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T7 Solving Eqn Using Change of variables

Here we assume t = fxn, which is coming multiple times in an Eqn.

Q $\sin^4(2x)+\cos^4(2x)=\sin 2x\cdot\cos 2x$ find G.S ?

$\sin^4\theta+\cos^4\theta = 1-2\sin^2\theta\cos^2\theta$

$$1-2\sin^2(2x)\cdot\cos^2(2x)=\sin 2x\cdot\cos 2x$$

$$1-2t^2 = t$$

$$2t^2+t-1=0$$

$$2t^2+2t-t-1=0$$

$$2t(t+1)-1(t+1)=0$$

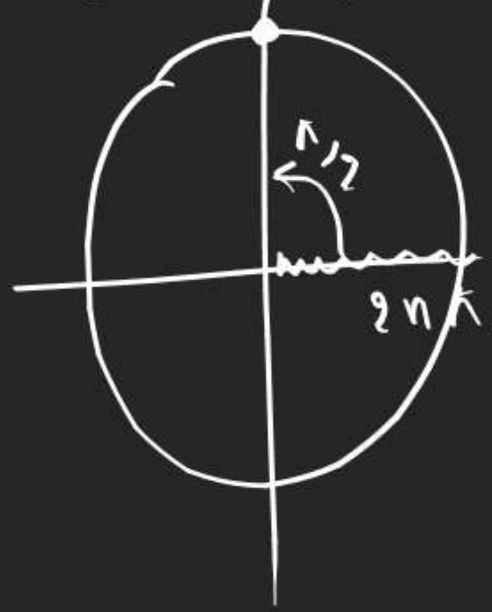
$$(2t-1)(t+1)=0$$

$$t=\tfrac{1}{2} \text{ or } t=-1$$

$\sin 2x\cdot\cos 2x=\frac{1}{2}$ OR $\sin 2x\cdot\cos 2x=-1$

$2\sin(2x)\cdot\cos(2x)=1$

$\sin 4x=1$



$$4x=2n\pi+\frac{\pi}{2}$$

$$\boxed{x=\frac{n\pi}{2}+\frac{\pi}{8}}$$

$2\sin(2x)\cdot\cos(2x)=-2$

$\sin 4x=-2$

X

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$(0, 2\pi, 4\pi, 6\pi, 8\pi, 10\pi, 12\pi \ldots) \cap (0, 4\pi, 8\pi, 12\pi) \cap (0, 6\pi, 12\pi, 18\pi) = (0, 12\pi, 24\pi \ldots)$

Q $\dfrac{\sin 6x}{\sin x} = 8 \cos x \cdot \cos 2x \cdot \cos 4x$ find G.S.?

$\sin 6x = 8 \sin x \cdot \cos x \cdot \cos 2x \cdot \cos 4x$

$= 4(\underbrace{2 \sin x \cos x}) \cos 2x \cdot \cos 4x$

$= 2 \times 2 \sin 2x \cdot \cos 2x \cos 4x$

$= 2(\sin 4x \cdot \cos 4x)$

$\sin 6x = \sin 8x$

$\sin 8x - \sin 6x = 0 \rightarrow \sin C - \sin D$

$2 \cos(7x) \cdot \boxed{\sin(x)} = 0$

$\cos 7x = 0$, $\sin x = 0$ → Consider Nahi as $\sin x$ is in Dr. in Qs.

$\cos 7x = 0$

$7x = (2n+1)\frac{\pi}{2} \Rightarrow \boxed{x = (2n+1)\frac{\pi}{14}}$ ✓

T8 Using Boundedness of $\sin x$ & $\cos x$ {f(x)}

In this type of Qs we always check Range of LHS & RHS & decide accordingly.

Q $\cos x + \cos \frac{x}{2} + \cos \frac{x}{3} = 3$ find G.S.

1) Here in LHS we have 3 cos

2) $\cos\theta$'s max value = 1 LCM(2, 4, 6) = 12

3) as we have 3 cos & every $\cos\theta$ has max = 1 $\Rightarrow$ all 3 cos can give max = 3

(4) RHS = 3 $\boxed{x = 12n\pi}$

$\Rightarrow \cos x = 1$ & $\cos \frac{x}{2} = 1$ & $\cos \frac{x}{3} = 1$

$x = 2n\pi$ $\cap$nd $\frac{x}{2} = 2n\pi$ $\cap$nd $\frac{x}{3} = 2n\pi$

$x = 4n\pi$ $x = 6n\pi$

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Q $\cos x+\cos 2x+\cos 3x=3$ find G.S.

3 Cos are added & giving 3.

$\Rightarrow$ Each $\cos\theta$ is giving its Max $=1$

$\cos x=1$ & $\cos 2x=1$ & $\cos 3x=1$

$x=2n\pi$ | $2x=2n\pi$, $x=n\pi$ | $3x=2n\pi$, $x=\frac{2n\pi}{3}$

$(0,2\pi,4\pi,6\pi\ldots)\cap(0,\pi,2\pi,3\pi,4\pi\ldots)\cap\left(0,\frac{2\pi}{3},\frac{4\pi}{3},2\pi,\frac{8\pi}{3},\frac{10\pi}{3},4\pi\ldots\right)$

$(0,2\pi,4\pi\ldots)$ $\therefore x=2n\pi$

M_2 LCM$\left(2,1,\frac{2}{3}\right)$

LCM$\left(\frac{2}{1},\frac{1}{1},\frac{2}{3}\right)$

$\frac{\text{LCM}(2,1,2)}{\text{HCF}(1,1,3)}=\frac{2}{1}=2$

$\therefore \boxed{x=2n\pi}$

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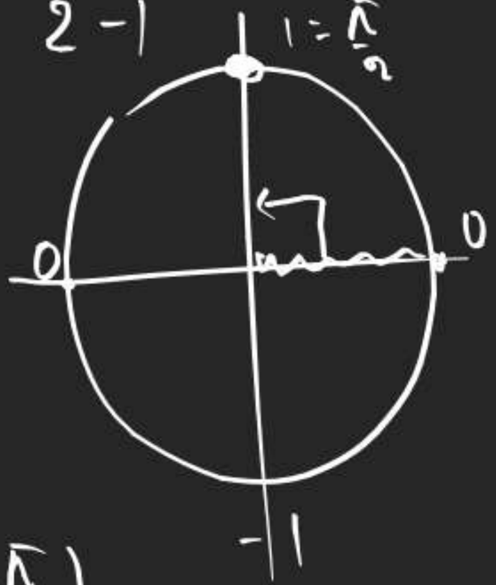
Q $\sin x\cdot\left(\cos\frac{x}{4}-2\sin x\right)+\left(\sin\frac{x}{4}-2\cos x\right)\cos x+1=0$ find GS ?

$$\sin x\cdot\cos\frac{x}{4}-2\sin^2 x+\cos x\cdot\sin\frac{x}{4}-2\cos^2 x=-1$$

$$\left\{\sin x\cdot\cos\frac{x}{4}+\cos x\cdot\sin\frac{x}{4}\right\}-2\left(\sin^2 x+\cos^2 x\right)=-1$$

$$\sin\left(x+\frac{x}{4}\right)=2-1$$

$$\sin\frac{5x}{4}=1$$



$$\frac{5x}{4}=2n\pi+\frac{\pi}{2}$$

$$x=\frac{4}{5}\left(2n\pi+\frac{\pi}{2}\right)$$

$$x=\frac{8n\pi}{5}+\frac{2\pi}{5}$$

Q If $x^2+2x+3=\sqrt{3}\sin y+\cos y$ find $(x,y)=$?

$a\cos\theta+b\sin\theta$

A. A $=\sqrt{\sqrt{3}^2+1^2}$
$=2$

$$(x^2+2x+1)+2=2\left(\frac{\sqrt{3}}{2}\sin y+\frac{1}{2}\cos y\right)$$

$$(x+1)^2+2=2\left(\cos y\cdot\cos\frac{\pi}{3}+\sin y\sin\frac{\pi}{3}\right)$$

LHS $\geq 0+2 = 2\cos\left(y-\frac{\pi}{3}\right)$

LHS ≥ 2 RHS ≤ 2

LHS $=2=$ RHS (Raaji hain)

$$(x+1)^2+\cancel{2}=\cancel{2}$$

$$(x+1)^2=0$$

$$\Rightarrow x+1=0$$

$$\boxed{x=-1}$$

$$\cancel{2}\cos\left(y-\frac{\pi}{3}\right)=\cancel{2}1$$

$$y-\frac{\pi}{3}=2n\pi$$

$$\boxed{y=2n\pi+\frac{\pi}{3}}$$

$$(x,y)\equiv\left(-1,\ 2n\pi+\frac{\pi}{3}\right)$$

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Concept:

1) $\sin x \in [-1,1]$

2) $\sin^2 x \in [0,1]$

$0 \le \sin^2 x \le 1$

3) $\frac{1}{0} > \frac{1}{\sin^2 x} \geq \frac{1}{1}$

$\infty > \frac{1}{\sin^2 x} \geq 1$

$\boxed{\frac{1}{\sin^2 x} \geq 1}$

$2^{\frac{1}{\sin^2 x}} \geq 2^1$

$2^{\frac{1}{\sin^2 x}} \geq 2$

Adv Lvl

Q $\boxed{2^{\frac{1}{\sin^2 x}}} \cdot \sqrt{y^2 - 2y + 2} \le 2$ find h.s.

$\Rightarrow 2 \times \sqrt{(y^2 - 2y + 1) + 1}$

$\sqrt{(y-1)^2 + 1}$

$\geq 0 + 1$

≥ 1

RHS ≤ 2

LHS ≥ 2 Set H least 2

LHS $= 2$

$2^{\frac{1}{\sin^2 x}} = 2^1$ & $\sqrt{(y-1)^2 + 1} = 1$

$\Rightarrow \frac{1}{\sin^2 x} = 1$

$\Rightarrow \sin^2 x = 1 = \sin^2 \frac{\pi}{2} \Rightarrow \boxed{x = n\pi \pm \frac{\pi}{2}}$

$\Rightarrow (y-1)^2 + 1 = 1$

$\Rightarrow y - 1 = 0 \Rightarrow y = 1$ $\therefore (x,y) = \left(n\pi \pm \frac{\pi}{2}, 1\right)$

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Tg Eqn of the form $f(x) = \sqrt{g(x)}$ type

(1) Sqr & Solve.

(2) Take only that value of x where $\sqrt{\ }$ is defined.

Q $\sqrt{1-\cos x} = \sin x$ find the G.S.?

Sqr $1-\cos x = \sin^2 x$

$(1-\cos x) = (1-\cos x)(1+\cos x)$

$(1-\cos x)(1+\cos x) - (1-\cos x) = 0$

$(1-\cos x)\{1+\cos x - 1\} = 0$

$\underset{0}{\underset{\shortparallel}{(\cos x)}}\ \underset{0}{\underset{\shortparallel}{(1-\cos x)}} = 0$

$\sqrt{1-0} = 1 \Rightarrow 1 = 1$

$\cos x = 0 \Rightarrow \sin x = 1 \Rightarrow \boxed{x = 2n\pi + \frac{\pi}{2}}$ ✓

$x = 0$ $\sin 0 = 0$

$\boxed{1 = \cos x} \Rightarrow \sqrt{1-1} = 0 \Rightarrow 0 = 0$ ✓

$\boxed{x = 2n\pi + 0}$ ✓

$\cos x = 0 \Rightarrow \sin x = -1$ $\sqrt{1-0} = -1$

$1 = -1$ ✗

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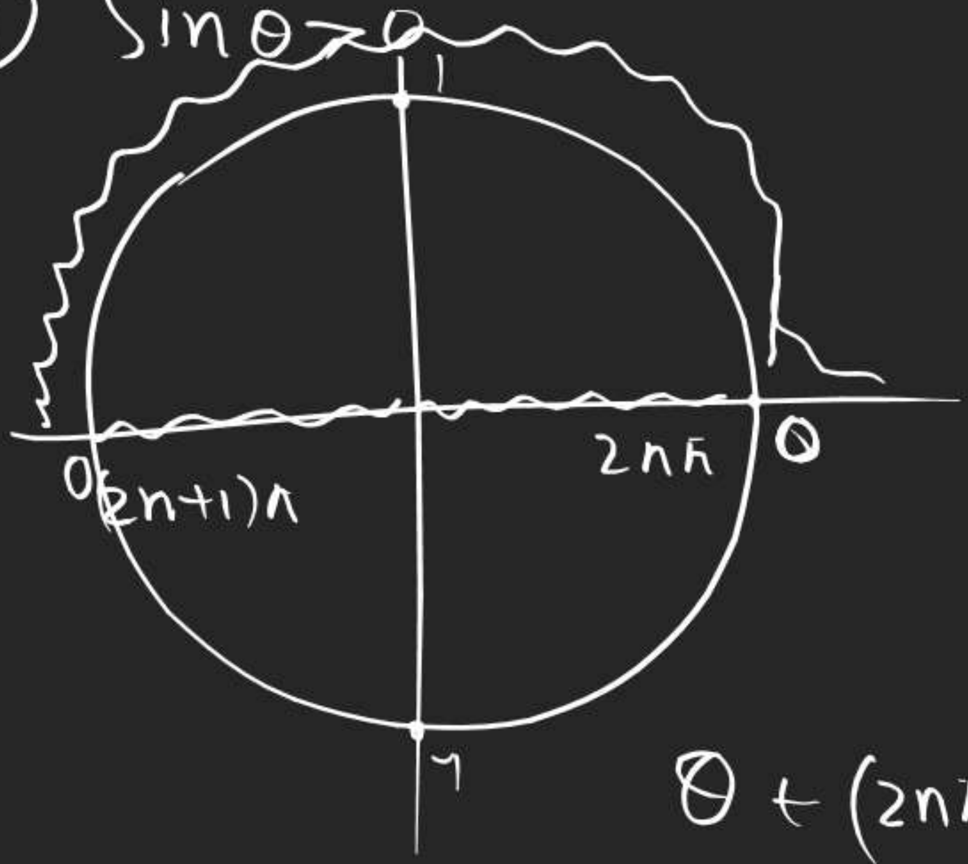
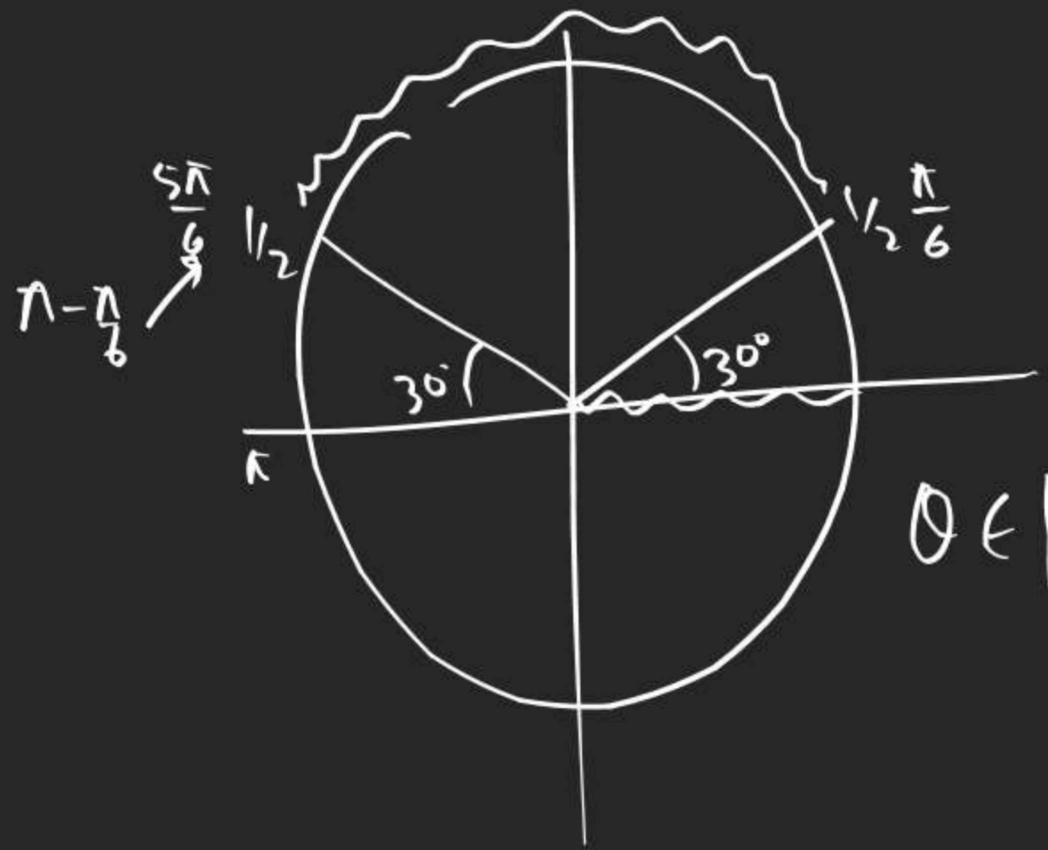
T10 Trigo Inequality

① $\sin\theta > 0$



$\theta \in (2n\pi, (2n+1)\pi)$

② $\sin\theta > \frac{1}{2} = \frac{\pi}{6}$ ऊपर



$\theta \in \left[2n\pi + \frac{\pi}{6}, 2n\pi + \frac{5\pi}{6}\right]$

Q $\log_2(\sin x) < -1$ find x?

$2 > 0$

$2 \neq 1$

$\boxed{\sin x > 0}$

log की 3 Cond^n

Base > 0

Base $\neq 1$

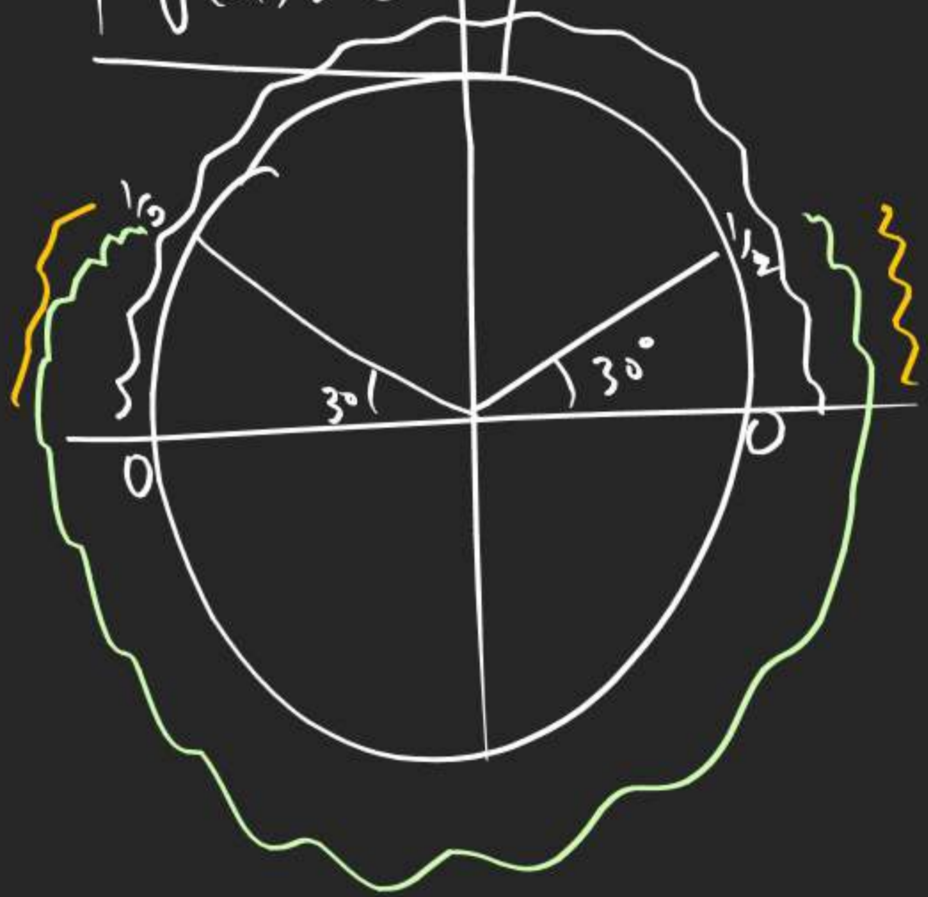
$f(x) > 0$

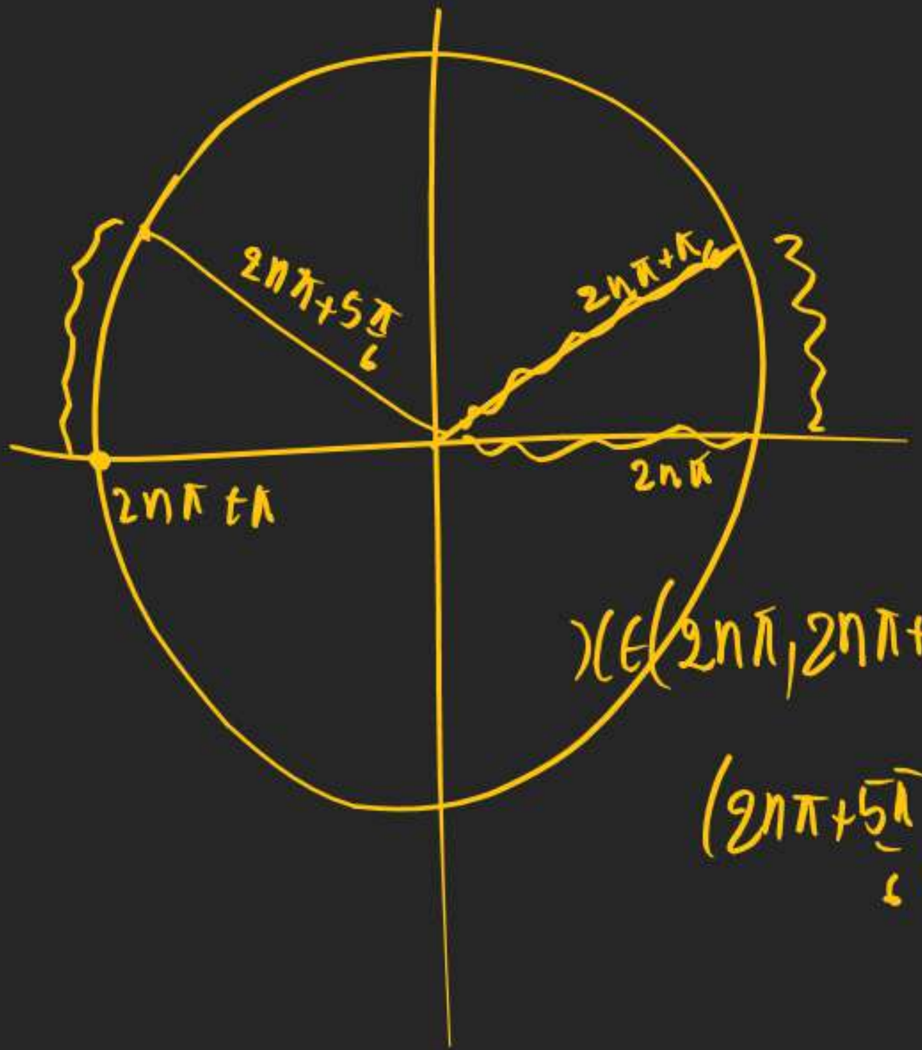
$\log_2(\sin x) < -1$

$\sin x < 2^{-1}$

$\boxed{\sin x < \frac{1}{2}}$

क्यों





$x \in \left(2n\pi, 2n\pi+\frac{\pi}{6}\right) \cup$

$\left(2n\pi+\frac{5\pi}{6}, 2n\pi+\pi\right)$

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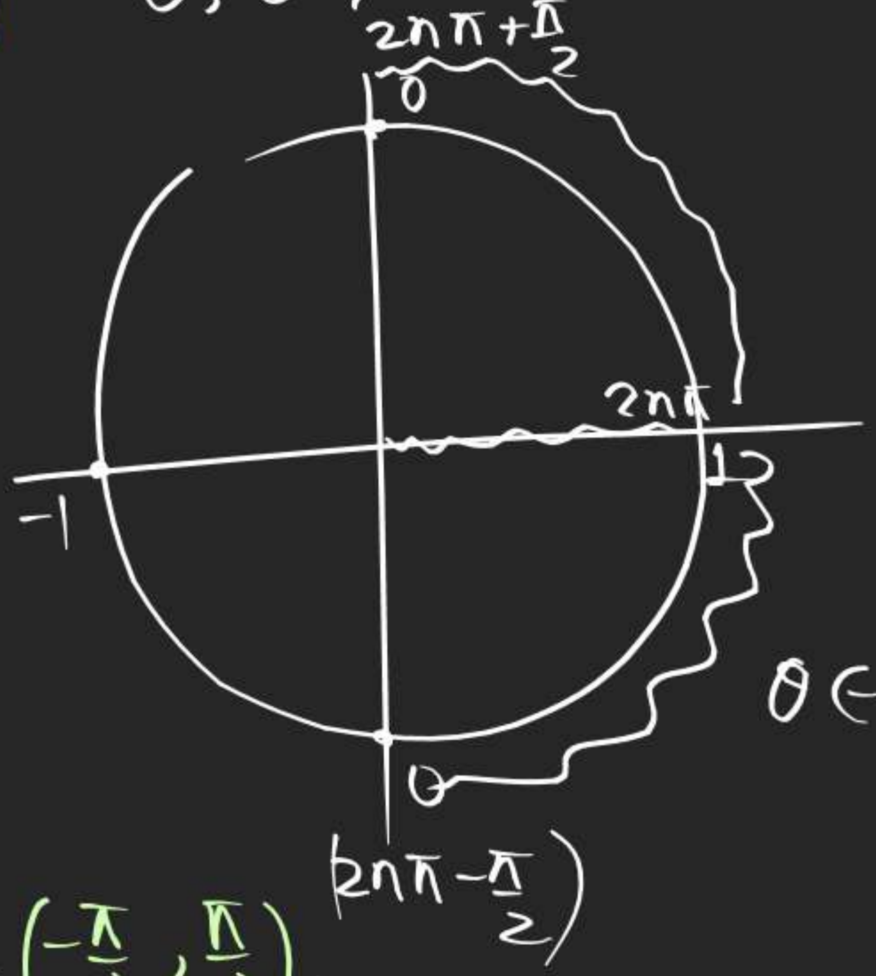
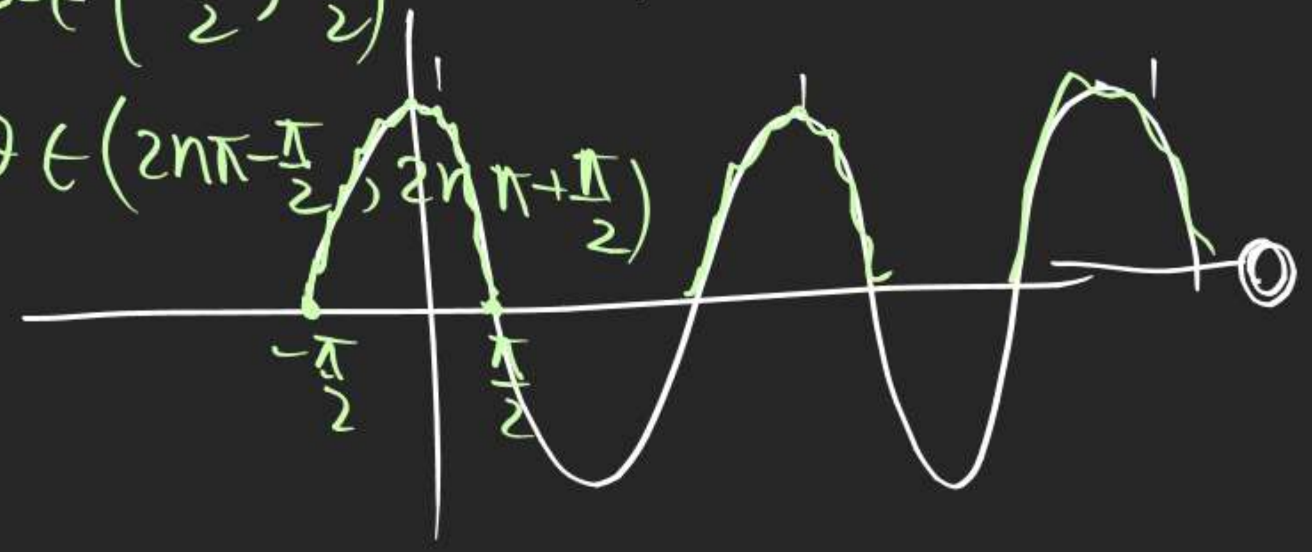
Q $\cos\theta > 0$



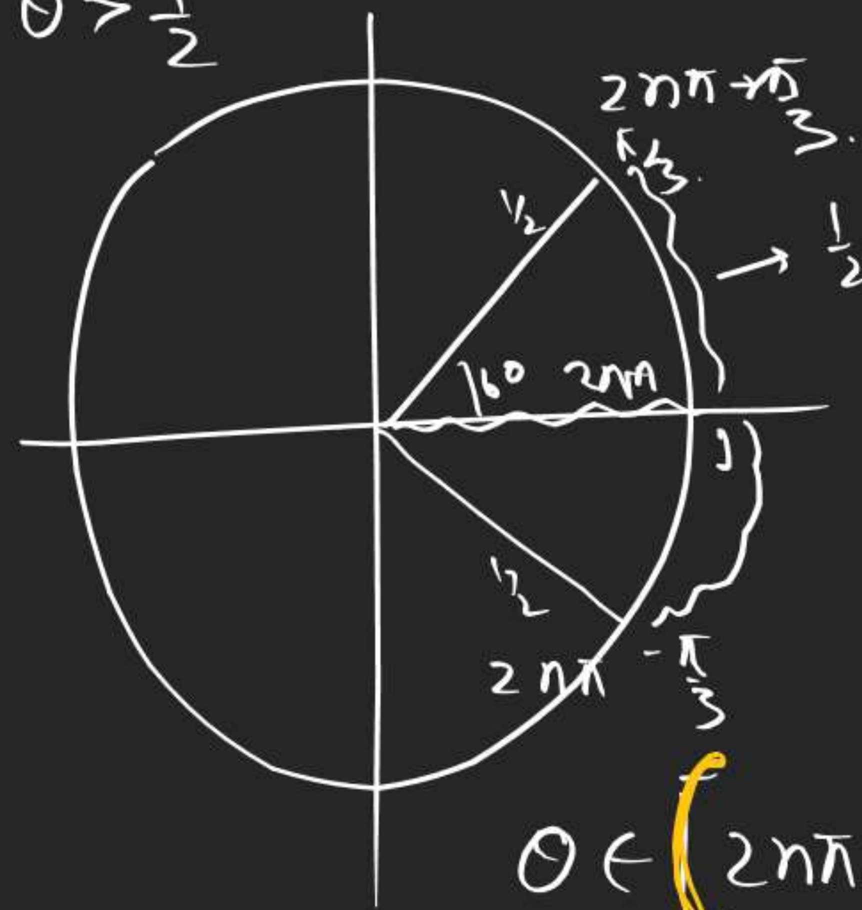
$\theta \in \left(2n\pi - \frac{\pi}{2}, 2n\pi + \frac{\pi}{2}\right)$

$\theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$\theta \in \left(2n\pi - \frac{\pi}{2}, 2n\pi + \frac{\pi}{2}\right)$

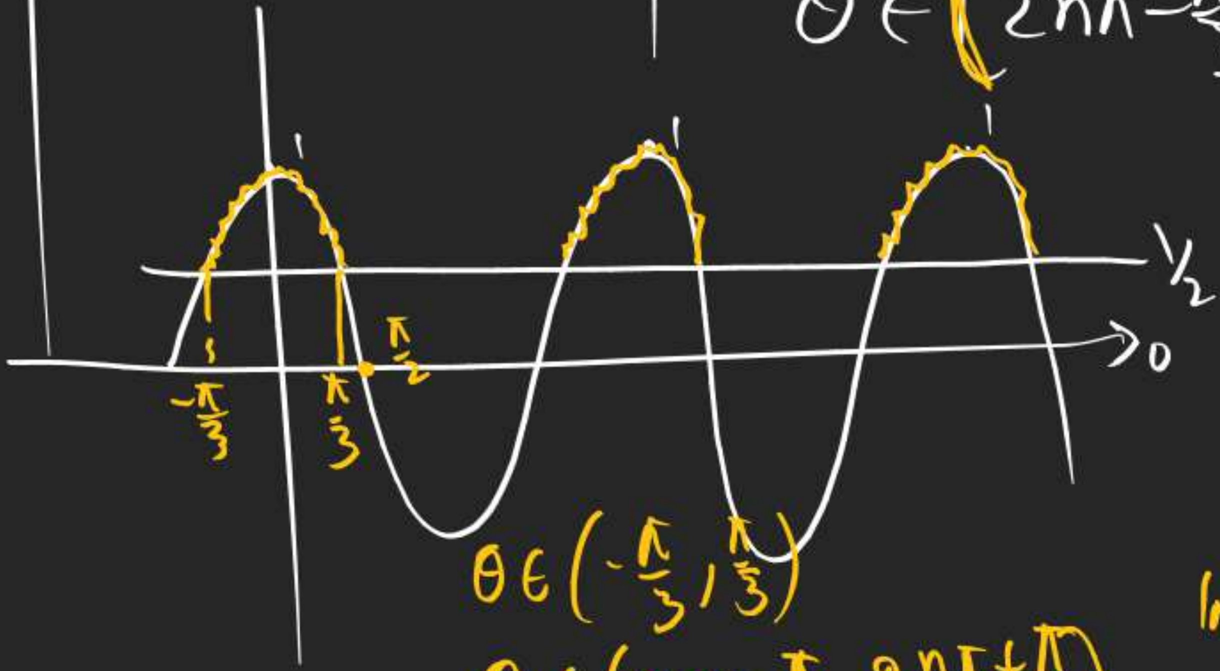


Q $\cos\theta > \frac{1}{2} \rightarrow \pi/3$



→ $\frac{1}{2}$ se 1 tak sari value $\frac{1}{2} \le a \le 1$

$\theta \in \left(2n\pi - \frac{\pi}{3}, 2n\pi + \frac{\pi}{3}\right)$



$\theta \in \left(-\frac{\pi}{3}, \frac{\pi}{3}\right)$

$\theta \in \left(2n\pi - \frac{\pi}{3}, 2n\pi + \frac{\pi}{3}\right)$

as $\cos\theta$ Repeats is graph at 2π distance So in Gen. Sol. We add $2n\pi$

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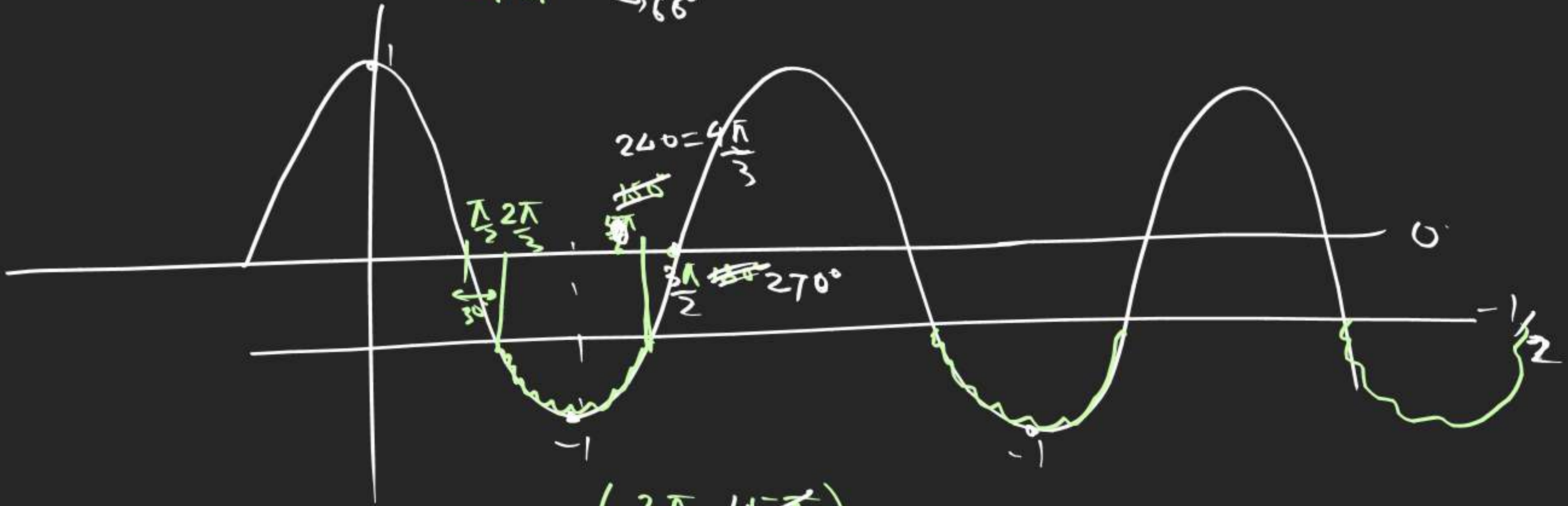
$\rightarrow 2^{nd}/3^{rd}$

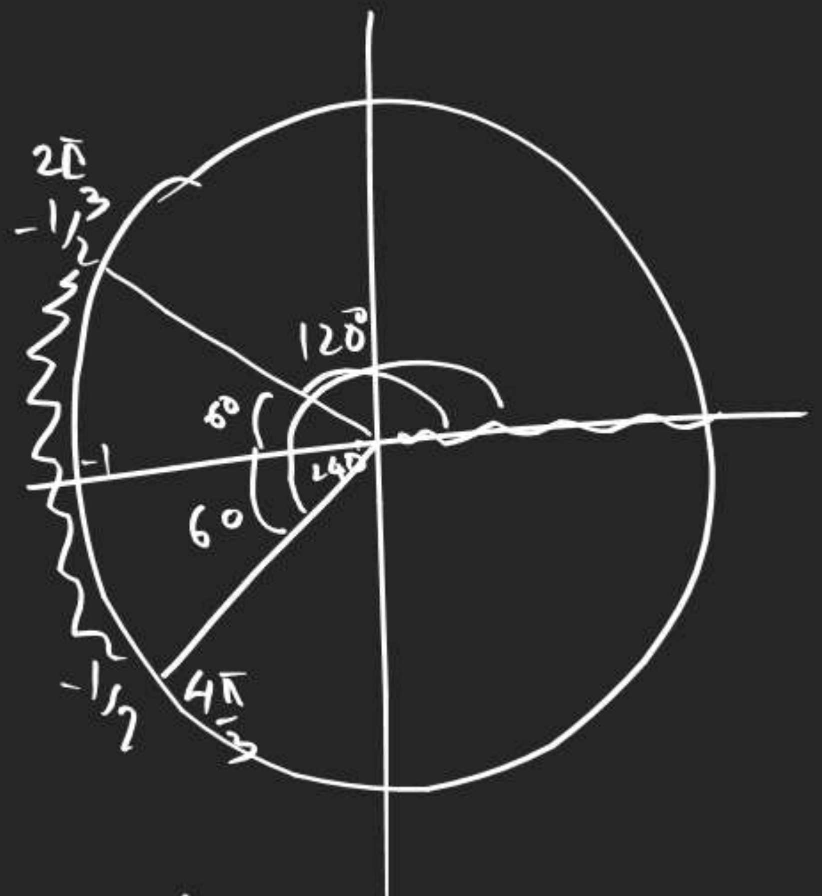
Q $\cos\theta < -\boxed{\frac{1}{2}}$ G.S ?

नीचे $\rightarrow 60^\circ$



$\theta \in \left(\frac{2\pi}{3}, \frac{4\pi}{3}\right)$

$\theta \in \left(2n\pi + \frac{2\pi}{3}, 2n\pi + \frac{4\pi}{3}\right)$



$\theta \in \left(2n\pi + \frac{2\pi}{3}, 2n\pi + \frac{4\pi}{3}\right)$

26 – 35

E 36, 37, 38, 39, E 40, E 41

48, 49