DPP 02

SOLUTION

1. Let the two ends of the object be at distance $u_1 = u - L/2$ and $u_2 = u + L/2$, respectively, so that $|u_1 - u_2| = L$. Let the image of the two ends be formed at v_1 and v_2, so that the image length would be

$L' = |v_1 - v_2|$. Since $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$ or $v = \frac{fu}{u - f}$

the image of the two ends will be at $v_1 = \frac{f(u-L/2)}{u-f-L/2}, v_2 = \frac{f(u+L/2)}{u-f+L/2}$

Hence $L' = |v_1 - v_2| = \frac{f^2 L}{(u-f)^2 \times L^2/4}$

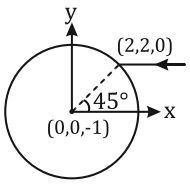
Since the object is short and kept away from focus, we have

$L^2/4 << (u - f)^2$

Hence finally

$L' = \frac{f^2}{(u-f)^2} L$

2. Direction of incident ray, $\hat{e} = -\hat{i}$



Direction of normal ray, $\hat{n} = \frac{2\hat{i}+2\hat{j}+\hat{k}}{3}$

So, direction of reflected ray,

$\hat{r} = \hat{e} - 2(\hat{e} \cdot \hat{n})\hat{n}$

$\Rightarrow \hat{r} = (-\hat{i}) - 2\left[\left(\frac{-2}{3}\right)\frac{2\hat{i} + 2\hat{j} + \hat{k}}{3}\right]$

$\Rightarrow \hat{r} = (-\hat{i}) + \frac{4}{9}(2\hat{i} + 2\hat{j} + \hat{k})$

$\Rightarrow \hat{r} = \frac{-\hat{i} + 8\hat{j} + 4\hat{k}}{9}$

According to the problem, the unit vector along the direction of reflected ray is $x\hat{i} + y\hat{j} + z\hat{k}$.

Now, on comparisons of two equations, we get the values of x, y and z as $-1/9, 8/9$ and $4/9$.

So, $\frac{yz}{x^2} = \frac{8/9 \times 4/9}{(-1/9)^2} = 32$

3. $u = -30$ cm, $v = 10$ cm

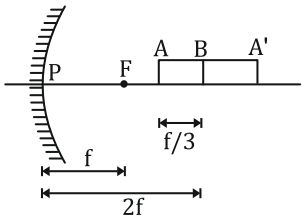
$v_{object} = 9$ cm/s

The velocity of image is given by

$$v_i = \frac{v^2}{u^2} \times v_0; v_i = \frac{10 \times 10}{-30 \times 30} \times 9$$

$$v_i = 1 \text{ cm/s}$$

4. The question deals with lateral magnification when rod is placed along optic axis of concave mirror. Image of A is formed at A′. Image of B is formed at B itself. B should therefore be at centre of curvature of mirror.



AB lies between f and 2f for real and elongated image.

Object distance for A = PA.

$$\therefore\ u = PB - AB = 2f - \frac{f}{3} = \frac{5f}{3}$$

Formula for mirror: $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

$$\therefore\ \frac{1}{v} + \frac{1}{-5f/3} = \frac{1}{-f}$$

$$\text{or } v = -\frac{5}{2}f = -2.5f\ \therefore PA' = 2.5f$$

$$\text{or } BA' = PA' - PB = 2.5f - 2f$$

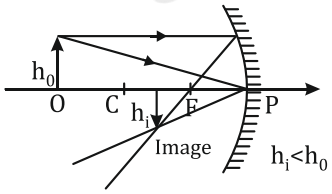
$$\text{or } BA' = 0.5f$$

$$\therefore\ \text{Magnification} = \frac{BA'}{BA}$$

$$\text{or } M = \frac{0.5f}{f/3} \text{ or } M = 1.5$$

5. The object is placed beyond the centre of curvature (C) of the concave mirror.

Therefore, the image formed is real, inverted and diminished or unmagnified.



6. When object is at 8 cm, image distance

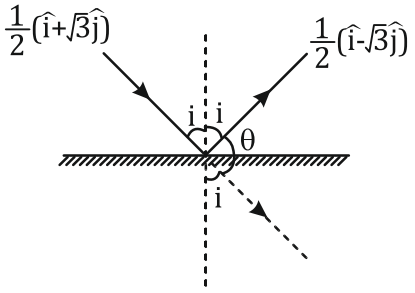
$$v_1 = \frac{f \times u}{u - f} = \frac{(-5)(-8)}{-8 + 5} = -\frac{40}{3} \text{ cm}$$

When object is at 12 cm, image distance

$$v_2 = -\frac{60}{7} \text{ cm}$$

$$\text{Separation} = |v_1 - v_2| = \frac{100}{21} \text{ cm}$$

7. Let θ be angle between the given rays.



$$\therefore \cos\theta = \frac{\left(\frac{\hat{i}+\sqrt{3}\hat{j}}{2}\right)\cdot\left(\frac{\hat{i}-\sqrt{3}\hat{j}}{2}\right)}{\left|\frac{\hat{i}+\sqrt{3}\hat{j}}{2}\right|\left|\frac{\hat{i}-\sqrt{3}\hat{j}}{2}\right|} = \frac{\frac{1-3}{4}}{1} = -\frac{1}{2}$$

$$\theta = \cos^{-1}\left(-\tfrac{1}{2}\right) = 120^\circ$$

$\therefore$ Angle of incidence,

$$i = \frac{180^\circ-\theta}{2} = \frac{180^\circ-120^\circ}{2} = 30^\circ$$

8. For the mirror, $\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$

Differentiate this equation w.r.t. t, we get

$$-\frac{1}{u^2}\frac{du}{dt}-\frac{1}{v^2}\frac{dv}{dt}=0 \Rightarrow \frac{dv}{dt}=-\frac{v^2}{u^2}\left(\frac{du}{dt}\right)$$

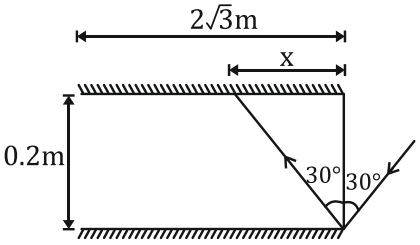
But $\frac{v}{u}=\frac{f}{u-f}$

$$\therefore \frac{dv}{dt} = -\left(\frac{f}{u-f}\right)^2\left(\frac{du}{dt}\right)$$

$$= \left(\frac{0.2}{-2.8-0.2}\right)^2 \times 15 = \frac{1}{15}\ ms^{-1}$$

9. $n = \frac{360^\circ}{\theta}-1 = \frac{360^\circ}{60^\circ}-1 = 5$

10. Let N = no. of reflections

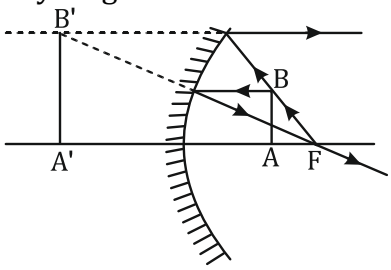


$\therefore$ $Nx = 2\sqrt{3}$ or $N = \frac{2\sqrt{3}}{x}$

or $N = \frac{2\sqrt{3}}{0.2\tan 30^\circ} = \frac{6}{0.2}$

or $N = 30$.

11. Ray diagram is not to scale.



Distance of point A from the mirror is f/2.
Using mirror formula,

$$\frac{1}{v}+\frac{1}{-f/2}=\frac{1}{-f}\Rightarrow\frac{1}{v}=\frac{2}{f}-\frac{1}{f}=\frac{1}{f}\therefore v=f$$

Image A′B′ of line AB should be perpendicular to the principal axis. Image of F will be formed at infinity.

Also light ray from infinity or towards infinity seems parallel to the principal axis of the mirror.

12. $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}\because -\frac{dv}{v^2}-\frac{du}{u^2}=0$

$$\because \frac{dv}{du}=\frac{-v^2}{u^2}=-\left(\frac{f}{u-f}\right)^2$$

$$\because \text{ image length } =\left(\frac{f}{f-u}\right)^2\times b$$

13. We know that the ray passing through focus after reflection get parallel to the principal axis. Thus, the ray 2 correctly represent the reflected ray.