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DPP – 1

Solution

1. moment of inertia of the ring where axis is in plane of the ring $I_1 = MR^2$

moment of inertia of 2^{nd} ring about axis passing through the center of the 1^{st} ring

$I_2 = \frac{MR^2}{2}$ (By perpendicular Theorem)

$I = I_1 + I_2 = MR^2 + \frac{MR^2}{2} = \frac{3}{2}MR^2$

so $p + q = 3 + 2 = 5$

2. Moment of Inertia of two spheres lying on the axis is $2 \times \frac{2}{5}ma^2 = \frac{4}{5}ma^2$

moment of Inertia of two sphere lying on the edge parallel to the axis at a distance of b :

$= 2\left[\frac{2}{5}ma^2 + mb^2\right] = \frac{4}{5}ma^2 + 2mb^2.$

So Total moment of Inertia $= \frac{4}{5}ma^2 + 2mb^2 + \frac{4}{5}ma^2 = \frac{8}{5}ma^2 + 2mb^2$

$\because \frac{\alpha + \gamma}{\beta} = \frac{8+2}{5} = 2$

3. Moment of inertia can be calculated by principle of superposition

$MI = \frac{1}{2} \times 9MR^2 - \left(\frac{MR^2}{18} + M \times \frac{4R^2}{9}\right)$

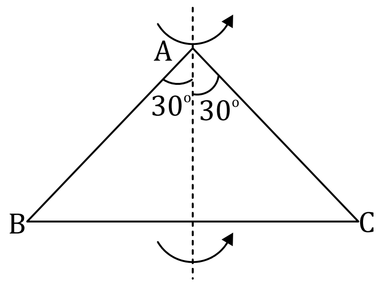
$= \frac{72MR^2}{18} = 4MR^2$

4. $I = I_{AB} + I_{BC} + I_{AC}$

$I_{AC} = \frac{ml^2}{3}\sin^2 \theta$

$= \frac{mb^2}{3}\frac{1}{4}$

$= \frac{mb^2}{12} = I_{AB}$



$I = 2\left(\frac{mb^2}{12}\right) + \frac{mb^2}{12}$

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$$I = \frac{mb^2}{4}$$

ie $\frac{a}{16} = \frac{1}{4}$

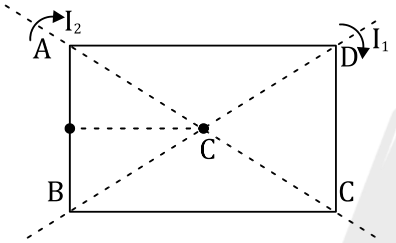
$a = 4$

5. As given in Question

$\sqrt{2}L = b$

$\because\ L = \frac{b}{\sqrt{2}}$

$L^2 = \frac{b^2}{2}$



Moment of Inertia of AB $= \frac{mL^2}{12}$ about C (MI)

$I' = \frac{mL^2}{12} + m\left(\frac{L}{2}\right)^2$

$I' = \frac{1}{3}ML^2$ parole axis

for < 114rod $= 4\left(\frac{1}{3}mL^2\right) = \frac{4}{3}mL^2 = I$

By perpendicular axis Theorem

$\because I_1 = I_2$ (because frame is symmetrical)

$I = I_1 + I_2 = 2I_1$

$$I_1 = I/2 = \frac{2mL^2}{3}$$

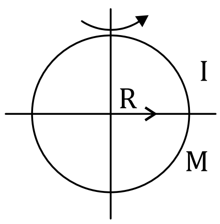
$$I_1 = \frac{2mb^2}{3 \times 2} = \frac{mb^2}{3}$$

ie $\cdot P = 6$.

6. **case 1**

Let

Aluminum density $= 0$



$I = \frac{2}{5}MR^2$

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$$M = \rho V = \rho\left(\frac{4}{3}\pi R^3\right) \rightarrow (1)$$

$$I' = \frac{2}{5}M'R'^2$$

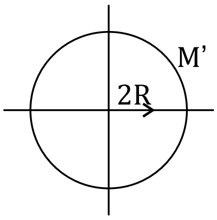
$$= \frac{2}{5}8M(2R)^2 = 32\left(\frac{2}{5}\right)MR^2$$

$$I' = 32\left[\frac{2}{5}MR^2\right] = 32I$$

$$\therefore I' = 32I$$

Thus option (E)

Case 2



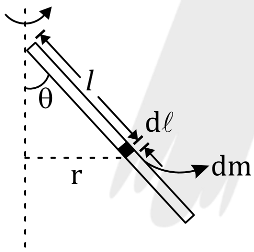
$$M' = \rho\left(\frac{4}{3}\pi(2R)^3\right)$$

$$M' = 8\cdot\frac{4}{3}\pi R^3 \rightarrow (2)$$

$$M' = 8M[\text{ From } 0]$$

7. We can observe that each and every element of rod is rotating with different radius about the axis of rotation

Take an elementary mass dm of the rod



$$dm = \frac{m}{l_0}dl$$

The moment of inertia of the elementary mass is given as $dI = (dm)r^2$

The moment of inertia of the rod

$$= I = \int dI \Rightarrow I = \int r^2 dm$$

Substituting $r = 1\sin\theta; dm = \frac{m}{l_0}$ we obtain

$$I = \int (1^2\sin^2\theta)\frac{m}{l_0}dI = \frac{m\sin^2\theta}{l_0}\int_0^{lom} l^2\,dl = \frac{ml_0^3}{3l_0}\sin^2\theta$$

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$$\Rightarrow I = \frac{ml_0^2 \sin^2 \theta}{3}$$

8. In the given diagrams, when the small piece Q is removed and glued to the centre of the plate, the mass comes closer to the z-axis; hence, moment of inertia decreases.

9. Divide the ring into infinitely small lengths of mass dm_i. Even though mass distribution is non-uniform, each mass dm_i is at the same distance R from origin.

10. $I_{yy'} = \frac{2}{5}M\left(\frac{R}{2}\right)^2 + M(2R)^2 + \frac{2}{5}M\left(\frac{R}{2}\right)^2$

$$= \frac{2}{5}\frac{MR^2}{4} + M(4R)^2 + \frac{2}{5}M\frac{R^2}{4}$$

$$= MR^2\left[\frac{1}{5} + 4\right]$$

$$I_{yy'} = \frac{21}{5}MR^2$$

11. When body is hinged at a distance x, then moment of inertia is given by:-

$\Rightarrow I = x^2 - 2x + 99$

As we know Moment of Inertia (I) about Centre of mass (com) is minimum

$\Rightarrow \frac{dI}{dx} =$ minimum about COM

$$\Rightarrow \frac{dI}{dx} = 2x - 2 = 0 \Rightarrow x = 1$$

Hence, Coordinate of Centre of mass is 1 unit.

12. Moment of Inertia about AB $\Rightarrow I_p$

moment of Inertia about AC $\Rightarrow I_B$

moment of Inertia about BC $= I_H$

Moment of Inertia about an axis perpendicular to point c $\Rightarrow I_c$

We know that,

Moment of Inertia is More when mass is farther from the axis. $(M \cdot O \cdot I \Rightarrow MR^2)$

ie BC

And Less when mass distribution is closest to it i.e AB.

So, the order will be.

$I_{AB} > I_{AC} > I_{BC}/I_p > I_B > I_H$.

Now, I_c

Let the distance of point c from COM of the plate be $\rightarrow y$

And of paint A from CoM of the plate by $\rightarrow x$

$I_c \Rightarrow I_{COM} + M_y^2$ (Parallel Axis Theorem)-(1)

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$I_A \Rightarrow I_{COM} + M_x^2$-(2)

From (1) & (2)

$I_C \Rightarrow I_A - mx^2 + my^2$

$I_C \Rightarrow I_A + m(y^2 - x^2)$

$I_A = I_P + I_B$ (Perpendicular Acis Theorem)

$I_C = I_p + I_B + m(y^2 - x^2)$

so, I_c will be the greatest one.

$I_C > I_P > I_B > I_H$.

Hence the correct option is (A.)