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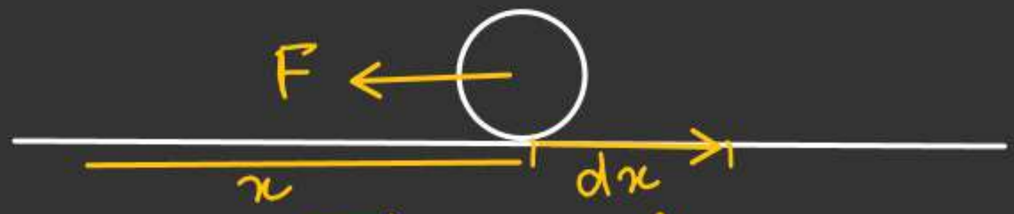
WORK POWER ENERGY

WORK

$$\boxed{W = \int \vec{F} \cdot d\vec{s}}$$



$$d\vec{s} = dx\,\hat{i}$$

Ex:- $\vec{F} = \left(-\frac{K}{x^2}\hat{i}\right)$ (K is a Constant)

Find work done by the force from $x = a$ to $x = 2a$

$$dW = \vec{F_x} \cdot d\vec{x}$$

$$dW = \frac{-K}{x^2}\hat{i} \cdot dx\,\hat{i}$$

$$\int_0^W dW = -K\int_a^{2a} \frac{dx}{x^2}$$

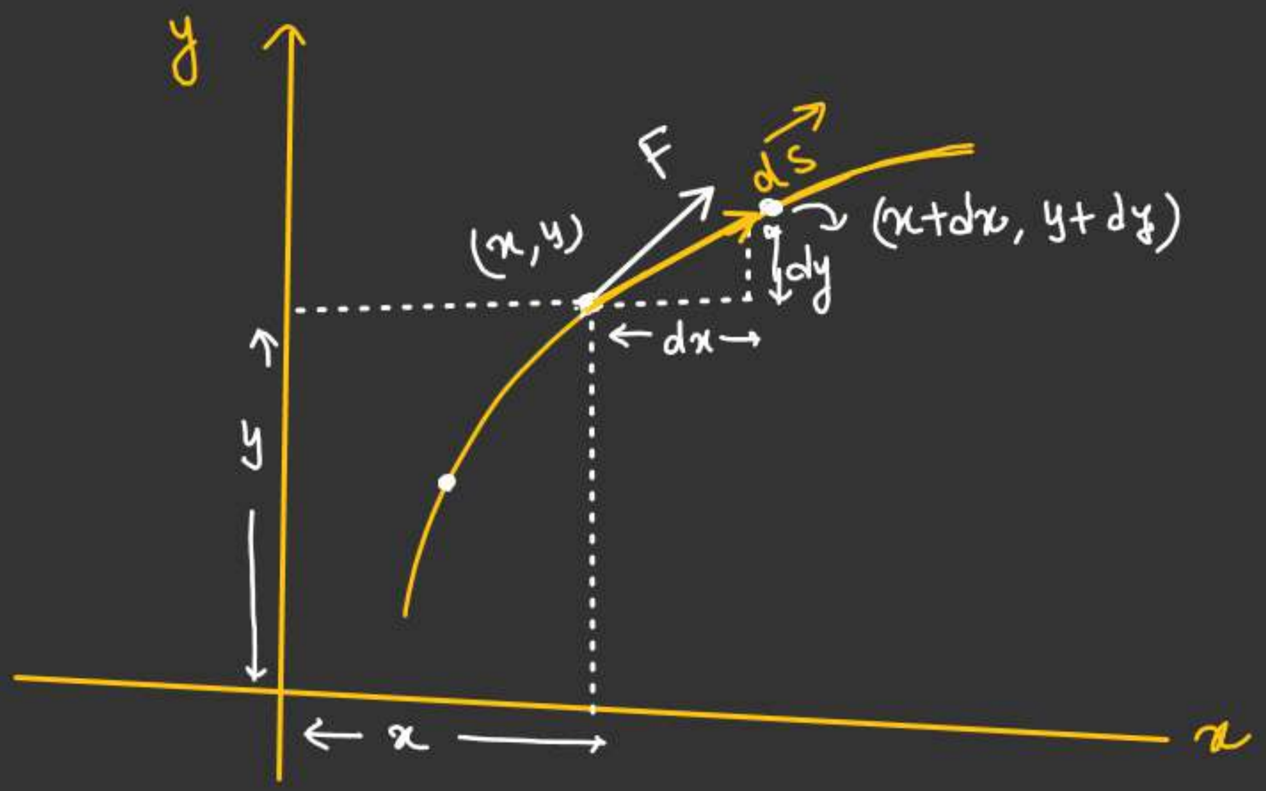
$$W = -K\left[-\frac{1}{x}\right]_a^{2a} = K\left[\frac{1}{2a} - \frac{1}{a}\right]$$

$$W = \left(\frac{-K}{2a}\right)$$

[-ve Sign tells that work done is -ve]

$$\vec{F} = x^2\hat{i} + y\hat{j}$$

Find work done by this force if particle is displaced from $(1, 2)$ to $(2, 3)$



Sol^n:-

$$d\vec{s} = dx\hat{i} + dy\hat{j}$$

$$dW = \vec{F} \cdot d\vec{s}$$

$$dW = (x^2\hat{i} + y\hat{j}) \cdot (dx\hat{i} + dy\hat{j})$$

$$\int_0^W dW = \int_1^2 x^2 dx + \int_2^3 y\,dy$$

$$W = \left[\frac{x^3}{3}\right]_1^2 + \left[\frac{y^2}{2}\right]_2^3$$

$$W = \frac{1}{3}(8-1) + \frac{1}{2}(9-4)$$

$$W = \frac{7}{3} + \frac{5}{2} = \frac{14+15}{6}$$

$$W = \frac{29}{6}\,J$$

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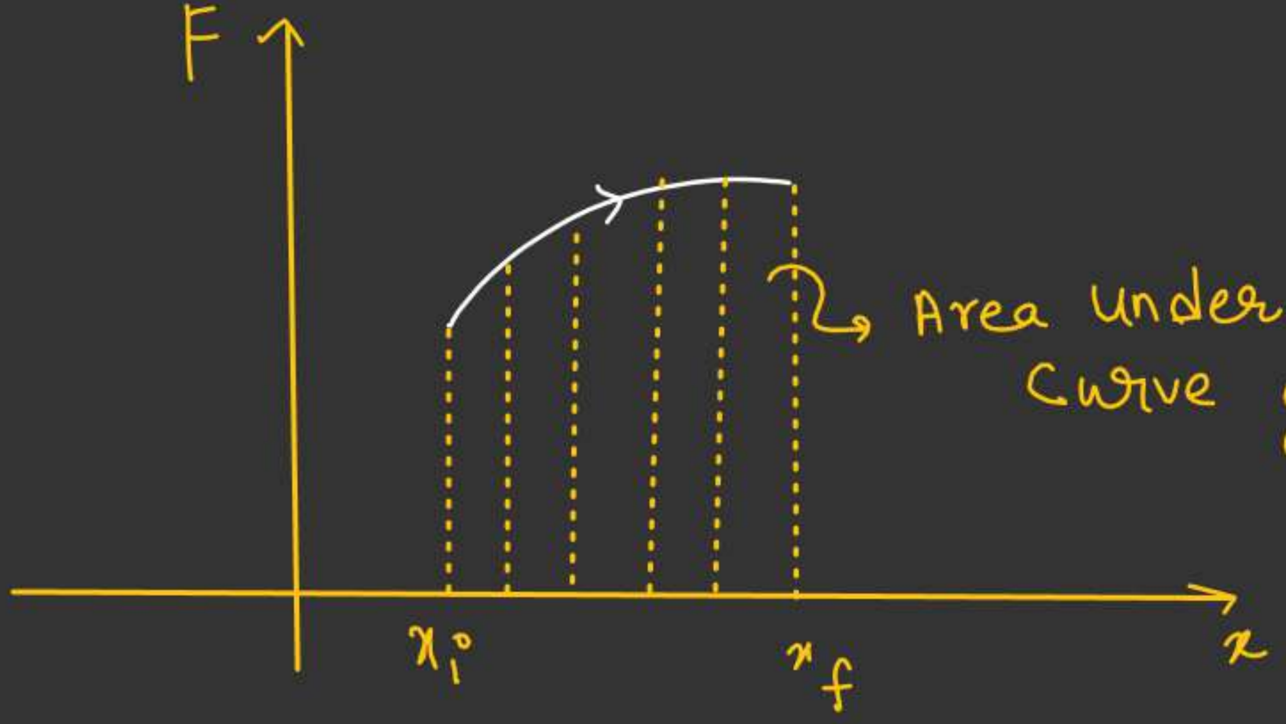
$$W = \left[\int_{x_i}^{x_f} F \cdot dx \right]$$

$F = f(x)$



Area under F Vs x curve gives total work done

$$\left[W = \int_{x_i}^{x_f} F \cdot dx \right]$$

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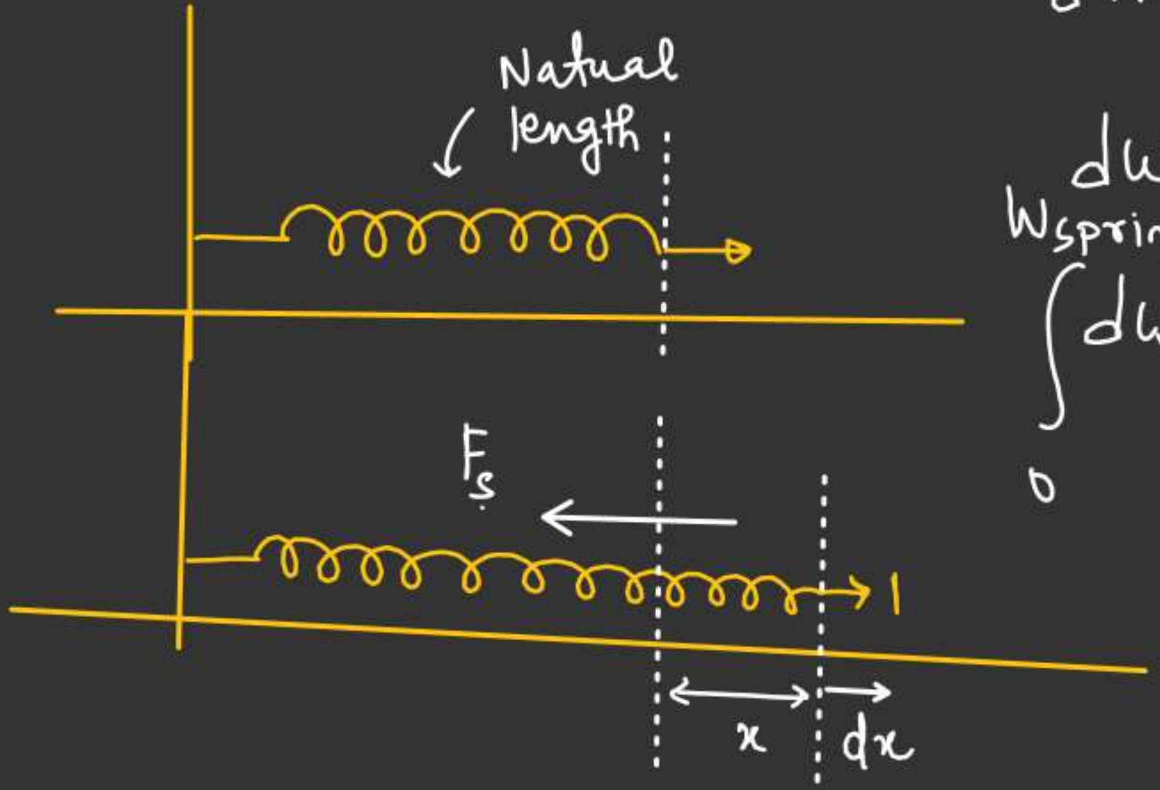
$F_s = -Kx$.
$\Downarrow$
Spring force.



M-1 Work done by Spring force

$$dW = F_s\, dx \cos\pi$$

$$dw = -F_s\, dx$$

$$\int_0^{W_{spring}} dw = -\int_0^{x} Kx\, dx$$

$$\boxed{W_{spring} = \frac{-Kx^2}{2}}$$

Since 'dx' is very small so Spring force at x elongation is Same as for (x+dx) elongation

$$\vec{F_s} = -kx\hat{i}$$

$$\vec{ds} = dx\,\hat{i}$$

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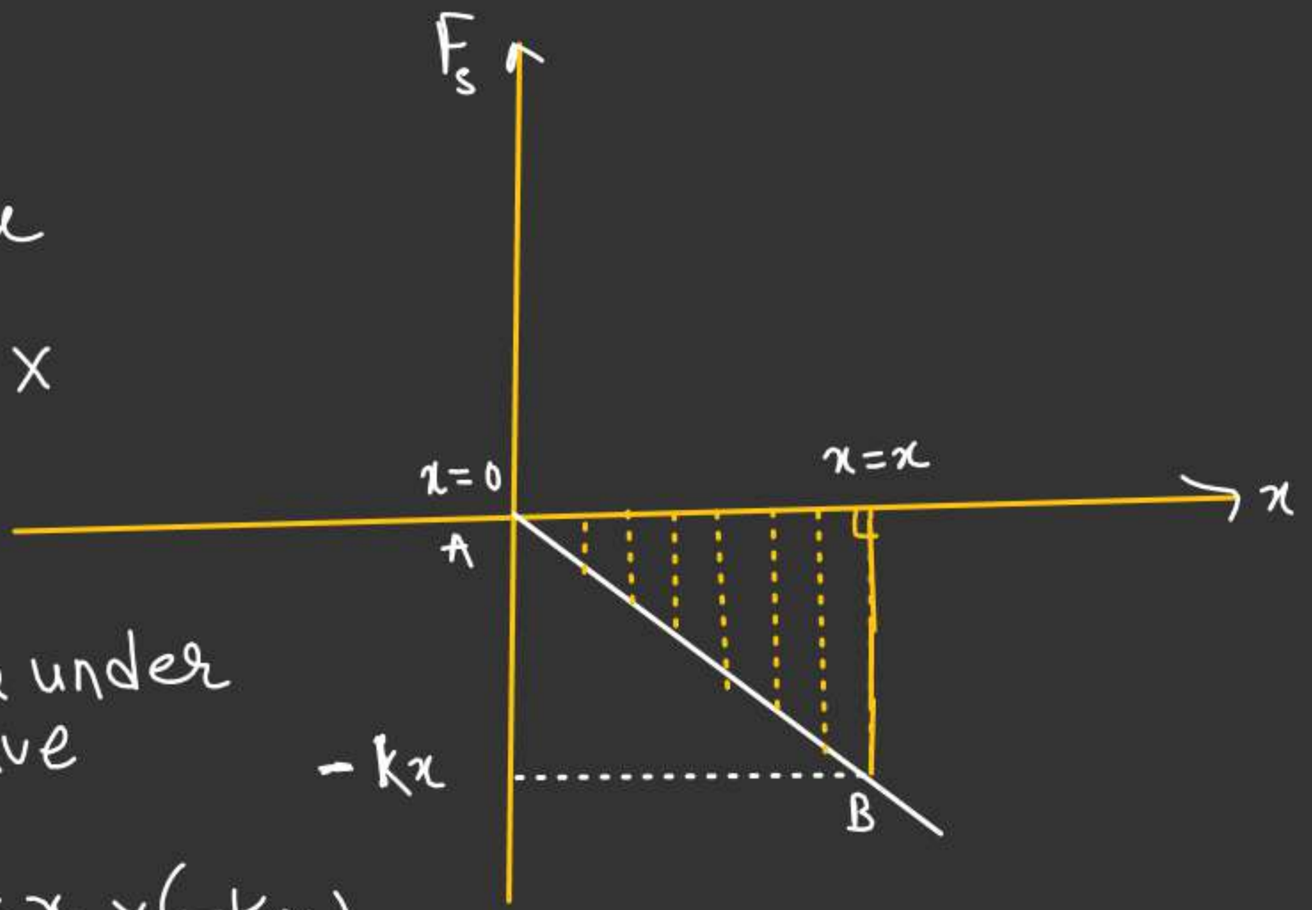
Work done by Spring force

graphically

$$F_s = -kx$$

$$y = -mx$$



$$W_{\text{spring force}} = \text{Area under Curve}$$

$$= \frac{1}{2} \times x \times (-kx)$$

$$= -\frac{1}{2} kx^2$$

Work done $-ve$

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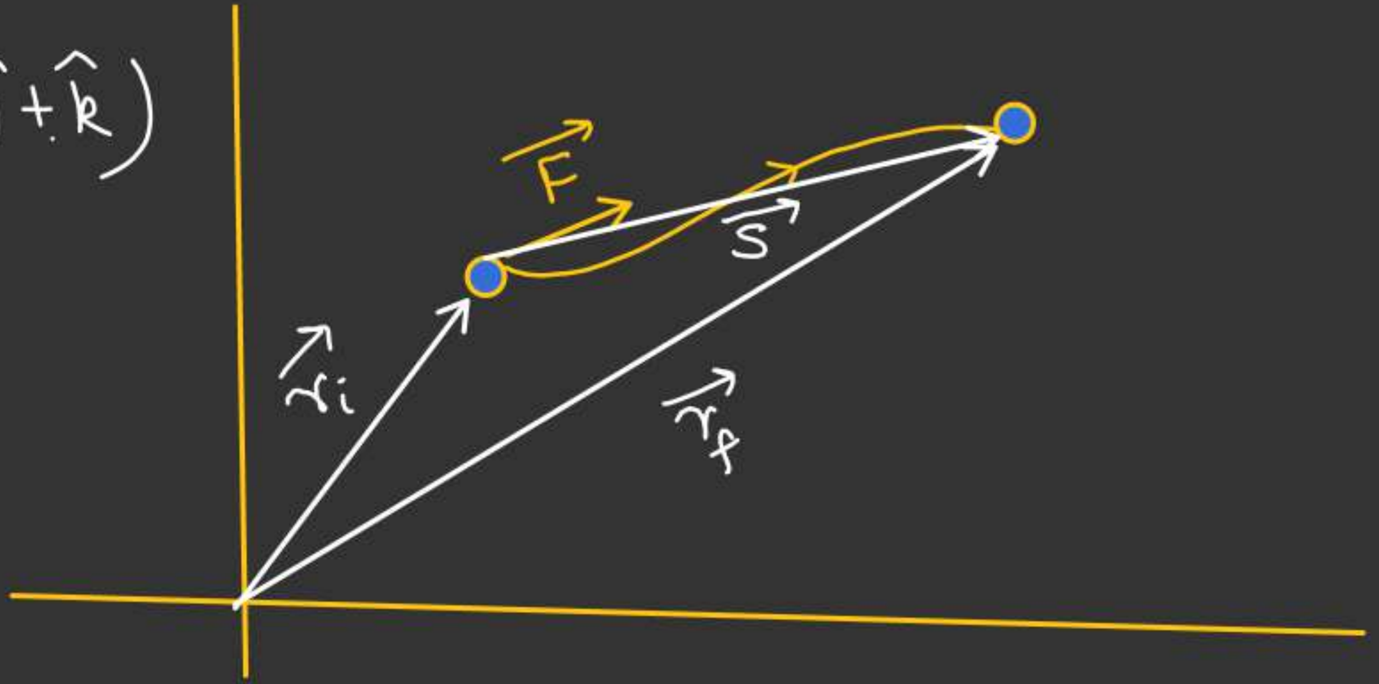
Q: A Constant force $\vec{F} = 2\hat{i} + 3\hat{j} + 4\hat{k}$ acting on a particle and displaces the particle from its initial position vector $2\hat{i} - \hat{j} + \hat{k}$ to final position vector $\hat{i} + 3\hat{j} + 2\hat{k}$

<u>Sol^n</u> :-

$$\vec{S} = \vec{r_f} - \vec{r_i}$$

$$\vec{S} = (\hat{i} + 3\hat{j} + 2\hat{k}) - (2\hat{i} - \hat{j} + \hat{k})$$

$$\vec{S} = (-\hat{i} + 4\hat{j} + \hat{k})$$



$$W = \vec{F} \cdot \vec{S}$$

$$= (2\hat{i} + 3\hat{j} + 4\hat{k}) \cdot (-\hat{i} + 4\hat{j} + \hat{k})$$

$$W = \underline{-2 + 12 + 4} = 14 \text{ J}$$ <u>Ans</u>

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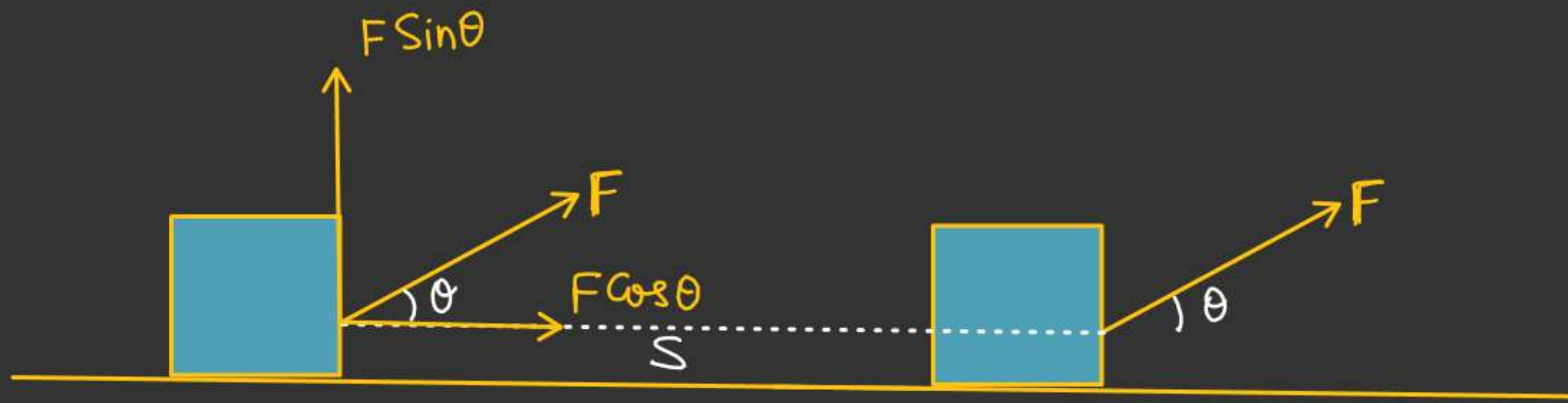
Work done by a Constant force

$$W_F = (F\cos\theta \cdot S)$$

$$\boxed{W_F = \vec{F} \cdot \vec{S}}$$

$$\boxed{W_{F\sin\theta} = 0}$$



$$W = (F\cos\theta)\,S$$

$$\Downarrow$$

[It is Component of force along the direction of displacement of body

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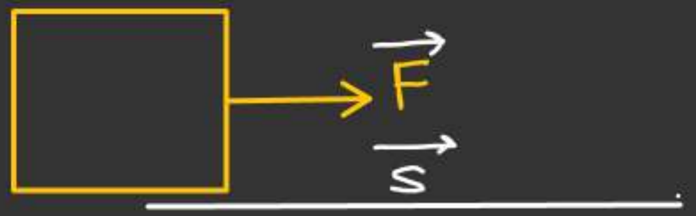
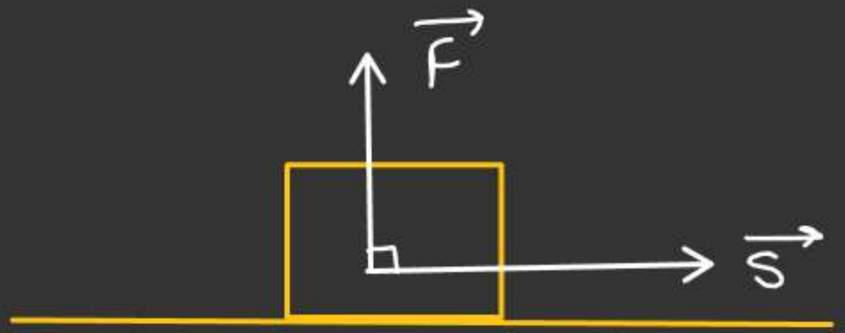
Case of Maximum work of a Constant force



$\vec{F} \parallel \vec{S}$ $\quad \theta = 0^\circ$

$$\boxed{W_{max} = F\,S}$$



$$\boxed{W_F = 0 \qquad \theta = \frac{\pi}{2}}$$

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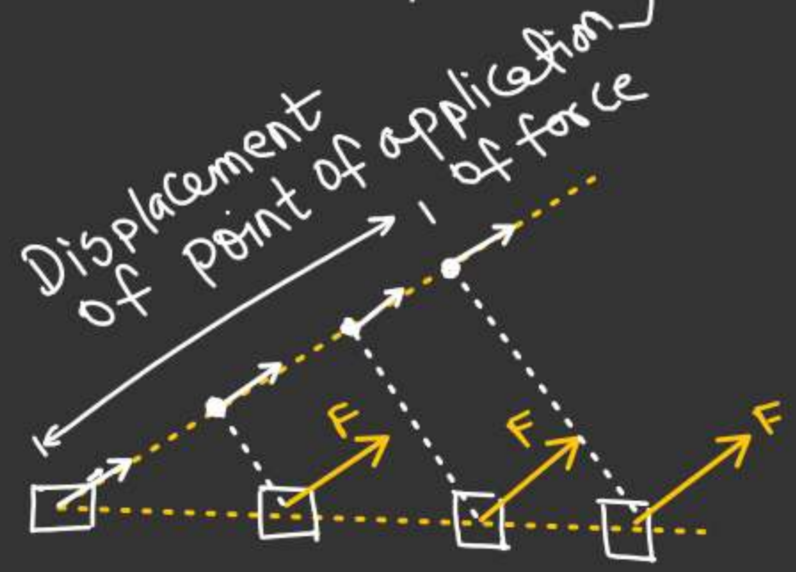
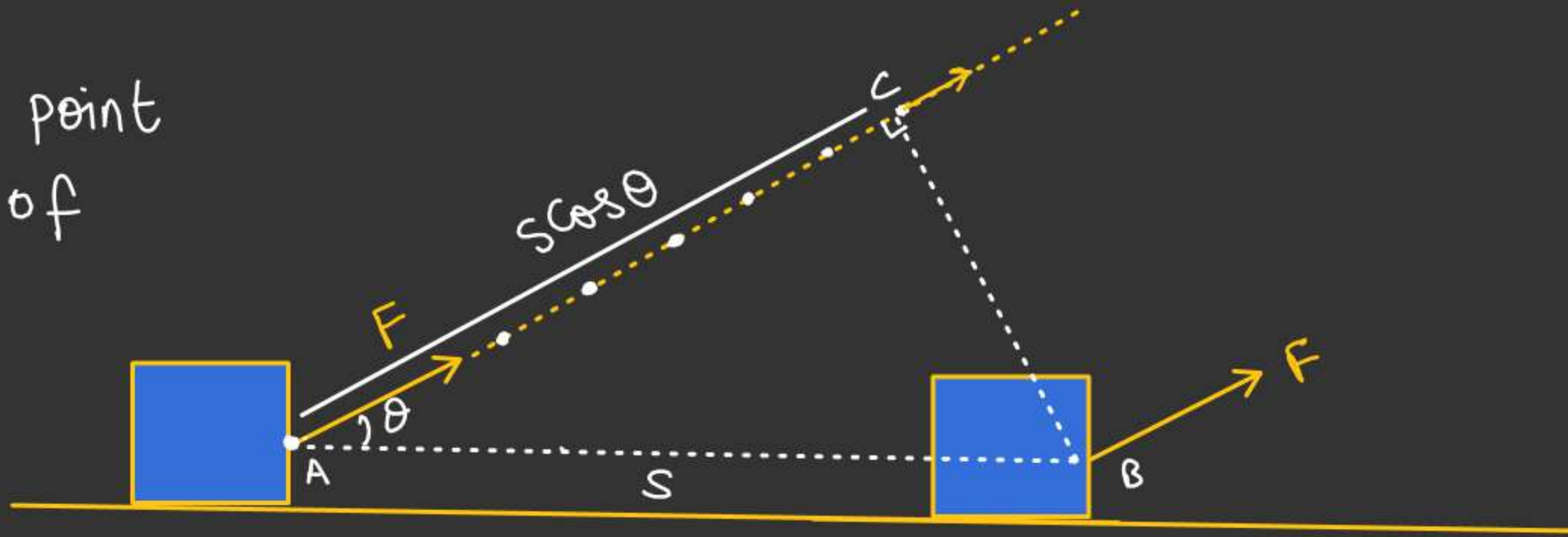
$$W = F\,(\underbrace{S\cos\theta})$$

$\Downarrow$

[Displacement of point of application of force]





In $\triangle ABC$

$$\cos\theta = \frac{AC}{AB}$$

$$AC = AB\cos\theta$$

$$= (S\cos\theta)$$

$\Downarrow$

Displacement of body along the direction of applied force.

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(Q) Find work done by F & mg when bob displaced from A to B.

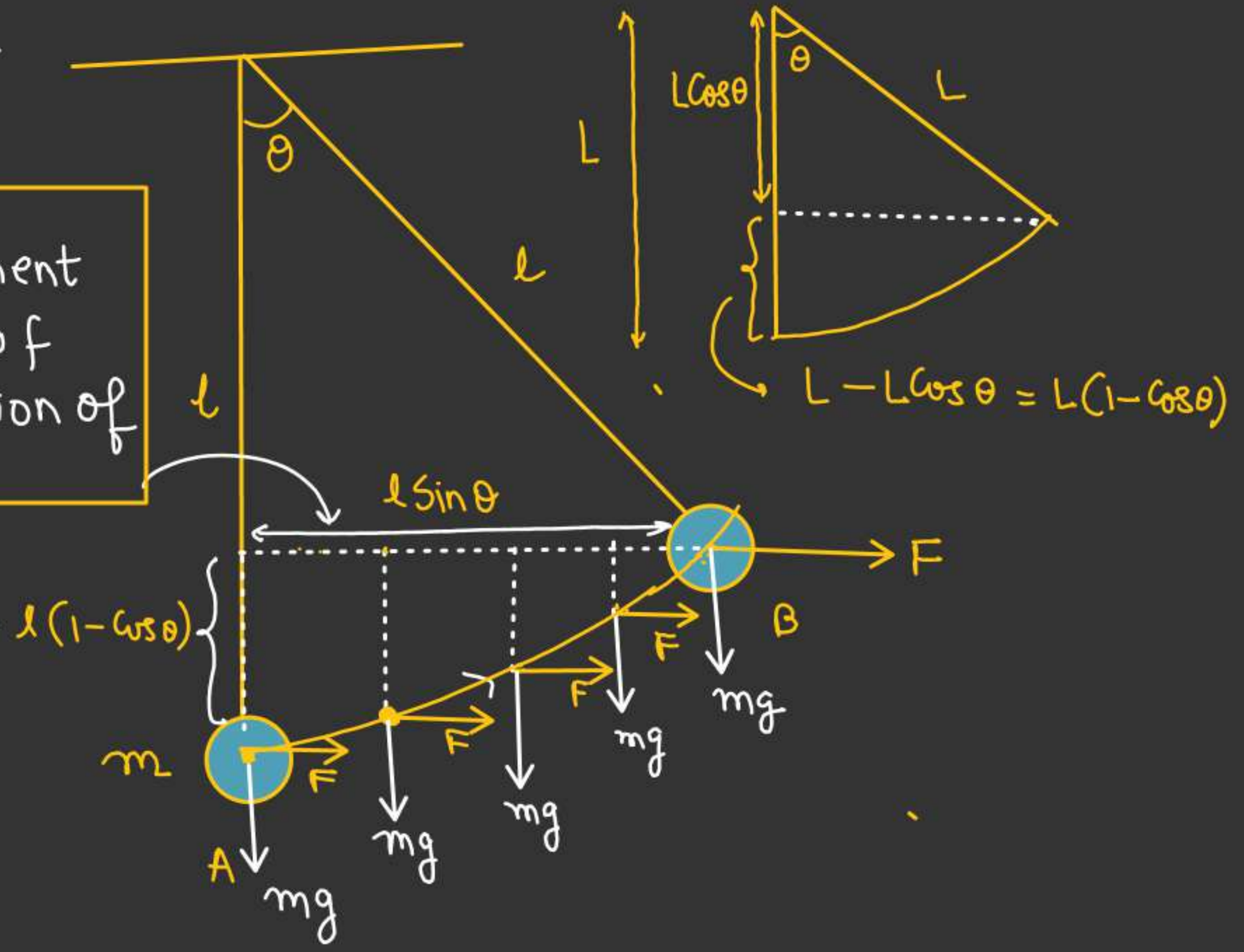
$$(W_F)_{A-B} = (F\, l \sin\theta)$$

Displacement of point of application of F

$$(W_{mg})_{A\to B} = -mg\, l(1-\cos\theta)$$

(opposite to applied force)

Displacement of point of application of Mg → $l(1-\cos\theta)$



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(Q) Find work done by F & mg when bob displaced from A to B.

Normal Method