

$$C_1 \rightarrow C_1 + C_3 - 2\omega \sin \alpha C_2$$

$$R_1 \rightarrow R_1 + R_2 + R_3$$

$$\begin{array}{c} \sum A + P = 0 \\ \tan(A+P) \tan(B+Q) \tan(C+R) \end{array}$$

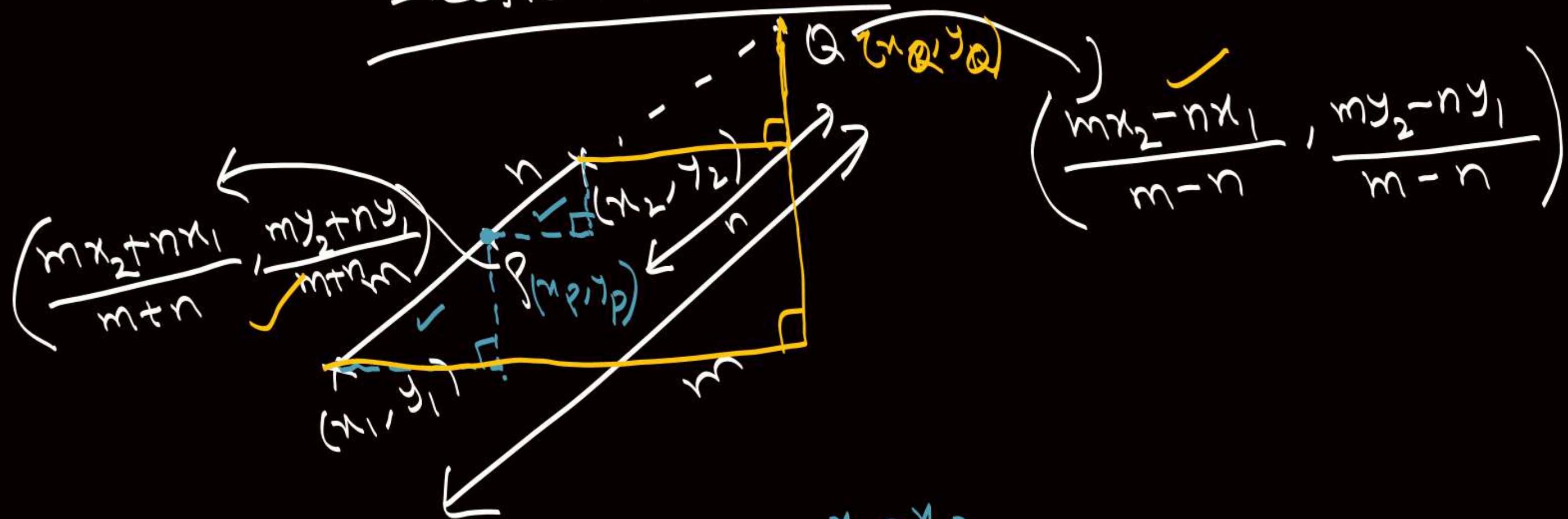
$\left| \begin{array}{c} \tan(A+P) + \tan(B+Q) + \tan(C+R) \\ (A+B+C) \end{array} \right| \quad \left| \begin{array}{ccc} 1 & 1 & 1 \end{array} \right|$

$\left| \begin{array}{c} \operatorname{sgn}(\vec{a}) - (\vec{P}^3) \\ \operatorname{sgn}(\vec{a} - 3abc) \end{array} \right| \quad \left| \begin{array}{c} \vec{a} \\ abc \end{array} \right|$

$C_1 \rightarrow C_1 - bC_3, \quad C_2 \rightarrow C_2 + aC_3 \quad C_2 \rightarrow C_2 - C_1$   
 $C_3 \rightarrow C_3 + 2bC_1 \quad C_3 \rightarrow C_3 - C_1$

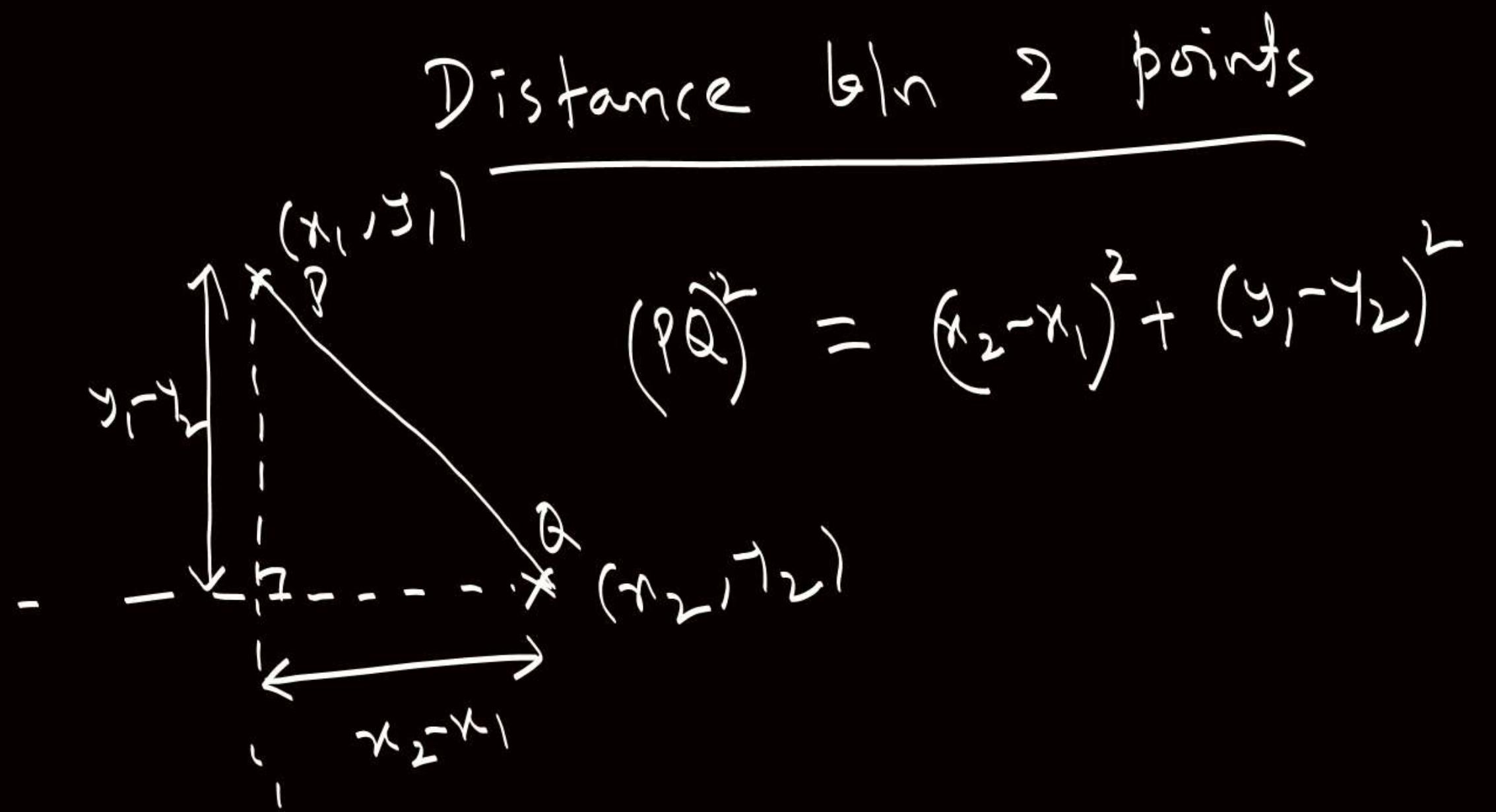
$\left( 1 + q_{AB}^2 \right) \left| \begin{array}{ccc} 1 & 0 & -2b \\ 0 & 1 & b \\ b & -a & 1 \end{array} \right|$

# Section Formula

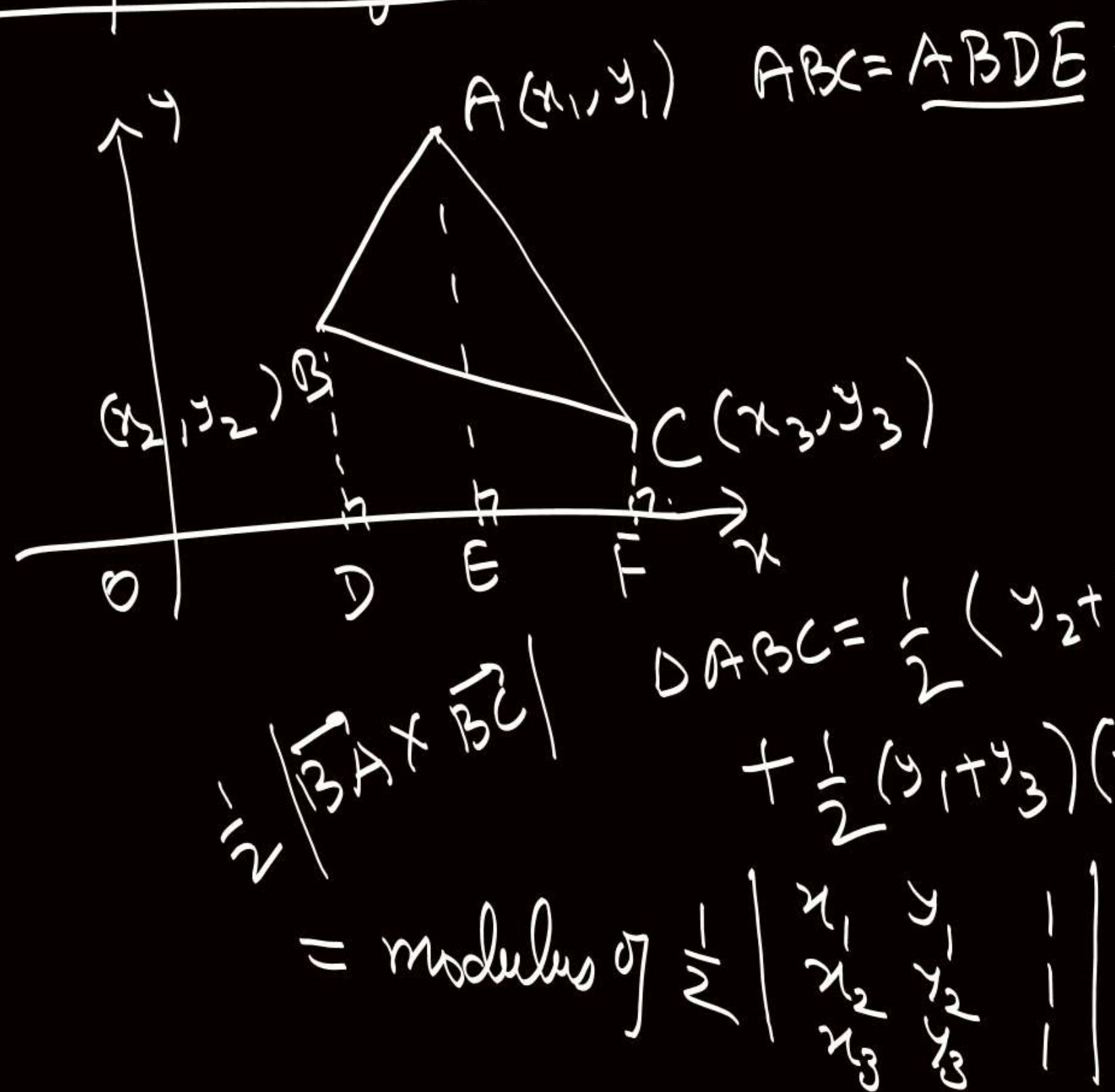


$$\frac{y_2 - y_p}{y_p - y_1} = \frac{n}{m} = \frac{x_2 - x_p}{x_p - x_1}$$

$$\frac{x_Q - x_2}{x_Q - x_1} = \frac{n}{m} = \frac{y_Q - y_2}{y_Q - y_1}$$



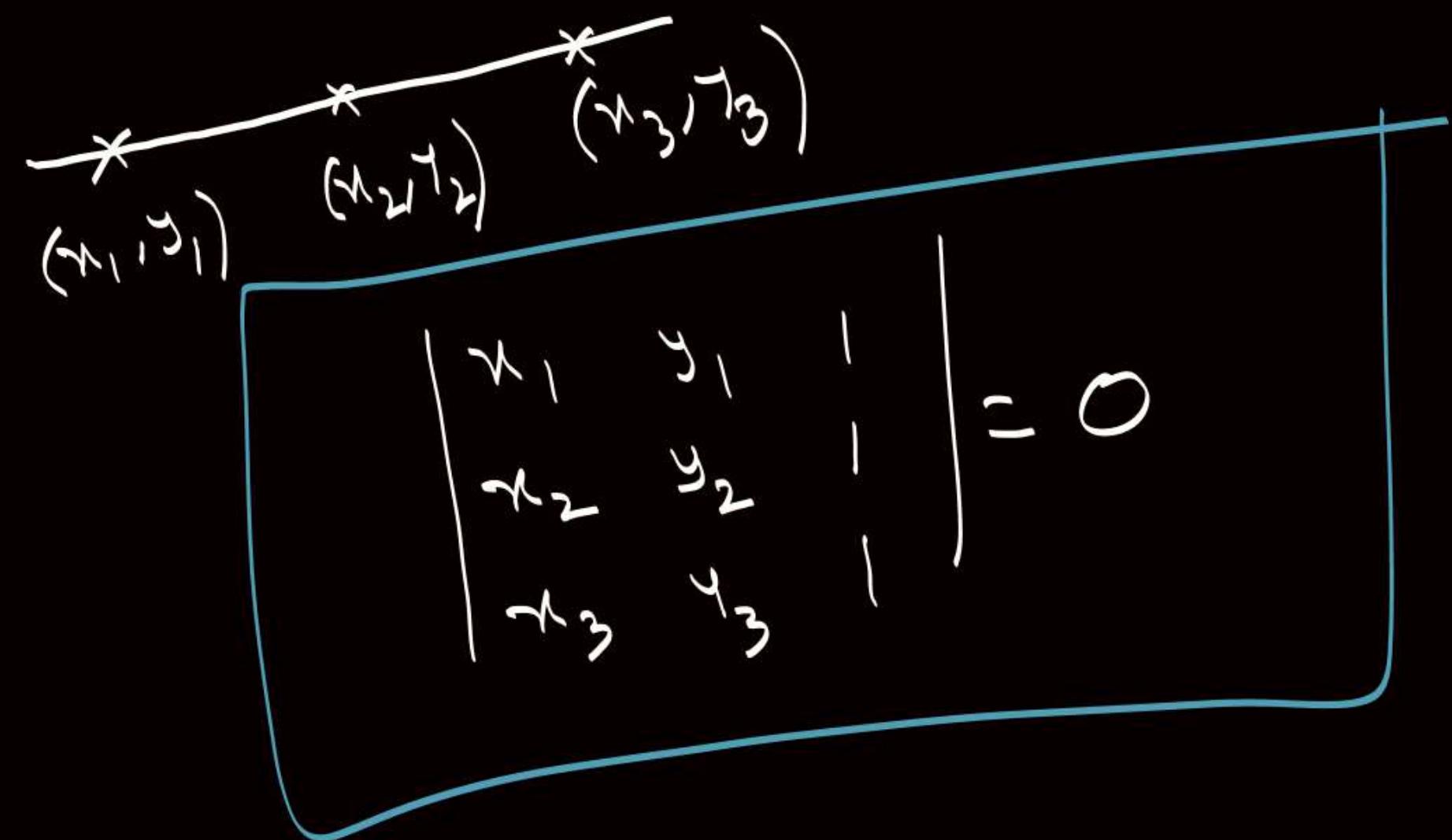
Area of triangle



$$\begin{aligned} \Delta ABC &= \frac{1}{2} (y_2 + y_1)(x_1 - x_2) \\ &\quad + \frac{1}{2} (y_1 + y_3)(x_3 - x_1) - \frac{1}{2} (y_2 + y_3)(x_3 - x_2) \end{aligned}$$

$$= \text{modulus of } \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

# Condition for Collinear of 3 points



A diagram illustrating the condition for three points to be collinear. Three points are shown on a single horizontal line:  $(x_1, y_1)$ ,  $(x_2, y_2)$ , and  $(x_3, y_3)$ . A blue bracket encloses the first two points, and another blue bracket encloses all three points. Below the points, a determinant equation is written:

$$\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$$

# Harmonic Conjugates

$$\Rightarrow \frac{AB}{AP} - \frac{(AQ-AB)+AQ}{AQ} \Rightarrow \frac{AB}{AP} = 2 - \frac{AB}{AQ} \Rightarrow \frac{1}{AP} + \frac{1}{AQ} = \frac{2}{AB}$$

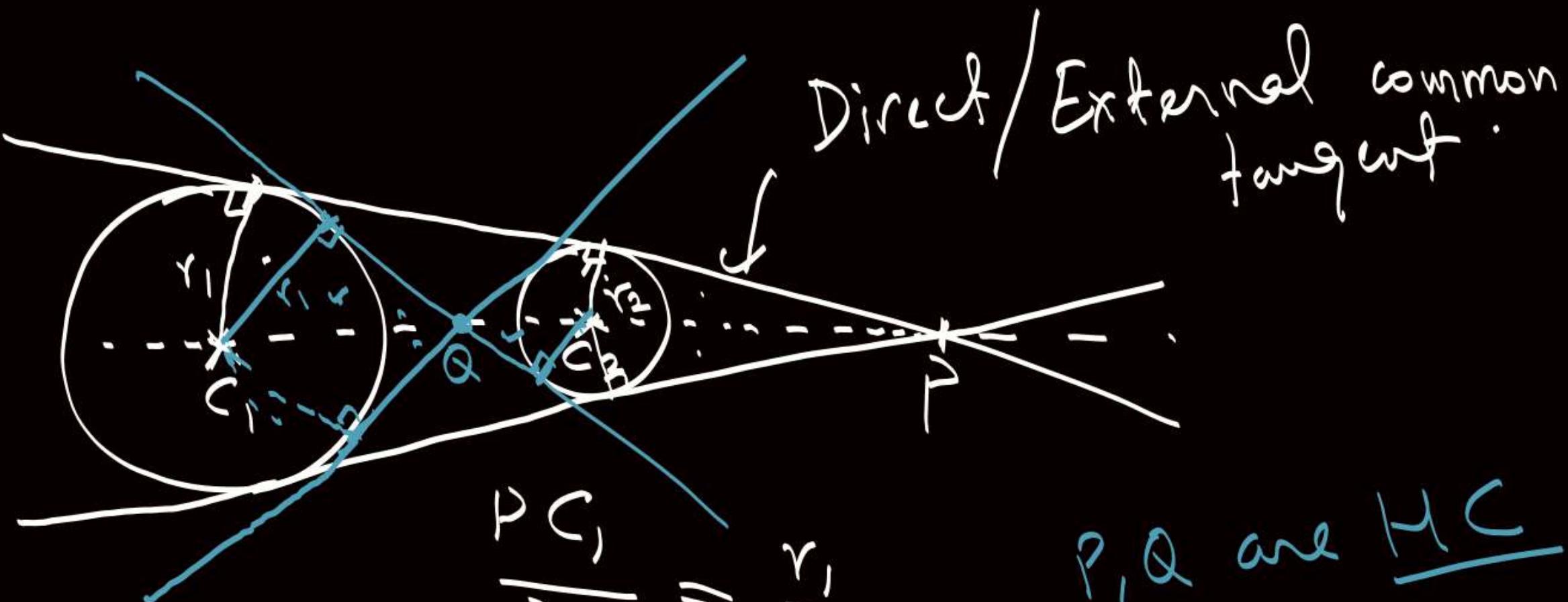
$$1 + \frac{PB}{PA} = \frac{QB}{QA} + 1 \Leftrightarrow \frac{PA}{PB} = \frac{QA}{QB}$$

$\frac{1}{AP}, \frac{1}{AB}, \frac{1}{AQ}$  will be in HP



HC to B, C

BD, BC, BE will form harmonic progression

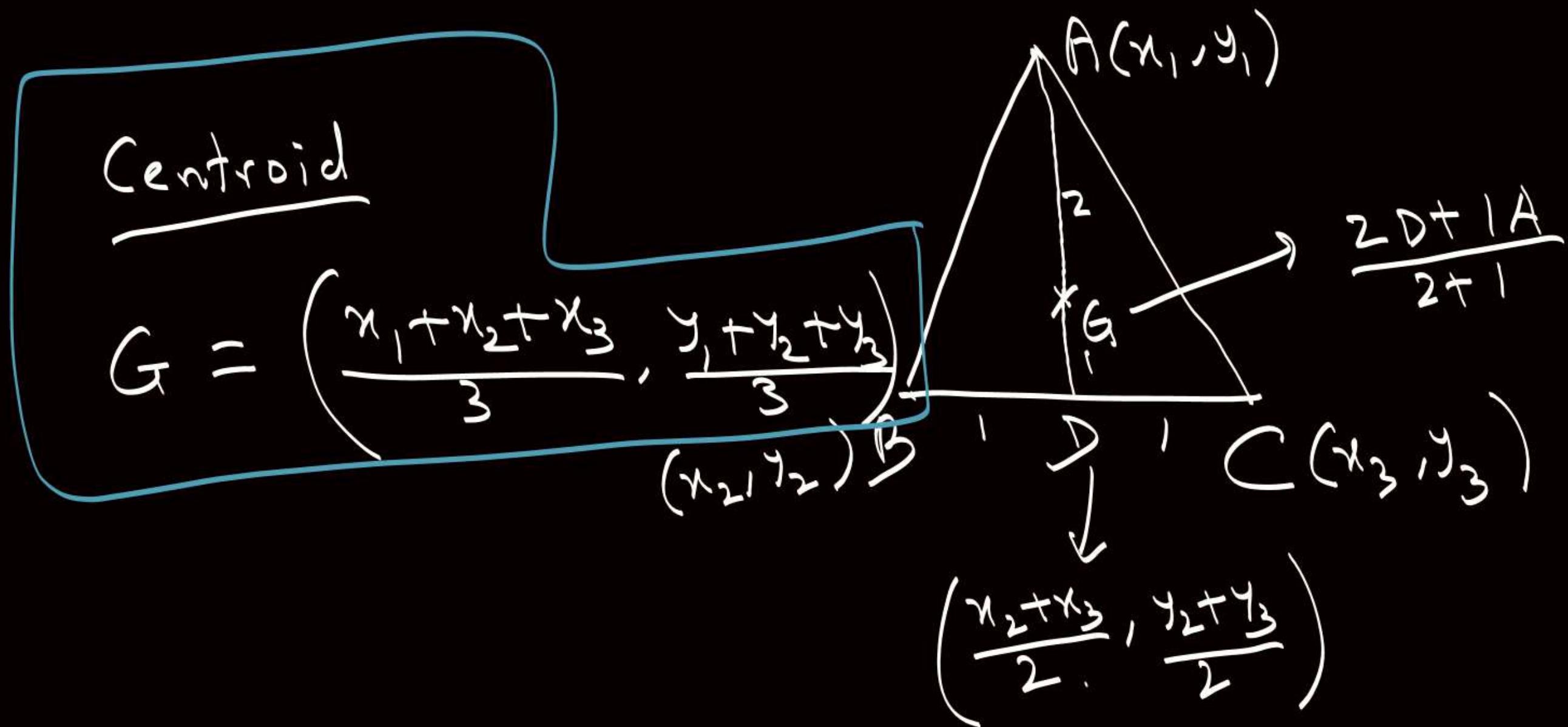


$$\frac{PC_1}{PC_2} = \frac{r_1}{r_2}$$

$P, Q$  are HC to  
 $C_1, C_2$

$$\frac{QC_1}{QC_2} = \frac{r_1}{r_2}$$

$C_1Q, C_1C_2, C_1P$   
will form HP

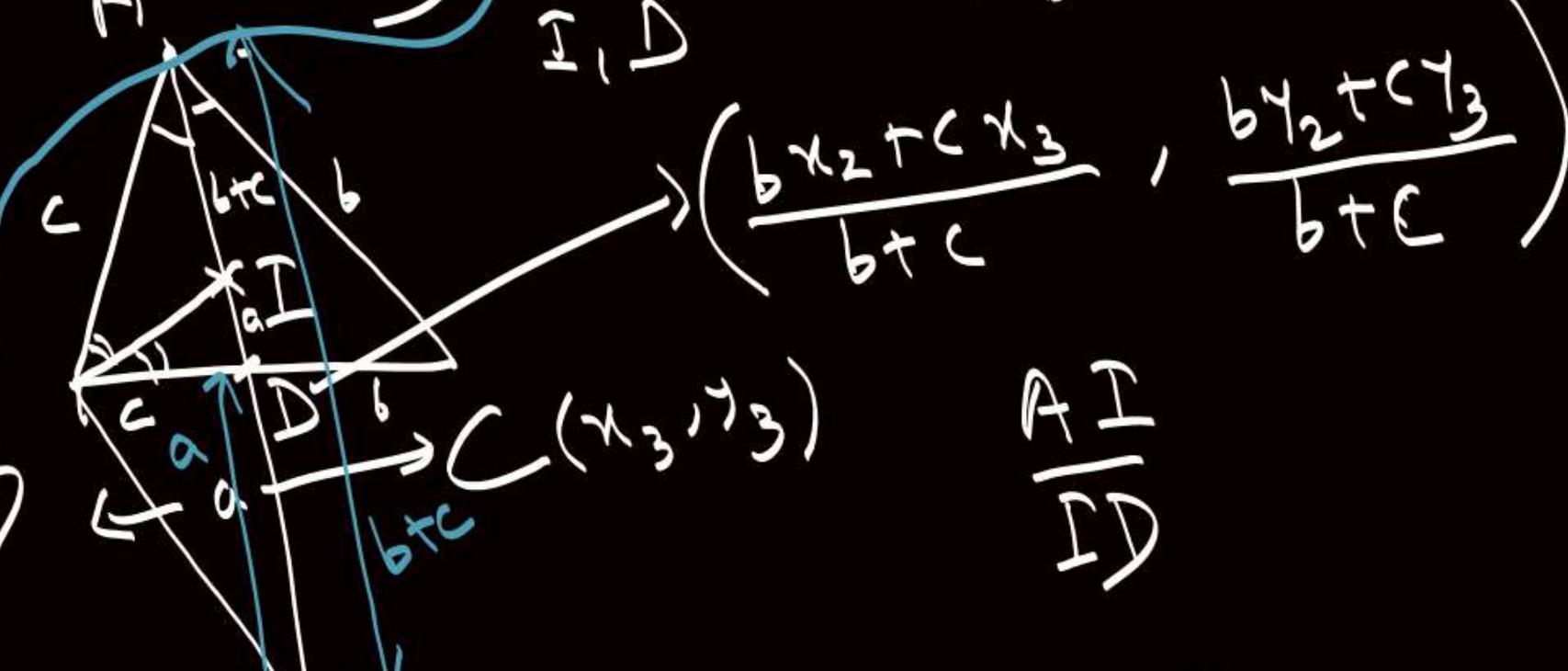


# Incentre / Excentre

$$\bar{I} = \left( \frac{ax_1 + bx_2 + cx_3}{a+b+c}, \frac{ay_1 + by_2 + cy_3}{a+b+c} \right)$$

$$\bar{I}_1 = \left( \frac{-ax_1 + bx_2 + cx_3}{-a+b+c}, \frac{-ay_1 + by_2 + cy_3}{-a+b+c} \right)$$

$$\frac{AI}{ID} = \frac{AI}{ID} = \frac{AB}{BD} = \frac{c}{\frac{ca}{c+b}} = \frac{c}{a} = \frac{b+c}{a}$$



$$I \Rightarrow a \rightarrow -a$$

$$b \rightarrow -b \Rightarrow I_1$$

$$c \rightarrow -c \Rightarrow I_2$$

$$I = \frac{(b+c)D + aA}{(b+c)+a}$$

$$I_1 = \frac{(b+c)D - aA}{(b+c)-a}$$

Orthocentre

$$\begin{aligned}
 H &= \frac{(\tan A)x_1 + (\tan B)x_2 + (\tan C)x_3}{\tan A + \tan B + \tan C} \\
 &\quad - \frac{\cos B \cos C x_1 + \cos A \cos B x_2 + \cos A \cos C x_3}{\cos B \cos C + \cos A} \\
 &\quad = \frac{\sin A \cos B \cos C x_1 + \sin B \cos A \cos C x_2 + \sin C \cos B \cos A x_3}{\sin A \sin B \sin C}
 \end{aligned}$$

$$\begin{aligned}
 H &= \frac{\sum \tan A x_i}{\sum \tan A} - \frac{\sum \tan A y_i}{\sum \tan A} \\
 &= \left( \frac{\cos B x_3 + b \cos C x_2}{\cos B \cos C + \cos A}, \right. \\
 &\quad \left. \frac{\cos A x_1 + a \cos C x_3}{\cos B \cos C + \cos A} \right)
 \end{aligned}$$