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RELATION FUNCTION

Q. $f(x)=\log\left(\frac{1-x}{1+x}\right)$, $g(x)=\frac{2x}{1+x^2}$ find $fog(x)$?

$fog(x)=f(g(x))=\log\left(\frac{1-g(x)}{1+g(x)}\right)$

$=\log\left(\frac{1-\frac{2x}{1+x^2}}{1+\frac{2x}{1+x^2}}\right)=\log\left(\frac{1+x^2-2x}{1+x^2+2x}\right)$

$=\log\left(\frac{(1-x)^2}{(1+x)^2}\right)=\log\left(\frac{1-x}{1+x}\right)^2$

$=2\log\left(\frac{1-x}{1+x}\right)=2f(x)$

Q. $f(g(x))=3x+4$, $g(x)=5x+4$.

$f(x)=?$

Sol

$f(5x+4)=3x+4$

$5x+4=t$

$x=\frac{t-4}{5}$

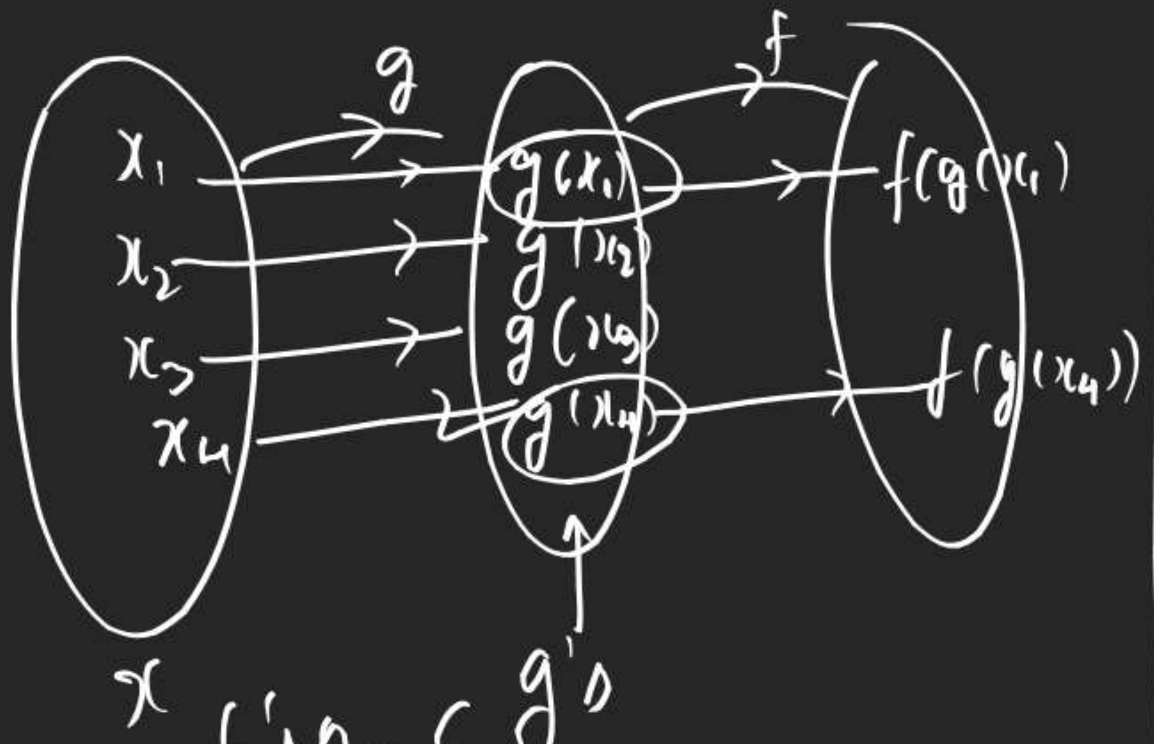
$f(t)=3\left(\frac{t-4}{5}\right)+4$

$f(t)=\frac{3t+8}{5}$

$f(x)=\frac{3x+8}{5}$

Rk :-

$f \circ g(x) = f(g(x))$



f's Dom $\subset$ g's Rf

(2) $f: A \to B$ & $g: B \to C$

A) $g \circ f$

$f: A \to B$ Matched

$g: B \to C$ $\Rightarrow g \circ f: A \to C$

B) $f \circ g$

$g: B \to C$ Not matched

$f: A \to B$ fog is not defined

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3) $f: A \to B$, $g: C \to D$

$f \circ g$

$g: C \to D$

$f: A \to B$

In case of not matching we consider 3 pts.

A) D must be matched to A

or.

B) D should be subset of A

or.

C) Range of $g \subset$ Domain of f

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Q If $f: N \to R$ $f(x) = 2x+1$, $g: Z \to R$ $g(\boxed{x}) = \dfrac{3x}{x^2+1}$

then $gof(x) = ?$

for $gof(x)$ $\{3, 5, 7, 9 \ldots\}$

$f: N \to R$ Matching possible

$g: \boxed{I} \to R$ gof will exist

$\boxed{I \subset R}$

$$gof(x) = g(f(x)) = \frac{3f(x)}{f^2(x)+1}$$

$$= \frac{3(2x+1)}{(2x+1)^2+1} = \frac{6x+3}{4x^2+4x+2}$$

$f: \boxed{N} \to R$

$x =$ Natural No

$= \{\underline{1, 2, 3, 4 \ldots \ldots}\}$

$f(x) = 2x+1$

$Y = \{3, 5, 7, 9 \ldots \ldots\}$

RELATION FUNCTION

Inverse fxn.

1) Rep. by $\boxed{y = f^{-1}(x)}$ → Notation

2) $f^{-1}(x) \neq \frac{1}{f(x)}$, $f^{-1}(x) \neq (f(x))^{-1}$

3) let $\boxed{f: A \to B}$, $y = f(x)$ be a one one & onto fxn then there exist a Unique fxn "g" $\boxed{g: B \to A}$, such that if $\boxed{f(x) = y} \Rightarrow \boxed{g(y) = x}$ $\forall\, x \in A$ & $y \in B$ then g is said to be inverse of f

$g = f^{-1}$

★ $f: \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \to [-1, 1]$, $y = \sin x$

then f^{-1} $[-1, 1] \to \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, $y = \sin^{-1} x$

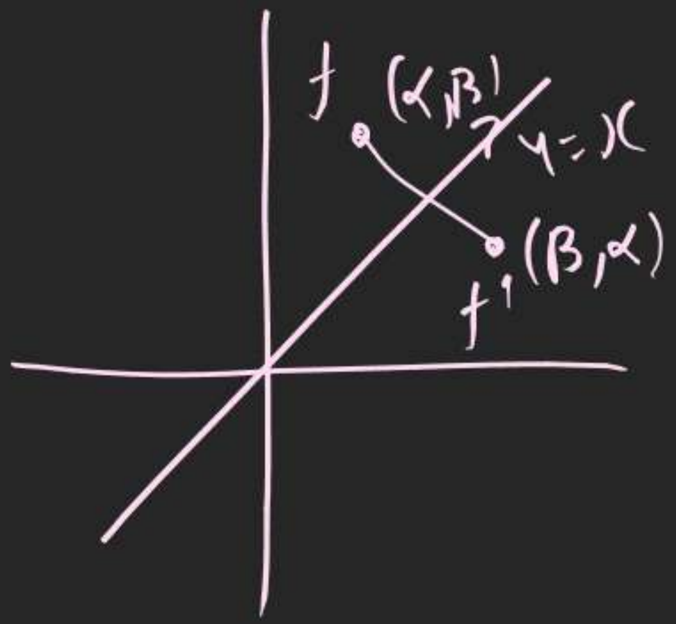
Domain & Codomain swapping.

★ 2) for Inverse fxn f(x) should be Bijective ⇒ & Inverse of f(x) will also be Bijective

★ 3) Inverse of any fxn is always Unique.

★ 4) for inverse fxn find x in terms of y.

* 5) $f : \to (\alpha, \beta)$ then $f^{-1} : (\beta, \alpha)$



* 6) $f^{-1}(x)$ is Image of $y=f(x)$ in Line

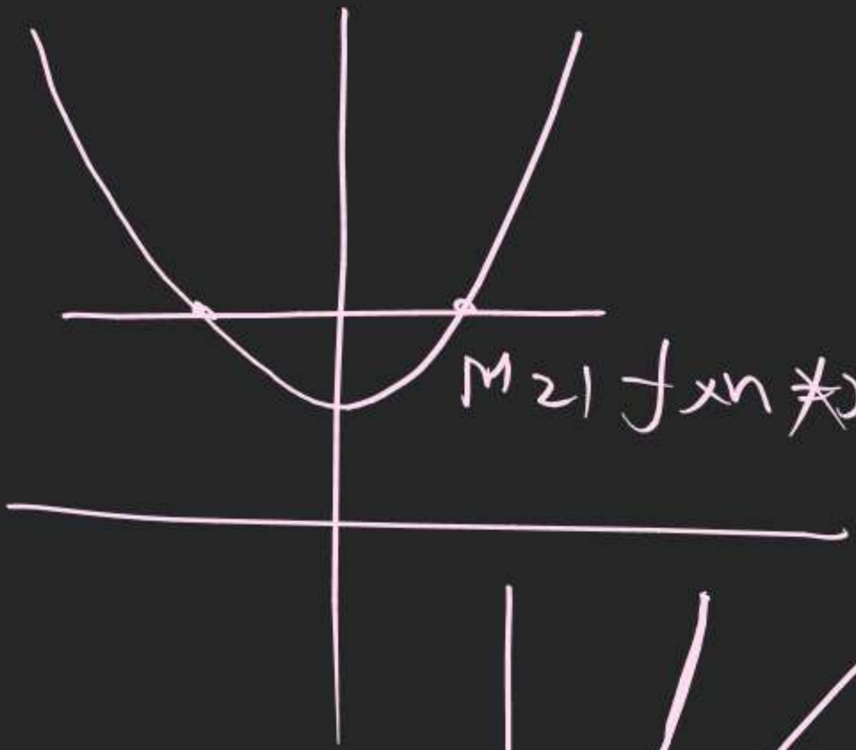
$y=x$

$f : [0,\infty) \to [1,\infty)$ $f(x) = x^2+1$

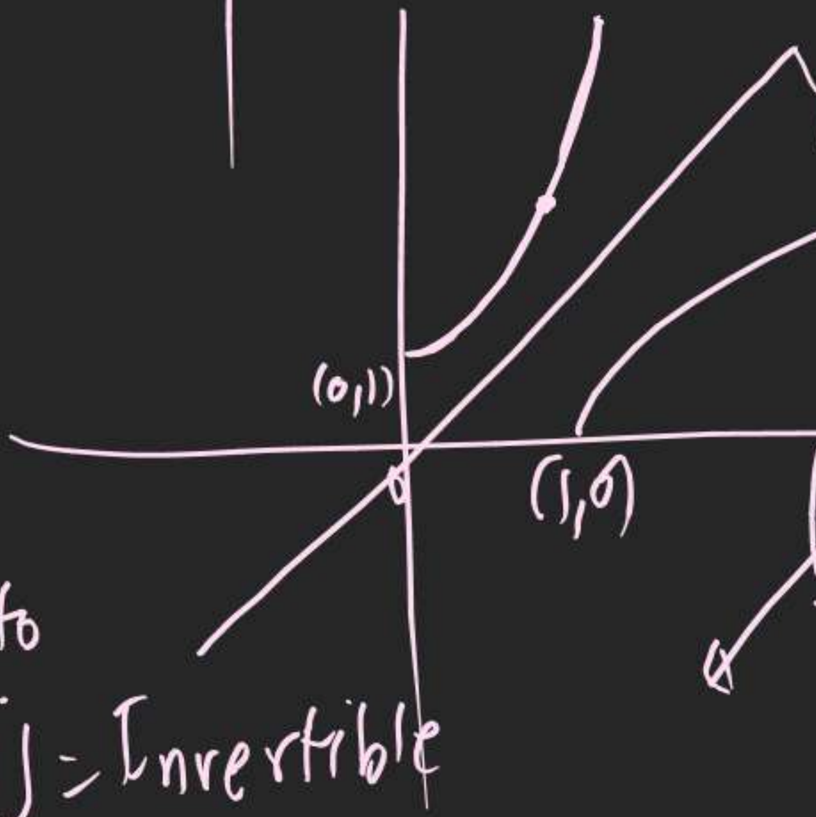
1-1 f $\left.\begin{array}{l} R_f = [1,\infty) \\ \text{Cod} \in [1,\infty) \end{array}\right\}$ onto

Bij = Invertible

$y = x^2+1$



M 2 1 fxn ⇒ Bij ⇒ Not Invertible.



$y = x^2+1$

$x^2 = y-1$

$x = \pm\sqrt{y-1}$

$\boxed{g(y) = x} = \sqrt{y-1}$

$f^{-1}(x) = \sqrt{x-1}$

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RELATION FUNCTION

(7) $g(y) = x$

$f^{-1}(y) = x$

$$\boxed{f^{-1}(f(x)) = x = f(f^{-1}(x))}$$

$f^{-1} o f(x) = f o f^{-1}(x) = x$

$f o g(x) = g o f(x) = x$

$\Rightarrow$ then Understood that f & g are Inverse of each other.

8) $f: A \to B \quad y = f(x)$

$f^{-1}: B \to A \quad y = f^{-1}(x)$

$f o f^{-1}(x)$

$f^{-1}: B \to A$

$f: A \to B$

$f o f^{-1}: B \to B = \underline{I_B}$

$f^{-1} o f(x)$

$f: A \to B$

$f^{-1}: B \to A$

$\Rightarrow f^{-1} o f: A \to A \Rightarrow \underline{I_A}$

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RELATION FUNCTION

Q find Inverse of following.

A) $f: R \to R$ $\quad f(x) = 3x - 5$

$y = 3x - 5$

$\boxed{x = \frac{y+5}{3}}$

$f^{-1}(x) = \frac{x+5}{3}$

(B) $f: \boxed{(4,6)} \to (6,8)$ $\quad f(x) = x + \left[\frac{x}{2}\right]$ ← [illegible]

$x \in (4,6)$

$\frac{x}{2} \in (2,3)$

$\left[\frac{x}{2}\right] = 2$

$f(x) = x + 2$

$y = x + 2$

$x = y - 2$

$f^{-1}(x) = x - 2$

(1) $f(x) = (1 - (x-4)^5)^{\frac{1}{7}}$ $\quad f^{-1}$?

$y = (1 - (x-4)^5)^{1/7}$

$y^7 = 1 - (x-4)^5$

$(x-4)^5 = 1 - y^7$

$x - 4 = (1 - y^7)^{\frac{1}{5}}$

$x = 4 + (1 - y^7)^{\frac{1}{5}}$

$y = 4 + (1 - x^7)^{\frac{1}{5}}$

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$$\frac{a+b}{a-b} \to C \& D$$

(D) $f(x) = \log_e\left(x+\sqrt{1+x^2}\right)$

$f^{-1}(x) = \dfrac{e^{x}-e^{-x}}{2}$

$y = \log_e\left(x+\sqrt{1+x^2}\right)$

$e^{y} = x+\sqrt{x^2+1}$

$e^{y}-x = \sqrt{x^2+1}$

Sqr $(e^{y}-x)^2 = x^2+1$

$e^{2y}+\cancel{x^2}-2x\cdot e^{y} = \cancel{x^2}+1$

$e^{2y}-1 = 2xe^{y}$

$x = \dfrac{e^{2y}-1}{2e^{y}} = \dfrac{e^{y}-e^{-y}}{2}$

(E) $f(x) = \dfrac{10^{x}+10^{-x}}{10^{x}-10^{-x}}+2$

$y = \dfrac{10^{x}+10^{-x}}{10^{x}-10^{-x}}+2$

$\dfrac{y-2}{1} = \dfrac{10^{x}+10^{-x}}{10^{x}-10^{-x}}$ C & D.

$\dfrac{y-2+1}{y-2-1} = \dfrac{(10^{x}+\cancel{10^{-x}})+(10^{x}-\cancel{10^{-x}})}{(\cancel{10^{x}}+10^{-x})-(\cancel{10^{x}}-10^{-x})} = 10^{2x}$

$\dfrac{y-1}{y-3} = 10^{2x} \Rightarrow \log_{10}\left(\dfrac{y-1}{y-3}\right) = \log_{10}10^{2x}$

$2x = \log_{10}\left(\dfrac{y-1}{y-3}\right) \Rightarrow f^{-1}(x) = \dfrac{1}{2}\log_{10}\left(\dfrac{x-1}{x-3}\right)$

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RELATION FUNCTION

Q If $f(x) = \frac{e^x - e^{-x}}{2}$ & $f(g(x)) = x$

then $g\left(\frac{e^{1002}-1}{2e^{501}}\right) = ?$

$g\left(\frac{e^{501} - e^{-501}}{2}\right)$

$g(f(501))$

$= 501$

$f(g(x)) = x$

$g(x) = f^{-1}(x)$

$f(x) = g^{-1}(x)$

$g(f(x)) = x$

Q $f : \boxed{(-\infty, 1]} \to (-\infty, 1]$ & $f(x) = 2x - x^2$

then $f^{-1}(x)$?

$y = 2x - x^2$

$\Rightarrow x^2 - 2x + y = 0$

$\Rightarrow x = \frac{2 \pm \sqrt{4-4y}}{2}$

$x = 1 \pm \sqrt{1-y}$

$x = 1 - \sqrt{1-y}$

$f^{-1}(x) = 1 - \sqrt{1-x}$

$x \in (-\infty, 1]$

$x \leq 1$

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Q $f : [1, \infty) \to \underset{\text{Range}}{\boxed{[2, \infty)}}$ $f(x) = x + \frac{1}{x}$. $\boxed{f^{-1}(x)}$?

one-one
onto

$y = x + \frac{1}{x}$

$x^2 - xy + 1 = 0$

$x = \frac{y \pm \sqrt{y^2 - 4}}{2}$

$x = \frac{y + \sqrt{y^2 - 4}}{2}$

$f^{-1}(x) = \frac{x + \sqrt{x^2 - 4}}{2}$

$x \in [1, \infty)$

$y \in [2, \infty)$
Put $y = 3$

$x \geq 1$
which taken (+)

Q $f(x) = 2x + 1$, $g(x) = x^3$.

$(gof)^{-1}(64)$?

$\boxed{(gof)^{-1}(x) = f^{-1} o g^{-1}(x)}$

let $(gof)^{-1} 64 = t = 3/2$

$64 = gof(t)$

$= g(f(t))$

$= g(2t + 1)$

$4^3 = 64 = (2t + 1)^3$

$2t + 1 = 4$

$2t = 3 \Rightarrow t = \frac{3}{2}$

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RELATION FUNCTION

D.E

Homogeneous fxn

$f(tx, ty) = t^n f(x,y)$

then fxn is Hom fxn of deg $= n$.

Q $f(x,y) = x^2 + y^2 \cos\left(\frac{y}{x}\right)$ is it a Hom fxn?

$f(tx, ty) = t^2x^2 + t^2y^2 \cos\left(\frac{\cancel{t}y}{\cancel{t}x}\right)$

$= t^2\left(\underbrace{x + y^2 \cos\left(\frac{y}{x}\right)}\right)$

$f(tx, ty) = t^2 f(x,y)$

Yes it is Hom fxn of deg 2.

Bounded fxn

If R_f = Limited & do not Contain ∞, or $-\infty$ in Range then fxn is Bounded

$Y = \sin x \rightarrow R_f = [-1, 1]$ Bounded

$Y = e^x$ $R_f \in (0, \infty)$ Not Bounded

$Y = \operatorname{Sgn} x \rightarrow R_f \in \{-1, 0, 1\}$ Bounded

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Implicit & Explicit fxn.

Implicit fx	Explicit (in term of y)
$x^3+y^3=1$ $\longrightarrow$	$y=(1-x^3)^{1/3}$.
$y-x=0$ $\longrightarrow$	$y=x$

R_1:→ A) $f(x)\cdot f\left(\frac{1}{x}\right)=f(x)+f\left(\frac{1}{x}\right)$ then take $\boxed{f(x)=1\pm x^n}$

B) $f(x+y)=f(x)+f(y) \rightarrow f(x)=Kx$

C) $f(x+y)=f(x)\cdot f(y) \rightarrow f(x)=K^x$

D) $f(x\cdot y)=f(x)+f(y) \rightarrow f(x)=K\log x$

E) $f(x\cdot y)=f(x)\cdot f(y) \Rightarrow f(x)=x^n$.

Q If $f(x)$ is a Poly fxn & $f(x)\cdot f\left(\frac{1}{x}\right)=f(x)+f\left(\frac{1}{x}\right)$

Key

& $\boxed{f(10)=1001}$ then find $f(20)=?$

$f(x)=1+x^n$

$f(10)=1+10^n=1001$

$10^n=1000=10^3$

$\boxed{n=3}$

$f(x)=1+x^3$

$f(20)=1+20^3=8001$

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RELATION FUNCTION

ITF = Inverse Trigo fxn.

A) all Trigo fxn are $\boxed{m\text{-}2\text{-}1}$ fxn.
Periodic

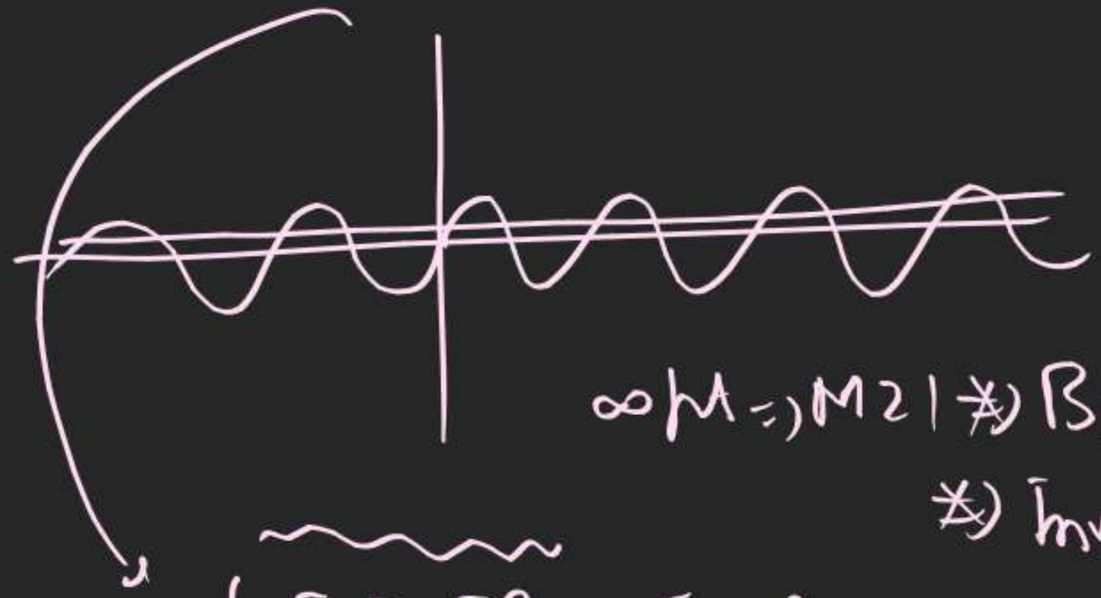
⇒ Not Bijective ⇒ Not Invertible

B) for making Inverse Possible

We Regulate Domain & Codomain

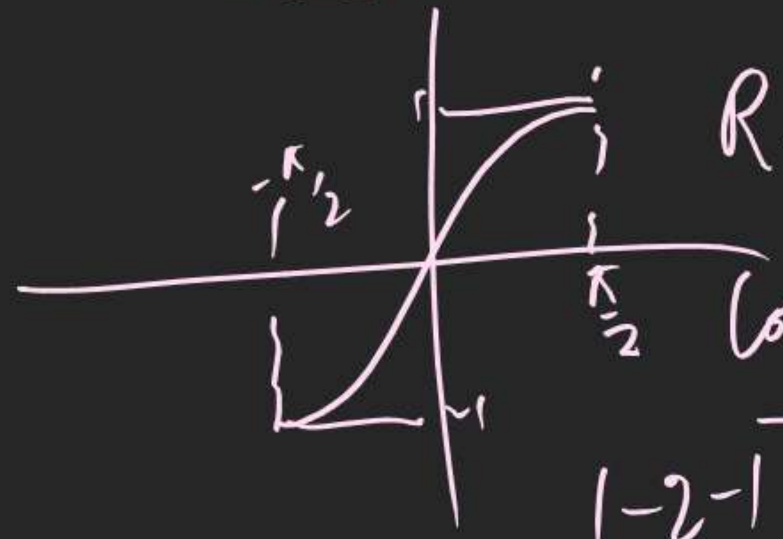
So that it becomes Bijective ⇒ Invertible

Ex: $f : \boxed{R} \to R$, $y = \sin x$



$\infty M \Rightarrow M21 \not\Rightarrow$ Bij
$\not\Rightarrow$ Invertible

$f : \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \to [-1, 1]$ $f(x) = \sin x$



$R_f \in [-1, 1]$
Cod $\in [-1, 1]$

1-2-1 onto
Invertible

RELATION FUNCTION

(1) $f : \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \to [-1, 1]$; $f(x) = \sin x$

$\Rightarrow f^{-1} [-1, 1] \to \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$; $f^{-1}(x) = \sin^{-1} x$

$\frac{\pi}{6} = \sin^{-1}\left(\frac{1}{2}\right)$

$\cancel{\sin\left(\frac{5\pi}{6}\right) = \frac{1}{2}}$ $\quad \frac{5\pi}{6} \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

(2) We have 6 ITF

$\sin^{-1} x, \cos^{-1} x, \tan^{-1} x, \cot^{-1} x, \csc^{-1} x, \sec^{-1} x$

Raaz-1

E) all ITF are θ

$\sin^{-1}\left(\frac{1}{2}\right) \Rightarrow$ find "θ" where sine value $= \frac{1}{2}$

$\sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$