Nishant Jindal

QUADRATIC EQUATION

Q1 $6x^2 - 7x + K = 0$ Root Rat

$D = 49 - 24K = $ Per sq

$K = -1 \quad 49 + 24 \times$

$K = 1 \quad 49 - 24 = 25 \checkmark$

$K = 2 \quad 49 - 48 = 1 \checkmark$

Q Eqn + D = 0

$x^2 - 2mx + 8m - 15 = 0$

Q3 $ax^2 - bx - c = 0 \begin{smallmatrix} \nearrow \alpha \\ \searrow \beta \end{smallmatrix}$

$\alpha + \beta = \frac{b}{a}, \alpha \cdot \beta = -\frac{c}{a}$

Demand

$(\alpha^2 + \beta^2) - \alpha\beta$

$(\alpha+\beta)^2 - 2\alpha\beta - \alpha\beta$

$(\alpha+\beta)^2 - 3\alpha\beta$

Q4 $ax^2 + bx + c = 0$

① a + b = 0, c = -ve

$D = b^2 - 4ac$

$= 0 - 4 \times \oplus \ominus = +ve$

a + b = 0 c > 0 ✓

(5) $x^2 + px + q = 0 \begin{smallmatrix} \nearrow \alpha \\ \searrow \beta \end{smallmatrix} \quad \frac{\alpha}{\beta}, \frac{\beta}{\alpha}$

$\alpha + \beta = -p, \alpha \cdot \beta = q$

$x^2 - \left(\frac{\alpha}{\beta} + \frac{\beta}{\alpha}\right)x + \frac{\alpha}{\beta} \times \frac{\beta}{\alpha} = 0$

$x^2 - \frac{(\alpha^2 + \beta^2)}{\alpha\beta} x + 1 = 0$

$x^2 - \frac{((\alpha+\beta)^2 - 2\alpha\beta)}{\alpha\beta} \cdot x + 1 = 0$

$x^2 - \left(\frac{p^2 - 2q}{q}\right)x + 1 = 0$

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QUADRATIC EQUATION

(6) $x^2+px+q=0 \begin{matrix}\nearrow p \\ \searrow q\end{matrix}$

$p+q=-p, \quad | \quad p\cdot q=q$

$q=-2 \quad | \quad p=1$

Q7 $ax^2+bx+c=0 \begin{matrix}\nearrow \alpha \\ \searrow \beta\end{matrix} \quad \alpha\cdot\beta=\frac{c}{a}$

$a\alpha^2+b\alpha+c=0 \rightarrow a\alpha^2+b\alpha=-c \Rightarrow a\alpha+b=\frac{-c}{\alpha}$

$a\beta^2+b\beta+c=0 \rightarrow a\beta^2+b\beta=-c \Rightarrow a\beta+b=-\frac{c}{\beta}$

Demand $= \frac{\alpha}{a\beta+b}+\frac{\beta}{a\alpha+b}$

$= \frac{\alpha}{-c/\beta}+\frac{\beta}{-\frac{c}{\alpha}} \Rightarrow \frac{\alpha\beta}{-c}+\frac{\alpha\beta}{-c}=-\frac{2\alpha\beta}{c}=-\frac{2c}{a\cdot c}=-\frac{2}{a}.$

8) $ax^2+x+b=0$ Real / Diff.

$D>0$

$1-4ab>0 \Rightarrow 4ab<1$

$16ab<4$

Eq2 → $x^2-4\sqrt{ab}\cdot x+1=0$

$D'=\underbrace{16ab}-4$

$D'=\left(\begin{matrix}4 \\ \text{less than } 4\end{matrix}\right)-4=-ve$

Imag. Roots

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QUADRATIC EQUATION

9) $x^2 - 2x + 3 = 0 \to \alpha, \beta$

$\alpha + \beta = 2, \quad \alpha \cdot \beta = 3$

$\frac{\alpha - 1}{\alpha + 1}, \frac{\beta - 1}{\beta + 1}$

$y = \frac{\alpha - 1}{\alpha + 1} \Rightarrow \alpha y + y = \alpha - 1$

$\alpha(y - 1) = -y - 1$

$\alpha = \frac{y + 1}{1 - y}$

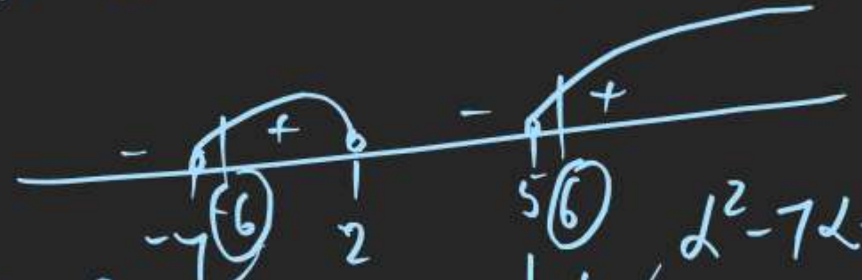
$\left(\frac{y+1}{1-y}\right)^2 - 2\left(\frac{1+y}{1-y}\right) + 3 = 0$

10) $x^2 - 3x + 1 = 0 \to \alpha, \beta \quad \frac{1}{\alpha - 2}, \frac{1}{\beta - 2}$

$y = \frac{1}{\alpha - 2} \Rightarrow \alpha - 2 = \frac{1}{y} \Rightarrow \alpha = \frac{1}{y} + 2$

$\left(\frac{1 + 2y}{y}\right)^2 - 3\left(\frac{1 + 2y}{y}\right) + 1 = 0$ ✓

Q 11 $\frac{x - 5}{x^2 + 5x - 14} > 0 \Rightarrow \frac{x - 5}{(x + 7)(x - 2)} > 0$



least Integer $= 6, -6$

(C) $\alpha^2 + 5\alpha - 6$

$(-6)^2 - 30 - 6 = 0$ ✓

(d) ✓ $\alpha^2 - 7\alpha + 6$

$36 - 42 + 6$

$\alpha^2 + 3\alpha - 4$

$36 + 18 - 4$ ✗

$36 - 18 - 4$ ✗

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QUADRATIC EQUATION

12) $x^2 - 3Kx + 2e^{2\log_e K} - 1 = 0$

$e^{2\log_e 2}$

-ve

$e^{2\log_e(-2)}$ ✗

$x^2 - 3Kx + 2e^{\log_e K^2} - 1 = 0$

$x^2 - 3Kx + (2K^2 - 1) = 0 \to \alpha, \beta$

Real Roots.

$D \geq 0$

$(-3K)^2 - 4 \times 1 \times (2K^2 - 1) \geq 0$

$9K^2 - 8K^2 + 4 \geq 0$

$K^2 + 4 \geq 0$

$\alpha \cdot \beta = \frac{2K^2 - 1}{1} = 7$

$2K^2 = 8$

$K = \boxed{2}, \boxed{-2}$

✓ ✗

13) $x^2 + px + \frac{3p}{4} = 0 \to \alpha, \beta$

$\alpha + \beta = -p, \ \alpha \cdot \beta = \frac{3p}{4}$

$|\alpha - \beta| = \sqrt{10}$

$\sqrt{(\alpha + \beta)^2 - 4\alpha\beta} = \sqrt{10}$

$(p)^2 - \frac{\cancel{12}^{3} p}{\cancel{4}} = 10$

~~$4p^2 - 12p = 40$~~

$p^2 - 3p - 10 = 0$

$(p - 5)(p + 2) = 0$

QUADRATIC EQUATION

(14)

$\alpha^3+\beta^3=-p$, $\alpha\beta=q$

$\downarrow$

$(\alpha+\beta)^2-3\alpha\beta(\alpha+\beta)=-p$

$\frac{\alpha^2}{\beta}, \frac{\beta^2}{\alpha}$

$x^2-\left(\frac{\alpha^2}{\beta}+\frac{\beta^2}{\alpha}\right)\cdot x+\frac{\alpha^2}{\beta}\times\frac{\beta^2}{\alpha}=0$

$x^2-\left(\frac{\alpha^3+\beta^3}{\alpha\beta}\right)x+\alpha\beta=0$

$x^2-\left(\frac{-p}{q}\right)x+q=0$

(15) $\frac{x^2-bx}{ax-c}=\frac{m-1}{m+1}$

Roots equal in Mag But opp in Sign.

$\alpha, -\alpha$

$(x^2-bx)(m+1)=(m-1)(ax-c)$

$(m+1)x^2-b(m+1)x-a(m-1)x-c(m-1)$

$(m+1)x^2-x(bm+b+am-a)-c(m-1)=0$

$\alpha+-\alpha=\frac{bm+b+am-a}{m+1}=0$

$(b+a)m=a-b$

$m=\frac{a-b}{a+b}$

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QUADRATIC EQUATION

16) $y = x^2 + ax + 25$ touches X Axis

$\downarrow$

$D = 0$

$a^2 - 100 = 0$

$a = 10, -10$

17) $a^2 x^2 + bx + 1 > 0$

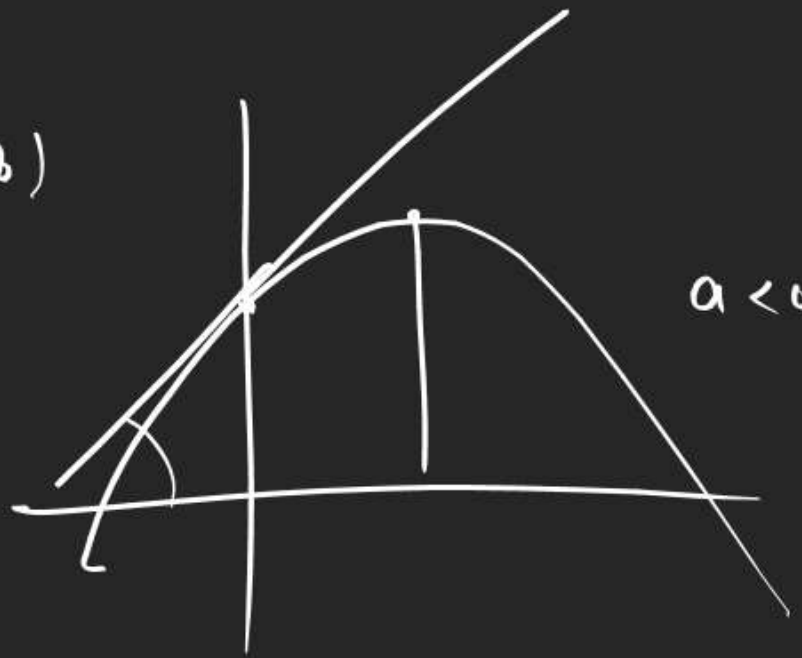
$\oplus$

$D < 0$

$b^2 - 4a^2 < 0$

$b^2 < 4a^2$

18)



$D > 0$

$a < 0,\ b > 0\ c > 0$

(B)

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QUADRATIC EQUATION

Lecture-11

Q If α & β are Roots of $ax^2+bx+c=0$ &

$S_n=\alpha^n+\beta^n$ find value of

$a\cdot S_n+bS_{n-1}+cS_{n-2}=?$

11^{th}

$S_n=\alpha^n+\beta^n$

$S_{n-1}=\alpha^{n-1}+\beta^{n-1}$

$S_{n-2}=\alpha^{n-2}+\beta^{n-2}$

$\alpha+\beta=-\frac{b}{a}$

$\alpha\beta=\frac{c}{a}$

$ax^2+bx+c=0 \rightarrow \alpha, \beta$

$a\alpha^2+b\alpha+c=0$

$a\beta^2+b\beta+c=0$

Demand $aS_n+b\cdot S_{n-1}+cS_{n-2}$

chhota

$=a\cdot(\alpha^n+\beta^n)+b(\alpha^{n-1}+\beta^{n-1})+c\cdot(\alpha^{n-2}+\beta^{n-2})$

$=\alpha^{n-2}(\cancel{a\alpha^2+b\alpha+c}^{\,0})+\beta^{n-2}(\cancel{a\beta^2+b\beta+c}^{\,0})$

$=0+0=0$

Newton Theorem.

$ax^2+bx+c=0 \rightarrow \alpha, \beta$

$S_n=\alpha^n+\beta^n$

$$\boxed{aS_n+bS_{n-1}+cS_{n-2}=0}$$

Advice

$\beta^{n-2}(a\beta^2+b\beta+c)$

$a\cdot\beta^{n-2}\cdot\beta^2+b\beta^{n-2}\cdot\beta^1+c\cdot\beta^{n-2}$

$a\cdot\beta^{n-\cancel{2}+\cancel{2}}+b\cdot\beta^{n-2+1}+c\beta^{n-2}$

$a\beta^n+b\beta^{n-1}+c\beta^{n-2}$

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QUADRATIC EQUATION

Adv 2011
Mains 2015
2020
2022

Q let α & β be roots of $x^2 - 6x - 2 = 0$ $(\alpha > \beta)$

If $a_n = \alpha^n - \beta^n$ $(n \geq 1)$ then value of $\frac{a_{10} - 2a_8}{2a_9} = ?$

$a_{10} = \alpha^{10} - \beta^{10}$

$a_8 = \alpha^8 - \beta^8$

$a_9 = \alpha^9 - \beta^9$

Demand $= \frac{a_{10} - 2a_8}{2a_9}$

M1

$= \frac{(\alpha^{10} - \beta^{10}) - 2(\alpha^8 - \beta^8)}{2(\alpha^9 - \beta^9)}$

$= \frac{\alpha^8(\alpha^2 - 2) - \beta^8(\beta^2 - 2)}{2(\alpha^9 - \beta^9)}$

$= \frac{\alpha^8(6\alpha) - \beta^8(6\beta)}{2(\alpha^9 - \beta^9)} = \frac{6\cancel{(\alpha^9 - \beta^9)}}{2\cancel{(\alpha^9 - \beta^9)}} = 3$

$\alpha^2 - 6\alpha - 2 = 0$

$\beta^2 - 6\beta - 2 = 0$

$\alpha^2 - 2 = 6\alpha$

$\beta^2 - 2 = 6\beta$

M2

$a \cdot S_n + b S_{n-1} + c S_{n-2} = 0$

$1 \cdot x^2 - 6x - 2 = 0$ $\quad a_n = \alpha^n - \beta^n$

$1 \cdot a_n - 6a_{n-1} - 2a_{n-2} = 0$

$n = 10$

$a_{10} - 6a_9 - 2a_8 = 0$

$a_{10} - 2a_8 = 6a_9$

$a_{10} - 2a_8 = 3 \times (2a_9)$

$$\boxed{\frac{a_{10} - 2a_8}{2a_9} = 3}$$

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QUADRATIC EQUATION

Q. α, β are Roots $5x^2+6x+2=0$

Mains 2020

If $S_n = \alpha^n+\beta^n$, $n=1,2,3\ldots$ then

A) $5S_6+6S_5+2S_4=0$

B) $6S_6+5S_5=2S_4$

C) $6S_6+5S_5=-2S_4$

D) $5S_6+6S_5=2S_4$

$5S_n+6S_{n-1}+2S_{n-2}=0$ $n=6$

$5S_6+6S_5+2S_4=0$

Q. If α, β are distinct Roots of $x^2+3^{1/4}x+3^{1/2}=0$ then value of $\alpha^{96}(\alpha^{12}-1)+\beta^{96}(\beta^{12}-1)=?$

Mains 2021

$\alpha^2+3^{1/4}\alpha+\sqrt{3}=0$

$\alpha^2+\sqrt{3}=-3^{1/4}\alpha$

$(\alpha^2+\sqrt{3})^2=\sqrt{3}\alpha^2 \Rightarrow \alpha^4+2\sqrt{3}\alpha^2+3=\sqrt{3}\alpha^2$

$\alpha^4+3=-\sqrt{3}\alpha^2 \Rightarrow (\alpha^4+3)^2=3\alpha^4$

$\alpha^8+6\alpha^4+9=3\alpha^4$

$\alpha^8=-9-3\alpha^4 \} \times \alpha^4$

$\alpha^{12}=-9\alpha^4-3\alpha^8=-9\cancel{\alpha^4}-3(-9-\cancel{3\alpha^4})$

$\boxed{\alpha^{12}=27}$ Sly $\boxed{\beta^{12}=27}$

Demand

$\alpha^{96}(27-1)+\beta^{96}(27-1)$

$\Rightarrow 26(\alpha^{96}+\beta^{96})$

$26((\alpha^{12})^8+(\beta^{12})^8)=26(27^8+27^8)$

$=26\times2\times27^8=52\times27^8$

Q. α, β are Roots of $x^2+20^{1/4}x+5^{1/2}=0$ then $\alpha^8+\beta^8=?$

Mains 21

$\alpha^2+20^{1/4}\alpha+\sqrt{5}=0$; $\beta^2+20^{1/4}\beta+\sqrt{5}=0$

$\alpha^2+\sqrt{5}=-20^{1/4}\alpha$

$(\alpha^2+\sqrt{5})^2=20^{1/2}\alpha^2 \Rightarrow \alpha^4+\cancel{2\sqrt{5}\alpha^2}+5=\cancel{\sqrt{20}\alpha^2}$

$\alpha^4=-5 \Rightarrow \alpha^8=(-5)^2=25$

$\beta^8=25$

$\alpha^8+\beta^8=50$

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QUADRATIC EQUATION

Q α, β are Roots of $x^2 + 5\sqrt{2}x + 10 = 0$ $\alpha > \beta$

Main 2021 & $\boxed{P_n = \alpha^n - \beta^n}$ for each +ve Integer n, then value of

$$\frac{P_{17} \cdot P_{20} + 5\sqrt{2}\, P_{17} \cdot P_{19}}{P_{18} \cdot P_{19} + 5\sqrt{2}\, P_{18}^2} = ?$$

$$1 \cdot P_n + 5\sqrt{2} P_{n-1} + 10\, P_{n-2} = 0$$

$n = 20$

$P_{20} + 5\sqrt{2} P_{19} + 10 P_{18} = 0$

$P_{20} + 5\sqrt{2} P_{19} = -10 P_{18}$

$n = 19$

$P_{19} + 5\sqrt{2} P_{18} = -10\, P_{17}$

$$\text{Demand} = \frac{P_{17}\left(P_{20} + 5\sqrt{2} P_{19}\right)}{P_{18}\left(P_{19} + 5\sqrt{2} P_{18}\right)}$$

$$= \frac{\cancel{P_{17}}\left(\cancel{-10}\,\cancel{P_{18}}\right)}{\cancel{P_{18}} \times \left(\cancel{-10}\,\cancel{P_{17}}\right)} = 1$$