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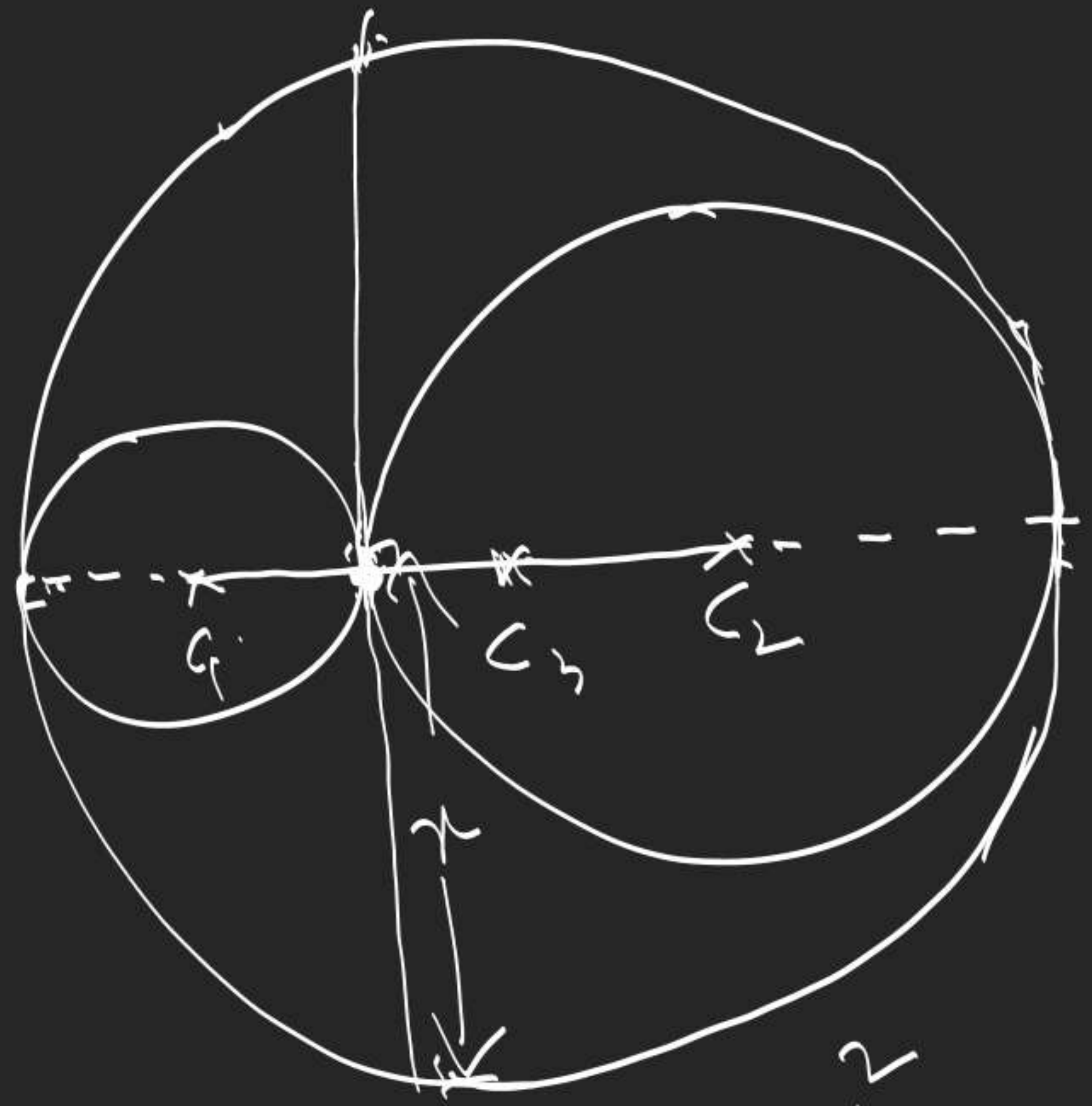

$8 \times 20 = x^2$

$2x = ?$

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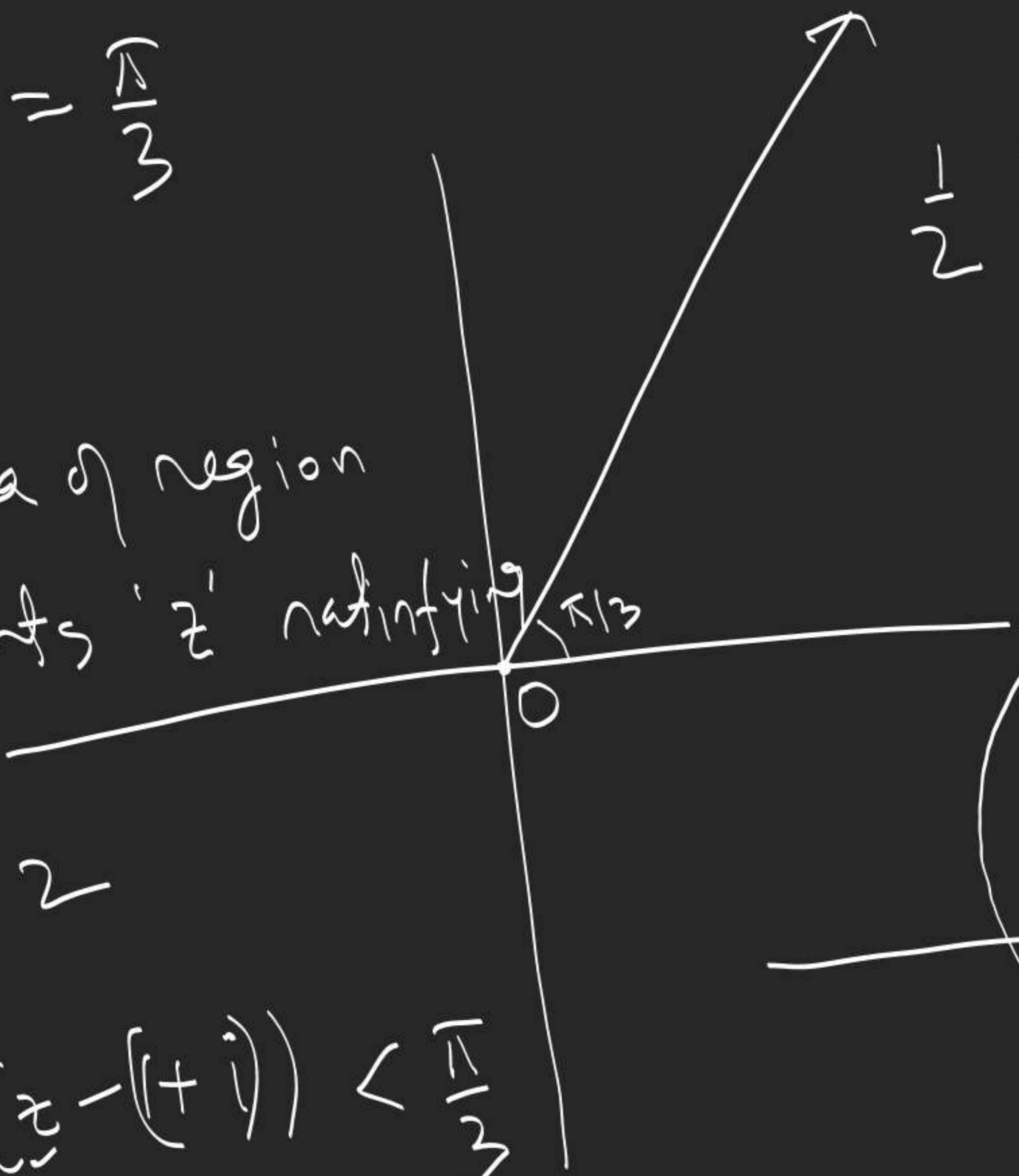
1. $\arg z = \frac{\pi}{3}$



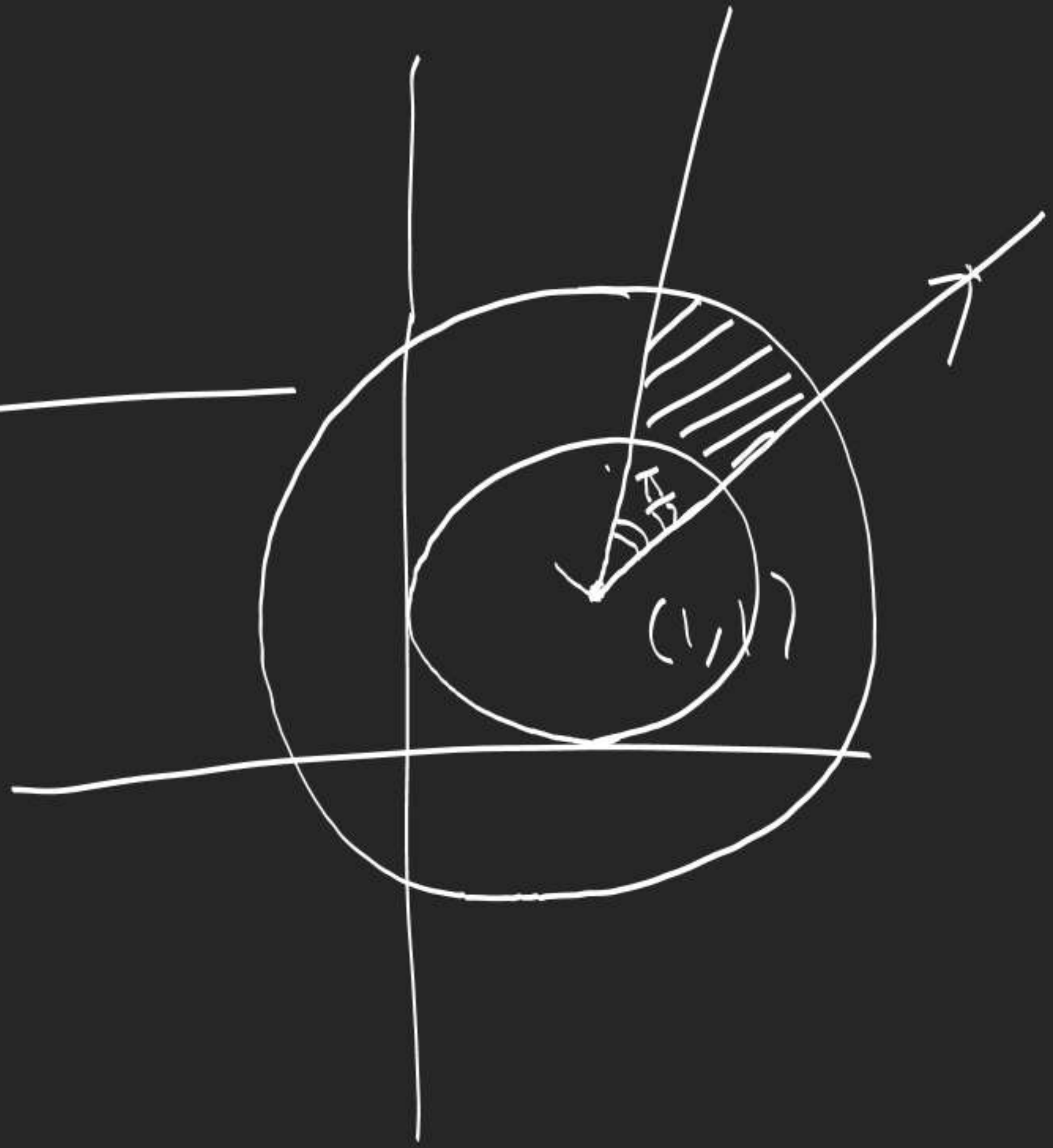
2. Find area of region enclosed by points 'z' satisfying

$$1 < |z-1-i| < 2$$

&

$$\frac{\pi}{4} < \arg(z-(1+i)) < \frac{\pi}{3}$$

$$\frac{1}{2}\cdot\frac{\pi}{12}(2^2-1^2) = \boxed{\frac{\pi}{8}}$$



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Conjugate of Complex Number

$$z = a + ib \qquad a, b \in R.$$

$$\bar{z} = a - ib$$

- $\boxed{\begin{aligned} &\text{Re}(z) = \frac{z+\bar{z}}{2} \\ &\text{Im}(z) = \frac{z-\bar{z}}{2i} \\ &z\bar{z} = |z|^2 \end{aligned}}$

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Algebra of Complex No.s

$$z_1 + z_2 = (x_1 + iy_1) + (x_2 + iy_2) = (x_2 + x_1) + i(y_1 + y_2)$$

$$z_1 z_2 = (x_1 + iy_1)(x_2 + iy_2) = (x_1 x_2 - y_1 y_2) + i(y_1 x_2 + y_2 x_1)$$

$$\frac{z_1}{z_2} = \frac{x_1 + iy_1}{x_2 + iy_2} = \frac{(x_1 + iy_1)(x_2 - iy_2)}{(x_2 + iy_2)(x_2 - iy_2)} = \frac{x_1 x_2 + y_1 y_2}{x_2^2 + y_2^2} + i\frac{(y_1 x_2 - y_2 x_1)}{x_2^2 + y_2^2}$$

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Equality of Complex Numbers

$$z_1 = z_2$$

$$\Rightarrow \quad \mathrm{Re}(z_1) = \mathrm{Re}(z_2) \ \& \ \mathrm{Im}(z_1) = \mathrm{Im}(z_2)$$

OR

$$|z_1| = |z_2| \ \& \ \arg(z_1) = \arg(z_2)$$

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Inequality

$$1+i \nless 2+i$$

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Properties of $\bar{z}$

$$\overline{(z_1+z_2)} = \bar{z}_1 + \bar{z}_2$$

$$\overline{z_1+(z_2+z_3)} = \bar{z}_1+\bar{z}_2+\bar{z}_3$$

$$\overline{z_1-z_2} = \bar{z}_1 - \bar{z}_2$$

$$\overline{z_1 z_2} = \bar{z}_1 \bar{z}_2$$

$$\overline{z_1 \underline{z_2 z_3}} = \bar{z}_1 \bar{z}_2 \bar{z}_3$$

$$\overline{\left(\frac{z_1}{z_2}\right)} = \frac{\bar{z}_1}{\bar{z}_2}$$

$$\frac{z_1}{z_2} = \frac{(x_1+iy_1)(x_2-iy_2)}{(x_2+iy_2)(x_2-iy_2)} = \frac{x_1x_2+y_1y_2}{x_2^2+y_2^2} + i\frac{(y_1x_2-y_2x_1)}{x_2^2+y_2^2}$$

$$LHS = \frac{x_1x_2+y_1y_2}{x_2^2+y_2^2} - i\frac{(y_1x_2-y_2x_1)}{x_2^2+y_2^2}$$

$z_2 \neq 0$

$$RHS = \frac{(x_1-iy_1)}{(x_2-iy_2)}\frac{(x_2+iy_2)}{(x_2+iy_2)} = \frac{x_1x_2+y_1y_2}{x_2^2+y_2^2} + i\frac{(y_2x_1-y_1x_2)}{x_2^2+y_2^2}$$

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$$\overline{\left(\frac{z_1 + z_2^2 - z_3 z_4 z_5^3}{z_6 z_7 + z_8}\right)} = \frac{\bar{z}_1 + (\bar{z}_2)^2 - \bar{z}_3 \bar{z}_4 (\bar{z}_5)^3}{\bar{z}_6 \bar{z}_7 + \bar{z}_8}$$

$$\overline{z_5^3} = \bar{z}_5 \bar{z}_5 \bar{z}_5$$

$$= (\bar{z}_5)^3$$

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Properties of $|z|$

$$\boxed{\begin{aligned} |z_1 z_2| &= |z_1||z_2| \\ \left|\frac{z_1}{z_2}\right| &= \frac{|z_1|}{|z_2|} \qquad z_2 \neq 0 \end{aligned}}$$

$$|z_1 \underbrace{z_2 z_3}| = |z_1||z_2||z_3|$$

$$|z^5| = |z|^5$$

$$\left|\frac{z_1}{z_2}\right|^2 = \left(\frac{z_1}{z_2}\right)\overline{\left(\frac{z_1}{z_2}\right)} = \left(\frac{z_1}{z_2}\right)\left(\frac{\bar{z}_1}{\bar{z}_2}\right) = \frac{|z_1|^2}{|z_2|^2}$$

$$\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}$$

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1. If $\dfrac{x-2+i(y-3)}{1+i} = 1-3i$, $x, y \in R$

find $\boxed{(x, y) = (6, 1)}$

$\dfrac{(x-2+i(y-3))(1-i)}{2} = \dfrac{x-2+y-3+i(y-3-x+2)}{2}$

$\dfrac{x+y-5}{2} = 1$ & $\dfrac{y-x-1}{2} = -3$

2. Find 'θ' s.t. $\boxed{\dfrac{3+2i\sin\theta}{1-2i\sin\theta}}$ is purely real.

$\dfrac{(3+2i\sin\theta)(1+2i\sin\theta)}{1+4\sin^2\theta} = \dfrac{(3-4\sin^2\theta)+i\,8\sin\theta}{1+4\sin^2\theta}$

$\sin\theta = 0$ $\boxed{\theta = n\pi}$, $n \in I$.

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3. Find m, $m \in R$ s.t. eqn.

$z^3 + (3+i)z^2 - 3z - (m+i) = 0$ has atleast one real root.

$z = k$, $k \in R$

$(k^3 + 3k^2 - 3k - m) + i(k^2 - 1) = 0$

$k^3 + 3k^2 - 3k - m = 0$ & $k^2 - 1 = 0$

$$\boxed{\begin{matrix} k=1, & m=1 \\ k=-1, & m=5 \end{matrix}}$$

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4. Convert the complex number

$$z = \frac{i-1}{\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}}$$ in polar form.

$$\cos\frac{\pi}{3} + i\sin\frac{\pi}{3} = \frac{1}{2} + i\frac{\sqrt{3}}{2}$$

$$\boxed{z = |z|(\cos\theta + i\sin\theta)}$$

$$= \frac{2(i-1)}{(1+i\sqrt{3})}$$

$|i-1| = \sqrt{2}$

$\theta = \frac{3\pi}{4}$

$$= \frac{\sqrt{2}\, e^{i\frac{3\pi}{4}}}{e^{i\pi/3}} = \boxed{\sqrt{2}\, e^{i\frac{5\pi}{12}}}$$

$$= \frac{2(i-1)(1-i\sqrt{3})}{4} = \frac{(\sqrt{3}-1) + i(\sqrt{3}+1)}{2}$$

$$= \sqrt{2}\left(\frac{\sqrt{3}-1}{2\sqrt{2}} + \frac{i(\sqrt{3}+1)}{2\sqrt{2}}\right)$$

$$\boxed{\sqrt{2}\left(\cos\frac{5\pi}{12} + i\sin\frac{5\pi}{12}\right)}$$

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5:

If $x - iy = \sqrt{\dfrac{a-ib}{c-id}}$, then P.T. $(x^2+y^2)^2 = \dfrac{a^2+b^2}{c^2+d^2}$.

$$z = \sqrt{\frac{z_1}{z_2}}$$

$$z^2 = \frac{z_1}{z_2}$$

$$|z^2| = \left|\frac{z_1}{z_2}\right| \Rightarrow |z|^2 = \frac{|z_1|}{|z_2|}$$

$$x^2+y^2 = \frac{\sqrt{a^2+b^2}}{\sqrt{c^2+d^2}}$$

$$(x^2+y^2)^2 = \frac{a^2+b^2}{c^2+d^2}.$$

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$a, b \in R \quad \sqrt{ab} = \sqrt{a}\sqrt{b}$ if atleast one of a, b is non negative.

$\boxed{\text{Ex-II } 1-16}$

$\sqrt{-4} = \sqrt{4(-1)} = \sqrt{4}\sqrt{-1} = 2i$

$1 = \sqrt{1} = \sqrt{(-1)^2} \cancel{=} \sqrt{-1}\sqrt{-1} \qquad i \cdot i = i^2 = -1$