

$$Q \lim_{x \rightarrow 0} \frac{\tan^3 x + \tan^3 2x + \tan^3 3x}{\tan^3 4x + \tan^3 5x + \tan^3 (6x)} = \frac{P}{Q}$$

$$\frac{x^3 + (2x)^3 + (3x)^3}{(4x)^3 + (5x)^3 + (6x)^3} = \frac{1+8+27}{64+125+216}$$

$$= \frac{36}{405} = \frac{4}{45}$$

$$Q \lim_{x \rightarrow 0} \frac{6x - \sqrt{1+6\sin^2 x}}{1 - \sqrt{1+\tan^2 x}} \quad \frac{0}{0} \text{ Rat.}$$

$$\lim_{x \rightarrow 0} \frac{(6x) - 1 - \sin^2 x}{(6x) + 1 + \sin^2 x} \times \frac{1 + \sqrt{1+\tan^2 x}}{x - x - \tan^2 x} = \frac{2}{2} \lim_{x \rightarrow 0} \frac{x - \sin^2 x - x - \sin^2 x}{-\tan^2 x} = \frac{2 \sin^2 x}{\sin^2 x} \times \frac{6x}{6x} = 2 \times 1 = 2$$

$$Q \lim_{x \rightarrow \cot^{-1}(-1)} \frac{\tan^3 x - 2 \tan x - 1}{\tan^5 x - 2 \tan x - 1} \quad \begin{matrix} x^3 \rightarrow 3x^2 \\ \tan^3 x \rightarrow 3 \end{matrix}$$

$$\cot x \rightarrow -1$$

$$x \rightarrow \frac{3\pi}{4}$$

$$\frac{\tan^3 x - 2 \tan x - 1}{\tan^5 x - 2 \tan x - 1} \quad \frac{0}{0} \text{ DL}$$

$$\frac{3 \tan^2 x \sec^2 x - 2 \sec^2 x}{5 \tan^4 x \sec^2 x - 2 \sec^2 x}$$

$$\lim_{x \rightarrow \frac{3\pi}{4}} \frac{\sec^2 x (3 \tan^2 x - 2)}{\sec^2 x (5 \tan^4 x - 2)} = \frac{3(-1)^2 - 2}{5(-1)^2 - 2} = \frac{1}{3}$$

Q $\lim_{x \rightarrow \infty} \sqrt{\frac{x - \sin x}{x(1 + \cos^2 x)}}$

Algebraic \rightarrow Trigo

$$\lim_{x \rightarrow \infty} \sqrt{\frac{x(1 - \frac{\sin x}{x})}{x(1 + \frac{\cos^2 x}{x})}} \quad \frac{\sin \infty}{\infty} = \frac{(-1 \text{ to } 1)}{\infty} = 0$$

$$\frac{\cos^2 \infty}{\infty} = \frac{(0 \text{ to } 1)}{\infty} = 0$$

$$\sqrt{\frac{1-0}{1+0}} = 1$$

Q $\lim_{x \rightarrow 0^+} \sqrt{\frac{\tan^{-1} x}{x} - \frac{\sin^{-1} x}{x}}$

$\frac{1}{\sqrt{1+x^2}} - \frac{1}{\sqrt{1-x^2}}$

$\sqrt{-1/e} = \text{LDNE}$

Q If $l = \lim_{n \rightarrow \infty} \sum_{r=2}^n (r+1) \cdot \sin \frac{\pi}{r+1} - r \cdot \sin \frac{\pi}{r}$

all Subjective

then $\{l\} = ?$

Ex

$$\begin{aligned} & 3 \sin \frac{\pi}{3} - 2 \sin \frac{\pi}{2} \\ & + 4 \sin \frac{\pi}{4} - 3 \sin \frac{\pi}{3} \\ & + 5 \sin \frac{\pi}{5} - 4 \sin \frac{\pi}{4} \\ & \vdots \\ & + (n+1) \sin \frac{\pi}{(n+1)} - n \sin \frac{\pi}{n} \end{aligned}$$

$\infty \times 0$
 $\infty \times \sin 0$

$$l = (n+1) \sin \frac{\pi}{n+1} - 2 \sin \frac{\pi}{2}$$

$$l = \lim_{n \rightarrow \infty} \left(\frac{\sin \left(\frac{\pi}{n+1} \right)}{\frac{\pi}{n+1}} - 2 \times 1 \right)$$

$$\begin{aligned} \{l\} &= \{ \pi - 2 \} \\ &= \{ \pi \} \\ &= \{ 3.14 \} \\ &= \underline{\underline{14}} \end{aligned}$$

$$\begin{aligned} l &= \frac{1}{\pi} - 2 \\ l &= \pi - 2 \end{aligned}$$

Q
mans
5 times

$$\lim_{x \rightarrow 0} \frac{\sin(\pi \cos^2 x)}{x^2} \xrightarrow{\text{L'Hôpital}} \lim_{x \rightarrow 0} \frac{\sin(\pi \cos^2 x)}{\pi \cos^2 x} \times \frac{\pi \cos^2 x}{x^2}$$

(1)

$$\lim_{x \rightarrow 0} \frac{\sin(\pi(1 - \sin^2 x))}{x^2} \xrightarrow{\text{L'Hôpital}} \lim_{x \rightarrow 0} \frac{\sin(\pi - \pi \sin^2 x)}{x^2}$$

(2)

$$\lim_{x \rightarrow 0} \frac{\sin(\pi - \pi \sin^2 x)}{x^2} \xrightarrow{\text{L'Hôpital}} \lim_{x \rightarrow 0} \frac{\cos(\pi - \pi \sin^2 x) \cdot (-2\pi \sin x \cos x)}{2x}$$

(3)

$$\lim_{x \rightarrow 0} \frac{\cos(\pi - \pi \sin^2 x) \cdot (-2\pi \sin x \cos x)}{2x} \xrightarrow{\text{L'Hôpital}} \lim_{x \rightarrow 0} \frac{\cos(\pi - \pi \sin^2 x) \cdot (-2\pi \sin x \cos x)}{2x}$$

(4)

$$\lim_{x \rightarrow 0} 1 \times \pi \times \left(\frac{|\sin x|}{x} \right)^2 = \pi$$

(5)

$$\lim_{x \rightarrow 0} \frac{\sin(\pi \cos^2 x)}{x^2} = \pi$$

$$\lim_{x \rightarrow 0} \frac{\sin(\pi \cdot \cos^2(\tan(\sin x)))}{x^2}$$

$$\lim_{x \rightarrow 0} \frac{\sin(\pi \cdot (1 - \sin^2(\tan(\sin x))))}{x^2} \xrightarrow{\text{L'Hôpital}} \lim_{x \rightarrow 0} \frac{\cos(\pi - \pi \sin^2(\tan(\sin x))) \cdot (-2\pi \sin(\tan(\sin x)) \cdot \cos(\tan(\sin x)) \cdot \cos x)}{2x}$$

(6)

$$\lim_{x \rightarrow 0} \frac{\cos(\pi - \pi \sin^2(\tan(\sin x))) \cdot (-2\pi \sin(\tan(\sin x)) \cdot \cos(\tan(\sin x)) \cdot \cos x)}{2x} \xrightarrow{\text{L'Hôpital}} \lim_{x \rightarrow 0} \frac{\cos(\pi - \pi \sin^2(\tan(\sin x))) \cdot (-2\pi \sin(\tan(\sin x)) \cdot \cos(\tan(\sin x)) \cdot \cos x)}{2x}$$

(7)

Ans

$$\lim_{x \rightarrow 0} \frac{\sin(\pi \cos^2(\tan(\sin x)))}{x^2}$$

$$\lim_{x \rightarrow 0} \frac{\sin(\pi \cos^2(\tan(\sin x)))}{x^2} \xrightarrow{\text{L'Hôpital}} \lim_{x \rightarrow 0} \frac{\cos(\pi \cos^2(\tan(\sin x))) \cdot (-2\pi \cos(\tan(\sin x)) \cdot \sin(\tan(\sin x)) \cdot \cos x)}{2x}$$

(8)

$$1 - \cos 2x = 2 \sin^2 x, \quad \sin 2x = 2 \sin x \cos x$$

Q $\lim_{x \rightarrow 0} \frac{x \boxed{\tan 2x} - 2x \boxed{\tan x}}{(1 - \cos 2x)^2}$

Adv

$$\lim_{x \rightarrow 0} \frac{x \cdot \frac{\sin 2x}{\cos 2x} - \frac{2x \sin x}{\cos x}}{4 \sin^4 x}$$

$$\lim_{x \rightarrow 0} \frac{2x \cancel{\sin x} \left[\frac{\cos x}{\cos 2x} - \frac{1}{\cos x} \right]}{24 \cdot \cancel{\sin x} \cancel{\cos x} \cdot \sin^2 x}$$

$$\frac{1}{2} \lim_{x \rightarrow 0} \left[\frac{\cos^2 x - \boxed{\cos 2x}}{\boxed{\cos x} \boxed{\cos 2x}} \right] \sin^2 x$$

$$\frac{1}{2} \lim_{x \rightarrow 0} \frac{\cos^2 x - (2 \cos^2 x - 1)}{\underbrace{\cos 0} \cdot \underbrace{\cos 0} \cdot \sin^2 x} = \frac{1}{2} \lim_{x \rightarrow 0} \frac{1 - \cos^2 x}{\sin^2 x} = 1 \times \frac{1}{2} = \frac{1}{2}$$

Q $\lim_{x \rightarrow \infty} x \left[\tan^{-1} \left(\frac{x+1}{x+2} \right) - \tan^{-1} \left(\frac{x}{x+2} \right) \right]$

Adv

$\tan^{-1} A - \tan^{-1} B = \tan^{-1} \frac{A-B}{1+AB}$

$$\lim_{x \rightarrow \infty} x \tan^{-1} \left[\frac{\frac{x+1}{x+2} - \frac{x}{x+2}}{1 + \left(\frac{x+1}{x+2} \right) \left(\frac{x}{x+2} \right)} \right]$$

$$\lim_{x \rightarrow \infty} x \tan^{-1} \left(\frac{\frac{1}{x+2}}{\frac{2x^2+5x+4}{(x+2)^2}} \right)$$

$$\lim_{x \rightarrow \infty} x \cdot \tan^{-1} \left(\frac{\frac{x+2}{2x^2+5x+4}}{\left(\frac{x+2}{2x^2+5x+4} \right)} \right) \times \left(\frac{x+2}{2x^2+5x+4} \right)$$

$$\lim_{x \rightarrow \infty} x \cdot \left(\frac{x+2}{2x^2+5x+4} \right)^2 = \frac{1}{2}$$

$$\boxed{\lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2} = \frac{1}{2}} \quad \text{L'Hôpital's Rule}$$

$$\frac{1 - \cos(\text{Same})}{(\text{Same})^2}$$

$$\frac{1 - \cos 0}{0^2}$$

$$Q \lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2} \left(\frac{0}{0} \right) \text{DL}$$

$$\lim_{x \rightarrow 0} \frac{0 + \sin x}{2x} = \frac{1}{2}$$

$$Q \lim_{x \rightarrow 0} \frac{1 - \cos(mx)}{x^2}$$

$$\lim_{x \rightarrow 0} \frac{1 - \cos(mx)}{x^2} \times \frac{m^2 x}{m^2 x}$$

$$\frac{1}{2} \times 1 = \frac{1}{2}$$

$$Q \lim_{x \rightarrow 0} \frac{1 - \cos(1 - \cos x)}{x^4}$$

$$\lim_{x \rightarrow 0} \frac{1 - \cos(1 - \cos x)}{(1 - \cos x)^2} \times \frac{(1 - \cos x)^2}{x^4}$$

$$\frac{1}{2} \times \lim_{x \rightarrow 0} \left(\frac{1 - \cos x}{x^2} \right)^2$$

$$\frac{1}{2} \times \left(\frac{1}{2} \right)^2 = \frac{1}{8}$$

$$Q \lim_{x \rightarrow 0} \frac{1 - \cos 3x}{x^2}$$

$$\lim_{x \rightarrow 0} \frac{1 - \cos(3x)}{9x^2} \times 9$$

$$\frac{1}{2} \times 9 = \frac{9}{2}$$

$$Q \lim_{x \rightarrow 0} \frac{1 - \cos mx}{x^2}$$

$$\lim_{x \rightarrow 0} \frac{1 - \cos(mx)}{m^2 x^2} \times m^2 = \frac{m^2}{2}$$

$$Q \lim_{x \rightarrow 0} \frac{1 - \cos 7x}{x^2} = \frac{7^2}{2} = \frac{49}{2}$$

$$Q \lim_{x \rightarrow 0^+} \frac{\pi(1 - \sqrt{1-x^2})}{\sqrt{1+x^2}(\sin^2 x)^2}$$

$$\lim_{t \rightarrow 0^+} \frac{\pi(1 - \sqrt{1-\sin^2 t})}{\sqrt{1+\sin^2 t}(t)^2}$$

$$\frac{1}{1} \lim_{t \rightarrow 0^+} \frac{\pi(1 - \cos t)}{t^2} = \frac{\pi}{2}$$

Q If $ax^2+bx+c=0$ has 2 Root α & β then.

$$\text{find } \lim_{x \rightarrow \alpha} \frac{1 - \cos(ax^2+bx+c)}{(x-\alpha)^2}$$

Single Party Inverse method

take that = t

$$\begin{array}{|l} \sin^2 x = t \\ x = \sin t \\ x \rightarrow 0^+ \\ \sin t \rightarrow 0^+ \\ t \rightarrow 0^+ \end{array}$$

calculation

$$\sqrt{\cos^2 t} = |\cos t| = \cos t$$

$ax^2+bx+c=0$ has Root α & β .

$$ax^2+bx+c = a(x-\alpha)(x-\beta)$$

$$\lim_{x \rightarrow \alpha} \frac{1 - \cos(a(x-\alpha)(x-\beta))}{a^2(x-\alpha)^2(x-\beta)^2} = \frac{1}{2} \times a^2(\alpha-\beta)^2$$

$$Q \lim_{x \rightarrow 0} \frac{(t \sin x - \sin x)}{x^3}$$

$$\lim_{x \rightarrow 0} \left(\sin x \right) \left(\frac{1}{\cos x} - 1 \right) = \lim_{x \rightarrow 0} \frac{1 - \cos x}{\cos x \times x^2} = 1 \times \frac{1}{1} \times \frac{1}{2} = \frac{1}{2}$$

$$\textcircled{1} \lim_{x \rightarrow \boxed{\frac{\pi}{2}}} \frac{(\sin x - \cos x)}{(\pi - 2x)^3}$$

$x = \text{constant}$

$x = \text{constant} - h$

$$\boxed{x = \frac{\pi}{2} - h}$$

$$\lim_{h \rightarrow 0} \frac{(\sin(\frac{\pi}{2} - h) - \cos(\frac{\pi}{2} - h))}{(\pi - (\pi - 2h))^3}$$

$$\frac{1}{8} \lim_{h \rightarrow 0} \frac{\tan h - \sin h}{h^3}$$

$$= \frac{1}{8} \times \frac{1}{2} = \frac{1}{16}$$

$$\textcircled{2} \lim_{x \rightarrow \frac{\pi}{2}} \boxed{\frac{(1 - \tan x)}{(1 + \tan x)}} \cdot \frac{(1 - \sin x)}{(\pi - 2x)^3}$$

Adv

$$\boxed{x = \frac{\pi}{2} - h}$$

$$\lim_{x \rightarrow \frac{\pi}{2}} \frac{\tan(\frac{\pi}{4} - \frac{x}{2}) \cdot (1 - \sin x)}{(\pi - 2x)^3}$$

$$\lim_{h \rightarrow 0} \frac{\tan(\frac{\pi}{4} - \frac{\pi}{4} + \frac{h}{2}) (1 - \sin(\frac{\pi}{2} - h))}{(\pi - \pi + 2h)^3}$$

$$\frac{1}{8} \lim_{h \rightarrow 0} \frac{\cancel{\tan(\frac{h}{2})} (1 - \cancel{\cos h})}{\cancel{\frac{h}{2} \times 2} \cancel{h^2}} = \frac{1}{8} \times 1 \times \frac{1}{2} \times \frac{1}{2}$$

$$= \frac{1}{32}$$

$$\textcircled{3} \lim_{x \rightarrow 0} \frac{1 - \cos x \sqrt{\cos 2x}}{\tan^2 x} \quad \boxed{\text{Rat.}}$$

$$\lim_{x \rightarrow 0} \frac{1 - \cos^2 x (\cos 2x)}{x^2 (1 + \cos x \sqrt{\cos 2x})} \rightarrow \text{limit not}$$

$$\lim_{x \rightarrow 0} \frac{1 - \cos^2 x (1 - 2 \sin^2 x)}{x^2}$$

$$\frac{1}{2} \lim_{x \rightarrow 0} \frac{1 - \cos^2 x + 2 \sin^2 x \cos^2 x}{x^2} = \frac{1}{2} \left(\frac{1}{2} + 2 \times 1 \times (\cos^2 0) \right) = \frac{5}{4}$$

$$Q \lim_{x \rightarrow 0} \frac{1 - \cos x \cdot \cos 2x \cdot \cos 3x}{8x^2(2x)}$$

$$\lim_{x \rightarrow 0} \frac{1 - (\cos x \cdot \cos 2x \cdot \cos 3x)}{4x^2}$$

$$\frac{1 - \frac{1}{4}(1 + \cos 2x + \cos 4x + \cos 6x)}{4x^2} = \frac{7}{4}$$

$$\frac{1}{2} (2 \cos x \cdot \cos 2x) \cos 3x$$

$$\frac{1}{2} ((\cos(3x) + \cos(-x))) \cdot \cos 3x$$

$$\frac{1}{2} (2 \cos^2(3x) + 2 \cos x \cdot \cos 3x)$$

$$\frac{1}{4} (1 + \cos 6x + \cos 4x + \cos 2x)$$

$$2 \cos A \cdot \cos B = \cos(A+B) + \cos(A-B)$$

$$16) \lim_{\substack{x \rightarrow -1 \\ |x| = -1}} \frac{\ln 2 - \ln 2x}{x^2 - 1x}$$

$$\lim_{x \rightarrow -1} \frac{\ln 2 - \ln 2x}{x^2 + x} = \frac{0}{0}$$

$$\frac{0 + 2 \ln 2x}{(2x + 1)}$$

$$\frac{2 \ln(-2)}{-2 + 1}$$

$$\frac{+2 \ln 2}{+1} = 2 \ln 2$$

$$\left[\overset{\text{Gr. than}}{\text{less than } -5} \right] + \left[\text{less than } 0 \right]$$

$$-5 + 5 = 0$$

$$\left[\frac{\ln x}{x} \right] + \left[\frac{2 \ln 2x}{2x} \right] + \left[\frac{3 \ln 3x}{3x} \right]$$

$$\left[< 1 \right] + \left[4 \left(\frac{\ln 2x}{2x} \right) \right]$$

$$\left[< 1 \right] + \left[< 4 \right] + \left[< 9 \right] + \left[< 16 \right] + \left[< 25 \right] + \dots \left[< 100 \right]$$

$$0 + 3 + 8 + 15 + \dots$$

$$\frac{\ln x}{x} < 1$$

$$\frac{-5 \ln x}{x} > -5$$

$$\frac{-5}{-6} \quad \frac{-5}{-5}$$

$$Q19) f(5+h) = \lim_{h \rightarrow 0} \frac{\ln \{5+h\}}{(5+h)^2 + a(5+h) + b} = \lim_{h \rightarrow 0} \frac{\cancel{\ln h} h}{(5+h)^2 + a(5+h) + b}$$

$$\Rightarrow \lim_{h \rightarrow 0} \frac{h}{h \left(h \left(\frac{5}{h} + 1 \right)^2 + 5a + b \right)}$$

$$f(5+h) = \lim_{h \rightarrow 0} \frac{\delta f \{5+h\}}{(5+h)^2 + a(5+h) + b} = \lim_{h \rightarrow 0} \frac{\delta f h \xrightarrow{0}}{(5+h)^2 + a(5+h) + b \xrightarrow{25+5a+b=0}} = \frac{0}{0}$$

$5a+b+25=0$

$$f(3+h) \rightarrow 0 \text{ as } h \rightarrow 0 \xrightarrow{5a+b+25=0}$$