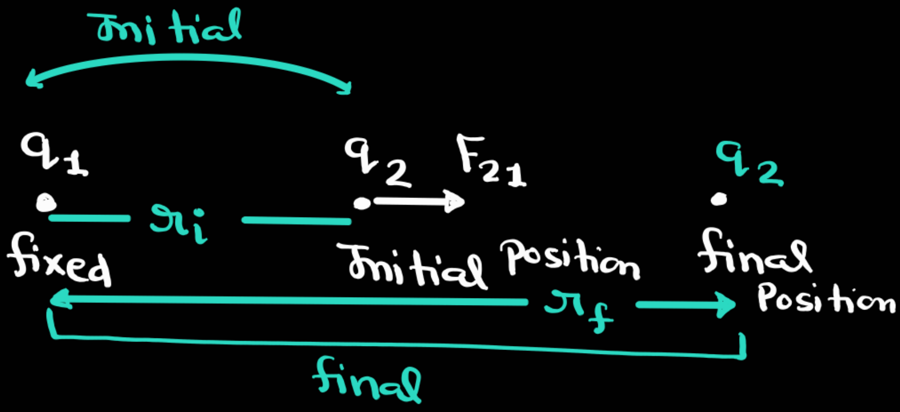
Nishant Jindal

Potential Energy b/w two point charges



$$\frac{\Delta U}{q} = \Delta V$$

$$\frac{U}{q} = V$$

$$U = qV$$

$$U_i = V_{q_1} \times q_2$$

$$= \frac{Kq_1}{r_i} \times q_2$$

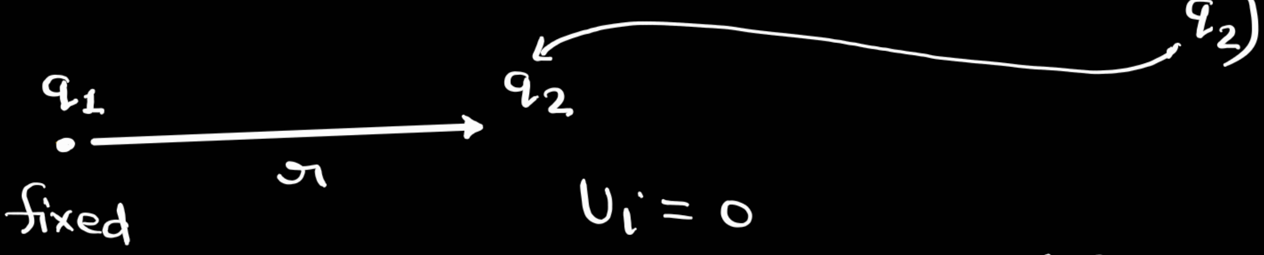
$$U_i = \frac{Kq_1 q_2}{r_i}$$

$$U_f = \left(\frac{Kq_1}{r_f}\right) q_2$$

$$U_f = \frac{Kq_1 q_2}{r_f}$$

In formula of Potential & Potential Energy, charges put with sign.

Mutual potential energy of a system of charge particles:-



$U_\infty = 0$

$V_\infty = 0$

$$U_i = 0$$

$$U_f = q_2 \left(\frac{Kq_1}{r}\right) \quad \left(\begin{array}{c}\text{Put } q_1 \text{ \& } q_2 \\ \text{with sign}\end{array}\right)$$

$$U_f = \frac{Kq_1 q_2}{r^2}$$

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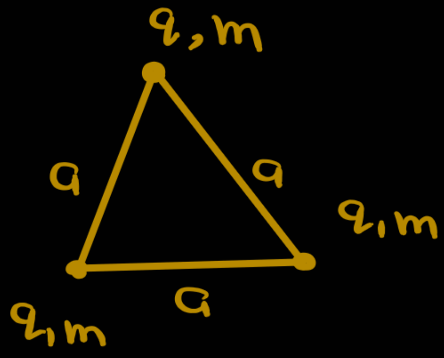
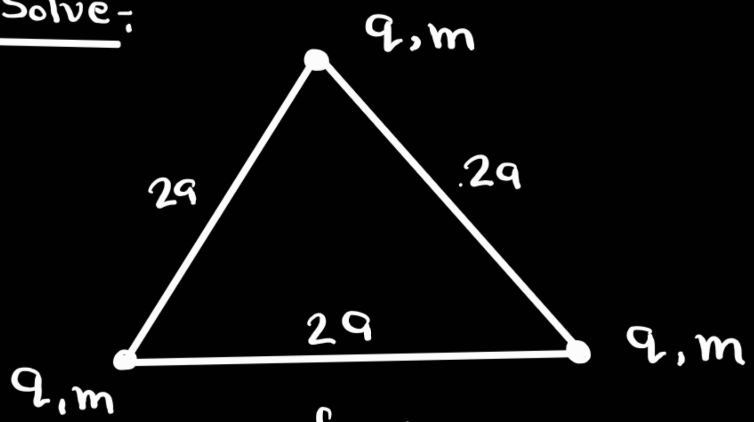
Q: find the speed of the charge particles. when they are at seperation of $2a$ If all are released of a initially as shown in figure.



Initially.

Solve:



finally

$$U_i = \frac{3Kq^2}{a} \qquad U_{final} = \frac{3Kq^2}{2a}$$

$$U_i + K_i = U_f + K_f$$

$$\frac{3Kq^2}{a} + 0 = \frac{3Kq^2}{2a} + 3\left[\frac{1}{2}mv^2\right]$$

$$\frac{\cancel{3}Kq^2}{\cancel{2}a} = \frac{\cancel{3}}{\cancel{2}}mv^2$$

$$v^2 = \frac{Kq^2}{ma} \Rightarrow v = q\sqrt{\frac{K}{ma}}$$

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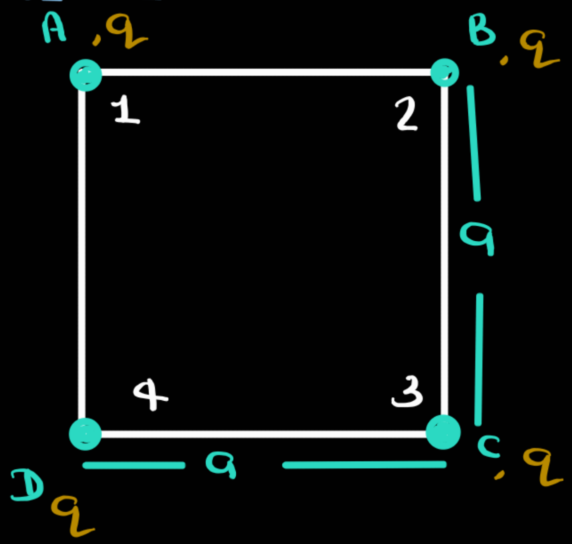


find work done by the ext agent in buiding the system.

Solve:- $\Delta U = W_{ext\ agent}$

$U_{1-2} = \frac{Kq^2}{a}$

$U_{3-2} = \frac{Kq^2}{a}$

$U_{3-1} = \frac{Kq^2}{\sqrt{2}a}$

$U_{4-1} = \frac{Kq^2}{a}$

$U_{4-2} = \frac{Kq^2}{\sqrt{2}a}$

$U_{4-3} = \frac{Kq^2}{a}$

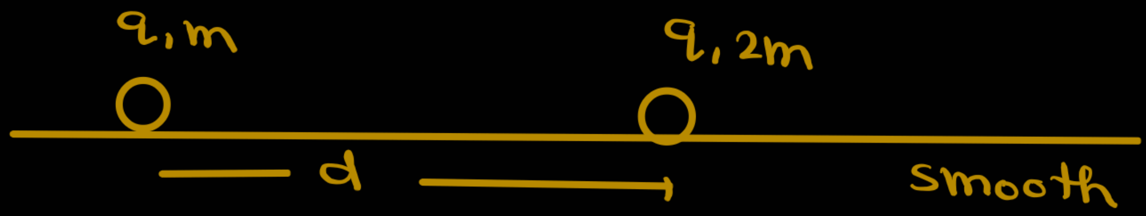
$\Delta U = \frac{4Kq^2}{a} + \frac{2Kq^2}{\sqrt{2}a}$

$\Delta U = \frac{Kq^2}{a}(\sqrt{2}+4)$

$\downarrow$

$W_{ext\ agent}$.

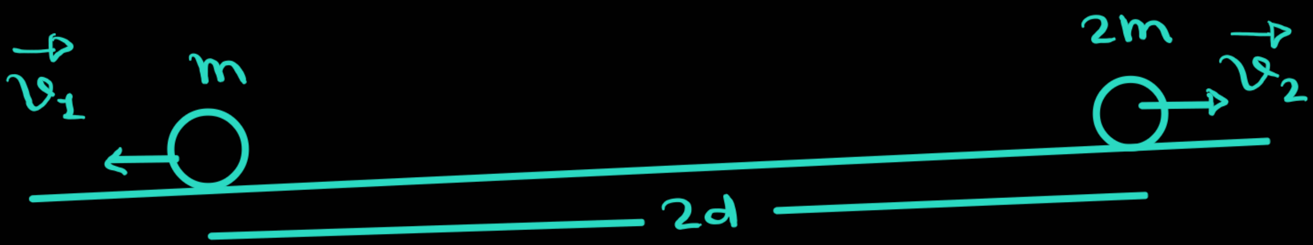
3) Find the speed of both the charge particles when they are at a seperation of 2d. Initially both are released at a seperation of d from rest. whole system is kept on a smooth horizontal surface.



Solve)

$$(\vec{F}_{net})_{\text{on system}} = 0$$

$$\vec{P}_1 + \vec{P}_2 = 0$$



$$m\vec{v}_1 + 2m\vec{v}_2 = 0$$

$$v_1 = 2v_2 \quad \text{[In magnitude]}$$

$$\vec{v}_1 = -2\vec{v}_2$$

$$U_i + K_i = U_f + K_f \quad \text{[MEC]}$$

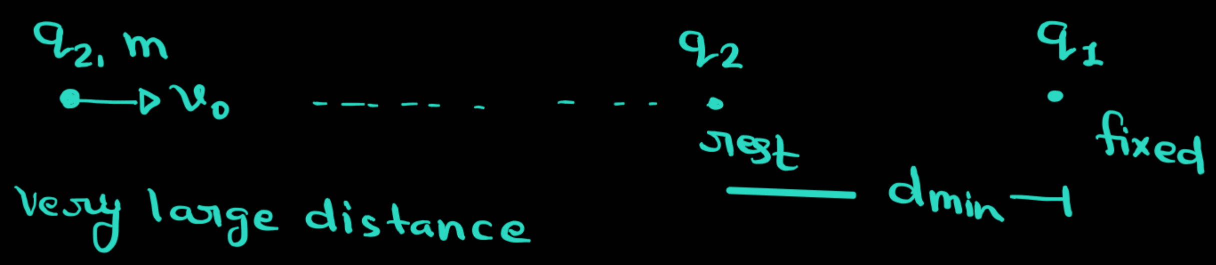
$$\frac{Kq^2}{d} + 0 = \frac{Kq^2}{2d} + \frac{1}{2}mv_1^2 + \frac{1}{2}2m(v_2)^2$$

After solving

$$v_1 = \sqrt{\frac{2Kq^2}{3md}} \qquad v_2 = \sqrt{\frac{Kq^2}{6md}}$$

closest distance of approach b/w two charges :

Case I : One charge is fixed.



$$U_i + K_i = U_f + K_f$$

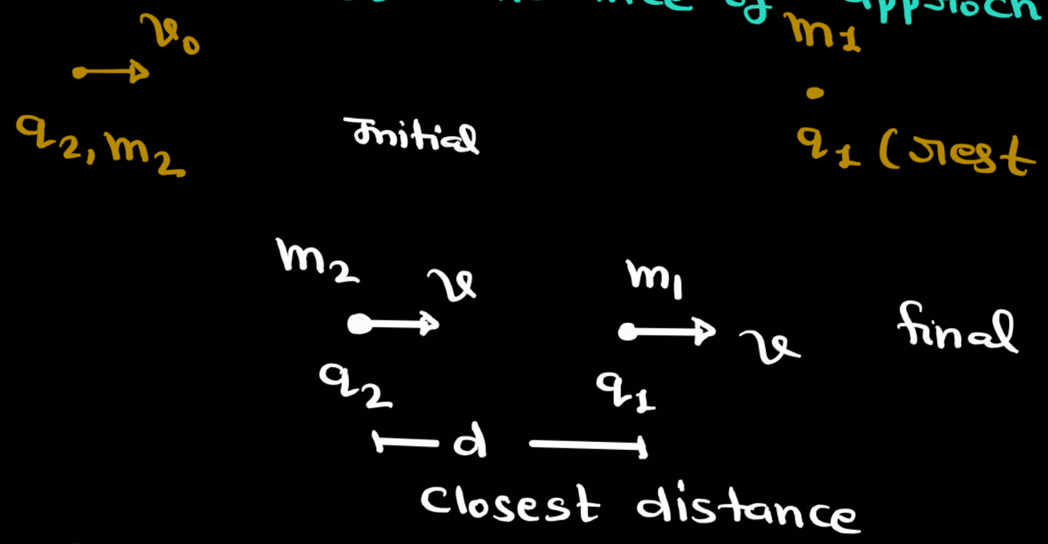
$$0 + \frac{1}{2}mv_0^2 = \frac{Kq_1q_2}{d} + 0$$

$$\boxed{d = \frac{2Kq_1q_2}{mv_0^2}}$$

→ Both the charges are moving :

At very large distance

→ q_2 is projected towards q_1 with velocity v_0. find the closest distance of approach.



By Energy Conservation

$$U_i + K_i = U_f + K_f$$

$$0 + \frac{1}{2}m_2v_0^2 = \frac{Kq_1q_2}{d} + \frac{1}{2}(m_1+m_2)v^2$$

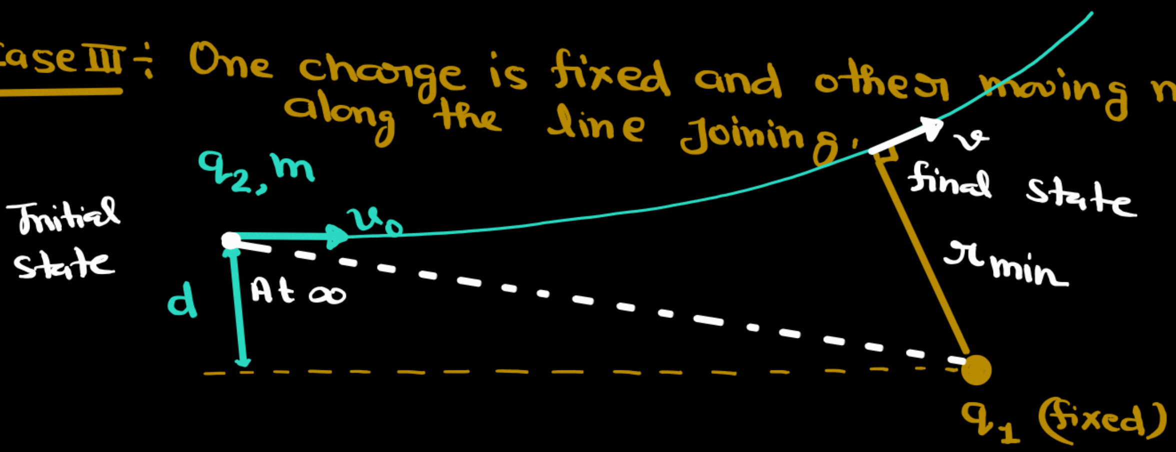
$$m_2 v_0 = m_2 v + m_1 v$$

$$v = \left(\frac{m_2}{m_1+m_2}\right) v_0$$

$$\Rightarrow \quad \frac{1}{2} m v_0^2 = \frac{K q_1 q_2}{d} + \frac{1}{2}(m_1+m_2)\frac{m_2^2 \cdot v_0^2}{(m_1+m_2)^2}$$

$$\frac{1}{2} m v_0^2 - \frac{1}{2}\frac{m_2^2 v_0^2}{(m_1+m_2)} = \frac{K q_1 q_2}{d}.$$

Case III :- One charge is fixed and other moving not along the line joining.



$$U_i + K_i = U_f + K.E_f$$

$$0 + \frac{1}{2} m v_0^2 = \frac{K q_1 q_2}{r} + \frac{1}{2} m v^2$$

Angular momentum Conservation.

$$m v_0 d = m v r$$

$$r = \frac{v_0 d}{v}$$

$$\Rightarrow \quad \frac{1}{2} m v_0^2 = \frac{K q_1 q_2 \cdot v}{v_0 d} + \frac{1}{2} m v^2$$

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Potential difference due to infinite line charge

$$\left[E = \frac{\lambda}{2\pi\varepsilon_0 r} \right]$$

$\vec{E}_r \parallel \vec{dr}$

$\vec{E}_r \cdot \vec{dr} = E_r dr$

$$\int_{V_A}^{V_B} \underbrace{dV}_{\Downarrow} = -\int_{r_i}^{r_f} E_r \, dr$$

Change in potential for 'dr' displacement

$$V_B - V_A = -\frac{\lambda}{2\pi\varepsilon_0}\int_{r_i}^{r_f} \frac{dr}{r}$$

$$\boxed{V_B - V_A = -\frac{\lambda}{2\pi\varepsilon_0} \ln\left(\frac{r_f}{r_i}\right)}$$ **

$V_B < V_A$

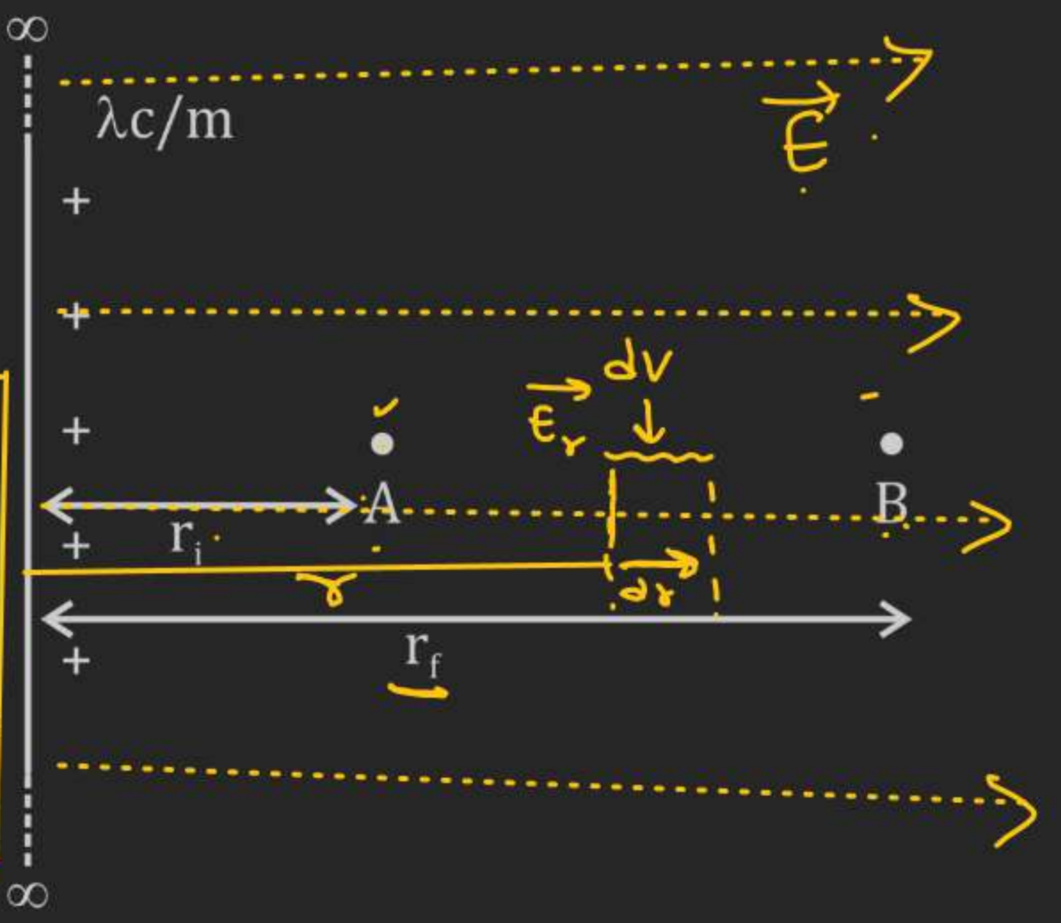
Note

Is '∞' is treated as to be a zero potential point for infinite line charge.

$r_f = r$. $r_i + 8$

$$\int_0^{V(r)} dV = -\frac{\lambda}{2\pi\varepsilon_0}\int_{\infty}^{r} \frac{dr}{r}$$

↳ $\ln(\infty) \to$ undefined



⟹ For infinite line charge ∞ is not taken as zero potential point

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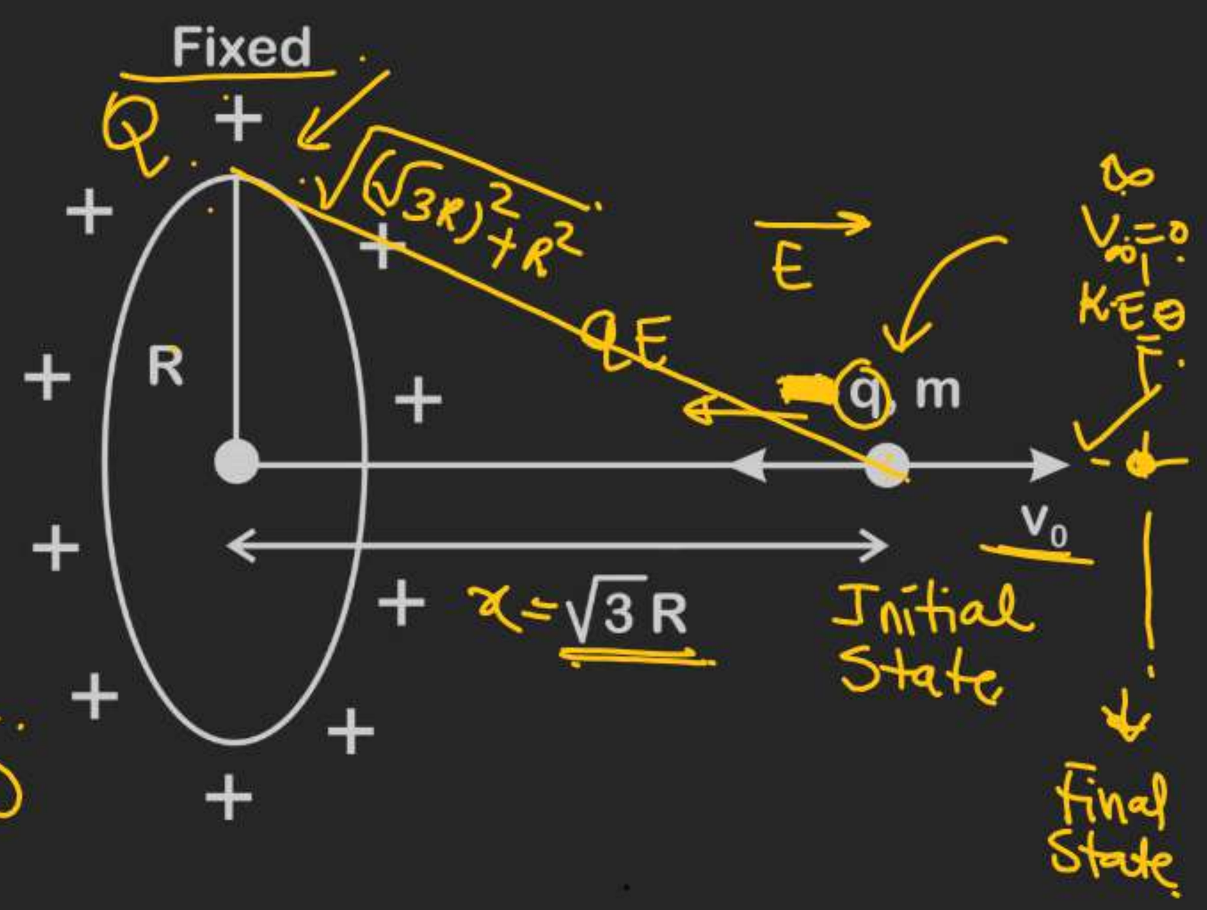
ELECTROSTATIC POTENTIAL AND POTENTIAL ENERGY

Concept of escape velocity

$$V = \frac{KQ}{\sqrt{x^2+R^2}} = \frac{KQ}{2R}$$

\# Find min velocity given to charge particle so that it Can escape from electric field of ring.



Sol^n:- For $-q_0$ to escape from electric field of ring it should reach at ∞. and for $(v_0)_{min}$ K.E of charge particle at infinity should be zero.

$$U_i + K.E_i = U_f + K.E_f$$

$$(V_{ring})(-q_0) + \frac{1}{2}mv_0^2 = 0 + 0$$

$$\frac{KQ}{2R}(-q_0) + \frac{1}{2}mv_0^2 = 0$$

$$\frac{1}{2}mv_0^2 = \frac{KQq_0}{2R}$$

$$v_0 = \sqrt{\frac{KQq_0}{mR}}$$

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ELECTROSTATIC POTENTIAL AND POTENTIAL ENERGY

Concept of escape velocity

$$\boxed{V_{inside} = \frac{KQ}{2R^3}(3R^2 - r^2)}$$

↳ [distance from center]

(a) Find min velocity given to a negative Charge particle having magnitude ' q ' and mass m. so that it Can escape from electric field of non-conducting solid-sphere. (Uniformly Charge) → $\rho = C$

(b) Find Speed of charge particle when it is at a height of R from surface of sphere.

a)

$$\boxed{U_i + K.E_i = U_f + K.E_f}$$

$U_i = (V_{r=R/2})(-q_0)$

$V_{r=\frac{R}{2}} = \frac{KQ}{2R^3}\left(3R^2 - \frac{R^2}{4}\right)$

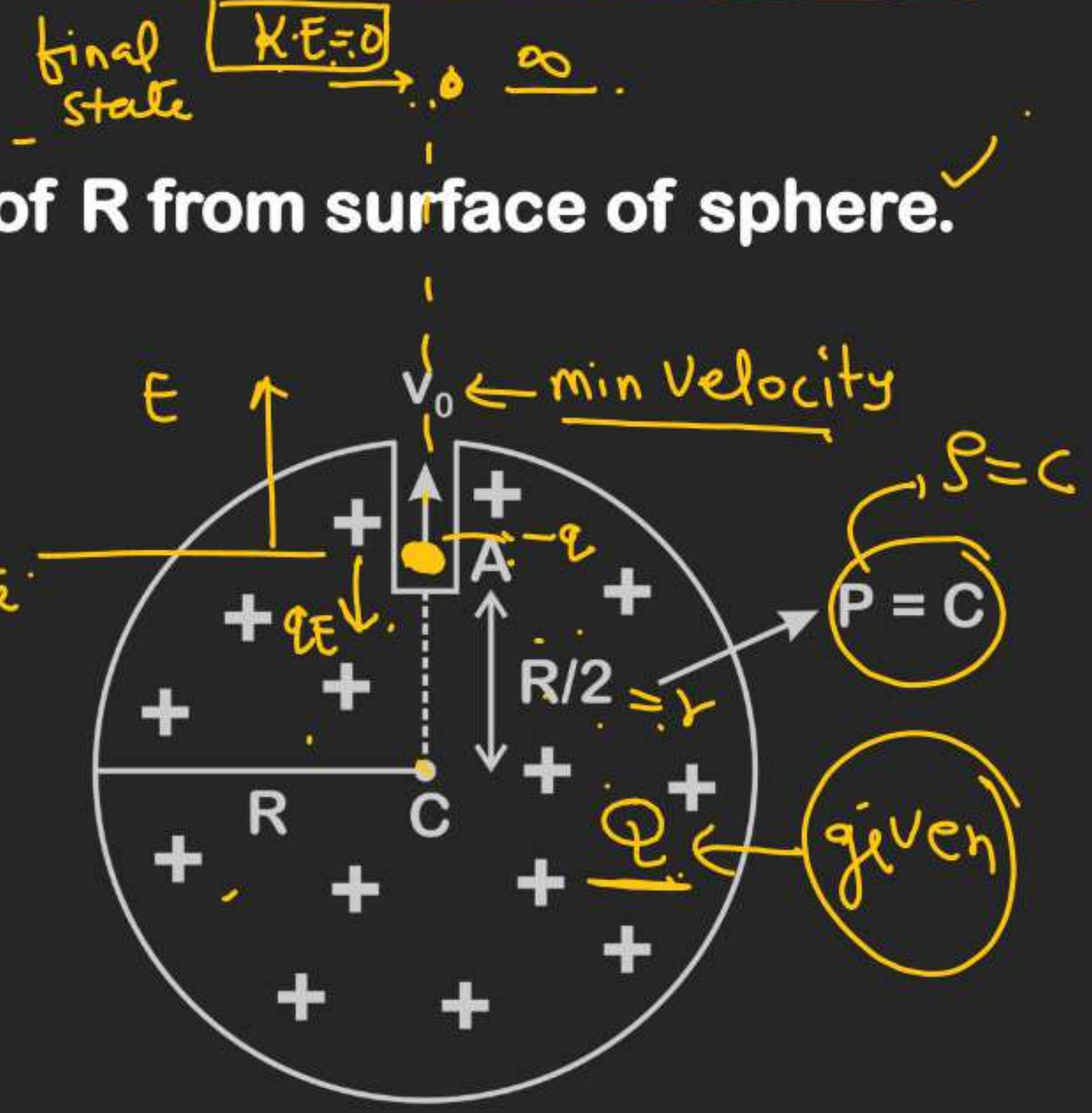
$= \left(\frac{11KQ}{8R}\right)$

$U_i = \left(\frac{-11KQq_0}{8R}\right)$

$-\frac{11KQq_0}{8R} + \frac{1}{2}mv_0^2 = 0 + 0$

$\frac{1}{2}mv_0^2 = \frac{11KQq_0}{8R}$

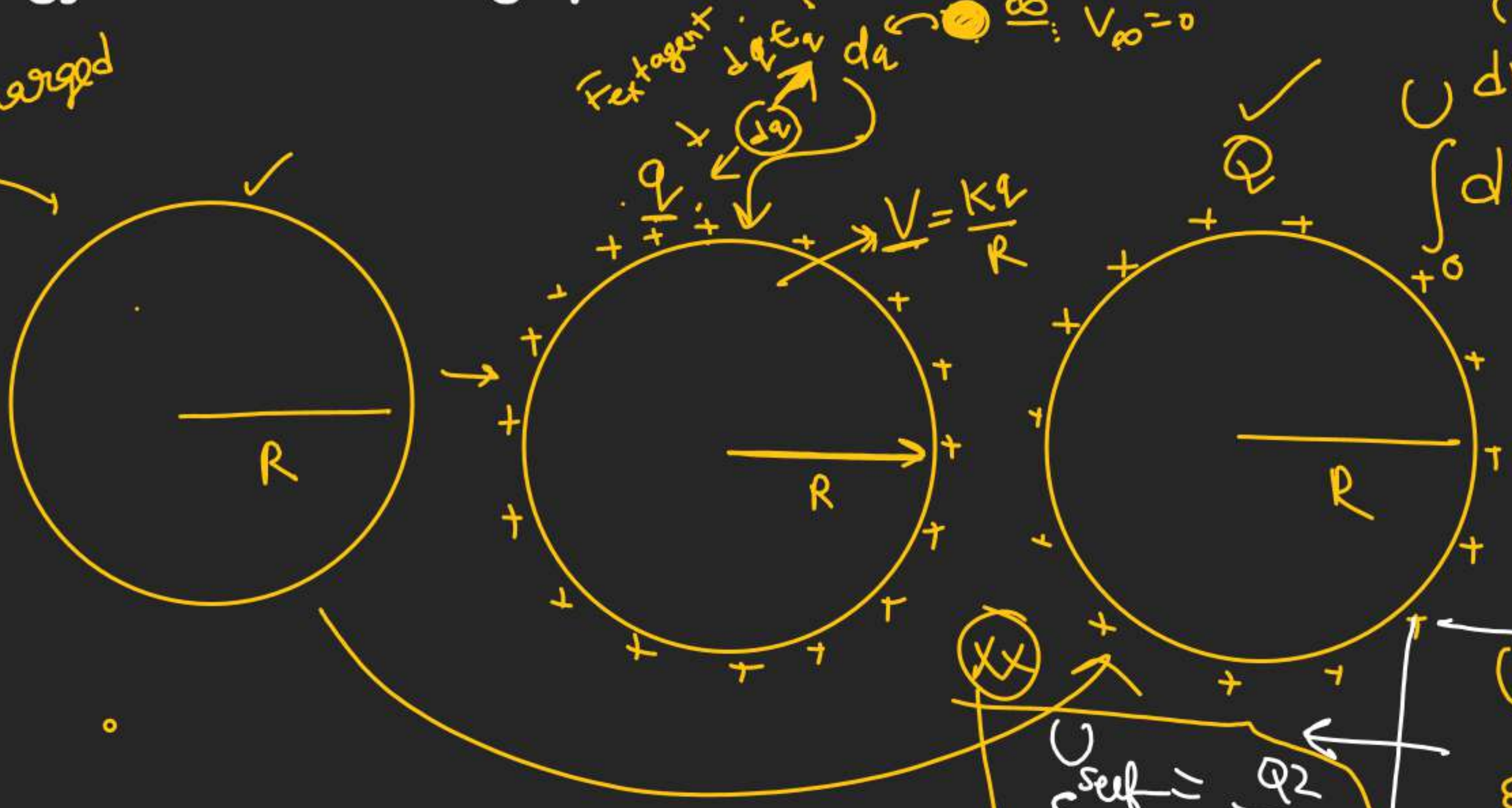
$v_0 = \sqrt{\frac{11KQq_0}{4mR}}$



Concept of Self Energy →

Self Energy of a charged conducting sphere – (Work done in building a System).

Metallic
Initially uncharged



$$dW_{extagent} = dU = V \cdot dq$$

$$\int_0^Q dU = \int_0^Q \frac{Kq}{R} \cdot dq$$

$$U = \frac{K}{R} \int_0^Q q\, dq$$

$$U = \frac{K}{R} \left[\frac{q^2}{2}\right]_0^Q$$

$$\boxed{U_{\text{Self Energy}} = \frac{KQ^2}{2R}}$$

$$\boxed{U_{\text{Self Energy}} = \frac{Q^2}{8\pi\varepsilon_0 R}}$$

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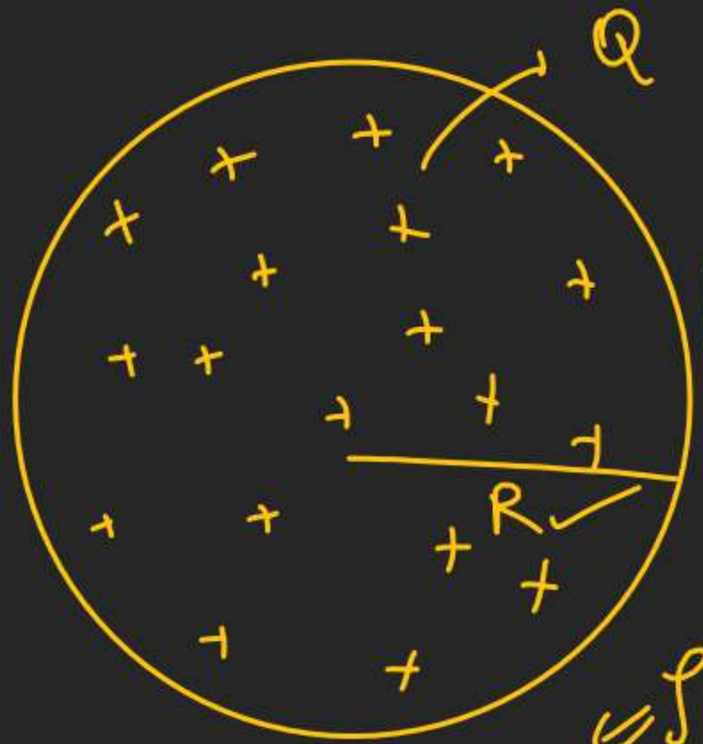
ELECTROSTATIC POTENTIAL AND POTENTIAL ENERGY

Self Energy of a non-conducting uniformly Charged solid Sphere.



During the bulding process let a non-conducting sphere of radius 'r' having charge (+q) has been made.

$$\left(V_r = V_{(r+dr)} = \frac{Kq}{r}\right)$$

$$\rho = \frac{Q}{\frac{4}{3}\pi R^3}$$

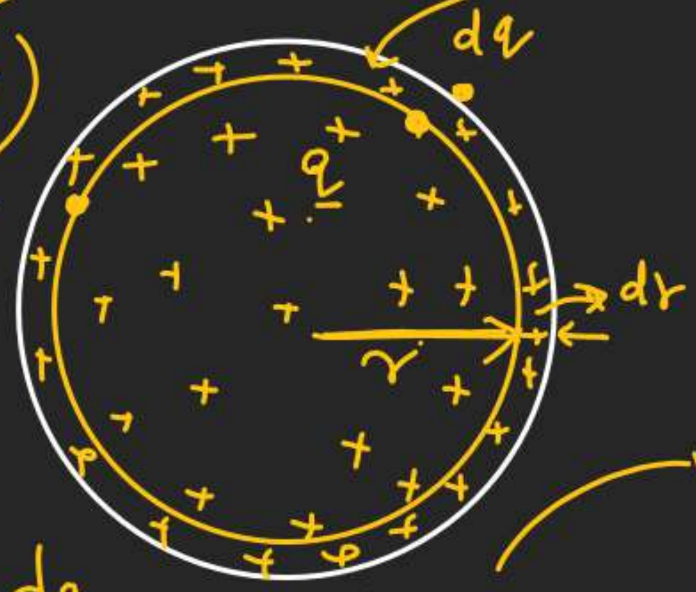
$$\rho = \left(\frac{3Q}{4\pi R^3}\right)$$

Constant

$$q = \rho \cdot \frac{4}{3}\pi r^3$$

$$q = \frac{3Q}{\cancel{4\pi} R^3}\times\frac{\cancel{4}}{\cancel{3}}\cancel{\pi} r^3 = \left(\frac{Q}{R^3}r^3\right)$$

$$V_r = \frac{K}{\cancel{r}}\times\frac{Q}{R^3}\times \cancel{r^3} = \frac{KQ}{R^3}\cdot r^2$$



$$dq = \rho\, dV$$

↳ Differential Volume of Shell having radius r & thickness dr

Power form

Potential of non-conducting Solid sphere of radius 'r'.

$$dW_{ext\,agent} = dU_{self} = dq(V)$$

$$dU_{self} = \left(dq \cdot \frac{Kq}{r}\right)$$

$$dq = \left(\frac{3Q}{\cancel{4\pi} R^3}\right)\times \cancel{4\pi} r^2 dr$$

$$\boxed{dq = \frac{3Q}{R^3}\, r^2 dr}$$

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$$dU_{self} = dq\,(V_{+q})$$

$$dU_{self} = \left[\frac{3Q}{R^3}(r^2 dr)\right]\left(\frac{KQ}{R^3}r^2\right)$$

$$\int_0^{U_{self}} dU_{self} = \frac{K3Q^2}{R^6}\int_0^R r^4\,dr$$

$$U_{self} = \frac{3KQ^2}{\cancel{R^6}} \times \frac{\cancel{R^5}}{5}$$

$$\boxed{U_{self} = \frac{3}{5}\frac{KQ^2}{R}}$$

$$K = \frac{1}{4\pi\varepsilon_0}$$

$$U_{self} = \frac{3}{5} \times \frac{1}{4\pi\varepsilon_0} \times \frac{Q^2}{R}$$

**

$$\boxed{U_{self} = \frac{3Q^2}{20\pi\varepsilon_0 R}}$$

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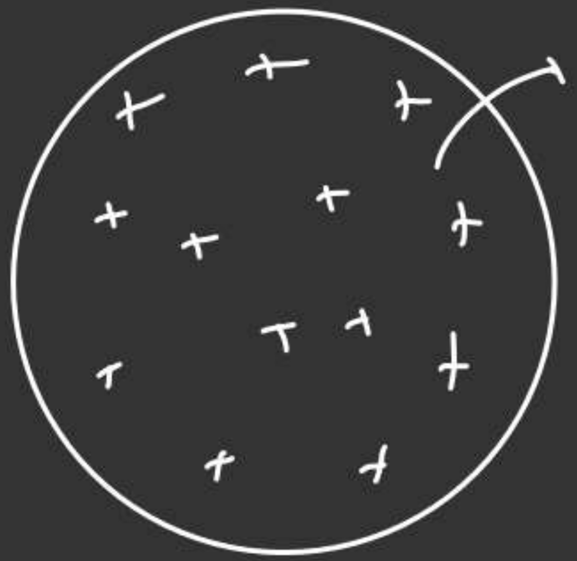
Find Self Energy of a non-conducting & non-uniformly charged.

#



$\boxed{\rho = \rho_0 r}$ —.

ρ_0 is a constant

$r \rightarrow$ radial distance.

$U_{self} = ??$

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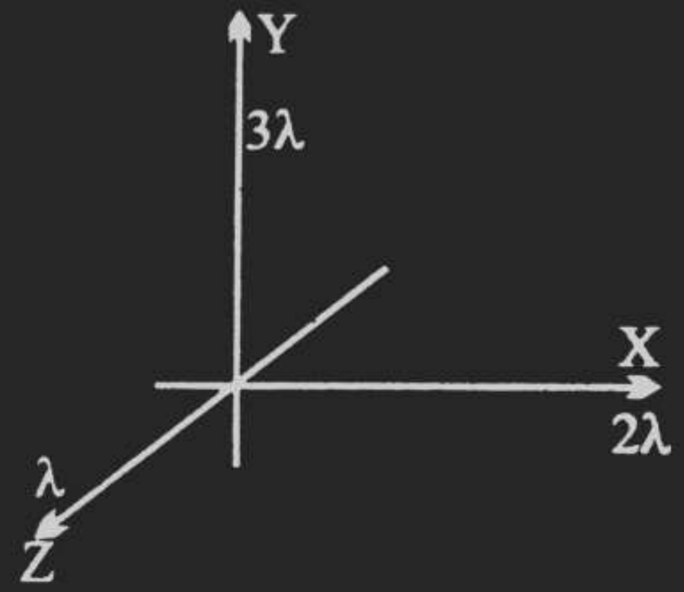
ELECTROSTATIC POTENTIAL AND POTENTIAL ENERGY

H.W

Total Electrostatic Energy

POTENTIAL ENERGY

Q. The diagram shows three infinitely long uniform line charges placed on the X, Y and Z axis. The work done in moving a unit positive charge from $(1, 1, 1)$ to $(0, 1, 1)$ is equal to:



(A) $(\lambda \ln 2)/2\pi\varepsilon_0$

(B) $(\lambda \ln 2)/\pi\varepsilon_0$

(C) $(3\lambda \ln 2)/2\pi\varepsilon_0$

(D) None of these

POTENTIAL ENERGY

H.W.

Q. A charged particle of charge Q is held fixed and another charged particle of mass m and charge q (of the same sign) is released from a distance r. The impulse of the force exerted by the external agent on the fixed charge by the time distance between Q and q becomes $2r$ is:

(A) $\sqrt{\frac{Qq}{4\pi\varepsilon_0 mr}}$

(B) $\sqrt{\frac{Qqm}{4\pi\varepsilon_0 r}}$

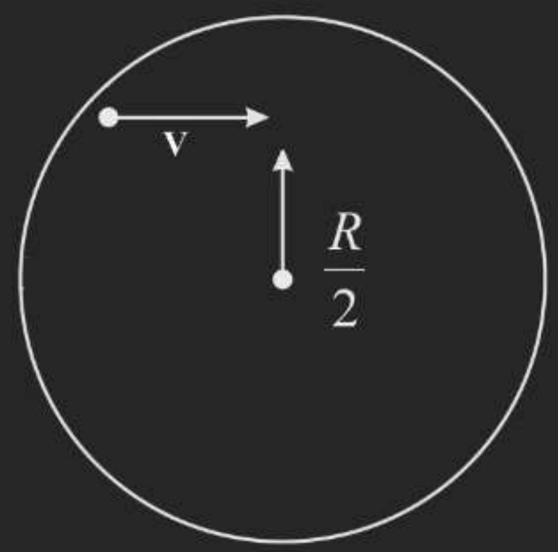
(C) $\sqrt{\frac{Qqm}{\pi\varepsilon_0 r}}$

(D) $\sqrt{\frac{Qqm}{2\pi\varepsilon_0 r}}$

POTENTIAL ENERGY

H.W

Q. A unit positive point charge of mass m is projected with a velocity v inside the tunnel as shown. The tunnel has been made inside a uniformly charged non-conducting sphere. The minimum velocity with which the point charge should be projected such it can it reach the opposite end of the tunnel, is equal to:



(A) $\left[\sigma R^2/4m\varepsilon_0\right]^{1/2}$

(B) $\left[\sigma R^2/24m\varepsilon_0\right]^{1/2}$

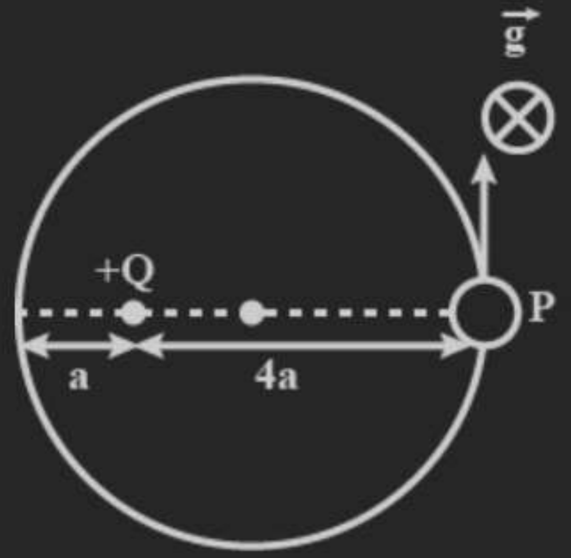
(C) $\left[\sigma R^2/6m\varepsilon_0\right]^{1/2}$

(D) zero because the initial and the final points are at same potential

POTENTIAL ENERGY

H.W :

Q. The diagram shows a small bead of mass m carrying charge q. The bead can freely move on the smooth fixed ring placed on a smooth horizontal plane. In the same plane a charge $+Q$ has also been fixed as shown. The potential at the point P due to $+Q$ is V. The velocity with which the bead should projected from the point P so that is can complete a circle should be greater than:



(A) $\sqrt{\frac{6qV}{m}}$

(B) $\sqrt{\frac{qV}{m}}$

(C) $\sqrt{\frac{3qV}{m}}$

(D) none of these

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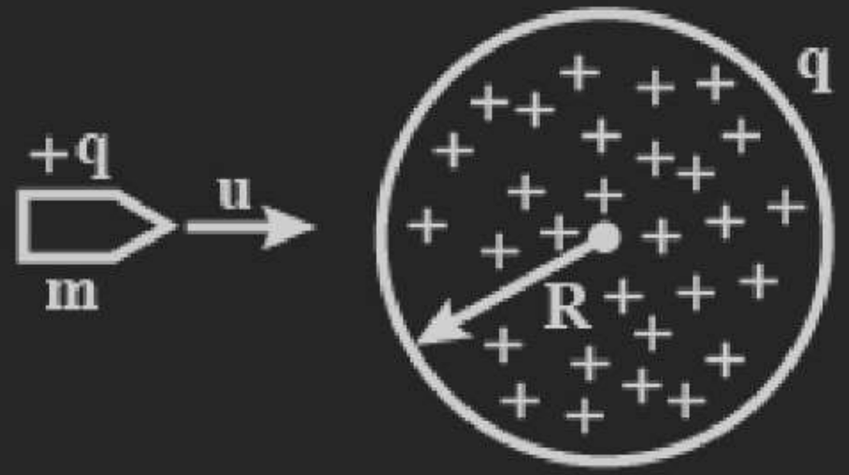
POTENTIAL ENERGY

H.w

Q. A bullet of mass m and charge q is fired towards a solid uniformly charged sphere of radius R and total charge $+q$. If it strikes the surface of sphere with speed u, find the minimum speed u so that it can penetrate through the sphere. (Neglect all resistance forces or friction acting on bullet except electrostatic forces.):



(A) $\frac{q}{\sqrt{2\pi\varepsilon_0 mR}}$

(B) $\frac{q}{\sqrt{4\pi\varepsilon_0 mR}}$

(C) $\frac{q}{\sqrt{2\pi\varepsilon_0 mR}}$

(D) $\frac{\sqrt{3}q}{\sqrt{4\pi\varepsilon_0 mR}}$

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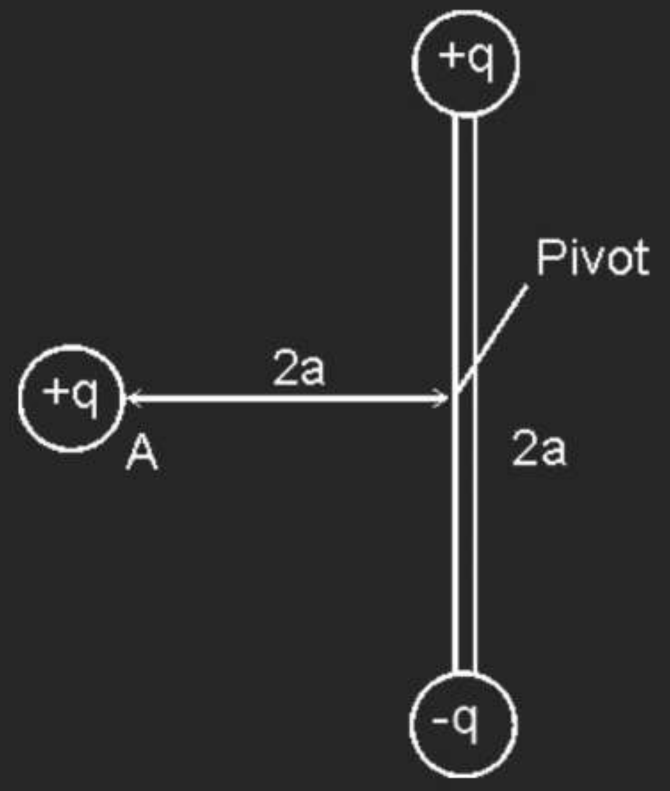
POTENTIAL ENERGY

H.W :

Q. Fig. shows a ball having a charge q fixed at a point A. Two identical balls of mass m having charge $+q$ and $-q$ are attached to the end of a light rod of length $2a$. The system is released from the situation shown in fig. Find the angular velocity of the rod when the rod turns through $90°$:



(A) $\dfrac{\sqrt{2}q}{3\pi\varepsilon_0 ma^3}$

(B) $\dfrac{q}{\sqrt{3\pi\varepsilon_0 ma^3}}$

(C) $\dfrac{q}{\sqrt{6\pi\varepsilon_0 ma^3}}$

(D) $\dfrac{\sqrt{2}q}{4\pi\varepsilon_0 ma^3}$

POTENTIAL ENERGY

Q. The arc AB with the center C and the infinitely long wire having linear charge density λ are lying in same plane. The minimum amount of work to be done to move a point charge q_0 from point A to B through a circular path AB of radius a is equal to:

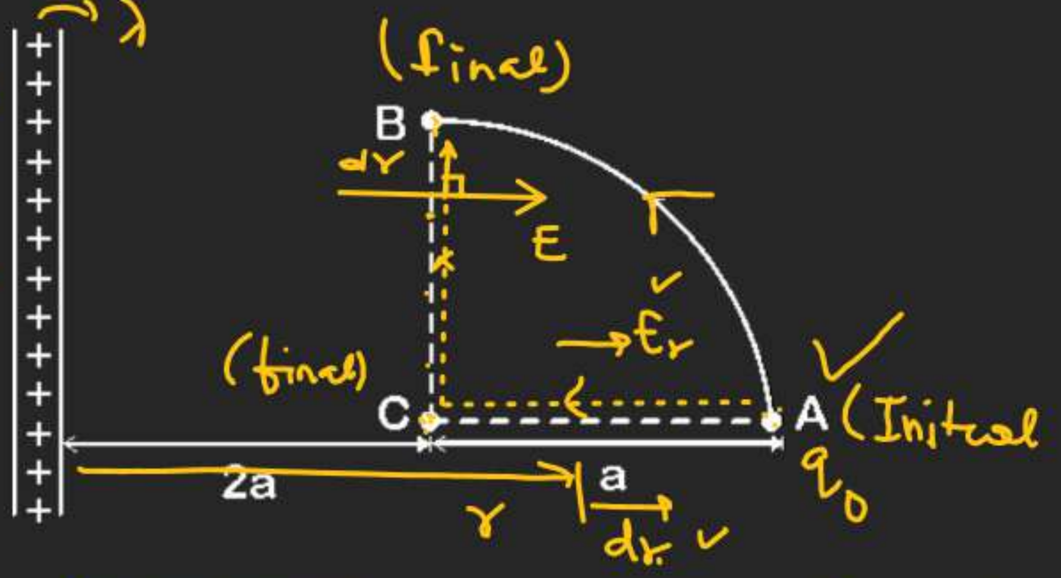
(A) $\frac{q_0^2}{2\pi\varepsilon_0}\ln\left(\frac{2}{3}\right)$

(B) $\frac{q_0\lambda}{2\pi\varepsilon_0}\ln\left(\frac{3}{2}\right)$

(C) $\frac{q_0\lambda}{2\pi\varepsilon_0}\ln\left(\frac{2}{3}\right)$

(D) $\frac{q_0\lambda}{\sqrt{2}\pi\varepsilon_0}$

Infinite line charge.



$W_{ext\,agent} = \Delta U$

$\frac{\Delta U}{q_0} = \Delta V$

$\Delta U = q_0(\Delta V)$

$\Delta U = q_0(V_B - V_A)$

$\Delta U = \frac{q_0\lambda}{2\pi\varepsilon_0}\ln\left(\frac{3}{2}\right)$

$W_{ext\,agent} = \frac{q_0\lambda}{2\pi\varepsilon_0}\ln\left(\frac{3}{2}\right)$

$\Delta V_{AB} = \Delta V_{ACB} = \Delta V_{AC} + \Delta V_{CB}$

$\Delta V_{CB} = 0$, $E \perp dr$

$\Delta V_{AC} = ??$

$\int dV = -\int E_r\,dr = -\frac{\lambda}{2\pi\varepsilon_0}\int_{3a}^{2a}\frac{dr}{r}$

$V_C - V_A = -\frac{\lambda}{2\pi\varepsilon_0}\ln\left(\frac{2}{3}\right)$

$(V_B - V_A) = V_C - V_A = \frac{\lambda}{2\pi\varepsilon_0}\ln(3/2)$

$\boxed{-\ln\left(\frac{2}{3}\right) = \ln(3/2) \qquad \ln\left(\frac{a}{b}\right) = -\ln(b/a)}$

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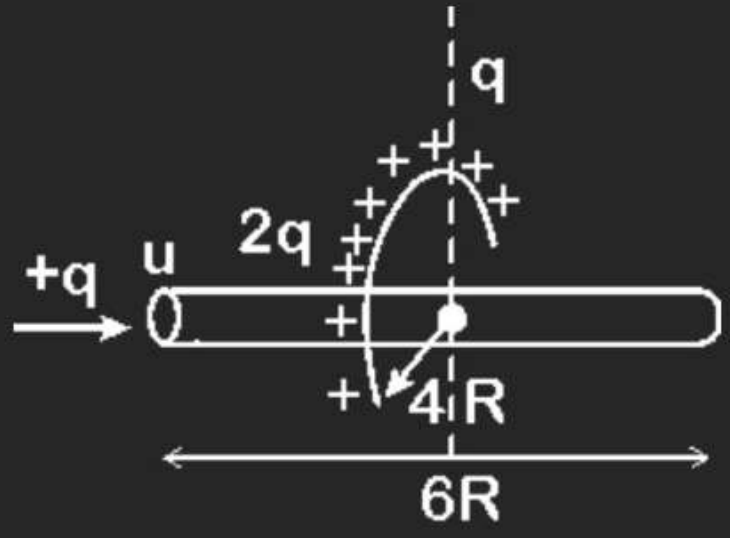
POTENTIAL ENERGY

H.W.:

Q. On a semicircular ring of radius $= 4R$, charge $+3q$ is distributed in such a way that on one quarter $+q$ is uniformly distributed and on another quarter $+2q$ is uniformly distributed. Along its axis a smooth non-conducting and uncharged pipe of length $6R$ is fixed axially as shown. A small ball of mass m and charge $+q$ is thrown from the other end of pipe. The ball can come out of the pipe if:



(A) $u > \sqrt{\dfrac{7q^2}{40\pi\varepsilon_0 Rm}}$

(B) $u > \sqrt{\dfrac{3q^2}{40\pi\varepsilon_0 Rm}}$

(C) $u \geq \sqrt{\dfrac{3q^2}{40\pi\varepsilon_0 Rm}}$

(D) $u > \sqrt{\dfrac{9q^2}{40\pi\varepsilon_0 Rm}}$