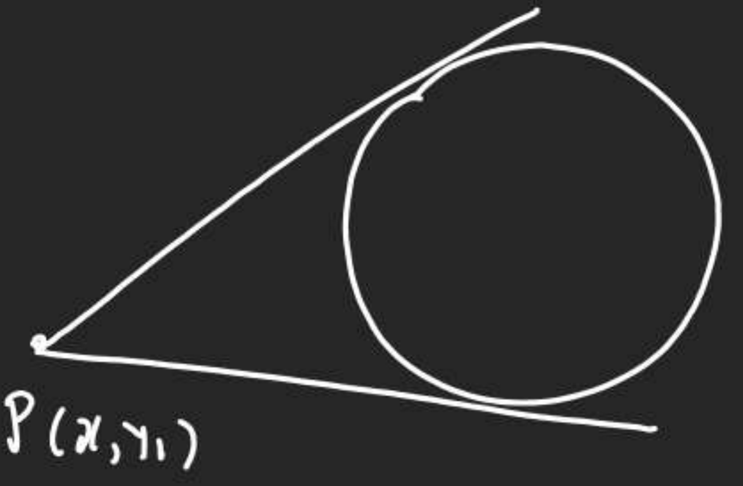
Nishant Jindal

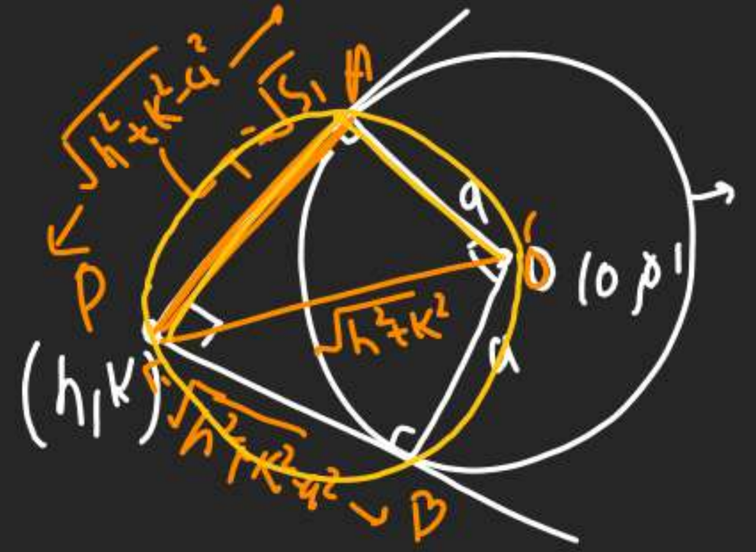
Pair of tangent



1) Writing Combine Eqⁿ of tangent is known as Pair of Tangent

2) Pair of tangents from $(x_1, y_1) \Rightarrow SS_1 = T^2$

Q If tangent lines from (h,k) to $x^2+y^2=a^2$ are making Rt. angle at centre then Sh. that $h^2+k^2=2a^2$

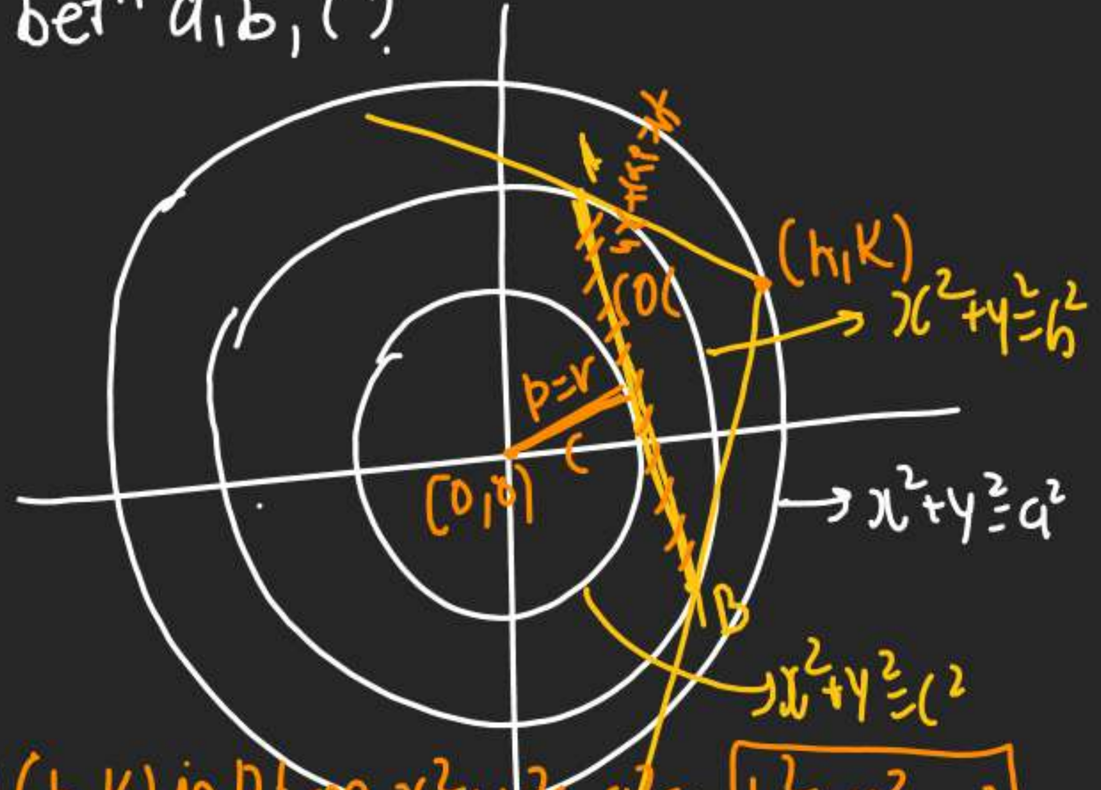


OPAB is a ~~Rectangle~~/Sqʳ

$\Rightarrow \sqrt{h^2+k^2-a^2} = a$

$h^2+k^2 = 2a^2$

Q If tangents are drawn from a pt. on $x^2+y^2=a^2$ to another circle $x^2+y^2=b^2$ & chord of contact of $x^2+y^2=b^2$ touches 3ʳᵈ circle $x^2+y^2=c^2$ Relation betⁿ a, b, c?



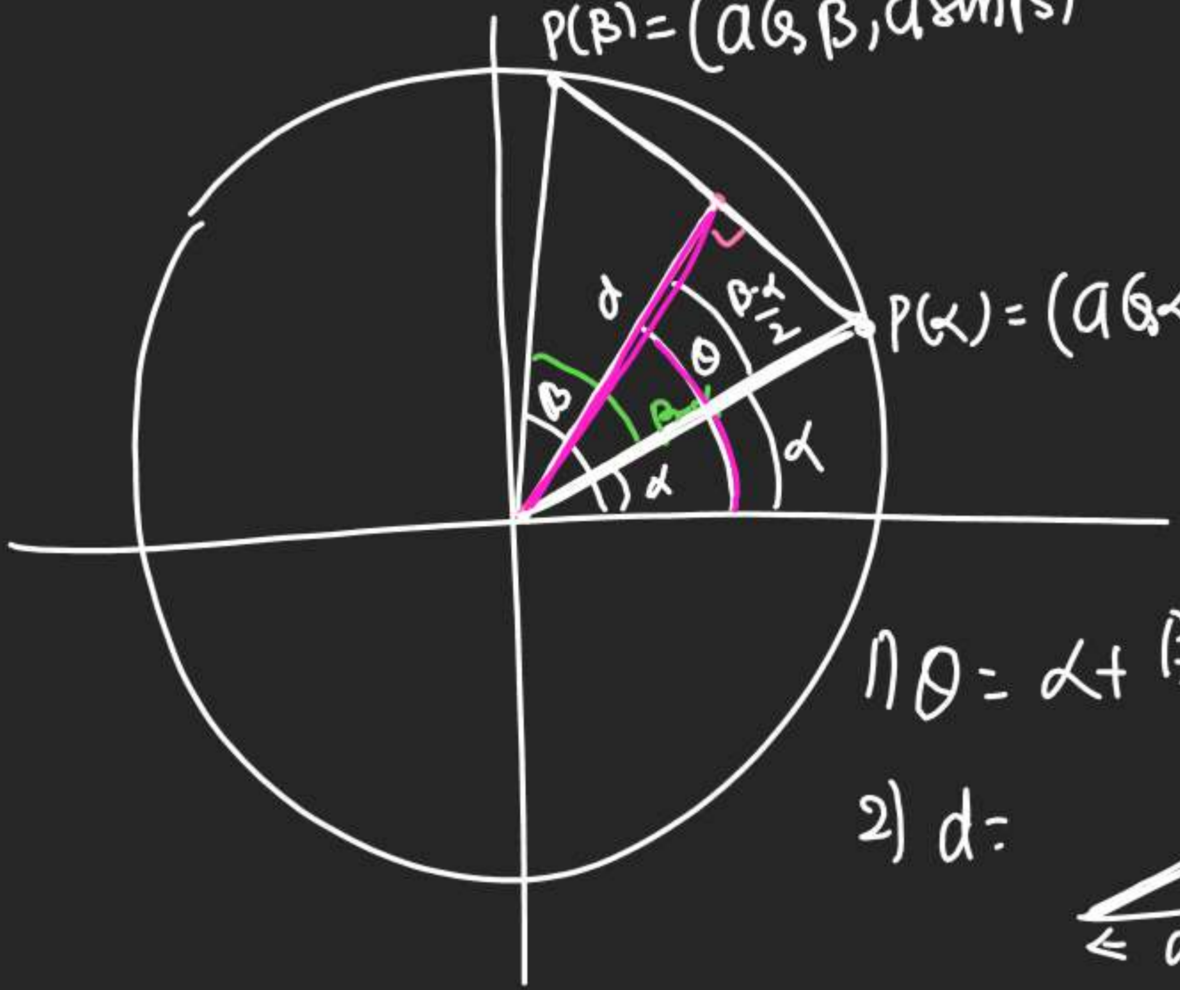
1) let (h,k) in pt on $x^2+y^2=a^2 \Rightarrow \boxed{h^2+k^2=a^2}$

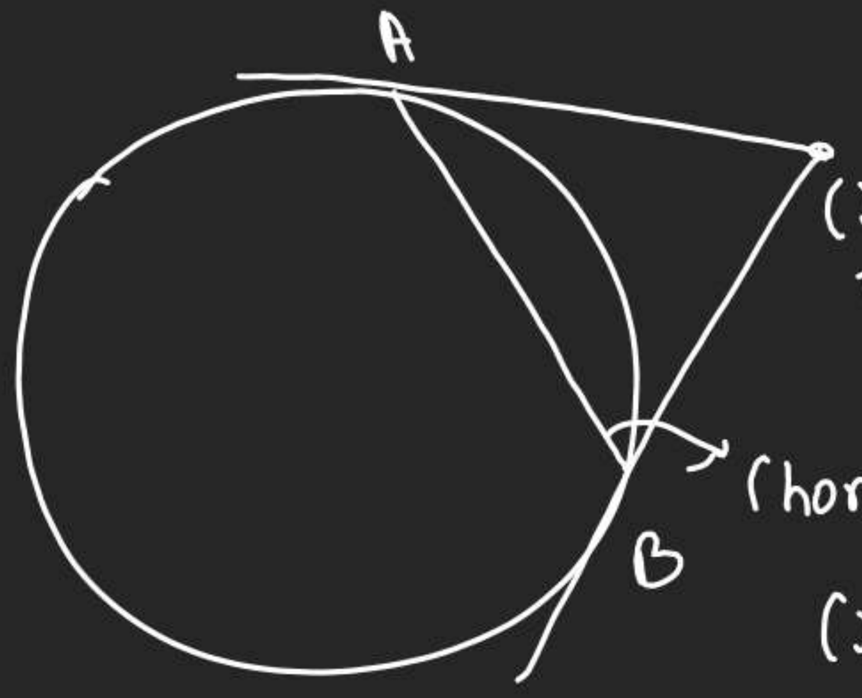
2) AB is COC from (h,k) to $x^2+y^2=b^2$ GP

$hx+ky=b^2$

(3) Using $p=r \Rightarrow c = \dfrac{|0+0-b^2|}{\sqrt{h^2+k^2}} \Rightarrow \boxed{ac=b^2} \rightarrow a, b, c$

Eqⁿ of Chord Joining $P(\alpha)$ & $P(\beta)$





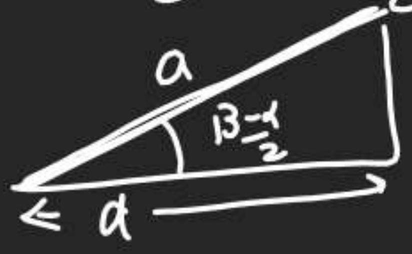
(x_1, y_1) छुपा होगा

Chord Eqⁿ दूंगा

(x_1, y_1) पूछेगा?

1) $\theta = \alpha + \frac{\beta-\alpha}{2} = \frac{2\alpha+\beta-\alpha}{2} = \frac{\alpha+\beta}{2}$

2) $d =$



$a\cos\left(\frac{\beta-\alpha}{2}\right) = d$

(3) $\therefore$ Line Joining $P(\alpha)$ & $P(\beta)$

$$x\cos\left(\frac{\alpha+\beta}{2}\right) + y\sin\left(\frac{\alpha+\beta}{2}\right) = a\cos\left(\frac{\beta-\alpha}{2}\right)$$

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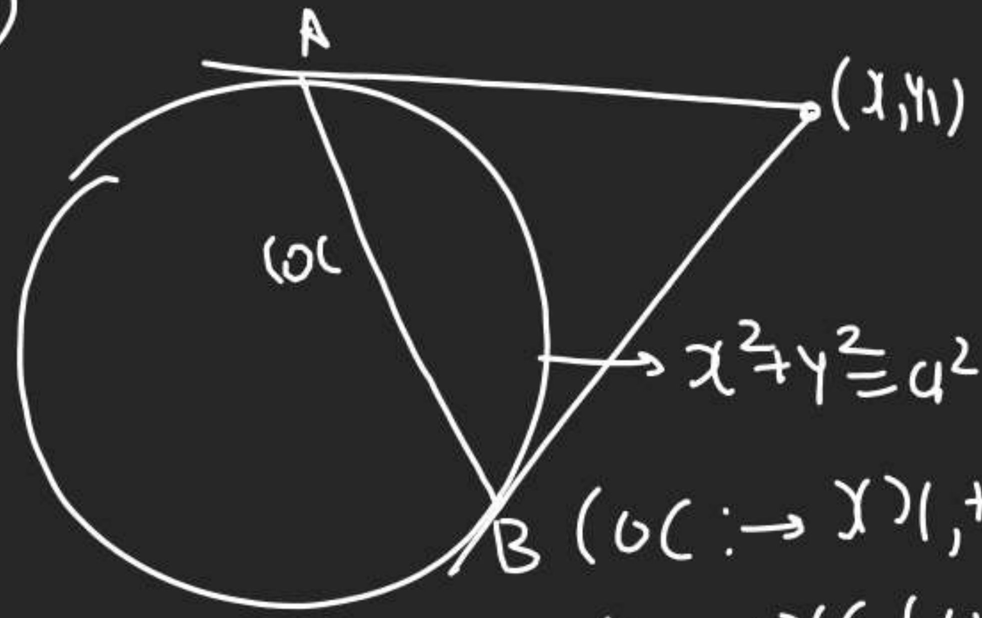
PoI of tangents made at End Pts of C.O.C.



$\to x^2+y^2=a^2$

COC :$\to xx_1+yy_1=a^2$

Chord $\to x\cos\left(\frac{\alpha+\beta}{2}\right)+y\sin\left(\frac{\alpha+\beta}{2}\right)=a\cos\left(\frac{\alpha-\beta}{2}\right)$

But for circle I & II Chord

But Both are same Line.

$\cos(-\theta) = \cos\theta$

$$\frac{x_1}{\cos\left(\frac{\alpha+\beta}{2}\right)}=\frac{y_1}{\sin\left(\frac{\alpha+\beta}{2}\right)}=\frac{a^2}{a\cos\left(\frac{\alpha-\beta}{2}\right)}$$

$$\boxed{x_1=\frac{a\cos\left(\frac{\alpha+\beta}{2}\right)}{\cos\left(\frac{\alpha-\beta}{2}\right)} \quad \Big| \quad y_1=\frac{a\sin\left(\frac{\alpha+\beta}{2}\right)}{\cos\left(\frac{\alpha-\beta}{2}\right)}}$$

Q3 If $m(x-2)+\sqrt{1-m^2}\cdot y=3$ is tangent to Circle $\forall\, m\in[-1,1]$ then Radius of Circle?

Solve

Rsta $\to (x-2)\cos\theta+y\sin\theta=3$.

$(x-2)x_1+yy_1=3$ & match

$\to (x-2)^2+(y)^2=(\sqrt{3})^2$

Q4 2 tangents are drawn from $x^2+y^2=16$ to circle $x^2+y^2=8$ then angle betn tangents?

$x^2+y^2=16$ is D.C. of $x^2+y^2=8$

$\Rightarrow$ tangents are $\perp^r$

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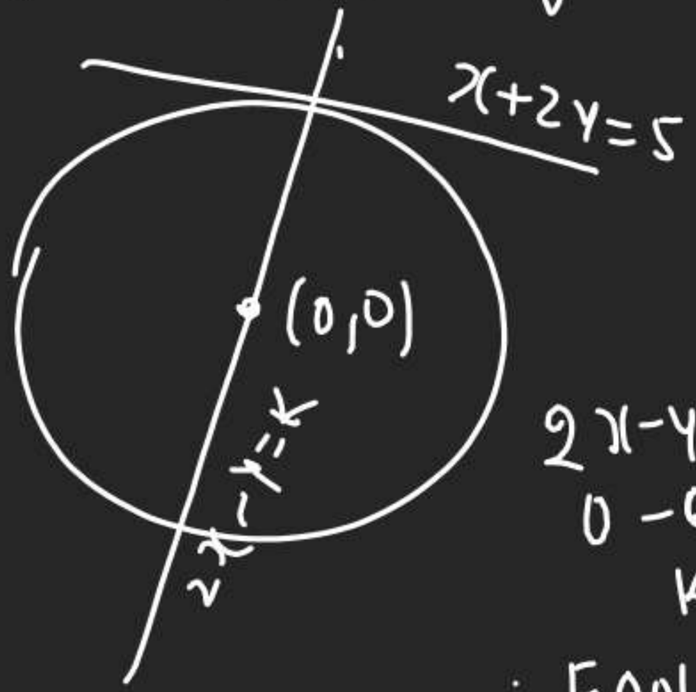
Q If $x+2y=5$ is tangent to $x^2+y^2=5$ then Eq^n of Normal at Pt. of contact is?



$2x-y=K$

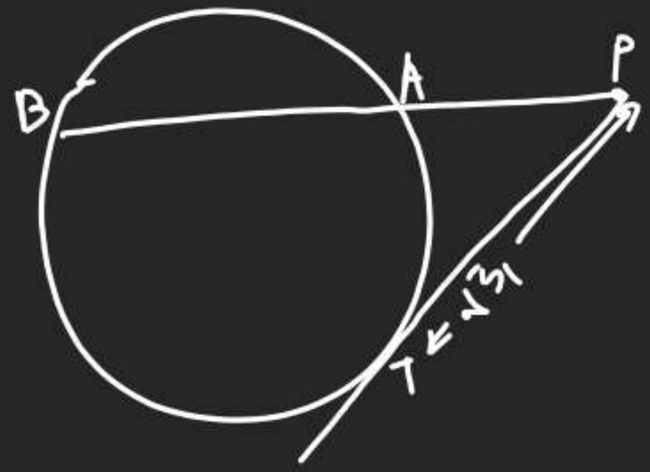
$0-0=K$

$K=0$

$\therefore$ EON

$2x-y=0$

Power of Point



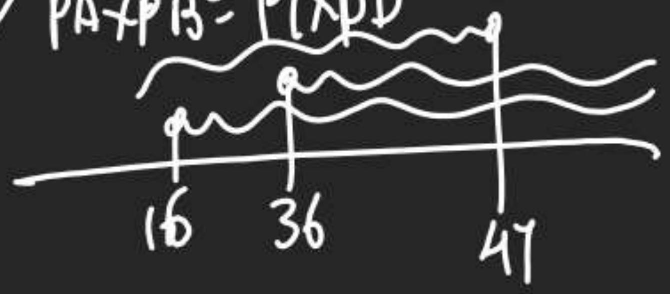
1) $PA\times PB = PT^2$

This $(PT)^2$ is Power of Pt.

2) Power of Pt. may be +ve, −ve & Zero.

(3)

$PA\times PB = PC\times PD$



(4) Power of Pt is S_1

(5) Acc to Position of Pt. →
- Inside = PoP = −ve
- Outside = PoP = +ve
- on Circle = PoP = 0

6 Q If Power of Pt. for (2,5) to the circle $x^2+y^2-8x-12y+P=0$ is −ve & circle do not touch any axes find P?

① PoP = −ve

$\Rightarrow S_1<0 \Rightarrow 4+25-16-60+P<0$

$P<47 \rightarrow$ (A)

(2) Circle → X Axis Not touching $\quad g^2-c<0 \Rightarrow (-4)^2-P<0$

$P>16 \rightarrow$ (B)

→ Y Axis Not touching $\quad f^2-c<0 \Rightarrow (-6)^2-P<0$

$P>36 \rightarrow$ (C)

$P\in(36,47)$ Ans.

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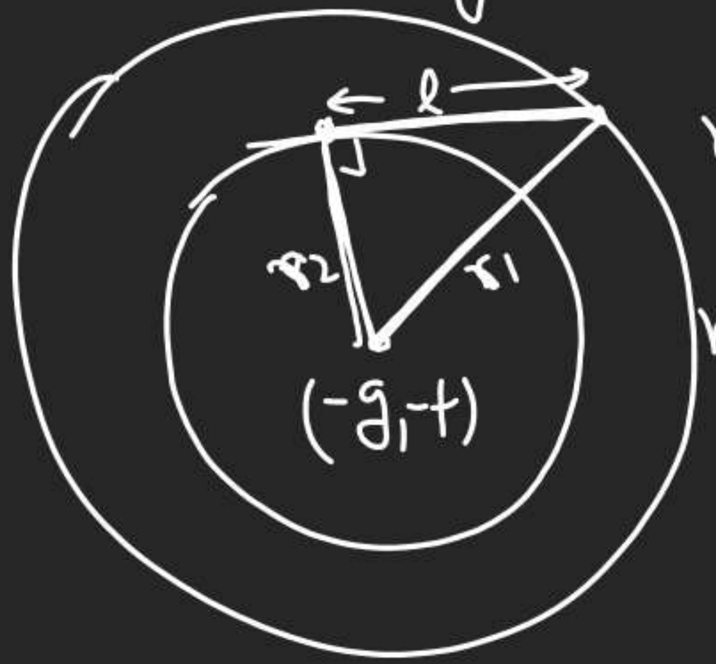
Q Find Length of tangent from a pt "P" on circle S: $x^2+y^2+2gx+2fy+\lambda=0$ to circle S: $x^2+y^2+2gx+2fy+K=0$?



$r_1=\sqrt{g^2+f^2-\lambda}$

$r_2=\sqrt{g^2+f^2-K}$

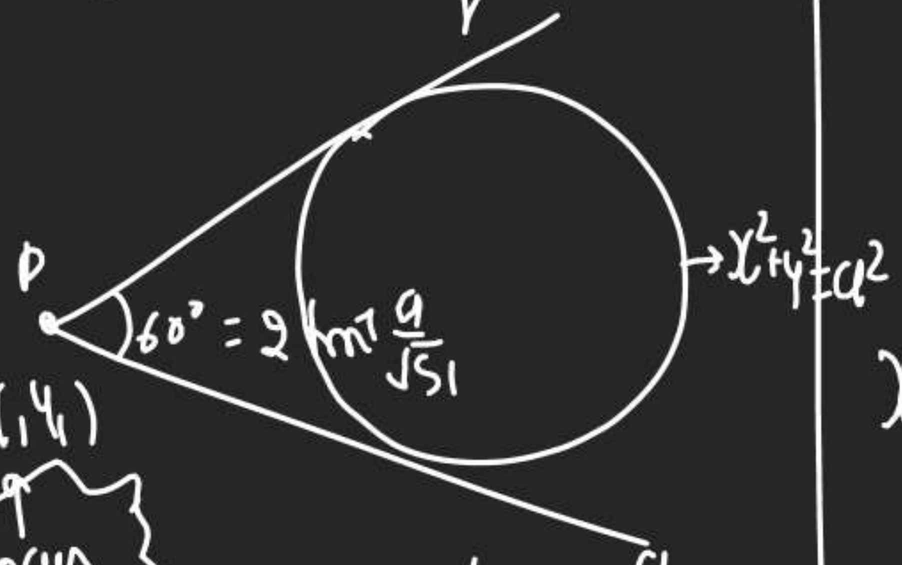
$r_1^2-r_2^2=l^2$

$l=\sqrt{r_1^2-r_2^2}$

$=\sqrt{(\cancel{g^2}+\cancel{f^2}-\lambda)-(\cancel{g^2}+\cancel{f^2}-K)}$

$=\sqrt{K-\lambda}$ Ans.

Q If Pair of tangents drawn from a pt P to the circle $x^2+y^2=a^2$ are at 60° find Locus of pt. P.



Locus चाहिए
$x_1 \to x$
$y_1 \to y$

$60° = 2\tan^{-1}\frac{a}{\sqrt{x_1^2+y_1^2-a^2}}$

$\tan 30° = \frac{a}{\sqrt{x_1^2+y_1^2-a^2}}$

$\frac{1}{\sqrt3} = \frac{a}{\sqrt{x_1^2+y_1^2-a^2}} \Rightarrow \sqrt{x_1^2+y_1^2-a^2}=a\sqrt3$

$\Rightarrow x_1^2+y_1^2=4a^2 \to x^2+y^2=4a^2$ Ans.

Q 9 If PoI of 2 tangents drawn at circle $x^2+y^2=a^2$ at $P(\alpha)$ & $P(\beta)$ such that $|\alpha-\beta|=120°$ find Locus of Po Intersection.

Locus of (x_1, y_1) Maang Rahah!!

$x_1=\frac{a\cos\left(\frac{\alpha+\beta}{2}\right)}{\cos\left(\frac{\alpha-\beta}{2}\right)}$, $y_1=\frac{a\sin\left(\frac{\alpha+\beta}{2}\right)}{\cos\left(\frac{\alpha-\beta}{2}\right)}$

$x_1=\frac{a\cos\left(\frac{\alpha+\beta}{2}\right)}{\cos 60°}$ $\quad y_1=\frac{a\sin\left(\frac{\alpha+\beta}{2}\right)}{\cos 60°}$

$\cos\left(\frac{\alpha+\beta}{2}\right)=\frac{x_1}{2a}$, $\sin\left(\frac{\alpha+\beta}{2}\right)=\frac{y_1}{2a}$

$c^2+s^2=1$

$\frac{x_1^2}{4a^2}+\frac{y_1^2}{4a^2}=1$

$x^2+y^2=4a^2$

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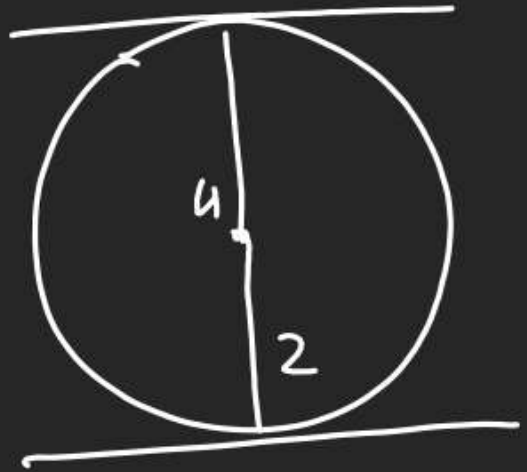
$y=\sqrt{3}x+c_1$

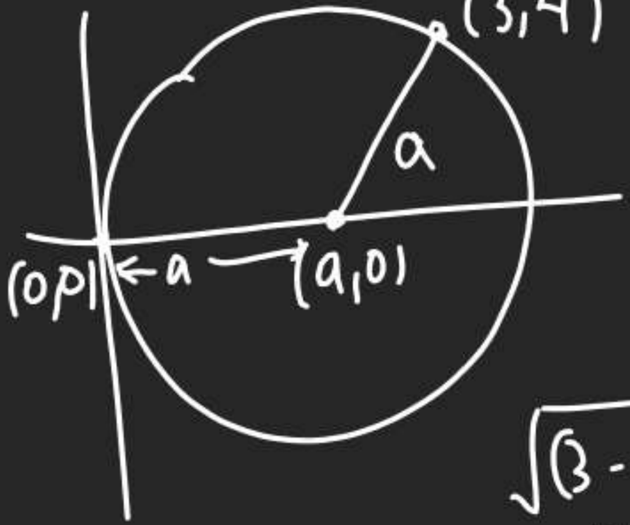
$y=\sqrt{3}x+c_2$



$$\frac{|c_1-c_2|}{\sqrt{\sqrt{3}^2+(1)^2}}=4$$

$$|c_1-c_2|=8$$

(8)

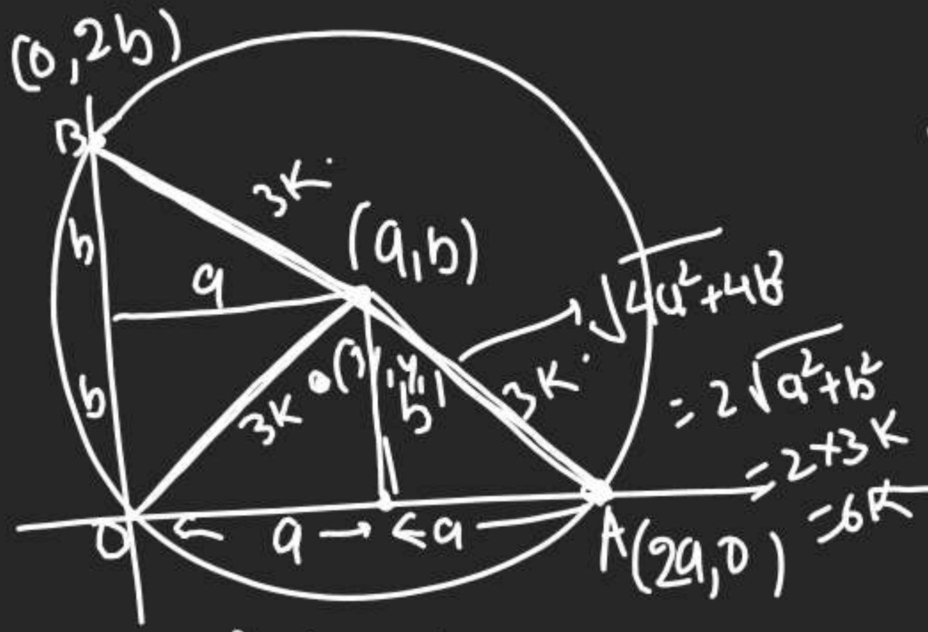


$$\sqrt{(3-a)^2+(4-0)^2}=a$$

$$a^2-6a+9+16=\cancel{a^2}$$

$$6a=25$$

(11)



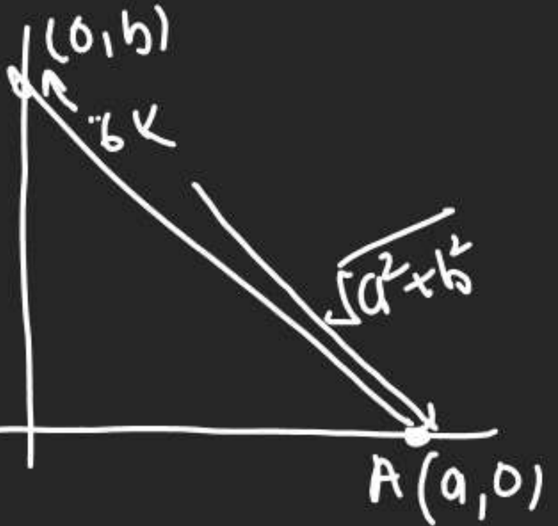
$$a^2+b^2=9k^2$$

$$x_1=\frac{2a+0+0}{3} \quad \Big| \quad y_1=\frac{0+0+2b}{3}$$

$$a=\frac{3x_1}{2},\ b=\frac{3y_1}{2}$$

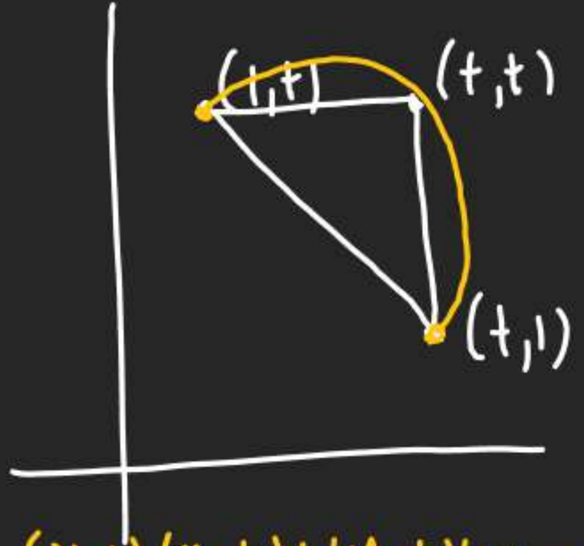
$$a^2+b^2=9k^2$$

$$\frac{9x_1^2}{4}+\frac{9y_1^2}{4}=9k^2$$



$$\frac{x}{a}+\frac{y}{b}=1$$

(12)



$$(x-1)(x-t)+(y-t)(y-1)=0$$