Nishant Jindal

LIMIT

Methods to Solve Qs of Limit

[ ] { } | | Sgn def / Char.

Limit Cont^{y} Diff^{y}

Limit1 → Limit2 → Limit3

When we do not need to check LHL = RHL then we follow following Method to solve Qs of limit

(A) Factorisation (B) Rationalisation

(C) $\lim_{n\to\infty}$ type (D) B.T. (E) Sandwich Theorem.

(F) Using Expansion (G) L. Hospital Rule.

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Factorisation

Make factors in Nr & Dr & Cancell common factor then Put Limit

Q $\lim_{x\to 2} \frac{x^2-5x+6}{x^2-3x+2}$ $\frac{0}{0}$

DL

$\lim_{x\to 2} \frac{2x-5+0}{2x-3+0}$

$\frac{4-5}{4-3} = -1$

$\lim_{x\to 2} \frac{\cancel{(x-2)}(x-3)}{\cancel{(x-2)}(x-1)} = \frac{2-3}{2-1} = -1$

Q. $\lim_{x\to 1} \frac{(2x+3)(\sqrt{x}-1)}{2x^2-5x+3}$ $\frac{0}{0}$

$\lim_{x\to 1} \frac{(2x+3)\cancel{(\sqrt{x}-1)}}{\cancel{(x-1)}(2x-3)(\sqrt{x}+1)} = \frac{5}{2x-1}$

$= -\frac{5}{2}$

$(a-b) = (\sqrt{a}-\sqrt{b})(\sqrt{a}+\sqrt{b})$

$\boxed{\sqrt{a}-\sqrt{b} = \frac{a-b}{\sqrt{a}+\sqrt{b}}}$

Q $\lim_{x\to \tan^{-1}3} \frac{\tan^2 x - 2\tan x - 3}{\tan^2 x - 4\tan x + 3}$

$\frac{9-6-3}{9-12+3}$ $\frac{0}{0}$

$\lim_{\tan x\to 3} \frac{\tan^2 x - 2\tan x - 3}{\tan^2 x - 4\tan x + 3}$ $\frac{0}{0}$

$\lim_{\tan x\to 3} \frac{\cancel{(\tan x-3)}(\tan x+1)}{\cancel{(\tan x-3)}(\tan x-1)} = \frac{3+1}{3-1} = 2$

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Q $\lim_{x\to 1} \frac{x^3 - x^2\log x + \log x - 1}{x^2-1}$ $\frac{0}{0}$

$\lim_{x\to 1} \frac{(x^3-1) - \log x\,(x^2-1)}{(x^2-1)}$

$\lim_{x\to 1} \frac{\cancel{(x-1)}\{(x^2+x+1) - (x+1)\log x\}}{\cancel{(x-1)}(x+1)}$

$\frac{3-(2)\times 0}{2} = \frac{3}{2}$

Q $\lim_{x\to 2a} \frac{\sqrt{x-2a} + (\sqrt{x} - \sqrt{2a})}{\sqrt{x^2-4a^2}}$ $\frac{0}{0}$ $\left(\sqrt{a}-\sqrt{b} = \frac{a-b}{\sqrt{a}+\sqrt{b}}\right)$

$\lim_{x\to 2a} \frac{\sqrt{x-2a} + \frac{(x-2a)}{\sqrt{x}+\sqrt{2a}}}{\sqrt{(x-2a)(x+2a)}}$

$\lim_{x\to 2a} \frac{\cancel{\sqrt{x-2a}}\left\{1 + \frac{\sqrt{x-2a}}{\sqrt{x}+\sqrt{2a}}\right\}}{\sqrt{\cancel{(x-2a)}(x+2a)}}$

$\frac{1+\frac{0}{(\;)}}{\sqrt{4a}} = \frac{1}{2\sqrt{a}}$

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Q $\lim_{x\to 2a} \dfrac{\sqrt{x-2a}+\sqrt{x}-\sqrt{2a}}{\sqrt{x^2-4a^2}}$ $\left(\dfrac{0}{0}\right)$

$$\lim_{x\to 2a} \dfrac{\left(\dfrac{1}{\cancel{2}\sqrt{x-2a}}\times 1+\dfrac{1}{2\sqrt{x}}\right)-0}{\dfrac{1}{2\sqrt{x^2-4a^2}}\times 2x}$$

$$\lim_{x\to 2a} \dfrac{\sqrt{x}+\sqrt{x-2a}}{\sqrt{x}\,\cancel{\sqrt{x-2a}}}\times\dfrac{\sqrt{(x+2a)}\,\cancel{\sqrt{x^2-4a^2}}}{2x}$$

$$\dfrac{\cancel{\sqrt{2a}}+0}{\cancel{\sqrt{2a}}}\times\dfrac{\sqrt{4a}}{4a}=\dfrac{\cancel{2}\cancel{\sqrt{a}}}{{}_2\cancel{4}\cancel{a}\sqrt{a}}$$

$$=\dfrac{1}{2\sqrt{a}}$$

{DL. Hospital Rule}

$\lim_{x\to a}\dfrac{f(x)}{g(x)}=\dfrac{0}{0}$ or $\dfrac{\infty}{\infty}$ form then We Can Use DL Rule.

$\Rightarrow \lim_{x\to a}\dfrac{f'(x)}{g'(x)}$ & put Limit

(1) $(\sqrt{x})'=\dfrac{1}{2\sqrt{x}}$

(2) $(\sqrt{x-2a})'\to\dfrac{1}{2\sqrt{x-2a}}\times(1-0)=\dfrac{1}{2\sqrt{x-2a}}$

(3) $(\sqrt{x^2-9a^2})'=\dfrac{1}{2\sqrt{x^2-9a^2}}\times(2x-0)=\dfrac{2x}{2\sqrt{x^2-9a^2}}$

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Q $\lim_{h\to 0}\left[\frac{1}{h(8+h)^{1/3}}-\frac{1}{2h}\right]$ $\infty-\infty$ form.

Funda → first try LCM.

$\lim_{h\to 0}\left[\frac{2-(8+h)^{1/3}}{2h(8+h)^{1/3}}\right]$

3rd Method

B.T.

$\lim_{h\to 0}\frac{2\left\{1-\left(1+\frac{h}{8}\right)^{1/3}\right\}}{4h\left(1+\frac{h}{8}\right)^{1/3}}$ → Shakl $(1+x)^n$ जैसी

$$\boxed{(1+x)^n = 1+nx+\frac{(n)(n-1)}{1\cdot 2}x^2}$$

$\lim_{h\to 0}\frac{\left\{1-\left(1+\frac{h}{24}\right)\right\}}{2h\left(1+\frac{h}{24}\right)}$

90% QS $(1+x)^n = 1+nx$

$=\frac{-\frac{1}{24}}{2(1+0)}=-\frac{1}{48}$

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Q $\lim_{x\to 2} \dfrac{3^x+3^{3-x}-12}{3^{-x/2}-3^{1-x}} = \dfrac{0}{0}$ DL

Main 20

$$\lim_{x\to 2} \frac{3^x\ln 3+3^{3-x}\ln 3\times -1-0}{3^{-\frac{x}{2}}\ln 3\times\frac{-1}{2}+3^{1-x}\ln 3(+1)}$$

$$\lim_{x\to 2} \frac{\cancel{\ln 3}(3^x-3^{3-x})}{\cancel{\ln 3}\left(-\frac{3^{-x/2}}{2}+3^{1-x}\right)} = \frac{9-3^1}{-\frac{3^{-1}}{2}+3^{-1}}$$

$$= \frac{6}{\left(-\frac{1}{6}+\frac{1}{3}\right)} = \frac{6}{\frac{1}{6}} = 36$$

diff^n

$(a^x)' \to a^x \ln a$

$(2^x)' \to 2^x \ln 2$

$(2^{2-x})' \to 2^{(2-x)}\ln 2\times(0-1)$

$= -2^{2-x}\ln 2$

$(2^{-x})' \to 2^{-x}\ln 2\times -1$

$(2^{-\frac{x}{2}})' \to 2^{-\frac{x}{2}}\ln 2\times\frac{-1}{2}$

$(2^{-x^2})' \to 2^{-x^2}\ln 2\times(-2x)$

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Rationalisation

When Qs has $(\sqrt{\ } - \sqrt{\ })$ or $(\square - \sqrt{\ })$ type then Use Rationalisation

Sometime Double Rationalisation

Q $\lim_{x\to 0} \frac{x^2}{\left(\sqrt{1+x^2}-\sqrt{1-x^2}\right)}$ $\frac{0}{0}$ Rat.

$$\lim_{x\to 0} \frac{(x^2)\left(\sqrt{1+x^2}+\sqrt{1-x^2}\right)}{(1+x^2)-(1-x^2)}$$

$$\lim_{x\to 0} \frac{(\cancel{x^2})\left(\sqrt{1+x^2}+\sqrt{1-x^2}\right)}{2\cancel{x^2}} = \frac{\sqrt{1}+\sqrt{1}}{2} = 1$$

Q $\lim_{x\to\infty} \left(\sqrt{x^2+x}-\sqrt{x^2-x}\right)$ $(\infty-\infty)$ form → Rationalisation

$$\Rightarrow \lim_{x\to\infty} \frac{(x^2+x)-(x^2-x)}{\left(\sqrt{x^2+x}+\sqrt{x^2-x}\right)}$$

$$\lim_{x\to\infty} \frac{2x}{\sqrt{x^2+x}+\sqrt{x^2-x}}$$

$$\lim_{x\to\infty} \frac{2x}{x\left\{\sqrt{1+\frac{1}{x}}+\sqrt{1-\frac{1}{x}}\right\}}$$

$$\frac{2}{\sqrt{1+0}+\sqrt{1-0}} = \frac{2}{2} = 1$$

In Qs of $\lim_{n\to\infty}$ always try to take max term common from Nr & Dr.

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Q $\lim_{x\to 0} \dfrac{\sqrt[n]{1+x}-\sqrt[m]{1+x}}{x}$

$\lim_{x\to 0} \dfrac{(1+x)^{1/n}-(1+x)^{1/m}}{x}$

B.T. Shakl $(1+x)^n = 1+nx$

$\lim_{x\to 0} \dfrac{\left(\cancel{1}+\frac{x}{n}\right)-\left(\cancel{1}+\frac{x}{m}\right)}{x}$

$\dfrac{\cancel{x}\left(\frac{1}{n}-\frac{1}{m}\right)}{\cancel{x}} = \frac{1}{n}-\frac{1}{m}$

$\sqrt{\ }-\sqrt{\ }$ Nahi hai Rationalisation mt lagaoo

Q $\lim_{x\to 1} \dfrac{\sqrt[3]{x}-1}{(x-1)}$

$x = 1+h$

$\lim_{h\to 0} \dfrac{(1+h)^{\frac{1}{3}}-1}{(\cancel{1}+h)-\cancel{1}}$

$\dfrac{\left(\cancel{1}+\frac{h}{3}\right)-\cancel{1}}{\cancel{h}}$

$= \frac{1}{3}$

$\lim_{x\to 1} \dfrac{x^{1/3}-1}{x-1}$ $\frac{0}{0}$

$\lim_{x\to 1} \dfrac{\frac{1}{3}x^{-\frac{2}{3}}}{1}$

$\frac{1}{3}(1)^{-2/3}$

$= \frac{1}{3}$

Funda

$\lim$ $\boxed{x\to \text{Constant}}$ Diya ho

U can Use

$x = \text{Constant}+h$

or

$x = \text{Constant}-h$

any time

$x \neq 0, \infty$

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Q $\lim_{\boxed{x\to -1}} \frac{\sqrt[3]{7-x}-2}{x+1}$

$\boxed{x = -1-h}$

$$\lim_{h\to 0} \frac{\left(7-(-1-h)\right)^{\frac{1}{3}}-2}{(\cancel{-1}-h)+\cancel{1}}$$

$$\lim_{h\to 0} \frac{(8+h)^{\frac{1}{3}}-2}{-h} \rightarrow \text{BT}$$

$$\lim_{h\to 0} \frac{2\left\{\left(1+\frac{h}{8}\right)^{\frac{1}{3}}-1\right\}}{-h}$$

$$\frac{2\left\{\cancel{1}+\frac{\cancel{h}}{24}-\cancel{1}\right\}}{-\cancel{h}} = -\frac{1}{12}$$

Q $\lim_{x\to 0} \frac{(\cos x)^{\frac{1}{3}}-(\cos x)^{\frac{1}{2}}}{1-\cos^2 x}$

$$\lim_{x\to 0} \frac{\left(\sqrt{1-\sin^2 x}\right)^{\frac{1}{3}}-\left(\sqrt{1-\sin^2 x}\right)^{\frac{1}{2}}}{\sin^2 x} \quad \boxed{\text{BT}}$$

$$\lim_{x\to 0} \frac{(1-\sin^2 x)^{\frac{1}{6}}-(1-\sin^2 x)^{\frac{1}{4}}}{\sin^2 x}$$

$$\frac{\left(\cancel{1}-\frac{\sin^2 x}{6}\right)-\left(\cancel{1}-\frac{\sin^2 x}{4}\right)}{\sin^2 x} = \frac{\cancel{\sin^2 x}\left(\frac{1}{4}-\frac{1}{6}\right)}{\cancel{\sin^2 x}}$$

$$= \frac{1}{12}$$

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Q $\lim_{x\to\infty}\left(\sqrt{(x+a)(x+b)}-x\right)$ $\infty-\infty$

x^2 comm

$$\lim_{x\to\infty}\left(\left(x^2+(a+b)x+ab\right)^{\frac{1}{2}}-x\right)$$

$(1+x)^n$

$$\lim_{x\to\infty} x\left(\left(1+\boxed{\frac{(a+b)}{x}+\frac{ab}{x^2}}\right)^{\frac{1}{2}}-1\right)$$

$$x\left(\cancel{1}+\left(\frac{a+b}{2x}\right)+\frac{ab}{2x^2}-\cancel{1}\right)$$

$$\lim_{x\to\infty}\frac{a+b}{2}+\cancel{\frac{ab}{2x}}=\frac{a+b}{2}$$

Q $\lim_{x\to\infty}\left\{\sqrt[3]{(x+a)(x+b)(x+c)}-x\right\}$

x^3 comm

$$\lim_{x\to\infty}\left\{\left(x^3+(a+b+c)x^2+\Sigma ab\,x+abc\right)^{\frac{1}{3}}-x\right\}$$

BT

yad

$$x\left\{\left(1+\boxed{\frac{a+b+c}{x}+\frac{\Sigma ab}{x^2}+\frac{abc}{x^3}}\right)^{\frac{1}{3}}-1\right\}$$

$$x\left\{\cancel{1}+\frac{a+b+c}{3x}+\frac{\Sigma ab}{3x^2}+\frac{abc}{3x^3}-\cancel{1}\right\}$$

$$\lim_{x\to\infty}\frac{a+b+c}{3}+\cancel{\frac{\Sigma ab}{3x}}+\cancel{\frac{abc}{3x^2}}$$

$$=\frac{a+b+c}{3}$$

Q $\lim_{x\to\infty}\sqrt[3]{(x+1)(x+2)(x+3)}-x$

$=2$

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very Major Role.

Q $\lim_{x\to a} \boxed{x^2}\left\{\sqrt{\frac{x+2}{x}}-\sqrt[3]{\frac{x+3}{x}}\right\}.$

<u>100% Case</u>

$(1+x)^n = 1+nx+\frac{(n)(n-1)}{2\cdot 1}\cdot x^2$

$\lim_{x\to\infty} x^2\left\{\left(1+\frac{2}{x}\right)^{\frac{1}{2}}-\left(1+\frac{3}{x}\right)^{\frac{1}{3}}\right\}$

$\lim_{x\to\infty} \cancel{x^2}\left\{\left(\cancel{1}+\cancel{\frac{1}{x}}+\frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{1\cdot 2}\cdot\left(\frac{2}{\cancel{x}}\right)^2\right)-\left(\cancel{1}+\cancel{\frac{1}{x}}+\frac{\left(\frac{1}{3}\right)\left(-\frac{2}{3}\right)}{1\cdot 2}\cdot\left(\frac{3}{\cancel{x}}\right)^2\right)\right\}$

$\left(-\frac{1}{8}\times 4\right)+\left(+\frac{1}{9}\times 9\right)=1+-\frac{1}{2}=\frac{1}{2}$

Q $\lim_{x\to 1} \frac{(3-\sqrt{8x+1})}{(5-\sqrt{24x+1})}$ $\frac{0}{0}$

Double Rat

$$\lim_{x\to 1} \frac{9-(8x+1)}{(3+\sqrt{8x+1})} \times \frac{(5+\sqrt{24x+1})}{25-(24x+1)}$$

$$\frac{5+\sqrt{24+1}}{3+\sqrt{8+1}} \times \lim_{x\to 1} \frac{8-8x}{24-24x}$$

$$\frac{10}{6} \times \lim_{x\to 1} \frac{\cancel{8}(\cancel{1-x})}{\underset{3}{\cancel{24}}(\cancel{1-x})}$$

$$= \frac{5}{9}$$

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$(\sqrt{3}, \sqrt{6})$

DPP2 $\cancel{Y = \sqrt{\frac{\pi}{2}+2} + \sqrt{1-\frac{\pi}{2}}}$

Range

5) $f(x) = \sqrt{\sin^{-1}x + 2} + \sqrt{1 - \sin^{-1}x}$ → -1 to 1

$Y = \sqrt{x-6} + \sqrt{5-x}$

$\sin^{-1}x + 2 \geq 0$ $\qquad$ $1 - \sin^{-1}x \geq 0$

$\boxed{\sin^{-1}x \geq -2}$ $\qquad$ $\sin^{-1}x \leq 1$

$x \cancel{\geq} \sin 2$ $\qquad$ $x \leq \sin 1$ → $\sin 60° = .85$

$\boxed{-\sin 120 = -.8}$

$x \in [-1, \sin 1]$ $\quad$ -1

$Y = \sqrt{-\frac{\pi}{2}+2} + \sqrt{1+\frac{\pi}{2}}$

$\sin 1$ $\quad$ $Y = \sqrt{3} + 0 = \sqrt{3}$ Min

$$\frac{dy}{dx} = \frac{1}{2\sqrt{\sin^{-1}x+2}} \times \frac{1}{\sqrt{1-x^2}} + \frac{1}{2\sqrt{1-\sin^{-1}x}} \times \frac{-1}{\sqrt{1-x^2}} = 0$$

$$\sqrt{\sin^{-1}x+2}\,\sqrt{1-x^2} = \sqrt{1-\sin^{-1}x}\,\sqrt{1-x^2}$$

$$(\sin^{-1}x+2)(1-x^2) = (1-\sin^{-1}x)(1-x^2)$$

$$(1-x^2)(\sin^{-1}x + 2 + \sin^{-1}x - 1) = 0$$

$x = 1, -1$ $\qquad$ $\sin^{-1}x = -\frac{1}{2}$

$x = \sin\left(-\frac{1}{2}\right) = -\sin\left(\frac{1}{2}\right)$

$Y = \sqrt{-\frac{1}{2}+2} + \sqrt{1+\frac{1}{2}}$

$= 2\sqrt{\frac{3}{2}} = \sqrt{6}$ M