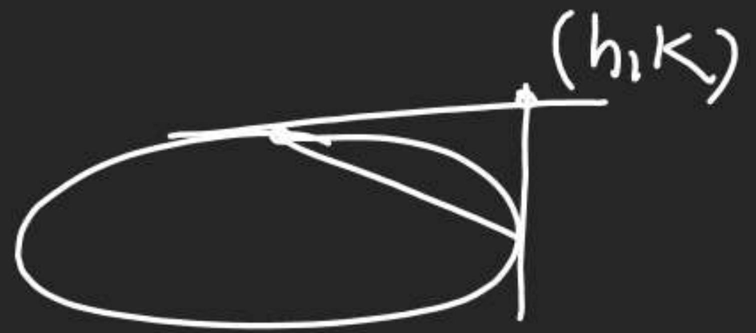
Nishant Jindal

E:



$$(h,K) = \left\{ \frac{a\cos\left(\frac{\alpha+\beta}{2}\right)}{\cos\left(\frac{\alpha-\beta}{2}\right)}, \frac{b\sin\left(\frac{\alpha+\beta}{2}\right)}{\cos\left(\frac{\alpha-\beta}{2}\right)} \right\}$$

H:

$$(h,K) = \left\{ \frac{a\cos\left(\frac{\alpha-\beta}{2}\right)}{\cos\left(\frac{\alpha+\beta}{2}\right)}, \frac{b\sin\left(\frac{\alpha+\beta}{2}\right)}{\cos\left(\frac{\alpha+\beta}{2}\right)} \right\}$$

Aise mila Kr yaad Rakhen....

$$\frac{x}{a}\cos\left(\frac{\alpha-\beta}{2}\right) - \frac{y}{b}\sin\left(\frac{\alpha+\beta}{2}\right) = \cos\left(\frac{\alpha+\beta}{2}\right)$$

$-$ $\quad -$ $\quad +$ $\quad +$

(24) If Chord joining $P(\alpha), Q(\beta)$ P.T. $(d,0)$

E: $$\frac{d-a}{d+a} = \tan\frac{\alpha}{2}\cdot\tan\frac{\beta}{2}$$

Similarly for Hyp.

$$\frac{d-a}{d+a} = -\tan\frac{\alpha}{2}\cdot\tan\frac{\beta}{2}$$

(25) If Chord P.T. Focus $(ae,0)$

$$\frac{e-1}{e+1} = -\tan\frac{\alpha}{2}\cdot\tan\frac{\beta}{2}$$

Q Find Eqn of Com. tangent to Parabola $y^2=8x$ & $H: 3x^2-y^2=3$?

$a=2$

EOT $\Rightarrow y = mx + \frac{2}{m}$

$H: \frac{x^2}{1}-\frac{y^2}{3}=1$

EOT: $y = mx \pm \sqrt{1m^2-3}$

Both tangents are Same.

$$\frac{2}{m} = \pm\sqrt{m^2-3}$$

$$\frac{4}{m^2} = m^2-3 \Rightarrow m^4-3m^2-4=0$$

$$(m^2-4)(m^2+1)=0$$

$$m=\pm 2$$

EOT $\Rightarrow y = \pm 2x \pm 1$

$y=2x+1$

$y=-2x-1$

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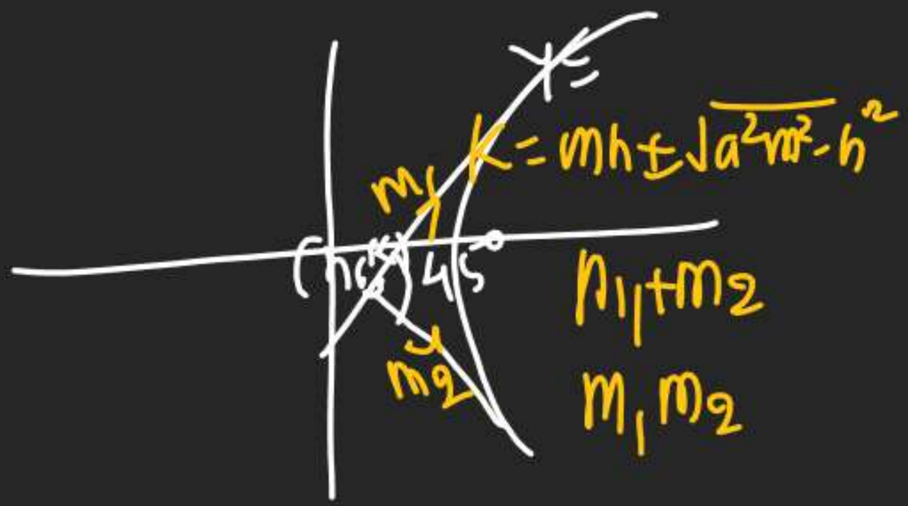
Q Tangents are drawn to Hyp. $x^2-y^2=a^2$ enclosed an angle of 45°. P.T. Locus of their Point of Int. is $(x^2+y^2)^2+4a^2(x^2-y^2)=4a^4$?

$H: x^2-y^2=a^2 \Rightarrow e=\sqrt{2}$



m_1+m_2

m_1m_2

$\tan 45^\circ = \frac{|m_1-m_2|}{1+m_1m_2} = 1$

$(1+m_1m_2)^2 = (m_1-m_2)^2$

$\Rightarrow (1+m_1m_2)^2 = (m_1+m_2)^2 - 4m_1m_2$

$\Rightarrow \left(1+\frac{k^2+a^2}{h^2-a^2}\right)^2 = \left(\frac{2kh}{h^2-a^2}\right)^2 - 4\frac{(k^2+a^2)}{(h^2-a^2)}$

$\Rightarrow \underbrace{(h^2+k^2)^2 = 4k^2h^2 - 4(k^2+a^2)(h^2-a^2)}$

H.P.

Q Find Eqn of tangent to $H: \frac{x^2}{36}-\frac{y^2}{9}=1$ P.T. $(0,4)$

$y = mx \pm \sqrt{36m^2-9}$ P.T. $(0,4)$

$4 = 0 \pm \sqrt{36m^2-9}$

$\Rightarrow 16 = 36m^2-9 \Rightarrow m^2 = \frac{25}{36} \Rightarrow m = \pm\frac{5}{6}$

$y = \pm\frac{5}{6}x+4$ as it is P.T. $(0,4)$

Q

Q P.T. 2 tangents drawn from any pt on hyp. $x^2-y^2=a^2-b^2$ to E: $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ makes complementary angle with x Axis?

To Prove → $\boxed{m_1 m_2 = 1}$
करना है

$m_1 = \frac{1}{m_2}$

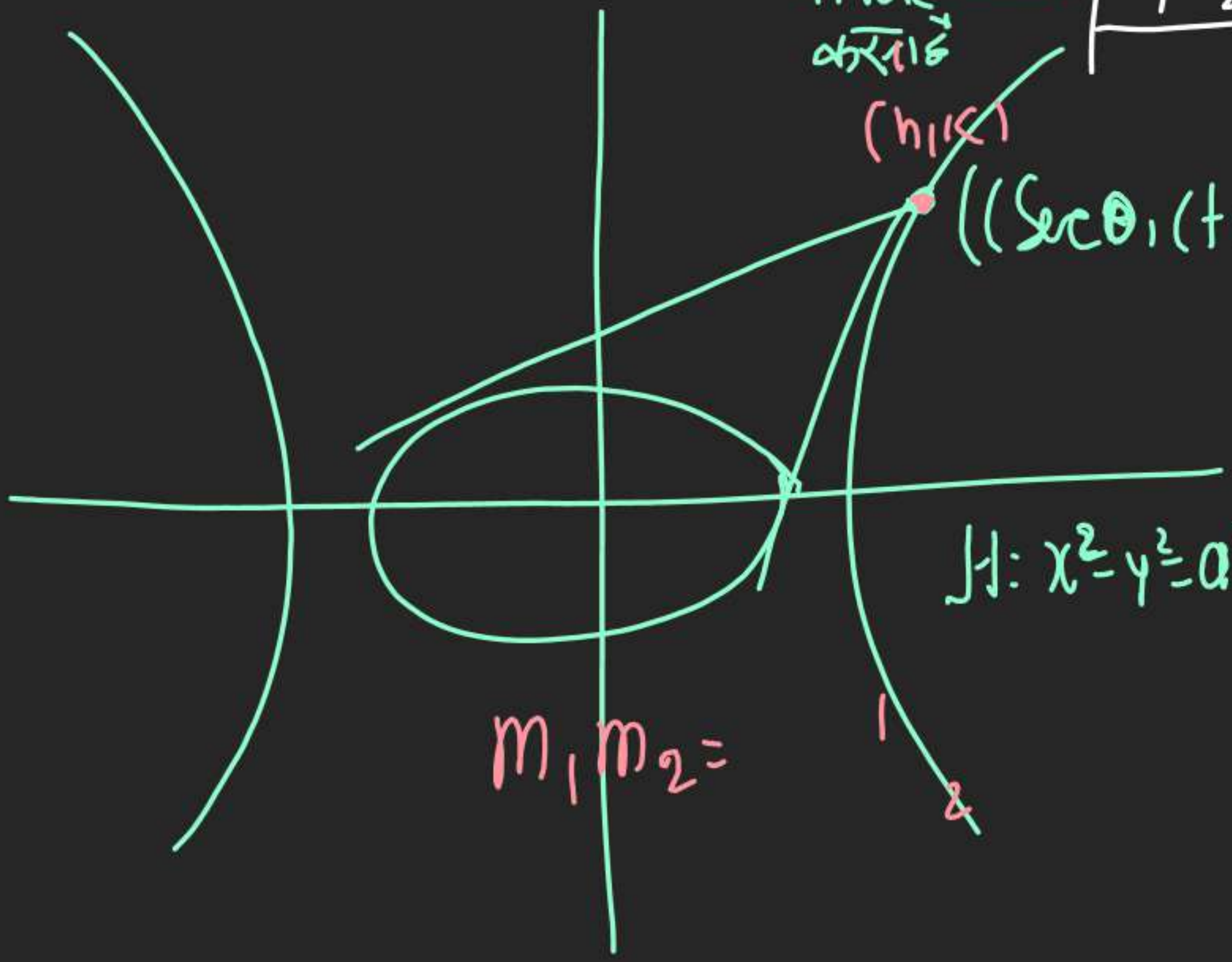
$\tan\alpha = \frac{1}{\tan\beta}$

$\tan\alpha = \cot\beta$

$\tan\alpha = \tan\left(\frac{\pi}{2}-\beta\right)$

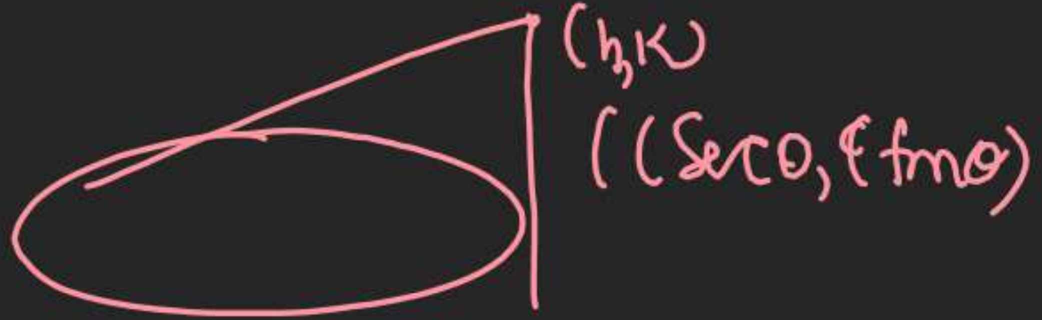
$\alpha = \frac{\pi}{2}-\beta$

$\boxed{\alpha+\beta = 90^\circ}$



$H: x^2-y^2=a^2-b^2=c^2$

$m_1 m_2 =$



$$m_1 m_2 = \frac{k^2-b^2}{h^2-a^2}$$

$$m_1 m_2 = \frac{c^2\tan^2\theta-b^2}{c^2\sec^2\theta-a^2}$$

$$= \frac{(a^2-b^2)\tan^2\theta-b^2}{(a^2-b^2)\sec^2\theta-a^2}$$

$$= \frac{a^2\tan^2\theta-b^2-b^2\tan^2\theta}{a^2\sec^2\theta-a^2-b^2\sec^2\theta}$$

$$= \frac{\cancel{a^2\tan^2\theta-b^2\sec^2\theta}}{\cancel{a^2\tan^2\theta-b^2\sec^2\theta}} = 1$$

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Q Find Eqn of Com. tangent $\underbrace{\frac{x^2}{a^2}-\frac{y^2}{b^2}=-1}$

to $H_1: \frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ & $H_2: \underbrace{\frac{x^2}{b^2}-\frac{y^2}{a^2}=-1}$

Not C.H.

$y=mx\pm\sqrt{a^2m^2-b^2}$

इसकी tangent के लिए do a change

$$\frac{x^2}{(-b^2)}-\frac{y^2}{(-a^2)}=1$$

$$y=mx\pm\sqrt{-b^2m^2-(-a^2)}$$

Compare

$$\pm\sqrt{a^2m^2-b^2}=\pm\sqrt{-b^2m^2+a^2}$$

$$a^2m^2-b^2=-b^2m^2+a^2$$

$$a^2m^2+b^2m^2=a^2+b^2 \Rightarrow m^2=1 \Rightarrow m=\pm 1$$

$$y=\pm x\pm\sqrt{a^2-b^2}$$

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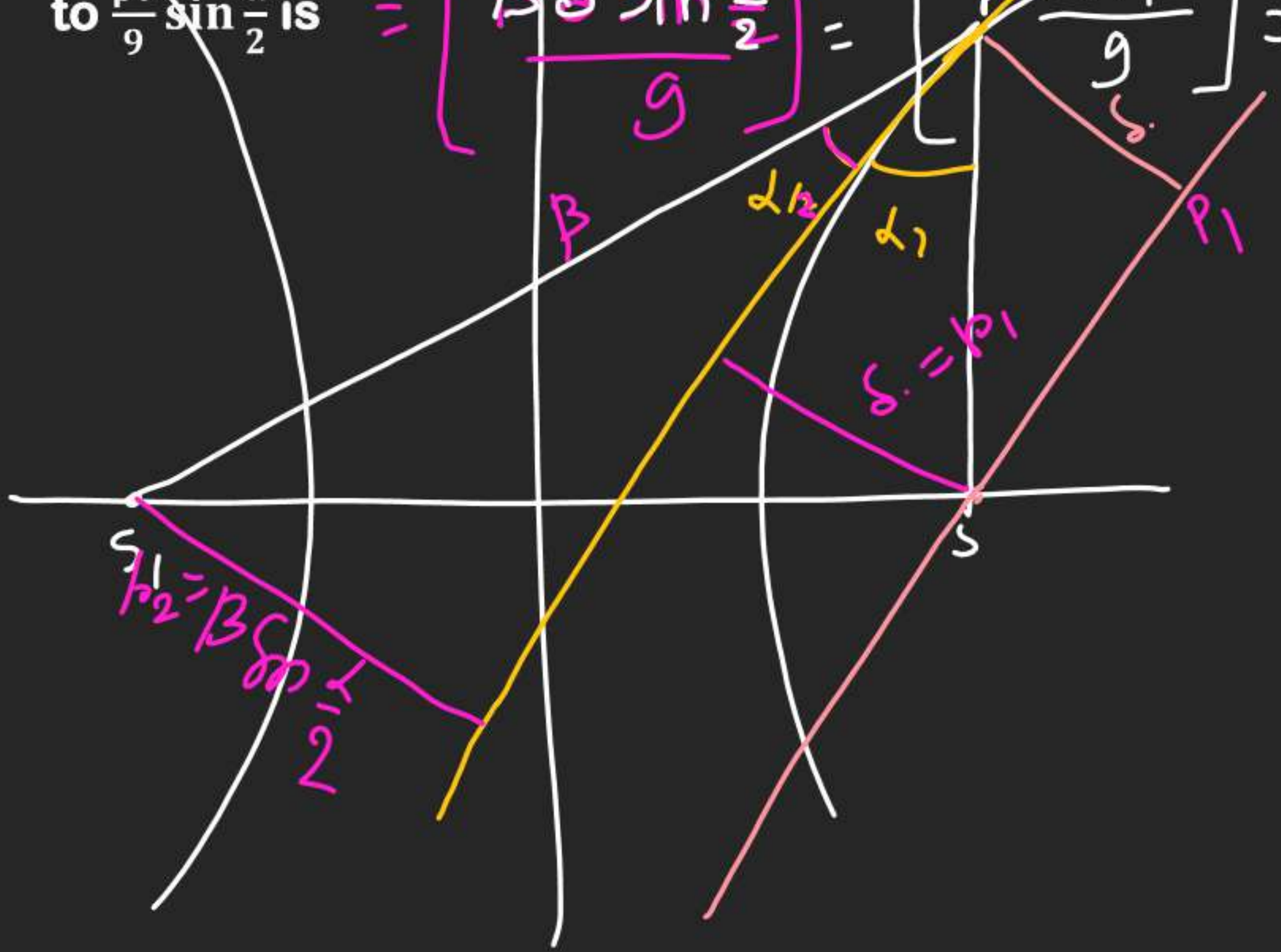
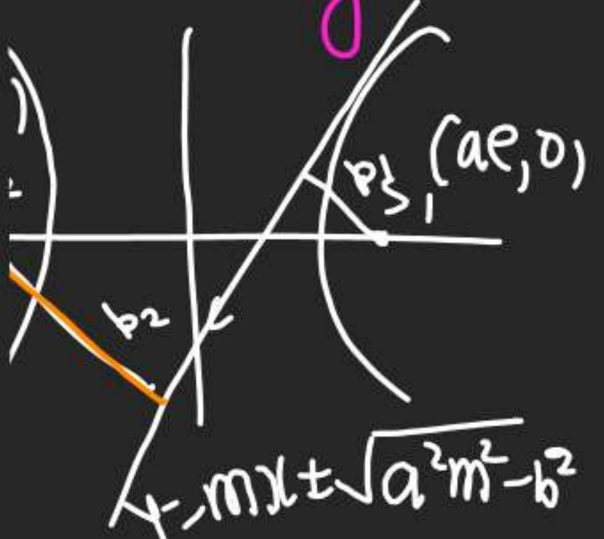
Q. Consider the hyperbola $\frac{x^2}{100} - \frac{y^2}{64} = 1$ with foci at S and S_1, where S lies on the positive X-axis, Let P be a point on the hyperbola, in the first quadrant. Let $\angle SPS_1 = \alpha$, with $\alpha < \frac{\pi}{2}$. The straight line passing through the point S and having the same slope as that of the tangent at P to the hyperbola, intersects the straight line S_1P at P_1. Let δ be the distance of P from the straight line SP_1, and $\beta = S_1P$. Then the greatest integer less than or equal to $\frac{\beta\delta}{9}\sin\frac{\alpha}{2}$ is

$$= \left[\frac{\beta\delta\sin\frac{\alpha}{2}}{9}\right] = \left[\frac{P_2 \cdot P_1}{9}\right] = \left[\frac{b^2}{9}\right] = \left[\frac{64}{9}\right] = [7.\ldots] = 7$$



...d Prod of ⊥r distances from Both foci of hyp. to tangent



$y = mx \pm \sqrt{a^2m^2 - b^2}$

$= \frac{(aem + 0 \pm \sqrt{a^2m^2 - b^2})}{\sqrt{m^2+1}} \times \frac{(-aem \pm \sqrt{a^2m^2 - b^2})}{\sqrt{m^2+1}}$

$= \frac{-a^2e^2m^2 + a^2m^2 - b^2}{m^2+1} = \frac{a^2m^2(-\frac{b^2}{a^2}) - b^2}{m^2+1}$

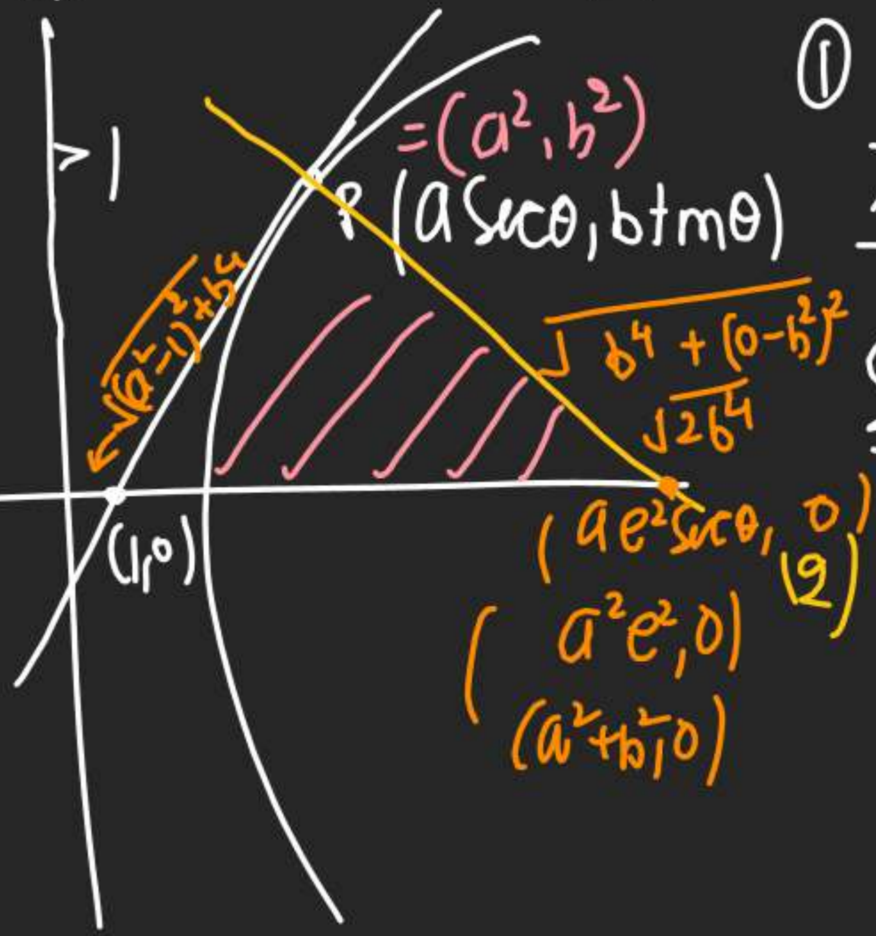
$= \frac{-b^2(m^2+1)}{(m^2+1)} = |-b^2| = b^2$

Q. Let a and b be positive real numbers such that $a > 1$ and $b < a$. Let P be a point in the first quadrant that lies on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. Suppose the tangent to the hyperbola at P passes through the point $(1, 0)$, and suppose the normal to the hyperbola at P cuts off equal intercepts on the coordinate axes. Let Δ denote the area of the triangle formed by the tangent at P, the normal at P and the x-axis. If e denotes the eccentricity of the hyperbola, then which of the following statements is/are TRUE?

$m = -1$

$\frac{a^2x}{x_1} + \frac{b^2y}{y_1} = a^2 + b^2$

(A) $1 < e < \sqrt{2}$ (B) $\sqrt{2} < e < 2$ (C) $\Delta = a^4$ (D) $\Delta = b^4$



$e^2 = 1 + \frac{b^2}{a^2}$

$e^2 < 2$

$\Rightarrow e < \sqrt{2}$

① EOT

$\frac{x\sec\theta}{a} - \frac{y\tan\theta}{b} = 1$ P.T. (1,0)

$\frac{\sec\theta}{a} = 1 \Rightarrow \boxed{a = \sec\theta}$

② EON

$ax\cos\theta + by\cot\theta = a^2e^2$

$m = \frac{-a\cos\theta}{b\cot\theta} = -1 \Rightarrow a\sin\theta = b$

$\Rightarrow \boxed{b = \tan\theta}$

③ $\Delta = \frac{1}{2} \times \sqrt{(a^2-1)^2 + b^4} \times \sqrt{2b^4}$

$\sec^2\theta - 1 = \tan^2\theta$

$a^2 - 1 = b^2 \Rightarrow a^2 - b^2 = 1$

$= \frac{1}{2}\sqrt{2b^4} \times \sqrt{2b^4} = b^4$

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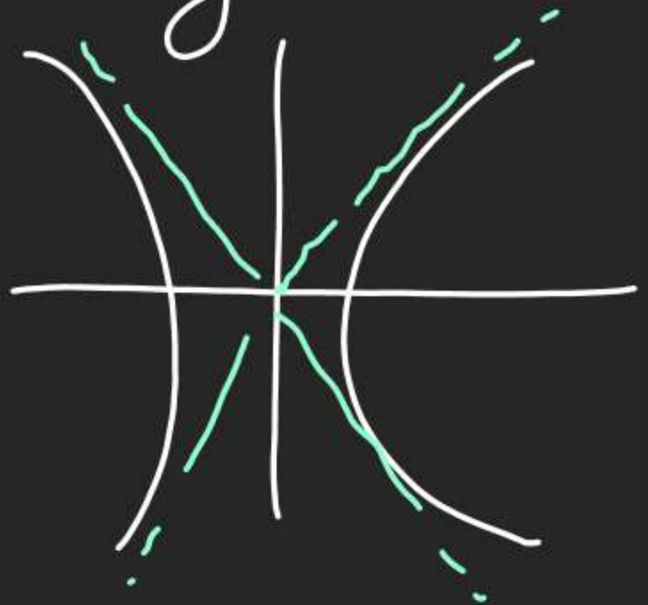
25) Assymptotes.

1) tangent at ∞

2)

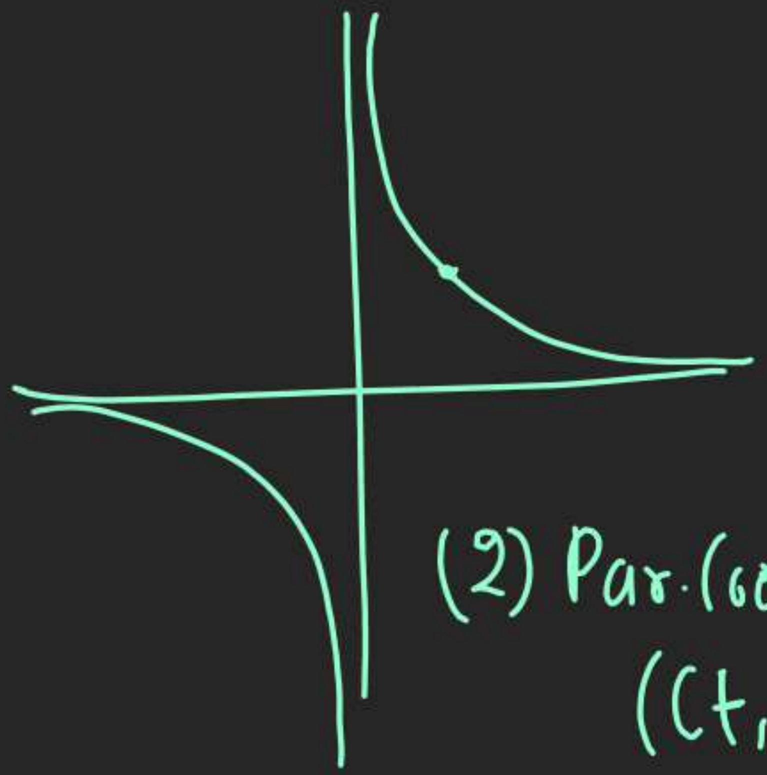


(3) Hyp $\Rightarrow \dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$

Assy $\Rightarrow y = \pm \dfrac{b}{a} x$

(26) Special R. H.

1) $xy = c^2$ is R. H.



(2) Par. Coord

$\left(ct, \dfrac{c}{t}\right)$

(3) EOT $\Rightarrow xy = c^2$

$\hookrightarrow \dfrac{xy_1 + yx_1}{2} = c^2$

H.N.Y