PROBLEM SET-06

SOLUTION

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Q.1 Find the number of permutations of the word "AUROBIND" in which vowels appear in an alphabetical order.

Ans. **($^{8}C_4 \cdot 4\,!$)**

Sol. A, I, O, U → treat them alike.

Now find the arrangement of 8 letters in which 4 alike and 4 different $= \frac{8!}{4!}$

Q.2 There are 10 different books in a shelf. If m denotes the number of ways of selecting 3 books when no two of them are consecutive and n denotes the corresponding figure when exactly two are consecutive then

(A) $8\,m = 7n$ (B) $6m = 5n$

(C) $7\,m = 5n$ (D) $m = n$

Ans. **(D)**

Sol. $B_1\ B_2\ B_3\ B_4\ B_5\ B_6\ B_7\ B_8\ B_9\ B_{10}$

No of ways of selecting 3 books out of 10 different books $= {}^{10}C_3$

$=$ all three are consecutive + exactly two are consecutive + no two are consecutive

$120 = 8 \quad + \quad 'x'(n) \quad + \quad {}^{10-3+1}C_3(m)$

$\therefore\ x = 56 \Rightarrow m = 56; n = 56$

$$\begin{tikzcd}[row sep=small] & m=56 \arrow[dr] & \\ \boxed{x=56} \arrow[ur] \arrow[dr] & & \boxed{m=n} \\ & n=56 \arrow[ur] & \end{tikzcd}$$

$\therefore\ m = n$

Note: Selecting " r " things out of " n " things when no two are consecutive $= {}^{n-r+1}C_r$

Q.3 If as many more words as possible be formed out of the letters of the word "DOGMATIC" then compute the number of words in which the relative order of vowels and consonants remain unchanged.

Ans. **(719)**

Sol.

	O			A		I	

$3! \times 5! - 1 = 719$

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Q.4 Number of ways in which 9 different toys be distributed among 4 children belonging to different age groups in such a way that distribution among the 3 elder children is even and the youngest one is to receive one toy more, is:

(A) $\frac{(5!)^2}{8}$ (B) $\frac{9!}{2}$ (C) $\frac{9!}{3!(2!)^3}$ (D) none

Ans. **(C)**

Sol. distribution 2, 2,2 and 3 to the youngest. Now 3 toys for the youngest can be selected in ${}^{9}C_3$ ways, remaining 6 toys can be divided into three equal groups in

$\frac{6!}{(2!)^3 \cdot 3!}$ way and can be distributed in 3! ways $\Rightarrow$ ${}^{9}C_3 \cdot \frac{6!}{(2!)^3} = \frac{9!}{3!(2!)^3}$]

Q.5 There are five different peaches and three different apples. Number of ways they can be divided into two packs of four fruits if each pack must contain atleast one apple, is

(A) 35 (B) 65 (C) 60 (D) 30

Ans. **(D)**

Sol. Peaches 5 p_1, p_2, p_3, p_4, p_5

Apples 3 a_1, a_2, a_3

Hence number of ways $=$ ${}^{3}C_1 \times {}^{5}C_3 = 30$ (think!) Ans.

Note that both the groups of 4 fruits must contain atleast one apple.]

Q.6 Let P_n denotes the number of ways in which three people can be selected out of ' n ' people sitting in a row, if no two of them are consecutive. If, $P_{n+1} - P_n = 15$ then the value of ' n ' is:

(A) 7 (B) 8 (C) 9 (D) 10

Ans. **(B)**

Sol. $P_n = {}^{n-2}C_3;\ P_{n+1} = {}^{n-1}C_3;$

Hence ${}^{n-1}C_3 - {}^{n-2}C_3 = 15$

${}^{n-2}C_3 + {}^{n-2}C_2 - {}^{n-2}C_3 = 15$

or

${}^{n-2}C_2 = 15 \Rightarrow n = 8 \Rightarrow C$]

Q.7 Number of ways in which 7 green bottles and 8 blue bottles can be arranged in a row if exactly 1 pair of green bottles is side by side, is (Assume all bottles to be alike except for the colour).

(A) 84 (B) 360 (C) 504 (D) none

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Ans. **(C)**

Sol. $^9C_6 \times 6 = 504$

(6 bottles in 6 gaps 1 in each and the remaining in any one of these 6 gaps)

Q.8 Ahas 3 maps and B has 9 maps. All the 12 maps being distinct. Determine the number of ways in which they can exchange their maps if each keeps his initial number of maps.

Ans. **219**

Sol. A $\to M_1, M_2, M_3$ B $\to B_1, B_2, B_3$

$^3C_1 \cdot {}^9C_1 + {}^3C_2 \cdot {}^9C_2 + {}^3C_3 \cdot {}^9C_3$]

Q.9 Number of three-digit number with atleast one 3 and at least one 2 is

(A) 58 (B) 56 (C) 54 (D) 52

Ans. **(D)**

Sol. When exactly one 2, exactly one 3 and

1 other non-zero digit $= 7 \times 3! = 42$

one 2 , one 3 and one $0 = 4$

two 2 's and one $3 = 3$

two 3 's and one $2 = 3$

Total $= 52$

Q.10 Number of ways in which 3 mangoes, 3 apples and 2 oranges can be distributed in 8 children so that every child gets exactly one fruit, is (Assume fruits of the same species to be alike.)

(A) $\frac{8!3!}{3!3!2!2!}$ (B*) $\frac{8!2!}{3!3!2!2!}$ (C) $\frac{3\cdot 8!}{3!3!2!2!}$ (D) $\frac{8!}{3!3!2!2!}$

Ans. **(B)**

Sol.

M	M	M	A	A	A	O	O
C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8

$= \frac{8!}{3!3!2!}$

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Q.11 Shoma has ten apples, and wants to give them to three of her friends. Number of different ways can she can given them is (Assume that the apples are all identical.)

(A) 36 (B) 63 (C)66 (D) 1000

Ans. **(C)**

Sol. $\underbrace{00\ldots\ldots0}_{\text{10 Apples}}\ \underbrace{\emptyset,\emptyset}_{\text{2 fake apples}} \leftarrow \boxed{{}^{n+p-1}C_{P-1}}$

$\left.\begin{array}{l} n=10 \\ p=3 \end{array}\right\} \Rightarrow {}^{10+3-1}C_{3-1}$

$= {}^{12}c_2 = 66$

Q.12 Number of divisors of the number $N = 2^3 \cdot 3^5 \cdot 5^7 \cdot 7^9$ which are perfect square is

(A) 24 (B) 60 (C) 119 (D) 120

Ans. **(D)**

Sol. $N = 2 \cdot 2^2 \cdot 3 \cdot 3^4 \cdot 5 \cdot 5^6 \cdot 77^8$

$N = 2 \cdot 3 \cdot 5 \cdot 7 \cdot \{(2^2)^1 \cdot (3^2)^2 \cdot (5^2)^3 \cdot (7^2)^4\}$

$\rightarrow$ No of divisors $= (1+1)(2+1)(3+1)(4+1)$

$= 2 \cdot 3 \cdot 4 \cdot 5$

$= 120$

Paragraph for question nos. 13 to 15

Consider the word W = MISSISSIPPI

Q.13 If N denotes the number of different selections of 5 letters from the word W = MISSISSIPPI then N belongs to the set

(A) {15,16,17,18,19} (B) {20,21,22,23,24}

(C) {25,26,27,28,29} (D) {30,31,32,33,34}

Ans. **(C)**

Q.14 Number of ways in which the letters of the word W can be arranged if atleast one vowel is separated from rest of the vowels

(A) $\frac{8!\cdot161}{4!\cdot4!\cdot2!}$ (B) $\frac{8!\cdot161}{4\cdot4!\cdot2!}$ (C) $\frac{8!\cdot161}{4!\cdot2!}$ (D) $\frac{8!}{4!\cdot2!}\cdot\frac{165}{4!}$

Ans. **(B)**

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Q.15 If the number of arrangements of the letters of the word W if all the S's and P's are separated is (K) $\left(\frac{10!}{4!\cdot 4!}\right)$ then Kequals

(A) $\frac{6}{5}$ (B) 1 (C) $\frac{4}{3}$ (D) $\frac{3}{2}$

Ans. **(B)**

Sol. (i) SSSS; IIII; PP; M

4 alike $+$ 1 different $= 2\cdot 3 = 6$
3 alike $+$ 2 other alike $= 2\cdot 2 = 4$
3 alike $+$ 2 different $= 2\cdot 3 = 6$
2 alike $+$ 2 other alike $+$ 1 different $= 3\cdot 2 = 6$
2 alike $+$ 3 different $= 3 \frac{= 3}{25}$

(ii) Atleast one vowel is separated from the rest

$= $ total $-$ when all vowels together

$$= \frac{11!}{4!\cdot 4!\cdot 2!} - \frac{8!}{4!\cdot 2!} = \frac{11\cdot 10\cdot 9\cdot 8!}{4!\cdot 4!\cdot 2!} - \frac{8!}{4!\cdot 2!}$$

$$= \frac{8!}{4!\cdot 2!}\left[\frac{11\cdot 10\cdot 9}{24} - 1\right] = \frac{8!}{4!\cdot 2!}\left[\frac{11\cdot 15}{4} - 1\right] = \frac{8!}{4!\cdot 2!}\left[\frac{165-4}{4}\right] = \frac{8!\cdot 161}{4\cdot 4!\cdot 2!} \text{ Ans.}$$

(iii) Separate S first

|I|I|I|I|P|P|M| (8 gaps)

$= \frac{7!}{4!2!}\ {}^8C_4$ (S's separated and P's may be together or may be repeated)

|I|I|I|I|PP|M |

Both S's and P's separated

$$= \frac{7!}{4!\cdot 2!}\cdot {}^8C_4 - \frac{6!}{4!}\cdot {}^7C_4$$

[(S's separated but P's may be together or may be separated) - when S's separated and P's together)] simplifies to $\frac{6!\cdot 7!}{4!\cdot 4!} = \frac{10!}{4!\cdot 4!} \Rightarrow K = 1$ Ans.]

Q.16 There are nine different books on a shelf, four red and five are green. Number of ways in which it is possible to arrange these books if

Column-I	Column-II
(A) the red books must be together and green books together	(P) (4) 6 !
(B) the red books must be together whereas the green books may or may not be together	(Q) (8) 6!

(C) no two books of the same colour are adjacent (R) (20) 6 !

(D) if the books are arranged on a round table and no two of the three specified books are together (S) (24) 6 !

Ans. **(A) Q, (B) S; (C) P; (D) R**

Sol. (A) $R_1R_2R_3R_4\ G_1G_2G_3G_4G_5$

$= 4! \cdot 5! \cdot 2 = 8 \cdot 6! \Rightarrow \quad \Rightarrow \qquad$ (**Q**)

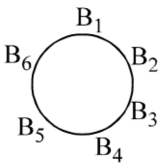
(B) $R_1R_2R_3R_4\ G_1G_2G_3G_4G_5$

$(6!)(4!) = (24)6\,! \ \Rightarrow \qquad$ (**S**)

(C) $G_1 \times G_2 \times G_3 \times G_4 \times G_5$

$= 5! \cdot 4! = (4)6! \ \ \Rightarrow \qquad$ (**P**)

(D) $5! \cdot {}^6C_3 \cdot 3! = (20)6! \Rightarrow \qquad$ (**R**)



Q.17 Find the number of five digit numbers that can be formed by using two 2 's, three 3 's, one zero and one 5 .

Ans. **212**

Sol. 22,333,0,5

3 alike +2 others alike

$33322 = \dfrac{5!}{3!\,2!} = 10$

3 alike + 2 different

$33325 \rightarrow \dfrac{5!}{3!} = 20$

$\begin{matrix}33302\\33305\end{matrix} \rightarrow 2\left(\frac{5!}{3!} - \frac{4!}{3!}\right) = 2(20-4) = 32$

2 alike + 3 different

$22035 \ \rightarrow \dfrac{5!}{2!} - \dfrac{4!}{2!}$

$33025 \ \rightarrow \dfrac{5!}{2!} - \dfrac{4!}{2!}$

$= 2(60-12) = 96$

2 alike +**2** other alike +1 different

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$$\left.\begin{array}{l} 22330 \rightarrow \dfrac{5!}{2!\,2!} - \dfrac{4!}{2!\,2!} \\ 22335 \rightarrow \dfrac{5!}{2!\,2!} \end{array}\right\} = 54$$

Hence Total $= 10 + 20 + 32 + 96 + 54 = 212$ Ans.