22.

$$\log_a^2(xyz) = 144$$

$$\log_a(xyz) = \pm \boxed{12}$$

$$\log_a x = 4 \quad , \quad \log_a y = 1 \quad , \quad \log_a z = 7$$

$$\log_a x = -4 \quad , \quad \log_a y = -1 \quad , \quad \log_a z = -7$$

$$\log_y x = 3, \tfrac{1}{3}$$

$$x = y^3 \quad , \quad x = y^{1/3}$$

$$\text{antilog}_{2401} 0.75 = (2401)^{3/4}$$

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3. $\left(2^{\log_2(\ln x)}\right)^2 - 1 + \ln^3 x - 3\ln^2 x - 5\ln x + 7$.

cyphers = zeroes

$-2\ln^2 x + \ln^3 x - 5\ln x + 6 = 0$

$(t-1)(t^2 - t - 6) = 0$

$(t-1)(t-3)(t+2) = 0$

$\ln x = 1, -2, 3$

$x = e^1, e^{-2}, e^3$

5. $\log_{10} \frac{18}{100}$ $\quad 3^2 \; 2$

$\log_{10} N = \boxed{2\log_{10} 3 + \log_{10} 2 - 2}$

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7.

$$12^{\frac{1-\log_{60}3-\log_{60}5}{2(1-\log_{60}5)}} = 12^{\frac{\log_{60}4}{2\log_{60}12}}$$

$$\left(12^{\log_{12}4}\right)^{\frac{1}{2}}$$

$$\log_2 3 = a$$

$$\frac{2\log_2 3+1}{2+\log_2 3} = b$$

$$\frac{3\log_2 3+1}{3+\log_2 3}$$

$$2t^2-4t+1=0 \quad \log_{10}x_1,\ \log_{10}x_2$$

$$\log_{10}x_1+\log_{10}x_2 = 2$$

$$\boxed{x_1x_2 = 10^2}$$

$$\frac{1}{2}+\left(\log_{2010}x\right)^2 = 2\log_{2010}x$$

$$4^{\frac{1}{2}}$$

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11. $\log_{10} x + \log_{10} y + \log_{10} z = 81$

$\log_{10} x \log_{10} y + \log_{10} x \log_{10} z + \log_{10} y \log_{10} z = 468$

$\sum (\log_{10} x)^2 + 2(468) = (81)^2$

12. $(3\log_{10} x)^2 - 10\log x + 1 = 0$

$(9t-1)(t-1) = 0$

$\log_{10} x = \frac{1}{9}, 1$

$\boxed{x = 10^{\frac{1}{9}}, 10}$

$\boxed{x = 10}$

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$$\sum \tan^2\frac{A}{2} \geq \sum \tan\frac{A}{2}\tan\frac{B}{2} = 1$$

$$a^2+b^2+c^2 \geq ab+bc+ca$$

$$\sum a^2 - \sum ab = \frac{1}{2}\left((a-b)^2+(b-c)^2+(c-a)^2\right) \geq 0$$

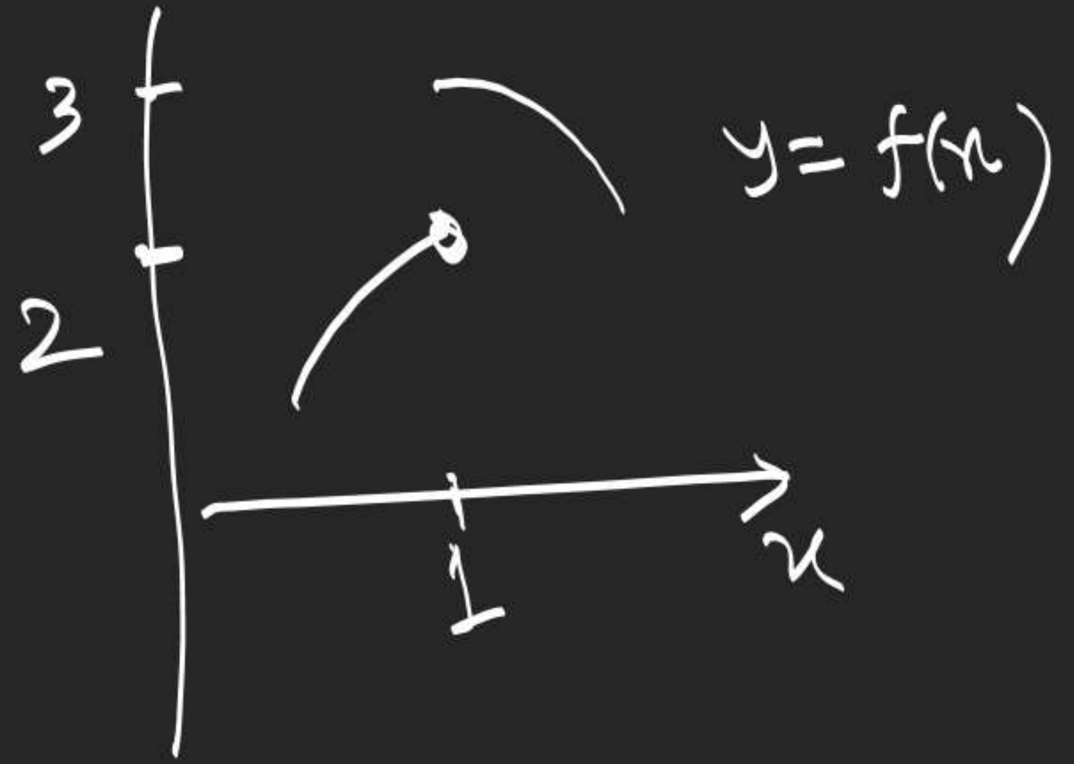
$\cot A + \cot B + \cot C = \sqrt{3}$ iff $ABC$ is equilateral

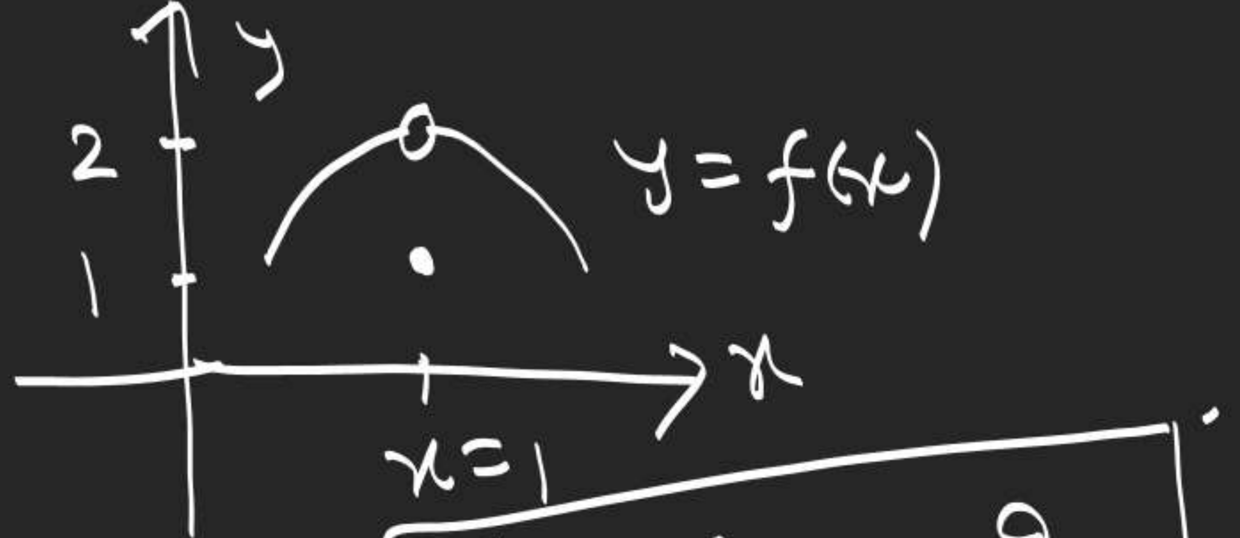
$$\sum a^2 = \sum ab \quad \text{iff } a=b=c.$$

$$\boxed{\sum \cot^2 A} + 2\sum \cot A \cot B = 3$$

$$\sum \cot^2 A \geq \sum \cot A \cot B$$

$$\sum \cot^2 A + 2\sum \cot A \cot B \geq 3\sum \cot A \cot B = 3.$$




$$\boxed{\lim_{x\to 1} f(x) = 2}$$

$\lim_{x\to 1} f(x)$ not exist.

At $x=a$

If $LHL = RHL = \text{finite} = l$

$\Rightarrow \lim_{x\to a} f(x)$ exist

& $\lim_{x\to a} f(x) = l$

Left hand limit $= LHL = \lim_{x\to 1^-} f(x) = 2$

Right hand limit $= RHL = \lim_{x\to 1^+} f(x) = 3$

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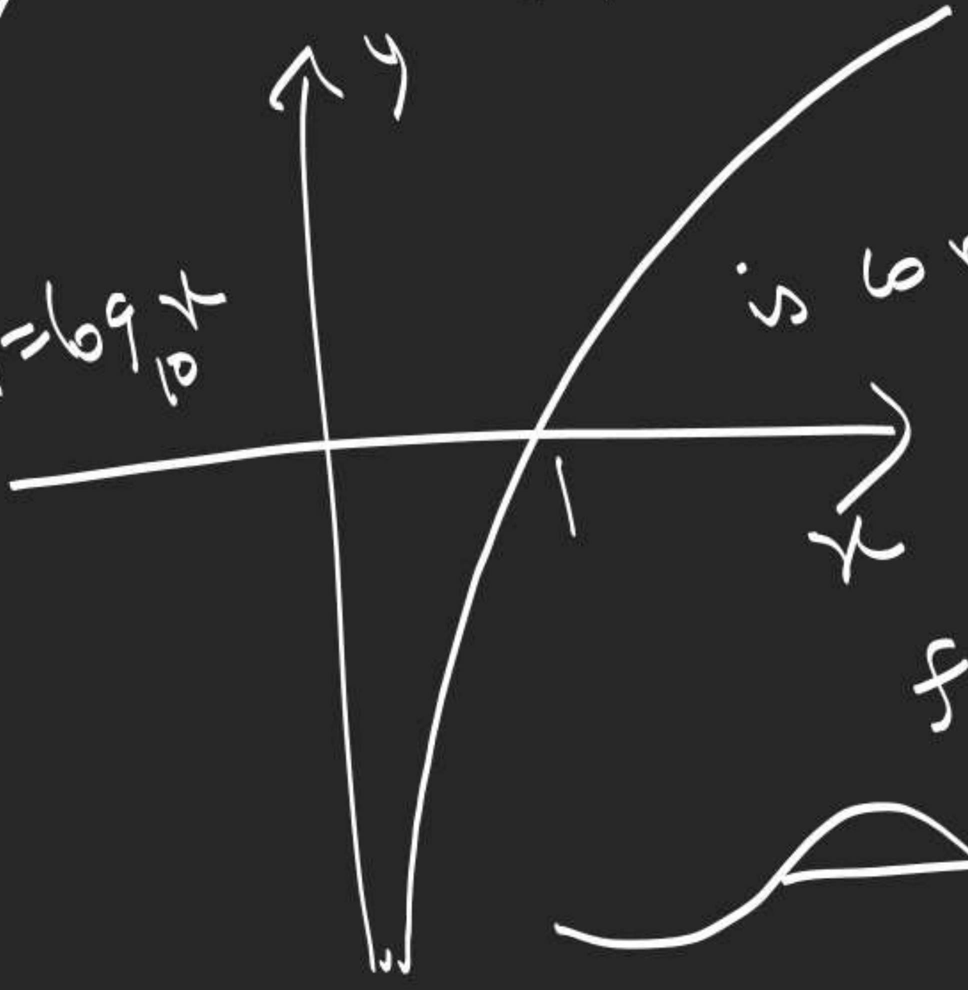
# Continuity of function

If we can draw the graph of $f(x)$ without raising pen, then $f(x)$ is said to be continuous function.

$f(x)=\log_{10} x$ is continuous




$\underline{x \in D}$.

$f(x) = \sin x \quad \forall x \in R$ is cont.

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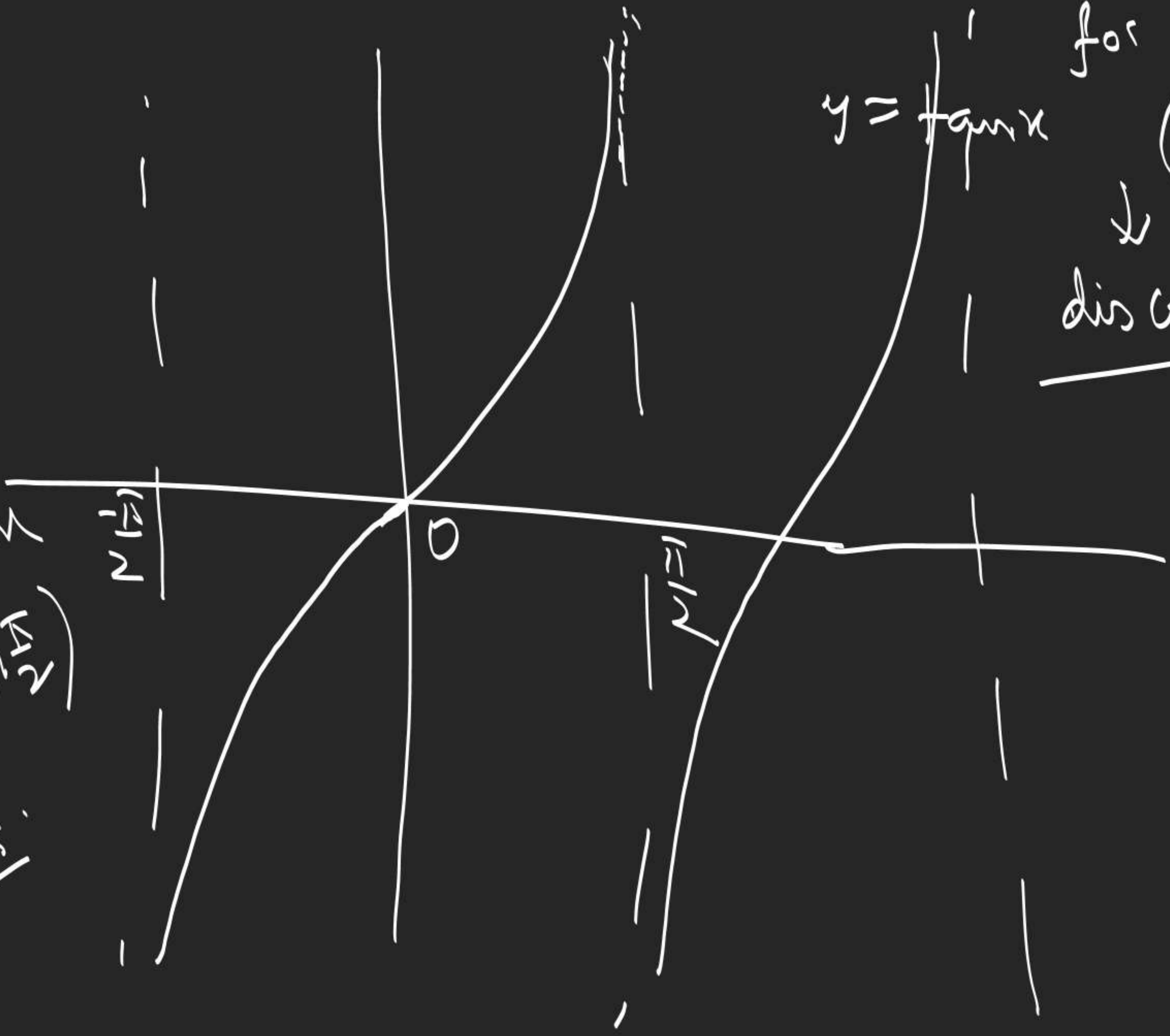
$f(x) = \tan x$

$\forall\, x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$\downarrow$

is cont.




$y = \tan x$ for all $\forall x \in \mathbb{R}$.

$\downarrow$

discontinuous

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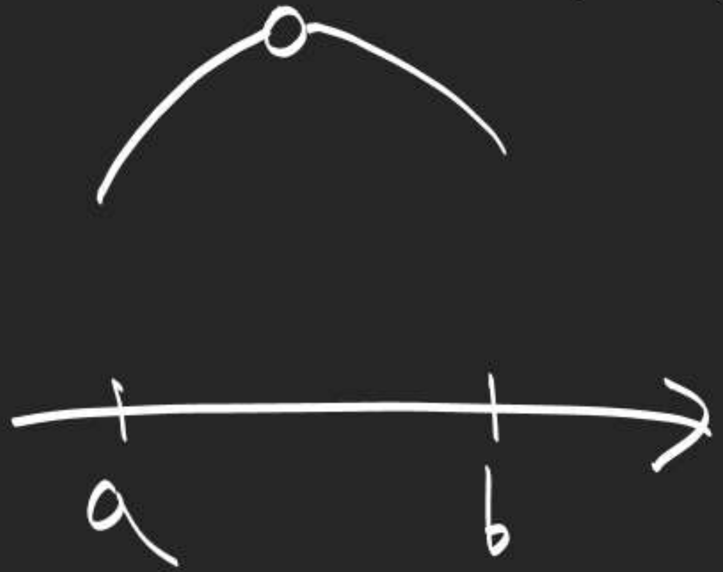
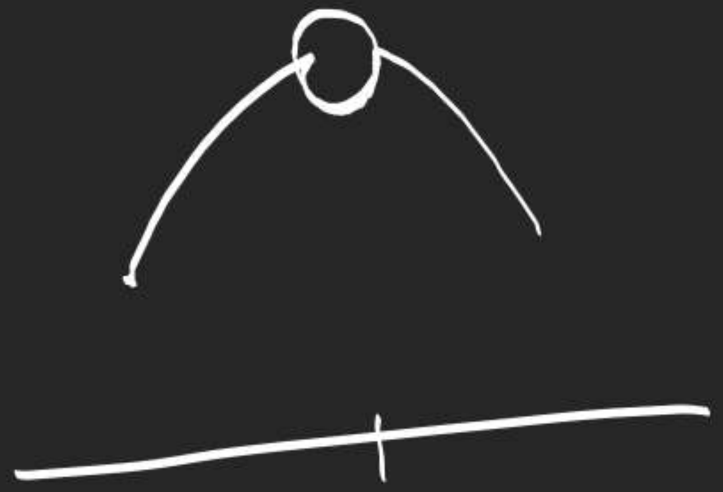
$y = f(x)$ in $[a, b]$

discontinuous




Discontinuous

Discont



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# Intermediate Value Theorem (IVT)


If $f(x)$ is continuous,

$f(1) = -3$ ✓ , $f(5) = 12$ ✓

there exist

$\exists\, c \in (1, 5)$ , such that $f(c) = 1$

$(\sin x)_{min} = -1$

$(\sin x)_{max} = 1$

$[-1, 1]$

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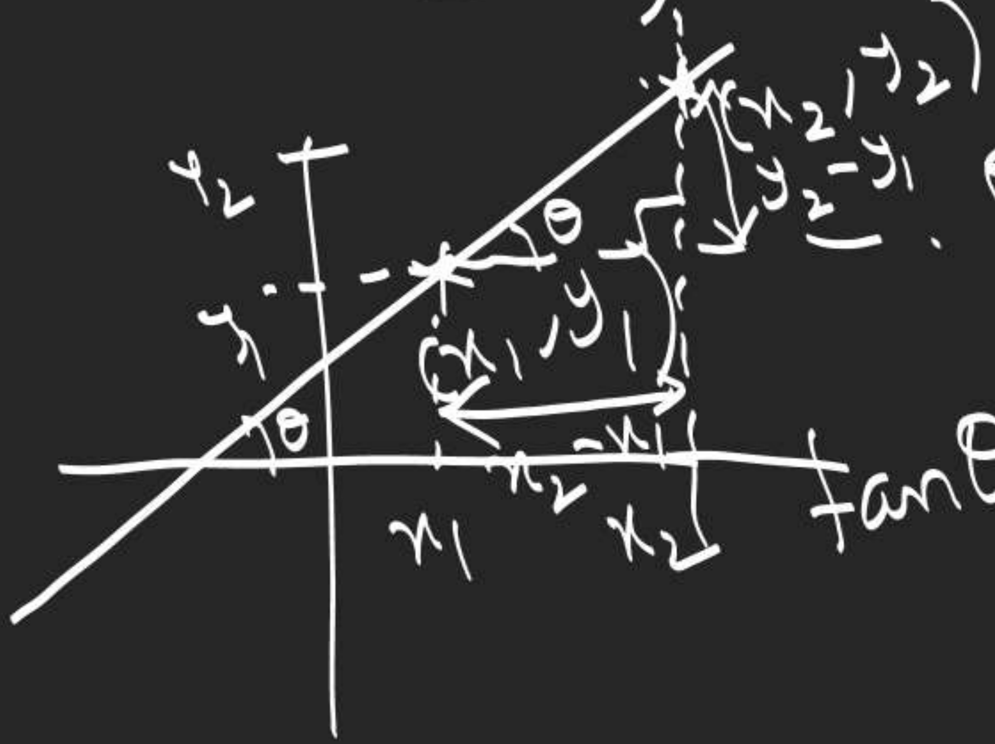
# Differentiation / Derivative of function

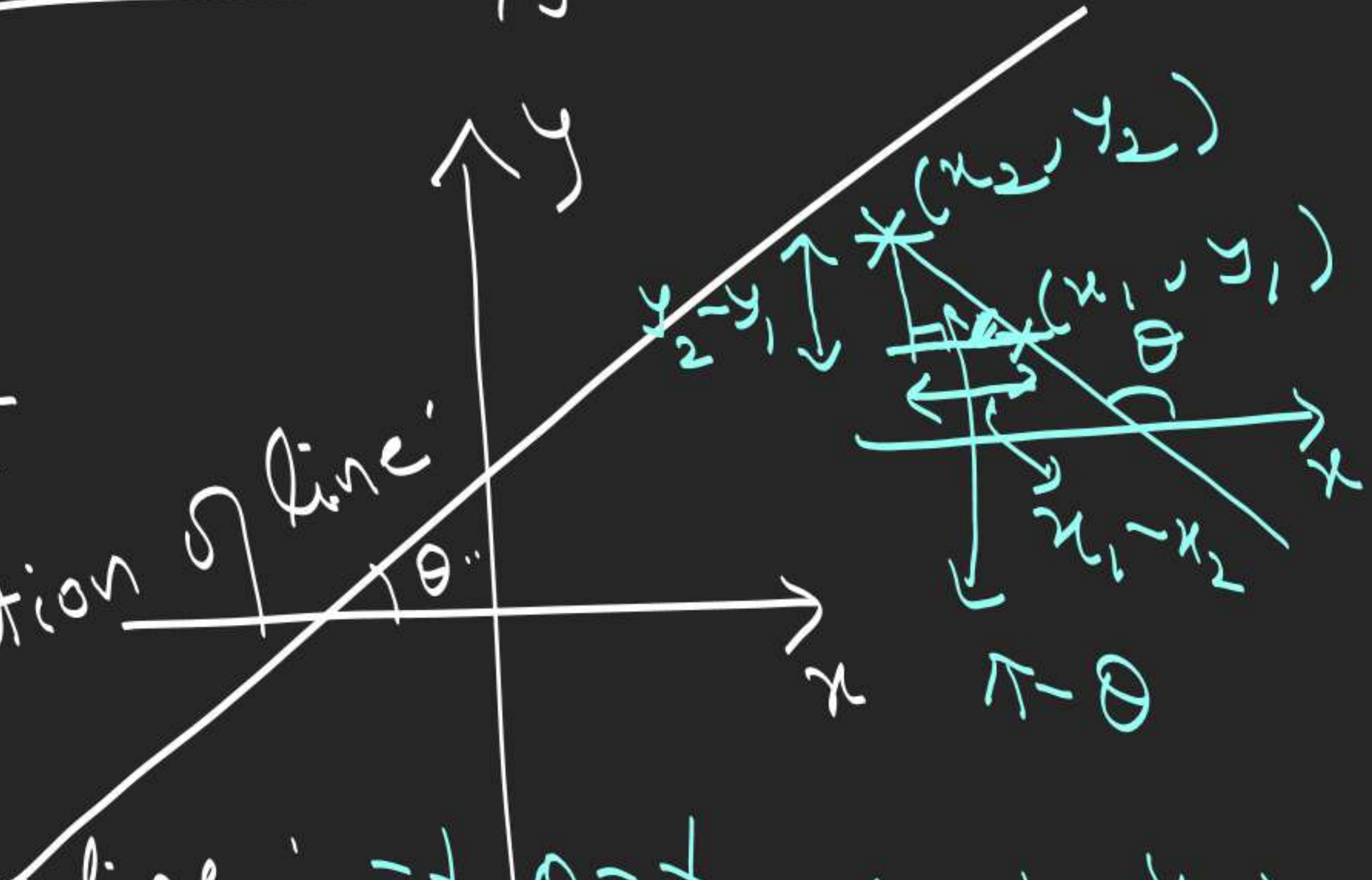
## Gradient / slope of line




$\theta$ = inclination of line

$\tan\theta$ = slope of line

$$= \frac{y_2 - y_1}{x_2 - x_1}$$




$$-\tan\theta = \tan(\pi - \theta) = \frac{y_2 - y_1}{x_1 - x_2}$$

$$\tan\theta = \frac{y_2 - y_1}{x_2 - x_1}$$

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# Derivative / Differentiation of function

Logarithm {Ex-II (1-13)}