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Rk:- Max^m / Min^m Value can be find out using AM ≥ GM also.

1) If $a, b, f(x) > 0$ & Q's is of $a f(x) + \frac{b}{f(x)}$ type We use AM ≥ GM

(2) If Min of Sum is asked

(3) If Max of Prod is asked

Q If x, y are 2 variables such that $x > 0, y > 0$ & $xy = 1$ find Min of $x+y$?

Elements x, y

AM ≥ GM

$$\frac{x+y}{2} \geq (x \cdot y)^{1/2}$$

$$x+y \geq 2\sqrt{xy}$$

$$x+y \geq 2 \times 1$$

$$x+y \geq 2$$

$$\text{Min}(x+y) = 2$$

Q If $ax + \frac{b}{x} = c$, a, b, c +ve constants & $x > 0$ then

$ab \geq c^2$ $\quad ab \leq c^2$ $\quad ab \geq \frac{c^2}{4}$ $\quad ab \leq \frac{c^2}{4}$

AM ≥ GM

$$\frac{ax + \frac{b}{x}}{2} \geq \sqrt{a\cancel{x} \times \frac{b}{\cancel{x}}}$$

$$c \geq 2\sqrt{ab}$$

$$\frac{c^2}{4} \geq ab$$

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Q If $a^2x^4+b^2y^4=c^6$ then Max. of $x\cdot y$ is? → Product Max.

AM $\geq$ GM

$$\frac{a^2x^4+b^2y^4}{2}\geq\sqrt{a^2x^4\cdot b^2y^4}$$

$$\frac{c^6}{2}\geq abx^2y^2$$

$$\frac{c^6}{2ab}\geq(xy)^2$$

$$xy\leq\frac{c^3}{\sqrt{2ab}}$$

$$(xy)_{Max}=\frac{c^3}{\sqrt{2ab}}$$

Q If $a,b=+$Constant $x\in(0,\frac{\pi}{2})$ then Min. Value of $a\tan x+b\cot x$.

$$a\tan x+\frac{b}{\tan x}$$

AM $\geq$ GM

$$\frac{a\tan x+\frac{b}{\tan x}}{2}\geq\sqrt{a\cancel{\tan x}*\frac{b}{\cancel{\tan x}}}$$

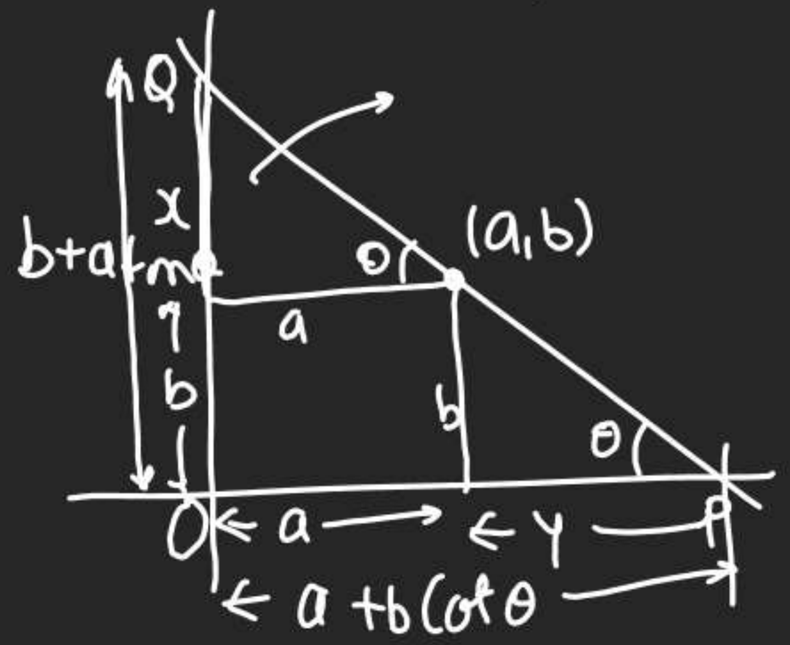
$$a\tan x+b\cot x\geq2\sqrt{ab}$$

$$(a\tan+b\cot x)_{Min}=2\sqrt{ab}$$

* $(a^2\tan^2x+b^2\cot^2x)_{Min}=?$

$2ab$

Q If a Line P.T. a fixed Pt. (a,b), $a>0$ $b>0$ & cuts axis at P & Q then Min. Value of $OP+OQ$.



$\frac{b}{y}=\tan\theta$

$y=b\cot\theta$

$\frac{x}{a}=\tan\theta$

$x=a\tan\theta$

$$OP+OQ=a+b\cot\theta+b+a\tan\theta$$

$$=(a+b)+(a\tan\theta+b\cot\theta)_{Min.}$$

$$=(a+b)+2\sqrt{ab}=(\sqrt{a}+\sqrt{b})^2$$

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Q Min value of $f(x) = a^2\sec^2 x + b^2\operatorname{cosec}^2 x$

$$f(x) = a^2(1+\tan^2 x) + b^2(1+\cot^2 x)$$

$$= (a^2+b^2) + (a^2\tan^2 x + b^2\cot^2 x)_{Min}$$

$$= (a^2+b^2) + 2ab$$

$$= (a+b)^2$$

Q Min. value of $f(x) = a\sec x + b\operatorname{cosec} x$? $a, b > 0$

$$\frac{dy}{dx} = a\sec x\tan x - b\operatorname{cosec} x\cot x = 0$$

$$\frac{a\cdot\sin x}{\cos^2 x} = b\cdot\frac{\cos x}{\sin^2 x}$$

$$\left(\frac{\sin x}{\cos x}\right)^3 = \frac{b}{a} \Rightarrow \tan x = \sqrt[3]{\frac{b}{a}}$$

$$\boxed{x = \tan^{-1}\sqrt[3]{\frac{b}{a}}}$$

$$f(x)_{Min} = \frac{a}{\frac{a^{1/3}}{\sqrt{a^{2/3}+b^{2/3}}}} + \frac{b}{\frac{b^{1/3}}{\sqrt{a^{2/3}+b^{2/3}}}}$$

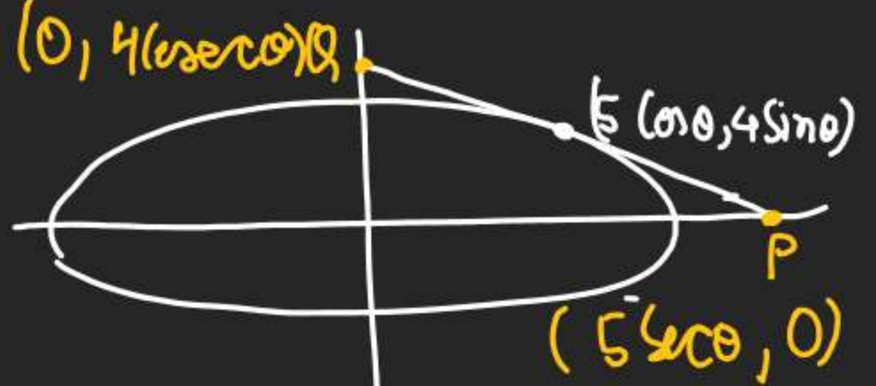
$$= a^{2/3}\sqrt{a^{2/3}+b^{2/3}} + b^{2/3}\sqrt{a^{2/3}+b^{2/3}}$$

$$= \sqrt{a^{2/3}+b^{2/3}}\,(a^{2/3}+b^{2/3})$$

$$f(x)_{Min} = (a^{2/3}+b^{2/3})^{3/2}$$

$$(a\sec x + b\operatorname{cosec} x)_{Min} = (a^{2/3}+b^{2/3})^{3/2}$$

Q If a tangent is drawn from any pt. of $\frac{x^2}{25}+\frac{y^2}{16}=1$ find Min length of Intercepted Part betn coordinates? PQ



(1) $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ | EOT: $\frac{xx_1}{a^2}+\frac{yy_1}{b^2}=1$

EOT. $\frac{x\cdot\cancel{5}\cos\theta}{\cancel{25}\,5} + \frac{y\cdot\cancel{4}\sin\theta}{\cancel{16}\,4} = 1$

$$\Rightarrow \frac{x}{5\sec\theta} + \frac{y}{4\operatorname{cosec}\theta} = 1$$

(2) Length of PQ $= \sqrt{25\sec^2\theta + 16\operatorname{cosec}^2\theta}$

Min. $= \sqrt{(5+4)^2} = 9$

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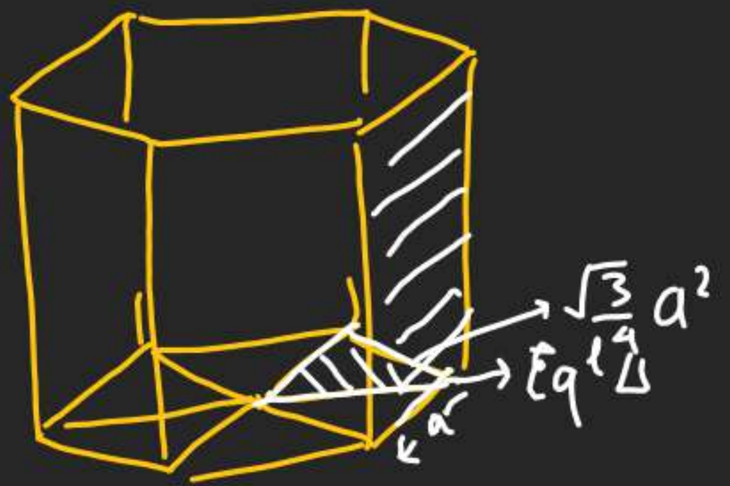
AM ≥ GM

	Min	Min at	Max
(1) $a\tan x + b\cot x$	$2\sqrt{ab}$	$a\tan x = b\cot x$	∞
(2) $a^2\sec^2 x + b^2\csc^2 x$	$(a+b)^2$	$a^2\tan^2 x = b^2\cot^2 x$	∞
(3) $a\sec x + b\csc x$	$(a^{2/3}+b^{2/3})^{3/2}$	$f'(x)=0$	∞
(4) $a\sin x + b\cos x$	$-\sqrt{a^2+b^2}$	$f'(x)=0$	$\sqrt{a^2+b^2}$

(5) Hexagonal Prism.



Base area $= 6\cdot\frac{\sqrt{3}}{4}a^2$

Volume $= \frac{6\sqrt{3}}{4}a^2\times h$.

Lateral Surface Area $= 6ah$

Total Surface Area $= 6ah + 2\times\frac{6\sqrt{3}}{4}a^2$

(6) Triangular Pyramid.

Base area $= \frac{\sqrt{3}}{4}a^2$

Volume $= \frac{1}{3}\cdot\frac{\sqrt{3}}{4}a^2h$

L.S.A $= \frac{3\sqrt{3}}{4}a^2$

T.S.A $= \frac{4\sqrt{3}}{4}a^2$

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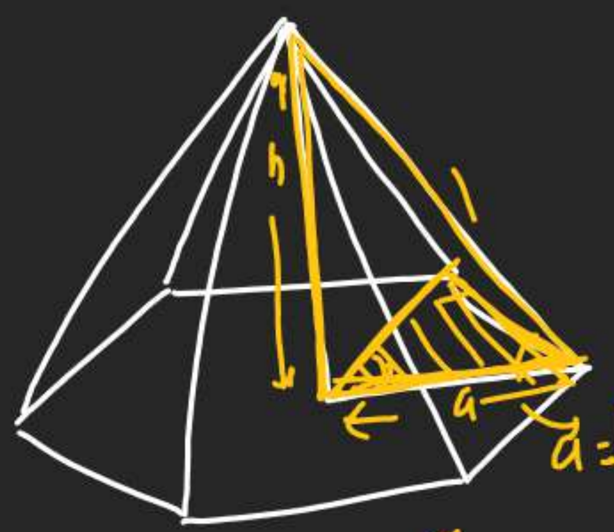
Q Lateral Edge of Regular hexagonal Pyramid is 1 cm. If volume is max. then its ht?



$a = \sqrt{1-h^2}$

$V = \frac{1}{\cancel{3}} \times \cancel{6}^{2} \frac{\sqrt{3}}{\cancel{4}_{2}} a^2 \times h$

$V = \frac{\sqrt{3}}{2} a^2 h$

$V = \frac{\sqrt{3}}{2}(1-h^2)\cdot h = \frac{\sqrt{3}}{2}(h-h^3)$

$\frac{dV}{dh} = \frac{\sqrt{3}}{2}(1-3h^2) = 0$

$h^2 = \frac{1}{3}$

$h = \frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}$

$\left.\frac{d^2V}{dh^2}\right| = \frac{\sqrt{3}}{2}(-6h) = -ve$

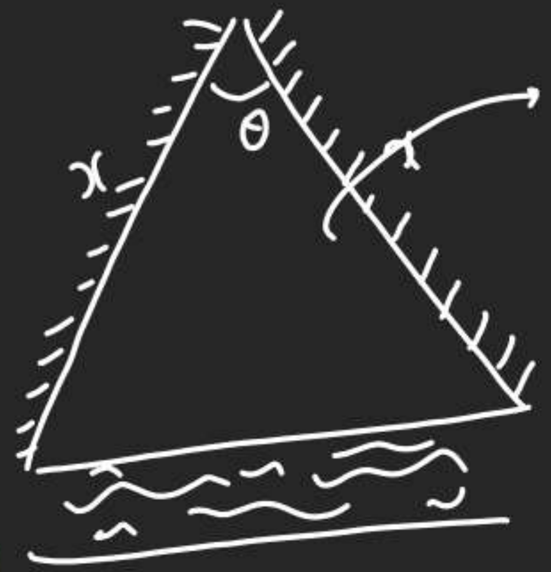
$h = \frac{1}{\sqrt{3}}$ Max.

$Vol = \frac{\sqrt{3}}{2}\left(\frac{1}{\sqrt{3}} - \frac{1}{3\sqrt{3}}\right)$

$= \frac{\sqrt{3}}{2}\left(\frac{3-1}{3\sqrt{3}}\right) = \frac{1}{3}.$

Max at $h = \frac{1}{\sqrt{3}}$

Q A trianglar Park is Enclosed on 2 Sides by fences and 3rd Side a Straight River bank. The 2 Sides having fences of Same Length x then max^m area enclosed by Park = ?



$A = \frac{1}{2} x \cdot x \cdot \sin\theta$

$A = \frac{x^2}{2} \sin\theta$

A Max. When $\sin\theta = 1$

$\therefore A_{Max} = \frac{x^2}{2}$

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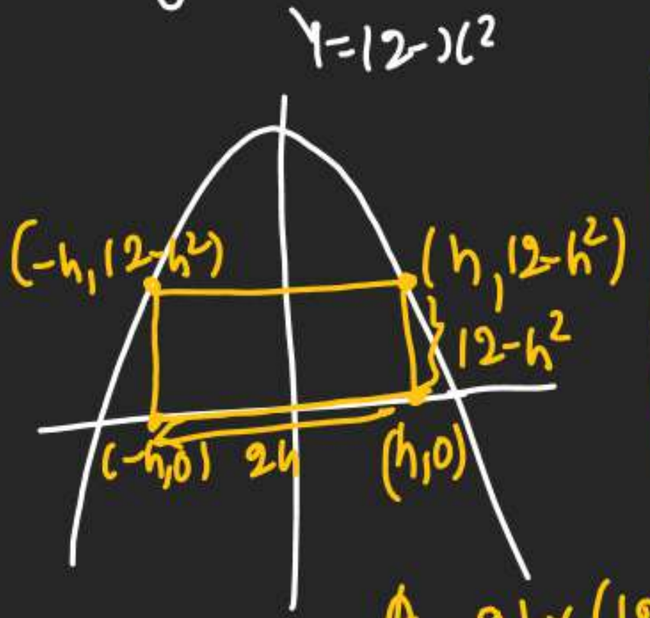
Q Max^m Area of Rectangle having Base on X Axis & other 2 sides On Parabola $y=12-x^2$ such that Rectangle lies inside Parabola, is?



Max Area

$A=24\times2-2\times2^3$

$=48-16$

$=32$

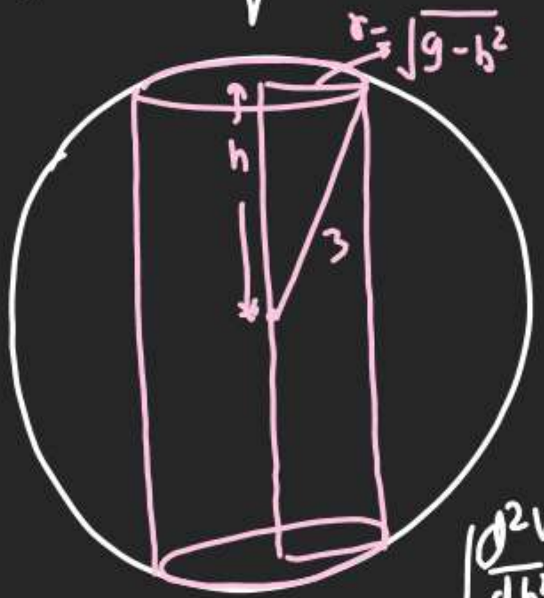
$A=2h\times(12-h^2)$

$A=24h-2h^3$

$\frac{dA}{dh}=24-6h^2=0 \Rightarrow h=2,-2$

$\frac{d^2A}{dh^2}=-12h \Rightarrow$ Max $h=2$

Q ht. of Rt. Circular Cylinder of max. Volume inscribed in sphere of Rad=3.



Volume $=\pi r^2\cdot h$

$V=\pi\cdot(9-h^2)\cdot h$

$V=9\pi h-\pi h^3$

$\frac{dV}{dh}=9\pi-3\pi h^2=0$

$h^2=3, h=\sqrt{3},-\sqrt{3}$.

$\frac{d^2V}{dh^2}=-6\pi h$

$h=\sqrt{3}$ Max.

$ht=2h$

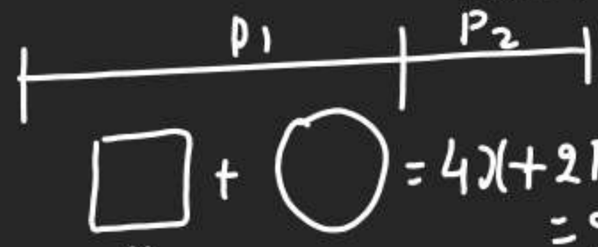
$=2\sqrt{3}$

Ans

Q A wire of length 2 cm is cut into 2 pieces which are bent to form a sqr of side x units & a Circle of Rad=r. If sum of area of sqr & Circle so formed is Min then

$2x=(\pi+4)r$ $(4-\pi)x=\pi r$ $x=2r$

$2x=r$



$=4x+2\pi r$
$=2$

$2x+\pi r=1 \to x=\frac{1-\pi r}{2}$

$A=x^2+\pi r^2=\left(\frac{1-\pi r}{2}\right)^2+\pi\cdot r^2$

$\frac{dA}{dr}=2\left(\frac{1-\pi r}{2}\right)\times-\frac{\pi}{2}+2\pi r=0$

$2r=\frac{1-\pi r}{2}$

$4r=1-\pi r$

$r(4+\pi)=1 \to r=\frac{1}{4+\pi}$

$x=\frac{1-\frac{\pi}{4+\pi}}{2}$

$=\frac{4+\pi-\pi}{2(4+\pi)}$

$x=\frac{2}{4+\pi}$

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A Rectangular sheet of fixed Perimeter with sides having their lengths in Ratio 8:15 is converted into an open rectangular box by folding after removing sq^r of equal areas from all 4 corners. If TSA of Removed sq^r is 100, the Resulting box has max^m vol. The lengths of the sides of Rectangular Sheet are?

$$4y^2 = 100 \Rightarrow y = 5$$

$$V = (15x-2y)(8x-2y)y$$

$$V = (120x^2 - 46xy + 4y^2)y$$

$$\frac{dV}{dy} = 120x^2 - 92xy + 12y^2$$

$$\left.\frac{dV}{dy}\right|_{y=5} = 120x^2 - 460x + 300 = 0$$

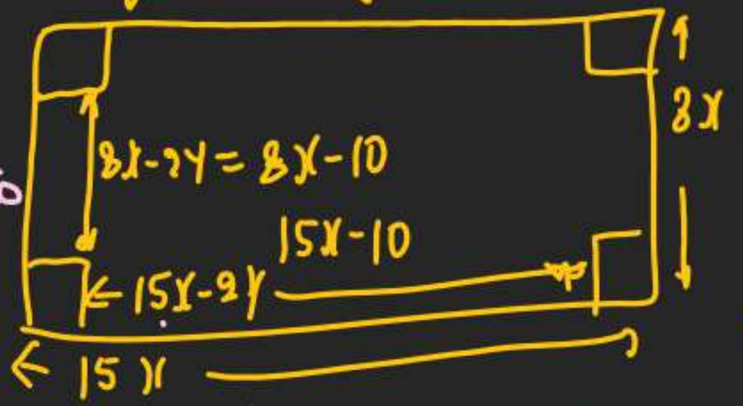
$$12x^2 - 46x + 30 = 0$$

$$\Rightarrow 6x^2 - 23x + 15 = 0$$

$$\Rightarrow 6x^2 - 18x - 5x + 15 = 0$$

$$(6x-5)(x-3) = 0$$

$$x = 3, \cancel{\tfrac{5}{6}}$$



$$\frac{75}{6} \times \frac{40}{6} = \frac{3000}{36} < 100$$

lengths = 45, 24 //