DPP-04 (INTEGRATION BY SUBSTITUTION)

1. Evaluate the following:

Ans. (i) $\log|3+\tan x|+C$ (ii) $\log_e(e^x+e^{-x})+C$ (iii) $\log_e|\cos x+\sin x|+C$

(iv) $\log_e(1+e^x)+C$

(i) $\int \frac{\sec^2 x}{3+\tan x}dx$

Sol. Let $3+\tan x=t$

$\sec^2 x dx = dt$

$\therefore \int \frac{\sec^2 x}{3+\tan x}dx=\int \frac{dt}{t}$ $=\log|t|+c$

$=\log|3+\tan x|+c$

(ii) $\int \frac{e^x-e^{-x}}{e^x+e^{-x}}dx$

Sol. Explanation:

Note that $\frac{d}{dx}(e^x+e^{-x})=e^x-e^{-x}$ so

$\frac{e^x-e^{-x}}{e^x+e^{-x}}=\frac{d}{dx}\log(e^x+e^{-x})$ and finally

$\int \frac{e^x-e^{-x}}{e^x+e^{-x}}dx=\int \frac{d}{dx}\log(e^x+e^{-x})dx=\log(e^x+e^{-x})+C$

(iii) $\int \frac{1-\tan x}{1+\tan x}dx$

Sol. $I=\int \frac{1+\tan x}{1-\tan x}dx$ $=\int \frac{1+\frac{\sin x}{\cos x}}{1-\frac{\sin x}{\cos x}}dx$

$=\int \frac{\cos x+\sin x}{\cos x-\sin x}dx$

Let $\cos x-\sin x=t$

$\Rightarrow (-\sin x+\cos x)dx=dt$

$\Rightarrow (\sin x+\cos x)dx=-dt$

$\therefore I=-\int \frac{dt}{t}$

$=-\log|t|+C$

$=-\log|\cos x-\sin x|+C$

(iv) $\int \frac{1}{1+e^{-x}}dx$

2. Evaluate the following:

Ans. (i) $\frac{2}{5}\left(x+\frac{1}{x}\right)^{\frac{5}{2}}+C$ (ii) $\frac{2(2+\log x)^{\frac{3}{2}}}{3}+c$ (iii) $\frac{(\sin^{-1}x)^4}{4}+c$

(i) $\int \left(x+\frac{1}{x}\right)^{\frac{1}{2}}\left(\frac{x^2-1}{x^2}\right)dx$

(ii) $\int \frac{\sqrt{2+\log x}}{x} dx$

Let $2 + \log x = t^2$

$= dx/x = 2tdt$ $= \int (t.2t)dt = 2\int t^2 dt$

$= 2(t^3/3) + c$ $= (2/3)\left[(2 + \log x)^{3/2}\right] + c$

(iii) $\int \frac{(\sin^{-1} x)^3}{\sqrt{1-x^2}} dx$

$\int \frac{(\sin^{-1} x)^3}{\sqrt{1-x^2}} dx$

Let $\sin^{-1} x = t$
Hence

$\frac{1}{\sqrt{1-x^2}} dx = dt$

Therefore, the above integral transforms to

$\int t^3 dt$

$= \frac{t^4}{4} + c$

$= \frac{(\sin^{-1} x)^4}{4} + c$

3. Evaluate $\int \frac{1}{1-\tan x} dx$

Ans. $-\frac{1}{2}\log_e|\cos x - \sin x| + \frac{1}{2}x + c$

Sol. Let $I = \int \frac{1}{(1-\tan x)} dx = \int \frac{1}{1-\frac{\sin x}{\cos x}} dx = \int \frac{1}{\frac{\cos x-\sin x}{\cos x}} dx$

$= \frac{1}{2}\int \frac{2(\cos x)dx}{(\cos x-\sin x)} = \frac{1}{2}\int \frac{\cos x+\cos x+\sin x-\sin x}{(\cos x-\sin x)} dx$

[add and subtract $\cos x$ in numerator]

$= \frac{1}{2}\left[\int \frac{(\cos x-\sin x)}{(\cos x-\sin x)} dx + \int \frac{(\cos x+\sin x)}{(\cos x-\sin x)} dx\right]$

$= \frac{1}{2}\left[\int 1dx + \int \frac{(\cos x+\sin x)}{(\cos x-\sin x)} dx\right]$

Let $\cos x - \sin x = t$

$\Rightarrow -\sin x - \cos x = \frac{dt}{dx} \Rightarrow -[\sin x + \cos x] = \frac{dt}{dx}$

$\Rightarrow dx = \frac{dt}{-[\sin x+\cos x]}$

$\therefore I = \frac{1}{2}\left[\int 1dx + \int \frac{\cos x+\sin x\, dt}{t} - [\sin x + \cos x] = \frac{1}{2}\left[\int 1dx - \int \frac{1}{t}dt\right]\right.$

$= \frac{1}{2}[x - \log|t|] + C = \frac{1}{2}[x - \log|\cos x - \sin x|] + C$

4. Evaluate $\int \frac{\log\left(\tan\frac{x}{2}\right)}{\sin x} dx$

Ans. $\frac{\left[\log\left(\tan\frac{x}{2}\right)\right]^2}{2} + C$

Sol. $\frac{d}{dx}\left[\log\left(\tan\frac{x}{2}\right)\right] = \frac{\frac{1}{2}\sec^2\frac{x}{2}}{\tan\frac{x}{2}} = \frac{1}{\sin x}$

Now $\int \frac{\log\left(\tan\frac{x}{2}\right)}{\sin x} dx = \int \log\left(\tan\frac{x}{2}\right)\frac{d}{dx}\left[\log\left(\tan\frac{x}{2}\right)\right] dx = \frac{\left[\log\left(\tan\frac{x}{2}\right)\right]^2}{2} + C$

5. Evaluate $\int \sec^p x \tan x \, dx$

Ans. $\frac{\sec^p x}{p} + c$

Sol. Let $I = \int \sec^p x\tan x dx = \int \sec^{p-1} x\sec x\tan x dx$
Put $\sec x = t \Rightarrow \sec x\tan x dx = dt$
Therefore

$I = \int t^{p-1}dt = t^p/p = (1/p)\sec^p x$

6. Evaluate $\int \frac{\log_e\left(x + \sqrt{x^2+1}\right)}{\sqrt{x^2+1}} dx.$

Ans. $\frac{\left(\log_e\left(x + \sqrt{x^2+1}\right)\right)^2}{2} + c$

7. Evaluate $\int \frac{2x - \sqrt{\sin^{-1}x}}{\sqrt{1-x^2}} dx.$

Ans. $2(1-x^2)^{\frac{1}{2}} - \frac{2}{3}(\sin^{-1}x)^{\frac{3}{2}} + c$

Sol. $\int \frac{2x-\sqrt{\sin^{-1}x}}{\sqrt{1-x^2}} dx.$

$\sin^{-1} x = t -; x = \sin t.$

$\frac{1}{\sqrt{1-x^2}} dx = dt$

$\int \frac{2\sin t - \sqrt{t}dt}{1}$

$= -2\cos t - 2/3t^{3/2} + C.$

$= -2\sqrt{1-\sin^2} t - 2/3(\sin^{-1} x)^{3/2} + C.$

$= -2\sqrt{1 - x^2 - 2/3}[(\sin^{-1} x)^3]^{1/2} + C$

$= C - 2\sqrt{1-x^2} - 2/3\sqrt{(\sin^{-1} x)^3} = C - 2\sqrt{1-x^2} - 2/3\sqrt{f(x)}$

$\therefore f(x) = (\sin^{-1} x)^3 *$

∴ Hence, option (c) is correct

8. Evaluate $\int (x^6 + x^4 + x^2)\sqrt{2x^4 + 3x^2 + 6}dx.$

Ans. $\frac{1}{18}(2x^6 + 3x^4 + 6x^2)^{\frac{3}{2}} + c$

Sol. $I = \int (x^6 + x^4 + x^2)\sqrt{2x^4 + 3x^2 + 6}$

$= \int (x^5 + x^3 + x)\sqrt{2x^6 + 3x^4 + 6x^2}dx$

Let $2x^6 + 3x^4 + 6x^2 = r^2$

$\therefore 12(x^5 + x^3 + x)dx = 2tdt$

$\therefore I = \frac{1}{12}\int 2t^2dt = \frac{1}{18}(2x^6 + 3x^4 + 6x^2)^{3/2} + c$

INTEGRATION OF FUNCTION $\mathbf{f(g(x)).g'(x)}$

9. Evaluate $\int \cos^3 x\sqrt{\sin x}\, dx,$

Ans. $\frac{2}{3}\sin^{\frac{3}{2}} x - \frac{2}{7}\sin^{\frac{7}{2}} x + c$

10. Evaluate $\int 2^{2^{2^x}} 2^{2^x} 2^x dx$

Ans. $\frac{1}{(\log 2)^3} 2^{2^{2^x}} + C$

Sol. $\int 2^{2^{2^x}} 2^{2^x} 2^x dx \qquad \Rightarrow$ Let $2^x = t$

$\frac{dt}{dx} = 2^x \ln 2 \qquad \Rightarrow 2^x dx = \frac{dt}{\ln 2}$

$2^{2^t} 2^t \frac{dt}{\ln 2} \qquad \Rightarrow$ Let $2^t = m$

$\frac{dm}{dt} = 2^t \ln 2 \qquad \Rightarrow 2^t dt = \frac{(dm)}{\ln 2}$

$\int \frac{2^m dm}{(\ln 2)^2} = \frac{2^m}{(\ln 2)^3} = \frac{2^{2^{2^s}}}{(\ln 2)^3} + C$

11. Evaluate $\int \frac{e^{\sqrt{x}}\cos\left(e^{\sqrt{x}}\right)}{\sqrt{x}}dx$

Ans. $2 \sin e^{\sqrt{x}} + c$

Sol. Substitute $e^{\sqrt{x}} = t$

$\therefore e^{\sqrt{x}} \cdot \frac{1}{2\sqrt{(x)}}dx = dt.$

$I = 2\int \cos t dt = 2\sin t + c = 2\sin e^{\sqrt{x}} + c.$

12. Find $\int \frac{e^x(1 + x)}{\cos^2(xe^x)}dx$

Ans. $\tan z + c = \tan(xe^{r}) + c$

Sol. Substitute $xe^{x} = t$

$e^{x}(1 + x)dx = dt$

$\int \frac{dt}{\cos^2 t} = \int \sec^2 t \cdot dt$

$= \tan(t) + c$

$= \tan(xe^{x}) + c$

So, $\int \frac{e^{x}(1+x)}{\cos^2(xe^{x})} \cdot dx = \tan(xe^{x}) + c$

13. $\int 5^{x + \tan^{-1} x} \cdot \left(\frac{x^2 + 2}{x^2 + 1}\right) dx$

Ans. $\frac{5^{x + \tan^{-1} x}}{\log_e 5} + c$

Sol. Let

$t = x + \tan^{-1} x$

$\Rightarrow dt = \left(1 + \frac{1}{1+x^2}\right) dx = \frac{x^2+2}{1+x^2} dx$

$\int 5^{x+\tan^{-1} x} \left(\frac{x^2+2}{1+x^2}\right) dx = \int 5^{t} dt$

Let $u = 5^{t} \Rightarrow du = 5^{t} \log 5 dt$

$= \frac{1}{\log 5} \int du = \frac{1}{\log 5} u + c$

$= \frac{1}{\log 5} 5^{t} + c$ where $u = 5^{t}$ $= \frac{5^{x+\tan^{-1} x}}{\log 5} + c$ where $t = x + \tan^{-1} x$

14. $\int \frac{a^{\sqrt{x}}}{\sqrt{x}} dx$ is equal to

(A) $\frac{a^{\sqrt{x}}}{\sqrt{x}} + c$ (B) $\frac{2a^{\sqrt{x}}}{\ln a} + c$ (C) $2a^{\sqrt{x}} \cdot \ln a + c$ (D) $\frac{a^{\sqrt{x}}}{\ln a} + c$

Ans. **(B)**

Sol. $\int \frac{a^{\sqrt{x}}}{\sqrt{x}} dx$

put $\sqrt{x} = t \Rightarrow \frac{1}{2\sqrt{x}} dx = dt \Rightarrow \frac{dx}{\sqrt{x}} = 2dt$

$= 2\int a^{t} dt = \frac{2a^{t}}{\ell na} + c = 2\frac{a^{\sqrt{x}}}{\ell na} + c$

15. $\int 5^{5^{5^x}} \cdot 5^{5^x} \cdot 5^x dx$ is equal to

(A) $\frac{5^{5^x}}{(\log 5)^3} + c$ (B) $5^{5^{5^x}}(\ln 5)^3 + c$ (C) $\frac{5^{5^{5^x}}}{(\log 5)^3} + c$ (D) $\frac{5^{5^{5^x}}}{\ln 5} + c$

Ans. **(C)**

Sol. $I = \int 5^{5^{5^x}} \cdot 5^{5^x} \cdot 5^x dx$ Let $5^{5^{5^x}} = t$

$5^{5^{5^x}} \cdot \ln 5.5^{5^x} \ln 5.5^x \ln 5 dx = dt$

$5^{5^{5^x}} \cdot 5^{5^x} \cdot 5^x dx = \frac{dt}{(\ell \ln 5)^3}$

$I = \int \frac{dt}{(\ell \ln 5)^3} = \frac{t}{(\ell \ln 5)^3} + c = \frac{5^{5^{5^x}}}{(\ell \ln 5)^3} + c$

16. $\int \frac{\sqrt{\tan x}}{\sin x \cos x} dx$ is equal to

(A) $2\sqrt{\tan x} + c$ (B) $2\sqrt{\cot x} + c$ (C) $\frac{\sqrt{\tan x}}{2} + c$ (D) $\frac{\sqrt{\sec x}}{2} + c$

Ans. **(A)**

Sol. $\int \frac{\sqrt{\tan x}}{\sin x \cos x} dx$

$\int \frac{\sqrt{\tan x} \sec^2 x}{\tan x} dx$

$\tan x = t^2 \Rightarrow \sec^2 x dx = 2t dt$

$\int \frac{t.2tdt}{t^2} = 2t + c = 2\sqrt{\tan x} + c$

17. If $\int \frac{2^x}{\sqrt{1-4^x}} dx = K \sin^{-1}(2^x) + C$, then K is equal to

(A) $\ln 2$ (B) $\frac{1}{2} \ln 2$ (C) $\frac{1}{2}$ (D) $\frac{1}{\ln 2}$

Ans. **(D)**

Sol. $\int \frac{2^x}{\sqrt{1-4^x}} dx$

$2^x = t \Rightarrow 2^x \ln 2 dx = dt \Rightarrow 2^x dx = \frac{dt}{\ln 2}$

$\frac{1}{\ln 2} \int \frac{dt}{\sqrt{1-t^2}} = \frac{1}{\ln 2} \sin^{-1}(2^x) + c$

18. $\int \sqrt{\frac{e^x-1}{e^x+1}} dx$ is equal to

(A) $\ln\left(e^x + \sqrt{e^{2x}-1}\right) - \sec^{-1}(e^x) + C$ (B) $\ln\left(e^x + \sqrt{e^{2x}-1}\right) + \sec^{-1}(e^x) + C$

(C) $\ln\left(e^x - \sqrt{e^{2x}-1}\right) - \sec^{-1}(e^x) + C$ (D) A and B both

Ans. **(A)**

Sol. $\int \sqrt{\frac{e^x-1}{e^x+1}} dx = \int \frac{e^x-1}{\sqrt{e^{2x}-1}} dx \quad = \int \frac{e^x}{\sqrt{e^{2x}-1}} dx - \int \frac{dx}{\sqrt{e^{2x}-1}}$

$= \int \frac{dt}{\sqrt{t^2-1}} - \int \frac{e^x}{e^x\sqrt{e^{2x}-1}} dx \quad = \int \frac{dt}{\sqrt{t^2-1}} - \int \frac{du}{u\sqrt{u^2-1}}$

$= \ln\left(e^x + \sqrt{e^{2x}-1}\right) - \sec^{-1}(e^x) + c$

19. $\int \sqrt{\sec x - 1}\, dx$ is equal to

(A) $2\ln\left(\cos\frac{x}{2} + \sqrt{\cos^2\frac{x}{2} - \frac{1}{2}}\right) + C$

(B) $2\ln\left(\cos\frac{x}{2} + \sqrt{\cos^2\frac{x}{2} - \frac{1}{2}}\right) + C$

(C) $-2\ln\left(\cos\frac{x}{2} + \sqrt{\cos^2\frac{x}{2} - \frac{1}{2}}\right) + C$

(D) $\ln\left(\cos\frac{x}{2} + \sqrt{\cos^2\frac{x}{2} - \frac{1}{2}}\right) + C$

Ans. **(C)**

Sol. $I = \int \sqrt{\sec x - 1}\,dx$ $\Rightarrow I = \int \sqrt{\frac{1-\cos x}{\cos x}}\,dx$

$I = \int \sqrt{\frac{2\sin^2\frac{x}{2}}{2\cos^2\frac{x}{2}-1}}\,dx$ $\Rightarrow I = \int \frac{\sin x}{\sqrt{\cos^2\frac{x}{2}-\frac{1}{2}}}\,dx$

put $\cos\frac{x}{2} = t$ $\Rightarrow -\sin\frac{x}{2}\frac{1}{2}dx = dt$

$\sin\frac{x}{2}dx = -2dt$ $\Rightarrow I = -2\int \frac{dt}{\sqrt{t^2-\frac{1}{2}}}$

$I = -2\int \frac{dt}{\sqrt{t^2-\frac{1}{2}}}$ $\Rightarrow I = -2\ln\left|t + \sqrt{t^2 - \frac{1}{2}}\right| + C$

$I = -2\ln\left|\cos\frac{x}{2} + \sqrt{\cos^2\frac{x}{2} - \frac{1}{2}}\right| + C$

20. $\int \frac{1}{\cos^6 x + \sin^6 x}\, dx$ is equal to

(A) $\tan^{-1}(\tan x + \cot x) + c$

(B) $-\tan^{-1}(\tan x + \cot x) + c$

(C) $\tan^{-1}(\tan x - \cot x) + c$

(D) $-\tan^{-1}(\tan x - \cot x) + c$

Ans. **(C)**

Sol. $\int \frac{dx}{\sin^6 x + \cos^6 x}$

$= \int \frac{dx}{\sin^4 x + \cos^4 x - \sin^2 x\cos^2 x}$

$= \int \frac{\sec^4 x\,dx}{\tan^4 x + 1 - \tan^2 x}$

$= \int \frac{(1+\tan^2 x)\sec^2 x\,dx}{\tan^4 x - \tan^2 x + 1}$

$= \int \frac{(1+t^2)dt}{t^4 - t^2 + 1} = \int \frac{1+\frac{1}{t^2}}{\left(t-\frac{1}{t}\right)^2+1}dt$ $= \tan^{-1}\left(\tan x - \frac{1}{\tan x}\right) + c$

$= \tan^{-1}(\tan x - \cot x) + c$

Let $t - \frac{1}{t} = u \Rightarrow \left(1 + \frac{1}{t^2}\right)dt = du$

$= \int \frac{du}{1+u^2} = \tan^{-1} u + c = \tan^{-1}\left(t - \frac{1}{t}\right) + c$

$= \tan^{-1}\left(\tan x - \frac{1}{\tan x}\right) + c$

$= \tan^{-1}(\tan x - \cot x) + c$

21. $\int \frac{dx}{\cos^3 x \cdot \sqrt{\sin 2x}}$ is equal to

(A) $\frac{\sqrt{2}}{5}(\tan x)^{\frac{5}{2}} + 2\sqrt{\tan x} + c$ (B) $\frac{\sqrt{2}}{5}(\tan^2 x + 5)\sqrt{\tan x} + c$

(C) $\frac{\sqrt{2}}{5}(\tan^2 x + 5)\sqrt{2\tan x} + c$ (D) $\sqrt{2}(\tan^2 x + 5)\sqrt{2\tan x} + c$

Ans. **(B)**

Sol. $\frac{1}{\sqrt{2}}\int \frac{dx}{\cos^{7/2} x \sin^{1/2} x}$

$= \frac{1}{\sqrt{2}}\int \frac{dx}{\cos^{7/2} x \frac{\sin^{1/2} x}{\cos^{1/2} x}\cos^{1/2} x}$ $= \frac{1}{\sqrt{2}}\int \frac{dx}{\tan^{1/2} x \cos^4 x}$

$= \frac{1}{\sqrt{2}}\int \frac{\sec^2 x(1+\tan^2 x)}{\sqrt{\tan x}} dx$

Put $\tan x = t^2$

$\text{Sec}\ ^2 x dx = 2t dt$

$= \frac{2}{\sqrt{2}}\int \frac{(1+t^4)\cdot t dt}{t} = \sqrt{2}\left(1 + \frac{t^5}{5}\right) = \frac{\sqrt{2}}{5}t(5 + t^2) = \frac{\sqrt{2}}{5}\sqrt{\tan x}(5 + \tan^2 x) + C$

22. If $\int \frac{dx}{\sqrt{\sin^3 x \cos^5 x}} = a\sqrt{\cot x} + b\sqrt{\tan^3 x} + c$ where c is an arbitrary constant of integration then the values of ' a ' and ' b ' are respectively

(A) -2 & $\frac{2}{3}$ (B) 2 & $-\frac{2}{3}$ (C) 2 & $\frac{2}{3}$ (D) 2, 2

Ans. **(A)**

Sol. $\int \frac{dx}{\sqrt{\sin^3 x \cos^5 x}} = a\sqrt{\cot x} + b\sqrt{\tan^3 x} + c$

$= \int \frac{\sec^4 x dx}{\sqrt{\tan^3 x}}$

$\tan x = t^2 \Rightarrow \sec^2 x dx = 2t dt$

$\int \frac{(1+\tan^2 x)}{\tan^{3/2} x}\sec^2 x dx$

$= \int \left(\frac{1+t^4}{t^3}\right)2t dt = 2\int \left(\frac{1}{t^2} + t^2\right) dt$

$= -\frac{2}{t} + \frac{2}{3}t^3 + c$

$= -2\sqrt{\cot x} + \frac{2}{3}\sqrt{\tan^3 x} + c$

23. $\int \frac{1}{\sqrt{\sin^3 x\cos x}}dx$ is equal to

(A) $\frac{-2}{\sqrt{\tan x}} + c$ (B) $2\sqrt{\tan x} + c$ (C) $\frac{2}{\sqrt{\tan x}} + c$ (D) $-2\sqrt{\tan x} - c$

Ans. **(A)**

Sol. $\int \frac{dx}{\sqrt{\sin^3 x\cos x}} \quad = \int \frac{dx}{\sqrt{\tan^3 x \cdot \cos^4 x}}$

$= \int \frac{\sec^2 xdx}{\sqrt{\tan^3 x}}$ $\tan x = t \Rightarrow \sec^2 xdx = dt$

$= \int \frac{dt}{t^{3/2} = \frac{t^{-3/2+1}}{1-3/2}} + c = \frac{-2}{\sqrt{\tan x}} + c$

24. $\int \frac{\ln|x|}{x\sqrt{1+\ln|x|}}dx$ is equal to

(A) $\frac{2}{3}\sqrt{1+\ln|x|}(\ln|x| - 2) + c$ (B) $\frac{2}{3}\sqrt{1+\ln|x|}(\ln|x| + 2) + c$

(C) $\frac{1}{3}\sqrt{1+\ln|x|}(\ln|x| - 2) + c$ (D) $\frac{1}{3}\sqrt{1+\ln|x|}(3\ln|\mathbf{x}| + 2) + c$

Ans. **(A)**

Sol. $\int \frac{\ell n|x|}{x\sqrt{1+\ell nx}}dx$

$1 + \ell nx = t^2 \Rightarrow \frac{1}{x}dx = 2tdt$

$\int \frac{(t^2-1)2tdt}{t} = 2\int (t^2-1)dt \quad = 2\left[\frac{t^3}{3} - t\right] + c = \frac{2}{3}t[t^2-3] + c$

$= \frac{2}{3}\sqrt{1+\ell nx}[1 + \ell nx - 3] + c$

$= \frac{2}{3}\sqrt{1+\ell nx}[\ell nx - 2] + c$

25. $\int \frac{\cos 2x}{(\sin x+\cos x)^2}dx$ is equal to

(A) $\frac{-1}{\sin x+\cos x} + c$ (B) $\ln(\sin x + \cos x) + c$

(C) $\ln(\sin x - \cos x) + c$ (D) $\ln(\sin x + \cos x)^2 + c$

Ans. **(B)**

Sol. $\int \frac{\cos 2x}{(\cos x+\sin x)^2}dx$

$\int \frac{\cos x-\sin x}{\cos x+\sin x}dx$

$\cos x + \sin x = t \Rightarrow (-\sin x + \cos x)dx = dt$

$\int \frac{dt}{t} = \ln(\cos x + \sin x) + c$

26. $\int \sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}}\,dx$ is equal to

(A) $\sqrt{x}\sqrt{1-x}-2\sqrt{1-x}+\cos^{-1}(\sqrt{x})+c$ (B) $\sqrt{x}\sqrt{1-x}+2\sqrt{1-x}+\cos^{-1}(\sqrt{x})+c$

(C) $\sqrt{x}\sqrt{1-x}-2\sqrt{1-x}+\cos^{-1}(\sqrt{x})+c$ (D) $\sqrt{x}\sqrt{1-x}+2\sqrt{1-x}-\cos^{-1}(\sqrt{x})+c$

Ans. **(A)**

Sol. $\int \sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}}\,dx \qquad = \int \frac{1-\sqrt{x}}{\sqrt{1-x}}\,dx$

$= \int \frac{dx}{\sqrt{1-x}} - \int \frac{\sqrt{x}}{\sqrt{1-x}}\,dx$

Let $I_1 = \int \sqrt{\frac{x}{1-x}}\,dx = \int \frac{\sqrt{x}}{\sqrt{1-(\sqrt{x})^2}}\,dx$

$= \int \frac{2t^2dt}{\sqrt{1-t^2}} = 2\int \frac{t^2+1-1}{\sqrt{1-t^2}}\,dt \qquad = \int \frac{2t^2dt}{\sqrt{1-t^2}} = 2\int \frac{t^2+1-1}{\sqrt{1-t^2}}\,dt$

$= 2\int \frac{dt}{\sqrt{1-t^2}} - 2\int \sqrt{1-t^2}\,dt$

$= 2\sin^{-1} t - 2\left[\frac{t}{2}\sqrt{1-t^2}+\frac{1}{2}\sin^{-1} t\right]+c$

$I = -2\sqrt{1-x}-2\sin^{-1}\sqrt{x}+\sqrt{x}\sqrt{1-x}+\sin^{-1}\sqrt{x}+c$

$= -2\sqrt{1-x}-\sin^{-1}\sqrt{x}+\sqrt{x}\sqrt{1-x}+c \qquad = -2\sqrt{1-x}+\cos^{-1}\sqrt{x}+\sqrt{x}\sqrt{1-x}+c$

27. If $\int \frac{1}{x\sqrt{1-x^3}}\,dx = a\ln\left|\frac{\sqrt{1-x^3}-1}{\sqrt{1-x^3}+1}\right| + b$, then a is equal to

(A) 1/3 (B) 2/3 (C) −1/3 (D) −2/3

Ans. **(A)**

Sol. $\int \frac{1}{x\sqrt{1-x^3}}\,dx$

$1-x^3 = t^2$

(Aliter put $x^3 = \sin^2\theta$)

$x^2dx = -\frac{2}{3}tdt \;= \int \frac{x^2dx}{x^3\sqrt{1-x^3}}$

$= -\frac{2}{3}\int \frac{tdt}{(1-t^2)t} = \frac{2}{3}\int \frac{dt}{(t^2-1)} = \frac{2}{3}\ln\frac{t-1}{t+1}+c = \frac{1}{3}\ln\frac{\sqrt{1-x^3}-1}{\sqrt{1-x^3}+1}+c$

28. $\int \frac{xdx}{\sqrt{1+x^2+\sqrt{(1+x^2)^3}}}\,dx$ is equal to

(A) $\frac{1}{2}\ln\left(1+\sqrt{1+x^2}\right)+c$ (B) $2\sqrt{1+\sqrt{1+x^2}}+c$

(C) $2\left(1+\sqrt{1+x^2}\right)+c$ (D) None of these

Ans. **(B)**

Sol. $\int \frac{xdx}{\sqrt{1+x^2+\sqrt{(1+x^2)^3}}}$

Let $1+x^2=t^2 \Rightarrow xdx=tdt$

$\int \frac{tdt}{\sqrt{t^2+t^3}}=\int \frac{dt}{\sqrt{1+t}}=2\sqrt{1+t}+c=2\sqrt{1+\sqrt{1+x^2}}+c$

29. $\int \tan^3 2x \sec 2x\, dx$ is equal to

(A) $\frac{1}{3}\sec^3 2x-\frac{1}{2}\sec 2x+c$

(B) $-\frac{1}{6}\sec^3 2x-\frac{1}{2}\sec 2x+c$

(C) $\frac{1}{6}\sec^3 2x-\frac{1}{2}\sec 2x+c$

(D) $\frac{1}{3}\sec^3 2x+\frac{1}{2}\sec 2x+c$

Ans. **(C)**

Sol. $\int \tan^3 2x\sec 2xdx$

$\int \tan 2x(\sec^2 2x-1)\sec 2xdx$

$=\int \frac{\sin 2x}{\cos^4 2x}dx-\int \frac{\sin 2x}{\cos^2 2x}dx$

put $\cos 2x=t$

$\sin 2xdx=-\frac{dt}{2}$

$=-\frac{1}{2}\int \frac{dt}{t^4}+\frac{1}{2}\int \frac{dt}{t^2}$

$=-\frac{1}{2}\left[\frac{t^{-3}}{-3}\right]-\frac{1}{2}\frac{1}{t}+c$

$=\frac{1}{6}\sec^3 2x-\frac{1}{2}\sec 2x+c$

30. If $\int x^{13/2}\cdot\left(1+x^{5/2}\right)^{1/2}dx=A\left(1+x^{5/2}\right)^{7/2}+B\left(1+x^{5/2}\right)^{5/2}+C\left(1+x^{5/2}\right)^{3/2}$, then

(A) $A=-\frac{4}{35}, B=-\frac{8}{25}, C=\frac{4}{15}$

(B) $A=\frac{4}{35}, B=-\frac{8}{25}, C=-\frac{4}{15}$

(C) $A=\frac{4}{35}, B=-\frac{8}{25}, C=\frac{4}{15}$

(D) $A=-\frac{4}{35}, B=-\frac{8}{25}, C=-\frac{4}{15}$

Ans. **(C)**

Sol. $\int x^{13/2}\left(1+x^{5/2}\right)^{1/2}dx$

$1+x^{5/2}=t^2 \Rightarrow x^{3/2}dx=\frac{4}{5}tdt$

$\int x^5\cdot x^{3/2}\left(1+x^{5/2}\right)^{1/2}dx$

$=\frac{4}{5}\int (t^2-1)^2\cdot t^2dt=\frac{4}{5}\int (t^4-2t^2+1)t^2dt$

$=\frac{4}{5}\int (t^6-2t^4+t^2)dt=\frac{4}{5}\left[\frac{t^7}{7}-\frac{2}{5}t^5+\frac{t^3}{3}\right]+c$

$= \frac{4}{35}\left(1+x^{5/2}\right)^{7/2} - \frac{8}{25}\left(1+x^{5/2}\right)^{5/2} + \frac{4}{15}\left(1+x^{5/2}\right)^{3/2} + c$

31. $\int \sqrt{\frac{1-\cos x}{\cos\alpha - \cos x}}\,dx$ where $0 < \alpha < x < \pi$, is equal to

(A) $2\ln\left(\cos\frac{\alpha}{2} - \cos\frac{x}{2}\right) + c$

(B) $\sqrt{2}\ln\left(\cos\frac{\alpha}{2} - \cos\frac{x}{2}\right) + c$

(C) $2\sqrt{2}\ell\ln\left(\cos\frac{\alpha}{2} - \cos\frac{x}{2}\right) + c$

(D) $-2\sin^{-1}\left(\frac{\cos\frac{x}{2}}{\cos\frac{\alpha}{2}}\right) + c$

Ans. (D)

Sol. $\int \sqrt{\frac{1-\cos x}{\cos\alpha - \cos x}}\,dx \qquad = \int \sqrt{\frac{1-\left(1-2\sin^2\frac{x}{2}\right)}{\cos\alpha - \left(2\cos^2\frac{x}{2}-1\right)}}\,dx$

$= \sqrt{2}\int \frac{\sin\frac{x}{2}}{\sqrt{\cos\alpha + 1 - 2\cos^2\frac{x}{2}}}\,dx \qquad = \sqrt{2}\int \frac{\sin\frac{x}{2}}{\sqrt{2\cos^2\frac{\alpha}{2} - 2\cos^2\frac{x}{2}}}\,dx$

$= \int \frac{\sin\frac{x}{2}}{\sqrt{\cos^2\frac{\alpha}{2} - \cos^2\frac{x}{2}}}\,dx \qquad \cos\frac{x}{2} = t \Rightarrow -\frac{1}{2}\sin\frac{x}{2}\,dx = dt$

$\sin\frac{x}{2}\,dx = -2dt \qquad = -2\int \frac{dt}{\sqrt{\cos^2\frac{\alpha}{2} - t^2}} = -2\sin^{-1}\frac{t}{\cos\frac{\alpha}{2}} + c$

$= -2\sin^{-1}\frac{\cos\frac{x}{2}}{\cos\frac{\alpha}{2}} + c$

32. $\int \frac{1}{\left[(x-1)^3(x+2)^5\right]^{1/4}}\,dx$ is equal to

(A) $\frac{4}{3}\left(\frac{x-1}{x+2}\right)^{1/4} + c$ (B) $\frac{4}{3}\left(\frac{x+1}{x-2}\right)^{1/4} + c$ (C) $\frac{1}{3}\left(\frac{x-1}{x+2}\right)^{1/4} + c$ (D) $\frac{1}{3}\left(\frac{x+1}{x-2}\right)^{1/4} + c$

Ans. (A)

Sol. $\int \frac{dx}{\left[(x-1)^3(x+2)^5\right]^{1/4}} \qquad = \int \frac{dx}{(x-1)^{3/4}(x+2)^{5/4}}$

$x - 1 = t^4 \Rightarrow dx = 4t^3dt$

$\int \frac{4t^3dt}{t^3(t^4+3)^{5/4}} = 4\int \frac{dt}{(t^4+3)^{5/4}} \qquad = 4\int \frac{dt}{t^5(1+3t^{-4})^{5/4}}$

$1 + 3t^{-4} = z^4 \Rightarrow -\frac{4}{t^5}dt = \frac{4}{3}z^3dz$

$= -\frac{4}{3}\int \frac{z^3dz}{z^5} = \frac{4}{3}\frac{1}{z} = \frac{4}{3}\left(\frac{x-1}{x+2}\right)^{1/4} + c$

ANSWER KEY

1. (i) $\log|3+\tan x|+C$ (ii) $\log_e(e^x+e^{-x})+C$ (iii) $\log_e|\cos x+\sin x|+C$
(iv) $\log_e(1+e^x)+C$

2. (i) $\frac{2}{5}\left(x+\frac{1}{x}\right)^{\frac{5}{2}}+C$ (ii) $\frac{2(2+\log x)^{\frac{3}{2}}}{3}+c$ (iii) $\frac{(\sin^{-1}x)^4}{4}+c$

3. $-\frac{1}{2}\log_e|\cos x-\sin x|+\frac{1}{2}x+c$ **4.** $\frac{\left[\log\left(\tan\frac{x}{2}\right)\right]^2}{2}+C$ **5.** $\frac{\sec^p x}{p}+c$

6. $\frac{\left(\log_e\left(x+\sqrt{x^2+1}\right)\right)^2}{2}+c$ **7.** $2(1-x^2)^{\frac{1}{2}}-\frac{2}{3}(\sin^{-1}x)^{\frac{3}{2}}+c$

8. $\frac{1}{18}(2x^6+3x^4+6x^2)^{\frac{3}{2}}+c$ **9.** $\frac{2}{3}\sin^{\frac{3}{2}}x-\frac{2}{7}\sin^{\frac{7}{2}}x+c$ **10.** $\frac{1}{(\log 2)^3}2^{2^{2^x}}+C$

11. $2\sin e^{\sqrt{x}}+c$ **12.** $\tan z+c=\tan(xe^r)+c$ **13.** $\frac{5^{x+\tan^{-1}x}}{\log_e 5}+c$

14.	**(B)**	**15.**	**(C)**	**16.**	**(A)**	**17.**	**(D)**	**18.**	**(A)**	**19.**	**(C)**	**20.**	**(C)**
21.	**(B)**	**22.**	**(A)**	**23.**	**(A)**	**24.**	**(A)**	**25.**	**(B)**	**26.**	**(A)**	**27.**	**(A)**
28.	**(B)**	**29.**	**(C)**	**30.**	**(C)**	**31.**	**(D)**	**32.**	**(A)**				