

~~DATA~~

# COM of a Solid Cone

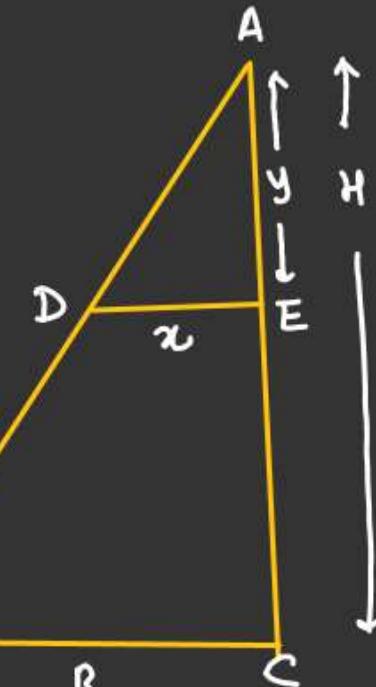
In  $\triangle ADE \& \triangle ABC$ 

$$\frac{x}{R} = \frac{y}{H}$$

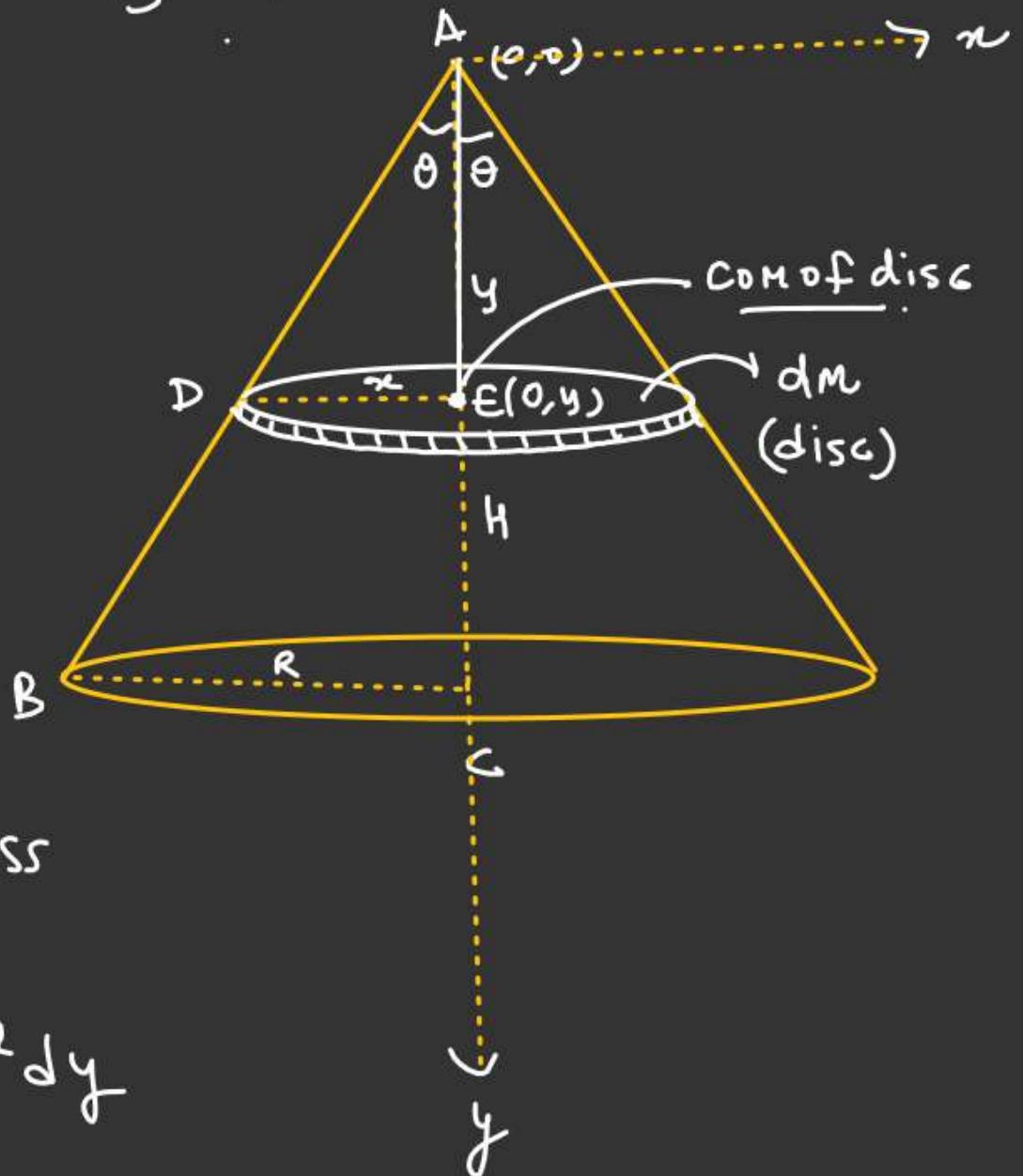
$$x = \left(\frac{R}{H}\right)y$$

$$dm = \left(\frac{3M}{\pi R^2 H}\right) \times \frac{dV}{\parallel}$$

differential  
Volume of disc



$$\rho = \left(\frac{M}{\frac{1}{3}\pi R^2 H}\right)$$



$dV = (\text{Area of differential element}) \times \text{thickness}$

$$= (\pi x^2) dy$$

$$= \left(\frac{\pi R^2}{H^2} y^2 dy\right)$$

$$dm = \frac{3M}{\pi R^2 H} \times \frac{\pi x^2}{H^2} y^2 dy$$

$$dm = \left(\frac{3M}{H^3} y^2 dy\right) \checkmark$$

$$dm = \frac{3M}{H^3} y^2 dy$$

$$y_{com} = \frac{\int dm y}{\int dm} = \frac{\frac{3M}{H^3} \int_0^H y^3 dy}{M}$$

$\circlearrowleft \int dm \rightarrow M$

$$y_{com} = \frac{3}{H^3} \times \frac{H^4}{4}$$

$$y_{com} = \frac{3H}{4} \Rightarrow \text{from Apex}$$

$$y_{com} = H - \frac{3H}{4} = \frac{H}{4}$$

From base

## COM of triangular lamina

Lamina = [Two dimensional having negligible thickness]

$$\tan \theta = \frac{y/2}{x} = \frac{b/2}{h}$$

$$x = \left(\frac{b}{h}\right)y$$

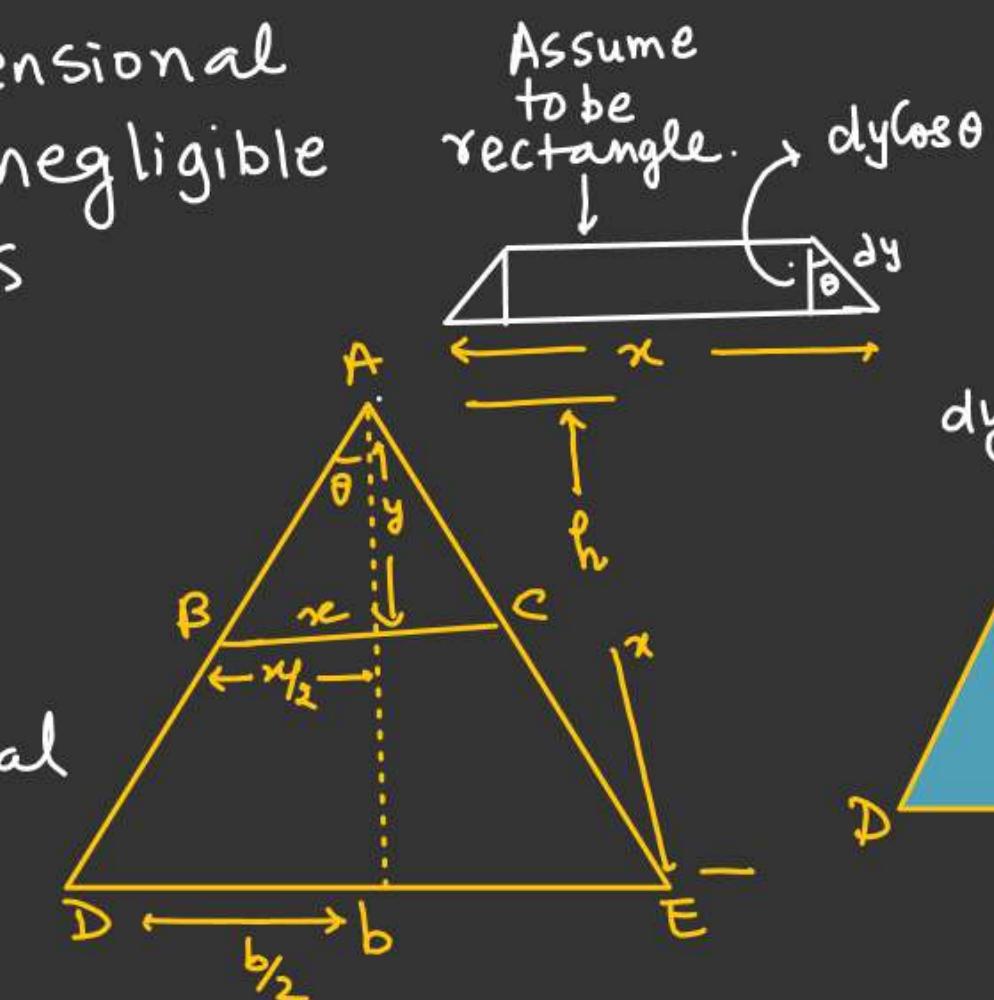
$$dm = \frac{M}{A} (dA)$$

↳ differential area of strip.

$$= \frac{M}{\frac{1}{2}bh} \times (x dy)$$

$$= \left(\frac{2M}{bh}\right) \times \frac{b}{h} \times y dy$$

$$= \left(\frac{2M}{h^2}\right) y dy$$

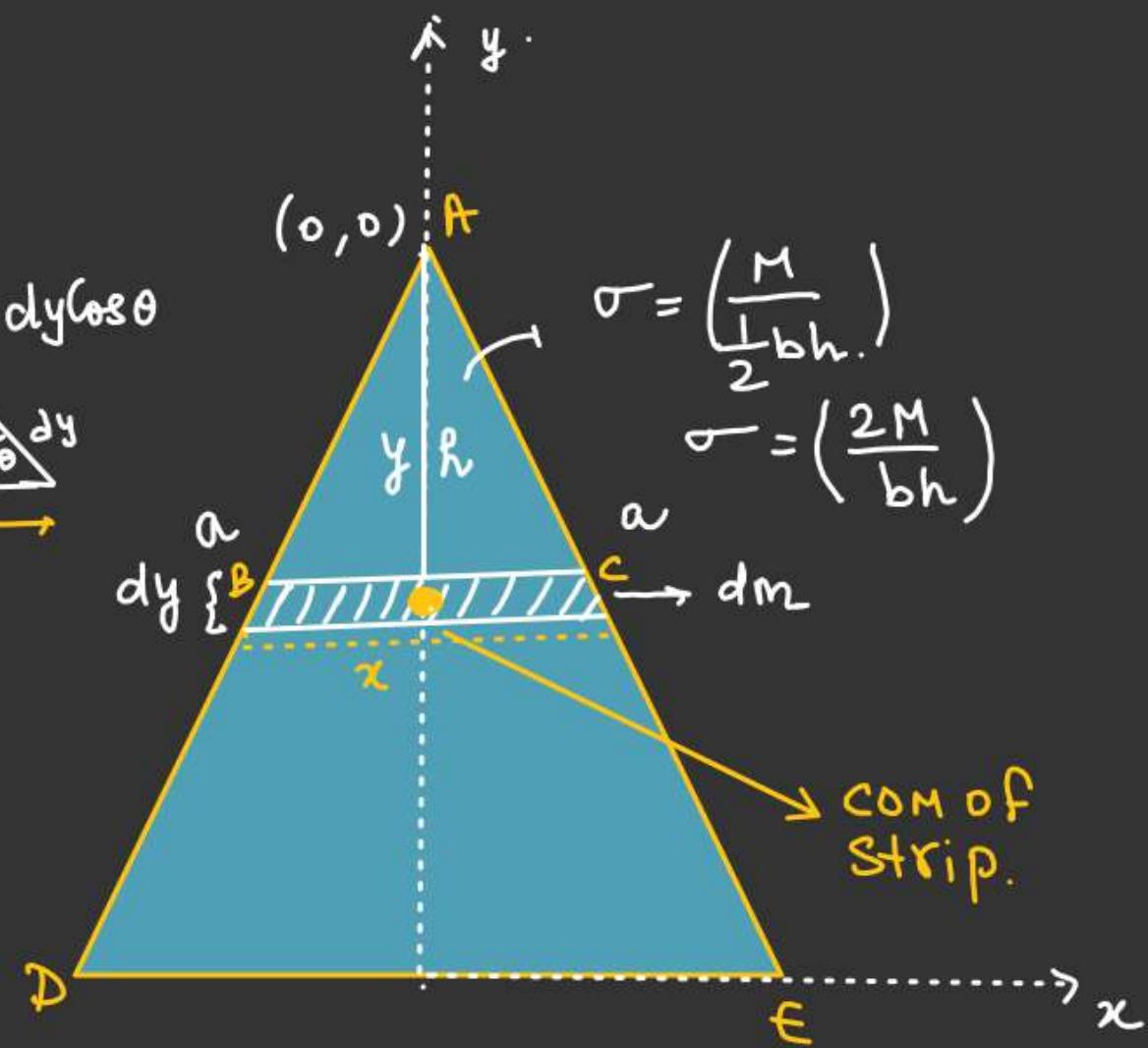


$$y_{com} = \frac{\int dm \cdot y}{M}$$

(From A)

$$= \frac{2M}{h^2 \times M} \int y^2 dy = \left(\frac{2h}{3}\right)$$

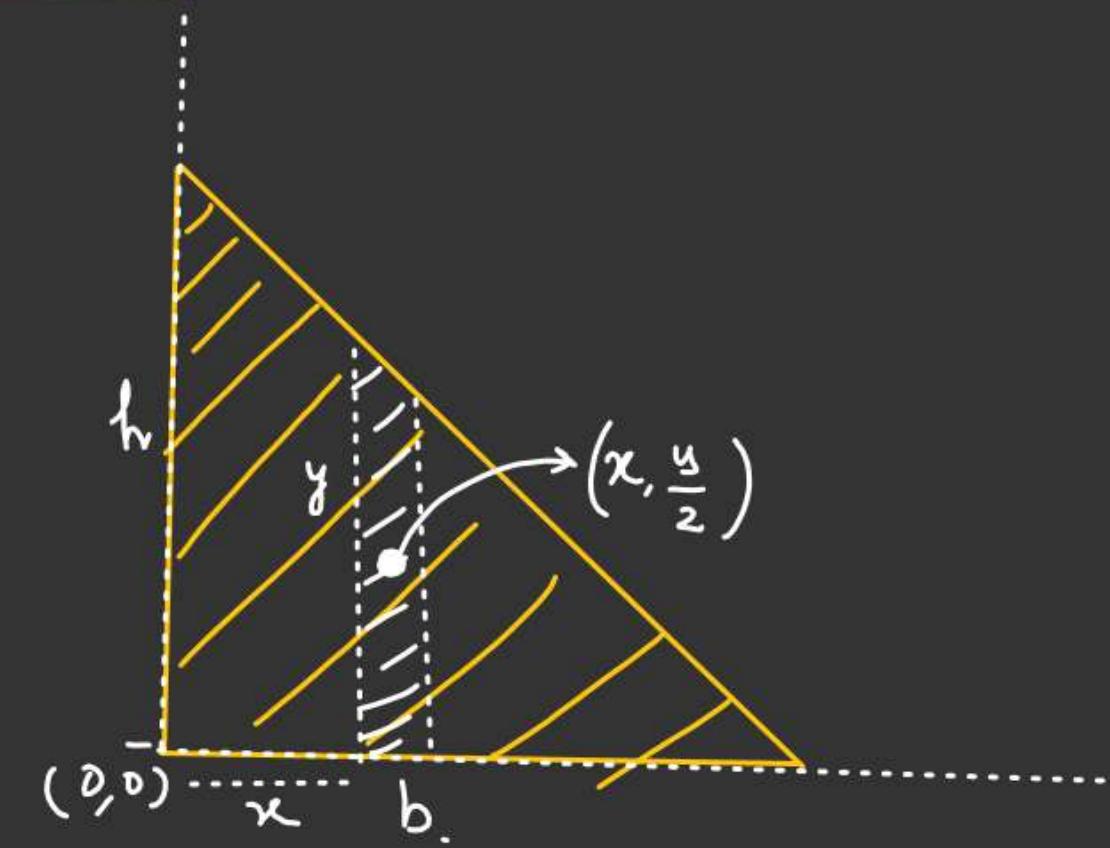
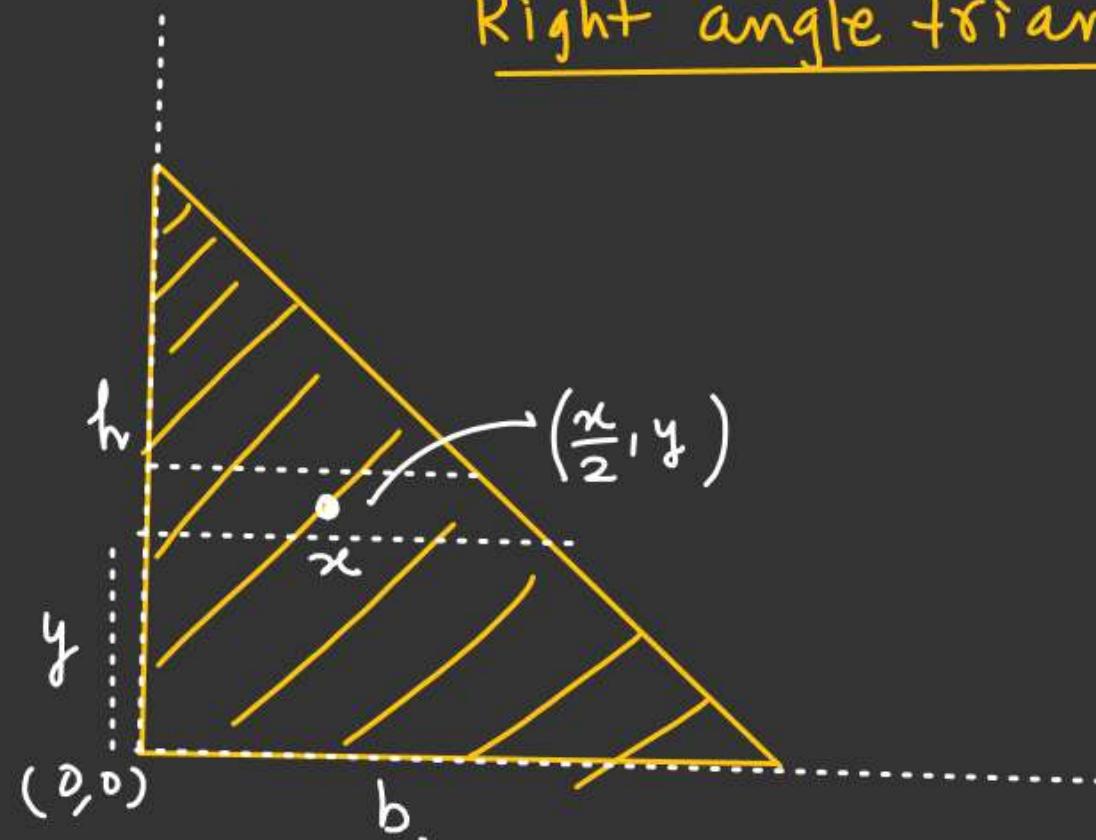
From base =  $h/3$  → (Same as hollow cone)



$$\sigma = \left(\frac{M}{\frac{1}{2}bh}\right)$$

$$\sigma = \left(\frac{2M}{bh}\right)$$

## Right angle triangular lamina



$$\text{CoM} = \left[ \frac{b}{3}, \frac{h}{3} \right] \checkmark$$

# Find COM of the figure.

Both Cone and Hemisphere have same density.

$\rho$  = density

$M_1$  = Mass of Solid Cone.

$$M_1 = \int \frac{1}{3} \pi R^2 (2R) = \frac{2}{3} (\rho \pi R^3) \checkmark$$

$M_2$  = Mass of Solid hemisphere.

$$M_2 = (\rho \cdot \frac{2}{3} \pi R^3) \checkmark$$

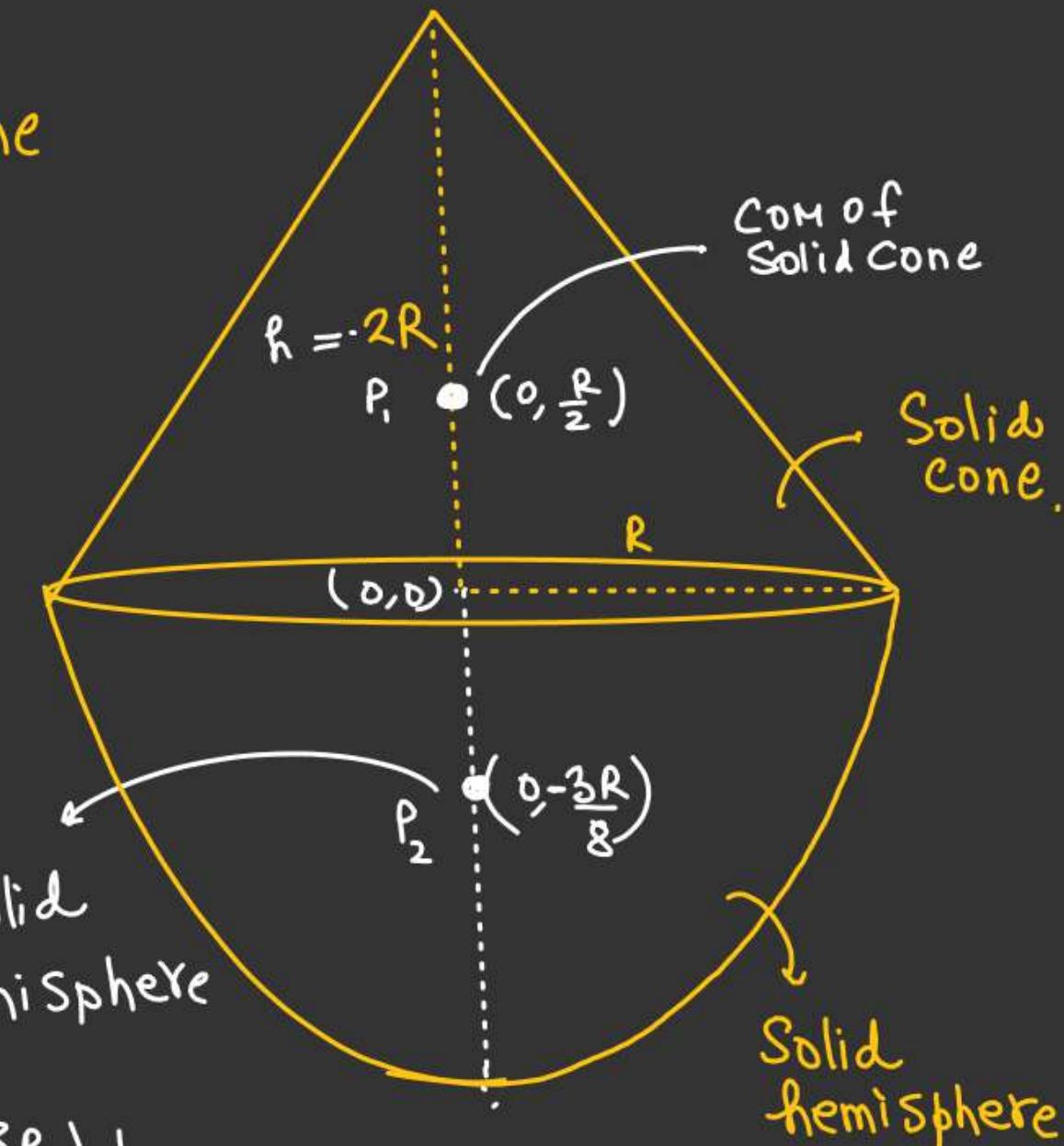
$$y_{\text{com}} = \frac{M_1 y_1 + M_2 y_2}{M_1 + M_2}$$

$$= \left( \frac{2}{3} \rho \pi R^3 \right) \frac{R}{2} + \left( \frac{2}{3} \rho \pi R^3 \right) \left( -\frac{3R}{8} \right)$$

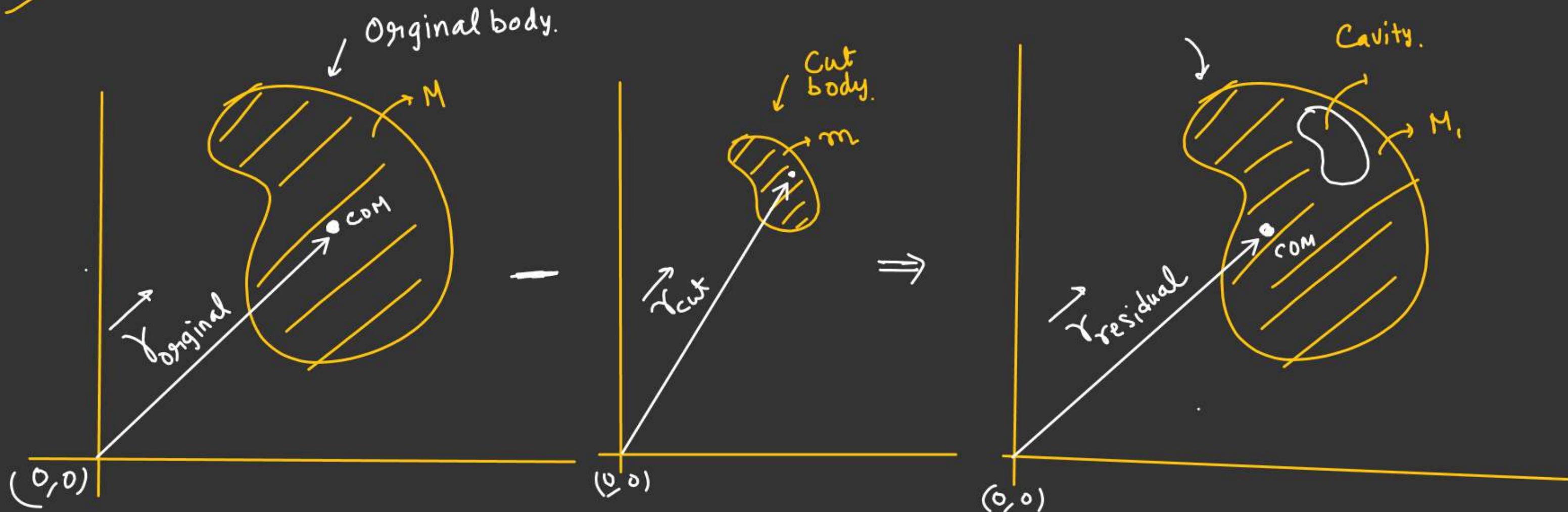
$$= \frac{\left( \frac{2}{3} \rho \pi R^3 \times 2 \right)}{\left( \frac{2}{3} \rho \pi R^3 \times 2 \right)} = \left( \frac{R}{2} - \frac{3R}{8} \right) \frac{1}{2}$$

$$= \frac{4R - 3R}{8} \times \frac{1}{2} = \left( \frac{R}{16} \right)$$

$$= R + \frac{R}{16} = \frac{17R}{16} \checkmark$$



From base of hemisphere

~~ΔΔ~~COM of Residual bodies.Residual body → (Remaining body.)

$$\vec{\gamma}_{\text{residual}} = \left( \frac{M \vec{\gamma}_{\text{original}} - m \vec{\gamma}_{\text{cut}}}{(M-m)} \right) = \left( \frac{M \vec{\gamma}_{\text{original}} + (-m) \vec{\gamma}_{\text{cut}}}{M + (-m)} \right)$$

If body is Lamina i.e 2-dimensional

$$M = \sigma A_1$$

$$m = \sigma A_2$$

$$\vec{r}_{\text{residual}} = \frac{M \vec{r}_{\text{original body}} - m \vec{r}_{\text{cut body}}}{M - m}$$

$$\vec{r}_{\text{residual}} = \left( \frac{A_1 \vec{r}_1 - A_2 \vec{r}_2}{A_1 - A_2} \right) \quad \text{=} \quad \boxed{\text{}}$$

$A_1$  = Area of original body

$A_2$  = Area of cut body

$\vec{r}_1$  = Position vector of COM of  
Original body

$\vec{r}_2$  = Position vector of COM of cut body.

If body is

3-dimensional

$$M = \rho V_1$$

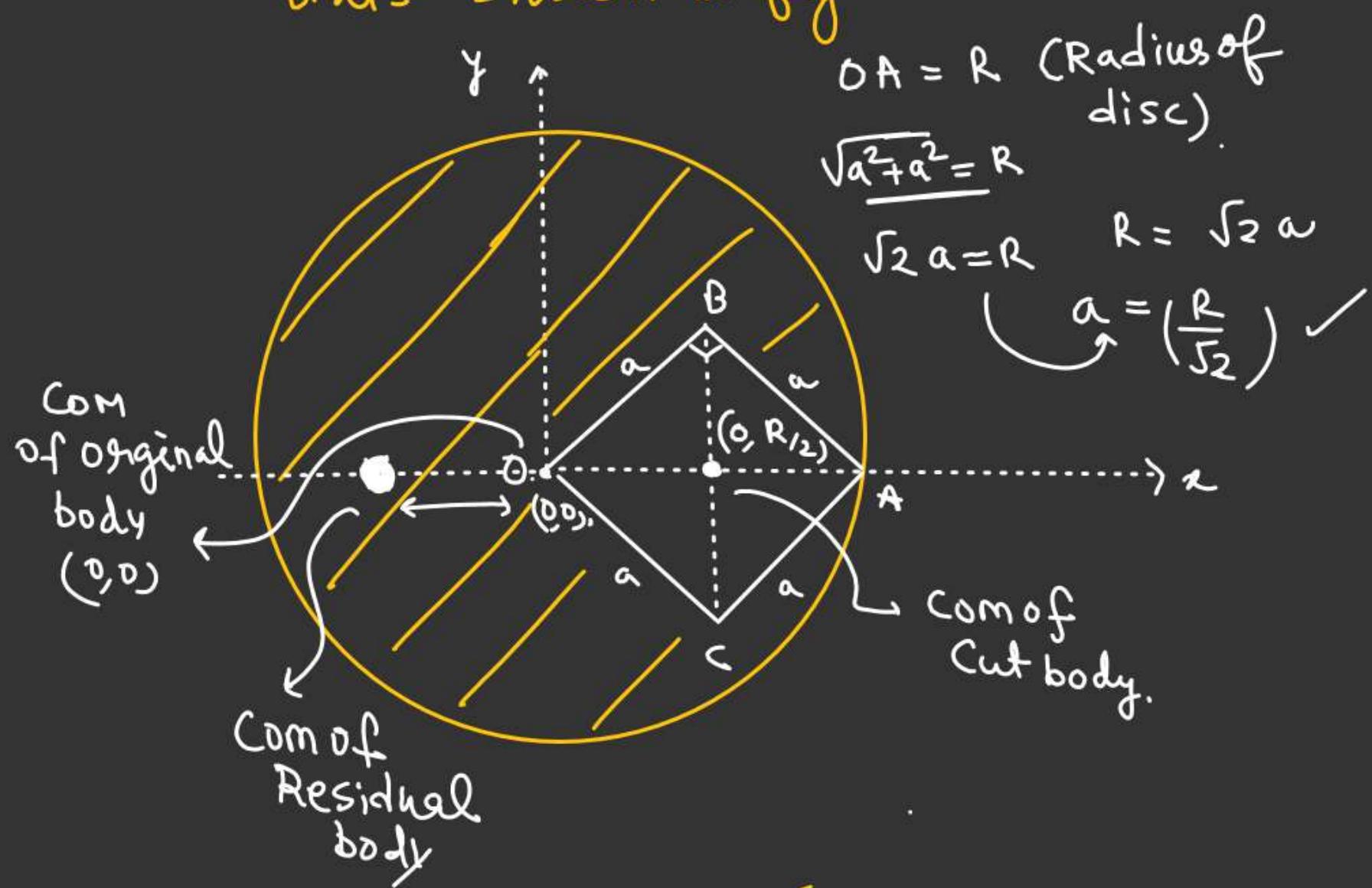
$$m = \rho V_2$$

~~AA~~

$$\vec{r}_{\text{residual}} = \left( \frac{V_1 \vec{r}_1 - V_2 \vec{r}_2}{V_1 - V_2} \right)$$

# From a Circular lamina a Square is cut so that diagonal of Square be the radius of Circular lamina.

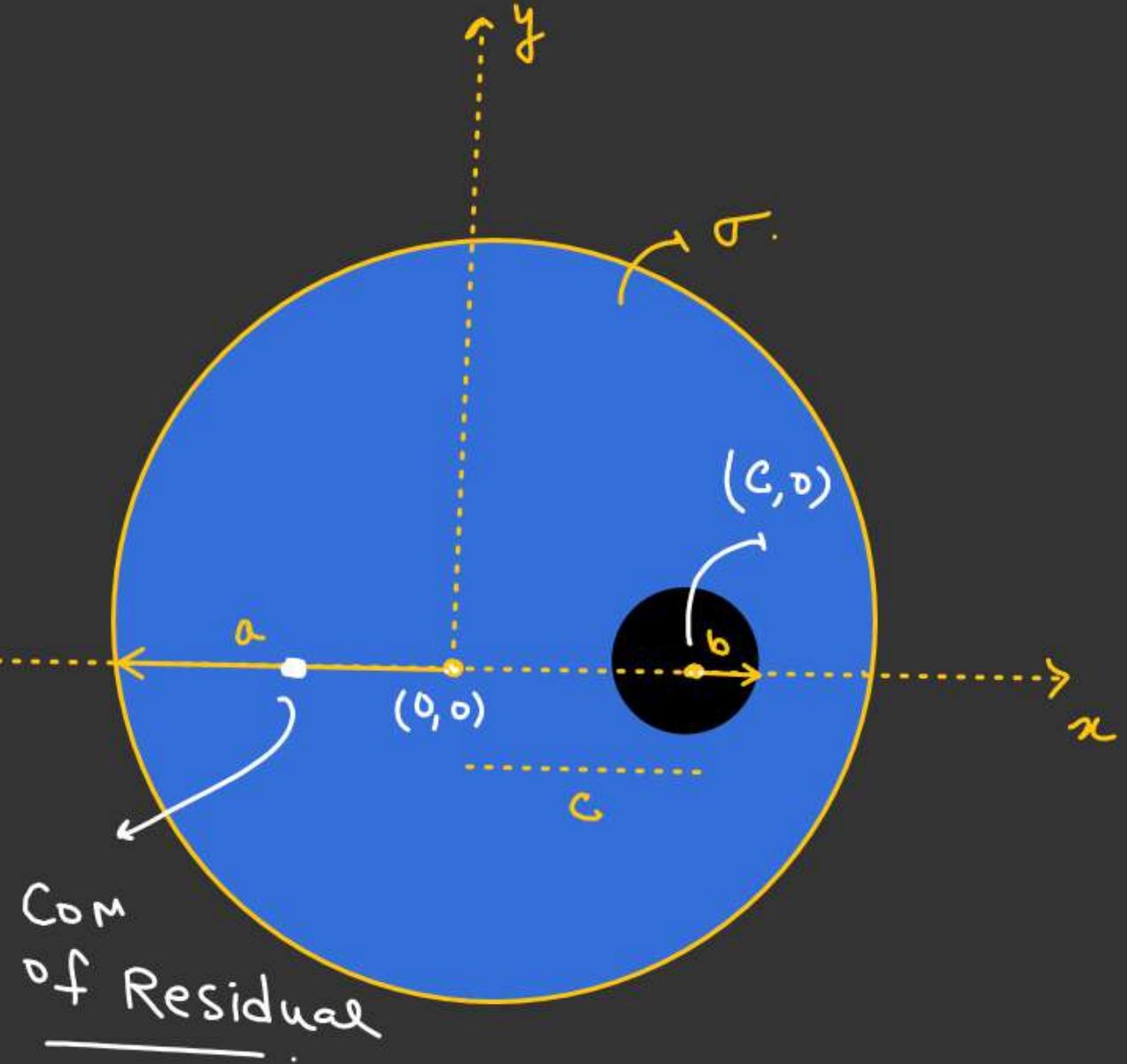
Find COM of residual body from the axis shown in fig.



$$\begin{aligned}\vec{r}_{\text{residual}} &= \frac{\vec{A}_1 \vec{r}_1 - \vec{A}_2 \vec{r}_2}{A_1 - A_2} \\ &= \frac{(\pi R^2)(0\hat{i}) - (a^2 \cdot \frac{R}{2}\hat{i})}{(\pi R^2 - a^2)} \\ &= \frac{-\left(\frac{R}{\sqrt{2}}\right)^2 \times \frac{R}{2}\hat{i}}{\pi R^2 - \left(\frac{R}{\sqrt{2}}\right)^2} = \frac{\frac{R}{4}(-\hat{i})}{\left(\pi - \frac{1}{2}\right)} \\ &= \frac{R}{2(2\pi - 1)}(-\hat{i}).\end{aligned}$$

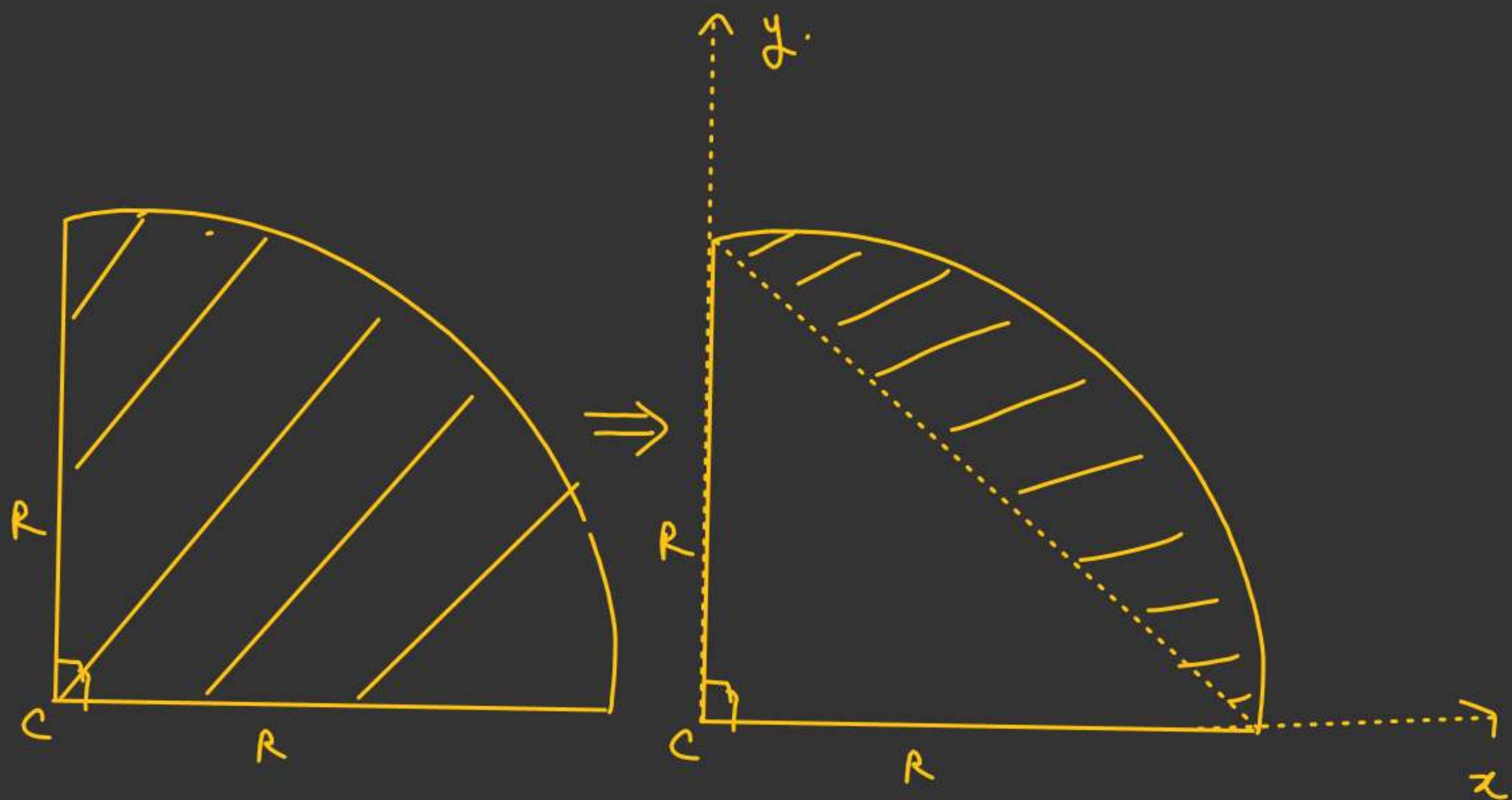
- # A small disc of radius  $b$  is cut from a larger disc of radius  $a$ .  
find COM of residual body.

$$\begin{aligned}\vec{\gamma}_{\text{residual}} &= \frac{A_1 \vec{\gamma}_1 - A_2 \vec{\gamma}_2}{A_1 - A_2} \\ &= \frac{-A_2 \vec{\gamma}_2}{A_1 - A_2} \\ &= \left( \frac{\pi b^2}{\pi a^2 - \pi b^2} \right) (-c \hat{i}) \\ &= \left( \frac{b^2 c}{a^2 - b^2} \right) (-\hat{i})\end{aligned}$$



#H.W.: A triangle is cut from a Sector  
of radius R.

Find COM of  
residual body.

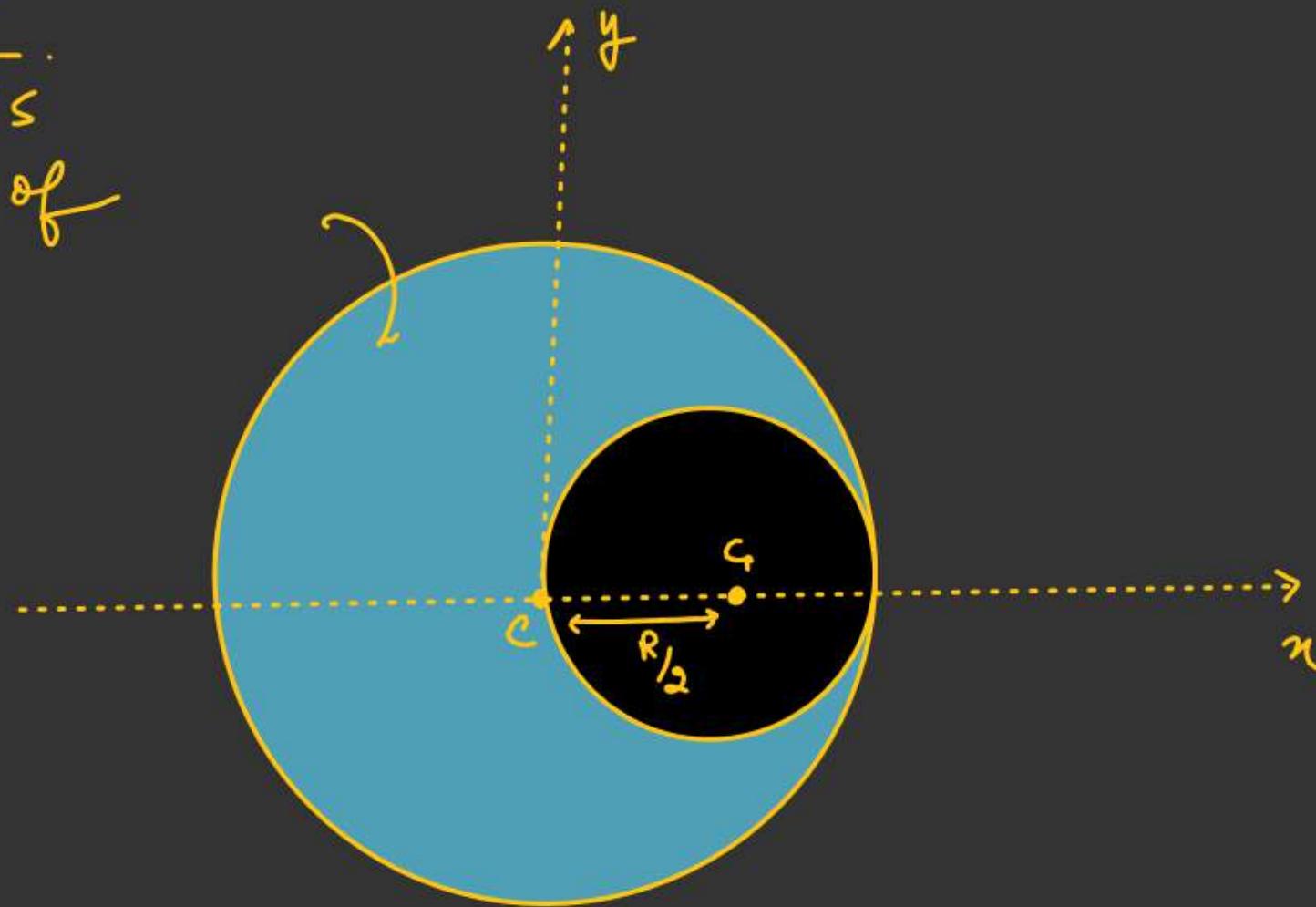


~~H.W~~

$$\# M = \frac{\text{Mass of Residual body}}{}$$

A Solid Sphere of radius  $R/2$  is  
Cut from a Solid Sphere of  
radius  $R$ .

Find COM of Residual body.





Nishant Jindal

$d^m =$