LIMIT

INTRODUCTION :

The concept of limit of a function is one of the fundamental ideas that distinguishes calculus from algebra and trigonometry. We use limits to describe the way a function ƒ varies. Some functions vary continuously; small changes in x produce only small changes in ƒ(x). Other functions can have values that jump or vary erratically. We also use limits to define tangent lines to graphs of functions. This geometric application leads at once to the important concept of derivative of a function. **2.**

DEFINITION :

Let ƒ(x) be defined on an open interval about „a" except possibly at „a" itself. If ƒ(x) gets arbitrarily close to L (a finite number) for all x sufficiently close to „a" we say that ƒ(x) approaches the limit L as x approaches „a" and we write ƒ(x) = L and say "the limit of ƒ(x), as x approaches a, equals L". This implies if we can make the value of ƒ(x) arbitrarily close to L (as close to L as we like) by taking x to be sufficiently close to a (on either side of a) but not equal to a. x a Lim

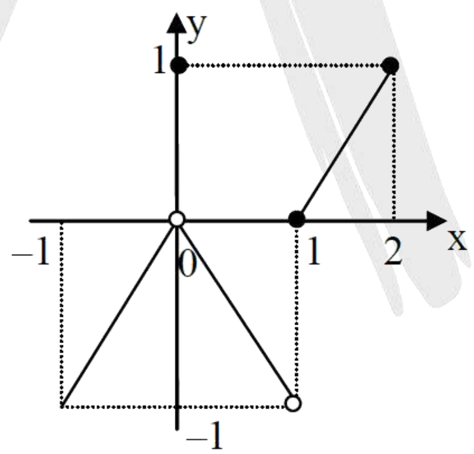
LEFT HAND LIMIT AND RIGHT HAND LIMIT OF A FUNCTION :

The value to which f(x) approaches, as x tends to ' a ' from the left hand side $(x \rightarrow a^-)$is called left hand limit of f(x) at x = a. Symbolically, LHL $= \mathrm{Lim}_{x\rightarrow a^-}\ f(x) = \mathrm{Lim}_{h\rightarrow 0}\ f(a-h)$.

The value to which f(x) approaches, as x tends to ' a ' from the right hand side $(x \rightarrow a^+)$is called right hand limit of f(x) at x = a. Symbolically, RHL $= \mathrm{Lim}_{x\rightarrow a^+}\ f(x) = \mathrm{Lim}_{h\rightarrow 0}\ f(a+h)$.

Limit of a function $\mathbf{f(x)}$ is said to exist as, $\mathbf{x \rightarrow a}$ when $\mathbf{Lim_{x\rightarrow a^-}\ f(x) = Lim_{x\rightarrow a^+}\ f(x)}$ = Finite quantity.

Example: Graph of $\mathbf{y = f(x)}$



$\mathrm{Lim}_{x\rightarrow -1^+}\ f(x) = \mathrm{Lim}_{h\rightarrow 0}\ f(-1+h) = f(-1^+) = -1$

$\mathrm{Lim}_{x\rightarrow 0^-}\ f(x) = \mathrm{Lim}_{h\rightarrow 0}\ f(0-h) = f(0^-) = 0$

$\mathrm{Lim}_{x\rightarrow 0^+}\ f(x) = \mathrm{Lim}_{h\rightarrow 0}\ f(0+h) = f(0^+) = 0$

$\mathrm{Lim}_{x\rightarrow 1^-}\ f(x) = \mathrm{Lim}_{h\rightarrow 0}\ f(1-h) = f(1^-) = -1$

$\mathrm{Lim}_{x\rightarrow 2^-}\ f(x) = \mathrm{Lim}_{h\rightarrow 0}\ f(2-h) = f(2^-) = 1$

$\mathrm{Lim}_{x\rightarrow 0}\ f(x) = 0$ and $\mathrm{Lim}_{x\rightarrow 1}\ f(x)$ does not exist.

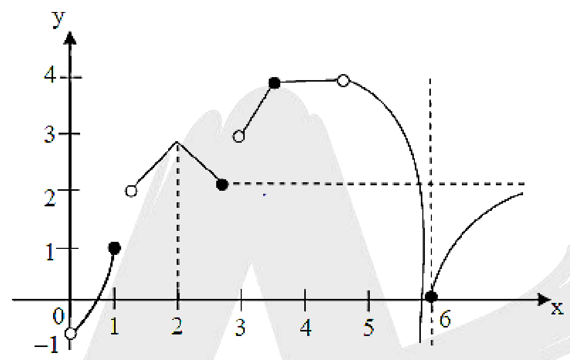
Important note :

In $\text{Lim}_{x\to a}\ f(x)$, $x \to a$ necessarily implies $x \neq a$. That is while evaluating limit at $x = a$, we are not concerned with the value of the function at $x = a$. In fact the function may or may not be defined at $x = a$.

Also it is necessary to note that if $f(x)$ is defined only on one side of '$\mathbf{x = a}$', one sided limits are good enough to establish the existence of limits, and if $f(x)$ is defined on either side of 'a' both sided limits are to be considered.

As in $\lim_{x\to 1}\cos^{-1} x = 0$, though $f(x)$ is not defined for $x > 1$, even in it's immediate vicinity.

Illustration 1 : Consider the adjacent graph of $y = f(x)$ Find the following :



(a) $\lim_{x\to 0^-} f(x)$ (b) $\lim_{x\to 0^+} f(x)$ (c) $\lim_{x\to 1^-} f(x)$

(d) $\lim_{x\to 1^+} f(x)$ (e) $\lim_{x\to 2^-} f(x)$ (f) $\lim_{x\to 2^+} f(x)$

(g) $\lim_{x\to 3^-} f(x)$ (h) $\lim_{x\to 3^+} f(x)$ (i) $\lim_{x\to 4^-} f(x)$

(j) $\lim_{x\to 4^+} f(x)$ (k) $\lim_{x\to\infty} f(x) = 2$ (1) $\lim_{x\to 6} f(x) = -\infty$

Solution : (a) As $x \to 0^-$: limit does not exist (the function is not defined to the left of $x = 0$)

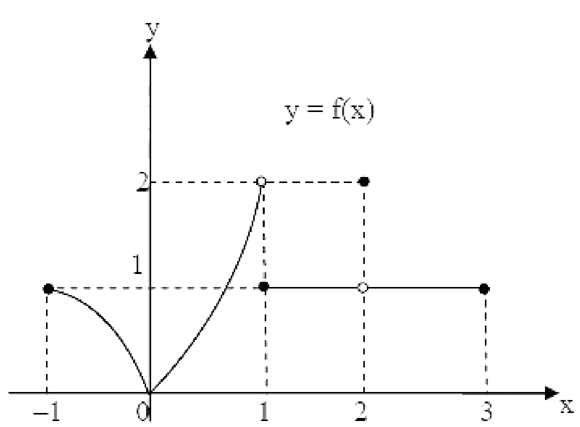
(b) As $\to 0^+ : f(x) \to -1 \Rightarrow \lim_{x\to 0^+} f(x) = -1$. (c) As $x \to 1^- : f(x) \to 1 \Rightarrow \lim_{x\to 1^-} f(x) = 1$.

(d) As $x \to 1^+ : f(x) \to 2 \Rightarrow \lim_{x\to 1^+} f(x) = 2$. (e) As $x \to 2^- : f(x) \to 3 \Rightarrow \lim_{x\to 2^-} f(x) = 3$.

(f) As $x \to 2^+ : f(x) \to 3 \Rightarrow \lim_{x\to 2^-} f(x) = 3$. (g) As $x \to 3^- : f(x) \to 2 \Rightarrow \lim_{x\to 3^-} f(x) = 2$.

Do yourself-1:

(i) Which of the following statements about the function $y = f(x)$ graphed here are true, and which are false ?



(a) $\lim_{x \to -1^+} f(x) = 1$ (b) $\lim_{x \to 2} f(x)$ does not exist

(c) $\lim_{x \to 2} f(x) = 2$ (d) $\lim_{x \to l^-} f(x) = 2$

(e) $\lim_{x \to 1} f(x)$ does not exist (f) $\lim_{x \to 0^+} f(x) = \lim_{x \to 0^-} f(x)$

(g) $\lim_{x \to c} f(x)$ exists at every $c \in (-1,1)$ (h) $\lim_{x \to c} f(x)$ exists at every $c \in (1,3)$

(i) $\lim_{x \to l^-} f(x) = 0$ (j) $\lim_{x \to 3^+} f(x)$ does not exist.

4. FUNDAMENTAL THEOREMS ON LIMITS :

Let $\lim_{x \to a} f(x) = \ell$ and $\lim_{x \to a} g(x) = m$. If ℓ and m exist finitely then :

(a) Sum rule : $\mathrm{Lim}_{x \to a} \{f(x) + g(x)\} = \ell + m$

(b) Difference rule : $\mathrm{Lim}_{x \to a} \{f(x) - g(x)\} = \ell - m$

(c) Product rule : $\mathrm{Lim}_{x \to a}\ f(x) \cdot g(x) = \ell \cdot m$

(d) Quotient rule : $\mathrm{Lim}_{x \to a} \frac{f(x)}{g(x)} = \frac{\ell}{m}$, provided $m \neq 0$

(e) Constant multiple rule : $\mathrm{Lim}_{x \to a}\ kf(x) = k\mathrm{Lim}_{x \to a}\ f(x)$; where k is constant.

(f) Power rule : If m and n are integers then $\underset{x \to a}{}[f(x)]^{m/n} = \ell^{m/n}$ provided $\ell^{m/n}$ is a real number.

(g) $\mathrm{Lim}_{x \to a}\ f[g(x)] = f(\mathrm{Lim}_{x \to a}\ g(x)) = f(m)$; provided f(x) is continuous at $x = m$.

For example : $\mathrm{Lim}_{x \to a}\ \ell n(g(x)) = \ell n[\mathrm{Lim}_{x \to a}\ g(x)]$

$= \ell n(m)$; provided ℓnx is continuous at $x = m, m = \lim_{x \to a} g(x)$.

5. INDETERMINATE FORMS :

$\frac{0}{0}, \frac{\infty}{\infty}, \infty - \infty, 0 \times \infty, 1^\infty, 0^0, \infty^0$

Initially we will deal with first five forms only and the other two forms will come up after we have gone through differentiation.

Note : (i) Here 0,1 are not exact, infact both are approaching to their corresponding values.

(ii) We cannot plot ∞ on the paper. Infinity (∞) is a symbol and not a number It does not obey the laws of elementary algebra,

(a) $\infty + \infty \to \infty$ (b) $\infty \times \infty \to \infty$ (c) $\infty^\infty \to \infty$ (d) $0^\infty \to 0$

6. GENERAL METHODS TO BE USED TO EVALUATE LIMITS :

(A) Factorization :

Important factors :

(i) $x^n - a^n = (x - a)(x^{n-1} + ax^{n-2} + \cdots \ldots \ldots .. + a^{n-1}), n \in N$

(ii) $x^n + a^n = (x + a))(x^{n-1} - ax^{n-2} + \cdots \ldots \ldots \ldots .. + a^{n-1})$, n is an odd natural number.

Note : $\mathrm{Lim}_{x \to a} \frac{x^n - a^n}{x - a} = na^{n-1}$

Note : $\mathrm{Lim}_{x \to a} \frac{x^n - a^n}{x - a} = na^{n-1}$

Illustration 2: Evaluate : $\lim_{x\to 2}\left[\frac{1}{x-2}-\frac{2(2x-3)}{x^3-3x^2+2x}\right]$

Solution : We have

$$\lim_{x\to 2}\left[\frac{1}{x-2}-\frac{2(2x-3)}{x^3-3x^2+2x}\right]=\lim_{x\to 2}\left[\frac{1}{x-2}-\frac{2(2x-3)}{x(x-1)(x-2)}\right]=\lim_{x\to 2}\left[\frac{x(x-1)-2(2x-3)}{x(x-1)(x-2)}\right]$$

$$=\lim_{x\to 2}\left[\frac{x^2-5x+6}{x(x-1)(x-2)}\right]=\lim_{x\to 2}\left[\frac{(x-2)(x-3)}{x(x-1)(x-2)}\right]=\lim_{x\to 2}\left[\frac{x-3}{x(x-1)}\right]=-\frac{1}{2}$$

Do yourself -2 :

(a) Evaluate : $\lim_{x\to 1}\frac{x-1}{2x^2-7x+5}$

(b) Rationalization or double rationalization :

Illustration 3: Evaluate : $\lim_{x\to 1}\frac{4-\sqrt{15x+1}}{2-\sqrt{3x+1}}$

Solution : $\lim_{x\to 1}\frac{4-\sqrt{15x+1}}{2-\sqrt{3x+1}}=\lim_{x\to 1}\frac{(4-\sqrt{15x+1})(2+\sqrt{3x+1})(4+\sqrt{15x+1})}{(2-\sqrt{3x+1})(4+\sqrt{15x+1})(2+\sqrt{3x+1})}$

$$\lim_{x\to 1}\frac{(15-15x)}{(3-3x)}\times\frac{2+\sqrt{3x+1}}{4+\sqrt{15x+1}}=\frac{5}{2}$$

Illustration 4: Evaluate : $\lim_{x\to 1}\left(\frac{\sqrt{x^2+8}-\sqrt{10-x^2}}{\sqrt{x^2+3}-\sqrt{5-x^2}}\right)$

Solution : This is of the form $\frac{3-3}{2-2}=\frac{0}{0}$ if we put $x=1$

To eliminate the $\frac{0}{0}$ factor, multiply by the conjugate of numerator and the conjugate of the denominator

$$\therefore \text{Limit } =\lim_{x\to 1}\left(\sqrt{x^2+8}-\sqrt{10-x^2}\right)\frac{\left(\sqrt{x^2+8}+\sqrt{10-x^2}\right)}{\left(\sqrt{x^2+8}+\sqrt{10-x^2}\right)}$$

$$\times\frac{\left(\sqrt{x^2+3}+\sqrt{5-x^2}\right)}{\left(\sqrt{x^2+3}+\sqrt{5-x^2}\right)\left(\sqrt{x^2+3}-\sqrt{5-x^2}\right)}$$

$$\lim_{x\to 1}\frac{\sqrt{x^2+3}+\sqrt{5-x^2}}{\sqrt{x^2+8}+\sqrt{10-x^2}}\times\frac{(x^2+8)-(10-x^2)}{(x^2+3)-(5-x^2)}=\lim_{x\to 1}\left(\frac{\sqrt{x^2+3}+\sqrt{5-x^2}}{\sqrt{x^2+8}+\sqrt{10-x^2}}\right)\times 1=\frac{2+2}{3+3}=\frac{2}{3}$$

Do yourself - 3 :

(i) Evaluate : $\lim_{x\to 0}\frac{\sqrt{p+x}-\sqrt{p-x}}{\sqrt{q+x}-\sqrt{q-x}}$

(ii) Evaluate : $\lim_{x\to a}\frac{\sqrt{a+2x}-\sqrt{3x}}{\sqrt{3a+x}-2\sqrt{x}}$, $a\neq 0$

(iii) If $G(x)=-\sqrt{25-x^2}$, then find the $\lim_{x\to 1}\left(\frac{G(x)-G(1)}{x-1}\right)$

(c) Limit when $x\to\infty$:

(i) Divide by greatest power of x in numerator and denominator.

(ii) Put $x=1/y$ and apply $y\to 0$

Illustration 5: Evaluate : $\text{Lim}_{x\to\infty}\frac{x^2+x+1}{3x^2+2x-5}$

Solution : $\text{Lim}_{x\to\infty}\frac{x^2+x+1}{3x^2+2x-5}\left(\frac{\infty}{\infty}\text{ form }\right)$

Put $x=\frac{1}{y}$

$$\text{Limit } = \text{Lim}_{y\to 0}\frac{1+y+y^2}{3+2y-5y^2}=\frac{1}{3}$$

Illustration 6: If $\lim_{x\to\infty}\left(\frac{x^3+1}{x^2+1}-(ax+b)\right)=2$, then

(A) $a=1, b=1$ (B) $a=1, b=2$ (C) $a=1, b=-2$ (D) none of these

Solution : $\lim\limits_{x\to\infty}\left(\frac{x^3+1}{x^2+1}-(ax+b)\right)=2\Rightarrow\lim\limits_{x\to\infty}\frac{x^3(1-a)-bx^2-ax+(1-b)}{x^2+1}=2$

$$\lim_{x\to\infty}\frac{x(1-a)-b-\frac{a}{x}+\frac{(1-b)}{x^2}}{1+\frac{1}{x^2}}=2\Rightarrow 1-a=0,-b=2\Rightarrow a=1,b=-2$$

$\Rightarrow 1-a=0,-b=2\Rightarrow a=1,b=-2$ Ans. (C)

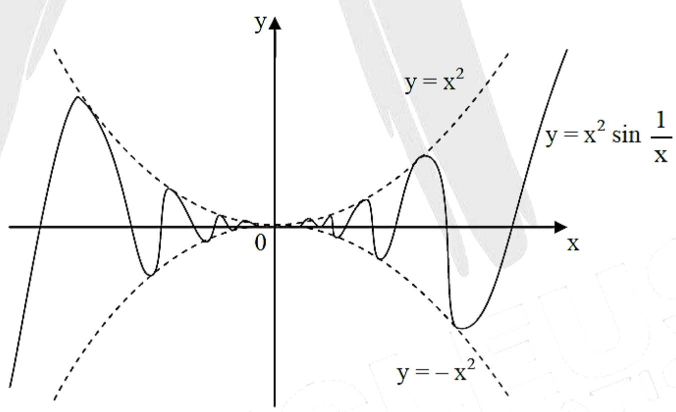
Do yourself 4 :

(i) Evaluate : $\lim_{n\to\infty}\frac{\underline{|n+2}+\underline{|n+1}}{\underline{|n+2}-\underline{|n+1}}$

(ii) Evaluate : $\lim_{n\to\infty}\left(n-\sqrt{n^2+n}\right)$

(d) Squeeze play theorem (Sandwich theorem) :

Statement : If $f(x)\le g(x)\le h(x)$; $\forall x$ in the neighbourhood at $x=a$ and



$\text{Lim}_{x\to a} f(x)=\ell=\text{Lim}_{x\to a}\ h(x)$ then $\text{Lim}_{x\to a}\ g(x)=\ell$

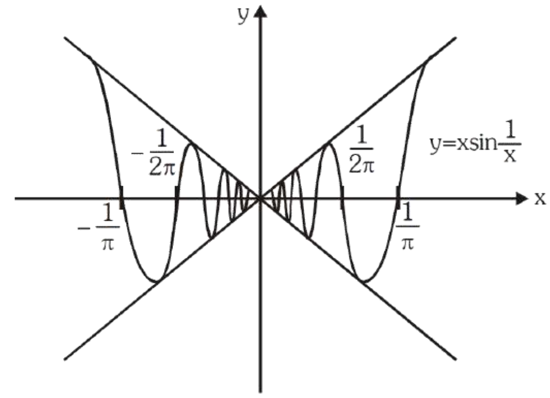
Ex.1 $\text{Lim}_{x\to 0}\ x^2\sin\frac{1}{x}=0,$

$\sin\left(\frac{1}{x}\right)$ lies between -1 and 1

As $\Rightarrow -x^2\le x^2\sin\frac{1}{x}\le x^2\ \Rightarrow \text{Lim}_{x\to 0}\ x^2\sin\frac{1}{x}=0$

$\text{Lim}_{x\to 0}(-x^2)=\text{Lim}_{x\to 0}\ x^2=0$

Ex. 2



$$\lim_{x\to 0} x\sin\frac{1}{x} = 0$$

$\because \sin\left(\frac{1}{x}\right)$ lies between -1 and 1

$$\Rightarrow -x \le x\sin\frac{1}{x} \le x$$

$$\Rightarrow \text{Lim}_{x\to 0}\ x\sin\frac{1}{x} = 0 \text{ as } \text{Lim}_{x\to 0}(-x) = \text{Lim}_{x\to 0}\ x = 0$$

Illustration 7: Evaluate: $\lim_{n\to\infty}\frac{\lfloor x\rfloor+\lfloor 2x\rfloor+\lfloor 3x\rfloor+\cdots..\lfloor nx\rfloor}{n^2}$ (where [.] denotes the greatest integer function.)

Solution: We know that $x-1<[x]\le x$

$$\Rightarrow x+2x+\cdots\ldots nx-n<\sum_{r=1}^{n}[rx]\le x+2x+\cdots\ldots.+nx$$

$$\Rightarrow \frac{xn}{2}(n+1)-n<\sum_{r=1}^{n}[rx]\le\frac{x\cdot n(n+1)}{2}\Rightarrow\frac{x}{2}\left(1+\frac{1}{n}\right)-\frac{1}{n}<\frac{1}{n^2}\sum_{r=1}^{n}[rx]\le\frac{x}{2}\left(1+\frac{1}{n}\right)$$

Now, $\lim_{n\to\infty}\frac{x}{2}\left(1+\frac{1}{n}\right)=\frac{x}{2}$ and $\lim_{n\to\infty}\frac{x}{2}\left(1+\frac{1}{n}\right)-\frac{1}{n}=\frac{x}{2}$

Thus, $\lim_{n\to\infty}\frac{[x]+[2x]+\cdots\ldots+[nx]}{n^2}=\frac{x}{2}$

7. LIMIT OF TRIGONOMETRIC FUNCTIONS :

$$\text{Lim}_{x\to 0}\frac{\sin x}{x}=1=\text{Lim}_{x\to 0}\frac{\tan x}{x}=\text{Lim}_{x\to 0}\frac{\tan^{-1}x}{x}=\text{Lim}_{x\to 0}\frac{\sin^{-1}x}{x}$$

[where x is measured in radians]

(a) If $\text{Lim}_{x\to a} f(x)=0$, then $\text{Lim}_{x\to a}\frac{\sin f(x)}{f(x)}=1$, e.g. $\text{Lim}_{x\to 1}\frac{\sin(\ell nx)}{(\ell nx)}=1$

Illustration 8: Evaluate : $\lim_{x\to 0}\frac{x^3\cot x}{1-\cos x}$

Solution : $\lim_{x\to 0}\frac{x^3\cos x}{\sin x(1-\cos x)}=\lim_{x\to 0}\frac{x^3\cos x(1+\cos x)}{\sin x\cdot\sin^2 x}=\lim_{x\to 0}\frac{x^3}{\sin^3 x}\cdot\cos x(1+\cos x)=2$

Illustration 9: Evaluate : $\lim_{x\to 0}\frac{(2+x)\sin(2+x)-2\sin 2}{x}$

Solution : $\lim_{x\to 0}\frac{2(\sin(2+x)-\sin 2)+x\sin(2+x)}{x}=\lim_{x\to 0}\left(\frac{2.2\cos\left(2+\frac{x}{2}\right)\sin\frac{x}{2}}{x}+\sin(2+x)\right)$

$$=\lim_{x\to 0}\frac{2\cos\left(2+\frac{x}{2}\right)\sin\frac{x}{2}}{\frac{x}{2}}+\lim_{x\to 0}\sin(2+x)=2\cos 2+\sin 2$$

Illustration 10: Evaluate : $\lim_{n\to\infty}\frac{\sin\frac{a}{n}}{\tan\frac{b}{n+1}}$

Solution : As $n\to\infty,\frac{1}{n}\to 0$ and $\frac{a}{n}$ also tends to zero $\sin\frac{a}{n}$ should be written as $\frac{\sin\frac{a}{n}}{\frac{a}{n}}$ so that it looks like

$$\lim_{\theta\to 0}\frac{\sin\theta}{\theta}$$

The given limit $= \lim_{n\to\infty}\left(\frac{\sin\frac{a}{n}}{\frac{a}{n}}\right)\left(\frac{\frac{b}{n+1}}{\tan\frac{b}{n+1}}\right)\cdot\frac{a(n+1)}{n\cdot b}$

$$= \lim_{n\to\infty}\left(\frac{\sin\frac{a}{n}}{\frac{a}{n}}\right)\left(\frac{\frac{b}{n+1}}{\tan\frac{b}{n+1}}\right)\cdot\frac{a}{b}\left(1+\frac{1}{n}\right) = 1\times 1\times\frac{a}{b}\times 1 = \frac{a}{b}$$

Do yourself – 5

(i) Evaluate :

(a) $\lim_{x\to 0}\frac{\sin\alpha x}{\tan\beta x}$ (b) $\lim_{x\to y}\frac{\sin^2 x-\sin^2 y}{x^2-y^2}$ c) $\lim_{h\to 0}\frac{(a+h)^2\sin(a+h)-a^2\sin a}{h}$

8. LIMIT OF EXPONENTIAL FUNCTIONS :

(a) $\text{Lim}_{x\to 0}\frac{a^x-1}{x} = \ell\text{n } a\ (a>0)$ In particular $\text{Lim}_{x\to 0}\frac{e^x-1}{x} = 1.$

In general if $\text{Lim}_{x\to a} f(x) = 0$, then $\text{Lim}_{x\to a}\frac{a^{f(x)}-1}{f(x)} = \ell\text{n}a, a>0$

Illustration 11: Evaluate : $\lim_{x\to 0}\frac{e^{\tan x}-e^x}{\tan x-x}$

Solution : $\lim\limits_{x\to 0}\frac{e^{\tan x}-e^x}{\tan x-x} = \lim\limits_{x\to 0}\frac{e^x\times e^{(\tan x-x)}-e^x}{\tan x-x}$

$= \lim\limits_{x\to 0}\frac{e^x(e^{\tan x-x}-1)}{\tan x-x} = \lim\limits_{x\to 0 y\to 0}\frac{e^x(e^y-1)}{y}$ where $y = \tan x - x$ and $\lim\limits_{y\to 0}\frac{e^y-1}{y} = 1$

$= e^0\times 1$

$[\text{as } x\to 0, \tan x - x\to 0]$

$= 1\times 1 = 1$

Do yourself -6 :

(i) Evaluate : $\lim_{x\to a}\frac{e^x-e^a}{x-a}$

(ii) Evaluate : $\lim_{x\to 0}\frac{2^x-1}{(1+x)^{1/2}-1}$

(b) (i) $\text{Lim}_{x\to 0}(1+x)^{1/x} = e = \text{Lim}_{x\to\infty}\left(1+\frac{1}{x}\right)^n$ (Note: The base and exponent depends on the same (variable.) In general, if $\text{Lim}_{x\to a} f(x) = 0$, then $\text{Lim}_{x\to a}(1+f(x))^{1/f(x)} = e$

(ii) $\text{Lim}_{x\to 0}\frac{\ell n(1+x)}{x} = 1$

(iii) If $\text{Lim}_{x\to a} f(x) = 1$ and $\text{Lim}_{x\to a}\phi(x) = \infty$, then; $\text{Lim}_{x\to a}[f(x)]^{\phi(x)} = e^k$ where $k = \text{Lim}_{x\to a}\phi(x)[f(x)-1]$

Illustration 12: Evaluate $\text{Lim}_{x\to 1}(\log_3 3x)^{\log_x 3}$

Solution : $\text{Lim}_{x\to 1}(\log_3 3x)^{\log_x 3} = \text{Lim}_{x\to 1}(\log_3 3+\log_3 x)^{\log_x 3}$

$= \lim\limits_{x\to 1}(1+\log_3 x)^{1/\log_3 x} = e \because \log_b a = \frac{1}{\log_a b}$

Illustration 13: Evaluate : $\text{Lim}_{x\to 0}\frac{x\ell n(1+2\tan x)}{1-\cos x}$

Solution : $\text{Lim}_{x\to 0}\frac{x\ln(1+2\tan x)}{1-\cos x}=\text{Lim}_{x\to 0}\frac{x\ln(1+2\tan x)}{\frac{1-\cos x}{x^2}\cdot x^2}\cdot\frac{2\tan x}{2\tan x}=4$

Illustration 14 : Evaluate : $\lim_{x\to\infty}\left(\frac{2x^2-1}{2x^2+3}\right)^{4x^2+2}$

Solution : Since it is in the form of 1^∞

$$\lim_{x\to\infty}\left(\frac{2x^2-1}{2x^2+3}\right)^{4x^2+2}=e^{\lim\limits_{x\to\infty}}\left(\frac{2x^2-1-2x^2-3}{2x^2+3}\right)(4x^2+2)=e^{-8}$$

Do yourself -7 :

(i) Evaluate : $\lim_{x\to\infty}x\{\ell n(x+a)-\ell nx\}$

(ii) Evaluate : $\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{pn+q}$

(iii) Evaluate : $\lim_{x\to 0}\left(1+\tan^2\sqrt{x}\right)^{\frac{1}{2x}}$

(iv) Evaluate : $\lim_{x\to\infty}\left(\frac{x+6}{x+1}\right)^{x+4}$

(c) If $\text{Lim}_{x\to a}f(x)=A>0$ and $\text{Lim}_{x\to a}\phi(x)=B$, then $\text{Lim}_{x\to a}[f(x)]^{\phi(x)}=e^{B\ln A}=A^B$

Illustration 15: Evaluate : $\lim_{x\to\infty}\left(\frac{7x^2+1}{5x^2-1}\right)^{\frac{x^5}{1-x^3}}$

$$\text{Here } f(x)=\frac{7x^2+1}{5x^2-1},\phi(x)=\frac{x^5}{1-x^3}=\frac{x^2\cdot x^3}{1-x^3}=\frac{x^2}{\frac{1}{x^3}-1}$$

Solution : $\therefore\lim\limits_{x\to\infty}f(x)=\frac{7}{5}$ and $\lim\limits_{x\to\infty}\phi(x)\to\infty$

$\Rightarrow\lim\limits_{x\to\infty}(f(x))^{\phi(x)}=\left(\frac{7}{5}\right)=0$

Do yourself -8 :

(i) Evaluate : $\lim_{x\to\infty}\left(\frac{1+5x^2}{1+3x^2}\right)^{-x^2}$

9. LIMIT USING SERIES EXPANSION :

Expansion of function like binomial expansion, exponential and logarithmic expansion, expansion of sin x , cos x, tanx should be remembered by heart which are given below :

(a) $a^x=1+\frac{x\ell na}{1!}+\frac{x^2\ell n^2a}{2!}+\frac{x^3\ell n^3a}{3!}+\cdots\ldots..a>0$ (b) $e^x=1+\frac{x}{1!}+\frac{x^2}{2!}+\frac{x^3}{3!}+\cdots\ldots$

(c) $\text{lm}(1+x)=x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+\cdots..$ for $-1<x\le 1$ (d) $\sin x=x-\frac{x^3}{3!}+\frac{x^5}{5!}-\frac{x^7}{7!}+\cdots\ldots$

(e) $\cos x=1-\frac{x^2}{2!}+\frac{x^4}{4!}-\frac{x^6}{6!}+\cdots.$ (f) $\tan x=x+\frac{x^3}{3}+\frac{2x^5}{15}+\cdots$

(g) $\tan^{-1}x=x-\frac{x^3}{3}+\frac{x^5}{5}-\frac{x^7}{7}+\cdots$

(h) $\sin^{-1}x=x+\frac{1^2}{3!}x^3+\frac{1^2\cdot 3^2}{5!}x^5+\frac{1^2\cdot 3^2\cdot 5^2}{7!}x^7+\cdots$

(i) $\sec^{-1} x = 1 + \frac{x^2}{2!} + \frac{5x^4}{4!} + \frac{61x^6}{6!} + \cdots$..

(j) $(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \cdots \ldots \ldots \ldots . n \in Q$

Illustration 16: $\lim_{x\to 0} \frac{e^x - e^{-x} - 2x}{x - \sin x}$

Solution : $\lim_{x\to 0} \frac{e^x - e^{-x} - 2x}{x - \sin x} \Rightarrow \lim_{x\to 0} \frac{1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots \ldots - 1 - x + \frac{x^3}{2!} - \frac{x^3}{3!} + \cdots \ldots - 2x}{x - \left(x - \frac{x^3}{3!} + \frac{x^5}{5!} \ldots\ldots\right)}$

$$\Rightarrow \lim_{x\to 0} \frac{2 \cdot \frac{x^3}{6} + 2 \cdot \frac{x^5}{5!} + \cdots .}{\frac{x^3}{6} + \frac{x^5}{5!} \ldots \ldots} \Rightarrow \lim_{x\to 0} \frac{x^3\left(\frac{1}{3} + \frac{1}{60}x^2 + \cdots\right)}{x^3\left(\frac{1}{6} + \frac{1}{120}x^2 + \cdots .\right)} = \frac{1/3}{1/6} = 2$$

Do yourself - 9:

(i) Evaluate : $\text{Lim}_{x\to 0} \frac{x - \sin x}{\sin (x^3)}$

(ii) Evaluate : $\text{Lim}_{x\to 0} \frac{x - \tan^{-1} x}{x^3}$

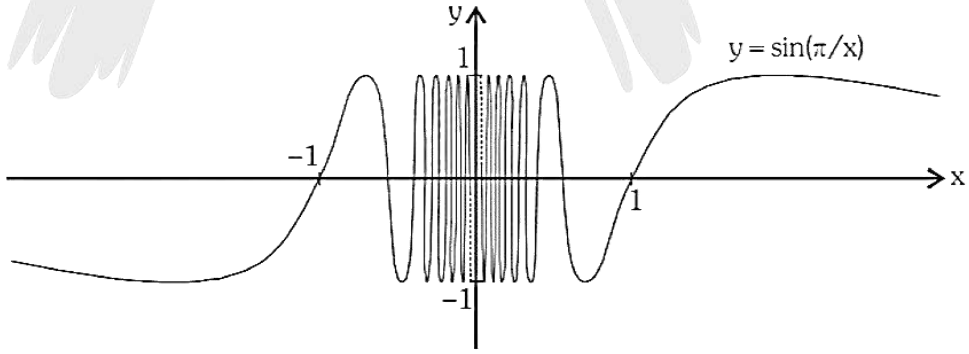
Miscellaneous Illustrations:

Illustration 17 : Evaluate $\lim_{x\to 0} \sin \frac{\pi}{x}$.

Solution: Again the function $f(x) = \sin(\pi/x)$ is undefined at 0 . Evaluating the function for some Small values of x, we get $f(1) = \sin \pi = 0, f\left(\frac{1}{2}\right) = \sin 2\pi = 0$,

$f(0.1) = \sin 10\pi = 0, f(0.01) = \sin 100\pi = 0$.

On the basis of this information we might be tempted to guess that $\lim_{x\to 0} \sin \frac{\pi}{x} = 0$ but this time our guess is wrong. Note that although $f(1/n) = \sin n\pi = 0$ for any integer n, it is also true that $f(x) = 1$ for infinitely many values of x that approach 0 . [In fact, $\sin(\pi/x) = 1$ when $\frac{\pi}{x} = \frac{\pi}{2} + 2n\pi$ and solving for x, we get $x = 2/(4n+1)$]. The graph of f is given in following figure



The dashed line indicate that the values of $\sin(\pi/x)$ oscillate between 1 and -1 infinitely often as x approaches 0 . Since the values of f(x) do not approach a fixed number as x approaches 0 ,

$\Rightarrow \lim_{x\to 0} \sin \frac{\pi}{x}$ does not exist.

ANSWERS FOR DO YOURSELF

1. (i) (a) T (b) F (c) F (d) T (e) T (f) T (g) T (h) T (i) F (j) T **2.** (i) $-\frac{1}{3}$

3: (i) $\frac{\sqrt{q}}{\sqrt{p}}$ (ii) $\frac{2}{3\sqrt{3}}$ (iii) $\frac{1}{\sqrt{24}}$ **4:** (i) 1 (ii) $-\frac{1}{2}$

5 : (i) (a) $\frac{\alpha}{\beta}$ (b) $\frac{\sin 2y}{2y}$ (c) $2a\sin a + a^2\cos a$

6. (i) e^a (ii) $2\ln 2$ **7.** (i) a (ii) e^p (iii) $e^{\frac{1}{2}}$ (iv) e^5

8. (i) 0 **9.** (i) $\frac{1}{6}$ (ii) $\frac{1}{3}$

LIMIT

EXERCISE – 1

[SINGLE CORRECT CHOICE TYPE]

1. $\lim_{x\to1}\left(\frac{1}{1-x}-\frac{3}{1-x^3}\right)$ is equal to

(A) -1 (B) 0 (C) 1 (D) D.N.E

2. $\lim_{x\to0}\frac{\sqrt{1+x}-\sqrt{1-x}}{2x}$ is equal to

(A) 0 (B) 1 (C) $\frac{1}{2}$ (D) $\frac{1}{4}$

3. $\lim_{x\to2}\frac{\sqrt{1+\sqrt{2+x}}-\sqrt{3}}{x-2}$ is equal to

(A) $\frac{1}{\sqrt{3}}$ (B) $\sqrt{3}$ (C) $\frac{1}{4\sqrt{3}}$ (D) $\frac{1}{8\sqrt{3}}$

4. $\lim_{x\to1}\frac{\sqrt[n]{x}-1}{\sqrt[m]{x}-1}$ (m and n integers) is equal to

(A) 0 (B) 1 (C) $\frac{m}{n}$ (D) $\frac{n}{m}$

5. If $\lim_{x\to a}\frac{2x-\sqrt{x^2+3a^2}}{\sqrt{x+a}-\sqrt{2a}}=\sqrt{2}$ (where $a\in R^+$), then a is equal to -

(A) $\frac{1}{3}$ (B) $\frac{1}{2\sqrt{2}}$ (C) $\frac{1}{3\sqrt{2}}$ (D) $\frac{1}{9}$

6. $\lim_{x\to0}\frac{\ell n(\sin 3x)}{\ln(\sin x)}$ is equal to

(A) 0 (B) 1 (C) 2 (D) Non existent

7. $\lim_{x\to0}\frac{\sqrt[3]{1+x^2}-\sqrt[4]{1-2x}}{x+x^2}$ is equal to

(A) $\frac{1}{4}$ (B) $\frac{1}{2}$ (C) 1 (D) D.N.E

8. $\lim_{x\to1}\frac{\sqrt[3]{7+x^3}-\sqrt{3+x^2}}{x-1}$ is equal to

(A) $\frac{1}{4}$ (B) $\frac{1}{6}$ (C) $-\frac{1}{4}$ (D) $-\frac{1}{6}$

9. $\lim_{n\to\infty}\frac{(n+1)^4-(n-1)^4}{(n+1)^4+(n-1)^4}$ is equal to

(A) -1 (B) 0 (C) 1 (D) D.N.E.

10. $\lim_{x\to\infty}\frac{(x+1)^{10}+(x+2)^{10}+\cdots+(x+100)^{10}}{x^{10}+10^{10}}$ is equal to

(A) 1 (B) 100 (C) 200 (D) 10

11. $\lim_{x\to-\infty}\left(\sqrt{x^2-2x-1}-\sqrt{x^2-7x+3}\right)$ is equal to :

(A) $-\frac{5}{2}$ (B) $\frac{5}{2}$ (C) 0 (D) D.N.E.

12. If $\lim_{n\to\infty}\left(\sqrt{2n^2+n}-\lambda\sqrt{2n^2-n}\right)=\frac{1}{\sqrt{2}}$ (where λ is a real number), then -

(A) $\lambda=1$ (B) $\lambda=-1$ (C) $\lambda=\pm1$ (D) $\lambda\in(-\infty,1)$

13. Let $U_n = \frac{n!}{(n+2)!}$ where $n \in N$. If $S_n = \sum_{n=1}^{n} U_n$ then $\lim_{n\to\infty} S_n$ equals

(A) 2 (B) 1 (C) ½ (D) Non existent

14. For $n \in N$, let $a_n = \sum_{k=1}^{n} 2k$ and $b_n = \sum_{k=1}^{n}(2k-1)$. Then $\lim_{n\to\infty}\left(\sqrt{a_n} - \sqrt{b_n}\right)$ is equal to -

(A) 1 (B) $\frac{1}{2}$ (C) 0 (D) 2

15. Let $P_n = \prod_{k=2}^{n}\left(1 - \frac{1}{{}^{k+1}C_2}\right)$. If $\lim_{n\to\infty} P_n$ can be expressed as lowest rational in the form $\frac{a}{b}$, then value of $(a+b)$ is:

(A) 4 (B) 8 (C) 10 (D) 12

16. $\lim_{x\to -1}\frac{\cos 2 - \cos 2x}{x^2 - |x|}$ is equal to

(A) 0 (B) $\cos 2$ (C) $2\sin 2$ (D) $\sin 1$

17. $\lim_{x\to 0}\left(\left[\frac{-5\sin x}{x}\right] + \left[\frac{6\sin x}{x}\right]\right)$ (where [.] denotes greatest integer function) is equal to -

(A) 0 (B) -12 (C) 1 (D) 2

18. Let $f(x) = \left[\frac{\sin x}{x}\right] + \left[\frac{2\sin 2x}{x}\right] + \cdots \ldots + \left[\frac{10\sin 10x}{x}\right]$ (where $[y]$ is the largest integer $\leq y$). The value of $\lim_{x\to 0} f(x)$ equals :

(A) 55 (B) 164 (C) 165 (D) 375

19. Let $f(x) = \frac{\sin\{x\}}{x^2+ax+b}$. If $f(5^+)$and $f(3^+)$exists finitely and are not zero, then the value of $(a+b)$ is (where {.}representsfractionalpartfunction) $-$

(A) 7 (B) 10 (C) 11 (D) 20

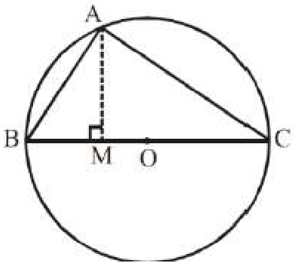
20. $\lim_{x\to 0}\frac{|\cos(\sin(3x))| - 1}{x^2}$ equals

(A) $\frac{-9}{2}$ (B) $\frac{-3}{2}$ (C) $\frac{3}{2}$ (D) $\frac{9}{2}$

21. Let $a = \min\{x^2 + 2x + 3, x \in R\}$ and $b = \lim_{\theta\to 0}\frac{1-\cos\theta}{\theta^2}$. then value of $\sum_{r=0}^{n} a^r \cdot b^{n-r}$ is :

(A) $\frac{2^{n+1}-1}{3.2^n}$ (B) $\frac{2^{n+1}+1}{3.2^n}$ (C) $\frac{4^{n+1}-1}{3.2^n}$ (D) none of these

22. Let BC is diameter of a circle centred at O. Point A is a variable point, moving on the circumference of circle. if $BC = 1$ unit, then $\lim_{A\to B}\frac{BM}{(\text{Area of sector OAB})^2}$ is equal to -



(A) 1 (B) 2 (C) 4 (D) 16

23. $\lim_{x\to\infty}\left(\frac{x^2-2x+1}{x^2-4x+2}\right)^x$ is equal to

(A) 1 (B) e (C) $\frac{1}{e^2}$ (D) e^2

24. $\lim_{x\to 0}(1+\sin x)^{\cos x}$ is equal to

(A) 0 (B) e (C) 1 (D) $\frac{1}{e}$

25. $\lim_{x\to0}(\cos x + a\sin bx)^{1/x}$ is equal to :

(A) e^a (B) e^{ab} (C) e^b (D) $e^{a/b}$

26. $\lim_{x\to0}\left(\tan\left(\frac{\pi}{4}+x\right)\right)^{1/x}$ is equal to

(A) e^{-2} (B) $\frac{1}{e}$ (C) e (D) e^2

27. $\lim_{n\to\infty}(4^n+5^n)^{1/n}$ is equal to

(A) 5 (B) 4 (C) 0(D) D.N.E.

28. $\lim_{x\to\infty}\left(\frac{1^{1/x}+2^{1/x}+3^{1/x}+\cdots+n^{1/x}}{n}\right)^{nx}$ is equal to

(A) n ! (B) 1 (C) $\frac{1}{n!}$ (D) 0

29. If $\lim_{x\to\lambda}\left(2-\frac{\lambda}{x}\right)^{\lambda\tan\left(\frac{\pi x}{2\lambda}\right)}=\frac{1}{e}$, then λ is equal to -

(A) $-\pi$ (B) π (C) $\frac{\pi}{2}$ (D) $-\frac{2}{\pi}$

30. If $\lim_{x\to0}(1+ax+bx^2)^{2/x}=e^3$, then

(A) $a=\frac{3}{2}$ and $b\in R$ (B) $a=\frac{3}{2}$ and $b\in R^+$

(C) $a=0$ and $b=1$ (D) $a=1$ and $b=0$

31. If f(x) is a polynomial of least degree, such that $\lim_{x\to0}\left(1+\frac{f(x)+x^2}{x^2}\right)^{1/x}=e^2$, then f(2) is -

(A) 2 (B) 8 ss(C) 10 (D) 12

32. Let $f(x)=\frac{\tan x}{x}$, then the value of $\lim_{x\to0}([f(x)]+x^2)^{\frac{1}{\{f(x)\}}}$ is equal to (where [], . {.}denotes greatest integer function and fractional part function respectively)-

(A) e^{-3} (B) e^3 (C) e^2 (D) non-existent

33. $\lim_{n\to\infty}\frac{e^n}{\left(1+\frac{1}{n}\right)^{n^2}}$ equals -

(A) 1 (B) $\frac{1}{2}$ (C) e (D) $\sqrt{e}$

34. If f(x) is odd linear polynomial with $f(1)=1$, then $\lim_{x\to0}\frac{2^{f(\tan x)}-2^{f(\sin x)}}{x^2 f(\sin x)}$ is :

(A) 1 (B) $\ell n2$ (C) $\frac{1}{2}\ell n2$ (D) $\cos 2$

35. $\lim_{x\to0}\frac{x(1+a\cos x)-b\sin x}{x^3}=1$ then

(A) $a=-5/2$ (B) $a=-3/2, b=-1/2$

(C) $a=-3/2, b=-5/2$ (D) $a=-5/2, b=-3/2$

36. $\lim_{h\to0}\frac{\sin(a+3h)-3\sin(a+2h)+3\sin(a+h)-\sin a}{h^3}$ is equal to

(A) $\cos a$ (B) $-\cos a$ (C) $\sin a$ (D) $\sin a\cos a$

37. $\lim_{x\to\frac{\pi}{2}}\tan^2 x\left(\sqrt{2\sin^2 x+3\sin x+4}-\sqrt{\sin^2 x+6\sin x+2}\right)$ is equal to

(A) $\frac{3}{4}$ (B) $\frac{1}{6}$ (C) $\frac{1}{12}$ (D) $\frac{5}{12}$

38. $\lim_{x\to\infty}x\left(\arctan\frac{x+1}{x+2}-\arctan\frac{x}{x+2}\right)$ is equal to

(A) $\frac{1}{2}$ (B) $-\frac{1}{2}$ (C) 1 (D) D.N.E.

39. $\lim_{h\to 0}\frac{\tan(a+2h)-2\tan(a+h)+\tan a}{h^2}$ is equal to

(A) $\tan a$ (B) $\tan^2 a$ (C) $\sec a$ (D) $2(\sec^2 a)(\tan a)$

40. $\lim_{x\to 0}\left(2^{x-1}+\frac{1}{2}\right)^{1/x}$ equals

(A) $\sqrt{2}$ (B) $\frac{1}{2}\ell n2$ (C) $\ln 2$ (D) 2

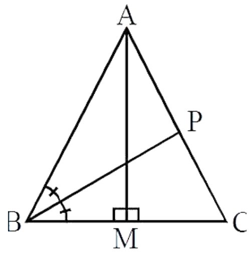
41. If $\lim_{x\to 0}(\cos x+a^3\sin(b^6x))^{\frac{1}{x}}=e^{512}$, then the value of ab^2 is equal to

(A) -512 (B) 512 (C) 8 (D) $8\sqrt{8}$

42. The value of $\lim_{x\to 0}\frac{\sin(\sqrt[3]{x})\ln(1+3x)}{(\tan^{-1}\sqrt{x})^2\left(e^{5(\sqrt[3]{x})}-1\right)}$ is equal to

(A) $\frac{1}{5}$ (B) $\frac{3}{5}$ (C) $\frac{2}{5}$ (D) $\frac{4}{5}$

43. The figure shows an isosceles triangle ABC with $\angle B=\angle C$. The bisector of angle B intersects the side AC at the point P. Suppose that BC remains fixed but the altitude AM approaches 0 , so that $A\to M$ (mid-point of BC). Limiting value of BP, is



(A) $\frac{a}{3}$ (B) $\frac{a}{2}$ (C) $\frac{2a}{3}$ (D) $\frac{3a}{4}$

where a is fixed side BC.

44. The value of $\lim_{x\to 2}\frac{\sec^x\theta-\tan^x\theta-1}{x-2}$ is equal to-

(A) $\sec^2\theta.\ell n\sec\theta+\tan^2\theta\cdot\ell n\tan\theta$ (B) $\sec^2\theta\cdot\ell n\tan\theta+\tan^2\theta\cdot\ell n\sec\theta$ (C) $\sec^2\theta\cdot\ell n\tan\theta-\tan^2\theta\cdot\ell n\sec\theta$ (D) $\sec^2\theta\cdot\ell n\sec\theta-\tan^2\theta\cdot\ell n\tan\theta$

45. Consider the function $f(x)=\begin{cases}1-x, 0\le x\le 1\\ x+2, 1<x<2\\ 4-x, 2\le x\le 4\end{cases}$. Let $\lim_{x\to 1}f(f(x))=\ell$ and $\lim_{x\to 2}f(f(x))=m$ then which one of the following hold good?

(A) ℓ exists but m does not. (B) m exists but ℓ does not.

(C) Both ℓ and m exist (D) Neither ℓ nor m exist.

46. If $f(x)=e^x$, then $\lim_{x\to 0}(f(f(x)))^{\frac{1}{\{f(x)\}}}$ is equal to (where $\{.\}$ denotes fractional part of x).

(A) f(1) (B) f(0) (C) 0 (D) does not exist

47. Let f(x) be a quadratic function such that $f(0)=f(1)=0$ and $f(2)=1$, then $\lim_{x\to 0}\frac{\cos\left(\frac{\pi}{2}\cos^2 x\right)}{f^2(x)}$ is equal to-

(A) $\frac{\pi}{2}$ (B) π (C) 2π (D) 4π

EXERCISE – 2

1. $\lim_{x\to 1}\frac{x^2-x\cdot\ln x+\ln x-1}{x-1}$

2. $\lim_{x\to 1}\frac{\left[\sum_{K=1}^{100}x^k\right]-100}{x-1}$

3. $\lim_{x\to\frac{\pi}{4}}\frac{1-\tan x}{1-\sqrt{2}\sin x}$

4. $\lim_{x\to 0}\frac{8}{x^8}\left[1-\cos\frac{x^2}{2}-\cos\frac{x^2}{4}+\cos\frac{x^2}{2}\cos\frac{x^2}{4}\right]$

5. $\lim_{\theta\to\frac{\pi}{4}}\frac{\sqrt{2}-\cos\theta-\sin\theta}{(4\theta-\pi)^2}$

6. $\lim\limits_{h\to 0}\frac{\sin\left(\frac{\pi}{3}+4h\right)-4\sin\left(\frac{\pi}{3}+3h\right)+6\sin\left(\frac{\pi}{3}+2h\right)-4\sin\left(\frac{\pi}{3}+h\right)+\sin\frac{\pi}{3}}{h^4}$

7. $\lim_{x\to\infty}x^2\left(\sqrt{\frac{x+2}{x}}-\sqrt[3]{\frac{x+3}{x}}\right)$

8. $\lim_{x\to-\infty}\frac{(3x^4+2x^2)\sin\frac{1}{x}+|x|^3+5}{|x|^3+|x|^2+|x|+1}$

9. If $\ell=\lim_{n\to\infty}\sum_{r=2}^{n}\left((r+1)\sin\frac{\pi}{r+1}-r\sin\frac{\pi}{r}\right)$ then find $\{\ell\}$. 3

(where {} denotes the fractional part function)

10. Find a and b if : (i) $\lim_{x\to\infty}\left[\frac{x^2+1}{x+1}-ax-b\right]=0$ (ii) $\lim_{x\to-\infty}\left[\sqrt{x^2-x+1}-ax-b\right]=0$

11. $\lim_{x\to 0}[\ell n(1+\sin^2 x)\cdot\cot(\ell\ln^2(1+x))]$

12. $\lim_{x\to 0}\frac{27^x-9^x-3^x+1}{\sqrt{2}-\sqrt{1+\cos x}}$

13. (a) $\lim_{x\to 0}\tan^{-1}\frac{a}{x^2}$, where $a\in R$; (b) Plot the graph of the function $f(x)=\lim_{t\to 0}\left(\frac{2x}{\pi}\tan^{-1}\frac{x}{t^2}\right)$

14. Let $\{a_n\},\{b_n\},\{c_n\}$ be sequences such that

(i) $a_n+b_n+c_n=2n+1$; (ii) $a_nb_n+b_nc_n+c_na_n=2n-1$;

(iii) $a_nb_nc_n=-1$; (iv) $a_n<b_n<c_n$

Then find the value of $\mathrm{Lim}_{n\to\infty}(na_n)$.

15. Let $f(x)=ax^3+bx^2+cx+d$ and $g(x)=x^2+x-2$.

If $\lim_{x\to 1}\frac{f(x)}{g(x)}=1$ and $\lim_{x\to-2}\frac{f(x)}{g(x)}=4$, then find the value of $\frac{c^2+d^2}{a^2+b^2}$.

16. $\lim_{x\to\infty}\left[\frac{2x^2+3}{2x^2+5}\right]^{8x^2+3}$

17. $\lim_{x\to\infty}\left(\frac{x+c}{x-c}\right)^x=4$ then find c

18. $\lim_{x\to 1}\left(\tan\frac{\pi x}{4}\right)^{\tan\frac{\pi x}{2}}$

19. $\lim_{x\to 0}\left(\frac{x-1+\cos x}{x}\right)^{\frac{1}{x}}$

20. If $n\in N$ and $a_n=2^2+4^2+6^2+\cdots\ldots+(2n)^2$ and $b_n=1^2+3^2+5^2+\cdots\ldots+(2n-1)^2$. Find the value $\lim_{n\to\infty}\frac{\sqrt{a_n}-\sqrt{b_n}}{\sqrt{n}}$

EXERCISE – 3

1. Let $f: R \to R$ be a positive increasing function with $\lim_{x\to\infty} \frac{f(3x)}{f(x)} = 1$. Then $\lim_{x\to\infty} \frac{f(2x)}{f(x)} =$

(1) 1 (2) $\frac{2}{3}$ (3) $\frac{3}{2}$ (4) 3 **[AIEEE-2010]**

2. $\lim_{x\to 2} \left(\frac{\sqrt{1-\cos\{2(x-2)\}}}{x-2}\right)$ **[AIEEE-2011]**

(1) equals $-\sqrt{2}$ (2) equals $\frac{1}{\sqrt{2}}$

(3) does not exist (4) equals $\sqrt{2}$

3. Let $f: R \to [0, \infty)$ be such that $\lim_{x\to 5} f(x)$ exists and $\lim_{x\to 5} \frac{(f(x))^2-9}{\sqrt{|x-5|}} = 0$ Then $\text{Lim}_{x\to 5}\ f(x)$ equal -

(1) 3 (2) 0 (3) 1 (4) 2 **[AIEEE-2011]**

4. $\lim_{x\to 0} \frac{\sin(\pi\cos^2 x)}{x^2}$ is equal to : **[JEE Mains Offline-2014]**

(1) $\frac{\pi}{2}$ (2) 1 (3) $-\pi$ (4) π

5. If $\lim_{x\to 2} \frac{\tan(x-2)\{x^2+(k-2)x-2k\}}{x^2-4x+4} = 5$ then k is equal to **[JEE Mains Online-2014]**

(1) 3 (2) 1 (3) 0 (4) 2

6. Let $p = \lim_{x\to 0+} \left(1+\tan^2\sqrt{x}\right)^{\frac{1}{2x}}$ then log p is equal to - **[JEE(Main)-2016]**

(1) $\frac{1}{4}$ (2) 2 (3) 1 (4) $\frac{1}{2}$

7. Let $p = \lim_{x\to 0+} \left(1+\tan^2\sqrt{x}\right)^{\frac{1}{2x}}$ then log p is equal to - **[JEE(Main)-2016]**

(1) $\frac{1}{4}$ (2) 2 (3) 1 (4) $\frac{1}{2}$

8. For each $t \in \mathbf{R}$, let $[t]$ be the greatest integer less than or equal to t.

Then $\lim_{x\to 0+} x\left(\left[\frac{1}{x}\right] + \left[\frac{2}{x}\right] + \cdots\ldots \left[\frac{15}{x}\right]\right)$ **[JEE(Main)-2018]**

(1) does not exist (in **R**). (2) is equal to 0 .

(3) is equal to 15 . (4) is equal to 120 .

9. $\lim_{y\to 0} \frac{\sqrt{1+\sqrt{1+y^4}}-\sqrt{2}}{y^4}$ **[JEE(Main) Online -2019]**

(1) exists and equals $\frac{1}{4\sqrt{2}}$ (2) does not exist

(3) exists and equals $\frac{1}{2\sqrt{2}}$ (4) exists and equals $\frac{1}{2\sqrt{2}(\sqrt{2}+1)}$

10. For each $x \in R$, let $[x]$ be the greatest integer less than or equal to x. Then $\lim_{x\to 0^-} \frac{x([x]+|x|)\sin[x]}{|x|}$

(1) $-\sin 1$ (2) $\sin 1$ (3) 1 (4) 0

[JEE(Main) Online -2019]

11. For each $t \in R$, let $[t]$ be the greatest integer less than or equal to t. Then,

$\lim_{x\to 1^+} \frac{(1-|x|+\sin|1-x|)\sin\left(\frac{\pi}{2}[1-x]\right)}{|1-x|[1-x]}$ **[JEE(Main) Online -2019]**

(1) equals 0 (2) does not exist (3) equals -1 (4) equals 1

12. Let $[x]$ denote the greatest integer less than or equal to x. Then : **[JEE(Main) Online -2019]**

$\lim_{x\to 0} \frac{\tan(\pi\sin^2 x) + (|x| - \sin(x[x]))^2}{x^2}$

(1) does not exist (2) equals π (3) equals 0 (4) equals $\pi + 1$

13. $\lim_{x\to 0} \frac{x\cot(4x)}{\sin^2 x\cot^2(2x)}$ is equal to: **[JEE(Main) Online -2019]**

(1) 1 (2) 4 (3) 0 (4) 2

14. $\lim_{x\to \pi/4} \frac{\cot^3 x - \tan x}{\cos(x+\pi/4)}$ is : **[JEE(Main) Online -2019]**

(1) $4\sqrt{2}$ (2) 8 (3) 4 (1) $8\sqrt{2}$

15. $\lim_{x\to 1^-} \frac{\sqrt{\pi}-\sqrt{2\sin^{-1}x}}{\sqrt{1-x}}$ is equal to: **[JEE(Main) Online -2019]**

(1) $\sqrt{\pi}$ (2) $\sqrt{\frac{\pi}{2}}$ (3) $\sqrt{\frac{2}{\pi}}$ (4) $\frac{1}{\sqrt{2\pi}}$

EXERCISE – 4 (JA)

SECTION-1

1. If $\lim_{x\to 0}[1 + x\ln(1 + b^2)]^{\frac{1}{x}} = 2b\sin^2\theta, b > 0$ and $\theta \in (-\pi, \pi]$, then the value of θ is-

[JEE 2012,3M, -1M]

(A) $\pm\frac{\pi}{4}$ (B) $\pm\frac{\pi}{3}$ (C) $\pm\frac{\pi}{6}$ (D) $\pm\frac{\pi}{2}$

2. If $\lim_{x\to\infty}\left(\frac{x^2+x+1}{x+1} - ax - b\right) = 4$, then -

(A) $a = 1, b = 4$ (B) $a = 1, b = -4$ (C) $a = 2, b = -3$ (D) $a = 2, b = 3$

3. Let $\alpha(a)$ and $\beta(a)$ be the roots of the equation $(\sqrt[3]{1 + a} - 1)x^2 + (\sqrt{1 + a} - 1)x + (\sqrt[6]{1 + a} - 1) = 0$ where $a > -1$. then $\lim_{a\to 0^+}\alpha(a)$ and $\lim_{a\to 0^+}\beta(a)$ are **[JEE 2012, 3m, -1M]**

(A) $-\frac{5}{2}$ and 1 (B) $-\frac{1}{2}$ and -1 (C) $-\frac{7}{2}$ and 2 (D) $-\frac{9}{2}$ and 3

SECTION-2

4. Let $L = \lim_{x\to 0}\frac{a-\sqrt{a^2-x^2}-\frac{x^2}{4}}{x^4}$, $a > 0$. If L is finite, then - **[JEE 2009, 4]**

(A) $a = 2$ (B) $a = 1$ (C) $L = \frac{1}{64}$ (D) $L = \frac{1}{32}$

5. The largest value of the non-negative integer a for which $\lim_{x\to 1}\left\{\frac{-ax+\sin(x-1)+a}{x+\sin(x-1)-1}\right\}^{\frac{1-x}{1-\sqrt{x}}} = \frac{1}{4}$ is

[JEE (Advanced)-2014, 3]

6. Let $\alpha, \beta \in \mathbb{R}$ be such that $\lim_{x\to 0}\frac{x^2\sin(\beta x)}{\alpha x-\sin x} = 1$. Then $6(\alpha + \beta)$ equals **[JEE (Advanced)-2016, 3(0)]**

7. Let $f(x) = \frac{1-x(1+|1-x|)}{|1-x|}\cos\left(\frac{1}{1-x}\right)$ for $x \neq 1$, Then

(A) $\lim_{x\to 1^+}f(x)$ does not exist (B) $\lim_{x\to 1^-}f(x)$ does not exist

(C) $\lim_{x\to 1^+}f(x) = 0$ (D) $\lim_{x\to 1^-}f(x) = 0$

EXERCISE - 5

1. If $\ell=\lim_{x\to a}\frac{\sqrt{3x^2+a^2}-\sqrt{x^2+3a^2}}{(x-a)}$ then -

(A) $\ell=1\forall a\in R$ (B) $\ell=1\forall a>0$

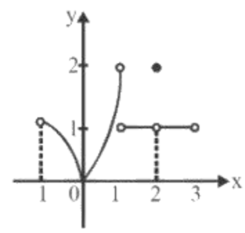
(C) $\ell=-1\forall a<0$ (D) $\ell=$ D.N.E if $a=0$

2. Which of the following limits vanish ?

(A) $\lim_{x\to\infty}\frac{\sin x}{x}$ (B) $\lim_{x\to\infty}\frac{\arctan x}{x}$

(C) $\lim_{x\to\infty}\frac{x+\sin x}{x+\cos x}$ (D) $\lim_{x\to 1}\frac{\arcsin x}{\tan\frac{\pi x}{2}}$

3. Which of the following statement are true for the function f defined for $-1\le x\le 3$ in the figure shown.



(A) $\lim_{x\to -1^+}f(x)=1$ (B) $\lim_{x\to 2}f(x)$ does not exist

(C) $\lim_{x\to 1^-}f(x)=2$ (D) $\lim_{x\to 0^+}f(x)=\lim_{x\to 0^-}f(x)$

4. Let $f(x)=x+\sqrt{x^2+2x}$ and $g(x)=\sqrt{x^2+2x}-x$, then

(A) $\lim_{x\to\infty}g(x)=1$ (B) $\lim_{x\to\infty}f(x)=1$

(C) $\lim_{x\to -\infty}f(x)=-1$ (D) $\lim_{x\to -\infty}g(x)=-1$

5. If $A=\lim_{x\to 0}\frac{\sin^{-1}(\sin x)}{\cos^{-1}(\cos x)}$ and $B=\lim_{x\to 0}\frac{[|x|]}{x}$, then (where [.] denotes greatest integer function)

(A) $A=1$ (B) A does not exist

(C) $B=0$ (D) $B=1$

6. Which of the following limit tends to unity?

(A) $\lim_{x\to 0}\frac{1-\cos x+2\sin x-\sin^3 x-x^2+3x^4}{\tan^3 x-6\sin^2 x+x-5x^3}$ (B) $\lim_{x\to\infty}\frac{x}{[x]}$

(C) $\lim_{x\to\infty}\frac{1}{(\sqrt{x+\sqrt{x+\sqrt{x}}}-\sqrt{x})}$ (D) $\lim_{x\to\infty}\left(\frac{\sqrt{x}}{\sqrt{x+\sqrt{x+\sqrt{x}}}}\right)$

7. Which of the following limits does not exist?

(A) $\lim_{x\to 1^+}([x])^{\frac{1}{x-1}}$ (B) $\lim_{x\to 3}\frac{(x^2-9-\sqrt{x^2-6x+9})}{|x-1|-2}$

(C) $\lim_{x\to 0^+}(x)^{(nx}$ (D) $\lim_{x\to 0^+}\left(\frac{1-\cos(\sin^2 x)}{x^2}\right)^{\frac{\ln(1-2x^2)}{\sin^2 x}}$

(where [.] represents greatest integer function)

8. The value (s) of ' n ' for which $\lim_{x\to 1}\frac{e^{x-1}-x}{(x-1)^n}$ exists is / are -

(A) 1 (B) 2 (C) 3 (D) 4

9. Let $f:\left(-\frac{\pi}{2},\frac{\pi}{2}\right) \rightarrow R, f(x) = \begin{cases} \lim_{n\to\infty}\left(\frac{(\tan x)^{2n}+x^2}{\sin^2 x+(\tan x)^{2n}}\right); & x \neq 0 \\ 1, & x = 0 \end{cases}$, $n \in N$. Which of the following

(A) $f\left(-\frac{\pi^-}{4}\right) = f\left(\frac{\pi^+}{4}\right)$ (B) $f\left(-\frac{\pi^-}{4}\right) = f\left(-\frac{\pi^+}{4}\right)$

(C) $f\left(\frac{\pi^-}{4}\right) = f\left(\frac{\pi^+}{4}\right)$ (D) $f(0^+) = f(0) = f(0^-)$

10. Let $f(x) = \left|\begin{matrix} \frac{\tan^2\{x\}}{x^2-[x]^2} & \text{for } x > 0 \\ 1 & \text{for } x = 0 \\ \sqrt{\{x\}\cot\{x\}} & \text{for } x < 0 \end{matrix}\right|$ where $[x]$ is the step up function and $\{x\}$ is the fractional part function of x, then -

(A) $\lim_{x\to0^+} f(x) = 1$ (B) $\lim_{x\to0^-} f(x) = 1$

(C) $\cot^{-1}\left(\lim_{x\to0^-} f(x)\right)^2 = 1$ (D) None

11. $\lim_{x\to c} f(x)$ does not exist when (where $[x]$ is the step up function, $\{x\}$ is fractional part function of x and sgn (x) denotes signum function), then-

(A) $f(x) = [[x]] - [2x-1]; c = 3$ (B) $f(x) = [x] - x, c = 1$

(C) $f(x) = \{x\}^2 - \{-x\}^2, c = 0$ (D) $f(x) = \frac{\tan(\text{sgn } x)}{\text{sgn } x}, c = 0$

12. Which of the following limits does not exist?

(A) $\lim_{x\to\infty} \operatorname{cosec}^{-1}\left(\frac{x}{x+7}\right)$ (B) $\lim_{x\to1} \sec^{-1}(\sin^{-1} x)$

(C) $\lim_{x\to0^+} x^{\frac{1}{x}}$ (D) $\lim_{x\to0}\left(\tan\left(\frac{\pi}{8}+x\right)\right)^{\cot x}$

13. Which of the following statement(s) is (are) INCORRECT ?

(A) If $\lim_{x\to c} f(x)$ and $\lim_{x\to c} g(x)$ both does not exist then $\lim_{x\to c} f(x)g(x)$ also does not exist.

(B) If $\lim_{x\to c} f(x)$ and $\lim_{x\to c} g(x)$ both does not exist then $\lim_{x\to c} f(g(x))$ also does not exist.

(C) If $\lim_{x\to c} f(x)$ exists and $\lim_{x\to c} g(x)$ does not exist then $\lim_{x\to c} g(f(x))$ does not exist.

(D) If $\lim_{x\to c} f(x)$ and $\lim_{x\to c} g(x)$ both exist then $\lim_{x\to c} f(g(x))$ and $\lim_{x\to c} g(f(x))$ also exist.

14. Consider following statements and identify correct options:

(i) $\lim_{x\to4}\left(\frac{2x}{x-4}-\frac{8}{x-4}\right) = \lim_{x\to4}\frac{2x}{x-4} - \lim_{x\to4}\frac{8}{x-4}$

(ii) $\lim_{x\to1}\frac{x^2+6x-7}{x^2+5x-6} = \frac{\lim_{x\to1}(x^2+6x-7)}{\lim_{x\to1}(x^2+5x-6)}$

(iii) $\lim_{x\to1}\frac{x-3}{x^2+2x-4} = \frac{\lim_{x\to1}(x-3)}{\lim_{x\to1}(x^2+2x-4)}$

(iv) If $\lim_{x\to5} f(x) = 2$ and $\lim_{x\to5} g(x) = 0$, then $\lim_{x\to5}\frac{f(x)}{g(x)}$ does not exist.

(v) If $\lim_{x\to5} f(x) = 0$ and $\lim_{x\to5} g(x) = 2$, then $\lim_{x\to5}\frac{f(x)}{g(x)}$ does not exist.

(A) Only one is true. (B) Only two are true.

(C) Only three are false. (D) Only two are false.

15. Which of the following limits equal to $\frac{1}{2}$

(A) $\lim_{n\to\infty}\left(\frac{1}{1.3}+\frac{1}{3.5}+\cdots.+\frac{1}{(2n-1)(2n+1)}\right)$

(B) $\lim_{x\to\infty}\left[\frac{3x^2}{2x+1}-\frac{(2x-1)(3x^2+x+2)}{4x^2}\right]$

(C) $\lim_{n\to\infty}\frac{1}{n^2}(1+2+3+\cdots\ldots.+n)$

(D) $\lim_{n\to\infty}\frac{(n+2)!+(n+1)!}{(n+2)!-(n+1)!}$

16. Let $f(x)=\begin{cases}\sin x; \text{where} x = \text{integer}\\ 0 \quad ; \quad \text{otherwise}\end{cases}$: $g(x)=\begin{cases}x^2+1 & ; & x\neq 0,2\\ 4 & ; & x=0\\ 5 & ; & x=2\end{cases}$, then

(A) $\lim_{x\to0} g(f(x)) =$

(B) $\lim_{x\to0} f(g(x)) = 0$

(C) $\lim_{x\to1} f(g(x)) = 0$

(D) $\lim_{x\to1} g(f(x)) = 5$

17. If $f(x)=\begin{cases}x^2, & \text{if x is rational}\\ 0, & \text{if x is irrational}\end{cases}$, then

(A) $\lim_{x\to0} f(x) = 0$

(B) $\lim_{x\to0} f(x)$ does not exist

(C) $\lim_{x\to2} f(x) = 4$

(D) $\lim_{x\to2} f(x)$ does not exist

18. Let $f(\beta)=\lim_{\alpha\to\beta}\frac{\sin^2\alpha-\sin^2\beta}{\alpha^2-\beta^2}$, then $f\left(\frac{\pi}{4}\right)$ is greater than -

(A) $\lim_{x\to0}\frac{1-\cos^3 x}{x\sin 2x}$

(B) $\lim_{x\to\pi/2}\frac{\cot x-\cos x}{(\pi-2x)^3}$

(C) $\lim_{x\to\infty}(\cos\sqrt{x+1}-\cos\sqrt{x})$

(D) $\lim_{x\to a}\frac{\sqrt{a+2x}-\sqrt{3x}}{\sqrt{3a+x}-2\sqrt{x}}$ where $a>0$

19. If $\frac{\sin x+ae^x+be^{-x}+c\ln(1+x)}{x^3}$ has a finite limit L as $x\to0$, then

(A) $a=-\frac{1}{2}$

(B) $b=\frac{1}{2}$

(C) $c=0$

(D) $L=-\frac{1}{3}$

20. Let $\ell=\lim_{x\to\infty}\frac{a^x-a^{-x}}{a^x+a^{-x}}(a>0)$, then

(A) $\ell=1\forall a>0$

(B) $\ell=-1\forall a\in(0,1)$

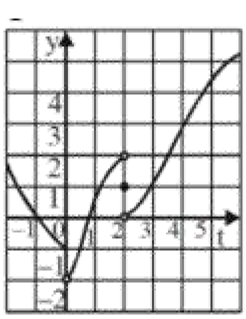
(C) $\ell=0$, if $a=1$

(D) $\ell=1\forall\ a>1$

[MATCH THE COLUMN TYPE]

21. For the function g(t) whose graph is given, match the entries of column-I to column-II

Column-I	Column-II
(A) $\lim_{t\to0^+}g(t)+\lim_{t\to2^-}g(t)$	(P) $\lim_{t\to2^+}g(t)$
(B) $\lim_{t\to0^-}g(t)+g(2)$	(Q) does not exist
(C) $\lim_{t\to0}g(t)$	(R) 0
(D) $\lim_{t\to2}g(t)$	(S) $\lim_{t\to4}g(t)$



22. Column-I / Column – II

(A) $\lim_{n\to\infty} n\sin\left(\frac{\pi}{4n}\right)\cos\left(\frac{\pi}{4n}\right)$ is equal to — (P) 0

(B) $\lim_{x\to 0}\frac{\sin x^\circ}{x}$ is equal to — (Q) $\frac{1}{2}$

(C) $\lim_{x\to 0}\left(\frac{1}{\sin x}-\frac{1}{\tan x}\right)$ is equal to — (R) $\frac{\pi}{4}$

(D) $\lim_{x\to\pi/2}\frac{1+\cos 2x}{(\pi-2x)^2}$ is equal to — (S) $\frac{\pi}{180}$

23. Column-I / Column II

(A) $\lim_{x\to\infty}\frac{a^x}{a^x+1}\,(a>0)$ can be equal to — (P) $\lim_{x\to\infty} x\left(e^{\frac{1}{x}}-1\right)$

(B) $\lim_{x\to 2}\frac{\sin(e^{x-2}-1)}{\log(x-1)}$ is equal to — (Q) $\lim_{x\to 0}\frac{a^x+b^x+c^x-3}{x}\,(a,b,c>0 \text{ and } abc=1)$

(C) $\lim_{x\to c}\frac{(\ln x-1)e}{x-e}$ is equal to — (R) $\lim_{x\to 0}\frac{e^{4x}-e^{3x}}{x}$

(D) $\lim_{x\to 0}\frac{x(5^x-1)}{(1-\cos x)4\ell n5}$ is equal to — (S) $\frac{1}{2}$

(T) 0

EXERCISE – 6

1. $\lim_{n\to\infty}\left(\frac{\sqrt{n^2+n}-1}{n}\right)^{2\sqrt{n^2+n}-1}$

2. $\lim_{x\to\infty}\left(\frac{a_1^{\frac{1}{x}}+a_2^{\frac{1}{x}}+a_3^{\frac{1}{x}}+\cdots\ldots+a_n^{\frac{1}{x}}}{n}\right)^{nx}$, where $a_1, a_2, \ldots\ldots\ldots a_n > 0$

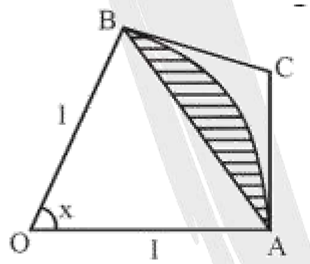
3. $\lim_{x\to 0}\left[\frac{(1+x)^{1/x}}{e}\right]^{1/x}$

4. If $\lim_{x\to\infty}\frac{a(2x^3-x^2)+b(x^3+5x^2-1)-c(3x^3+x^2)}{a(5x^4-x)-bx^4+c(4x^4+1)+2x^2+5x} = 1$, then the value of $(a+b+c)$ can be expressed in the lowest form as $\frac{p}{q}$. Find the value of $(p+q)$.

5. $\lim_{x\to 0}\left[\frac{\ell n(1+x)^{1+x}}{x^2}-\frac{1}{x}\right]$

6. Let $L = \prod_{n=3}^{\infty}\left(1-\frac{4}{n^2}\right)$; $M = \prod_{n=2}^{\infty}\left(\frac{n^3-1}{n^3+1}\right)$ and $N = \prod_{n=1}^{\infty}\frac{(1+n^{-1})^2}{1+2n^{-1}}$, then find the value of $L^{-1}+M^{-1}+N^{-1}$

7. A circular arc of radius 1 subtends an angle of x radians, $< x < \frac{\pi}{2}$ as shown in the figure. The point C is the intersection of the two tangent lines at A and B. Let T(x) be the area of the triangle ABC and let S(x) be the area of the shaded region. Compute :



(a) T(x) (b) S(x) and (c) the limit of $\frac{T(x)}{S(x)}$ as $x\to 0$.

8. Let $f(x) = \lim_{n\to\infty}\sum_{n=1}^{n}3^{n-1}\sin^3\frac{x}{3^n}$ and $g(x) = x-4f(x)$. Evaluate $\lim_{x\to 0}(1+g(x))^{\cot x}$.

9. If $f(n,\theta) = \prod_{r=1}^{n}\left(1-\tan^2\frac{\theta}{2^r}\right)$, then compute $\lim_{n\to\infty}f(n,\theta)$

10. Evaluate $\lim_{x\to\infty}\left(\frac{x}{e}-x\left(\frac{x}{x+1}\right)^x\right)$

11. f(x) is the function such that $\lim_{x\to 0}\frac{f(x)}{x} = 1$. $\lim_{x\to 0}\frac{x(1+a\cos x)-b\sin x}{(f(x))^3} = 1$, then find the value of a and b.

12. Through a point A on a circle, a chord AP is drawn and on the tangent at A a point T is taken such that AT = AP. If TP produced meet the diameter through A at Q, prove that the limiting value of AQ when P moves upto A is double the diameter of the circle.

13. At the end points A, B of the fixed segment of length L, lines are drawn meeting in C and making angles θ and 2θ respectively with the given segment. Let D be the foot of the altitude CD and let x represents the length of AD. Find the value of x as θ tends to zero i.e. $\text{Lim}_{\theta\to 0}\ x$.

14. Let $f(x) = \lim_{n\to\infty} \frac{2x^{2n}\sin\frac{1}{x}+x}{1+x^{2n}}$, then find

(A) $\lim_{x\to\infty} x(x)$

(B) $\lim_{x\to 1} f(x)$

(C) $\lim_{x\to 0} f(x)$,

(D) $\lim_{x\to -\infty} f(x)$

15. Using Sandwich theorem, evaluate

(a) $\lim_{n\to\infty}\left(\frac{1}{\sqrt{n^2}}+\frac{1}{\sqrt{n^2+1}}+\frac{1}{\sqrt{n^2+2}}+\cdots..+\frac{1}{\sqrt{n^2+2n}}\right)$

s(b) $\lim_{n\to\infty}\frac{1}{1+n^2}+\frac{2}{2+n^2}+\cdots..+\frac{n}{n+n^2}$

ANSWER KEY

LIMIT EXERCISE - 1

1.	A	**2.**	C	**3.**	D	**4.**	C	**5.**	D	**6.**	B	**7.**	B
8.	C	**9.**	B	**10.**	B	**11.**	A	**12.**	A	**13.**	C	**14.**	B
15.	A	**16.**	C	**17.**	A	**18.**	D	**19.**	A	**20.**	A	**21.**	C
22.	D	**23.**	D	**24.**	C	**25.**	B	**26.**	D	**27.**	A	**28.**	A
29.	C	**30.**	A	**31.**	D	**32.**	B	**33.**	D	**34.**	C	**35.**	D
36.	B	**37.**	C	**38.**	A	**39.**	D	**40.**	A	**41.**	C	**42.**	B
43.	C	**44.**	D	**45.**	A	**46.**	D	**47.**	C				

EXERCISE – 2

1. 2 **2.** 5050 **3.** 2 **4.** $\frac{1}{32}$ **5.** $\frac{1}{16\sqrt{2}}$ **6.** $\frac{\sqrt{3}}{2}$ **7.** $\frac{1}{2}$

8. -2 **9.** $\pi - 3$ **10.** (i) $a = 1, b = -1$ (ii) $a = -1, b = \frac{1}{2}$ **11.** 1 **12.** $8\sqrt{2}(\ell n3)^2$

13. (a) $\pi/2$ if $a > 0$; 0 if $a = 0$ and $-\pi/2$ if $a < 0$; (b) $f(x) = |x|$ **14.** $-1/2$ **15.** 16

16. e^{-8} **17.** $c = \ell n2$ **18.** e^{-1} **19.** $e^{-1/2}$ **20.** $\frac{\sqrt{3}}{2}$

EXERCISE – 3 (JM)

1.	1	**2.**	3	**3.**	1	**4.**	4	**5.**	1	**6.**	4	**7.**	3	**8.**	4
9.	1	**10.**	1	**11.**	1	**12.**	1	**13.**	1	**14.**	2	**15.**	3		

EXERCISE - 4 (JA) SECTION-1

1. D **2.** B **3.** B

SECTION-2

4. AC **5.** 0 **6.** 7 **7.** AD

EXERCISE – 5

1.	BCD	**2.**	ABD	**3.**	ACD	**4.**	AC	**5.**	BC	**6.**	BD	**7.**	BCD
8.	AB	**9.**	AD	**10.**	AC	**11.**	BC	**12.**	AD	**13.**	ABCD	**14.**	BC
15.	AC	**16.**	ABC	**17.**	AD 1	**8.**	BCD	**19.**	ABCD	**20.**	BCD		

21. (A) → (P,R); (B) → (P,R); (C) → (Q); (D) → (Q) **22.** (A) → (R);(B) → (S);(C) → (P);(D) → (Q)

23. (A) → (P,Q,R,S,T);(B) → (P,R);(C) → (P,R); (D) → (S)

EXERCISE – 6

1. e^{-1} **2.** $(a_1 \cdot a_2 \cdot a_3 \dots\dots a_n)$ **3.** $e^{-\frac{1}{2}}$ **4.** 167 **5.** ½

6. 8 **7.** $T(x) = \frac{1}{2}\tan^2\frac{x}{2} \cdot \sin x$ or $\tan\frac{x}{2} - \frac{\sin x}{2}$, $S(x) = \frac{1}{2}x - \frac{1}{2}\sin x$, limit $= \frac{3}{2}$

8. $g(x) = \sin x$ and $\ell = e$ **9.** $\frac{\theta}{\tan\theta}$ **10.** $-\frac{1}{2e}$ **11.** $a = -5/2, b = -3/2$

13. $\frac{2L}{3}$ **14.** (a) 2 , (b) D.N.E., (c) 0 , (d) 0 **15.** (a) 2 ; (b) 1/2