frac{\alpha}{\beta}\frac{GM^{\gamma}}{L}$. Find $\alpha + \beta + \gamma$?

5. **Statement-1 :** Assuming zero potential at infinity, the gravitational potential at a pont can never be positive.

Statement-2 : The magnitude of gravitational force between two particles has inverse square dependence on the distance between two particles.

(A) Statement-1 is true, statement-2 is true, statement-2 is a correct explanation for statement-1

(B) Statement-1 is true, statement-2 is true, statement-2 is NOT a correct explanation for statement-1

(C) Staement-1 is true, statement-2 is false

(D) Statement-1 is false, statement-2 is true

6. Two satellites are launched simultaneously into orbits of radius R and 4R. At an instant the two satellite are on the same radial line. The time after which they have maximum separation is $\frac{\alpha\pi}{\beta}\sqrt{\frac{R^3}{GM_e}}$. Find $\alpha + \beta$? (mass of earth = M_e)

7. Two satellites S_1 and S_2 revolve around a planet in coplanar circular orbits in the same sense. Their periods of revolution are 1 hour and 8 hours respectively. The radius of the orbit of S_1 is 10^4 km. When S_1 is closest to S_2, the angular speed of S_2 as observed by an astronaut in S_1 is $\frac{\pi}{n}$ rad / hr. Find n?

8. A binary star system consists of two stars having masses M and 2M. the distance between their centres is equal to R. they revolve about their common centre of mass under their mutual gravitational force of interaction. Which of the following is/are correct?

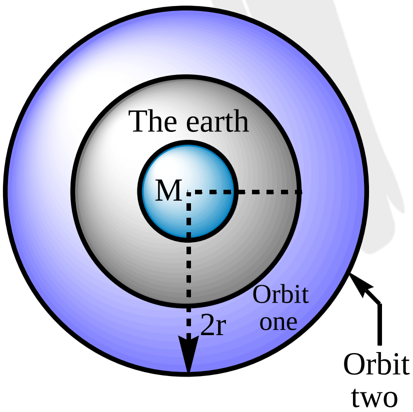
(A) Heavier star revolves in orbit of radius $\frac{2R}{3}$

(B) Both the stars will revolve with the same time period which is equal to $\frac{2\pi}{\sqrt{3GM}}R^{3/2}$

(C) Kinetic energy of lighter star is twice that of the other

(D) Kinetic energy of lighter star is 4 times that of the other

9. A satellite of mass m is moving in a circular orbit of radius 'r' around the earth. This satellite is shifted in circular orbit of radius 2r. (Mass of the earth is M)



(A) The minimum energy required to shift the satellite in new orbit is $\frac{GMm}{2r}$

(B) Time period of satellite in new orbit will be double that time period of previous orbit

(C) The acceleration of satellite in new orbit becomes $\frac{1}{4}$of acceleration in previous orbit

(D) The minimum energy required to shift the satellite in new orbit is $\frac{GMm}{4r}$

10. Two point objects of masses m and 4m are at rest at an infinite separation. They move towards each other under mutual gravitational attraction. If G is the universal gravitational constant, then at a separation r

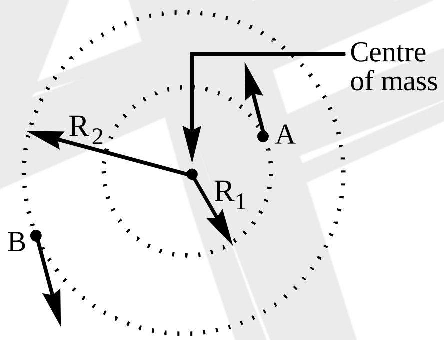
(A) The total mechanical energy of the two objects is zero

(B) their relative velocity is $\sqrt{\frac{10Gm}{r}}$

(C) the total kinetic energy of the objects is $\frac{2Gm^2}{3r}$

(D) their relative velocity is zero

11. Two stars A and B travel in circular orbit around their common centre of mass with radii R_1 and R_2 respectively as shown in figure. The time for the star B to move through one orbit is 2 days with it orbital radius $R_2 = 2\times10^9$ m. The star A has a constant speed of $3.64\times10^4\ ms^{-1}$ in its orbit



(A) According to Kepler's law $T^2 \propto r^3$, so both have different time periods

(B) The radius of the orbit of the star A, $R_1 = 2\times10^9$m

(C) The mass of star A is 3.57×10^{29}kg

(D) The mass of star B is 1.79×10^{29}kg

12. A satellite revolves around a planet in circular orbit of radius R (much larger than the radius of the planet) with a time – period of revolution T. if the satellite is stopped and then released in its orbit. (Assume that the satellite experiences gravitational force due to the planet only)

(A) It will fall into the planet

(B) The time of fall of the satellite is nearly $\frac{T}{\sqrt{8}}$

(C) The time of fall of the satellite into the planet is nearly $\frac{\sqrt{2}T}{8}$

(D) It cannot fall into the planet so time of fall of the satellite is meaningless.

13. An asteroid orbiting around a planet in circular orbit suddenly explodes into two fragments in mass ratio 1:4. If immediately after the explosion, the smaller fragment starts orbiting the planet in reverse direction in the same orbit, what will happen with the heavier fragment?

(A) Fall on the planet

(B) Start orbiting the planet in a larger orbit

(C) It's Angular momentum remains conserve during motion

(D) Escape from the gravitational interaction with the planet.

14. Two Earth's satellites move in a common plane along circular orbits. The orbital radius of one satellite is r while that of the other satellite is $r-\Delta r$ (Here $\Delta r << r$).

(A) Time interval separates the periodic approaches of the satellites to each other over the minimum distance is $\frac{4\pi r^{5/2}}{3(GM)^{1/2}\Delta r}$

(B) Time interval separates the periodic approaches of the satellites to each other over the minimum distance is $\frac{2\pi r^{5/2}}{3(GM)^{1/2}\Delta r}$

(C) Angular velocity of approach between two satellite is $\frac{3(GM)^{1/2}\Delta r}{r^{5/2}}$

(D) Angular velocity of approach between two satellite is $\frac{3(GM)^{1/2}\Delta r}{2r^{5/2}}$

15. A student can trow a ball at a speed on earth which can just cross a river of width 10 m. He reaches on an imaginary planet whose mean density is twice of the earth. Find out the maximum possible radius of planet so that if he throws the ball at same speed it may escape from planet. Given radius of earth $=6.4\times10^{6}$ m.

(A) 2×10^{3} m (B) 3×10^{3} m (C) 4×10^{3} m (D) 5×10^{3} m

EXERCISE-V

1. A cavity of radius $\frac{R}{2}$ is made inside a solid sphere of radius R. The centre of the cavity is located at a distance $\frac{R}{2}$ from the centre of the sphere. The gravitational force on a particle of mass 'm' at a distance $\frac{R}{2}$ from the centre of the sphere on the line joining both the centres of sphere and cavity is $\frac{\alpha mg}{\alpha\beta}$. Find $\alpha+\beta$? (opposite to the centre of cavity) :

[Here $g=\frac{GM}{R^2}$, where M is the mass of the solid sphere]

2. The Sun travels in approximately circular orbit of radius R around the center of the galaxy and completes one revolution in time T. The Earth also revolves around the Sun in time t. Assume orbit of the Earth to be a circle of radius $r\left(r<<R\right)$ and whole mass of the galaxy centered on its center. By using only these given informations, find an expression for the ratio of the mass of the galaxy to that of the Sun :

(A) $\left(\frac{R}{r}\right)^3\left(\frac{t}{T}\right)^2$ (B) $\left(\frac{R}{r}\right)^3\left(\frac{T}{t}\right)^2$

(C) $\left(\frac{R}{r}\right)^2\left(\frac{t}{T}\right)^3$ (D) $\left(\frac{R}{r}\right)^2\left(\frac{T}{t}\right)^3$

3. A smooth tunnel is dug along the radius of the earth that ends at the centre and a ball is released from the surface of earth along the tunnel. If the coefficient of restitution is 0.2 between the surface and ball then the distance travelled by the ball before second collision at the centre is :

(A) $\frac{6}{5}R$ (B) $\frac{7}{5}R$ (C) $\frac{9}{5}R$ (D) $\frac{3}{2}R$

4. A satellite of mass m is orbiting a planet of mass M at a radial distance r_0 from the centre of the planet. The satellite expels a small mass in a direction opposite to its orbital velocity. The immediate recoil velocity of the satellite exceeds the initial orbital velocity by Δv. The largest possible value of Δv for which the satellite remains within the gravitational field of the planet is $\left(\sqrt{\alpha}-1\right)\sqrt{\frac{GM}{r_0}}$.

Find α ?

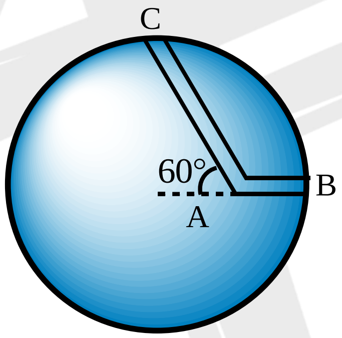
5. Consider a spherical homogeneous cloud of mass M made by an explosion. Due to energy received from the explosion, it is expansing and the expansion is spherically symmetric. At an instant its radius is R_0 and all the particle on the surface are moving away from its center with velocity V_0. Its radius when expansion ceases is $\frac{\alpha GMR_0}{\beta GM - R_0 V_0^{\gamma}}$. Find $\alpha + \beta + \gamma$?

6. A star is modeled as a uniform spherical distribution of matter. How gravitational pressure on surface depends on volume of the star?

(A) $P \propto V$ (B) $P \propto V^{-1/3}$ (C) $P \propto V^{-2/3}$ (D) $P \propto V^{-4/3}$

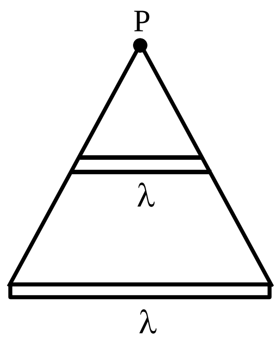
7. Two frictionless tunnels AB and AC are dug inside the earth as shown in the figure. A is at $\frac{R}{2}$ distance from centre of earth along the line connecting centre and B. A particle of mass m_1 is projected along AB another particle of mass m_2 is projected along AC such that both escape. Then ratio of their minimum velocities, so that the particle escapes the gravitational field of earth :

(Assume tunnel to be small and their is no loss of energy of particle during its collision with tunnel)



(A) 1 : 1 (B) 2 : 1 (C) 1 : 2 (D) None of these

8. The two sticks shown in diagram have same linear mass density λ, and they subtend same angle at point P. Their distances from point P are ℓ and 2ℓ. Which stick creates a larger force on a point mass placed at point P?



(A) The top stick (B) The bottom stick

(C) They produce equal forces (D) The relative magnitude of forces depends on ℓ

9. Two spherical bodies of masses M and 2M and radii R and 2R, respectively, start approaching each other at time t = 0 from rest, due to mutual gravitational attraction. They were initially very far away from each other. They collide at time t = T.

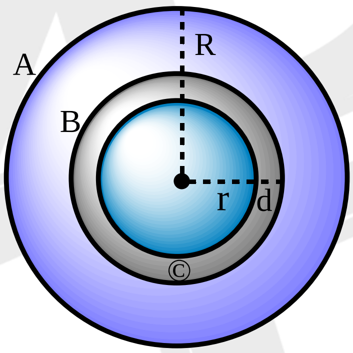
(A) Distance travelled by their COM till time t = T is zero (COM – centre of mass)

(B) Their relative velocity at time t = T is $\frac{1}{3}\sqrt{\frac{2GM}{R}}$

(C) PE of the system decreases as the bodies approach each other

(D) In COM frame, speeds of the bodies are always equal.

10. 'A' is a huge planet with uniform mass distribution, mass M, radius R. B is a circular tunnel made in it, concentric with planet. Radius of tunnel is r, $\left(r=\frac{R}{2}\right)$ cross – sectional diameter is d $(d<<r)$. C is a small ball of mass m $(m<<M)$ which is moving freely inside the tunnel without friction in a uniform circular motion, take acceleration due to gravity on surface of A as g



(A) If contact force on C is double the gravitational force, its time period is $2\pi\sqrt{\frac{R}{3g}}$

(B) If contact force on C is double the gravitational force, its time period is $2\pi\sqrt{\frac{R}{g}}$

(C) If contact force is zero, time period T, radius of tunnel r, then $T \propto r$

(D) If contact force is zero, time period T, radius of tunnel r, then T would be independent of r.

11. A satellite is moving in circular orbit of r with speed v_0 and time period T_1. A particle of small mass is projected from satellite in direction of it's motion with relative speed $\left(\frac{2}{\sqrt{3}}-1\right)v_0$

(A) Maximum distance of particle from centre of earth is 2r

(B) Maximum distance of particle from centre of earth is $\left(\sqrt{5}-1\right)r$

(C) Time period of particle around earth $>T_1$

(D) Time period of particle around earth $<T_1$

12. Mass density of a planet varies as $p(r)=\frac{\rho_0 r}{R}$, where R is radius of planet and r is distance from centre of planet

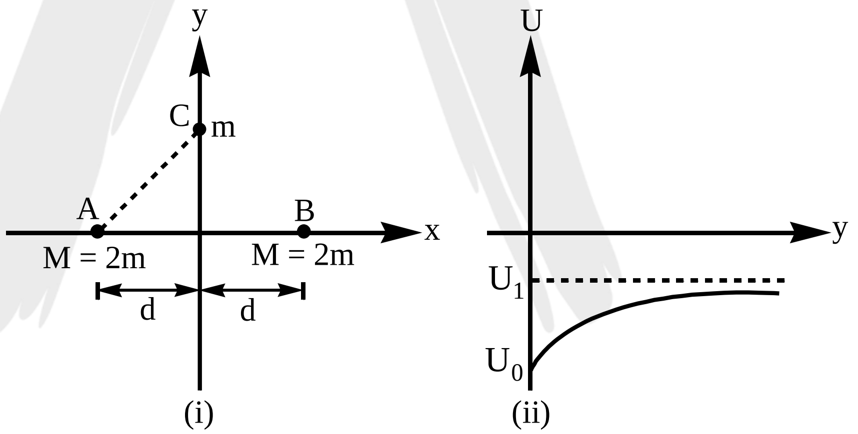
(A) A particle of mass m is to be projected from surface so as to escape the planet, its velocity of projection relative to surface is $\sqrt{2\pi G p_0 R^2}$

(B)) A particle of mass m is to be projected from surface so as to escape the planet, its velocity of projection relative to surface is $2\sqrt{2\pi G p_0 R^2}$

(C) Gravitational intensity on surface of planet is $2\pi G p_0 R$

(D) Time period of a satellite in circular orbit very close to planet is $2\pi\sqrt{\frac{R}{\pi G p_0}}$

13. Two point masses, each of mass M are fixed at points A and B respectively. A third point m is released from infinity, so that it can move along y – axis under the influence of mutual gravitational attraction on it due to point masses kept at A and B respectively as shown in the figure – 1. Figure – 2 represents the potential energy of system (includes m, M at A and M at B) with position of m at y – axis. (neglect any forces other than gravity on particle C) (Given: $Gm^2/d=12\,\text{joule}$, $m=6\,\text{kg}$). Choose the correct option(s)



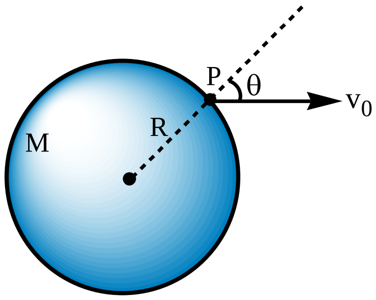
(A) Point mass m will perform periodic motion

(B) Value of U_1 must be equal to -24 joule if gravitational potential energy of two point mass system is taken to be zero when separation between them is infinite

(C) Maximum speed of particle is 24 m/s

(D) Maximum speed of particle is 4 m/s

14. Consider a planet of mass M and radius R as shown in the figure. A particle is projected from the surface of the planet at an angle θ from radial direction and with velocity equal to the orbital velocity of circular orbit at that point then choose the correct statement(s)



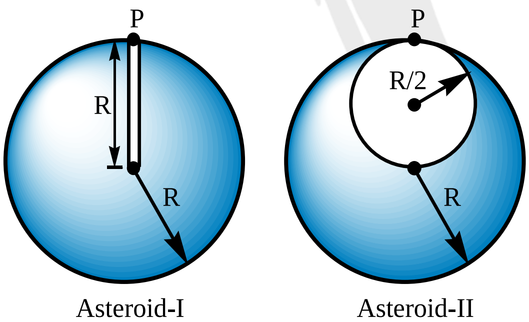
(A) Maximum distance of particle from centre of the planet is $(R + R\cos\theta)$

(B) Path of the particle is parabola

(C) Time taken by the particle to return to the surface of planet is $\sqrt{\frac{R^3}{GM}}$

(D) Time taken by the particle to reach the surface of planet is equal to $\pi\sqrt{\frac{R^3}{GM}}$

15. Consider two solid spherical asteroid of uniform density of mass M and radius R. in one asteroid a tunnel of very small size of depth R is bored to the centre and in other asteroid a spherical cavity of radius R/2 is made as shown in the figure. Now, identical particles of mass m dropped into the cavities of both asteroids from the top most point P. if force experienced by particle is F_I and F_{II} respectively in cavities of asteroids I and II, when they are x distance away from the centre of asteroids. If the time taken by particles to reach the centre of asteroids is T_I and T_{II} respectively then



(A) the ratio of $\frac{F_I}{F_{II}}$ is equal to $\frac{2x}{R}$

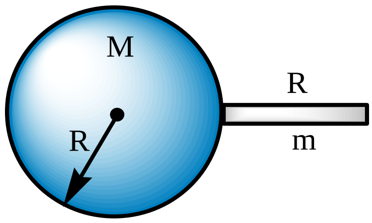
(B) the ratio of $\frac{F_I}{F_{II}}$ is equal to $\frac{x}{R}$

(C) the ratio of $\frac{T_I}{T_{II}}$ is equal to $\frac{\pi}{4}$

(D) the ratio of $\frac{T_I}{T_{II}}$ is equal to $\frac{\pi}{2}$

PROFICIENCY TEST-I

1. A uniform thin rod of mass m and length R is placed normally on surface of earth as shown. The mass of earth is M and its radius is R. Then the magnitude of gravitational force exerted by earth on the rod is :



(A) $\frac{GMm}{2R^2}$ (B) $\frac{GMm}{4R^2}$ (C) $\frac{4GMm}{9R^2}$ (D) $\frac{GMm}{8R^2}$

2. A tunnel is dug along the diameter of the earth (Radius R and mass M). There is a particle of mass 'm' at the centre of the tunnel. Find the minimum velocity given to the particle so that it just reaches to the surface of the earth :

(A) $\sqrt{\frac{GM}{R}}$

(B) $\sqrt{\frac{GM}{2R}}$

(C) $\sqrt{\frac{2GM}{R}}$

(D) it will reach with the help of negligible velocity

3. GG sir once went on the spherical planet named B-612, the density of which is 5200 kg/m^3. He noticed that if he quickens his pace, he feels himself lighter. When he reached the speed of 2 m/s he became weightless, and began to orbit about the planet as a satellite. The escape speed on the surface of planet is $n\sqrt{2}$ m/s. Find n ?

4. In older tmes, people used to think that the Earth was flat. Imagine that the Earth is indeed not a sphere of radius R, but an infinite plate of thickness H. What value of H is needed to allow the same gravitational acceleration to be experienced as on the surface of the actual Earth? (Assume that the Earth's density is uniform and equal in the two models.)

(A) $\frac{2R}{3}$ (B) $\frac{4R}{3}$

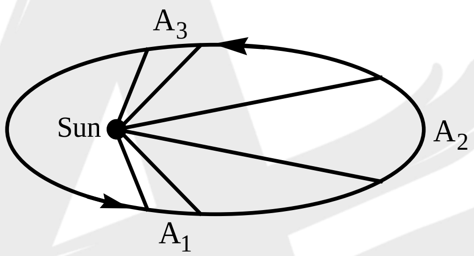
(C) $\frac{8R}{3}$ (D) $\frac{R}{3}$

5. Six starts of equal mass m moving about the centre of mass of the system such that they are always on the vertices of a regular hexagon of side length a. Their common time period of revolution around centre will be :

(A) $4\pi\sqrt{\frac{a^3}{Gm}}$ (B) $2\pi\sqrt{\frac{4\sqrt{3}a^3}{Gm\left(5\sqrt{3}+4\right)}}$

(C) $2\pi\sqrt{\frac{3a^3}{Gm\left(5\sqrt{3}+4\right)}}$ (D) $2\pi\sqrt{\frac{3a^3}{Gm\left(5\sqrt{3}-4\right)}}$

6. A planet moving around sun sweeps area A_1 in 2 days, A_2 in 3 days and A_3 in 6 days. Then the relation between A_1, A_2 and A_3 is :



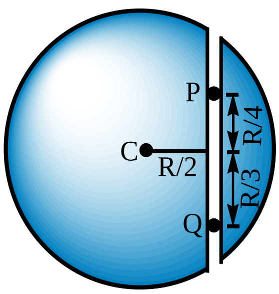
(A) $3A_1 = 2A_2 = A_3$ (B) $2A_1 = 3A_2 = 6A_3$

(C) $3A_1 = 2A_2 = 6A_3$ (D) $6A_1 = 3A_2 = 2A_3$

7. A satellite is revolving round the earth in an orbit of radius r with time period T. If the satellite is revolving round the earth in an orbit of radius $r+\Delta r$ $(\Delta r << r)$ with time period $T+\Delta T$ then :

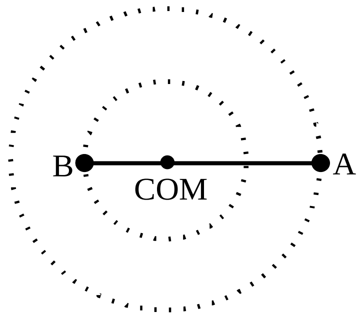
(A) $\frac{\Delta T}{T} = \frac{3}{2}\frac{\Delta r}{r}$ (B) $\frac{\Delta T}{T} = \frac{2}{3}\frac{\Delta r}{r}$ (C) $\frac{\Delta T}{T} = \frac{\Delta r}{r}$ (D) $\frac{\Delta T}{T} = -\frac{\Delta r}{r}$

8. A tunnel is dug along the chord of the earth at a perpendicular distance $\frac{R}{2}$ from the earth's centre. Walls of tunnel are frictionless. A particle of mass m is released into tunnel. Find the ratio of normal reactions at point P and Q between tunnel and particle as shwon in the figure.



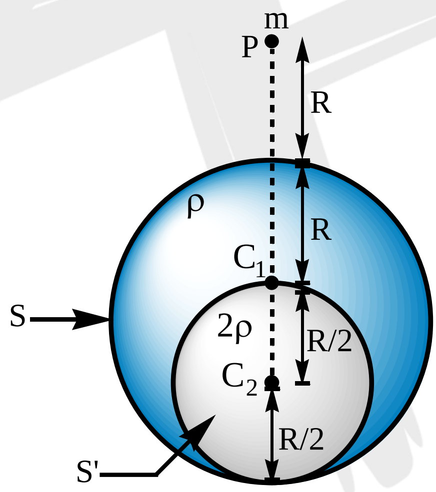
(A) 1 (B) 2 (C) 3 (D) 4

9. Figure shows a binary star system revolving about their COM. The masses of star A and B are 15×10^{30} kg and 45×10^{30} kg respectively. Find the ratio of area swept by star A to area swept by star B in a common time interval.



(A) 3 (B) 6 (C) 9 (D) 12

10. A composite solid sphere S of radius R has density 2ρ in spherical region S' of radius $\frac{R}{2}$ while remaining region has density ρ. Gravitational force exerted by this composite body on a point mass (m) placed as shown in the figure is $\pi R\rho Gm\left(\frac{K}{25}\right)$. Find K.



(A) 3 (B) 6 (C) 9 (D) 12

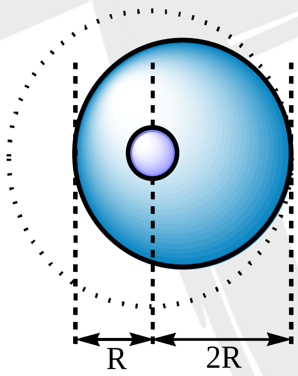
PROFICIENCY TEST-II

1. A satellite is seen after each 8 hours over equator at a place on the earth when its sense of rotation is opposite to the earth. The time (in hours) interval after which it can be seen at the place when the sense of rotation of earth and satellite is same will be :

2. Masses and radii of Earth and Moon are M_1, M_2 and R_1, R_2 respectively. The distance between their centre is 'd'. The minimum velocity given to mass 'M' from the midpoint of line joining their centre so that it will escape is $\sqrt{\frac{\alpha G(M_1+M_2)}{\beta d}}$. Find $\alpha+\beta$?

3. If a smooth straight tunnel is cut at any orientation through earth, then a ball released from one end will reach the other end in time $\frac{423}{n}$ minutes. Find n? (neglect earth rotation) :

4. Spaceman Fred's spaceship (which has negligible mass) is in an elliptical orbit about planet Bob. The minimum distance between the spaceship and the planet is R; the maximum distance between the spaceship and the planet is 2R. At the point of maximum distance, Spaceman Fred is travelling at speed v_0. He then fires his thrusters so that he enters a circular orbit of radius 2R. The new speed is $\sqrt{\frac{\alpha}{\beta}}v_0$. Find $\alpha+\beta$?



5. A planet moves round the sun in an elliptical orbit such that its K.E. is k_1 and k_2 when it is nearest to the sun and farthest from the sun respectively.

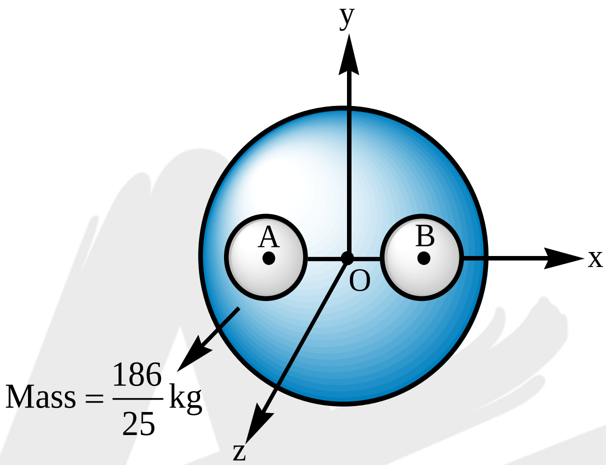
(A) If total energy of the planet is u, then ratio of largest distance (r_2) and smallest distance (r_1) between planet and the sun is $\frac{u-k_1}{u-k_2}$

(B) If total energy of the planet is u, then ratio of largest distance (r_2) and smallest distance (r_1) between planet and the sun is $\frac{u-k_2}{u-k_1}$

(C) If $r_2 = 2r_1$, the total energy of planet in terms of k_1 and k_2 is $2k_1 - k_2$

(D) If $r_2 = 2r_1$, the total energy of planet in terms of k_1 and k_2 is $2k_2 - k_1$

6. A solid sphere of uniform density and radius 4 m is loacted with its centre at the origin O of coordinate system. Two spheres of equal radii 1 m each, with their centres at $A(-2m, 0, 0)$ and $B(2m, 0, 0)$ are taken out of the solid sphere leaving behind spherical cavities as shown in the figure and remaining body has mass of $\frac{186}{23}$kg. Find the gravitational potential at the centre of the sphere is



(A) – 24
(B) – 34
(C) 24
(D) 34

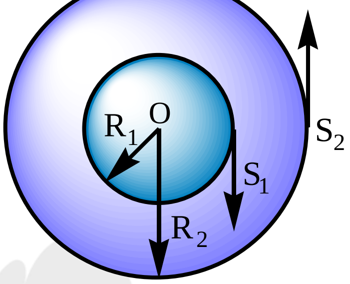
7. A solid sphere of mass M of radius R is moving with speed $\sqrt{\frac{GM}{R}} = 100\sqrt{10}\ ms^{-1}$ in free space. Due too application of external impulsive force, it breaks into eight spheres of same radius in such a way that the total mechanical energy before breaking and after breaking of sphere remains same. Find the speed of each sphere when separation between them is very large (assume at that instant speed of each sphere is same)

(A) 25 m/s
(B) 50 m/s
(C) 75 m/s
(D) 100 m/s

8. In some hypothetical space the gravitational field intensity is given by $\vec{g} = -2\vec{r}$, where $\vec{r}$ is position vector. If x-z plane is a rigid elastic plane and a particle of 1 kg mass is released from (1, 1, 0) position, find the magnitude of change in momentum of the particle during its first eventual elastic collision with plane,

(A) 2 N-m
(B) $2\sqrt{2}$ N-m
(C) 3 N-m
(D) $2\sqrt{3}$ N-m

9. Two satellites S_1 and S_2 revolve around a planet in coplanar circular orbit in the opposite sense. The periods of revolutions are $T = \frac{\pi}{3}$ seconds and ηT, where η is equal to 8 respectively. Find the angular speed of S_2 as observed by an astronaut in S_1, when they are closest to each other.



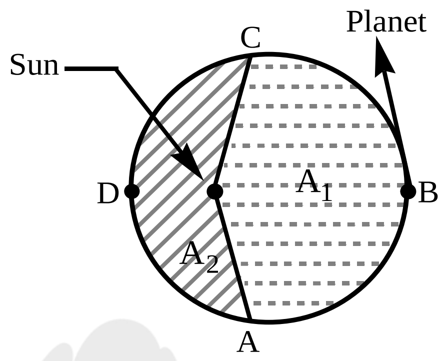
(A) 2 rad/s (B) 3 rad/s (C) 4 rad/s (D) 5 rad/s

10. A satellite of mass m is in a circular orbit around an airless spehrical planet of radius R. An astroid of equal mass m falls radially towards the planet starting at zero velocity from a very large distance. The satellite and the asteroid collide inelastically and stick together, moving in a new orbit such that it just misses the planet's surface. Find the radius of the satellite's original circular orbit is

(A) $\left(4+\sqrt{7}\right)R$ (B) $\left(4+\sqrt{11}\right)R$ (C) $\left(2+\sqrt{7}\right)R$ (D) $\left(2+\sqrt{11}\right)R$

PROFICIENCY TEST-III

1. Time taken by the planet to cover path ABC is t_1 time taken by the planet to cover path CDA is t_2.



(A) $t_1 = t_2$ (B) $t_2 > t_1$ (C) $t_1 > t_2$ (D) $t = 2t_2$

2. A hypothetical spherical planet of radius R and its density varies as $\rho = Kr$, where K is constant and r is the distance from the center. The pressure caused by gravitational pull inside (r < R) the planet at a distance r measured from its center given by :

(A) $\frac{1}{4}K^2G\pi\left(R^4 - r^4\right)$

(B) $\frac{1}{2}K^2G\pi\left(R^4 - r^4\right)$

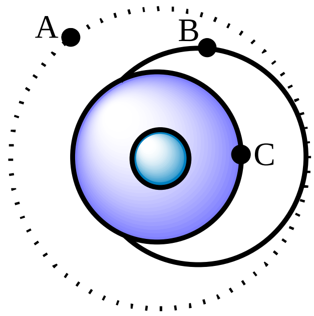
(C) $2K^2G\pi\left(R^4 - r^4\right)$

(D) $4K^2G\pi\left(R^4 - r^4\right)$

3. A small satellite of mass 'm' is revolving around earth in a circular orbit of radius r_0 with speed v_0. At certaion point of its orbit, the direction of motion of satellite is suddenly changed by angle $\theta = \cos^{-1}\left(\frac{3}{5}\right)$ by turning its velocity vector in the same plane of motion, such that speed remains constant. The satellite, consequently goes to elliptical orbit around earth. The ratio of speed at perigee to speed at apogee is :

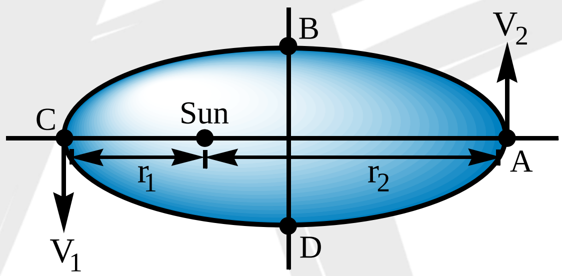
4. Assuming nothing else blocks their view, approximately how far (in km) can two people stand from each other in clear atmosphere until tey cannot longer see each other due to the curvature of the Earth? [Take : radius of earth = 6400 km]

5. Three equal mass satellites A, B and C are in coplanar orbits around a planet as shown in the figure. The magnitudes of the angular momenta of the satellites as measured about the planet are L_A, L_B and L_C. Which of the following statements is correct?



(A) $L_A > L_B > L_C$ (B) $L_C > L_B > L_A$ (C) $L_B > L_C > L_A$ (D) $L_B > L_A > L_C$

6. A planet moves around the sun in an elliptical orbit as shown. Ecentricity of ellipse is $\frac{1}{2}$. Time taken by planet to move from D to B $(D \to A \to B)$ and B to D $(B \to C \to D)$ are respectively T_{DAB} and T_{BCD} :



(A) $\frac{T_{DAB}}{T_{BCD}} = \frac{\pi + 2}{\pi - 2}$

(B) $\frac{T_{DAB}}{T_{BCD}} = \frac{2\pi + 1}{2\pi - 1}$

(C) Velocity of planet of point B is $\sqrt{3}V_2$

(D) Velocity of planet of point B is $\sqrt{2}V_2$

7. If gravitational forces alone prevent a spherical rotating neutron star from disintegrating. Find minimum time period (in sec) of star that has a mean density of magnitude $\left(\frac{9\pi}{20}\right) \times 10^{11}\ \text{kg/m}^3$, $G = \frac{20}{3} \times 10^{-11}\ \text{N-m}^2/\text{kg}^2$:

8. A planet of mass 'm' revolves around the sun (of mass M) in an elliptical orbit. The semi major axis of the orbit is 'a'. The total mechanical energy of the planet is $-\frac{\alpha GMm}{\beta a}$. Find $\alpha + \beta$?

9. In figure A, a stationary spacecraft of mass M is passed by asteroid A of mass m, asteroid B of the same mass m, and asteroid C of mass 2m. The asteroids move along the indicated straight paths at the same speed; the perpendicular distances between the spacecraft and the paths are given as multiples of R. Figure B gives the gravitational potential energy U(t) of the spacecraft-asteroid system during the passage of each asteroid treating time t = 0 as the moment when separation is minimum. Which asteroid corresponds to which plot of U(t)?

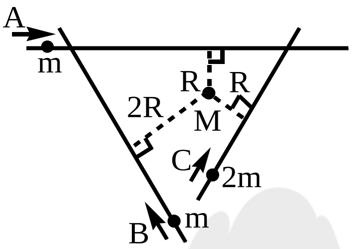


Figure A

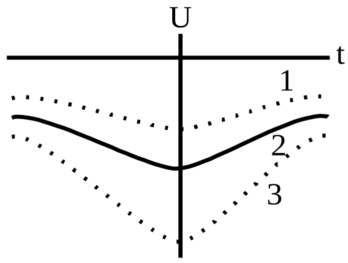


Figure B

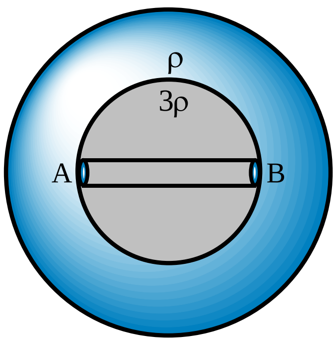
(A) A – 1, B – 2, C – 3 (B) B – 1, A – 3, C – 2

(C) B – 1, A – 2, C – 3 (D) None of these

10. The orbital properties of a geostationary satellite include (i) its orbit is directly over the Earth equator, and (ii) its orbital period is the same as the Earth's rotation period. Suppose a lunar stationary satellite (its orbital period is the same as the Moon's rotation period) is placed over the Moon equator, what is the value of the ratio $\frac{(R_{Earth}+H_{Earth})}{(R_{Moon}+H_{Moon})}$? R_{Earth} and R_{Moon} are the Earth and the Moon radii, H_{Earth} and H_{Moon} are the satellite heights from the Earth and from the Moon surfaces, respectively. You may assume $\frac{(\text{Earth mass})}{(\text{Moon mass})} = 81$ and rotation period of the Moon = 27 days.

(A) $\left(\frac{1}{9}\right)^{1/3}$ (B) $(81\times27)^2$ (C) $\left(\frac{81}{27}\right)^3$ (D) $\left(\frac{1}{9}\right)^3$

11. A planet of core denity 3ρ and outer crust of density ρ has small tunnel in core. A small particle of mass m is released from end A then time required to reach end B :



(A) $\sqrt{\frac{\pi}{\rho G}}$ (B) $\frac{1}{2}\sqrt{\frac{\pi}{\rho G}}$ (C) $\pi\sqrt{\frac{1}{\rho G}}$ (D) $2\pi\sqrt{\frac{1}{\rho G}}$

ANSWER KEY

EXERCISE-I_KEY

1	2	3	4	5	6	7	8	9	10
C	A	D	100	D	BC	B	A	B	A
11	**12**	**13**	**14**	**15**					
D	D	C	D	A					

EXERCISE-II_KEY

1	2	3	4	5	6	7	8	9	10
D	2	B	4	A	45	ACD	AB	ABC	A
11	**12**	**13**	**14**	**15**					
C	C	D	C	D					

EXERCISE-III_KEY

1	2	3	4	5	6	7	8	9	10
8	12	B	B	5	9	B	A	AC	AB
11	**12**	**13**	**14**	**15**					
AB	AD	AC	B	A					

EXERCISE-IV_KEY

1	2	3	4	5	6	7	8	9	10
B	4	40	7	B	15	3	BC	CD	AB
11	**12**	**13**	**14**	**15**					
CD	AC	CD	AD	C					

EXERCISE-V_KEY

1	2	3	4	5	6	7	8	9	10
7	A	B	2	6	D	A	A	AC	AD
11	**12**	**13**	**14**	**15**					
AC	AD	ABD	AD	AC					

PROFICIENCY TEST-I_KEY

1	2	3	4	5	6	7	8	9	10
A	A	2	A	B	A	A	A	C	C

PROFICIENCY TEST-II_KEY

1	2	3	4	5	6	7	8	9	10
24	5	10	5	AD	B	D	B	B	B

PROFICIENCY TEST-III_KEY

1	2	3	4	5	6	7	8	9	10
C	A	9	10	A	C	1	3	C	A
11									
B									