CIRCLE

SINGLE CORRECT ANSWER TYPE

1. S1: The locus of the centre of a circle which cuts a given circle orthogonally and also touches a given straight line is a parabola.

S2: Two circles $x^2 + y^2 + 2ax + c = 0$ and $x^2 + y^2 + 2by + c = 0$ touches iff $\frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{c^2}$.

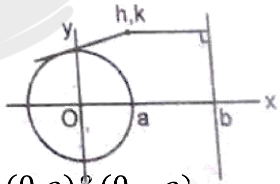
S3: The two circles which passes through $(0, a)$ and $(0, -a)$ and touch the straight line $y = mx + c$, will cut orthogonally if $c^2 = a^2(2 + m^2)$.

S4: The length of the common chord of the circles $(x - a)^2 + y^2 = a^2$ and $x^2 + (y - b)^2 = b^2$ is $\frac{ab}{\sqrt{a^2-b^2}}$.

(A) TFTF (B) TTFF (C) TFTT (D) FFTT

Ans. **(A)**

Hint. S1: Assume circle as $x^2 + y^2 = a^2$ and the given line as $x - b = 0$ Let (h, k) be the centre of the required circle. Then length of tangent from (h, k) to the circle and distance of (h, k) from the line should be equal.



Hence $\sqrt{h^2 + k^2 - a^2} = |h - b|$

S2: Apply $c_1c_2 = |r_1 \mp r_2|$

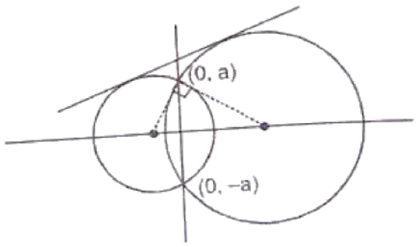
S3: Let $x^2 + y^2 + 2gx + 2fy + c = 0$ be a circle passing through the points $(0, a)\&(0, -a)$

$\therefore\ a^2 + 2af + c = 0\ \&\ a^2 - 2af + c = 0$

$\therefore f = 0$ and $c = -a^2$

$\therefore$ Equation of the circle is

$x^2 + y^2 + 2gx - a^2 = 0$



Since it touches the line $y = mx + c$ then use P=r

it gives two values of g, therefore, there are two circles

further these two circles cut each other orthogonally

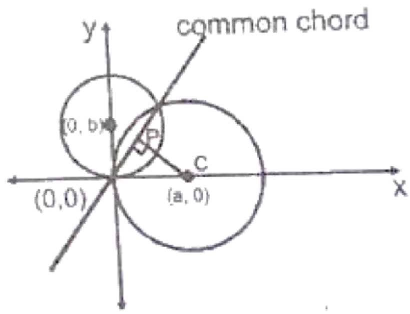
then use $2\,g_1\,g_2 = f_1+f_2$

S4: Equation of common chord is $S_1 - S_2 = 0$

$\Rightarrow\ ax - by = 0$

from the figure, $CP = P_1$(let)

$\therefore$ length of common chord $= 2\sqrt{{r_1}^2 - {P_1}^2}$



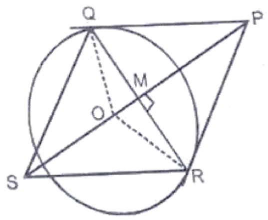
2. P is a variable point on the line $L = 0$. Tangents are drawn to the circle $x^2 + y^2 = 4$ from P to touch it at Q and R. The parallelogram PQSR is completed.

If $L = 2x + y - 6 = 0$, then the locus of circumcentre of $\triangle$ PQR is

(A) $2x - y - 4$ (B) $2x + y = 3$ (C) $x - 2y = 4$ (D) $x + 2y = 3$

Ans. **(B)**

Sol. $\because$ $PQ = PR$ i.e. parallelogram PQRS is a rhombus



$\therefore$ Mid point of QR = Midpoint of PS and $QR \perp PS$

$\therefore$ S is the mirror image of P w.r.t. QR

$\therefore$ S is the mirror image

$\because L \equiv 2x + y = 6$ Let $P \equiv (k, 6 - 2k)$

$\because \angle PQO = \angle PRO = \frac{\pi}{2}$

$\therefore$ OP is diameter of circumcircle PQR, then centre is $\left(\frac{k}{2}, 3 - k\right)$

$\therefore x = \frac{k}{2} \Rightarrow k = 2x$

$y = 3 - k \therefore 2x + y = 3$

PARABOLA

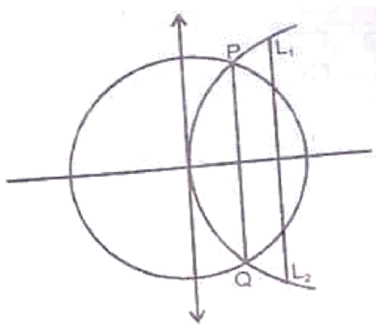
SINGLE CORRECT ANSWER TYPE

3. A circle is described whose centre is the vertex and whose diameter is three-quarters of the latus rectum of the parabola $y^2 = 4ax$. If PQ is the common chord of the circle and the parabola and L_1L_2 is the latus rectum, then the area of the trapezium $PL_1\,L_2Q$ is

(A) $3\sqrt{2}a^2$ (B) $2\sqrt{2}a^2$ (C) $4a^2$ (D) $\left(\frac{2+\sqrt{2}}{2}\right)a^2$

Ans. **(D)**

Sol. Centre (0,0), radius $= \frac{1}{2} \cdot \frac{3}{4} \cdot 4a = \frac{3a}{2}$



Equation of the circle is $4(x^2 + y^2) = 9a^2$ ...(i)

Equation of the parabola is $y^2 = 4ax$ ...(ii)

Solving (i) and (ii) $x^2 + 4ax - \frac{9a^2}{4} = 0$

$$x = \frac{-4a \pm \sqrt{16a^2 + 9a^2}}{2} = \frac{-4a \pm 5a}{2}$$

$\therefore$ $x = a/2$

For $x = a/2$ $y^2 = 4ax = 4aa/2 = 2a^2$

$\Rightarrow$ $y = \pm\sqrt{2}a$

$\therefore$ The double ordinate $= 2\sqrt{2}\,a$

$\therefore$ area of the trapezium $PL_1L_2Q = \frac{1}{2}\left(a - \frac{a}{2}\right)(4a + 2\sqrt{2}a) = \left(\frac{2+\sqrt{2}}{2}\right)a^2$

MATRIX - MATCH TYPE

4. **Column-I** **Column-II**

(A) Area of a triangle formed by the tangents drawn from a point $(-2,2)$ to the parabola $y^2 = 4(x+y)$ and their corresponding chord of contact is — (P) 8

(B) Length of the latus rectum of the conic $25\{(x-2)^2 + (y-3)^2\} = (3x+4y-6)^2$ is — (Q) $4\sqrt{3}$

(C) If focal distance of a point on the parabola $y = x^2 - 4$ is $25/4$ and points are of the form $(\pm\sqrt{a}, b)$ then value of $a + b$ is

(D) Length of side of an equilateral triangle inscribed in a parabola $y^2 - 2x - 2y - 3 = 0$ whose one angular point is vertex of the parabola, is — (R) $\frac{12}{5}$ (T) $\frac{24}{5}$

Ans. **(**$(A) \to (r), (B) \to (t), (C) \to (p), (D) \to (q)$**)**

Sol. (A) Point of contact of tangent drawn from $(-2,2)$ on $y^2 = 4(x+y)$ are $(0,4)$ and $(0,0)$

$\therefore$ Area $= 4$

(B) The conic is a parabola having focus is $(2,3)$ & Directrix

$3x + 4y - 6 = 0]$ $\quad\therefore$ Latus rectum $= 2$ ($\perp$ distance of focus from the directrix)

$= 2\left(\frac{6+12-6}{5}\right) = \frac{24}{5}$(C) $y + 4 = x^2 \quad x^2 = 4 \cdot \frac{1}{4}(y+4) \quad$ focal distance $= \frac{25}{4}$

$\therefore$ distance from directrix $\left(y = \frac{-17}{4}\right) =$ ordinate of points on the parabola whose focal distance is

$\frac{25}{4} = \frac{-17}{4} + \frac{25}{4} = 2 \Rightarrow$ Points are $\left(\pm\sqrt{6}, 2\right)$ $\;a + b = 8$

(D) Length of side $= 8\sqrt{3}a = 8\sqrt{3}\frac{1}{2} = 4\sqrt{3}$

ELLIPSE

MULTIPLE CORRECT ANSWER TYPE

5. If P is a point of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, whose focii are S and S′. Let $\angle PSS' = \alpha$ and $\angle PS'S = \beta$, then

(A) $PS + PS' = 2a$, if $a > b$

(B) $PS + PS' = 2b$, if $a < b$

(C) $\tan\frac{\alpha}{2}\tan\frac{\beta}{2} = \frac{1-e}{1+e}$

(D) $\tan\frac{\alpha}{2}\tan\frac{\beta}{2} = \frac{\sqrt{a^2-b^2}}{b^2}\left[a - \sqrt{a^2-b^2}\right]$ when $a > b$

Ans. **(ABC)**

Sol. Focal property of ellipse

$PS + PS' = 2a;$ if $a > b$

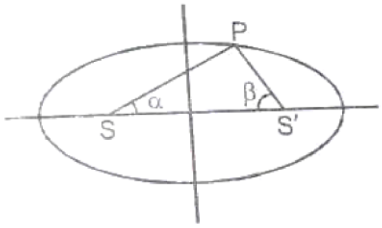
$PS + PS' = 2b$; if $a < b$

$PS\cos\alpha + PS'\cos\beta = 2ae$ ...(i)

$PS\sin\alpha - PS'\sin\beta = 0$ ...(ii)

$PS + PS' = 2a$ ...(iii)

from (i) and (ii), we get $PS = \frac{2ae\sin\beta}{\sin(\alpha+\beta)}, P' = \frac{2ae\sin\alpha}{\sin(\alpha+\beta)}$



from (iii) and (iv)

$e(\sin\alpha + \sin\beta) = \sin(\alpha + \beta)$

$\therefore$ $e.\,2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2} = 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha+\beta}{2}$

$\therefore$ $e\left(\cos\frac{\alpha}{2}\cos\frac{\beta}{2} + \sin\frac{\alpha}{2}\sin\frac{\beta}{2}\right) = \cos\frac{\alpha}{2}\cos\frac{\beta}{2} - \sin\frac{\alpha}{2}\sin\frac{\beta}{2}$

$\therefore$ $\tan\frac{\alpha}{2}\tan\frac{\beta}{2} = \frac{1-e}{1+e} = \frac{2a\left(a-\sqrt{a^2-b^2}\right)-b^2}{b^2}$

$\therefore$ C is correct option & D is incorrect

INTEGER TYPE

6. Origin O is the centre of two concentric circles whose radii are a & b respectively, $a < b$. A line OPQ is drawn to cut the inner circle in P& the outer circle in Q. PR is drawn parallel to the y-axis & QR is drawn parallel to the x-axis. The locus of R is an ellipse touching the two circles. If the focii of this ellipse lie on the inner circle, if eccentricity is $\sqrt{2}\lambda$, then find λ

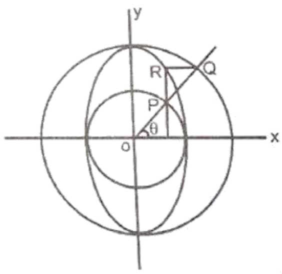
Ans. **(1)**

Sol. Let line OPQ makes angle θ with x-axis so $P \equiv (a\cos\theta, a\sin\theta), Q(b\cos\theta, b\sin\theta)$

and Let $R(x, y)$

So $X = a\cos\theta$ $Y = b\sin\theta$

eliminating θ, we get



$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, locus of R is an ellipse.

Also $a < b$ so vertices are $(0, b)$ and $(0, -b)$

and extremities of minor axis are $(\pm a, 0)$.

So ellipse touches both inner circle and outer circle

if focii are $(0, \pm a)$

$\Rightarrow$ $a = be$ i.e $e = a/b$Also $e = \sqrt{1 - e^2}$ $\Rightarrow$ $e^2 = 1 - e^2$

$\Rightarrow$ $e = 1/\sqrt{2}$

and ratio of radii, is $\frac{a}{b} = e = \frac{1}{\sqrt{2}}$.

HYPERBOLA

COMPREHENSION TYPE (7-8)

For the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ the normal at P meets the transverse axis AA' in G and the conjugate axis BB' in g and CF be perpendicular to the normal from the centre.

7. PF . PG = K CB^2 , then K =

(A) 2 (B) 1 (C) $\frac{1}{2}$ (D) 4

Ans. **(B)**

Sol. Equation of normal at $P(a\sec\theta, b\tan\theta)$ is $ax\cos\theta + by\cot\theta = a^2 + b^2$

$G\left(\frac{a^2+b^2}{a}\sec\theta, 0\right)$, $g\left(0, \frac{a^2+b^2}{b}\tan\theta\right)$

equation of CF is $bx\cot\theta - ay\cos\theta = 0$

$\therefore PF = \frac{ab}{\sqrt{b^2\sec^2\phi + a^2\tan^2\phi}}$

and $PG^2 = \frac{b^2}{a^2}(b^2\sec^2\phi + a^2\tan^2\phi)$

$\therefore PF.PG = b^2$.

8. PF. Pg equals to

(A) CA^2 (B) CF^2 (C) CB^2 (D) CA . CB

Ans. **(A)**

Sol. $Pg^2 = \frac{a^2}{b^2}(b^2\sec^2\theta + a^2\tan^2\theta) \Rightarrow PF.Pg = a^2 = CA^2$

QUADRATIC EQUATION

SINGLE CORRECT ANSWER TYPE

9. A quadratic equation, product of whose roots x_1 and x_2 is equal to 4 and satisfying the relation $\frac{x_1}{x_1-1}+\frac{x_2}{x_2-1}=2$, is

(A) $x^2 - 2x + 4 = 0$ (B) $x^2 - 4x + 4 = 0$

(C) $x^2 + 2x + 4 = 0$ (D) $x^2 + 4x + 4 = 0$

Ans. **(A)**

Sol. Since $x_1x_2 = 4$

$$x_2 = \frac{4}{x_1} \therefore \frac{x_1}{x_1-1} + \frac{\frac{4}{x_1}}{\frac{4}{x_1}-1} = 2 \Rightarrow \frac{x_1}{x_1-1} + \frac{4}{4-x_1} = 2$$

$4x_1 - x_1^2 + 4x_1 - 4 = 2(x_1-1)(4-x_1)$

$\Rightarrow x_1^2 - 2x_1 + 4 = 0$

MULTIPLE CORRECT ANSWER TYPE

10. If the quadratic equation $(ab-bc)x^2+(bc-ca)x+ca-ab=0$, $a, b, c \in R$, has both the roots equal, then

(A) both roots are equal to 0

(B) both roots are equal to 1

(C) a, c, b are in harmonic progression

(D) $ab^2c^2, b^2a^2c, a^2c^2b$ are in arithmetic progression

Ans. **(B, C, D)**

Sol. $x=1$ is a root of the equation, $\therefore \frac{ca-ab}{ab-bc}=1 \Rightarrow 2ab=bc+ac$

$\therefore$ other root is also 1 i.e $c=\frac{2ab}{a+b}$ $\therefore$ a, c, b are in H.P. Since $\frac{b^2a^2c-ab^2c^2}{a^2c^2b-b^2a^2c}=\frac{ab-bc}{ac-ab}=1$

$\therefore ab^2c^2, b^2a^2c, a^2c^2b$ are in A.P.

SEQUENCE & SERIES

MULTIPLE CORRECT ANSWER TYPE

11. For the series $S=1+\frac{1}{(1+3)}(1+2)^2+\frac{1}{(1+3+5)}(1+2+3)^2+\frac{1}{(1+3+5+7)}(1+2+3+4)^2+\cdots$

(A) 7^{th} term is 16

(B) 7^{th} term is 18

(C) sum of first ten terms is $\frac{505}{4}$

(D) sum of first ten term is $\frac{405}{4}$

Ans. **(A, C)**

Sol. $S=1+\frac{1}{(1+3)}(1+2)^2+\frac{1}{(1+3+5)}(1+2+3)^2+\frac{1}{(1+3+5+7)}(1+2+3+4)^2+\cdots.$

r^{th} term $T_r=\frac{1}{r^2}(1+2+\cdots\ldots+r)^2$

$=\frac{1}{r^2}\left\{\frac{r(r+1)}{2}\right\}^2=\frac{r^2+2r+1}{4}$

$\therefore T_7=16$ and $S_{10}=\sum_{r=1}^{10}T_r=\frac{1}{4}\left\{\frac{(10)(10+1)(20+1)}{6}+(10)(10+1)+10\right\}=\frac{505}{4}$

INTEGER TYPE

12. The sum of the terms of an infinitely decreasing GP is equal to the greatest value of the function $f(x)=x^3+3x-9$ on the interval $[-4,3]$ and the difference between the first and second terms is 3. Then find the value of 27r where r is common ratio.

Ans. **(18)**

Sol. f is increasing

so its greatest value is $f(3)=27$. Let the GP be $a, ar, ar^2 \ldots\ldots.$ with, $-1<r<1$

$\frac{a}{1-r}=27$ and $a-ar=3 \Rightarrow r=\frac{4}{3}$ or $r=\frac{2}{3}$

but $-1<r<1$ so $r=\frac{2}{3} \Rightarrow 27r=18$

BINOMIAL THEOREM

MULTIPLE CORRECT ANSWER TYPE

13. The value of $\frac{^{50}C_0}{3} - \frac{^{50}C_1}{4} + \frac{^{50}C_2}{5} \ldots . + \frac{^{50}C_{50}}{53}$ is equal to

(A) $\int_0^1 x^3(1-x)^{50}dx$ (B) $\int_0^1 x(1-x)^{50}dx$

(C) $\frac{1}{51} - \frac{2}{52} + \frac{1}{53}$ (D) $\frac{1}{70278}$

Ans. **(C, D)**

Sol. $\frac{^{50}C_0}{3} - \frac{^{50}C_1}{4} + \frac{^{50}C_2}{5} \ldots .. + \frac{^{50}C_{50}}{53}$

$= \int_0^1 {}^{50}C_0x^2 - {}^{50}C_1x^3 + {}^{50}C_2x^4 \ldots . + {}^{50}C_{50}x^{52}dx$

$= \int_0^1 x^2(1-x)^{50}x = \int_0^1 (1-x)^2x^{50}x = \int_0^1 (1-2x+x^2)x^{50}dx = \frac{1}{51} - \frac{2}{52} + \frac{1}{53}$

INTEGER TYPE

14. The value of $\frac{y_1 \cdot y_2 \cdot y_3}{501(y_1-x_1)(y_2-x_2)(y_3-x_3)}$ when $(x_i, y_i), i = 1,2,3$ satisfy both

$x^3 - 3xy^2 = 2005$ & $y^3 - 3x^2y = 2004$ is

Ans. **(2)**

Sol. $x^3 - 3xy^2 = 2005 \Rightarrow \left[\left(\frac{x}{y}\right)^3 - 3\left(\frac{x}{y}\right) = \frac{2005}{y^3}\right] \times 2004 \ldots \ldots (1)$

$y^3 - 3x^2y = 2004 \Rightarrow \left[1 - 3\left(\frac{x}{y}\right)^2 = \frac{2004}{y^3}\right] \times 2005 \ldots \ldots (2)$

subtract (1) & (2) & put $\frac{x}{y} = t$

$2004t^3 + 6015t^2 - 6012t - 2005 = 0 \begin{matrix} t_1 \\ t_2 \\ t_3 \end{matrix}$

$\frac{y_1 \cdot y_2 \cdot y_3}{(y_1-x_1)(y_2-x_2)(y_3-y_3)} = \frac{1}{(1-t_1)(1-t_2)(1-t_3)}$

$= \frac{1}{1 + (t_1t_2 + t_2t_3 + t_3t_1) - (t_1 + t_2 + t_3) - t_1t_2t_3}$

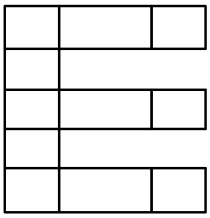
put values

$= 1002$

PERMUTATION & COMBINATION

SINGLE CORRECT ANSWER TYPE

15. Number of ways in which A A A B B B can be placed in the squares of the figure as shown so that no row remains empty, is



(A) 2430 (B) 2160 (C) 1620 (D) none

Ans. **(C)**

Sol. A A A B B B can be arranged at 6 places in $\frac{6!}{3!3!} = 20$ ways 6 places can be selected in

$3 \times {}^3C_1 \times {}^3C_1 \times {}^3C_2 = 81 \therefore$ total number of ways $= 20 \times 81 = 1620$

MULTIPLE CORRECT ANSWER TYPE

16. The number of ways of arranging the letters AAAAA, BBB, CCC, D, EE & F in a row if the letters C are separated from one another is:

(A) ${}^{13}C_3 \cdot \frac{12!}{5!3!2!}$ (B) $\frac{13!}{5!3!3!2!}$

(C) $\frac{14!}{3!3!2!}$ (D) $\frac{15!}{5!(3!)^2 2!} - \frac{13!}{5!3!2!} - \frac{12!}{5!3!}\ {}^{13}C_2$

Ans. **(A, D)**

Sol. All AAAAA BBB D EEF can be arranged in $\frac{12!}{5!3!2!}$ ways

Between the gaps C can be arranged in ${}^{13}C_3$ ways Total ways $= {}^{13}C_3 \times \frac{12!}{5! \times 3! \times 2!}$

Number of ways = without considering separation of C – in which all C's are together - in which exactly two C′ 's are together $= \frac{15!}{5!(3!)^2 2!} - \frac{13!}{5!3!2!} - \frac{12!}{5!3!}\ {}^{13}C_2$

PROBABILITY

SINGLE CORRECT ANSWER TYPE

17. S_1 : Two persons each make a single throw with a die. The probability they get equal values is P_1. Four persons each make a single throw and probability of exactly three being equal is P_2. Then P_1 greater than P_2.

S_2 : Each of A & B throw 2 dice, if A throws 9 , then B 's probability of throwing a higher number is $\frac{1}{6}$

S_3: If $P(A_1 \cup A_2) = 1 - P(A_1{}^C) \cdot P(A_2{}^C)$, then A_1 and A_2 are independent

S_4 : If the events A, B, C are independent, then A, B, $\bar{C}$ are independent

(A) T T T T (B) TTFT (C) TFTF (D) F TTF

Ans. **(A)**

Sol. S_1: $P_1 = \frac{1}{6}$

$P_2 = {}^4C_3 \times {}^6C_1 \times \frac{1}{6} \times \frac{1}{6} \times \frac{1}{6} \times \frac{5}{6} = \frac{5}{54}$ $\qquad \therefore P_1 > P_2$

$\therefore$ S_1 is true.

S_2 : B throws 10,11, or 12. The cases are

$6 + 4, 5 + 5, 4 + 6$

$6 + 5, 5 + 6, 6 + 6$

$\therefore$ prob. $= \frac{6}{36} = \frac{1}{6}$

$\therefore$ S_2 is true

S_3: $P(A_1 \cup A_2) = 1 - P(A_1 \cup A_2)' = 1 - P(A_1{}' \cap A_2{}')$

also $P(A_1 \cup A_2) = 1 - P(A_1')P(A_2')$

$\therefore$ $P(A_1' \cap A_2') = P(A_1{}')P(A_2{}')$

$\therefore$ A_1' and A_2' are independent

$\therefore$ A_1 and A_2 are independent

S_A: $P(A \cap B \cap C) = P(A)P(B)P(C)$

$P(A \cap B \cap \bar{C}) = P(A \cap B) - C) = P((A \cap B) - (A \cap B \cap C))$

$= P(A)P(B) - P(A) \cdot P(B)P(C) = P(A)P(B)(1 - P(C)) = P(A)P(B)P(\bar{C})$

$\therefore$ $A, B, \bar{C}$ are independent.

MULTIPLE CORRECT ANSWER TYPE

18. A bag initially contains one red & two blue balls. An experiment consisting of selecting a ball at random, noting its colour & replacing it together with an additional ball of the same colour. If three such trials are made, then:

(A) probability that atleast one blue ball is drawn is 0.9

(B) probability that exactly one blue ball is drawn is 0.2

(C) probability that all the drawn balls are red given that all the drawn balls are of same colour is 0.2

(D) probability that atleast one red ball is drawn is 0.6 .

Ans. **(A,B,C,D)**

Sol. (i) $P(E_1) = 1 - P(RRR)$

$= 1 - \left[\frac{1}{3} \cdot \frac{2}{4} \cdot \frac{3}{5}\right] = 0.9$

(ii) $P(E_2) = 3P(BRR) = 3 \cdot \frac{2}{3} \cdot \frac{1}{4} \cdot \frac{2}{5} = 0.2$

(iii) $P(E_3) = P\left(\frac{RRR}{RRR} \cup BBB = \frac{P(RRR)}{P(RRR)+P(BBB)}\right.$ but $P(B\,B\,B\,) = \frac{2}{3} \cdot \frac{3}{4} \cdot \frac{4}{5} = \frac{8}{20}$

$\Rightarrow P(E_3) = \frac{0.1}{0.1+0.4} = 0.2$(iv) $P(E_4) = 1 - P(B\,B\,B\,) = 1 - \frac{2}{5} = 0.6$

COMPLEX NUMBER

SINGLE CORRECT ANSWER TYPE

19. S1: If (z_1, z_2) and (z_3, z_4) are two pairs of non zero conjugate complex numbers then $\arg\left(\frac{z_1}{z_3}\right) + \arg\left(\frac{z_2}{z_4}\right) = \pi/2$

S2: If ω is an imaginary fifth root of unity, then $\log_2\left|1+\omega+\omega^2+\omega^3-\frac{1}{\omega}\right| = 1$

S3: If z_1 and z_2 are two of the 8th roots of unity, such that $\arg\left(\frac{z_1}{z_2}\right)$ is least positive, then $\frac{z_1}{z_2} = \frac{1+i}{\sqrt{2}}$

S4: The product of all the fifth roots of -1 is equal to -1

(A) TTFT (B) TFFT (C) FFTF (D)FTIT

Ans. **(D)**

Sol. S1. $\bar{z}_1 = z_2, \bar{z}_3 = z_4 \quad \arg\left(\frac{z_1}{z_3}\right) + \arg\left(\frac{z_2}{z_4}\right) = \arg\left(\frac{z_1 z_2}{z_3 z_4}\right) = \arg\left(\frac{z_1 \bar{z}_1}{z_3 \bar{z}_3}\right) = 0$

S2. $\log_2|1+\omega+\omega^2+\omega^3-\omega^4| = \log_2|-2\omega^4| = \log_2 2 = 1 \because |\omega^4| = 1$

S3. Since z_1, z_2 are 8^{th} roots of unity

$\therefore \arg\left(\frac{z_1}{z_2}\right) = \frac{\pi}{4}$ (least positive) $\therefore \frac{z_1}{z_2} = e^{i\pi/4} = \frac{1+i}{\sqrt{2}}$. S4. $z^5 + 1 = 0$

$\therefore$ product of all the roots $= (-1)^5 \cdot 1 = -1$

20. **Match the column :**

If z_1, z_2, z_3, z_4 are the roots of the equation $z^4 + z^3 + z^2 + z + 1 = 0$ then

Column-I	**Column - II**
(A) $\left|\sum_{i=1}^{4} z_i^4\right|$ is equal to	(p) 0
(B) $\sum_{i=1}^{4} z_i^5$ is equal to	(q) 4
(C) $\prod_{i=1}^{4}(z_i+2)$ is equal to	(r) 1
(D) least value of $[|z_1+z_2|]$ is	(s) 11
(Where [] represents greatest integer function)	(t) $\left|4\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)\right|$

Ans. $\mathbf{(A) \to (r), (B) \to (q,t), (C) \to (s), (D) \to (p)}$

Sol. The given equation is $\frac{z^5-1}{z-1} = 0$ which means that z_1, z_2, z_3, z_4 are four out of five roots of unity except 1 .

(A) $z_1{}^4 + z_2{}^4 + z_3{}^4 + z_4{}^4 + 1^4 = 0 \Rightarrow \left|\sum_{i=1}^{4} z_i{}^4\right| = 1$

(B) $z_1^5 + z_2^5 + z_3^5 + z_4^5 + 1^5 = 5 \Rightarrow \sum_{i=1}^{4} z_i^5 = 4$

(C) $z^4 + z^3 + z^2 + z + 1 = (z-z_1)(z-z_2)(z-z_3)(z-z_4)$.

Putting $z = -2$ both the sides and we get $\prod_{i=1}^{4}(z_i+2) = 11$

(D) $|z_1+z_2| = \sqrt{2+2\cos 144^\circ}$ for minimum $= 2\cos 72^\circ = \frac{\sqrt{5}-1}{2}$ whose greatest integer is 0 .

TRIGONOMETRIC IDENTITIES & EQUATION

MULTIPLE CORRECT ANSWER TYPE

21. The solution of the equation $(\tan^2 x - 1)^{-1} = 1 + \cos 2x$ satisfy the inequality $2^{x+1} - 8 > 0$ are

(A) $x = n\pi - \frac{\pi}{2}, n \in I$ (B) $x = n\pi + \frac{\pi}{3}$

(C) $x = n\pi - \frac{\pi}{3}$ (D) None of these

Ans. **(B,C)**

Sol. $(1 + \cos 2x)\left(1 + \frac{1}{2\cos 2x}\right) = 0$

$x = n\pi - \frac{\pi}{2}, n\pi \pm \frac{\pi}{3} n \in I$

but $\cos x \neq 0$ for $2^{x+1} - 8 > 0$

so $x = n\pi \pm \frac{\pi}{3} n \in I$

SOLUTION OF TRIANGLES & HEIGHT DISTANCE

SINGLE CORRECT ANSWER TYPE

22. In triangle ABC, $a: b: c = (1 + x): 1: (1 - x)$, where $x \in (0,1)$. If $\angle A = \frac{\pi}{2} + \angle C$, then x is equal to

(A) $\frac{1}{\sqrt{6}}$ (B) $\frac{1}{2\sqrt{6}}$ (C) $\frac{1}{\sqrt{7}}$ (D) $\frac{1}{2\sqrt{7}}$

Ans. **(C)**

Sol. $a = (1 + x)h, b = h, c = (1 - x)h, \frac{A}{2} - \frac{C}{2} = \frac{\pi}{4}$

$\Rightarrow \cos\frac{A}{2} \cdot \cos\frac{c}{2} + \sin\frac{A}{2} \sin\frac{C}{2} = \frac{1}{\sqrt{2}}$

$\Rightarrow \sqrt{\frac{s^2(s-a)(s-c)}{bc \cdot ab}} + \sqrt{\frac{(s-b)(s-c)(s-a)(s-b)}{bc \cdot ab}} = \frac{1}{\sqrt{2}}$

$\Rightarrow \frac{s}{b}\sqrt{\frac{(s-a)(s-c)}{ac} + \frac{(s-b)}{b}\sqrt{\frac{(s-a)(s-c)}{ac}}} = \frac{1}{\sqrt{2}}$

$\Rightarrow \left(\frac{2s-b}{b}\right)\sqrt{\frac{(s-a)(s-c)}{ac}} = \frac{1}{\sqrt{2}}$

$\Rightarrow \left(\frac{a+c}{b}\right)\sqrt{\frac{(s-a)(s-c)}{ac}} = \frac{1}{\sqrt{2}}$

$\Rightarrow \frac{ac}{(s-a)(s-c)}$

Now $a + c = 2h, b = h$

$\Rightarrow 2, s = \frac{a+b+c}{2} = \frac{3h}{2}$

$\Rightarrow s - a = \frac{(1-2x)h}{2}, (s - c) = \frac{(1+2x)h}{2}$

$\Rightarrow 8 = \frac{4(1-x^2)}{(1-4x^2)} \Rightarrow x = \frac{1}{\sqrt{7}}$

MULTIPLE CORRECT ANSWER TYPE

23. If in a triangle ABC, p, q and r are the altitudes drawn from the vertices A, B, C respectively to the opposite sides, then which of the following hold(s) good.

(A) $(\Sigma p)\left(\Sigma \frac{1}{p}\right) = (\Sigma a)\left(\Sigma \frac{1}{a}\right)$

(B) $(\Sigma p)(\Sigma a) = \left(\Sigma \frac{1}{p}\right)\left(\Sigma \frac{1}{a}\right)$

(C) $(\Sigma p)(\Sigma pq)(\Pi a) = (\Sigma a)(\Sigma ab)(\Pi p)$

(D) $\left(\Sigma \frac{1}{p}\right)\Pi\left(\frac{1}{p}+\frac{1}{q}-\frac{1}{r}\right)\Pi a^2 = 16R^2$, where R is the circum-radius of $\triangle$ ABC.

Ans. **(A,C,D)**

Sol. $p = \frac{2\Delta}{a}, q = \frac{2\Delta}{b}, r = \frac{2\Delta}{c}$

(A) $(\Sigma p)\left(\Sigma \frac{1}{p}\right) = 2\Delta\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)\left(\frac{a+b+c}{2\Delta}\right) = (\Sigma a)\left(\Sigma \frac{1}{a}\right)$

(C) $(\Sigma p)(\Sigma pq)(\Pi a)$

$= \left(\frac{2\Delta}{a}+\frac{2\Delta}{b}+\frac{2\Delta}{c}\right)\left(\frac{4\Delta^2}{ab}+\frac{4\Delta^2}{bc}+\frac{4\Delta^2}{ca}\right)abc$

$= \frac{2\Delta(ab+bc+ca)}{abc}, 4\Delta^2\frac{(a+b+c)}{abc}\cdot abc = (\Sigma a)(\Sigma ab)(\Pi p)$

(D) $\left(\Sigma \frac{1}{p}\right)\Pi\left(\frac{1}{p}+\frac{1}{q}-\frac{1}{r}\right)\Pi a^2$

$= \left(\frac{a+b+c}{2\Delta}\right)\left(\frac{a+b-c}{2\Delta}\right)\left(\frac{a-b+c}{2\Delta}\right)\left(\frac{b+c-a}{2\Delta}\right)\cdot a^2b^2c^2$

$=\cdot \frac{(s-b)}{\Delta}\cdot\frac{(s-a)}{\Delta}(abc)^2 = \left(\frac{abc}{\Delta}\right)^2$

$= 16R^2$

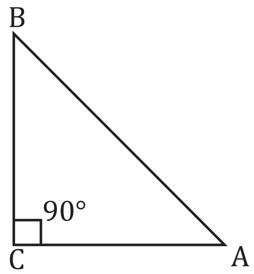
24. **Match the following**

Column – I	**Column - II**
(A) In a $\triangle$ ABC, let $\angle C = \frac{\pi}{2}$, r = in-radius and R = circum-radius, then $2(r+R)$ is equals to	(p) $a+b+c$
(B) IF ℓ, m, n are perpendicular drawn from the vertices of triangle having sides a , b and c, then $\sqrt{2R\left(\frac{b\ell}{c}+\frac{cm}{a}+\frac{an}{b}\right)+2ab+2bc+2ca}$ equals to	(q) $a-b$
(C) In a $\triangle$ ABC, $R(b^2\sin 2C + c^2\sin 2B)$ equals to	(r) $a+b$
(D) In a right angle triangle ABC if $\angle C = \frac{\pi}{2}$, then	(s) abc
$4R\sin\frac{(A+B)}{2}\cdot\sin\frac{(A-B)}{2}$ equals to	(t) $\frac{a+b+c}{2}$

Ans. **(A) → (r).(B) → (p),(C) → (s),(D) → (q)**

Sol. (A) $2(r+R)=2\left((s-c)\tan\frac{C}{2}+\frac{c}{2}\right)$

$2\left(s-\frac{c}{2}\right)=a+b$



(B) $\sin C=\frac{\ell}{b}\ldots\ldots\ldots\ldots....(i)$

$\sin B=\frac{n}{a}\quad\ldots\ldots\ldots\ldots....(ii)$

$\sin A=\frac{m}{c}\ldots\ldots\ldots\ldots....(iii)$

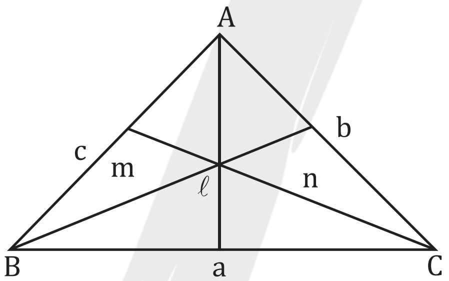
$2R\left(\frac{b\ell}{c}\right)+2R\left(\frac{cm}{a}\right)+2R\left(\frac{an}{b}\right)$

$\frac{c}{\sin C}\cdot\frac{b\ell}{c}+\frac{a}{\sin A}\frac{cm}{a}+\frac{b}{\sin B}\frac{an}{b}$

From (iii). (ii). (iii)

$2R\left(\frac{b\ell}{c}+\frac{cm}{a}+\frac{an}{b}\right)=a^2+b^2+c^2$

$\therefore\sqrt{2R\left(\frac{bt}{c}+\frac{cm}{a}+\frac{an}{b}\right)+2ab+2bc+2ac}=a+b+c$



(C) $R=\frac{b}{2\sin B}=\frac{c}{2\sin C}$

$\therefore Rb^2\sin 2C+Rc^2\sin 2B=b^2c\cos C+c^2b\cos B$

$=bc(b\cos C+c\cos B)$

$=abc$

(D) $2R(\cos B-\cos A)$

$=2R\left(-\frac{b}{\sqrt{a^2+b^2}}+\frac{a}{\sqrt{a^2+b^2}}\right)$

$\because\ R=\sqrt{a^2+b^2}/2$

$=2R\left(-\frac{b}{\sqrt{a^2+b^2}}+\frac{a}{\sqrt{a^2+b^2}}\right)=a-b$