Nishant Jindal

Position of Pt. WRT Ellipse

$E(P) > 0$ Outside

$= 0$ on Ellipse

< 0 Inside Ellipse.

Q Varify Position of $(3,2)$ WRT. $\frac{x^2}{25} + \frac{y^2}{16} = 1$

$$E(3,2) = \frac{9}{25} + \frac{4}{16} - 1 = \frac{144+100-400}{25\times 16} < 0$$

Inside.

Q $E: \frac{x^2}{9} + \frac{y^2}{4} = 1$ Circle $x^2+y^2=9$

$P(1,2)$, $Q(2,1)$ are 2 pts.

find Position of P, Q WRT. both Curves

$E(1,2)$ $\quad \frac{1}{9} + \frac{4}{4} - 1 > 0$

$C(1,2): \quad 1+4-9<0$

$(1,2)$ Inside Circle outside Ellipse

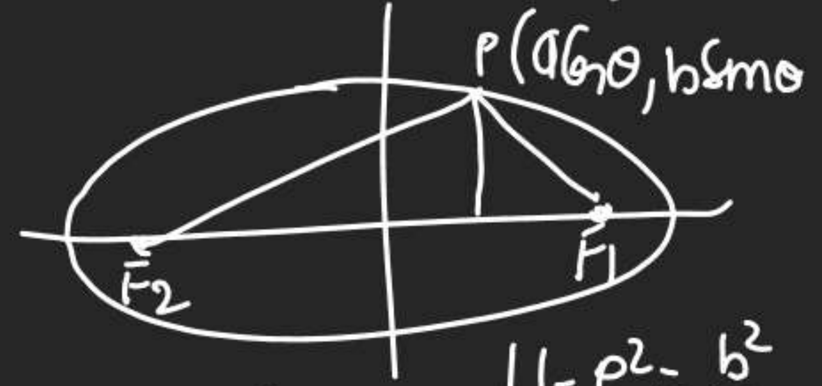
$E(2,1): \quad \frac{4}{9} + \frac{1}{4} - 1 < 0$

$C(2,1): \quad 4+1-9<0$

Inside both.

Q P is a variable Pt. on $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ with Foci F_1, F_2

Find Max Area of ΔPF_1F_2



$E: \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$1-e^2 = \frac{b^2}{a^2}$

$b^2 = a^2(1-e^2)$

$a^2e^2 = a^2-b^2$

$\Delta = \frac{1}{2} \times 2ae \times b\sin\theta$

$\Delta = bae(\sin\theta)_{Max} = bae$

Max.

$= b\sqrt{a^2-b^2}$

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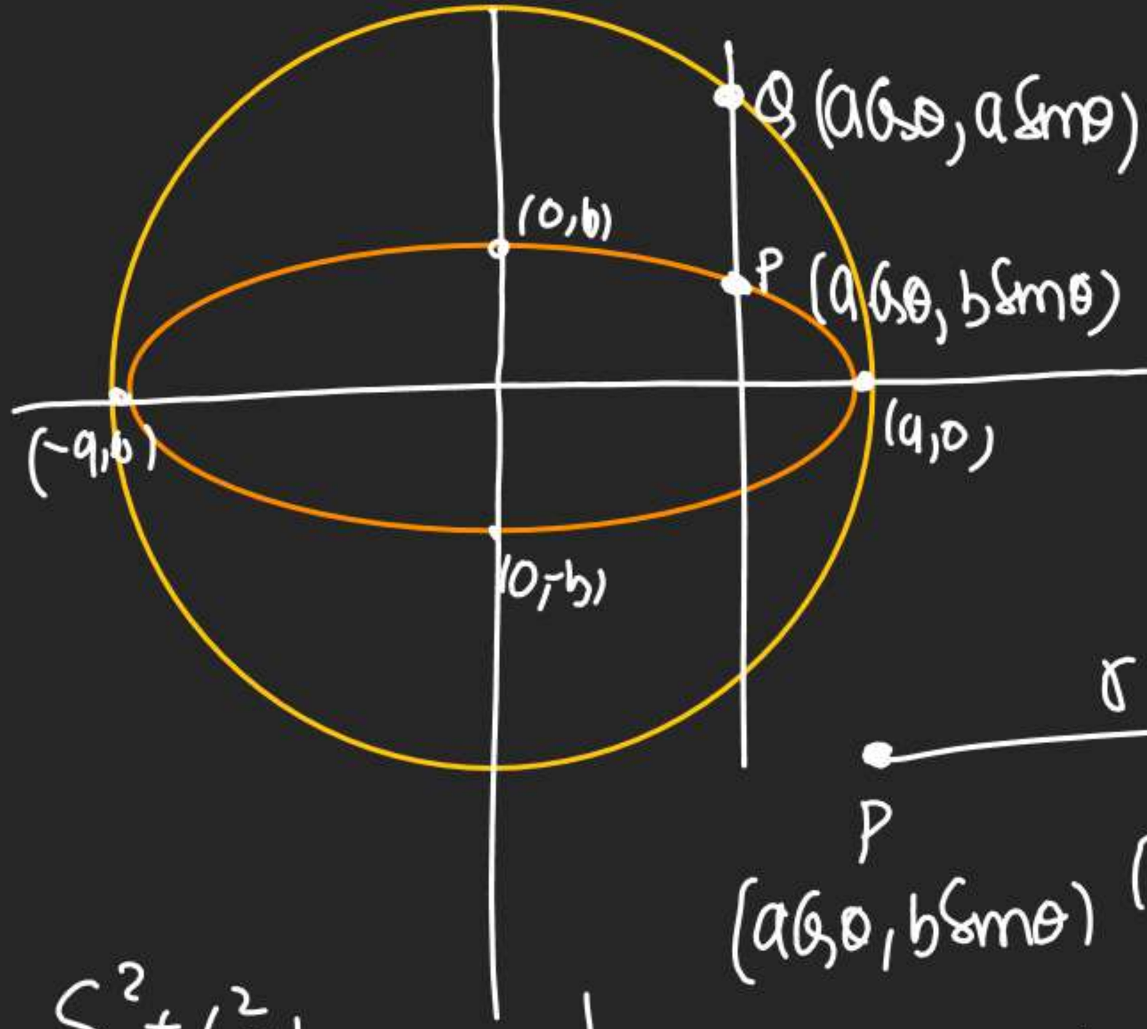
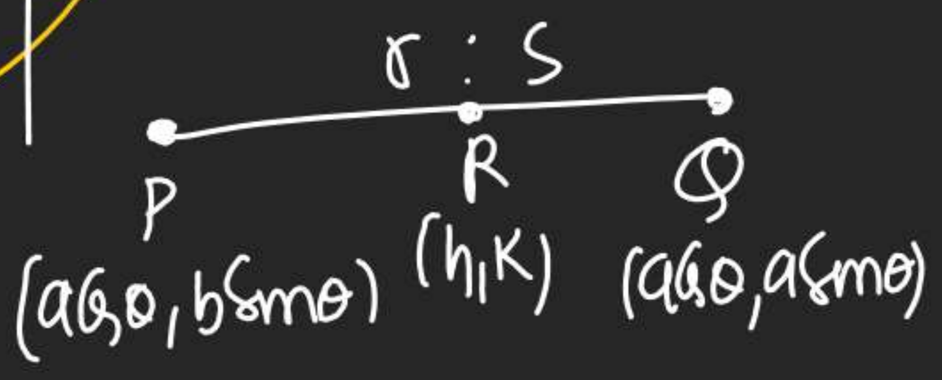
Q Let P be a Pt. on $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1;\ 0<b<a$

Let a Line ||re to y Axis P.T. P meet Circle $x^2+y^2=a^2$ at Pt. Q such that P & Q are on same side of x Axis for 2 +ve Real No. r & s. find Locus of Pt. R on PQ such that $PR : RQ = r : s$ as P varies over ellipse.

$$\frac{PR}{RQ}=\frac{r}{s}$$





$$h=\frac{ar\cos\theta+as\cos\theta}{r+s} \Rightarrow a\cos\theta=h$$
$$\cos\theta=\frac{h}{a}$$

$$k=\frac{ar\sin\theta+bs\sin\theta}{r+s} \Rightarrow \sin\theta=\frac{k(r+s)}{ar+bs}$$

$$S^2+C^2=1$$

$$\frac{x^2}{a^2}+\frac{y^2(r+s)^2}{(ar+bs)^2}=1$$

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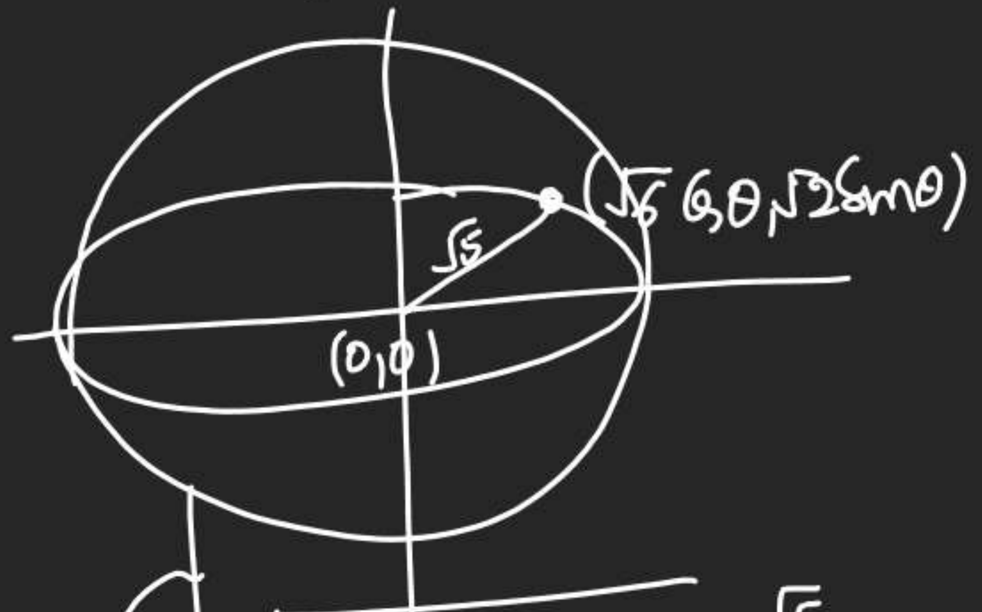
Q Find ecc. angle of $\frac{x^2}{6}+\frac{y^2}{2}=1$

Whose dist. from Centre is $\sqrt{5}$



$\sqrt{6\cos^2\theta+2\sin^2\theta}=\sqrt{5}$

$2+4\cos^2\theta=5$

$4\cos^2\theta=3$

$\cos\theta=\sqrt{\frac{3}{2}}, -\frac{\sqrt{3}}{2}$

$\theta=\frac{\pi}{6}, \frac{11\pi}{6}, \sim$

Q Find Eqn of Curve whose

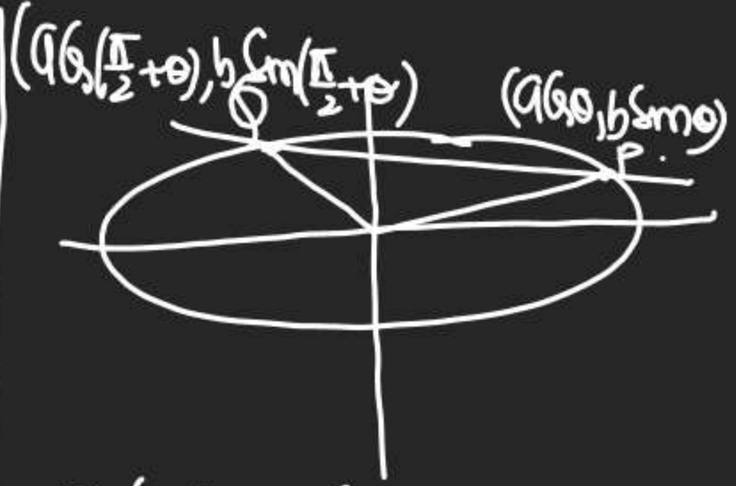
Par. Eqn are $x=1+4\cos\theta$

$y=2+3\sin\theta, \theta\in R$.

$\cos\theta=\frac{x-1}{4}, \sin\theta=\frac{y-2}{3}$

$\left(\frac{x-1}{4}\right)^2+\left(\frac{y-2}{3}\right)^2=1$

Q Line $lx+my=n$ Cuts Ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ in P & Q whose ecc. angle is differ by $\frac{\pi}{2}$, then find value of $\frac{a^2l^2+b^2m^2}{n^2}$



P: $(a\cos\theta, b\sin\theta)$

Q: $(-a\sin\theta, b\cos\theta)$

Line

$al\cos\theta+bm\sin\theta=n$

$-al\sin\theta+bm\cos\theta=n$

SAA

$a^2l^2+b^2m^2=2n^2$

$\frac{a^2l^2+b^2m^2}{n^2}=2$

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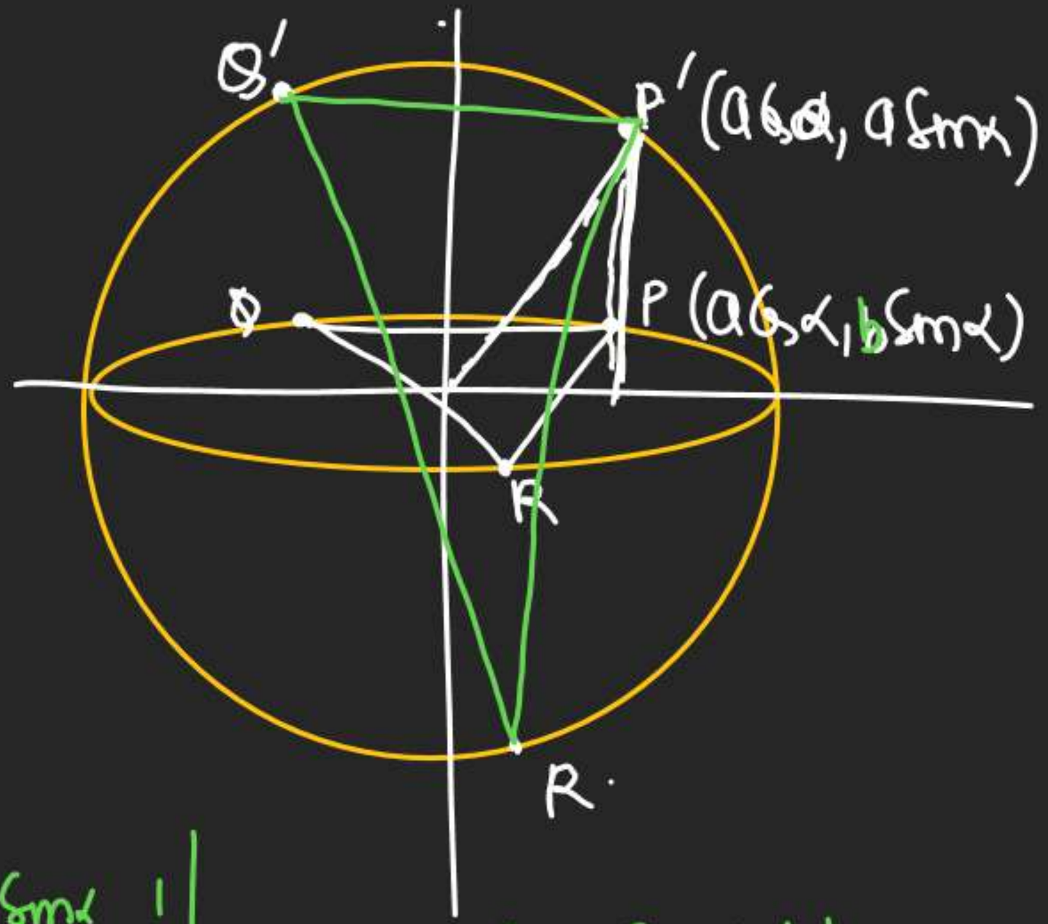
Q P.T. Ratio of area of any ΔPQR inscribed in an Ellipse to the Δ formed by Corresponding Pts on Aux. Circle is equal to Ratio of semi minor Axis to semi major Axis. $\rightarrow \frac{b}{a}$



$$\frac{\Delta PQR}{\Delta P'Q'R'} = \frac{\frac{1}{2}\begin{vmatrix} a\cos\alpha & b\sin\alpha & 1 \\ a\cos\beta & b\sin\beta & 1 \\ a\cos\gamma & b\sin\gamma & 1 \end{vmatrix}}{\frac{1}{2}\begin{vmatrix} a\cos\alpha & a\sin\alpha & 1 \\ a\cos\beta & a\sin\beta & 1 \\ a\cos\gamma & a\sin\gamma & 1 \end{vmatrix}} = \frac{ab\begin{vmatrix} \cos\alpha & \sin\alpha & 1 \\ \cos\beta & \sin\beta & 1 \\ \cos\gamma & \sin\gamma & 1 \end{vmatrix}}{a^2\begin{vmatrix} \cos\alpha & \sin\alpha & 1 \\ \cos\beta & \sin\beta & 1 \\ \cos\gamma & \sin\gamma & 1 \end{vmatrix}} = \frac{b}{a}$$

Q If Ratio of area of Δ inscribed in El. to the Δ formed by Corresponding pts on Aux. Circle is 5 find Ecc of Ellipse?

$\frac{b}{a} = \frac{1}{2}$

$1 - e^2 = \frac{b^2}{a^2}$

$e^2 = 1 - \frac{1}{4} = \frac{3}{4}$

$e = \frac{\sqrt{3}}{2}$

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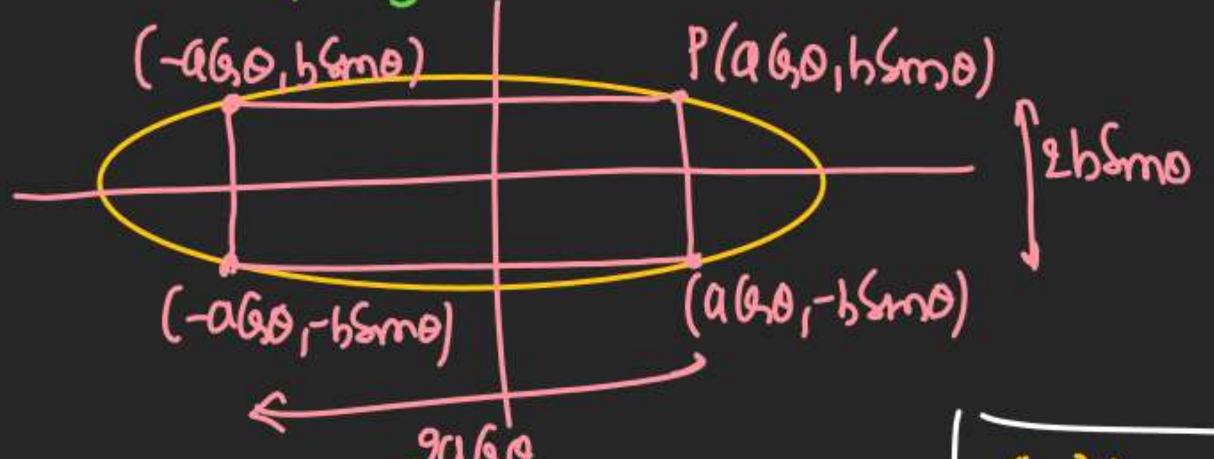
Q Find Mx. area of Rectangle Inscribed in Ellipse

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$



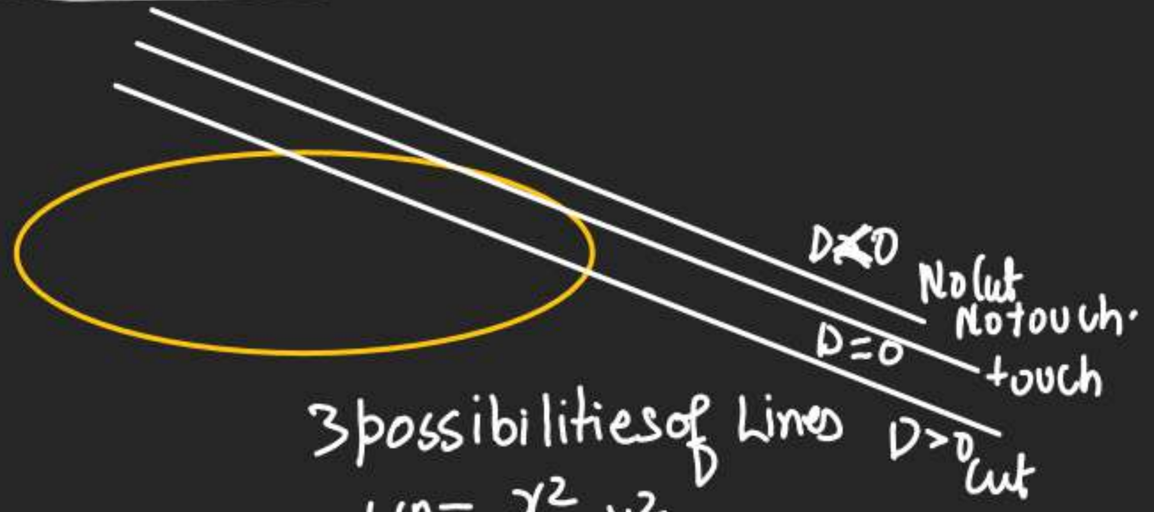
$\Delta = 2a\cos\theta \times 2b\sin\theta$

$= 4ab\sin\theta\cos\theta$

$\Delta_{Max} = 2ab(\sin 2\theta)_{Max}$

$= 2ab\times 1 = 2ab.$

Line & Ellipse



3 possibilities of Lines WRT $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

Line: $\Rightarrow y = mx + c$

$$\frac{x^2}{a^2}+\frac{(mx+c)^2}{b^2}=1$$

$$b^2x^2+a^2m^2x^2+2a^2mcx+a^2c^2=a^2b^2$$

$$(b^2+a^2m^2)x^2+\underbrace{2a^2mc}\,x+a^2c^2-a^2b^2=0$$

Cond^n of Tangency $D=0$

$$4a^4m^2c^2-4(b^2+a^2m^2)(a^2c^2-a^2b^2)=0$$

$$\cancel{a^4m^2c^2}-b^2a^2c^2+a^2b^4-\cancel{a^4m^2c^2}+a^4m^2b^2=0$$

$$-c^2+b^2+a^2m^2=0$$

$$\boxed{c=\pm\sqrt{a^2m^2+b^2}}$$

Cond^n of tangency

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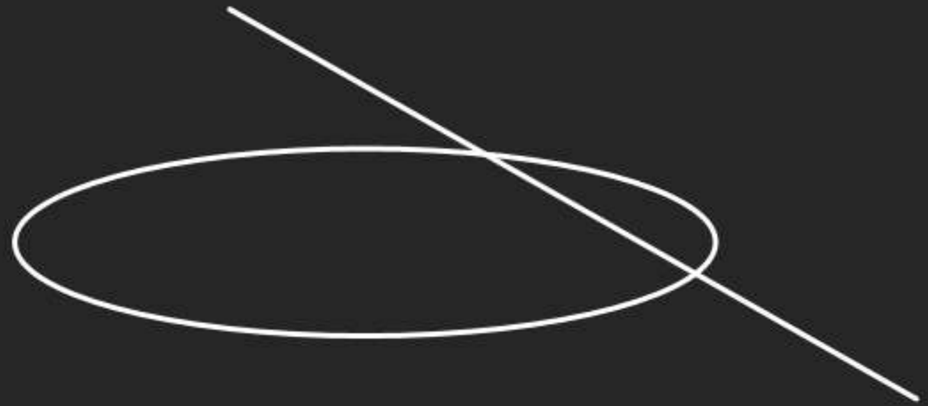
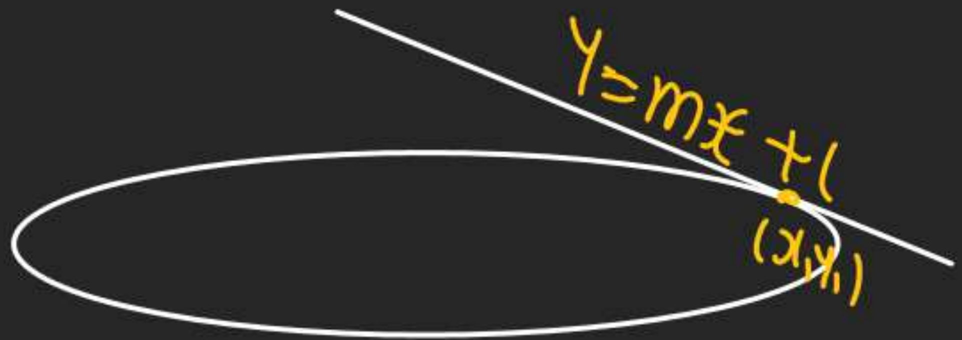


$C^2 < a^2m^2 + b^2$



$$\boxed{c^2 = a^2m^2 + b^2}$$

$$c = \pm\sqrt{a^2m^2+b^2}$$

$C^2 > a^2m^2 + b^2$

Eqn of tangent

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Slope form

$$\boxed{y = mx \pm \sqrt{a^2m^2+b^2}}$$

Cart. form

$T = 0$

Pt. of tangency (x_1, y_1)

$$\boxed{\frac{xx_1}{a^2} + \frac{yy_1}{b^2} = 1}$$

Par. form

$(x_1, y_1) = (a\cos\theta, b\sin\theta)$

$$\frac{x \cdot a\cos\theta}{a^2} + \frac{y \cdot (b\sin\theta)}{b^2} = 1$$

$$\boxed{\frac{x\cos\theta}{a} + \frac{y\sin\theta}{b} = 1}$$

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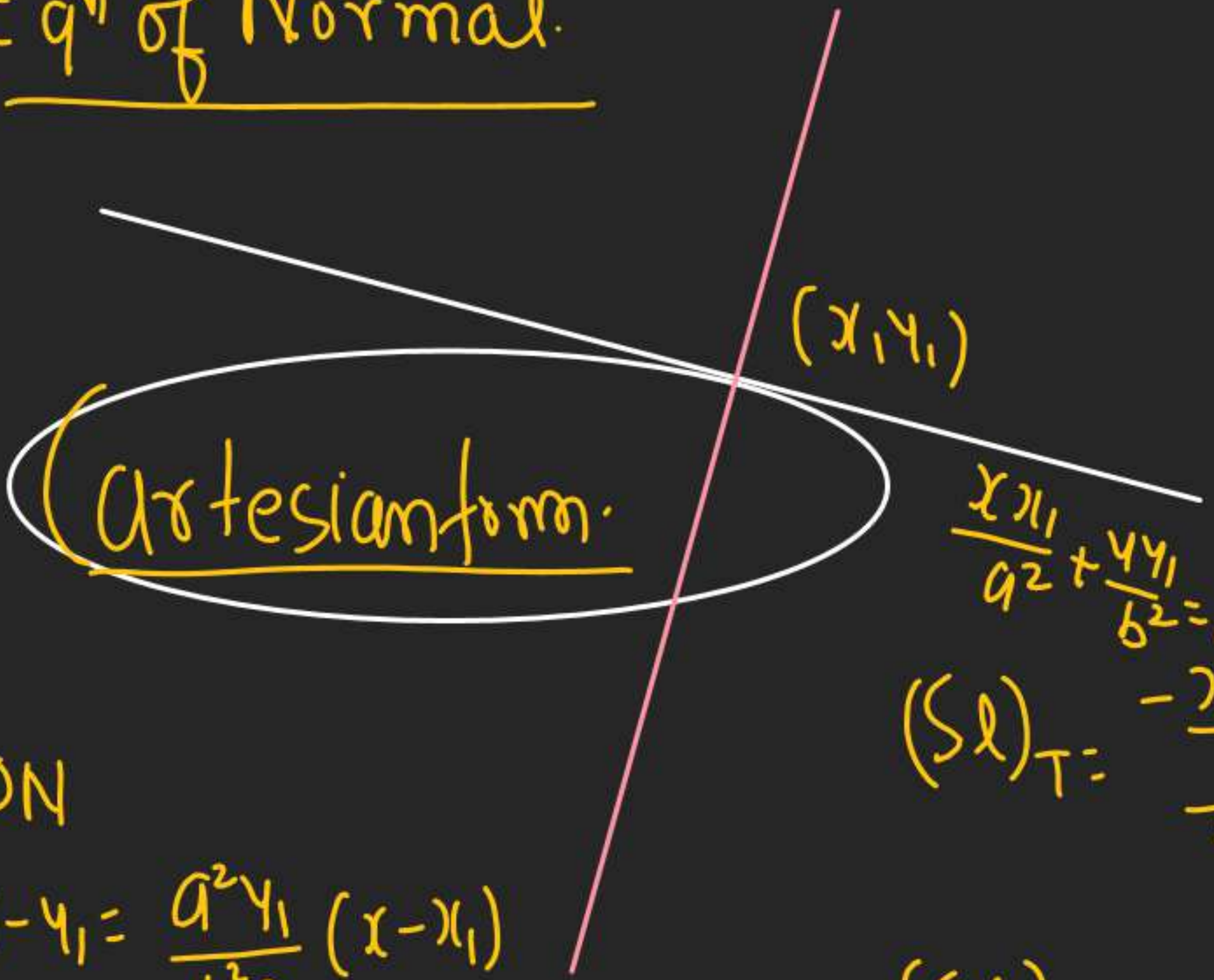
Eqⁿ of Normal.



Cartesian form.

$$\frac{xx_1}{a^2}+\frac{yy_1}{b^2}=1$$

$$(Sl)_T=\frac{-\frac{x_1}{a^2}}{\frac{y_1}{b^2}}=-\frac{b^2x_1}{a^2y_1}$$

$$(Sl)_N=\frac{a^2y_1}{b^2x_1}$$

EON

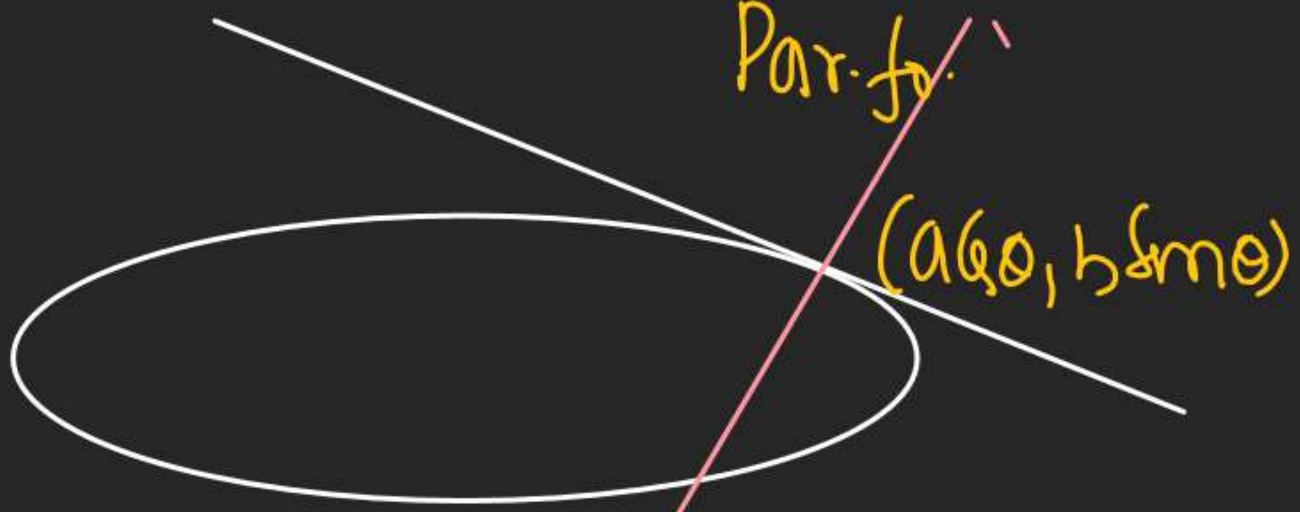
$$y-y_1=\frac{a^2y_1}{b^2x_1}(x-x_1)$$

$$\frac{y}{a^2y_1}-\frac{y_1}{a^2y_1}=\frac{x}{b^2x_1}-\frac{x_1}{b^2x_1}$$

$$\frac{x}{b^2x_1}-\frac{y}{a^2y_1}=\frac{1}{b^2}-\frac{1}{a^2}$$

$$\boxed{\frac{a^2x}{x_1}-\frac{b^2y}{y_1}=a^2-b^2}$$

Par. fo.



$$\frac{a^{\cancel{2}}x}{\cancel{a}\cos\theta}-\frac{b^{\cancel{2}}y}{\cancel{b}\sin\theta}=a^2-b^2$$

$$\boxed{ax\sec\theta-by\,\mathrm{cosec}\,\theta=a^2-b^2}$$

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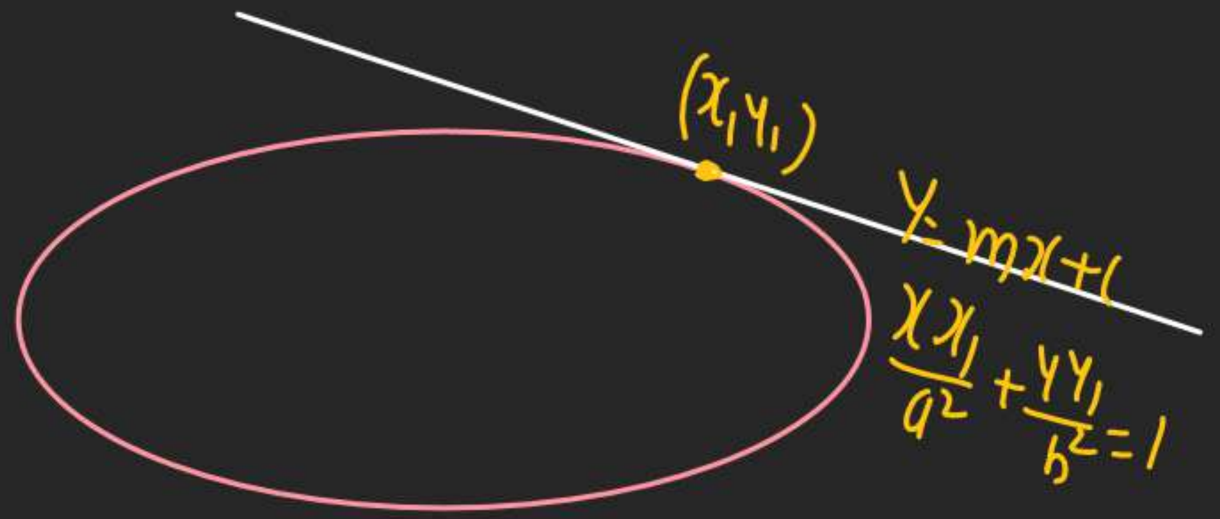
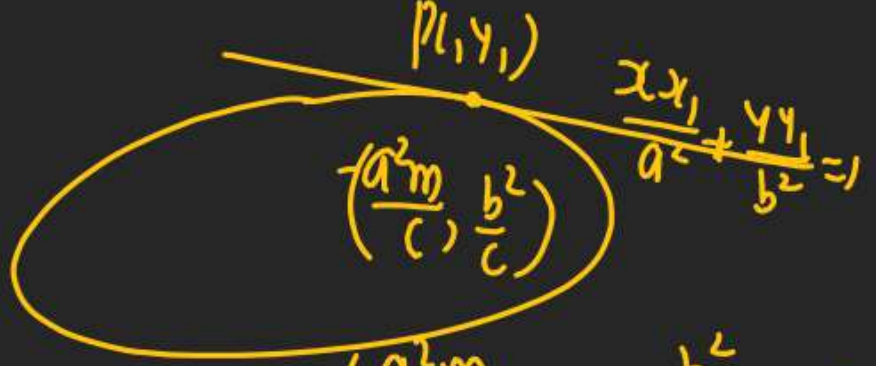
Pt. of tangency.





$$= \left(\frac{-a^2m}{\pm\sqrt{a^2m^2+b^2}}, \frac{b^2}{\pm\sqrt{a^2m^2+b^2}}\right)$$

$$mx - y + c = 0$$

$$\frac{xx_1}{a^2} + \frac{yy_1}{b^2} - 1 = 0$$

$$\frac{\frac{x_1}{a^2}}{m} = \frac{\frac{y_1}{b^2}}{-1} = \frac{-1}{c}$$

$$x_1 = \frac{-a^2m}{c}$$

$$y_1 = \frac{b^2}{c}$$

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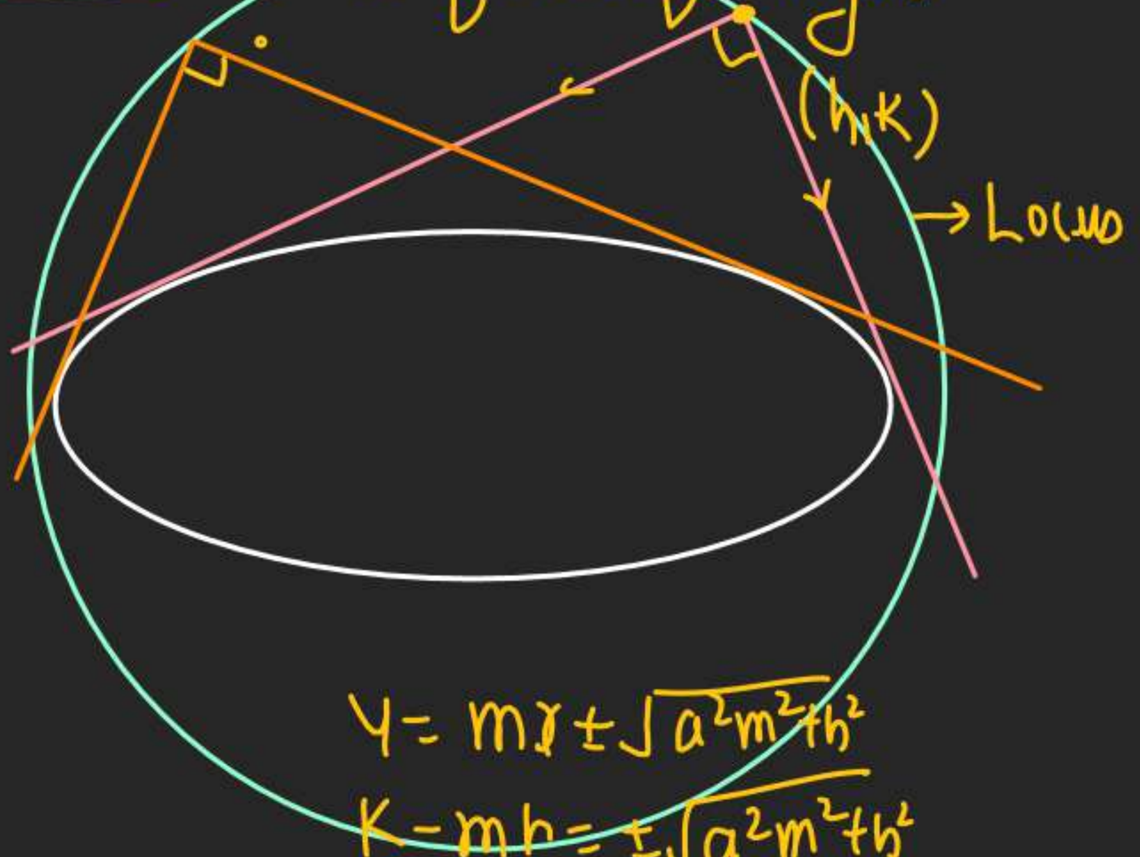
Director Circle → Locus of PoI of ⊥r tangent



$$y = mx \pm \sqrt{a^2m^2+b^2}$$

$$k - mh = \pm\sqrt{a^2m^2+b^2}$$

$$k^2 + m^2h^2 - 2mkh = a^2m^2 + b^2$$

$$m^2(h^2-a^2) - 2mkh + (k^2-b^2) = 0 \;<\; \begin{matrix} m_1 \\ m_2 \end{matrix}$$

$$\boxed{m_1+m_2 = \frac{2kh}{h^2-a^2}} \quad \boxed{m_1m_2 = \frac{k^2-b^2}{h^2-a^2}}$$

If (h, k) follows ⊥r tangents

$$m_1m_2 = -1$$

$$\frac{k^2-b^2}{h^2-a^2} = -1$$

$$h^2 - a^2 = +b^2 - k^2$$

$$h^2 + k^2 = a^2 + b^2$$

$$\boxed{x^2+y^2 = \left(\sqrt{a^2+b^2}\right)^2}$$

Eqn D.C.

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Q Find Dir. Circle of:

A) $\frac{x^2}{16}+\frac{y^2}{9}=1$

B) $\frac{(x-2)^2}{11}+\frac{(y+3)^2}{7}=1$

$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ का

D.C

$x^2+y^2=a^2+b^2$

D.C ⇒ $x^2+y^2=16+9$

$x^2+y^2=25$

(B) D.C : $(x-2)^2+(y+3)^2=11+7$

$(x-2)^2+(y+3)^2=18$

Q Find E.O.T. to Ell: $9x^2+16y^2=144$

P.T. $(2,3)$.

Position of $(2,3)$

$9\times4+16\times9-144>0 \Rightarrow (2,3)$ बाहर

$y=mx\pm\sqrt{16m^2+9}$ $(2,3)$

$3=2m\pm\sqrt{16m^2+9} \Rightarrow 3-2m=\pm\sqrt{16m^2+9}$

$\cancel{9}+4m^2-12m=16m^2+\cancel{9}$

$12m^2+12m=0$

$m=0,-1$

$m=0$

$(y-3)=0(x-2)$

$m=-1$

$(y-3)=-1(x-2)$

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① If $x+y=\lambda$ touches

$9x^2+16y^2=144$.

then $\lambda=?$

$y=-x+\lambda \Rightarrow m=-1,\ c=\lambda$

$E: \frac{x^2}{16}+\frac{y^2}{9}=1$

$c=\pm\sqrt{a^2m^2+b^2}$

$=\pm\sqrt{16\times 1^2+9}$

$\lambda=c=\pm 5$