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34. $x^2 + 2a\sqrt{a^2-3}\ x + 4 = 0$

$D = 0 \Rightarrow a^2(a^2-3) - 4 = 0$

$(a^2-4)(a^2+1) = 0$

$\boxed{a = \pm 2}$ ✓

39. i) $\boxed{a = 0}$ ✓ $0.5 > 0 \quad \forall x \in R$

~~OR~~

$a \neq 0$ $a > 0$ & $D < 0$

$a^2 - 3 \geq 0$

$4a^2 - 2a < 0$

$\boxed{a \in \left(0, \frac{1}{2}\right)}$ ✓

Ans $\boxed{a \in \left[0, \frac{1}{2}\right)}$

38. ~~$4 = 0$~~ $\boxed{D < 0}$ ✓

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43.

$D \geq 0.$

$$(2^a - 1)^2 + 12(4^{a-1} - 2^{a-2}) \geq 0.$$

$$4 \cdot 4^a - 5 \cdot 2^a + 1 \geq 0$$

$$4t^2 - 5t + 1 \geq 0$$

$$-4t - t$$

$$(4t-1)(t-1) \geq 0$$

$$t \in \left(-\infty, \frac{1}{4}\right] \cup [1, \infty)$$

$$2^a \in \left(0, \frac{1}{4}\right] \cup [1, \infty)$$

$2^{-\infty}$, 2^{-2}, 2^0, 2^∞

$$\boxed{0 < 2^a \leq \frac{1}{4}}$$

$$2^{-\infty} < 2^a \leq 2^{-2}$$

$$\boxed{a \in (-\infty, -2] \cup [0, \infty)}$$

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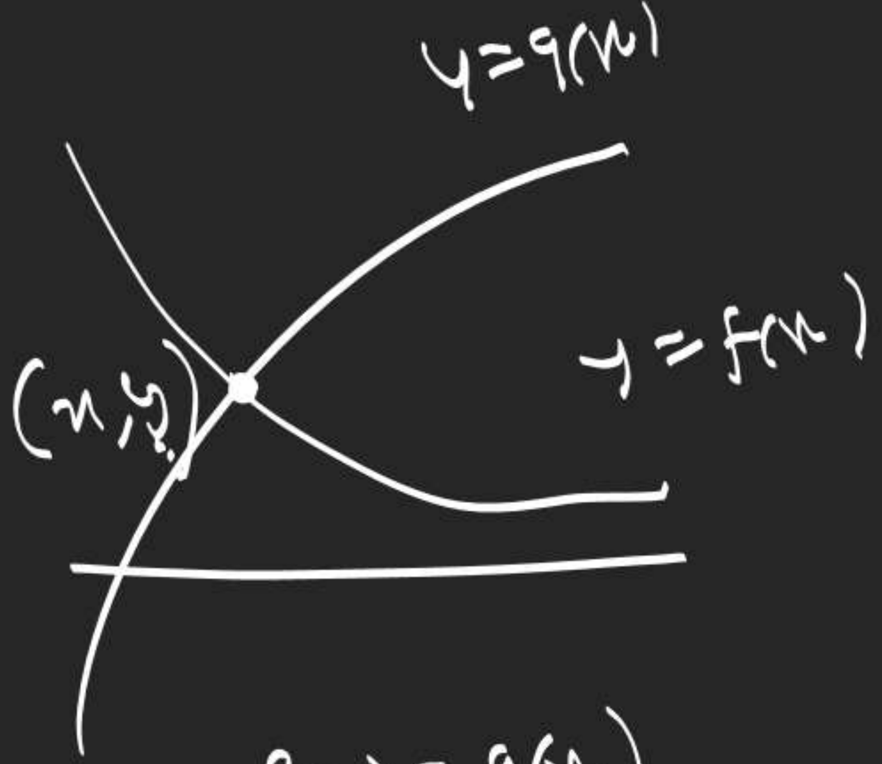
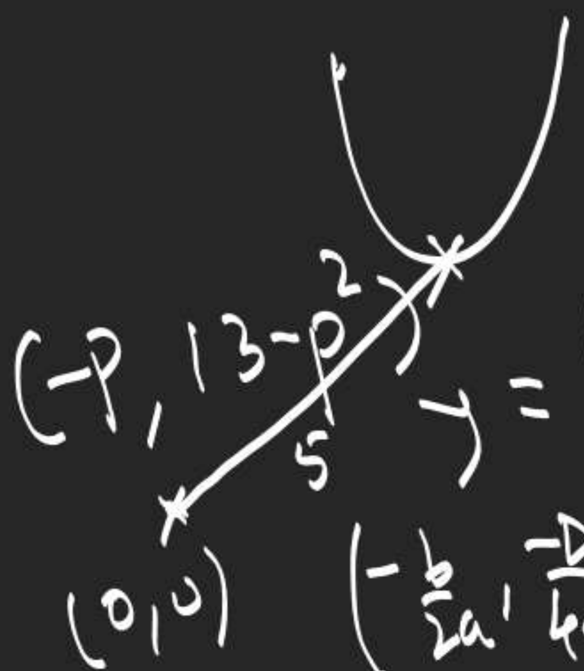
44.



$f(x)=g(x)$



$y=(x+p)^2+13-p^2$

$\left(-\frac{b}{2a}, \frac{-D}{4a}\right)$

$2ax+1=(a-6)x^2-2$

$(a-6)x^2-2ax-3=0$

$D<0$

$25=p^2+(13-p^2)^2$

$0=p^4-25p^2+144$

$(p^2-9)(p^2-16)=0$

$\boxed{p=\pm 3, \pm 4}$

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$\underline{x = 3+\sqrt{5}}$,

$x^4 - 12x^3 + 44x^2 - 49x + 17$.

$= (x^2-6x+4)(x^2-6x+4) - x + 1$

$= 0 - (3+\sqrt{5}) + 1$

$= -2 - \sqrt{5}$

$x^2 - 6x + 4 = 0$

$x - 3 = \sqrt{5}$

$x^2 - 6x + 9 = 5$

$x^2-6x+4 \overline{\big)\, x^4 \ldots}$

$\boxed{5 = 2(2) + 1}$

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1. If $p(q-r)x^2 + q(r-p)x + r(p-q) = 0$ has equal roots, then P.T. $\frac{2}{q} = \frac{1}{p} + \frac{1}{r}$.

$x = 1$ satisfy

$$\frac{r(p-q)}{p(q-r)} = 1$$

$$2pr = qr + pq$$

$$\boxed{\frac{2}{q} = \frac{1}{p} + \frac{1}{r}}$$

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Q. If $f(x) = ax^2 + bx + c > 0 \ \forall x \in R$, then P.T.

$g(x) = f(x) + f'(x) + f''(x) > 0 \ \forall x \in R$.

If $a = 0$

$bx + c > 0 \ \forall x \in R$

$b = 0$

$\boxed{c > 0}$

$f(x) = c$

$g(x) = c$

$g(x) = \overbrace{f(x+1)}^{>0} + \overbrace{a}^{\geq 0} > 0$

$\boxed{g(x) = f(x+1) + a}$

$0 > 0 \ \forall x \in R$

$a > 0, \ D < 0 \Rightarrow \boxed{a > 0 \ \& \ b^2 - 4ac < 0}$

$= (a(x+1)^2 + b(x+1) + c) + a$

$-2x + 3 > 0$

$\boxed{x < 3/2}$

$g(x) = \underline{ax^2} + bx + c + \underline{2ax} + b + a = ax^2 + (b+2a)x + (c+b+2a)$

$+ a$

$a > 0, \quad D = (b+2a)^2 - 4a(c+b+2a)$

$2x > 0$

$\boxed{g(x) > 0 \ \forall x \in R}$

$= b^2 - 4ac - 4a^2$

$= (\underbrace{b^2 - 4ac}_{<0}) + (\underbrace{-4a^2}_{<0}) < 0$

Nishant Jindal

Condition for $ax^2+bx+c=0$ to have more than 2 roots $a,b,c \in R$

Identity

$\boxed{a=b=c=0}$ ✓

$(x-2)(x-i)(x+i)$

$0=0$

↓

Infinite

$x \in R$

$(x-1)(x-2)(x-3)(x-4)$

Nishant Jindal

Condition for $a_n x^n + a_{n-1} x^{n-1} + \cdots + a_2 x^2 + a_1 x + a_0 = 0 \quad a_0, a_1, a_2, \ldots a_n \in R.$

to have more than n solutions.

$$a_0 = a_1 = a_2 = a_3 = \ldots = a_{n-1} = a_n = 0$$

Nishant Jindal

1. Find 'p' for which equation $(p+2)(p-1)x^2 + (p-1)(2p+1)x + p^2-1=0$ has more than two roots.

$$\boxed{p=1}$$

Nishant Jindal

Q. Solve for x

(i) $\dfrac{(x-a)(x-b)}{(c-a)(c-b)} + \dfrac{(x-b)(x-c)}{(a-b)(a-c)} + \dfrac{(x-c)(x-a)}{(b-c)(b-a)} = 1$

$\boxed{x \in R}$

$x = a$.

$x = b$

$x = c$

(ii) $\dfrac{a^2(x-b)(x-c)}{(a-b)(a-c)} + \dfrac{b^2(x-c)(x-a)}{(b-c)(b-a)} + \dfrac{c^2(x-a)(x-b)}{(c-a)(c-b)} = x^2$.

$\boxed{x \in R}$

$\cancel{\alpha} x^2 + \beta x + \gamma = 0$

$0 = 0$

a b c

Nishant Jindal

Q: Find the (i) sum of squares
(ii) sum of cubes of
the roots of equation $x^3-px^2+qx-r=0$

$x^3-px^2+qx-r=0 \;<\; \alpha, \beta, \gamma$ (i) $\alpha^2+\beta^2+\gamma^2$

$= (\alpha+\beta+\gamma)^2-2(\alpha\beta+\beta\gamma+\gamma\alpha)$

$= p^2-2q$

(ii) $\alpha^3+\beta^3+\gamma^3-3\alpha\beta\gamma = \underline{(\alpha+\beta+\gamma)}(\underbrace{\alpha^2+\beta^2+\gamma^2}-\alpha\beta-\beta\gamma-\gamma\alpha)$

$\alpha^3+\beta^3+\gamma^3-3r = p(p^2-2q-q)$

$\alpha^3+\beta^3+\gamma^3 = p^3-3pq+3r$

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Q: If a, b, c are roots of cubic $x^3 - x^2 + 1 = 0$, find the value of $\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2}$.

$$= \frac{a^2b^2 + b^2c^2 + c^2a^2}{(abc)^2} = \frac{(ab+bc+ca)^2 - 2(ab^2c + bc^2a + a^2bc)}{(abc)^2}$$

$$= \frac{(ab+bc+ca)^2 - 2abc(a+b+c)}{(abc)^2}$$

$$= \frac{0 - 2(-1)(1)}{(-1)^2} = 2.$$

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3: If a, b, c are roots of $x^3 - x^2 + 1 = 0$,

find (i) $(a+2)(b+2)(c+2)$

$x^3 - x^2 + 1 = (x-a)(x-b)(x-c)$

Put $x = -2$

(ii) $(a^2-4)(b^2-4)(c^2-4)$

$-8 - 4 + 1 = (-2-a)(-2-b)(-2-c)$

$11 = (a+2)(b+2)(c+2)$

$\big((a-2)(b-2)(c-2)\big)\big((a+2)(b+2)(c+2)\big)$

$abc + 2(ab+bc+ca) + 4(a+b+c) + 8$

$= -1 + 2(0) + 4(1) + 8 = 11$

$= (-5)(11) = \boxed{-55}$

Put $x = 2$, $8 - 4 + 1 = (2-a)(2-b)(2-c)$

Nishant Jindal

H.W

Hall & Knight → Examples - IX (a)

$\boxed{7, 10, 13-29}$

17: a, b, c are rational number.