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{ Indefinite Integration } (4 Chapters)

II, DI, AUC, DE

← 30s →

(1) Integration & Diff^n are Reverse process to each other.

$$\frac{d(x^2)}{dx} = 2x$$

$$\frac{d(x^2+5)}{dx} = 2x$$

$$\frac{d(x^2-5)}{dx} = 2x$$

$$\int 2x\,dx = \boxed{x^2} + \boxed{C} \quad C \in R$$

x^2 : Antiderivative of $2x$, Premitive of $2x$

C : Arbitrary Const. take any value

$\Rightarrow$ So we call this Indefinite Int.

(2) $$\int \underbrace{f(x)}_{\text{Integrand}}\,dx = \underbrace{F(x)}_{\text{Integral}} + C$$

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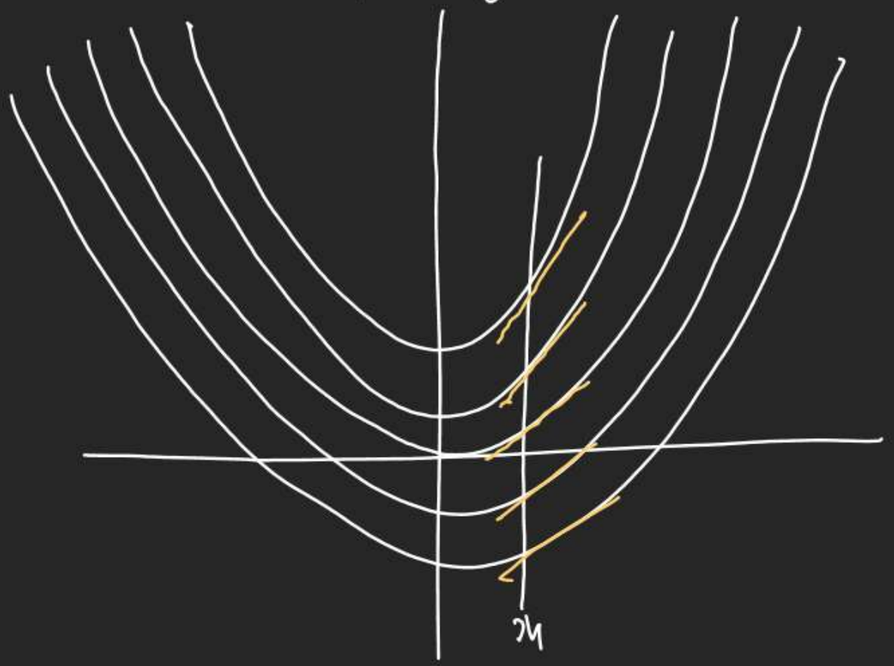
(3) Graph of Ind. Int.

Graph of $\int 2x\,dx$

= Graph of $x^2 + c$



(4) Integral of Periodic fxn need not to be Periodic..

(5) Many fxn's antiderivative can not be find out

$\int \frac{\sin x}{x}\,dx$ $\int \frac{\cos x}{x}\,dx$ $\int \sqrt{\sin x}\,dx$ $\int \sin x^2\,dx$

$\int \cos x^2\,dx$ $\int x \cdot \tan x\,dx$ $\int e^{-x^2}\,dx$ $\int e^{x^2}\,dx$

$\int \frac{x^3}{1+x^5}\,dx$ $\int \frac{dx}{\ln x}$ $\int (1+x^2)^{1/3}\,dx$

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(6) List of Integration

1) $\int x^n dx = \frac{x^{n+1}}{n+1} + C ; n \neq -1$

2) $\int x^{-1} dx = \int \frac{1}{x} dx = \ln|x| + C$

3) $\int \frac{1}{x^2} dx = -\frac{1}{x} + C$

4) $\int \sqrt{x}\, dx = \frac{2}{3} x^{3/2} + C$

(5) $\int \frac{1}{\sqrt{x}} dx = 2\sqrt{x} + C$

$\frac{d}{dx}(\ln|\sec x|)$

$= \frac{1}{|\sec x|} \times \frac{|\sec x|}{\sec x} \times \sec x \tan x$

$= \tan x$

$\int x^{1/2} dx = \frac{x^{1/2+1}}{\frac{1}{2}+1} + C$

$= \frac{2}{3} x^{3/2}$

$\int x^{-1/2} dx = \frac{x^{-1/2+1}}{-\frac{1}{2}+1} = 2\sqrt{x} + C$

6) $\int a^x . dx = \frac{a^x}{\ln a} + C$

7) $\int e^x dx = \frac{e^x}{\ln e} + C = e^x + C$

8) $\int \sin x\, dx = -\cos x + C$ | $\frac{d(-\cos x)}{dx} = \sin x$

9) $\int \cos x . dx = \sin x + C$

10) $\int \tan x\, dx = \ln|\sec x| + C$

11) $\int \cot x\, dx = \ln|\sin x| + C$

12) $\int \sec x\, dx = \ln|\sec x + \tan x| + C$

$= \ln\left|\tan\left(\frac{\pi}{4} + \frac{x}{2}\right)\right| + C$

13) $\int \operatorname{cosec} x\, dx = \ln|\operatorname{cosec} x - \cot x|$

$= \ln\left|\tan\frac{x}{2}\right| + C$

14) $\int \sec^2 x\, dx = \tan x + C$

15) $\int \operatorname{cosec}^2 x\, dx = -\cot x + C$

16) $\int \sec x \tan x\, dx = \sec x + C$

17) $\int \operatorname{cosec} x \cot x\, dx = -\operatorname{cosec} x + C$

$\frac{\sec^2\theta}{2\tan\theta} = \frac{1}{\cos^2\theta} \times \frac{\cos\theta}{2\sin\theta} = \sec 2\theta$

11) $\frac{d(\ln|\sec x + \tan x|)}{dx} = \sec 2\left(\frac{\pi}{4} + \frac{x}{2}\right)$

$= \sec\left(\frac{\pi}{2} + x\right)$

$= \operatorname{cosec} x$

$= \frac{1}{\sec x + \tan x} \times (\sec x \tan x + \sec^2 x)$

$= \frac{\sec x(\sec x + \tan x)}{(\sec x + \tan x)}$

12) $\frac{d(\ln(\tan(\frac{\pi}{4} + \frac{x}{2})))}{dx}$

$= \frac{1}{2\tan(\frac{\pi}{4} + \frac{x}{2})} \times \sec^2\left(\frac{\pi}{4} + \frac{x}{2}\right)$

$= \sec x$

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18) $\int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a}+C$

19) $\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right|+C$

20) $\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left|\frac{x+a}{x-a}\right|+C$

21) $\int \frac{dx}{\sqrt{x^2+a^2}} = \ln\left|x+\sqrt{x^2+a^2}\right|+C$

22) $\int \frac{dx}{\sqrt{x^2-a^2}} = \ln\left|x+\sqrt{x^2-a^2}\right|+C$

23) $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\frac{x}{a}+C$

24) $\int \sqrt{x^2+a^2}\,dx = \frac{x}{2}\sqrt{x^2+a^2}+\frac{a^2}{2}\ln\left|x+\sqrt{x^2+a^2}\right|+C$

25) $\int \sqrt{x^2-a^2}\,dx = \frac{x}{2}\sqrt{x^2-a^2}-\frac{a^2}{2}\ln\left|x+\sqrt{x^2-a^2}\right|+C$

26) $\int \sqrt{a^2-x^2}\,dx = \frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\frac{x}{a}+C$

$\int \frac{x\,dx}{\sqrt{a^2-x^2}} = -\sqrt{a^2-x^2}+C$

$\int \frac{x\,dx}{\sqrt{x^2+a^2}} = \sqrt{x^2+a^2}+C$

$\int \frac{x\,dx}{\sqrt{x^2-a^2}} = \sqrt{x^2-a^2}+C$

$\int x^n\,dx = \frac{x^{n+1}}{n+1}+C$

$\int (ax+b)^n\,dx = \frac{(ax+b)^{n+1}}{a(n+1)}+C$

* In ∫ treat $ax+b$ as x

Q) $\int \sin x\,dx = -\cos x+C$

Q) $\int \sin(4x-7)\,dx = -\frac{\cos(4x-7)}{4}+C$

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Q $\int (2x-3)^7 \cdot dx$ → $\int x^7 \cdot dx = \frac{x^8}{8} + C$

$\Rightarrow \frac{(2x-3)^8}{8 \times 2} + C$

Q $\int (4-3x)^7 dx$

$\Rightarrow \frac{(4-3x)^8}{8 \times -3} + C$

Q $\int \frac{dx}{4-3x}$ → $\int \frac{1}{x} d(x \text{ जोड़ा}) = \ln|x|$

$= \frac{\ln|4-3x|}{-3} + C$

Q $\int \sqrt{4-3x} \cdot dx$ → $\int \sqrt{x}\, dx = \frac{2}{3} x^{3/2}$

$\Rightarrow \frac{2}{3} \frac{(4-3x)^{3/2}}{x-3} + C$

Q $\int \frac{dx}{\sqrt{4-3x}} \rightarrow \int \frac{dx}{\sqrt{x}} = 2\sqrt{x} + C$

$= \frac{2\sqrt{4-3x}}{-3} + C$

$\int \sec^2 x = \tan$.

Q $\int \sec^2(1-x)\, dx$

$= \frac{\tan(1-x)}{-1} + C$

Q $\int \sec(2+3x) \cdot dx$

$\int \sec x \cdot dx = \ln|\sec x + \tan x| + C$

$= \ln\left|\tan\left(\frac{\pi}{4} + \frac{x}{2}\right)\right| + C$

$= \frac{1}{3}\ln\left|\tan\left(\frac{\pi}{4} + \frac{(2+3x)}{2}\right)\right| + C$

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Q $\int m\sqrt{x^n}\,dx$

$= \int (x^n)^{\frac{1}{m}}\,dx$

$= \int x^{\frac{n}{m}}\,dx$

$= \dfrac{x^{\frac{n}{m}+1}}{\frac{n}{m}+1} + C$

Q $\int e^{1+\ln\sqrt{x}}\,dx$

$\int e^1 \cdot e^{\ln\sqrt{x}}\,dx$

$e\int e^{\ln\sqrt{x}}\,dx$

$e\int \sqrt{x}\,dx$

$e \times \frac{2}{3} x^{3/2} + C$

Q $\int 5^{\ln x}\,dx$

$a^{\log_e c} = c^{\log_e a}$

$= \int 5^{\log_e x}\,dx$

$= \int x^{\log_e 5}\,dx$

$= \int x^{\ln 5}\,dx$

$= \dfrac{x^{\ln 5+1}}{\ln 5+1} + C$

Q $\int \dfrac{dx}{\sqrt{2x+3}-\sqrt{2x-3}}$

$\int \dfrac{1}{\sqrt{2x+3}-\sqrt{2x-3}} \times \dfrac{(\sqrt{2x+3}+\sqrt{2x-3})}{(\sqrt{2x+3}+\sqrt{2x-3})}$

$\int \dfrac{\sqrt{2x+3}+\sqrt{2x-3}\,dx}{(2x+3)-(2x-3)}$

$\frac{1}{6}\int \sqrt{2x+3}+\sqrt{2x-3}\,dx$

$\frac{1}{6}\left\{\int \sqrt{2x+3}\,dx + \int \sqrt{2x-3}\,dx\right\}$ ($\sqrt{x}$)

$\frac{1}{6}\left\{\frac{2}{3}\dfrac{(2x+3)^{3/2}}{2} + \frac{2}{3}\dfrac{(2x-3)^{3/2}}{2}\right\} + C$

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Q $\int 3^{3x} \cdot 5^{-2x} dx$

$\int \frac{3^{3x}}{5^{2x}} dx$

$\Rightarrow \int \frac{(3^3)^x}{(5^2)^x} dx$

$\Rightarrow \int \frac{(27)^x}{(25)^x} dx$

$\Rightarrow \int \left(\frac{27}{25}\right)^x dx$ $\rightarrow \int a^x dx$

$\Rightarrow \frac{\left(\frac{27}{25}\right)^x}{\ln\left(\frac{27}{25}\right)} + C$

Q $\int (2^x + 3^{-x})^2 dx$

$\int (2^x)^2 + (3^{-x})^2 + 2 \cdot 2^x \cdot 3^{-x} dx$

$\int 2^{2x} + 3^{-2x} + 2\left(\frac{2}{3}\right)^x dx$

$\int 2^{2x} dx + \int 3^{-2x} dx + 2\int \left(\frac{2}{3}\right)^x dx$

$\frac{2^{2x}}{2 \ln 2} + \frac{3^{-2x}}{-2 \ln 3} + 2 \cdot \frac{\left(\frac{2}{3}\right)^x}{\ln\left(\frac{2}{3}\right)} + C$

Q $\int \frac{2^{x+1} - 5^{x+1}}{10^x} dx$

$\int \frac{2 \cdot 2^x - 5 \cdot 5^x}{10^x} dx$

$\int 2 \cdot \frac{2^x}{10^x} - 5 \cdot \frac{5^x}{10^x} dx$

$2\int \left(\frac{2}{10}\right)^x dx - 5\int \left(\frac{5}{10}\right)^x dx$

$2\int \left(\frac{1}{5}\right)^x dx - 5\int \left(\frac{1}{2}\right)^x dx$

$2\int 5^{-x} dx - 5\int 2^{-x} dx$

$\frac{2 \times 5^{-x}}{-1 \times \ln 5} + \frac{5 \times 2^{-x}}{\ln 2} + C$ ✓

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Q $\int a^x \, da = ?$

→ 1) Integration w.r.t a
2) a = variable
3) x = const.

$\int x^n \, dx$ जैसा

$\Rightarrow \frac{a^{x+1}}{x+1} + C$

$\int 1 \cdot dx = x + C$

Q $\int \sin x \, d(\cos x)$

$d(\cos x)$ → $\frac{d(\cos x)}{dx} = -\sin x$

$d(\cos x) = -\sin x \cdot dx$

$-\int \sin x \cdot \sin x \, dx$

$= -\int \sin^2 x \, dx$

$= -\int \frac{1-\cos 2x}{2} dx$ $\Rightarrow \int -\frac{1}{2} + \frac{\cos 2x}{2} dx$

$\Rightarrow -\frac{x}{2} + \frac{\sin 2x}{2 \times 2} + C$

$\int \sin^2 x \, dx = \int \frac{1-\cos 2x}{2} dx$

$\int \cos^2 x \, dx = \int \frac{1}{2} + \frac{\cos 2x}{2} dx$

$\int \tan^2 x \, dx = \int \sec^2 x - 1 \cdot dx$

$\int \cot^2 x \, dx = \int \operatorname{cosec}^2 x - 1 \cdot dx$

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Q $\int \frac{dx}{2\sqrt{x}}$

Q $\int 3.4\, x^{-0.17}\, dx$

Q $\int (1-2v)\, dv$

Q $\int \frac{\sqrt{x} - x^3 \cdot e^x + x^2}{x^3}\, dx$

Q $\int \frac{(1+\sqrt{x})^3}{\sqrt[3]{x}}\, dx$

Q $\int \frac{dx}{\sqrt{3-3x^2}}$

Q $\int \frac{3\cdot 2^x - 2\cdot 3^x}{2^x}\, dx$

Q $\int \frac{\cos 2x\, dx}{\cos^2 x \cdot \sin^2 x}$

Q $\int \tan^2 x\, dx$

Q $\int \operatorname{arc}\sin x + \operatorname{arc}\cos x\, dx$

Q $\int \sin x\, d(\sin x)$

Q $\int \tan^3 x\, d(\tan x)$

Q $\int (x+1)^{15}\, dx$

Q $\int \frac{dx}{(2x-3)^5}$

Det
1) Ex 2
Indefinite
2) formula
Pad
3) Yeqs