Nishant Jindal

ELECTROSTATICS

$$\boxed{E_{\perp \text{ to line charge}} = \frac{K\lambda}{d}(\sin\alpha + \sin\beta)}$$

$$\boxed{E_{\parallel \text{ to line charge}} = \frac{K\lambda}{d}\left[\cos\beta - \cos\alpha\right]}$$

$$E_{\parallel \text{ to line charge}} = \int_{-\beta}^{+\alpha} \underline{dE}\cdot\sin\theta = \frac{K\lambda}{d}\int_{-\beta}^{+\alpha} \sin\theta\cdot d\theta$$

$$E_{\parallel \text{ to line charge}} = \frac{K\lambda}{d}\left[-\cos\theta\right]_{-\beta}^{\alpha}$$

$$= \frac{K\lambda}{d}\left[-\cos\alpha + \cos\beta\right]$$

$$= \frac{K\lambda}{d}\left[\cos\beta - \cos\alpha\right]$$

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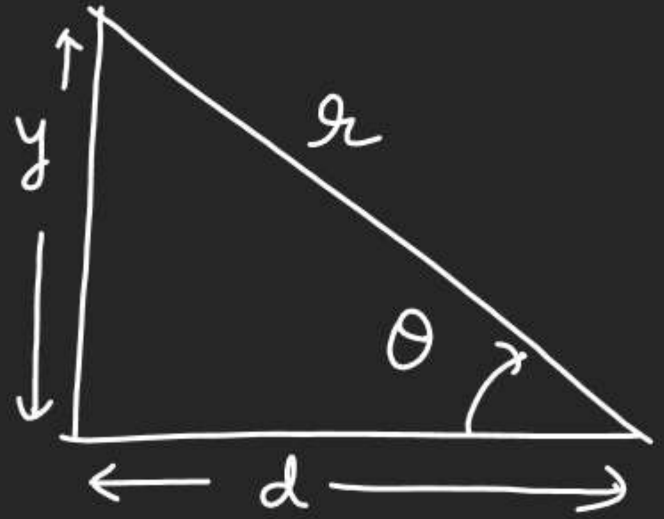
ELECTROSTATICS

$$dE = \frac{K\lambda\,dy}{r^2}$$



$$\tan\theta = \frac{y}{d} \qquad \cos\theta = \frac{d}{r}$$

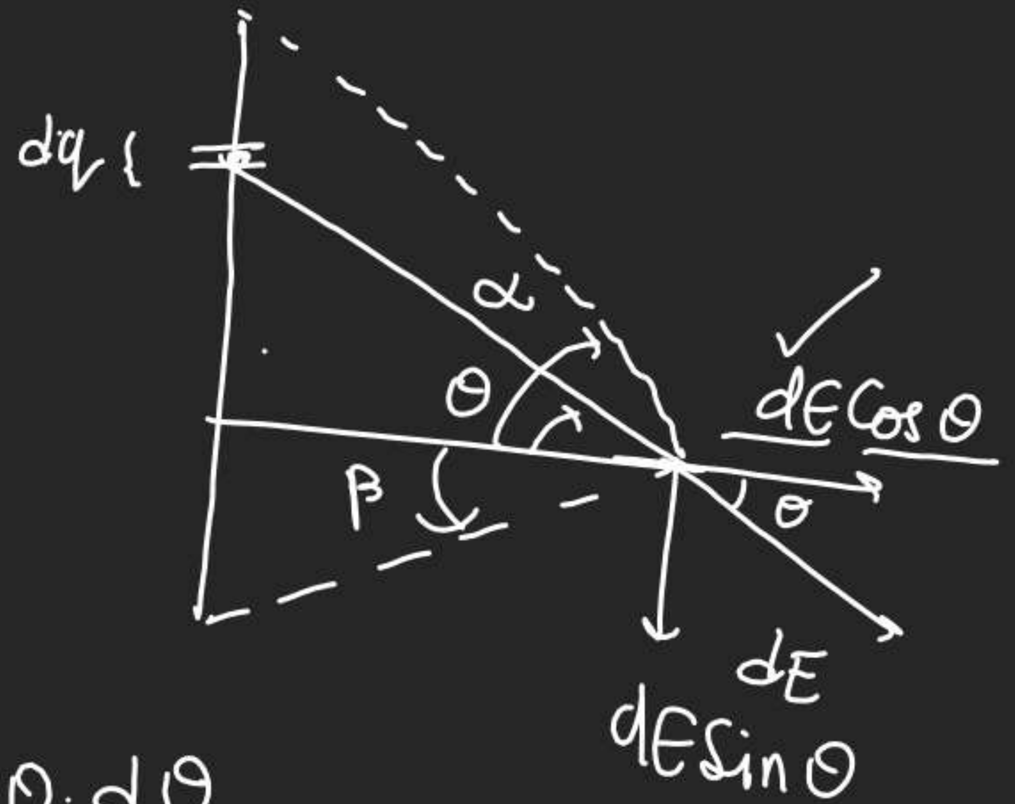
$$y = d\tan\theta \qquad r = d\sec\theta$$

$$\frac{dy}{d\theta} = d\sec^2\theta$$

$$dy = d\sec^2\theta \cdot d\theta$$

$$dE = \frac{K\lambda\,\cancel{d}\,\cancel{\sec^2\theta}\cdot d\theta}{\cancel{d^2}\,\cancel{\sec^2\theta}}$$

$$dE = \left(\frac{K\lambda}{d}\right)d\theta$$

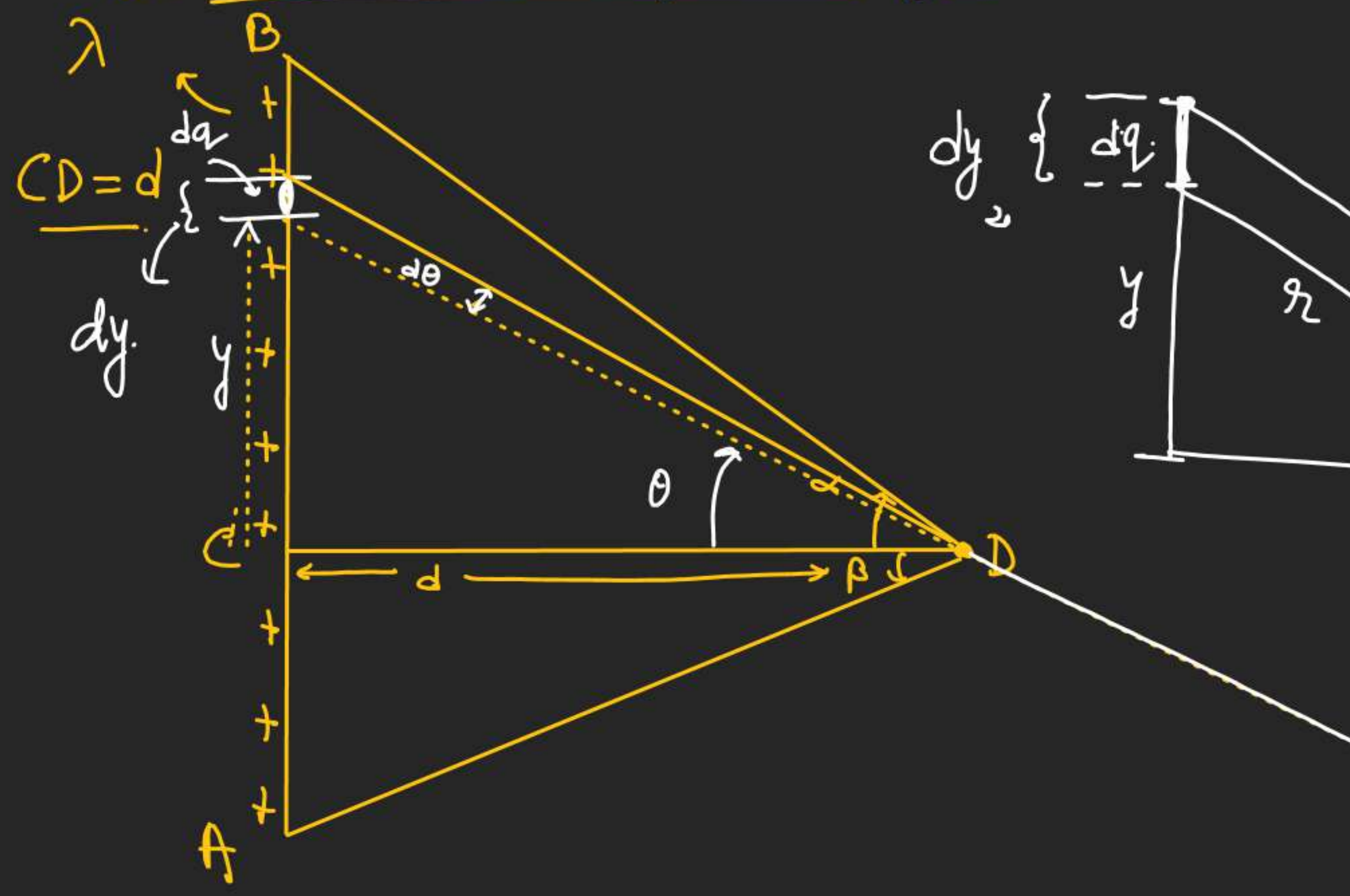


$$E_{\perp\text{ to line charge}} = \int_{-\beta}^{+\alpha} \frac{K\lambda}{d}\cos\theta\cdot d\theta$$

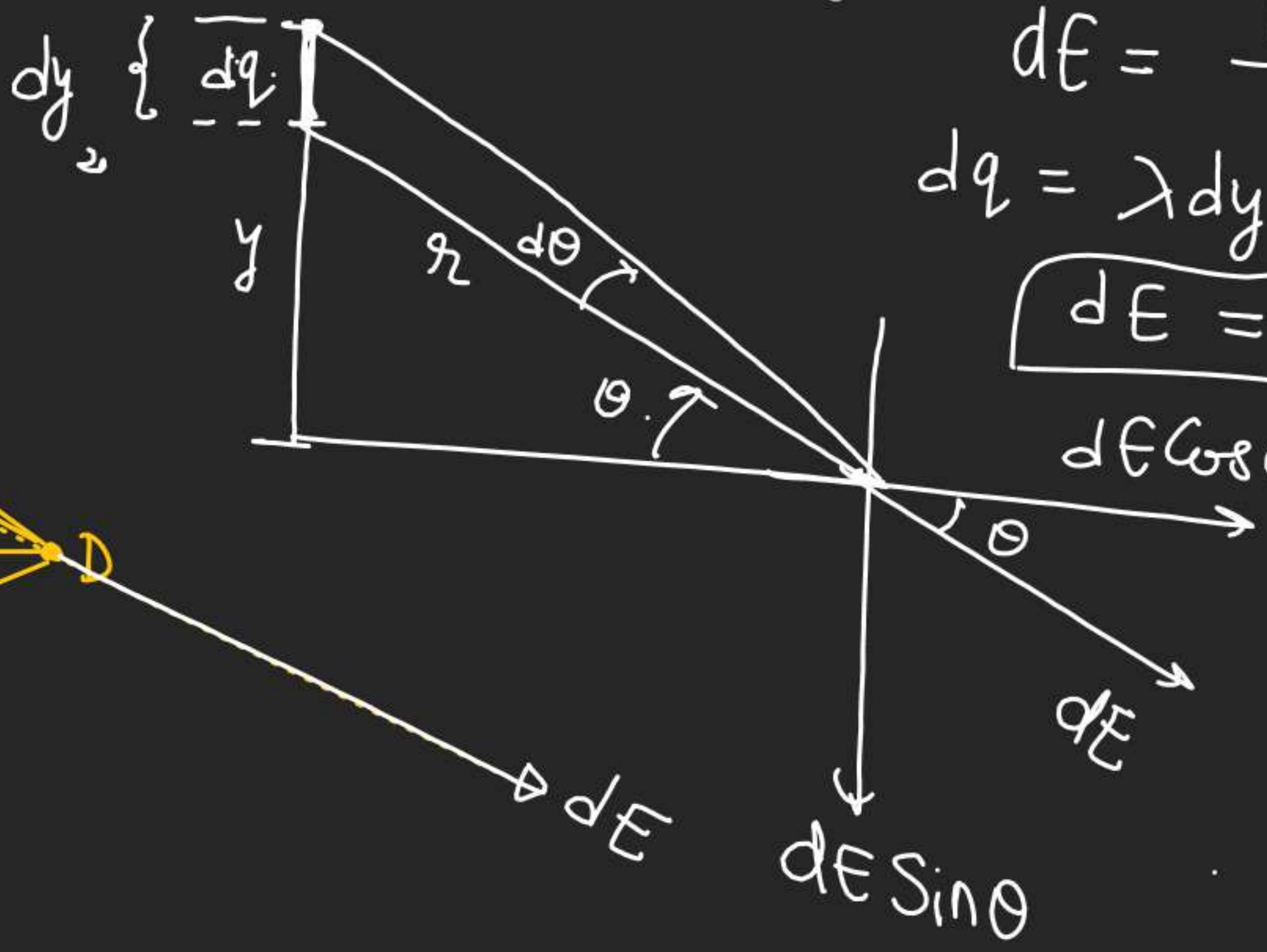
$$= \frac{K\lambda}{d}\int_{-\beta}^{\alpha}\cos\theta\cdot d\theta = \frac{K\lambda}{d}\left[\sin\theta\right]_{-\beta}^{\alpha}$$

$$E_{\perp\text{ to line charge}} = \frac{K\lambda}{d}\left[\sin\alpha - \sin(-\beta)\right]$$

Electric field due to Uniformly Charge rod at any poin P as shown in fig.



$d\theta \rightarrow$ very small



$$dE = \frac{K\,dq}{r^2}$$

$$dq = \lambda\, dy$$

$$\boxed{dE = \frac{K\lambda\, dy}{r^2}}$$

$$dE\cos\theta =$$

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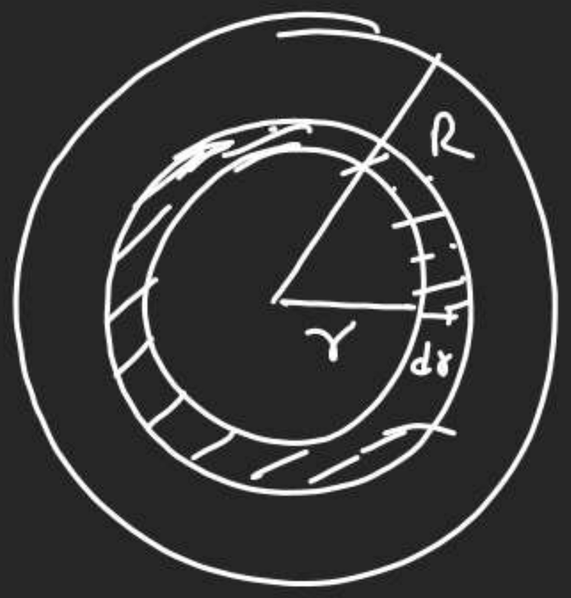
$$\underline{dV} = (4\pi r^2)dr$$

$$dq = \rho_r \, dV$$

$$Q = \int_0^R \rho_0\left(1-\frac{r}{R}\right)4\pi r^2 dr$$

$$Q = \rho_0 4\pi\left[\int_0^R r^2 dr - \frac{1}{R}\int_0^R r^3 dr\right]$$

$$Q = \rho_0 4\pi\left[\frac{R^3}{3} - \frac{1}{\cancel{R}}\left(\frac{R^{\cancel{4}}}{4}\right)\right]$$



$$\rho = \rho_0\left(1-\frac{r}{R}\right)$$

$$\int x^n dx = \frac{x^{n+1}}{n+1}$$

$$Q = \rho_0 4\pi\left[\frac{R^3}{3} - \frac{R^3}{4}\right]$$

$$Q = \rho_0 4\pi\left(\frac{4R^3 - 3R^3}{12}\right)$$

$$Q = \frac{\rho_0 4\pi R^3}{12} = \frac{\rho_0 \pi R^3}{3}$$

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ELECTROSTATICS

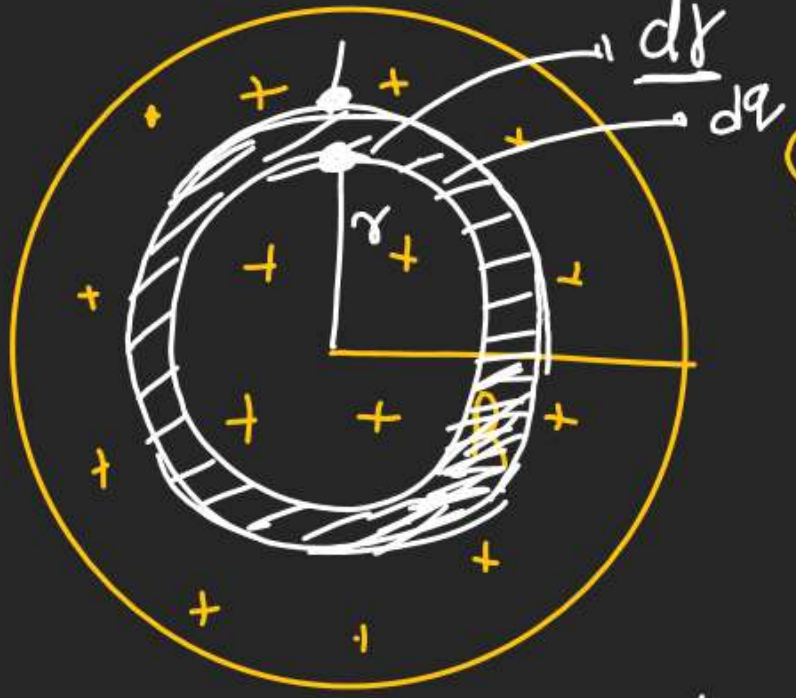
Volume Charge distribution

$$\boxed{\rho = \frac{Q}{V}}$$

[Charge per Unit Volume

If ρ = Constant
↳ Uniformly distributed.

$\rho \neq$ Constant ⇒ Non-uniform distribution.



ρ = Constant

$$Q = \rho \cdot \frac{4}{3}\pi R^3$$

If $\rho = \rho_0\left(1 - \frac{r}{R}\right)$ find total Charge in the Sphere. ρ_0 & R constant
$r \to$ distance from center.

$$\rho_r = \rho_0\left(1 - \frac{r}{R}\right)$$

$$\rho_{r+dr} = \rho_0\left(1 - \frac{r+dr}{R}\right)$$

$\boxed{r+dr \simeq r}$ ↓ Very Small.

For 'dr' thickness ρ is assumed to be constant.

$$dq = \rho_r \cdot dV$$

$$dV = (\text{Area of differential element}) \times (\text{thickness})$$

$dV \to$ differential volume of hollow Sphere having radius r and thickness dr

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$$dq = \sigma_r \cdot dA$$

$$dq = (\sigma_0 r)(2\pi r)dr$$

$$\int_0^Q dq = \sigma_0 2\pi \int_0^R r^2 dr$$

$$Q = \frac{\sigma_0 2\pi R^3}{3}$$

ELECTROSTATICS

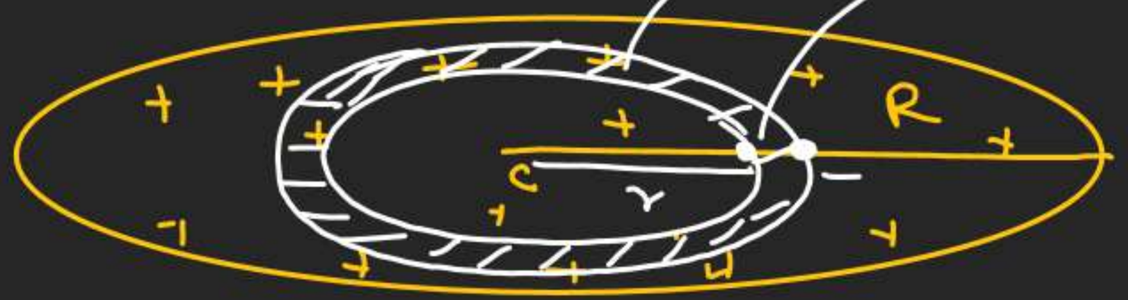
⇒ Surface Charge distribution

$$\sigma = \left(\frac{Q}{A}\right)$$

if σ = Constant
↓ Uniformly distributed.

if $\sigma \neq$ Constant
Non - uniformly distributed

[If $\sigma = (\sigma_0 r)$ where 'r' is radial distance. then find total Charge.



$\sigma_r \approx \sigma_{(r+dr)}$

$dq = (\sigma_r)(dA)$ σ is ie for dr thickness assumed to be constant

dA = (Length of the differential element) × (thickness)

$$\boxed{dA = (2\pi r)dr}$$

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ELECTROSTATICS

$$\int_0^Q dq = \lambda_0 \int_0^L x^2\,dx$$

$$Q = \lambda_0 \left[\frac{x^3}{3}\right]_0^L$$

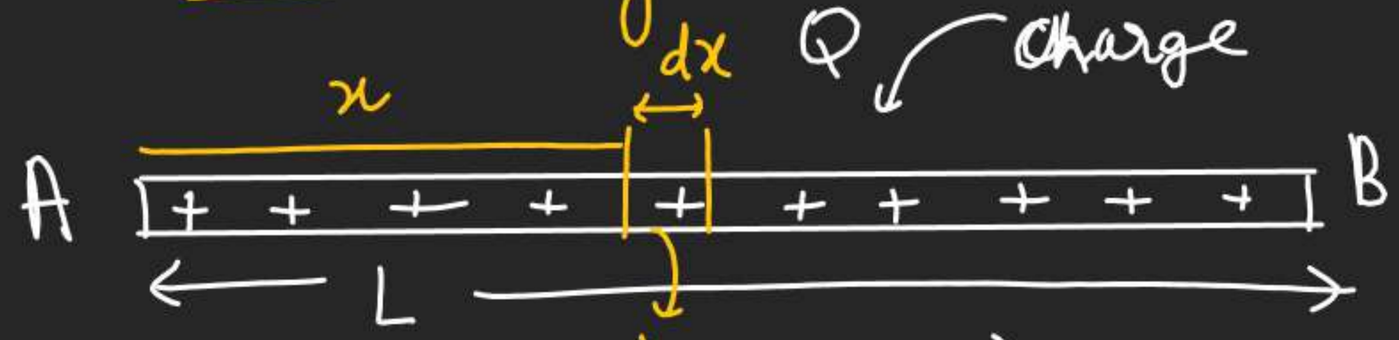
$$\boxed{Q = \frac{\lambda_0 L^3}{3}}$$

$$\boxed{\int x^n\,dx = \frac{x^{n+1}}{n+1}}$$

ELECTROSTATICS

Electric field due to continious charge distribution

Linear charge distribution



$\lambda = \left(\frac{Q}{L}\right)$ ⇓ Linear Charge density

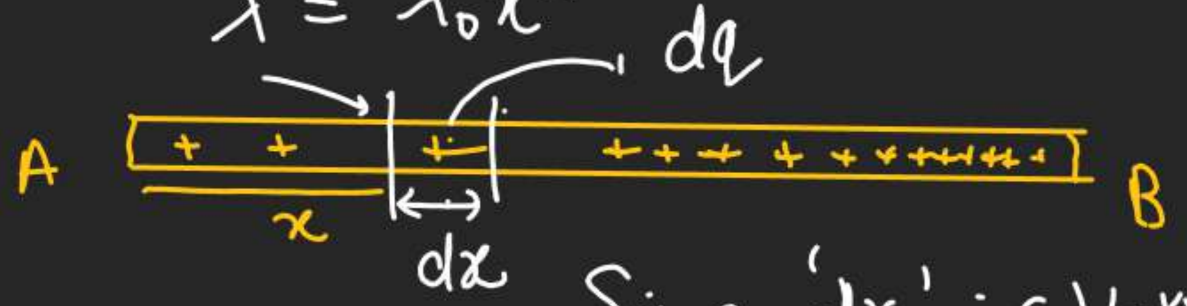
if $\lambda = C$ ↳ Uniformly distributed

$$\boxed{Q_x = \frac{Q}{L}x}$$

$$dq = \frac{Q}{L}dx = \lambda dx$$

If λ is not uniform

$\lambda = \lambda_0 x^2$



dq

Since 'dx' is very small λ_x is same as $\lambda(x+dx)$

$\lambda_x \simeq \lambda(x+dx)$

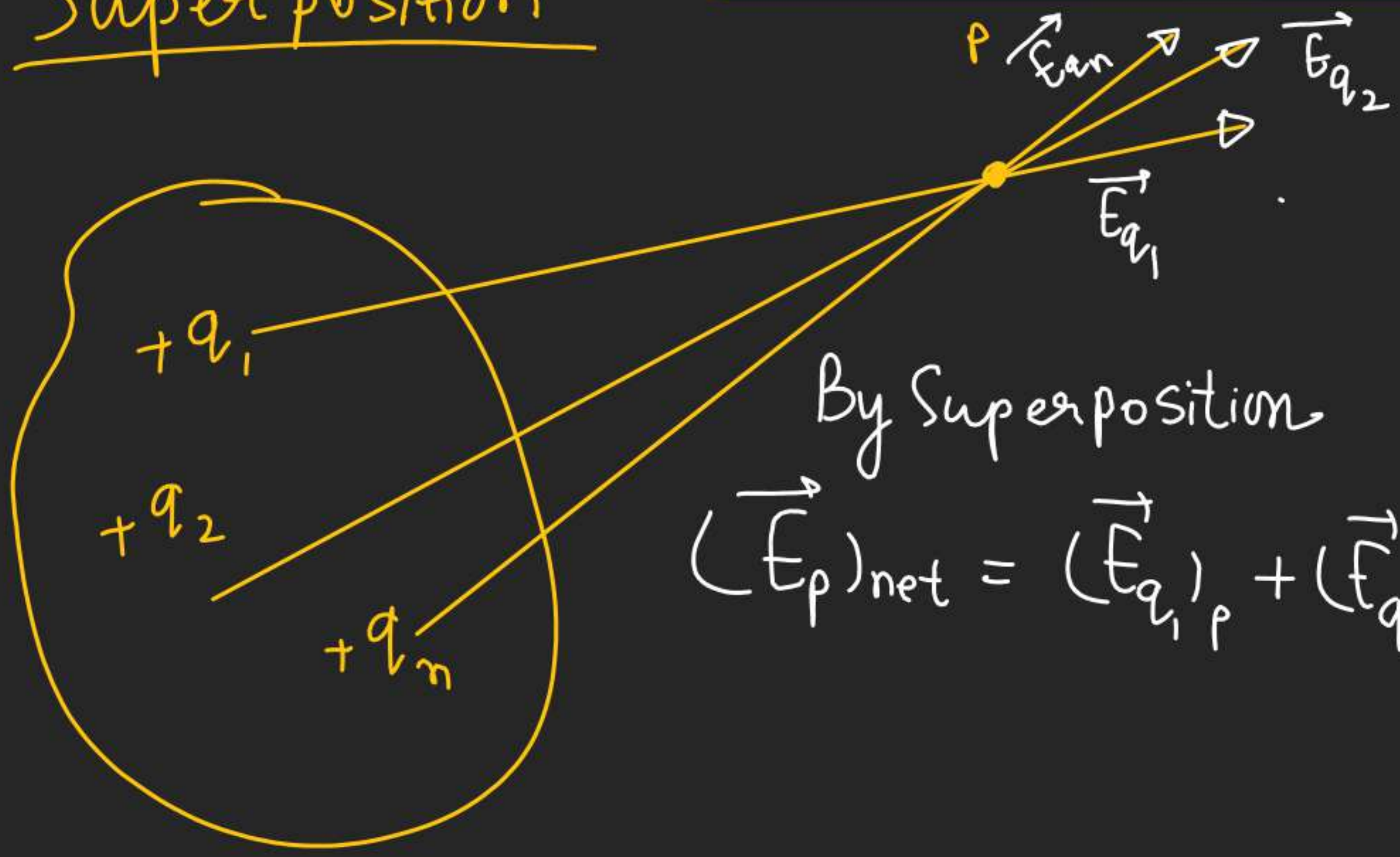
$x \simeq x + dx$ — very small

$$dq = \lambda_x dx$$

$$\int_0^Q dq = \lambda_0 \int_0^L x^2 dx$$

ELECTROSTATICS

Superposition



By Superposition

$$(\vec{E}_p)_{net} = (\vec{E}_{q_1})_p + (\vec{E}_{q_2})_p \cdots + (\vec{E}_{q_n})_p$$

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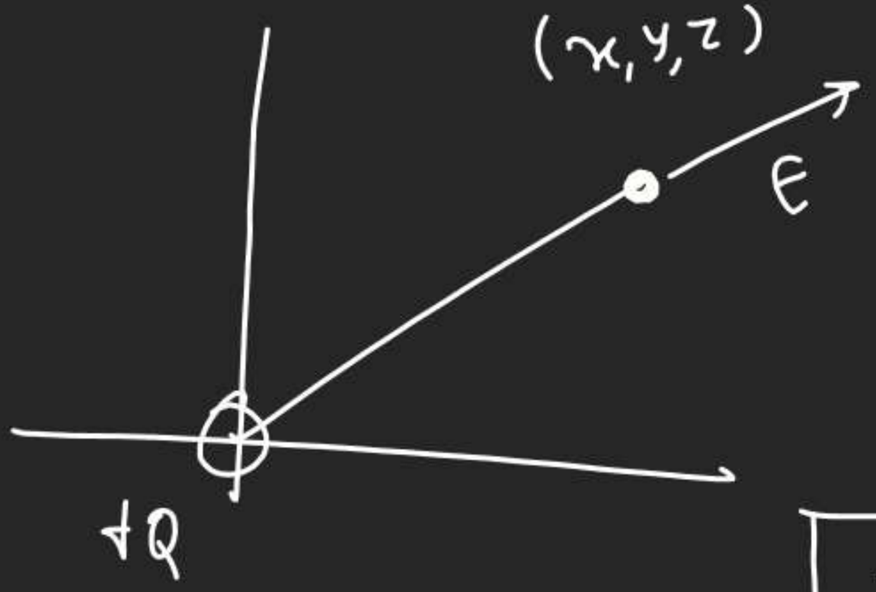
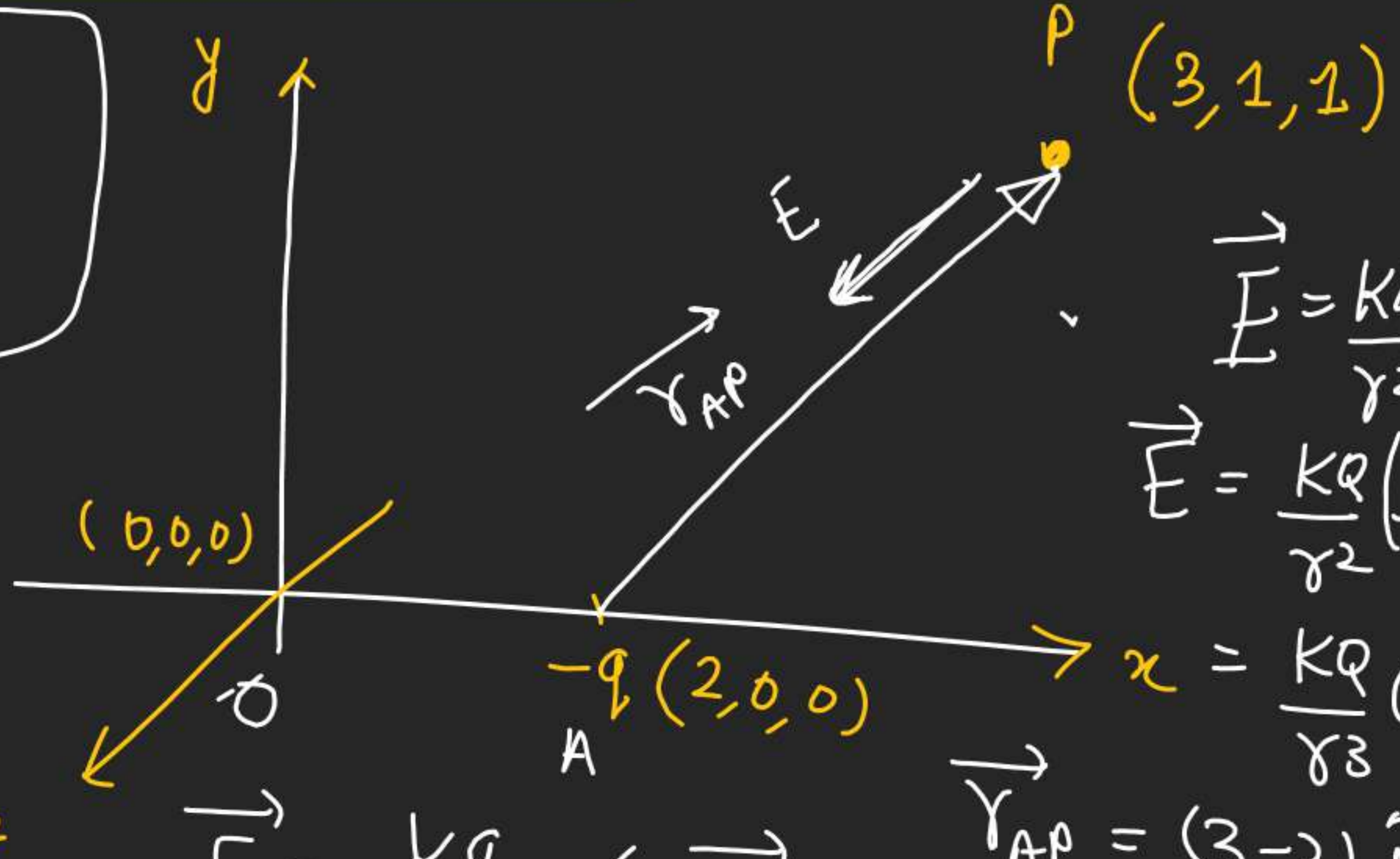
ELECTROSTATICS

$$\boxed{\vec{E} = \frac{KQ}{(x^2+y^2+z^2)^{3/2}}\left[x\hat{i}+y\hat{j}+z\hat{k}\right]}$$





$$\vec{E} = \frac{KQ}{r^2}\hat{r}$$

$$\vec{E} = \frac{KQ}{r^2}\left(\frac{\vec{r}}{|\vec{r}|}\right) = \frac{KQ}{r^3}(\vec{r})$$

$$\vec{r}_{AP} = (3-2)\hat{i} + (1-0)\hat{j} + (1-0)\hat{k}$$

$$\vec{r}_{AP} = \underline{\hat{i}+\hat{j}+\hat{k}}$$

$$\vec{E} = \frac{Kq}{|\vec{r}_{AP}|^3}\left(-\vec{r}_{AP}\right)$$

$$\boxed{\vec{E} = \frac{-Kq}{(3)^{3/2}}\left(\hat{i}+\hat{j}+\hat{k}\right)}$$

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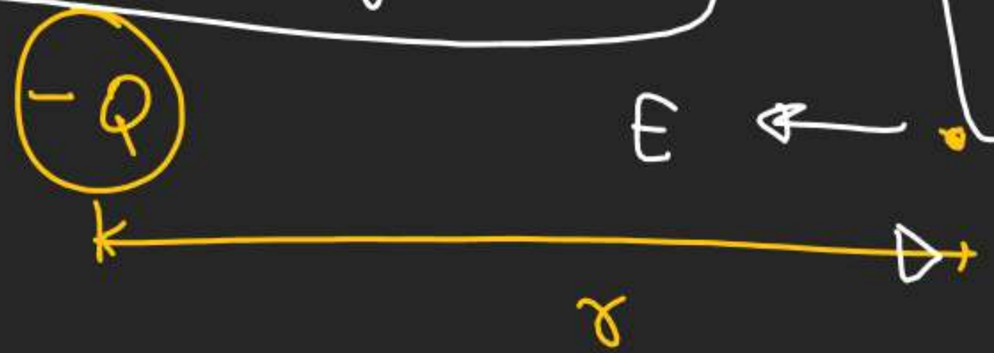
ELECTROSTATICS

Electric field due to point Charge

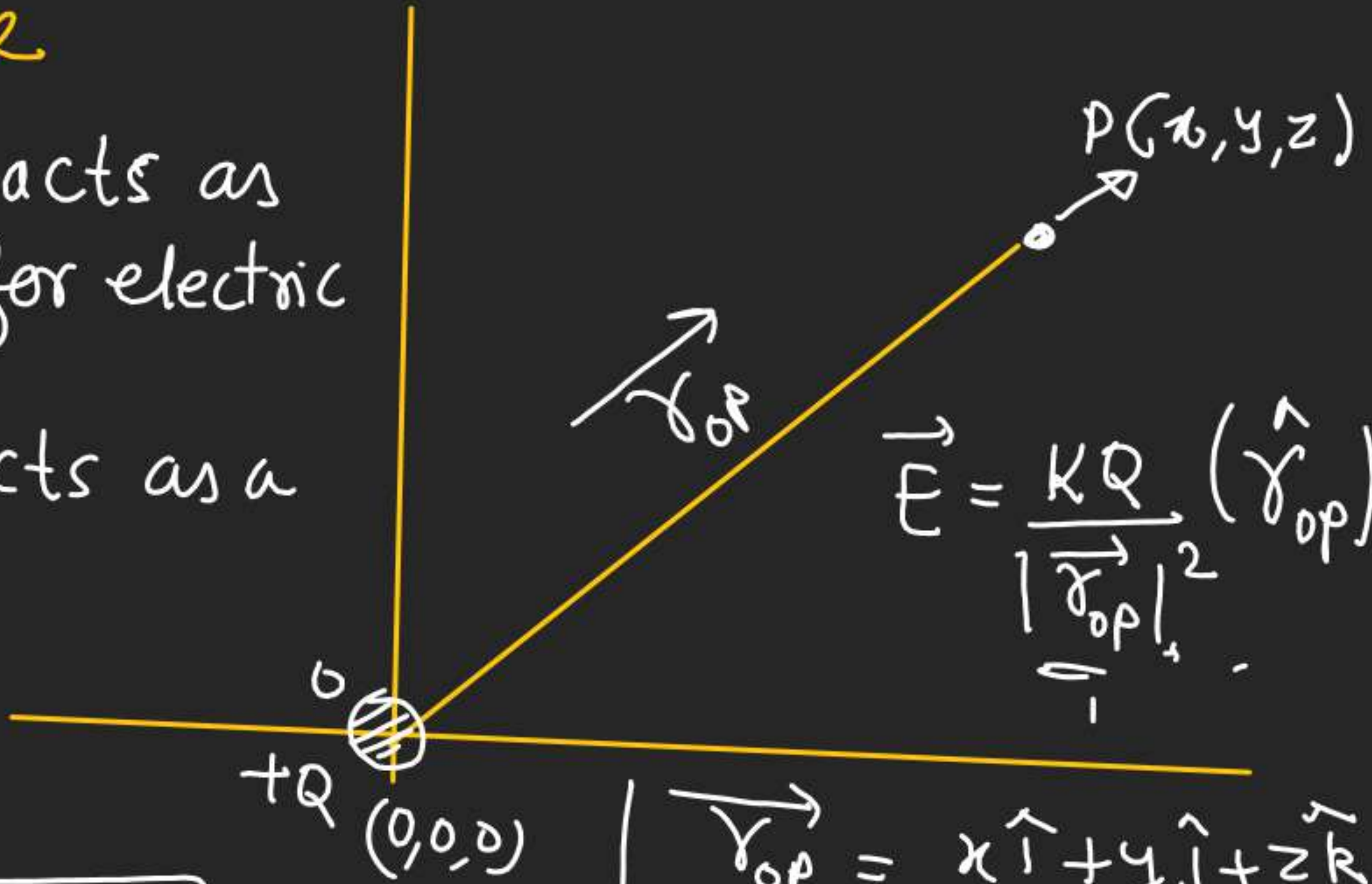
$+Q \rightarrow$ Always acts as Source for electric field.

$-Q \rightarrow$ Always acts as a Sink.

$$\boxed{\vec{E} = \left[\frac{KQ}{r^2}\right]\hat{r}}$$



$$\boxed{\vec{E} = \left(\frac{KQ}{r^2}\right)(-\hat{r})}$$



$$\vec{E} = \frac{KQ}{|\vec{r}_{op}|^2}(\hat{r}_{op})$$

$$\vec{r}_{op} = x\hat{i} + y\hat{j} + z\hat{k}$$

$$|\vec{r}_{op}| = \sqrt{x^2+y^2+z^2}$$

$$\hat{r}_{op} = \frac{\vec{r}_{op}}{|\vec{r}_{op}|}$$

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ELECTROSTATICS

Electric field

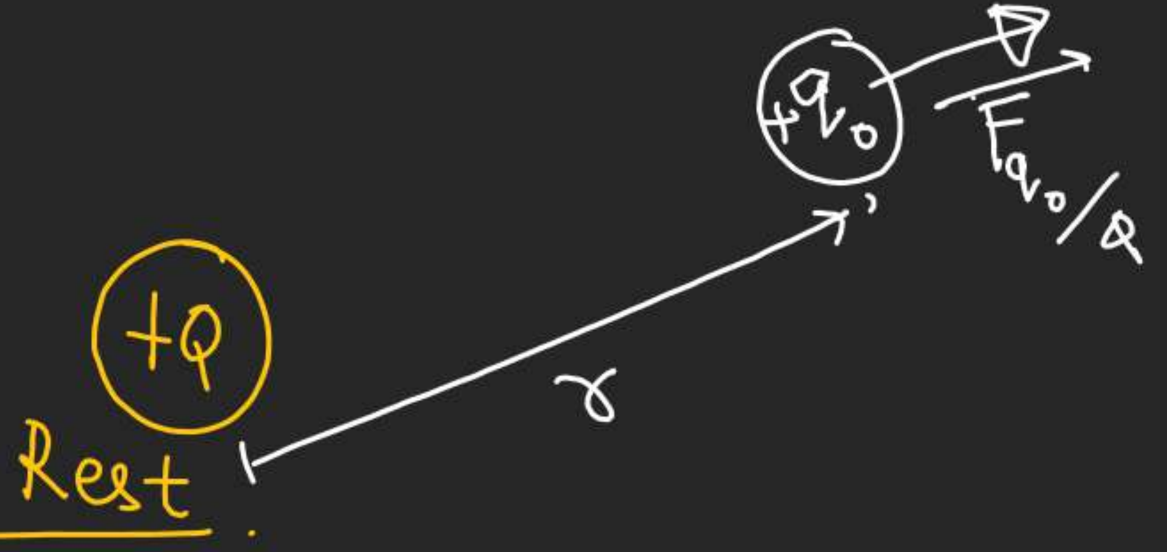
↳ It is defined as force acting per unit Charge

$$\vec{E} = \lim_{q \to 0} \frac{\vec{F}}{q}$$

$$\boxed{\vec{E} = \frac{\vec{F}}{q_0}}$$

$$\boxed{\vec{F} = q_0 \vec{E}}$$

$q \to 0 \Rightarrow$ Test charge (q_0)



$$\left|\vec{F}_{q_0/Q}\right| = \frac{KQq_0}{r^2}$$

$$\frac{\left|\vec{F}_{q_0/Q}\right|}{q_0} = \frac{KQ}{r^2} \to \vec{E}_Q$$