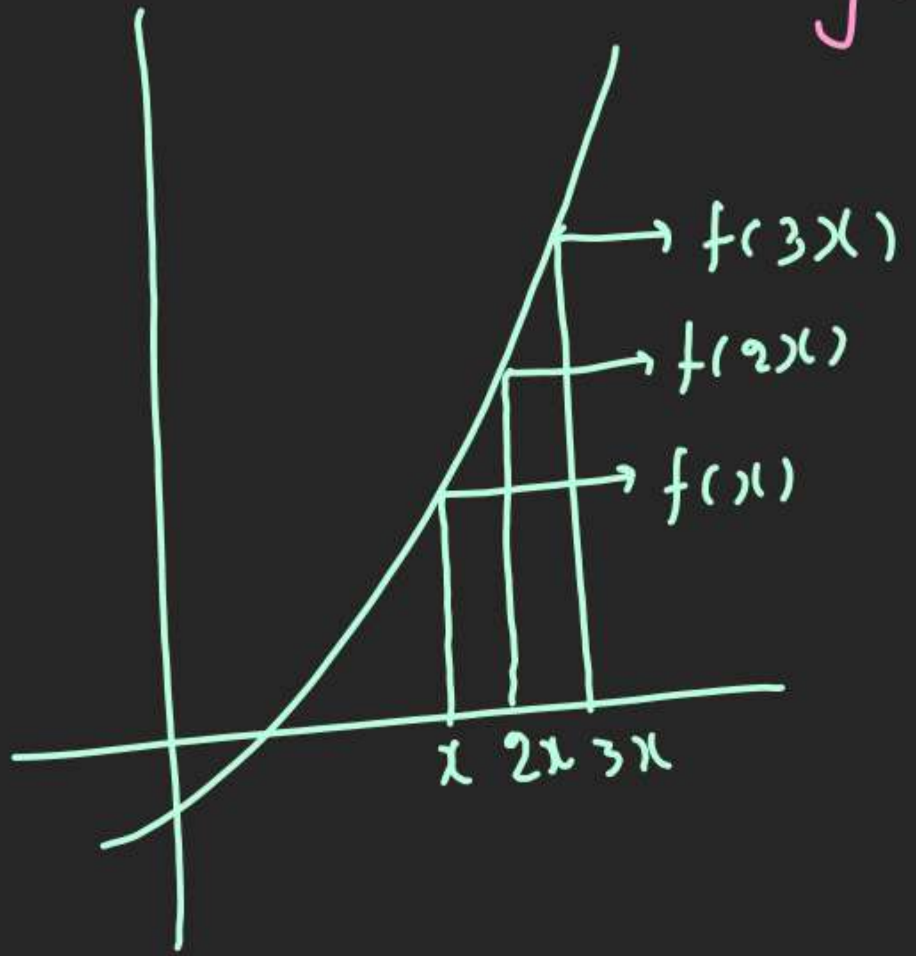
Nishant Jindal

# LIMIT

Q Let $f: R \to R$ be +ve Increasing fxn. with $\lim\limits_{x\to\infty} \dfrac{f(3x)}{f(x)} = 1$ then find $\lim\limits_{x\to\infty} \dfrac{f(2x)}{f(x)}$ ?

Main




$$f(3x) > f(2x) > f(x)$$

Inequality

Sandwich.

$$\boxed{\lim_{x\to\infty} \frac{f(3x)}{f(x)}} > \lim_{x\to\infty} \frac{f(2x)}{f(x)} > \lim_{x\to\infty} \frac{\cancel{f(x)}}{\cancel{f(x)}} \quad 1$$

$$1 > \lim_{x\to\infty} \frac{f(2x)}{f(x)} > 1$$

$$\lim_{x\to\infty} \frac{f(2x)}{f(x)} = 1$$

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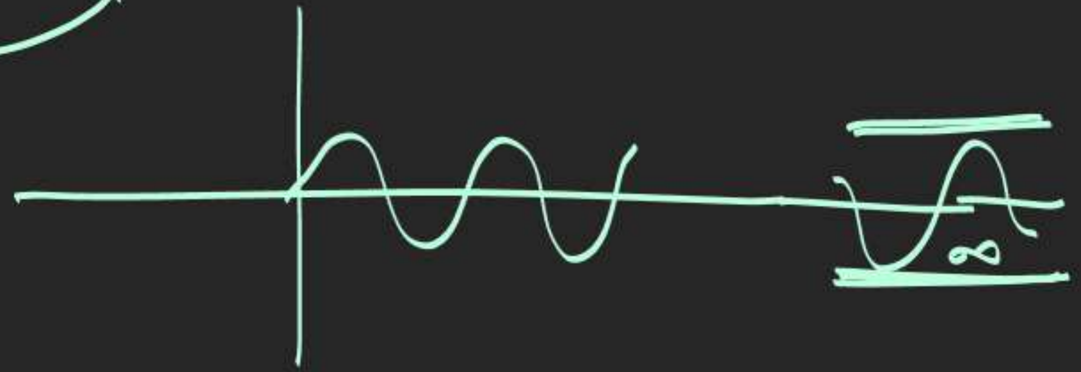
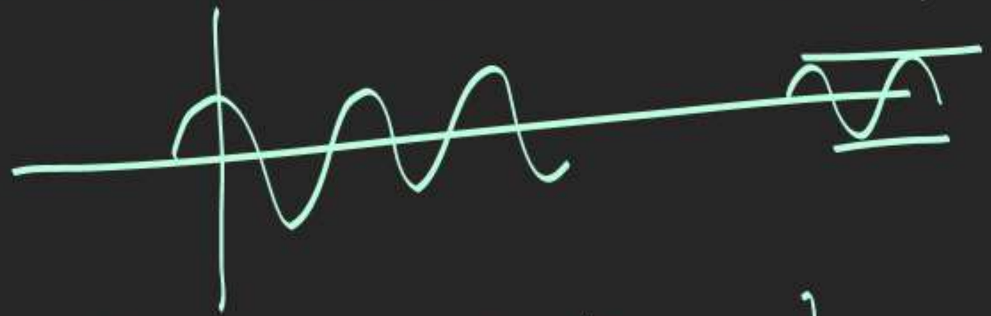
# LIMIT

Concepts:

1) $\lim_{x\to\infty} \sin x = \sin\infty = \sin \text{ at } \infty = [\text{Any value bet}^n -1 \text{ to } +1]$



2) $\lim_{x\to\infty} \cos x = \cos\infty = \cos \text{ at } \infty = [\text{Any value bet}^n -1 \text{ to } +1]$



3) $\lim_{x\to\infty} \frac{\sin x}{x} = \frac{[\text{Any value bet}^n -1 \text{ to } +1]}{\infty} = 0$

(4) $\lim_{x\to\infty} \frac{\cos x}{x} = \frac{\cos\infty}{\infty} = \frac{[-1 \text{ to } +1]}{\infty} = 0$

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# LIMIT

Q $\lim_{n\to\infty} \sin\left(\pi\boxed{\sqrt{n^2+1}}\right)$ ; $n \in N$.

Assume $\lim_{n\to\infty} \frac{\sqrt{n^2+1} - n}{1}$ & Solve.

Rat.

$$\lim_{n\to\infty} \frac{(\cancel{n^2}+1) - \cancel{n^2}}{\sqrt{n^2+1}+n} = \frac{1}{\infty} \to 0$$

Now $\lim_{n\to\infty} \sqrt{n^2+1} - n = 0$

$$\lim_{n\to\infty} \sqrt{n^2+1} = \lim_{n\to\infty} n$$

$$\lim_{n\to\infty} \sin(\pi n) = 0$$

Q $\lim_{n\to\infty} \sin \pi\left(\sqrt{n^2+n+1}\right)$ ; $n \in N$

$\lim_{n\to\infty} \frac{\sqrt{n^2+n+1} - n}{1}$ & Solve

Ration

$$\lim_{n\to\infty} \frac{(\cancel{n^2}+n+1) - \cancel{n^2}}{\sqrt{n^2+n+1}+n} = \frac{1}{1+1} = \frac{1}{2}$$

Now $\lim_{n\to\infty} \sqrt{n^2+n+1} - n = \frac{1}{2}$

$$\lim_{n\to\infty} \sqrt{n^2+n+1} = \lim_{n\to\infty} n + 1/2$$

$$\lim_{n\to\infty} \sin \pi\left(n+\frac{1}{2}\right) = \sin\left(\frac{\pi}{2} + n\pi\right)$$

$\lim_{n\to\infty} \cos n\pi = \cos\infty =$ Any value bet$^n$ 1 to 1

$= \underline{LDNE}$

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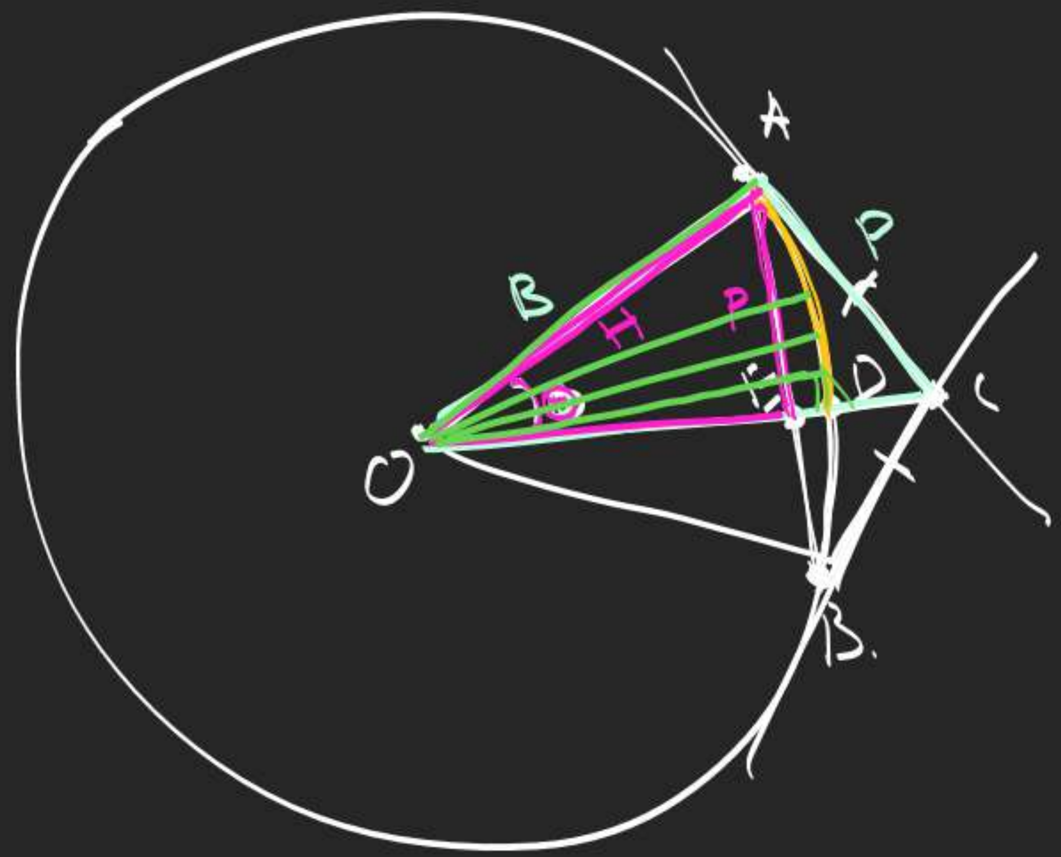
LIMIT

# Trigo fxn's Standard Limit




## Concept of $\sin\theta < \theta < \tan\theta$

$$AC + BC > \overset{\frown}{AB} > \text{Chord } AB$$

$$\frac{AC+BC}{2} > \frac{\overset{\frown}{AB}}{2} > \frac{\text{Chord } AB}{2}$$

$$AC > \overset{\frown}{AD} > AE$$

$$\frac{AC}{OA} > \frac{\overset{\frown}{AD}}{OA} > \frac{AE}{OA}$$

$$\boxed{\tan\theta > \theta > \sin\theta}$$

$$\boxed{\lim_{\theta\to 0} \tan\theta \simeq \theta \simeq \sin\theta}$$

$$\boxed{\lim_{\theta\to 0} \frac{\tan\theta}{\theta} = 1 \qquad \lim_{\theta\to 0} \frac{\sin\theta}{\theta} = 1}$$

Without limit

$$\boxed{\frac{\sin\theta}{\theta} < 1 \qquad \frac{\tan\theta}{\theta} > 1}$$

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## Without limit

| | |
|---|---|
| $\frac{\sin x}{x} < 1$ | $\frac{\tan x}{x} > 1$ |
| $\frac{\sin^{-1} x}{x} > 1$ | $\frac{\tan^{-1} x}{x} < 1$ |

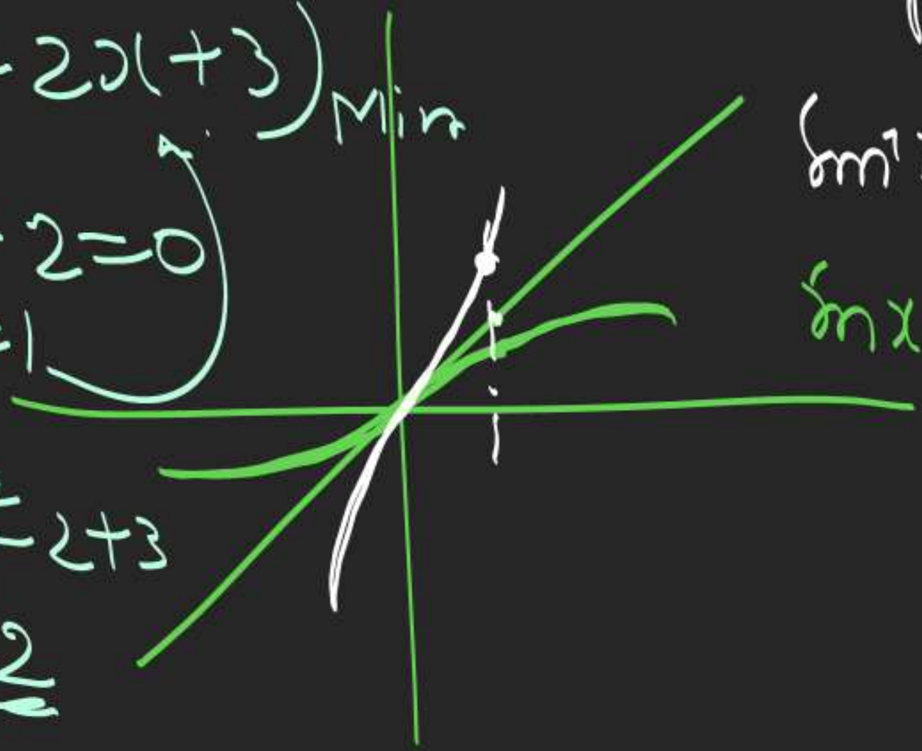
$y = (x^2 - 2x + 3)_{Min}$

$\frac{dy}{dx} = 2x - 2 = 0$

$x = 1$

$y_{Min} = 1^2 - 2 + 3$

$= \underline{2}$



By Graph

$\sin^{-1} x > x \Rightarrow \frac{\sin^{-1} x}{x} > 1$

$\sin x < x \Rightarrow \frac{\sin x}{x} < 1$

## Concept No 2 VV Imp

at $\boxed{\lim_{x \to 0}} \rightarrow \boxed{\sin x = \tan x = \sin^{-1} x = \tan^{-1} x = x}$

---

Q $\lim_{x \to 0} \left[ \frac{\sin x}{x} \right]$

$\frac{\sin x}{x} < 1$

$\frac{2x}{\sin x} > 2$

$\lim_{x \to 0} [\text{less than } 1] = 0$

Q $\lim_{x \to 0} \left[ \frac{\text{Min}(x^2 - 2x + 3) \cdot x}{\sin x} \right] = \left[ \frac{\text{Min}(x^2 - 2x + 1 + 2)\, x}{\sin x} \right]$

$\left[ \frac{\text{Min}((x-1)^2 + 2)\, x}{\sin x} \right] = \left[ \frac{2x}{\sin x} \right] = [\text{Gr. than } 2]$

$= 2$

# LIMIT

Q $\lim_{x\to 0}\left[\frac{100\sin x}{x}\right]+\left[\frac{100x}{\tan x}\right]=?$

$\frac{\sin x}{x}<1$

$\frac{100\sin x}{x}<100$

$\frac{\tan x}{x}>1$

$\frac{x}{\tan x}<1$

$\frac{100x}{\tan x}<100$

$[\text{less than } 100]+[\text{less than } 100]$

$=99+99=198$

Q $\lim_{x\to 0}\left[\frac{100x}{\sin^{-1}x}\right]+\left[\frac{100\tan^{-1}x}{x}\right]$

$\frac{\sin^{-1}x}{x}>1$

$\frac{x}{\sin^{-1}x}<1$

$\frac{100x}{\sin^{-1}x}<100$

$\frac{\tan^{-1}x}{x}<1$

$\frac{100\tan^{-1}x}{x}<100.$

$[\text{less than } 100]+[\text{less than } 100]$

$=99+99=198$

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# LIMIT

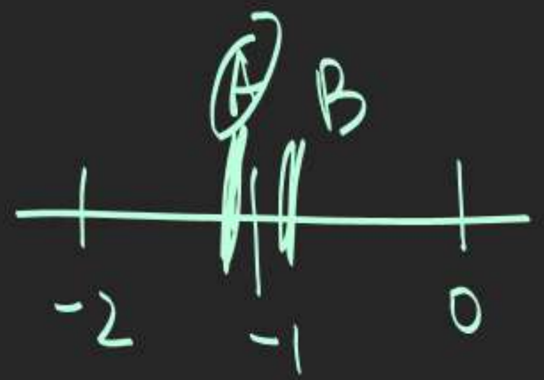
V. good

Q $\lim_{x \to \frac{\pi}{2}} \left[\frac{x-\frac{\pi}{2}}{\cos x}\right] = -2$

$\lim_{x \to 0}$ Nahi hai

Chokdi Kum Nhi Karogi

[ ] aakarshit kr Rha hai.



LHL $x = \frac{\pi}{2} - h$

$\lim_{h \to 0} \left[\frac{\cancel{\frac{\pi}{2}} - h - \cancel{\frac{\pi}{2}}}{\cos\left(\frac{\pi}{2} - h\right)}\right]$

$\left[\frac{-h}{+\sin h}\right]$

[less than -1]

$= -2$

RHL $x = \frac{\pi}{2} + h$.

$\lim_{h \to 0} \left[\frac{\cancel{\frac{\pi}{2}} + h - \cancel{\frac{\pi}{2}}}{\cos\left(\frac{\pi}{2} + h\right)}\right]$

$\left[\frac{h}{-\sin h}\right]$

[less than -1]

$= -2$

$\frac{\sin h}{h} < 1$

$\frac{h}{\sin h} > 1$

$\frac{-h}{\sin h} < -1$

# LIMIT

[Trigo]

Q $\lim_{x\to 0} \sin^{-1}[\sin x] + \cos^{-1}[\cos x] - 2\tan^{-1}[\tan x]$ $= \frac{\pi}{2}$

(hokdi Useless
[ ] hai => Aakarshan

LHL $\lim_{x\to 0^-} \sin^{-1}[\sin x] + \cos^{-1}[\cos x] - 2\tan^{-1}[\tan x]$

$<0 \qquad <1 \qquad <0$

$= \sin^{-1}(-1) + \cos^{-1}(0) - 2\tan^{-1}(-1)$

$-\frac{\pi}{2} + \frac{\pi}{2} + 2\times\frac{\pi}{4} = \frac{\pi}{2}$

RHL $\lim_{x\to 0^+} \sin^{-1}[\sin x] + \cos^{-1}[\cos x] - 2\tan^{-1}[\tan x]$

$>0 \qquad <1 \qquad >0$

$\sin^{-1}(0) + \cos^{-1}(0) - 2\tan^{-1}(0)$

$0 + \frac{\pi}{2} - 2\times 0 = \frac{\pi}{2}$

---

Q $\lim_{x\to 0} \dfrac{x^2 + 2\tan x - \tan^3 x - 3x^4}{\tan^3 x - 6\sin^2 x + x - 5x^3}$

$\lim_{x\to 0} \dfrac{x^2 + 2x - x^3 - 3x^4}{x^3 - 6x^2 + x - 5x^3} = \frac{2}{1} = 2$

Q $\lim_{x\to 0} \dfrac{\sin ax}{x} = \dfrac{a\cancel{x}}{\cancel{x}} = a$

Q $\lim_{x\to 0} \dfrac{\sin ax}{\tan bx} = \dfrac{a\cancel{x}}{b\cancel{x}} = \dfrac{a}{b}$

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Q $\lim_{x\to0} \dfrac{(\sin x-\tan x)^2-(1-\cos 2x)^4+x^5}{7(\tan^7 x)^7+(\sin^7 x)^6+3\sin^5 x}=?$

$(2\sin^2 x)^4$

$\dfrac{(\cancel{x}-\cancel{x})^2-(2x^2)^4+x^5}{7x^7+x^6+3x^5}=\dfrac{1}{3}$

Q $\lim_{x\to0} \dfrac{x+5\tan x}{x-5\sin x}=\dfrac{x+5x}{x-5x}=\dfrac{6x}{-4x}=-\dfrac{3}{2}$

Q $\lim_{x\to0} x\cdot\sin\frac{1}{x}=0\cdot\sin\infty=0\times(\text{Any value bet}^n\ -1\ \text{to}\ 1)$

$=0$

Concepts

A) $\lim_{x\to0}\dfrac{\sin x}{x}=\lim_{x\to0}\dfrac{x}{\sin x}=\lim_{x\to0}\dfrac{\tan x}{x}=\lim_{x\to0}\dfrac{x}{\tan x}$

$=\lim_{x\to0}\dfrac{\sin^{-1}x}{x}=\lim_{x\to0}\dfrac{x}{\sin^{-1}x}=\lim_{x\to0}\dfrac{\tan^{-1}x}{x}=\lim_{x\to0}\dfrac{x}{\tan^{-1}x}$

$=1$

** IMMM

(B) $\lim_{x\to0}\dfrac{\sin x}{x}=1 \rightarrow \dfrac{\sin(\text{Same})}{(\text{Same})}$

$\dfrac{\sin 0}{0}$

$\lim_{x\to0}\dfrac{\tan x}{x} \rightarrow \dfrac{\tan(\text{Same})}{(\text{Same})}=1$

$\dfrac{\tan 0}{0}$

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Q $\lim_{x\to 2} (x-2)\csc(x-2) \to 0\times \underset{\sim}{\csc}\,0 = 0\times\infty$ form.

$\boxed{\text{L Party Niche Jayegi}}$

$\lim_{x\to 2} \frac{(x-2)}{\sin(x-2)}$ → $\frac{\sin(\text{Same})}{(\text{Same})}$ → $\frac{0}{\sin 0}$

$=1$

Q $\lim_{x\to\infty} x\cdot\sin\frac{1}{x} \to \infty\cdot\sin 0 = \boxed{\infty\times 0}$

$\lim_{x\to\infty} \frac{\sin\left(\frac{1}{x}\right)}{\left(\frac{1}{x}\right)}$ → $\frac{\sin(\text{Same})}{(\text{Same})}$ → $\frac{\sin 0}{0}$

$=1$

Pichhli Barrrr~----

$\lim_{x\to 0} x\cdot\sin\frac{1}{x} = 0\times\sin\infty \neq 0\times\infty$

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Q $\lim_{x\to\infty} x \cdot \sin\left(\frac{\pi}{8x}\right) \cdot \cos\left(\frac{\pi}{8x}\right) = \infty \times \sin 0 \times \cos 0 = \infty \times 0$

$$\lim_{x\to\infty} \frac{\sin\left(\frac{\pi}{8x}\right)}{\frac{\pi}{8x}} \times \frac{8}{\pi} \cos\left(\frac{\pi}{8x}\right)$$

$\frac{\sin \text{Same}}{\text{Same}}$

$\frac{\sin 0}{0}$

$$\frac{1 \times \cos(0)}{\frac{8}{\pi}} = \frac{1}{\frac{8}{\pi}} = \frac{\pi}{8}$$

Q $\lim_{x\to-\infty} \frac{x^4 \sin\left(\frac{1}{x}\right) - x^2}{1+|x|^3}$ 2 Concept

① $x = -ve$

$|x| = -x$

QS Correct Kro

② $x \to \boxed{-\infty}$

$x =$ Very large No

$\frac{1}{x} \to 0$

$\sin\left(\frac{1}{x}\right) \simeq \frac{1}{x}$

Top

$$\lim_{x\to-\infty} \frac{x^{\cancel{4}\,3} \times \frac{1}{\cancel{x}} - x^2}{1+(-x)^3} = \frac{1}{-1}$$

$= -1$

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Q $\lim\limits_{\boxed{x\to-\infty}} \dfrac{x^2 \sin\frac{1}{x}}{\sqrt{8x^2+7x+1}}$

$\frac{1}{x}\to 0$

$\lim\limits_{x\to-\infty} \dfrac{x^2\times\frac{1}{x}}{|x|\sqrt{8+\frac{7}{x}+\frac{1}{x^2}}}$

$x=-ve$

$\lim\limits_{x\to-\infty} \dfrac{\cancel{x}}{-\cancel{x}\sqrt{8+\cancel{\frac{7}{x}}+\cancel{\frac{1}{x^2}}}}$

$-\dfrac{1}{2\sqrt{2}}$

Q Adv $\lim\limits_{x\to 0} \dfrac{\sin nx\,[(a-n)nx-\tan x]}{x^2} \Rightarrow 0$ then $\boxed{a}$?

$\lim\limits_{x\to 0} \dfrac{n\cancel{x}}{\cancel{x}}\left[\dfrac{(a-n)nx-x}{x}\right]=0$

$n\left[\dfrac{(a-n)n\cancel{x}}{\cancel{x}}-\dfrac{\cancel{x}}{\cancel{x}}\right]=0$

$(n)[(a-n)n-1]=0$

$\boxed{n=0}$ | $(a-n)n-1=0$

$(a-n)n=1 \Rightarrow (a-n)=\frac{1}{n} \Rightarrow \boxed{a=n+\frac{1}{n}}$

Ex 1

16, 17, 18, 19

20, 21

Ex 2 3