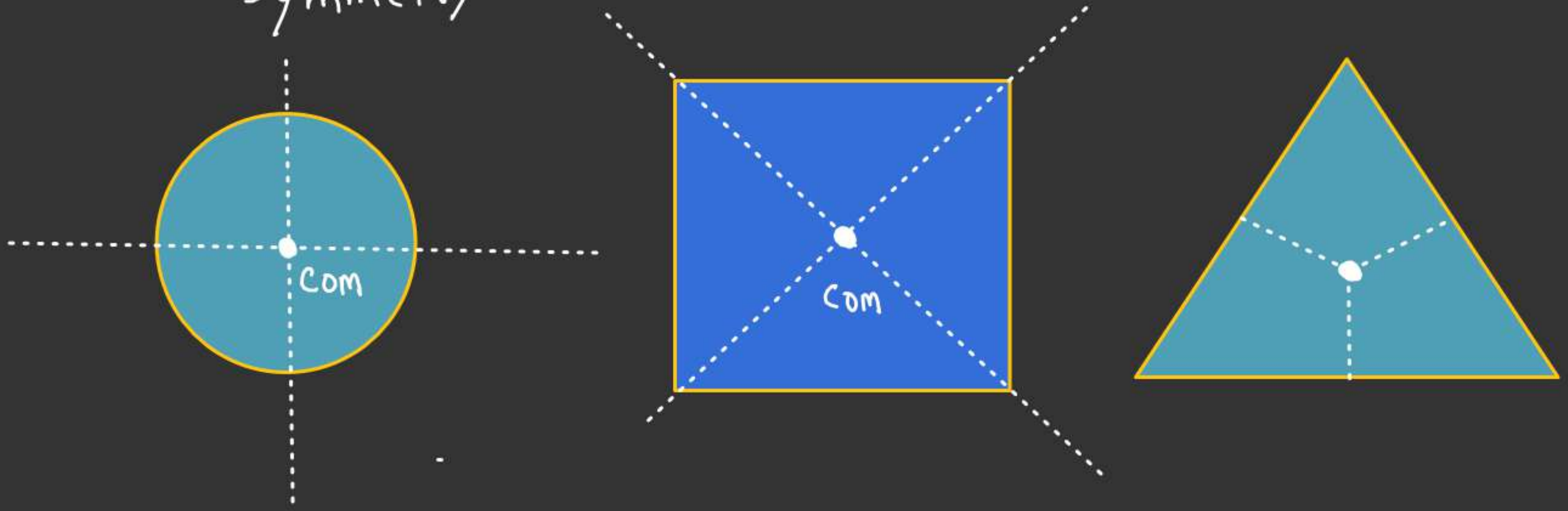
Nishant Jindal

COM

Def^n:- [An imaginary point either inside or outside the body where the whole mass of System or body assumed to be Concentrated.

- "COM and center of gravity are Same only when g is uniform"

- For Symmetrical bodies COM always lie at the axis of Symmetry



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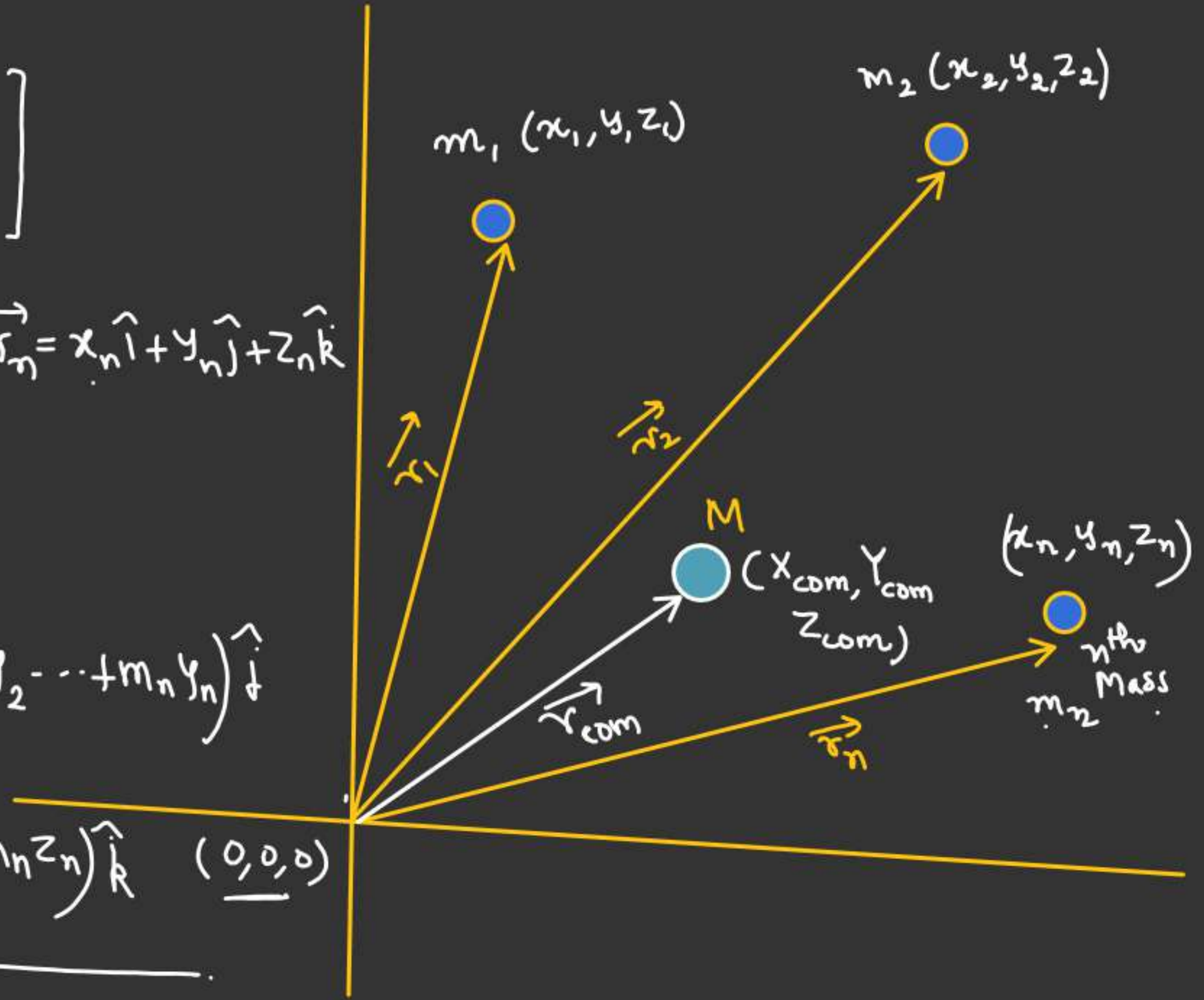
COM of n- particle System.

$$\vec{r}_{com} = \left[\frac{m_1\vec{r_1} + m_2\vec{r_2} \cdots + m_n\vec{r_n}}{m_1+m_2 \cdots + m_n}\right]$$

$\vec{r_1} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$, $\vec{r_2} = x_2\hat{i} + y_2\hat{j} + z_2\hat{k}$, $\vec{r_n} = x_n\hat{i} + y_n\hat{j} + z_n\hat{k}$

$\vec{r}_{com} = X_{com}\hat{i} + Y_{com}\hat{j} + Z_{com}\hat{k}$

$$\left(X_{com}\hat{i} + Y_{com}\hat{j} + Z_{com}\hat{k}\right) = \frac{\left(m_1x_1 + m_2x_2 \cdots + m_nx_n\right)\hat{i} + \left(m_1y_1 + m_2y_2 \cdots + m_ny_n\right)\hat{j} + \left(m_1z_1 + m_2z_2 \cdots + m_nz_n\right)\hat{k}}{\left(m_1 + m_2 \cdots + m_n\right)}$$



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$$X_{com} = \frac{m_1x_1 + m_2x_2 - - - + m_nx_n}{m_1 + m_2 - - - + m_n} = \left[\frac{\sum_{i=1}^{n} m_i x_i}{\sum_{i=1}^{n} m_i}\right].$$

$$Y_{com} = \left(\frac{m_1y_1 + m_2y_2 - - - + m_ny_n}{m_1 + m_2 - - - + m_n}\right) = \left[\frac{\sum_{i=1}^{n} m_i y_i}{\sum_{i=1}^{n} m_i}\right]$$

$$Z_{com} = \left(\frac{m_1z_1 + m_2z_2 - - + m_nz_n}{m_1 + m_2 - - - + m_n}\right) = \left[\frac{\sum_{i=1}^{n} m_i z_i}{\sum_{i=1}^{n} m_i}\right]$$

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Q: Locate COM of the system

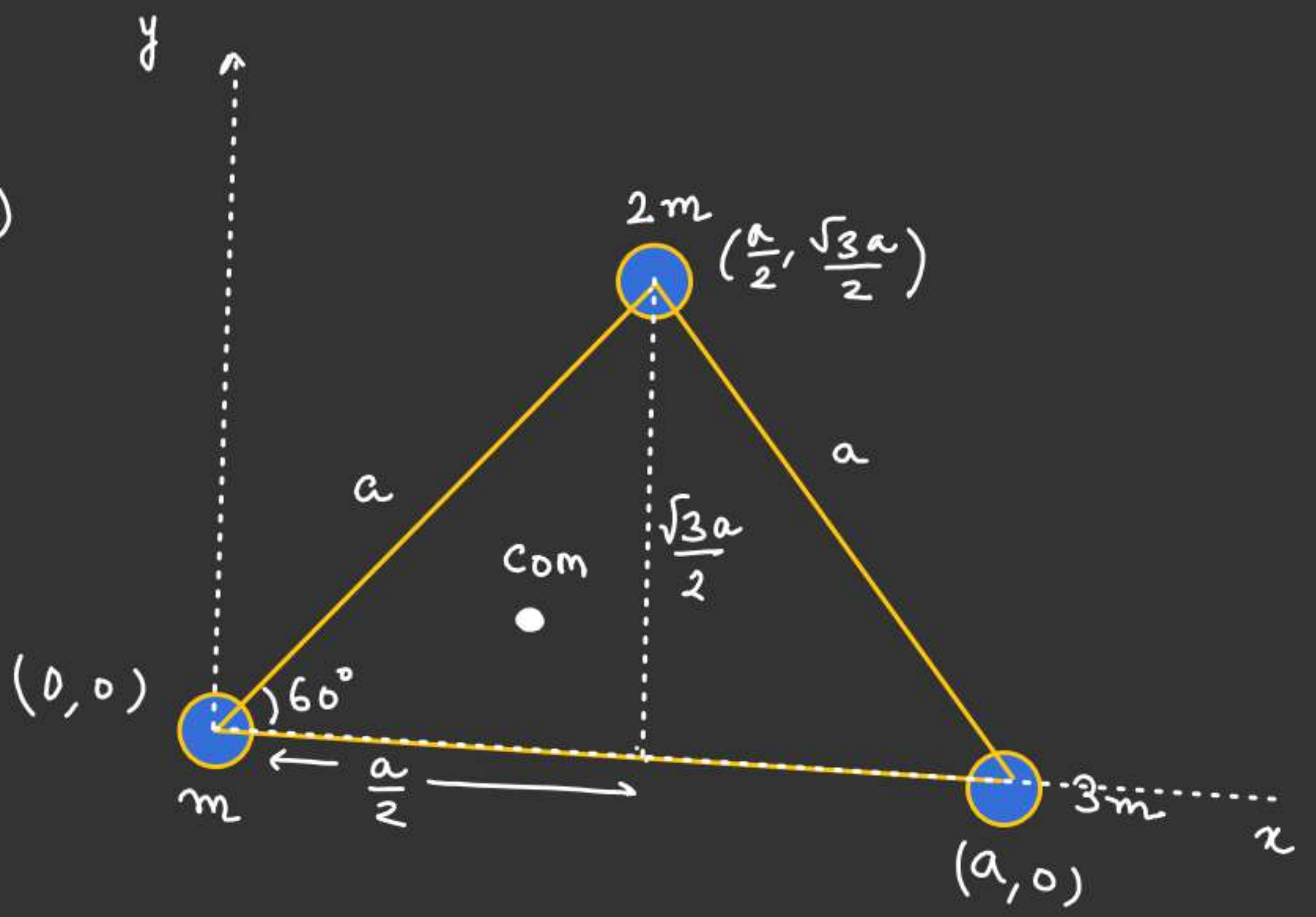
$$X_{com} = \frac{m(0) + (2m)\left(\frac{a}{2}\right) + 3m(a)}{6m}$$

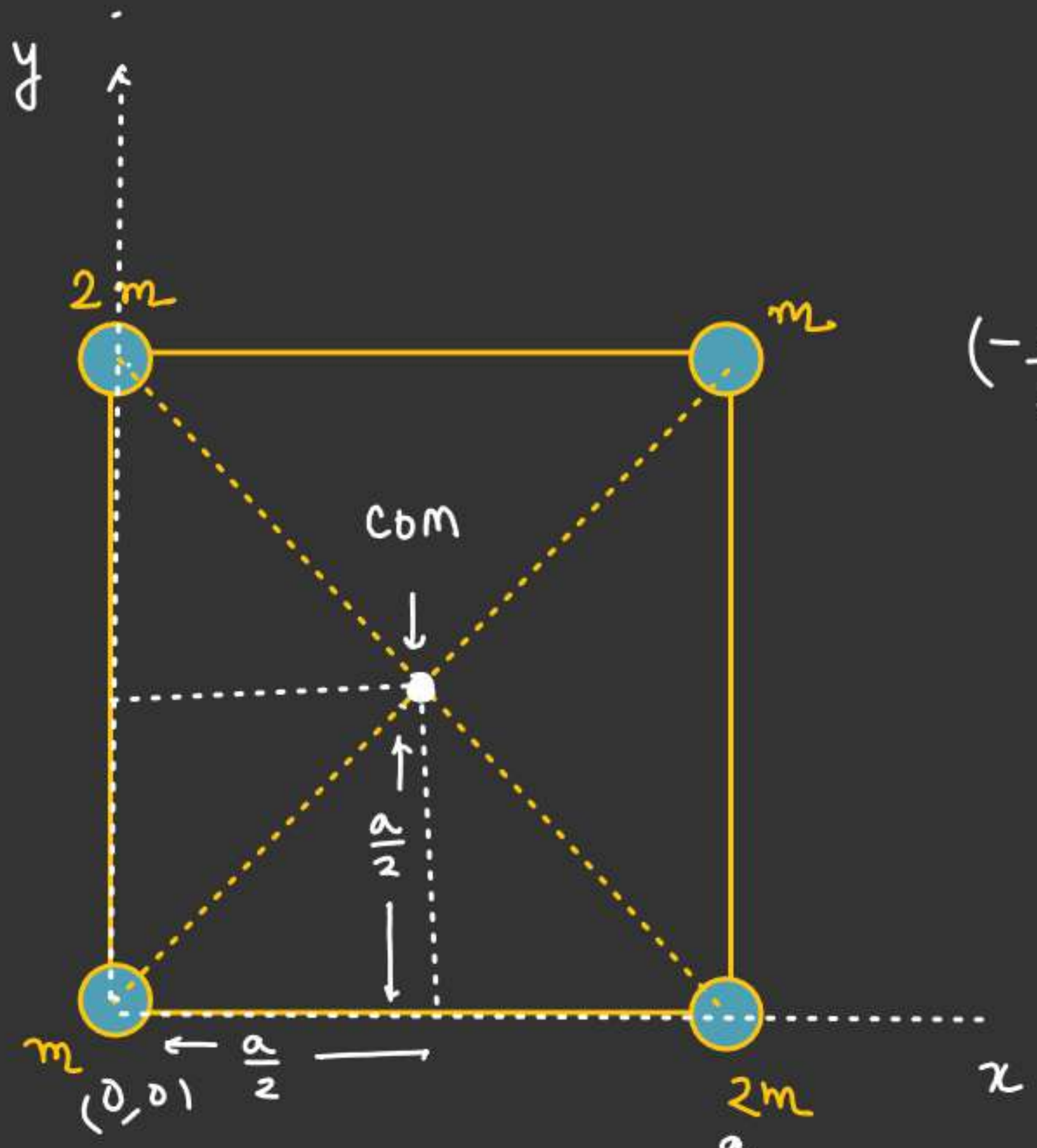
$$X_{com} = \frac{4a}{6} = \frac{2a}{3}$$

$$Y_{com} = \frac{m(0) + 3m(0) + 2m\left(\frac{\sqrt{3}a}{2}\right)}{6m}$$

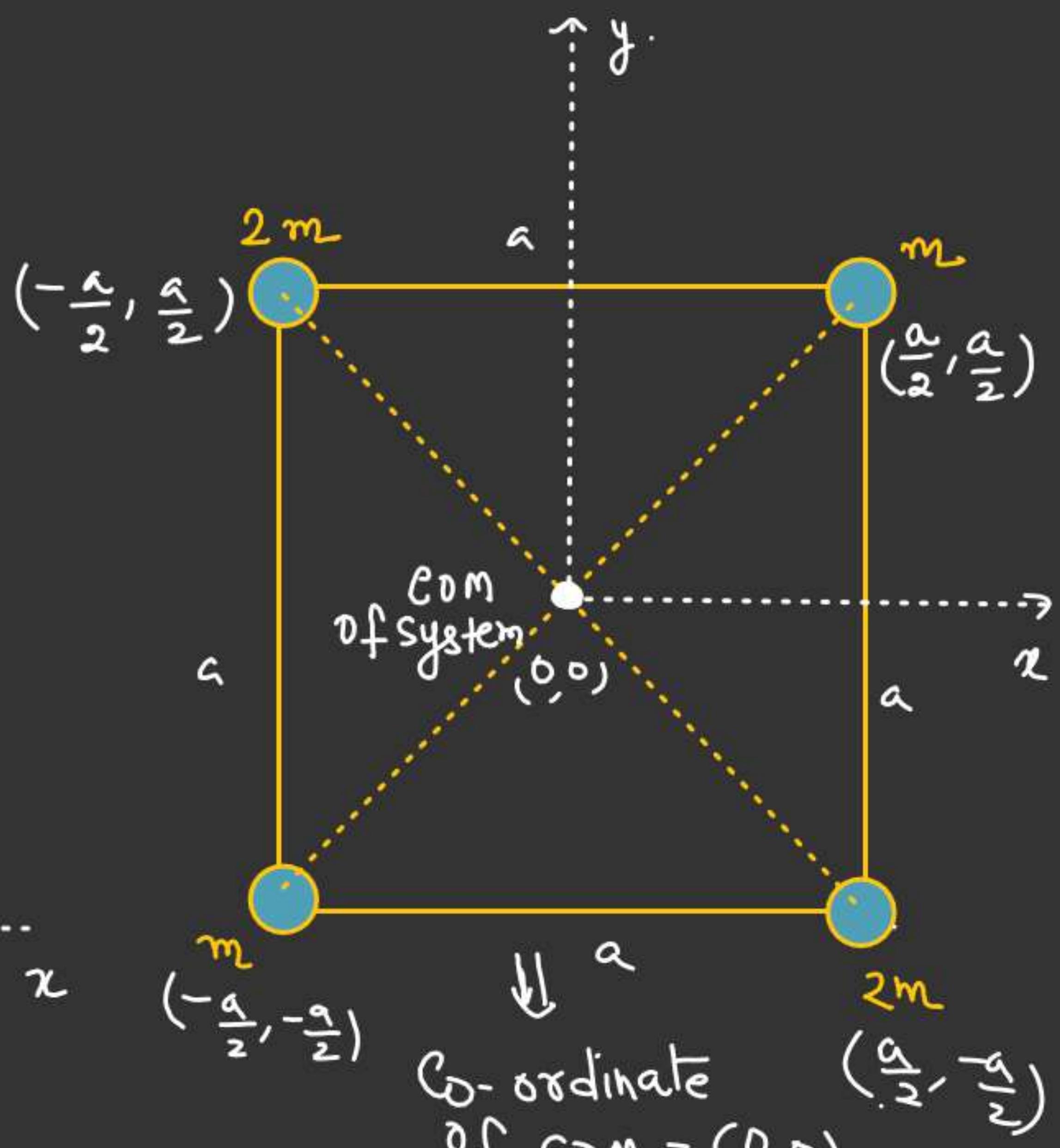
$$Y_{com} = \frac{\sqrt{3}a}{6}$$

$$COM = \left(\frac{2a}{3}, \frac{\sqrt{3}a}{6}\right)$$





Co-ordinate of COM $= \left(\frac{a}{2}, \frac{a}{2}\right)$



$\Downarrow$

Co-ordinate of COM $= (0,0)$

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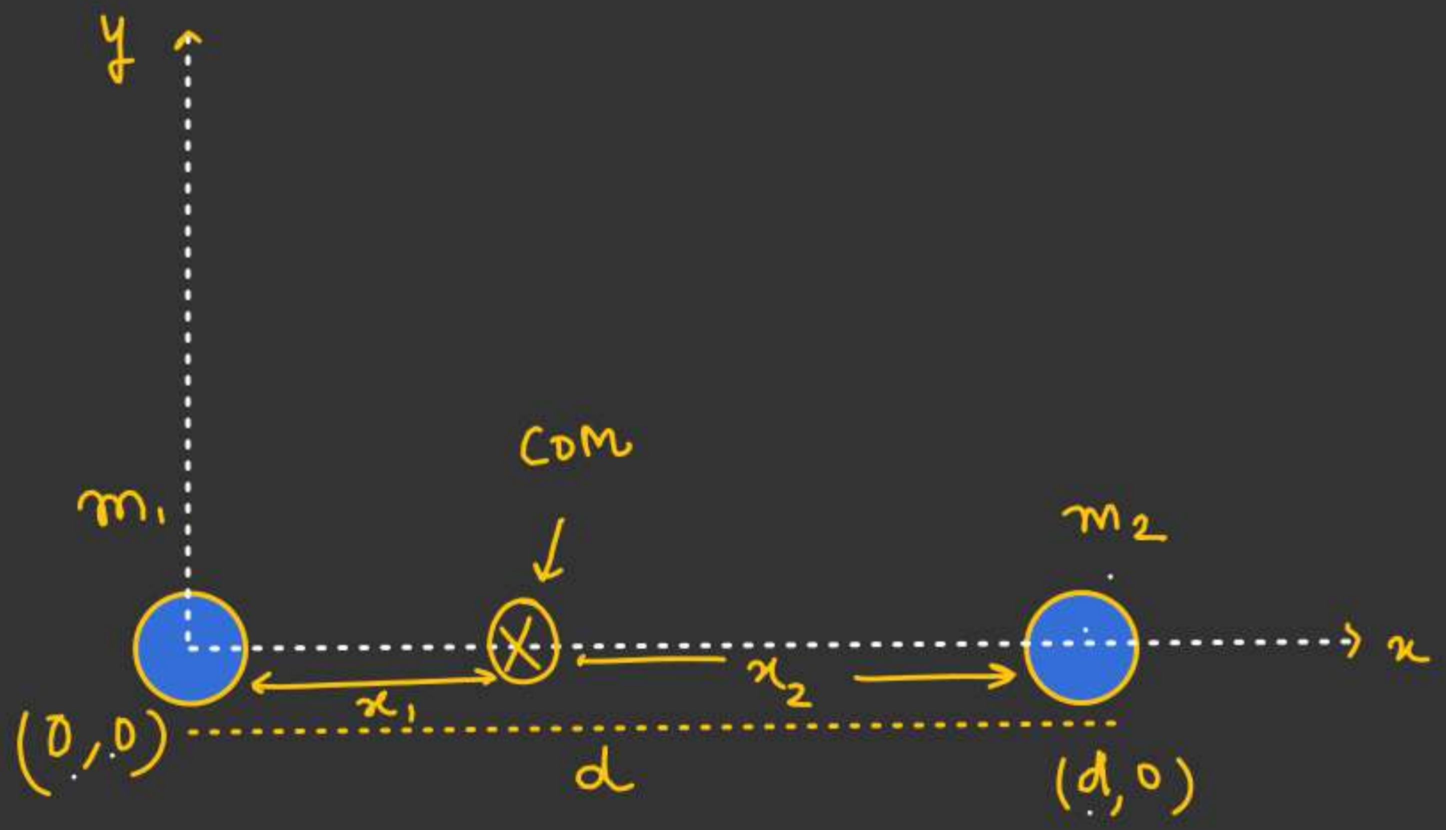
COM of two point System



$$X_{com} = \frac{m_1(0) + m_2 d}{m_1+m_2}$$

$$X_{com} = \left(\frac{m_2 d}{m_1+m_2}\right)$$

$$Y_{com} = 0, \quad Z_{com} = 0.$$

if $m_2 >> m_1$

$$X_{com} = \frac{\cancel{m_2} d}{\cancel{m_2}\left(1+\frac{m_1}{m_2}\right)}$$

$$X_{com} \to d. \qquad \frac{m_1}{m_2} \to 0$$

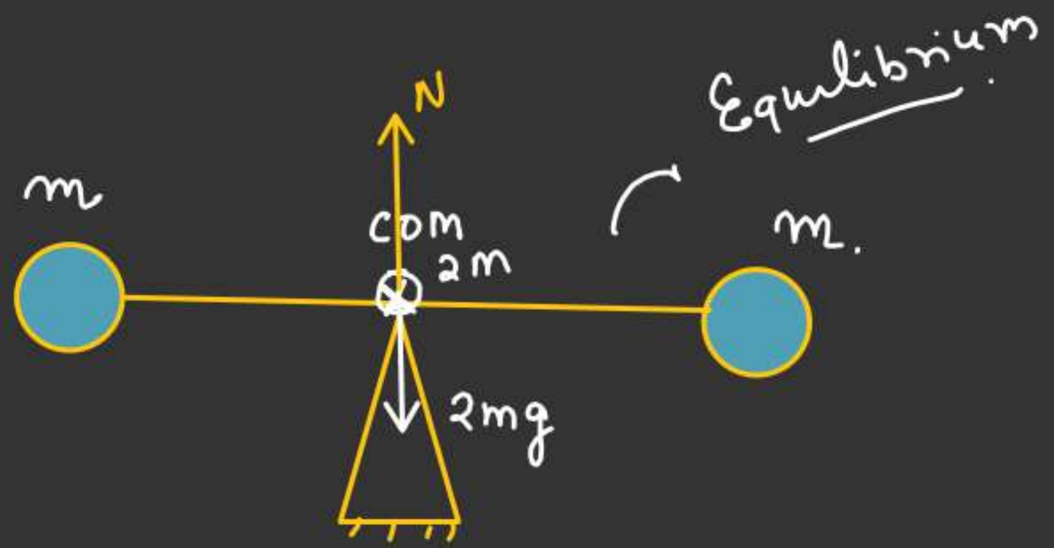
✓ $$x_1 = \left(\frac{m_2 d}{m_1+m_2}\right)$$

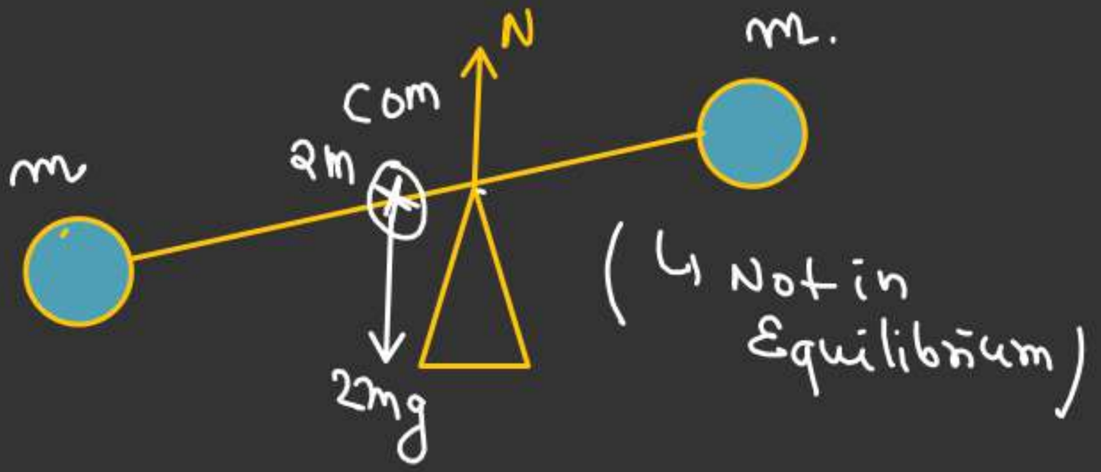
✓ $$x_2 = d - x_1 = \left(\frac{m_1 d}{m_1+m_2}\right)$$

$$\boxed{\frac{x_1}{x_2} = \frac{m_2}{m_1}}$$

if $m_1 = m_2$

COM at $\frac{d}{2}$





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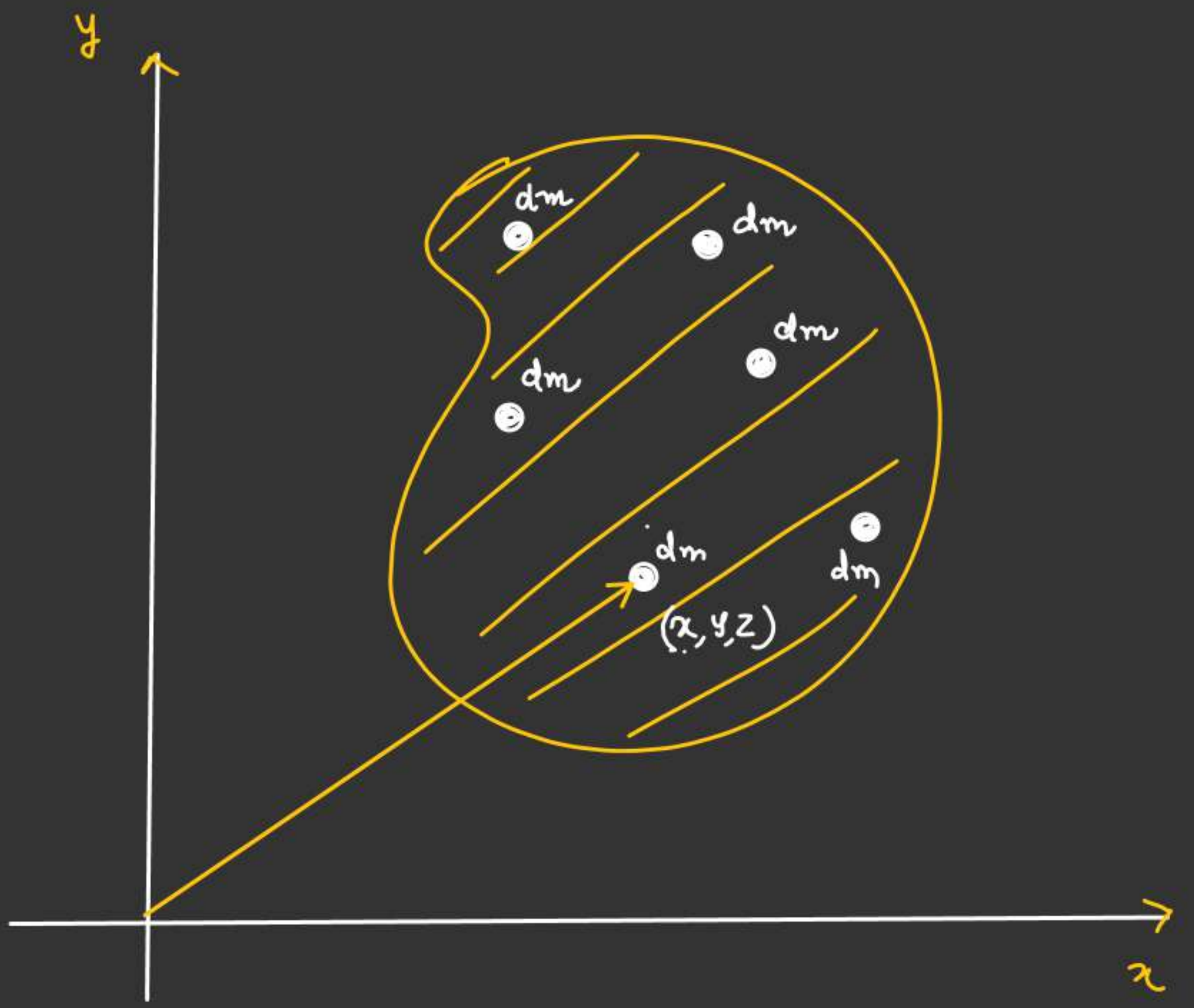
COM of Continious Mass distribution

$$\left(X_{com}\right)_{body} = \left[\frac{\int dm\, x}{\int dm}\right]$$

$$\left(Y_{com}\right)_{body} = \frac{\int dm \cdot y}{\int dm}$$

$$\left(Z_{com}\right)_{body} = \frac{\int dm \cdot z}{\int dm}$$



Note

For Uniform body

$\int dm = M$

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Continious Mass distribution

(1D) Linear Mass density

$$\left(\frac{M}{L} = \lambda\right)$$

If λ = Constant.

Uniform mass distribution.

(2D) Areal Mass density

$$\frac{M}{A} = \sigma$$

if σ = constant

uniform mass distribution.

(3D) Volume Mass density

$$\frac{M}{V} = \rho$$

ρ = Constant.

(Uniform mass distribution)

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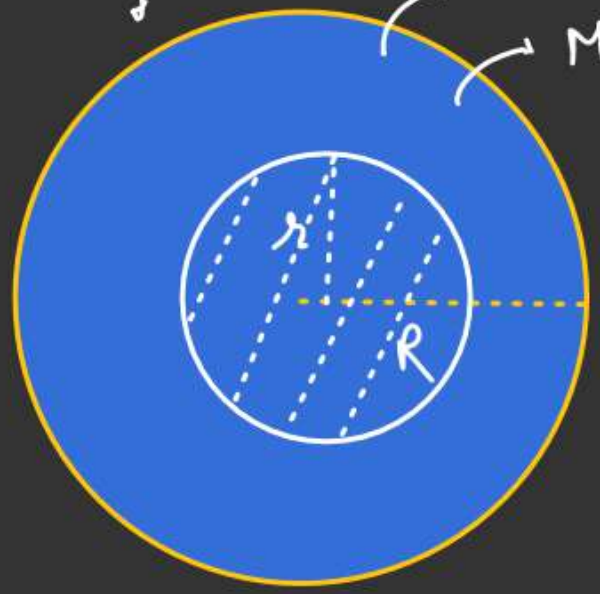
Disc

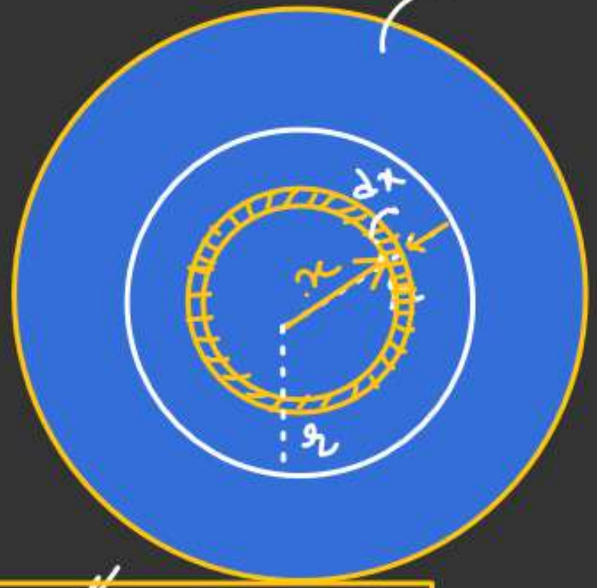
$\sigma =$ Constant.

M



$\sigma = \sigma_0 r$ (Non-Uniform)

$r =$ radial distance.



$\sigma_x = \sigma_{x+dx}$

as dx is very small.

for dx σ assumed to be constant.

$$\frac{M}{\pi R^2} = \sigma.$$

$m_r =$ mass of disc having radius r.

$$\underline{\underline{m_r}} = (\sigma \times \pi r^2)$$

$$= \left(\frac{M}{\pi R^2} \times \pi r^2\right)$$

$$= \frac{M r^2}{R^2}.$$

$$\boxed{dA = \text{length of differential element} \times \text{Thickness}}$$

let, dm be the mass of ring having thickness dx and radius x.

$$dm = \sigma_x \cdot dA$$

$$dA = (\text{differential area of ring})$$

$$dA = (2\pi x)\, dx.$$

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$$\int_0^{m_r} dm = \int_0^r (\sigma_0 x)(2\pi x\, dx) \qquad dm = \sigma_x \cdot dA$$

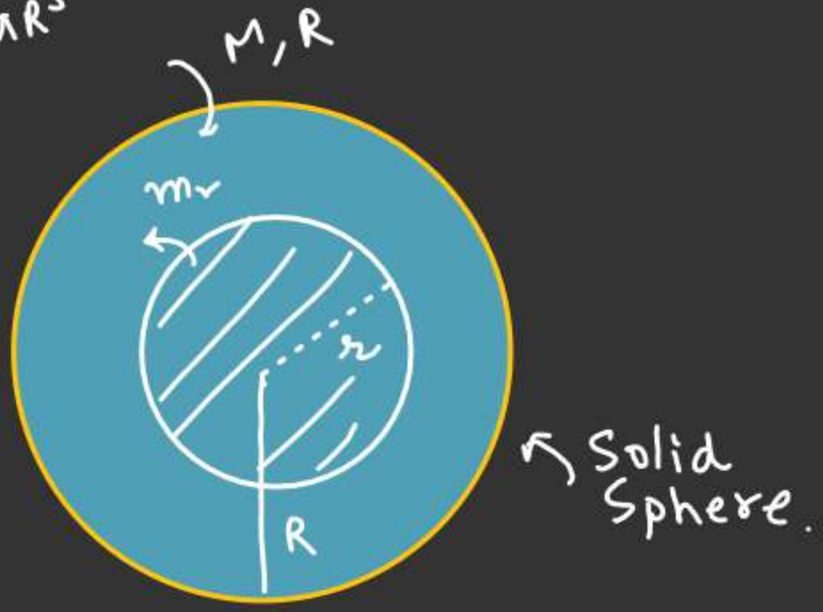
$$m_r = \sigma_0 2\pi \int_0^r x^2\, dx$$

$$m_r = \left(\frac{\sigma_0 \cdot 2\pi\ r^3}{3}\right)$$

$$m_r = \left(\frac{2\pi\sigma_0 r^3}{3}\right)$$

ρ = Constant $\quad \rho = \frac{M}{\frac{4}{3}\pi R^3}$

$m_r = ??$



$$m_r = \left(\frac{M}{\cancel{\frac{4}{3}\pi} R^3}\right) \times \left(\cancel{\frac{4}{3}\pi} r^3\right)$$

$$m_r = \left(\frac{M}{R^3} r^3\right)$$

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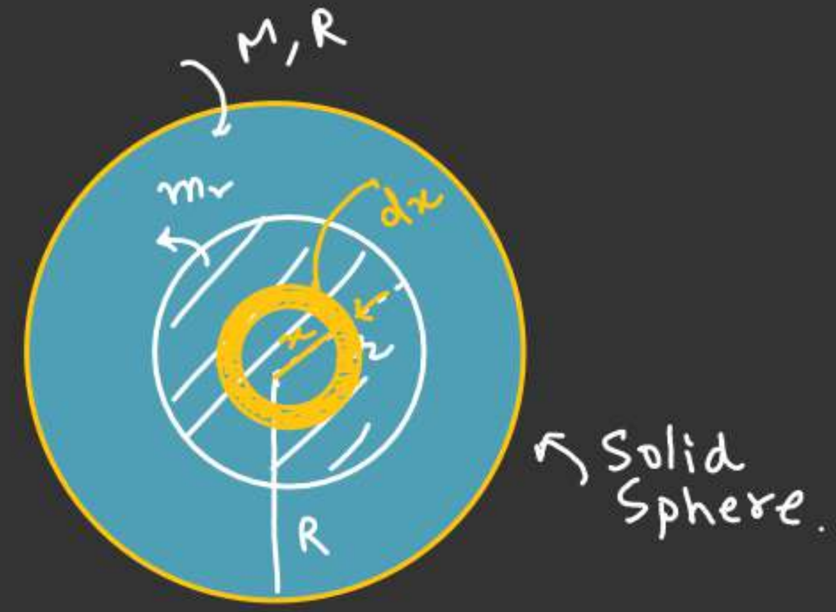
Case-2 ρ = Non-Uniform

$\rho = \rho_0 r$ (ρ_0 = Constant)

$m_r = ??$



let, dm be the mass of shell having radius x (hollow sphere) and thickness dx

$$\boxed{dV = \left(\begin{array}{l}\text{Area of}\\ \text{differential}\\ \text{element}\end{array}\right) \times \text{thickness}}$$

$$dm = \rho_x \underbrace{dV}_{}$$

differential Volume of the hollow sphere having radius x & thickness dx

$$dV = (4\pi x^2).dx$$

$\rho_x = \rho_{x+dx}$
i.e for dx thickness ρ assumed to be constant

$$dm = (\rho_0 x)\, 4\pi x^2 . dx$$

$$\int_0^{m_r} dm = \rho_0 4\pi \int_0^r x^3 dx$$

$$m_r = \frac{\rho_0 4\pi r^4}{4} \Rightarrow \underline{m_r = (\rho_0 \pi r^4)}$$

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$dV = ??$

$\Downarrow$

Differential volume

$$V = \frac{4}{3}\pi r^3$$

$$\frac{dV}{dr} = \frac{4}{3}\pi \frac{d}{dr}(r^3)$$

$$\frac{dV}{dr} = \frac{4}{\cancel{3}}\pi \times \cancel{3} r^2$$

$$\boxed{dV = \underbrace{(4\pi r^2)}_{\text{Volume of differential element}} \underbrace{dr}_{\text{Thickness}}}$$

$dA = ??$ (Differential Area)

$$A = \pi r^2$$

$$\frac{dA}{dr} = \pi \frac{d}{dr}(r^2)$$

$$\frac{dA}{dr} = 2\pi r$$

$$\boxed{dA = \underbrace{(2\pi r)}_{\text{Length of differential element}} \underbrace{(dr)}_{\text{thickness}}}$$