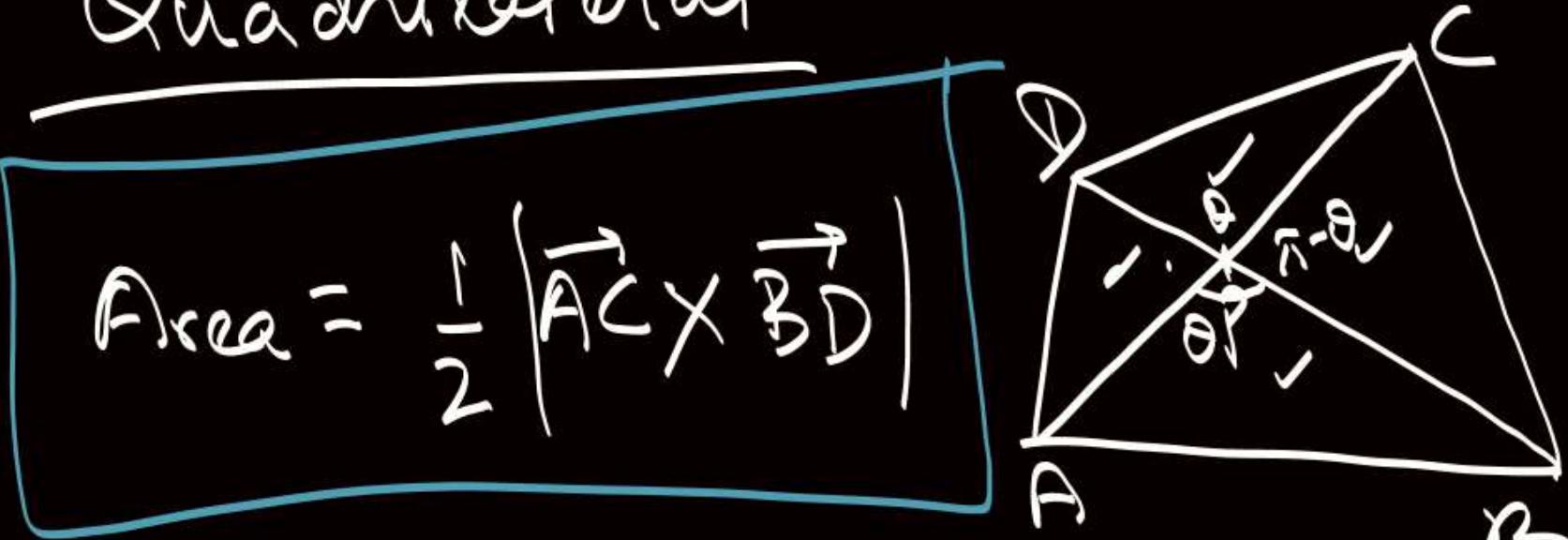


# Quadrilateral

$$\text{Area} = \frac{1}{2} |\vec{AC} \times \vec{BD}|$$



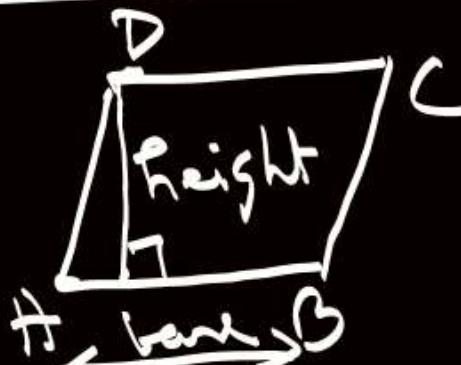
$$= \frac{1}{2} \sin\theta \left( (AP)(PB) + (PB)(PC) + (PC)(PD) + (PD)(PA) \right)$$

$$= \frac{1}{2} \sin\theta \left( (PB)(AC) + (PD)AC \right)$$

$$= \frac{1}{2} \sin\theta (AC)(BD)$$

# Quadrilateral

↓  
Parallelogram



- Both opposite pairs of sides are parallel
- Both diagonals are equal
- One opposite pair of sides is || & equal
- Diagonals bisect each other

$$\begin{aligned} \text{Area} &= |\vec{AB} \times \vec{AD}| \\ &= \text{base} \times \text{height} \\ &= P_1 P_2 \cos \theta \end{aligned}$$

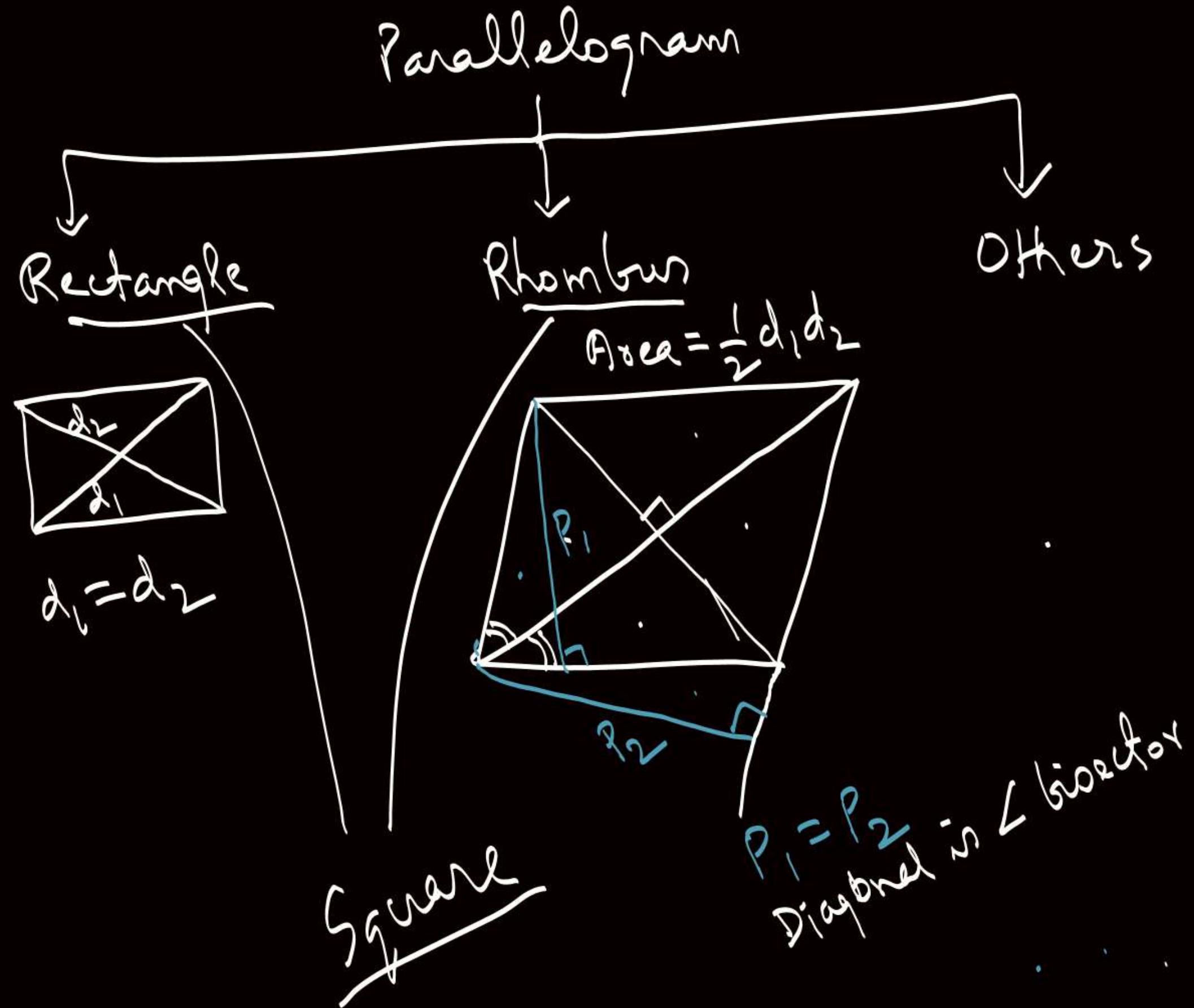
↓  
Trapezium

↓  
Inscribed  
Others

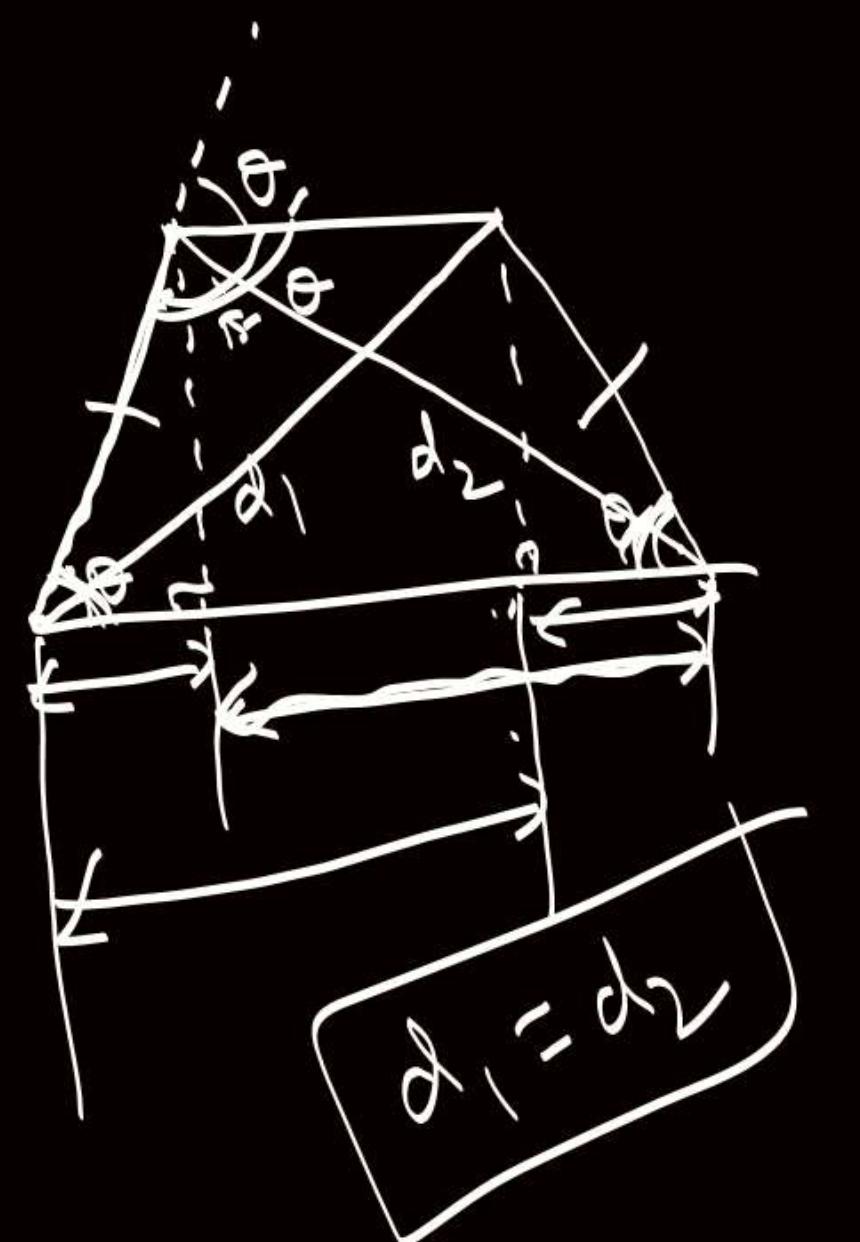
↓  
Cyclic Quadrilateral

↓  
Others

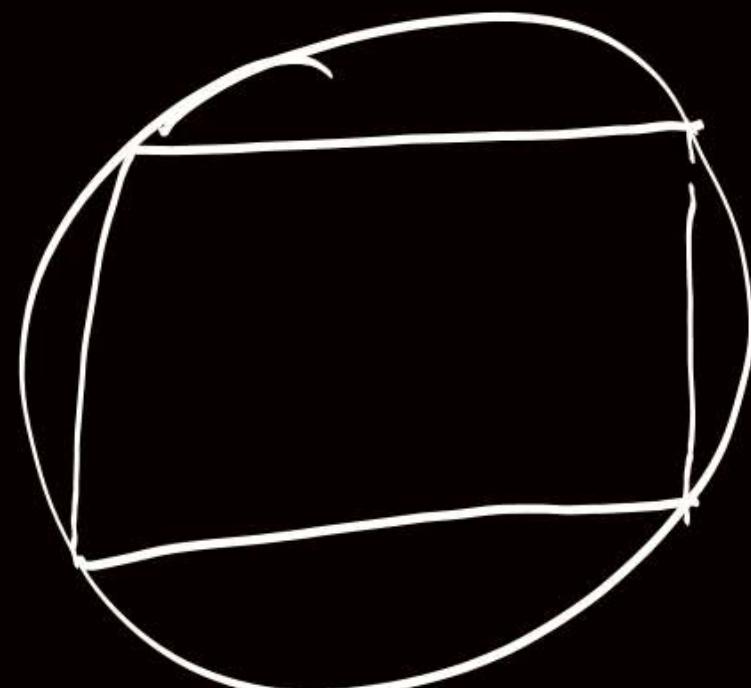
$$\begin{aligned} \text{Area} &= 2 \times \frac{1}{2} \times P_1 (AB) \\ &= 2 \times \frac{1}{2} P_1 \frac{P_2}{\sin \theta} \end{aligned}$$



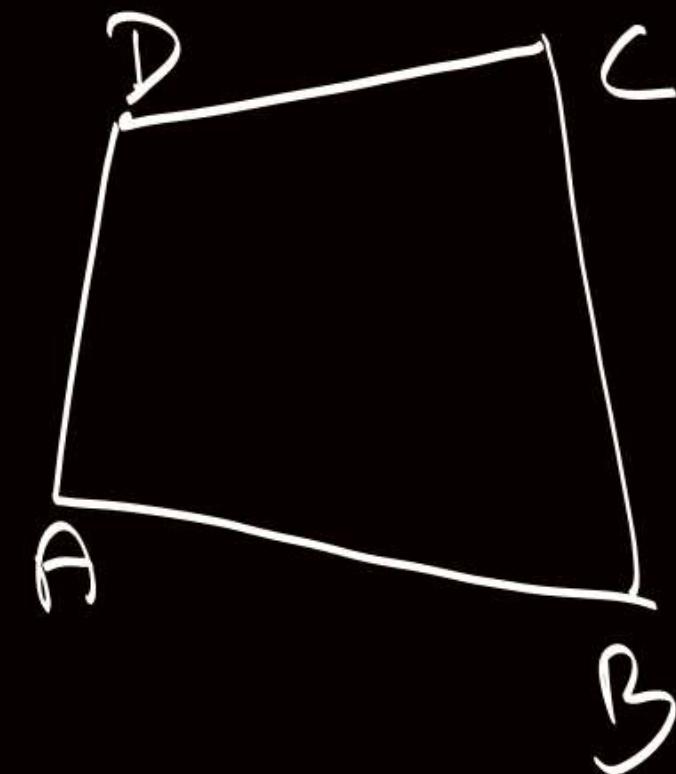
Incongruent Trapezium



Cyclic Quadrilateral



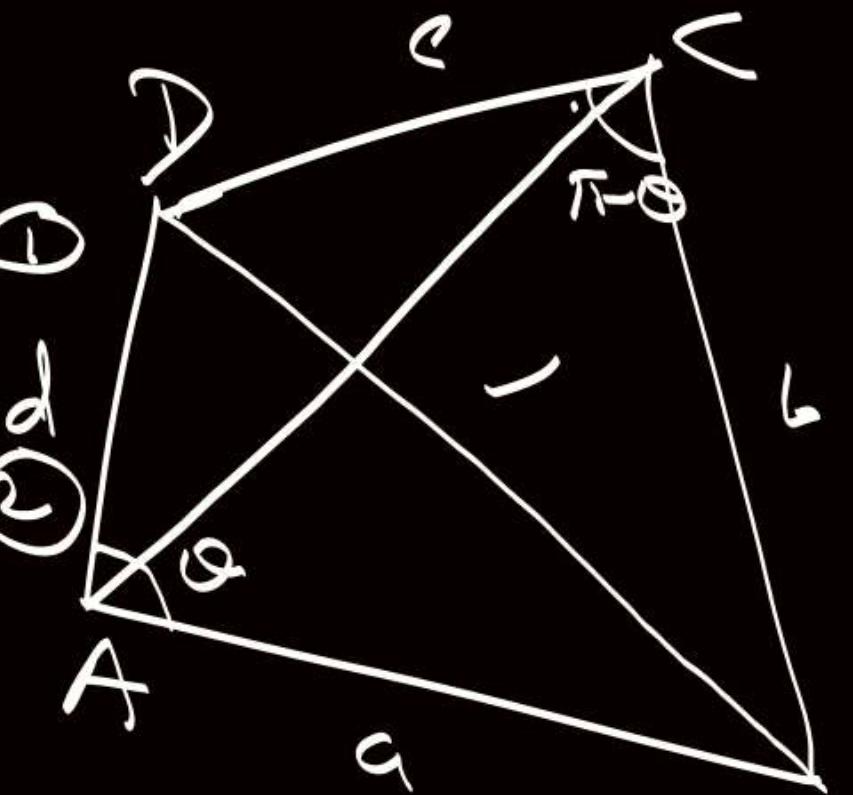
# Cyclic Quadrilateral



$$A + C = \pi = B + D$$

Ptolemy's Theorem

$$(AB)(CD) + (BC)(AD) = (AC)(BD)$$



$$BD^2 = a^2 + d^2 - 2ad \cos \theta \quad \textcircled{1}$$

$$BD^2 = b^2 + c^2 + 2bc \cos \theta \quad \textcircled{2}$$

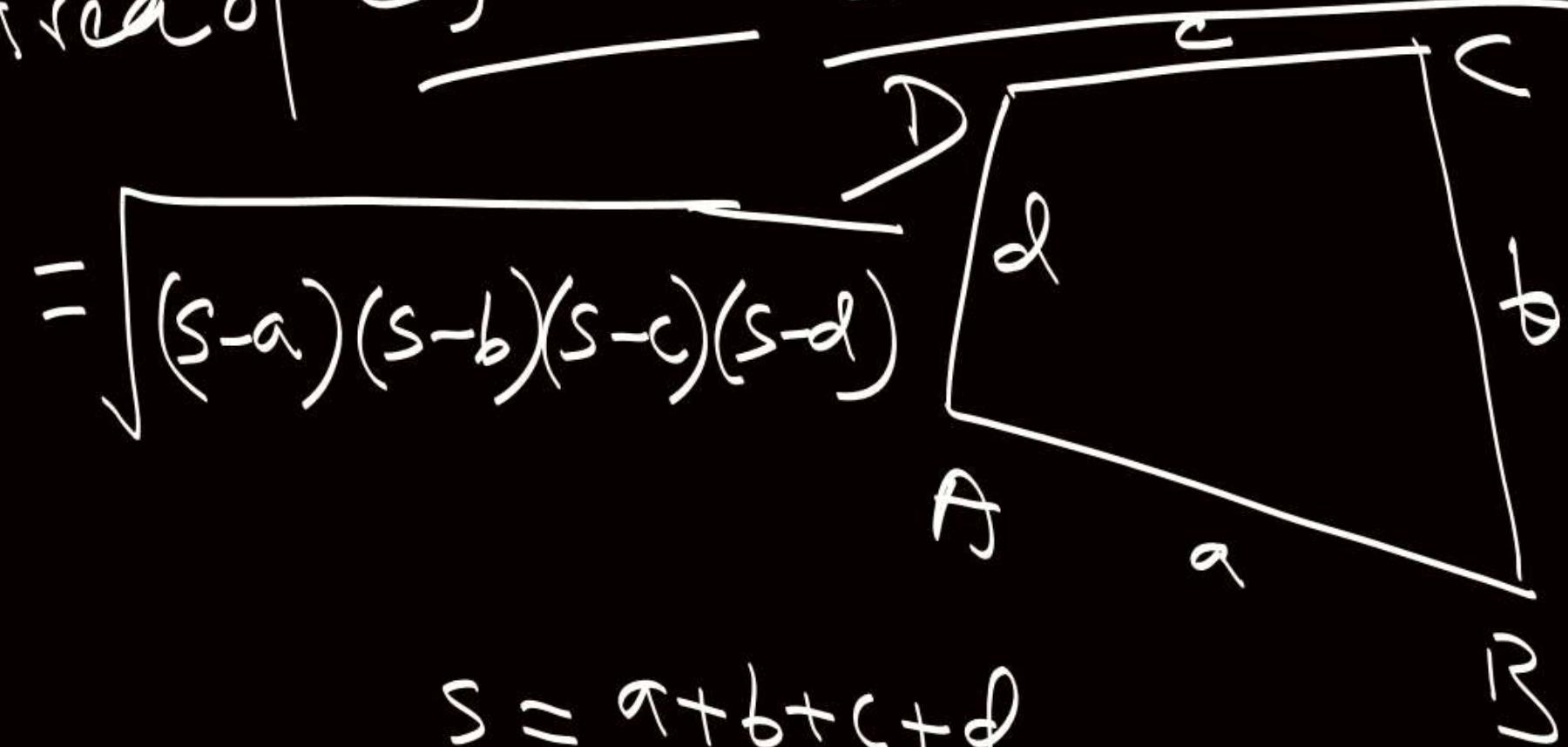
$$\textcircled{1} \times bc + \textcircled{2} \times ad$$

$$(bc + ad) BD^2 = (a^2 + d^2) bc + (b^2 + c^2) ad$$

$$BD = \sqrt{\frac{(ab+cd)(ac+bd)}{(bc+ad)}}$$

$$AC = \sqrt{\frac{(bc+ad)(ac+bd)}{(ab+cd)}}$$

Area of Cyclic Quadrilateral



$$= \sqrt{(s-a)(s-b)(s-c)(s-d)}$$

$$s = \frac{a+b+c+d}{2}$$

Determinant

$\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$
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