ATOMIC STRUCTURE

Q.9 Consider a hydrogen-like ionized atom with atomic number Z with a single electron. In the emission spectrum of this atom, the photon emitted in the $n = 2$ to $n = 1$ transition has energy 74.8eV higher than the photon emitted in the $n = 3$ to $n = 2$ transition. The ionization energy of the hydrogen atom is 13.6eV. The value of Z is

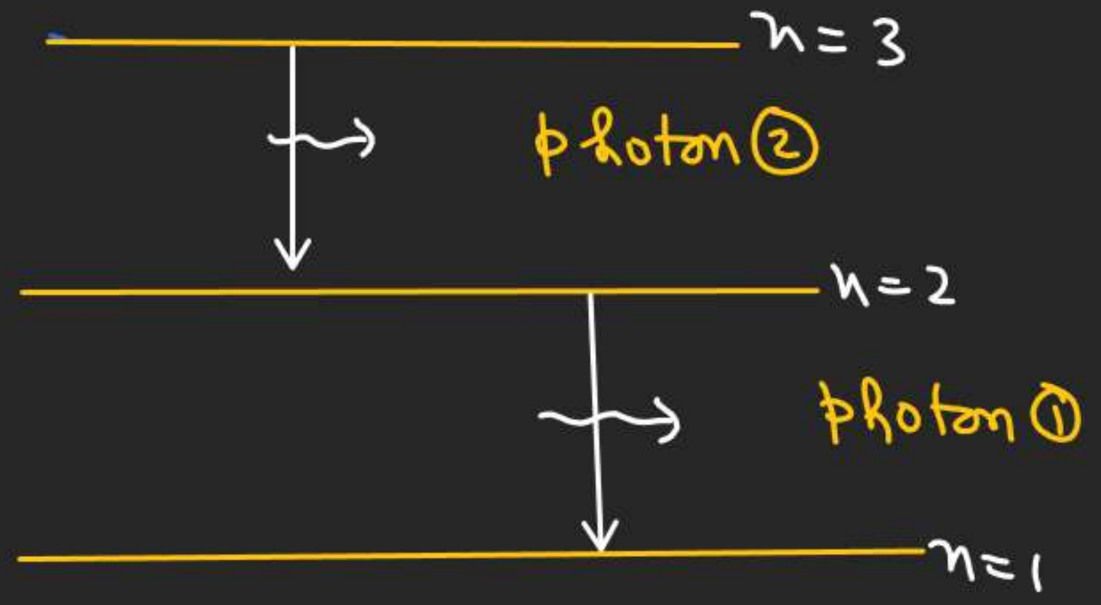
(2018)

$$E = h\nu = \frac{hc}{\lambda}$$

$$+13.6 Z^2 \left[1 - \frac{1}{2^2}\right] = 13.6 Z^2 \left[\frac{1}{4} - \frac{1}{9}\right] + 74.8$$

$$\Delta E_{2-1} = \text{photon } 1 \qquad \boxed{Z = 3}$$

$$\Delta E_{3-2} = \text{photon } 2$$



ATOMIC STRUCTURE

Q.10 An electron in a hydrogen atom undergoes a transition from an orbit with quantum number n_i to another with quantum number n_f. V_i and V_f are respectively the initial and final potential energies of the electron. If $\frac{V_i}{V_f} = 6.25$, then the smallest possible n_f is **(2017)**

$$|E_i| = \frac{V_i}{2} \qquad \frac{V_i}{V_f} = \frac{E_i}{E_f} = \frac{n_f^2}{n_i^2}$$

$$|E_f| = \frac{V_f}{2}$$

$$E_i = -\frac{13.6}{n_i^2} \qquad \frac{n_f}{n_i} = \sqrt{6.25} = 2.5$$

$$E_f = -\frac{13.6}{n_f^2} \qquad \frac{n_f}{n_i} = \frac{25}{10} = \frac{5}{2}, \frac{25}{4}, \ldots$$

$$n_f = 5$$

ATOMIC STRUCTURE

Q.11 A hydrogen atom in its ground state is irradiated by light of wavelength 970Å. Taking $hc/e = 1.237 \times 10^{-6}\ \text{V m}$ and the ground state energy of hydrogen atom as -13.6eV, the number of lines present in the emission spectrum is **(2016)**

λ

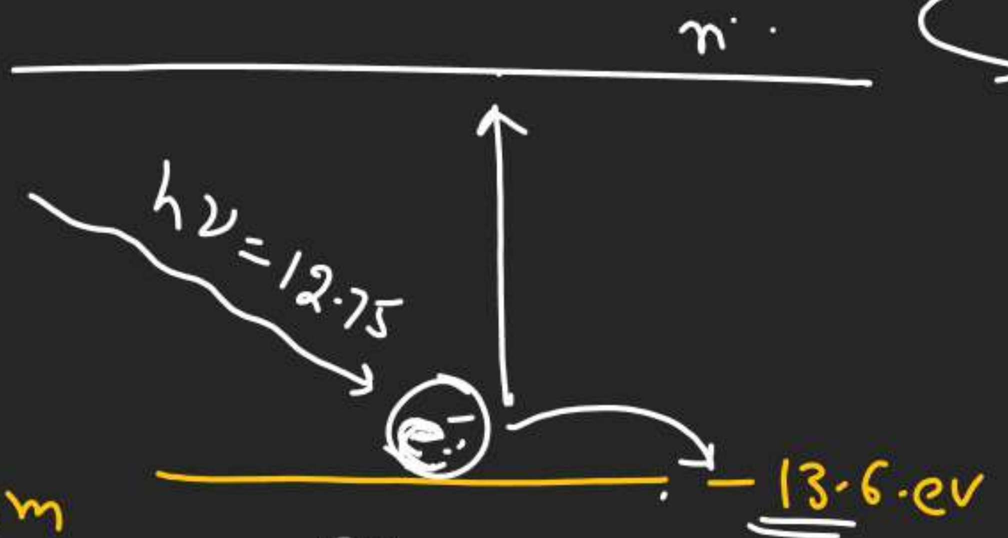
Energy of photon $= \dfrac{hc}{\lambda} = \left(\dfrac{12370}{970}\right) = \dfrac{1237}{97} = 12.75\ \text{e.V}$ — ΔE b/w ground & n^{th} level

$\dfrac{hc}{e} = 1.237 \times 10^{-6}\ \text{V}$

$hc = 1.237 \times 10^{-6}\ \text{eV}$

$hc = 1237 \times 10^{-9}\ \text{eV-nm}$

$hc = 12370\ \text{eV-Å}$



$12.75 = 13.6\left[1 - \dfrac{1}{n^2}\right]$

$-13.6 + 12.75 = -\dfrac{13.6}{n^2}$ ✓

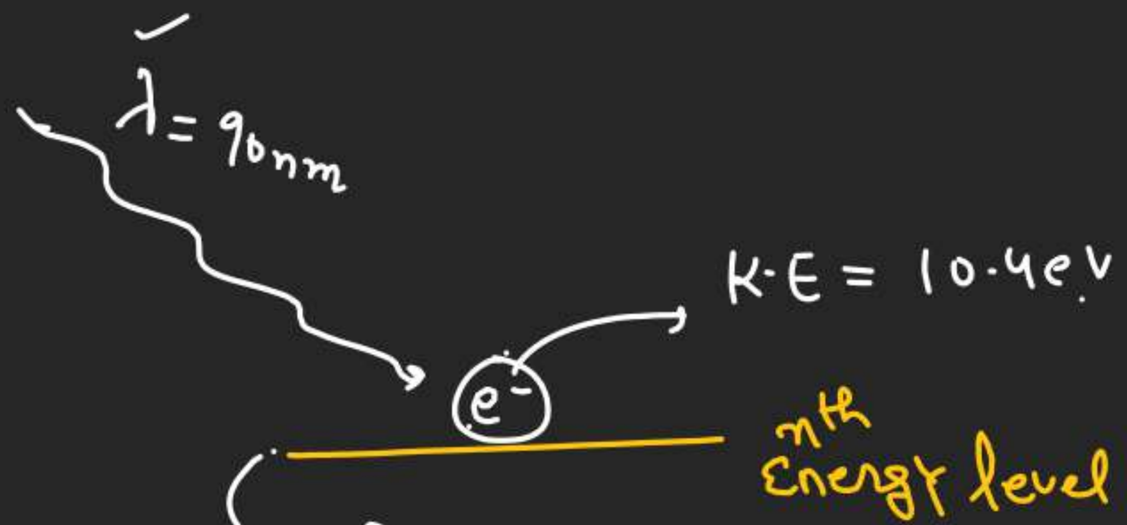
$+0.85 = +\dfrac{13.6}{n^2}$

$n^2 = \left(\dfrac{13.6}{0.85}\right)$

$n = 4$

$6 = {}^4C_2 = {}^nC_2 =$ No of lines in emission spectrum

Q.12 Consider a hydrogen atom with its electron in the n^{th} orbital. An electromagnetic radiation of wavelength 90 nm is used to ionize the atom. If the kinetic energy of the ejected electron is 10.4eV, then the value of n is $(hc = 1242\,\text{eVnm})$ **(2015)**



$$E_n = \left(-\frac{13.6}{n^2}\right)$$

For ionisation of e^- from nth energy level energy required $= +\frac{13.6}{n^2}$

$$\frac{hc}{\lambda} = \frac{13.6}{n^2} + (10.4)$$

$$\left(\frac{1242}{90} - 10.4\right) = \frac{13.6}{n^2}$$

$$\Downarrow$$

$$n^2 = 4$$

$$n = \underline{2}\ \text{Ans.}$$

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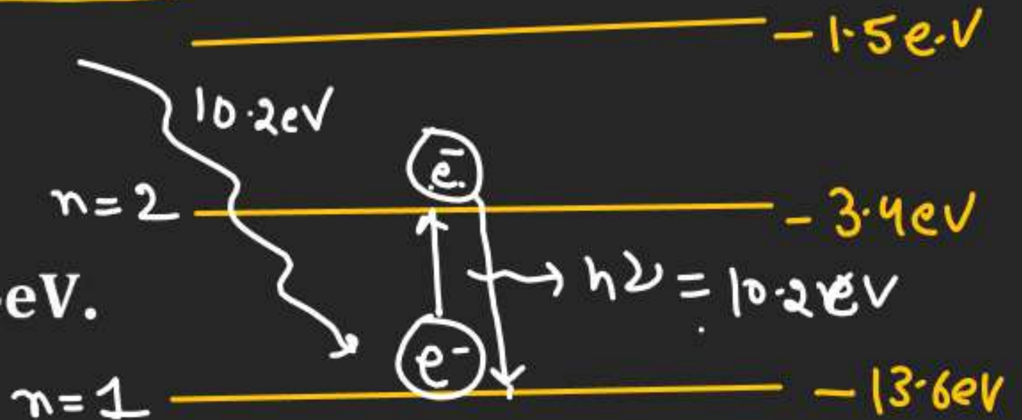
Q.23 A photon collides with a stationary hydrogen atom in ground state inelastically. Energy of the colliding photon is $10.2eV$. After a time interval of the order of micro second another photon collides with same hydrogen atom inelastically with an energy of $15eV$. What will be observed by the detector? **(2005)**

(A) One photon of energy $10.2eV$ and an electron of energy $1.4eV$

(B) Two photons of energy $1.4eV$

(C) Two photons of energy $10.2eV$

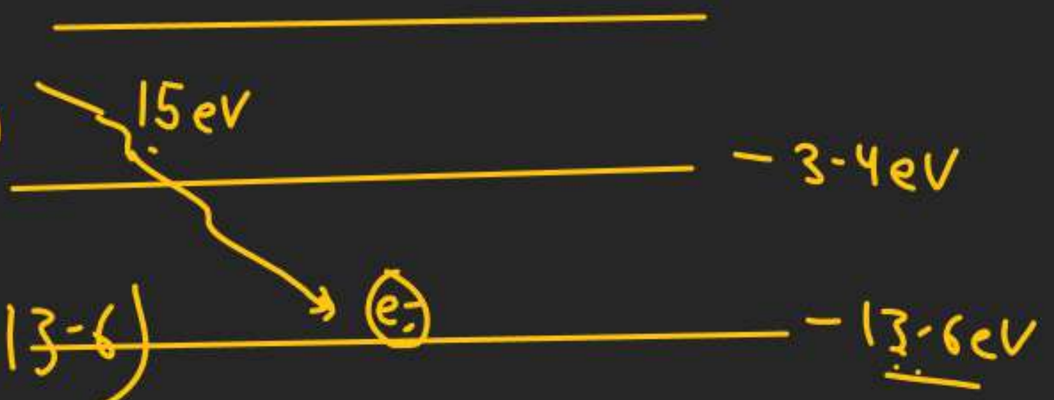
(D) One photon of energy $10.2eV$ and another photon of $1.4eV$.



$(13.6 - 3.4) = 10.2eV$

For photon of 15 e.V electron Ionise and have energy $= (15 - 13.6)$

$= 1.4 e.V$



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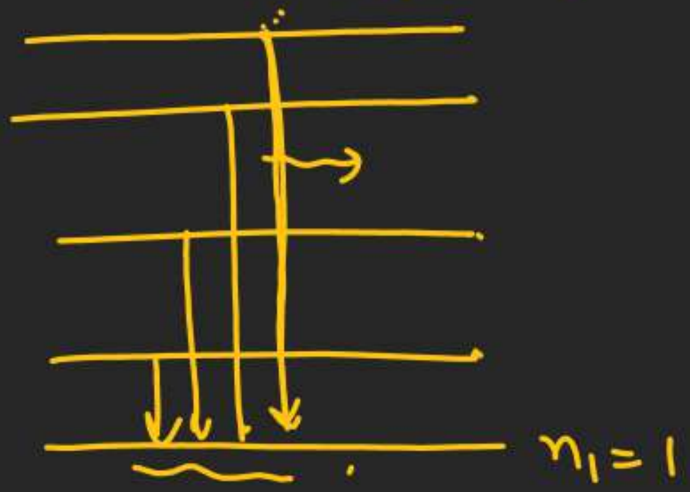
ATOMIC STRUCTURE

Q.13 In hydrogen-like atom $(Z = 11)$, n^{th} line of Lyman series has wavelength λ. The de-Broglie's wavelength of electron in the level from which it originated is also λ. Find the value of n. **(2006)**

For Lyman Series

$n_1 = 1$



$$\frac{1}{\lambda} = RZ^2\left[1 - \frac{1}{n^2}\right] \quad \text{— (1)}$$

De-broglie's wavelength — λ'

$$\lambda' = \frac{h}{mv} = \frac{hr}{mvr} \Rightarrow \lambda' = \frac{hr(2\pi)}{nh}$$

$$\left(mvr = \frac{nh}{2\pi}\right)$$

r = radius of n^{th} orbit

$$r = (0.529)\left(\frac{n^2}{Z}\right)$$

$$\lambda' = \left(\frac{h2\pi}{nh}\right) r$$

$$\lambda' = \frac{\cancel{h}\cdot 2\pi}{\cancel{n}\cancel{h}} \times (0.529)\frac{n^{\cancel{2}}}{Z}$$

$$\lambda' = \frac{2\pi(0.529)\times n}{Z} \quad \text{— (2)}$$

$$\frac{1}{\lambda} = \frac{1}{\lambda'}$$

$$RZ^{\cancel{2}}\left(1 - \frac{1}{n^2}\right) = \frac{Z}{2\pi(0.529)\times n}$$

$$\left[\frac{n^2-1}{n} = \left(\frac{1}{2\pi \times 0.529 \times R \times Z}\right)\right]$$

Å

$$\frac{n^2-1}{n} = 0.0249 \times 10^3$$

$$= 24.9$$

$$n^2 - 1 = 25n$$

$$n^2 = 25n \Rightarrow n = 25$$

ATOMIC STRUCTURE

Q.15 The potential energy of a particle of mass m is given by

$$V(x) = \begin{cases} E_0; & 0 \le x \le 1 \\ 0; & x > 1 \end{cases}$$

λ_1 λ_2

λ_1 and λ_2 are the de-Broglie wavelengths of the particle, when $0 \le x \le 1$ and $x > 1$ respectively. If the total energy of particle is $2E_0$, find λ_1/λ_2. = ??

(2005)

Sol^n

$|E_T| = P.E + K.E$

$K.E = E_T - P.E$

$= 2E_0 - E_0$

$= E_0$

$0 \le x \le 1$

$K.E_1 = E_T = 2E_0$

$\lambda_1 = \dfrac{h}{\sqrt{2m(K.E)_1}} = \dfrac{h}{\sqrt{2mE_0}}$

$x > 1$

$P.E = 0.$

$E_T = (K.E)_2 + \cancel{P.E}^{\,0}$

$K.E_2 = E_T = 2E_0$

$\lambda_2 = \dfrac{h}{\sqrt{2m(K.E)_2}} = \dfrac{h}{\sqrt{4mE_0}}$

$\dfrac{\lambda_1}{\lambda_2} = \sqrt{2}$ A

ATOMIC STRUCTURE

Q.16 A hydrogen-like atom (described by the Bohr model) is observed to emit six wavelengths, originating from all possible transitions between a group of levels. These levels have energies between -0.85eV and -0.544eV (including both these values).

(a) Find the atomic number of the atom.

(b) Calculate the smallest wavelength emitted in these transitions.

(Take $\text{hc} = 1240\text{eV}$-nm, ground state energy of hydrogen atom $= -13.6\text{eV}$)

(2002)

→ 6 Wavelength

n = No of Energy level

$^{n}C_2 = 6$

$\frac{n!}{2!(n-2)!} = 6$

$\frac{n(n-1)}{2} = 6$

$n^2 - n - 12 = 0$

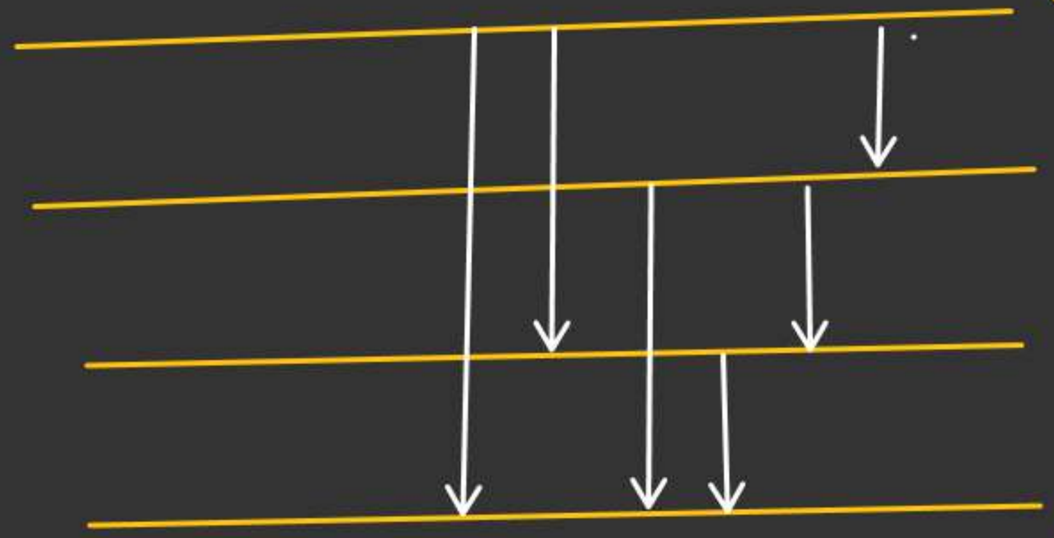
$n^2 - 4n + 3n - 12 = 0$

$n(n-4) + 3(n-4) = 0$

$n = -3$ ✗ $\quad \underbrace{n = 4}$ ✓

⇓

No of energy level



$(m+3)^{th}$ Energy level $= -0.544\,eV$

m^{th} Energy level $= -0.85\,eV$

$$Z = 3$$
$$m = 12$$

Ans

$$-0.544 = \frac{-13.6 Z^2}{(m+3)^2} \Rightarrow \left(\frac{Z}{m+3}\right)^2 = \left(\frac{0.544}{13.6}\right) = \frac{1}{25} \Rightarrow \left(\frac{Z}{m+3} = \frac{1}{5}\right)$$

$$-0.85 = \frac{-13.6 Z^2}{m^2} \Rightarrow \left(\frac{Z}{m}\right)^2 = \left(\frac{0.85}{13.6}\right) = \frac{1}{16} \Rightarrow \left(\frac{Z}{m} = \frac{1}{4}\right)$$

Q.17 A hydrogen-like atom of atomic number Z is in an excited state of quantum number $2n$. It can emit a maximum energy photon of $204 eV$. If it makes a transition to quantum state n, a photon of energy $40.8 eV$ is emitted. Find n, Z and the ground state energy (in eV) for this atom. Also calculate the minimum energy (in eV) that can be emitted by this atom during de-excitation. Ground state energy of hydrogen atom is $-13.6 eV$. **(2000)**

$$204 = 13.6 z^2 \left[1 - \frac{1}{4n^2}\right]$$

$$204 = 13.6 z^2 \left[\frac{4n^2 - 1}{4n^2}\right] \quad - (1)$$

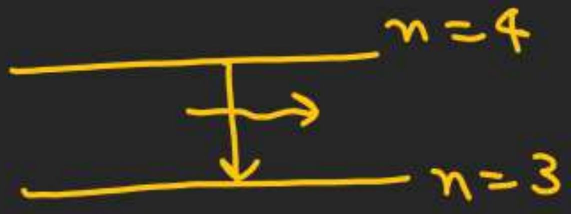
$n = 2$

$Z = 4$

Ans

$$40.8 = 13.6 z^2 \left[\frac{1}{n^2} - \frac{1}{4n^2}\right]$$

$$40.8 = 13.6 z^2 \left[\frac{3}{4n^2}\right] \quad - (2)$$



$$E_{min} = 13.6 (4)^2 \left[\frac{1}{3^2} - \frac{1}{4^2}\right]$$

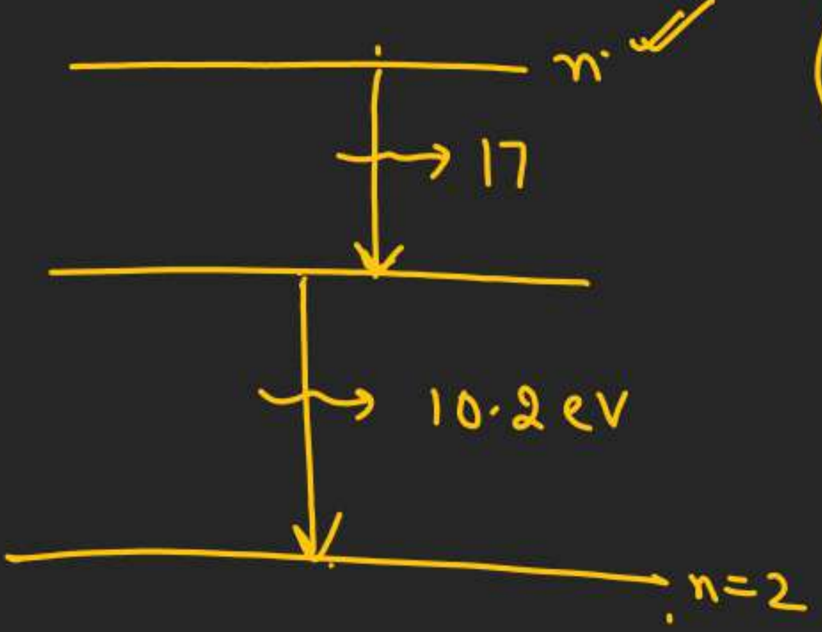
$\hookrightarrow$ = ✓.

Q.18 A hydrogen like atom (atomic number Z) is in a higher excited state of quantum number n. The excited atom can make a transition to the first excited state by successively emitting two photons of energy 10.2eV and 17.0eV respectively. Alternately, the atom from the same excited state can make a transition to the second excited state by successively emitting two photons of energies 4.25eV and 5.95eV respectively. Determine the values of n and Z. (Ionization energy of H-atom 13.6eV)

n = 2

n = 3

(1994)



$$(17+10.2) = 13.6Z^2\left[\frac{1}{4}-\frac{1}{n^2}\right] \quad -①$$

$$(4.25+5.95) = 13.6Z^2\left[\frac{1}{9}-\frac{1}{n^2}\right] \quad -②$$

① ÷ ②

$n = 6$ Ans.

Put $n=6$ in ①

$Z = 3$ Ans.

ATOMIC STRUCTURE

Q.19 An electron, in a hydrogen-like atom, is in an excited state. It has a total energy of -3.4eV. Calculate (i) the kinetic energy and (ii) the de Broglie wavelength of the electron. **(1996)**

$K.E = |E_T| = 3.4\ e.V.$

$1\ e.V = 1.6 \times 10^{-19}\ J$

$\lambda = \frac{h}{mv} = \frac{h}{p} = \frac{h}{\sqrt{2m(K.E)}}$

$$\lambda = \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times (3.4 \times 1.6 \times 10^{-19})}}$$

$\lambda = 6.63\ \text{Å}$ ✓

Q.20 A free hydrogen atom after absorbing a photon of wavelength λ_a gets excited from the state $n = 1$ to the state $n = 4$. Immediately after that the electron jumps to $n = m$ state by emitting a photon of wavelength λ_e. Let the change in momentum of atom due to the absorption and the emission are Δp_a and Δp_e respectively. If $\lambda_a/\lambda_e = 1/5$, which of the option(s) is/are correct?

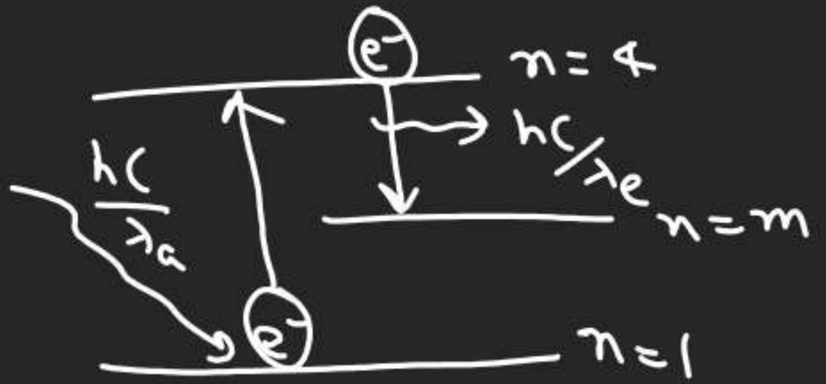
[Use $hc = 1242\,\text{eVnm}$; $1\text{ nm} = 10^{-9}\text{ m}$, h and c are Planck's constant and speed of light, respectively]

(a) The ratio of kinetic energy of the electron in the state $n = m$ to the state $n = 1$ is $1/4$

(b) $m = 2$

(c) $\Delta p_a/\Delta p_e = 1/2$

(d) $\lambda_e = 418\text{ nm}$



(2019)

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$$|E_T| = K.E = \frac{|P.E|}{2}$$

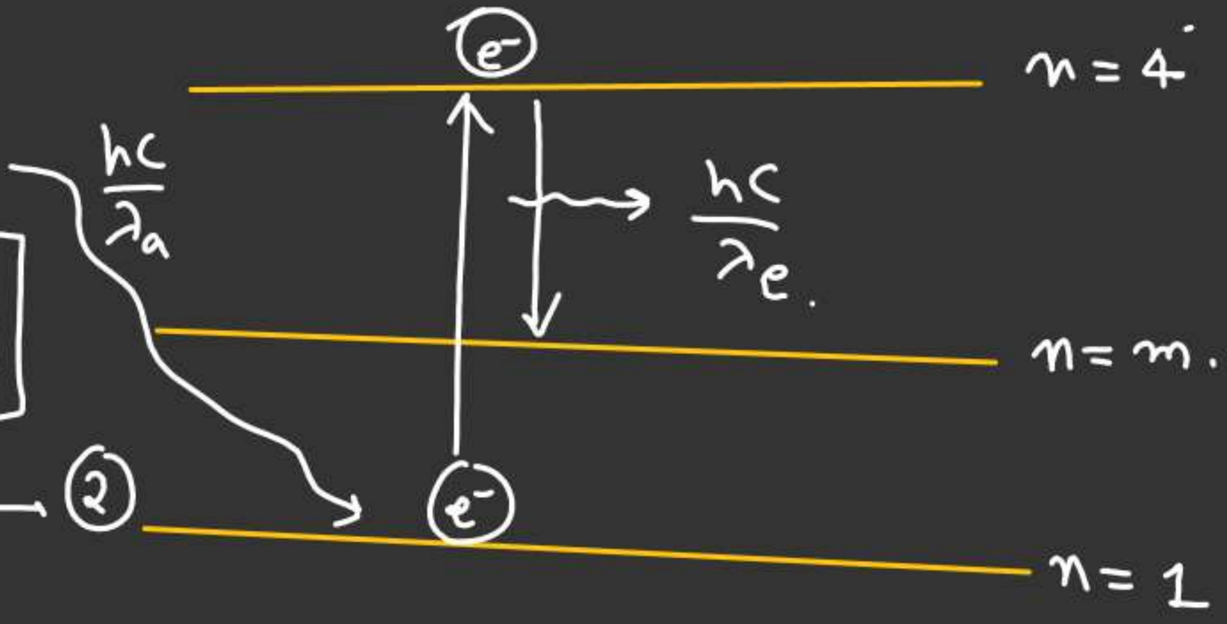
$$E_4 - E_1 = \frac{hc}{\lambda_a}$$

$$-\frac{13.6}{4^2} - \left(\frac{-13.6}{1^2}\right) = \frac{hc}{\lambda_a}$$

$$13.6\left[1 - \frac{1}{16}\right] = \frac{hc}{\lambda_a}$$

$$13.6\left[\frac{15}{16}\right] = \frac{hc}{\lambda_a} \quad -①$$

$$\frac{hc}{\lambda_e} = 13.6\left[\frac{1}{m^2} - \frac{1}{16}\right] \quad \llcorner ②$$



$$② \div ①$$

$$\frac{\lambda_a}{\lambda_e} = \left(\frac{\frac{1}{m^2} - \frac{1}{16}}{\frac{15}{16}}\right)$$

$$\frac{1}{\cancel{5}} \times \frac{\cancel{15}}{16} = \frac{1}{m^2} - \frac{1}{16}$$

$$\frac{3}{16} + \frac{1}{16} = \frac{1}{m^2}$$

$$\frac{1}{m^2} = \frac{4}{16} = \frac{1}{4}$$

$$\underline{m = 2} \ \checkmark$$

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$n=1$

$$K\cdot E_1 = |E_T| = \frac{13\cdot 6}{(1)^2}$$

$n=m=2$

Deexcitation
$n=m$

$$K\cdot E_2 = |E_T| = \frac{13\cdot 6}{(2)^2}$$

$$\frac{K\cdot E_1}{K\cdot E_2} = 4$$

$$\frac{1}{\lambda_a} = R\left[1-\frac{1}{(4)^2}\right]$$

$\lambda_a = \_\_$ ✓

By De brogile Equation

$$\lambda = \frac{h}{mv} = \frac{h}{p}$$

$$K\cdot E = \frac{p^2}{2m}$$

$$p = \sqrt{2m(K\cdot E)}$$

$$p_{\lambda_a} = \sqrt{2m\,(K\cdot E)_1}$$

$$p_{\lambda_e} = \sqrt{2m(K\cdot E_2)}$$

$$\frac{p_{\lambda_a}}{\lambda_{\lambda_e}} = \sqrt{\frac{(K\cdot E)_1}{(K\cdot E)_2}} = \sqrt{\frac{4}{1}} = 2$$

$Rhc = 13\cdot6$

$$R = \left(\frac{13\cdot 6}{hc}\right)$$

$hc = 1242$ eV. nm

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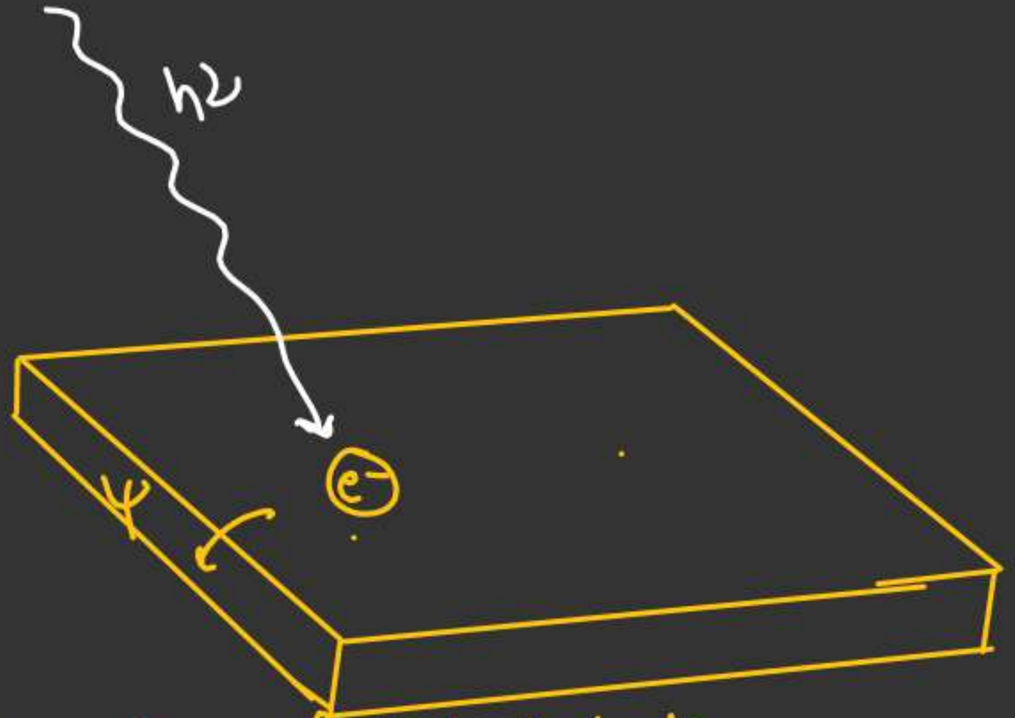
PHOTOELECTRIC EFFECT

Defⁿ :- When light of sufficiently small wavelength incident on metal surface then electrons ejected from the metal plate. This phenomenon is called photoelectric effect.

if $h\nu > \psi \Rightarrow$ Electron may eject

- may eject because electron loose $(h\nu - \psi)$ in successive collision
- only surface electron utalize the $(h\nu - \psi)$ energy most ecnomically then exit with maximum K.E



ψ = work function of metal plate depends on material of plate only.

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$$\psi = \left(\frac{hc}{\lambda_0}\right)$$

λ_0 = threshold wavelength.

For photoelectric emission.

$$\boxed{\lambda \le \lambda_0}$$

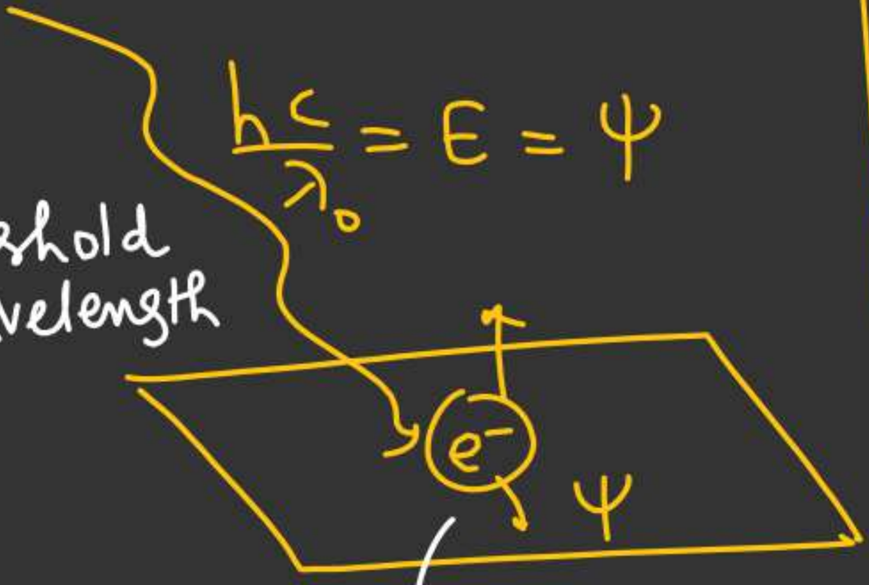
if $\lambda \le \lambda_0 \Rightarrow (\underbrace{E_\lambda}_{\substack{\text{Energy of photon} \\ \text{having wavelength} \\ (\lambda)}} \ge \psi)$

Photo Electric Equation

$$\boxed{K.E_{max} = h\nu - \psi}$$

$$\frac{hc}{\lambda_0} = E = \psi$$

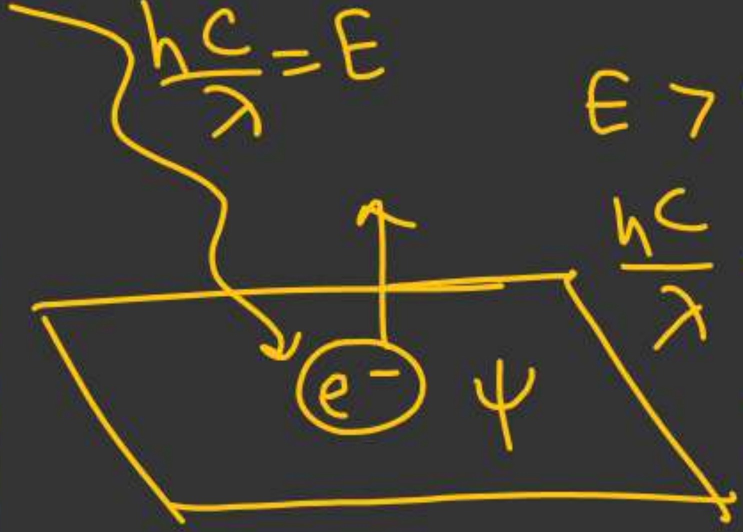
λ_0 = Threshold Wavelength



Photoelectron just manage to over come the binding energy of metal plate ie work function

$K.E = 0$

$$\frac{hc}{\lambda} = E$$



$\Downarrow$

$K.E > 0$

$E > \psi$

$$\frac{hc}{\lambda} > \frac{hc}{\lambda_0}$$

$\lambda < \lambda_0$

↳ For photoemission

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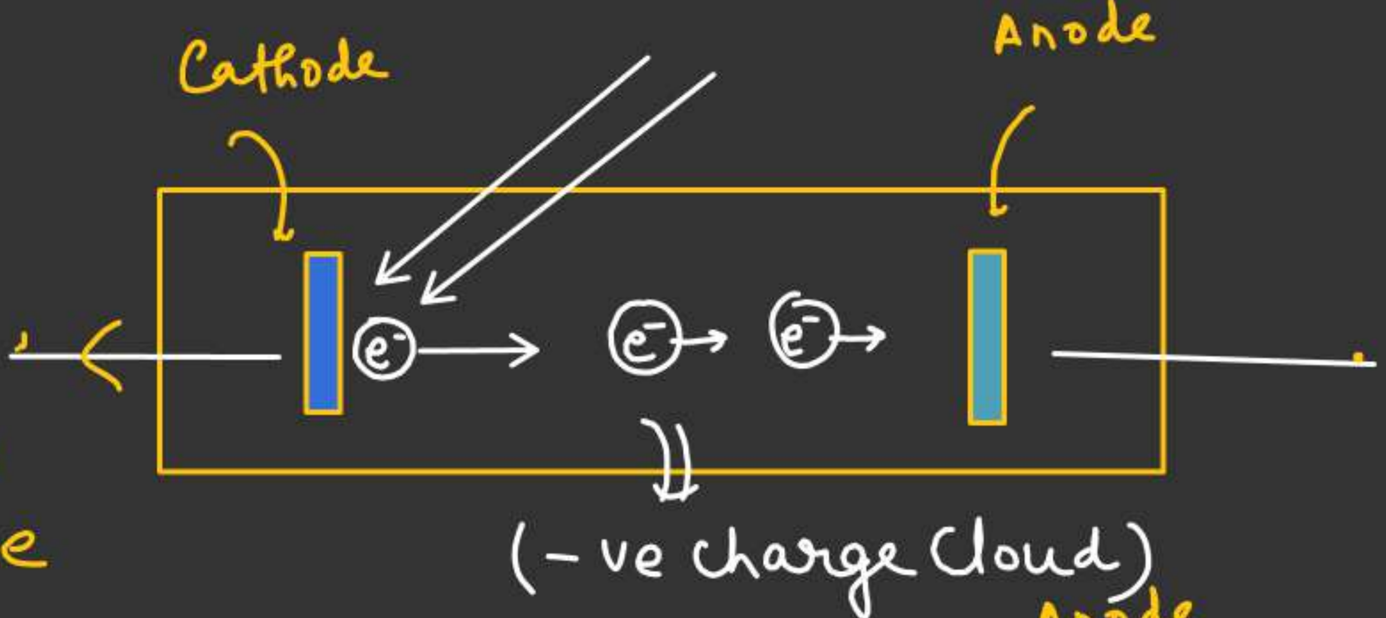
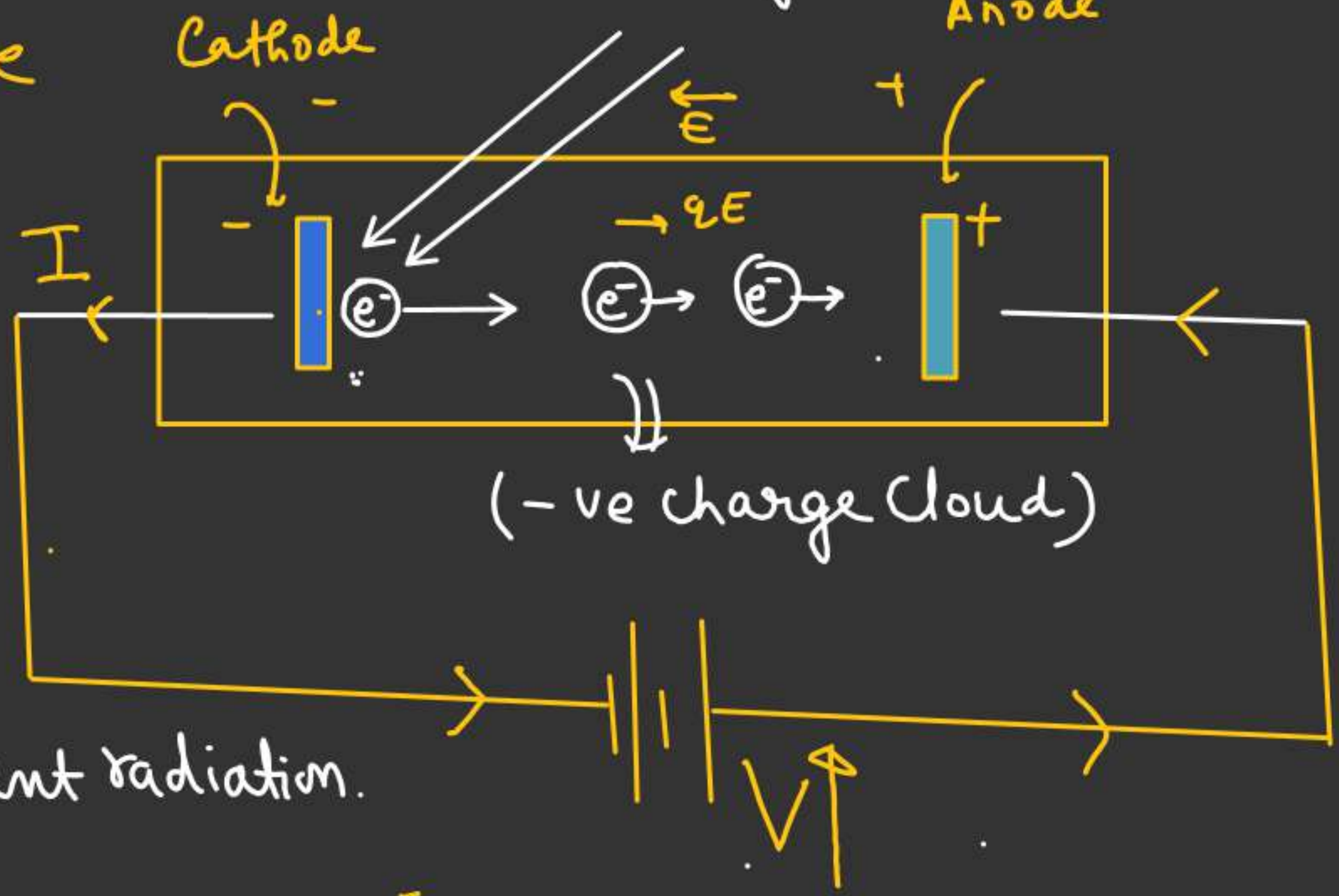
Concept of Saturation Current

Due to charge cloud b/w cathode and anode further photo electrons doesn't reach to anode plate

After connecting with a potential source at a particular value of potential source V all the photoelectrons emitted from cathode reaches to anode. Current corresponding to this stage is called saturation current.





(*) (Saturation current doesn't depend on applied potential)

(*) Saturation current increases with increasing the intensity of incident radiation.

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Concept of Stopping Potential

↳ Min −ve potential applied to anode plate So that photo electrons having maximum Kinetic energy not reaches to anode plate.

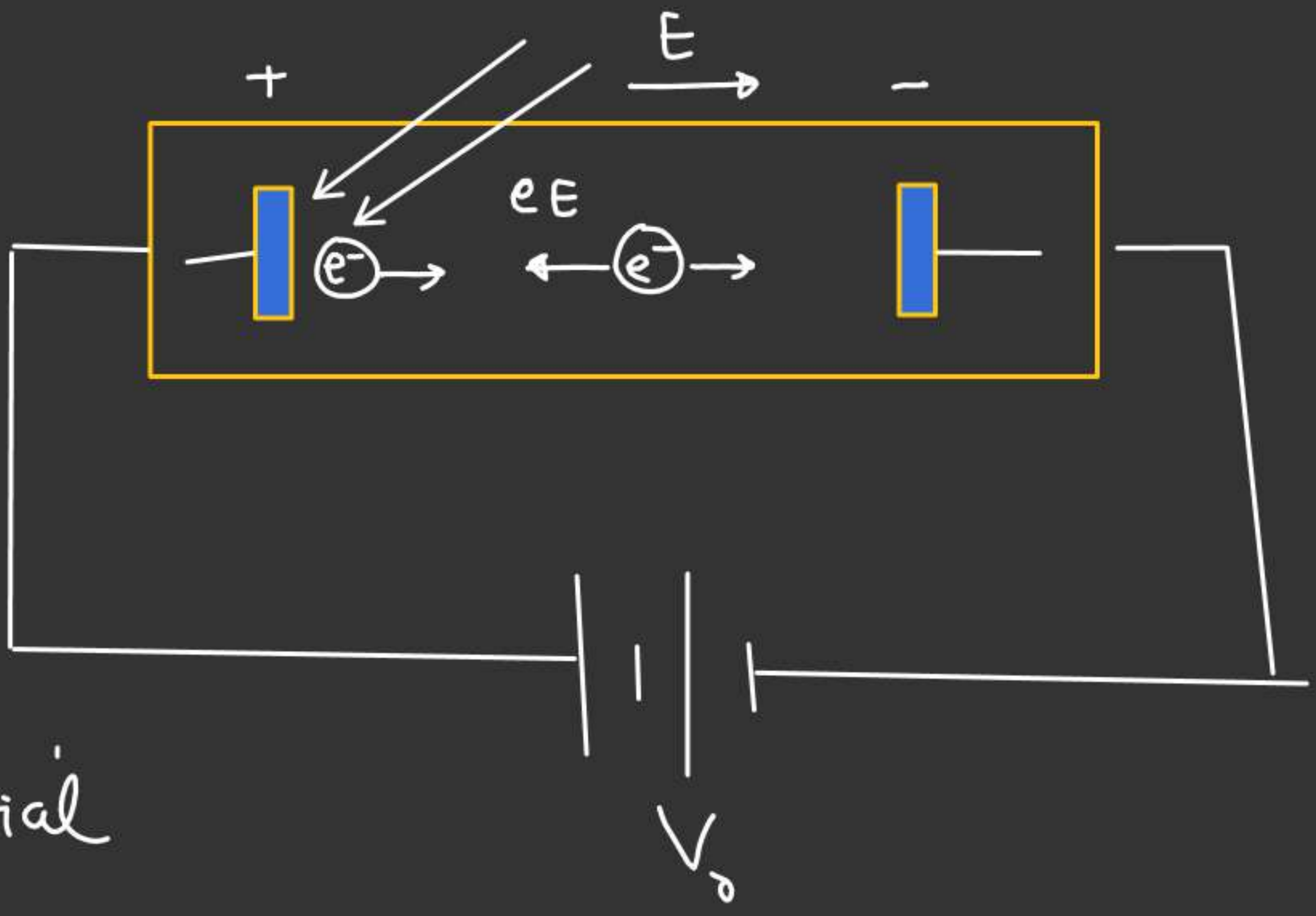
$$\boxed{eV_0 = (K.E)_{max}}$$

$K.E_{max} = h\nu - \psi$

⇓

$$\boxed{eV_0 = h\nu - \psi}$$

V_0 = Stopping potential



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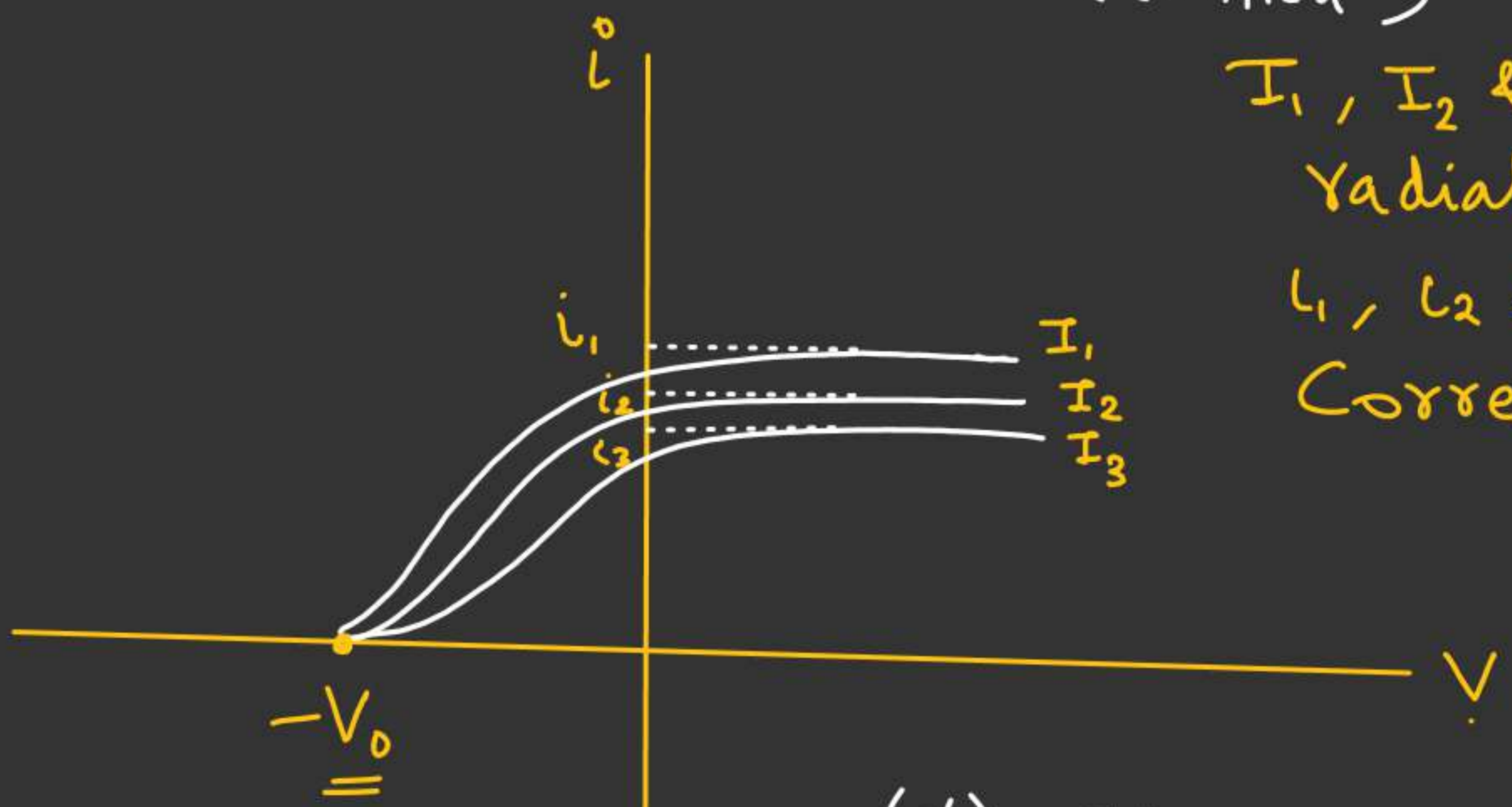
Note :- (Intensity related with no of photoelectrons emitted)



I_1, I_2 & I_3 be the intensity of radiation

i_1, i_2 & i_3 be the Saturation Current Corresponding to I_1, I_2 & I_3.

$(I_1 > I_2 > I_3)$

(*) Stopping potential doesn't depend on intensity of light.

(*) Stopping potential depends on
→ work function of Metal plate
→ Energy of incident photon

$$eV_0 = K.E_{max} = (h\nu - \psi)$$