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Differentiation

First Principle / Delta Method / Ab-initio

$$\boxed{D(f(x)) = \frac{d}{dx} f(x) = f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}}$$

$$D(\ln x) = \lim_{\Delta x \to 0} \frac{\ln(x+\Delta x) - \ln x}{\Delta x} = \lim_{\Delta x \to 0} \frac{\ln\left(1+\frac{\Delta x}{x}\right)}{x \, \frac{\Delta x}{x}}$$

$$= \frac{1}{x}.$$

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$$D\left(e^{x^2}\sin x\right) = \lim_{\Delta x\to 0} \frac{e^{(x+\Delta x)^2}\sin(x+\Delta x) - e^{x^2}\sin x}{\Delta x}$$

$$= \lim_{\Delta x\to 0} \frac{e^{(x+\Delta x)^2}\sin(x+\Delta x) - e^{(x+\Delta x)^2}\sin x + e^{(x+\Delta x)^2}\sin x - e^{x^2}\sin x}{\Delta x}$$

$$= \lim_{\Delta x\to 0}\left(\frac{e^{(x+\Delta x)^2}\left(2\sin\frac{\Delta x}{2}\cos\left(x+\frac{\Delta x}{2}\right)\right)}{\Delta x} + \frac{\sin x\, e^{x^2}\left(e^{2x\Delta x+(\Delta x)^2}-1\right)}{2x\Delta x+(\Delta x)^2} \times (2x+\Delta x)\right)$$

$$e^{x^2}\cos x + e^{x^2}\sin x(2x) =$$

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$$D(\sin^{-1}x) = \lim_{\Delta x\to 0}\frac{\sin^{-1}(x+\Delta x)-\sin^{-1}x}{\Delta x} = \lim_{\Delta x\to 0}\frac{(y+\Delta y)-y}{\Delta x} = \lim_{\Delta x\to 0}\frac{\Delta y}{(x+\Delta x)-x}$$

$(x+\Delta x, y+\Delta y)$

$f(x+\Delta x) = y+\Delta y$

$(x, y) = f(x)$

$-\frac{\pi}{2}, \frac{\pi}{2}$

$\sin^{-1}x = y \Rightarrow \boxed{x = \sin y}$

$\sin^{-1}(x+\Delta x) = y+\Delta y \Rightarrow x+\Delta x = \sin(y+\Delta y)$

$\Delta x \to 0, \ \Delta y \to 0$

$$= \lim_{\Delta y\to 0}\frac{\Delta y}{\sin(y+\Delta y)-\sin y} = \lim_{\Delta y\to 0}\frac{\Delta y}{2\sin\left(\frac{\Delta y}{2}\right)\cos\left(y+\frac{\Delta y}{2}\right)}$$

$$= \frac{1}{\cos y}$$

$$= \frac{1}{\sqrt{1-x^2}}$$

$$\boxed{D(\sin^{-1}x) = \frac{1}{\sqrt{1-x^2}} \quad x\in(-1,1)}$$

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$$D(\tan^{-1}x) = \lim_{\Delta y\to 0} \frac{\Delta y}{\tan(y+\Delta y) - \tan y} = \lim_{\Delta y\to 0} \frac{\Delta y}{\tan(\Delta y)\left(1+\tan y\tan(y+\Delta y)\right)}$$

$$= \frac{1}{1+\tan^2 y} = \frac{1}{1+x^2}.$$

$$\boxed{\sec^{-1}x = y} \in [0,\pi] - \left\{\frac{\pi}{2}\right\}$$

$$\boxed{\tan^2 y = x^2 - 1}$$

$$D(\sec^{-1}x) = \lim_{\Delta y\to 0} \frac{\Delta y \cos y\cos(y+\Delta y)}{\cos y - \cos(y+\Delta y)} = \lim_{\Delta y\to 0} \frac{\Delta y \cos y \cos(y+\Delta y)}{2\sin\frac{\Delta y}{2}\sin\left(y+\frac{\Delta y}{2}\right)}.$$

$$= \frac{\cos^2 y}{\sin y} = \frac{1}{\tan y\sec y} = \begin{cases} \dfrac{1}{x\sqrt{x^2-1}}, & y\in\left(0,\frac{\pi}{2}\right), x\in(1,\infty) \\ \dfrac{1}{-x\sqrt{x^2-1}}, & y\in\left(\frac{\pi}{2},\pi\right), x\in(-\infty,-1) \end{cases}$$

$$\boxed{D(\sec^{-1}x) = \frac{1}{|x|\sqrt{x^2-1}}}$$

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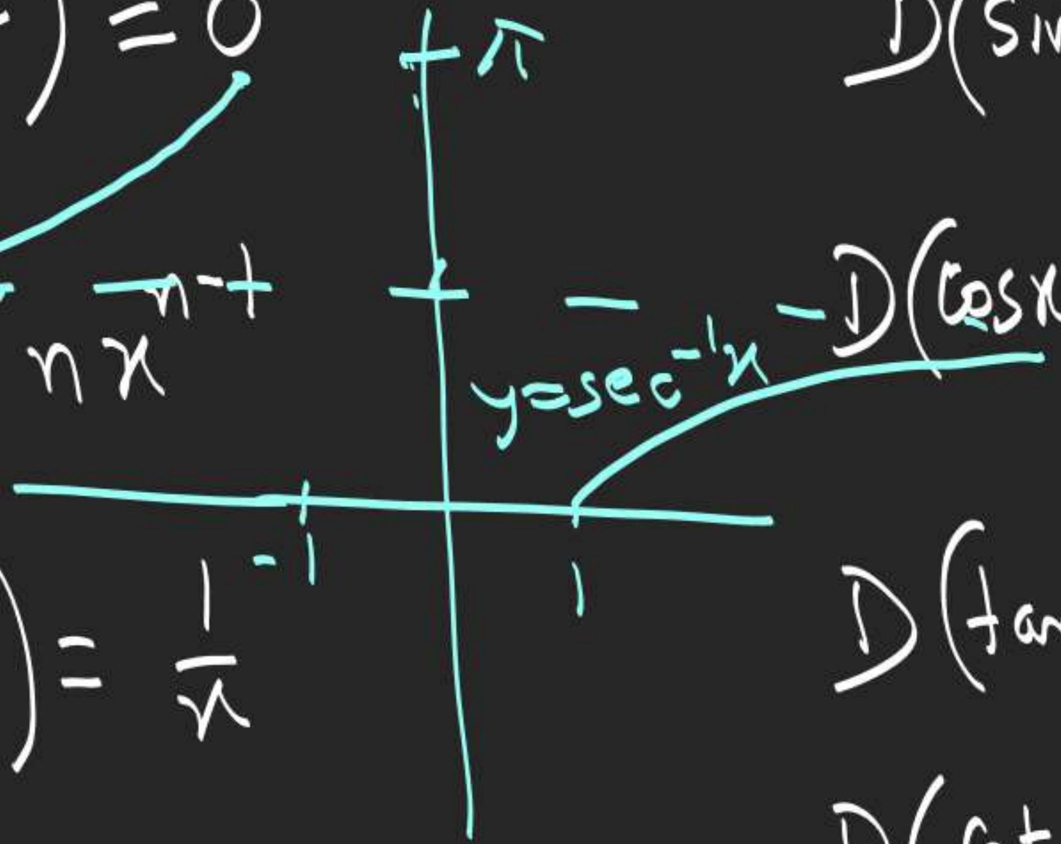
$$D(\text{constant}) = 0$$

$$D(x^n) = n x^{n-1}$$

$$D(\ln|x|) = \frac{1}{x}$$

$$D(e^x) = e^x$$

$$D(a^x) = a^x \ln a, \quad a>0$$



$$D(\sin x) = \cos x$$

$$D(\cos x) = -\sin x$$

$$D(\tan x) = \sec^2 x$$

$$D(\cot x) = -\operatorname{cosec}^2 x$$

$$D(\sec x) = \sec x \tan x$$

$$D(\operatorname{cosec} x) = -\operatorname{cosec} x \cot x$$

$$D(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$$

$$D(\cos^{-1} x) = \frac{-1}{\sqrt{1-x^2}}$$

$$D(\tan^{-1} x) = \frac{1}{1+x^2}$$

$$D(\cot^{-1} x) = \frac{-1}{1+x^2}$$

$$D(\sec^{-1} x) = \frac{1}{|x|\sqrt{x^2-1}}$$

$$D(\operatorname{cosec}^{-1} x) = \frac{-1}{|x|\sqrt{x^2-1}}$$

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Product Rule

$$D\left(f(x)g(x)\right) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x)g(x+\Delta x) - f(x)g(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{f(x+\Delta x)g(x+\Delta x) - f(x+\Delta x)g(x) + f(x+\Delta x)g(x) - f(x)g(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \left(f(x+\Delta x)\left(\frac{g(x+\Delta x) - g(x)}{\Delta x}\right) + g(x)\left(\frac{f(x+\Delta x) - f(x)}{\Delta x}\right) \right)$$

$$= f(x)g'(x) + g(x)f'(x)$$

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$$D\left(f_1\underbrace{(f_2 f_3 f_4 \cdots f_n)}\right)(x) = f_1' f_2 f_3 \cdots f_n + f_1 (\underbrace{f_2 f_3 f_4 \cdots f_n})'$$

$$= f_1' f_2 f_3 \cdots f_n + f_1 f_2' f_3 f_4 \cdots f_n + f_1 f_2 (f_3 f_4 \cdots f_n)'$$

$$\vdots$$

$$\boxed{D(f_1 f_2 \cdots f_n) = f_1' f_2 f_3 \cdots f_n + f_1 f_2' f_3 \cdots f_n + f_1 f_2 f_3' f_4 \cdots f_n + \cdots + f_1 f_2 \cdots f_{n-1} f_n'}$$

$$F(x) = (f_1 f_2 f_3 \cdots f_n)(x)$$

$$\boxed{\frac{F'(x)}{F(x)} = \frac{f_1'(x)}{f_1(x)} + \frac{f_2'(x)}{f_2(x)} + \frac{f_3'(x)}{f_3(x)} + \cdots + \frac{f_n'(x)}{f_n(x)}}$$

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$$\frac{1}{x-1}+\frac{1}{x-2}+\cdots+\frac{1}{x-n}=\frac{f'(x)}{f(x)}$$

$$f(x)=(x-1)(x-2)\cdots(x-n)$$

Quotient Rule

$$D\left(\frac{f(x)}{g(x)}\right)=\frac{g(x)f'(x)-f(x)g'(x)}{g^2(x)}$$

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Chain Rule

$y = f(u)$, $u = g(v)$, $v = h(x)$

$\Delta x \to 0$, $\Delta v \to 0$, $\Delta u \to 0$

$\Delta y \to 0$

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \left(\frac{\Delta y}{\Delta x} \right)$$

$$= \lim_{\Delta x \to 0} \left(\frac{\Delta y}{\Delta u} \frac{\Delta u}{\Delta v} \frac{\Delta v}{\Delta x} \right) = \left(\lim_{\Delta u \to 0} \frac{\Delta y}{\Delta u} \right) \left(\lim_{\Delta v \to 0} \frac{\Delta u}{\Delta v} \right) \left(\lim_{\Delta x \to 0} \frac{\Delta v}{\Delta x} \right)$$

$$= \left(\frac{dy}{du} \right) \left(\frac{du}{dv} \right) \left(\frac{dv}{dx} \right)$$

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PT

Differentiability

Ex-I