Nishant Jindal

$$w = z + \frac{1}{z} \qquad |z| = a \cdot \quad a > 0, \neq 1 \cdot$$

$$h + ik = x_1 + iy_1 + \frac{x_1 - iy_1}{a^2}$$

$$h = x_1\left(1 + \frac{1}{a^2}\right), \quad k = y_1\left(1 - \frac{1}{a^2}\right)$$

$$\frac{h^2}{\left(1 + \frac{1}{a^2}\right)^2} + \frac{k^2}{\left(1 - \frac{1}{a^2}\right)^2} = a^2$$

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1. Let $z_1, z_2, z_3, \ldots z_n$ are complex no.'s s.t.

$|z_1| = |z_2| = |z_3| = \ldots = |z_n| = 1$. If $z = \left(\sum_{k=1}^{n} z_k\right)\left(\sum_{k=1}^{n} \frac{1}{z_k}\right)$

then P.T. z is real & $0 \leq z \leq n^2$.

$|z_1|^2 = z_1\bar{z}_1 = 1$

$(z_1 + z_2 + \ldots z_n)\left(\overline{z_1 + z_2 + \ldots + z_n}\right)$

$|z_1 + z_2| \leq |z_1| + |z_2|$

$|z_1 + (z_2 - z_n)| \leq |z_1| + |z_2 + (z_3 + \ldots z_n)|$

$= |z_1 + z_2 + \ldots + z_n|^2 \leq \left(|z_1| + \ldots |z_n|\right)^2 = n^2$.

$\leq |z_1| + |z_2| + \ldots + |z_n|$

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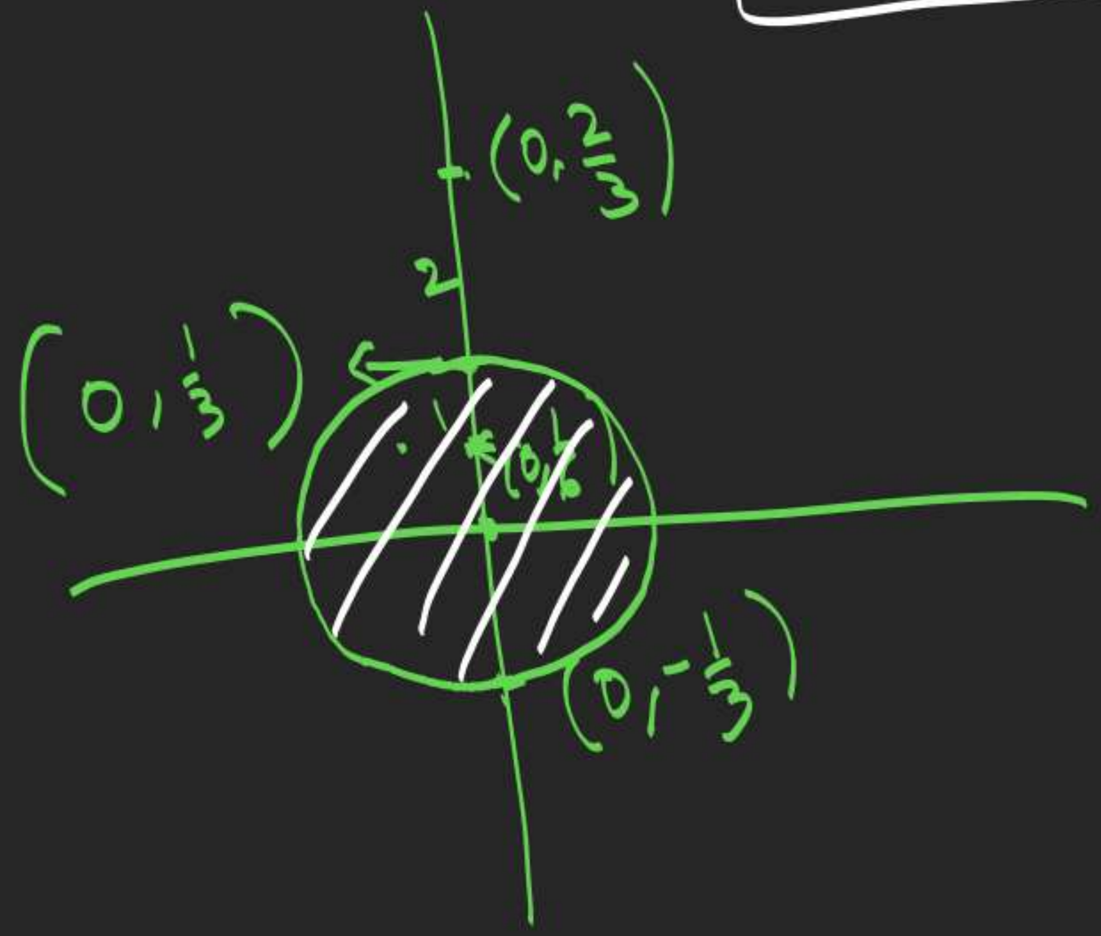
2. If $\left|\dfrac{6z-i}{2+3iz}\right| \le 1$, then P.T. $|z| \le \frac{1}{3}$.

$$|6z-i|^2 \le |2+3iz|^2 \Rightarrow (6z-i)(6\bar{z}+i) \le (2+3iz)(2-3i\bar{z})$$

$$27|z|^2 \le 3 \Rightarrow |z|^2 \le \frac{1}{9} \Rightarrow \boxed{|z| \le \frac{1}{3}}$$

$$\left|\frac{z-\frac{i}{6}}{z-\frac{2i}{3}}\right| \le \frac{1}{2}$$



3. Find the greatest and the least values of $|z|$ if z satisfies $\left|z - \frac{4}{z}\right| = 2$.

$$\left||z| - \frac{4}{|z|}\right| \le \left|z + \left(-\frac{4}{z}\right)\right| = 2 \le |z| + \frac{4}{|z|}$$

$$\left(\frac{|z|^2-4}{|z|}\right)^2 \le 4 \Rightarrow \left(|z|^2-4\right)^2 - 4|z|^2 \le 0$$

$$\left(|z|^2-2|z|-4\right)\left(|z|^2+2|z|-4\right) \le 0$$

$+$ $\quad -1-\sqrt{5}$ $\quad -$ $\quad 1-\sqrt{5}$ $\quad +$ $\quad -1+\sqrt{5}$ $\quad -$ $\quad 1+\sqrt{5}$ $\quad +$

$$|z| \in \left[\sqrt{5}-1,\ 1+\sqrt{5}\right]$$

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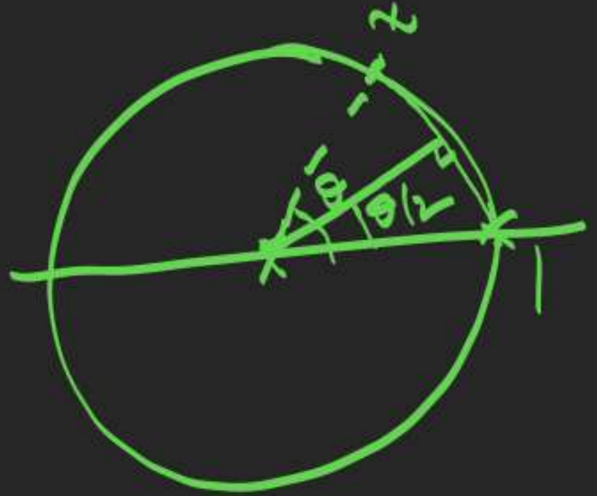
4. P.T. for all complex no.n 'z' with $|z|=1$,

$$\sqrt{2} \le |1-z| + |1+z^2| \le 4$$

$|1-z| =$ ($z = e^{i\theta}$)

$2\sin\frac{\theta}{2} +$



rem: Probability

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5. Let a, b, c be distinct non zero complex numbers with $|a| = |b| = |c|$. P.T. if a root of eqn. $az^2 + bz + c = 0$ has modulus equal to 1, then $b^2 = ac$.