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5. $\underbrace{\sin A + \sin B} - \sin C = 2\sin\overset{\frac{\pi}{2}-\frac{C}{2}}{\frac{A+B}{2}}\cos\frac{A-B}{2} - \sin C$

$= 2\cos\frac{C}{2}\cos\frac{A-B}{2} - 2\sin\frac{C}{2}\cos\frac{C}{2}$

$= 2\cos\frac{C}{2}\left(\cos\frac{A-B}{2} - \sin\left(\frac{C}{2}\right)\right)$

$\frac{C}{2} \rightarrow \frac{\pi}{2} - \frac{A+B}{2}$

$= 2\cos\frac{C}{2}\left(\cos\frac{A-B}{2} - \cos\frac{A+B}{2}\right)$

$= 2\cos\frac{C}{2}\left(2\sin\frac{B}{2}\sin\frac{A}{2}\right)$

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7. $\sin^2 A + (\sin^2 B - \sin^2 C) = (\sin(B+C) + \sin(B-C))\sin A$

9. $\cos^2 A + \sin^2 C - \sin^2 B = 1 - \sin^2 A + \sin(C-B)\sin A$

$A+B+B = \pi - (C-B)$

$B+C+C = \pi - (A-C)$

$C+A+A = \pi - (B-A)$

$= 1 - \sin A\left(\sin(C+B) - \sin(C-B)\right)$

15. $\sin(A-C) + \sin(B-A) + \sin(C-B)$

$= 2\sin\left(\frac{B-C}{2}\right)\cos\left(\frac{2A-C-B}{2}\right) - 2\sin\frac{B-C}{2}\cos\frac{B-C}{2}$

$= 2\sin\left(\frac{B-C}{2}\right)\left(\cos\frac{2A-B-C}{2} - \cos\frac{B-C}{2}\right)$

$= 2\sin\frac{B-C}{2}\, 2\sin\frac{B-A}{2}\sin\frac{A-C}{2}$

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$$\frac{\cancel{4}\sin A \sin B \sin C}{\cancel{4}\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}} = 8\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$$

19.

$$B+C-A = \pi - 2A$$

$$\sin 2A + \sin 2B + \sin 2C$$

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$$A+B+C=\pi$$

$$\sum \cot\frac{A}{2} = \prod \cot\frac{A}{2}$$

$$\sum \tan\frac{A}{2}\tan\frac{B}{2} = 1 \checkmark$$

$$\tan\left(\frac{A}{2}+\frac{B}{2}+\frac{C}{2}\right) = \frac{\pi}{2} \text{ not defined.}$$

$$\Rightarrow \frac{S_1 - S_3}{1 - S_2} \text{ not defd.}$$

$$1 - S_2 = 0$$

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If $\tan A + \tan B + \tan C = \tan A \tan B \tan C$

$$\Rightarrow \frac{S_1 - S_3}{1 - S_2} = 0$$

$$\Rightarrow \tan(A+B+C) = 0$$

$$\boxed{A+B+C = n\pi \quad , n \in I}$$

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1. If $A+B+C=2S$, then P.T.

$$\left(\cancel{4}\,2\sin S\sin(S-A)\right)\left(\sin(S-B)\sin(S-C)\right)\times 2 = 1-\cos^2 A-\cos^2 B-\cos^2 C+2\cos A\cos B\cos C$$

$$=\left(\cos(A)-\cos(\underline{2S-A})\right)\left(\cos(B-C)-\cos(\underline{2S-B-C})\right)$$

$$=\left(\cos A-\cos(B+C)\right)\left(\cos(B-C)-\cos A\right)$$

$$=-\cos^2 A+\cos A\left(\cos(B+C)+\cos(B-C)\right)-\cos(B+C)\cos(B-C)$$

$$=-\cos^2 A+2\cos A\cos B\cos C-\left(\cos^2 B-\sin^2 C\right)$$

$$=1-\cos^2 A-\cos^2 B-\cos^2 C+2\cos A\cos B\cos C.$$

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2.

If $\alpha+\beta+\gamma+\delta=2\pi$, then P.T.

$$\sum \tan\alpha = \left(\prod \tan\alpha\right)\left(\sum \cot\alpha\right)$$

$$\tan(\alpha+\beta+\gamma+\delta) = \tan 2\pi$$

$$\frac{S_1-S_3}{1-S_2+S_4} = 0 \Rightarrow S_1 = S_3$$

$$\tan\alpha+\tan\beta+\tan\gamma+\tan\delta = \tan\alpha\tan\beta\tan\gamma+\tan\alpha\tan\beta\tan\delta + \tan\alpha\tan\gamma\tan\delta+\tan\beta\tan\gamma\tan\delta$$

$$= \tan\alpha\tan\beta\tan\gamma\tan\delta\left(\frac{1}{\tan\delta}+\frac{1}{\tan\gamma}+\frac{1}{\tan\beta}+\frac{1}{\tan\alpha}\right)$$

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Series

$$T_1 + T_2 + T_3 + T_4 + T_5 + \cdots + T_n$$

$$= (\cancel{T_1'} - \underline{T_1''}) + (\cancel{T_2'} - \cancel{T_2''}) + (\cancel{T_3'} - \cancel{T_3''}) + (T_4' - \cancel{T_4''}) + \cdots + (\underline{T_n'} - \cancel{T_n''})$$

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$$S = \sin\theta + \sin(\theta+\alpha) + \sin(\theta+2\alpha) + \sin(\theta+3\alpha) + \cdots + \sin(\theta+(n-1)\alpha)$$

$$2S\sin\frac{\alpha}{2} = 2\sin\frac{\alpha}{2}\sin\theta + 2\sin\frac{\alpha}{2}\sin(\theta+\alpha) + 2\sin\frac{\alpha}{2}\sin(\theta+2\alpha) + 2\sin\frac{\alpha}{2}\sin(\theta+3\alpha) + \cdots + 2\sin\frac{\alpha}{2}\sin(\theta+(n-1)\alpha)$$

$$= \left(\cos\left(\theta-\frac{\alpha}{2}\right) - \cos\cancel{\left(\theta+\frac{\alpha}{2}\right)}\right) + \left(\cos\cancel{\left(\theta+\frac{\alpha}{2}\right)} - \cos\cancel{\left(\theta+\frac{3\alpha}{2}\right)}\right) + \left(\cos\cancel{\left(\theta+\frac{3\alpha}{2}\right)} - \cos\cancel{\left(\theta+\frac{5\alpha}{2}\right)}\right)$$
$$+ \left(\cos\cancel{\left(\theta+\frac{5\alpha}{2}\right)} - \cos\cancel{\left(\theta+\frac{7\alpha}{2}\right)}\right) + \cdots + \left(\cos\cancel{\left(\theta+\left(n-\frac{3}{2}\right)\alpha\right)} - \cos\left(\theta+\left(n-\frac{1}{2}\right)\alpha\right)\right)$$

$$2S\sin\frac{\alpha}{2} = \cos\left(\theta-\frac{\alpha}{2}\right) - \cos\left(\theta+\left(n-\frac{1}{2}\right)\alpha\right) = 2\sin\frac{n\alpha}{2}\sin\left(\frac{2\theta+(n-1)\alpha}{2}\right)$$

$$S = \frac{\sin\left(\frac{n\alpha}{2}\right)\sin\left(\frac{2\theta+(n-1)\alpha}{2}\right)}{\sin\left(\frac{\alpha}{2}\right)}$$

Nishant Jindal

$$\sin(\theta)+\sin(\theta+\alpha)+\sin(\theta+2\alpha)+\sin(\theta+3\alpha)+\cdots+\sin(\theta+(n-1)\alpha)$$

$$=\frac{\sin\left(\frac{n\alpha}{2}\right)}{\sin\left(\frac{\alpha}{2}\right)}\sin\left(\frac{2\theta+(n-1)\alpha}{2}\right)$$

$$=\frac{\sin\left(\frac{(\text{no. of terms})\,\text{difference}}{2}\right)}{\sin\left(\frac{\text{difference}}{2}\right)}\sin\left(\frac{(1^{st}+\text{last})\,\text{angle}}{2}\right)$$

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$$\cos\theta + \cos(\theta+\alpha) + \cos(\theta+2\alpha) + \cos(\theta+3\alpha) + \cdots + \cos(\theta+(n-1)\alpha)$$

$$= \frac{\sin\left(\frac{n\alpha}{2}\right)}{\sin\left(\frac{\alpha}{2}\right)} \cos\left(\frac{2\theta+(n-1)\alpha}{2}\right)$$

$$= \frac{\sin\left(\frac{(\text{no. of terms})\,\text{difference}}{2}\right)}{\sin\left(\frac{\text{difference}}{2}\right)} \cos\left(\frac{(1^{st}+\text{last})\,\text{angle}}{2}\right)$$

Nishant Jindal

1. $\cos\theta + \cos 2\theta + \cos 3\theta + \cos 4\theta + \cdots + \cos(n\theta)$

$$= \frac{\sin\left(\frac{n\theta}{2}\right)}{\sin\left(\frac{\theta}{2}\right)} \cos\left(\frac{\theta + n\theta}{2}\right)$$

2. $\cos\frac{\pi}{11} + \cos\frac{3\pi}{11} + \cos\frac{5\pi}{11} + \cos\frac{7\pi}{11} + \cos\frac{9\pi}{11}$

$$= \frac{\sin\left(\frac{5\pi}{11}\right)}{\sin\frac{\pi}{11}} \cos\left(\frac{10\pi}{2\times 11}\right) = \frac{\sin\frac{5\pi}{11}\cos\frac{5\pi}{11}}{\sin\frac{\pi}{11}} = \frac{\sin\frac{10\pi}{11}}{2\sin\frac{\pi}{11}}$$

$\pi - \frac{\pi}{11}$

$$= \frac{1}{2}$$

Nishant Jindal

3. $\sin^2\theta + \sin^2 2\theta + \sin^2 3\theta + \sin^2 4\theta + \cdots + \text{upto } n \text{ terms}$

$$= \frac{1-\cos 2\theta}{2} + \frac{1-\cos 4\theta}{2} + \frac{1-\cos 6\theta}{2} + \frac{1-\cos 8\theta}{2} + \cdots + \frac{1-\cos(2n\theta)}{2}$$

$$= \frac{n}{2} - \frac{1}{2}\left(\cos 2\theta + \cos 4\theta + \cos 6\theta + \cdots + \cos(2n\theta)\right)$$

$$= \frac{n}{2} - \frac{1}{2}\,\frac{\sin(n\theta)}{\sin\theta}\cos(n+1)\theta$$

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4. $\cos^3\theta + \cos^3 3\theta + \cos^3 5\theta + \cos^3 7\theta + \cdots + \cos^3(\theta + 2\theta(n-1))$

$\theta + 2\theta(n-1)$

$\cos 3\theta = 4\underline{\underline{\cos^3\theta}} - 3\cos\theta$

$$\frac{1}{4}\frac{\sin(3n\theta)}{\sin(3\theta)}\cos\left(\frac{6\theta+6\theta(n-1)}{2}\right) + \frac{3}{4}\frac{\sin(n\theta)}{\sin\theta}\cos\left(\frac{2\theta+2\theta(n-1)}{2}\right)$$

$$= \frac{\cos 3\theta + 3\cos\theta}{4} + \frac{\cos 9\theta + 3\cos 3\theta}{4} + \frac{\cos 15\theta + 3\cos 5\theta}{4} + \frac{\cos 21\theta + 3\cos 7\theta}{4} + \cdots + \frac{\cos(3\theta + 6\theta(n-1)) + 3\cos(\theta + 2\theta(n-1))}{4}$$

H.W Ex-20 (20, 24, 25, 34, 35)

$$= \frac{1}{4}\Big(\cos 3\theta + \cos 9\theta + \cos 15\theta + \cdots + \cos(3\theta + 6\theta(n-1))\Big) + \frac{3}{4}\Big(\cos\theta + \cos 3\theta + \cos 5\theta + \cdots + \cos(\theta + 2\theta(n-1))\Big)$$