

$$3h = x_1 + x_2 + x_3 = \lambda x_1$$

$$3k = \frac{x_1}{2} - \frac{x_2}{3} + x_3 = \mu x_1$$

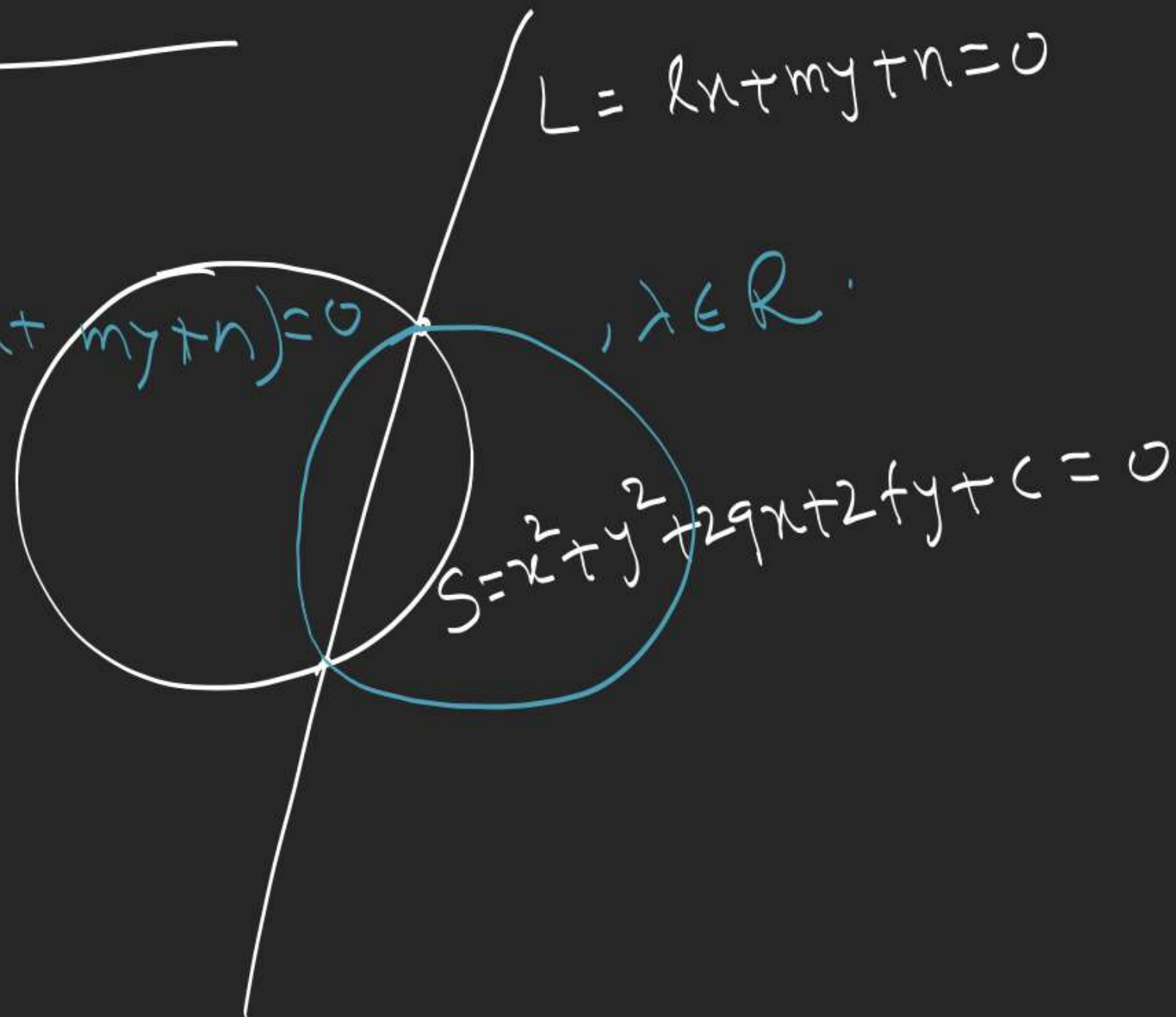
$$\frac{\frac{x_1}{2} + \frac{x_2}{3}}{x_1 - x_2} = -1 \Rightarrow x_2 = f(x_1)$$

$$\frac{x_3 - \frac{x_1}{2}}{x_3 - x_1} = 3 \Rightarrow x_3 = g(x_1)$$

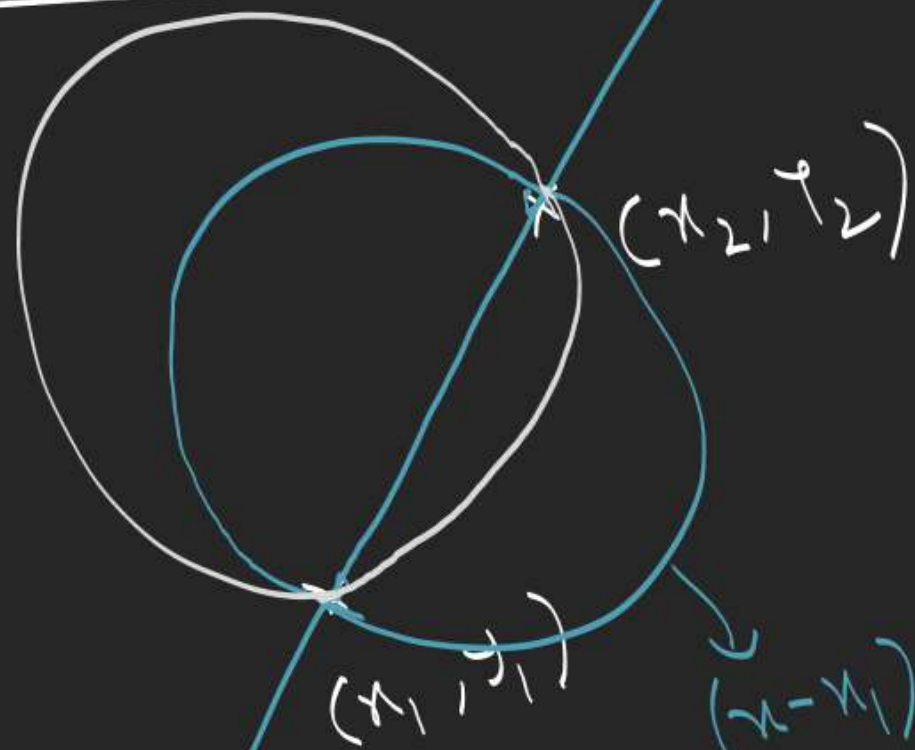
Family of Circles through intersection
of ^{given} a line & a circle

$$x^2 + y^2 + 2gx + 2fy + c + \lambda(lx + my + n) = 0, \lambda \in \mathbb{R}$$

$$\boxed{S + \lambda L = 0, \lambda \in \mathbb{R}}$$



Family of Circles thru two given points



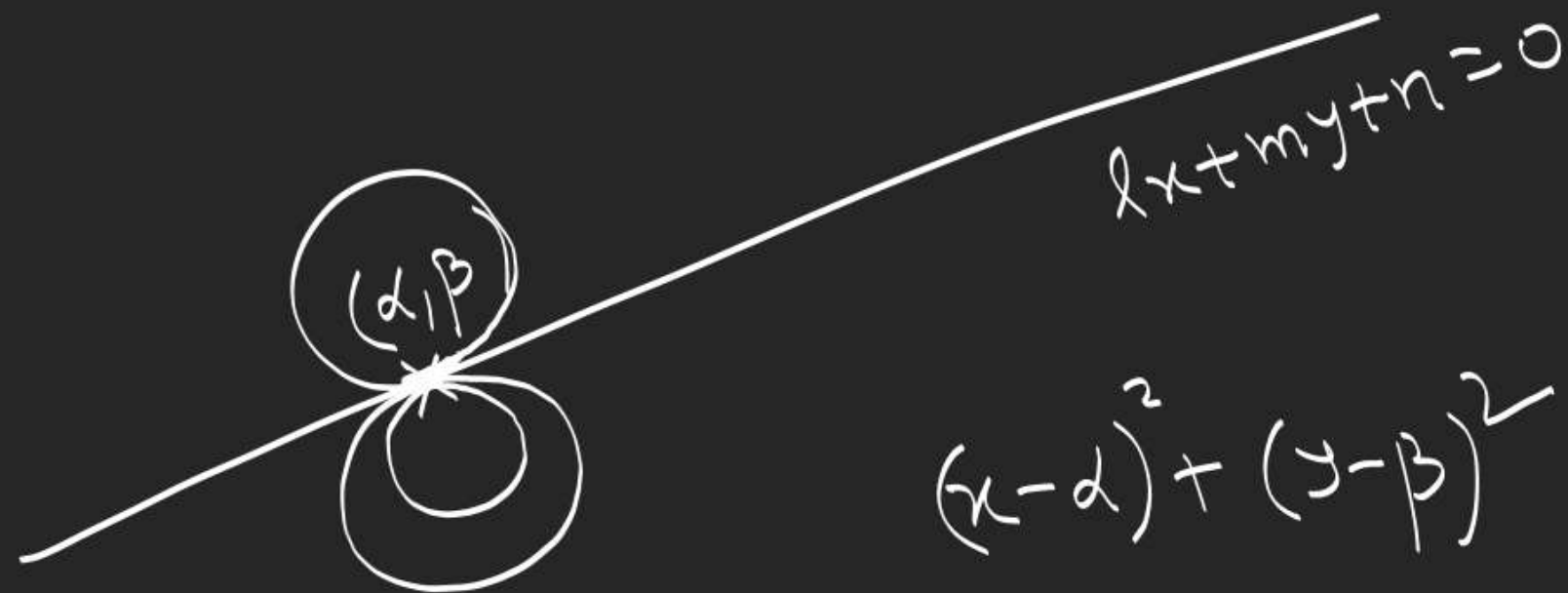
$$(x-x_1)(x-x_2) + (y-y_1)(y-y_2) + \lambda \left((y-y_1) - \left(\frac{y_2-y_1}{x_2-x_1} \right) (x-x_1) \right) = 0$$

$$(x-x_1)(x-x_2) + (y-y_1)(y-y_2) = 0$$

$$\lambda \in \mathbb{R}$$

$$y-y_1 = \left(\frac{y_2-y_1}{x_2-x_1} \right) (x-x_1)$$

Family of Circles touching a line at a given point on it

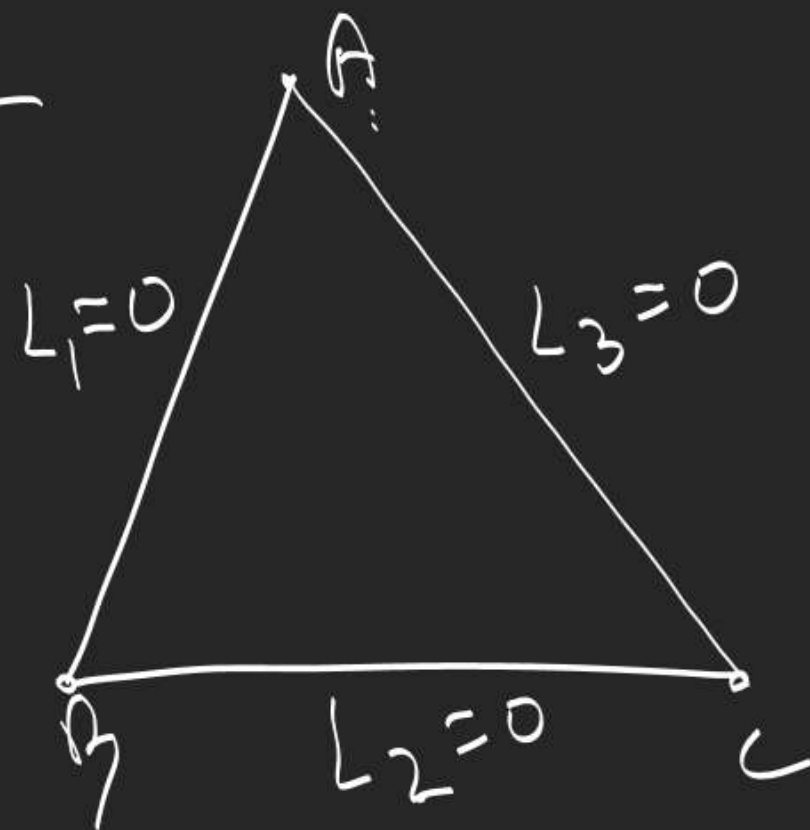


$$(x - \alpha)^2 + (y - \beta)^2 + \lambda (lx + my + n) = 0$$

$$\lambda \in \mathbb{R}$$

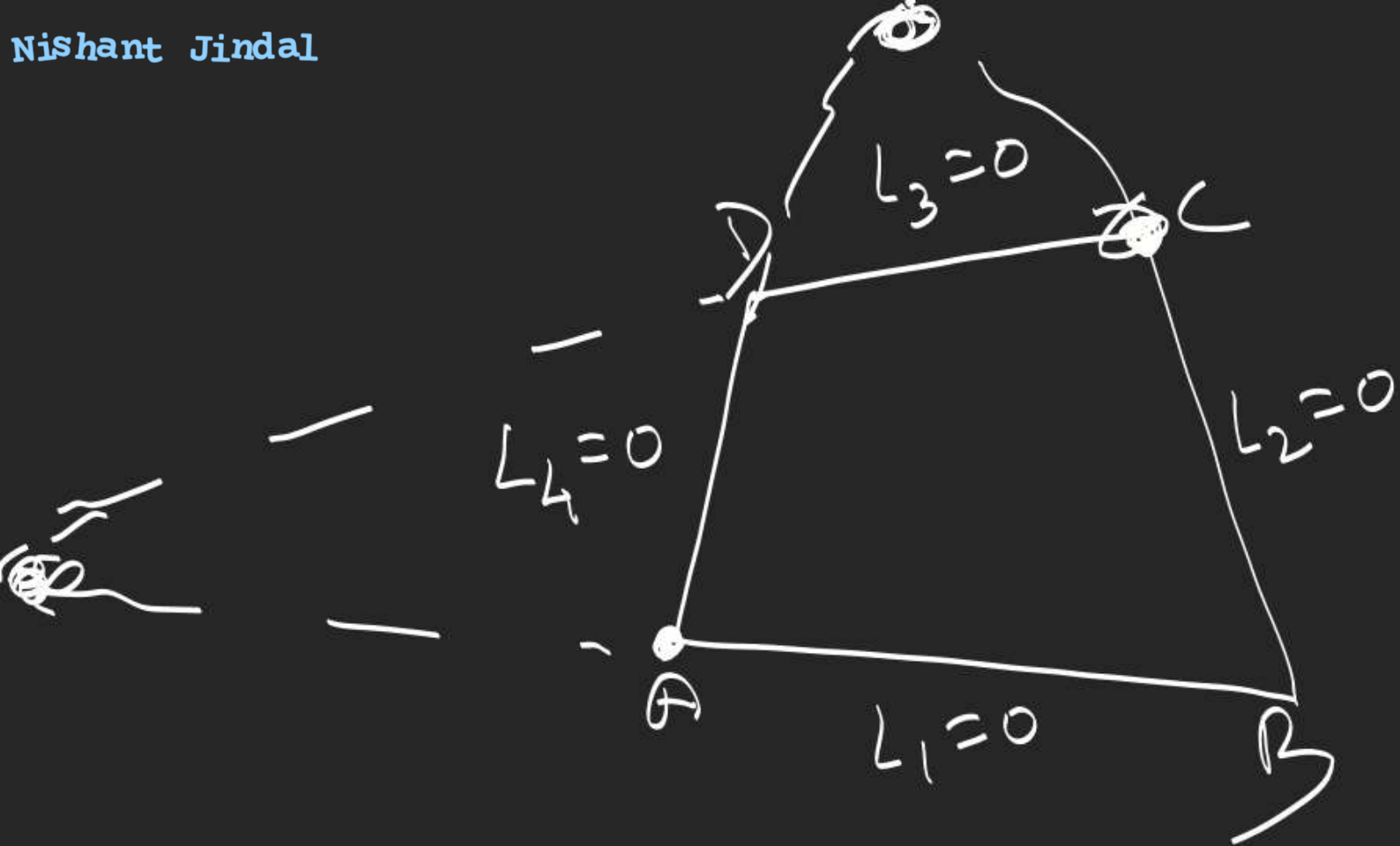


Circumcircle



$$L_1 L_2 + \lambda L_2 L_3 + \mu L_3 L_1 = 0$$

$$\left. \begin{array}{l} \text{Coeff. of } x^2 = \text{Coeff. of } y^2 \\ \text{Coeff. of } xy = 0 \end{array} \right\} \lambda, \mu = ?$$



Circumcircle ✓

$$L_1 L_3 + \lambda L_2 L_4 = 0$$

$$\text{Coeff. of } x^2 = \text{Coeff. of } y^2$$

$$\text{Coeff. of } xy = 0$$

$$\lambda = ?$$

$$L_1 L_2 + \lambda L_3 L_4 = 0$$

X

Find the eqn. of circle which touches the line $2x - y = 4$ at point $(1, -2)$ and

(i) passes through $(3, 4)$

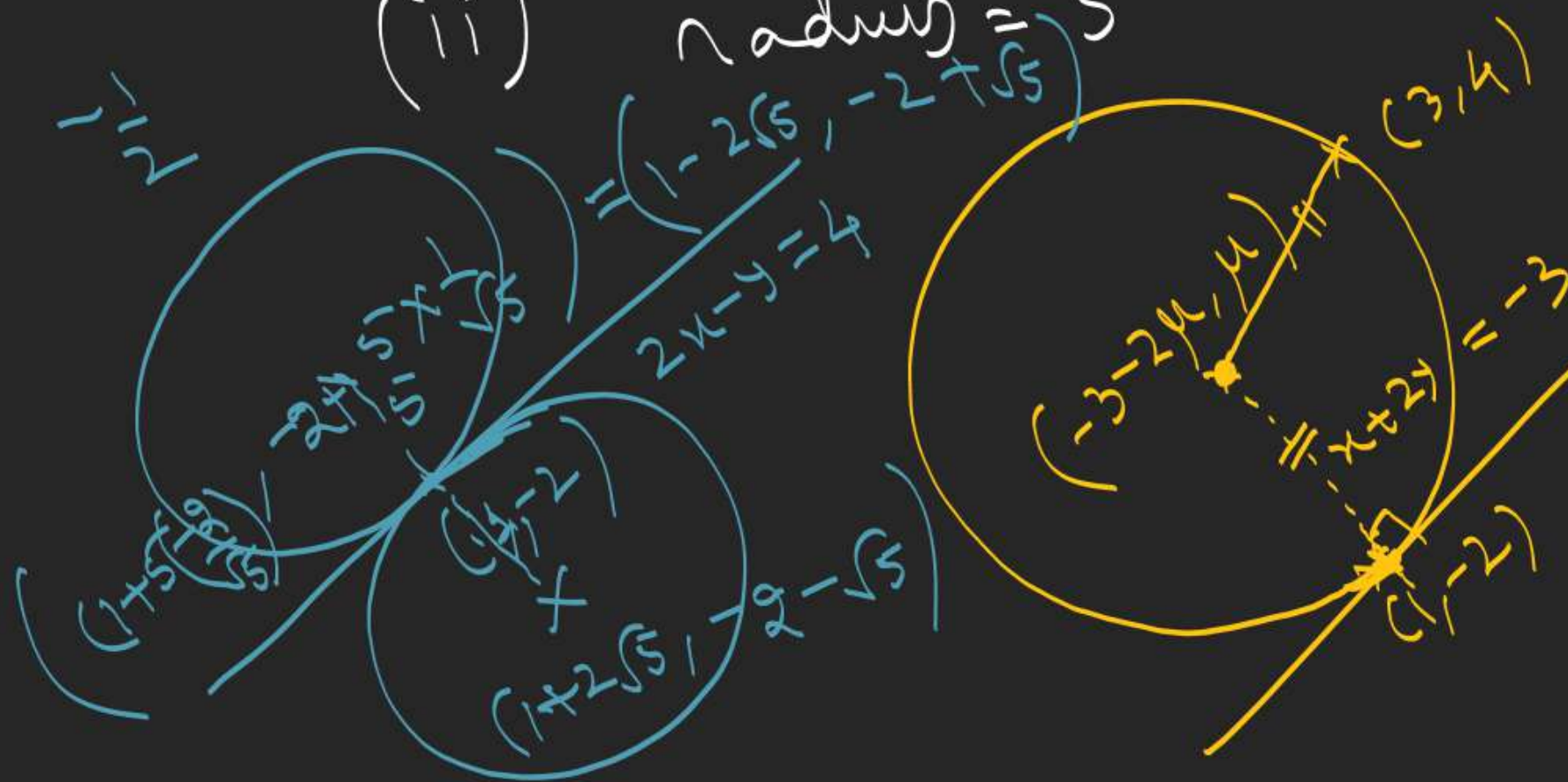
$$(x-1)^2 + (y+2)^2 + \lambda(2x-y-4) = 0$$

Put $(3, 4)$

$$4 + 36 + \lambda(-2) = 0$$

$$\lambda = 20$$

(ii) radius = 5

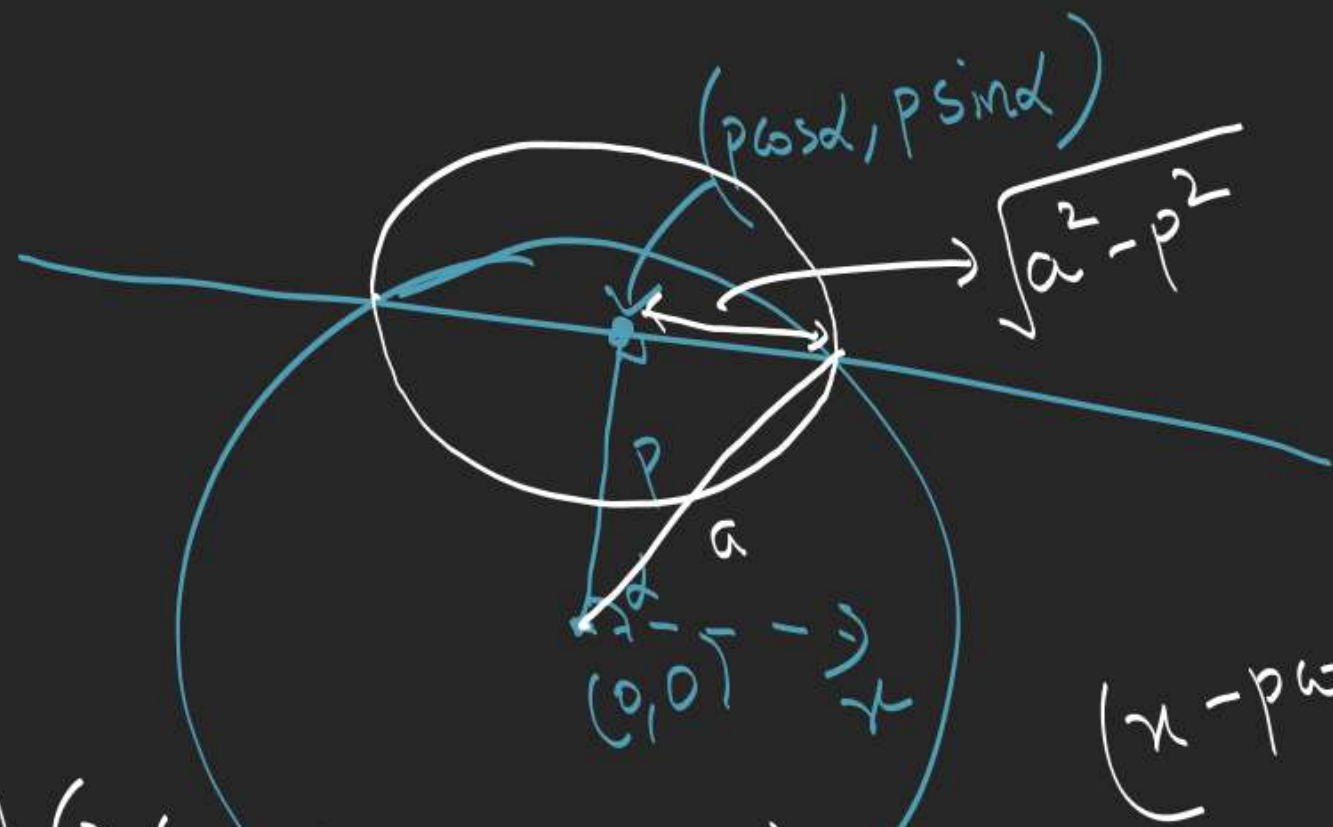


$$2x - y - 4 = 0$$

$$(6 + 2\mu)^2 + (4 - \mu)^2 = (4 + 2\mu)^2 + (\mu + 2)^2$$

$\mu = ?$

2. Find the eqn. of circle drawn on chord $x \cos \alpha + y \sin \alpha = p$ of the circle $x^2 + y^2 = a^2$ as its diameter.



$$(x - p \cos \alpha)^2 + (y - p \sin \alpha)^2 = a^2 - p^2$$

$$x^2 + y^2 - a^2 + \lambda(x \cos \alpha + y \sin \alpha - p) = 0$$

Put $\left(-\frac{1}{\lambda} \cos \alpha, -\frac{1}{\lambda} \sin \alpha\right)$ to $x \cos \alpha + y \sin \alpha = p$, $\lambda = ?$

3. Find the equation of circle which passes through origin and through the point of contact of tangents drawn from origin to the circle $x^2 + y^2 - 11x + 13y + 17 = 0$.

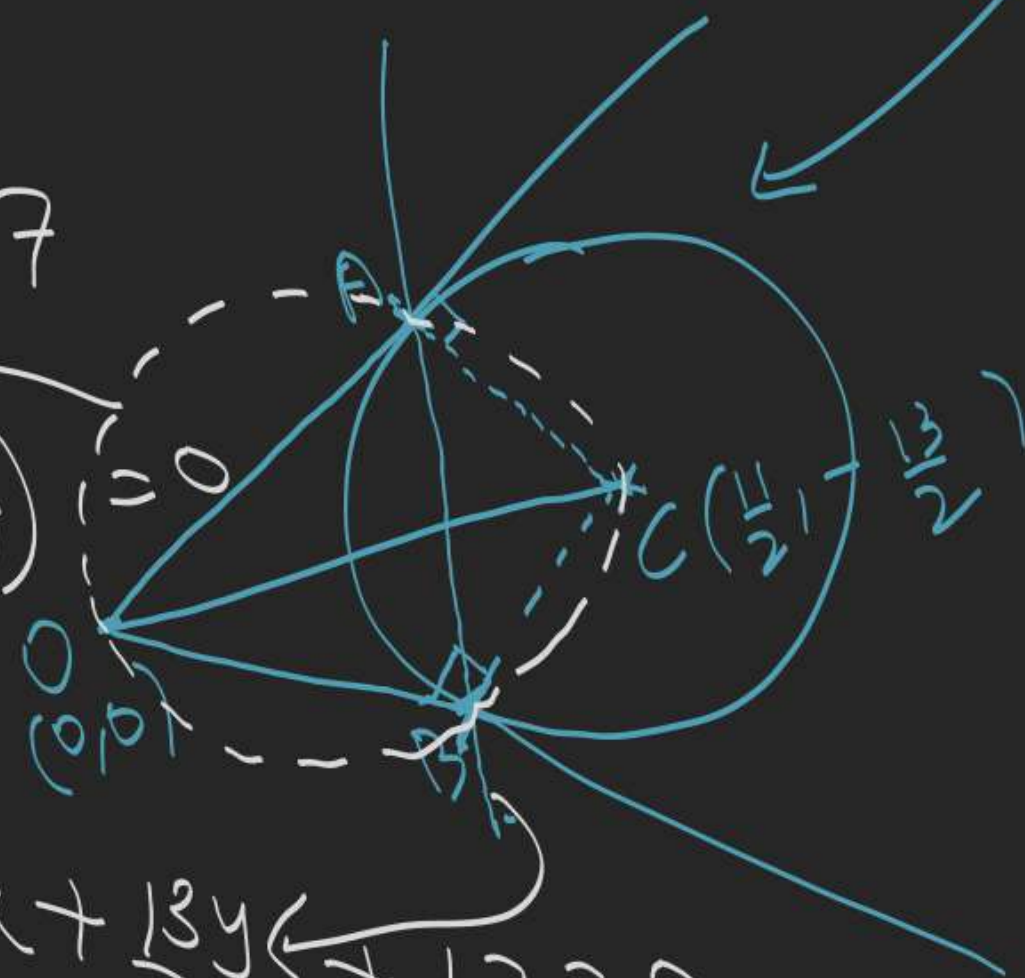
$$x^2 + y^2 - 11x + 13y + 17$$

$$+ \lambda(11x - 13y - 34) = 0$$

Put $(0,0)$

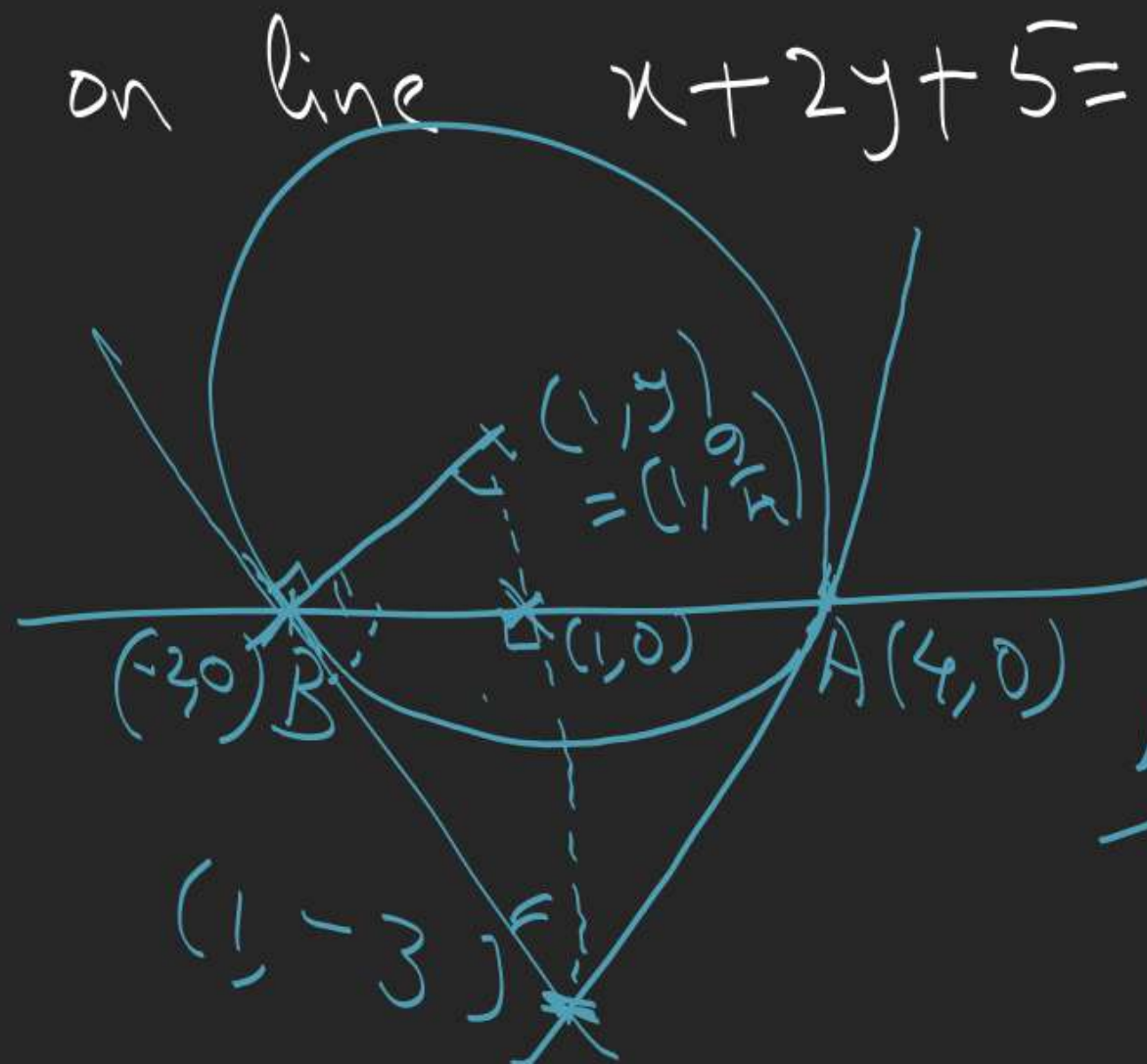
$\lambda = ?$

$$-\frac{11}{2}x + \frac{13}{2}y + 17 = 0$$



$$x\left(x - \frac{11}{2}\right) + y\left(y + \frac{13}{2}\right) = 0$$

4. S.T. eqn $x^2 + y^2 - 2x - 2\lambda y - 8 = 0$ represents for different values of λ , a system of circles passing through fixed points A, B . Also find the eqn. of that circle of the system, the tangent to which at A and B meet on line $x + 2y + 5 = 0$.



$$\begin{aligned} (x^2 + y^2 - 2x - 8) - 2\lambda(y) &= 0 \\ x=0 \quad x^2 - 2x - 8 &= 0 \end{aligned}$$

$$\boxed{A(4, 0), B(-2, 0)}$$

$$\frac{4}{3} = \frac{3}{y}$$

$$\frac{a}{h} = \frac{3}{4}$$