Nishant Jindal

Electric Potential

⇒ **Electric potential due to a point charge.**

d

$$\boxed{V_{+Q} = \frac{KQ}{r}} \;**$$

Put Q with Sign (*)

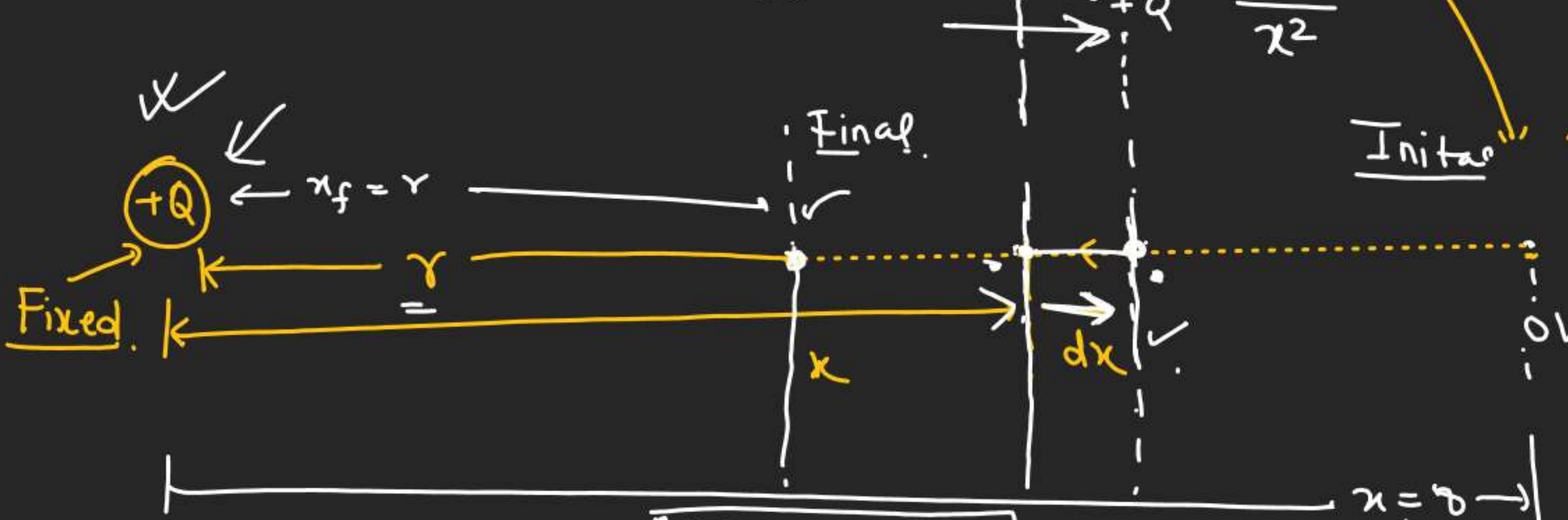
$\boxed{V_\infty = 0}$ Refrance potential for V_{+Q}.

$dV = -\vec{E}\cdot d\vec{x}$

⇓

Potential due to +Q at x

$E_{+Q} = \frac{KQ}{x^2}$



$\boxed{V_\infty = 0}$

$$\int_{V(\infty)}^{V(r)} dV = -\int_{\infty}^{r} E_x\, dx$$

$$\int dV = -\int \frac{KQ}{x^2}\, dx$$

$$V(r) - \cancel{V(\infty)}^{\,0} = -KQ\left[\frac{-1}{x}\right]_\infty^r$$

$$\boxed{V(r) = \frac{KQ}{r}}$$

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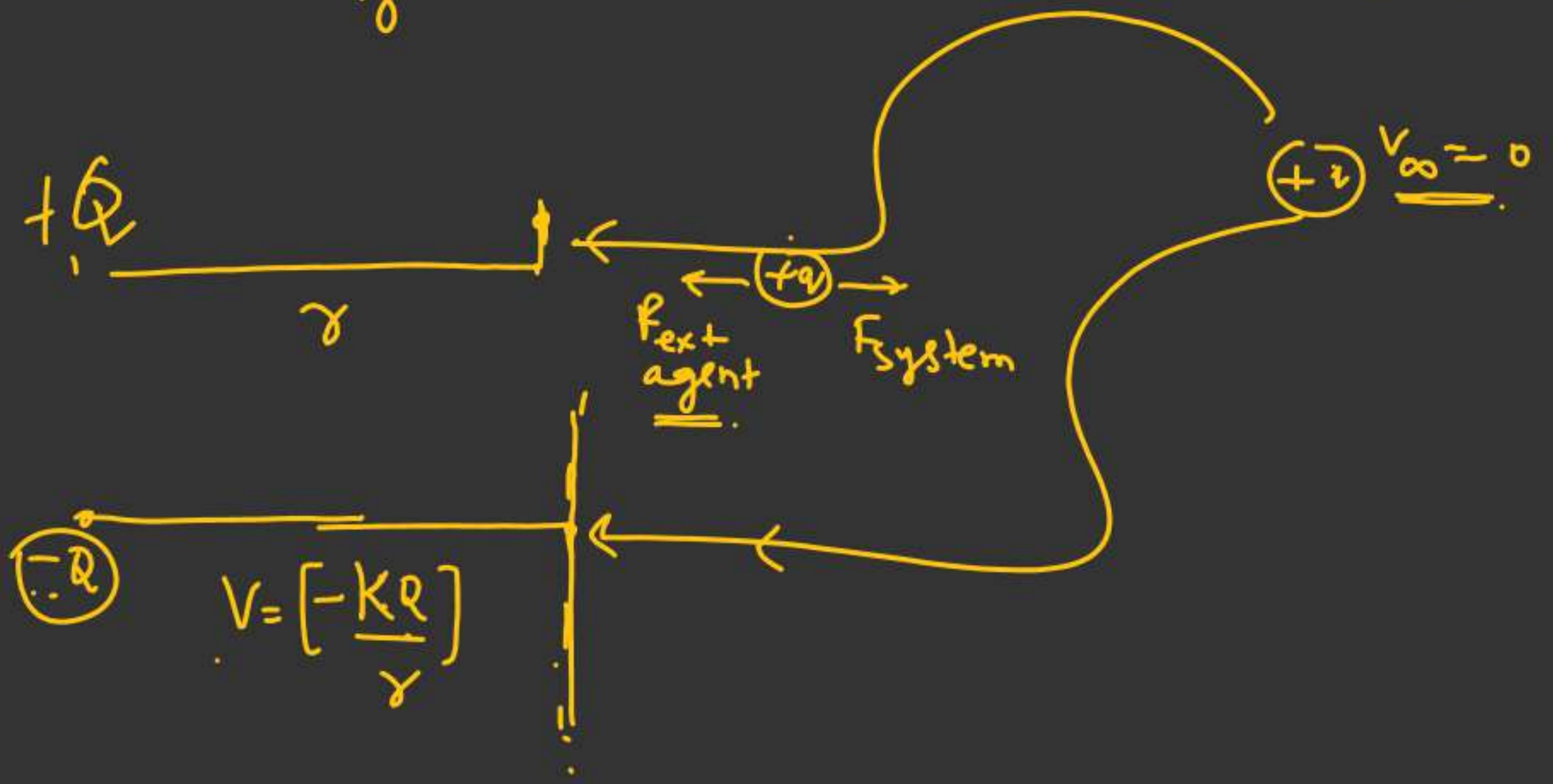
$V = \frac{KQ}{r}$

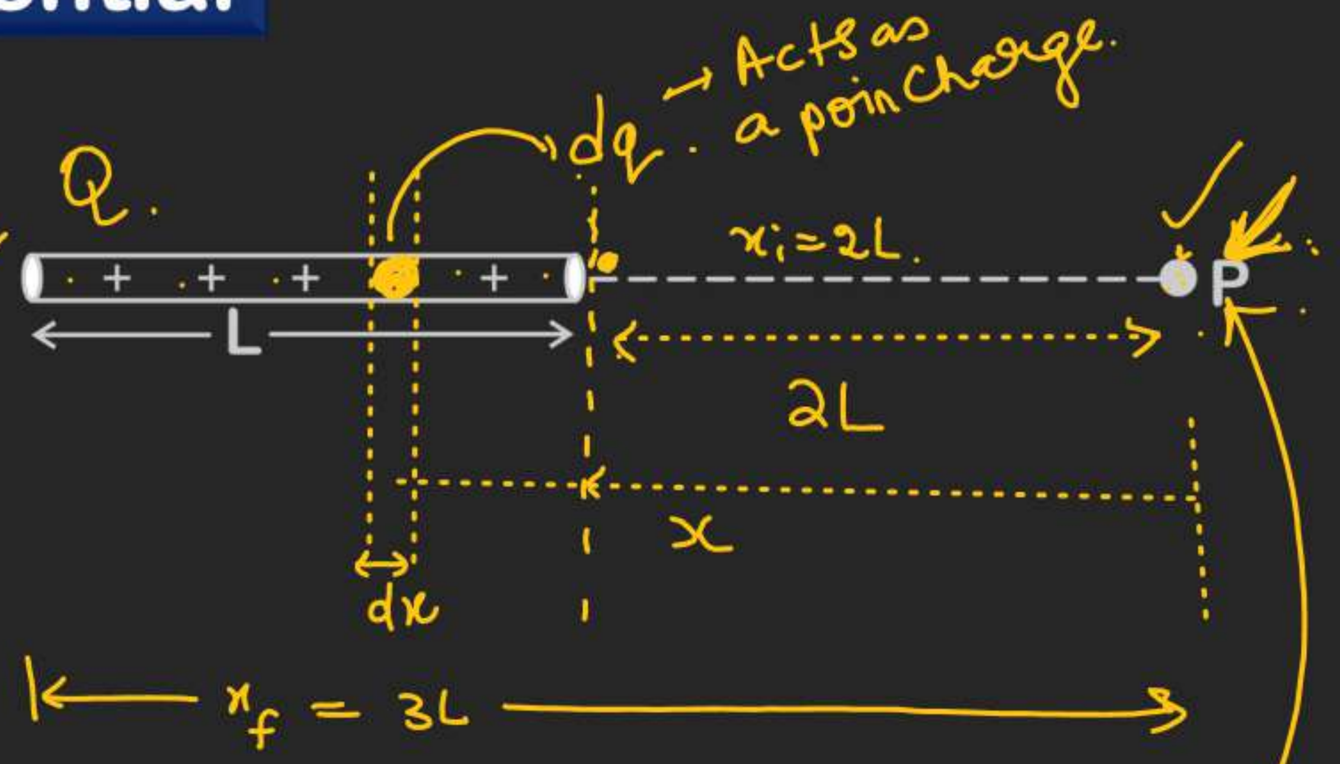
$\boxed{V = \frac{U}{q}}$ $\boxed{V = \frac{W_{ext\,agent}}{q}}$



Electric Potential

$k = 1/4\pi\varepsilon_0$

⇒ **Electric potential due to a charge rod.**



$\lambda = \left(\frac{Q}{L}\right)$

$V = \frac{kQ}{r}$ → dq

V → dV, r → x

$dq = \lambda dx = \left(\frac{Q}{L} dx\right)$

$dV = \left[\frac{k\,dq}{x}\right]$

Potential due to dq

$\int_0^V dV = \frac{KQ}{L}\int_{2L}^{3L}\frac{dx}{x}$

$V = \frac{KQ}{L}\ln[x]_{2L}^{3L}$

$\boxed{V = \frac{kQ}{L}\ln\left(\frac{3}{2}\right)}$

$\boxed{V_\infty = 0}$

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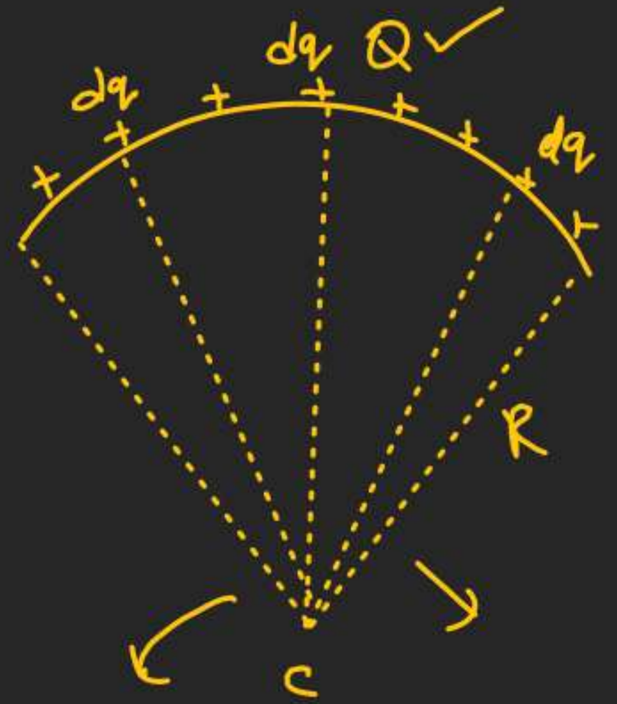
Electric Potential

⇒ **Electric potential due to a charge ring at its center.**

$$\int dV_c = \int \frac{K\,dq}{R}$$

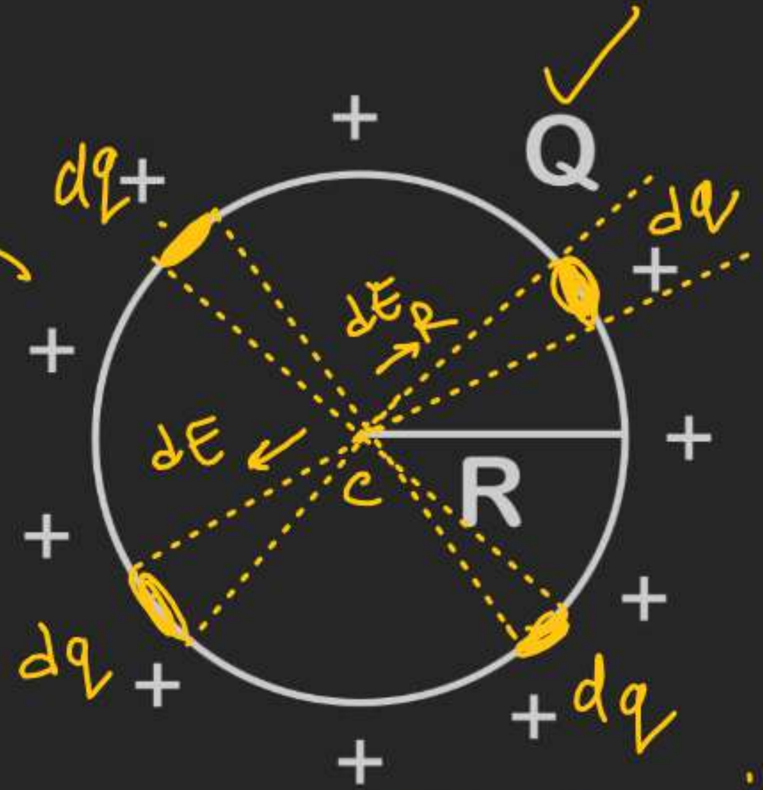
$$V_c = \frac{K}{R}\left(\int dq\right)$$

$$\boxed{V_c = \frac{KQ}{R}}$$



$$\int dV_c = \frac{K}{R}\int dq$$

$$\boxed{(V_c)_{arc} = \frac{K\bar{Q}}{R}}$$



$$\boxed{E_c = 0}$$

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Electric Potential

⇒ **Electric potential due to a charge ring at its axis point.**

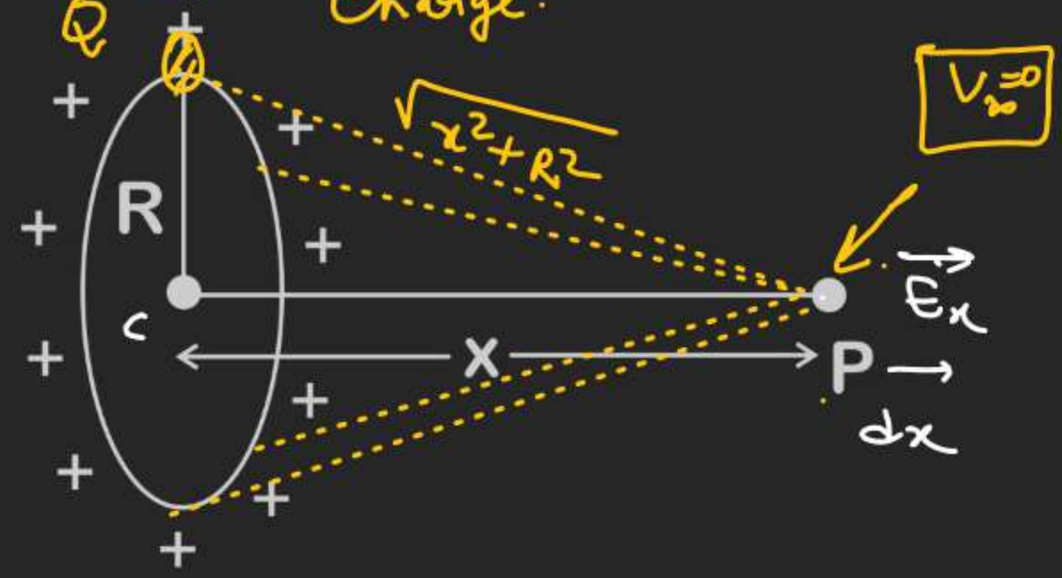
let 'dV' be the potential at P.

$$\int_0^V dV = \int \frac{K\,dq}{\sqrt{x^2+R^2}}$$

dq.

$$\int dV = \frac{K}{\sqrt{x^2+R^2}} \int dq$$

$$\boxed{V = \frac{KQ}{\sqrt{x^2+R^2}}}$$ **



Another Method

$$E_{ring} = \frac{KQx}{(x^2+R^2)^{3/2}}$$

$$\int_0^{V(x)} dV = -KQ \int_\infty^x \frac{x}{(x^2+R^2)^{3/2}}$$

$$\boxed{x^2+R^2 = t}$$

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Electric Potential

Electric potential due to a uniformly Charge disc at it's center:-

$$dV = \frac{K\,dq}{\sqrt{x^2+r^2}}$$

$dq = \sigma . dA$ ↳ differential area of ring.

$dq = \sigma (2\pi r) dr$

σ = Surface Charge density

$$dV = \frac{1}{\cancel{4}\pi\varepsilon_0} \times \sigma \times \cancel{2\pi} \frac{r\,dr}{\sqrt{x^2+r^2}}$$

$$\int_0^V dV = \frac{\sigma}{2\varepsilon_0}\int_0^R \frac{r\,dr}{\sqrt{x^2+r^2}}$$

put $\boxed{x^2+r^2 = t}$

$$\frac{d}{dt}(x^2+r^2) = \frac{d}{dt}(t)$$

$$\frac{d}{dt}(x^2) + \frac{d}{dt}r^2 = 1$$

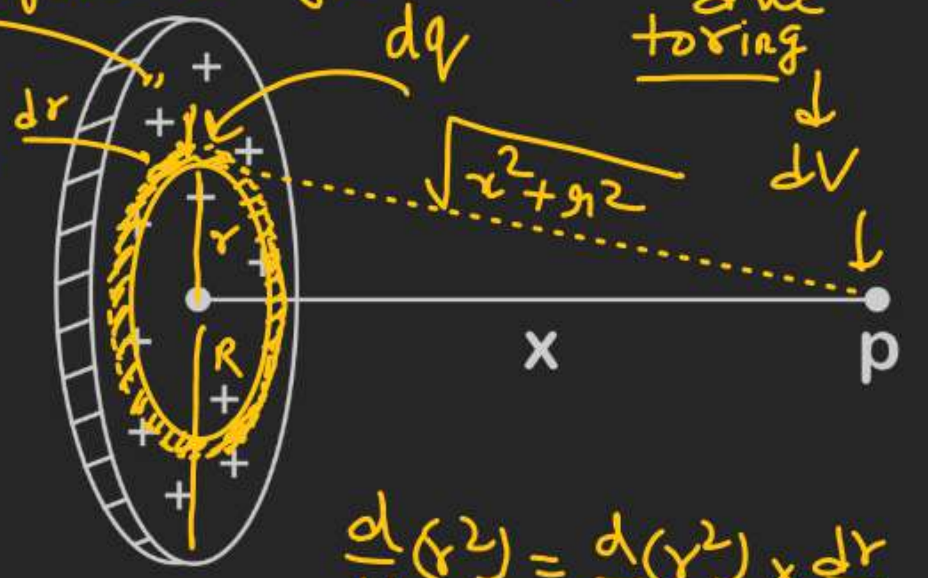
$$0 + 2r\left(\frac{dr}{dt}\right) = 1$$

$$r\,dr = \left(\frac{dt}{2}\right)$$

$$\frac{d}{dt}(r^2) = \frac{d}{dr}(r^2) \times \frac{dr}{dt}$$

$$\left(2r\frac{dr}{dt}\right)$$

$$\int_0^V dV = \frac{\sigma}{2\varepsilon_0}\int_0^R \frac{dt}{2\sqrt{t}} = \frac{\sigma}{4\varepsilon_0}\int_0^R t^{-1/2}\,dt$$



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ELECTROSTATIC POTENTIAL AND POTENTIAL ENERGY

$$V = \frac{\sigma}{4\varepsilon_0} \int_0^R t^{-1/2} \cdot dt$$

$$\int x^n dx = \frac{x^{n+1}}{n+1}$$

$$V = \frac{\sigma}{4\varepsilon_0} \frac{\left[t^{1/2}\right]_0^R}{1/2}$$

$$\boxed{V = \frac{\sigma}{2\varepsilon_0} \left[\sqrt{x^2+R^2} - x\right]}$$

A+

$$V = \frac{\sigma}{2\varepsilon_0} \left[\sqrt{t}\right]_0^R$$

At center of disc

$$\boxed{x = 0}$$

$$V = \frac{\sigma}{2\varepsilon_0} \left[\sqrt{x^2+r^2}\right]_0^R$$

XX

$$\boxed{V_{\text{center of disc}} = \frac{\sigma R}{2\varepsilon_0}}$$

ELECTROSTATIC POTENTIAL AND POTENTIAL ENERGY

Potential due to uniformly charged conducting Sphere-

① $r < R$ (Inside).

$V = ??$

$$\int_{V_c}^{V_A} dV = -\int_0^r E\,dr$$

$E \to 0$

$$V_A - V_c = 0$$

$$\boxed{V_A = V_c}$$

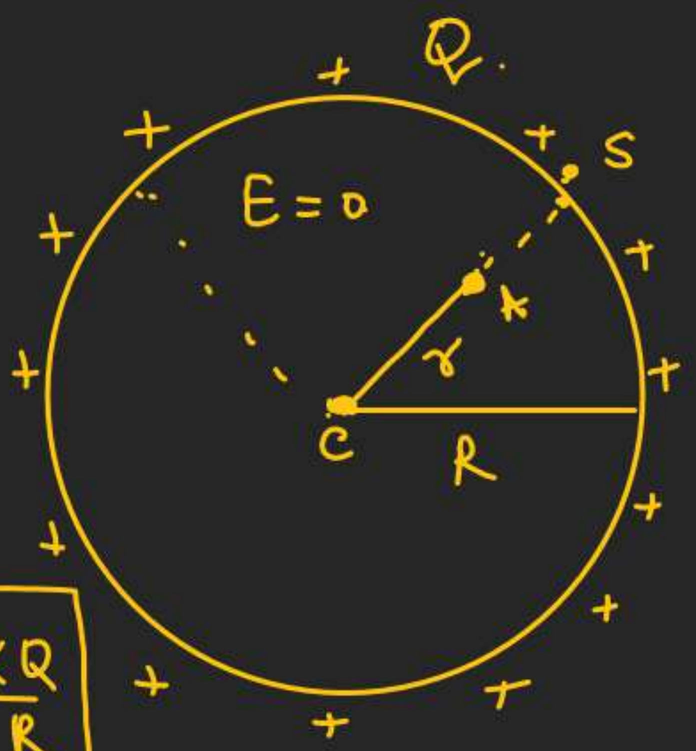
$$\int_{V_c}^{V_s} dV = -\int_0^R E\,dr$$

$E \to 0$

$$V_s - V_c = 0$$

$$\boxed{V_s = V_c}$$

$$\boxed{V_c = V_s = V_A = \frac{KQ}{R}}$$



② $r > R$

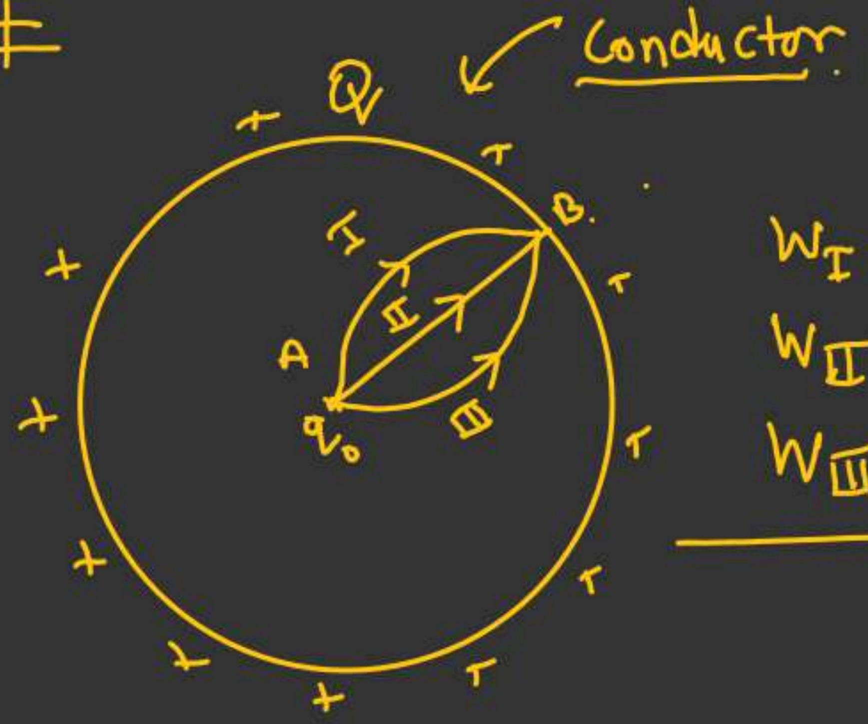
$E = \frac{KQ}{r^2}$

Behave as a point Charge.

$$\boxed{V = \frac{KQ}{r}}$$

put with Sign

#



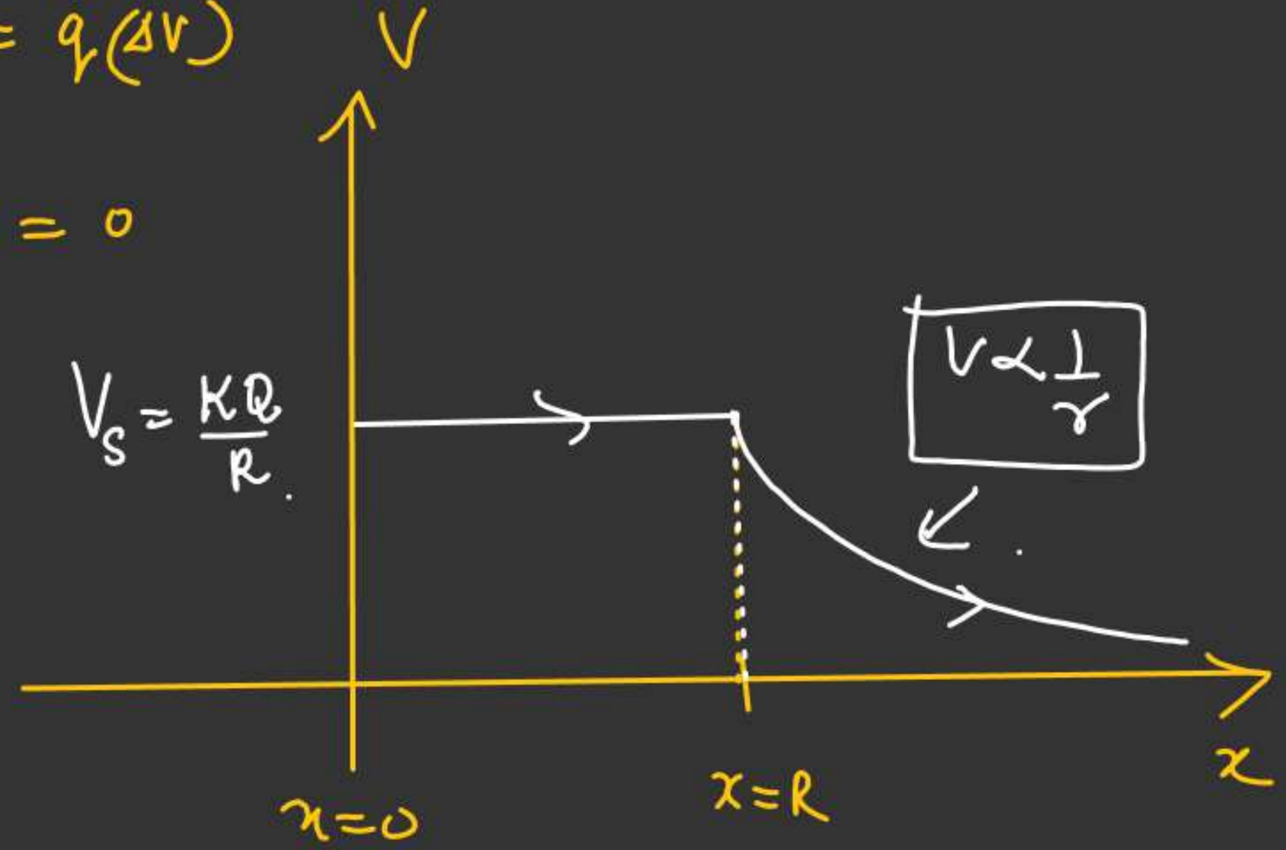
$W = \Delta U = q(\Delta V)$

V

$W_{I} = q_0 \, (\underbrace{V_B - V_A}_{0}) = 0$

$W_{II} = 0$

$W_{III} = 0$



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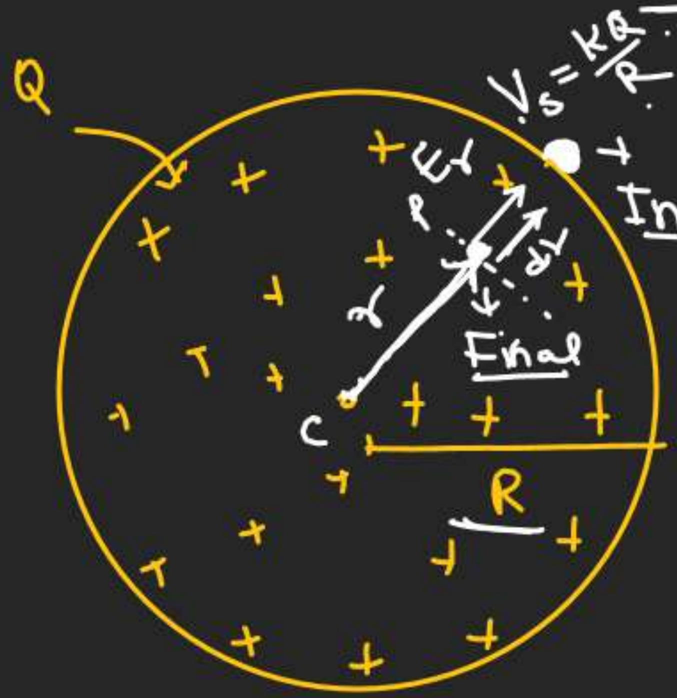
ELECTROSTATIC POTENTIAL AND POTENTIAL ENERGY

Potential due to uniformly charge now -conducting Uniformly charged solid Sphere.

$0 < r < R$ (Inside)

$\rho = \frac{Q}{\frac{4}{3}\pi R^3} = \frac{3Q}{4\pi R^3}$

$E = \frac{\rho r}{3\varepsilon_0} = \frac{3Q}{4\pi R^3} \times \frac{r}{3\varepsilon_0} = \left(\frac{Q}{4\pi\varepsilon_0}\frac{r}{R^3}\right)$



$\int_{V_s}^{V(r)} dV = -\int E_r \cdot dr$

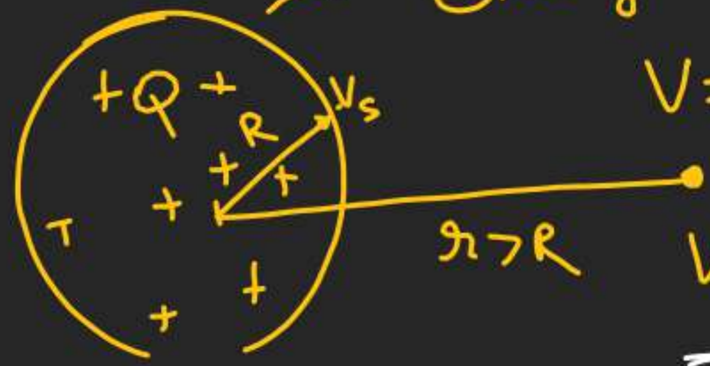
$\int dV = -\frac{Q}{4\pi\varepsilon_0 R^3}\int_R^r r\,dr$

$V(r) - V(s) = -\frac{Q}{4\pi\varepsilon_0 R^3}\left[\frac{r^2}{2}\right]_R^r$

$V(r) - V(s) = -\frac{KQ}{2R^3}\left[r^2 - R^2\right]$

$r > R$

Acts as a point charge



$V = \left[\frac{KQ}{r}\right]$

$V_s = \left[\frac{KQ}{R}\right]$

$V_r = V_{(s)} - \frac{KQ}{2R^3}r^2 + \frac{KQ}{2R}$

$V_r = \left(\frac{KQ}{R} + \frac{KQ}{2R}\right) - \left(\frac{KQ}{2R^3}\right)r^2$

$V(r) = \frac{3KQ}{2R} - \left(\frac{KQ}{2R^3}\right)r^2$

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$$V(r) = \left(\frac{3KQ}{2R}\right) - \frac{KQ}{2R^3} r^2.$$

$$V(r) = \frac{KQ}{2R^3}\left[\frac{3\cancel{KQ}}{\cancel{2R}} \times \frac{\cancel{2}R^3}{\cancel{KQ}} - r^2\right]$$

$$\boxed{V_{(r)} = \frac{KQ}{2R^3}\left[3R^2 - r^2\right]}$$

quadratic
↳ Parabola.
↳ opening downward.

Inside
$[0 \le r < R]$

$r = 0,$
$V_c = \frac{3KQ}{2R}$

$r = R$.
$\left(V_c = \frac{KQ}{R}\right)$