DPP 01

SOLUTION

1. At equilibrium, $F_{net} = 0$

$\therefore$ Conservation of momentum,

$mv = m_1v_1 + m_2v_2$

$mv = \frac{m}{2}v_1 + \frac{m}{2}v_2 \Rightarrow v = \frac{v_1 + v_2}{2}$ ... (i)

By conservation of energy,

$\frac{1}{2}mv^2 = \frac{1}{2}\frac{m}{2}v_1^2 + \frac{1}{2}\frac{m}{2}v_2^2$ or $v^2 = \frac{v_1^2 + v_2^2}{2}$ ... (ii)

By (i) and (ii)

$2v^2 = v_1^2 + (2v - v_1)^2$

$2v^2 = v_1^2 + 4v^2 + v_1^2 - 4vv_1$ or $2v_1^2 - 4vv_1 + 2v^2 = 0$

Solving, we get $v = v_1 = v_2$ ... (iii)

According to the question, Initial amplitude $= A$, final amplitude $= fA$

From (iii), $v = v_1$ or $A\omega = fA\omega_f$

$A\sqrt{\frac{k}{m}} = fA\sqrt{\frac{2k}{m}}$ or $f = \frac{1}{\sqrt{2}}$.

2. According to the question

Potential energy = Kinetic energy

$\frac{1}{2}m\omega^2x^2 = \frac{1}{2}m\omega^2(A^2 - x^2)$ or, $2x^2 = A^2; x = \frac{A}{\sqrt{2}}$

3. For SHM, speed, $v = \omega\sqrt{A^2 - x^2}$

Acceleration, $a = -\omega^2 x$

As $|v| = |a|$

$\omega\sqrt{A^2 - x^2} = \omega^2 x$

$A^2 - x^2 = \omega^2x^2 \Rightarrow \omega^2 = \frac{A^2 - x^2}{x^2}$

$= \frac{5^2 - 4^2}{4^2} = \left(\frac{3}{4}\right)^2; \omega = \frac{3}{4} \Rightarrow T = \frac{2\pi}{\omega} = \frac{8\pi}{3}$

4. Restoring force due to pressing the bottle with small amount x,

$F = -(\rho Ax)g$

$$a = -\left(\frac{\rho Ag}{m}\right)x \;\therefore\; \omega^2 = \frac{\rho Ag}{m} = \frac{\rho(\pi r^2)g}{m}$$

$$\omega = \sqrt{\frac{10^3 \times \pi \times (2.5 \times 10^{-2})^2 \times 10}{310 \times 10^{-3}}} \simeq 7.95\text{rad/s}$$

5. Maximum K.E. of the particle $= \frac{1}{2}m(A^2\omega^2)$

P.E. of the particle at any time $t = \frac{1}{2}m\omega^2x^2$

By energy conservation

$$\Rightarrow \frac{KE}{PE} = \frac{KE_{max}}{PE} - 1 \Rightarrow \frac{KE}{PE} = \frac{\frac{1}{2}mA^2\omega^2}{\frac{1}{2}m\omega^2x^2} - 1$$

$$= \frac{A^2}{A^2\sin^2\frac{\pi}{90}\times 210} - 1 = \frac{1}{\left[\sin\left(2\pi + \frac{\pi}{3}\right)\right]^2} - 1 = \frac{1}{3}$$

6. Here, $x = a\sin\omega t$

$y = a\sin 2\omega t$

$y = 2a\sin\omega t\cos\omega t$

$$y = 2x\sqrt{1 - \frac{x^2}{a^2}} \text{ or } y = \frac{2}{a}x\sqrt{(a-x)(a+x)}$$

$y = 0$, at $x = 0, \pm a$

7. $x_1 = A\sin(\omega t + \phi_1)$

$x_2 = A\sin(\omega t + \phi_2)$

$$x_1 - x_2 = A\left[2\sin\left[\omega t + \frac{\phi_1 + \phi_2}{2}\right]\sin\left[\frac{\phi_1 - \phi_2}{2}\right]\right]$$

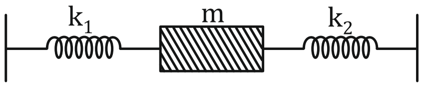
$$A = 2A\sin\left(\frac{\phi_1 - \phi_2}{2}\right)$$

$$\frac{\phi_1 - \phi_2}{2} = \frac{\pi}{6}$$

$$\phi_1 = \frac{\pi}{3}$$

8. In given arrangement two springs are connected in parallel. So, the effective spring constant will be

$k_{eff} = k_1 + k_2$



Frequency of oscillation,

$$f = \frac{1}{2\pi}\sqrt{\frac{k_{eff}}{m}} = \frac{1}{2\pi}\sqrt{\frac{k_1 + k_2}{m}} \ldots \text{(i)}$$

As k_1 and k_2 are increased four times

New frequency, $f' = \frac{1}{2\pi}\sqrt{\frac{4(k_1 + k_2)}{m}} = 2f \quad \text{(using (i))}$

9. For a particle to perform simple harmonic motion its displacement at any time t is given by

$x(t) = a(\cos\omega t + \phi)$

a = amplitude, ω = angular frequency, ϕ = phase constant.

Let us choose $\phi = 0 \therefore x(t) = a\cos\ \omega t$

Velocity of a particle $v = \frac{dx}{dt} = -a\omega\sin\ \omega t$

K.E. of a particle is

$$K = \frac{1}{2}mv^2 = \frac{1}{2}ma^2\omega^2\sin^2\ \omega t$$

Average K. E. $< K > = < \frac{1}{2}ma^2\omega^2\sin^2\omega t >$

$$= \frac{1}{2}m\omega^2a^2 < \sin^2\omega t >$$

$$= \frac{1}{2}m\omega^2a^2\left(\frac{1}{2}\right)\left[\because < \sin^2\ \theta > = \frac{1}{2}\right]$$

$$= \frac{1}{4}ma^2(2\pi\upsilon)^2\ [\because \omega = 2\pi\upsilon]$$

$$= \pi^2ma^2\upsilon^2$$

10. For a SHM, acceleration,

$a = -\omega^2x$, where ω is a constant $= \frac{2\pi}{T}$.

$$a = -\frac{4\pi^2}{T^2}\cdot x \Rightarrow \frac{aT}{x} = -\frac{4\pi^2}{T}$$

The period of oscillation T is a constant.

$\therefore \frac{aT}{x}$ is a constant.