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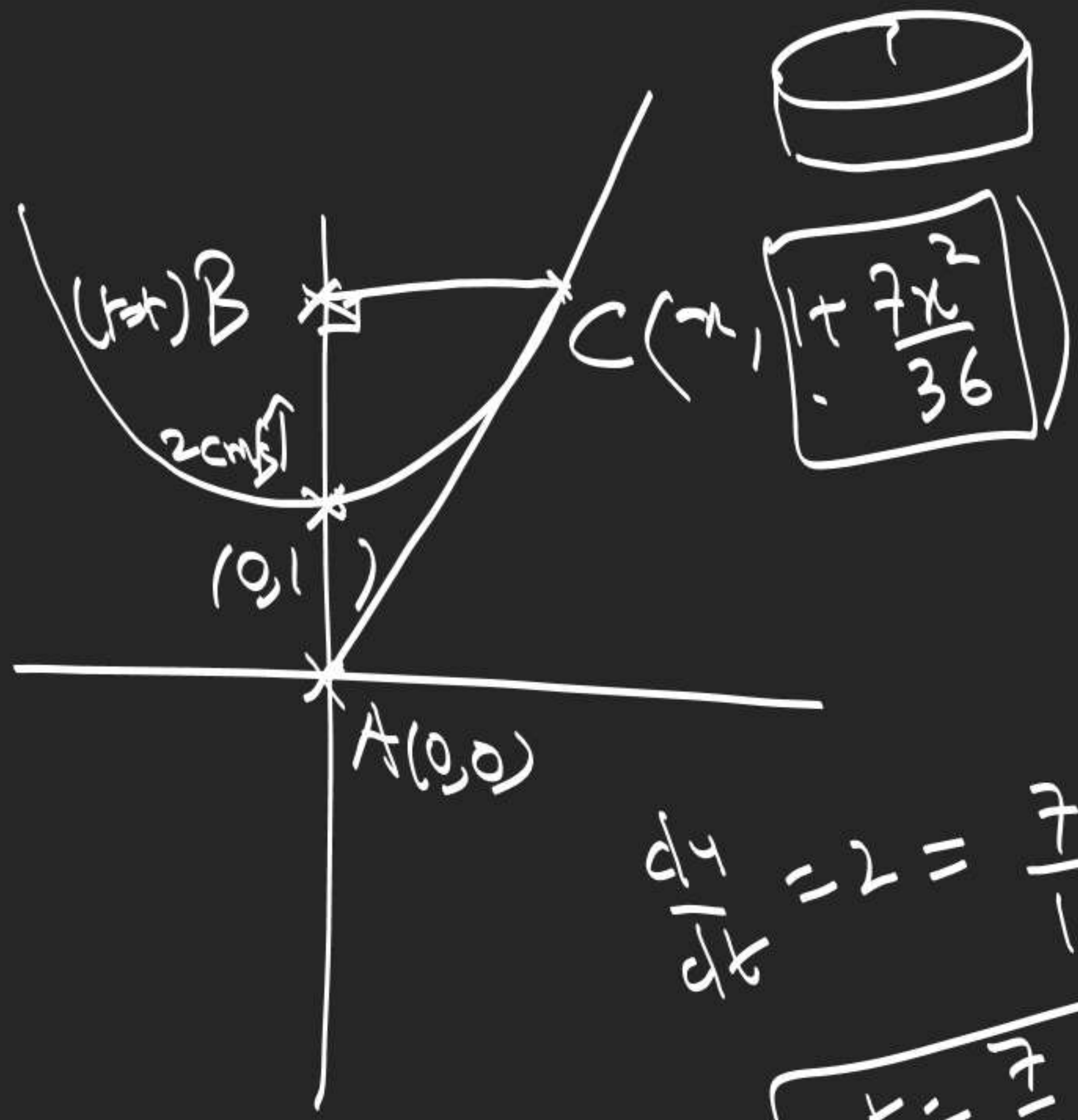

$$A = \frac{1}{2}\left(x + \frac{7x^3}{36}\right)$$

$$\frac{dA}{dt} = \frac{1}{2}\left(1 + \frac{7}{12}x^2\right)\frac{dx}{dt}$$

$$\frac{dy}{dt} = 2 = \frac{7}{18}x\frac{dx}{dt} \Rightarrow \left(\frac{dx}{dt}\right)_{t=\frac{7}{2}\,sec} = \frac{36}{7\times 6}$$

$$\boxed{t = \frac{7}{2}\,sec} \to x$$

$$y = 8 = 1 + \frac{7x^2}{36} \Rightarrow \boxed{x = 6}$$

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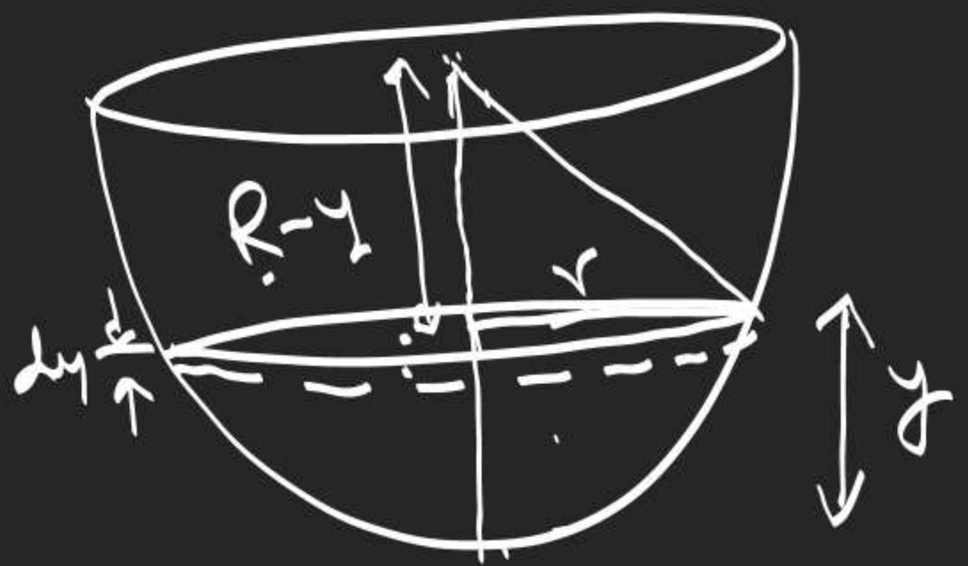

$$r^2 = R^2 - (R-y)^2$$
$$= y(2R-y)$$

$$2r\frac{dr}{dt} = (2R-2y)\frac{dy}{dt}.$$

$$\int_0^y dV = \int_0^y \pi r^2\, dy = \int_0^y \pi y(2R-y)\, dy$$

$$\int_0^t dt \qquad \int_1^r r^3\, dr \qquad K>0$$

$$V(y) =$$

$$V = \frac{4}{3}\pi r^3$$

$$4\pi r^2 \frac{dr}{dt} = \frac{K}{r}$$

$$(r^4-1) = \frac{K}{\pi}t.$$

$$t=15,\ r=2.\quad \boxed{\frac{K}{\pi}=1}$$

$$\boxed{r^4 = 1+t}$$

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Cubic Function

$$f(x) = ax^3 + bx^2 + cx + d \quad , \; a, b, c, d \in R \; , \; a \neq 0 .$$

$$f'(x) = 3ax^2 + 2bx + c = 0$$

→ 2 distinct real solutions, x_1, x_2

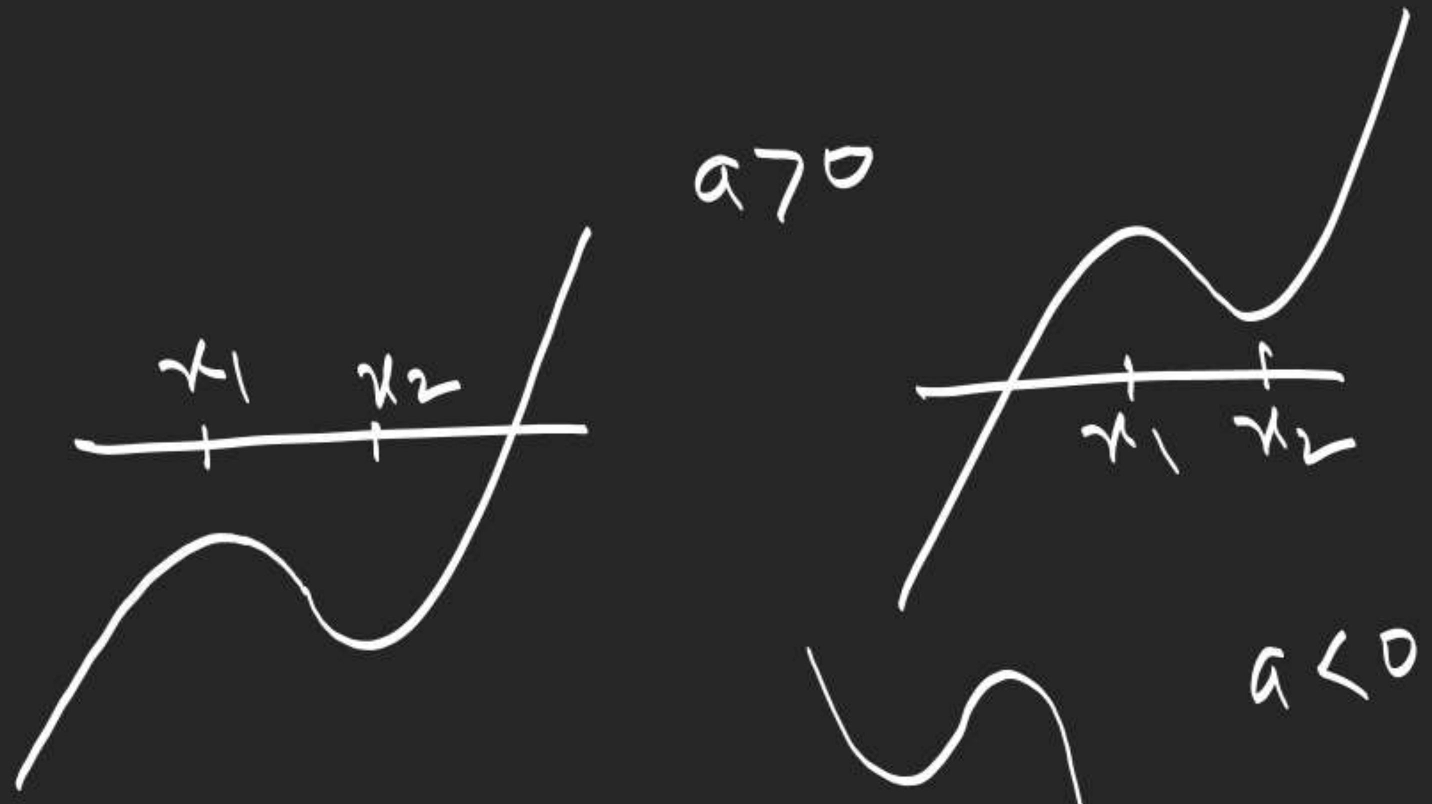
→ 2 equal real solutions

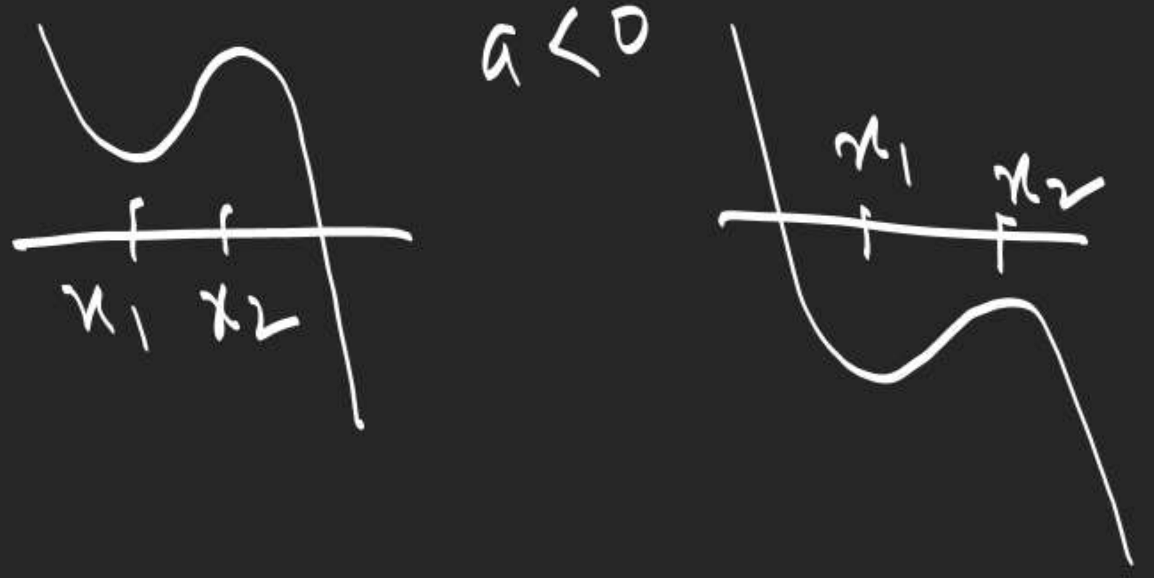
→ 2 imaginary solutions

Case I. $f'(x)=0$ has 2 distinct real solutions x_1, x_2

Condition for $f(x)$ to have 1 distinct real roots

$$\boxed{f(x_1)\,f(x_2) > 0}$$





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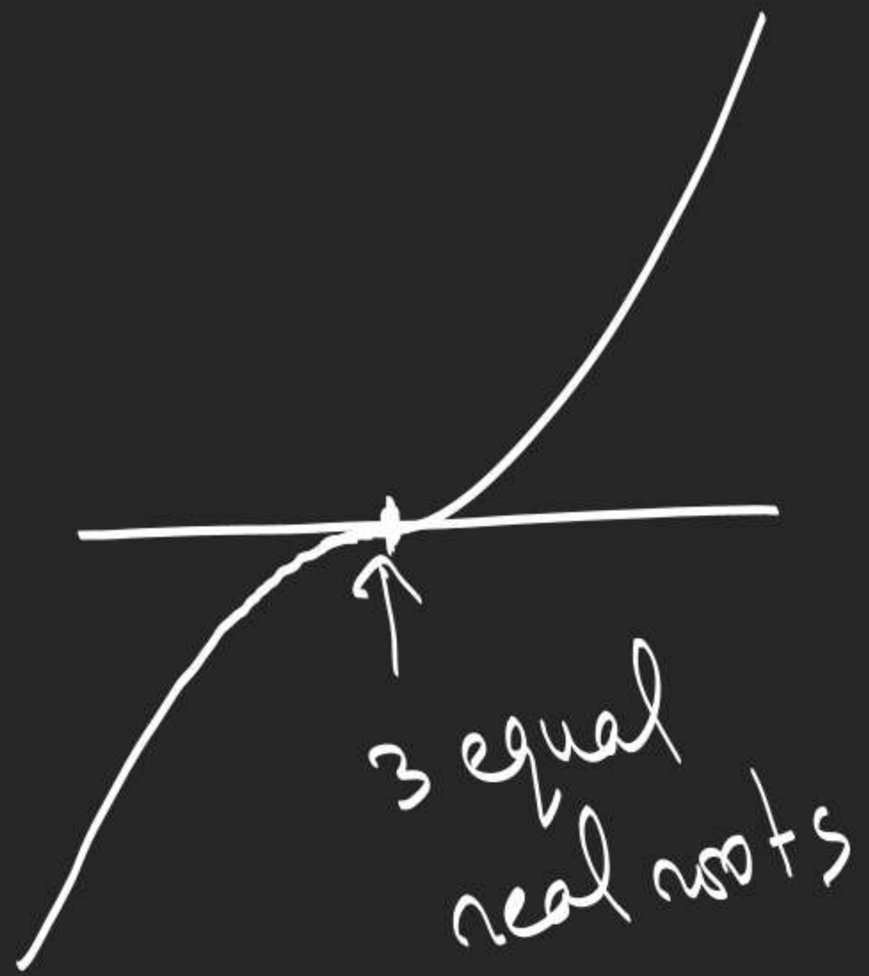
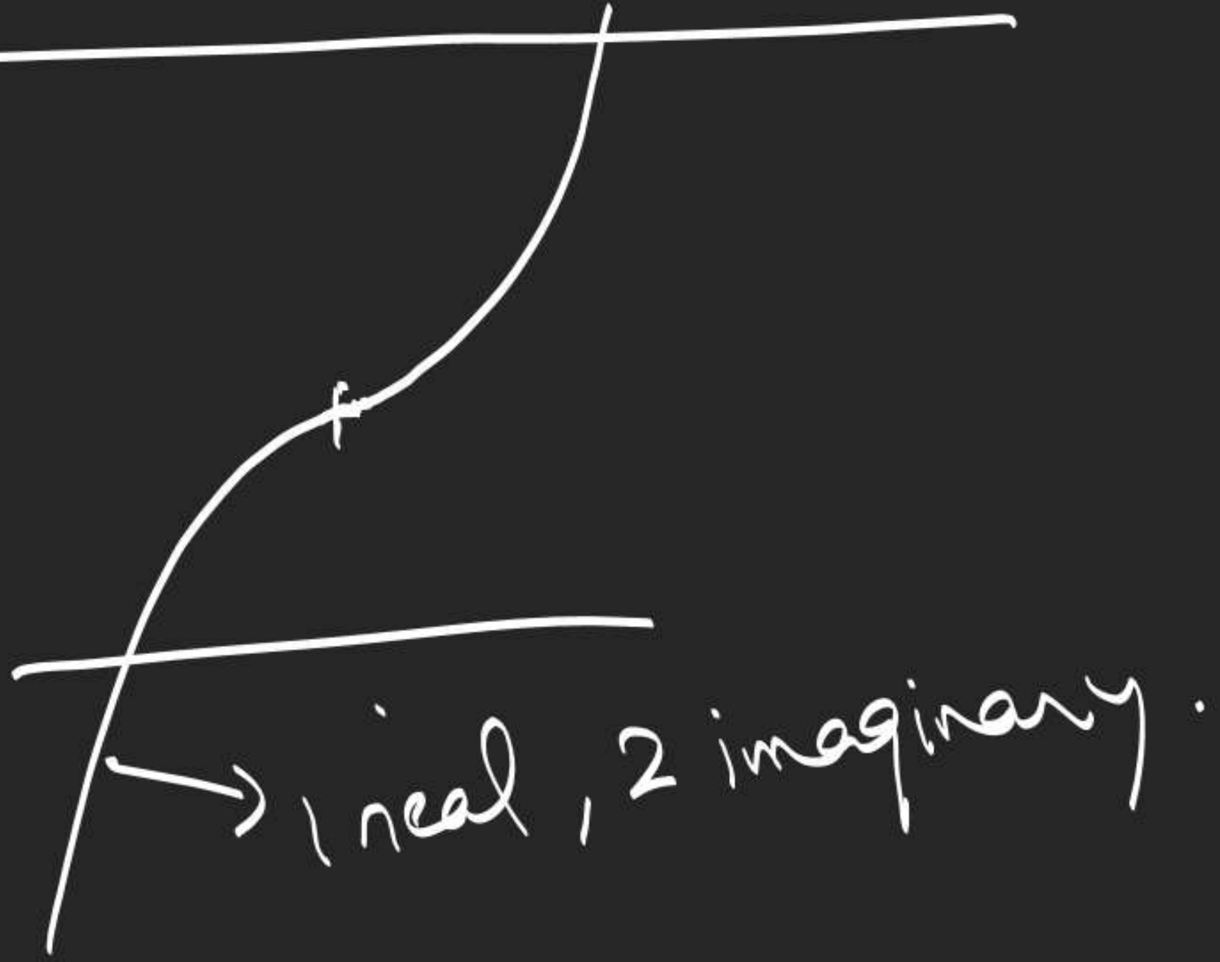
Case II

$f'(x)=0$ has 2 equal real roots $x_1=x_2$

$f'(x)=3a(x-x_1)^2$





Case III $f'(x)=0$ has 2 imaginary roots

1 real,
2 imaginary

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1. If the cubic $y = x^3 + px + q$ has 3 distinct real roots, then P.T. $4p^3 + 27q^2 < 0$.

$3x^2 + p = 0 \; < \begin{matrix} x_1 \\ x_2 \end{matrix}$

$f(x_1) = x_1^3 + px_1 + q = (3x_1^2 + p)\frac{x_1}{3} + \frac{2px_1}{3} + q$

$f(x_1)f(x_2) < 0$

$\Rightarrow \left(\frac{2px_1}{3} + q\right)\left(\frac{2px_2}{3} + q\right) < 0$

$\frac{4p^2}{9}x_1x_2 + \frac{2pq}{3}\cancel{(x_1+x_2)}^{\,0} + q^2 < 0$

$\frac{4p^2}{9}\frac{p}{3} + q^2 < 0$

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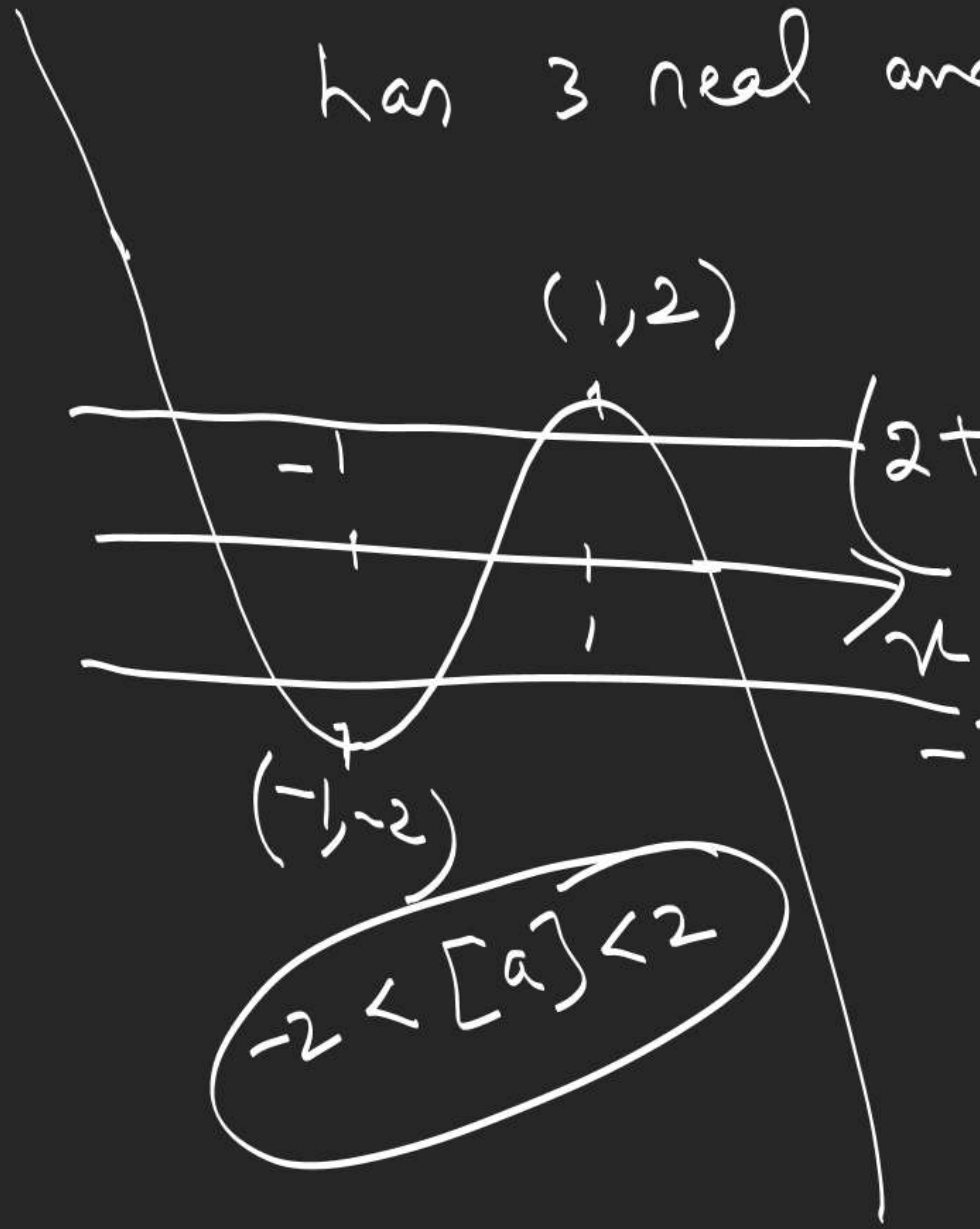
Q. Find 'a' so that $f(x) = x^3 - 3x + [a]$, $[\cdot] = \text{G.I.F}$ has 3 real and distinct roots.

$$[a] = \boxed{3x - x^3}$$



± 1

$$(2+[a])([a]-2) < 0$$

$$-2 < [a] < 2$$

$$-2 < [a] < 2$$

$\{-1, 0, 1\}$

$$\boxed{a \in [-1, 2)}$$

JEE Main

Adv

Ex-3, 4

Ex-3

99.5+

$(-2,3)$ A

$(-1,1)$ B

D $(2,7)$

$\frac{7-1}{2x+1} = 2$

$C(x, x^2+x+1) = \left(\frac{1}{2}, \frac{7}{4}\right)$

$y = x^2+x+1$

$2x+1 = 2$