Nishant Jindal

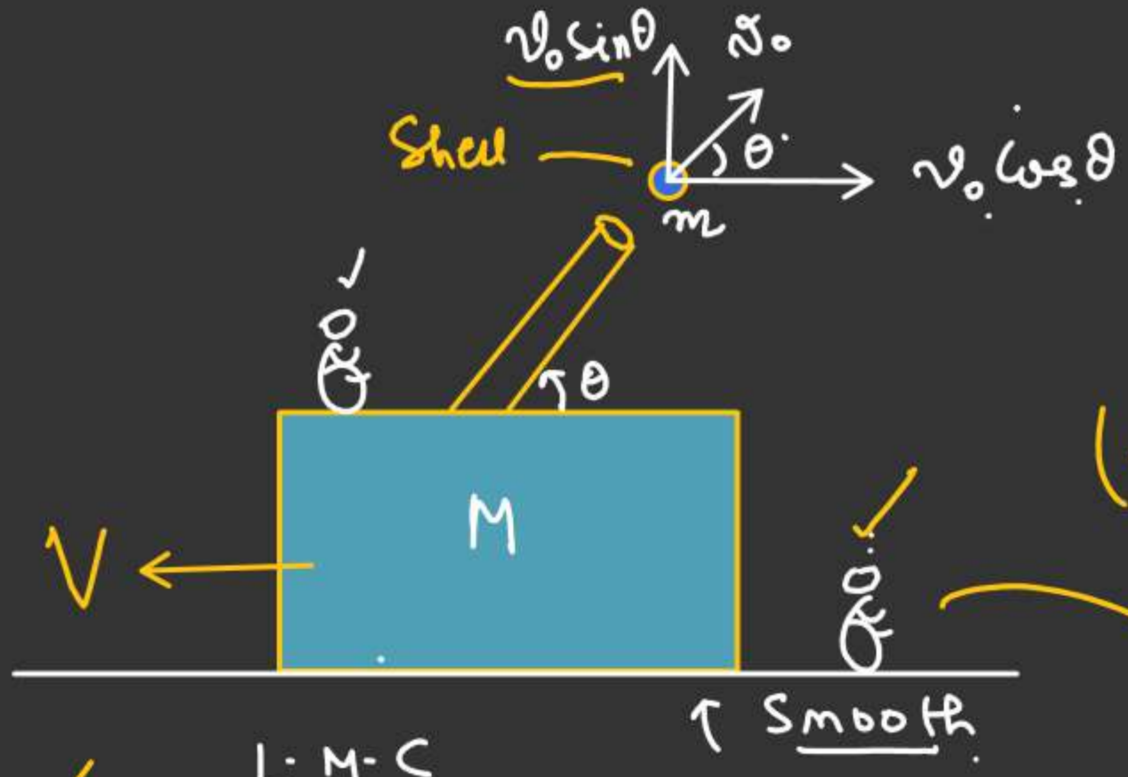
Case of firing of Shell through a Cannon

v_0 = velocity of shell w.r.t Cannon ✓

$$(\vec{v}_{shell/E})_x = (\vec{v}_{shell/Cannon})_x + (\vec{v}_{Cannon/E})_x$$

$$(\vec{v}_{shell/E})_x = (v_0\cos\theta\,\hat{i} - V\hat{i})$$ ✓

$$(\vec{v}_{shell/E})_y = v_0\sin\theta\,\hat{j}$$



L.M.C

$$(\vec{P}_i)_x = (\vec{P}_f)_x$$

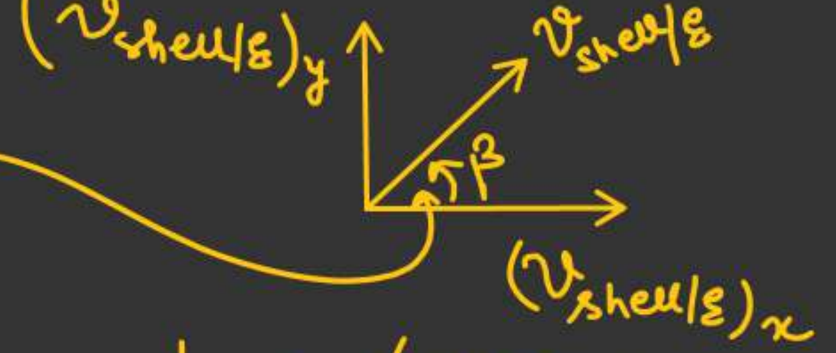
$$0 = m(v_0\cos\theta - V) - MV$$

$$(M+m)V = mv_0\cos\theta$$

$$V = \left(\frac{mv_0\cos\theta}{M+m}\right)$$ ✓

w.r.t earth

Angle made by Shell with horizontal



$$\tan\beta = \left(\frac{v_0\sin\theta}{v_0\cos\theta - V}\right)$$

β = ??

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L·M·C IN COM FRAME

By Δ Law.

$$\vec{r}_{P/E} = \vec{r}_{P/com} + \vec{r}_{com/E}$$

Differentiating both side w.r.t time.

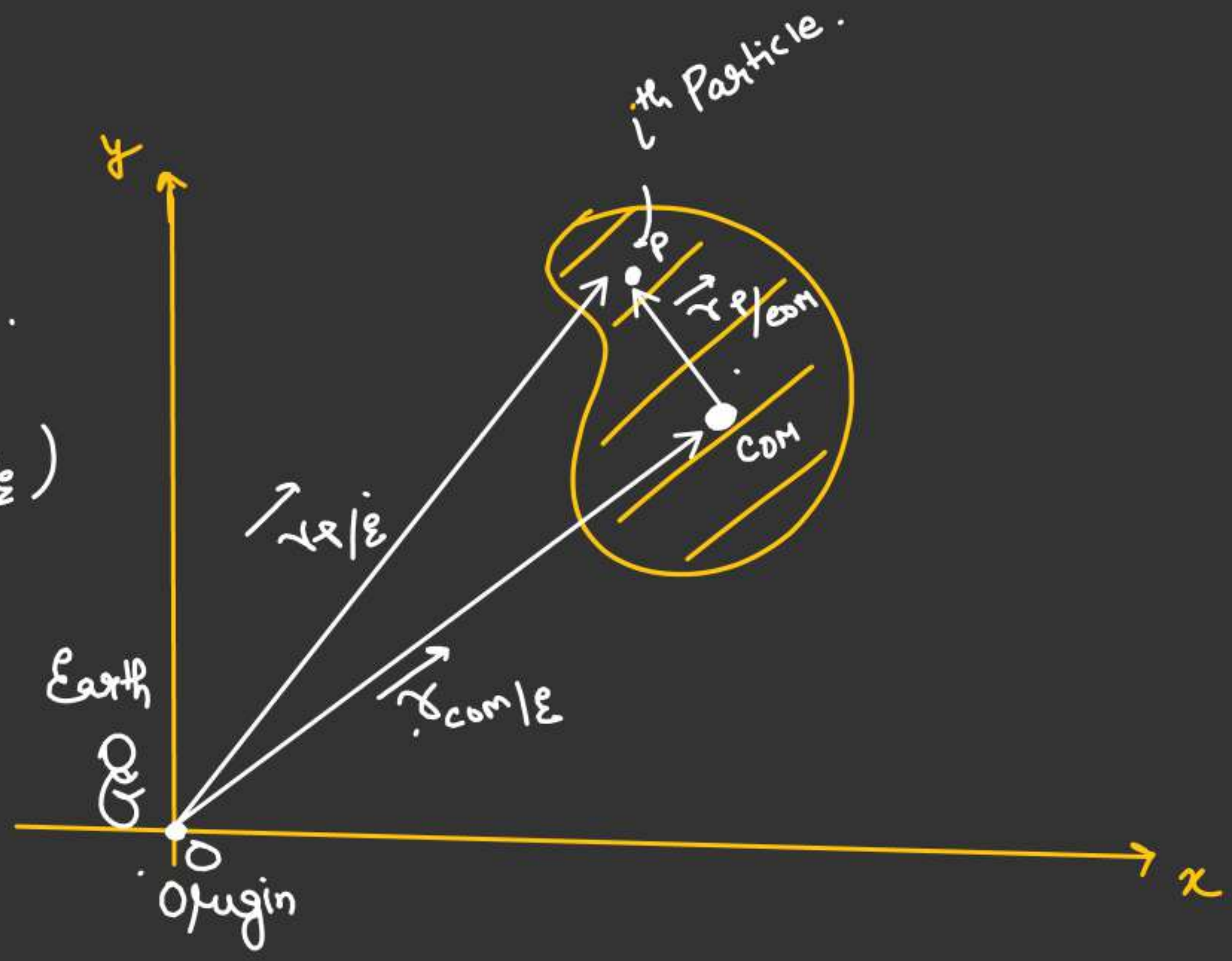
$$\frac{d}{dt}(\vec{r}_{P/E}) = \frac{d}{dt}(\vec{r}_{P/com}) + \frac{d}{dt}(\vec{r}_{com/E})$$

$\Downarrow$

$$\boxed{\vec{v}_{P/E} = \underbrace{\vec{v}_{P/com}}_{} + \vec{v}_{com/E}}$$

$\Downarrow$

Relative velocity of P w.r.t com



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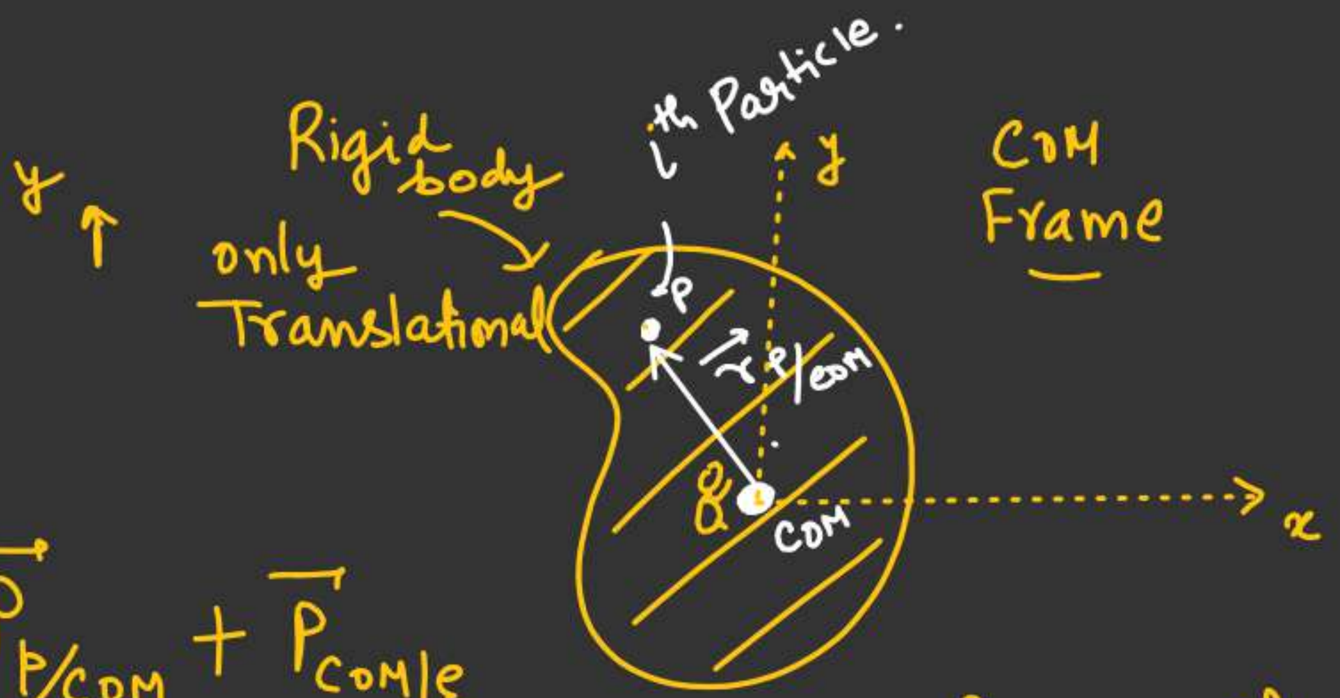
L·M·C IN COM FRAME

$$\boxed{\vec{v}_{P/E} = \underbrace{\vec{v}_{P/com}}_{\text{Relative velocity of P w.r.t COM}} + \vec{v}_{com/E}}$$

w.r.t COM frame

$$\vec{v}_{P/com} = 0 \checkmark$$

$$\boxed{\vec{v}_{P/E} = \vec{v}_{com/E}}$$



$$\vec{P}_{P/E} = \underbrace{\vec{P}_{P/COM}}_{0} + \vec{P}_{COM/E}$$

$$\boxed{\vec{P}_{P/E} = \vec{P}_{COM/E}} \checkmark$$

⇒ when particle P at rest w.r.t COM frame OR both particle & COM move with same velocity w.r.t Earth fram

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L.M.C of two mass system in COM FRAME

$$\vec{V}_{com} = \frac{m_2 v_2 \hat{i} + m_1 v_1 \hat{i}}{(m_1+m_2)}$$

$$\vec{V}_{com} = \left(\frac{m_2 v_2 + m_1 v_1}{m_1+m_2}\right)\hat{i}$$

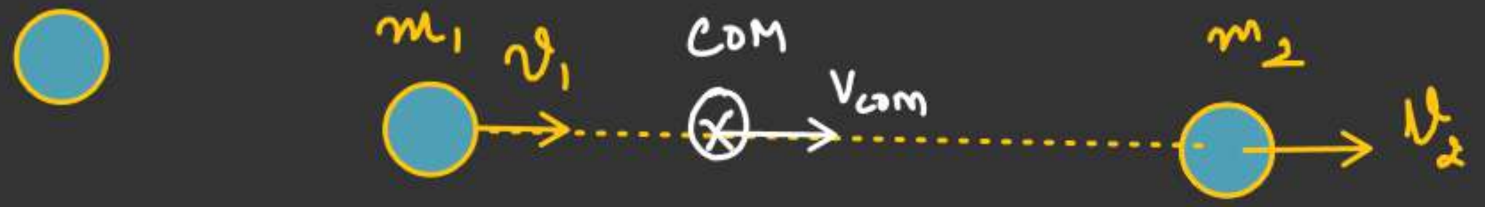
$$\vec{v}_{m_1/com} = \vec{v}_{m_1/E} - \vec{v}_{com/E}$$

$$= \left[v_1 - \left(\frac{m_2 v_2 + m_1 v_1}{m_1+m_2}\right)\right]\hat{i}$$

$$= \left(\frac{m_2 v_1 - m_2 v_2}{m_1+m_2}\right)\hat{i}$$

$$\vec{V}_{m_1/com} = \frac{m_2}{m_1+m_2}(v_1 - v_2)\,\hat{i}$$

$$\vec{V}_{m_1/com} = \left(\frac{m_2}{m_1+m_2}\right)\left(\vec{v}_{m_1/m_2}\right)$$



W.r.t COM

$$\vec{p}_{m_1/com} = m_1 \vec{V}_{m_1/com}$$

$$= \underbrace{\left(\frac{m_1 m_2}{m_1+m_2}\right)}_{\mu}\left(\vec{V}_{m_1/m_2}\right)$$

$$\boxed{\vec{p}_{m_1/com} = \mu\, \vec{v}_{rel}}$$

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:

L.M.C of two mass system in COM FRAME

$$\vec V_{com} = \frac{m_2 v_2\,\hat i + m_1 v_1\,\hat i}{(m_1+m_2)}$$

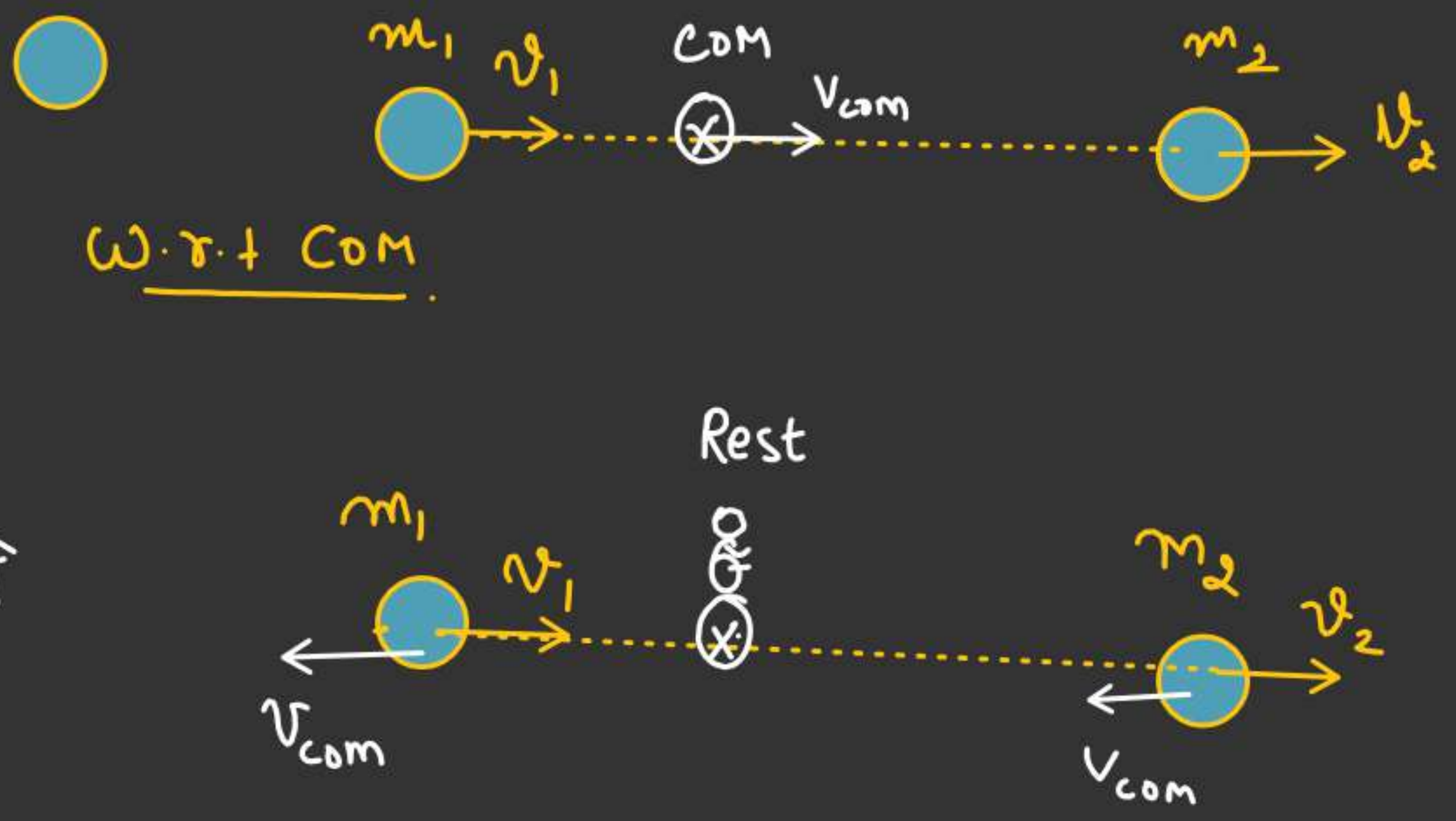
$$\vec V_{com} = \left(\frac{m_2 v_2 + m_1 v_1}{m_1+m_2}\right)\hat i$$

$$\vec v_{m_2/com} = \vec v_{m_2/E} - \vec V_{com/E}$$

$$= v_2\,\hat i - \left(\frac{m_2 v_2 + m_1 v_1}{m_1+m_2}\right)\hat i$$

$$= \frac{m_1}{m_1+m_2}(v_2 - v_1)\,\hat i$$

$$\vec p_{m_2/com} = m_2 \vec v_{m_2/com} = \underbrace{\left(\frac{m_1 m_2}{m_1+m_2}\right)}_{\mu}\left(\vec v_{m_2/m_1}\right) = \left(\mu\, \vec v_{m_2/m_1}\right)$$



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TRICK

For two point mass system.

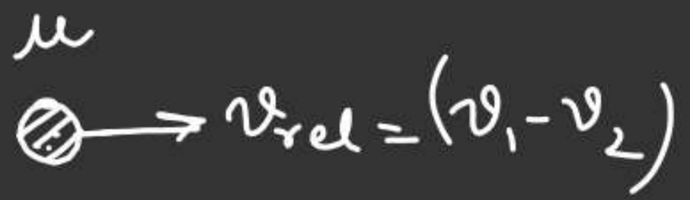
$$\vec{p}_{m_1/COM} = \mu \vec{V}_{rel}$$

$$\vec{p}_{m_2/COM} = -\mu \vec{V}_{rel}$$

$$\vec{v}_{rel} = (v_1 - v_2)\hat{i}$$

$$\boxed{\mu = \frac{m_1 m_2}{m_1 + m_2}}$$

(Reduced Mass)



$$\boxed{\left|\vec{p}_{m_1/com}\right| = \left|\vec{p}_{m_2/com}\right| = \mu(V_{rel})}$$

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K.E in COM frame

K.E of m_1 w.r.t COM =

$$\frac{1}{2}m_1\left(v_{m_1/COM}\right)^2$$

$$= \frac{1}{2}m_1\left[\frac{m_2}{m_1+m_2}(v_1-v_2)\right]^2$$

$$= \frac{1}{2}\frac{m_1^2}{m_1}\times\frac{m_2^2}{(m_1+m_2)^2}\times(v_{rel})^2$$

$$= \frac{1}{2m_1}\left(\frac{m_1m_2}{m_1+m_2}\right)^2 \cdot v_{rel}^2$$

$$\Downarrow$$

$$= \frac{1}{2m_1}\mu^2 v_{rel}^2$$

$$|v_{rel}| = |v_{m_1/m_2}| = |v_{m_2/m_1}|$$

K.E of m_2 w.r.t COM

$$\frac{1}{2}m_2\left(v_{m_2/COM}\right)^2$$

$$= \frac{1}{2m_2}\mu^2 v_{rel}^2$$

$$K.E_{system/COM} = \frac{1}{2}m_1\underbrace{\left(v_{m_1/COM}\right)^2}+\frac{1}{2}m_2\underbrace{\left(v_{m_2/COM}\right)^2}$$

$$= \frac{\mu^2 V_{rel}^2}{2}\left(\frac{1}{m_1}+\frac{1}{m_2}\right)$$

$$= \left(\frac{m_1m_2}{m_1+m_2}\right)^2\times\frac{m_1+m_2}{m_1m_2}\times\frac{1}{2}V_{rel}^2$$

$$\boxed{K.E_{system/COM} = \frac{1}{2}\mu V_{rel}^2}$$

$h_{max} = ??$

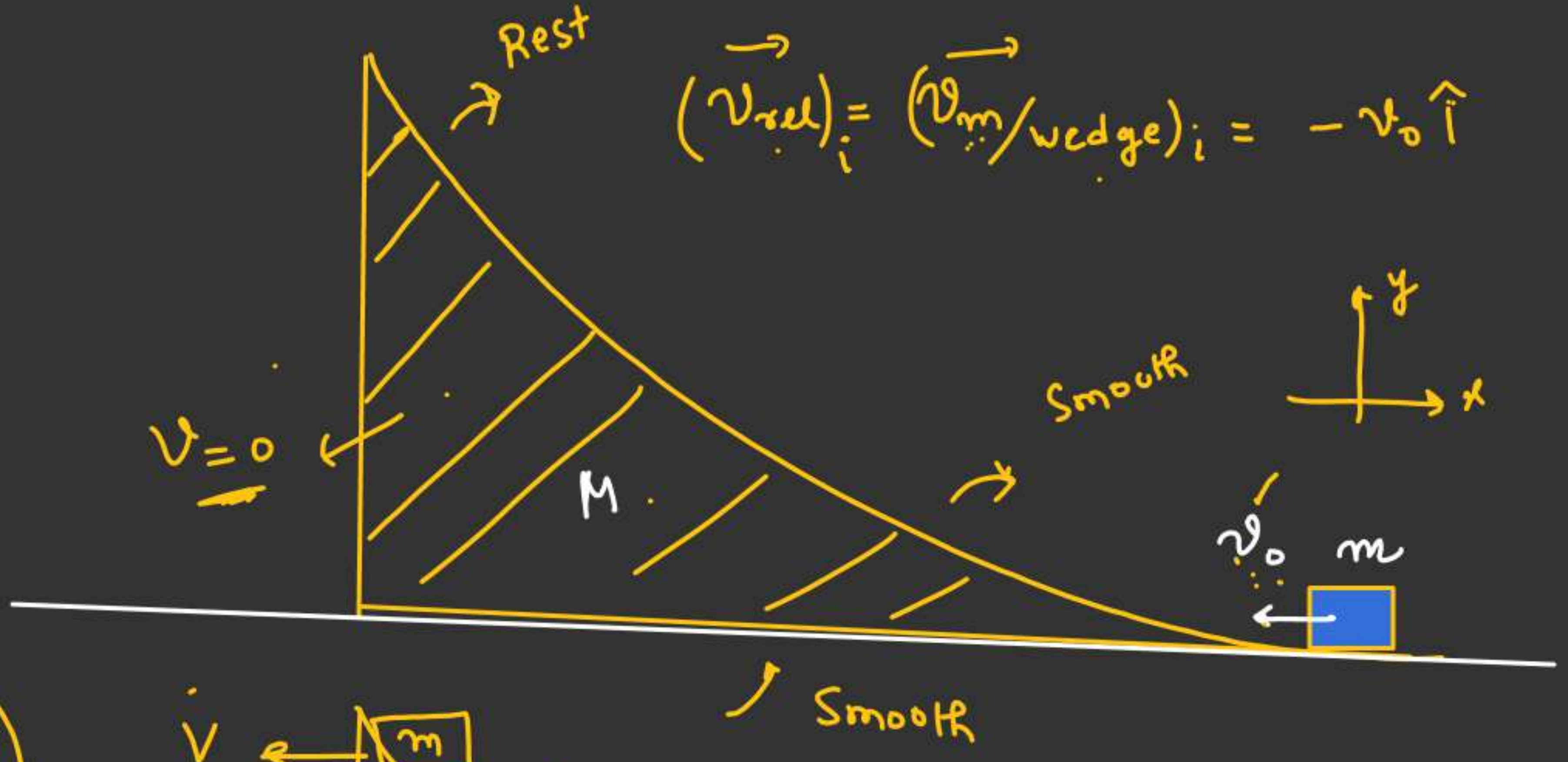
$$\frac{1}{2}\mu V_{rel}^2 = mg\,h_{max}$$

$$\mu = \left(\frac{Mm}{M+m}\right)$$

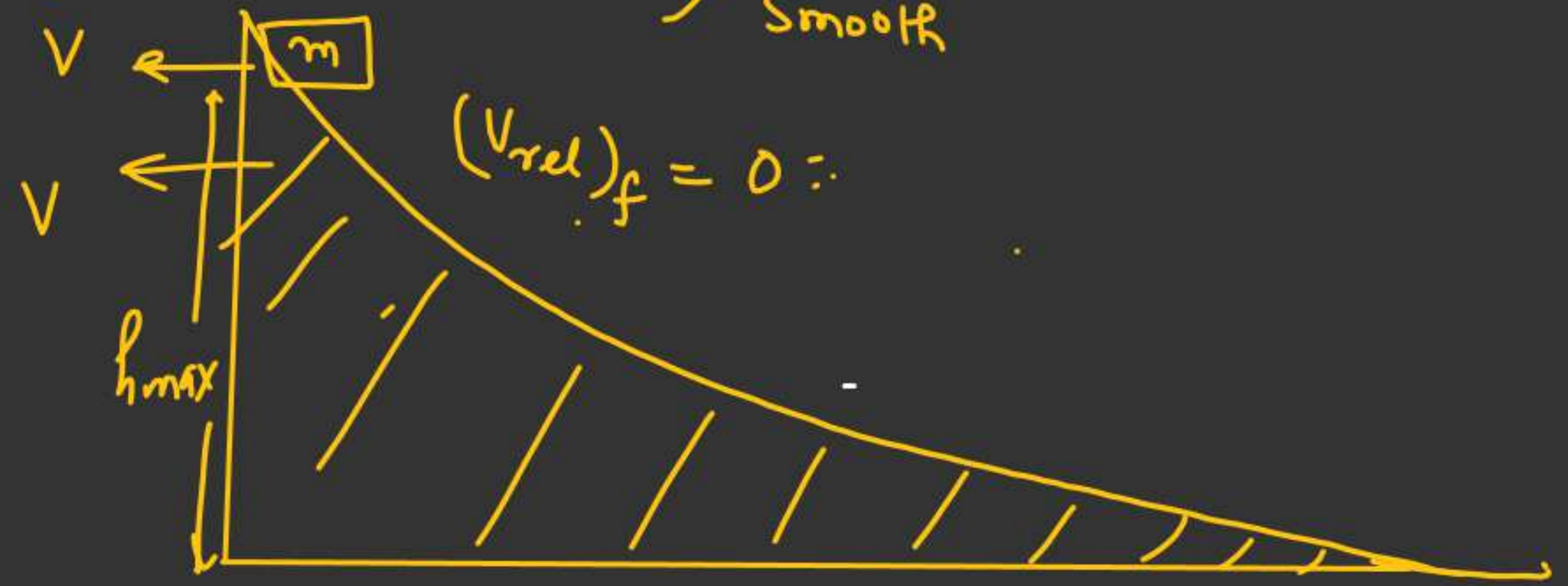
$$\frac{1}{2}\left(\frac{Mm}{M+m}\right)v_0^2 = mg\,h_{max}$$

$$h_{max} = \frac{v_0^2}{2g}\left(\frac{M}{M+m}\right)$$

$$(\vec{v}_{rel})_i = (\vec{v}_{m/wedge})_i = -v_0\hat{i}$$



$$(V_{rel})_f = 0$$



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L·M·C IN SPRING BLOCK SYSTEM

Maximum Compression in the Spring

For two blocks and Spring as System kx is an internal force So no net external force in x-direction

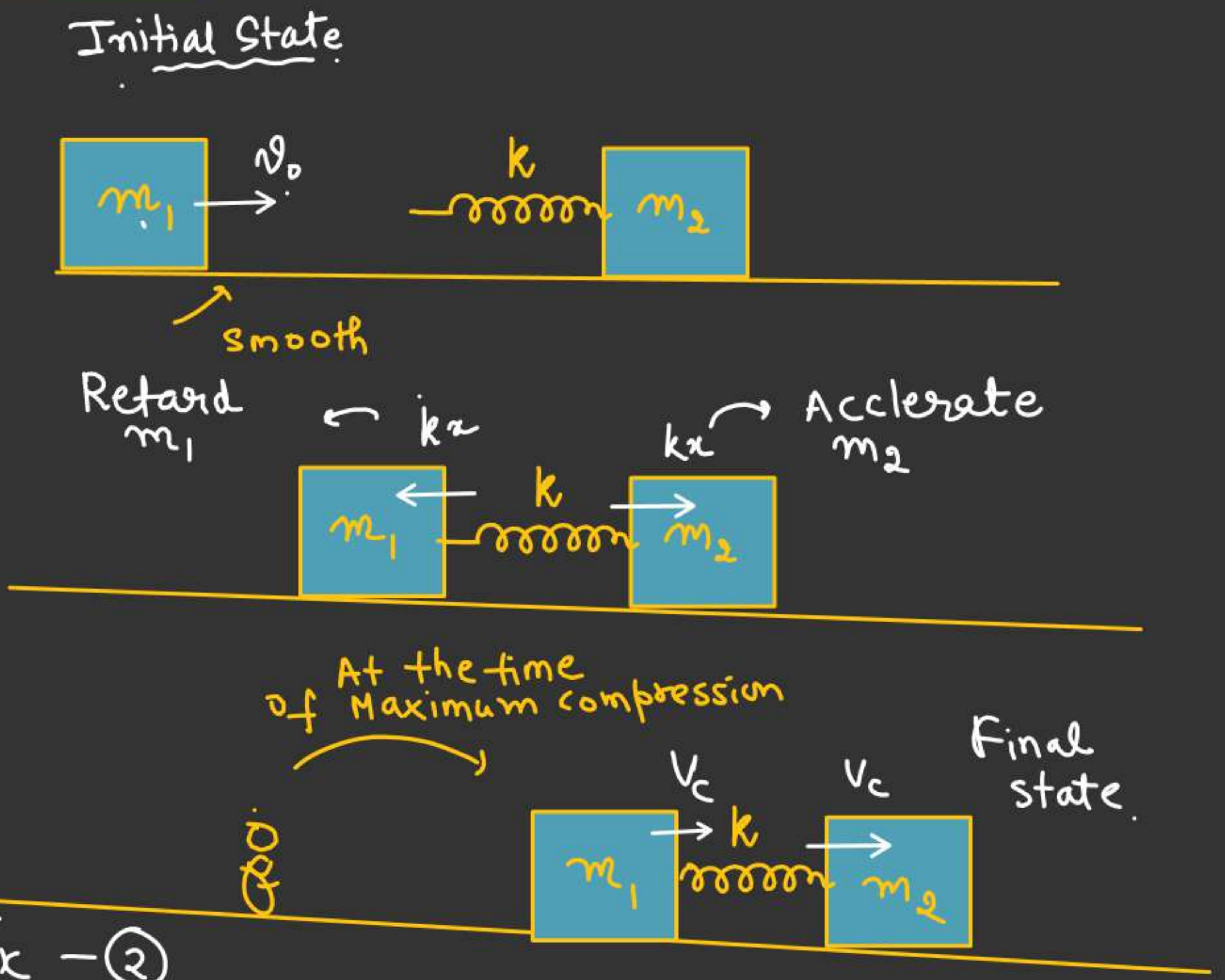
L·M·C

$$m_1 v_0 = (m_1+m_2)V_c$$

$$V_c = \left(\frac{m_1 v_0}{m_1+m_2}\right) \checkmark \quad \text{— (1)}$$

Energy Conservation

$$\frac{1}{2}m_1 v_0^2 = \frac{1}{2}(m_1+m_2)V_c^2 + \frac{1}{2}k x_{max}^2 \quad \text{— (2)}$$



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$$v_c = \frac{m_1 v_0}{m_1+m_2} \quad - (1)$$

$$\frac{1}{2} m_1 v_0^2 = \frac{1}{2}(m_1+m_2)V_c^2 + \frac{1}{2} k x_{max}^2 \quad - (2)$$

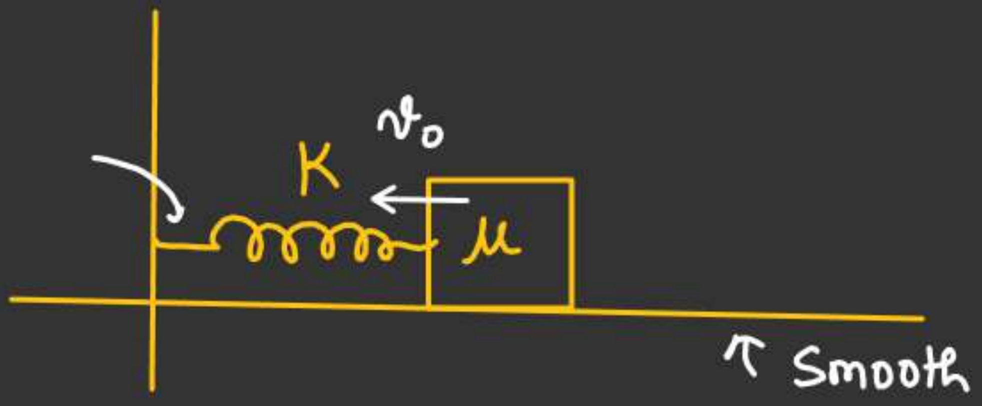
$$\frac{1}{2} K x_{max}^2 = \frac{1}{2} m_1 V_0^2 - \frac{1}{2}\cancel{(m_1+m_2)} \frac{m_1^2 v_0^2}{(m_1+m_2)^{\cancel{2}}}$$

$$\frac{1}{2} K x_{max}^2 = \frac{1}{2} m_1 V_0^2 \left[1 - \frac{m_1}{m_1+m_2}\right]$$

$$\cancel{\frac{1}{2}} K x_{max}^2 = \cancel{\frac{1}{2}}\left(\frac{m_1 m_2}{m_1+m_2}\right) V_0^2$$

$$x_{max} = \sqrt{\frac{\left(\frac{m_1 m_2}{m_1+m_2}\right) V_0^2}{K}} \Rightarrow \boxed{x_{max} = \sqrt{\frac{\mu}{K}}\, v_0}$$

μ = Reduced Mass



$$\frac{1}{2}\mu v_0^2 = \frac{1}{2} k x_{max}^2$$

$$x_{max} = \sqrt{\frac{\mu}{K}}\, v_0$$

Rest

At the time of Maximum Compression

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$$v_c = \frac{m_1 v_0}{m_1+m_2} \quad - (1)$$

$$\frac{1}{2}m_1 v_0^2 = \frac{1}{2}(m_1+m_2)V_c^2 + \frac{1}{2}k x_{max}^2 \quad - (2)$$

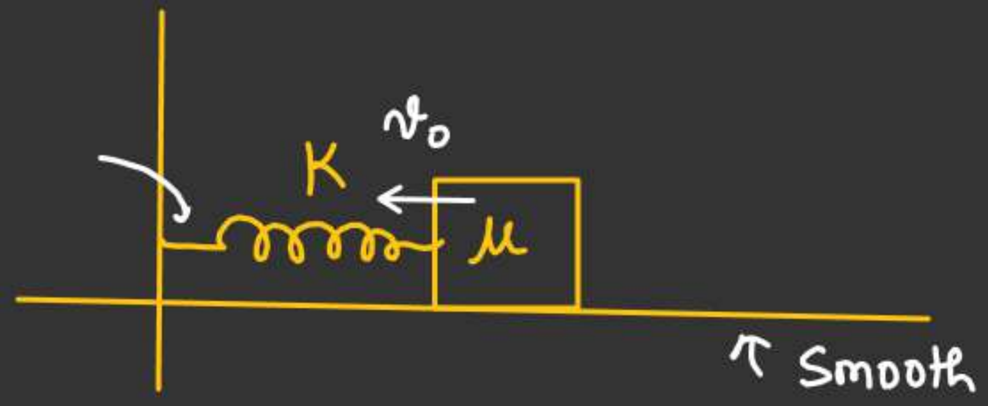
$$\frac{1}{2}K x_{max}^2 = \frac{1}{2}m_1 v_0^2 - \frac{1}{2}(m_1+\cancel{m_2})\frac{m_1^2 v_0^2}{\cancel{(m_1+m_2)^2}}$$

$$\frac{1}{2}K x_{max}^2 = \frac{1}{2}m_1 v_0^2\left[1 - \frac{m_1}{m_1+m_2}\right]$$

$$\cancel{\frac{1}{2}}K x_{max}^2 = \cancel{\frac{1}{2}}\left(\frac{m_1 m_2}{m_1+m_2}\right)v_0^2$$

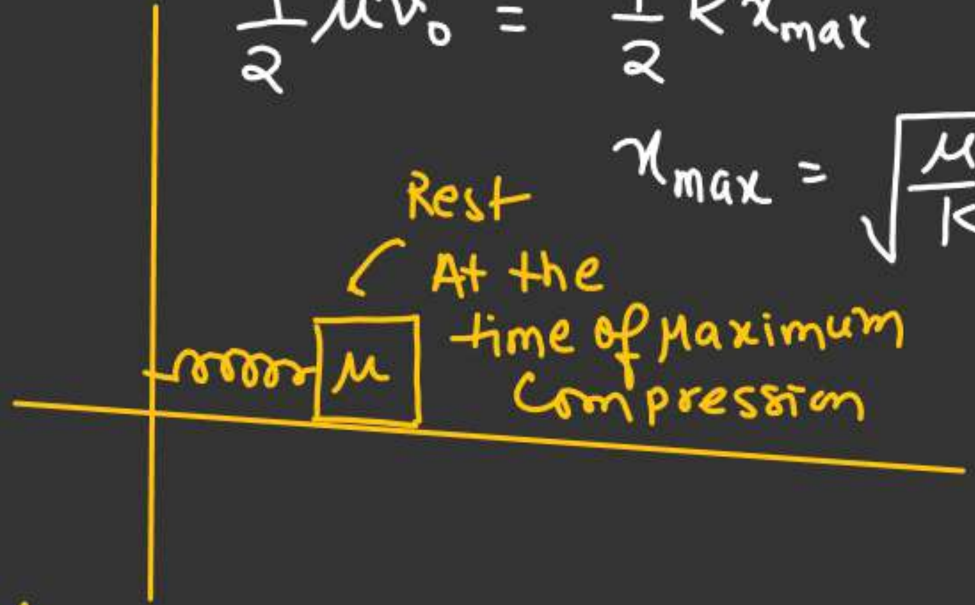
$$x_{max} = \sqrt{\frac{\left(\frac{m_1 m_2}{m_1+m_2}\right)v_0^2}{K}} \Rightarrow \boxed{x_{max} = \sqrt{\frac{\mu}{K}}\, v_0}$$

μ = Reduced Mass



$$\frac{1}{2}\mu v_0^2 = \frac{1}{2}k x_{max}^2$$

$$x_{max} = \sqrt{\frac{\mu}{K}}\, v_0$$



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Find $x_{max} = ??$

M-2 H.W Solve w.r.t Earth frame

M-1 W.r.t. COM ✓

$$\frac{1}{2}\mu v_{rel}^2 = \frac{1}{2}K x_{max}^2$$

$$\Downarrow$$

$(V_{rel})_i$:

$$\frac{1}{2}\mu (v_{rel})_i^2 + \frac{1}{2}\mu (v_{rel})_f^2 = \frac{1}{2}K x_{max}^2$$

$(v_{rel})_f \Rightarrow 0$

$$\frac{1}{2}\left(\frac{2m \cdot m}{2m+m}\right) v_0^2 = \frac{1}{2}K x_{max}^2$$

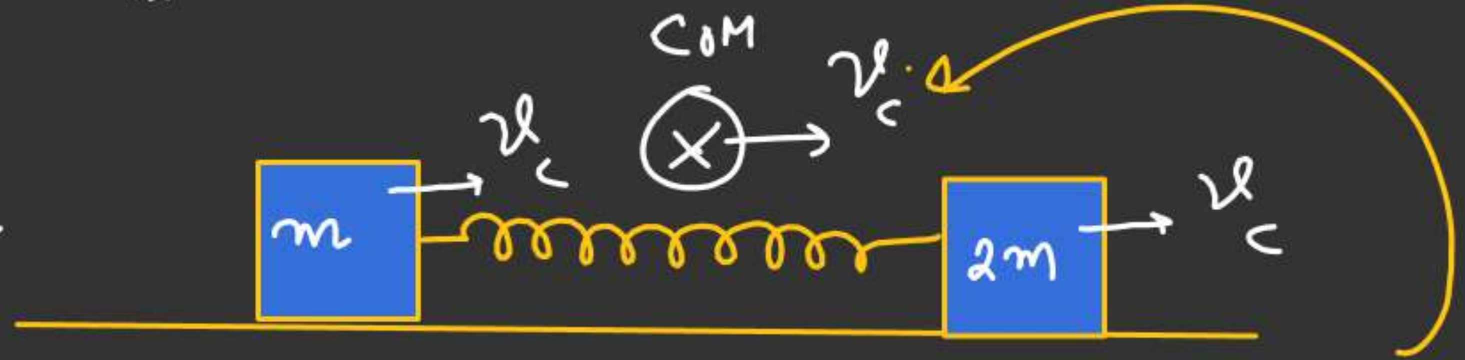
$$\frac{m}{3} v_0^2 = \frac{1}{2}K x_{max}^2$$

$$x_{max} = \left(\sqrt{\frac{2}{3}\frac{m}{K}}\right) v_0$$

At $t = 0$. Spring at its Natural length.

$\Downarrow$

At the time of Maximum elogation both move with common velocity.



$$v_{cm} = \frac{2m v_c + m v_c}{(2m+m)} = v_c$$