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QUADRATIC EQUATION

Common Root

M1

$x^2 - 3x + 2 = 0$

$x^2 - 5x + 6 = 0$

$2x - 4 = 0$

$\boxed{x = 2}$ is common Root.

Cross Multiplication

M2 let α is a Com. Root

$\alpha^2 - 3\alpha + 2 = 0$

$\alpha^2 - 5\alpha + 6 = 0$

$$\frac{\alpha^2}{\begin{vmatrix} -3 & 2 \\ -5 & 6 \end{vmatrix}} = \frac{-\alpha}{\begin{vmatrix} 1 & 2 \\ 1 & 6 \end{vmatrix}} = \frac{1}{\begin{vmatrix} 1 & -3 \\ 1 & -5 \end{vmatrix}}$$

$-18 - (-10)$ $\quad (6 - 2)$ $\quad (-5) - (-3)$

$$\frac{\alpha^2}{-8} = \frac{+\alpha}{4} = \frac{1}{+2}$$

$$\alpha = \frac{4}{2} = 2$$

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QUADRATIC EQUATION

Theorey

1) One Common Root

$$a_1x^2+b_1x+c_1=0 \quad (\beta,\ \alpha)$$
$$a_2x^2+b_2x+c_2=0 \quad (\alpha,\ \gamma)$$

$$a_1\alpha^2+b_1\alpha+c_1=0$$
$$a_2\alpha^2+b_2\alpha+c_2=0$$

$$\frac{\alpha^2}{\begin{vmatrix} b_1 & c_1 \\ b_2 & c_2 \end{vmatrix}} = \frac{-\alpha}{\begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}} = \frac{1}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}}$$

We use this method when we need a cond^n for common Root

(2) 2 Common Roots

has 2 Roots com.

$$\left.\begin{aligned} a_1x^2+b_1x+c_1=0 \\ a_2x^2+b_2x+c_2=0 \end{aligned}\right\}(\alpha,\beta)$$

Coff are Proportionate

$$\boxed{\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}}$$

QUADRATIC EQUATION

Q If $3x^2+4mx+2=0$ & $\boxed{2x^2+3x-2=0}$ have One Com. Root then $m=?$

कुछ भी

$2x^2+3x-2=0$

$2x^2+4x-x-2=0$

$2x(x+2)-1(x+2)=0$

$x=+\frac{1}{2}, -2$

$D=3^2-4\times2\times-2$

$=9+16$

$=25$

$D=5^2$

$=$ Per sqr

It is factorisable

1) Complete Eqn is given then check D

2) If $D<0$ then Roots will be in Conjugate Pair

3) If $D>0$ then check $D \neq$ Per sqr as Roots will be irrational $\to \alpha+\sqrt{\beta}, \alpha-\sqrt{\beta}$

let's $x=\frac{1}{2}$ is Com. Root

$\frac{3}{4}+\frac{4m}{2}+2=0$

$2m=-2-\frac{3}{4}$

$=-\frac{11}{4} \Rightarrow \boxed{m=-\frac{11}{8}}$

let's $x=-2$ is Com Root

$3(-2)^2+4m(-2)+2=0$

$-8m=-14$

$\boxed{m=\frac{7}{4}}$

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QUADRATIC EQUATION

Q If both Roots of $K(6x^2+3)+rx+2x^2-1=0$

& $6K(2x^2+1)+px+4x^2-2=0$ are Com.

then $2r-p=?$

Eqn ढंग से लिखो।

$$\left.\begin{aligned} x^2(6K+2)+rx+3K-1=0 \\ x^2(12K+4)+px+6K-2=0 \end{aligned}\right\} \text{Both Root Com.}$$

$$\frac{2\cancel{(12K+4)}}{\cancel{6K+2}}=\frac{p}{r}=\frac{\cancel{6K-2}}{\cancel{3K-1}}$$

$$\frac{p}{r}=2$$

$2r=p \Rightarrow 2r-p=0$ Ans

Q $a,b,c \in N$ & Eqn $x^2+3x+5=0$ → complete Eqn दी है।

& $ax^2+bx+c=0$ has 1 Com Root

then min value of $a+b+c=?$

होगा

$D=3^2-4\times5$
$=-11$
$=-ve$

Conjugate Pair.

No, it must be of 2 Com.

$$\frac{a}{1}=\frac{b}{3}=\frac{c}{5}=K$$

$a=K,\ b=3K,\ c=5K$

$a,b,c \in N$
$\{1,2,3,\ldots\}$
सबसे छोटा Nat No लेते हैं
$a+b+c$ will give min Value

$=1$ $b=3$ $c=5$

$\therefore$ Min $(a+b+c)=1+3+5$
$=9$

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QUADRATIC EQUATION

Q If Eq^n $x^2-ax+b=0$ & $x^2+bx-a=0$

have a Com. Root th.

$a=b$ $\quad a+b=0$ $\quad a-b=1$ $\quad a-b+1=0$

$x^2-ax+b=0$ $\quad \alpha$

$x^2+bx-a=0$

$\alpha^2-a\alpha+b=0$

$\alpha^2+b\alpha-a=0$

$$\frac{\alpha^2}{\begin{vmatrix}-a & b\\ b & -a\end{vmatrix}}=\frac{-\alpha}{\begin{vmatrix}1 & b\\ 1 & -a\end{vmatrix}}=\frac{1}{\begin{vmatrix}1 & -a\\ 1 & b\end{vmatrix}}$$

$$\Rightarrow \frac{\alpha^2}{a^2-b^2}=\frac{-\alpha}{-a-b}=\frac{1}{b+a}$$

$$\frac{\alpha^{\cancel{2}}}{a^2-b^2}=\frac{\cancel{\alpha}}{a+b}$$

$$\alpha=\frac{\cancel{a^2-b^2}}{\cancel{a+b}}\ a-b$$

$$\frac{\alpha}{a+b}=\frac{1}{a+b}$$

$$\alpha=\frac{\cancel{a+b}}{\cancel{a+b}}=1$$

$a-b=1$

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QUADRATIC EQUATION

Q Eq^n $ax^2+bx+c=0$ & $cx^2+bx+a=0$ has one com Root find cond^n?

↳ let 1 com. Root is α.

$a\alpha^2+b\alpha+c=0$

$c\alpha^2+b\alpha+a=0$

$$\frac{\alpha^2}{\begin{vmatrix} b & c \\ b & a \end{vmatrix}} = \frac{-\alpha}{\begin{vmatrix} a & c \\ c & a \end{vmatrix}} = \frac{1}{\begin{vmatrix} a & b \\ c & b \end{vmatrix}}$$

$$\frac{\alpha^2}{ab-bc} = \frac{-\alpha}{a^2-c^2} = \frac{1}{ab-bc}$$

$$\begin{matrix} a & & b & & c \\ c & & b & & a \end{matrix}$$

$(ab-bc)(ab-bc)=(a^2-c^2)^2$

$b^2\cancel{(a-c)^2}=(a+c)^2\cancel{(a-c)^2}$

$(a+c)^2=b^2$

$(a+c)^2-b^2=0$

$(a+c-b)(a+c+b)=0$

$a+c-b=0$ or $a+c+b=0$

$$\frac{-\alpha}{a^2-c^2}=\frac{1}{b(a-c)} \Rightarrow \alpha=-\frac{\cancel{(a^2-c^2)}}{b\cancel{(a-c)}}=\frac{-(a+c)}{b}$$

$$\frac{\alpha^2}{b(a-c)}=\frac{-\alpha}{(a^2-c^2)}$$

$$\alpha=-\frac{b\cancel{(a-c)}}{\cancel{(a^2-c^2)}_{a+c}} \Rightarrow \alpha=\frac{-b}{a+c}$$

$$-\frac{(a+c)}{b}=-\frac{b}{(a+c)}$$

$(a+c)^2=b^2 \Rightarrow (a+c)^2-b^2=0$

$\Rightarrow (a+c-b)=0$ or $(a+c+b)=0$

QUADRATIC EQUATION

Q If $ax^2+bx+c=0$ & $bx^2+cx+a=0$ have 1 Com. Root then $\frac{a^3+b^3+c^3}{abc}=?$

$$a^3+b^3+c^3-3abc=(a+b+c)(a^2+b^2+c^2-ab-bc-ca)$$

$$\begin{array}{ccc} a & b & c \\ b & c & a \end{array}$$

$$(ac-b^2)(ab-c^2)=(a^2-bc)^2$$

$$a^2bc-ab^3-ac^3+\cancel{b^2c^2}=a^4+\cancel{b^2c^2}-2a^2bc$$

a com. लेकर cancel

$$abc-b^3-c^3=a^3-2abc$$

$$3abc=a^3+b^3+c^3 \Rightarrow \frac{a^3+b^3+c^3}{abc}=3$$

Q If $x^2+bx+c=0$ & $bx^2+cx+1=0$ have One Com. Root then P.T. $b+c+1=0$

or $b^2+c^2+1=b+c+bc$

$$\begin{array}{ccc} 1 & b & c \\ b & c & 1 \end{array}$$

$$(c-b^2)(b-c^2)=(1-bc)^2$$

$$bc-b^3-c^3+\cancel{b^2c^2}=1+\cancel{b^2c^2}-2bc$$

$$3bc=b^3+c^3+1^3$$

$$\Rightarrow b^3+c^3+1^3-3\cdot1\cdot b\cdot c=0$$

$$(b+c+1)(b^2+c^2+1^2-bc-c-b)=0$$

$$b+c+1=0 \text{ or } b^2+c^2+1=bc+b+c$$

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QUADRATIC EQUATION

Q If $x^2+px+q=0$ & $x^2+qx+p=0$ $(p\neq q)$ have 1 Com. Root then Sh. That $\boxed{1+p+q=0}$ & also show that Uncommon Roots are Roots of $x^2+x+pq=0$.

$$x^2+px+q=0 \xrightarrow{x=1} 1+p+q=0$$

$$x^2+qx+p=0$$

$$p+q=-1$$

$$(p-q)x+(q-p)=0$$

$$(p-q)x=(p-q)$$

$$x=\frac{p-q}{p-q}=1 \Rightarrow \text{Com.}$$

$x^2+px+q=0$ — roots β, 1

$x^2+qx+p=0$ — roots 1, γ

Uncomm. Root

$\beta\cdot 1=q \Rightarrow \beta=q$

$\gamma\cdot 1=p \Rightarrow \gamma=p$

$$x^2-(\beta+\gamma)x+\beta\gamma=0$$

$$x^2-(p+q)x+p\cdot q=0$$

$$x^2-(-1)x+pq=0$$

$$x^2+x+pq=0$$

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QUADRATIC EQUATION

Quad Eqⁿ of 2 Variable.

1) $f(x) = ax^2 + bx + c$ q Eqⁿ in x

$f(x, y)$ में Q Eqⁿ maang lito?

2) $f(x, y) = ax^2 + by^2 + 2hxy + 2gx + 2fy + c = 0$

is known as Q Eqⁿ in 2 Variable.

3) It is Product of 2 Linear Eqⁿ

$(a_1x + b_1y + c_1)(a_2x + b_2y + c_2) = ax^2 + by^2 + 2hxy + 2gx + 2fy + c$

(4) We will study how to Resolve Q Eqⁿ in 2 variable into 2 Linear Eqⁿ

QUADRATIC EQUATION

Ex L, 2

Q find Linear factors of

$$(x^2-3xy+2y^2)-2x-3y-35=0$$

1) Resolve Q. Part first

$$(x-2y)(x-y)$$

$\therefore$ L factor = $(x-2y-7)$
$(x-y+5)$

2) Add constant and compare

$$(x-2y+c_1)(x-y+c_2) = \cancel{x^2-3xy+2y^2}-2x-3y-35$$

$c_1=-7$
$c_2=5$

$$x^2\cancel{-3xy+2y^2}+c_1x+c_2x-c_1y-2c_2y+c_1c_2$$

$$(c_1+c_2)x-y(c_1+2c_2)+c_1c_2 = -2x-3y-35$$

$c_1+c_2=-2$	$c_1+2c_2=3$	$c_1c_2=-35$
$-c_1 \pm 2c_2=-3$	$c_1=-7$	$-7\times5=-35$
$-c_2=-5$		