KINEMATICS

(※). <u>Distance travelled by the particle in n^{th} Second:—</u>

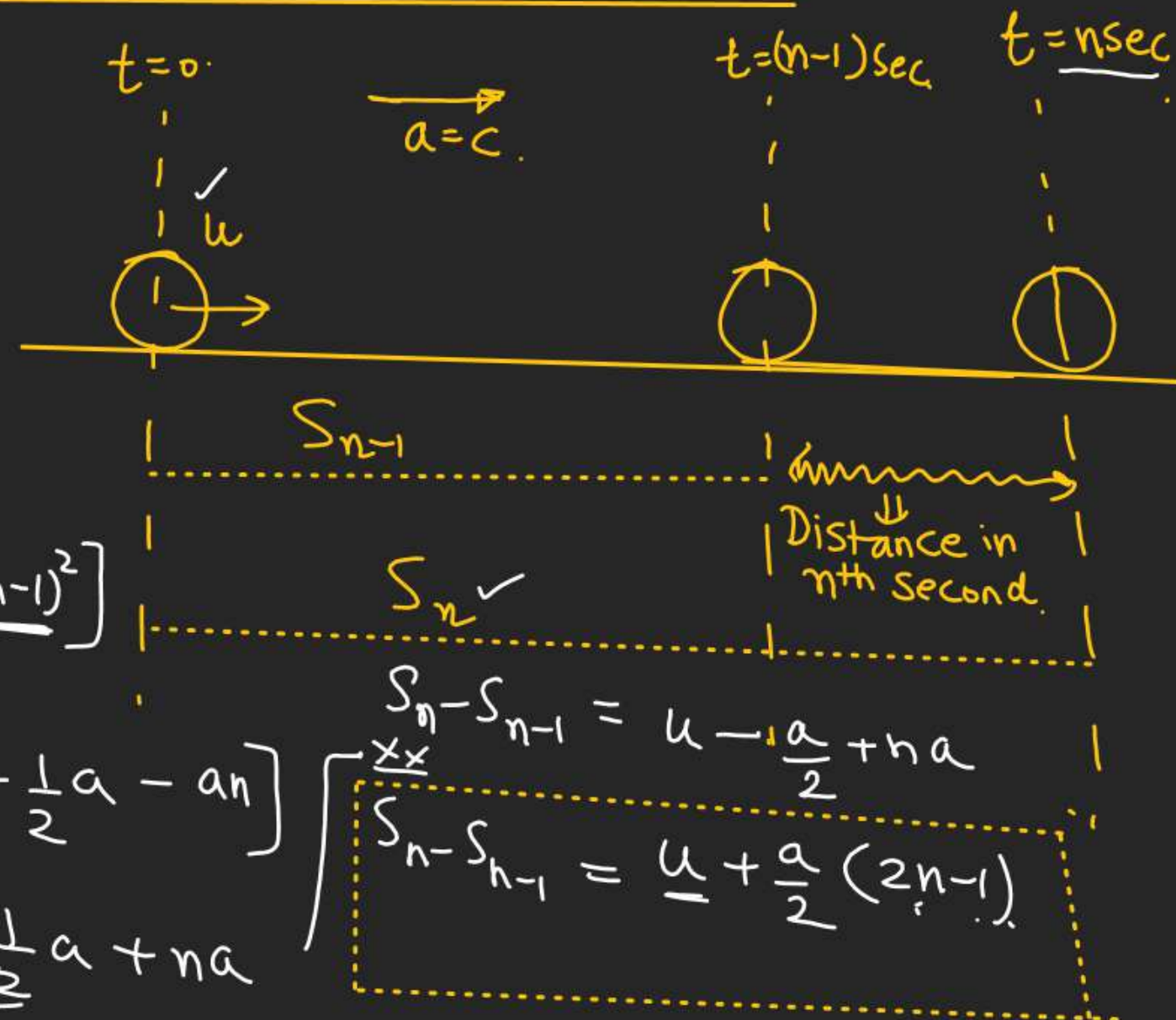Distance travelled in n^{th} Second

$$= (S_n - (S_{n-1}))$$

$$S_n = u(n) + \frac{1}{2}a(n)^2$$

$$S_{n-1} = u(n-1) + \frac{1}{2}a(n-1)^2$$

$$S_n - S_{n-1} = \left[u(n) + \frac{1}{2}an^2\right] - \left[u(n-1) + \frac{1}{2}a(n-1)^2\right]$$

$$= \left(un + \frac{1}{2}an^2\right) - \left[un - u + \frac{1}{2}an^2 + \frac{1}{2}a - an\right]$$

$$= un + \frac{1}{2}an^2 - un + u - \frac{1}{2}an^2 - \frac{1}{2}a + na$$

$t = 0$

$\overrightarrow{a = c}$

u

$t = (n-1) sec \quad t = n\,sec$

S_{n-1}

Distance in n^{th} second.

S_n

$$S_n - S_{n-1} = u - \frac{a}{2} + na$$

$$S_n - S_{n-1} = u + \frac{a}{2}(2n-1)$$

(A) Motion under gravity

Assumption:-

→ ① 'g' → accleration due to gravity always constant.

$$g = 9.8 \, m\bar{s}^2 \simeq 10 \, m\bar{s}^2$$

$$F = \dfrac{GM_e m}{R_e^2}$$

$$\boxed{g = a = \dfrac{F}{m} = \dfrac{GM_e}{R_e^2}}$$

m

R_e

M_e

$$[R_e = \text{Radius of earth}]$$

$\downarrow g$

$h << R_e$

↳ f ⇒ Constant.

$\downarrow g$
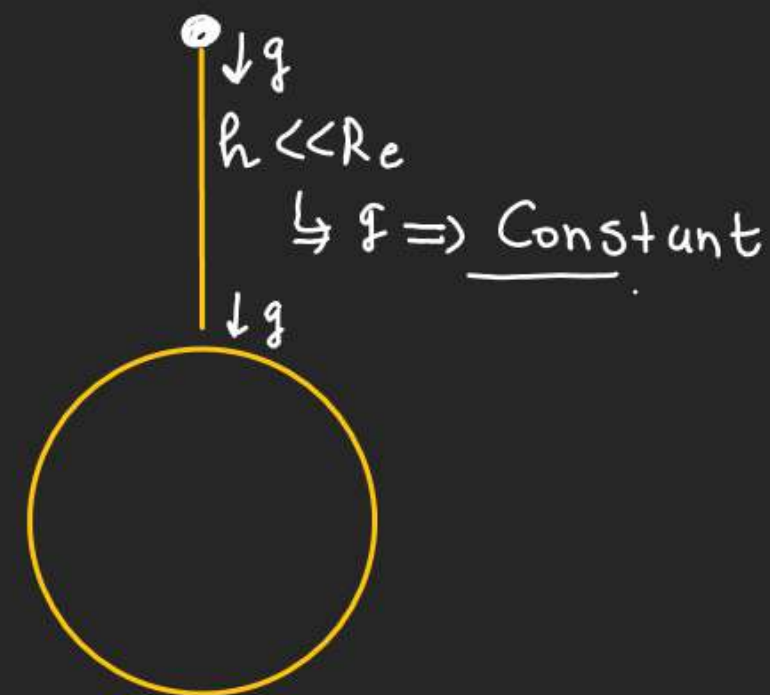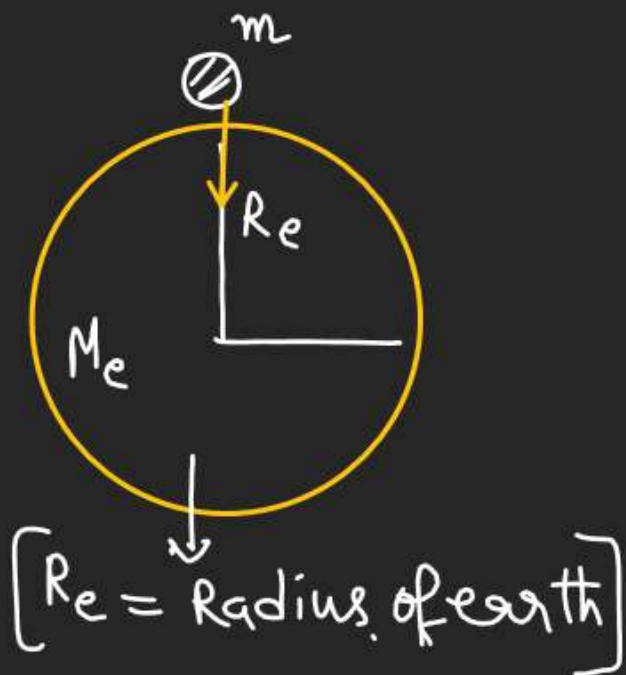
② Air resistance neglected

KINEMATICS

(A)

ground to ground projection

$v_B = 0$

upward Motion

Down-ward Motion

h_{max}

g

u

u_C

$t = 0$

A

$A \rightarrow B$:— upward Motion

$t = t$

v

h

$\downarrow g \rightarrow \left(\begin{array}{c} \text{Acts as a} \\ \text{retardation} \end{array} \right)$

$t = 0$

u
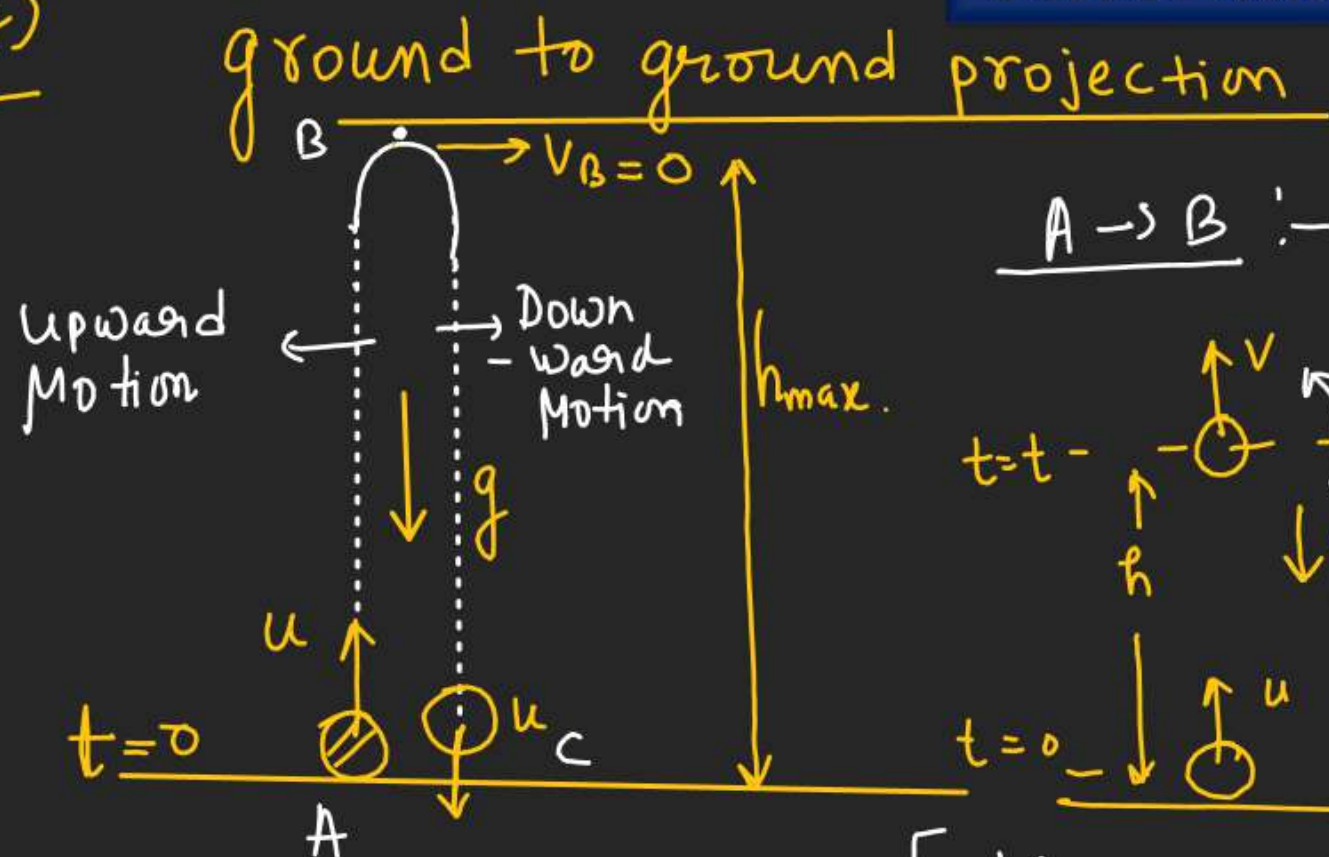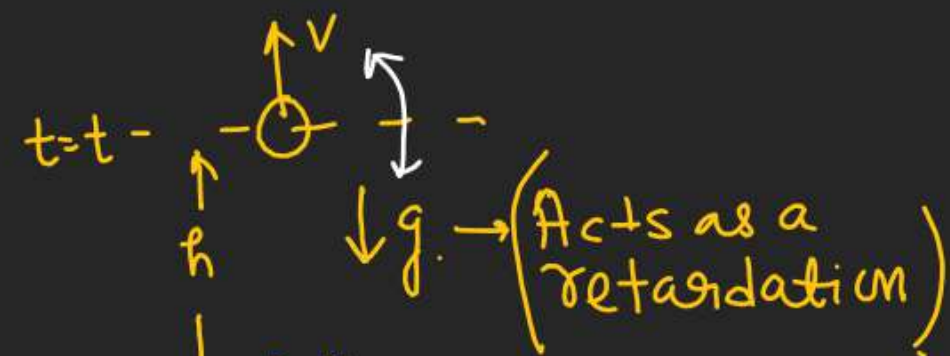
$$\left[\begin{array}{l} v = u - gt \\ h = ut - \dfrac{1}{2} gt^2 \\ v^2 = u^2 - 2gh \end{array} \right]$$

$t_{AB} = ??$, $h_{max} = ??$

$0 = u - g t_{AB}$

$$\boxed{t_{AB} = \left[\dfrac{u}{g} \right]}$$

$v^2 = u^2 - 2gh$

$0 = u^2 - 2g h_{max}$

$$\boxed{h_{max} = \dfrac{u^2}{2g}}$$
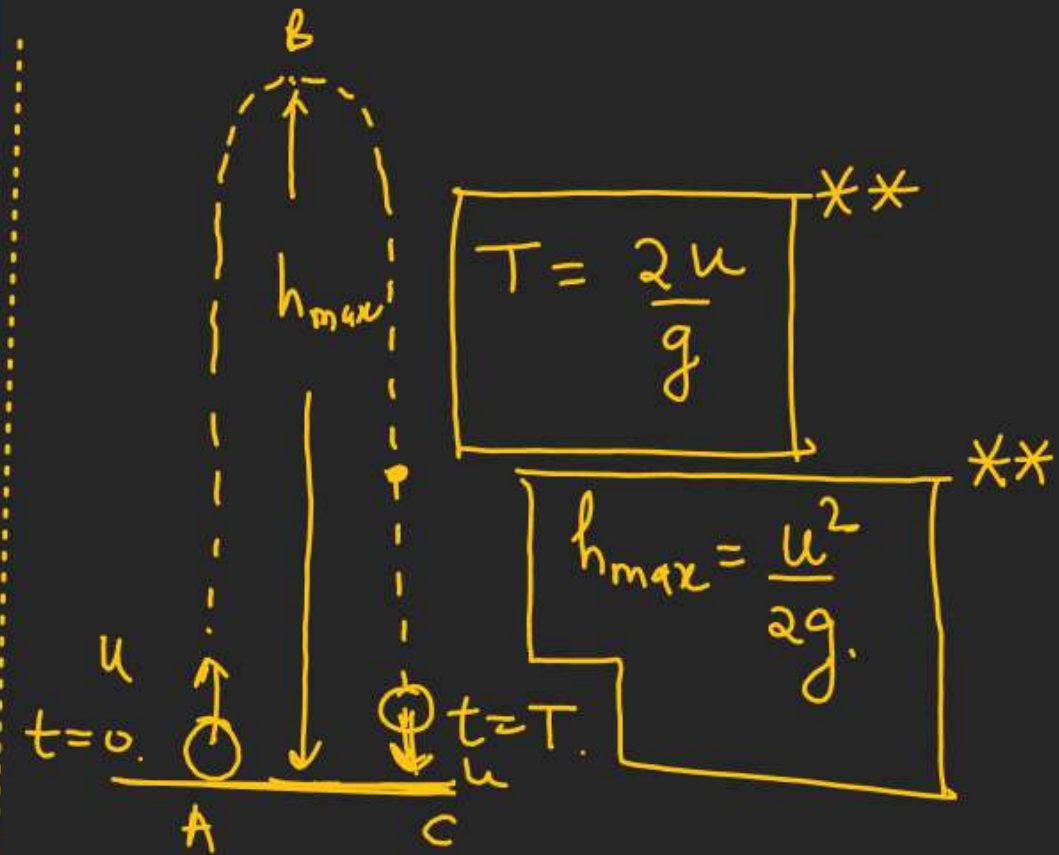
KINEMATICS

 # Downward journey :-
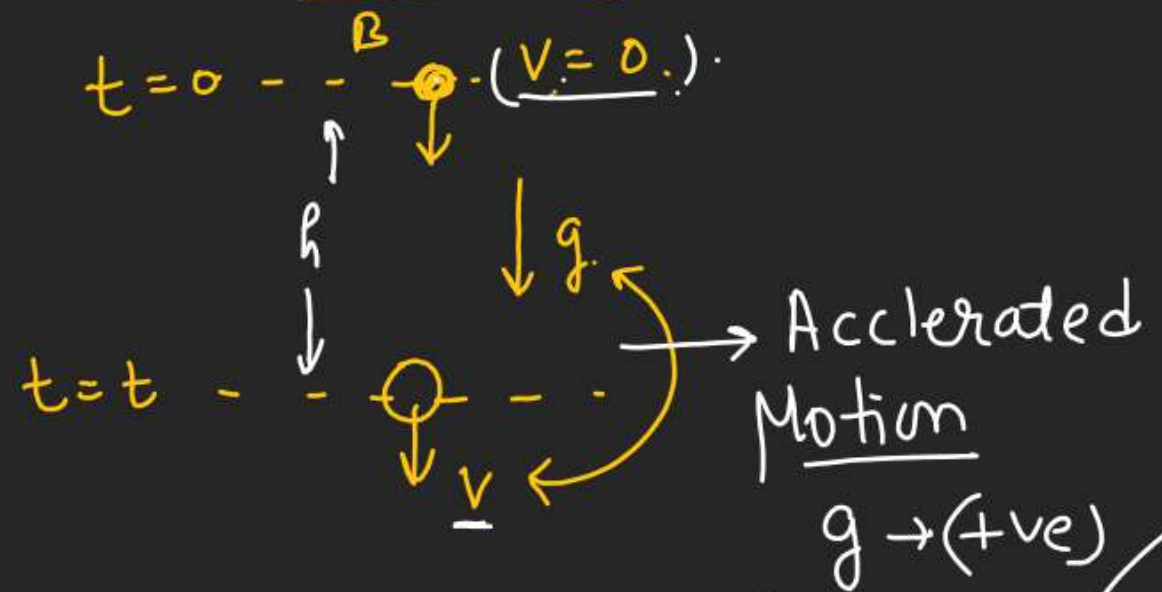
$t = 0 \quad \overset{B}{-} - \overset{\circ}{\phi} \cdot (V = 0.)$

h

$\downarrow g$

$t = t \quad - \quad - \quad \phi - - -$

$\downarrow v$

$\longrightarrow$ Acclerated Motion

$g \rightarrow (+ve)$

$t = t_{BC} < \phi u = v_f$

$\begin{bmatrix} V = gt \\ h = \frac{1}{2}gt^2 \\ v^2 = 2gh \end{bmatrix}$

$t_{BC} = ??$

$u = g\, t_{BC}$

$\boxed{t_{BC} = \dfrac{u}{g}}$

B

h_{max}

$\boxed{T = \dfrac{2u}{g}} \;**$

$\boxed{h_{max} = \dfrac{u^2}{2g}} \;**$

u

$t = 0. \; \phi \qquad \phi \, t = T.$

u

$A \qquad\qquad C$

KINEMATICS

graph :-

$$V = u - gt$$

$(0, u)$

$t = \frac{2u}{g}$

$t = \frac{u}{g}$

$(0, -u)$

$|V|$

u

$\frac{u}{g}$

B $v = 0$.
$t = \frac{u}{g}$

g

u

$ut = \frac{2u}{g}$

A   C

Displacement = 0.

Distance = 2 × Area of △
$$= 2 \times \frac{1}{2} \times \frac{u}{g} \times u$$
$$= \left(\frac{u^2}{g} \right)$$
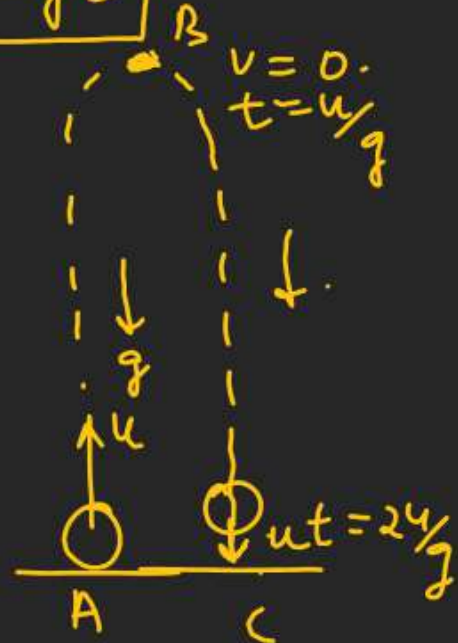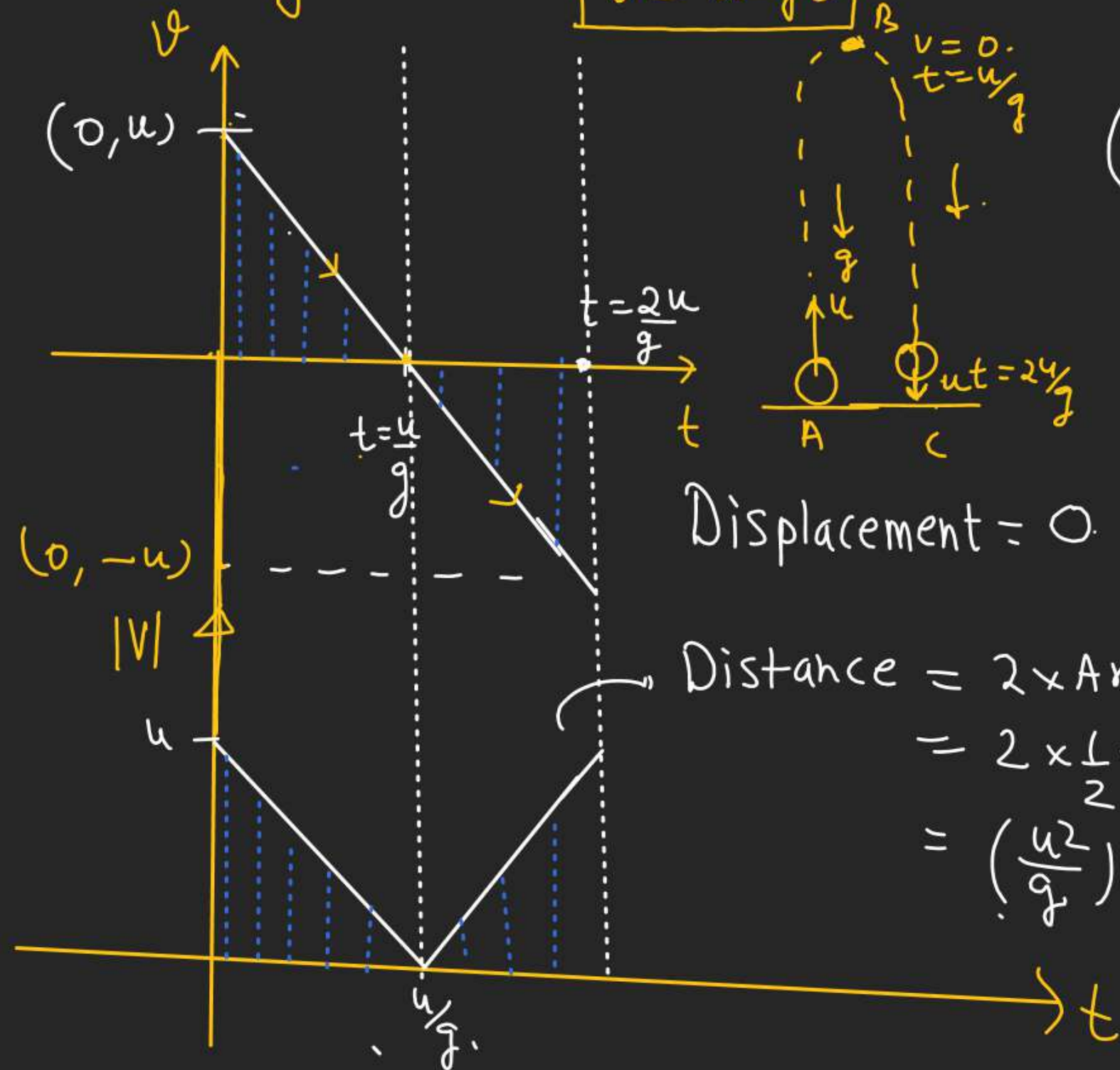
S-t (Displacement Vs time graph)

$$h = ut - \frac{1}{2}gt^2, \qquad h = 0.$$
$$t\left(u - \frac{1}{2}gt\right) = 0$$
$$t = 0, \quad t = \frac{2u}{g}$$

(Displacement) h

$h_{max} = \frac{u^2}{2g}$

$\frac{dh}{dt} = 0, \ V_B = 0.$

B

A

$t = 0$

(Distance)

$\frac{u^2}{g}$

$\frac{u^2}{2g}$

$t = \frac{u}{g}$

$t = \frac{2u}{g}$

KINEMATICS

A ball

\#

$t=0$ A $u=0$

$g \downarrow$

$t=t$

$[V=gt]$

$V \downarrow$

+ve $\uparrow$

B V_0 C $\uparrow V_0$ V_0 $\uparrow V_0$

$\downarrow$

−ve

elastic collision (→ Ball rebound with same velocity.)

$\uparrow V$

−   − $t=t$ $(V=V_0-gt)$

$\downarrow g$

$t=0$ $\uparrow V_0$ $Y = C - mx$ $m = -g$

H·W

$(S-t) \to (graph)$

V

$(0,+V_0)$

$t=0$ $t=\frac{u}{g}$ $t=\frac{2u}{g}$ $t=\frac{3u}{g}$ t

$(0,-V_0)$
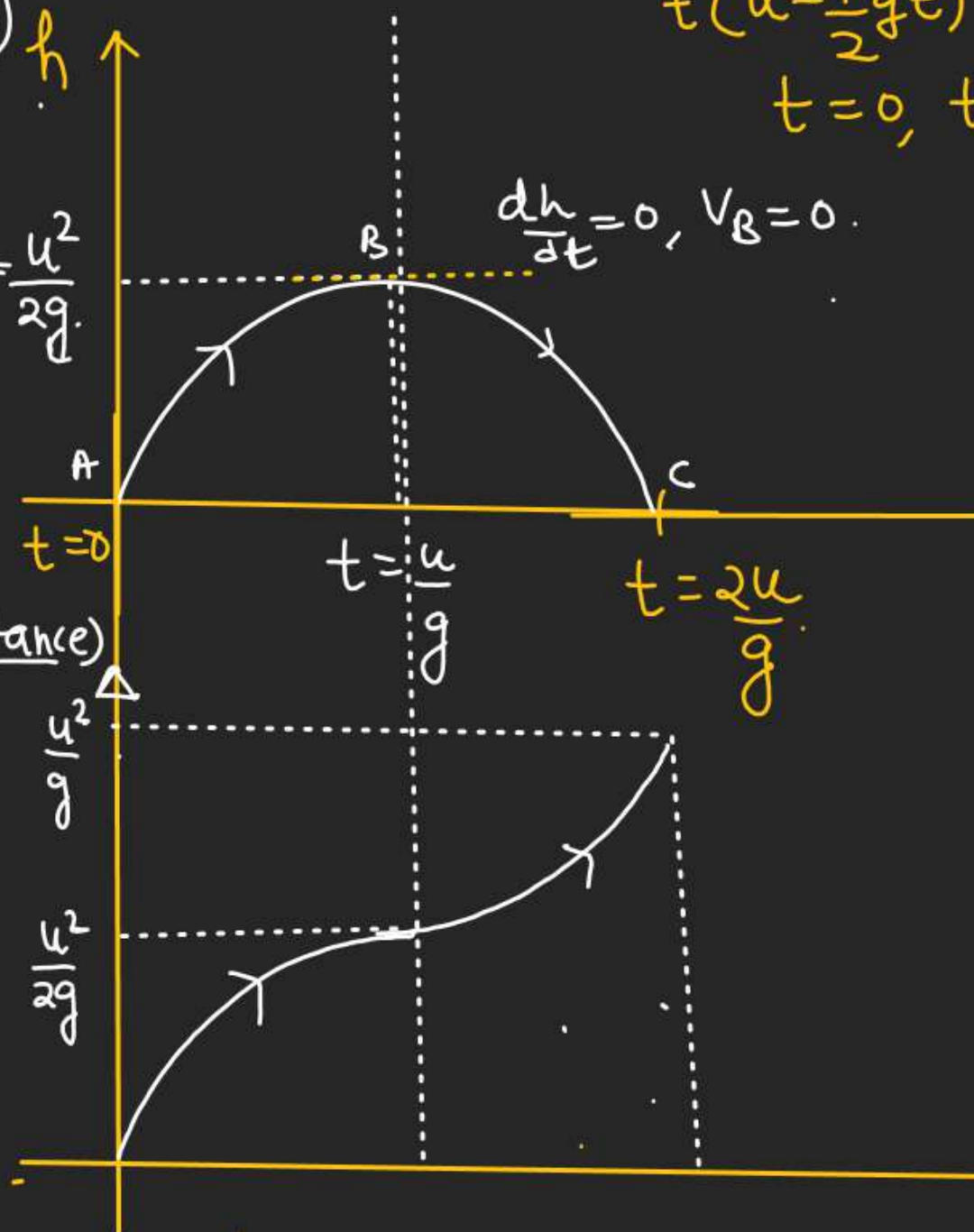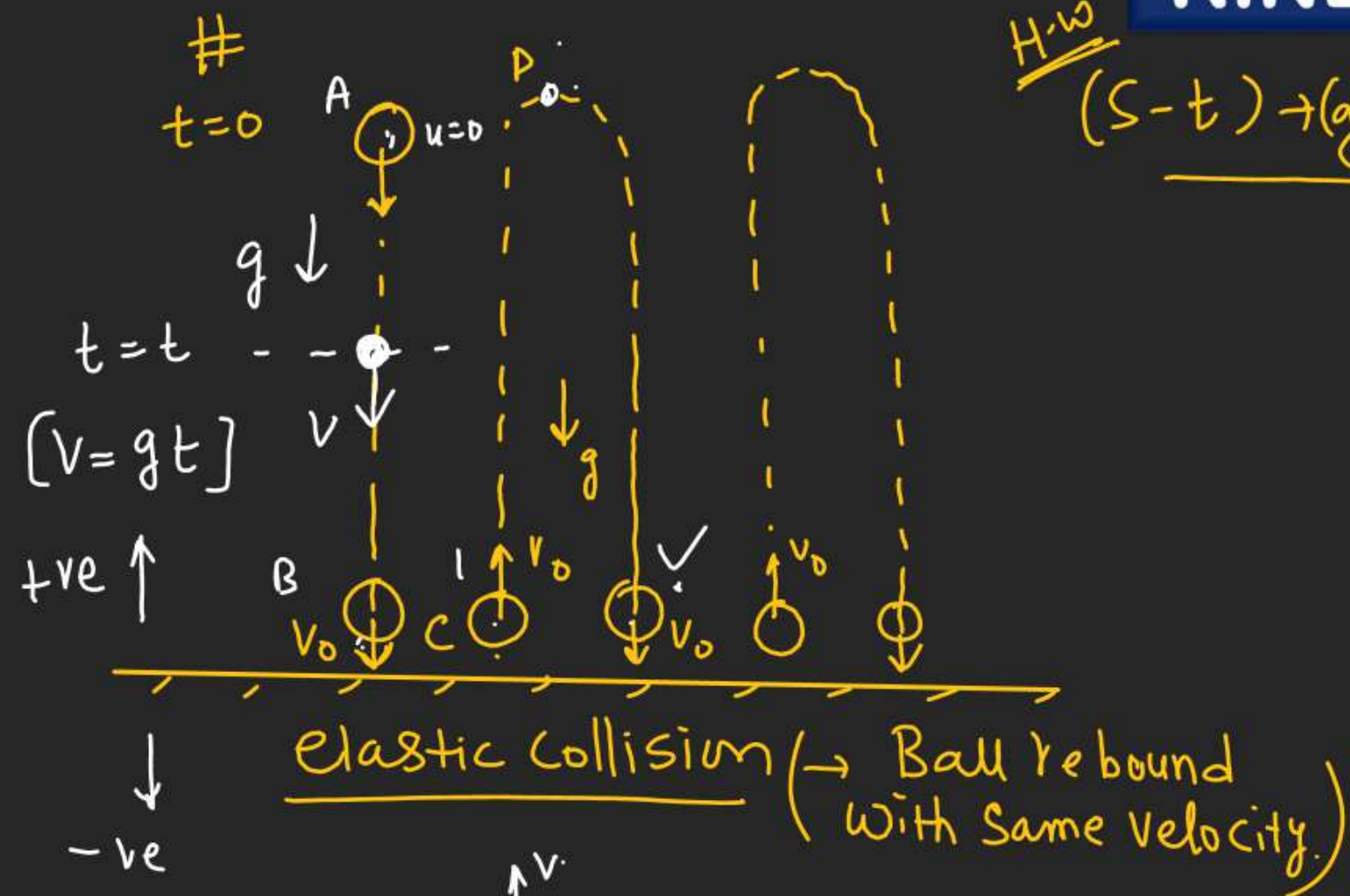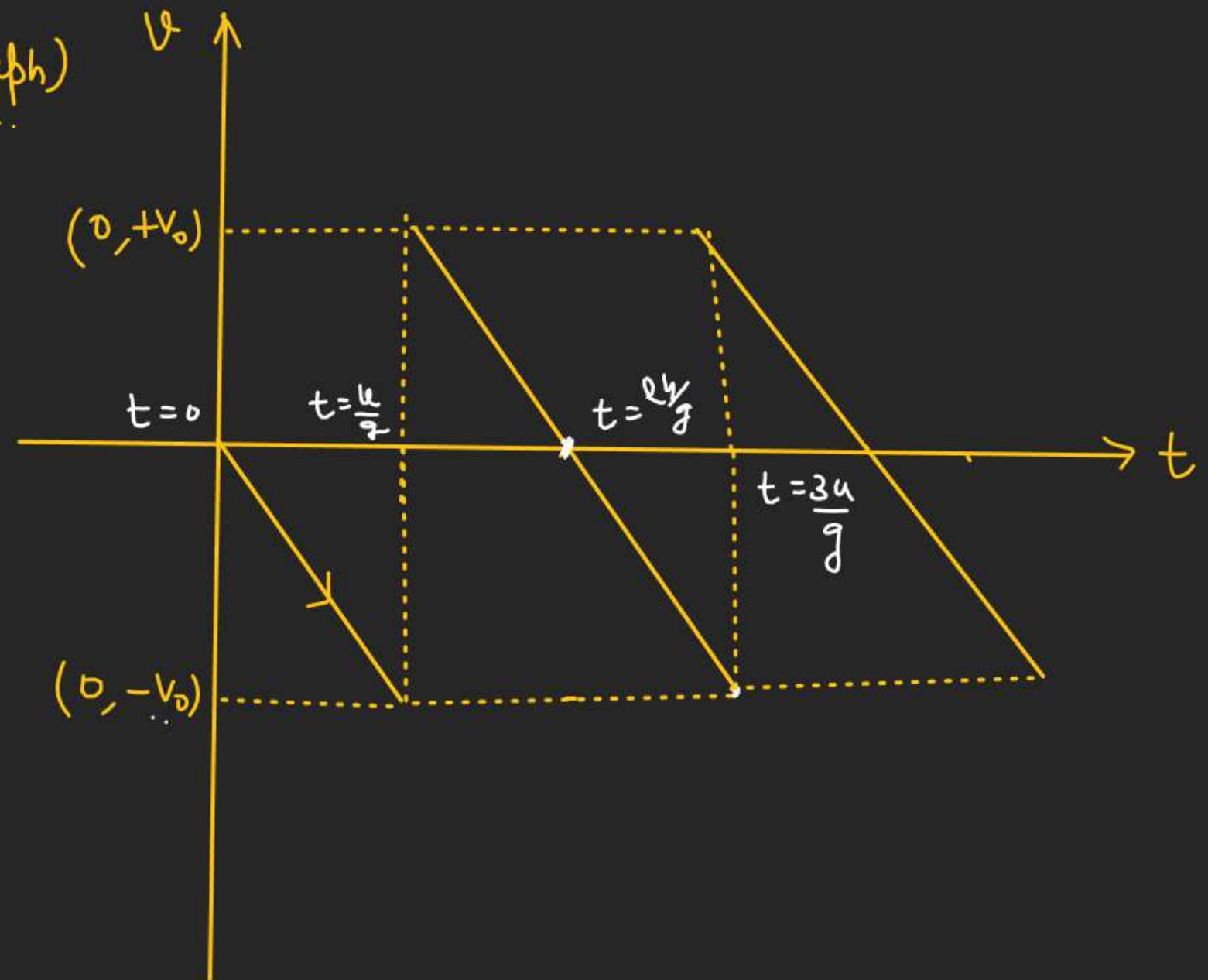
KINEMATICS

(⋆). Time of flight in motion under gravity:—
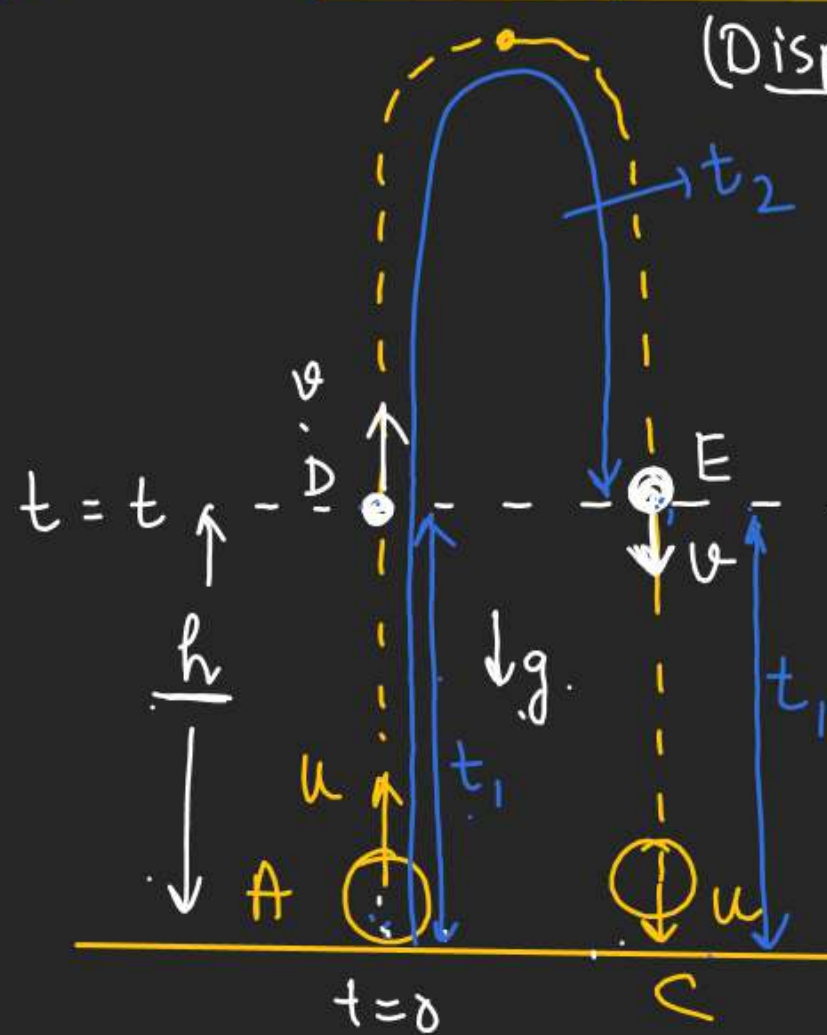
(Displacement)

$$h = ut - \frac{1}{2}gt^2$$

$$2h = (2u)t - gt^2$$

$$\frac{2h}{g} = \left(\frac{2u}{g}\right)t - t^2$$

$$\boxed{(1)t^2 - \left(\frac{2u}{g}\right)t + \frac{2h}{g} = 0}$$

$$t_{AD} = t_{EC} = t_1 \checkmark$$

$$t_{ADBE} = t_2 \checkmark$$

[Let, t_1 and t_2 be two roots. i·e for both t_1 and t_2 displacement of particle is (h)]

$$t_1 + t_2 = \left(\frac{2u}{g}\right)$$

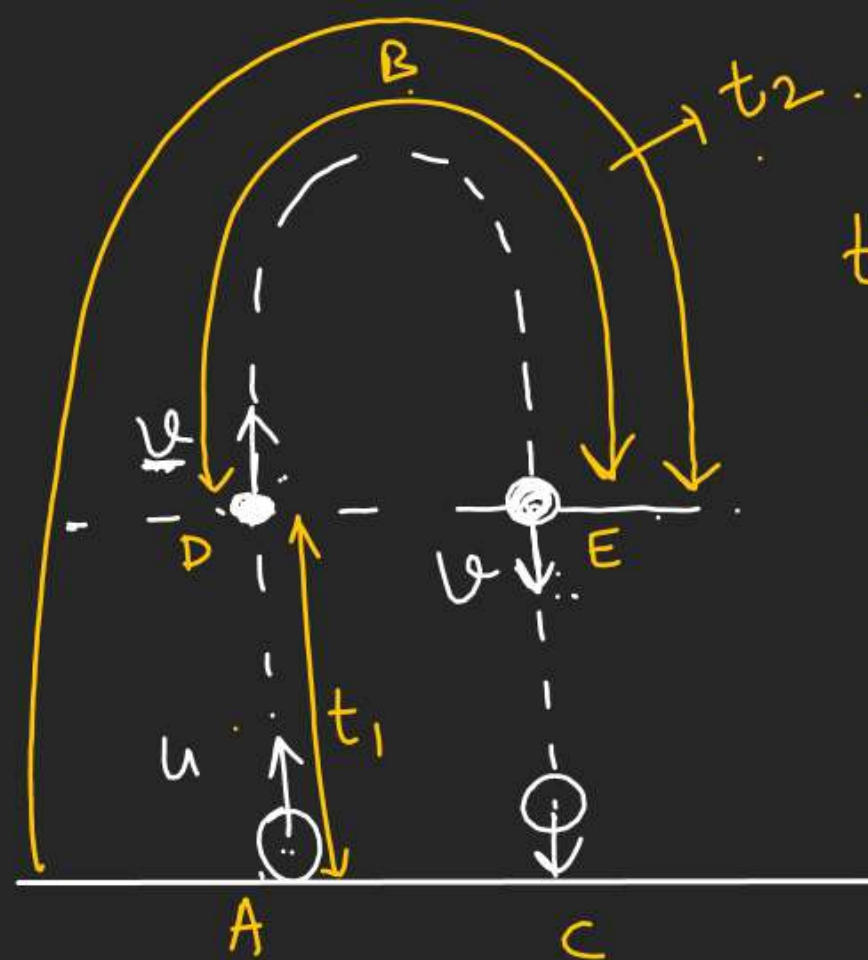$$t_1 t_2 = \left(\frac{2h}{g}\right)$$

$$t_1 > 0, \ t_2 > 0$$

let, $t_1 < t_2$

$$\boxed{ax^2 + bx + c = 0}$$

$$\begin{bmatrix} \text{Sum of} \\ \text{roots} = \frac{-b}{a} \\ \text{product} \\ \text{of root} = \frac{c}{a} \end{bmatrix}$$

KINEMATICS

t_2

$t_{DBE} = ??$

$t_{DBE} = (t_2 - t_1)$

$t^2 - \dfrac{2u}{g} t + \dfrac{2h}{g} = 0$

$$\begin{cases} t_1 + t_2 = \dfrac{2u}{g} \\[2mm] t_1 t_2 = \dfrac{2h}{g} \end{cases}$$

$(t_2 - t_1)^2 = (t_1 + t_2)^2 - 4 t_1 t_2$

$(t_2 - t_1) = \sqrt{(t_1 + t_2)^2 - 4 t_1 t_2}$

KINEMATICS

(a) Vertical projection from a tower $\rightarrow$.
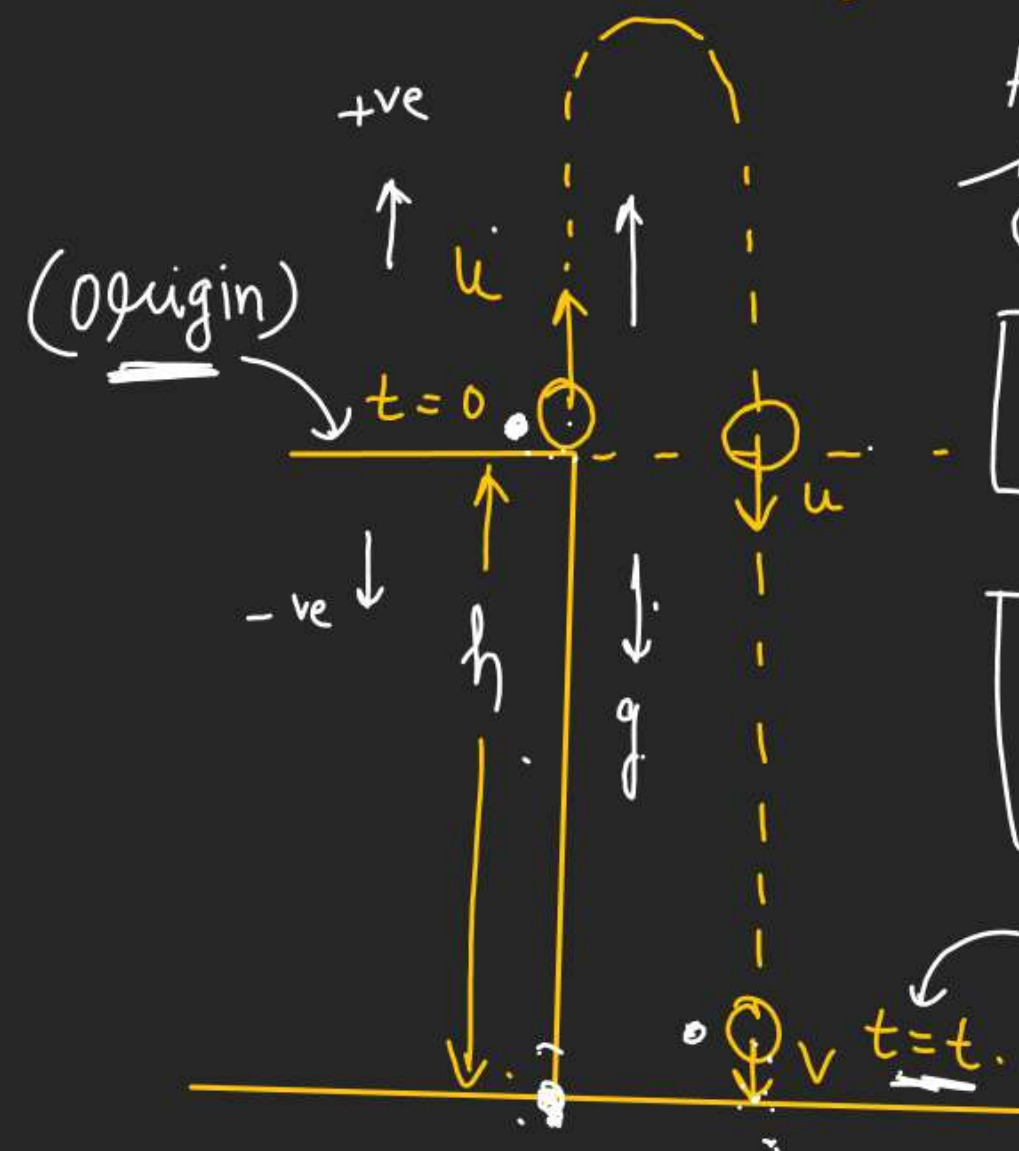
At $t = t$ ball reaches at ground.

Displacement at $t = (-h)$

$$\boxed{-h = ut - \frac{1}{2}gt^2}$$

$$\boxed{t^2 - \frac{2u}{g}t - \frac{2h}{g} = 0}$$

let, t_1 and t_2 be two roots.

$$\begin{bmatrix} t_1 t_2 = \left(\dfrac{-2h}{g}\right) < 0. \\ t_1 + t_2 = \left[\dfrac{2u}{g}\right] > 0. \end{bmatrix}$$

[t_1 & t_2 both are of opposite Sign.

[+ve root will give time of flight]