Nishant Jindal

128.

$$\log_3\left(3^x-8\right)=2-x$$

$$3^x-8=3^{2-x}=\frac{9}{3^x}$$

$$3^x=t$$

$$t^2-8t-9=0$$

$$(t-9)(t+1)=0$$

$$3^x=9\ ,\ \cancel{-1}$$

$$3^x=3^2 \Rightarrow \boxed{x=2}$$

130.

$$\log_{5-x}\left(x^2-2x+65\right)=2$$

$$x^2-2x+65=(5-x)^2$$

$$8x+40=0$$

$$\boxed{x=-5}$$

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131

$\log_9 x + \frac{1}{2} + \cancel{9^x} = \cancel{3^{2x}}$

$\log_9 x = -\frac{1}{2}$

$\Rightarrow x = 9^{-1/2} = \frac{1}{3}$

$\boxed{x = 0}$

$\log_{10} 8 = \log_{10}\left(\frac{1}{2} x\, 5^{\frac{1}{3}}\right)$

32

$8 = \frac{x}{3}\left(5^{\frac{1}{3}}\right)$

$\log_{10} 5 + \log_{10}(x+10) - \log_{10} 10$

34

$\log_{10} \frac{5(x+10)}{10} = \log_{10}\left(\frac{21x-20}{2x-1}\right)$

$\frac{x+10}{2} = \frac{21x-20}{2x-1}$

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136

$$2\log x - \log\left(x-\frac{1}{2}\right) = 2\log\left(x+\frac{1}{2}\right) - \log\left(x+\frac{1}{8}\right)$$

$$\frac{x^2}{x-\frac{1}{2}} = \frac{\left(x+\frac{1}{2}\right)^2}{x+\frac{1}{8}}$$

$$x^3 + \frac{1}{8}x^2 = \left(x^2+\frac{1}{4}+x\right)\left(x-\frac{1}{2}\right)$$

$x =$

37

$$3^{\log_3\left(\log_{10}\sqrt{x}\right)} = \log_{10}\sqrt{x} = \frac{1}{2}\log_{10}x$$

Nishant Jindal

138

$$\boxed{\log_b a^n = n\log_b a}$$

$$(x-2)^{\log_{10}^2(x-2)+5\log_{10}(x-2)-12} = 10^{2\log_{10}(x-2)}$$

$$\log_{10}\left((x-2)^{\log_{10}^2(x-2)+5\log_{10}(x-2)-12}\right) = \log_{10}\left(10^{2\log_{10}(x-2)}\right)$$

$$\log_{10}(x-2) = t$$

$$(t^2+5t-12)t = 2t$$

$$t(t^2+5t-14) = 0$$

$$t(t+7)(t-2) = 0$$

$$ab = ac$$

$$\Rightarrow b = c \text{ or } a = 0$$

$$\log_{10}(x-2) = 0, -7, 2$$

$$x-2 = 1, 10^{-7}, 100$$

Nishant Jindal

139.

$$\underline{9}^{\log_3(1-2x)} = \left(3^{\log_3(1-2x)}\right)^2 = (1-2x)^2.$$

$$9 = 3^2$$

1. If $\log_7 12 = a$ and $\log_{12} 24 = b$, find

$$\log_{54}(168) = \frac{\log_2(2^3\cdot 3\cdot 7)}{\log_2(3^3\cdot 2)} = \frac{3+\log_2 3+\log_2 7}{1+3\log_2 3} = \frac{3+\frac{3-2b}{b-1}+\frac{1}{a(b-1)}}{1+\frac{9-6b}{b-1}}$$

$$= \frac{a(3b-3)+3a-2ab+1}{a(8-5b)} = \boxed{\frac{ab+1}{a(8-5b)}}$$

$$a = \frac{\log_2(2^2\, 3)}{\log_2 7} = \frac{2+\log_2 3}{\log_2 7} \Rightarrow \log_2 7 = \frac{2+\frac{3-2b}{b-1}}{a} = \frac{1}{a(b-1)}$$

$$b = \frac{\log_2(2^3\, 3)}{\log_2(2^2\, 3)} = \frac{3+\log_2 3}{2+\log_2 3} \Rightarrow \log_2 3\,(b-1) = 3-2b$$

$$\boxed{\log_2 3 = \frac{3-2b}{b-1}}$$

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$$x>0 \Rightarrow a^x > 1 \quad \text{if } a>1 \checkmark$$

$$x<0 \Rightarrow a^x < 1 \quad , \quad a>1 \checkmark$$

$\longrightarrow > 0$

$\cancel{2^x = -8}$

$$\left(\frac{1}{2}\right)^1 = \frac{1}{2}, \left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

If $0<a<1$

$$\text{if } x>0 \Rightarrow a^x < 1$$

$$x<0 \Rightarrow a^x > 1$$

$$\left(\frac{2}{3}\right)^{-3} = \left(\frac{3}{2}\right)^3 = \frac{27}{8} > 1$$

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$$(1.7)^{\frac{3}{8}} = \left(\underbrace{1.7^3}_{>1}\right)^{\frac{1}{8}}$$

$$0.00\cdots01 = 10^{-}$$

$$N^{(100\cdots0)} = 2$$

$$\text{Let } N = 2^{\left(\frac{1}{1000\cdots0}\right)}$$

$$\cancel{<1}$$

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$a>1$ $\quad a^x$ increases as x increases.
& $\log_a x$ increases — " ————

$0<a<1$ $\quad a^x$ decreases as x increases.
& $\log_a x$ decreases — " ————

$a^x = y$

$\log_a y = x$

x increases, y increases

$\boxed{a>1}$

$x_1 > x_2$

$\Rightarrow a^{x_1} > a^{x_2}$ $\quad a>1$

$a^{x_1} < a^{x_2}$ $\quad 0<a<1$

$x_1 - x_2 > 0$

$a^{x_1 - x_2} > 1$ if $a>1$

$\Rightarrow a^{x_1} > a^{x_2}$

$a^{x_1 - x_2} < 1$, $0<a<1$

$a^{x_1} < a^{x_2}$

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. TJ $\quad x_1 > x_2 > 0$

$$\Rightarrow \begin{cases} \log_a x_1 > \log_a x_2 & \text{if } a>1 \\ \log_a x_1 < \log_a x_2 & \text{if } 0<a<1 \end{cases}$$

$$\log_{\frac{1}{2}} x < \log_{\frac{1}{2}} y$$

$$\Rightarrow \quad x > y > 0$$

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$$\log_3 x < \log_3 y$$

$$\Rightarrow \quad x < y$$

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$\underline{a>1}$, $b>1$ $\Rightarrow$ $\log_a b > \log_a 1 = 0$

$\Rightarrow$ $\log_a b > 0$

$\log_a b > 0$
if a, b lie on same side of unity

$\log_a b < 0$
if a, b lie on opposite side of unity

$a>1$, $0<b<1$ $\Rightarrow$ $\log_a b < 0$ ✓

$\log_a b < 0$
$-\log_a b > 0$

$\underline{0<a<1}$, $0<b<1$ $\Rightarrow$ $\log_a b > 0$

$b<1 \Rightarrow$ $\log_a b > \log_a 1 = 0$

$0<a<1$, $b>1 \Rightarrow$ $\log_a b < 0$ ✓

Nishant Jindal

$$\log_3\left(\frac{3}{5}\right) < 0$$

$$\log_{(2-\sqrt{3})}(0.5) > 0$$

$\cdot\ x > y$

$a^x > a^y$ if $a > 1$

$a^x < a^y$ if $0 < a < 1$

$\log_a x > \log_a y$ if $a > 1$

$\log_a x < \log_a y$ if $0 < a < 1$

$\boxed{141-160}$