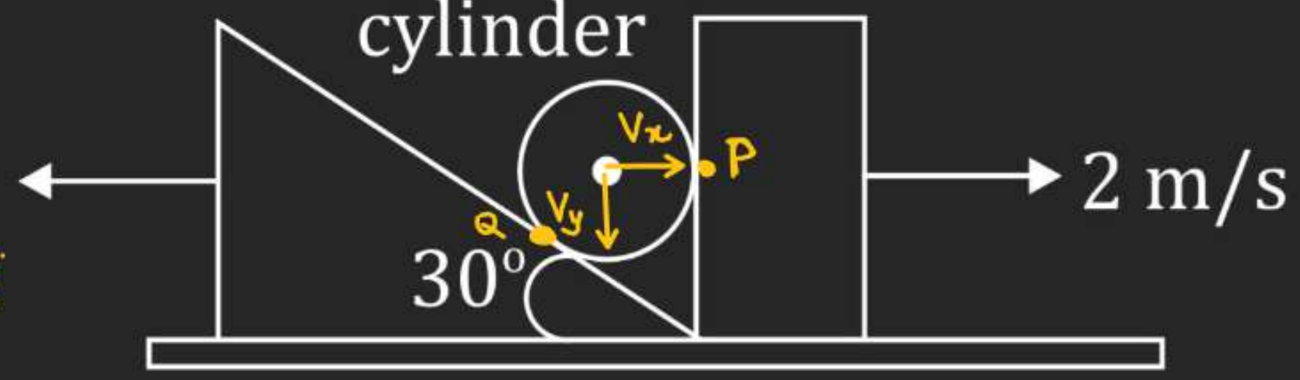
Rigid body constrain

Law of Motion

Q.1 A system shown in the figure. Assume that cylinder remains in constant with the wedge and block hence the velocity of cylinder is



(A) $\frac{\sqrt{19-4\sqrt{3}}}{2}$ m/s

(B) $\frac{\sqrt{13}}{2}$ m/s

(C) $\sqrt{3}$ m/s

(D) $\sqrt{7}$ m/s

$1\sin 30^\circ = V_y \sin 60^\circ - 2\cos 60^\circ$

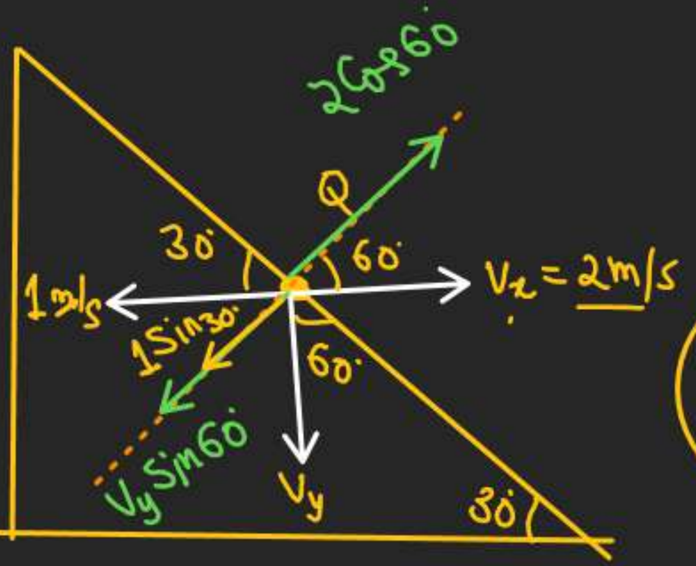
$\frac{1}{2} = V_y \frac{\sqrt{3}}{2} - 1$

$\frac{3}{2} = \frac{\sqrt{3}\, V_y}{2}$

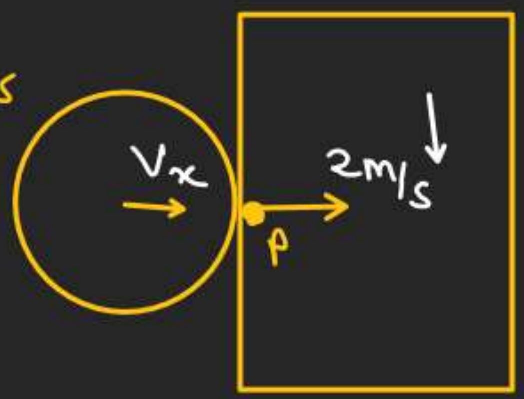
$V_y = \sqrt{3}$

$V_{cylinder} = \sqrt{(\sqrt{3})^2 + (2)^2}$

$= \sqrt{7}$ m/s



$V_x = 2$ m/s.



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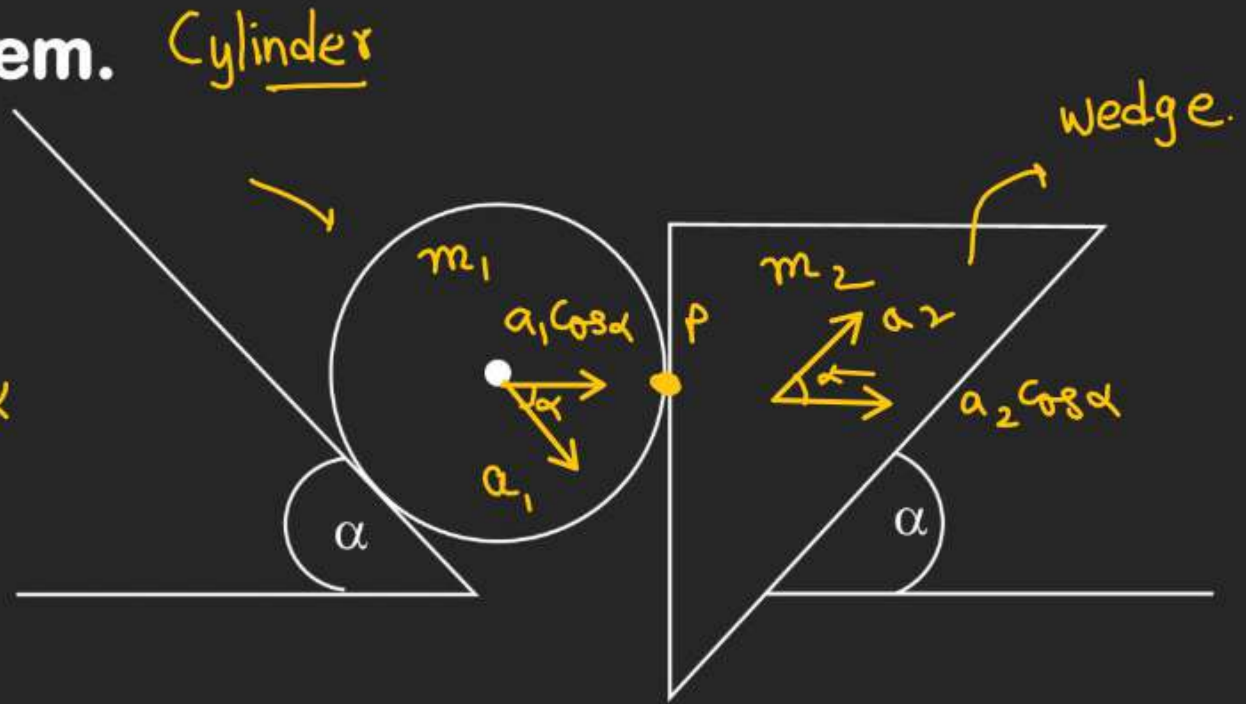
Rigid body constrain

Law of Motion

Q.3 A cylinder and a wedge with a vertical face, touching each other. move along two smooth inclined planes forming the same angle α with the horizontal (see figure). The masses of the cylinder and the wedge are m_1 and m_2 respectively. Determine the force of normal pressure N exerted by the wedge on the cylinder, neglecting the friction between them.

$a_1 \cos\alpha = a_2 \cos\alpha$

$\boxed{a_1 = a_2}$ — (1)



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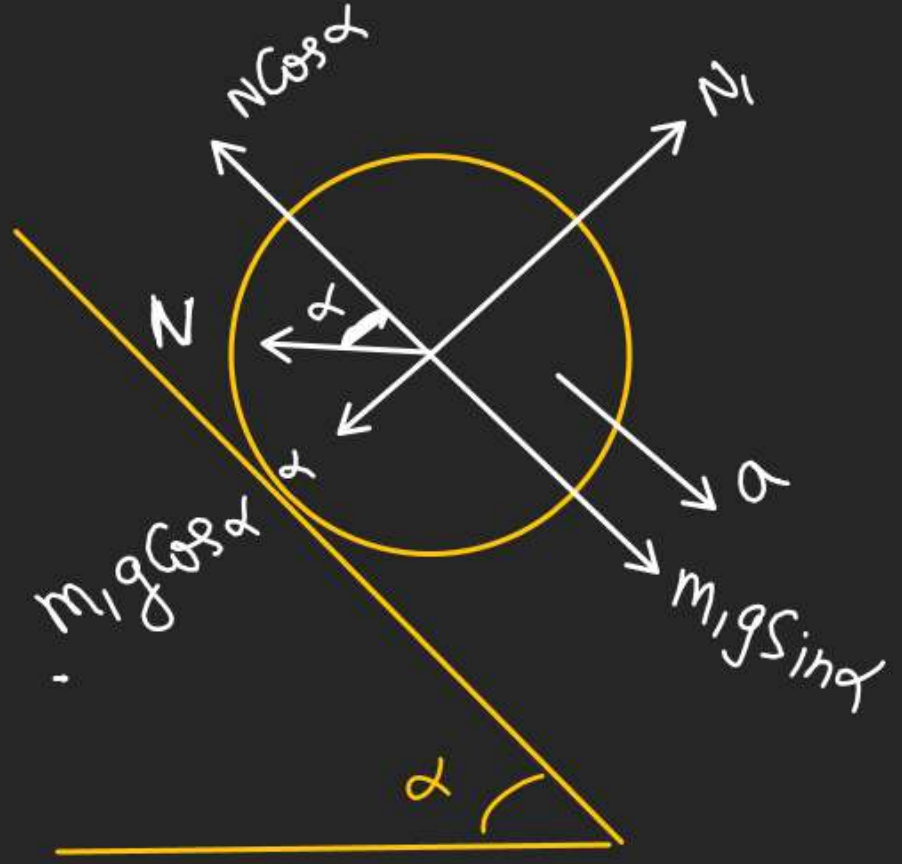
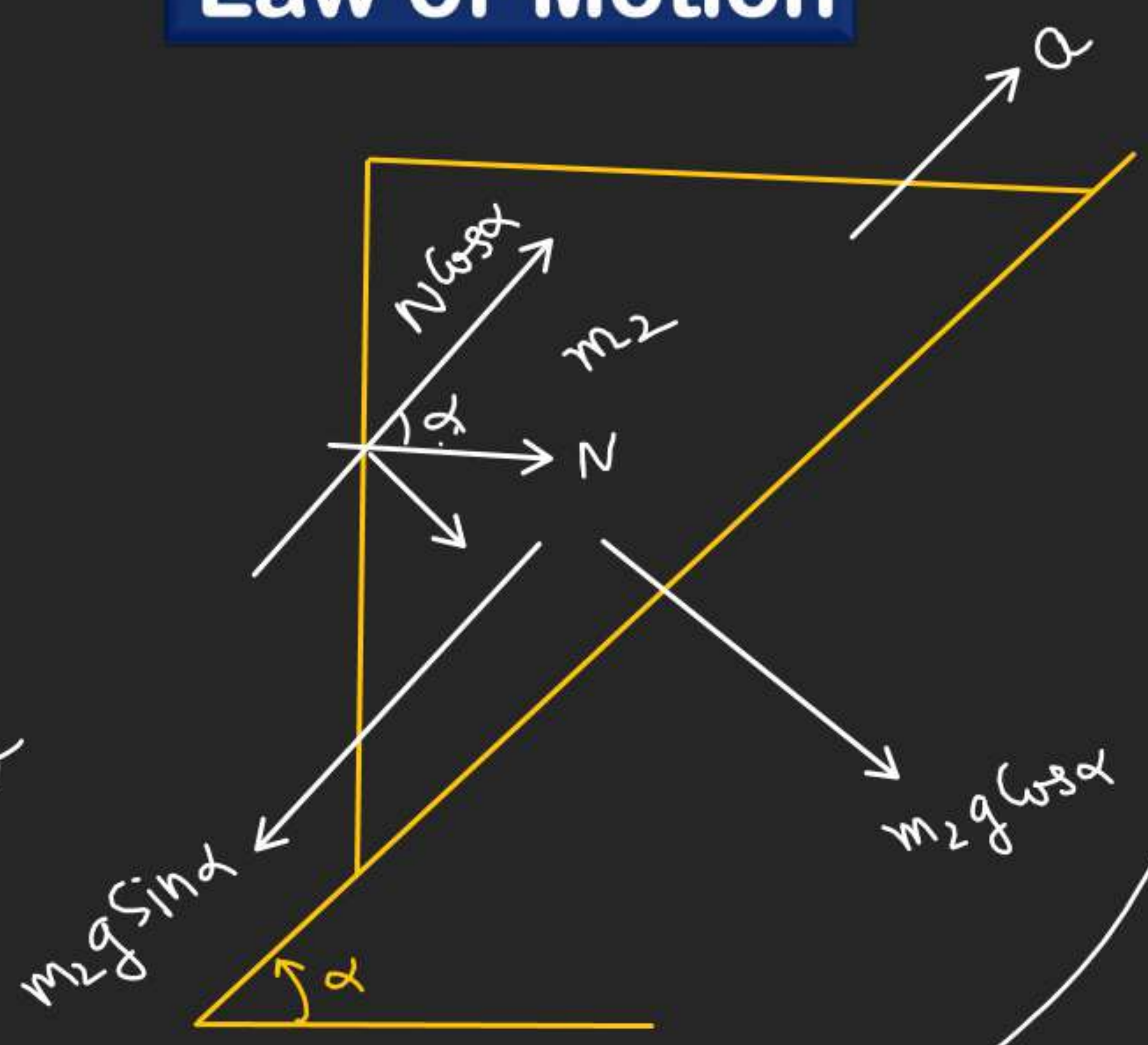
Law of Motion





For Cylinder

$$m_1 g\sin\alpha - N\cos\alpha = m_1 a \quad (1)$$

$$g\sin\alpha - \frac{N\cos\alpha}{m_1} = a$$

For wedge

$$N\cos\alpha - m_2 g\sin\alpha = m_2 a \quad (2)$$

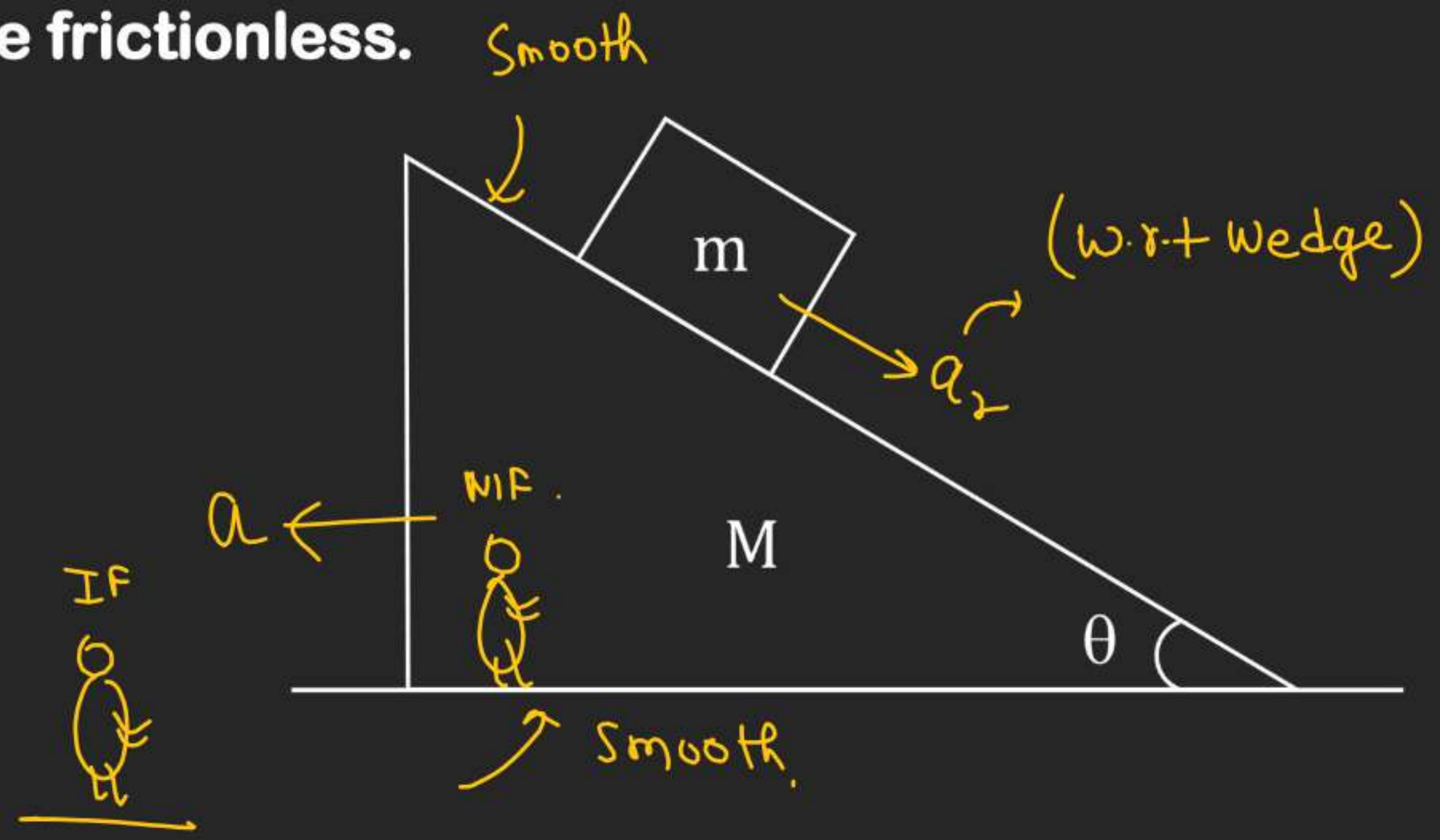
$$a = \frac{N\cos\alpha}{m_2} - g\sin\alpha$$

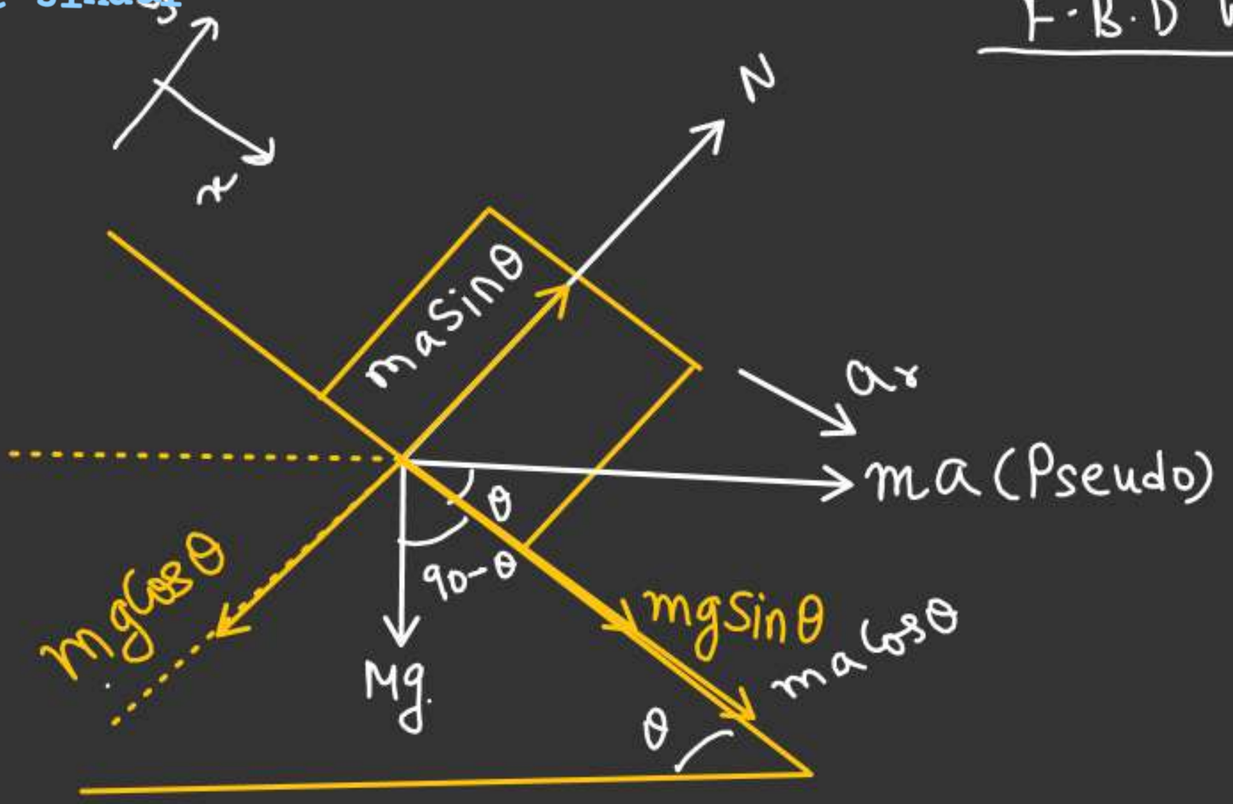
$$g\sin\alpha - \frac{N\cos\alpha}{m_1} = \frac{N\cos\alpha}{m_2} - g\sin\alpha$$

$$2g\sin\alpha = N\cos\alpha\left[\frac{1}{m_1} + \frac{1}{m_2}\right]$$

$$\frac{2g\tan\alpha}{\left[\frac{1}{m_1} + \frac{1}{m_2}\right]} = N$$

Q.6 A block of mass m is placed on the inclined surface of a wedge as shown in Fig. Calculate the acceleration of the wedge and the block when the block is released. Assume all surfaces are frictionless.





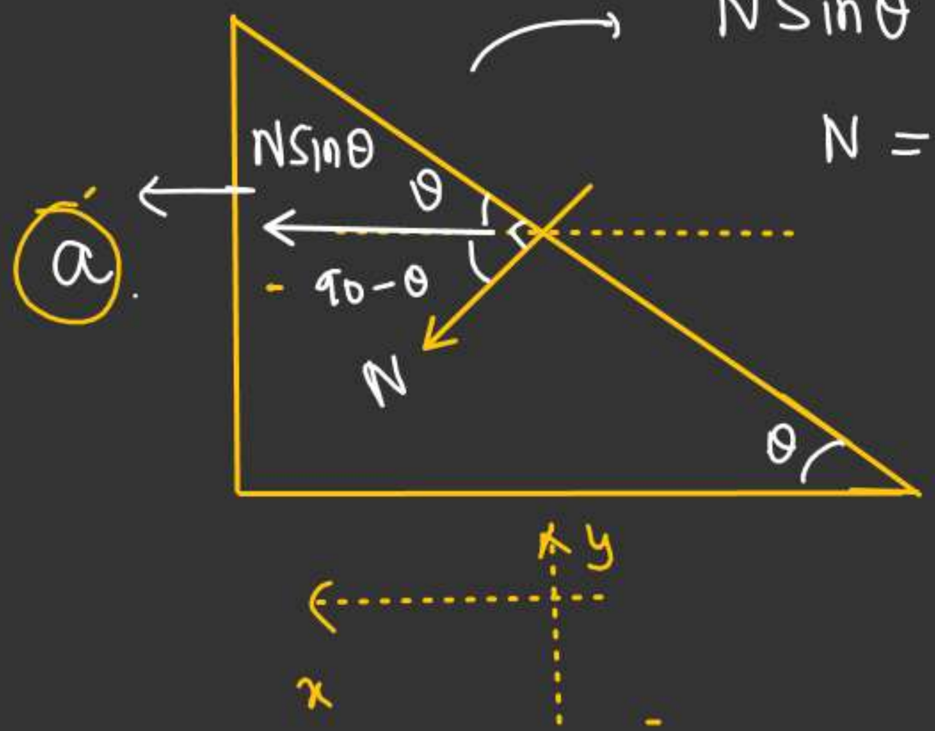
F.B.D w.r.t Wedge

Along the Inclined plane

$$mg\sin\theta + ma\cos\theta = ma_r \quad -(1)$$

Perpendicular to Inclined plane.

$$N + ma\sin\theta = mg\cos\theta \quad -(2)$$



$$N\sin\theta = Ma \quad -(3)$$

$$N = \left(\frac{Ma}{\sin\theta}\right) \Rightarrow \text{put in (2)}$$

$$\frac{Ma}{\sin\theta} + ma\sin\theta = mg\cos\theta$$

$$(M + m\sin^2\theta)a = mg\cos\theta\cdot\sin\theta$$

$$\boxed{a = \left(\frac{mg\sin\theta\cdot\cos\theta}{M+m\sin^2\theta}\right)}$$

Put in (1)

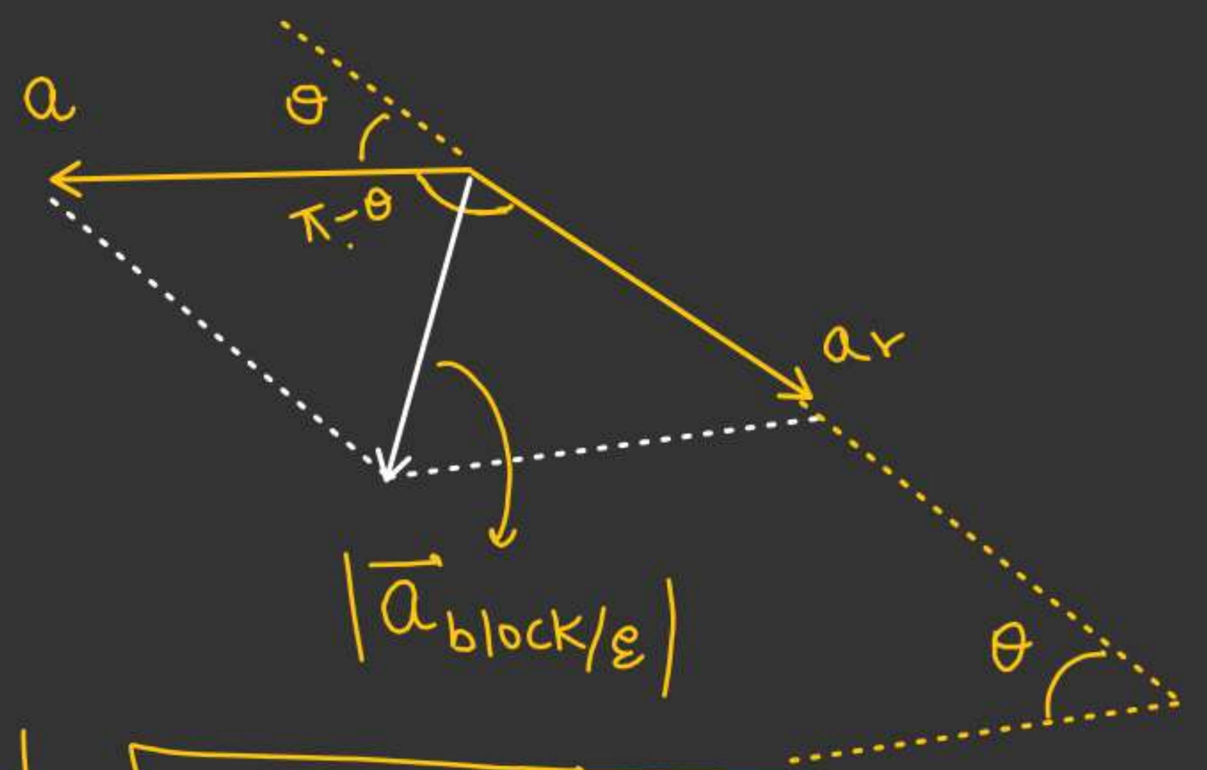
$$ma_r = mg\sin\theta + m\cos\theta\left(\frac{mg\sin\theta\cdot\cos\theta}{M+m\sin^2\theta}\right)$$

$$\cancel{m}a_r = \cancel{m}g\sin\theta\left[1 + \frac{m\cos^2\theta}{M+m\sin^2\theta}\right]$$

$$a_r = g\sin\theta\left[\frac{M+m}{M+m\sin^2\theta}\right]$$

$$\boxed{a_r = \frac{(M+m)g\sin\theta}{M+m\sin^2\theta}}$$

$$|\vec{a}_{block/E}| = ??$$



$$|\vec{a}_{block/E}| = \sqrt{a_r^2 + a^2 - 2aa_r\cos\theta}$$

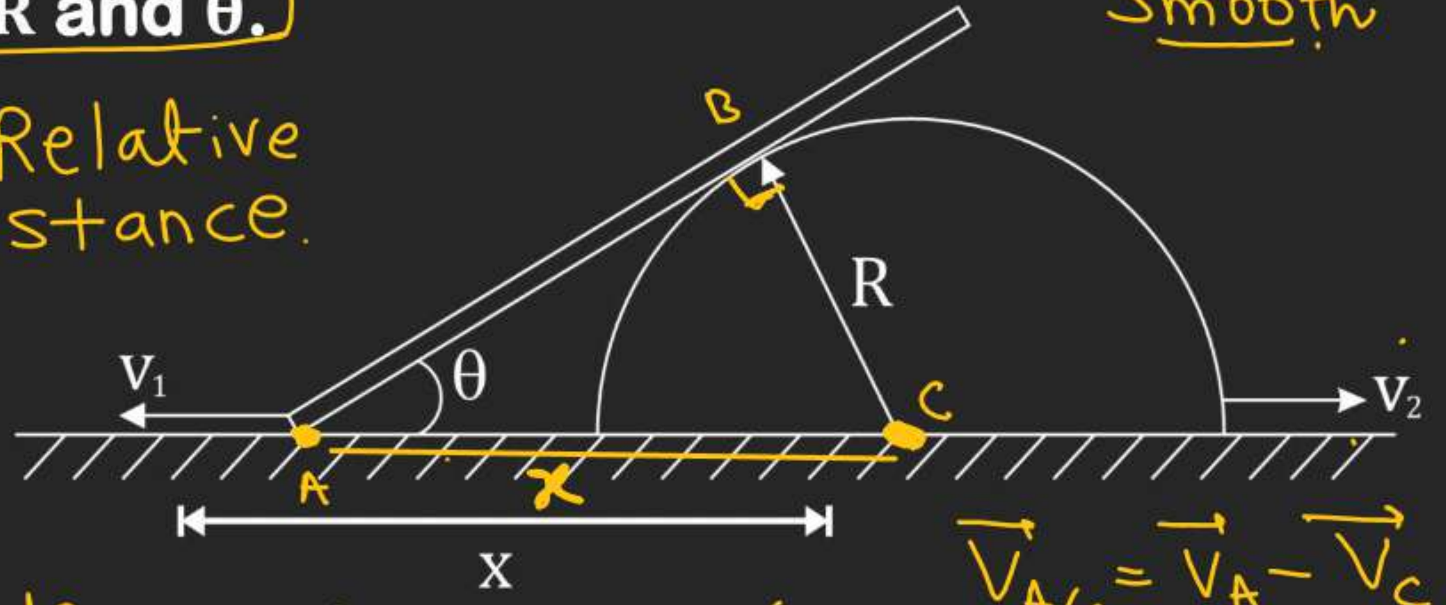
Rigid body constrain

Law of Motion

$\omega = \frac{d\theta}{dt}$ ⇐

Q.8 Shows a hemisphere and a supported rod. Hemisphere is moving in right direction with a uniform velocity v_2 and the end of rod which is in contact with ground is moving in left direction with a velocity v_1. Find the rate at which the angle θ is changing in terms of $\boxed{v_1, v_2, R \text{ and } \theta.}$

Smooth



Sol^n

$$\sin\theta = \frac{R}{x}$$

$x \rightarrow$ Relative distance.

$$x = R\,\mathrm{cosec}\,\theta.$$

$$\left(\frac{dx}{dt}\right) = R\,\frac{d}{d\theta}(\mathrm{cosec}\,\theta) \times \left(\frac{d\theta}{dt}\right)$$

$\Downarrow$

$$v_{A/C} = R[-\mathrm{cosec}\,\theta \cdot \cot\theta]\left(\frac{d\theta}{dt}\right) \Rightarrow \left(+\frac{d\theta}{dt}\right) = \frac{v_{A/C}}{R\,\mathrm{cosec}\,\theta \cdot \cot\theta} = \frac{+(v_1+v_2)\sin^2\theta}{R\cos\theta}.$$

$$\vec{V}_{A/C} = \vec{V}_A - \vec{V}_C = -V_1\hat{i} - V_2\hat{i} = -(V_1+V_2)\hat{i}$$

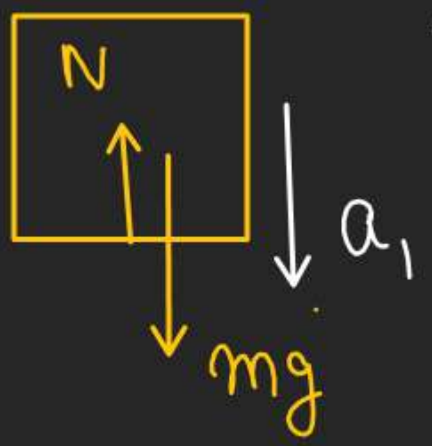
$$\boxed{\omega = \frac{(v_1+v_2)\sin^2\theta}{R\cos\theta}}$$

✓

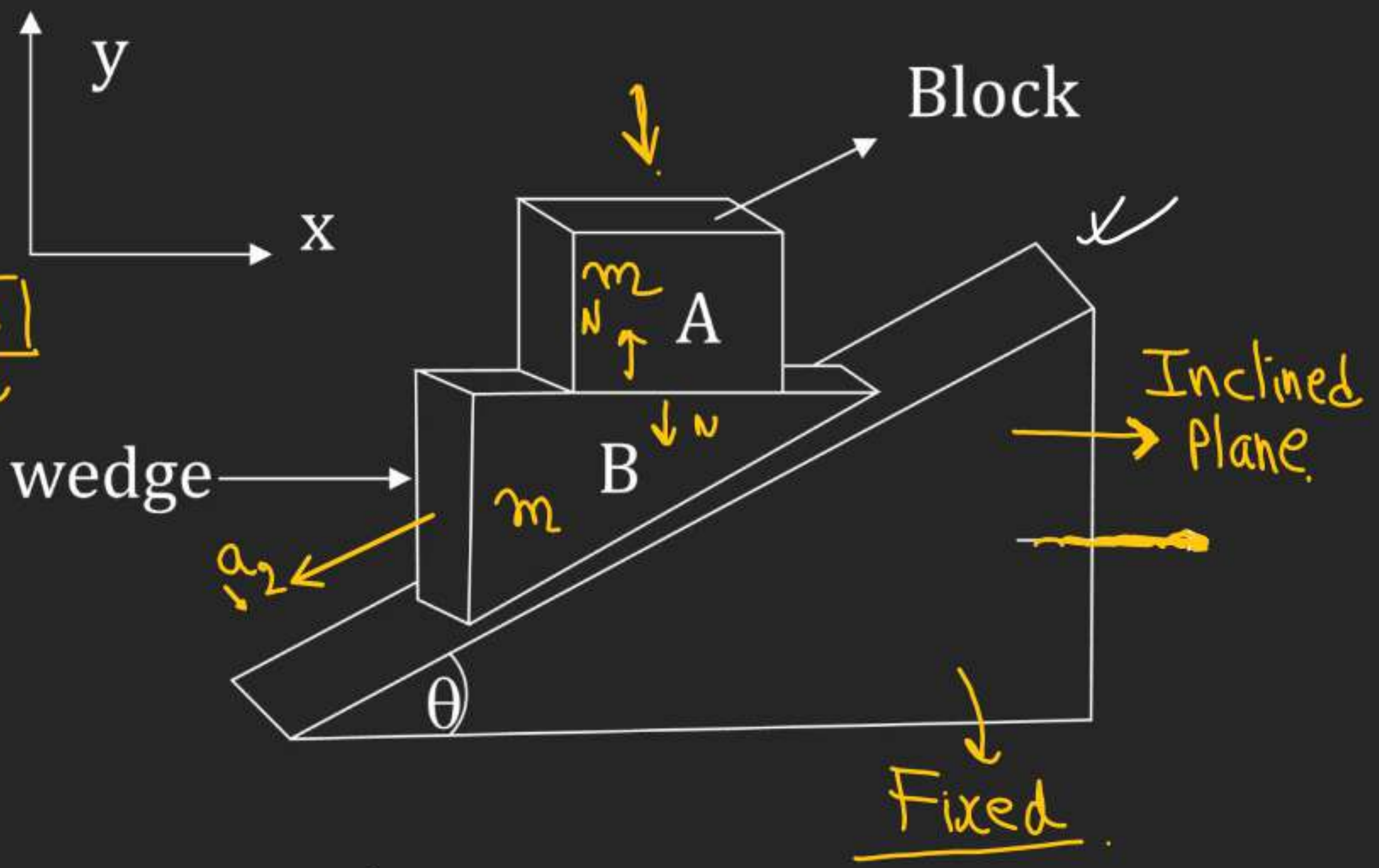
Q.14 Block A of mass m is placed over a wedge of same mass m. Both the block and wedge are placed on a fixed inclined plane. Assuming all surfaces to be smooth, calculate the displacement of the block A in ground frame in 1 s.

$a_A = ?$ $a_B = ?$

F.B.D of A w.r.t earth



$mg - N = ma_1$ (1)



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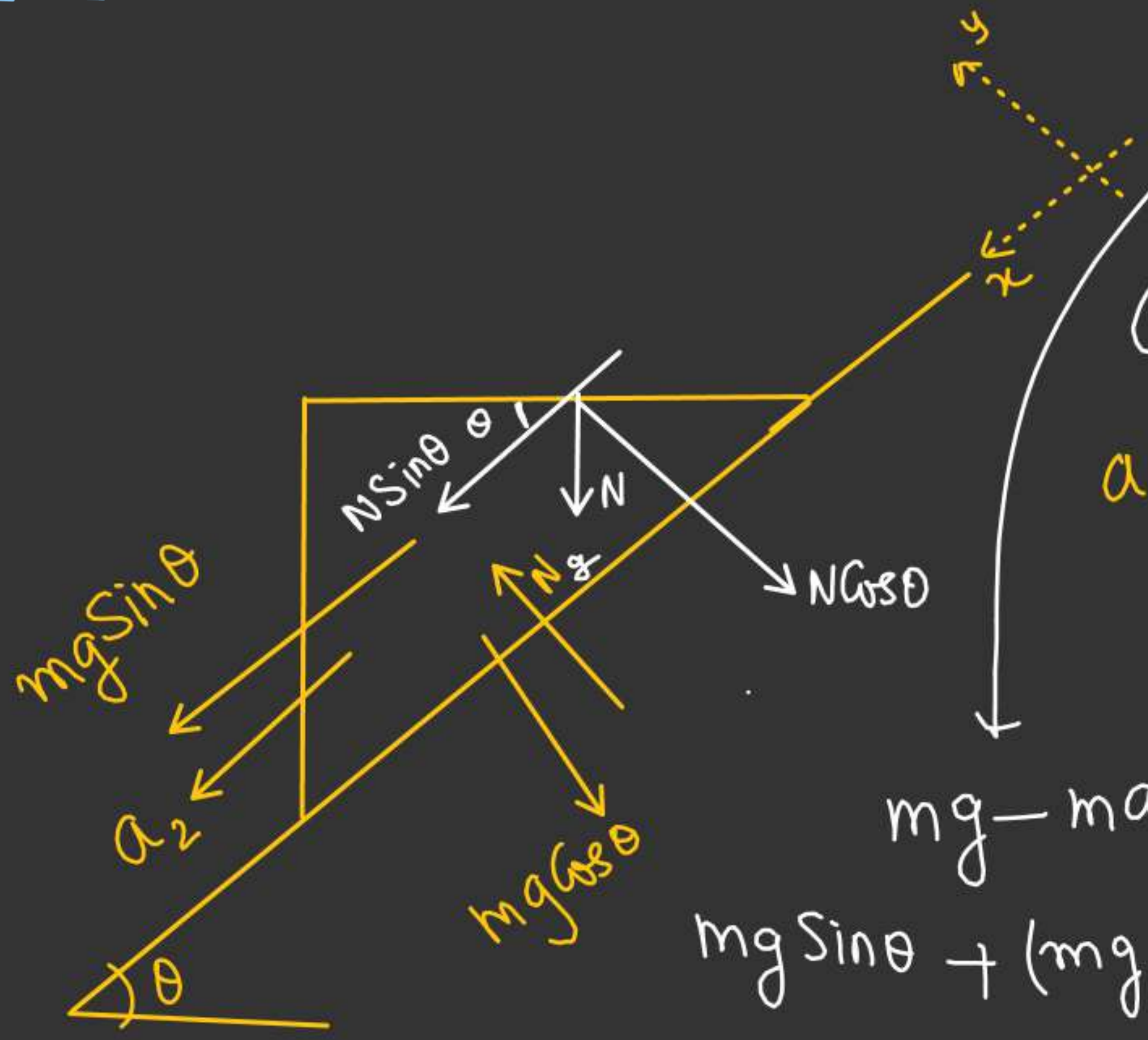
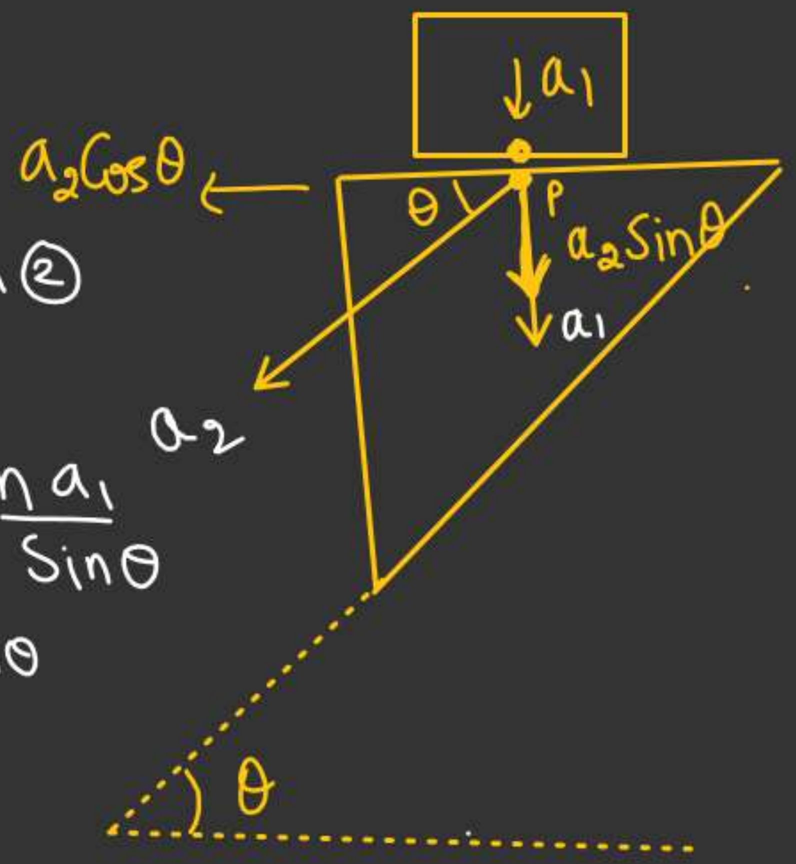


$$mg - N = ma_1 \quad —(1)$$

$$mg\sin\theta + N\sin\theta = ma_2 \quad —(2)$$

Constrain relation

$$a_2\sin\theta = a_1 \quad —(3)$$



$mg - ma_1 = N \rightarrow$ Put in (2)

$$mg\sin\theta + (mg - ma_1)\sin\theta = \frac{ma_1}{\sin\theta}$$

$$2mg\sin\theta = \frac{ma_1}{\sin\theta} + ma_1\sin\theta$$

$$2g\sin\theta = a_1\left[\frac{1+\sin^2\theta}{\sin\theta}\right]$$

$$\boxed{a_2 = \frac{2g\sin\theta}{1+\sin^2\theta}, \quad a_1 = \frac{2g\sin^2\theta}{1+\sin^2\theta}}$$

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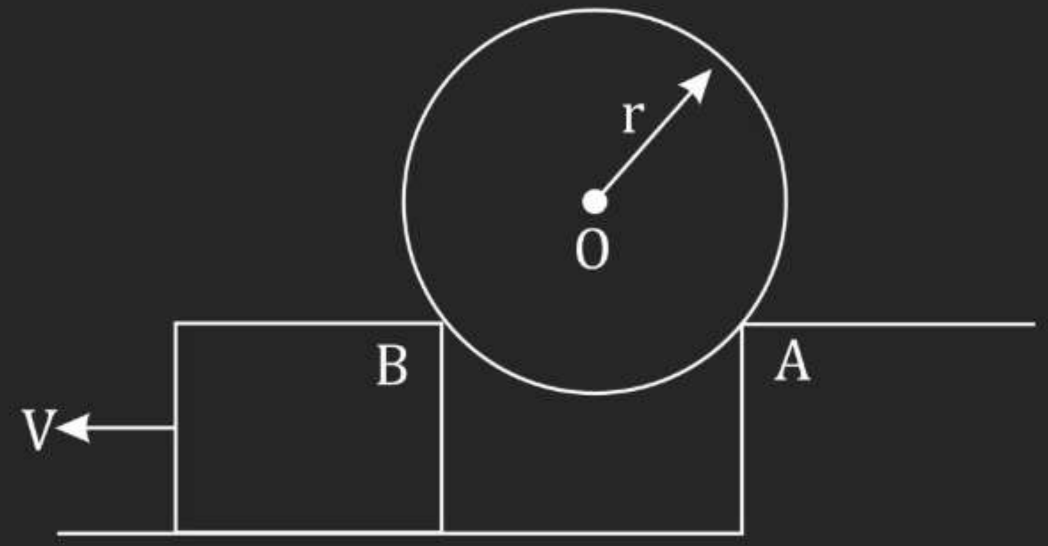
Law of Motion

Q.2 A cylinder of mass m and radius r rests on two supports of the same height (see figure). One support is stationary, while the other slides from under the cylinder at a velocity v. Determine the force of normal pressure N exerted by the cylinder on the stationary support at the moment when the distance between the points A and B of the supports is $AB = r\sqrt{2}$, assuming that the supports were very close to each other at the initial instant. The friction between the cylinder and the supports should be neglected.

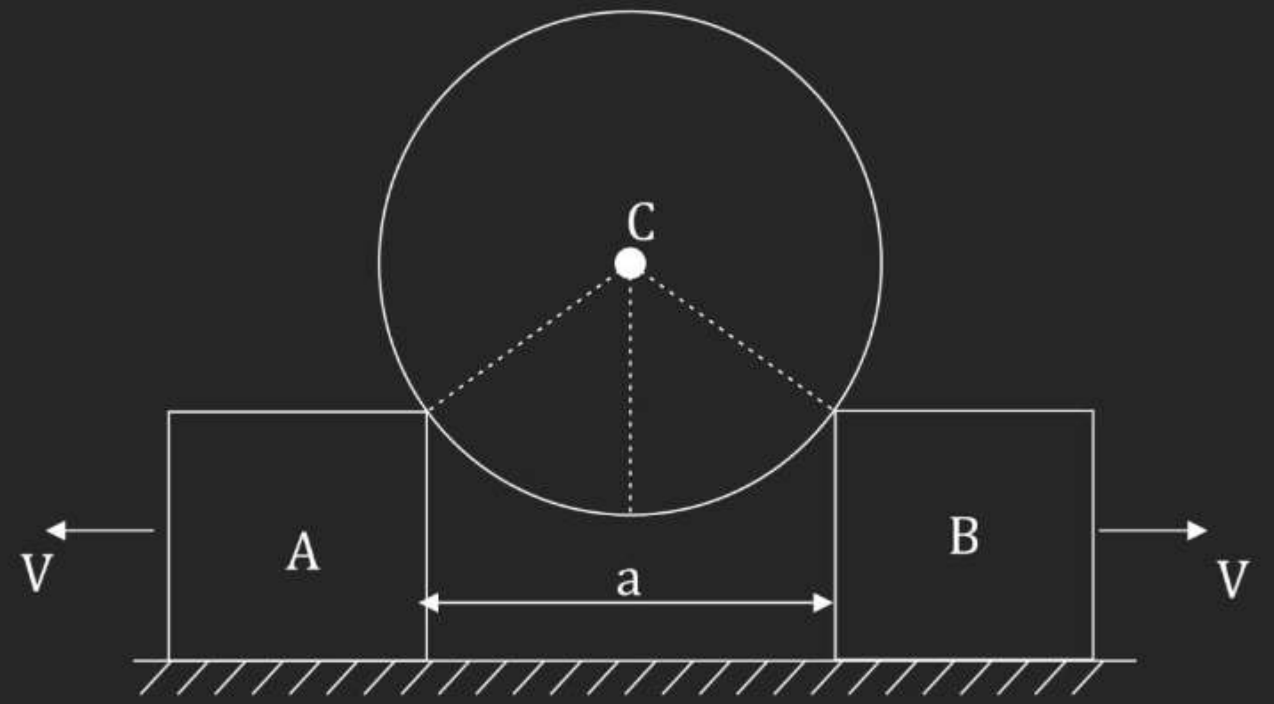
H-W



Law of Motion

Q.7 A smooth spherical ball of mass $M = 2\ \text{kg}$ is resting on two identical blocks A and B as shown in the figure. The blocks are moved apart with same horizontal velocity $V = 1\ \text{m/s}$ in opposite directions (see figure).

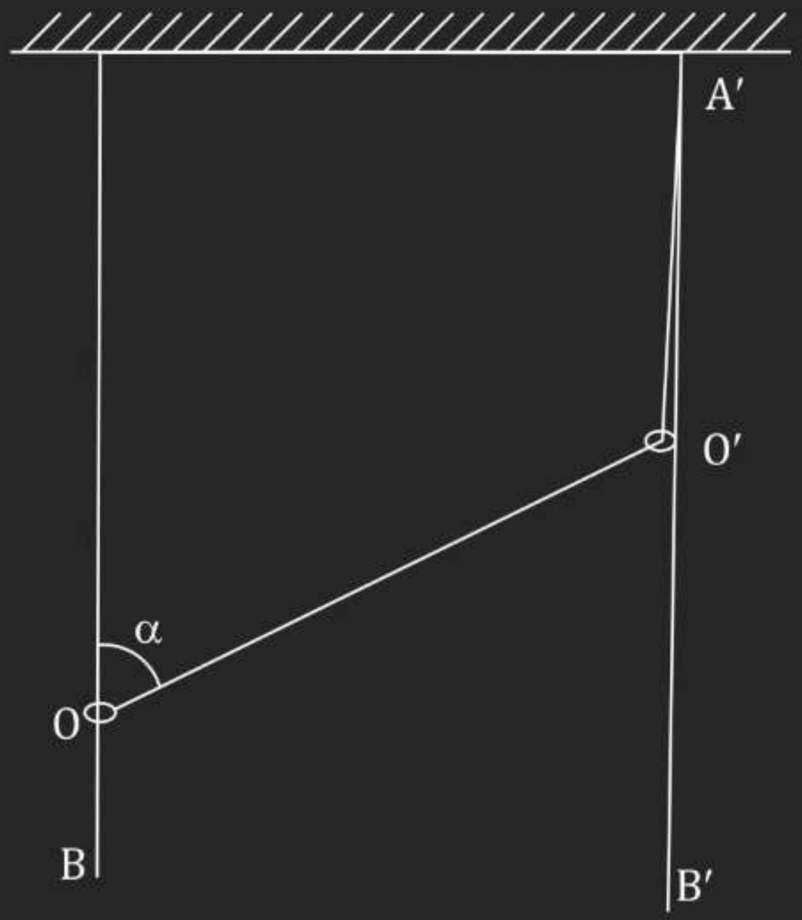
(a) Find the normal force applied by each of the blocks on the sphere at the instant separation between the blocks is $a = \sqrt{2}R$; $R = 1.0\ \text{m}$ being the radius of the ball.



Q.9 (ii) Two rings O and O' are put on two vertical stationary rods AB and $A'B'$ respectively as shown in figure. An inextensible string is fixed at point A' and on ring O and is passed through O'. Assuming that ring O' moves downwards at a constant speed v, find the velocity of the ring O in terms of α.

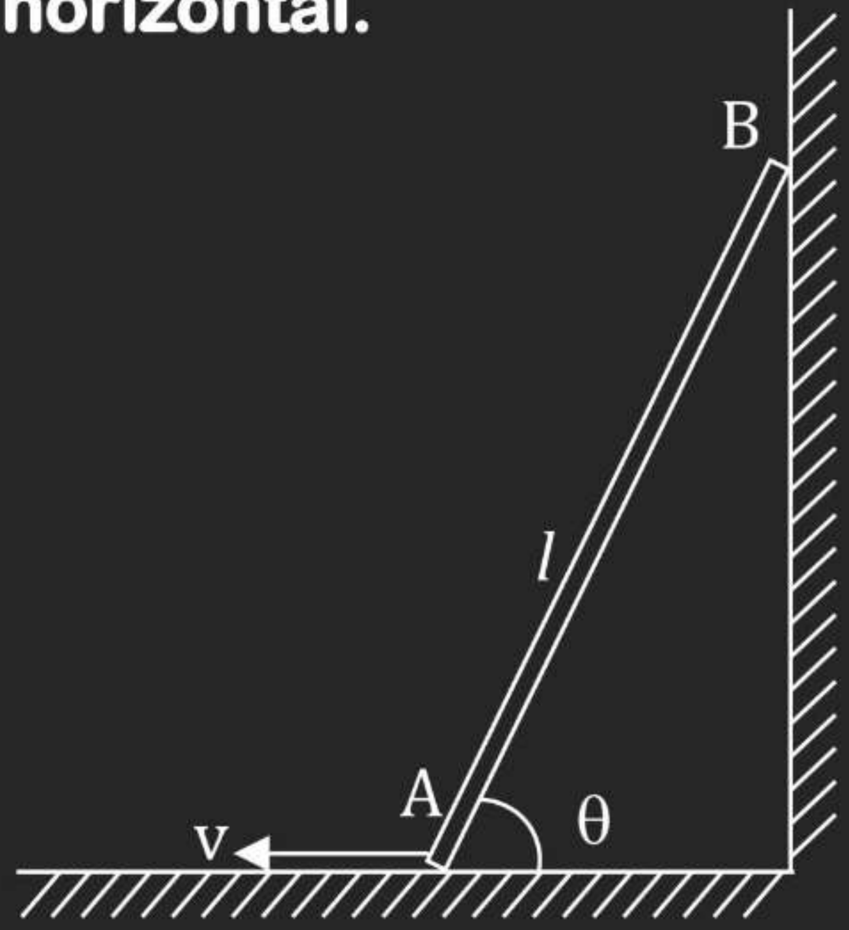
H.W

$\left[\frac{v(1-\cos\alpha)}{\cos\alpha}\right]$



Q.10 Shows a rod of length l resting on a wall and the floor. Its lower end A is pulled towards left with a constant velocity u. Find the velocity of the other end B downward when the rod makes an angle θ with the horizontal.

H.W



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Rigid body constrain

Law of Motion

Q.11 Two lines AB and CD intersect at O at an inclination α, as shown in figure If they move out parallel to themselves with the speed v, find the speed of O.

H.W.