Nishant Jindal

Complex No.

(1) $Z = x + iy$; $i = \sqrt{-1}$

$Z = (x, y) \rightarrow x, y \in R.$

(2) $i = \sqrt{-1}$

$i^2 = -1$

$i^3 = i^2 \times i = -i$

$i^4 = i^2 \times i^2 = 1$

Repeatation Start

$i^5 = i^4 \times i = i$

$i^6 = i^4 \times i^2 = i^2 = -1$

$i^7 = i^4 \times i^3 = i^3 = -i$

$i^{98} = (i^4)^{24} \times i^2 = i^2 = -1$

$i^{1026} = (i^4)^{256} \cdot i^2 = i^2 = -1$

$i^{57} = (i^4)^{14} \cdot i^1 = i$

$$\boxed{\begin{aligned} i^{4K} &= 1 \\ i^{4K+1} &= i \\ i^{4K+2} &= i^2 \\ i^{4K+3} &= i^3 \end{aligned}}$$

$i^{4K+4} = (i^4)^{K+1} = 1$

* Sum of any 4 deg of Iota gives Zero

$i^3 + i^4 + i^5 + i^6 = 0$

(3) $\frac{1}{i} = \frac{1}{i} \times \frac{-i}{-i} = \frac{-i}{-i^2} = \frac{-i}{+(1)} = -i$

Q2 $\frac{1}{1+i} = ?$

$\frac{1}{1+i} \times \frac{1-i}{1-i} = \frac{1-i}{1^2 - i^2} = \frac{1-i}{1-(-1)} = \frac{1-i}{2}$

Q(3) $\frac{1}{3-4i} = ?$

$\frac{1}{3-4i} \times \frac{3+4i}{3+4i} = \frac{3+4i}{3^2 - (4i)^2} = \frac{3+4i}{9 - 16(-1)}$

$= \frac{3+4i}{25}$

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4) $Re(z)$ & $Im(z)$

$Z = x + iy.$

then $Re(z) = x$

& $Im(z) = y$

Q 4 $Re\left(\frac{1+i}{1-i}\right) = ?$

$$\frac{1+i}{1-i} \times \frac{1+i}{1+i} = \frac{(1+i)^2}{1^2-(i)^2}$$

$$= \frac{1^2+i^2+2\times1\times i}{1-(-1)}$$

$$= \frac{\cancel{1}-\cancel{1}+2i}{2} = i$$

$Re\left(\frac{1+i}{1-i}\right) = Re(\ i)$

$= 0$ (Noniota कुछ है ही नहीं)

*1) $\frac{1}{i} = -i$

2) $\frac{1+i}{1-i} = i$

3) $\frac{1-i}{1+i} = -i$

(4) $(1+i)^2 = 2i$

(5) $(1-i)^2 = -2i$

Q 5 if $\left(\frac{1+i}{1-i}\right)^m = 1$ then Mim value of m?

$m \in N$

$(i)^m = 1$

$m = 4$ as we know $i^4 = 1$

Q 6 $Z = 5-3i$ find $Re(z)$ & $Im(z)$

$Re(z) = 5$

$Im(z) = -3$

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(5) Purely Real / Imaginary

$Z = x + iy$

$x = 0$	$y = 0$	$x = y = 0$
$Z = iy$	$Z = x + 0 \cdot i$	$Z = 0 + 0i$
Purely Imaginary	$= x$	Purely Real & Purely Img.
When $Re(z) = 0$	Purely Real	
	$Im(Z) = 0$	

Q 7 $1 + \sqrt{-2}$ is Purely Real / Imag.

No. as $x = 1$, $y = \sqrt{2}$

$Z = 1 + \sqrt{2}i$

Q 8 $Z = 1 + \sqrt{2}$ is Purely Real / Img

i wali Part is missing it is Purely Real.

$Z = (1 + \sqrt{2}) + 0 \cdot i$

Q 9 $Z = -5i$ Purely Real / Img?

$Z = 0 - 5i$

$x = 0$ ∴ it Purely Img.

Q 10 $x^2 + 16 = 0$ then $x = ?$

$x^2 = -16$

$x = \pm\sqrt{-16} = \pm\sqrt{16}\sqrt{-1}$

$x = \pm 4i$

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Q Real Part of $(1+i)^{50}$?

11 $\left((1+i)^2\right)^{25} = (2i)^{25}$

$= 2^{25}(i^4)^6 \cdot i$

$= 2^{25} \cdot i$

$Re(Z) = 0$

Q If $z+z^2=0$ then

12 A) $Re(z)<0$ ×

B) $Re(z)>0$ ×

C) $Re(z)=0$ ×

D) $Im(z)=0$ ✓

$z+z^2=0$

$z(z+1)=0$

$z=0$ & $z=-1$

$z=0+0i$ & $z=-1+0i$

$Im(z)=0$ in Both Cases

Q If it were like $z+z^3=0$

13 $z(1+z^2)=0$

$z=0$ or $z^2=-1$

$z=\pm i$

$z=0+0i$ $z=0+i$ / $0-i$ $Re(z)=0$

Q Find No. of Integral values of

14 n for which $(n+i)^4$ is an Integer?

① $(n+i)^4$

$= {}^4C_0 \cdot n^4 + {}^4C_1 n^3 i + {}^4C_2 \cdot n^2 i^2 + {}^4C_3 n \cdot i^3 + {}^4C_4 i^4$

$= n^4 + 4n^3 i - 6n^2 - 4ni + 1$

$= (n^4-6n^2+1) + i(4n^3-4n)$ is Integer.

(2) Int. ਮੇਂ Iota hota hi nahi

$\Rightarrow 4n^3-4n=0$

$4n(n^2-1)=0$

$n=0, n=1, n=-1$

Ans = 3 Integral values of n.

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Q $1+z+z^2+z^3+\dots+z^{17}=0$

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GP of 18 term.

& $1+z+z^2+z^3+\dots+z^{13}=0$

find z?

$$\frac{z^{18}-1}{z-1}=0 \quad \& \quad \frac{z^{14}-1}{z-1}=0$$

$$z^{18}=1 \quad \& \quad z^{14}=1$$

$$z=\pm 1$$

$z=1$ & $z=-1$

✗ ✓

Q find $f(3+2i)$

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if $f(x)=x^4-4x^3+4x^2+10x+45$

$$f(x)=\cancel{(x^2-6x+13)}(x^2+2x+3)+2x+6$$

$$f(x)=0+2x+6$$

$$f(3+2i)=2(3+2i)+6$$

$$=4i+12$$

$$x=3+2i$$

$$x-3=2i$$

$$(x-3)^2=-4$$

$$x^2-6x+9=-4$$

$$x^2-6x+13=0$$

$x^2-6x+13\,)\,x^4-4x^3+4x^2+10x+45\,(\,x^2+2x+3$

$$\begin{array}{l}
\underline{x^4-6x^3+13x^2} \\
2x^3-9x^2+10x \\
\underline{2x^3-12x^2+26x} \\
3x^2-16x+45 \\
\underline{3x^2-18x+39} \\
2x+6
\end{array}$$

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Q If $|\text{Re}\, z| + |\text{Im}\, z| = 1$ then
19
Locus of z?

If $z = x+iy$

$$|x| + |y| = 1$$

$x+y=1,\ x-y=1$

$-x+y=1,\ -x-y=1$

Square

$$\frac{\frac{(2m)\cancel{(2m+1)}}{2}}{\cancel{(2m+1)}}$$

$$x^3-6x^2+11x-6=(x-1)(x-2)(x-3)$$

$$\sum \frac{1}{z_r-1} = \frac{1}{z_1-1}+\frac{1}{z_2-1}+\frac{1}{z_3-1}\cdots\frac{1}{z_{2m}-1}$$

Q Sq^r Root of
20

$$x^2+\frac{1}{x^2}-\frac{4}{i}\left(x-\frac{1}{x}\right)-6$$

$x \in R$ is

$$\pm\sqrt{x^2+\frac{1}{x^2}+4i\left(x-\frac{1}{x}\right)-6}$$

$$\pm\sqrt{\left(x-\frac{1}{x}\right)^2+4i\left(x-\frac{1}{x}\right)+(2i)^2}$$

$$\pm\sqrt{\left(\left(x-\frac{1}{x}\right)+2i\right)^2}$$

$$\pm\left\{\left(x-\frac{1}{x}\right)+2i\right\}$$

diff^te

Base Idea $x^3-6x^2+11x-6=0$
has Roots 1, 2, 3

Q If z_r, $r=1,2,3\ldots 2m$, $m \in N$
21
are Roots of Eq^n

$z_1, z_2, z_3 \ldots z_{2m}$ Roots

$$z^{2m}+z^{2m-1}+z^{2m-2}\ldots+z+1=0$$

then find $\boxed{\sum_{r=1}^{2m}\frac{1}{z_r-1}} = ?$ $-m$

log.

$$z^{2m}+z^{2m-1}+z^{2m-2}+\ldots+z^2+z+1=(z-z_1)(z-z_2)(z-z_3)\ldots(z-z_{2m})$$

$$\log\{z^{2m}+z^{2m-1}+z^{2m-2}\ldots z^2+z+1\}=\log(z-z_1)+\log(z-z_2)\ldots\log(z-z_{2m})$$

$$\Rightarrow \left\{\frac{2m z^{2m-1}+(2m-1)z^{2m-2}+(2m-2)z^{2m-3}+\ldots 2z+1}{z^{2m}+z^{2m-1}+z^{2m-2}+\ldots z^2+z+1}\right\}=\frac{1}{z-z_1}+\frac{1}{z-z_2}+\frac{1}{z-z_3}\ldots\frac{1}{z-z_{2m}}$$

$z=1$

$$\Rightarrow \left\{\frac{1+2+3+\ldots+(2m-2)+(2m-1)+2m}{(2m+1)}\right\} = -\sum\frac{1}{z_r-1}$$

$$\sum\frac{1}{z_r-1} = -m.$$

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Q Dividing $f(z)$ by $(z-i)$, we get Rem i & dividing it by $(z+i)$ we get Rem $= 1+i$

22

find Rem upon division of $f(z)$ by z^2+1?

① $f(i) = i$

(2) $f(-i) = 1+i$

(3) $f(z) = (z^2+1)Q(z) + (az+b)$

$f(z) = (z-i)(z+i)Q(z) + (az+b)$

$z=i$ $\quad f(i) = 0 + ai + b = i \rightarrow ①$

$z=-i$ $\quad f(-i) = 0 - ai + b = 1+i \rightarrow ②$

find $\underline{a \,\&\, b}$

Rem $= az+b$

Geometrical Interpretation of C.N

1) We Rep. C.N. at Argand Plane.

2) $z = x+iy = (x,y)$

$z = 2-3i = (2,-3)$

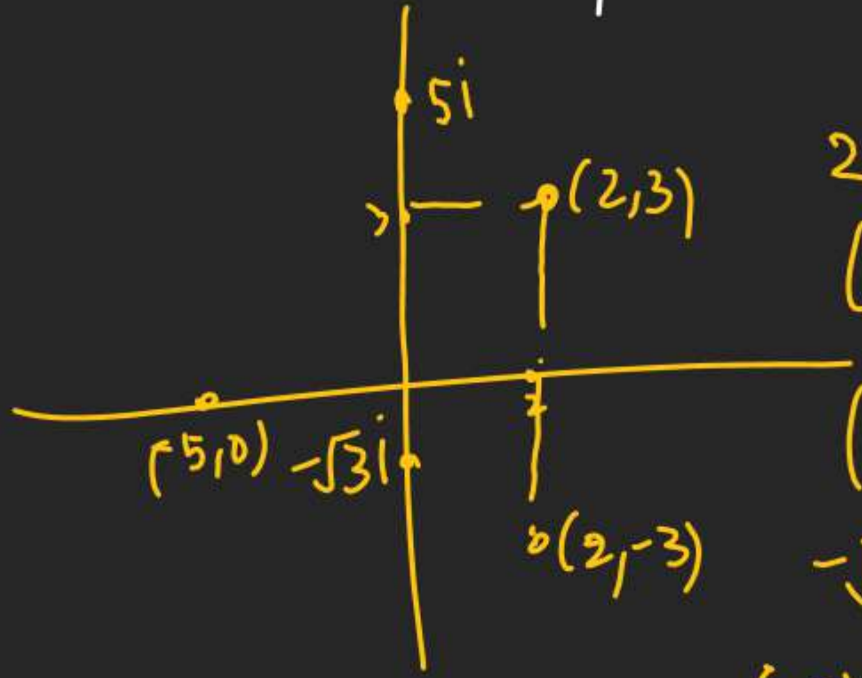
$z = -3-4i = (-3,-4)$

$z = 5i = (0,5)$

$z = 5 = (5,0)$

(3) x Axis is Real Axis

y Axis is Img Axis



$2+3i = (2,3)$

$(2-3i) = (2,-3)$

$(5i) = (0,5)$

$-\sqrt{3}i = (0,-\sqrt{3})$

$(-5) = (-5,0)$

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Q Mark

23 (A) $1+0i$, (B) $-1+0i$, (C) $3+4i$

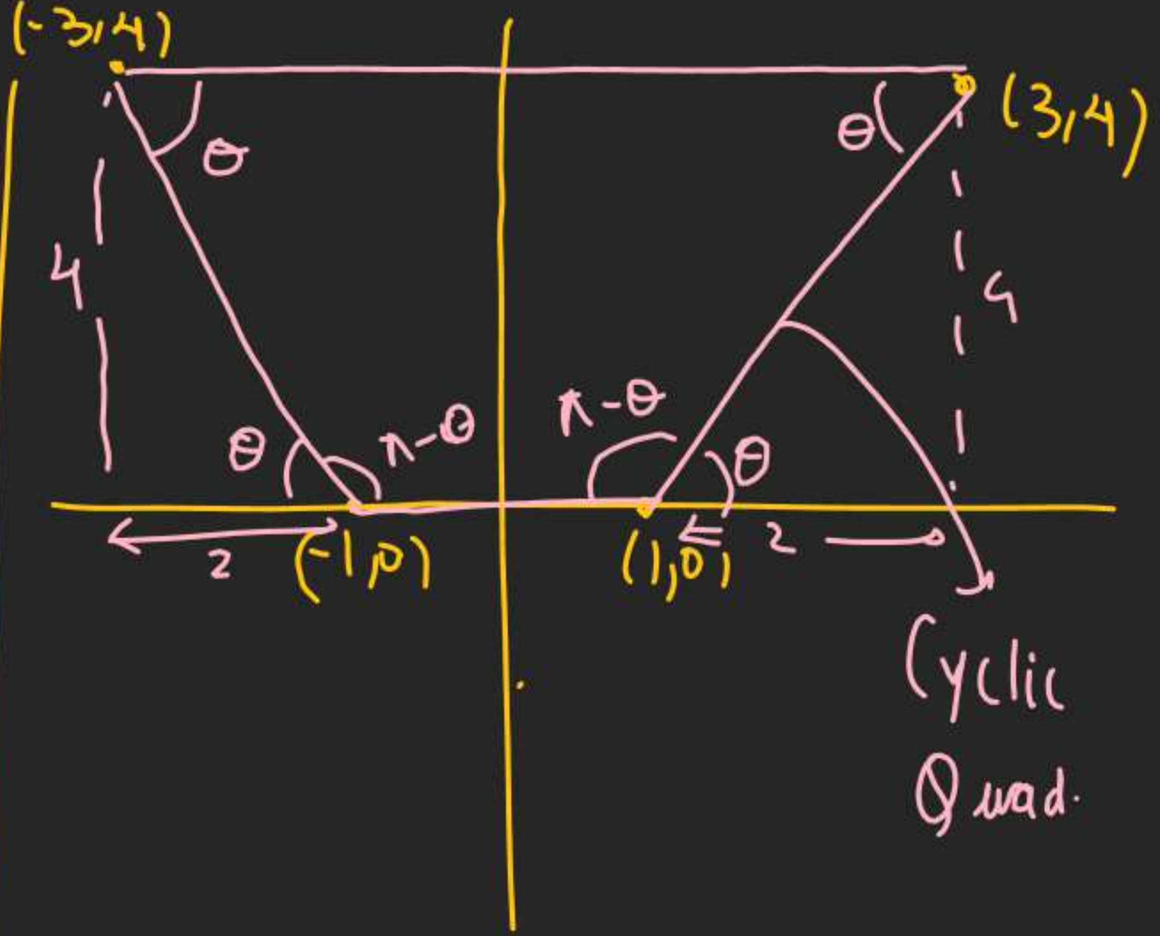
& $\dfrac{25}{-3-4i}$ at Argand Plane

$$\frac{25}{-3-4i} \times \frac{-3+4i}{-3+4i}$$

$$= \frac{25(-3+4i)}{(-3)^2-(4i)^2}$$

$$= \frac{\cancel{25}(-3+4i)}{\cancel{9+16}} = -3+4i$$

(D)



$1+0i = (1,0)$ | $3+4i = (3,4)$

$-1+0i = (-1,0)$ | $-3+4i = (-3,4)$ (D)