

Calculus based questions

KINEMATICS

$$\int x^n dx = \frac{x^{n+1}}{n+1}$$

Q.3 A particle retards from a velocity v_0 while moving in a straight line. If the magnitude of deceleration is directly proportional to the square root of the speed of the particle, find its average velocity for the total time of its motion.

$$a = -K\sqrt{v}$$

$K \rightarrow$ proportionality constant $\xleftarrow{t=0}$

$$t = t$$

$$v = 0$$

$$v_{\text{avg}} = \left(\frac{\text{Total distance}}{\text{Total time}} \right)$$

$$\frac{dv}{dt} = -K\sqrt{v}$$

$$\int_{v_0}^0 \frac{dv}{\sqrt{v}} = -K \int_0^t dt$$

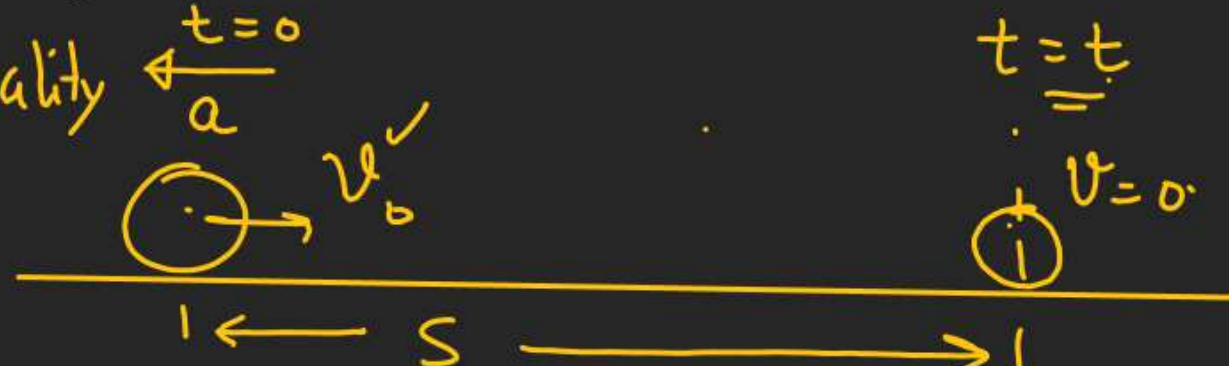
$$\Rightarrow \int_{v_0}^0 v^{-1/2} dv = -K \int_0^t dt$$

$$\Rightarrow \left[\frac{v^{-1/2+1}}{(-1/2+1)} \right]_{v_0}^0 = -K(t)_0^t$$

$$\Rightarrow 2[\sqrt{v}]_{v_0}^0 = -Kt$$

$$2\sqrt{v_0} = Kt$$

$$\frac{2\sqrt{v_0}}{K} = t$$



$$a = -k\sqrt{v} \quad \begin{matrix} a \rightarrow f(v) \\ v \rightarrow f(s) \end{matrix}$$

$$\frac{v dv}{ds} = -k\sqrt{v}$$

$$\int_{v_0}^0 \sqrt{v} dv = -k \int_0^S ds$$

$$\int_{v_0}^0 v^{1/2} dv = -k \int_0^S ds$$

$$\frac{[v^{3/2}]_{v_0}^0}{3/2} = -kS$$

$$\frac{2}{3} [0 - v_0^{3/2}] = -kS$$

$$\underline{S} = \left(\frac{2}{3k} v_0^{3/2} \right)$$

$$\text{Avg velocity} = \frac{S}{t} = \left(\frac{\frac{2}{3k} (v_0^{3/2})}{\frac{2 v_0^{1/2}}{k}} \right)$$

$$= \left(\frac{v_0}{3} \right) \underline{\underline{\text{Ans}}} \checkmark$$

$$\Rightarrow [a = 6t - 6]$$

$$\downarrow$$

$$\frac{dv}{dt} = (6t - 6)$$

$$\int_2^v dv = \int_0^t (6t - 6) dt$$

$$[v]_2^v = 6 \int_0^t t dt - 6 \int_0^t dt$$

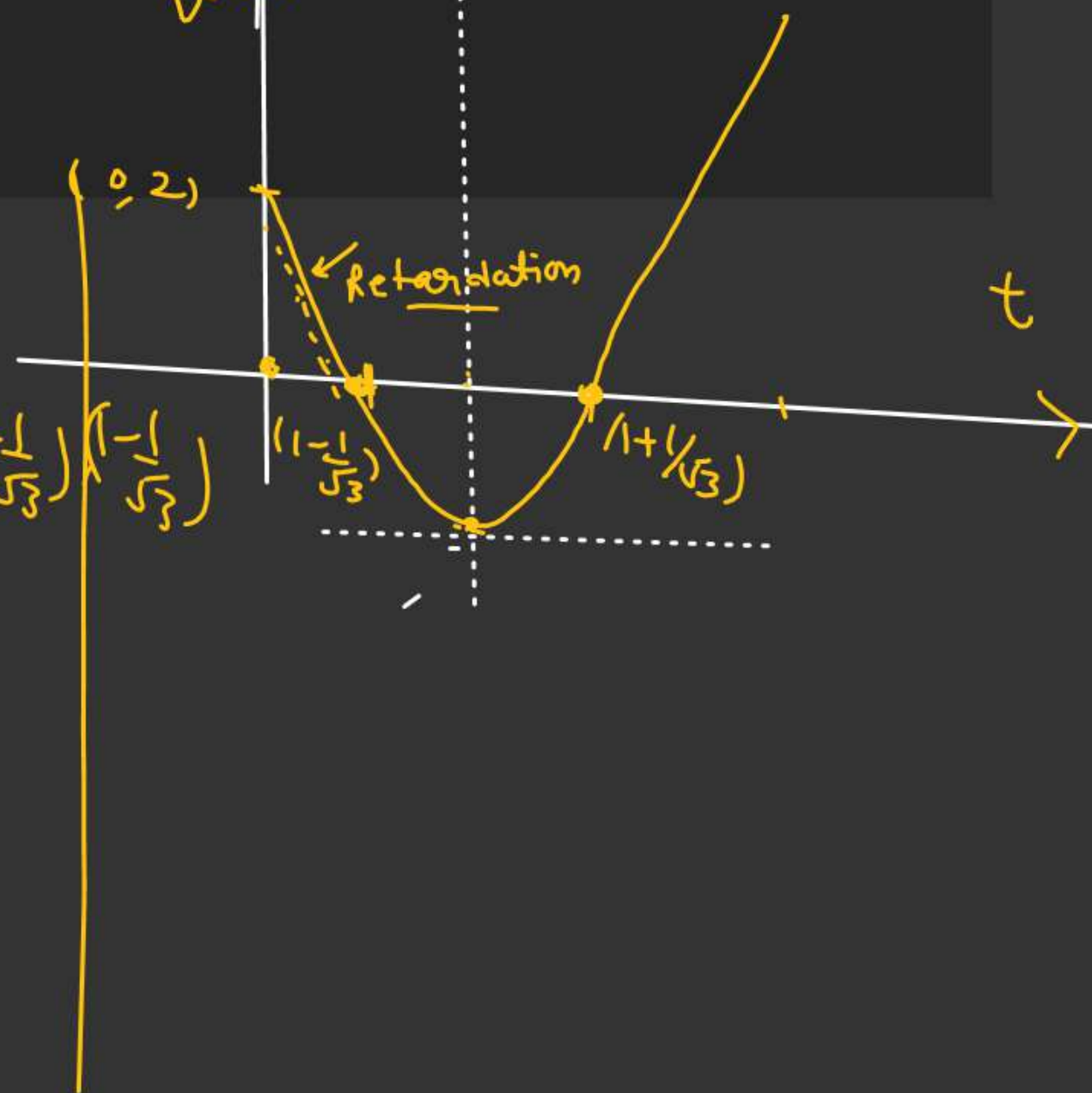
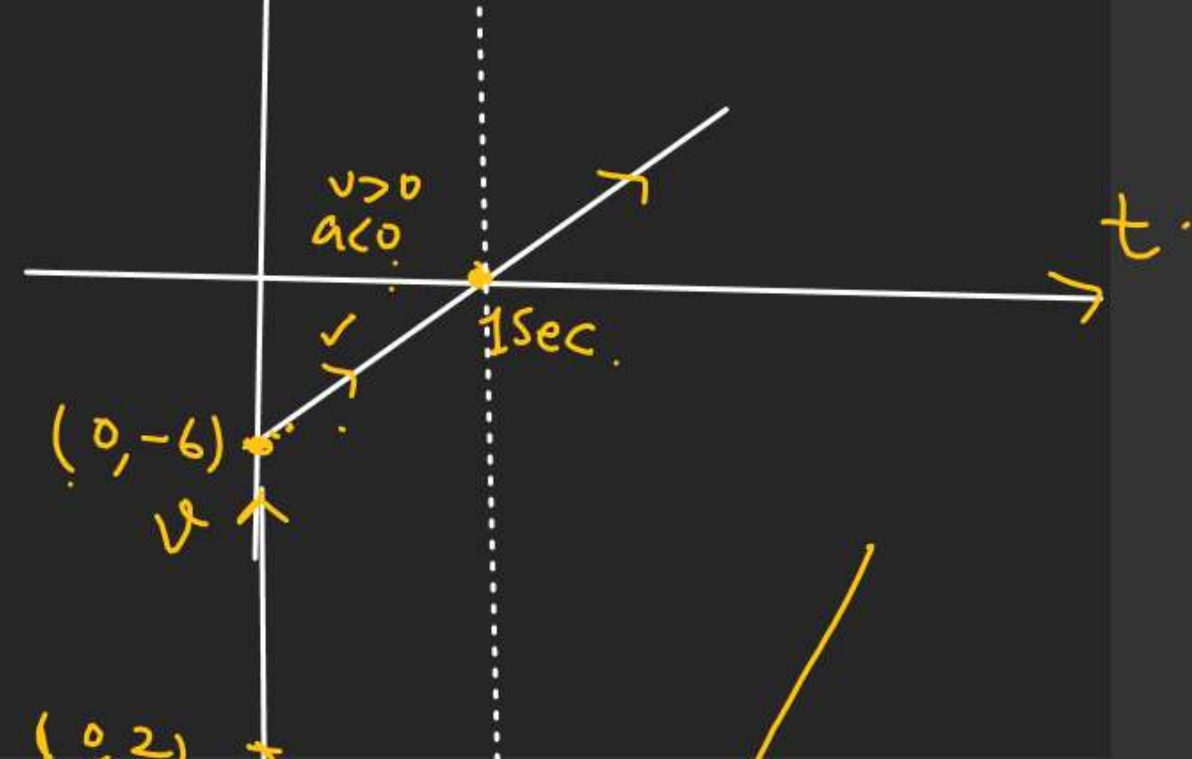
$$v - 2 = 6 \left[\frac{t^2}{2} \right]_0^t - 6[t]_0^t$$

$$v - 2 = 3t^2 - 6t$$

$$\boxed{v = 3t^2 - 6t + 2}$$

$$t = \frac{6 \pm \sqrt{36 - 4 \times 3 \times 2}}{6} = \frac{6 \pm \sqrt{36 - 24}}{6}$$

$$t = \frac{6 \pm \sqrt{12}}{6} = \left(\frac{6 \pm 2\sqrt{3}}{6} \right) = \left(1 \pm \frac{1}{\sqrt{3}} \right) \Rightarrow \left(1 + \frac{1}{\sqrt{3}} \right), \left(1 - \frac{1}{\sqrt{3}} \right)$$



Q.5 The position x of a particle varies with time (t) as $x = at^2 - bt^3$. The acceleration at time t of the particle will be equal to zero, where t is equal to

(A) $\frac{2a}{3b}$

(B) $\frac{a}{b}$

(C) $\frac{a}{3b}$ ✓✓

(D) zero

$$x = at^2 - bt^3$$

↓

$$v = \frac{dx}{dt} = 2at - 3bt^2$$

↓

$$a = \frac{dv}{dt} = 2a - 3b \times (2t)$$

$$a = 2a - 6bt$$

$$a = 0$$

$$2a - 6bt = 0$$

$$t = \frac{a}{3b}$$

(ii) Find velocity and displacement of the particle when $a = 0$

$$v = 2at - 3bt^2$$

$$t = \frac{a}{3b} = 2a \times \frac{a}{3b} - 3b \left(\frac{a}{3b} \right)^2$$

$$\frac{2a^2}{3b} - \frac{a^2}{3b} = \frac{a^2}{3b}$$

$$\frac{a^2}{3b}$$

Q.6 A particle is moving along the x-axis whose instantaneous speed is given by

$v^2 = 108 - 9x^2$. The acceleration of the particle is

(A) $-9x \text{ m s}^{-2}$ ✓

(B) $-18x \text{ m s}^{-2}$

(C) $\frac{-9x}{2} \text{ m s}^{-2}$

(D) None of these

$v \rightarrow f(x), \quad a = \left(v \frac{dv}{dx} \right)$
 $v^2 = 108 - 9x^2$
 Differentiating both side w.r.t x
 $\frac{d}{dx}(v^2) = \frac{d}{dx}(108) - 9 \frac{d}{dx}(x^2)$
 $\downarrow \quad \downarrow$
 $\left[\frac{d}{dv}(v^2) \times \frac{dv}{dx} \right] = 0 - 18x$
 $2 \left\{ v \frac{dv}{dx} \right\} = -18x$
 \downarrow
 $2a = -18x$
 $a = -9x$

Q.7 The relation between time t and distance x is $t = \alpha x^2 + \beta x$ where α and β are constants. The retardation is

(A) $2\alpha v^3$

(B) $2\beta v^3$

(C) $2\alpha\beta v^3$

(D) $2b^2 v^3$

Q.8 The displacement x of a particle moving in one dimension under the action of a constant force is related to time t by the equation $t = \sqrt{x} + 3$, where x is in meters and t is in seconds. Find the displacement of the particle when its velocity is zero.

(A) Zero ✓✓

(B) 12 m

(C) 6 m

(D) 18 m

$t = \sqrt{x} + 3$

$\sqrt{x} = (t - 3)$


$x = (t - 3)^2 = (t^2 + 9 - 6t)$

$\leftarrow \underline{v} = \frac{dx}{dt} = \underline{(2t - 6)}$

$v = 0$

$2t - 6 = 0$

$t = 3 \text{ Sec}$



Q.9 The deceleration experienced by a moving motor boat, after its engine is cut-off is given by $\frac{dv}{dt} = -kv^3$, where k is constant. If v_0 is the magnitude of the velocity at cut-off, the magnitude of the velocity at a time t after the cut-off is

(A) $v_0/2$

(B) v

(C) $v_0 e^{-kt}$

(D) $\frac{v_0}{\sqrt{2v_0^2 kt + 1}}$

$$a = -kv^3$$

$$\frac{dv}{dt} = -kv^3$$

$$\int_{v_0}^v \frac{dv}{v^3} = -k \int_0^t dt$$

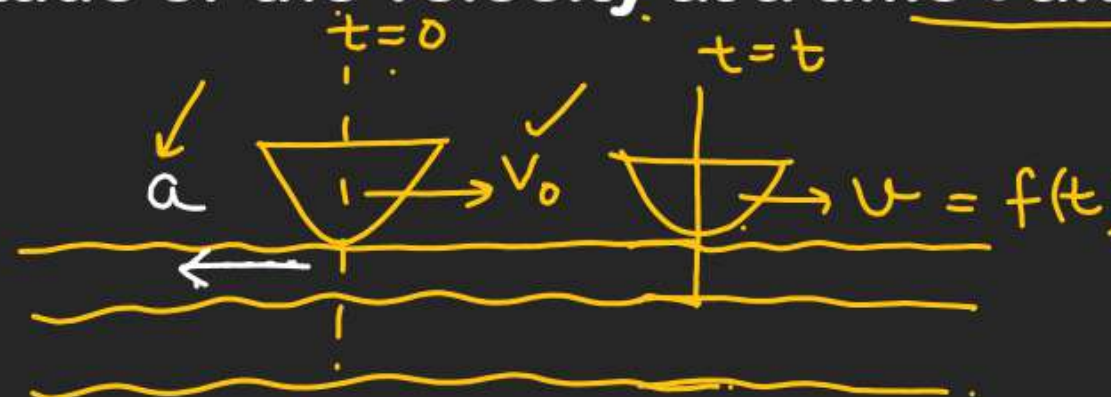
$$\Rightarrow \left[\frac{v^{-2}}{-2} \right]_{v_0}^v = -kt$$

$$\Rightarrow \frac{1}{v^2} - \frac{1}{v_0^2} = (2kt)$$

$$\frac{1}{v^2} = \frac{1}{v_0^2} + 2kt$$

$$\frac{1}{v^2} = \frac{(1 + 2kv_0^2 t)}{v_0^2}$$

$$v^2 = \left(\frac{v_0^2}{1 + 2kv_0^2 t} \right)$$



$$a \rightarrow f(v)$$

$$\frac{dv}{dt} = f(v)$$

Q.10 For motion of an object along the x-axis, the velocity v depends on the displacement x as $v = 3x^2 - 2x$, then what is the acceleration at $x = 2$ m.

H.W.

(A) 48 m s^{-2}

(B) 80 m s^{-2}

(C) 18 m s^{-2}

(D) 10 m s^{-2}

Q.11 A point moves in a straight line so that its displacement x metre at time t second is given by $x^2 = 1 + t^2$. Its acceleration in ms^{-2} at time t second is

(A) $\frac{1}{x^3}$

(B) $\frac{-t}{x^3}$

(C) $\frac{1}{x} - \frac{t^2}{x^3}$

(D) $\frac{1}{x} - \frac{1}{x^2}$