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$$1+\cos^2\alpha-\underline{\sin^2(\alpha+120)}+\underline{\cos^2(\alpha-120)}$$

$$=1+\cos^2\alpha+\cos 2\alpha\cos\underset{\substack{\parallel\\180+60}}{240}$$

$$=1+\cos^2\alpha-\frac{1}{2}\cos 2\alpha$$

$$=1+\cos^2\alpha-\frac{1}{2}(2\cos^2\alpha-1)$$

$$2\left(\cos^4\frac{\pi}{8}+\cancel{\cos^4\frac{3\pi}{8}}^{\sin^4\frac{\pi}{8}}\right)=2\left(1-\frac{1}{2}\sin^2\frac{\pi}{4}\right)$$

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$$\left(\frac{\tan 3A + \tan A}{1 - \tan 3A \tan A} + \frac{\tan 3A - \tan A}{1 + \tan 3A \tan A}\right)(1 - \tan^2 3A \tan^2 A)$$

$$2 \tan 3A + 2\tan^2 A \tan 3A$$

$$= 2 \tan 3A \sec^2 A$$

$$22 \quad \boxed{\frac{2\tan\frac{\alpha}{2}}{\cos^2\frac{\alpha}{2}}} \cos\frac{\alpha}{2} = 2\sin\alpha \sec^2\frac{\alpha}{2}$$

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23.

2 cos

$= 2\cos\frac{A}{2} = +\sqrt{1-\sin A} - \sqrt{1+\sin A}$ $\boxed{\frac{A}{2} = 278^\circ}$

$\pm\left|\underbrace{\cos\frac{A}{2}-\sin\frac{A}{2}}_{>0}\right| \pm \left|\underbrace{\cos\frac{A}{2}+\sin\frac{A}{2}}_{<0}\right|$

cos 7

$2\cos\frac{A}{2} = \pm\left(\cos\frac{A}{2}-\sin\frac{A}{2}\right) \pm \left(-\cos\frac{A}{2}-\sin\frac{A}{2}\right)$

$= \left(\cos\frac{A}{2}-\sin\frac{A}{2}\right) - \left(-\cos\frac{A}{2}-\sin\frac{A}{2}\right)$

$\cos 278^\circ + \sin 278^\circ$

$= \sin 8^\circ - \cos 8^\circ < 0$

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3.

$$\cos 12^\circ + \underline{\cos 60^\circ + \cos 84^\circ}$$

$$= \cos 12^\circ + 2\cos 72^\circ \cos 12^\circ$$

$$= \cos 12^\circ \left(1 + 2\sin 18^\circ\right)$$

$$= \cos 12^\circ \left(1 + \frac{\sqrt{5}-1}{2}\right)$$

$$= \cos 12^\circ \left(\frac{\sqrt{5}+1}{2}\right) = 2\cos 12^\circ \cos 36^\circ = \cos 24^\circ + \cos 48^\circ$$

5.

$$\sin\frac{\pi}{10}+\sin\frac{13\pi}{10} = 2\sin\left(\frac{7\pi}{10}\right)\cos\frac{6\pi}{10}$$

$$= 2\sin\frac{3\pi}{10}\cos\frac{3\pi}{5}$$

$54^\circ$ $\quad$ $108^\circ = 90+18$

$$= -2\cos 36^\circ \sin 18^\circ$$

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7.

$$\frac{\tan 6^\circ \tan 66^\circ \tan 54^\circ}{\tan 54^\circ} \quad \frac{\tan 18^\circ \tan 42^\circ \tan 78^\circ}{\tan 18^\circ}$$

$$= \frac{\tan 18^\circ}{\tan 54^\circ} \quad \frac{\tan 54^\circ}{\tan 18^\circ} = 1$$

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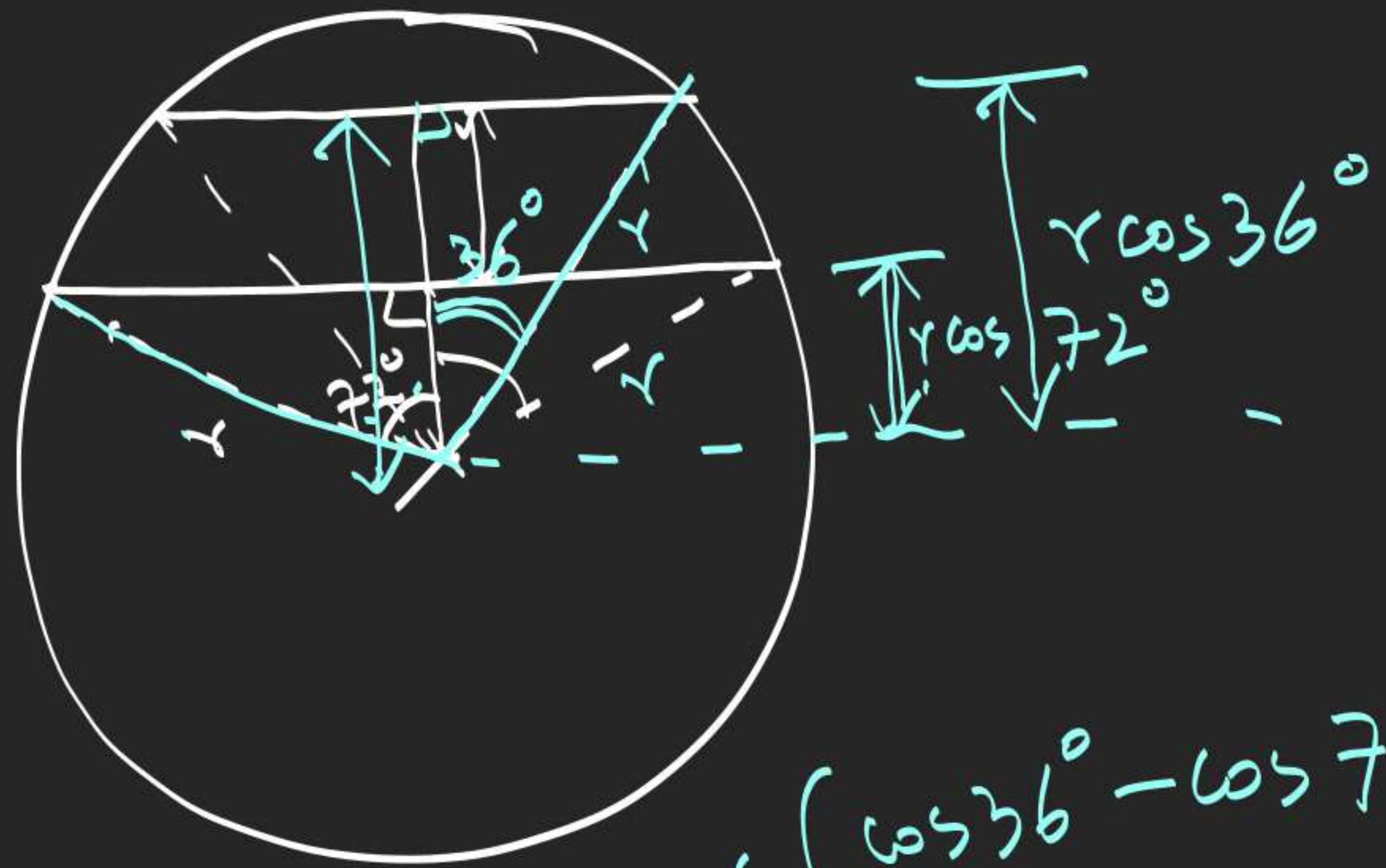
8.

$$\cos\frac{2\pi}{5}\cos\frac{\pi}{5}\cos\frac{\pi}{3}\left(\cos\frac{\pi}{15}\cos\frac{2\pi}{15}\cos\frac{4\pi}{15}\cos\frac{8\pi}{15}\right)$$

$\downarrow$ $\sin 18^\circ$ $\quad$ $\downarrow$ $\cos 36^\circ$

$$\frac{(\sqrt{5}-1)}{4}\frac{(\sqrt{5}+1)}{4}\frac{1}{2}\left(\frac{-\sin\frac{16\pi}{15}}{16\sin\frac{\pi}{15}}\right)$$




$$d = r(\cos 36^\circ - \cos 72^\circ)$$
$$= r\left(\frac{\sqrt{5}+1}{4} - \frac{\sqrt{5}-1}{4}\right)$$

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# Conditional Identity

If $A+B+C=\pi$, P.T.

$$\sin 2A+\sin 2B+\sin 2C=4\sin A\sin B\sin C$$

$$2\sin(\underbrace{A+B}_{\pi-C})\cos(A-B)+\sin 2C$$

$$=2\sin C\cos(A-B)+2\sin C\cos C$$

$$=2\sin C\left(\cos(A-B)+\cos \underset{\pi-(A+B)}{\underset{\downarrow}{C}}\right)=2\sin C\left(\cos(A-B)-\cos(A+B)\right)$$

$$=2\sin C\,(2\sin A\sin B)$$

$$\tan(A+(B+C)) = \frac{\tan A + \tan(B+C)}{1 - \tan A \tan(B+C)} = \frac{\tan A + \dfrac{\tan B + \tan C}{1 - \tan B \tan C}}{1 - \dfrac{\tan A(\tan B + \tan C)}{(1 - \tan B \tan C)}}$$

$$= \frac{\tan A + \tan B + \tan C - \tan A \tan B \tan C}{1 - (\tan A \tan B + \tan B \tan C + \tan C \tan A)}$$

$$\tan(A+B+C+D) = \frac{\sum \tan A - \sum \tan A \tan B \tan C}{1 - \sum \tan A \tan B + \tan A \tan B \tan C \tan D}$$

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$$\boxed{\tan(\theta_1+\theta_2+\theta_3+\cdots+\theta_n)=\frac{S_1-S_3+S_5-S_7+\cdots}{1-S_2+S_4-S_6+\cdots}}$$

$$S_1=\sum\tan\theta_1$$

$$S_2=\sum\tan\theta_1\tan\theta_2$$

$$S_3=\sum\tan\theta_1\tan\theta_2\tan\theta_3$$

$$\tan(A+B)=\frac{S_1}{1-S_2}$$

$$\tan(A+B+C)=\frac{S_1-S_3}{1-S_2}$$

$$\tan(A+B+C+D)=\frac{S_1-S_3}{1-S_2+S_4}$$

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If $A+B+C=\pi$, then P.T.

2. $\cos A+\cos B-\cos C = 4\cos\frac{A}{2}\cos\frac{B}{2}\sin\frac{C}{2}-1$

$= 2\cos\frac{A+B}{2}\cos\frac{A-B}{2} - \cos C = 2\sin\frac{C}{2}\cos\frac{A-B}{2} - \left(1-2\sin^2\frac{C}{2}\right)$

$\frac{\pi}{2}-\frac{C}{2}$

$= 2\sin\frac{C}{2}\left(\cos\frac{A-B}{2}+\sin\frac{C}{2}\right)-1$

$\frac{\pi}{2}-\frac{A+B}{2}$

$= 2\sin\frac{C}{2}\left(\cos\frac{A-B}{2}+\cos\frac{A+B}{2}\right)-1 = 4\sin\frac{C}{2}\cos\frac{A}{2}\cos\frac{B}{2}-1$

3. $\sin^2A+\sin^2B+\sin^2C = 2+2\cos A\cos B\cos C$.

$1-\left(\cos^2B-\sin^2A\right)+\sin^2C = 1-\cos(B-A)\cos(B+A)+\sin^2C$

$\pi-C$

$= 1+\cos(B-A)\cos C+1-\cos^2C$

$= 2+\left(\cos(B-A)-\cos C\right)\cos C$

$\pi-(A+B)$

$2+2\cos A\cos B\cos C = 2+\left(\cos(B-A)+\cos(A+B)\right)\cos C$

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4. $$\sin\frac{A}{2}+\sin\frac{B}{2}+\sin\frac{C}{2}-1=4\sin\left(\frac{\pi-A}{4}\right)\sin\left(\frac{\pi-B}{4}\right)\sin\left(\frac{\pi-C}{4}\right)$$

$$2\sin\left(\frac{A+B}{4}\right)\cos\frac{A-B}{4}+\sin\frac{C}{2}-1=2\sin\left(\frac{\pi-C}{4}\right)\cos\frac{A-B}{4}+\cos\left(\frac{\pi}{2}-\frac{C}{2}\right)-1$$

$$=2\sin\left(\frac{\pi-C}{4}\right)\cos\frac{A-B}{4}-2\sin^2\left(\frac{\pi-C}{4}\right)$$

$$=2\sin\left(\frac{\pi-C}{4}\right)\left(\cos\frac{A-B}{4}-\sin\frac{\pi-C}{4}\right)=2\sin\left(\frac{\pi-C}{4}\right)\left(\cos\frac{A-B}{4}-\sin\frac{A+B}{4}\right)$$

$$=2\sin\left(\frac{\pi-C}{4}\right)\left(\cos\frac{A-B}{4}-\cos\left(\frac{\pi}{2}-\frac{A+B}{4}\right)\right)$$

$$=2\sin\left(\frac{\pi-C}{4}\right)\left(2\sin\left(\frac{\frac{\pi}{2}-\frac{A}{2}}{2}\right)\sin\left(\frac{\frac{\pi}{2}-\frac{B}{2}}{2}\right)\right)$$

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If $A+B+C=\pi$,

- $\tan A+\tan B+\tan C=\tan A\tan B\tan C$
- $\cot A\cot B+\cot B\cot C+\cot C\cot A=1$
- $\tan\frac{A}{2}\tan\frac{B}{2}+\tan\frac{B}{2}\tan\frac{C}{2}+\tan\frac{C}{2}\tan\frac{A}{2}=1$
- $\cot\frac{A}{2}+\cot\frac{B}{2}+\cot\frac{C}{2}=\cot\frac{A}{2}\cot\frac{B}{2}\cot\frac{C}{2}$.

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$A+B+C=\pi$

$\sum \tan A = \prod \tan A$

$\boxed{\text{Ex-20} \quad \text{Q } 1-19}$

$\tan(A+B+C) = \tan\pi = 0.$

$\frac{S_1 - S_3}{1 - S_2} = 0 \Rightarrow S_1 = S_3$

$\tan A + \tan B + \tan C = \tan A \tan B \tan C.$

$\cot B \cot C + \cot C \cot A + \cot A \cot B = (\tan A + \tan B + \tan C) / \tan A \tan B \tan C = 1$