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1. Find the eqn. of common tangent to $y^2 = 8x$ and $3x^2 - y^2 = 3$.

$ty = x + 2t^2$

$y = \frac{1}{t}x + 2t$

$y = 2x + 1, \quad y = -2x - 1$

$4t^2 = 1\left(\frac{1}{t^2}\right) - 3$

$4t^4 + 3t^2 - 1 = 0$

$(4t^2 - 1)(t^2 + 1) = 0$

$t = \pm\frac{1}{2}$

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2. Find eqn. of tangent to $\frac{x^2}{36}-\frac{y^2}{9}=1$ passing thru $(0,4)$:

$$y = mx \pm \sqrt{m^2(36)-9}$$

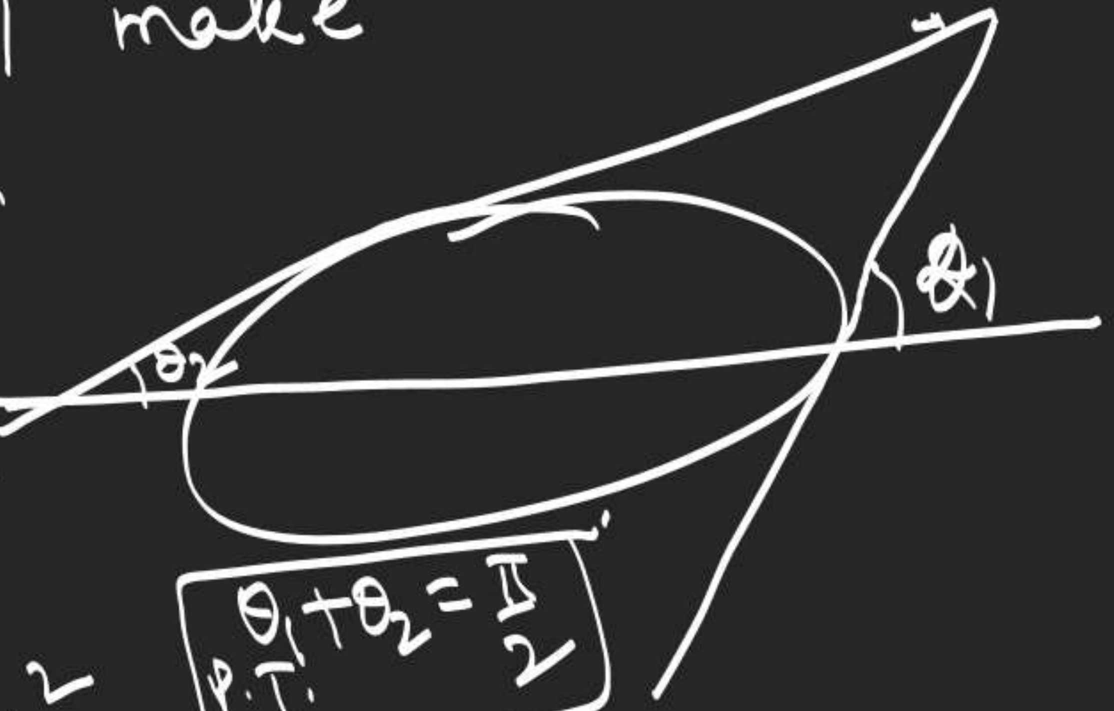
$$16 = 36m^2 - 9$$

$$m = \pm \frac{5}{6}$$

$$\boxed{\begin{aligned} y-4 &= \frac{5}{6}x \\ y-4 &= -\frac{5}{6}x \end{aligned}}$$

3: P.T. two tangents drawn from any point on the hyperbola $x^2-y^2=a^2-b^2$ to ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ make complementary angle with x-axis

(α,β) ✓ $\alpha^2-\beta^2=a^2-b^2$



$$y=mx\pm\sqrt{a^2m^2+b^2}$$

$$(\beta-m\alpha)^2=a^2m^2+b^2$$

P.T. $\boxed{\theta_1+\theta_2=\frac{\pi}{2}}$

$$m^2(\alpha^2-a^2)-2\alpha\beta m+\beta^2-b^2=0 \quad m_1,\ m_2$$

$$m_1m_2=\frac{\beta^2-b^2}{\alpha^2-a^2}=\frac{\alpha^2-a^2+b^2-b^2}{\alpha^2-a^2}=1$$

4. Find the eqn. and length of common tangent to hyperbolas $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ ✓ & $\frac{y^2}{a^2}-\frac{x^2}{b^2}=1$ ✓ $\rightarrow \frac{x^2}{(-b^2)}-\frac{y^2}{(-a^2)}=1$

$P(x_1,y_1)=\left(\frac{-a^2}{\sqrt{a^2-b^2}},\frac{-b^2}{\sqrt{a^2-b^2}}\right)$

$y=mx\pm\sqrt{a^2m^2-b^2}$

$a^2m^2-b^2=-b^2m^2-(-a^2)$

$\boxed{m=\pm 1}$

$y-x=\sqrt{a^2-b^2}$

$\frac{xx_1}{a^2}-\frac{yy_1}{b^2}=1$

$\frac{yy_2}{a^2}-\frac{xx_2}{b^2}=1$

$PQ=\frac{(a^2+b^2)\sqrt{2}}{\sqrt{a^2-b^2}}$

$\left(\frac{b^2}{\sqrt{a^2-b^2}},\frac{a^2}{\sqrt{a^2-b^2}}\right)=Q(x_2,y_2)$

$\frac{x_2}{b^2}=\frac{y_2}{a^2}=\frac{x_1}{-a^2}=\frac{y_1}{-b^2}=\frac{1}{\sqrt{a^2-b^2}}$

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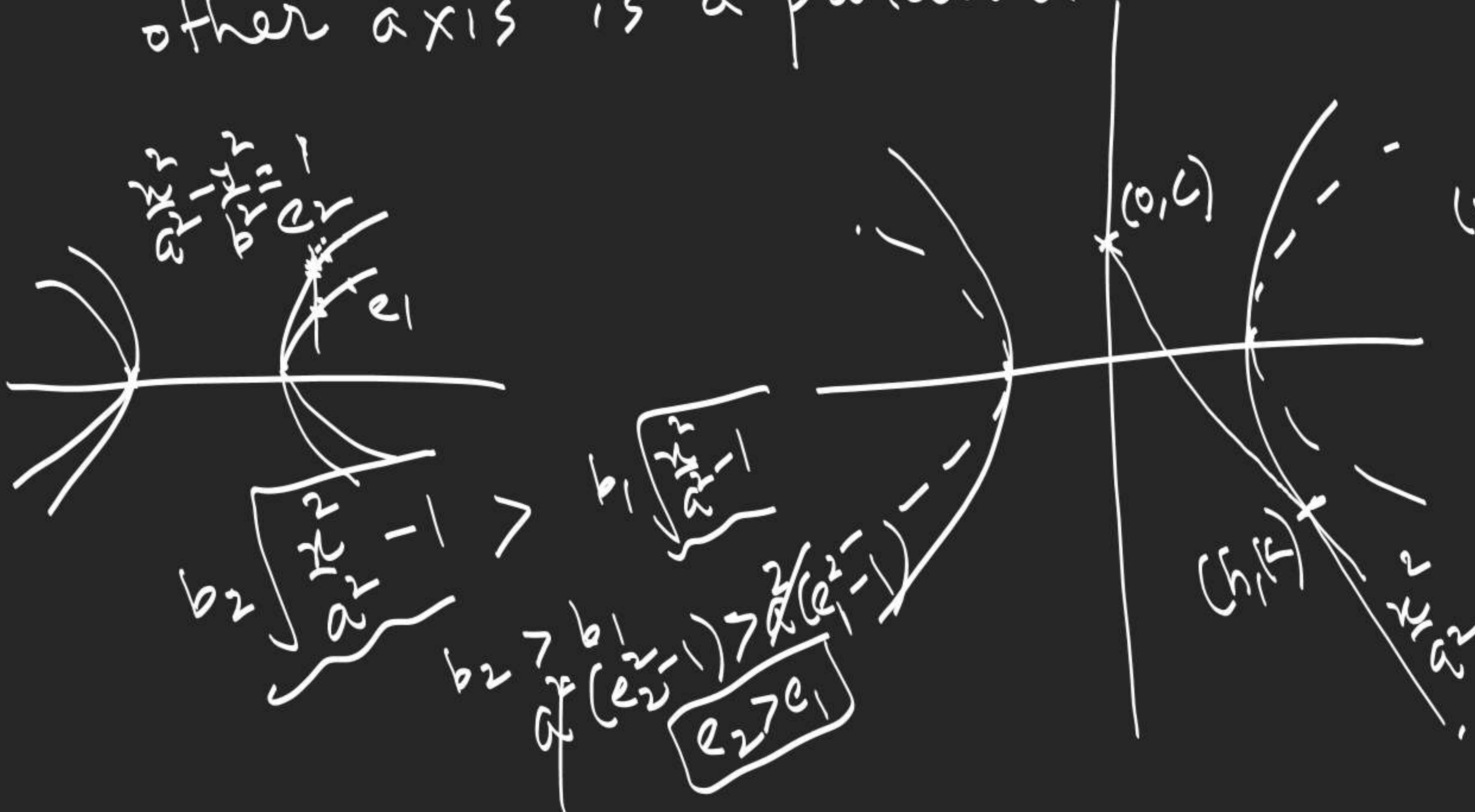
5: If one axis of varying hyperbola be fixed in magnitude and position. P.T. locus of point of contact of a tangent drawn to it from a fixed point on other axis is a parabola.



$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ e_2 e_1

$b_2\sqrt{\frac{x^2}{a^2}-1} > b_1\sqrt{\frac{x^2}{a^2}-1}$

$b_2 > b_1$

$a^2(e_2^2-1) > a^2(e_1^2-1)$

$\boxed{e_2 > e_1}$

$(0,c)$, (h,k)

$\frac{xh}{a^2}-\frac{yk}{b^2}=1$

$(0,c)$, $-\frac{kc}{b^2}=1$

$\frac{h^2}{a^2}-\frac{k^2}{b^2}=1$

$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

$$\boxed{\frac{h^2}{a^2}+\frac{k^2}{kc}=1}$$

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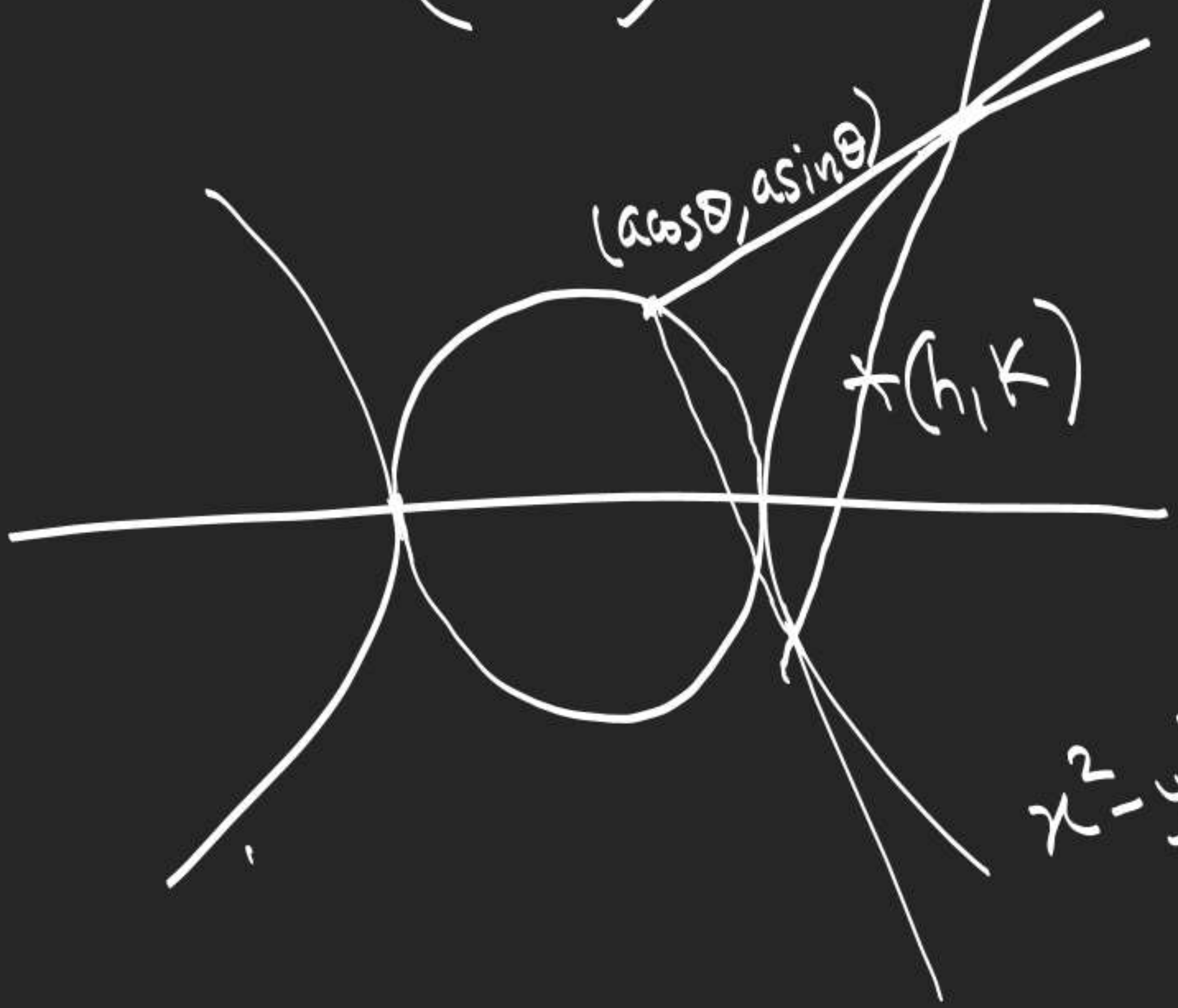
6: From points on circle $x^2+y^2=a^2$, tangents are drawn to hyperbola $x^2-y^2=a^2$. P.T. locus of middle points of chord of contact is the curve $(x^2-y^2)^2 = a^2(x^2+y^2)$.



$$x\cos\theta - y\sin\theta = a$$

$$xh - yk = h^2-k^2$$

$$\frac{\cos\theta}{h} = \frac{\sin\theta}{k} = \frac{a}{h^2-k^2}$$

$$x^2-y^2=a^2$$

$$\boxed{\frac{a^2}{(h^2-k^2)^2}(h^2+k^2)=1}$$

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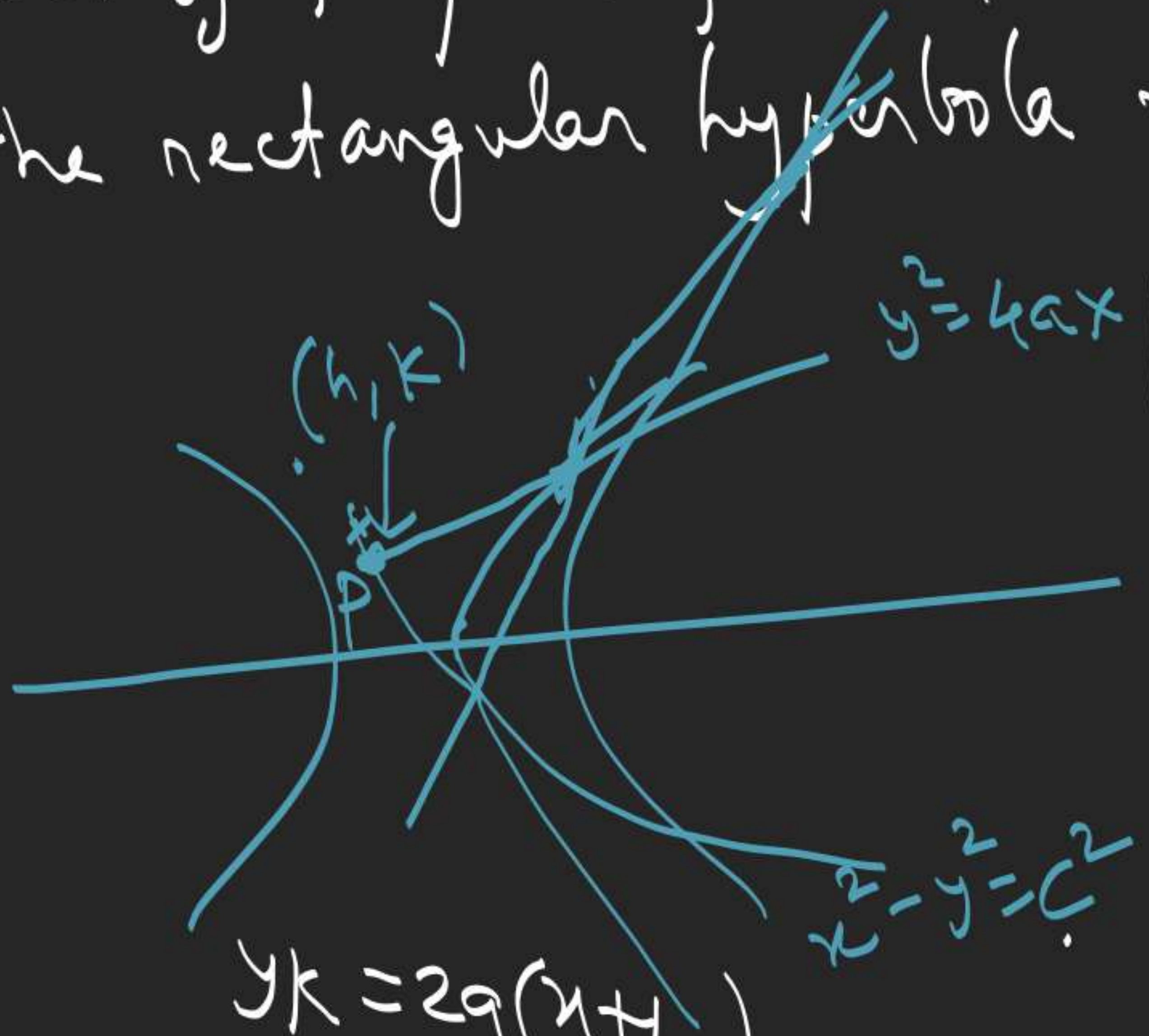
7. A point 'P' moves such that chord of contact of pair of tangents from P on parabola $y^2=4ax$ touches the rectangular hyperbola $x^2-y^2=c^2$. Find the locus of 'P'.



$yk = 2a(x+h)$

$y = \frac{2a}{k}x + \frac{2ah}{k}$

$$\boxed{\frac{4a^2h^2}{k^2} = c^2\left(\frac{4a^2}{k^2}-1\right)}$$

asymptote

$xy = c^2$

~~Ellipse~~ ← Ex-III (Complete)

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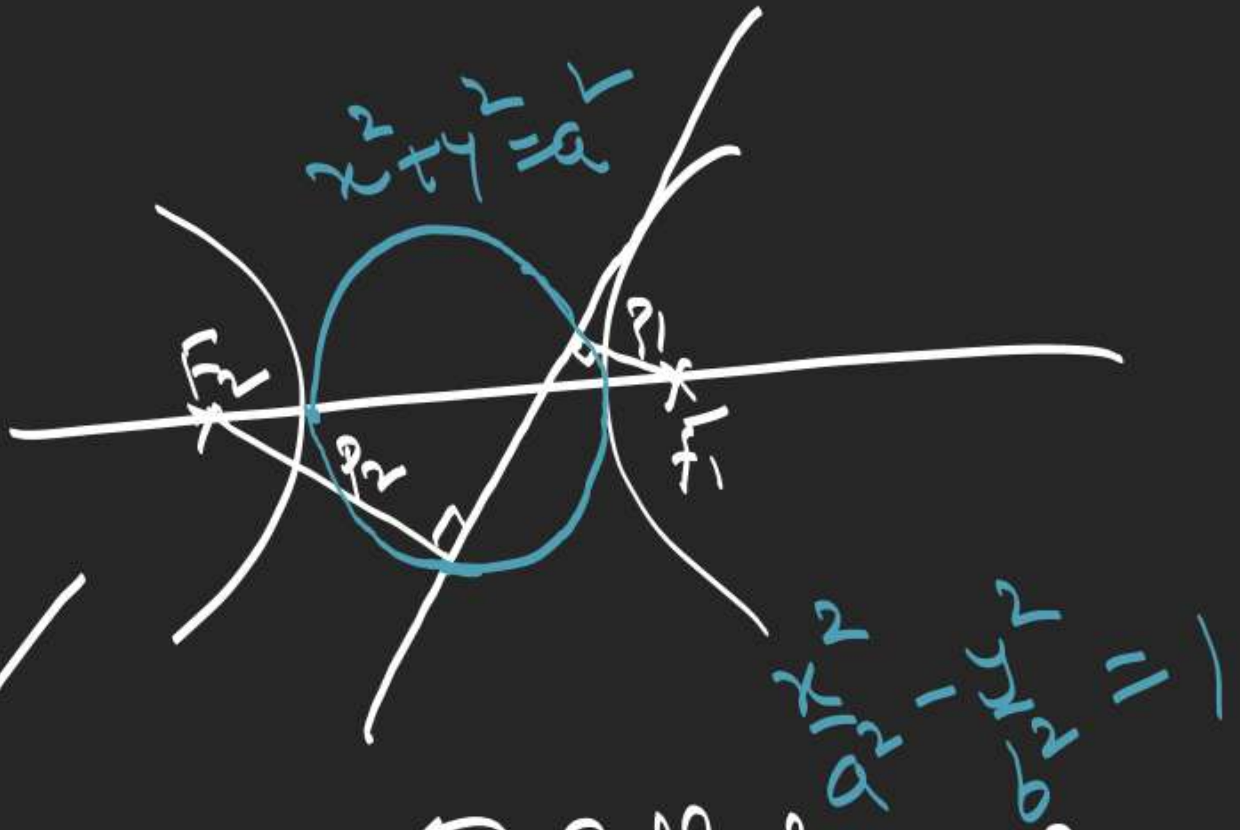
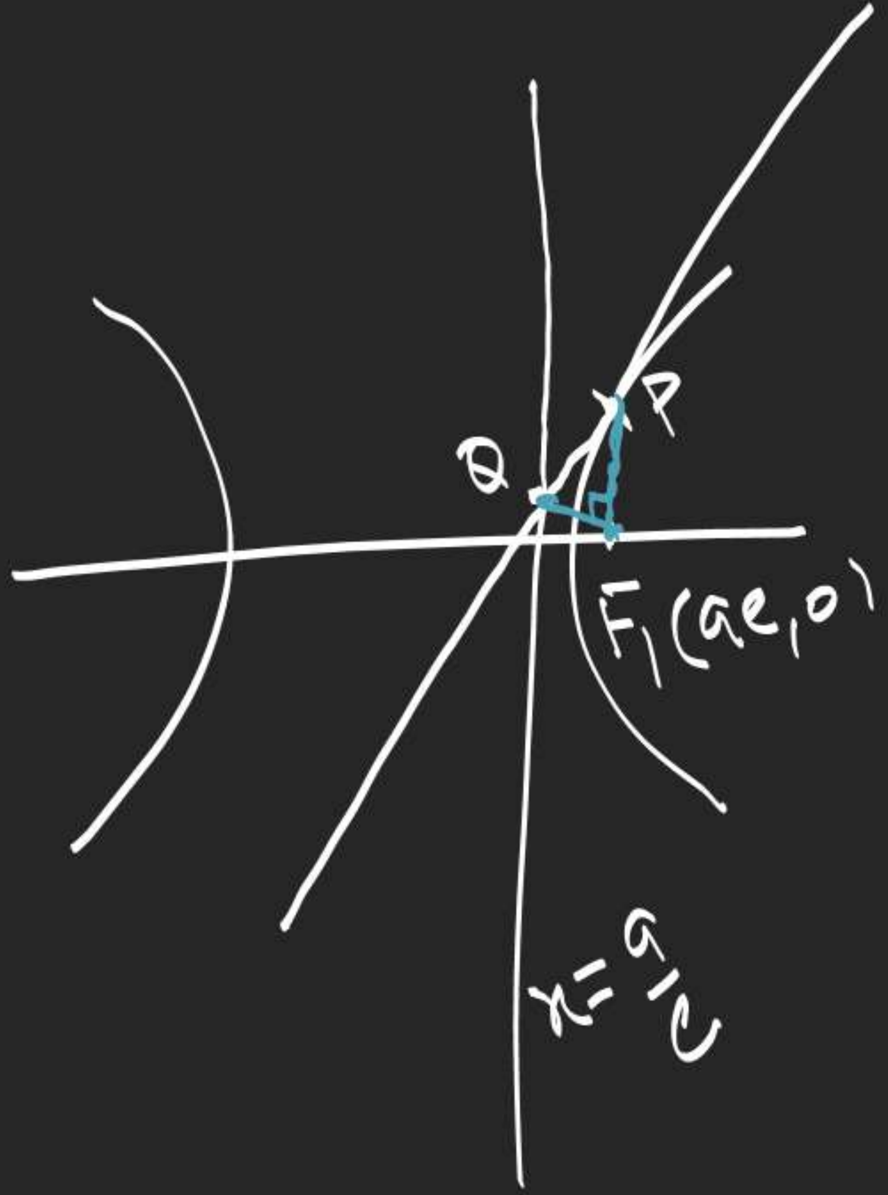
Note → ①

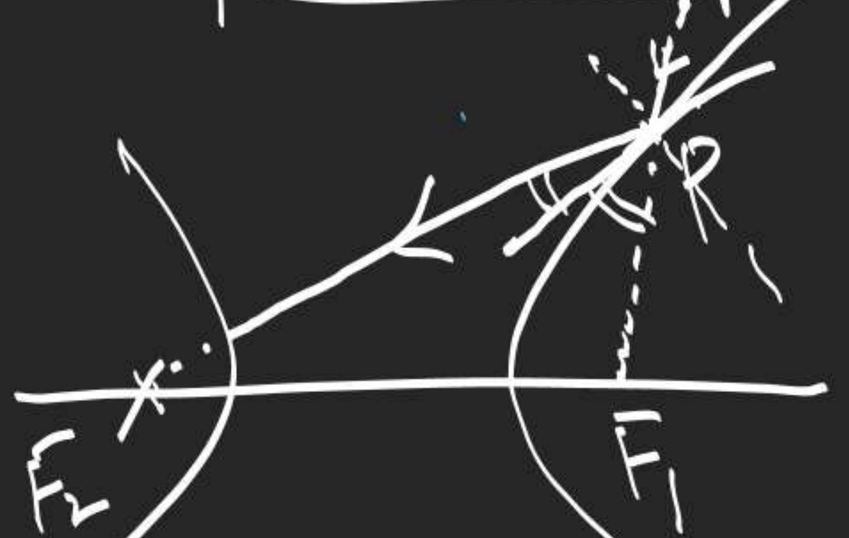
$P_1 P_2 = (\text{semi CA})^2$



②



③ Reflection Prop



Confocal Ellipse & Hyperbola are orthogonal