EXERCISE - 1

Objective Problems | JEE Main

SECTION - A

QUESTIONS BASED ON BASIC DEFINITION & PARAMETRIC REPRESENTATION

1. The eccentricity of the hyperbola $4x^2 - 9y^2 - 8x = 32$ is

(A) $\frac{\sqrt{5}}{3}$ (B) $\frac{\sqrt{13}}{3}$ (C) $\frac{\sqrt{13}}{9}$ (D) $\frac{3}{2}$

2. The locus of the point of intersection of the lines $\sqrt{3}x - y - 4\sqrt{3}k = 0$ and $\sqrt{3}kx + ky - 4\sqrt{3} = 0$ for different values of k is

(A) ellipse (B) parabola (C) circle (D) hyperbola

3. If the latus rectum of an hyperbola be 8 and eccentricity be $\frac{3}{\sqrt{5}}$ then the equation of the hyperbola is

(A) $4x^2 - 5y^2 = 100$ (B) $5x^2 - 4y^2 = 100$

(C) $4x^2 + 5y^2 = 100$ (D) $5x^2 + 4y^2 = 100$

4. If the centre, vertex and focus of a hyperbola be $(0, 0)$, $(4,0)$ and $(6,0)$ respectively, then the equation of the hyperbola is

(A) $4x^2 - 5y^2 = 8$ (B) $4x^2 - 5y^2 = 80$

(C) $5x^2 - 4y^2 = 80$ (D) $5x^2 - 4y^2 = 8$

5. The equation of the hyperbola whose foci are $(6,5)$, $(-4,5)$ and eccentricity $5/4$ is

(A) $\frac{(x-1)^2}{16} - \frac{(y-5)^2}{9} = 1$ (B) $\frac{x^2}{16} - \frac{y^2}{9} = 1$

(C) $\frac{(x-1)^2}{16} - \frac{(y-5)^2}{9} = -1$ (D) none of these

6. The vertices of a hyperbola are at $(0,0)$ and $(10,0)$ and one of its foci is at $(18,0)$. The equation of the hyperbola is

(A) $\frac{x^2}{25} - \frac{y^2}{144} = 1$ (B) $\frac{(x-5)^2}{25} - \frac{y^2}{144} = 1$

(C) $\frac{x^2}{25} - \frac{(y-5)^2}{144} = 1$ (D) $\frac{(x-5)^2}{25} - \frac{(y-5)^2}{144} = 1$

7. Given the family of hyperbolas $\frac{x^2}{\cos^2\alpha} - \frac{y^2}{\sin^2\alpha} = 1$ for $\alpha \in (0, \pi/2)$ which of the following does not change with varying α ?

(A) abscissa of foci (B) eccentricity

(C) equations of directrices (D) abscissa of vertices

SECTION – B

POSITION OF POINT & CHORD JOINING TWO POINTS

8. Locus of the middle points of the parallel chords with gradient m of the rectangular hyperbola $xy = c^2$ is

(A) $y + mx = 0$ (B) $y - mx = 0$

(C) $my - mx = 0$ (D) $my + x = 0$

SECTION – C

TANGENT TO PARABOLA

9. The equation of the tangent lines to the hyperbola $x^2 - 2y^2 = 18$ which are perpendicular to the line $y = x$ are

(A) $y = x \pm 3$ (B) $y = -x \pm 3$ (C) $2x + 3y + 4 = 0$ (D) none of these

10. The line $2x + \sqrt{6}y = 2$ is a tangent to the curve $x^2 - 2y^2 = 4$. The point of contact is

(A) $(4, -\sqrt{6})$ (B) $(7, -2\sqrt{6})$ (C) $(2,3)$ (D) $(\sqrt{6}, 1)$

11. The equation of the common tangent to the parabola $y^2 = 8x$ and the hyperbola $3x^2 - y^2 = 3$ is

(A) $2x \pm y + 1 = 0$ (B) $x \pm y + 1 = 0$ (C) $x \pm 2y + 1 = 0$ (D) $x \pm y + 2 = 0$

SECTION - D NORMAL TO PARABOLA

12. Let P(asec θ, btan θ) and Q(asec ϕ, btan ϕ), where $\theta + \phi = \frac{\pi}{2}$, be two points on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. If (h, k) is the point of intersection of the normals at P&Q, then k is equal to

(A) $\frac{a^2+b^2}{a}$ (B) $-\left(\frac{a^2+b^2}{a}\right)$ (C) $\frac{a^2+b^2}{b}$ (D) $-\left(\frac{a^2+b^2}{b}\right)$

13. If the normal to the rectangular hyperbola $xy = c^2$ at the point ' t ' meets the curve again at ' t_1 ' then t^3t_1 has the value equal to

(A) 1 (B) -1 (C) 0 (D) none

SECTION - E & F

SUBTANGENT & NORMAL / CHORD OF CONTACT, LENGTH, CHORD WITH A GIVEN MID POINT

14. If $x = 9$ is the chord of contact of the hyperbola $x^2 - y^2 = 9$, then the equation of the corresponding pair of tangents, is

(A) $9x^2 - 8y^2 + 18x - 9 = 0$ (B) $9x^2 - 8y^2 - 18x + 9 = 0$

(C) $9x^2 - 8y^2 - 18x - 9 = 0$ (D) $9x^2 - 8y^2 + 18x + 9 = 0$

15. Equation of the chord of the hyperbola $25x^2 - 16y^2 = 400$ which is bisected at the point (6,2) is

(A) $16x - 75y = 418$ (B) $75x - 16y = 418$

(C) $25x - 4y = 400$ (D) none of these

SECTION - G & H HIGHLIGHTS & BASED RESULTS / MIXED PROBLEMS

16. The ellipse $4x^2 + 9y^2 = 36$ and the hyperbola $4x^2 - y^2 = 4$ have the same foci and they intersect at right angles then the equation of the circle through the points of intersection of two conics is

(A) $x^2 + y^2 = 5$ (B) $\sqrt{5}(x^2 + y^2) - 3x - 4y = 0$

(C) $\sqrt{5}(x^2 + y^2) + 3x + 4y = 0$ (D) $x^2 + y^2 = 25$

17. The asymptotes of the hyperbola $xy - 3x - 2y = 0$ are

(A) $x - 2 = 0$ and $y - 3 = 0$ (B) $x - 3 = 0$ and $y - 2 = 0$

(C) $x + 2 = 0$ and $y + 3 = 0$ (D) $x + 3 = 0$ and $y + 2 = 0$

18. If the product of the perpendicular distances from any point on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ of eccentricity $e = \sqrt{3}$ on its asymptotes is equal to 6 , then the length of the transverse axis of the hyperbola is

(A) 3 (B) 6 (C) 8 (D) 12

EXERCISE - 2 (LEVEL-I)

Objective Problems | JEE Main

QUESTIONS BASED ON BASIC DEFINITION & PARAMETRIC REPRESENTATION

1. The equation $9x^2 - 16y^2 - 18x + 32y - 151 = 0$ represent a hyperbola

(A) The length of the transverse axes is 4

(B) Length of latus rectum is 9

(C) Equation of directrix is $x = \frac{21}{5}$ and $x = -\frac{11}{5}$

(D) none of these

2. The length of the transverse axis of a hyperbola is 7 and it passes through the point $(5, -2)$. The equation of the hyperbola is

(A) $\frac{4}{49}x^2 - \frac{196}{51}y^2 = 1$ (B) $\frac{49}{4}x^2 - \frac{51}{196}y^2 = 1$

(C) $\frac{4}{49}x^2 - \frac{51}{196}y^2 = 1$ (D) none of these

3. If the eccentricity of the hyperbola $x^2 - y^2\sec^2\ \alpha = 5$ is $\sqrt{3}$ times the eccentricity of the ellipse $x^2\sec^2\ \alpha + y^2 = 25$, then a value eof α is

(A) $\pi/6$ (B) $\pi/4$ (C) $\pi/3$ (D) $\pi/2$

4. The foci of the ellipse $\frac{x^2}{16} + \frac{y^2}{b^2} = 1$ and the hyperbola $\frac{x^2}{144} - \frac{y^2}{81} = \frac{1}{25}$ coincide. Then the value of b^2 is-

(A) 9 (B) 1 (C) 5 (D) 7

5. AB is a double ordinate of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ such that $\triangle$ AOB (where ' O ' is the origin) is an equilateral triangle, then the eccentricity e of the hyperbola satisfies

(A) $e > \sqrt{3}$ (B) $1 < e < \frac{2}{\sqrt{3}}$ (C) $e = \frac{2}{\sqrt{3}}$ (D) $e > \frac{2}{\sqrt{3}}$

POSITION OF POINT & CHORD JOINING TWO POINTS

6. The equation to the chord joining two points (x_1, y_1) and (x_2, y_2) on the rectangular hyperbola $xy = c^2$ is

(A) $\frac{x}{x_1+x_2} + \frac{y}{y_1+y_2} = 1$ (B) $\frac{x}{x_1-x_2} + \frac{y}{y_1-y_2} = 1$

(C) $\frac{x}{y_1-y_2} + \frac{y}{x_1-x_2} = 1$ (D) $\frac{x}{y_1-y_2} + \frac{y}{x_1-x_2} = 1$

TANGENT TO PARABOLA

7. The locus of the foot of the perpendicular from the centre of the hyperbola $xy = c^2$ on a variable tangent is

(A) $(x^2 - y^2)^2 = 4c^2xy$ (B) $(x^2 + y^2)2 = 2c^2xy$

(C) $(x^2 - y^2) = 4c^2xy$ (D) $(x^2 + y^2)^2 = 4c^2xy$

8. The equation to the common tangents to the two hyperbolas $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ and $\frac{y^2}{a^2}-\frac{x^2}{b^2}=1$ are

(A) $y=\pm x\pm\sqrt{b^2-a^2}$ (B) $y=\pm x\pm(a^2-b^2)$

(C) $y=\pm x\pm\sqrt{a^2-b^2}$ (D) $y=\pm x\pm\sqrt{a^2+b^2}$

9. Locus of the feet of the perpendiculars drawn from either foci on a variable tangent to the hyperbola $16y^2-9x^2=1$ is

(A) $x^2+y^2=9$ (B) $x^2+y^2=1/9$

(C) $x^2+y^2=7/144$ (D) $x^2+y^2=1/16$

10. Area of triangle formed by tangent to the hyperbola $xy=16$ at (16,1) and co-ordinate axes equals

(A) 8 (B) 16 (C) 32 (D) 64

NORMAL TO PARABOLA

11. The tangent to the hyperbola $xy=c^2$ at the point P intersects the x-axis at T and the y-axis at T′. The normal to the hyperbola at P intersects the x-axis at N and the y-axis at N′. The areas of the triangles PNT and PN′T′ are Δ and Δ′ respectively, then $\frac{1}{\Delta}+\frac{1}{\Delta'}$ is

(A) equal to 1 (B) depends on t (C) depends on c (D) equal to 2

CHORD OF CONTACT, LENGTH, CHORD WITH A GIVEN MID POINT

12. The locus of the mid points of the chords passing through a fixed point (α,β) of the hyperbola, $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is

(A) a circle with centre $\left(\frac{\alpha}{2},\frac{\beta}{2}\right)$ (B) an ellipse with centre $\left(\frac{\alpha}{2},\frac{\beta}{2}\right)$

(C) a hyperbola with centre $\left(\frac{\alpha}{2},\frac{\beta}{2}\right)$ (D) straight line through $\left(\frac{\alpha}{2},\frac{\beta}{2}\right)$

13. From the points of the circle $x^2+y^2=a^2$, tangents are drawn to the hyperbola $x^2-y^2=a^2$; then the locus of the middle points of the chords of contact is

(A) $(x^2-y^2)^2=a^2(x^2+y^2)$ (B) $(x^2-y^2)^2=2a^2(x^2+y^2)$

(C) $(x^2+y^2)^2=a^2(x^2-y^2)$ (D) $2(x^2-y^2)^2=3a^2(x^2+y^2)$

MIXED PROBLEMS

14. Variable circles are drawn touching two fixed circles externally then locus of centre of variable circle is

(A) parabola (B) ellipse (C) hyperbola (D) circle

15. The asymptote of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ form with any tangent to the hyperbola a triangle whose area is $a^2\tan\lambda$ in magnitude then its eccentricity is

(A) $\sec\lambda$ (B) $\operatorname{cosec}\lambda$ (C) $\sec^2\lambda$ (D) $\operatorname{cosec}^2\lambda$

16. From any point on the hyperbola $H_1: (x^2/a^2) - (y^2/b^2) = 1$ tangents are drawn to the hyperbola $H_2: (x^2/a^2) - (y^2/b^2) = 2$. The area cut-off by the chord of contact on the asymptotes of H_2 is equal to

(A) ab/2 (B) ab (C) 2ab (D) 4ab

17. The tangent at P on the hyperbola $(x^2/a^2) - (y^2/b^2) = 1$ meets the asymptote $\frac{x}{a} - \frac{y}{b} = 0$ at Q. If the locus of the mid point of PQ has the equation $(x^2/a^2) - (y^2/b^2) = k$, then k has the value equal to

(A) 1/2 (B) 2 (C) 3/4 (D) 4/3

EXERCISE - 2 (LEVEL-II)

Multiple Correct | JEE Advanced

QUESTIONS BASED ON BASIC DEFINITION & PARAMETRIC REPRESENTATION

1. If (5,12) and (24,7) are the focii of a conic passing through the origin then the eccentricity of conic is

(A) $\sqrt{386}/12$ (B) $\sqrt{386}/13$ (C) $\sqrt{386}/25$ (D) $\sqrt{386}/38$

2. Equation $(2+\lambda)x^2 - 2\lambda xy + (\lambda-1)y^2 - 4x - 2 = 0$ represents a hyperbola if

(A) $\lambda = 4$ (B) $\lambda = 1$ (C) $\lambda = 4/3$ (D) $\lambda = -1$

TANGENT TO PARABOLA

3. The point of contact of $5x + 12y = 19$ and $x^2 - 9y^2 = 9$ will lie on

(A) $4x + 15y = 0$ (B) $7x + 12y = 19$

(C) $4x + 15y + 1 = 0$ (D) $7x - 12y = 19$

MIXED PROBLEMS

4. The tangent to the hyperbola, $x^2 - 3y^2 = 3$ at the point $(\sqrt{3}, 0)$ when associated with two asymptotes constitutes.

(A) isosceles triangle (B) an equilateral triangle

(C) a triangle whose area is $\sqrt{3}$ sq. unit (D) a right isosceles triangle

5. If θ is the angle between the asymptotes of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ with eccentricity e, then $\sec\ \theta/2$ can be

(A) e (B) e/2 (C) e/3 (D) 1/e

EXERCISE - 3

| Subjective | JEE Advanced

1. Find the equation to the hyperbola whose directrix is $2x + y = 1$, focus $(1,1)$& eccentricity $\sqrt{3}$. Find also the length of its latus rectum.

2. The hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ passes through the point of intersection of the lines $7x + 13y - 87 = 0$ and $5x - 8y + 7 = 0$& the latus rectum is $32\sqrt{2}/5$. Find ' a ' & ' b '.

3. For the hyperbola $\frac{x^2}{100} - \frac{y^2}{25} = 1$, prove that

(i) eccentricity $= \sqrt{5}/2$

(ii) $SA.S'A = 25$, where S& S′ are the foci & A is the vertex.

4. Find the centre, the foci, the directrices, the length of the latus rectum, the length & the equations of the axes of the hyperbola $16x^2 - 9y^2 + 32x + 36y - 164 = 0$.

5. If C is the centre of a hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, S, S′ its foci and P a point on it. Prove that $SP.S'P = CP^2 - a^2 + b^2$.

6. If θ_1&θ_2 are the parameters of the extremities of a chord through $(ae, 0)$ of a hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, then show that $\tan\frac{\theta_1}{2} \cdot \tan\frac{\theta_2}{2} + \frac{e-1}{e+1} = 0$.

7. Chords of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ are tangents to the circle drawn on the line joining the foci as diameter. Find the locus of the point of intersection of tangents at the extremities of the chords.

8. Show that the locus of the middle points of normal chords of the rectangular hyperbola $x^2 - y^2 = a^2$ is $(y^2 - x^2)^3 = 4a^2x^2y^2$.

9. Find the equation of the tangent to the hyperbola $x^2 - 4y^2 = 36$ which is perpendicular to the line $x - y + 4 = 0$.

10. Tangents are drawn to the hyperbola $3x^2 - 2y^2 = 25$ from the point $(0,5/2)$. Find their equations.

11. Tangents are drawn from the point (α, β) to the hyperbola $3x^2 - 2y^2 = 6$ and are inclined at angles θ and ϕ to the x-axis. If $\tan\theta \cdot \tan\phi = 2$, prove that $\beta^2 = 2\alpha^2 - 7$.

12. Let 'p' be the perpendicular distance from the centre C of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ to the tangent drawn at a point R on the hyperbola. If S&S′ are the two foci of the hyperbola, then show that $(RS + RS')^2 = 4a^2\left(1 + \frac{b^2}{p^2}\right)$.

13. The tangents & normal at a point on $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ cut the y-axis at A& B. Prove that the circle on AB as diameter passes through the foci of the hyperbola.

14. The perpendicular from the centre upon the normal on any point of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ meets at R. Find the locus of R.

15. If the normal at a point P to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ meets the x-axis at G, show that $SG = e.SP$, S being the focus of the hyperbola.

16. The normal to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ drawn at an extremity of its latus rectum is parallel to an asymptote. Show that the eccentricity is equal to the square root of $(1 + \sqrt{5})/2$.

17. Prove that the locus of the middle point of the chord of contact of tangents from any point of the circle $x^2 + y^2 = r^2$ to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is given by the equation $\left(\frac{x^2}{a^2} - \frac{y^2}{b^2}\right)^2 = \frac{(x^2+y^2)}{r^2}$.

18. Find the equations of the tangents to the hyperbola $x^2 - 9y^2 = 9$ that are drawn from (3,2). Find the area of the triangle that these tangents from with their chord of contact.

19. An ellipse and a hyperbola have their principal axes along the coordinate axes and have a common foci separated by a distance $2\sqrt{13}$, the difference of their focal semi axes is equal to 4 . If the ratio of their eccentricities is 3/7. Find the equation of these curves.

20. An ellipse has eccentricity 1/2 and one focus at the point P(1/2,1). Its one directrix is the common tangent, nearer to the point P, to the circle $x^2 + y^2 = 1$ and the hyperbola $x^2 - y^2 = 1$. Find the equation of the ellipse in the standard form.

21. Prove that the part of the tangent at any point of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ intercepted between the point of contact and the transverse axis is a harmonic mean between the lengths of the perpendiculars drawn from the foci on the normal at the same point.

22. Find the length of the diameter of the ellipse $\frac{x^2}{25} + \frac{y^2}{9} = 1$ perpendicular to the asymptote of the hyperbola $\frac{x^2}{16} - \frac{y^2}{9} = 1$ passing through the first & third quadrants.

23. The tangent at P on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ meets one of the asymptote in Q. Show that the locus of the mid point of PQ is a similar hyperbola.

24. From any point of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, tangents are drawn to another hyperbola which has the same asymptotes. Show that the chord of contact cuts off a constant area from the asymptotes.

25. If two points P&Q on the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ whose centre is C be such that CP is perpendicular to CQ & $a<b$, then prove that

$\frac{1}{CP^2}+\frac{1}{CQ^2}=\frac{1}{a^2}-\frac{1}{b^2}$.

COMPREHENSION

The difference between the second degree curve and pair of asymptotes is constant. If second degree curve represented by a hyperbola $S=0$, then the equation of its asymptotes is $S+\lambda=0$ where λ is constant. Which will be a pair of straight lines, then we get λ. Then equation of asymptotes is $A\equiv S+\lambda=0$ and if equation of conjugate hyperbola of S represented by S_1, then A is the arithmetic mean of S and S_1.

26. The asymptotes of a hyperbola having centre at the point (1,2) are parallel to the lines $2x+3y=0$ and $3x+2y=0$. If the hyperbola passes through the point (5,3), then its equation is

(A) $(2x+3y-3)(3x+2y-5)=256$ (B) $(2x+3y-7)(3x+2y-8)=156$

(C) $(2x+3y-5)(3x+2y-3)=252$ (D) $(2x+3y-8)(3x+2y-7)=154$

27. A hyperbola passing through origin has $3x-4y-1=0$ and $4x-3y-6=0$ as its asymptotes. Then the equation of its transverse and conjugate axes are

(A) $x-y-5=0$ and $x+y+1=0$ (B) $x-y=0$ and $x+y+5=0$

(C) $x+y-5=0$ and $x-y-1=0$ (D) $x+y-1=0$ and $x-y-5=0$

28. The tangent at any point of a hyperbola $16x^2-25y^2=400$ cuts off a triangle from the asymptotes and that the portion of it intercepted between the asymptotes, then the area of this triangle is

(A) 10 sq unit (B) 20 sq unit (C) 30 sq unit (D) 40sq unit

MATRIX MATCH TYPE

29.

Column – I	**Column - II**
(A) The foci of the hyperbola $12x^2-4y^2-24x+32y-127=0$ are	(P) (10,5)
(B) The foci of the hyperbola $8x^2-y^2-64x+10y+71=0$ are	(Q) $(2,-2)$
(C) The foci of the hyperbola $9x^2-16y^2-36x+96y+36=0$ are	(R) $(-4,4)$
(S) $(-2,5)$	(T) (6,4)

EXERCISE - 4

| Level-I Previous Year | JEE Main

1. The locus of a point $P(\alpha, \beta)$ moving under the condition that the line $y = \alpha x + \beta$ is a tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is- **[AIEEE-2005]**

(A) an ellipse (B) a circle (C) a parabola (D) a hyperbola

2. For the hyperbola $\frac{x^2}{\cos^2 \alpha} - \frac{y^2}{\sin^2 \alpha} = 1$, which of the following remains constant when α varies?

(A) eccentricity (B) directrix **[AIEEE-2007]**

(C) abscissa of vertices (D) abscissa of foci

3. The eccentricity of the hyperbola whose length of the latus rectum is equal to 8 and the length of its conjugate axis is equal to half of the distance between its foci, is: **[JEE main 2016]**

(A) $\frac{4}{\sqrt{3}}$ (B) $\frac{2}{\sqrt{3}}$ (C) $\sqrt{3}$ (D) $\frac{4}{3}$

4. Tangents are drawn to the hyperbola $4x^2 - Y^2 = 36$ at the points P and Q. If the tangents intersect at the point T(0,3) then the area (in sq. units) of $\triangle$ PTQ is : **[JEE main 2018]**

(A) $36\sqrt{5}$ (B) $45\sqrt{5}$ (C) $54\sqrt{3}$ (D) $60\sqrt{3}$

EXERCISE - 4

| Level-II Previous Year | JEE Advanced

1. If a hyperbola passes through the focus of the ellipse $\frac{x^2}{25}+\frac{y^2}{16}=1$ and its transverse and conjugate axis coincides with the major and minor axis of the ellipse, and product of their eccentricities is 1 , then **[JEE 2006, 5]**

(A) equation of hyperbola $\frac{x^2}{9}-\frac{y^2}{16}=1$ (B) equation of hyperbola $\frac{x^2}{9}-\frac{x^2}{25}=1$

(C) focus of hyperbola (5,0) (D) focus of hyperbola is $(5\sqrt{3},0)$

Comprehension : (3 questions)

2. Let ABCD be a square of side length 2 units. C_2 is the circle through vertices A, B, C, D and C_1 is the circle touching all the sides of the square ABCD. L is a line through A **[JEE 2006, 5 + 5 + 5]**

(a) If P is a point on C_1 and Q is another point on C_2, then $\frac{PA^2+PB^2+PC^2+PD^2}{QA^2+QB^2+QC^2+QD^2}$ is equal to

(A) 0.75 (B) 1.25 (C) 1 (D) 0.5

(b) A circle touches the line L and the circle C_1 externally such that both the circles are on the same side of the line, then the locus of centre of the circle is

(A) ellipse (B) hyperbola (C) parabola (D) parts of straight line

(c) A line M through A is drawn parallel to BD. Point S moves such that its distances from the line BD and the vertex A are equal. If locus of S cuts. M at T_2 and T_3 and AC at T_1, then area of $\Delta T_1\ T_2\ T_3$ is

(A) 1/2 sq. units (B) 2/3 sq. units (C) 1 sq. unit (D) 2 sq. units

3. **(a)** A hyperbola, having the transverse axis of length 2 sin θ, is confocal with the ellipse $3x^2+4y^2=12$. Then its equation is **[JEE 2007, 3 + 6]**

(A) $x^2\text{cosec}^2\ \theta-y^2\sec^2\ \theta=1$ (B) $x^2\sec^2\ \theta-y^2\text{cosec}^2\ \theta=1$

(C) $x^2\sin^2\ \theta-y^2\cos^2\ \theta=1$ (D) $x^2\cos^2\ \theta-y^2\sin^2\ \theta=1$

(b) Match the statements in Column I with the properties in Column II.

Column – I	**Column - II**
(A) Two intersecting circles	(P) have a common tangent
(B) Two mutually external circles	(Q) have a common normal
(C) Two circles, one strictly	(R) do not have a inside the other common tangent
(D) Two branches of a hyperbola	(S) do not have a common normal

4. **(a)** Let a and b be non-zero real numbers. Then, the equation $(ax^2 + by^2 + c)(x^2 - 5xy + 6y^2) = 0$ represents **[JEE 2008, 3 + 3]**

(A) four straight lines, when $c = 0$ and a, b are of the same sign.

(B) two straight lines and a circle, when $a = b$, and c is of sign opposite to that of a.

(C) two straight lines and a hyperbola, when a and b are of the same sign and c is of sign opposite to that of a

(D) a circle and an ellipse, when a and b are of the same sign and c is of sign opposite to that of a.

(b) Consider a branch of the hyperbola, $x^2 - 2y^2 - 2\sqrt{2}x - 4\sqrt{2}y - 6 = 0$ with vertex at the point A. Let B be one of the end points of its latus rectum. If C is the focus of the hyperbola nearest to the point A, then the area of the triangle ABC is

(A) $1 - \sqrt{\frac{2}{3}}$ (B) $\sqrt{\frac{3}{2}} - 1$ (C) $1 + \sqrt{\frac{2}{3}}$ (D) $\sqrt{\frac{3}{2}} + 1$

5. Match the conics in column I with statements/expressions in Column II. **[JEE 2009]**

Column-I	**Column-II**
(A) Circle	(P) The locus of the point (h, k) for which the line $hx + ky = 1$ touches the circle $x^2 + y^2 = 4$
(B) Parabola	(Q) Points z in the complex plane satisfying $\lvert z + 2\rvert - \lvert z - 2\rvert = \pm 3$
(C) Ellipse	(R) Points of the conic have parametric representation $x = \sqrt{3}(1 - t^2/1 + t^2), y = 2t/1 + t^2$
(D) Hyperbola	(S) The eccentricity of the conic lies in the interval $1 < x < \infty$
	(T) Points z in the complex plane satisfying $\text{Re}\,(z + 1)^2 = \lvert z\rvert^2 + 1$

6. An ellipse intersects the hyperbola $2x^2 - 2y^2 = 1$ orthogonally. The eccentricity of the ellipse is reciprocal of that of the hyperbola. If the axes of the ellipse are along the coordinates axes, then

(A) equation of ellipse is $x^2 + 2y^2 = 2$ (B) the foci of ellipse are $(\pm 1, 0)$ **[JEE 2009]**

(C) equation of ellipse is $x^2 + 2y^2 = 4$ (D) the foci of ellipse are $(\pm\sqrt{2}, 0)$

Paragraph for Questions 7 to 8

The circle $x^2 + y^2 - 8x = 0$ and hyperbola $\frac{x^2}{9} - \frac{y^2}{4} = 1$ intersect at the points A and B.

[JEE 2010]

7. Equation of a common tangent with positive slope to the circle as well as to the hyperbola is

(A) $2x - \sqrt{5}y - 20 = 0$ (B) $2x - \sqrt{5}y + 4 = 0$

(C) $3x - 4y + 8 = 0$ (D) $4x - 3y + 4 = 0$

8. Equation of the circle with AB as its diameter is

(A) $x^2 + y^2 - 12x + 24 = 0$ (B) $x^2 + y^2 + 12x + 24 = 0$

(C) $x^2 + y^2 + 24x - 12 = 0$ (D) $x^2 + y^2 - 24x - 12 = 0$

9. The line $2x + y = 1$ is tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. If this line passes through the point of intersection of the nearest directrix and the x-axis, then the eccentricity of the hyperbola is

[JEE 2010]

10. Let the eccentricity of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ be reciprocal to that of the ellipse $x^2 + 4y^2 = 4$. If the hyperbola passes through a focus of the ellipse, then

(A) the equation of the hyperbola is $\frac{x^2}{3} - \frac{y^2}{2} = 1$

(B) a focus of the hyperbola is (2,0)

(C) the eccentricity of the hyperbola is $\sqrt{\frac{5}{3}}$

(D) the equation of the hyperbola is $x^2 - 3y^2 = 3$

11. Let P(6,3) be a point on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. If the normal at the point P intersects the x-axis at (9,0), then the eccentricity of the hyperbola is

(A) $\sqrt{\frac{5}{2}}$ (B) $\sqrt{\frac{3}{2}}$ (C) $\sqrt{2}$ (D) $\sqrt{3}$ **[JEE 2011]**

12. Consider the hyperbola H: $x^2 - y^2 = 1$ and a circle S with center $N(x_2, 0)$. Suppose that H and S touch each other at a point $P(x_1, y_1)$ with $x_1 > 1$ and $y_1 > 0$. The common tangent to H and S at P intersects the X-axis at point M. If (1, m) is the centroid of the triangle $\triangle$ PMN, then the correct expression(s) is(are) **[JEE 2015]**

(A) $\frac{d\ell}{dx_1} = 1 - \frac{1}{3x_1^2}$ for $x_1 > 1$

(B) $\frac{dm}{dx_1} = \frac{x}{3\left(\sqrt{x_1^2 - 1}\right)}$ for $x_1 > 1$

(C) $\frac{d\ell}{dx_1} = 1 + \frac{1}{3x_1^2}$ for $x_1 > 1$

(D) $\frac{dm}{dy_1} = \frac{1}{3}$ for $y_1 > 0$

13. Let H: $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, where $a > b > 0$, be a hyperbola in xy - plane whose conjugate axis LM subtends an angle of $60°$ at one of its vertices N. Let the area of the trignale LMN be $4\sqrt{3}$.

List – I	**List – II**
P. The length of the conjugate axis of H is	1. 8
Q. The eccentricity of H is	2. $\frac{4}{\sqrt{3}}$
R. The distance between the foci of H is	3. $\frac{2}{\sqrt{3}}$
S. The length of the latus rectum of H is	4. 4

The correct option is :

(A) $P \to 4; Q \to 2; R \to 1; S \to 3$ (B) $P \to 4; Q \to 3; R \to 1; S \to 2$

(C) $P \to 4; Q \to 1; R \to 3; S \to 2$ (D) $P \to 3; Q \to 4; R \to 2; S \to 1$

ANSWER KEY

EXERCISE - I JEE Main

1.	B	**2.**	D	**3.**	A	**4.**	C	**5.**	A	**6.**	B	**7.**	A
8.	A	**9.**	B	**10.**	A	**11.**	A	**12.**	D	**13.**	B	**14.**	B
15.	B	**16.**	A	**17.**	A	**18.**	B						

EXERCISE - II JEE Advance Single correct Option - type Questions

1.	C	**2.**	C	**3.**	B	**4.**	D	**5.**	D	**6.**	A	**7.**	D
8.	C	**9.**	D	**10.**	C	**11.**	C	**12.**	C	**13.**	A	**14.**	C
15.	A	**16.**	D	**17.**	C								

Multiple correct Option - type Questions

1.	A,D	**2.**	B,D	**3.**	A,B	**4.**	B,C	**5.**	A,D

EXERCISE - III Subjective - type Questions

1. $7x^2 + 12xy - 2y^2 - 2x + 4y - 7 = 0; \sqrt{\frac{48}{5}}$ **2.** $a^2 = 25/2; b^2 = 16$

4. $(-1,2); (4,2) \& (-6,2); 5x - 4 = 0 \& 5x + 14 = 0; \frac{32}{3}; 6; 8; y - 2 = 0$

$x + 1 = 0; 4x - 3y + 10 = 0; 4x + 3y - 2 = 0.$

7. $\frac{x^2}{a^4} + \frac{y^2}{b^4} = \frac{1}{a^2+b^2}$ **9.** $x + y \pm 3\sqrt{3} = 0$

10. $3x + 2y - 5 = 0; 3x - 2y + 5 = 0$

14. $(x^2 + y^2)^2(a^2y^2 - b^2x^2) = x^2y^2(a^2 + b^2)^2$ **18.** $y = \frac{5}{12}x + \frac{3}{4}; x - 3 = 0; 8$ sq. unit

19. $\frac{x^2}{49} + \frac{y^2}{36} = 1; \frac{x^2}{9} - \frac{y^2}{4} = 1$ **20.** $\frac{\left(x-\frac{1}{3}\right)^2}{1/9} + \frac{(y-1)^2}{1/12} = 1$

22. $\frac{150}{\sqrt{481}}$ **23.** $4\left(\frac{x^2}{a^2} - \frac{y^2}{b^2}\right) = 3$

Comprehension - based Questions

26.	D	**27.**	C	**28.**	B

Matrix Match - type Questions

29. (A) R,T ; (B) P,Q ; (C) Q

EXERCISE - IV Previous Year's Question JEE Main

1.	D	**2.**	D	**3.**	B	**4.**	B

JEE Advanced

1.	A, C	**2.**	**(a)** A, **(b)** C, **(c)** C,	**3.**	**(a)** A ; **(b)** (A)–P, Q; (B)–P, Q; (C)–Q, R; (D)–Q, R						
4.	(a) B ; (b) B			**5.**	(A)–P ; (B)– T ; (C)–R ; (D)–Q,S	**6.**	A,B				
7.	B	**8.**	A	**9.**	2	**10.**	B,D	**11.**	B	**12.**	A,B,D	**13.**	B