Nishant Jindal

Types of Matrix

Matrix

(5) Trace of Matrix

A) Tr(A)

B) Sum of values of Pr. diag

C) $Tr(A) = \sum_{i=1}^{n} a_{ii} = a_{11} + a_{22} + a_{33} \cdots a_{nn}$.

D) Prop.

(1) $Tr(KA) = K\,Tr(A)$

(2) $Tr(A+B) = Tr(A) + Tr(B)$

(3) $Tr(AB) = Tr(BA)$

(4) $Tr(A) = Tr(A^T)$

Q $A = \begin{bmatrix} 1 & 2 & 3 \\ 5 & 0 & 9 \\ -1 & 2 & -3 \end{bmatrix}$ $Tr(2A) = ?$

$Tr(2A) = 2\,Tr(A)$

$= 2(1 + 0 + -3) = -4$

Nishant Jindal

Jeemains/Adv Qs Machine

Q If A, B are 2 Matrices Such that

$$A+2B=\begin{bmatrix}1 & 2 & 0\\ 6 & -3 & 3\\ -5 & 3 & 1\end{bmatrix} \quad \& \quad A-2B=\begin{bmatrix}2 & -1 & 5\\ 2 & -1 & 6\\ 0 & 1 & 2\end{bmatrix}$$

find $Tr(A)-Tr(B)$?

$Tr(A+2B)= 1+(-3)+1=-1$

$Tr\,A + Tr(2B)=-1$

$Tr(A)+2Tr(B)=-1 \rightarrow (1)$

$Tr(A-2B)=3$

$Tr(A)-Tr(2B)=3$

$Tr(A)-2Tr(B)=3 \rightarrow (2)$

$$\begin{aligned}Tr(A)+2Tr(B)&=-1\\ Tr(A)-2Tr(B)&=3\\ \hline 2Tr(A)&=2\end{aligned}$$

$Tr(A)=1 \quad \therefore Tr(B)=-1$

demand $Tr(A)-Tr(B)=1-(-1)=2$

Q_3 If $\alpha, \beta, \gamma \in R$ $A=\begin{bmatrix}\alpha^2 & 6 & 8\\ 3 & \beta^2 & 9\\ 4 & 5 & \gamma^2\end{bmatrix}$

$\& B=\begin{bmatrix}2\alpha & 3 & 5\\ 2 & 2\beta & 6\\ 1 & 4 & 2\gamma-3\end{bmatrix}$ $\& \; tr(A)=tr(B)$

then $(\alpha^{-1}+\beta^{-1}+\gamma^{-1})=?$

$Tr(A)=Tr(B)$

$\alpha^2+\beta^2+\gamma^2 = 2\alpha+2\beta+2\gamma-3$

$(\alpha^2-2\alpha+1)+(\beta^2-2\beta+1)+(\gamma^2-2\gamma+1)=0$

$(\alpha-1)^2+(\beta-1)^2+(\gamma-1)^2=0$

$\alpha-1=0 \;\&\; \beta-1=0 \;\&\; \gamma-1=0$

$\alpha=\beta=\gamma=1$

Demand

$\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma}=\frac{1}{1}+\frac{1}{1}+\frac{1}{1}$

$=3$

Nishant Jindal

Q If trace of Matrix

$$A = \begin{bmatrix} 2\sin x & 1 & 0 \\ 0 & 5\cos y & 3 \\ 7 & 4 & 3\sin z \end{bmatrix}$$

is 10 find $\boxed{\text{Trace of } diag\left(8\tan\frac{x}{2}, 5\sin^2 y, -6\cos z\right)}$

where $x, y, z \in [0, \pi]$

diag. Matrix (a, b, c)

$$= \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$$

$Tr(A) = 2\boxed{\sin x} + 5\cos y + 3\sin z = \underline{10}$

$\leq 2 \quad \leq 5 \quad \leq 3$

Sab Max value denge

$2 + 5 + 3 = 10$ Ayega

$\sin x = 1$ & $\cos y = 1$ & $\sin z = 1$

$x = \frac{\pi}{2} \quad y = 0 \quad z = \frac{\pi}{2}$

demand

$diag\left(8\tan\frac{x}{2}, 5\sin^2 y, 6\cos z\right)$

$$= \begin{bmatrix} 8\tan\frac{x}{2} & 0 & 0 \\ 0 & 5\sin^2 y & 0 \\ 0 & 0 & -6\cos z \end{bmatrix}$$

Trace $= 8\tan\frac{x}{2} + 5\sin^2 y - 6\cos z$

$= 8\tan\frac{\pi}{4} + 5\sin^2 0 - 6\cos\frac{\pi}{2}$

$= 8 + 0 - 0 = 8$

Nishant Jindal

6) Types of Sq^n Matrix.

diagonal Matrix

$\text{diag}(d_1, d_2, d_3)$ = Upper & lower Δ^r Both

$$\begin{bmatrix} d_1 & 0 & 0 \\ 0 & d_2 & 0 \\ 0 & 0 & d_3 \end{bmatrix}$$

सार Zero नहीं होने देना

Scalar Matrix

$$A = \begin{bmatrix} K & 0 & 0 \\ 0 & K & 0 \\ 0 & 0 & K \end{bmatrix}$$

Identity Matrix (I_2, I_3)

$$I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$a_{ii} = 1$

Δ^r Matrix.

Upper Δ^r

$$A = \begin{bmatrix} a & b & c \\ 0 & d & e \\ 0 & 0 & f \end{bmatrix}$$

Lowe Δ^r Mat

$$A = \begin{bmatrix} a & 0 & 0 \\ b & c & 0 \\ d & e & f \end{bmatrix}$$

Nishant Jindal

Max Min type Qs.

Q Min No. of Zeroes in diag. Matrix?

Sq Mat

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ a_{31} & a_{32} & a_{33} & \cdots & a_{3n} \\ \vdots & & & & \\ a_{n1} & a_{n2} & a_{n3} & \cdots & a_{nn} \end{bmatrix}$$

x elements
n elements
x elements

n order.

No of total Elements $= n^2$

total Elements

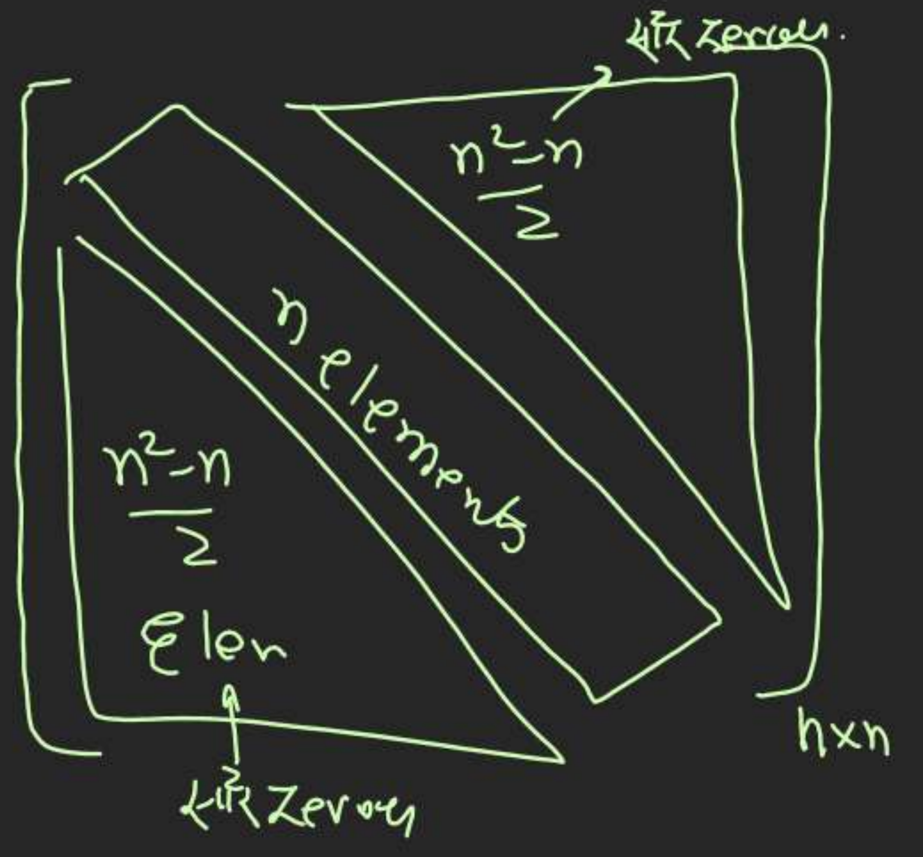
$= x + n + x = n^2$

$2x = n^2 - n \Rightarrow x = \dfrac{n^2-n}{2}$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix}$$

16

16 - 4



Min No of Zeroes $= \dfrac{n^2-n}{2} + \dfrac{n^2-n}{2}$

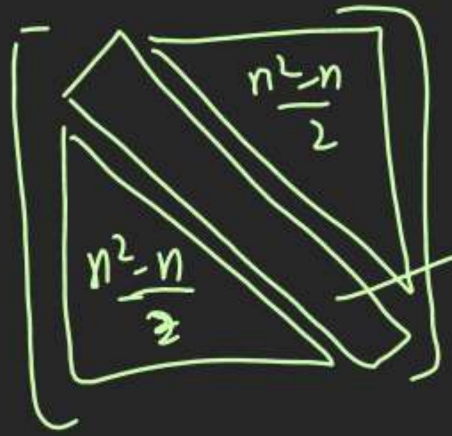
$= n^2 - n$ Zeroes

Nishant Jindal

Q Max. No. of Zeros?

Max Kitne Zeroes diagonal Matr Jhel lega.

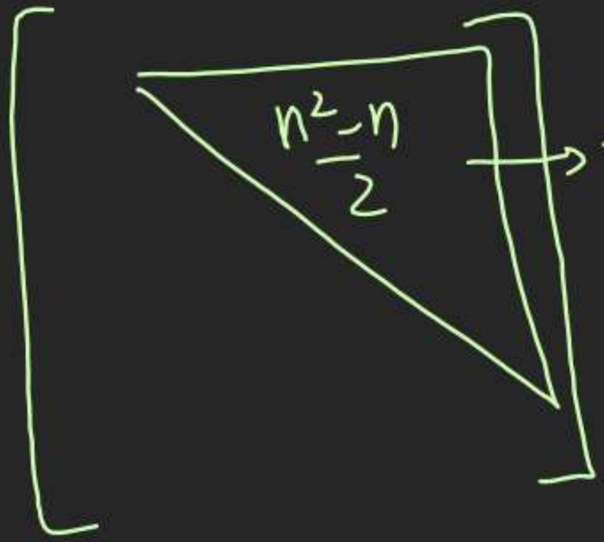
$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 3 & 0 \\ 4 & 5 & 1 \end{bmatrix}$$



→ n elements में 1 को Chhod Kar Baki Sare Zeroes Jhel Jayega

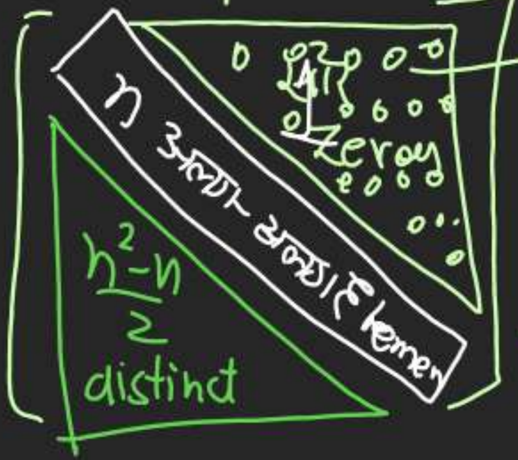
Ans $= n^2 - 1$

Q Min No of Zeros in Lower Δ^r Matrix?



$\therefore$ Min No of Zeros $= \frac{n^2-n}{2}$

Q Max. No. of distinct Elements in Lower Δ^r Matrix?



→ 1 है element भरे जा सके हैं

$$= \frac{n^2-n}{2} + n + 1$$

$$= \frac{n^2 - n + 2n + 2}{2} = \frac{n^2+n+2}{2}$$

Nishant Jindal

Comparable Matrix.

order of A = order of B.

Q If $A_{3\times5}$ & $B_{m\times n}$ are Comparable.

A) $(m,n) \equiv (5,3)$

B) $(m,n) \equiv (10,6)$

C) $(m,n) \equiv (15,15)$

D) $(m,n) \equiv (3,5)$

Equal Matrix.

1) When order of A = order of B

2) When elements of ~~both matrix~~ are Identical.

Q If $A = \begin{bmatrix} x & y & z \\ a & b & c \end{bmatrix}_{2\times3}$ & $B = \begin{bmatrix} 3 & 1 & 5 \\ 2 & 0 & 4 \end{bmatrix}_{2\times3}$

are Equal Matrix then $\frac{x+y+z}{a+b+c} = ?$

order of A = 2x3 = order of B

$x=3,\ y=1,\ z=5,\ a=2,\ b=0,\ c=4$

$$\frac{x+y+z}{a+b+c} = \frac{3+1+5}{2+0+4} = \frac{9}{6} = \frac{3}{2}$$

Nishant Jindal

Q If $\begin{bmatrix} \underline{2x+1} & 3y \\ 0 & y^2-5y \end{bmatrix} = \begin{bmatrix} \underline{x+3} & y^2+2 \\ 0 & 6 \end{bmatrix}$

then $\left[\frac{x+2y}{3}\right] = ?$

$2x+1 = x+3$

$x = 2$

$3y = y^2+2$

$y^2-3y+2=0$

$(y-1)(y-2)=0$

$y=1, 2$

$0=0$

$y^2-5y=6$

$y^2-5y-6=0$

$(y-6)(y+1)=0$

$\underline{y=-1}, y=6.$

$y = \phi$

Ans ϕ

Q $\begin{bmatrix} 2x+1 & 3y \\ 0 & y^2-5y \end{bmatrix} = \begin{bmatrix} x+3 & y^2+2 \\ 0 & -6 \end{bmatrix}$

$x=2$

$y^2-3y+2=0$

$y=1, 2$

$y^2-5y=-6$

$y^2-5y+6=0$

$(y-2)(y-3)=0$

$y=2, 3$

$y=2$

$\left[\frac{x+2y}{3}\right] = \left[\frac{2+2\times2}{3}\right]$

$= [2] = 2$

Nishant Jindal

Algebra of Matrices

1) Sum / difference of Matrix.

A) Sum / difference of 2 Matrix is Possible only when order of Both is Same.

B) Corresponding elements can be added or subtracted

① $A = \begin{bmatrix} 3 & 2 \\ 5 & 9 \\ 1 & 6 \end{bmatrix}_{3\times 2}$ & $B = \begin{bmatrix} 6 & 1 \\ 3 & 7 \\ 4 & 8 \end{bmatrix}_{3\times 2}$

$$A - B = \begin{bmatrix} 3-6 & 2-1 \\ 5-3 & 9-7 \\ 1-4 & 6-8 \end{bmatrix} = \begin{bmatrix} -3 & 1 \\ 2 & 2 \\ -3 & -2 \end{bmatrix}$$

Q. $\begin{pmatrix} x^2+x & x \\ 3 & 2 \end{pmatrix} + \begin{pmatrix} 0 & -1 \\ -x+1 & x \end{pmatrix} = \begin{pmatrix} 0 & -2 \\ 5 & 1 \end{pmatrix}$ find x?

$$\begin{pmatrix} x^2+x & x-1 \\ -x+4 & x+2 \end{pmatrix} = \begin{pmatrix} 0 & -2 \\ 5 & 1 \end{pmatrix}$$

$\Rightarrow x^2+x=0$	$x-1=-2$	$-x+4=5$	$x+2=1$
$x(x+1)=0$	$x=-1$	$-x=1$	$x=-1$
$x=0,-1$		$x=-1$	

$$\boxed{x=-1}$$

Nishant Jindal

(1) $A = \begin{bmatrix} 8 & 0 \\ 4 & -2 \\ 3 & 6 \end{bmatrix}$ & $B = \begin{bmatrix} 2 & -2 \\ 4 & 2 \\ -5 & 1 \end{bmatrix}$

find Matrix X such that $2A + 3X = 5B$

$$2A + 3X = 5B$$

$$\begin{bmatrix} 16 & 0 \\ 8 & -4 \\ 6 & 12 \end{bmatrix} + 3X = \begin{bmatrix} 10 & -10 \\ 20 & 10 \\ -25 & 5 \end{bmatrix}$$

$$3X = \begin{bmatrix} -6 & -10 \\ 12 & 14 \\ -31 & -7 \end{bmatrix} \Rightarrow X = \begin{bmatrix} -6/3 & -10/3 \\ 12/3 & 14/3 \\ -31/3 & -7/3 \end{bmatrix}$$

(B) Scalar Multiplication:

When a Scalar is Multiplied to a Matrix then it is Multiplied to every element of it.

$$A = \begin{bmatrix} 3 & 0 \\ 5 & 9 \\ 1 & 2 \end{bmatrix}$$

$$\frac{A}{3} = \begin{bmatrix} 3/3 & 0/3 \\ 5/3 & 9/3 \\ 1/3 & 2/3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 5/3 & 3 \\ 1/3 & 2/3 \end{bmatrix}$$

Nishant Jindal

Q If $A+B = \begin{bmatrix} 2 & -1 \\ 1 & 3 \\ -3 & -2 \end{bmatrix}$ & $A-2B = \begin{bmatrix} -2 & 1 \\ 3 & -1 \\ 4 & -2 \end{bmatrix}$

find A & B ?

let this is X

let this is Y

$$A + B = X$$
$$A - 2B = Y$$
$$3B = X - Y$$
$$B = \frac{1}{3}(X - Y)$$

$$2A + 2B = 2X$$
$$A - 2B = Y$$

Add

$$3A = 2X + Y$$
$$A = \frac{1}{3}(2X + Y)$$

$$A = \frac{1}{3}\left(\begin{bmatrix} 4 & -2 \\ 2 & 6 \\ -6 & -4 \end{bmatrix} + \begin{bmatrix} -2 & 1 \\ 3 & -1 \\ 4 & -2 \end{bmatrix}\right)$$

$$A = \frac{1}{3}\begin{bmatrix} 2 & -1 \\ 5 & 5 \\ -2 & -6 \end{bmatrix} = \begin{bmatrix} 2/3 & -1/3 \\ 5/3 & 5/3 \\ -2/3 & -2 \end{bmatrix}$$

$$B = \frac{1}{3}(X - Y) = \frac{1}{3}\left(\begin{bmatrix} 2 & -1 \\ 1 & 3 \\ -3 & -2 \end{bmatrix} - \begin{bmatrix} -2 & 1 \\ 3 & -1 \\ 4 & -2 \end{bmatrix}\right)$$

$$= \frac{1}{3}\begin{pmatrix} 4 & -2 \\ -2 & 4 \\ -7 & 0 \end{pmatrix} = \begin{pmatrix} 4/3 & -2/3 \\ -2/3 & 4/3 \\ -7/3 & 0 \end{pmatrix}$$

Nishant Jindal

Q find x & y

if $2x+y = \begin{pmatrix} 6 & -6 & 0 \\ -4 & 2 & 1 \end{pmatrix}$

& $x+2y = \begin{pmatrix} 3 & 2 & 5 \\ -2 & 1 & -7 \end{pmatrix}$

Q $A = \begin{bmatrix} 4 & 6 & -1 \\ 1 & -2 & 3 \end{bmatrix}$ & $B = \begin{bmatrix} 0 & -2 & 3 \\ 1 & -1 & 4 \end{bmatrix}$

find (1) $3A+2B$ (2) $2A-3B$.

Q $A = \begin{bmatrix} 1 & -3 & 2 \\ 2 & 0 & 2 \end{bmatrix}$ & $B = \begin{bmatrix} 2 & -1 & -1 \\ 1 & 0 & 1 \end{bmatrix}$

find matrix C such that $A+B+C$ is a Null Matrix.

Transpose of a Matrix.

(1) Rep. by A^T & A'

(2) $A = [a_{ij}]_{m\times n}$ then $A^T = [a_{ji}]_{n\times m}$

(3) Prop. of Transpose.

(A) $(A^T)^T = A$ (B) $(KA)^T = K(A^T)$

(C) $(A+B)^T = A^T+B^T \Rightarrow (A+B-C)^T = (A^T+B^T-C^T)$

(D) $(A\cdot B)^T = B^T \cdot A^T$

$(A\cdot B\cdot C)^T = C^T\cdot B^T\cdot A^T$

Nishant Jindal

Q $A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$ then $A + A^T = I$

find $\alpha = ?$

$$A + A^T = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} + \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$$

$$= \begin{bmatrix} 2\cos\alpha & 0 \\ 0 & 2\cos\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

$$2\cos\alpha = 1 \Rightarrow \cos\alpha = \frac{1}{2} = \cos\frac{\pi}{3}.$$

$$\boxed{\alpha = 2n\pi \pm \frac{\pi}{3}}$$

Nishant Jindal

Remaining Part of diff^y

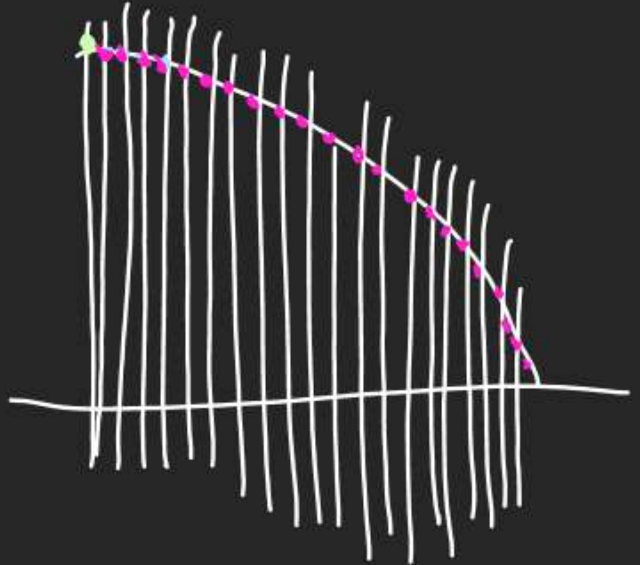
Max / Min f(t) type

Q $f(x) = \cos x$

$$g(x) = \begin{cases} \text{Min } f(t) & 0 \le t \le x \ ; \ 0 \le x \le \frac{\pi}{2} \\ 3 - x & \frac{\pi}{2} < x < \pi \end{cases}$$

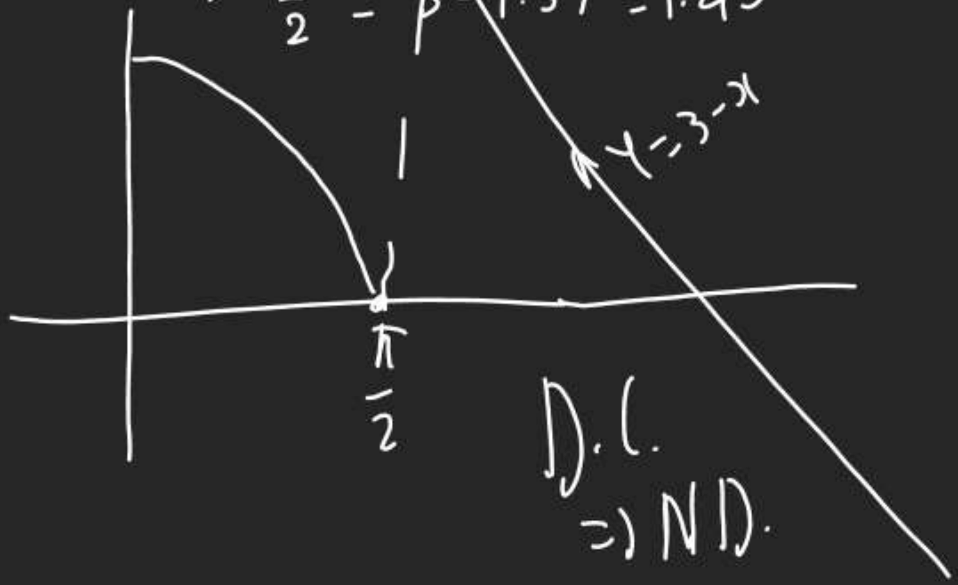
Check diff^y at $x = \frac{\pi}{2}$

$f(t) = \cos t$



$$g(x) = \begin{cases} \cos x & 0 \le x \le \frac{\pi}{2} \\ 3 - x & \frac{\pi}{2} < x < \pi \end{cases}$$

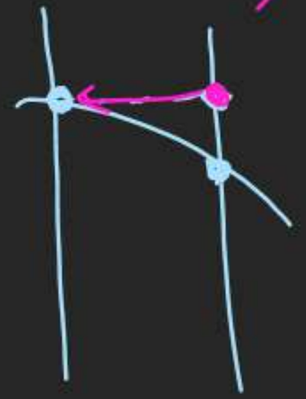
$3 - \frac{\pi}{2} = 3 - 1.57 = 1.43$

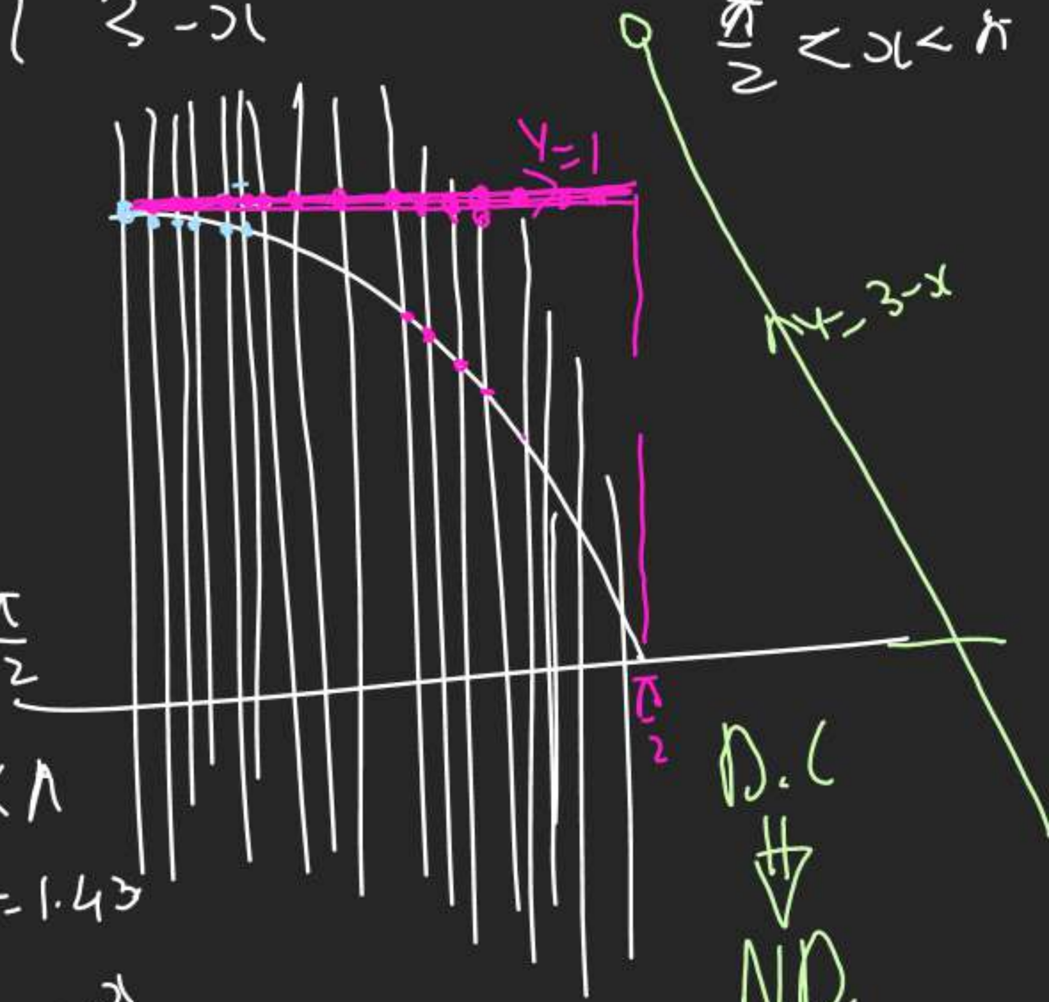


D.C.
$\Rightarrow$ N.D.

Q $f(x) = \cos x$

$$g(x) = \begin{cases} \text{Max } f(t) & 0 \le t \le x & 0 \le x \le \frac{\pi}{2} \\ 3 - x & & \frac{\pi}{2} < x < \pi \end{cases}$$





D.C
$\Downarrow$
N.D.

Nishant Jindal

Q $f(x) = \underline{x^3 - x^2 + x + 1} \to f(0) = 1$

$f(1) = 1 - 1 + 1 + 1 = 2$

$$g(x) = \begin{cases} \text{Max } f(t) : 0 \le t \le x & 0 \le x \le 1 \\ 3 - x + x^2 & 1 < x \le 2 \end{cases}$$

Check diff.

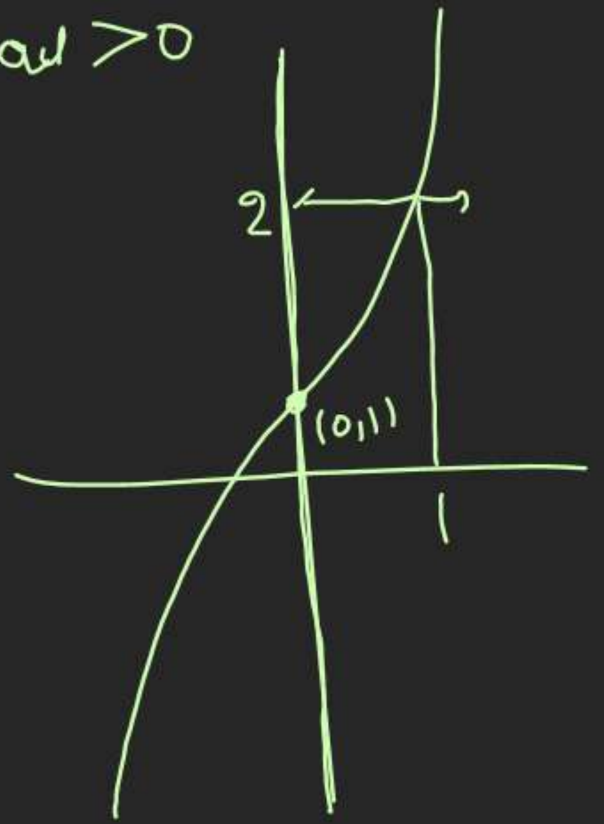
$\frac{dy}{dx} = 3x^2 - 2x + 1 \to$ Q.uad > 0

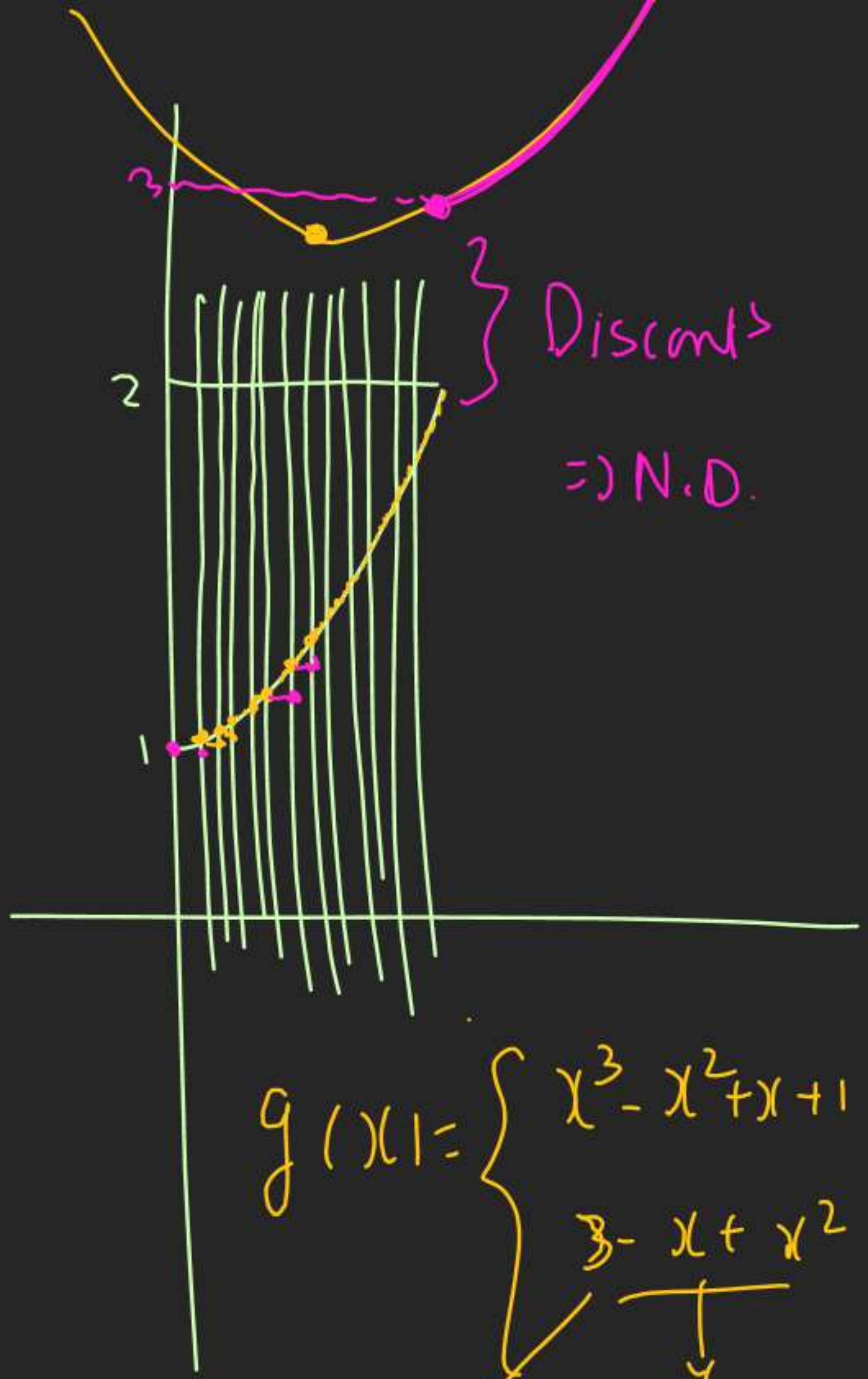
$D = (-2)^2 - 4 \times 3$

$= 4 - 12$

$= -8$ -ve

$\frac{dy}{dx} > 0 \Rightarrow \underline{f \times n \uparrow}$





} Discont's

$\Rightarrow$ N.D.

$$g(x) = \begin{cases} x^3 - x^2 + x + 1 & 0 \le x \le 1 \\ 3 - x + x^2 & 1 < x \le 2 \end{cases}$$

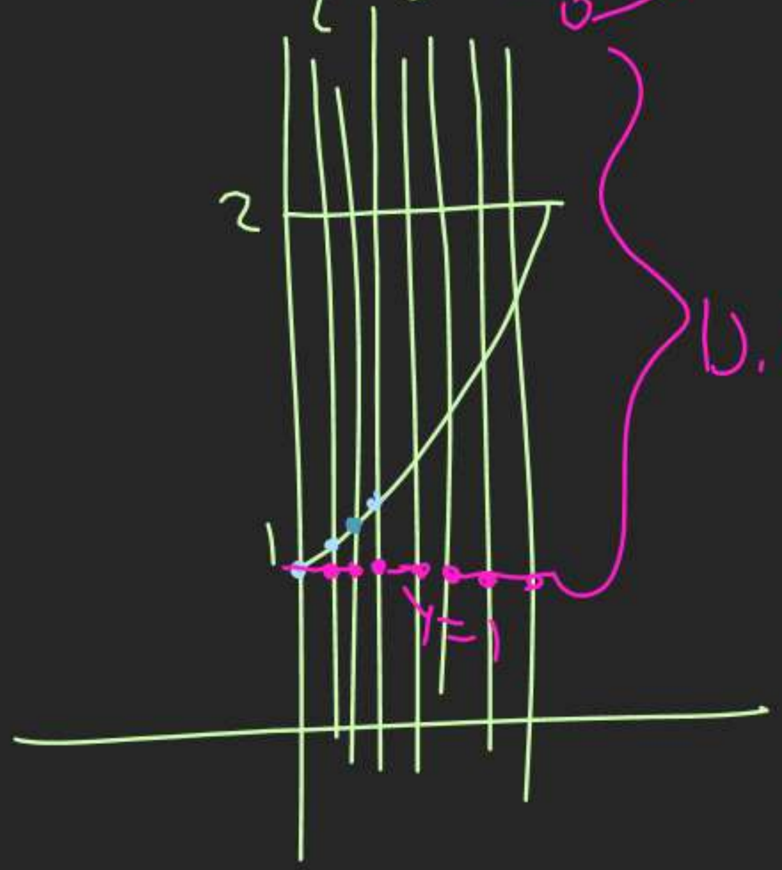
$3 - 1 -$

Vertex $= \left(\frac{1}{2}, \frac{11}{4}\right)$

Nishant Jindal

Q $f(x) = x^3 - x^2 + x + 1$

$$g(x) = \begin{cases} \text{Min } f(t) & 0 \le t \le x, \quad 0 \le x \le 1 \\ 3 - x + x^2 & 1 < x \le 2 \end{cases}$$



D. C⇒ND

Ex1

H.W.

Diff^n

Q $f(x) = x^2 - 2|x|$

$$g(x) = \begin{cases} \text{Min } f(t) & -2 \le t \le x, \quad -2 \le x \le 0 \\ \text{Max } f(t) & 0 \le t \le x, \quad 0 < x \le 3. \end{cases}$$

find $g(x)$?