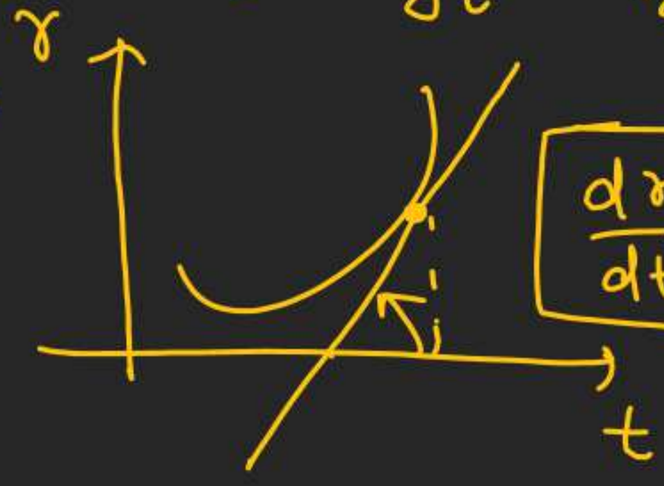
Nishant Jindal

KINEMATICS

$$\vec{v}_{inst} = \frac{d\vec{s}}{dt} \Big/ \frac{d\vec{r}}{dt}$$

$$v_{avg} = \left(\frac{\Delta\vec{s}}{\Delta t}\right) \Big/ \left(\frac{\Delta\vec{r}}{\Delta t}\right)$$



$$\boxed{\frac{dr}{dt} = v_{inst}}$$

↳ Slope of displacement Vs time Curve gives int. Velocity

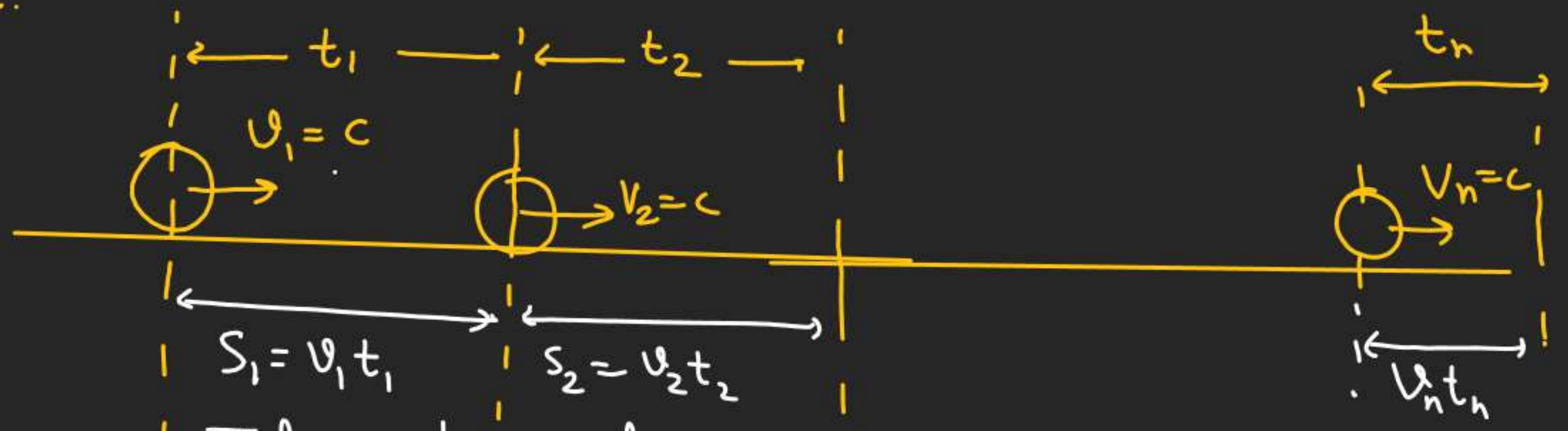
KINEMATICS

(*) Different Cases of avg velocity in 1-dimensional and unidirectional motion :-

→ [Distance = Displacement]

Avg velocity or Avg speed $\Big\} = \dfrac{\text{Total displacement}}{\text{Total time taken}}$

Case-1



$\langle V \rangle = V_{avg} = ?$

$$V_{avg} = \left[\frac{v_1 t_1 + v_2 t_2 \cdots + v_n t_n}{t_1 + t_2 \cdots + t_n}\right]$$

If Intervals are same
$t_1 = t_2 = t_3 = t_n = t$

$$V_{avg} = \frac{\cancel{t}\,(v_1 + v_2 \cdots + v_n)}{n\cancel{t}} = \left(\frac{v_1 + v_2 \cdots + v_n}{n}\right)$$

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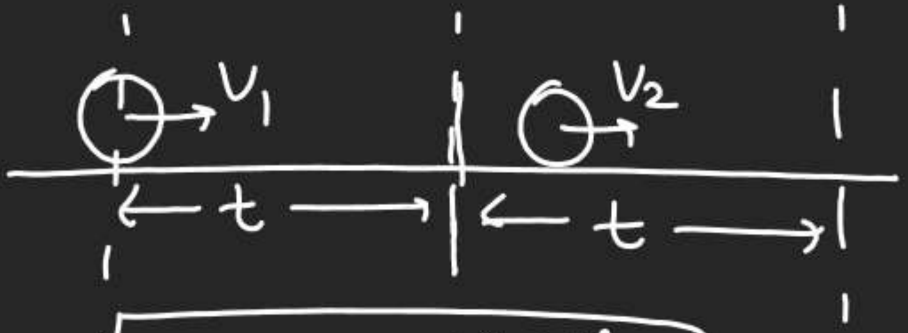
KINEMATICS

✓

If $n = 2$

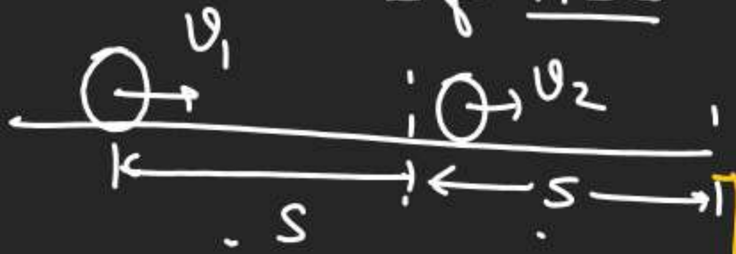


$a \,\&\, b$

$$\left\{ \begin{aligned} A \cdot M &= \frac{a+b}{2} \\ G \cdot M &= \sqrt{ab} \end{aligned} \right\}$$

$$\boxed{V_{avg} = \frac{v_1 + v_2}{2}}$$

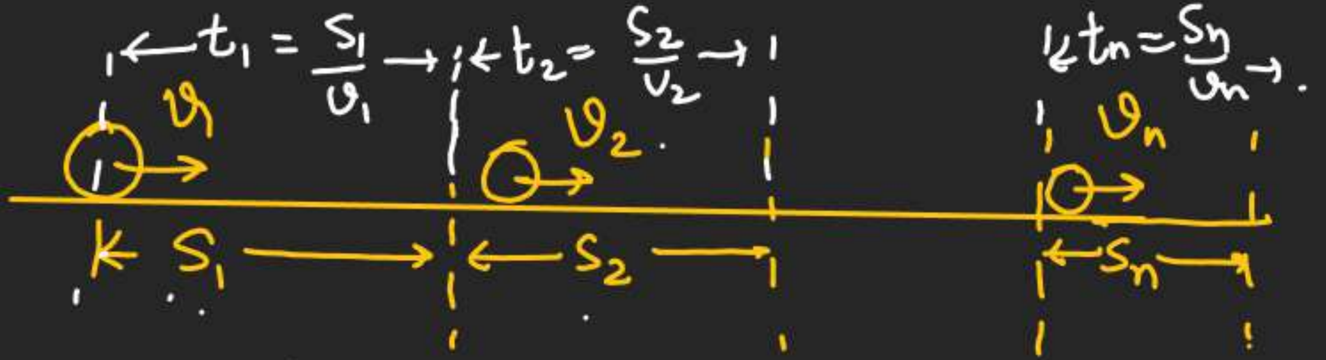
If $n = 2$



$$V_{avg} = \frac{s + s}{\frac{s}{v_1} + \frac{s}{v_2}}$$

$$\boxed{V_{avg} = \frac{2 v_1 v_2}{v_1 + v_2}} \; **$$

Case-2



$$V_{avg} = \left[\frac{\text{Total Displacement}}{\text{Total time taken}} \right]$$

Case

$$= \left[\frac{S_1 + S_2 + S_3 - - - + S_n}{\frac{S_1}{v_1} + \frac{S_2}{v_2} - - - + \frac{S_n}{v_n}} \right]$$

$\Leftarrow$ $\ni$ $S_1 = S_2 = S_3 = S_n = S$

$$V_{avg} = \frac{n\cancel{S}}{\cancel{S}\left[\frac{1}{v_1} + \frac{1}{v_2} - - - + \frac{1}{v_n}\right]} = \left[\frac{n}{\frac{1}{v_1} + \frac{1}{v_2} - - + \frac{1}{v_n}} \right]$$

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(v) General approach to find avg of any quantity:—

$y = f(x)$

$$y_{avg} = \frac{\int_{x_i}^{x_f} y\,dx}{\int_{x_i}^{x_f} dx}$$

$$\int x^n dx = \frac{x^{n+1}}{n+1}$$

Ex:— $v = (t-2)$

Find avg velocity in the interval $t = 1\,sec$ to $t = 5\,sec$.

Sol^n

$$V_{avg} = \frac{\int_1^5 v\,dt}{\int_1^5 dt} = \frac{\int_1^5 (t-2)\,dt}{\int_1^5 dt} = \frac{\int_1^5 t\,dt - 2\int_1^5 dt}{\int_1^5 dt}$$

$$V_{avg} = \frac{\left[\frac{t^2}{2}\right]_1^5 - 2\left[t\right]_1^5}{\left[t\right]_1^5}$$

$$\Rightarrow V_{avg} = \frac{\frac{1}{2}[25-1] - (5-1)\times 2}{(5-1)} = \frac{12-8}{4} = \frac{4}{4} = 1\,m/s$$

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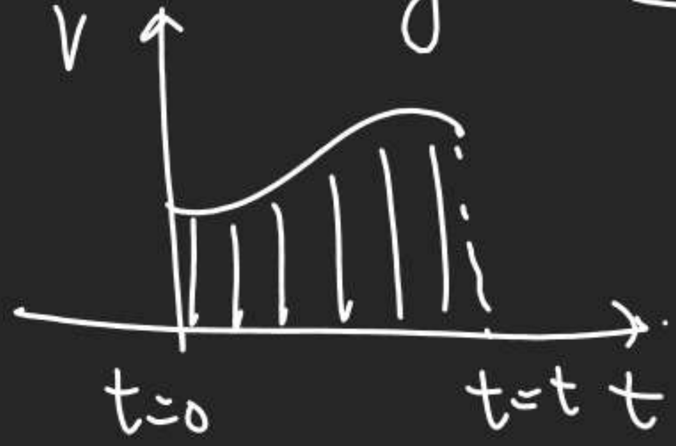
$$\Rightarrow \underline{v} = \left(\frac{ds}{dt}\right)$$

$$v \rightarrow f(t)$$

$$v = \frac{ds}{dt}$$

$$\int_0^t v\,dt = \int_0^s ds$$

[Area under v Vs t curve gives displacement.]

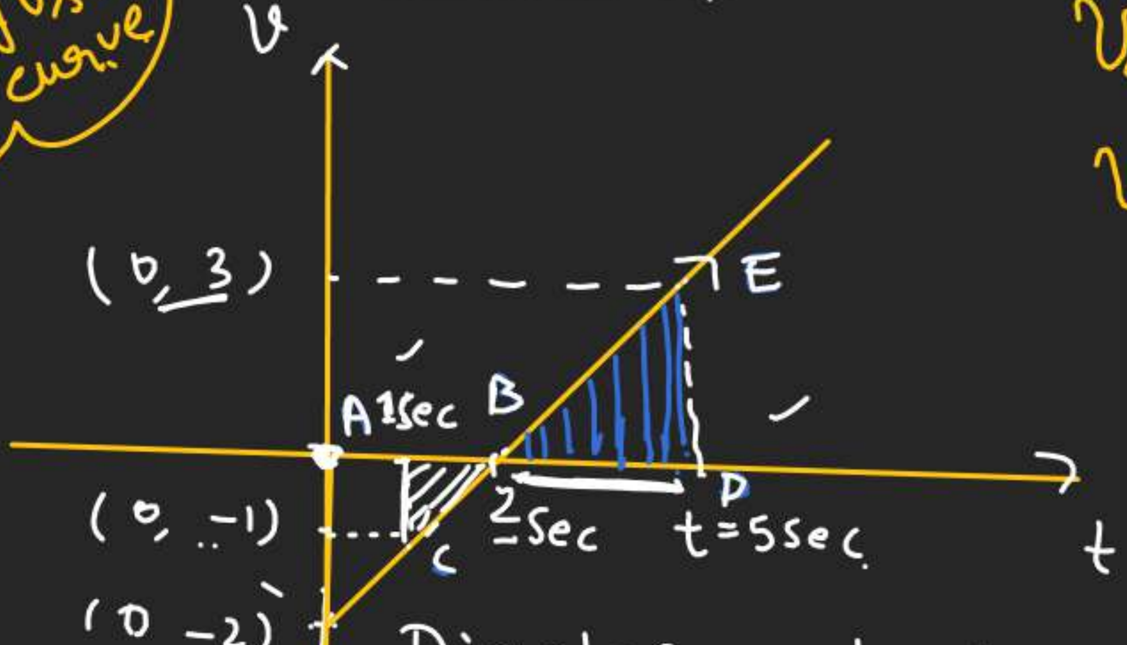


$$\int_{x_i}^{x_f} y \cdot dx = \text{Area under } y \text{ Vs } x \text{ curve}$$

$v = mx + c$ ($m = 1$)

$(v = t - 2)$

[$t = 1$ to $t = 5$ sec]



$$v_{avg} = \frac{\text{Total displacement}}{\text{Total time}}$$

$$v_{avg} = \frac{\int_{t_1}^{t_2} v\,dt}{\int_{t_1}^{t_2} dt} \quad \frac{\text{Total Displacement}}{\text{Total time}}$$

Displacement $= (\text{Area of } \triangle ABC) + (\text{Area of } \triangle BDE)$

$= \frac{1}{2} \times 1 \times (-1) + \frac{1}{2} \times 3 \times 3$

$= -\frac{1}{2} + \frac{9}{2} = \frac{8}{2} = 4\text{ m}$

$\Delta t = (5-1)$

$= 4$ sec

$v_{avg} = \left(\frac{4}{4}\right) = 1\text{ m/s}$

KINEMATICS

(*) **Avg Speed:** $\rightarrow$ $= \left(\dfrac{\text{Total Distance}}{\text{Total time}}\right)$

$$\boxed{|v| = \frac{ds}{dt}}$$

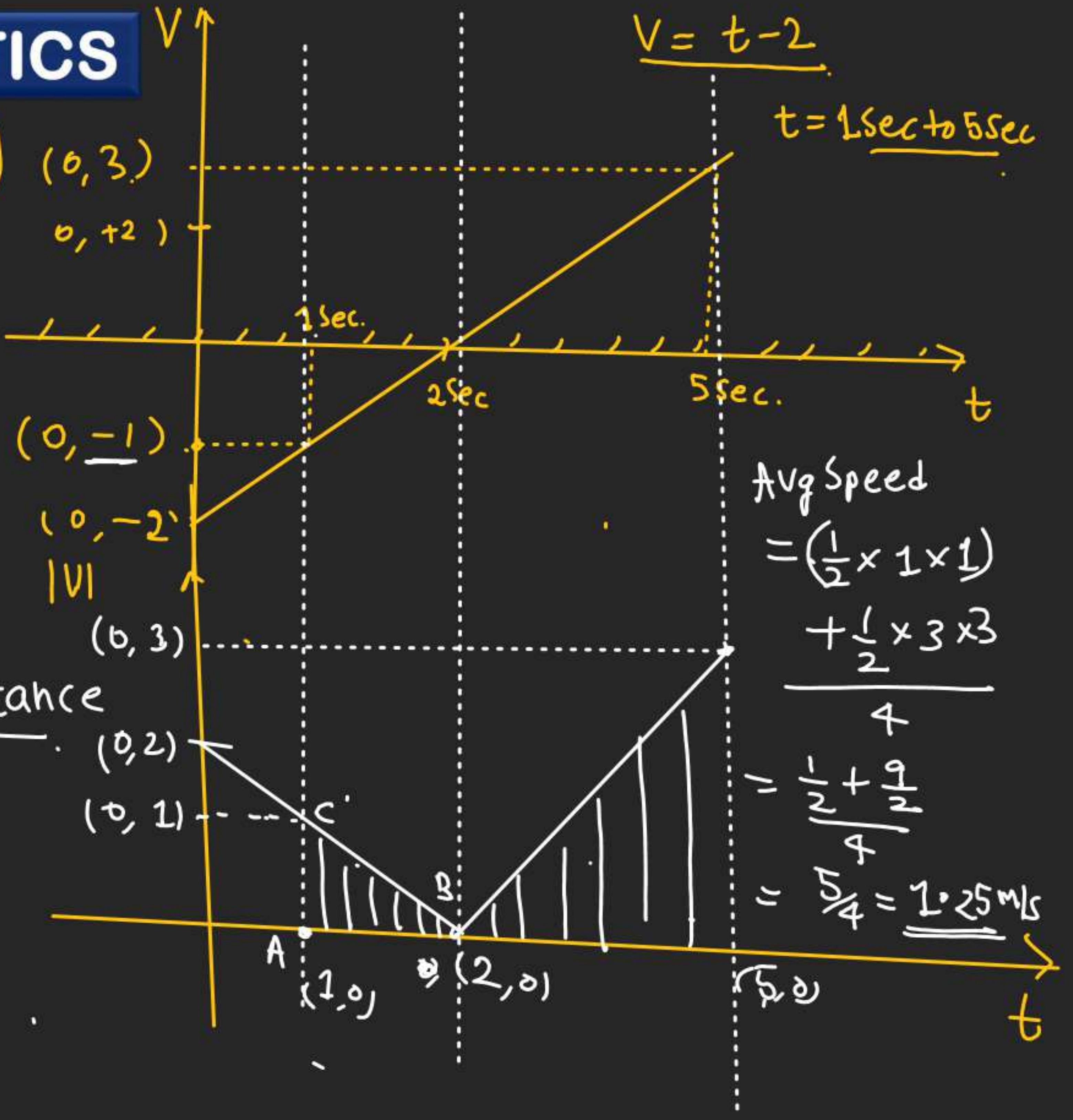
$$\int_0^s ds = \int_0^t |v|\,dt$$

$\Downarrow$

Area under Speed vs time graph = Distance

$|\vec{v}| \Rightarrow$ Magnitude of velocity $\Rightarrow$ Speed

Note:- To Convert v-t graph into $|v|$ vs t graph reflect the $-$ve part of the graph w.r.t time axis



Avg Speed

$$= \frac{\left(\frac{1}{2}\times 1\times 1\right) + \frac{1}{2}\times 3\times 3}{4}$$

$$= \frac{\frac{1}{2} + \frac{9}{2}}{4}$$

$$= \frac{5}{4} = 1.25\,\text{m/s}$$

KINEMATICS

⟹ How to find Avg Speed ⟶

$$|v| = \frac{ds}{dt}$$

$$\int_0^s ds = \int_{t_1}^{t_2} |v|\, dt$$

Distance

$$|x| = \begin{cases} x & \text{if } x > 0 \\ -x & \text{if } x < 0 \end{cases}$$

$t - 2 = 0$

$\boxed{t = 2\,sec}$

$$\text{Distance} = \int_1^5 |v|\, dt$$

Ex:- $(v = t - 2)$. Avg Speed = ??

$t = 1\,sec$ to $t = 5\,sec$

$$|v| = |t-2| = \begin{cases} -(t-2) & 1 \le t \le 2 \\ (t-2) & 2 < t < 5 \end{cases}$$

$$= \int_1^2 (2-t)\,dt + \int_2^5 (t-2)\,dt$$

$$= \left(2\int_1^2 dt - \int_1^2 t\,dt\right) + \int_2^5 t\,dt - 2\int_2^5 dt$$

$$= 5$$

$$\text{Avg Speed} = \left(\frac{5}{4}\right)$$

$$= \underline{\qquad}$$

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KINEMATICS

H.W.

Find avg speed and avg velocity of a particle whose displacement as a function of time is given as.

$$\Rightarrow S = \left(\frac{t^3}{3} - 4t\right)$$

in the interval $t = 1$ sec to $t = 3$ sec.

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KINEMATICS

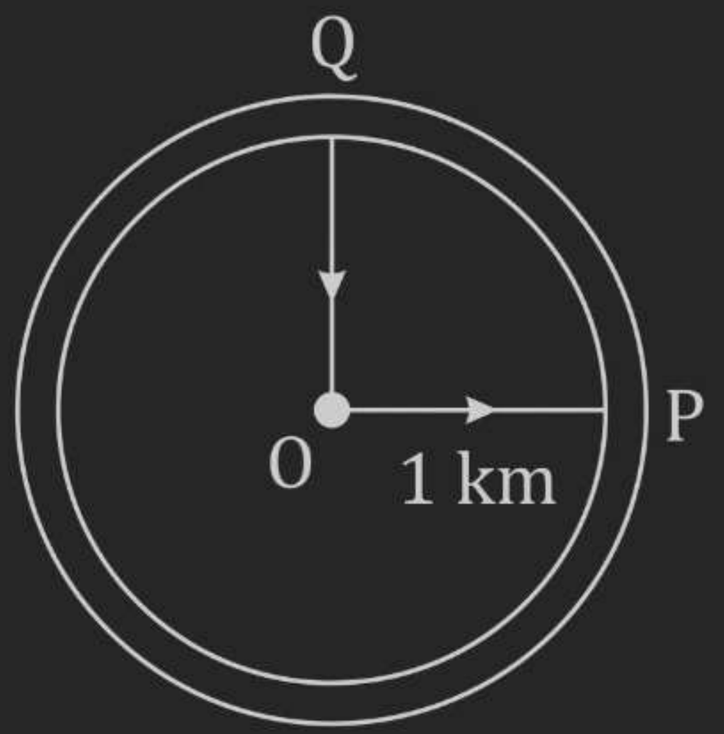
H.W. :

Q. A cyclist travels from centre O of a circular park of radius 1 km and reaches point P. After cycling 1/4th of the circumference along PQ, he returns to the centre of the park QO. If the total time taken is 10 minute, calculate

(i) net displacement Fig.

(ii) average velocity and

(iii) average speed of the cyclist.



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KINEMATICS

H.W :

Q. A point traversed half the distance with a velocity v_0. The remaining part of the distance was covered with velocity v_1 for half the time, and with velocity v_2 for the other half of the time. Find the mean velocity of the point averaged over the whole time of motion.

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KINEMATICS

H-W

Q. A man walks on a straight road from his home to a market 2.5 km away with a speed of 5 km/h. Finding market closed, he instantly turns and walks back home with a speed of 7.5 km/h. What is the

(a) magnitude of average velocity,

(b) average speed of the man over the interval of time

(i) 0 to 30 min., (ii) 0 to 50 min., (iii) 0 to 40 min. ?

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KINEMATICS

H.W.

Q. A particle is moving at a speed of 5 m/s along east. After 10 s its velocity changes and becomes 5 m/s along north. What is the average acceleration during this interval?

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KINEMATICS

H.W.:

Q. If a point moves in a straight line with uniform acceleration and covers successive equal distances in times t_1, t_2, t_3, then show that

$$\frac{1}{t_1} - \frac{1}{t_2} + \frac{1}{t_3} = \frac{3}{t_1 + t_2 + t_3}$$