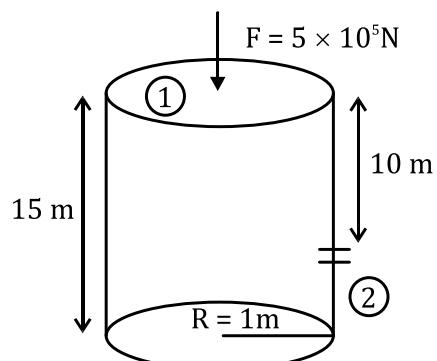


## DPP 2

## SOLUTION

1. Say there is no atmospheric pressure on piston,  $P_1 = \frac{F}{\pi R^2}$  and  $P_2 = P_0$ .



Bernoulli's equation at point 1 and 2,

$$p_1 + \rho gh_1 + \frac{1}{2} \rho v_1^2 = p_2 + \rho gh_2 + \frac{1}{2} \rho v_2^2$$

$$\Rightarrow \frac{F}{\pi R^2} + \rho gh_1 - h_2 = P_0 + \frac{1}{2} \rho v_2^2$$

$$\Rightarrow v_2 = \sqrt{\frac{2F}{\pi R^2 \rho} - \frac{2P_0}{\rho} + 2gh_1 - h_2}$$

$$v_2 = \sqrt{\frac{2 \times 5 \times 10^5}{3.14 \times 1^2 \times 1000} - \frac{2 \times 1.01 \times 10^5}{1000} + 2 \times 10 \times 10}$$

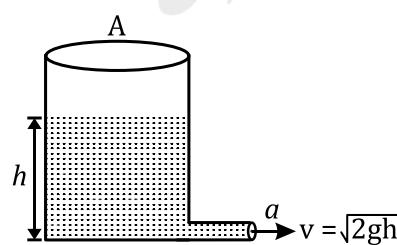
$$\approx 17.8 \text{ m s}^{-1}$$

2. Say the coefficient of friction is  $\mu$ .

$$\text{Mass of fluid} = m = (Ah)\rho$$

$$\text{The velocity of efflux, } v = \sqrt{2gh}$$

where  $h$  is the height of liquid in vessel.



Force due to emerging liquid

$$f = \rho av^2$$

$$\text{So, for preventing from sliding } \mu mg \geq \rho a^2$$

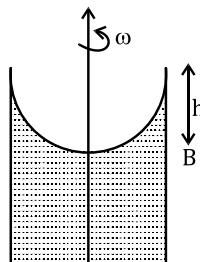
$$\therefore \mu \times Ah\rho g \geq \rho av^2$$

$$\text{or } \mu Ahg \geq a \times 2gh$$



$$\therefore \mu \geq \frac{2a}{A}$$

3. By Bernoulli's theorem



$$P_A + \frac{1}{2} \rho v_A^2 + \rho g h_A = P_B + \frac{1}{2} \rho v_B^2 + \rho g h_B$$

$$\text{As } h_A = h_B \therefore P_A + \frac{1}{2} \rho v_A^2 = P_B + \frac{1}{2} \rho v_B^2$$

$$\Rightarrow P_A - P_B = \frac{1}{2} \rho (v_B^2 - v_A^2)$$

Now,  $v_A = 0$ ,  $v_B = r\omega$  and  $P_A - P_B = h\rho g$

$$\therefore h\rho g = \frac{1}{2} \rho (r^2 \omega^2 - 0)$$

$$\text{or } h = \frac{25\omega^2}{2g} \text{ cm [Given, } \omega r = 5 \text{ cm ]}$$

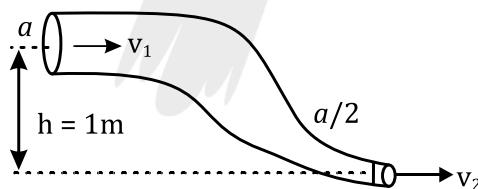
4. The momentum per second carried by liquid per second is  $\rho v^2 A$ .  $A$  is area of cross section of pipe

The force exerted due to reflected back molecules is  $2 \left( \frac{1}{4} \rho a v^2 \right)$ .

So, the resultant pressure is

$$\frac{\left( \frac{1}{2} \rho A v^2 + \frac{1}{4} \rho A v^2 \right)}{A} = \frac{3}{4} \rho v^2$$

5. Density,  $\rho = 500 \text{ kg/m}^3$



$$A_1 = a, A_2 = a/2; P_1 - P_2 = 4100 \text{ Pa}$$

$$v_1 = \frac{\sqrt{x}}{6}$$

By Bernoulli's theorem,

$$P_1 + \frac{1}{2} \rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2} \rho v_2^2 + \rho g h_2$$

$$P_1 - P_2 = \frac{1}{2} \rho v_2^2 - \frac{1}{2} \rho v_1^2 - \rho g h_1$$

By equation of continuity

$$A_1 v_1 = A_2 v_2$$

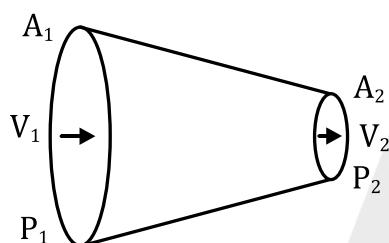
$$a \cdot \frac{\sqrt{x}}{6} = \frac{a}{2} \times v_2; v_2 = \frac{\sqrt{x}}{3}$$

$$4100 = \frac{1}{2} \times 800 \left( \frac{x}{9} - \frac{x}{36} \right) - 800 \times 10 \times 1$$

$$4100 = 400 \left( \frac{3x}{36} \right) - 8000$$

$$\Rightarrow \frac{(4100 + 8000) \times 36}{400 \times 3} = x; x = 363$$

6.  $A_2 = \frac{A_1}{2}$



$$P_1 - P_2 = 4500 \text{ Pa}$$

$$P_1 + \frac{1}{2} \rho V_1^2 + \rho gh = P_2 + \frac{1}{2} \rho V_2^2 + \rho gh$$

$$P_1 - P_2 = \frac{1}{2} \rho (V_2^2 - V_1^2)$$

And  $A_1 V_1 = A_2 V_2$

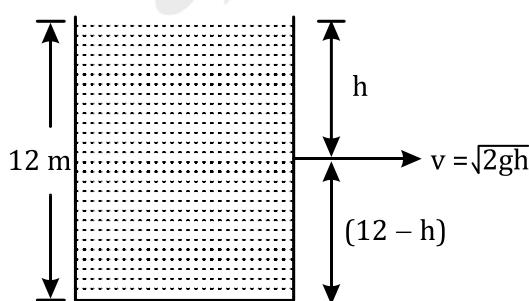
$$\Rightarrow V_2 = 2 V_1$$

$$4500 = \frac{1}{2} \times 750 \times 3 V_1^2$$

$$V_1 = 2 \text{ m/s}$$

$$\text{Volume flow rate} = A_1 V_1 = 24 \times 10^{-3} \text{ m}^3 \text{ s}^{-1}$$

7. The velocity of efflux is,  $v = \sqrt{2gh}$



Let it takes time t sec in covering a horizontal range R.

$$\text{Then, } 12 - h = \frac{1}{2} g t^2 \Rightarrow t = \sqrt{\frac{2(12-h)}{g}}$$



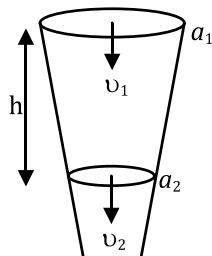
$$\text{Range, } R = vt = \sqrt{2gh} \sqrt{\frac{2(12-h)}{g}}$$

$$\Rightarrow R = \sqrt{4h(12-h)}$$

$$\text{For maximum range } \frac{dR}{dh} = 0$$

$$\Rightarrow h = 6 \text{ m.}$$

8. Here,



$$d_1 = 8 \times 10^{-3} \text{ m}$$

$$v_1 = 0.4 \text{ m s}^{-1}, h = 0.2 \text{ m}$$

$$sv_2 = \sqrt{v_1^2 + 2gh} = \sqrt{(0.4)^2 + 2 \times 10 \times 0.2}$$

$$\approx 2 \text{ m s}^{-1}$$

$\therefore$  As per equation of continuity

$$a_1 v_1 = a_2 v_2$$

$$\pi \times \left(\frac{8 \times 10^{-3}}{2}\right)^2 \times 0.4 = \pi \times \left(\frac{d_2}{2}\right)^2 \times 2$$

$$d_2 = 3.6 \times 10^{-3} \text{ m}$$

9. Let  $v$  = velocity of efflux

$$\text{then } v^2 = 2gh 1 - \left(\frac{a}{2}\right)^2$$

$$h = 3.0 - 0.525 = 2.475 \text{ m}$$

$$v^2 = \frac{2 \times 10 \times 2.475}{1 - (0.1)^2} = \frac{49.50}{0.99}$$

$$v^2 = 50 \text{ m}^2 \text{ s}^{-2}$$

10.  $v = \sqrt{2gh} = \sqrt{2 \times 10 \times 20} = 20 \text{ m/s}$