Nishant Jindal

① Send your Notes on Group/W/P

Pr 6 $\int_0^{2a} f(x)\,dx = \begin{cases} 2\int_0^{a} f(x)\,dx & f(2a-x) = f(x) \\ 0 & f(2a-x) = -f(x) \end{cases}$

UL − x

Q $\int_0^{2\pi} \cos^4 x\,dx \xrightarrow{Pr\,6} (\cos(2\pi - x))^4 = (\cos x)^4 = \cos^4 x$

$\Rightarrow 2\int_0^{\pi} \cos^4 x\,dx \xrightarrow{Pr\,6} (\cos(\pi - x))^4 = (-\cos x)^4 = \cos^4 x$

$= 2\times 2\int_0^{\pi/2} \cos^4 x\,dx = 4 \times \frac{3\cdot 1}{4\times 2} \times \frac{\pi}{2} = \frac{3\pi}{4}$

Walli

Q $\int_0^{2\pi} \sin^5 x\,dx$ $\quad (\sin(2\pi - x))^5 = (-\sin x)^5 = -\sin^5 x$

Pr 6

$= 0$

$\frac{\pi}{2}$ multiple

Q Trick $\int_0^{64\pi} \sin^6 x\,dx$ ← Even

$\Rightarrow \int_0^{128\times\frac{\pi}{2}} \sin^6 x\,dx = 128\int_0^{\pi/2} \sin^6 x\,dx$

Walli

$= \cancel{128}^{4} \times \frac{5\cdot 3\cdot 1}{6\cdot 4\cdot \cancel{2}_{2}} \times \frac{\pi}{\cancel{2}} = 20\pi$

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Basic

Qs. ① $\int_0^{2a} f(x)\cdot dx = \int_0^a f(x)\cdot dx + \int_0^a f(2a-x)\,dx$ [P.T.]

$a = 2a - t$, $t = a$

$2a = 2a - t$, $t = 0$

LHS $\int_0^{2a} f(x)\,dx = \int_0^a f(x)\cdot dx + \int_a^{2a} f(x)\cdot dx$ P(a)

$x = 2a - t$, $dx = -dt$

x	t
a	a
2a	0

$+ -\int_a^0 f(2a-t)\,dt$ P1, 2

$\int_0^{2a} f(x)dx = \int_0^a f(x)\cdot dx + \int_0^a f(2a-x)dx$

$f(2a-x) = f(x)$

$f(2a-x) = -f(x)$

$\int_0^a f(x)dx + \int_0^a f(x)dx = 2\int_0^a f(x)dx$ (Pr6 1st Proved)

$= \int_0^a \cancel{f(x)dx} + \int_0^a \cancel{-f(x)dx} = 0$

Q $I = \int_0^\pi x\cdot \sin^4 x\,dx$

x हटाएंगे

Limit $\frac{\pi}{2}$ आएंगी

Walli लगाएंगे

$\int_0^{\pi/2} \frac{\sin^n x}{\sin^n x + \cos^n x} = \frac{\pi}{4}$

$= \frac{\pi}{2}\int_0^\pi \sin^4 x\,dx$ Pr6 $2\times\frac{\pi}{2}$

$= \frac{\pi}{2}\int \sin^4 x\,dx = 2\times\frac{\pi}{2}\int_0^{\pi/2} \sin^4 x\,dx = \pi\times\frac{3\times 1}{4\times 2}\times\frac{\pi}{2}$

$= \frac{3\pi^2}{16}$

Adv Main Q3 $I = \int_0^{2\pi} \frac{x\cdot \sin^{2n} x\,dx}{\sin^{2n} x + \cos^{2n} x}$

① x हटाएंगे

② Limit $\frac{\pi}{2}$ लगाएंगे

③ Set 1 (Pr 4) लगाएंगे

$I = \pi\int_0^{2\pi} \frac{\sin^{2n} x\,dx}{\sin^{2n} x + \cos^{2n} x}$ Pr6

$= 4\pi \boxed{\int_0^{\pi/2} \frac{\sin^{2n} x\,dx}{\sin^{2n} x + \cos^{2n} x}} = 4\pi\times\frac{\pi}{4} = \pi^2$

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Result 4 → (a/c) ... Q 4 $I = \int_0^{\pi/2} \ln \sin x \, dx \rightarrow A$

$\downarrow$ Pr 4 $x \to \left(\frac{\pi}{2} - x\right)$

$I = \int_0^{\pi/2} \ln \cos x \, dx \rightarrow B$

A+B

$2I = \int_0^{\pi/2} \ln\left(\frac{2\sin x \cdot \cos x}{2}\right) dx$

$2I = \int_0^{\pi/2} \ln(\sin \boxed{2x}) - \ln 2 \, dx$

$2x = t \Rightarrow dx = \frac{dt}{2}$

$= \frac{1}{2}\int_0^{\pi} \ln \sin t \, dt - \int_0^{\pi/2} \ln 2 \, dx$

$\downarrow$ Pr 6, 1

$= \frac{2}{2}\int_0^{\pi/2} \ln \sin x \, dx - \ln 2 \, (x)_0^{\pi/2}$

$2I = I - \ln 2 \times \frac{\pi}{2} \Rightarrow I = -\frac{\pi}{2}\ln 2$

Results

1) $\int_0^{\pi/2} \ln \sin x \, dx = \int_0^{\pi/2} \ln \cos x \, dx = -\frac{\pi}{2}\ln 2$

2) $\int_0^{\pi/2} \ln \operatorname{cosec} x \, dx = \int_0^{\pi/2} \ln \sec x \, dx = \frac{\pi}{2}\ln 2$

3) $\int_0^{\pi/2} \ln \tan x \, dx = \int_0^{\pi/2} \ln \cot x \, dx = 0$

4) $\int_0^{\pi/2} \ln \sin 2x \, dx = -\frac{\pi}{2}\ln 2$

Q $I = \int_0^{\pi/4} \ln \sin 2x \, dx$

Q $I = \int_0^{\pi/2} \ln(\tan x + \cot x) \, dx$

Q $I = \int_0^{\infty} \ln\left(x + \frac{1}{x}\right) dx$

Q $\int_{-\pi/4}^{\pi/4} \ln(\sin x + \cos x) \, dx$

Q $\int_{-\pi/2}^{\pi/2} \ln\left(\frac{px^2 - qx + r}{px^2 + qx + r}\right) \cdot (a+b)\,|\sin x| \, dx$

(5) $2x = t \Rightarrow dx = \frac{dt}{2}$

x	t
0	0
$\frac{\pi}{4}$	$\frac{\pi}{2}$

$I = \frac{1}{2}\int_0^{\pi/2} \ln(\sin t) \, dt = \frac{1}{2} \times -\frac{\pi}{2}\ln 2$

$= -\frac{\pi}{4}\ln 2$

Q 6 $I = \int_0^{\pi/2} \ln\left(\frac{1 + \tan^2 x}{\tan x}\right) dx$

$= \int_0^{\pi/2} \ln \sec^2 x - \ln \tan x \, dx$

$= 2 \times \frac{\pi}{2}\ln 2 - 0 = \pi \ln 2$

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Q7 $\int_0^{\infty} \ln\left(x+\frac{1}{x}\right) \frac{dx}{1+x^2}$ $\quad x = \tan\theta$

$dx = \sec^2\theta\, d\theta$

x	θ
0	0
∞	$\frac{\pi}{2}$

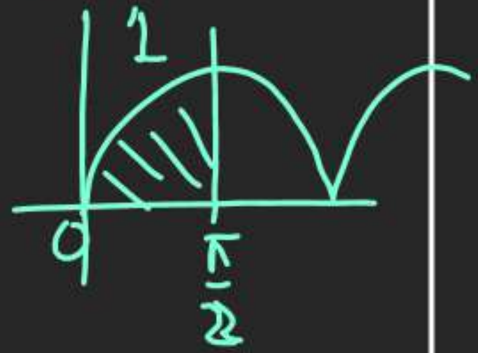
$$I = \int_0^{\pi/2} \ln(\tan\theta + \cot\theta) \frac{\cancel{\sec^2\theta}\, d\theta}{\cancel{1+\tan^2\theta}}$$

$$= \pi \ln 2$$

Q9 $$I = \int_{-\pi/2}^{\pi/2} \cancel{\ln\left(\frac{px^2-qx+r}{px^2+qx+r}\right)} + \int_{-\pi/2}^{\pi/2} (a+b)\,|\sin x|\, dx$$

Even.

$$= 2(a+b)\int_0^{\pi/2} |\sin x|\, dx$$



$$= 2(a+b) \times 1 = 2(a+b)$$

Q8 $\int_{-\pi/4}^{\pi/4} \ln(\sin x + \cos x)\, dx$

Q $\int_{-\pi/2}^{\pi/2} \boxed{\ln\left(\frac{px^2-qx+r}{px^2+qx+r}\right)} (a+b)\,|\sin x|\, dx$

→ x की Jagah $-x$ Socho

→ odd

Q7 $\int_{-\pi/4}^{\pi/4} \ln(\sin x + \cos x)\, dx$

O + E = NE NO

$$= \int_0^{\pi/4} \ln(\sin x + \cos x) + \ln(\cos x - \sin x)\, dx$$

$$= \int_0^{\pi/4} \ln(\cos^2 x - \sin^2 x)\, dx = \int_0^{\pi/4} \ln\cos \boxed{2x}\, dx \qquad 2x = t$$

$$= \frac{1}{2}\int_0^{\pi/2} \ln\cos t\, dt = -\frac{\pi}{4}\ln 2$$

(5) $2x = t \Rightarrow dx = \frac{dt}{2}$

x	t
0	0
$\frac{\pi}{4}$	$\frac{\pi}{2}$

$$I = \frac{1}{2}\int_0^{\pi/2} \ln(\sin t)\, dt = \frac{1}{2} \times -\frac{\pi}{2}\ln 2$$

$$= -\frac{\pi}{4}\ln 2$$

Q $I = \int_0^{\pi/4} \ln \sin 2x\, dx$

Q $I = \int_0^{\pi/2} \ln(\tan x + \cot x)\, dx$

Q $I = \int_0^{\infty} \ln\left(x + \frac{1}{x}\right) \frac{dx}{1+x^2}$

Q6 $$I = \int_0^{\pi/2} \ln\left(\frac{1+\tan^2 x}{\tan x}\right) dx$$

$$= \int_0^{\pi/2} \ln \sec^2 x - \ln\tan x \cdot dx$$

$$= 2 \times \frac{\pi}{2}\ln 2 - 0 = \pi \ln 2$$

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Q10 $I = \int_0^{\pi} x\left(\sin^2(\sin x) + \cos^2(\cos x)\right)dx$

① x हटाओ
② Any how.
$\sin^2(\theta)+\cos^2\theta = 1$
→ Use करो

$= \frac{\pi}{2}\int_0^{\pi} \sin^2(\sin x)+\cos^2(\cos x)\,dx$

↓ Pr6 $f(\pi - x) = f(x)$ Same रहा

3Qs

$I = \frac{\pi}{2}\times 2\int_0^{\pi/2} \sin^2(\sin x)+\cos^2(\cos x)\,dx \to (A)$

↓ Pr4 $\left(x \to \frac{\pi}{2} - x\right)$

$I = \pi\int_0^{\pi/2} \sin^2(\cos x)+\cos^2(\sin x)\,dx \to (B)$

$2I = \pi\int_0^{\pi/2} \underbrace{\sin^2(\cos x)+\cos^2(\cos x)} + \underbrace{\sin^2(\sin x)+\cos^2(\sin x)}\,dx$

$= 2\pi(x)_0^{\pi/2} = \pi^2 \Rightarrow I = \frac{\pi^2}{2}$

Q11 $I = \int_0^1 \ln x\cdot dx$

$= x\ln x - x\Big|_0^1$

$\boxed{= (1\ln 1 - 1) - (0\ln 0 - 0)}$ Hold
$0\times\infty$

$= (1\cdot\ln 1 - 1) - \left(\lim_{x\to 0} x\ln x - 0\right)$

$= (0-1) - \left(\lim_{x\to 0}\frac{\ln x}{1/x} - 0\right)$

$- \left(\lim_{x\to 0}\frac{\frac{1}{x}}{-\frac{1}{x^2}} - 0\right)$

$= -1 - (0-0)$

$= -1$

Q $I = \int_0^{\pi/2}\left(\frac{\theta}{\sin\theta}\right)^2 d\theta$ → Pr1

Q $I = \int_0^1 \frac{\sin^{-1}x}{x}\cdot dx$

Q12 $I = \int_0^{\pi/2} x^2\cdot\operatorname{cosec}^2 x\cdot dx$

$= x^2(-\cot x)\Big|_0^{\pi/2} + 2x(+\ln\sin x)\Big|_0^{\frac{\pi}{2}} +$

$2\int_0^{\pi/2} -\ln\sin x\, dx$

$= \left(0 + \lim_{x\to 0} x^2\cot x\right) + 2\left(0 - \lim_{x\to 0} x\ln\sin x\right)$
$0\times\infty$ $\quad$ $0\times\infty$

$+ 2\times\frac{\pi}{2}\ln 2$

$= \pi\ln 2$

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Prop 7 Periodic fxn Based Prop

A) $\int_0^{nT} f(x)dx = n\int_0^{T} f(x)\,dx$

B) $\int_a^{a+T} f(x)\cdot dx = \int_0^{T} f(x)\,dx$ [Independent of a]

A)

$\int_0^{nT} f(x)dx = n$ Jhopadiyon ki Area $= n\int_0^{T} f(x)dx$

(B)

Q13 $I = \int_0^1 \frac{\sin^{-1}x}{x}\cdot dx$

$x = \sin\theta$, $dx = \cos\theta\, d\theta$ | x: 0, 1 | θ: 0, $\frac{\pi}{2}$

$= \int_0^{\pi/2} \frac{\theta\cdot\cos\theta\cdot d\theta}{\sin\theta}$

$= \int_0^{\pi/2} \theta\cdot\cot\theta\cdot d\theta$ IBP-(Tricks)

$= \theta(-\ln\sin\theta)\Big|_0^{\frac{\pi}{2}} + \int_0^{\pi/2} 1\times\ln\sin\theta\, d\theta$

$0\times\infty$

$= 0 - \frac{\pi}{2}\ln 2$

Q $I = \int_0^{\pi/2}\left(\frac{\theta}{\sin\theta}\right)^2 d\theta$ → Pr1

Q $I = \int_0^1 \frac{\sin^{-1}x}{x}\cdot dx$

Q12 $I = \int_0^{\pi/2} x^2\cdot\operatorname{cosec}^2x\cdot dx$

$= x^2(-\cot x)\Big|_0^{\pi/2} + 2x(+\ln\sin x)\Big|_0^{\frac{\pi}{2}} +$

$2\int_0^{\pi/2} -\ln\sin x\, dx$

$= \left(0 + \lim_{x\to 0} x^2\cot x\right) + 2\left(0 - \lim_{x\to 0} x\ln\sin x\right)$

$0\times\infty$ $\quad$ $0\times\infty$

$+ 2\times\frac{\pi}{2}\ln 2$

$= \pi\ln 2$

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Prop 7 Periodic fxn Based Prop

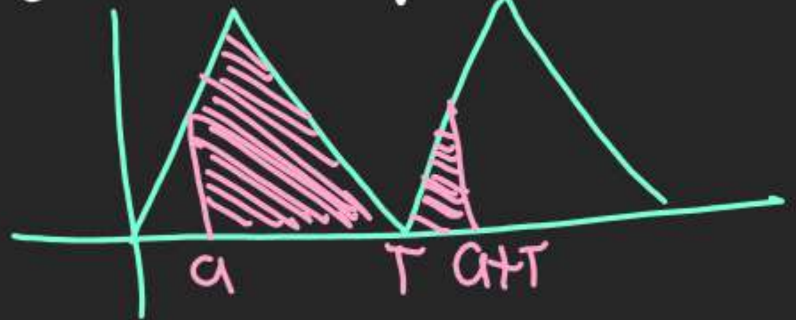
A) $\int_0^{nT} f(x)\,dx = n\int_0^{T} f(x)\,dx \; ; \; n \in I$

B) $\int_a^{a+T} f(x)\cdot dx = \int_0^{T} f(x)\,dx$ [Independent of a]

A)

$\int_0^{nT} f(x)\,dx = n$ Jhopadiyon ka Area $= n\int_0^{T} f(x)\,dx$

(B)



Q $\int_0^{9\pi} |\sin x|\,dx \rightarrow T=\pi$

$= 9\int_0^{\pi} |\sin x|\,dx$

$= 9\times 2 = 18.$

Q1: $\int_0^{9\pi} \sqrt{\frac{1-\cos 2x}{2}}\,dx$

$= \int_0^{9\pi} \sqrt{\frac{2\sin^2 x}{2}}$

$= \int_0^{9\pi} |\sin x|\,dx = 18$

Q16 $\int_0^{400\pi} \sqrt{\frac{1-\cos 2x}{2}}\,dx$

$= \int_0^{400\pi} |\sin x|\,dx = 400\times 2$

$= 800$

Q $I = \int_0^{\frac{16\pi}{3}} |\sin x|\,dx$; $16\frac{\pi}{3} = 5\pi + \frac{\pi}{3}.$

$= \int_0^{5\pi} |\sin x|\,dx + \int_{5\pi}^{5\pi+\frac{\pi}{3}} |\sin x|\,dx$

$= 5\times 2 + \int_0^{\pi/3} \sin x\,dx$ 1 Quad

$= 10 - (\cos x)_0^{\frac{\pi}{3}}.$

$= 10 - \left(\frac{1}{2} - 1\right) = \frac{21}{2}$

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Q 18 $\displaystyle\int_0^{10} \frac{2^x}{2^{[x]}}\,dx$ $\quad \{x\}$ T=1

$$= \int_0^{10\times 1} 2^{x-[x]}\,dx$$

$$= 10\int_0^1 2^{x-0}\,dx$$

$x\in(0,1)$, $[x]=0$

$$= 10\times \left.\frac{2^x}{\ln 2}\right|_0^1$$

$$= 10\left(\frac{2}{\ln 2}-\frac{1}{\ln 2}\right)$$

$$= \frac{10}{\ln 2}$$

Q $\displaystyle I=\int_0^{[x]} \frac{2^x}{2^{[x]}}\,dx$ $\quad$ Int, T=1

$$= \int_0^{[x]\times 1} 2^{x-[x]}\,dx$$

$$= [x]\int_0^1 2^{x-0}\,dx$$

$$= \frac{[x]}{\ln 2}$$

Q 20 $\displaystyle I=\int_0^{10.7} \frac{2^x}{2^{[x]}}\,dx$ $\quad$ T=1

$$= \int_0^{10.7} 2^{x-[x]}\,dx$$

$$= \int_0^{10\times 1} 2^{x-[x]} + \int_{10}^{10.7}$$

$$= 10\int_0^1 2^{x-0}\,dx + \int_0^{.7} 2^{x-0}\,dx$$

$x\in(0,.7)$, $[x]=0$

$$= 10\times\left.\frac{2^x}{\ln 2}\right|_0^{1} + \left.\frac{2^x}{\ln 2}\right|_0^{.7}$$

$$= 10\times\frac{(2^{10}-1)}{\ln 2} + \frac{2^{.7}-1}{\ln 2}$$

Q $\displaystyle I=\int_0^{\frac{41}{2}} \frac{e^{2x}}{e^{[2x]}}\,dx$ $\quad \{2x\}$ $T=\frac{1}{2}$

$$= \int_0^{41\times\frac{1}{2}} e^{2x-[2x]}$$

$$= 41\int_0^{1/2} e^{2x-0}\,dx$$

$x\in(0,\frac{1}{2})$, $[2x]=0$

$$= \frac{41}{2}\times \left. e^{2x}\right|_0^{1/2}$$

$$= \frac{41}{2}(e-1)$$

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Q21 $I = \int_0^{n\pi+v} |\cos x|\,dx$

$\frac{\pi}{2} < v < \pi$; $n \in N$ $\rightarrow$ $v = 2^{nd}$ Quad $= 130^\circ$

$\cos x = -ve$

$|\cos x| = -\cos x$

$$= \int_0^{n\pi} |\cos x|\,dx + \int_{n\pi}^{n\pi+v} |\cos x|\,dx$$

$$= n\int_0^{\pi} |\cos x|\,dx + \int_0^{v} |\cos x|\,dx$$

$$= 2n + \int_0^{\pi/2} \cos x\,dx + \int_{\pi/2}^{v} -\cos x\,dx$$

$$= 2n + (\sin x)_0^{\pi/2} - (\sin x)_{\pi/2}^{v}$$

$$= 2n + 1 - (\sin v - 1)$$

Doubt

56-58

Subjective (Kal Upload hoga)

Q1- 30

Q1, 2, 3, 4, 5, 6.

8, 9, 10, 11, 14, 15, 16. } Jee main