KEY CONCEPTS

BINOMIAL THEOREM

1. BINOMIAL THEOREM :

The formula by which any positive integral power of a binomial expression can be expanded in the form of a series is known as **Binomial Theorem** .

If $x, y \in R$ and $n \in N$, then ;

$(x + y)^n = {}^nC_0\, x^n + {}^nC_1\, x^{n-1} y + {}^nC_2\, x^{n-2}y^2 + ... + {}^nC_r\, x^{n-r}y^r + ... + {}^nC_n y^n = \sum_{r=0}^{n} {}^nC_r\, x^{n-r} y^r$.

This theorem can be proved by Induction .

OBSERVATIONS :

(i) The number of terms in the expansion is $(n + 1)$ i.e. one or more than the index .

(ii) The sum of the indices of x & y in each term is n .

(iii) The binomial coefficients of the terms ${}^nC_0$, ${}^nC_1$ .... **equidistant** from the beginning and the end are equal.

2. IMPORTANT TERMS IN THE BINOMIAL EXPANSION ARE:

(i) General term   **(ii)** Middle term

(iii) Term independent of x   &   **(iv)** Numerically greatest term

(i) The general term or the (r + 1)th term in the expansion of $(x + y)^n$ is given by : $T_{r+1} = {}^nC_r\, x^{n-r} . y^r$

(ii) The middle term(s) is the expansion of $(x + y)^n$ is (are) :

(a) If n is even , there is only one middle term which is given by ;

$T_{(n+2)/2} = {}^nC_{n/2} . x^{n/2} . y^{n/2}$

(b) If n is odd , there are two middle terms which are :

$T_{(n+1)/2}$ & $T_{[(n+1)/2]+1}$

(iii) Term independent of x contains no x ; Hence find the value of r for which the exponent of x is zero.

(iv) To find the Numerically greatest term is the expansion of $(1 + x)^n$, $n \in N$ find $\frac{T_{r+1}}{T_r} = \frac{{}^nC_r x^r}{{}^nC_{r-1} x^{r-1}} = \frac{n-r+1}{r}\, x$. Put the absolute value of x & find the value of r Consistent with the inequality $\frac{T_{r+1}}{T_r} > 1$.

Note that the Numerically greatest term in the expansion of $(1 - x)^n$, $x > 0$, $n \in N$ is the same as the greatest term in $(1 + x)^n$.

3. If $\left(\sqrt{A}+B\right)^n = I + f$, where I & n are positive integers, n being odd and $0 < f < 1$, then $(I + f) . f = K^n$ where $A - B^2 = K > 0$ & $\sqrt{A} - B < 1$.

If n is an even integer, then $(I + f)(1 - f) = K^n$.

4. BINOMIAL COEFFICIENTS :

(i) $C_0 + C_1 + C_2 + ... + C_n = 2^n$

(ii) $C_0 + C_2 + C_4 + ... = C_1 + C_3 + C_5 + ... = 2^{n-1}$

(iii) $C_0^2 + C_1^2 + C_2^2 + ... + C_n^2 = {}^{2n}C_n = \frac{(2n)!}{n!\, n!}$

(iv) $C_0.C_r + C_1.C_{r+1} + C_2.C_{r+2} + ... + C_{n-r}.C_n = \frac{(2n)!}{(n+r)(n-r)!}$

REMEMBER :

(i) $(2n)! = 2^n . n! [1. 3. 5 ...... (2n - 1)]$

5. BINOMIAL THEOREM FOR NEGATIVE OR FRACTIONAL INDICES :

If $n \in Q$, then $(1 + x)^n = 1 + n\,x + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + ...\infty$ Provided $| x | < 1$.

Note:

(i) When the index n is a positive integer the number of terms in the expansion of $(1 + x)^n$ is finite i.e. $(n + 1)$ & the coefficient of successive terms are: ${}^nC_0 , {}^nC_1 , {}^nC_2 , {}^nC_3 \ldots\ldots {}^nC_n$

(ii) When the index is other than a positive integer such as negative integer or fraction, the number of terms in the expansion of $(1 + x)^n$ is infinite and the symbol ${}^nC_r$ cannot be used to denote the Coefficient of the general term .

(iii) Following expansion should be remembered ($|x| < 1$).

(a) $(1 + x)^{-1} = 1 - x + x^2 - x^3 + x^4 - \ldots. \infty$

(b) $(1 - x)^{-1} = 1 + x + x^2 + x^3 + x^4 + \ldots. \infty$

(c) $(1 + x)^{-2} = 1 - 2x + 3x^2 - 4x^3 + \ldots. \infty$

(d) $(1 - x)^{-2} = 1 + 2x + 3x^2 + 4x^3 + \ldots.. \infty$

(iv) The expansions in ascending powers of x are only valid if x is 'small'. If x is large i.e. $|x| > 1$ then we may find it convinient to expand in powers of $\frac{1}{x}$, which then will be small.

PROFICIENCY TEST - 1

1. The 5^{th} term of the expansion of $(x-2)^8$ is -

(A) ${}^8C_5x^3(-2)^5$ (B) ${}^8C_5x^3\,2^5$ (C) ${}^8C_4x^4\,(-2)^4$ (D) ${}^8C_6x^2\,(-2)^6$

2. If the 4^{th} term in the expansion of $\left(ax+\frac{1}{x}\right)^n$ is $\frac{5}{2}$, then the values of a and n are-

(A) 1/2 , 6 (B) 1, 3 (C) 1/2, 3 (D) can not be found

3. The coefficient of $(3r)^{th}$ term and coefficient of $(r+2)^{th}$ term in the expansion of $(1+x)^{2n}$ are equal then (where $r > 1$, $n > 2$, positive integer)-

(A) r = n/2 (B) r = n/3 (C) $r = \frac{n+1}{2}$ (D) $r = \frac{n-1}{2}$

4. The coefficient of x^5 in the expansion of $(2+3x)^{12}$ is -

(A) ${}^{12}C_5 2^5.3^7$ (B) ${}^{12}C_6 2^6.3^6$ (C) ${}^{12}C_5 2^7.\,3^5$ (D) None of these

5. If in the expansion of $\left(x^2-\frac{1}{4}\right)^n$,the coefficient of third term is 31, then the value of n is-

(A) 30 (B) 31 (C) 29 (D) 32

6. The number of terms in the expansion of $(1+5\sqrt{2}\,x)^9+(1-5\sqrt{2}\,x)^9$ is -

(A) 5 (B) 7 (C) 9 (D) 10

7. The coefficient of x^{-26} in the expansion of $\left(x^2-\frac{2}{x^4}\right)^{11}$ is

(A) 330×2^6 (B) -330×2^6 (C) 330×2^7 (D) -330×2^7

8. The term independent of y in the binomial expansion of $\left(\frac{1}{2}y^{1/3}+y^{-1/5}\right)^8$ is -

(A) sixth (B) seventh (C) fifth (D) None of these

9. The term independent of x in $\left(2x+\frac{1}{3x}\right)^6$ is -

(A) 160/9 (B) 80/9 (C) 160/27 (D) 80/3

10. If in the expansion of $(1+y)^n$, the coefficient of 5^{th}, 6^{th} and 7^{th} terms are in A.P., then n is equal to-

(A) 7, 11 (B) 7, 14 (C) 8, 16 (D) None of these

PROFICIENCY TEST - 2

1. The number of integral terms in the expansion of $(5^{1/2} + 7^{1/6})^{642}$ is -

(A) 106 (B) 108 (C) 103 (D) 109

2. If the middle term in the expansion of $\left(x^2 + \frac{1}{x}\right)^n$ is $924\,x^6$, then **n** =

(A) 10 (B) 12 (C) 14 (D) None of these

3. The middle term in the expansion of $(1 - 3x + 3x^2 - x^3)^6$ is -

(A) ${}^{18}C_{10}x^{10}$ (B) ${}^{18}C_9(-x)^9$ (C) ${}^{18}C_9x^9$ (D) $-{}^{18}C_{10}\,x^{10}$

4. The 5^{th} term from the end in the expansion of $\left(\frac{x^3}{2} - \frac{2}{x^3}\right)^9$ is -

(A) $63x^3$ (B) $-\frac{252}{x^3}$ (C) $\frac{672}{x^{18}}$ (D) None of these

5. If $(1+x)^n = C_0 + C_1x + C_2x^2 + ... + C_nx^n$, then the value of $C_0 + 2C_1 + 3C_2 + ... + (n+1)C_n$ is -

(A) $2^n(n+1)$ (B) $2^{n-1}(n+1)$ (C) $2^{n-1}(n+2)$ (D) $2^n(n+2)$

6. If the coefficients of r^{th} and $(r+1)^{th}$ terms in the expansion of $(3+7x)^{29}$ are equal, then r equals-

(A) 15 (B) 21 (C) 14 (D) None of these

7. The coefficient of x^{49} in the expansion of $(x-1)\left(x-\frac{1}{2}\right)\left(x-\frac{1}{2^2}\right).....\left(x-\frac{1}{2^{49}}\right)$ is equal to

(A) $-2\left(1-\frac{1}{2^{50}}\right)$ (B) coefficient of x (C) $2\left(1-\frac{1}{2^{50}}\right)$ (D) $-2\left(1-\frac{1}{2^{49}}\right)$

8. The sum of the binomial coefficients of $\left[2x + \frac{1}{x}\right]^n$ is equal to 256. The constant term in the expansion is

(A) 1120 (B) 2110 (C) 1210 (D) none

9. Middle term in the expansion of $(x^2 - 2x)^{10}$ will be -

(A) ${}^{10}C_4x^{17}\,2^4$ (B) $-{}^{10}C_5\;2^5x^{15}$ (C) $-{}^{10}C_4\,2^4x^{17}$ (D) ${}^{10}C_5\;2^4x^{15}$

10. The middle term in the expansion of $\left(\frac{3}{x^2} - \frac{x^3}{6}\right)^9$ is -

(A) $\frac{189}{8}x^2, \frac{21}{16}x^7$ (B) $\frac{189}{8}x^2, -\frac{21}{16}x^7$ (C) $-\frac{189}{8}x^2, -\frac{21}{16}x^7$ (D) None of these

PROFICIENCY TEST - 3

1. $n!\left(\frac{1}{n!}+\frac{1}{2!(n-2)!}+\frac{1}{4!(n-4)!}+.....+\frac{1}{n!}\right)$ is equal to -

(A) 2^n (B) 2^{n-1} (C) 2^{n+1} (D) 2^{-n+1}

2. If $(1+x)^n = C_0 + C_1x + C_2x^2 + ...+ C_nx^n$, then $C_0C_1 + C_1C_2 + C_2C_3 + ...+ C_{n-1}C_n$ is equal to-

(A) $\frac{2n!}{n!\ n!}$ (B) $\frac{2n!}{n!(n+1)!}$ (C) $\frac{2n!}{(n-1)!(n+1)!}$ (D) $\frac{2n!}{(n-1)!\ n!}$

3. If $(1+x+x^2)^{2n} = a_0 + a_1x + a_2x^2 + ...$then the value of $a_0 - a_1 + a_2 - a_3 + ...$ is-

(A) 2^n (B) 3^n (C) 1 (D) 0

4. The sum of the coefficients in the expansion of $(a+2b+c)^{10}$ is -

(A) 4^{10} (B) 3^{10} (C) 2^{10} (D) 10^4

5. The sum of the coefficient of the terms of the expansion of polynomial $(1+x-3x^2)^{2143}$ is-

(A) 2^{2143} (B) 1 (C) –1 (D) 0

6. If $|x| < 1/2$, then expansion of $(1-2x)^{1/2}$ is-

(A) $1-x-\frac{1}{2}x^2 ....$ (B) $1-x+\frac{1}{2}x^2 ....$ (C) $1+x-\frac{1}{2}x^2 ....$ (D) None of these

7. The tenth term in the expansion of $(1+x)^{-3}$ is -

(A) $-55x^9$ (B) $55x^9$ (C) $-66x^{10}$ (D) $66x^{10}$

8. The value of $\sqrt{99}$ upto three decimals is -

(A) 9.949 (B) 9.958 (C) 9.944 (D) 9.939

9. The remainder, when $(15^{23} + 23^{23})$ is divided by 19, is

(A) 4 (B) 15 (C) 0 (D) 18

10. In the expansion of $\left(3^{-\frac{x}{4}}+3^{\frac{5x}{4}}\right)^n$ the sum of the binomial coefficients is 64 and the term with the greatest binomial coefficient exceeds the third term by $(n-1)$, then the value of x must be

(A) 1 (B) 2 (C) 0 (D) – 1

11. The last two digits of the number 3^{400} are :

(A) 81 (B) 43 (C) 29 (D) 01

EXERCISE-I

1. Find the coefficients : (i) x^7 in $\left(ax^2 + \frac{1}{bx}\right)^{11}$ (ii) x^{-7} in $\left(ax - \frac{1}{bx^2}\right)^{11}$

(iii) Find the relation between a & b , so that these coefficients are equal.

2. If the coefficients of the r^{th}, $(r + 1)^{th}$ & $(r + 2)^{th}$ terms in the expansion of $(1 + x)^{14}$ are in AP, find *r*.

3. Given that $(1 + x + x^2)^n = a_0 + a_1x + a_2x^2 + ... + a_{2n}x^{2n}$, find the values of :
(i) $a_0 + a_1 + a_2 + ... + a_{2n}$; (ii) $a_0 - a_1 + a_2 - a_3 ..... + a_{2n}$; (iii) $a_0^2 - a_1^2 + a_2^2 - a_3^2 + ... + a_{2n}^2$

4. Prove that : ${}^{n-1}C_r + {}^{n-2}C_r + {}^{n-3}C_r + ... + {}^{r}C_r = {}^{n}C_{r+1}$.

5. (a) Which is larger : $(99^{50} + 100^{50})$ or $(101)^{50}$.

(b) Show that ${}^{2n-2}C_{n-2} + 2.{}^{2n-2}C_{n-1} + {}^{2n-2}C_n > \frac{4n}{n+1}$, $n \in N$, $n > 2$

6. Find numerically the greatest term in the expansion of :

(i) $(2 + 3x)^9$ when $x = \frac{3}{2}$ (ii) $(3 - 5x)^{15}$ when $x = \frac{1}{5}$

7. Given $s_n = 1 + q + q^2 + ... + q^n$ & $S_n = 1 + \frac{q+1}{2} + \left(\frac{q+1}{2}\right)^2 + ... + \left(\frac{q+1}{2}\right)^n$, $q \neq 1$,
prove that ${}^{n+1}C_1 + {}^{n+1}C_2.s_1 + {}^{n+1}C_3.s_2 + ... + {}^{n+1}C_{n+1}.s_n = 2^n . S_n$.

8. If the coefficient of a^{r-1} , a^r , a^{r+1} in the expansion of $(1 + a)^n$ are in arithmetic progression, prove that $n^2 - n(4r + 1) + 4r^2 - 2 = 0$.

9. If ${}^{n}J_r = \frac{(1-x^n)(1-x^{n-1})(1-x^{n-2}).................(1-x^{n-r+1})}{(1-x)(1-x^2)(1-x^3).................(1-x^r)}$, prove that ${}^{n}J_{n-r} = {}^{n}J_r$.

10. The expressions $1 + x$, $1+x + x^2$, $1 + x + x^2 + x^3$,............. $1 + x + x^2 +.......... + x^n$ are multiplied together and the terms of the product thus obtained are arranged in increasing powers of x in the form of $a_0 + a_1x + a_2x^2 +$................, then,
(a) how many terms are there in the product.
(b) show that the coefficients of the terms in the product, equidistant from the beginning and end are equal.
(c) show that the sum of the odd coefficients = the sum of the even coefficients = $\frac{(n+1)!}{2}$

11. Find the coeff. of
(a) x^6 in the expansion of $(ax^2 + bx + c)^9$.
(b) $x^2 y^3 z^4$ in the expansion of $(ax - by + cz)^9$.
(c) $a^2 b^3 c^4 d$ in the expansion of $(a - b - c + d)^{10}$.

12. Find the coefficient of x^r in the expression of :
$(x + 3)^{n-1} + (x + 3)^{n-2} (x + 2) + (x + 3)^{n-3} (x + 2)^2 + \ldots + (x + 2)^{n-1}$

13. (a) Find the index n of the binomial $\left(\frac{x}{5} + \frac{2}{5}\right)^n$ if the 9th term of the expansion has numerically the greatest coefficient $(n \in N)$.
(b) For which positive values of x is the fourth term in the expansion of $(5 + 3x)^{10}$ is the greatest.

14. **(a)** Find the number of divisors of the number
$N = {}^{2000}C_1 + 2 \cdot {}^{2000}C_2 + 3 \cdot {}^{2000}C_3 + \ldots + 2000 \cdot {}^{2000}C_{2000}$.

(b) Find the sum of the roots (real or complex) of the equation $x^{2001} + \left(\frac{1}{2} - x\right)^{2001} = 0$.

15. **(a)** Show that the integral part in each of the following is odd. $n \in N$

(A) $\left(5 + 2\sqrt{6}\right)^n$ (B) $\left(8 + 3\sqrt{7}\right)^n$ (C) $\left(6 + \sqrt{35}\right)^n$

(b) Show that the integral part in each of the following is even. $n \in N$

(A) $\left(3\sqrt{3} + 5\right)^{2n+1}$ (B) $\left(5\sqrt{5} + 11\right)^{2n+1}$

16. If $\left(7 + 4\sqrt{3}\right)^n = p+\beta$ where n & p are positive integers and β is a proper fraction, show that $(1 - \beta)(p + \beta) = 1$.

17. If $\left(6\sqrt{6} + 14\right)^{2n+1} = N$ and F be the fractional part of N, prove that $NF = 20^{2n+1}$ $(n \in N)$

Direction: Q.18 to Q. 19

If $C_0, C_1, C_2, \ldots, C_n$ are the combinatorial coefficients in the expansion of $(1 + x)^n$, $n \in N$, then prove the following :

18. $C_0^2 + C_1^2 + C_2^2 + \ldots + C_n^2 = \frac{(2n)!}{n!\, n!}$

19. $C_1 + 2C_2 + 3C_3 + \ldots + n \cdot C_n = n \cdot 2^{n-1}$

20. If P_n denotes the product of all the coefficients in the expansion of $(1 + x)^n$, $n \in N$, show that, $\frac{P_{n+1}}{P_n} = \frac{(n+1)^n}{n!}$.

EXERCISE-II

1. Prove that : $\sum_{r=0}^{n}(-1)^r \cdot {}^nC_r\left[\frac{1}{2^r}+\frac{3^r}{2^{2r}}+\frac{7^r}{2^{3r}}+\frac{15^r}{2^{4r}}+\text{.....up to m terms}\right]=\frac{(2^{mn}-1)}{(2^n-1)(2^{mn})}$

2. Prove that $\sum_{K=0}^{n} {}^nC_K \sin Kx.\cos(n-K)x = 2^{n-1}\sin nx$.

3. If $\sum_{r=0}^{2n} a_r(x-2)^r = \sum_{r=0}^{2n} b_r(x-3)^r$ & $a_k = 1$ for all $k \ge n$, then show that $b_n = {}^{2n+1}C_{n+1}$.

4. Prove that $\frac{(72)!}{(36!)^2} - 1$ is divisible by 73.

5. Prove that the integer next above $\left(\sqrt{3}+1\right)^{2n}$ contains 2^{n+1} as factor ($n \in N$)

6. Let I denotes the integral part and F the proper fractional part of $\left(3+\sqrt{5}\right)^n$ where $n \in N$ and if ρ denotes the rational part and σ the irrational part of the same, show that $\rho = \frac{1}{2}(I+1)$ and $\sigma = \frac{1}{2}(I + 2F - 1)$.

7. Prove that $\frac{{}^{2n}C_n}{n+1}$ is an integer, $\forall$ $n \in N$.

Direction: Q 8 to Q 13

If $C_0, C_1, C_2, \ldots, C_n$ are the combinatorial coefficients in the expansion of $(1 + x)^n$, $n \in N$, then prove the following :

8. $2.C_o + \frac{2^2.C_1}{2} + \frac{2^3.C_2}{3} + \frac{2^4.C_3}{4} + \ldots\ldots + \frac{2^{n+1}.C_n}{n+1} = \frac{3^{n+1}-1}{n+1}$

9. $C_oC_r + C_1C_{r+1} + C_2C_{r+2} + \ldots + C_{n-r}C_n = \frac{2n!}{(n-r)!\,(n+r)!}$

10. $(n-1)^2 . C_1 + (n-3)^2 . C_3 + (n-5)^2 . C_5 + \ldots = n(n+1)2^{n-3}$

11. $1 . C_o^2 + 3 . C_1^2 + 5 . C_2^2 + \ldots + (2n+1) C_n^2 = \frac{(n+1)\,(2n)!}{n!\,n!}$

12. $\sqrt{C_1}+\sqrt{C_2}+\sqrt{C_3}+\ldots\ldots+\sqrt{C_n} \le 2^{n-1}+\frac{n-1}{2}$

13. $\sqrt{C_1}+\sqrt{C_2}+\sqrt{C_3}+\ldots\ldots+\sqrt{C_n} \le \left[n\left(2^n-1\right)\right]^{1/2}$ for $n \ge 2$.

14. If $a_0, a_1, a_2, \ldots$ be the coefficients in the expansion of $(1 + x + x^2)^n$ in ascending powers of x, then prove that: **(i)** $a_0a_1 - a_1a_2 + a_2a_3 - \ldots = 0$; **(ii)** $a_0a_2 - a_1a_3 + a_2a_4 - \ldots + a_{2n-2}a_{2n} = a_{n+1}$ or a_{n-1}.

(iii) $E_1 = E_2 = E_3 = 3^{n-1}$; where $E_1 = a_0 + a_3 + a_6 + \ldots$; $E_2 = a_1 + a_4 + a_7 + \ldots$ & $E_3 = a_2 + a_5 + a_8 + \ldots$

15. If $(1+x)^n = C_0 + C_1x + C_2x^2 + \ldots + C_nx^n$, then show that the sum of the products of the C_i's taken two at a time, represented by $\underset{0 \le i < j \le n}{\sum\sum} C_i\,C_j$ is equal to $2^{2n-1} - \frac{2n!}{2(n!)^2}$.

EXERCISE-III

1. If in the expansion of $\left(2^{1/3}+\frac{1}{3^{1/3}}\right)^n$, the ratio of 6th terms from beginning and from the end is 1/6, then the value of n is -

(A) 5 (B) 7 (C) 9 (D) None of these

2. $\frac{1}{1!(n-1)!}+\frac{1}{3!(n-3)!}+\frac{1}{5!(n-5)!}+ ....=$

(A) $\frac{2^n}{n!}$ (B) $\frac{2^{n-1}}{n!}$ (C) 0 (D) None of these

3. If $(1+x)^n = C_0 + C_1x + C_2x^2 + ... + C_nx^n$, then $\frac{(C_0+C_1)(C_1+C_2)...(C_{n-1}+C_n)}{C_1C_2...C_n}$ equals-

(A) $\frac{n^n}{(n+1)!}$ (B) $\frac{(n+1)^n}{n!}$ (C) $\frac{n^n}{n!}$ (D) None of these

4. $1+\frac{1}{5}+\frac{1.3}{5.10}+\frac{1.3.5}{5.10.15}+ ....$ is equal to -

(A) $\frac{1}{\sqrt{5}}$ (B) $\frac{1}{\sqrt{2}}$ (C) $\sqrt{\frac{5}{3}}$ (D) $\sqrt{5}$

5. The coefficient of x^4 in the expansion of $\frac{1+2x+3x^2}{(1-x)^2}$ is-

(A) 13 (B) 14 (C) 20 (D) 22

6. If the fourth term in the expansion of $(px + 1/x)^n$ is 5/2 then the value of n and p are respectively-

(A) 6, 1/2 (B) 1/2, 6 (C) 3,1 (D) 3, 1/2

7. The coefficient of x^4 in the expansion of $(1+x+x^2+x^3)^n$ is -

(A) ${}^nC_4$ (B) ${}^nC_4 + {}^nC_2$

(C) ${}^nC_1 + {}^nC_2 + {}^nC_4 . {}^nC_2$ (D) ${}^nC_4 + {}^nC_2 + {}^nC_1 . {}^nC_2$

8. If $(2-x-x^2)^{2n} = a_0 + a_1x + a_2x^2 + a_3x^3 + ...$, then the value of $a_0 + a_2 + a_4 + ...$ is-

(A) 2^{n-1} (B) 2^{2n} (C) 2^{2n-1} (D) None of these

9. If the third term in the expansion of $\left[x+x^{\log_{10}x}\right]^5$ is equal to 10,00,000 , then x equals-

(A) 10 (B) 10^2 (C) 10^3 (D) No such x exists

10. If $(1+x)^n = C_0 + C_1x + C_2x^2 + ... + C_nx^n$, then $3C_0 - 5C_1 + 7C_2 + ... + (-1)^n (2n+3) C_n$ equals-

(A) 1 (B) $2(2n+3)2^n$ (C) $(2n+3)2^{n-1}$ (D) 0

11. If $6^{83} + 8^{83}$ is divided by 49, then the remainder is

(A) 35 (B) 5 (C) 14 (D) 2

12. If C_0, C_1, C_2 ..... denotes the combinatorial coefficients in the expansion of $(1 + x)^{10}$, then the value of $\frac{C_0}{1} + \frac{C_1}{2} + \frac{C_2}{3} + ... + \frac{C_{10}}{11}$ is equal to

(A) $\frac{2^{11}}{11}$ (B) $\frac{2^{11}-1}{11}$ (C) $\frac{3^{11}}{11}$ (D) $\frac{3^{11}-1}{11}$

13. Last three digits of the number $N = 7^{100} - 3^{100}$ are

(A) 100 (B) 300 (C) 500 (D) 000

14. Let $\left(5 + 2\sqrt{6}\right)^n = p + f$ where $n \in N$ and $p \in N$ and $0 < f < 1$ then the value of, $f^2 - f + pf - p$ is

(A) a composite number
(B) a negative integer
(C) a prime number
(D) an irrational number

15. Number of rational terms in the expansion of $\left(\sqrt{2} + \sqrt[4]{3}\right)^{100}$ is :

(A) 25 (B) 26 (C) 27 (D) 28

EXERCISE-IV

1. The coefficient of the middle term in the binomial expansion in powers of x of $(1 + \alpha x)^4$ and of $(1 - \alpha x)^6$ is the same if α equals : **[AIEEE 2004]**

(A) $-\frac{5}{3}$ (B) $\frac{3}{5}$ (C) $\frac{-3}{10}$ (D) $\frac{10}{3}$

2. If $S_n = \sum_{r=0}^{n} 1/{}^nC_r$ and $t_n = \sum_{r=0}^{n} r/{}^nC_r$, then t_n/S_n is equal to **[AIEEE 2004]**

(A) $\frac{1}{2}n$ (B) $\frac{1}{2}n - 1$ (C) $n - 1$ (D) $\frac{2n-1}{2}$

3. If the coefficients of r^{th}, $(r + 1)^{th}$, and $(r + 2)^{th}$ terms in the binomial expansion of $(1 + y)^m$ are in A.P., then m and r satisfy the equation **[AIEEE 2005]**

(A) $m^2 - m(4r - 1) + 4r^2 - 2 = 0$ (B) $m^2 - m(4r + 1) + 4r^2 + 2 = 0$
(C) $m^2 - m(4r + 1) + 4r^2 - 2 = 0$ (D) $m^2 - m(4r - 1) + 4r^2 + 2 = 0$

4. The value of ${}^{50}C_4 + \sum_{r=1}^{6} {}^{56-r}C_3$ is **[AIEEE 2005]**

(A) ${}^{55}C_4$ (B) ${}^{55}C_3$ (C) ${}^{56}C_3$ (D) ${}^{56}C_4$

5. If the coefficient of x^7 in $[ax^2 + (1/bx)]^{11}$ equals the coefficient of x^{-7} in $[ax - (1/bx^2)]^{11}$, then a and b satisfy the relation **[AIEEE 2005]**

(A) $a - b = 1$ (B) $a + b = 1$ (C) $\frac{a}{b} = 1$ (D) $ab = 1$

6. If x is so small that x^3 and higher powers of x may be neglected, then $\dfrac{(1+x)^{3/2} - \left(1+\frac{1}{2}x\right)^3}{(1-x)^{1/2}}$ may be approximated as **[AIEEE 2005]**

(A) $1 - \frac{3}{8}x^2$ (B) $3x + \frac{3}{8}x^2$ (C) $-\frac{3}{8}x^2$ (D) $\frac{x}{2} - \frac{3}{8}x^2$

7. If the expansion of powers of x of the function $\dfrac{1}{(1-ax)(1-bx)}$ is $a_0 + a_1x + a_2x^2 + a_3x^3 + \ldots\ldots$, then a_n is **[AIEEE 2006]**

(A) $\frac{b^n - a^n}{b-a}$ (B) $\frac{a^n - b^n}{b-a}$ (C) $\frac{a^{n+1} - b^{n+1}}{b-a}$ (D) $\frac{b^{n+1} - a^{n+1}}{b-a}$

8. For natural numbers m and n, if $(1 - y)^m (1 + y)^n = 1 + a_1y + a_2y^2 + \ldots\ldots$, and $a_1 = a_2 = 10$, then (m, n) is **[AIEEE 2006]**

(A) (20, 45) (B) (35, 20) (C) (45, 35) (D) (35, 45)

9. In the binomial expansion of $(a - b)^n$, $n \geq 5$, the sum of the fifth and sixth terms is zero, then a/b equals **[AIEEE 2007]**

(A) $\frac{5}{n-4}$ (B) $\frac{6}{n-5}$ (C) $\frac{n-5}{6}$ (D) $\frac{n-4}{5}$

10. The sum of the series ${}^{20}C_0 - {}^{20}C_1 + {}^{20}C_2 - {}^{20}C_3 + \ldots\ldots + {}^{20}C_{10}$ is : **[AIEEE 2007]**

(A) $-{}^{20}C_{10}$ (B) $\frac{1}{2}\,{}^{20}C_{10}$ (C) 0 (D) ${}^{20}C_{10}$

11. **Statement 1 :** $\sum_{r=0}^{n}(r+1)\,{}^nC_r = (n+2)\times 2^{n-1}$ **[AIEEE 2008]**

Statement 2 : $\sum_{r=0}^{n}(r+1)\,{}^nC_r\,x^r = (1+x)^n + nx(1+x)^{n-1}$

(A) Statement 1 is false, statement 2 is true.
(B) Statement 1 is true, statement 2 is true ; statement 2 is a correct explanation for statement 1.
(C) Statement 1 is true, statement 2 is true; statement 2 is not a correct explanation for statement 1.
(D) Statement 1 is true, statement 2 is false.

12. The remainder left out when $8^{2n} - (62)^{2n+1}$ is divided by 9 is **[AIEEE 2009]**
(A) 0 (B) 2 (C) 7 (D) 8

13. Let $S_1 = \sum_{j=1}^{10} j(j-1)\,{}^{10}C_j$, $S_2 = \sum_{j=1}^{10} j.\,{}^{10}C_j$, and $S_3 = \sum_{j=1}^{10} j^2.\,{}^{10}C_j$ **[AIEEE 2010]**

Statement 1 : $S_3 = 55 \times 2^9$.
Statement 2 : $S_1 = 90 \times 2^8$ and $S_2 = 10 \times 2^8$.
(A) Statement 1 is false, statement 2 is true.
(B) Statement 1 is true, statement 2 is true ; statement 2 is a correct explanation for statement 1.
(C) Statement 1 is true, statement 2 is true; statement 2 is not a correct explanation for statement 2.
(D) Statement 1 is true, statement 2 is false.

14. The coefficient of x^7 in the expansion of $(1 - x - x^2 + x^3)^6$ is **[AIEEE 2011]**
(A) 132 (B) 144 (C) –132 (D) –144

15. If n is a positive integer then $(\sqrt{3}+1)^{2n} - (\sqrt{3}-1)^{2n}$ is **[AIEEE 2012]**
(A) an irrational number (B) an odd positive integer
(C) an even positive integer (D) a rational number other than positive integers

16. The term independent of x in expansion of $\left(\dfrac{x+1}{x^{2/3} - x^{1/3} + 1} - \dfrac{x-1}{x - x^{1/2}}\right)^{10}$ is : **[JEE Main 2013]**
(A) 310 (B) 4 (C) 120 (D) 210

17. If the coefficients of x^3 and x^4 in the expansion of $(1 + ax + bx^2)(1 - 2x)^{18}$ in powers of x are both zero, then (a, b) is equal to **[JEE Main 2014]**

(A) $\left(16, \dfrac{272}{3}\right)$ (B) $\left(16, \dfrac{251}{3}\right)$ (C) $\left(14, \dfrac{251}{3}\right)$ (D) $\left(14, \dfrac{272}{3}\right)$

18. The sum of coefficents of integral powers of x in the binomial expansion of $(1-2\sqrt{x})^{50}$ is : **[JEE Main 2015]**

(A) $\dfrac{1}{2}(2^{50}+1)$ (B) $\dfrac{1}{2}(3^{50}+1)$ (C) $\dfrac{1}{2}(3^{50})$ (D) $\dfrac{1}{2}(3^{50}-1)$

19. If the number of terms in the expansion of $\left(1 - \dfrac{2}{x} + \dfrac{4}{x^2}\right)^n$, $x \neq 0$, is 28, then the sum of the coefficients of all the terms in this expansion, is : **[JEE Main 2016]**
(A) 64 (B) 2187 (C) 243 (D) 729

20. The value of **[JEE Main 2017]**
$({}^{21}C_1 - {}^{10}C_1) + ({}^{21}C_2 - {}^{10}C_2) + ({}^{21}C_3 - {}^{10}C_3) + ({}^{21}C_4 - {}^{10}C_4) + \ldots\ldots.. + ({}^{21}C_{10} - {}^{10}C_{10})$ is
(A) $2^{20} - 2^{10}$ (B) $2^{21} - 2^{11}$ (C) $2^{21} - 2^{10}$ (D) $2^{20} - 2^9$

21. The sum of the co-efficients of all odd degree terms in the expansion of $\left(x + \sqrt{x^3 - 1}\right)^5 + \left(x - \sqrt{x^3 - 1}\right)^5, (x > 1)$ is : **[JEE Main 2018]**
(A) 2 (B) –1 (C) 0 (D) 1

EXERCISE-V

1. If in the expansion of $(1 + x)^m (1 - x)^n$, the co-efficients of x and x^2 are 3 and – 6 respectively, then m is : **[JEE '99, 2 (Out of 200)]**

(A) 6 (B) 9 (C) 12 (D) 24

2. For $2 \le r \le n$, $\binom{n}{r} + 2\binom{n}{r-1} + \binom{n}{r-2} =$

(A) $\binom{n+1}{r-1}$ (B) $2\binom{n+1}{r+1}$ (C) $2\binom{n+2}{r}$ (D) $\binom{n+2}{r}$

3. For any positive integers m , n (with $n \ge m$), let $\binom{n}{m} = {}^nC_m$. Prove that

$$\binom{n}{m} + \binom{n-1}{m} + \binom{n-2}{m} + \ldots\ldots.. + \binom{m}{m} = \binom{n+1}{m+1}$$

Hence or otherwise prove that ,

$$\binom{n}{m} + 2\binom{n-1}{m} + 3\binom{n-2}{m} + \ldots\ldots.. + (n-m+1)\binom{m}{m} = \binom{n+2}{m+2}.$$

[JEE '2000 (Mains), 6]

4. Find the largest co-efficient in the expansion of $(1 + x)^n$, given that the sum of co-efficients of the terms in its expansion is 4096 . **[REE '2000 (Mains)]**

5. In the binomial expansion of $(a - b)^n$, $n \geq 5$, the sum of the 5th and 6th terms is zero. Then a/b equals

(A) $\frac{n-5}{6}$ (B) $\frac{n-4}{5}$ (C) $\frac{5}{n-4}$ (D) $\frac{6}{n-5}$

[JEE '2001 (Screening), 3]

6. Find the coeffcient of x^{49} in the polynomial **[REE '2001 (Mains) , 3]**

$$\left(x - \frac{C_1}{C_0}\right)\left(x - 2^2 \cdot \frac{C_2}{C_1}\right)\left(x - 3^2 \cdot \frac{C_3}{C_2}\right) \ldots\ldots\ldots\ldots\ldots\ldots \left(x - 50^2 \cdot \frac{C_{50}}{C_{49}}\right)$$ where $C_r = {}^{50}C_r$.

7. The sum $\sum_{i=0}^{m} \binom{10}{i}\binom{20}{m-i}$, (where $\binom{p}{q} = 0$ if $P < q$) is maximum when m is **[JEE '2002 (Screening), 3]**

(A) 5 (B) 10 (C) 15 (D) 20

8. (a) Coefficient of t^{24} in the expansion of $(1+ t^2)^{12} (1 + t^{12}) (1 + t^{24})$ is

(A) ${}^{12}C_6 + 2$ (B) ${}^{12}C_6 + 1$ (C) ${}^{12}C_6$ (D) none

[JEE 2003, Screening 3 out of 60]

(b) Prove that : $2^K \cdot \binom{n}{0}\binom{n}{K} - 2^{K-1}\binom{n}{1}\binom{n-1}{K-1} + 2^{K-2}\binom{n}{2}\binom{n-2}{K-2} \ldots\ldots (-1)^K\binom{n}{K}\binom{n-K}{0} = \binom{n}{K}$.

[JEE 2003, Mains-2 out of 60]

9. ${}^{n-1}C_r = (K^2 - 3).{}^nC_{r+1}$, if $K \in$

(A) $[-\sqrt{3}, \sqrt{3}]$ (B) $(-\infty, -2)$ (C) $(2, \infty)$ (D) $(\sqrt{3}, 2]$

[JEE 2004 (Screening)]

10. The value of $\binom{30}{0}\binom{30}{10}-\binom{30}{1}\binom{30}{11}+\binom{30}{2}\binom{30}{12}........+\binom{30}{20}\binom{30}{30}$ is, where $\binom{n}{r}={}^nC_r$.

(A) $\binom{30}{10}$ (B) $\binom{30}{15}$ (C) $\binom{60}{30}$ (D) $\binom{31}{10}$

[JEE 2005 (Screening)]

11. For r = 0, 1, ..., 10, let A_r, B_r and C_r denote, respectively, the coefficient of x^r in the expansions of $(1 + x)^{10}$, $(1 + x)^{20}$ and $(1 + x)^{30}$. Then $\sum_{r=1}^{10} A_r(B_{10}B_r - C_{10}A_r)$ is equal to **[JEE 2010]**

(A) $B_{10} - C_{10}$ (B) $A_{10}(B_{10}^2 - C_{10}A_{10})$ (C) 0 (D) $C_{10} - B_{10}$

12. The coefficients of three consecutive terms of $(1 + x)^{n+5}$ are in the ratio 5 : 10 : 14. Then n =

[JEE Advanced 2013]

13. Coefficient of x^{11} in the expansion of $(1 + x^2)^4 (1 + x^3)^7 (1 + x^4)^{12}$ is : **[JEE Advanced 2014]**

(A) 1051 (B) 1106 (C) 1113 (D) 1120

14. The coefficient of x^9 in the expansion of $(1 + x)(1 + x^2)(1 + x^3)\ .....\ (1 + x^{100})$ is **[JEE Advanced 2015]**

15. Let m be the smallest positive integer such that the coefficient of x^2 in the expansion of $(1 + x)^2 + (1 + x)^3 + ... + (1 + x)^{49} + (1 + mx)^{50}$ is $(3n + 1)\ {}^{51}C_3$ for some positive integer n. Then the value of n is : **[JEE Advanced 2016]**

16. Let $X = \left({}^{10}C_1\right)^2 + 2\left({}^{10}C_2\right)^2 + 3\left({}^{10}C_3\right)^2 + ... + 10\left({}^{10}C_{10}\right)^2$, where ${}^{10}C_r$, r ∈ {1, 2, ..., 10} denote binomial coefficients. Then, the value of $\frac{1}{1430}X$ is ______. **[JEE Advanced 2018]**

17. Suppose **[JEE Advanced 2019]**

$$\det\begin{bmatrix} \sum_{k=0}^{n} k & \sum_{k=0}^{n} {}^nC_k k^2 \\ \sum_{k=0}^{n} {}^nC_k k & \sum_{k=0}^{n} {}^nC_k 3^k \end{bmatrix} = 0$$

holds for some positive integer n. Then $\sum_{k=0}^{n} \frac{{}^nC_k}{k+1}$ equals___.

18. Let

$S_1 = \{(i, j, k) : i, j, k \in \{1, 2, ..., 10\}\}$,

$S_2 = \{(i, j) : 1 \le i < j + 2 \le 10,\ i, j \in \{1, 2, ..., 10\}\}$,

$S_3 = \{(i, j, k, l) : 1 \le i < j < k < l,\ i, j, k, l \in \{1, 2, ..., 10\}\}$

and

S_4 = {(i, j, k, l): i, j, k and l are distinct elements in {1,2,.....,10}}.

If the total number of elements in the set S_r is n_r, r =1, 2, 3, 4 then which of the following statements is (are) True ? **[JEE Advanced 2021]**

(A) $n_1 = 1000$ (B) $n_2 = 44$ (C) $n_3 = 220$ (D) $\frac{n_4}{12} = 420$

ANSWER KEY

PROFICIENCY TEST - 1

1.	C	**2.**	A	**3.**	A	**4.**	C	**5.**	D	**6.**	A	**7.**	C
8.	A	**9.**	C	**10.**	B								

PROFICIENCY TEST - 2

1.	B	**2.**	B	**3.**	B	**4.**	B	**5.**	C	**6.**	B	**7.**	A
8.	A	**9.**	B	**10.**	B								

PROFICIENCY TEST - 3

1.	B	**2.**	C	**3.**	C	**4.**	A	**5.**	C	**6.**	A	**7.**	A
8.	A	**9.**	C	**10.**	C	**11.**	D						

EXERCISE-I

1. **(i)** ${}^{11}C_5 \frac{a^6}{b^5}$ **(ii)** ${}^{11}C_6 \frac{a^5}{b^6}$ **(iii)** $ab = 1$ **2.** $r = 5$ or 9

3. (i) 3^n **(ii)** 1, **(iii)** a_n **4.** $x = 0$ or 2

5. **(a)** 101^{50} (Prove that $101^{50} - 99^{50} = 100^{50}$ + some +ve qty)

6. **(i)** $T_7 = \frac{7.3^{13}}{2}$ **(ii)** 455×3^{12} **10. (a)** $\frac{n^2+n+2}{2}$

11. **(a)** $84b^6c^3 + 630ab^4c^4 + 756a^2b^2c^5 + 84a^3c^6$; **(b)** $-1260 \cdot a^2b^3c^4$; **(c)** -12600

12. ${}^nC_r(3^{n-r} - 2^{n-r})$ **13.** (a) $n = 12$ (b) $\frac{5}{8} < x < \frac{20}{21}$ **14.** (a) 8016; (b) 500

EXERCISE-III

1.	B	**2.**	B	**3.**	B	**4.**	C	**5.**	D	**6.**	A	**7.**	D
8.	C	**9.**	A	**10.**	D	**11.**	A	**12.**	B	**13.**	D	**14.**	B
15.	B												

EXERCISE–IV

1.	C	**2.**	A	**3.**	C	**4.**	D	**5.**	D	**6.**	C	**7.**	D
8.	D	**9.**	D	**10.**	B	**11.**	B	**12.**	B	**13.**	B	**14.**	D
15.	A	**16.**	D	**17.**	A	**18.**	B	**19.**	Bonus	**20.**	A	**21.**	A

EXERCISE–V

1.	C	**2.**	D	**4.**	${}^{12}C_6$	**5.**	B	**6.**	-22100	**7.**	C		
8.	**(a)** A	**9.**	D	**10.**	A	**11.**	D	**12.**	6	**13.**	C		
14.	8	**15.**	5	**16.**	646	**17.**	6.20	**18.**	A,B,D				