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Special Case

MAGNETIC FIELD

Motion of charge particle in a magnetic field

. $\vec{E} \perp \vec{B}$, A Charge particle is released.

$\vec{E} = E\hat{j}$

$\vec{B} = B\hat{k}$

Sol^n let at any time t, velocity of charge particle be v

$\vec{v} = v_x\hat{i} + v_y\hat{j}$

S.H.M

$a = -\omega^2 x$

$\frac{d^2x}{dt^2} = -\omega^2 x$

$$\boxed{\frac{d^2x}{dt^2} + \omega^2 x = 0}$$

$\Downarrow$

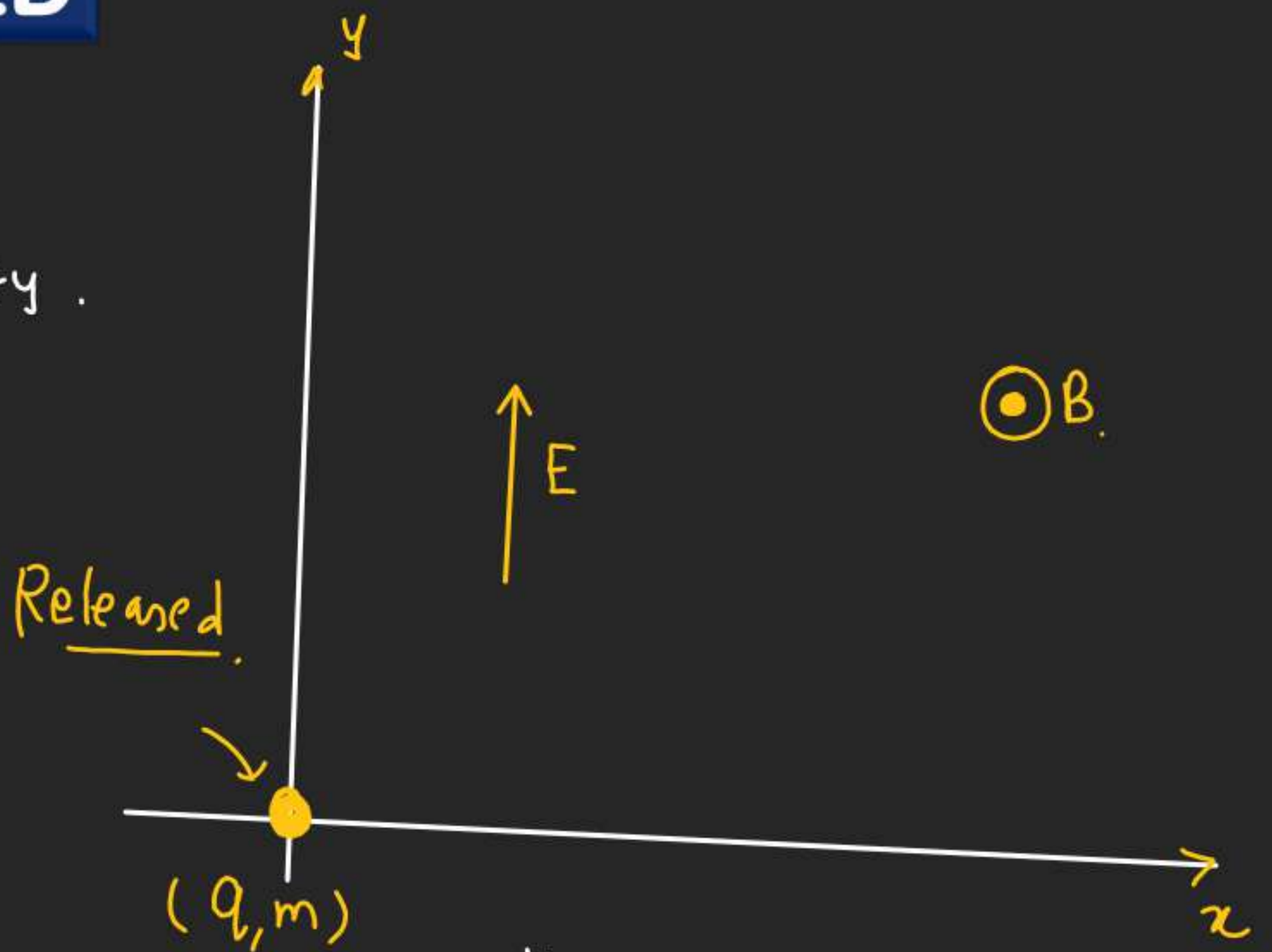
$x = A\sin(\omega t + \phi)$

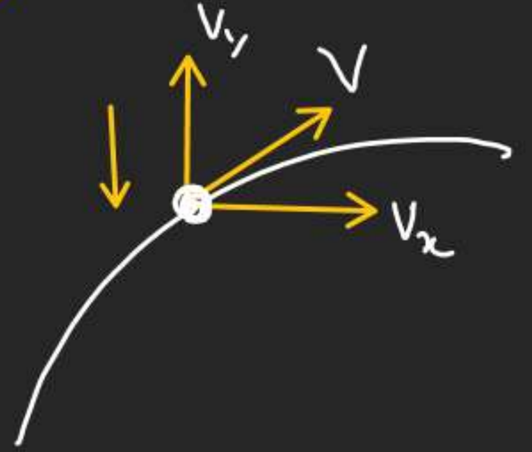
$\vec{F} = q\vec{E} + q(\vec{v} \times \vec{B})$

$\vec{F} = (qE)\hat{j} + [q(v_x\hat{i} + v_y\hat{j}) \times B(\hat{k})]$

$\vec{F} = qE\hat{j} + [qv_xB(-\hat{j}) + qBv_y\hat{i}]$

$\vec{F} = [qE - qBv_x]\hat{j} + (qBv_y)\hat{i}$





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$X = f(t),\ Y \rightarrow f(t)$

$$\vec{a} = \frac{\vec{F}}{m} = \frac{(qE - qBV_x)}{m}\hat{j} + \left(\frac{qBV_y}{m}\right)\hat{i}$$

$$a_x = \left(\frac{qB}{m}\right)V_y$$

$$\boxed{\frac{dV_x}{dt} = \left(\frac{qB}{m}\right)V_y} \quad \text{--- (1)}$$

Differentiating both Side w.r.t time of eqn (1)

$$\frac{d^2V_x}{dt^2} = \frac{qB}{m}\left(\frac{dV_y}{dt}\right)$$

$$\frac{d^2V_x}{dt^2} = \frac{qB}{m}\left[\frac{q}{m}(E - BV_x)\right]$$

$$a_y = \frac{q}{m}(E - BV_x)$$

$$\Downarrow$$

$$\boxed{\frac{dV_y}{dt} = \frac{q}{m}(E - BV_x)} \quad \text{--- (2)}$$

$\Downarrow$ Differentiating both Side w.r.t time of Eqn (2)

$$\frac{d^2V_y}{dt^2} = -\frac{qB}{m}\left(\frac{dV_x}{dt}\right)$$

From (1) $\frac{dV_x}{dt} = \frac{qB}{m}V_y$

$$\boxed{\frac{d^2V_y}{dt^2} = -\left(\frac{qB}{m}\right)^2 V_y}$$

$$\frac{d^2x}{dt^2} = -\omega^2 x$$

$$\Downarrow$$

$$V_y = V_0 \sin[\omega t + \phi] \qquad \left[\omega = \frac{qB}{m}\right]$$

At $t = 0$, $V_y = 0$

$0 = V_0 \sin\phi \Rightarrow \phi = 0$

$$\boxed{V_y = V_0 \sin\omega t} \rightarrow (3)$$

$$\frac{d^2V_x}{dt^2} = \frac{q^2B}{m^2}(E - BV_x)$$

$$\frac{d^2V_x}{dt^2} = -\frac{q^2B^2}{m^2}V_x + \frac{q^2BE}{m^2}$$

Constant

Continue in next lecture

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From ③ $v_y = v_0 \sin\omega t$.

$$\frac{dv_y}{dt} = v_0\omega \cos\omega t$$

$$a_y = v_0\omega \cos\omega t.$$

At $t=0$

$$a_y = v_0\omega.$$

Also, From-②

$$\frac{dv_y}{dt} = \frac{q}{m}(E - v_x B).$$

At $t=0$, $v_x = 0$. so,

$$a_y = \left(\frac{qE}{m}\right)$$

$$\Downarrow$$

$$v_0\omega = \frac{qE}{m}$$

$$v_0 = \frac{qE}{m\omega} = \frac{qE}{\cancel{m} \times \frac{\cancel{q}B}{\cancel{m}}}$$

$$\boxed{v_0 = \frac{E}{B}}$$ ✓.

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$$\left(V_y = \frac{E}{B}\sin\omega t\right)$$

$$\frac{dy}{dt} = \frac{E}{B}\sin\omega t$$

$$\int_0^y dy = \frac{E}{B}\int_0^t \sin\omega t \cdot dt$$

$$y = \frac{E}{B}\left[\frac{-\cos\omega t}{\omega}\right]_0^t$$

$$\boxed{y = \frac{E}{B\omega}\left[1-\cos\omega t\right]}$$ ✓

From Eq^n ①

$$\frac{dV_x}{dt} = \frac{qB}{m}V_y$$

$$\frac{dV_x}{dt} = \frac{qB}{m}\left(\frac{E}{B}\sin\omega t\right)$$

$$\frac{dV_x}{dt} = \frac{qE}{m}\left(\sin\omega t\right)$$

$$\int_0^{V_x} dV_x = \frac{qE}{m}\int_0^t \sin\omega t \cdot dt$$

$$V_x = \frac{qE}{m}\left[\frac{-\cos\omega t}{\omega}\right]_0^t$$

$$V_x = \frac{qE}{m\omega}\left[1-\cos\omega t\right] \Rightarrow V_x = \frac{qE}{m\times\frac{qB}{m}} = \frac{E}{B}\left(1-\cos\omega t\right)$$

$$V_x = \frac{E}{B}\left(1-\cos\omega t\right)$$

$$\frac{dx}{dt} = \frac{E}{B}\left(1-\cos\omega t\right)$$

$$\int_0^x dx = \frac{E}{B}\int_0^t (1-\cos\omega t)\cdot dt$$

$$x = \frac{E}{B}\left[\int_0^t dt - \int_0^t \cos\omega t \cdot dt\right]$$

$$x = \frac{E}{B}\left[t - \frac{[\sin\omega t]_0^t}{\omega}\right]$$

$$\boxed{x = \frac{E}{B\omega}\left[\omega t - \sin\omega t\right]}$$ ✓

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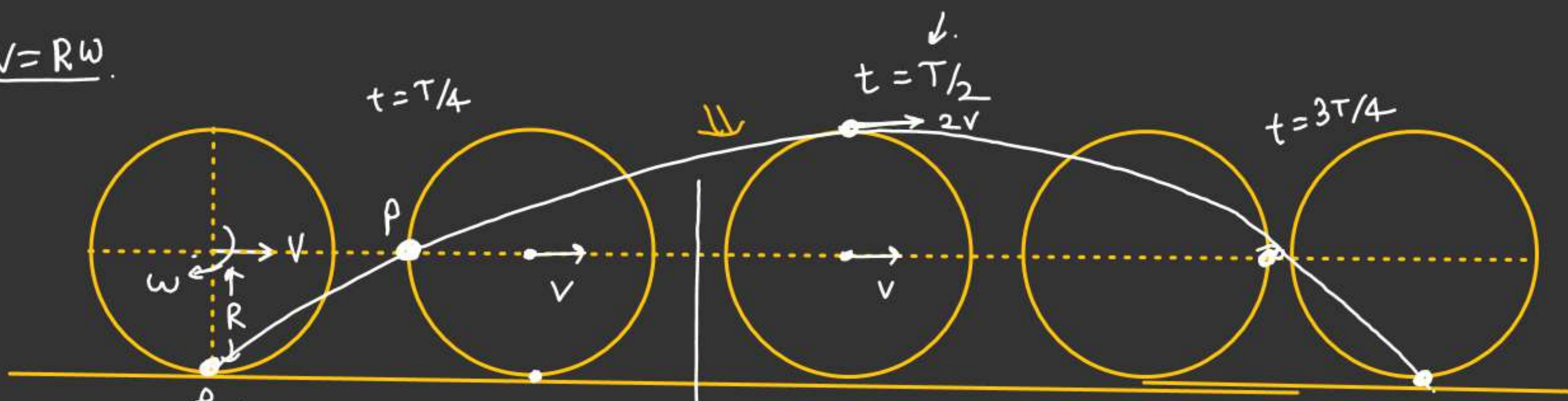
Remember

$y_{max} = ??$

$$\boxed{y_{max} = \frac{E}{B\omega}}$$

$V = R\omega$



$x = 8R$

t = T

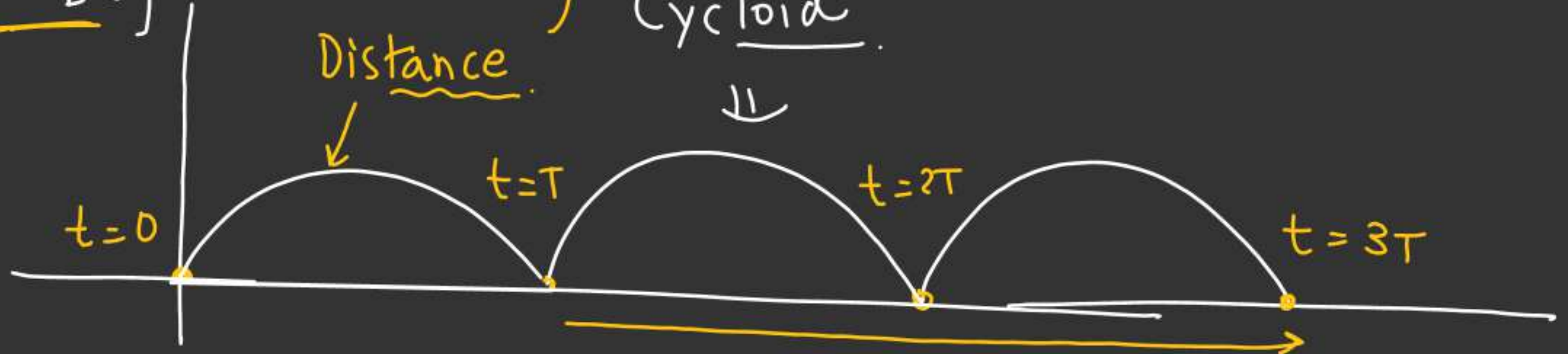
Cycloid

When $\cos\omega t = 0$

$\omega t = \frac{\pi}{2}$

$t = \left(\frac{\pi}{2\omega}\right)$

$\left[R = y_{max} = \frac{E}{B\omega}\right]$



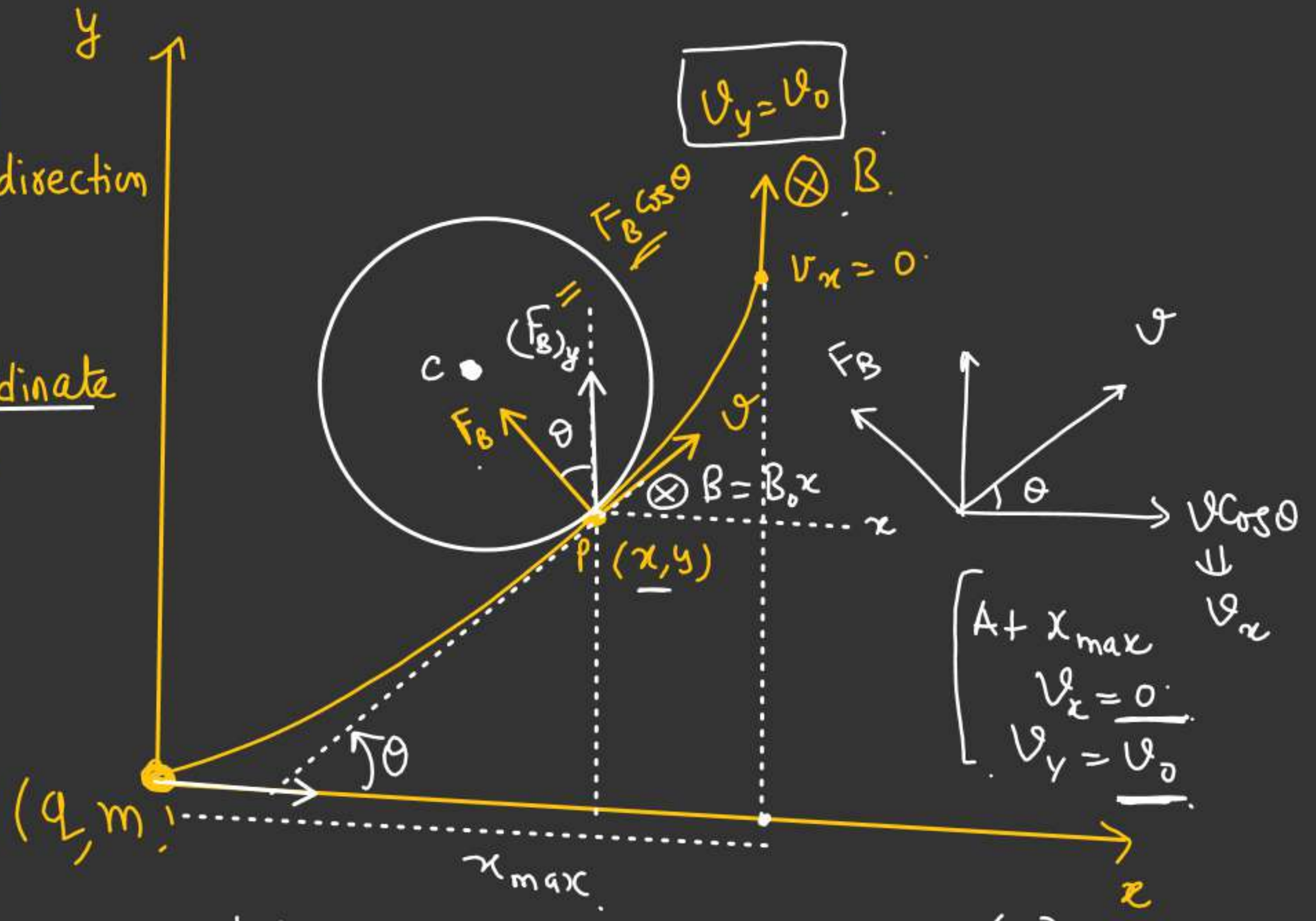
Charge particle distance in 1 time-period $= 8\frac{E}{B\omega} = \frac{8E}{B\omega} = \frac{8E}{B\left(\frac{qB}{m}\right)} = \frac{8mE}{qB^2}$

Non-Uniform Magnetic field

$\vec{v} = v_0 \hat{i}$

$\vec{B} = (-B_0 x)\hat{k}$

\# A Charge is projected with velocity v_0 in x-direction in a non-uniform magnetic field.
Find maximum x-co-ordinate of the Charge particle.



At x_{max}
$v_x = 0$
$v_y = v_0$

$$F_{B_y} = F_B \cos\theta = q v B_x \cos\theta$$

$$F_{B_y} = +q v B_0 x \cos\theta$$

$$a_y = +\frac{q v B_0 \cos\theta}{m} x$$

$$\frac{dv_y}{dt} = +\frac{q B_0}{m}(v\cos\theta) x$$

$$\frac{dv_y}{dx} \times \left(\frac{dx}{dt}\right) = +\frac{q B_0}{m} x (v\cos\theta)$$

$$\left(\frac{dv_y}{dx}\right)\cancel{v_x} = +\frac{q B_0 x}{m}(\cancel{v_x})$$

$$\left[\frac{dy}{dx} = \tan\theta\right]$$

$$\int_0^{v_0} dv_y = \frac{q B_0}{m}\int_0^{x_{max}} x\, dx$$

$$v_0 = \frac{q B_0}{m}\left(\frac{x_{max}^2}{2}\right)$$

$$x_{max} = \sqrt{\frac{2 m v_0}{q B_0}} \quad \text{Ans}$$

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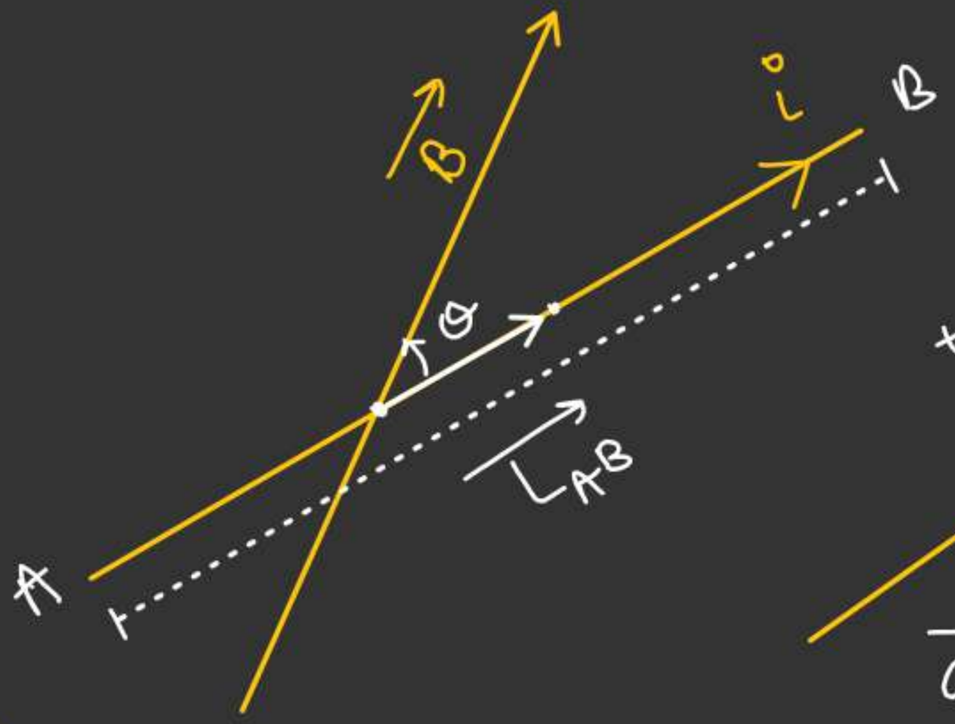
A1:

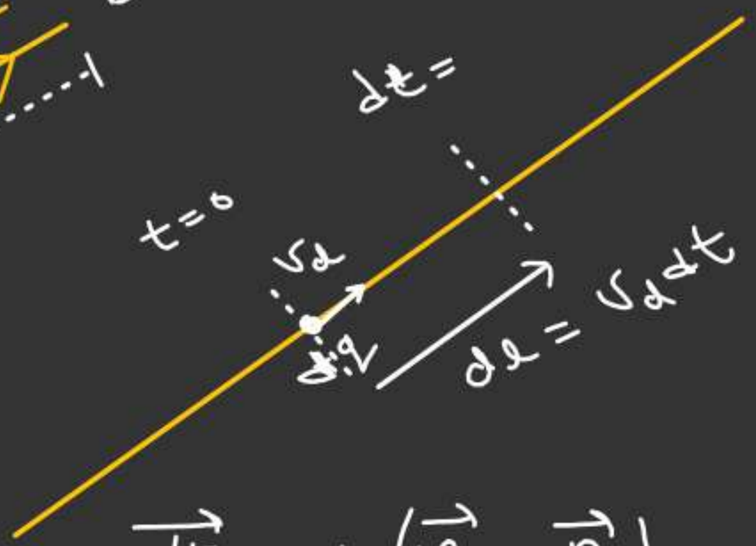
Force acting on a Current Carrying wire placed in a magnetic field. (External magnetic field)

$$\boxed{\vec{F} = i\left[\vec{L} \times \vec{B}\right]}$$

$$\boxed{|\vec{F}| = iLB\sin\theta}$$

$\vec{L}$ = length vector always taken along the direction of current flow.





$$\vec{dF_B} = dq(\vec{v_d} \times \vec{B})$$

$$\vec{dF_B} = dq\,\frac{\vec{dl}}{dt} \times \vec{B}$$

$$\Downarrow$$

$$\boxed{\vec{dF_B} = i\,\vec{dl} \times \vec{B}}$$

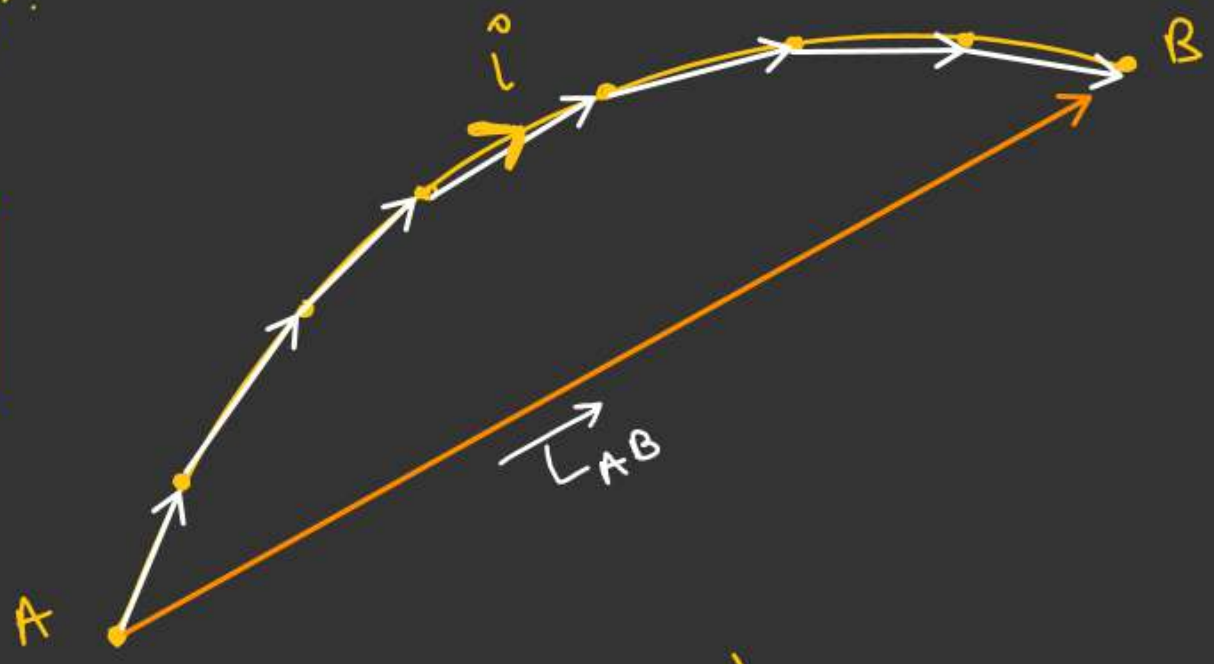
$$\vec{dF_B} = i\,(\vec{dl} \times \vec{B})$$

$$\vec{F_B} = i\left[\int \vec{dl}\right] \times \vec{B}$$

$$\Downarrow$$

$$\boxed{\vec{F_B} = i\left[\vec{l_{AB}} \times \vec{B}\right]}$$

$\vec{l_{AB}}$ = (Effective length vector joining initial point to final point.)



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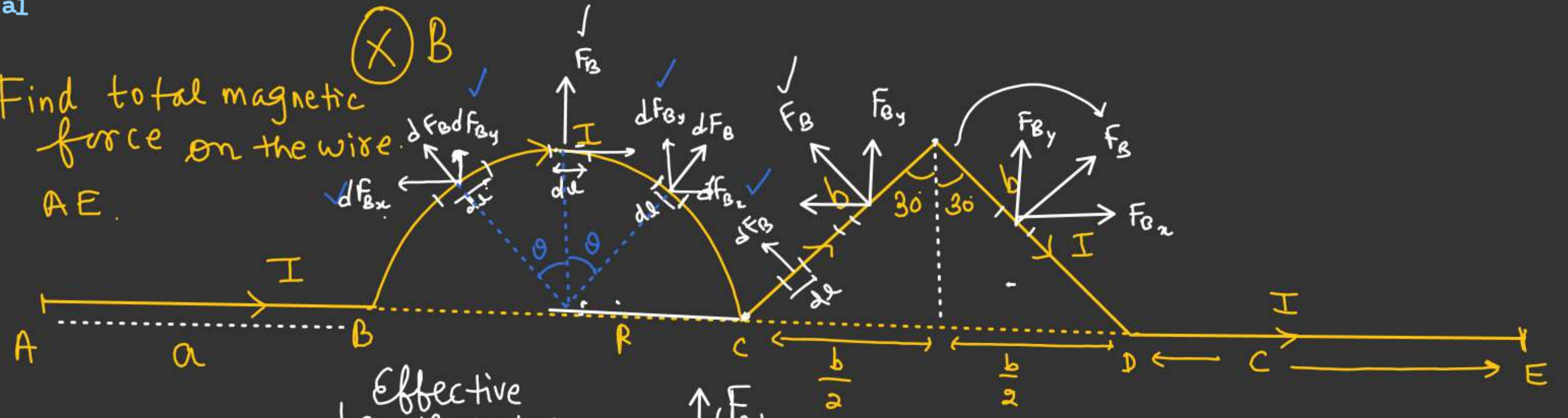
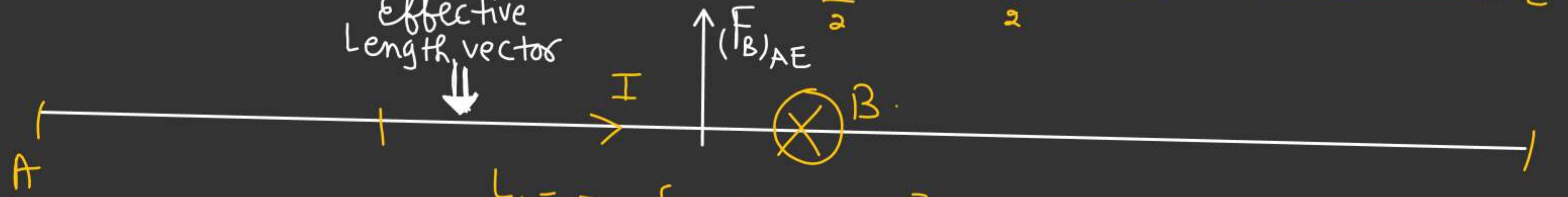
Find total magnetic force on the wire AE.





$$L_{AE} = [a+2R+b+c]$$

$$(F_B)_{AE} = I\,L_{AE}\cdot B = I(a+b+c+2R)B$$

Ans.