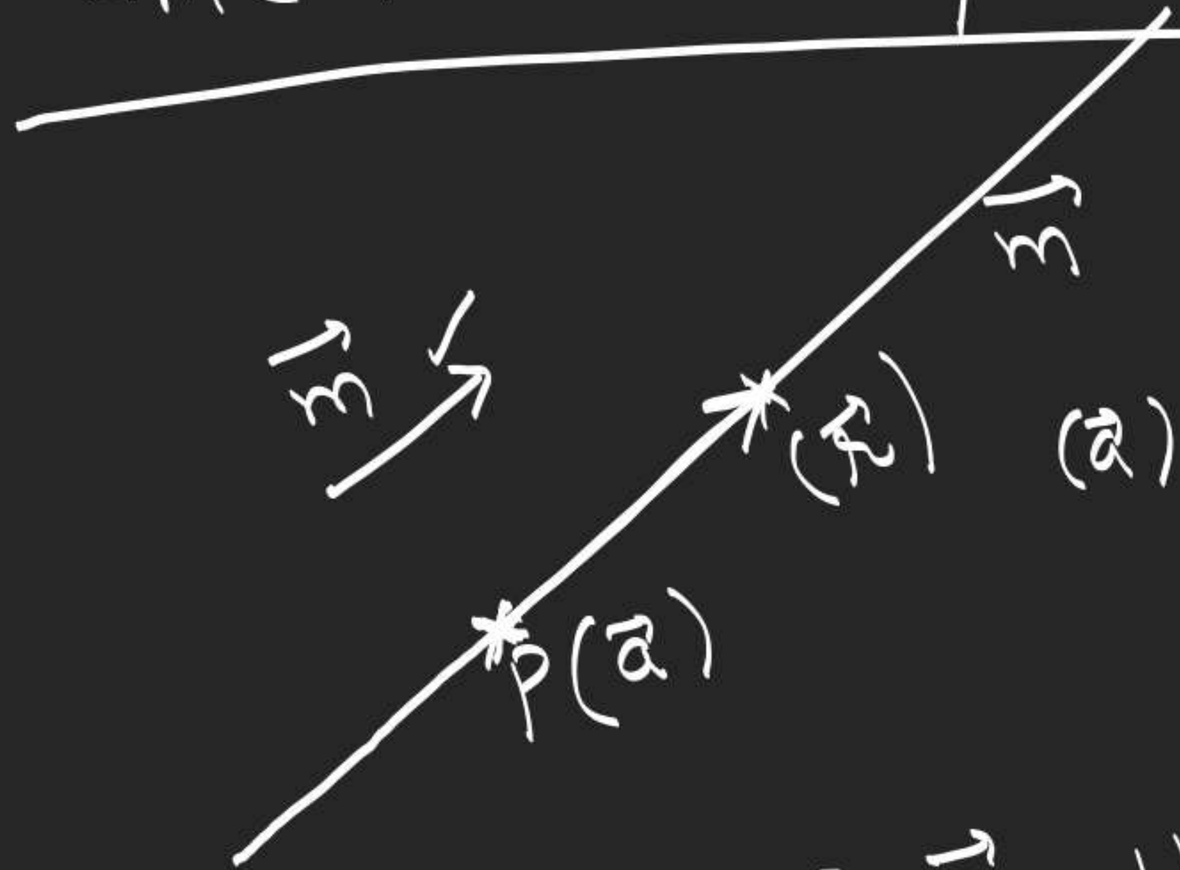
# Line in 3D Space




$\vec{m} \parallel$ to line.

$(\vec{a})$ p.v. of point on line.

$\vec{r} - \vec{a} \parallel \vec{m}$

$\vec{r} - \vec{a} = \lambda \vec{m}$ , $\lambda \in R$.

$$\boxed{\vec{r} = \vec{a} + \lambda \vec{m}}$$ Vector form.

$\vec{r} = \hat{i} - \hat{j} + \lambda(2\hat{i} - \hat{j} + \hat{k})$

$\vec{m} = 2\hat{i} - \hat{j} + \hat{k}$.

# 2 lines in 3D Space

- Coplanar
  - intersecting
  - Parallel
- Non coplanar
  - neither intersecting nor parallel → **Skew lines**

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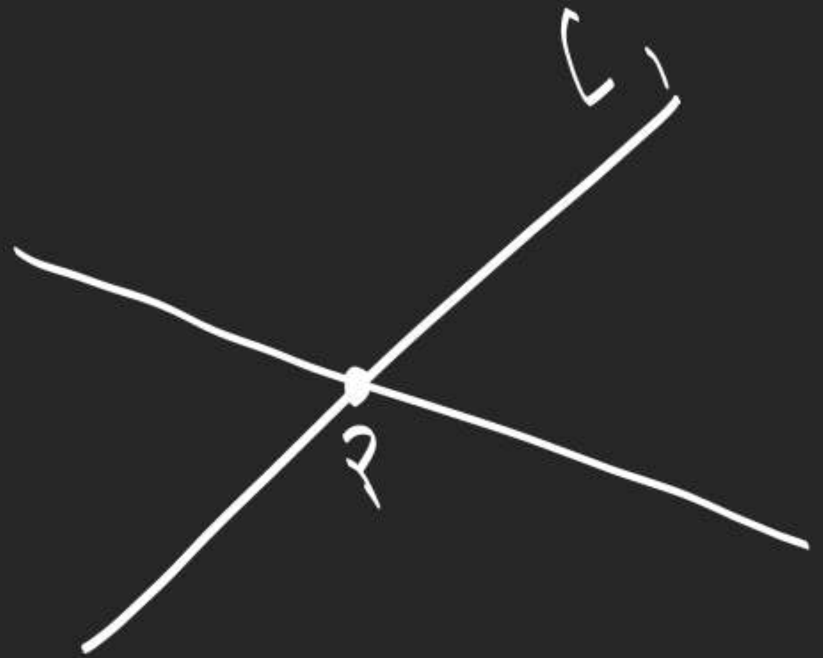
$\vec{r} = \vec{a_1} + \lambda \vec{m_1} \rightarrow L_1$

$\vec{r} = \vec{a_2} + \mu \vec{m_2} \rightarrow L_2$

find point of intersection if exist.




$\underline{P}$ : $\vec{a_1} + \lambda \vec{m_1} = \vec{a_2} + \mu \vec{m_2}$

$\hat{i}, \hat{j}, \hat{k}$

$\lambda, \mu = ?$

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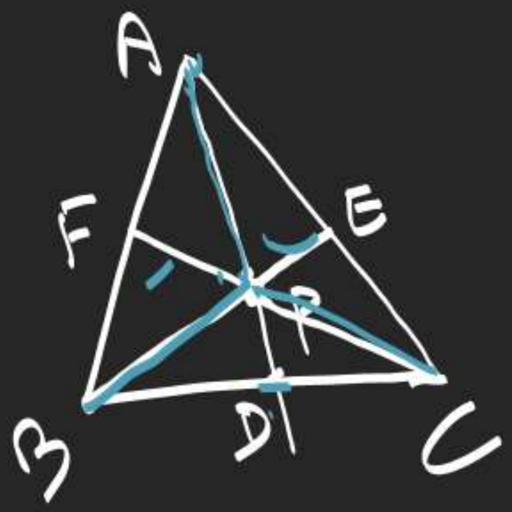
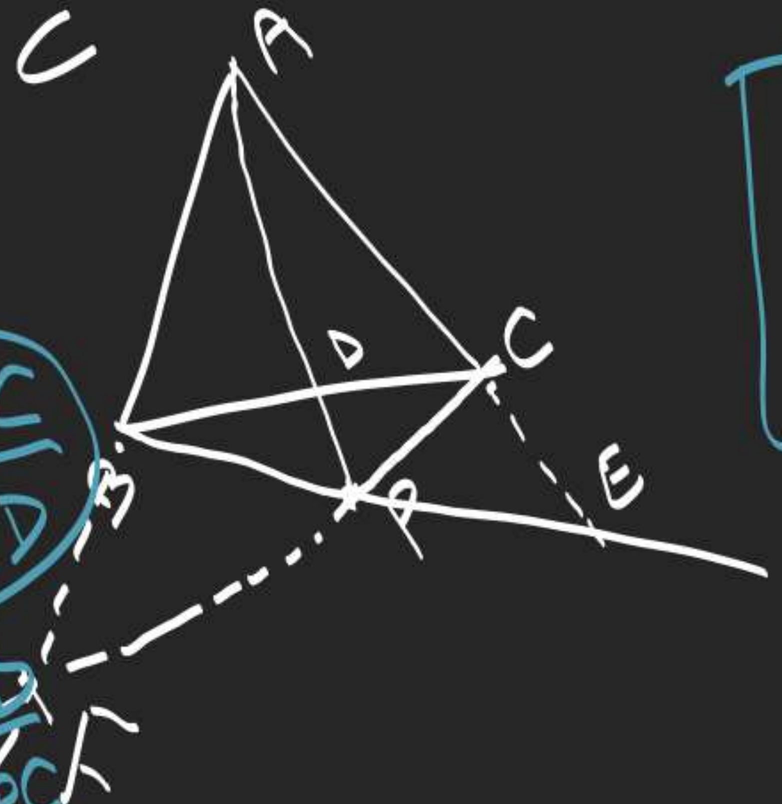
# Ceva's Theorem




$$\frac{\cancel{\Delta APB}}{\cancel{\Delta APC}} \times \frac{\cancel{\Delta BPC}}{\cancel{\Delta APB}} \times \frac{\cancel{\Delta APC}}{\cancel{\Delta BPC}}$$

$$\boxed{\frac{BD}{DC} \times \frac{CE}{AE} \times \frac{AF}{FB} = 1}$$

$$\boxed{\frac{BD}{CD} = \frac{\Delta APB}{\Delta APC}}$$




$$\frac{BD}{DC} = \frac{\Delta BPD}{\Delta CPD}$$

$$\frac{PA}{PD} = \frac{\Delta APB}{\Delta BPD} = \frac{\Delta APC}{\Delta CPD}$$

$$\Rightarrow \frac{\Delta BPD}{\Delta CPD} = \frac{\Delta APB}{\Delta APC}$$

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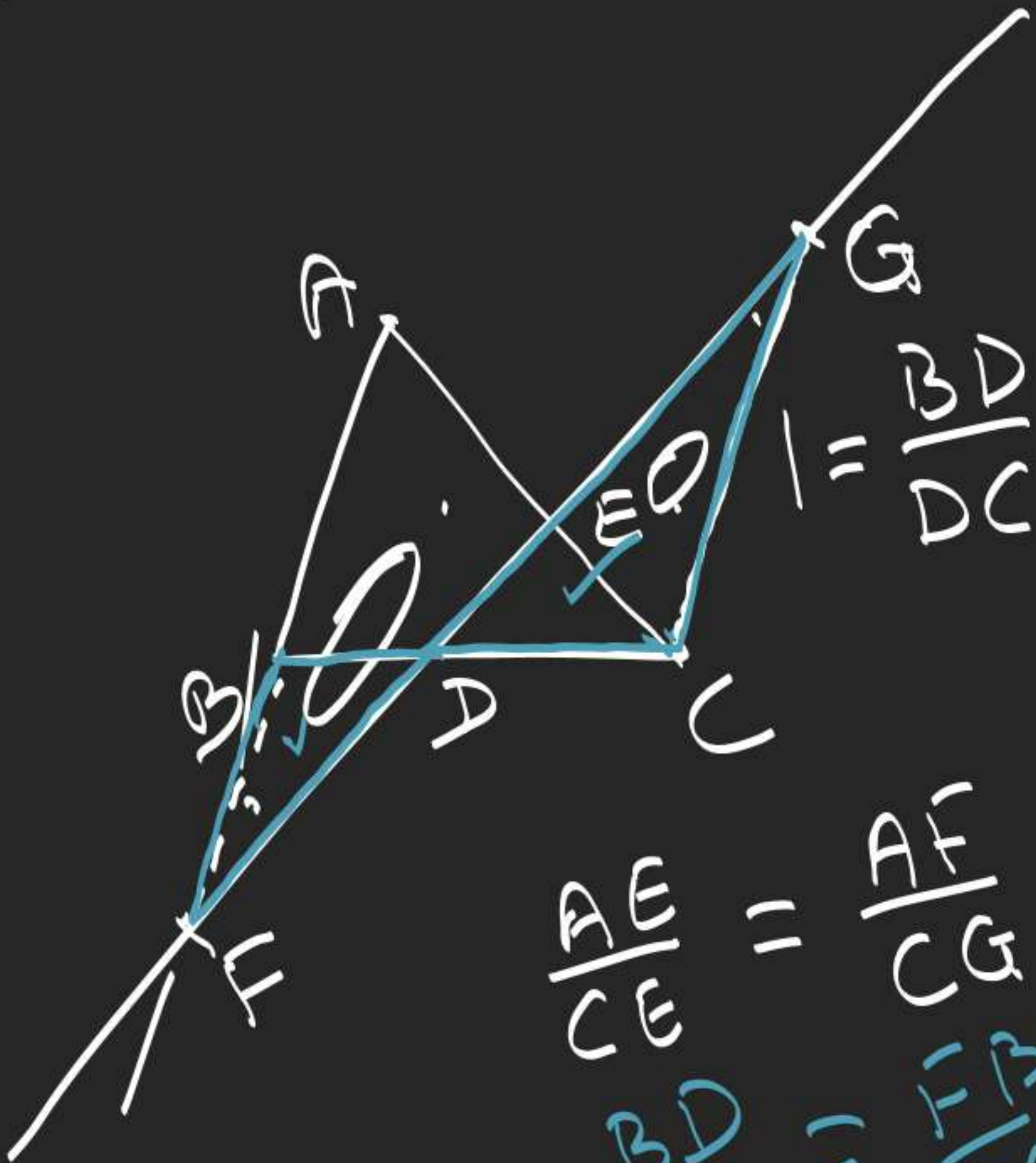
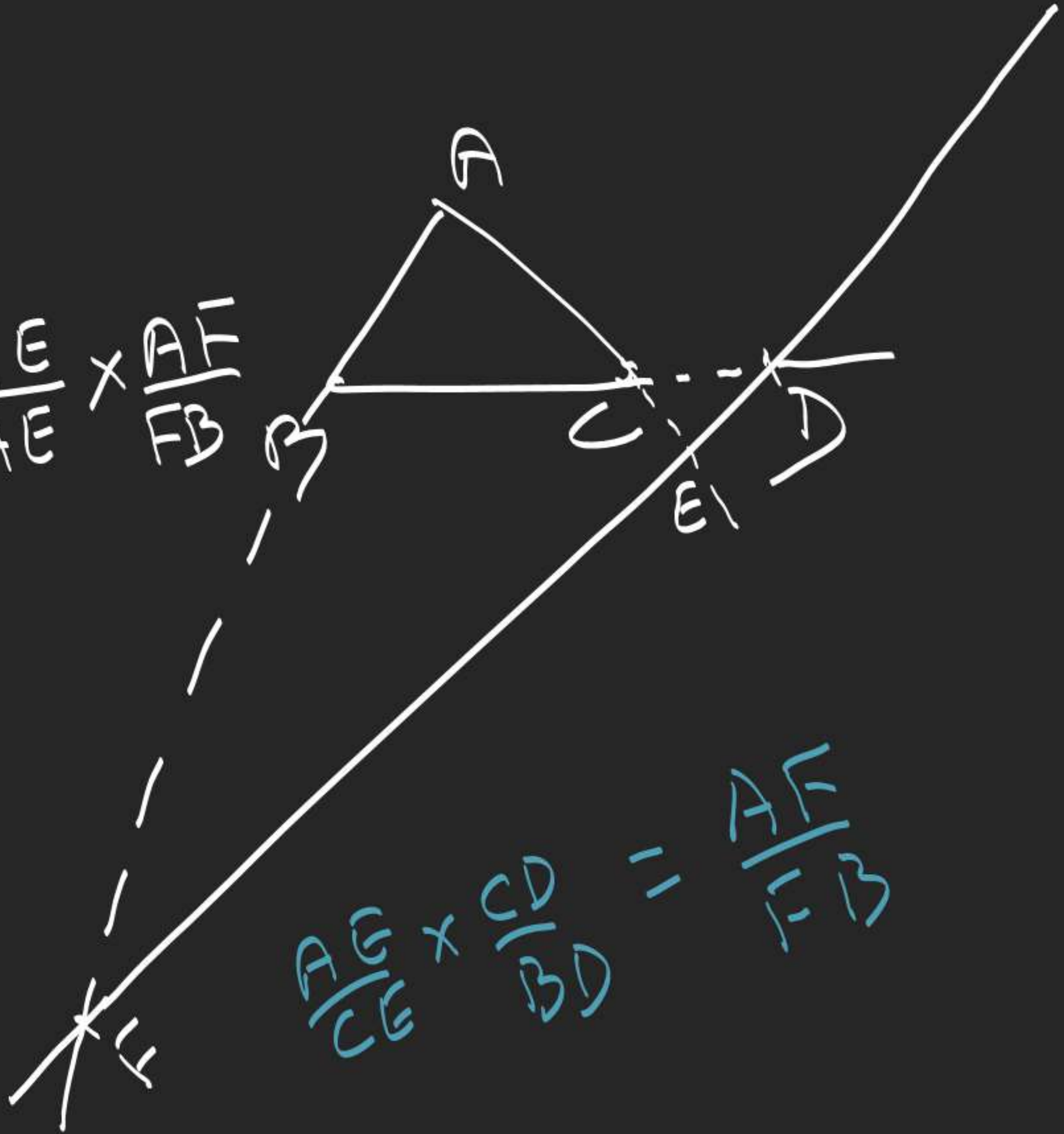
# Menalau's Theorem




$$1 = \frac{BD}{DC} \times \frac{CE}{AE} \times \frac{AF}{FB}$$

$$\frac{AE}{CE} = \frac{AF}{CG}$$

$$\frac{BD}{CD} = \frac{FB}{CG}$$




$$\frac{AE}{CE} \times \frac{CD}{BD} = \frac{AF}{FB}$$

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1. Find the p.v. of point of intersection of lines

(i) $\vec{r} = \hat{i} - \hat{j} - 10\hat{k} + \lambda(2\hat{i} - 3\hat{j} + 8\hat{k})$

$\vec{r} = 4\hat{i} - 3\hat{j} - \hat{k} + \mu(\hat{i} - 4\hat{j} + 7\hat{k})$ $\rightarrow (5, -7, 6)$

$1 + 2\lambda = 4 + \mu$

$-1 - 3\lambda = -3 - 4\mu$ $\quad \lambda = 2, \mu = 1$

$-10 + 8\lambda = -1 + 7\mu$

(ii) $\vec{r} = -3\hat{i} + 6\hat{j} + \lambda(-4\hat{i} + 3\hat{j} + 2\hat{k})$

$\vec{r} = -2\hat{i} + 7\hat{k} + \mu(-4\hat{i} + \hat{j} + \hat{k})$ skew

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(iii) $\vec{r} = \lambda(3\hat{i} - \hat{j} + \hat{k})$ parallel & distinct

$(0,0,0)$

$\vec{r} = 2\hat{i} + \mu(-6\hat{i} + 2\hat{j} - 2\hat{k})$

$(2,0,0)$

(iv) $\vec{r} = 2\hat{k} + \lambda(3\hat{i} + 2\hat{j} + \hat{k})$ coincident

$\vec{r} = 3\hat{i} + 2\hat{j} + 3\hat{k} + \mu(6\hat{i} + 4\hat{j} + 2\hat{k})$

$(3,2,3)$

$\vec{r} = 2\hat{k} + 3\hat{i} + 2\hat{j} + \hat{k} + 2\mu(3\hat{i} + 2\hat{j} + \hat{k})$

$\vec{r} = 2\hat{k} + (2\mu + 1)(3\hat{i} + 2\hat{j} + \hat{k})$

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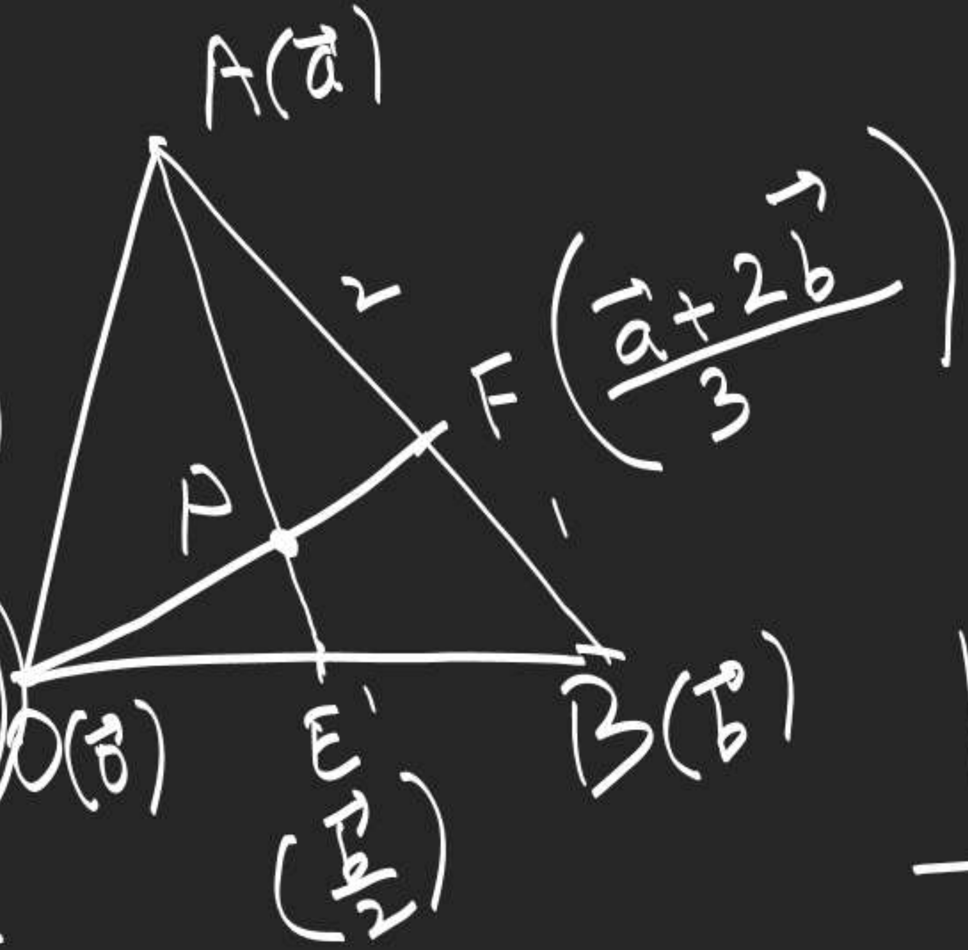
2. In $\Delta AOB$, E is the midpoint of OB and F divides BA in the ratio 1 : 2. AE & OF intersect at P. Find the ratio $\frac{OP}{PF}$.




$\underline{AE} \rightarrow \vec{r} = \vec{a} + \lambda(\vec{b} - 2\vec{a})$

$\underline{OF} \rightarrow \vec{r} = \vec{0} + \mu(\vec{a} + 2\vec{b})$

$\underline{P}$ $\quad \vec{a} + \lambda(\vec{b} - 2\vec{a}) = \mu(\vec{a} + 2\vec{b})$

$$\left.\begin{aligned} 1 - 2\lambda &= \mu \\ \lambda &= 2\mu \end{aligned}\right\} \begin{aligned} &\lambda = \\ &\mu = \tfrac{1}{5} \end{aligned}$$

$\frac{1}{3} - \frac{1}{5} = \frac{2}{15}$

$P = \frac{\vec{a} + 2\vec{b}}{5}$

$$\frac{|\overrightarrow{OP}|}{|\overrightarrow{PF}|} = \frac{\left|\frac{\vec{a}+2\vec{b}}{5}\right|}{\left|\frac{2}{15}(\vec{a}+2\vec{b})\right|} = \frac{3}{2}$$

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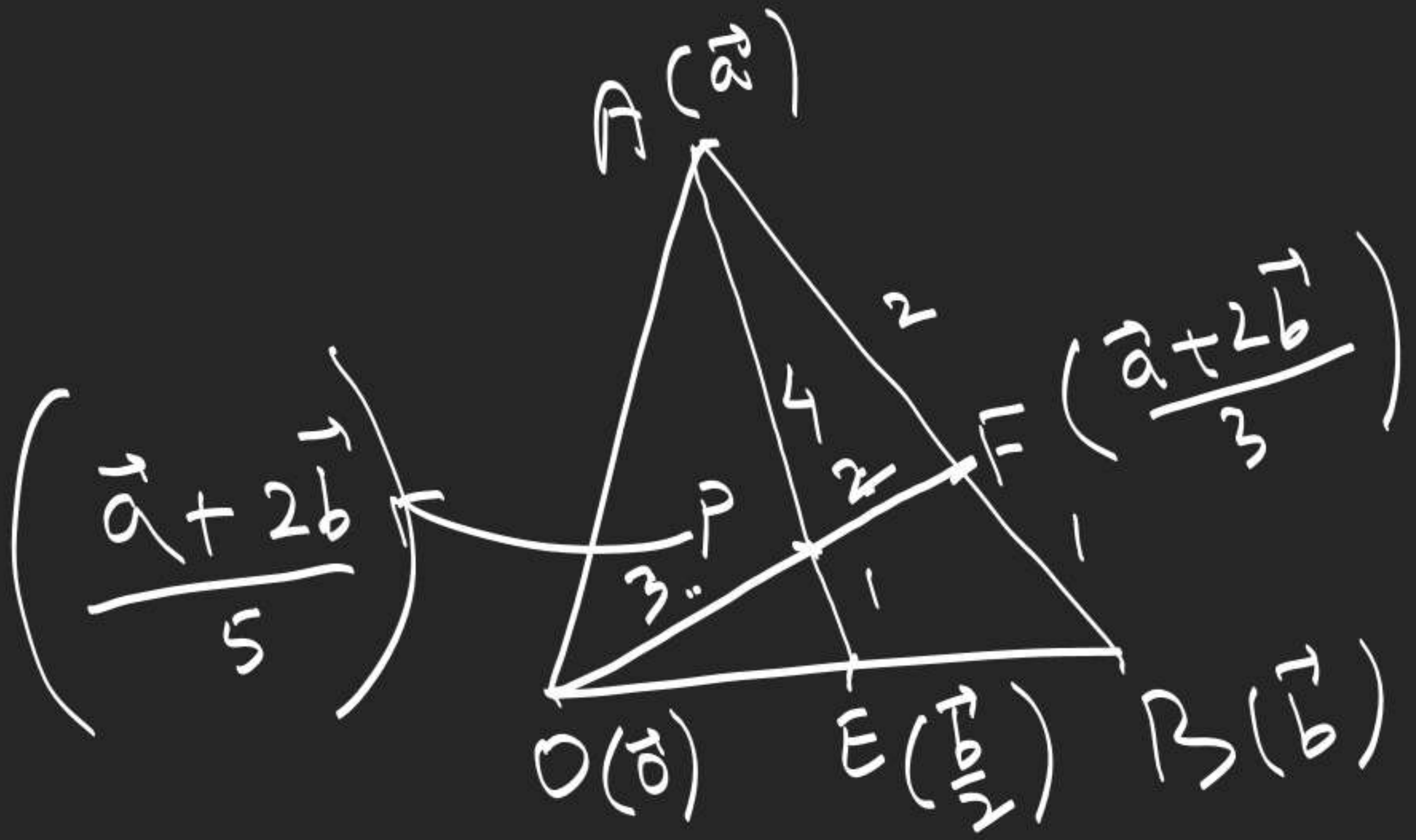

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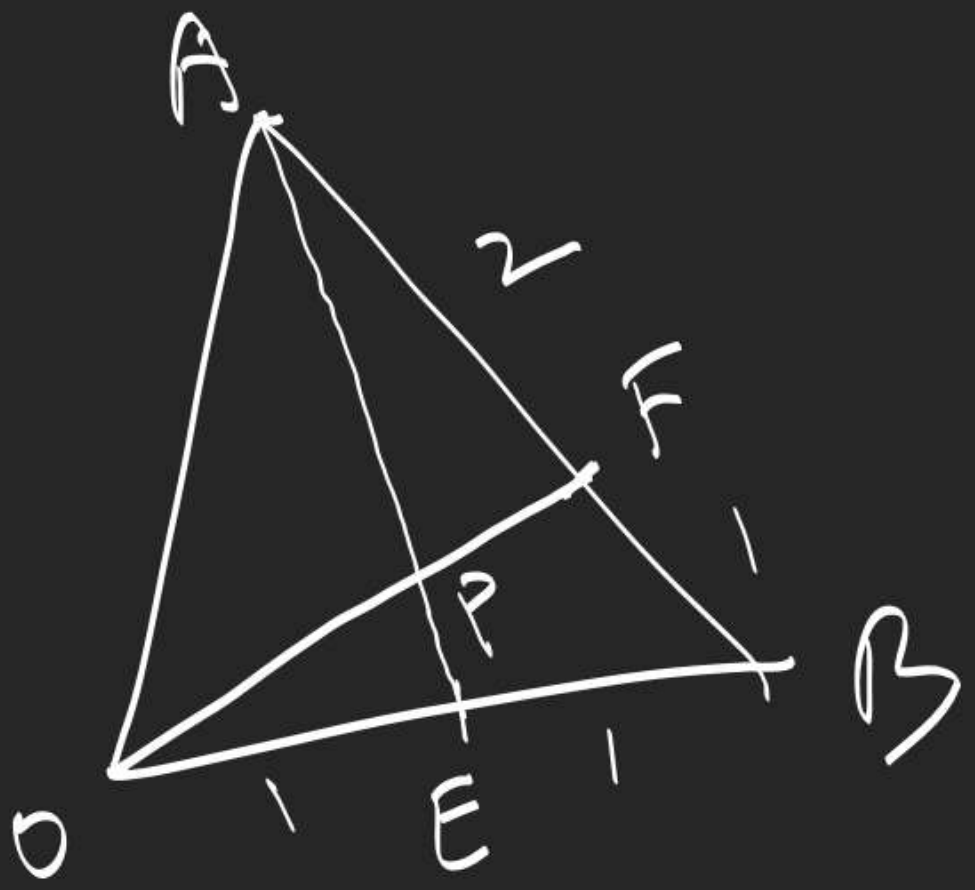


$\triangle OFB$

$$\frac{OP}{PF} \times \frac{2}{3} \times \frac{1}{1} = 1$$

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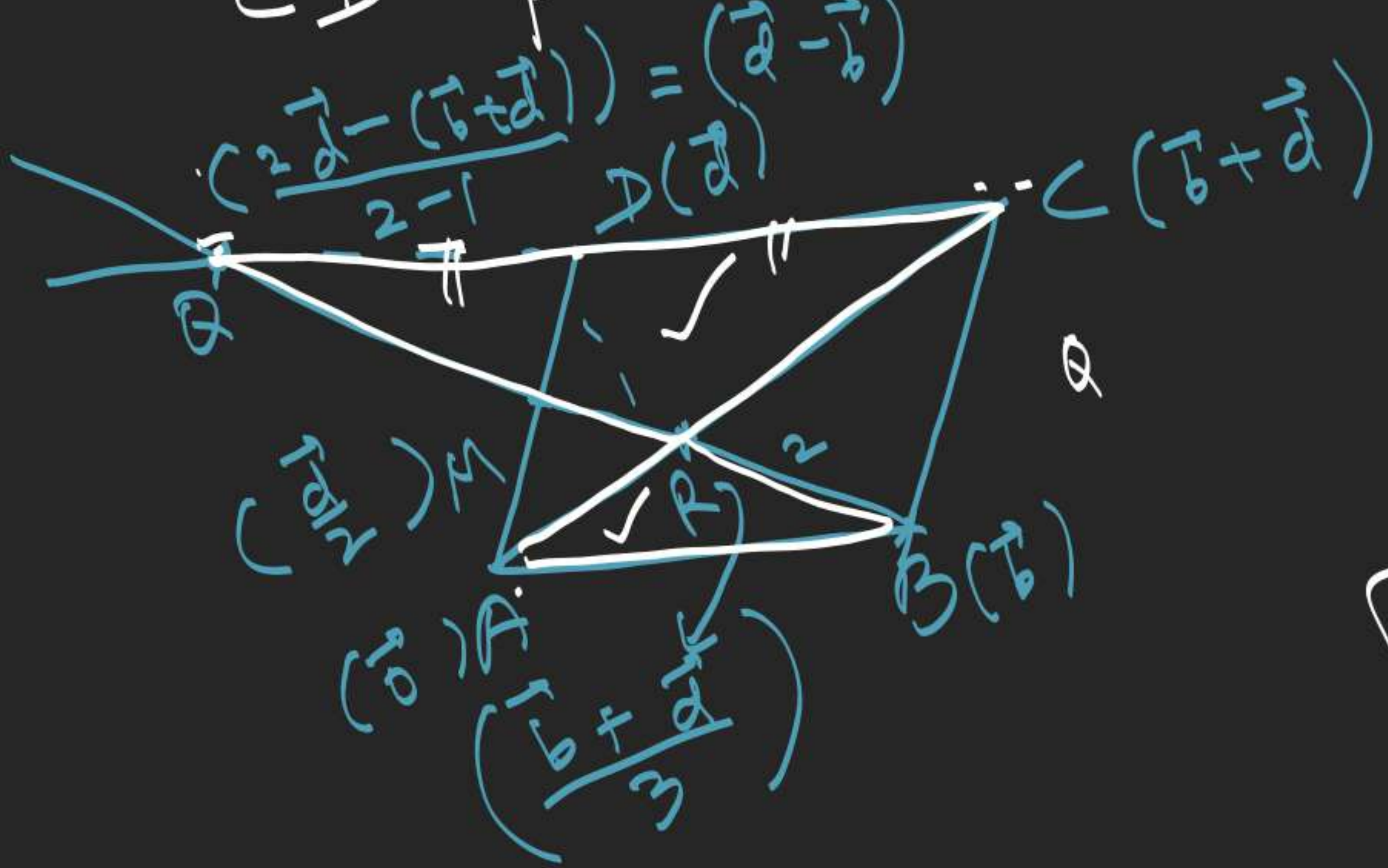
3. Thru the middle point M of the side AD of parallelogram ABCD, a straight line BM is drawn intersecting AC at R and CD produced at Q. Find the ratio $\frac{QR}{RB}$.




$$\frac{|\vec{QR}|}{|\vec{RB}|} = 2$$

$$\frac{QR}{RB} = \frac{QC}{AB} = 2$$

$\boxed{6-30} \rightarrow DI$

P-1