

165.

$$4 \cdot 3^x - 1 = 3^{2x+1} = 3 \cdot 3^{2x}$$

$$(t+4)(t+2) - 21(t-1)(t+2) + 10(t-1)(t+4) = 0$$

$$(3t-1)(t-1) = 0 \Rightarrow 3^x = 3^{-1}, 3^0$$

$$\log_2 x = 0 \Rightarrow \boxed{x=1}$$

$$\log_2 \left(\log_2 x + 2 \right)^2 + \left(2 \log_2 x - 3 \right) = 8$$

63.

$$\frac{2 \log_2 x}{\log_2 x - 1} - \frac{14(3 \log_2 x)}{\log_2 x + 4} + \frac{20 \log_2 x}{\log_2 x + 2} = 0$$

174

$$\left(\frac{4 \cdot 3^x - 6}{9^x - 6} \right) = 2$$

$$2 = \log_{10} 10^2$$

$$4t - 6 = 2(t^2 - 6)$$

75

$$\log_{10}(5x-4) + \log_{10}(\sqrt{x+1})^2 = \log_{10}(100 \times 0.18)$$

$$\frac{6}{2} \cdot 2 \cdot \frac{x-2}{4} - 1 = 5^{2x-4}$$

$$2t - 1 = t^2$$

$$(5x-4)(x+1) = 18$$

$$5x^2 + x - 22 = 0 = (5x+11)(x-2)$$

$$-10 \pm 11$$

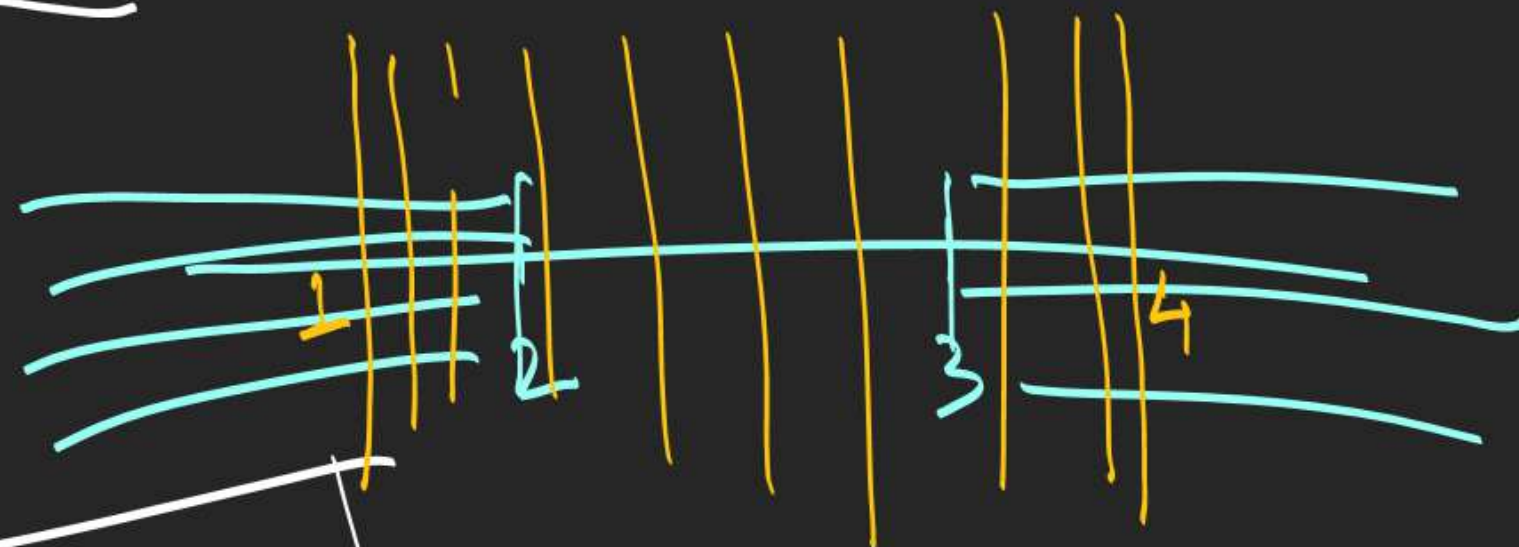
$$x = \left(-\frac{11}{5}, 2 \right)$$

reject

66. $0 < x^2 - 5x + 6 < 2$

$x \in (-\infty, 2) \cup (3, \infty)$

$x \in (1, 4)$



$x \in (1, 2) \cup (3, 4) \Rightarrow \text{Ans}$

$(x - \frac{5}{2})^2 + 7 - \frac{25}{4} > 0$

$0 < x^2 - 5x + 7 < 1$

$x \in (2, 3) \Rightarrow \text{Ans}$

$$4(\cos^3 \theta - \sin^3 \theta) - 3(\cos \theta - \sin \theta) = a$$

$$b \left(\frac{4(1 + \sin \theta \cos \theta) - 3}{1} \right) = a$$

$$b \left(1 + 2(1 - (\cos \theta - \sin \theta)^2) \right) = a$$

$$b(3 - 2b^2) = a$$

$$\left. \begin{aligned} 2 \sin\left(\frac{x+y}{2}\right) \cos\left(\frac{x-y}{2}\right) &= a \\ 2 \cos\left(\frac{x+y}{2}\right) \cos\left(\frac{x-y}{2}\right) &= b \end{aligned} \right\} \frac{a}{b} = \tan \frac{x+y}{2}$$

$$a^2 + b^2 = 4 \cos^2\left(\frac{x-y}{2}\right) = c$$

$$\frac{2 \sin(x+y)}{2 \cos x \cos y}$$

$$\frac{2 \sin(x+y)}{2 \cos x \cos y} = c$$

$$\frac{\cos(x+y) + \cos(x-y)}{2 \left(\frac{2 \frac{a}{b}}{1 + \frac{a^2}{b^2}} \right)} = c$$

$$\frac{1 - \frac{a^2}{b^2}}{1 + \frac{a^2}{b^2}}$$

Characteristic & Mantissa

$$\log_a b = I + F$$

I is an integer & $0 \leq F < 1$

$$\pi = 3 + (\pi - 3)$$

\downarrow \downarrow
 F F

$I = \text{Characteristic of } \log \text{ of } b \text{ to base } a$

$$-13.685 = -14 + 0.315$$

$F = \text{Mantissa of } \log \text{ of } b \text{ to base } a$

$$(-13)^+ \underbrace{(-0.685)}_{e(-1, 0)}$$

$$\log_a b = \underline{143} \cdot 8987$$

$$\text{Char.} = 143$$

$$\text{Mantissa} = 0.8987$$

$$\log_a b = -13.685 = \overline{14} \cdot 315$$

$$\text{Ch} = -14$$

$$\text{Mantissa} = 0.315$$

1. find N if characteristic of \log of N to

base 5 is 3 $N \in [125, 625)$

$$[3, 6]$$

$$\log_5 N = \text{Ch.} + \text{Mant.}$$

$$\log_5 N = 3 + \underline{M}$$

$$(\log_5 5^3 \leq \log_5 N < \log_5 5^4)$$

$$0 \leq M < 1$$

$$3 \leq \log_5 N < 4$$

$$127.012$$

no. of integral
values of N
 $= 625 - 125$

Use $\log_{10} 2 = 0.301$ and $\log_{10} 3 = 0.4771$,

find the number of digits in

(i) 6^{50}

(ii) $3^{12} \times 2^8$

$$10^{38} \times 1.12 \times 10$$

$$2.13 \times 10^{38}$$

Ans $\rightarrow 39$

$$N = 6^{50}$$

$$\log_{10} N = 50 \log_{10} 6 = 50 (0.301 + 0.4771) = 50 \times 0.7781$$

$$\log_{10} N = 38.905 = 38 + 0.905$$

$$0 < \downarrow < 1$$

$$1 < 10^{0.905} < 10$$

$$N = \left(\frac{10^{38}}{10} \right) \times 10^{0.905}$$

$$10^0 < 10^{0.905} < 10^1$$

$$1.2 \times 10^3 = 1200$$

$$8.96 \times 10^3 = 8960$$

$$9.99 \times 10^3 = 9890$$

$$11.2 \times 10^3 = 11200$$

$$\underline{8.292893 \times 10^3}$$

Fin ↓

$$N = 3^{12} 2^8$$

$$\log_{10} N = 12 \log_{10} 3 + 8 \log_{10} 2$$

$$= 8.1332$$

$$N = \boxed{10^8} \times \boxed{10^{0.1332}}$$

\downarrow
 $1 < \quad < 10$

9

2. Find the number of zeroes after decimal before a significant figure starts in

$$(i) \left(\frac{9}{8}\right)^{-100} = N$$

Ans \rightarrow 5

$$(ii) 3^{-50}$$

$$10^{-3} = [0.001]$$

$$\log_{10} N = -100(2\log_{10} 3 - 3\log_{10} 2) = -5.12 = -6 + 0.88$$

$$N = 10^{0.88} \times 10^{-6}$$

$1 < 10^{0.88} < 10$

0.000001

$$2.5 \times 10^{-3}$$

0.0025

$$N = 3^{-50}$$

$$\log_{10} N = -50 \log_{10} 3 = -50 (0.4771)$$

$$21.000003087$$

4

$$= -2.3855 \times 10$$

Ans → 23

Logarithm

Proficiency Test 1, 2

$$= -23.855$$

$$= -24 + 0.145$$

$$N = 10^{0.145} \times 10^{-24}$$