Nishant Jindal

Imp. Title

Proving Inequality Using Calculus.

Q $x \in (0, \frac{\pi}{2})$ Check $x > \sin x$ or not?

① Assume fxn taking difference

$f(x) = x - \sin x$

② Check Monotonic Behaviour & make graph. (NKLI)

$f'(x) = 1 - \cos x = 2\sin^2\frac{x}{2} \geq 0$

$f(x)$ ↑ing

(3) Now Compare $f(x)$, & $f(0)$

$f(x) > f(0)$

$x - \sin x > 0 - \sin 0$

$x > \sin x$ is True.

Q for $x \leq 0$

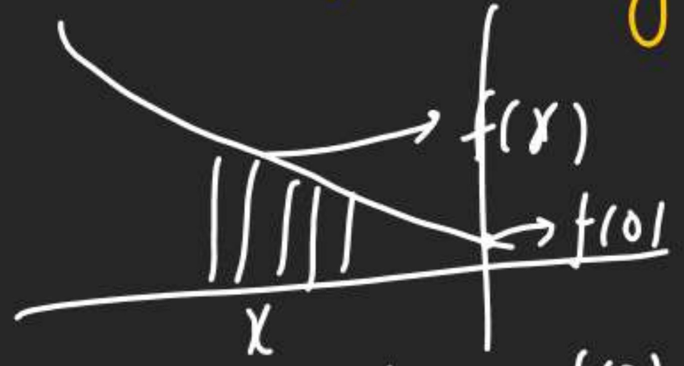
$e^x \geq x+1$ [T/F]

① $f(x) = e^x - x - 1$

② $f'(x) = e^x - 1 = -ve$ $\quad x \leq 0$

$f'(x) \leq 0$

$f(x)$ is ↓ing



$f(x) \geq f(0)$

$e^x - x - 1 \geq e^0 - 0 - 1$

$e^x \geq x + 1$

Q $f(x) = x\ln x - x + 1$ is +ve in $(1, \infty)$?

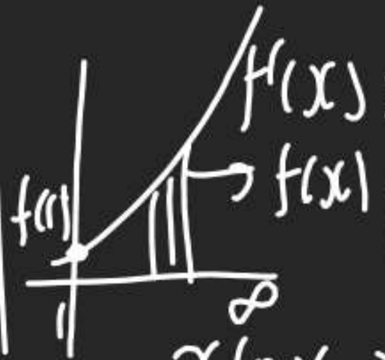
$f(x) > 0$

$x\ln x > x - 1$ [To prove]

① $f(x) = x\ln x - x + 1$

② $f'(x) = \frac{x}{x} + \ln x - 1 = \ln x$ +ve $\quad x \in (1, \infty)$

$f'(x) > 0 \Rightarrow f(x)\uparrow$



③ $f(x) > f(1)$

$x\ln x - x + 1 > 1\cdot\ln 1 - \cancel{1} + \cancel{1}$

$x\ln x - x + 1$ is +ve in $(1, \infty)$

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Q $f(x) = x\ln x - x + 1$ is +ve in $x \in (0,1)$?

① $f'(x) = \frac{\cancel{x}}{\cancel{x}} + \ln x - 1 = \ln x$, $x \in (0,1)$ $= -ve$

② $f(x)$ ↓ing in $x \in (0,1)$

$f(x) > f(1)$

$x\ln x - x + 1 > 1\ln 1 - \cancel{1} + \cancel{1}$

$x\ln x - x + 1 > 0$ for $x \in (0,1)$

By Q3,4 We can say that $f(x) = x\ln x - x + 1$ is +ve for $x \in (0,1)$

Q $f(x) = \frac{x}{\sin x}$ is ↑ or ↓ in $x \in (0,1)$?

① $f'(x) = \frac{\sin x - x\cos x}{\sin^2 x} = \frac{\oplus}{\oplus} = +ve$

$\Rightarrow f(x)$ is ↑

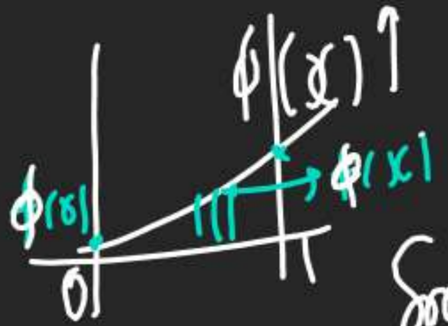
② Nr is +ve or -ve we dont know

$\phi(x) = \sin x - x\cos x$

$\phi'(x) = \cancel{\cos x} + \boxed{x}\,\boxed{\sin x} - \cancel{\cos x}$

$x \in (0,1)$

$x \in (0°, 57°)$



$\phi(x) > \phi(0)$

$\sin x - x\cos x > \sin 0 - 0\cos 0$

$\sin x - x\cos x > 0$

$\sin x > x\cos x$

Recycling of famous Inequality Result

Q 6 $x \in (0, \frac{\pi}{2})$ Prove that $\sin(\cos x) < \cos(\sin x)$

① Result we know $\sin x < x$

$x \to \cos x$ Put $\sin(\cos x) < \cos x \to A$

② Putting Inequality in $\cos x$ ↓ing

$\cos(\sin x) > \cos(x) \to B$

$A \& B \Rightarrow \cos(\sin x) > \cos x > \sin(\cos x)$

$\cos(\sin x) > \sin(\cos x)$

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Q P.T.

$$e^x+\sqrt{1+e^{2x}} \geq (1+x)+\sqrt{2+2x+x^2}$$

Carefully see

$$e^x+\sqrt{1+(e^x)^2} \geq (1+x)+\sqrt{1+(1+x)^2}$$

$$f(t)=t+\sqrt{1+t^2} \quad ; t>0$$

$$f'(t)=1+\frac{2t}{2\sqrt{1+t^2}}=\frac{t+\sqrt{1+t^2}\ \oplus}{\sqrt{1+t^2}\ \oplus} \text{ +ve}$$

$f(t)\uparrow$ ing. Inc. fxn में $e^x \geq x+1$ को Q② लगाकर

$$f(e^x) \geq f(x+1)$$

$$e^x+\sqrt{1+e^{2x}} \geq (1+x)+\sqrt{1+(1+x)^2}$$

HP

Q P.T.

$$1+x\cdot\ln\left(x+\sqrt{1+x^2}\right) \geq \sqrt{1+x^2} \quad x\geq 0$$

$$f(x)=1+x\ln\left(x+\sqrt{1+x^2}\right)-\sqrt{1+x^2}$$

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Q $1 + x\ln(x+\sqrt{1+x^2}) \geq \sqrt{1+x^2},\ x \geq 0$

$f(x) = 1 + x\ln(x+\sqrt{1+x^2}) - \sqrt{1+x^2}$

$f'(x) = \frac{\cancel{x}}{\sqrt{1+x^2}} + \ln(x+\sqrt{1+x^2}) - \frac{\cancel{x}}{\sqrt{1+x^2}}$

$x \geq 0$

$f'(x) \geq 0 \Rightarrow f(x) \uparrow$ ing

$f(x) \geq f(0)$

$1 + x\ln(x+\sqrt{1+x^2}) - \sqrt{1+x^2} \geq \cancel{1} + 0 - \cancel{1}$

$1 + x\ln(x+\sqrt{1+x^2}) \geq \sqrt{1+x^2}$ [H.P.]

① $e^x \cdot f'(x) + e^x \cdot f(x) = \frac{d}{dx}(e^x \cdot f(x))$

② $e^{-x} \cdot f'(x) - e^{-x} \cdot f(x) = \frac{d}{dx}(e^{-x} \cdot f(x))$

$\frac{d(e^{g(x)} \cdot f(x))}{dx} = e^{g(x)} \cdot f'(x) + f(x) \cdot e^{g(x)} \cdot g'(x)$

Q $\lim_{x \to 0}\left[\frac{\sin x \cdot \tan x}{x^2}\right]_{\to \text{GIF}} = [1 \text{ से बड़ा}] = 1$ A

$f(x) = \sin x \cdot \tan x - x^2$

$f'(x) = \sin x \cdot \sec^2 x + \tan x \cdot \cos x - 2x$

$= \tan x \cdot \sec x + \tan x \cdot \cos x - 2x$

$= \underbrace{\tan x}_{> x}\underbrace{\left(\cos x + \frac{1}{\cos x}\right)}_{> 2} - 2x$

$> 2x \qquad = +ve$

$f'(x) = +ve \Rightarrow f(x) \uparrow$ ing

$f(x) > f(0)$

$\sin x \cdot \tan x - x^2 > 0$

$\sin x \tan x > x^2$

$\frac{\sin x \tan x}{x^2} > 1$

Expression	Multiply
① $f'(x) + f(x)$	e^x
2) $f'(x) - f(x)$	e^{-x}
3) $f'(x) + g'(x) \cdot f(x)$	$e^{g(x)}$
4) $f''(x) + 2f'(x) + f(x)$	e^x
5) $f''(x) - 2f'(x) + f(x)$	e^{-x}

$\frac{d^2(e^x \cdot f(x))}{dx^2} \Rightarrow$ ① $e^x \cdot f'(x) + f(x) \cdot e^x$

② $e^x \cdot f''(x) + f'(x) e^x + f(x) \cdot e^x + e^x \cdot f'(x)$

$e^x\{f''(x) + 2f'(x) + f(x)\}$

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Q If $P(1)=0$, & $\frac{d(P(x))}{dx} > P(x)$ for all $x \geq 1$ then P.T. $P(x) > 0$ for all $x > 1$

① given $P'(x) > P(x)$

$P'(x) - P(x) > 0$ $\quad e^{-x}$

$e^{-x} \cdot P'(x) - e^{-x} \cdot P(x) > 0$

$\frac{d(e^{-x} \cdot P(x))}{dx} > 0$

$e^{-x} \cdot P(x)$ is ↑ fxn $x \geq 1$

$e^{-1} \cdot P(1)$

$e^{-x} \cdot P(x)$

$e^{-x} P(x) \geq e^{-1} \cdot P(1)$

$e^{-x} P(x) \geq 0$

$P(x) \geq 0$

[H.P.]

Q $f : [0,1] \to R$

Suppose fxn is twice diff^ble

$f(0) = f(1) = 0$ Satisfies

$f''(x) - 2f'(x) + f(x) \geq e^x$, $x \in [0,1]$

then WOTF is True for $x \in (0,1)$

A) $0 \leq f(x) < \infty$ B) $-\frac{1}{4} < f(x) < 1$

C) $-\frac{1}{2} < f(x) < \frac{1}{2}$ (D) $-\infty < f(x) < 0$

① $(f''(x) - 2f'(x) + f(x)) \geq e^x$

$(f''(x) - 2f'(x) + f(x)) e^{-x} \geq 1$

$\frac{d^2}{dx^2}(e^{-x} \cdot f(x)) \geq 1$

Practive ②*

$\frac{d^2}{dx^2}(e^{-x} f(x)) \geq 0$ $\quad x \in [0,1]$

$\Rightarrow e^{-x} \cdot f(x)$ is Con. Upward $x \in [0,1]$

let $g(x) = e^{-x} \cdot f(x)$ is Con. up

$g(0) = e^{-0} \cdot f(0) = 0$

$g(1) = e^{-1} \cdot f(1) = 0$

Correct diagram

Possible नहीं है।

$\Rightarrow$ $g(x)$ in $(0,1)$ Below X Axis

$\Rightarrow g(x) < 0 \Rightarrow e^{-x} \cdot f(x) < 0$

$f(x) < 0$ in $(0,1)$

$\Rightarrow$

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Q Let $f(x)=(1-x)^2\sin^2 x+x^2 \ \forall x\in R$

$$g(x)=\int_1^x\left(\frac{2(t-1)}{t+1}-\ln t\right)f(t)\,dt \ \forall x\in(1,\infty)$$

A) $g\uparrow$ in $(1,\infty)$ B) $g\downarrow$ $(1,\infty)$

C) $g\uparrow(1,2)\downarrow$ in $(2,\infty)$

D) $g\downarrow(1,2)\uparrow$ in $(2,\infty)$

① $g'(x)=\left(\underbrace{\frac{2(x-1)}{x+1}-\ln x}\right)\underbrace{f(x)}_{\oplus}$

$f(x)$
$=(1-x)^2.\sin^2 x+x^2$
$\oplus \quad \oplus \quad \oplus$

② $\phi(x)=\frac{2(x-1)}{x+1}-\ln x$

$$\phi'(x)=\frac{(x+1)2-2(x-1)}{(x+1)^2}-\frac{1}{x}=\frac{4}{(x+1)^2}-\frac{1}{x}$$

$$\phi'(x)=\frac{4x-(x+1)^2}{x(x+1)^2}$$

$$\phi'(x)=-\frac{(x-1)^2}{(x)(x+1)^2}$$

$x\in(1,\infty)$

$\phi'(x)=-ve$

$\phi(x)\downarrow$ing in $(1,\infty)$