Nishant Jindal

Binomial Theorem

$\left(1-\frac{1}{x}\right)^n = C_0 - \frac{C_1}{x} + \frac{C_2}{x^2} - \frac{C_3}{x^3} + \frac{C_4}{x^4} - - -$

$\Rightarrow (x-1)^n = C_0x^n - C_1x^{n-1} + C_2x^{n-2} - C_3x^{n-3} - - -$

Q. If $(1+x)^n = C_0 + C_1x + C_2x^2 + C_3x^3 + \cdots + C_nx^n$ then find the following

(A) $C_0^2 + C_1^2 + C_2^2 + \cdots Cn^2 = ? \rightarrow C_0 \cdot C_0 = {}^nC_0 \times \boxed{{}^nC_0} = {}^nC_0 \times {}^nC_n = {}^{2n}C_n$

(B) $C_0C_1 + C_1C_2 + C_2C_3 + \cdots C_{n-1}C_n = C_0C_1 = {}^nC_0 \times {}^nC_1 = {}^nC_0 \times {}^nC_{n-1} = {}^{2n}C_{n-1}$

(C) $C_0C_2 + C_1C_3 + \cdots C_{n-2}C_n = C_0C_2 = {}^nC_0\,{}^nC_2 = {}^nC_0\,{}^nC_{n-2} = {}^{2n}C_{n-2}$

(D) $C_0^2 - C_1^2 + C_2^2 - C_3^2 + \cdots =$ Coff of x^n in $(x^2-1)^n = {}^nC_r\,(x^2)^{n-r}(-1)^r = \begin{cases} 0 \\ {}^nC_{\frac{n}{2}}(-1)^{n/2} \end{cases}$

$2n-2r=n$

$2r=n \Rightarrow r=n/2$

$n=$ odd

$n=$ Even

① $(1+x)^n = C_0 + C_1x + C_2x^2 + C_3x^3 - - - C_nx^n \rightarrow (A)$

$x = \frac{1}{x}$

$\left(1+\frac{1}{x}\right)^n = C_0 + \frac{C_1}{x} + \frac{C_2}{x^2} + \frac{C_3}{x^3} - - - \frac{C_n}{x^n}$

$(x+1)^n = C_0x^n + C_1x^{n-1} + C_2x^{n-2} + C_3x^{n-3} - - + C_n \rightarrow (B)$

${}^nC_0\,{}^nC_2 = {}^nC_0\,{}^nC_{n-2}$

A×B ② $(1+x)^n(x+1)^n = \{C_0 + C_1x + C_2x^2 + C_3x^3 + - - C_nx^n\}\{C_0x^n + C_1x^{n-1} + C_2x^{n-2} + C_3x^{n-3} - - C_n\}$

${}^nC_1\,{}^nC_3 = {}^nC_1\,{}^nC_{n-3}$

$n-2$

Q1 $C_0 \cdot C_0 + C_1 \cdot C_1 + C_2 \cdot C_2 + C_3 \cdot C_3 = - - = $ Coff of x^n of $(1+x)^{2n} = {}^{2n}C_n$

Q2 $C_0 \cdot C_1 + C_1C_2 + C_2C_3 + C_3C_4 - - = $ Coff of x^{n-1} in $(1+x)^{2n} = {}^{2n}C_{n-1}$

Q3 $C_0C_2 + C_1C_3 + C_2C_4 - = $ Coff of x^{n-2} in $(1+x)^{2n} = {}^{2n}C_{n-2}$

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Binomial Theorem

Q. Find ${}^{30}C_0\,{}^{20}C_0 + {}^{30}C_1\,{}^{20}C_1 + \cdots + {}^{30}C_{20}\,{}^{20}C_{20}$.

$$\underbrace{{}^{30}C_0 \cdot {}^{20}C_{20}}_{\text{Const}} + \underbrace{{}^{30}C_1 \cdot {}^{20}C_{19}}_{\text{Const}} + \quad + \underbrace{{}^{30}C_{20} \cdot {}^{20}C_0}_{\text{Const}}$$

Coff of x^{20} in $(1+x)^{30}(1+x)^{20}$

$\Rightarrow$ Coff of x^{20} in $(1+x)^{50} = {}^{50}C_{20}$ Ans

Binomial Theorem

Q. Find $^{30}C_0\,^{30}C_{10} + {}^{30}C_1\,^{30}C_{11} + {}^{30}C_2\,^{30}C_{12} \dots + {}^{30}C_{20}\,^{30}C_{30}$.

$$\underbrace{^{30}C_0 \cdot {}^{30}C_{20}}_{20} + \underbrace{^{30}C_1 \cdot {}^{30}C_{19}}_{20} + \underbrace{^{30}C_2 \cdot {}^{30}C_{18}}_{20} \quad + \underbrace{^{30}C_{20} \cdot {}^{30}C_0}_{20}$$

$= \text{Coff of } x^{20} \text{ in } (1+x)^{30}(1+x)^{30}$

$= \text{Coff of } x^{20} \text{ in } (1+x)^{60}$

$\therefore {}^{60}C_{20}$

Q. Find ${}^{m}C_{r}\,\underbrace{\ {}^{n}C_{0}}_{r} + {}^{m}C_{r-1}\,\underbrace{\ {}^{n}C_{1}}_{r} + {}^{m}c_{r-2}\,\underbrace{\ {}^{n}C_{2}}_{r} + \cdots + {}^{m}c_{0}\,\underbrace{\ {}^{n}C_{r}}_{r}$.

Coff of x^r in $(1+x)^m \cdot (1+x)^n$

Coff of x^r in $(1+x)^{m+n}$

$= {}^{m+n}C_r$.

Q. Prove that $\sum_{r=0}^{2n} r \cdot \left({}^{2n}C_r\right)^2 = {}^{4n-1}C_{2n+1}$.

$$\sum_{r=0}^{2n} r \times {}^{2n}C_r \times {}^{2n}C_r$$

$$\sum \cancel{r} \times \frac{2n}{\cancel{r}} \cdot {}^{2n-1}C_{r-1} \times {}^{2n}C_r$$

$$2n \sum \underbrace{{}^{2n-1}C_{r-1} \cdot {}^{2n}C_r}_{\text{Sum} = 2r-1}$$

$$2n \sum \underbrace{{}^{2n-1}C_{r-1} \times {}^{2n}C_{2n-r}}_{\substack{\text{Sum} = \cancel{r}-1+2n-\cancel{r} \\ = 2n-1}} = 2n \times {}^{4n-1}C_{2n-1}$$

$${}^{4n-1}C_{2n}$$

Q. Find $\sum_{r=0}^{n} {}^{n+r}C_r$

$${}^{n}C_0 + {}^{n+1}C_1 + {}^{n+2}C_2 + {}^{n+3}C_3 + \cdots + {}^{n+n}C_n$$

$$= {}^{n}C_n + {}^{n+1}C_n + {}^{n+2}C_n + {}^{n+3}C_n + \cdots + {}^{2n}C_n$$

Coff of x^n in $(1+x)^n + (1+x)^{n+1} + (1+x)^{n+2} + \cdots + (1+x)^{2n}$

$$\frac{(1+x)^n\{(1+x)^{n+1} - 1\}}{\cancel{1}+x-\cancel{1}} = \frac{(1+x)^{2n+1} - (1+x)^n}{x}$$

x^{n+1} Mangoga

Coff of x^n = ${}^{2n+1}C_{n+1} - 0$

$= {}^{2n+1}C_{n+1}$

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$$(1+x^2+x^4)^n = a_0+a_1x^2+a_2x^4+a_3x^6 \; a_{n-1}x^{2n-2}+a_nx^{2n}$$

$$(1+x+x^2)^n = a_0+a_1x+a_2x^2+a_3x^3+\dots a_nx^n+\dots a_{2n}x^{2n}$$

$$\dots + a_nx^n+a_{n+1}x^{n+1}\dots a_{2n}\cdot x^{2n}$$

$$-\boxed{a_0a_3}x^{2n-3}$$

→ Put $x=1 \Rightarrow 3^n = a_0+a_1+a_2+a_3\dots$

→ Put $x=-1 \Rightarrow (1-1+1)^n = a_0-a_1+a_2-a_3+a_4\dots = 1$

$$\longrightarrow 2(a_0+a_2+a_4+\dots) = 3^n+1 \Rightarrow a_0+a_2+a_4+a_6\dots = \frac{3^n+1}{2}$$

$$\xrightarrow{\text{Sub}} 2(a_1+a_3+a_5\dots) = 3^n-1 \Rightarrow a_1+a_3+a_5\dots = \frac{3^n-1}{2}$$

5) $a_0^2-a_1^2+a_2^2-a_3^2\dots$

6) $a_0a_2-a_1a_3+a_2a_4-a_3a_5\dots$

7) $a_0a_3-a_1a_4+a_2a_5\dots$

When Prod is asked with alternate + − Sign

$x \to -\frac{1}{x}$

A)

$$\left(1-\frac{1}{x}+\frac{1}{x^2}\right)^n = a_0-\frac{a_1}{x}+\frac{a_2}{x^2}-\frac{a_3}{x^3}+\frac{a_4}{x^4}\dots a_{2n}\cdot\frac{1}{x^{2n}}$$

$$\frac{(x^2-x+1)^n}{\cancel{x^{2n}}} = \frac{a_0x^{2n}-a_1x^{2n-1}+a_2x^{2n-2}-a_3x^{2n-3}\dots+a_{2n}}{\cancel{x^{2n}}}$$

(B)

$$(1+x+x^2)^n(1-x+x^2)^n = \{a_0+a_1x+a_2x^2+a_3x^3\dots a_{2n}x^{2n}\}\{a_0x^{2n}-a_1x^{2n-1}+a_2x^{2n-2}-a_3x^{2n-3}\dots+a_{2n}\}$$

$$(1+x^2+x^4)^n$$

(C) (5) $a_0\cdot a_0-a_1\cdot a_1+a_2\cdot a_2-a_3\cdot a_3\dots = $ Coff of x^{2n} in $(1+x^2+x^4)^{2n}$

x^{2n} Coff, x^{2n} Coff, x^{2n} Coff

$= a_n$

(6) $a_0a_2-a_1a_3+a_2a_4-a_3a_5\dots = $ Coff of $x^{2n-2} = a_{n-1}$

(7) $a_0a_3-a_1a_4+a_2a_5\dots = $ Coff of $x^{2n-3} = 0$

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Multinomial Theorem.

1) $(x_1+x_2)^n = \sum {}^nC_r \cdot (x_1)^{n-r} \cdot (x_2)^r = \sum \frac{n!}{r!(n-r)!} \cdot (x_1)^{\overset{=K_1}{n-r}} \cdot (x_2)^{\overset{=K_2}{r}} = \sum_{K_1+K_2=n} \frac{n!}{K_2!\,K_1!} \cdot (x_1)^{K_1} \cdot (x_2)^{K_2}$

2) $(x_1+x_2)^n = \sum \frac{n!}{K_1!\,K_2!} \, x_1^{K_1} \cdot x_2^{K_2}$

$$\boxed{3)\ (x_1+x_2+x_3)^n = \sum_{K_1+K_2+K_3=n} \frac{n!}{K_1!\,K_2!\,K_3!} (x_1)^{K_1} (x_2)^{K_2} \cdot (x_3)^{K_3}.}$$

(4) Total No of terms when x_1, x_2, x_3 are not linked to each other $= {}^{n+r-1}C_{r-1}$

Q1 No of distinct terms in $(x+y-z)^{16}$

$r=3,\ n=16$

$= {}^{16+3-1}C_{3-1} = {}^{18}C_2$

Q If No of terms in exp. of $(x+y+z)^n$ is 36 find n?

$r=3,\ n=n$

total No of terms $= {}^{n+3-1}C_{3-1} = {}^{n+2}C_2$

${}^{n+2}C_2 = 36 \Rightarrow \frac{(n+2)(n+1)}{1\cdot 2} = 36$

$n^2+3n+2 = 72 \Rightarrow n^2+3n-70=0 \rightarrow n = 7, -10$

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Q Find Coff of $a^8 \cdot b^6 \cdot c^4$ in $(a+b+c)^{18}$?

$$(x_1+x_2+x_3)^n = \sum \frac{n!}{K_1!\,K_2!\,K_3!} \boxed{(x_1)^{K_1}(x_2)^{2}\cdot(x_3)^{K_3}}$$

$\cancel{5}!$ Coff

In $(a+b+c)^{18}$ Coff $a^8 \cdot b^6 \cdot c^4 = \dfrac{18!}{8!\,6!\,4!}$

$8+6+4=18$

Q Coff of $x^3 y^4 z^2$ in $(2x-3y+4z)^9$

$$\frac{9!}{3!\,4!\,2!}(2x)^3\cdot(-3y)^4\cdot(4z)^2$$

$$\text{Coff} = \frac{9 \times 8 \times 81 \times 16}{3!\,4!\,2!} = \frac{9! \times 8 \times 81 \times 16}{6 \times 24 \times 2} = 36 \cdot 9!$$

Q Coff of x^3 in $(1-x+x^2)^5$

$$G.T. = \sum \frac{5!}{K_1!\,K_2!\,K_3!}\cdot(1)^{K_1}\cdot(-x)^{K_2}\cdot(x^2)^{K_3}$$

$$= \sum \frac{5!}{K_1!\,K_2!\,K_3!}(x)^{K_2+2K_3}\cdot(-1)^{K_2}$$

$K_1+K_2+K_3=5$

$\boxed{K_2+2K_3=3}$

$$\text{Coff} = \frac{5!}{3!\,1!\,1!}(-1)^1 + \frac{5!}{2!\,3!\,0!}\cdot(-1)^3$$

K_1	K_2	K_3
3	1	1
2	3	0

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Binomial Coeff for -ve & fractional Index.

1) $\boxed{\begin{array}{l} \text{If } n \in Q - W \ \& \ |x| < 1 \\ \text{then } (1+x)^n = 1 + nx + \frac{(n)(n-1)}{2!}x^2 + \frac{(n)(n-1)(n-2)}{3!}\cdot x^3 + \cdots \end{array}}$

2) No of terms are ∞

3) $\boxed{G.T. = \frac{(n)(n-1)(n-2)\cdots(n-r+1)}{r!}\cdot x^r}$

Q $\frac{1}{1-x} = ?$

$(1-x)^{-1} = 1 + (-1)(-x) + \frac{(-1)(-2)}{2!}(-x)^2 + \frac{(-1)(-2)(-3)}{3!}(-x)^3 + \cdots$

$\boxed{(1-x)^{-1} = 1 + x + x^2 + x^3 + x^4 + \cdots}$

Q Evaluate $\frac{1}{\sqrt[3]{6-3x}}$

$= (6-3x)^{-\frac{1}{3}}$

$= 6^{-\frac{1}{3}}\left(1 - \frac{x}{2}\right)^{-1/3}$

$= 6^{-\frac{1}{3}}\left\{1 + \left(-\frac{1}{3}\right)\left(-\frac{x}{2}\right) + \frac{\left(-\frac{1}{3}\right)\left(-\frac{1}{3}-1\right)}{2!}\cdot\left(-\frac{x}{2}\right)^2 + \frac{\left(-\frac{1}{3}\right)\left(-\frac{1}{3}-1\right)\left(-\frac{1}{3}-2\right)}{3!}\left(-\frac{x}{2}\right)^3 + \cdots\right\}$

Q $(1-x)^{-2} = ?$

$= 1 + 2x + \frac{(-2)(-3)}{1\cdot 2}(-x)^2 + \frac{(\cancel{-2})(-\cancel{3})(-4)}{1\cdot\cancel{2}\cdot\cancel{3}}(-x)^3 + \cdots$

$= 1 + 2x + 3x^2 + 4x^3 + \cdots$

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Q Find range of x for which $\frac{1}{\sqrt{5+4x}}$ can be expanded in ring Powers of x.

$= (5+4x)^{-1/2}$

$= \frac{1}{\sqrt{5}}\left(1+\frac{4x}{5}\right)^{-1/2}$

Expand hogo only when $\left|\frac{4x}{5}\right|<1$

$\Rightarrow \frac{4}{5}|x|<1$

$|x|<\frac{5}{4}$

$-\frac{5}{4}<x<\frac{5}{4}$ $\therefore$ R of $x \in \left(-\frac{5}{4},\frac{5}{4}\right)$

Q Find 6^{th} term in $(1-2x)^{-3}$?

$T_6 = \frac{(-3)(-3-1)(-3-2)(-3-3)(-3-4)}{5!}\cdot(-2x)^5$