

Q If $\vec{a}, \vec{b}, \vec{c}$ are unit vectors.

$$\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{c} = \vec{c} \cdot \vec{a} = \frac{\pi}{3}$$

then $|\vec{a} + 2\vec{b} - 3\vec{c}| = ?$

$$|\vec{a} + 2\vec{b} - 3\vec{c}|^2 = |\vec{a}|^2 + 4|\vec{b}|^2 + 9|\vec{c}|^2 + 2|\vec{a}||\vec{b}|\cos\frac{\pi}{3} - 12|\vec{b}||\vec{c}|\cos\frac{\pi}{3} - 6|\vec{a}||\vec{c}|\cos\frac{\pi}{3}$$

$$= 1 + 4 + 9 + 2 - 6 - 3$$

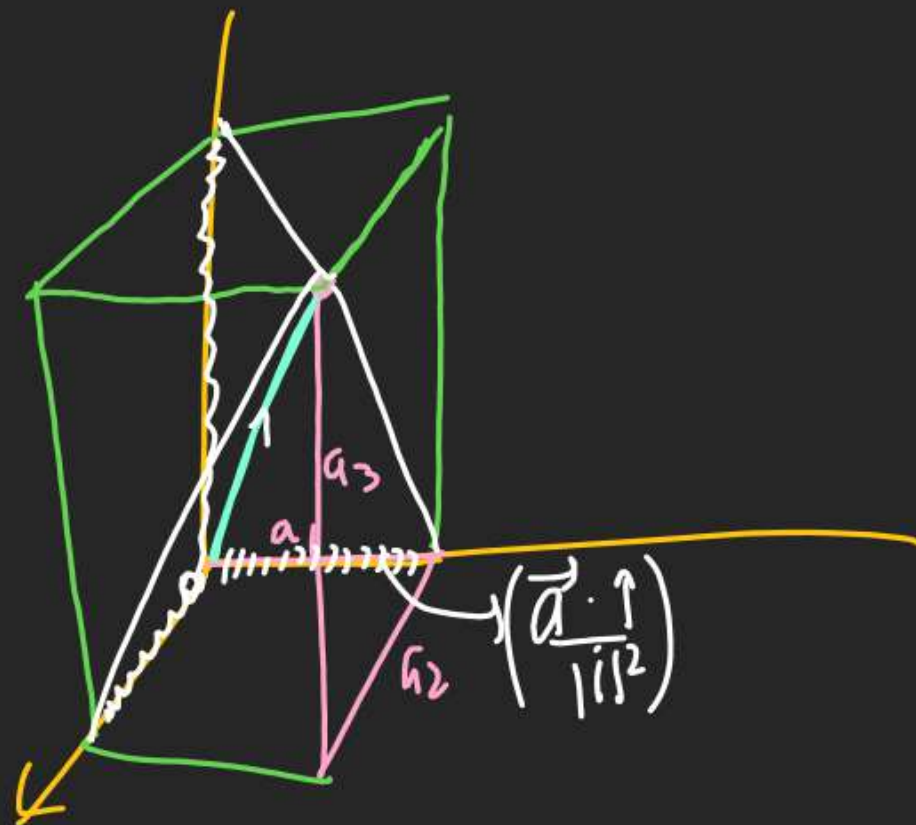
$$|\vec{a} + 2\vec{b} - 3\vec{c}| = \sqrt{7}$$

5 \vec{a}

$$(\vec{a} \cdot \hat{i})\hat{i} + (\vec{a} \cdot \hat{j})\hat{j} + (\vec{a} \cdot \hat{k})\hat{k}$$

$$\left(\frac{\vec{a} \cdot \hat{i}}{|\hat{i}|^2}\right)\hat{i} + \left(\frac{\vec{a} \cdot \hat{j}}{|\hat{j}|^2}\right)\hat{j} + \left(\frac{\vec{a} \cdot \hat{k}}{|\hat{k}|^2}\right)\hat{k} = \vec{a}$$

↓ Proj. of \vec{a} on X Axis
 ↓ Proj. of \vec{a} on Y Axis
 ↑ Proj. of \vec{a} on Z Axis



1) dot Product Base

2) Repeat \vec{a} then max. No of Qs try \vec{a}

3) Bahut weak
 होजे पर उम्मेद Example
Revise \vec{a} Self

If you are feeling good
 then try Adv. + \vec{a}

Q If \vec{e}_1, \vec{e}_2 such that $|\vec{e}_1|=2, |\vec{e}_2|=1$

$\vec{e}_1 \wedge \vec{e}_2 = 60^\circ$ If angle betⁿ $\vec{V}_1 = 2t\vec{e}_1 + 7\vec{e}_2$ &

$\vec{V}_2 = \vec{e}_1 + t\vec{e}_2$ lies betⁿ $(\frac{\pi}{2}, \pi)$ find

Range of t ?

Obtuse Angle
 $\vec{V}_1 \cdot \vec{V}_2 < 0$

$$\vec{V}_1 \wedge \vec{V}_2 = \pi$$

$$\frac{2t}{1} = \frac{7}{t}$$

$$t^2 = \frac{7}{2}$$

$$t = \sqrt{\frac{7}{2}}, -\sqrt{\frac{7}{2}}$$

$$(2t\vec{e}_1 + 7\vec{e}_2) \cdot (\vec{e}_1 + t\vec{e}_2) < 0$$

$$2t|\vec{e}_1|^2 + 7|\vec{e}_1||\vec{e}_2|\cos 60^\circ + 2t^2|\vec{e}_1||\vec{e}_2|\cos 60^\circ + 7t|\vec{e}_2|^2 < 0$$

(K-10)

$$8t + 7 + 2t^2 + 7t < 0$$

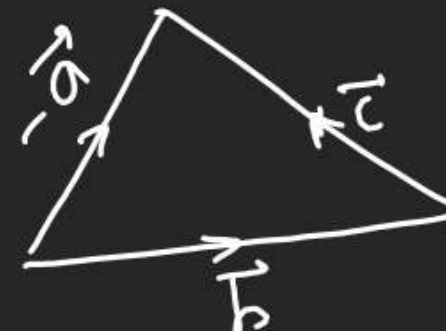
$$2t^2 + 15t + 7 < 0$$

$$2t^2 + 14t + 1 + 7 < 0$$

$$(2t+1)(t+7) < 0$$

$$t \in (-7, -\frac{1}{2}) - \{-\sqrt{\frac{7}{2}}\}$$

Q If $\vec{a} + \vec{b} + \vec{c} = 0$ then
angle betⁿ \vec{b} & \vec{c}
 $\vec{b} + \vec{c} = -\vec{a}$



$$\vec{b} + \vec{c} = -\vec{a}$$

$$(\vec{b} + \vec{c})^2 = (-\vec{a})^2$$

$$|\vec{b}|^2 + |\vec{c}|^2 + 2|\vec{b}||\vec{c}|\cos\theta = |\vec{a}|^2$$

$$\cos\theta = \frac{|\vec{a}|^2 - |\vec{b}|^2 - |\vec{c}|^2}{2|\vec{b}||\vec{c}|}$$

Q Proof of Cosine for

$$\vec{a} + \vec{b} + \vec{c} = 0$$

$$(\vec{a} + \vec{b})^2 = (-\vec{c})^2$$

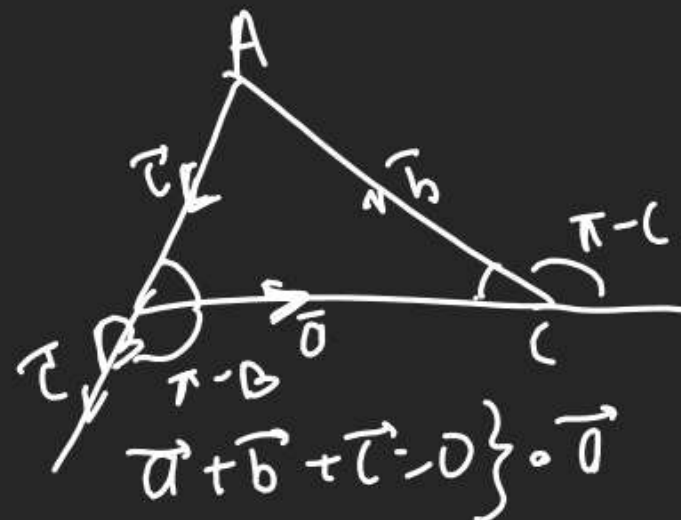
$$|\vec{a}|^2 + |\vec{b}|^2 + 2|\vec{a}||\vec{b}|\cos(\pi - \theta) = |\vec{c}|^2$$

$$-2|\vec{a}||\vec{b}|\cos\theta = |\vec{c}|^2 - |\vec{a}|^2 - |\vec{b}|^2$$

$$\cos\theta = \frac{|\vec{a}|^2 + |\vec{b}|^2 - |\vec{c}|^2}{2|\vec{a}||\vec{b}|}$$

Q Proof of Projection formula.

$$a = b \cos C + c \cos B$$



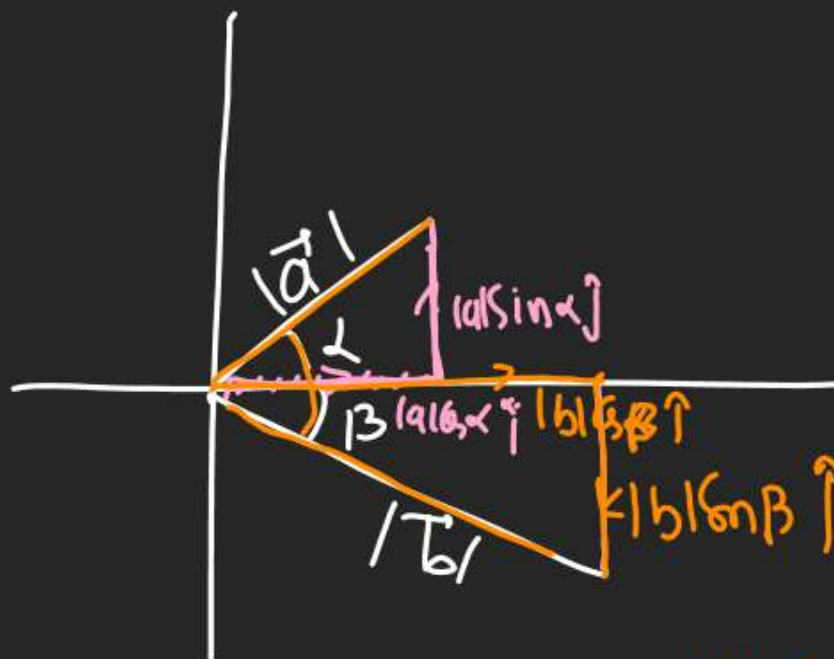
$$\vec{a} \cdot \vec{a} + \vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c} = 0$$

$$|\vec{a}|^2 + |\vec{a}||\vec{b}|\cos(\pi - C) + |\vec{a}||\vec{c}|\cos(\pi - B) = 0$$

$$|\vec{a}| - |\vec{b}|\cos C - |\vec{c}|\cos B = 0$$

$$a = b \cos C + c \cos B \text{ (H.P.)}$$

Q Proof $\cos(A+B) = \cos A \cos B - \sin A \sin B$



$$\vec{a} = |\vec{a}|\cos\alpha\hat{i} + |\vec{a}|\sin\alpha\hat{j}$$

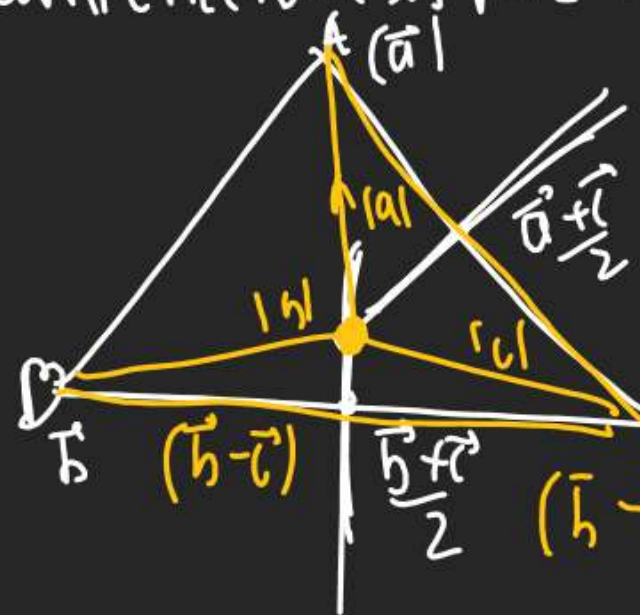
$$\vec{b} = |\vec{b}|\cos\beta\hat{i} - |\vec{b}|\sin\beta\hat{j}$$

$$\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}|\cos\alpha\cos\beta - |\vec{a}||\vec{b}|\sin\alpha\sin\beta$$

$$|\vec{a}||\vec{b}|\cos(\alpha + \beta) = |\vec{a}||\vec{b}|\cos\alpha\cos\beta - |\vec{a}||\vec{b}|\sin\alpha\sin\beta$$

$$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta \text{ (H.P.)}$$

Q P.T. Circumcentre is P.O.I of \perp^r Bisector.



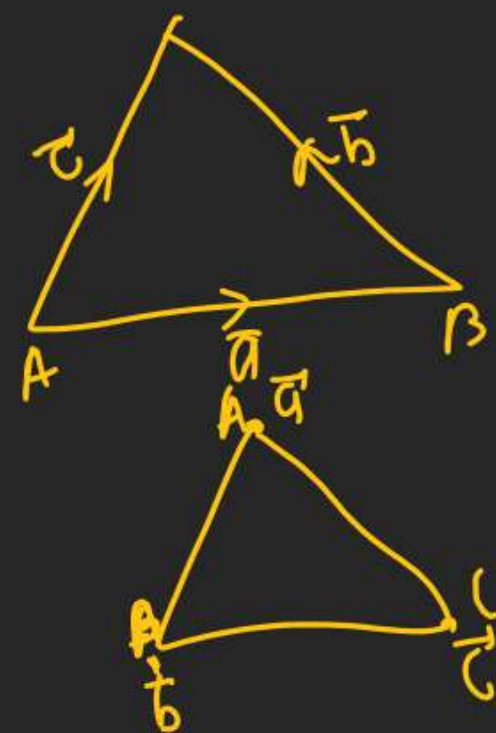
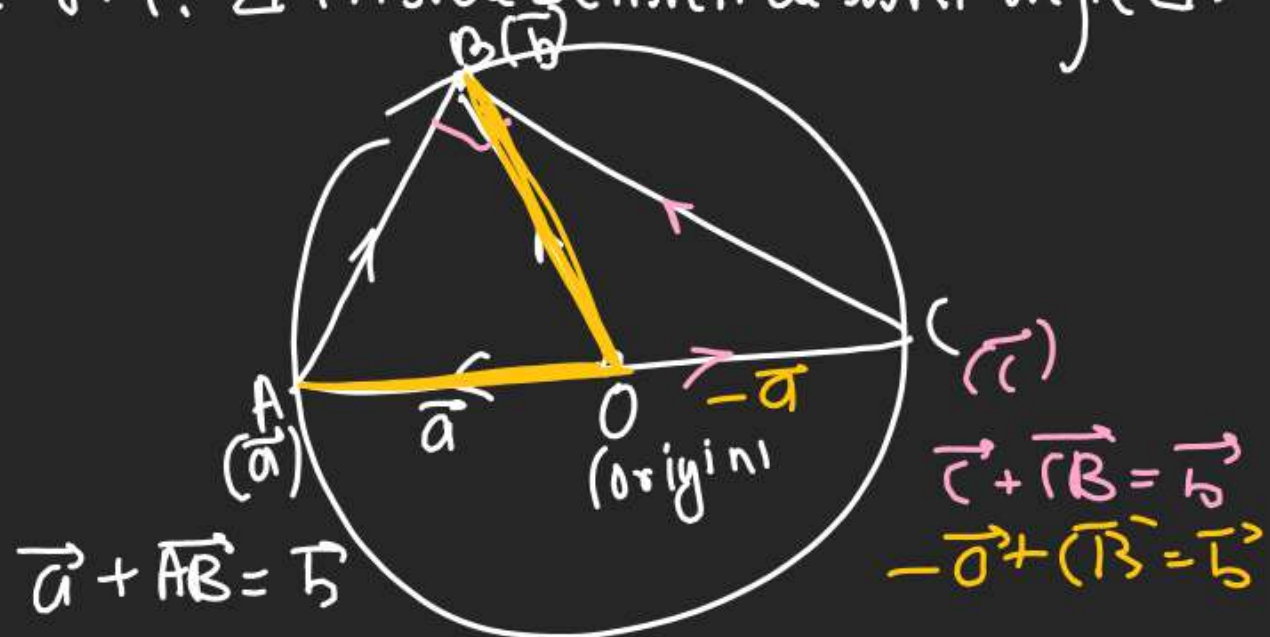
$$(\vec{a} - \vec{c}) \cdot (\vec{a} + \vec{c}) = 0$$

$$|\vec{a}|^2 = |\vec{c}|^2$$

$$(\vec{b} - \vec{c}) \cdot (\vec{b} + \vec{c}) = 0 \Rightarrow |\vec{b}|^2 = |\vec{c}|^2$$

$$\Rightarrow |\vec{a}| = |\vec{b}| = |\vec{c}|$$

Q. P.T. Δ inside Semicircle is Rt. angle Δ .



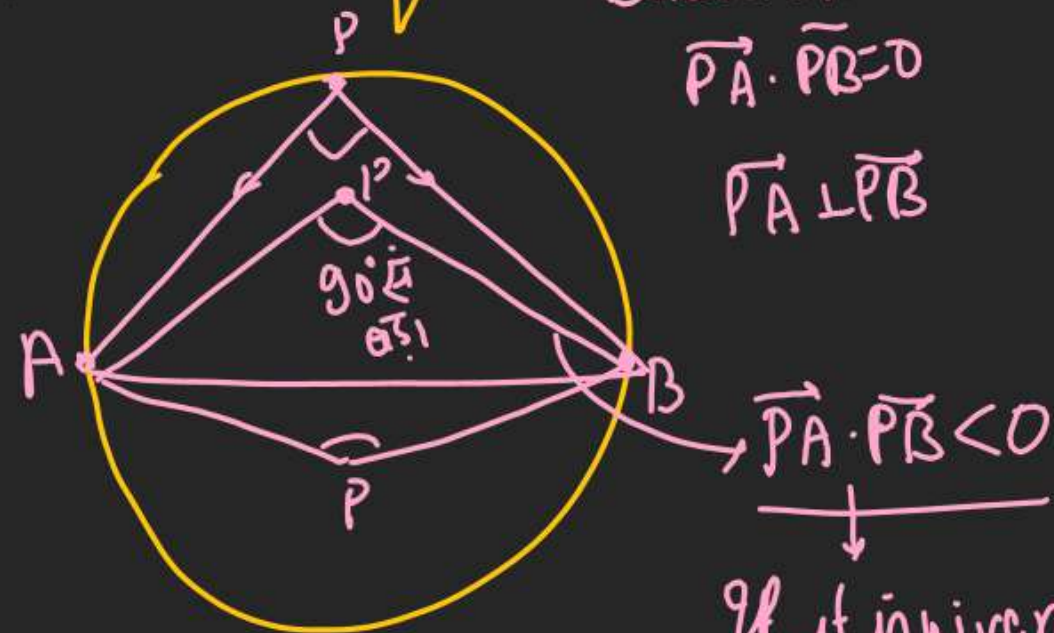
$$\begin{aligned}\vec{AB} &= \vec{b} - \vec{a} \\ \vec{CB} &= \vec{b} - \vec{r} \\ &= \vec{b} - (-\vec{a}) \\ &= \vec{b} + \vec{a}\end{aligned}$$

$$\begin{aligned}\vec{AB} \cdot \vec{CB} &= (\vec{b} + \vec{a}) \cdot (\vec{b} - \vec{a}) \\ &= |\vec{b}|^2 - |\vec{a}|^2 \\ &= 0\end{aligned}$$

Q. If P is pt in Space & $\vec{PA} \cdot \vec{PB} < 0$

Where A & B are 2 fixed Pts.

find locus of P . \rightarrow Based on $\vec{PA} \cdot \vec{PB} = 0$



If it is given we should understand then P can lie inside circle of diameter AB

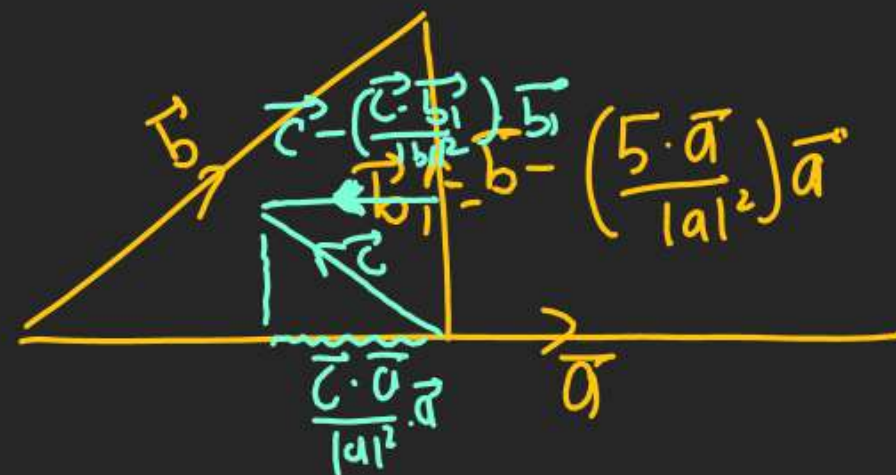
$$\underline{0} \quad \vec{b}_1 = \left(\vec{b} - \frac{\vec{b} \cdot \vec{a}}{|\vec{a}|^2} \vec{a} \right), \quad \vec{a} \text{ are } \perp$$

$$\vec{c}_2 = \vec{c} - \left(\frac{\vec{c} \cdot \vec{a}}{|\vec{a}|^2} \right) \vec{a} - \frac{\vec{c} \cdot \vec{b}_1}{|\vec{b}_1|^2} \vec{b}_1$$

$$\vec{b}_1 \perp \vec{c}_2 \text{ (TIF)?}$$

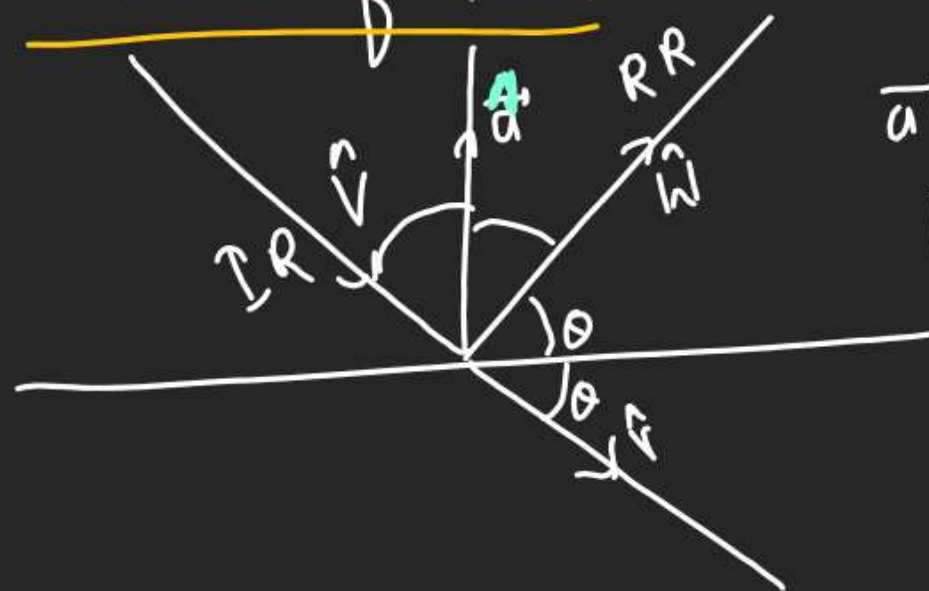
$$\vec{b}_1 \cdot \vec{c}_2 = 0$$

$$\vec{b} \cdot \vec{c} - \left(\frac{\vec{c} \cdot \vec{a}}{|\vec{a}|^2} \right) \vec{a} \cdot \vec{b} - \left(\frac{\vec{c} \cdot \vec{b}_1}{|\vec{b}_1|^2} \right) \vec{b} \cdot \vec{b}_1 - \left(\frac{\vec{b} \cdot \vec{a}}{|\vec{a}|^2} \right) \left(\vec{c} \cdot \vec{a} \right) + \left(\frac{\vec{b} \cdot \vec{a}}{|\vec{a}|^2} \right) \left(\frac{\vec{c} \cdot \vec{a}}{|\vec{a}|^2} \right) |\vec{a}|^2 + \frac{(\vec{c} \cdot \vec{b}_1)}{|\vec{b}_1|^2} \left(\frac{\vec{b} \cdot \vec{a}}{|\vec{a}|^2} \right) \vec{a} \cdot \vec{b}_1$$



Solve करते को try करना

Q Incident Ray is along unit vector \hat{v} &
 Reflected Ray along \hat{w} , Normal vector
 along Unit vector \hat{a} outward. Express
 \hat{w} in terms of \hat{a} & \hat{v} .



\vec{a} is External
 Angle Bisector

$$\vec{a} = \lambda(\hat{v} - \hat{w})$$

$$t\vec{a} = \hat{v} - \hat{w}$$

$$(\hat{w})^2 = (\hat{v} - t\vec{a})^2$$

$$\hat{w} = \hat{v} - t\vec{a}$$

$$\hat{w} = \hat{v} - 2(\hat{a} \cdot \hat{v})\vec{a}$$

$$|\hat{w}|^2 = |\hat{v}|^2 + t^2|\vec{a}|^2 - 2t(\hat{a} \cdot \hat{v})$$

$$0 = t(t - 2(\hat{a} \cdot \hat{v}))$$

$$t = 0, t = 2(\hat{a} \cdot \hat{v})$$

Max/Min type Qs

$$Q \quad \oint f^2(x) + g^2(x) + h^2(x) \leq 9$$

& $u(x) = 3f(x) + 4g(x) + 10h(x)$ find Max. value
 of $u(x)$?

$$(1) \text{ Let } \vec{a} = 3\hat{i} + 4\hat{j} + 10\hat{k} \quad | \vec{b} = f(x)\hat{i} + g(x)\hat{j} + h(x)\hat{k}$$

$$|\vec{a}| = \sqrt{25 + 100} = 5\sqrt{5}$$

$$(2) \cos \theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|} = \frac{u(x)}{5\sqrt{5}|\vec{b}|}$$

$$(3) \cos^2 \theta = \frac{u^2(x)}{125|\vec{b}|^2} \leq 1$$

$$u(x)_{\text{Max}} = 15\sqrt{5}$$

$$u^2(x) \leq 125(f^2(x) + g^2(x) + h^2(x))$$

$$\leq 125 \times 9$$

$$|u(x)| \leq 15\sqrt{5}$$

Q If $a > 2$ A, B, C are Var. Angles of ΔABC

$$S.I. \boxed{\sqrt{a^2-4} \tan A + a \tan B + \sqrt{a^2+4} \tan C = 6a}$$

then find MIN value of $\sum \tan^2 A$.

$$\vec{L} = \sqrt{a^2-4} \hat{i} + a \hat{j} + \sqrt{a^2+4} \hat{k}$$

$$|\vec{L}| = \sqrt{a^2-4 + a^2 + a^2+4} = a\sqrt{3}$$

$$\vec{M} = \tan A \hat{i} + \tan B \hat{j} + \tan C \hat{k}$$

$$|\vec{M}| = \sqrt{\sum \tan^2 A}$$

$$\vec{L} \cdot \vec{M} = 6a \Rightarrow \cos \theta = \frac{\vec{L} \cdot \vec{M}}{|\vec{L}| |\vec{M}|}$$

$$\sum \tan^2 A \geq 12$$

$$\sum \tan^2 A = 12$$

$$\cos \theta = \frac{6a \cdot 2\sqrt{3}}{a\sqrt{3} \sqrt{\sum \tan^2 A}}$$

$$\cos^2 \theta \leq 1 \Rightarrow \frac{12}{\sum \tan^2 A} \leq 1$$

Let $x, y \in \mathbb{R}$ Such that

$$2 \sin x \cdot \sin y + 3 \cos y + 6 \cos x \cdot \sin y = 7 \quad \text{find } \tan^2 x + 2 \tan^2 y = ?$$