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Q. (i) $\alpha^2\left(\frac{\alpha^2-\beta^2}{\beta}\right)+\beta^2\left(\frac{\beta^2-\alpha^2}{\alpha}\right)$

$(\alpha^2-\beta^2)\left(\frac{\alpha^3}{\beta}-\frac{\beta^3}{\alpha}\right)$

$(\alpha+\beta)^2-\alpha\beta$

$(\alpha+\beta)(\alpha-\beta)^2(\alpha^2+\beta^2+\alpha\beta)$

$\alpha\beta$

$+ \quad - \quad +$

$2m \quad 2n$

$(-\infty, 2m] \cup [2n, \infty)$

$$\boxed{(y-2m)(y-2n)\geq 0}$$

$$y=\frac{(x+m)^2-4mn}{2(x-n)} \Rightarrow x^2+(2m-2y)x+m^2-4mn+2ny=0$$

$D\geq 0$

$y^2-2(m+n)y+4mn\geq 0$

$(y^2+\cancel{m^2}-2my)-(\cancel{m^2}-4mn+2ny)\geq 0$

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9.

$\alpha, \beta \qquad \alpha', \beta'$

$$(\alpha+\beta)^2 - 4\alpha\beta = (\alpha'+\beta')^2 - 4\alpha'\beta'$$

$$\frac{4b^2}{a^2} - 4\frac{c}{a} = 4\frac{B^2}{A^2} - 4\frac{C}{A}$$

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Find range.

1. $y = \dfrac{(x+1)(x-2)}{x(x+3)}$

$x^2(y-1) + (3y+1)x + 2 = 0$

$\boxed{y=1}$ ✓ OR $y \neq 1$

$4x+2=0$

$x = -\frac{1}{2}$

$9y^2+6y+1-8(y-1) \geq 0$

$\underbrace{9y^2-2y+9}_{>0} \geq 0$

$4-4(99)<0$

$\boxed{y \in R - \{1\}}$

$y^2 - \frac{2}{9}y + \frac{1}{81}$

$\left(y-\frac{1}{9}\right)^2 + \frac{80}{81} > 0$

$\boxed{y \in R}$ ✓

y

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$$f(x) = \frac{(x+1)(x-2)}{x(x+3)} = \frac{x^2-x-2}{x^2+3x} = 1 - \frac{4x+2}{x^2+3x}$$

$$= 1 - \frac{4}{x+3} - \frac{2}{x(x+3)}$$

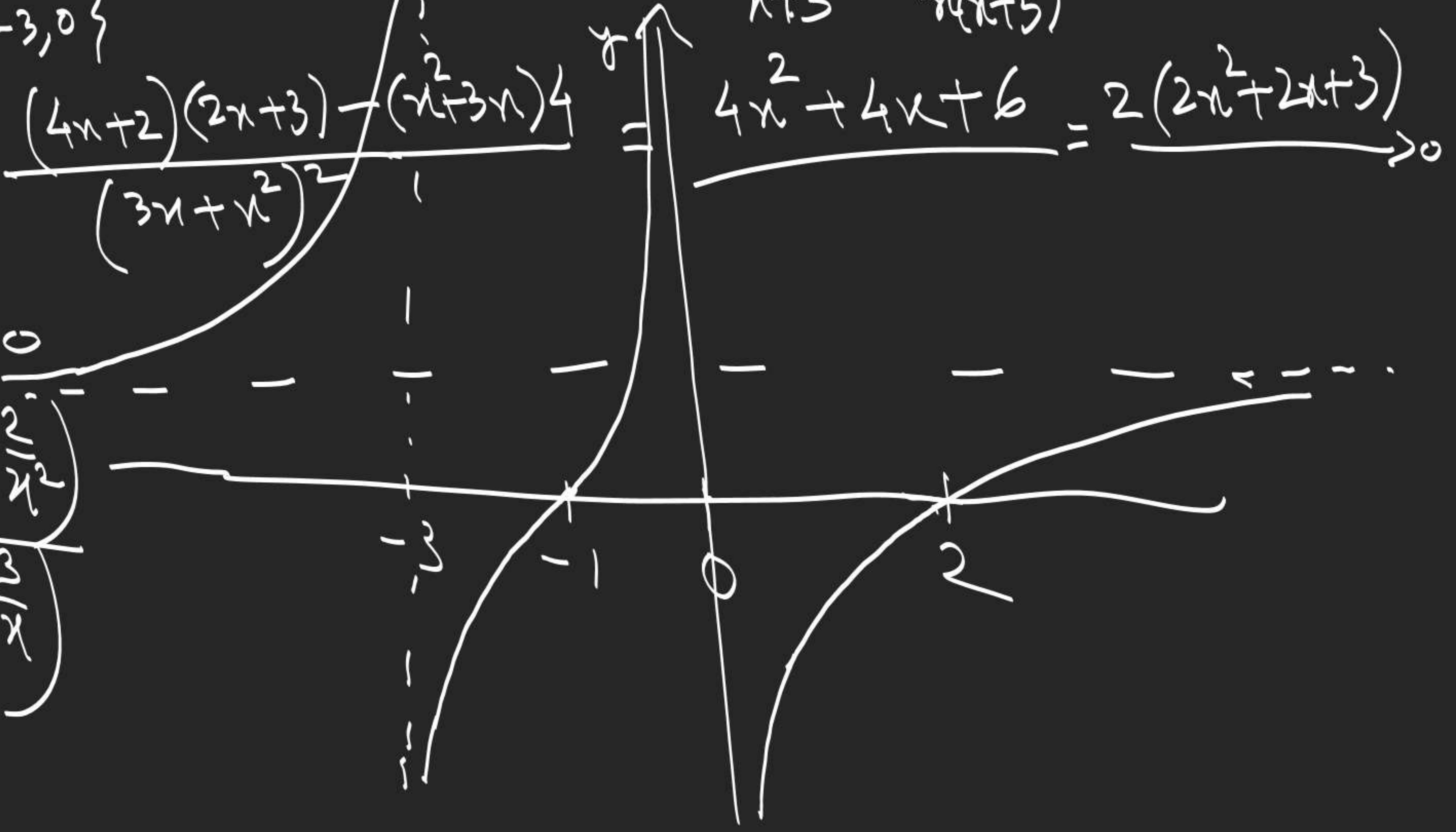
① $D_f = R - \{-3, 0\}$

② $f'(x) = \dfrac{(4x+2)(2x+3) - (x^2+3x)4}{(3x+x^2)^2} = 4x^2+4x+6 = 2(2x^2+2x+3) > 0$

$f'(x) > 0$

$\boxed{x \to -\infty, +\infty, \; y \to 1}$

$$y = \frac{\cancel{x^2}\left(1 - \frac{1}{x} - \frac{2}{x^2}\right)}{\cancel{x^2}\left(1 + \frac{3}{x}\right)}$$



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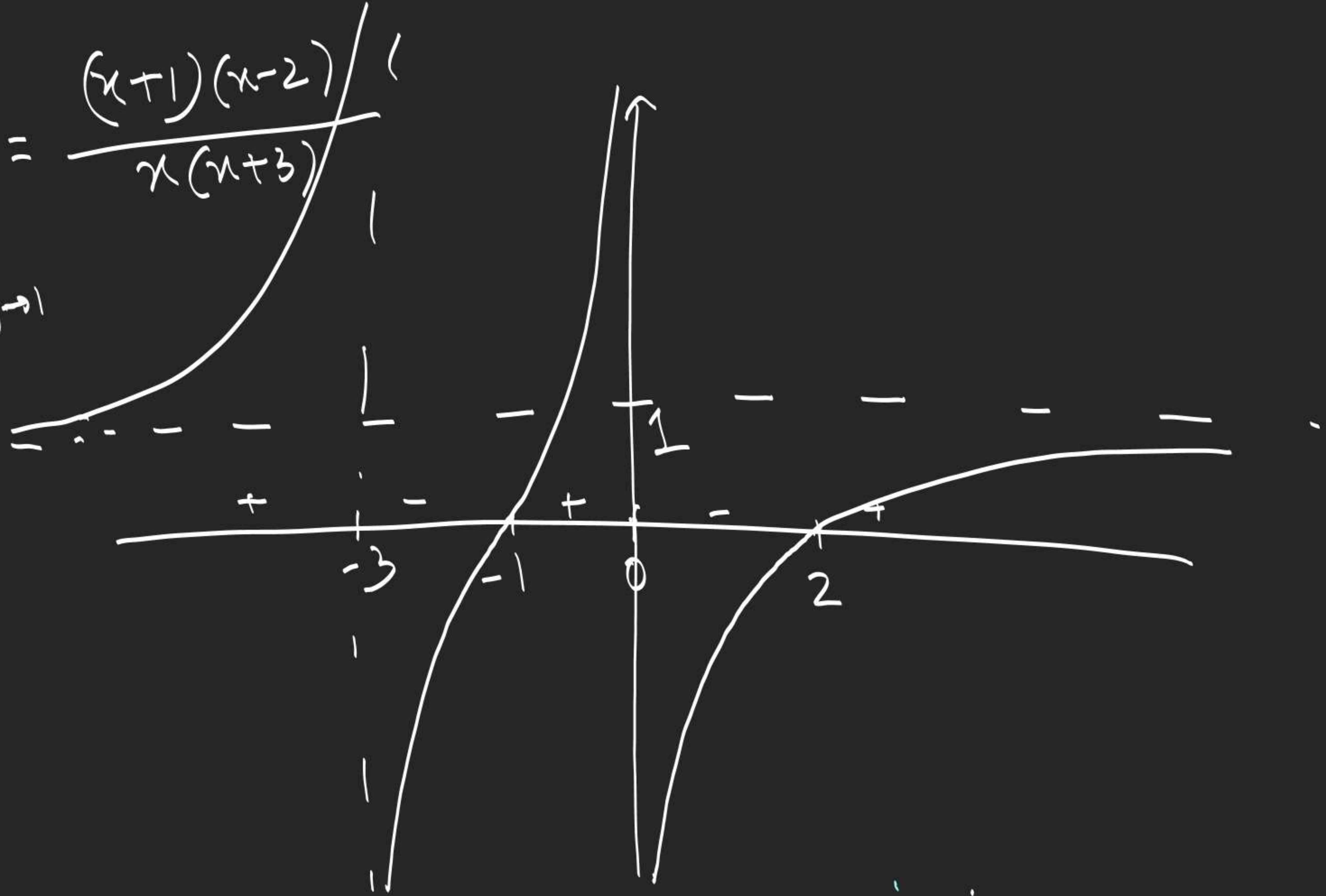
$$y = \frac{(x+1)(x-2)}{x(x+3)}$$

$x \to -\infty,\ y \to 1$



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$$y = \frac{x^2+14x+9}{x^2+2x+3}$$

$(\ )x^2 + (\ )x + (\ ) = 0$

$D \geq 0$

$$\boxed{R_f = [-5, 4]}$$

$$y = \frac{x^2+2x-11}{2(x-3)}$$

$(-\infty, 2] \cup [6, \infty)$

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$$y = \frac{ax^2 - 7x + 5}{5x^2 - 7x + a}$$ common root, $a = ?$

$ax^2 - 7x + 5 = 0$

$5x^2 - 7x + a = 0$

$(a-5)x^2 + 5 - a = 0 \Rightarrow \boxed{x = \pm 1}$

$a = 5$

$\boxed{x = 1,\ a = 2;\ x = -1,\ a = -12}$

$a = 5,\ y = 1$ ✗

$(2a^2 + 20a - 48)(2a - 20a + 50) \le 0$

$(a+12)(a-2)(a-5)^2 \le 0$

$+ \quad - \quad + \quad +$

$-12 \quad 2 \quad 5$

$\boxed{a \in (-12, 2)}$

$(5y - a)x^2 + (7 - 7y)x + ay - 5 = 0$

Ans $\boxed{a \in [-12, 2]} \Leftarrow [-12, 2] \cup \{5\} \Leftarrow$

$D \ge 0$

$\Rightarrow 49(y^2 - 2y + 1) - 4(5ay^2 - 25y - a^2y + 5a) \ge 0$

$(49 - 20a)y^2 + (2 + 4a^2)y + (49 - 20a) \ge 0 \quad \forall y \in R$

$\boxed{a < \frac{49}{20}} \Leftarrow 49 - 20a > 0$ ✓ $\ \&\ D \le 0$ ✓ $\Rightarrow (2a^2 + 1)^2 - (49 - 20a)^2 \le 0$

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$$\frac{ax^2-7x+5}{5x^2-7x+a}=\frac{a(x-\alpha)(x-\beta)}{5(x-\alpha)(x-\gamma)}=\frac{a(x-\beta)}{5(x-\gamma)}$$

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Condition for expression $ax^2+2hxy+by^2+2gx+2fy+c$ to be resolved into product of two linear factors

$$ax^2+2hxy+by^2+2gx+2fy+c = (\alpha_1 x+\beta_1 y+\gamma_1)(\alpha_2 x+\beta_2 y+\gamma_2)$$

$$\boxed{abc+2fgh-af^2-bg^2-ch^2 = 0}$$

$$ax^2+2hxy+by^2+2gx+2fy+c=0$$

$$ax^2+(2hy+2g)x+by^2+2fy+c=0$$

$$x = \frac{-\cancel{2}(hy+g) \pm \sqrt{\cancel{4}(h^2y^2+g^2+2ghy)-\cancel{4}a(by^2+2fy+c)}}{\cancel{2}a}$$

$$= \frac{-(hy+g) \pm \sqrt{(h^2-ab)y^2+2(gh-af)y+g^2-ac}}{a}$$

$\rightarrow$ perfect square $\Rightarrow$ $D=0$

$$(gh-af)^2-(h^2-ab)(g^2-ac)=0$$

$$g^2h^2+a^2f^2-2ghaf-(h^2-ab)(g^2-ac)=0$$

$$\boxed{a(abc+2fgh-af^2-bg^2-ch^2)=0}$$

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1. P.T. expression $2x^2+3xy+y^2+2y+3x+1$ can be factorised into two linear factors. Also find them.

$$2x^2+(3y+3)x+\underline{y^2+2y+1}=0$$

$$x=\frac{-3(y+1)\pm\sqrt{9(y+1)^2-8(y+1)^2}}{4}=\frac{-3(y+1)\pm(y+1)}{4}$$

$$x=\frac{-(y+1)}{2} \quad \text{or} \quad x=-(y+1) \Rightarrow \underline{2x+y+1},\ \underline{x+y+1}$$

$$2(1)(1)+2(1)\left(\frac{3}{2}\right)\left(\frac{3}{2}\right)-2(1)^2-1\left(\frac{3}{2}\right)^2-1\left(\frac{3}{2}\right)^2=0$$