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10.

$$\frac{1-\underbrace{\cos A+\cos B}_{-(\cos A-\cos B)}-\cos(A+B)}{1+\cos A-\cos B-\cos(A+B)}$$

$$=\frac{2\sin^2\left(\frac{A+B}{2}\right)-2\sin\left(\frac{B-A}{2}\right)\sin\left(\frac{A+B}{2}\right)}{2\sin^2\left(\frac{A+B}{2}\right)+2\sin\frac{B-A}{2}\sin\frac{A+B}{2}}$$

$$=\frac{\sin\frac{A+B}{2}-\sin\frac{B-A}{2}}{\sin\left(\frac{A+B}{2}\right)+\sin\frac{B-A}{2}}=\frac{2\sin\frac{A}{2}\cos\frac{B}{2}}{2\sin\frac{B}{2}\cos\frac{A}{2}}$$

$$=\boxed{\tan\frac{A}{2}\cot\frac{B}{2}}$$

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12.

$$\frac{\sec 8A - 1}{\sec 4A - 1} = \left(\frac{1-\cos 8A}{1-\cos 4A}\right)\frac{\cos 4A}{\cos 8A} = \frac{2\sin 4A \cos 4A}{2\sin^2 2A \cos 8A}\sin 4A$$

$$= \frac{\tan 8A \; 2\sin 2A \cos 2A}{2\sin^2 2A}$$

$$\frac{\tan 8A}{\tan 2A}$$

13.

$$\frac{2\left(\cos(A+15)\cos(A-15) - \sin(A-15)\sin(A+15)\right)}{2\sin(A+15)\cos(A-15)}$$

$$= \frac{2\cos 2A}{\sin 2A + \sin 30^\circ}$$

16.

$$\frac{\sin 2\theta}{\cos\left(\frac{\pi}{4}-\theta\right)\cos\left(\frac{\pi}{4}+\theta\right)}$$

$$\frac{\sin 2\theta}{\frac{1}{2} - \sin^2\theta}$$

$$\frac{2\sin 2\theta}{\cos 2\theta} = \frac{2\sin 2\theta}{1-2\sin^2\theta} =$$

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20.

$$\frac{\boxed{1+\sin\theta}-\cos\theta}{1+\sin\theta+\cos\theta} = \frac{2\sin^2\frac{\theta}{2}+2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{2\cos^2\frac{\theta}{2}+2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}$$

$$\frac{\left(\cos\frac{\theta}{2}+\sin\frac{\theta}{2}\right)^2-\left(\cos^2\frac{\theta}{2}-\sin^2\frac{\theta}{2}\right)}{\left(\cos\frac{\theta}{2}+\sin\frac{\theta}{2}\right)^2+\left(\cos^2\frac{\theta}{2}-\sin^2\frac{\theta}{2}\right)}$$

$$= \frac{\sin\frac{\theta}{2}\cancel{\left(\sin\frac{\theta}{2}+\cos\frac{\theta}{2}\right)}}{\cos\frac{\theta}{2}\cancel{\left(\cos\frac{\theta}{2}+\sin\frac{\theta}{2}\right)}}$$

$$= \tan\frac{\theta}{2}$$

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22. $$\frac{2\sin nA\cos A+2\sin nA}{2\sin A\sin nA}=\frac{\cos A+1}{\sin A}=\cot\frac{A}{2}.$$

25. $2\sin A\cos 2A+2\sin A\cos A$

$= 2\sin A\left(\cos 2A+\cos A\right)$

26. $$\frac{2\tan A}{1-\tan^2 A}=\frac{2\cos^2 A\sqrt{\sec^2 A-1}}{\cos^2 A-\sin^2 A}$$

$$=\frac{(1+\cos 2A)\sqrt{\sec^2 A-1}}{\cos 2A}$$

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27. $\cos^3 2\theta + 3\cos 2\theta = (\cos^2\theta - \sin^2\theta)\left((\cos^2\theta - \sin^2\theta)^2 + 3\right)$

$$= (\cos^2\theta - \sin^2\theta)^3 + 3(\cos^2\theta - \sin^2\theta)(\cos^2\theta + \sin^2\theta)^2$$

$$(\cos^2\theta - \sin^2\theta)\left((\cos^2\theta - \sin^2\theta)^2 + 3(\cos^2\theta + \sin^2\theta)^2\right)$$

$$(\cos^2\theta - \sin^2\theta)\left(4\sin^4\theta + 4\cos^4\theta + \cancel{4\sin^2\theta\cos^2\theta}\right)$$

$$= \cos^6\theta - \sin^6\theta - 3\sin^2\theta\cos^2\theta(\cos^2\theta - \sin^2\theta)$$

$$+ 3(\cos^2\theta - \sin^2\theta)(\cos^4\theta + \sin^4\theta + 2\sin^2\theta\cos^2\theta)$$

$$= 4(\cos^6\theta - \sin^6\theta)$$

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30.

$$\frac{1}{\sin A} - \frac{2\cos 2A\cos A}{\sin 2A} = \frac{1}{\sin A} - \frac{\cos 2A}{\sin A}$$

$\sin 2A = 2\sin A\cos A$

$$= \frac{1-\cos 2A}{\sin A}$$

$\tan\alpha = t$

34.

$$\frac{1}{t} + \frac{1-\sqrt{3}t}{\sqrt{3}+t} - \frac{1+\sqrt{3}t}{\sqrt{3}-t}$$

$$= \frac{3-9t^2}{t(3-t^2)} = \frac{3(1-3t^2)}{3t-t^3} = \frac{3}{\tan 3\alpha}$$

36. $(\sin 20^\circ \sin 40^\circ \sin 80^\circ)\sin 60^\circ$

$40^\circ = 60^\circ - 20^\circ$, $80^\circ = 60^\circ + 20^\circ$

$$= \frac{1}{4}\sin^2 60^\circ$$

40. $\tan(2A+A)\tan 2A\tan A$.

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41.

39.

$$\left(2\cos^2 3\alpha - 1\right) = 2\left(4\cos^3\alpha - 3\cos\alpha\right)^2 - 1$$

31.

$$\frac{1-\tan^2\frac{A}{2}}{2\tan\frac{A}{2}} = \frac{1}{2}\left(\cot\frac{A}{2} - \tan\frac{A}{2}\right)$$

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41. $\dfrac{(2\cos 2^{n}\theta+1)}{(2\cos\theta+1)} = \dfrac{2(2\cos^{2}2^{n-1}\theta-1)+1}{2\cos\theta+1} = \dfrac{4\cos^{2}2^{n-1}\theta-1}{2\cos\theta+1}$

$= \dfrac{(2\cos 2^{n-1}\theta-1)\boxed{(2\cos 2^{n-1}\theta+1)}}{2\cos\theta+1}$

$(2\cos\theta+1) = (2\cos\frac{\theta}{2}-1)(2\cos\frac{\theta}{2}+1)$

$= \dfrac{(2\cos 2^{n-1}\theta-1)(2\cos 2^{n-2}\theta-1)\boxed{(2\cos 2^{n-2}\theta+1)}}{(2\cos\theta+1)}$

$= \dfrac{(2\cos 2^{n-1}\theta-1)(2\cos 2^{n-2}\theta-1)(2\cos 2^{n-3}\theta-1)(2\cos 2^{n-3}\theta+1)}{(2\cos\theta+1)}$

$\vdots$

$\dfrac{(2\cos 2^{n-1}\theta-1)(2\cos 2^{n-2}\theta-1)\,-\,-\,-\,(2\cos 2\theta-1)(2\cos\theta-1)\cancel{(2\cos\theta+1)}}{\cancel{(2\cos\theta+1)}}$

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$$\tan 3A = \frac{\sin 3A}{\cos 3A} = \frac{3\sin A - 4\sin^3 A \;/\cos^3 A}{4\cos^3 A - 3\cos A \;/\cos^3 A}$$

$$= \frac{3\tan A\sec^2 A - 4\tan^3 A}{4 - 3\sec^2 A}$$

$$\boxed{\tan 3A = \frac{3\tan A - \tan^3 A}{1 - 3\tan^2 A}}$$

$$= \frac{3\sin A(\sin^2 A + \cos^2 A) - 4\sin^3 A}{4\cos^3 A - 3\cos A(\cos^2 A + \sin^2 A)}$$

$$\tan 2A = \frac{2\tan A}{1-\tan^2 A}$$

$$= \frac{3\sin A\cos^2 A - \sin^3 A \;/\cos^3 A}{\cos^3 A - 3\cos A\sin^2 A \;/\cos^3 A}$$

$$\tan(A + 2A) = \frac{\tan A + \tan 2A}{1 - \tan A\tan 2A} = \frac{3\tan A - \tan^3 A}{1 - 3\tan^2 A}$$

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1.

$$\sin 6^\circ \sin 42^\circ \sin 66^\circ \boxed{\sin 78^\circ}$$

$$\left(-\cos(90+6^\circ)\right)\cos(90-42^\circ)\ \cos 24^\circ \quad \cos 12^\circ$$

$$\frac{\left(\sin 18^\circ \sin 42^\circ \sin 78^\circ\right)}{\sin 18^\circ} \qquad \frac{\sin 6^\circ \sin 66^\circ \sin 54^\circ}{\sin 54^\circ}$$

$$= \frac{1}{4}\frac{\cancel{\sin(3\times 18^\circ)}}{\cancel{\sin 18^\circ}} \quad \frac{1}{4}\frac{\sin(\cancel{3}\times 6)^\circ}{\cancel{\sin 54^\circ}} = \frac{1}{16}.$$

$180+12$

$$-\cos 12^\circ \cos 24^\circ \cos 48^\circ \cos 96^\circ = \frac{-\sin(192^\circ)}{2^4 \sin 12^\circ} = \frac{-(-\sin 12^\circ)}{2^4 \sin 12^\circ}$$

$$= \boxed{\frac{1}{16}}$$

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2.

$\sin\frac{\pi}{16}\sin\frac{3\pi}{16}\sin\frac{5\pi}{16}\boxed{\sin\frac{7\pi}{16}} \to \cos\frac{\pi}{16}$

$8-3$ $\quad 8-1$

Ex-18

$\boxed{7, 8, 9, 10, 11, 12, 13, 14, 15}$

$\frac{\pi}{2}-\frac{3\pi}{16} \qquad \frac{\pi}{2}-\frac{\pi}{16}$

$$= \left(\sin\frac{\pi}{16}\cos\frac{\pi}{16}\right)\left(\sin\frac{3\pi}{16}\cos\frac{3\pi}{16}\right)$$

$$= \left(\frac{1}{2}\sin\frac{\pi}{8}\right)\left(\frac{1}{2}\sin\frac{3\pi}{8}\right) = \frac{1}{4}\sin\frac{\pi}{8}\sin\frac{3\pi}{8}$$

$\frac{\pi}{2}-\frac{\pi}{8}$

$$= \frac{1}{4}\left(\sin\frac{\pi}{8}\cos\frac{\pi}{8}\right) = \frac{1}{8}\sin\frac{\pi}{4} = \frac{1}{8\sqrt{2}}$$