Q 92

$(\sqrt{3}+1)^{2n}$ contains 2^{n+1} as factor.

meaning

$\Rightarrow \boxed{(\sqrt{3}+1)^{2n} \text{ का } G.I.F.} \div \boxed{2^{n+1}}$

A) G.I.F. वाले Q s.

B) $\div$ वाले Q s.

* 2^{n+1} से divisible hua then It must be Even Int

$(\sqrt{3}+1)^{2n} = I + f \qquad 0 < f < 1$

$(\sqrt{3}-1)^{2n} = f' \qquad 0 < f' < 1$

$(\sqrt{3}+1)^{2n} + (\sqrt{3}-1)^{2n} = I + \boxed{f + f'}$

$((\sqrt{3}+1)^2)^n + ((\sqrt{3}-1)^2)^n = I + 1 \text{ Even}$

$(4 + 2\sqrt{3})^n + (4 - 2\sqrt{3})^n = I + 1$

$2^n \{(2+\sqrt{3})^n + (2-\sqrt{3})^n\}$

$P+Q \qquad + P - Q$

$2^n \{2(T_1 + T_3 + \cdots)\}$

$2^{n+1} (\qquad " \qquad) \div \text{ by } 2^{n+1} (H.P.)$

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91 ✓

85 (oby (lec.)

64 (Trick)

61 $(4 + \sqrt{15})^n = I + f \qquad \cancel{I} + f + f' = \cancel{I} + 1$

$(4 - \sqrt{15})^n = f' \qquad\qquad f' = 1 - f$

$(4 + \sqrt{15})^n + (4 - \sqrt{15})^n = I + \boxed{f + f'}$

$2[\,^nC_0 \sqrt{15}^0 + \,^nC_2 \cdot 4^{n-2} (\sqrt{15})^2 + \,^nC_4 \, 4^{n-4} (\sqrt{15})^4 - - - \,] = I + 1$

$(I + f) \cdot (1 - f)$

$(I + f) \cdot f' = (4 + \sqrt{15})^n \times (4 - \sqrt{15})^n$

$= (16 - 15)^n$

$= 1^n = 1$

Q Let $Z = (6\sqrt{6} + 14)^{2n+1}$ $n \in N$ then $Z\{Z\}$

$\{ \} = $ Fraction Part
$= f'$

$Z = (6\sqrt{6} + 14)^{2n+1} = I + f \rightarrow \{ \} = f$

$(6\sqrt{6} - 14)^{2n+1} = f'$

$0 < f < 1$ $\}$ $0 < f < 1$
$0 < f' < 1$ $\}$ $-1 < -f' < 0$

$\overline{(6\sqrt{6} + 14)^{2n+1} - (6\sqrt{6} + 14)^{2n+1}} = I + f - f'$

$-1 < f - f' < 1$

$2[T_2 + T_4 + \cdots] = $

$\text{Even In} = \dfrac{I + 0}{f - f' = 0}$

$\overset{2n+1}{C_1}(6\sqrt{6})^{2n}$

$f' = f$

$x - \lceil x \rceil = \{x\}$

Demand $= Z \cdot \{z\}$ f'

$= (I + f) f' = (6\sqrt{6} + 14)^{2n+1} \times (6\sqrt{6} - 14)^{2n+1}$

$= (216 - 196)^{2n+1} = (20)^{2n+1}$

$(a + \sqrt{b})^n$ type Qs. me

Vo Kya Kya Puch Saktah?

✓① $(a + \sqrt{b})^n$ की पूरी value IF n odd/Even

② $(I + f) \cdot (1 - f)$?

$\boxed{\begin{array}{c} f + f' = L \\ f - f' = 0 \end{array}}$

(3) $(Z)\{z\}$

(4) $(z)(z - \lceil z \rceil)$

Q $(5+2\sqrt{6})^n = I+f$ then $I =$

A) $f - \frac{1}{f}$ (B) $\boxed{\frac{1}{1-f} - f}$ $f + f' = 1$

(C) $\frac{1}{f} - f$ (D) $f - \frac{1}{1-f}$

$(5+2\sqrt{6})^n = I+f$ $0 < f < 1$

$(5-2\sqrt{6})^n = f'$ $0 < f' < 1$

$(I+f) \cdot f' = (5+2\sqrt{6})^n \cdot (5-2\sqrt{6})^n$ $\boxed{f + f' = 1}$

$\boxed{f - f' = 0}$

$= (25-24)^n$

$= 1^n = 1$

$(I+f) \cdot f' = 1 \Rightarrow If + f = \frac{f}{f} \Rightarrow I = \frac{1}{f} - f = \boxed{\frac{1}{1-f} - f}$

Q $x = (2+\sqrt{3})^n$ then $x - x^2 + x[x]$

$x = (2+\sqrt{3})^n = I + f$

$(2-\sqrt{3})^n = f'$

$x(1 - x + [x])$

$x(1 - (x - [x]))$

$\underline{\underline{x}}(1 - \{x\})$

$(I+f)(1-f)$

$(I+f) \cdot f'$

$(2+\sqrt{3})^n (2-\sqrt{3})^n$

$= 1^n = 1$

$f = f' \to$ बराबर

$\dfrac{f + f' = 1}{f' = 1 - f}$

Q find $\left[(\sqrt{2}+1)^7 \right] \to$ G.I.F

G.I.F. वाला

I Demanded

$\overbrace{}^{8 \text{ terms}}$

$(\sqrt{2}+1)^7 = I + f$

$(\sqrt{2}-1)^7 = f'$

$\overline{(\sqrt{2}+1)^7 - (\sqrt{2}-1)^7 = I + \boxed{f - f'}}$

$2\left[T_2 + T_4 + T_6 + T_8 \right] = I$

$2\left[\binom{7}{1}(\sqrt{2})^6 + \binom{7}{3}(\sqrt{2})^4 + \binom{7}{5}(\sqrt{2})^2 + \binom{7}{7}(\sqrt{2})^0 \right] = I$

$I = 2\left[7\times 8 + 35\times 4 + 21\times 2 + 1 \right] = I$

① $n_{C_0}, n_{C_1}, n_{C_2}, n_{C_3} \cdots$ Bin Coff are Integer.

Ex:- $7_{C_2} = \dfrac{7 \cdot \cancel{6} 3}{1 \cdot \cancel{2}} = 21$

② $(x+1)^n = \underbrace{n_{C_0} x^n + n_{C_1} x^{n-1} + n_{C_2} \cdot x^{n-2} + \cdots + n_{C_{n-1}} x + n_{C_n} \cdot x^0}$

$= x \left\{ n_{C_0} \cdot x^{n-1} + n_{C_1} \cdot x^{n-2} + n_{C_2} \cdot x^{n-3} + \cdots + n_{C_{n-1}} \right\} + 1$

$(x+1)^n = \lambda \cdot x + 1$

③ $(x-1)^n = n_{C_0} x^n (-1)^0 + n_{C_1} \cdot x^{n-1} \cdot (-1)^1 + n_{C_2} x^{n-2} (-1)^2 + n_{C_3} \cdot x^{n-3} (-1)^3 + \cdots + n_{C_n} (-1)^n$

$= \left\{ n_{C_0} \cdot x^n - n_{C_1} x^{n-1} + n_{C_2} x^{n-2} - n_{C_3} \cdot x^{n-3} \right.$

$\left. \cdots + n_{C_{n-1}} \cdot x (-1)^{n-1} \right\} + (-1)^n$

$= x \left\{ n_{C_0} \cdot x^{n-1} - n_{C_1} \cdot x^{n-2} + n_{C_2} \cdot x^{n-3} \cdots \right.$

$\left. + n_{C_{n-1}} (-1)^{n-1} \right\} + (-1)^n$

$(x-1)^n = \lambda \cdot x + (-1)^n = \begin{cases} \lambda x - 1 & n = \text{odd} \\ \lambda x + 1 & n = \text{Even} \end{cases}$

4 Qs के बाद समझ आने लगेगा

Q find Remainder when.

$(x+1)^n - \lambda x + 1$

$(x-1)^n - \lambda x - 1$ $(n = odd)$

① $\dfrac{5^{99}}{4} = \dfrac{(4+1)^{99}}{4} = \dfrac{4\lambda+1}{4}$

$\begin{array}{r} \lambda \\ 4\overline{)4\lambda+1} \\ 4\lambda \\ \hline 1 \end{array}$

5 में 4 देखोंकरके try

∴ Rem = 1

Q2 $\dfrac{5^{99}}{6}$ find Rem.

$n = 99 = odd$

$\dfrac{(6-1)^{99}}{6} = \dfrac{6\lambda-1}{6}$

$= \dfrac{6\lambda-1+6-6}{6}$

$= \dfrac{6(\lambda-1)+5}{6}$

$\begin{array}{r} x \\ 6\overline{)6\lambda-1} \\ 6\lambda \\ \hline -1 \end{array}$ Rem -ve नहीं होता

$\begin{array}{r} \lambda-1 \\ 6\overline{)6(x-1)+5} \\ 6(\lambda-1) \\ \hline 5 \end{array}$

∴ Rem = 5

$\begin{array}{r} 3 \\ 5\overline{)14} \\ 15 \\ \hline -1 \end{array}$ Ⓐ

Q $\dfrac{5^{99}}{12}$ find Rem.

$= \dfrac{(5^{-2})^{49} \cdot 5}{12}$

$= \dfrac{5 \cdot (24+1)^{49}}{12}$ → $(x+1)^n$

$= \dfrac{5(24\lambda+1)}{12}$

$\begin{array}{r} 12\overline{\smash)24\lambda+1}\ \ (2\lambda \\ \underline{24\lambda} \\ +1 \end{array}$

Rem $= 5$

1) $(x+1)^n = \lambda x + 1$

$(x-1)^n = \lambda x - 1 \quad (n = odd)$

$\qquad\qquad \lambda x + 1 \quad (n = \text{even})$

Q $\dfrac{5^{99}}{13}$

$\dfrac{(5^2)^{49} \cdot 5}{13} = \dfrac{5 \cdot (25)^{49}}{13} = \dfrac{5(26-1)^{49}}{13}$ → $(x-1)^n$ odd

$= \dfrac{5(26\lambda-1)}{13} = \dfrac{5\times 26\lambda - 5 + 8 - 8}{13}$

$= \dfrac{5\times 26\lambda - 13 + 8}{13}$

$= \dfrac{13(10\lambda-1) + 8}{13}$ ∴ Rem $= 8$

Q5 $\dfrac{2^{32}}{3}$ find Rem. $n = Even$

$\longrightarrow (a-1)^n = \lambda x + 1.$

$\dfrac{(3-1)^{32}}{3}$

$\dfrac{-3\lambda + 1}{3}$ ∴ Rem $= 1$

Q6 $\dfrac{2^{123}}{9}$ find Rem? $n = odd$

$\longrightarrow (a-1)^n = \lambda x - 1$

$\dfrac{(2^3)^{41}}{9} = \dfrac{(9-1)^{41}}{9}$

$= \dfrac{9\lambda - 1}{9} = \dfrac{9\lambda - 1 + 8 - 8}{9}$

$= \dfrac{9(\lambda - 1) + 8}{9}$ ∴ Rem $= 8$.

① If $(106)^{85} - (85)^{106} + 1$ is divided by 7 find Rem?

$$(106)^{85} - (85)^{106} + 1$$

15×7
105　　12×7
=84

$(x+1)^n = \lambda x + 1$

$$(105+1)^{85} - (84+1)^{106} + 1$$

$$(105\lambda + 1) - (84\mu + 1) + 1$$

$$\underbrace{105\lambda - 84\mu} + 1$$

Completely
dir.
by 7

$\therefore$ Rem = 1

$$7(\underline{15\lambda} - \underline{12\mu}) + 1$$

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② Find Remainder if

$$Y = 1 + 2 + 2^2 + 2^3 + \cdots + 2^{1999} \div 5$$

GP → $a = 1, r = 2, n = 2000$

$$\frac{a(r^n - 1)}{r - 1} = \frac{1 \cdot (2^{2000} - 1)}{2 - 1}$$

$$= (2^{2000} - 1) \div 5$$

$$= \frac{2^{2000} - 1}{5} = \frac{(2^2)^{1000} - 1}{5} = \frac{(4)^{1000} - 1}{5}$$

→ $(x-1)^n \; n = $ even → $\lambda x + 1$

$$= \frac{(5-1)^{1000} - 1}{5} = \frac{5\lambda + 1 - 1}{5} \Rightarrow \text{Rem} = 0$$

Q Rem when 2^{2005} divided by 17.

$$\frac{2 \cdot (2^4)^{501}}{17} = \frac{2(17-1)^{501}}{17}$$

$(x-1)^n = \lambda x - 1$ [odd]

$$= \frac{2(17\lambda - 1)}{17} = \frac{34\lambda - 2 + 15 - 15}{17}$$

$$= \frac{17(2\lambda - 1) + \overset{Rem}{\boxed{15}}}{17}$$

$$Rem = \underline{15}$$

Q Rem when $2^{32^{32}}$ divisible by 7

$$2^{(65)^{32}} = 2^{2^{160}} \quad ???$$

Q Find Rem when. $6^n - 5n$ divided by 25?

$(1+5)^n - 5n$.

$\left({}^n C_0 \cdot 1^n \cdot 5^0 + {}^n C_1 \cdot 1^{n-1} \cdot 5^1 + {}^n C_2 \cdot 1^{n-2} \cdot 5^2 + {}^n C_3 \cdot 1^{n-3} \cdot 5^3 + \cdots \right) - 5n$

$\left(1 + 5n + \underbrace{25 \cdot {}^n C_2 + 125 \cdot {}^n C_3 + \cdots} \right) - 5n$

$$\frac{25(\lambda) + 1}{25}$$

$$\therefore Rem = \underline{1}$$

Q $\dfrac{7^{103}}{25}$

$= \dfrac{7 \cdot (7^2)^{51}}{25}$

$= \dfrac{7 \cdot (50-1)^{51}}{25}$

$= \dfrac{7 \cdot (50\lambda - 1)}{25} = \dfrac{7 \times 50\lambda - \overset{\bullet}{7} + 18 - \overset{\bullet}{18}}{25}$

$= \dfrac{25(14\lambda - 1) + 18}{25}$

$\Rightarrow \therefore \text{Rem} = 18.$