

Fundamentals of Mathematics

D) PP-2

Q1 Prod of all sol. of $x^{\log_{10} x} = 100 + 2\sqrt{\log_2 3} - 3\sqrt{\log_3 2}$ is

$$\log_{10} x = 100/x$$

$$\log_{10} x (\log_{10} x) = \log_{10} (100 \cdot x)$$

$$\log_{10} x \cdot \log_{10} x = \log_{10} 100 + \log_{10} x$$

$$t \cdot t = 2 + t$$

$$t^2 - t - 2 = 0 \Rightarrow (t-2)(t+1) = 0$$

Note: $\sqrt{\log_a b} = \sqrt{\log_b a}$

* Try Prove

$$t=2 \quad \& \quad t=-1$$

$$\log_{10} x = 2 \quad | \quad \log_{10} x = -1$$

$$x = 10^2 \\ = 100$$

$$2^{\frac{\sqrt{3}}{3}} = (\sqrt{2} \cdot \sqrt{2})^{\frac{\sqrt{3}}{3}}$$

$$= \sqrt{2} \cdot \frac{\sqrt{3}}{2}$$

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Q2 If $\log_7 2 = m$ then $\boxed{\log_{49} 28}$ is

$$2(1+2m) \quad \Rightarrow \quad 1 + \frac{2m}{2} \quad \frac{2}{1+2m} \quad 1+m.$$

$$\log_{49} 28 = \log_{7^2} 7^2 \times 2^2 = \frac{1}{2} \log_7 (7^2 \times 2^2)$$

$$3^{C-P-405}$$

$$3^C = 3^4 \Rightarrow \boxed{(-4)}$$

$$= \frac{1}{2} (\log_7 7 + \log_7 2^2)$$

$$= \frac{1}{2} (1 + 2 \log_7 2) = \frac{1}{2} (1 + 2m)$$

Q4 If $P = \log_5 (\log_5 3)$ & $3^{C+5^{-P}} = 405$ Then $C = ?$

$$3^{C+5^{-P}} = 405$$

$$\Rightarrow 3^C \times 3^{\boxed{5^{-P}}} = 405 \Rightarrow 3^C \cdot 3^{\log_5 5^{-1}} = 405$$

$$5^{-P} = 5 - (\log_5 (\log_5 3)) = \log_5 (\log_5 3)^{-1}$$

$$= (\log_5 3)^{-1} = \frac{1}{\log_5 3}$$

$$= \log_3 5$$

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Q4 If $\frac{a + \log_4 3}{a + \log_2 3} = \frac{a + \log_8 3}{a + \log_4 3} = b$ then $b = ?$

$$\left| \begin{array}{l} \frac{a}{b} = \frac{c}{d} = \frac{c-a}{d-b} \end{array} \right.$$

$$= \frac{(a + \log_8 3) - (a + \log_4 3)}{(a + \log_4 3) - (a + \log_2 3)} = b$$

$$\frac{m}{n} - \frac{x}{y} = \frac{x-m}{y-n}$$

$$b = \frac{\log_2 3 - \log_2 2^{\frac{1}{3}}}{\log_{2^2} 3 - \log_2 3} = \frac{\frac{1}{3} \log_2 3 - \frac{1}{2} \log_2 3}{\frac{1}{2} \log_2 3 - \log_2 3} = \frac{\log_2 \left(\frac{1}{3} - \frac{1}{2} \right)}{\log_2 \left(\frac{1}{2} - 1 \right)}$$

$$b = \frac{\frac{1}{3} - \frac{1}{2}}{\frac{1}{2} - 1} = \frac{-\frac{1}{6}}{-\frac{1}{2}} = \frac{1}{3}$$

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Ka mtlb r ki value ek ek kr k chdaoo 8 + + Lgaate Ju o
 $L = \sum_{r=7}^{2400} \log_7 \left(\frac{r+1}{r} \right), M = \prod_{r=2}^{1023} \log_r (r+1), N = \sum_{r=2}^{2011} \frac{1}{\log_r P}$

$L + M = 13 \quad M^2 + N^2 = 101 \quad L - M + N = 6 \times 3 \times 10 \times 1 \quad LMN = 30$

$L = \log_7 \frac{8}{7} + \log_7 \frac{9}{8} + \log_7 \frac{10}{9} + \dots + \log_7 \frac{2401}{2400}$

$L = \log_7 \left\{ \frac{8}{7} \times \frac{9}{8} \times \frac{10}{9} \times \frac{11}{10} \times \dots \times \frac{2401}{2400} \right\}$

$L = \log_7 \frac{2401}{7} = \log_7 7^3 = 3$

$M = \log_2 (2+1) \times \log_3 (3+1) \times \log_4 (4+1) \dots \log_{1023} (1023+1)$

$M = \frac{\log 3}{\log 2} \times \frac{\log 4}{\log 3} \times \frac{\log 5}{\log 4} \times \dots \times \frac{\log 1024}{\log 1023}$

$M = \log_2 1024 = \log_2 1024 = \log_2 10 = 10$

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Q11 $\prod_{r=3}^{26} \log_r(r+1) = 3^x$ find x ?

$$\log_3 \frac{(3+1)}{1} \times \log_4 \frac{(4+1)}{1} \times \dots \times \log_{26} \frac{(26+1)}{1}$$

$$\log_3 4 \times \log_4 5 \times \log_5 6 \dots \log_{26} 27 = 3^x$$

$$\frac{\log 4}{\log 3} \times \frac{\log 5}{\log 4} \times \frac{\log 6}{\log 5} \times \dots \times \frac{\log 27}{\log 26}$$

$$\begin{aligned} \cancel{\log 27} - 3^0 &= \cancel{\log 3^3} - 3^0 \\ &= \cancel{\log 3^3} - 3^0 \\ &= 3^0 - 3^0 \\ &= 0 \end{aligned}$$

$\log 6^{-2} = \log K \Rightarrow K = 6^{-2} = \frac{1}{36}$

K is Unique +ve value satisfying Eqn $(4K)^{\log_2} = (9K)^{\log_3}$ then $72K$?

$\log_{10}(4K)^{\log_2} = \log_{10}(9K)^{\log_3} \left[72 \times \frac{1}{36} \right]$

$\log_2 \log(4 \cdot K) = \log_3 \log(9 \cdot K)$

$\log_2 (\log 4 + \log K) = \log_3 (\log 9 + \log K)$

$\log_2 \log 2^2 + \log_2 \log K = \log_3 \log 3^2 + \log_3 \log K$

$2(\log 2)^2 + \log_2 \log K = 2(\log 3)^2 + \log_3 \log K$

$2(\log^2 2 - \log^2 3) = \log K (\log 3 - \log 2)$

$-2(\log 2 + \log 3)(\log 2 + \log 3) = \log K (\log 3 - \log 2)$

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$$(\log 2)^2 + \log 2^2$$

$$Q. x^{(1+\log x)} = 10x \text{ from } dx$$

$$\log_{10} x^{(1+\log_{10} x)} = \log_{10} (10 \cdot x)$$

$$(1+\log_{10} x) \log_{10} x = \log_{10} 10 + \log_{10} x$$

$$(1+t)t = (1+t)$$

$$(1+t)(t-1)=0$$

$$t=1, t=-1 \Rightarrow \log_{10} x = -1 \quad \left| \begin{array}{l} \log_{10} x = -1 \\ x = 10^{-1} \end{array} \right.$$

$$Q. x^{\frac{\log x+s}{s}} = 10^{s+\log x} \text{ from } dx ?$$

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Questions Based on Tricotomy.

As $A \in (-\infty, 0) \cup (0, \infty)$
 $\therefore B > A$

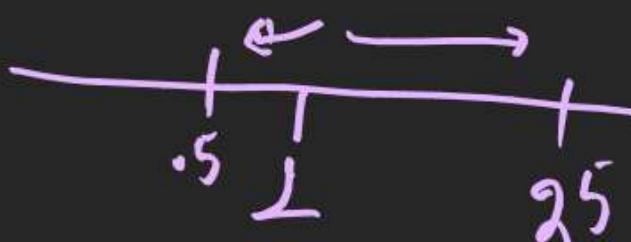
Which is greater?

Q) $A = \log_{.5} 25$, $B = \log_2 3$

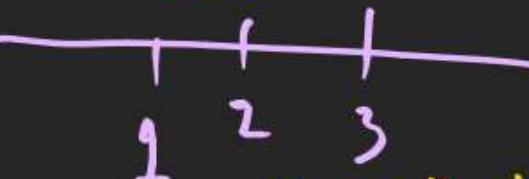
Check Base & exponent w.r.t 1

If Both are on Same Side of 1
 then log is +ve otherwise -ve

$$A = \log_{.5} 25 = -\text{ve}$$

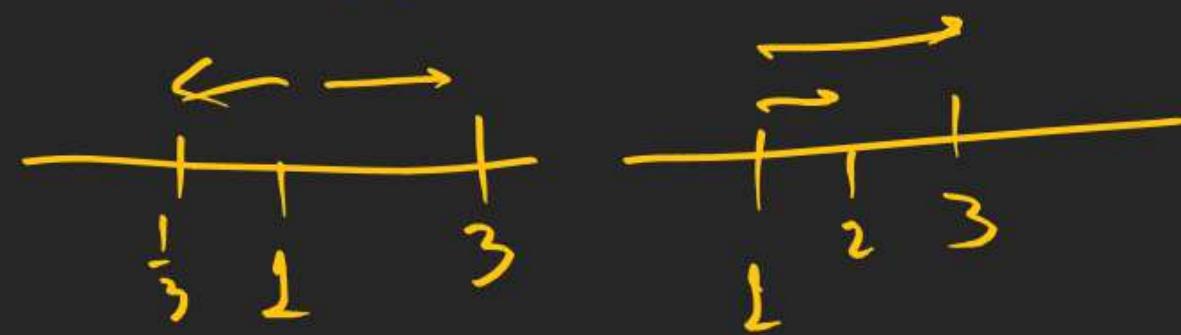


$$B = \log_2 3 \rightarrow +\text{ve}$$



3, 2 Both are on
Same Side

Q) $A = \log_{\frac{1}{3}} 3$ $B = \log_2 3$
 $= -\text{ve}$ $= +\text{ve}$



$B > A$

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$$\log_3 3 = 1 \quad \log_3 4 > 1$$

On Behalf of values hr. thus

$$Q \quad A = \log_{\underline{3}} \underline{3.1} \quad \& \quad B = \log_{\underline{3}} \underline{1.3}$$

which is hr.

$$\text{Exp} > \text{Base}$$

$$\text{then } \log \text{Value} > 1$$

$$A > B$$

$$\text{Exp} = 3$$

$$\text{Base} = 3.1$$

$$\Rightarrow \text{Exp} < \text{Base}$$

$$B = \log \text{Value} < 1$$

$$A = \log_{\underline{3}} \underline{4} \quad B = \log_{\underline{4}} \underline{3}$$

$$\text{Exp} = 4$$

$$\text{Base} = 3.$$

$$\text{Exp} > \text{Base}$$

$$\therefore A = \log_{\underline{3}} \underline{4} > 1$$

$$\underline{A > B}$$

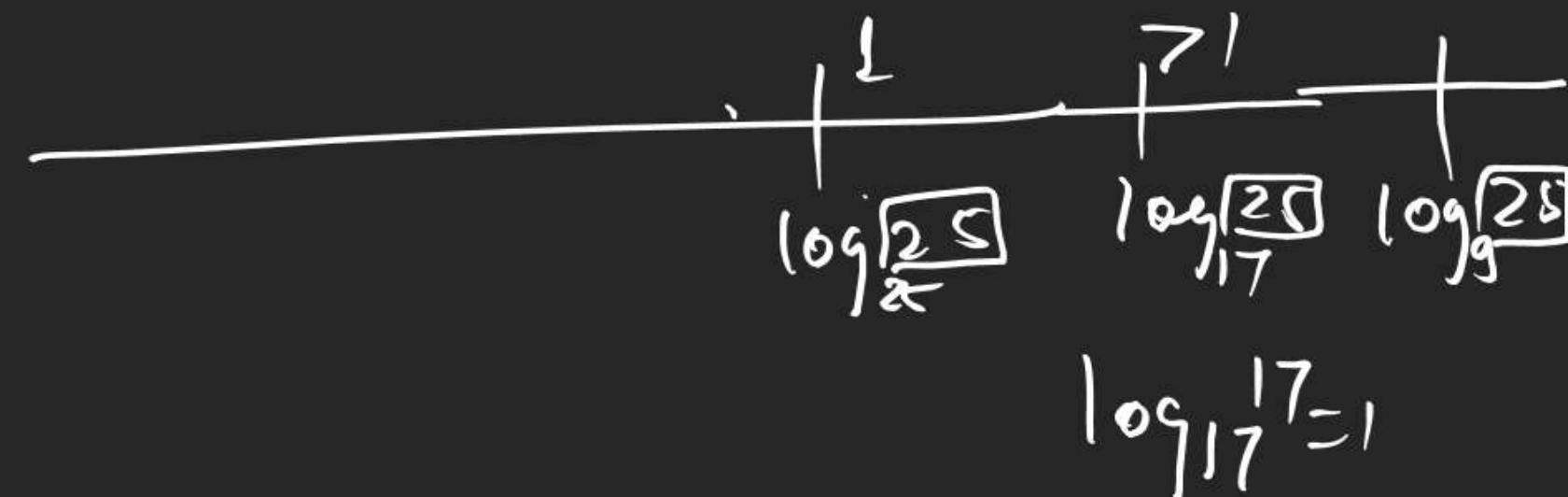
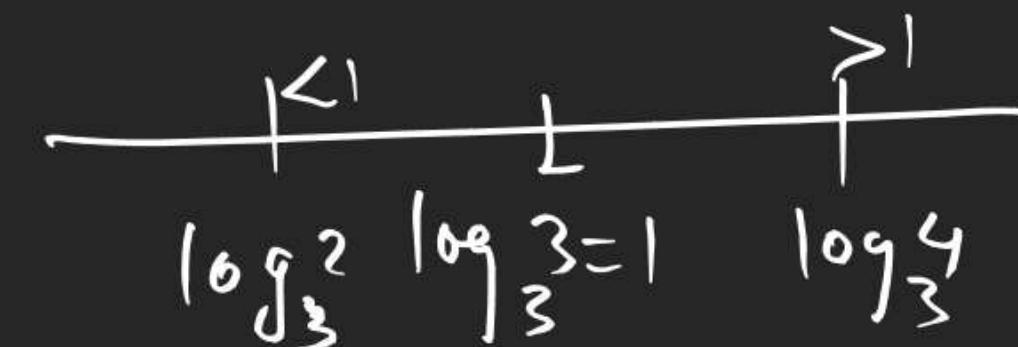
$$\text{Exp} = 3$$

$$\text{Base} = 4$$

$$\text{Exp} < \text{Base}$$

$$B = \log_{\underline{4}} \underline{3} < 1$$

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Q) $A = \underline{\log_3 5}$ & $B = \log_{17} 25$
which is larger.

$$A = \underline{\log_3 25} \quad | \quad B = \log_{17} 25$$

$$A > B$$



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Some Imp. ht.

$$\textcircled{1} \quad \log_e e = 1$$

$$\textcircled{2} \quad \log_e x = \ln x$$

$$\textcircled{3} \quad \log_{10} 3 = .4771$$

$$\textcircled{4} \quad \log_{10} 2 = .3010$$

$$\textcircled{5} \quad \ln 2 = .693$$

$$\log_{10} 3456 = 1.5386$$

$$= 1 + .5386$$

Int.
Part

fractional
Part

Mantissa.

(characteristic)

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2 New fns.

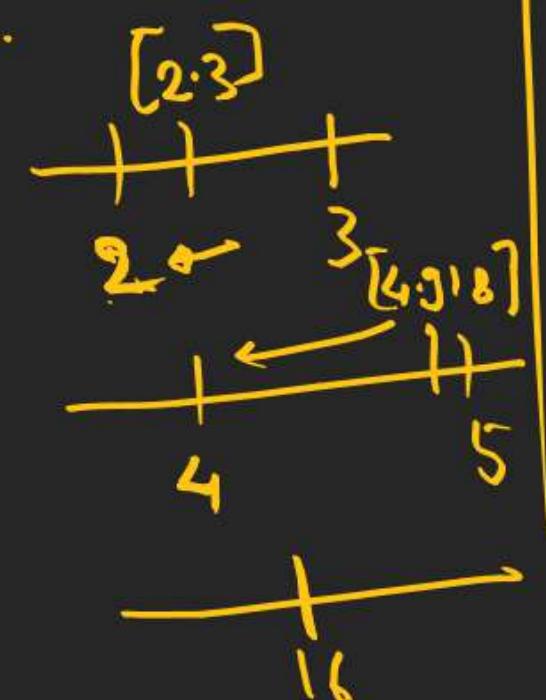
(1) Greatesst Integer fn.

$$1) f(x) = [x] \text{ Rep.}$$

$$2) [2.3] = 2$$

$$[4.918] = 4$$

$$[16] = 16$$



$$[-17] = -17$$

A horizontal number line with tick marks. The first tick mark is labeled -17 with three vertical bars above it. An arrow points from the -17 tick mark to the next tick mark, which is labeled -16 .

$$[-17.4] = -18$$

A horizontal number line with tick marks. The first tick mark is labeled -18 with three vertical bars above it. The second tick mark is labeled -17 . The third tick mark is labeled -16 . An arrow points from the -18 tick mark to the -17 tick mark, indicating that the interval $[-18, -17)$ is excluded. The fourth tick mark is labeled -17.4 with three vertical bars above it. The fifth tick mark is labeled -17 . The sixth tick mark is labeled -16 . The seventh tick mark is labeled -15 . The eighth tick mark is labeled -14 . The ninth tick mark is labeled -13 . The tenth tick mark is labeled -12 . The eleventh tick mark is labeled -11 . The twelfth tick mark is labeled -10 . The thirteenth tick mark is labeled -9 . The fourteenth tick mark is labeled -8 . The fifteenth tick mark is labeled -7 . The sixteenth tick mark is labeled -6 . The seventeenth tick mark is labeled -5 . The eighteenth tick mark is labeled -4 . The nineteenth tick mark is labeled -3 . The twentieth tick mark is labeled -2 . The twenty-first tick mark is labeled -1 . The twenty-second tick mark is labeled 0 . The twenty-third tick mark is labeled 1 . The twenty-fourth tick mark is labeled 2 . The twenty-fifth tick mark is labeled 3 . The twenty-sixth tick mark is labeled 4 . The twenty-seventh tick mark is labeled 5 . The twenty-eighth tick mark is labeled 6 . The twenty-ninth tick mark is labeled 7 . The thirtieth tick mark is labeled 8 . The thirty-first tick mark is labeled 9 . The thirty-second tick mark is labeled 10 . The thirty-third tick mark is labeled 11 . The thirty-fourth tick mark is labeled 12 . The thirty-fifth tick mark is labeled 13 . The thirty-sixth tick mark is labeled 14 . The thirty-seventh tick mark is labeled 15 . The thirty-eighth tick mark is labeled 16 . The thirty-ninth tick mark is labeled 17 . The forty-th tick mark is labeled 18 . The forty-first tick mark is labeled 19 . The forty-second tick mark is labeled 20 . The forty-third tick mark is labeled 21 . The forty-fourth tick mark is labeled 22 . The forty-fifth tick mark is labeled 23 . The forty-sixth tick mark is labeled 24 . The forty-seventh tick mark is labeled 25 .

$$[-2023.29] = -2024$$

A horizontal number line with tick marks. The first tick mark is labeled -2024 with three vertical bars above it. An arrow points from the -2024 tick mark to the next tick mark, which is labeled -2023 .

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(3)

$$\lceil x \rceil = 4$$

$$x \in [4, 5)$$

$$\lceil x \rceil = 7$$

$$x \in [7, 8)$$

$$\lceil x \rceil = 29$$

$$x \in [29, 30)$$

final $\lceil x \rceil = n$

$$x \in [n, n+1)$$

$$\lceil x \rceil = -11$$

$$x \in [-11, -11+1)$$

$$x \in [-11, -10)$$

Q

$$\lceil \log_{10} x \rceil = 4$$

then $x \in ?$

$$\log_{10} x \in [4, 5)$$

$$4 \leq \log_{10} x < 5$$

$$10^4 \leq x < 10^5$$

$$10000 \leq x < 100000$$

$$x \in [10000, 100000)$$