Nishant Jindal

QUADRATIC EQUATION

HW

(1) Draw graph.

A) $y = x^2 + 2x + 5$

2) $y = -2x^2 + 3x - 5$

3) $y = 2x^2 - x + 1$

Q. If α, β are Roots of Eqⁿ. $2x^2 - 5x + 3 = 0$ then $\alpha^2\beta + \beta^2\alpha = ?$

$\rightarrow \alpha+\beta = 5/2, \alpha\beta = 3/2$

$\frac{15}{2}, -\frac{15}{4}, \frac{15}{4}, -\frac{15}{2}$

$\alpha\beta(\alpha+\beta)$

$\frac{3}{2}\left(\frac{5}{2}\right)$

Q. α, β are Roots of $P(x^2+n^2) + pnx + qn^2x^2 = 0$ then value of $P(\alpha^2+\beta^2) + P\alpha\beta + q\alpha^2\beta^2 = ?$

$x^2(P+qn^2) + Pnx + Pn^2 = 0$ $\alpha+\beta = -\frac{Pn}{P+qn^2}$

$\alpha\cdot\beta = \frac{Pn^2}{P+qn^2}$

$P((\alpha+\beta)^2 - 2\alpha\beta) + P\alpha\beta + q(\alpha\beta)^2$

$\alpha+\beta$ $\quad 0 \quad$ $p+q$ $\quad$ $\alpha+\beta+p+q$

Q. α & β are Roots of $ax^2 - bx + c = 0$ then value of $(\alpha+1)(\beta+1) = ?$

$\alpha\beta + (\alpha+\beta) + 1$

$\alpha+\beta = b/a$ $\alpha\beta = c/a$

$\frac{c}{a} + \frac{b}{a} + 1$

$\frac{a+b+c}{a}$

$\frac{a-b+c}{a}, \frac{a+b-c}{a}, \frac{a+b+c}{a}, \frac{b-a+c}{a}$

Q If DOR of $x^2 - px + q = 0$ is 1 then $p^2 + 4q^2 = ?$

$4q^2 + 1 + 4q$

$(2q+1)^2$

$2q+3 \quad (1-2q)^2 \quad (1+2q)^2 \quad 2q-3$

$\frac{\sqrt{p^2-4q}}{1} = 1 \Rightarrow p^2 - 4q = 1 \Rightarrow p^2 = 1 + 4q$

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QUADRATIC EQUATION

Q If Roots α & β of Eq^n $x^2+px+q=0$ are Such that $3\alpha+4\beta=7$ & $5\alpha-\beta=4$ then $(p,q)=?$

$\rightarrow$ $p=-(\alpha+\beta)$

$=-2$

$q=\alpha\cdot\beta=1$

$3\alpha+4\beta=7$

$20\alpha-4\beta=16$

$\alpha=1=\beta$

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QUADRATIC EQUATION

Q Eqⁿ $x^2+ax+b=0$ has distinct non Zero Roots a & b then Min. value of $x^2+ax+b=?$

$x^2+ax+b=0 \to a, b$

$a+b=-\frac{a}{1}$ | $a\cdot b=b$

$\boxed{a=1}$

$1+b=-1$

$\boxed{b=-2}$

Expression $Z=x^2+1\cdot x-2$

$Z=x^2+x-2$ $\frac{1}{2}$

$=\left(x+\frac{1}{2}\right)^2-\left(\frac{1}{2}\right)^2-2$

$-2-\frac{1}{4}$

$=-\frac{9}{4}$

$\text{Exp}=\underbrace{\left(x+\frac{1}{2}\right)^2}_{\text{Min}=0}-\frac{9}{4}$

$\text{Min}=0-\frac{9}{4}$

$=-\frac{9}{4}$

1) For Max / Min Value of Algebric Exp. do $\frac{dZ}{dx}$ & Put $\frac{dz}{dx}=0$

2) Now put value of x in Z

M2

$Z=x^2+x-2$

$\frac{dZ}{dx}=2x+1-0=0$

$x=-\frac{1}{2}$

$Z_{Min}=\left(-\frac{1}{2}\right)^2+\left(-\frac{1}{2}\right)-2$

$=\frac{1}{4}-\frac{1}{2}-2$

$=\frac{1-2-8}{4}=-\frac{9}{4}$

Min

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QUADRATIC EQUATION

Q $Y = -2x^2 - 6x + 9$ has Max^m value $\frac{27}{2}$ at $x = -\frac{3}{2}$?

Max^m

$$\frac{dy}{dx} = -2 \times 2x - 6 \times 1 + 0$$

$$= -4x - 6 = 0$$

$$x = \frac{6}{-4} = -\frac{3}{2}$$

$$Y = -2\left(-\frac{3}{2}\right)^2 + \cancel{6}^{3} \times \left(+\frac{3}{\cancel{2}}\right) + 9$$

$$= -\frac{9}{2} + 9 + 9 = 18 - \frac{9}{2} = \frac{27}{2}$$

$$Y = -2\left(x^2 + 3x - \frac{9}{2}\right)$$

$$= -2\left(\left(x + \frac{3}{2}\right)^2 - \left(\frac{3}{2}\right)^2 - \frac{9}{2}\right)$$

$$= -2\left\{\left(x + \frac{3}{2}\right)^2 - \frac{9}{4} - \frac{9}{2}\right\}$$

$$= -2\left\{\left(x + \frac{3}{2}\right)^2 - \frac{27}{4}\right\}$$

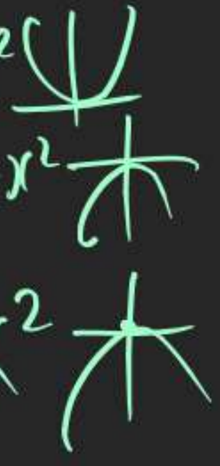
$$Y = -2\left(x + \frac{3}{2}\right)^2 + \frac{27}{2}$$

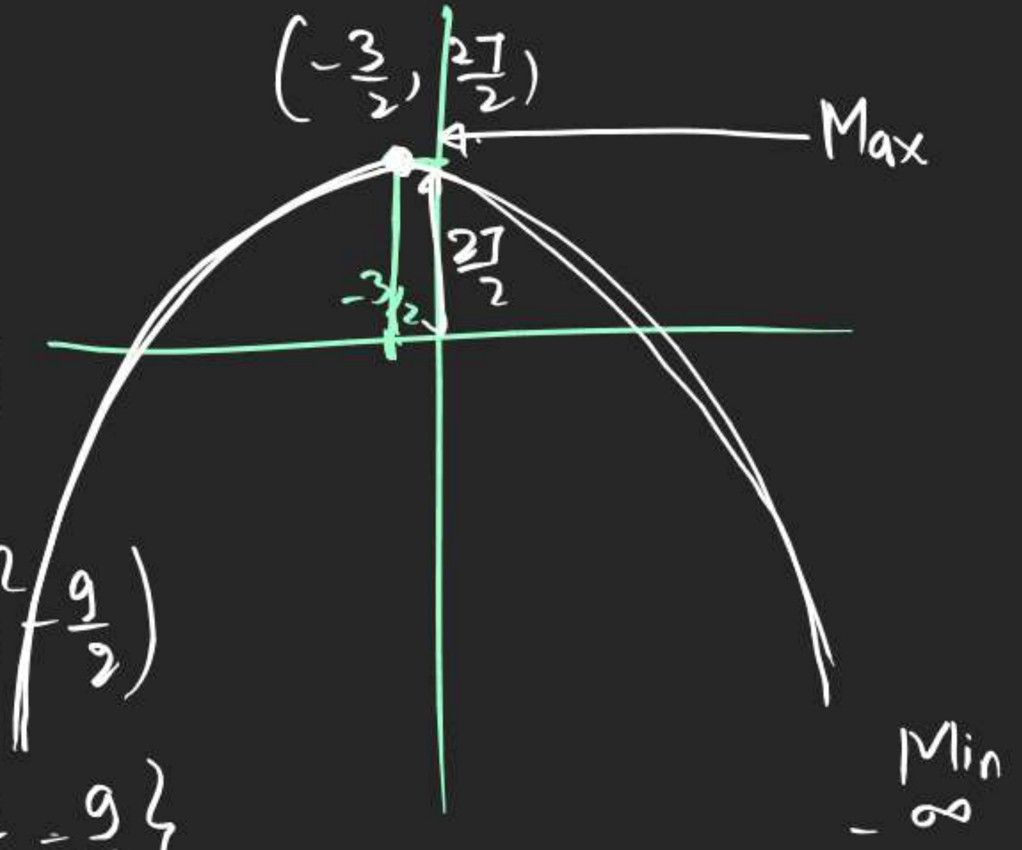
$$\left(Y - \frac{27}{2}\right) = -2\left(x + \frac{3}{2}\right)^2 \rightarrow Y = -2X^2$$

$$\left(-\frac{3}{2}, \frac{27}{2}\right)$$

$Y = x^2$

$Y = -x^2$





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QUADRATIC EQUATION

Q If $a>0, b>0, c>0$ then Roots of $ax^2+bx+c=0$ are?

$$\left.\begin{array}{l}\alpha+\beta=-\frac{b}{a}=-\frac{(+)}{(+)}=-ve \\ \alpha\cdot\beta=\frac{c}{a}=\frac{(+)}{(+)}=+ve\end{array}\right\}$$ Both Root $-ve$

Q. α, β are Roots of $ax^2+bx+c=0 \longrightarrow \alpha+\beta=-\frac{b}{a}$

then Eqn whose Roots are $2\alpha+3\beta$ & $3\alpha+2\beta$?

$$SOR=(2\alpha+3\beta)+(3\alpha+2\beta)=5(\alpha+\beta)=-\frac{5b}{a}$$

$$POR=(2\alpha+3\beta)(3\alpha+2\beta)=6\alpha^2+6\beta^2+13\alpha\beta$$

$$=6(\alpha^2+\beta^2)+13\alpha\beta$$

$$=6((\alpha+\beta)^2-2\alpha\beta)+13\alpha\beta=6(\alpha+\beta)^2+\alpha\beta$$

$$=\frac{6b^2}{a^2}+\frac{c}{a}$$

$$x^2-(SOR)x+POR=0$$

$$x^2+\frac{5b}{a}x+\frac{6b^2+ac}{a^2}=0$$

$$\alpha^3+\beta^3=(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)$$

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QUADRATIC EQUATION

Q If Q Eqⁿ $x^2+px+q=0$ has Roots $\tan 30°$ & $\tan 15°$ then $2+q-p=?$

$$x^2+px+q=0 \begin{cases} \to \tan 30° \\ \to \tan 15° \end{cases}$$

$SOR = \tan 30° + \tan 15° = -p$

$POR = \tan 30° \cdot \tan 15° = q$

$\tan(45°) = 1$

$\tan(30°+15°) = 1$

$$\frac{\tan 30° + \tan 15°}{1 - \tan 30° \cdot \tan 15°} = 1 \Rightarrow \frac{-p}{1-q} = 1$$

$-p = 1 - q \Rightarrow q - p = 1$

Demand

$2+(q-p)$

$2+1=3$

Q. Q Eqⁿ $(a^2-5a+3)x^2+(3a-1)x+2=0$ has 1 Root twice of other then $a=?$

$\alpha, 2\alpha$

$$\frac{b^2}{ac} = \frac{(k+1)^2}{k}$$

$$\frac{(3a-1)^2}{(a^2-5a+3)\,2} = \frac{(2+1)^2}{2}$$

$9\cancel{a^2}-6a+1 = 9(\cancel{a^2}-5a+3)$

$39a = 26$

$a = \frac{2}{3}$

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QUADRATIC EQUATION

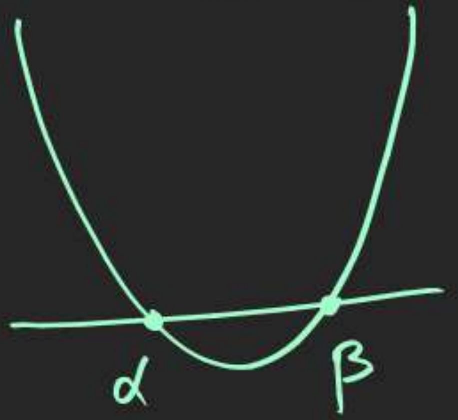
Nature of graph of Q Eqn

(1) $y = ax^2 + bx + c$

$a > 0$

Real & Unequal Roots

$D > 0$



(2) $a > 0$

Real & Equal Roots

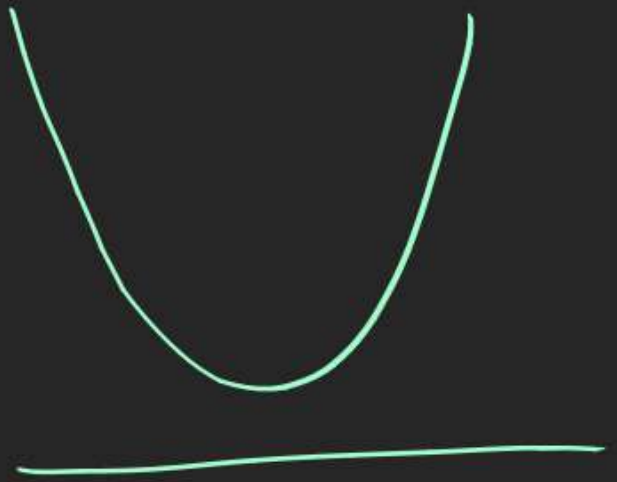
$D = 0$

(3) $a > 0$

Imaginary Roots

↓

$D < 0$



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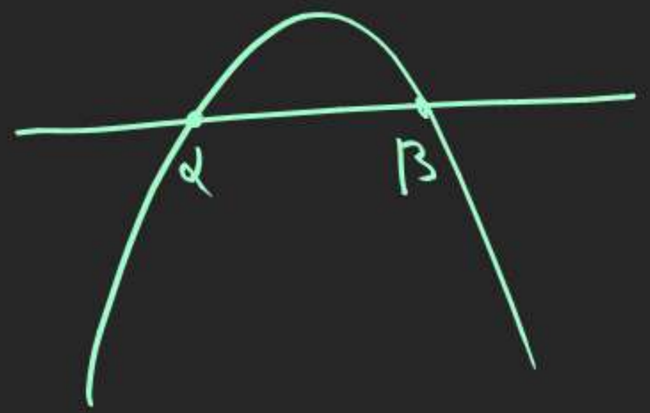
QUADRATIC EQUATION

$y = ax^2+bx+c$

1) When $a<0$

Real & unequal Roots

$D>0$



2) When $a<0$

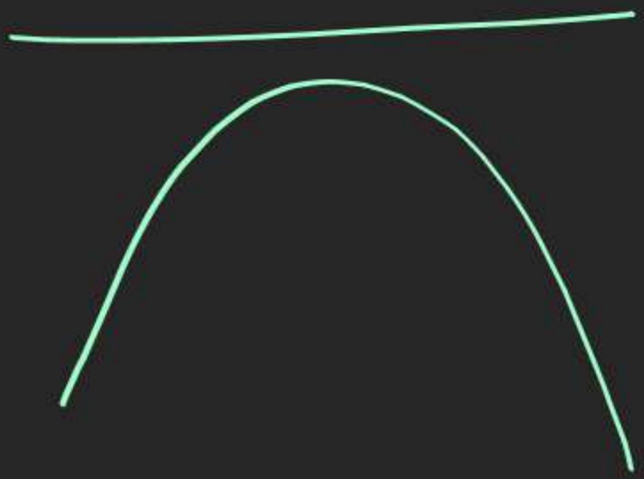
Real Roots & Equal

$D=0$

3) $a \leq 0$

Imaginary Roots.

$D<0$



Q. Why graph of Q Eqn depends on "a"?

Ans 1) It is because of Concavity —

- Concavity Up  $\rightarrow \frac{d^2y}{dx^2} > 0$
- Concavity down  $\rightarrow \frac{d^2y}{dx^2} < 0$

2) If $\frac{d^2y}{dx^2} > 0$ then graph of expression y is Con Up.

If $\frac{d^2y}{dx^2} < 0$ then graph of expression "y" is Con. down.

3) $y = ax^2 + bx + c$

$\frac{dy}{dx} = 2ax + b$

$\frac{d^2y}{dx^2} = \underline{2a}$

4) If $\underline{a > 0} \rightarrow \underline{2a > 0} \Rightarrow \frac{d^2y}{dx^2} > 0$  Upward Parabola.

If $\underline{a < 0} \Rightarrow 2a < 0 \Rightarrow \frac{d^2y}{dx^2} < 0$  downward Parabola

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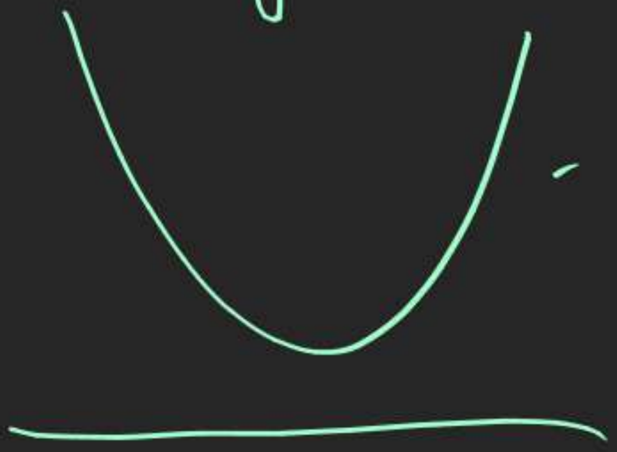
QUADRATIC EQUATION

2 Imp graph. $Y = ax^2+bx+c$

$a > 0$

$\boxed{D < 0}$

Imag. Roots



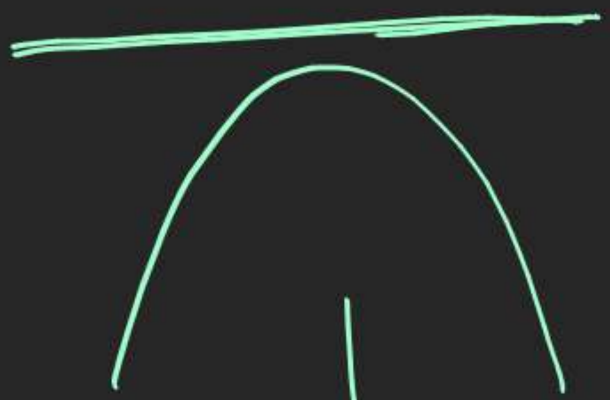
1) all Points of graph above X Axis $\rightarrow y > 0$

(2) $ax^2+bx+c > 0$

$a < 0$

$D < 0$

Imag. Roots



1) all Point of this graph is below X Axis $y < 0$

2) $ax^2+bx+c < 0$ for all x

$ax^2+bx+c > 0$ then $a > 0$
सारा graph X Axis के Upar $D < 0$

$ax^2+bx+c < 0 \rightarrow a < 0$ & $D < 0$
सारा graph below Axis

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QUADRATIC EQUATION

1) Expression ax^2+bx+c will be of same sign for all Real values of x if $D<0$

2) ax^2+bx+c will be +ve if $D<0$ & $a>0$

3) If $D>0$ then sign of expression betn Roots will be opp. to that of a.

4) If $a>0$ then Minm of $f(x)$ occurs at $x=-\frac{b}{2a}$.

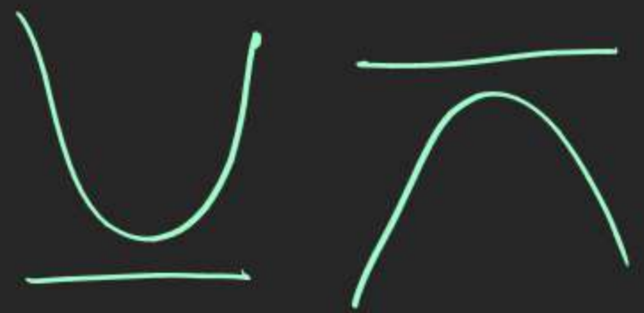
5) If $a<0$ then Maxm of $f(x)$ occurs at $x=-\frac{b}{2a}$.

6) Max & Minm values of fxn is $-\frac{D}{4a}$ always.

7) If ax^2+bx+c is -ve for all x then $a<0$ & $D<0$

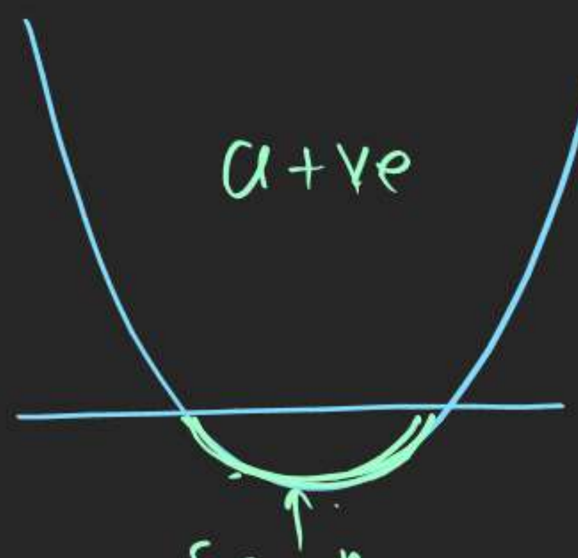
① If $D<0$ then How many graph are Possible.

②

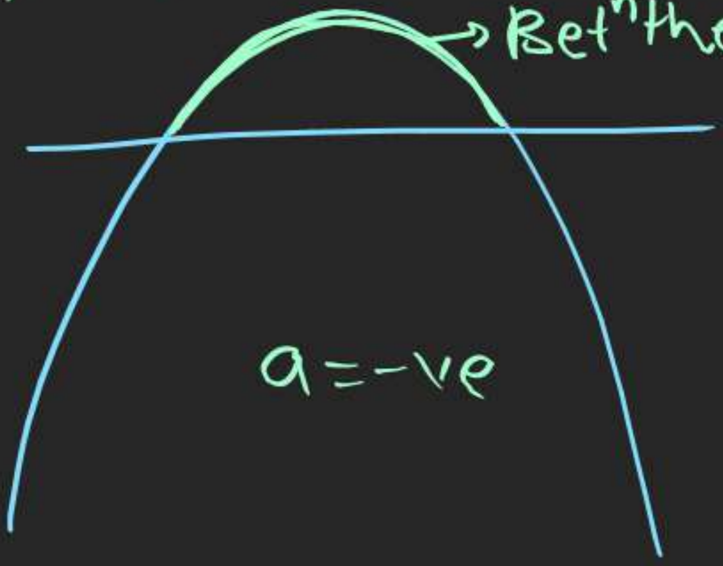


QUADRATIC EQUATION

Q. $D > 0$ How many Graphs are Possible?

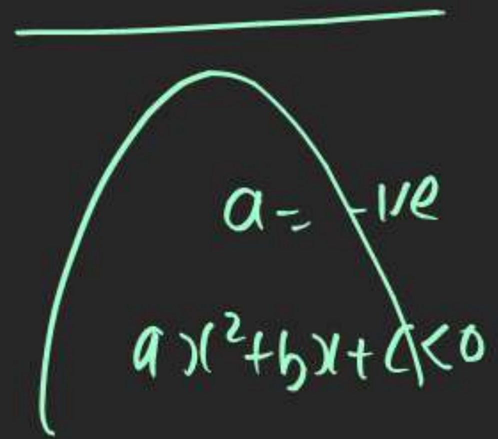


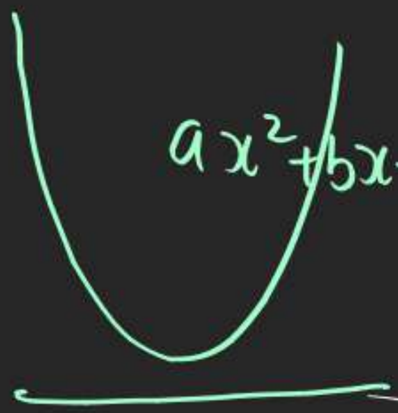
Bet^n the Roots वाला Part is Below X Axis } -ve



Bet^n the Roots वाला Part above X Axis } +ve

Q $D<0$





a +ve

Q If $\boxed{D<0}$ then Sign of $a(ax^2+bx+c) = ?$

$(-)(-) = +ve$ $(+)(+) = +ve$

+ve