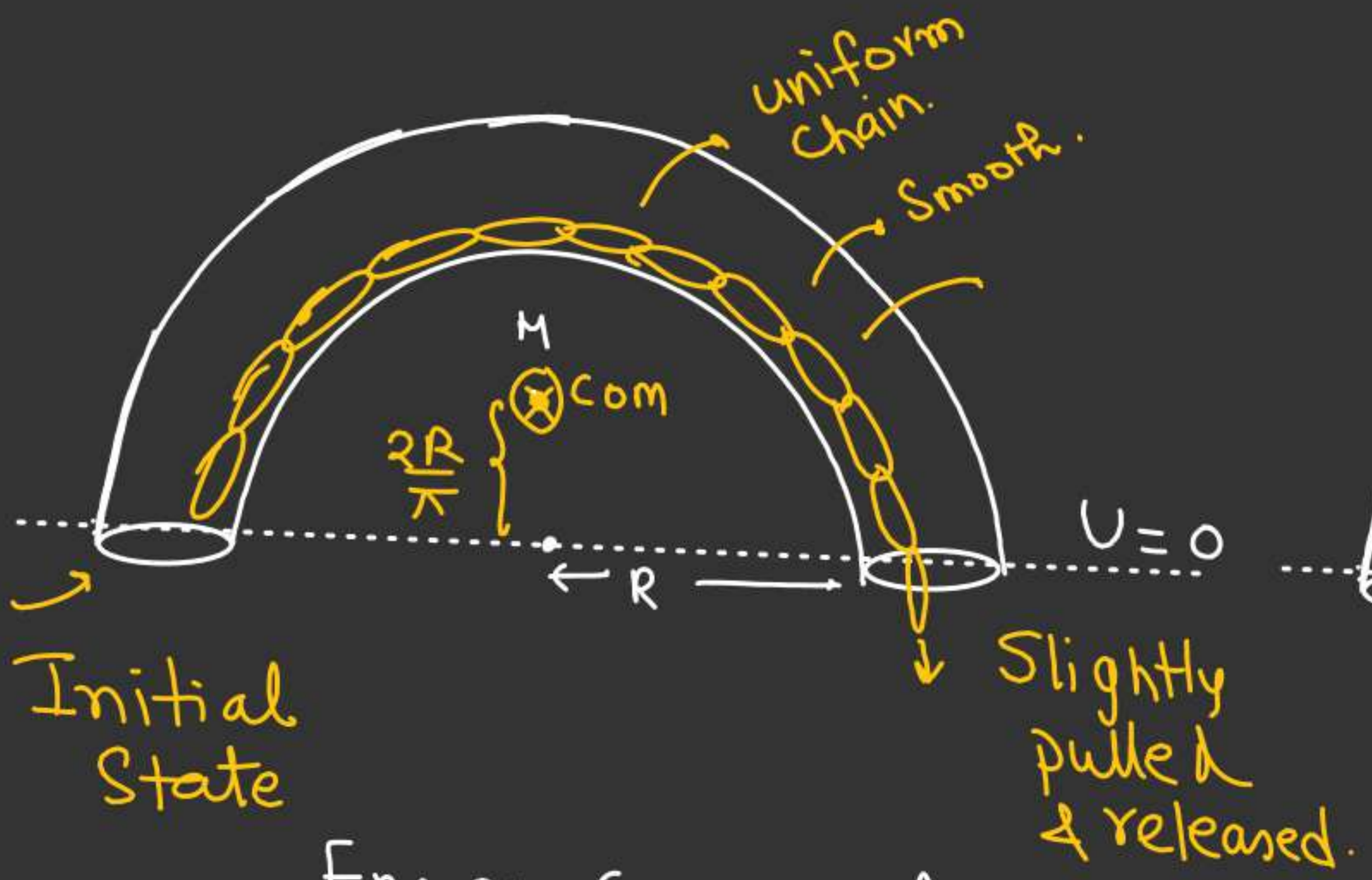
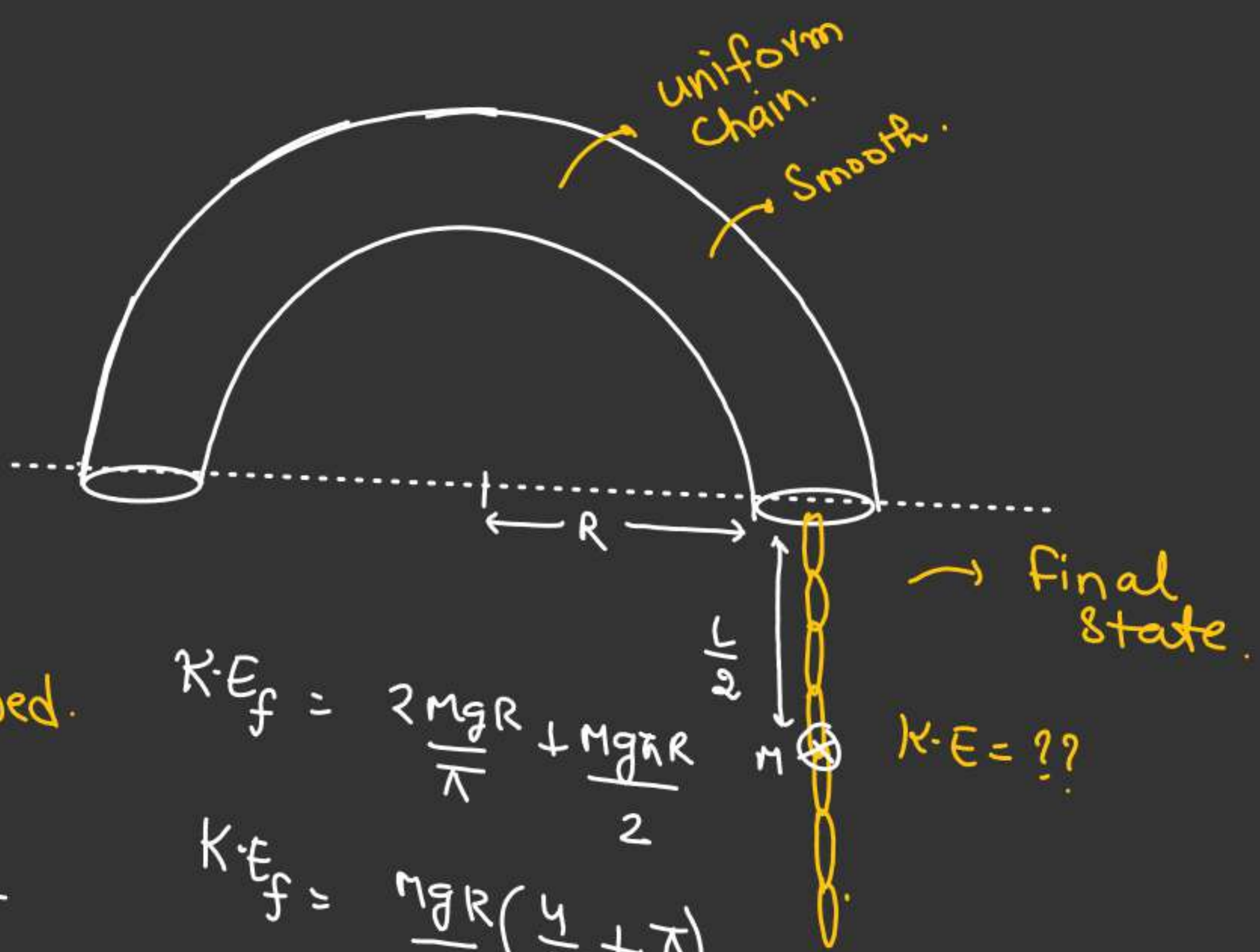
Nishant Jindal

COM

$L = \pi R$ ✓ $M = \lambda L$.





Energy Conservation.

$$U_i + K.E_i = U_f + K.E_f$$

$$Mg\left(\frac{2R}{\pi}\right) + 0 = -Mg\left(\frac{\pi R}{2}\right) + K.E_f$$

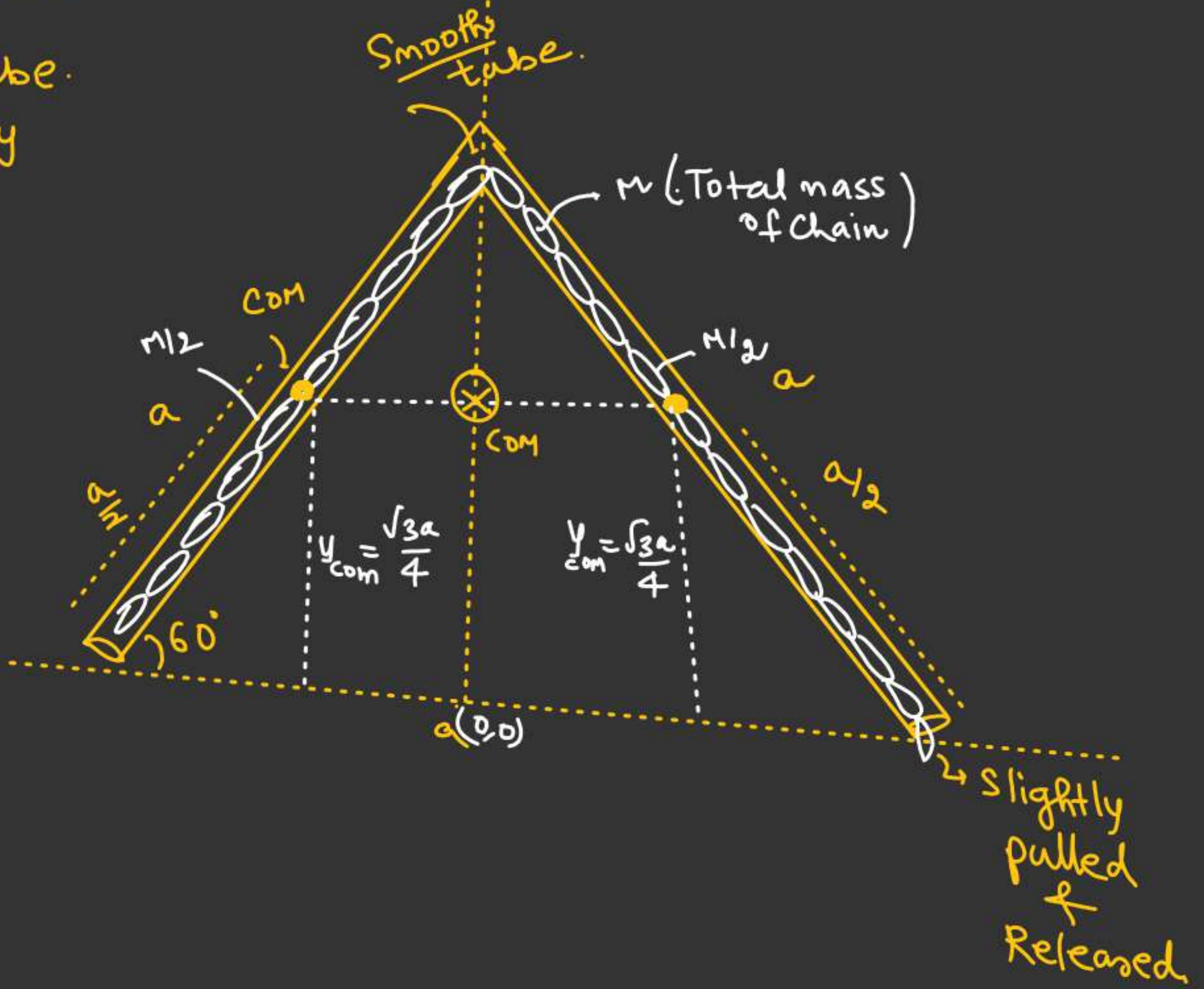
$$K.E_f = \frac{2MgR}{\pi} + \frac{Mg\pi R}{2}$$

$$K.E_f = \frac{MgR}{2}\left(\frac{4}{\pi} + \pi\right)$$

Nishant Jindal

COM

Find K.E of chain when
chain just about to leave the tube.
λ = linear mass density
Chain is uniform.



$$(y_{com})_{system} = \frac{(M/2)\cdot y_{com} + (M/2)\, y_{com}}{M}$$

$$= y_{com} = \left(\frac{\sqrt{3}a}{4}\right)$$

$$M = (\lambda 2a)$$

-y
pulled

Nishant Jindal

COM

Energy Conservation.

$$U_i + K.E_i = U_f + K.E_f$$

$\Downarrow$

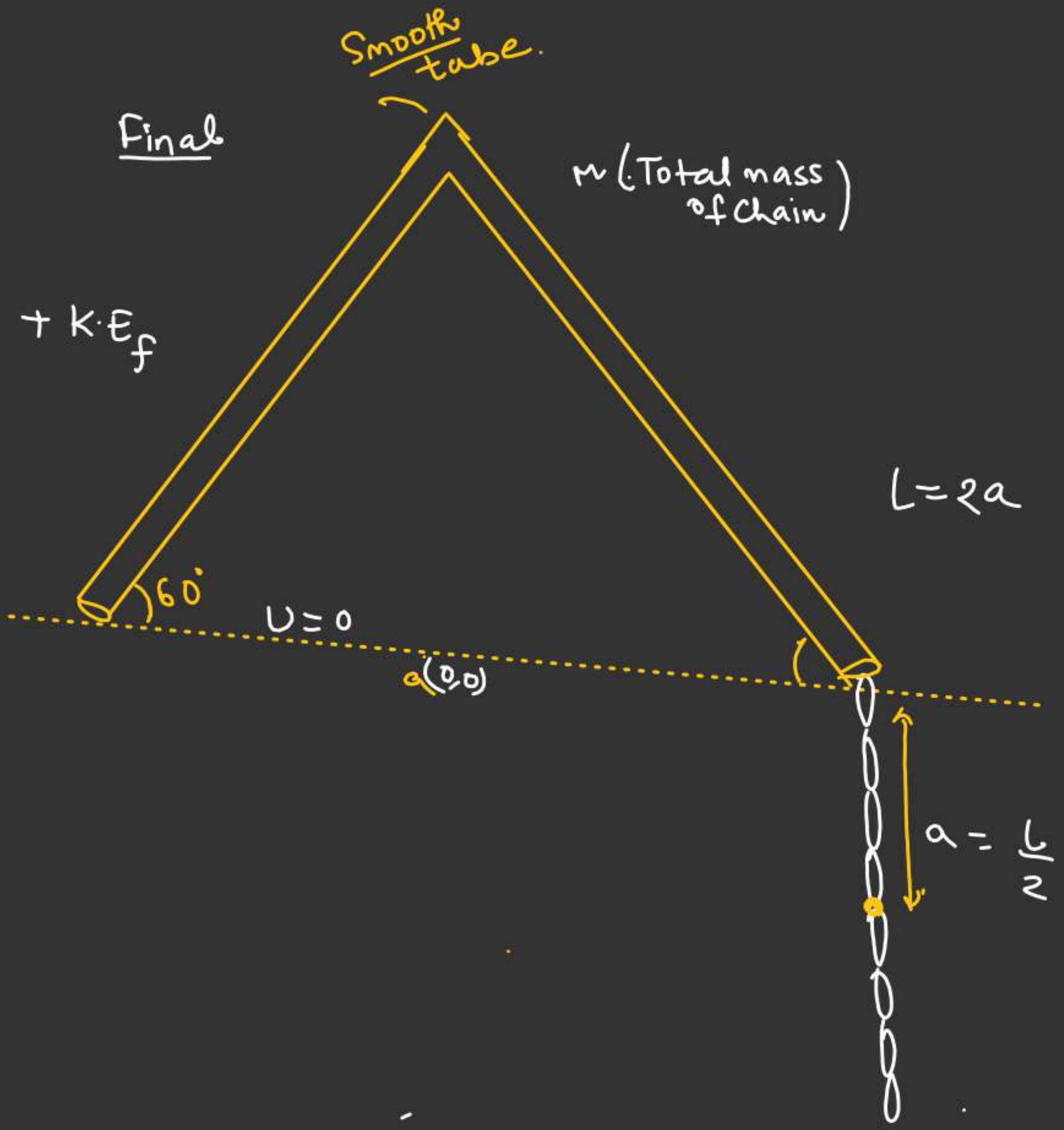
$$(\lambda 2a)g\left(\frac{\sqrt{3}a}{4}\right) + 0 = -(\lambda 2a)ga + K.E_f$$

$$\frac{\sqrt{3}\lambda a^2 g}{2} + 2\lambda a^2 g = K.E_f$$

$$\frac{4\lambda a^2 g + \sqrt{3}\lambda a^2 g}{2} = K.E_f$$

$$K.E_f = \lambda a^2 g\left(\frac{\sqrt{3}}{2} + 2\right) \checkmark$$

Final



-y
pulled

Nishant Jindal

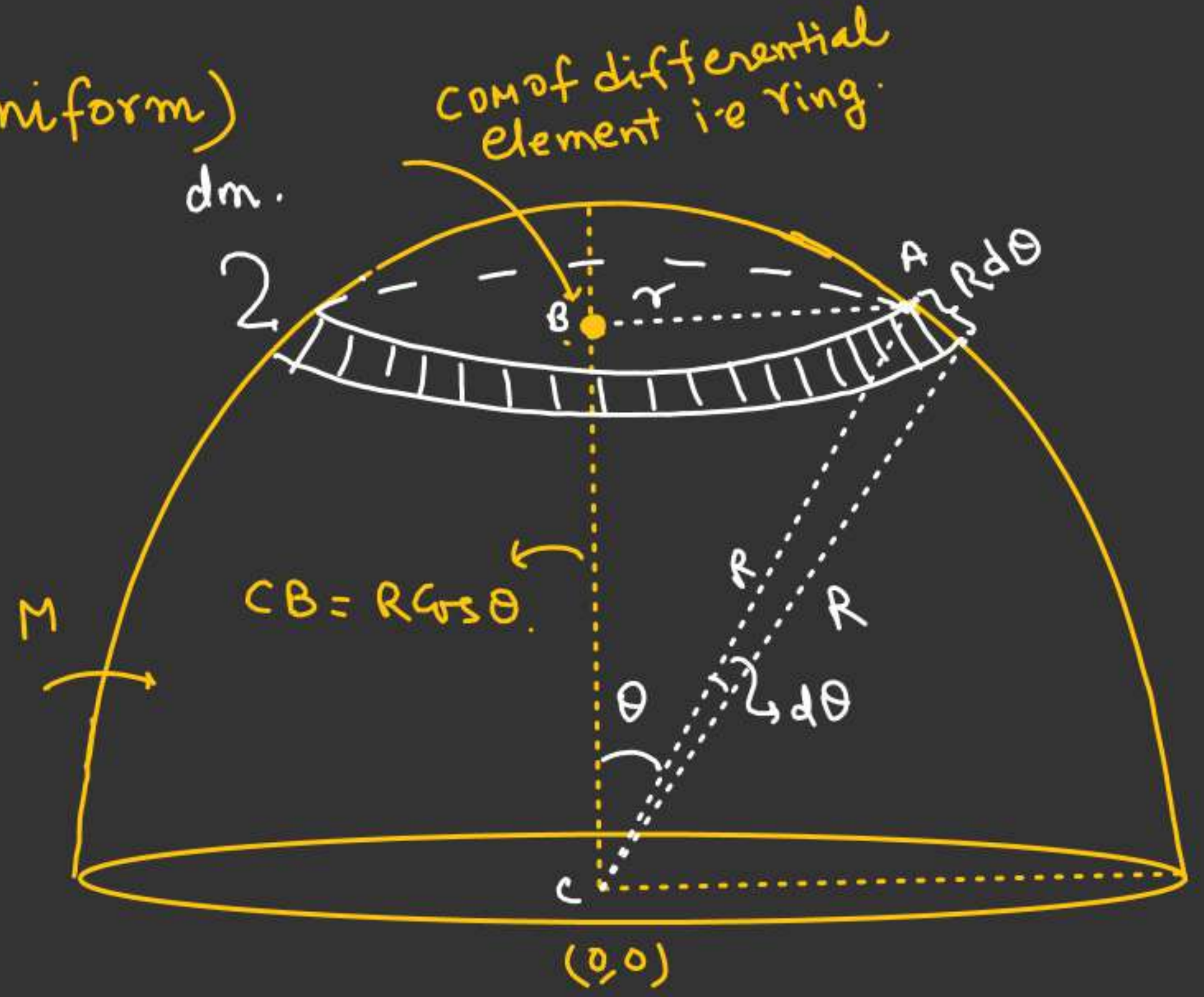
COM

COM of hollow hemisphere (Uniform)

In ΔCAB. $\quad \sin\theta = \dfrac{r}{R}$

dm = Mass of differential element ie ring $\qquad r = \underline{R\sin\theta}$.

$$dM = \left(\frac{M}{A}\cdot dA\right)$$



A = Area of hemisphere = $2\pi R^2$

dA = (length of differential element) × thickness

$$dA = (2\pi r)R\,d\theta \quad (r = R\sin\theta)$$

$$= \left(2\pi R^2 \sin\theta \cdot d\theta\right)$$

$$dM = \frac{M}{2\pi R^2} \times 2\pi R^2 \cdot \sin\theta \cdot d\theta$$

$$dM = \left(M \sin\theta \cdot d\theta\right)$$

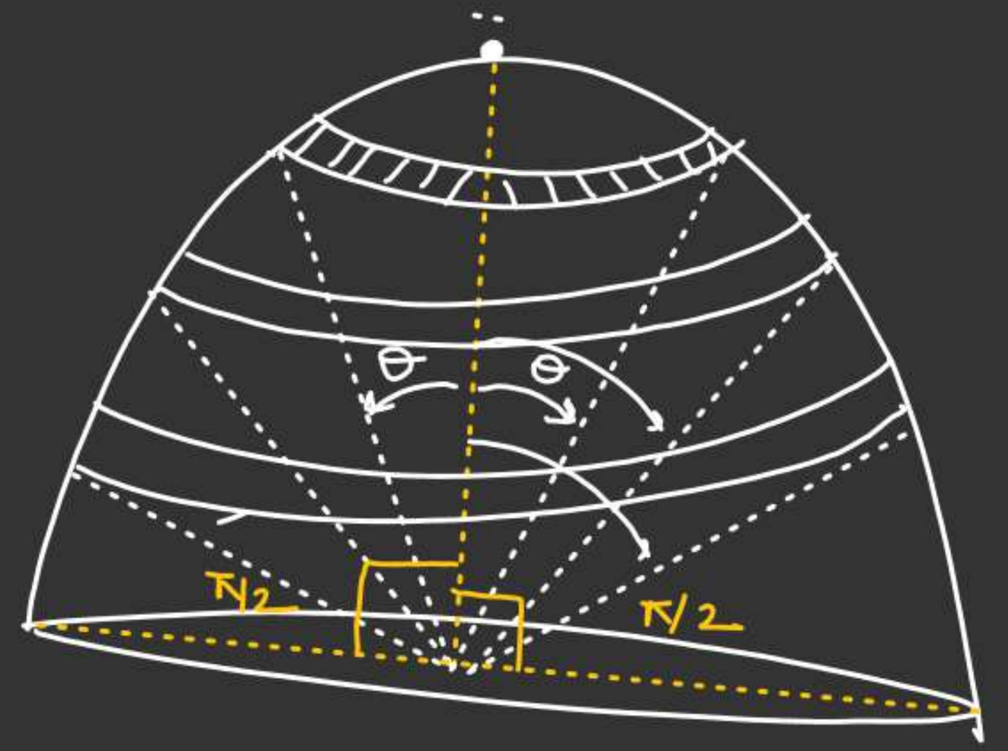
Nishant Jindal

COM

$$y_{com} = \frac{\int dm \cdot y}{\int dM} = \frac{\int_0^{\pi/2} \overbrace{(M\sin\theta \cdot d\theta)}^{dm} \cdot \overbrace{(R\cos\theta)}^{y}}{M}$$

$\int dM \to M$ (uniform)

$$\int \sin\alpha\theta \cdot d\theta = \left[\frac{-\cos\alpha\theta}{\alpha}\right]$$



$$y_{com} = \frac{\cancel{M}R}{\cancel{M}} \int_0^{\pi/2} \sin\theta \cdot \cos\theta \cdot d\theta$$

$$y_{com} = \frac{R}{2} \int_0^{\pi/2} (2\sin\theta \cdot \cos\theta) \cdot d\theta$$

$$y_{com} = \frac{R}{2} \int_0^{\pi/2} \sin 2\theta \cdot d\theta$$

$$y_{com} = \frac{R}{2} \left[\frac{-\cos 2\theta}{2}\right]_0^{\pi/2}$$

$$y_{com} = \frac{R}{4}\left[-\cos 2\left(\frac{\pi}{2}\right) - \left[-\cos 2(0)\right]\right]$$

$$\boxed{y_{com} = \frac{R}{2}}$$

Nishant Jindal

COM

COM of a Solid hemisphere (Uniform)

$\rho = \frac{M}{V}$

$\rho = \frac{M}{\frac{2}{3}\pi R^3} = \left(\frac{3M}{2\pi R^3}\right)$

Let, a differential element in the form of hollow sphere of radius r and mass dm.

dr = thickness of hollow hemisphere.

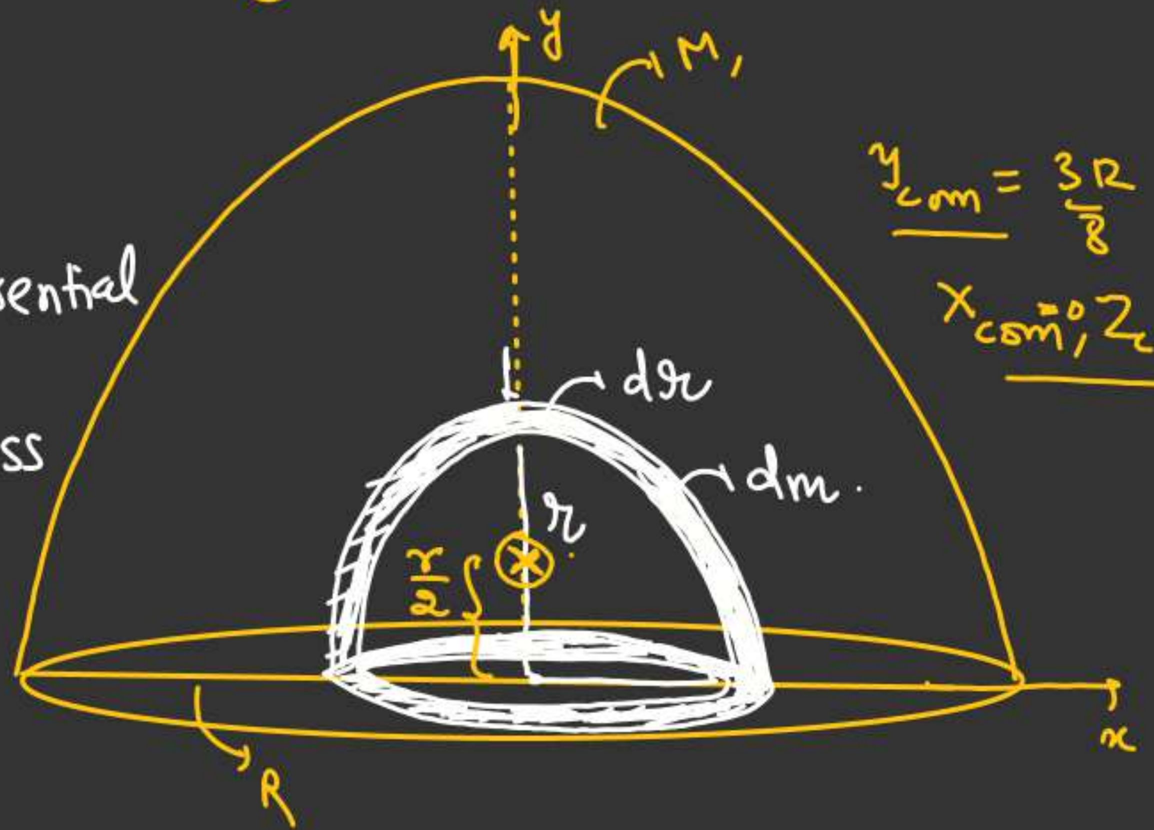
$dm = \rho dV$

dV = (Area of differential element) × thickness

$dV = (2\pi r^2) \cdot dr$

$dm = \frac{3M}{\cancel{2}\cancel{\pi}R^3} \times \cancel{2}\cancel{\pi}r^2 dr$

$dm = \left(\frac{3M}{R^3}\right) r^2 dr$



$y_{com} = \frac{3R}{8}$

$x_{com} = 0;\ z_{com} = 0$

$$y_{com} = \frac{\int_0^R dm \cdot \left(\frac{r}{2}\right)}{M} = \frac{\frac{3\cancel{M}}{2R^3}}{\cancel{M}} \int_0^R r^3 \cdot dr = \frac{3}{2\cancel{R^3}} \times \frac{\cancel{R^4}}{4} = \left(\frac{3R}{8}\right)$$

$\frac{r}{2}$ → COM of differential element ie hollow hemisphere.

$$\boxed{y_{com} = \frac{3R}{8}}$$

Nishant Jindal

COM

H.W

Find COM of a Solid non-uniform hemisphere whose ρ is

a) $\rho = \rho_0 r^2$ (ρ_0 is a constant)

(r = radial distance)

b) $\rho = r_0 + ar$ (a & r_0 is a constant)

Nishant Jindal

COM

COM of a hollow cone

Note :- [For hollow cone take element along slant height of the cone]

Uniform

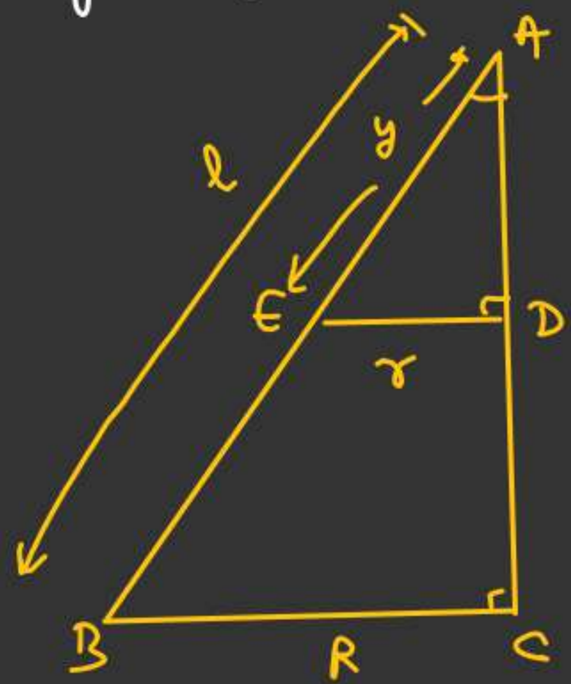
σ = constant

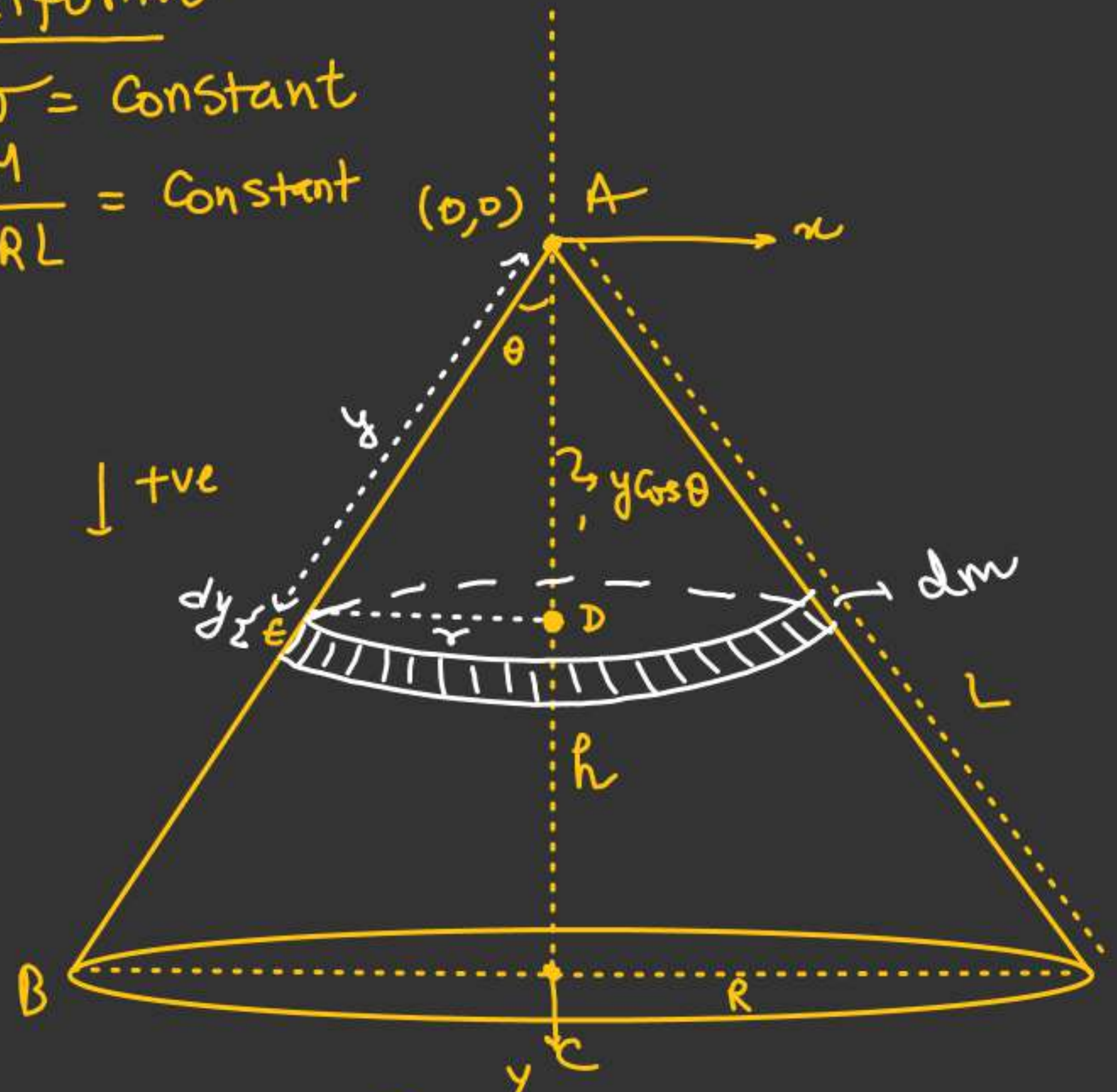
$\frac{M}{\pi R L}$ = constant

For $\triangle AED$ & $\triangle ABC$

$$\frac{r}{R} = \frac{y}{l}$$

$$r = \left(\frac{y}{l} \times R\right)$$





$$y_{com} = \frac{\int_0^l dm\cdot[y\cos\theta]}{M}$$

$$y_{com} = \frac{\int_0^l \left(\frac{2M}{l^2}\, y\, dy\right) \times y\cos\theta}{M}$$

$$= \frac{2\cos\theta}{l^2}\int_0^l y^2 dy = \frac{2\cos\theta}{l^2} \times \frac{l^3}{3}$$

$$y_{com} = \left(\frac{2l}{3}\cos\theta\right) \longrightarrow y_{com} = \frac{2h}{3}$$

$$dm = \frac{M}{\pi R l} \times dA$$

$$= \frac{M}{\pi R l} \times (2\pi r)dy$$

$$= \frac{2M}{Rl} \times \left(\frac{y}{l} \times R\right)\cdot dy = \left(\frac{2M}{l^2}\, y\, dy\right)$$

D → com of ring

Co-ordinates = $(0, y\cos\theta)$

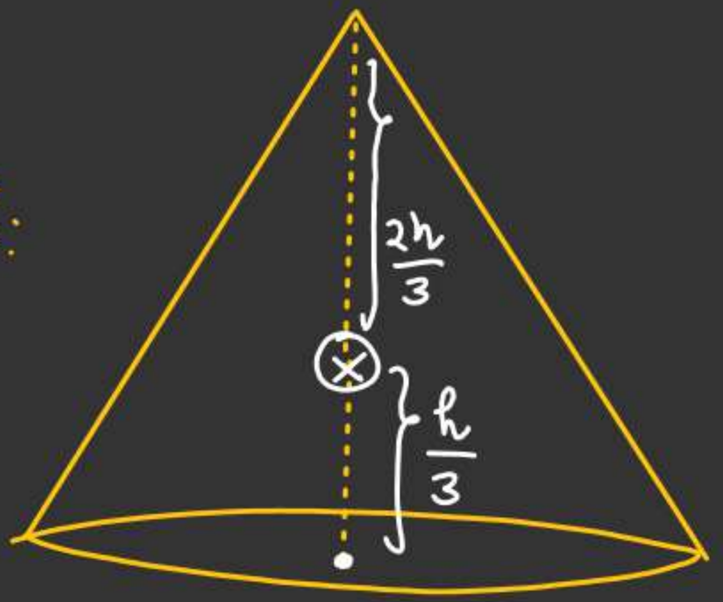
$h = l\cos\theta$

$$y_{com} = \frac{2h}{3}$$

Nishant Jindal

$y_{com} = \frac{2h}{3}$ from Apex

$$\boxed{y_{com} = \frac{h}{3} \text{ from the Base}}$$



Nishant Jindal

COM

H.W.

COM of a Solid Cone

$\left(\frac{h}{4}\right)$ ✓

Isosceles △ Lamina

Find COM = ??

$\left(0, \frac{a}{3}\right)$ Ans