HOMEWORK from PRILEPKO

1. Find the first term a_1 and the common difference d of the arithmetic progression in which
$\begin{cases} a_2 + a_5 - a_3 = 10, \\ a_2 + a_9 = 17. \end{cases}$

2. The sum of the first n terms of the sequence $\{a_n\}$ is defined by the formula $S_n = 2n^2 + 3n$. Prove that it is an arithmetic progression.

3. The sum S_n of the terms of an arithmetic progression is defined by the formula $S_n = 4n^2 - 3n$ for any n. Write the first three terms of the progression.

4. Find the sum of 20 terms of an arithmetic progression if its first term is 2 and the seventh term is 20 .

5. Find the first term and the difference of an arithmetic progression if the sum of its first five even terms is equal to 15 and the sum of the first three terms is equal to (-3).

6. The sum of the first and the fifth term of an arithmetic progression is 26 and the product of the second by the fourth term is 160 . Find the sum of the first six terms of the progression.

7. The sum of the third and the ninth term of an arithmetic progression is 8 . Find the sum of the first 11 terms of the progression.

8. The sum of the squares of the fifth and the eleventh term of an arithmetic progression is 3 and the product of the second by the fourteenth term is equal to k. Find the product of the first by the fifteenth term of the progression.

9. The sum of the second and the fifth term of an arithmetic progression is 8 and that of the third and the seventh term is 14. Find the progression.

10. How many terms of an arithmetic progression must be taken for their sum to be equal to 91 , if its third term is 9 and the difference between the seventh and the second term is 20?

11. Put five terms between the numbers 1 and 1.3 so that together with the given terms they will form an arithmetic progression.

12. Find four numbers between the numbers 4 and 40 such that an arithmetic progression results.

13. Find the sum of all three-digit natural numbers which leave a remainder 2 when they are divided by 3 .

14. Solve the equation $5^2 \cdot 5^4 \cdot 5^6 \ldots 5^{2x} = (0.04)^{-28}$.

15. Find an increasing arithmetic progression in which the sum of the first three terms is 27 and the sum of their squares is 275.

16. The sum of four numbers which form an arithmetic progression is 1 and the sum of the squares of those numbers is 0.3. Find the numbers.

17. An arithmetic progression consists of 12 terms whose sum is 354. The ratio of the sum of the even terms to the sum of the odd terms is 32: 27. Find the common difference of the progression.

18. The product of the third by the sixth term of an arithmetic progression is 406 . The division of the ninth term of the progression by the fourth term gives a quotient 2 and a remainder 6 . Find the first term and the difference of the progression.

19. The first term of an arithmetic progression $a_1, a_2, a_3, \ldots$ is equal to unity. At what value of the difference of the progression is $a_1a_3 + a_2a_3$ at a minimum?

20. In an arithmetic progression, $a_7 = 9$. At what value of its difference is the product $a_1a_2a_7$ the least?

21. Two arithmetic progressions contain the same number of terms. The ratio of the last term of the first progression to the first term of the second is equal to the ratio of the last term of the second progression to the first term of the first progression and is equal to 4. The ratio of the sum of the first progression to that of the second is 2 . Find the ratio of the differences of the progressions.

22. All terms of an arithmetic progression are natural numbers. The sum of its nine successive terms, beginning with the first, is larger than 200 and smaller than 220 . Find the progression if its second term is 12 .

23. Each of the two triplets of numbers loga , logb , logc and loga − log2b , log2b − log3c , log3c − loga is an arithmetic progression. Can the numbers a, b, c be the lengths of the sides of a triangle? If they can, then what triangle is it? Find the angles of the triangle provided that it exists.

24. The fourth term of an arithmetic progression is 4. At what value of the difference of the progression is the sum of the pairwise products of the first three terms of the progression the least?

25. The sixth term of an arithmetic progression is 3 and the difference exceeds 0.5 . At what value of the difference of the progression is the product of the first, the fourth, and the fifth term of the progression the greatest?

26. Certain numbers appear in both arithmetic progressions 17,21, … and 16,21, …. Find the sum of the first hundred numbers appearing in both progressions.

27. Three numbers form an arithmetic progression. The sum of the numbers is equal to 3 and the sum of their cubes is equal to 4 . Find the numbers.

28. Prove that the numbers $\frac{1}{\log_3 2}, \frac{1}{\log_6 2}$, and $\frac{1}{\log_{12} 2}$ form an arithmetic progression.

29. At what values of the parameter a are there values of x such that the numbers $5^{1+x} + 5^{1-x}, \frac{a}{2}, 25^x + 25^{-x}$ form an arithmetic progression?

ANSWER KEY

1. $a_1 = 13, d = -1$ **3.** 1,9,17 **4.** 610 **5.** $a_1 = -2, d = 1$

6. 69, 87 **7.** 44 **8.** $(116k - 39)/90$

9. $a_1 = -1, d = 2$ **10.** 7 **11.** 1.05 ,1.1 ,1.15 ,1.2 ,1.25

12. 11.2 ,18.4 ,25.6 , 32.8 **13.** 164850

14. 7 **15.** $a_1 = 5$, $d = 4$ **16.** 0.1 ,0.2 ,0.3 ,0.4

17. 5 **18.** $a_1 = 4$, $d = 5$ or $a_1 = -79/7$, $d = -37/14$

19. $d = -5/4$ **20.** $d = 33/20$ **21.** 26 **22.** $a_1 = 8$, $d = 4$

23. Yes they can ; obtuse angled triangle ; $\angle A = \cos^{-1}(-29/48) = \pi - \text{arccos}(29/48)$, $\angle B = \cos^{-1}(61/72), \angle C = \cos^{-1}(101/108)$

24. $d = 24/11$. **25.** $d = 12/5$ **26.** 101100

27. $(6 - \sqrt{6})/6$,1 , $(6 + \sqrt{6})/6$ **29.** $a \in [12, +\infty)$