

# Fundamentals of Mathematics

$$[x] = 5$$

$$x \in [5, 6)$$

$$[x] = -7$$

$$x \in [-7, -6)$$

Q [log<sub>3</sub>x] = 5 then x ∈ ?

$$\log_3 x \in [5, 6)$$

$$5 \leq \log_3 x < 6$$

$$3^5 \leq x < 3^6$$

$$x \in [3^5, 3^6)$$

Q No of Integer from

$$[7, 12) ?$$

$$= 12 - 7 \text{ Int.}$$

$$= 5 \text{ Int}$$



# Fundamentals of Mathematics

Q  $(81, 243)$  No of Int?

$$243 - 81 = 162 \text{ Int.}$$

$$\begin{array}{r} 3 \\ 4771 \\ \hline 23855 \\ \hline 5 \end{array}$$

**Use of characteristic**

- ↳ Mantissa is basically 0
- ↳ to find No. of digits before Decimal & No of zeros After Decimal

Q No of Digits in  $2^{100}$ ?

take  $\log_{10}$

$$\log_{10} 2^{100}$$

$$= 100 \times \log_{10} 2$$

$$= 100 \times 0.3010$$

$$= 30.10 \rightarrow h=30$$

$$\text{Mant}=.10$$

$$\text{No of Digits} = 1(h+1)$$

$$= 1(30+1) = 31$$

$$= 31$$

Q No of digits in  $3^{50}$ ?

$$\log_{10} 3^{50}$$

$$50 \log_{10} 3$$

$$50 \times 0.4771$$

$$= 23.855 \rightarrow \begin{cases} h=23 \\ \text{Mant} = .855 \end{cases}$$

$$\text{No of Digits} = |23+1|$$

$$= |24|$$

$$= 24$$

# Fundamentals of Mathematics

Q) No of digits in  $6^{100}$ ?

$$\log_{10} 6^{100}$$

$$100 \log_{10} 6$$

$$100 \times \log_{10}(2 \times 3)$$

$$100 \times (\log_{10} 2 + \log_{10} 3)$$

$$100 \times (0.3010 + 0.4771)$$

$$100 \times (0.7781)$$

$$= 77.81 - \begin{matrix} \text{Ch: 71} \\ \text{Mant: .81} \end{matrix}$$

$$\text{No of digit: } |77| + 1 = 78$$

## Fractional Part fxn.

① It is Rep. by  $f(x) = \{x\}$

② It gives fractional value of No.

$$\{6.22\} = .22$$

$$\{4.031\} = .031$$

$$\{4\} = 0$$

$$\{-17\} = 0$$

$$\{\text{Integer}\} = 0$$

$$\{-6.22\} = 1 - .22 = .78$$

$$\{-4.013\} = 1 - .013$$

$$\{-6.29\} = 1 - .29$$

$$= .71$$

# Fundamentals of Mathematics

$$\left\{ -1.12 \right\} = 1 - .12 \\ - .88$$

$$\left\{ -12 \right\} = 0$$

$$\left\{ 4.28 \right\} = .28$$

No. of cyphers

No of zero after decimal  
before any significant digit  
is always  $= \lceil ch_i + 1 \rceil$

Q No of cyphers in  $2^{-100}$

$$\log_{10} 2^{-100}$$

$$-100 \times \log_{10} 2$$

$$-100 \times -3.010$$

$$= -30.10$$

$$(h_a = \left[ -30.10 \right] > -31) \\ \left[ -30.10 \right]$$



$$\text{No of zeros} = |-31| + 1 \\ = |-30| = 30$$

Q No of zeros in  $\left(\frac{2}{3}\right)^{100}$  ?

$$\log_{10} \left(\frac{2}{3}\right)^{100}$$

$$= 100 \times \log \frac{2}{3}$$

$$= 100 \left( \log_{10} 2 - \log_{10} 3 \right)$$

$$= 100 \left( .3010 - .4771 \right)$$

$$= 100 \times (-.1761)$$

$$= -17.61$$

$$ch = \left[ -17.61 \right] = -18$$

$$\text{No of zeros} = |-18| + 1 \geq 17$$

# Fundamentals of Mathematics

Q Given that  $\log_3 N = \alpha + \beta$  where  $\beta \in [0, 1)$  ① Smj Jang  
Ki  $\beta$  = Mantissa.

Now find No of Integers in  $N$  if  $\alpha = 4$  ② yha  $\alpha$  = characteristic.

$$\alpha = 4 \Rightarrow \text{char} = 4$$

$$[\log_3 N] = 4$$

$$\log_3 N \in [4, 5)$$

$$4 \leq \log_3 N < 5$$

$$3^4 \leq N < 3^5$$

$$81 \leq N < 243$$

No of Integer in  $N$

$$= 243 - 81$$

$$= \underline{\underline{162}}$$

# Fundamentals of Mathematics

Q Given that  $\log_5 N = \alpha + \beta$ ;  $\beta \in [0,1)$

Now find the No of Integral values of  $N$

$$\text{if } \alpha = 3$$

$\alpha = 3$  - characteristic

$$[\log_5 N] = 3$$

$$\log_5 N \in [3,4)$$

$$3 \leq \log_5 N < 4$$

$$5^3 < N < 5^4$$

$$125 \leq N < 625 \Rightarrow N: 625 - 125 = 500 \text{ Int}$$

# Fundamentals of Mathematics

## Modulus fn / Absolute Value fn.

①  $f(x) = |x|$  is Modulus fn.

②  $| -2 | = -(-2) = 2$

$$|x| = \begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases}$$

$$|2x-1| = \begin{cases} (2x-1) & 2x-1 \geq 0 \\ -(2x-1) & 2x-1 < 0 \end{cases}$$

Aur Better

$$|2x-1| = \begin{cases} (2x-1) & x \geq \frac{1}{2} \\ -(2x-1) & x < \frac{1}{2} \end{cases}$$

# Fundamentals of Mathematics

$$|7x-4| = \begin{cases} (7x-4) & 7x-4 \geq 0 \\ -(7x-4) & 7x-4 < 0 \end{cases}$$

$$|7x-4| = \begin{cases} (7x-4) & x > \frac{4}{7} \\ -(7x-4) & x < \frac{4}{7} \end{cases}$$

Q Find  $x$  if  $|x|=3$   
 $x = \pm 3$

$$|3|=3$$

$$|-3|=-(-3)=3$$

Q If  $|4x-1|=3$  then  $x=?$

$$4x-1=\pm 3$$

$$4x-1=3 \quad \text{OR} \quad 4x-1=-3$$

$$4x=4 \quad 4x=-3+1$$

$$x=1 \quad 4x=-2$$

$$x=-\frac{1}{2}$$

$$x = -\frac{1}{2}, 1$$

# Fundamentals of Mathematics

Q |  $|2x+3|=5$  find  $x$

$$2x+3=5$$

$$2x+3=5 \text{ or } 2x+3=-5$$

$$2x=-8$$

$$2x=2$$

$$x=1$$

$$x=1, -4$$

Q find value of  $|x-2|$  when  $x < 2$

$$|x-2| = -(x-2)$$

$$= 2-x$$

$x=1.9$  (Assume)

$$|1.9-2|$$

| -ve |

Q find value of  $|x+5|$  when  $x > -5$

$$|x+5|=x+5$$

$$x=-4$$

$$|-4+5|$$

| +ve |

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$$|4+3| + |4-3|$$

Q Find value of  $|4x-3|$  when  $x < \frac{3}{4}$

$$\begin{aligned} |4x-3| &= -(4x-3) & x &= 0 \\ &= 3-4x & |4 \times 0 - 3| &= 3 \\ & & |0-3| &= 3 \\ & & |-ve| &= 3 \end{aligned}$$

Q Find value of  $y = |x+2| + |x-2|$  when  $x > 2$

$$y = (x+2) + (x-2)$$

$$y = 2x$$

Q Find value of  $y = |x+3| + |x-3|$

When  $x \geq 2$

$$y = (x+3) - (x-3)$$

$$y = \underline{\underline{6}}$$

If  $x \geq 3$

$$x \geq 2$$

$$x > 3$$

$$|3+2| + |3-2|$$

$$|4+3| + |4-3|$$

$$\oplus$$

$$|+ve| + |-ve|$$

$$\begin{cases} x > 2 \\ x = 2 \\ [2, 3] \end{cases}$$

$$\begin{cases} |2 \cdot 1 + 3| + |2 \cdot 1 - 3| \\ |+ve| + |-ve| \end{cases}$$

$$|4+3| + |4-3|$$

$$\oplus$$

$$y = |x+3| + |x-3|$$

$$= x+3 + x-3$$

$$y = 2x$$

# Fundamentals of Mathematics

Q  $y = |3x-4| + |5x+1|$  has r.h.t?

$$\begin{array}{l} \downarrow \\ 3x-4=0 \\ x=\frac{4}{3} \end{array}$$

$$\begin{array}{l} \downarrow \\ 5x+1=0 \\ x=-\frac{1}{5} \end{array}$$

$$x = -\frac{1}{5}$$

Q  $y = |x+3| + |x-3|$  has r.h.t?

$$\begin{array}{l} \downarrow \\ -3 \end{array}$$

$$\begin{array}{l} \downarrow \\ 3 \end{array}$$

$$x < -3$$

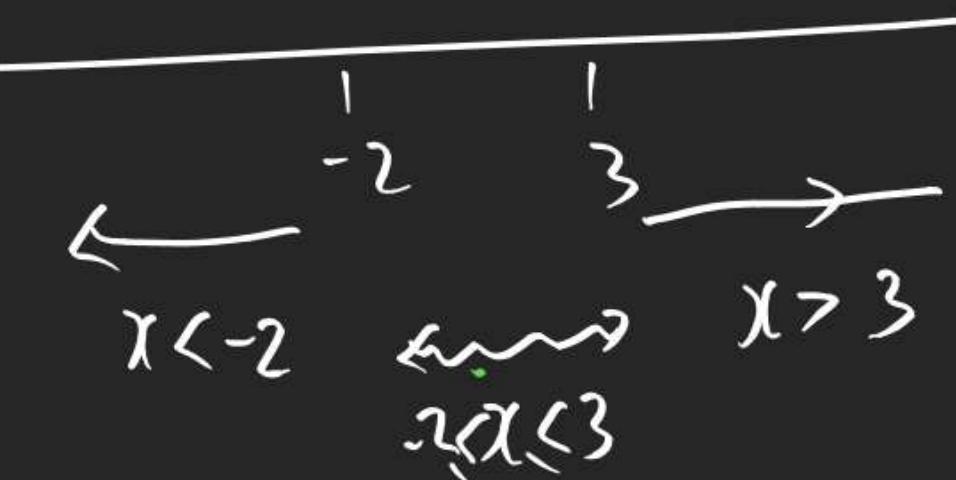
$$-3 < 0 < 3$$

$$x > 3$$

Q  $y = |x+2| + |x-3|$  has r.h.t

& Interval

(r.h.t = -2, 3)



# Fundamentals of Mathematics

Q Solve  $|x+2| + |x-2| = 5$

$$(x = -2 \text{ or } x)$$

$$\begin{array}{c} \text{---} \\ | \quad | \\ -2 \quad 2 \\ \swarrow \quad \searrow \\ x > 2 \end{array}$$

$$\begin{aligned} \text{① When } x > 2 \rightarrow x = 2.1 \\ |2.1+2| + |2.1-2| \\ (x+x) + (x-x) = 5 \\ 2x = 5 \Rightarrow x = \frac{5}{2} = 2.5 \end{aligned}$$

② When  $-2 \leq x \leq 2$

$$\begin{array}{c} x+2 - (x-2) = 5 \\ 4 = 5 \\ \emptyset \end{array}$$

$$x = \frac{5}{2}, -\frac{5}{2}$$

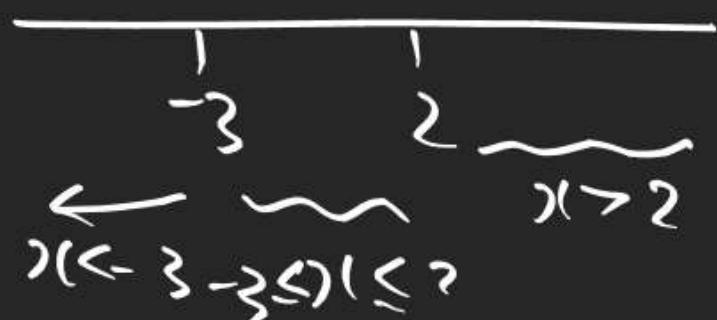
③ When  $x < -2$

$$\begin{array}{c} x < -2 \\ -(x+x) - (-x-x) = 5 \\ -2x = 5 \\ x = -\frac{5}{2} = -2.5 \end{array}$$

# Fundamentals of Mathematics

Q

$$|x+3| + |x-2| = 7 \text{ Solve}$$



② In the range  $-3 \leq x \leq 2$

$$(x+3) - (x-2) = 7$$

$$5 = 7$$

∅

$$x = -2.5$$

$$|-2.5+3| + |-2.5-2|$$

⊕

⊖

① In the range  $x < -3$

$$-(x+3) - (x-2) = 7$$

$$-2x = 8$$

$$x = -4$$

$$\begin{aligned} &x = -3.1 \\ &| -3.1 + 3 | + | -3.1 - 2 | \\ &\Theta \quad \Theta \end{aligned}$$

③  $x > 2$  ✓

$$(x+3) + (x-2) = 7$$

$$2x = 6$$

$$\boxed{x = 3}$$

$$x = 2.1$$

$$|2.1+3| + |2.1-2|$$

⊕ ⊕

$$\boxed{x = 3, -4}$$

# Fundamentals of Mathematics

Q If  $|x+1| = -4$  find  $x=?$

$$\begin{aligned} |x+1| &= -4 \\ \text{R.H.S.} &\neq \text{L.H.S.} \end{aligned}$$

$\emptyset$

Q Find values of  $x$  satisfying  $|x-4| + |x-3| = 7$

$$Q \quad |x-1| + |x-3| = 4$$

$$Q \quad |2x-1| + |2x| = 3.$$