DPP 05

Angular momentum & Energy conservation

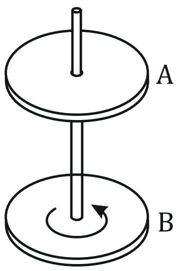
SOLUTION

1. (a) Since there are no external torques acting, we may apply the conservation of angular momentum.

$I_f\omega_f = I_i\omega_i$

$(6\text{kgm}^2)\omega_f = (4\text{kgm}^2)(3\text{rads}^{-1})$

Thus, $\omega_f = 2$ rads $^{-2}$



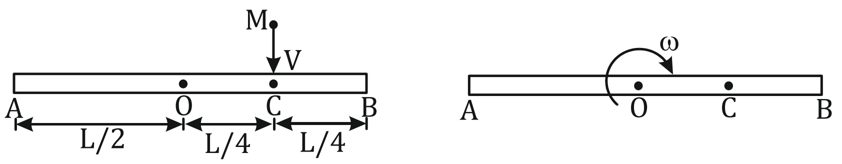
(b) The kinetic energies before and after the collision are

$K_i = \frac{1}{2}I_i\omega_i^2 = 18$ J

$K_f = \frac{1}{2}I_f\omega_f^2 = 12$ J

The change is $\Delta K = K_f - K_i = -6$ J.

2. In this problem we will denote angular momentum by its standard symbol H because L has been used for length of the rod.



Angular momentum of the system (rod + insect) about the centre of the rod O will remain conserved just before collision and after collision i.e., $H_i = H_f$.

$$\Rightarrow \ MV\frac{L}{4} = I\omega = \left[\frac{ML^2}{12} + M\left(\frac{L}{4}\right)^2\right]\omega$$

$$\Rightarrow\ MV\frac{L}{4}=\frac{7}{48}ML^2\omega$$

$$\Rightarrow\ \omega=\frac{12}{7}\frac{V}{L}\ldots(1)$$

Let at time t the insect be at a distance x from O and by then the rod has rotated through an angle θ. Then, angular momentum at that moment,

$$H=\left[\frac{ML^2}{12}+Mx^2\right]\omega$$

$$\Rightarrow\ \frac{dH}{dt}=2M\omega x\frac{dx}{dt}\qquad\{\omega=\ \text{constant}\}$$

$$\Rightarrow\ \tau=2M\omega x\frac{dx}{dt}\qquad\left\{\because\frac{dH}{dt}=\tau\right\}$$

$$\Rightarrow\ Mgx\cos\theta=2M\omega x\frac{dx}{dt}$$

$$\Rightarrow\ dx=\left(\frac{g}{2\omega}\right)\cos\theta dt\{\because\ \theta=\omega t\}$$

$$\Rightarrow\ \frac{dx}{d\theta}\frac{d\theta}{dt}=\frac{g}{2\omega}\cos\ \theta$$

$$\Rightarrow\ \omega\frac{dx}{d\theta}=\frac{g}{2\omega}\cos\ \theta$$

$$\Rightarrow\ \int_{\frac{L}{4}}^{\frac{L}{2}}dx=\frac{g}{2\omega^2}\int_0^{\frac{\pi}{2}}\cos\ \theta d\theta$$

$$\Rightarrow\ x\Big|_{\frac{L}{4}}^{\frac{L}{2}}=\frac{g}{2\omega^2}\sin\ \theta\Big|_0^{\frac{\pi}{2}}$$

$$\Rightarrow\ \omega=\sqrt{\frac{2g}{L}}$$

Substituting in equation (1), we get

$$\sqrt{\frac{2g}{L}}=\frac{12}{7}\frac{V}{L}$$

$$\Rightarrow V=\frac{7}{12}\sqrt{2gL}=\frac{7}{12}\sqrt{2\times10\times1.8}=3.5\ ms^{-1}$$

$\Rightarrow V = 3.5\ ms^{-1}$

3. By Law of Conservation of Angular Momentum about centre of disc, we have

$I_1\omega_1 = I_2\omega_2$

$\Rightarrow \omega_2 = \left(\frac{I_1}{I_2}\right)\omega_1 = \left[\frac{\frac{1}{2}ma^2}{\frac{1}{2}ma^2+2m\left(\frac{a}{2}\right)^2}\right]\omega$

$\Rightarrow \omega_2 = \frac{\omega}{2}$

4. Since, net external torque on the system is zero. Therefore, angular momentum will remain conserved.

So, $I_1\omega_1 = I_2\omega_2$

$\Rightarrow \omega_2 = \frac{I_1\omega_1}{I_2}$

where, $I_1 = I, \omega_1 = \omega_0, I_2 = I + mR^2$

$$\Rightarrow \omega_2 = \frac{I\omega_0}{I + mR^2}$$

5. No external torque is acting on the system, so angular momentum is conserved. Further there exists no non conservative forces in the system, so total energy is also conserved.

6. By Law of Conservation of Energy

Loss in Potential energy = Gain in Kinetic energy

$$mg\ell(1 - \cos\theta) = \frac{1}{2}mv^2$$

$$\Rightarrow \frac{mv^2}{\ell} = 2mg(1 - \cos\theta)$$

At point of breaking

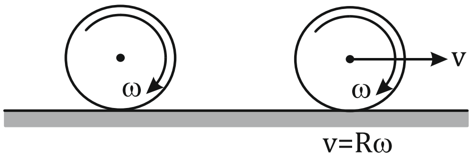
$$mg\cos\theta = \frac{mv^2}{\ell}$$

$\Rightarrow mg\cos\theta = 2mg(1 - \cos\theta)$

$$\Rightarrow \cos\,\theta = \frac{2}{3}$$

7. Applying Conservation of Angular Momentum about point of contact, we get

$$I\omega_0 = I\omega + mvR$$



$$\Rightarrow\ I\omega_0 = I\left(\frac{v}{R}\right) + mvR$$

$$\Rightarrow\ v = \frac{I\omega_0}{\frac{I}{R} + mR}$$

$$\Rightarrow\ v = \frac{\omega_0}{\frac{1}{R} + \frac{mR}{I}}$$

Now $I_{\text{solid sphere}} < I_{\text{hollow}}$

$$\Rightarrow\ v_{\text{solid}} < v_{\text{hollow}}$$

$$\Rightarrow\ v_1 < v_2$$

8. Given that bullet is getting embedded at a distance x from O.

Then from angular momentum conservation

$$Mvx = \left(\frac{ML^2}{3} + Mx^2\right)\omega$$

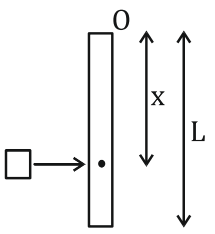
$$\text{or } \omega = \frac{vx}{\left(\frac{L^2}{3} + x^2\right)} = \frac{3vx}{L^2 + 3x^2}$$

For maximum angular velocity, $\frac{d\omega}{dx} = 0$

$$\frac{d\omega}{dx} = \frac{3v(L^2 + 3x^2) - 3vx(6x)}{(L^2 + 3x^2)^2} = 0$$

or $L^2 + 3x^2 - 6x^2 = 0$ or $3x^2 = L^2$

$$\Rightarrow x = x_M = \frac{L}{\sqrt{3}}$$



$$\therefore \omega_{max} = \omega_M = \frac{3vx_M}{L^2+3x_M^2}$$

9. From the law of conservation of mechanical energy,

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

As there is no slipping, so $v = \omega r$

$$mgh = \frac{1}{2}m(\omega r)^2 + \frac{1}{2}\frac{mr^2}{2}\omega^2 \qquad \left[\because \text{ For disk, } I = \frac{1}{2}mr^2\right]$$

$$mgh = \frac{3}{4}m\omega^2r^2 \Rightarrow \omega = \frac{1}{r}\sqrt{\frac{4}{3}gh}$$

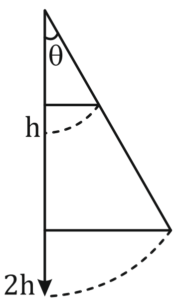
10. Using energy conservation principle,

loss in potential energy = gain in kinetic energy

$$mgl\sin\alpha = \frac{1}{2}\frac{ml^2}{3}\omega^2 \Rightarrow 6gl\sin\alpha = v^2$$

$$\Rightarrow v = \sqrt{6gl\sin\alpha} \text{ or } v \propto \sqrt{\sin\alpha}$$

11. The uniform rod of length l and mass m is swinging about an axis passing through the end. When the centre of mass is raised through h, the increase in potential energy is mgh. This is equal to the kinetic energy



$= \frac{1}{2} I\omega^2.$

$\Rightarrow mgh = \frac{1}{2}\left(m\frac{l^2}{3}\right) \cdot \omega^2 \Rightarrow h = \frac{l^2 \cdot \omega^2}{6g}$