Nishant Jindal

Q $\lim_{x \to 0} \dfrac{a - \sqrt{a^2 - x^2} - \frac{x^2}{4}}{x^4}$ $a > 0$, $L$ $\boxed{\text{is finite}}$

A) $a = 2$ B) $a = 1$ C) $L = \frac{1}{64}$ (D) $\frac{1}{32}$

$(1+x)^n = 1 + nx + \frac{(n)(n-1)}{1 \cdot 2} \cdot x^2$

$\left(1 - \frac{x^2}{a^2}\right)^{1/2} = \underbrace{1 + \left(\frac{1}{2}\right)\left(-\frac{x^2}{a^2}\right)} + \frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{1 \cdot 2} \times \left(-\frac{x^2}{a^2}\right)^2$

BT

$\lim_{x \to 0} \dfrac{a - a\left(1 - \frac{x^2}{a^2}\right)^{1/2} - \frac{x^2}{4}}{x^4}$ $\to 1$

$\lim_{x \to 0} \dfrac{a - a\left(1 - \frac{x^2}{2a^2}\right) - \frac{x^2}{4}}{x^4}$

$\lim_{x \to 0} \dfrac{a - a + \frac{x^2}{2a} - \frac{x^2}{4}}{x^4} = \lim_{x \to 0} \dfrac{x^2\left(\frac{1}{2a} - \frac{1}{4}\right) + \frac{1}{8} \times \frac{x^4}{a^3} - \frac{x^2}{4}}{x^4}$

$a = 2$

$\frac{1}{8 \times 2^3} = \frac{1}{64}$

# LIMIT

Q $f(1)=3,\ f'(1)=6$

$$\lim_{x\to 0}\left[\frac{f(1+x)}{f(1)}\right]^{\frac{1}{x}}=?\qquad \left(\frac{f(1)}{f(1)}\right)^{\infty}$$

$f(x)\to f'(x)$

$f(1+x)\to f'(1+x)\times 1$

$$e^{\lim_{x\to 0}\frac{1}{x}\left[\frac{f(1+x)}{f(1)}-1\right]}$$

$$e^{\frac{1}{f(1)}\lim_{x\to 0}\left[\frac{f(1+x)-f(1)}{x}\right]}\ \frac{0}{0}\ \text{DL}$$

$$e^{\frac{1}{f(1)}\lim_{x\to 0}\frac{f'(1+x)-0}{1}}=e^{\frac{f'(1)}{f(1)}}=e^{\frac{6}{3}}=e^{2}$$

Q Let $\alpha(a)$ & $\beta(a)$ be the Roots of

Eqn $\left(\sqrt[3]{1+a}-1\right)x^2+\left(\sqrt{1+a}-1\right)x+\left(\sqrt[6]{1+a}-1\right)=0$

Where $a>-1$ Then $\lim_{\alpha\to 0^+}\alpha(a)$ & $\lim_{\alpha\to 0^-}\beta(a)$ are?

BT

$$\left((1+a)^{\frac{1}{3}}-1\right)x^2-\left((1+a)^{\frac{1}{2}}-1\right)x+\left((1+a)^{\frac{1}{6}}-1\right)=0$$

$$\left(\cancel{1}+\frac{a}{3}-\cancel{1}\right)x^2-\left(\cancel{1}+\frac{a}{2}-\cancel{1}\right)x+\left(\cancel{1}+\frac{a}{6}-\cancel{1}\right)=0$$

$$\frac{ax^2}{3}-\frac{ax}{2}+\frac{a}{6}=0$$

$$\frac{x^2}{3}-\frac{x}{2}+\frac{1}{6}=0\Rightarrow 2x^2-3x+1=0$$

$$2x^2-2x-x+1=0\Rightarrow 2x(x-1)-1(x-1)=0$$

$$x=\underbrace{\boxed{\frac{1}{2}}}_{\alpha}\ \&\ \underbrace{\boxed{1}}_{\beta}$$

Nishant Jindal

$\log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3}$

$\log(1-x) = -x - \frac{(-x)^2}{2} + \frac{(-x)^3}{3}$

$m^n = e^{n \log m}$

Q $\lim_{x\to 0} \left[1 + x \cdot \log(1+b^2)\right]^{\frac{1}{x}} = 2b\sin^2\theta ;\ b>0$

Ans

$\theta \in (-\pi, \pi]$ then value of $\theta$ = ? $\rightarrow \frac{\pi}{2}$ or $-\frac{\pi}{2}$

$1^\infty$

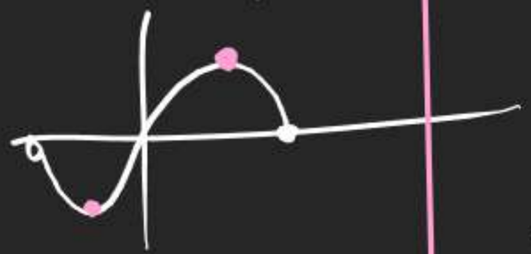
$e^{\lim_{x\to 0} \frac{1}{x}\left[x + x\log(1+b^2) - x\right]}$

$e^{\log_e(1+b^2)} = 2b\sin^2\theta$

$2\sin^2\theta = 2$

$\sin^2\theta = 1$

$\sin\theta = 1, -1$



$a^{\log_a x} = x$

$1+b^2 = 2b\sin^2\theta$

$2\sin^2\theta \le 2$

$\frac{b^2+1}{b} = 2\sin^2\theta \Rightarrow b + \frac{1}{b} = 2\boxed{\sin^2\theta}$

$\ge 2 \quad \boxed{2} \quad \le 2$

Q let e denotes the base of Natural log The value of real No $\boxed{a}$ for which the RHL $\lim_{x\to 0^+} \frac{(1-x)^{\frac{1}{x}} - e^{-1}}{x^a}$ is equal to a Non Zero No, is?

$\lim_{x\to 0^+} \frac{e^{\frac{1}{x}\log(1-x)} - e^{-1}}{x^a}$

$\lim_{x\to 0^+} \frac{e^{\frac{1}{x}\left(-x - \frac{x^2}{2} - \frac{x^3}{3}\right)} - e^{-1}}{x^a}$

$\lim_{x\to 0^+} \frac{e^{-1-\frac{x}{2}-\frac{x^2}{3}} - e^{-1}}{x^a}$

Nishant Jindal

$\log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3}$

$\log(1-x) = -x - \frac{(-x)^2}{2} + \frac{(-x)^3}{3}$

$m^n = e^{n\log m}$

$(0)^{a-1} \to 0^2 = 0$

$\to 0^0$ ⊗

$\to 0^{-2} = \frac{1}{0^2} = \frac{1}{0} \to \infty$

$$\lim_{x\to 0^+} \frac{e^{-1}\left\{e^{-\frac{x}{2}-\frac{x^2}{3}} - 1\right\}}{x^a\left(-\frac{x}{2}-\frac{x^2}{3}\right)} \times \left(-\frac{x}{2}-\frac{x^2}{3}\right)$$

$$e^{-1}\lim_{x\to 0^+} \frac{-\frac{x}{2}-\frac{x^2}{3}}{x^a}$$

$$e^{-1}\lim_{x\to 0^+} \frac{x\left(-\frac{1}{2}-\frac{x}{3}\right)}{x^a}$$

$a = 1$

$$e^{-1}\lim_{x\to 0^+} \frac{\cancel{x}\left(-\frac{1}{2}-\frac{x}{3}\right)}{\cancel{x}} = e^{-1}\cdot\frac{-1}{2} = \frac{-1}{2e}$$

Q let e denotes the base of Natural log. The value of real No $\boxed{a}$ for which the RHL $\lim_{x\to 0^+} \frac{(1-x)^{\frac{1}{x}} - e^{-1}}{x^a}$ is equal to a Non zero No, is?

$$\lim_{x\to 0^+} \frac{e^{\frac{1}{x}\log(1-x)} - e^{-1}}{x^a}$$

$$\lim_{x\to 0^+} \frac{e^{\frac{1}{x}\left(-x-\frac{x^2}{2}-\frac{x^3}{3}\right)} - e^{-1}}{x^a}$$

$$\lim_{x\to 0^+} \frac{e^{-1-\frac{x}{2}-\frac{x^2}{3}} - e^{-1}}{x^a}$$

# LIMIT

Q. Let $f(x)=\frac{\sin\{x\}}{x^2+ax+b}$. If $f(5^+)$ and $f(3^+)$ exists finitely and are not zero, then the value of $(a+b)$ is (where $\{.\}$ represents fractional part function) –

(A) 7 (B) 10 (C) 11 (D) 20

Q. $\lim_{x\to 0}\frac{|\cos(\sin(3x))|-1}{x^2}$ equals

$0^+ \to 1^{st}$ $0^- \to 4^{th}$

$\lim_{x\to 0}\frac{\cos(\sin 3x)-1}{x^2} = -\left(\frac{1-\cos(3x)}{x^2}\right) = -\frac{9}{2}$

(A) $\frac{-9}{2}$ (B) $\frac{-3}{2}$ (C) $\frac{3}{2}$ (D) $\frac{9}{2}$

Q. Let $a=\min\{x^2+2x+3, x\in R\}$ and $b=\lim_{\theta\to 0}\frac{1-\cos\theta}{\theta^2}$. then value of $\sum_{r=0}^{n} a^r\cdot b^{n-r}$ is :

(A) $\frac{2^{n+1}-1}{3.2^n}$ (B) $\frac{2^{n+1}+1}{3.2^n}$ (C) $\frac{4^{n+1}-1}{3.2^n}$ (D) none of these

# LIMIT

$$\lim_{\theta \to 0} \frac{\frac{1}{2}(1-\cos\theta)}{\frac{1}{64}\theta^2} = \frac{\frac{1}{4}}{\frac{1}{64}} = 16$$

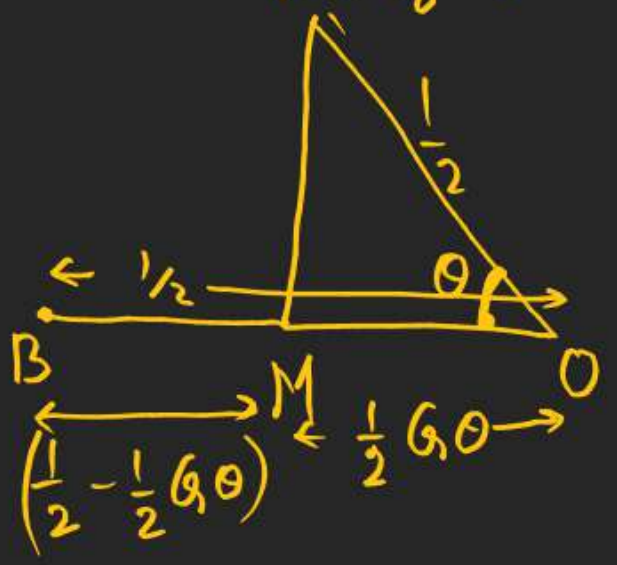
**Q.** Let $\mathrm{BC}$ is diameter of a circle centred at $\mathrm{O}$. Point $\mathrm{A}$ is a variable point, moving on the circumference of circle. if $\mathrm{BC} = 1$ unit, then $\lim_{A \to B} \frac{\mathrm{BM}}{(\text{Area of sector } \mathrm{OAB})^2}$ is equal to -

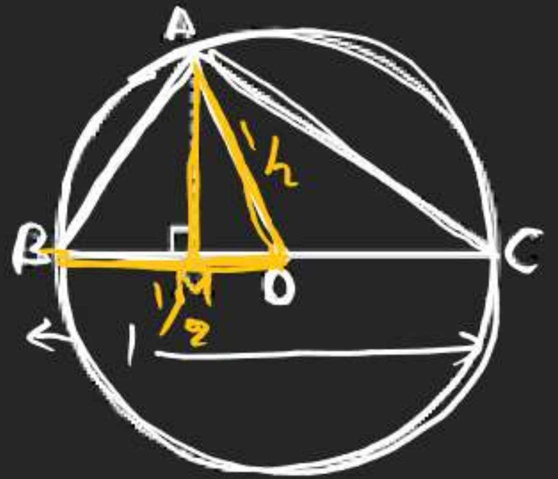
$\mathrm{BM} = \frac{1}{2}(1-\cos\theta)$

(A) 1 (B) 2 (C) 4 (D) 16

$\frac{1}{2}r^2\theta = \frac{1}{2}\left(\frac{1}{2}\right)^2\theta$

$A\left(\frac{1}{8}\theta\right)^2$





**Q.** $\lim_{x \to \infty}\left(\frac{x^2-2x+1}{x^2-4x+2}\right)^x$ is equal to

$\to 1^\infty$

(A) 1 (B) e (C) $\frac{1}{e^2}$ (D) $e^2$

$$e^{x\left(\frac{x^2-2x+1}{x^2-4x+2}-1\right)}$$

# LIMIT

**Q.** $\lim_{x\to 0}(1+\sin x)^{\cos x}$ is equal to

$(1+0)^1$

(A) 0 (B) e (C) 1 (D) $\frac{1}{e}$

**Q.** $\lim_{x\to 0}(\cos x + a\sin bx)^{1/x}$ is equal to :

$(1+0)^\infty$

(A) $e^a$ (B) $e^{ab}$ (C) $e^b$ (D) $e^{a/b}$

$$e^{\frac{1}{x}[\cos x + a\sin bx - 1]} = e^{\lim_{x\to 0} -\frac{(1-\cos x)}{x^2}\cdot x + a\frac{\sin bx}{x}} = e^{-0\times\frac{1}{2} + \frac{a\cdot bx}{x}} = e^{ab}$$

**Q.** $\lim_{x\to 0}\left(\tan\left(\frac{\pi}{4}+x\right)\right)^{1/x}$ is equal to

[D]

(A) $e^{-2}$ (B) $\frac{1}{e}$ (C) e (D) $e^2$

# LIMIT

Q. $\lim_{n\to\infty}(4^n+5^n)^{1/n}$ is equal to

DY

(A) 5 (B) 4 (C) 0( D) D.N.E.

$\to 5$

Q. $\lim_{x\to\infty}\left(\frac{1^{1/x}+2^{1/x}+3^{1/x}+\dots+n^{1/x}}{n}\right)^{nx}$ is equal to

$\left(\lim_{n\to\infty}\left(\frac{1^{\frac{1}{x}}+2^{\frac{1}{x}}+3^{\frac{1}{x}}+\dots+n^{\frac{1}{x}}}{n}\right)^{x}\right)^{n} = \left(\left(Ln\right)^{\frac{1}{x}}\right)^{x}$

(A) $n!$ (B) 1 (C) $\frac{1}{n!}$ (D) 0

Q. If $\lim_{x\to\lambda}\left(2-\frac{\lambda}{x}\right)^{\lambda\tan\left(\frac{\pi x}{2\lambda}\right)}=\frac{1}{e}$, then $\lambda$ is equal to - Copy

(A) $-\pi$ (B) $\pi$ (C) $\frac{\pi}{2}$ (D) $-\frac{2}{\pi}$

# LIMIT

$e^{\lim_{x\to0}\frac{1}{x}\left[x+\frac{f(x)+x^2}{x^2}-x\right]} = e^{\lim_{x\to0}\frac{f(x)+x^2}{\boxed{x^3}}} = e^{\lim_{x\to0}\frac{ax^3+bx^2+cx+d+x^2}{x^3}}$

$c=0,\ d=0$

$= e^{\lim_{x\to0}\frac{a\cancel{x^3}}{\cancel{x^3}}+\frac{(b+1)x^2}{x^3}+\frac{cx}{x^3}+\frac{d}{x^3}} = e^2$

$b+1=0$

$b=-1,\ a=2$

**Q.** If $\lim_{x\to0}\left(1+ax+bx^2\right)^{2/x}=e^3$, then

(A) $a=\frac{3}{2}$ and $b\in R$ (B) $a=\frac{3}{2}$ and $b\in R^+$

(C) $a=0$ and $b=1$ (D) $a=1$ and $b=0$

Adv

**Q.** If $f(x)$ is a polynomial of least degree, such that $\lim_{x\to0}\left(1+\boxed{\frac{f(x)+x^2}{x^2}}\right)^{1/x}=e^2$, then $f(2)$ is -

1) Achha hota limit put karne par $1^\infty$ Bnta

(2) $1^\infty$ can be proceeded only when $\lim_{x\to0}\frac{f(x)+x^2}{x^2}$ can become 0.

(A) 2 (B) 8 (C) 10 (D) 12

$\lim_{x\to0}\left(1+x^2\right)^{\frac{1}{\frac{\tan x}{x}-1}} \to 1^\infty$

$f(x)=2x^3-x^2+0\cdot x+0 \Rightarrow f(2)=2\cdot2^3-2^2$
$=12$

**Q.** Let $f(x)=\frac{\tan x}{x}$, then the value of $\lim_{x\to0}\left([f(x)]+x^2\right)^{\frac{1}{\{f(x)\}}}$ is equal to (where $[\,],\{.\}$ denotes greatest integer function and fractional part function respectively)-

$\left[\frac{\tan x}{x}\right]=[>1]=$

$\lim_{x\to0}\left(1+x^2\right)^{\frac{1}{\frac{\tan x}{x}-\left[\frac{\tan x}{x}\right]}} = \lim_{x\to0}\left(1+x^2\right)^{\frac{1}{\frac{\tan x}{x}-1}}$

(A) $e^{-3}$ (B) $e^3$ (C) $e^2$ (D) non-existent

# LIMIT

$$\lim_{n\to\infty}\frac{e^n}{e^{n-1/2}}=\frac{\cancel{e^n}}{\cancel{e^n}\cdot e^{-1/2}}=\frac{1}{\frac{1}{\sqrt{e}}}=\sqrt{e}$$

$(1-x)^{\frac{1}{x}}$

$\ln(1+x)=\left(x-\frac{x^2}{2}+\frac{x^3}{3}\right)$

**Q.** $\lim_{n\to\infty}\frac{e^n}{\left(1+\frac{1}{n}\right)^{n^2}}$ **equals -**

good

$$e^{n^2\ln\left(1+\frac{1}{n}\right)}=e^{n^2\left(\frac{1}{n}-\frac{\left(\frac{1}{n}\right)^2}{2}\right)}=e^{n-\frac{1}{2}}$$

$$\boxed{a^x=e^{x\cdot\ln a}}$$

**(A)** 1 **(B)** $\frac{1}{2}$ **(C)** e **(D)** $\sqrt{e}$

**Q.** **If $f(x)$ is odd linear polynomial with $f(1)=1$, then** $\lim_{x\to0}\frac{2^{f(\tan x)}-2^{f(\sin x)}}{x^2f(\sin x)}$ **is :**

$f(x)=x$

$f(\tan x)=\tan x,\ f(\sin x)=\sin x$

**(A)** 1 **(B)** $\ell n2$ **(C)** $\frac{1}{2}\ell n2$ **(D)** $\cos 2$

$$\lim_{x\to0}\frac{2^{\tan x}-2^{\sin x}}{x^2\cdot\sin x}=\lim_{x\to0}\frac{2^{\sin x}\left(2^{\tan x-\sin x}-1\right)}{x^3}$$

$$\lim_{x\to0}\frac{2^{\sin x}\left(2^{\tan x-\sin x}-1\right)}{(\tan x-\sin x)}\times\left(\frac{\tan x-\sin x}{x^3}\right)$$

$2^{\sin 0}\cdot\ln 2\times$

**Q.** $\lim_{x\to0}\frac{x(1+a\cos x)-b\sin x}{x^3}=1$ **then**

**(A)** $a=-5/2$ **(B)** $a=-3/2,\ b=-1/2$

**(C)** $a=-3/2,\ b=-5/2$ **(D)** $a=-5/2,\ b=-3/2$

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# LIMIT

**Q.** $\lim_{h\to 0} \frac{\sin(a+3h)-3\sin(a+2h)+3\sin(a+h)-\sin a}{h^3}$ **is equal to** → 3 बार DL

(A) $\cos a$  (B) $-\cos a$  (C) $\sin a$  (D) $\sin a \cos a$

→ Rationalisation

**Q.** $\lim_{x\to\frac{\pi}{2}} \tan^2 x\left(\sqrt{2\sin^2 x + 3\sin x + 4} - \sqrt{\sin^2 x + 6\sin x + 2}\right)$ **is equal to**

(A) $\frac{3}{4}$  (B) $\frac{1}{6}$  (C) $\frac{1}{12}$  (D) $\frac{5}{12}$

**Q.** $\lim_{x\to\infty} x\left(\arctan\frac{x+1}{x+2} - \arctan\frac{x}{x+2}\right)$ **is equal to** Copy

(A) $\frac{1}{2}$  (B) $-\frac{1}{2}$  (C) 1  (D) D.N.E.

Nishant Jindal

# LIMIT

Q. $\lim_{h\to 0} \frac{\tan(a+2h)-2\tan(a+h)+\tan a}{h^2}$ is equal to

DL → $\tan(a+2h) \to \sec^2(a+2h)\times 2$

(A) $\tan a$ (B) $\tan^2 a$ (C) $\sec a$ (D) $2(\sec^2 a)(\tan a)$

$2\times 2 \cdot 2\sec(a+2h)\cdot\sec(a+2h)\cdot\tan(a+2h)$

Q. $\lim_{x\to 0}\left(2^{x-1}+\frac{1}{2}\right)^{1/x}$ equals

(A) $\sqrt{2}$ (B) $\frac{1}{2}\ell n2$ (C) $\ln 2$ (D) 2

Q. If $\lim_{x\to 0}\left(\cos x + a^3\sin(b^6x)\right)^{\frac{1}{x}} = e^{512}$, then the value of $ab^2$ is equal to

(A) -512 (B) 512 (C) 8 (D) $8\sqrt{8}$

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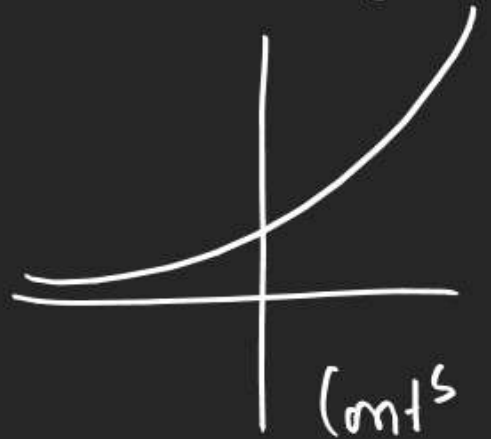
# Continuity [Limit-2]

Prelepco
→ 80-90

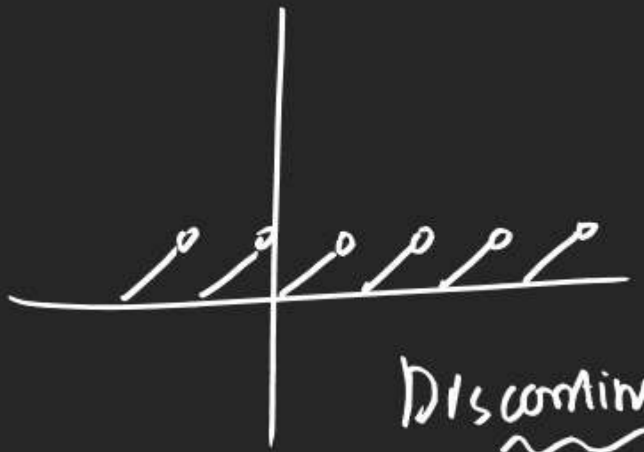
1) While drawing graph of a fxn if you need to Pick up Pen then fxn is not continuous.

$Y = e^x$



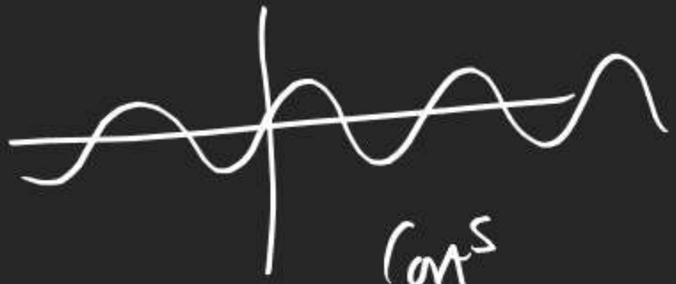
Conts

$Y = \{x\}$



Discontinuous at every Integer.

$Y = \sin x$

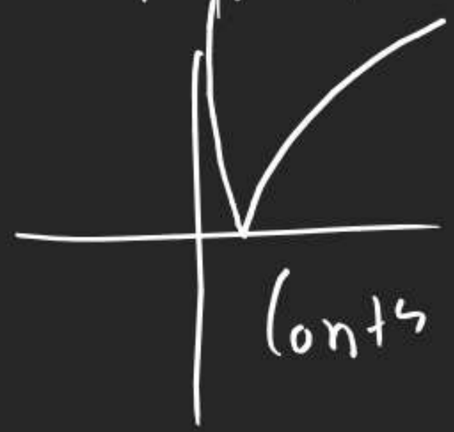


Conts

$Y = \ln x$

Cont.

$Y = |\ln x|$



Conts

$Y = |\sin x|$

Conts

$$f(x) = \begin{cases} x^2 & x \geq 1 \\ x & x < 1 \end{cases}$$

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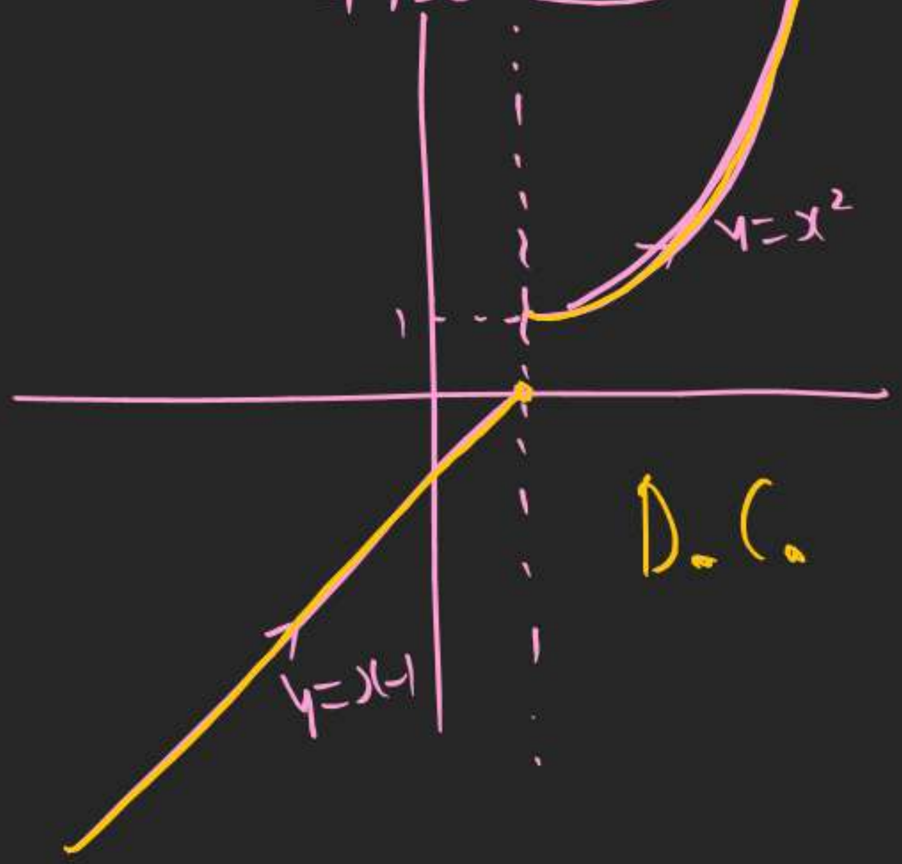
$$f(x) = \begin{cases} x^2 & x \geq 1 \\ y = x - 1 & \boxed{x < 1} \end{cases}$$ Defined fxn

$1^2 = 1$

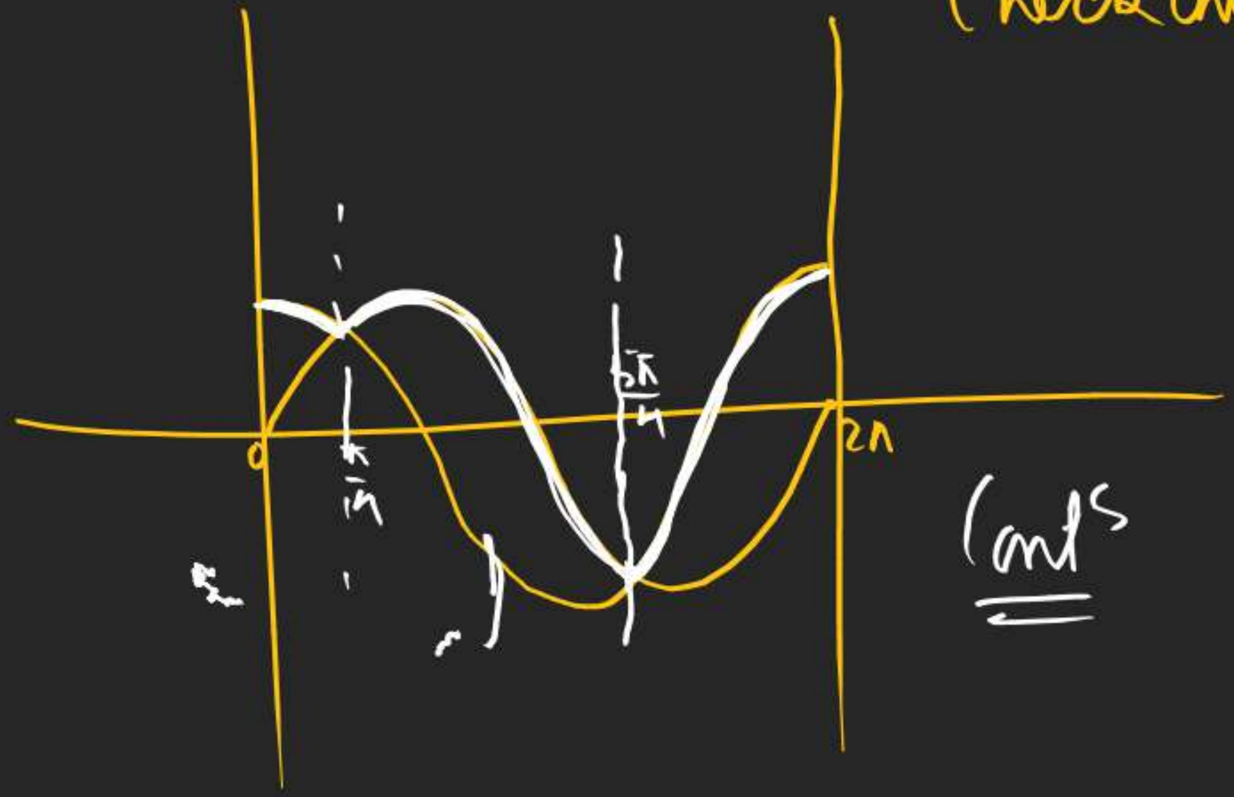
$1 - 1 = 0$




D. C.

$y = \text{Max}\{\sin x, \cos x\}$ $x \in [0, 2\pi]$

Check Conts.




Conts

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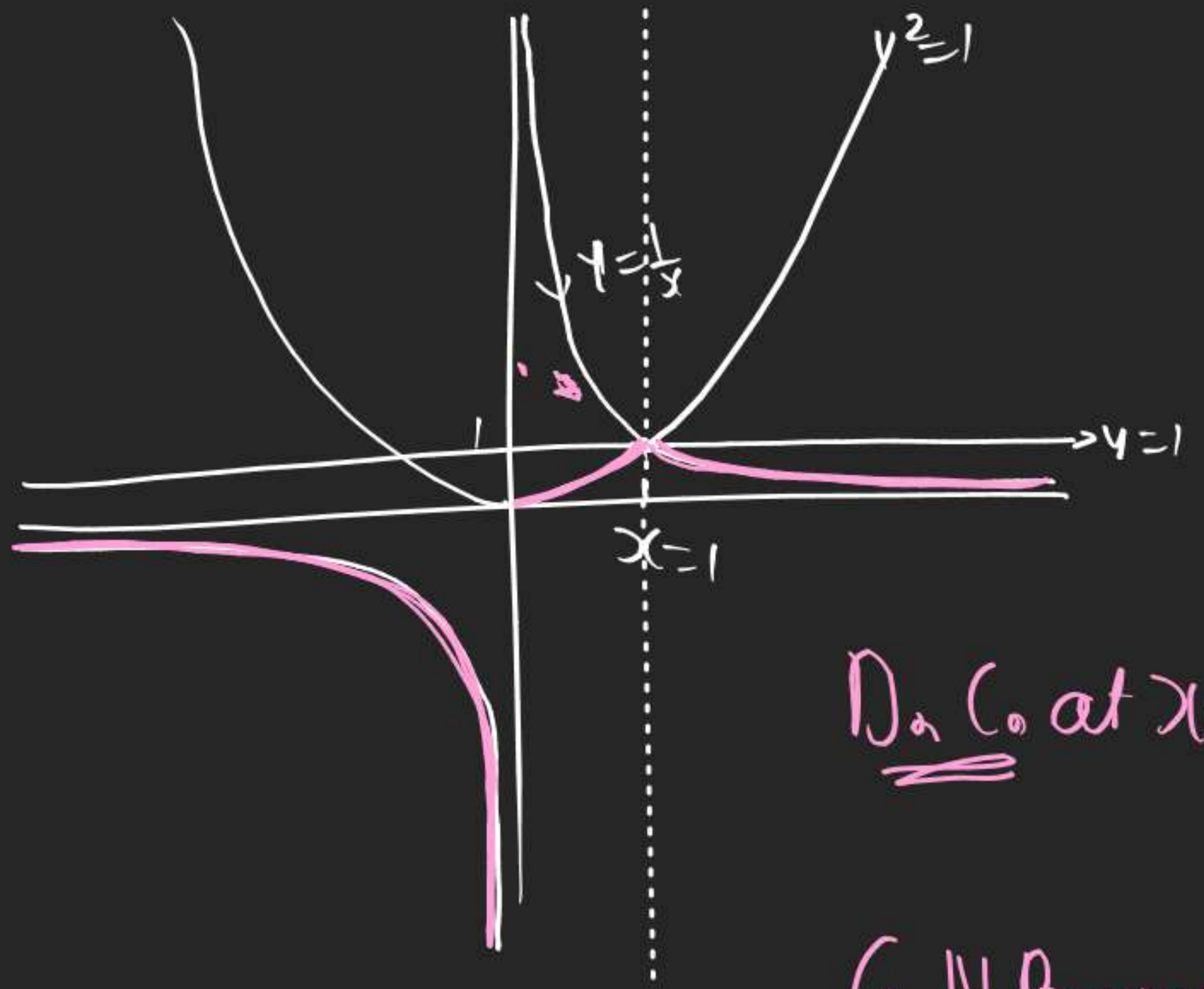
Q $y = \text{Min}\{\boxed{x^2}, \boxed{\frac{1}{x}}, 1\}$




D. C. at $x=0$

G.N. Bermal

Limit Qs → 100 QS

$$\lim_{x\to 2} \frac{\sqrt{1-\cos 2(x-2)}}{x-2}$$

$$1-\cos 2\theta = 2\sin^2\theta$$

$$1-\cos 2(x-2) = 2\sin^2(x-2)$$

$$\frac{\sqrt{2}\,|\sin(x-2)|}{(x-2)} \xrightarrow{\text{LHL}} \frac{\sqrt{2}\,|\sin(-h)|}{-h} = \frac{\sqrt{2}\sin h}{-h} = -\sqrt{2}$$

RHL

$$\frac{\sqrt{2}\,|\sin(h)|}{h} = \frac{\sqrt{2}\sin h}{h} = \sqrt{2}$$

LDNE