Nishant Jindal

ANGULAR MOMENTUM

⇒ Def^n :- Moment of linear momentum is angular momentum.

⇒ Angular Momentum in Case of translational

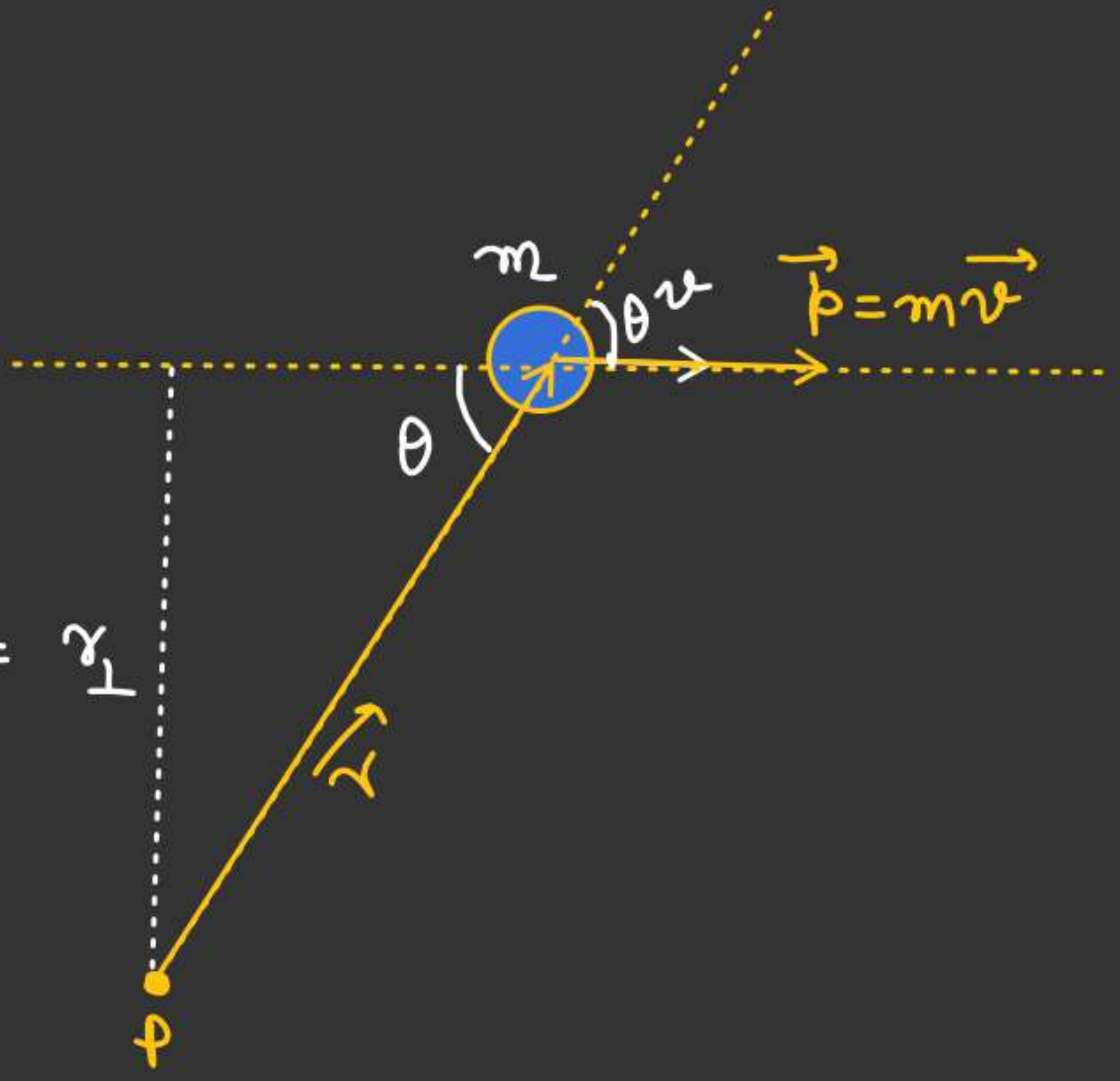
$$\boxed{\vec{L} = \vec{r} \times \vec{p}}$$

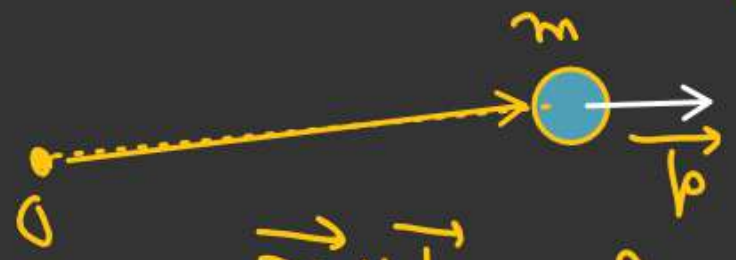
$$|\vec{L}| = p(r\sin\theta)$$

θ = Angle b/w $\vec{r}$ & $\vec{p}$

$r\sin\theta = r_{\perp}$

$$\boxed{|\vec{L}| = r_{\perp}\, p}$$





$\vec{r} \parallel \vec{p} \Rightarrow \theta = 0 \quad L_O = 0$

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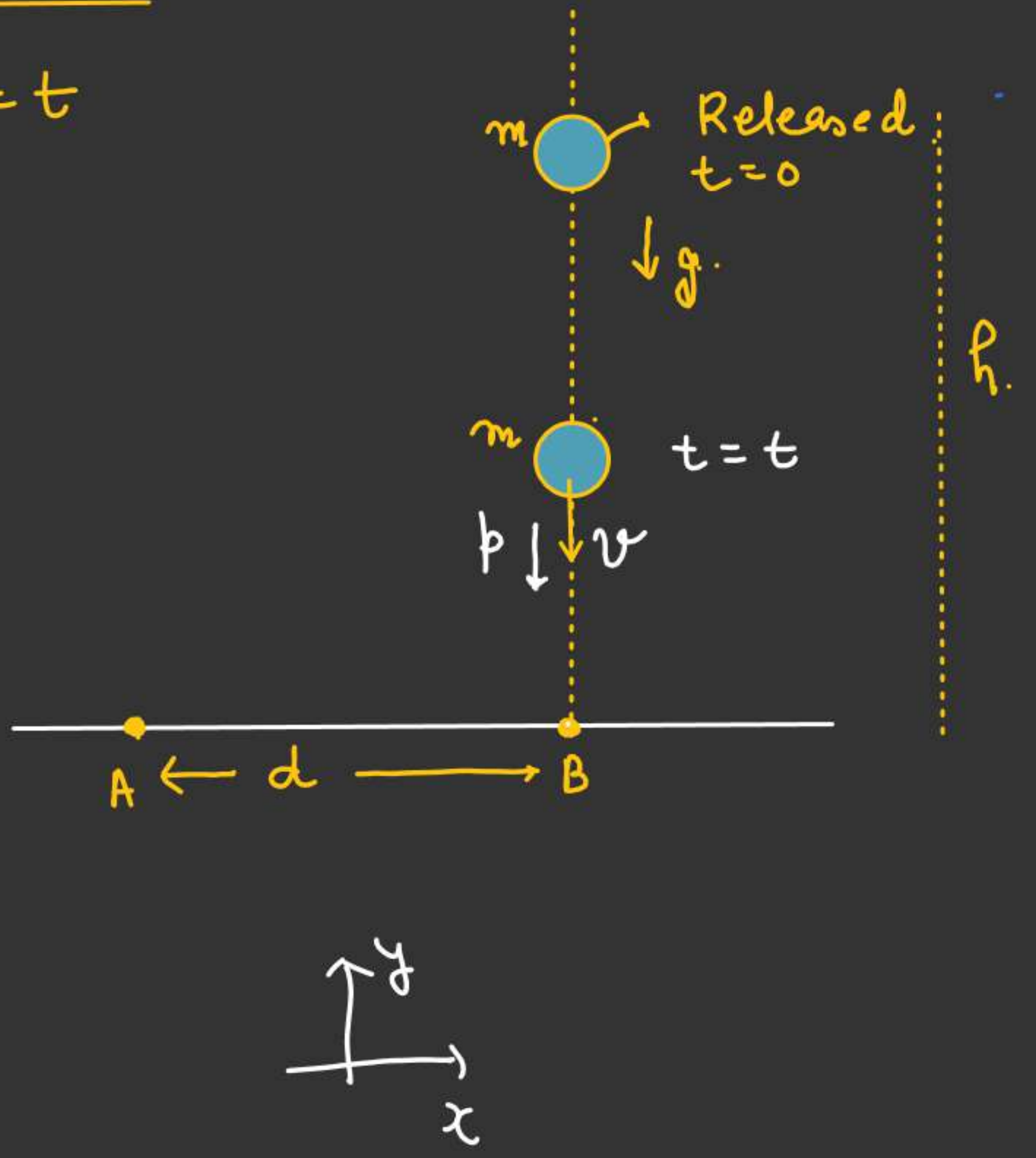
ANGULAR MOMENTUM

Find Angular Momentum of ball at $t = t$ w.r.t point A & B.

<u>M-1.</u>

<u>M-2.</u>



$$L_{ball/B} = 0$$

$$v = \cancel{u}^{\,0} + gt$$

$$v = gt$$

$$p = mv = (mgt)$$

$$\vec{L}_{ball/A} = (mgt)d(-\hat{k})$$

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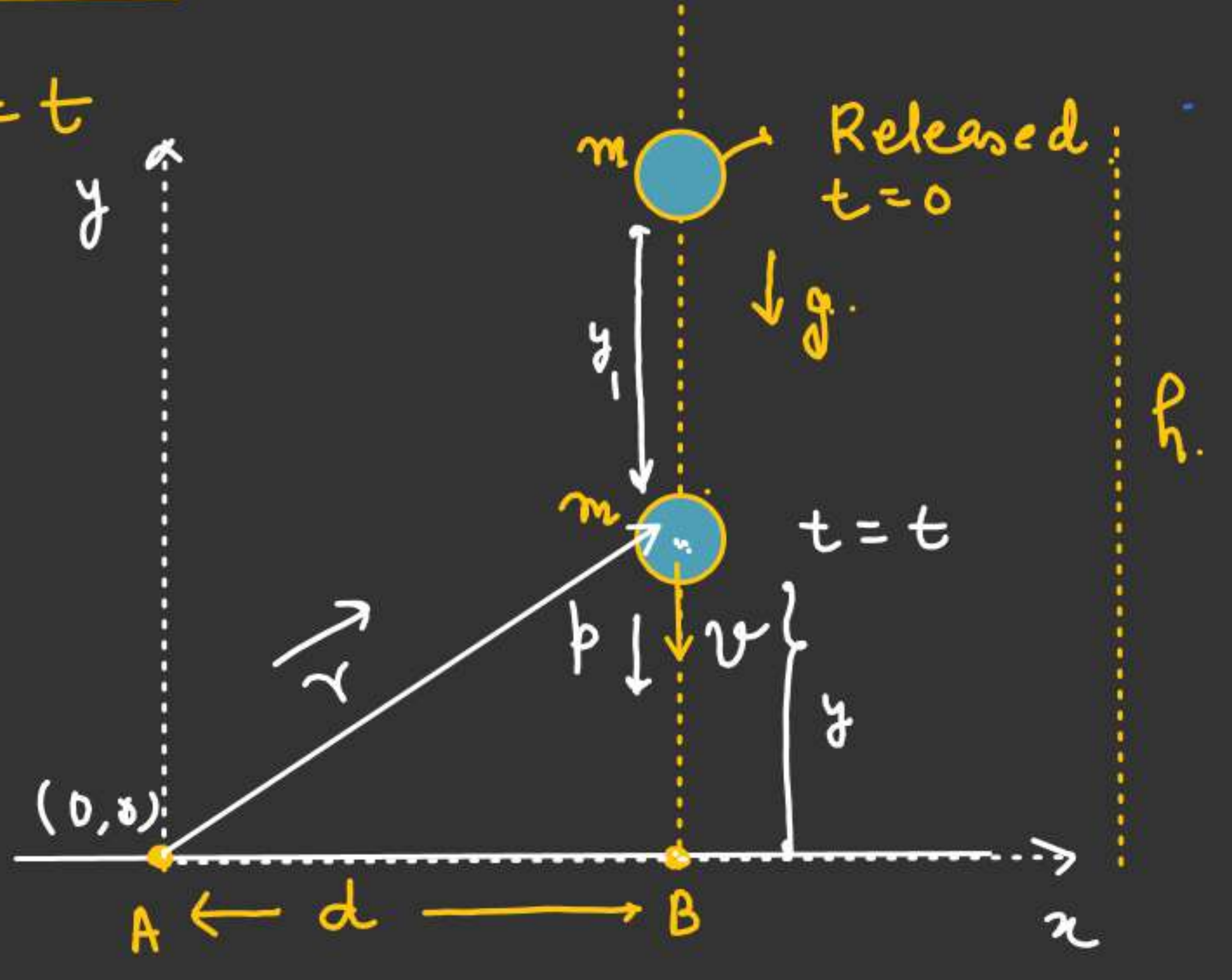
ANGULAR MOMENTUM

Find Angular Momentum of ball at $t = t$ w.r.t point A & B.



M-2

$$y_1 = \frac{1}{2}gt^2 \qquad y = h - y_1$$

$$y = \left(h - \frac{1}{2}gt^2\right)$$

$$\vec{r} = d\hat{i} + y\hat{j}$$

$$\vec{r} = d\hat{i} + \left(h - \frac{1}{2}gt^2\right)\hat{j}$$

$$\vec{p} = m\vec{v} = mgt(-\hat{j})$$

$$\vec{L}_A = \vec{r} \times \vec{p} = \left[d\hat{i} + \left(h - \frac{1}{2}gt^2\right)\hat{j}\right] \times (mgt)(-\hat{j})$$

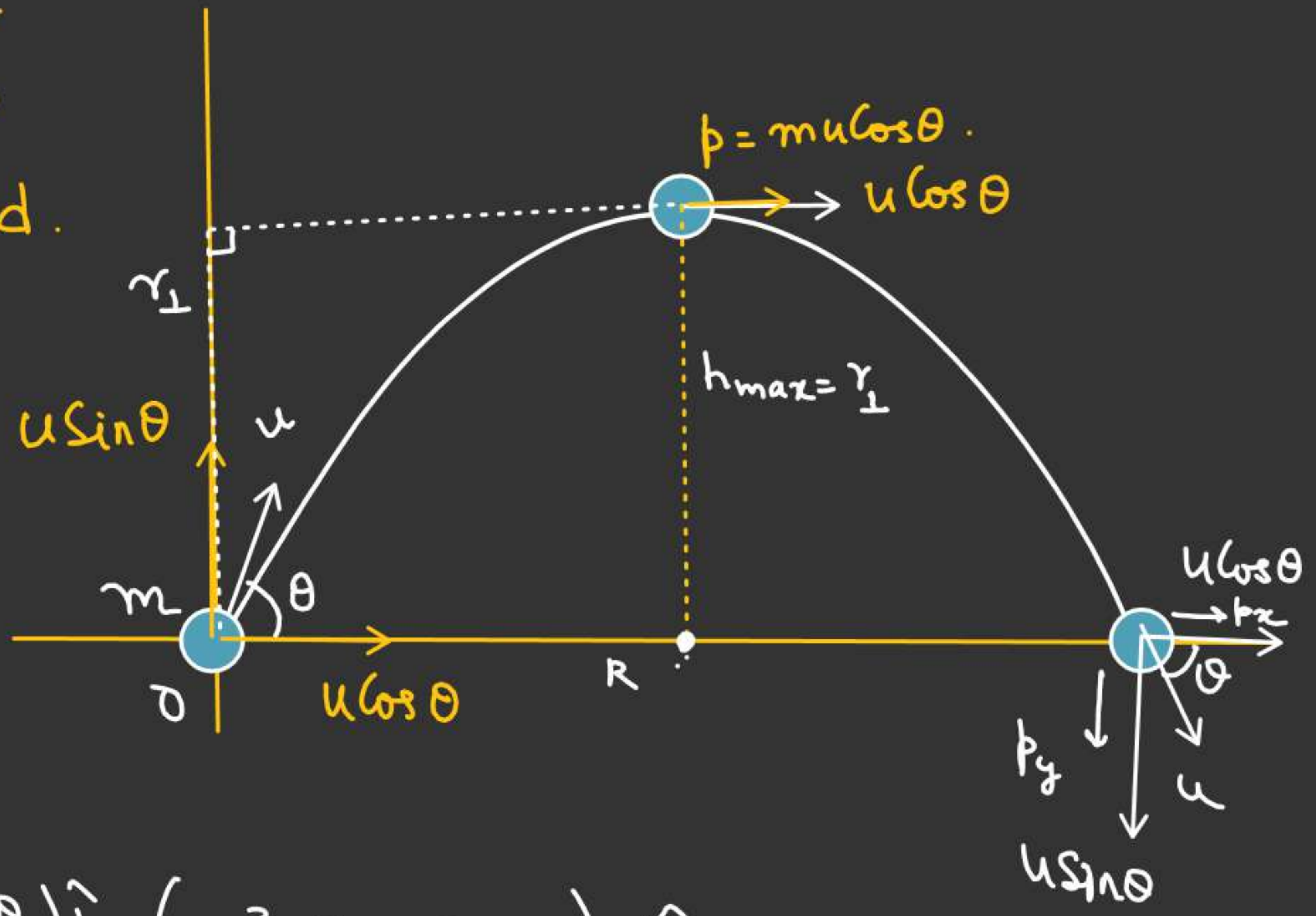
$$= (mgdt)(-\hat{k})$$

✓

ANGULAR MOMENTUM

\# Find Angular momentum of projectile about origin when.

1) projectile at it's highest point.
2) projectile just reaches to ground.



1) $\vec{L}_o = (mu\cos\theta) \times h_{max}\,(-\hat{k})$

$= (mu\cos\theta) \times \left(\frac{u^2 \sin^2\theta}{2g}\right)$

$= \left(\frac{mu^3 \sin^2\theta \cdot \cos\theta}{2g}\right)$

2) $\vec{L}_o = p_y \cdot R = (mu\sin\theta)\left(\frac{u^2 \sin 2\theta}{g}\right)\hat{k} = \left(\frac{mu^3}{g} \sin\theta \cdot \sin 2\theta\right) -\hat{k}$

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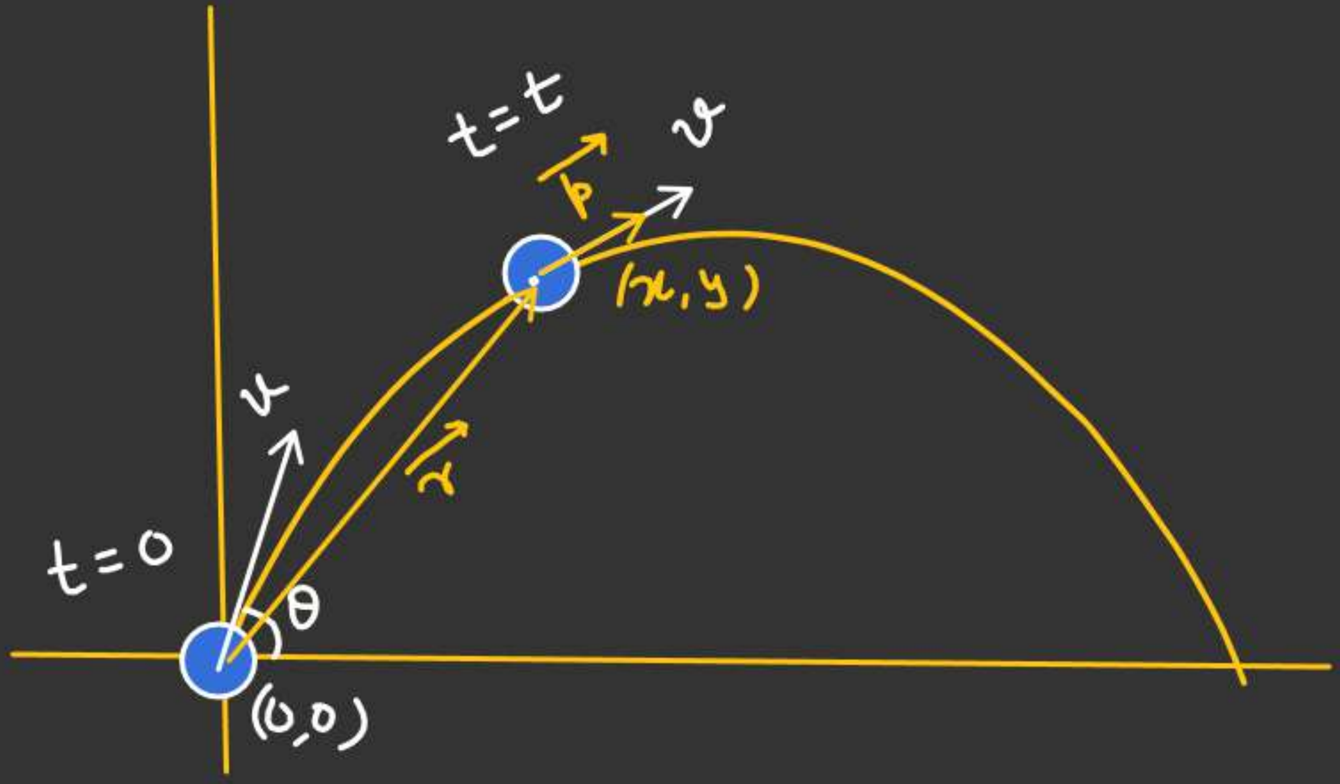
ANGULAR MOMENTUM

Angular Momentum at any time $t = t$.



$$\vec{r} = x\hat{i} + y\hat{j}$$

$$\vec{r} = [(u\cos\theta)t]\hat{i} + \left((u\sin\theta)t - \frac{1}{2}gt^2\right)\hat{j}$$

$$\vec{p} = m\vec{v}.$$

$$\vec{V} = V_x\hat{i} + V_y\hat{j}$$

$$\vec{p} \quad mu(\cos\theta)\hat{i} + m(u\sin\theta - gt)\hat{j}$$

$$\vec{L}_o = \vec{r} \times \vec{p} = \begin{array}{ccc} +\hat{i} & -\hat{j} & \hat{k} \end{array}$$

$$\begin{vmatrix} (u\cos\theta)t & (u\sin\theta)t - \frac{1}{2}gt^2 & 0 \\ mu\cos\theta & (mu\sin\theta - mgt) & 0 \end{vmatrix} = \left[(u\cos\theta)t\,(mu\sin\theta - mgt)\right] - \left[mu\cos\theta\left\{(u\sin\theta)t - \frac{1}{2}gt^2\right\}\right]$$

$$=$$

ANGULAR MOMENTUM

4. Angular Momentum of a body rotating about any axis of rotation

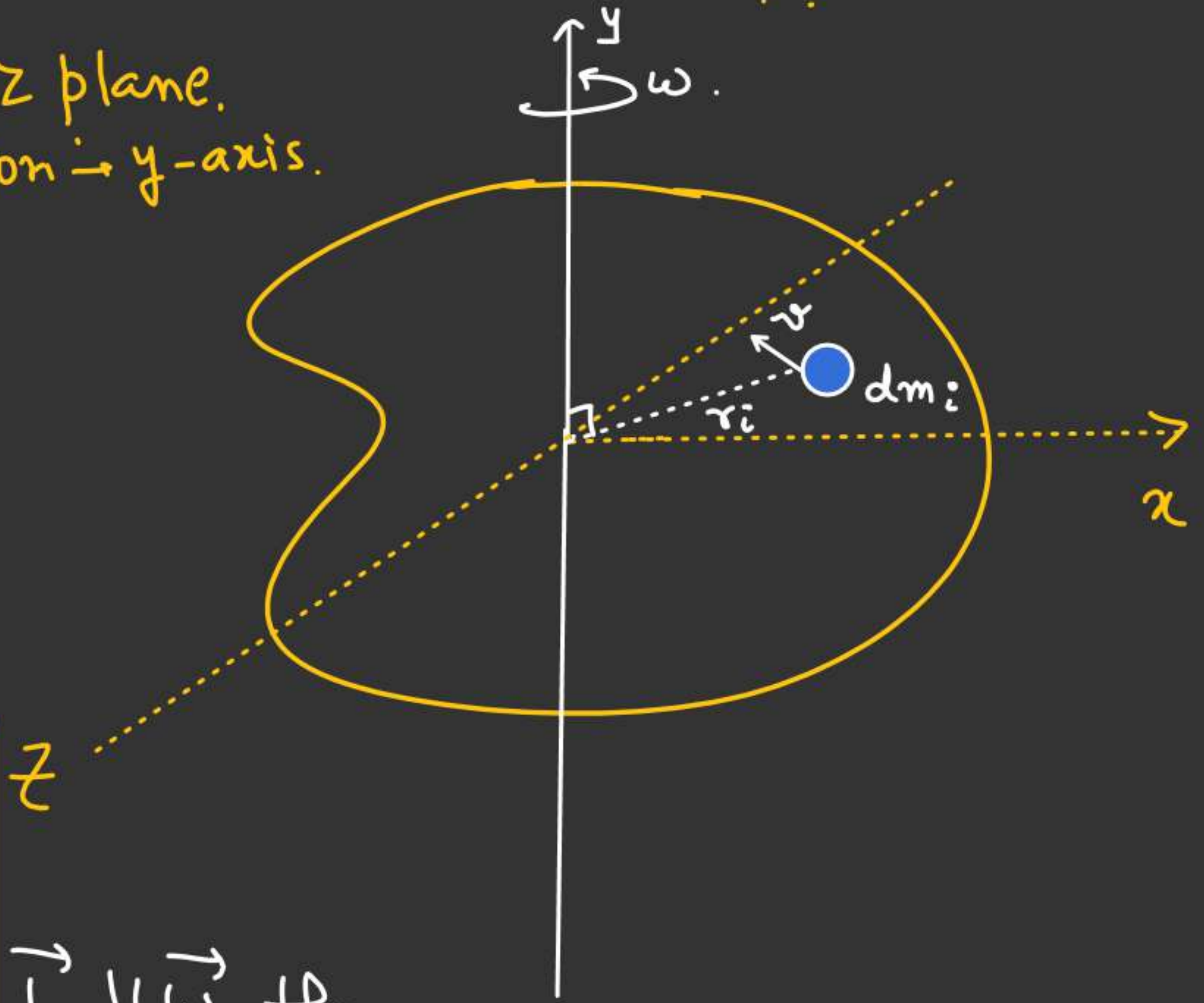
Body in x-z plane.
Axis of rotation → y-axis.

$$dL_i = dm_i v r_i \qquad v = r_i\omega$$

$$\int dL_i = \int \underbrace{dm_i r_i^2}_{\Downarrow} \cdot \omega$$

$(I_{body})_{\text{axis of Rotation}}$

$$\boxed{\vec{L}_{body} = I_{\substack{body \\ \text{about axis of Rotation}}} \cdot \vec{\omega}}$$



$\vec{L} \parallel \vec{\omega}$ then only applicable.

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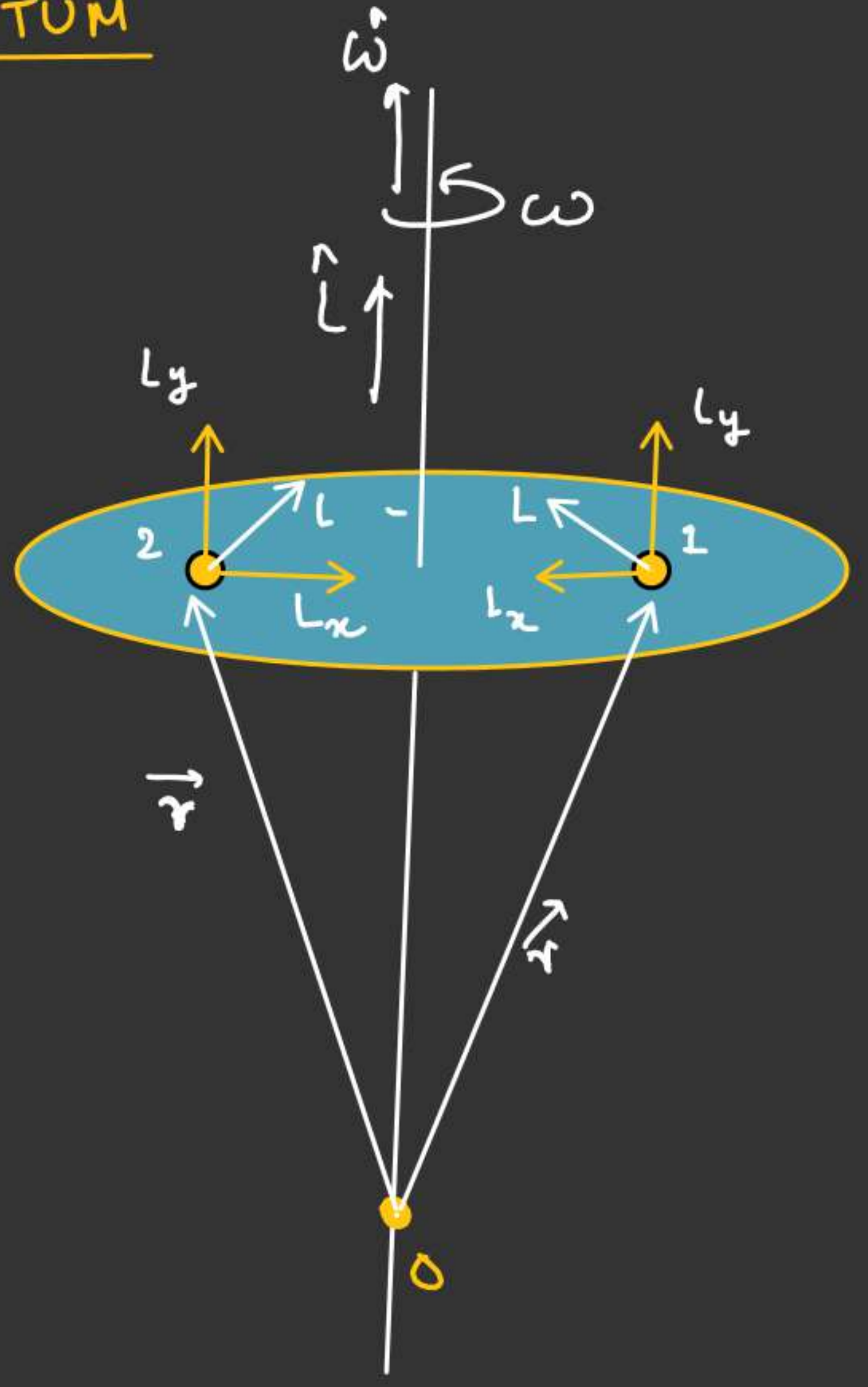
ANGULAR MOMENTUM

When body rotating about axis of Symmetry then only

$$\boxed{\vec{L} = I\vec{\omega}}$$



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ANGULAR MOMENTUM

$$\boxed{\vec{\tau} = \frac{d\vec{L}}{dt}}$$

$$\vec{F} = \frac{d\vec{p}}{dt} \rightarrow \text{2nd Law.}$$

$$\vec{L} = \vec{r} \times \vec{p}.$$

Differentiating both side w.r.t time.

$$\frac{d\vec{L}}{dt} = \vec{r} \times \left(\frac{d\vec{p}}{dt}\right) + \vec{p} \times \left(\frac{d\vec{r}}{dt}\right)$$

$$\frac{d\vec{L}}{dt} = (\vec{r} \times \vec{F}) + (\underbrace{\vec{p} \times \vec{v}}_{=0})$$

$\vec{v} \parallel \vec{p}$

$$\boxed{\frac{d\vec{L}}{dt} = \vec{\tau}}$$

$$\vec{\tau} = \frac{d\vec{L}}{dt}$$

$$\vec{L} = I\vec{\omega}$$

$$\vec{\tau} = \frac{d}{dt}(I\vec{\omega})$$

$$\vec{\tau} = I\frac{d\vec{\omega}}{dt}$$

$$\boxed{\vec{\tau} = I\vec{\alpha}}$$

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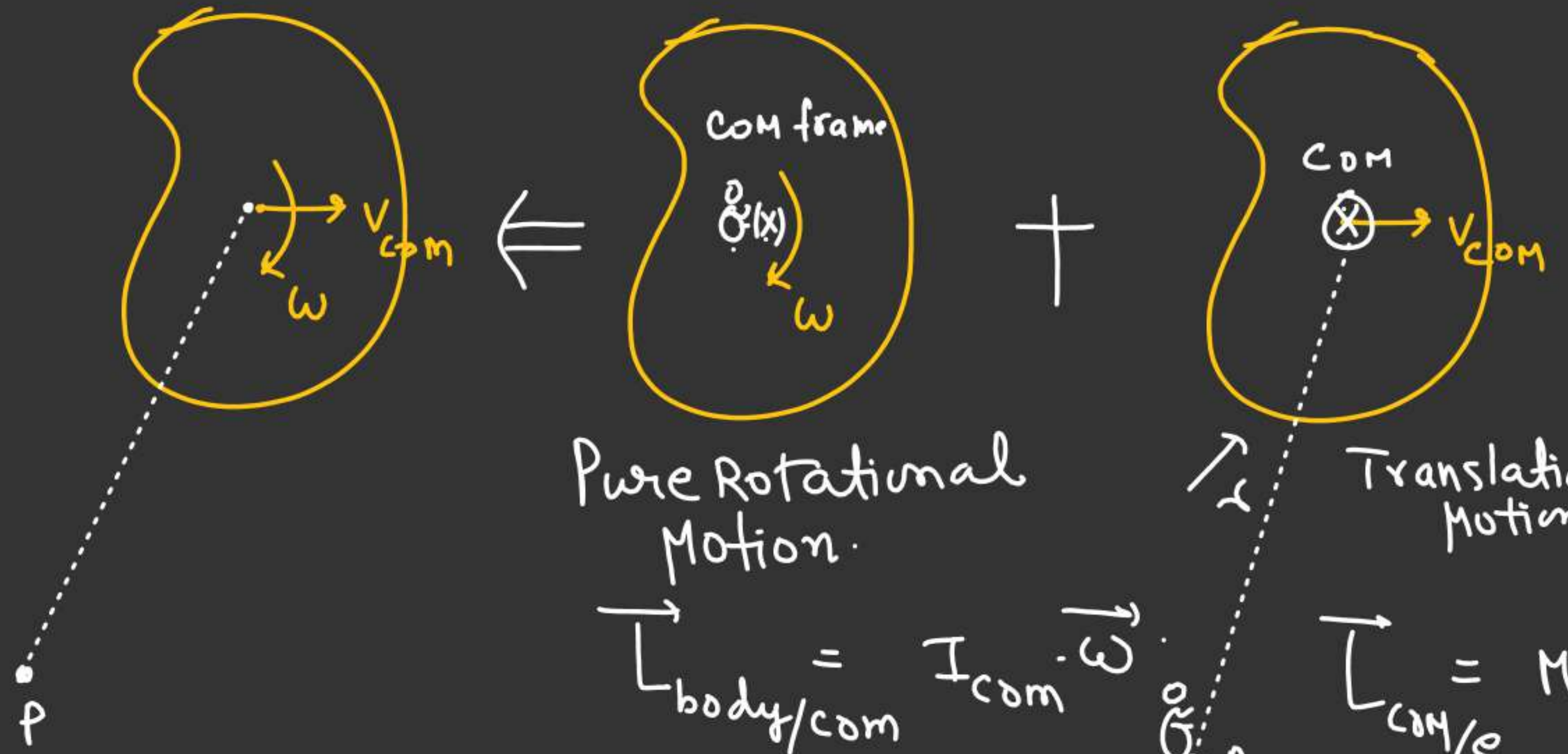
ANGULAR MOMENTUM

Angular Momentum when body has translational as well as rotational Motion



Pure Rotational Motion.

Translational Motion

$$\vec{L}_{body/com} = I_{com}\cdot\vec{\omega}$$

$$\vec{L}_{COM/\varepsilon} = M(\vec{r}\times\vec{V}_{com})$$

$$\boxed{\begin{aligned}\vec{L}_{body/\varepsilon} &= \vec{L}_{body/COM} + \vec{L}_{COM/\varepsilon}\\ &= I_{com}\vec{\omega} + M(\vec{r}\times\vec{V}_{com})\end{aligned}}$$

ANGULAR MOMENTUM

Rod moving in a smooth horizontal plane.

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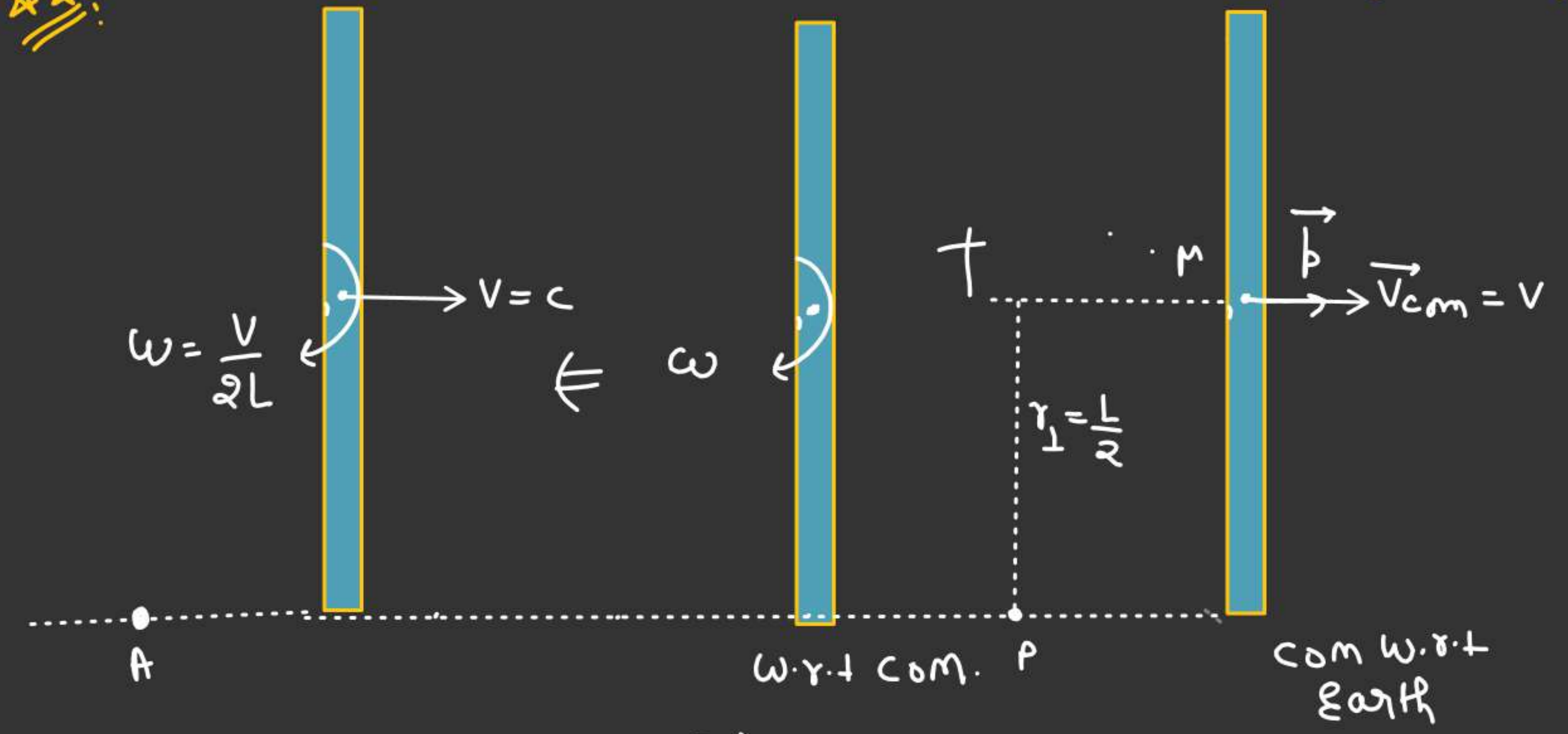

$$\vec{L}_{Rod/com} = \left(\frac{ML^2}{12}\cdot\omega\right)(-\hat{k}) \qquad \vec{L}_{com/E} = MV\frac{L}{2}(-\hat{k})$$

$$\vec{L}_{Rod/E} = \vec{L}_{R/com} + \vec{L}_{com/E} = \left[\frac{ML^2}{12}\times\frac{V}{2L} + \frac{MVL}{2}\right](-\hat{k}) = \left(\frac{MVL}{24} + \frac{MVL}{2}\right)(-\hat{k})$$

$$= \underline{\frac{13}{24}MVL(-\hat{k})}$$