Nishant Jindal

Fundamentals of Mathematics

Q. $LCM(\sqrt{2}, \sqrt{3}) = ?$

Q' Q'

$LCM(\sqrt{2}, \sqrt{3}) = \sqrt{6}$ let

Check.

$\frac{\sqrt{6}}{\sqrt{2}}, \frac{\sqrt{6}}{\sqrt{3}}$

$\sqrt{3}, \sqrt{2}$

$\notin I \quad \notin I$

$\Rightarrow \sqrt{6}$ is not LCM of $\sqrt{2}$ & $\sqrt{3}$

$LCM(\sqrt{2}, \sqrt{3})$ Not Possible.

funda: $LCM(a, b) = c$

then $\frac{c}{a}, \frac{c}{b}$ should be Int.

* RK: Product of HCF & LCM

= Product of 2 No

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Q. Find Product of 2 No. if their HCF is 4 & LCM is 36?

Product of $(a, b) = a \times b = \boxed{HCF \times LCM}$

$a \times b = 4 \times 36$

$\boxed{a \times b = 144}$

Q. Find Product of 2 No if their HCF is 25 & LCM is 5?

Rem $HCF < LCM$

HCF is always lesser to LCM.

HCF 25 & LCM 5 not Possible

$\Rightarrow$ No Answer

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Adv.

Remainder Theorem :- If $P(x)$ is Polynomial of deg ≥ 1 & a is a Real No ($a \in R$). If $P(x)$ is divided by $\boxed{(x-a)}$ then Remainder $= P(a)$

Q Find Remainder of $\underbrace{x^3+x+1}_{\text{Poly}}$ is divided by $\boxed{(x-1)}$?

$a=1$

$P(x) = x^3+x+1$

here Remainder $= P(1) = 1^3+1+1$

$= 3$.

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Mains + Adv.

Q Find Remainder if x^2-3x+5 is divided by $(x+2)$?

$a=-2$

$P(x)=x^2-3x+5$

$Rem=P(a)=P(-2)=(-2)^2+3\times 2+5$

$=4+6+5$

$=15$

Q Find Remainder if x^2-x+3 is divided by $(2x-1)$.

$(a=\frac{1}{2})$

$P(x)=x^2-x+3$

$Rem=P(\frac{1}{2})=(\frac{1}{2})^2-\frac{1}{2}+3=3-\frac{1}{4}=\frac{11}{4}$

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Fundamentals of Mathematics

Factor Theorem :- If Polynomial $P(x)$ of degree ≥ 1 is divided by $(x-a)$ & $(x-a)$ is factor of $P(x)$ then $P(a)=0$.

Q Check whether $(x-3)$ is factor of x^3-3x^2+5x-7?

$\Rightarrow a=3$.

$$P(x) = x^3-3x^2+5x-7$$

$$P(3) = \cancel{3^3} - \cancel{3\times 3^2} + 5\times 3 - 7$$

$$= 15-7 = 8 \neq 0$$

$\therefore$ $(x-3)$ is not factor of x^3-3x^2+5x-7

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Q Check $(x+2)$ is Factor of $x^2 - x + 6$?

$a = -2$

$$P(x) = x^2 - x + 6$$

$$\left.\begin{aligned} P(-2) &= (-2)^2 - (-2) + 6 \\ &= 4 + 6 + 6 \neq 0 \end{aligned}\right\} \underline{\underline{\text{Not}}}$$

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Rem $= ax+b$

$= 2x+3$

Q. If $P(x)$ is divided by $(x-1)$ & Rem $=5$ again when $P(x)$ is divided by $(x+2)$ Rem $= -1$ find Rem $=$? when $P(x)$ is divided by $(x-1)(x+2)$

Most difficult

$P(x) = (x-1)Q_1(x) + 5 \rightarrow P(1) = 5$

$P(x) = (x+2)Q_2(x) + (-1) \rightarrow P(-2) = -1$

$P(x) = \underbrace{(x-1)(x+2)}_{\text{Quad}} Q_3(x) + \underbrace{ax+b}_{\text{Linear}} \xrightarrow[x=1]{} P(1) = \cancel{(1-1)(1+2)Q_1(x)} + a+b$

$5 = 0 + a + b$

$-1 = 0 + -2a + b$

$6 = 3a \Rightarrow a = 2$, $b = 3$

Dividend = Divisor × Quotient + Remainder

$P(x) = (x-a) \times Q(x) + R(x)$

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Q If $P(x)$ is divided by $(x-1)$ then Remainder is 2. If it is divided by $(x-2)$ then Rem = 1. Then find Remainder when $P(x)$ is divided by $(x-1)(x-2)$?

$P(1) = 2$ & $P(x) = (x-1)Q_1(x) + 2$

$P(2) = 1$ & $P(x) = (x-2)Q_2(x) + 1$

Useless.

Rem Hoga

$P(x) = (x-1)(x-2)Q_3(x) + \boxed{ax+b}$

$P(1) = 0 + a+b \Rightarrow \boxed{a+b = 2}$

$P(2) = 0 + 2a+b \Rightarrow 2a+b = 1$

$-a = 1 \Rightarrow a = -1 \quad b = 3$

Rem $= ax+b$

$= -x+3$

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Fundamentals of Mathematics

Q If Polynomial $P(x)$ is divided by $(x-1)$ then Rem $=1$ & If it is divided by $(x-4)$ then Rem $=10$. Find Remainder when Poly is divided by $(x-1)(x-4)$?

$$\text{Rem} = ax+b$$
$$= 3x-2$$

$P(1)=1$ & $P(x)=(x-1)Q_1(x)+1$

$P(4)=10$ & $P(x)=(x-4)Q_2(x)+10$

$$P(x)=(x-1)(x-4)Q_3(x)+\boxed{ax+b}$$

$P(1)=0+a+b \Rightarrow a+b=1$

$P(4)=0+4a+b \Rightarrow 4a+b=10$

$-3a=-9 \Rightarrow a=3,\ b=-2$

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Fundamentals of Mathematics

Q Find Remainder of $x^{135}+x^{125}-x^{115}+x^5+1$ is divided by x^3-x?

$\boxed{\text{Ans} = 2x+1}$

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Ratio & Proportion.

A) If a & b are Quantities of same type then Ratio is denoted by $a:b$ or $\frac{a}{b}$.

B) If A & B are same Quantities then Ratio = A : B

Ratio A : B

then $A^2 : B^2 \Rightarrow$ Duplicate Ratio

$A^3 : B^3 \Rightarrow$ Triplicate Ratio.

$A^{1/2} : B^{1/2} \Rightarrow$ Sub Duplicate Ratio

$A^{1/3} : B^{1/3} \Rightarrow$ Sub Triplicate Ratio

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(C) Ratios can also be Compounded.

if $x:y$, $a:b$, $m:n$ are 3 Ratio

then Compound Ratio = $\frac{x}{y} \times \frac{a}{b} \times \frac{m}{n}$.

(D) $\frac{a}{b} = \frac{2a}{2b} = \frac{17a}{17b} \Rightarrow \boxed{\frac{a}{b} = \frac{ma}{mb} = \frac{na}{nb}}$

(E) If a, b & c, d are in Proportion then

it means $a:b::c:d \Rightarrow \frac{a}{b} = \frac{c}{d}$

$1:2::2:4$

$\frac{1}{2} = \frac{2}{4}$

$\boxed{2:4::3:6}$

$\frac{2}{4} = \frac{3}{6}$

2, 4 & 3, 6 are in Proportion.

Q 1, 4 & 13, 52 are in Proport?

$\frac{1}{4} = \frac{13}{52}$

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(F) (1) If a, b, c are in Continued Proportion

$$a : b :: b : c$$

$$\Rightarrow \frac{a}{b} = \frac{b}{c} \Rightarrow \boxed{a \cdot c = b^2}$$

(2) If a, b, c, d are in Continued Proportion

$$\frac{a}{b} = \frac{b}{c} = \frac{c}{d}$$

(G) Componendo

$$\frac{a}{b} = \frac{c}{d}$$

then $\frac{a}{b} + 1 = \frac{c}{d} + 1$

$$\Rightarrow \frac{a+b}{b} = \frac{c+d}{d}$$

$$\frac{a}{b} = \frac{c}{d} \xRightarrow{\text{Com}} \frac{a+b}{b} = \frac{c+d}{d}$$

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Dividendo

$$\frac{a}{b} = \frac{c}{d} \Rightarrow \frac{a}{b} - 1 = \frac{c}{d} - 1$$

If $\frac{a}{b} = \frac{c}{d}$ then $\frac{a-b}{b} = \frac{c-d}{d}$

Niche Wale Ko Jodna

Niche Wale Ko Ghtaana

Componendo & Dividendo

$$\Rightarrow \boxed{\frac{a}{b} = \frac{c}{d} \Rightarrow \frac{a+b}{a-b} = \frac{c+d}{c-d}}$$

$$\frac{x}{y} = \frac{3}{4}$$

C&D

$$\boxed{\frac{x+y}{x-y} = \frac{3+4}{3-4}}$$

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(J) If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f}$

Teeno ka Har tareekey Ka Linear combination BInke equal hoga.

Ex $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \frac{a+c+e}{b+d+f} = \frac{bk+dk+fk}{b+d+f} \Rightarrow \frac{k(\cancel{b+d+f})}{\cancel{b+d+f}}$

Ex: $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \frac{a-c+e}{b-d+f}$

Ex: $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \frac{2a-3c+5e}{2b-3d+5f}$

Q $\frac{x}{y} = \frac{m}{n}$ then $\frac{x+3m}{y+3n}$ is equal to their Ratio [T/F]

True

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K-method.

Ratio me hr Kese Ka Jbab K method.

(1) $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \boxed{\frac{2a-3c+5e}{2b-3d+5f}}$ P.T.?

let $\Rightarrow \frac{a}{b} = \frac{c}{d} = \frac{e}{f} = K \Rightarrow$ $\boxed{\begin{array}{l} a = bK \\ c = dK \\ e = fK \end{array}}$

$\frac{2a-3c+5e}{2b-3d+5f} = \frac{2bK-3dK+5fK}{2b-3d+5f}$

$\Rightarrow \frac{K\cancel{(2b-3d+5f)}}{\cancel{(2b-3d+5f)}} = K$