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Eqn of Tangents.

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

$y=mx\pm\sqrt{a^2m^2+b^2}$

Slope form.

Pt. form.

Pt. (x_1,y_1)

$T=0$

$$\boxed{\frac{xx_1}{a^2}+\frac{yy_1}{b^2}=1}$$

if Pt. lies on Ellipse use this form.

Par. form.

Pt. $(a\cos\theta, b\sin\theta)$

$$\frac{ax\cos\theta}{a^2}+\frac{by\sin\theta}{b^2}=1$$

$$\boxed{\frac{x\cos\theta}{a}+\frac{y\sin\theta}{b}=1}$$

When we need to take coord. on Ellipse itself

Min Value of $a\sec\theta+b\csc\theta=(a^{2/3}+b^{2/3})^{3/2}$ at $\theta=\tan^{-1}\left(\frac{b}{a}\right)^{1/3}$ (12th में पढ़ा था)

Q Value of θ for which Sum of intercepts made by tangents $(3\sqrt{3}\cos\theta, \sin\theta)$; $\theta\in\left(0,\frac{\pi}{2}\right)$ to E: $x^2+27y^2=27$ on coord axes is Min?

E: $\frac{x^2}{27}+\frac{y^2}{1}=1$

E.O.T:- $\frac{\cancel{3\sqrt{3}}\cos\theta\cdot x}{\cancel{3\sqrt{3}}\,27}+\frac{y\cdot\sin\theta}{1}=1$

Intercept form $\frac{x}{3\sqrt{3}\sec\theta}+\frac{y}{\csc\theta}=1$

Min of $3\sqrt{3}\sec\theta+\csc\theta$ at $\theta=\tan^{-1}\left(\frac{1}{3\sqrt{3}}\right)^{1/3}$

$\theta=\tan^{-1}\left(\frac{1}{(\sqrt{3})^3}\right)^{1/3}$

$=\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)=\frac{\pi}{6}$

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Q Area of quad. formed by tangents at the ends of LR of $5x^2+9y^2=45$ is?

$a=3$ $\qquad \frac{x^2}{9}+\frac{y^2}{5}=1$

$b=\sqrt{5}$

LR's end pt. $\left(ae, \frac{b^2}{a}\right)$

$1-e^2=\frac{b^2}{a^2}=\frac{5}{9}$

$e^2=\frac{4}{9} \Rightarrow e=\frac{2}{3}$

LR: $\left(2, \frac{5}{3}\right)$

EOT: $\frac{2x}{9}+\frac{\cancel{5}y}{3\cdot\cancel{5}}=1$

Intercept form: $\frac{x}{9/2}+\frac{y}{3}=1$

Δ's Area $=\frac{1}{2}\times\frac{9}{2}\times 3=\frac{27}{4}$

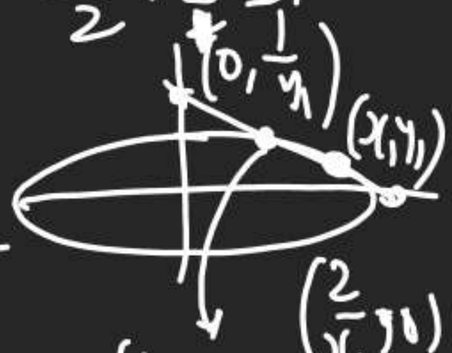
Quad Area $=4\times\frac{27}{4}=27$

Q If tangents are drawn to E: $x^2+2y^2=2$ then Locus of Midpt. of Intercept made by tangents betn coord axis is?

Let Pt. is (x_1, y_1) $\qquad$ E: $\frac{x^2}{2}+\frac{y^2}{1}=1$

EOT $\Rightarrow \frac{x\cdot x_1}{2}+\frac{y\cdot y_1}{1}=1$ $\qquad \Rightarrow \frac{x_1^2}{2}+\frac{y_1^2}{1}=1$

Int. form $\Rightarrow \frac{x}{2/x_1}+\frac{y}{1/y_1}=1$



$h=\frac{\frac{2}{x_1}+0}{2}$ $\quad\Big|\quad$ $k=\frac{0+\frac{1}{y_1}}{2} \Rightarrow y_1=\frac{1}{2k}$

$x_1=\frac{1}{h}$

$\frac{1}{2h^2}+\frac{1}{4k^2}=1 \Rightarrow \boxed{\frac{1}{2x^2}+\frac{1}{4y^2}=1}$

Q Let d be ⊥r distance from centre to E: $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ to tangent at a Pt. P on ellipse. If F_1 & F_2 are Focii of Ellipse then S.T. $(PF_1-PF_2)^2 = 4a^2\left(1-\frac{d^2}{a^2}\right)$

$(a-ex_1-a-ex_1)^2 = 4e^2x_1^2$

EOT. $\frac{xx_1}{a^2}+\frac{yy_1}{b^2}-1=0$

$$d = \frac{|-1|}{\sqrt{\frac{x_1^2}{a^4}+\frac{y_1^2}{b^4}}} \Rightarrow \frac{d^2}{a^2} = \frac{1}{a^2\left(\frac{x_1^2}{a^4}+\frac{y_1^2}{b^4}\right)}$$

here $\frac{x_1^2}{a^2}+\frac{y_1^2}{b^2}=1$

$\frac{y_1^2}{b^2} = 1-\frac{x_1^2}{a^2}$

$$\frac{d^2}{a^2} = \frac{1}{\left(\frac{x_1^2}{a^2}+\frac{a^2y_1^2}{b^4}\right)} = \frac{1}{\frac{x_1^2}{a^2}+\frac{a^2}{b^2}\left(1-\frac{x_1^2}{a^2}\right)}$$

$$-\frac{d^2}{a^2} = \frac{-1}{\frac{x_1^2}{a^2}+\frac{a^2}{b^2}-\frac{x_1^2}{b^2}} = \frac{-1}{x_1^2\left(\frac{1}{a^2}-\frac{1}{b^2}\right)+\frac{a^2}{b^2}} = \frac{+1}{x_1^2\left(\frac{a^2-b^2}{a^2b^2}\right)-\frac{a^2}{b^2}}$$

$$-\frac{d^2}{a^2} = \frac{b^2}{x_1^2e^2-a^2} \Rightarrow 1-\frac{d^2}{a^2} = 1+\frac{b^2}{x_1^2e^2-a^2}$$

$$\text{RHS} = 4a^2\left(1-\frac{d^2}{a^2}\right) = 4a^2\left(\frac{x_1^2e^2-a^2+a^2(1-e^2)}{x_1^2e^2-a^2}\right)$$

$$= 4a^2\left(\frac{e^2(x_1^2-a^2)}{x_1^2e^2-a^2}\right)$$