DPP-1

1. Let $g: R \to \left(0, \frac{\pi}{3}\right]$ is defined by $g(x) = \cos^{-1}\left(\frac{x^2-k}{1+x^2}\right)$.

Then the possible values of ' k ' for which g is surjective function, is

(A) $\left\{\frac{1}{2}\right\}$ (B) $\left(-1, -\frac{1}{2}\right]$ (C) $\left\{-\frac{1}{2}\right\}$ (D) $\left[-\frac{1}{2}, 1\right)$

2. Number of values of x satisfying the equation $\cos(3\text{arc}\cos(x-1)) = 0$ is equal to

(A) 0 (B) 1 (C) 2 (D) 3

3. If range of the function $f(x) = \tan^{-1}(3x^2 + bx + 3), x \in R$ is $\left[0, \frac{\pi}{2}\right)$, then square of sum of all possibl values of b will be

(A)0 (B) 18 (C) 72 (D) None of these

4. Range of the function $f(x) = \cos^{-1}\left(\frac{1}{e^x+e^{-x}}\right)$ is

(A) $(0, \pi)$ (B) $\left[\frac{\pi}{6}, \frac{\pi}{2}\right)$ (C) $\left[\frac{\pi}{3}, \frac{\pi}{2}\right)$ (D) $\left[\frac{\pi}{2}, \frac{2\pi}{3}\right)$

5. The number of ordered triplets (x, y, z) satisfy the equation $(\sin^{-1} x)^2 = \frac{\pi^2}{4} + (\sec^{-1} y)^2 + (\tan^{-1} z)^2$ is

(A) 2 (B) 4 (C) 6 (D) 8

6. If x_1, x_2 and x_3 are the positive roots of the equation $x^3 - 6x^2 + 3px - 2p = 0, p \in R$ then the value of $\sin^{-1}\left(\frac{1}{x_1} + \frac{1}{x_2}\right) + \cos^{-1}\left(\frac{1}{x_2} + \frac{1}{x_3}\right) - \tan^{-1}\left(\frac{1}{x_3} + \frac{1}{x_1}\right)$ is equal to

(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{2}$ (C) $\frac{3\pi}{4}$ (D) π

7. The domain of the function $f(x) = \frac{1}{\sqrt{\log_{\frac{\pi}{4}}(\sin^{-1} x) - 1}}$, is

(A) $\left(-1, \frac{1}{\sqrt{2}}\right)$ (B) $\left(0, \frac{1}{\sqrt{2}}\right)$ (C) $\left[0, \frac{1}{\sqrt{2}}\right)$ (D) $\left(-1, \frac{1}{\sqrt{2}}\right]$

8. If α and β are the two zeroes of the equation $3\cos^{-1}\left(x^2 - 5x - \frac{11}{2}\right) = \pi$, then $(\alpha^3 + \beta^3)$ equals

(A) 255 (B) 215 (C) -215 (D) -217

9. Let $\alpha = \arcsin\frac{\sqrt{63}}{8}$ then the value of $\sin^2\left(\frac{\alpha}{4}\right)$ is

(A) $\frac{1}{2}$ (B) $\frac{1}{4}$ (C) $\frac{1}{8}$ (D) $\frac{1}{16}$

10. Number of values of x satisfying the equation $\cos\left(\frac{4\pi}{3} - \cos^{-1} x\right) = x$, is

(A) 4 (B) 3 (C) 2 (D) 1

DPP-1

Answer Key

1.	(C)	**2.**	(D)	**3.**	(A)	**4.**	(C)	**5.**	(A)	**6.**	(A)	**7.**	(B)
8.	(B)	**9.**	(C)	**10.**	(D)								

DPP-2

1. Let $f: R \to \left[\frac{\pi}{6}, \frac{\pi}{4}\right)$ defined by $f(x) = \tan^{-1}\left(\frac{x^2+1}{x^2+\sqrt{3}}\right)$. Then f(x) is

(A) injective and subjective (B) injective but not subjective

(C) subjective but not injective (D) neither injective nor subjective

2. The value of $3\sin\left(\frac{1}{2}\arccos\frac{1}{9}\right) + 4\cos\left(\frac{1}{2}\arccos\frac{1}{8}\right)$ is equal to

(A) 5 (B) 4 (C) 1 (D) 0

3. The true set of values of p for which the equation $\cos^{-1}\left(\frac{1}{1+\cos^2 x}\right) = \frac{p\pi}{3}$ have a solution is

(A) [0,1] (B) [0,2] (C) [1,2] (D) $\left[1, \frac{3}{2}\right]$

4. The value of $p \in R$ for which the equation

$\sin^{-1}((\log_{10} x)^2 - 2\log_{10} x + 2) + \tan^{-1}((\log_{10} x)^2 - 2\log_{10} x + 2)$

$+ \cos^{-1}\left((\log_{10} x)^2 - 2(\log_{10} x)\right) = p,$

possess solution is

(A) $\frac{5\pi}{4}$ (B) $\frac{3\pi}{4}$ (C) $\frac{3\pi}{2}$ (D) $\frac{7\pi}{4}$

5. The domain of definition of f(x) is

(A) $[-1,1]$ (B) $[\sin 1, 1]$ (C*) $[-1, \sin 1]$ (D) $[-1,0]$

6. The range of f(x) is

(A) $[0, \sqrt{3}]$ (B) $[1, \sqrt{3}]$ (C) $[1, \sqrt{6}]$ (D*) $[\sqrt{3}, \sqrt{6}]$

7. Statement-1 : The number of solution of the equations $\sec^{-1}\left(\sqrt{-x^2 + 6x - 8}\right) = \sin^{-1}\left(\frac{3-x}{4}\right)$ is exactly one.

Statement-2 : The domain of $\sec^{-1} x$ is $(-\infty, -1] \cup [1, \infty)$

(A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.

(B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1.

(C) Statement-1 is true, statement-2 is false.

(D) Statement-1 is false, statement-2 is true.

8. Let function f(x) be defined as $f(x) = |\sin^{-1} x| + \cos^{-1}\left(\frac{1}{x}\right)$. Then which of the following is/are TRUE?

(A) f(x) is injective in its domain.

(B) f(x) is many-one in its domain.

(C) Range of f is a singleton set.

(D) $\operatorname{sgn}(f(x)) = 1$ where $\operatorname{sgn} x$ denotes signum function of x.

9. Column I contains functions and column II contains their range. Match the entries of column I with the entries of column II.

Column-I	**Column-II**
(A) $f(x) = \sin^{-1}\left(\frac{x}{1+\lvert x\rvert}\right)$	(P) $(0, \pi)$
(B) $g(x) = \cos^{-1}\left(\frac{x}{1+\lvert x\rvert}\right)$	(Q) $\left(\frac{\pi}{4}, \frac{3\pi}{4}\right)$
(C) $h(x) = \tan^{-1}\left(\frac{x}{1+\lvert x\rvert}\right)$	(R) $\left(-\frac{\pi}{4}, \frac{\pi}{4}\right)$
(D) $k(x) = \cot^{-1}\left(\frac{x}{1+\lvert x\rvert}\right)$	(S) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

10. If the range of function $f(x) = \left(\pi\sqrt{2} + \cos^{-1}\alpha\right)x^2 + 2(\cos^{-1}\beta)x + \pi\sqrt{2} - \cos^{-1}\alpha$ is $[0, \infty)$ then find the value of $|\alpha - \beta| + 2\alpha\beta + 1$.

DPP-2

Answer Key

1.	(C)	**2.**	(A)	**3.**	(A)	**4.**	(D)	**5.**	(C)	**6.**	(D)	**7.**	(A)
8.	(A, D)	**9.**	(A) S, (B) P, (C) R, (D)Q			**10.**	(0003)						

DPP-3

1. The value of $\sec\left[\sin^{-1}\left(-\sin\frac{50\pi}{9}\right)+\cos^{-1}\cos\left(-\frac{31\pi}{9}\right)\right]$ is equal to

(A) $\sec\frac{10\pi}{9}$ (B) $\sec\frac{\pi}{9}$ (C) 1 (D) -1

2. If $f(x)=x^{11}+x^{9}-x^{7}+x^{3}+1$ and $f(\sin^{-1}(\sin 8))=\alpha$, α is constant, then $f(\tan^{-1}(\tan 8))$ is equal to

(A) α (B) $\alpha-2$ (C) $\alpha+2$ (D) $2-\alpha$

3. There exists a positive real number x satisfying $\cos(\tan^{-1}x)=x$. The value of $\cos^{-1}\left(\frac{x^2}{2}\right)$ is

(A) $\frac{\pi}{10}$ (B) $\frac{\pi}{5}$ (C) $\frac{2\pi}{5}$ (D) $\frac{4\pi}{5}$

4. The range of values of p for which the equation $\sin\cos^{-1}\left(\cos(\tan^{-1}x)\right)=p$ has a solution is:

(A) $\left(-\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}\right]$ (B) $[0,1)$ (C) $\left[\frac{1}{\sqrt{2}},1\right)$ (D) $(-1,1)$

5. The range of function $f(x)=\log_2\left(\frac{\pi+2\sin^{-1}\left(\frac{3-x}{7}\right)}{\pi}\right)$ is equal to

(A) $(-\infty,1)$ (B) $(1,\infty)$ (C) $(-\infty,1]$ (D) $[1,\infty)$

6. If the equation $x^3+ax^2+bx+216=0$ has three real roots in G.P., then the value of $\tan^{-1}\left(\tan\frac{b}{a}\right)$ is equal to

(A) $2\pi-5$ (B) $2\pi-6$ (C) $5-2\pi$ (D) $6-2\pi$

7. The product of all real values of x satisfying the equation

$$\cos^{-1}\left(\cos\left(\frac{2x^2+10|x|+4}{x^2+5|x|+3}\right)\right)+\cot\left(\cot^{-1}\left(\frac{2}{9|x|}-2\right)\right)=0 \text{ is}$$

(A) 9 (B) -9 (C) -3 (D) -1

8. Which of the following is/are correct?

(A) $\cos\left(\cos(\cos^{-1}1)\right)<\sin(\sin^{-1}(\sin(\pi-1))<\sin(\cos^{-1}(\cos(2\pi-2))$

(B)$\cos(\cos(\cos^{-1}1))<\sin(\cos^{-1}(\cos(2\pi-2)))<\sin(\sin^{-1}(\sin(\pi-1)))<$
$\tan(\cot^{-1}(\cot 1))$

(C)$\sum_{t=1}^{5000}\cos^{-1}(\cos(2t\pi-1))=\sum_{t=1}^{2500}\cot^{-1}(\cot(t\pi+2))$ where $t\in I$

(D) $\cot^{-1}\cot\ \mathrm{cosec}^{-1}\mathrm{cosec}\ \sec^{-1}\sec\ \tan\ \tan^{-1}\cos\ \cos^{-1}\sin^{-1}\sin 4=4-\pi$

9. If $x\in\left(0,\frac{\pi}{2}\right)$ satisfies the inequality $|\tan x-\sqrt{3}|+|4\sin^2 x-3|+\left|\tan(\tan^{-1}x)-\frac{\pi}{3}\right|\le 0$, then find the value of $\left[\tan\left(\cot^{-1}\left(\frac{\sqrt{2}}{30x}\cos\left(\frac{3x}{4}\right)\right)\right)\right]$.

[Note: [·] denotes greatest integer function.]

10. If all the roots of the equation $x^3-3x=0$ satisfy the equation $(\alpha-\sin^{-1}(\sin 2))x^2-(\beta-\tan^{-1}(\tan 1))x+\gamma^2-2\gamma+1=0$, then find the value of $|\cot(\beta+\gamma)+\cot\alpha|$.

DPP-3

Answer Key

1.	(D)	**2.**	(D)	**3.**	(C)	**4.**	(B)	**5.**	(C)	**6.**	(D)	**7.**	(A)
8.	(A, C, D)	**9.**	(0031)	**10.**	(0)								

DPP–4

1. If the equation $5\text{arc}\tan(x^2+x+k)+3\text{arc}\cot(x^2+x+k)=2\pi$, has two distinct solutions, then the range of k, is

(A) $\left(0,\frac{5}{4}\right]$ (B) $\left(-\infty,\frac{5}{4}\right)$ (C) $\left(\frac{5}{4},\infty\right)$ (D) $\left(-\infty,\frac{5}{4}\right]$

2. If $\sin^{-1}\left(x^2-\frac{x^4}{3}+\frac{x^6}{9}\ldots\ldots\ldots.\right)+\cos^{-1}\left(x^4-\frac{x^8}{3}+\frac{x^{12}}{9}\ldots\ldots\ldots\right)=\frac{\pi}{2}$, where $0\le|x|<\sqrt{3}$, then number of values of ' x ' is equal to

(A) 1 (B) 2 (C) 3 (D) 4

3. A value of α for which $\sin(\cot^{-1}(1+\alpha))=\cos(\tan^{-1}\alpha)$, is

(A) $\frac{-1}{2}$ (B) 0 (C) $\frac{1}{2}$ (D) 1

4. If maximum value of $(\sin^{-1}x)^2+(\cos^{-1}x)^2$ is equal to $\frac{a\pi^2}{b}$ (a and b are coprime), then $(a+b)$ equals

(A) 1 (B) 5 (C) 4 (D) 9

5. The value of $\tan^{-1}\left(\frac{1}{2}\tan 2A\right)+\tan^{-1}(\cot A)+\tan^{-1}(\cot^3 A)$ for $0<A<(\pi/4)$ is

(A) $4\tan^{-1}(1)$ (B) $2\tan^{-1}(2)$ (C) 0 (D) none

6. The value of the angle $\tan^{-1}(\tan 65^\circ-2\tan 40^\circ)$ in degrees is equal to

(A) -20° (B) 20° (C) 25° (D) 40°

7. The value of α so that $\sin^{-1}\frac{2}{\sqrt{5}},\sin^{-1}\frac{3}{\sqrt{10}},\sin^{-1}\alpha$ are the angles of a triangle is

(A) $\frac{-1}{\sqrt{2}}$ (B) $\frac{1}{2}$ (C) $\frac{1}{\sqrt{2}}$ (D) $\frac{1}{\sqrt{3}}$

8. If $\alpha,\beta(\alpha>\beta)$ are the two solutions of the equation $\tan^{-1}x+\cot^{-1}(-|x|)=2\tan^{-1}(6x)$ then $(2\alpha+3\beta)$ is equal to

(A) 4 (B) 3 (C) 2 (D) 1

9. The value of $\tan^{-1}\left(\sec\left(\cos^{-1}\left(\sin\frac{\alpha}{2}\right)\right)-1\right)$ is equal to

(A*) $\frac{\alpha}{4}$ (B) $\frac{\alpha}{2}$ (C) $\frac{\beta}{2}$ (D) $\frac{\beta}{4}$

10. The range of function $f(x)=\cot^{-1}(x^2+bx)$ is equal to

(A) $\left(0,\frac{\pi}{4}\right]$ (B) $(0,\pi)$ (C) $\left(0,\frac{3\pi}{4}\right]$ (D) $\left[\frac{3\pi}{4},\pi\right)$

11. The number of solutions(s) of the equation $|b|\sin^{-1}x=(a-b)x$, is equal to

(A) 0 (B) 1 (C) 2 (D) 3

DPP-4

Answer Key

1.	(B)	**2.**	(C)	**3.**	(A)	**4.**	(D)	**5.**	(A)	**6.**	(C)	**7.**	(C)
8.	(C)	**9.**	(A)	**10.**	(C)	**11.**	(D)						

DPP-5

1. The sum $\sum_{n=1}^{\infty}\tan^{-1}\left(\frac{4n}{n^4-2n^2+2}\right)$ is equal to

(A) $\tan^{-1}\frac{1}{2}+\tan^{-1}\frac{2}{3}$ (B) $4\tan^{-1}1$

(C) $\frac{\pi}{2}$ (D) $\sec^{-1}(-\sqrt{2})$

2. If $\tan\left(\sum_{r=1}^{n}\tan^{-1}\left(\frac{2r-1}{(r^2+r+1)(r^2-r+1)-2r^3}\right)\right)=961$ then the value of n is equal to

(A) 31 (B) 30 (C) 60 (D) 61

3. If the solution set of inequality $(\operatorname{cosec}^{-1}x)^2-2\operatorname{cosec}^{-1}x\geq\frac{\pi}{6}(\operatorname{cosec}^{-1}x-2)$ is $(-\infty,m]\cup[n,\infty)$ then $(m+n)$ equals

(A) 0 (B) 1 (C) 2 (D) -3

4. If $\alpha=\sin\left(\frac{\sin^{-1}\frac{1}{\sqrt{3}}}{3}\right),\beta=\cos\left(\cos^{-1}\left(\frac{1}{\sqrt{5}}\right)-\sin^{-1}\left(\frac{2}{\sqrt{5}}\right)\right)$ then $\frac{\beta^2}{(3\alpha-4\alpha^3)^2}$ is equal to

(A) 1 (B) 2 (C) 3 (D) 4

5. The set of values of x, satisfying the equation $\tan^2(\sin^{-1}x)>1$ is

(A) $[-1,1]$ (B) $\left[-\frac{\sqrt{2}}{2},\frac{\sqrt{2}}{2}\right]$

(C) $(-1,1)-\left[-\frac{\sqrt{2}}{2},\frac{\sqrt{2}}{2}\right]$ (D) $[-1,1]-\left(-\frac{\sqrt{2}}{2},\frac{\sqrt{2}}{2}\right)$

6. The values of x satisfying the equation $2\tan^{-1}(3x)=\sin^{-1}\left(\frac{6x}{1+9x^2}\right)$ is equal to

(A) $\left[\frac{1}{3},\infty\right)$ (B) $\left(-\infty,\frac{-1}{3}\right]$ (C) $\left[\frac{-1}{3},\frac{1}{3}\right]$ (D) $[-1,1]$

7. The number of solution of the equation $2\sin^{-1}\left(\frac{2x}{1+x^2}\right)-\pi x^3=0$ is equal to

(A) 0 (B) 1 (C) 2 (D) 3

8. Find the number of solutions of the equation $\tan\left(\sum_{r=1}^{5}\cot^{-1}(2r^2)\right)=\frac{5x+6}{6x+5}$.

9. If $\sin(30^\circ+\arctan x)=\frac{13}{14}$ and $0<x<1$, the value of x is $\frac{a\sqrt{3}}{b}$, where a and b are positive integers with no common factors. Find the value of $\left(\frac{a+b}{2}\right)$.

10. Consider $f(x)=\cos^{-1}x+\cos^{-1}\left(\frac{\sqrt{3}x+\sqrt{1-x^2}}{2}\right)$. If $\sum_{r=1}^{100}f\left(1-\left(\frac{1}{10}\right)^r\right)=\frac{p}{q}\pi$ where p and q are relatively prime number, then find the value of $(p-16q)$.

DPP-5

Answer Key

1.	(D)	**2.**	(A)	**3.**	(B)	**4.**	(C)	**5.**	(C)	**6.**	(C)	**7.**	(D)
8.	(0)	**9.**	(8)	**10.**	(0002)								

EXERCISE-1

1. Find the domain of definition the following functions.

(i) $f(x) = \arccos \frac{2x}{1+x}$

(ii) $f(x) = \sqrt{\cos(\sin x)} + \sin^{-1} \frac{1+x^2}{2x}$

(iii) $f(x) = \sin^{-1}(2x + x^2)$

(iv) $f(x) = \sqrt{3-x} + \cos^{-1}\left(\frac{3-2x}{5}\right) + \log_6(2|x| - 3) + \sin^{-1}(\log_2 x)$

(v) $f(x) = \log_{10}\left(1 - \log_7(x^2 - 5x + 13)\right) + \cos^{-1}\left(\frac{3}{2+\sin\frac{9\pi x}{2}}\right)$

2. Identify the pair(s) of functions which are identical. Also plot the graphs in each case.

(a) $y = \tan(\cos^{-1} x); y = \frac{\sqrt{1-x^2}}{x}$ (b) $y = \tan(\cot^{-1} x); y = \frac{1}{x}$

(c) $y = \sin(\arctan x); y = \frac{x}{\sqrt{1+x^2}}$ (d) $y = \cos(\arctan x); y = \sin(\text{arccot } x)$

3. Let $y = \sin^{-1}(\sin 8) - \tan^{-1}(\tan 10) + \cos^{-1}(\cos 12) - \sec^{-1}(\sec 9) + \cot^{-1}$

$(\cot 6) - \text{cosec}^{-1}(\text{cosec } 7)$. If y simplifies to $a\pi + b$ then find $(a - b)$.

4. Show that: $\sin^{-1}\left(\sin\frac{33\pi}{7}\right) + \cos^{-1}\left(\cos\frac{46\pi}{7}\right) + \tan^{-1}\left(-\tan\frac{13\pi}{8}\right) + \cot^{-1}\left(\cot\left(-\frac{19\pi}{8}\right)\right) = \frac{13\pi}{7}$

5. (i) If α and β are the roots of the equation $x^2 + 5x - 49 = 0$ then find the value of $\cot(\cot^{-1}\alpha + \cot^{-1}\beta)$.

(ii) If $a > b > c > 0$ then find the value of : $\cot^{-1}\left(\frac{ab+1}{a-b}\right) + \cot^{-1}\left(\frac{bc+1}{b-c}\right) + \cot^{-1}\left(\frac{ca+1}{c-a}\right)$.

6. Find all values of k for which there is a triangle whose angles have measure

$\tan^{-1}\left(\frac{1}{2}\right), \tan^{-1}\left(\frac{1}{2} + k\right)$, and $\tan^{-1}\left(\frac{1}{2} + 2k\right)$

7. Find the simplest value of

(a) $f(x) = \arccos x + \arccos\left(\frac{x}{2} + \frac{1}{2}\sqrt{3 - 3x^2}\right), x \in \left(\frac{1}{2}, 1\right)$

(b) $f(x) = \tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right), x \in R - \{0\}$

8. (a) Let $f(x) = \cot^{-1}(x^2 + 4x + \alpha^2 - \alpha)$ be a function defined $R \to (0, \pi/2]$ then find the complete set of real values of α for which $f(x)$ is onto.

(b) Let $f: R \to \left(0, \frac{3\pi}{4}\right]$ be defined as $f(x) = \cot^{-1}(x^2 + x + a)$. If $f(x)$ is surjective, then find the range of a.

9. (i) Find the sum of the series:

(a) $\tan^{-1}\frac{1}{3} + \tan^{-1}\frac{2}{9} + \cdots .. + \tan^{-1}\frac{2^{n-1}}{1+2^{2n-1}} + \cdots .. \infty$

(b) $\tan^{-1}\frac{1}{x^2+x+1} + \tan^{-1}\frac{1}{x^2+3x+3} + \tan^{-1}\frac{1}{x^2+5x+7} + \tan^{-1}\frac{1}{x^2+7x+13}$ to n terms. where $x > 0$

(ii) If the sum $\sum_{n=1}^{10}\sum_{m=1}^{10}\tan^{-1}\left(\frac{m}{n}\right) = k\pi$, find the value of k.

10. Solve the following equations:

(a) $\sin^{-1} x + \sin^{-1} 2x = \frac{\pi}{3}$

(b) $\tan^{-1}(x-1) + \tan^{-1}(x) + \tan^{-1}(x+1) = \tan^{-1}(3x)$

(c) $\tan^{-1}\frac{x-1}{x+1} + \tan^{-1}\frac{2x-1}{2x+1} = \tan^{-1}\frac{23}{36}$

(d) $\cos^{-1}\frac{x^2-1}{x^2+1} + \tan^{-1}\frac{2x}{x^2-1} = \frac{2\pi}{3}$

11. Let $f(x) = ax^2 + 2bx + c - 1, a, b, c \in R$ such that $f(1) = f(-1) = f(2) = 4$.

If $g(x) = \sin^{-1}\left(\frac{-1+x^2}{1+x^2}\right) - 2\tan^{-1} x$, then find the value of $|g(f(10)) + g(f(-10))|$.

12. Consider the functions $f(x) = \sin^{-1}\left(\frac{2x}{1+x^2}\right), g(x) = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ and $h(x) = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$.

(a) If $x \in (-1,1)$, then find the solution of the equation $f(x) + g(x) + h(x) = \frac{\pi}{2}$.

(b) Find the value of $f(2) + g(2) + h(2)$.

13. Solve the following inequalities

(a) $\text{arc } \cot^2 x - 5\text{arc } \cot x + 6 > 0$

(b) $\text{arc } \sin x > \arccos x$

(c) $\tan^2(\arcsin x) > 1$

14. Consider $f(x) = \cot^{-1}\left(\frac{1+\sin 2x+\cos 2x}{1+\sin 2x-\cos 2x}\right)$ and $\sum_{r=1}^{5} f(r) = 5a - b\pi$, where $a, b \in N$, find $(a + b)$.

15. Let $f(x) = x^2 - 2ax + a - 2$ and $g(x) = \left[2 + \sin^{-1}\frac{2x}{1+x^2}\right]$. If the set of real values of 'a' for which $f(g(x)) < 0 \forall x \in R$ is (k_1, k_2) then find the value of $(10k_1 + 3k_2)$.

[Note : [k] denotes greatest integer less than or equal to k.]

EXERCISE-1

Answer Key

1. (i) $-1/3 \le x \le 1$ (ii) $\{1, -1\}$ (iii) $[-(1+\sqrt{2}), (\sqrt{2}, -1)]$
(iv) $(3/2, 2]$ (v) $\{7/3, 25/9\}$

2. (a), (b), (c) and (d) all are identical. **3.** (53) **4.** $\frac{13\pi}{7}$

5. (i) 10 ; (ii) π **6.** $k = \frac{11}{4}$ **7.** (a) $\frac{\pi}{3}$; (b) $\frac{\tan^{-1} x}{2}$

8. (a) $\frac{1 \pm \sqrt{17}}{2}$; (b) $\left\{\frac{-3}{4}\right\}$

9. (i) (a) $\frac{\pi}{4}$, (b) $\arctan(x+n) - \arctan x$; (ii) 25 ;

10. (a) $x = \frac{1}{2}\sqrt{\frac{3}{7}}$; (b) $x = 0, \frac{1}{2}, -\frac{1}{2}$; (c) $x = \frac{4}{3}$; (d) $x = 2 - \sqrt{3}$ or $\sqrt{3}$

11. $-\pi$ **12.** (a) $2 - \sqrt{3}$; (b) $\cot^{-1}\left(\frac{-3}{4}\right)$

13. (a) $(\cot 2, \infty) \cup (-\infty, \cot 3)$ (b) $\left(\frac{\sqrt{2}}{2}, 1\right]$ (c) $\left(\frac{\sqrt{2}}{2}, 1\right) \cup \left(-1, -\frac{\sqrt{2}}{2}\right)$ **14.** (5)

EXERCISE-2

SECTION-A

(JEE-ADVANCE Previous Year's Questions)

1. Let (x, y) be such that

$$\sin^{-1}(ax) + \cos^{-1}(y) + \cos^{-1}(bxy) = \frac{\pi}{2}$$

Match the statements in Column I with statements in Column II and indicate your answer by darkening the appropriate bubbles in the 4×4 matrix given in the OMR **[JEE 2007, 6]**

Column I	**Column II**
(A) If $a = 1$ and $b = 0$, then (x, y)	(P) lies on the circle $x^2 + y^2 = 1$
(B) If $a = 1$ and $b = 1$, then (x, y)	(Q) lies on $(x^2 - 1)(y^2 - 1) = 0$
(C) If $a = 1$ and $b = 2$, then (x, y)	(R) lies on $y = x$
(D) If $a = 2$ and $b = 2$, then (x, y)	(S) lies on $(4x^2 - 1)(y^2 - 1) = 0$

2. If $0 < x < 1$, then $\sqrt{1+x^2}[\{x\cos(\cot^{-1} x) + \sin(\cot^{-1} x)\}^2 - 1]^{1/2} =$

(A) $\frac{x}{\sqrt{1+x^2}}$ (B) x (C) $x\sqrt{1+x^2}$ (D) $\sqrt{1+x^2}$ **[JEE 2008, 3]**

3. The value of $\cot\left(\sum_{n=1}^{23} \cot^{-1}\left(1 + \sum_{k=1}^{n} 2k\right)\right)$ is **[JEE Adv. 2013, 2]**

(A) $\frac{23}{25}$ (B) $\frac{25}{23}$ (C) $\frac{23}{24}$ (D) $\frac{24}{23}$

4. Match List-I with List-II and select the correct answer using the code given below the lists:

List I		List II
P. $\left(\frac{1}{y^2}\left(\frac{\cos(\tan^{-1} y) + y\sin(\tan^{-1} y)}{\cot(\sin^{-1} y) + \tan(\sin^{-1} y)}\right)^2 + y^4\right)^{\frac{1}{2}}$ takes value	1.	$\frac{1}{2}\sqrt{\frac{5}{3}}$
Q. If $\cos x + \cos y + \cos z = 0 = \sin x + \sin y + \sin z$ then possible value of $\cos\left(\frac{x-y}{2}\right)$, is	2.	$\sqrt{2}$
R. If $\cos\left(\frac{\pi}{4} - x\right)\cos 2x + \sin x \sin 2x \sec x = \cos x \sin 2x \sec x + \cos\left(\frac{\pi}{4} + x\right)\cos 2x$	3.	$\frac{1}{2}$

then possible value of $\sec x$ is

S. If $\cot\left(\sin^{-1}\sqrt{1-x^2}\right) = \sin\left(\tan^{-1}(x\sqrt{6})\right), x \neq 0,$ 4. 1

then possible value of x is

Codes: **[JEE Adv. 2013, 3]**

	P	Q	R	S
(A)	4	3	1	2
(B)	4	3	2	1
(C)	3	4	2	1
(D)	3	4	1	2

5. Let $f: [0,4\pi] \to [0,\pi]$ be defined by $f(x) = \cos^{-1}(\cos x)$. The number of points $x \in [0,4\pi]$ satisfying the equation $f(x) = \frac{10-x}{10}$ is **[JEEAdv. 2014, 3]**

6. If $\alpha = 3\sin^{-1}\left(\frac{6}{11}\right)$ and $\beta = 3\cos^{-1}\left(\frac{4}{9}\right)$, where the inverse trigonometric functions take only the principal values, then the correct option(s) is(are) **[JEE Adv. 2015, 4]**

(A) $\cos\beta > 0$ (B) $\sin\beta < 0$ (C) $\cos(\alpha+\beta) > 0$ (D) $\cos\alpha < 0$

SECTION-B (JEE-MAIN Previous Year's Questions)

1. If $\cos^{-1}x - \cos^{-1}\frac{y}{2} = \alpha$, then $4x^2 - 4xy\cos\alpha + y^2$ is equal to **[AIEEE-2005]**

(A) $2\sin 2\alpha$ (B) 4 (C) $4\sin^2\alpha$ (D) $-4\sin^2\alpha$

2. If $\sin^{-1}\left(\frac{x}{5}\right) + \operatorname{cosec}^{-1}\left(\frac{5}{4}\right) = \frac{\pi}{2}$ then a value of x is- **[AIEEE-2007]**

(A) 1 (B) 3 (C) 4 (D) 5

3. The value of $\cot\left(\operatorname{cosec}^{-1}\frac{5}{3} + \tan^{-1}\frac{2}{3}\right)$ is **[AIEEE-2008]**

(A) $\frac{3}{17}$ (B) $\frac{4}{17}$ (C) $\frac{5}{17}$ (D) $\frac{6}{17}$

4. If x, y, z are in A.P. and $\tan^{-1}x, \tan^{-1}y$ and $\tan^{-1}z$ are also in A.P., then **[JEE Main 2013]**

(A) $2x = 3y = 6z$ (B) $6x = 3y = 2z$ (C) $6x = 4y = 3z$ (D) $x = y = z$

5. Let $\tan^{-1}y = \tan^{-1}x + \tan^{-1}\left(\frac{2x}{1-x^2}\right)$, where $|x| < \frac{1}{\sqrt{3}}$. Then a value of y is **[JEE Main 2015]**

(A) $\frac{3x-x^3}{1+3x^2}$ (B) $\frac{3x+x^3}{1+3x^2}$ (C) $\frac{3x-x^3}{1-3x^2}$ (D) $\frac{3x+x^3}{1-3x^2}$

EXERCISE-2 SECTION-A

(JEE-ADVANCE Previous Year's Questions)

ANSWER KEY

1. Ans. (A) P; (B) Q; (C) P; (D)S **2.** (C) **3.** (B) **4.** (B) **5.** (3)

6. (B, C, D)

SECTION-B (JEE-MAIN Previous Year's Questions)

1. (C) **2.** (B) **3.** (D) **4.** (D) **5.** (C)