Nishant Jindal

$(ad) \perp (bc)$

$\downarrow$

$$(\vec{a}\times\vec{d})\cdot(\vec{b}\times\vec{c})=0 \Rightarrow (\vec{a}\cdot\vec{b})(\vec{d}\cdot\vec{c}) - (\vec{d}\cdot\vec{b})(\vec{a}\cdot\vec{c})=0$$

$$(\vec{b}\times\vec{d})\cdot(\vec{c}\times\vec{a})=0 \Rightarrow (\vec{b}\cdot\vec{c})(\vec{d}\cdot\vec{a}) - (\vec{d}\cdot\vec{c})(\vec{b}\cdot\vec{a})=0$$

$$(\vec{b}\cdot\vec{c})(\vec{d}\cdot\vec{a}) - (\vec{d}\cdot\vec{b})(\vec{a}\cdot\vec{c})=0$$

T.P.T.

$$(\vec{c}\times\vec{d})\cdot(\vec{a}\times\vec{b})$$

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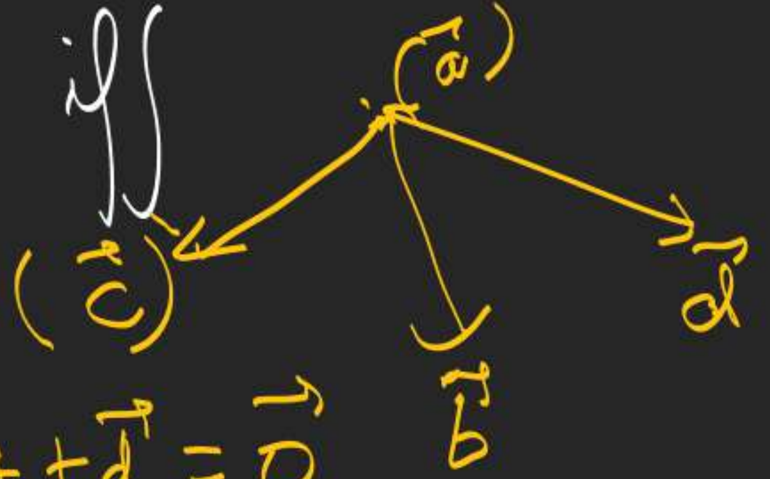
Condition for Coplanarity of 4 points

4 points with p.v. $\vec{a}, \vec{b}, \vec{c}, \vec{d}$ are coplanar iff



$\exists$ scalars x, y, z, t (not all zero) satisfying

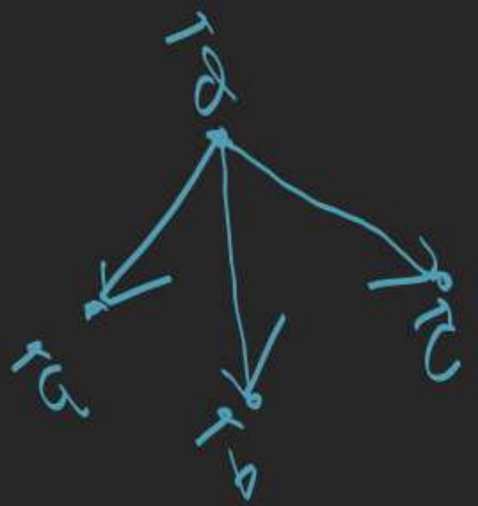
$$(-y-z-t)\vec{a} + y\vec{b} + z\vec{c} + t\vec{d} = \vec{0}$$

$z \neq 0$,

$$x\vec{a} + y\vec{b} + z\vec{c} + t\vec{d} = \vec{0} \quad \text{and} \quad x+y+z+t = 0$$

$$y(\vec{b}-\vec{a}) + z(\vec{c}-\vec{a}) + t(\vec{d}-\vec{a}) = \vec{0}$$

$$\Rightarrow \boxed{\vec{c}-\vec{a} = -\frac{y}{z}(\vec{b}-\vec{a}) - \frac{t}{z}(\vec{d}-\vec{a})}$$



$$\vec{d}-\vec{a} = \lambda(\vec{d}-\vec{b}) + \mu(\vec{d}-\vec{c})$$

$$(\lambda+\mu-1)\vec{d} + (1)\vec{a} + (-\lambda)\vec{b} + (-\mu)\vec{c} = \vec{0}$$

$$(\lambda+\mu-1) + (1) + (-\lambda) + (-\mu) = 0$$

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Reciprocal System of Vectors

If $\vec{a}, \vec{b}, \vec{c}$ be non coplanar vectors, then vectors reciprocal to them are $\vec{a}' = \dfrac{\vec{b}\times\vec{c}}{[\vec{a}\ \vec{b}\ \vec{c}]}$, $\vec{b}' = \dfrac{\vec{c}\times\vec{a}}{[\vec{a}\ \vec{b}\ \vec{c}]}$, $\vec{c}' = \dfrac{\vec{a}\times\vec{b}}{[\vec{a}\ \vec{b}\ \vec{c}]}$

$$\left((\vec{b}\times\vec{c})\times(\vec{c}\times\vec{a})\right)\cdot(\vec{a}\times\vec{b})$$

$$\left([\vec{b}\ \vec{c}\ \vec{a}]\vec{c} - 0\right)\cdot(\vec{a}\times\vec{b})$$

1. $\vec{a}'\times\vec{b}' + \vec{b}'\times\vec{c}' + \vec{c}'\times\vec{a}' = \dfrac{\vec{a}+\vec{b}+\vec{c}}{[\vec{a}\ \vec{b}\ \vec{c}]}$ $= [\vec{a}\ \vec{b}\ \vec{c}]^2$

$$\frac{(\vec{b}\times\vec{c})\times(\vec{c}\times\vec{a})}{[\vec{a}\ \vec{b}\ \vec{c}]^2} = \frac{[\vec{b}\ \vec{c}\ \vec{a}]\vec{c}}{[\vec{a}\ \vec{b}\ \vec{c}]^2} = \frac{\vec{c}}{[\vec{a}\ \vec{b}\ \vec{c}]}$$

2. $[\vec{a}'\ \vec{b}'\ \vec{c}'] = \dfrac{1}{[\vec{a}\ \vec{b}\ \vec{c}]}$

$$\frac{\left[(\vec{b}\times\vec{c})\quad \vec{c}\times\vec{a}\quad \vec{a}\times\vec{b}\right]}{[\vec{a}\ \vec{b}\ \vec{c}]^3}$$

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1.

$$\left[\vec{a}\times\vec{b} \quad \vec{b}\times\vec{c} \quad \vec{c}\times\vec{a}\right] = \left[\vec{a} \quad \vec{b} \quad \vec{c}\right]^2$$

$$c_{31}\hat{i} + c_{32}\hat{j} + c_{33}\hat{k}$$

$$= \begin{vmatrix} \vec{a}\cdot\vec{a} & \vec{a}\cdot\vec{b} & \vec{a}\cdot\vec{c} \\ \vec{b}\cdot\vec{a} & \vec{b}\cdot\vec{b} & \vec{b}\cdot\vec{c} \\ \vec{c}\cdot\vec{a} & \vec{c}\cdot\vec{b} & \vec{c}\cdot\vec{c} \end{vmatrix}$$

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} \qquad \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ \hat{i} & \hat{j} & \hat{k} \end{vmatrix}$$

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Q. If $\vec{a}, \vec{b}, \vec{c}$ are non coplanar vectors, then

P.T. $$\vec{r} = \frac{(\vec{r}\cdot\vec{a})(\vec{b}\times\vec{c}) + (\vec{r}\cdot\vec{b})(\vec{c}\times\vec{a}) + (\vec{r}\cdot\vec{c})(\vec{a}\times\vec{b})}{[\vec{a}\ \vec{b}\ \vec{c}]}$$

$$\vec{r} = x(\vec{b}\times\vec{c}) + y(\vec{c}\times\vec{a}) + z(\vec{a}\times\vec{b})$$

$$\vec{r}\cdot\vec{a} = x[\vec{b}\ \vec{c}\ \vec{a}]$$

$$\vec{r}\cdot\vec{b} = y[\vec{b}\ \vec{c}\ \vec{a}]$$

$$\vec{r}\cdot\vec{c} = z[\vec{a}\ \vec{b}\ \vec{c}]$$

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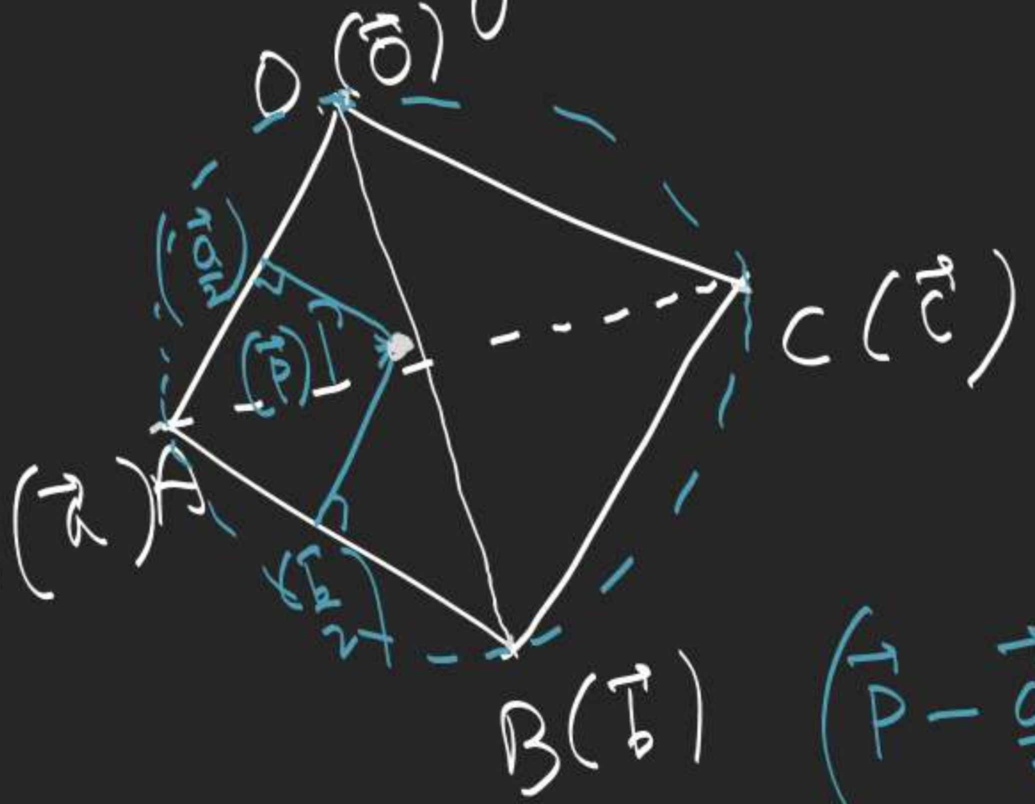
3. P.T. p.v. of centre of sphere circumscribing a tetrahedron OABC is

$$\frac{|a|^2(\vec{b}\times\vec{c})+|\vec{b}|^2(\vec{c}\times\vec{a})+|\vec{c}|^2(\vec{a}\times\vec{b})}{2[\vec{a}\ \vec{b}\ \vec{c}]}$$



$$\left(\vec{p}-\frac{\vec{a}}{2}\right)\cdot\vec{a}=0$$

$$\left(\vec{p}-\frac{\vec{b}}{2}\right)\cdot\vec{b}=0$$

$$\vec{p}=\frac{(\vec{p}\cdot\vec{a})(\vec{b}\times\vec{c})+(\vec{p}\cdot\vec{b})(\vec{c}\times\vec{a})+(\vec{p}\cdot\vec{c})(\vec{a}\times\vec{b})}{[\vec{a}\ \vec{b}\ \vec{c}]}$$

$$|\vec{p}|^2=|\vec{p}-\vec{a}|^2=|\vec{p}-\vec{b}|^2=|\vec{p}-\vec{c}|^2$$

$$\cancel{|\vec{p}|^2}=\cancel{|\vec{p}|^2}-2\vec{a}\cdot\vec{p}+|\vec{a}|^2$$

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4. Given that $\vec{a}$ & $\vec{b}$ are orthogonal to each other, find $\vec{v}$ in terms of $\vec{a}$, $\vec{b}$ satisfying $\vec{v} \cdot \vec{a} = 0$, $\vec{v} \cdot \vec{b} = 1$ & $[\vec{v}\ \vec{a}\ \vec{b}] = 1$

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Vector Equations

Dot / Cross

$\boxed{\text{VTP}}$

1. Solve for $\vec{x}$ satisfying

$$\vec{x}\cdot\vec{a} = c$$

and $\vec{a}\times\vec{x} = \vec{b}$

$$\vec{a}\times(\vec{a}\times\vec{x}) = \vec{a}\times\vec{b}$$

$$(\vec{a}\cdot\vec{x})\vec{a} - |\vec{a}|^2\vec{x} = \vec{a}\times\vec{b}$$

$$\frac{c\,\vec{a} - (\vec{a}\times\vec{b})}{|\vec{a}|^2} = \vec{x}$$

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2. Find unknown vector $\vec{R}$ satisfying

$$k\vec{R} + \vec{A}\times\vec{R} = \vec{B}$$

$$k(\vec{A}\cdot\vec{R}) + 0 = \vec{A}\cdot\vec{B}$$

$$k(\vec{A}\times\vec{R}) + \vec{A}\times(\vec{A}\times\vec{R}) = \vec{A}\times\vec{B}$$

$$k(\vec{B}-k\vec{R}) + (\vec{A}\cdot\vec{R})\vec{A} - |\vec{A}|^2\vec{R} = \vec{A}\times\vec{B}$$

$$k\vec{B} + \left(\frac{\vec{A}\cdot\vec{B}}{k}\right)\vec{A} - \left(k^2 + |\vec{A}|^2\right)\vec{R} = \vec{A}\times\vec{B}$$

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3. Solve for $\vec{x}, \vec{y}$ satisfying

$\vec{a}\cdot\vec{x} + \vec{a}\cdot\vec{y} = |\vec{a}|^2$

$\vec{x} + \vec{y} = \vec{a}$ — (1)

$\vec{x} \times \vec{y} = \vec{b}$ $\quad (\vec{a}\cdot\vec{y})\vec{x} - (\vec{a}\cdot\vec{x})\vec{y} = \vec{a}\times\vec{b}$

& $\vec{x}\cdot\vec{a} = 1$

Ex-II (remaining)

$$\left(|\vec{a}|^2 - 1\right)\vec{x} - \vec{y} = \vec{a}\times\vec{b} \quad \text{— (2)}$$

(1) + (2)

$$|\vec{a}|^2\vec{x} = \vec{a} + \vec{a}\times\vec{b}$$

Wed → Area (Ex-I) + Test

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4. Solve for $\vec{x}$ satisfying

$$\vec{x} \times \vec{a} + (\vec{x} \cdot \vec{b})\vec{a} = \vec{c}$$