

RELATION FUNCTION

$$Q \quad f(x) = \left(\frac{a^x - 1}{a^x + 1} \right) \cdot x^2 \in \mathbb{E}/\mathbb{O}?$$

\downarrow

= Odd \times Even

= Odd

$$Q \quad f(x) = \left(\frac{a^x - 1}{a^x + 1} \right) \cdot x^3 \in \mathbb{E}/\mathbb{O}?$$

\downarrow

= Odd \times Odd

= Even

$$Q \quad f(x) = \left(\frac{a^x - 1}{a^x + 1} \right) \boxed{x^n} \text{ odd then } n ?$$

1) 1 2) $\boxed{2}$ 3) 3 4) NOT.

\downarrow

Odd \times Even = Odd

$n = \text{Even deg.}$

$$Q \quad f(x) = \left(\frac{a^x - 1}{a^x + 1} \right) \cdot \boxed{x^n} \text{ odd then } n = ?$$

odd \times even = odd

1) $\frac{1}{3}$ 2) $\frac{2}{3}$ 3) $\frac{3}{3}$ 4) NOT.

$n = \text{Even}$

$$(x)^{\frac{2}{3}} = (x^2)^{\frac{1}{3}} = \text{Even}$$

(E)

RELATION FUNCTION

$$Q \quad f(x) = \begin{cases} x & |x| \\ [x+1] + [-1-x] & -1 < x < 1 \\ -x & |x| \end{cases}$$

 $x \leq -1$

$$\boxed{-1 < x < 1}$$

 $x \geq 1$

$$[x] + [-x] = \begin{cases} 0 & x=0 \\ -1 & x \neq 0 \end{cases}$$

RK:

If Q is difficult or seems like
NEVEN fcn Try $\boxed{f(x) + f(-x) = 0}$
 $x=0$
 $x \neq 0$

Odd Ki Nishani

$$E/09$$

$$f(-x) = \begin{cases} -x & | -x | \\ [1-x] + [x+1] & -1 < -x < 1 \\ +(x) & | -x | \end{cases}$$

 $-x \leq -1$ $-1 < -x < 1$ $-x > 1$

Q

$$f(x) = [x] + \frac{1}{2}, \boxed{x \notin I} \quad E/09$$

$$f(-x) = [-x] + \frac{1}{2}$$

$$\overline{f(x) + f(-x)} = \boxed{[x] + [-x] + 1}$$

$$f(-x) = \begin{cases} -x & |x| \\ [1-x] + [x+1] & x > -1 \\ x & |x| \end{cases}$$

$$\begin{matrix} x \geq 1 \\ \boxed{x > -1} \\ x \leq -1 \end{matrix} = f(x)$$

Even

$$\begin{aligned} &= -1 + 1 \\ &+ 0 + f(-x) = 0 \\ &f(-x) = f(x) \text{ odd} \end{aligned}$$

RELATION FUNCTION

Q) $f(x) = \log_e (x + \sqrt{1+x^2})$ E/02

$$f(-x) = \log_e (-x + \sqrt{1+x^2})$$

$$f(x) + f(-x) = \log_e \left\{ \left(x + \sqrt{1+x^2} \right) \left(-x + \sqrt{1+x^2} \right) \right\}$$

$$= \log_e \left\{ (\sqrt{1+x^2})^2 - (x)^2 \right\}$$

$$= \log_e \left\{ 1 + x^2 - x^2 \right\}$$

$$f(-x) + f(x) = 0$$

\Rightarrow odd

$$[x] + [-x] = -1$$

$$\left[\frac{x}{\pi} \right] + \left[-\frac{x}{\pi} \right] = -1$$

$$\left[-\frac{x}{\pi} \right] = -1 - \left[\frac{x}{\pi} \right]$$

Q) Find whether given $f(x) = \frac{x(\sin x + \tan x)}{\left[\frac{x}{\pi} \right] - \frac{1}{2}}$ is

E/0; $x \neq n\pi$?

$$f(x) = \frac{x(\sin x + \tan x)}{\left[\frac{x}{\pi} \right] - \frac{1}{2}} = \frac{x(\sin x + \tan x)}{\left[\frac{x}{\pi} \right] + 1 - \frac{1}{2}}$$

$$f(x) = \boxed{\frac{x(\sin x + \tan x)}{\left[\frac{x}{\pi} \right] + \frac{1}{2}}}$$

$$f(-x) = \frac{-x(\sin(-x) + \tan(-x))}{\left[-\frac{x}{\pi} \right] + \frac{1}{2}}$$

$x \neq n\pi$

$$\begin{cases} \frac{x}{\pi} + n \Rightarrow \frac{x}{\pi} \neq \frac{1}{2} \\ \left[\frac{x}{\pi} \right] + \left[-\frac{x}{\pi} \right] = -1 \end{cases}$$

$$\begin{aligned} &= \frac{x(\sin x + \tan x)}{-1 - \left[\frac{x}{\pi} \right] + \frac{1}{2}} = \frac{x(\sin x + \tan x)}{-1 - \left[\frac{x}{\pi} \right] + \frac{1}{2}} \\ &f(-x) = -f(x) \end{aligned}$$

ODD

RELATION FUNCTION

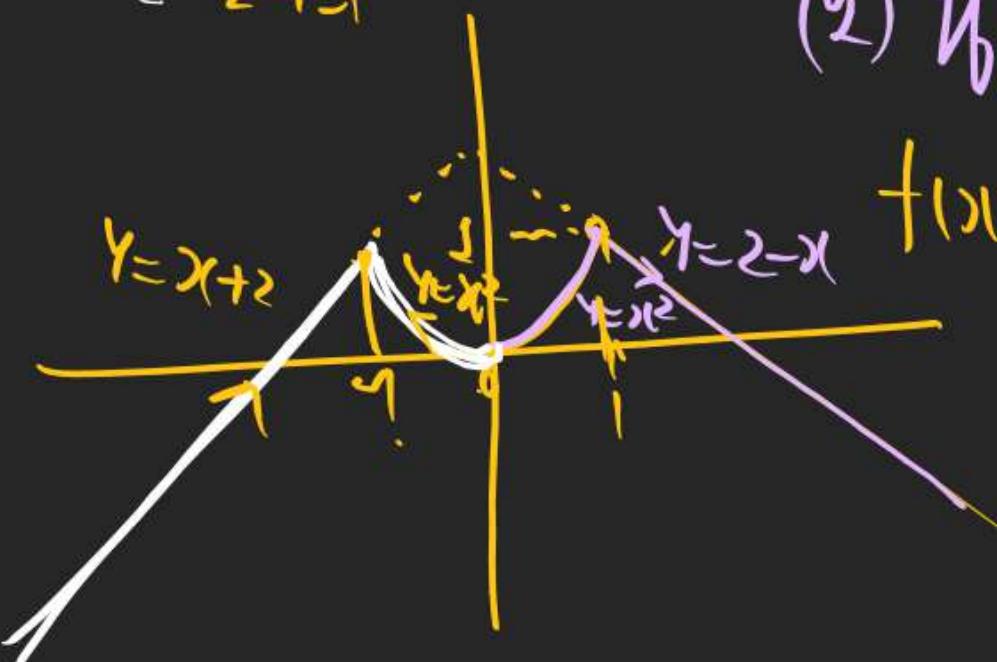
$$(1) \text{ If } f(x) = \begin{cases} x^2 & [x \in [-1, 1)] \\ 2-x & x \in [1, \infty) \end{cases}$$

concept $\rightarrow f(x)$ Even \rightarrow Symm to y Axis
 $f(x)$ odd \rightarrow Symm to origin.

from $f(1/x)$ for $x < 0$ if

- 1) $f(x)$ is odd 2) $f(x)$ is even.

$$f(x) = \begin{cases} x^2 & 0 \leq x < 1 \\ 2-x & 1 \leq x < \infty \\ -x-1 & x < -1 \end{cases}$$



(2) If $f(x)$ Even

$$f(x) = \begin{cases} x+2 & -\infty < x \leq -1 \\ x^2 & x \in (-1, 0] \end{cases}$$



$$f(x) = \begin{cases} -x^2 & x \in [-1, 0] \\ -x-2 & x \in (-\infty, -1] \end{cases}$$

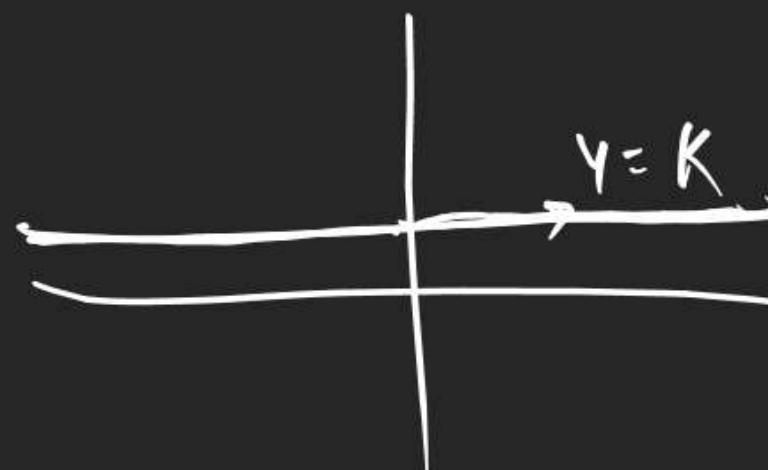
RELATION FUNCTION

Properties of E/O fxn

1) 2 **popular** Non fxn $\Rightarrow f(x) = 1 + a^x$
 $f(x) = 1 + \sin x$

(2) $f(x) = K$ (constant fxn) are always

Even fxn $f(x) = \frac{f(x) + f(-x)}{2}$ Even + odd.



(3) $f(x)=0$ is only fxn which is E&O both.



(4) If odd fxn contains Zero in domain
then graph of fxn will pass thro
origin.

(5) Every fxn containing Zero in domain
(can) be Represented as Sum of
Even & Odd fxn.

RELATION FUNCTION

$$\text{Q) } f(x) = \left[\frac{|\sin x|}{2} \right] \text{ E/O?}$$

$$y = |\sin x| \in [0, 1]$$



$$\left[\frac{|\sin x|}{2} \right] \in \left[0, \frac{1}{2} \right]$$

$$f(x) = \left[\frac{|\sin x|}{2} \right] = 0$$

Even & odd

$$\text{Q) } f(x) = a + \sin x = \text{odd then } a=?$$

1) 0 2) 1 3) 2 4) 3

$$f(x) = \boxed{a} + \underbrace{\sin x}_{\text{const}} = \text{odd fn}$$

E + odd = NE NO hone me hai

$$\begin{aligned} f(x) &= 0 + \sin x \\ &= \sin x \\ &\text{odd} \end{aligned}$$

RELATION FUNCTION

(6)

| $f(x)$ | $g(x)$ | $f(x) + g(x)$ | $f(x) \times g(x)$ |
|--------|--------|---------------|--------------------|
| E | E | E | E |
| O | O | O | E |
| E | O | NEN | O |

(7)

| | |
|------------|---|
| $f(g(x))$ | $\{ f(x) = \lim_{n \rightarrow \infty} (6x) E/O \}$ |
| $E(E) = E$ | $O(E) \neq E$ |
| $E(O) = E$ | $f(x) = \lim_{n \rightarrow \infty} (6x + 6) E/O$ |
| $O(E) = E$ | $O + E = NEN$ |
| $O(O) = O$ | |

RELATION FUNCTION

$$\begin{aligned}
 Q & \quad f(x) = \left\{ \begin{array}{l} \frac{\sin^3 x \cdot 6x^2 \cdot \log\left(\frac{1-x}{1+x}\right)}{\operatorname{sgn} x \cdot \tan\left(\frac{a^x - 1}{a^x + 1}\right)} \\ \end{array} \right\}^{2026/1} E/0 \\
 &= \left\{ \begin{array}{l} 0 \times E \times \text{odd} \\ \text{odd} \times 0(0) \end{array} \right\}^{2026/1} \\
 &= \left\{ \begin{array}{l} 0 \times 0 \times 0 \\ 0 \times 0 \end{array} \right\}^{2026/1} \\
 &= E^{2026/1} \in \mathbb{R}_{\text{Even}}
 \end{aligned}$$

$$\begin{aligned}
 Q &= \left\{ \begin{array}{l} \sin^3 x \cdot 6x^2 \cdot \log\left(\frac{1-x}{1+x}\right) \\ \operatorname{sgn} x \cdot \tan\left(\frac{1+\tan x}{1+\tan x}\right) \end{array} \right\}^{2026/1} \\
 &= \text{NE NO}
 \end{aligned}$$

RELATION FUNCTION

Q f: $[-10, 10] \rightarrow \mathbb{R}$; $f(x) = x + \sin x + \left[\frac{x^2}{a} \right]$ = odd fxn.
 Domain
 \downarrow

$$x \in [-10, 10] \quad f(x) = x + \sin x + \left[\frac{x^2}{a} \right] = \text{odd function}$$

$0 + 0 + \underbrace{\text{Even}}_{\text{No const}}$

$$x^2 \in [0, 100]$$

ROKO (const)

$$a \in (100, \infty) \quad \left[\frac{x^2}{a} \right] = 0$$

$$0 < \frac{x^2}{a} < 1$$

$$x^2 < a \Rightarrow a > (x^2)_{\text{Top}}$$

$a > 100$

RELATION FUNCTION

Periodic fxn

A) Any fxn who attends same height after a certain interval or who repeats himself after a certain interval is known as Periodic.



(2) That certain Interval is known as Period of fxn.

(3) If Period is Min. & +ve than that is known as fundamental Period (T)

(4) If $f(x)$ Satisfies

$$\boxed{f(x+T) = f(x)}$$

then $f(x)$ is Periodic &
F.P. is T

RELATION FUNCTION

$$f(x+T) = f(x) \rightarrow T = \text{FP}$$

$$\left. \begin{array}{l} \sin(2\pi+x) = \sin x \\ \sin(4\pi+x) = \sin x \\ \sin(6\pi+x) = \sin x \end{array} \right\} \begin{array}{l} \text{Period of sine} \\ = \boxed{2\pi}, 4\pi, 6\pi, 8\pi, \dots \end{array}$$

$\therefore 2\pi$ is Min as well as T

$$\Rightarrow T = 2\pi \text{ for sine}$$

Q If $f(x) + f(x+3) = 0$ then Period of $f(x)$?

$$\begin{array}{ll} x \rightarrow x+3 & \begin{array}{l} f(x+3) = -f(x) \longrightarrow f(x+6) = +f(x) \\ f(x+6) = -f(x+3) \\ f(x+3) = -f(x+6) \end{array} \\ & f(x+T) = f(x) \\ & T = 6 \end{array}$$

RELATION FUNCTION

Q If $f(x) + f(x+4) = f(x+2) + f(x+6)$
find T?

$$\begin{aligned}x \rightarrow x+2 \\ f(x) + f(x+4) &= \cancel{f(x+2)} + f(x+6) \\ f(x+2) + \cancel{f(x+6)} &= f(x+4) + f(x+8)\end{aligned}$$

~~$$f(x) + f(x+4) = f(x+4) + f(x+8)$$~~

$$\begin{aligned}f(x+8) &= f(x) \quad \boxed{T=8} \\ f(x+T) &= f(x)\end{aligned}$$

Q If $f(x+a) = \frac{1}{2} + \sqrt{f(x) - f^2(x)}$ find T of f(x)

$$f(x+a) = \frac{1}{2} + \sqrt{\frac{1}{4} - (f(x) - \frac{1}{2})^2} \quad x \rightarrow x+a$$

$$f(x+2a) = \frac{1}{2} + \sqrt{\frac{1}{4} - (f(x+4) - \frac{1}{2})^2}$$

$$= \frac{1}{2} + \sqrt{\frac{1}{4} - \left(\frac{1}{2} + \sqrt{\frac{1}{4} - (f(x) - \frac{1}{2})^2} - \frac{1}{2} \right)^2}$$

$$= \frac{1}{2} + \sqrt{\frac{1}{4} - \left(\frac{1}{2} - \left(f(x) - \left(f(x) - \frac{1}{2} \right)^2 \right) \right)}$$

$$= \frac{1}{2} + f(x) - \frac{1}{2}$$

$$f(x+2a) = f(x)$$

T-24

RELATION FUNCTION

Table Based Qs.

| | | |
|---|-------------------|---------------------|
| 1) $\sin^n x / \cos^n x$ $\sec^n x / \csc^n x$ | $n = \text{odd}$ | $T = 2\pi$ |
| | $n = \text{Even}$ | $T = \pi$ |
| 2) $\tan^n x / \cot^n x$ | $n = \infty/0$ | $T = \pi$ |
| 3) $ \sin^n x , \cos^n x $ $ \tan^n x , \cot^n x $ $ \sec^n x , \csc^n x $ | $n = \infty/0$ | $T = \pi$ |
| (4) $ \sin x + \cos x $ $ \sec x + \csc x $ | | $T = \frac{\pi}{2}$ |
| (5) $\{x\}$ | | $T = 1$ |

| | | |
|------------------|---|-----------------------|
| (6) Constant fn | | Periodic But No FP |
| (7) Dirichlet fn | $f(x) = \begin{cases} 1 & x \in \mathbb{Q} \\ -1 & x \in \mathbb{Q}' \end{cases}$ | Periodic But No FP |
| [x], Poly, Rest | | |
| all fn | are Non Periodic | |
| $\sin^2 x$ | $T = \pi$ | $\sec^3 x = 2\pi$ |
| $\sin^3 x$ | $T = 2\pi$ | $\tan^3 x = \pi$ |
| $\cos^{32} x$ | $T = \pi$ | $ \sin^{32} x = \pi$ |

RELATION FUNCTION

$$\delta m^3 x \rightarrow 2n$$

$$\delta m^2 x \rightarrow n$$

$$G^{31} x \rightarrow 2n$$

$$G^{204} x \rightarrow n$$

$$\delta e^3 x \rightarrow 2n$$

$$fm^4 x \rightarrow n$$

$$(xt^6)x \rightarrow n$$

$$fm^5 x \rightarrow n$$

$$(xt^3)x \rightarrow n$$

$$\delta ec^3 x \rightarrow 2n$$

$$Gec^8 x \rightarrow n$$

$$\delta m^2 x \rightarrow n$$

$$\left| \begin{array}{l} \delta m^3 x \rightarrow 2n \\ \{x\} \rightarrow \perp \end{array} \right.$$

$$\left| \begin{array}{l} fm^3 x \rightarrow n \\ \{x\} = 1 \end{array} \right.$$

$$\delta m^4 x + 64x \rightarrow \frac{n}{2}$$

$$|6n|x| + |6s|x| \rightarrow \frac{n}{2}$$

$$|fm|x| + |16r|x| \rightarrow \frac{n}{2}$$

$$|6c|x| \rightarrow n$$

$$|(8e^4)x| \rightarrow n$$

$$|\delta m^{39} x| \rightarrow n$$

$$\delta m^{39} x \rightarrow 2n$$

$$G^{38} x \rightarrow n$$

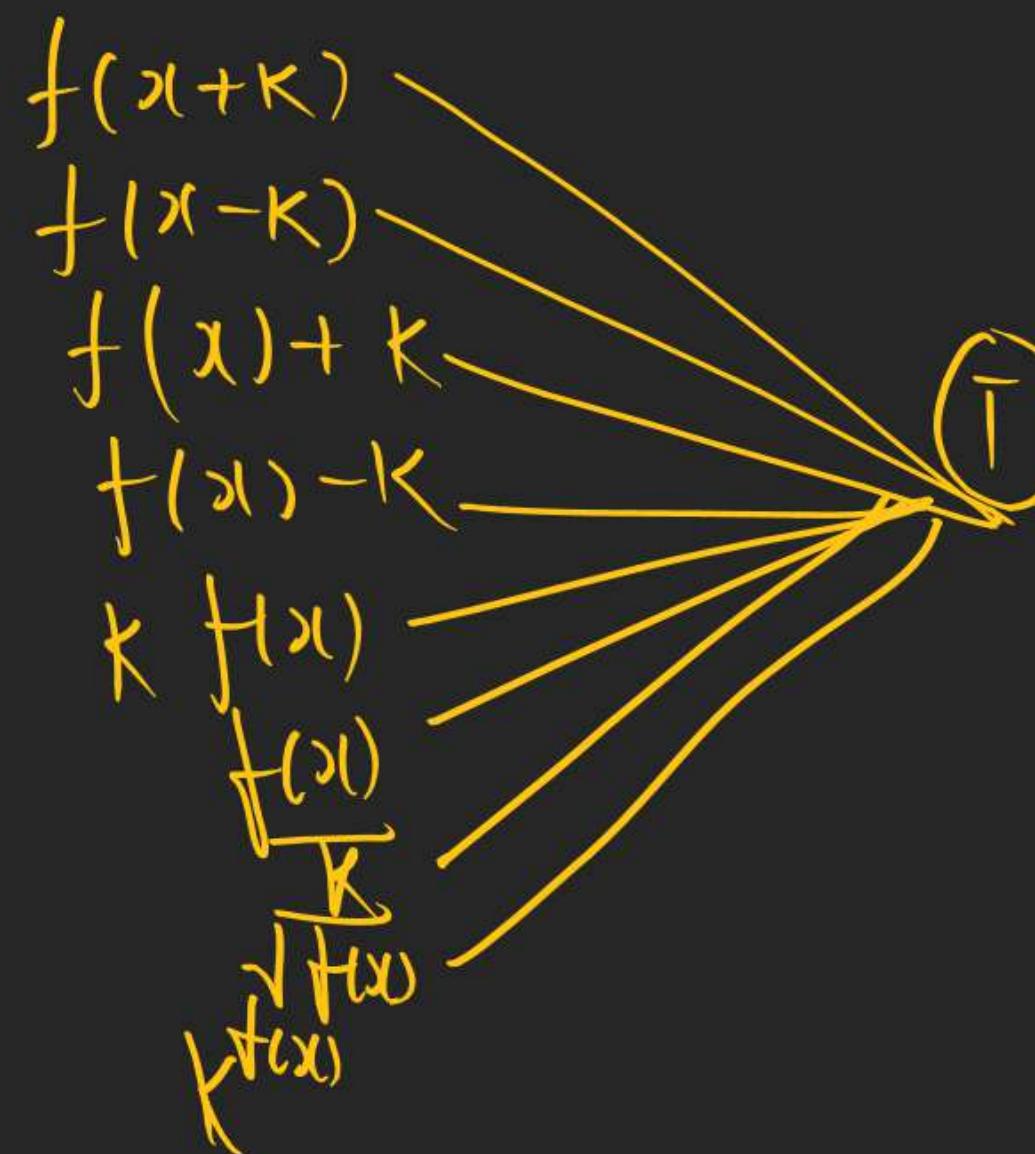
$$\boxed{f(x) = 6}$$

No FP

RELATION FUNCTION

Type 2 When Constant Introduced with x

Let $f(x) \rightarrow T$



$$\delta m^3 x \rightarrow 2\kappa$$

$$\delta m^3 x + 7 \rightarrow 2\kappa$$

$$7\delta m^3 x \rightarrow 2\kappa$$

$$7 \frac{\delta m^3 x}{\kappa} \rightarrow 2\kappa$$

$$\sqrt{\delta m^3 x} \rightarrow 2\kappa$$

$$\delta m^3 (x+4) \rightarrow 2\kappa$$

$$\delta m^3 (\sqrt{x}+4) + 5 \rightarrow 2\kappa$$