Nishant Jindal

$$\int \frac{(x^4+1)x}{x^4-1}\,dx$$

$$\int \frac{(t+1)\,dt}{(t^2+1)^3} = \int \frac{t\,dt}{(t^2+1)^2} + \int \frac{dt}{(t^2+1)^3}$$

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$$\int \frac{\cos x\,dx}{\sqrt{8-\sin 2x}} = \frac{1}{2}\int \frac{(\cos x+\sin x)\,dx}{\sqrt{7+(\sin x-\cos x)^2}} + \frac{1}{2}\int \frac{(\cos x-\sin x)\,dx}{\sqrt{9-(\sin x+\cos x)^2}}$$

$$= \frac{1}{2}\ln\left|(\sin x-\cos x)+\sqrt{8-\sin 2x}\right| + \frac{1}{2}\sin^{-1}\left(\frac{\sin x+\cos x}{3}\right)$$

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Integral of form

- $\int f(\alpha - x, x - \beta)\, dx \rightarrow x = \alpha\cos^2\theta + \beta\sin^2\theta$ ✓

- $\int f(x - \alpha, x - \beta)\, dx \rightarrow x = \alpha\sec^2\theta - \beta\tan^2\theta$

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$$\int \frac{dx}{(ax+b)\sqrt{px+q}} \longrightarrow px+q=t^2$$

$$\int \frac{dx}{(ax^2+bx+c)\sqrt{px+q}} \longrightarrow px+q=t^2$$

$$\int \frac{dx}{(ax+b)\sqrt{px^2+qx+r}} \longrightarrow ax+b=\frac{1}{t}$$

✓

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- $\displaystyle\int \frac{dx}{a(x-\alpha)(x-\beta)\sqrt{px^2+qx+r}} = \int \frac{A\,dx}{(x-\alpha)\sqrt{px^2+qx+r}} + \int \frac{B\,dx}{(x-\beta)\sqrt{px^2+qx+r}}$

  $x-\alpha = \frac{1}{t}$ $\qquad$ $x-\beta = \frac{1}{t}$

- $\displaystyle\int \frac{dx}{(x-\alpha)^2\sqrt{px^2+qx+r}} \longrightarrow x-\alpha = \frac{1}{t}$

- $\displaystyle\int \frac{dx}{(ax^2+b)\sqrt{px^2+q}} \longrightarrow x = \frac{1}{t}$

Trigonometric Substitutions

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1\. $$\int \frac{dx}{(x-\alpha)\sqrt{(x-\alpha)(\beta-x)}}$$

$$x = \alpha\cos^2\theta + \beta\sin^2\theta$$

$$x-\alpha = (\beta-\alpha)\sin^2\theta$$

$$\beta - x = (\beta-\alpha)\cos^2\theta$$

$$dx = 2(\beta-\alpha)\sin\theta\cos\theta\,d\theta$$

$$\frac{2}{(\alpha-\beta)}\sqrt{\frac{\beta-x}{x-\alpha}} + C = \frac{1}{\alpha-\beta}\int\frac{(\alpha-\beta)\,dx}{(x-\alpha)^2\sqrt{\frac{\beta-x}{x-\alpha}}}$$

$$\frac{\beta-\alpha}{x-\alpha} - 1$$

$$-\frac{(\beta-\alpha)}{(x-\alpha)^2}dx$$

$$= \int \frac{2(\beta-\alpha)\sin\theta\cos\theta\,d\theta}{(\beta-\alpha)\sin^2\theta\,(\beta-\alpha)\sin\theta\cos\theta}$$

$$= \frac{2}{(\beta-\alpha)}\int \operatorname{cosec}^2\theta\,d\theta = -\frac{2}{(\beta-\alpha)}\cot\theta + C$$

$$= -\frac{2}{(\beta-\alpha)}\sqrt{\frac{\beta-x}{x-\alpha}} + C$$

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2. $\displaystyle\int \frac{dx}{(x+1)\sqrt{1+x-x^2}}$

$x+1 = \frac{1}{t}$

$dx = -\frac{1}{t^2}dt$

$$= \int \frac{-\frac{dt}{t^2}}{\frac{1}{t}\sqrt{\frac{1}{t}-\left(\frac{1}{t}-1\right)^2}} = \int \frac{-dt}{\sqrt{3t-1-t^2}}$$

$\frac{3}{t}-\frac{1}{t^2}-1$

$$= -\int \frac{dt}{\sqrt{\frac{5}{4}-\left(t-\frac{3}{2}\right)^2}}$$

$$= -\sin^{-1}\left(\frac{t-\frac{3}{2}}{\frac{\sqrt{5}}{2}}\right) + C$$

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3.
$$\int \frac{dx}{(x^2+5x+2)\sqrt{x-2}} \qquad x-2=t^2$$

$$= \int \frac{2t\,dt}{(t^4+4t^2+4+5t^2+10+2)\,t} = \int \frac{\frac{2}{t^2}\,dt}{t^2+9+\frac{16}{t^2}} \qquad t,\ \frac{4}{t}$$

$$= \frac{1}{4}\int \frac{\left(1+\frac{4}{t^2}\right)dt}{\left(t-\frac{4}{t}\right)^2+17} - \frac{1}{4}\int \frac{\left(1-\frac{4}{t^2}\right)dt}{\left(t+\frac{4}{t}\right)^2+1}$$

$$= \frac{1}{4\sqrt{17}}\tan^{-1}\left(\frac{t-\frac{4}{t}}{\sqrt{17}}\right) - \frac{1}{4}\tan^{-1}\left(t+\frac{4}{t}\right)+C.$$

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4. $\displaystyle\int \frac{dx}{(x^2-x-2)\sqrt{x^2+x+1}}$.

$$= \frac{1}{3}\int \frac{dx}{(x-2)\sqrt{x^2+x+1}} - \frac{1}{3}\int \frac{dx}{(x+1)\sqrt{x^2+x+1}}$$

$\downarrow$ $x-2=\frac{1}{t}$

$\downarrow$ $x+1=\frac{1}{t}$

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5.

$$\int \frac{x^2\,dx}{(x\sin x+\cos x)^2} = \int \frac{x\cos x \quad x \quad dx}{\cos x\,(x\sin x+\cos x)^2}$$

II

$$= -\frac{x}{(x\sin x+\cos x)\cos x} + \int \frac{\left(\cos x - \cancel{x}(-\sin x)\right)dx}{\cancel{(x\sin x+\cos x)}\cos^2 x}$$

$$= -\frac{x}{(x\sin x+\cos x)\cos x} + \tan x + C$$

$$\tan\left(x - \tan^{-1} x\right) + C$$

$$= \frac{-x + \sin x(x\sin x+\cos x)}{(x\sin x+\cos x)\cos x} + C = \frac{\sin x\cos x - x\cos^2 x}{(x\sin x+\cos x)\cos x} + C$$

$$= \frac{\tan x - x}{x\tan x + 1} + C.$$

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6.
$$\int \frac{dx}{(\sin x + 2\sec x)^2} = \int \frac{\sec^2 x\, dx}{(\tan x + 2 + 2\tan^2 x)^2}$$

$$\frac{1}{4\times 2}\int \frac{2t\, dt}{\left(t^2 + \frac{15}{16}\right)^2 t} = \frac{1}{4}\int \frac{\sec^2 x\, dx}{\left(\left(\tan x + \frac{1}{4}\right)^2 + \frac{15}{16}\right)^2}$$

$$\tan x + \frac{1}{4} = t$$

$$\tan x + \frac{1}{4} = \frac{\sqrt{15}}{4}\tan\theta$$

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$$\left.\begin{matrix} x\sin x \pm \cos x \\ \\ x\cos x \pm \sin x \end{matrix}\right\rangle \text{Put } x = \tan\theta$$

$x = \tan\theta$

$$\int \frac{x^2\,dx}{(x\sin x + \cos x)^2} = \int \frac{\tan^2\theta\,\sec^2\theta\,d\theta}{\left(\frac{\sin\theta}{\cos\theta}\sin(\tan\theta) + \cos(\tan\theta)\right)^2}$$

$$= \int \frac{\tan^2\theta\,d\theta}{\cos^2(\tan\theta - \theta)} = \tan(\tan\theta - \theta) + C$$

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7. $\int \sqrt[3]{\frac{1-x}{1+x}}\,\frac{dx}{x}$

$$\frac{1-x}{1+x} = t^3$$

$$x = \frac{1-t^3}{1+t^3} = \frac{2}{1+t^3} - 1$$

$$dx = \frac{-6t^2\,dt}{(1+t^3)^2}$$

$$\int \frac{-6t^3\,dt}{(1+t^3)(1-t^3)} = -3\int\left(\frac{1}{1-t^3} - \frac{1}{1+t^3}\right)dt$$

$t^2 - (t^2 - 1)$

2068 to 2075

Tuesday

2090 – 2131

2175 – 2230

Thursday

Wed → 2076, 2079, 2081, 2085, 2087, 2088, 2089