

1. Find the eqn. of common tangent to  $y^2 = 8x$  and  $3x^2 - y^2 = 3$ .

$$ty = x + 2t^2$$

$$y = \frac{1}{t}x + 2t$$

$$y = 2x + 1, \quad y = -2x - 1$$

$$4t^2 = 1\left(\frac{1}{t}\right) - 3$$

$$4t^4 + 3t^2 - 1 = 0$$

$$(4t^2 - 1)(t^2 + 1) = 0$$

$$t = \pm \frac{1}{2}$$

2. Find eqn. of tangent to  $\frac{x^2}{36} - \frac{y^2}{9} = 1$  passing through  $(0, 4)$ :

$$y = mx \pm \sqrt{m^2(36) - 9}$$

$$16 = 36m^2 - 9$$

$$m = \pm \frac{5}{6}$$

$$y - 4 = \frac{5}{6}x$$

$$y - 4 = -\frac{5}{6}x$$

3. P.T. two tangents drawn from any point on the hyperbola  $x^2 - y^2 = a^2 - b^2$  to ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  make complementary angle with  $x$ -axis.

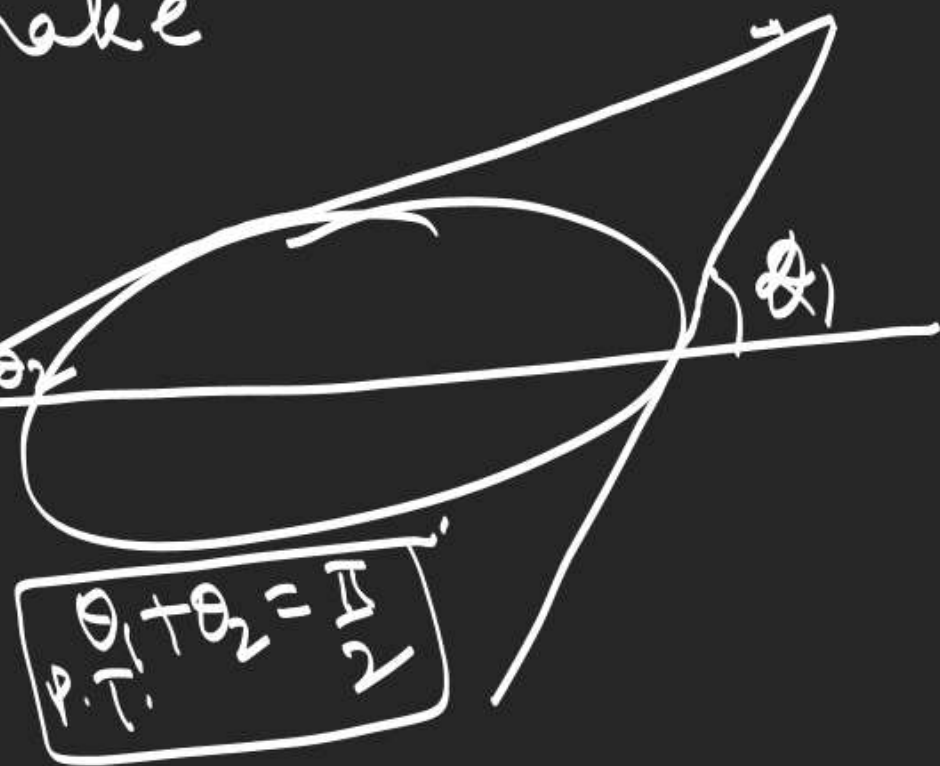
$(\alpha, \beta)$   $\rightarrow$   $x^2 - \beta^2 = a^2 - b^2$

$$y = mx \pm \sqrt{a^2 m^2 + b^2}$$

$$(\beta - m\alpha)^2 = a^2 m^2 + b^2$$

$$m^2(\alpha^2 - a^2) - 2\alpha\beta m + \beta^2 - b^2 = 0 \quad \begin{matrix} m_1 \\ m_2 \end{matrix}$$

$$m_1 m_2 = \frac{\beta^2 - b^2}{\alpha^2 - a^2} = \frac{\alpha^2 - a^2 + b^2 - b^2}{\alpha^2 - a^2} = 1$$



Find the eqn. and length of common tangent to hyperbolas

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$$

$$\frac{x^2}{(-b^2)} - \frac{y^2}{(-a^2)} = 1$$

$$PQ = \frac{(a^2 + b^2)\sqrt{2}}{\sqrt{a^2 - b^2}}$$

$$P(x_1, y_1) = \left( \frac{-a^2}{\sqrt{a^2 - b^2}}, \frac{-b^2}{\sqrt{a^2 - b^2}} \right)$$

$$y = mx \pm \sqrt{a^2 m^2 - b^2}$$

$$a^2 m^2 - b^2 = -b^2 m^2 - (-a^2)$$

$$m = \pm 1$$

$$\left( \frac{b^2}{\sqrt{a^2 - b^2}}, \frac{a^2}{\sqrt{a^2 - b^2}} \right) = Q(x_2, y_2)$$

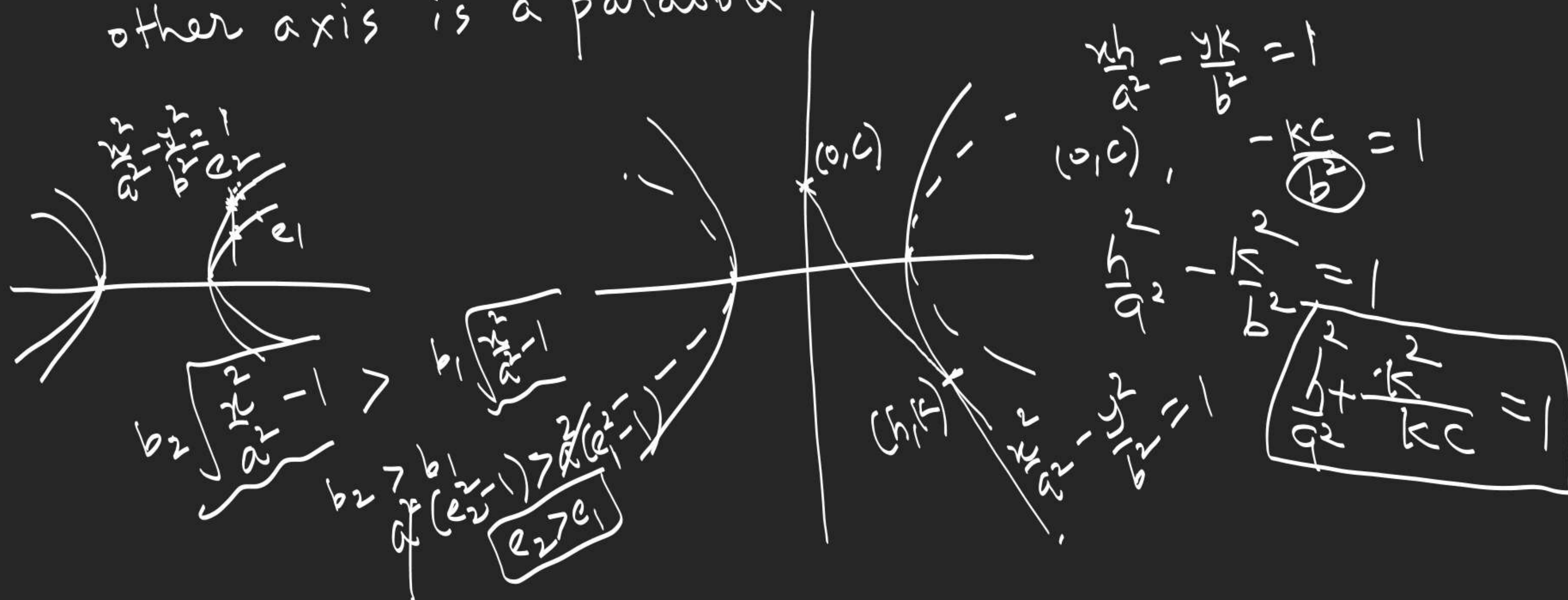
$$y - x = \sqrt{a^2 - b^2}$$

$$\frac{x_1 x_2}{a^2} - \frac{y_1 y_2}{b^2} = 1$$

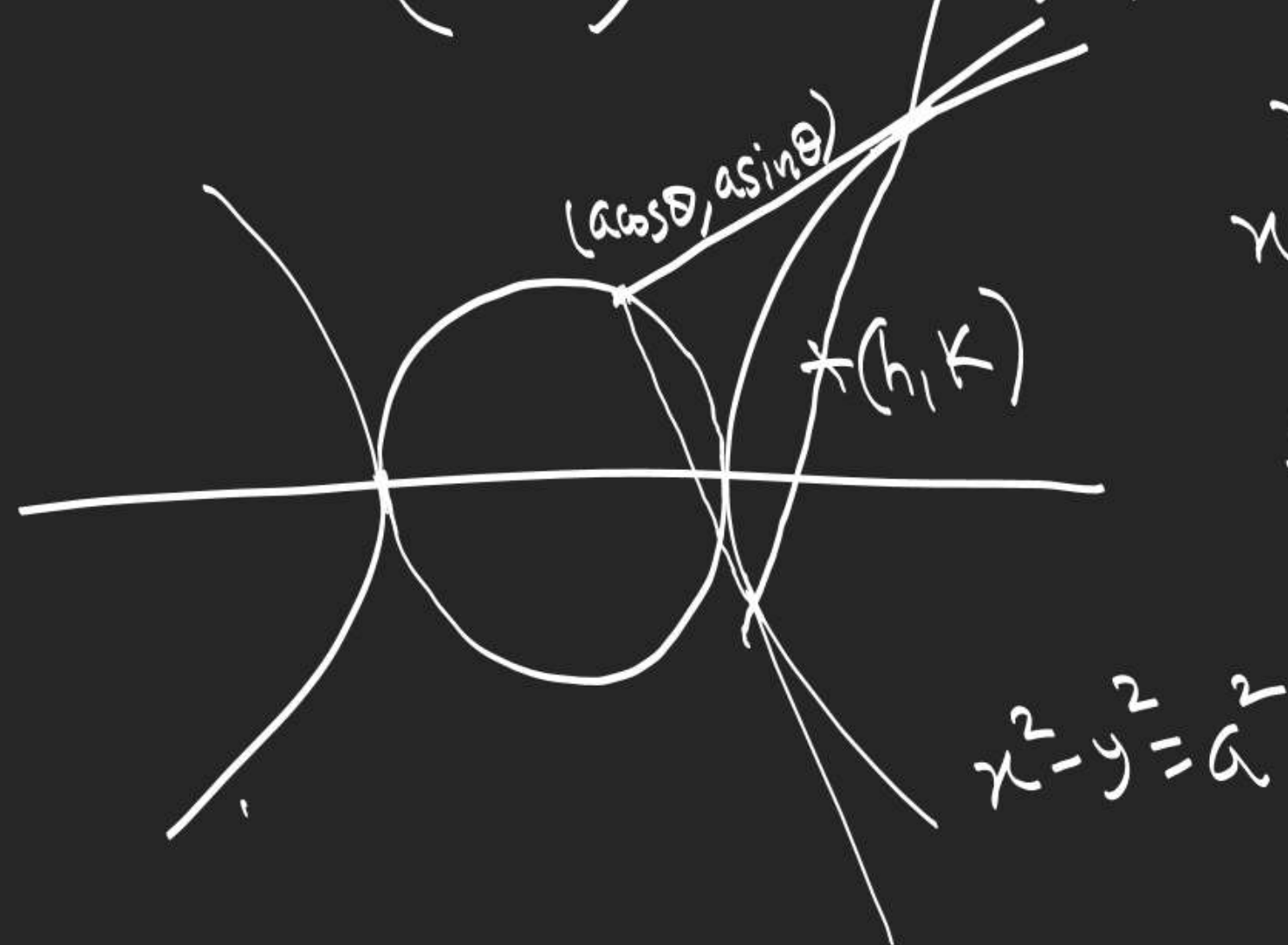
$$\frac{y_1 y_2}{a^2} - \frac{x_1 x_2}{b^2} = 1$$

$$\frac{x_1}{b^2} = \frac{y_1}{a^2} = \frac{x_2}{b^2} = \frac{y_2}{a^2} = \frac{1}{\sqrt{a^2 - b^2}}$$

5. If one axis of varying hyperbola be fixed in magnitude and position. P.T. locus of point of contact of a tangent drawn to it from a fixed point on other axis is a parabola.



6. From points on circle  $x^2 + y^2 = a^2$ , tangents are drawn to hyperbola  $x^2 - y^2 = a^2$ . P.T. locus of middle points of chord of contact is the curve  $(x^2 - y^2)^2 = a^2(x^2 + y^2)$ .



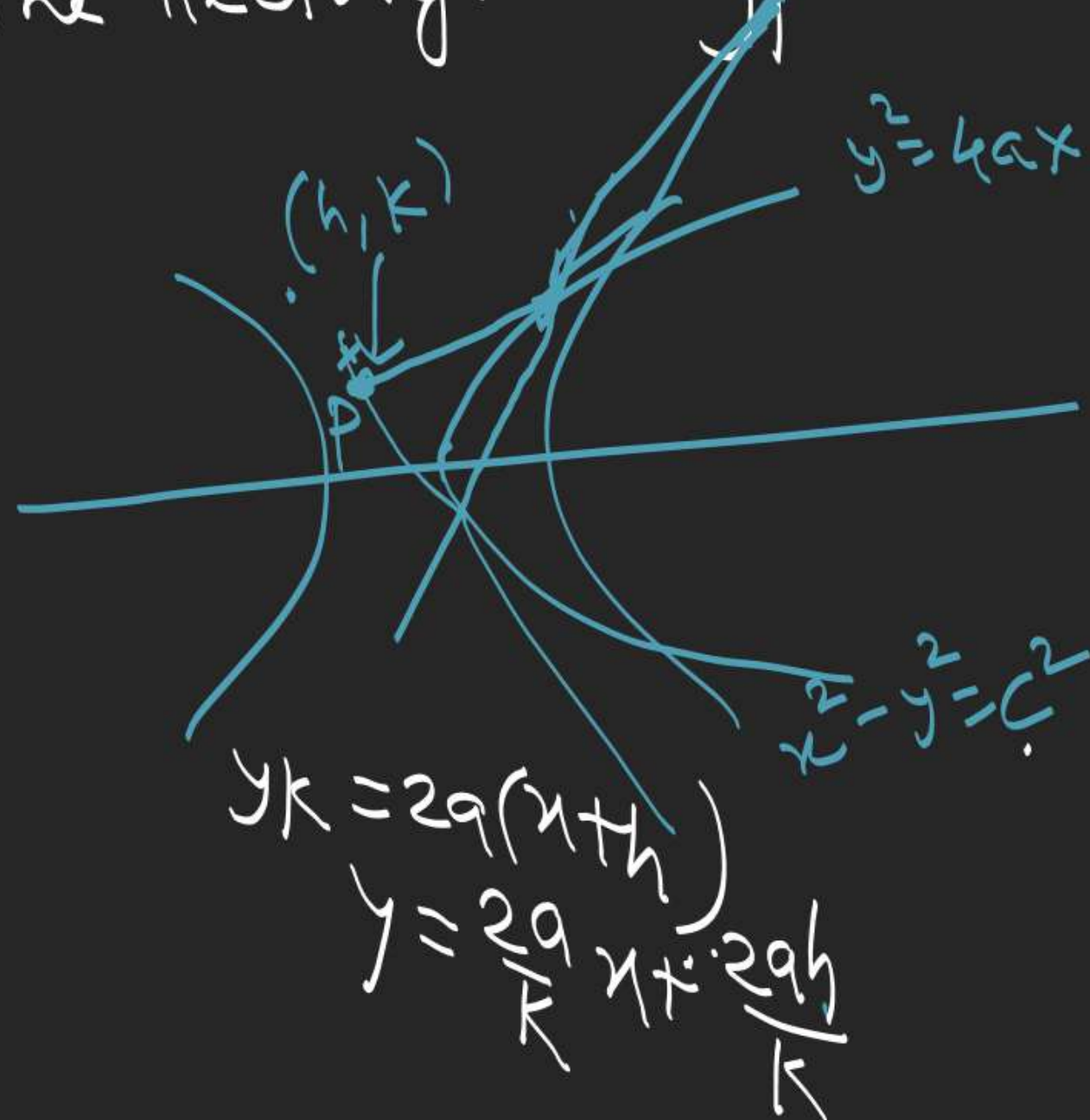
$$x \cos \theta - y \sin \theta = a$$

$$xh - yk = h^2 - k^2$$

$$\frac{\cos \theta}{h} = \frac{\sin \theta}{k} = \frac{a}{h^2 - k^2}$$

$$\frac{a^2}{(h^2 - k^2)^2} (h^2 + k^2) = 1$$

7. A point 'P' moves such that chord of contact of pair of tangents from P on parabola  $y^2 = 4ax$  touches the rectangular hyperbola  $x^2 - y^2 = c^2$ . Find the locus of 'P'.



$$\frac{4a^2 h^2}{k^2} = c^2 \left( \frac{4a^2}{k^2} - 1 \right)$$

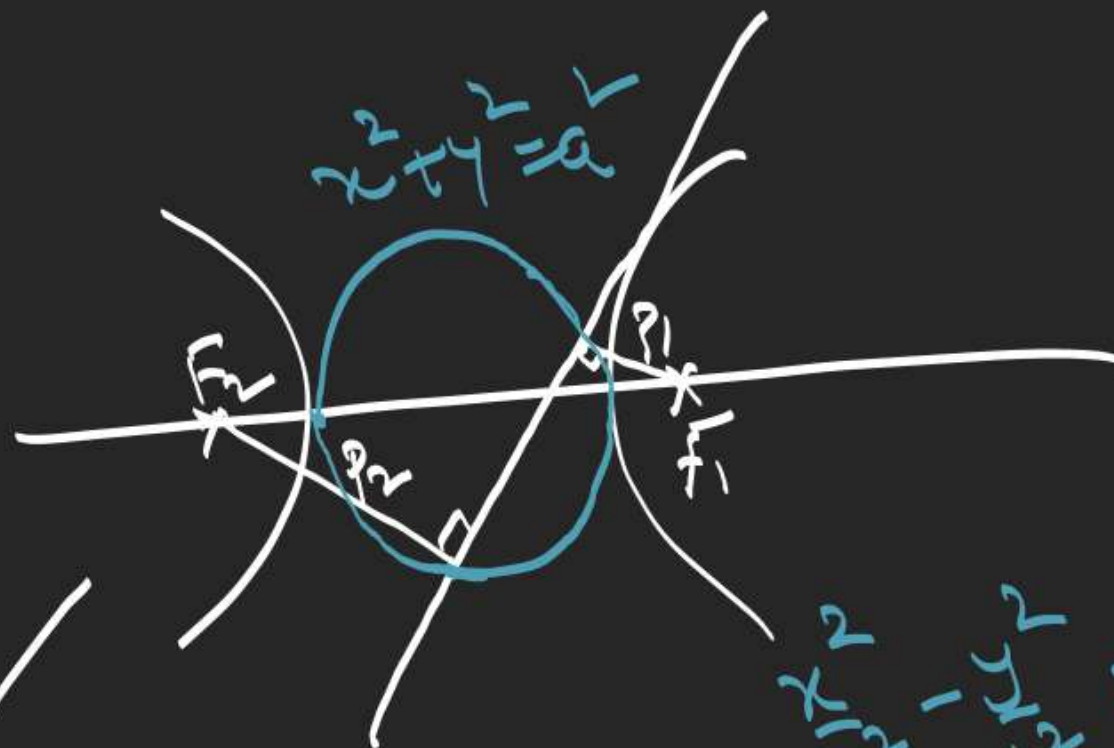
asymptote

$$xy = c^2$$

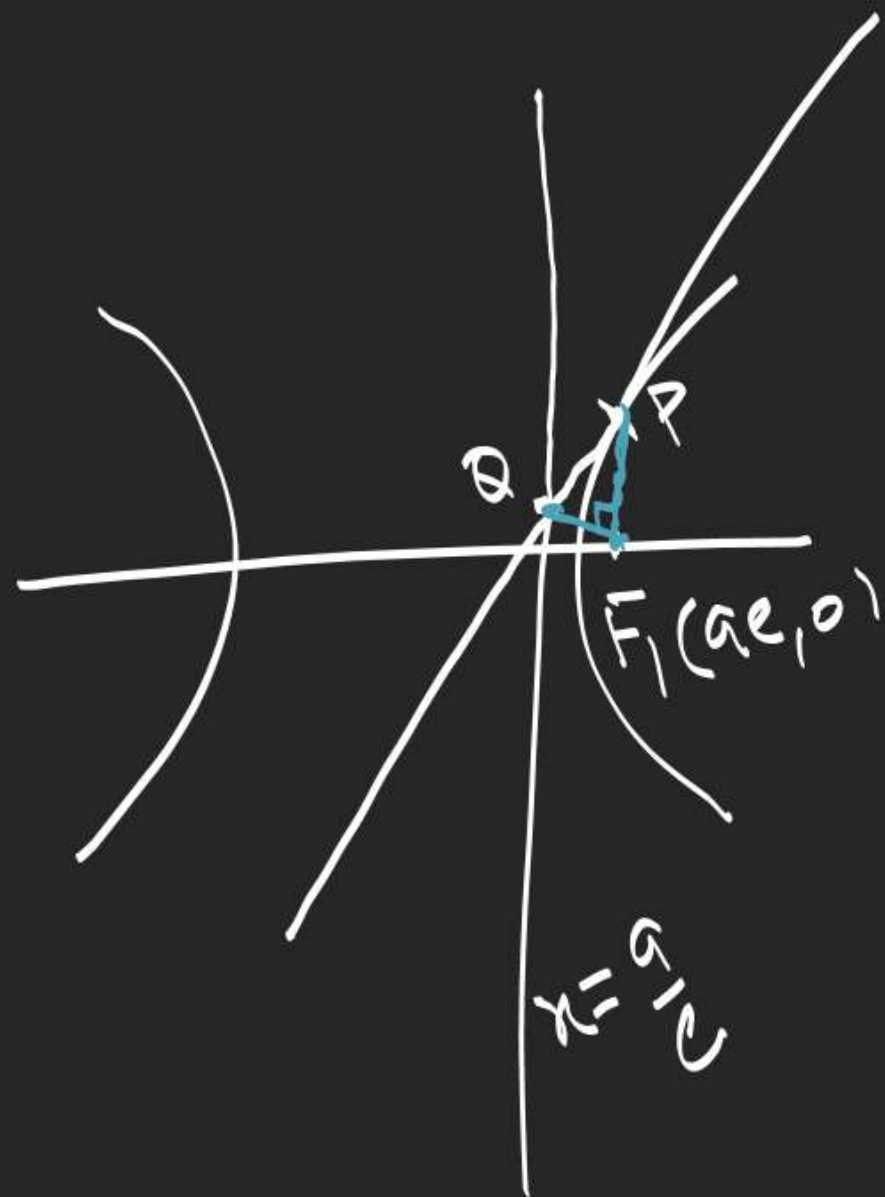
~~Ellipse~~  $\leftarrow$  Ex-III (Complete)

Note → ①

$$P_1 P_2 = (\text{semi latus rectum})^2$$



②



③ Reflection Prop



Con focal Ellipse & Hyperbola are orthogonal