Nishant Jindal

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New CHAPTER

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Sequence & Progression

Sequence

A) A sequence is a fxn of natural No whose codomain is set of Real No.

B) Seq can be finite or Infinite according to the No of terms.

C) Seq is an ordered list of object or events or No.

1, 3, 5, 7 ...

1, 22, 333, 4444 ...

1, 4, 9, 16 ...

3, 9, 27, 81 ...

Q Can you find Next No in Seq?

0, 3, 8, 15, 24, ?

0, 3, 8, 15, 24, 35

3, 5, 7, 9, 11 ← 1^{st} order diff

2, 2, 2, 2 ← 2^{nd} order diff

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Seq :- A fxn $f: N \to R$ defined as

$f(n) = \{t_n\}$; $n \in N$ is called Real seq^n

$\boxed{\begin{array}{l} t_1 = 1^{st} \text{ term.} \\ t_2 = 2^{nd} \text{ term.} \\ t_3 = 3^{rd} \text{ term} \\ \vdots \\ t_r = r^{th} \text{ term.} \\ \vdots \\ t_n = n^{th} \text{ term.} \end{array}}$

$\underbrace{f : N \to R}$

fxn is taking only Natural No & answers are coming to Real No. only.

$f_n = 2n - 1$; $n \in N$

$= 2\times1-1, 2\times2-1, 2\times3-1, 2\times4-1 \ldots \ldots$

$= 1, 3, 5, 9, 11, 13 \ldots \ldots$

$\underline{\infty \text{ Seq}^n}$

$\therefore$ If it were like 1, 3, 5, 7, 9 only then finite seq^n

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Series.

1) A series is sum of No. of terms of seq^n

2) If $\{a_n\}$ is seq^n containing the terms $a_1, a_2, a_3, a_4 \ldots$ then Series will be

$a_1 + a_2 + a_3 + a_4 \cdots$

Ex: $t_r = \frac{1}{r}$ then ① Seq^n ② Series?

$t_1 = \frac{1}{1}$

$t_2 = \frac{1}{2}$

$t_3 = \frac{1}{3}$

$t_4 = \frac{1}{4}$

① Seq^n $\frac{1}{1}, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots$

② Series $\rightarrow \frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \cdots$

(3) And Sum of all n terms of series is Rep. by S_n

$S_n =$ Sum of n terms.

$S_1 =$ Sum of 1st term.

$S_2 =$ " of 1st two terms.

$S_2 = T_1 + T_2$

$S_3 =$ Sum of 1st 3 terms.

$S_3 = T_1 + T_2 + T_3$

$$\boxed{\begin{aligned} \therefore S_n &= T_1 + T_2 + T_3 + \cdots + T_n \\ S_n &= \sum_{r=1}^{n} T_r \end{aligned}}$$

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Q find $S_n = \sum_{r=1}^{n} \frac{1}{r}$ if $n=3$?

$$S_3 = \sum_{r=1}^{3} \frac{1}{r}$$

$$S_3 = \frac{1}{1} + \frac{1}{2} + \frac{1}{3}.$$

Q $f_n = \{t_n\}$; $t_n = 2^n$ find $S_n = ?$

We know $\Rightarrow S_n = \sum_{r=1}^{n} t_r$

$$S_n = \sum_{r=1}^{n} 2^r = 2^1 + 2^2 + 2^3 + \dots + 2^n$$

(4) $S_1 = T_1$

$S_2 = T_1 + T_2$

$S_3 = T_1 + T_2 + T_3.$

Q find T_3?

$T_3 = S_3 - S_2$

Q find T_5?

$T_5 = S_5 - S_4$

$T_5 = (T_1 + T_2 + T_3 + T_4 + T_5) - (T_1 + T_2 + T_3 + T_4)$

n^{th} term

$$\therefore \boxed{T_n = S_n - S_{n-1}}$$

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Q find n^{th} term?

A) $Seq^n \rightarrow 1, 2, 3, 4, 5, \ldots$

$T_n = n \; ; \; n \in N$

(B) $Seq^n \rightarrow 1, 3, 5, 7, 9, 11 \ldots$

$T_n = 2n - 1, \; n \in N$

$T_1 = 2\times1-1 = 1$

$T_2 = 2\times2-1 = 3$

$T_3 = 2\times3-1 = 5$

$2n+1$

$2\times1+1 = 3$

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(C) $Seq^n \rightarrow 2, 4, 6, 8, 10 \ldots$

$T_n = 2n \; ; \; n \in N$

(D) $Seq^n \rightarrow 1, 2, 4, 8, 16, 32 \ldots$

$T_n = 2^{n-1}$ ✓

$T_1 = 2^{1-1} = 2^0 = 1$

$T_2 = 2^{2-1} = 2^1 = 2$

$T_3 = 2^{3-1} = 2^2 = 4$

$T_n \; 2^n$ ×

$T_1 = 2^1 = 2$

$T_2 = 2^2 = 4$

(E) $Seq^n \rightarrow 1, 3, 9, 27, 81 \ldots$

$T_n = 3^{n-1}$

$T_1 = 3^{1-1} = 3^0 = 1$

$T_2 = 3^{2-1} = 3^1 = 3$

$T_3 = 3^{3-1} = 3^2 = 9$

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F) Seqⁿ → $1, \frac{1}{3}, \frac{1}{9}, \frac{1}{27} \ldots$

$$\boxed{T_n = \frac{1}{3^{n-1}}}$$

$T_1 = \frac{1}{3^{1-1}} = \frac{1}{3^0} = 1$

$T_2 = \frac{1}{3^{2-1}} = \frac{1}{3^1} = \frac{1}{3}$

$T_3 = \frac{1}{3^{3-1}} = \frac{1}{3^2} = \frac{1}{9}$

(G) $1\cdot 2^0, 2\cdot 2^1, 3\cdot 2^2, 4\cdot 2^3, 5\cdot 2^4, 6\cdot 2^5 \ldots$

$T_n = n\cdot 2^{n-1}$

$T_1 = 1\cdot 2^{1-1} = 1\cdot 2^0 = 1\cdot 1 = 1$

$T_2 = 2\cdot 2^{2-1} = 2\cdot 2^1 = 2\cdot 2$

$T_3 = 3\cdot 2^{3-1} = 3\cdot 2^2$

Q find Seqⁿ if $a_n = \begin{cases} \frac{1}{n} & n = \text{odd} \\ -\frac{1}{n} & n = \text{Even} \end{cases}$?

Seq → $a_1, a_2, a_3, a_4, a_5 \ldots$

$\frac{1}{1}, -\frac{1}{2}, \frac{1}{3}, -\frac{1}{4}, \frac{1}{5}, -\frac{1}{6}, \frac{1}{7}, -\frac{1}{8} \ldots$

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$\boxed{L_1}$

Q Write down seq^n whose n^{th} term is : $\frac{3+(-1)^n}{3^n}$,

$$T_n = \frac{3+(-1)^n}{3^n}$$

Seq^n $\rightarrow$ T_1 , T_2 , T_3 , T_4 - - - -

$$\rightarrow \frac{3+(-1)^1}{3^1}, \frac{3+(-1)^2}{3^2}, \frac{3+(-1)^3}{3^3}, \frac{3+(-1)^4}{3^4}, - - -$$

$$\rightarrow \frac{2}{3}, \frac{4}{9}, \frac{2}{27}, \frac{4}{81} ———$$

Q Writedown seq^n whose n^{th} term is : $\frac{2^n}{n}$,

$$T_n = \frac{2^n}{n}.$$

T_1 , T_2 , T_3 , T_4 - - -

$$\frac{2^1}{1}, \frac{2^2}{2}, \frac{2^3}{3}, \frac{2^4}{4} - - - -$$

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Q If $t_1 = t_2 = 1$ & $t_n = t_{n-1} + t_{n-2}$; $n > 2$

Write 1^{st} 5 terms?

First 2 terms.

$t_3 = t_{3-1} + t_{3-2} = t_2 + t_1 = 1 + 1 = 2$

$t_4 = t_{4-1} + t_{4-2} = t_3 + t_2 = 2 + 1 = 3$

$t_5 = t_{5-1} + t_{5-2} = t_4 + t_3 = 2 + 3 = 5$

$\therefore$ Seq^n $\rightarrow 1, 1, 2, 3, 5$ $\rightarrow$ Fibonacci seq^n

Q If Sum of n terms of a Series is $S_n = 3n^2 - 5n$ find Sum of 1^{st} 10 terms?

Sum of 1^{st} 10 terms $= S_{10} = 3 \times 10^2 - 5 \times 10 = 250$

Q If Sum of n terms of Series is $S_n = 3n^2 - 5n$. Find 10^{th} term?

$T_{10} = ?$ $\quad T_n = S_n - S_{n-1}$

$\therefore T_{10} = S_{10} - S_9$

$= (3 \times 10^2 - 5 \times 10) - (3 \times 9^2 - 5 \times 9)$

$= 250 - 198 = 52$

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Q If Sum of n terms of a Series is

$$S_n = \frac{(n)(n+1)(n+2)}{6}$$ find n^{th} term?

Demand = $T_n = S_n - S_{n-1}$

$$T_n = \frac{(n)(n+1)(n+2)}{6} - \frac{(n-1)(n)(n+1)}{6}$$

$$= \frac{(n)(n+1)}{6}\{(\cancel{n}+2) - (\cancel{n}-1)\}$$

$$= \frac{(n)(n+1)}{\cancel{6}\,2} \times \cancel{3} = \frac{(n)(n+1)}{2}$$

Concept → Which term ... Jab bhi Poochhe

Write 1st line → let n^{th} term.....

Q Sum of n terms of seq^n is $3n^2+5n$ then which term equals 164?

$S_n = 3n^2+5n$

$S_{n-1} = 3(n-1)^2+5(n-1)$

let n^{th} term = 164

$\Rightarrow T_n = 164$

$\Rightarrow S_n - S_{n-1} = 164$

$$(3n^2+5n) - (3(n-1)^2+5(n-1)) = 164$$

$$3(n^2-(n-1)^2)+5(\cancel{n}-(\cancel{n}-1)) = 164$$

$$3(\cancel{n^2}-\cancel{n^2}+2n-1) = 159 \Rightarrow 6n = 162 \Rightarrow n = \frac{\cancel{162}^{\,27}}{\cancel{6}} = 27$$

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Q let the seq^n $\{t_n\}$ be defined as $t_1 = 1$

& $t_n = t_{n-1} + 2$ $(n \geq 2)$ then value of S_5 is?

$t_2 = t_{2-1} + 2 = t_1 + 2 = 1 + 2 = 3$

$t_3 = t_{3-1} + 2 = t_2 + 2 = 3 + 2 = 5$

$t_4 = t_{4-1} + 2 = t_3 + 2 = 5 + 2 = 7$

$t_5 = t_{5-1} + 2 = t_4 + 2 = 7 + 2 = 9$

S_5 = Sum of 1st 5 terms.

$= t_1 + t_2 + t_3 + t_4 + t_5$

$= 1 + 3 + 5 + 7 + 9$

$= 25$

Q $1, 3, 5, 7, 9.$ ← odd No Series.

$S_1 = 1$

$S_2 = 1 + 3 = 4 = 2^2$

$S_3 = 1 + 3 + 5 = 9 = 3^2$

$S_4 = 1 + 3 + 5 + 7 = 16 = 4^2$

$S_5 = 1 + 3 + 5 + 7 + 9 = 25 = 5^2$

⋮

nth term

$S_n = 1 + 3 + 5 + 7 + 9 + \dots + (2n-1)$

$= n^2$

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Arithmatic Progression.

= A.P.

1) The seq^n whose terms follow a certain Pattern is called Progression.

2) If difference of consecutive terms Remains constant then Progression is Arithmatic Prog

$-5, -3, -1, 1, 3 \ldots$ — AP?

$+2 \quad +2 \quad +2 \quad +2$ ← 1st order difference = Const

$0 \quad 0 \quad 0$ ← 2nd Order diff = 0

Q $121, 114, 107, 100 \ldots$ AP?

$-7 \quad -7 \quad -7$ ← Common diff = d

(3) A.P. normally denoted by

$a, a+d, a+2d, a+3d \ldots a+(n-1)d$

$T_1 \quad T_2 \quad T_3 \quad T_4 \quad T_n$

a = 1st term

d = Com. diff

n = No of term

$$\boxed{(4)\ n^{th} \text{ term of AP} = T_n = a+(n-1)d}$$

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Q If 12th term of AP is 25 & 7th term is 10 find 20th term?

$T_{12} = a + 11d = 25$

$T_7 = a + 6d = 10$

$5d = 15$

$\boxed{d = 3}$

$a = 25 - 33 = -8$

Demand 20th term $= T_{20} = a + 19d$

$= -8 + 19 \times 3$

$= 49$

Q In an AP $a_2 + a_5 - a_3 = 10$ & $a_2 + a_9 = 17$ find a, d?

$(a+d) + (a+4d) - (a+2d) = 10$

$a + 3d = 10$

$(a+d) + (a+8d) = 17$

$2a + 9d = 17$

$2a + 6d = 20$

$2a + 9d = 17$

$-3d = 3$

$\boxed{d = -1}$

$\boxed{a = 13}$