DPP-2

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1. If a, b, c are positive numbers such that $a^{\log_3 7} = 27, b^{\log_7 11} = 49, c^{\log_{11} 25} = \sqrt{11}$, then the sum of digits of $S = a^{(\log_3 7)^2} + b^{(\log_7 11)^2} + c^{(\log_{11} 25)^2}$ is :

(A) 15 (B) 17 (C) 19 (D) 21

Ans. **(C)**

Sol. $a^{\log,27} = 27 \Rightarrow a = (27)^{\frac{1}{\log,_7 7}}$

Similarly, $b = (49)^{\frac{1}{109,11}}, c = (11)^{\frac{1}{2\log_{22} 25}}$

Now, $S = (27)^{\frac{1}{\log_3 7}\cdot(100,7)^2} + (49)^{\frac{1}{\log,11}\cdot(\log,11)^2} + (11)^{\frac{1}{2\log_{11} 25}=(\log_{11} 25)^2}$

$S = (27)^{\log_3 7} + (49)^{\log_7 11} + 11^{\frac{1}{2}\log_1 25} \Rightarrow S = 7^3 + 11^2 + 5 = 343 + 121 + 5 = 469$

$4 + 6 + 9 = 19$

2. Let $P = \dfrac{5}{\frac{1}{\log_2 x}+\frac{1}{\log_3 x}+\frac{1}{\log_4 x}+\frac{1}{\log_5 x}}$ and $(120)^p = 32$, then the value of x be:

(A) 1 (B) 2 (C) 3 (D) 4

Ans. **(B)**

Sol. $P = \dfrac{5}{\log_x (2\times3\times4\times5)} \quad \Rightarrow \quad P\log_x(120) = 5 \Rightarrow 120^{P^x} = x^5 \Rightarrow x = 2$

3. If $\log_{12}27 = a$, then $\log_6 16 =$

(A) $2\left(\frac{3-a}{3+a}\right)$ (B) $3\left(\frac{3-a}{3+a}\right)$ (C) $4\left(\frac{3-a}{3+a}\right)$ (D) None of these

Ans. **(C)**

Sol. $3\log_{12} 3 = a$

$\dfrac{3\log_3 3}{\log_3 12} = a \Rightarrow a = \dfrac{3}{1+\log_3 4}$ ....(1)

$\log_6 16 = \dfrac{\log_4 16}{\log_4 6} = \dfrac{4}{2\log_4 3+1}$ ....(2)

from (1) & (2), $\log_6 16 = \dfrac{4(3-a)}{(3+a)}$

4. Suppose that a and b are positive real numbers such that $\log_{27}a + \log_9 b = \frac{7}{2}$ and $\log_{27}b + \log_9 a = \frac{2}{3}$. Then the value of $a\cdot b$ is:

(A) 81 (B) 243 (C) 27 (D) 729

Ans. **(B)**

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Sol. $\frac{1}{3}\log_3 a + \frac{1}{2}\log_3 b = \frac{7}{2}$

$2\log_3 a + 3\log_3 b = 21$ ....(1)

similarly, $\frac{1}{3}\log_3 b + \frac{1}{2}\log_3 a = \frac{2}{3}$

$2\log_3 b + 3\log_3 a = 4$ ....(2)

Solve (1) & (2)

$\log_3 a = -6$ $\quad$ $\log_3 3_b = -11$

$a = 3^{-6}, b = 3^{11}$

$a \cdot b = 3^5 = 243$

5. The number of zeros after decimal before the start of any significant digit in the number $N = (0.15)^{20}$ are :

(A) 15 (B) 16 (C) 17 (D) 18

Ans. **(B)**

Sol. No zeros after decimal before start of any significant digit in $N = (0.15)^{20}$.

Taking log to base 10 on both sides

$\therefore \log N = \log(0.15)^{20} = 20\log\left(\frac{15}{100}\right)$

$= 20(\log 15 - \log 100) = 20(\log 3 \times 5 - 2)$

$= 20(\log 3 + \log 5 - 2)$ using log table to base 10

$= 20(0.4771 + 0.6990 - 2) = 20(-0.8239) = -16.478$ -16 $\quad$ 0.478

$\therefore N = 10^{-16.478} = 16 \times 10^{0.478} \rightarrow$ significant digit

we know $10^3 = \frac{1}{1000} = 0.001$

$\therefore 10^{-16}$ means no. of zeros after decimal are $(16 - 1) = 15$

6. $\log_{(x-1)}(3) = 2$

(A) $\sqrt{3}$ (B) $1-\sqrt{3}$ (C) 1 (D) None of these

Ans. **(D)**

Sol. $\log_{x-1} 3 = 2$

$\Rightarrow 3 = (x-1)^2 \Rightarrow x^2 - 2x - 2 = 0$

$\Rightarrow x = \frac{2\pm\sqrt{4+8}}{2} \Rightarrow x = \frac{2\pm2\sqrt{3}}{2}$

$\Rightarrow x = 1+\sqrt{3}$ $(\because x \neq 1-\sqrt{3})$

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7. $\log_2 [\log_4 (\log_{10} 16^4 + \log_{10} 25^8)]$ simplifies to

(A) an irrational (B) an odd prime (C) a composite (D) unity

Ans **(D)**

Sol. $\log_2 \{\log_4 [\log_{10} (16^4 \cdot 25^8)]\}$

$\log_2 \{\log_4 [\log_{10} (100^8)]\}$

$\log_2 [\log_4 (\log_{10} 10^{16})]$

$\log_2 (\log_4 16) = \log_2 \left(\log_4^{4^2}\right)$

$= \log_2^2 = 1$

8. The sum of all the solutions to the equation $2\log x - \log (2x - 75) = 2$:

(A) 30 (B) 350 (C) 75 (D) 200

Ans. **(D)**

Sol. $2\log_{10} x - \log (2x - 75) = 2$

$\log_{10} x^2 - \log_{10} (2x - 75) = 2$

$\log_{10} \left(\frac{x^2}{2x-75}\right) = 2$

$\frac{x^2}{2x-75} = 100$

$x^2 - 200x + 7500 = 0$

sum of solutions $= \frac{-b}{a} = \frac{-(-200)}{1} = 200$

9. Product of all values of x satisfying the equation $\sqrt{2^x \sqrt[3]{4^x(0.125)^{1/x}}} = 4\,(\sqrt[3]{2})$ is :

(A) $\frac{14}{5}$ (B) 3 (C) $-\frac{1}{5}$ (D) $-\frac{3}{5}$

Ans. **(D)**

Sol. $\sqrt{2^x\left(4^x\left(\frac{125}{1000}\right)^{\frac{1}{x}}\right)^{1/3}} = 4(2)^{1/3}$

take $\log_2$ both side :

$\frac{5x}{3} - \frac{1}{x} = \frac{14}{3}$

$5x^2 - 14x - 3 = 0$ x_1, x_2 $\Rightarrow x_1x_2 = \frac{-3}{5}$

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10. If $a^x = b^y = c^z = d^w$, then $\log_a(bcd) =$

(A) $z\left(\frac{1}{x}+\frac{1}{y}+\frac{1}{w}\right)$ (B) $y\left(\frac{1}{x}+\frac{1}{z}+\frac{1}{w}\right)$

(C) $x\left(\frac{1}{y}+\frac{1}{z}+\frac{1}{w}\right)$ (D) $\frac{xyz}{w}$

Ans. **(C)**

Sol. Let $a^x = b^y = c^z = d^w = k$

$a = k^{1/x}, b = k^{1/y}, c = k^{1/z}, d = k^{1/w}$ now $bcd = k^{\left(\frac{1}{y}+\frac{1}{z}+\frac{1}{w}\right)}$

$\log_a(bcd) = \left(\frac{1}{y}+\frac{1}{z}+\frac{1}{w}\right)\log_w(k)$

also, $\log_a a = \frac{1}{x}\log_a k$ $\log_a k = x$

11. If $(4)^{\log_9 3} + (9)^{\log_2 4} = (10)^{\log_x 83}$, then x is equal to:

(A) 2 (B) 3 (C) 10 (D) 30

Ans. **(C)**

Sol. $4^{\log 3} + 9^{\log 4} = 10^{\log 83}$

$2 + 9^2 = 83^{\log 10}$

$83 = 83^{\log 10} \rightarrow \log_x 10 = 1$

$x = 10$

12. $x^{\log_{10}\left(\frac{y}{z}\right)} \cdot y^{\log_{10}\left(\frac{z}{x}\right)} \cdot z^{\log_{10}\left(\frac{x}{y}\right)}$ is equal to:

(A) 0 (B) 1 (C) -1 (D) 2

Ans. **(B)**

Sol. Let $k = x^{\log_{10}\left(\frac{y}{z}\right)} \cdot y^{\log_{10}\left(\frac{z}{x}\right)} \cdot z^{\log_{10}\left(\frac{x}{y}\right)}$

$\log_{10} k = \log_{10}\left(\frac{y}{z}\right)\log_{10} x + \log_{10}\left(\frac{z}{x}\right)\log_{10} y + \log_{10}\left(\frac{x}{y}\right)\log_{10} z$

$\log_{10} k = \log_{10} x\log_{10} y - \log_{10} x\log_{10} z +$

$\log_{10} z\log_{10} y - \log_{10} x\log_{10} y + \log_{10} \times \log_{10} z -$

$\log_{10} y\log_{10} z$

$k = 10^0 = 1$

13. $\log_3(3^x - 8) = 2 - x$

(A) 1 (B) 3 (C) 4 (D) 2

Ans. **(D)**

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Sol. $\log_3 (3^x - 8) = 2 - x$

$3^x - 8 = 3^{2-x}$

$3^x - 8 = \frac{3^2}{3^x}$

$(3^x)^2 - 8 \cdot (3^x) - 9 = 0$

$(3^x - 9)(3^x + 1) = 0$

$3^x - 9 = 0$

$3^x = 9 = 3^2$

$x = 2$