Nishant Jindal

No of terms in Multinomial

$$\frac{(n-1)(n)}{2} = {}^{n}C_2$$

$$\Rightarrow {}^{8}C_2 = \frac{8\cdot 7}{1\cdot 2}$$

$$^{n+2}C_2 = \frac{(n+1)(n+2)}{1\cdot 2}$$

$$(a+b+c)^n$$

$$= \left((a+b)+c\right)^n$$

$$= {}^{n}C_0 \underbrace{(a+b)^n}_{(n+1)\text{ term}} \cdot (c)^0 + {}^{n}C_1 \cdot \underbrace{(a+b)^{n-1}}_{n\text{ terms}} \cdot (c)^1 + {}^{n}C_2 \cdot \underbrace{(a+b)^{n-2}}_{(n-1)\text{ term}} \cdot c^2 + {}^{n}C_3 \cdot \underbrace{(a+b)^{n-3}}_{(n-2)\text{ term}} \cdot c^3 + \cdots + {}^{n}C_n \cdot \underbrace{(a+b)^0}_{1} \cdot c^n$$

$(n+1)$ term $+$ n terms $+$ $(n-1)$ term $+$ $(n-2)$ term $+$ $3+2+1$

$$\text{Total Terms} = 1+2+3+\cdots+(n)+(n+1) = \frac{(n+1)(n+1+1)}{2} = \frac{(n+1)(n+2)}{1\cdot 2} = {}^{(n+2)}C_2 = {}^{\text{deg}+\text{term}-1}C_{\text{term}-1}$$

Q No of terms in $(x+y+z)^6$

deg $= 6$

No of terms $= 3$ | No of terms $= {}^{6+3-1}C_{3-1} = {}^{8}C_2 = \frac{\overset{4}{\cancel{8}}\cdot 7}{1\cdot\cancel{2}} = 28$ terms

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Q $\left(x^2+\frac{1}{x}\right)^9$ find M.T.?

$n = 9$ odd

$T_{\frac{9+1}{2}}$ $\quad T_{\frac{9+3}{2}}$

$T_5 = {}^9C_4 (x^2)^5 \left(\frac{1}{x}\right)^4 = {}^9C_4 \times x^6$

$T_6 = {}^9C_5 (x^2)^4 \left(\frac{1}{x}\right)^5 = {}^9C_5 \cdot x^3$

Q Coff of $(r-1)^{th}$ term in Exp. of $(1+x)^{21}$

$T_{r-1} = {}^{21}C_{r-2} (1)^{21-r+2} \cdot (x)^{r-2} \Rightarrow$ Coff $= {}^{21}C_{r-2}$

Q If Coff. of r^{th} term & $(r-1)$ term of $(1+x)^{21}$ are Eql then $r = ?$

$T_r = {}^{21}C_{r-1} (1)^{21-r+1} \cdot (x)^{r-1} \rightarrow$ Coff $= {}^{21}C_{r-1}$

$T_{r-1} = {}^{21}C_{r-2} (1)^{21-r+2} (x)^{r-2} \rightarrow$ Coff $= {}^{21}C_{r-2}$

Acc QS $\rightarrow$ ${}^{21}C_{r-1} = {}^{21}C_{r-2}$

$r-1+r-2 = 21$

$2r = 24$

$\boxed{r = 12}$

Q. $\left(\frac{3}{x^2} - \frac{x^3}{6}\right)^9$ find M.T.?

$n = 9 =$ odd (2 M.T.)

$T_{\frac{9+1}{2}}$ $\quad T_{\frac{9+3}{2}}$

$T_5 = {}^9C_4 \left(\frac{3}{x^2}\right)^5 \left(-\frac{x^3}{6}\right)^4 = {}^9C_4 \frac{3^5}{6^4} \times \frac{\cancel{x^{12}}}{\cancel{x^{10}}} x^2$

$T_6 = {}^9C_5 \left(\frac{3}{x^2}\right)^4 \left(-\frac{x^3}{6}\right)^5 = {}^9C_5 \times \frac{3^4}{6^5} \times \frac{-\cancel{x^{15}}}{\cancel{x^8}} x^7$

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Q If 4^{th} term in Expansion of $\left(ax+\frac{1}{x}\right)^n$ is $\frac{5}{2}$ then $a, n = ?$

$$T_4 = {}^nC_3 \cdot (ax)^{n-3} \cdot \left(\frac{1}{x}\right)^3 = \frac{5}{2}$$

$$= {}^nC_3 \cdot (a)^{n-3} \cdot \frac{x^{n-3}}{x^3} = \frac{5}{2}$$

$$= {}^nC_3 \cdot (a)^{n-3} \cdot (x)^{n-6} = \frac{5}{2} \cdot (x)^0$$

Maha oros.

$$\boxed{\begin{aligned} n-6&=0 \\ n&=6 \end{aligned}}$$

$${}^6C_3 \cdot a^{6-3} = \frac{5}{2}$$

$$\frac{\cancel{6}^{2} \cdot 5 \cdot 4}{1 \cdot \cancel{2 \cdot 3}} \cdot (a)^3 = \frac{\cancel{5}}{2}$$

$$a^3 = \frac{1}{8} \Rightarrow \boxed{a = \frac{1}{2}}$$

Q If Coff. of $\boxed{x^7}$, $\boxed{x^8}$ in Exp of $\left(2+\frac{x}{3}\right)^n$ are Eql. then $n = ?$

$$\boxed{\frac{{}^nC_r}{{}^nC_{r-1}} = \frac{n-r+1}{r}}$$

$$T_{r+1} = {}^nC_r (2)^{n-r} \cdot \left(\frac{x}{3}\right)^r$$

$$= {}^nC_r \cdot (2)^{n-r} \cdot (3)^{-r} \cdot (x)^r$$

$$\text{Coff of } x^r = {}^nC_r \cdot 2^{n-r} \cdot (3)^{-r}$$

$$\text{Coff of } x^7 = {}^nC_7 \cdot 2^{n-7} \cdot (3)^{-7}$$

$$\text{Coff of } x^8 = {}^nC_8 \cdot 2^{n-8} (3)^{-8}$$

$${}^nC_7 \cdot \frac{\cancel{2^{n-7}}\,2}{\cancel{3^7}} = {}^nC_8 \cdot \frac{\cancel{2^{n-8}}}{\cancel{3^8}\,3} \Rightarrow \frac{{}^nC_8}{{}^nC_7} = 2 \times 3$$

$$\Rightarrow \frac{n-8+1}{8} = 6$$

$$n-7 = 48$$

$$\boxed{n = 55}$$

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Q Find 5^{th} term from End in $(x+2x^2)^8$?

Interesting Tariku.

$\boxed{5^{th} \text{ term from End in } (x+2x^2)^8 = 5^{th} \text{ term from Beginning of } (2x^2+x)^8}$

$T_5 = {}^8C_4 (2x^2)^4 (x)^4$

Q Find term independant to x in $\left(x+\frac{1}{x}\right)^6$?

$\underbrace{\text{term independant to } x}_{\text{देखो } x \text{ का deg}=0}$

$T_{r+1} = {}^6C_r (x)^{6-r} \left(\frac{1}{x}\right)^r$

$= {}^6C_r \cdot (x)^{6-2r=0}$ $\boxed{r=3}$

4^{th} term $= T_4$ is Ind. of x

Q Find term independant to x in $\left(\sqrt{\frac{x}{3}}+\sqrt{\frac{3}{2x^2}}\right)^{10}$

$T_{r+1} = {}^{10}C_r \left(\sqrt{\frac{x}{3}}\right)^{10-r} \cdot \left(\sqrt{\frac{3}{2x^2}}\right)^r$

$= {}^{10}C_r \cdot \left(\frac{x}{3}\right)^{\frac{10}{2}-\frac{r}{2}} \cdot \left(\frac{3}{2x^2}\right)^{\frac{r}{2}}$

$= {}^{10}C_r \cdot \frac{1}{3^{5-r/2}} \cdot \frac{(3)^{r/2}}{(2)^{r/2}}$

$x^{\boxed{5-\frac{r}{2}-r}=0}$

$\frac{3r}{2}=5$

$r=\frac{10}{3}$ (Int. X?) ✗

$\boxed{\text{No term is Ind. of } x}$

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Q Find coff of x^7 in $\left(x^2+\frac{1}{x}\right)^{11}$?

$$T_{r+1} = {}^{11}C_r \cdot (x^2)^{11-r} \cdot \left(\frac{1}{x}\right)^r$$

$$= {}^{11}C_r \cdot (x)^{22-2r-r}$$

$$= {}^{11}C_r \cdot (x)^{22-3r=7}$$

$3r = 15$

$r = 5$

$$\text{Coff} = {}^{11}C_r = {}^{11}C_5 = \frac{11 \cdot \cancel{10}^{2} \cdot \cancel{9}^{3} \cdot \cancel{8}^{2} \cdot 7}{1 \cdot \cancel{2} \cdot \cancel{3} \cdot \cancel{4} \cdot \cancel{5}}$$

$= 462$

Q Coff of x^{-6} in $\left(x^2+\frac{3a}{x}\right)^{15}$ → $n = 15$, $\alpha = 2$, $\beta = 1$

$m = -6$

$$r = \frac{15 \times 2 - (-6)}{2+1} = 12$$

$$T_{r+1} = {}^{15}C_r \cdot (x^2)^{15-r} \cdot \left(\frac{3a}{x}\right)^r$$

$$= {}^{15}C_r \cdot (x)^{30-2r-r} \cdot (3a)^r$$

$$= {}^{15}C_r \cdot (3)^r \cdot (a)^r \cdot (x)^{30-3r} = -6$$

$3r = 36$

$r = 12$

$$\text{Coff} = {}^{15}C_{12} \cdot (3)^{12} \cdot a^{12}$$

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Q Which term is Ind. of x in

$$\left(3x^2+\frac{2}{x}\right)^9$$

$$T_{r+1} = {}^9C_r \cdot (3x^2)^{9-r} \cdot \left(\frac{2}{x}\right)^r.$$

$$= {}^9C_r \cdot (3)^{9-r} (2)^r \cdot (x)^{18-2r-r=0}$$

$3r = 18$

$r = 6$

7^{th} term is Ind. of x.

$$\boxed{\begin{array}{l}\left(x^\alpha \pm \frac{1}{x^\beta}\right)^n \text{ me } x^m \text{ वाला term.} \\ r = \dfrac{n\alpha - m}{\alpha+\beta}\end{array}}$$

Q. term Ind of x in $\left(3x^2+\frac{2}{x}\right)^9$.

x^0

$\alpha = 2, \beta = 1, n = 9 \therefore m = 0$

$$r = \frac{9 \times 2 - 0}{2+1} = 6$$

Nishant Jindal

Q Find coff of term Ind. of x

in Exp. of $\left(\frac{\cancel{x+1}}{\cancel{x^{2/3}-x^{1/3}+1}}-\frac{x-1}{x-\sqrt{x}}\right)^{10}$?

$$=\left(x^{1/3}+1-\frac{\overset{(\sqrt{x}+1)}{\cancel{x-1}}}{\sqrt{x}\,\cancel{(\sqrt{x}-1)}}\right)^{10}$$

$$=\left(x^{1/3}+\cancel{1}-\left(\cancel{1}+\frac{1}{\sqrt{x}}\right)\right)^{10}$$

$$=\left(x^{1/3}-\frac{1}{x^{1/2}}\right)^{10}$$

$\alpha=\frac{1}{3},\ \beta=\frac{1}{2},\ n=10$

$m=0$

$$r=\frac{10\times\frac{1}{3}-0}{\frac{1}{3}+\frac{1}{2}}=\frac{\overset{2}{\cancel{10}}}{\cancel{3}}\times\frac{\overset{2}{\cancel{6}}}{\cancel{5}}=4$$

$\therefore \boxed{\text{Coff}={}^{10}C_4}$

$A^3+B^3=(A+B)(A^2-AB+B^2)$

$x+1$

$=(x^{1/3})^3+(1^{1/3})^3$

$=(x^{1/3}+1)\left((x^{1/3})^2-(x^{1/3})+1\right)$

$x+1=(x^{1/3}+1)(x^{2/3}-x^{1/3}+1)$

$$\frac{x+1}{x^{2/3}-x^{1/3}+1}=x^{1/3}+1$$

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Q Find n if coff of $2^{nd}, 3^{rd}, 4^{th}$ terms are in AP in Exp. of $(1+y)^n$.

$$\boxed{\begin{array}{l} T_{r+1} = {}^nC_r \cdot (1)^{n-r} \cdot (y)^r \\ \\ T_{r+1} = {}^nC_r \cdot y^r \end{array}}$$

① $T_2 = {}^nC_1 \cdot y$ | $T_3 = {}^nC_2 \cdot y^2$ | $T_4 = {}^nC_3 \cdot y^3$

② ${}^nC_1, {}^nC_2, {}^nC_3$ AP

$$\frac{{}^nC_1 + {}^nC_3}{2} = {}^nC_2 \Rightarrow n + \frac{(n)(n-1)(n-2)}{1\cdot 2\cdot 3} = \cancel{2}\cdot\frac{(n)(n-1)}{\cancel{1\cdot 2}}$$

$$\Rightarrow \frac{n(n^2-3n+2)+6n}{6} = n^2-n$$

$$n^3-3n^2+8n = 6n^2-6n$$

$$\cancel{n}(n^2-3n+8) = 6\cancel{n}(n-1)$$

$$n^2-3n-6n+8+6=0$$

$$n^2-9n+14=0$$

$$n = \cancel{2}, 7$$

$${}^2C_3 \quad ⊗$$

$$\boxed{n=7}$$

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Q Find coff of $x^{\boxed{15}}$ in $(1+x)^{15}+(1+x)^{16}+(1+x)^{17}+ - - +(1+x)^{30}$?

$(1+x)^{15} \xrightarrow{G.T.} {}^{15}C_r \cdot x^{\boxed{r}}$, $(1+x)^{16} \xrightarrow{G.T} {}^{16}C_r \cdot x^{\boxed{r}}$, $(1+x)^{17} \to {}^{17}C_r \cdot x^r$, $(1+x)^{30} \to {}^{30}C_r \cdot x^r$

$r=15$

$$\text{Coff} = \boxed{{}^{15}C_{15}} + {}^{16}C_{15} + {}^{17}C_{15} + {}^{18}C_{15} + - - + {}^{30}C_{15}$$

$${}^{16}C_{16} + {}^{16}C_{15} + {}^{17}C_{15} + {}^{18}C_{15} + - - + {}^{30}C_{15}$$

$$= {}^{31}C_{16} \text{ A}$$

1, 3, 4, 5, 6, 7, 8

9, 10, 11, 12, 13, 14

15, 16, 17, 18, 19

20, ~~21~~, 22 | 64, 65, 66, 67

Q Coff of x^{53} in $\sum_{m=0}^{100} {}^{100}C_m \cdot (x-3)^{100-m} \cdot 2^m$

$\Rightarrow$ Coff of x^{53} in $\left((x-3)+2\right)^{100}$

in $(x-1)^{100} \longrightarrow T_{r+1} = {}^{100}C_r \cdot (x)^{100-r} \cdot (-1)^r$

$100-r=53$

$r=47$

$\therefore$ Coff $= {}^{100}C_{47}(-1)^{47} = -{}^{100}C_{47} = -{}^{100}C_{53}$