Nishant Jindal

Product of 2 Determinants

1) $\boxed{R\times R,\ R\times C}$, $C\times R$, $C\times C$ all 4 ways Psbl

↳ mainly

$R\times C$

2) $\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} \times \begin{vmatrix} l_1 & m_1 \\ l_2 & m_2 \end{vmatrix}$

$\begin{vmatrix} a_1 l_1 + b_1 l_2 & a_1 m_1 + b_1 m_2 \\ a_2 l_1 + b_2 l_2 & a_2 m_1 + b_2 m_2 \end{vmatrix}$

(3) $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \times \begin{vmatrix} l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ l_3 & m_3 & n_3 \end{vmatrix}$

$= \begin{vmatrix} a_1 l_1 + b_1 l_2 + c_1 l_3 & a_1 m_1 + b_1 m_2 + c_1 m_3 & a_1 n_1 + b_1 n_2 + c_1 n_3 \\ a_2 l_1 + b_2 l_2 + c_2 l_3 & a_2 m_1 + b_2 m_2 + c_2 m_3 & a_2 n_1 + b_2 n_2 + c_2 n_3 \\ a_3 l_1 + b_3 l_2 + c_3 l_3 & a_3 m_1 + b_3 m_2 + c_3 m_3 & a_3 n_1 + b_3 n_2 + c_3 n_3 \end{vmatrix}$

Q $\begin{vmatrix} 1 & -2 & 4 \\ 5 & 0 & -6 \\ -3 & 7 & 1 \end{vmatrix} \times \begin{vmatrix} 6 & -1 & 3 \\ -4 & 2 & 8 \\ 0 & -9 & 5 \end{vmatrix}$

$$= \begin{vmatrix} 6+8+0 & -1-4-36 & 3-16+20 \\ 30+0+0 & -5+0+54 & 15+0-30 \\ -18-28+0 & 3+14-9 & -9+56+5 \end{vmatrix}$$

$$= \begin{vmatrix} 14 & -41 & 7 \\ 30 & 49 & -15 \\ -46 & 8 & 52 \end{vmatrix}$$

Nishant Jindal

ULLi Soch Apply

Q — Jee Adv 2015

Which of the following values of α satisfy Eqn.

$$\begin{vmatrix} (1+\alpha)^2 & (1+2\alpha)^2 & (1+3\alpha)^2 \\ (2+\alpha)^2 & (2+2\alpha)^2 & (2+3\alpha)^2 \\ (3+\alpha)^2 & (3+2\alpha)^2 & (3+3\alpha)^2 \end{vmatrix} = -648\alpha$$

$-4 \quad 9 \quad -9 \quad 4$

$$\Rightarrow \begin{vmatrix} 1+2\alpha+\alpha^2 & 1+4\alpha+4\alpha^2 & 1+6\alpha+9\alpha^2 \\ 4+4\alpha+\alpha^2 & 4+8\alpha+4\alpha^2 & 4+12\alpha+9\alpha^2 \\ 9+6\alpha+\alpha^2 & 9+12\alpha+4\alpha^2 & 9+18\alpha+9\alpha^2 \end{vmatrix} = \begin{vmatrix} 1 & \alpha & \alpha^2 \\ 4 & 2\alpha & \alpha^2 \\ 9 & 3\alpha & \alpha^2 \end{vmatrix} \times \begin{vmatrix} 1 & 1 & 1 \\ 2 & 4 & 6 \\ 1 & 4 & 9 \end{vmatrix}$$

$$\{(2\alpha^3+9\alpha^3+12\alpha^3)-(18\alpha^3+\alpha^3+4\alpha^3)\}$$

$$\{(36+6+8)-(4+24+18)\}$$

$$\Rightarrow (-2\alpha^3)\times 4 = +648\alpha$$

$$\alpha^2 = 81 \Rightarrow \alpha = 9, -9$$

Q If $\alpha, \beta \neq 0$ $f(n) = \alpha^n + \beta^n$

$$\begin{vmatrix} 3 & 1+f(1) & 1+f(2) \\ 1+f(1) & 1+f(2) & 1+f(3) \\ 1+f(2) & 1+f(3) & 1+f(4) \end{vmatrix}$$

$= K(1-\alpha)^2(1-\beta)^2(\alpha-\beta)^2$ then $K =$

(1) 1 $\quad$ -1 $\quad$ $\alpha\beta$ $\quad$ $\frac{1}{\alpha\beta}$

$\{ \times 1 \to Q3$

$f(n) = \alpha^n + \beta^n$

$f(1) = \alpha^1 + \beta$

$f(2) = \alpha^2 + \beta^2$

$f(3) = \alpha^3 + \beta^3$

$$\begin{vmatrix} 1+1+1 & 1+\alpha+\beta & 1+\alpha^2+\beta^2 \\ 1+\alpha+\beta & 1+\alpha^2+\beta^2 & 1+\alpha^3+\beta^3 \\ 1+\alpha^2+\beta^2 & 1+\alpha^3+\beta^3 & 1+\alpha^4+\beta^4 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 1 & 1 \\ 1 & \alpha & \beta \\ 1^2 & \alpha^2 & \beta^2 \end{vmatrix} \times \begin{vmatrix} 1 & 1 & 1^2 \\ 1 & \alpha & \alpha^2 \\ 1 & \beta & \beta^2 \end{vmatrix}$$

$$= (1-\alpha)^2(\alpha-\beta)^2(1-\beta)^2$$

$\therefore K = 1$

Nishant Jindal

Q9 $\begin{vmatrix} \sin 2\alpha & \sin(\alpha+\beta) & \sin(\alpha+\gamma) \\ \sin(\beta+\alpha) & \sin(2\beta) & \sin(\gamma+\beta) \\ \sin(\gamma+\alpha) & \sin(\gamma+\beta) & \sin 2\gamma \end{vmatrix} = ?$

$$= \begin{vmatrix} \sin\alpha\cos\alpha+\sin\alpha\cos\alpha & \sin\alpha\cos\beta+\cos\alpha\sin\beta & \sin\alpha\cos\gamma+\cos\gamma\sin\alpha \\ \sin\beta\cos\alpha+\cos\beta\sin\alpha & \sin\beta\cos\beta+\sin\beta\cos\beta & \sin\gamma\cos\beta+\cos\gamma\sin\beta \\ \sin\gamma\cos\alpha+\cos\gamma\sin\alpha & \sin\gamma\cos\beta+\cos\gamma\sin\beta & \sin\gamma\cos\gamma+\cos\gamma\sin\alpha \end{vmatrix}$$

$$= \begin{vmatrix} \sin\alpha & \cos\alpha & K \\ & & \\ & & \end{vmatrix} \times \begin{vmatrix} \cos\alpha & \cos\beta & \cos\gamma \\ \sin\alpha & \sin\beta & \sin\gamma \\ 0 & 0 & 0 \end{vmatrix}$$

$$\sim \times 0 = 0$$

Nishant Jindal

System of Linear Eq^n.

$ax = b$; $x = \frac{b}{a}$

- $a = 0$
  - $0 \cdot x = b$
    - $b = 0$
      - $0 \cdot x = 0$
      - ∞ Sol.
    - $b \neq 0$
      - $0 \cdot x = 5$
      - No sol.
      - Inconsistent
- $a \neq 0$; $x = \frac{b}{a}$
  - $b = 0$
    - $x = \frac{0}{5}$
    - $x = 0$
    - Trivial Sol.
  - $b \neq 0$
    - $x \neq 0$
    - x — other than Zero
    - Non Trivial sol

System of Linear Eq^n.

- Consistent Sol
  - Unique Sol.
    - all variable Zero (Trivial)
    - at least one $\neq 0$ then Non Trivial.
  - ∞ Sol.
- Inconsistent

Nishant Jindal

2 Variables

Use Staright Line → $a_1x+b_1y=c_1$, $a_2x+b_2y=c_2$

others than origin → Non Trivial

origin पर होगा $x=0, y=0$ Trivial

Intersecting

$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

1 Sol. / Unq Sol.

Consistent

$\parallel$ rd

$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

No sol.

Inconsystent

Coincident

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

∞ Sol.

Consistent

Nishant Jindal

3 Variable → Planes

$a_1x+b_1y+c_1z=d_1$

$a_2x+b_2y+c_2z=d_2$

$a_3x+b_3y+c_3z=d_3$

4 Things findout

(1) $D=\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$

(2) $D_1=\begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix}$

(3) $D_2=\begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix}$

(4) $D_3=\begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix}$

$\boxed{x=\frac{D_1}{D},\ y=\frac{D_2}{D},\ z=\frac{D_3}{D}}$

$\boxed{D\cdot x=D_1},\ D\cdot y=D_2,\ D\cdot z=D_3$

$x=\frac{D_1}{D}$

$D=0$

$0\cdot y=D_2,\ 0\cdot x=D_1,\ 0\cdot z=D_3$

- $D_1=0=D_2=D_3$

  $0\cdot x=0,\ 0\cdot y=0,\ 0\cdot z=0$

  ∞ Sol

- D_1, D_2, D_3 not all Zero

  एकभी बागी

  $0\cdot y=5,\ 0\cdot x=0,\ 0\cdot z=0$

  No sol.

$D\neq 0$ $\left(x=\frac{D_1}{D}, y=\frac{D_2}{D}, z=\frac{D_3}{D}\right)$

- $D_1=D_2=D_3=0$

  $x=\frac{0}{D}, y=\frac{0}{D}, z=\frac{0}{D}$

  $x=0=y=z$

  (x,y,z)

  $(0,0,0)$

  Trivial Sol.

- $D=5$

  D_1, D_2, D_3 not all Zero

  $x=\frac{2}{5}, y=\frac{3}{5}, z=\frac{4}{6}$

  not all Zero

  Non Trivial

Nishant Jindal

Sol → Cons, Incons

Cons → Unq, ∞

Unq → Triv., Non Tri

$D\cdot x = D_1$, $D\cdot y = D_2$

$0\cdot x = 0$ $0\cdot y = 0$

∞ Sol.

Q $x + 2y = 3$.
$2x + 4y = 6$ find sol.?

$$D = \begin{vmatrix} 1 & 2 \\ 2 & 4 \end{vmatrix} = 0$$

$$D_1 = \begin{vmatrix} 3 & 2 \\ 6 & 4 \end{vmatrix} = 0$$

$$D_2 = \begin{vmatrix} 1 & 3 \\ 2 & 6 \end{vmatrix} = 0$$

Let $x = t$, $y = \frac{3-t}{2}$ $\Rightarrow \boxed{(x,y) = \left(t, \frac{3-t}{2}\right)}$; $t \in R$ → $\frac{9 - x^2}{y} = ?$

$$\frac{\cancel{9 - t^2}^{(3+t)}}{\cancel{(3-t)}} \times 2 = 6 + 2t$$

Q $2x + y = 3$.
$4x + 2y = 5$ Sol.?

$$D = \begin{vmatrix} 2 & 1 \\ 4 & 2 \end{vmatrix} = 4 - 4 = 0$$

$$D_1 = \begin{vmatrix} 3 & 1 \\ 5 & 2 \end{vmatrix} = 6 - 5 = 1$$

$$D_2 = \begin{vmatrix} 2 & 3 \\ 4 & 5 \end{vmatrix} = 10 - 12 = -2$$

$D\cdot x = D_1$, $D\cdot y = D_2$

$0\cdot x = 1$ $0\cdot y = -2$

$x, y =$ No Sol.

Incons.

Nishant Jindal

Q Solve Eq^n

$$5x - 7y + z = 11$$
$$6x - 8y - z = 15$$
$$3x + 2y - 6z = 7$$

$$D = \begin{vmatrix} 5 & -7 & 1 \\ 6 & -8 & -1 \\ 3 & 2 & -6 \end{vmatrix} = 55$$

$$D_1 = \begin{vmatrix} 11 & -7 & 1 \\ 15 & -8 & -1 \\ 7 & 2 & -6 \end{vmatrix} = 55$$

$$D_2 \begin{vmatrix} 5 & 11 & 1 \\ 6 & 15 & -1 \\ 3 & 7 & -6 \end{vmatrix} = -55$$

$$D_3 = \begin{vmatrix} 5 & -7 & 11 \\ 6 & -8 & 15 \\ 3 & 2 & 7 \end{vmatrix} = -55$$

$D \cdot x = D_1$, $D \cdot y = D_2$, $D \cdot z = D_3$

$55\,x = 55$ | $55 \cdot y = -55$ | $55 \cdot z = -55$

$x = 1$, $y = -1$, $z = -1$

$(x, y, z) = (1, 1, -1)$

Non Trivial
uniq
Consistent

Nishant Jindal

Q Solve Eqn. $x+y+z=1$

$x+2y+3z=4$

$x+3y+5z=7$

$$D=\begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & 5 \end{vmatrix}=0$$

$$D_1=\begin{vmatrix} 1 & 1 & 1 \\ 4 & 2 & 3 \\ 7 & 3 & 5 \end{vmatrix}=0$$

$$D_2=\begin{vmatrix} 1 & 1 & 1 \\ 1 & 4 & 3 \\ 1 & 7 & 5 \end{vmatrix}=0$$

$(10+3+3)-(2+9+5)$

$$D_3=\begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 3 & 7 \end{vmatrix}=0$$

$D.x=D_1$, $D.y=D_2$, $D.z=D_3$

$0.x=0$ $\quad 0.y=0$ $\quad 0.z=0$

$(x,y,z)\to\infty$ Sol.

Q2 find sol. $x=t$

$y+z=1-t$ $\times 2 \to 2y+2z=2-2t$

$2y+3z=4-t$ $\quad \to 2y+3z=4-t$

$-z=-2-t$

$y=1-t-2-t=-1-2t$

$(x,y,z)=(t,-1-2t,+2+t)$

$t\in R$

Nishant Jindal

Q let λ be a real No. for which system of linear Eqn

Mains

$x+y+z=6,\ 4x+\lambda y-\lambda z=2,\ 3x+2y-4z=-5$

has $\boxed{\infty \text{ many sol}}$ then λ is Root of Q Eqn

$\to D=0$

$\lambda^2-3\lambda-4=0$ ×

$9-9-4=0$

$\lambda^2+3\lambda-4=0$

$9+9-4=0$ ×

$\boxed{\lambda^2-\lambda-6=0}$

$9-3-6=0$

$\lambda^2+\lambda-6=0$

$9+3-6=0$ ×

$$\begin{vmatrix} 1 & 1 & 1 \\ 4 & \lambda & -\lambda \\ 3 & 2 & -4 \end{vmatrix}=0$$

$=(-4\lambda-3\lambda+8)-(3\lambda-2\lambda-16)=0$

$\Rightarrow -8\lambda=-24$

$\boxed{\lambda=3}$

Q If the system of line eqn

$x+y+z=5$

$x+2y+2z=6$

$x+3y+\lambda z=\mu \quad (\lambda,\mu\in R)$

has $\boxed{\infty \text{ may sol.}}$ $\lambda+\mu=?$

$D=0$

$$D=\begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 3 & \lambda \end{vmatrix}=0$$

$(2\lambda+2+3)-(2+6+\lambda)=0$

$\boxed{\lambda=1}$

$$D_3=\begin{vmatrix} 1 & 1 & 5 \\ 1 & 2 & 6 \\ 1 & 3 & \mu \end{vmatrix}=0$$

$(2\mu+6+15)-(10+18+\mu)=0$

$\boxed{\mu=7}$

$\boxed{\lambda+\mu=8}$