Nishant Jindal

# Heat transfer in case of Variable Crossectional area

At a radial distance $r$, a spherical Shell of 'dr' thickness is cut

$T_1 > T_2$

if $\left(\frac{dQ}{dt}\right)$ be the heat flow at the time of Steady state

dT be the temp difference for dr thickness of the shell

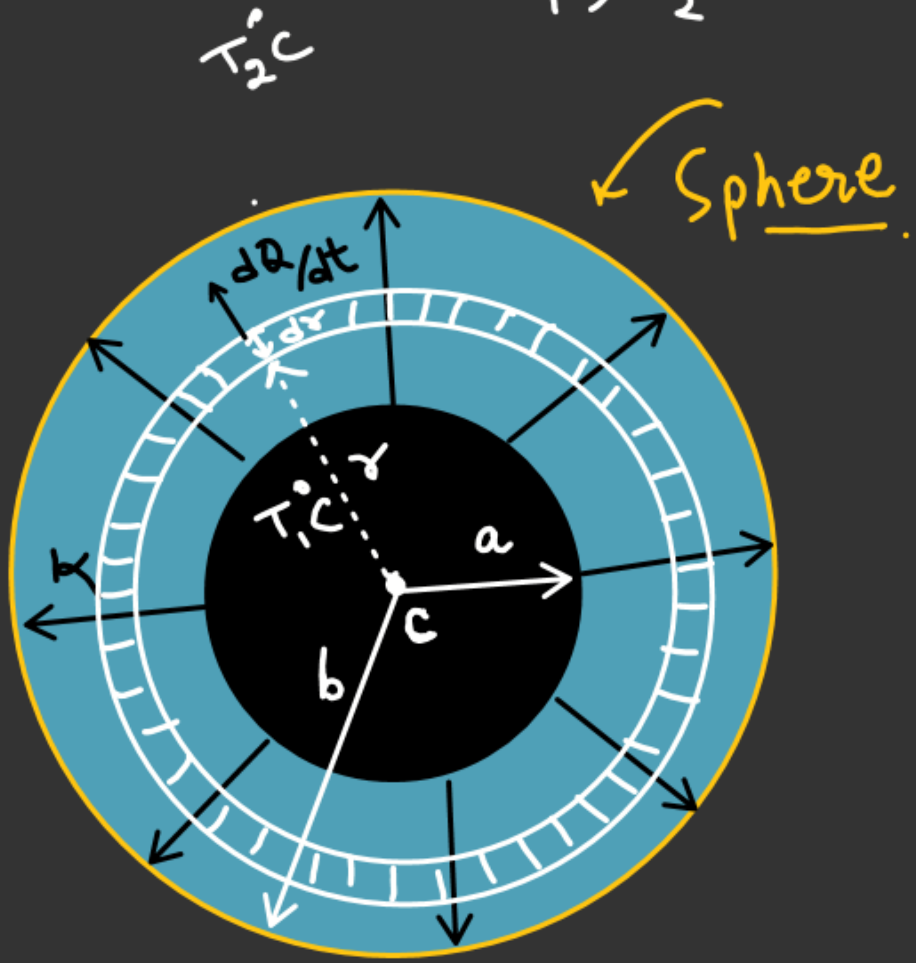
$$\text{J/s} \Leftarrow \left(\frac{dQ}{dt}\right) = -K\underbrace{4\pi r^2}_{A}\left(\frac{dT}{dr}\right)$$

⇓

??

$$P = -K \cdot 4\pi r^2 \left(\frac{dT}{dr}\right)$$

$$P\int_a^b \frac{dr}{r^2} = -4\pi K \int_{T_1}^{T_2} dT$$



Nishant Jindal

# Heat transfer in case of Variable Crossectional area

$$\left(i = \frac{\Delta V}{R}\right)$$

$$P\int_a^b \frac{dr}{r^2} = -4\pi K \int_{T_1}^{T_2} dT$$

$$P\left[-\frac{1}{r}\right]_a^b = -4\pi K(T_2 - T_1)$$

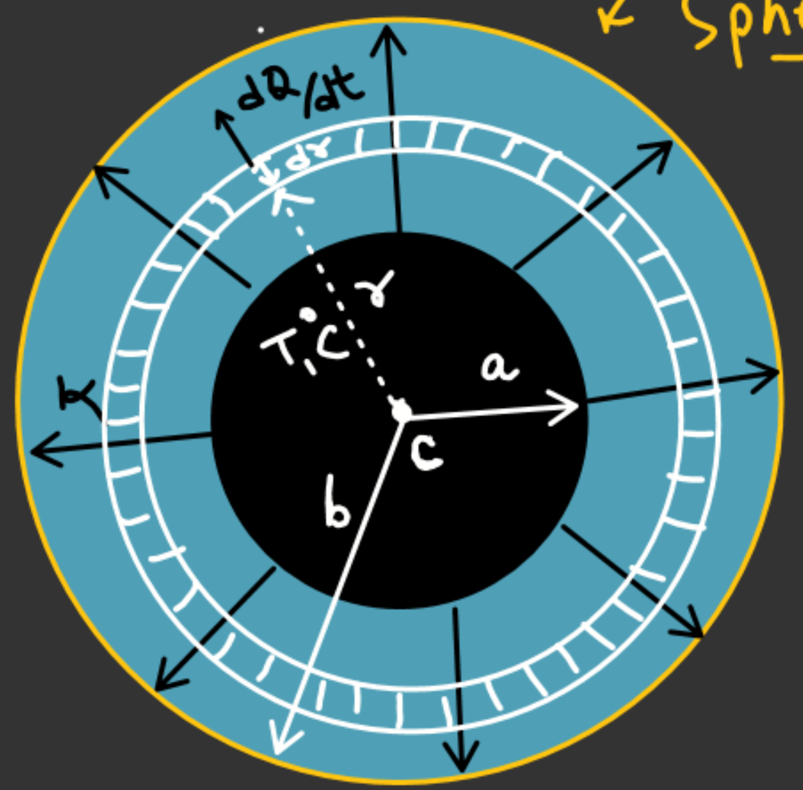
$$P\left[-\frac{1}{b} + \frac{1}{a}\right] = 4\pi K(T_1 - T_2)$$

$$\underset{\substack{\Downarrow \\ i_{th}}}{P} = \left[\frac{(T_1 - T_2)}{\underbrace{\frac{1}{4\pi K}\left(\frac{1}{a} - \frac{1}{b}\right)}_{\Downarrow\ R_{th}}}\right]$$

$$P \Leftarrow \frac{dQ}{dt}$$

$$\boxed{R_{th} = \frac{1}{4\pi K}\left(\frac{1}{a} - \frac{1}{b}\right)}$$

$T_1 > T_2$



Nishant Jindal

# Heat transfer in case of Variable Crossectional area

if K is Variable, $K = K_0 r$ $\quad r \to$ radial distance

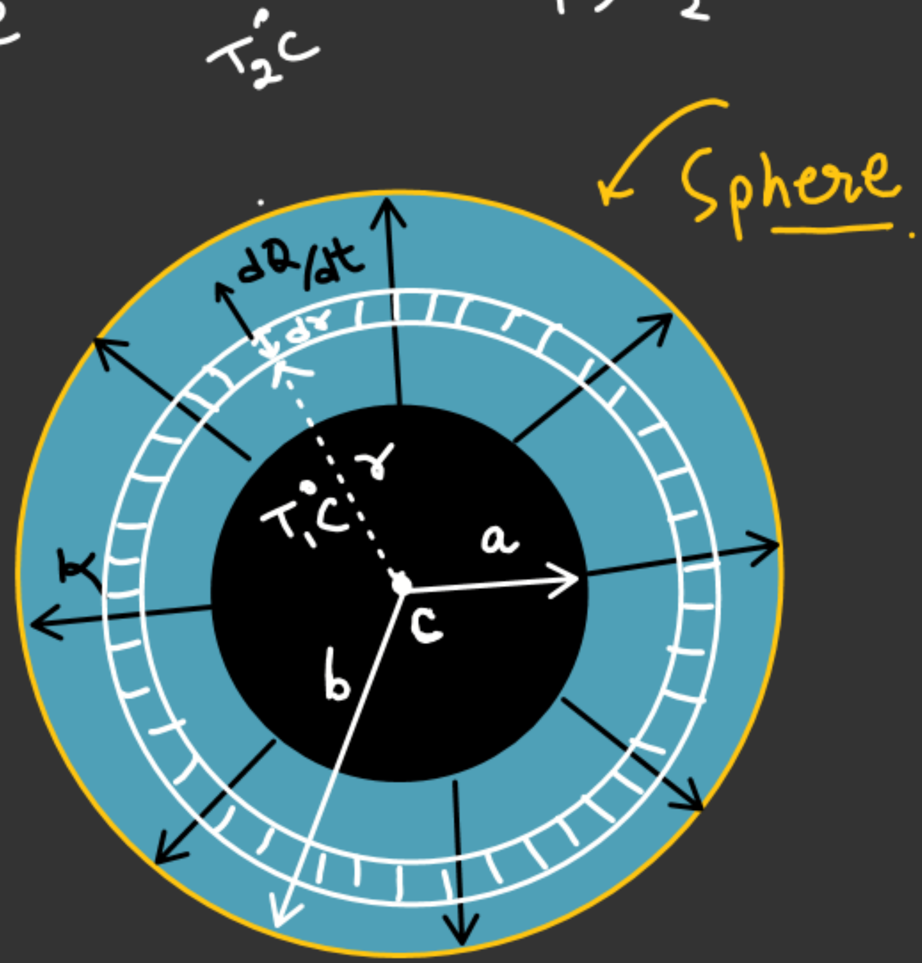
$T_1 > T_2$

$$\frac{dQ}{dt} = -K_r 4\pi r^2 \left(\frac{dT}{dr}\right)$$

$$P = -K_0 4\pi r^3 \frac{dT}{dr}$$

$$P\int_a^b \frac{dr}{r^3} = -K_0 4\pi \int_{T_1}^{T_2} dT$$

$$P = \checkmark$$



Nishant Jindal

# Heat flow through Cylindrical Crossectional area

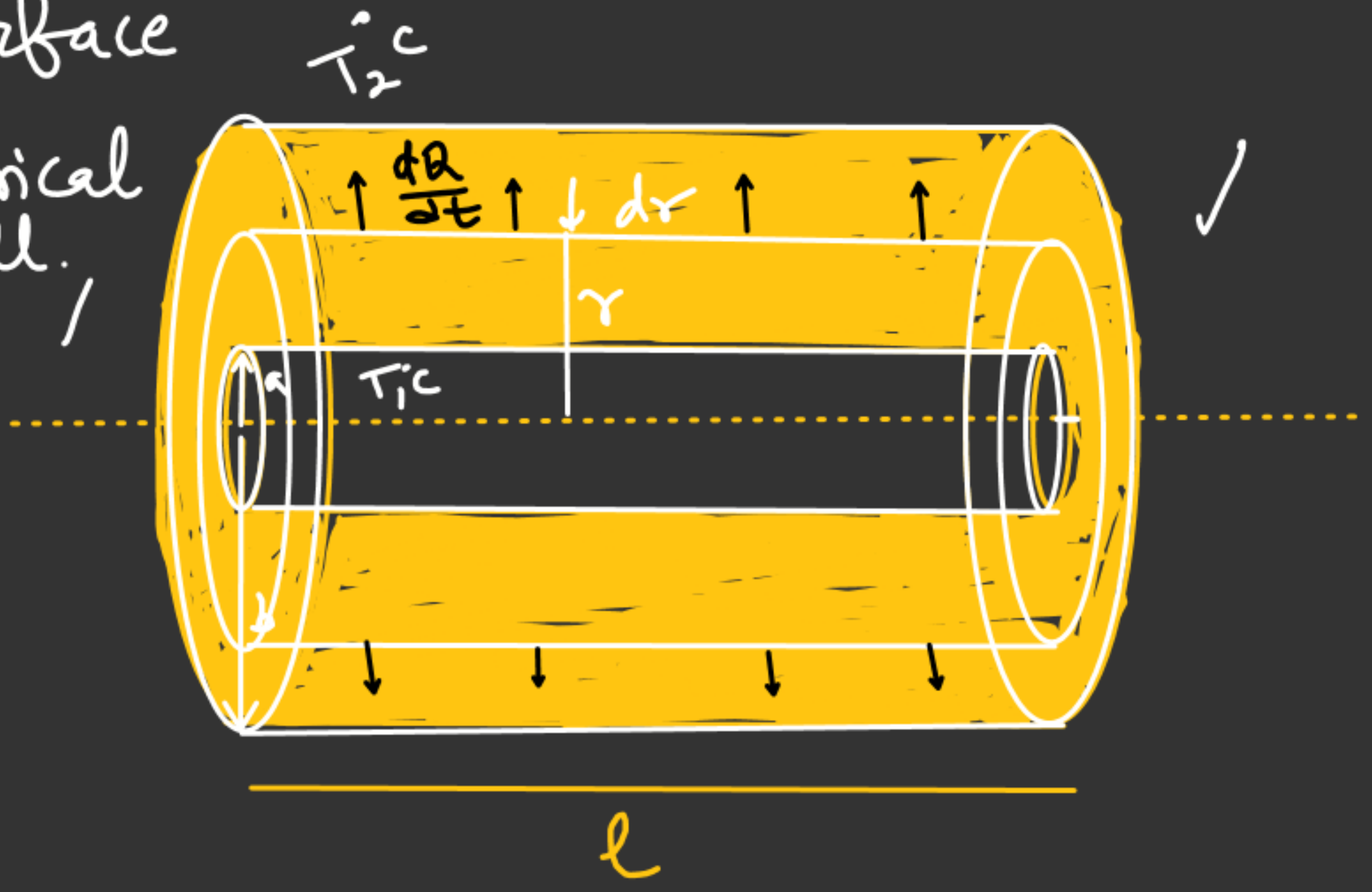
$A \rightarrow$ Curve surface area of Cylindrical shell.

$$P\ J/s \Leftarrow \frac{dQ}{dt} = -K(2\pi r l)\frac{dT}{dr}$$

$dT \rightarrow$ temp difference for $dr$ thickness of shell.

$$P = -K2\pi l\left(\frac{r}{dr}\right)dT$$

$$P\int_a^b \frac{dr}{r} = -(2\pi l)K\int_{T_1}^{T_2} dT$$



Nishant Jindal

$$P\int_a^b \frac{dr}{r} = -(2\pi l)K\int_{T_1}^{T_2} dT$$

$$P\ln\left(\frac{b}{a}\right) = -2\pi K l\,(T_2-T_1)$$

$$P = \frac{(T_1-T_2)}{\frac{1}{2\pi K l}\ln\left(\frac{b}{a}\right)}$$

$i_{th}$

$R_{th}$

$$\boxed{R_{th} = \frac{1}{2\pi K l}\ln\left(b/a\right)}$$

Thermal resistance of a Cylindrical Conductor.

H.W

$K = K_0 r$

$K = K_0/r$

$K = K_0 + \alpha r$

$r \rightarrow$ radial distance

Find $\frac{dQ}{dt} = ??$

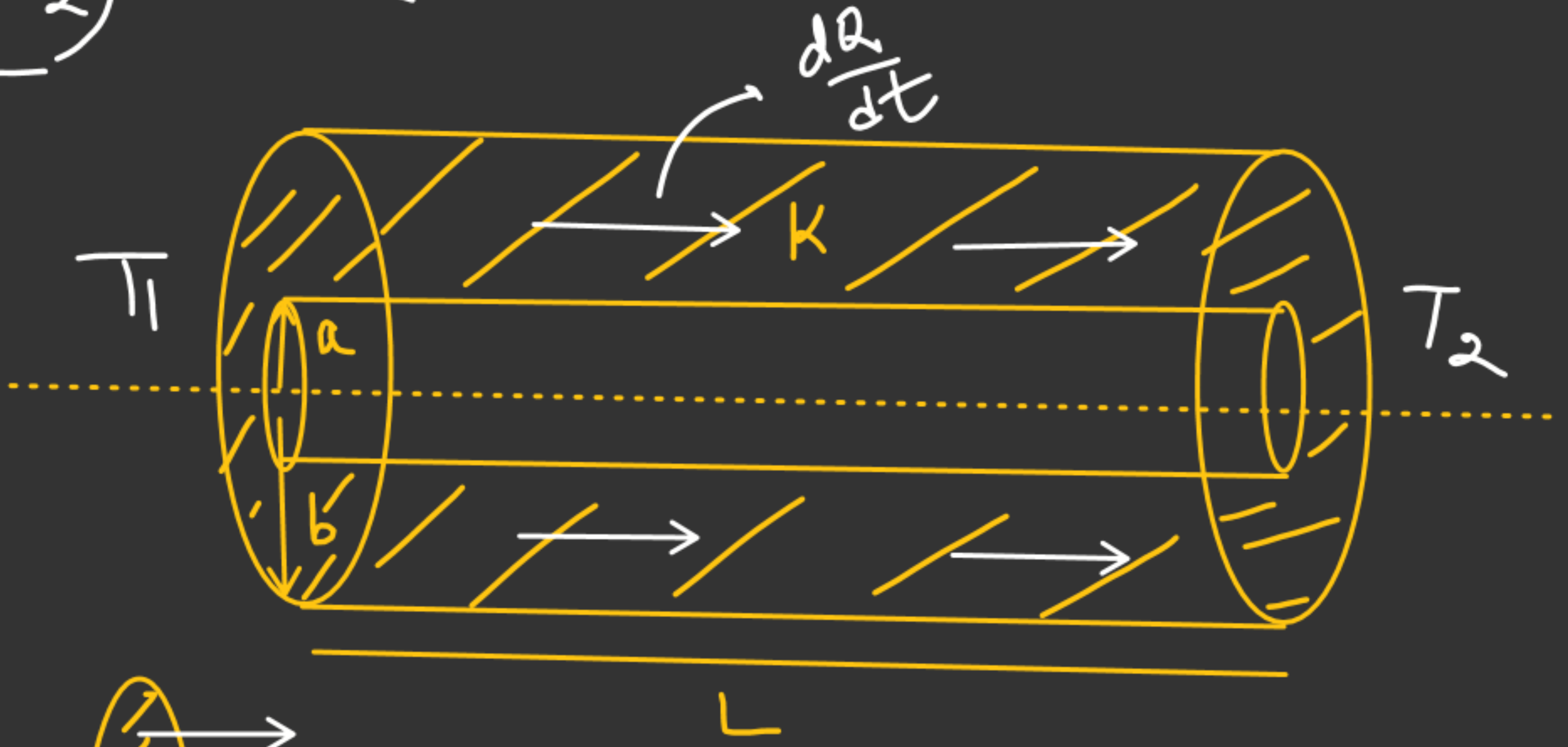
Nishant Jindal

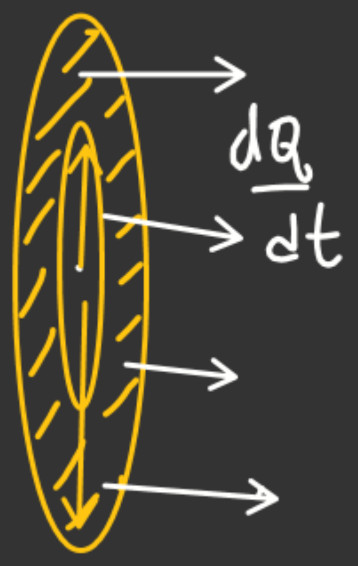
$$\frac{\Delta Q}{\Delta t} = \frac{dQ}{dt} = \frac{K\pi(b^2-a^2)\overbrace{(T_1-T_2)}^{T_H-T_L}}{L} \quad T_1 > T_2$$

$\Downarrow$

P J/s

$$P = \frac{T_1 - T_2}{\left(\underbrace{\frac{L}{K\pi(b^2-a^2)}}_{\Downarrow\, R_{th}}\right)}$$




$$\frac{dQ}{dt} = \text{Power}$$

$dQ \to$ heat

Nishant Jindal

## Case when one end of temperature of rod is a function of time

$T_B = (20 + 2t)$

$t \to$ time.

K, A, L


Total heat flow from A to B

$$\left(\frac{dQ}{dt}\right) = \frac{KA(100^\circ - (20+2t))}{L}$$

$$\frac{dQ}{dt} = \frac{KA}{L}(80 - 2t)$$

$$\int_0^Q dQ = \frac{KA}{L}\int_0^{40} (80-2t)\,dt$$

Heat Continue to flow from A to B untill & unless

$T_A = T_B$

$100 = 20 + 2t$

$\frac{80}{2} = t$

$t = 40\,sec$ ✓

Nishant Jindal

$$\int_0^Q dQ = \frac{KA}{L}\int_0^{40}(80-2t)\,dt$$

$$Q = \frac{KA}{L}\left\{80\left[t\right]_0^{40} - \cancel{\frac{2}{2}}\left[t^2\right]_0^{40}\right\}$$

$$Q = \frac{KA}{L}\left\{3200 - 1600\right\}$$

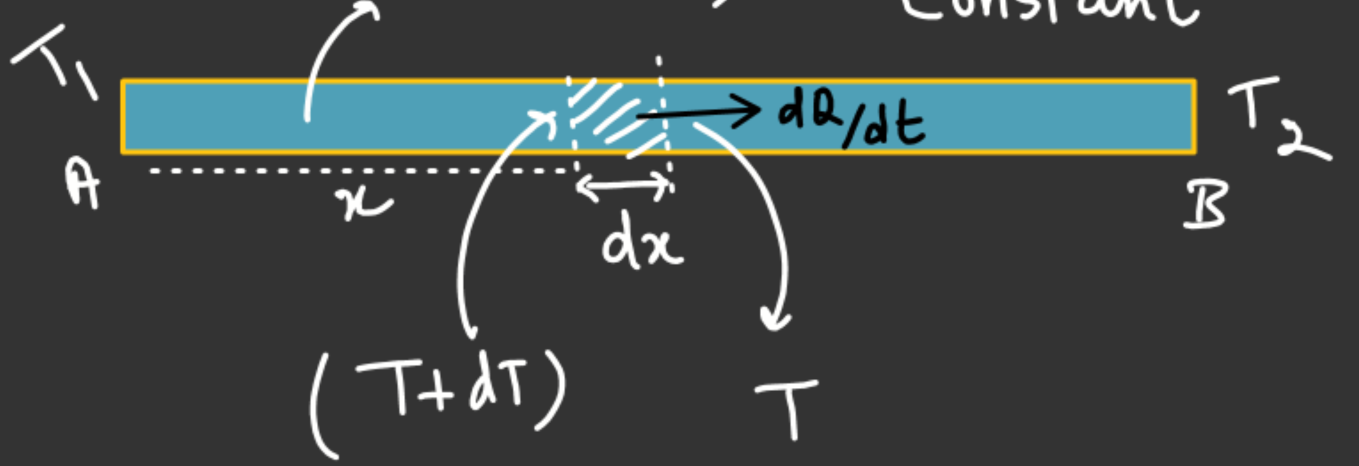
$$Q = \left(1600\frac{KA}{L}\right)J \checkmark$$

Nishant Jindal

## Case when conductive of rod is a function of distance from end A.

$T_1 > T_2$

$K = (a+bx)$ where a & b are Constant




For dx length of rod.

$$\frac{dQ}{dt} = -\underline{K_x A}\left(\frac{dT}{dx}\right)$$

$\Downarrow$

$$P = -(a+bx)A\cdot\left(\frac{dT}{dx}\right)$$

$$P\int_0^L \frac{dx}{a+bx} = -A\int_{T_1}^{T_2} dT$$

$$P\,\frac{\ln[a+bx]_0^L}{b} = -A(T_2 - T_1)$$

$$P = \left[\frac{bA(T_1 - T_2)}{\ln\left(\frac{a+bL}{a}\right)}\right] \qquad \left[\int \frac{dx}{a+bx} = \frac{\ln(a+bx)}{b}\right]$$

$\Downarrow$

Total rate of heat flow =:

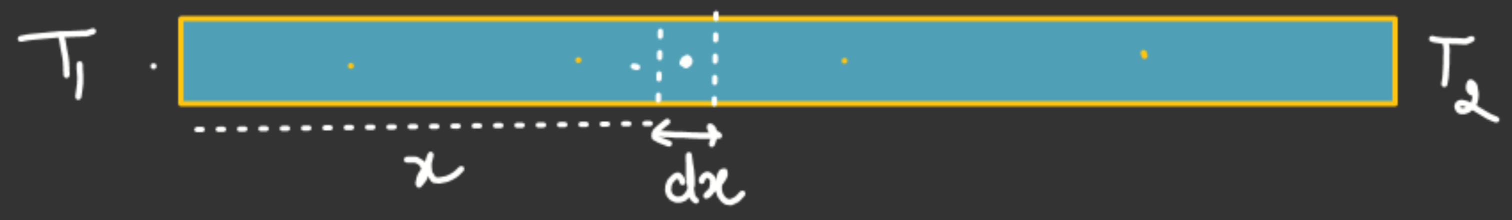
Nishant Jindal

## Case when thermal conductivity of rod as a function of temp of the rod

$K = \left(\frac{\alpha}{T}\right)$ $\alpha =$ Constant

$T_1 > T_2$

A, L




$T_x = T$

$$\frac{dQ}{dt} = -KA\left(\frac{dT}{dx}\right)$$

$$P = -\frac{\alpha}{T} A \frac{dT}{dx}$$

$$P\int_0^L dx = -\alpha A \int_{T_1}^{T_2} \left(\frac{dT}{T}\right)$$

$$P(L) = -\alpha A \ln\left(\frac{T_2}{T_1}\right)$$

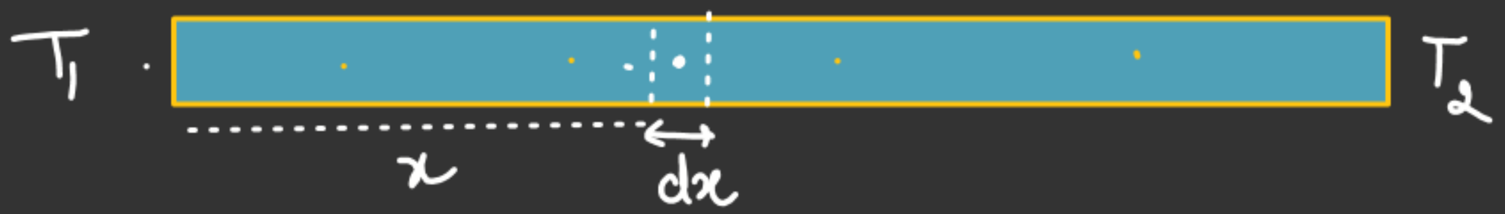
$$P = \frac{\alpha A}{L} \ln\left(\frac{T_1}{T_2}\right)$$

Nishant Jindal

## Case when thermal Conductivity of rod as a function of temp of the rod

$T_1 > T_2$

$A, L$




$K = \left(\frac{\alpha}{T}\right)$ $\alpha =$ Constant,

$T \rightarrow f(x) = ??$

$$\frac{dQ}{dt} = -KA\left(\frac{dT}{dx}\right)$$

$$P = -\frac{\alpha}{T} A \frac{dT}{dx}$$

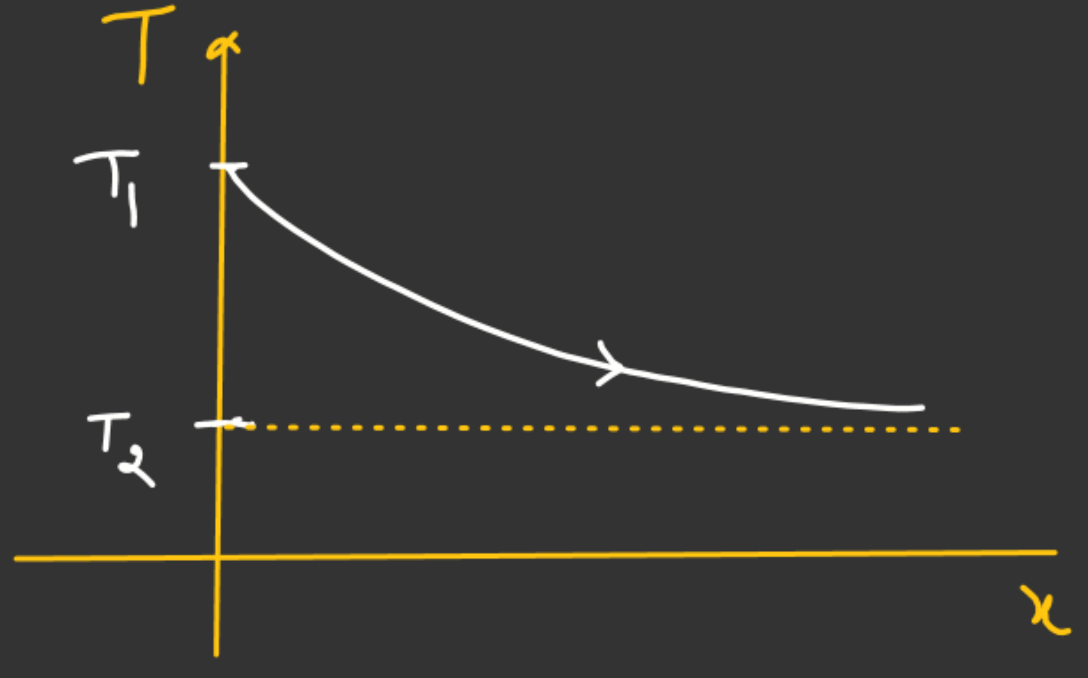
$$P\int_0^x dx = -\alpha A\int_{T_1}^{T}\left(\frac{dT}{T}\right)$$

$$Px = -\alpha A \ln\left(\frac{T}{T_1}\right)$$

$$-\frac{P}{\alpha A}x = \ln\left(T/T_1\right)$$

$$\frac{T}{T_1} = e^{-\frac{P}{\alpha A}x}$$

$$\boxed{T = T_1 e^{\frac{-P}{\alpha A}x}}$$




$x \rightarrow L$

# Heat flow in a Sink

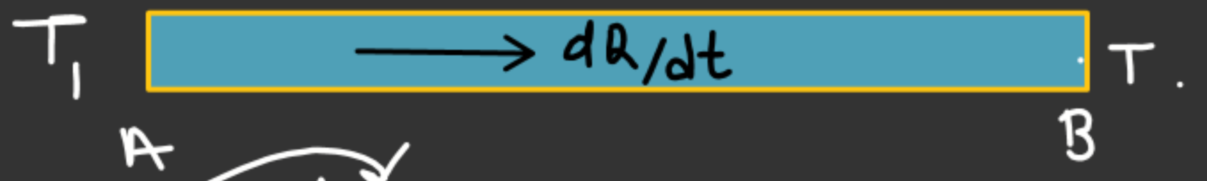
$T_1 > T_2$

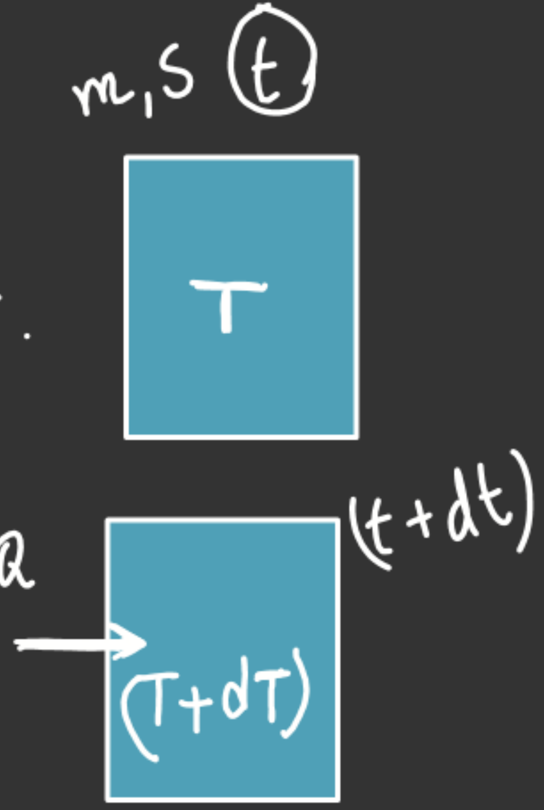
At $t=0$, $T_1$ & $T_2$ be the temp of end A & B.

let, at any time 't' temp of end B or sink be T.

## Equation of Rod

K, A, L




$$\frac{dQ}{dt} = \frac{KA}{L}(T_1 - T) \quad -(1)$$

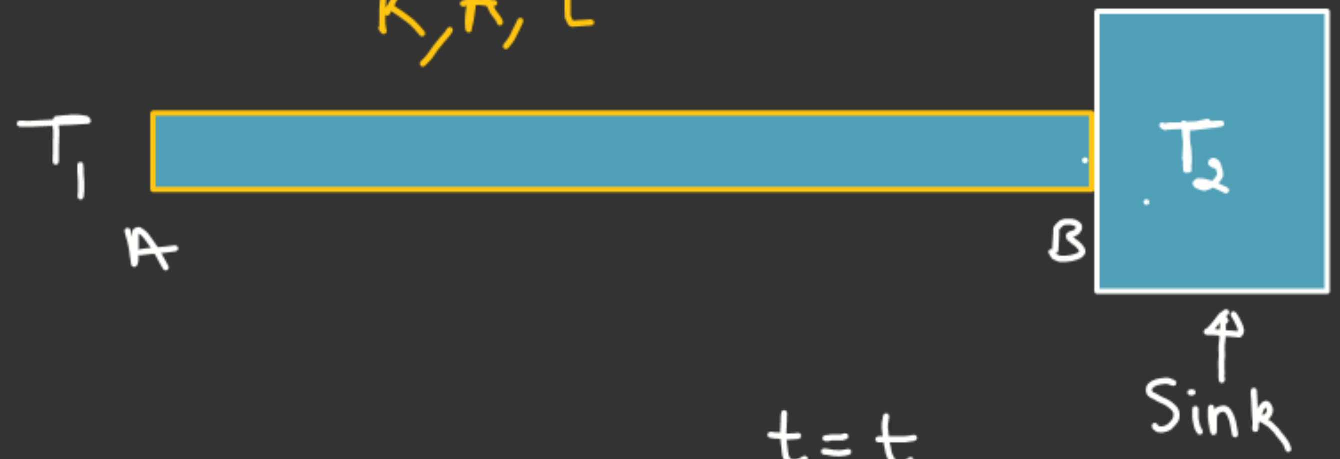
## Equation of Sink

$$dQ = (ms\,dT) \quad -(2)$$

$t=0$

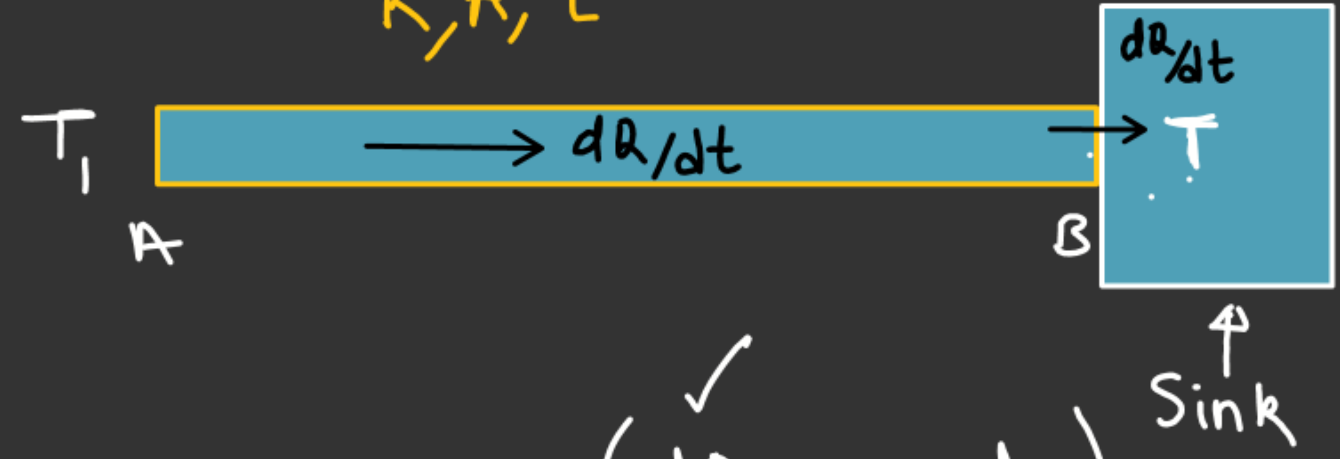
m, s

K, A, L




$t=t$

K, A, L




$$(dQ = ms\,dT)$$

s → specific heat

ms → heat capacity

Nishant Jindal

$$\frac{dQ}{dt} = \frac{KA}{L}(T_1 - T) \quad -(1)$$

Equation of Sink

$$dQ = (ms\,dT) \quad -(2)$$

From (1) & (2)

$$ms\frac{dT}{dt} = \frac{KA}{L}(T_1 - T)$$

$$\int_{T_2}^{T} \frac{dT}{T_1 - T} = \frac{KA}{msL}\int_0^t dt$$

$$\frac{\ln\left[T_1 - T\right]_{T_2}^{T}}{(-1)} = \frac{KA}{msL}t$$

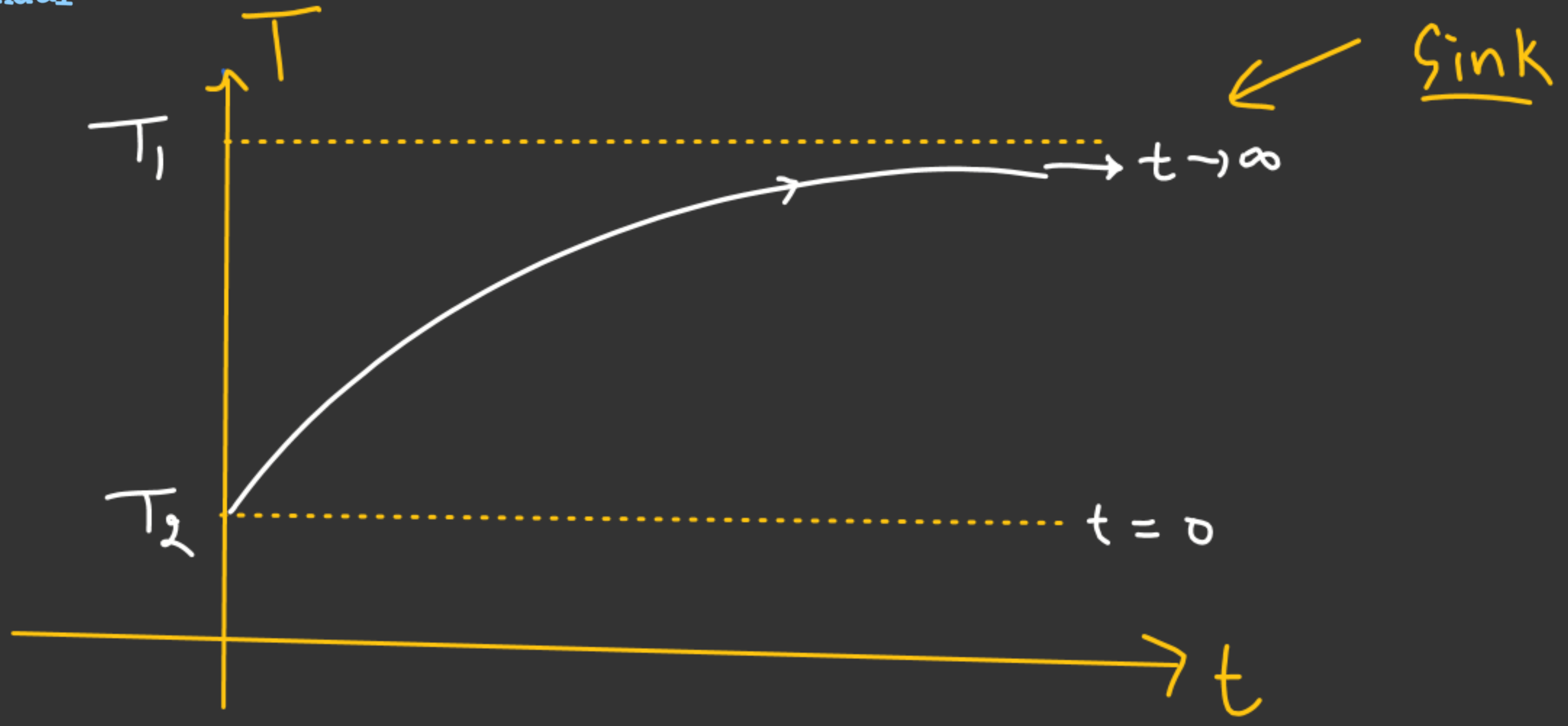
$$\ln(T_1 - T) - \ln(T_1 - T_2) = \frac{-KA}{msL}t$$

$$\ln\left(\frac{T_1 - T}{T_1 - T_2}\right) = \frac{-KA}{msL}t$$

$$T_1 - T = (T_1 - T_2)\,e^{-\frac{KA}{msL}t}$$

$$\boxed{T = T_1 - (T_1 - T_2)\,e^{-\frac{KA}{msL}t}}$$

Nishant Jindal



H.W

Find time 't' to melt the ice cube.


$$t = (t_1 + t_2)$$

phase Change

$\downarrow Q$

$$-10^\circ C \text{ ice} \xrightarrow{t_1} 0^\circ C \text{ ice} \xrightarrow{t_2} 0^\circ C \text{ Water}$$

$(Q = mL_f)$