

$$Q \quad S = \frac{1}{1 \cdot 3 \cdot 5} + \frac{1}{3 \cdot 5 \cdot 7} + \frac{1}{5 \cdot 7 \cdot 9} + \dots n \text{ terms.}$$

$\underbrace{1, 3, 5, 7, 9}_{\text{odd } 2s+1}$

$$S = \sum_{r=1}^n T_r = \sum_{r=1}^n \frac{1}{(2r-1)(2r+1)(2r+3)}$$

$\underbrace{\hspace{10em}}_{\text{diff} = 4} \quad \text{Rearrish.}$

$$= \frac{1}{4} \sum_{r=1}^n \left(\frac{1}{(2r-1)(2r+1)} - \frac{1}{(2r+1)(2r+3)} \right)$$

$$= \frac{1}{4} \left\{ \begin{array}{l} \frac{1}{1 \cdot 3} - \cancel{\frac{1}{3 \cdot 5}} \\ + \cancel{\frac{1}{3 \cdot 5}} - \cancel{\frac{1}{5 \cdot 7}} \\ \vdots \\ \cancel{\frac{1}{(2n-1)(2n+1)}} - \frac{1}{(2n+1)(2n+3)} \end{array} \right\} = \frac{1}{4} \left\{ \frac{1}{1 \cdot 3} - \frac{1}{(2n+1)(2n+3)} \right\}$$

$$- \sum_{r=1}^n \frac{1}{(r)(r+1)(r+2)}$$

diff = 2

$$\sum_{r=1}^n \frac{1}{(r)(r+1)(r+2)(r+3)}$$

$$= \sum_{r=1}^n \frac{(r+2)^2}{(r)(r+1)(r+2)(r+3)}$$

$$- \frac{1}{2} \left(\frac{1}{(r)(r+1)} - \frac{1}{(r+1)(r+2)} \right) = \sum_{r=1}^n \frac{r^2 + 4r + 4}{(r)(r+1)(r+2)(r+3)}$$

$$- \frac{1}{2} \left\{ \begin{array}{l} \frac{1}{1 \cdot 2} - \frac{1}{2 \cdot 3} \\ + \frac{1}{2 \cdot 3} - \frac{1}{3 \cdot 4} \\ \vdots \\ \frac{1}{(n)(n+1)} - \frac{1}{(n+1)(n+2)} \end{array} \right\}$$

$$= \frac{1}{2} \left(\frac{1}{1 \cdot 2} - \frac{1}{(n+1)(n+2)} \right)$$

$$\Rightarrow \sum_{r=1}^n \frac{(r^2 + 3r)}{(r)(r+1)(r+2)(r+3)} + \frac{r}{(r)(r+1)(r+2)(r+3)} + \frac{4}{(r)(r+1)(r+2)(r+3)}$$

$$\Rightarrow \sum_{r=1}^n \frac{1}{(r+1)(r+2)} + \sum_{r=1}^n \frac{1}{(r+1)(r+2)(r+3)} + 4 \sum_{r=1}^n \frac{1}{(r)(r+1)(r+2)(r+3)}$$

$$\frac{1}{1} \left(\frac{1}{r+1} - \frac{1}{r+2} \right) + \frac{1}{2} \left(\frac{1}{(r+1)(r+2)} - \frac{1}{(r+2)(r+3)} \right) + \frac{4}{3} \left(\frac{1}{r(r+1)(r+2)} - \frac{1}{(r+1)(r+2)(r+3)} \right)$$

$$Q \quad S = \frac{4}{1 \cdot 2 \cdot 3} + \frac{5}{2 \cdot 3 \cdot 4} + \frac{6}{3 \cdot 4 \cdot 5} + \dots - n \text{ terms.}$$

$$S_n = \sum T_r = \sum \frac{(r+3)}{(r)(r+1)(r+2)}$$

$$= \sum \frac{\cancel{(r+2)}}{(r)(r+1)\cancel{(r+2)}} + \frac{1}{\underbrace{(r)(r+1)(r+2)}_2}$$

$$\Rightarrow \sum \frac{1}{\underbrace{(r)(r+1)}_1} + \frac{1}{2} \left(\frac{1}{(r)(r+1)} - \frac{1}{(r+1)(r+2)} \right)$$

$$= \frac{1}{1} \left(\frac{1}{r} - \frac{1}{r+1} \right) + \frac{1}{2} \left(\frac{1}{1 \cdot 2} - \frac{1}{(n+1)(n+2)} \right)$$

$$= \frac{1}{1} \left(\frac{1}{1} - \frac{1}{n+1} \right) + \frac{1}{2} \left(\frac{1}{1 \cdot 2} - \frac{1}{(n+1)(n+2)} \right)$$

$$Q \quad \frac{1}{\boxed{2} \cdot 3 \cdot 4} + \frac{1}{\boxed{3} \cdot 4 \cdot 5} + \frac{1}{\boxed{4} \cdot 5 \cdot 6} + \dots + \frac{1}{100 \cdot 101 \cdot 102} = \frac{K}{101} \text{ find } K.$$

$$\begin{aligned} \sum_{r=1}^{99} T_r &= \sum_{r=1}^{99} \frac{1}{(r+1)(r+2)(r+3)} \\ &= \frac{1}{2} \sum_{r=1}^{99} \left(\frac{1}{(r+1)(r+2)} - \frac{1}{(r+2)(r+3)} \right) \\ &= \frac{1}{2} \left(\frac{1}{2 \cdot 3} - \frac{1}{(101)(102)} \right) = \frac{K}{101} \\ K &= 2 \end{aligned}$$

Q Let n^{th} term of any seqⁿ is given by $T_n = \frac{-1^3 + 2^3 - 3^3 + 4^3 - \dots + (2n)^3}{(n)(4n+3)}$ then $\sum_{r=1}^{120} T_r = ?$

$\sum n = \frac{n(n+1)}{2} = \frac{15 \times 16}{2} = 120$

$$T_n = \frac{2(2^3 + 4^3 + 6^3 - \dots - (2n)^3) - (1^3 + 2^3 + 3^3 + \dots + (2n)^3)}{(n)(4n+3)}$$

$$= \frac{2 \times 8(1^3 + (2)^3 + 3^3 + \dots + (n)^3) - (1^3 + 2^3 + \dots + (2n)^3)}{(n)(4n+3)}$$

$$= \frac{164 \frac{(n)^3(n+1)^2}{4} - \frac{(2n)^2(2n+1)^2}{4}}{(n)(4n+3)} = \frac{n^4((2n+2)^2 - (2n+1)^2)}{(n)(4n+3)}$$

$$= \frac{n(4n+3)}{(4n+3)} = 1$$

Himmat

$$Q \quad \frac{2^3-1^3}{1 \times 7} + \frac{4^3-3^3+2^3-1^3}{2 \times 11} + \frac{6^3-5^3+4^3-3^3+2^3-1^3}{3 \times 15} + \dots + \frac{30^3-29^3+28^3-27^3 \dots + 2^3-1^3}{15 \times 63} = ?$$

$$\frac{8-1}{7} + \frac{64-27+8-1}{2 \times 11} + \frac{216-125+64-27+8-1}{3 \times 15} \dots \times \dots$$

$$1 + 2 + 3 + \dots + 15 = \frac{15 \times 16}{2} = 120 \neq$$

$$Q \quad \text{Value of } \sum_{n=1}^n \frac{3}{(4n-1)(4n+3)}$$

$$\frac{3}{4} \left(\frac{1}{4n-1} - \frac{1}{4n+3} \right)$$

$$\frac{3}{4} \left(\frac{1}{3} - \frac{1}{11} \right) = \frac{1}{3} \left(\frac{1}{3} - \frac{1}{4n+3} \right)$$

$$Q \quad \frac{1}{3^2-1} + \frac{1}{5^2-1} + \frac{1}{7^2-1} \dots + \frac{1}{(201)^2-1} = ?$$

$$T_n = \sum \frac{1}{(2r+1)^2-1^2} = \sum \frac{1}{(2r-1+1)(2r-1-1)}$$

$$\sum \frac{1}{(2r)(2r-2)} = \frac{1}{4} \sum \frac{1}{(r)(r-1)}$$

$$\frac{1}{4} \times \frac{1}{1} \sum \frac{1}{r-1} = \frac{1}{8} \text{ open \& sol.}$$

1) There are some series which are directly not AP or HP

2) $S = t_1 + t_2 + t_3 + t_4 + t_5 + t_6 + \dots + t_n$

$\underbrace{t_1, t_2, t_3, t_4, t_5}_{K_1, K_2, K_3, K_4, K_5} \rightarrow 1^{\text{st}} \text{ Order diff}$

$\underbrace{K_1, K_2, K_3, K_4}_{S_1, S_2, S_3, S_4} \rightarrow 2^{\text{nd}} \text{ Order diff}$

$\underbrace{S_1, S_2, S_3}_{P_1, P_2, P_3} \rightarrow 3^{\text{rd}} \text{ Order diff}$

(3) If 3^{rd} Order difference is same then n^{th} term

will be $T_n = t_1 + K_1(n-1) + S_1 \frac{(n-1)(n-2)}{1 \cdot 2} + P_1 \frac{(n-1)(n-2)(n-3)}{1 \cdot 2 \cdot 3}$

(4) If P^{th} order difference in h.p. in which C.R. = r then

$T_n = a \cdot r^{n-1} + f(n)$

where $f(n)$ is Poly of degree $(P-1)$

Ex $\boxed{P=3} \rightarrow 2^{\text{nd}} \text{ Order Poly.}$
 $\Rightarrow 3^{\text{rd}} \text{ order difference in h.p.}$

$T_n = a \cdot r^{n-1} + \boxed{bn^2 + cn + d}$

Q Find Sum of n terms of

$3 + 7 + 13 + 21 + \dots$

$S = 3 + 7 + 13 + 21 + 31 + 43 + \dots$

$\underbrace{3, 7, 13, 21, 31, 43}_{4, 6, 8, 10, 12}_{2, 2, 2, 2} \rightarrow 2^{\text{nd}} \text{ Order diff} = \text{constant}$

$S_n = \sum T_n = \sum 3 + 4(n-1) + \frac{2 \cdot (n-1)(n-2)}{1 \cdot 2}$

$= \sum 3 + 4n - 4 + n^2 - 3n + 2$

$= \sum n^2 + n + 1$ Solve

$\Rightarrow \frac{(n)(n+1)(2n+1)}{6} + \frac{(n)(n+1)}{2} + n$

Q Find sum of n terms of Series.

$$1 + 4 + 10 + 20 + \dots$$

$$1 + 4 + 10 + 20 + \dots$$

$\underbrace{3 \quad 6 \quad 12}_{r=2} \rightarrow \boxed{1st \text{ order diff}} \xrightarrow{(1-1) \rightarrow 0 \text{ order Poly}} \text{const}$
 G.P.

$$\therefore S_n = \sum T_n = \sum 3 \cdot 2^{n-1} - 1 = 3 \sum 2^{n-1} - \sum 1$$

$$T_n = a \cdot (2)^{n-1} + b$$

$$T_1 = a \cdot 2^0 + b \Rightarrow a + b = 1$$

$$T_2 = a \cdot 2^{2-1} + b \quad \left| \begin{array}{l} 2a + b = 4 \\ -a = -3 \end{array} \right.$$

$$4 = 2a + b$$

$$a = 3$$

$$b = 2$$

$$T_n = 3 \cdot 2^{n-1} - 2$$

$$3(2^0 + 2^1 + 2^2 + \dots + 2^{n-1}) - 2 \sum 1$$

$$3 \cdot \frac{(1 \cdot (2)^n - 1)}{(2-1)} - 2 \times n$$

$$3 \cdot 2^n - 3 - 2n$$

Q $S = 2 + 12 + 36 + 80 + 150 + 252 + \dots$ - n terms.

$$\begin{array}{ccccccc} & & 10 & 24 & 44 & 70 & 102 \\ & & \swarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ & 14 & 20 & 26 & 32 & & \\ & \swarrow & \downarrow & \downarrow & & & \\ & 6 & 6 & 6 & & & \end{array}$$

$\rightarrow \text{const (3rd order)}$
 $2^{nd} \text{ order poly}$

$$T_n = 2 + 10(n-1) + \frac{14(n-1)(n-2)}{1 \cdot 2} + \frac{6(n-1)(n-2)(n-3)}{1 \cdot 2 \cdot 3}$$

$$T_n = n^3 + n^2$$

$$S_n = \sum n^3 + \sum n^2$$

$$\textcircled{1} S = \textcircled{9} + \textcircled{16} + 29 + 54 + 103 + \dots$$

$\underbrace{\quad\quad\quad}_{7} \quad \underbrace{\quad\quad\quad}_{13} \quad \underbrace{\quad\quad\quad}_{25} \quad \underbrace{\quad\quad\quad}_{49}$
 $\underbrace{\quad\quad}_{6} \quad \underbrace{\quad\quad}_{12} \quad \underbrace{\quad\quad}_{24} \rightarrow GP \rightarrow 2^{na} \text{ or } 1^{st} \text{ deg poly.}$

$$T_n = a(2)^{n-1} + bn + c$$

$$T_1 = a \cdot 2^0 + b + c = 9 \Rightarrow \overset{6+1+2}{a+b+c=9}$$

$$T_2 = a \cdot 2^1 + 2b + c = 16 \Rightarrow \overset{12+2+2}{2a+2b+c=16}$$

$$T_3 = a \cdot 2^2 + 3b + c = 29 \Rightarrow \overset{8+6+2}{4a+3b+c=29}$$

$$\overset{24+3+2}{\text{find } a, b, c = (6, 1, 2)}$$

$$\text{find } \underline{\underline{S_n}}$$

Relation betⁿ AM, GM & HM.

$$\textcircled{1} G^2 = AH$$

$$\textcircled{2} \boxed{AM \geq GM \geq HM} \rightarrow \text{for +ve No. only}$$

$$\frac{a_1 + a_2 + \dots + a_n}{n} \geq (a_1 a_2 \dots a_n)^{\frac{1}{n}} \geq \frac{n}{\left(\frac{1}{a_1} + \frac{1}{a_2} + \frac{1}{a_3} + \dots + \frac{1}{a_n}\right)}$$

Q If $x \geq 0$ then P.T. $1+x \geq 2\sqrt{x}$

if 1, x are 2 elements then

$$AM \geq HM$$

$$\frac{1+x}{2} \geq (1 \cdot x)^{\frac{1}{2}}$$

$$1+x \geq 2\sqrt{x} \quad \text{J.I.P.}$$

Q If $x > 0$ then P.T. $\boxed{x + \frac{1}{x}} \geq 2$

elements $\rightarrow x, \frac{1}{x}$

$$AM \geq HM$$

$$\frac{x + \frac{1}{x}}{2} \geq \left(x \cdot \frac{1}{x}\right)^{\frac{1}{2}}$$

$$x + \frac{1}{x} \geq 2 \times 1$$

$$x + \frac{1}{x} \geq 2$$

Q If $x, y \in \mathbb{R}^+$ then P.T.

$$x^2 + y^2 \geq 2xy$$

elements x^2, y^2

$$A.M. \geq H.M.$$

$$\frac{x^2 + y^2}{2} \geq (x^2 \cdot y^2)^{\frac{1}{2}}$$

$$x^2 + y^2 \geq 2xy \quad \text{(J.I.P.)}$$

Q. If $x, y \in \mathbb{R}^+$ then P.T.

$$2(x^2 + y^2) \geq (x + y)^2$$

We already know.

$$x^2 + y^2 \geq 2xy \quad (\text{adding } x^2 + y^2)$$

$$x^2 + y^2 + (x^2 + y^2) \geq 2xy + x^2 + y^2$$

$$2(x^2 + y^2) \geq (x + y)^2 \quad \text{(J.I.P.)}$$

Q If $x, y \in \mathbb{R}^+$ then P.T. $\frac{1}{x} + \frac{1}{y} \geq \frac{4}{x+y}$

elements $\rightarrow \frac{1}{x}, \frac{1}{y}$

AM \geq HM.

$$\frac{\frac{1}{x} + \frac{1}{y}}{2} \geq \frac{2}{\left(\frac{1}{x}\right) + \left(\frac{1}{y}\right)}$$

$$\frac{\frac{1}{x} + \frac{1}{y}}{(2)} \geq \frac{2}{x+y}$$

$$\frac{1}{x} + \frac{1}{y} \geq \frac{4}{x+y} \text{ (H.P.)}$$

Q $a, b, x \in \mathbb{R}^+$ then $ax + \frac{b}{x} \geq \dots$?

elements $\rightarrow ax, \frac{b}{x}$

AM \geq HM.

$2\sqrt{ab}$

$$\frac{ax + \frac{b}{x}}{2} \geq \left(ax \cdot \frac{b}{x}\right)^{\frac{1}{2}}$$

$$ax + \frac{b}{x} \geq 2\sqrt{ab}$$

Q If $x, y, z \in \mathbb{R}^+$ then P.T.

$$x^2 + y^2 + z^2 \geq xy + yz + zx$$

from 3rd last Pr. up we come to know

$$x^2 + y^2 \geq 2xy$$

$$y^2 + z^2 \geq 2yz$$

$$z^2 + x^2 \geq 2zx$$

(I.P.)

$$2(x^2 + y^2 + z^2) \geq 2(xy + yz + zx)$$

Q $x, y, z \in \mathbb{R}^+$ P.T.

$$(x+y)(y+z)(z+x) \geq 8xyz$$

$$x+y \geq 2\sqrt{xy} \quad (\text{AM} \geq \text{GM})$$

$$y+z \geq 2\sqrt{yz}$$

$$z+x \geq 2\sqrt{zx}$$

$$\text{Multiply } (x+y)(y+z)(z+x) \geq 2 \times 2 \times 2 \sqrt{xy} \times \sqrt{yz} \times \sqrt{zx}$$

$$\geq 8(xyz)$$

H.P.