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Trigonometric Eqn – (8-10)

Imp. chapter for Jee mains/Adv.

① Identity & Eqn

A) $\sin^2 x + \cos^2 x = 1$ is an Identity

as this is satisfied by all values of x.

(B) $\cos x = 1$ is an Eqn as this is satisfied

by some particular value of x.

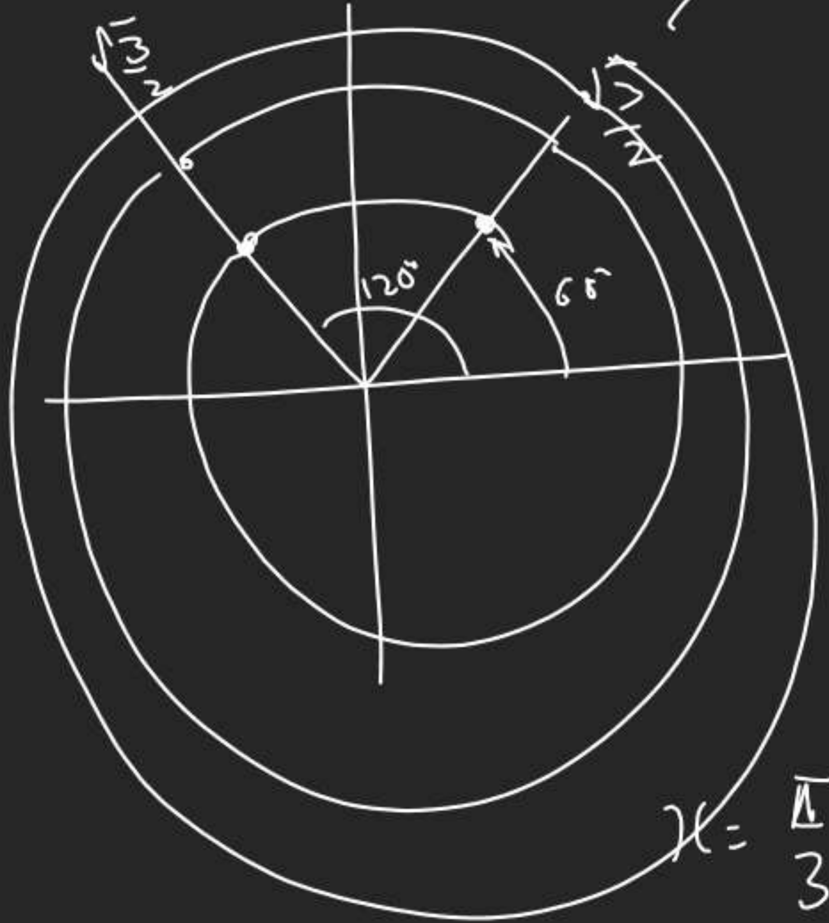
② Trigo Eqn is an eqn involving one or more trigo fxn of variable.

Trigo Eqn

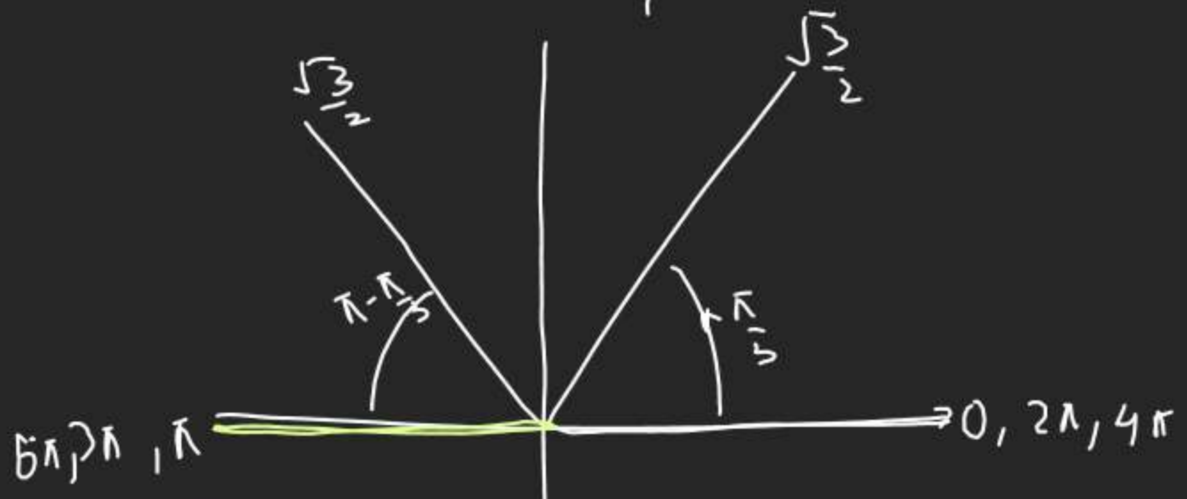
- Sin
- Cos x
- tan x
- cot
- Sec
- Cosec

Q $\sin x = \frac{\sqrt{3}}{2}$ Solve.

$x = 60^\circ = \frac{\pi}{3}$



$\sin x = \frac{\sqrt{3}}{2}$



$\pi - \frac{\pi}{3}, 3\pi - \frac{\pi}{3}, 5\pi - \frac{\pi}{3}$ → $\boxed{\text{Odd}\,\pi - \frac{\pi}{3}}$

$x = \frac{\pi}{3}, \frac{2\pi}{3}, 2\pi + \frac{\pi}{3}, 2\pi + \frac{2\pi}{3}$

$(2n+1)\pi - \frac{\pi}{3}$

$x = \frac{\pi}{3}$

$\frac{\pi}{3}, 2\pi + \frac{\pi}{3}, 4\pi + \frac{\pi}{3} \ldots$

$\text{Even}\,\pi + \frac{\pi}{3}$

$2n\pi + \frac{\pi}{3}$

Merge

$x = \text{odd}\,\pi - \frac{\pi}{3}$ $\quad$ $x = \text{Even}\,\pi + \frac{\pi}{3}$

$(2n+1)\pi - \frac{\pi}{3}$ $\quad$ $2n\pi + \frac{\pi}{3}$

$$\boxed{x = n\pi + (-1)^n \frac{\pi}{3}}$$

$n = 2$

$x = 2\pi + (-1)^2 \frac{\pi}{3} = 2\pi + \frac{\pi}{3}$

$n = 5$

$x = 5\pi + (-1)^5 \frac{\pi}{3}$

$= 5\pi - \frac{\pi}{3}$

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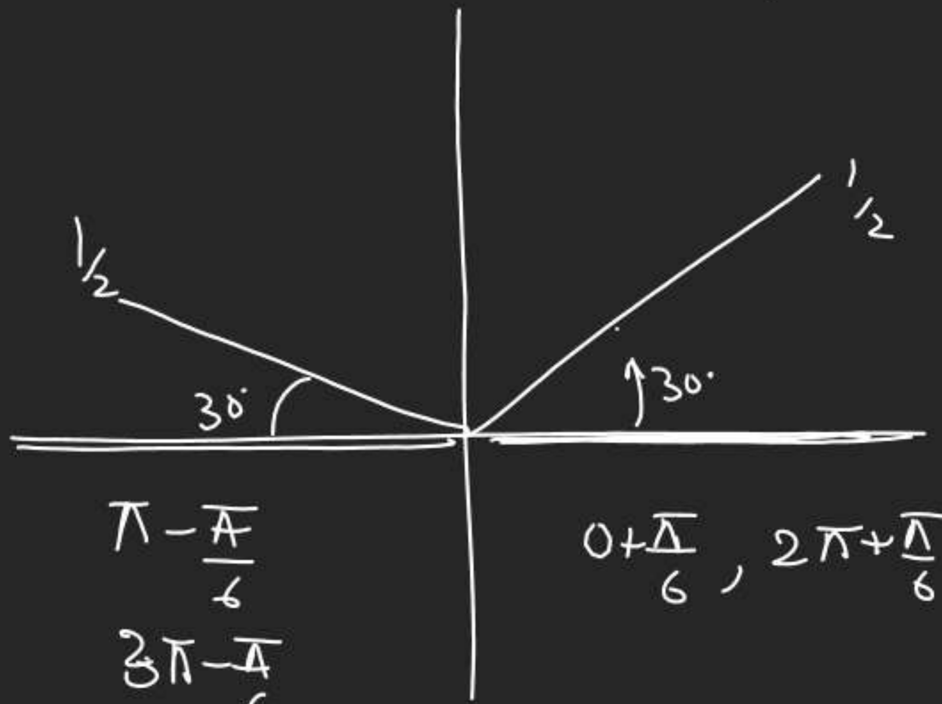
$\sin x = \sin\frac{\pi}{6}$

Q $\sin x = \frac{1}{2}$

$x = n\pi + (-1)^n \cdot \frac{\pi}{6}$

$\sin x \oplus \begin{cases} \to 1^{st} Q \\ \to 2^{nd} Q \end{cases}$



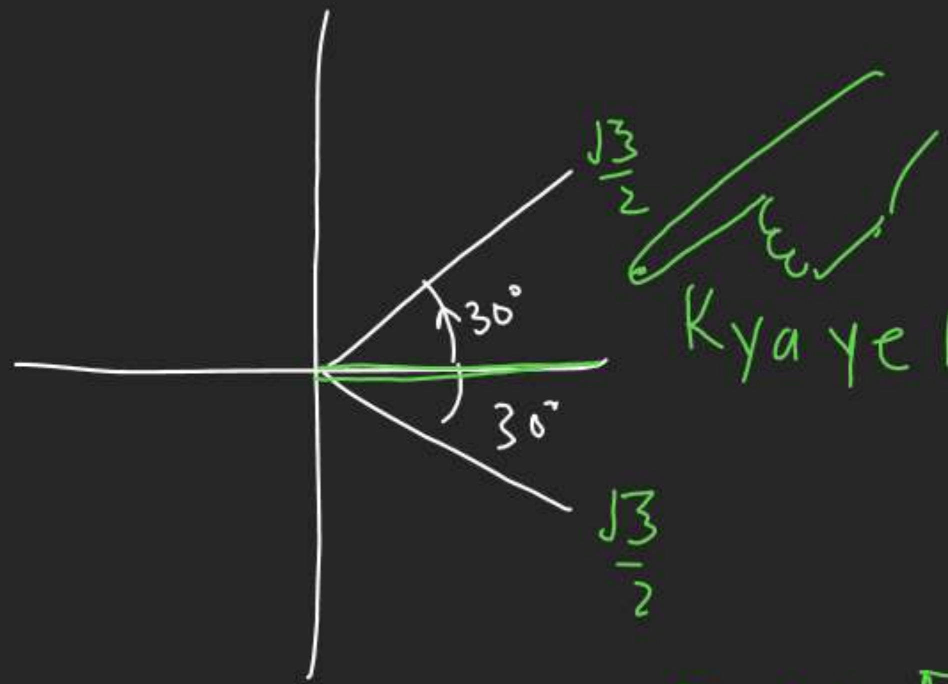
$\pi - \frac{\pi}{6}$, $3\pi - \frac{\pi}{6}$, $5\pi - \frac{\pi}{6}$

$0 + \frac{\pi}{6}, 2\pi + \frac{\pi}{6}, 4\pi + \frac{\pi}{6}$

$\cos x = \cos\frac{\pi}{6}$

Q $\cos x = \frac{\sqrt{3}}{2} \Rightarrow x = \frac{\pi}{6}$

$\cos x = \oplus \begin{cases} \to 1^{st} \\ \to 4^{th} \end{cases}$



Kya ye Axis even & odd Dono hai?

$x = 2\pi + \frac{\pi}{6}, 2\pi - \frac{\pi}{6}, 4\pi + \frac{\pi}{6}, 4\pi - \frac{\pi}{6}$

$6\pi + \frac{\pi}{6}, 6\pi - \frac{\pi}{6} \ldots$

$$\boxed{x = 2n\pi \pm \frac{\pi}{6}}$$

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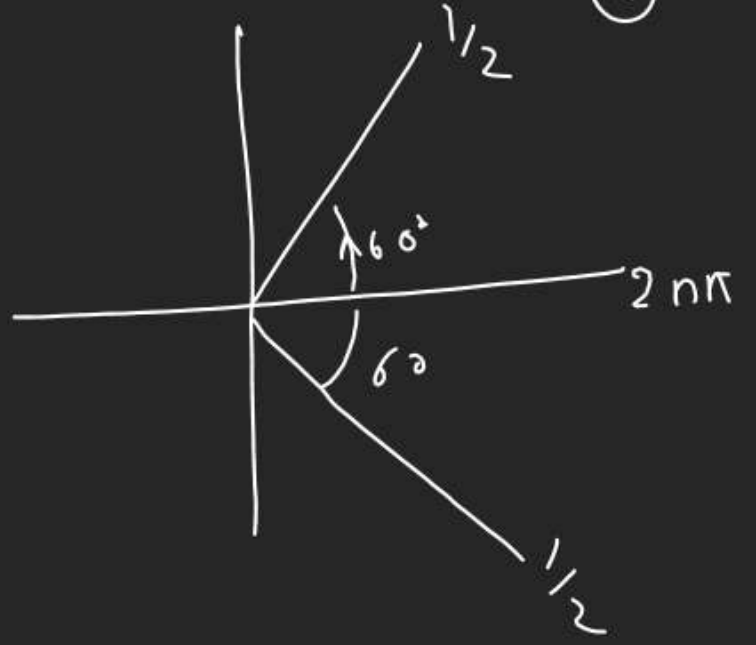
$\cos x = \cos\frac{\pi}{3}$

Q $\boxed{\cos x = \frac{1}{2}}$ find x?

$x = 60^\circ$

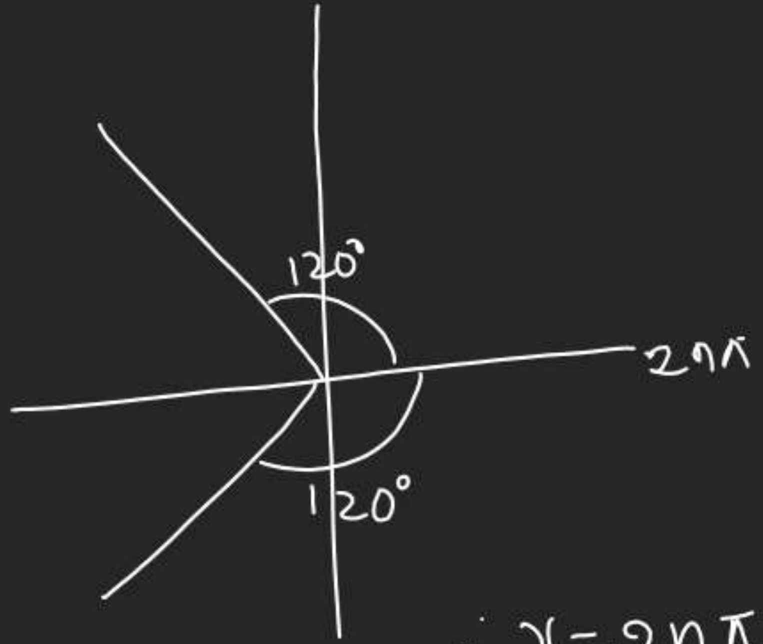
$\oplus \rightarrow$ ①, ④



$x = 2n\pi \pm \frac{\pi}{3}$.

Q $\cos x = -\frac{1}{2}$

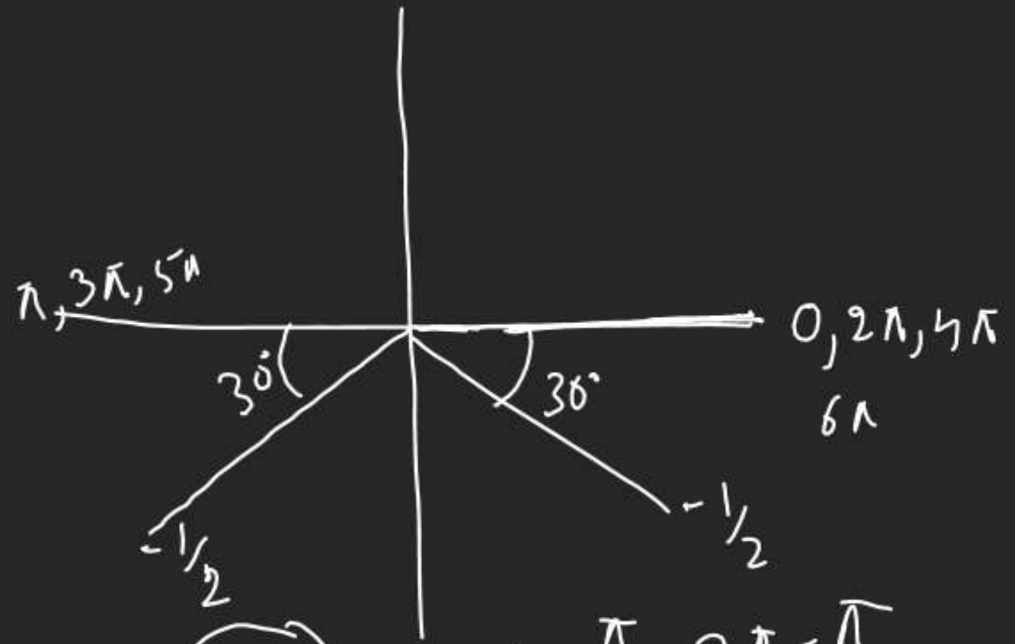
$\ominus \rightarrow$ 2nd, 3rd



$\therefore x = 2n\pi \pm \frac{2\pi}{3}$.

Q $\sin x = -\frac{1}{2}$ find x $\rightarrow 30^\circ$

$\ominus \rightarrow$ 3rd, 4th



$\pi + \frac{\pi}{6}$, $3\pi + \frac{\pi}{6}$

$5\pi + \frac{\pi}{6}$

$0 - \frac{\pi}{6}$, $2\pi - \frac{\pi}{6}$

$4\pi - \frac{\pi}{6}$, $6\pi - \frac{\pi}{6}$

$x = n\pi + (-1)^n \cdot \left(-\frac{\pi}{6}\right)$

$n = 3$

$x = 3\pi + (-1)^3 \cdot \left(-\frac{\pi}{6}\right)$

$= 3\pi + \frac{\pi}{6}$ Hurreyyy

$n = 4$

$x = 4\pi + (-1)^4 \cdot \left(-\frac{\pi}{6}\right)$

$x = 4\pi - \frac{\pi}{6}$

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Direct System

$\sin\theta = 0$ | $\cos\theta = 0$

$\theta = n\pi$ | $\theta = (2n+1)\frac{\pi}{2}$

① $\sin\theta = \sin\alpha$

$\sin\theta - \sin\alpha = 0$

$2\cos\left(\frac{\theta+\alpha}{2}\right)\cdot\sin\left(\frac{\theta-\alpha}{2}\right) = 0$

$\cos\left(\frac{\theta+\alpha}{2}\right) = 0$ or $\sin\left(\frac{\theta-\alpha}{2}\right) = 0$

$\frac{\theta+\alpha}{2} = (2n+1)\frac{\pi}{2}$ | $\frac{\theta-\alpha}{2} = n\pi$

$\theta = (2n+1)\pi - \alpha$. | $\theta = 2n\pi + \alpha$.

$\boxed{\theta = n\pi + (-1)^n\cdot\alpha}$.

② $\cos\theta = \cos\alpha$.

$\cos\theta - \cos\alpha = 0$

$-2\sin\left(\frac{\theta+\alpha}{2}\right)\cdot\sin\left(\frac{\theta-\alpha}{2}\right) = 0$

$\sin\left(\frac{\theta+\alpha}{2}\right) = 0$ or $\sin\left(\frac{\theta-\alpha}{2}\right) = 0$

$\frac{\theta+\alpha}{2} = n\pi$ | $\frac{\theta-\alpha}{2} = n\pi$

$\theta = 2n\pi - \alpha$. | $\theta = 2n\pi + \alpha$.

$\boxed{\theta = 2n\pi \pm \alpha}$

③ $\tan\theta = \tan\alpha$.

$\frac{\sin\theta}{\cos\theta} = \frac{\sin\alpha}{\cos\alpha}$.

$\sin\theta\cdot\cos\alpha = \sin\alpha\cdot\cos\theta$

$\sin\theta\cdot\cos\alpha - \cos\theta\cdot\sin\alpha = 0$

$\sin(\theta-\alpha) = 0$

$\theta - \alpha = n\pi$

$\boxed{\theta = n\pi + \alpha}$.

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Special Angle.

① $\sin\theta = 0$

$\theta = n\pi$

(2) $\cos\theta = 0$

When $\theta = (2n+1)\frac{\pi}{2}$

General value of θ

$$\boxed{\begin{array}{l} ①\ \sin\theta = \sin\alpha. \\ \quad \theta = n\pi + (-1)^n\alpha. \\ 2)\ \cos\theta = \cos\alpha. \\ \quad \theta = 2n\pi \pm \alpha. \\ 3)\ \tan\theta = \tan\alpha. \\ \quad \theta = n\pi + \alpha. \end{array}}$$

Q $\sin x = \frac{\sqrt{3}}{2}$ $\rightarrow \frac{\sqrt{3}}{2} = 60°$

$\sin x = \sin\frac{\pi}{3}$

$x = n\pi + (-1)^n \cdot \frac{\pi}{3}$

Q $\sin x = \frac{1}{2}$

$\sin x = \sin\frac{\pi}{6}$

$x = n\pi + (-1)^n \frac{\pi}{6}$

Q $\cos x = \frac{\sqrt{3}}{2}$

$\cos x = \cos\left(\frac{\pi}{6}\right)$ α.

$x = 2n\pi \pm \frac{\pi}{6}$

Q $\tan x = 1$

$\tan x = \tan\frac{\pi}{4}$ α.

$\theta = n\pi + \frac{\pi}{4}$

Q $\cot x = \sqrt{3}$.

$\tan x = \frac{1}{\sqrt{3}} = \tan\frac{\pi}{6}$

$x = n\pi + \frac{\pi}{6}$.

Q $\sec x = \frac{2}{\sqrt{3}}$.

$\cos x = \frac{\sqrt{3}}{2} = \cos\frac{\pi}{6}$.

$x = 2n\pi + \frac{\pi}{6}$.

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Principle Solution.

If value of $\theta \in [0, 2\pi)$ then values are known as Principle Solution

How to select α? (Very Imp.)

① $\sin\theta = \sin\alpha$

We take $\alpha \in [-90^\circ, 90^\circ]$

$\alpha \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

(2) $\cos\theta = \cos\alpha$.

$\alpha = [0, \pi]$

③ $\tan\theta = \tan\alpha$.

$\theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Q $\sin\theta = -\frac{1}{2}$

$\sin\left(-\frac{\pi}{6}\right) = -\sin\frac{\pi}{6}$

$= -\frac{1}{2}$

$\sin\theta = \sin\left(-\frac{\pi}{6}\right)$

$\downarrow$

$-\frac{\pi}{6} = -30^\circ$

$\in [-90^\circ, 90^\circ]$

Q $\sin\theta = -\frac{1}{\sqrt{2}}$

$\sin\theta = \sin\left(-\frac{\pi}{4}\right)$

$\downarrow$

$-\sin\frac{\pi}{4}$

$-\frac{1}{\sqrt{2}}$

Q $\cos\theta = -\frac{1}{2}$

$\cos\theta = \cos\left(\frac{2\pi}{3}\right)$

$\downarrow$

$\cos\frac{2\pi}{3} = \cos 120^\circ = -\frac{1}{2}$

$120^\circ \in [0, \pi]$

Q $\cos\theta = -\frac{\sqrt{3}}{2}$ $\quad \frac{\pi}{6} \rightarrow \pi - \frac{\pi}{6}$

$\cos\theta = \cos\left(\frac{5\pi}{6}\right)$

$\cos\left(\pi - \frac{\pi}{6}\right) = -\cos\frac{\pi}{6} = -\frac{\sqrt{3}}{2}$

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	$0°$	$30°$	$45°$	$60°$	$90°$	$120°$	135	$150°$	$180°$
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{\sqrt{3}}{2}$	-1
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞	$-\sqrt{3}$	-1	$-\frac{1}{\sqrt{3}}$	0

If Unknown value is given?

Q $\sin\theta = \frac{1}{3}$.

$\sin\theta = \sin\alpha$ let $\sin\alpha = \frac{1}{3} \Rightarrow \alpha = \sin^{-1}\frac{1}{3}$

$\theta = n\pi + (-1)^n \alpha$

$= n\pi + (-1)^n \sin^{-1}\frac{1}{3}$

Q $\cos\theta = \frac{2}{7}$ find gen. value

$\cos\theta = \cos\alpha$ let $\cos\alpha = \frac{2}{7} \in [-1, 1]$

$\theta = 2n\pi \pm \alpha$. where $\alpha = \cos^{-1}\frac{2}{7}$

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Q $\sin\theta = \frac{1}{\sqrt{2}}$ find G.V.

$\sin\theta = \sin\frac{\pi}{4}$

$\sin\theta = \sin\alpha$

$\theta = n\pi + (-1)^n\alpha$

$\theta = n\pi + (-1)^n\frac{\pi}{4}$

Q $\cos\theta = -\boxed{\frac{1}{\sqrt{2}}}$ find G.V.

$\cos\theta$ है $-ve$

$[0, 180°]$

$135°$

$\cos\theta = \cos\frac{3\pi}{4}$

$\theta = 2n\pi \pm \frac{3\pi}{4}$

Q $\sin\theta = -\frac{1}{\sqrt{2}}$

$\sin\theta$ है neg value

$[-90°, 90°]$ है select

$\sin\theta = \sin\left(-\frac{\pi}{4}\right)$

$-45° \in [-90°, 90°]$

$\theta = n\pi + (-1)^n\left(-\frac{\pi}{4}\right)$

Q $\tan\theta = -\sqrt{3}$ find G.V.

$\frac{\pi}{3}$

$(-90°, 90°)$

$\tan\theta = \tan\left(-\frac{\pi}{3}\right)$

$-60° \in (-90, 90)$

$\theta = n\pi - \frac{\pi}{3}$

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$\underline{\underline{2^{nd}\text{ Case.}}}$ $[T^2(\theta)=T^2(\alpha)]$

$\sin^2\theta=\sin^2\alpha$ $\quad$ $\cos^2\theta=\cos^2\alpha$ $\quad$ $\tan^2\theta=\tan^2\alpha$

$$\boxed{\begin{array}{l} \sin^2\theta=\sin^2\alpha \\ \cos^2\theta=\cos^2\alpha \\ \tan^2\theta=\tan^2\alpha \end{array} \rightarrow \theta=n\pi\pm\alpha}$$

Q $4\sin^2\theta=3$ find h.v.

$\sin^2\theta=\frac{3}{4}$

$\sin^2\theta=\left(\frac{\sqrt{3}}{2}\right)^2$

$\sin^2\theta=\sin^2\frac{\pi}{3}$

$\theta=n\pi\pm\frac{\pi}{3}$

Q $4\cos^2\theta=3$ find h.v.

$\cos^2\theta=\frac{3}{4}$

$\cos^2\theta=\left(\frac{\sqrt{3}}{2}\right)^2$

$\cos^2\theta=\cos^2\frac{\pi}{6}$

$\theta=n\pi\pm\frac{\pi}{6}$

Q $\tan^2\theta=3$ find h.v.

$\tan^2\theta=(\sqrt{3})^2$

$\tan^2\theta=\tan^2\frac{\pi}{3}$

$\theta=n\pi\pm\frac{\pi}{3}$

Example No 11

Q 1-18

SL Loney

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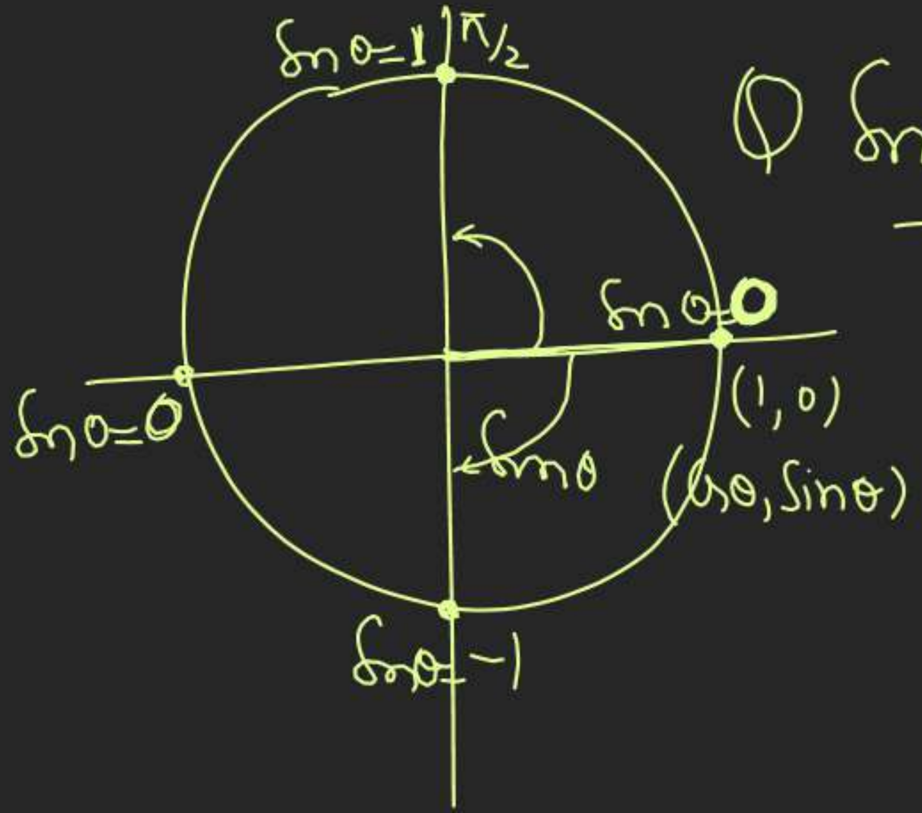
Special Angle.

$\sin\theta=1$, $\sin\theta=-1$, $\cos\theta=1$, $\cos\theta=-1$

$\sin\theta=0$, $\cos\theta=0$



Q $\sin\theta=1$ find G.V.

$\theta=2n\pi+\frac{\pi}{2}$ ✓

यहाँ $n\pi+(-1)^n\frac{\pi}{2}$ Nhi likhte

Q $\sin\theta=-1$ (One time Only)

→ $-90°$ पे

$\theta=2n\pi-\frac{\pi}{2}$

Q $\sin\theta=0$

[Coming twice]

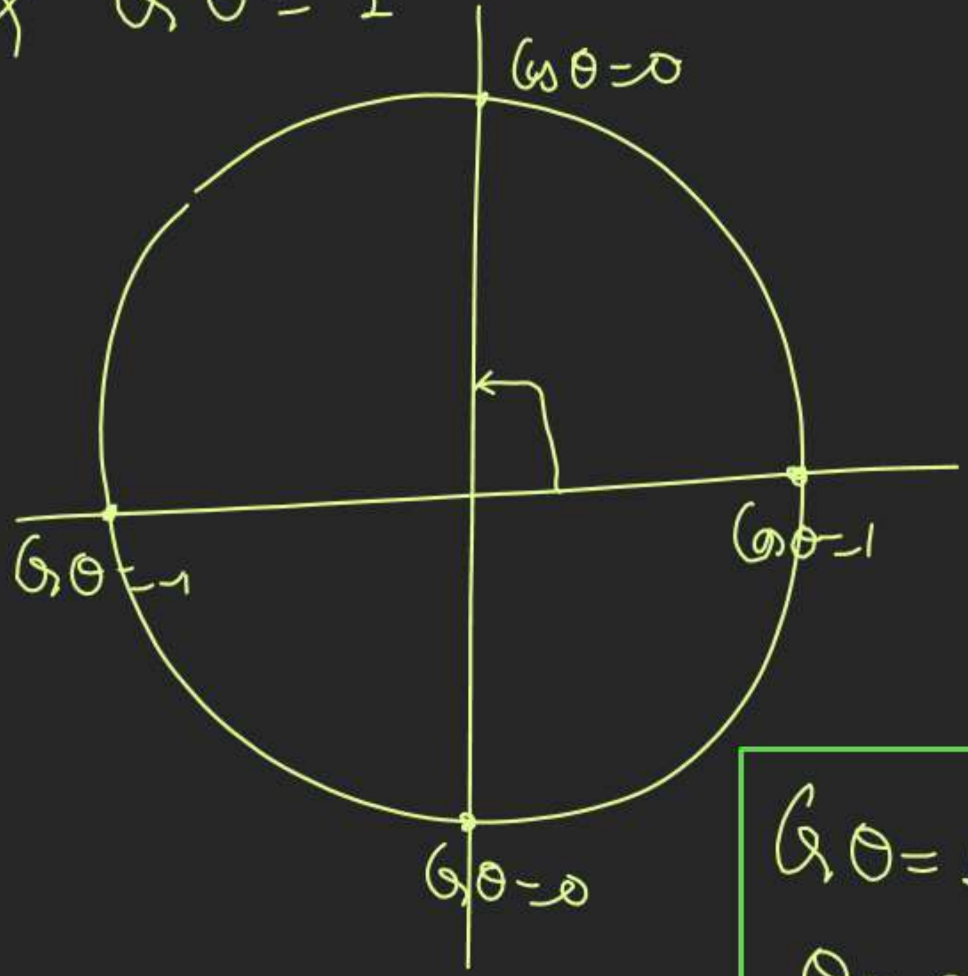
Not an Spl Case

$\theta=n\pi+(-1)^n\cdot 0$

$\theta=n\pi$

Q $Eq^n \rightarrow Ex\ 1$

Q $\cos\theta = 1$



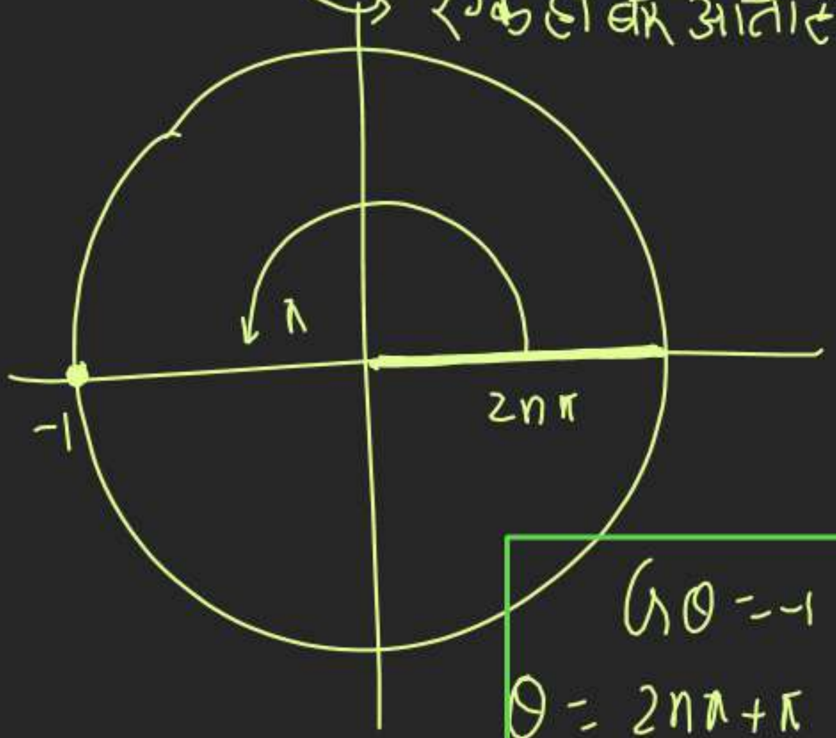
$\cos\theta = 1 \rightarrow 0°$ पर

$\theta = 2n\pi + 0$

$\theta = 2n\pi$

Q $\cos\theta = -1$ G.V.?

एक ही बार आता हैं



$\cos\theta = -1$

$\theta = 2n\pi + \pi$

$\theta = (2n+1)\pi$

Q $\cos\theta = 0$ find G.V.

$\theta = 2n\pi + \frac{\pi}{2}$