Nishant Jindal

Fundamentals of Mathematics

DPP-2

Q1 Prod of all sol. of $x^{\log_{10} x} = \left(100 + 2^{\sqrt{\log_2 3}} - 3^{\sqrt{\log_3 2}}\right) x$ is

$2^{\sqrt{3}} = (\sqrt{2}\cdot\sqrt{2})^{\sqrt{3}} = \sqrt{2}^{\sqrt{3}} \cdot \sqrt{2}^{\sqrt{3}}$

Note
$$\boxed{a^{\sqrt{\log_a b}} = b^{\sqrt{\log_b a}}}$$
Try Prove

$$x^{\log_{10} x} = 100x$$

$$\log_{10} x^{(\log_{10} x)} = \log_{10}(100 \cdot x)$$

$$\log_{10} x \cdot \log_{10} x = \log_{10} 100 + \log_{10} x$$

$$t \cdot t = 2 + t$$

$$t^2 - t - 2 = 0 \Rightarrow (t-2)(t+1) = 0$$

$t = 2$ & $t = -1$

$\log_{10} x = 2$ | $\log_{10} x = -1$

$x = 10^2$ | $x = 10^{-1}$

$= 100$ | $= \frac{1}{10}$

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Q2 If $\log_7 2 = m$ then $\boxed{\log_{49} 28}$ is

$2(1+2m)$ $\quad \frac{1+2m}{2}$ $\quad \frac{2}{1+2m}$ $\quad 1+m.$

$$\log_{49} 28 = \log_{7^2} 7\times 2^2 = \frac{1}{2}\log_7 (7\times 2^2)$$

$$= \frac{1}{2}\left(\log_7 7 + \log_7 2^2\right)$$

$$= \frac{1}{2}\left(1 + 2\log_7 2\right) = \frac{1}{2}(1+2m)$$

$3^{c}\cdot \cancel{5} = \cancel{405}\ 81$

$3^c = 3^4 \Rightarrow \boxed{c=4}$

Q3 If $p = \log_5(\log_5 3)$ & $\boxed{3^{c+5^{-p}} = 405}$ Then $c = ?$

$$3^{c+5^{-p}} = 405 \qquad a^{m+n} = a^m \times a^n$$

$$\Rightarrow 3^c \times 3^{\boxed{5^{-p}}} = 405 \Rightarrow 3^c \cdot 3^{\log_3 5} = 405$$

$$5^{-p} = 5^{-(\log_5(\log_5 3))} = 5^{\log_5(\log_5 3)^{-1}}$$

$$= (\log_5 3)^{-1} = \frac{1}{\log_5 3}$$

$$= \log_3 5$$

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Q4 If $\dfrac{a+\log_4 3}{a+\log_2 3} = \dfrac{\boxed{a+\log_8 3}}{a+\log_4 3} = b$ then $b = ?$

$$\boxed{\frac{a}{b} = \frac{c}{d} = \frac{c-a}{d-b}}$$

$$\frac{m}{n} = \frac{x}{y} = \frac{x-m}{y-n}$$

$$= \frac{(a+\log_8 3)-(a+\log_4 3)}{(a+\log_4 3)-(a+\log_2 3)} = b$$

$$b = \frac{\log_{2^3} 3 - \log_{2^2} 3}{\log_{2^2} 3 - \log_2 3} = \frac{\frac{1}{3}\log_2 3 - \frac{1}{2}\log_2 3}{\frac{1}{2}\log_2 3 - \log_2 3} = \frac{\cancel{\log_2 3}\left(\frac{1}{3}-\frac{1}{2}\right)}{\cancel{\log_2 3}\left(\frac{1}{2}-1\right)}$$

$$b = \frac{\frac{1}{3}-\frac{1}{2}}{\frac{1}{2}-1} = \frac{+\frac{1}{6}}{+\frac{1}{2}} = \frac{1}{3}$$

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Σ Ka mtlb r Ki Value ek ek Kr K chdao & + + Lgaate Jao

Π x xx

Q $L = \sum_{r=7}^{2400} \log_7\left(\frac{r+1}{r}\right)$, $M = \prod_{r=2}^{1023} \log_r(r+1)$, $N = \sum_{r=2}^{2011} \frac{1}{\log_r P}$; $\{P = 1\cdot 2\cdot 3\cdot 4 \cdots 2011\}$

$L+M = 13$

$3+10 = 13$

$M^2+N^2 = 101$ ✓

$10^2+1^2 = 101$

$3-10+1 \neq 6$

$L-M+N = 6$ ✗

$3\times 10\times 1$

$\boxed{LMN = 30}$ ✓

$L = \log_7 \frac{7+1}{7} + \log_7 \frac{8+1}{8} + \log_7 \frac{9+1}{9} + \cdots + \log_7 \frac{2400+1}{2400}$

$= \log_7 \frac{8}{7} + \log_7 \frac{9}{8} + \log_7 \frac{10}{9} + \cdots + \log_7 \frac{2401}{2400}$

$L = \log_7\left\{\frac{\cancel{8}}{7} \times \frac{\cancel{9}}{\cancel{8}} \times \frac{\cancel{10}}{\cancel{9}} \times \frac{11}{\cancel{10}} \times \cdots \times \frac{2401}{\cancel{2400}}\right\}$

$L = \log_7 \frac{2401}{7} = \log_7 343 = \log_7 7^3 = 3$

$N = \frac{1}{\log_2 P} + \frac{1}{\log_3 P} + \frac{1}{\log_4 P} + \cdots + \frac{1}{\log_{2011} P}$

$N = \log_P 2 + \log_P 3 + \log_P 4 + \cdots + \log_P 2011$

$N = \log_P(\boxed{2\times 3\times 4\times \cdots 2011}) = \log_P P = 1$

$M = \log_2(2+1) \times \log_3(3+1) \times \log_4(4+1) \log_5(5+1) \cdots \log_{1023}(1023+1)$

$M = \frac{\cancel{\log 3}}{\log 2} \times \frac{\cancel{\log 4}}{\cancel{\log 3}} \times \frac{\cancel{\log 5}}{\cancel{\log 4}} \times \frac{\cancel{\log 6}}{\cancel{\log 5}} \cdots \frac{\log 1024}{\cancel{\log 1023}}$

$M = \frac{\log 1024}{\log 2} = \log_2 1024 = \log_2 2^{10} = 10$

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Q11 $\prod_{r=3}^{26} \log_r (r+1) = 3^x$ find x?

$\log_3 (3+1) \times \log_4 (4+1) \times \cdots \log_{26} (26+1)$

$\log_3 \boxed{4} \times \log_4 5 \times \log_5 6 \cdots \log_{26} 27 = 3^x$

$\frac{\log 4}{\log 3} \times \frac{\log 5}{\log 4} \times \frac{\log 6}{\log 5} \times \cdots \times \frac{\log 27}{\log 26}$

$\frac{\log 27}{\log 3} = 3^x \Rightarrow \log_3 3^3 = 3^x$

$\Rightarrow 3 = 3^x$

$\Rightarrow \boxed{x = 1}$

Q K is unique +ve value satisfying Eqn $(4K)^{\log 2} = (9K)^{\log 3}$ then $72K$?

$\log_{10} (4K)^{\boxed{\log_{10} 2}} = \log_{10} (9K)^{\boxed{\log_{10} 3}}$

$\log 2 \log(4 \cdot K) = \log_{10} 3 \log(9K)$

$\log 2 (\log 4 + \log K) = \log 3 (\log 9 + \log K)$

$\log 2 \cdot \log 2^{\boxed{2}} + \log 2 \cdot \log K = \log 3 \cdot \log 3^2 + \log 3 \log K$

$2(\log 2)^2 + \log 2 \cdot \log K = 2(\log 3)^2 + \log 3 \cdot \log K$

$2(\log^2 2 - \log^2 3) = \log K(\log 3 - \log 2)$

$-2(\cancel{\log 2 - \log 3})(\log 2 + \log 3) = \log K (\cancel{\log 3 - \log 2})$

$-2 \log 6 = \log K$

$\log 6^{-2} = \log K \Rightarrow K = 6^{-2} = \frac{1}{36}$

$\boxed{72 \times \frac{1}{36} = 2}$

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$(\log 2)^2 \neq \log 2^2$

Q $x^{(1+\log x)} = 10x$ find x?

$\log_{10} x^{(1+\log_{10} x)} = \log_{10}(10 \cdot x)$

$(1+\log_{10} x)\log_{10} x = \log_{10} 10 + \log_{10} x$

$(1+t)t = (1+t)$

$(1+t)(t-1) = 0$

$t=-1,\ t=1 \Rightarrow \log_{10} x = -1 \ \big|\ \log_{10} x = 1$

$x = 10^{-1} \ \big|\ x = 10$

Q $x^{\frac{\log x + 5}{3}} = 10^{5+\log x}$ find x?

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Questions Based on Tricotomy.

Which is greater?

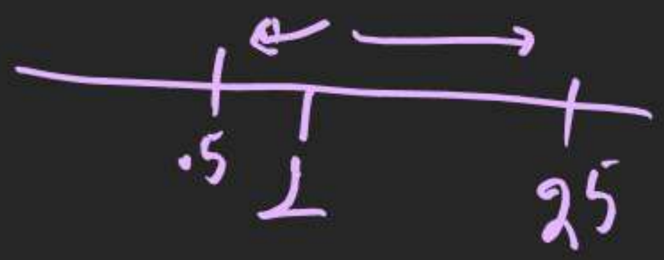
(1) $A = \log_{.5} 25 < 0$, $B = \log_2 3 > 0$

Check Base & exponent WRT 1

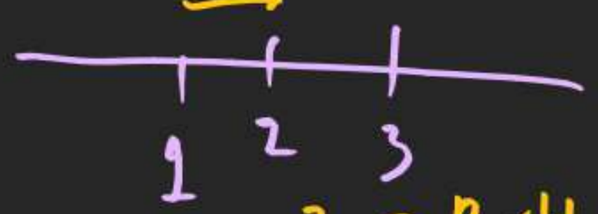
if Both are on Same Side of 1

then log is +ve otherwise -ve

$A = \log_{.5} 25 = -ve$



$B = \log_2 3 = +ve$



3, 2 Both are on Same Side

As A is < 0 & B is > 0

So B > A

Q $A = \log_{\frac{1}{3}} 3 = -ve$ & $B = \log_2 3 = +ve$

B > A

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$\log_3 3 = 1$ $\log_3 4 > 1$

On Behalf of values Gr. thums.

Q $A = \log_{\boxed{3}} \boxed{3.1}$ & $B = \log_{3.1} \boxed{3}$

Which is Gr.

Exp > Base

then log value > 1

Exp = 3

Base = 3.1

⇒ Exp < Base

B = log value < 1

A > B

$A = \log_3 4$ $B = \log_4 3$

Exp = 4

Base = 3.

E > B

∴ $A = \log_3 4 > 1$

Exp = 3

Base = 4

E < B.

$B = \log_4 3 < 1$

A > B

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<1 $\quad 1 \quad$ >1

$\log_3 2 \quad \log_3 3 = 1 \quad \log_3 4$

$1 \quad >1$

$\log_2 \boxed{25} \quad \log_{17} \boxed{25} \quad \log_9 \boxed{25}$

$\log_{17} 17 = 1$

Q $A = \underline{\log_3 5}$ & $B = \log_{17} 25$

Which is hr.

$A = \underline{\log_9 25}$ | $B = \log_{17} 25$

$A > B$

$1 \quad 2$

$\boxed{\log_{25} 25} \quad \log_{17} 25 \quad \log_9 25 \quad \log_5 25$

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Some Imp. pt.

(1) $\log_e e = 1$

(2) $\log_e x = \ln x$

(3) $\log_{10} 3 = .4771$

(4) $\log_{10} 2 = .3010$

(5) $\ln 2 = .693$

$\log_{10} 34.56 = 1.5386$

$= \underbrace{1}_{\text{Int. Part}} + \underbrace{.5386}_{\text{fractional Part}}$

Int. Part — Characteristic

$\boxed{\text{fractional Part}}$ Mantissa.

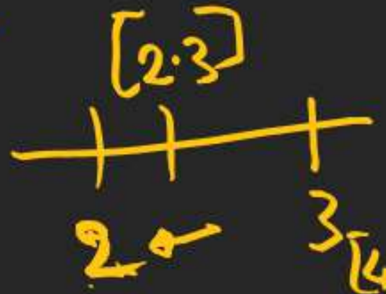
Fundamentals of Mathematics

2 New fxns.

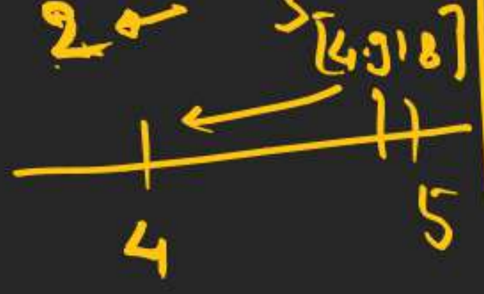
① Greastest Integer fxn.

1) $f(x) = [x]$ Rep.

2) $[2.3] = 2$



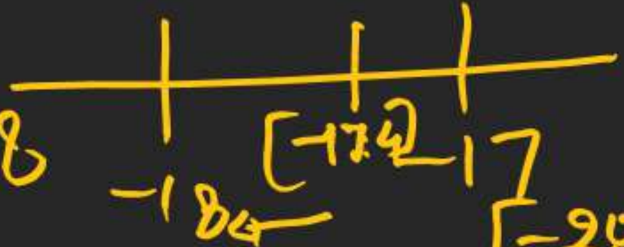
$[4.918] = 4$



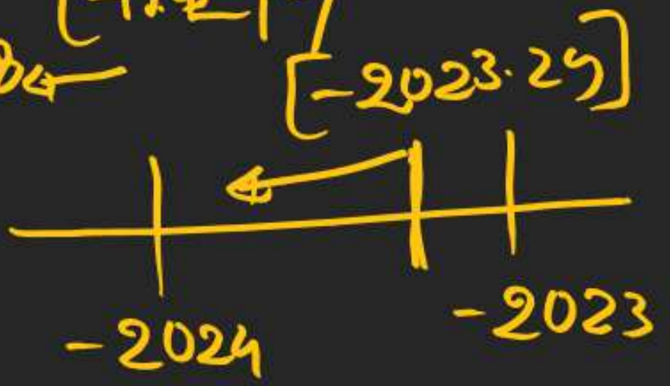
$[16] = 16$

$[-17] = -17$

$[-17.4] = -18$



$[-2023.29]$



$= -2024$

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(3)

$[x] = 4$

$x \in [4, 5)$

$[x] = 7$

$x \in [7, 8)$

$[x] = 29$

$x \in [29, 30)$

final :- $[x] = n$

$x \in [n, n+1)$

$[x] = -11$

$x \in [-11, -11+1)$

$x \in [-11, -10)$

Q $[\log_{10} x] = 4$

then $x \in$?

$\log_{10} x \in [4, 5)$

$4 \leq \log_{10} x < 5$

$10^4 \leq x < 10^5$

$10000 \leq x < 100000$

$x \in [10000, 100000)$