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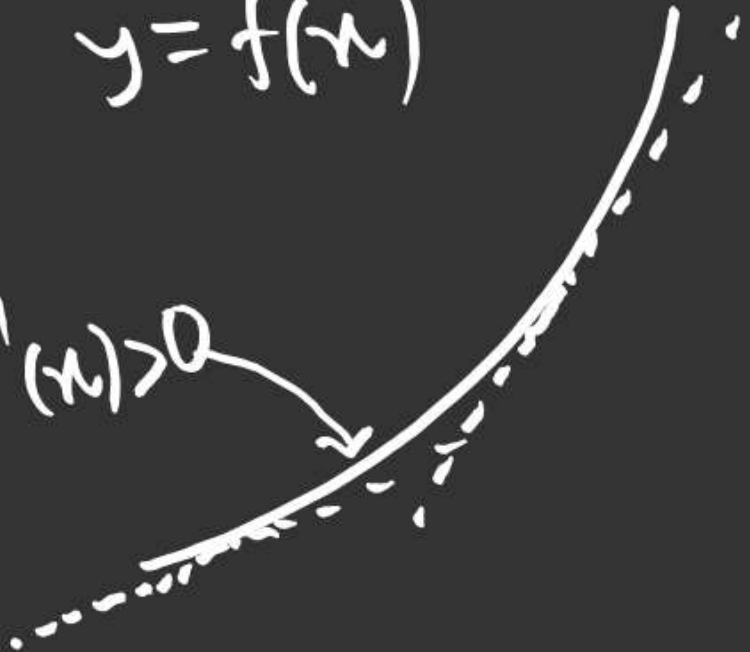
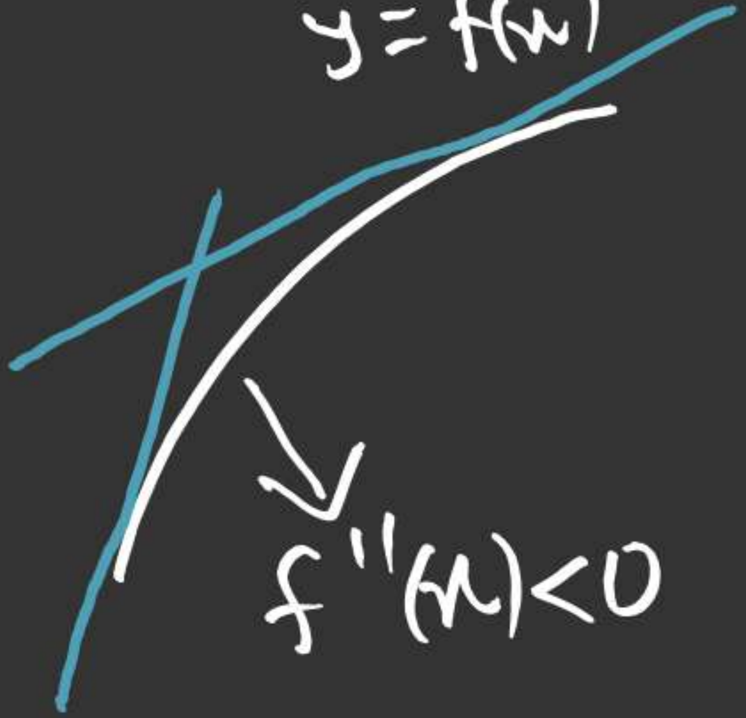
# FUNCTIONS

$y = f(x)$



$f''(x) > 0$

$y = f(x)$

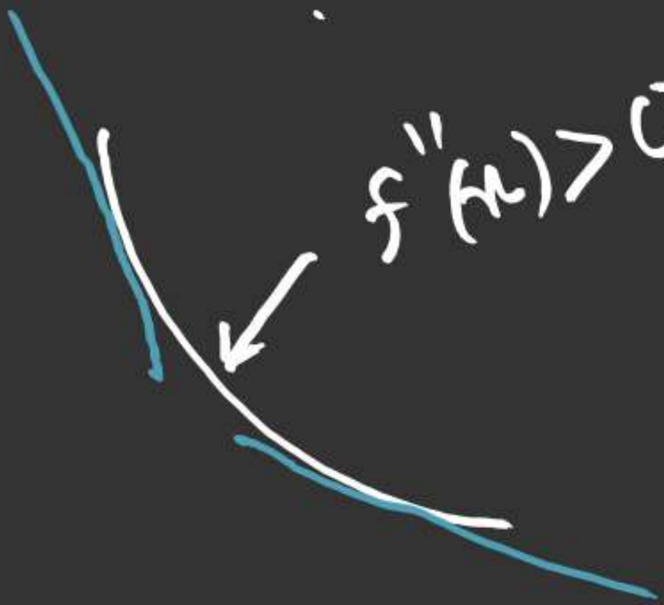


$f''(x) < 0$

$f''(x) = 0$

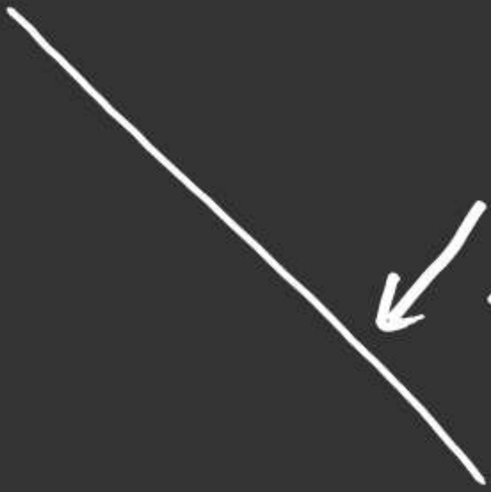
$f''(x) = \left(f'(x)\right)' > 0$

$\Rightarrow$ $f'(x)$ increas-es



$f''(x) > 0$

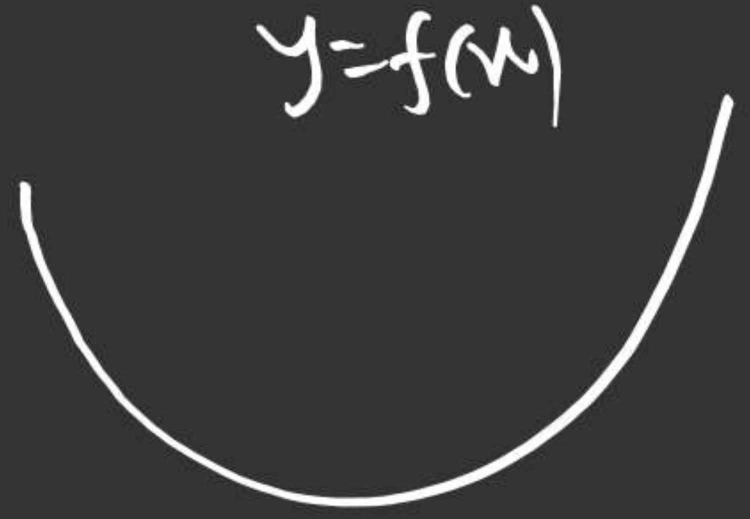
$f''(x) < 0$



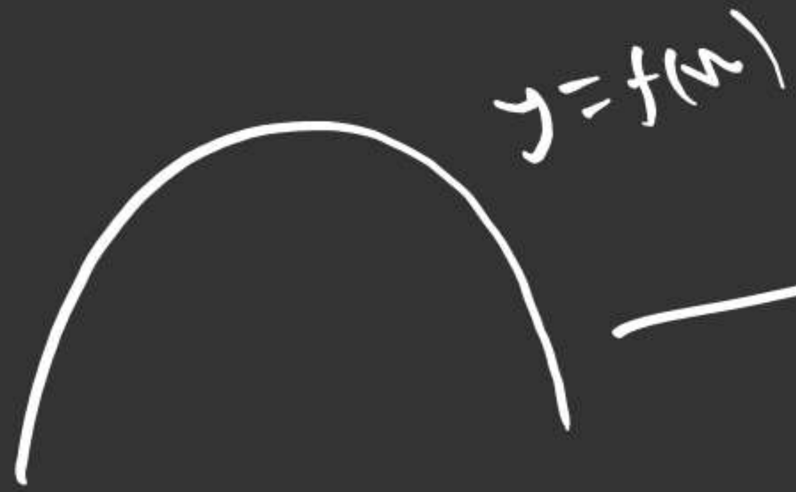
$f''(x) = 0$

$\tan\theta$ $\quad -\infty, 0$

# FUNCTIONS




$\longrightarrow f''(x) > 0$




$\longrightarrow f''(x) < 0$

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# Graph

1. Domain
2. Intervals of increase/decrease
3. Concavity
4. Sketch

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# FUNCTIONS

Domain $f: A \rightarrow B$ CoDomain

$$y = f(x)$$

$$R_f \subseteq B$$

Range

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$f(x) = \sin x$ $\qquad x \in [0, 2\pi)$

$f'(x) = \cos x$

$f \uparrow \quad (0, \pi/2) \cup (3\pi/2, 2\pi)$

$\quad \downarrow \quad (\pi/2, 3\pi/2)$


$f''(x) = -\sin x$

$f''(x) < 0 \qquad (0, \pi)$

$f''(x) > 0 \qquad (\pi, 2\pi)$

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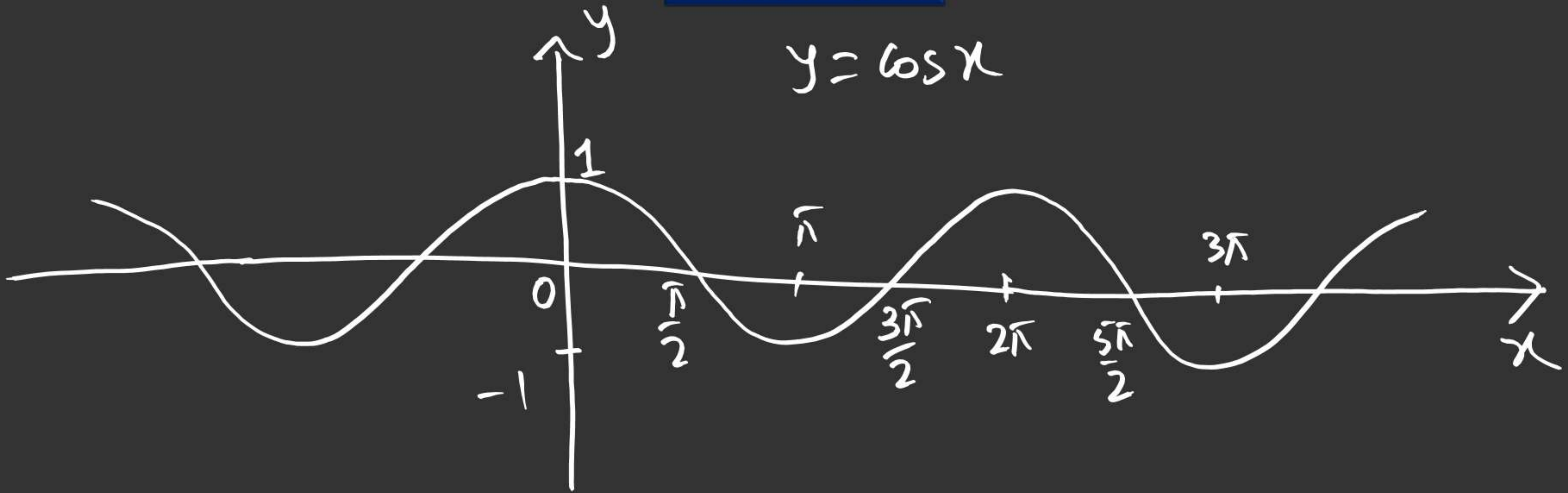
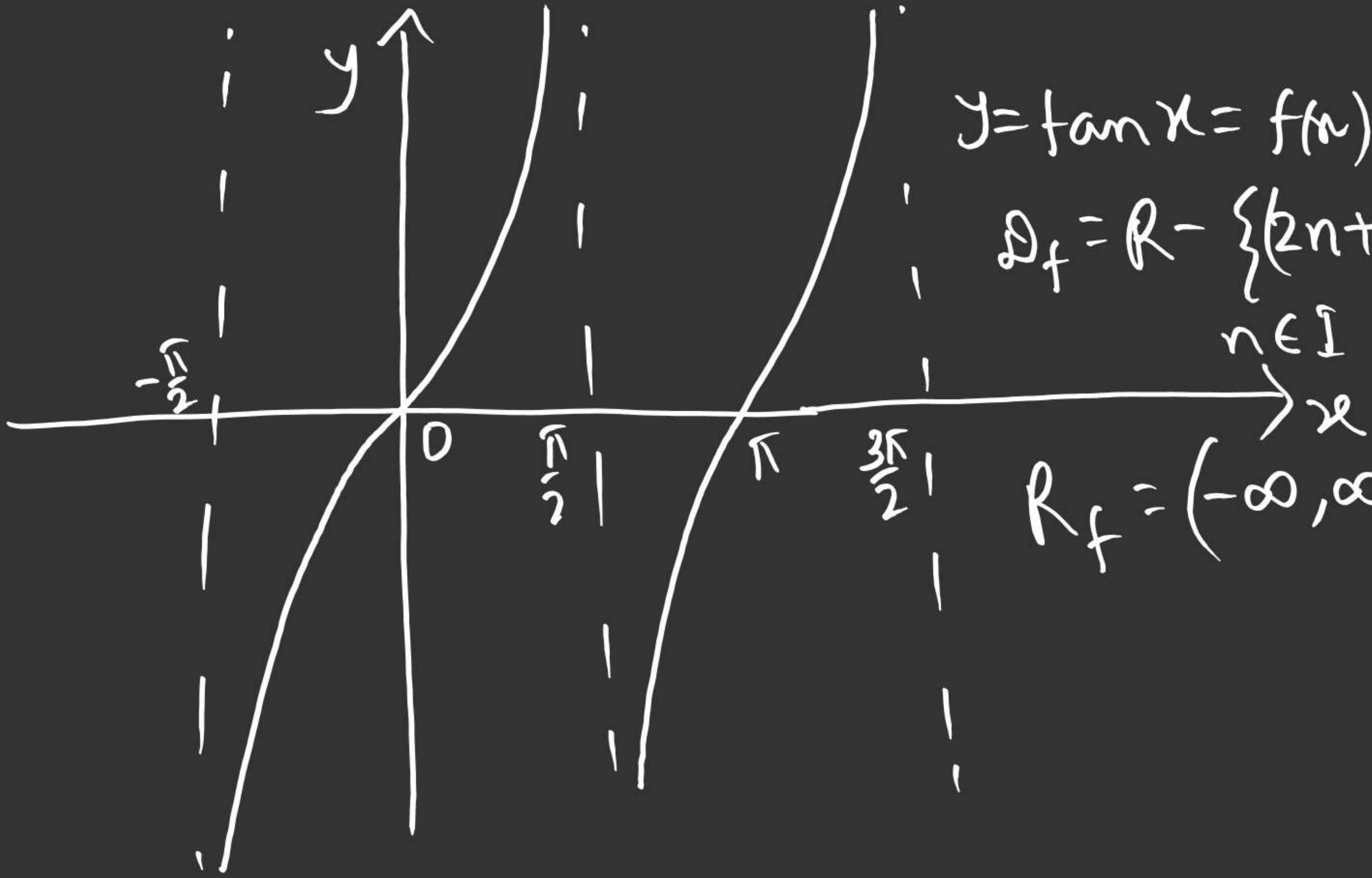
# FUNCTIONS

$y = \cos x$






$$y = \tan x = f(x)$$

$$D_f = R - \left\{(2n+1)\frac{\pi}{2}\right\}$$

$$n \in I$$

$$R_f = (-\infty, \infty)$$

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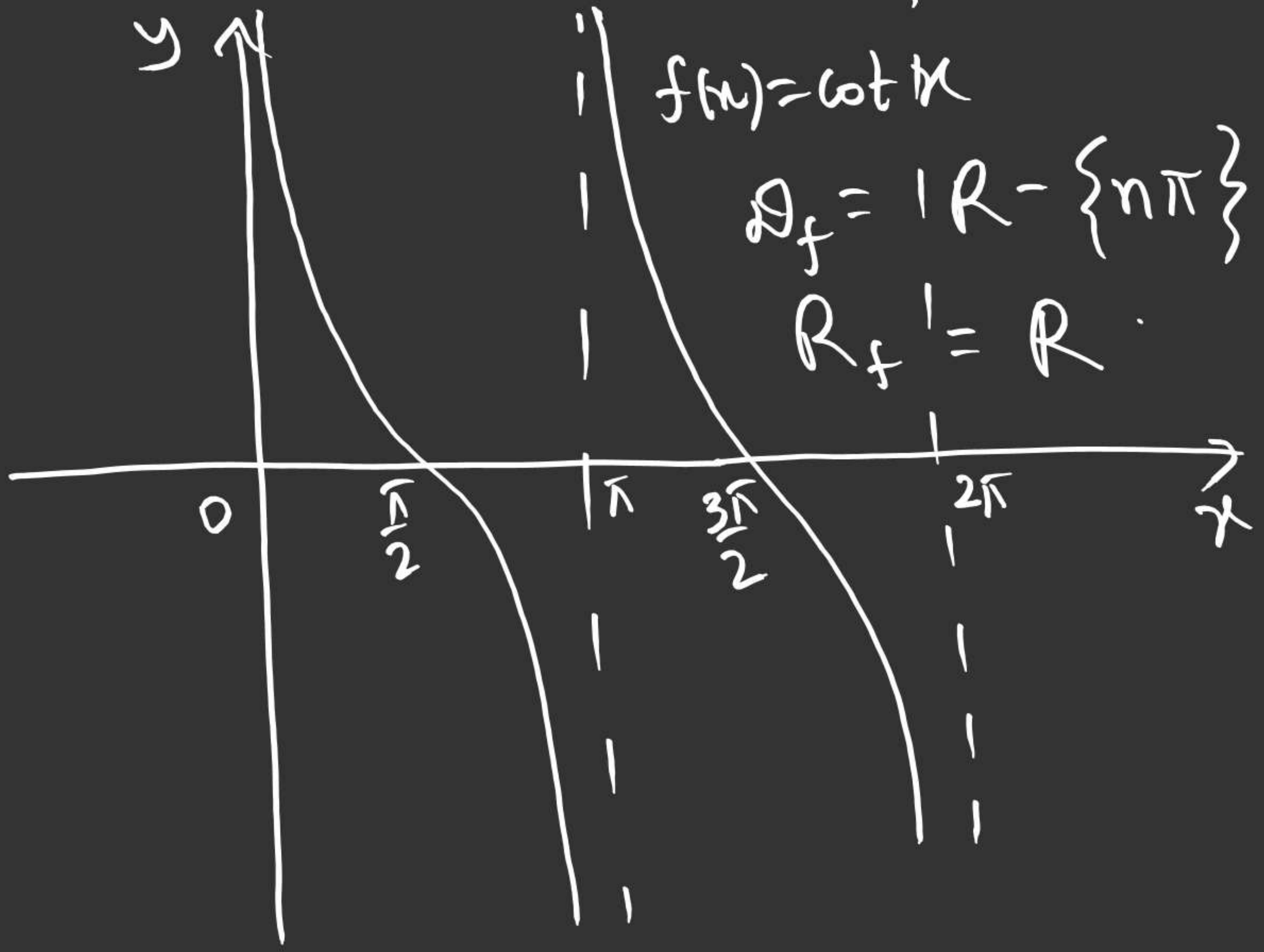
# FUNCTIONS




$f(x) = \cot x$

$D_f = \mathbb{R} - \{n\pi\}, n \in I$

$R_f = \mathbb{R}$

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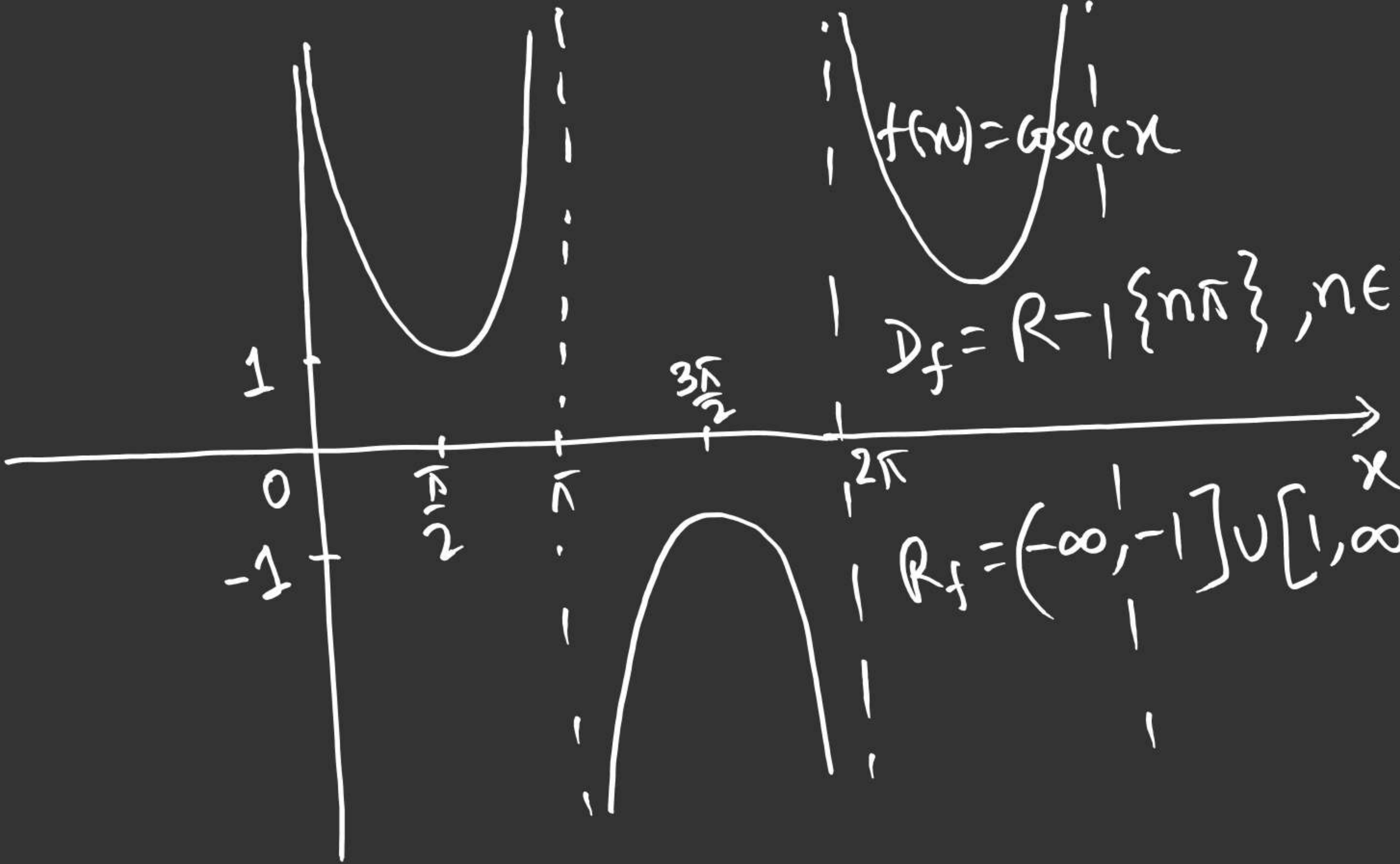


$f(x) = \operatorname{cosec} x$

$D_f = R - \{n\pi\},\ n \in I$

$R_f = (-\infty, -1] \cup [1, \infty)$

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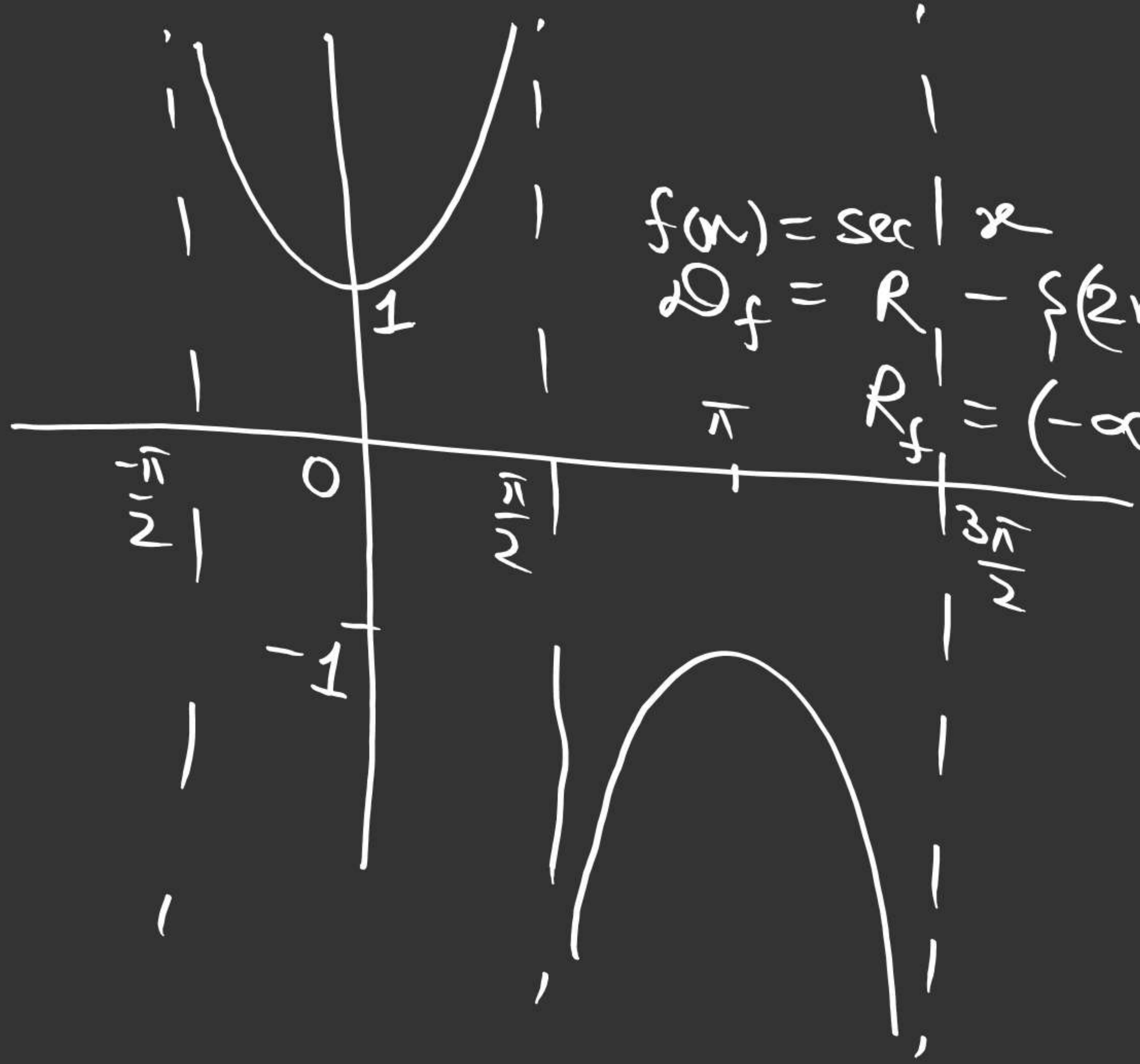
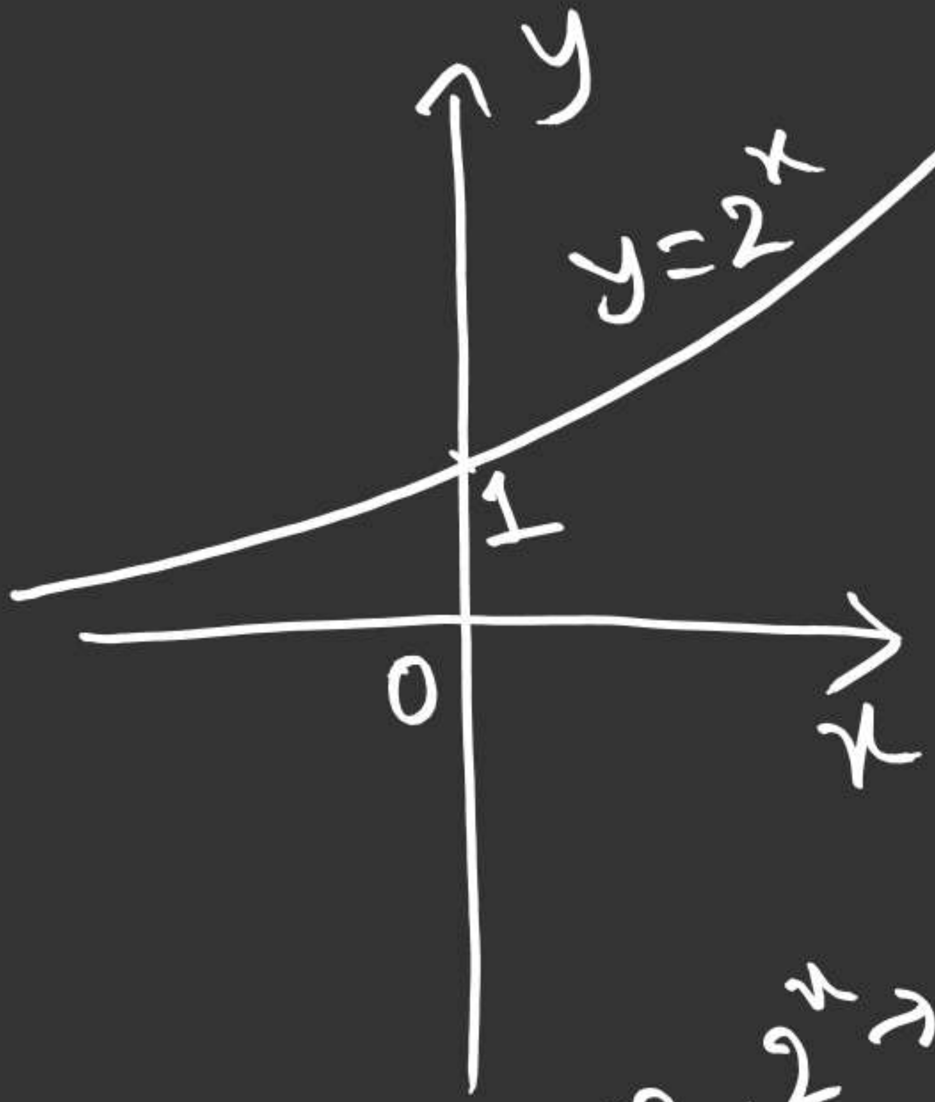



$f(x) = \sec x$

$D_f = R - \{(2n+1)\frac{\pi}{2}\}, n \in I.$

$R_f = (-\infty, -1] \cup [1, \infty)$

# Exponential & Logarithm




$$f(x) = 2^x = e^{x\ln 2}$$

① $D_f = R$

② $f'(x) = 2^x \ln 2 > 0$

③ $f''(x) = 2^x \ln^2 2 > 0$

$x \to -\infty,\ 2^x \to 0$

$x \to \infty,\ 2^x \to \infty$

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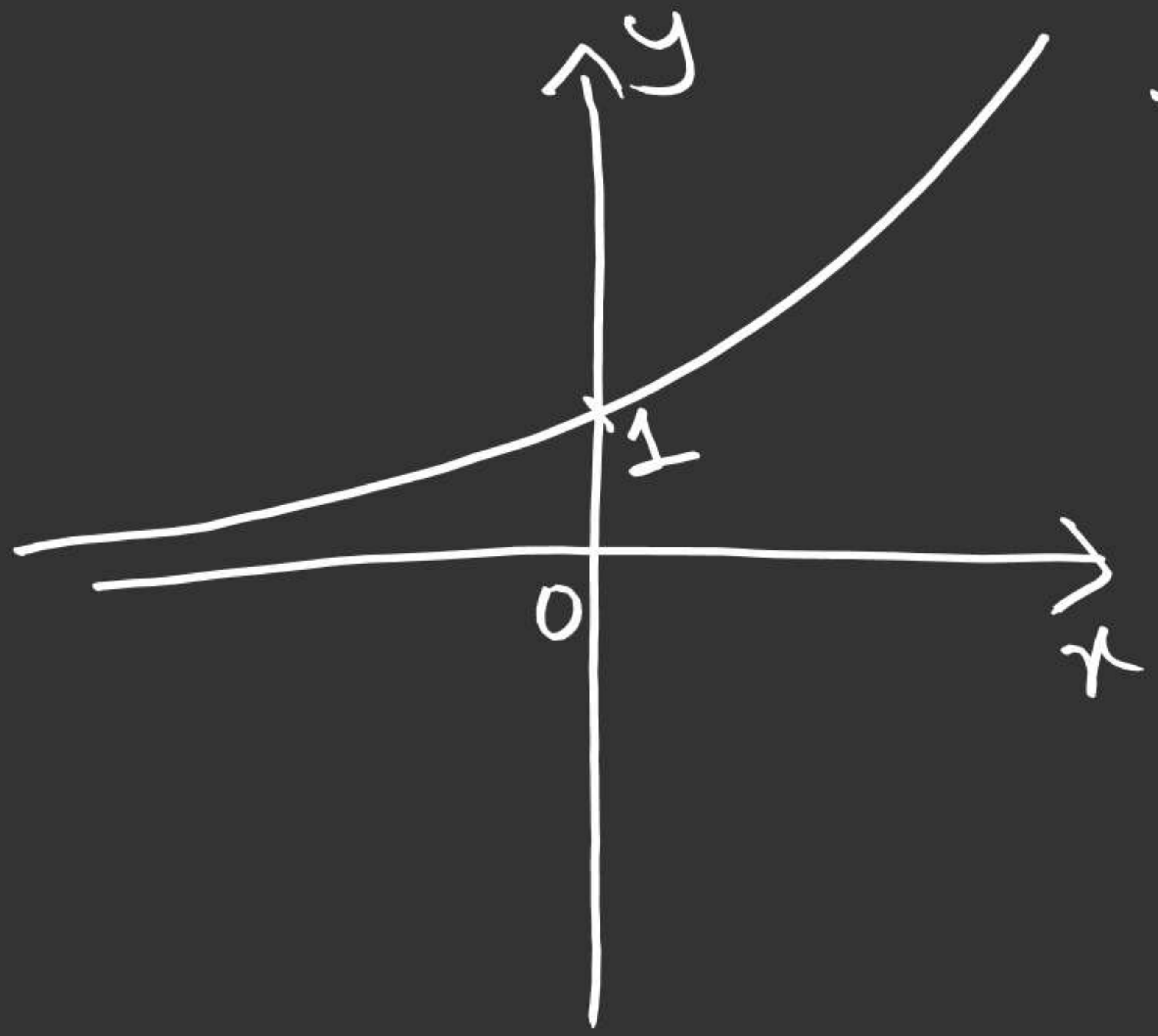


$$y = a^x \quad , a > 1$$

$$\frac{dy}{dx} = a^x \ln a \quad > 0$$

$$\frac{d^2y}{dx^2} = a^x \ln^2 a > 0$$

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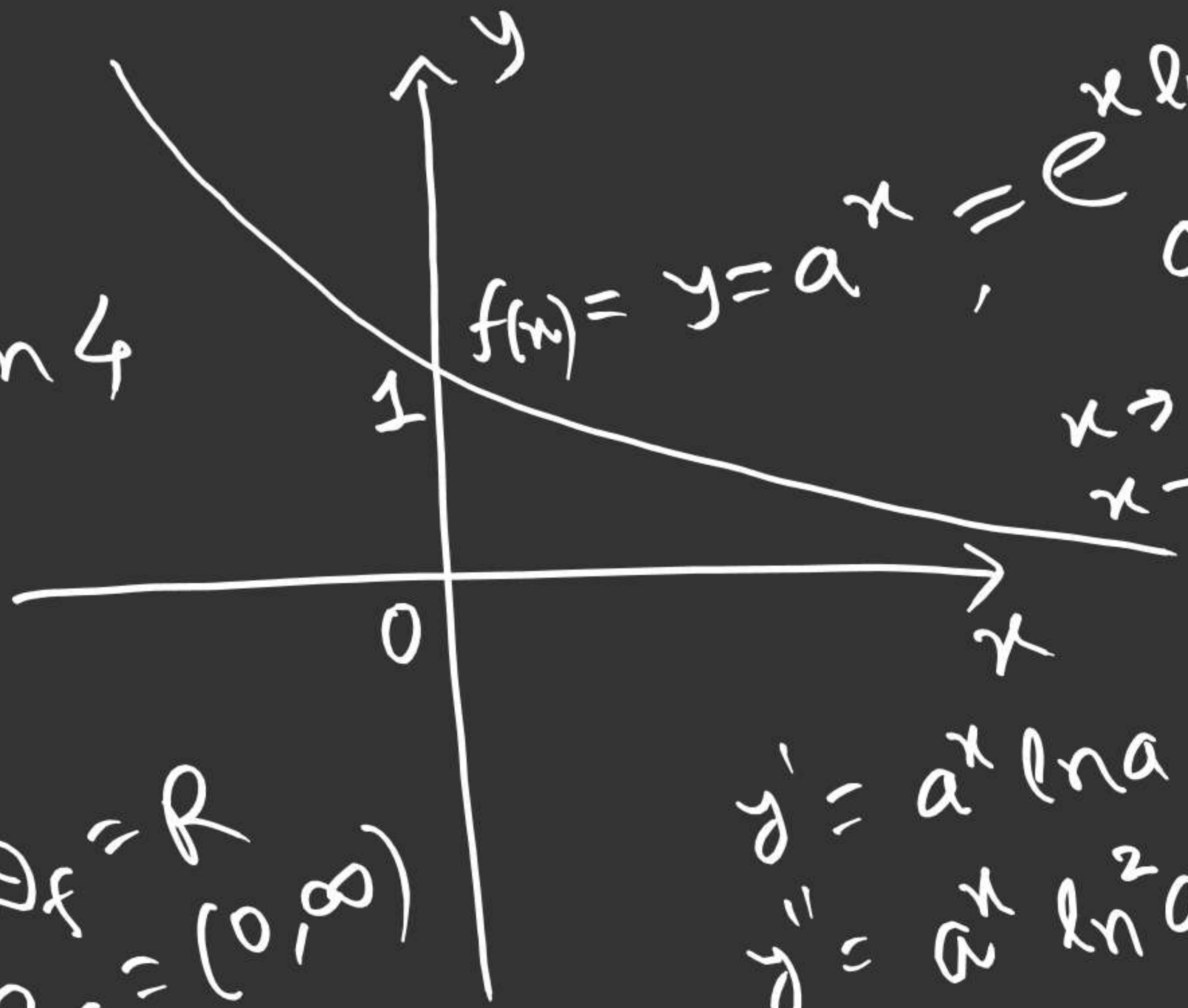
$$f(x) = y = a^x = e^{x \ln a}, \quad 0 < a < 1$$

$$4 = e^{\ln 4}$$




$x \to -\infty, \; y \to \infty$

$x \to \infty, \; y \to 0$

$D_f = \mathbb{R}$

$R_f = (0, \infty)$

$y' = a^x \ln a < 0$

$y'' = a^x \ln^2 a > 0$

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$$f(x) = \log_a x = \frac{\ln x}{\ln a}$$

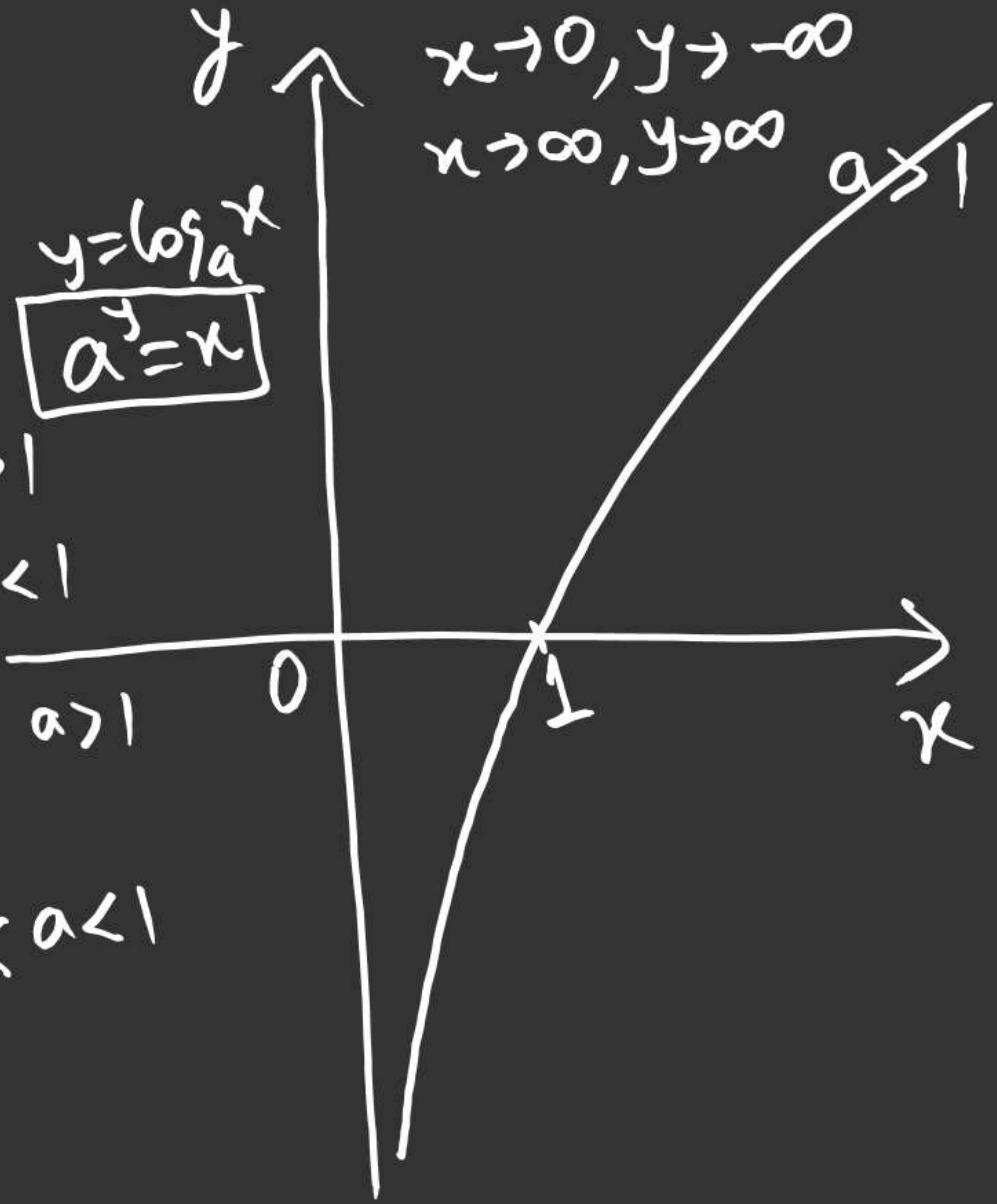
① $D_f = (0, \infty)$

② $f'(x) = \dfrac{1}{x \ln a}$ $> 0$, $a > 1$

$< 0$, $0 < a < 1$

③ $f''(x) = -\dfrac{1}{x^2 \ln a} < 0$ $a > 1$

$> 0$ $0 < a < 1$



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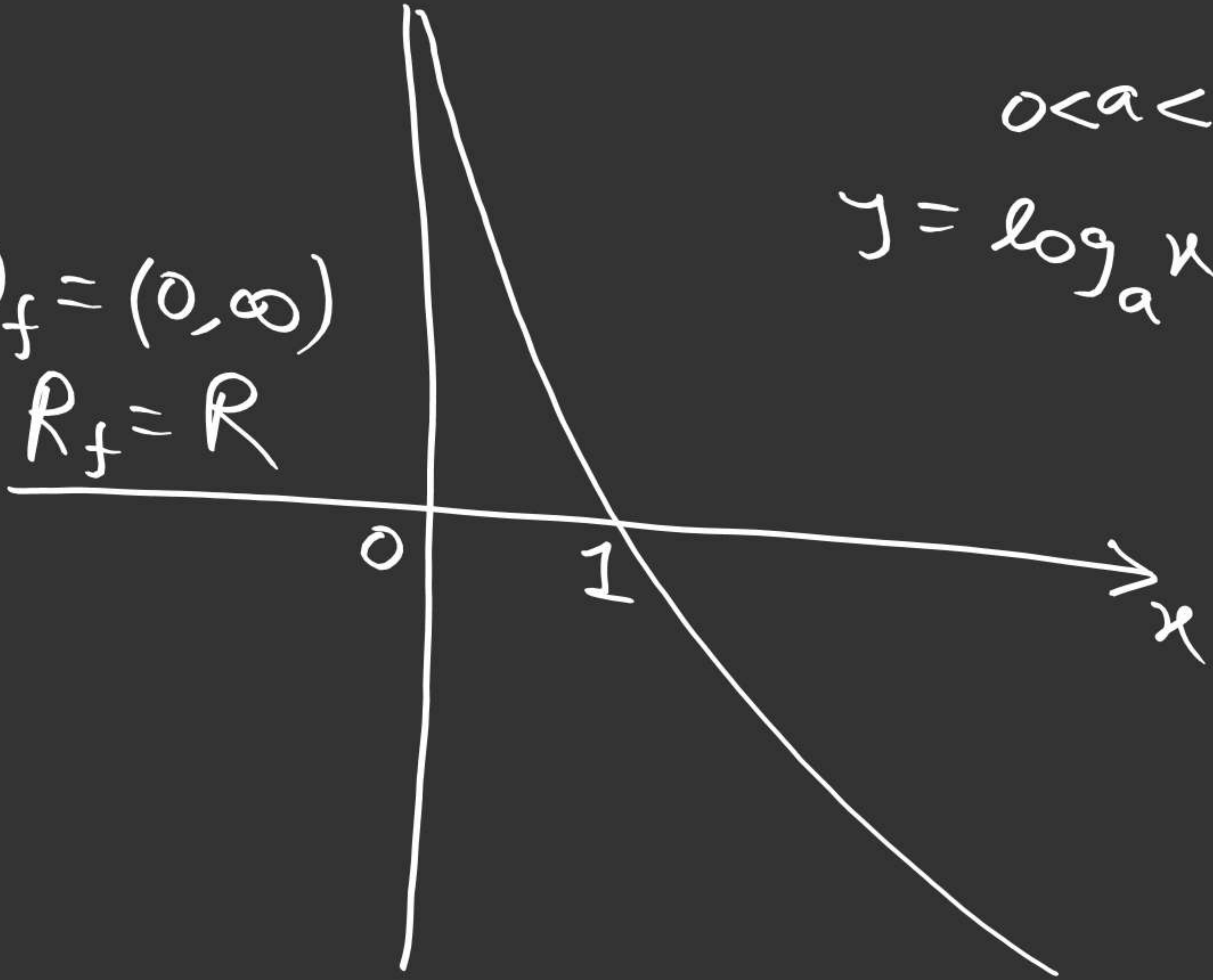
$0 < a < 1$

$y = \log_a x$

$D_f = (0, \infty)$

$R_f = \mathbb{R}$



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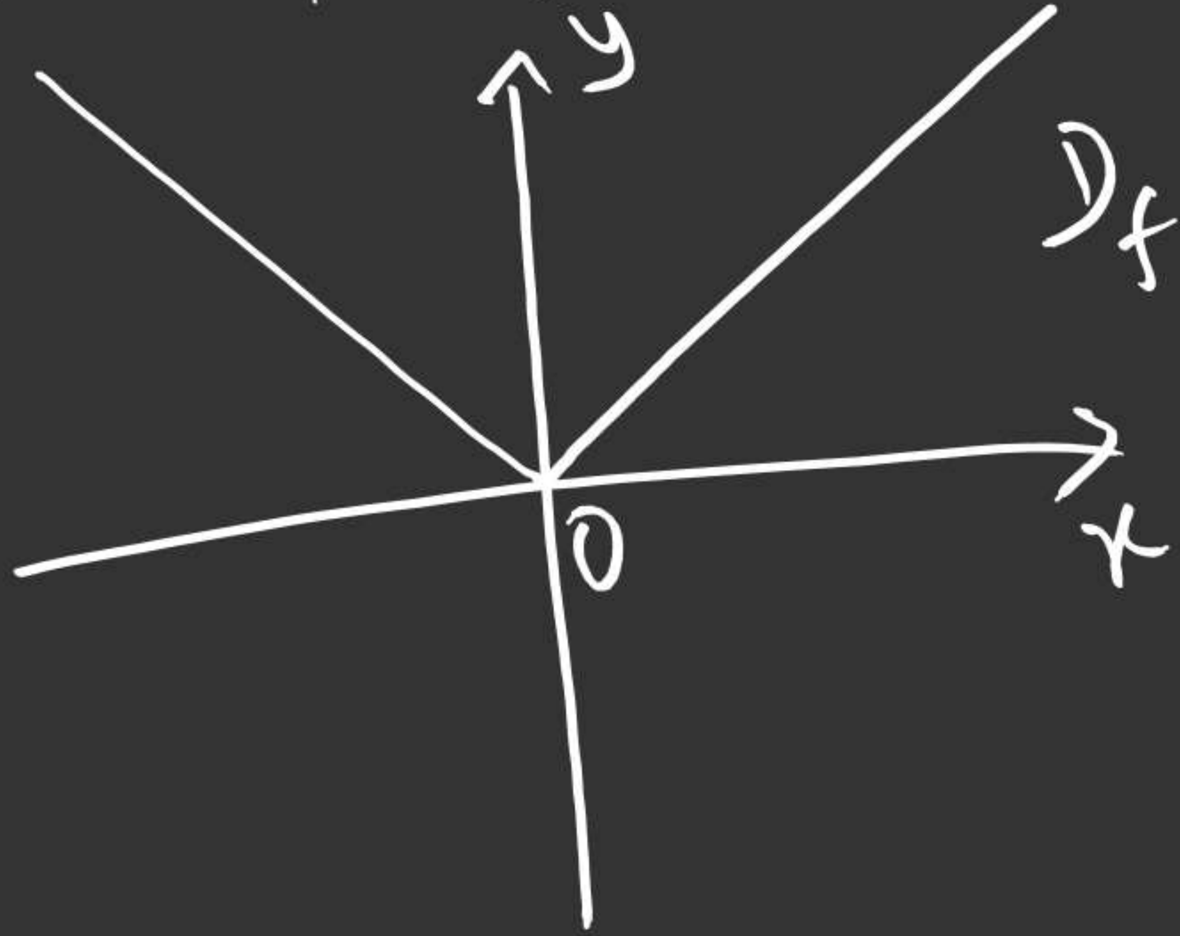
# Modulus/Absolute Value Function

$$f(x) = |x| = \sqrt{x^2}$$




$D_f = R$

$R_f = [0, \infty)$

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# FUNCTIONS

## Properties

$$|ab| = |a||b|$$

$$\left|\frac{a}{b}\right| = \frac{|a|}{|b|}$$

Equality holds if $ab \le 0$

Equality holds if $ab \geq 0$

$$\big||a|-|b|\big| \le |a+b| \le |a|+|b|$$

$a, b \in R$

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# Jensen's Inequality

If $f''(x) > 0$ in $[a,b]$, $x_1, x_2, \ldots, x_n \in (a,b)$ and $\lambda_1, \lambda_2, \ldots, \lambda_n > 0$, then

$$\frac{\lambda_1 f(x_1) + \lambda_2 f(x_2) + \cdots + \lambda_n f(x_n)}{\lambda_1 + \lambda_2 + \cdots + \lambda_n} \geq f\left(\frac{\lambda_1 x_1 + \lambda_2 x_2 + \cdots + \lambda_n x_n}{\lambda_1 + \lambda_2 + \cdots + \lambda_n}\right)$$

equality holds if $x_1 = x_2 = x_3 = \cdots = x_n$.

Inequality get reversed

if $f''(x) < 0$

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For any triangle ABC,
find maximum value of

(1) $\sin A + \sin B + \sin C$

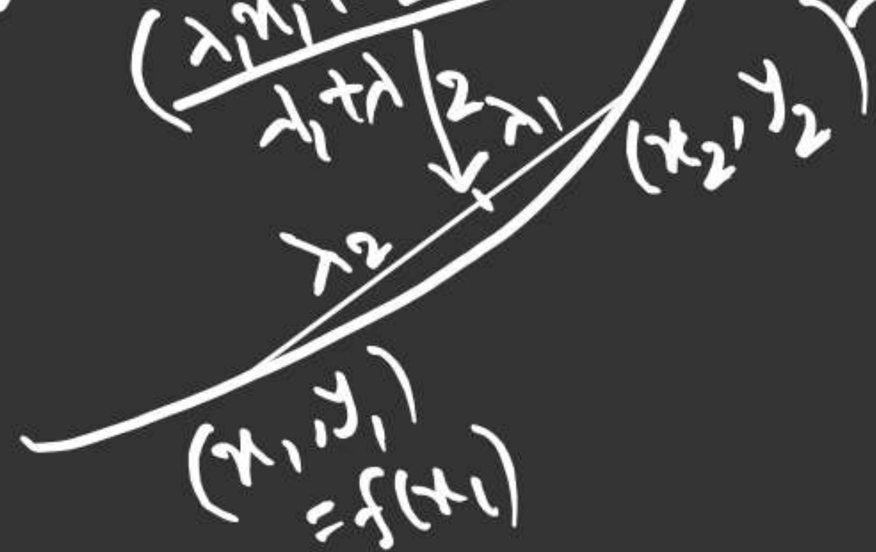
(2) $\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$

# FUNCTIONS

$f'$

$f''(x) > 0$

$$\left(\frac{\lambda_1 x_1+\lambda_2 x_2}{\lambda_1+\lambda_2}, \frac{\lambda_1 f(x_1)+\lambda_2 f(x_2)}{\lambda_1+\lambda_2}\right)$$




$f''(x) > 0$

$f''(x) < 0$

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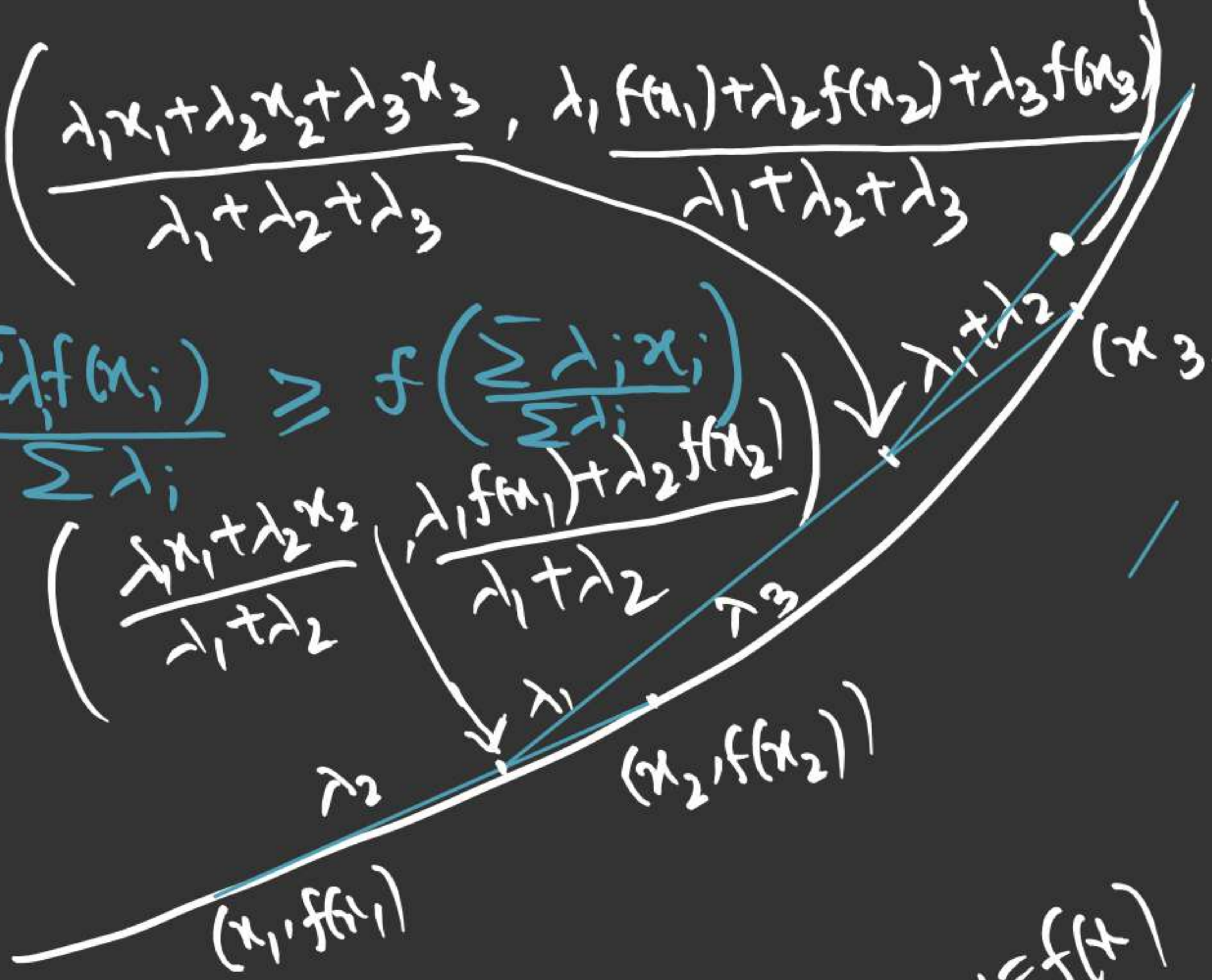
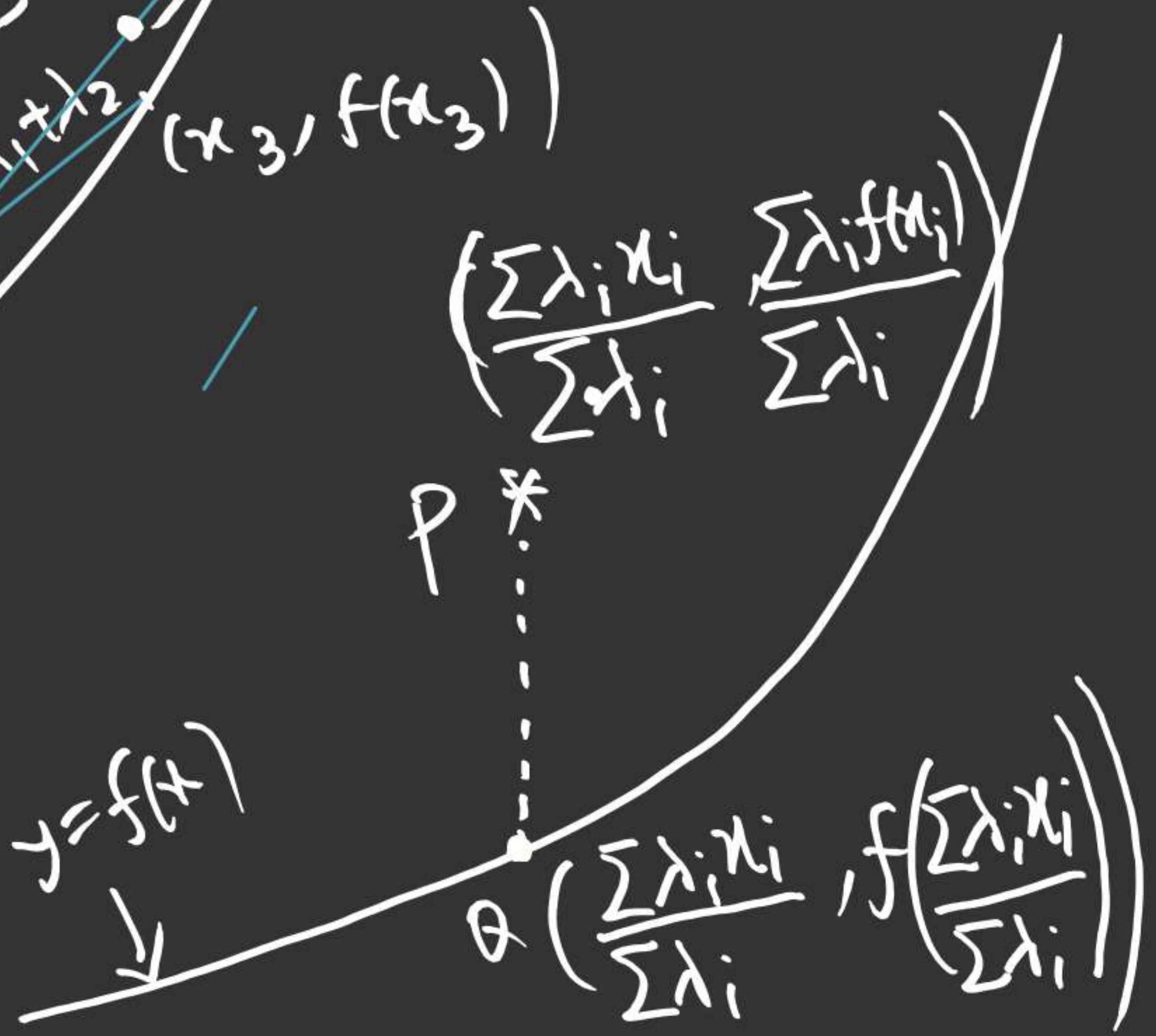
$$\frac{\sum \lambda_i f(x_i)}{\sum \lambda_i} \geq f\left(\frac{\sum \lambda_i x_i}{\sum \lambda_i}\right)$$



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# Concavity of function

$y = f(x)$

$\boxed{\begin{array}{l} \text{If } f''(x) > 0 \Rightarrow f \text{ is concave upwards} \\ \text{If } f''(x) < 0 \Rightarrow f \text{ is concave downwards} \end{array}}$

$\frac{dy}{dx} = f'(x)$

$\frac{d}{dx}\left(\frac{dy}{dx}\right) = f''(x) = \frac{d^2y}{dx^2}$