

I: Draw the graph of $f(x) = \left(1 + \frac{1}{x}\right)^x$.

$$(-\infty, -1) \cup (0, \infty)$$

$$f'(x) = \underbrace{\left(1 + \frac{1}{x}\right)^x}_{>0} \left(\ln\left(1 + \frac{1}{x}\right) - \frac{1}{x+1} \right) > 0$$

$$g(x) = \ln\left(1 + \frac{1}{x}\right) - \frac{1}{x+1}$$

$$g'(x) = -\frac{1}{x(x+1)} + \frac{1}{(x+1)^2}$$

$$= \frac{-1}{x(x+1)^2}$$

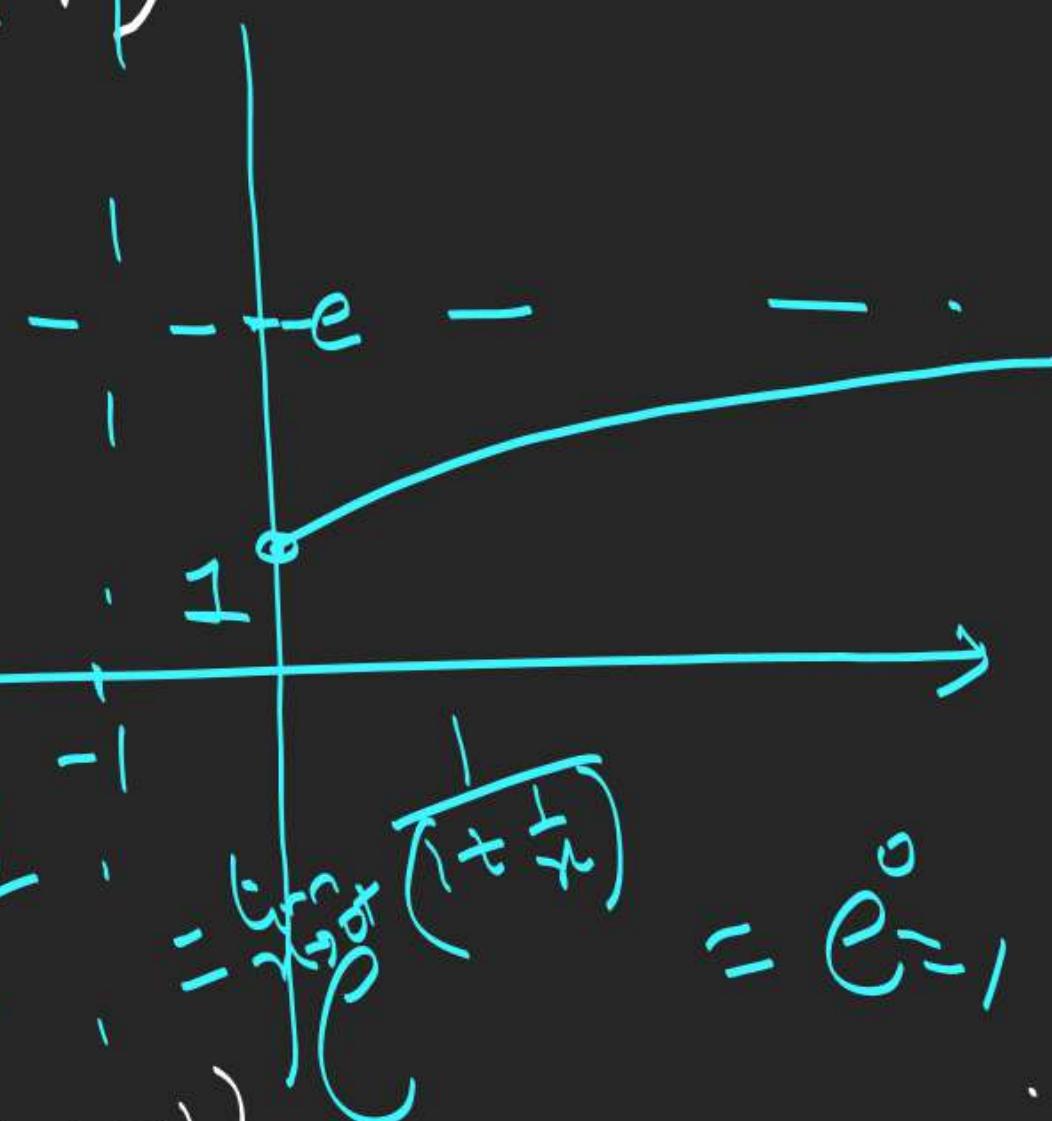
$$\begin{aligned} g'(\underset{x \in (-\infty, -1)}{x}) &> 0 \\ \downarrow x \in (0, \infty) &\Rightarrow g(x) > \lim_{x \rightarrow -\infty} g(x) = 0 \end{aligned}$$

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$$\lim_{x \rightarrow \infty} f(x) = e$$

$$\lim_{x \rightarrow -\infty} f(x) = \infty$$

$$\lim_{x \rightarrow 0^+} f(x) = e^{\lim_{x \rightarrow 0^+} \frac{x \ln(1 + \frac{1}{x})}{x}} = e^{\lim_{x \rightarrow 0^+} \frac{\ln(1 + \frac{1}{x})}{1 + \frac{1}{x}}} = e^{\lim_{x \rightarrow 0^+} \frac{\frac{1}{1 + \frac{1}{x}}}{-\frac{1}{(1 + \frac{1}{x})^2}}} = e^{-1}$$

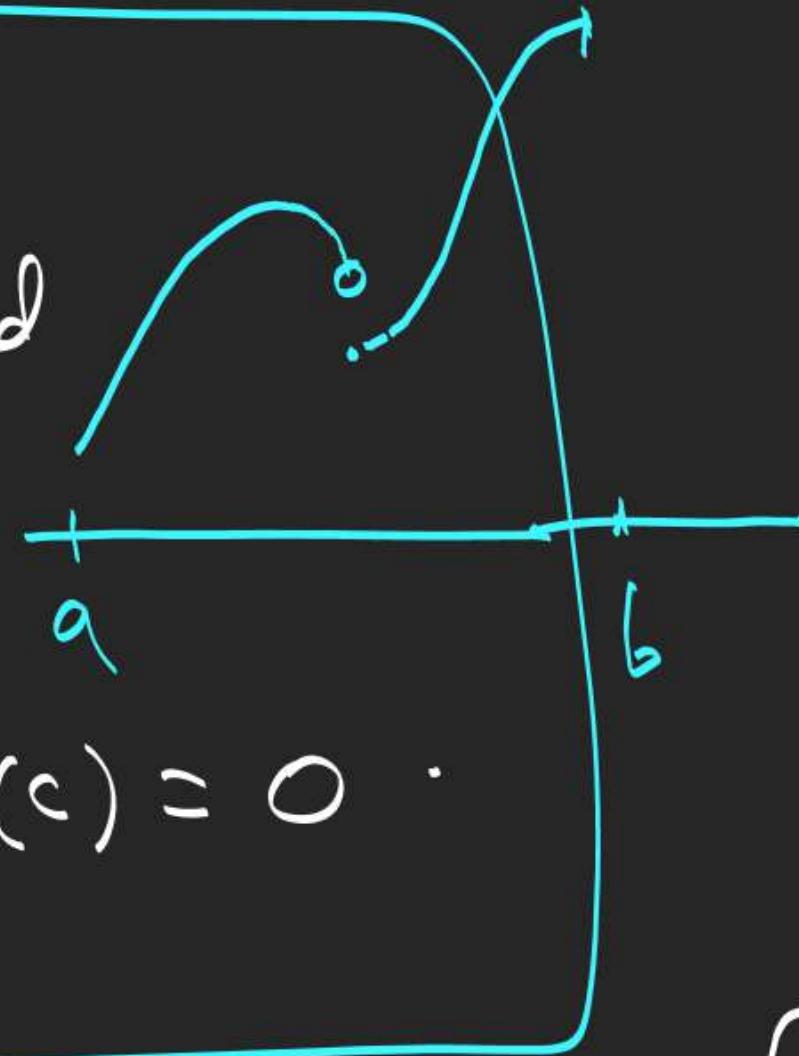
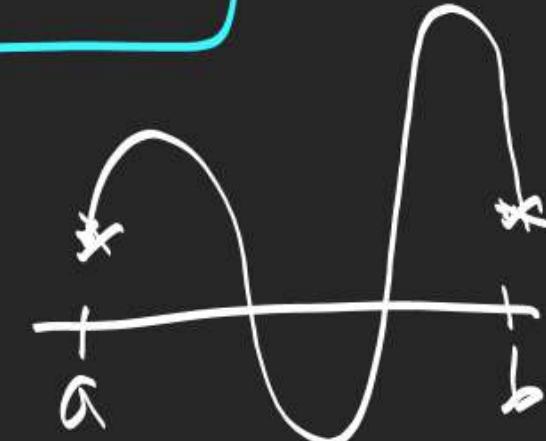
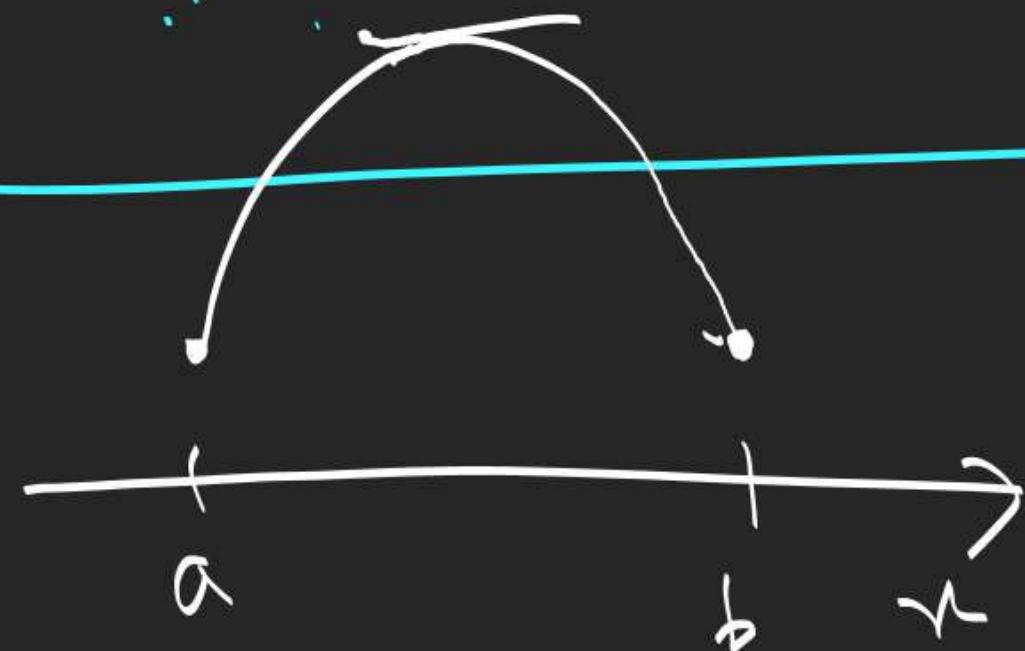


$$R_f = (-\infty, 1) \cup (e, \infty)$$

Rolle's Theorem

- ① $f(x)$ is continuous in $[a, b]$
- ② $f(x)$ is differentiable in (a, b) , and
- ③ $f(a) = f(b)$

$\Rightarrow \exists c \in (a, b)$, such that $f'(c) = 0$.



Lagrange's Mean Value Theorem (LMVT)

- If $f(x)$ is continuous in $[a, b]$, and
- $f(x)$ is differentiable in (a, b) ,

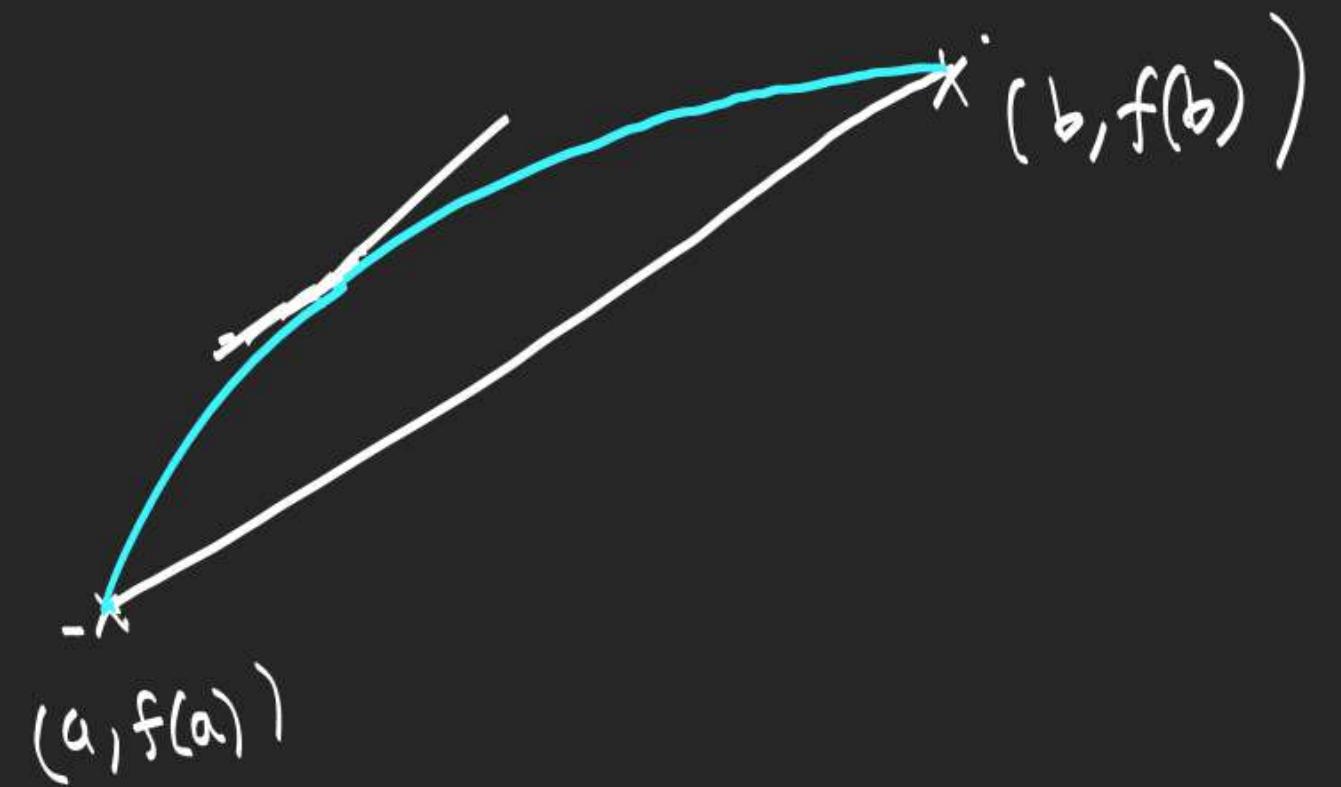
$$\Rightarrow \exists c \in (a, b) \text{, s.t. } f'(c) = \frac{f(b) - f(a)}{b - a}$$

⇒ Using Rolle's

$$g(b) - g(a) = (f(b) - f(a)) - \left(\frac{f(b) - f(a)}{b - a} \right)(b - a) = 0.$$

$$g(x) = f(x) - \left(\frac{f(b) - f(a)}{b - a} \right)x$$

$\therefore g$ is cont. in $[a, b]$
 $\therefore g$ is diff. in (a, b)
 $\therefore g(b) = g(a)$



$$\frac{f(b) - f(a)}{b - a}$$

Generalized Mean Value Theorem

11-15 (Int.)
 Diff. - Ex-4 (1-10)

(Cauchy's Mean Value Theorem)

- $f(x), g(x)$ are cont. in $[a, b]$
- $f(x), g(x)$ are diff. in (a, b)

$$\Rightarrow \exists c \in (a, b) \text{ s.t. } (f(b) - f(a))g'(c) = (g(b) - g(a))f'(c).$$