Nishant Jindal

QUADRATIC EQUATION

Q If $a<0$ & $D<0$ then for $f(x)=ax^2+bx+c$

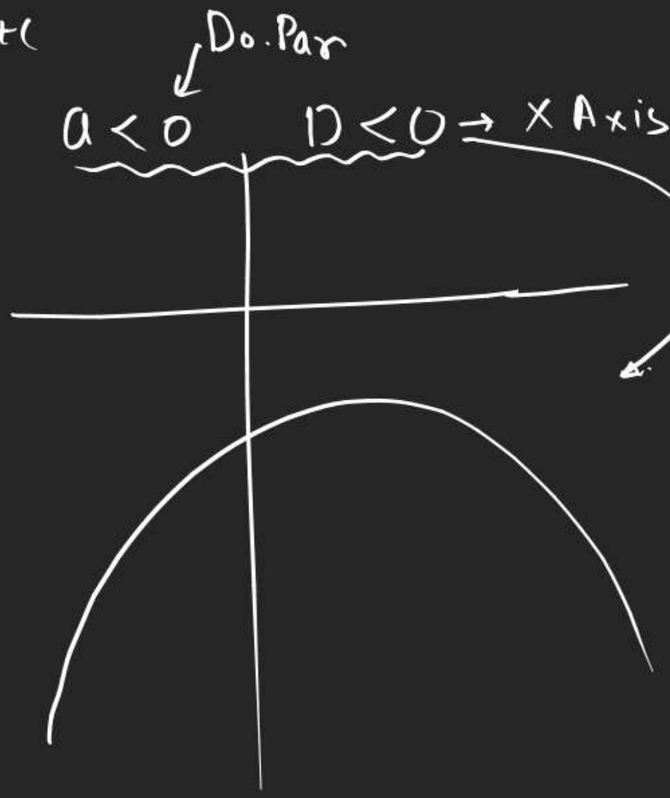
(1) Sign of $a+b+c$:- -ve

(2) Sign of $a-b+c$:- -ve

(3) Sign of c :- -ve

(4) Sign of $4a+2b+c$:- ?

:- -ve



1) as graph is Completely Below X Axis

$\Rightarrow f(x)=ax^2+bx+c<0$

2) $f(0)=0+0+c<0$

$\boxed{c<0}$

3) $f(1)=a+b+c<0$

4) $f(-1)=a(-1)^2+b(-1)+c<0$ ($x=-1$)

$=a-b+c<0$

(5) $4a+2b+c<0$ ($x=2$)

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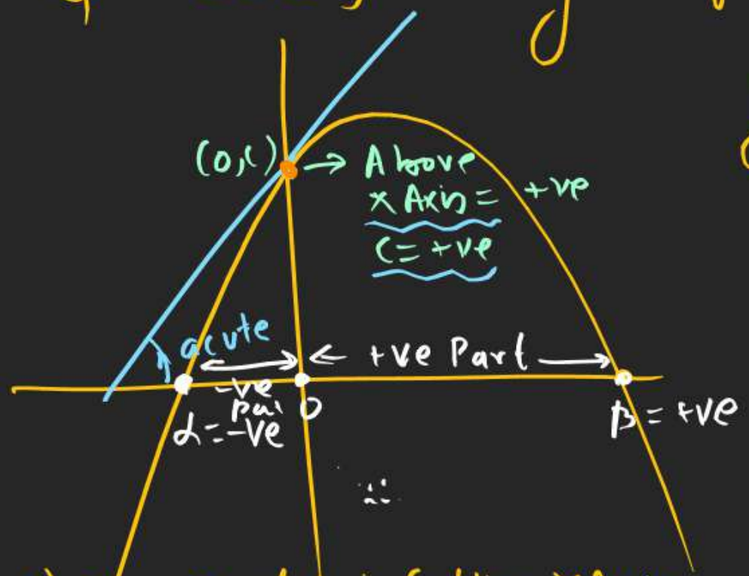
QUADRATIC EQUATION

Q See following Graph.



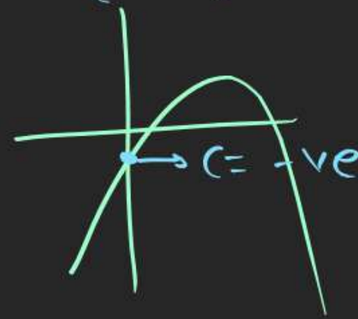
$f(x) = ax^2 + bx + c$.

$f'(x) = 2ax + b$.

$f'(0) = b$ → y Axis पर graph का slope = b.

$\tan\theta = b$

$\tan(\text{acute}) = +ve = b$



$x = 0$

for $f(x) = ax^2 + bx + c$

Sign of

1) a 2) B 3) C

4) D 5) POR 6) SOR

4) D = graph is Cutting XAxis at 2 distinct Point

=) ① D = +ve ② Roots = Real & Distinct

(5) Prod of Root = $\alpha \cdot \beta = \frac{c}{a} = \frac{\oplus}{\ominus} = -ve$

(6) SOR of Root = α is more closer to origin as compare to β = +ve

1) Downward Parabola → a < 0

2) Sign of b → "make tangent at Pt. where graph is Intersecting YAxis (2) If tangent is making acute Angle => b⊕ (3) If tangent is making obtuse angle then b = -ve

(3) c = f(0) → y Axis पर fxn Cutting above x Axis = +ve

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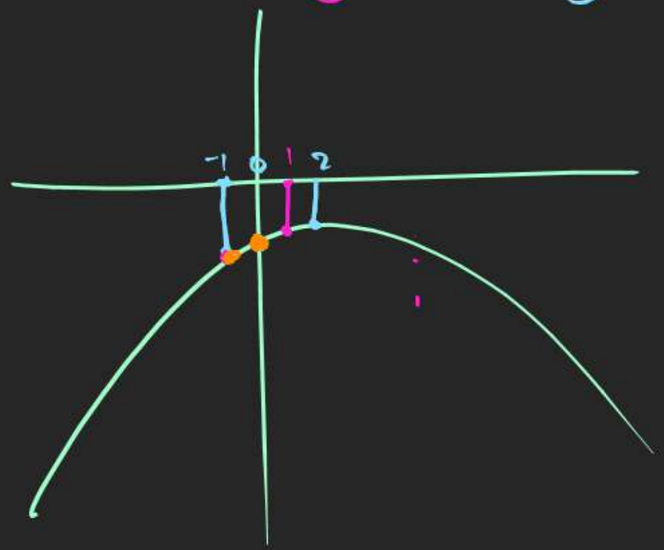
QUADRATIC EQUATION

Q If trinomial $f(x)=ax^2+bx+c$ has no Real Zeroes & $a+b+c<0$ then Sign of

$f(1)<0$

1) c 2) $a-b+c$ 3) $4a+2b+c$

4) $a^2+c^2-b^2+2ac$ (+) (5) $4a-b+c=?$ (−)



यहाँ तो सब +ve हो!

$f(1)<0$ इसी में Psbl ✓

(1) $c=f(0)=$ Below X Axis $=-ve$

(2) $a-b+c=f(-1)=-ve$

(3) $4a+2b+c=f(2)=-ve$

(4) $a^2+c^2-b^2+2ac=(a^2+c^2+2ac)-b^2$

$=(a+c)^2-b^2=(a+c-b)(a+c+b)$

$=(a+b+c)(a-b+c)$

$=f(1)\,f(-1)=-ve\times-ve=+ve$

(5) $4a-b+c=3a+(a-b+c)$

$=3a+f(-1)$

$=-ve+-ve$

$=-ve$

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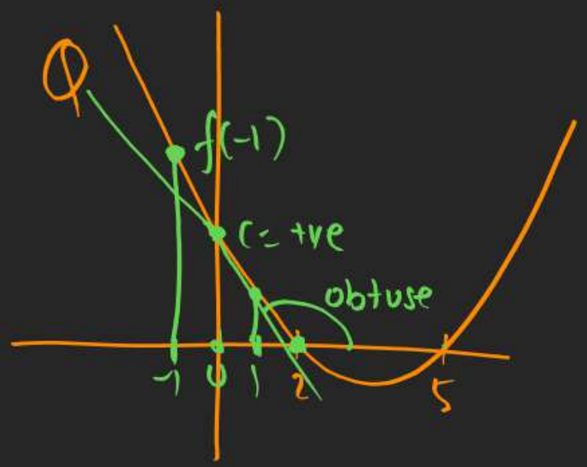
QUADRATIC EQUATION



Sign of

1) a = +ve
2) b = -ve
3) c = +ve
4) D = +ve
5) SOR = $\frac{7}{1}$ = +ve
6) POR = $\frac{10}{1}$ = +ve
7) $a - b + c = f(-1)$ = +ve
8) $a + b + c = f(1)$ = +ve
9) $4a + 2b + c = f(2) = 0$

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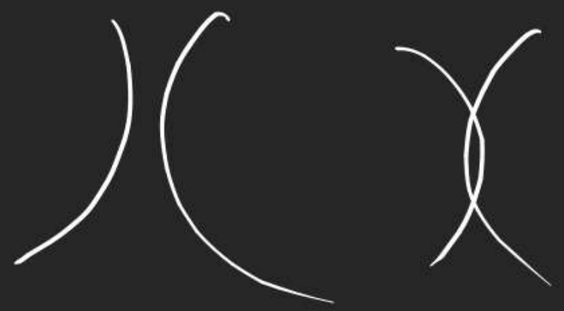
QUADRATIC EQUATION

Q Find value of a for which graph of $y = 2ax + 1$ & $y = (a-6)x^2 - 2$ does not Intersect



1) If 2 fxn are not Intersecting then Both fxn do not has Real solution.

2) Solving Both Eqn i will not give any Real Sol.

$$2ax + 1 = (a-6)x^2 - 2$$

$$(a-6)x^2 - 2ax - 3 = 0 \quad \text{Do not have Real Roots}$$

$$D < 0$$

$$(-2a)^2 + 4 \times (a-6)(+3) < 0$$

$$4a^2 + 12a - 72 < 0 \Rightarrow a^2 + 3a - 18 < 0 \Rightarrow (a+6)(a-3) < 0$$

$$-6 < a < 3$$

$\therefore a \in (-6, 3)$

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QUADRATIC EQUATION

Q Find K for which $x^2-2(4K+1)x+(15K^2-2K-7)>0$

$Ax^2+ \quad Bx+ \quad C \quad >0$

1) $A=1>0$

2) $D<0$ Hona chaiye

$$(-2(4K+1))^2-4(1)(15K^2-2K-7)<0$$

$$4(4K+1)^2-4(15K^2-2K-7)<0$$

$$(4K+1)^2-(15K^2-2K-7)<0$$

$$16K^2+8K+1-15K^2+2K+7<0$$

$$K^2+6K+8<0$$

$$(K+2)(K+4)<0$$

$$-4<K<-2 \rightarrow K\in(-4,-2)$$

Concept

If $ax^2+bx+c>0$ then $a>0$ $D<0$

If $ax^2+bx+c<0$ then $a<0$ $D<0$

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QUADRATIC EQUATION

Q If $mx^2-(m+1)x+(2m-1)=0$ has no Real Roots then $m \in ?$

no Real Roots $\rightarrow D<0$

$(-(m+1))^2-4\times m(2m-1)<0$

$m^2+2m+1-8m^2+4m<0$

$-7m^2+6m+1<0$

$7m^2-6m-1>0$

$7m^2-7m+m-1>0$

$7m(m-1)+1(m-1)>0$

$(7m+1)(m-1)>0$

$m<-\frac{1}{7} \cup m>1 \Rightarrow m \in (-\infty,-\frac{1}{7}) \cup (1,\infty)$

m Bde ki Bdq
m chhote ki chhota

Q If

$(\lambda^2+\lambda-2)x^2+(\lambda+2)x<1 \quad \forall x \in R$ then $\lambda \in ?$

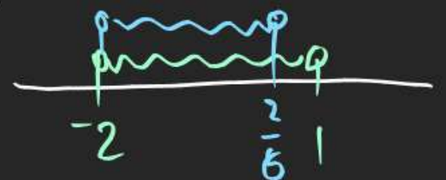
$(\lambda^2+\lambda-2)x^2+(\lambda+2)x-1<0$

Given $Ax^2+Bx+C<0 \rightarrow a<0, D<0$

① $\lambda^2+\lambda-2<0$

$(\lambda+2)(\lambda-1)<0$

$-2<\lambda<1$



② $D<0$

$(\lambda+2)^2+4(\lambda^2+\lambda-2)(+1)<0$

$\lambda^2+4\lambda+4+4\lambda^2+4\lambda-8<0$

$5\lambda^2+8\lambda-4<0$

$5\lambda^2+10\lambda-2\lambda-4<0$

$(5\lambda-2)(\lambda+2)<0 \Rightarrow -2<\lambda<\frac{2}{5}$

$\lambda \in (-2,\frac{2}{5})$

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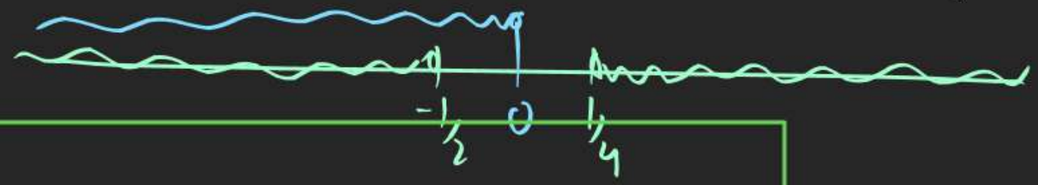
QUADRATIC EQUATION

Q Range of a for which

$$f(x)=\frac{ax^2+2(a+1)x+9a+4}{x^2-8x+32} \text{ is always -ve.}$$

Ans → $a \in (-\infty, -\frac{1}{2})$



$$\frac{ax^2+2(a+1)x+(9a+4)}{\underbrace{(x^2-8x+32)}_{+ve}}<0$$

Here

$x^2-8x+32 \rightarrow a=1>0,\ b=-8,\ c=32$

$D=(-8)^2-4\times1\times32$

$=64-128$

D = -ve

$a>0$ & $D<0$ then

$x^2-8x+32>0$

$\Rightarrow ax^2+2(a+1)x+(9a+4)<0$

(general $Ax^2+Bx+C<0$, $a<0, b<0$)

1) $a<0$ (Ans 1)

2) $D<0$

$4(a+1)^2-4a(9a+4)<0$

$a^2+2a+1-9a^2-4a<0$

$-8a^2-2a+1<0 \Rightarrow 8a^2+2a-1>0$

$8a^2+4a-2a-1>0$

$(4a-1)(2a+1)>0$

$a<-\frac{1}{2} \cup a>\frac{1}{4}$

$a \in (-\infty, -\frac{1}{2}) \cup (\frac{1}{4}, \infty)$ (Ans 2)

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QUADRATIC EQUATION

Q Value of a for which

$(a+4)x^2 - 2ax + 2a - 6 < 0 \quad \forall x \in R.$

$A \quad x^2 + \quad Bx + C < 0$

$A < 0$ and $D < 0$

$a + 4 < 0$

$a < -4$

$B^2 - 4AC < 0$

$(-2a)^2 - 4(a+4)(2a-6) < 0$

$4a^2 - 4(2a^2 + 2a - 24) < 0$

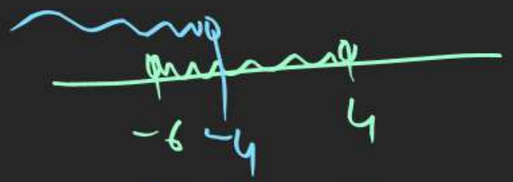
$a^2 - (2a^2 + 2a - 24) < 0$

$-a^2 - 2a + 24 < 0$

$a^2 + 2a - 24 < 0$

$(a+6)(a-4) < 0$

$-6 < a < 4$



$a \in (-6, -4)$

Ex 1 Q 1 - 18

Ex 5 → Q 8, 9