Work done by constant force :

- When a constant force $\vec{F}$ acts on a particle and the particle moves through a displacement $\vec{S}$, then the force is said to do work W on the particle.

$W = \vec{F}\cdot\vec{S}$

The scalar (dot) product of $\vec{F}$ and $\vec{S}$, can be evaluated as $W = \vec{F}\cdot\vec{S} = FS\cos\theta$

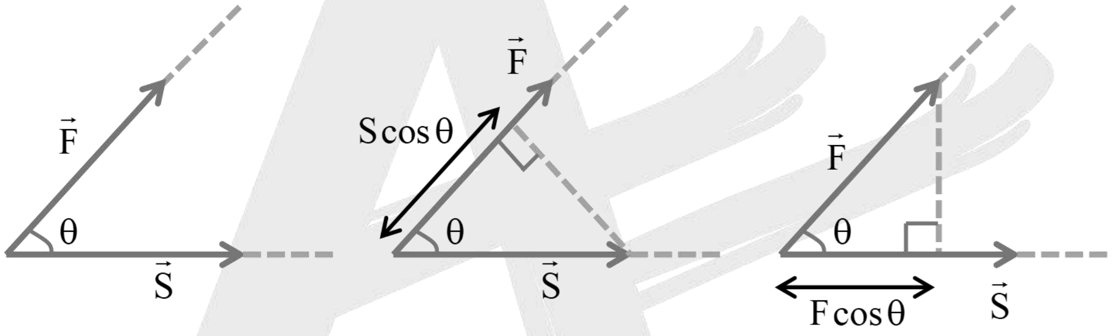
Where F is the magnitude of $\vec{F}$, S is the magnitude of $\vec{S}$ and θ is the angle between $\vec{F}$ and $\vec{S}$.

$W = FS\cos\theta = F(S\cos\theta)$

= magnitude of the force X component of displacement in the direction of force

$W = (F\cos\theta)S$

= component of the force in the direction of displacement X magnitude of the displacement



- Work is a scalar quantity.
- SI Unit is Nm or joule (J).
- CGS unit is erg.

  $1J \times 1N \times 1m; 1erg = 1dyne \times 1cm$

  Dimensional formula of work is $[ML^2T^{-2}]$.

- Relation between joule and erg : 1 joule $= 10^7$ erg

Other units of work :

Electron Volt $(eV) = 1.6 \times 10^{-19}$ J

Kilowatt hour /

- **Work done by multiple forces :**

  If a number of forces act on a body or particle then :

  $W = W_1 + W_2 + W_3 + \ldots\ldots$

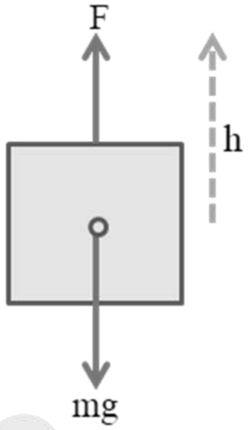
  or $W = \int \overline{F}_1 \cdot \overline{ds} + \int \overline{F}_2 \cdot \overline{ds} + \ldots\ldots$ or $W = \int (\overline{F}_1 + \overline{F}_2 + \ldots\ldots) \cdot \overline{ds}$

  or $W = \int \overline{F}_R \cdot \overline{ds}\ [as \overline{F}_R = \sum \overline{F}]$

  Work done in displacing a particle under the action of a number of forces is equal to the work done by the resultant force.

- **Nature of Work :** Work done by a force may be positive or negative or zero.

Ex : (a) If we lift a body from rest to a height h



- Work done by lifting force F

$$W_1 = Fh\cos 0° = Fh \quad (+ve)$$

- Work done by gravitational force

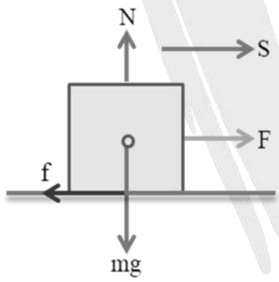
$$W_2 = mgh\cos 180° = -mgh \quad (-ve)$$

So, network

$$W = W_1 + W_2 = Fh - mgh = (F - mg)h$$

Now, if the body is in equilibrium $F = mg,\ W = 0$

Ex : (b) If a body is pulled on a rough horizontal road through a displacement S



- Work done by normal reaction and gravity $W_1 = 0$ as force is $\perp$ to S

- Work done by pulling force F,

$$W_2 = FS\cos 0° = FS \quad (+ve)$$

- Work done by frictional force f, $W_3 = fs\cos 180° = -\mu mgs \quad (-ve)$

Net work $W = W_1 + W_2 + W_3 = 0 + FS - fS = (F - f)S$

Now, if the body is in dynamic equilibrium $f = F$

So, W = 0

Zero Work :

- Work done is zero if
  1. Force and displacement are perpendicular.
  2. Displacement of point of application of force is zero.
  3. Net force acting on the body is zero.

- As $W = \int \vec{F} \cdot d\vec{s}$ so, if $d\vec{s} = 0$, $W = 0$ i.e., if the displacement of a particle or body is zero whatever be the force, work done is zero (except non-conservative force)

  (a) When a person exerts a force to displace a wall or stone but the object, specifically its center of mass, remains stationary, the work done by the person is zero.

  (b) A weight lifter performs work when lifting a weight from the ground, but no work is done in holding the weight in a static position.

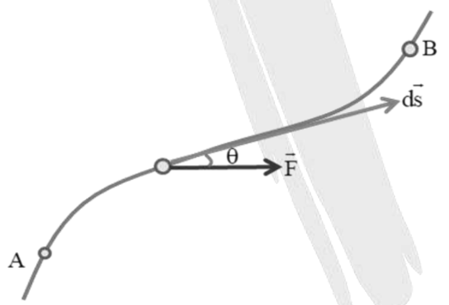
  As $W = \int F ds \cos\theta$, so $W = 0$, if $\theta = 90°$, i.e., if force is always perpendicular to motion, work done by the force will be zero though neither force nor displacement is zero. This is why :

  (a) When a porter walks with a suitcase on his head along a horizontal level road, the work done by the lifting force (against gravity) is zero.

  (b) When a body moves in a circular path, the work done by the centripetal force is always zero.

  (c) When the bob of a simple pendulum swings back and forth, the work done by the tension in the string is zero.

WORK DONE BY VARIABLE FORCE :



- When the magnitude and direction of a force varies with position, then the work done by such a force for an infinitesimal displacement ds is given by $dW = \vec{F} \cdot \vec{ds}$

  The total work done in going from A to B is

$$W_{AB} = \int_A^B \vec{F} \cdot \vec{ds} = \int_A^B (F\cos\theta) ds$$

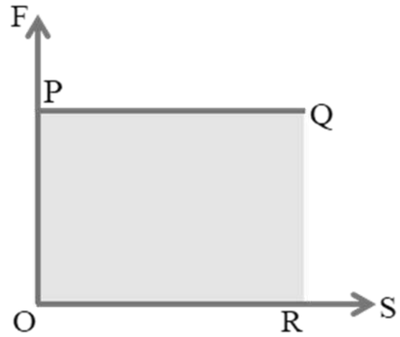
  In terms of rectangular components

  $\vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}; \vec{ds} = dx\hat{i} + dy\hat{j} + dz\hat{k}$

  $W = \int_{x_1}^{x_2} F_x dx + \int_{y_1}^{y_2} F_y dy + \int_{z_1}^{z_2} F_z dz$

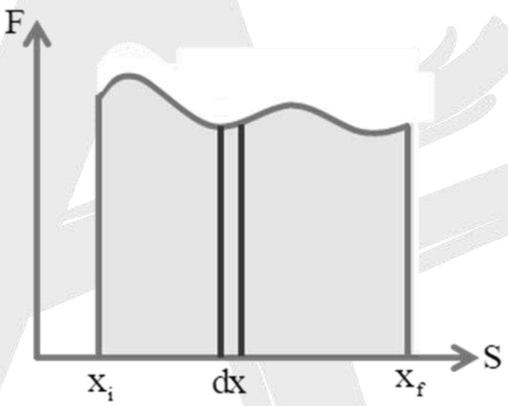
Graphical representation of work done :

- The area enclosed by the F-S graph and displacement axis gives the amount of work done by the force.



Work = FS = Area of OPQR

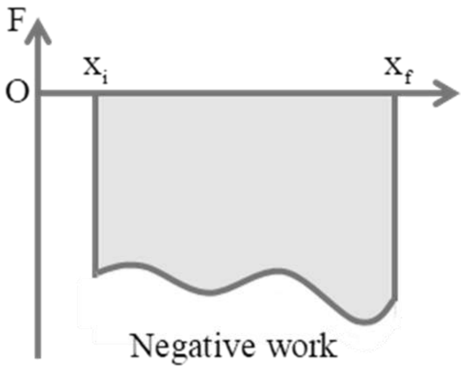
- Work done by variable force.



- For a small displacement dx the work done will be the area of the strip of width dx

$$W = \int_{x_i}^{x_f} dW = \int_{x_i}^{x_f} F\,dx$$

- If area enclosed above X-axis, work done is +ve and if the area enclosed below X-axis, work done is –ve.

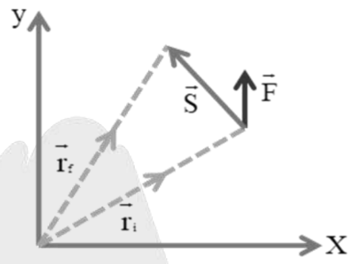


Application on work

- If a force is changing linerly from F_1 to F_2 over a displacement S then work done is

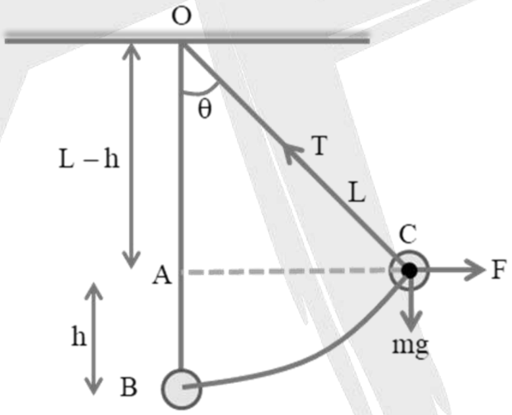
$$W = \left(\frac{F_1 + F_2}{2}\right)S$$

- If a force displaces the particle from its initial position $\vec{r}_i$ to final position $\vec{r}_f$ then displacement vector is $\vec{S} = \vec{r}_f - \vec{r}_i$.



$$W = \vec{F}\cdot\vec{S} = \vec{F}\cdot\left(\vec{r}_f - \vec{r}_i\right)$$

- Work done in pulling the bob of mass m of a simple pendulum of length L through an angle θ to vertical by means of a horizontal force F.



$$\cos\theta = \frac{L-h}{L} = 1 - \frac{h}{L};\ \frac{h}{L} = 1 - \cos\theta$$

$$h = L(1-\cos\theta)$$

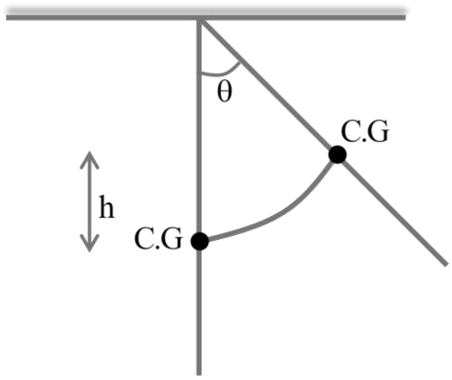
Work done by gravitational force

$$W = -mgh = -mgL(1-\cos\theta)$$

Work done by horizontal force F is $W = FL\sin\theta$

Work done by tension T in the string is zero.

- Work done by gravitational force in pulling a uniform rod of mass m and length ℓ through an angle θ is given by

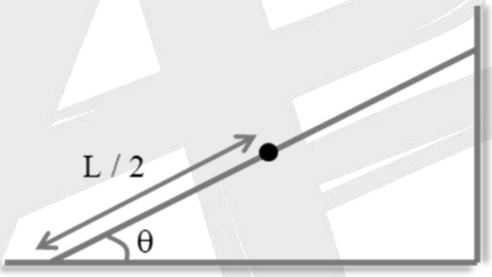


$W = -mg\frac{\ell}{2}(1 - \cos\theta)$, where $\frac{\ell}{2}$ is the distance of centre of mass from the support.

- A ladder of mass 'm' and length 'L' resting on a level floor is lifted and held against a wall at an angle θ with the floor

  Work done by gravitational force is

  $$W_g = -mgh = -mg\left(\frac{L}{2}\right)\sin\theta$$



- A bucket full of water of total mass M is pulled by using a uniform rope of mass m and length ℓ.

  Work done by pulling force

  $$W = Mg\ell + mg\frac{\ell}{2}$$

- A block of mass m is suspended vertically using a rope of negligible mass. If the rope is used to lift the block vertically up with uniform acceleration 'a', work done by tension in the rope is

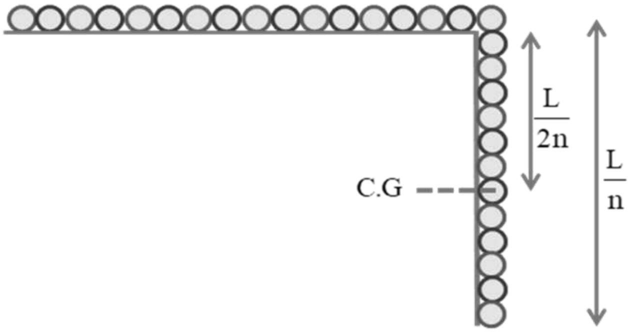
  $$W = m(g + a)h$$

  (h = height)

  If block is lowered with acceleration 'a', then $W = -m(g - a)h$

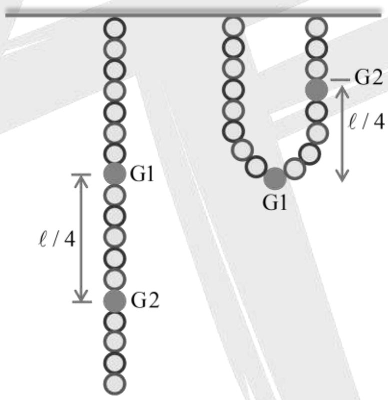
- A uniform chain of mass M and length L is kept on smooth horizontal table such that $\frac{1}{n^{th}}$ of its length is hanging over the edge of the table.

  The work done by the pulling force to bring the hanging part onto the table is



$W = \left(\frac{M}{n}\right)gh = \left(\frac{M}{n}\right)g\left(\frac{L}{2n}\right) = \frac{MgL}{2n^2}$ Mass of hanging part is $\frac{M}{n}$

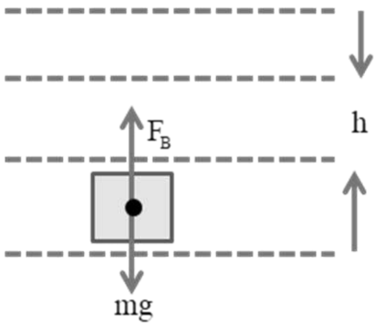
- A uniform chain of mass M and length L rests on a smooth horizontal table with $\frac{1}{n_1^{th}}$ part of its length is hanging from the edge of the table. Work done in pulling the chain partially such that $\frac{1}{n_2^{th}}$ part is hanging from the edge of the table is given by $W = \frac{MgL}{2}\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$
- A uniform chain of mass 'M' and length L is suspended vertically. The lower end of the chain is lifted upto point of suspension



$h = \frac{L}{4} + \frac{L}{4} = \frac{L}{2}$ = raise in centre of mass of lower half of the chain.

Work done by gravitational force is $W_g = -\frac{M}{2}g\frac{L}{2} = -\frac{MgL}{4}$

- The Work done in lifting a body of mass 'm' having density $'d_1'$ inside a liquid of density $'d_2'$ through a height 'h' is



$W = mgh = mgh\left[1 - \frac{d_2}{d_1}\right]$

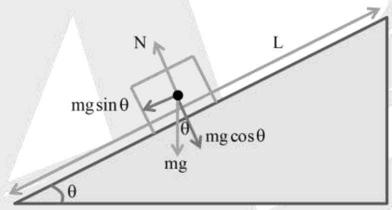
- A body of mass 'm' is placed on a frictionless horizontal surface. A force F acts on the body parallel to the surface such that it moves with an acceleration 'a', through a displacement 'S'. The work done by the force is $W = FS = maS \quad (\because \quad \theta = 0^\circ)$
- A body of mass 'm' is placed on a rough horizontal surface of coefficient of friction μ. A force F acts on the body parallel to the surface such that it moves with an acceleration 'a', through a displacement 'S'. The work done by the frictional force is $f = \mu mg\cos\theta; \quad \text{but } \theta = 0^\circ$

$$\therefore \quad f = \mu mg\cos 0^\circ = \mu mg \Rightarrow W_f = \mu mgs$$

$$W_{net} = (f + ma)S = (\mu mg + ma)S = m(\mu g + a)S$$

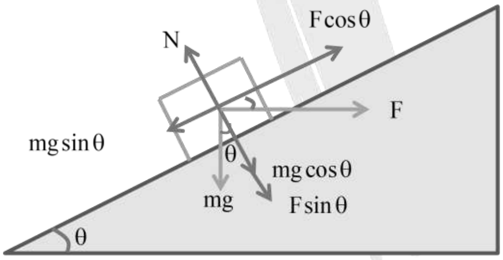
If the body moves with uniform velocity then $W = FS = \mu mgS$

- A body of mass m is sliding down on a smooth inclined plane of inclination θ. If L is length of inclined plane then work done by gravitational force is

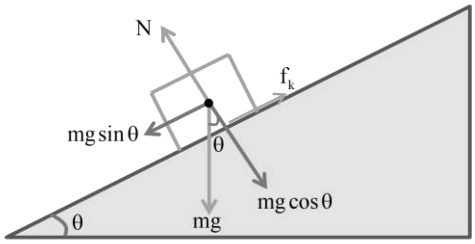


$$W_g = FS = mg\sin\theta L$$

- A body of mass 'm' is moved up the smooth inclined plane of inclination θ and length L by a constant horizontal force F then work done by the resultnt force is $W = (F\cos\theta - mg\sin\theta)L$



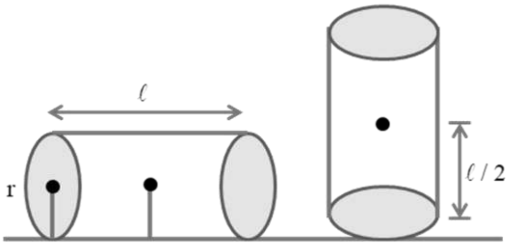
- A body of mass 'm' is sliding down on rough inclined plane of inclination θ. If L is the length of incline and v is the coefficient of kinetic friction then work done by the resultant force on the body is



$$W = (mg\sin\theta - f_k)L = (mg\sin\theta - \mu_k mg\cos\theta)L$$

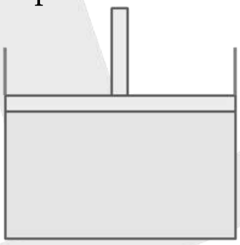
$$= mgL(\sin\theta - \mu_k\cos\theta)$$

➢ A uniform solid cylinder of mass m, length ℓ and radius r is lying on ground with curved surface in contact with ground. If it is turned such that its circular face is in contact with ground then work done by applied force is



$$W = mgh = mg\left(\frac{\ell}{2} - r\right) \qquad \left(\because h = \frac{\ell}{2} - r\right)$$

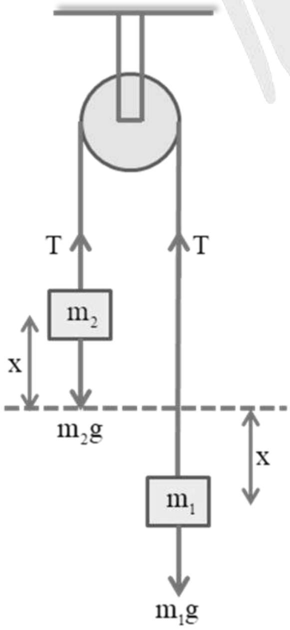
➢ A gas at a pressure P is enclosed in a cylinder with a movable piston. Work done by the gas in producing small displacement dx of the piston is



$$dW = Fdx = PAdx = PdV$$

Total work done by the gas during the change in its volume from V_1 to V_2 is $W = \int_{V_1}^{V_2} PdV$

➢ Two blocks of masses m_1 and m_2 $(m_1 > m_2)$ connected by an inextensible string are passing over a smooth, massless pulley. The two blocks are released from the same level. At any instant 't', if 'x' is the displacement of each block then



Work done by gravity on block m_1, $W_1 = +m_1gx$

Work done by gravity on block m_2, $W_2 = -m_2gx$

Work done by gravitational force on the system,

$$m_2,\ W_2 = -m_2gx$$

$$W_g = (m_1 - m_2)gx = (m_1 - m_2)g\left(\frac{1}{2}at^2\right) \qquad [\because\ v^2 - u^2 = 2as]$$

$$W_g = \frac{(m_1 - m_2)^2 g^2t^2}{2(m_1 + m_2)} \qquad \left[\because\ a = \frac{(m_1 - m_2)g}{m_1 + m_2}\right]$$

Note : In this case work done on the two blocks by tension is zero.

$$W = T(x) + T(-x) = 0$$

Energy:

- Energy is the capacity or ability to perform work. The greater the amount of energy a body possesses, the more work it can do.
- Energy is the cause of work, and work is the effect of energy.
- Energy is a scalar quantity, and it shares the same units and dimensions as work.
- There are various forms of energy, including mechanical energy, light energy, heat energy, sound energy, electrical energy, nuclear energy, and more.
- Mechanical energy can be classified into two types:
  1. potential energy
  2. kinetic energy

Potential energy (U)

- It is the energy possessed by a body based on its position or configuration in a field.
- Potential energy is applicable only to conservative forces and does not exist for non-conservative forces.

In case of conservative forces.

$$F = -\left(\frac{dU}{dr}\right) \quad \therefore dU = -\vec{F}\cdot\vec{dr} \Rightarrow \int_{U_1}^{U_2} dU = -\int_{r_1}^{r_2} \vec{F}\cdot\vec{dr}$$

$$U_2 = U_1 = -\int_{r_1}^{r_2} \vec{F}\cdot\vec{dr} = -W$$

$$\text{If } r_1 = \infty,\ U_1 = 0 \quad \therefore\ U = \int_{\infty}^{r} \vec{F}\cdot\vec{dr} = -W$$

- P.E. can be +ve or –ve or can be zero.
- P.E. depends on frame of reference.
- Ex : Water stored in a dam, A stretched bow,
  A loaded spring etc., possesses P.E
- In the case of a conservative force or field, potential energy is defined as the negative of the work done in moving a body from a reference position to a specific position.
- The presence of potential energy in a moving body depends on the specific situation and the forces acting upon it.
- Consider potential energy as a property of the entire system, rather than attributing it to individual particles.

Kinetic energy

- Kinetic energy is the energy possessed by a body by virtue of its motion.
- Kinetic energy of a body of mass 'm' moving with a velocity 'v', $KE = \frac{1}{2}mv^2$
- Kinetic energy is a scalar quantity.
- The kinetic energy of an object is a measure of the work an object can do by the virtue of its motion.

Examples for bodies having K.E

(1) A vehicle in motion.

(2) Water flowing in a river.

(3) A bullet fired from a gun.

- Kinetic energy depends on frame of reference.
  Ex : kinetic energy of a person of mass m sitting in a train moving with speed v is zero in the frame of train but $\frac{1}{2}mv^2$ in the frame of earth.

Relation between K.E. and linear momentum

- $KE = \frac{1}{2}mv^2 = \frac{P^2}{2m} = \frac{1}{2}Pv (\because P = mv)$
- If two bodies of different masses have same momentum then lighter body will have greater KE $\left(\because KE \alpha \frac{1}{m}\right)$
- When a bullet is fired from a gun the momentum of the bullet and gun are equal and opposite.
  i.e $\frac{KE_{bullet}}{KE_{gun}} = \frac{M_{gun}}{M_{bullet}}$
  Hence, the KE of the bullet is greater than that of the gun

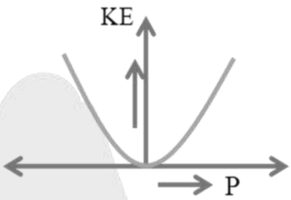
- A body can have energy without momentum. But it can not have momentum without energy.
- A bullet of mass 'm' moving with velocity 'v' stops in wooden block after penetrating through a distance 'x'. If F is resistance offered by the block to the bullet

  (Assuming F is constant inside the block)

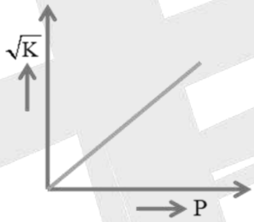
  $$\frac{1}{2}mv^2 = Fx \ ; \quad F = \frac{mv^2}{2x} \qquad \therefore \ v^2 \alpha x$$

- **For a given body**

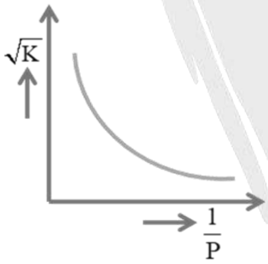
  (1) The graph between KE and P is a parabola.



(2) The graph between $\sqrt{KE}$ and P is a straight line passing throught the origin. Its slope = $\frac{1}{\sqrt{2m}}$

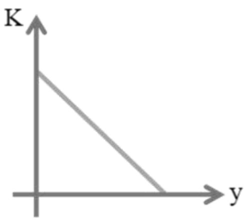


(3) The graph between $\sqrt{KE}$ and $\frac{1}{P}$ is a rectangular hyperbola.



- A particle is projected up from a point at an angle $'\theta'$ with the horizontal. At any time 't' if 'P' is linear momentum, 'y' is vertical displacement and 'x' is horizontal displacement, then nature of the curves drawn for KE of the particle (K) against these parameters are

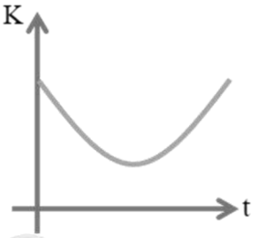
  (i) <u>K – y graph :</u> $K = K_i - mgy$; It is a straight line



(ii) K – t graph

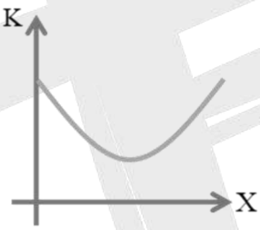
$$K = K_i - mg\left(u_y t - \frac{1}{2}gt^2\right)$$

$\because \; y = u_y t - \frac{1}{2}gt^2$; It is a parabola



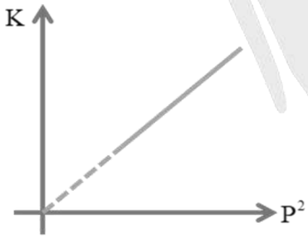
(iii) K – x graph $K = K_i - mg\left(x\tan\theta - \frac{gx^2}{2u_x^2}\right)$

$\because \; y = (\tan\theta)x - \left(\frac{g}{2u_x^2}\right)x^2$; It is also a parabola



iv) K – P graph

It is a straight line passing through origin and slope $= \frac{1}{2m}$ $P^2 = 2mK$



$P^2 \alpha K$

Conservative and non-Conservative forces

- A force is considered to be a conservative force if the work done by the force around a closed path is zero and is independent of the path taken.
- The property of a force being conservative indicates that the work done by the force depends only on the initial and final positions of an object, rather than the specific path taken.

- Conservative forces are characterized by the absence of energy dissipation and the potential for the conservation of mechanical energy.
- Under conservative force $F = -\frac{dU}{dr}$ where U is Potential Energy.

$$U = \int dU = -\int \vec{F} \cdot \overrightarrow{dr}$$

$\left(\vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}\right)$ and

$\left(\vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k} \text{ and } \overrightarrow{dr} = dx\,\hat{i} + dy\,\hat{j} + dz\,\hat{k}\right)$

$$\vec{F} = -\left(\frac{\partial u}{\partial x}\hat{i} + \frac{\partial u}{\partial y}\hat{j} + \frac{\partial u}{\partial z}\hat{k}\right)$$

Ex 1 : Gravitational force is a conservative force

Ex 2 : Elastic force in a stretched spring is a conservative force

Non-Conservative Forces :

- If the work done by a force around a closed path is not equal to zero and is dependent on the path then the force is non-conservative force

Ex :- Force of friction, Viscous force.

- Work done by the non-conservative force will not be stored in the form of Potential energy.
- Potential energy is defined only for conservative forces.

Spring force

- Spring force is an example of a variable force which is conservative.
- In an ideal spring, the spring force F_s is directly proportional to 'x'. Where x is the displacement of the block from equilibrium position. i.e., $F_s = -Kx$. The constant K is calle spring constant.
- The work done on the block by the spring force as the block moves from underformed position $x = 0$ to $x = x_1$

$$dW = \vec{F} \cdot \overrightarrow{dx} = -Kxdx$$

$$W = \int dW = \int_0^{x_1} -Kxdx = -\frac{1}{2}K(x^2)_0^{x_1} = -\frac{1}{2}Kx_1^2$$

- If the block moves from $x = x_1$ to $x = x_2$ the work done by spring force is $W = \int_{x_1}^{x_2} -Kxdx$

$$W = \frac{1}{2}K(x_1^2 - x_2^2) = \frac{1}{2}Kx_1^2 - \frac{1}{2}Kx_2^2$$

Potential energy stored in a spring :

- The change in potential energy of a system corresponding to a conservative internal force is

$$dU = -\int_0^x \vec{F}\cdot\overrightarrow{dx},$$

dU = - (work done by the spring force)

$$dU = -\left(\frac{-Kx^2}{2}\right) \; ; \; U_f - U_i = \frac{1}{2}Kx^2$$

Since U_i is zero when spring is at its natural length

$$\therefore \quad U_f = \frac{1}{2}Kx^2$$

Work – energy theorem

- Work done by all forces acting on a body is equal to change in its kinetic energy.

$$\text{i.e., } W = K_f - K_i = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$$

Where K_f and K_i are the final and initial kinetic energies of the body.

- The work-energy theorem is not limited to a single particle but can also be applied to a system of particles.
- When applied to a system of two or more particles, the change in kinetic energy of the system is equal to the work done on the system by both external and internal forces.
- The work-energy theorem is applicable to systems experiencing variable forces, pseudo forces, conservative forces, as well as non-conservative forces.

Applications of work-energy theorem :

- A body of mass m starting from rest acquire a velocity 'v' due to constant force F. Neglecting air resistance.

Work done = change in Kinetic energy $= \frac{1}{2}mv^2$

- A particle of mass 'm' is thrown vertically up with a speed 'u'. Neglecting the air friction, the work done by gravitational force, as particle reaches maximum height is $W_g = \Delta K = K_f - K_i$

$$W_g = \frac{1}{2}m(0)^2 - \frac{1}{2}m\times u^2 = -\frac{1}{2}mu^2$$

- A particle of mass 'm' falls freely from a height 'h' in air medium onto the ground. If 'v' is the velocity with which it reaches the ground, the work done by air friction is W_f and work done by gravitational force W_g then, $W_g + W_f = \frac{1}{2}mv^2 - 0 = \frac{1}{2}mv^2$
- A block of mass 'm' slides down a frictionless incline of inclination 'θ' to the horizontal. If h is the height of incline, the velocity with which body reaches the bottom of incline is

$$W_g = \Delta K \;;\; mgh = \frac{1}{2}mv^2 - 0 \Rightarrow mgh = \frac{1}{2}mv^2 \;;\; v = \sqrt{2gh}$$

- A body of mass 'm' starts from rest from the top of a rough inclined plane of inclination 'θ' and length 'ℓ'. The velocity 'v' with which it reaches the bottom of incline if μ_k is the coefficient of kinetic friction is $W_g + W_f = \Delta k$

$$(mg\sin\theta)\ell + (-\mu_k mg\cos\theta)\ell = \tfrac{1}{2}mv^2 - 0$$

$$v = \sqrt{2g\ell(\sin\theta - \mu_k\cos\theta)}$$

- A bob of mass m suspended from a string of length ℓ is given a speed u at its lowest position then the speed of the bob v when it makes an angle θ with the vertical is

$$W_g + W_T = \Delta K \Rightarrow -mg\ell(1-\cos\theta) + 0 = \frac{1}{2}m(v^2 - u^2)$$

$$v = \sqrt{u^2 - 2g\ell(1-\cos\theta)}$$

- A bullet of mass 'm' moving with velocity 'v' stops in a wooden block after penetrating through a distance x. If 'f' is the resistance offered by the block to the bullet.

$$W_f = K_f - K_i; -fx = 0 - KE_i$$

i.e., stopping distance $x = \frac{KE_i}{f} = \frac{mv^2}{2f} = \frac{P^2}{2mf}$

- A block of mass 'm' attached to a spring of spring constant 'K' oscillates on a smooth horizontal table. The other end of the spring is fixed to a wall. It has a speed 'v' when the spring is at natural length. The distance it moves on a table before it comes to rest is calculated as below

$W_{S\cdot F} + W_g + W_N = \Delta K$ (S.F = spring force)

Let the mass be oscillating with amplitude 'x'.

On compressing the spring $W_{S\cdot F} = -\frac{1}{2}Kx^2$

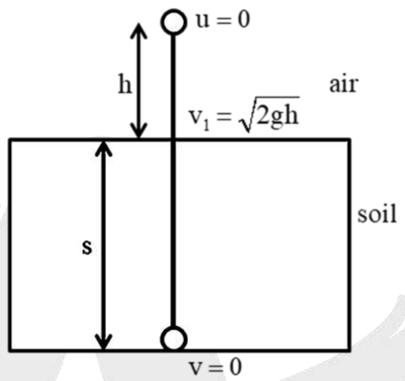
$W_g = FS\cos 90° = 0; W_N = NS\cos 90° = 0$

$$W_{S\cdot F} = K_f - K_i \Rightarrow -\frac{1}{2}Kx^2 = 0 - \frac{1}{2}mv^2 \Rightarrow x = v\sqrt{\frac{m}{K}}$$

- A pile driver of mass 'm' is dropped from a height 'h' above the ground. On reaching the ground it pierces through a distance 's' and then stops finally. If R is the average resistance offered by ground then

$$W_g + W_R = K_f - K_i = \frac{1}{2}mu^2 - \frac{1}{2}mv^2$$

$$mg(h+s)+(-Rs)=0; \;\; R = mg\left(1+\frac{h}{s}\right)$$



Here time of penetration is given by impulse equation $(R-mg)t = 0 - m\sqrt{2gh}$

- A body of mass 'm' is initially at rest. By the application of a constant force, its velocity changes to v_0 in time t_0 the kinetic energy of the body at time 't' is

$$W = \Delta K = K_f - K_i = K_f - 0$$

$$K_f = W = mas = ma\left(\frac{1}{2}at^2\right) = \frac{1}{2}ma^2t^2$$

Since $a = \frac{v_0}{t_0}$; $\quad K_f = \frac{1}{2}m\left(\frac{v_0}{t_0}\right)^2 t^2$

Types of Equilibrium

A body is said to be in translatory equilibrium, if net force acting on the body is zero i.e., $\vec{F}_{net} = 0$

If the forces are conservative $F = -\frac{dU}{dr}$

and for equilibrium F = 0,

so $-\frac{dU}{dr} = 0$ or $\frac{dU}{dr} = 0$.

$\therefore$ At equilibrium position, slope of U-r graph is zero or the potential energy is optimum (maximum or minimum or constant)

There are three types of equilibrium

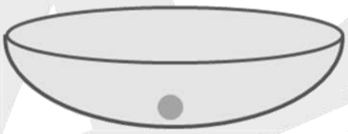
(i) Stable equilibrium

(ii) Unstable equilibrium

(iii) Neutral equilibrium.

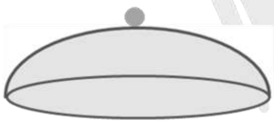
Stable equilibrium

1. Net force is zero
2. $\frac{dU}{dr} = 0$ or slope of U-r graph is zero
3. When displaced from its equilibrium position, a net retarding forces starts acting on the body, which has a tendency to bring the body back to its equilibrium position
4. PE in equilibrium position is minimum as compared to its neighbouring points as $\frac{d^2U}{dr^2}$ is positive
5. When displaced from equilibrium position the centre of gravity of the body comes down



Unstable equilibrium

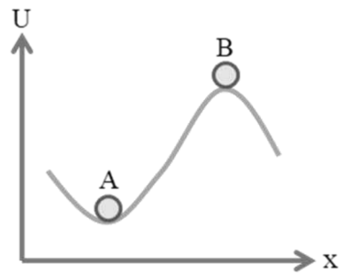
1. Net force is zero
2. $\frac{dU}{dr} = 0$ (or) slope of U-r graph is zero
3. When a body is displaced from its equilibrium position, a net force arises, causing the body to move in the direction of the displacement or away from the equilibrium position.
4. PE in equilibrium position is maximum as compared to other positions as $\frac{d^2U}{dr^2}$ is negative
5. When displaced from equilibrium position the centre of gravity of the body goes up



Neutral equilibrium

1. Net force is zero
2. $\frac{dU}{dr} = 0$ or slope of U-r graph is zero
3. When displaced from its equilibrium position, the body does not exhibit any tendency to return to its original position or move away from it.
4. PE remains constant even if the body is moving to neighbouring points $\frac{d^2U}{dr^2} = 0$
5. When displaced from equilibrium position the centre of gravity of the body remains constant

Potential energy and Equilibrium



In the figure, at A :

$\frac{dU}{dx} = 0$ and $\frac{d^2U}{dx^2}$ is positive

Thus at A the particle is in stable equilibrium.

At B; $\frac{dU}{dx} = 0$ and $\frac{d^2U}{dx^2}$ is negative

Thus at B the particle is in unstable equilibrium

Law of conservation of Mechanical energy :

➢ Total mechanical energy of a system remains constant, if only conservative forces are acting on a system of particles and the work done by all other forces is zero.

$\therefore \quad U_f - U_i = -W$

From work energy theorem $W = k_f - k_i$

$\therefore \quad U_f - U_i = -(k_f - k_i)$

$\therefore \quad U_f + k_f = U_i + k_i \Rightarrow U + K$ = constant

The sum of potential energy and kinetic energy remains constnt in any state.

➢ A body is projected vertically up from the ground.

When it is at height 'h' above the ground, its potential and kinetic energies are in the ratio $x : y$. If H is the maximum height reached by the body, then $\frac{x}{y} = \frac{h}{H-h}$ or $\frac{h}{H} = \frac{x}{x+y}$

POWER

➢ The rate of doing work is called power.

Power or average power is given by $P_{avg} = \frac{\text{work done}}{\text{time}}$, Power is a scalar

SI Unit : watt (W) (or) J/s, CGS Unit : erg/sec

Other Units : kilo watt, mega watt and horse power

One horse power (H.P) = 746 watt

- **Instantaneous Power :**

$$P = \underset{\Delta t \to 0}{Lt}\left(\frac{\Delta W}{\Delta t}\right)$$

It is also calculated by $P = FV\cos\theta = \vec{F}\cdot\vec{V}$

- **Relation Between P_{avg} and P_{ins} :**

$$P_{avg} = \frac{W}{t} = \frac{mv^2}{2t} = \frac{1}{2}mv\left(\frac{v}{t}\right) = \frac{1}{2}mav = \frac{1}{2}\vec{F}\cdot\vec{V}$$

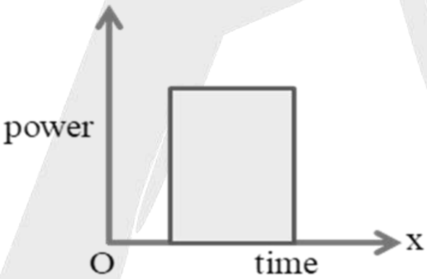
$$P_{avg} = \frac{1}{2}P_{inst}$$

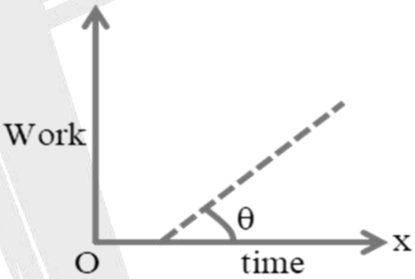
- The area undere P-t graph gives work done

$$P = \frac{dW}{dt} \qquad \therefore\ W = \int P\cdot dt$$

The slope of W-t curve gives instantaneous power

$$P = \frac{dW}{dt} = \tan\theta$$





Applications on power

- The power of a machine gun firing 'n' bullets each of mass 'm' with a velocity 'v' in a time interval 't' is given by $P = \dfrac{n\left(\frac{1}{2}mv^2\right)}{t} = \dfrac{nmv^2}{2t}$
- A crane lifts a body of mass 'm' with a constant velocity v from the ground, its power is

$$P = Fv = mgv$$

- Power of lungs of a boy blowing a whistle is $P = \frac{1}{2}$ (mass of air blown per sec) (velocity)^2
- Power of a heart pumping blood = (pressure) (volume of blood pumped per sec)

- A conveyor belt is moving with a constant speed 'v' horizontally and gravel is falling on it at a rate of $\frac{dm}{dt}$. Then additional force required to maintain speed v is $F = v\frac{dm}{dt}$ and additional power required to drive the belt is, $P = Fv = v^2\frac{dm}{dt}$
- When a liquid of density 'ρ' coming out of a hose pipe of area of cross section 'A' with a velocity 'v' strikes the wall normally and stops dead. Then power exerted by the liquid is $P = \frac{1}{2}\frac{mv^2}{t} = \frac{1}{2}\rho A v^3$

  ($\because$ mass = density $\times$ volume = m = $\rho \times A \times \ell$)
- A vehicle of mass 'm' is driven with constant acceleration along a straight level road against a constant external resistance 'R' when the velocity is 'v', power of engine is $P = Fv = (R + ma)v$
- If P is a rated power of a device and it its efficiency is x%, useful power is (output power) $P^1 = \frac{x}{100}P$
- If a motor lifts water from a well of depth 'h' and delivers with a velocity 'v' in a time t then power of the motor $P = \frac{mgh + \frac{1}{2}mv^2}{t}$
- If a body of mass 'm' starts from rest and accelerated uniformly to a velocity V_0 in a time t_0, then the work done on the body in a time 't' is given by

  $$W = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{v_0 t}{t_0}\right)^2; \ a = \frac{v_0}{t_0}; \quad v = at = \left(\frac{v_0}{t_0}\right)t$$

  Instantaneous power, $P = Fv = mav$

  $$\therefore \quad P = m\frac{v_0}{t_0}\left(\frac{v_0}{t_0}\right)t = m\frac{v_0^2}{t_0^2}t$$
- A motor pump is used to deliver water at a certain rate from a given pipe. To obtain 'n' times water from the same pipe in the same time by what amount of (a) force and (b) power of the motor should be increased.

If a liquid of density $'\rho'$ is flowing through a pipe of cross section 'A' at speed 'v' the mass coming out per second will be $\frac{dm}{dt} = Av\rho$.

To get 'n' time water in the same time

$$\left(\frac{dm}{dt}\right)^1 = n\left(\frac{dm}{dt}\right) \Rightarrow A'v'\rho' = n(Av\rho)$$

As the pipe and liquid are not changed,

$$\rho' = \rho\ ;\ A' = A\ \&\ v' = nv$$

$$\text{as } F = v\frac{dm}{dt} \Rightarrow \frac{F'}{F} = \frac{v^{|}\left(\frac{dm}{dt}\right)^1}{v\left(\frac{dm}{dt}\right)} = \frac{(nv)\left(n\frac{dm}{dt}\right)}{v\left(\frac{dm}{dt}\right)} = n^2$$

as $P = Fv \Rightarrow$

$$\frac{P'}{P} = \frac{F'v'}{Fv} = \frac{(n^2F)(nv)}{Fv} = n^3$$

$$\therefore \quad F' = n^2F \qquad \therefore \quad P' = n^3P$$

To get 'n' times of water force must be increased

n^2 times while power n^3 times.

Position and velocity of an automobile w.r.t. time :

An automobile of mass 'm' accelerates starting from rest, while the engine supplies constant power, its position and velocity changes w.r.t. time as

Velocity : As F v = P = constant

i.e. $m\frac{dv}{dt}v = P \qquad \left(F = m\frac{dv}{dt}\right)$

or $\int v\,dv = \int \frac{P}{m}dt$ on integrating we get $\Rightarrow \frac{v^2}{2} = \frac{P}{m}t + C_1$

As initially the body is at rest,

i.e. $v = 0$ at t = 0 $\Rightarrow\ C_1 = 0$;

$$v = \left(\frac{2Pt}{m}\right)^{1/2} \Rightarrow v\,\alpha\,t^{1/2}$$

Position : From the above expression

$$v=\left(\frac{2Pt}{m}\right)^{1/2}\ (\text{or})\ \frac{ds}{dt}=\left(\frac{2Pt}{m}\right)^{1/2}$$

$$\int ds=\int\left(\frac{2Pt}{m}\right)^{1/2}dt=\left(\frac{2p}{m}\right)^{\frac{1}{2}}\int t^{\frac{1}{2}}dt$$

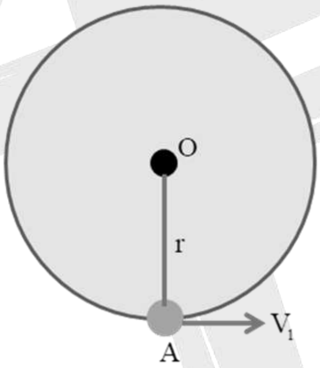
Integrating on both sides we get

$$S=\left(\frac{2P}{m}\right)^{1/2}\frac{2}{3}\cdot t^{3/2}+C_2$$

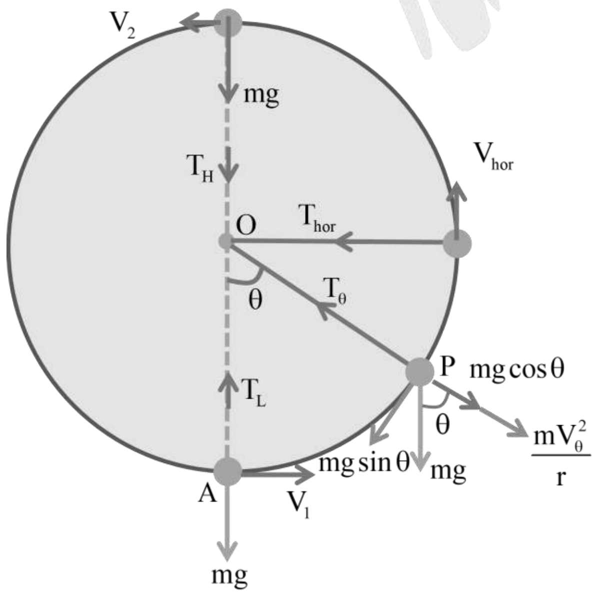
Now at t = 0, S = 0 $\Rightarrow C_2=0$

$$S=\left(\frac{8P}{9m}\right)^{1/2}t^{3/2},\quad \therefore S\alpha\, t^{3/2}$$

Vertical circular motion with variable speed :



Consider a body of mass 'm' tied at one end of a string of length 'r' and is whirled in a vertical circle by fixing the other end at 'O'. Let V_1 be the velocity of the body at the lowest point.



- **Velocity of the body at any point on the vertical circle :**

$$TE_A = TE_P \ ; \ \frac{1}{2}mV_1^2 + 0 = \frac{1}{2}mV^2 + mgh$$

$V^2 = V_1^2 - 2gh$, but $h = r(1-\cos\theta)$

$$V^2 = V_1^2 - 2gr(1-\cos\theta); \ \ V = \sqrt{V_1^2 - 2gr(1-\cos\theta)}$$

If V_2 is the velocity of the body at highest point $(\theta = 180°)$

$$V_2 = V_1^2 - 2gr(1+1) = \sqrt{V_1^2 - 4gr}$$

Tension in the string at any point :

- Let T be the tension in the string when the string makes an angle with vertical.

$$T = \frac{mV^2}{r} + mg\cos\theta$$

- (1) At the lowest point $\theta = 0°$ tension in the string is $T_L = \frac{mV_1^2}{r} + mg$ (maximum).
- (2) At the highest point $\theta = 180°$.

  The tension in the string is $T_H = \frac{mV_2^2}{r} - mg$ (minimum)
- (3) When the string is horizontal, $\theta = 90°$, tension in the string at this position is

$$T_{(hor)} = \frac{mV_{horz}^2}{r}$$

- (4) The difference in maximum and minimum tension in the string is

$$T_{max} - T_{min} = \frac{mV_1^2}{r} + mg - \frac{mV_2^2}{r} + mg$$

$$= \frac{m}{r}\left(V_1^2 - V_2^2\right) + 2mg$$

$$= \frac{m}{r}(4gr) + 2mg = 4mg + 2mg = 6mg$$

- (5) Ratio of maximum tension to minimum tension in the string is

$$\frac{T_{max}}{T_{min}} = \frac{\frac{mV_1^2}{r} + mg}{\frac{mV_2^2}{r} - mg} = \frac{V_1^2 + rg}{V_2^2 - rg}$$

- **When the particle is at 'P'**

(a) Tangential force acting on the particle is $F_t = mg\sin\theta$.

Tangential acceleration $a_t = g\sin\theta$

(b) Centripetal force acting on the particle is $F_c = \left(\frac{mV^2}{r}\right) = T - mg\cos\theta$.

Centripetal acceleration $a_c = \dfrac{V^2}{r}$

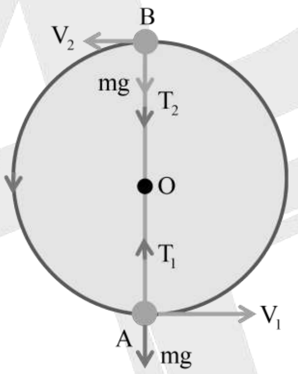
(c) Net acceleration of the particle at the point 'P' is $a = \sqrt{a_t^2 + a_c^2}$.

(d) The net force acting on the particle at point 'P' is $F = \sqrt{F_t^2 + F_c^2}$

- Angle made by net force or net acceleration with centripetal component is and

$$\tan\theta = \frac{F_t}{F_c} = \frac{a_t}{a_c}$$

Condition for vertical circular motion of a body



We know that $T_2 = \dfrac{mV_2^2}{r} - mg$

The body will complete the vertical circular path when tension at highest point is such that $T_2 \geq 0, \frac{mV_2^2}{r} - mg \geq 0; V\sqrt{gr}_{2min}$

Hence the minimum speed at highest point to just complete the vertical circle is $\sqrt{gr}$

From the law of conservation of mechanica energy total energy at lowest point A = total energy at highest point B

$U_A + KE_A = U_B + KE_B$

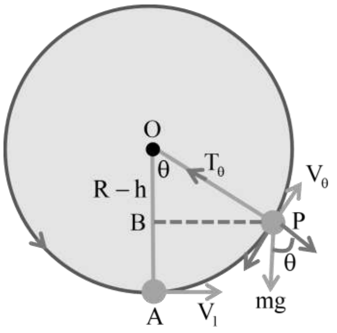
$0 + \frac{1}{2}mV_1^2 = mg(2r) + \frac{1}{2}mV_2^2$

$\frac{1}{2}mV_1^2 = 2mgr + \frac{1}{2}mgr[\because V_2 = \sqrt{gr}]$

$= \dfrac{5}{2}mgr \Rightarrow V_1 = \sqrt{5gr}$

For the body to continue along a circular path the critical velocity at lowest point is $\sqrt{5gr}$

Critical velocity at any point on the vertical circle :



From the Law of conservation of energy total energy at point 'A' = total energy at point P

$$U_A + KE_A = U_P + KE_P$$

$$O + \frac{1}{2}mV_1^2 = mgh + \frac{1}{2}mV_\theta^2$$

$$\frac{1}{2}m(5gR) = mgR(1-\cos\theta) + \frac{1}{2}mV_\theta^2$$

$$\frac{5gmR}{2} = mgR - mgR\cos\theta + \frac{1}{2}mV_\theta^2$$

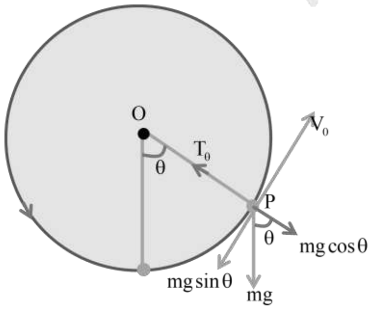
$$\frac{5gmR}{2} - mgR + mgR\cos\theta = \frac{1}{2}mV_\theta^2$$

$$\frac{mgR}{2}[3+2\cos\theta] = \frac{1}{2}mV_\theta^2$$

$$V_\theta = \sqrt{gR(3+2\cos\theta)}$$

Minimum tension in the string to just complete vertical circle :

- Let T_θ be the tension in the string when the string is making an angle θ from lowest point



$$T_\theta = mg\cos\theta + \frac{mV_\theta^2}{R} = mg\cos\theta + \frac{m}{R}gR(3+2\cos\theta)$$

$$= mg\cos\theta + 3mg + 2mg\cos\theta$$

$$= 3mg\cos\theta + 3mg = 3mg(1+\cos\theta)$$

- In case of non uniform circular motion in a vertical plane if velocity of the body at the lowest point is less than $\sqrt{5gr}$, the particle will not complete the circle in vertical plane, the particle may either oscillate about the lowest point or it leaves the circle with out looping.

Condition for oscillating about the lowest position :

(1) If $0 < V_L < \sqrt{2gr}$, in this case, velocity becomes zero before tension vanishes and the particle oscillates about its lowest position with angular amplitude $0° < \theta < 90°$

(2) If velocity of the body at the lowest point $V_L < \sqrt{2gr}$, then the maximum height reached by the body just before its velocity becomes zero is given by $h = \frac{V_L^2}{2g}$.

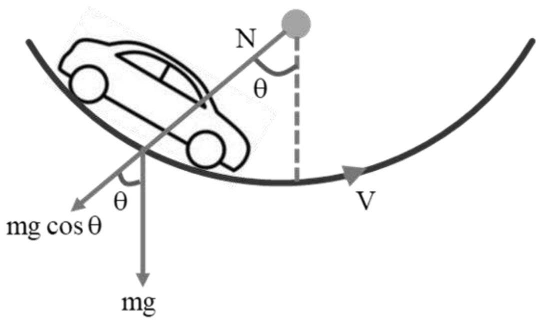
(3) The angle made by the string with the vertical when its velocity becomes zero is given by

$$\cos\theta = 1 - \frac{V_L^2}{2gr}$$

Note : If $0 < V_L \le \sqrt{2gr}$ then the particle oscillates such that $0° < \theta \le 90°$

Condition for leaving the circular path without looping :

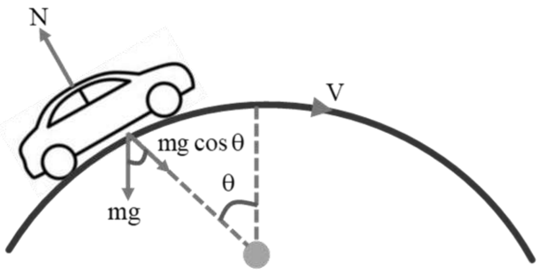
- If $\sqrt{2gr} < V_L < \sqrt{5gr}$, the particle is not able to complete the vertical circle, it goes to certain height and leaves the circular path $(90° < \theta < 180°)$ while leaving the circular path $T = 0$ but $V \ne 0$
- The angle made by the string with downward vertical when the tension in the string becomes zero is given by $\cos\theta = \frac{2}{3} - \frac{V_L^2}{3gr}$
- The height at which the tension in the string becomes zero is given by $h = \frac{V_L^2+gr}{3g}$
- When car moves on a concave bridge of radius



Centripetal force $= N - mg\cos\theta = \frac{mv^2}{r}$

and normal reaction $N = mg\cos\theta + \frac{mv^2}{r}$

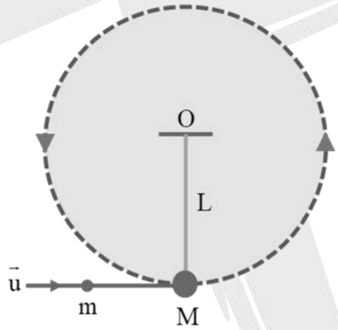
➢ When car moves on a convex bridge of radius r



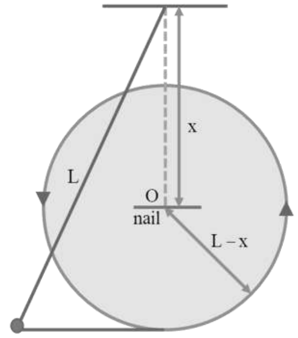
Centripetal force $N = mg\cos\theta + \frac{mv^2}{r}$

and normal reaction $N = mg\cos\theta - \frac{mv^2}{r}$

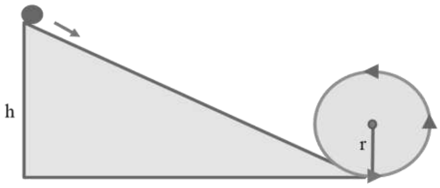
➢ A ball of mass 'M' is suspended vertically by a string of length 'L'. A bullet of mass 'm' is fired horizontally with a velocity 'u' onto the ball, sticks to it. For the system to complete the vertical circle, the minimum value of 'u' is given by $u = \frac{(M+m)}{m}\sqrt{5gL}$



➢ A nail is fixed at a certain ditance 'x' vertically below the point of suspension of a simple pendulum of length L. The bob is released when the string makes and angle θ with vertical. The bob reaches the lowest position then describes a vertical circle whose centre coincides with the nail. Then $x_{min} = \frac{L(3+2\cos\theta)}{5}$



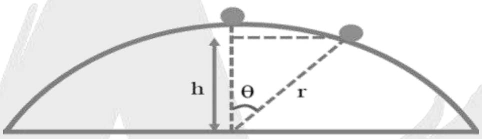
- A body of mass 'm' is allowed to slide down from rest, from the top of a smooth incline of height 'h'. For the body to move in a loop of radius 'r' on arriving at the bottom.



(a) Minimum height of smooth incline $h = \left(\frac{5r}{2}\right)$

(b) 'h' is independent of mass of the body

- A small body is freely sliding down from the top of a smooth convex hemisphere of radius r, placed on a table with its flat face on the table then



(a) Normal reaction on the body is zero at the instant the body leaves the hemisphere.

(b) the vertical height from table at which the body leaves the hemisphere is h = 2r/3

(c) If the position vector of the body with respect to the centre of curvature makes an angle θ with vertical when the body leaves the hemisphere, then $\cos\theta = 2/3$

(d) velocity of block at that instant is $V = \sqrt{\frac{2gr}{3}}$

(e) If the block is given a horizontal velocity 'u' from the top of the smooth convex hemisphere then the angle θ with vertical at which the block leaves hemisphere is $\cos\theta = \frac{2}{3} + \frac{u^2}{3gr}$

EXERCISE – 1

1. The total work done on a particle is equal to the change in its kinetic energy:

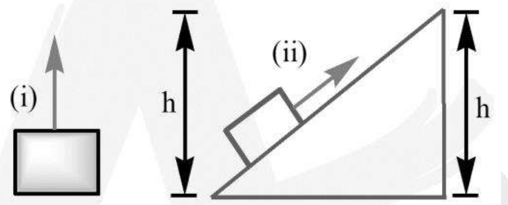
(A) always

(B) only if the forces acting on the body are conservative

(C) only if the forces acting on the body are gravitational

(D) only if the forces acting on the body are elastic

2. You wish to lift a heavy block through a height h by attaching a string of negligible mass to it and pulling so that it moves at a constant velocity. You have the choice of lifting it either by pulling the string (i) vertically upward or (ii) along a frictionless inclined plane (see diagram). Which one of the following statements is true?



(A) The magnitude of the tension force in the string is smaller in case (i) then in case (ii)

(B) The work done on the block by the tension force is the same in both cases

(C) The work done on the block by the tension force is smaller in case (ii) than in case (i)

(D) The work done on the block by the gravitational force is smaller in case (ii) than in case (i)

3. When the momentum of a body increases by 100%, its KE increases by:

(A) 400% (B) 100% (C) 300% (D) none of these

4. A ball is released from the top of a tower. The work done by force of gravity in 1^{st} second, 2^{nd} second, 3^{rd} second of the motion of the ball is

(A) 1 : 2 : 3 (B) 1 : 4 : 16 (C) 1 : 3 : 5 (D) 1 : 9 : 25

5. **Statement – 1**: while running on a straight track with increasing speed work done by friction is not zero.

Statement – 2: work done by all the forces on a system is equal to the change in kinetic energy of the system.

(A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1

(B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1

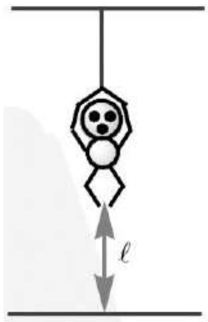
(C) Statement-1 is true, statement-2 is false

(D) Statement-1 is false, statement-2 is true

6. A Particle moves along the y-axis of a coordinate system, With a force component $F_y = \left(2N/m^3\right)y^3$ acting on it .As the particle moves from the origin toy=3m, how much work is done on it by the force?

(A) -162 J (B) 40.5 J (C) -40.5 J (D) 162 J

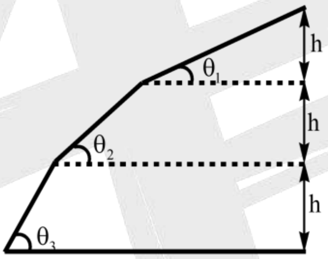
7. One end of a light rope is tied directly to the ceiling. Aman initially at rest on the ground starts climbing the rope hand over hand upto a height ℓ.



From the time he starts at rest on the ground to the time he is hanging at rest at a height ℓ, how much work was done on the man by the rope?

(A) 0
(B) $Mg\ell$
(C) $-Mg\ell$
(D) It depends on how fast the man goes up.

8. A block slides over 3 rough inclined plane having equal coefficient of friction and equal height but different slopes. The increase in kinetic energy will be greatest for:

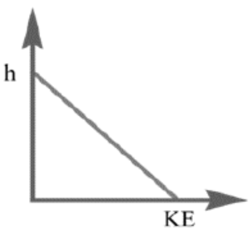


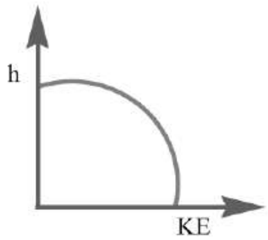
(A) slope-1 (B) slope-2 (C) slope-3 (D) equal for all

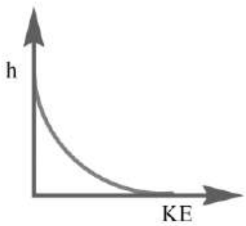
9. A body of mass m, accelerates uniformaly from rest to v_1 in time t_1. The instantaneous power delivered to the body as a function of time t is:

(A) $\frac{mv_1^2 t}{t_1^2}$ (B) $\frac{mv_1^2 t}{2t_1}$ (C) $\frac{mv_1 t^2}{t_1}$ (D) $\frac{mv_1^2 t}{t_1}$

10. Which of the following graphs depict the variation of kinetic energy of a ball bouncingelastically on a horizontalfloor, with height? (Neglect air resistance)

(A)


(B)


(C)


(D) None of These

11. An elevator of total mass (elevartor + passenger) 1800 kg is moving up with a constant speed of 2m/s. A frictional force of 4000N opposes its motion. Determine the instantaneous power (kW) delivered by the motor to the elevator.

12. A body is moving with a speed of 1m/s and constant force F is needed to stop it in distance x. If the speed of body is 3m/s then constant force needed to stop it in same distance x would be:

(A) 1.5 F (B) 3 F (C) 6 F (D) 9 F

13. The cause of increase in kinetic energy when a man start running without his feet slipping on ground is asked from two students.

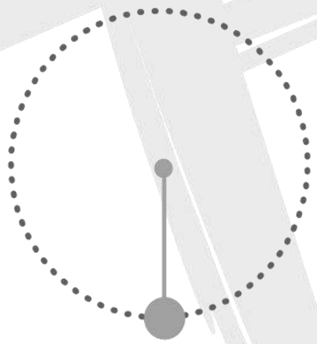
Ricky : Cause of increase in kinetic energy is work done by friction force without which he cannot run.

Anil : Cause of increase in kinetic energy is work done by internal force of the body.

(A) Ricky is correct, Anil is wrong (B) Anil is correct, Ricky is wrong

(C) Both are correct (D) Both are wrong

14. A body is tied to one end of a light inextensible string and is moving in a vertical circle, the other end of the string being fixed at the centre. Then (at all instants) (In the absence of air resistance)



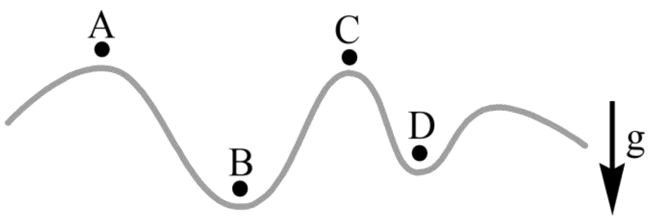
(A) Acceleration of the body is directed towards the centre

(B) Total mechanical energy of the body-earth system remains constant

(C) Tension in the string remains constant

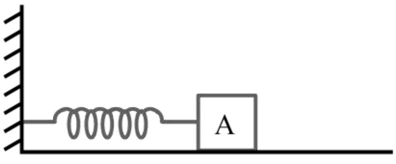
(D) Acceleration of the body remains constant

15. Different locations of a cyclist moving with constant speed on a road of hilly region are as shown in the figure below. At which of these locations would he feels heaviest?



(A) A (B) B (C) C (D) D

16. A block of mass 1 kg is pressed against a spring of force constant 400 N/m. The spring is compressed by 10 cm and block is released. Which of the following is a possible velocity of the block during subsequent motion ?



(A) 2 m/s (B) 1 m/s (C) 3 m/s (D) 4 m/s

17. An elevator is rising at constant speed. Select the correct statement(s) :

(A) The upward cable force is constant

(B) The kinetic energy of the elevator is constant

(C) The gravitational potential energy of the Earth-elevator system is constant

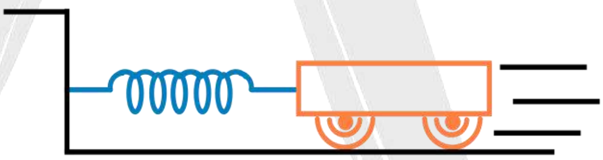
(D) The mechanical energy of the Earth-elevator system is constant

18. An escalator is moving down with constant speed. You are moving on it such that you remain at rest with respect to ground. Choose correct statements from ground frame :

(A) work done by you is zero (B) work done by escalator on you is zero

(C) work done by gravity on you is zero (D) work done by escalator on man is negative

19. A cart is rolling at constant speed towards a fixed spring. What can you say about the velocity and acceleration of the cart after the cart hits the spring, but before it stop instantaneously?



(A) The velocity is to the right (B) The velocity is to the left

(C) The acceleration is to the right (D) The acceleration is to the left

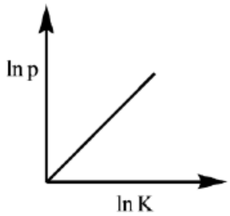
20. The potential energy (in joules) function of a particle in a region of space is given as : $U = \left(2x^2 + 3y^2 + 2z\right)$

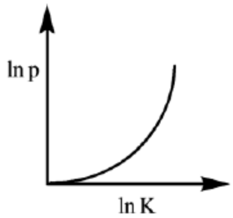
Here x, y and z are in metres. Find the magnitude of x component of force acting on the particle at point P (1 m, 2 m, 3 m).

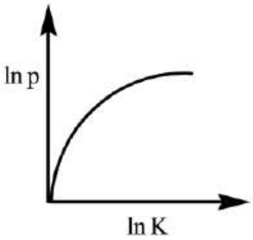
(A) 2 N (B) 4 N (C) 6 N (D) 8 N

EXERCISE – 2

1. Which of the following graphs best represents the graphical relation between momentum (p) and kinetic energy (K) for a body in motion?

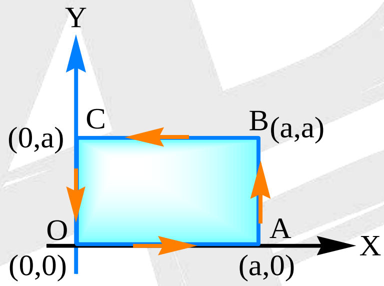
(A)


(B)


(C)


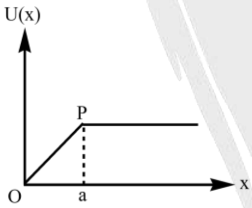
(D) None of these

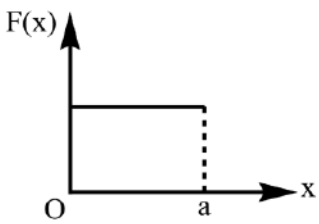
2. The work done by the force $\vec{F} = x\hat{i} + y\hat{j}$ around the path shown in the figure is:

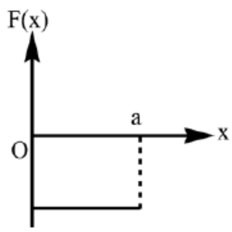


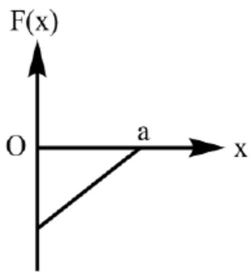
(A) $\frac{2}{3}a^3$ (B) zero (C) a^3 (D) $\frac{4}{3}a^3$

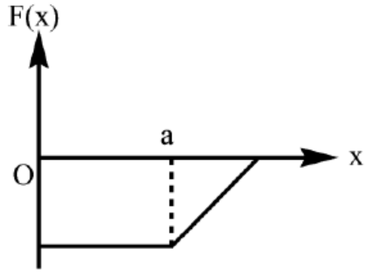
3. The $U(x) - x$ curve for system is shown in the figure. Its force curve will be:



(A)


(B)


(C)


(D)


4. Two conservation force, $\vec{F}_1$ and $\vec{F}_2$, act on an object. What is the relationship between $W_+ = \oint(\vec{F}_1 + \vec{F}_2).d\vec{s}$ and $W_- = \oint(\vec{F}_1 - \vec{F}_2).d\vec{s}$?

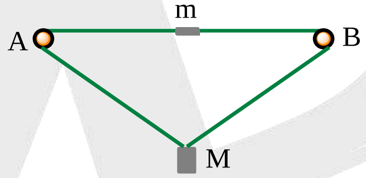
(The circle on the tangential symbol means that the integral is to be evaluated around a closed path)

(A) $W_+ > W_-$ (B) $W_+ = W_- \neq 0$ (C) $W_+ = W_- = 0$ (D) $W_+ < W_-$

5. A particle moving along the x-axis is acted upon by a single force $F = F_0 e^{-kx}$, where F_0 and k are constants. The particle is released from rest at x = 0. It will attain a maximum kinetic energy of:

(A) $\frac{F_0}{k}$ (B) $\frac{F_0}{e^k}$ (C) kF_0 (D) $\frac{1}{2}(kF_0)^2$

6. An endless inextensible string passes over two smooth pegsA and B, AB being horizontal. Two particles of mass M and m(M>m) are tied symmetrically to the two parts of the strings as shown. The particle of mass m is released when it is in level with the pegs. As it comes down M rises up vertically and when they cross each other.



(A) The speed of M > the speed of m (B)The speed of M = the speed of m

(C) the speed of M< the speed of m (D) The ratio of their speeds is M: m

7. Potential energy of a system is given by $U(x) = (x+1)(x+2)$. Then:

(A) Point $x = -\frac{3}{2}$ corresponds to equilibrium position of the system

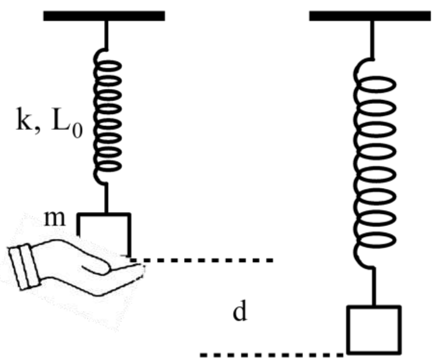
(B) Point $x = -1$ and $x = -2$ corresponds to equilibrium position of the system

(C) System is in stable equilibrium position at the $x = -\frac{3}{2}$

(D) System is in unstable equilibrium position at the $x = -\frac{3}{2}$

8. A spring having a relaxed length $L_0 = 25\text{cm}$ has a block attached to one end, while the other end is attached to a ceiling as shown. A person slowly lowers the block a distance d until the block just hangs without moving, as shown in the second diagram. Which statements below are true ?

(Mark ALL that apply. Use k = 2 N/cm and m = 600 g.)



(A) The distance d is 6 cm

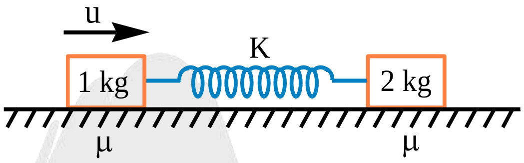
(B) The potential energy stored in the spring when the block is simply hanging from it is 0.09 J

(C) The change in gravitational potential energy of the earth-block system is $-0.18\,\text{J}$

(D) The decrease in gravitational potential energy is equal (in magnitude) to the increase in potential energy stored in the spring

9. Two blocks of masses 1 kg and 2 kg are connected by an ideal unstretched spring of spring constant 2 N/m and is kept on a rough horizontal surface with friction coefficient $\mu = \frac{1}{2}$.

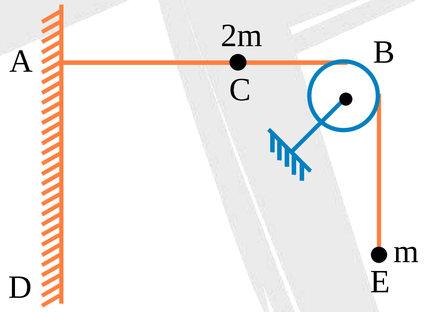
Now the left block is imparted a velocity u towards right as shown. The largest value of u such that the block of mass 2 kg never moves is



Fixed rough horizontal surface

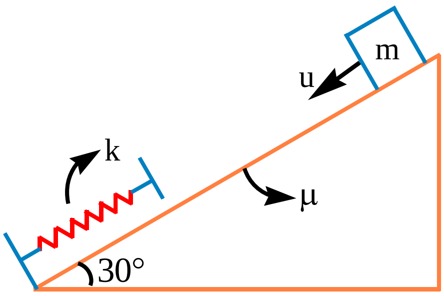
(A) 2.5 mn/s (B) 5 m/s (C) 7.5 m/s (D) 10 m/s

10. In the figure shwon two small spheres are arranged with string and pulleys. Part ACB is horizontal and BE vertical. AC is 3 meters and CB is 1 meter. The system is released from rest. When the heavier sphere is about to strike the vertical wall AD, find the sum of kinetic energies of the two spheres is



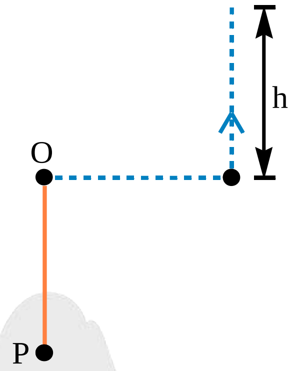
(A) mg (B) 2 mg (C) 3 mg (D) 4 mg

11. A block of mass 1 kg is released from top of a rough incline having $\mu = \frac{1}{\sqrt{3}}$. The initial speed of block is 2 m/s. The incline plane is of unknown length and has a spring of constant k = 1N/m connected at base as in figure. Find the maximum compression of spring (answer in meter).



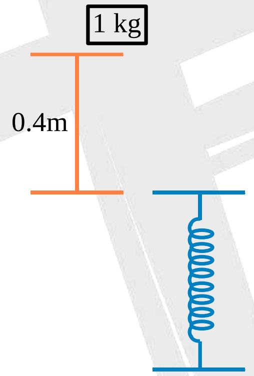
(A) 1 m (B) 2 m (C) 3 m (D) 4 m

12. A mass is attached to one end of a massless string, the other end of which is attached to a fixed support. Length of the pendulum is 25 cm. At lower most position P, pendulum is given velocity 5 m/s. When string becomes horizontal, string is cut. Find maximum height, achieved by the pendulum from the horizontal position of the string. $\left(g = 10\,m/s^2\right)$



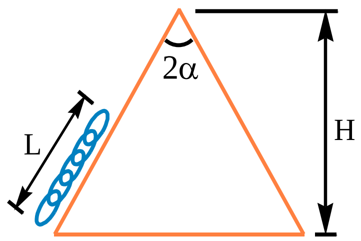
(A) 1 m (B) 1.5 m (C) 2 m (D) 2.5 m

13. A block of mass m = 1 kg falls from a height h = 0.4 m on a massless spring of stiffness constant k = 300 Nm. If $g = 10\,m/s^2$, then, find the maximum acceleration of the block is



(A) $25\ m/s^2$ (B) $50\ m/s^2$ (C) $75\ m/s^2$ (D) $100\ m/s^2$

14. An inextensible, flexible and homogeneous chain length L = 2 m can move along the incline having a shape of an isosceles triangle with an apex angle 2α (where $\alpha = 37°$) and located in a vertical plane. There is no friction between the chain and the incline. Find the lowest initial speed of the chain necessary to overcome the inclined hill of height H = 3 m. At the initial moment the position of the chain is shown in the figure.

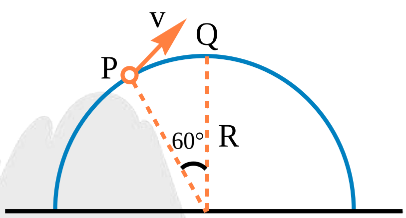


(A) 2 m/s (B) 4 m/s (C) 6 m/s (D) 8 m/s

15. A single conservative force F(x) acts on 1 kg particle that moves along x-axis. The potential energy U(x) is given by $U(x)=(x-2)^2-20$. Where x is in meters and U(x) in joules. If particle has a kinetic energy of 20 J at x = 5 m, find mechanical energy at x = 2 m.

(A) 3 J (B) 6 J (C) 9 J (D) 12 J

16. A particle is given a certain velocity v at a point P as shown on a hemispherical smooth surface. The value of v, such that particle when reaches Q the normal reaction of surface is equal to particle's weight, is : $\left[R=1.6\text{m},\ g=10\text{m/s}^2\right]$



(A) 2 m/s (B) 4 m/s (C) 6 m/s (D) 8 m/s

17. A mass of 2 kg slides down $\frac{1}{4}$ circular track of radius 1 m. If the speed of mass at the bottom is 4ms^{-1}, find the work done by the frictional force is $\left[g=10\text{ms}^2\right]$

(A) -2J (B) $+2\text{J}$ (C) -4J (D) $+4\text{J}$

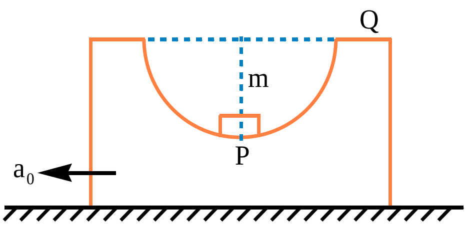
18. A pendulum bob has a speed of 3ms^{-1} at its lowest position. The pendulum is 0.5 m long. Find the speed of the bob, when the length makes an angle of $60°$ to the vertical. $\left(g=10\text{ms}^{-2}\right)$

(A) 1 m/s (B) 2 m/s (C) 3 m/s (D) 4 m/s

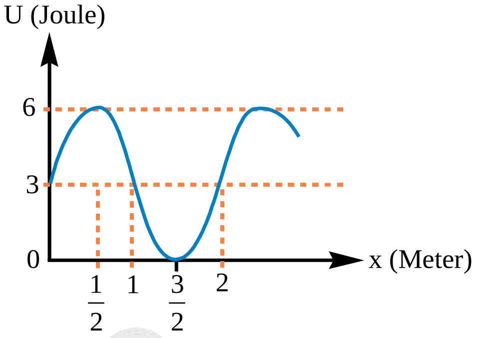
19. A small ball of mass m is attached to the end of the string of length $\ell=1\text{m}$ whose other end is fixed. From its lowest position, the ball is given a kinetic energy $mg\ell/5$. The net acceleration of the ball at the instant when the string makes an angle θ of $37°$ with the vertical is

(A) 2m/s^2 (B) 4m/s^2 (C) 8m/s^2 (D) 6m/s^2

20. A small block of mass m is lying at rest at point P of a wedge having a smooth semi circular track of radius R. The minimum value of horizontal acceleration a_0 of wedge so that mass can just reach the point Q. Find the value of $\left(\frac{a_0}{g}\right)$.



21. Potential energy (sinusoidal curve) is shown graphically for a particle. The potential energy does not depend on y and z coordinates. For range $0 < x < 2$ maximum value of conservative force (in magnitude) is $\beta\pi$. Find the value of β. [Here this force is corresponding to above potential energy and all units are in S.I.]



(A) 1 (B) 2 (C) 3 (D) 4

22. Two small body of mass $m_2 = 6\,kg$ and $m_1 = 2\,kg$ is connected by a massless thread and placed on a fixed smooth cylindrical surface of radius R = 0.5 m as shown in Figure (a). After the body is released from rest, they begin to move and attain the position shown in figure (b). Find the decrease in gravitational potential energy of the system.

Take $\alpha = 53°;\ \cos 53° = 0.6,\ \cos 26.5° = 0.9$.

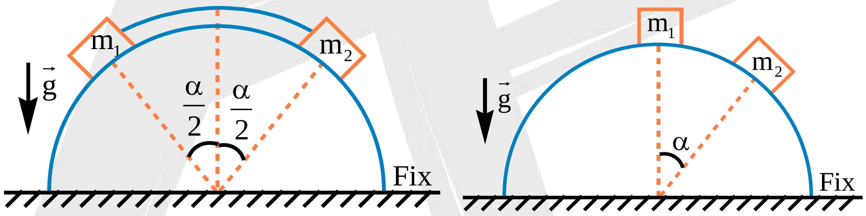
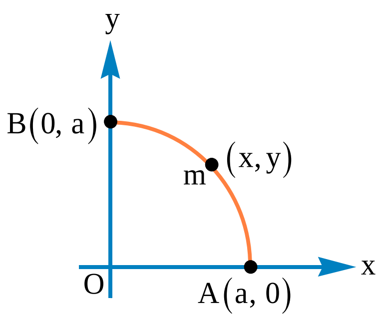


Figure (a) Figure (b)

(A) 8 J (B) 6 J (C) 4 J (D) 2 J

23. A particle of mass ‘m’ moves along the quarter section of the circular path whose centre is at the origin. The radius of the circular path is ‘a’. A force $\vec{F} = y\hat{i} - x\hat{j}$ newton acts on the particle, where x, y denote the coordinates of position of the particle. Find the magnitude of work done by this force in taking the particle from point A (a, 0) to point B (0, a) along the circular path is



(A) $\frac{\pi a^2}{2}$ (B) $\frac{\pi a^2}{3}$ (C) $\frac{2\pi a^2}{3}$ (D) $\frac{3\pi a^2}{2}$

24. A body of mass m = 0.1 kg is connected to a massless and intextensible thread of length $\ell = 1\text{m}$. It rotates in a vertical circle such that the center of circle is 2 m above the floor. When passing through the lowest position, the thread breaks and the body falls on the floor at a distance of 4 m (horizontal) from the point of breakage. Determine the tension in thread just before it breaks.

(A) 3 N (B) 12 N (C) 6 N (D) 9 N

25. From the top of tower of height 80 m, a body is projected up with velocity 50 m/s at an angle of inclination of $37°$. If mass of the body is m = 0.02 kg and acceleration due to gravity is $10\,\text{m/s}^2$, then find instantaneously power supplied by gravitational force on the body just after the 7 s of projection.

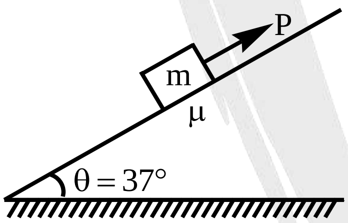
(A) 8 W (B) 1 W (C) 6 W (D) 2 W

26. A train of mass 100 metric tons is ascending uniformly on an incline of 1 in 250, and the resistance due to friction, etc is equal to 60 kg per metric ton. If the engine be of 7.84×10^4 watts and be working at full power, find the speed at which the train is going.

(A) 1 m/s (B) 9 m/s (C) 8 m/s (D) 3 m/s

27. A block of mass m is being pulled up the rough incline, inclined at an angle $37°$ with horizontal by an agent delivering constant power P. The coefficient of friction between the block and the incline is μ. Find the maximum speed of the block during the course of ascent.

[Take : $P = 60\,\text{W},\ m = 1\text{kg},\ \mu = 0.5$]



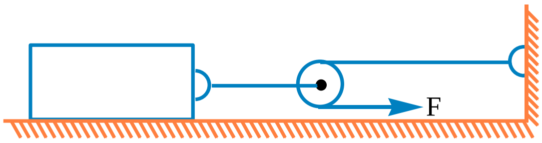
(A) 6 m/s (B) 2 m/s (C) 1 m/s (D) 7 m/s

28. Power supplied to a particle of mass 2 kg varies with time as P = 2t watt, t is in second. Velocity of particle at t = 0 is 6 m/s. What is the velocity of particle at time $t = \sqrt{13}\,\text{s}$?

(A) 3.5 m/s (B) 4.5 m/s (C) 6 m/s (D) 7 m/s

29. The block bas mass m and rests on a surface for which the coefficient of friction μ. If a force $F = kt^2$ is applied to the cable (see figure), find the power 'P' developed by the force F at $t = t_2$ is

(Given : $M = 20\text{kg},\ \mu = 0.4,\ k = 40\,\text{N/s}^2,\ t_2 = 3\text{sec}$.)



(A) 16.2 kW (B) 14.2 kW (C) 17.2 kW (D) 19.2 W

30. The total mass of a cyclist and her bicycle is 120 kg. While pedalling, she generates power of 640 W. Her motion is opposed by road resistance of magnitude 16 N and air resistance of magnitude 8v N. Where v in m/s is her speed. Find the greatest speed that she can maintain on a horizontal road.

(A) 1 m/s (B) 6 m/s (C) 8 m/s (D) 3 m/s

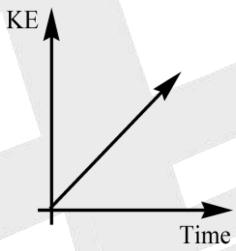
EXERCISE – 3

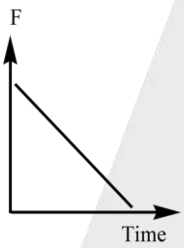
1. A body is moving uni-directionally under the influence of a source of constant power. Its displacement in time t is proportional to $t^{\alpha/\beta}$. Find $\alpha+\beta$?

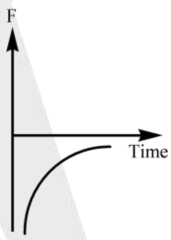
2. A particel is projected along a horizontal field whose coefficient of friction varies as $\mu=\frac{A}{r^2}$ where r is the distance from the origin in meters and A is a positive constant. The initial distance of the particle is 1m from the origin and its velocity is radially outwards. The minimum initial velocity at this point so that particle never stops is (if the given friction condition continues):

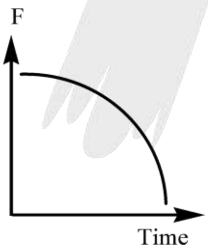
(A) ∞ (B) $2\sqrt{gA}$ (C) $\sqrt{2gA}$ (D) $4\sqrt{gA}$

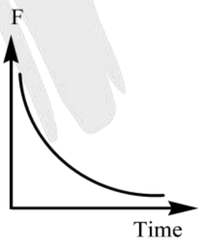
3. The kinetic energy (KE) vs time graph for a particle moving along a straight line is shown in the figure. The force vs time graph for the partice may be:



(A)


(B)


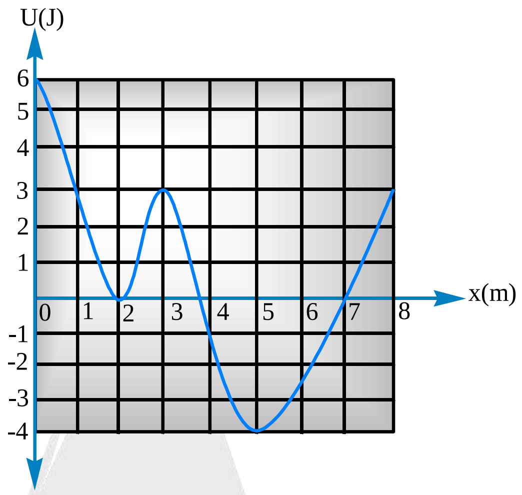
(C)


(D)


4. A Cannon ball of mass m is fired with an initial velocity $\vec{u}=u_x\hat{i}+u_y\hat{j}$, which makes an angle $\theta=\tan^{-1}\left(\frac{u_y}{u_x}\right)$ with respect to the horizontal. What is the work done by gravity on the cannon ball till it reaches the peak (i.e., highest elevation) of its trajectory? (Consider y-axis along vertical)

(A) $\frac{1}{2}mu_y^2$ (B) $\frac{1}{2}mu_x^2$ (C) $-\frac{1}{2}mu_y^2$ (D) $-\frac{1}{2}mu_x^2$

5. potential energy curve U of a particle as function of the position of a particle is shown. The particle has total mechanical energy E of 3.0 J. Then select correct alternative



(A) It can never be present at $x = 0\,m$

(B) It can never be present at $x = 5\,m$

(C) At x = 2 its kinetic energy is 0 J

(D) At x = 1 its kinetic energy 3 J

6. The potential energy of a body is given by $U = \frac{9}{x^2} - \frac{2}{x}$. The position at which it's speed can be maximum is:

(A) $x = +3m$ (B) $x = -3m$ (C) $x = 9m$ (D) $x = -9m$

7. A ball is suspended from the top of a cart by a string of length 1.0 m. The cart and the ball are initiallymoving to the right at constant speed V, as show in figure

(1). The cart comes to rest after colliding and sticking to a fixed bumper, as in figure

(2). The suspended ball swings through a maximum angle $60°$.

The initial speed V is $\sqrt{\alpha}\ m/s$. Find α ? (Take g = $10\ m/s^2$)

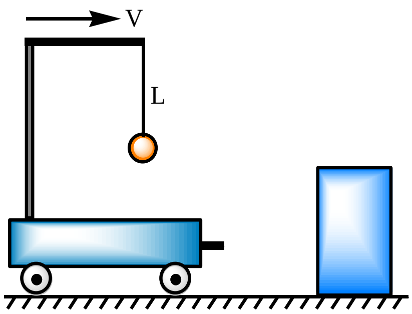


Figure (1)

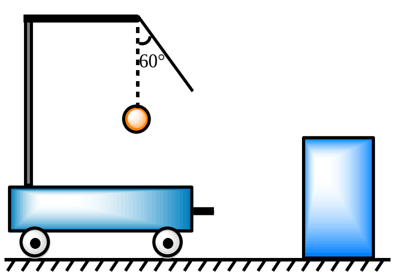
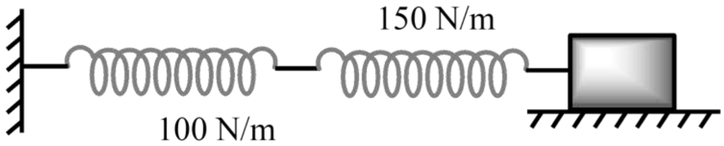


Figure (2)

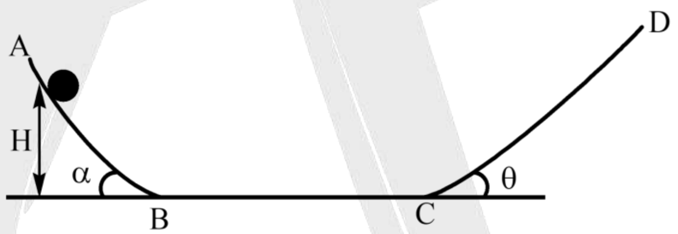
8. Two springs of force constant 100 N/m and 150 N/m are in series as shown. The block is pulled by a distance of 2.5 cm to the right from equilibrium position. What is the ratio of work done by the spring at left to the work done by the spring at right?



(A) $\frac{3}{2}$ (B) $\frac{2}{3}$ (C) 0.2 (D) None of these

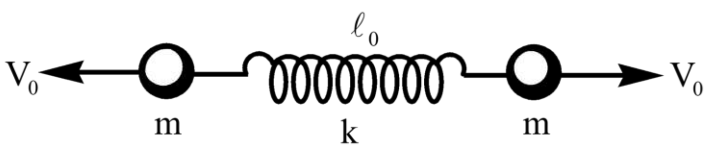
9. A block of mass m is pushed up against a spring, compressing it a distance x, and the block is then released. The spring projects the block along a frictionless horizontal surface, giving the block a speed v. The same spring projects a second block of mass 4m, giving it a speed $3v$. The distance was the spring compressed in the second case is nx. Find n?

10. A body is released from rest at height from the bottom of the crate shown. The portion AB of crate has fixed inclination α. The very long portion CD can be set into inclinations $\theta_1 = 30°, \theta_2 = 45°$ and $\theta_3 = 60°$. The body always remain in contact with the crate and rises upto heights h_1, h_2, h_3 respectively from the bottom for the given angles. All the surfaces are frictionless.Which of the follwing is correct?



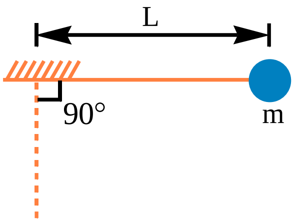
(A) $h_1 > h_2 > h_3$ (B) $h_1 < h_2 < h_3$ (C) $h_1 = h_3 < h_2$ (D) $h_1 = h_2 = h_3$

11. A sysetm comprises of two small spheres with same masses 'm'. The spring is non deformed. The spheres are set in motion in gravity free space at the velocities as shown in the diagram. The maximum elastic potential energy stored in the system is :



(A) $\frac{mv_0^2}{2\sqrt{2}}$ (B) mv_0^2 (C) $\frac{1}{2}mv_0^2$ (D) $2mv_0^2$

12. A mass m is pulled outward until the string of length L to which it is attached makes a $90°$ angle with the vertical. The mass is released from rest and swings through a circular arc. What is the tension in the string when the mass swings through the bottom of the arc?

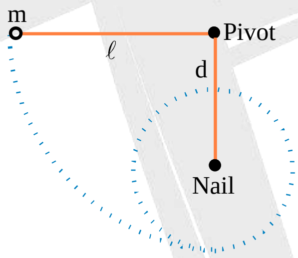


(A) 0 (B) mg (C) 2 mg (D) 3 mg

13. A ball of mass m is hung on a thread. The thread is held taut and horizontal, and the ball is released as shown. At what angle between the thread and vertical will the tension in thread be equal to weight in magnitude?

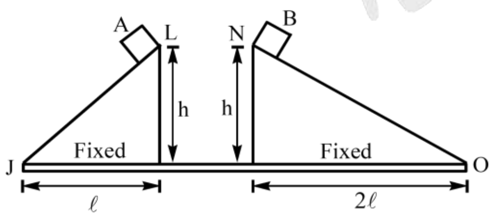
(A) 30° (B) $\cos^{-1}\left(\frac{2}{3}\right)$ (C) $\cos^{-1}\left(\frac{1}{3}\right)$ (D) Never

14. A pendulum of mass m and length ℓ is released from rest in a horizontal position. A nail, a distance d below the pivot, causes the mass to move along the path indicated by the dotted line. The minimum distance such that the mass will swing completely round in the circle shown in figure is d then $\frac{5d}{L}$ is:



(A) 2 (B) 3 (C) 4 (D) 5

15. Two identical blocks A and B are placed on two inclined planes as shown in diagram. Neglect air resistance and friction.



Read the following statement and choose the correct options:

STATEMENT-1: Kinetic energyof "A" on sliding to J will be greater than the kinetic energy of B on falling vertically to M.

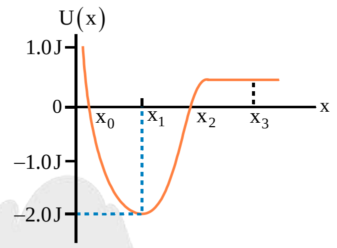
STATEMENT-2: Acceleration of "A" will be greater than acceleration of "B" when both are released to slide on inclined planeds.

STATEMENT-3: Work done by external agent to move block slowlyfrom position B to O is negative.

(A) Statement I is true (B) Statement II is true

(C) Statement III is true (D) No statement is true

16. Aconservative force has the potential energy function U(x) as shown by the graph.A particle moving in one dimension under the influence of this forece has kinetic energy 1.0 J when it is at position x_1. Which of the following is/are correct statement(s) about the motion of the particle?



(A) It oscillates (B) It moves to the right of x_3 and never returns

(C) It comes to rest at either x_0 or x_2

(D) It cannot reach either x_0 or x_2

17. A particle of mass 2 kg starts moving in a straight line with an initial velocity of 2 m/s and a constant acceleration of $2 m/s^2$. Then rate of change of KE

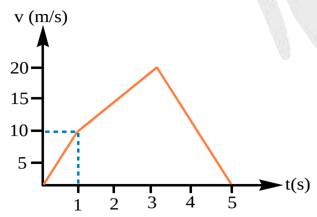
(A) it four times the velocity at any moment

(B) is two times the displacement at any moment

(C) is four times the rate fo charge of velocity at any moment

(D) is constant throughout

18. Figure given below shows the plot of velocity of a body moving rectilinearly under the influence of certain forces. Choose the correct statement(s) :



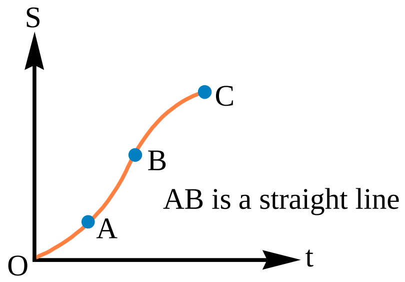
(A) The net work done by the forces for the first five seconds is zero

(B) The average velocity of the body during the first five seconds is zero

(C) The average acceleration of the body during the first five seconds is zero

(D) The average force acting on the body during the first five seconds is zero

19. Displacement time graph of a particle moving in a straight line is as shown in figure. Select the correct alternative(s) :



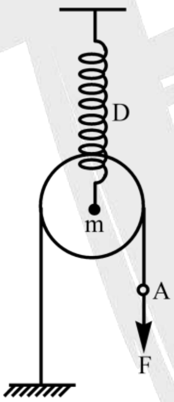
(A) Work done by all the force in region OA and BC is positive

(B) Work done by all the force in region AB is zero

(C) Work done by all the force in region BC is negative

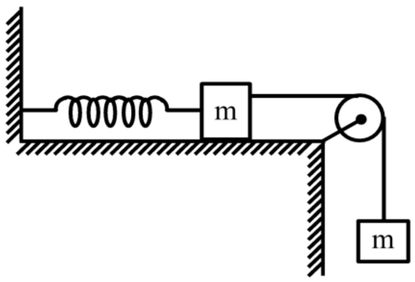
(D) Work done by all the force in region OA is negative

20. The axle of a pulley of mass m = 1 kg is attached to the end of a spring of spring constant k = 200 N/m whose other end is fixed to the ceiling. A rope of negligible mass is placed on the pulley such that its left end is fixed to the ground and its right end is hanging freely from the pulley which is at rest in equilibrium. We begin to pull the endpoint A at the right end of the rope by a constant vertical force of F = 15 N. Friction can be neglected between the rope and the pulley. Find the maximum displacement of point A after applying F.



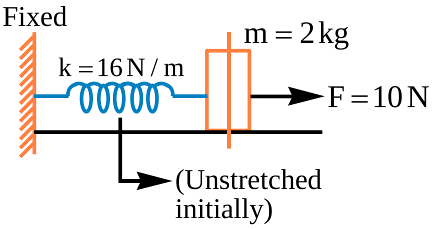
(A) 20 cm (B) 40 cm (C) 60 cm (D) 80 cm

21. Consider the system shown in figure, with two equal masses m = 1 kg and a spring with spring constant 800 N/m. The coefficient of kinetic friction is $\mu = 0.2$ and the pulley is frictionless. The system is held with the spring at its relaxed length and then released. Find maximum displacement (in cm) of the block in subsequent motion.



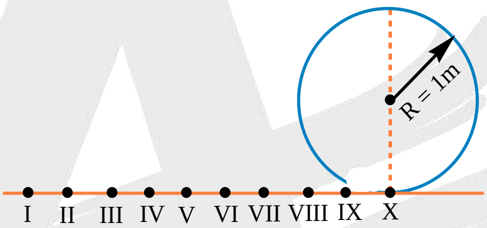
(A) 1 (B) 2 (C) 3 (D) 4

22. A block-spring system is placed on a smooth horizontal plane as shown. What initial velocity should be given to the block along a direction parallel to the force as shown in the figure, simultaneously when then force is applied so that the maximum elongation occurred in the spring is twice of the maximum compression occurred in it.



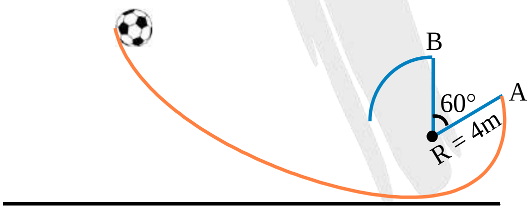
(A) 2.5 m/s (B) 5 m/s (C) 7.5 m/s (D) 10 m/s

23. There are 10 small identical elastic balls placed at rest on a smooth horizontal surface as shown in figure. Find the least velocity which should be provided to the first ball such that 10^{th} ball completes the circle.



(A) 7 m/s (B) 4 m/s (C) 3 m/s (D) 6 m/s

24. As shown in the figure, a ball is released from a certain height which has to perform circular motion on the vertical track of radius 4 m. The track is absent between points A and B. Calculate the height from where the ball has to be released so that it will reach at highest point B of the circular track.



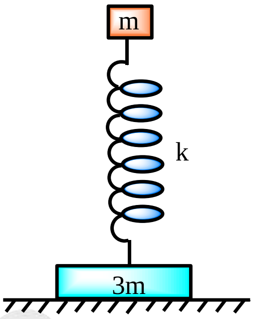
(A) 3 m (B) 12 m (C) 6 m (D) 9 m

25. A bob of mass 'm' is suspended by a light inextensible string of length $'\ell'$ from a fixed point such that it is free to rotate in a vertical plane. The bob is given a speed of $\sqrt{4g\ell}$ horizontally where $\ell = 3\text{m}$. Find the height of the bob from lower most point where the string just becomes slacked.

(A) 2 m (B) 1 m (C) 5 m (D) 4 m

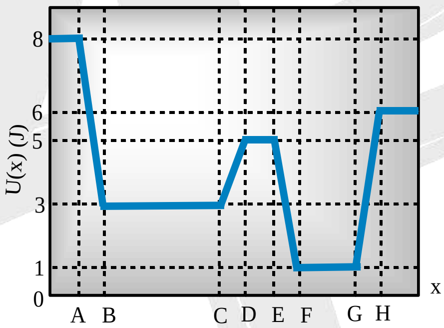
EXERCISE – 4

1. In the figure shown the spring constant is k. The mass of the upper disc is m and that of the lower disc is 3m. The upper block is depressed down from its equilibrium position by a distance $\delta = 5mg/k$ and released at $t = 0$. Find the velocity of 'm' when normal reaction on 3m is mg.



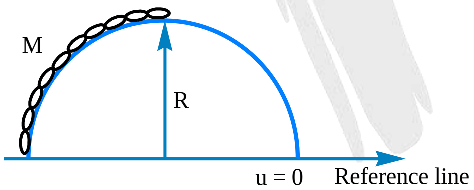
(A) zero (B) $g[m/k]^{1/2}$ (C) $2g[m/k]^{1/2}$ (D) $4g[m/k]^{1/2}$

2. A potential energy curve U(x) is shown in the figure. What value must the mechanical energy E_{mec} of the particle not exceed, if the particle is to be trapped within the region shown in graph?



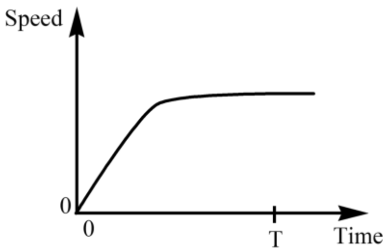
(A) 3J (B) 5J (C) 6J (D) 8J

3. A chain of mass M is kept on a hemisphere as shown. Find out potential energy of the chain assuming reference line as a zero potential energy level.



(A) $Mg\frac{2R}{\pi}$ (B) $Mg\frac{R}{\pi}$ (C) $Mg\frac{R}{2\pi}$ (D) $Mg\frac{3R}{2\pi}$

4. The variation of the vertical speed with time of a ball falling in air is shown below.



During the time from 0 to T, the ball gains kinetic energy and loses gravitational potential energy ΔE_p. Which of the following statements must be correct?

(A) ΔE_p is equal to the gain in kinetic energy

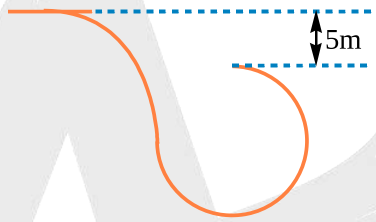
(B) ΔE_p is equal to the work done against air resistance

(C) ΔE_p is greater than the gain in kinetic energy

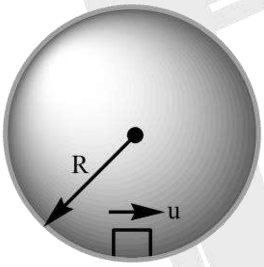
(D) ΔE_p is less than the work done against air resistance

5. A particle is rotated in a vertical circle by connecting it to a light rod of length ℓ and keeping the other end of the rod fixed. The minimum speed of the particle when the light rod is horizontal for which the particle will complete the circle is $\sqrt{\alpha g \ell}$. Find α ?

6. Consider a roller coaster with a circular loop. A roller coaster car starts from rest from the top of a hill which is 5m higher than the stop of the loop. It rolls down the hill and through the loop. What must the radius (in m) of the loop be so that the passengers of the car will fell at highest point, as if they have their normal weight?

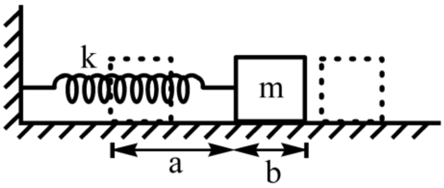


7. A particle is given an initial speed u from the lowest point inside a fixed smooth spherical shell of radius $R = 1m$ that it is just able to complete the circle. Acceleration of the particle when its velocity is vertical is:



(A) $g\sqrt{10}$ (B) g (C) $g\sqrt{2}$ (D) $g\sqrt{6}$

8. The spring is compressed by a distance a and released. The block again comes to rest when the spring is elongated by a distance b. During this process :



(A) work done by the spring on the block$= \frac{1}{2}k(a^2 + b^2)$

(B) work done by the spring on the block $= \frac{1}{2}k(a^2 - b^2)$

(C) coefficient of friction $= \frac{k(a-b)}{2mg}$

(D) coefficient of friction$= \frac{k(a+b)}{2mg}$

9. A particle of mass m is at rest in a train moving with constant velocity with respect to ground. Now the particle is accelerated by a constant force F_0 acting along the direction of motion oftrain for time t_0. A girl in the train and a boy on the ground measure the work done by this force. Which of the following are **incorrect?**

(A) Both will measure the same work.

(B) Boy will measure higher value than the girl.

(C) Girl will measure higher value than the boy.

(D) Data are insufficient for the measurement of work done by the force F_0

10. A single conservative force acts on a 1 kg particle that moves along x-axis. The potential energy of the particle varies with x as $U = 20 + (x-2)^2$, here U is in joules and x is in meters. When the particle is at $x = 5m$, its kinetic energy is 20 J. Then which of the following is/are correct ?

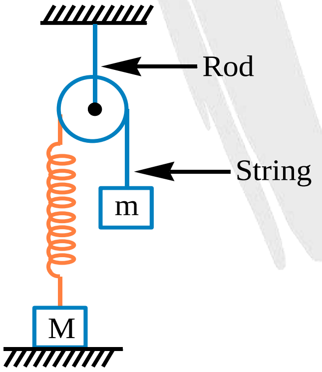
(A) Mechanical energy of particle is 49 J

(B) Least and greatest value of x between which particle can move is $(2-\sqrt{29})m$ and $(2+\sqrt{29})m$ respectively

(C) Maximum kinetic energy of the particle is 29 J

(D) At x = 2, the body is in equilibrium

11. In figure, a block of mass m is released from rest when spring was in its natural length. The pulley also has mass m but it is frictionless. Suppose the value of m is such that finally it is just able to lift the block M up after releasing it.



(A) The weight of m required to just lift M is $\frac{M}{2}g$

(B) The tension in the rod, when m has zero acceleration $\frac{M}{2}g$

(C) The normal force acting on M when m has zero acceleration $\frac{M}{2}g$

(D) The tension in the string when displacement of m is maximum possible is Mg

12. A moving particle is acted by several forces $\vec{F}_1$, $\vec{F}_2$ ...etc. One of the force is chosen say $\vec{F}_2$, then which of the following statement about work done by $\vec{F}_2$ will be true.

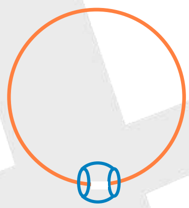
(A) Work done by $\vec{F}_2$ will be negative if speed of particle is decreasing

(B) Work done by $\vec{F}_2$ will be positive if speed of particle is increasing

(C) Work done by $\vec{F}_2$ will be equal in magnitude to the sum of work done by all other forces if speed of particle remains constant

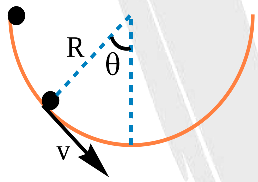
(D) If $\vec{F}_2$ is conservative force, then work done by all other forces will be equal to change in P.E. due to $\vec{F}_2$ if speed remains constant

13. A small bead is threaded in a smooth fixed ring of radius 2.5 m kept in a vertical plane. What can be the velocity of the bead at lowermost point so that it can complete the circle?



(A) 9 m/s (B) 12 m/s (C) 10.5 m/s (D) 14 m/s

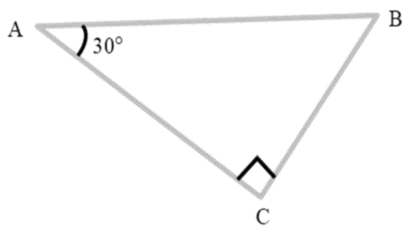
14. A particle of mass m is going along surface of smooth hemisphere of radius R in vertical plane. At the moment shown its speed is v. Choosen correct option(s).



(A) $mg - N\cos\theta = m\left(g\sin^2\theta - \frac{v^2}{R}\cos\theta\right)$ (B) $N - mg\cos\theta = \frac{mv^2}{R}$

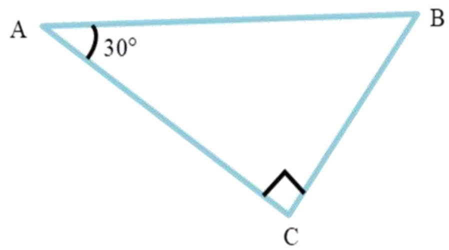
(C) $mg - N\sin\theta = \frac{mv^2}{R}$ (D) $N\sin\theta = m\left(g\sin\theta\cos\theta - \frac{v^2}{R}\sin\theta\right)$

15. Two light string AC and BC attached to a ceiling at point A and B holds the mass m in the position as shown, at t = 0 the string AC is cut. The tension in BC just after the string AC is cut :



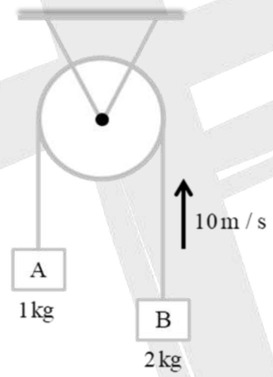
(A) $\frac{mg}{\sqrt{3}}$ (B) mg (C) $\frac{mg\sqrt{3}}{2}$ (D) None of these

16. Two light string AC and BC attached to a ceiling at point A and B holds the mass m in the position as shown, at t = 0 the string AC is cut. The acceleration of mass m, when the string BC becomes vertical :



(A) $g\left(2-\sqrt{3}\right)$ (B) $g\left(2+\sqrt{3}\right)$ (C) g (D) $\frac{2g}{\sqrt{3}}$

17. Two blocks of A and B of mass 1 kg and 2 kg are hung from light pulley. Initially the block B is held stationary. At t = 0 block B is given velocity 10 m/s in upward direction. String and pulley are light and there is no friction anywhere. Velocity of block 'B' as block 'A' has ascended by distance 5m from its original position :



(A) $10\sqrt{7}\,m/s$ (B) $\frac{10}{2}\sqrt{2}\,m/s$ (C) $10\sqrt{\frac{5}{3}}\,m/s$ (D) $10\frac{\sqrt{7}}{3}\,m/s$

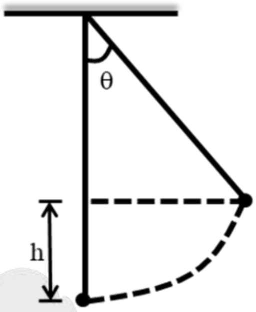
18. A small ball hanging by a thread undergoes circular motion in vertical plane. At the uppermost point of its path its velocity is 6 m/s. The tension in the thread is three times at the lowermost point than at the uppermost point. What is the maximum velocity of the ball ?

(A) $6\sqrt{5}\,m/s$ (B) 30 m/s (C) $6\sqrt{2}\,m/s$ (D) None of these

19. A small ball of mass 1 kg hanging on a thread revolves in a vertical plane. At the uppermost point of its path its velocity is 6 m/s. The tension force stretching the thread at the lowermost point is four times as much as that at the uppermost one. What is tension in the string at the lowermost point ? $\left(g=10\,m/s^2\right)$

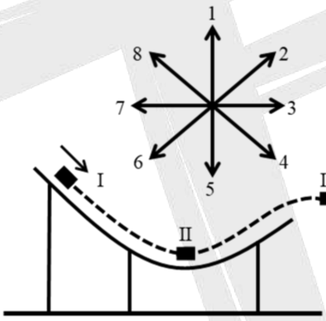
(A) 20 N (B) 70 N (C) 80 N (D) None of these

20. Figure here shows a ball of weight W hanging at the end of a light and inextensible string of length L. A force F, which is always kept horizontal, is applied to push the ball sideways, and its value is very slowly varied corresponding to rise h of the ball (for angle θ shown). Such a process is called quasistatic process. The force is increased so slowly that the ball may be treated to be in equilibrium under the forces T, W and F all the time, where T is the tension in the string. At any state magnitude of tension in string is equal to :



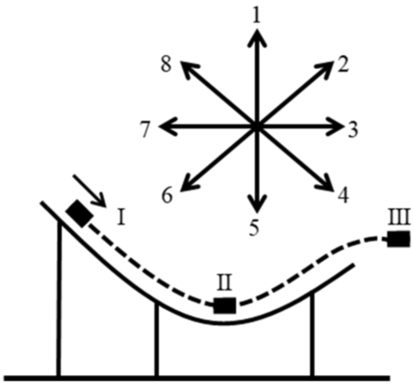
(A) $W\cos\theta$ (B) $W\sin\theta$ (C) $\dfrac{W}{\cos\theta}$ (D) $\dfrac{W}{\sin\theta}$

21. This diagram depicts a block sliding along a frictionless ramp in vertical plane. The eight numbered arrows in the diagram represent directions to be referred when the direction of the acceleration of the block, when in position I, is best represented by which of the arrows in the diagram ?



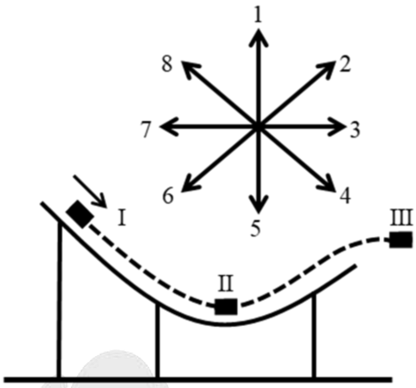
(A) 2 (B) 4
(C) 5 (D) None of the arrows, the acceleration is zero

22. This diagram depicts a block sliding along a frictionless ramp in vertical plane. The eight numbered arrows in the diagram represent directions to be referred when the direction of the acceleration of the block when in position II is best represented by which of the arrows in the diagram ?



(A) 1 (B) 3 (C) 5 (D) 8

23. This diagram depicts a block sliding along a frictionless ramp in vertical plane. The eight numbered arrows in the diagram represent directions to be referred when the direction of the acceleration of the block (after leaving the ramp) at position III is best represented by which of the arrows in the diagram?



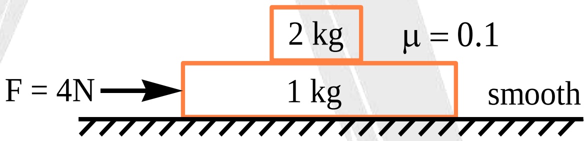
(A) 2 (B) 5 (C) 6

(D) None of the arrows, the acceleration is zero

24. A block of mass $m = \frac{1}{3}$ kg is kept on a rough horizontal plane. Friction coefficient is $\mu = 0.75$. Find the work done by minimum force required to drag the block along the plane by a distance 5 m, is

(A) 2 J (B) 4 J (C) 6 J (D) 8 J

25. 2 kg block is kept on 1 kg block as shown. The friction between 1 kg block and fixed surface is absent and the coefficient of friction between 2 kg block and 1 kg block is $\mu = 0.1$. A constant horizontal force F = 4 N is applied on 1 kg block. Find the work done by the friction on 1 kg blcok in 2s is



(A) 8 J (B) 6 J (C) 2 J (D) 4 J

26. A long vertical ladder of mass 100 kg is moving downward with constant speed 3 m/s. A boy of mass 50 kg is climbing up on ladder with constant speed of 5 m/s w.r.t. ladder. Work done by gravity on ladder and boy in 2 sec. are respectively W_1 and W_2. Find the value of $W_1 - W_2$ is

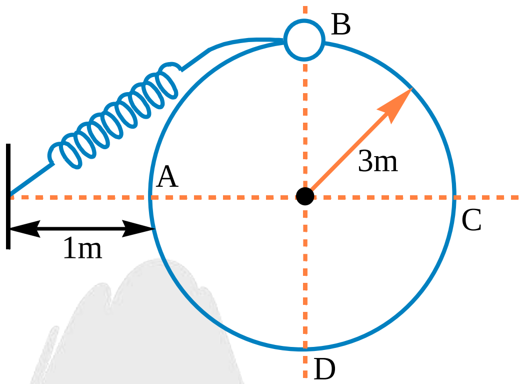
(A) 4 kJ (B) 6 kJ (C) 8 kJ (D) 10 kJ

27. An object of mass 4 kg falls from rest through a vertical distance of 10 m and reaches ground with a speed of 14 m/s. How much work is done against air friction is

$\left[g = 10\ m/s^2\right]$

(A) 6 J (B) 7 J (C) 8 J (D) 9 J

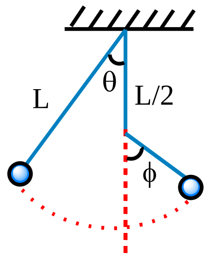
28. A bead of mass 150 g is constrained to move along a vertical smooth and fixed circular track of radius 3 m as shown in figure is released at B. The spring is in the plane of the track has natural length of 1m and spring constant of $\frac{3}{16}$ N/m. It starts from rest at B. What is the normal force exerted by the track on the bead when it passes through A?



(A) 2 N (B) 6 N (C) 4 N (D) 7 N

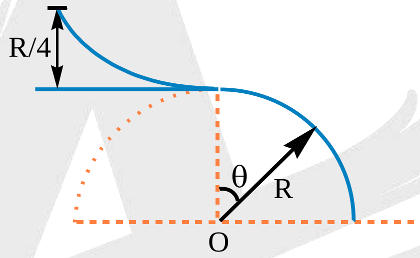
EXERCISE – 5

1. Figure shows a pendulum of length L suspended from the top of a flat beam of height L/2. The bob is pulled away from the beam so it makes an angle θ with the vertical. Now, it is released from rest. If ϕ is the maximum angular deflection to the right, then:



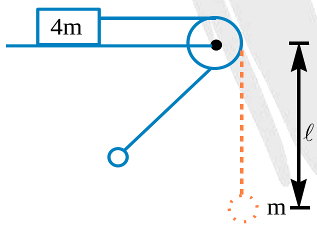
(A) $\phi = \theta$ (B) $\phi < \theta$ (C) $\theta < \phi \le 2\theta$ (D) $\phi > 2\theta$

2. A skier plans to ski a smooth fixed hemisphere of radius R. He starts from rest from a curved smooth surface of height (R/4). The angle θ at which he leaves the hemisphere is:

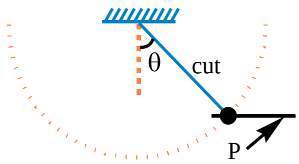


(A) $\cos^{-1}\left(\frac{2}{3}\right)$ (B) $\cos^{-1}\left(\frac{5}{\sqrt{3}}\right)$ (C) $\cos^{-1}\left(\frac{5}{6}\right)$ (D) $\cos^{-1}\left(\frac{5}{2\sqrt{3}}\right)$

3. Two bodies of mass m and 4m are attached to a string as shown in the figure. The body of mass m hanging from string is executing oscillation with angular amplitude $60°$ while other body is at rest on a horizontal surface. The minimum coefficient of friction between the mass 4m and the horizontal surface is $\frac{5}{n}$. Find n?



4. Consider a pendulum, consisting of a massless string with a mass on the end. The mass is held with the string horizontal, and then released. The mass swings down, and then on its way back up, the string is cut at point P when it makes an angle of θ with the vertical.What should θ be, so that the mass travels the largest horizontal distance from P by the time it returns to the height it had when the string was cut?

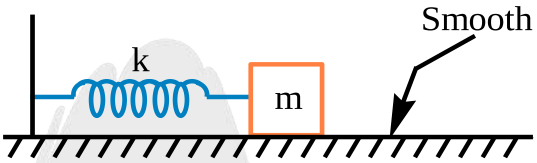


(A) $\tan^{-1}\sqrt{3}$ (B) $\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$ (C) $\tan^{-1}\left(\frac{1}{\sqrt{2}}\right)$ (D) $\tan^{-1}(1)$

5. A block of mass 1 kg kept on a rough horizontal surface $(\mu = 0.4)$ is attached to a light spring (force constant = 200 N/m) whose other end is attached to a vertical wall. The block is pushed to compress the spring by a distance d and released. Find the value(s) of 'd' for which (spring + block) system loses its entire mechanical energy in form of heat.

(A) 4 cm (B) 6 cm (C) 8 cm (D) 10 cm

6. Figure shows an ideal spring block system, force constantt of spring is k which has been compressed by an amount x_0. If x is instantaneous deflection of spring from its natural length, mark the correct option(s).



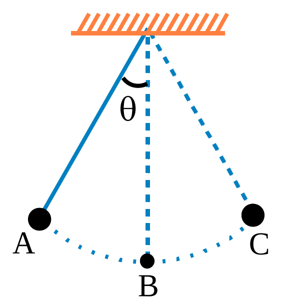
(A) Instantaneous power developed by spring is $P = kx\sqrt{\frac{k}{2m}\left(x_0^2 - x^2\right)}$

(B) Maximum power of spring is $\frac{k}{2}\sqrt{\frac{k}{m}}x_0^2$

(C) Maximum power occurs at $x = \frac{x_0}{\sqrt{2}}$

(D) Maximum power occurs at $x = \frac{x_0}{2}$

7. On the figure shown, a stone tied to a light string is oscillating between extreme points A and C in a vertical plane. Acceleration of strone has magnitude a_A, a_B and a_C at the respective points then : $\left(\text{Given} : \sin\theta = \frac{4}{5}\right)$



(A) $a_A = a_B$ (B) $a_B = 2a_A$

(C) $2a_B = a_A + a_C$ (D) $a_A = a_C$

8. A heavy particle is attached to one end of a string 1m long whose other end is fixed at O. It is projected from its lowest position horizontally with a velocity V :

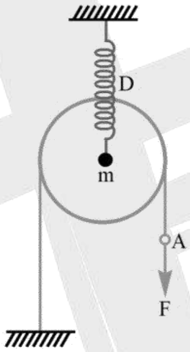
(A) If $V^2 > 5g$ the particle will describe complete circular motion in the vertical plane

(B) If $V^2 = 3.5g$ the tension in the string will becomes zero after the string has turned through $120°$

(C) If $V^2 = 2g$, the tension in the string becomes zero the velocity of the particle also becomes zero

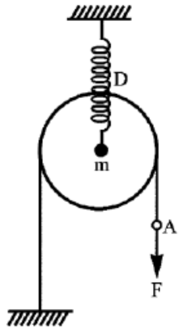
(D) If $V^2 = g$ the velocity of the particle becomes zero after the string turns through $60°$

9. The axle of a pulley of mass m = 1 kg is attached to the end of a spring of spring constant k = 200N/m whose other end is fixed to the ceiling. A rope of negligible mass is placed on the pulley such that its left end is fixed to the ground and its right end is hanging freely from the pulley which is at rest in equilibrium. We begin to pull the endpoint A at the right end of the rope by a constant vertical force of F = 15 N. Friction can be neglected between the rope and the pulley. What is the elongation of the spring before applying force F ?



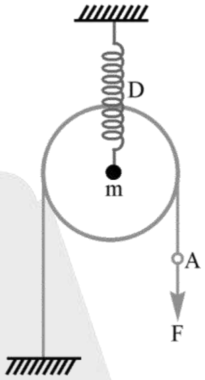
(A) 50 cm (B) 0.5 cm (C) 0.05 cm (D) 5 cm

10. The axle of a pulley of mass m = 1 kg is attached to the end of a spring of spring constant $k = 200N/m$ whose other end is fixed to the ceiling. A rope of negligible mass is placed on the pulley such that its left end is fixed to the ground and its right end is hanging freely from the pulley which is at rest in equilibrium. We begin to pull the endpoint A at the right end of the rope by a constant vertical force of F = 15 N. Friction can be neglected between the rope and the pulley. Find the greatest elongation of the spring.



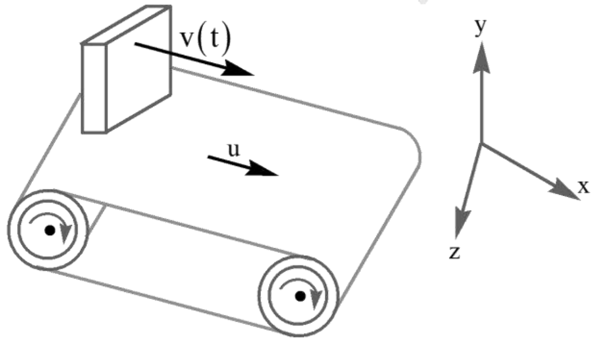
(A) 25 cm (B) 20 cm (C) 35 cm (D) None of these

11. The axle of a pulley of mass m = 1 kg is attached to the end of a spring of spring constant $k = 200\,N/m$ whose other end is fixed to the ceiling. A rope of negligible mass is placed on the pulley such that its left end is fixed to the ground and its right end is hanging freely from the pulley which is at rest in equilibrium. We begin to pull the endpoint A at the right end of the rope by a constant vertical force of F = 15 N. Friction can be neglected between the rope and the pulley. Find the maximum displacement of point A after applying F.



(A) 60 cm (B) 30 cm (C) 40 cm (D) None of these

12. A suitcase of mass M is placed on a level conveyor belt at an airport. The coefficient of static friction between the suitcase and the conveyor belt is μ_s, and the coefficient of kinetic friction is μ_k, with $\mu_k < \mu_s$. The conveyor belt moves with constant speed u, and at time t = 0 the suitcase is placed on the conveyor with speed v = 0. At a time t_f, after moving a distance ℓ, the suitcase catches up with the conveyor belt, and starts to move at speed u with the conveyor belt. Gravity acts downward with acceleration g > 0. Work can depend on one's frame of reference, so be sure to answer the following two parts in the frame of reference of the airport. How much work does friction do on the suitcase during this period?



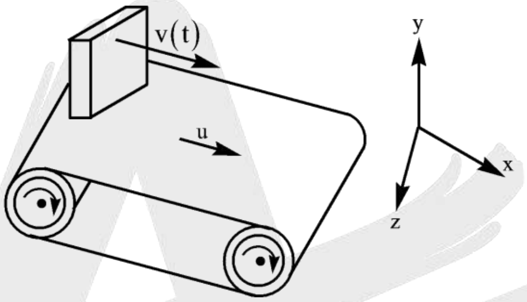
(A) $\frac{1}{2}Mu^2$

(B) $-\frac{1}{2}Mu^2$

(C) $Mg\ell$

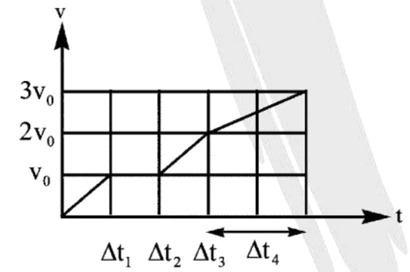
(D) $\mu_s Mg\ell$

13. A suitcase of mass M is placed on a level conveyor belt at an airport. The coefficient of static friction between the suitcase and the conveyor belt is μ_s, and the coefficient of kinetic friction is μ_k, with $\mu_k < \mu_s$. The conveyor belt moves with constant speed u, and at time t = 0 the suitcase is placed on the conveyor with speed v = 0. At a time t_f, after moving a distance ℓ, the suitcase catches up with the conveyor belt, and starts to move at speed u with the conveyor belt. Gravity acts downward with acceleration g > 0. Work can depend on one's frame of reference, so be sure to answer the following two parts in the frame of reference of the airport. How much work does the force of friction from the suitcase do on the belt, during this time period?



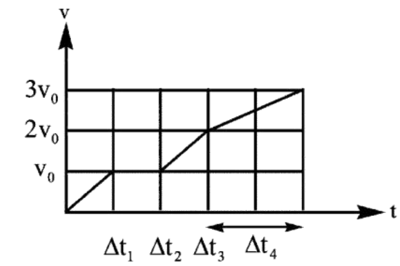
(A) $\mu_k Mg\ell$ (B) $-\mu_k Mg\ell$ (C) $\mu_k Mgut_f$ (D) $-\mu_k Mgut_f$

14. Figure gives the velocity v versus time t graph of a carriage of constant mass being moved along an axis by applying force. The time axis shows four time periods, with $\Delta t_1 = \Delta t_2 = \Delta t_3$ and $\Delta t_4 = 2\Delta t_1$. The work done by the force is maximum during which time period :



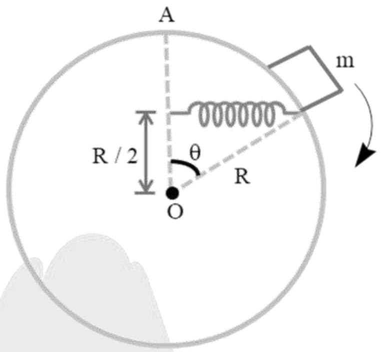
(A) Δt_3 and Δt_4 (B) $\Delta t_1, \Delta t_3$ and Δt_4 (C) Only Δt_3 (D) Only Δt_4

15. Figure gives the velocity v versus time t graph of a carriage of constant mass being moved along an axis by applying force. The time axis shows four time periods, with $\Delta t_1 = \Delta t_2 = \Delta t_3$ and $\Delta t_4 = 2\Delta t_1$. The rate at which work done is maximum :



(A) Only Δt_1 (B) Δt_1 and Δt_3 (C) Only Δt_3 (D) Δt_4

16. A small block of mass m, can move without friction on the outside of a fixed vertical circular track of radius R. The block is attached to a spring of natural length R/2 and spring constant k. The other end of spring is connected to a point at height R/2 directly above the centre of track. If the block is released from rest when the spring is in horizontal state (see figure) then at that moment :



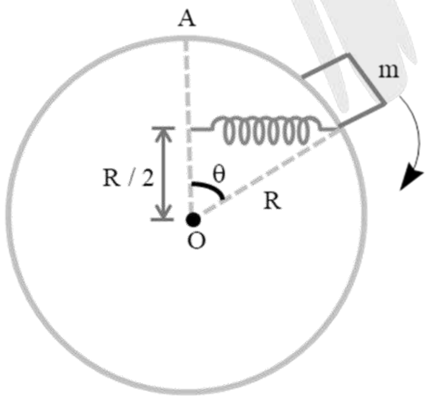
(A) tangential acceleration is $g\frac{\sqrt{3}}{2} - \frac{kR}{4m}(\sqrt{3}-1)$

(B) radial acceleraton is $\frac{g}{2} + \frac{kR\sqrt{3}}{4m}(\sqrt{3}-1)$

(C) tangential acceleration is $\frac{g}{2} - \frac{kR\sqrt{3}}{4m}(\sqrt{3}-1)$

(D) radial acceleration is $g\frac{\sqrt{3}}{2} - \frac{kR}{4m}(\sqrt{3}-1)$

17. A small block of mass m, can move without friction on the outside of a fixed vertical circular track of radius R. The block is attached to a spring of natural length R/2 and spring constant k. The other end of spring is connected to a point at height R/2 directly above the centre of track. Consider block to be at rest at top most point A of track. If the block is slowly pushed from rest at the highest point A. When the spring reaches in horizontal state, then :



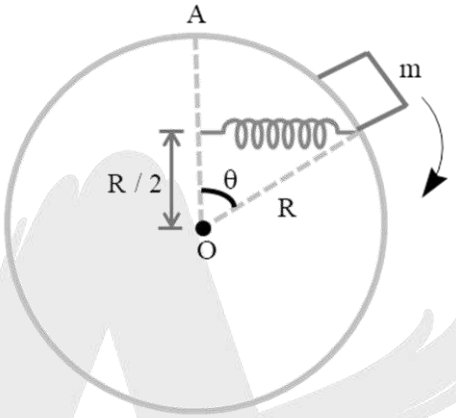
(A) spring potential energy is $\left(\frac{3kR^2}{4}\right)(2-\sqrt{3})$

(B) spring potential energy is $\left(\frac{kR^2}{8}\right)(\sqrt{3}-1)^2$

(C) gravitational potential energy (taking U = 0 at $\theta = 0$) is $\left(\frac{mgR}{2}\right)$

(D) gravitational potential energy (taking U = 0 at $\theta = 0$) is $\left(\frac{3mgR}{8}\right)$

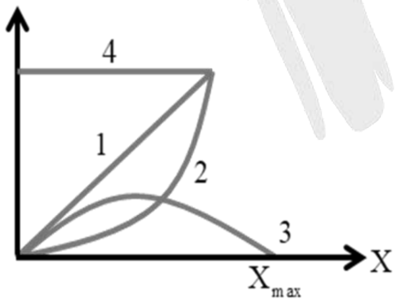
18. A small block of mass m, can move without friction on the outside of a fixed vertical circular track of radius R. The block is attached to a spring of natural length R/2 and spring constant k. The other end of spring is connected to a point at height R/2 directly above the centre of track. If the complete setum is in a gravity free space, then the minimum speed (v_0) required at the highest point A to just reach the lowest point is :



(A) $2R\sqrt{\frac{k}{m}}$ (B) $\frac{3R}{2}\sqrt{\frac{k}{m}}$

(C) $R\sqrt{\frac{k}{m}}$ (D) motion not possible in gravity free space.

19. A block suspended from a spring is released from rest when spring is unstretched. 'x' represents stretch in spring. Select the appropriate graph taking quantities in column-I as y-axis.

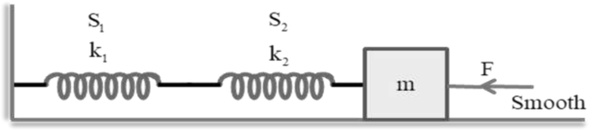


	Column – I		Column – II
(A)	The KE of block	(P)	1
(B)	The work done on the block by gravity	(Q)	2
(C)	The magnitude of work done on the block by spring	(R)	3
(D)	The total mechanical energy of block earth spring system	(S)	4

(A) $(A-R);(B-P);(C-Q);(D-S)$ (B) $(A-R);(B-P);(C-Q);(D-S)$

(C) $(A-P);(B-R);(C-Q);(D-S)$ (D) $(A-R);(B-P);(C-S);(D-Q)$

20. Initially spring are in natural length. An application of external varying force F causes the block to move slowly distance x towards wall on smooth floor :

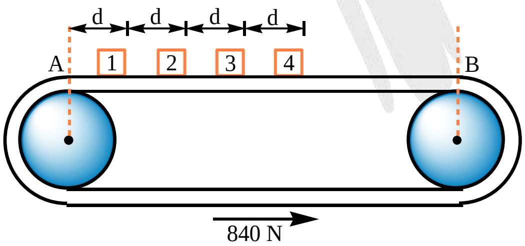


	Column – I		Column – II
(A)	Work done by S_2 on block	(P)	Zero
(B)	Work done by S_2 on S_1	(Q)	$-\frac{1}{2}\left(\frac{k_1k_2}{k_1+k_2}\right)x^2$
(C)	Work done by F on block	(R)	$\frac{1}{2}\left(\frac{k_1k_2}{k_1+k_2}\right)x^2$
(D)	Work done by S_1 on wall	(S)	$\frac{1}{2}\frac{k_1k_2^2x^2}{(k_1+k_2)^2}$

(A) $(A-Q);(B-S);(C-P);(D-R)$ (B) $(A-S);(B-Q);(C-R);(D-P)$

(C) $(A-Q);(B-S);(C-R);(D-P)$ (D) $(A-R);(B-S);(C-Q);(D-P)$

22. Four packages each having a mass of 4 kg are attached on the belt at equal distances d = 200 mm as shown in the figure. Initially belt is at rest. If a constant force of magnitude 840 N is applied to the belt, determine the velocity of package 2 as it falls off the belt at point A. Assume that the mass of the belt and pulleys is small compared with the mass of the packages. Assume that the radius of pulley is negligible in comparison to width d.

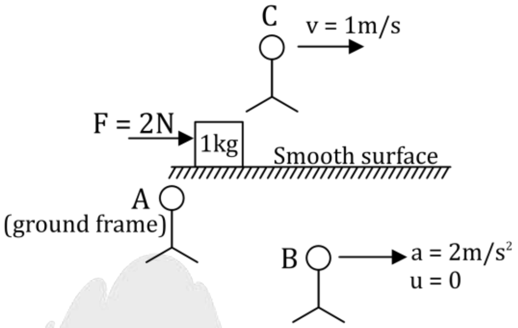


(A) 5 m/s (B) 6 m/s (C) 7 m/s (D) 8 m/s

EXERCISE(ALP)

ONE OR MORE THAN ONE ANSWER TYPE

1. An observer A is at rest in ground frame, observer B is moving with constant acceleration of $2m/s^2$ and another observer C is moving with constant velocity of 1 m/s. A constant force F = 2N is applied on the block of mass 1 kg. Comment on work on block of mass 1 kg.



(A) Net work done by all forces (both real and pseudo) in 1 sec. in frame of A is 2 J

(B) Net work done by all forces (both real and pseudo) in 1 sec. in frame of B is 0 J

(C) Net work done by all forces (both real and pseudo) in 2 sec. in frame of C is 4 J

(D) Change in kinetic energy in frame of C in 1 sec. is zero

2. Which of the following statements are correct regarding work energy theorem

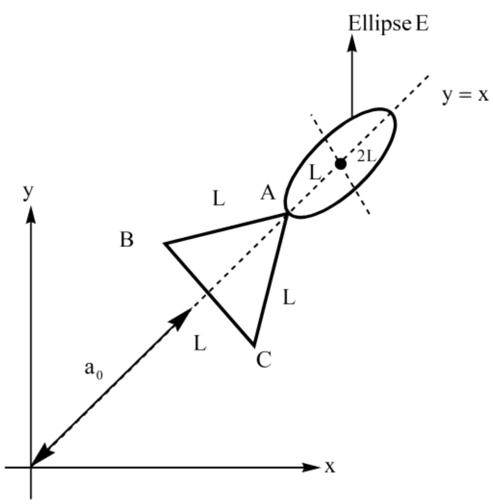
(A) The work energy theorem is an invariant law of physics

(B) Work energy theorem is also applicable in non inertial frame of reference with modification

(C) For the system of particles it is change in kinetic energy is equal to work done by external forces.

(D) Work done by kinetic friction on a system of two blocks is always negative

3. In the x-y plane there exist an equilateral triangle ABC and ellipse E. side length of equilateral triangle is L and length of major and minor axis of the ellipse is given by 4L and 2L respectively. If work done by the $\vec{F} = K(y\hat{i} - x\hat{j})$ in complete rotation in anti clock wise sense for triangle ABC and ellipse E is given by W_1 and W_2 respectively.



Choose the **CORRECT** option(s) :

(A) $W_1 = \frac{\sqrt{3}KL^2}{2}$

(B) $W_2 = 4\pi KL^2$

(C) total work done remains same even if orientation of equilateral triangle ABC and ellipse E are changed.

(D) The given force is a non-conservative force

4. No work is done by a force on an object if

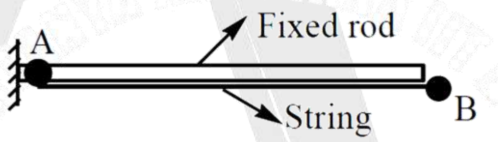
(A) the force is always perpendicular to its velocity

(B) the force is always perpendicular to its acceleration

(C) the object is stationary but the point of application of the force moves on the object

(D) the object moves in such a way that the point of application of the force remains fixed

5. A ball A of mass m = 2kg can slide without friction on a fixed horizontal rod which is led through a diametric hole across the ball. There is another ball B of the same mass 'm' attached to the first ball by a thin thread of the length l = 1.6m. Initially the balls are at rest. The thread is taut and is initially oriented in horizontal direction as shown in figure. Then, the ball B is released with zero initial velocity. At the instant when the string becomes vertical.



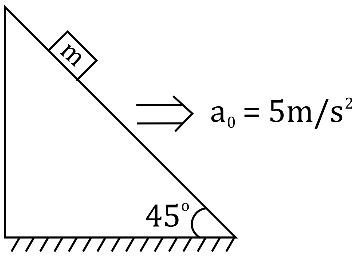
(A) Velocity of A is $\sqrt{gl}$ towards right

(B) Acceleration of A is zero

(C) Acceleration of B is g in vertically upward direction

(D) Tension in the string is 2mg

6. A block of mass m = 2kg is kept on a wedge as shown in figure. Coefficient of friction between the block and the incline surface of the wedge is $\mu = 0.2$. At time 't' = 0, the block is at rest and the wedge starts moving towards right, from rest, with a constant acceleration of $a_0 = 5m/s^2$. During the interval t = 0 to 1s



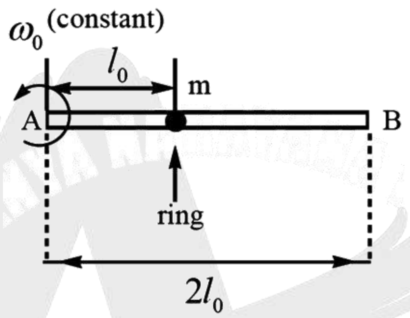
(A) Work done by normal contact force acting on the block (from wedge) is 37.5 J

(B) Total work done by internal friction between the block and the wedge is 3 J

(C) Total work done by interval normal contact force between the block and the wedge is zero

(D) Mechanical energy dissipated at the interface between the block and the wedge is 3 J

7. A ring of mass 'm' is just loosely fit on a frictionless rod AB of length $2l_0$ which rotates in horizontal plane with a constant angular velocity ω_0. Initially the ring is located at a distance l_0 from the end A. Now, the ring is left free and is allowed to slide along the rod. In the subsequent motion,



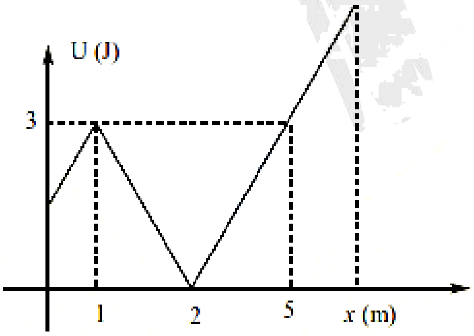
(A) Component of acceleration of the ring parallel to rod is zero

(B) Component of velocity of the ring parallel to rod as a function of its radial distance $r(>l_0)$ is $v_r = \omega_0\sqrt{(r^2 - l_0^2)}$

(C) Component of acceleration of the ring perpendicular to rod is zero

(D) work done by the rod, on the ring when the ring leaves the rod is $\frac{3}{2}m\omega_0^2 l_0^2$

8. A particle of mass m = 2kg with potential energy shown in the graph is moving towards positive x-axis with a speed 1 m/s at x = 1m.



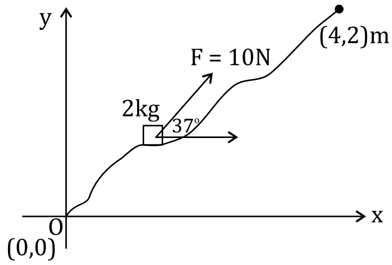
(A) Particle reverses its direction of motion at x = 2m

(B) Particle reverses its direction of motion at x = 6m

(C) Force acting on the particle at the instant of its direction of reversal is 1N towards negative x direction

(D) Maximum speed of the particle before it reverses its direction is 3 m/s

9. A body of mass 2 kg is moving on a rough curved surface under a constant force F = 10N which acts at an angle of 37° from the x-axis. Initially the object is at origin and is moving with a speed of 10m/sec. At point 'P' the speed of body is 6m/sec. Choose correct option(s). $\left(g = 10\ m/s^2\right)$



(A) Normal reaction and friction force on the body are constant in magnitude

(B) Normal reaction and friction force on the body are variable

(C) The work done by friction force on the body when it moves from O to P is 68 J

(D) The work done by friction force cannot be calculated as friction force is not known

10. Which of the following is/are conservative force(s)?

(A) $\vec{F} = 2r^3\hat{r}$ (B) $\vec{F} = -\frac{5}{r}\hat{r}$ (C) $\vec{F} = \frac{3(x\hat{i}+y\hat{j})}{(x^2+y^2)^{3/2}}$ (D) $\vec{F} = \frac{3(y\hat{i}+x\hat{j})}{(x^2+y^2)^{3/2}}$

11. There are two massless springs A and B of spring constant K_A and K_B respectively and $K_A > K_B$. If W_A and W_B be denoted as work done on A and work done on B respectively, then

(A) If they are compressed to same distance, $W_A > W_B$

(B) If they are compressed by same force (upto equilibrium state) $W_A < W_B$

(C) If they are compressed by same distance, $W_A = W_B$

(D) If they are compressed by same force (upto equilibrium state) $W_A > W_B$

12. Work done by a force on an rigid object having no rotational motion will be zero, if:

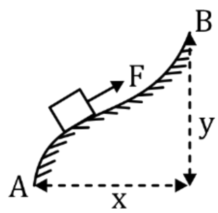
(A) the force is always perpendicular to acceleration of object.

(B) the object is at rest relative to ground but the point of application of force moves on the object.

(C) the force is always perpendicular to velocity of object.

(D) The point of application of force is fixed relative to ground but the object moves

13. A body of mass 'm' is slowly halved up a rough hill by a force F, which at each point is directed along the tangent to the surface of hill. Let μ be the coefficient of friction at the interface. Then



(A) work done by friction on the hill $W_f = -\mu mgx$

(B) work done by friction on the block is $W_f = -\mu mgx$

(C) work done by F is $W_F = \mu mgx + mgy$

(D) Mechanical energy supplied by F to the block is $\mu mgx + mgy$

14. Choose the correct statement(s)

(A) Total work done by action and reaction pair is zero

(B) Total work done by action and reaction pair is frame invarient

(C) Total work done by internal static friction is zero in any frame of reference

(D) Total work done by internal kinetic friction is negative

(E) A normal reaction which does not cause deformations at the interface beyond elastic limit is non-dissipative

15. Which of the following is/are correct expression work-energy theorem for a particle?

(A) Work done by all the forces acting on the particle =change in KE of the particle.

(b) Work done by non-conservative forces acting on the particle = change in potential energy of the particle

(C) work done by non-conservative forces acting on the particle = change in mechanical energy of the particle (steady surroundings)

(D) Work done by conservative forces acting on the particle = - ΔU of the particle (steady surroundings)

16. Which of the following statements is TRUE for a system comprising of two bodies in contact exerting frictional force on each other:

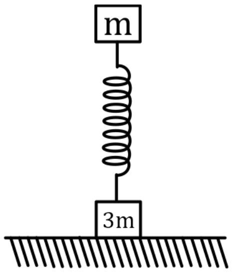
(A) Total work done by static friction on whole system is always zero

(B) Work done by static friction on a body is always zero

(C) Work done by kinetic friction on a body is always negative

(D) Total work done by internal kinetic friction on whole system is always negative

17. In the figure shown, the spring constant is k. The mass of upper block is m and that of the lower block is 3m. The upper block is depressed from its equilibrium position by a distance δ and released at t = 0



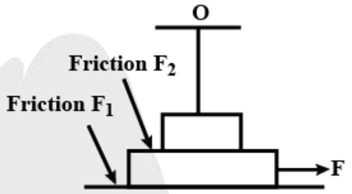
(A) The minimum value of δ for which the lower block loses contact with the ground is $\frac{4mg}{k}$

(B) The minimum value of δ for which the lower block loses contact with the ground is $\frac{2mg}{k}$

(C) The value of δ for which the minimum normal reaction on 3m from ground, is mg is $\frac{3mg}{k}$

(D) The value of δ for which the minimum normal reaction on 3m from ground, is mg is $\frac{mg}{2k}$

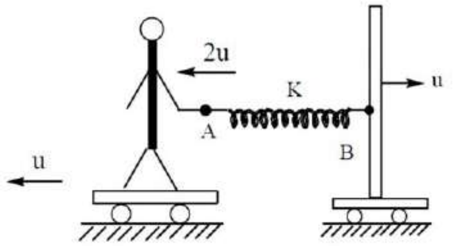
18. A horizontal plane supports a plank with a block placed on it. A light elastic string is attached to the block which is attached to a fixed-point O. Initially the string is unstretched and vertical. The plank is slowly shifted to right until the block starts sliding over it. It occurs at the moment when the string deviates from vertical by an angle θ. Work done by force F equals



(A) Energy lost against friction f_1 plus strain energy in string
(B) Work done against total friction acting on the plank alone
(C) Work done against total friction acting on plank plus strain
(D) work done against total friction acting on the plank plus the difference of strain energy in the spring and work done by friction acting on the block.

19. A block of mass 1 kg is kept on a rough horizontal surface $(\mu = 0.4)$ is attached to a horizontal light spring (k = 400 N/m) whose other end is attached to a vertical wall. The block is pushed to compress the spring by a distance d and released. Find the value(s) of 'd' for which (spring + block) system loses its entire mechanical energy in form of heat
(A) 4 cm (B) 6cm (C) 8cm (D) 10 cm

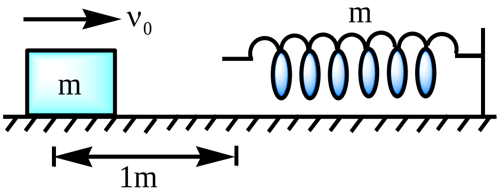
20. End 'B' of an ideal spring is attached to a wall moving with a constant velocity 'u' towards right. The order end A is held by a man who is travelling on a trolley with a constant velocity 'u' towards left. At time "t=0", the spring is in relaxed state and the man starts pulling end 'A' towards left with a constant velocity 2u.



Stiffness constant of the spring is $K = 10\text{N}/\text{cm}$. When deformation in the spring becomes $\Delta\ell = 6\text{cm}$, (Assume no slip between the man and trolley)
(A) work done by man on the spring from t = 0 is 1.2 J
(B) work done by man on the trolley from t = 0 is -0.6J
(C) work done by wall on the spring from t = 0 is 0.6 J
(D) Total work done by the spring is -1.8 J

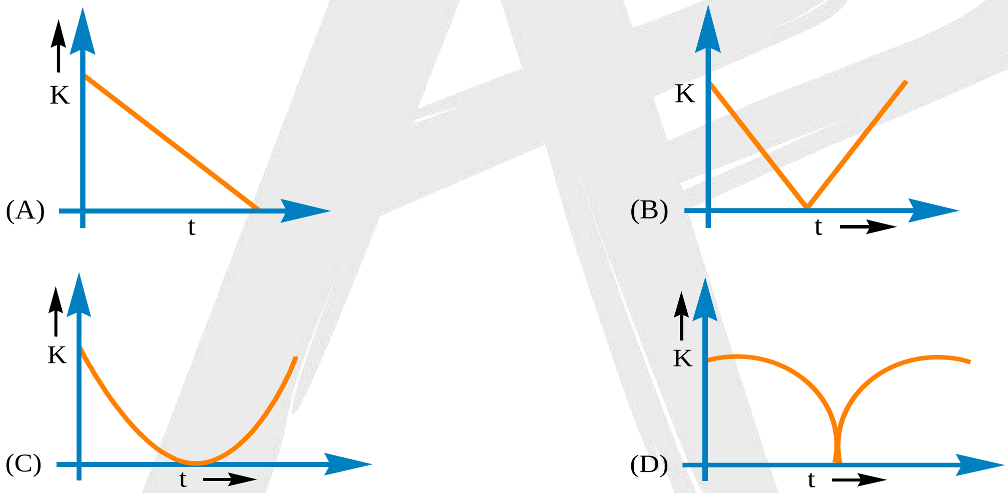
PROFICIENCY TEST – 1

1. A block of mass m = 2kg is moving with velocity v_0 towards a massless unstretched spring of force constant k = 10N/m. Coefficient of friction between the block and the ground is $\mu = \frac{1}{5}$. Find the maximum value of v_0, so that after pressing the spring the block does not return back but stops there permanently.

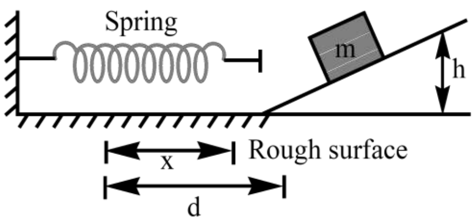


(A) 6 m/s (B) 12 m/s (C) 10 m/s (D) $\sqrt{6.4}$ m/s

2. A ball is projected upwards with an initial velocity. Which of the following graphs best represent the kinetic energy of the ball as a function of time till it reaches back the point of projection.



3. A block of mass m starts at rest at heigt h on a frictionless inclined plane. The block slides down the plane. On a rough horizontal surface with coefficient of kinetic friction μ, and compresses a spring with force constant k by distance x and travels total distance d on horizontal surface before momentarily coming to rest. Then the spring extends and the block travels back across the rough surface, sliding up the plane. The correct expression for the maximum height h' that the block reaches on its return is:

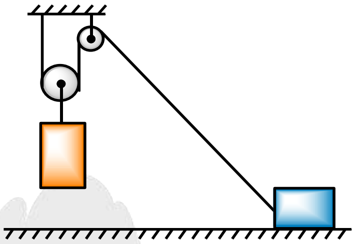


(A) $mgh' = mgh - 2\mu mgd$ (B) $mgh' = mgh + 2\mu mgd$

(C) $mgh' = mgh + 2\mu mgd + kx^2$ (D) $mgh' = mgh - 2\mu mgd - kx^2$

4. In the figure shown, the mass of the hanging block is m, while that of the block resting on the floor is 3m. The floor is horizontal an frictionless and all pulleys ideal. The system is initally held stationary with the inclined thread making an angle $\theta = 30°$ with the horizontal. The blocks are now released from rest and allowed to move. The hanging block falls through a height (49/5)m before hitting the floor. It is found that the value of θ becomes $60°$, when the hanging block hits the floor.

The speed (in m/s) with which the hanging block hits the floor.



5. The potential energy of a 1 kg particle free to move along the x-axis is given by :

$$V(x) = \left(\frac{x^4}{4} - \frac{x^2}{2}\right) J$$

The total mechanical energy of the particle is 2 J. Then the maximum speed (in m/s) is :

(A) $\frac{3}{\sqrt{2}}$ (B) $3\sqrt{2}$ (C) $\frac{9}{2}$ (D) 2

6. Amachine delivers power given by $P = \frac{P_0 t_0^2}{(t+t_0)^2}$ where P_0 and t_0 are constants. The machine starts at t = 0 and runs forever. What is maximum work that the machine can perform?

(A) Infinite

(B) zero

(C) $P_0 t_0$

(D) Cannot be predicted, data insufficient

7. **Statement-1:** The position of maximum potential energy is always the positon of unstable equilibrium.

Statement-2: A conservative force is equal to negative of slope of potential energy vs position graph.

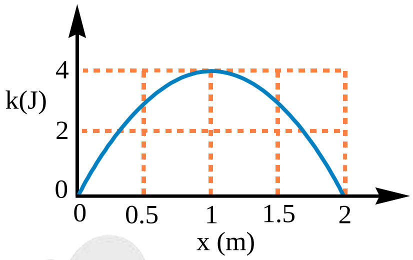
(A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1

(B) Statement-1 is true, statement-2 is true and statement -2 is not the correct explanation for statement-1

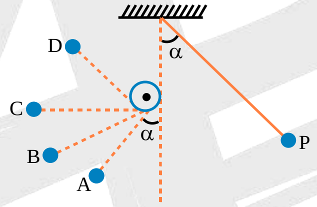
(C) Statement-1 is true, statement-2 is false

(D) Statement-1 is false, statement-2 is true

8. A block of mass m lies on a horizontal frictionless surface and is attached to one end of a horizontal spring (with spring constant k) whose other end is fixed. The block is initially at rest at the position where the spring is unstretched ($x = 0$) when a constant horizontal force $\vec{F}$ in the positive direction of the x-axis is applied to it. A plot of the resulting kinetic energy of the block versus its position x is shown in figure. What is the magnitude of $\vec{F}$ (in N)?

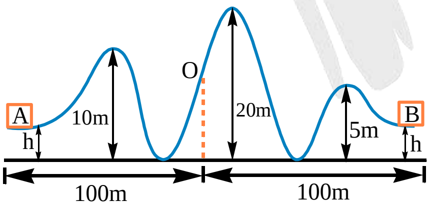


9. A stone on a rope is released at point P (refer figure). A pencil is firmly located in the way of the string. As a result the stone will be deflected from its normal path. At which of the following points the stone will stop and retrace its path?



(A) Point A (B) Point B (C) Point C (D) Point D

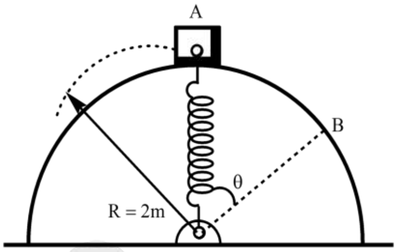
10. A point object of mass 2 kg is moved from point A to point B very slowly on a curved path by applying a tangential force on a curved path as shown in figure. Then find the work done by external force in moving the body. Given that $\mu_s = 0.3$, $\mu_k = 0.1$ for the part OB while AO is smooth.



(A) 50 J (B) 100 J (C) 150 J (D) 200 J

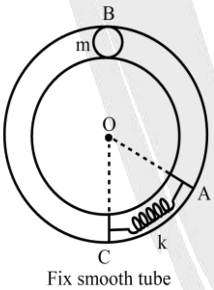
PROFICIENCY TEST – 2

1. A 2 kg block is gently pushed from rest at A and it slides down along the fixed smooth circular surface. If the attached spring has a force constant k =20N/m. What is unstretched length of spring so that it does not allow the block to leave the surface until angle with the vertical is $\theta = 60°$

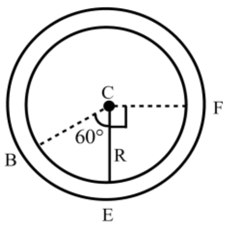


(A) 1m (B) 1.5 m (C) 0.5 m (D) 0.8 m

2. In the figure shown, there is a smooth tube of radius 'R', fixed in the vertical plane. A ball 'B' of mass 'm' is released from the top of the tube. B slides down due to gravity and compresses the spring. The end 'C' of the spring is fixed and the end A is free. Initially the line OA makes an angle of $60°$ with OC and finally it makes an angle of $30°$ after compression. The spring constant of the spring is $\dfrac{nmg\left(\alpha+\sqrt{\beta}\right)}{\pi^2 R}$. Find $n+\alpha+\beta$?



3. A shown in figure BEF is a fixed vertical circular tube. A block of mass m starts moving in the tube at point B with velocity V towards E. It is just able to complete the vertical circle, then :



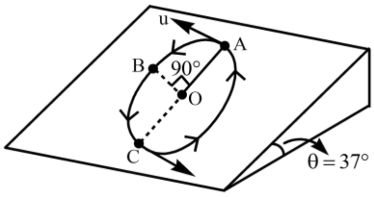
(A) velocity at B must be $\sqrt{3Rg}$

(B) velocity at F must be $\sqrt{2Rg}$

(C) normal reaction at point F is 2 mg

(D) the normal reaction at point E is 6 mg

4. A small sphere of mass m is connected by a string to a nail at O and moves in a circle of radius r on the smooth plane inclined at an angle θ with the horizontal. If the sphere has a velocity u at the top position A. Mark the correct option(s).



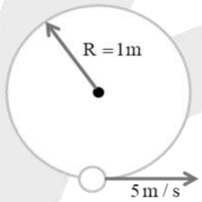
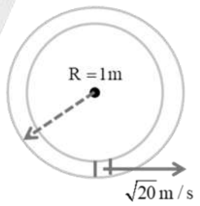
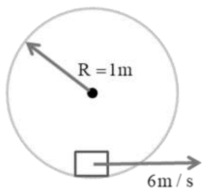
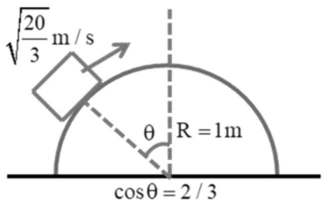
(A) Minimum velocity at A so that string does no get slack instantaneously is $\sqrt{\frac{3}{5}gr}$

(B) Tension at B if sphere has required velocity to just complete circle A is $\frac{11}{5}$mg $\frac{11}{5}$mg

(C) Tension at C if sphere has required velocity to just complete circle is $\frac{21}{5}$mg

(D) Centripetal force at point A is $\frac{3}{5}$mg if it just get slack instantaneously

5. In column-I, a situation is depicted each of which is in vertical plane. The surfaces are frictionless. Match with appropriate entries in column-II.

	Column – I		Column – II
(A)	Bead is threaded on a circular fixed wire and is projected from the lowest point. 	(P)	Normal force is zero at top most point of its trajectory.
(B)	Block loosely fits inside the fixed small tube and is projected from lowest point. 	(Q)	Velocity of the body is zero at top most point of its trajectory.
(C)	Block is projected horizontally from lowest point of a smooth fixed cylinder. 	(R)	Acceleration of the body is zero at the top most point of its trajectory.
(D)	Block is projected on a fixed hemispherical from angular position θ. 	(S)	Normal force is radially outward at top most point of trajectory.

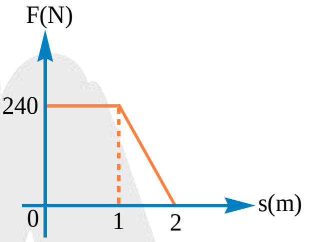
(A) $(A-Q,S);(B-P,Q);(C-P);(D-Q,R,S)$

(B) $(A-P,Q);(B-P,Q,S);(C-Q);(D-Q,S)$

(C) $(A-P,Q);(B-P);(C-P,Q);(D-P,Q)$

(D) $(A-Q,R,S);(B-P);(C-P);(D-Q,S)$

6. A block of mass 45 kg resting on a horizontal surface is acted upon by a force F which varies as shown in the figure. If the coefficient of friction between the blcok and surface is 0.2, find the displacement when the block will come to rest.

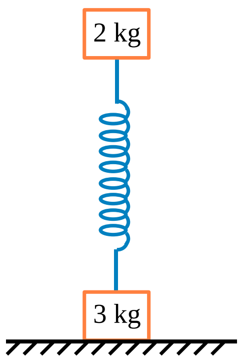


(A) 2 m (B) 4 m (C) 6 m (D) 8 m

7. A 0.5 kg block slides from point A on a horizontal track with an initial speed of 3 m/s towards a weightless spring of length 1 m and having a force constant 2 N/m as shown in figure. The part AB of the track is the frictionless and the part BC has co-efficient of static and kinetic friction as 0.22 amd 0.20 respectively. If the distance AB and BD are 2 m and 2.14 m respectively. Find the total distance through which the block moves before it comes to rest completely. $[g = 10m/s^2]$

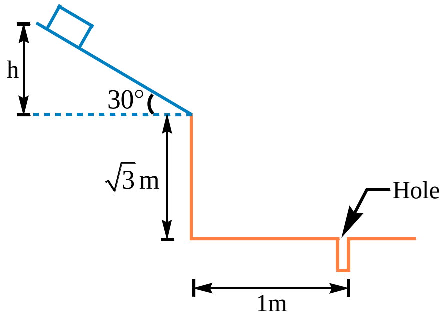
(A) 124 cm (B) 224 cm (C) 324 cm (D) 424 cm

8. The ends of a spring are attached to blocks of mass 3 kg and 2 kg as shown in the figure. The 3 kg block rests on a horizontal surface and the 2 kg block which is vertically above it is in equilibrium producing a compression of 2 mm of the spring. It is found that the 2 kg mass must be compressed further by at least x (in mm) so that when it is released, the 3 kg block may be lifted off the ground. Find the value of x in



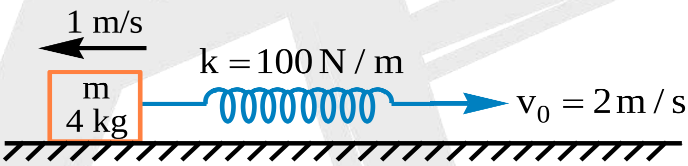
(A) 5 mm (B) 4 mm (C) 2.5 mm (D) 1.5 mm

9. A small block is placed at height h on a frictionless ramp having inclination 30°. When released from height 'h', the block slides down the ramp and then fall 1 m away in the hole. In order to fall directly in the hole, find the height h



(A) $\frac{1}{3\sqrt{2}}$ (B) $\frac{1}{2\sqrt{3}}$ (C) $\frac{3}{2\sqrt{3}}$ (D) $\frac{3}{3\sqrt{2}}$

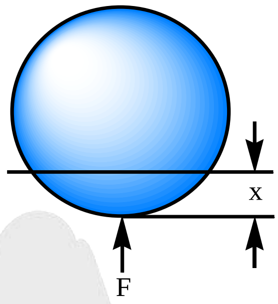
10. The spring block system lies on a smooth horizontal surface. The free end of the spring is being pulled towards right with constant speed $v_0 = 2 m/s$. At t = 0 sec, the spring of constant $k = 100 N/cm$ is unstretched and the block has a speed 1 m/s to left. Find the maximum extension of the spring is.



(A) 2 cm (B) 4 cm (C) 6 cm (D) 8 cm

PROFICIENCY TEST – 3

1. Experimentally it has been found that the force F needed to compress elastically a ball through a distance x (as shown in the figure) follows the formula $F(x) = ax + bx^2 + cx^3$ where a, b and c are constants. The small ball of mass m at rest is dropped from a great height h. It bounces elastically off the floor and is compressed a maximum distance 'd' during the bounce. The height h is:



(A) $\frac{1}{mg}\left(\frac{1}{2}ad^2 + \frac{1}{3}bd^3 + \frac{1}{4}cd^4\right)$ (B) $\frac{1}{mg}\left(ad + bd^2 + cd^3\right)$

(C) $\frac{1}{mg}\left(ad^2 + bd^3 + cd^4\right)$ (D) $\frac{1}{mg}\left(ad^2 + 2bd^3 + 3cd^4\right)$

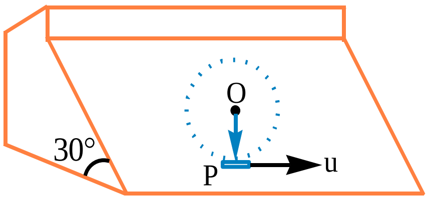
Where 'g' is the acceleration due to gravity.

2. Two smooth tracks of equal length have "bumps" – A up, and B down, both of the same curvature and size. The two balls start simultaneously with the same initial speed of 3 m/s. If the speed of the ball at the bottom of the curve on track B is 4 m/s, then the speed of the ball at the top of the curve on track A is:

(A) $= 2\,m/s$ (B) $> 2\,m/s$

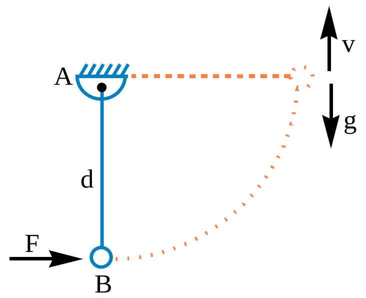
(C) $< 2\,m/s$ (D) not enough information given

3. A particle is attached with a string of length ℓ which is fixed at point O on an inclined plane. What minimum velocity should be given (at the lowest point) to the particle along the incline so that it may complete a circle on inclined plane (plane is smooth and initially particle was resting on the inclined plane)?



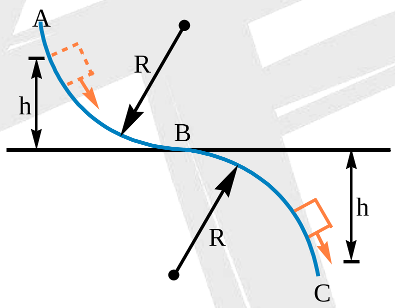
(A) $\sqrt{5g\ell}$ (B) $\sqrt{\frac{5g\ell}{2}}$ (C) $\sqrt{\frac{5\sqrt{3}g\ell}{2}}$ (D) None of these

4. A small sphere 'B' of mass M is connected at one end of a light rigid rod whose other end is hinged so that the sphere hangs vertically. At some instant of time a strong wind begins to apply a constant horizontal force to sphere 'B'. As a result, the sphere rotates about A in a vertical plane. The speed of sphere B at the instant when the rod becomes horizontal:

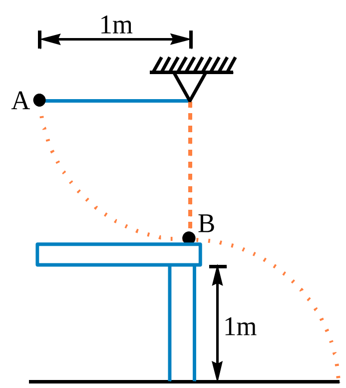


(A) $\sqrt{\left(\frac{F}{M}-g\right)\pi d}$ (B) $\sqrt{\frac{Fd-2Mgd}{M}}$ (C) $\sqrt{\frac{2Fd-2Mgd}{M}}$ (D) $\sqrt{\frac{Fd\pi-Mgd}{M}}$

5. Figure shows a smooth track in a vertical plane, consisting of two circular arcs AB and BC of the same radius $R = 2\text{m}$. The common tangent to the arcs, BD, is horizontal. A small block placed on the track at a height h above BD leaves the track at exactly the same depth below. Determine h in cm.

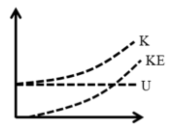
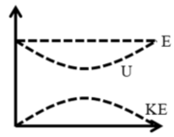
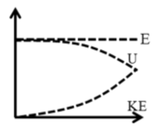
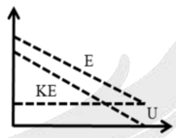


6. A small steel ball B is at rest on the edge of a table of height 1m. Another steel ball A, used as the bob of a metre long simple pendulum, is released from rest with the pendulum suspension horizontal, and swing against B as shwon in the figure. The masses of the balls are identical and the collision is elastic. Consider the motion of B only up until it first hits the ground.



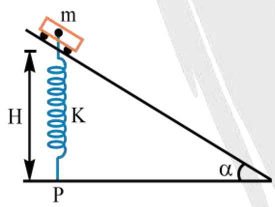
(A) Ball A is in motion for longer time (B) Ball B is in motion for longer time
(C) Ball A has greater path length (D) Ball B has greater path length

7. Match the physical situation on the left with the graph on the right. The graphs depict the variation of total energy (E), potential energy (U) and kinetic energy (KE) with time.

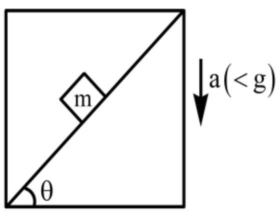
	Column – I		Column – II
(A)	A mass attached to an unstretched ideal spring. Released in vertical plane from rest until it reaches its maximum extension.	(P)	
(B)	An object undergong free fall.	(Q)	
(C)	An object being pulled on a level, frictionless surface by a constant force in the horizontal direction.	(R)	
		(S)	

(A) (A – R); (B – P); (C – Q)
(B) (A – Q); (B – P); (C – R)
(C) (A – Q); (B – R); (C – P)
(D) (A – R); (B – Q); (C – Q)

8. On a slope of height h a small body is sliding down without friction. It is connected to a point P at the bottom of the slope with a push-and-pull pring of an elastic constant of K, which was initially not extended. The body just stops at the bottom of the slope. If 'a' is the acceleration when the body of mass m starts sliding back, then what will be the value of $\frac{a}{6}$.

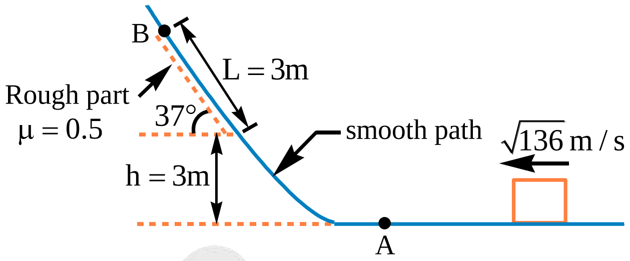


9. In the arrangement shown a blcok of mass m is kept over an incline plane and inclination θ and coefficient of friction $\mu(\mu > \tan\theta)$. Whole system is kept in a lift and lift starts moving in vertical downward direction with constant acceleration $a(< g)$ with zero initial velocity. Work done by net contact force on the block from t = 0 to $t = t_0$ is W. Find the value of 'a' for which value of |W| is maximum, is :



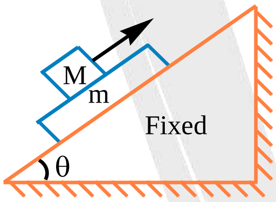
(A) $2\,m/s^2$
(B) $3\,m/s^2$
(C) $4\,m/s^2$
(D) $5\,m/s^2$

10. A small block slides along a patth which is smooth until the block reaches the section of length L = 3m, which begins at height h = 3m on a flat incline of angle $37°$, as shown in the figure. In that section, the coefficient of kinetic friction is 0.50. The block passes through point A with a speed of $\sqrt{136}\,m/s$. Find the speed of the block as it passes through point B where the friction ends.



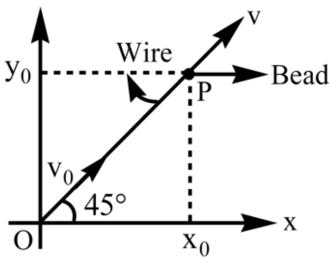
(A) 1 m/s (B) 2 m/s (C) 3 m/s (D) 4 m/s

11. A board of mass m is placed on a frictionless inclined plane that makes an angle θ with the horizontal. A block of mass M is placed on the board and is given a quick push up the board with initial velocity v. There is sufficient friction between board and block. Find the distance d covered by the block by the time its velocity drops to $\frac{v}{2}$. The board does not move relative to the plane. $\left[\text{Given}: M=2\,kg,\ m=4\,kg,\ \theta=30°,\ v=4\sqrt{5}\,ms^{-1}\right]$



(A) 1 m (B) 2 m (C) 3 m (D) 4 m

12. A bead slides along a frictionless wire lying on a horizontal plane. It makes an angle of $45°$ with x-axis as shown in figure. In addition to any normal forces exerted by the wire, the bead is subject to an external force that depends on position according to formula $\vec{F} = F_0\left(\frac{x}{x_0}\right)^2 \hat{i} + F_0\left(\frac{y}{y_0}\right)^2 \hat{j}$. Find the work done by the force $\vec{F}$ on bead till it reaches at end P of the wire. (Given data : $F_0 = 1N, x_0 = 6m, y_0 = 6m$ mass of bead = 2 kg.)



(A) 2 J (B) 4 J (C) 6 J (D) 8 J

EXERCISE – 1 - KEY

1	2	3	4	5	6	7	8	9	10
A	B	C	C	D	B	A	C	A	A
11	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
44	D	B	B	D	AB	AB	CD	BC	B

EXERCISE – 2 - KEY

1	2	3	4	5	6	7	8	9	10
D	B	B	C	A	B	AC	BC	D	B
11	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
B	A	B	C	C	B	C	B	D	1
21	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
C	A	A	D	A	C	A	D	D	C

EXERCISE – 3 - KEY

1	2	3	4	5	6	7	8	9	10
5	C	D	C	A	C	10	A	6	D
11	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
B	D	C	B	BC	AD	A	ACD	BC	C
21	**22**	**23**	**24**	**25**					
B	B	A	D	C					

EXERCISE – 4 - KEY

1	2	3	4	5	6	7	8	9	10
D	D	A	C	2	5	A	B,C	A,C	A,B,C,D
11	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
A,C,D	D	B,C,D	A,B	C	A	D	C	C	C
21	**22**	**23**	**24**	**25**	**26**	**27**	**28**		
B	A	B	B	A	C	C	C		

EXERCISE – 5 - KEY

1	2	3	4	5	6	7	8	9	10
C	C	10	C	A,C	B,C	A,C,D	A,B,C,D	D	C
11	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
A	A	D	D	C	A	B	C	B	C
21	**22**								
C	C								

EXERCISE(ALP)
ONE OR MORE THAN ONE ANSWER_KEY

1	2	3	4	5	6	7	8	9	10
A,B,C,D	A,B,D	A,B,C,D	A,C,D	A,B	A,C,D	A,B	B,C	B,C	A,B,C
11	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
A,B	B,C	B,C,D	B,C,D,E	A,C,D	A,D	A,C	A,B,D	A,C	A,B,C,D

PROFICIENCY TEST – 1 - KEY

1	2	3	4	5	6	7	8	9	10
D	C	A	2	A	C	D	8	B	D

PROFICIENCY TEST – 2 - KEY

1	2	3	4	5	6	7	8	9	10
B	17	ABC	AD	A	B	D	A	B	C

PROFICIENCY TEST – 3 - KEY

1	2	3	4	5	6	7	8	9	10
A	C	B	C	40	AD	C	7	D	D
11	**12**								
B	B								