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$$x\tan^{-1}\sqrt{x} - \int \frac{\sqrt{x}\, dx}{2(1+x)}$$

$$x = t^2$$

$$\frac{t^2\,dt}{1+t^2}$$

$$x\ln(x^2+1) - \int \frac{2x^2\, dx}{x^2+1}$$

$$\int x \cdot \frac{x}{(1+x^2)^2}, \quad \frac{x^2}{x(1+x^2)^{1/2}}$$

I II

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$$-\frac{\ln x}{2x^2} + \int \frac{dx}{2x^3}$$

$$\int \frac{-2\sqrt{1-x}}{\sqrt{1-x}} \frac{1}{2\sqrt{x}}$$

$$\int \frac{1\, dx}{1+x^2}$$

$$\int x\sqrt{1+x^2}$$

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$$\ln\left|x+\sqrt{x^2+1}\right| + \frac{1}{\sqrt{x^2+1}} + C .$$

$$\frac{1}{2}\int x^2\,dx + \frac{1}{2}\int \underset{\mathrm{I}}{x^2} \underset{\mathrm{II}}{\cos 2x}\,dx .$$

$$\frac{(a^m b^n)^x}{\ln(a^m b^n)} + C$$

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$$\int \sqrt{x^2+a^2}\,dx = \frac{x}{2}\sqrt{x^2+a^2} + \frac{a^2}{2}\ln\left|x+\sqrt{x^2+a^2}\right| + C$$

$$\int \sqrt{x^2-a^2}\,dx = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}\ln\left|x+\sqrt{x^2-a^2}\right| + C$$

$$\int \sqrt{a^2-x^2}\,dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} + C$$

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$$I = \int \overset{\text{II}}{1}\cdot\overset{\text{I}}{\sqrt{a^2-x^2}}\,dx = x\sqrt{a^2-x^2} + \int \frac{(x^2-a^2)+a^2}{\sqrt{a^2-x^2}}\,dx$$

$$x\sqrt{a^2-x^2} - \int \sqrt{a^2-x^2}\,dx + a^2\int \frac{dx}{\sqrt{a^2-x^2}}$$

$$I = x\sqrt{a^2-x^2} - I + a^2 \sin^{-1}\left(\frac{x}{a}\right)$$

$$I = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right) + C$$

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1. $\int 1 \cdot \ln\left(x+\sqrt{x^2+a^2}\right) dx = x \ln\left(x+\sqrt{x^2+a^2}\right) - \frac{1}{2}\int \frac{2x\, dx}{\sqrt{x^2+a^2}}$

$\int \sqrt{1+t^2}\, dt$

$x\ln\left(x+\sqrt{x^2+a^2}\right) - \sqrt{x^2+a^2} + C$

$\int \sqrt{1+\tan^2 x}\, \sec^2 x\, dx$

$= \frac{\tan x}{2}\sqrt{\tan^2 x+1} + \frac{1}{2}\ln\left|\tan x+\sqrt{\tan^2 x+1}\right| + C$

2. $I = \int \sec^3 x\, dx = \int \underset{\mathrm{I}}{\sec x}\ \underset{\mathrm{II}}{\sec^2 x}\, dx = \sec x \tan x - \int \sec x \underbrace{\tan^2 x}_{\sec^2 x - 1} dx$

$I = \sec x \tan x - \underbrace{\int \sec^3 x\, dx}_{I} + \int \sec x\, dx$

$I = \frac{1}{2}\sec x \tan x + \frac{1}{2}\ln|\sec x + \tan x| + C.$

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3. $I = \int 1 \cdot \sin(\ln x)\,dx = x\sin(\ln x) - \int 1 \cdot \cos(\ln x)\,dx$

$$= x\sin(\ln x) - x\cos(\ln x) - \int \sin(\ln x)\,dx$$

$$I = x\sin(\ln x) - x\cos(\ln x) - I$$

$$I = \frac{x}{2}\left(\sin(\ln x) - \cos(\ln x)\right) + C$$

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Integrals of form

$$\int e^x\left(f(x)+f'(x)\right)dx = e^x f(x) + C$$

$$\int \left(x f'(x) + f(x)\right) dx = x f(x) + C$$

$$\int \underset{\mathrm{I}}{x}\, \underset{\mathrm{II}}{f'(x)}\, dx + \int f(x)\, dx$$

$$x f(x) - \cancel{\int 1 \cdot f(x)\, dx} + \cancel{\int f(x)\, dx}$$

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1\. $\displaystyle\int \frac{x e^x\,dx}{(1+x)^2} = \int \frac{x+1-1}{(1+x)^2} e^x\,dx = \int\Big(\underbrace{\frac{1}{1+x}}_{f(x)} + \underbrace{\frac{-1}{(1+x)^2}}_{f'(x)}\Big)e^x\,dx$

$\displaystyle = \frac{e^x}{1+x} + C.$

2\. $\displaystyle\int\Big(\underbrace{\sin(\ln x)}_{f(x)} + \underbrace{\cos(\ln x)}_{x f'(x)}\Big)dx = x\sin(\ln x) + C.$

$\ln x = t \Rightarrow x = e^t$, $dx = e^t\,dt$

$\displaystyle\int(\underbrace{\sin t}_{f(t)} + \underbrace{\cos t}_{f'(t)})e^t\,dt = e^t \sin t + C.$

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3. $\int \frac{e^{2x}(\sin 4x - 2)}{(1-\cos 4x)}dx = \frac{1}{2}\int e^{t}\left(\frac{\sin 2t}{1-\cos 2t} - \frac{2}{1-\cos 2t}\right)dt$

$2x = t$

$dx = \frac{1}{2}dt$

$\frac{1}{2}\int e^{t}(\underbrace{\cot t}_{f(t)} - \operatorname{cosec}^2 t)dt$

$= \frac{1}{2}e^{t}\cot t + C = \frac{1}{2}e^{2x}\cot 2x + C.$

4. $\int \frac{e^{\tan^{-1}x}(1+x+x^2)}{(1+x^2)}dx = \int\left(\underbrace{e^{\tan^{-1}x}}_{f(x)} + \underbrace{\frac{x}{1+x^2}e^{\tan^{-1}x}}_{x f'(x)}\right)dx$

$\tan^{-1}x = t.$

$\int e^{t}(\tan t + \sec^2 t)\,dx.$

$= x e^{\tan^{-1}x} + C.$

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5. $$\int \frac{e^x (x^2+5x+\overset{6+1}{7})\,dx}{(x+3)^2} = \int e^x \left(\underbrace{\frac{x+2}{x+3}}_{f(x)} + \underbrace{\frac{1}{(x+3)^2}}_{f'(x)} \right) dx$$

$$= e^x \left(\frac{x+2}{x+3}\right) + C$$

$$\int e^x \, \frac{(x+3)^2 - (\overset{x+3-1}{x+2})}{(x+3)^2}\, dx = \int e^x\,dx - \int e^x \left(\underbrace{\frac{1}{x+3}}_{f(x)} - \frac{1}{(x+3)^2} \right) dx$$

$$1 - \frac{1}{x+3}$$

$$= e^x - \frac{e^x}{x+3} + C$$

6. $$\int \left(\underline{\ln(\ln x)} + \frac{1}{\ln^2 x} \right) dx$$

$$= \int \left(\underbrace{\ln(\ln x)}_{f(x)} + \underbrace{\frac{1}{\ln x}}_{x f'(x)} \right) dx - \int \left(\underbrace{\frac{1}{\ln x}}_{f(x)} + \underbrace{\frac{-1}{\ln^2 x}}_{x f'(x)} \right) dx = x \ln(\ln x) - \frac{x}{\ln x} + C$$

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$$\int \ln(\ln x)\,dx + \int \frac{dx}{\ln^2 x}$$

$$= x\ln(\ln x) - \int \frac{1\cdot dx}{\ln x} + \int \frac{dx}{\ln^2 x}$$

$$= x\ln(\ln x) - \frac{x}{\ln x} - \cancel{\int \frac{dx}{\ln^2 x}} + \cancel{\int \frac{dx}{\ln^2 x}}$$

$$x\ln(\ln x) - \cancel{\int \frac{1}{\ln x}dx} - \frac{x}{\ln x} + \cancel{\int \frac{dx}{\ln x}}$$

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$$I = \int \underset{\mathrm{II}}{e^{ax}} \underset{\mathrm{I}}{\sin(bx+c)}\, dx = \frac{e^{ax}}{a}\sin(bx+c) - \frac{b}{a}\int e^{ax}\cos(bx+c)\,dx.$$

$$= \frac{e^{ax}}{a}\sin(bx+c) - \frac{b}{a^2}e^{ax}\cos(bx+c) - \frac{b^2}{a^2}\int e^{ax}\sin(bx+c)\,dx$$

$$I = \frac{e^{ax}}{(a^2+b^2)}\left(a\sin(bx+c) - b\cos(bx+c)\right) + C$$

$$\int e^{ax}\sin(bx+c)\,dx = e^{ax}\left(A\sin(bx+c) + B\cos(bx+c)\right) + C$$

$$\frac{d}{dx}\left[e^{ax}\left(A\sin(bx+c) + B\cos(bx+c)\right)\right] = e^{ax}\sin(bx+c)$$

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$\boxed{\text{II}}$

$$\int e^{ax}\sin(bx+c)\,dx = \frac{e^{ax}}{a^2+b^2}\Big(-b\cos(bx+c)+a\sin(bx+c)\Big)$$

$$\int e^{ax}\, e^{i(bx+c)}\,dx = \int e^{ax}\Big(\cos(bx+c)+i\sin(bx+c)\Big)dx$$

$$= \int e^{ax}\cos(bx+c)\,dx + i\int e^{ax}\sin(bx+c)\,dx$$

$$\int e^{x}\sin x\,dx$$

$$\boxed{\int e^{x}e^{ix}\,dx}$$

$$\alpha+i\beta$$

$$\int e^{(a+ib)x+ic}\,dx = e^{ic}\int e^{(a+ib)x}\,dx = e^{ic}\,\frac{e^{(a+ib)x}}{(a+ib)} + C$$

$$= \frac{e^{ax}\,e^{i(bx+c)}}{(a+ib)} = \frac{e^{ax}(a-ib)\Big(\cos(bx+c)+i\sin(bx+c)\Big)}{a^2+b^2}$$

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$\boxed{\begin{array}{c} 1856 - 1868 \\ 1951 - 2011 \end{array}}$