

$$\underline{9.} \quad (ii) \quad \cos\left(\frac{\sin^{-1} x}{y}\right) = 0$$

$$\frac{\sin^{-1} x}{y} = (2n+1)\frac{\pi}{2} \quad n \in \mathbb{I}$$

161.

$$y=1 \quad \sin^{-1} x = (2n+1)\frac{\pi}{2} = -\frac{\pi}{2}, \frac{\pi}{2}$$

$$x = -1, 1$$

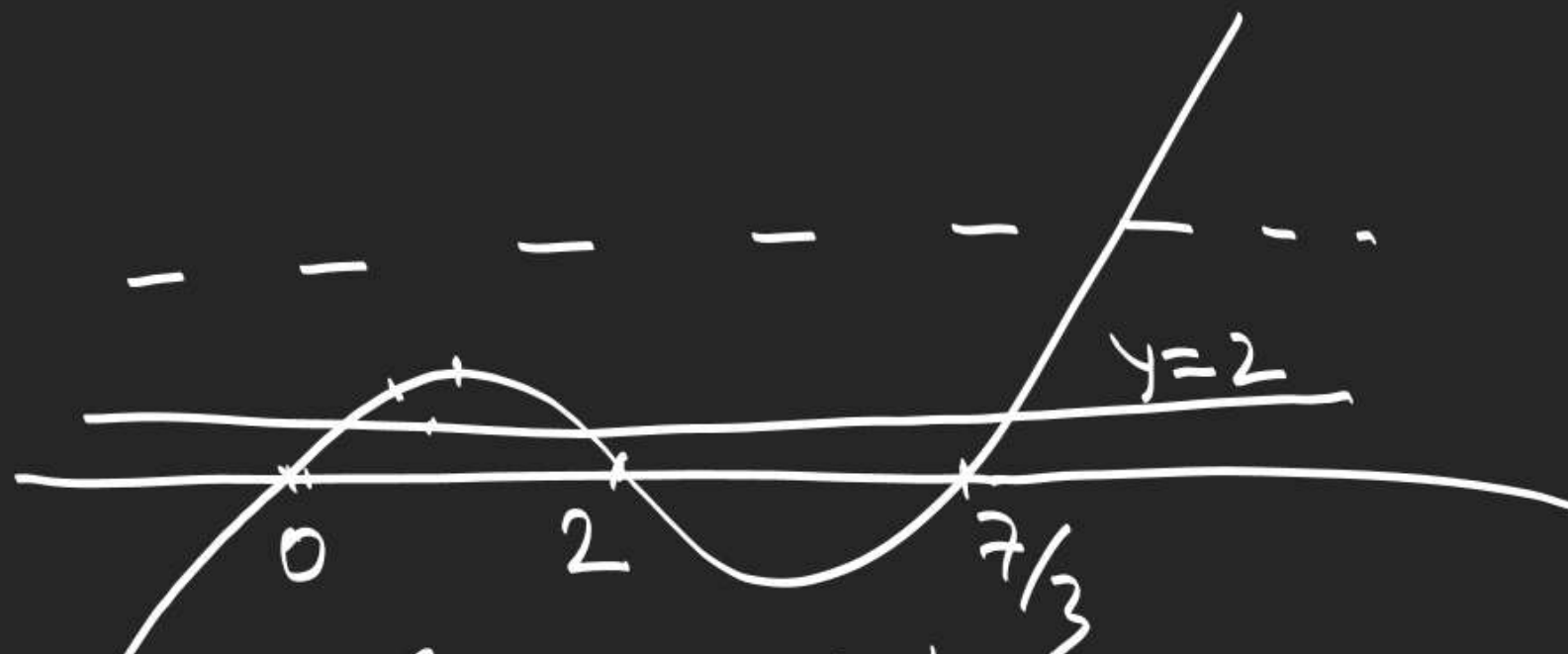
$$y = -1 \quad -\sin^{-1} x = (2n+1)\frac{\pi}{2} \\ = -\frac{\pi}{2}, \frac{\pi}{2}$$

$$x = -1, 1$$

$$y=2 \quad \frac{\sin^{-1} x}{2} = (2n+1)\frac{\pi}{2}$$

$$(x, y) = (-1, 1), (1, 1), (-1, -1), (1, -1)$$

11.



$$f(x) = x(x-2)(3x-7) \quad \checkmark$$

$$f(x) = 2$$

$$f(x) = 1(-1)(-4) = 4 \quad \checkmark$$

$$r, s, t > 0$$

$$x(x-2)(3x-7) - 2 = 0$$

$$y=2 \quad \theta_1 + \theta_2 + \theta_3 \in (0, \frac{3\pi}{2})$$

$$\tan(\tan^{-1}r + \tan^{-1}s + \tan^{-1}t) = \frac{s_1 - s_3}{1 - s_2}$$

$$\boxed{\tan^{-1}r + \tan^{-1}s + \tan^{-1}t = \frac{3\pi}{4}}$$

$$= \frac{\left(\sum r\right) - rst}{1 - \sum rs} = -1$$

13. $2(\cos^{-1}x)^2 = a \cos^{-1}x + a^2$

$$2t^2 - at - a^2 = 0 \quad -2at + at \quad \frac{\pi}{2} < 2 < \frac{2\frac{\pi}{2} + 4}{1+x^2} = 2 + \frac{2}{1+x^2} \leq 4 < \frac{3\pi}{2}$$

$$(2t+a)(t-a) = 0$$

$$\cos^{-1}x = -\frac{a}{2}, \text{ or } \cos^{-1}x = a$$

$$-\frac{a}{2} \in (0, \pi] \text{ or } a \in (0, \pi]$$

$$a \in [-2\pi, 0) \cup (0, \pi]$$

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots \infty, \quad x \in \mathbb{R}.$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots \infty, \quad x \in (-1, 1]$$

$$\ln(1-x) = -x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} - \dots \infty, \quad x \in [-1, 1)$$

$$\sin x = \frac{x}{1!} - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots \rightarrow \infty, \quad x \in \mathbb{R}.$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots \infty, \quad x \in \mathbb{R}.$$

$$\tan x = x + \frac{x^3}{3} + \frac{2}{15}x^5 + \dots \infty$$

Binomial theorem for any index

$n \in \mathbb{I}$,
 $\mathbb{Q} - \{\mathbb{I}\}$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \dots \infty$$

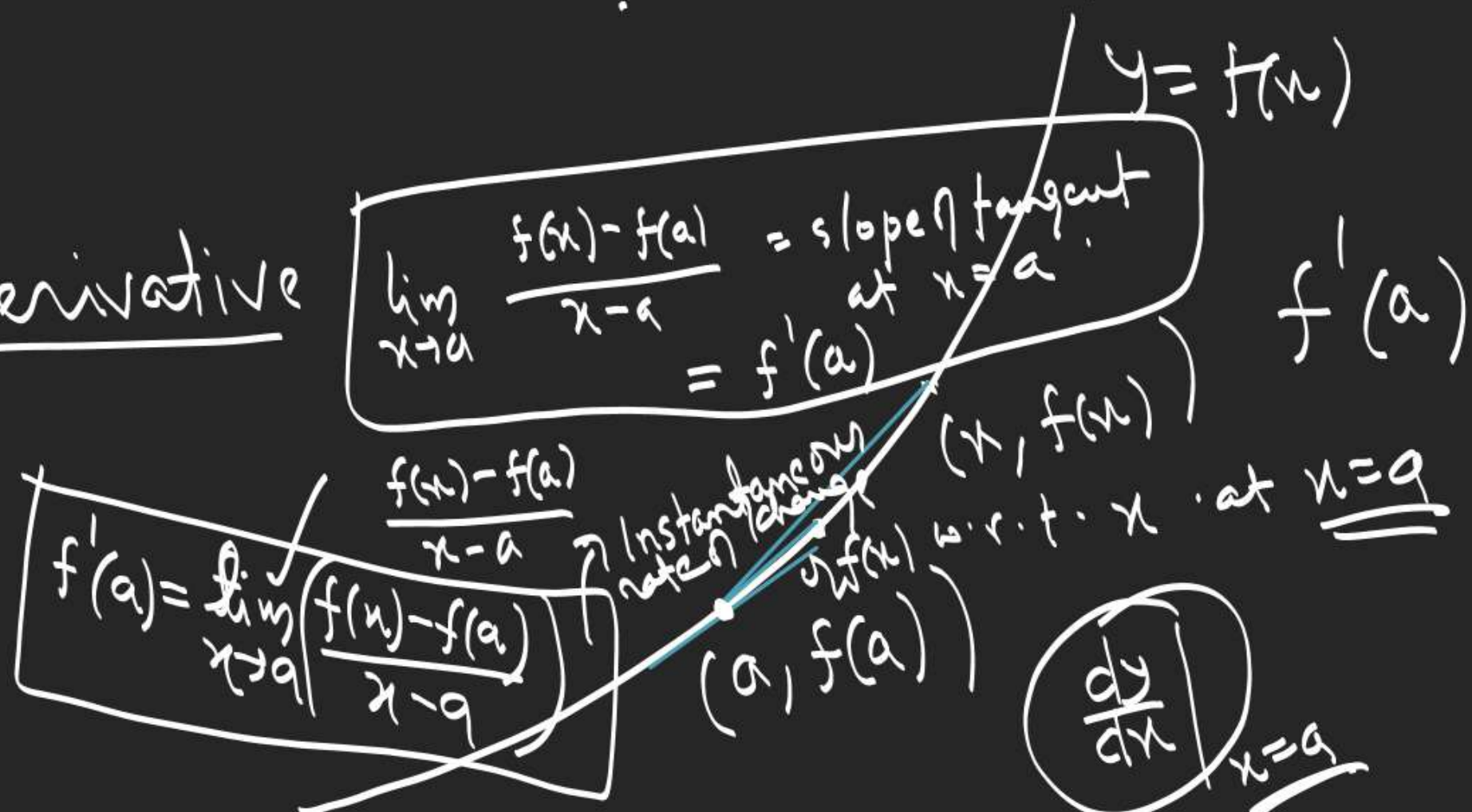
Derivative

$$\lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a} = \text{slope of tangent at } x = a = f'(a)$$

$$f'(a) = \lim_{x \rightarrow a} \left(\frac{f(x) - f(a)}{x - a} \right)$$

Instantaneous rate of change of $f(x)$ w.r.t. x at $x = a$

$$\left. \frac{dy}{dx} \right|_{x=a}$$



$$\begin{aligned}
 \underline{1.} \quad \lim_{x \rightarrow \frac{\pi}{4}} \left(\frac{1 - \cot^3 x}{2 - \cot x - \cot^3 x} \right) &= \lim_{x \rightarrow \frac{\pi}{4}} \frac{(1 - \cancel{\cot x})(1 + \cot^2 x + \cot x)}{(1 - \cancel{\cot x})(\cot^2 x + \cot x + 2)} \\
 &= \frac{1 + 1 + 1}{1 + 1 + 2} = \frac{3}{4}
 \end{aligned}$$

$$\cot x = t$$

$$\begin{aligned}
 \underline{2.} \quad \lim_{x \rightarrow 2} \frac{2^x + 2^{3-x} - 6}{\sqrt{2^{-x}} - 2^{1-x}} \\
 \lim_{x \rightarrow 2} \frac{2^{2x} + 8 - 6 \cdot 2^x}{2^{x/2} - 2}
 \end{aligned}$$

$$\begin{aligned}
 \lim_{t \rightarrow 1} \frac{1 - t^3}{2 - t - t^3} \\
 = \lim_{x \rightarrow 2} \frac{(2^x - 4)(2^x - 2)}{2^{x/2} - 2} = \lim_{x \rightarrow 2} \frac{(2^{x/2} + 2)(2^x - 2)}{2^{x/2} - 2} \\
 = 4 \times 2 = \boxed{8}
 \end{aligned}$$

$$\underline{3.} \quad \lim_{x \rightarrow 9} \left(\frac{3 - \sqrt{x}}{4 - \sqrt{2x-2}} \right)$$

$$\lim_{x \rightarrow 9} \frac{(9-x)(4+\sqrt{2x-2})}{(3+\sqrt{x})(16-(2x-2))}$$

$$= \lim_{x \rightarrow 9} \frac{4 + \sqrt{2x-2}}{2(3+\sqrt{x})} = \frac{8}{12} = \frac{2}{3}.$$

$$4. \lim_{n \rightarrow \infty} \left(\frac{\sqrt{n^3 - 2n^2 + 1} + \sqrt[3]{n^4 + 1}}{\sqrt[4]{n^6 + 6n^5 + 2} - \sqrt[5]{n^7 + 3n^3 + 1}} \right)$$

$$\lim_{n \rightarrow \infty} \left(\frac{n^{3/2} \sqrt{1 - \frac{2}{n} + \frac{1}{n^3}} + n^{4/3} \sqrt[3]{1 + \frac{1}{n^4}}}{n^{3/2} \sqrt[4]{1 + \frac{6}{n} + \frac{2}{n^6}} - n^{7/5} \sqrt[5]{1 + \frac{3}{n^4} + \frac{1}{n^7}}} \right) = \lim_{n \rightarrow \infty} \frac{\cancel{n^{3/2}} \left(\sqrt{1 - \frac{2}{n} + \frac{1}{n^3}} + \frac{1}{n^{1/6}} \sqrt[3]{1 + \frac{1}{n^4}} \right)}{\cancel{n^{3/2}} \left(\sqrt[4]{1 + \frac{6}{n} + \frac{2}{n^6}} - \frac{1}{n^{1/10}} \sqrt[5]{1 + \frac{3}{n^4} + \frac{1}{n^7}} \right)}$$

$$= \frac{1 + 0}{1 - 0} = 1$$

$$\begin{aligned} \underline{5.} \quad \lim_{x \rightarrow -\infty} \frac{\sqrt{x^2+1}}{(3x-6)} &= \lim_{x \rightarrow -\infty} \frac{(|x|) \sqrt{1+\frac{1}{x^2}}}{x \left(3 - \frac{6}{x}\right)} \\ &= -\frac{1}{3} \end{aligned}$$

$$\sqrt{x^2} = |x|$$

$$\underline{6.} \quad \lim_{x \rightarrow \pm \infty} \left(\sqrt{x^2 - 2x - 1} - \sqrt{x^2 - 7x - 3} \right)$$

$$= \lim_{x \rightarrow \pm \infty} \frac{5x + 2}{\sqrt{x^2 - 2x - 1} + \sqrt{x^2 - 7x - 3}}$$

$$= \lim_{x \rightarrow \pm \infty} \frac{\overbrace{x}^{\text{}} \left(5 + \frac{2}{x} \right)}{\underbrace{(|x|)}_{\text{}} \left(\sqrt{1 - \frac{2}{x} - \frac{1}{x^2}} + \sqrt{1 - \frac{7}{x} - \frac{3}{x^2}} \right)}$$

$$\left. \begin{array}{l} \xrightarrow{x \rightarrow -\infty, l = -\frac{5}{2}} \\ \xrightarrow{x \rightarrow \infty, l = \frac{5}{2}} \end{array} \right\}$$

$$\begin{aligned}
 \underline{7.} \quad \lim_{x \rightarrow \infty} \left(\sqrt{4x^2 + x} - \sqrt{\frac{4x^3}{x+2}} \right) &= \lim_{x \rightarrow \infty} \left(\frac{\sqrt{4x^3 + 9x^2 + 2x} - \sqrt{4x^3}}{\sqrt{x+2}} \right) \\
 &= \lim_{x \rightarrow \infty} \frac{(9x^2 + 2x)}{\sqrt{x+2} \left(\sqrt{4x^3 + 9x^2 + 2x} + \sqrt{4x^3} \right)} = x^2 \left(9 + \frac{2}{x} \right)
 \end{aligned}$$

$$\underline{8.} \quad \lim_{n \rightarrow \infty} \left(\frac{1^2 + 2^2 + 3^2 + \dots + n^2}{n^3} \right) = \lim_{n \rightarrow \infty} \frac{x^2 \left(9 + \frac{2}{x} \right)}{\cancel{x} \sqrt{1 + \frac{2}{x}} \left(\sqrt{4 + \frac{9}{x} + \frac{2}{x^2}} + \sqrt{4} \right)}$$

$$\lim_{n \rightarrow \infty} \frac{n(n+1)(2n+1)}{6n^3} = \frac{2}{3} = \frac{1}{3} = \boxed{\frac{9}{4}}$$

$$9. \lim_{x \rightarrow \infty} \left(\sqrt[3]{x^3 + 3x^2} - \sqrt{x^2 - 2x} \right)$$

$$\lim_{x \rightarrow \infty} x \left(\left(1 + \frac{3}{x} \right)^{\frac{1}{3}} - \left(1 - \frac{2}{x} \right)^{\frac{1}{2}} \right)$$

G.N. Berman

$$\frac{245 - 270}{2x - 4} \rightarrow \frac{1 - 10}{1 - 10}$$

$$\lim_{x \rightarrow \infty} x \left[\left(1 + \frac{1}{3} \left(\frac{3}{x} \right) + \frac{\frac{1}{3} \left(\frac{1}{3} - 1 \right) \left(\frac{3}{x} \right)^2}{2!} + \dots \infty \right) - \left(1 + \frac{1}{2} \left(\frac{-2}{x} \right) + \frac{\frac{1}{2} \left(\frac{1}{2} - 1 \right) \left(\frac{-2}{x} \right)^2}{2!} + \dots \infty \right) \right]$$

$$\lim_{x \rightarrow \infty} x \left(\frac{2}{x} + \frac{1}{x^2} (\dots) \right) = \lim_{x \rightarrow \infty} \left(2 + \frac{1}{x} (\dots) \right) = 2$$