Nishant Jindal

COLLISION

Case of head-on elastic Collision

L.M.C.

$$m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2' \quad -(1)$$

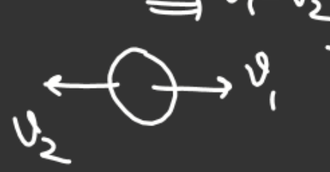
Equation of e

$$e = \frac{\text{Relative Speed of Seperation}}{\text{Relative Speed of approach.}}$$

(Relative Speed of Seperation → After Collision)

(Relative Speed of approach → Before Collision)

$$\Rightarrow v_2' - v_1' = \text{Relative speed of seperation}$$



(Relative Speed of approach)

Just before Collision

$v_1 > v_2$

During Collision

$N\delta t = J_N$ ← → $J_N = N\delta t$

Just after Collision

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COLLISION

L.M.C

$$m_1u_1 + m_2u_2 = m_1v_1' + m_2v_2' \quad -①$$

$$1 = e = \frac{v_2' - v_1'}{u_1 - u_2}$$

$$u_1 - u_2 = v_2' - v_1' \quad -②$$

② m_1 + ①

$$m_1u_1 - m_1u_2 = m_1v_2' - m_1v_1'$$

$$2m_1u_1 + (m_2 - m_1)u_2 = (m_1 + m_2)v_2'$$

$$\boxed{v_2' = \frac{(m_2 - m_1)}{(m_1 + m_2)}u_2 + \frac{2m_1u_1}{m_1 + m_2}}$$

$1 \leftrightarrow 2$

② m_2

$$m_2u_1 - m_2u_2 = m_2v_2' - m_2v_1'$$

② m_2 − ①

$$(m_2 - m_1)u_1 - 2m_2u_2 = -(m_1 + m_2)v_1'$$

$$\boxed{v_1' = \frac{(m_1 - m_2)}{m_1 + m_2}u_1 + \left(\frac{2m_2}{m_1 + m_2}\right)u_2}$$

Head on Elastic Collision in 1D

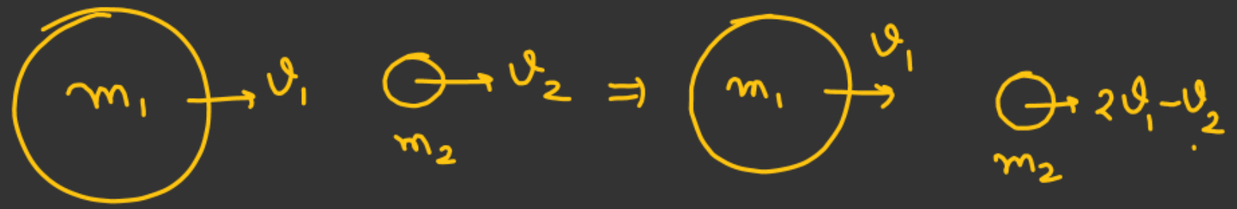
COLLISION

$$v_1' = \frac{(m_1-m_2)}{m_1+m_2}u_1 + \left(\frac{2m_2}{m_1+m_2}\right)u_2$$

$$v_2' = \frac{(m_2-m_1)}{(m_1+m_2)}v_2 + \frac{2m_1u_1}{m_1+m_2}$$

Special Case

Case-1 $m_1 >> m_2$



$$\frac{m_1-m_2}{m_1+m_2} \simeq \frac{m_1}{m_1} \simeq 1$$

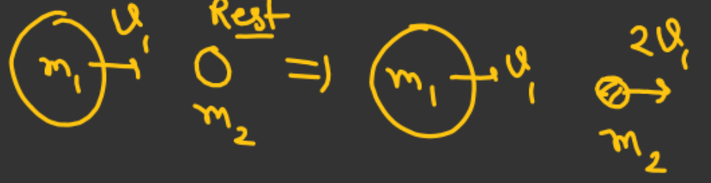
$$\frac{m_2}{m_1+m_2} \simeq \frac{m_2}{m_1} \to 0$$

$$\frac{m_2-m_1}{m_1+m_2} \simeq \frac{-m_1}{m_1} \simeq -1$$

$$v_1' = u_1$$

$$v_2' = -u_2 + 2u_1$$

if $u_2 = 0$. (Imp)



Head on Elastic Collision in 1D

COLLISION

$$v_1' = \frac{(m_1 - m_2)}{m_1 + m_2} v_1 + \left(\frac{2m_2}{m_1 + m_2}\right) v_2$$

$$v_2' = \frac{(m_2 - m_1)}{(m_1 + m_2)} v_2 + \frac{2m_1 v_1}{m_1 + m_2}$$

Special Case

$m_1 << m_2$

$$\frac{m_1 - m_2}{m_1 + m_2} \simeq \frac{-m_2}{m_2} \simeq -1$$

$$\frac{m_2}{m_1 + m_2} \simeq \frac{m_2}{m_2} \simeq 1$$

$$\frac{m_1}{m_1 + m_2} \simeq \frac{m_1}{m_2} \to 0$$

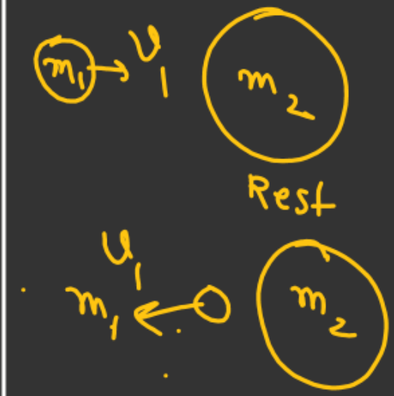
$$\frac{m_2 - m_1}{m_1 + m_2} \simeq +1$$

$$v_1' = 2v_2 - v_1$$

$$\Rightarrow v_2' = v_2$$

Imp

If $v_2 = 0$. [Heavier body at Rest]



Head on Elastic Collision in 1D

COLLISION

Imp.

$$v_1' = \frac{(m_1 - m_2)}{m_1 + m_2} v_1 + \left(\frac{2m_2}{m_1 + m_2}\right) v_2$$

$$v_2' = \frac{(m_2 - m_1)}{(m_1 + m_2)} v_2 + \frac{2m_1 v_1}{m_1 + m_2}$$

If $m_1 = m_2$

$$v_1' = v_2$$
$$v_2' = v_1$$

$\Rightarrow$ (Velocity interchange just after Collision)

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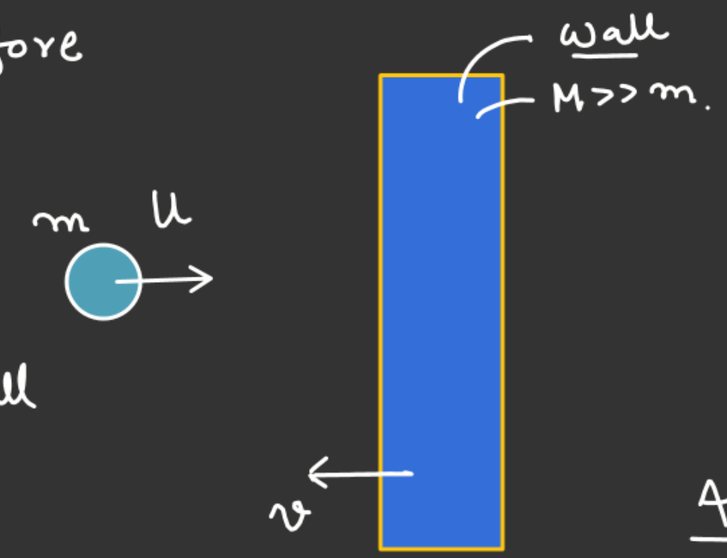
COLLISION

Case of Collision of ball with a moving wall

Just before Collision.



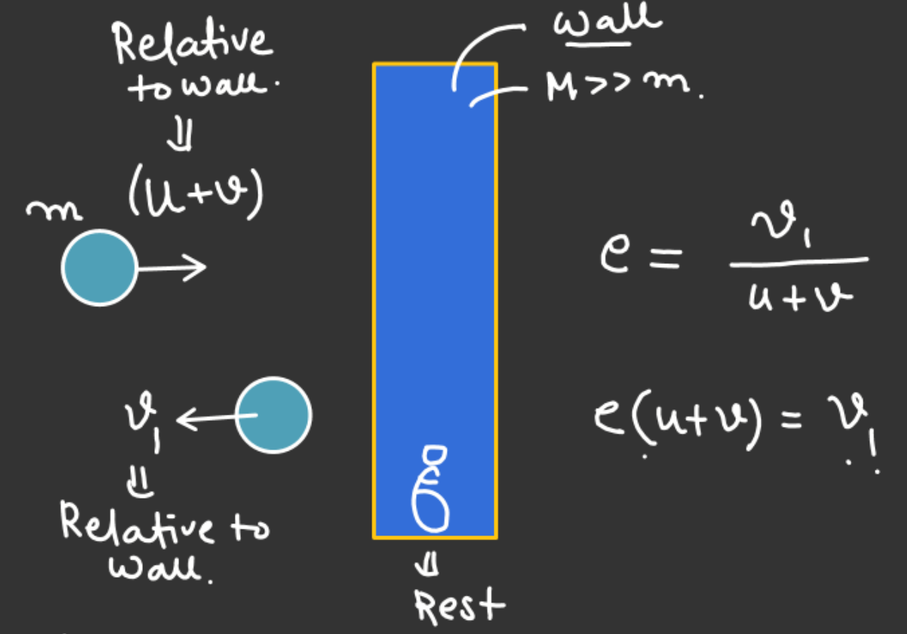
e = Coff^{n} of Restitution b/w wall & ball

If $e = 1$

$$|v_{ball/wall}| = (u+v) \checkmark$$

$$|v_{ball/\mathcal{E}}| = (u+2v) \checkmark$$

After Collision



$$e = \frac{v_1}{u+v}$$

$$e(u+v) = v_1$$

$$\vec{v}_{ball/\mathcal{E}} = \vec{v}_{ball/wall} + \vec{v}_{wall/\mathcal{E}}$$

$$= -e(u+v)\hat{i} - u\hat{i}$$

$$= -[eu + (e+1)u]\hat{i}$$

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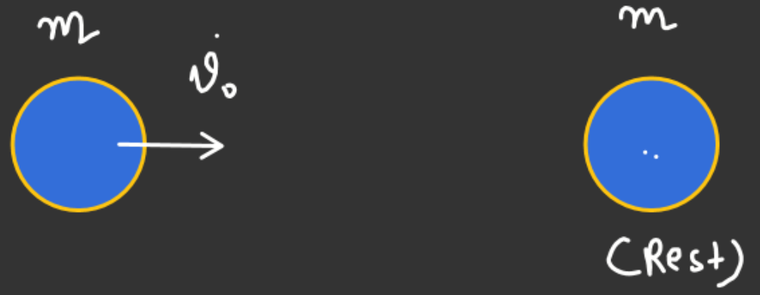
COLLISION

Just before Collision



Just after Collision

After Collision K.E of the System become $\frac{3}{4}^{th}$ of initial K.E of the System.

then find e = ??

L.M.C

$$mv_0 = mv_1 + mv_2$$

$$v_0 = v_1 + v_2 \quad -(1)$$

Equation of e

$$e = \frac{v_2 - v_1}{v_0}$$

$$ev_0 = v_2 - v_1 \quad -(2)$$

From Eqn (1) & (2)

$$\frac{(e+1)}{2} v_0 = v_2$$

$$\frac{(1-e)}{2} v_0 = v_1$$

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COLLISION

According to question

$$(K.E_f) = \frac{3}{4}(K.E_i)$$

$$\frac{1}{2}m(v_1^2 + v_2^2) = \frac{3}{4}\left(\frac{1}{2}mv_0^2\right)$$

$$v_1^2 + v_2^2 = \frac{3}{4}v_0^2$$

$$\left[\left(\frac{1-e}{2}\right)v_0\right]^2 + \left[\left(\frac{1+e}{2}\right)v_0\right]^2 = \frac{3}{4}v_0^2$$

$$(1-e)^2 + (1+e)^2 = 3$$

$$1 + e^2 - 2e + 1 + e^2 + 2e = 3$$

$$2e^2 = 1$$

$$\left(e = \frac{1}{\sqrt{2}}\right)$$ Ans

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COLLISION

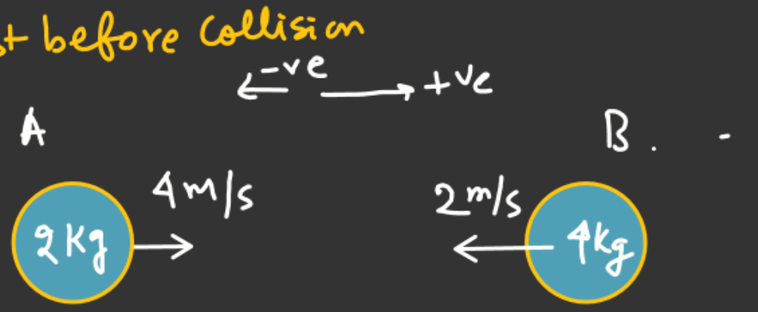
If Collision is inelastic

Find a) $v_B = ?$ ✓

b) $e = ?$ ✓

c) Impulse of deformation

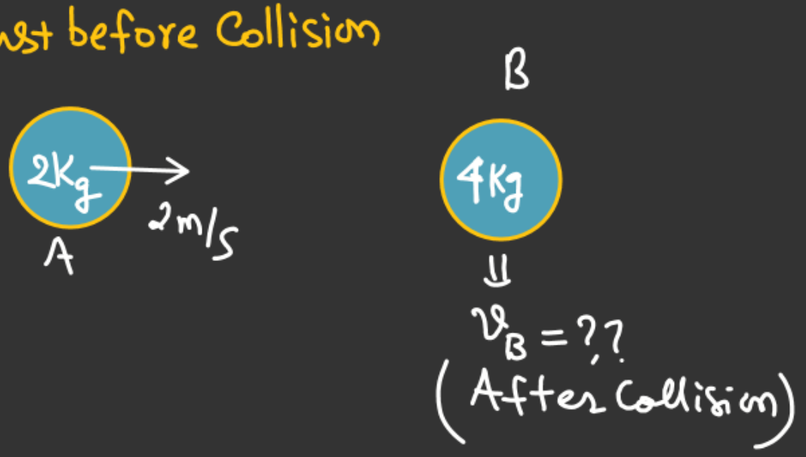
d) Impulse of Reformation



L.M.C a) $(4 \times 2) - (4 \times 2) = (2 \times 2) + 4 v_B$

$0 = 4 + 4 v_B$

$v_B = -1 \text{ m/s}$



b) $$e = \frac{\text{Relative Speed of Seperation}}{\text{Relative Speed of approach}} = \frac{3}{6} = \frac{1}{2}$$

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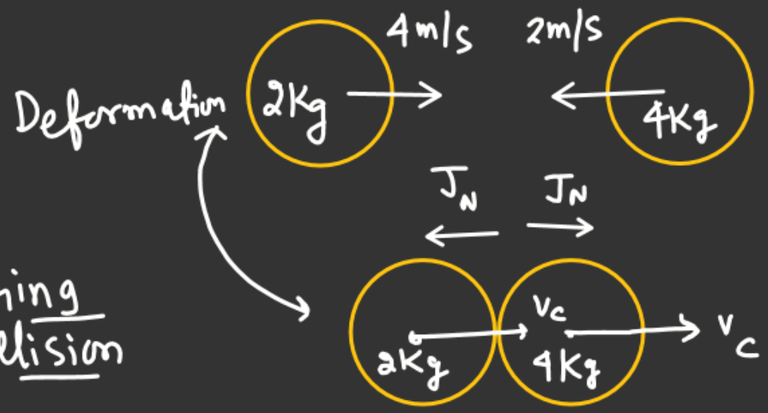
COLLISION

Just before Collision



V_c = At the time of maximum deformation

L·M·C

$$(2\times4) - (4\times2) = 6V_c$$

$$V_c = 0$$

Impulse during deformation

$$-J_N\hat{\imath} = (\Delta\vec{p})_{2Kg}$$

$$-J_N\hat{\imath} = [2\times V_c - (2\times4)]\hat{\imath}$$

($V_c \to 0$)

$$J_N = 8\ \text{Kg m/s}$$

$\Downarrow$

Only Magnitude

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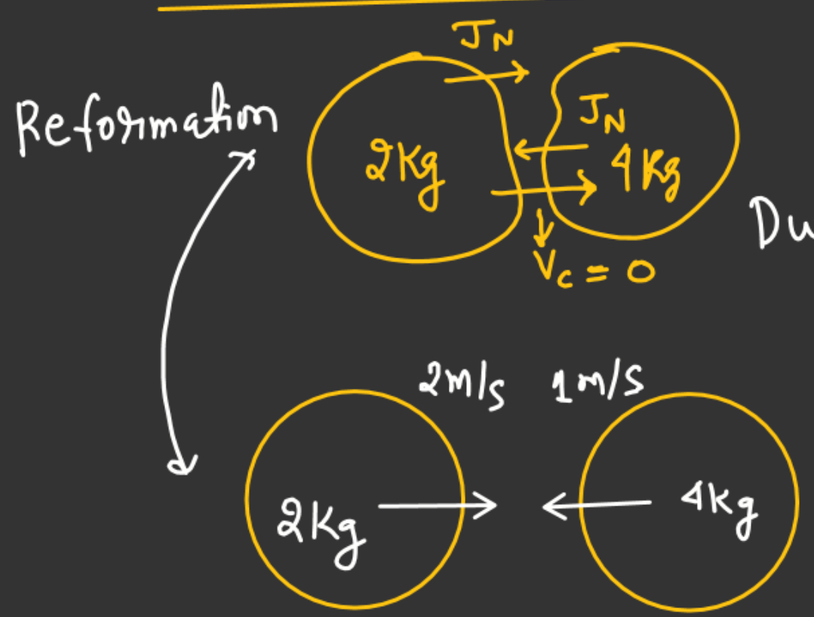
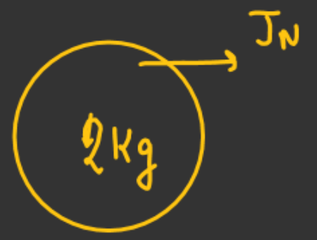
COLLISION

Impulse of Reformation



J_N = Impulse of Reformation

During Collision



$$\vec{J}_N = (\vec{\Delta p})_{2Kg}$$

$$= (2\times 2)\hat{i} - (2\times V_c)\hat{i}$$

$$\left[\vec{J}_N = 4\hat{i}\right]$$

$\Downarrow$

(Impulse of Reformation) ✓