Nishant Jindal

1. $$\frac{\sin\frac{\theta}{2}}{\cancel{\cos\frac{\theta}{2}}}\overbrace{\left(\frac{1+\cos\theta}{\cos\theta}\right)}^{2\cos^2\theta/2}\overbrace{\left(\frac{1+\cos 2\theta}{\cos 2\theta}\right)}^{2\cos^2\theta}\overbrace{\left(\frac{1+\cos 4\theta}{\cos 4\theta}\right)}^{2\cos^2 2\theta}\overbrace{\left(\frac{1+\cos 8\theta}{\cos 8\theta}\right)}^{2\cos^2 4\theta}\cdots\overbrace{\left(\frac{1+\cos 2^n\theta}{\cos 2^n\theta}\right)}^{2\cos^2 2^{n-1}\theta}$$

$$= \frac{\sin\frac{\theta}{2}\left(2\cos\frac{\theta}{2}\right)\left(2\cos\theta\right)\left(2\cos 2\theta\right)\left(2\cos 4\theta\right)\cdots\left(2\cos 2^{n-1}\theta\right)}{\cos 2^n\theta}$$

$$= \frac{\sin 2^n\theta}{\cos 2^n\theta} = \tan 2^n\theta = f_n(\theta)$$

4. $\alpha+\beta=\frac{\pi}{2}$

$\tan\alpha = \tan\left(\frac{\pi}{2}-\beta\right) = \cot\beta$.

$\beta+\gamma=\alpha$

$\gamma = \alpha-\beta$

$\tan\gamma = \frac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta} = \frac{\tan\alpha-\tan\beta}{2}$

(b) $\frac{\cot x\cot y+1}{\cot y-\cot x}=1$

$\boxed{\cot x(2-\cot x)+1 = 2-2\cot x}$

$a^m a^n = a^{m+n}$

$(a^m)^n = a^{mn}$

3. (a) 3

$3\cos 2x + 4\sin 2x$

5. $\frac{\pi}{2} < \theta+\phi < \frac{2\pi}{3}$

$\theta = \frac{\pi}{6}$

$\cos\phi = \frac{1}{3}$

$0 < \frac{1}{3} < \frac{1}{2}$

$0 \quad \frac{\pi}{3} \quad \frac{\pi}{2}$

$\boxed{\frac{\pi}{3} < \phi < \frac{\pi}{2}}$

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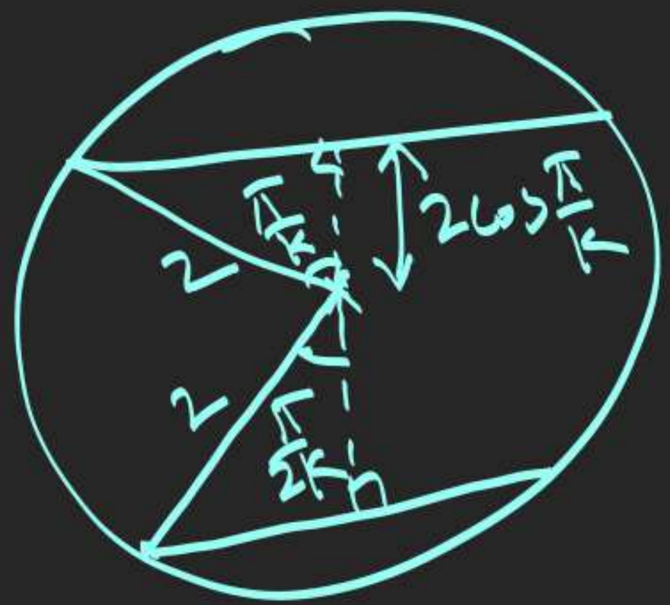
$$\frac{1}{1+2(1+\cos 2\theta)+\frac{3}{2}\sin 2\theta} = \frac{2}{6+(4\cos 2\theta+3\sin 2\theta)} \le \frac{2}{6-5}$$



$$\left(\cos\frac{\pi}{k}+\boxed{\cos\frac{\pi}{2k}}^{\,t}\right) = \frac{\sqrt{3}}{2}+\frac{1}{2}$$

$$\boxed{k=3}$$

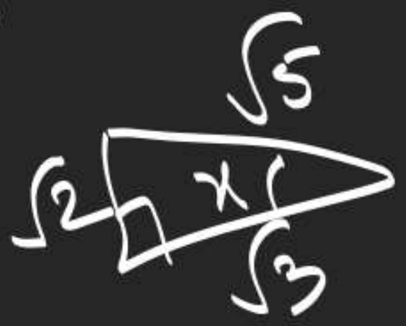
$$2t^2-1+t = \frac{\sqrt{3}}{2}+\frac{1}{2}.$$

$$\frac{1}{2}\tan^4 x + \frac{1}{3} = \frac{1}{5}\sec^4 x = \frac{1}{5}\left(1+\tan^2 x\right)^2$$

$$5\left(3\tan^4 x + 2\right) = 6\left(1 + 2\tan^2 x + \tan^4 x\right)$$

$$9\tan^4 x - 12\tan^2 x + 4 = 0$$

$$\left(3\tan^2 x - 2\right)^2 = 0$$



$$\tan^2 x = \frac{2}{3}$$

$$\sin^2 x = \frac{2}{5}$$

$$\cos^2 x = \frac{3}{5}$$

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7: $\left(0, \frac{\pi}{4}\right)$

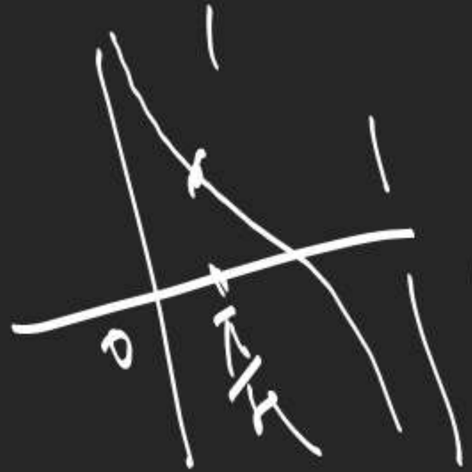
$0 < \tan\theta < 1$

$\cot\theta > 1$

$$(\tan\theta)^{\cot\theta} < (\tan\theta)^{\tan\theta} < (\cot\theta)^{\tan\theta} < (\cot\theta)^{\cot\theta}$$

$$(\tan\theta)^{\tan\theta} > (\tan\theta)^{\cot\theta}$$

$$2^{-1} < 2^{0} < 2^{1} < 2^{3/2} < 2^{2}$$



$$\left(\frac{1}{2}\right)^{-1} > \left(\frac{1}{2}\right)^{0} > \left(\frac{1}{2}\right)^{1} > \left(\frac{1}{2}\right)^{3/2} > \left(\frac{1}{2}\right)^{2} >$$

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$a>0,\ a\neq 1,\ m\in\mathbb{R}.$

$a^m > 1$

$a^m < 1$

If $0<a<1$, $a^m>1$, $m<0$
$a^m<1$, $m>0$
$a^m=1$, $m=0$

If $a>1$, $a^m>1$ if $m>0$
$a^m<1$ if $m<0$
$a^m=1$ if $m=0$

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If $m > n$, $a^m > a^n$

$a > 1$

$\boxed{m > n}$

$a^{m-n} > 1$

$0 < a < 1$

If $m > n$, $a^m < a^n$.

$a^{m-n} < 1$

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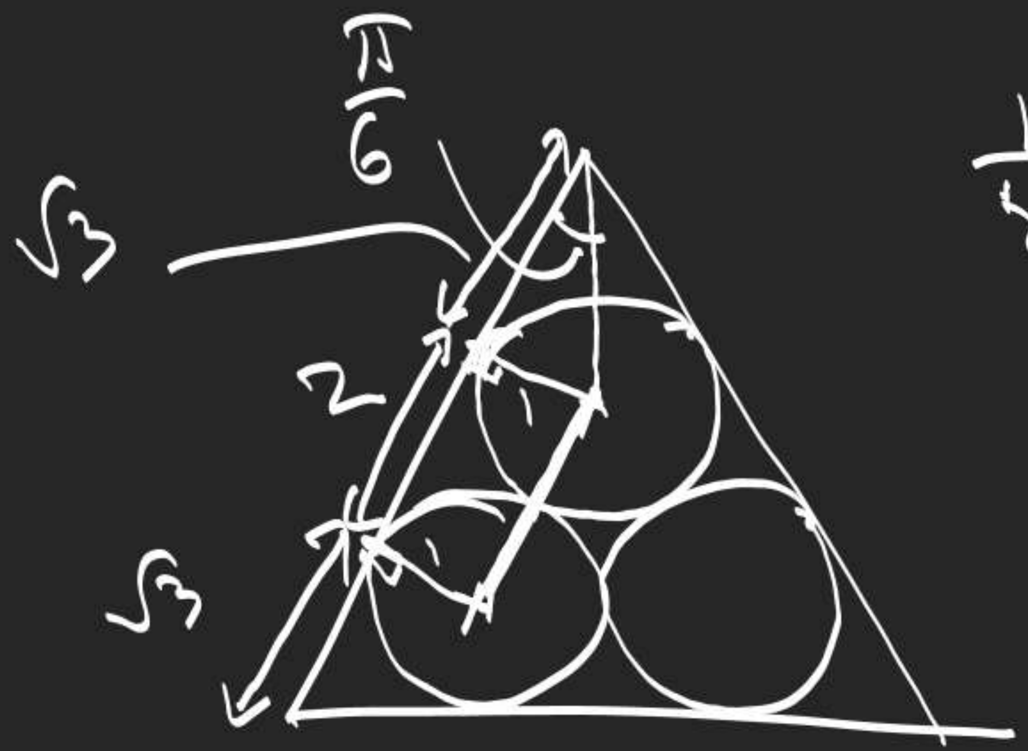
Q.



$$\frac{1}{x} = \tan\frac{\pi}{6} = \frac{1}{\sqrt{3}}$$

$$2 + 2\sqrt{3}$$

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$$\frac{1}{\sin\frac{\pi}{6}}\sum_{k=1}^{13}\frac{\sin\left(\left(\frac{\pi}{4}+\frac{k\pi}{6}\right)-\left(\frac{\pi}{4}+(k-1)\frac{\pi}{6}\right)\right)}{\sin\left(\frac{\pi}{4}+(k-1)\frac{\pi}{6}\right)\sin\left(\frac{\pi}{4}+\frac{k\pi}{6}\right)}$$

$$\frac{12}{}$$

$$2\left(1-\left(2-\sqrt{3}\right)\right)$$

$$2\pi+\frac{\pi}{6}$$

$$2\sum_{k=1}^{13}\left(\cot\left(\frac{\pi}{4}+(k-1)\frac{\pi}{6}\right)-\cot\left(\frac{\pi}{4}+\frac{k\pi}{6}\right)\right)$$

$$2\left(1-\cot\left(\frac{\pi}{4}+\frac{13\pi}{6}\right)\right)=2$$

$$=2\left(1-\cot\left(\frac{\pi}{4}+\frac{\pi}{6}\right)\right)$$

$$\left\{\left(\cot\frac{\pi}{4}-\cancel{\cot\left(\frac{\pi}{4}+\frac{\pi}{6}\right)}\right)+\left(\cancel{\cot\left(\frac{\pi}{4}+\frac{\pi}{6}\right)}-\cancel{\cot\left(\frac{\pi}{4}+\frac{2\pi}{6}\right)}\right)\right.$$

$$\left.+\left(\cancel{\cot\left(\frac{\pi}{4}+\frac{2\pi}{6}\right)}-\cot\left(\frac{\pi}{4}+\frac{3\pi}{6}\right)\right)+\cdots\left(\cancel{\cot\left(\frac{\pi}{4}+\frac{12\pi}{6}\right)}-\cot\left(\frac{\pi}{4}+\frac{13\pi}{6}\right)\right)\right\}$$

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$$2\cos\beta - \underline{2\cos\alpha} + \underline{\cos\alpha\cos\beta} = 1$$

$$(\cos\alpha + 2)(\cos\beta - 2) = 1 - 4 = -3$$

$$(2+\cos\alpha)(2-\cos\beta) = 3$$

$$\alpha, \beta, \alpha\beta \left(2 + \frac{1-\tan^2\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}}\right)\left(2 - \frac{1-\tan^2\frac{\beta}{2}}{1+\tan^2\frac{\beta}{2}}\right) = 3$$

$$3\tan^2\frac{\beta}{2} = \tan^2\frac{\alpha}{2}$$

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14.

$$\frac{\sum_{k=0}^{n} \cos\frac{\pi}{n+2} - \sum_{k=0}^{n} \cos\frac{(2k+3)\pi}{n+2}}{\sum_{k=0}^{n} 1 - \sum_{k=0}^{n} \cos\frac{(2k+2)\pi}{n+2}}$$

$$\sum_{r=1}^{n} (\sin r\theta - \cos r\theta)$$

$$= (\sin\theta - \cos\theta) + (\sin 2\theta - \cos 2\theta) + \cdots + (\sin n\theta - \cos n\theta)$$

$$= (\sin\theta + \sin 2\theta + \cdots \sin n\theta) - (\cos\theta + \cos 2\theta + \cdots + \cos n\theta)$$

$$= \sum_{r=1}^{n} \sin r\theta - \sum_{r=1}^{n} \cos r\theta$$

$$\sum_{k=0}^{n} \cos\frac{\pi}{n+2}$$

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$$\sum_{k=0}^{n}\left(\cos\frac{\pi}{n+2}-\cos\frac{(3k+3)\pi}{n+2}\right) = (n+1)\cos\frac{\pi}{n+2}$$

$$\left(\cos\frac{\pi}{n+2}-\cos\frac{3\pi}{n+2}\right)+\left(\cos\frac{\pi}{n+2}-\cos\frac{6\pi}{n+2}\right)+\left(\cos\frac{\pi}{n+2}-\cos\frac{9\pi}{n+2}\right)+$$

$$\cdots+\left(\cos\frac{\pi}{n+2}-\cos\frac{(3n+3)\pi}{n+2}\right)$$

$$=(n+1)\cos\frac{\pi}{(n+2)}-\left(\cos\frac{3\pi}{n+2}+\cos\frac{6\pi}{n+2}+\cos\frac{9\pi}{n+2}+\cdots+\cos\frac{(3n+3)\pi}{n+2}\right)$$

$$=(n+1)\cos\frac{\pi}{n+2}-\frac{\sin\frac{(n+1)3\pi}{2(n+2)}}{\sin\frac{3\pi}{2(n+2)}}\cos\left(\frac{(3n+6)\pi}{2(n+2)}\right) \rightarrow \frac{3\pi}{2}$$

$$=\boxed{(n+1)\cos\frac{\pi}{n+2}}$$

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$$\sum_{k=0}^{n}\left(1-\cos\frac{(2k+2)\pi}{(n+2)}\right)$$

$$=\left(1-\cos\frac{2\pi}{n+2}\right)+\left(1-\cos\frac{4\pi}{n+2}\right)+\left(1-\cos\frac{6\pi}{n+2}\right)+\cdots\left(1-\cos\frac{(2n+2)\pi}{n+2}\right)$$

$$\pi-\frac{\pi}{n+2} \qquad\qquad \pi$$

$$=(n+1)-\frac{\sin\frac{(n+1)\pi}{(n+2)}}{\sin\frac{\pi}{n+2}}\cos\left(\frac{(2n+4)\pi}{2(n+2)}\right)$$

$$=n+1-(-1)$$

$$=n+2.$$

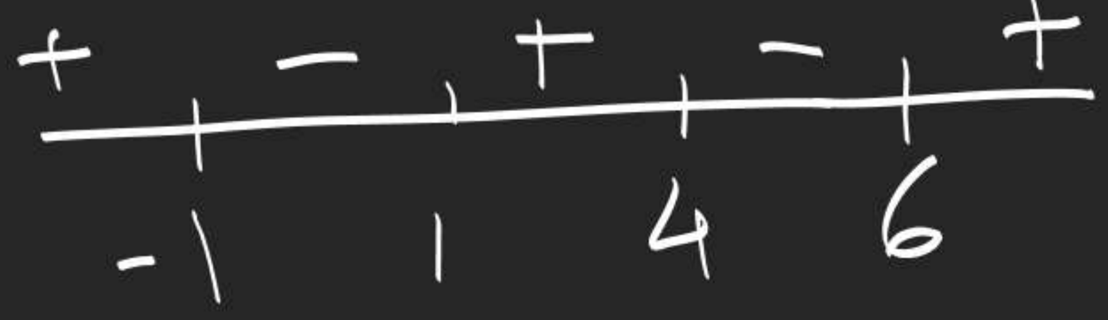
Nishant Jindal

1. $$\frac{14x}{x+1} - \frac{9x-30}{x-4} < 0$$

$x = ?$

$$\frac{14x(x-4) - (9x-30)(x+1)}{(x+1)(x-4)} < 0$$

$$\boxed{x \in (-1, 1) \cup (4, 6)}$$



$$\frac{5x^2 - 35x + 30}{(x+1)(x-4)} < 0$$

$$\frac{x^2 - 7x + 6}{(x+1)(x-4)} < 0 \Rightarrow \frac{(x-1)(x-6)}{(x+1)(x-4)} < 0$$

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2.

$$\frac{20}{(x-3)(x-4)} + \frac{10}{x-4} + 1 > 0$$

$$\frac{20 + 10(x-3) + (x-3)(x-4)}{(x-3)(x-4)} > 0$$

$$\Rightarrow \quad \frac{x^2 + 3x + 2}{(x-3)(x-4)} > 0 \Rightarrow \frac{(x+1)(x+2)}{(x-3)(x-4)} > 0$$

$$\boxed{x \in (-\infty,-2) \cup (-1,3) \cup (4,\infty)}$$