

H.W.

WORK POWER ENERGY

- Q.4** A particle free to move along x -axis has potential energy given by $U(x) = K[1 - \exp(-x)^2]$ for $-\infty \leq x \leq \infty$, where K is a positive constant of appropriate dimensions. Then:
- (A) At point away from the origin, the particle is in unstable equilibrium
 - (B) For any finite non-zero value of x , there is force directed away from the origin
 - (C) If its total mechanical energy is $K/2$, it has its minimum KE at the origin
 - (D) For small displacements from $x = 0$, the motion is simple harmonic.

Q.7 The potential energy of a 2 kg particle free to move along the x -axis is given by

$$U(x) = \left[\frac{x}{b}\right]^4 - 5 \left[\frac{x}{b}\right]^2 \text{ J}, \quad b=1, \quad U(x) = x^4 - 5x^2.$$

Where $b = \underline{1 \text{ m}}$. Plot this potential energy, identifying the extremum points.

Identify the regions where particle may be found and its maximum speed.

Given that the total mechanical energy is (i) 36 J ; (ii) -4 J .

$$E_T \geq U$$

$$U(x) = x^4 - 5x^2$$

$$U(x) = 0$$

$$x^4 - 5x^2 = 0$$

$$x^2(x^2 - 5) = 0$$

$$x = 0, x = \pm\sqrt{5}$$

$$\left(\frac{d^2U}{dx^2}\right)_{x=\sqrt{5}/2} > 0$$

$$x = \pm\sqrt{5}/2 \text{ Point of}$$

Minima or
Stable Equilibrium.

For Maxima or minima

$$\frac{dU}{dx} = 0$$

$$\frac{d^2U}{dx^2} = (12x^2 - 10)$$

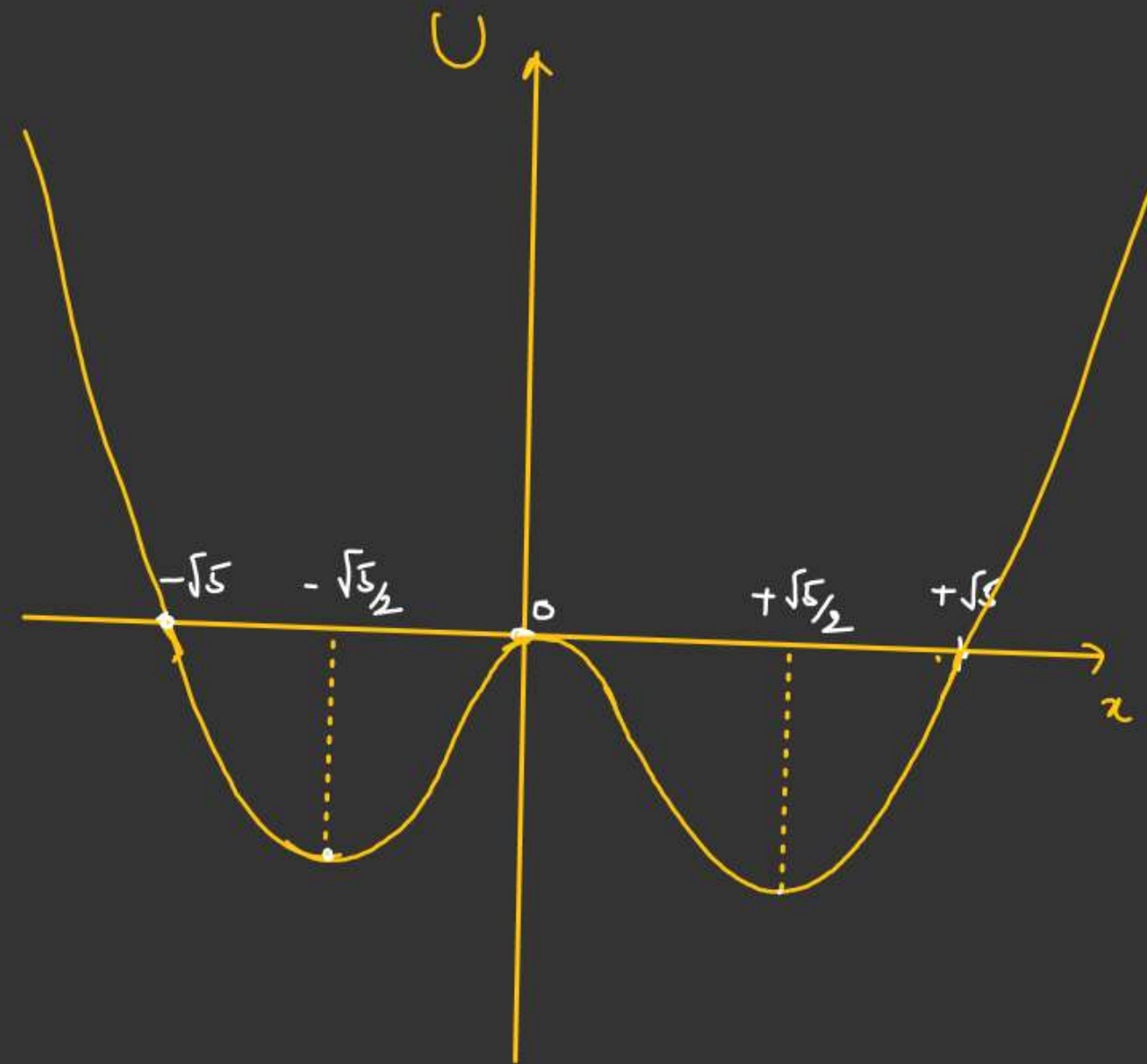
$$4x^3 - 10x = 0$$

$$2x(2x^2 - 5) = 0$$

$$x = 0, x = \pm\sqrt{5}/2$$

$$\left(\frac{d^2U}{dx^2}\right)_{x=0} = (-10) < 0$$

Point of Maxima
or Unstable Equilibrium.



a) $E_T = 36 \text{ J}$

$$-3 \leq x \leq 3$$

$$E_T = U_{\max} = 36 \text{ J}$$

$$x^4 - 5x^2 = 36$$

$$x^4 - 5x^2 - 36 = 0$$

put $x^2 = t$

$$t^2 - 5t - 36 = 0$$

$$t^2 - 9t + 4t - 36 = 0$$

$$t(t-9) + 4(t-9) = 0$$

$$t = -4$$

$$t = 9$$

↓

$$x^2 = 9$$

$$(x = \pm 3)$$

b) $E_T = -4 \text{ J} \checkmark$

$$U_{\max} = E_T = -4$$

$$x^4 - 5x^2 = -4$$

$$x^4 - 5x^2 + 4 = 0$$

$$(t = x^2)$$

$$t^2 - 5t + 4 = 0$$

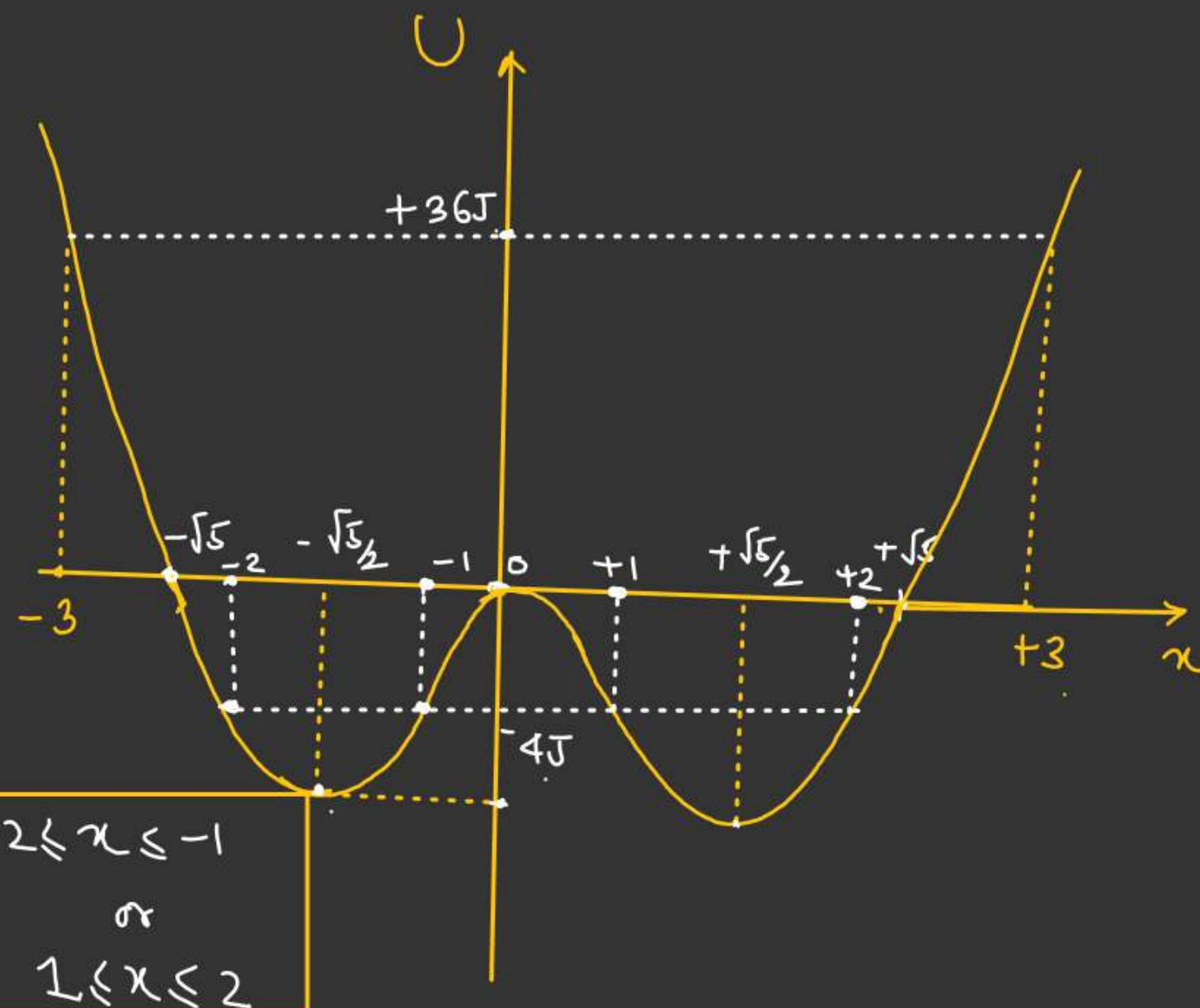
$$t^2 - 4t - t + 4 = 0$$

$$t(t-4) - 1(t-4) = 0$$

$$t = 1, t = 4$$

$$x^2 = 1, x^2 = 4$$

$$(x = \pm 1) (x = \pm 2)$$



Maximum Speed of the particle

a) $E_T = 36 \text{ J}$

$$E_T = P.E + (K.E)$$

$$(K.E) = E_T - (P.E)$$

For $(K.E)_{\max}$ $(P.E)$ will be minimum.

For $E_T = 36 \text{ J}$

$$U_{\min} \text{ at } x = \pm \sqrt{5/2}$$

$$U = (\sqrt{5/2})^4 - 5(\sqrt{5/2})^2$$
$$= \frac{25}{4} - \frac{25}{2} = \left(-\frac{25}{4} \text{ J}\right)$$

$$\frac{1}{2} m v_{\max}^2 = \left(\frac{169}{4}\right)$$

$$\frac{1}{2} \times 2 \times v_{\max}^2 = \left(\frac{169}{4}\right)$$

$$v_{\max} = \left(\frac{13}{2} \text{ m/s}\right)$$

$$(K.E)_{\max} = 36 - \left(-\frac{25}{4}\right)$$

$$= \left(\frac{144 + 25}{4}\right)$$

$$= \left(\frac{169}{4}\right) \text{ J}$$

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Q.8 A single conservative force $F(x)$ acts on a 1.0 kg particle that moves along the x -axis. The potential energy $U(x)$ is given by:

$$U(x) = 20 + (x - 2)^2$$

where x is in meters. At $x = 5.0 \text{ m}$ the particle has a kinetic energy of 20 J .

- (A) What is the mechanical energy of the system?
- (B) Make a plot of $U(x)$ as a function of x for $-10\text{m} \leq x \leq 10\text{m}$.
- (C) The least value of x and
- (D) The greatest value of x between which the particle can move.
- (E) The maximum kinetic energy of the particle and
- (F) The value of x at which it occurs.
- (G) Determine the equation for $F(x)$ as a function of x .
- (H) For what (finite) value of x does $f(x) = 0$?

Q.10 A particle of mass 2 kg is moving under the influence of a force which always acts towards the centre and whose potential energy is given by $U(r) = 2r^3 \text{ joule}$. If the body is moving in a circular orbit of radius 5 m , then find its energy. ✓

$$U(r) = 2r^3$$

$$F = -\left(\frac{dU}{dr}\right)$$

$$F = -\frac{d(2r^3)}{dr}$$

$$F = -2 \frac{d}{dr}(r^3)$$

$$F = -6r^2$$

$$\vec{F} = (6r^2)(-\hat{r})$$

$$F = \frac{mv^2}{r}$$

$$6r^2 = \frac{mv^2}{r}$$

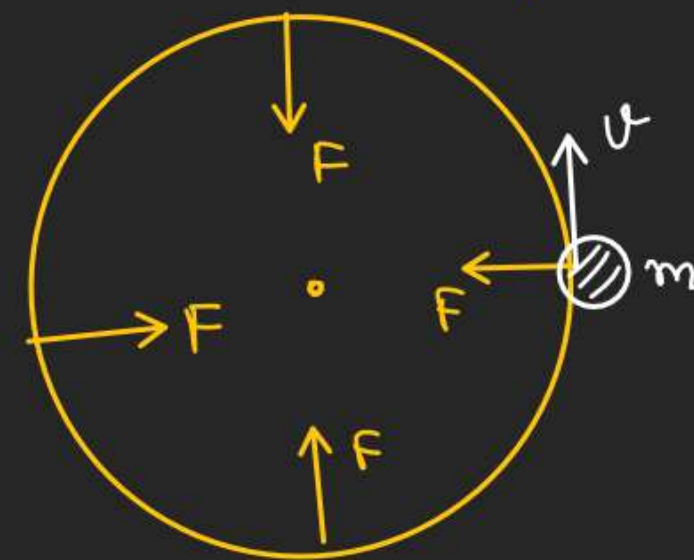
$$mv^2 = 6r^3$$

$$\frac{1}{2}mv^2 = (3r^3)$$

$$E_T = P.E + K.E$$

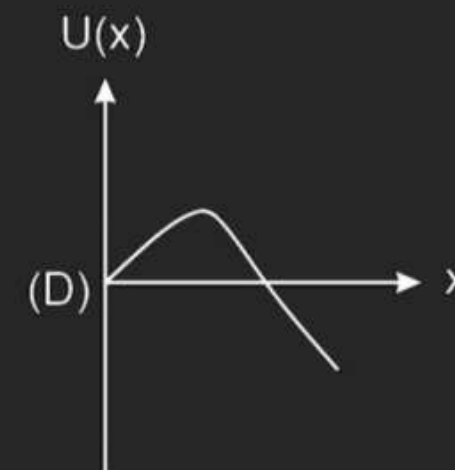
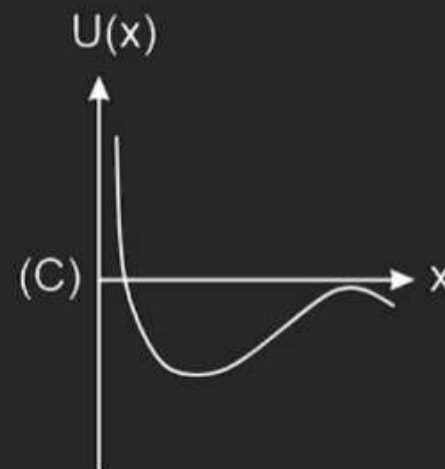
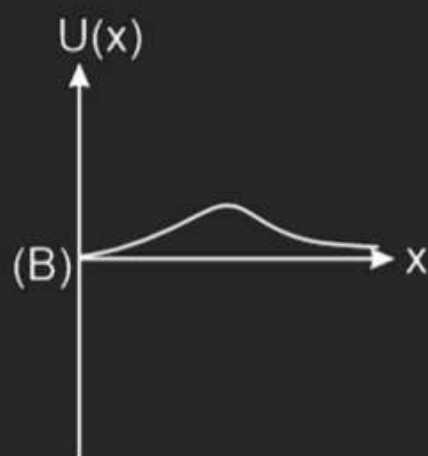
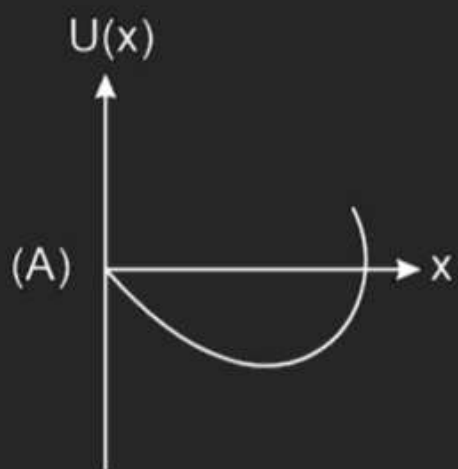
$$= (2r^3 + 3r^3) = (5r^3)$$

$$r = 5$$



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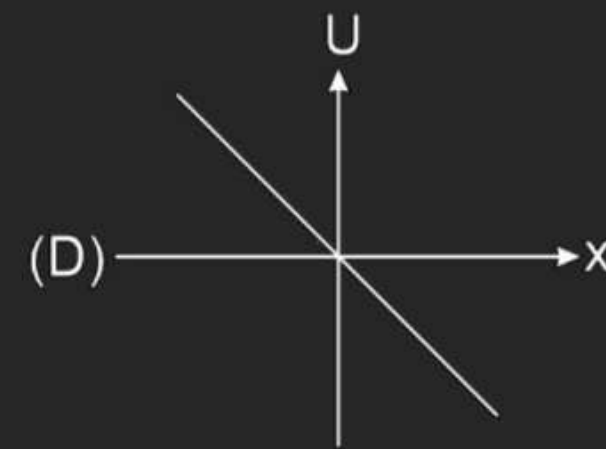
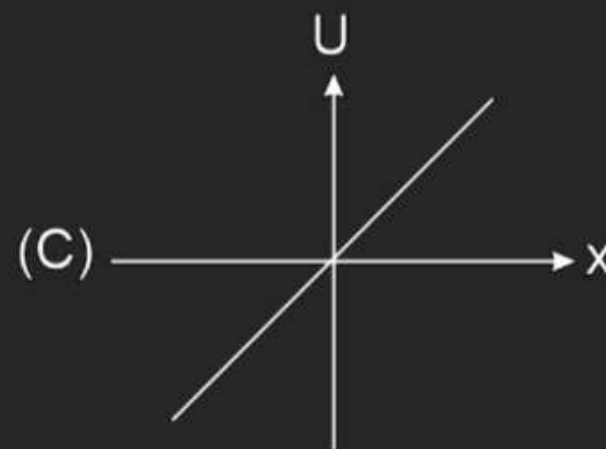
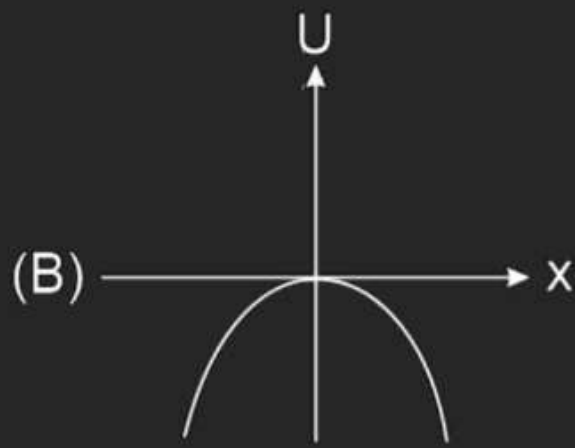
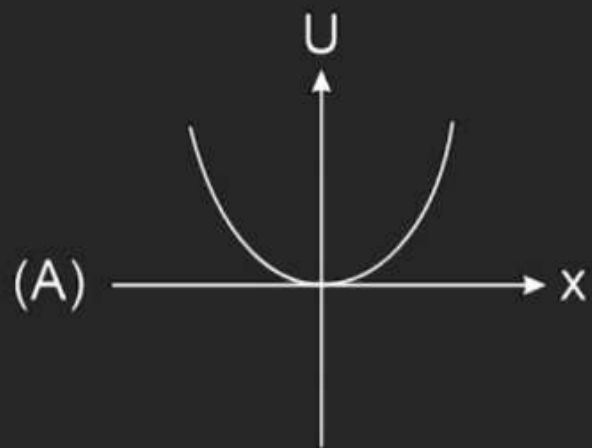
Q.11 A particle, which is constrained to move along the x -axis, is subjected to a force in the same direction which varies with the distance x of the particle from the origin as $F(x) = -kx + ax^3$. Here k and a are positive constants. For $x \geq 0$, the functional graphically form of the potential energy $U(x)$ of the particle is



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Q.12 A particle is acted by a force $F = kx$, where k is a +ve constant. Its potential energy at $x = 0$ is zero. Which curve correctly represents the variation of potential energy of the block with respect to x ?



Q.13 A particle of mass $m = 1 \text{ kg}$ is free to move along x axis under influence of a conservative force. The potential energy function for the particle is

$$U = a \left[\left(\frac{x}{b} \right)^4 - 5 \left(\frac{x}{b} \right)^2 \right] \text{ joule}$$

Where $b = 1.0 \text{ m}$ and $a = 1.0 \text{ J}$. If the total mechanical energy of the particle is zero, find the co-ordinates where we can expect to find the particle and also calculate the maximum speed of the particle.

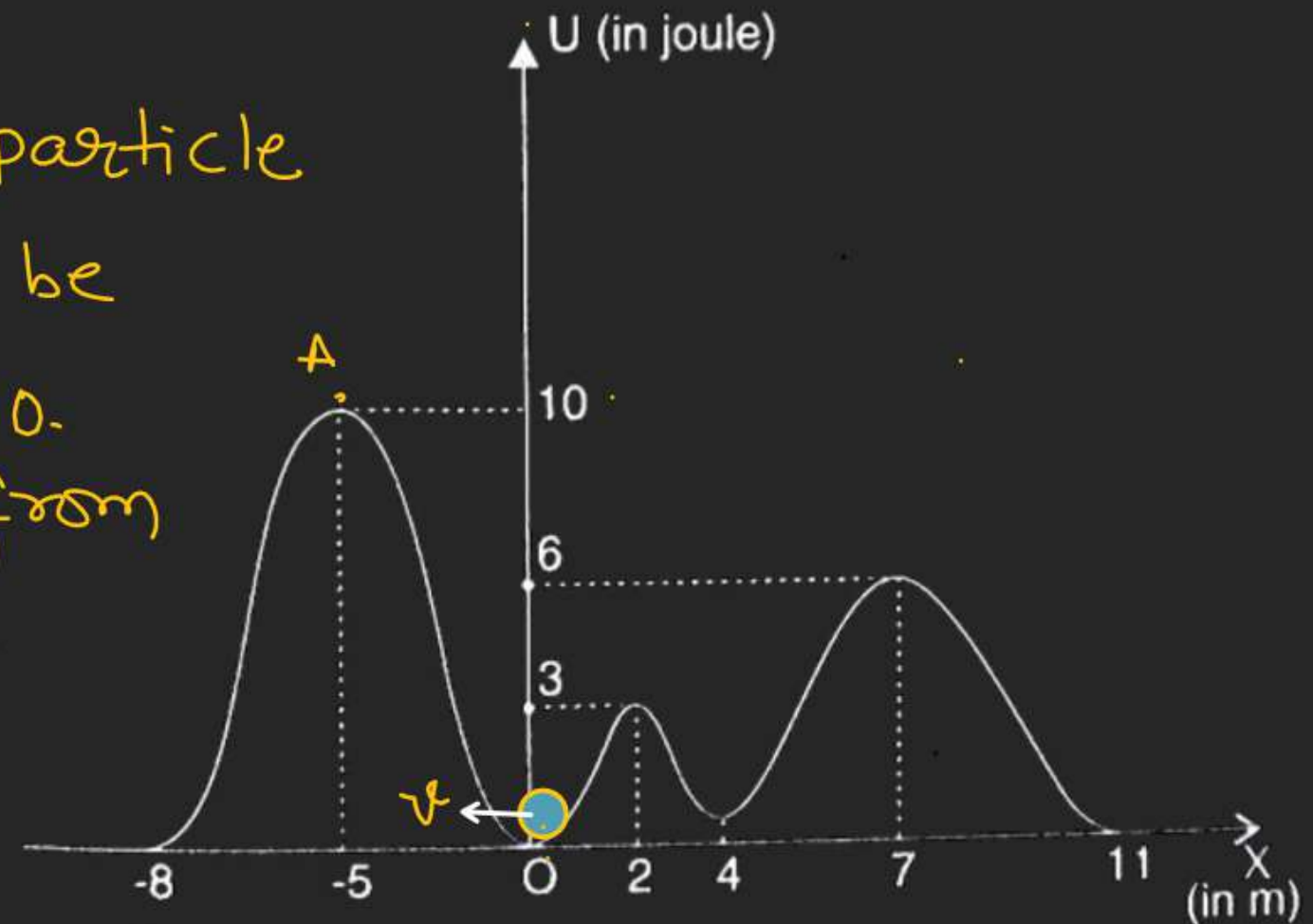
Q.14 A particle of mass $m = 1.0$ kg is free to move along the x axis. It is acted upon by a force which is described by the potential energy function represented in the graph below. The particle is projected towards left with a speed v , from the origin. Find minimum value of v for which the particle will escape far away from the origin.

For v_{\min} , K.E of particle at $x = -5$ will be assumed to be zero.
Energy Conservation from 0 to A.

$$U_0 + K.E_0 = U_A + (K.E)_A$$

$$0 + \frac{1}{2} m v_{\min}^2 = 10$$

$$v_{\min} = \sqrt{20} = 2\sqrt{5} \text{ m/s}$$



Q.15 A particle of mass m moves under the action of a central force. The potential energy function is given by $U(r) = mkr^3$

Where k is a positive constant and r is distance of the particle from the centre of attraction.

(a) What should be the kinetic energy of the particle so that it moves in a circle of radius a_0 about the centre of attraction?

Special case of work-done by friction

$$f_k = \mu N = (\mu mg \cos \theta)$$

$$dW_{f_k} = (\mu mg \cos \theta \cdot dl) \cdot \cos \pi$$

$$dW_{f_k} = -\mu mg (dl \cos \theta)$$

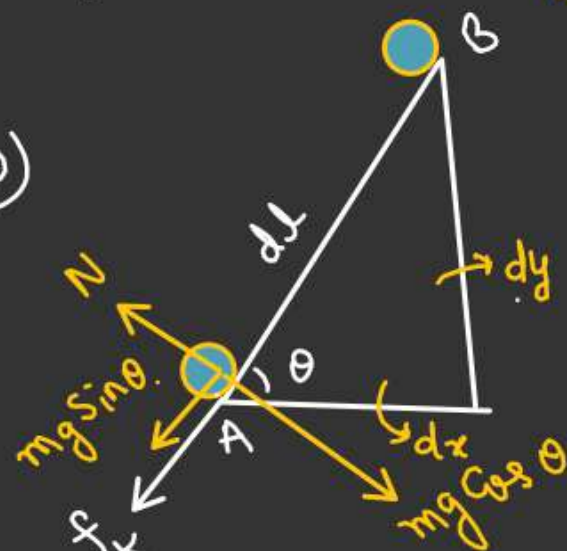
\downarrow
 dx

$$\int_0^d dW_{f_k} = -\mu mg \int_0^d dx$$

$$W_{f_k} = -\mu mg(d)$$

μ = Coefficient of friction b/w ball and Rough Surface.

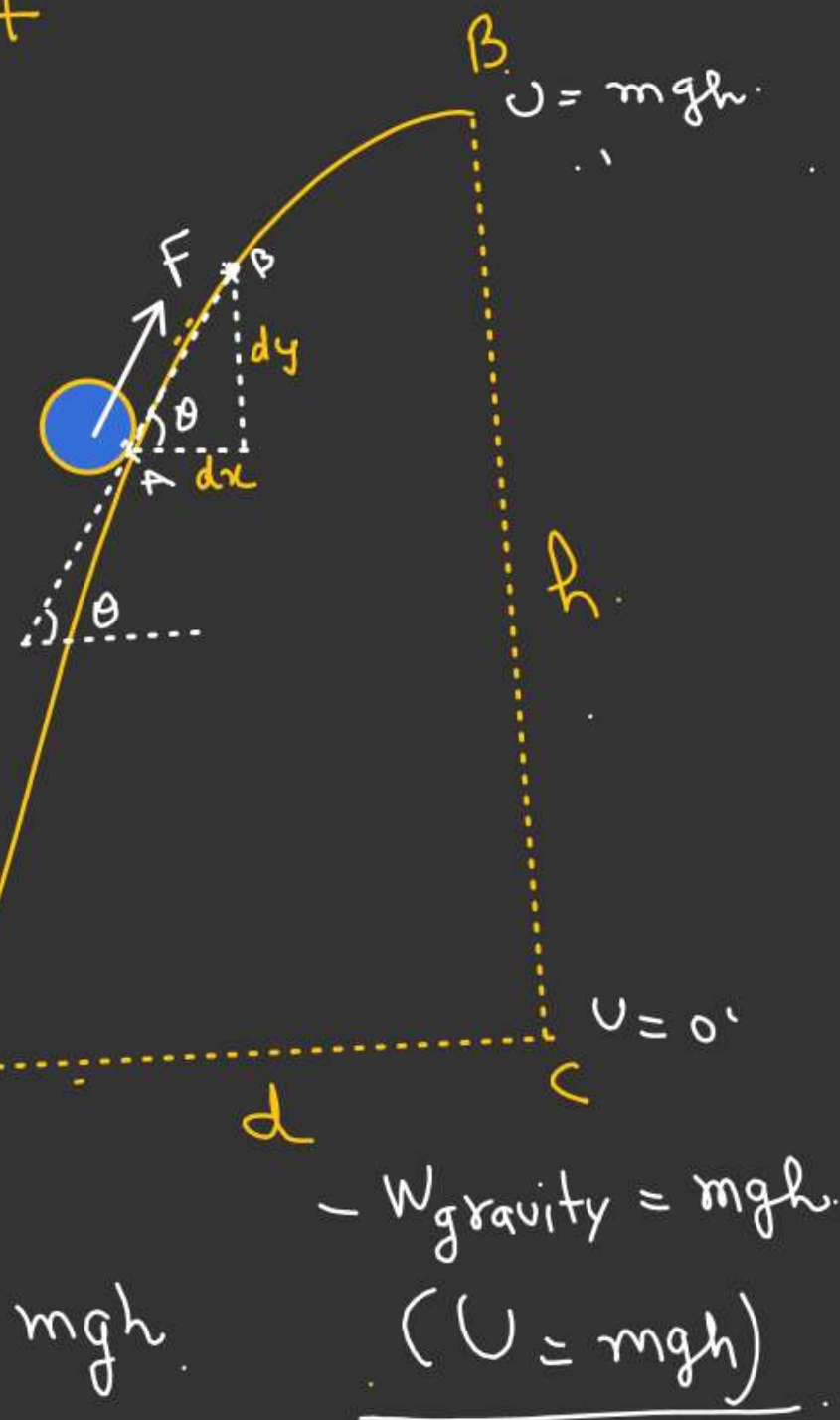
For dl displacement θ assume to be constant.



Rough

W_{gravity}

$$\begin{aligned} \int_0^d dW_{\text{gravity}} &= \int_0^d -mg \sin \theta \cdot dl \\ &\quad \downarrow dy \\ &= -mg \int_0^h dy = -mgh \end{aligned}$$



Force on a Curve :-

$$f_k = -\mu mg d$$

True only when
particle move very
slowly on the curve
ie $v=0$

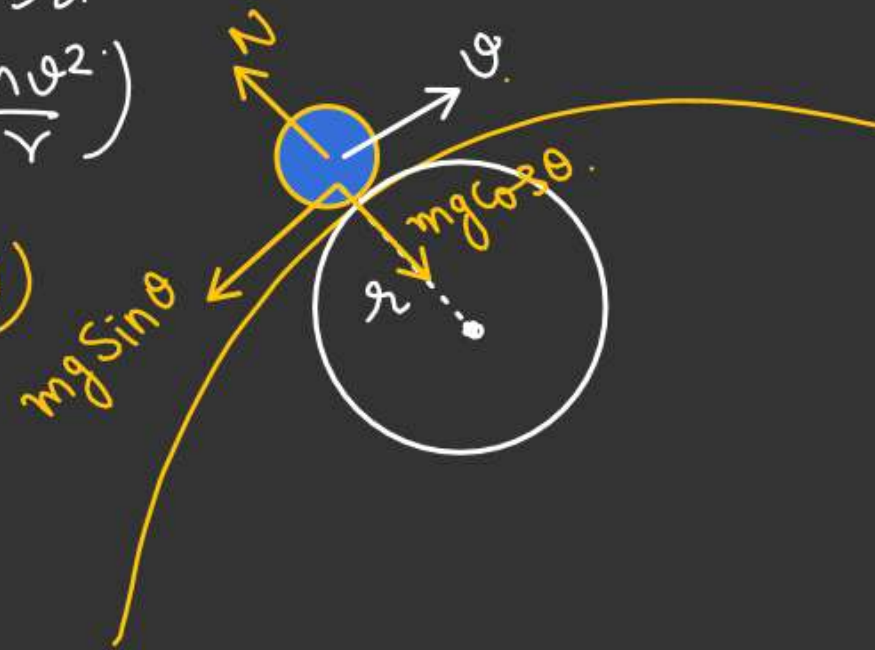
$d = \left(\text{Projection of Curve in} \right)$
horizontal direction)

$$mg \cos \theta - N = \frac{mv^2}{r}$$

$$N = \left(mg \cos \theta - \frac{mv^2}{r} \right)$$

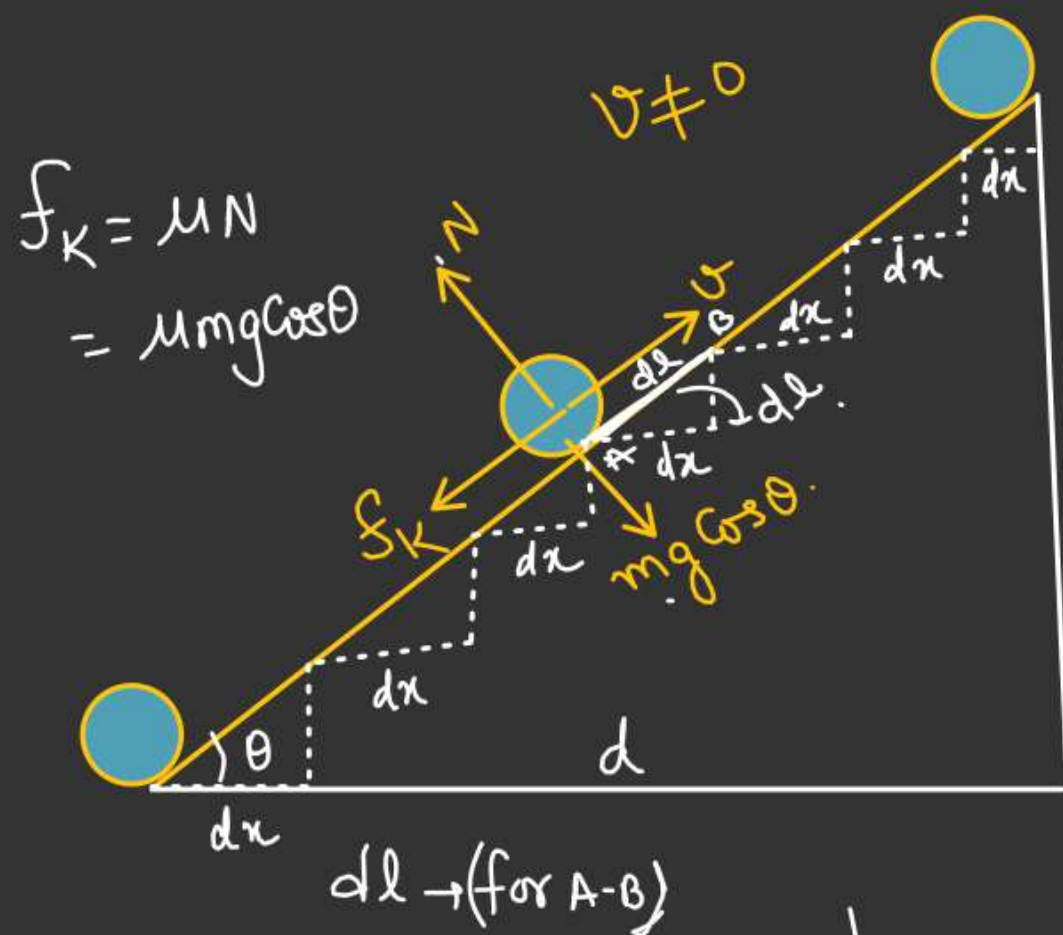
$$\Downarrow$$

$$(N \neq mg \cos \theta)$$



$f_k = (-\mu mg d)$ not valid if
particle moving with velocity
on the curve
as $N \neq mg \cos \theta$

For Inclined plane.



$$\int dx = d$$

$R \rightarrow \infty$
 \Downarrow
 Radius of Curvature in infinite

$$mg \cos \theta - N = \frac{mv^2}{R \rightarrow \infty}$$

$$N = mg \cos \theta$$

$$dW_{A-B} = -\mu mg (dl \cos \theta)$$

W_{net}

$$\int_0^d dW_{A-B} = -\mu mg \int_0^d dx$$

$$W_{f_k} = -\mu mg d \quad \checkmark$$