Q 1 If $g(x)=f(x)+f(1-x)$ & $f''(x)<0$

$0\le x\le 1$ then Show that $g(x)$ is

↑ in $[0,\frac{1}{2}]$ & ↓ in $[\frac{1}{2},1]$

① $f''(x)<0 \Rightarrow f(x)$ is ↓

$\Rightarrow f''(x)<0 \Rightarrow f'(x)$ is ↓

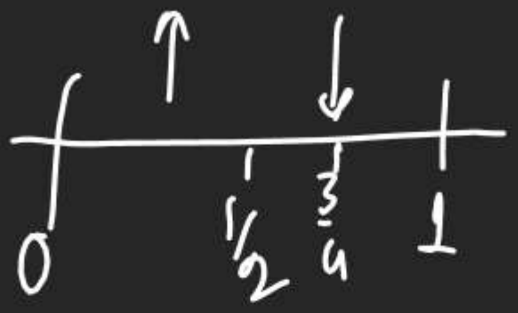
value of f(x) will get lesser as x Increases.

② $g'(x)=f'(x)-f'(1-x)$ $x\in[0,1]$

$x=1-x$

$2x=1$

$x=\frac{1}{2}$



$g'(x)$

$d'(\frac{3}{4})-f'(\frac{1}{4})$

Chhota Bda

$g(x)$ is ↑ in $[0,\frac{1}{2}]$ & ↓ in $[\frac{1}{2},1]$ H.P.

Q 2 Main 2021 Let $f:[0,2]\to R$ be a twice diff'ble fxn such that

$f''(x)>0\ \forall x\in(0,2)$ If

$\phi(x)=f(x)+f(2-x)$ then ϕ

is ↑ in $(0,1)$, ↓ in $(1,2)$ [T/F]

① $f'(x)>0 \Rightarrow f(x)$ is ↑

$f''(x)>0 \Rightarrow f'(x)$ is ↑

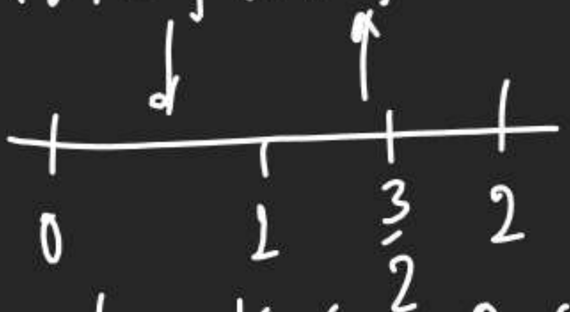
as Values of x will rise value of f'(x) will also be rising.

(2) $\phi'(x)=f'(x)-f'(2-x)$

$x=2-x$

$2x=2$

$x=1$



$f'(\frac{3}{2})-f'(\frac{1}{2})$

Bda - Chhota

$\phi'(x)$ ⊕

$\phi(x)$ ↓ in $(0,1]$ & ↑ in $(1,2)$

Q 3 Find Interval of Concavity for f(x)

$f(x)=x^4-6x^3-108x^2+57x+2$

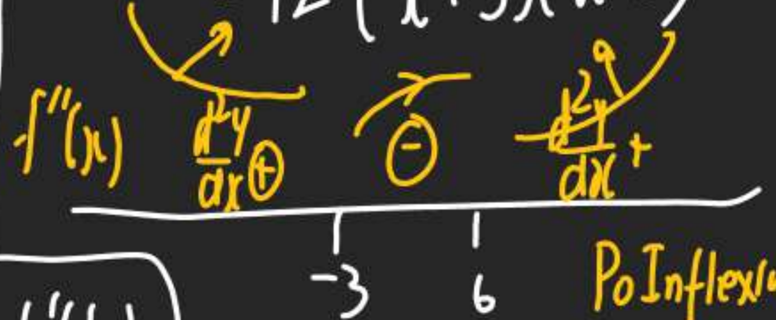
also find Inflection Pt.

$f'(x)=4x^3-18x^2-216x+57$

$f''(x)=12x^2-36x-216$

$=12(x^2-3x-18)$

$=12(x+3)(x-6)$



$f''(x)=0$ $f''(x)=0$

Po Inflexion $x=-3$ & 6

Con. up → $x\in(-\infty,-3)\cup(6,\infty)$

Con down → $x\in(-3,6]$

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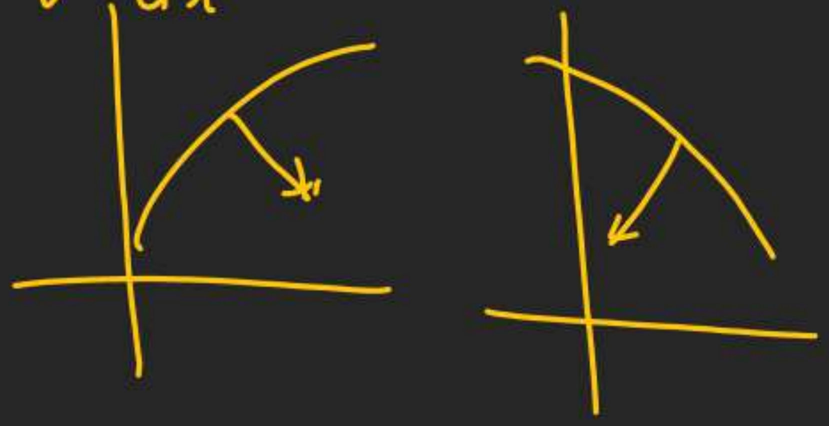
Rem:-

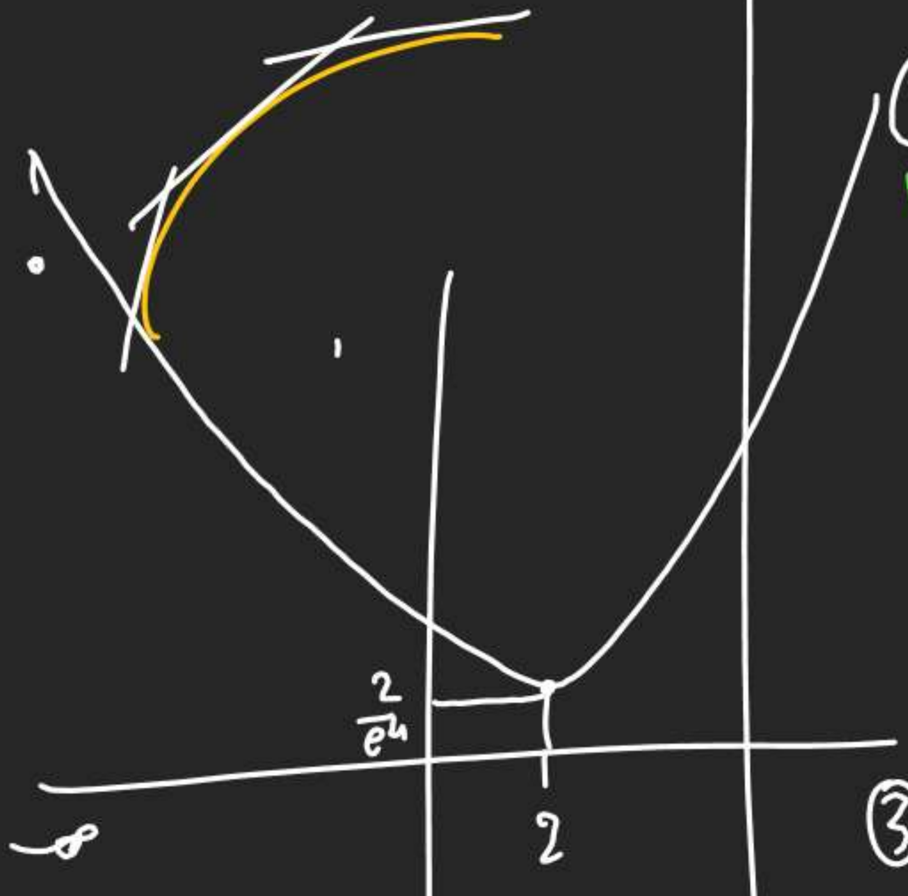
1) If $\frac{d^2y}{dx^2}$ ⊕ then Con. up.

2) If $\frac{d^2y}{dx^2}$ ⊖ then Con. down



3) Where $\frac{d^2y}{dx^2}$ Starts changing its Sign is Pt. Inflection $\Rightarrow$ there $\frac{d^2y}{dx^2}=0$

(4) If Concavity is down. then at every Pt. tangent is lying above Curve.



$f(x)_{Min} = \frac{2}{e^4}$ at $x=2$

Plotting Curve.

① Domain of fxn ② $\frac{dy}{dx}$'s sign ③ value of fxn at Cr. pt + End Pts

④ Time waste करना हो then $\frac{d^2y}{dx^2}$ का Sign

Q Sketch: $f(x) = 2 \cdot e^{x^2-4x}$.

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① Domain → $x \in R \Rightarrow x \in (-\infty, \infty)$

② $f'(x) = 2e^{x^2-4x} \times (2x-4) = 0$

$x = 2$

$f'(3) = ⊕$

$f'(1) = ⊖$

③ $f(-\infty) = 2e^{(-\infty)^2 + 4\times(+\infty)} \xrightarrow{} \infty$

$f(\infty) = 2e^{\infty^2 - 4\infty} = \infty$

$f(2) = 2e^{2^2-4\times2} = 2e^{-4} = \frac{2}{e^4}$

Q. $f(x) = \frac{1}{1+x^2}$

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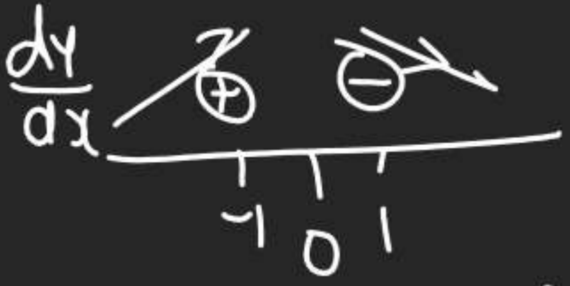
1) Domain $1+x^2 \neq 0$, $x^2 \neq -1$ $\}$ $x \in R_\infty$

$x \in (-\infty, \infty)$

2) $\frac{dy}{dx} = -\frac{1 \times 2x}{(1+x^2)^2} = -\frac{2x}{(1+x^2)^2}$

$\frac{dy}{dx} = 0 \Rightarrow \frac{-2x}{(1+x^2)^2} = 0 \Rightarrow x = 0 \ \forall \ \frac{dy}{dx} = 0$



$f'(1) = \ominus$

$f'(-1) = \oplus$

(3) $f(-\infty) = \frac{1}{1+(-\infty)^2} = 0^+$

$f(+\infty) = \frac{1}{1+(\infty)^2} = 0^+$

$f(0) = 1$

Q $f(x) = x^2 \cdot e^{-x}$'s graph

6

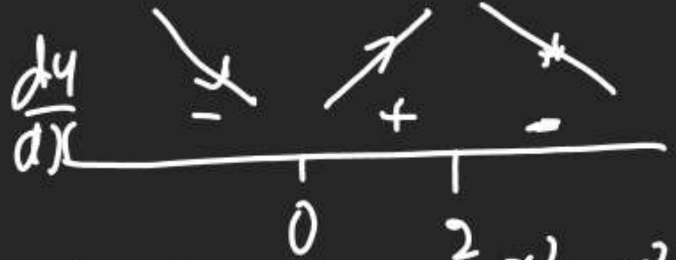
$\downarrow \cdot \downarrow$

① Dom $R \cap R = R = (-\infty, \infty)$

② $\frac{dy}{dx} = -x^2 e^{-x} + e^{-x} \cdot 2x$

$= e^{-x} \cdot x\{2-x\}$

$= -e^{-x}(x)(x-2)$

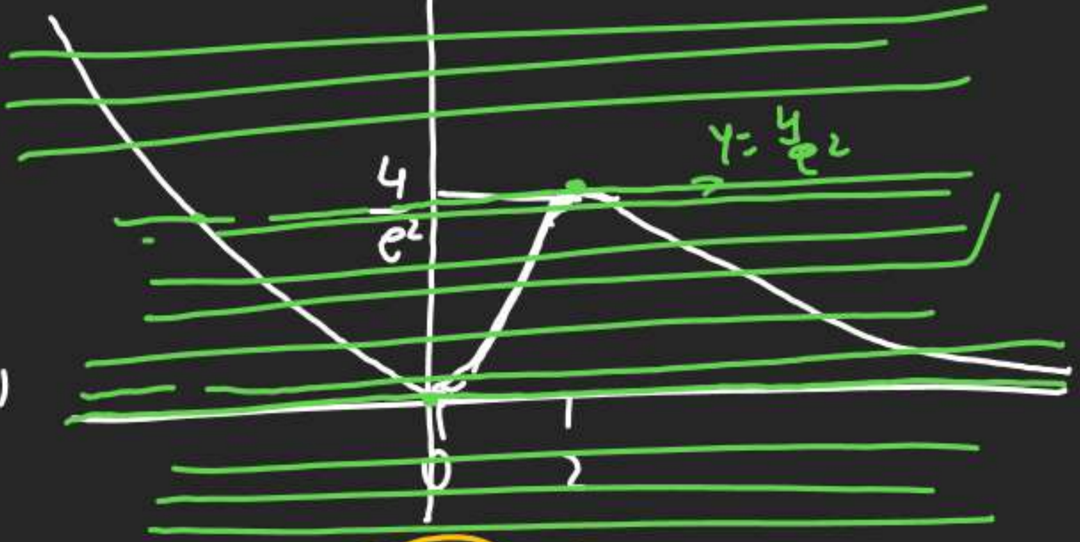


(3) $f(-\infty) = \lim_{x \to -\infty} \frac{x^2}{e^x} \ \frac{\infty^2}{e^{-\infty}} \to \infty$

$f(\infty) \xrightarrow{\quad} \lim_{x \to \infty} \frac{x^2}{e^x} \ \frac{\infty}{\infty} \overset{DL}{\ } \frac{|\underline{2}}{e^x} = \frac{|\underline{2}}{e^\infty} = 0$

$f(0) = \frac{x^2}{e^x} = \frac{0}{e^0} = 0$

$f(2) = \frac{4}{e^2} = \frac{4}{e^2}$



Q $\underbrace{x^2 \cdot e^{-x}}_{y=} = \underbrace{K}_{y=}$ has Exactly 2 Solution for $K \in$?

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Exactly 1 Sol $\Rightarrow K \in (\frac{4}{e^2}, \infty) \cup \{0\}$

$Y = 0 \Rightarrow K = 0 \ \forall$ Exactly 1 Sol.

Exactly 3 solution $K \in (0, \frac{4}{e^2})$

Exactly 2 Solution when $K \in \{\frac{4}{e^2}\}$

Exactly 1 Solution when $K \in (\frac{4}{e^2}, \infty)$

Q Find graph of $f(x) = 2x^2 - \ln|x|$

8

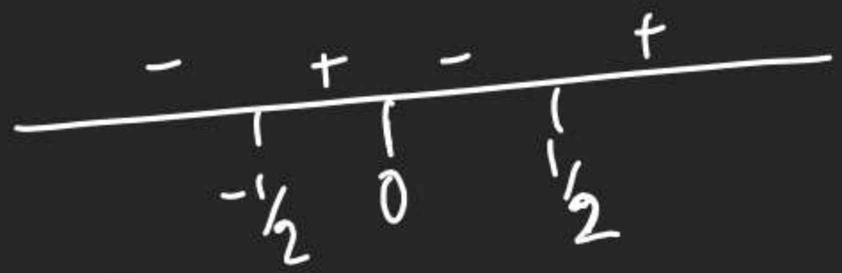
$$f(x) = \underset{\downarrow}{2x^2} - \underset{\downarrow}{\ln|x|}$$

① Dom $R \cap x \neq 0 \Rightarrow x \in R - \{0\}$

$x \in (-\infty, 0) \cup (0, \infty)$

② $$\frac{dy}{dx} = 4x - \frac{1}{\cancel{|x|}} \times \frac{\cancel{|x|}}{x} = 4x - \frac{1}{x} = \frac{4x^2-1}{x}$$

$$= \frac{(2x-1)(2x+1)}{x}$$



(3) $f(x) = 2x^2 - \ln|x|$ is Even fxn

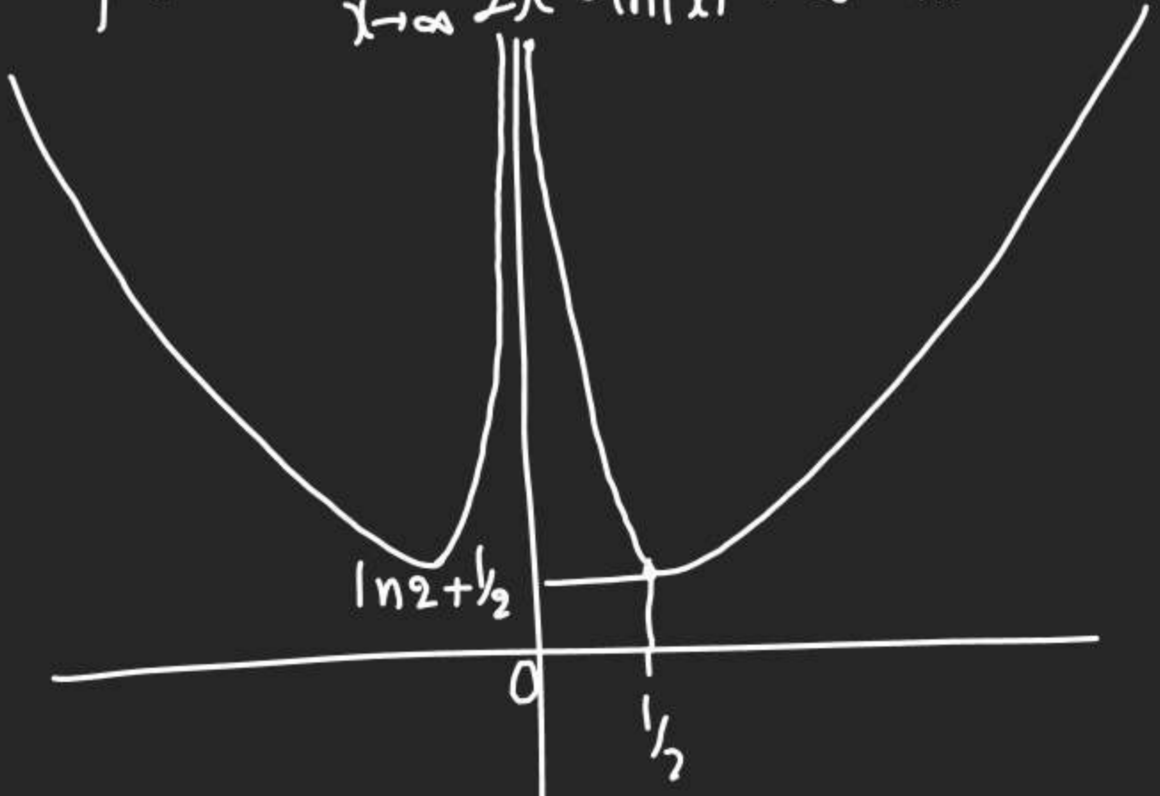
RHS of graph is enough LHS will copy it.

$$f(0+h) = \lim_{x \to 0^+} 2x^2 - \ln|x| = 2(\cancel{0+h})^2 - \ln|0+h|$$

$$= +\infty$$

$$f(1/2) = 2 \times \frac{1}{4} - \ln|1/2| = 1/2 + \ln 2$$

$$f(\infty) = \lim_{x \to \infty} 2x^2 - \ln|x| \to \infty^2 - \ln\infty = \infty$$



Q $$f(x) = \frac{x^2-x+1}{x^2+x+1}$$

$D \to (-\infty, \infty)$

$$\frac{dy}{dx} = \frac{(x^2+x+1)(2x-1) - (x^2-x+1)(2x+1)}{(x^2+x+1)^2}$$

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Composite fxn's Monotonocity

★

$f(x)$	$g(x)$	$f(g(x))$	$g(f(x))$	$f(f(x))$	$g(g(x))$
MI	MI	MI	MI	MI	MI
MD	MD	MI	MI	MI	MI
MI	MD	MD	MD	MI	MI

$x_1 > x_2$

$g(x_1) < g(x_2)$

$f(g(x_1)) < f(g(x_2))$

Q9 $f(x)=e^x-x$, $g(x)=x^2-x$ then Set of all $x \in R$ where $h(x)=f(g(x))$ is ↑ in

$[0,\infty)$ $\quad [-1,-\frac{1}{2}]\cup[\frac{1}{2},\infty)$ $\quad [-\frac{1}{2},0]\cup[1,\infty)$

✓ $[0,\frac{1}{2}]\cup[1,\infty)$

$h'(x)=f'(g(x))\times g'(x)$

$=(e^{x^2-x}-1)\times(2x-1)$

↓ $\quad$ ↓

$x^2-x=0 \qquad x=\frac{1}{2}$

$x(x-1)=0$

$x=0,1$

− + − +

0 $\quad \frac{1}{2}$ $\quad$ 1

↑ in $[0,\frac{1}{2}]\cup[1,\infty)$

① $f'(x)=e^x-1$

② $f'(g(x))=e^{x^2-x}-1$

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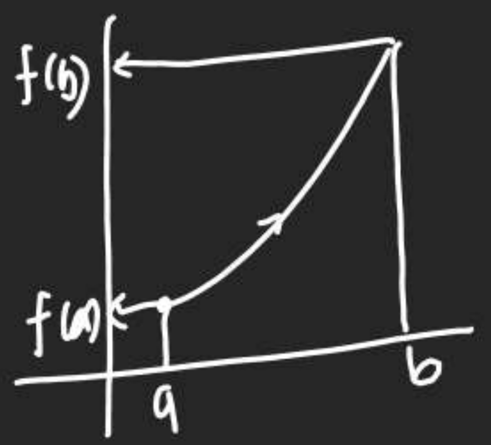
Rx:- Application of Monotonocity is to find Range of fxn also.

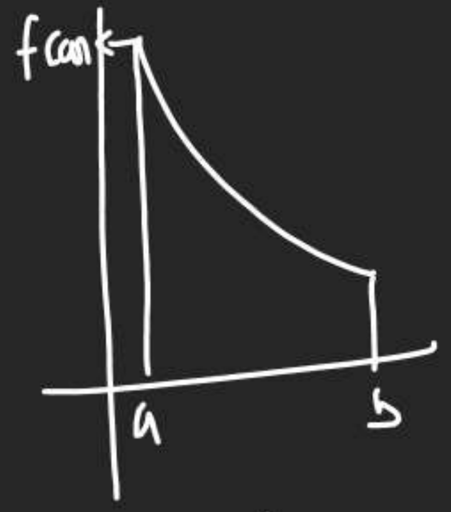
↓ 3 Kinds of fxn.

fxn is Strictly ↑ in [a, b]



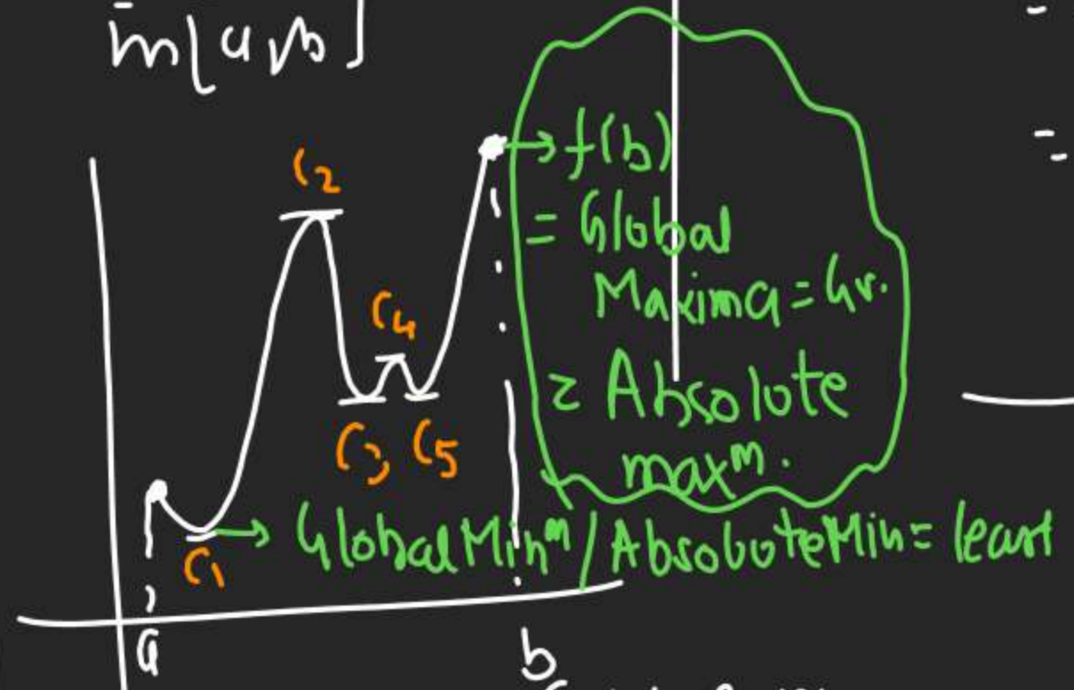
Min f(a)
Max f(b)

fxn is strictly ↓ in [a, b]



Max = f(a)
Min = f(b)

fxn is Non Monotonic in [a, b]



→ f(b) = Global Maxima = Gr. = Absolute max^m.

→ Global Min^m / Absolute Min = least

Check pt = Cr pt + End Pt

Max $\{f(a), f(b), f(c_1), f(c_2), f(c_3), f(c_4) f(c_5)\}$ = Max of f(x)

Min $\{f(a), f(b), f(c_1), f(c_2), f(c_3), f(c_4), f(c_5)\}$ = Min. of f(x)

$\frac{3}{16}-\frac{4}{16}-\frac{24}{16}+\frac{48}{16}+\frac{16}{16}$ $\underline{\underline{10}}$

$= -25+64$

Q $f(x) = 3x^4 - 2x^3 - 6x^2 + 6x + 1$ has Max^m value in [0, 2] at?

↑ End Pt = 0, 2

$f'(x) = 12x^3 - 6x^2 - 12x + 6$

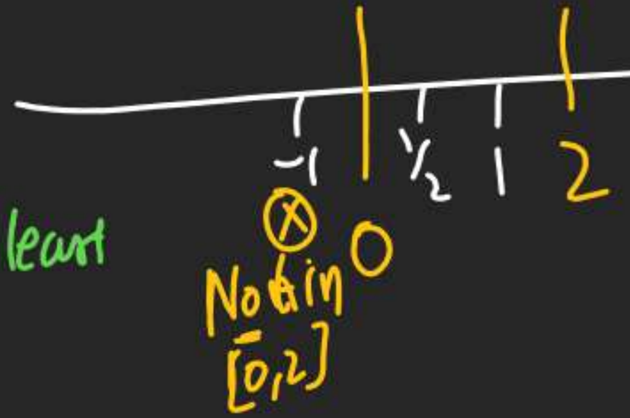
$= 6\{2x^3 - x^2 - 2x + 1\}$

$= 6\{x^2(2x-1) - (2x-1)\}$

$= 6\{(x^2-1)(2x-1)\} = 6(x-1)(x+1)(2x-1)$

48 - 16 - 24 + 12 + 1

3 - 2 - ~~6~~ + ~~6~~ + 1



f(0) =	1 (Chhota)
f(1/2) =	39/16
f(1) =	2
f(2) =	21 (Max.)

Max^m at x = 2
(21)