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1.

$$y \sin x \frac{dy}{dx} = \cos x (\sin x - y^2)$$

$$y^2 = t \Rightarrow 2y\frac{dy}{dx} = \frac{dt}{dx}$$

$$\sin x\left(2y\frac{dy}{dx}\right) = 2\sin x\cos x - 2\cos x\, y^2$$

$$\frac{dt}{dx} + (2\cot x)t = 2\cos x$$

$$I.F = \sin^2 x$$

$$t\sin^2 x = \int 2\sin^2 x\cos x\,dx = \frac{2}{3}\sin^3 x + C$$

$$y^2\sin^2 x = \frac{2}{3}\sin^3 x + C$$

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2. $e^y \frac{dy}{dx} = e^x \left(e^x - e^y\right)$.

$e^y = t$

$\frac{dt}{dx} + e^x t = e^{2x}$

$I.F = e^{e^x}$ $\Rightarrow t e^{e^x} = \int e^{e^x} e^x e^x \, dx = \int e^u u \, du$

$e^x = u$

$= e^u (u-1) + C$

$$\boxed{e^y e^{e^x} = e^{e^x}(e^x - 1) + C}$$

3. $x \frac{dy}{dx} + y \ln y = x y e^x$

$\frac{x}{y} \frac{dy}{dx} + \ln y = x e^x$

$\ln y = t$

$\frac{dt}{dx} + \frac{t}{x} = e^x$

$I.F = x$ $\Rightarrow t x = \int x e^x \, dx = (x-1) e^x + C$

$x \ln y = (x-1) e^x + C$.

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4. $$\left(\frac{y+\sin x\cos^2(xy)}{\cos^2(xy)}\right)dx+\frac{x\,dy}{\cos^2(xy)}+\sin y\,dy=0.$$

$$\frac{y\,dx+x\,dy}{\cos^2(xy)}+\sin x\,dx+\sin y\,dy=0.$$

$$\int \sec^2(xy)\,d(xy)+\sin x\,dx+\sin y\,dy=0$$

$$\Rightarrow \tan(xy)-\cos x-\cos y=C$$

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5. $\left(\frac{1}{y}\sin\frac{x}{y} - \frac{y}{x^2}\cos\frac{y}{x} + 1\right)dx + \left(\frac{1}{x}\cos\frac{y}{x} - \frac{x}{y^2}\sin\frac{x}{y} + \frac{1}{y^2}\right)dy = 0$.

$$\underbrace{\left(\frac{y\,dx - x\,dy}{y^2}\right)}_{d\left(\frac{x}{y}\right)}\sin\frac{x}{y} + \underbrace{\left(\frac{x\,dy - y\,dx}{x^2}\right)}_{d\left(\frac{y}{x}\right)}\cos\frac{y}{x} + dx + \frac{dy}{y^2} = 0$$

$$-\cos\frac{x}{y} + \sin\frac{y}{x} + x - \frac{1}{y} = C$$

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6. $$\left(\frac{1}{x}-\frac{y^2}{(x-y)^2}\right)dx+\left(\frac{x^2}{(x-y)^2}-\frac{1}{y}\right)dy=0$$

$$\frac{dx}{x}-\frac{dy}{y}+\frac{x^2dy-y^2dx}{(x-y)^2}=0$$

$$+\frac{\frac{dy}{y^2}-\frac{dx}{x^2}}{\left(-\frac{1}{y}+\frac{1}{x}\right)^2}$$

$$\ln x-\ln y-\frac{1}{\left(\frac{1}{x}-\frac{1}{y}\right)}=C$$

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DE of form

$$y\,f(xy)\,dx + x\,g(xy)\,dy = 0$$

Put $xy = t$

$$x\,dy + y\,dx = dt$$

$$\frac{t}{x} f(t)\,dx + \left(dt - \frac{t}{x}dx\right) g(t) = 0$$

$$\frac{t}{x}\Big(f(t) - g(t)\Big)dx + g(t)\,dt = 0$$

$$\int \frac{g(t)\,dt}{t\big(g(t) - f(t)\big)} = \int \frac{dx}{x}$$

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7. $(x^3y^3 + x^2y^2 + xy + 1)\,y\,dx + (x^3y^3 - x^2y^2 - xy + 1)\,x\,dy = 0$.

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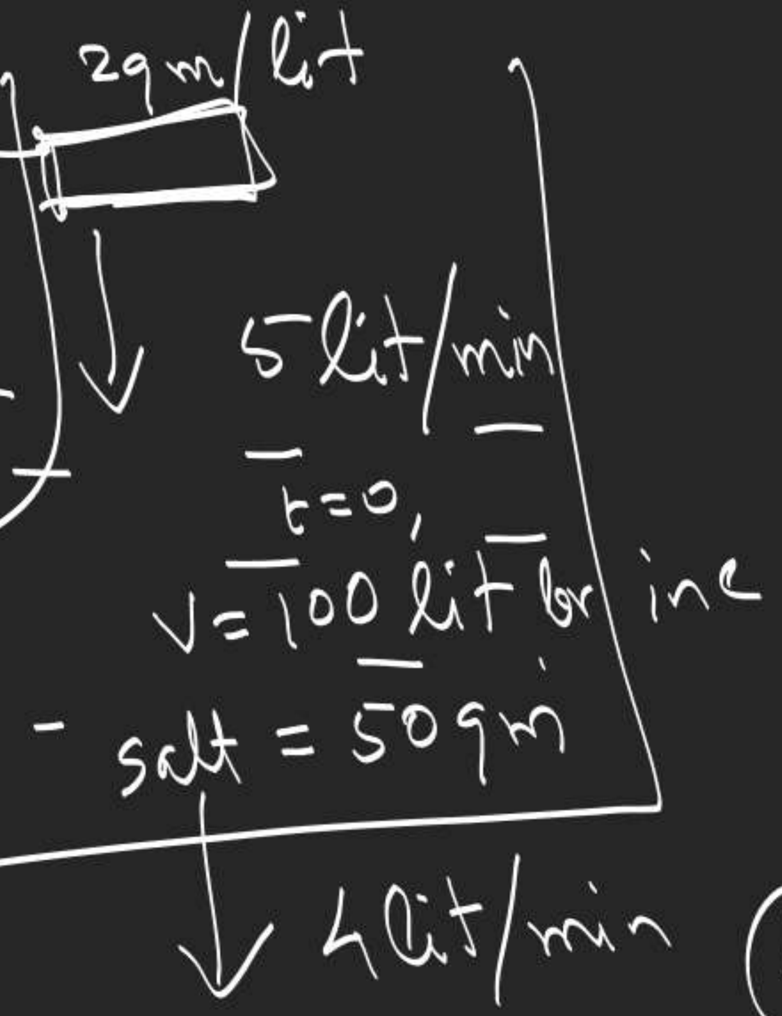
8.

$$\boxed{y = 2(100+t) - \frac{150\times(100)^4}{(100+t)^4}}$$



$t=t.$

$V=(100+t)$ lit

salt $= y$ gm

① 10 gm/min

② $(100+t)$ lit

③ $\frac{4y}{100+t}$ gm/min

④ $\frac{dy}{dt} = \frac{dy_{in}}{dt} - \frac{dy_{out}}{dt}$

$$= 10 - \frac{4y}{100+t}$$

$\frac{dy}{dt}$

$$I.F = (100+t)^4$$

$$y(100+t)^4 = 2(100+t)^5 + c$$

$t=0,\ y=50 \Rightarrow\ 50\times 100^4 = 200(100)^4 + c$