EXERCISE - I

GENERAL TERM

1. If the coefficients of $x^7 \& x^8$ in the expansion of $\left[2+\frac{x}{3}\right]^n$ are equal, then the value of n is

(A) 15 (B) 45 (C) 55 (D) 56

2. Number of rational terms in the expansion of $(\sqrt{2}+\sqrt[4]{3})^{100}$ is

(A) 25 (B) 26 (C) 27 (D) 28

3. The expression $\frac{1}{\sqrt{4x+1}}\left[\left[\frac{1+\sqrt{4x+1}}{2}\right]^7-\left[\frac{1-\sqrt{4x+1}}{2}\right]^7\right]$ is a polynomial in x of degree

(A) 7 (B) 5 (C) 4 (D) 3

4. Find n, if the ratio of the fifth term form the beginning to the fifth term from the end in the expansion of $\left(\sqrt[4]{2}+\frac{1}{\sqrt[4]{3}}\right)^n$ is $\sqrt{6}: 1$

(A) 9 (B) 10 (C) 11 (D) 12

5. find the coefficient of x^4 in the expansion of $(x/2-3/x^2)^{10}$.

(A) $\frac{405}{202}$ (B) $\frac{405}{206}$ (C) $\frac{403}{202}$ (D) None of These

6. If the coefficients of rth, $(r+1)$ th and $(r+2)$ th terms in the binomial expansion of $(1+y)^m$ are in A.P., then m and r satisfy the equation-

(A) $m^2-m(4r-1)+4r^2-2=0$ (B) $m^2-m(4r+1)+4r^2+2=0$

(C) $m^2-m(4r+1)+4r^2-2=0$ (D) $m^2-m(4r-1)+4r^2+2=0$

7. If the coefficient of x^7 in $\left[ax^2+\left(\frac{1}{bx}\right)\right]^{11}$ equals the coefficient of x^{-7} in $\left[ax-\left(\frac{1}{bx^2}\right)\right]^{11}$, then a and b satisfy the relation -

(A) $a-b=1$ (B) $a+b=1$ (C) $\frac{a}{b}=1$ (D) $ab=1$

8. In the expansion of $\left(x^{2/3}-\frac{1}{\sqrt{x}}\right)^{30}$, a term containing the power x^{13}

(A) does not exist

(B) exists & the co-efficient is divisible by 29

(C) exists & the co-efficient is divisible by 63

(D) exists & the co-efficient is divisible by 65

9. In the expansion of $\left(x^3+3\cdot 2^{-\log_{\sqrt{2}}\sqrt{x^3}}\right)^{11}$

(A) there appears a term with the power x^2

(B) there does not appear a term with the power x^2

(C) there appears a term with the power x^{-3}

(D) the ratio of the co-efficient of x^3 to that of x^{-3} is $\frac{1}{3}$

MIDDLE TERM

10. Middle term in the expansion of $(x^2 - 2x)^{10}$ will be -

(A) ${}^{10}C_4 x^{17} 2^4$ (B) $-{}^{10}C_5 2^5 x^{15}$ (C) $-{}^{10}C_4 2^4 \times 17$ (D) ${}^{10}C_5 2^4 x^{15}$

11. The middle term in the expansion of $\left(\frac{3}{x^2} - \frac{x^3}{6}\right)^9$ is-

(A) $\frac{189}{8}x^2, \frac{21}{16}x^7$ (B) $\frac{189}{8}x^2, -\frac{21}{16}x^7$ (C) $-\frac{189}{8}x^2, -\frac{21}{16}x^7$ (D) None of these

12. If the middle term in the expansion of $\left(x^2 + \frac{1}{x}\right)^n$ is $924x^6$, then n =

(A) 10 (B) 12 (C) 14 (D) None

13. The middle term in the expansion of $(1 - 3x + 3x^2 - x^3)^6$ is -

(A) ${}^{18}C_{10}x^{10}$ (B) ${}^{18}C_9(-x)^9$ (C) ${}^{18}C_9 x^9$ (D) $-{}^{18}C_{10}x^{10}$

14. The middle term in the expansion of $\left(x + \frac{1}{2x}\right)^{2n}$ is-

(A) $\frac{1.3\cdot5......(2n-3)}{n!}$ (B) $\frac{1.3\cdot5.....(2n-1)}{n!}$ (C) $\frac{1.3\cdot5\cdots..(2n+1)}{n!}$ (D) None of these

15. The coefficient of the middle term in the binomial expansion in powers of x of $(1 + \alpha x)^4$ and of $(1 - \alpha x)^6$ is the same if α equals-

(A) $-\frac{5}{3}$ (B) $\frac{10}{3}$ (C) $-\frac{3}{10}$ (D) $\frac{3}{5}$

16. The middle term in the expansion of $\left(x^3 - \frac{1}{x^3}\right)^{10}$ is-

(A) 252 (B) -252 (C) 210 (D) -210

17. If the middle term in the expansion of $(x^2 + 1/x)^n$ is $924x^6$, then the value of n is less than

(A) 11 (B) 6 (C) 13 (D) 14

18. The middle term in the expansion of $\left(x^2 + \frac{1}{x^2} + 2\right)^n$ is :

(A) $\frac{(2n)!}{(n!)^2}$ (B) $\frac{2^n(1.3.5.......(2n-1))}{n!}$ (C) $\frac{(2n+1)!}{n!^2}$ (D) 2n !

TERM INDEPENDENT OF X

19. Given that the term of the expansion $\left(x^{1/3} - x^{-1/2}\right)^{15}$ which does not contain x is 5 m where $m \in N$, then m equals

(A) 1100 (B) 1010 (C) 1001 (D) none

20. The binomial expansion of $\left(x^k + \frac{1}{x^{2k}}\right)^{3n}$, $n \in N$ contains a term independent of x

(A) only if k is an integer (B) only if k is a natural number

(C) only if k is rational (D) for any real k

21. Find the term independent of x in the expansion of $(1 + x + 2x^3)\left(\frac{3x^2}{2} - \frac{1}{3x}\right)^9$.

(A) $\frac{24}{9}$ (B) $\frac{17}{54}$ (C) $\frac{27}{4}$ (D) None

22. The ratio of the coefficient of x^{10} in $(1-x^2)^{10}$& the term independent of x in $\left(x-\frac{2}{x}\right)^{10}$ is 1: 32.

(A) $\frac{1}{6}$ (B) $\frac{1}{4}$ (C) $\frac{1}{32}$ (D) None

NGT

23. The greatest terms of the expansion $(2x+5y)^{13}$ when $x=10, y=2$ is

(A) ${}^{13}C_5 \cdot 20^8 \cdot 10^5$ (B) ${}^{13}C_6 \cdot 20^7 \cdot 10^4$

(C) ${}^{13}C_4 \cdot 20^9 \cdot 10^4$ (D) none of these

24. Let n be a positive integer. Then of the following, the greatest term is

(A) $\left(1+\frac{1}{4n}\right)^{4n}$ (B) $\left(1+\frac{1}{3n}\right)^{3n}$

(C) $\left(1+\frac{1}{2n}\right)^{2n}$ (D) $\left(1+\frac{1}{n}\right)^{n}$

25. Find the index n of the binomial $\left(\frac{x}{5}+\frac{2}{5}\right)^n$ if the 9 th term of the expansion has numerically the greatest coefficient $(n \in N)$.

(A) 11 (B) 12 (C) 13 (D) None

26. For which positive values of x is the fourth term in the expansion of $(5+3x)^{10}$ is the greatest.

(A) $x \in \left(\frac{5}{8}, \frac{20}{21}\right)$ (B) $x \in \left(-\infty, \frac{5}{8}\right) \cup \left(\frac{20}{21}, \infty\right)$

(C) $x \in \left(\frac{5}{8}, \frac{20}{21}\right)$ (D) None

27. The greatest term in the expansion of $\sqrt{3}\left(1+\frac{1}{\sqrt{3}}\right)^{20}$ is

(A) $\frac{2480}{13}$ (B) $\frac{25840}{9}$ (C) Obtained at $r=7$(D) None

28. The greatest coefficient in the expansion of $(1+2x/3)^{15}$ is

(A) obtained for 6^{th} term (B) obtained for 7^{th} term

(C) ${}^{15}C_6(2/3)^6$ (D) ${}^{15}C_9(4/9)^3$

E & F :

PROBLEMS BASED ON BINOMIAL COEFFICIENT

&

COLLECTION OF BINOMIAL COEFFICIENT

If $C_0, C_1, C_2, \ldots \ldots .., C_n$ are the combinatorial coefficients in the expansion of $(1+x)^n, n \in N$, then

29. $C_0^2+C_1^2+C_2^2+\cdots \ldots+C_n^2=$

(A) ${}^{2n}C_n$ (B) ${}^{2n}C_{n+1}$ (C) ${}^{2n}C_n+2$ (D) None

30. $C_1+2C_2+3C_3+\cdots \ldots \ldots+n \cdot C_n=n \cdot 2^{n-1}$

(A) $n. 2^n$ (B) $n. 2^{n-1}$ (C) $n \cdot 2^{n+1}$ (D) None

31. The sum of the binomial coefficients of $\left[2x+\frac{1}{x}\right]^n$ is equal to 256. The constant term in the expansion is

(A) 1120 (B) 2110 (C) 1210 (D) none

32. Set of values of r for which, ${}^{18}C_{r-2}+2\cdot{}^{18}C_{r-1}+{}^{18}C_r\geq{}^{20}C_{13}$ contains

(A) 4 elements (B) 5 elements (C) 7 elements (D) 10 elements

33. $\frac{C_0}{1}+\frac{C_1}{2}+\frac{C_2}{3}+\cdots\ldots.+\frac{C_{10}}{11}=$

(A) $\frac{2^{11}}{11}$ (B) $\frac{2^{11}-1}{11}$ (C) $\frac{3^{11}}{11}$ (D) $\frac{3^{11}-1}{11}$

34. If $\sum_{k=1}^{n-r}{}^{n-k}C_r={}^{x}C_y$ then

(A) $x=n+1; y=r$ (B) $x=n; y=r+1$

(C) $x=n; y=r$ (D) $x=n+1; y=r+1$

35. If $(1+x)^n=C_0+C_1x+C_2x^2+\cdots.+C_nx^n$, then for n odd, $C_1{}^2+C_3{}^2+C_5{}^2+\cdots..+C_n{}^2$ is equal to

(A) 2^{2n-2} (B) 2^n (C) $\frac{(2n)!}{2(n!)^2}$ (D) $\frac{(2n)!}{(n!)^2}$

If $C_0, C_1, C_2, \ldots\ldots, C_n$ are the combinatorial coefficients in the expansion of $(1+x)^n, n\in N$,

36. $C_0+2C_1+3C_2+\cdots\ldots.+(n+1)C_n=$

(A) $(n+2)2^n$ (B) $n.2n+2$ (C) $(n+222^{n-1}$ (D) $(n+2)2^n$

37. $C_0+3C_1+5C_2+\ldots\ldots+(2n+1)C_n=$

(A) $(n+1)2^n$ (B) $n.2^{n+1}$ (C) $(n-1)2^n$ (D) None

38. $\frac{C_1}{C_0}+\frac{2C_2}{C_1}+\frac{3C_3}{C_2}+\cdots.+\frac{n\cdot C_n}{C_{n-1}}=$

(A) $\frac{n(n+1)}{3}$ (B) $\frac{n(n+1)}{2}$ (C) $\frac{(n-1)n}{2}$ (D) None

39. In the expansion of $(1+x)^n(1+y)^n(1+z)^n$, the sum of the co-efficients of the terms of degree ' r ' is

(A) ${}^{n^3}C_r$ (B) ${}^{n}C_{r^3}$ (C) ${}^{3n}C_r$ (D) $3.\ {}^{2n}C_r$

40. The co-efficient of x^4 in the expansion of $(1-x+2x^2)^{12}$ is

(A) ${}^{12}C_3$ (B) ${}^{13}C_3$ (C) ${}^{14}C_4$ (D) ${}^{12}C_3+3\,{}^{13}C_3+{}^{14}C_4$

41. The coefficient of x^4 in the expansion of $\frac{1+2x+3x^2}{(1-x)^2}$ is-

(A) 13 (B) 14 (C) 20 (D) 22

42. The sum $3\,{}^{n}C_0-8\,{}^{n}C_1+13\,{}^{n}C_2-18\times{}^{n}C_3+\cdots$ is less than

(A) 1 (B) 3 (C) 4 (D) 5

43. $\frac{^nC_1}{2} - \frac{2(^nC_2)}{3} + \frac{3(^nC_3)}{4} - \cdots . + (-1)^{n+1}\frac{n(^nC_n)}{n+1} =$

(A) $\frac{1}{n+1}$ (B) $\frac{2}{n+2}$

(C) $1 - \frac{1}{n+1}[\,^{n+1}C_1 - \,^{n+1}C_0]$ (D) $\frac{n^2-1}{n+1}$

DIVISIBILITY CONCEPT & REMAINDER CONCEPT

44. The remainder when 5^{99} is divides by 13 is -

(A) 6 (B) 8 (C) 9 (D) 10

45. When 2^{301} is divided by 5 , the least positive remainder is

(A) 4 (B) 8 (C) 2 (D) 6

46. $(1+x)^n - nx - 1$ is divisible by (where $n \in N$) $-$

(A) $2x^3$ (B) $2x$ (C) x^2 (D) All of these

47. Find the remainder when $6^n - 5n$ is divided by 25 .

(A) 1 (B) 2 (C) 3 (D) 4

48. Find the remainder when $1690^{2608} + 2608^{1690}$ is divided by 7 .

(A) 1 (B) 2 (C) 3 (D) 4

BINOMIAL THEOREM FOR ANY INDEX

49. Find the condition for which the formula $(a+b)^m = a^m + ma^{m-1}\,b + \frac{m(m-1)}{1\times 2}a^{m-2}\,b^2 + \cdots$ holds.

(A) $a^2 > b$ (B) $b^2 < a$ (C) $|b| < |a|$ (D) None

50. Find the values of x, for which $1/(\sqrt{5} + 4x)$ can be expanded as infinite series.

(A) $x < \frac{5}{4}$ (B) $|x| < \frac{5}{4}$ (C) $x > \frac{5}{4}$ (D) None

51. The coefficient of x^r in the expansion of $(1-2x)^{-1/}\,2$ is

(A) $\frac{2r}{2^r(r!)}$ (B) $\frac{(2r)!}{2^r(r!)}$ (C) $\frac{(2r)!}{2^r(r!)^2}$ (D) None

52. Find the sum $1 - \frac{1}{8} + \frac{1}{8} \times \frac{3}{16} - \frac{1\times3\times5}{8\times16\times24} + \cdots$

(A) $\frac{1}{5}$ (B) $\frac{2}{5}$ (C) $\frac{3}{\sqrt{5}}$ (D) $\frac{2}{\sqrt{5}}$

53. If x is so small that x^3 and higher powers of x may be neglected, then $\frac{(1+x)^{3/2} - \left(1+\frac{1}{2}x\right)}{(1-x)^{1/2}}$ may be approximated as

(A) $\frac{x}{2} - \frac{3}{8}x^2$ (B) $-\frac{3}{8}x^2$ (C) $3x + \frac{3}{8}x^2$ (D) $1 - \frac{3}{8}x^2$

54. In the expansion, powers of x in the function $\frac{1}{(1-ax)(1-bx)}$ is $a_0 + a_1x + a_2x^2 + a_3x^3 + \cdots$, then a_n is equal to

(A) $\frac{a^n - b^n}{b-a}$ (B) $\frac{a^{n+1} - b^{n+1}}{a-b}$ (C) $\frac{b^{n+1} - a^{n+1}}{b-a}$ (D) $\frac{b^n - a^n}{b-a}$

55. Let $(1+x^2)^2(1+x)^n = A_0 + A_1x + A_2x^2 + \cdots \ldots \ldots \ldots$ If A_0, A_1, A_2 are in A.P. then the value of n is

(A) 2 (B) 3 (C) 5 (D) 7

MIXED PROBLEM

56. Number of terms free from radical sign in the expansion of $\left(1 + 3^{1/3} + 7^{1/7}\right)^{10}$ is

(A) 4 (B) 5 (C) 6 (D) 8

57. The co-efficient of x^{401} in the expansion of $(1 + x + x^2 + \cdots \ldots .. + x^9)^{-1}$, $(|x| < 1)$ is

(A) 1 (B) -1 (C) 2 (D) -2

58. Let $(5 + 2\sqrt{6})^n = p + f$ where $n \in N$ and $p \in N$ and $0 < f < 1$ then the value, $f^2 - f + pf - p$ is

(A) a natural number
(B) a negative integer
(C) a prime number
(D) an irrational number

59. If $(r+1)^{th}$ term is $\frac{3.5\ldots(2r-1)}{r!}\left(\frac{1}{5}\right)^r$, then this is the term of binomial expansion-

(A) $\left(1-\frac{2}{5}\right)^{1/2}$ (B) $\left(1-\frac{2}{5}\right)^{-1/2}$ (C) $\left(1+\frac{2}{5}\right)^{-1/2}$ (D) $\left(1+\frac{2}{5}\right)^{1/2}$

60. For natural numbers m, n if $(1-y)^m(1+y)^n = 1 + a_1y + a_2y^2 + \cdots$, and $a_1 = a_2 = 10$, then (m, n) is-

(A) (35,20) (B) (45,35) (C) (35,45) (D) (20,45)

61. If $(4 + \sqrt{15})^n = I + f$. where n is an odd natural number, I is an integer and $0 < f < 1$, then

(A) I is an odd interger
(B) I is an even interger
(C) $(I + f)(1 - f) = 1$
(D) $1 - f = (4 - \sqrt{5})^n$

62. In the expansion of $\left(7^{1/3} + 11^{1/9}\right)^{6561}$,

(A) there are exactly 730 rational terms
(B) there are exactly 5831 irrational terms
(C) the term which involves greatest binomial coefficients is irrational
(D) None of these

63. In the expansion of $\left(x^2 + 1 + \frac{1}{x^2}\right)^n$, $n \in N$,

(A) number of terms is $2n + 1$
(B) coefficient of constant term is 2^{n-1}
(C) coefficient of x^{2n-2} is n
(D) coefficient of x^2 in n

EXERCISE - II

64. Find the coefficients

(i) x^7 in $\left(ax^2+\frac{1}{bx}\right)^{11}$

(ii) x^{-7} in $\left(ax-\frac{1}{bx^2}\right)^{11}$

(iii) Find the relation between a and b, so that these coefficients are equal.

65. If the coefficients of the r^{th}, $(r+1)^{th}$ and $(r+2)^{th}$ terms in the expansion of $(1+x)^{14}$ are in A.P., find r.

66. Find the value of x for which the fourth term in the expansion, $\left(5^{\log_5 \sqrt[2]{4^x+44}}+\frac{1}{5^{\log_5 \sqrt[3]{2^{x-1}+7}}}\right)^8$ is 336 .

67. Find the term independent of x in the expansion of

(a) $\left[\sqrt{\frac{x}{3}}+\frac{\sqrt{3}}{2x^2}\right]^{10}$

(b) $\left[\frac{1}{2}x^{1/3}+x^{-1/5}\right]^8$

68. In the expansion of $\left(1+x+\frac{7}{x}\right)^{11}$ find the term not containing x.

69. Find numerically the greatest term in the expansion of

(i) $(2+3x)^9$ when $x=\frac{3}{2}$

(ii) $(3-5x)^{15}$ when $x=\frac{1}{5}$

70. Prove that : ${}^{n-1}C_r+{}^{n-2}C_r+{}^{n-3}C_r+\cdots\ldots\ldots\ldots+{}^{r}C_r={}^{n}C_{r+1}$

71. Given $s_n=1+q+q^2+\cdots.+q^n$ and $S_n=1+\frac{q+1}{2}+\left(\frac{q+1}{2}\right)^2+\cdots\ldots+\left(\frac{q+1}{2}\right)^n$, $q\neq 1$.

Prove that ${}^{n+1}C_1+{}^{n+1}C_2\cdot s_1+{}^{n+1}C_3\cdot s_2+\cdots\ldots\ldots+{}^{n+1}C_{n+1}\cdot s_n=2^n\cdot S_n$.

72. If $C_0, C_1, C_2, \ldots\ldots, C_n$ are the combinatorial coefficients in the expansion of $(1+x)^n, n\in N$, then prove the following:

$(C_0+C_1)(C_1+C_2)(C_2+C_3)\ldots(C_{n-1}+C_n)=\frac{C_0\cdot C_1\cdot C_2\ldots\ldots C_{n-1}(n+1)^n}{n!}$.

73. If P_n denotes the product of all the coefficients in the expansion of $(1+x)^n, n\in N$, show that,
$\frac{P_{n+1}}{P_n}=\frac{(n+1)^n}{n!}$.

74. Prove that $C_0+\frac{C_1}{2}+\frac{C_2}{3}+\cdots.+\frac{C_n}{n+1}=\frac{2^{n+1}-1}{n+1}$

75. Prove that $2\cdot C_0+\frac{2^2\cdot C_1}{2}+\frac{2^3\cdot C_2}{3}+\frac{2^4\cdot C_3}{4}+\cdots\ldots+\frac{2^{n+1}\cdot C_n}{n+1}=\frac{3^{n+1}-1}{n+1}$

76. Prove that $C_0-\frac{C_1}{2}+\frac{C_2}{3}-\cdots\ldots.+(-1)^n\frac{C_n}{n+1}=\frac{1}{n+1}$

77. Prove that $C_0 - 2C_1 + 3C_2 - 4C_3 + \cdots + (-1)^n (n+1) C_n = 0$

78. Prove that $1 \cdot C_0^2 + 3 \cdot C_1^2 + 5 \cdot C_2^2 + \cdots + (2n+1) C_n{}^2 = \frac{(n+1)(2n)!}{n!n!}$

79. If $(1+x)^n = C_0 + C_1 x + C_2 x^2 + \cdots \ldots \ldots + C_n x^n$, then show that the sum of the products of the C_i 's taken two at a time, represented by $\Sigma\Sigma C_i C_j$ is equal to $2^{2n-1} - \frac{2n!}{2(n!)^2}$.

80. Prove that $\sqrt{C_1} + \sqrt{C_2} + \sqrt{C_3} + \cdots \ldots + \sqrt{C_n} \le 2^{n-1} + \frac{n-1}{2}$.

81. Prove that $\sqrt{C_1} + \sqrt{C_2} + \sqrt{C_3} + \cdots \ldots + \sqrt{C_n} \le [n(2^n - 1)]^{1/2}$ for $n \ge 2$.

82. Show that coefficient of x^5 in the expansion of $(1+x^2)^5 \cdot (1+x)^4$ is 60 .

83. Find the coefficient of x^4 in the expansion of

(i) $(1 + x + x^2 + x^3)^{11}$

(ii) $(2 - x + 3x^2)^6$

84. Prove that $\frac{(72)!}{(36!)^2} - 1$ is divisible by 73 .

85. Find the sum of the series $\sum_{r=0}^{n} (-1)^r \, {}^nC_r \left[\frac{1}{2^r} + \frac{3^r}{2^{2r}} + \frac{7^r}{2^{3r}} + \frac{15^r}{2^{rr}} + \cdots \text{ up to m terms}\right]$

86. Given that $(1 + x + x^2)^n = a_0 + a_1 x + a_2 x^2 + \cdots \ldots + a_{2n} x^{2n}$, find the values of

(i) $a_0 + a_1 + a_2 + \cdots .. + a_{2n}$;

(ii) $a_0 - a_1 + a_2 - a_3 \ldots \ldots . + a_{2n}$;

(iii) $a_0^2 - a_1^2 + a_2{}^2 - a_3^2 + \cdots + a_{2n}{}^2$

87. If ${}^nJ_r = \frac{(1-x^n)(1-x^{n-1})(1-x^{n-2}) \ldots\ldots (1-x^{n-r+1})}{(1-x)(1-x^2)(1-x^3) \ldots\ldots (1-x^r)}$,

prove that ${}^nJ_{n-r} = {}^nJ_r$.

88. Prove that $\sum_{K=0}^{n} C_K \cdot \sin Kx \cdot \cos (n-K)x = 2^{n-1} \sin nx$.

89. Find the coefficients of

(a) x^6 in the expansion of $(ax^2 + bx + c)^9$

(b) $x^2 y^3 z^4$ in the expansion of $(ax - by + cz)^9$.

(c) $a^2 b^3 c^4 d$ in the expansion of $(a - b - c + d)^{10}$

90. If $\sum_{r=0}^{2n} a_r (x-2)^r = \sum_{r=0}^{2n} b_r (x-3)^r$ and $a_k = 1$ for all $k \ge n$, then show that $b_n = {}^{2n+1}C_{n+1}$.

91. (a) Show that the integral part in each of the following is odd. $n \in N$.

(i) $(5 + 2\sqrt{6})^n$

(ii) $(8 + 3\sqrt{7})^n$

(iii) $(6 + \sqrt{35})^n$

(b) Show that the integral part in each of the following is even. $n \in N$.

(i) $(3\sqrt{3} + 5)^{2n+1}$

(ii) $(5\sqrt{5} + 11)^{2n+1}$

92. Prove that the integer next above $(\sqrt{3}+1)^{2n}$ contains 2^{n+1} as factor $(n \in N)$

COMPREHENSION (31-33)

If m, n, r are positive integers and if $r < m, r < n$, then

${}^{m}C_r + {}^{m}C_{r-1} \cdot {}^{n}C_1 + {}^{m}C_{r-2} \cdot {}^{n}C_2 + \cdots + {}^{n}C_r$

$=$ Coefficient of x^r in $(1+x)^m(1+x)^n$

$=$ Coefficient of x^r in $(1+x)^{m+n}$

$= {}^{m+n}C_r$

On the basis of the above information, answer the following questions.

93. The value of ${}^{n}C_0 \cdot {}^{n}C_n + {}^{n}C_1 \cdot {}^{n}C_{n-1} + \cdots + {}^{n}C_n \cdot {}^{n}C_0$ is

(A) ${}^{2n}C_{n-1}$ (B) ${}^{2n}C_n$ (C) ${}^{2n}C_{n+1}$ (D) ${}^{2n}C_2$

94. The value of r for which ${}^{30}C_r \cdot {}^{20}C_0 + {}^{30}C_{r-1} \cdot {}^{20}C_1 + \cdots + {}^{30}C_0 \cdot {}^{20}C_r$ is maximum, is

(A) 10 (B) 15 (C) 20 (D) 25

95. The value of $r (0 \le r \le 30)$ for which ${}^{20}C_r \cdot {}^{10}C_0 + {}^{20}C_{r-1} \cdot {}^{10}C_1 + \cdots + {}^{20}C_0 \cdot {}^{10}C_r$ is minimum, is

(A) 0 (B) 1 (C) 5 (D) 15

MATRIX MATCH TYPE

96.

Column-I	Column-II
(A) If λ be the number of terms in the expansion of $(1+5x+10x^2+10x^2+5x^4+x^5)^{20}$ and if unit's place and ten's place digits in 3^λ are O and T, then	(P) $O+T=3$
(B) If λ be the number of terms in the expansion of $\left(x^2+1+\frac{1}{x^2}\right)^{100}$ and if unit's place and ten's place digits in 7^λ are O and T, then	(Q) $O+T=7$
	(R) $O+T=9$
(C) If λ be the number of terms in the expansion of $(1+x)^{101}(1+x^2-x)^{100}$ and if unit's place and ten's place digits in 9^λ are O and T, then	(S) $T-O=7$
	(T) $O-T=7$

97. Prove the following identities using the theory of permutation where $C_0, C_1, C_2, \ldots, C_n$ are the combinatorial coefficients in the expansion of $(1+x)^n, n \in N$:

(a) $C_0{}^2 + C_1{}^2 + C_2{}^2 + \cdots .. + C_n{}^2 = \frac{(2n)!}{n!n!}$

(b) $C_0C_1 + C_1C_2 + C_2C_3 + \cdots + C_{n-1}C_n = \frac{(2n)!}{(n+1)!(n-1)!}$

(c) $C_0C_r + C_1C_{r+1} + C_2C_{r+2} + \cdots . + C_{n-r}C_n = \frac{2n!}{(n-r)!(n+r)!}$

(d) $\sum_{r=0}^{n-2}({}^nC_r \cdot {}^nC_{r+2}) = \frac{(2n)!}{(n-2)!(n+2)!}$

(e) ${}^{100}C_{10} + 5 \cdot {}^{100}C_{11} + 10 \cdot {}^{100}C_{12} + 10 \cdot {}^{100}C_{13} + 5 \cdot {}^{100}C_{14} + {}^{100}C_{15} = {}^{105}C_{90}$

98. If $C_0, C_1, C_2, \ldots, C_n$ are the combinatorial coefficients in the expansion of $(1+x)^n, n \in N$, then prove the following:

(a) $C_1 + 2C_2 + 3C_3 + \cdots . + n \cdot C_n = n \cdot 2^{n-1}$

(b) $C_0 + 2C_1 + 3C_2 + \cdots . + (n+1)C_n = (n+2)2^{n-1}$

(c) $C_0 + 3C_1 + 5C_2 + \cdots . + (2n+1)C_n = (n+1)2^n$

(d) $(C_0 + C_1)(C_1 + C_2)(C_2 + C_3) \ldots . (C_{n-1} + C_n) = \frac{C_0 \cdot C_1 \cdot C_2 \ldots\ldots C_{n-1}(n+1)^n}{n!}$

(e) $1 \cdot C_0{}^2 + 3 \cdot C_1{}^2 + 5 \cdot C_2{}^2 + \cdots \ldots + (2n+1)C_n{}^2 = \frac{(n+1)(2n)!}{n!n!}$

99. Prove that

(a) $\frac{C_1}{C_0} + \frac{2C_2}{C_1} + \frac{3C_3}{C_2} + \cdots \ldots + \frac{n \cdot C_n}{C_{n-1}} = \frac{n(n+1)}{2}$

(b) $C_0 + \frac{C_1}{2} + \frac{C_2}{3} + \cdots \ldots + \frac{C_n}{n+1} = \frac{2^{n+1}-1}{n+1}$

(c) $2 \cdot C_0 + \frac{2^2 \cdot C_1}{2} + \frac{2^3 \cdot C_2}{3} + \frac{2^4 \cdot C_3}{4} + \cdots \ldots \frac{2^{n+1} \cdot C_n}{n+1} = \frac{3^{n+1}-1}{n+1}$

(d) $C_0 - \frac{C_1}{2} + \frac{C_2}{3} - \cdots \ldots + (-1)^n \frac{C_n}{n+1} = \frac{1}{n+1}$

ANSWER KEY

EXERCISE - I

1.	(C)	**2.**	(B)	**3.**	(D)	**4.**	(B)	**5.**	(D)	**6.**	(C)	**7.**	(D)
8.	(BCD)	**9.**	(BCD)	**10.**	(B)	**11.**	(B)	**12.**	(B)	**13.**	(B)	**14.**	(B)
15.	(C)	**16.**	(B)	**17.**	(CD)	**18.**	(AB)	**19.**	(C)	**20.**	(D)	**21.**	(B)
22.	(C)	**23.**	(C)	**24.**	(A)	**25.**	(B)	**26.**	(A)	**27.**	(BC)	**28.**	(BCD)
29.	(A)	**30.**	(B)	**31.**	(A)	**32.**	(C)	**33.**	(B)	**34.**	(B)	**35.**	(C)
36.	(A)	**37.**	(A)	**38.**	(B)	**39.**	(C)	**40.**	(D)	**41.**	(D)	**42.**	(ABC)
43.	(AC)	**44.**	(B)	**45.**	(C)	**46.**	(C)	**47.**	(A)	**48.**	(A)	**49.**	(C)
50.	(B)	**51.**	(C)	**52.**	(D)	**53.**	(B)	**54.**	(B)	**55.**	(AB)	**56.**	(C)
57.	(B)	**58.**	(B)	**59.**	(B)	**60.**	(C)	**61.**	(ACD)	**62.**	(AC)	**63.**	(AC)

EXERCISE - II

64. **(i)** ${}^{11}C_5 \frac{a^6}{b^5}$ **(ii)** ${}^{11}C_6 \frac{a^5}{b^6}$ **(iii)** $ab = 1$

65. $r = 5$ or 9

66. $x = 0$ or 1

67. (a) $T_3 = \frac{5}{12}$, (b) $T_6 = 7$

68. $1 + \sum_{k=1}^{5} {}^{11}C_{2k} \cdot {}^{2k}C_k \cdot 7^k$

69. (i) $T_7 = \frac{7.3^{13}}{2}$, (ii) 455×3^{12}

70. $x = 0$ or 2

83. (i) 990, (ii) 3660

85. $\frac{(2^{mn}-1)}{(2^n-1)(2^{mn})}$

86. (i) 3^n,(ii) 1 , (iii) a_n

89. (a) $84b^6c^3 + 630ab^4c^4 + 756a^2b^2c^5 + 84a^3c^6$, (b) $-1260 \cdot a^2b^3c^4$, (c) -12600

93. (B) **94.** (D) **95.** (A) **96.** (A)-P; (B)-Q,T; (C)-R,S