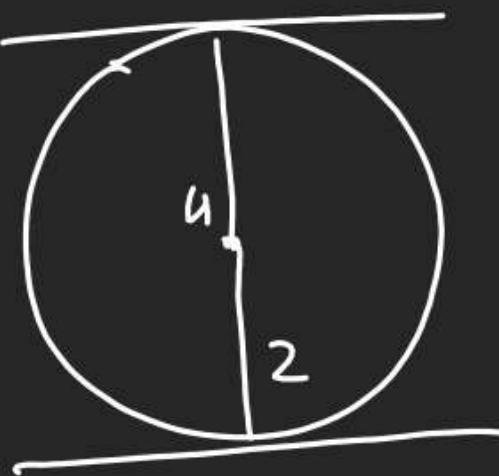




HAPPY
Birthday
GB SIR

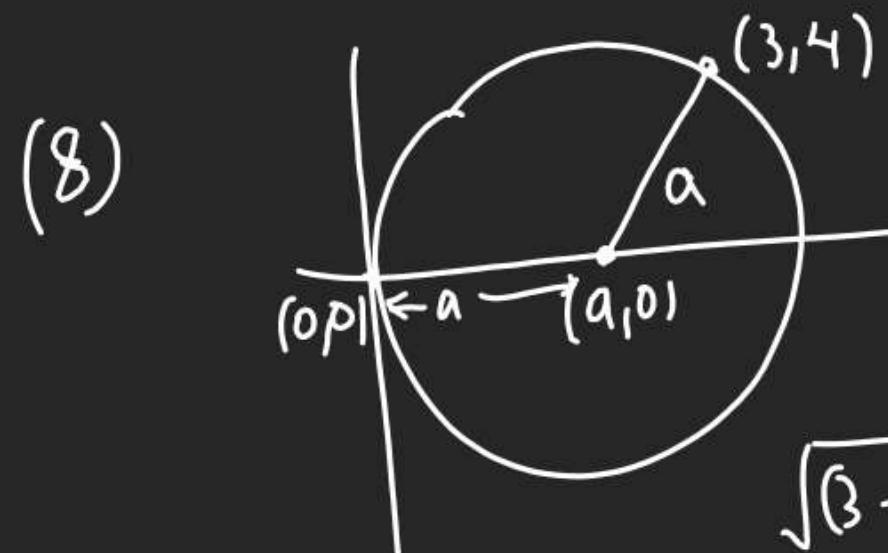
$$y = \sqrt{3}x + 1$$

$$y = \sqrt{3}x + 2$$



$$\frac{|C_1 - C_2|}{\sqrt{\sqrt{3}^2 + 1^2}} = 4$$

$$|C_1 - C_2| = 8$$

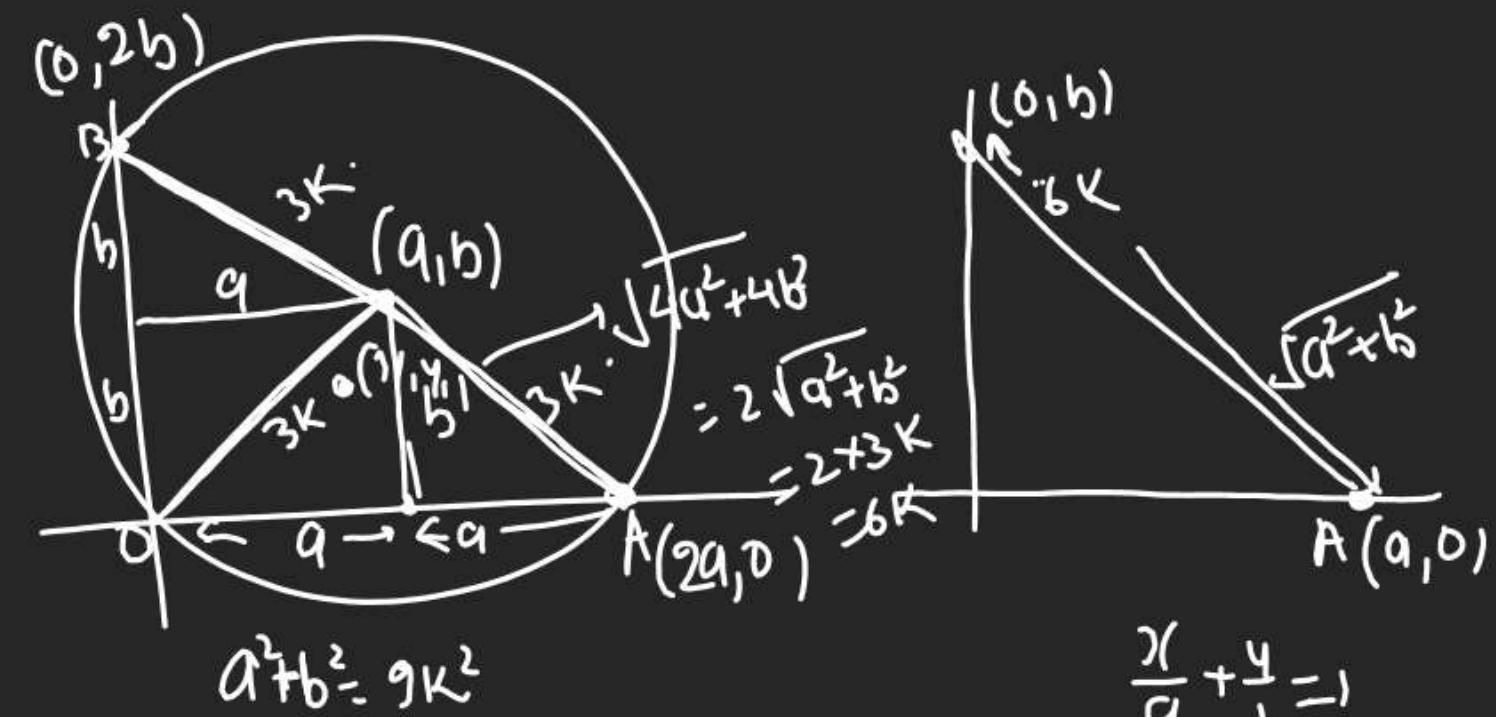


$$\sqrt{(3-a)^2 + (4-0)^2} = a$$

$$a^2 - 6a + 9 + 16 = a^2$$

$$6a = 25$$

(11)



$$a^2 + b^2 = 9K^2$$

$$x_1 = \frac{2y_1 + 0 + a}{3} \quad | \quad y_1 = 0 + 0 + 2b$$

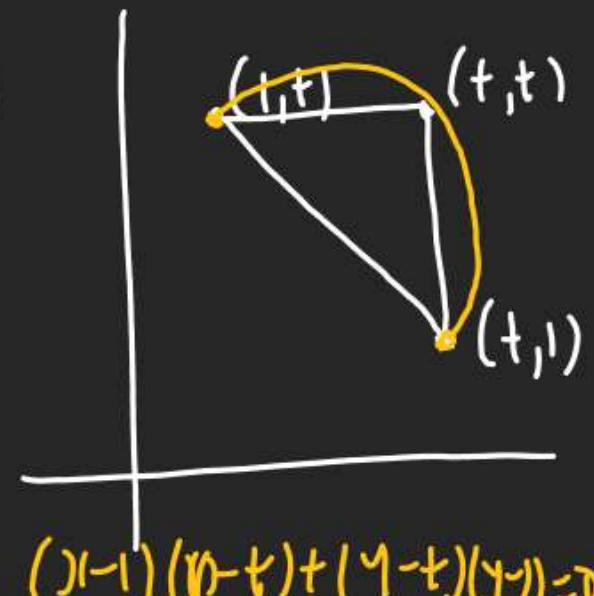
$$a = \frac{3x_1}{2}, \quad b = \frac{3y_1}{2}$$

$$a^2 + b^2 = 9K^2$$

$$\frac{9x_1^2}{4} + \frac{9y_1^2}{4} = 9K^2$$

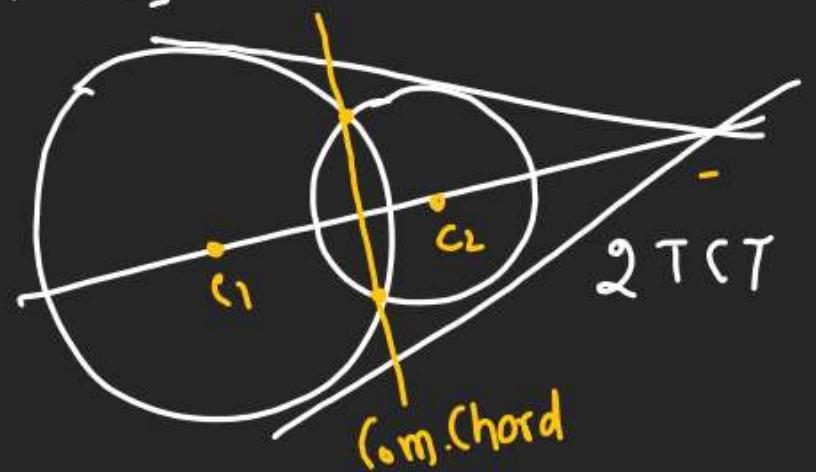
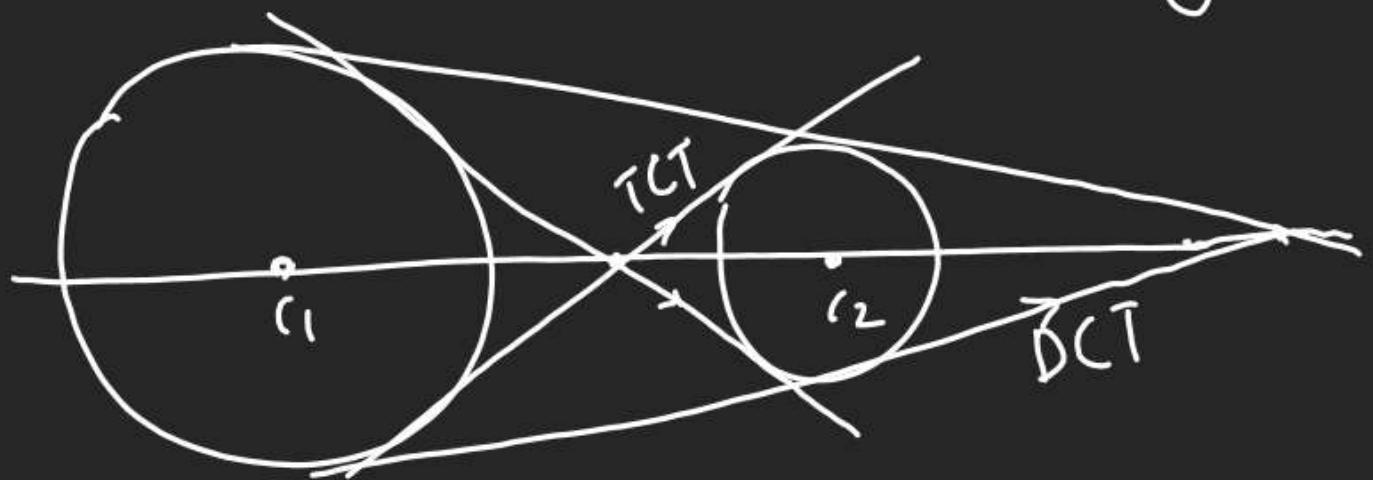
$$\frac{x}{a} + \frac{y}{b} = 1$$

(12)

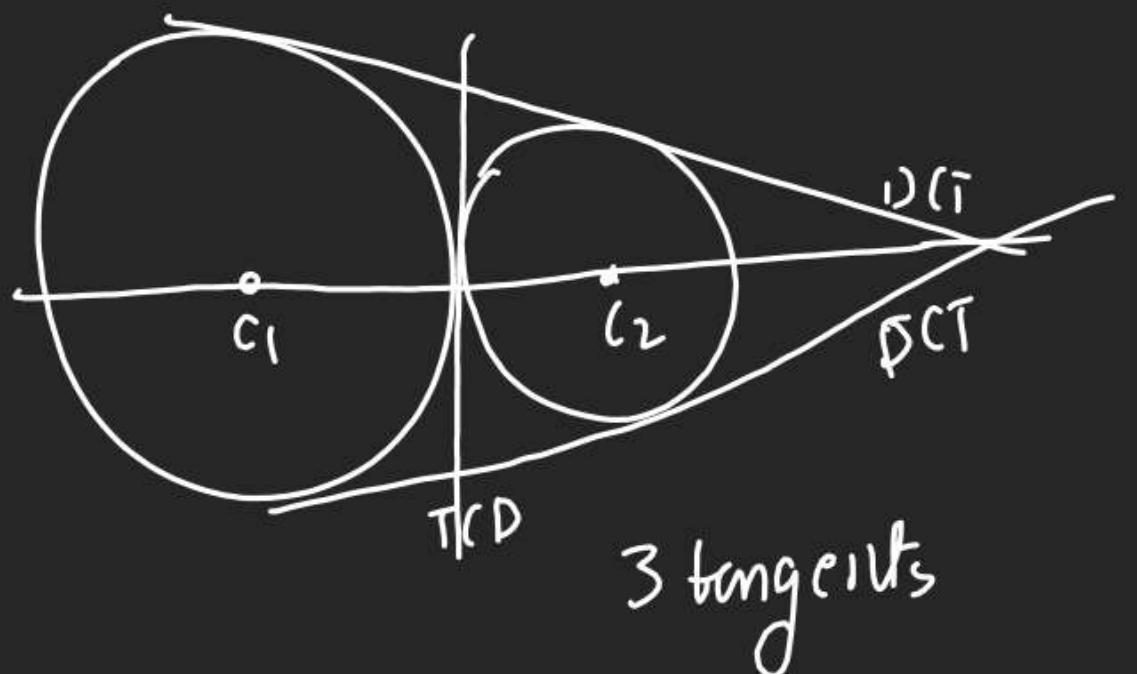


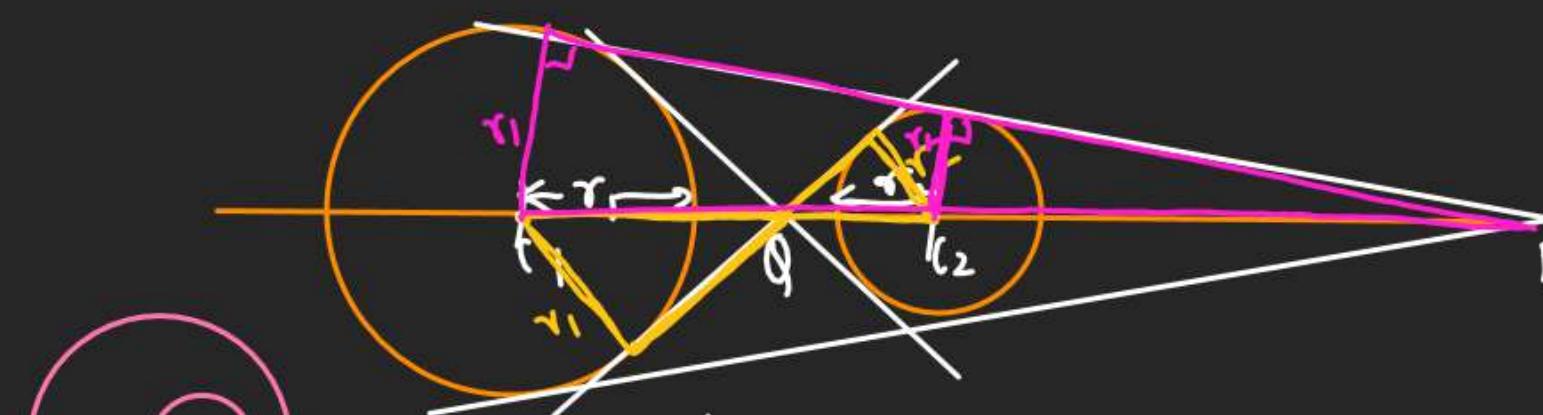
$$(1-t)(1-t) + (1-t)(1-t) = 0$$

Position of 2 circles [Common tangent B/w 2 circles]



No Com.
Chord +
No Com
tmgnt





(circles separated)

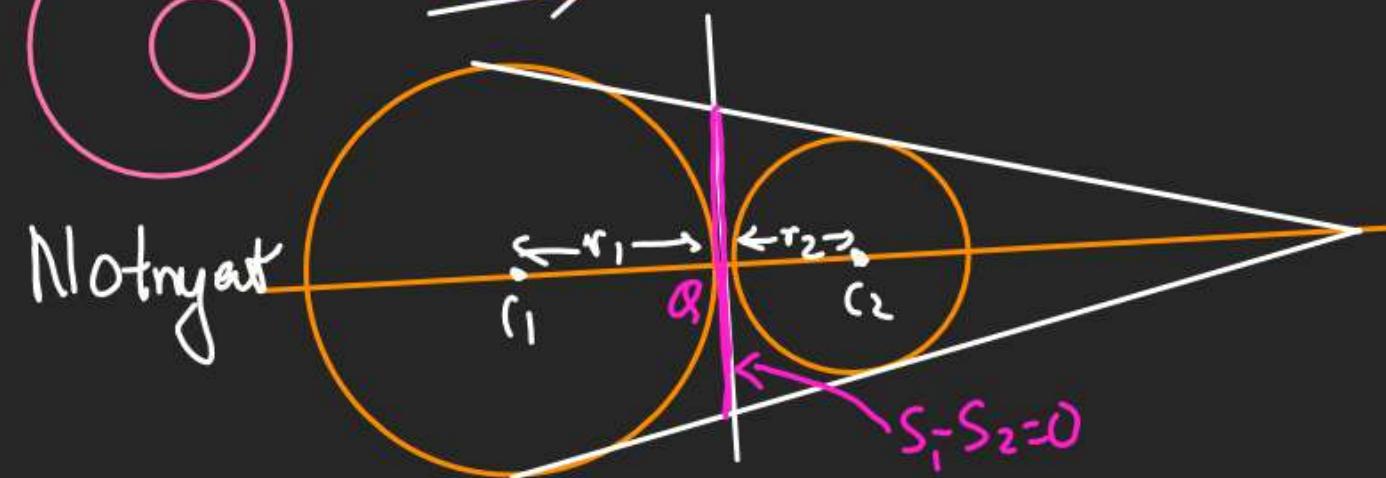
$$|C_1C_2| > r_1 + r_2$$

$$4 \text{ tangent} = 2\bar{T}(\bar{T} + 2D\bar{T})$$

$$Q \text{ is POI of } TCT \rightarrow \frac{C_1Q}{C_2Q} = \frac{r_1}{r_2}$$

$$\text{POI of } DCT \quad \frac{C_1P}{C_2P} = \frac{r_1}{r_2} \rightarrow P \text{ divides } r_1 : r_2 \text{ externally}$$

$$3 \text{ tangents} \quad 2D\bar{T} + 1 \text{ (com. tangent)}$$



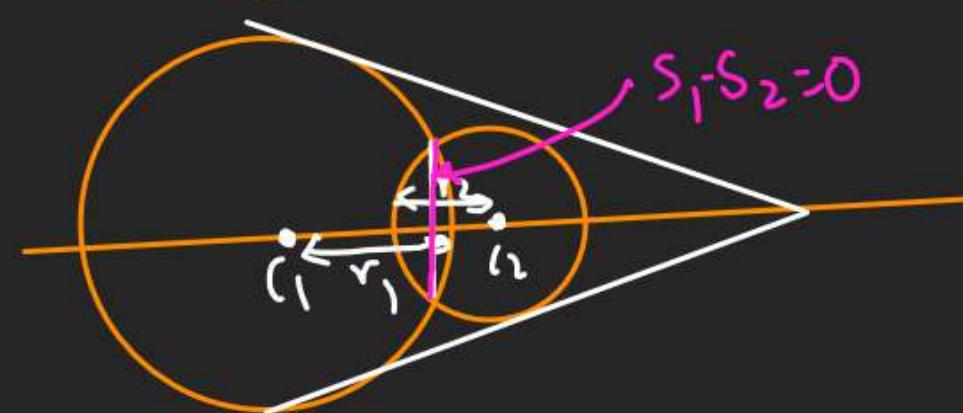
(circles touching)

$$|C_1C_2| = r_1 + r_2$$

$$2 \text{ tangents} \rightarrow 2D\bar{T} + 1 \text{ (com. chord)}$$

(circle Intersecting)

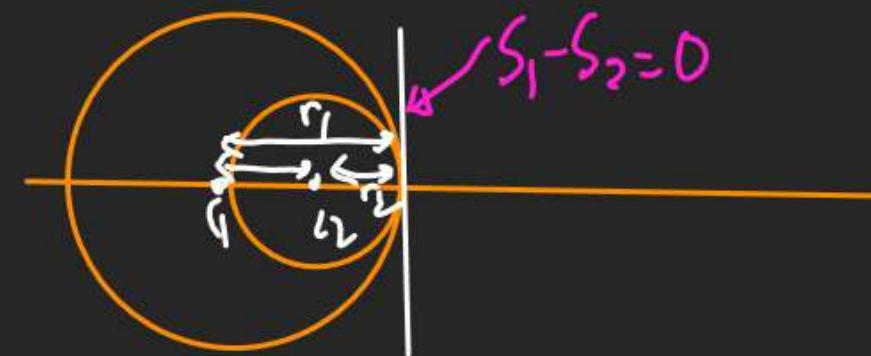
$$|r_1 - r_2| < |C_1C_2| < r_1 + r_2$$



(circles touching Internally)

$$|C_1C_2| = |r_1 - r_2|$$

1 tangent \rightarrow common tangent



Q) Check Position of Circles

$$S_1: x^2 + y^2 = 2$$

$$S_2: x^2 + y^2 - 6x - 6y + 10 = 0$$

Find No of Tangents.

$$S_1: x^2 + y^2 = 2$$

$$C_1: \text{Centre} = (0,0), \text{Rad.} = \sqrt{2} = r_1$$

$$S_2: x^2 + y^2 - 6x - 6y + 10 = 0$$

$$C_2: \text{Centre} = (3,3), \text{Rad.} = \sqrt{9+9-10} \\ = \sqrt{8} = 2\sqrt{2} = r_2$$



$$C_1 C_2 = \sqrt{(3-0)^2 + (3-0)^2} = 3\sqrt{2}$$

$$r_1 + r_2 = \sqrt{2} + 2\sqrt{2} = 3\sqrt{2}$$

$C_1 C_2 = r_1 + r_2$ (touching each other)
3 tangents

Q) Find (and) if

$$S_1: x^2 + y^2 + 2ax + c = 0$$

$$S_2: x^2 + y^2 + 2by + c = 0$$

touches each other.

touching

$$C_1 C_2 = r_1 + r_2$$

$$C_1: (-a, 0), r_1 = \sqrt{a^2 + 0 - c}$$

$$C_2: (0, b), r_2 = \sqrt{0 + b^2 - c}$$

$$C_1 C_2 = \sqrt{a^2 + b^2} = \sqrt{a^2 - c + b^2 - c}$$

$$\Rightarrow a^2 + b^2 = (a^2 - c) + (b^2 - c) \pm 2\sqrt{a^2 - c + b^2 - c}$$

$$2c = \pm \sqrt{(a^2 - c)(b^2 - c)}$$

$$c^2 = (a^2 - c)(b^2 - c) = a^2 b^2 - ((a^2 + b^2) + c^2)$$

$$\Rightarrow a^2 b^2 = c(a^2 + b^2)$$

$$\frac{1}{c} = \frac{a^2 + b^2}{a^2 b^2} \Rightarrow \boxed{\frac{1}{c} = \frac{1}{a^2} + \frac{1}{b^2}}$$

Q) Check Position of

$$S_1: x^2 + y^2 - 4x - 10y + 4 = 0$$

$$S_2: x^2 + y^2 - 6x - 12y - 55 = 0$$

$$1) C_1: (2, 5), r_1 = \sqrt{4+25-4} = 5$$

$$C_2: (3, 6), r_2 = \sqrt{9+36+55} = 10$$

$$2) C_1 C_2 = \sqrt{(-1)^2 + (-1)^2} = \sqrt{2}$$

$$r_1 + r_2 = 5 + 10 = 15 > C_1 C_2$$

$$(3) |r_1 - r_2| = |10 - 5| = 5$$

$$|r_1 - r_2| > C_1 C_2$$

Circle Neither touching
Externally nor Internally

But Inside Somewhere.



Q Find Eqn of com. tangent
for circle.

$$S_1: x^2 + y^2 = 2$$

$$S_2: x^2 + y^2 - 6x - 6y + 10 = 0$$

$$\begin{aligned} \textcircled{1} \quad C_1: (0,0) \\ C_2: (3,3) \end{aligned} \quad \left\{ \begin{aligned} C_1C_2 &= \sqrt{3^2 + 3^2} = 3\sqrt{2} = 3\sqrt{2} = 3\sqrt{2} \\ &= 4\cdot 2 \end{aligned} \right.$$

$$\begin{aligned} \textcircled{2} \quad r_1 &= \sqrt{2} \\ r_2 &= \sqrt{3^2 + 3^2 - 10} = \sqrt{8} = 2\sqrt{2} \\ r_1 + r_2 &= 3r_2 = C_1C_2 \quad \text{QED} \end{aligned}$$

(3) Com. tangent $\rightarrow S_1 - S_2 = 0$

$$\begin{aligned} (x^2 + y^2 - 2) - (x^2 + y^2 - 6x - 6y + 10) &= 0 \\ 6x + 6y - 12 &= 0 \\ x + y &= 2 \end{aligned}$$

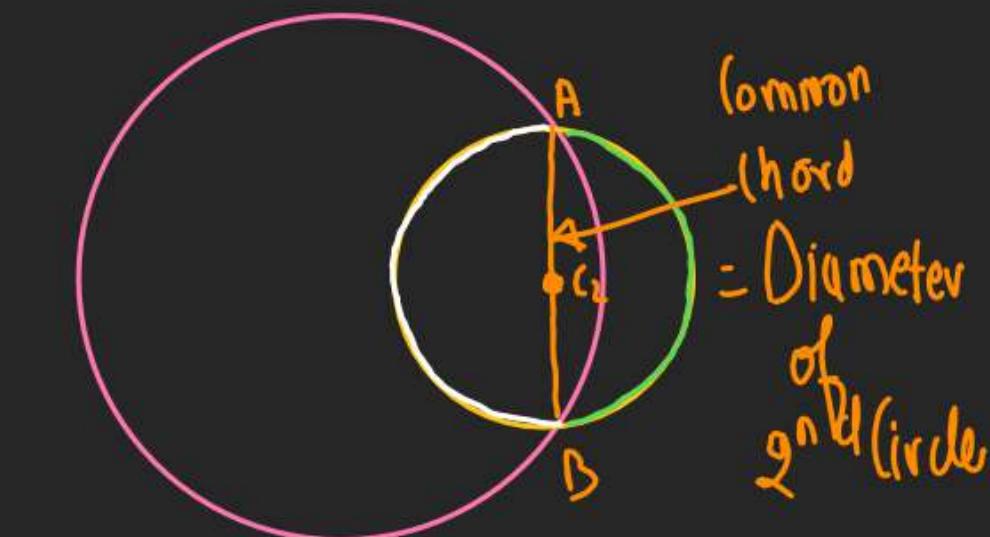


Q If circle $x^2 + y^2 + 2gx_1 + 2fy_1 + c_1 = 0$
Bisects circumference
 $x^2 + y^2 + 2g_2x + 2fy_2 + c_2 = 0$ then
 $2g_2(g_1 - g_2) + 2f_2(f_1 - f_2) = ?$

Q Com. chord is satisfied by
centre of 2nd circle (C_2)
($-g_2, -f_2$)

$$\begin{aligned} -2g_2(g_1 - g_2) - 2f_2(f_1 - f_2) &= -c_2 - c_1 \\ 2g_2(g_1 - g_2) + 2f_2(f_1 - f_2) &= c_1 - c_2 \end{aligned}$$

$c_1 - c_2$

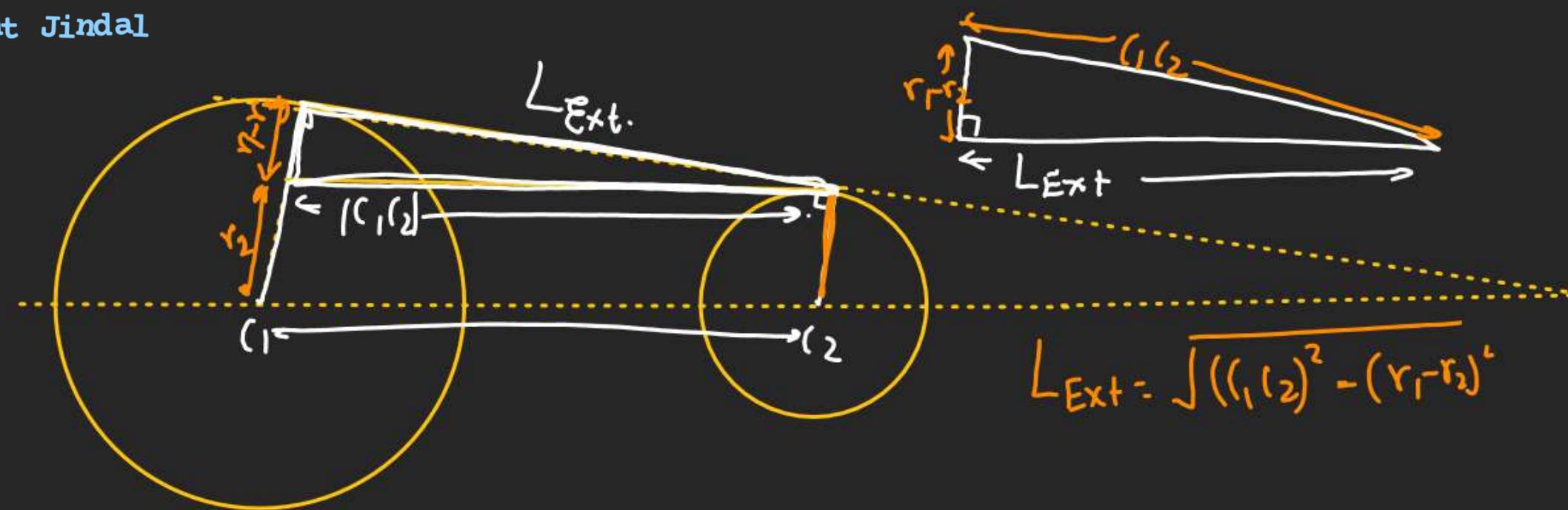


Q Com. chord $\rightarrow S_1 - S_2 = 0$

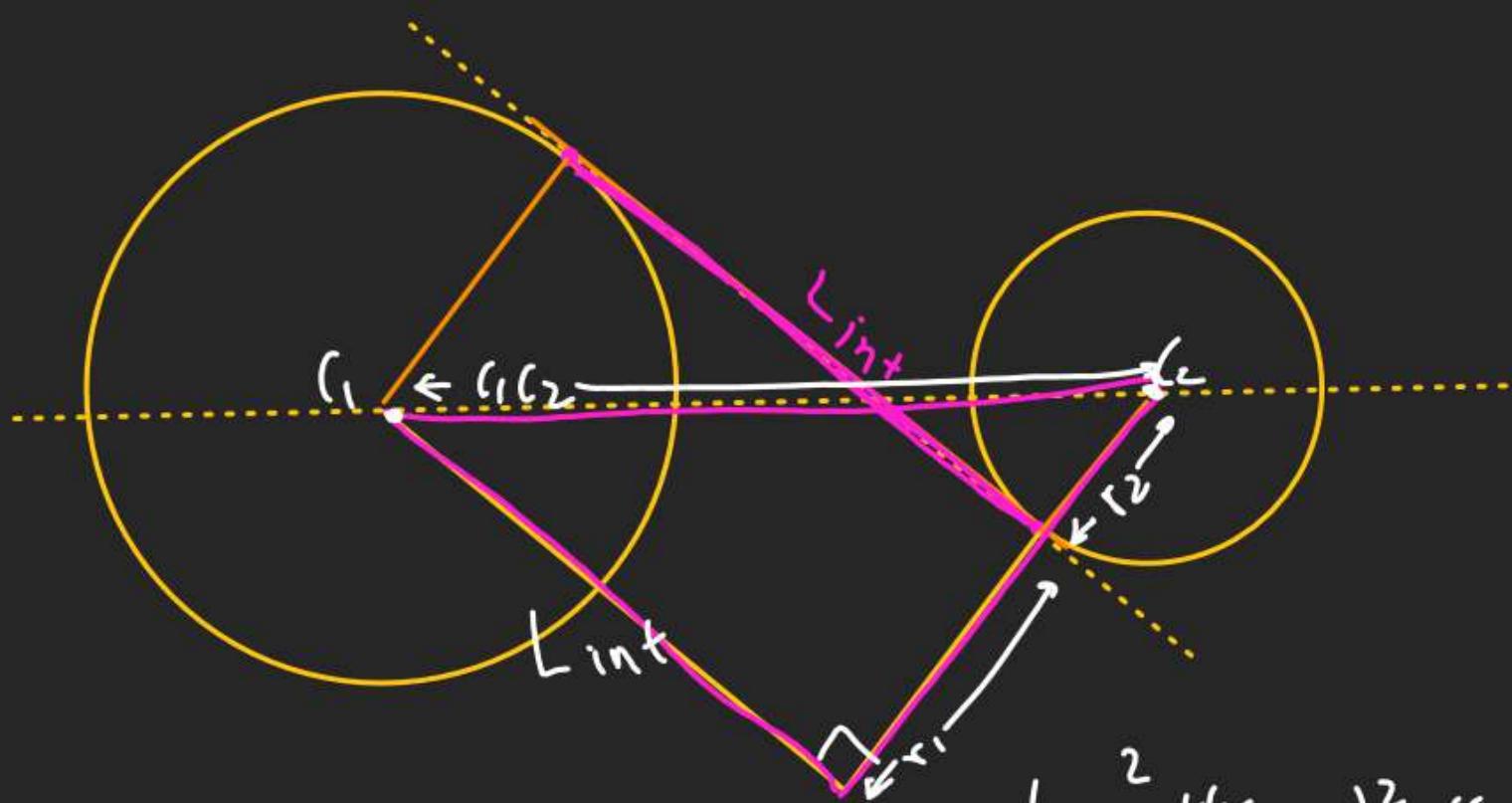
$$x^2 + y^2 + 2g_1x + 2f_1y + c_1 = 0$$

$$x^2 + y^2 + 2g_2x + 2f_2y + c_2 = 0$$

$$2x(g_1 - g_2) + 2y(f_1 - f_2) = c_1 - c_2$$



$$L_{Ext} = \sqrt{(c_1 - c_2)^2 + (r_1 + r_2)^2}$$



$$L_{Int}^2 + (r_1 - r_2)^2 = ((c_1 - c_2))^2$$

$$L_{Int} = \sqrt{(c_1 - c_2)^2 - (r_1 - r_2)^2}$$

Angle of Intersection of 2 circles