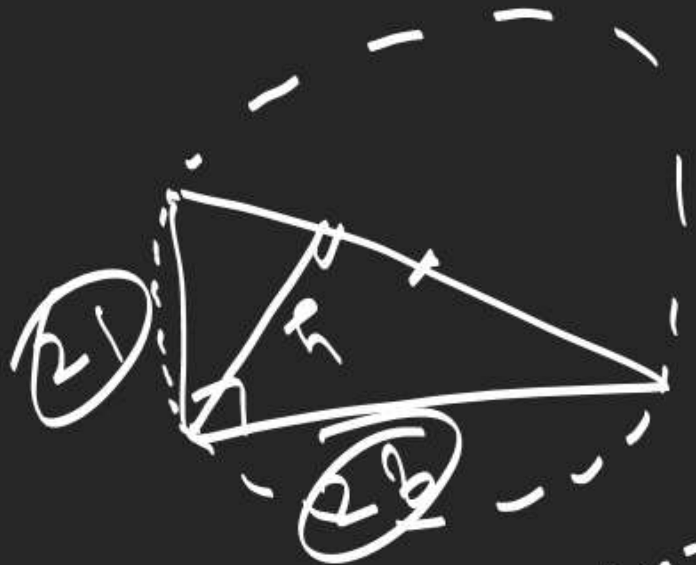
$$\sum a^2\, 2R \sin(B+C)\cos(B-C) = \sum a^2 R\,(\sin 2B + \sin 2C)$$

$$= a^2 b\cos B + a^2 c\cos C + b^2 a\cos A + b^2 c\cos C + c^2 a\cos A + c^2 b\cos B$$

$$= 3abc$$




$$\frac{1}{2}\times 21\times 28 = \frac{1}{2}\times h\times \sqrt{\phantom{x}}$$

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$$\boxed{n\left(A_1 \cup A_2 \cup A_3 \cup \cdots \cup A_m\right) = S_1 - S_2 + S_3 - S_4 + \cdots + (-1)^{m-1} S_m}$$

$$S_1 = \sum n(A_1)$$

$$S_2 = \sum n(A_1 \cap A_2)$$

$$S_3 = \sum n(A_1 \cap A_2 \cap A_3)$$

$\vdots$

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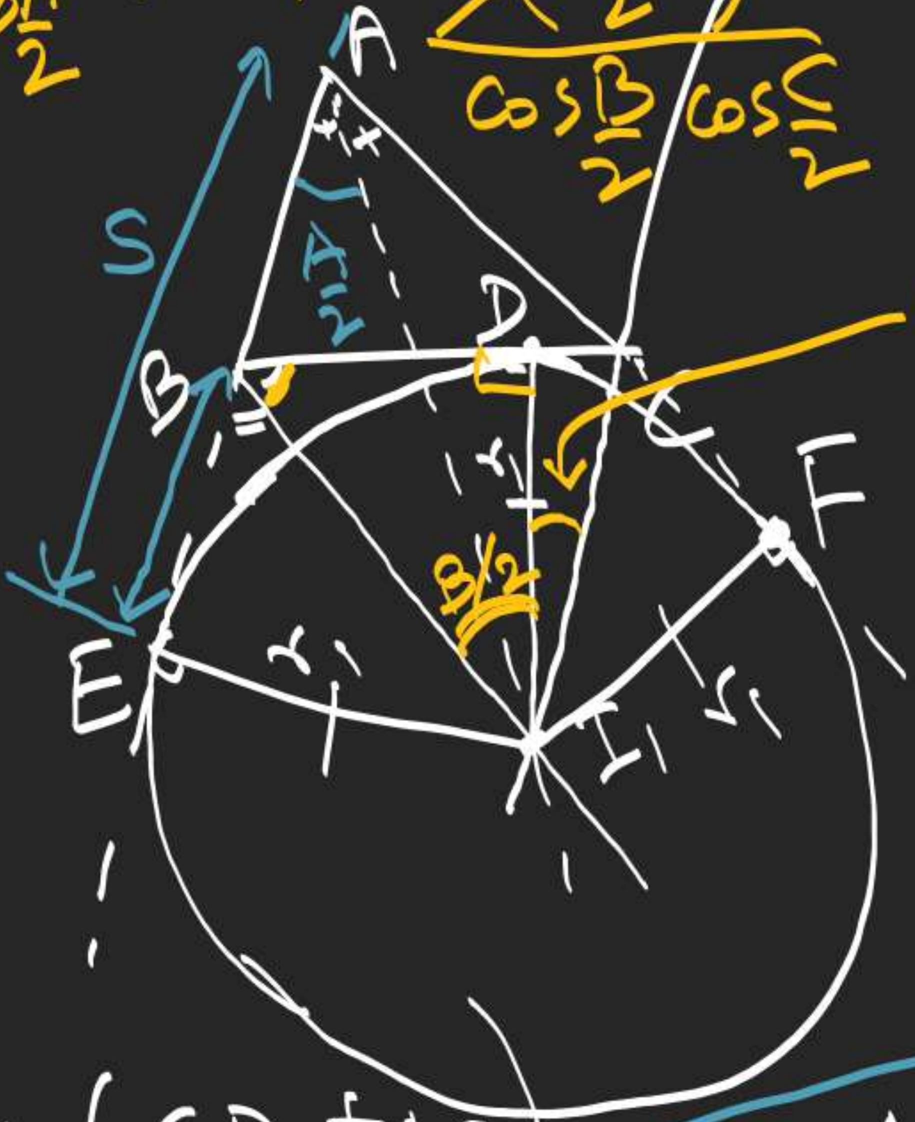
# Excircle.

$BC = a = 2R\sin A = BD + DC = r_1\left(\tan\frac{B}{2} + \tan\frac{C}{2}\right)$

$4R\sin\frac{A}{2}\cancel{\cos\frac{A}{2}} = r_1 \frac{\sin\left(\cancel{\frac{B+C}{2}}\right)}{\cos\frac{B}{2}\cos\frac{C}{2}}$




$\tan\frac{A}{2} = \frac{r_1}{S}$

$$\boxed{\begin{aligned} r_1 &= s\tan\frac{A}{2} \\ r_2 &= s\tan\frac{B}{2} \\ r_3 &= s\tan\frac{C}{2} \end{aligned}}$$

$2AE = AE + AF$

$= (AB + BD) + (CD + AC)$

$= AB + BC + AC = 2s$

$\Delta AI_1B + AI_1C - BI_1C$

$= \Delta ABC$

$\frac{1}{2} r_1(c + b - a) = \Delta$

$\frac{1}{2} r_1(2s - 2a) = \Delta$

$r_1 = \frac{\Delta}{s-a}$

$r_3 = \frac{\Delta}{s-c}$ $\quad r_2 = \frac{\Delta}{s-b}$

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$$r_1 = 4R \sin\frac{A}{2} \cos\frac{B}{2} \cos\frac{C}{2}$$

$$r_2 = 4R \cos\frac{A}{2} \sin\frac{B}{2} \cos\frac{C}{2}$$

$$r_3 = 4R \cos\frac{A}{2} \cos\frac{B}{2} \sin\frac{C}{2}$$

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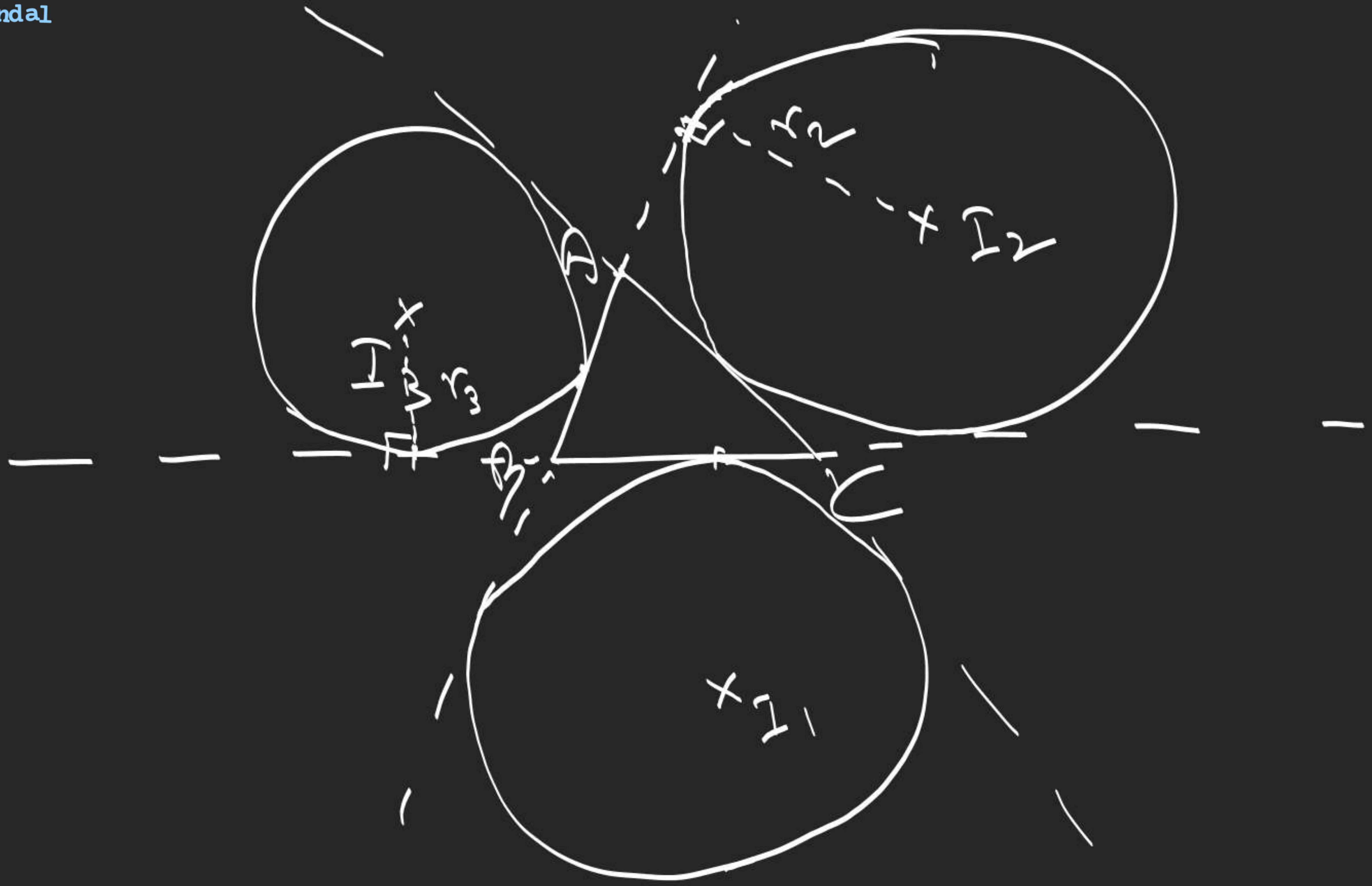

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P.T.

1. $r r_1 r_2 r_3 = \Delta^2$

$\frac{\Delta}{s}\frac{\Delta}{s-a}\frac{\Delta}{s-b}\frac{\Delta}{s-c} = \Delta^2.$

2. $r_2 r_3 + r_3 r_1 + r_1 r_2 = s^2$

$s^2\left(\tan\frac{B}{2}\tan\frac{C}{2} + \tan\frac{C}{2}\tan\frac{A}{2} + \tan\frac{A}{2}\tan\frac{B}{2}\right)$

$= s^2.$

3. $\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} = \frac{1}{r}$

$\frac{s-a}{\Delta} + \frac{s-b}{\Delta} + \frac{s-c}{\Delta}$

$= \frac{s}{\Delta} = \frac{1}{r}.$

$\frac{\Delta^2}{(s-b)(s-c)} + \frac{\Delta^2}{(s-c)(s-a)} + \frac{\Delta^2}{(s-a)(s-b)}$

$= \frac{s\Delta^2\left(s-a+s-b+s-c\right)}{s(s-a)(s-b)(s-c)} = \frac{\Delta^2 s s}{\Delta^2}$

Nishant Jindal

4. $\cos A + \cos B + \cos C = 1 + \dfrac{r}{R}$.

$2\sin\frac{C}{2}\cos\frac{A-B}{2} + 1 - 2\sin^2\frac{C}{2} = 2\sin\frac{C}{2}\left(\cos\frac{A-B}{2} - \cos\frac{A+B}{2}\right) + 1$

$= 1 + 4\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$.

$= 1 + \frac{r}{R} \le 1 + \frac{1}{2}$.

5. $\dfrac{r_1}{bc} + \dfrac{r_2}{ca} + \dfrac{r_3}{ab} = \dfrac{1}{r} - \dfrac{1}{2R}$.

$\dfrac{\Delta}{(s-a)bc} + \dfrac{\Delta}{(s-b)ca} + \dfrac{\Delta}{(s-c)ab}$

$= \dfrac{\Delta}{abc}\left(\dfrac{a}{s-a} + \dfrac{b}{s-b} + \dfrac{c}{s-c}\right) = \dfrac{\Delta}{abc}\left(\left(\dfrac{a}{s-a}+1\right) + \left(\dfrac{b}{s-b}+1\right) + \dfrac{c}{s-c} - 2\right)$

$ab$

$= \dfrac{\Delta}{abc}\left(\underbrace{\dfrac{s}{s-a} + \dfrac{s}{s-b}} + \dfrac{c}{s-c}\right) - \dfrac{1}{2R} = \dfrac{\Delta}{abc}\left(\dfrac{sc}{(s-a)(s-b)} + \dfrac{c}{s-c}\right) - \dfrac{1}{2R}$

$\dfrac{1}{r} - \dfrac{1}{2R} = \dfrac{\Delta s}{\Delta^2} - \dfrac{1}{2R}$. $= \dfrac{s\,\Delta c}{abc}\left(\dfrac{s(s-c) + (s-a)(s-b)}{s(s-a)(s-b)(s-c)}\right) - \dfrac{1}{2R}$.

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6. $r^2+r_1^2+r_2^2+r_3^2 = 16R^2-(a^2+b^2+c^2)$

$$= 16R^2\left(s^2\frac{A}{2}\left(s^2\frac{B}{2}s^2\frac{C}{2}+c^2\frac{B}{2}c^2\frac{C}{2}\right)+c^2\frac{A}{2}\left(s^2\frac{B}{2}c^2\frac{C}{2}+c^2\frac{B}{2}s^2\frac{C}{2}\right)\right)$$

$$= 16R^2\left[s^2\frac{A}{2}\left(\sin^2\frac{A}{2}+2s\frac{B}{2}s\frac{C}{2}c\frac{B}{2}c\frac{C}{2}\right)+c^2\frac{A}{2}\left(\cos^2\frac{A}{2}-2s\frac{B}{2}c\frac{C}{2}c\frac{B}{2}s\frac{C}{2}\right)\right]$$

$$16R^2-2a^2-\boxed{2bc\cos A} = 16R^2-2a^2-(b^2+c^2-a^2)$$

$$= 16R^2\left[1-\frac{1}{2}\sin^2 A-\frac{1}{2}\sin B\sin C\left(\cos^2\frac{A}{2}-\sin^2\frac{A}{2}\right)\right]$$

$$= 16R^2\left[1-\frac{1}{2}\sin^2 A-\frac{1}{2}\sin B\sin C\cos A\right]$$

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7. If $\left(1-\frac{r_1}{r_2}\right)\left(1-\frac{r_1}{r_3}\right)=2$, P.T. triangle is right angled.

$$\left(1-\frac{s-b}{s-a}\right)\left(1-\frac{s-c}{s-a}\right)=2.$$

$$(b-a)(c-a)=2(s-a)^2$$

$$bc-a(b+c)+a^2=$$

$$2s^2-4as+2a^2.$$

$$\Rightarrow bc=a^2+a(b+c)+2s^2-4as=2s^2-2as$$

$$\frac{s(s-a)}{bc}=\frac{1}{2}.$$

$$\cos^2\frac{A}{2}=\frac{1}{2}$$

$$\frac{A}{2}=\frac{\pi}{4}$$

$$\boxed{A=\frac{\pi}{2}}$$

$$\boxed{\text{Ex-27} \rightarrow 26-39}$$