Nishant Jindal

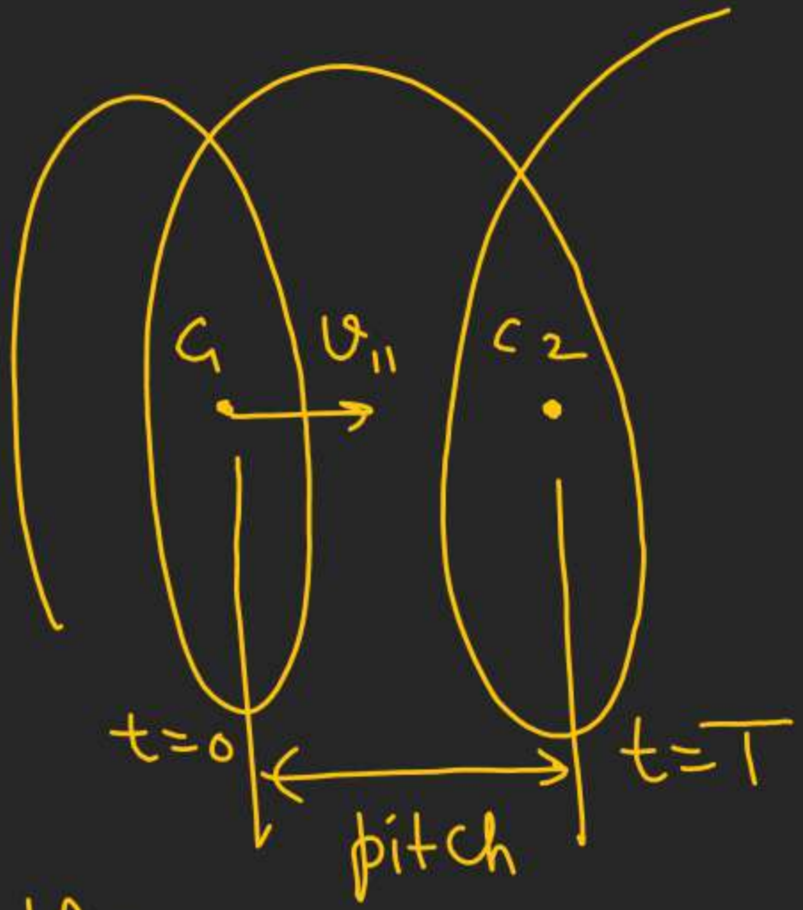
MAGNETIC FIELD

Motion of charge particle in a magnetic field

pitch $= \left[\underline{v_{\parallel} \times T} \right]$

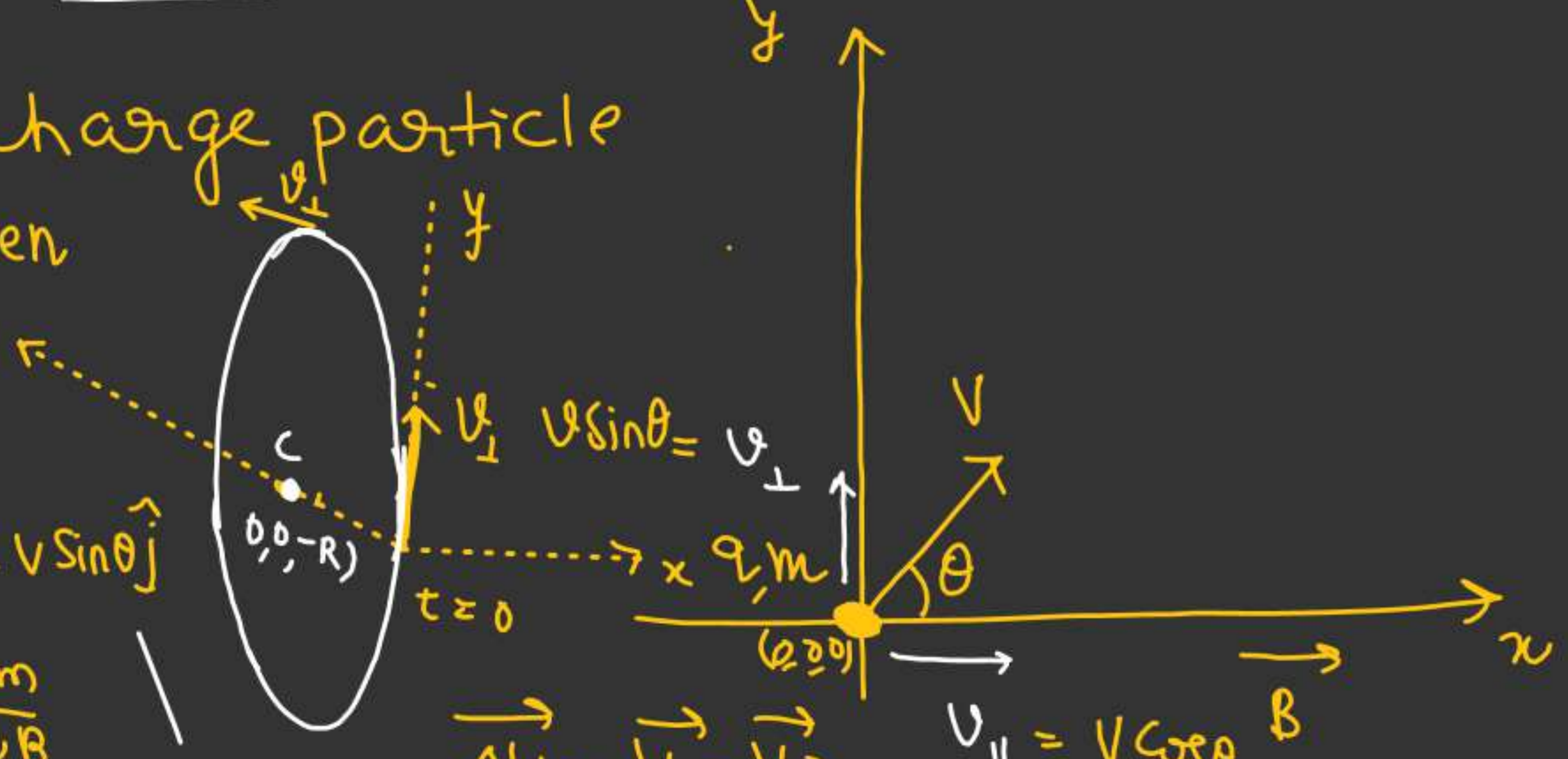
Uniform pitch $\Rightarrow$ when $v_{\parallel}$ is constant.

(If $v_{\parallel}$ is non uniform then variable pitch)



a) Find Change in velocity of a Charged particle in the interval $t=0$ to $t=\frac{\pi m}{2qB}$ when Charge particle is projected as shown in fig.

b) Also find $\vec{v}$ and $\vec{r}$ of the Charge particle at $t=\frac{\pi m}{3qB}$. $[\theta = 30^\circ]$ given



$$T = \frac{2\pi m}{qB}$$

$$t = \frac{\pi m}{2qB} \times \frac{2}{2} = \frac{T}{4}$$

$$\vec{V_i} = v\cos\theta\,\hat{i} + v\sin\theta\,\hat{j}$$

$(\vec{V_f})$ at $t = \frac{\pi m}{2qB}$

$$\vec{V_f} = v\cos\theta\,\hat{i} - v\sin\theta\,\hat{k}$$

$$\vec{\Delta V} = \vec{V_f} - \vec{V_i}$$

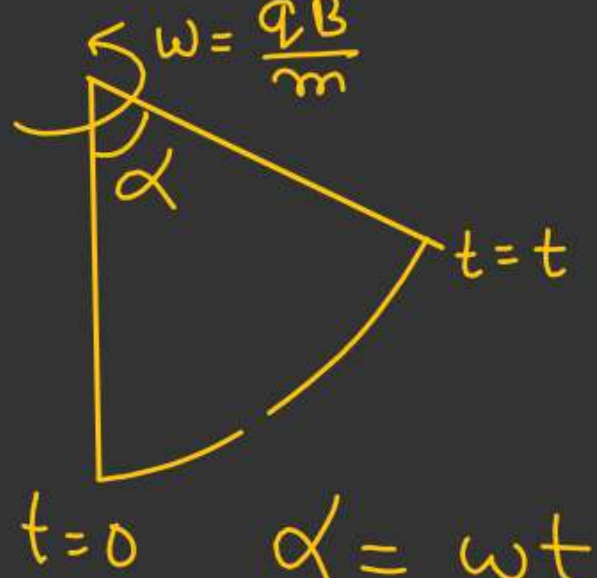
$$= -v\sin\theta\,\hat{k} - v\sin\theta\,\hat{j}$$

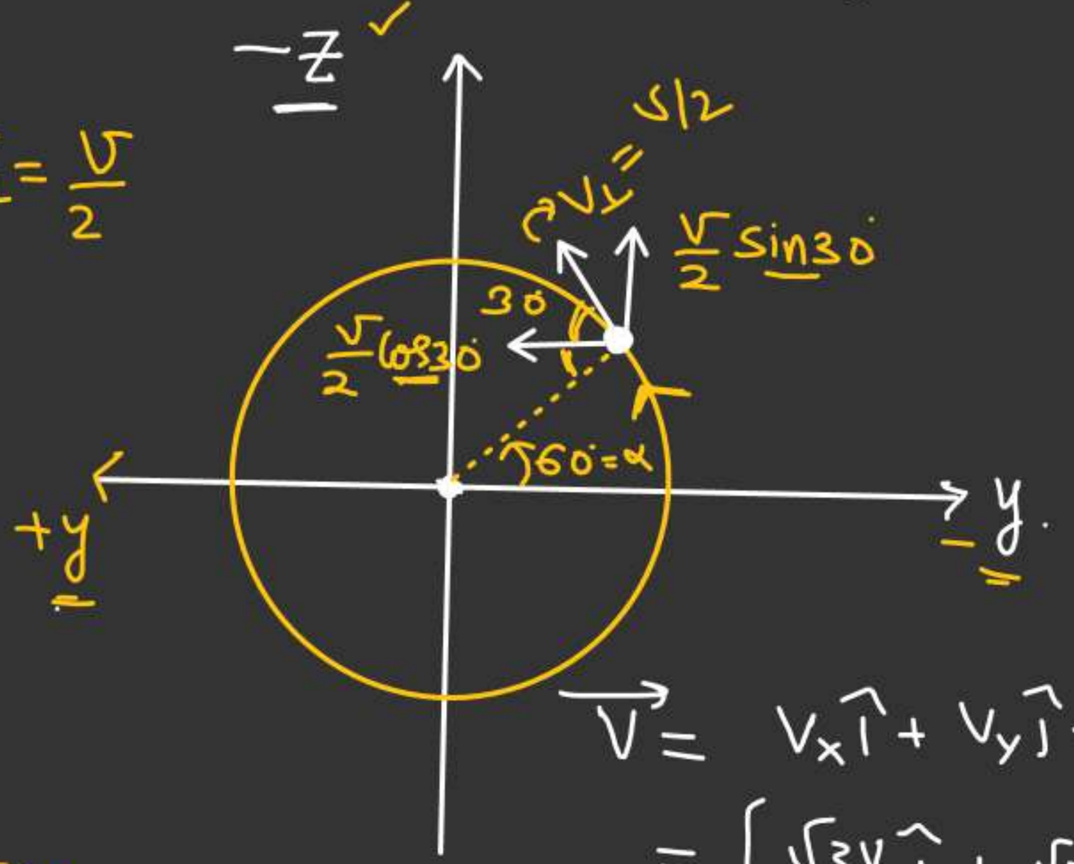
$$|\vec{\Delta V}| = \sqrt{2}\,v\sin\theta \quad \text{Ans.}$$

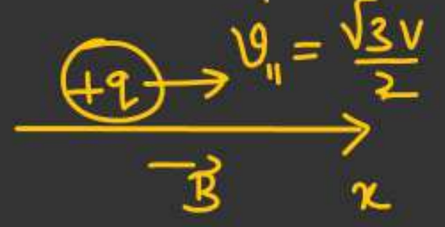
(b) $t = \left(\frac{\pi m}{3qB} \right)$ — $\vec{V}$ at $t = \frac{\pi m}{3qB}$.

$\vec{r}$, $\vec{v}$

$v_\perp = \frac{v}{2}$







$\alpha = \omega t$

$\alpha = \frac{qB}{m} \times \frac{\pi m}{3qB}$

$\alpha = \frac{\pi}{3}$

$\vec{V} = V_x\hat{i} + V_y\hat{j} + V_z\hat{k}$

$= \left[\frac{\sqrt{3}v}{2}\hat{i} + \frac{\sqrt{3}v}{4}\hat{j} - \frac{v}{4}\hat{k} \right]$

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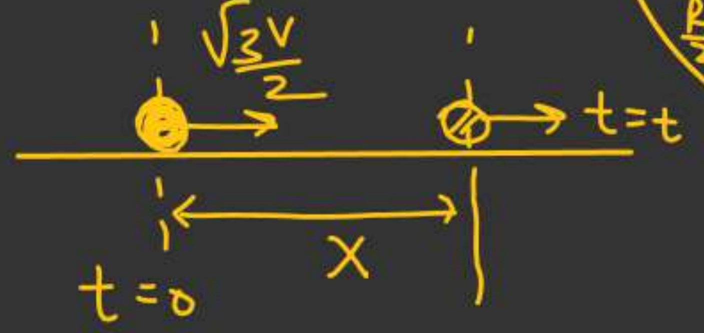
$$\vec{r} = X\hat{i} + Y\hat{j} - Z\hat{k}$$

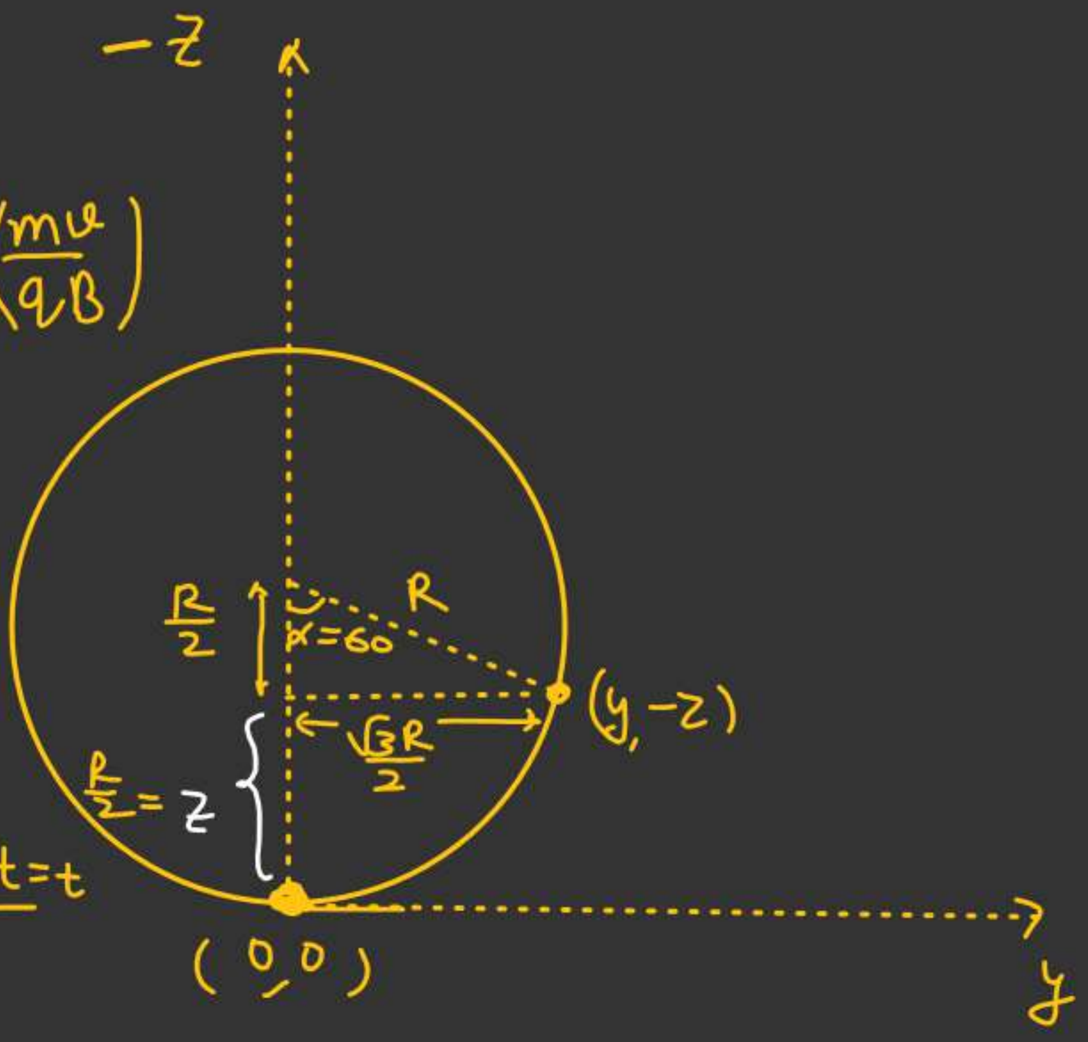
$$\left[\ \vec{r} = \frac{\pi m v}{2\sqrt{3}qB}\hat{i} + \frac{\sqrt{3}}{2}\left(\frac{mv}{qB}\right)\hat{j} - \left(\frac{mv}{2qB}\right)\hat{k}\ \right]$$

$$y = \left(\frac{\sqrt{3}R}{2}\right) = \frac{\sqrt{3}}{2}\left(\frac{mv}{qB}\right)$$

$$|Z| = \left(\frac{R}{2}\right)$$

$$= \frac{mv}{2qB}$$





$$X = \frac{\sqrt{3}v}{2} \times t$$

$$X = \frac{\sqrt{3}v}{2} \times \frac{\pi m}{3qB} = \left(\frac{\pi m v}{2\sqrt{3}qB}\right)$$

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MAGNETIC FIELD

Motion of charge particle in a magnetic field

- Case of Non-Uniform pitch

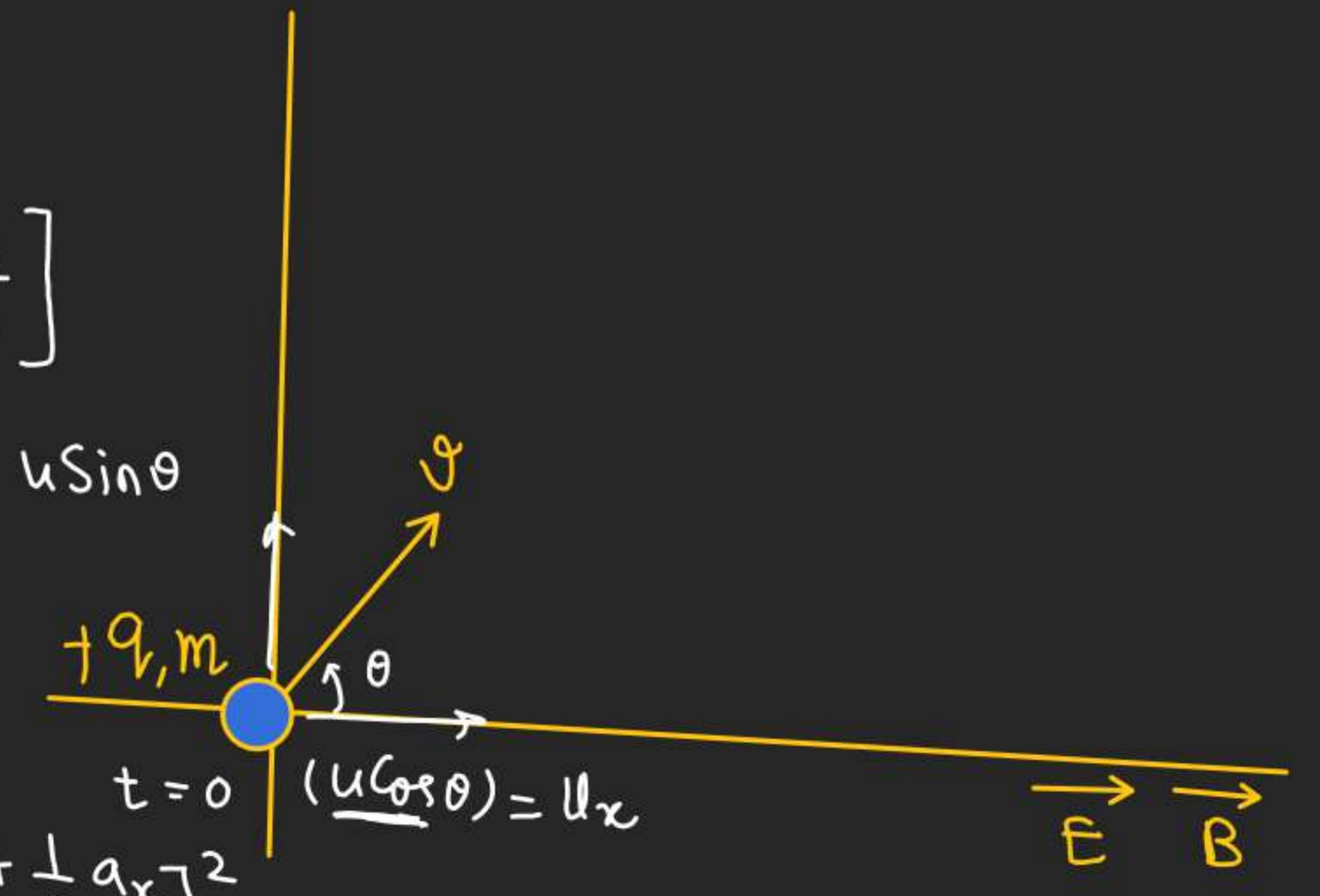
$\vec{E} = E\hat{i}, \quad \vec{B} = B\hat{i}$

$\vec{a}_x = \dfrac{qE}{m}\hat{i}$

$[S = ut + \frac{1}{2}at^2]$

$\Downarrow$

Distance in x-direction



$$\left[p_1 = (u\cos\theta)T + \frac{1}{2}a_x T^2\right]$$

V_x in time $t = T$

$V_x = (u\cos\theta) + a_x T$

$\Downarrow$

[Initial velocity for next pitch]

$$p_2 = V_x T + \frac{1}{2}a_x T^2$$

$$p_2 = [u\cos\theta + a_x T]T + \frac{1}{2}a_x T^2$$

$$p_2 = (u\cos\theta)T + a_x T^2 + \frac{1}{2}a_x T^2$$

$$p_2 = \left[(u\cos\theta)T + \frac{1}{2}a_x T^2\right] + a_x T^2$$

$$p_2 = (p_1 + a_x T^2)$$

MAGNETIC FIELD

Motion of charge particle in a magnetic field

$p_3 = ?$

Total time

$(V_x)_2$ in the time $t = 2T$

interval

$(V_x)_2 = (V_x)_1 + a_x T$

$(V_x)_2 = (u\cos\theta + a_x T) + a_x T$

$(V_x)_2 = [u\cos\theta + 2a_x T]$

$\Downarrow$

Initial velocity of 3rd pitch.

$p_3 = (V_{x_2})T + \frac{1}{2}a_x T^2$

$p_3 = (u\cos\theta + 2a_x T)T + \frac{1}{2}a_x T^2$

$p_3 = \left[(u\cos\theta)T + \frac{1}{2}a_x T^2\right] + 2a_x T^2$

$\Downarrow$

$p_3 = p_1 + 2a_x T^2$

$\Downarrow$

$$\boxed{p_n = p_1 + (n-1)a_x T^2}$$ **

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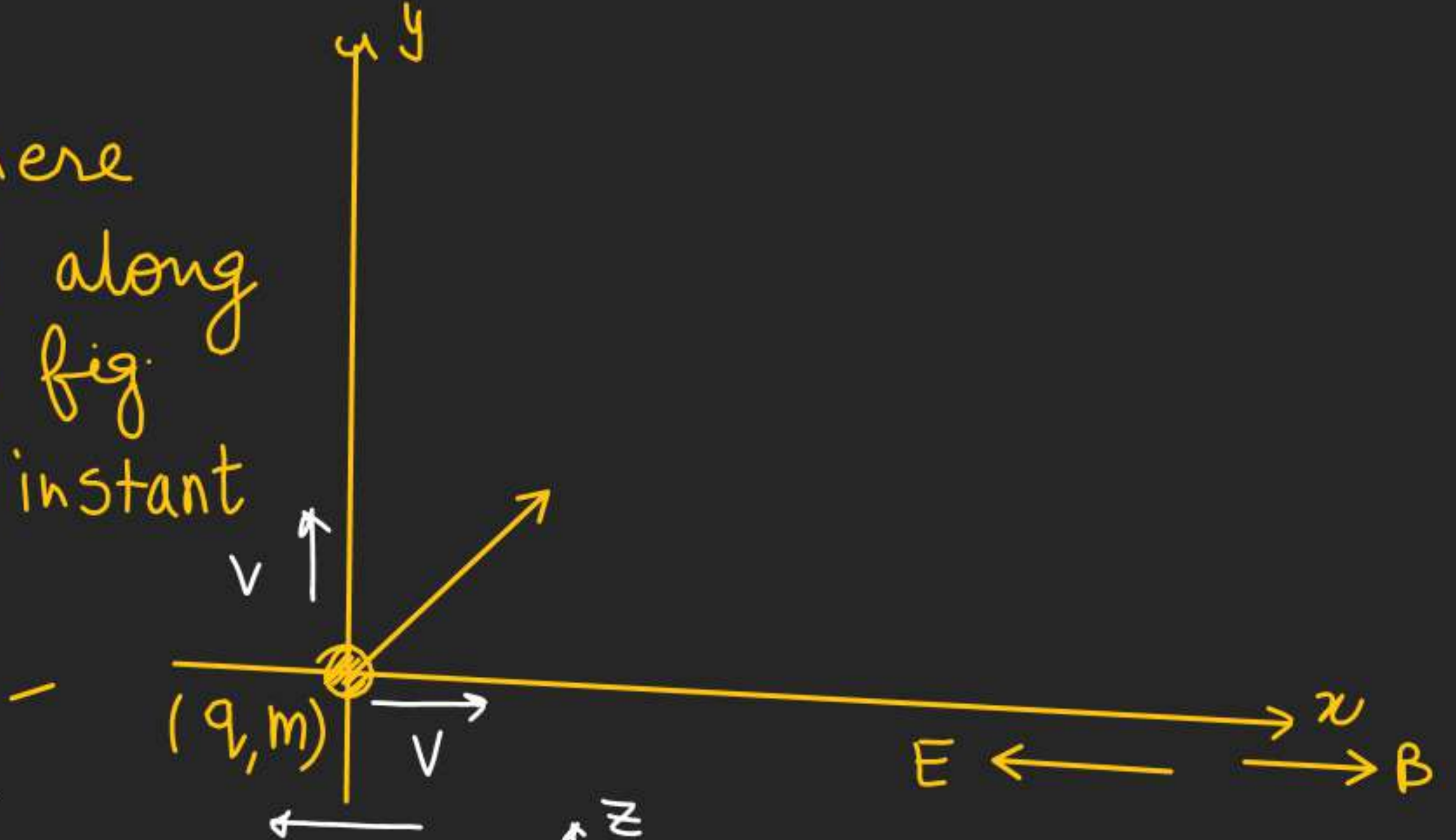
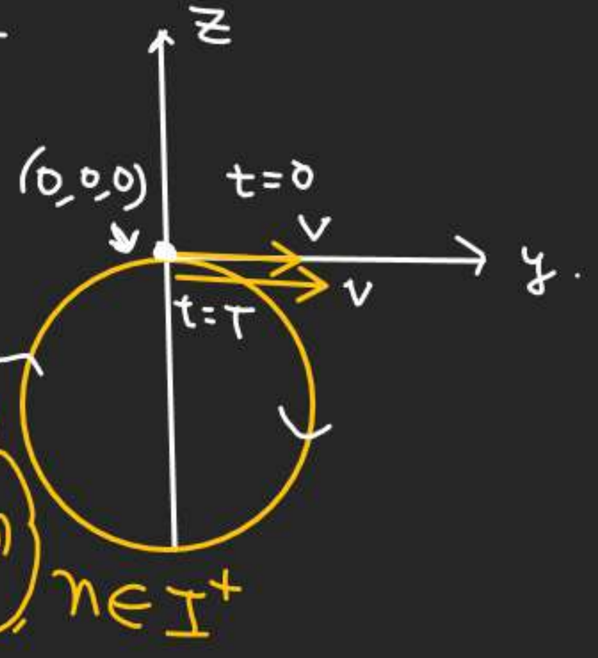
MAGNETIC FIELD

Motion of charge particle in a magnetic field

** A Charge particle having velocity $\vec{V} = v\hat{i} + v\hat{j}$ is projected where Electric field and magnetic field along $-x$ & $+x$ direction as shown in fig. Find possible value of $\frac{E}{B}$ if at any instant net velocity of charge partic is $[v\hat{j}]$



$a_x = \frac{qE}{m}$



Sol^n For net velocity to become $v\hat{j}$.

$$\left[T = \left(\frac{2\pi m}{qB}\right)\right]$$

$$0 = V_x = V - \frac{qE}{m}t$$

$$\frac{qE}{m}t = V$$

$$t = \left(\frac{mV}{qE}\right)$$

$$t = (nT)$$

$$\frac{\cancel{m}v}{\cancel{q}E} = n\frac{2\pi \cancel{m}}{\cancel{q}B} \Rightarrow \frac{E}{B} = \frac{v}{2\pi n}$$

$n \in I^+$

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MAGNETIC FIELD

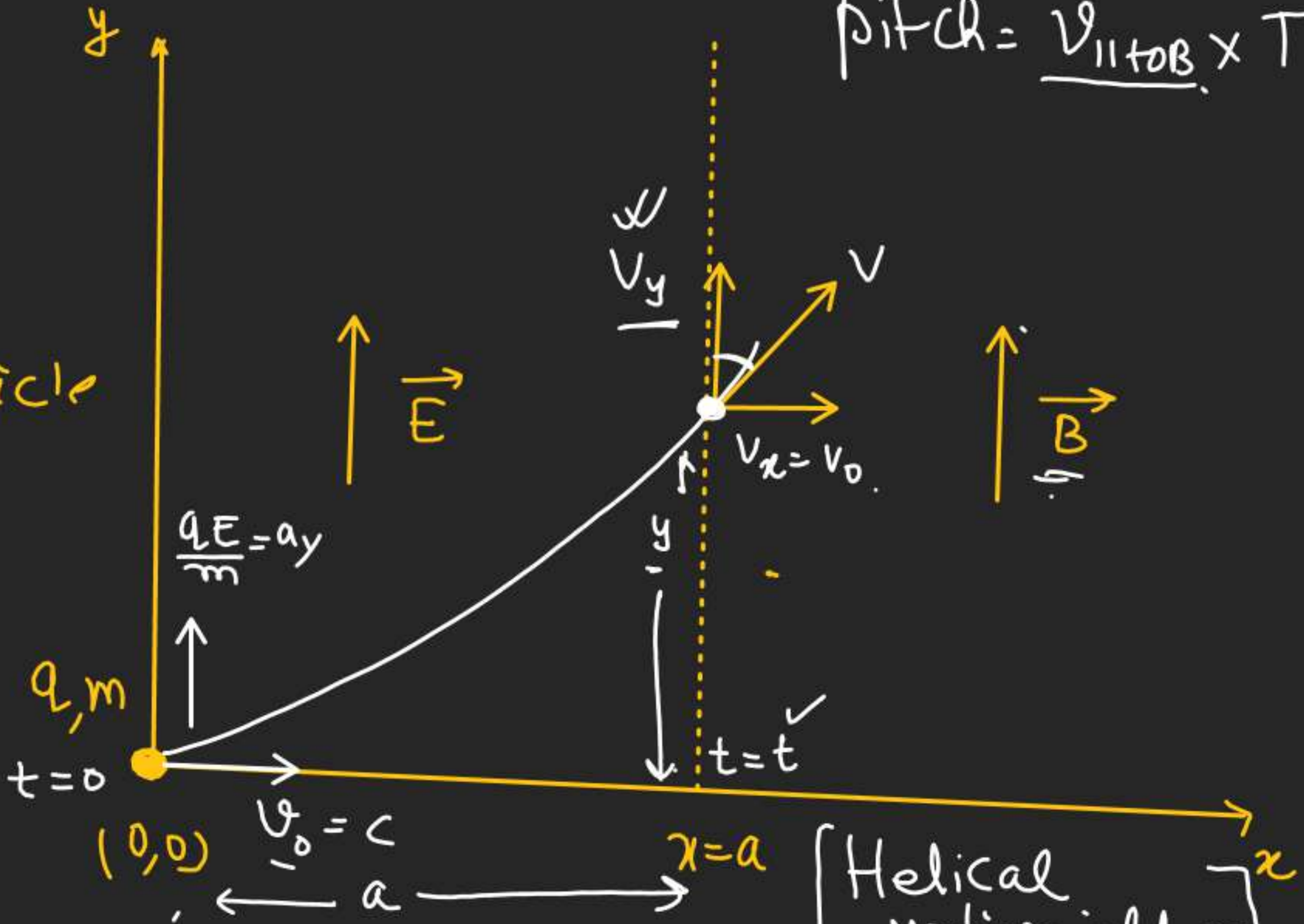
Motion of charge particle in a magnetic field

Charge particle is accleerated by a potential difference $\underline{V}$ from origin along $+x$ axis.

a) Find the point where charge particle crosses the line $x=a$.

b) Find $\underline{\text{pitch}}$ of the helix after it crosses the line $x=a$.

pitch $= \underline{v_{\parallel \text{ to } B}} \times T$



[Helical motion after $x=a$]

Sol^n $\quad qV = \frac{1}{2} m v_0^2$

$$v_0 = \sqrt{\frac{2qV}{m}}$$

In horizontal direction

$$a = v_0 t$$

$$t = \left(\frac{a}{v_0}\right) \quad t \to \text{when it crosses } x=a \text{ line.}$$

$$y = \frac{1}{2} a_y t^2 = \frac{1}{2}\left(\frac{qE}{m}\right)\left(\frac{a}{v_0}\right)^2 = \left(\frac{qEa^2}{2m}\right)\frac{1}{v_0^2} = \frac{\cancel{q}Ea^2}{2\cancel{m}} \times \frac{\cancel{m}}{2\cancel{q}V}$$

$$y = \left(\frac{Ea^2}{4V}\right)$$

MAGNETIC FIELD

Motion of charge particle in a magnetic field

$$\text{pitch} = v_y \times T$$

$$v_y = a_y t$$

$$v_y = \left(\frac{qE}{m}\right)\left(\frac{a}{v_0}\right)$$

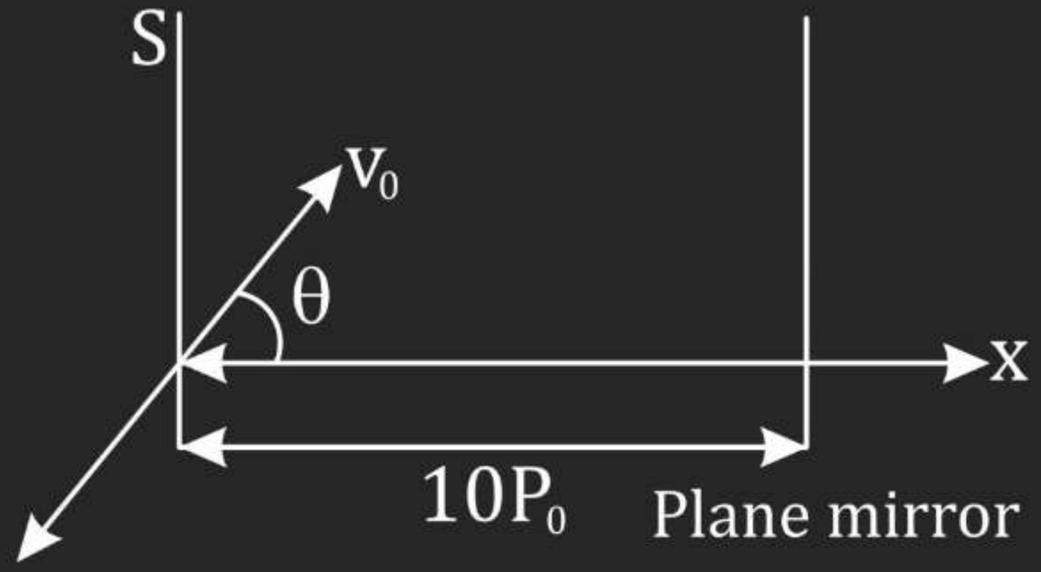
$$\text{pitch} = \frac{\cancel{q}Ea}{\cancel{m}v_0} \times \left(\frac{2\pi \cancel{m}}{\cancel{q}B}\right)$$

$$\text{pitch} = \frac{2\pi Ea}{B} \times \sqrt{\frac{m}{2qV}}$$

MAGNETIC FIELD

Motion of charge particle in a magnetic field

Q.10 In the plane mirror, the coordinates of image of a charged particle (initially at origin as shown in figure) after two and a half time periods are (Initial velocity of charge particle is v_0 in the xy-plane and the plane mirror is perpendicular to the x-axis. A uniform magnetic field $B\hat{i}$ exists in the space. P_0 is pitch of helix, R_0 is radius of helix):



(A) $17P_0, 0, -2R_0$

(B) $3P_0, 0, -2R_0$

(C) $17.5P_0, 0, -2R_0$

(D) $3P_0, 0, 2R_0$

MAGNETIC FIELD

Motion of charge particle in a magnetic field

Q.11 A charged particle of specific charge α is released from origin at time $t = 0$ with velocity $\vec{v} = V_0\hat{i} + V_0\hat{j}$ in magnetic field $\vec{B} = B_0\hat{i}$. The coordinates of the particle at time $t = \frac{\pi}{B_0\alpha}$ are (specific charge $\alpha = q/m$)

(A) $\left(\frac{V_0}{2B_0\alpha}, \frac{\sqrt{2}V_0}{aB_0}, \frac{-V_0}{B_0\alpha}\right)$

(B) $\left(\frac{-V_0}{2B_0\alpha}, 0, 0\right)$

(C) $\left(0, \frac{2V_0}{B_0\alpha}, \frac{V_0 p}{2B_0\alpha}\right)$

(D) $\left(\frac{V_0\pi}{B_0\alpha}, 0, -\frac{2V_0}{B_0\alpha}\right)$

MAGNETIC FIELD

Motion of charge particle in a magnetic field

Q.12 A particle of specific charge (charge/mass) α starts moving from the origin under the action of an electric field $\vec{E} = E_0\hat{i}$ and magnetic field $\vec{B} = B_0\hat{k}$. Its velocity at $(x_0, y_0, 0)$ is $(4\hat{i} + 3\hat{j})$. The value of x_0 is:

(A) $\frac{13}{2}\frac{\alpha E_0}{B_0}$

(B) $\frac{16\alpha B_0}{E_0}$

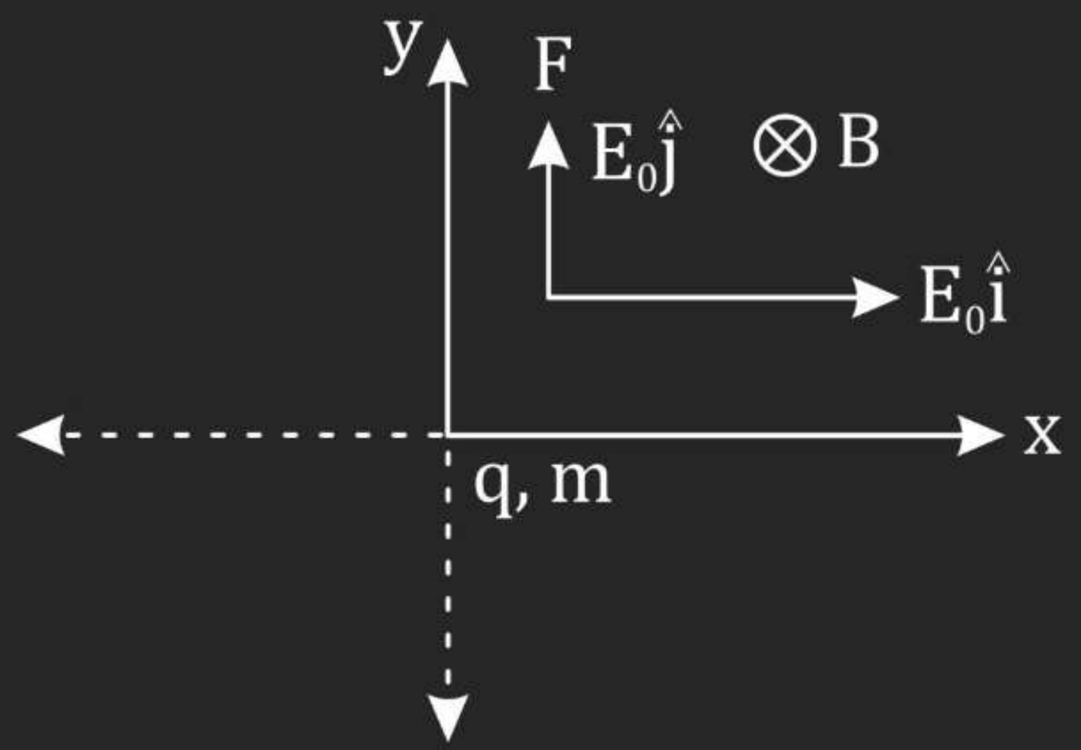
(C) $\frac{25}{2\alpha E_0}$

(D) $\frac{5\alpha}{2B_0}$

MAGNETIC FIELD

Motion of charge particle in a magnetic field

Q.13 A free point charge q and mass m is at rest at the origin as shown in the figure. A constant electric field $E_0\hat{i} - E_0\hat{j}$ and constant magnetic filed $B(-\hat{k})$ is present in whole region. When this charge reaches at position $(2, 20)$, its speed will be :



(A) $\sqrt{\frac{8qE_0}{m}}$

(B) $e\sqrt{\frac{4qE_0}{m}}$

(C) $\sqrt{\frac{qE_0}{Bm}}$

(D) $\sqrt{\frac{2qB}{m}}$

MAGNETIC FIELD

Motion of charge particle in a magnetic field

Q.15 A particle of charge per unit mass α is released from origin with velocity $\vec{v} = v_0\hat{i}$ in a magnetic field

$\vec{B} = -B_0\hat{k}$ for $x \le \frac{\sqrt{3}}{2}\frac{v_0}{B_0\alpha}$ and $\vec{B} = 0$ for $x > \frac{\sqrt{3}}{2}\frac{v_0}{B_0\alpha}$

The x-coordinate of the particle at time $t\left(> \frac{\pi}{3B_0\alpha}\right)$ would be :

(A) $\frac{\sqrt{3}}{2}\frac{v_0}{B_0\alpha} + \frac{\sqrt{3}}{2}v_0\left(t - \frac{\pi}{B_0\alpha}\right)$

(B) $\frac{\sqrt{3}}{2}\frac{v_0}{B_0\alpha} + v_0\left(t - \frac{\pi}{3B_0\alpha}\right)$

(C) $\frac{\sqrt{3}}{2}\frac{v_0}{B_0\alpha} + \frac{v_0}{2}\left(t - \frac{\pi}{3B_0\alpha}\right)$

(D) $\frac{\sqrt{3}}{2}\frac{v_0}{B_0\alpha} + \frac{v_0 t}{2}$