Nishant Jindal

$$\lim_{n\to\infty}\int_{-\sqrt[3]{a}}^{\sqrt[3]{a}}\underbrace{\left(1-\frac{t^3}{n}\right)^n t^2}\,dt = \int_{-\sqrt[3]{a}}^{\sqrt[3]{a}}\underline{\lim_{n\to\infty}\left(1-\frac{t^3}{n}\right)^n} t^2\,dt$$

$$= \int_{-\sqrt[3]{a}}^{\sqrt[3]{a}} e^{-t^3}\, t^2\, dt$$

$$\lim_{n\to c}\int_{\alpha}^{\beta} f(t,n)\,dt = \lim_{n\to c}\lim_{m\to\infty}\sum_{r=1}^{m} f\left(\alpha+\frac{r}{m}(\beta-\alpha),n\right)\left(\frac{\beta-\alpha}{m}\right)$$

$$= \lim_{m\to\infty}\lim_{n\to c}$$

Nishant Jindal

$$h(x) = f^2(x) + \left(f'(x)\right)^2$$

$f'(c) \neq 0$.

$$\boxed{f(c) + f''(c) = 0}$$

$c_1 \in (-4, 0)$

$$-1 \le \frac{f(0) - f(-4)}{0-(-4)} = \underline{f'(c_1)} \le 1$$

impossible

$h(c_1) \le 5$

$\underline{f'(c) = 0}$, $h(c) = f^2(c) \le 4$

$c_2 \in (0, 4)$

$$-1 \le \frac{f(4) - f(0)}{4-0} = f'(c_2) \le 1, \quad h(c_2) \le 5$$

$h(0) = 85$.

global max in $[c_1, c_2]$ at $\underline{c}$

$$h'(c) = 0 = 2f'(c)\left(\underbrace{f(c) + f''(c)}\right) = 0$$

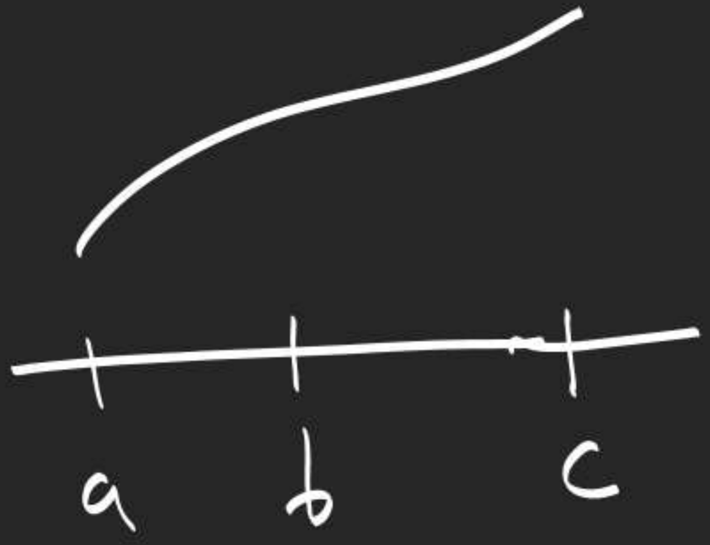
Nishant Jindal

Properties

- $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$

- $\int_a^b f(x)\,dx = \int_a^b f(t)\,dt$

- $\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$



$$\int_a^c = \int_a^b + \int_b^c$$

$$\Rightarrow \int_a^b = \int_a^c - \int_b^c$$

$$= \int_a^c + \int_c^b$$

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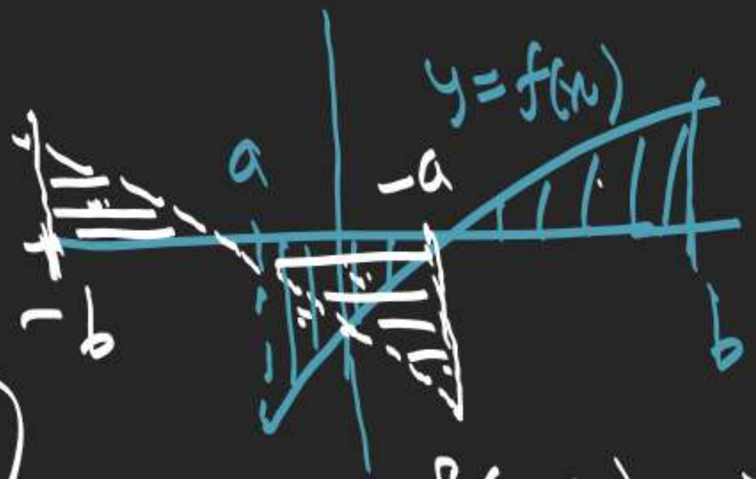
- $$\int_a^b f(x)\,dx = \int_{-b}^{-a} f(-x)\,dx$$

$x = -t$

$$\int_{-b}^{-a} f(-t)\,dt = -\int_{-a}^{-b} f(-t)\,dt$$

$f(x) \to (\alpha, \beta)$

$f(-x) \to (-\alpha, \beta)$

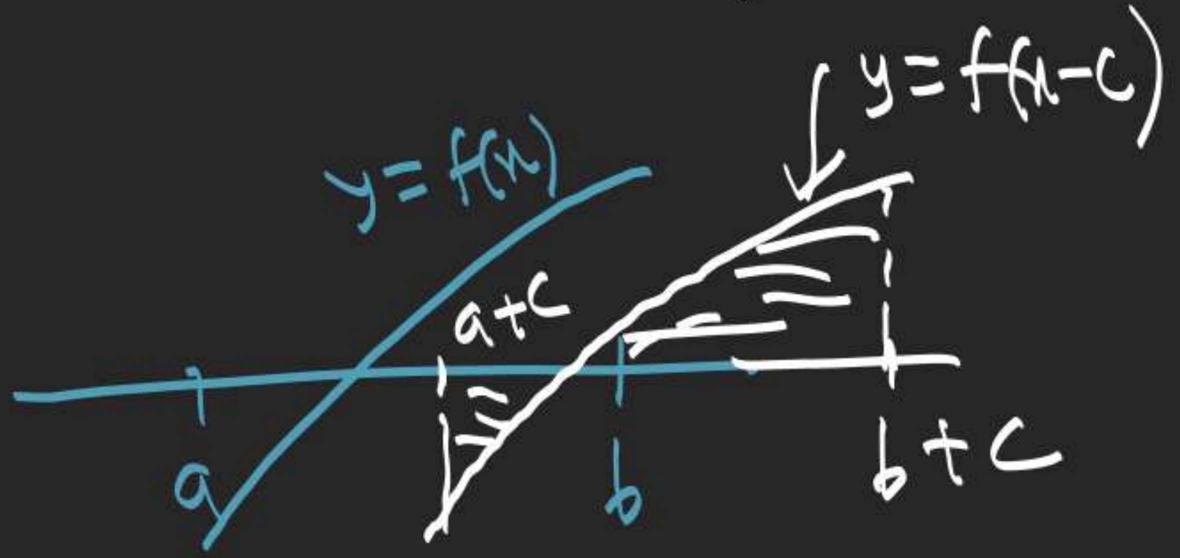


$f(-x) = y$

- $$\int_a^b f(x)\,dx = \int_{a+c}^{b+c} f(x-c)\,dx$$

$x = t - c$

$$\int_{c+a}^{c+b} f(t-c)\,dt$$



- $$\int_a^b f(x)\,dx = \int_{a-c}^{b-c} f(x+c)\,dx$$

$f(x) \to (\alpha, \beta)$

$f(x-c) \to (\alpha + c, \beta)$

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- $$\int_{-a}^{a} f(x)\,dx = \int_{0}^{a}\Big(f(x)+f(-x)\Big)\,dx$$

$$= \begin{cases} 0 & \text{if } f \text{ is odd} \\ 2\int_{0}^{a} f(x)\,dx & \text{if } f \text{ is even} \end{cases}$$

$$\int_{-a}^{a} f(x)\,dx = \int_{-a}^{0} f(x)\,dx + \int_{0}^{a} f(x)\,dx = \int_{-(0)}^{-(-a)} f(-x)\,dx + \int_{0}^{a} f(x)\,dx$$

$$= \int_{0}^{a}\Big(f(-x)+f(x)\Big)\,dx$$

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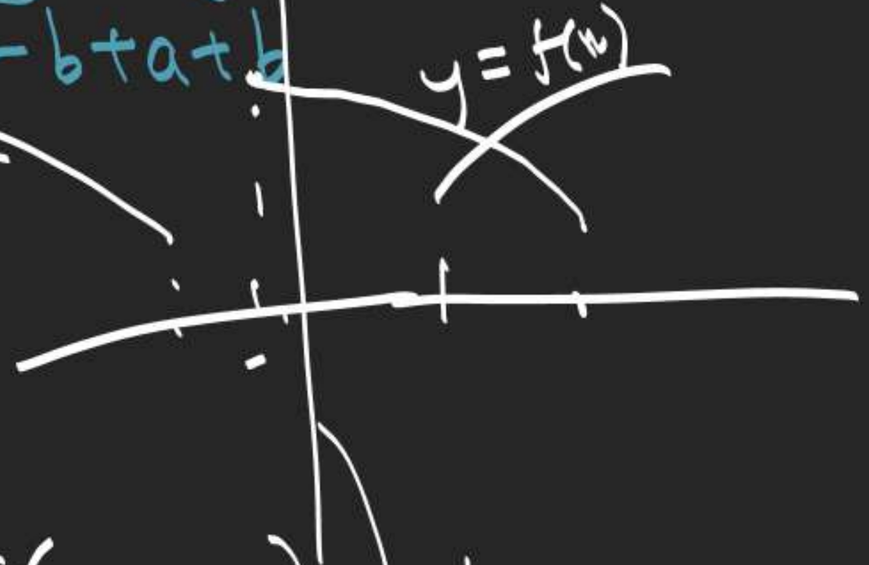
- $\displaystyle\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$

$x \to x-(a+b)$

$\displaystyle = \int_{-b}^{-a} f(-x)\,dx = \int_{-b+a+b}^{-a+a+b} f(-(x-a-b))\,dx = \int_a^b f(a+b-x)\,dx$

- $\displaystyle\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$



$y = f(3-x)$

- $\displaystyle\int_0^{2a} f(x)\,dx = \int_0^a \big(f(x)+f(2a-x)\big)\,dx$

$\displaystyle = \int_0^a f(x)\,dx + \int_a^{2a} f(x)\,dx$

$\displaystyle \int_{-2a}^{-a} f(-x)\,dx = \int_{-2a+2a}^{-a+2a} f(-(x-2a))\,dx$

$\displaystyle = \begin{cases} 2\int_0^a f(x)\,dx & f(x) = f(2a-x) \\ 0 & f(x) = -f(2a-x) \end{cases}$

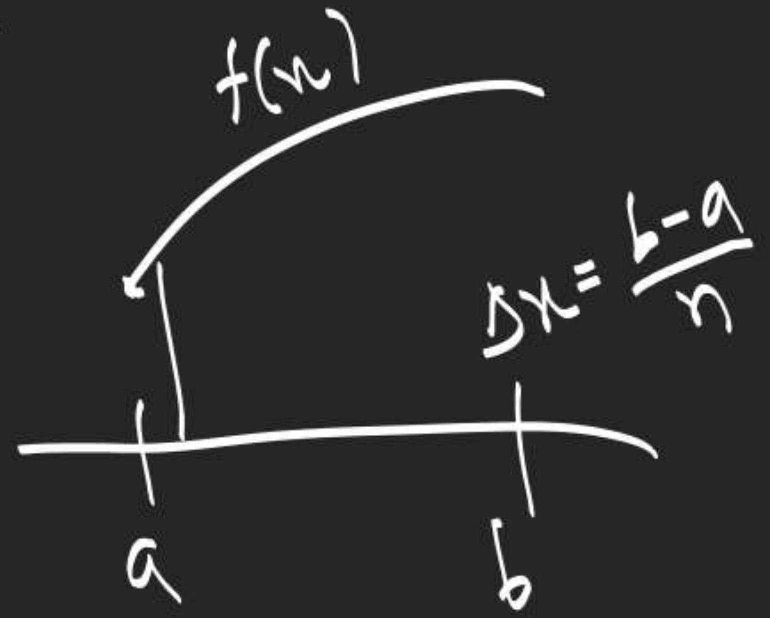
Nishant Jindal

- $\int_a^b f(x)\,dx = k\int_{a/k}^{b/k} f(kx)\,dx$

$$\int_{a/k}^{b/k} f(kx)\,dx = \frac{1}{k}\int_a^b f(x)\,dx$$

- $\int_a^b f(x)\,dx = \frac{1}{k}\int_{ak}^{bk} f\left(\frac{x}{k}\right)dx$

$\boxed{\text{Ex-I (Max/Min)}}$



$$\lim_{n\to\infty}\sum_{i=1}^{n} y_i\left(\frac{b-a}{n}\right) = \int_a^b f(x)\,dx$$

$$\lim_{n\to\infty}\sum_{i=1}^{n} y_i\,\frac{b-a}{nk} = \int_{a/k}^{b/k} f(kx)\,dx$$