$$\cot\theta + \tan\theta = 4 \Rightarrow \tan\theta + \frac{1}{\tan\theta} = 4 \Rightarrow \frac{\tan^2\theta + 1}{\tan\theta} = 4 \Rightarrow \tan^2\theta - 4\tan\theta + 1 = 0$$

$$Q \quad \sum_{m=1}^{6} \sec\left(\theta + \frac{(m-1)\pi}{4}\right) \cdot \sec\left(\theta + \frac{m\pi}{4}\right) = \boxed{4\sqrt{2}} \; ; \; 0 < \theta < \frac{\pi}{2} \quad \text{find the } \theta \text{?}$$

① diff = $\left(\theta + \frac{m\pi}{4}\right) - \left(\theta + \frac{(m-1)\pi}{4}\right) = \frac{\pi}{4}$ ② $\sin\frac{\pi}{4}$ Multiply/div.

$$\sum \frac{1}{\sin\left(\theta + \frac{(m-1)\pi}{4}\right) \cdot \sin\left(\theta + \frac{m\pi}{4}\right)} \times \sin\frac{\pi}{4}$$

$$\frac{1}{\sin\frac{\pi}{4}} \sum \left(\frac{\sin\left\{\left(\theta + \frac{m\pi}{4}\right) - \left(\theta + \frac{(m-1)\pi}{4}\right)\right\}}{\sin\left(\theta + \frac{(m-1)\pi}{4}\right) \cdot \sin\left(\theta + \frac{m\pi}{4}\right)} \right) = \sqrt{2} \sum_{m=1}^{6} \frac{\sin\left(\theta + \frac{m\pi}{4}\right) \cos\left(\theta + \frac{(m-1)\pi}{4}\right) - \cos\left(\theta + \frac{m\pi}{4}\right) \sin\left(\theta + \frac{(m-1)\pi}{4}\right)}{\sin\left(\theta + \frac{(m-1)\pi}{4}\right) \cdot \sin\left(\theta + \frac{m\pi}{4}\right)}$$

$$+ \quad \sum_{m=1}^{6} \cot\left(\theta + \frac{(m-1)\pi}{4}\right) - \cot\left(\theta + \frac{m\pi}{4}\right) = \sqrt{2} \begin{cases} \cot(\theta + 0) - \cot\left(\theta + \frac{\pi}{4}\right) \\ + \cot\left(\theta + \frac{\pi}{4}\right) - \cot\left(\theta + \frac{2\pi}{4}\right) \\ + \cot\left(\theta + \frac{2\pi}{4}\right) - \cot\left(\theta + \frac{3\pi}{4}\right) \\ \vdots \\ + \cot\left(\theta + \frac{5\pi}{4}\right) - \cot\left(\theta + \frac{6\pi}{4}\right) \end{cases}$$

$$\frac{3\pi}{2}$$

$$= \sqrt{2}\left(\cot\theta - \cot\left(\theta + \frac{6\pi}{4}\right) \right) = 4\sqrt{2}$$

$$\cot\theta - \cot\left(\frac{3\pi}{2} + \theta\right) = 4$$

$\mathcal{E}x.$ XIII

$\mathcal{E}xample 13$ S.L.Loney. $\underline{15}$

1) ✓ Copy

$\begin{array}{r} 56 \\ 45 \\ \hline 280 \\ 224 \\ \hline 2520 \end{array}$

$\begin{array}{r} {}^{1}8\,4 \\ 6\cdot4 \\ \hline 9\,24 \end{array}$

2) $\sin \alpha = \dfrac{45}{53}$ & $\sin \beta = \dfrac{33}{65}$ $\sin(\alpha - \beta)$ & $\sin(\alpha + \beta)$

$\widehat{1943}$

$\cos \alpha = \sqrt{1 - \sin^2 \alpha} = \sqrt{1 - \left(\dfrac{45}{53}\right)^2}$

① demand $= \sin(\alpha - \beta)$

$= \sin \alpha \cos \beta - \cos \alpha \sin \beta$

$\sin^2 \alpha + \cos^2 \alpha = 1$

$= \dfrac{28}{53}$

$= \dfrac{45}{53} \times \dfrac{56}{65} - \dfrac{28}{53} \times \dfrac{33}{65} = \dfrac{45 \times 56 - 28 \times 33}{53 \times 65}$

$\cos \beta = \sqrt{1 - \sin^2 \beta}$

$= \sqrt{1 - \dfrac{(33)^2}{(65)^2}} = \dfrac{56}{65}$

(2) $\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$

$= \dfrac{2520 + 924}{53 \times 65}$

$= \dfrac{2520 - 924}{53 \times 65} = \dfrac{1596}{53 \times 65}$

Q3 $\sin \alpha = \dfrac{15}{17}$, $\cos \beta = \dfrac{12}{13}$.

$\cos \alpha = \sqrt{1 - \sin^2 \alpha} = \sqrt{1 - \dfrac{225}{289}} = \dfrac{8}{17}$ $\bigg|$ $\sin \beta = \sqrt{1 - \cos^2 \beta}$

$= \sqrt{1 - \dfrac{144}{169}} = \dfrac{5}{13}$

$\sin(\alpha + \beta)$, $\cos(\alpha - \beta)$

① $\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$.

$= \dfrac{15}{17} \cdot \dfrac{12}{13} + \dfrac{8}{17} \cdot \dfrac{5}{13} = \dfrac{220}{17 \times 13}$.

(2) $\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$

$= \dfrac{8}{17} \times \dfrac{12}{13} + \dfrac{15}{17} \times \dfrac{5}{13} = \dfrac{96 + 75}{17 \times 13} = \dfrac{171}{17 \times 13}$

4) $\cos(45° - A) \cdot \cos(45° - B) - \sin(45° - A) \cdot \sin(45° - B)$

$\cos a \cdot \cos b - \sin a \cdot \sin b = \cos(a+b)$

$= \cos\{(45° - A) + (45° - B)\}$

$= \cos(90 - (A+B))$

$= \sin(A+B)$

Q) $\sin(45° + A) \cdot \cos(45° - B) + \cos(45° + A) \sin(45° - B)$

$\sin a \cdot \cos b + \cos a \sin b = \sin(a+b)$

$= \sin\{(45 + A) + (45° - B)\}$

$= \sin(90 + (A - B)) = \sin\left(\frac{\pi}{2} + \theta\right)$ ②

$= +\cos\theta = \cos(A - B)$

Q. $\dfrac{\sin(A-B)}{\cos A \cos B} + \dfrac{\sin(B-C)}{\cos B \cos C} + \dfrac{\sin(C-A)}{\cos C \cos A} = ?$

$\left(\dfrac{\sin A \cos B - \cos A \sin B}{\cos A \cos B} \right) + \left(\dfrac{\sin B \cos C}{\cos B \cos C} - \dfrac{\cos B \sin C}{\cos B \cos C} \right) + \left(\dfrac{\sin C \cos A}{\cos C \cos A} - \dfrac{\cos C \sin A}{\cos C \cos A} \right)$

$(\tan A - \tan B) + (\tan B - \tan C) + (\tan C - \tan A)$

$$= 0$$

$\dfrac{\sin(A-B)}{\cos A \cos B} = \dfrac{\sin A \cos B - \cos A \sin B}{(\cos A \cos B)}$

Q $\sin 105° + \cos 105° = \cos\boxed{45°}$ check.

45° Prove krna hai to 105° ko 45° me socho.

$105° = 45° + \_\_\_$

$105° = 45° + 60°$
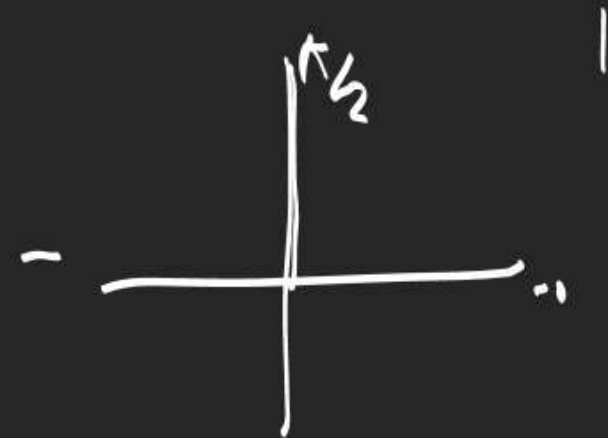
$\sin(60 + 45°) + \cos(60 + 45°)$

$\sin 60 \cdot \cos 45° + \cos 60 \cdot \sin 45° + \cos 60 \cdot \cos 45° - \sin 60 \cdot \sin 45°$

$\dfrac{\sqrt{3}}{2} \cdot \dfrac{1}{\sqrt{3}} + \dfrac{1}{2} \cdot \dfrac{1}{\sqrt{2}} + \dfrac{1}{2} \cdot \dfrac{1}{\sqrt{2}} - \dfrac{\sqrt{3}}{2} \cdot \dfrac{1}{\sqrt{2}} = \dfrac{1}{2\sqrt{2}} + \dfrac{1}{2\sqrt{2}} = 2 \times \dfrac{1}{2\sqrt{2}} = \dfrac{1}{\sqrt{2}}$

$= \cos 45° = RHS$

Q $\sin 75° - \sin 15° = \cos 105° + \cos 15°$ [T1F]

RHS $= \cos 105° + \cos 15°$

$$= \cos(90+15°) + \cos(90-75°)$$ ①

$$= -\sin 15° + \sin 75 = LHS.$$

$-\sin\theta = \cos\left(\dfrac{\pi}{2}+\theta\right)$

Q9 $\cos\alpha \cdot \cos(\gamma-\alpha) - \sin\alpha \cdot \sin(\gamma-\alpha) = \cos\gamma$ (P.T)

$\cos A \cdot \cos B - \sin A \sin B = \cos(A+B)$

$$= \cos(\alpha + \gamma - \alpha) = \cos\gamma = RHS.$$

Q $\sin\boxed{(n+1)A}\cdot\sin\boxed{(n-1)A}+\cos(n+1)A\cos(n-1)A$

$\sin A\cdot\sin B+\cos A\cos B=\cos(A-B)$

$$=\cos((n+1)A-(n-1)A)$$

$$=\cos(nA+A-nA+A)$$

$$=\cos 2A$$

Q $\cos(n+1)A\cdot\cos(n+2)A+\cos(n+1)A\cos(n+2)A=?$

$\cos a\cdot\cos b+\cos a\cdot\cos b=\cos(a-b)$

$$=\cos((n+1)A-(n+2)A)$$

$$=\cos(nA+A-nA+2A)$$

$$=\cos(-A)=\cos A$$

Sheets

$\theta \in \left(0, \frac{\pi}{2}\right)$

Q $\sin\theta + \csc\theta = 2$ then $\sin^{20}\theta + \csc^{20}\theta = ?$

$\sin\theta + \dfrac{1}{\sin\theta} = 2$

Concept

$\boxed{\begin{array}{l} x + \dfrac{1}{x} = 2 \\[2mm] x = 1 \end{array}}$ $(x + \text{ve})$

$x + \dfrac{1}{x} \geqslant 2$

$\Rightarrow \sin\theta = 1 \Rightarrow \csc\theta = 1$

$\text{Demand} = (\sin\theta)^{20} + (\csc\theta)^{20}$

$= 1^{20} + 1^{20} = 2$

$a\sin\theta + b\cos\theta$ TypeQs.

$\rightarrow$ SAA Lgaate $\Rightarrow$ Sqr & Add.

Q. If $a\cos\theta + b\sin\theta = 3$ & $a\sin\theta - b\cos\theta = 4$

find value of $a^2 + b^2 = ?$

$a\cos\theta + b\sin\theta = 3 \xrightarrow{Sqr} a^2\cos^2\theta + b^2\sin^2\theta + 2ab\sin\theta\cos\theta = 9$

$a\sin\theta - b\cos\theta = 4 \Rightarrow a^2\sin^2\theta + b^2\cos^2\theta - 2ab\sin\theta\cos\theta = 16$

add $(a^2\cos^2\theta + a^2\sin^2\theta) + (b^2\sin^2\theta + b^2\cos^2\theta) = 25$

$a^2(s^2 + c^2) + b^2(s^2 + c^2) = 25$

$a^2 + b^2 = 25$

Q If $\cos x + \sin x = \sqrt{2}\cos x$ then P.T.

$\cos x - \sin x = \sqrt{2}\sin x$

← Photo abnorbha

Given $\cos x + \sin x = \sqrt{2}\cos x$

$Sq^n \rightarrow (\cos x + \sin x)^2 = (\sqrt{2}\cos x)^2$

$\cos^2 x + \sin^2 x + 2\sin x \cos x = 2\cos^2 x$

$\sin^2 x = 2\cos^2 x - \cos^2 x - 2\sin x \cdot \cos x$

$\sin^2 x + \sin^2 x = \cos^2 x - 2\sin x \cdot \cos x + \sin^2 x$ ← $(a-b)^2$ Jesa

$2\sin^2 x = (\cos x - \sin x)^2$

$\cos x - \sin x = \sqrt{2}\sin x$ ⇐

(H.P)

Q If $3\sin x + 4\cos x = 5$ then find value of $4\sin x - 3\cos x = ?$

$= \bigcirc$

$a\sin\theta + b\cos\theta$ ←

Photo

$(3\sin x + 4\cos x)^2 = (5)^2$

$9\sin^2 x + 16\cos^2 x + 24\sin x \cos x = 25$

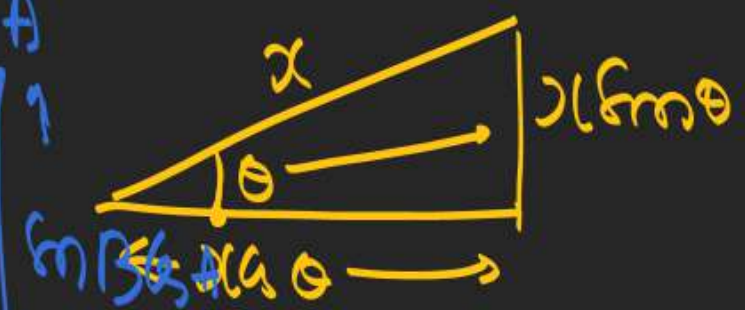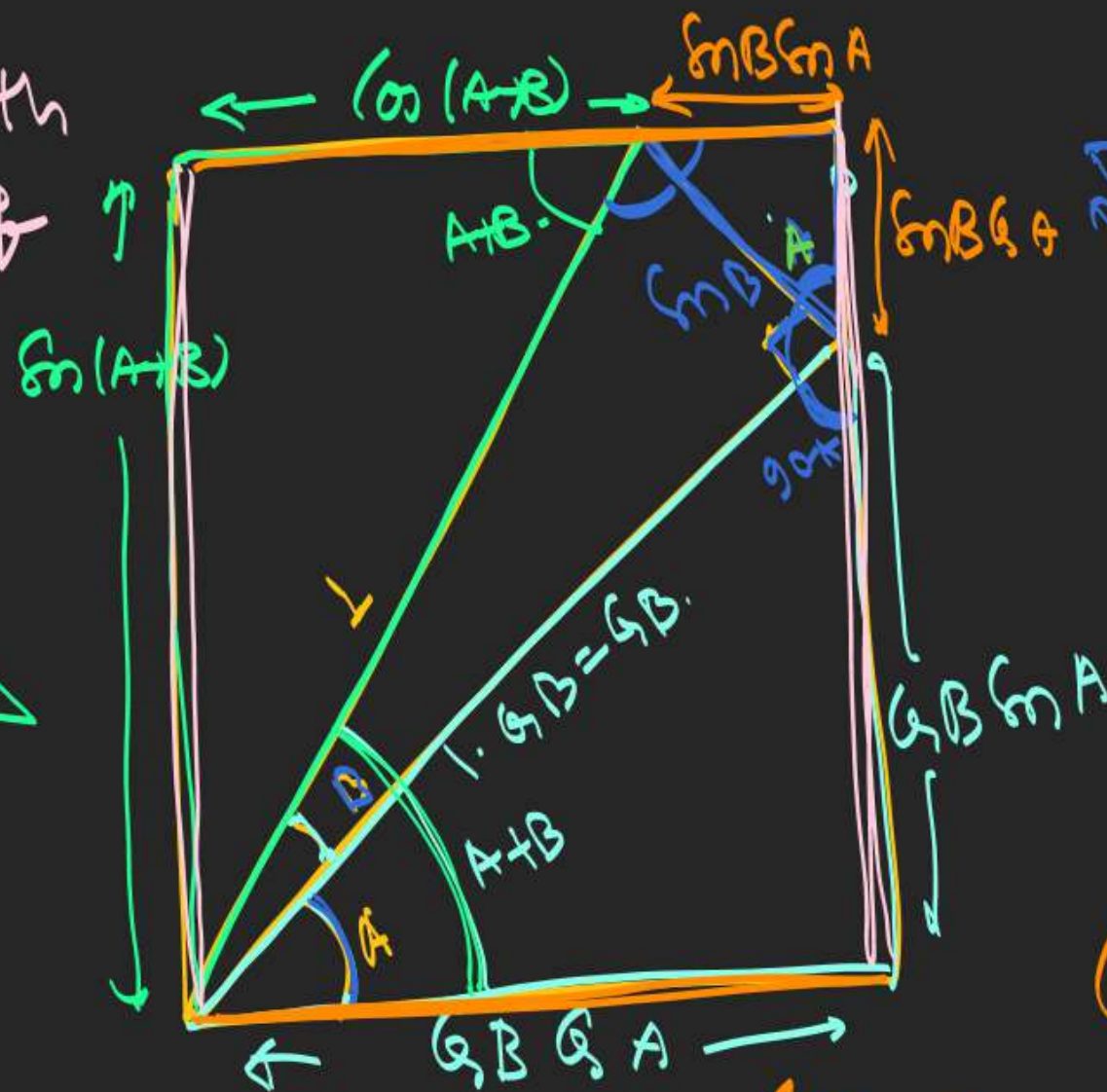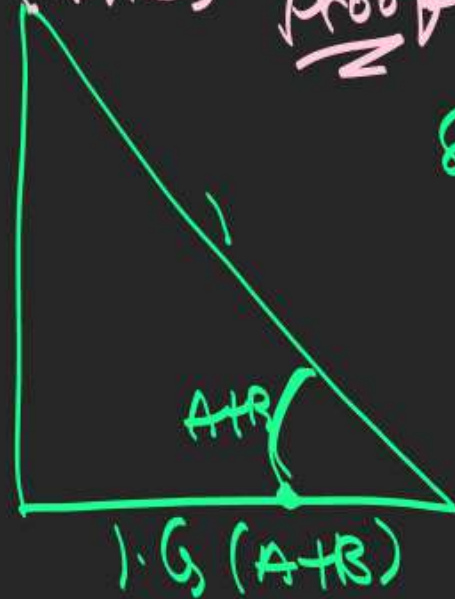$9(1 - \cos^2 x) + 16(1 - \sin^2 x) + 24\sin x \cos x = 25$

$9 + 16 - 9\cos^2 x - 16\sin^2 x + 24\sin x \cos x = 25$

$\Rightarrow 16\sin^2 x - 24\sin x \cdot \cos x + 9\cos^2 x = 0$

$\Rightarrow (4\sin x - 3\cos x)^2 = 0$

$4\sin x - 3\cos x = 0$

$\sin(A+B)$ Both

$\cos(A+B)$ Proof

$\sin(A+B)$

$\cos(A+B)$

$A+B$

$1 \cdot \cos(A+B)$

$\cos(A+B)$

$\sin B \sin A$

$\sin B \cos A$

$A+B$

$\cos B \cdot A$

$1 \cdot \cos B = \cos B$

$A+B$

A

$\cos B \sin A$

$\cos B \cos A$

$\sin B \sin A$

$\sin B$

A

$\cos B$

x

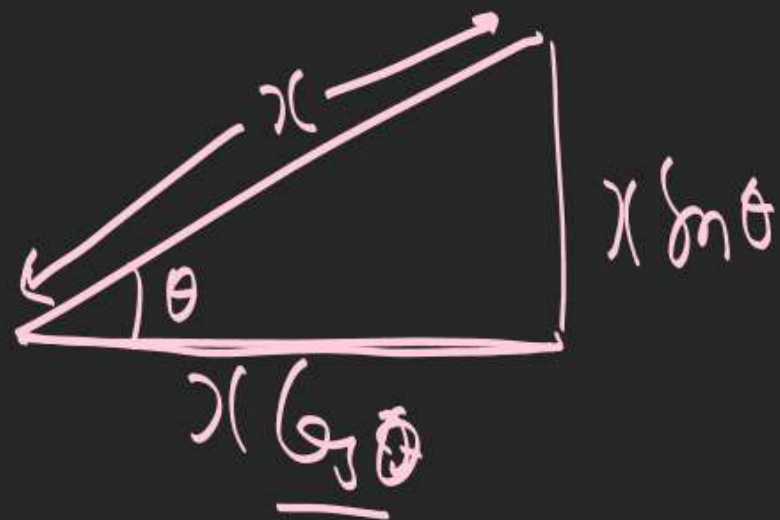$x (\sin \theta)$

θ

$\sin B \cos B \cos \theta$

$(3) \ \tan(A+B) = \dfrac{\sin(A+B)}{\cos(A+B)}$

$\cos B \cos A = \cos(A+B) + \sin B \sin A$

$\cos B \cdot \cos A - \sin B \sin A = \cos(A+B) \quad ①$

$② \sin(A+B) = \cos B \sin A + \sin B \cos A$

x

$x \sin \theta$

θ

$x (\cos \theta)$

$$\tan(A+B) = \frac{\sin(A+B)}{\cos(A+B)} = \frac{\sin A \cos B + \cos A \sin B}{\boxed{\cos A \cos B} - \sin A \sin B} \qquad \div \cos A \cos B.$$

$$= \frac{\dfrac{\sin A \cos B}{\cos A \cos B} + \dfrac{\cos A \sin B}{\cos A \cos B}}{\dfrac{\cos A \cos B}{\cos A \cos B} - \dfrac{\sin A \sin B}{\cos A \cos B}} = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

(3) $\boxed{\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}}$

(4) $\boxed{\tan(A-B) = \dfrac{\tan A - \tan B}{1 + \tan A \tan B}}$

(5) $\sin(A+B) \cdot \sin(A-B) = (\sin A \cos B + \cos A \sin B) \cdot (\sin A \cos B - \cos A \sin B)$

$$= (\sin A \cos B)^2 - (\cos A \sin B)^2$$

$$= \sin^2 A \cos^2 B - \cos^2 A \sin^2 B$$

$$= \sin^2 A (1 - \sin^2 B) - (1 - \sin^2 A) \cdot \sin^2 B$$

$$= \sin^2 A - \sin^2 A \cdot \sin^2 B - \sin^2 B + \sin^2 A \sin^2 B$$

$$\boxed{\begin{aligned} \sin(A+B) \cdot \sin(A-B) &= \sin^2 A - \sin^2 B \\ \sin(A+B) \cdot \sin(A-B) &= \cos^2 B - \cos^2 A \end{aligned}}$$

$\longrightarrow (1 - \cos^2 A) - (1 - \cos^2 B)$

(6) $C_s (A + B) \cdot C_s (A - B) = \ ?$