Nishant Jindal

Q Comhute

$$1+2x+3x^2+4x^3+\cdots+\infty$$

AP

AP

$$S_n = 1+2x+3x^2+4x^3+\cdots$$

$$x\cdot S_n = \quad x+2x^2+3x^3+\cdots$$

$$S_n(1-x) = 1+x+x^2+x^3+\cdots\infty$$

∞ GP

$$S_\infty(1-x) = \frac{1}{1-x}$$

$$S_\infty = \frac{1}{(1-x)^2}$$

Q $1+3x+6x^2+10x^3+\cdots\infty$

$$S_\infty = 1+3x+6x^2+10x^3+\cdots$$

$$x\cdot S_\infty = \quad x+3x^2+6x^3+\cdots$$

$$S(1-x) = 1+2x+3x^2+4x^3+\cdots$$

$$x\cdot S(1-x) = \quad x+2x^2+3x^3+\cdots$$

$$S(1-x)(1-x) = 1+x+x^2+x^3+\cdots$$

$\leftarrow \infty$ GP

$$S(1-x)^2 = \frac{1}{1-x}$$

$$S = \frac{1}{(1-x)^3} \text{ Ans}$$

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** Q Find sum of.

$$2017+\frac{1}{4}\left(2016+\frac{1}{4}\left(2015+\cdots+\frac{1}{4}\left(2+\frac{1}{4}(1)\right)\cdots\right)\right)$$

2017 log, $\frac{1}{4}$ 2016 का

A.G.P.

$$S = 2017+\frac{1}{4}\cdot 2016+\frac{1}{4^2}\cdot 2015+\frac{1}{4^3}\cdot 2014 \cdots \frac{1}{4^{2015}}\cdot 2+\frac{1}{4^{2016}}\cdot 1$$

$$\frac{S}{4} = \frac{2017}{4}+\frac{2016}{4^2}+\frac{2015}{4^3}+\cdots+\frac{1}{4^{2016}}\cdot 2+\frac{1}{4^{2017}}\cdot 1$$

$$\frac{3S}{4} = 2017-\left(\frac{1}{4}+\frac{1}{4^2}+\frac{1}{4^3}+\cdots+\frac{1}{4^{2016}}+\frac{1}{4^{2017}}\right)$$

← 2017 terms GP →

$$\frac{3S}{4} = 2017-\frac{\frac{1}{4}\left(1-\left(\frac{1}{4}\right)^{2017}\right)}{\left(1-\frac{1}{4}\right)} = 2017-\frac{\left(1-\left(\frac{1}{4}\right)^{2017}\right)}{3} \Rightarrow S=\frac{4}{3}\left(2017-\frac{\left(1-\left(\frac{1}{4}\right)^{2017}\right)}{3}\right)$$

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Q $1+\frac{4}{5}+\frac{7}{5^2}+\frac{10}{5^3}+ \dots n$ terms & $S\infty$?

$S = 1+\frac{4}{5}+\frac{7}{5^2}+\frac{10}{5^3}+\dots$

$\frac{S}{5} = \quad \frac{1}{5}+\frac{4}{5^2}+\frac{7}{5^3}+\dots$

$\frac{4S}{5} = 1+\left(\frac{3}{5}+\frac{3}{5^2}+\frac{3}{5^3}+\dots\right)$

$\leftarrow (n-1$ terms GP

$\frac{4S}{5} = 1+3\left(\frac{1}{5}+\frac{1}{5^2}+\frac{1}{5^3}+\dots\right)$

$= 1+\frac{3}{5}\frac{\left(1-\left(\frac{1}{5}\right)^{n-1}\right)}{\frac{4}{5}}$

$S = \frac{5}{4}\left(1+\frac{3}{4}\left(1-\left(\frac{1}{5}\right)^{n-1}\right)\right)$

$\frac{4S}{1} = 1+\frac{\frac{3}{5}}{1-\frac{1}{5}}$

$= 1+\frac{3}{4}$

$\frac{4S}{5} = \frac{7}{4}$

$S = \frac{35}{16}$

Q $S = 1+(1+b)r+(1+b+b^2)r^2+\dots\infty \quad |br|<1$

$r\cdot S = \quad r \quad +(1+b)r^2+\dots$

$S(1-r) = 1+br+b^2r^2+b^3r^3+\dots$

$S(1-r) = \frac{1}{1-(br)}$

$S = \frac{1}{(1-br)(1-r)}$

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Miscelleneous Series.

(1) $\sum_{n=1}^{n} 1 = \underbrace{1+1+1+\cdots+1}_{n} = n.$

(2) $\sum_{r=1}^{n} r = \frac{(n)(n+1)}{2}$ | $\sum_{r=1}^{n} r = 1+2+3+\cdots+n$

(3) $\sum_{r=1}^{n} r^2 = 1^2+2^2+3^2+4^2+\cdots+n^2 = \frac{(n)(n+1)(2n+1)}{6}$

(4) $\sum_{r=1}^{n} r^3 = 1^3+2^3+3^3+\cdots+n^3 = \left(\frac{(n)(n+1)}{2}\right)^2$

(5) $\sum_{r=1}^{n} (2r-1) = \underbrace{1+3+5+\cdots+(2n-1)}_{n \text{ odd No. of Sum}} = n^2$

(6) $\sum_{r=1}^{n} 2r = \underbrace{2+4+6+\cdots+2n}_{n \text{ Even No of Sum}} = 2(1+2+3+\cdots+n)$

$= 2\frac{(n)(n+1)}{2}$

$= n(n+1)$

Q $1+2+3+\cdots+\overset{n}{\boxed{23}}$?

$1+2+3+\cdots+n = \frac{(n)(n+1)}{2}$

$= \frac{23 \times \overset{12}{\cancel{24}}}{\cancel{2}}$

$= 276$

Q $1^2+2^2+3^2+\cdots+23^2$?

$1^2+2^2+\cdots+n^2 = \frac{(n)(n+1)(2n+1)}{6}$

$\frac{(23)(\overset{4}{\cancel{24}})(47)}{\cancel{6}}$

$= 92 \times 47$

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Q3 $1^3+2^3+3^3+ \dots +23^3$

$$\left(\frac{(23)\times \cancel{24}^{12}}{\cancel{2}}\right)^2$$

$$= 276^2$$

$1^3+2^3+3^3+ \dots + n^3$

$$= \left(\frac{(n)(n+1)}{2}\right)^2$$

Q4 $31^2+32^2+33^2+ \dots + 50^2 = ?$

$$\Rightarrow (1^2+2^2+3^2+ \dots + \overset{n=50}{50^2}) - (1^2+2^2+ \dots \overset{n=30}{30^2})$$

$$\frac{(50)(51)(101)}{6} - \frac{(30)(31)(61)}{6}$$

Q $3^2+7^2+11^2+ \dots$ n terms. AP

$$= \Sigma T_n = \Sigma (4n-1)^2$$

3, 7, 11, 15 ... nth term

$a+(n-1)d$ se

$3+(n-1)4$

$4n-1$

$$\Sigma (4n-1)^2 = \Sigma 16n^2 - 8n + 1.$$

$$= \Sigma 16n^2 - \Sigma 8n + \Sigma 1$$

(3)

$$= 16 \Sigma n^2 - 8 \Sigma n + \Sigma 1$$

$$\Rightarrow 16 \times \frac{(n)(n+1)(2n+1)}{6} - 8 \times \frac{(n)(n+1)}{2} + n$$

LCM

Q6 If $T_n = 3n-1+2^n$ find Sum of 10 terms

$$S_n = \Sigma T_n = \Sigma 3n-1+2^n$$

$$= \Sigma 3n - \Sigma 1 + \Sigma 2^n.$$

$$\Rightarrow 3\Sigma n - \Sigma 1 + \Sigma 2^n \rightarrow 2^1+2^2+2^3 \dots 2^n$$

GP

$$S_n = 3\left(\frac{(n)(n+1)}{2}\right) - n + \frac{2(2^n-1)}{(2-1)}$$

$$S_{10} = 3\left(\frac{\cancel{10}^{5} \times 11}{\cancel{2}}\right) - 10 + 2(1024-1)$$

$$= 165 - 10 + 2046.$$

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Q7 $\overset{T_1}{1} + \overset{T_2}{(1+2)} + \overset{T_3}{(1+2+3)} + \overset{T_4}{(1+2+3+4)} - - -$ Sum of 15 terms.

$S_n = \sum T_n = \sum (1+2+3+4+ - - - +n)$

$= \sum \frac{(n)(n+1)}{2} = \frac{1}{2} \sum n^2 + n.$

$= \frac{1}{2} \left\{ \sum n^2 + \sum n \right\}$

$= \frac{1}{2} \left\{ \frac{(n)(n+1)(2n+1)}{6} + \frac{(n)(n+1)}{2} \right\}$

$= \frac{(n)(n+1)}{4} \left\{ \frac{2n+1}{3} + 1 \right\}$

$= \frac{(n)(n+1)}{4} \times \frac{(2n+4)}{3}$

$S_n = \frac{(n)(n+1)(n+2)}{6} \Rightarrow S_{15} = \frac{\overset{5}{\cancel{15}} \times \overset{8}{\cancel{16}} \times 17}{\cancel{6}\cancel{2}} = 680$

Q8 $\overset{T_1}{\frac{1^3}{1}} + \overset{T_2}{\frac{1^3+2^3}{1+3}} + \overset{T_3}{\frac{1^3+2^3+3^3}{1+3+5}} + - -$ Sum of 70 term.

$S_n = \sum T_n = \sum \frac{1^3+2^3+3^3+ - +n^3}{1+3+5+ - - - (2n-1)}$ 5

$= \sum \frac{\frac{\cancel{n^2}(n+1)^2}{4}}{\cancel{n^2}} = \frac{1}{4} \sum (n+1)^2$

$= \frac{1}{4} \sum n^2 + 2n + 1$

$= \frac{1}{4} \left\{ \sum n^2 + 2 \sum n + \sum 1 \right\}$

$= \frac{1}{4} \left\{ \frac{(n)(n+1)(2n+1)}{6} + \frac{\cancel{2}(n+1)(n)}{\cancel{2}} + n \right\}$

$= \frac{n}{4} \left\{ \frac{2n^2+3n+1+6n+6+6}{6} \right\} = \frac{n(2n^2+9n+13)}{24}$

$S_{70} = \frac{70(9800+630+13)}{24}$

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Q $\overset{T_1}{\boxed{1\cdot 2}}+\overset{T_2}{\boxed{2\cdot 3}}+\overset{T_3}{\boxed{3\cdot 4}}+4\cdot 5 - - - n\text{tem}$

OR

Q If $t_r=(r)(r+1)$ then $S_n=?$

$$S_n=\sum t_r=\sum r^2+r$$

$$=\sum r^2+\sum r$$

$$=\frac{(n)(n+1)(2n+1)}{6}+\frac{(n)(n+1)}{2}$$

$$=\frac{(n)(n+1)}{2}\left(\frac{2n+1}{3}+1\right)$$

$$S_n=\frac{(n)(n+1)(n+2)}{3}$$

Q $\overset{T_1}{\boxed{1\cdot 2\cdot 3}}+\overset{T_2}{\boxed{2\cdot 3\cdot 4}}+\overset{T_3}{\boxed{3\cdot 4\cdot 5}}+4\cdot 5\cdot 6+- - - +n\text{term}$

$$S_n=\sum T_r=\sum r(r+1)(r+2)$$

$$=\sum r(r^2+3r+2)$$

$$=\sum r^3+3r^2+2r$$

$$=\sum r^3+3\sum r^2+2\sum r$$

$$=\frac{(n)^2(n+1)^2}{4}+\cancel{3}\frac{(n)(n+1)(2n+1)}{\cancel{6}\,2}+2\frac{(n)(n+1)}{2}$$

$$=\frac{(n)(n+1)}{2}\left\{\frac{(n)(n+1)}{2}+\frac{2n+1}{2}+2\right\}$$

$$=\frac{(n)(n+1)}{2}\left\{\frac{n^2+n+4n+2+4}{2}\right\}=\frac{(n)(n+1)(n+2)(n+3)}{4}$$

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Trick

t_r	S_n
$\sum r$	$\frac{(n)(n+1)}{2}$
$\sum (r)(r+1)$	$\frac{(n)(n+1)(n+2)}{3}$
$\sum (r)(r+1)(r+2)$	$\frac{(n)(n+1)(n+2)(n+3)}{4}$
$\sum (r)(r+1)(r+2)(r+3)$	$\frac{(n)(n+1)(n+2)(n+3)(n+4)}{5}$

Q $\overset{T_1}{\boxed{1\cdot 3}}+\overset{T_2}{\boxed{2\cdot 4}}+\overset{T_3}{\boxed{3\cdot 5}}+4\cdot 6+5\cdot 7 - - - n$ ter.

$$\sum T_r = \sum (r)(r+2)$$

$$= \sum (r)((r+1)+1)$$

$$= \sum (r)(r+1) + r$$

$$= \sum (r)(r+1) + \sum r$$

$$= \frac{(n)(n+1)(n+2)}{3} + \frac{(n)(n+1)}{2}$$

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Q $\underbrace{\boxed{1\cdot 2\cdot 5}}_{}^{T_1} + \boxed{2\cdot 3\cdot 6}^{T_2} + \boxed{3\cdot 4\cdot 7}^{T_3} + \cdots + n$ terms?

$$\sum T_r = \sum (r)(r+1)(r+4)$$

$$= \sum (r)(r+1)((r+2)+2)$$

$$= \sum (r)(r+1)(r+2) + 2(r)(r+1)$$

$$= \sum (r)(r+1)(r+2) + 2\sum (r)(r+1)$$

$$= \frac{(n)(n+1)(n+2)(n+3)}{4} + 2\,\frac{(n)(n+1)(n+2)}{3}$$

Q $1\cdot 2^2\cdot 3 + 2\cdot 3^2\cdot 4 + 3\cdot 4^2\cdot 5 + \cdots + n$ terms.

$$\sum T_r = \sum (r)(r+1)^2(r+2)$$

$$= \sum (r)(r+1)(r+2)(r+1)$$

$$= \sum (r)(r+1)(r+2)((r+3)-2)$$

$$= \sum (r)(r+1)(r+2)(r+3) - 2\sum (r)(r+1)(r+2)$$

$$= \frac{(n)(n+1)(n+2)(n+3)(n+4)}{5} - \frac{2(n)(n+1)(n+2)(n+3)}{4}$$

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Q $\frac{1}{1\cdot 2}+\boxed{\frac{1}{2\cdot 3}}^{T_2}+\boxed{\frac{1}{3\cdot 4}}^{T_3}+\dots$ n terms ∞ terms

Trick. S_∞

$=\frac{1}{\text{diff}\times 1^{st}\text{ term}}$

$=\frac{1}{1\times 1}=1$

$\sum T_r=\sum\frac{1}{\underbrace{(r)(r+1)}_{\text{diff}=1}}$ $\quad\frac{1}{\text{diff}}\left(\frac{1}{\text{chh}}-\frac{1}{\text{Bda}}\right)$

$=\frac{1}{1}\sum_{r=1}^{n}\frac{1}{r}-\frac{1}{(r+1)}$ Baarish.

$=\frac{1}{1}-\cancel{\frac{1}{2}}$
$+\cancel{\frac{1}{2}}-\cancel{\frac{1}{3}}$
$+\cancel{\frac{1}{3}}-\cancel{\frac{1}{4}}$
$\vdots$
$+\cancel{\frac{1}{n}}-\frac{1}{n+1}$

$S_n=1-\frac{1}{n+1}=\frac{n}{n+1}$

Q $\frac{1}{1\cdot 2\cdot 3}+\boxed{\frac{1}{2\cdot 3\cdot 4}}^{T_2}+\boxed{\frac{1}{3\cdot 4\cdot 5}}^{T_3}+\dots+n$ term.

$S_\infty=\frac{1}{2\times(1\cdot 2)}$

$=\frac{1}{4}$

$\sum T_r=\sum\frac{1}{\underbrace{(r)(r+1)(r+2)}_{\text{diff}=2}}$

$=\frac{1}{2}\sum\frac{1}{1^{st}\text{ couple}}-\frac{1}{2^{nd}\text{ couple}}$

$=\frac{1}{2}\sum_{r=1}^{n}\frac{1}{(r)(r+1)}-\frac{1}{(r+1)(r+2)}$ Baarish

$=\frac{1}{2}\left\{\frac{1}{1\cdot 2}-\cancel{\frac{1}{2\cdot 3}}\right.$
$\cancel{\frac{1}{2\cdot 3}}-\cancel{\frac{1}{3\cdot 4}}$
$\vdots$
$\left.\cancel{\frac{1}{(n)(n+1)}}-\frac{1}{(n+1)(n+2)}\right\}$

$S_n=\frac{1}{2}\left\{\frac{1}{2}-\frac{1}{(n+1)(n+2)}\right\}$ Ans

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Q $S = \boxed{\overset{T_1}{\frac{1}{1\cdot 3\cdot 5}}} + \boxed{\overset{T_2}{\frac{1}{3\cdot 5\cdot 7}}} + \boxed{\overset{T_3}{\frac{1}{5\cdot 7\cdot 9}}} + \cdots + n\text{ term}$

$$S_n = \sum T_r = \sum_{r=1} \frac{1}{(2r-1)(2r+1)(2r+3)}$$

diff = 4

$$= \frac{1}{4} \sum_{r=1} \frac{1}{(2r-1)(2r+1)} - \frac{1}{(2r+1)(2r+3)}$$

$$= \frac{1}{4} \left\{ \begin{array}{l} \frac{1}{1\cdot 3} - \cancel{\frac{1}{3\cdot 5}} \\ \cancel{\frac{1}{3\cdot 5}} - \frac{1}{5\cdot 7} \\ \\ \cancel{\frac{1}{(2n-1)(2n+1)}} - \frac{1}{(2n+1)(2n+3)} \end{array} \right\}$$

$$S_n = \frac{1}{4}\left\{\frac{1}{1\cdot 3} - \frac{1}{(2n+1)(2n+3)}\right.$$