

# Matrices

Array/Collection of objects in rectangular form is called matrix.

$$A_{2 \times 3} = \begin{bmatrix} -a_1 & -a_2 & -a_3 \\ -b_1 & -b_2 & -b_3 \end{bmatrix}$$

$$A_{m \times n} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \dots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \dots & a_{2n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & a_{m3} & \dots & a_{mn} \end{bmatrix}$$

$$\begin{pmatrix} x & x \\ x & \\ x & x \end{pmatrix} \rightarrow \text{not a matrix.}$$

A

Construct a matrix 'A' of order  $2 \times 3$

$A = \{a_{ij}\}$  such that

$$a_{ij} = \frac{i^2 - j}{2}$$

$$A = \begin{bmatrix} 0 & -\frac{1}{2} & -1 \\ \frac{3}{2} & 1 & \frac{1}{2} \end{bmatrix}$$

Row matrix/vector

Matrix having only one row

$$A = \begin{pmatrix} -1 & 0 & 2 \end{pmatrix}$$

Column matrix/vector

only one column

$$\begin{pmatrix} x \\ x \\ x \\ x \end{pmatrix}$$

$$\vec{v} = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$$

Horizontal Matrix

no. of rows &lt; no. of columns

$$\begin{bmatrix} x & x & x & x \\ x & x & x & x \end{bmatrix}$$

Vertical matrix

rows &gt; columns

$$\begin{bmatrix} x & x & x \\ x & x & x \\ x & x & x \\ x & x & x \\ x & x & x \\ x & x & x \end{bmatrix}$$

Square matrix

rows = columns

$$\begin{bmatrix} x & x & x \\ x & x & x \\ x & x & x \end{bmatrix}$$

Null matrix  
all elements are zero

$$O_{3 \times 2} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$



# Square matrix

- Principle diagonal elements  
 $a_{ii}$

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

- Singular matrix

$$|A| = 0$$

- Non Singular matrix  
 $|A| \neq 0$

- Trace of matrix  
$$\text{Tr}(A) = \sum_{i=1}^n a_{ii}$$

$A_{n \times n}$

- Determinant of matrix

$$|A| = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11}(a_{22}a_{33} - a_{23}a_{32}) - a_{12}(a_{21}a_{33} - a_{23}a_{31}) + a_{13}(a_{21}a_{32} - a_{31}a_{22})$$

# Square matrix

Identity matrix =  $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$   
= I

Others

$\rightarrow (a_{ij} = 0 \text{ } \forall i \neq j)$

Diagonal matrix

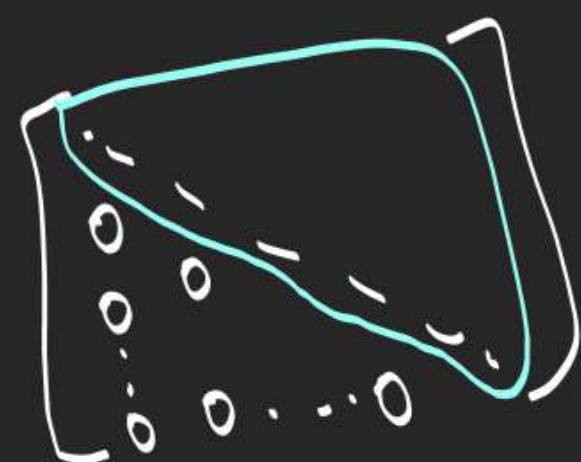
Triangular matrix

Upper triangular matrix

Lower triangular matrix

$(a_{ij} = 0 \text{ } \forall i > j)$

$(a_{ij} = 0 \text{ } \forall i < j)$



Scalar matrix

Others

$a_{ii} = k$

$\begin{bmatrix} k & 0 & 0 \\ 0 & k & 0 \\ 0 & 0 & k \end{bmatrix}$

diag (-1 0 2 3)  
=  $\begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \end{bmatrix}$