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MOD. L4

Differentiation of Parametric fxn.

Q $x = at^2$, $y = 2at$ then $\frac{dy}{dx} = ?$, $\frac{d^2y}{dx^2} = ?$

① $\frac{dx}{dt} = 2at$ | ② $\frac{dy}{dt} = 2a$.

③ $\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$

$= \frac{2a}{2at} = \frac{1}{t}$

(4) $\frac{d^2y}{dx^2} = \frac{d\left(\frac{dy}{dx}\right)}{dx} = \frac{d\left(\frac{1}{t}\right)}{dx}$

$= \frac{d\left(\frac{1}{t}\right)}{dt} \times \frac{dt}{dx}$

$= -\frac{1}{t^2} \times \frac{1}{2at} = -\frac{1}{2at^3}$

Q $x = a(\cos\theta + \theta\sin\theta)$

$y = a(\sin\theta - \theta\cos\theta)$ then Slope of tangent at θ?

① $\frac{dx}{d\theta} = a\{-\cancel{\sin\theta} + \theta\cdot\cos\theta + \cancel{\sin\theta}\}$

$\frac{dx}{d\theta} = a\theta\cos\theta$

② $\frac{dy}{d\theta} = a(\cancel{\cos\theta} + \theta\sin\theta - \cancel{\cos\theta})$

$= a\theta\sin\theta$

(3) $\frac{dy}{dx} = (Sl)_T = \frac{dy/d\theta}{dx/d\theta}$

$= \frac{\cancel{a\theta}\sin\theta}{\cancel{a\theta}\cos\theta} = \tan\theta$

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Q $x = \sin\left(t+\frac{7\pi}{12}\right) + \sin\left(t-\frac{\pi}{12}\right) + \sin\left(t+\frac{\cancel{3\pi}}{\cancel{12}_{4}}\right)$

$y = \cos\left(t+\frac{7\pi}{12}\right) + \cos\left(t-\frac{\pi}{12}\right) + \cos\left(t+\frac{\cancel{3\pi}}{\cancel{12}_{4}}\right)$ $\quad \frac{dy}{dx} = ?$

$x = 2\sin\left(t+\frac{\pi}{4}\right)\cdot \cancel{\cos\left(\frac{\pi}{3}\right)} + \sin\left(t+\frac{\pi}{4}\right)$

$x = 2\sin\left(t+\frac{\pi}{4}\right) \rightarrow \frac{dx}{dt} = 2\cos\left(t+\frac{\pi}{4}\right)$

$y = \cancel{2}\cos\left(t+\frac{\pi}{4}\right)\cancel{\cos\left(\frac{\pi}{3}\right)} + \cos\left(t+\frac{\pi}{4}\right)$

$y = 2\cos\left(t+\frac{\pi}{4}\right) \rightarrow \frac{dy}{dt} = -2\sin\left(t+\frac{\pi}{4}\right)$

$\frac{dy}{dx} = \frac{-\cancel{2}\sin\left(t+\frac{\pi}{4}\right)}{\cancel{2}\cos\left(t+\frac{\pi}{4}\right)} = -\tan\left(t+\frac{\pi}{4}\right)$

Q $\frac{d^2x}{dy^2} = ?$

① let $y = f(x)$

② $\frac{dy}{dx} = f'(x)$ ③ $\frac{dx}{dy} = \frac{1}{f'(x)}$

(4) $\frac{d^2x}{dy^2} = \frac{d\left(\frac{dx}{dy}\right)}{dy} = \frac{d\left(\frac{1}{f'(x)}\right)}{dy}$

$= \frac{d\left(\frac{1}{f'(x)}\right)}{dx} \times \frac{dx}{dy} = -\frac{1}{(f'(x))^2} \times f''(x) \times \frac{1}{f'(x)}$

$\frac{d^2x}{dy^2} = -\frac{f''(x)}{(f'(x))^3} = -\frac{\left(\frac{d^2y}{dx^2}\right)}{\left(\frac{dy}{dx}\right)^3}$

$$\boxed{\frac{d^2x}{dy^2} = -\frac{2^{n}d}{(1^{st})^3}}$$

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Q $y = e^x + \sin x \quad \frac{d^2x}{dy^2} = ?$

$$\frac{d^2x}{dy^2} = -\frac{2^{nd}}{(1^{st})^3} \Rightarrow \frac{d^2x}{dy^2} = -\frac{(e^x - \sin x)}{(e^x + \cos x)^3}$$

Q If $\boxed{x = \csc\theta - \sin\theta}$

$y = \csc^n\theta - \sin^n\theta$ then P.T.

$$(x^2+4)\left(\frac{dy}{dx}\right)^2 = n^2(y^2+4)$$

① $x^2+4 = (\csc\theta - \sin\theta)^2 + 4 = (\csc\theta + \sin\theta)^2$

② $y^2+4 = (\csc^n\theta - \sin^n\theta)^2 + 4 = (\csc^n\theta + \sin^n\theta)^2$

③ $$\frac{dy}{dx} = \frac{+n\csc^{n-1}\theta\cdot\csc\theta\cot\theta + n\sin^{n-1}\theta\cdot\cos\theta}{+\csc\theta\cdot\cot\theta + \cos\theta}$$

$$= \frac{n\cancel{\cos\theta}\left(\csc^{n-1}\theta\cdot\frac{1}{\sin^2\theta} + \sin^{n-1}\theta\right)}{\cancel{\cos\theta}\left(\csc\theta\times\frac{1}{\sin\theta} + 1\right)}$$

$$\left(\frac{dy}{dx}\right) = n\frac{\left(\csc^n\theta\cdot\frac{1}{\sin\theta} + \sin^{n-1}\theta\right)}{\left(\csc\theta\cdot\frac{1}{\sin\theta} + 1\right)} = n\frac{(\csc^n\theta + \sin^n\theta)}{(\csc\theta + \sin\theta)}$$

$$\left(\frac{dy}{dx}\right)^2 = \frac{n^2(y^2+4)}{x^2+4}$$

H.P

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$y_1\, y_2$ Based Qs.

$y_1 = \frac{dy}{dx}$, $y_2 = \frac{d^2y}{dx^2}$

(1) $y = (\sin^{-1}x)^2$ then $(1-x^2)y_2 - xy_1 = ?$

diff $y_1 = \frac{2\sin^{-1}x}{\sqrt{1-x^2}}$

$\sqrt{1-x^2}\cdot y_1 = 2\sin^{-1}x$

diff^n $\left.\sqrt{1-x^2}\cdot y_2 + y_1 \cdot \frac{-2x}{2\sqrt{1-x^2}} = \frac{2}{\sqrt{1-x^2}}\right\} \times\sqrt{1-x^2}$

$\underbrace{(1-x^2)y_2 - xy_1}_{\text{Demand}} = 2$

Q $y = (x+\sqrt{1+x^2})^n$ then $(1+x^2)y_2 + xy_1 = ?$

$y^{\frac{1}{n}} = x+\sqrt{1+x^2}$

$(y^{\frac{1}{n}}-x)^2 = 1+x^2$

$y^{\frac{2}{n}} - 2x\cdot y^{1/n} = 1$

diff^n $y_1 = n(x+\sqrt{1+x^2})^{n-1}\left(1+\frac{2x}{2\sqrt{1+x^2}}\right)$

$= n(x+\sqrt{1+x^2})^{n-1} \times \frac{(x+\sqrt{1+x^2})^1}{\sqrt{1+x^2}}$

$\boxed{\sqrt{1+x^2}\cdot y_1} = n(x+\sqrt{1+x^2})^n = ny$

diff $\left.\sqrt{1+x^2}\cdot y_2 + y_1\frac{2x}{2\sqrt{1+x^2}} = n\cdot y_1\right\}\sqrt{1+x^2}$

$(1+x^2)y_2 + xy_1 = n\,y_1\sqrt{1+x^2}$

$= n \times ny$

Demand $= n^2y$

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Infinite Series. Jahan Pehla admi dubara dikha htaa ke y Likh do.

$$Y = \sqrt{f(x) + \sqrt{f(x) + \sqrt{f(x) + \sqrt{f(x)} - \infty}}} \quad Y' = ?$$

$$Y = \sqrt{f(x) + y}$$

$$y^2 = f(x) + y.$$

$$2y \cdot \frac{dy}{dx} = f'(x) + \frac{dy}{dx}$$

$$\frac{dy}{dx}(2y-1) = f'(x)$$

$$\boxed{\frac{dy}{dx} = \frac{f'(x)}{2y-1}}$$

Q $y = \sqrt{\sin x + \sqrt{\sin x + \cdots \infty}}$

$$\frac{dy}{dx} = ?$$

$$\frac{dy}{dx} = \frac{\cos x}{(2y-1)}$$

Q $y = \sqrt{a^x + \sqrt{a^x + \sqrt{a^x + \cdots \infty}}}$

$$\frac{dy}{dx} = \frac{a^x \ln a}{(2y-1)}$$

Q $Y = (\sin x)^{\sin x^{\sin x \cdots \infty}}$ $\quad \frac{dy}{dx}$

$$y = (\sin x)^y$$

$$\ln y = y \ln \sin x.$$

$$\frac{1}{y} \cdot \frac{dy}{dx} = y \cdot \cot x + \ln \sin x \cdot \frac{dy}{dx}$$

$$\frac{dy}{dx}\left(\frac{1}{y} - \ln \sin x\right) = y \cot x$$

$$\frac{dy}{dx} = \left(\frac{y^2 \cot x}{1 - y \ln \sin x}\right)$$

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Q If $f(x) = x + \cfrac{1}{2x + \cfrac{1}{2x + \cfrac{1}{2x + \cdots \infty}}}$ then $f(2023) \cdot f'(2023) = ?$

$$y = x + \cfrac{1}{2x + \cfrac{1}{2x + \cfrac{1}{2x \cdots \infty}}}$$

$$y - x = \cfrac{1}{2x + \underbrace{\cfrac{1}{2x + \cfrac{1}{2x + \cdots}}}_{y-x}}$$

$$y - x = \frac{1}{2x + y - x}$$

$$y - x = \frac{1}{y + x} \Rightarrow y^2 - x^2 = 1$$

$$2yy' - 2x = 0 \Rightarrow yy' = x \Rightarrow f(x) \cdot f'(x) = x$$

$$f(2023) f'(2023) = 2023.$$

Inverse fxn Base.

(1) Formula (Direct) Based.

A) $\tan^{-1}\left(\frac{a-b}{1+ab}\right) = \tan^{-1} a - \tan^{-1} b.$

(2) Substitution Based

(3) Interval Based (I.T.F Knowledge)

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Q $y = \cot^{-1}\left(\frac{1+x}{1-x}\right) \quad y' = ?$

$y = \tan^{-1}\left(\frac{1-x}{1+x}\right)$

$= \tan^{-1}\left(\frac{1-x}{1+1.x}\right) \rightarrow \frac{a-b}{1+ab}$

$y = \tan^{-1}(1) - \tan^{-1}(x)$

$y' = 0 - \frac{1}{1+x^2}$

Q $y = \tan^{-1}\left(\frac{ax-b}{bx+a}\right) \quad \frac{dy}{dx} = ?$

1 Chahiye

$\div bx$

$= \tan^{-1}\left(\frac{\frac{a}{b} - \frac{1}{x}}{1 + \frac{a}{b}\cdot\frac{1}{x}}\right)$

$y = \tan^{-1}\left(\frac{a}{b}\right) - \tan^{-1}\left(\frac{1}{x}\right)$

$y = \tan^{-1}\left(\frac{a}{b}\right) - \cot^{-1} x$

$y' = 0 - \frac{(-1)}{1+x^2} = \frac{1}{1+x^2}$

$y = \tan^{-1}\left(\frac{1-3\log x}{1+3\log x}\right) + \tan^{-1}\left(\frac{4+3\log x}{1-\underbrace{12\log x}_{4\times 3\log x}}\right)$

$y' = ?$

$y = \tan^{-1}(1) - \cancel{\tan^{-1}(3\log x)} + \tan^{-1}(4)$

$+ \cancel{\tan^{-1}(3\log x)}$

$y' = 0$

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Q. $y = \tan^{-1}\left(\frac{1}{x^2+x+1}\right) + \tan^{-1}\left(\frac{1}{x^2+3x+3}\right)$ then $\frac{dy}{dx} = ?$

$$y = \tan^{-1}\left(\frac{1}{1+(x)(x+1)}\right) + \tan^{-1}\left(\frac{1}{1+(x+1)(x+2)}\right)$$

$$= \tan^{-1}\left(\frac{(x+1)-x}{1+(x)(x+1)}\right) + \tan^{-1}\left(\frac{(x+2)-(x+1)}{1+(x+1)(x+2)}\right)$$

$$= \tan^{-1}(\cancel{x+1}) - \tan^{-1}(x) + \tan^{-1}(x+2) - \cancel{\tan^{-1}(x+1)}$$

$$y = \tan^{-1}(x+2) - \tan^{-1} x$$

$$y' = \frac{1}{1+(x+2)^2} - \frac{1}{1+x^2}$$

(B) $\sin^{-1}(3x-4x^3) = 3\sin^{-1}x$.

$\cos^{-1}(4x^3-3x) = 3\cos^{-1}x$

$\sin^{-1}(2x\sqrt{1-x^2}) = 2\sin^{-1}x \rightarrow \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \Rightarrow \left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$

$\Rightarrow \left[-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right]$

$\cos^{-1}(2x^2-1) = 2\cos^{-1}x$

$\tan^{-1}\left(\frac{2x}{1-x^2}\right) = 2\tan^{-1}x = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$

Interval कैसे? (Trick)

$\sin^{-1}(3x-4x^3) = \boxed{3}\sin^{-1}x$ कब?

1) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

2) $\left[-\frac{\pi}{6}, \frac{\pi}{6}\right]$

3) $x \in \left[\sin\left(-\frac{\pi}{6}\right), \sin\left(\frac{\pi}{6}\right)\right]$

$x \in \left[-\frac{1}{2}, \frac{1}{2}\right]$

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Q $y = \cos^{-1}\left(4\frac{x^3}{27} - x\right)$ then $\frac{dy}{dx} = ?$

Aclulator → $\cos^{-1}(4x^3 - 3x) = 3\cos^{-1}x$

$$y = \cos^{-1}\left(4\left(\frac{x}{3}\right)^3 - 3\left(\frac{x}{3}\right)\right)$$

$$y = 3\cos^{-1}\left(\frac{x}{3}\right)$$

$$\frac{dy}{dx} = \frac{-3}{\sqrt{1-\left(\frac{x}{3}\right)^2}} \times \frac{1}{3} = \frac{-3}{\sqrt{9-x^2}}$$

Q $y = \sin^{-1}\left(x\sqrt{1-x} + \sqrt{x}\sqrt{1-x^2}\right)$ $\frac{dy}{dx} = ?$

: $\sin^{-1}\left(x\sqrt{1-y^2} + y\sqrt{1-x^2}\right) = \sin^{-1}x + \sin^{-1}y$

$$y = \sin^{-1}x + \sin^{-1}\sqrt{x}$$

$$y' = \frac{1}{\sqrt{1-x^2}} + \frac{1}{\sqrt{1-(\sqrt{x})^2}} \times \frac{1}{2\sqrt{x}}$$

(2)** Substitution

$1 - \sin^2\theta = \cos^2\theta$

$\sec^2\theta - 1 = \tan^2\theta$

$1 + \tan^2\theta = \sec^2\theta$

$\frac{1-\cos 2\theta}{1+\cos 2\theta} = \tan^2\theta$

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$\sqrt{\dfrac{a}{a-x}}$ $\quad x = a\sin^2\theta$

$\sqrt{\dfrac{a}{a+x}}$ $\quad x = a\tan^2\theta$

$\sqrt{\dfrac{a-x}{a+x}}$ $\quad x = a\cos 2\theta$

$\sqrt{a^2-x^2}$ $\quad x = a\sin\theta$

$\sqrt{x^2-a^2}$ $\quad x = a\sec\theta$

$\sqrt{a^2+x^2}$ $\quad x = a\tan\theta$

Q. $y = \tan^{-1}\left(\dfrac{\sqrt{1+x^2}-1}{x}\right)$ then $\dfrac{dy}{dx} = ?$

$x = 1\cdot\tan\theta \Rightarrow \theta = \tan^{-1}x$

$y = \tan^{-1}\left(\dfrac{\sqrt{1+\tan^2\theta}-1}{\tan\theta}\right)$

$= \tan^{-1}\left(\dfrac{\sec\theta-1}{\tan\theta}\right)$

$= \tan^{-1}\left(\dfrac{1-\cos\theta}{\sin\theta}\right) = \tan^{-1}\left(\dfrac{\cancel{2}\sin^2\theta/2}{\cancel{2}\sin\theta/2\cos\theta/2}\right)$

$y = \tan^{-1}(\tan\frac{\theta}{2}) = \frac{\theta}{2} = \frac{1}{2}\tan^{-1}x$

$$\boxed{y' = \frac{1}{2}\times\frac{1}{1+x^2}}$$

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Q Find diff^n of $\tan^{-1}\left(\frac{2x}{1-x^2}\right)$ WRT $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$ at $x=-3$.

$$\tan^{-1}\left(\frac{2x}{1-x^2}\right)=\begin{cases}\pi+2\tan^{-1}x & x<-1\\ 2\tan^{-1}x & -1<x<1\\ -\pi+2\tan^{-1}x & x>1\end{cases}$$

$\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$

$\left(-\frac{\pi}{4},\frac{\pi}{4}\right)$

$(-1,1)$

$$\sin^{-1}\left(\frac{2x}{1+x^2}\right)=\begin{cases}-\pi-2\tan^{-1}x & x\le-1\\ 2\tan^{-1}x & -1\le x\le 1\\ \pi-2\tan^{-1}x & x>1\end{cases}$$

Find diff^n of $\pi+2\tan^{-1}x$ WRT. $-\pi-2\tan^{-1}x$ $(x=-3)$

$y=\pi+2\tan^{-1}x$ $\qquad$ $z=-\pi-2\tan^{-1}x$

$$\frac{dy}{dx}=\frac{2}{1+x^2} \qquad \frac{dz}{dx}=\frac{-2}{1+x^2} \qquad \frac{dy}{dz}=\frac{\frac{2}{1+x^2}}{\frac{-2}{1+x^2}}=-1$$