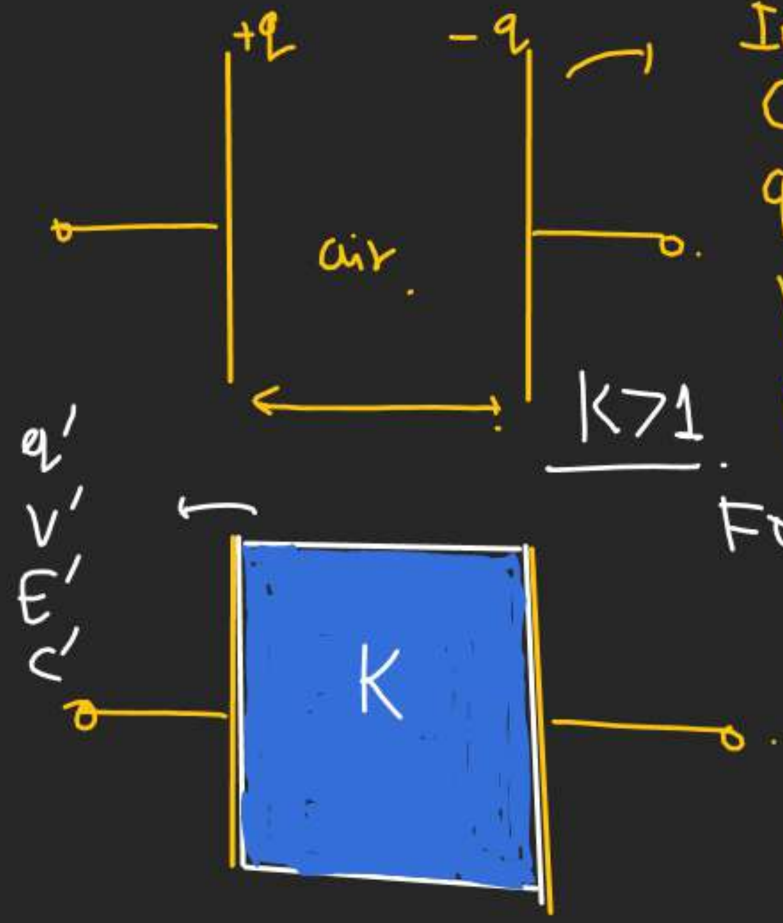
Nishant Jindal

Energy analysis

CAPACITOR

(*) Inserting a dielectric Slab in a parallel plate Capacitor:-

Case-1 :- Inserting in a isolated Capacitor:-



In air

C → Capacitance

q → Charge.

V → Potential difference.

E → Electric field.

For Isolated Capacitor

$\underline{q = \text{constant}}$

→ $q' = q$

→ $C' = KC$

→ $E' = \dfrac{q}{\varepsilon_m A} = \dfrac{q}{K\varepsilon_0 A} = \left(\dfrac{E}{K}\right)$

→ $V' = \dfrac{V}{K}$

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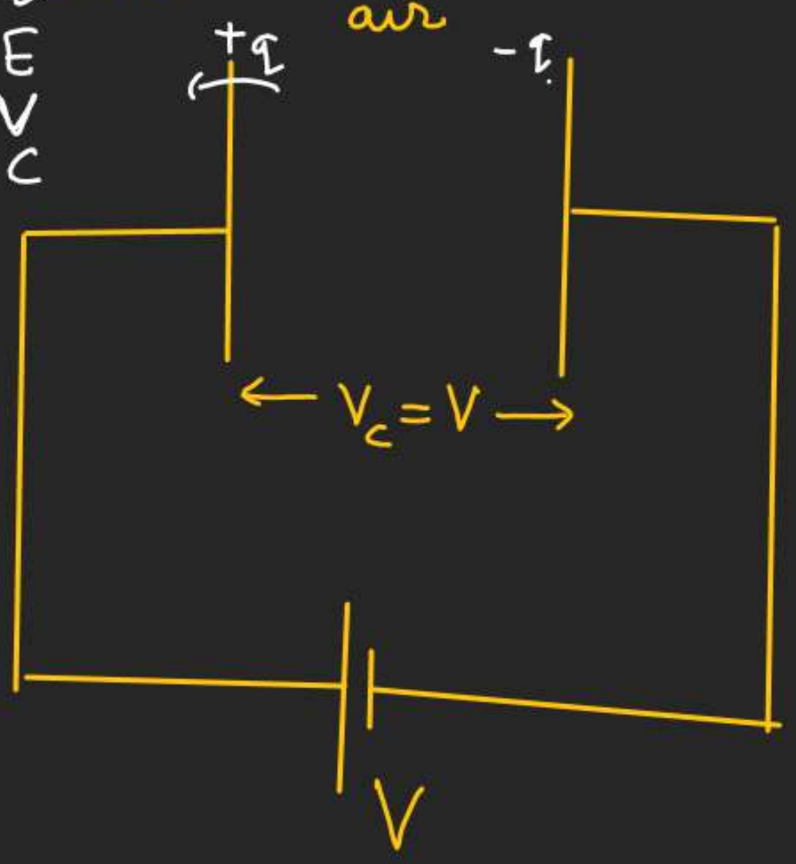
Case-2 :- Inserting dielectric slab when battery is connected :-

$q = CV$

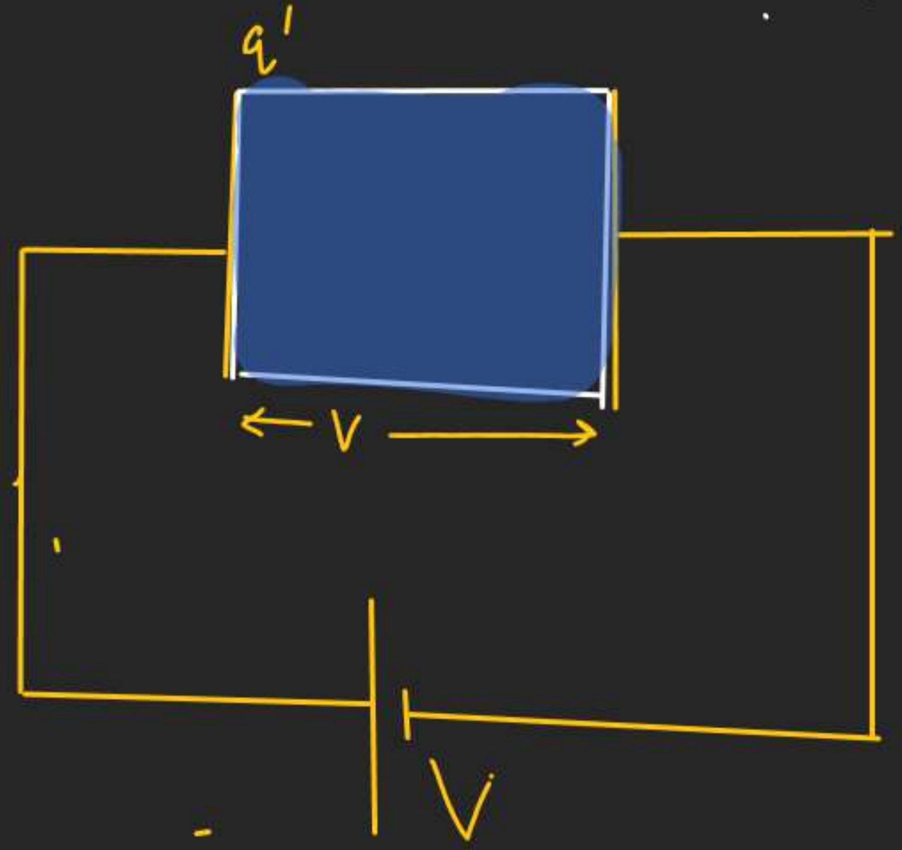
E

V

C



[If Capacitor is connected to battery then potential across the capacitor is equal to potential of battery]



$\boxed{C' = KC}$ ✓

$q' = C'V$

$q' = KCV$

$\boxed{q' = Kq}$ ✓

$V = E'd$

$E' = \frac{V}{d} = E$

$E' = \frac{q'}{\varepsilon_m A} = \frac{\cancel{K}q}{\cancel{K}\varepsilon_0 A} = E$

$W_{battery}$

$= (\Delta q)V$

$= (q_f - q_i)V$

$= (Kq - q)V$

$= (K-1)qV$

$= (K-1)CV^2$

Energy analysis

CAPACITOR

$\underline{Heat} = ??$

$W_b = \Delta U + heat$

$\Downarrow$

$\underline{\Delta U} = U_f - U_i$

$= \frac{1}{2}(KC)V^2 - \frac{1}{2}CV^2$

$= (K-1)\frac{CV^2}{2}$

$W_b = \Delta U + heat$

$\downarrow$

$(K-1)CV^2 = (K-1)\frac{CV^2}{2} + heat$

$$\boxed{heat = \frac{(K-1)CV^2}{2}}$$

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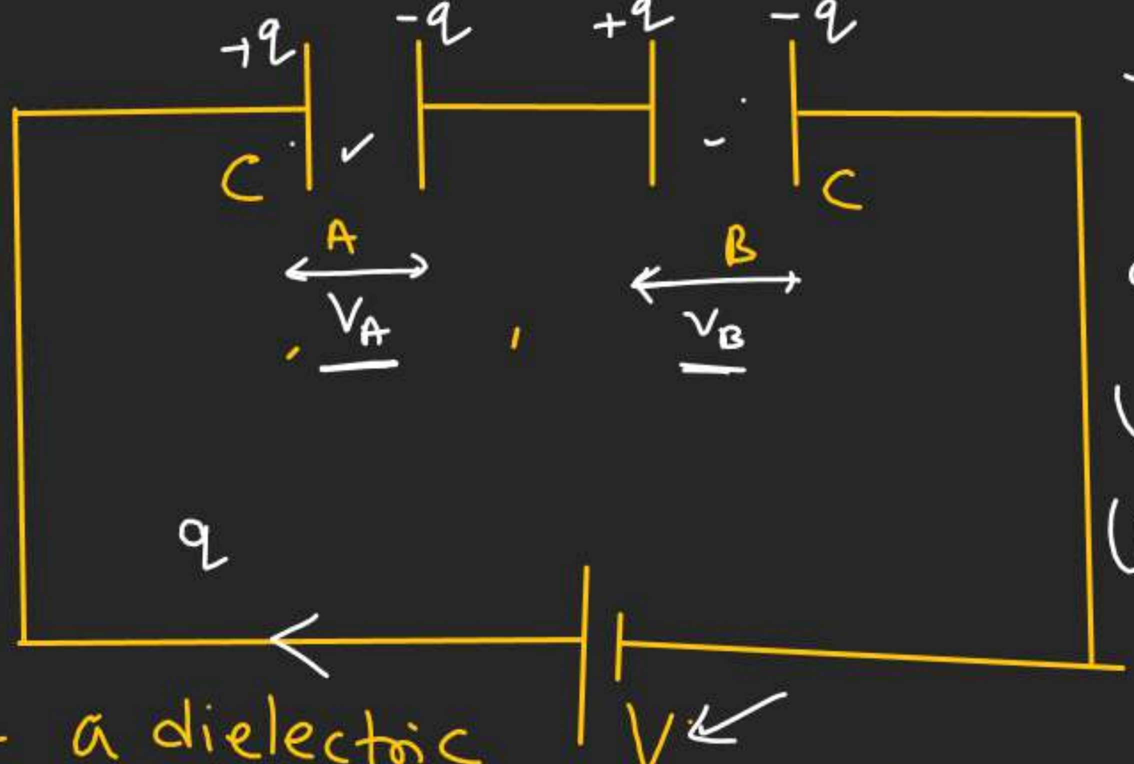
Energy analysis

CAPACITOR

\# Initially both the Capacitor A and B fully Charged.



⇒ If a dielectric having dielectric constant 'k' inserted in Capacitor B. Find heat = ??

Before dielectric

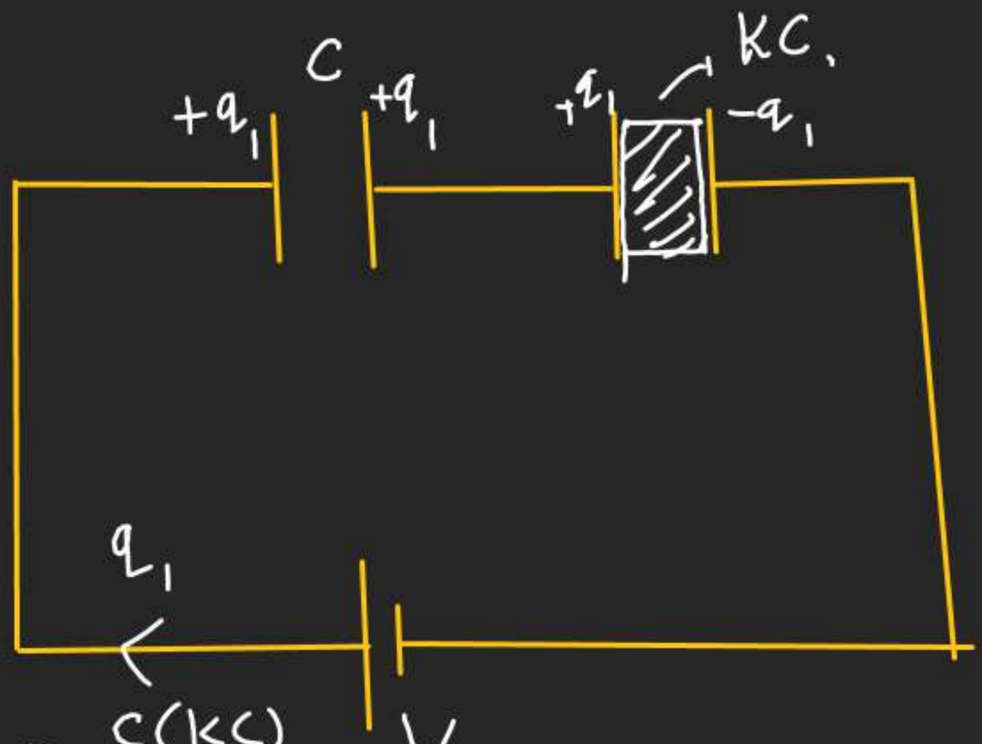
$$q = C_{eq} \cdot V$$

$$q = \left(\frac{C}{2}\right)V \quad (C_{eq})$$

$$U_i = \frac{1}{2}\left(\frac{C}{2}\right)V^2$$

$$U_i = \left[\frac{CV^2}{4}\right]$$

After dielectric.



$$\Rightarrow (C_{eq})_1 = \frac{C(KC)}{C+KC} = \left(\frac{KC}{K+1}\right)$$

$$\Rightarrow q_1 = (C_{eq})_1 \cdot V = \left(\frac{K}{K+1}\right)CV$$

$$\Rightarrow U_f = \frac{1}{2}\left(\frac{KC}{K+1}\right) \cdot V^2$$

$$U_f = \frac{K}{2(K+1)} \cdot CV^2$$

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Energy analysis

CAPACITOR

$$W_b = \Delta U + \text{heat}$$

$$\Downarrow$$

$$(q_1 - q_2)V = (U_f - U_i) + \text{heat}$$

$$V\left[\left(\frac{K}{K+1}\right)CV - \frac{CV}{2}\right] = \left[\left(\frac{K}{K+1}\right)\frac{CV^2}{2} - \frac{CV^2}{4}\right] + \text{heat}$$

$$CV^2\left[\frac{2K - K - 1}{2(K+1)}\right] - \frac{CV^2}{4}\left[\frac{2K}{K+1} - 1\right] = \text{heat}$$

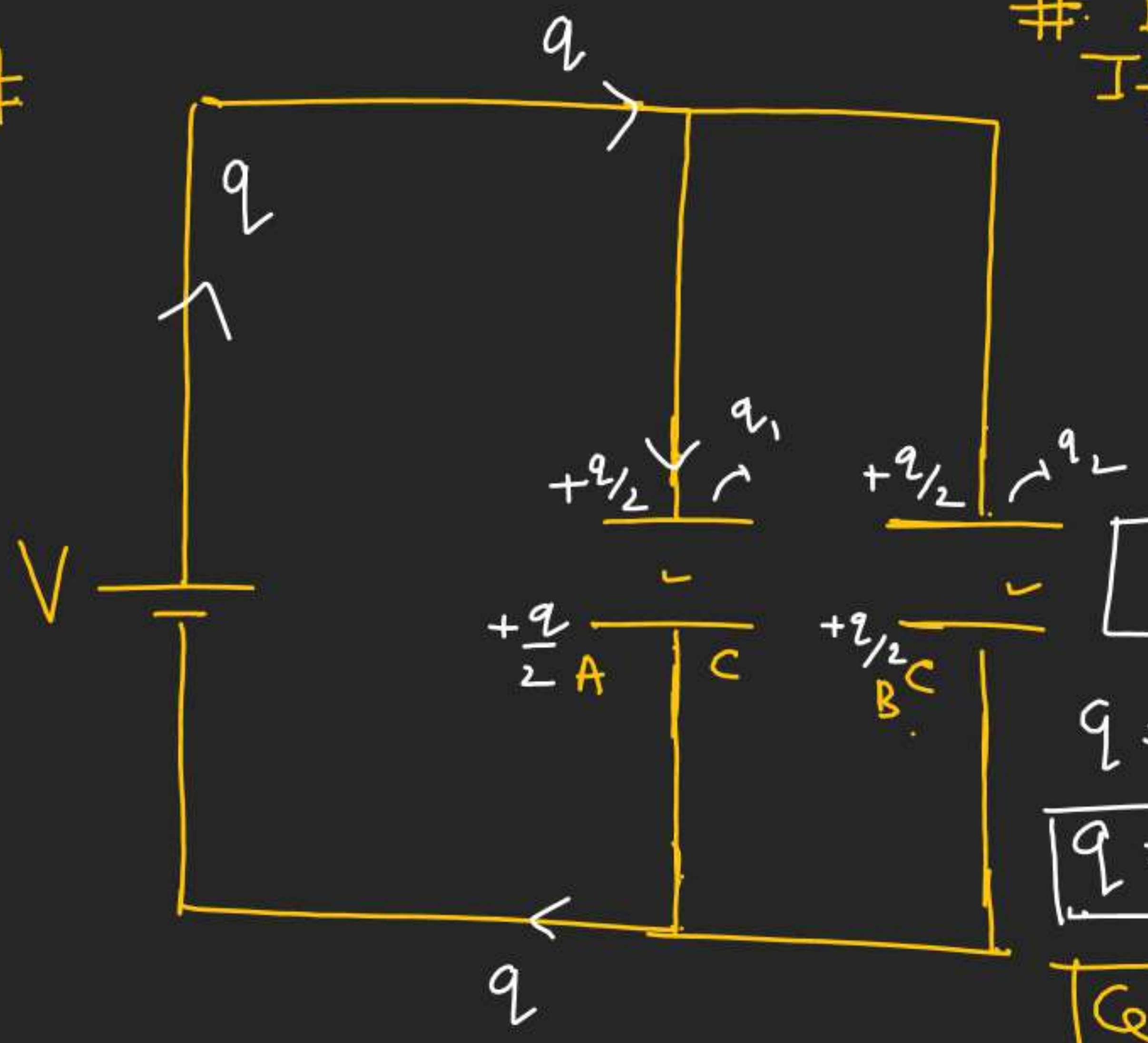
$$\frac{CV^2}{2}\left[\frac{K-1}{K+1}\right] - \frac{CV^2}{4}\left[\frac{(K-1)}{K+1}\right] = \text{heat}$$

$$\boxed{\text{heat} = \frac{CV^2}{4}\left(\frac{K-1}{K+1}\right)} \quad **$$

Energy analysis

CAPACITOR

#



#. Both the Capacitor A and B at steady state. If a dielectric having dielectric Constant 'K' inserted in B. find heat dissipiated.

$q_1 + q_2 = q$

$\frac{q_1}{c} = \frac{q_2}{c}$

$q_1 = q_2$

$\boxed{q_1 = q_2 = q/2}$

$q = C_{eq} \cdot V$

$\boxed{q = (2C)V}$

$\boxed{C_{eq} = 2C}$

$U_i = \frac{1}{2}(2C)V^2$ ← C_{eq}

$\underline{U_i = CV^2}$

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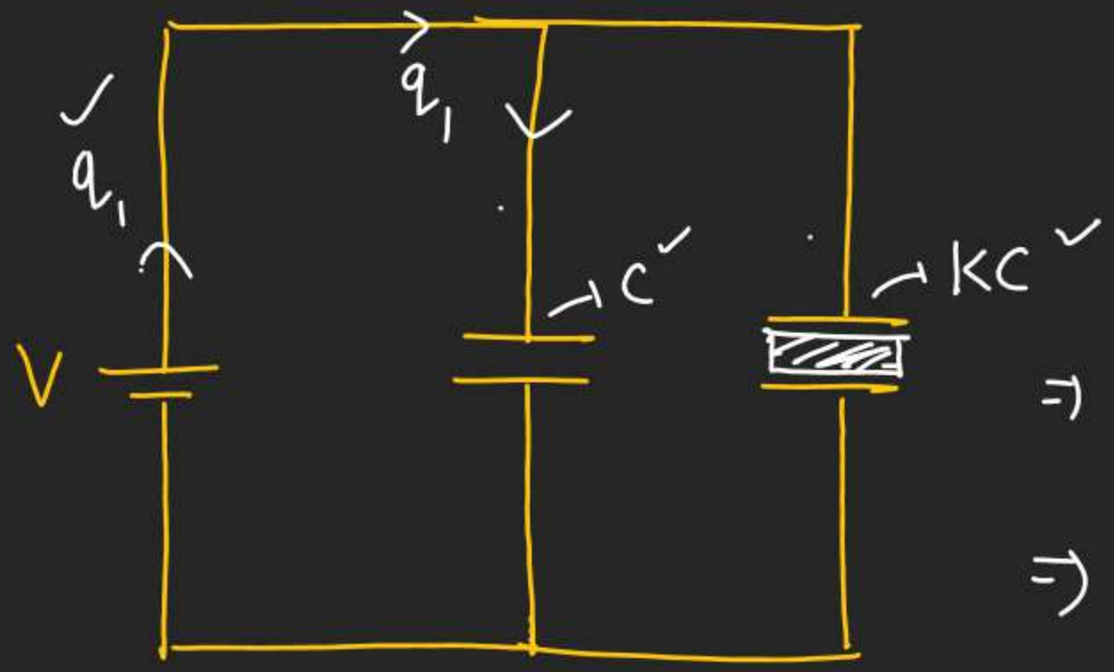
Energy analysis

CAPACITOR

After inserting dielectric



$C_{eq} = (K+1)C$

$q_1 = C_{eq} \cdot V = (K+1)CV$

$(U_f)_{system} = \frac{1}{2}(K+1)CV^2$

$W_b = \Delta U + \text{heat}$

$\Downarrow$

$(q_1 - q)V = (U_f - U_i) + \text{heat}$

$\Rightarrow \left[(K+1)CV - 2CV\right]V = \left[(K+1)\frac{CV^2}{2} - CV^2\right] + \text{heat}$

$\Rightarrow * \; (K-1)CV^2 - \frac{CV^2}{2}[K-1] = \text{heat}$

$$\boxed{(K-1)\frac{CV^2}{2} = \text{heat}}$$

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Energy analysis

CAPACITOR

H.W

Q.2 A battery of $10\ \mathrm{V}$ is connected to a capacitor of capacity $0.1\ \mathrm{F}$. The battery is now removed and this capacitor is connected to a second uncharged capacitor. If the charges are distributed equally on these two capacitors, find the total energy stored in the two capacitors. Find the ratio of final energy to initial energy stored in capacitors.

Energy analysis

H.W.

Q.3 For the circuit shown in figure, which of the following statements is true?

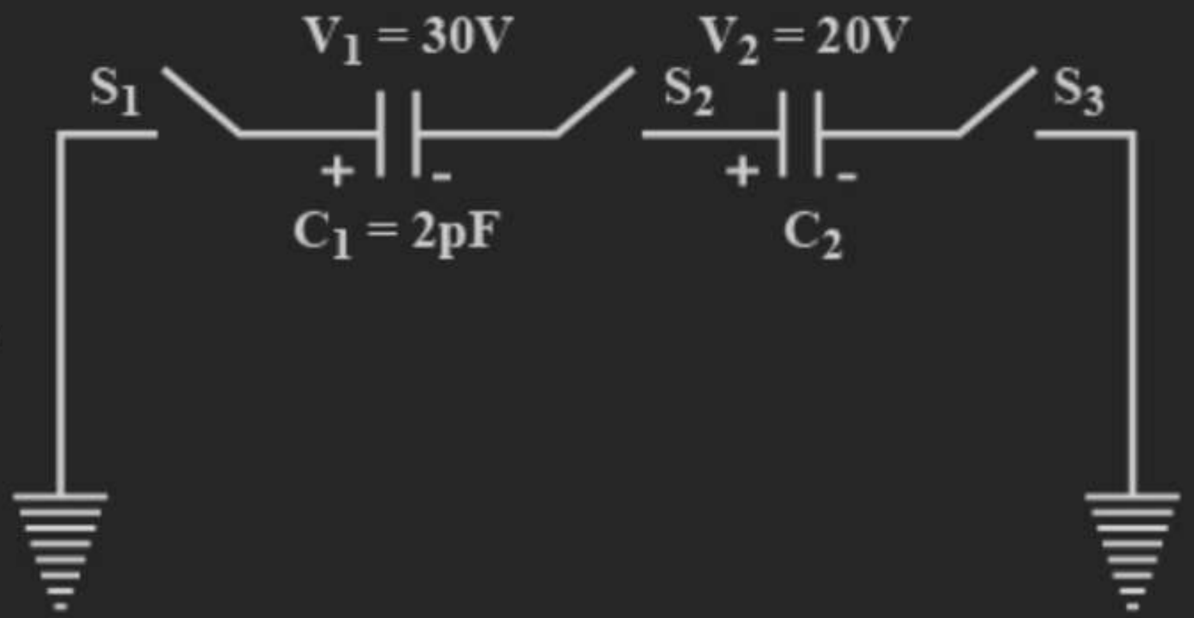
(1999)

(A) With S_1 closed, $V_1 = 15$ V, $V_2 = 20$ V

(B) With S_3 closed, $V_1 = V_2 = 25$ V

(C) With S_1 and S_2 closed, $V_1 = V_2 = 0$

(D) With S_1 and S_3 closed, $V_1 = 30$ V, $V_2 = 20$ V.



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Energy analysis HW.

CAPACITOR

Q.4 Two identical capacitors have the same capacitance C. One of them is charged to potential V_1 and the other V_2. The negative ends of the capacitors are connected together. When the positive ends are also connected, the decrease in energy of the combined system is (2002)

(A) $\frac{1}{4}C\left(V_1^2 - V_2^2\right)$

(B) $\frac{1}{4}C\left(V_1^2 + V_2^2\right)$

(C) $\frac{1}{4}C(V_1 - V_2)^2$

(D) $\frac{1}{4}C(V_1 + V_2)^2$

Energy analysis

CAPACITOR

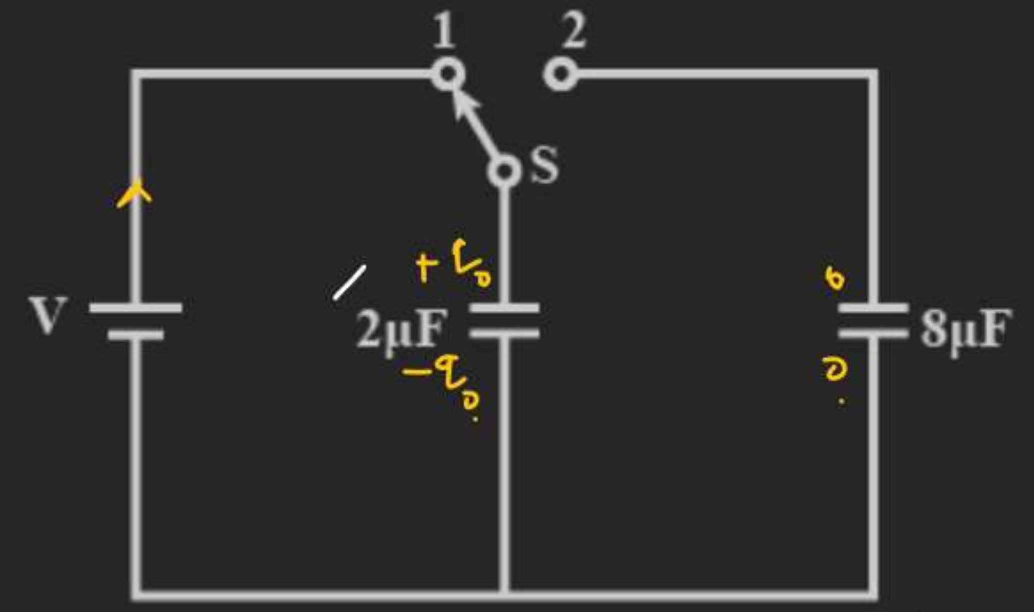
Q.5 A $2\mu F$ capacitor is charged as shown in figure. The percentage of its stored energy dissipated after the switch S is turned to position 2 is **(2011)**

(A) 0%

(B) 20%

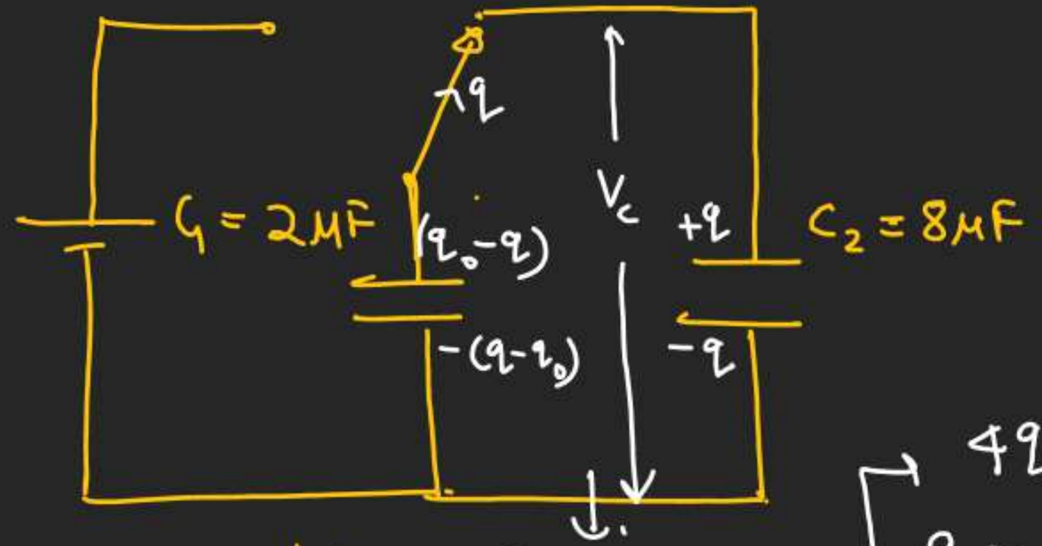
(C) 75%

(D) 80%



$q_0 = 2V$

$U_i = \frac{1}{2}\times 2\times V^2 = V^2$



$V_c \Rightarrow \frac{q_0-q}{2} = \frac{q}{8}$

$4(q_0-q) = q$

$4q_0 = 5q$

$q = \frac{4}{5}q_0 = \frac{4}{5}\times 2V$

$q = \frac{8}{5}V$

$U_f = \frac{(q_0-q)^2}{2\times 2} + \frac{q^2}{2\times 8}$

$= \left(\frac{2V}{5}\right)^2\times\frac{1}{4} + \left(8V/5\right)^2\times\frac{1}{16}$

$= \frac{V^2}{25} + \frac{4V^2}{25} = \frac{V^2}{5}$

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Energy analysis

CAPACITOR

$$U_f = \frac{V^2}{5}$$

$$U_i = V^2$$

$$|heat| = \left(V^2 - \frac{V^2}{5}\right)$$

$$= \left(\frac{4V^2}{5}\right)$$

$$\% \text{ of Energy dissipiated} = \frac{4V^2/5}{V^2} \times 100$$

$$= \underline{80\%}$$

Energy analysis

H.W.

CAPACITOR

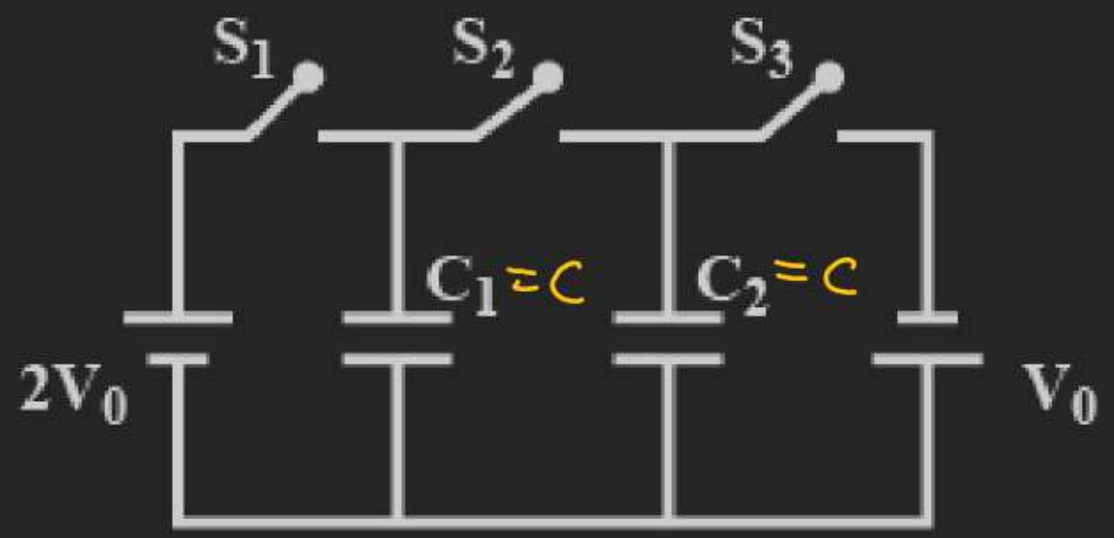
Q.6 In the circuit shown in the figure, there are two parallel plate capacitors each of capacitance C. The switch S_1 is pressed first to fully charge the capacitor C_1 and then released. The switch S_2 is then pressed to charge the capacitor C_2. After some time, S_2 is released and then S_3 is pressed. After some time, **(2013)**

(A) the charge on the upper plate of C_1 is $2CV_0$.

(B) the charge on the upper plate of C_1 is CV_0.

(C) the charge on the upper plate of C_2 is 0 .

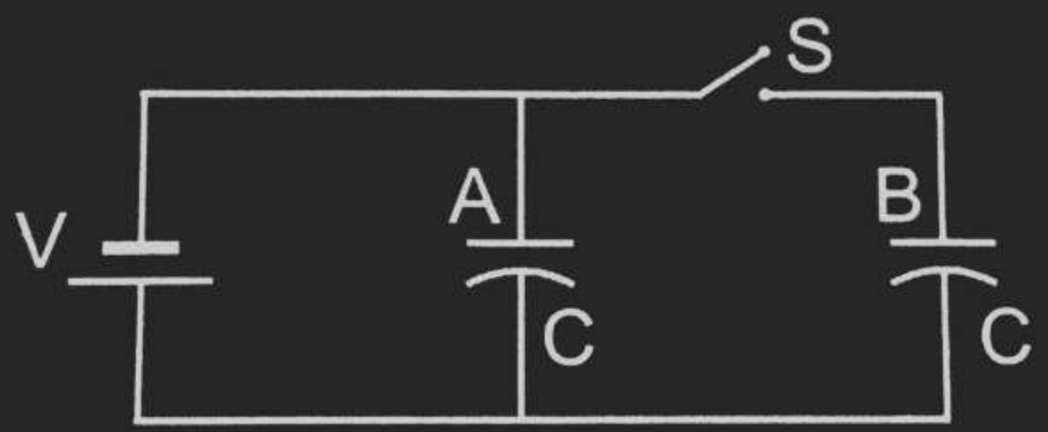
(D) the charge on the upper plate of C_2 is $-CV_0$.



Energy analysis H.W

CAPACITOR

Q.8 The figure shows two identical parallel plate capacitors to a battery with the switch S closed. The switch is now opened and the free space between the plates of the capacitors is filled with a dielectric of dielectric constant (or relative permittivity) 3. Find the ratio of the total electrostatic energy stored in both capacitors before and after the introduction of the dielectric. (1983)



Energy analysis

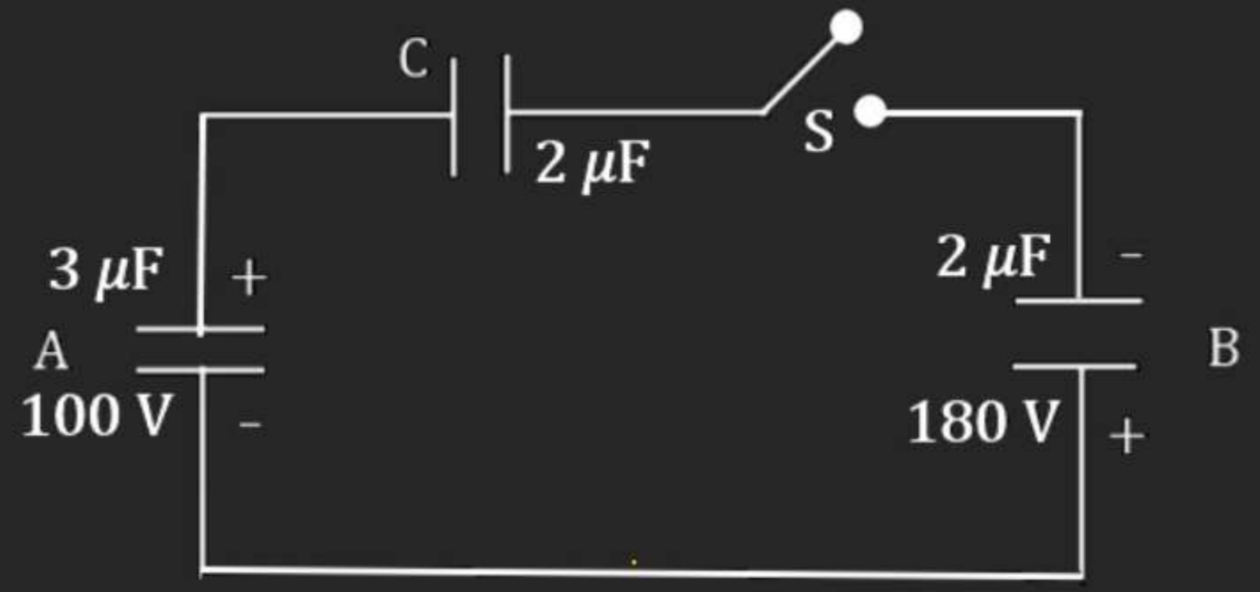
H.W

CAPACITOR

Q.10 Two capacitors A and B with capacities $3\mu F$ and $2\mu F$ are charged to a potential difference of $100\ V$ and $180\ V$ respectively. The plates of the capacitors are connected as shown in the figure with one wire from each capacitor free. The upper plate of A is positive and that of B is negative. An uncharged $2\mu F$ capacitor with lead wires falls on the free ends to complete the circuit. Calculate **(1997)**

(i) the final charge on the three capacitors and

(ii) the amount of electrostatic energy stored in the system before and after the completion of the circuit.



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Energy analysis

H.W

CAPACITOR

Q.11 Two isolated metallic solid spheres of radii R and $2R$ are charged such that both of these have same charge density σ. The spheres are located far away from each other, and connected by a thin conducting wire. Find the new charge density on the bigger sphere. (1996)

Energy analysis H.W.

CAPACITOR

Q.14 In the following circuit, $C_1 = 12\mu F, C_2 = C_3 = 4\mu F$ and $C_4 = C_5 = 2\mu F$. The charge stored in C_3 is ______ μC. **(2022)**