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1. $9x^2+4y^2-18x-16y-11=0$

$$\frac{(x-1)^2}{4}+\frac{(y-2)^2}{9}=1$$

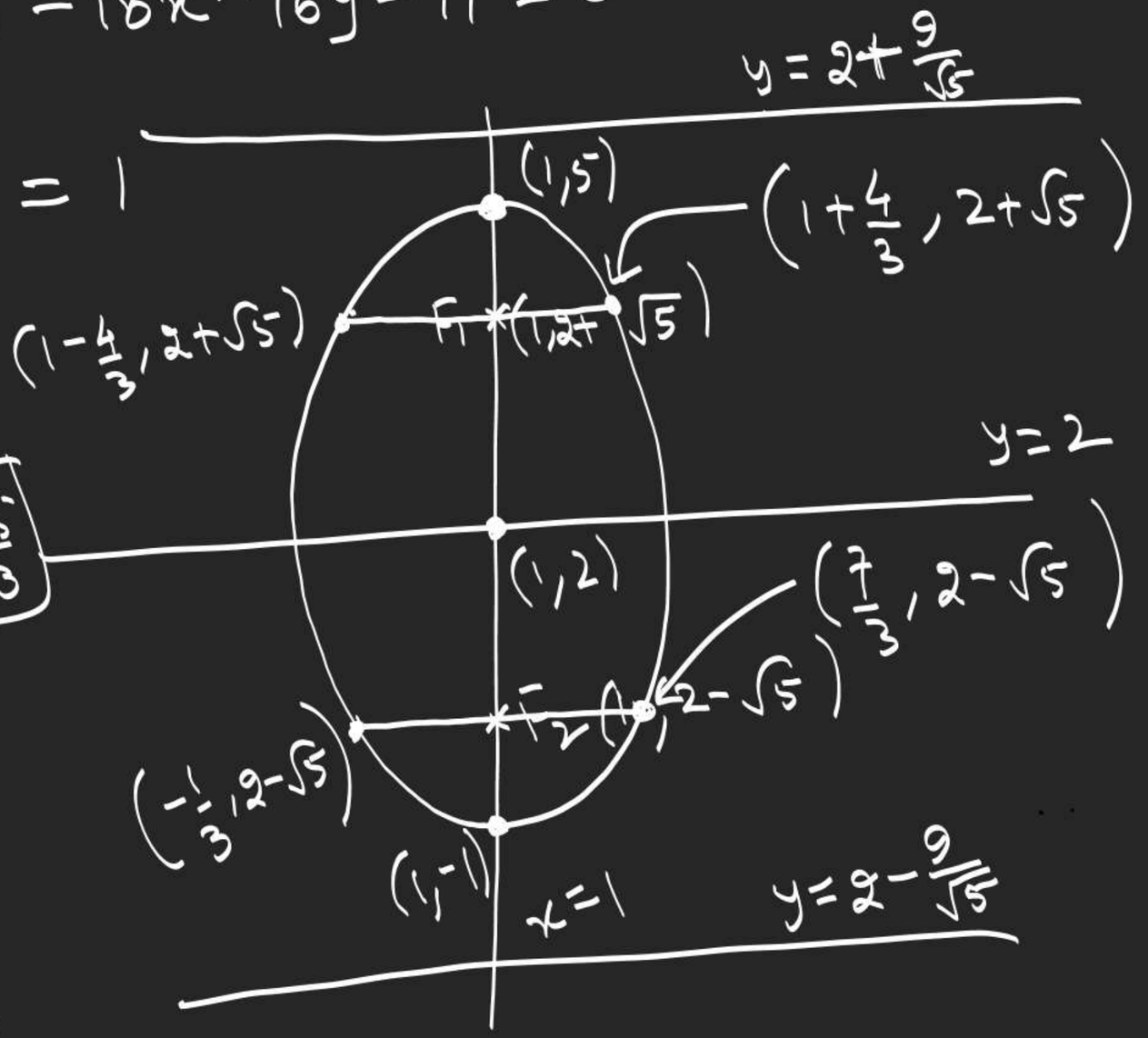
$4=9(1-e^2)$

$\sqrt{5}=3e \Rightarrow e=\frac{\sqrt{5}}{3}$

$ae=\frac{3}{\sqrt{5}/3}$

$\frac{b^2}{a}=\frac{4}{3}$

Length of LR $=\frac{8}{3}$



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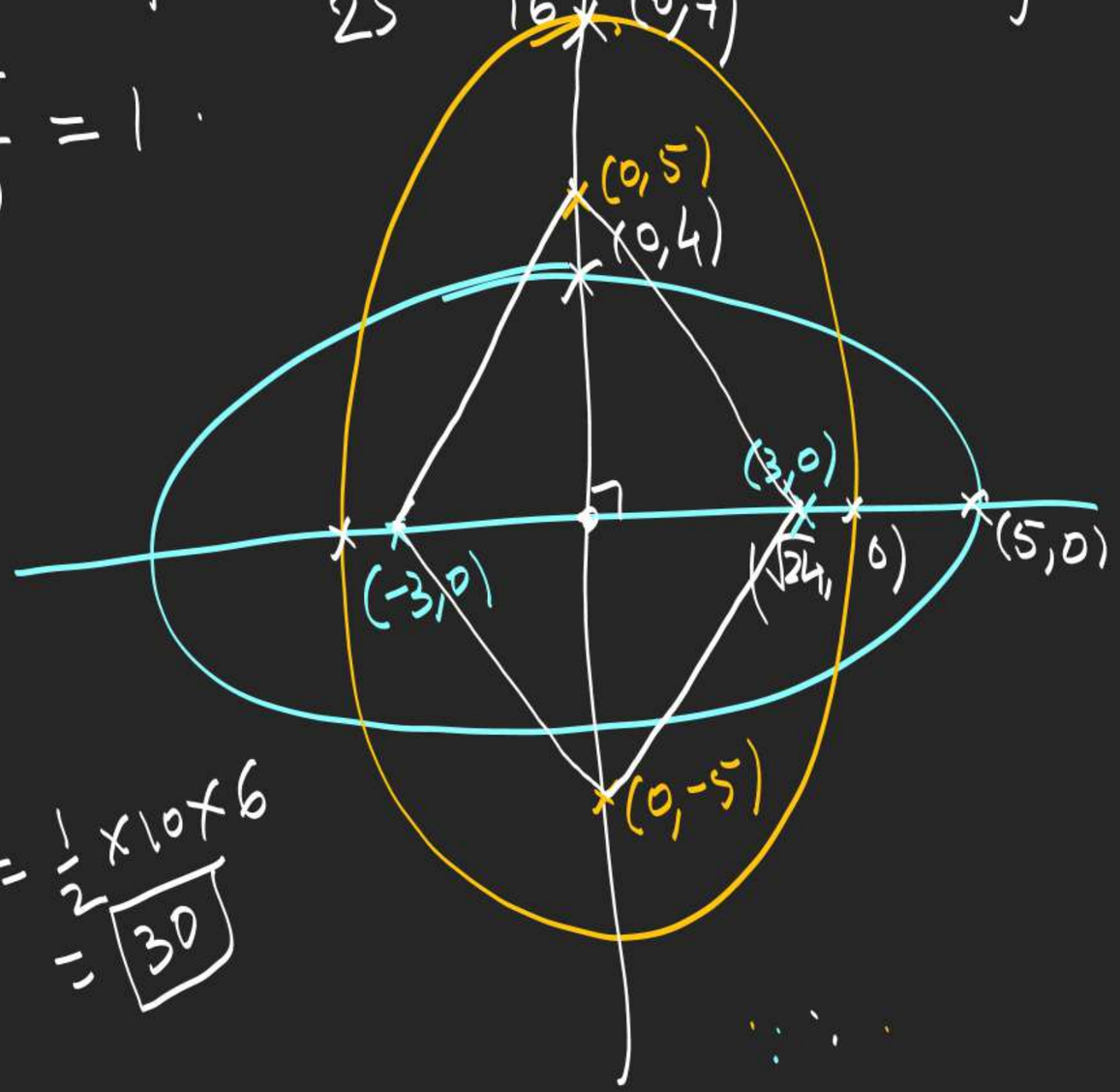
2. Find the area of quadrilateral formed by joining the foci of ellipse $\frac{x^2}{25}+\frac{y^2}{16}=1$ to the foci of ellipse $\frac{x^2}{24}+\frac{y^2}{49}=1$.



$16 = 25(1-e^2)$

$5e = 3$

$24 = 49(1-e'^2)$

$7e' = 5$

$A = \frac{1}{2} \times 10 \times 6$

$= \boxed{30}$

$\therefore$

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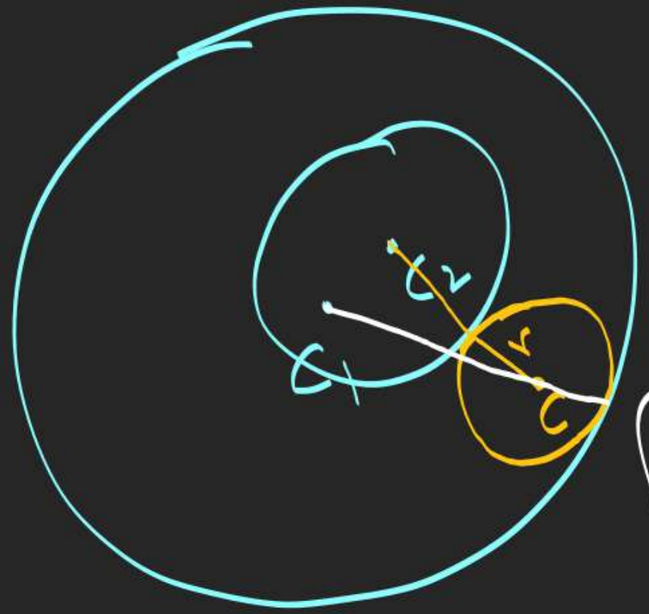
3: Let S_1, S_2 are two fixed circles with radii r_1 & r_2. such that S_2 is contained in S_1. A third circle moves in such a way that it touches S_1 internally and S_2 externally. Find the locus of its centre.



$CC_2 = r + r_2$

$CC_1 = r_1 - r$

$$CC_1 + CC_2 = r_1 + r_2$$

$C \rightarrow$ Ellipse

$C_1, C_2 \rightarrow$ foci

$r_1 + r_2 =$ length of major axis

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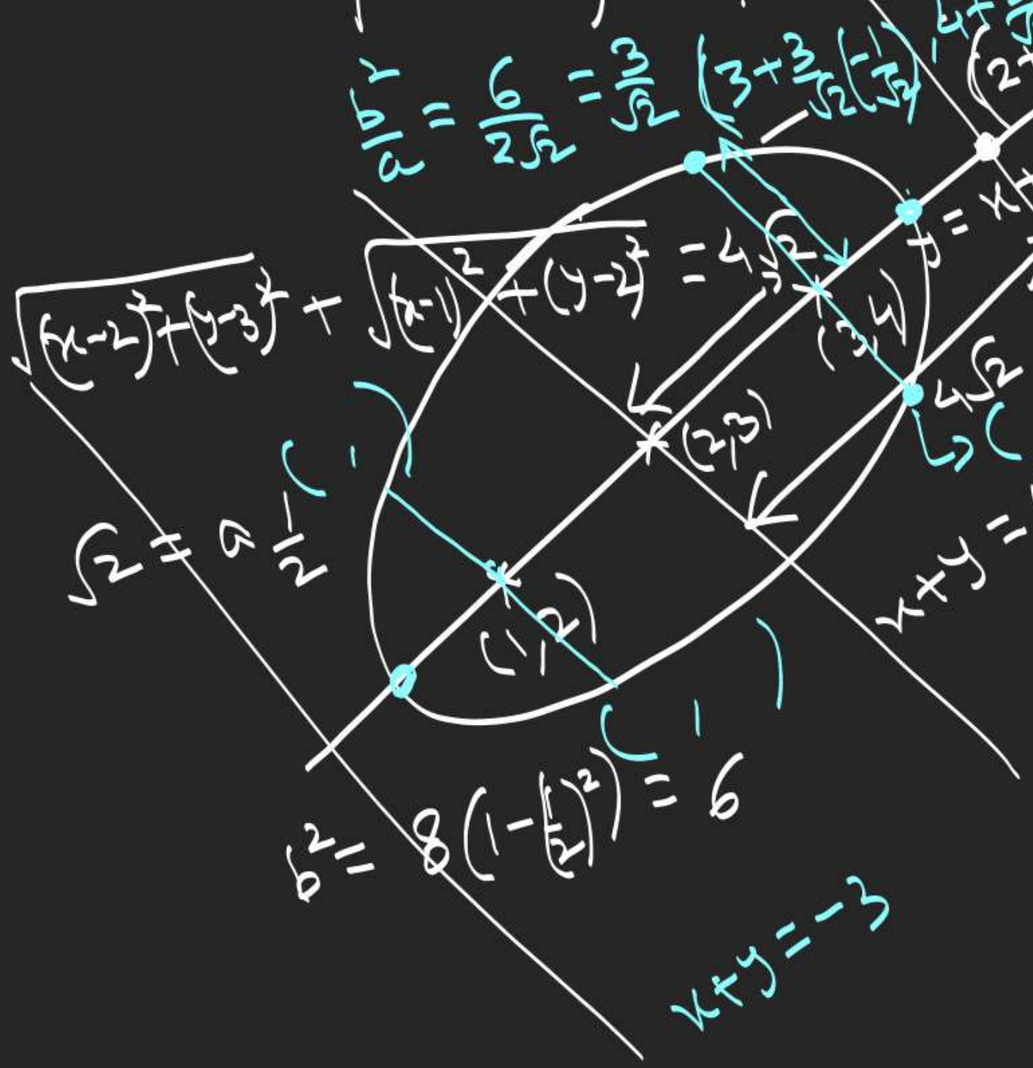
4. If the focus, centre and eccentricity of an ellipse are respectively $(3,4)$, $(2,3)$ and $\frac{1}{2}$. Find its equation & standard terms.



$$\frac{\left(\frac{y-x-1}{\sqrt{2}}\right)^2}{6} + \frac{\left(\frac{x+y-5}{\sqrt{2}}\right)^2}{8} = 1$$

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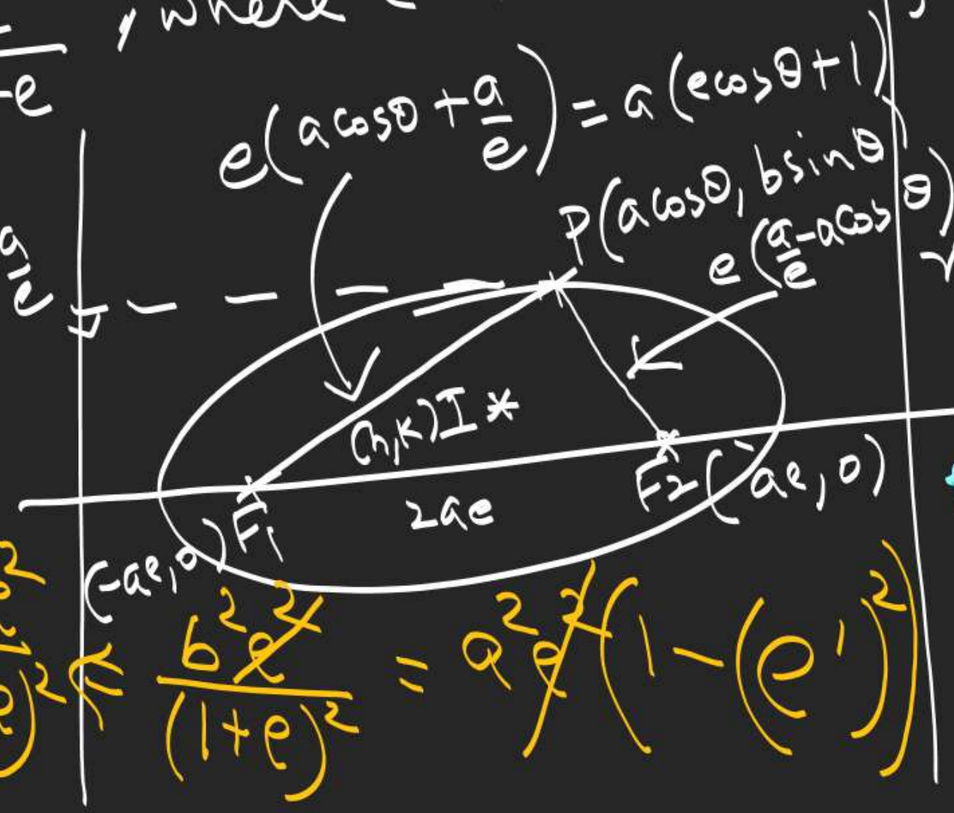
5. Show that locus of incentre of triangle PF_1F_2, where P is a variable point lying on ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and F_1, F_2 are its foci, is an ellipse whose eccentricity is $\sqrt{\frac{2e}{1+e}}$, where e is eccentricity of given ellipse

$(e')^2 = \frac{2e}{1+e}$

$\frac{x^2}{(ae)^2}+\frac{y^2}{\left(\frac{be}{1+e}\right)^2}=1$



$K=\frac{2ae\,b\sin\theta}{2a(1+e)}=\frac{be\sin\theta}{1+e}$

$h=\frac{-a^2(1-e\cos\theta)e+a^2e(e\cos\theta+1)+2a^2e\cos\theta}{2a(1+e)}$

$=\frac{2a^2e\cos\theta(1+e)}{2a(1+e)}=ae\cos\theta$

$1-(e')^2=\frac{1-e^2}{(1+e)^2}$

$\frac{b^2e^2}{(1+e)^2}=a^2e^2\left(1-(e')^2\right)$

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Position of Point w.r.t. Ellipse

$$S = \frac{x^2}{a^2} + \frac{y^2}{b^2} - 1$$

$P_x(x_1, y_1)$

$|y_1| > b\sqrt{1 - \frac{x_1^2}{a^2}}$

$\left(x_1, b\sqrt{1 - \frac{x_1^2}{a^2}}\right)$

$\Rightarrow \frac{x_1^2}{a^2} + \frac{y_1^2}{b^2} - 1 > 0$

$S_1 > 0 \Rightarrow$ point 'P' lies outside the ellipse

$S_1 < 0 \Rightarrow$ —"— inside —"— $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$PF_1 + PF_2 > 2a \Rightarrow$ P lies outside ellipse

$< 2a \Rightarrow$ —"— inside —"—

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