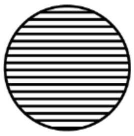
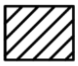
DPP – 01

SOLUTION

1.  – 

$x_{com} = 0$ $\qquad$ $x_{com} = \frac{a}{2}$

$m_1 = \sigma\pi a^2$ $\qquad$ $m_2 = \frac{\sigma a^2}{4}$

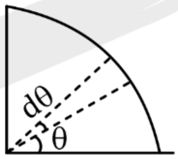
$$\left[\vec{x}_{com} = \frac{m_1\vec{x}_1 + m_2\vec{x_2}}{m_1 + m_2}\right] \text{ general}$$

$$\Rightarrow \; x_{com} = \frac{0-\left(\frac{\sigma a^2}{4}\right)\times\left(\frac{a}{2}\right)}{\sigma(\pi a^2)-\sigma\frac{a^2}{4}}$$

$$= -\left(\frac{a}{8}\right) \times \frac{1}{2.89}$$

$$x_{com} = -\frac{a}{23.12} = -\frac{a}{23.12}$$

2. mass of small segment, $dm = \sigma \times \frac{1}{2} a \times ad\theta$



$$= \sigma a^2 \frac{d\theta}{2}$$

Therefore x-co-ordinate of com

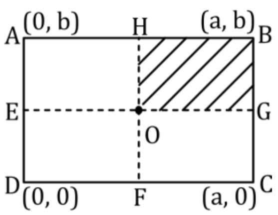
$$x_{cm} = \frac{\int xdm}{\int dm}$$

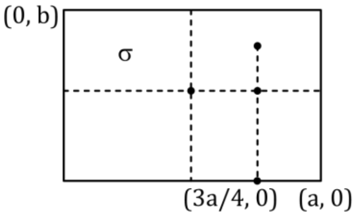
$$= \int_0^{\pi/2} \sigma \frac{a^2}{2} d\theta \cdot \frac{2a}{3} \cos\,\theta / \int_0^{\pi/2} \frac{\sigma a^2 d\theta}{2}$$

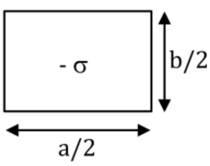
For triangular part com is at $\frac{2a}{3}$ because com lies at median

$$x_{com} = \frac{2a}{3}\frac{\int_0^{\pi/2} \cos\,\theta d\theta}{\int_0^{\pi/2} d\theta} = \frac{4a}{3\pi}$$

$x = 4$

3. 





$$x_{com} = \frac{\sigma\frac{ba}{4}\cdot\frac{a}{4}}{ab\sigma-\frac{ab}{4}\sigma} = \frac{-\sigma\frac{a}{4}\cdot\frac{ba}{q}}{\sigma\left(\frac{3ab}{4}\right)}$$ (distance between com of both figure on x-axis.)

$$x_{com} = -\frac{a}{12} = -\frac{a}{12}$$

$$x_{com} = \frac{a}{2} - \frac{a}{12} = \frac{5a}{12}$$ $\beta = 5$

4. $r_{com} = \frac{m_1\bar{r}_1+m_2\bar{r}_2}{m_1+m_2}$

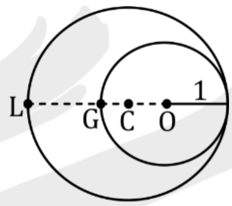
$$= \frac{1(i + 2j + k) + 3(-3i - 2j + k)}{1 + 3}$$

$$= \frac{i + 2j + k - 9i - 6j + 3k}{4}$$

$$= \frac{-8i - 4j + 4k}{4}$$

$r_{com} = -2i - j + k \Rightarrow |\, r\ com\ | = \sqrt{6}$

5. $LG = 2R - 2$



$GC = LC - LG$

$= R - 2R + 2$

$GC = 2 - R$

$CO = (R - 1)$

By COM

$m_1r_1 \quad = \quad m_2r_2$

mass of remaining part $\quad$ mass of cavity

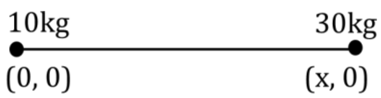
$$\left(\frac{4}{3}\pi R^3 - \frac{4}{3}\pi(1)^2\right)\rho(2 - R) = \frac{4}{3}\pi(1)^3\rho\cdot(R - 1)$$

$(R^3 - 1)(2 - R) = (R - 1)$

$(R - 1)(R^2 + R + 1)(2 - R) = (R - 1)$

$(R^2 + R + 1)(2 - R) = 1$

6. $dx_{com} = \frac{m_1dx_1+m_2dx_2}{m_1+m_2}$

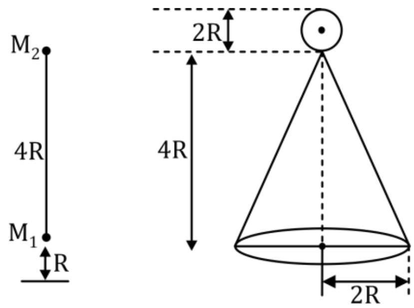


$$0 = \frac{10 \times 6 + 30dx_2}{10 + 30}$$

$dx_2 = -2$ cm

i.e. 2 cm towards 10 kg block.

7. say $\rho \rightarrow$ density of materid (cone)



$$M_1 = \frac{\rho\pi(2R)^2 \times 4R}{3} = \frac{16\pi R^3 \rho}{3}$$

$$M_2 = \frac{4}{3}\pi R^3 \cdot 12\sigma = 16\pi R^3 \rho$$

$$Y_{com}^0 = \frac{\frac{16\pi R^3\rho}{3} \times R + 16\pi R^3 \rho \times 5R}{\frac{16\pi R^3 \rho}{3} + 16\pi R^3 \rho}$$

$$= \frac{\frac{R}{3} + 5R}{\frac{1}{3} + 1} = \frac{\frac{16R}{3}}{\frac{4}{3}}$$

$$Y_{con} = 4R$$

8. Area of $\quad A_1 = \pi(4R)^2$

$A_2 = -\pi(R)^2$

$A_3 = -\pi(R)^2$

$$\text{position vector of com} \begin{cases} \vec{r}_1 = oi + oj \text{ (for 4R radius)} \\ \vec{r}_2 = 3Ri + oj \text{ (for R redius) } (x - \text{axis}) \\ \vec{r}_3 = oi + 3Rj \text{ (for R radius } (y - \text{axis}) \end{cases}$$

$$\vec{r}_{cm} = \frac{A_1\vec{r}_1 + A_2\vec{r}_2 + A_3\vec{r}_3}{A_1 + A_2 + A_3}$$

$$= \frac{\pi(4R)^2 \times 0 - \pi(R)^2 \times 3Ri - \pi(R)^2 \times 3Rj}{\pi(4R)^2 - \pi R^2 - \pi R^2}$$

$$= \frac{-3\pi R^3 i - 3\pi R^3 j}{14R^2\pi}$$

$$r_{cm} = \frac{-3R}{14}(i + j)$$

9. for disc $r_1 = \frac{4R}{3\pi} = \frac{4(12)}{3\pi} = \frac{48}{3\pi}$

for are $r_2 = \frac{2R'}{\pi}$

for com at origin $0 = \frac{\frac{48}{3\pi}\times m - \frac{2R'}{\pi}m}{m+m}$

$R' = 8$ cm

10. There is no externd force is present

so it will be on rest.