

**KEY CONCEPTS****THINGS TO REMEMBER :****1. LOGARITHM OF A NUMBER :**

The logarithm of the number  $N$  to the base '  $a$  ' is the exponent indicating the power to which the base '  $a$ ' must be raised to obtain the number  $N$ . This number is designated as  $\log_a N$ .

Hence:  $\log_a N = x \Leftrightarrow a^x = N, a > 0, a \neq 1 \& N > 0$

If  $a = 10$ , then we write  $\log_b$  rather than  $\log_{10}b$ .

If  $a = e$ , we write  $\ln b$  rather than  $\log_e b$ .

The existence and uniqueness of the number  $\log_a N$  follows from the properties of an exponential functions.

From the definition of the logarithm of the number  $N$  to the base '  $a$  ', we have an

identity :  $a^{\log_a N} = N, a > 0, a \neq 1 \& N > 0$

This is known as the **Fundamental Logarithmic Identity**.

**Note :**

$$\log_a 1 = 0 \quad (a > 0, a \neq 1)$$

$$\log_a a = 1 \quad (a > 0, a \neq 1) \text{ and}$$

$$\log_{1/a} a = -1 \quad (a > 0, a \neq 1)$$

**2. THE PRINCIPAL PROPERTIES OF LOGARITHMS :**

Let  $M$  &  $N$  are arbitrary positive numbers,  $a > 0, a \neq 1, b > 0, b \neq 1$  and  $\alpha$  is any real number then ;

$$(i) \log_a (M \cdot N) = \log_a M + \log_a N \quad (ii) \log_a (M/N) = \log_a M - \log_a N$$

$$(iii) \log_a M^\alpha = \alpha \cdot \log_a M \quad (iv) \log_b M = \frac{\log_a M}{\log_a b}$$

$$\text{Note: } \log_b a \cdot \log_a b = 1 \Leftrightarrow \log_b a = 1/\log_a b. \quad \log_b a \cdot \log_c b \cdot \log_a c = 1$$

$$\log_y x \cdot \log_z y \cdot \log_a z = \log_a x.$$

$$e^{\ln a^x} = a^x$$

**3. PROPERTIES OF MONOTONOCITY OF LOGARITHM :**

- (i)** For  $a > 1$  the inequality  $0 < x < y \& \log_a x < \log_a y$  are equivalent.
- (ii)** For  $0 < a < 1$  the inequality  $0 < x < y \& \log_a x > \log_a y$  are equivalent.
- (iii)** If  $a > 1$  then  $\log_a x < p \Rightarrow 0 < x < a^p$
- (iv)** If  $a > 1$  then  $\log_a x > p \Rightarrow x > a^p$
- (v)** If  $0 < a < 1$  then  $\log_a x < p \Rightarrow x > a^p$
- (vi)** If  $0 < a < 1$  then  $\log_a x > p \Rightarrow 0 < x < a^p$

**➤ NOTE THAT :**

If the number & the base are on one side of the unity, then the logarithm is positive ; If the number & the base are on different sides of unity, then the logarithm is negative.

- The base of the logarithm ' a ' must not equal unity otherwise numbers not equal to unity will not have a logarithm & any number will be the logarithm of unity.
- For a non negative number ' a ' &  $n \geq 2$ ,  $n \in N$      $\sqrt[n]{a} = a^{1/n}$ .
- **Will be covered in detail in quadratic equation**





## **PROFICIENCY TEST-01**

**PROFICIENCY TEST-02**

1. If  $\log_5 a \cdot \log_a x = 2$ , then x is equal to:
2.  $f(x) = \sqrt{\log_{10} x^2}$ . The set of all values of x for which f(x) is real is :
3. Value of  $x^{\log_x a} \cdot \log_a y \cdot \log_y z$  :
4. If  $\log_2 x + \log_2 y \geq 6$ , then the least value of x + y is :
5. A rational number which is 50 times its own logarithm to the base 10 is :
6. If  $\log_2 = 0.30103$  the no. of digits in  $2^{64}$  is :
7. The number of zeroes coming immediately after the decimal point in the value of  $(0.2)^{25}$  is :  
(Given  $\log_{10} 2: 0.30103$ )
8. Value of  $6 \frac{a^{\log_e b} (\log_{a^2} b)(\log_{b^2} a)}{e^{\log_e a \cdot \log_e b}}$
9. The solution set of  $\log_2 |4 - 5x| > 2$  is :
10. If  $\frac{1}{2} \leq \log_{0.1} x \leq 2$  then 'x' belongs to

**EXERCISE-I**

- 1.** Let A denotes the value of  $\log_{10} \left( \frac{ab + \sqrt{(ab)^2 - 4(a+b)}}{2} \right) + \log_{10} \left( \frac{ab - \sqrt{(ab)^2 - 4(a+b)}}{2} \right)$  when  $a = 43$  and  $b = 57$

and B denotes the value of the expression  $(2^{\log_6 18}) \cdot (3^{\log_6 3})$ .

Find the value of  $(A \cdot B)$ .

- 2.** (a) If  $x = \log_3 4$  and  $y = \log_5 3$ , find the value of  $\log_3 10$  and  $\log_3(1.2)$  in terms of x and y.  
 (b) If  $k^{\log_2 5} = 16$ , find the value of  $k^{(\log_2 5)^2}$ .

**Solve for x(Q. 3 to Q.5):**

- 3.** (a) If  $\log_{10} (x^2 - 12x + 36) = 2$

(b)  $9^{1+\log x} - 3^{1+\log x} - 210 = 0$ ; where base of log is 3.

- 4.** Simplify: (a)  $\log_{1/3} \sqrt[4]{729} \cdot \sqrt[3]{9^{-1} \cdot 27^{-4/3}}$ ; (b)  $a^{\frac{\log_b (\log_b N)}{\log_b a}}$

- 5.** (a) If  $\log_4 \log_3 \log_2 x = 0$ ;

(b) If  $\log_e \log_5 [\sqrt{2x-2} + 3] = 0$

- 6.** (a) Which is smaller? 2 or  $(\log_\pi 2 + \log_2 \pi)$ .

(b) Prove that  $\log_3 5$  and  $\log_2 7$  are both irrational.

- 7.** Let a and b be real numbers greater than 1 for which there exists a positive real number c, different from 1, such that

$2(\log_a c + \log_b c) = 9\log_{ab} c$ . Find the largest possible value of  $\log_a b$ .

- 8.** Find the square of the sum of the roots of the equation

$$\log_3 x \cdot \log_4 x \cdot \log_5 x = \log_3 x \cdot \log_4 x + \log_4 x \cdot \log_5 x + \log_5 x \cdot \log_3 x$$

- 9.** Find the value of the expression  $\frac{2}{\log_4(2000)^6} + \frac{3}{\log_5(2000)^6}$ .

- 10.** Calculate :  $4^{5\log_{4\sqrt{2}}(3-\sqrt{6})-6\log_8(\sqrt{3}-\sqrt{2})}$

- 11.** Simplify :  $\frac{81^{\frac{1}{\log_5 9}+3^{\frac{3}{\log_{\sqrt{6}} 3}}}}{409} \cdot \left( (\sqrt{7})^{\frac{2}{\log_{25} 7}} - (125)^{\log_{25} 6} \right)$

- 12.** Simplify:  $5^{\log_{1/5}(\frac{1}{2})} + \log_{\sqrt{2}} \frac{4}{\sqrt{7}+\sqrt{3}} + \log_{1/2} \frac{1}{10+2\sqrt{21}}$ .

- 13.** Find 'x' satisfying the equation  $4^{\log_{10} x+1} - 6^{\log_{10} x} - 2 \cdot 3^{\log_{10} x^2+2} = 0$ .

- 14.** Given that  $\log_2 a = s$ ,  $\log_4 b = s^2$  and  $\log_{c^2}(8) = \frac{2}{s^3+1}$ . Write  $\log_2 \frac{a^2 b^5}{c^4}$  as a function of 's'  
 ( $a, b, c > 0, c \neq 1$ )

- 15.** Find the value of  $49^{(1-\log_7 2)} + 5^{-\log_5 4}$ .

- 16.** Given that  $\log_2 3 = a$ ,  $\log_3 5 = b$ ,  $\log_7 2 = c$ , express the logarithm of the number 63 to the base 140 in terms of a, b &



17. Prove that  $\frac{\log_2 24}{\log_{96} 2} - \frac{\log_2 192}{\log_{12} 2} = 3$ .
18. Prove that  $a^x - b^y = 0$  where  $x = \sqrt{\log_a b}$  &  $y = \sqrt{\log_b a}$ ,  $a > 0, b > 0$  &  $a, b \neq 1$ .
19. Prove the identity :  $\log_a N \cdot \log_b N + \log_b N \cdot \log_c N + \log_c N \cdot \log_a N = \frac{\log_a N \cdot \log_b N \cdot \log_c N}{\log_{abc} N}$
20. (a) Solve for  $x$ ,  $\frac{\log_{10}(x-3)}{\log_{10}(x^2-21)} = \frac{1}{2}$   
 (b)  $\log(\log x) + \log(\log x^3 - 2) = 0$ ; where base of log is 10 everywhere.  
 (c)  $\log_x 2 \cdot \log_{2x} 2 = \log_{4x} 2$   
 (d)  $5^{\log x} + 5^{x \log 5} = 3$  ( $a > 0$ ); where base of log is  $a$ .
21. If  $x, y > 0$ ,  $\log_y x + \log_x y = \frac{10}{3}$  and  $xy = 144$ , then  $\frac{x+y}{2} = \sqrt{N}$  where  $N$  is a natural number, find the value of  $N$ .
22. Solve the system of equations:  
 $\log_a x \cdot \log_a (xyz) = 48$   
 $\log_a y \cdot \log_a (xyz) = 12$ ,  $a > 0, a \neq 1$   
 $\log_a z \cdot \log_a (xyz) = 84$
23. (a) Given:  $\log_{10} 34.56 = 1.5386$ , find  $\log_{10} 3.456$ ;  $\log_{10} 0.3456$  &  $\log_{10} 0.003456$ .  
 (b) Find the number of positive integers which have the characteristic 3, when the base of the logarithm is 7.  
 (c) If  $\log_{10} 2 = 0.3010$  &  $\log_{10} 3 = 0.4771$ , find the value of  $\log_{10} (2.25)$ .  
 (d) Find the antilogarithm of 0.75, if the base of the logarithm is 2401.
24. If  $\log_{10} 2 = 0.3010$ ,  $\log_{10} 3 = 0.4771$ . Find the number of integers in :  
 (a)  $5^{200}$       (b)  $6^{15}$       &      (c) the number of zeros after the decimal in  $3^{-100}$ .
25. Let 'L' denotes the antilog of 0.4 to the base 1024 .  
 and ' M ' denotes the number of digits in  $6^{10}$  (Given  $\log_{10} 2 = 0.3010$ ,  $\log_{10} 3 = 0.4771$ )  
 and ' N ' denotes the number of positive integers which have the characteristic 2, when base of the logarithm is 6.  
 Find the value of LMN.

**EXERCISE-II**

**Note :** From Q. 1 to Q. 8 , solve the equation for x :

1.  $\log_a(x) = x$  where  $a = x^{\log_4 x}$ .
2.  $x^{\log x + 4} = 32$ , where base of logarithm is 2.
3.  $\log_{x+1}(x^2 + x - 6)^2 = 4$
4.  $x + \log_{10}(1 + 2^x) = x \cdot \log_{10} 5 + \log_{10} 6$ .
5.  $5^{\log x} - 3^{\log x - 1} = 3^{\log x + 1} - 5^{\log x - 1}$ , where the base of logarithm is 10.

6.  $\frac{1+\log_2(x-4)}{\log_{\sqrt{2}}(\sqrt{x+3}-\sqrt{x-3})} = 1$
7.  $\log_5 120 + (x-3) - 2 \cdot \log_5(1 - 5^{x-3}) = -\log_5(0.2 - 5^{x-4})$

8.  $\log 4 + \left(1 + \frac{1}{2x}\right) \log 3 = \log(\sqrt[3]{3} + 27)$ .
9. The real x and y satisfy  $\log_8 x + \log_4 y^2 = 5$  and  $\log_8 y + \log_4 x^2 = 7$ , find xy.
10. Find the real solutions to the system of equations

$$\begin{aligned} & \log_{10}(2000xy) - \log_{10}x \cdot \log_{10}y = 4 \\ \text{and } & \log_{10}(2yz) - \log_{10}y \cdot \log_{10}z = 1 \\ & \log_{10}(zx) - \log_{10}z \cdot \log_{10}x = 0 \end{aligned}$$

11. If  $x = 1 + \log_a bc$ ,  $y = 1 + \log_b ca$ ,  $z = 1 + \log_c ab$ , then prove that  $xyz = xy + yz + zx$ .
12. If  $p = \log_a bc$ ,  $q = \log_b ca$ ,  $r = \log_c ab$ , then prove that  $pqr = p + q + r + 2$ .
13. If  $\log_b a \cdot \log_c a + \log_a b \cdot \log_c b + \log_a c \cdot \log_b c = 3$  (Where a, b, c are different positive real numbers  $\neq 1$  ), then find the value of abc.
14. Let  $y = \sqrt{\log_2 3 \cdot \log_2 12 \cdot \log_2 48 \cdot \log_2 192 + 16} - \log_2 12 \cdot \log_2 48 + 10$ . Find  $y \in \mathbb{N}$ .
15. If  $\frac{\log_a N}{\log_c N} = \frac{\log_a N - \log_b N}{\log_b N - \log_c N}$  where  $N > 0 \& N \neq 1$ ,  $a, b, c > 0 \&$  not equal to 1 , then prove that  $b^2 = ac$ .
16. Solve the equation  $\frac{3}{2} \log_4(x+2)^2 + 3 = \log_4(4-x)^3 + \log_4(6+x)^3$ .
17. If  $(x_1, y_1)$  and  $(x_2, y_2)$  are the solution of the system of equation  
 $\log_{225}(x) + \log_{64}(y) = 4$   
 $\log_x(225) - \log_y(64) = 1$   
then show that the value of  $\log_{30}(x_1 y_1 x_2 y_2) = 12$ .
18. Find x satisfying the equation  $\log^2\left(1 + \frac{4}{x}\right) + \log^2\left(1 - \frac{4}{x+4}\right) = 2\log^2\left(\frac{2}{x-1} - 1\right)$ .
19. Solve for x :  $\log^2(4-x) + \log(4-x) \cdot \log\left(x + \frac{1}{2}\right) - 2\log^2\left(x + \frac{1}{2}\right) = 0$
20. If  $\log_{3x} 45 = \log_{4x} 40\sqrt{3}$  then find the characteristic of  $x^3$  to the base 7 .





**ANSWER KEY****PROFICIENCY TEST-01**

1.  $x > y$    2. 10   3. 7   4.  $x = \frac{484}{49}$    5.  $x = 9$    6.  $\log 2$   
 7. B   8. 0.954   9. B   10. 2

**PROFICIENCY TEST-02**

1.  $x = 25$    2.  $(-\infty, -1] \cup [1, \infty)$    3. z   4. 16   5. 100  
 6. 20   7. 17   8.  $\frac{3}{2}$   
 9.  $(-\infty, 0) \cup (8/5, \infty)$    10.  $\left[\frac{1}{100}, \frac{1}{\sqrt{10}}\right]$

**EXERCISE-I**

1. 12   2. (a)  $\frac{xy+2}{2y}, \frac{xy+2y-2}{2y}$ ; (b) 625   3. (a)  $x = 16$  or  $x = -4$  (b)  $x = 5$   
 4. (a) -1 (b)  $\log_b N$    5. (a) 8 (b)  $x = 3$    6. (a) 2   7. 2  
 8. 3721   9. 1/6   10. 9   11. 1  
 12. 6   13.  $x = \frac{1}{100}$    14.  $2s + 10s^2 - 3(s^3 + 1)$   
 15.  $\frac{25}{2}$    16.  $\frac{1+2ac}{2c+abc+1}$   
 20. (a)  $x = 5$  (b)  $x = 10$  (c)  $x = 2^{\sqrt{2}}$  or  $2^{-\sqrt{2}}$  (d)  $x = 2^{-\log_a}$  where base of log is 5. 21. 507  
 22.  $(a^4, a, a^7)$  or  $\left(\frac{1}{a^4}, \frac{1}{a}, \frac{1}{a^7}\right)$    23. (a) 0.5386; 1.5386; 3.5386 (b) 2058 (c) 0.3522 (d) 343  
 24. (a) 140 (b) 12 (c) 47   25. 23040

**EXERCISE-II**

1.  $x = 2$    2.  $x = 2$  or  $\frac{1}{32}$    3.  $x = 1$    4.  $x = 1$   
 5.  $x = 100$    6.  $x = 5$    7.  $x = 1$    8.  $x \in \emptyset$   
 9.  $xy = 2^9$    10.  $x = 1, y = 5, z = 1$  or  $x = 100, y = 20, z = 100$   
 13.  $abc = 1$    14.  $y = 6$    16.  $x = 2$  or  $1 - \sqrt{33}$   
 18.  $x = \sqrt{2}$  or  $\sqrt{6}$    19.  $\left\{0, \frac{7}{4}, \frac{3+\sqrt{24}}{2}\right\}$    20. 2

**EXERCISE-III**

1. C   2. D   3. C   4. D   5. B   6. B   7. B  
 8. D   9. B   10. C   11. D   12. B

**EXERCISE-IV**

1.  $\{-10, 20\}, \{10/3, 20/3\}$    2.  $x = 8$    3.  $x = 3$  or  $-3$    4. B   5. C  
 6. 4   7. 8.00   8. 4.00