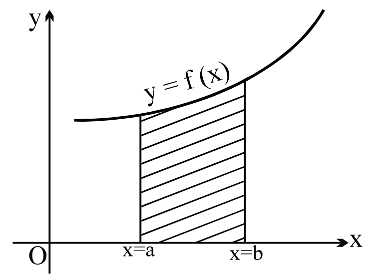
KEY CONCEPTS (AREA UNDER THE CURVE)

THINGS TO REMEMBER :

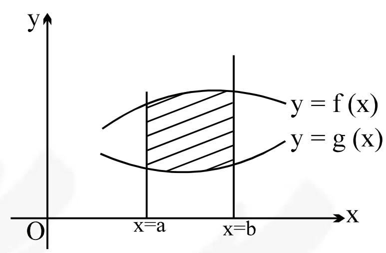
1. The area bounded by the curve y = f(x) , the x-axis and the ordinates at x = a & x = b is given by,

$$A = \int_a^b f(x)\,dx = \int_a^b y\,dx.$$



2. If the area is below the x–axis then A is negative. The convention is to consider the magnitude only i.e.

$$A = \left|\int_a^b y\,dx\right| \text{ in this case.}$$



3. Area between the curves y = f (x) & y = g (x) between the ordinates at x = a & x = b is given by,

$$A = \int_a^b f(x)\,dx - \int_a^b g(x)\,dx = \int_a^b [f(x) - g(x)]\,dx.$$

4. Average value of a function y = f (x) w.r.t. *x* over an interval $a \le x \le b$ is defined as :

$$y\,(av) = \frac{1}{b-a}\int_a^b f(x)\,dx.$$

5. The area function A_a^x satisfies the differential equation $\frac{dA_a^x}{dx} = f(x)$ with initial condition $A_a^a = 0$.

Note : If F (x) is any integral of f (x) then ,

$A_a^x = \int f(x)\,dx = F(x) + c$ $\qquad A_a^a = 0 = F(a) + c \Rightarrow c = -F(a)$

hence $A_a^x = F(x) - F(a)$. Finally by taking x = b we get , $A_a^b = F(b) - F(a)$.

6. CURVE TRACING :

The following outline procedure is to be applied in Sketching the graph of a function y = f (x) which in turn will be extremely useful to quickly and correctly evaluate the area under the curves.

(a) Symmetry : The symmetry of the curve is judged as follows :

(i) If all the powers of *y* in the equation are even then the curve is symmetrical about the axis of *x*.

(ii) If all the powers of *x* are even , the curve is symmetrical about the axis of *y*.

(iii) If powers of *x* & *y* both are even, the curve is symmetrical about the axis of *x* as well as *y*.

(iv) If the equation of the curve remains unchanged on interchanging *x* and *y*, then the curve is symmetrical about y = x.

(v) If on interchanging the signs of *x* & *y* both the equation of the curve is unaltered then there is symmetry in opposite quadrants.

(b) Find dy/dx & equate it to zero to find the points on the curve where you have horizontal tangents.

(c) Find the points where the curve crosses the x–axis & also the y–axis.

(d) Examine if possible the intervals when f (x) is increasing or decreasing. Examine what happens to 'y' when $x \to \infty$ or $-\infty$.

7. USEFUL RESULTS :

(i) Whole area of the ellipse, $x^2/a^2 + y^2/b^2 = 1$ is πab.

(ii) Area enclosed between the parabolas $y^2 = 4ax$ & $x^2 = 4by$ is 16ab/3.

(iii) Area included between the parabola $y^2 = 4ax$ & the line y = mx is $8a^2/3m^3$.

PROFICIENCY TEST-1

1. The area between the curve $xy = a^2$, x-axis, $x = a$ and $x = 2a$ is $(a > 0)$

(A) $a \log 2$ (B) $a^2 \log 2$ (C) $2a \log 2$ (D) None of these

2. Area under the curve $y = \sin 2x + \cos 2x$ between $x = 0$ and $x = \frac{\pi}{4}$, is

(A) 2 sq. units (B) 1 sq. units (C) 3 sq. units (D) 4 sq. units

3. The area bounded by the lines $y = x$, $y = 0$ and $x = 2$ is

(A) 1 (B) 2 (C) 4 (D) None of these

4. The area between the curve $y = \log x$ and x-axis which lies between $x = a$ and $x = b$ $(1 < a < b)$ is

(A) $b \log (b/e) - a \log (a/e)$ (B) $b \log (b/e) + a \log (a/e)$

(C) $\log ab$ (D) $\log (b/a)$

5. Area bounded by the curve $y = xe^{x^2}$, x- axis and the ordinates $x = 0$, $x = \alpha$ is $(\alpha > 0)$

(A) $\frac{e^{\alpha^2}+1}{2}$ sq. units (B) $\frac{e^{\alpha^2}-1}{2}$ sq.units (C) $e^{\alpha^2}+1$ sq. units (D) $e^{\alpha^2}-1$ sq.units

6. The area bounded between the curve $y = 2x^2 + 5$, x-axis and ordinates $x = -2$ and $x = 1$ is

(A) 21 (B) 29/5 (C) 23 (D) 24

7. The area bounded by the curve $y = x \sin x^2$, x-axis, $x = 0$ and $x = \sqrt{\frac{\pi}{2}}$ is

(A) 1/2 (B) $1/\sqrt{2}$ (C) 1/4 (D) $\pi/2$

8. The area bounded between the curve $\frac{x}{4} - \frac{y}{2} + 1 = 0$, $x = -2$, $x = 3$ and x-axis is

(A) 45/4 (B) 45/2 (C) 15 (D) 25/2

9. The area bounded by the curve $y = \frac{2}{1+\cos 2x}$, coordinate axes and $x = \pi/4$ is

(A) 1 (B) 2 (C) $\pi/4$ (D) Not defined

10. The area enclosed between the curve $x = 2y - y^2$ and y-axis is

(A) 9/4 (B) 4/3 (C) 9 (D) None of these

11. The area between the curve $y = \sin^3 x$, x-axis, and $x = \pi/2$ in first quadrant is

(A) 1 (B) 1/3 (C) 2/3 (D) 3/2

12. The value of a for which the area of the region bounded by the curve $y = \sin 2x$, the straight lines $x = \pi/6$, $x = a$ and x-axis is equal to 1/2 is $(a > \pi/6)$

(A) $\pi/2$ (B) $\pi/3$ (C) $2\pi/3$ (D) None of these

PROFICIENCY TEST-2

1. The area bounded by the curve $y = 4x^2$, $y = 1$ and $y = 4$ in the first quadrant is

(A) $2\frac{2}{3}$ (B) $3\frac{1}{3}$ (C) $2\frac{1}{3}$ (D) $3\frac{1}{2}$

2. The area between the curve $y = \sec x$ and y-axis when $1 \le y \le 2$ is

(A) $\frac{2\pi}{3} - \log(2 + \sqrt{3})$ (B) $\frac{2\pi}{3} + \log(2 + \sqrt{3})$

(C) $\frac{\pi}{3} - \frac{1}{2}\log(2 + \sqrt{3})$ (D) None of these

3. Area of the enclosed region bounded by the curve $y = x(x - 1)^2$ and x-axis is

(A) 4 (B) 1/3 (C) 1/12 (D) 1/2

4. The area bounded by the curve $y = \log(x)$, coordinate axes and $y = 1$ is

(A) e (B) e + 1 (C) e – 1 (D) 1/e

5. The area bounded between the curve $|y| = 1 - x^2$ is

(A) 2/3 (B) 4/3 (C) 8/3 (D) None of these

6. The whole area bounded by the curves $x = a\cos t$, $y = b\sin t$ is $(a, b > 0)$

(A) πab (B) $\frac{\pi}{2}ab$ (C) $\frac{\pi}{4}ab$ (D) None of these

7. The area between the curve $y = \tan x$ and x-axis, when $-\pi/4 \le x \le \pi/4$ is

(A) log 2 (B) log 4 (C) $\log\sqrt{2}$ (D) None of these

8. The area bounded between parabola $x^2 = 4y$ and $y = |x|$ is

(A) 2/3 (B) 4/3 (C) 8/3 (D) 16/3

9. The common area of the curves $y = \sqrt{x}$ and $x = \sqrt{y}$ is

(A) 3 (B) 5/3 (C) 1/3 (D) None of these

10. Area of the region bounded by the curves $y = e^x$, $y = e^{-x}$ and the straight line $y = 2$ is

(A) log (4/e) (B) 2 log (4/e) (C) 4 log (4/e) (D) None of these

11. The area bounded by $y = \tan x$, $y = \cot x$, x-axis in $0 \le x \le \frac{\pi}{2}$ is

(A) log 2 (B) 3 log 2 (C) 2 log 2 (D) 4 log 2

12. The area bounded by the curve $y = 2x - x^2$ and straight line $y = -x$ is

(A) 35/6 (B) 9/2 (C) 43/6 (D) None of these

13. Common area between the parabolas $y = 2x^2$ and $y = x^2 + 4$ is

(A) 16/3 (B) 8/3 (C) 32/3 (D) None of these

14. If A is the area between the curve $y = \sin x$ and x-axis in the interval $[0, \pi/2]$, then the area between $y = \sin 2x$ and x-axis in this interval will be

(A) A (B) 2A (C) A/2 (D) None of these

15. Find the area enclosed by the lines $2y = x$, $y = 2x$ and $x = 4$ is

(A) 1 (B) 2 (C) 12 (D) 16

EXERCISE–I

1. Compute the area of the region bounded by the curves y = e. x. *ln* x & y = *ln* x/(e. x) where *ln* e=1.

2. A figure is bounded by the curves $y = \left|\sqrt{2}\sin\frac{\pi x}{4}\right|$, y = 0, x = 2 & x = 4. At what angles to the positive x-axis straight lines must be drawn through (4 , 0) so that these lines partition the figure into three parts of the same size.

3. If the area enclosed by the parabolas $y = a - x^2$ and $y = x^2$ is $18\sqrt{2}$ sq. units. Find the value of 'a'.

4. The line 3x + 2y = 13 divides the area enclosed by the curve, $9x^2 + 4y^2 - 18x - 16y - 11 = 0$ into two parts. Find the ratio of the larger area to the smaller area.

5. Find the area of the region enclosed between the two circles $x^2 + y^2 = 1$ & $(x - 1)^2 + y^2 = 1$

6. Find the values of m (m > 0) for which the area bounded by the line y = mx + 2 and $x = 2y - y^2$ is , (i) 9/2 square units & (ii) minimum. Also find the minimum area.

7. Consider two curves $C_1 : y = \frac{1}{x}$ and $C_2 : y = \ln x$ on the xy plane. Let D_1 denotes the region surrounded by C_1, C_2 and the line x = 1 and D_2 denotes the region surrounded by C_1, C_2 and the line x = a. If $D_1 = D_2$. Find the value of 'a'.

8. Find the value (s) of the parameter 'a' (a > 0) for each of which the area of the figure bounded by the straight line, $y = \frac{a^2 - ax}{1 + a^4}$ & the parabola $y = \frac{x^2 + 2ax + 3a^2}{1 + a^4}$ is the greatest.

9. Find the value of 'c' for which the area of the figure bounded by the curve, $y = 8x^2 - x^5$, the straight lines x = 1 & x = c & the abscissa axis is equal to 16/3.

10. Compute the area included between the straight lines , x – 3 y + 5 = 0 ; x + 2 y + 5 = 0 and the circle $x^2 + y^2 = 25$.

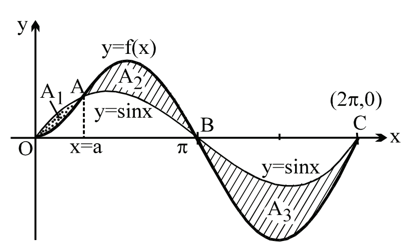
11. A polynomial function *f* (x) satisfies the condition *f* (x + 1) = *f* (x) + 2x + 1. Find *f* (x) if *f* (0) = 1. Find also the equations of the pair of tangents from the origin on the curve y = *f* (x) and compute the area enclosed by the curve and the pair of tangents.

12. Find the equation of the line passing through the origin and dividing the curvilinear triangle with vertex at the origin , bounded by the curves $y = 2x - x^2$, y = 0 and *x* = 1 into two parts of equal area.

13. Consider the curve $y = x^n$ where n > 1 in the 1st quadrant. If the area bounded by the curve, the x-axis and the tangent line to the graph of $y = x^n$ at the point (1, 1) is maximum then find the value of n.

14. Consider the collection of all curve of the form $y = a - bx^2$ that pass through the the point (2, 1), where *a* and *b* are positive constants. Determine the value of *a* and *b* that will minimise the area of the region bounded by $y = a - bx^2$ and x-axis. Also find the minimum area.

15. In the adjacent figure, graphs of two functions y = f(x) and y = sinx are given. y = sinx intersects, y = f(x) at A (a, f(a)); B(π, 0) and C(2π, 0). A_i (i = 1, 2, 3,) is the area bounded by the curves y = f (x) and y = sinx between x=0 and x= a; i = 1, between x = a and x = π; i = 2, between x = π and x = 2π; i = 3. If $A_1 = 1 - \sin a + (a - 1)\cos a$, determine the function f(x). Hence determine 'a' and A_1. Also calculate A_2 and A_3.



16. Show that the area bounded by the curve $y = \dfrac{\ln x - c}{x}$, the x-axis and the vertical line through the maximum point of the curve is independent of the constant c.

17. For what value of 'a' is the area of the figure bounded by the lines,
$y = \dfrac{1}{x}$, $y = \dfrac{1}{2x-1}$, x = 2 & x = a equal to $\ln\dfrac{4}{\sqrt{5}}$?

18. Compute the area of the loop of the curve $y^2 = x^2\,[(1+x)/(1-x)]$.

19. For the curve $f(x) = \dfrac{1}{1+x^2}$, let two points on it are $A(\alpha, f(\alpha))$, $B\left(-\dfrac{1}{\alpha}, f\left(-\dfrac{1}{\alpha}\right)\right)$ $(\alpha > 0)$. Find the minimum area bounded by the line segments OA, OB and $f(x)$, where 'O' is the origin.

20. Let 'c' be the constant number such that c > 1. If the least area of the figure given by the line passing through the point (1, c) with gradient 'm' and the parabola $y = x^2$ is 36 sq. units find the value of $(c^2 + m^2)$.

21. Let A_n be the area bounded by the curve $y = (\tan x)^n$ & the lines $x = 0$, $y = 0$ & $x = \pi/4$.
Prove that for $n > 2$, $A_n + A_{n-2} = 1/(n-1)$ & deduce that $1/(2n+2) < A_n < 1/(2n-2)$.

22. If f (x) is monotonic in (a, b) then prove that the area bounded by the ordinates at x = a ; x = b ; y = f (x) and y = f (c), $c \in (a, b)$ is minimum when $c = \dfrac{a+b}{2}$.

Hence if the area bounded by the graph of $f(x) = \dfrac{x^3}{3} - x^2 + a$, the straight lines x = 0, x = 2 and the x-axis is minimum then find the value of 'a'.

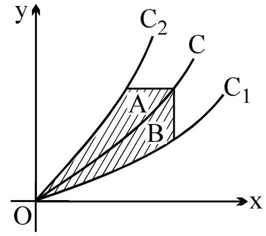
23. Consider the two curves $C_1 : y = 1 + \cos x$ & $C_2 : y = 1 + \cos(x - \alpha)$ for $\alpha \in (0, \pi/2)$; $x \in [0, \pi]$. Find the value of α, for which the area of the figure bounded by the curves C_1, C_2 & x = 0 is same as that of the figure bounded by C_2, y = 1 & $x = \pi$. For this value of α, find the ratio in which the line y = 1 divides the area of the figure by the curves C_1, C_2 & $x = \pi$.

24. Compute the area of the figure which lies in the first quadrant inside the curve
$x^2 + y^2 = 3a^2$ & is bounded by the parabola $x^2 = 2ay$ & $y^2 = 2ax$ $(a > 0)$.

25. Find the whole area included between the curve $x^2 y^2 = a^2 (y^2 - x^2)$ & its asymptotes (asymptotes are the lines which meet the curve at infinity).

26. For what values of $a \in [0, 1]$ does the area of the figure bounded by the graph of the function y = f (x) and the straight lines x = 0, x = 1 & y = f(a) is at a minimum & for what values it is at a maximum if $f(x) = \sqrt{1-x^2}$. Find also the maximum & the minimum areas.

27. Let C_1 & C_2 be two curves passing through the origin as shown in the figure. A curve C is said to "bisect the area" the region between C_1 & C_2, if for each point P of C, the two shaded regions A & B shown in the figure have equal areas. Determine the upper curve C_2, given that the bisecting curve C has
the equation $y = x^2$ & that the lower curve C_1 has the equation $y = x^2/2$.



28. Given $f(x) = \int_0^x e^t (\ln \sec t - \sec^2 t)\, dt$; $g(x) = -2e^x \tan x$. Find the area bounded by the curves y = f (x) and y = g (x) between the ordinates x = 0 and $x = \dfrac{\pi}{3}$.

EXERCISE–II

1. The area between the curve $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and the straight line $\frac{x}{a}+\frac{y}{b}=1$, in the first quadrant is (a, b > 0)

(A) $\frac{1}{4}\pi ab-\frac{1}{2}ab$ (B) $\frac{1}{4}ab$ (C) $\frac{1}{2}\pi ab$ (D) None of these

2. The area of the region consisting of all points (x, y) so that $x^2+y^2\le 1\le |x|+|y|$, is (in sq. units)

(A) π (B) $\pi-1$ (C) $\pi-2$ (D) $\pi-3$

3. The area of the region defined by the inequality $|x|+|y|+|x+y|\le 1$ is (in sq. units)

(A) $\frac{1}{2}$ (B) $\frac{3}{4}$ (C) $\frac{1}{8}$ (D) $\frac{1}{4}$

4. If the area enclosed between $f(x)=\min\left(\cos^{-1}(\cos x),\cot^{-1}(\cot x)\right)$ and x-axis in $x\in(\pi, 2\pi)$ is $\frac{\pi^2}{k}$ sq. units, where $k\in N$, then k is equal to

(A) 4 (B) 6 (C) 8 (D) 12

5. If the area enclosed by $y^2=4ax$ and line $y=ax$ is $\frac{1}{3}$ sq. unit, then the area enclosed by $y=4x$ with same parabola is (in sq. units)

(A) 8 (B) 4 (C) $\frac{4}{3}$ (D) $\frac{8}{3}$

6. The smaller area enclosed by y = f(x), when f(x) is a polynomial of least degree satisfying $\lim_{x\to 0}\left(1+\frac{f(x)}{x^3}\right)^{1/x}=e$, and the circle $x^2+y^2=2$, is (in sq. units)

(A) $\frac{\pi}{2}$ (B) $\frac{3}{5}$ (C) $\frac{\pi}{2}-\frac{3}{5}$ (D) $\frac{\pi}{2}+\frac{3}{5}$

7. The area bounded by the curve f(x) = x + sinx and its inverse function betweeen the ordinates x = 0 and $x=2\pi$ is (in sq. units)

(A) 4π (B) 8π (C) 4 (D) 8

8. If $f(x)=a+bx+cx^2$, where $c>0$ and $b^2-4ac<0$, then the area enclosed by the co-ordinate axes, the line x = 2 and the curve y = f(x) is given by (in sq. units)

(A) $\frac{1}{3}$ [4f(1) + f(2)] (B) $\frac{1}{2}$ [f(0) + 4f(1)]

(C) $\frac{1}{2}$ [f(0) + 4f(1) + f(2)] (D) $\frac{1}{3}$ [f(0) + 4f(1) + f(2)]

9. Area of region bounded by $[x]^2=[y]^2$ if $x\in[1, 5]$ (where [·] represents the greatest integer function), is (in sq. units)

(A) 10 (B) 8 (C) 6 (D) 4

10. The area bounded by the curve $y=x^2+2x+1$, tangent to the curve at (1, 4) and y-axis is (in sq. units)

(A) $\frac{2}{3}$ (B) $\frac{1}{3}$ (C) 2 (D) 1

11. Let $f(x) = \begin{cases} x^2, & x < 0 \\ x, & x \geq 0 \end{cases}$; area bounded by the curve y = f(x), y = 0 and x = ±3a is $\frac{9a}{2}$ sq unit, a > 0. Then a is equal to

(A) $\frac{1}{2}$ (B) $\frac{1}{4}$ (C) $\frac{3}{4}$ (D) 1

12. Area bounded by the curve $xy^2 = a^2(a - x)$ and y-axis (in sq. units), given $a \neq 0$, is :

(A) $\frac{\pi a^2}{2}$ (B) πa^2 (C) $2\pi a^2$ (D) Not defined

13. For $x \in \left[0, \frac{\pi}{3}\right]$, α equals to area between the curves y = cosx and y = cos2x ; β equals to area between the curve y = cos2x and x-axis ; then $(\alpha + \beta)$ equals to (in sq. units)

(A) 1 (B) $\frac{\sqrt{3}}{2}$ (C) $\frac{1}{2}$ (D) 2

14. The area of the region satisfying $|y| + \frac{1}{2} \leq e^{-|x|}$, (in sq. units), is

(A) (1 – log2) (B) 2(1 – log2) (C) e(1 – log2) (D) 4(1 – log2)

15. Let $f(x) = \min\left\{e^x, \frac{3}{2}, 1 + e^{-x}\right\}$, $\forall x \in R$. The area bounded by the curve y = f(x), co-ordinate axes and the line x = 1, is $\left(2 + \ln\frac{a}{b\sqrt{b}} - \frac{1}{e}\right)$ sq. units, where, a & b are single-digit natural numbers, then, (a + b) equals :

(A) 4 (B) 5 (C) 7 (D) 8

EXERCISE–III

1. If the area bounded by the x- axis, curve y = f(x) and the lines x = 1, x = b is equal to $\sqrt{b^2+1} - \sqrt{2}$ for all b > 1, then f(x) is **[AIEEE 2002]**

(A) $\sqrt{(x-1)}$ (B) $\sqrt{(x+1)}$ (C) $\sqrt{(x^2+1)}$ (D) $\frac{x}{\sqrt{1+x^2}}$

2. The area of the region bounded by the curves y = | x – 1| and y = 3 – |x| is- **[AIEEE 2003]**

(A) 6 sq. units (B) 2 sq. units (C) 3 sq. units (D) 4 sq. units

3. The area of the region bounded by the curves y = |x – 2|, x = 1, x = 3 and the x- axis is-

(A) 1 (B) 2 (C) 3 (D) 4 **[AIEEE 2004]**

4. The area enclosed between the curve $y = \log_e(x + e)$ and the coordinate axes is - **[AIEEE-2005]**

(A) 1 (B) 2 (C) 3 (D) 4

5. The parabolas $y^2 = 4x$ and $x^2 = 4y$ divide the square region bounded by the lines x = 4, y = 4 and the coordinate axes. If S_1, S_2, S_3 are respectively the areas of these parts numbered from top to bottom; then $S_1 : S_2 : S_3$ is - **[AIEEE-2005]**

(A) 1 : 2 : 1 (B) 1 : 2 : 3 (C) 2 : 1 : 2 (D) 1 : 1 : 1

6. Let f (x) be a non-negative continuous function such that the area bounded by the curve y = f (x), x-axis and the ordinates $x = \frac{\pi}{4}$ and $x = \beta > \frac{\pi}{4}$ is $\left(\beta \sin\beta + \frac{\pi}{4}\cos\beta + \sqrt{2}\beta\right)$. Then $f\left(\frac{\pi}{2}\right)$ is - **[AIEEE-2005]**

(A) $\left(\frac{\pi}{4} + \sqrt{2} - 1\right)$ (B) $\left(\frac{\pi}{4} - \sqrt{2} + 1\right)$ (C) $\left(1 - \frac{\pi}{4} - \sqrt{2}\right)$ (D) $\left(1 - \frac{\pi}{4} + \sqrt{2}\right)$

7. The area enclosed between the curves $y^2 = x$ and $y = |x|$ is **[AIEEE 2007]**

(A) 2/3 (B) 1 (C) 1/6 (D) 1/3

8. The area of the plane region bounded by the curves $x + 2y^2 = 0$ and $x + 3y^2 = 1$ is equal to -**[AIEEE 2008]**

(A) 1/3 (B) 2/3 (C) 4/3 (D) 5/3

9. The area of the region bounded by the parabola $(y - 2)^2 = x - 1$, the tangent to the parabola at the point (2, 3) and the x – axis is - **[AIEEE 2009]**

(A) 3 (B) 6 (C) 9 (D) 12

10. The area bounded by the curves y = cos x and y = sin x between the ordinates x = 0 and $x = \frac{3\pi}{2}$ is

(A) $4\sqrt{2}+1$ (B) $4\sqrt{2}-2$ (C) $4\sqrt{2}+2$ (D) $4\sqrt{2}-1$ **[AIEEE-2010]**

11. The area of the region enclosed by the curves y = x, x = e, $y = \frac{1}{x}$ and the positive x-axis is **[AIEEE-2011]**

(A) $\frac{1}{2}$ square units (B) 1 square units (C) $\frac{3}{2}$ sqaure units (D) $\frac{5}{2}$ square units

12. The area bounded between the parabolas $x^2 = \frac{y}{4}$ and $x^2 = 9y$ and the straight line y = 2 is

(A) $20\sqrt{2}$ (B) $\frac{10\sqrt{2}}{3}$ (C) $\frac{20\sqrt{2}}{3}$ (D) $10\sqrt{2}$ **[AIEEE-2012]**

13. The area (in square units) bounded by the curves $y = \sqrt{x}$, $2y - x + 3 = 0$, x-axis and lying in the first quadrant is : **[JEE Main 2013]**

(A) 27/4 (B) 9 (C) 36 (D) 18

14. The area of the region described by $A = \{(x, y) : x^2 + y^2 \le 1 \text{ and } y^2 \le 1 - x\}$ is **[JEE Main 2014]**

(A) $\frac{\pi}{2} + \frac{2}{3}$ (B) $\frac{\pi}{2} + \frac{4}{3}$ (C) $\frac{\pi}{2} - \frac{4}{3}$ (D) $\frac{\pi}{2} - \frac{2}{3}$

15. The area (in sq. units) of the region described by **[JEE Main 2015]**
$\{(x,y): y^2 \le 2x \text{ and } y \ge 4x - 1\}$ is :

(A) $\frac{9}{32}$ (B) $\frac{7}{32}$ (C) $\frac{5}{64}$ (D) $\frac{15}{64}$

16. The area (in sq. units) of the region $\{(x, y) : y^2 \ge 2x \text{ and } x^2 + y^2 \le 4x, x \ge 0, y \ge 0\}$ is : **[JEE Main 2016]**

(A) $\pi - \frac{4}{3}$ (B) $\pi - \frac{8}{3}$ (C) $\pi - \frac{4\sqrt{2}}{3}$ (D) $\frac{\pi}{2} - \frac{2\sqrt{2}}{3}$

17. The area (in sq. units) of the region $\{(x, y) : x \geq 0, x + y \leq 3, x^2 \leq 4y \text{ and } y \leq 1 + \sqrt{x}\}$ is **[JEE Main 2017]**

(A) $\frac{7}{3}$ (B) $\frac{5}{2}$ (C) $\frac{59}{12}$ (D) $\frac{3}{2}$

18. Let $g(x) = \cos x^2$, $f(x) = \sqrt{x}$, and α, β $(\alpha < \beta)$ be the roots of the quadratic equation $18x^2 - 9\pi x + \pi^2 = 0$. Then the area (in sq. uinits) bounded by the curve $y = (gof)(x)$ and the lines $x = \alpha$, $x = \beta$ and $y = 0$, is :

[JEE Main 2018]

(A) $\frac{1}{2}(\sqrt{2}-1)$ (B) $\frac{1}{2}(\sqrt{3}-1)$ (C) $\frac{1}{2}(\sqrt{3}+1)$ (D) $\frac{1}{2}(\sqrt{3}-\sqrt{2})$

EXERCISE–IV

1. (a) For which of the following values of m, is the area of the region bounded by the curve $y = x - x^2$ and the line $y = mx$ equals 9/2 ?

(A) – 4 (B) – 2 (C) 2 (D) 4

(b) Let f(x) be a continuous function given by $f(x) = \begin{cases} 2x & \text{for } |x| \leq 1 \\ x^2 + ax + b & \text{for } |x| > 1 \end{cases}$

Find the area of the region in the third quadrant bounded by the curves, $x = -2y^2$ and $y = f(x)$ lying on the left of the line $8x + 1 = 0$. **[JEE '99, 3 + 10 (out of 200)]**

2. Find the area of the region lying inside $x^2 + (y - 1)^2 = 1$ and outside $c^2x^2 + y^2 = c^2$ where $c = \sqrt{2} - 1$.

[REE '99, 6]

3. Find the area enclosed by the parabola $(y - 2)^2 = x - 1$, the tangent to the parabola at (2, 3) and the x-axis.

[REE 2000,3]

4. Let $b \neq 0$ and for j = 0, 1, 2,.........n, let S_j be the area of the region bounded by the y axis and the curve $xe^{ay} = \sin by$, $\frac{j\pi}{b} \leq y \leq \frac{(j+1)\pi}{b}$. Show that S_0, S_1, S_2,............S_n are in geometric progression. Also, find their sum for $a = -1$ and $b = \pi$. **[JEE'2001, 5]**

5. The area bounded by the curves $y = |x| - 1$ and $y = -|x| + 1$ is

(A) 1 (B) 2 (C) $2\sqrt{2}$ (D) 4 **[JEE'2002, (Scr)]**

6. Find the area of the region bounded by the curves $y = x^2$, $y = |2 - x^2|$ and $y = 2$, which lies to the right of the line $x = 1$. **[JEE '2002, (Main)]**

7. If the area bounded by $y = ax^2$ and $x = ay^2$, $a > 0$, is 1, then a =

(A) 1 (B) $\frac{1}{\sqrt{3}}$ (C) $\frac{1}{3}$ (D) $-\frac{1}{\sqrt{3}}$ **[JEE '2004, (Scr)]**

8. (a) The area bounded by the parabolas y = $(x + 1)^2$ and y = $(x - 1)^2$ and the line y = 1/4 is

(A) 4 sq. units (B) 1/6 sq. units
(C) 4/3 sq. units (D) 1/3 sq. units

[JEE '2005 (Screening)]

(b) Find the area bounded by the curves $x^2 = y$, $x^2 = -y$ and $y^2 = 4x - 3$.

(c) If $\begin{bmatrix} 4a^2 & 4a & 1 \\ 4b^2 & 4b & 1 \\ 4c^2 & 4c & 1 \end{bmatrix}\begin{bmatrix} f(-1) \\ f(1) \\ f(2) \end{bmatrix} = \begin{bmatrix} 3a^2+3a \\ 3b^2+3b \\ 3c^2+3c \end{bmatrix}$, $f(x)$ is a quadratic function and its maximum value occurs at a point V. A is a point of intersection of y = $f(x)$ with x-axis and point B is such that chord AB subtends a right angle at V. Find the area enclosed by $f(x)$ and chord AB. **[JEE '2005 (Main), 4 + 6]**

9. Match the following

(i) $\int_0^{\pi/2} (\sin x)^{\cos x}(\cos x \cot x - \ln(\sin x)^{\sin x})\,dx$ (A) 1

(ii) Area bounded by $-4y^2 = x$ and $x - 1 = -5y^2$ (B) 0

(iii) Cosine of the angle of intersection of curves $y = 3^{x-1}\ln x$ and $y = x^x - 1$ is (C) 6 $\ln$ 2

(D) 4/3 **[JEE 2006, 6]**

10. (a) The area of the region between the curves y = $\sqrt{\dfrac{1+\sin x}{\cos x}}$ and y = $\sqrt{\dfrac{1-\sin x}{\cos x}}$ bounded by the lines x = 0 and x = $\dfrac{\pi}{4}$ is

(A) $\int_0^{\sqrt{2}-1} \dfrac{t}{(1+t^2)\sqrt{1-t^2}}\,dt$ (B) $\int_0^{\sqrt{2}-1} \dfrac{4t}{(1+t^2)\sqrt{1-t^2}}\,dt$

(C) $\int_0^{\sqrt{2}+1} \dfrac{4t}{(1+t^2)\sqrt{1-t^2}}\,dt$ (D) $\int_0^{\sqrt{2}+1} \dfrac{t}{(1+t^2)\sqrt{1-t^2}}\,dt$

(b) **Comprehension (3 questions together):**
Consider the functions defined implicitly by the equation $y^3 - 3y + x = 0$ on various intervals in the real line.
If $x \in (-\infty, -2) \cup (2, \infty)$, the equation implicitly defines a unique real valued differentiable function y = $f(x)$.
If $x \in (-2, 2)$, the equation implicitly defines a unique real valued differentiable function y = g(x) satisfying g(0) = 0.

(i) If $f(-10\sqrt{2}) = 2\sqrt{2}$, then $f''(-10\sqrt{2})$ =

(A) $\dfrac{4\sqrt{2}}{7^3 3^2}$ (B) $-\dfrac{4\sqrt{2}}{7^3 3^2}$ (C) $\dfrac{4\sqrt{2}}{7^3 3}$ (D) $-\dfrac{4\sqrt{2}}{7^3 3}$

(ii) The area of the region bounded by the curve $y = f(x)$, the x-axis, and the lines $x = a$ and $x = b$, where $-\infty < a < b < -2$, is

(A) $\int_a^b \frac{x}{3\left((f(x))^2 - 1\right)} dx + b f(b) - a f(a)$ (B) $-\int_a^b \frac{x}{3\left((f(x))^2 - 1\right)} dx + b f(b) - a f(a)$

(C) $\int_a^b \frac{x}{3\left((f(x))^2 - 1\right)} dx - b f(b) + a f(a)$ (D) $-\int_a^b \frac{x}{3\left((f(x))^2 - 1\right)} dx - b f(b) + a f(a)$

(iii) $\int_{-1}^{1} g'(x)\, dx =$

(A) $2g(-1)$ (B) 0 (C) $-2g(1)$ (D) $2g(1)$

[JEE 2008, 3 + 4 + 4 + 4]

11. Area of the region bounded by the curve $y = e^x$ and lines $x = 0$ and $y = e$ is **[JEE 2009]**

(A) $e - 1$ (B) $\int_1^e \ln(e + 1 - y)\, dy$ (C) $e - \int_0^1 e^x\, dx$ (D) $\int_1^e \ln y\, dy$

12. Let the straight line $x = b$ divides the area enclosed by $y = (1 - x)^2$, $y = 0$ and $x = 0$ into two parts R_1 $(0 \le x \le b)$ and $R_2 (b \le x \le 1)$ such that $R_1 - R_2 = \frac{1}{4}$. Then b equals **[JEE 2011]**

(A) $\frac{3}{4}$ (B) $\frac{1}{2}$ (C) $\frac{1}{3}$ (D) $\frac{1}{4}$

13. Let $f : [-1, 2] \to [0, \infty)$ be a continuous function such that $f(x) = f(1 - x)$ for all $x \in [-1, 2]$. Let $R_1 = \int_{-1}^{2} x f(x) dx$, and R_2 be the area of the region bounded by $y = f(x)$, $x = -1$, $x = 2$, and the x-axis. Then

(A) $R_1 = 2R_2$ (B) $R_1 = 3R_2$ (C) $2R_1 = R_2$ (D) $3R_1 = R_2$ **[JEE 2011]**

14. Let S be the area of the region enclosed by $y = e^{-x^2}$, $y = 0$, $x = 0$ and $x = 1$. Then **[JEE 2012]**

(A) $S \ge \frac{1}{e}$ (B) $S \ge 1 - \frac{1}{e}$ (C) $S \le \frac{1}{4}\left(1 + \frac{1}{\sqrt{e}}\right)$ (D) $S \le \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{e}}\left(1 - \frac{1}{\sqrt{2}}\right)$

15. The area enclosed by the curves $y = \sin x + \cos x$ and $y = |\cos x - \sin x|$ over the interval $\left[0, \frac{\pi}{2}\right]$ is :

[JEE (Adv.) 2013]

(A) $4(\sqrt{2} - 1)$ (B) $2\sqrt{2}(\sqrt{2} - 1)$ (C) $2(\sqrt{2} + 1)$ (D) $2\sqrt{2}(\sqrt{2} + 1)$

16. Area of the region $\left\{(x, y) \in \mathbb{R}^2 : y \ge \sqrt{|x + 3|}, 5y \le x + 9 \le 15\right\}$ is equal to **[IIT JEE (Adv.) 2016]**

(A) $\frac{1}{6}$ (B) $\frac{4}{3}$ (C) $\frac{3}{2}$ (D) $\frac{5}{3}$

17. If the line $x = \alpha$ divides the area of region $R = \{(x, y) \in \mathbb{R}^2 : x^3 \le y \le x, 0 \le x \le 1\}$ into two equal parts, then **[IIT JEE Advance - 2017]**

(A) $\frac{1}{2} < \alpha < 1$ (B) $\alpha^4 + 4\alpha^2 - 1 = 0$ (C) $0 < \alpha \le \frac{1}{2}$ (D) $2\alpha^4 - 4\alpha^2 + 1 = 0$

18. A farmer F_1 has a land in the shape of a triangle with vertices at P(0, 0), Q(1, 1) and R(2, 0). From this land, a neighbouring farmer F_2 takes away the region which lies between the side PQ and a curve of the form $y = x^n$ $(n > 1)$. If the area of the region taken away by the farmer F_2 is exactly 30% of the area of ΔPQR, then the value of n is ______________ . **[JEE Advanced 2018]**

19. Let $f : [0, \infty) \to R$ be a continuous function such that **[JEE Advanced 2018]**

$$f(x) = 1 - 2x + \int_0^x e^{x-t} f(t)\,dt$$

for all $x \in [0, \infty)$. Then, which of the following statement(s) is(are) TRUE?

(A) The curve $y = f(x)$ passes through the point (1, 2)

(B) The curve $y = f(x)$ passes through the point (2, –1)

(C) The area of the region $\{(x, y) \in [0, 1] \times R : f(x) \le y \le \sqrt{1-x^2}\}$ is $\frac{\pi - 2}{4}$

(D) The area of the region $\{(x, y) \in [0, 1] \times R : f(x) \le y \le \sqrt{1-x^2}\}$ is $\frac{\pi - 1}{4}$

20. The area of the region $\{(x, y) : xy \le 8,\ 1 \le y \le x^2\}$ is **[JEE Advanced 2019]**

(A) $16\log_e 2 - 6$ (B) $8\log_e 2 - \frac{7}{3}$ (C) $16\log_e 2 - \frac{14}{3}$ (D) $8\log_e 2 - \frac{14}{3}$

21. Let the functions $f : R \to R$ and $g : R \to R$ be defined by **[JEE Advanced 2020]**

$f(x) = e^{x-1} - e^{-|x-1|}$ and $g(x) = \frac{1}{2}\left(e^{x-1} + e^{1-x}\right)$.

Then the area of the region in the first quadrant bounded by the curves $y = f(x)$, $y = g(x)$ and $x = 0$ is

(A) $\left(2-\sqrt{3}\right)+\frac{1}{2}\left(e-e^{-1}\right)$

(B) $\left(2+\sqrt{3}\right)+\frac{1}{2}\left(e-e^{-1}\right)$

(C) $\left(2-\sqrt{3}\right)+\frac{1}{2}\left(e+e^{-1}\right)$

(D) $\left(2+\sqrt{3}\right)+\frac{1}{2}\left(e+e^{-1}\right)$

22. Consider all rectangles lying in the region $\left\{(x,y) \in R \times R : 0 \le x \le \frac{\pi}{2} \text{ and } 0 \le y \le 2\sin(2x)\right\}$ and having one side on the x-axis. The area of the rectangle which has the maximum perimeter among all such rectangles, is : **[JEE Advanced 2020]**

(A) $\frac{3\pi}{2}$ (B) π (C) $\frac{\pi}{2\sqrt{3}}$ (D) $\frac{\pi\sqrt{3}}{2}$

23. The area of the region $\left\{(x, y) : 0 \le x \le \frac{9}{4},\ 0 \le y \le 1,\ x \ge 3y,\ x + y \ge 2\right\}$ is **[JEE Advanced 2021]**

(A) $\frac{11}{32}$ (B) $\frac{35}{96}$ (C) $\frac{37}{96}$ (D) $\frac{13}{32}$

ANSWER KEY

AREA UNDER THE CURVE

PROFICIENCY TEST-1

1. B	**2.** B	**3.** B	**4.** A	**5.** B	**6.** A	**7.** A
8. A	**9.** A	**10.** B	**11.** C	**12.** B		

PROFICIENCY TEST-2

1. C	**2.** A	**3.** C	**4.** C	**5.** C	**6.** A	**7.** A
8. D	**9.** C	**10.** B	**11.** A	**12.** B	**13.** C	**14.** A
15. C						

EXERCISE–I

1. $(e^2 - 5)/4e$ sq. units **2.** $\pi - \tan^{-1}\frac{2\sqrt{2}}{3\pi}$; $\pi - \tan^{-1}\frac{4\sqrt{2}}{3\pi}$ **3.** a = 9

4. $\frac{3\pi + 2}{\pi - 2}$ **5.** $\frac{2\pi}{3} - \frac{\sqrt{3}}{2}$ sq. units **6.** **(i)** m = 1, **(ii)** m = ∞ ; $A_{min} = 4/3$

7. e **8.** $a = 3^{1/4}$ **9.** C = – 1 or $\left(8 - \sqrt{17}\right)^{1/3}$

10. $\frac{5}{4}(5\pi + 14)$ sq. units **11.** $f(x) = x^2 + 1$; $y = \pm 2x$; $A = \frac{2}{3}$ sq. units **12.** $y = 2x/3$

13. $\sqrt{2} + 1$ **14.** b = 1/8, $A_{minimum} = 4\sqrt{3}$ sq. units

15. f(x) = x sinx, a = 1; $A_1 = 1 - \sin 1$; $A_2 = \pi - 1 - \sin 1$; $A_3 = (3\pi - 2)$ sq. units

16. 1/2 **17.** a = 8 or $\frac{2}{5}\left(6 - \sqrt{21}\right)$ **18.** $2 - (\pi/2)$ sq. units

19. $\frac{(\pi - 1)}{2}$ **20.** 104

22. $a = \frac{2}{3}$ **23.** $\alpha = \pi/3$, ratio = $2 : \sqrt{3}$

24. $\left[\frac{\sqrt{2}}{3} + \frac{3}{2}.\text{arc sin}\frac{1}{3}\right]a^2$ sq. units **25.** $4a^2$

26. a = 1/2 gives minima, $A\left(\frac{1}{2}\right) = \frac{3\sqrt{3} - \pi}{12}$; a = 0 gives local maxima $A(0) = 1 - \frac{\pi}{4}$;

a = 1 gives maximum value , $A(1) = \pi/4$

27. $(16/9)x^2$ **28.** $e^{\pi/3}\log 2$ sq. units

EXERCISE–II

1.	A	**2.**	C	**3.**	B	**4.**	A	**5.**	D	**6.**	D	**7.**	D
8.	D	**9.**	B	**10.**	B	**11.**	A	**12.**	B	**13.**	A	**14.**	B
15.	C												

EXERCISE–III

1.	D	**2.**	D	**3.**	A	**4.**	A	**5.**	D	**6.**	D	**7.**	C
8.	C	**9.**	C	**10.**	B	**11.**	C	**12.**	C	**13.**	B	**14.**	B
15.	A	**16.**	B	**17.**	B	**18.**	B						

EXERCISE–IV

1. **(a)** B, D **(b)** 257/192 ; a = 2 ; b = – 1

2. $\left(\pi - \frac{\pi - 2}{2\sqrt{2}}\right)$ sq. units

3. 9 sq. units

4. $\frac{S_j}{S_{j+1}} = e^{\frac{\pi a}{b}}$; $S_0 = \frac{b(e^{-\frac{a\pi}{b}} + 1)}{a^2 + b^2}$ for a = –1, b = π, $S_0 = \frac{\pi(e+1)}{\pi^2 + 1}$ and r = π

5. B **6.** $\left(\frac{20}{3} - 4\sqrt{2}\right)$ sq. units **7.** B

8. (a) D ; (b) $\frac{1}{3}$ sq. units ; (c) $\frac{125}{3}$ sq. units **9.** **(i)** A, (ii) D, (iii) A

10. **(a)** B, **(b)** (i) B, (ii) A, (iii) D **11.** B,C,D **12.** B **13.** C

14.	A, B, D	**15.**	B	**16.**	C	**17.**	A,D	**18.**	4
19.	BC	**20.**	C	**21.**	A	**22.**	C	**23.**	A