Nishant Jindal

Force on a dielectric slab

CAPACITOR

Q.4 A parallel plate capacitor is made by fixing two plates inside of a container as shown in figure. The plates are connected to a battery of voltage V. If at $t = 0$ the tap is opened from which a liquid of dielectric constant k starts filling in the container at a constant rate of $r\ m^3/s$, find the current in connecting wires as a function of time. Neglect any resistance in connecting wires.

Volume flow rate

Volume flow rate $\Leftarrow \dfrac{dV}{dt} = (r)$

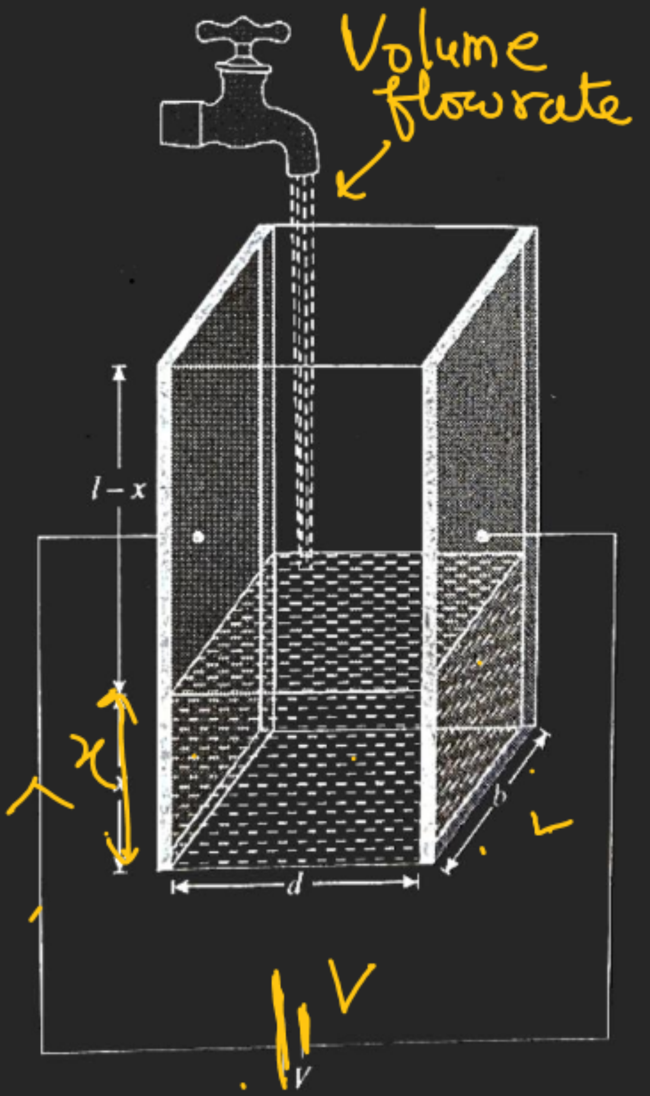
let, at $t = t$. x - height of liquid is raised in the capacitor.

$V_c = V$

$V_c = \dfrac{q\uparrow}{C\uparrow}$

$rt = xbd \Rightarrow x = \left(\dfrac{rt}{bd}\right)$

Volume of liquid in any t second



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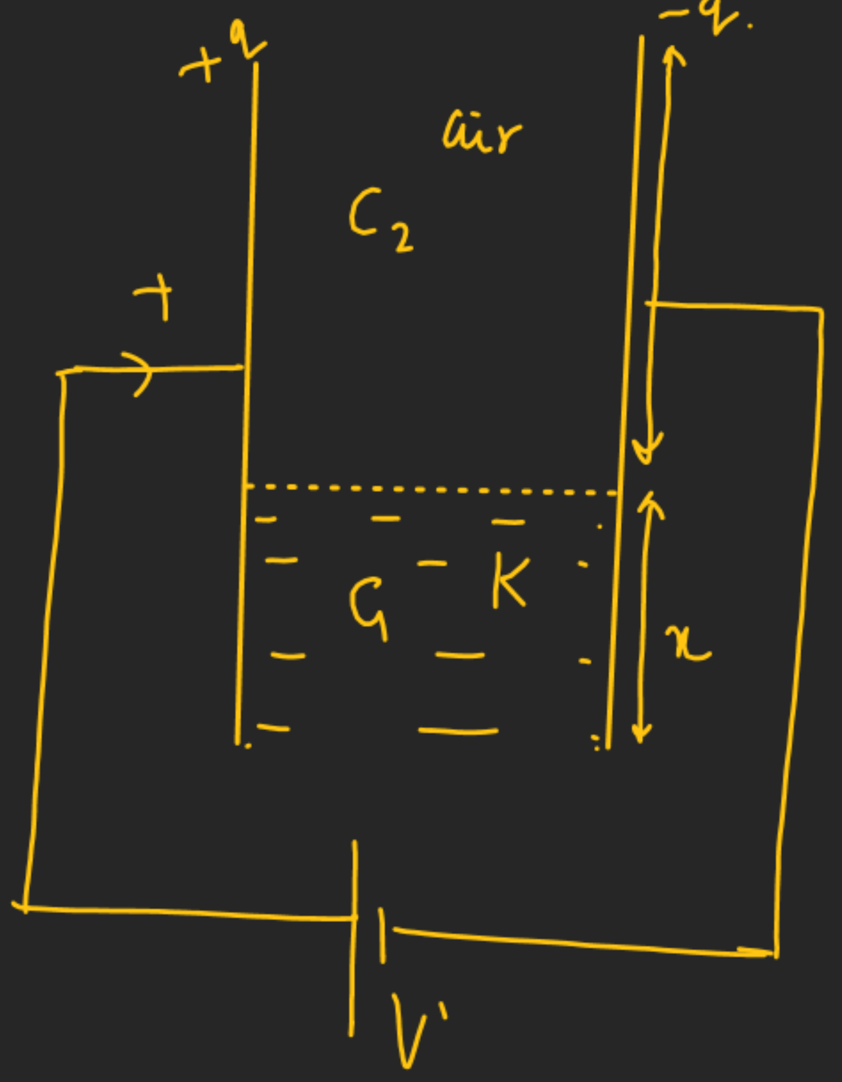
Force on a dielectric slab

CAPACITOR



$$C_{eq} = C_1 + C_2$$

$$C_{eq} = \frac{K\varepsilon_0(bx)}{d} + \frac{\varepsilon_0 b(l-x)}{d} = \left[\frac{\varepsilon_0 b x}{d}(K-1) + \frac{\varepsilon_0 b l}{d}\right]$$

$$q = C_{eq}V \longrightarrow i = \frac{dq}{dt} =$$

$$q = \left[\frac{\varepsilon_0 b(K-1)}{d}x + \frac{\varepsilon_0 b l}{d}\right]V$$

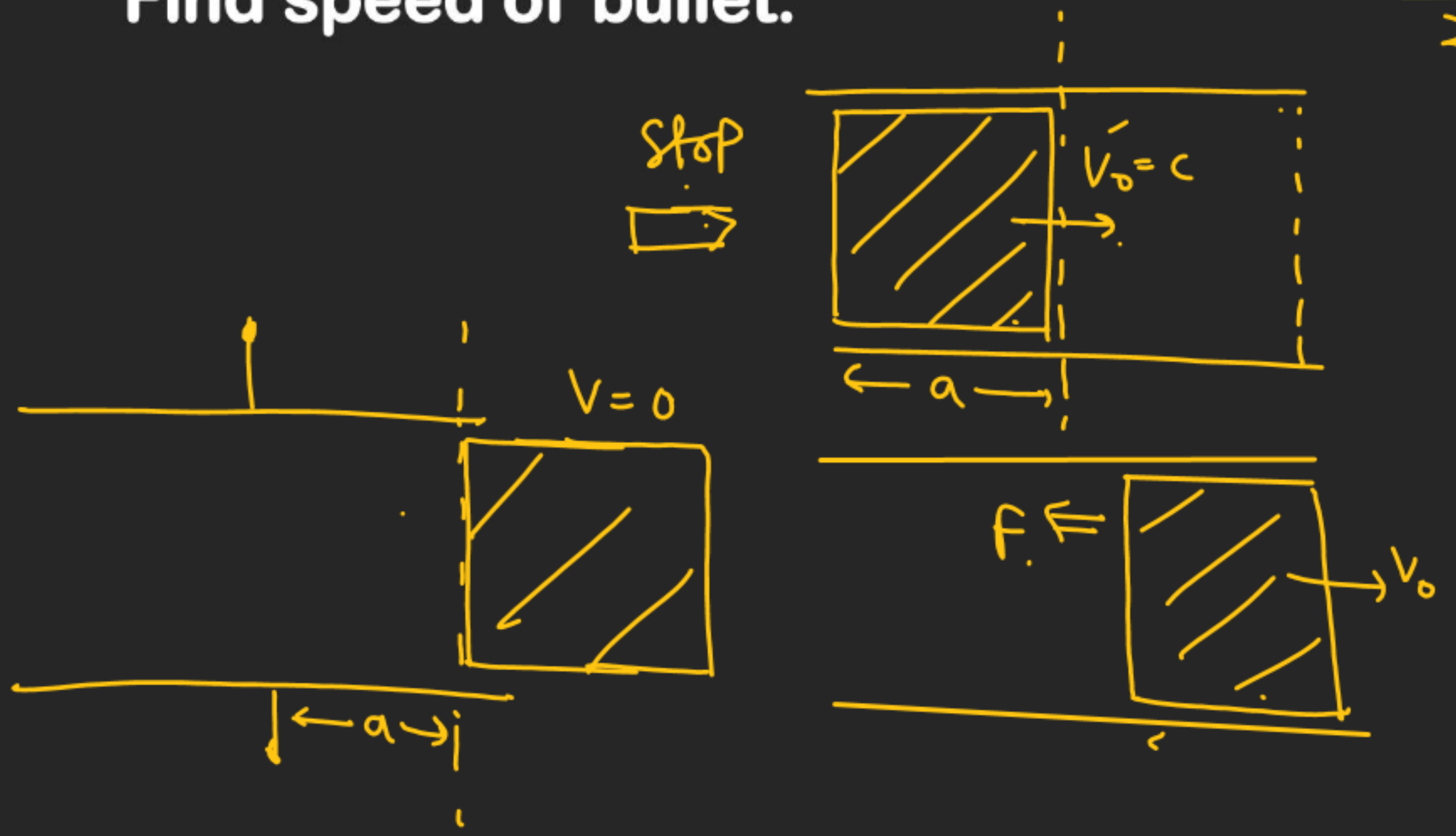
$$q = \left[\frac{\varepsilon_0 \cancel{b}(K-1)}{d} \times \frac{v t}{\cancel{b} d}\right]V + \left(\frac{\varepsilon_0 b V l}{d}\right)$$

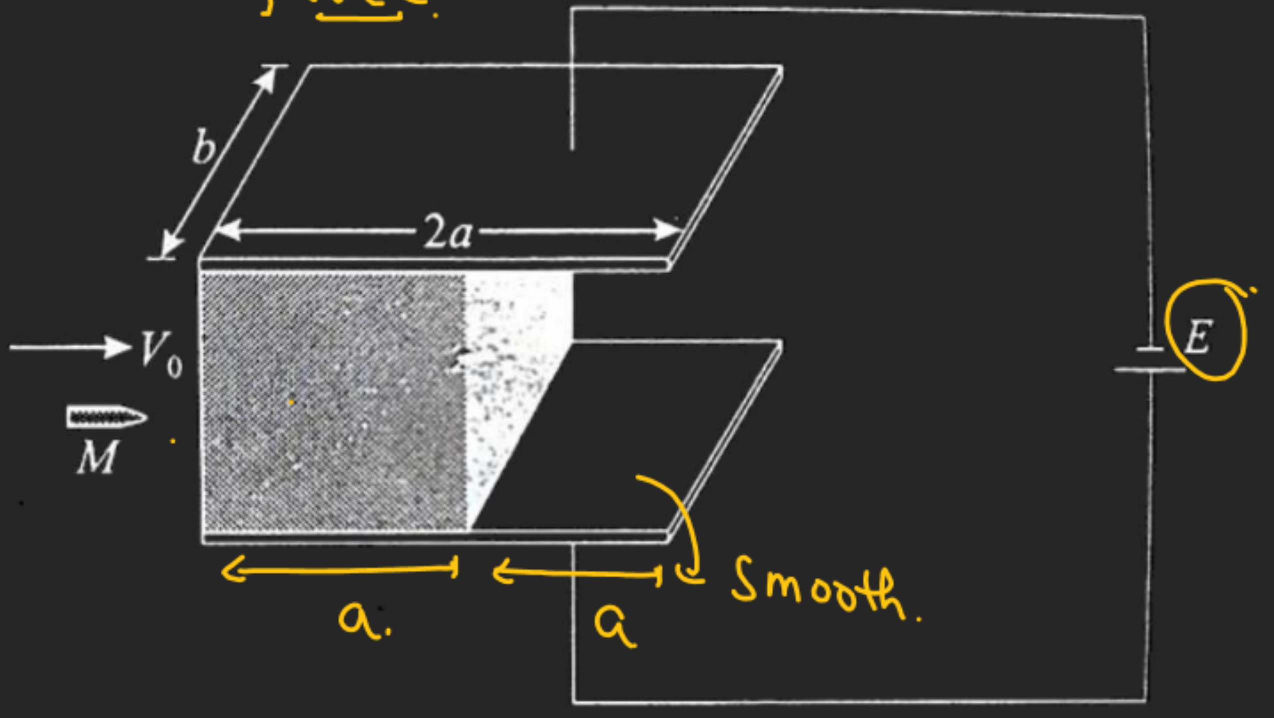
c.

$$i = \frac{dq}{dt} = \frac{\varepsilon_0 v V}{d^2}(K-1)\ A$$

Q.5 A parallel plate capacitor is half filled with a dielectric of relative permittivity k and of mass M. Capacitor is attached with a cell of voltage E. Plates are held fixed on smooth insulating horizontal surface. A bullet of equal mass M hits the dielectric elastically and its found that dielectric just leaves out the capacitor. Find speed of bullet.

$$F = \frac{\varepsilon_0 b V^2}{2d}(K-1) \leftarrow \text{constant force}$$





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Force on a dielectric slab

CAPACITOR

Work-Energy theorem

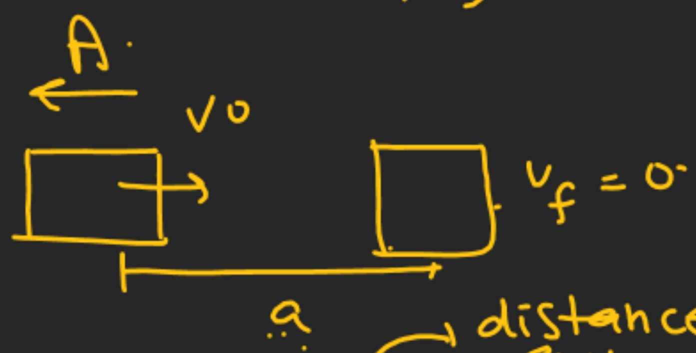
$$W_F = (\Delta K.E)$$

$$-\frac{\varepsilon_0 b V^2 (K-1)}{2d} \times a = 0 - \frac{1}{2} M v_0^2$$

$$v_0 = \sqrt{\frac{\varepsilon_0 b V^2 (K-1)}{M d}}$$

M-2

$$\left(A = \frac{F}{M}\right)$$



$$0 = v_0^2 - 2a.A$$

distance covered

Accleration

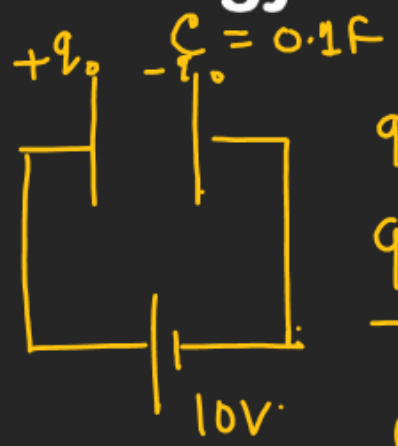
$$v_0 = \sqrt{2aA}$$

Nishant Jindal

Energy analysis

CAPACITOR

Q.2 A battery of $10\ \mathrm{V}$ is connected to a capacitor of capacity $0.1\ \mathrm{F}$. The battery is now removed and this capacitor is connected to a second uncharged capacitor. If the charges are distributed equally on these two capacitors, find the total energy stored in the two capacitors. Find the ratio of final energy to initial energy stored in capacitors.

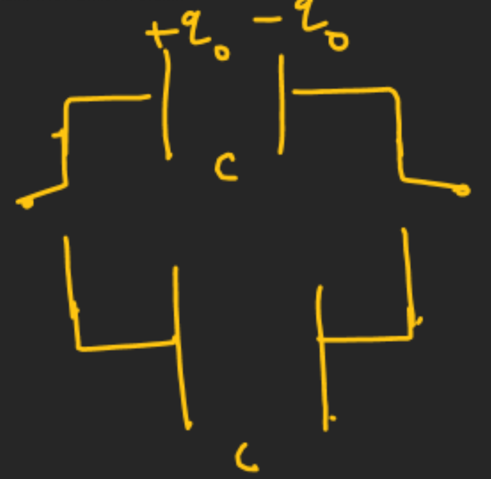


$q_0 = (0.1 \times 10)$

$q_0 = 1C$

$U_i = \frac{1}{2 \times 1 \times 10^{-1}}$

$U_i = 5J$



$\frac{U_f}{U_i} = \frac{1}{2}$

$\Rightarrow$

$(U_f)_T = \frac{(q_0/2)^2}{2C} \times 2$

$U_f = \frac{q_0^2}{4C} = \frac{1}{4 \times 1 \times 10^{-1}}$

$U_f = \left(\frac{10}{4}\right) = \frac{5}{2} J$

Energy analysis

CAPACITOR

Q.4 Two identical capacitors have the same capacitance C. One of them is charged to potential V_1 and the other V_2. The negative ends of the capacitors are connected together. When the positive ends are also connected, the decrease in energy of the combined system is **(2002)**

Heat $= \dfrac{C_1C_2}{2(C_1+C_2)}(V_1-V_2)^2$

$C_1 = C_2 = C$

$= \dfrac{C^2}{4C}(V_1-V_2)^2$

$= \dfrac{C}{4}(V_1-V_2)^2$

(A) $\frac{1}{4}C(V_1^2 - V_2^2)$

(B) $\frac{1}{4}C(V_1^2 + V_2^2)$

(C) $\frac{1}{4}C(V_1 - V_2)^2$ ✓

(D) $\frac{1}{4}C(V_1 + V_2)^2$

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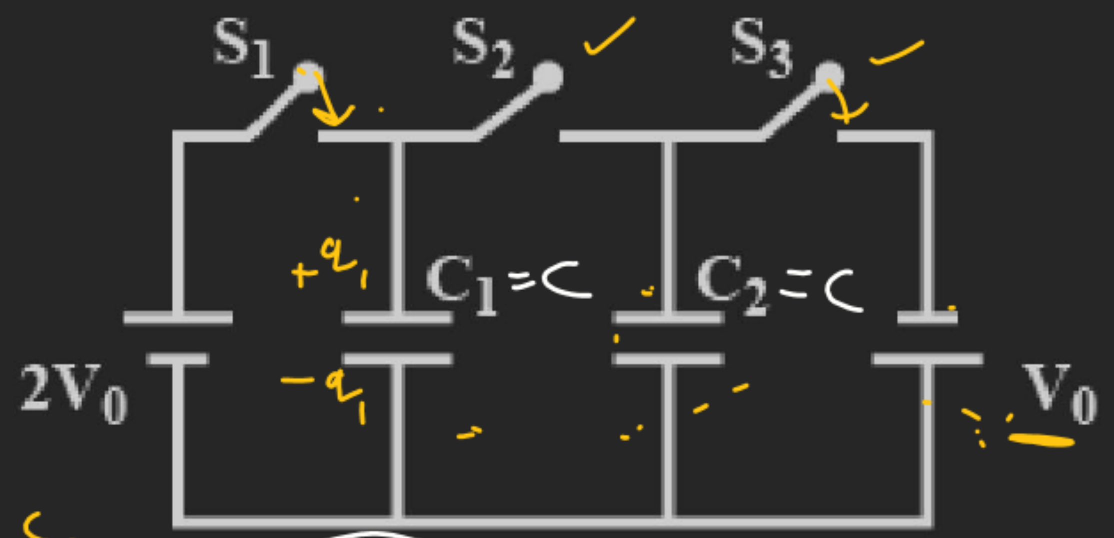
Q.6 In the circuit shown in the figure, there are two parallel plate capacitors each of capacitance C. The switch S_1 is pressed first to fully charge the capacitor C_1 and then released. The switch S_2 is then pressed to charge the capacitor C_2. After some time, S_2 is released and then S_3 is pressed. After some time, **(2013)**

(A) the charge on the upper plate of C_1 is $2CV_0$.

(B) the charge on the upper plate of C_1 is CV_0.

(C) the charge on the upper plate of C_2 is 0 .

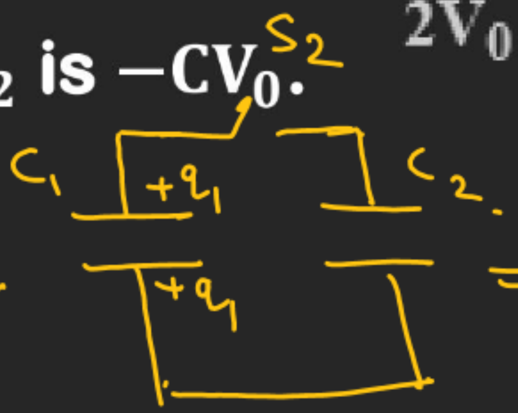
(D) the charge on the upper plate of C_2 is $-CV_0$.



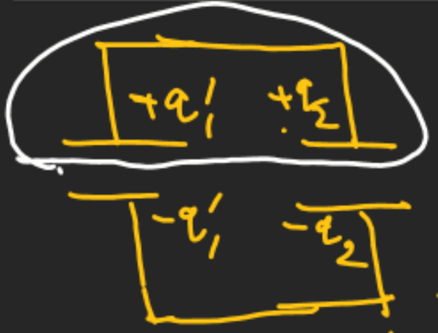
$q_1 = C_1(2V_0)$

$q_1' + q_2 = 2C_1V_0$

$q_1' = q_2 = \underline{C_1V_0} = CV_0$



$\Rightarrow$



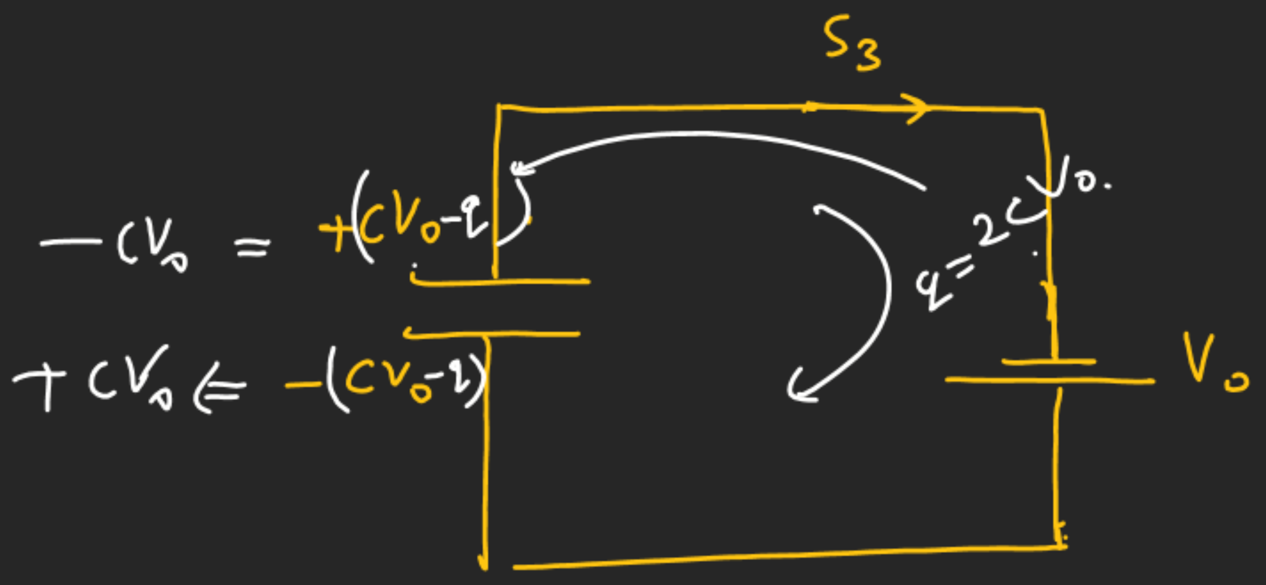
$\frac{q_1'}{C_1} = \frac{q_2}{C_2}$

$\frac{q_1'}{q_2} = \frac{C_1}{C_2}$ — (1)

$\underline{q_1' = q_2}$

Energy analysis

CAPACITOR



$$\frac{CV_0-q}{C}+V_0 = 0$$

$$CV_0 - q + CV_0 = 0$$

$$\boxed{q = 2CV_0}$$

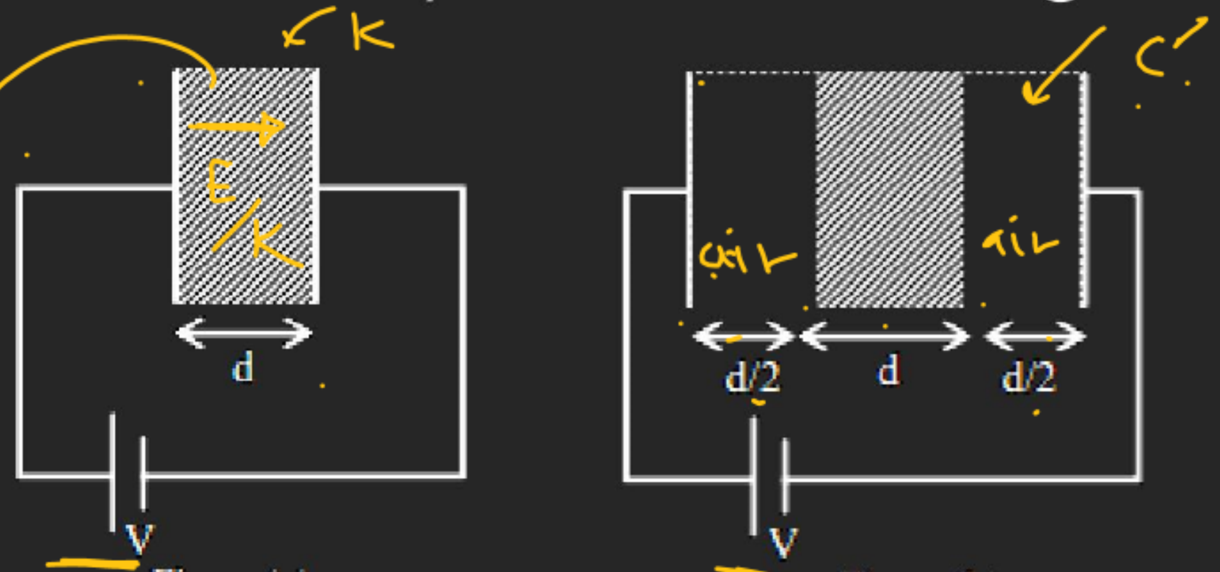
Q.7 A medium having dielectric constant $K > 1$ fills the space between the plates of a parallel plate capacitor. The plates have large area, and the distance between them is d. The capacitor is connected to a battery of voltage V, as shown in figure (a) Now, both the plates are moved by a distance of $d/2$ from their original positions, as shown in figure (b).

$C_0 = \frac{K\varepsilon_0 A}{d}$

$C' = \frac{\varepsilon_0 A}{\left[d + \frac{d}{K}\right]}$

$C' = \frac{K\varepsilon_0 A}{d(K+1)} \rightarrow C_0$

$C' = \left(\frac{C_0}{K+1}\right)$ ✓



Figure(a) Figure(b)

Energy analysis

CAPACITOR

In the process of going from the configuration depicted in figure (a) to that in figure (b), which of the following statement(s) is (are) correct?

(A) The electric field inside the dielectric material is reduced by a factor of $2K$.

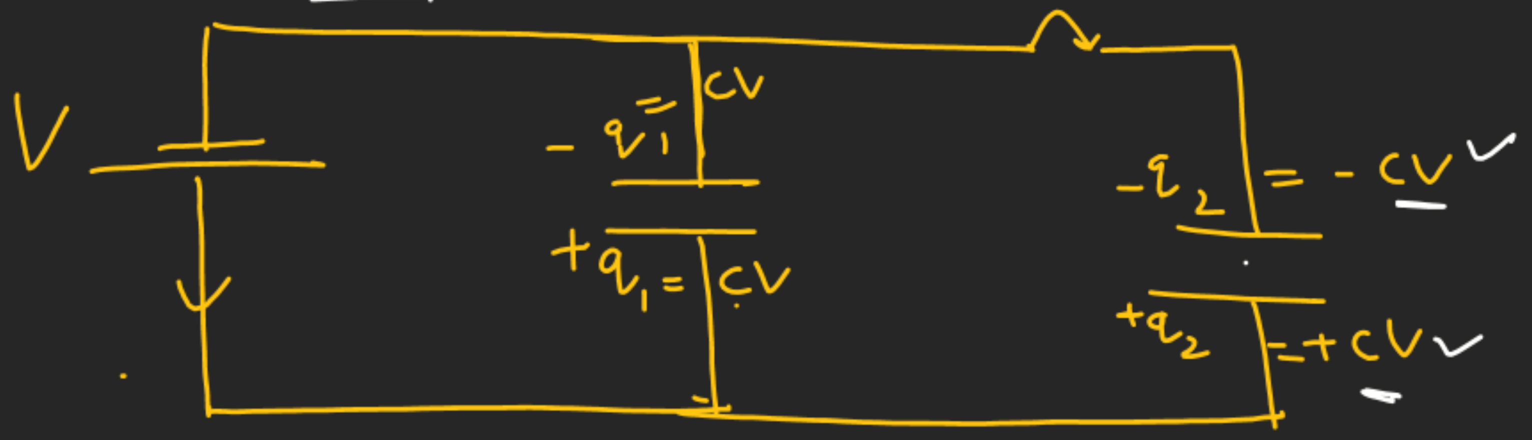
(B) The capacitance is decreased by a factor of $\frac{1}{K+1}$.

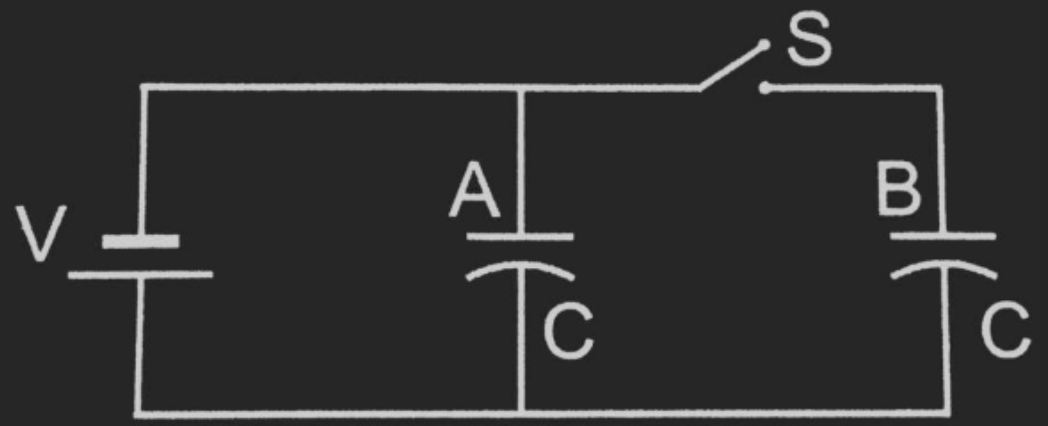
(C) The voltage between the capacitor plates is increased by a factor of $(K+1)$.

(D) The work done in the process DOES NOT depend on the presence of the dielectric material. **(2022)**

Energy analysis

Q.8 The figure shows two identical parallel plate capacitors to a battery with the switch S closed. The switch is now opened and the free space between the plates of the capacitors is filled with a dielectric of dielectric constant (or relative permittivity) 3. Find the ratio of the total electrostatic energy stored in both capacitors before and after the introduction of the dielectric. **(1983)**





$$U_i = \frac{1}{2}CV^2 \times 2 = CV^2$$

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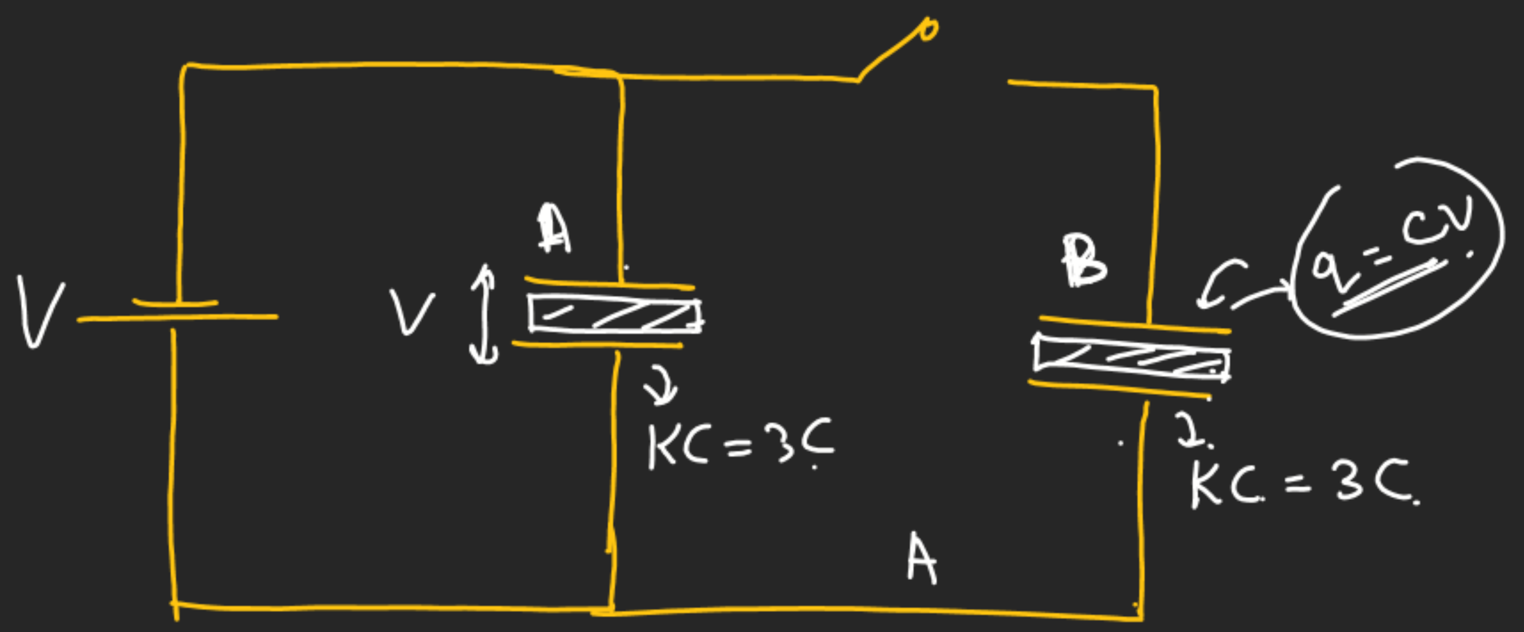
Energy analysis

CAPACITOR



$$U_f = \frac{1}{2}(3C)V^2 + \frac{(CV)^2}{2\times(3C)}$$

$$U_f = \frac{3CV^2}{2} + \frac{CV^2}{6}$$

$$= \left(\frac{10CV^2}{6}\right) J$$

$$\frac{U_i}{U_f} = \frac{CV^2}{\left(\frac{10CV^2}{6}\right)}$$

$$= \frac{6}{10} = \frac{3}{5} \text{ Ans.}$$