Practice Questions (Solutions)

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1. $\cos^4 A - \sin^4 A + 1 = 2\cos^2 A$.

Sol. $L \cdot H \cdot S := (\cos^2 A)^2 - (\sin^2 A)^2 + 1\{\because a^2 - b^2 = (a+b)(a-b)\}$

$= (\cos^2 A + \sin^2 A)(\cos^2 A - \sin^2 A) + 1$

$= (1) \cdot (\cos^2 A - \sin^2 A) + 1$

$= \cos^2 A - (1 - \cos^2 A) + 1$

$= \cos^2 A - 1 + \cos^2 A + 1$

$= 2\cos^2 A = R \cdot H \cdot S$

2. $(\sin A + \cos A)(1 - \sin A \cos A) = \sin^3 A + \cos^3 A$.

Sol. R.H. S $= \sin^3 A + \cos^3 A[\because a^3 + b^3 = (a+b)(a^2 + b^2 - ab)]$

$= (\sin A + \cos A)(\sin^2 A + \cos^2 A - \sin A \cdot \cos A)$

$= (\sin A + \cos A)(1 - \sin A \cos A)$

$=$ L.H.S.

3. $\frac{\sin A}{1+\cos A} + \frac{1+\cos A}{\sin A} = 2\operatorname{cosec} A$.

Sol. L.H.S. $= \frac{\sin A}{1+\cos A} \times \frac{1-\cos A}{1-\cos A} + \frac{1+\cos A}{\sin A}$

$$= \frac{\sin A(1-\cos A)}{(1^2 - \cos^2 A)} + \frac{1+\cos A}{\sin A}$$

$$= \frac{\sin A(1-\cos A)}{\sin^2 A} + \frac{1+\cos A}{\sin A}$$

$$= \frac{1-\cos A}{\sin A} + \frac{1+\cos A}{\sin A} = \frac{1-\cos A + 1 + \cos A}{\sin A} = \frac{2}{\sin A}$$

$= 2\operatorname{cosec} A$

4. $\cos^6 A + \sin^6 A = 1 - 3\sin^2 A \cos^2 A$.

Sol. L.H.S. $= (\cos^2 A)^3 + (\sin^2 A)^3 (a^3 + b^3 = (a+b)^3 - 3ab(a+b))$

$= (\cos^2 A + \sin^2 A)^3 - 3\cos^2 A \sin^2 A(\cos^2 A + \sin^2 A)$

$= (1)^3 - 3\cos^2 A \sin^2 A(1)$

$= 1 - 3\cos^2 A \sin^2 A =$ R.H.S.

5. $\sqrt{\frac{1-\sin A}{1+\sin A}} = \sec A - \tan A$

Sol. L.H.S. $= \sqrt{\frac{1-\sin A}{1+\sin A} \times \frac{1-\sin A}{1-\sin A}}$

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$$= \sqrt{\frac{(1-\sin A)^2}{1^2-\sin^2 A}} = \sqrt{\frac{(1-\sin)^2}{\cos^2 A}} = \sqrt{\left(\frac{1-\sin A}{\cos A}\right)^2}$$

$= \frac{1-\sin A}{\cos A} = \frac{1}{\cos A} - \frac{\sin A}{\cos A} = \sec A - \tan A$

6. $\frac{\operatorname{cosec} A}{\operatorname{cosec} A - 1} + \frac{\operatorname{cosec} A}{\operatorname{cosec} A + 1} = 2\sec^2 A$

Sol. Given $\frac{\operatorname{cosec} A}{\operatorname{cosec}\theta+1} + \frac{\operatorname{cosec} A}{\operatorname{cosec} A-1}$

Taking LCM

$\frac{\operatorname{cosec} A}{\operatorname{cosec} A+1} + \frac{\operatorname{cosec} A}{\operatorname{cosec} A-1} = \frac{\operatorname{cosec}^2 A-\operatorname{cosec} A+\operatorname{cosec}^2 A+\operatorname{cosec} A}{\operatorname{cosec}^2 A-1}$

$= \frac{2\operatorname{cosec}^2 A}{\operatorname{cosec}^2 A-1}$

As $\operatorname{cosec}^2 A - 1 = \cot^2 A$

$$= \frac{2\operatorname{cosec}^2 A}{\cot^2 A} = \frac{\frac{2}{\sin^2 A}}{\frac{\cos^2 A}{\sin^2 A}} = 2\sec^2 A$$

7. $\frac{\operatorname{cosec} A}{\cot A + \tan A} = \cos A$

Sol. Solving LHS of $\frac{\csc A}{\cot A+\tan A} = \cos A$

$$\frac{\csc A}{\cot A + \tan A} = \frac{\frac{1}{\sin A}}{\frac{\cos A}{\sin A} + \frac{\sin A}{\cos A}}$$

$$= \frac{1}{\sin A} \times \frac{\sin A \cos A}{\cos^2 A + \sin^2 A}$$

$$= \frac{1}{1} \times \frac{\cos A}{1}$$

$= \cos A$

8. $(\sec A + \cos A)(\sec A - \cos A) = \tan^2 A + \sin^2 A$

Sol. $(\sec A - \cos A)(\sec A + \cos A)$

$= \sec^2 A - \cos^2 A$

$= 1 + \tan^2 A - \cos^2 A$

$= 1 - \cos^2 A + \tan^2 A$

$= \sin^2 A + \tan^2 A$

9. $\frac{1}{\cot A + \tan A} = \sin A \cos A$

Sol. $\frac{1}{\tan A+\cot A} = \sin A \cos A$

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$$\frac{1}{\tan A + \cot A}$$

$$= \frac{1}{\frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}} = \frac{1}{\frac{\sin^2 A + \cos^2 A}{\sin A \cos A}}$$

$$= \frac{1}{\frac{1}{\sin A \cos A}} (\sin^2 A + \cos^2 A = 1) = \sin A \cos A$$

10. $\frac{1}{\sec A - \tan A} = \sec A + \tan A$

Sol. $\frac{1}{\sec A - \tan A} \times \frac{\sec A + \tan A}{\sec A + \tan A} = \frac{\sec A + \tan A}{1} = \sec A + \tan A$

11. $\frac{1-\tan A}{1+\tan A} = \frac{\cot A - 1}{\cot A + 1}$

Sol. $\text{L. H. S.} = \frac{1-\tan A}{1+\tan A} = \frac{1-\frac{1}{\cot A}}{1+\frac{1}{\cot A}} = \frac{\frac{\cot A - 1}{\cot A}}{\frac{\cot A + 1}{\cot A}}$

$$= \frac{\cot A - 1}{\cot A + 1} = \text{R.H.S.}$$

12. $\frac{1+\tan^2 A}{1+\cot^2 A} = \frac{\sin^2 A}{\cos^2 A}$

Sol. $\text{L.H. S} = \frac{1+\frac{\sin^2 A}{\cos^2 A}}{1+\frac{\cos^2 A}{\sin^2 A}} = \frac{\frac{(\cos^2 A + \sin^2 A) = 1}{(\cos^2 A}}{\frac{(\sin^2 A + \cos^2 A) = 1}{\sin^2 A}}$

$$= \frac{\sin^2 A}{\cos^2 A} = \text{R. H. S.}$$

13. $\frac{\sec A - \tan A}{\sec A + \tan A} = 1 - 2\sec A \tan A + 2\tan^2 A$

Sol. $\text{L.H.S.} = \frac{\sec A - \tan A}{\sec A + \tan A}$

$$= \frac{\sec A - \tan A}{\sec A + \tan A} \times \frac{\sec A - \tan A}{\sec A - \tan A}$$

$$= \frac{\sec^2 A - 2 \sec A \tan A + \tan^2 A}{\sec^2 A - \tan^2 A}$$

$$= 1 + \tan^2 A - 2 \sec A \tan A + \tan^2 A$$

$$= 1 - 2 \sec A \tan A + 2 \tan^2 A$$

14. $\frac{\tan A}{1-\cot A} + \frac{\cot A}{1-\tan A} = \sec A \operatorname{cosec} A + 1.$

Sol. $\text{LHS} = \frac{\frac{\sin A}{\cos A}}{1-\frac{\cos A}{\sin A}} + \frac{\frac{\cos A}{\sin A}}{1-\frac{\sin A}{\cos A}}$

$$= \frac{\sin A}{\cos A} \times \frac{\sin A}{\sin A - \cos A} + \frac{\cos A}{\sin A} \times \frac{\cos A}{\cos A - \sin A}$$

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$$= \frac{\sin^2 A}{\cos A(\sin A - \cos A)} + \frac{\cos^2 A}{\sin A(\cos A - \sin A)}$$

$$= \frac{\sin^2 A}{\cos A(\sin^2 A - \cos A)} - \frac{\cos^2 A}{\sin A(\sin^2 - \cos A)}$$

$$= \frac{\sin^3 A - \cos^3 A}{\sin A \cdot \cos A(\sin A - \cos A)}$$

$$= \frac{(\sin A - \cos A)(\sin^2 A + \cos^2 A + \sin A \cdot \cos A)}{\sin A \cdot \cos A(\sin A - \cos A)}$$

$$= \frac{1 + \sin A \cdot \cos A}{\sin A \cdot \cos A}$$

$$= \frac{1}{\sin A \cdot \cos A} + \frac{\sin A \cdot \cos A}{\sin A \cdot \cos A}$$

$= \sec A \cdot \operatorname{cosec} A + 1$

$=$ R.H.S.

15. $\frac{\cos A}{1-\tan A} + \frac{\sin A}{1-\cot A} = \sin A + \cos A$

Sol. We need to prove $\frac{\cos A}{1-\tan A} + \frac{\sin A}{1-\cot A} = \sin A + \cos$

Solving the L.H.S, we get

$$\frac{\cos A}{1-\tan A} + \frac{\sin A}{1-\cot A} = \frac{\cos A}{1-\frac{\sin A}{\cos A}} = \frac{\sin A}{1-\frac{\cos A}{\sin A}}$$

$$= \frac{\cos A}{\frac{\cos A-\sin A}{\cos A}} + \frac{\sin A}{\frac{\sin A-\cos A}{\sin A}}$$

$$= \frac{\cos^2 A}{\cos A-\sin A} = \frac{\sin^2 A}{\sin A-\cos A}$$

$$= \frac{\cos^2 A-\sin^2 A}{\cos A-\sin A}$$

$$= \frac{(\cos A+\sin A)(\cos A-\sin A)}{\cos A-\sin A} \text{ [using } a^2 - b^2 = (a+b)(a-b)]$$

$= \cos A + \sin A$

$=$ RHS

16. $(\sin A + \cos A)(\cot A + \tan A) = \sec A + \operatorname{cosec} A.$

Sol. L.H.S. $= (\sin A + \cos A)\left(\frac{\cos A}{\sin A} + \frac{\sin A}{\cos A}\right)$

$$= (\sin A + \cos A) \cdot \frac{\cos^2 A + \sin^2 A)}{\sin A \cdot \cos A}$$

$$= \frac{\sin A}{\sin A\cos A} + \frac{\cos A}{\sin A \cdot \cos A}$$

$= \sec A + \operatorname{cosec} A = \text{R. H. S.}$

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17. $\sec^4 A - \sec^2 A = \tan^4 A + \tan^2 A$

Sol. $\sec^4 A - \sec^2 A = \sec^2 A(\sec^2 A - 1)$

$= (1 + \tan^2 A)(1 + \tan^2 A - 1)$

$= \tan^2 A + \tan^4 A$

18. $\cot^4 A + \cot^2 A = \operatorname{cosec}^4 A - \operatorname{cosec}^2 A.$

Sol. To prove: $\operatorname{cosec}^4 A - \operatorname{cosec}^2 A = \cot^4 A + \cot^2 A$

LHS $= (\operatorname{cosec}^2 A)^2 - \operatorname{cosec}^2 A$

$= (1 + \cot^2 A)^2 - (1 + \cot^2 A)$

$= 1 + 2\cot^2 A + \cot^4 A - 1 - \cot^2 A[\operatorname{cosec}^2 A = 1 + \cot^2 A]$

$= \cot^2 A + \cot^4 A =$ RHS

19. $\sqrt{\operatorname{cosec}^2 A - 1} = \cos A \operatorname{cosec} A$

Sol. $\sqrt{\operatorname{cosec}^2 A - 1} = \cos A \operatorname{cosec} A$

LTTS: $\sqrt{\frac{1}{\sin^2 A} - 1}$

$= \sqrt{\frac{1-\sin^2 A}{\sin^2 A}}$

$= \sqrt{\frac{\cos^2 A}{\sin^2 A}}\,[1 - \sin^2 A = \cos^2 A]$

$= \frac{\cos A}{\sin A}$

$= \cos A \operatorname{cosec} A$

20. $\sec^2 A \operatorname{cosec}^2 A = \tan^2 A + \cot^2 A + 2.$

Sol. $\sec^2 A \operatorname{cosec}^2 A$

$= (1 + \tan^2 A)(1 + \cot^2 A)$

$= 1 + \cot^2 A + \tan^2 A + \tan^2 A \cdot \cot^2 A$

$= 1 + \cot^2 A + \tan^2 A + \frac{1}{\cot^2 A} \times \cot^2 A$

$= \tan^2 A + \cot^2 A + 2$

21. $\tan^2 A - \sin^2 A = \sin^4 A \sec^2 A.$

Sol. L.H.S $= \tan^2 A - \sin^2 A$

$= \frac{\sin^2 A}{\cos^2 A} - \sin^2 A$

$= \sin^2 A\left(\frac{1}{\cos^2 A} - 1\right)$

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$$= \sin^2A\left(\frac{1-\cos^2A}{\cos^2A}\right)$$

$$= \sin^2A\left(\frac{\sin^2A}{\cos^2A}\right)$$

$= \sin^4A\sec^2A =$ R.H.S

22. $(1+\cot A-\operatorname{cosec}A)(1+\tan A+\sec A)=2.$

Sol. L.H.S. $=(1+\cot A-\operatorname{cosec}A)(1+\tan A+\sec A)$

$$=\left(1+\frac{\cos A}{\sin A}-\frac{1}{\sin A}\right)\left(1+\frac{\sin A}{\cos A}+\frac{1}{\cos A}\right)$$

$$=\left(\frac{\sin A+\cos A-1}{\sin A}\right)\left(\frac{\cos A+\sin A+1}{\cos A}\right)$$

$$=\frac{(\sin A+\cos A)^2-1^2}{\sin A\cdot\cos A}$$

$$=\frac{\sin^2A+\cos^2A+2\sin A\cdot\cos A-1}{\sin A\cdot\cos A}$$

$$=\frac{1+2\sin A\cdot\cos A-1}{\sin A\cdot\cos A}$$

$= 2$

$=$ R.H.S.

23. $\frac{1}{\operatorname{cosec}A-\cot A}-\frac{1}{\sin A}=\frac{1}{\sin A}-\frac{1}{\operatorname{cosec}A+\cot A}.$

Sol. $\frac{1}{\operatorname{cosec}A-\cot A}-\frac{1}{\sin A}=\frac{1}{\sin A}-\frac{1}{\operatorname{cosec}A+\cot A}$

or $\frac{1}{\operatorname{cosec}A-\cot A}+\frac{1}{\operatorname{cosec}A+\cot A}=\frac{1}{\sin A}+\frac{1}{\sin A}=\frac{2}{\sin A}$

$$\text{LHS } =\frac{(\operatorname{cosec}A+\cot A)+(\operatorname{cosec}A-\cot A)}{\operatorname{cosec}A-\cot A}(\operatorname{cosec}A+\cot A)$$

$$=\frac{2\operatorname{cosec}A}{\operatorname{cosec}A-\cot^2A}$$

$$=\frac{2\operatorname{cosec}A}{1}$$

$$=\frac{2}{\sin A}$$

$=$ RHS

Hence proved.

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24. $\frac{\cot A \cos A}{\cot A+\cos A}=\frac{\cot A-\cos A}{\cot A \cos A}$

Sol. Simplifying the LHS of $\frac{\cot A \cos A}{\cot A+\cos A}=\frac{\cot A-\cos A}{\cot A \cos A}$.

$$\frac{\cot A \cos A}{\cot A+\cos A}=\frac{\frac{\cos A}{\sin A}\cos A}{\frac{\cos A}{\sin A}+\cos A}$$

$$=\frac{\cos^2 A}{\cos A+\cos A \sin A}$$

$$=\frac{1-\sin^2 A}{\cos A(1+\sin A)}$$

$$=\frac{(1-\sin A)(1+\sin A)}{\cos A(1+\sin A)}$$

$$=\frac{1-\sin A}{\cos A}$$

Now, simplifying the RHS of $\frac{\cot A-\cos A}{\cot A \cos A}=\frac{\cot A-\cos A}{\cot A \cos A}$.

$$\frac{\cot A-\cos A}{\cot A \cos A}=\frac{\frac{\cos A}{\sin A}-\cos A}{\frac{\cos A}{\sin A}\times\cos A}$$

$$=\frac{\cos A-\cos A \sin A}{\cos^2 A}$$

$$=\frac{\cos A(1-\sin A)}{\cos^2 A}$$

$$=\frac{1-\sin A}{\cos A}$$

This shows that LHS = RHS.

25. $\frac{\cot A+\tan B}{\cot B+\tan A}=\cot A \tan B.$

Sol. LHS $=\frac{\cot A+\tan B}{\cot B+\tan A}$

$$=\frac{\frac{1}{\tan A}+\tan B}{\frac{1}{\tan B}+\tan A}=\frac{\frac{1+\tan A \tan B}{\tan A}}{\frac{1+\tan A \tan B}{\tan B}}=\frac{\tan B}{\tan A}=\cot A \tan B= \text{ RHS}$$

26. $\left(\frac{1}{\sec^2\alpha-\cos^2\alpha}+\frac{1}{\operatorname{cosec}^2\alpha-\sin^2\alpha}\right)\cos^2\alpha \sin^2\alpha=\frac{1-\cos^2\alpha \sin^2\alpha}{2+\cos^2\alpha \sin^2\alpha}$

Sol. $\frac{1}{\sec^2\alpha-\cos^2\alpha}=\frac{\cos^2\alpha}{1-\cos^4\alpha}\left(\because \sec\alpha=\frac{1}{\cos\alpha}\right)$

$$\frac{1}{\csc^2\alpha-\sin^2\alpha}=\frac{\sin^2\alpha}{1-\sin^4\alpha}\left(\because \csc\alpha=\frac{1}{\sin\alpha}\right)$$

$$\frac{1}{\sec^2\alpha-\cos^2\alpha}+\frac{1}{\csc^2\alpha-\sin^2\alpha}=\frac{\cos^2\alpha}{1-\cos^4\alpha}+\frac{\sin^2\alpha}{1-\sin^4\alpha}$$

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$$= \frac{\cos^2\alpha}{(1-\cos^2\alpha)(1+\cos^2\alpha)} + \frac{\sin^2\alpha}{(1-\sin^2\alpha)(1+\sin^2\alpha)}$$

$$= \frac{\cos^2\alpha}{\sin^2\alpha(1+\cos^2\alpha)} + \frac{\sin^2\alpha}{\cos^2\alpha(1+\sin^2\alpha)}$$

$$= \frac{\cos^4\alpha(1+\sin^2\alpha)+\sin^4\alpha(1+\cos^2\alpha)}{\sin^2\alpha\cos^2\alpha(1+\cos^2\alpha)(1+\sin^2\alpha)}$$

$= \frac{\cos^4\alpha+\sin^4\alpha+\sin^2\alpha\cos^2\alpha(\sin^2\alpha+\cos^2\alpha)}{\sin^2\alpha\cos^2\alpha(1+\cos^2\alpha+\sin^2\alpha+\sin^2\alpha\cos^2\alpha)}$ $(\because \sin^2\alpha+\cos^2\alpha=1)(\because \cos^4\alpha+\sin^4\alpha =$
$(\cos^2\alpha+\sin^2\alpha)^2 - 2\cos^2\alpha\sin^2\alpha)$

$$= \frac{(\cos^2\alpha+\sin^2\alpha)^2 - 2\sin^2\alpha\cos^2\alpha + \sin^2\alpha\cos^2\alpha}{\sin^2\alpha\cos^2\alpha(2+\cos^2\alpha+\sin^2\alpha)}$$

$$\left[\frac{1}{\sec^2\alpha-\cos^2\alpha} + \frac{1}{\csc^2\alpha-\sin^2\alpha}\right]\sin^2\alpha\cos^2\alpha = \frac{1-\sin^2\alpha\cos^2\alpha}{2+\sin^2\alpha\cos^2\alpha}$$

27. $\frac{\cos A\,\mathrm{cosec}A-\sin A\sec A}{\cos A+\sin A} = \mathrm{cosec}A - \sec A$

Sol. LHS $= \frac{\cos A\,\mathrm{cosec}A-\sin A\sec A}{\cos A+\sin A}$

$$= \frac{\cos^2 A - \sin^2 A}{\sin A\cos A(\cos A+\sin A)}$$

$$= \frac{(\cos A-\sin A)(\cos A+\sin A)}{\sin A\cos A(\cos A+\sin A)}$$

$$= \frac{\cos A-\sin A}{\sin A\cos A}$$

$$= \frac{1}{\sin A} - \frac{1}{\cos A}$$

$= \mathrm{cosec}A - \sec A$

$= \text{RHS}$

Hence proved

28. $\frac{\tan A+\sec A-1}{\tan A-\sec A+1} = \frac{1+\sin A}{\cos A}$

Sol. L.H.S $= \frac{\tan A+\sec A-1}{\tan A-\sec A+1}$

$$= \frac{(\tan A+\sec A)-(\sec^2 A-\tan^2 A)}{\tan A-\sec A+1}$$

$$= \frac{(\tan A+\sec A)(1-(\sec A-\tan A))}{\tan A-\sec A+1}$$

$$= \frac{(\tan A+\sec A)(1-\sec A-\tan A)}{\tan A+1-\sec A}$$

$= \sec A + \tan A$

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$= \frac{1}{\cos A} + \frac{\sin A}{\cos A}$

$= \frac{1 + \sin A}{\cos A}$

R.H.S

29. $(\tan\alpha + \operatorname{cosec}\beta)^2 - (\cot\beta - \sec\alpha)^2 = 2\tan\alpha \cot\beta(\operatorname{cosec}\alpha + \sec\beta)$

Sol. L.H.S $= (\tan\alpha + \csc\beta)^2 - (\cot\beta - \sec\alpha)^2$

$= \tan^2\alpha + \csc^2\beta + 2\tan\alpha\csc\beta - \cot^2\beta - \sec^2\alpha + 2\cot\beta\sec\alpha$

$= (\tan^2\alpha - \sec^2\alpha) + (\csc^2\beta - \cot^2\beta) + 2\tan\alpha\csc\beta + 2\cot\beta\sec\alpha$

$= -1 + 1 + 2\tan\alpha\csc\beta + 2\cot\beta\sec\alpha$

$= 2\tan\alpha\cot\beta\left(\frac{1}{\sin\beta}\frac{\sin\beta}{\cos\beta} + \frac{1}{\cos\alpha}\frac{\cos\alpha}{\sin\alpha}\right)$

$= 2\tan\alpha\cot\beta(\sec\beta + \csc\alpha)$

$=$ R.H.S

30. $2\sec^2\alpha - \sec^4\alpha - 2\operatorname{cosec}^2\alpha + \operatorname{cosec}^4\alpha = \cot^4\alpha - \tan^4\alpha$

Sol. $R \cdot H \cdot S \cdot = (\cot^2\alpha)^2 - (\tan^2\alpha)^2$

$= (\operatorname{cosec}^2\alpha - 1)^2 - (\sec^2\alpha - 1)^2$

$= \operatorname{cosec}^4\alpha - 2\operatorname{cosec}^2\alpha + 1 - \sec^4\alpha + 2\sec^2\alpha - 1$

$= 2\sec^2\alpha - \sec^4\alpha - 2\operatorname{cosec}^2\alpha + \operatorname{cosec}^4\alpha$

$= L \cdot H \cdot S.$

31. $(\sin\alpha + \operatorname{cosec}\alpha)^2 + (\cos\alpha + \sec\alpha)^2 = \tan^2\alpha + \cot^2\alpha + 7$

Sol. L.H.S. $= \sin^2\alpha + \operatorname{cosec}^2\alpha + 2\sin\alpha \cdot \operatorname{cosec}\alpha + \cos^2\alpha + \sec^2\alpha + 2\cos\alpha \cdot \sec\alpha$

$= \operatorname{sen}^2\alpha + \cos^2\alpha) + (1 + \cot^2\alpha) + (1 + \tan^2\alpha) + 4$

$= 1 + 1 + \cot^2\alpha + 1 + \tan^2\alpha + 4$

$= \tan^2\alpha + \cot^2\alpha + 7 =$ R.H.S.

32. $(1 + \cot A + \tan A)(\sin A - \cos A) = \frac{\sec A}{\operatorname{cosec}^2 A} - \frac{\operatorname{cosec} A}{\sec^2 A}$

Sol. L.H.S. $= (\sin A - \cos A)\left(1 + \frac{\cos A}{\sin A} + \frac{\sin A}{\cos A}\right)$

$= \frac{1}{\sin A\cos A}(\sin A - \cos A)(\sin^2 A + \cos^2 A + \sin A - \cos A)$

$= \frac{\sin^3 A \cos^3 A}{\sin A \cos A} = \frac{\sin^2 A}{\cos A} - \frac{\cos^2 A}{\sin A}$

$= \frac{\sec A}{\csc^2 A} - \frac{\csc A}{\sec^2 A}.$