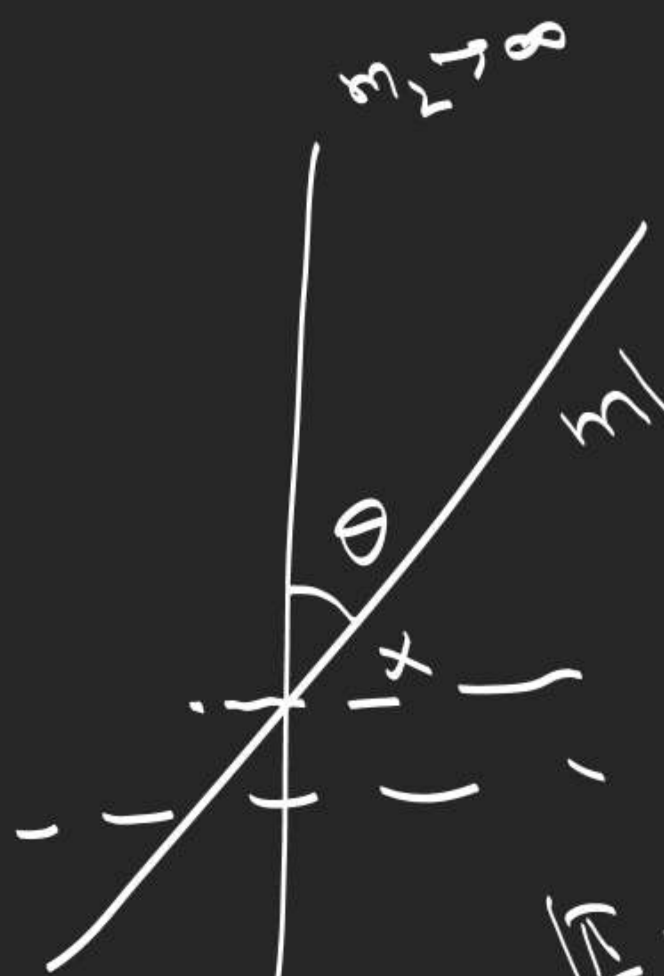
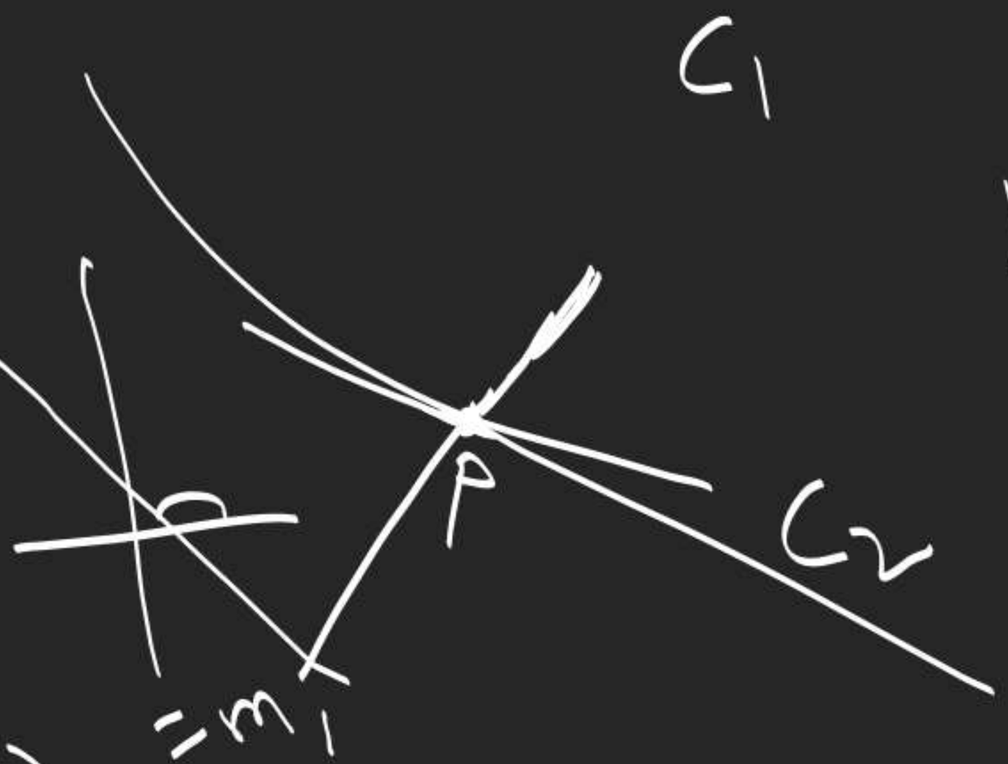
Nishant Jindal

# Angle b/n two Curves



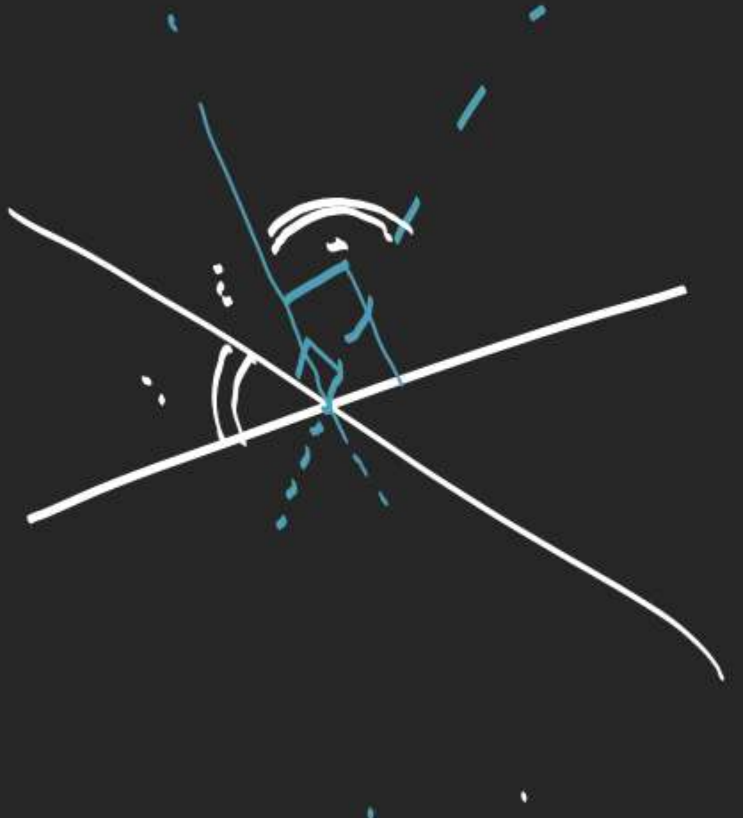

$\tan\left(\frac{\pi}{2}-\theta\right)$ or $\tan\left(\frac{\pi}{2}+\theta\right)$




$$\tan\theta = \left|\frac{m_1 - m_2}{1+m_1 m_2}\right|$$

If $m_2 \to \infty$

$$\left|-\frac{1}{m_1}\right| = \left|\frac{\frac{m_1}{m_2} - 1}{\frac{1}{m_2} + m_1}\right|$$



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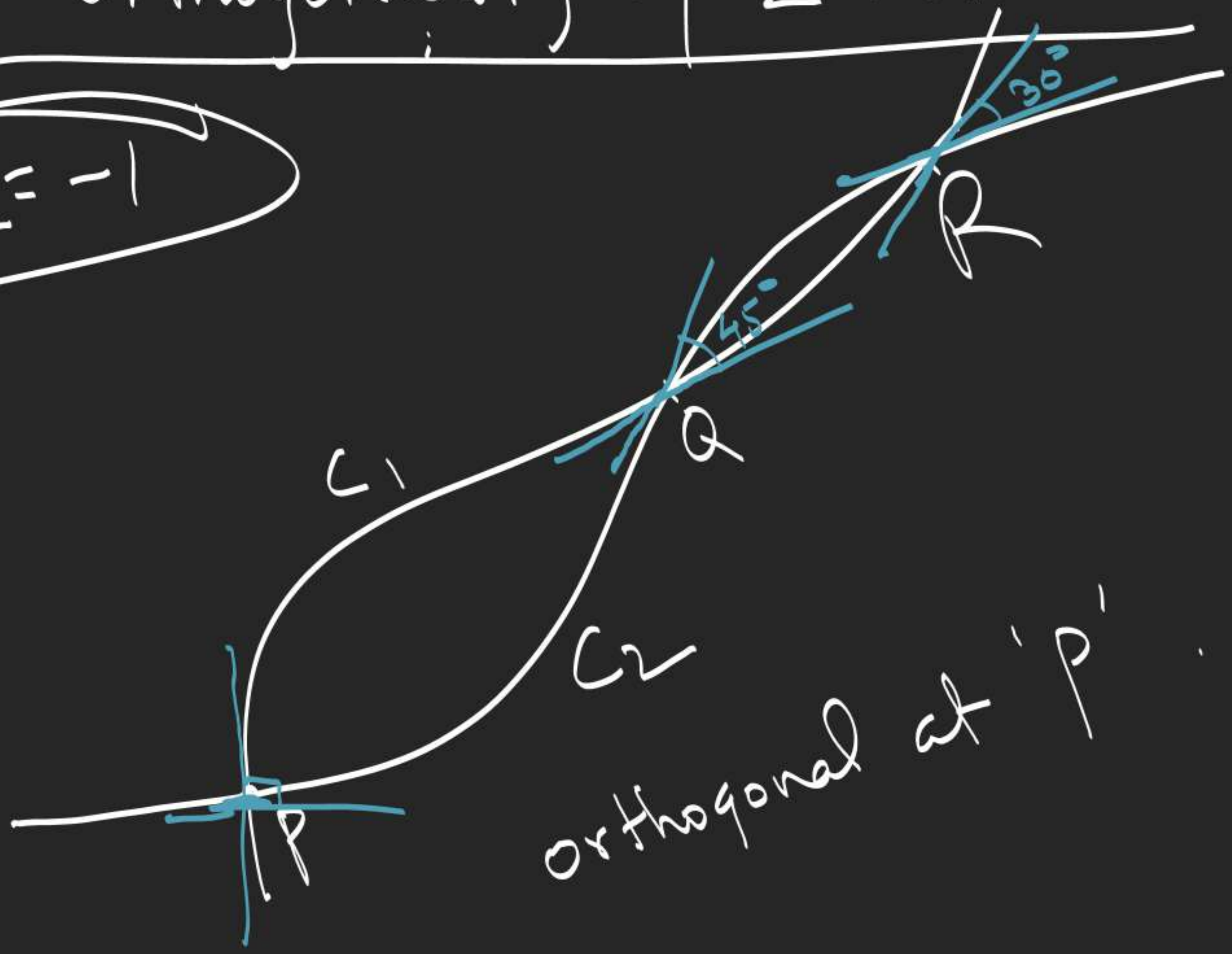
# Orthogonality of 2 curves

$m_1 m_2 = -1$




orthogonal at 'p'.

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# Isogonal Curves

$$\left|\frac{m_1 - m_2}{1 + m_1 m_2}\right| =$$

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1. Find the angle b/n curves

(i) $y = \sin x$ & $y = \cos x$

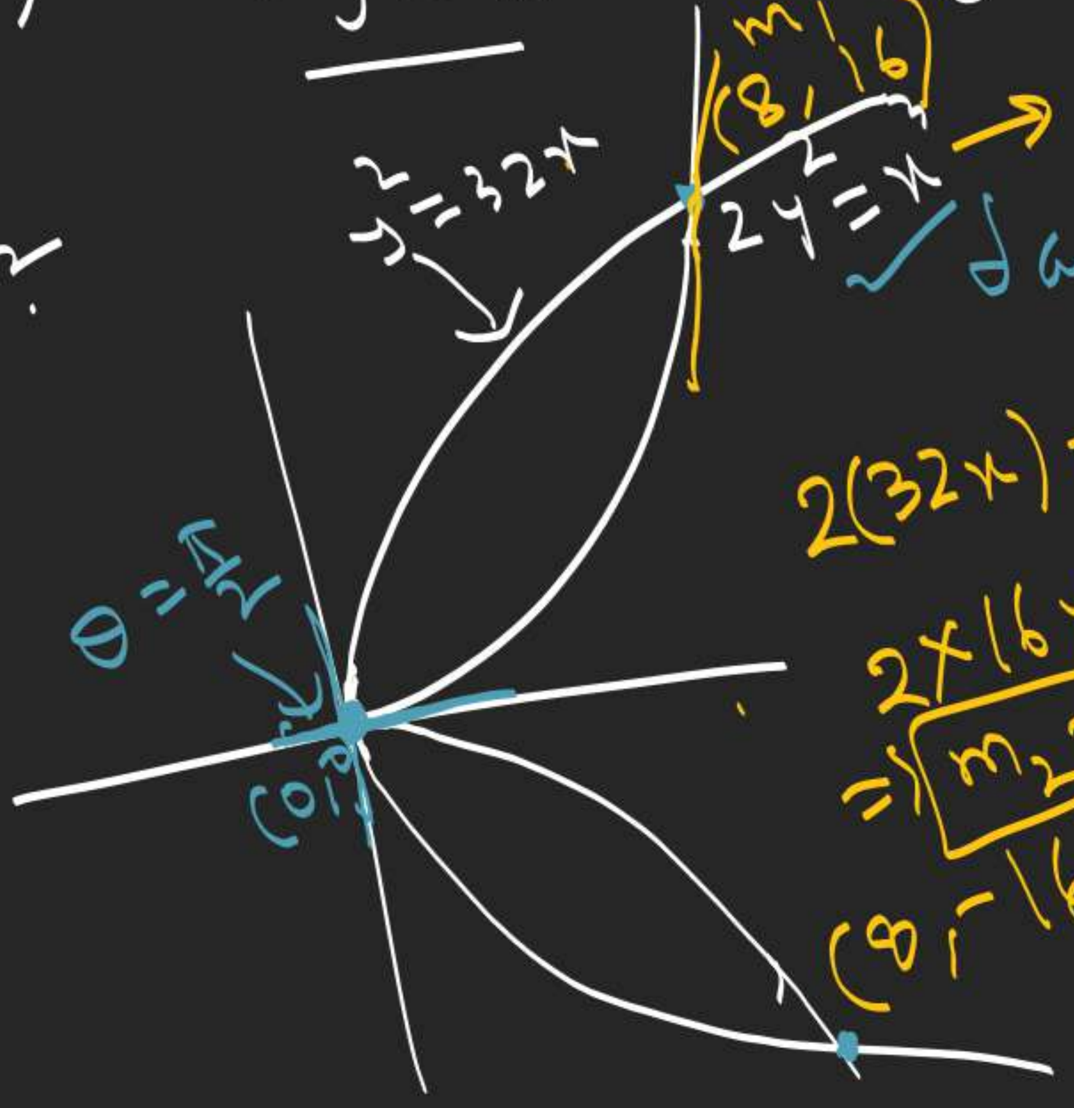
$\hookrightarrow m_1 = \frac{1}{\sqrt{2}}$ $\quad \hookrightarrow m_2 = -\frac{1}{\sqrt{2}}$


$$\tan\theta = \frac{\frac{2}{\sqrt{2}}}{1 + \frac{1}{\sqrt{2}}\left(-\frac{1}{\sqrt{2}}\right)} = 2\sqrt{2}$$

$$\tan^{-1}\left(2\sqrt{2}\right)$$

(ii) $2y^2 = x^3$ and $y^2 = 32x$

$$y = \pm \frac{1}{\sqrt{2}} x^{3/2}$$




$$2(32x) = x^3$$

$(8, 16)$ $\quad$ $(8, -16)$

$$\rightarrow 4(16)y' = 3(8)^2 \Rightarrow \boxed{m_1 = 3}$$

$$2 \times 16 y' = 32 \Rightarrow \boxed{m_2 = 1}$$

$$\tan\theta = \left|\frac{3-1}{1+3}\right| = \frac{1}{2}$$

$$\boxed{\theta = \tan^{-1}\frac{1}{2}}$$

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2. P.T. angle between curves $y^2 = x$ and $x^3 + y^3 = 3xy$ at point other than origin is $\tan^{-1}(16)^{\frac{1}{3}}$.

$$y^6 + y^3 = 3y^3 \Rightarrow y^3 = 2 \quad , \quad y = 0$$

$$\left(2^{2/3} , 2^{\frac{1}{3}}\right)$$

$$2yy' = 1 \Rightarrow m_1 = \frac{1}{2^{4/3}}$$

$$3\left(2^{4/3}\right) + 3\left(2^{\frac{2}{3}}\right)y' = 3\left(2^{2/3}\right)y' + 3\left(2^{\frac{1}{3}}\right)$$

$$\Rightarrow m_2 \to \infty.$$

$$\tan\theta = \left|\frac{1}{m_1}\right| = 2^{\frac{4}{3}}$$

$$= (16)^{1/3}.$$

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3: Find the condition for two curves $a_1x^2+b_1y^2=1$ and $a_2x^2+b_2y^2=1$ to intersect orthogonally.

$(\alpha,\beta)$

$$2a_1\alpha + 2b_1\beta y' = 0 \Rightarrow m_1 = -\frac{a_1\alpha}{b_1\beta}$$

$$m_2 = -\frac{a_2\alpha}{b_2\beta}$$

$$m_1m_2 = -1 = \frac{a_1a_2}{b_1b_2}\frac{\alpha^2}{\beta^2} \Rightarrow \boxed{\frac{a_1a_2}{b_1b_2}\frac{(b_1-b_2)}{(a_1-a_2)} = 1}$$

$$a_1\alpha^2+b_1\beta^2=1$$

$$a_2\alpha^2+b_2\beta^2=1$$

$$(a_1-a_2)\alpha^2+(b_1-b_2)\beta^2=0 \Rightarrow \frac{\alpha^2}{\beta^2} = -\left(\frac{b_1-b_2}{a_1-a_2}\right)$$

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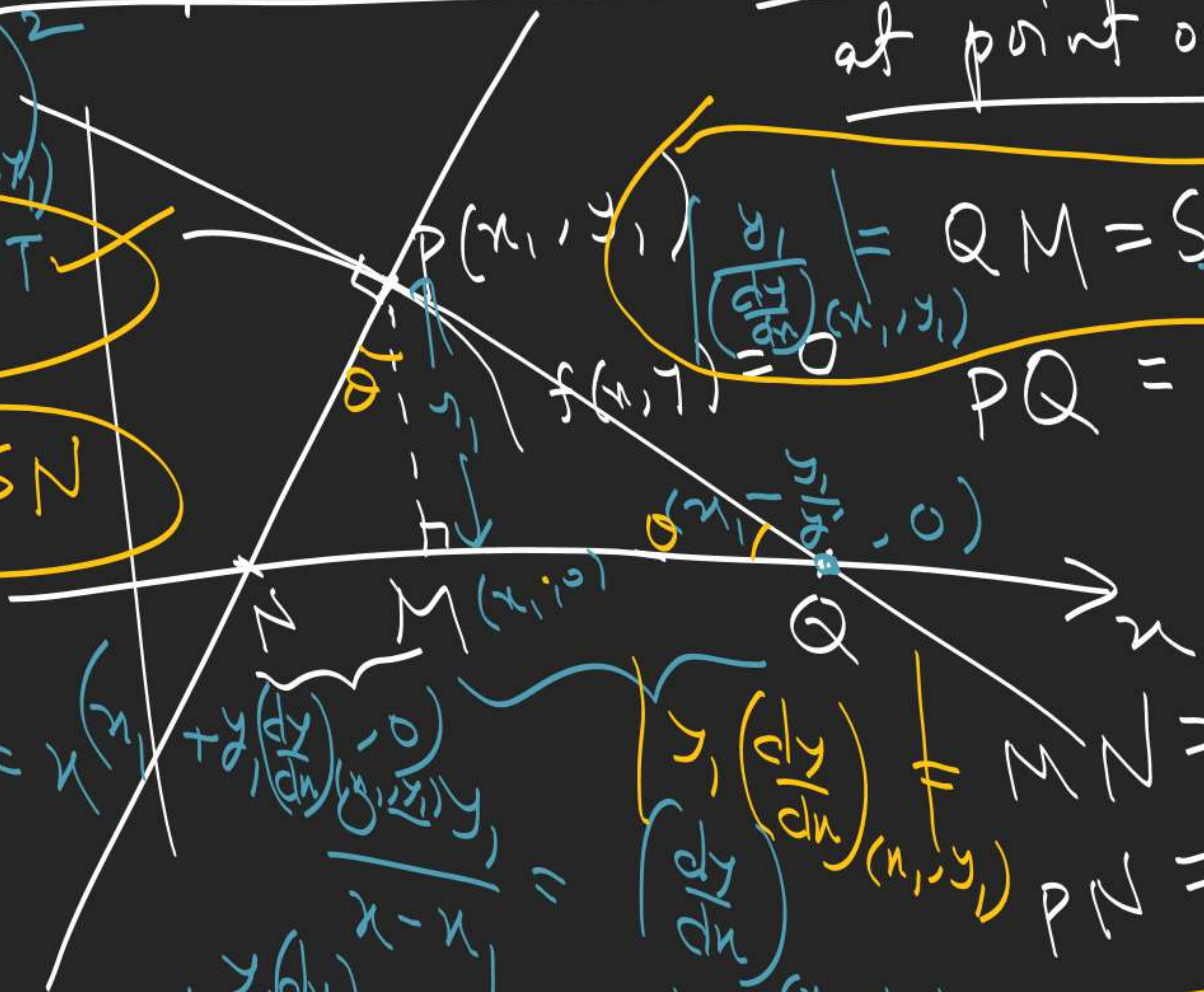
# Subtangent / Subnormal to Curve at point on it




$PQ = \sqrt{y_1^2 + \left(\frac{y_1}{\left(\frac{dy}{dx}\right)_{(x_1,y_1)}}\right)^2}$

$\left|\frac{y}{y'}\right| = ST$

$|yy'| = SN$

$\left|\frac{y_1}{\left(\frac{dy}{dx}\right)_{(x_1,y_1)}}\right| = QM$ = Subtangent at P

PQ = Length of tangent at P.

$x_1 - \frac{y_1}{\left(\frac{dy}{dx}\right)} = x$

$\left(x_1 + y_1\left(\frac{dy}{dx}\right)_{(x_1,y_1)}, 0\right)$

$\left|y_1\left(\frac{dy}{dx}\right)_{(x_1,y_1)}\right| = $ MN = Subnormal at P.

$\frac{y - y_1}{x - x_1} = -\frac{1}{\left(\frac{dy}{dx}\right)_{(x_1,y_1)}}$

$x = x_1 + y_1\left(\frac{dy}{dx}\right)_{(x_1,y_1)} \Leftarrow$

PN = Length of normal at P

$= \sqrt{y_1^2 + \left(y_1\left(\frac{dy}{dx}\right)_{(x_1,y_1)}\right)^2}$

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1. Show that for curve $by^2 = (x+a)^3$, the square of subtangent varies as the subnormal.

$$\frac{(ST)^2}{(SN)}$$

$$2b\boxed{yy'} = 3(x+a)^2 \Rightarrow y' = \frac{3(x+a)^2}{2by}$$

$$ST = \frac{y}{y'} = \frac{2by^2}{3(x+a)^2} = \frac{2(x+a)^3}{3(x+a)^2} = \frac{2}{3}(x+a)$$

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$$SN = \frac{3}{2b}(x+a)^2$$

$$\frac{(ST)^2}{SN} = \frac{\frac{4}{9}\cancel{(x+a)^2}}{\frac{3}{2b}\cancel{(x+a)^2}} = \frac{8b}{27}$$