

RELATION FUNCTION

$$Q \quad f(x) = \left(\frac{a^x - 1}{a^x + 1} \right) \cdot x^2 = \text{E/O?}$$

$$\downarrow \quad \downarrow$$

$$= \text{Odd} \times \text{Even}$$

$$= \text{Odd}$$

$$Q \quad f(x) = \left(\frac{a^x - 1}{a^x + 1} \right) \cdot x^3 = \text{E/O?}$$

$$\downarrow \quad \downarrow$$

$$= \text{Odd} \quad \text{Odd}$$

$$= \text{Even}$$

$$Q \quad f(x) = \left(\frac{a^x - 1}{a^x + 1} \right) \cdot \boxed{x^n} = \text{odd then } n?$$

$$1) 1 \quad \downarrow \quad 2) \boxed{2} \quad 3) 3 \quad 10) \text{NOT.}$$

$$\text{Odd} \times \text{Even} = \text{odd}$$

$$n = \text{Even deg.}$$

$$Q \quad f(x) = \left(\frac{a^x - 1}{a^x + 1} \right) \cdot \boxed{x^n} = \text{odd then } n = ?$$

$$\text{odd} \times \text{Even} = \text{odd}$$

$$1) \frac{1}{3} \quad \cancel{2) \frac{2}{3}} \quad 3) \frac{3}{3} \quad (4) \text{NOT.}$$

$$n = \text{Even}$$

$$(x)^{\frac{2}{3}} = (x^2)^{\frac{1}{3}} = \text{Even}$$

(E)

RELATION FUNCTION

$$Q \ f(x) = \begin{cases} x|x| \\ [x+1] + [1-x] \\ -x|x| \end{cases}$$

$$x \leq -1$$

$$-1 < x < 1$$

$$x \geq 1$$

$$[x] + [-x] = \begin{cases} 0 \\ -1 \end{cases}$$

Rk:-

If Qs is difficult or seems like
NEED fn Try $f(x) + f(-x) = 0$
then Answer.

$$x=1 \\ x+1$$

Odd Ki Nishani

E/O?

$$f(-x) = \begin{cases} -x|-x| \\ [1-x] + [x+1] \\ +(+x)|-x| \end{cases}$$

$$-x \leq -1$$

$$-1 < -x < 1$$

$$-x \geq 1$$

$$f(-x) = \begin{cases} -x|x| \\ [1-x] + [x+1] \\ x|x| \end{cases}$$

$$x \geq 1$$

$$1 > x > -1$$

$$x \leq -1$$

$$= f(x)$$

Even

Q

$$f(x) = [x] + \frac{1}{2}; x \notin \mathbb{I} \quad E/O?$$

$$f(-x) = [-x] + \frac{1}{2}$$

$$f(x) + f(-x) = [x] + [-x] + 1$$

$$= -1 + 1$$

$$f(x) + f(-x) = 0$$

$$f(-x) = -f(x) \text{ odd}$$

RELATION FUNCTION

ODD

$$Q \quad f(x) = \log_e (x + \sqrt{1+x^2}) \in \mathbb{R}$$

$$f(-x) = \log_e (-x + \sqrt{1+x^2})$$

$$f(x) + f(-x) = \log_e \left\{ \left(\underbrace{x}_{-B} + \underbrace{\sqrt{1+x^2}}_{\frac{A}{-B}} \right) \left(-\underbrace{x}_{-B} + \underbrace{\sqrt{1+x^2}}_{\frac{A}{-B}} \right) \right\}$$

$$= \log_e \left\{ (\sqrt{1+x^2})^2 - (x)^2 \right\}$$

$$= \log_e \{ 1 + x^2 - x^2 \}$$

$$f(-x) + f(x) = 0$$

\Rightarrow odd

$$[x] + [-x] = -1$$

$x \neq 1$

$$\left[\frac{x}{\pi} \right] + \left[-\frac{x}{\pi} \right] = -1$$

$$\left[-\frac{x}{\pi} \right] = -1 - \left[\frac{x}{\pi} \right]$$

$$Q \quad \text{Find whether give } f(x) = \frac{x(\sin x + \tan x)}{\left[\frac{x+\pi}{\pi} \right] - \frac{1}{2}} \text{ is}$$

$\in \mathbb{R}; x \neq n\pi$?

$$f(x) = \frac{x(\sin x + \tan x)}{\left[\frac{x}{\pi} + 1 \right] - \frac{1}{2}} = \frac{x(\sin x + \tan x)}{\left[\frac{x}{\pi} \right] + 1 - \frac{1}{2}}$$

$$f(x) = \frac{x(\sin x + \tan x)}{\left[\frac{x}{\pi} \right] + \frac{1}{2}}$$

$$f(-x) = \frac{-x(+\sin x + \tan x)}{\left[-\frac{x}{\pi} \right] + \frac{1}{2}}$$

$$= \frac{-x(\sin x + \tan x)}{-1 - \left[\frac{x}{\pi} \right] + \frac{1}{2}} = \frac{-x(\sin x + \tan x)}{-\left(\left[\frac{x}{\pi} \right] + \frac{1}{2} \right)} = -f(x)$$

$x \neq n\pi$

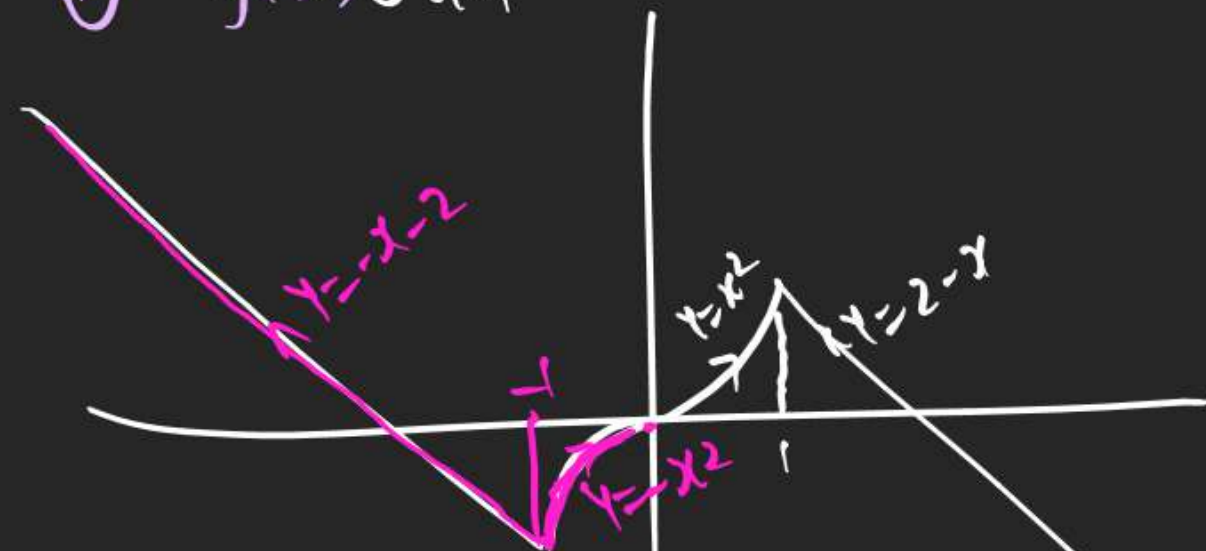
$$\frac{x}{\pi} \neq n \Rightarrow \frac{x}{\pi} \neq [n] \\ \left[\frac{x}{\pi} \right] + \left[-\frac{x}{\pi} \right] = -1$$

RELATION FUNCTION

Q If $f(x) = \begin{cases} x^2 & x \in [0, 1) \\ 2-x & x \in [1, \infty) \end{cases}$

Concept $\rightarrow f(x)$ Even \rightarrow Symmetry Axis
 $f(x)$ odd \rightarrow Symmetry origin.

① $f(x)$ odd



$$f(x) = \begin{cases} -x^2 & x \in [-1, 0] \\ -x-2 & x \in (-\infty, -1] \end{cases}$$

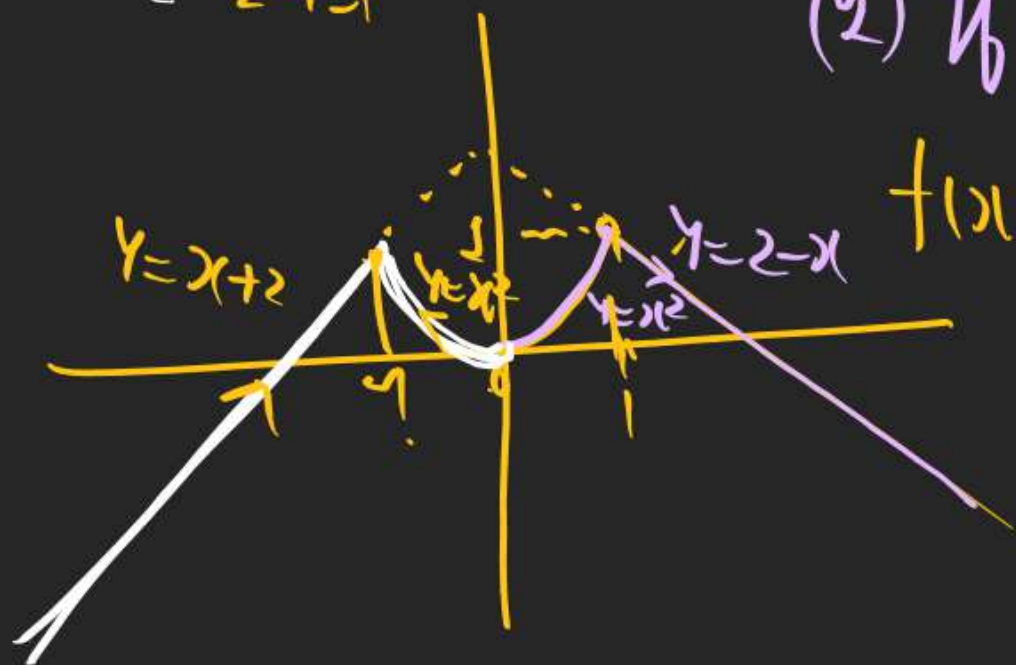
find $f(x)$ for $x < 0$ if

1) $f(x)$ is odd 2) if $f(x)$ is even.

$$f(x) = \begin{cases} x^2 & 0 \leq x < 1 \\ 2-x & 1 \leq x < \infty \end{cases}$$

(2) If $f(x)$ Even

$$f(x) = \begin{cases} x+2 & -\infty < x \leq -1 \\ x^2 & x \in (-1, 0] \end{cases}$$



RELATION FUNCTION

Properties of E/O fxn

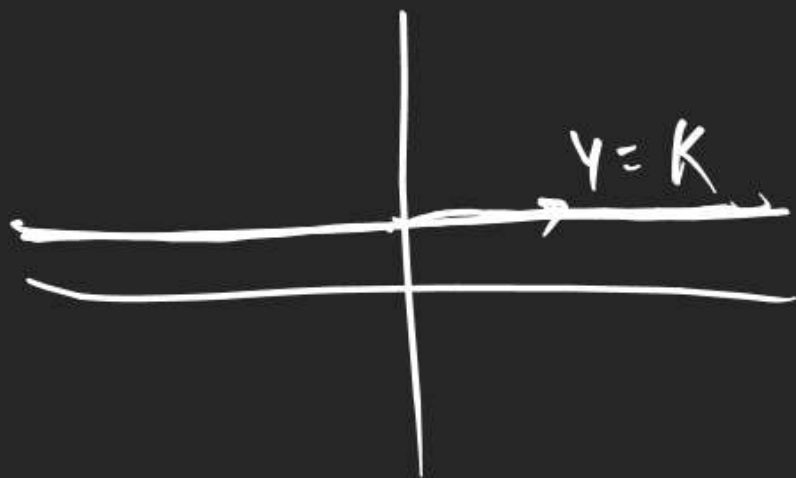
1) 2 popular Even fxn \Rightarrow $f(x) = 1 + a^x$
 $f(x) = 1 + \sin x$

(2) $f(x) = K$ (constant fxn) are always.

Even fxn

$$f(x) = \frac{f(x) + f(-x)}{2} + \frac{f(x) - f(-x)}{2}$$

Even + odd.



(3) $f(x) = 0$ is only fxn in which is E & O both.

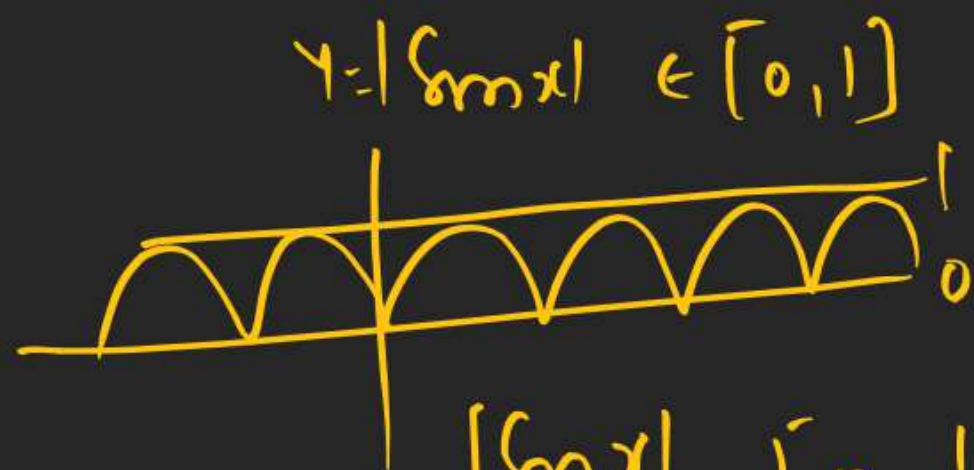


(4) If odd fxn contain Zero in domain then graph of fxn will pass thro origin.

(5) Every fxn containing Zero in domain can be Represented as Sum of Even & odd fxn.

RELATION FUNCTION

$$Q \ f(x) = \left\lceil \frac{|\sin x|}{2} \right\rceil \in \mathbb{Z}/0?$$



$$\frac{|\sin x|}{2} \in \left[0, \frac{1}{2}\right]$$

$$f(x) = \left\lceil \frac{|\sin x|}{2} \right\rceil = 0$$

Even & odd

$$Q \ f(x) = a + \sin x = \text{odd then } a = ?$$

1) 0 2) 1 3) 2 4) 3

$$f(x) = \overset{\text{const}}{\boxed{a}} + \sin x = \text{odd fcn?}$$

$E + \text{odd} = \text{NE}$ No hone me hai

$$\begin{aligned} f(x) &= 0 + \sin x \\ &= \sin x \\ &\text{odd} \end{aligned}$$

RELATION FUNCTION

(6)

$f(x)$	$g(x)$	$f(x) + g(x)$	$f(x) \times g(x)$
E	E	E	E
O	O	O	E
E	O	NEN	O

$E = +$
 $O = -$

7) $f(g(x))$

$$E(E) = E$$

$$E(O) = E$$

$$O(E) = E$$

$$O(O) = O$$

Q $f(x) = \sin(g(x))$ E/O?

$$O(E) = E$$

Q $f(x) = \sin x + \cos x$ E/O?

$$O + E = NEN$$

RELATION FUNCTION

$$Q \quad f(x) = \left\{ \frac{\sin^3 x \cdot \cos^2 x \cdot \log\left(\frac{1-x}{1+x}\right)}{\sin x \cdot \tan\left(\frac{a^{2x}-1}{a^{2x}+1}\right)} \right\}^{2026} \quad E/O?$$

$$= \left\{ \frac{O \times E \times O}{O \times O} \right\}^{2026}$$

$$= \left\{ \frac{- \times + \times -}{- \times -} \right\}^{2026}$$

$$= \boxed{+}^{2026} = \text{Even}$$

$$Q = \left\{ \frac{\sin^3 x \cdot \cos^2 x \cdot \log\left(\frac{1-x}{1+x}\right)}{\sin x \cdot \tan\left(\boxed{1+\sin x}\right)} \right\}^{2026}$$

NEN

= NEN

RELATION FUNCTION

Q $f: [-10, 10] \rightarrow \mathbb{R}$; $f(x) = x + \sin x + \left\lceil \frac{x^2}{a} \right\rceil = \text{odd fn.}$
 then $a \in ?$

Domain

$$x \in [-10, 10]$$

$$x^2 \in [0, 100]$$

$$f(x) = \underbrace{x}_{0} + \underbrace{\sin x}_{0} + \left\lceil \frac{x^2}{a} \right\rceil = \text{odd fn.}$$

$$0 + 0 + \underbrace{\left\lceil \frac{x^2}{a} \right\rceil}_{\text{No fn.}}$$

$$\left(\mathbb{R} \setminus \mathbb{K} \right)$$

$$a \in (100, \infty)$$

$$\left\lceil \frac{x^2}{a} \right\rceil = 0$$

$$0 \leq \frac{x^2}{a} \leq 1$$

$$x^2 < a \Rightarrow a > (x^2)_{\text{top}}$$

$$a > \underline{100}$$

RELATION FUNCTION

Periodic fxn

- A) Any fxn who attains same height after a certain interval or who repeats himself after a certain interval is known as Periodic fxn.



- (2) That certain interval is known as Period of fxn.

- (3) If Period is Min. & +ve then that is known as fundamental Period (T)

- (4) If $f(x)$ Satisfies

$$f(x+T) = f(x)$$

then $f(x)$ is Periodic & F.P. is T

RELATION FUNCTION

$$f(x+T) = f(x) \rightarrow T = FP$$

$$\left. \begin{aligned} \sin(2\pi + x) &= \sin x \\ \sin(4\pi + x) &= \sin x \\ \sin(6\pi + x) &= \sin x \end{aligned} \right\} \begin{aligned} &\text{Period of } \sin x \\ &= \boxed{2\pi}, 4\pi, 6\pi, 8\pi, \dots \end{aligned}$$

$\therefore 2\pi$ is Min as well as +ve

$$\Rightarrow T = 2\pi \text{ for } \sin x$$

Q If $f(x) + f(x+3) = 0$ then Period of $f(x)$?

$$\begin{aligned} x \rightarrow x+3 & \quad \begin{aligned} f(x+3) &= -f(x) \end{aligned} \longrightarrow \begin{aligned} +f(x+6) &= +f(x) \end{aligned} \\ f(x+6) &= -f(x+3) \\ f(x+3) &= -f(x+6) \end{aligned} \quad \begin{aligned} f(x+T) &= f(x) \\ T &= 6 \end{aligned}$$

RELATION FUNCTION

Q If $f(x) + f(x+4) = f(x+2) + f(x+6)$
find T ?

$$\begin{aligned} f(x) + f(x+4) &= f(x+2) + f(x+6) \\ x \rightarrow x+2 \quad f(x+2) + f(x+6) &= f(x+4) + f(x+8) \end{aligned}$$

$$f(x) + f(x+4) = f(x+4) + f(x+8)$$

$$\begin{aligned} f(x+8) &= f(x) \\ f(x+T) &= f(x) \end{aligned} \quad \boxed{T=8}$$

Q If $f(x+a) = \frac{1}{2} + \sqrt{f(x) - f^2(x)}$ find T of $f(x)$?

$$f(x+a) = \frac{1}{2} + \sqrt{\frac{1}{4} - \left(f(x) - \frac{1}{2}\right)^2} \quad x \rightarrow x+a$$

$$f(x+2a) = \frac{1}{2} + \sqrt{\frac{1}{4} - \left(f(x+a) - \frac{1}{2}\right)^2}$$

$$= \frac{1}{2} + \sqrt{\frac{1}{4} - \left(\frac{1}{2} + \sqrt{\frac{1}{4} - \left(f(x) - \frac{1}{2}\right)^2} - \frac{1}{2}\right)^2}$$

$$= \frac{1}{2} + \sqrt{\frac{1}{4} - \left(\sqrt{\frac{1}{4} - \left(f(x) - \frac{1}{2}\right)^2}\right)^2}$$

$$= \frac{1}{2} + f(x) - \frac{1}{2}$$

$$\begin{aligned} f(x+2a) &= f(x) \\ T &= 2a \end{aligned}$$

RELATION FUNCTION

Table Based Qs.

1) $\sin^n x / \cos^n x$ $\sec^n x / \csc^n x$	$n = \text{odd}$	$T = 2\pi$
	$n = \text{Even}$	$T = \pi$
2) $\tan^n x / \cot^n x$	$n = \mathbb{E}/\mathbb{O}$	$T = \pi$
3) $ \sin^n x , \cos^n x $ $ \tan^n x , \cot^n x $ $ \sec^n x , \csc^n x $	$n = \mathbb{E}/\mathbb{O}$	$T = \pi$
(4) $ \sin x + \cos x $ $\sin^E x + \cos^E x$		$T = \frac{\pi}{2}$
(5) $\{x\}$		$T = 1$

(6) Constant fxn

Periodic But
No FP

(7) Dirichlet
fxn

$$f(x) = \begin{cases} 1 & x \in \mathbb{Q} \\ -1 & x \notin \mathbb{Q} \end{cases}$$

Periodic But
No FP

$[x]$, Poly, Rest
all fxn

are Non Periodic

$$\sin^2 x \quad T = \pi \quad \left| \quad \sec^3 x = 2\pi$$

$$\sin^3 x \quad T = 2\pi \quad \left| \quad \tan^3 x = \pi$$

$$\cos^{32} x \quad T = \pi \quad \left| \quad |\sin^{32} x| = \pi$$

RELATION FUNCTION

$$\delta m^3 x \rightarrow 2\pi$$

$$\delta m^2 x \rightarrow \pi$$

$$G^{31} x \rightarrow 2\pi$$

$$G^{204} x \rightarrow \pi$$

$$G^{31} x \rightarrow 2\pi$$

$$f m^4 x \rightarrow \pi$$

$$G^{6} x \rightarrow \pi$$

$$f m^5 x \rightarrow \pi$$

$$G^{31} x \rightarrow \pi$$

$$\delta m^3 x \rightarrow 2\pi$$

$$\{x\} \rightarrow 1$$

$$G^{31} x \rightarrow 2\pi$$

$$G^{204} x \rightarrow \pi$$

$$\delta m^2 x \rightarrow \pi$$

$$f m^3 x \rightarrow \pi$$

$$\{x\} = 1$$

$$\delta m^4 x + G^4 x \rightarrow \frac{\pi}{2}$$

$$|f m x| + |G x| \rightarrow \frac{\pi}{2}$$

$$|f m x| + |G x| \rightarrow \frac{\pi}{2}$$

$$|G x| \rightarrow \pi$$

$$|G^{49} x| \rightarrow \pi$$

$$|f m^3 x| \rightarrow \pi$$

$$\delta m^3 x \rightarrow 2\pi$$

$$G^{38} x \rightarrow \pi$$

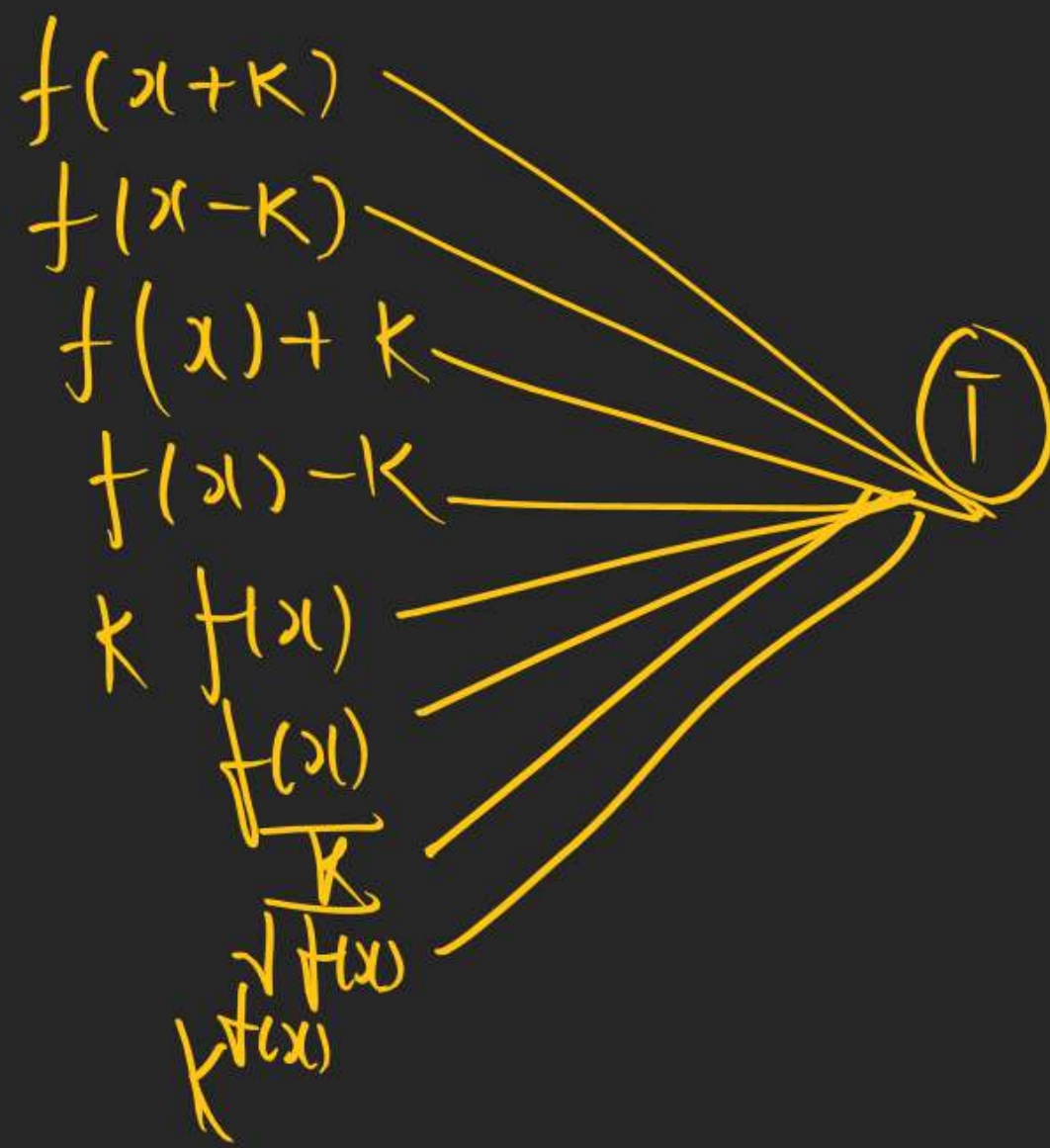
$$\boxed{f(x) = 0}$$

No FP

RELATION FUNCTION

Type 2 When Constant is Involved with x

Let $f(x) \rightarrow T$



$$\sin^3 x \rightarrow 2\pi$$

$$\sin^3 x + 7 \rightarrow 2\pi$$

$$7\sin^3 x \rightarrow 2\pi$$

$$7^{\sin^3 x} \rightarrow 2\pi$$

$$\frac{1}{\sin^3 x} \rightarrow 2\pi$$

$$\sqrt{\sin^3 x} \rightarrow 2\pi$$

$$\sin^3(x+4) \rightarrow 2\pi$$

$$\sin^3(\sqrt{x}+4) + 5 \rightarrow 2\pi$$