Nishant Jindal

1: $\log_{(x+3)}(x^2-x) < 1 = \log_{(x+3)}(x+3)$ $\qquad (-3,-2)\cup(-1,0)\cup(1,3)$

$0 < x+3 < 1 \Rightarrow \boxed{x \in (-3,-2)}$ OR $\qquad x+3 > 1 \Rightarrow \boxed{x \in (-2,\infty)}$ ✓

&

$x^2 - x > x+3 > 0 \qquad\qquad 0 < \boxed{x^2-x} < x+3 \qquad (-1,3)$

$(-\infty,0)\cup(1,\infty)$

$x^2-2x-3 > 0 \Rightarrow (x-3)(x+1) > 0$

$\boxed{x \in (-\infty,-1)\cup(3,\infty)} \qquad \boxed{x \in (-1,0)\cup(1,3)}$ ✓

$\boxed{x \in (-3,-2)} \qquad \boxed{x \in (-1,0)\cup(1,3)}$

$\boxed{x \in (-3,-2)\cup(-1,0)\cup(1,3)}$

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1. $A = \log_{10}\left(\frac{\cancel{4}(a+b)}{\cancel{4}}\right) = 2$

$\log_a(\log_b N)$

$\frac{4(b)}{}$

$a^{\log_a(\log_b N)} = \log_b N$

$2^{\log_6 18} \quad 3^{\log_6 3}$ $\quad 18 \rightarrow 3 \cdot 6$

$\frac{2(b)}{}$ $\left(16^{\log_2 5}\right)^{\log_2 5} = 16^{\log_2 5} = \frac{}{\left(2^{\log_2 5}\right)^4} = \boxed{5^4}$

$B = 2^{1+\log_6 3} \cdot 3^{\log_6 3} = 2 \cdot 2^{\log_6 3} \cdot 3^{\log_6 3}$

$= 2\left(2 \times 3\right)^{\log_6 3} = 2 \times 3 = 6$

$a^m b^m = (ab)^m$

$\frac{\log_c b}{\log_c a} = \log_a b$

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7. $2\left(\frac{1}{\log_c a}+\frac{1}{\log_c b}\right)=\frac{9}{\log_c(ab)}$

$c \neq 1$

$2\left(\log_c b+\log_c a\right)^2 = 9(\log_c a)\log_c b$

$2\log_c^2 b + 2\log_c^2 a = 5\log_c b \log_c a$

$2\log_a^2 b + 2 - 5\log_a b = 0$

$2t^2 - 4t - t + 2 = 0$

$(2t-1)(t-2) = 0 \Rightarrow \log_a b = \frac{1}{2}, 2.$

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8.

$$\frac{\log_{10}x}{\log_{10}3}\,\frac{\log_{10}x}{\log_{10}4}\,\frac{\log_{10}x}{\log_{10}5} = \frac{\log_{10}^2 x}{\log_{10}3\log_{10}4} + \frac{\log_{10}^2 x}{\log_{10}4\log_{10}5} + \frac{\log_{10}^2 x}{\log_{10}3\log_{10}5}$$

$\log_{10}x = 0$

$$\frac{\log_{10}x}{\cancel{\log_{10}3\log_{10}4\log_{10}5}} = \frac{\log_{10}5+\log_{10}3+\log_{10}4}{\cancel{\log_{10}3\log_{10}4\log_{10}5}}$$

$\log_{10}x = \log_{10}60$

$\boxed{x = 60}$

$(60+1)^2$

$\boxed{ab = ac}$

$2\log_{(2000)^6}4 + 3\log_{(2000)^6}5 = \boxed{\frac{1}{6}}$

$\log_{(2000)^6}(4^2\,5^3)$

$$4^{5\log_{2^{5/2}}(3-\sqrt{6})-6\log_{2^3}(\sqrt{3}-\sqrt{2})}$$

$$4^{\underline{2\log_2(3-\sqrt{6})}-\underline{2\log_2(\sqrt{3}-\sqrt{2})}}$$

$$2^{2+\frac{1}{2}}$$

$$= 4^{2\log_2\frac{\cancel{(\sqrt{3}-\sqrt{2})}\sqrt{3}}{\cancel{(\sqrt{3}-\sqrt{2})}}}$$

$$= 4^{2\log_2\sqrt{3}}$$

$$= \left(2^{\log_2\sqrt{3}}\right)^4 = (\sqrt{3})^4 = 9$$

Nishant Jindal

1. $\log_{\frac{x+4}{2}}\left(\underbrace{\log_2\left(\frac{2x-1}{x+3}\right)}\right) < 0 = \log_{\frac{x+4}{2}} 1.$

$\frac{x+4}{2} > 1 \Rightarrow x \in (-2, \infty)$

$0 < \frac{x+4}{2} < 1 \Rightarrow x \in (-4, -2)$ ✓

$\log_2 1 = 0 < \log_2\left(\frac{2x-1}{x+3}\right) < 1 = \log_2 2$

$\log_2\left(\frac{2x-1}{x+3}\right) > 1 = \log_2 2$

$\frac{2x-1}{x+3} > 2,$

$\boxed{x \in (-4, -3) \cup (4, \infty)}$

$1 < \boxed{\frac{2x-1}{x+3} < 2}$

$\boxed{x \in (4, \infty)}$

$\frac{2x-1}{x+3} - 2 > 0$

$\boxed{x > -3.}$

$\frac{-7}{x+3} > 0 \Rightarrow x < -3$ ✓

$\boxed{x \in (-4, -3)}$

$\frac{2x-1}{x+3} - 1 > 0$

$\frac{x-4}{x+3} > 0 \Rightarrow \boxed{x \in (-\infty, -3) \cup (4, \infty)}$

$x \in (4, \infty)$

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Q.

$$\log_{\frac{1}{2}}\left(\log_6\left(\frac{x^2+x}{x+4}\right)\right) < 0 = \log_{\frac{1}{2}} 1$$

$$\log_6\left(\frac{x^2+x}{x+4}\right) > 1 = \log_6 6$$

$$\frac{x^2+x}{x+4} > 6$$

$$\frac{x^2+x}{x+4} - 6 > 0 \Rightarrow \frac{x^2-5x-24}{x+4} > 0$$

$$\frac{(x-8)(x+3)}{x+4} > 0$$

$$\begin{array}{ccccccc} - & & + & & - & & + \\ & -4 & & -3 & & 8 & \end{array}$$

$$\boxed{x \in (-4,-3) \cup (8,\infty)}$$

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3. $\left(2\log_3^2 x - 3\log_3 x - 8\right)\left(2\log_3^2 x - 3\log_3 x - 6\right) \geq 3$

$\left(2y^2-3y-8\right)\left(2y^2-3y-6\right) \geq 3$

$\left(2\log_3^2 x - 3\log_3 x\right)^2 - 14\left(2\log_3^2 x - 3\log_3 x\right) + 45 \geq 0$

$t^2 - 14t + 45$

$\left(2\log_3^2 x - 3\log_3 x - 5\right)\left(2\log_3^2 x - 3\log_3 x - 9\right) \geq 0$

$+2\log_3 x - 5\log_3 x$ $\quad$ $+3\log_3 x - 6\log_3 x$

$\left(2\log_3 x - 5\right)\left(\log_3 x + 1\right)\left(\log_3 x - 3\right)\left(2\log_3 x + 3\right) \geq 0$

$+ \quad -3/2 \quad - \quad -1 \quad + \quad 5/2 \quad - \quad 3 \quad +$

$y \in \left(-\infty, -\frac{3}{2}\right] \cup \left[-1, \frac{5}{2}\right] \cup [3, \infty)$

$\log_3 x \in \left(-\infty, -\frac{3}{2}\right] \cup \left[-1, \frac{5}{2}\right] \cup [3, \infty)$

$\log_3 3^{-\infty} = -\infty < \log_3 x \leq \frac{3}{2} = \log_3 3^{3/2}$

$3^{-\infty} < x \leq 3^{3/2}$

$\boxed{0 < x \leq 3^{3/2}}$

$\log_3 3^{-1} = -1 \leq \log_3 x \leq \frac{5}{2} = \log_3 3^{5/2}$

$\frac{1}{3} \leq x \leq 3^{5/2}$

$\log_3 3^3 = 3 \leq \log_3 x < \infty = \log_3 3^{\infty}$

$\boxed{3^3 \leq x < \infty}$

$x \in \left(0, 3^{-3/2}\right] \cup \left[\frac{1}{3}, 3^{5/2}\right] \cup \left[3^3, \infty\right)$

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$\boxed{Ex\text{-}I\ (11-20)} \rightarrow$ Logarithm

2 Questions of $Ex\text{-}\underline{II}$

$\downarrow$

Compound Angles