

$$\begin{aligned}
 & \underline{24} \quad 1 + (\cos^2 A - \sin^2 B) + \cos^2 C + \underline{2 \cos A \cos B \cos C} \\
 &= 1 + \cos(A-B) \cos(A+B) + \underline{\cos^2 C} + \frac{(\cos(A-B) + \cos(A+B))}{\cos C} \\
 &= 1 + (\cos C + \cos(A+B)) (\cos C + \cos(A-B)) \\
 &= 1 + 2 \cos\left(\frac{A+B+C}{2}\right) \cos\left(\frac{A+B-C}{2}\right) \left(2 \cos\left(\frac{C+A-B}{2}\right)\right. \\
 &\quad \left. + 2 \cos\left(\frac{C-A+B}{2}\right)\right) \\
 &= 1 + 4 \cos S \cos(S-C) \cos(S-B) \cos(S-A) \left(2 \cos\left(\frac{S-C-A+B}{2}\right)\right)
 \end{aligned}$$

25. (i)

$$2 \cos \frac{\alpha+\beta}{2} \cos \frac{\alpha-\beta}{2} + 2 \cos \left(\frac{\gamma+\delta}{2} \right) \cos^2 \frac{\gamma-\delta}{2} + 4 \cos$$

$$\frac{2\pi - (\alpha+\beta)}{2} = \pi - \frac{\alpha+\beta}{2}$$

$$= 2 \cos \frac{\alpha+\beta}{2} \left(\cos \frac{\alpha-\beta}{2} - \cos \frac{\gamma-\delta}{2} \right) + 4 \cos \frac{\alpha+\beta}{2} \cos \frac{\alpha+\gamma}{2} \cos \frac{\delta+\beta}{2}$$

$$\pi - \frac{\alpha+\delta}{2}$$

$$= 4 \cos \frac{\alpha+\beta}{2}$$

$$\sin \frac{\gamma - \alpha - \delta + \beta}{4}$$

$$\sin \frac{\alpha - \beta + \gamma - \delta}{4}$$

$$\cos \frac{\alpha+\delta}{2}$$

$$\cos \frac{\alpha+\delta}{2}$$

$$\sin (\alpha+\gamma) - (2\pi - \alpha - \gamma) = -\sin \left(\frac{\alpha+\gamma-\pi}{2} \right)$$

$$= -\cos \left(\frac{\alpha+\gamma}{2} \right)$$

34: $x = \tan A, y = \tan B, z = \tan C$

$$\sum \tan A = \overline{\prod} \tan A$$

$$\Rightarrow A + B + C = n\pi \quad n \in \mathbb{I}$$

$$3A + 3B + 3C = 3n\pi = k\pi, k \in \mathbb{I}$$

$$\sum \tan 3A = \overline{\prod} \tan 3A$$

35: $\boxed{\sum \tan 2A = \overline{\prod} \tan 2A}$

$$\begin{aligned}
 & \frac{1}{\sin \theta} (\sin \theta - \sin 2\theta + \sin 3\theta - \sin 4\theta + \sin 5\theta - \sin 6\theta + \dots) \\
 &= \frac{\sin 3\theta - \sin 6\theta}{\sin \theta + 2(\theta - \pi)} \quad \text{upto } n \text{ terms.} \\
 & \sin \theta + \sin(\theta + (\theta + \pi)) + \sin(\theta + 2(\theta + \pi)) + \sin(\theta + 3(\theta + \pi)) \\
 & \quad + \sin(\theta + 4(\theta + \pi)) + \dots + \sin(\theta + (n-1)(\theta + \pi)) \\
 &= \frac{\sin \left(\frac{n(\theta + \pi)}{2} \right)}{\sin \left(\frac{\theta + \pi}{2} \right)} \sin \left(\frac{2\theta + (n-1)(\theta + \pi)}{2} \right)
 \end{aligned}$$

$$\begin{aligned}
 & \frac{1}{2} \cdot \tan \frac{x}{2} \sec x + \tan \frac{x}{2^2} \sec \frac{x}{2} + \tan \frac{x}{2^3} \sec \frac{x}{2^2} + \tan \frac{x}{2^4} \sec \frac{x}{2^3} + \dots \\
 & = \frac{\sin \frac{x}{2}}{\cos \frac{x}{2} \cos x} + \frac{\sin \frac{x}{2^2} = \frac{x}{2} - \frac{x}{2^2}}{\cos \frac{x}{2^2} \cos \frac{x}{2}} + \frac{\sin \frac{x}{2^3} = \frac{x}{2^2} - \frac{x}{2^3}}{\cos \frac{x}{2^3} \cos \frac{x}{2^2}} + \dots + \frac{\sin \frac{x}{2^n} = \frac{x}{2^{n-1}} - \frac{x}{2^n}}{\cos \frac{x}{2^n} \cos \frac{x}{2^{n-1}}} \\
 & \boxed{\frac{\sin \frac{x}{2}}{\cos \frac{x}{2} \cos x} = \frac{\sin \left(x - \frac{x}{2} \right)}{\cos \frac{x}{2} \cos x} = \tan x - \tan \frac{x}{2}}
 \end{aligned}$$

$$\begin{aligned}
 & = \left(\tan x - \tan \frac{x}{2} \right) + \left(\cancel{\tan \frac{x}{2}} - \cancel{\tan \frac{x}{2^2}} \right) + \left(\cancel{\tan \frac{x}{2^2}} - \cancel{\tan \frac{x}{2^3}} \right) \\
 & \quad + \left(\cancel{\tan \frac{x}{2^3}} - \cancel{\tan \frac{x}{2^4}} \right) + \dots + \left(\cancel{\tan \frac{x}{2^{n-2}}} - \cancel{\tan \frac{x}{2^{n-1}}} \right) + \left(\cancel{\tan \frac{x}{2^{n-1}}} - \cancel{\tan \frac{x}{2^n}} \right)
 \end{aligned}$$

$$\begin{aligned}
 & \text{Q: } \csc x + \csc(2x) + \csc(2^2 x) + \csc(2^3 x) + \dots + \csc(2^{n-1} x) \\
 & = \frac{\sin \frac{x}{2}}{\sin x \sin \frac{x}{2}} + \frac{\sin x}{\sin 2x \sin x} + \frac{\sin 2x}{\sin(2x) \sin(2x)} + \dots + \frac{\sin 2^{n-2} x}{\sin(2^{n-1} x) \sin(2^{n-2} x)} \\
 & = \frac{\sin\left(x - \frac{x}{2}\right)}{\sin x \sin \frac{x}{2}} + \frac{\sin(2x - x)}{\sin 2x \sin x} + \frac{\sin(2^2 x - 2x)}{\sin 2^2 x \sin(2x)} + \dots + \frac{\sin(2^{n-1} x - 2^{n-2} x)}{\sin 2^{n-1} x \sin 2^{n-2} x} \\
 & = \left(\cot \frac{x}{2} - \cot x\right) + \left(\cot x - \cot 2x\right) + \left(\cot 2x - \cot 2^2 x\right) + \dots + \left(\cot 2^{n-2} x - \cot 2^{n-1} x\right) \\
 & = \cot \frac{x}{2} - \cot 2^{n-1} x .
 \end{aligned}$$

$$\text{LHS} = \sin x \sec 3x + \sin 3x \sec 9x + \sin 9x \sec 27x + \sin 27x \sec 81x + \dots$$

$$\begin{aligned}
 &= \frac{\sin 2x}{2 \cos 3x \cos x} + \frac{\sin 6x}{2 \cos 9x \cos 3x} + \frac{\sin 18x}{2 \cos 27x \cos 9x} + \dots + \frac{\sin(3^{n-1}x) \sec(3^n x)}{2 \cos 3^n x \cos 3^{n-1}x} \\
 &= \frac{1}{2} \left(\frac{\sin(3x-x)}{\cos 3x \cos x} + \frac{\sin(9x-3x)}{\cos 9x \cos 3x} + \frac{\sin(27x-9x)}{\cos 27x \cos 9x} + \dots + \frac{\sin(3^{n-1}x-3^0x)}{\cos 3^n x \cos 3^{n-1}x} \right) \\
 &= \frac{1}{2} \left[(\cancel{\tan 3x - \tan x}) + (\cancel{\tan 9x - \tan 3x}) + (\cancel{\tan 27x - \tan 9x}) + \dots + \left(\frac{\tan 3^n x}{\cos 3^n x} - \frac{\tan 3^0 x}{\cos 3^0 x} \right) \right] \\
 &= \frac{1}{2} \left(\tan 3^n x - \tan x \right)
 \end{aligned}$$

$$a \sin x + b \cos x = \sqrt{a^2 + b^2} \left(\frac{a}{\sqrt{a^2 + b^2}} \sin x + \frac{b}{\sqrt{a^2 + b^2}} \cos x \right)$$

$$\frac{a}{\sqrt{a^2 + b^2}} = \cos \theta$$

$$\frac{b}{\sqrt{a^2 + b^2}} = \sin \theta$$

$$\alpha^2 + \beta^2 = 1$$

$$= \sqrt{a^2 + b^2} (\sin x \cos \theta + \sin \theta \cos x)$$

$$a \sin x + b \cos x = \sqrt{a^2 + b^2} \sin(x + \theta) \in [-\sqrt{a^2 + b^2}, \sqrt{a^2 + b^2}]$$

$$\sin x - \cos x = \sqrt{1^2 + 1^2} \left(\frac{1}{\sqrt{2}} \sin x - \frac{1}{\sqrt{2}} \cos x \right)$$

\downarrow

$$= \sqrt{2} \sin \left(x - \frac{\pi}{4} \right)$$

$$\sin x - \cos x \in [-\sqrt{2}, \sqrt{2}]$$

$$f(x) = \sqrt{3} \cos x + \sin x = 2 \left(\frac{\sqrt{3}}{2} \cos x + \frac{1}{2} \sin x \right) = 2 \sin \left(x + \frac{\pi}{3} \right)$$

$$R_f = [-2, 2]$$

$$2 \left(\left(\frac{\sqrt{3}}{2} \right) \cos x + \left(\frac{1}{2} \right) \sin x \right)$$

$\cos \frac{\pi}{6}$ $\sin \frac{\pi}{6}$

$$= 2 \cos \left(x - \frac{\pi}{6} \right)$$

