Nishant Jindal

$$\int x\cot x\,\mathrm{cosec}^2 x\,dx$$

1955

$$\int \frac{(a-x)\,dx}{\sqrt{(a-x)(x-b)}} = \int \frac{\frac{1}{2}\left((a+b)-2x\right)+\frac{a-b}{2}}{\sqrt{(a+b)x-x^2-ab}}\,dx$$

$$(a+b)x-x^2-ab = \frac{(a-b)^2}{4}-\left(x-\frac{a+b}{2}\right)^2$$

$$\sqrt{(a+b)x-x^2-ab}+\left(\frac{a-b}{2}\right)\sin^{-1}\left(\frac{x-\left(\frac{a+b}{2}\right)}{\left(\frac{a-b}{2}\right)}\right)+C$$

$$\frac{(a+b)^2}{4}-ab$$

$$\int\left(1-\frac{1}{1+\sin x}\right)dx \qquad \int\left(1-\frac{1-\sin x}{\cos^2 x}\right)dx$$

$$\int \frac{x^3}{(x^4+1)}$$

I II

$$\int \frac{x^4 \cdot x^3}{(1+x^4)^2}\,dx$$

$$\frac{1}{4}\int \frac{(t-1)\,dt}{t^2}$$

$$\int e^t\,\frac{?}{t}\,dt$$

$$\frac{3}{\sqrt{2}}\ln\left|x+\sqrt{1+x^2}\right| + \frac{1}{\sqrt{2}}\int \frac{x \cdot x}{\sqrt{1+x^2}}\,dx$$

II

$$\int x\sqrt{1+x^2}$$

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$$\int x^3 \frac{x\,dx}{(1-x^2)^{3/2}} = \frac{x^3}{(1-x^2)^{1/2}} - 3\int \frac{x^2-1+1}{\sqrt{1-x^2}}\,dx\,.$$

$$\int \frac{dx}{x^2\, x^3 \sqrt{\underbrace{1+\frac{4}{x^2}}_{=t^2}}}$$

$$\frac{x^3}{\sqrt{1-x^2}} + 3\int \sqrt{1-x^2}\,dx - 3\int \frac{dx}{\sqrt{1-x^2}}$$

$$K\int (t^2-1)\,dt$$

$$\int \frac{\left(\underbrace{x^2-a^2}_{=t^2}\right)^{5/2} \frac{x\,dx}{}}{x^2} = \int \frac{t^6 +a^6 - a^6\;dt}{t^2+a^2} = \int \left(t^4+a^4-a^2t^2 - \frac{a^6}{t^2+a^2}\right)$$

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$$\int \frac{x^{1/2}\,dx}{x^{3/4}+1} \qquad t^4 = x.$$

$$= 4\int \frac{t^2(1+t^3-1)\,dt}{t^3+1} = 4\int\left(t^2 - \frac{t^2}{t^3+1}\right)dt$$

$$4\left(\frac{t^3}{3} - \frac{1}{3}\ln\left|t^3+1\right|\right) + C.$$

Nishant Jindal

61.

$$\int e^{3x} e^{i2x}\,dx = \frac{e^{(3+2i)x}}{3+2i} = \frac{e^{3x}(\cos 2x + i\sin 2x)(3-2i)}{13}$$

61.

$$\frac{e^{3x}}{13}\Big[(3\sin 2x - 2\cos 2x) - (3\cos 2x + 2\sin 2x)\Big] + C$$

$$(x+2)^2 - 4\boxed{(x+2-1)}$$

$$\frac{x^2 - 4 + 4}{(x+2)^2}$$

$$\cancel{\frac{x-2}{x+2}} + \frac{4}{(x+2)^2}$$

$f(x)$

$$\int x^2 e^x e^{ix}\,dx = e^{(1+i)x}\left(\frac{x^2}{1+i} - \frac{2x}{\underbrace{(1+i)^2}_{=2i}} + \frac{2}{\underbrace{(1+i)^3}_{2i(1+i) = -2+2i}}\right)$$

$$e^x(\cos x + i\sin x)\left(\frac{x^2(1-i)}{2} + ix + \frac{-1-i}{2}\right)$$

$$I = e^x\left(\cos x\left(x - \frac{1}{2} - \frac{x^2}{2}\right) + \sin x\left(\frac{x^2}{2} - \frac{1}{2}\right)\right) + C$$

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$$\int \frac{\sin x}{\sin 4x}\,dx = \frac{1}{4}\int \frac{dx}{\cos 2x \cos x} = \frac{1}{4}\int \frac{\cos x\,dx}{(1-2\sin^2 x)(1-\sin^2 x)}$$

$$\frac{1}{4}\int \frac{2(1-\sin^2 x)-(1-2\sin^2 x)}{(1-2\sin^2 x)(1-\sin^2 x)}\cos x\,dx.$$

$$= \frac{1}{4}\int \left(\frac{\cos x}{\frac{1}{2}-\sin^2 x} - \sec x\right)dx.$$

$$= \frac{1}{4}\left[\frac{1}{\sqrt{2}}\ln\left|\frac{\frac{1}{\sqrt{2}}+\sin x}{\frac{1}{\sqrt{2}}-\sin x}\right| - \ln|\sec x+\tan x|\right] + C.$$

Nishant Jindal

1. $$\int \frac{x^7\,dx}{(1-x^2)^5} = -\frac{1}{2}\int \frac{-2\quad dx}{x^3\left(\frac{1}{x^2}-1\right)^5} = \frac{1}{8}\,\frac{1}{\left(\frac{1}{x^2}-1\right)^4}+C.$$

2. $$\int \frac{x\,dx}{(1-x^4)^{3/2}} = -\frac{1}{4}\int \frac{-4\quad dx}{x^5\left(\frac{1}{x^4}-1\right)^{3/2}}$$

$$= \frac{1}{2}\,\frac{1}{\left(\frac{1}{x^4}-1\right)^{1/2}} + C.$$

Nishant Jindal

3. $\displaystyle\int \frac{dx}{x^2\left(x+\sqrt{1+x^2}\right)} = \int \frac{dx}{x^3\left(1+\sqrt{1+\frac{1}{x^2}}\right)}$

$\displaystyle\int \frac{\sqrt{1+x^2}-x}{x^2}\,dx = -\int \frac{t\,dt}{1+t}$ $\qquad 1+\frac{1}{x^2}=t^2$

$\displaystyle = \int\left(\frac{1+x^2}{x^2\sqrt{1+x^2}} - \frac{1}{x}\right)dx = -t+\ln|1+t|+C.$

$\displaystyle\int\left(\frac{1}{x^3\sqrt{1+\frac{1}{x^2}}} + \frac{1}{\sqrt{1+x^2}} - \frac{1}{x}\right)dx = -\sqrt{1+\frac{1}{x^2}} + \ln\left|x+\sqrt{1+x^2}\right| - \ln|x| + C$

4. $\displaystyle\int \frac{(x-1)\,dx}{x^2\sqrt{2x^2-2x+1}} = \frac{1}{2}\int \frac{2\left(\frac{1}{x^2}-\frac{1}{x^3}\right)dx}{\sqrt{2-\frac{2}{x}+\frac{1}{x^2}}} = \sqrt{2-\frac{2}{x}+\frac{1}{x^2}}+C.$

Nishant Jindal

5. $$\int \frac{(x^4-1)\,dx}{x^2\sqrt{x^4+x^2+1}} = \frac{1}{2}\int \frac{2\left(x-\frac{1}{x^3}\right)dx}{\sqrt{x^2+1+\frac{1}{x^2}}}.$$

$$= \sqrt{x^2+1+\frac{1}{x^2}} + C.$$

6. $$\int \frac{(ax^2-b)\,dx}{x\sqrt{c^2x^2-(ax^2+b)^2}} = \int \frac{\left(a-\frac{b}{x^2}\right)dx}{\sqrt{c^2-\left(ax+\frac{b}{x}\right)^2}}.$$

$$= \sin^{-1}\left(\frac{ax+\frac{b}{x}}{c}\right) + C$$

1. $$\int \frac{dx}{(x^4-1)^2} = \frac{1}{4}\int \frac{4\overset{\mathrm{II}}{x^3}\,dx}{(x^4-1)^2\,x^3}$$

$$= -\frac{1}{4(x^4-1)x^3} - \frac{3}{4}\int \cancel{\frac{dx}{(x^4-1)\,x^4}} =$$

$$\int\left(\frac{1}{x^4-1} - \frac{1}{x^4}\right)dx$$

$$= \int\left(\frac{1}{2}\left(\frac{1}{x^2-1} - \frac{1}{x^2+1}\right) - \frac{1}{x^4}\right)dx$$

Nishant Jindal

2.
$$\int \frac{dx}{x^3\sqrt{(1+x)^3}} = \int \frac{2t\,dt}{(t^2-1)^3 \cdot t^3}$$

$1+x=t^2$

II

I

$$= \frac{1}{-2(t^2-1)^2 t^3} - \frac{3}{4}\int \frac{2t\,dt}{(t^2-1)^2 t^5}$$

$$-\frac{1}{(t^2-1)t^5} - 5\int \frac{dt}{(t^2-1)\,t^6}$$

$t^6-(t^6-1)$

$$\int\left(\frac{1}{t^2-1} - \frac{t^4+t^2+1}{t^6}\right)$$

Nishant Jindal

3. $\displaystyle\int \frac{(5x^2-12)\,dx}{(x^2-6x+13)^2} = \int \frac{5(x^2-6x+13)+15(2x-6)+13}{(x^2-6x+13)^2}\,dx$

$= \frac{5}{2}\tan^{-1}\frac{x-3}{2} - \frac{15}{x^2-6x+13} + 13\displaystyle\int \frac{dx}{\left((x-3)^2+4\right)^2}$

$x-3 = 2\tan\theta$

$\displaystyle\int \frac{2\sec^2\theta\,d\theta}{16\sec^4\theta}$

$= \frac{1}{8}\displaystyle\int \cos^2\theta\,d\theta.$

$\frac{1}{2}\displaystyle\int \frac{2t\,dt}{(t^2+4)^2 t} = -\frac{1}{2t(t^2+4)} - \frac{1}{2}\int \frac{dt}{(t^2+4)t^2}$

$\boxed{-2011}$