

(MATHEMATICS)

DEFINITE INTEGRATION

ELEMENTARY

1. If $\int_1^x \frac{dt}{|t|\sqrt{t^2-1}} = \frac{\pi}{6}$, then x can be equal to
 (A) $\frac{2}{\sqrt{3}}$ (B) $\sqrt{3}$ (C) 2 (D) None of these
2. $\int_0^{(\pi/2)^{1/3}} x^5 \cdot \sin x^3 dx$ equals to
 (A) 1 (B) 1/2 (C) 2 (D) 1/3
3. If $A = \int_0^{\pi} \frac{\cos x}{(x+2)^2} dx$, then $\int_0^{\pi/2} \frac{\sin 2x}{x+1} dx$ is equal
 (A) $\frac{1}{2} + \frac{1}{\pi+2} - A$ (B) $\frac{1}{\pi+2} - A$ (C) $1 + \frac{1}{\pi+2} - A$ (D) $A - \frac{1}{2} - \frac{1}{\pi+2}$
4. If $f(\pi) = 2$ and $\int_0^{\pi} (f(x) + f''(x)) \sin x dx = 5$ then $f(0)$ is equal to (It is given that $f(x)$ is continuous in $[0, \pi]$)
 (A) 7 (B) 3 (C) 5 (D) 1
5. If $x \in (0, 2)$ then the value of $\int_1^{2x-[2x]} e^{2x-[2x]} d(x - [x])$ is (where $[*]$ denotes the greatest integer function)
 (A) $e + 1$ (B) e (C) $2e - 2$ (D) $e - 1$
6. $\int_{\log \pi - \log 2}^{\log \pi} \frac{e^x}{1 - \cos(\frac{2}{3}e^x)} dx$ is equal to
 (A) $\sqrt{3}$ (B) $-\sqrt{3}$ (C) $\frac{1}{\sqrt{3}}$ (D) $-\frac{1}{\sqrt{3}}$
7. If $\int_0^{\infty} e^{-x^2} dx = \frac{\sqrt{\pi}}{2}$, then $\int_0^{\infty} e^{-ax^2} dx$ where $a > 0$ is
 (A) $\frac{\sqrt{\pi}}{2}$ (B) $\frac{\sqrt{\pi}}{2a}$ (C) $2 \frac{\sqrt{\pi}}{a}$ (D) $\frac{1}{2} \sqrt{\frac{\pi}{a}}$
8. Let $A = \int_0^1 \frac{e^t dt}{1+t}$ then $\int_{a-1}^a \frac{e^{-t}}{t-a-1} dt$ has the value
 (A) Ae^{-a} (B) $-Ae^{-a}$ (C) $-ae^{-a}$ (D) Ae^a
9. Let $\frac{d}{dx} F(x) = \left(\frac{e^{\sin x}}{x}\right)$, $x > 0$, If $\int_1^4 \frac{3}{x} e^{\sin x^3} dx = F(k) - F(1)$, then one of the possible values of k, is
 (A) 15 (B) 16 (C) 63 (D) 64
10. $\frac{1}{c} \int_{ac}^{bc} f\left(\frac{x}{c}\right) dx =$
 (A) $\frac{1}{c} \int_a^b f(x) dx$ (B) $\int_a^b f(x) dx$ (C) $c \int_a^b f(x) dx$ (D) $\int_{ac^2}^{bc^2} f(x) dx$
11. $\int_{5/2}^5 \frac{\sqrt{(25-x^2)^3}}{x^4} dx$ equals to
 (A) $\frac{\pi}{3}$ (B) $\frac{2\pi}{3}$ (C) $\frac{\pi}{6}$ (D) $\frac{\pi}{2}$

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12. The value of $\int_0^{[x]} \{x\} dx$ is
 (A) $\frac{1}{2}[x]$ (B) $2[x]$ (C) $\frac{1}{2[x]}$ (D) $[x]$
13. The value of $\int_0^{\pi/4} (\tan^n x + \tan^{n-2} x) d\left(x - \frac{[x]}{1!} + \frac{[x]^2}{2!} - \frac{[x]^3}{3!} + \dots\right)$, where $[x]$ is greatest integral function, is
 (A) $\frac{1}{n}$ (B) $\frac{1}{n+2}$ (C) $\frac{1}{n-1}$ (D) $\frac{1}{n-2}$
14. If $I = \int_0^1 \frac{dx}{1+x^{\pi/2}}$, then
 (A) $I > \ln 2$ (B) $I < \ln 2$ (C) $I < \pi/4$ (D) $I > \pi/4$

PROPERTY-01

15. The value of integral $\int_a^b \frac{[x]}{x} dx$, $a < b$ is
 (A) $b - a$ if $a > 0$ (B) $a - b$ if $b < 0$
 (C) $b + a$ if $a < 0 < b$ (D) $|b| - |a|$

PROPERTY-02

16. The value of the integral $I = \int_0^1 x(1-x)^n dx$ is -
 (A) $\frac{1}{n+1} + \frac{1}{n+2}$ (B) $\frac{1}{n+1}$ (C) $\frac{1}{n+2}$ (D) $\frac{1}{n+1} - \frac{1}{n+2}$
17. Let $I_1 = \int_0^1 \frac{e^x dx}{1+x}$ and $I_2 = \int_0^1 \frac{x^2 dx}{e^{x^3}(2-x^3)}$, then $\frac{I_1}{I_2}$ is to
 (A) $3/e$ (B) $e/3$ (C) $3e$ (D) $1/3e$
18. If $f(x)$ is a function satisfying $f\left(\frac{1}{x}\right) + x^2 f(x) = 0$ for all non-zero x , then $\int_{\sin\theta}^{\operatorname{cosec}\theta} f(x) dx$ equals
 (A) $\sin\theta + \operatorname{cosec}\theta$ (B) $\sin^2\theta$ (C) $\operatorname{cosec}^2\theta$ (D) 0

PROPERTY-03

19. If $\int_0^{100} f(x) dx = a$, then $\sum_{r=1}^{100} \left(\int_0^1 f(r-1+x) dx\right) =$
 (A) $100a$ (B) a (C) 0 (D) $10a$
20. If $f(x) = \begin{cases} x & ; x < 1 \\ x-1 & ; x \geq 1 \end{cases}$, then $\int_0^2 x^2 f(x) dx$ is equal to
 (A) 1 (B) $\frac{4}{3}$ (C) $\frac{5}{2}$ (D) $\frac{5}{3}$
21. Suppose for every integer n , $\int_n^{n+1} f(x) dx = n^2$. The value of $\int_{-2}^4 f(x) dx$ is
 (A) 16 (B) 14 (C) 19 (D) 21
22. $\int_0^\pi |1 + 2\cos x| dx$ equals to:
 (A) $\frac{2\pi}{3}$ (B) π (C) 2 (D) $\frac{\pi}{3} + 2\sqrt{3}$

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23. The value of $\int_{-1}^3 (|x - 2| + [x])dx$ is equal to (where $[*]$ denotes greatest integer function)
 (A) 7 (B) 5 (C) 4 (D) 3
24. If $\int_{-1}^{3/2} |x \sin \pi x| dx = \frac{k}{\pi^2}$, then the value of k is
 (A) $3\pi + 1$ (B) $2\pi + 1$ (C) 1 (D) 4
25. $\int_{\pi}^{10\pi} |\sin x| dx =$
 (A) 9 (B) 10 (C) 18 (D) 20
26. The value of $\int_{-2}^3 |1 - x^2| dx$ is-
 (A) $28/3$ (B) $14/3$ (C) $7/3$ (D) $1/3$
27. $\int_0^{\infty} [2e^{-x}] dx$ is equal to (where $[*]$ denotes the greatest integer function)
 (A) 0 (B) $\ln 2$ (C) e^2 (D) $2e^{-1}$
28. $\int_0^{\sqrt{2}} [x^2] dx =$
 (A) $\sqrt{2} - 1$ (B) $2(\sqrt{2} - 1)$ (C) $\sqrt{2}$ (D) $5 - \sqrt{3} - \sqrt{2}$
29. If $f(x) = \int_0^x \sin[2x] dx$ then $f(\pi/2)$ is (where $[*]$ denotes greatest integer function)
 (A) $\frac{1}{2} \{ \sin 1 + (\pi - 2) \sin 2 \}$ (B) $\frac{1}{2} \{ \sin 1 + \sin 2 + (\pi - 3) \sin 3 \}$
 (C) 0 (D) $\sin 1 + \left(\frac{\pi}{2} - 2 \right) \sin 2$
30. $f(x) = \text{Minimum} \{ \tan x, \cot x \} \forall x \in \left(0, \frac{\pi}{2} \right)$.
 Then $\int_0^{\pi/3} f(x) dx$ is equal to
 (A) $\ln \left(\frac{\sqrt{3}}{2} \right)$ (B) $\ln \left(\sqrt{\frac{3}{2}} \right)$ (C) $\ln(\sqrt{2})$ (D) $\ln(\sqrt{3})$
31. The value of $\int_1^2 ([x^2] - [x]^2) dx$ is equal to (where $[*]$ denotes the greatest integer function)
 (A) $4 + \sqrt{2} - \sqrt{3}$ (B) $4 - \sqrt{2} + \sqrt{3}$ (C) $4 - \sqrt{3} - \sqrt{2}$ (D) $4 + \sqrt{2} + \sqrt{3}$
32. If $f(x) = \begin{cases} e^{\cos x} \sin x, & |x| \leq 2 \\ 2, & \text{otherwise} \end{cases}$ then $\int_{-2}^3 f(x) dx =$
 (A) 0 (B) 1 (C) 2 (D) 3
33. The value of $\int_0^{\pi/3} [\sqrt{3} \tan x] dx$ (where $[*]$ denotes the greatest integer function)
 (A) $\frac{5\pi}{6}$ (B) $\frac{5\pi}{6} - \tan^{-1} \left(\frac{2}{\sqrt{3}} \right)$ (C) $\frac{\pi}{2} - \tan^{-1} \left(\frac{2}{\sqrt{3}} \right)$ (D) $\frac{\pi}{2} + \tan^{-1} \left(\frac{2}{\sqrt{3}} \right)$
34. The value of $\int_{-2}^1 \left[x \left[1 + \cos \left(\frac{\pi x}{2} \right) \right] + 1 \right] dx$ is (where $[*]$ denotes the greatest integer function)
 (A) 1 (B) $1/2$ (C) 2 (D) 3

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35. $\int_0^2 x^3 \left[1 + \cos \frac{\pi x}{2}\right] dx$ is (where $[*]$ denotes the greatest integer function)
 (A) $\frac{1}{2}$ (B) $\frac{1}{4}$ (C) 0 (D) $\frac{1}{3}$
36. $\int_0^{2n\pi} \left(|\sin x| - \left[\left|\frac{\sin x}{2}\right|\right]\right) dx$ is equal to (where $[*]$ denotes the greatest integer function)
 (A) 0 (B) $2n$ (C) $2n\pi$ (D) $4n$
37. The value of $\int_{-1}^3 \{ |x-2| + [x] \} dx$ is p , then $\frac{p}{2}$ is equal to (where $[x]$ stands for greatest integer less than or equal to x), is

PROPERTY-04

38. Let $f(x) = \text{minimum}(|x|, 1 - |x|, 1/4)$, $\forall x \in \mathbb{R}$, then the value of $\int_{-1}^1 f(x) dx$ is equal to
 (A) $\frac{1}{32}$ (B) $\frac{3}{16}$ (C) $\frac{4}{32}$ (D) $\frac{3}{8}$
39. If $[x]$ denotes the greatest integer less than or equal to x , then the value of $\int_1^5 [|x-3|] dx$ is
 (A) 1 (B) 2 (C) 4 (D) 8
40. Let $f: \mathbb{R} \rightarrow \mathbb{R}$, $g: \mathbb{R} \rightarrow \mathbb{R}$ be continuous functions. Then the value of integral

$$\int_{\ln \lambda}^{\ln 1/\lambda} \frac{f\left(\frac{x^2}{4}\right) [f(x) - f(-x)]}{g\left(\frac{x^2}{4}\right) [g(x) + g(-x)]} dx$$
 is
 (A) depend on λ (B) a non-zero constant
 (C) zero (D) can't be determined
41. $\int_{-1/2}^{1/2} \sqrt{\left\{ \left(\frac{x+1}{x-1}\right)^2 + \left(\frac{x-1}{x+1}\right)^2 - 2 \right\}} dx$ is
 (A) $\ln\left(\frac{256}{81}\right)$ (B) $4\ln\left(\frac{3}{4}\right)$ (C) $4\ln\left(\frac{4}{3}\right)$ (D) $-\ln\left(\frac{81}{256}\right)$

PROPERTY-05

42. $\int_{2-\log 3}^{3+\log 3} \frac{\log(4+x)}{\log(4+x)+\log(9-x)} dx$
 (A) cannot be evaluated (B) is equal to $\frac{5}{2}$
 (C) is equal to $1 + 2\log 3$ (D) is equal to $\frac{1}{2} + \log 3$
43. If $[x]$ stands for the greatest integer function, the value of $\int_4^{10} \frac{[x^2]}{[x^2-28x+196]+[x^2]} dx$ is
 (A) 0 (B) 1 (C) 3 (D) -1
44. If $f(x)$ and $g(x)$ are continuous functions satisfying $f(x) = f(a-x)$ and $g(x) + g(a-x) = 2$, then $\int_0^a f(x)g(x) dx$ is equal to
 (A) $\int_0^a g(x) dx$ (B) $\int_0^a f(x) dx$ (C) 0 (D) 1

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45. If $f(a + b - x) = f(x)$, then $\int_a^b xf(x)dx$ is equal to-
- (A) $\frac{a+b}{2} \int_a^b f(a + b - x)dx$ (B) $\frac{a+b}{2} \int_a^b f(b - x)dx$
 (C) $\frac{a+b}{2} \int_a^b f(x)dx$ (D) $\frac{b-a}{2} \int_a^b f(x)dx$
46. If $\int_0^\pi xf(\sin x)dx = A \int_0^{\pi/2} f(\sin x)dx$, then A is -
- (A) 0 (B) π (C) $\pi/4$ (D) 2π
47. If $f(x) = \frac{e^x}{1+e^x}$, $I_1 = \int_{f(-a)}^{f(a)} xg\{x(1-x)\}dx$ and $I_2 = \int_{f(-a)}^{f(a)} g\{x(1-x)\}dx$, then the value of $\frac{I_2}{I_1}$ is-
- (A) 2 (B) -3 (C) -1 (D) 1
48. The value of $\int_{-\pi}^\pi \frac{\cos^2 x}{1+a^x} dx$, $a > 0$, is -
- (A) $a\pi$ (B) $\frac{\pi}{2}$ (C) $\frac{\pi}{a}$ (D) 2π
49. $\int_{-\pi/2}^{\pi/2} \frac{|x|dx}{8\cos^2 2x+1}$ has the value
- (A) $\frac{\pi^2}{6}$ (B) $\frac{\pi^2}{12}$ (C) $\frac{\pi^2}{24}$ (D) $\frac{\pi^2}{4}$
50. $\int_0^{\pi/2} \frac{dx}{1+\tan^3 x}$ is equal to
- (A) 0 (B) $\pi/2$ (C) $\pi/3$ (D) $\pi/4$
51. $\int_{-1}^1 \frac{x^4}{1+e^{x^7}} dx$ is
- (A) $\frac{1}{2}$ (B) 0 (C) $\frac{1}{5}$ (D) $\frac{1}{7}$
- PROPERTY-06**
52. $\int_{-1}^1 \frac{\sin x + x^2}{3-|x|} dx$
- (A) 0 (B) $2 \int_0^1 \frac{\sin x}{3-|x|} dx$ (C) $2 \int_0^1 \frac{x^2}{3-|x|} dx$ (D) $2 \int_0^1 \frac{\sin x + x^2}{3-|x|} dx$
53. $\int_{-\pi/4}^{\pi/4} \frac{e^x \sec^2 x}{e^{2x}-1} dx =$
- (A) 0 (B) $\frac{\pi}{2}$ (C) $2e^{\pi/4}$ (D) $\frac{\pi}{4}$
54. $\int_{-\pi}^\pi \frac{2x(1+\sin x)}{1+\cos^2 x} dx =$
- (A) π^2 (B) $\pi^2/4$ (C) $\pi/8$ (D) $\pi^2/8$
55. The value of integral $\int_0^\pi xf(\sin x)dx$ is
- (A) $\frac{\pi}{2} \int_0^\pi f(\sin x)dx$ (B) $\pi \int_0^{\pi/2} f(\sin x)dx$
 (C) 0 (D) None of these

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PROPERTY-07

56. If $I = \int_0^{2\pi} \sin^2 x dx$, then
 (A) $I = 2 \int_0^{\pi} \sin^2 x dx$ (B) $I = 4 \int_0^{\pi/2} \sin^2 x dx$
 (C) $I = \int_0^{2\pi} \cos^2 x dx$ (D) $I = 8 \int_0^{\pi/4} \sin^2 x dx$
57. If $f(x) = 2^{\{x\}}$, where $\{x\}$ denotes the fractional part of x . Then which of the following is true ?
 (A) f is periodic (B) $\int_0^1 2^{\{x\}} dx = \frac{1}{\ln 2}$
 (C) $\int_0^1 2^{\{x\}} dx = \log_2 e$ (D) $\int_0^{100} 2^{\{x\}} dx = 100 \log_2 e$
58. The expression $\frac{\int_0^n [x] dx}{\int_0^n \{x\} dx}$ is equal to (where $[*]$ and $\{*\}$ denotes greatest integer function and fractional part function and $n \in \mathbb{N}$)
 (A) $\frac{1}{n-1}$ (B) $\frac{1}{n}$ (C) n (D) $n-1$

PROPERTY-08

59. If $f(x) = \int_0^x (\cos^4 t + \sin^4 t) dt$, $f(x + \pi)$ will be equal to
 (A) $f(x) + f(\pi)$ (B) $f(x) + 2(\pi)$ (C) $f(x) + f\left(\frac{\pi}{2}\right)$ (D) $f(x) + 2f\left(\frac{\pi}{2}\right)$
60. If $f(x) = \int_0^x (2\cos^2 3t + 3\sin^2 3t) dt$, $f(x + \pi)$ is equal to
 (A) $f(x) + f(\pi)$ (B) $f(x) + 2f\left(\frac{\pi}{2}\right)$ (C) $f(x) + 4f\left(\frac{\pi}{4}\right)$ (D) $f(x) + f\left(\frac{\pi}{4}\right)$

NEWTON'S LEBNITZ

61. Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be a differentiable function having $f(2) = 6$, $f'(2) = \left(\frac{1}{48}\right)$. Then $\lim_{x \rightarrow 2} \int_6^{f(x)} \frac{4t^3}{x-2} dt$ equals
 (A) 18 (B) 12 (C) 36 (D) 24
62. The value of $\lim_{x \rightarrow 0} \frac{\int_0^{x^2} \sec^2 t dt}{x \sin x}$ is
 (A) 3 (B) 2 (C) 1 (D) -1
63. $\lim_{x \rightarrow 0} \frac{\int_0^{x^2} \cos t^2 dt}{x \sin x}$ is equal to
 (A) -1 (B) 1 (C) 2 (D) -2
64. If $\int_a^y \cos^2 t dt = \int_a^{x^2} \frac{\sin t}{t} dt$, then the value of $\frac{dy}{dx}$ is
 (A) $\frac{2\sin^2 x}{x \cos y^2}$ (B) $\frac{2\sin x^2}{x \cos y^2}$ (C) $\frac{2\sin x^2}{x(1-2\sin^2 \frac{y^2}{2})}$ (D) $\frac{\sin x^2}{\cos y^2}$

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65. Let $f(x) = \int_0^x (t^2 - t + 1) dt \forall x \in (3, 4)$, then the difference between the greatest and the least values of the function is
 (A) $\frac{49}{6}$ (B) $\frac{59}{6}$ (C) $\frac{69}{8}$ (D) $\frac{59}{3}$
66. If $\int_{\sin x}^1 t^2(f(t)) dt = (1 - \sin x)$, then $f\left(\frac{1}{\sqrt{3}}\right)$ is
 (A) $1/3$ (B) $1/\sqrt{3}$ (C) 3 (D) $\sqrt{3}$
67. The value of function $f(x) = 1 + x + \int_1^x (\ln^2 t + 2 \ln t) dt$ where $f'(x)$ vanishes is
 (A) e^{-1} (B) 0 (C) $2e^{-1}$ (D) $1 + 2e^{-1}$
68. If $f(x) = e^{g(x)}$ and $g(x) = \int_2^x \frac{t dt}{1+t^4}$ then $f'(2)$ has the value equal to
 (A) $2/17$ (B) 0 (C) 1 (D) Cannot be determined
69. (a) If $\int_0^{t^2} x f(x) dx = \frac{2}{5} t^5, t > 0$, then $f\left(\frac{4}{25}\right) =$
 (A) $2/5$ (B) $5/2$ (C) $-2/5$ (D) 1
- (b) If $y(x) = \int_{\pi^2/16}^{x^2} \frac{\cos x \cdot \cos \sqrt{\theta}}{1 + \sin^2 \sqrt{\theta}} d\theta$ then find $\frac{dy}{dx}$ at $x = \pi$
 (A) π (B) 2π (C) 3π (D) $\pi/2$

ESTIMATION

70. The smallest interval $[a, b]$ such that $\int_0^1 \frac{dx}{\sqrt{(1+x^4)}} \in [a, b]$ is given by -
 (A) $[1/\sqrt{2}, 1]$ (B) $[0, 1]$ (C) $[1/2, 1]$ (D) $[3/4, 1]$

(R/N) FORM

71. $\lim_{n \rightarrow \infty} \sum_{r=1}^n \left(\frac{r^3}{r^4 + n^4} \right)$ equals
 (A) $\log 2$ (B) $\frac{1}{2} \log 2$ (C) $\frac{1}{3} \log 2$ (D) $\frac{1}{4} \log 2$
72. $\lim_{n \rightarrow \infty} \sum_{r=2n+1}^{3n} \frac{n}{r^2 - n^2}$ is equal to
 (A) $\log \sqrt{\frac{2}{3}}$ (B) $\log \sqrt{\frac{3}{2}}$ (C) $\log \frac{2}{3}$ (D) $\log \frac{3}{2}$
73. The value of $\lim_{n \rightarrow \infty} \left[\left(1 + \frac{1}{n^2}\right) \left(1 + \frac{2}{n^2}\right) \cdots \left(1 + \frac{n^2}{n^2}\right) \right]^{\frac{1}{n}}$ is
 (A) $\frac{\pi}{2e^2}$ (B) $2e^2 e^{\frac{\pi}{2}}$ (C) $\frac{2}{e^2} e^{\frac{\pi}{2}}$ (D) $\frac{2\pi}{e^2}$
74. $\lim_{n \rightarrow \infty} \frac{\pi}{n} \left[\sin \frac{\pi}{n} + \sin \frac{2\pi}{n} + \cdots + \sin \frac{(n-1)\pi}{n} \right]$ equals
 (A) 0 (B) π (C) 2 (D) 1

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75. $\lim_{n \rightarrow \infty} \left[\frac{1}{n^2} \sec^2 \frac{1}{n^2} + \frac{2}{n^2} \sec^2 \frac{4}{n^2} + \dots + \frac{1}{n} \sec^2 1 \right]$ equals
 (A) $\frac{1}{2} \sec 1$ (B) $\frac{1}{2} \operatorname{cosec} 1$ (C) $\tan 1$ (D) $\frac{1}{2} \tan 1$
76. $\lim_{n \rightarrow \infty} \left(\sin \frac{\pi}{2n} \cdot \sin \frac{2\pi}{2n} \cdot \sin \frac{3\pi}{2n} \dots \sin \frac{(n-1)\pi}{2n} \right)^{1/n}$ is equal to
 (A) $\frac{1}{2}$ (B) $\frac{1}{3}$ (C) $\frac{1}{4}$ (D) 1
77. $\lim_{n \rightarrow \infty} \sum_{r=1}^n \frac{\sqrt{n}}{\sqrt{r}(3\sqrt{r}+4\sqrt{n})^2} =$
 (A) $\frac{1}{7}$ (B) $\frac{1}{10}$ (C) $\frac{1}{14}$ (D) $\frac{2}{7}$
78. $\lim_{n \rightarrow \infty} \frac{1^p + 2^p + 3^p + \dots + n^p}{n^{p+1}}$ equals-
 (A) 1 (B) $\frac{1}{p+1}$ (C) $\frac{1}{p+2}$ (D) p^2
79. $\lim_{n \rightarrow \infty} \frac{1+2^4+3^4+\dots+n^4}{n^5} +$
 $\lim_{n \rightarrow \infty} \frac{1+2^3+3^3+\dots+n^3}{n^5}$ is equal to-
 (A) $1/5$ (B) $1/30$ (C) zero (D) $1/4$
80. $\lim_{n \rightarrow \infty} \sum_{r=1}^n \frac{1}{n} e^{\frac{r}{n}}$ is-
 (A) e (B) $e - 1$ (C) $1 - e$ (D) $e + 1$
81. If $f(x)$ is integrable over $[1, 2]$, then $\int_1^2 f(x) dx$ is equal to
 (A) $\lim_{n \rightarrow \infty} \frac{1}{n} \sum_{r=1}^n f\left(\frac{r}{n}\right)$ (B) $\lim_{n \rightarrow \infty} \frac{1}{n} \sum_{r=n+1}^{2n} f\left(\frac{r}{n}\right)$
 (C) $\lim_{n \rightarrow \infty} \frac{1}{n} \sum_{r=1}^n f\left(\frac{r+n}{n}\right)$ (D) $\lim_{n \rightarrow \infty} \frac{1}{n} \sum_{r=1}^{2n} f\left(\frac{r}{n}\right)$
82. Given that $\lim_{n \rightarrow \infty} \sum_{r=1}^n \frac{\log(n^2+r^2)-2\log n}{n} = \log 2 + \frac{\pi}{2} - 2$, then
 $\lim_{n \rightarrow \infty} \frac{1}{n^{2m}} [(n^2 + 1^2)^m (n^2 + 2^2)^m \dots (2n^2)^m]^{1/n}$ is equal to
 (A) $2^m e^{m(\pi/2-2)}$ (B) $2^m e^{m(2-\pi/2)}$ (C) $e^{m(\pi/2-2)}$ (D) $e^{2m(\pi/2-2)}$

DETERMINATION OF FUNCTION

83. If $\int_0^{x^2} f(t) dt = x \cos \pi x$, then the value of $f(4)$ is
 (A) 1 (B) $1/4$ (C) -1 (D) $-1/4$

MIXED PROBLEMS

84. If $\frac{C_0}{1} + \frac{C_1}{2} + \frac{C_2}{3} = 0$, where C_0, C_1, C_2 are all real, the equation $C_2 x^2 + C_1 x + C_0 = 0$ has
 (A) atleast one root in $(0, 1)$ (B) one root in $(1, 2)$ & other in $(3, 4)$
 (C) one root in $(-1, 1)$ & the other in $(-5, -2)$ (D) both roots are imaginary

(MATHEMATICS)

DEFINITE INTEGRATION

85. Let $I_1 = \int_1^2 \frac{dx}{\sqrt{1+x^2}}$ and $I_2 = \int_1^2 \frac{dx}{x}$
 (A) $I_1 > I_2$ (B) $I_2 > I_1$ (C) $I_1 = I_2$ (D) $I_1 > 2I_2$
86. If $I_n = \int_0^{\pi/4} \tan^n x dx$, then $\frac{1}{I_2+I_4}, \frac{1}{I_3+I_5}, \frac{1}{I_4+I_6}$ is
 (A) A.P. (B) G.P. (C) H.P. (D) does not form a progression
87. If $I_1 = \int_0^1 2^{x^2} dx, I_2 = \int_0^1 2^{x^3} dx, I_3 = \int_1^2 2^{x^2} dx$ and $I_4 = \int_1^2 2^{x^3} dx$ then -
 (A) $I_2 > I_1$ (B) $I_1 > I_2$ (C) $I_3 = I_4$ (D) $I_3 > I_4$
88. If $u_n = \int_0^{\pi/2} x^n \sin x dx, n \in \mathbb{N}$ then the value of $u_{10} + 90u_8$ is
 (A) $9 \left(\frac{\pi}{2}\right)^8$ (B) $\left(\frac{\pi}{2}\right)^9$ (C) $10 \left(\frac{\pi}{2}\right)^9$ (D) $9 \left(\frac{\pi}{2}\right)^9$
89. A function $f(x)$ which satisfies, $f'(\sin^2 x) = \cos^2 x$ for all real x & $f(1) = 1$ is
 (A) $f(x) = x - \frac{x^3}{3} + \frac{1}{3}$ (B) $f(x) = x^2 - \frac{x}{2} + \frac{1}{2}$
 (C) a polynomial of degree two (D) $f(0) = 1/2$
90. If $I_n = \int_0^1 \frac{dx}{(1+x^2)^n}; n \in \mathbb{N}$, then which of the following statements hold good?
 (A) $2nI_{n+1} = 2^{-n} + (2n-1)I_n$ (B) $I_2 = \frac{\pi}{8} + \frac{1}{4}$
 (C) $I_2 = \frac{\pi}{8} - \frac{1}{4}$ (D) $I_3 = \frac{\pi}{16} - \frac{5}{48}$
91. If $k \in \mathbb{N}$ and $I_k = \int_{-2k\pi}^{2k\pi} |\sin x| [\sin x] dx$, (where $[.]$ denotes the greatest integer function) then $\sum_{k=1}^{100} I_k$ equals to
 (A) -10100 (B) -40400 (C) 20200 (D) none of these
92. The value of $\int_{-6}^6 \max(|2-x|, 4-|x|, 3) dx$ is
 (A) 40 (B) 50 (C) 60 (D) 30

ANSWER KEY

1. (A) 2. (D) 3. (A) 4. (B) 5. (D) 6. (C) 7. (D)
 8. (B) 9. (D) 10. (B) 11. (A) 12. (A) 13. (C) 14. (AC)
 15. (ABCD) 16. (D) 17. (C) 18. (D) 19. (B) 20. (C)
 21. (C) 22. (D) 23. (A) 24. (A) 25. (C) 26. (A) 27. (B)
 28. (A) 29. (B) 30. (D) 31. (C) 32. (C) 33. (C) 34. (C)
 35. (B) 36. (D) 37. 3.5 38. (B) 39. (B) 40. (C) 41. (ABCD)
 42. (D) 43. (C) 44. (B) 45. (AC) 46. (B) 47. (A) 48. (B)
 49. (B) 50. (D) 51. (C) 52. (C) 53. (A) 54. (A) 55. (AB)
 56. (ABC) 57. (ABCD) 58. (D) 59. (AD) 60. (AB) 61. (A)
 62. (C) 63. (B) 64. (B) 65. (B) 66. (C) 67. (D) 68. (A)
 69. (a)(A),(b)(B) 70. (A) 71. (D) 72. (B) 73. (C) 74. (C)
 75. (D) 76. (A) 77. (C) 78. (B) 79. (A) 80. (B) 81. (BC)
 82. (A) 83. (B) 84. (A) 85. (B) 86. (A) 87. (B) 88. (C)
 89. (CD) 90. (AB) 91. (D) 92. (A)