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ELECTROSTATIC POTENTIAL AND POTENTIAL ENERGY

$V \rightarrow f(x, y, z) \rightarrow$ (given)

$E \rightarrow f(x, y, z)$

$$\vec{E} = -\left[\underbrace{\frac{\partial V}{\partial x}}\hat{i} + \underbrace{\frac{\partial V}{\partial y}}\hat{j} + \underbrace{\frac{\partial V}{\partial z}}\hat{k}\right]$$

(assume y & z as a constant) (x & z as constant) (x & y as constant)

$V = -2x^2$

Find E at $x = 2$.

$E = -\left(\frac{dV}{dx}\right) = -\left[\frac{d}{dx}(-2x^2)\right]$

$E = (2 \times 2x) = 4x$

$E_{x=2} = (4 \times 2) = 8\ (V/m)$

$E = -\frac{dV}{dr} \rightarrow V/m$

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ELECTROSTATIC POTENTIAL AND POTENTIAL ENERGY

\# If $V = (x^2y + 2yz)$

Find the value of electric field at $(1, 2, 1)$

$$\vec{E} = -\left[\frac{\partial V}{\partial x}\hat{i} + \frac{\partial V}{\partial y}\hat{j} + \frac{\partial V}{\partial z}\hat{k}\right]$$

$$\frac{\partial}{\partial x}(x^2y + 2yz) = y\frac{\partial}{\partial x}(x^2) + \frac{\partial}{\partial x}(2yz)$$

(y & z as constant)

$$= (2xy)$$

$$\frac{\partial V}{\partial y} = \frac{\partial}{\partial y}(x^2y + 2yz) = (x^2(1) + 2z)$$

(x & z as constant)

$$\frac{\partial V}{\partial z} = \frac{\partial}{\partial z}(x^2y + 2yz)$$

(x & y as constant) $= (2y)$

$$\vec{E} = -\left[(2xy)\hat{i} + (x^2 + 2z)\hat{j} + (2y)\hat{k}\right]$$

$$\vec{E}_{(1,2,1)} = -\left[4\hat{i} + 3\hat{j} + 4\hat{k}\right]$$

$$|\vec{E}| = \sqrt{16 + 16 + 9}$$

$$= \sqrt{32 + 9} = \sqrt{41}\ V/m$$

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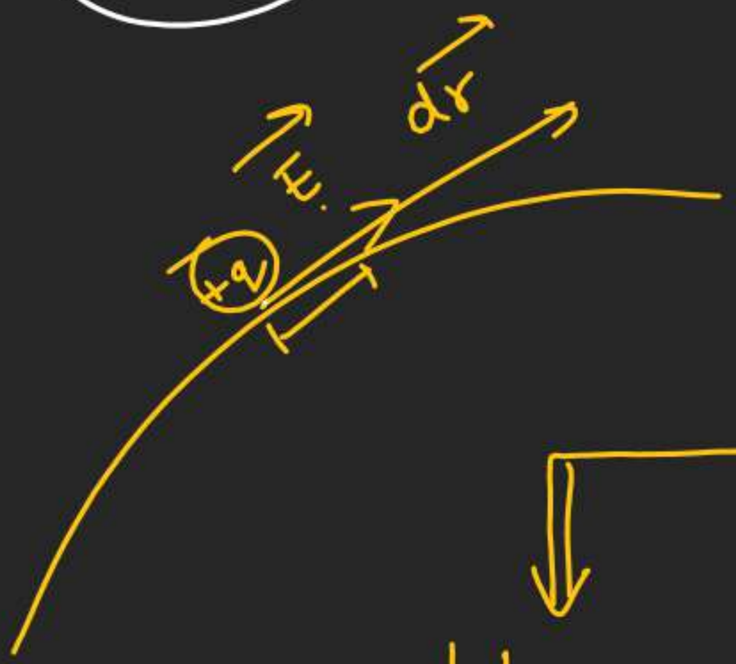
ELECTROSTATIC POTENTIAL AND POTENTIAL ENERGY

If $\vec{E} = (x^2\hat{i} + y\hat{j})$ Find work done in moving a Charge $+2\mu C$ from $(1, 2)$ to $(3, 4)$

$(1,2) \rightarrow A \rightarrow V_A$

$(3,4) \rightarrow B \rightarrow V_B$



$$\frac{\Delta U}{q} = \Delta V$$

$$\Delta U = (q\,\Delta V)$$

$$\Downarrow$$

$$W_{ext\,agent} = [q\,\Delta V]$$

$$W_{system\,force} = -\Delta U = -[q\,\Delta V]$$

$$\boxed{-W_{system} = \Delta U}$$

$$\vec{dr} = dx\,\hat{i} + dy\,\hat{j}$$

$$\int_{V_A}^{V_B} dV = -\int_{(1,2)}^{(3,4)} \vec{E}\cdot\vec{dr} = -\int_{(1,2)}^{(3,4)} (x^2\hat{i} + y\hat{j})\cdot(dx\,\hat{i} + dy\,\hat{j})$$

$$\int_{V_A}^{V_B} dV = -\left[\int_1^3 x^2\,dx + \int_2^4 y\,dy\right]$$

$$V_B - V_A = -\left[\frac{1}{3}(27-1) + \frac{1}{2}(16-4)\right]$$

$$V_B - V_A = -\left[\frac{26}{3} + 6\right]$$

$$= -\left[\frac{26+18}{3}\right] = -\frac{44}{3}\ \text{V/m}$$

$$V_B - V_A = \left(-\frac{44}{3}\right)V$$

$$\Delta U = q(\Delta V)$$

[put with Sign]

$$\Delta U = (+2)\times 10^{-6} \times \left(-\frac{44}{3}\right)$$

$$\boxed{\Delta U = -\frac{88}{3}\,\mu J}$$

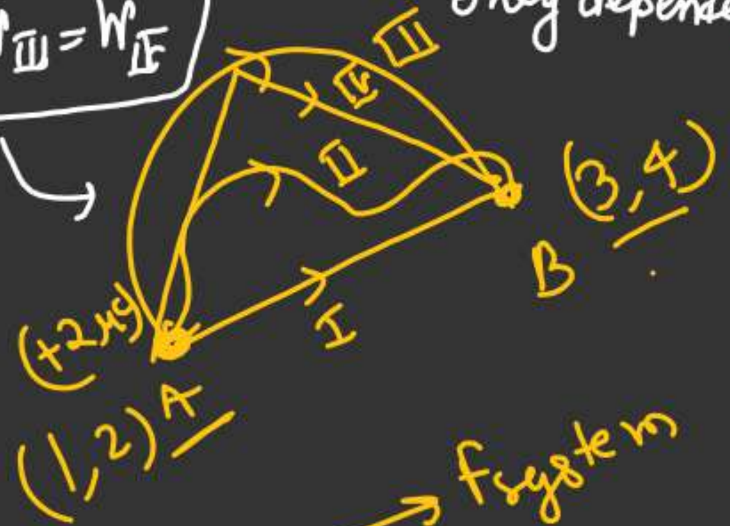
$-ve \Rightarrow$ (work done by System force)

$$W_{ext\,agent} = \Delta U = -\frac{88}{3}\,\mu J$$

$$W_{system} = -\Delta U = \frac{88}{3}\,\mu J$$

$$\boxed{W_{I} = W_{II} = W_{III} = W_{IV}}$$

Conservative force work only dependent on initial & final position



$$\boxed{+W_{ext\,agent} = -W_{system} = \Delta U}$$

$F_{ext\,agent} \leftarrow$ o $\rightarrow f_{system}$

(+2μC)

$\Downarrow$

very slowly

$$|F_{ext\,agent}| = |F_{system}|$$

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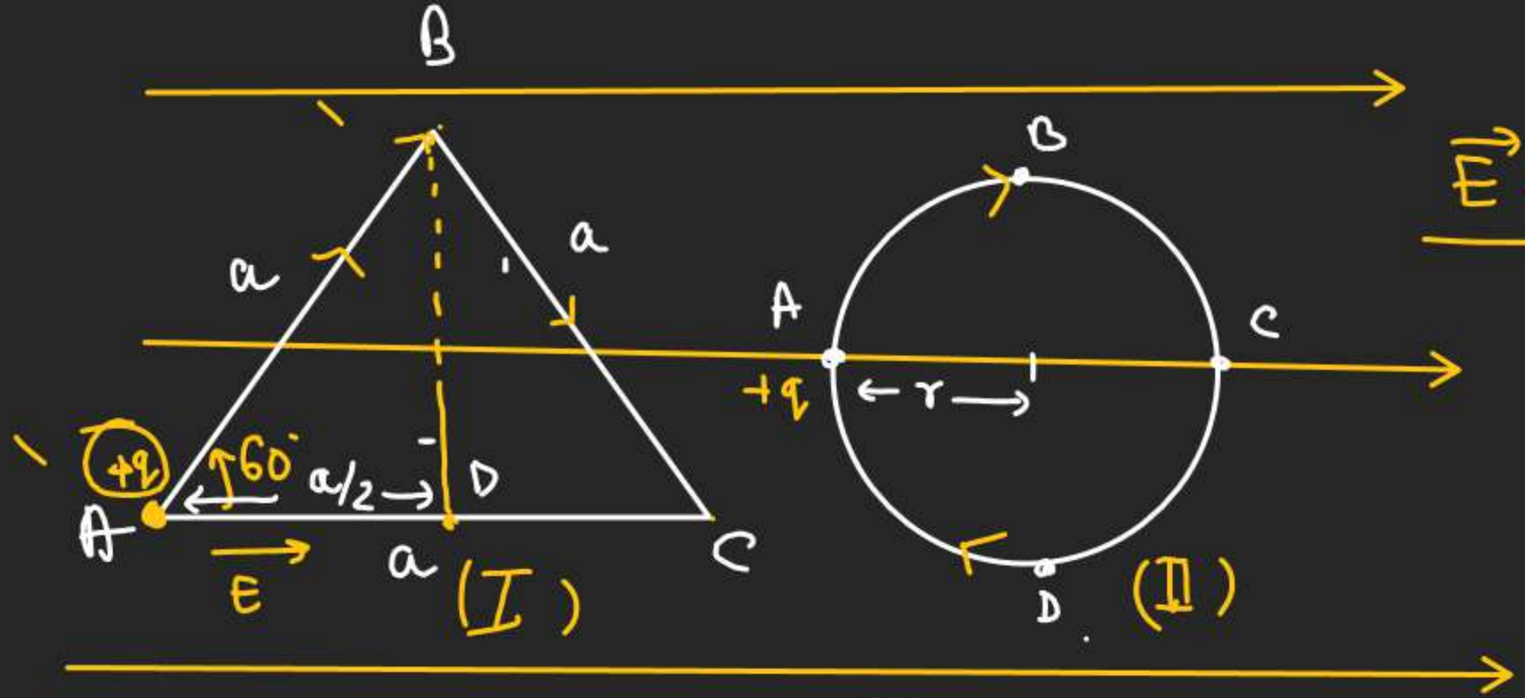
ELECTROSTATIC POTENTIAL AND POTENTIAL ENERGY

Potential difference in a Constant Electric field :-



For Fig (I)

by System

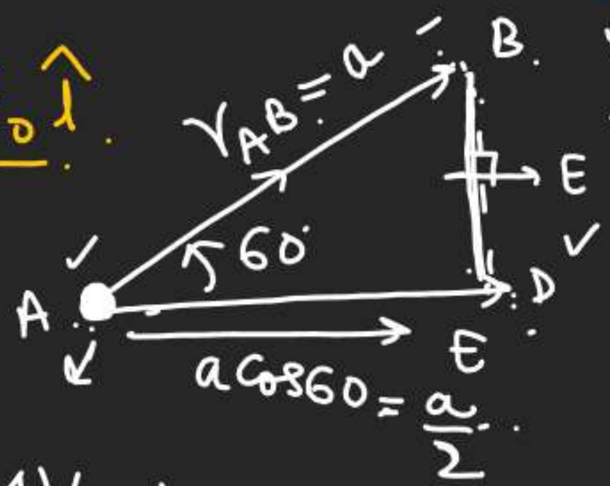
Find work done in moving the Charge from

i) A to B ✓

ii) A to C ✓

i) ABCA.

$\vec{E} = E_0 \hat{i}$



$V_D = V_B$

→ Moving along the electric field potential decreases.

Moving opposite to electric field potential Increases.

$$\Delta V = V_D - V_A = -\left(\frac{a}{2}E\right)$$

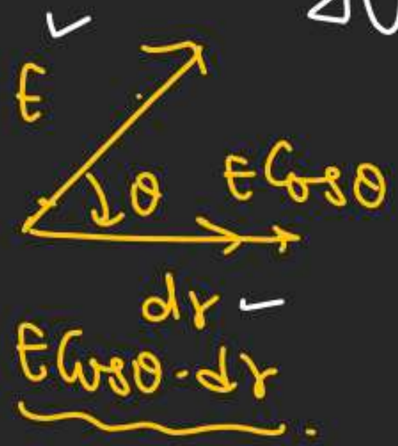
$$\Delta U = U_D - U_A = -\left(\frac{Ea}{2}\right)$$

$$W_{AB} = -\Delta U$$

$$= -\left(-\frac{Ea}{2}\right) = \left(\frac{Ea}{2}\right)$$

$$dV = -\left(\vec{E} \cdot \vec{dr}\right)$$

$$\boxed{dV = -E\,dr\cos\theta}$$



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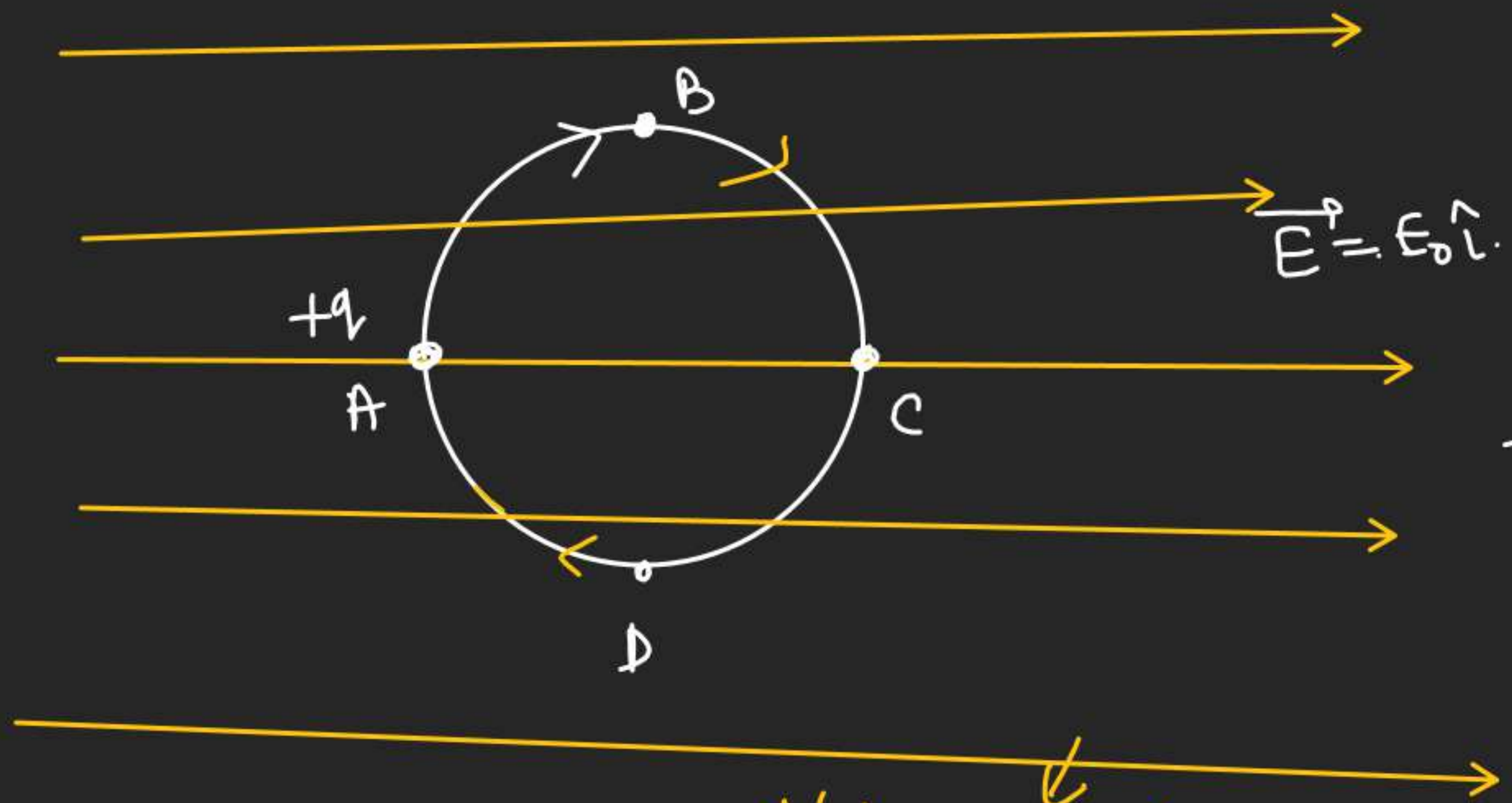
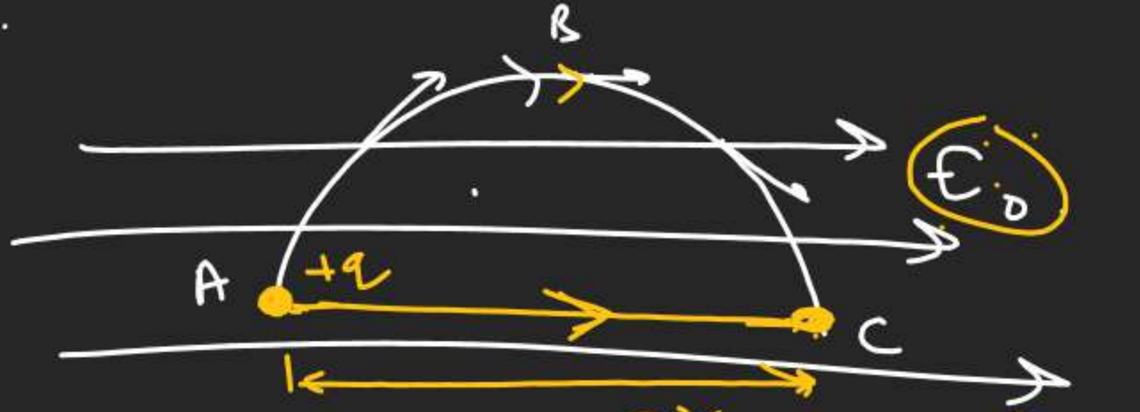


$(W_{ABC})_{System} = ??$

$W_{CDA} = -W_{ABC}$

$=$



$W_{ABC} = W_{AC}$

$\vec{E}$

2r

$\Delta U = (+q)(\Delta V)$

$(U_c - U_A) = (+q)(-E2r)$

$= -2qEr$

$W_{ABC} = -\Delta U$

$= -(-2qEr) = \boxed{2qEr}$

$\Delta V = (V_c - V_A) = -(E \cdot 2r)$

$V_c < V_A$

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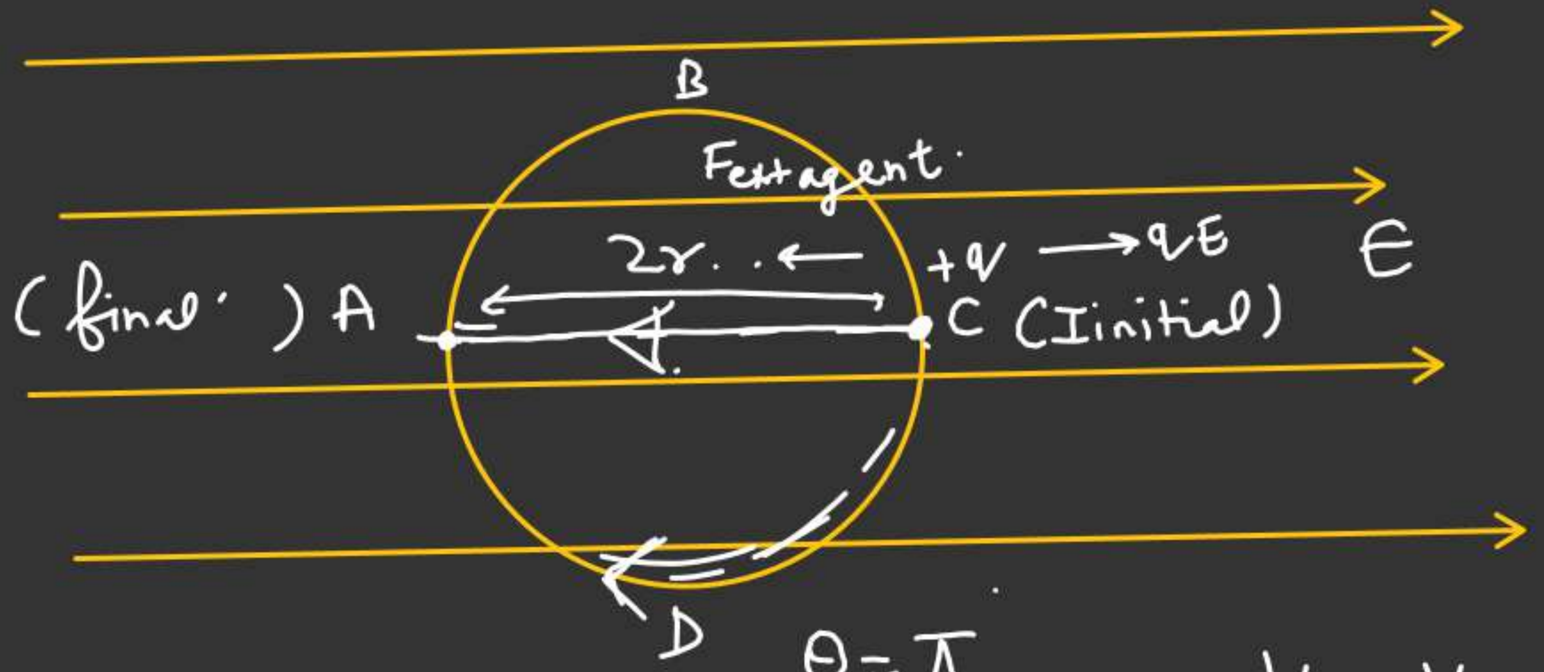
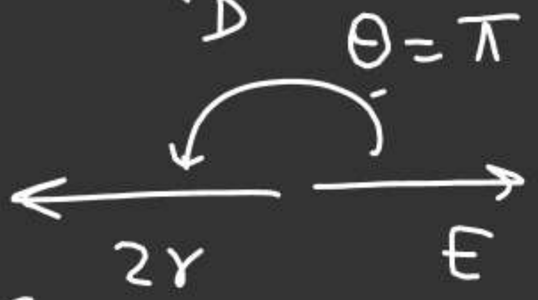




$$dV = -\vec{E}\cdot d\vec{r}$$

$$dV = -(E\, dr \cos\theta)$$

$$V_A - V_C = -E(2r)\cos\pi$$

$$V_A - V_C = (E\,2r)$$

$$\Delta U = +q(\Delta V)$$

$$\boxed{U_A - U_C = (qE2r)}$$

$$W_{system} = -\Delta U$$

$$= -qE2r$$

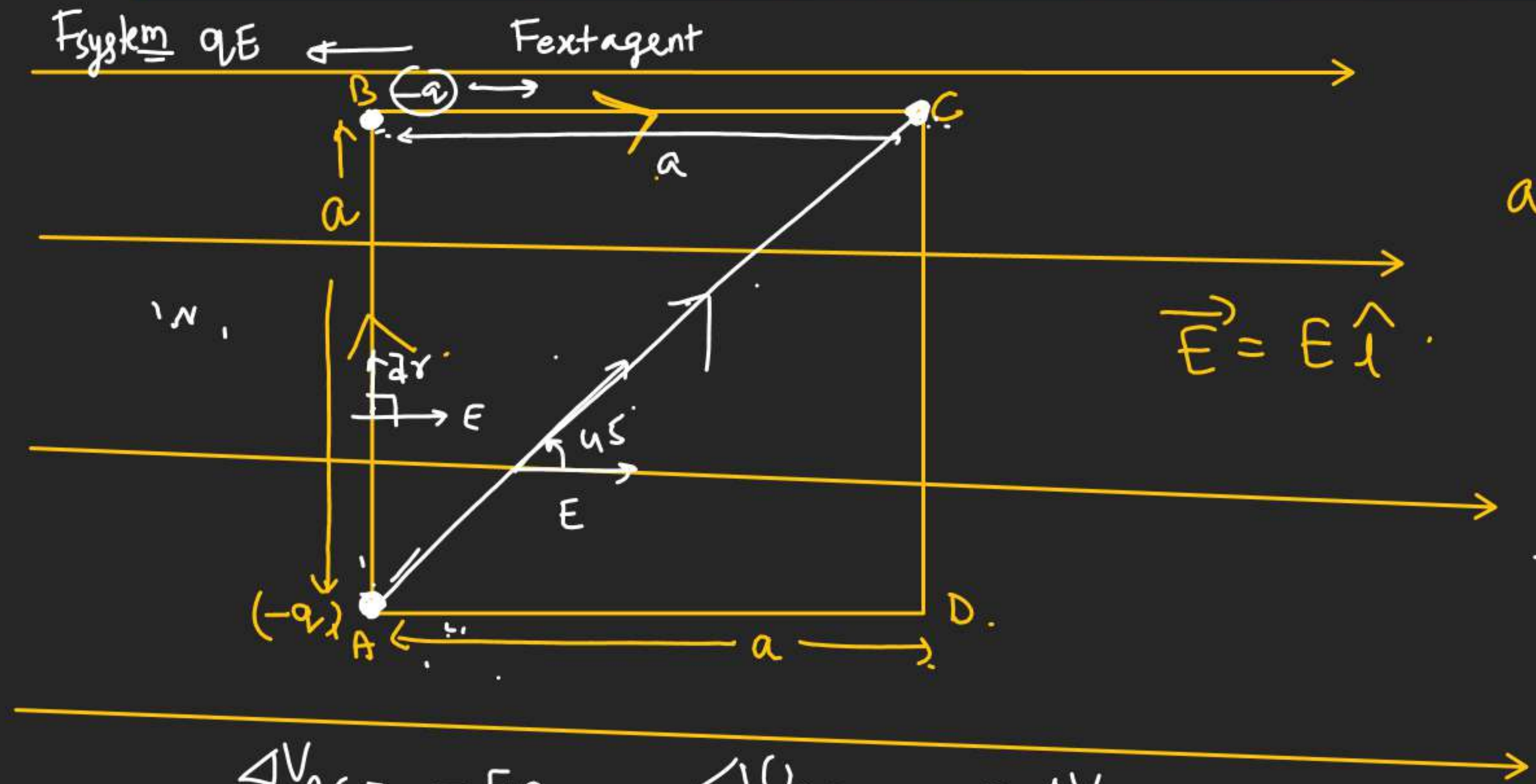
$$\boxed{(W_{net})_{\text{closed path}} = 0}$$

$$\boxed{\Delta V = 0}$$

$$\boxed{\Delta U = 0}$$

In a close path Electric field Uniform

ELECTROSTATIC POTENTIAL AND POTENTIAL ENERGY



Find work done in moving a charge (-q) along the path.

i) ABC = ??

ii) AC = ?? ← Same as W_{ABC}

$\vec{E} = E\hat{i}$

$W_{ABC} = W_{AB} + W_{AC}$

For AB path

$\vec{dr} \perp \vec{E}$

$dV = \vec{E}\cdot\vec{dr}$ $\quad \vec{E} \perp \vec{dr}$

$\boxed{dV = 0} \Rightarrow V_A = V_B$

$U_A = U_B$

$\Delta U_{AB} = 0$

$\Delta V_{BC} = -Ea$

$\Delta U_{BC} = -q\,\Delta V_{BC}$

$= -q(-Ea)$

$= qEa$

$(W_{BC}) = -\Delta U_{BC}$

$= \boxed{-qEa}$

$(W_{BC})_{extagent} = \Delta U = qEa$

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ELECTROSTATIC POTENTIAL AND POTENTIAL ENERGY

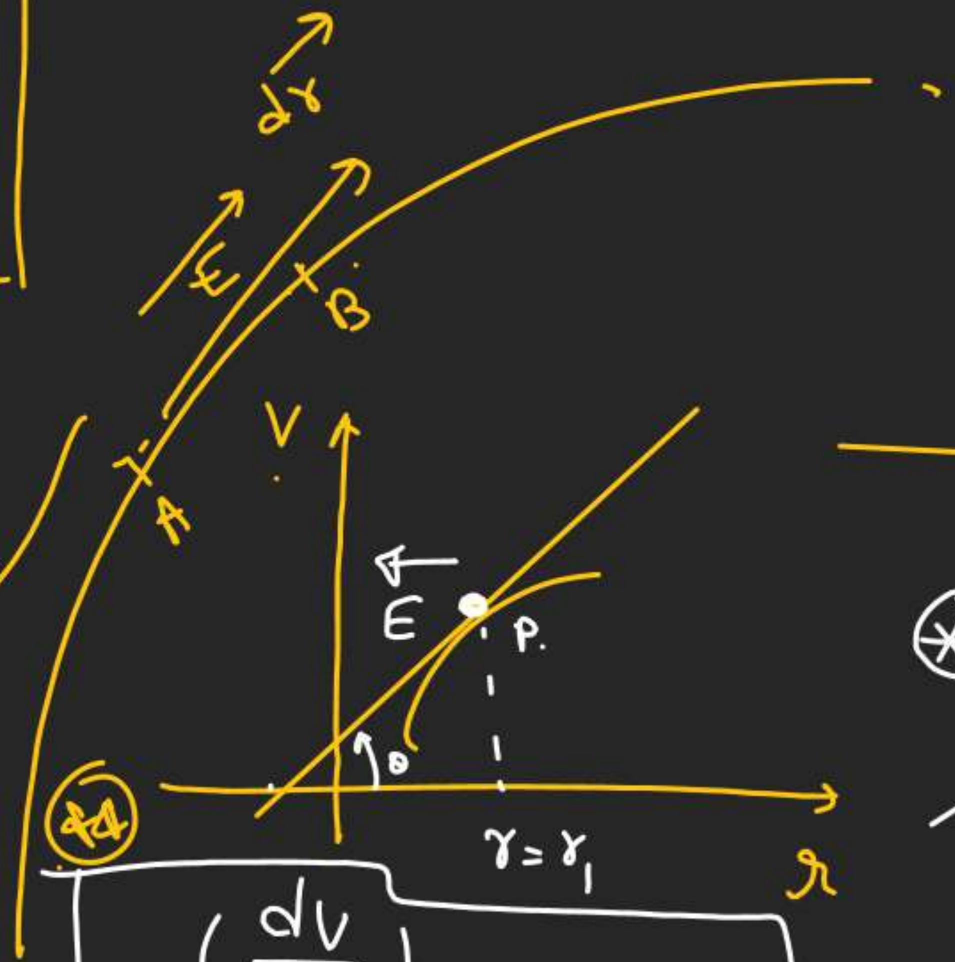
$$\boxed{\int_{V_i}^{V_f} dV = -\int_{r_i}^{r_f} \vec{E}\cdot\vec{dr}}$$

$$dV = -\vec{E}\cdot\vec{dr}$$

$$\vec{E} \parallel \vec{dr}$$

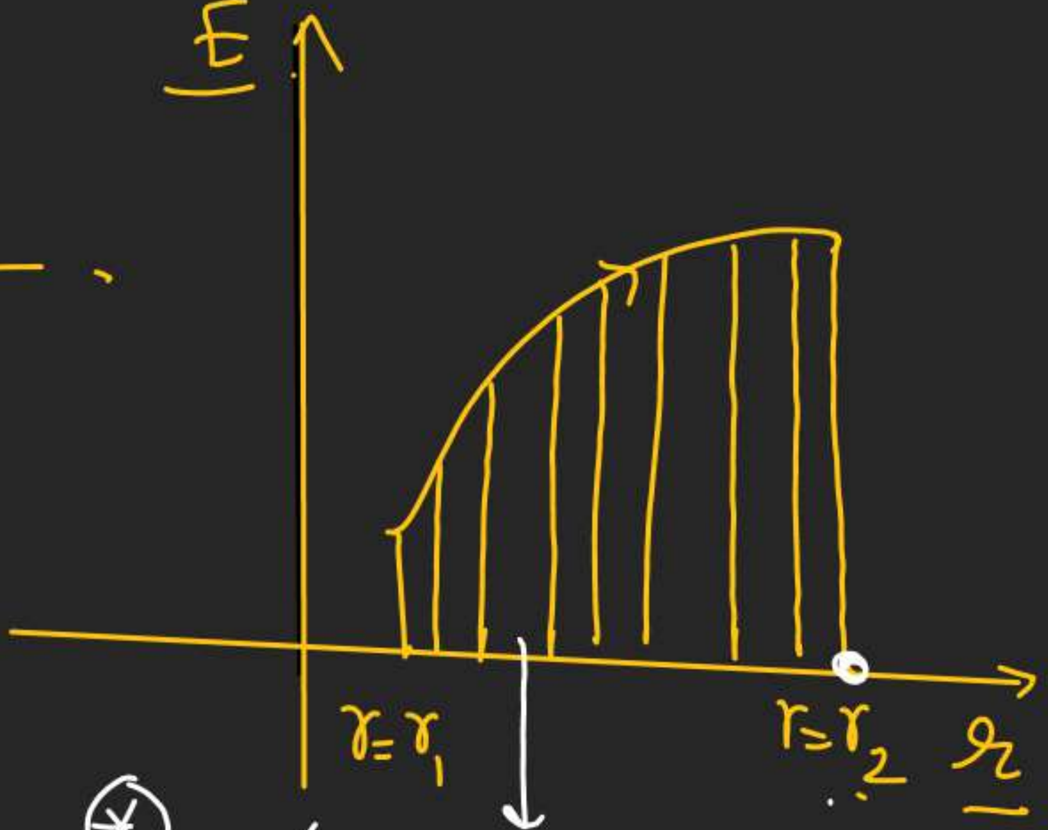
$$dV = -E\,dr$$

$$\boxed{E = -\frac{dV}{dr}}$$



$$\boxed{\left(\frac{dV}{dr}\right)_{r=r_1} = -E}$$

slope of V vs r graph gives electric field at $r=r_1$



(Area under E vs r graph gives potential difference)

$$\int_{r_1}^{r_2} E\,dr = V_{(r_2)} - V_{(r_1)}$$