Nishant Jindal

Khushiyan hi Khushiyan!!

Q Let $\{a_n\}$ be a G.P. Such that $\frac{a_4}{a_6} = \frac{1}{4}$ & $a_2 + a_5 = 216$ then $a_1 = ?$

1) $\frac{a_4}{a_6} = \frac{1}{4} \Rightarrow \frac{\cancel{a} \cancel{r^3}}{\cancel{a} \cancel{r^5} r^2} = \frac{1}{4}$

$r^2 = 4 \Rightarrow r = 2$ or -2

2) $a_2 + a_5 = 216$.

$ar + ar^4 = 216$.

$a(r + r^4) = 216$

$r = 2$

$a(2 + 2^4) = 216$

$a = \frac{\overset{72}{\cancel{216}}\ 12}{\cancel{18}\ 6}$

$r = -2$

$a(-2 + (-2)^4) = 216$.

$a = \frac{\overset{108}{\cancel{216}}}{\cancel{14}\ 7}$

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Q If each term of GP is +ve & $(p+q)^{th}$ term of GP is a & the $(p-q)^{th}$ term is b, Show that p^{th} term is $\sqrt{ab}$.

Given ① $T_{p+q} = a \Rightarrow A \cdot R^{p+q-1} = a$

② $T_{p-q} = b \Rightarrow A \cdot R^{p-q-1} = b$

(3) Demand

$\downarrow$

$T_p = A \cdot R^{p-1}$

$$A^2 (R)^{p+\cancel{q}-1+p-\cancel{q}-1} = ab$$

$$A^2 R^{2(p-1)} = ab$$

$$\left(A R^{(p-1)}\right)^2 = ab$$

$$A R^{p-1} = \sqrt{ab}$$

$$T_p = \sqrt{ab}$$

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Q P.T. $(b-c)^2+(c-a)^2+(d-b)^2=(a-d)^2$

if a, b, c, d are in GP.

$c^2=bd$

$b^2=ac$

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$2, 4, 8, 16 \to$ GP

32

LHS.

$(b-c)^2+(c-a)^2+(d-b)^2 = (a-d)^2$

$b^2+c^2+c^2+a^2+d^2+b^2-2bc-2ac-2bd=(a-d)^2$

$\cancel{b^2}+\cancel{c^2}+\cancel{c^2}+a^2+d^2+\cancel{b^2}-2bc-\cancel{2b^2}-\cancel{2c^2}=(a-d)^2$

$a^2+d^2-2bc=a^2+d^2-2ad$

$=(a-d)^2=$ RHS. H.P.

Concept

a, b, c GP.

$\frac{b}{a}=\frac{c}{b}$.

$\Rightarrow b^2=ac$

b, c, d GP

$c^2=bd$

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Q Let a, b, c, d be in G.P. If U, V, W satisfying $U+2V+3W=6$

$4U+5V+6W=12$, $6U+9V=4$

then S.T. roots of Eqn.

$$\left(\frac{1}{U}+\frac{1}{V}+\frac{1}{W}\right)x^2+\left[(b-c)^2+(c-a)^2+(d-b)^2\right]x+(U+V+W)=0 \quad (1)$$

& $20x^2+10(a-d)^2x-9=0$ are Reciprocal to each other.

$\times 2$

$$2U+4V+6W=12$$
$$4U+5V+6W=12$$
$$-2U+V=0$$

$V=2U$

$6U+18U=4 \Rightarrow -12U=4 \Rightarrow U=-\frac{1}{3}, V=\frac{2}{3}$

$W=\frac{5}{3}$

$\frac{1}{U}+\frac{1}{V}+\frac{1}{W}=-\frac{3}{1}+\frac{3}{2}+\frac{3}{5}$

$=-\frac{9}{10}$

$-\frac{9}{10}x^2+(a-d)^2x+2=0$

$-9x^2+10(a-d)^2x+20=0$ $\rightarrow \alpha, \beta$

$\frac{1}{\alpha}, \frac{1}{\beta}$ Reciprocal

$20x^2+10(a-d)^2x-9=0$ (H.P.)

Nishant Jindal

Q If $ab^2c^3,\ a^2b^3c^4,\ a^3b^4c^5$ are in AP Bola!!

then find the min. value of $a+b+c$, if $a, b, c \in N$.

Check GP

$$(a^2b^3c^4)^2 = (ab^2c^3)\times(a^3b^4c^5)$$

$$a^4b^6c^8 = a^4b^6c^8$$

G.P. bhi hai

AP bhi hai

$\Rightarrow ab^2c^3 = a^2b^3c^4 = a^3b^4c^5$ Pshbl.

If $a=b=c=1$

$\therefore$ Min $a+b+c = 1+1+1 = 3$

a, b, c can be in AP & GP both.

$a, a, a \rightarrow AP \rightarrow CD=0$

$a, a, a \rightarrow GP \rightarrow r=1$

$\rightarrow a=b=c$

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Sum of n term of GP.

$$S_n = a + ar + ar^2 + ar^3 + \cdots + ar^{n-1}$$

$$r \cdot S_n = \quad ar + ar^2 + ar^3 \quad \cdots \quad + ar^{n-1} + ar^n$$

$$S_n(1-r) = a - ar^n$$

$$\boxed{S_n = \frac{a(1-r^n)}{(1-r)}} \rightarrow |r|<1$$

$$\boxed{S_n = \frac{a(r^n-1)}{(r-1)}} \rightarrow |r|>1$$

If $r=1 \Rightarrow S_n = a+a+a+\cdots+a$
$= na$

∞ Series of GP

$$S = a + ar + ar^2 + ar^3 \cdots \infty$$

$$S = \lim_{n\to\infty} \frac{a(1-r^n)}{(1-r)} \qquad r^n \to 0, \quad |r|<1$$

$(\text{Base}<1)^{\infty} = 0$

$$= \frac{a(1-0)}{1-r}$$

$$\boxed{S_\infty = \frac{a}{1-r}}$$

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$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_n = \frac{a - ar^n}{1-r}$$

$$= \frac{a - r\cdot(ar^{n-1})}{1-r}$$

$$\boxed{S_n = \frac{a - lr}{1-r}}$$

जब No. of term नहीं पता है

(1) $S_n = \dfrac{a(1-r^n)}{1-r}$ Use when $r<-1$ or $r>1$

$$S_n = \frac{(1^{st}\text{term})\left(1 - (\text{Com. Ratio})^{\text{No. of terms}}\right)}{(1 - \text{C.R.})}$$

(2) $S_n = \dfrac{a(r^n-1)}{(r-1)}$ Use when $-1<r<1$

(3) $S_n = \dfrac{a - lr}{1-r}$ When No. of term is not known.

(4) $S_\infty = \dfrac{a}{1-r} = \dfrac{1^{st}\text{term}}{1 - \text{Com. Ratio}}$

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Q Find Sum of Prog.

$\frac{1}{3}+\frac{1}{9}+\frac{1}{27}+ \dots \infty \rightarrow S=\frac{a}{1-r}$

$\times\frac{1}{3}$ $\times\frac{1}{3}$

$a=\frac{1}{3}, \quad r=\frac{1}{3}$

$S_\infty = \frac{\frac{1}{3}}{1-\frac{1}{3}} = \frac{\frac{1}{\cancel{3}}}{\frac{2}{\cancel{3}}} = \frac{1}{2}$

Q Sum of $3+9+27+\dots$ Up to 7 terms.

$\times 3$ $\times 3$

$a=3, \quad r=3, \quad n=7$

$S_7 = \frac{a(r^n-1)}{r-1} = \frac{3(3^7-1)}{(3-1)} = \frac{3}{2}(3^7-1)$

Q find $\sum_{K=1}^{10} 2+3^K = ?$

Concept

$\sum f+g = \sum f + \sum g$

$\sum \lambda \cdot f$

$\lambda \sum f$

$\sum_{K=1}^{10} 2 + \sum_{K=1}^{10} 3^K$

$2\sum_{K=1}^{10} 1 + \{\underbrace{3^1+3^2+3^3+\dots+3^{10}}_{\leftarrow 10 \text{ लोग} \rightarrow}\}$

G.P.

$2\{\underbrace{1+1+1+\dots+1}_{\leftarrow 10 \text{ लोग} \rightarrow}\}$

$a=3, r=3, n=10$

$20 + \frac{3(3^{10}-1)}{3-1}$

$20+\frac{3}{2}(3^{10}-1)$

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Q How many terms of $1+3+3^2+3^3 \dots$ must be taken to make 3280?

let n terms will give sum 3280

$a=1, r=3, n=n$

$$\frac{1\cdot(3^n-1)}{3-1}=3280$$

$$3^n-1=6560$$

$$3^n=6561=(81)^2=(3^4)^2$$

$$3^n=3^8$$

$$\Rightarrow \boxed{n=8}$$

$$\left(\frac{1}{2}\right)^n<\frac{1}{1000}$$

$$\frac{1}{2^n}<\frac{1}{1000}$$

$$\boxed{\frac{1}{1024}<\frac{1}{1000}}$$

$n=10, 11, 12, 13 \dots$

$\frac{1}{512} \not< \frac{1}{1000}$

Q Given $S_n=\sum_{r=0}^{n}\frac{1}{2^r}$, $S=\sum_{r=0}^{\infty}\frac{1}{2^r}$ If $S-S_n<\frac{1}{1000}$ then least value of $n=?$

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① $S=\frac{1}{2^0}+\frac{1}{2^1}+\frac{1}{2^2}+\frac{1}{2^3}+\dots$ $\quad \frac{a}{1-r}$

$S=1+\frac{1}{2}+\frac{1}{2^2}+\frac{1}{2^3}+\dots\infty$ (∞ GP)

$$S=\frac{1}{1-1/2}=2$$

② $S_n=\frac{1}{2^0}+\frac{1}{2^1}+\frac{1}{2^2}+\dots+\frac{1}{2^n}=\frac{a(1-r^n)}{1-r}=\frac{1\left(1-\left(\frac{1}{2}\right)^{n+1}\right)}{1-\frac{1}{2}}$

$\leftarrow n+1$ पद $\rightarrow$

$r=\frac{1}{2}<1$ $\quad S_n=2\left(1-\left(\frac{1}{2}\right)^{n+1}\right)$

③ $S-S_n<\frac{1}{1000} \Rightarrow 2-2\left(1-\left(\frac{1}{2}\right)^{n+1}\right)<\frac{1}{1000} \Rightarrow \cancel{2}-\cancel{2}+\cancel{2}\left(\frac{1}{\cancel{2}}\right)\left(\frac{1}{2}\right)^n<\frac{1}{1000}$

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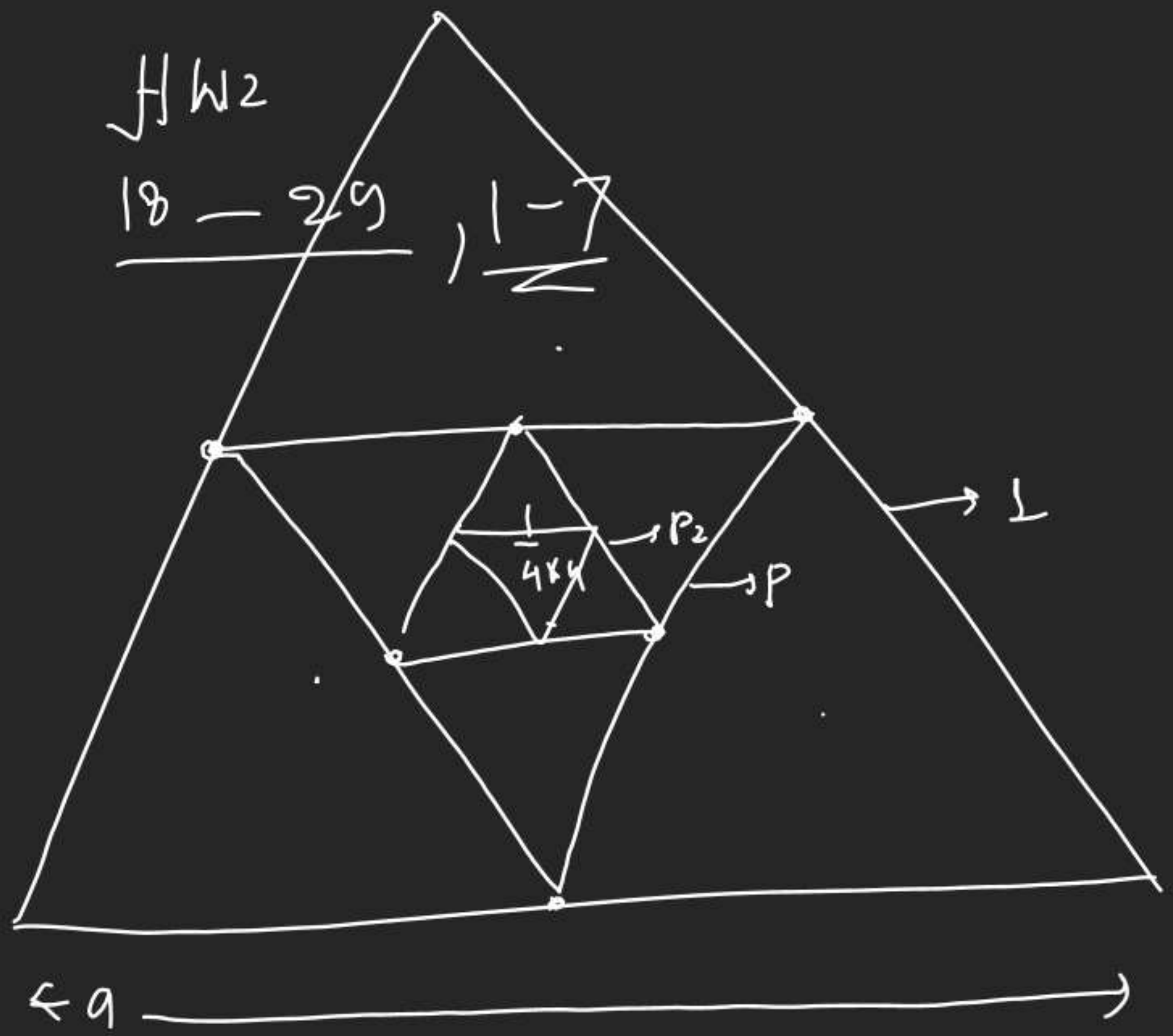
Q Area of an equilateral Δ is 1 sq unit

The mid Point of its Sides are joined to form another Δ P_1, hence dividing original Δ into 4 smaller Δ's. The mid Pts of sides of one of those smaller Δ are joined to form another Δ P_2. This Process goes continues infinitely. Find sum of areas of Δ's P_1, P_2, P_3 - - - .

HW2

18 — 29, 1 — 7



$$\Delta = \frac{\sqrt{3}}{4}a^2 = 1$$

$$\frac{1}{4} + \frac{1}{4^2} + \frac{1}{4^3} + \cdots \infty = \frac{1}{4-1} = \frac{1}{3}$$

$$\frac{1}{17} + \frac{1}{17^2} + \frac{1}{17^3} \cdots \infty = \frac{1}{17-1} = \frac{1}{16}$$

$$S = P_1 + P_2 + P_3 + P_4 \cdots$$

$$S = \frac{1}{4} + \frac{1}{4\times4} + \frac{1}{4\times4\times4} + \frac{1}{4\times4\times4\times4} + \cdots \infty \text{ GP}$$

$$= \frac{a}{1-r} = \frac{1/4}{1-1/4} = \frac{1}{3}$$