

$$Q \int \frac{\sin x}{\sin 3x} \cdot dx$$

$$\int \frac{dx}{3-4\sin^2 x}$$

$\div 6 \sin x$

$$\int \frac{\sec^2 x \cdot dx}{3+3\sin^2 x - 4\sin^2 x}$$

$$\int \frac{\sec^2 x \cdot dx}{3-\sin^2 x}$$

$$(67) \int \frac{6^3 x \cdot dx}{(\sin x + 6x)}$$

$$(T3) \int \frac{(a+b\sin x)dx}{(b+a\sin x)^2}$$

$\div 6^2 x$

$$\int \frac{a \sec^2 x + \sec x \tan x \cdot dx}{(b \sec x + a \tan x)^2}$$

$$b \sec x + a \tan x = t$$

$$b \sec x (\tan x + a \sec^2 x) dx = w$$

$$\int \frac{at}{t^2} = -\frac{1}{t}$$

$$Q \int \frac{\sin^2 x \cdot dx}{1+\sin^2 x}$$

$$\int dx \int \frac{1}{1+\sin^2 x} \cdot dx$$

$\div 6^2 \sin$

$$\sin 2x \rightarrow (1+\sin 2x) - 1$$

511

68, 69

71, 72

73, 74

76, 75

$$\sqrt{7}, 20, 25, 31, 32$$

$$33, 35, 36, 37, 38, 39$$

$$52, 54, 55, 56, 57$$

$$x^{1/6} = t$$

$$58, 59, 61, 63, 67$$

$$2x+2=3 \tan$$

$$\underline{Q_{25}} \int \frac{\sin x}{\sin^4 x} dx$$

$$\int \frac{\sin x \cdot dx}{2 \sin^2 x \cdot \cos^2 x}$$

$$\int \frac{\sin x \cdot dx}{4 \sin^2 x \cdot \cos x \cdot \cos^2 x}$$

$$\frac{1}{4} \int \frac{dx}{\sin x \cdot \cos^2 x}$$

$$\frac{1}{4} \int \frac{\cos x \cdot dx}{\cos^2 x \cdot \sin^2 x}$$

$$\frac{1}{4} \int \frac{\cos x \cdot dx}{(1 - \sin^2 x)(1 - 2\sin^2 x)}$$

$$\sin x = t$$

$$\frac{1}{4} \int \frac{dt}{(t^2-1)(2t^2-1)}$$

$$\frac{1}{8} \int \frac{dt}{(t^2-1)(t^2-\frac{1}{2})}$$

$$\frac{1}{8} \times \frac{1}{2} \int \left( \frac{1}{t^2-1} - \frac{1}{t^2-\frac{1}{2}} \right)$$

$$\frac{1}{16} \int \left( \frac{1}{t^2-1} - \frac{1}{t^2-\frac{1}{2}} \right) \frac{dt}{(2t)^2-1^2}$$

$$\frac{1}{16} \times \frac{1}{2} \ln \left| \frac{t-1}{t+1} \right|$$

$$\frac{1}{4} \int 4 \sin^2 x \cos^2 x$$

$$\frac{1}{4} \int (\sin^2 x) \cdot dx$$

$$\frac{1}{4} \int \frac{1 - \cos 4x}{2} \cdot dx$$

$$(35) \int \frac{4 \sin \phi \cos \phi - \cos \phi}{6 - (1 - \sin^2 \phi) - 4 \sin \phi}$$

$$\int \frac{(4 \sin \phi - 1) \cos \phi d\phi}{5 + \sin^2 \phi - 4 \sin \phi}$$

$$\int \frac{1}{9} \text{ Solve } \sin \phi = t$$



$$36) \int \frac{dx}{(1-\sin^2 x)(1+\sin^2 x)}$$

$$\int \frac{\sec^2 x \cdot dx}{1+\sin^2 x} \rightarrow (T_3) \div \sec^2 x$$

$$\int \frac{\sec^2 x \cdot \sec^2 x \cdot dx}{1+2\tan^2 x}$$

$$\int \frac{(1+\tan^2 x)(\sec^2 x) \cdot dx}{(1+2\tan^2 x)}$$

$$\tan x = t$$

$$38) \int \frac{x(\sec x + 1)}{(x^2 + 2x(\sec x + 1))^{\frac{3}{2}}}$$

$$\int \frac{x(\sec x + 1) \cdot dx}{x^3 \left(1 + \frac{2\sec x}{x} + \frac{1}{x^2}\right)^{\frac{3}{2}}}$$

$$\int \frac{\frac{\sec x}{x^2} + \frac{1}{x^3} \cdot dx}{\left(1 + \frac{2\sec x}{x} + \frac{1}{x^2}\right)^{\frac{3}{2}}}$$

$$\rightarrow t$$

$$\sec^4 x + \sec^2 x = 1 - 2\sin^2 x \sec^2 x$$

$$\sec^6 x + \sec^4 x = 1 - 3\sin^2 x \sec^2 x$$

$$Q \int \frac{dx}{\sec^6 x + \sin^6 x}$$

$$\Rightarrow \int \frac{dx}{1 - 3\sin^2 x \sec^2 x}$$

$$\Rightarrow \int \frac{\sec^4 x \cdot dx}{(1 + \tan^2 x)^2 - 3\tan^2 x} \div \sec^4 x$$

$$\Rightarrow \int \frac{(1 + \tan^2 x) \sec^2 x \cdot dx}{\tan^4 x - \tan^2 x + 1}$$

$$\tan x = t$$

$$\Rightarrow \int \frac{(t^2 + 1) \cdot dt}{t^4 - t^2 + 1}$$

10 lines

$$\begin{aligned} & \int \sec^{4/3} x \cdot \sec^{2/3} x \cdot dx \\ & \Rightarrow \int \frac{dx}{\frac{\sin^{4/3} x \cdot \cos^{2/3} x}{\cos^{4/3} x}} \quad \text{Dir. Substn} \\ & \quad \cos x = t \\ & = \int \frac{\sec^2 x \cdot dx}{(\tan x)^{4/3}} = \int \frac{dt}{t^{4/3}} \end{aligned}$$

$$Q \int \frac{\sqrt{\cot x}}{\sin x \cdot \cos x} \cdot dx$$

$$\int \frac{\cancel{\sqrt{63x}}}{8m^{3/2}x(\cancel{63x}\sqrt{63x})} = \int \frac{dx}{\frac{8m^{3/2}}{(\cancel{63}^{3/2})}x(\cancel{63}^{1/2}x)} = \int \frac{dx}{8m^{3/2}x^2}$$

$$\Rightarrow \int \frac{\sec^2 x}{(\tan x)^{3/2}} \cdot \tan x = t \Rightarrow \int \frac{dt}{t^{3/2}}$$

$$\int \frac{dx}{\sqrt{8m^2x \cdot 65x}}$$

$$\int \frac{dx}{\sin^{3/2} x \cdot e^{5/2 x} \cos^{3/2} x}$$

$$= \int \frac{(1+t^2) \cdot \sec^2 x \, dx}{(\tan x)^{3/2}} \quad \tan x = t$$

$$\int \frac{dx}{4\sqrt{8m^3x \cdot 65x}}$$

$$\int \frac{dx}{\ln^{3/4} x \cdot e^{5/4} x \cdot e^{3/4} x}$$

$$\frac{1}{(e^{3/4})^2} \int \frac{e^{2x}}{(\ln x)^{3/2}} = \int \frac{dt}{t^{3/2}}$$

$$Q \int \sec^{25/13} x \cdot \sec^{27/13} x$$

$$\int \frac{dx}{e^{25/13}x \cdot \frac{\sin^{27/13}x \cdot e^{27/13}x}{e^{27/13}x}}$$

$$\int \frac{(1 + \tan^2 x) \cdot \sec^2 x}{(\tan x)^{27/13}} = \int \frac{(1 + t^2)}{t^{27/13}}$$

$$\int \frac{6^4 x \cdot dx}{\sin^3 x (\underbrace{\sin^5 x + 6^5 x}_{\text{com.}})^{3/5}}$$

$$\int \frac{\frac{u+x}{u^6 x}}{\frac{\sin^6 x \cdot (1 + (\tan^5 x)^{3/5})^{3/5}}{\tan^6 x \cdot (1 + \frac{1}{\tan^5 x})^{3/5}}} \cdot \sec^2 x \cdot dx \quad \left| \quad 1 + \frac{1}{\tan^5 x} = t \right.$$



$$Q \int \frac{dx}{(x-1)^{5/4}(x+2)^{3/4}} \quad 5/4 + 3/4 = 2$$

निर्धारण के लिए deg 2 का मान है

$$\int \frac{dx}{(x-1)^{5/4} \cdot (x-1)^{3/4} \cdot (x+2)^{3/4}}$$

$$\int \frac{dx}{(x-1)^2 \cdot \left(\frac{x+2}{x-1}\right)^{3/4}}$$

$$\frac{x+2}{x-1} = t \quad \left\{ \begin{array}{l} -\frac{1}{3} \int \frac{dt}{t^{3/4}} \\ -\frac{1}{8} \int t^{-3/4} dt \\ -\frac{1}{8} \cdot \frac{t^{1/4}}{1/4} + C \end{array} \right.$$

$$\int \frac{dx}{(x-p)\sqrt{(x-p)(x-q)}}$$

$$\int \frac{dx}{(x-p)^{3/2} \cdot (x-q)^{1/2}} \quad \frac{3}{2} + \frac{1}{2} = 2$$

$$\int \frac{dx}{(x-p)^{3/2} \cdot (x-p)^{1/2} \cdot \left(\frac{x-q}{x-p}\right)^{1/2}}$$

$$\int \frac{dx}{(x-p)^2 \cdot \left(\frac{x-q}{x-p}\right)^{1/2}} \quad \frac{x-q}{x-p} = 1$$

$$\int \frac{2 dx}{(2-x)^2} \cdot 3 \sqrt{\frac{2-x}{2+x}} \cdot dx$$

$$\int \frac{2 \cdot dt}{(2-x)^2 \cdot \left(\frac{2+x}{2-x}\right)^{1/3}}$$

$$\frac{2}{4} \int \frac{dt}{(t)^{1/3}}$$

$$\frac{2+x}{2-x} = t$$

$$\frac{(2-x) + (2+x) \cdot dx}{(2-x)^2} = dt$$

$$\frac{dx}{(2-x)^2} = \frac{dt}{4}$$



$$\int \frac{a \sin x + b \cos x}{c \sin x + d \cos x} \cdot dx = \int \frac{a e^x + b e^{-x}}{c e^x + d e^{-x}} \cdot dx$$

$$Q \int \frac{\sin x + \cos x}{3 \sin x + 2 \cos x} \cdot dx$$

Nr Replale

$$\sin x + \cos x = \lambda (3 \sin x + 2 \cos x) + \mu (3 \cos x - 2 \sin x)$$

$$\sin x \quad \cos x$$

$$1 = 3\lambda - 2\mu \quad 6\lambda - 4\mu = 2$$

$$1 = 2\lambda + 3\mu \quad 6\lambda + 9\mu = 3$$

$$\mu = \frac{1}{13} \quad \lambda = \frac{5}{13}$$

$$\int \frac{\frac{5}{13} (3 \sin x + 2 \cos x) + \frac{1}{13} (3 \cos x - 2 \sin x)}{3 \sin x + 2 \cos x} dx$$

$$\frac{5}{13} \cdot x + \frac{1}{13} \ln |3 \sin x + 2 \cos x| + C$$

$$Q \int \frac{3 \cos x + 4}{\sin x + 2 \cos x + 3} \cdot dx$$

$$3 \cos x + 4 = \lambda (\sin x + 2 \cos x + 3) + \mu (\cos x - 2 \sin x) + K$$

$$\begin{array}{l} \sin x \\ \cos x \end{array} \quad \begin{array}{l} 0 = \lambda - 2\mu \\ 3 = 2\lambda + \mu \end{array}$$

$$4 = 3\lambda + K$$

$$\mu = \frac{3}{5}, \quad \lambda = \frac{6}{5}$$

$$K + \frac{18}{5} = \frac{20}{5} \Rightarrow K = \frac{2}{5}$$

$$= \int \frac{\frac{6}{5} (\sin x + 2 \cos x + 3) + \frac{3}{5} (\cos x - 2 \sin x) + \frac{2}{5}}{\sin x + 2 \cos x + 3} dx$$

$$\frac{6}{5} x + \frac{3}{5} \ln |\sin x + 2 \cos x + 3| + \frac{2}{5} \int \frac{dx}{\sin x + 2 \cos x + 3}$$

$$= \frac{6}{5} x + \frac{3}{5} \ln |\sin x + 2 \cos x + 3| + \frac{2}{5} \int \frac{\sec^2 \frac{x}{2} \cdot dx}{2 \tan^2 \frac{x}{2} + 1 - \tan^2 \frac{x}{2} + 5}$$

$\tan \frac{x}{2} = t$

$$Q \int \frac{1 + \cos x \cdot \cos x}{\cos x + \cos x} \cdot dx$$

$$1 + \cos x \cdot \cos x = \lambda (\cos x + \cos x) + \mu (-\sin x) + K$$

$$\int \frac{\text{Linear}}{\text{Linear}}, \int \frac{\text{Linear}}{\sqrt{\text{Linear}}}$$


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$$Q \int \frac{3x+4 \cdot dx}{4x+3}$$

$$\frac{3}{4} \int \frac{4x+3}{4x+3} + \left(4 - \frac{9}{4}\right) \int \frac{dx}{4x+3}$$

$$\frac{3x}{4} + \frac{7}{4} \cdot \frac{\ln|4x+3|}{4} + C$$

$$Q \int \frac{5x+7}{\sqrt{7x+9}} \cdot dx$$

$$\frac{5}{7} \int \frac{7x+9}{\sqrt{7x+9}} + \left(7 - \frac{45}{7}\right) \int \frac{dx}{\sqrt{7x+9}}$$

$$\frac{5}{7} \cdot \frac{2}{3} \cdot \frac{(7x+9)^{3/2}}{7} + \frac{4}{7} \times \frac{2\sqrt{7x+9}}{7} + C$$

$$\int \frac{\text{Linear}}{Q \text{ quad}} \quad \int \frac{\text{Linear}}{\sqrt{Q \text{ quad}}}$$

$$Q \int \frac{x+1 \cdot dx}{x^2+x+1}$$

$$\frac{1}{2} \int \frac{(2x+1) \overset{dx}{\rightarrow}}{x^2+x+1} + \left(1 - \frac{1}{2}\right) \int \frac{dx}{x^2+x+1}$$

$$\frac{1}{2} \ln|x^2+x+1| + \frac{1}{2} \int \frac{dx}{\left(x+\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2 + 1}$$

$$+ \frac{1}{2} \int \frac{dx}{\left(x+\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$$

$$+ \frac{1}{2} \times \frac{1}{\frac{\sqrt{3}}{2}} \tan^{-1} \left( \frac{x+\frac{1}{2}}{\frac{\sqrt{3}}{2}} \right) + C$$