Nishant Jindal

# Trigonometry

Q $\cos^2(A-B)+\cos^2B-2\cos(A-B)\cos A\cdot\cos B=?$ $=\sin^2 A$ [Last]

Q $\dfrac{4\left(\cos^3 10^\circ+\sin^3 20^\circ\right)}{\left(\cos 10^\circ+\sin 20^\circ\right)}=?$

Basic $\Rightarrow$ $\dfrac{a^3+b^3}{a+b}=\dfrac{\cancel{(a+b)}(a^2-ab+b^2)}{\cancel{(a+b)}}$

ULTA

$\sin^2A-\sin^2B=\sin(A+B)\cdot\sin(A-B)$

$4\left(\cos^2 10^\circ+\sin^2 20^\circ-\cos 10^\circ\cdot\sin 20^\circ\right)$

$4\left(1-\underline{\sin^2 \overset{B}{10^\circ}}+\underline{\sin^2 \overset{A}{20^\circ}}-\cos 10^\circ\cdot\sin 20^\circ\right)$

$4\left(1+\sin(10+20)\cdot\sin(20-10^\circ)-\cos 10^\circ\cdot\sin 20^\circ\right)$

$4\left(1+\frac{1}{2}\sin 10^\circ-\underline{\cos 10^\circ\cdot\sin 20^\circ}\right)$ (HOLD)

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# Sum & Difference of Sin & Cos.

## 4 formulae.

1) $\sin C + \sin D = 2\sin\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)$

2) $\sin C - \sin D = 2\cos\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$

3) $\cos C + \cos D = 2\cos\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)$

4) $\cos C - \cos D = -2\sin\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$

S +
S −
C +
C −

Q ① $\sin 75^\circ + \sin 15^\circ$

$2\cdot\sin\left(\frac{75+15^\circ}{2}\right)\cos\cdot\left(\frac{75^\circ-15^\circ}{2}\right)$

$2\sin(45^\circ)\cdot\cos(30^\circ)$

$\cancel{2}\,\frac{1}{\sqrt{2}}\cdot\frac{\sqrt{3}}{\cancel{2}} = \sqrt{\frac{3}{2}}$

Q $\cos 75^\circ + \cos 15^\circ$

$2\cos\left(\frac{75+15}{2}\right)\cos\cdot\left(\frac{75-15}{2}\right)$

$2\cos 45^\circ\cdot\cos 30^\circ$

$= \sqrt{\frac{3}{2}}$

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# Trigonometry

① S + S = 2 S C

② S − S = 2 C S

③ C + C = 2 C C

④ C − C = −2 S S

Q ② $\dfrac{\sin 7\theta - \sin 5\theta}{\cos 7\theta + \cos 5\theta}$ ③ $= \dfrac{\cancel{2}\cancel{\cos\left(\frac{7\theta+5\theta}{2}\right)}\sin\left(\frac{7\theta-5\theta}{2}\right)}{\cancel{2}\cancel{\cos\left(\frac{7\theta+5\theta}{2}\right)}\cos\left(\frac{7\theta-5\theta}{2}\right)} = \dfrac{\sin(\theta)}{\cos(\theta)} = \tan\theta$

Q ② $\dfrac{\sin 7\theta - \sin 5\theta}{\cos 7\theta - \cos 5\theta}$ ④ $= \dfrac{\cancel{2}\cos\left(\frac{7\theta+5\theta}{2}\right)\sin\cancel{\left(\frac{7\theta-5\theta}{2}\right)}}{-\cancel{2}\sin\left(\frac{7\theta+5\theta}{2}\right)\sin\cancel{\left(\frac{7\theta-5\theta}{2}\right)}}$

$= -\dfrac{\cos 6\theta}{\sin 6\theta} = -\cot 6\theta$

$+\cot(A+B)\cot(A-B)$ Ans

Q. ③ $\dfrac{\cos 2B + \cos 2A}{\cos 2B - \cos 2A}$ ④ $= \dfrac{\cancel{2}\cos\left(\frac{2B+2A}{2}\right)\cos\left(\frac{2B-2A}{2}\right)}{-\cancel{2}\ \left(\frac{2B+2A}{2}\right)\sin\left(\frac{2B-2A}{2}\right)}$

$= \dfrac{(B-A)}{(B-A)} = -\cot(B+A)\,\underline{\cot(B-A)}$ Ans

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$S+S = 2\ S\ C$

$S-S = 2\ C\ S$

$C+C = 2\ C\ C$

$C-C = -2\ S\ S$

Q Solve $\dfrac{(\cos\theta - \cos 3\theta)(\sin 8\theta + \sin 2\theta)}{(\sin 5\theta - \sin\theta)(\cos 4\theta - \cos 6\theta)}$

$$\frac{\left(-\cancel{2}\sin\left(\frac{\theta+3\theta}{2}\right)\sin\left(\frac{\theta-3\theta}{2}\right)\right)\left(\cancel{2}\sin\left(\frac{8\theta+2\theta}{2}\right)\cos\left(\frac{8\theta-2\theta}{2}\right)\right)}{\left(\cancel{2}\cos\left(\frac{5\theta+\theta}{2}\right)\sin\left(\frac{5\theta-\theta}{2}\right)\right)\left(-\cancel{2}\sin\left(\frac{4\theta+6\theta}{2}\right)\sin\left(\frac{4\theta-6\theta}{2}\right)\right)}$$

$$\frac{-2\theta}{2} = -\theta$$

$$\frac{\sin\cancel{2\theta}\cdot\sin(-\theta)\times\sin\cancel{5\theta}\cdot\cos 3\theta}{\cos(3\theta)\cdot\sin(\cancel{2\theta})\times\sin(\cancel{5\theta})\cdot\sin(-\theta)} = 1$$

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# Trigonometry

Q $\dfrac{\sin(A+B) - 2\sin A + \sin(A-B)}{\cos(A+B) - 2\cos A + \cos(A-B)} = ?$

$$\frac{2\sin A\cos B - 2\sin A}{2\cos A\cos B - 2\cos A}$$

$$\frac{\cancel{2}\sin A(\cancel{\cos B - 1})}{\cancel{2}\cos A(\cancel{\cos B - 1})} = \tan A$$

1) $\sin(A+B) = \sin A\cos B + \cancel{\cos A\sin B}$

$\sin(A-B) = \sin A\cos B - \cancel{\cos A\sin B}$.

$+$

$\sin(A+B) + \sin(A-B) = 2\sin A\cos B$

2) $\cos(A+B) = \cos A\cos B - \cancel{\sin A\sin B}$.

$\cos(A-B) = \cos A\cos B + \cancel{\sin A\sin B}$

$\cos(A+B) + \cos(A-B) = 2\cos A\cos B$

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# Trigonometry

Q Solve $\dfrac{\sin A + \sin 3A + \sin 5A + \sin 7A}{\cos A + \cos 3A + \cos 5A + \cos 7A} = ?$

1 $S + S = 2\,S\,C$

~~2 $S - S = 2\,C\,S$~~

3 $C + C = 2\,C\,C$

$$\frac{(\sin A + \sin 7A) + (\sin 3A + \sin 5A)}{(\cos A + \cos 7A) + (\cos 3A + \cos 5A)}$$

$$\frac{\left(2\sin\left(\frac{A+7A}{2}\right)\cos\left(\frac{A-7A}{2}\right)\right) + \left(2\sin\left(\frac{3A+5A}{2}\right)\cos\left(\frac{3A-5A}{2}\right)\right)}{\left(2\cos\left(\frac{A+7A}{2}\right)\cos\left(\frac{A-7A}{2}\right)\right) + \left(2\cos\left(\frac{3A+5A}{2}\right)\cos\left(\frac{3A-5A}{2}\right)\right)}$$

$$\frac{\sin(4A)\cos(\cancel{-}3A) + \sin(4A)\cdot\cos(\cancel{-}A)}{\cos(4A)\cos(\cancel{-}3A) + \cos(4A)\cos(\cancel{-}A)} = \frac{\sin 4A\,\cancel{(\cos 3A + \cos A)}}{\cos 4A\,\cancel{(\cos 3A + \cos A)}}$$

$$= \tan 4A$$

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# Trigonometry

$\sin\theta = \cos(90-\theta)$

Q $\cos(A+B) + \sin(A-B) = 2\sin(45^\circ+A)\cos(45^\circ+B)$ [Check]

Sin me change Kar Lo ⇒ $\cos\theta = \sin(90-\theta)$

LHS $\sin\left(\frac{\pi}{2}-(A+B)\right) + \sin(A-B)$ ← Sin C + Sin D

$$2\sin\left(\frac{\frac{\pi}{2}-\cancel{A}-B+\cancel{A}-B}{2}\right)\cos\left(\frac{\frac{\pi}{2}-A-\cancel{B}-A+\cancel{B}}{2}\right)$$

$$2\sin\left(\frac{\frac{\pi}{2}-2B}{2}\right)\cdot\cos\left(\frac{\frac{\pi}{2}-2A}{2}\right)$$

$$2\sin\left(\frac{\pi}{4}-B\right)\cdot\cos\left(\frac{\pi}{4}-A\right) = 2\cos\left(\frac{\pi}{2}-\left(\frac{\pi}{4}-B\right)\right)\cdot\sin\left(\frac{\pi}{2}-\left(\frac{\pi}{4}-A\right)\right)$$

$$= 2\cos(45^\circ+B)\sin(45^\circ+A)$$ RHS

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# Trigonometry

Q $\dfrac{\cos 3\theta + 2\cos 5\theta + \cos 7\theta}{\cos\theta + 2\cos 3\theta + \cos 5\theta} = ?\ \cos 2\theta - \sin 2\theta \tan 3\theta$

$$\frac{(\cos 3\theta + \cos 7\theta) + 2\cos 5\theta}{(\cos\theta + \cos 5\theta) + 2\cos 3\theta}$$

$$\frac{\cancel{2}\cos\left(\frac{3\theta+7\theta}{2}\right)\cos\left(\frac{3\theta-7\theta}{2}\right) + \cancel{2}\cos 5\theta}{\cancel{2}\cos\left(\frac{\theta+5\theta}{2}\right)\cos\left(\frac{\theta-5\theta}{2}\right) + \cancel{2}\cos 3\theta}$$

$$\frac{\cos 5\theta \cdot \cos(+2\theta) + \cos 5\theta}{\cos 3\theta \cdot \cos(+2\theta) + \cos 3\theta} = \frac{\cos 5\theta(\cancel{\cos 2\theta + 1})}{\cos 3\theta(\cancel{\cos 2\theta + 1})} = \frac{\cos 5\theta}{\cos 3\theta}$$

$$\frac{\cos 5\theta}{\cos 3\theta} = \frac{\cos(2\theta + 3\theta)}{\cos 3\theta}$$

$$= \frac{\cos 2\theta \cos 3\theta - \sin 2\theta \cdot \sin 3\theta}{\cos 3\theta}$$

$$= \frac{\cos 2\theta \cdot \cancel{\cos 3\theta}}{\cancel{\cos 3\theta}} - \frac{\sin 2\theta \cdot \sin 3\theta}{\cos 3\theta}$$

$$= \cos 2\theta - \sin 2\theta \tan 3\theta = \underline{\underline{RHS}}$$

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# Trigonometry

Q $\dfrac{\sin(\theta+\phi) - 2\sin\theta + \sin(\theta-\phi)}{\cos(\theta+\phi) - 2\cos\theta + \cos(\theta-\phi)}$

$$\frac{\{\sin(\theta+\phi)+\sin(\theta-\phi)\} - 2\sin\theta}{\{\cos(\theta+\phi)+\cos(\theta-\phi)\} - 2\cos\theta}$$

$$\frac{2\sin\left(\frac{\theta+\cancel{\phi}+\theta-\cancel{\phi}}{2}\right)\cos\left(\frac{\cancel{\theta}+\phi-\cancel{\theta}+\phi}{2}\right) - 2\sin\theta}{2\cos\left(\frac{\theta+\cancel{\phi}+\theta-\cancel{\phi}}{2}\right)\cos\left(\frac{\cancel{\theta}+\phi-\cancel{\theta}+\phi}{2}\right) - 2\cos\theta}$$

$$\frac{2\sin\theta\cdot\cos\phi - 2\sin\theta}{2\cos\theta\cdot\cos\phi - 2\cos\theta} = \frac{\cancel{2}\sin\theta\,(\cancel{\cos\phi-1})}{\cancel{2}\cos\theta\,(\cancel{\cos\phi-1})} = \tan\theta$$

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Q $(\cos 3A+\cos 5A)+(\cos 7A+\cos 15A)= 4\cos 4A\cos 5A\cdot\cos 6A$

$2\cos\left(\frac{3A+5A}{2}\right)\cos\left(\frac{3A-5A}{2}\right)+2\cos\left(\frac{7A+15A}{2}\right)\cos\left(\frac{7A-15A}{2}\right)$

$2\cos 4A\cdot\cos(+A)+2\cos(11A)\cdot\cos(+4A)$

$2\cos 4A\left(\cos A+\cos 11A\right)$

$2\cos 4A\times 2\cos\left(\frac{A+11A}{2}\right)\cdot\cos\left(\frac{A-11A}{2}\right)$

$4\cos 4A\cos(6A)\cos(+5A)$ RHS

Q If $\alpha=\frac{\pi}{19}$

find $\frac{\sin 23\alpha-\sin 3\alpha}{\sin 16\alpha+\sin 4\alpha}=?$

$\frac{\cancel{2}\cos(13\alpha)\cancel{\sin(10\alpha)}}{\cancel{2}\cancel{\sin(10\alpha)}\cos(6\alpha)}$

$\frac{\cos\left(\frac{13\pi}{19}\right)}{\cos\left(\frac{6\pi}{19}\right)}=\frac{\cos\left(\frac{19\pi-6\pi}{19}\right)}{\cos\left(\frac{6\pi}{19}\right)}$

$\boxed{2}$

$\Leftarrow\frac{\cos\left(\pi-\frac{6\pi}{19}\right)}{\cos\left(\frac{6\pi}{19}\right)}$

$-1\Leftarrow\frac{-\cancel{\cos\frac{6\pi}{19}}}{\cancel{\cos\frac{6\pi}{19}}}$

$$\left.\begin{aligned}
\sin C + \sin D &= 2\sin\left(\frac{C+D}{2}\right)\cdot\cos\left(\frac{C-D}{2}\right)\\
\sin C - \sin D &= 2\cos(\,,,\,)\cdot\sin(\,,,\,)\\
\cos C + \cos D &= 2\cos(\,,,,\,)\cdot\cos(\,,,,\,)\\
\cos C - \cos D &= -2\sin(\,,,\,)\cdot\sin(\,,,\,)
\end{aligned}\right\}\quad \boxed{+,\ - \longrightarrow \times}$$

Plus or Minus — Product

When Product is given Change it into Plus or Minus.

$$2\sin A\cdot\cos B = \sin(A+B) + \sin(A-B)$$

$$2\cos A\cdot\sin B = \sin(A+B) - \sin(A-B)$$

$$2\cos A\cdot\cos B = \cos(A+B) + \cos(A-B)$$

$$2\sin A\cdot\sin B = \cos(A-B) - \cos(A+B)\ {**}$$

Nishant Jindal

# Trigonometry

Example XIV

SLLONEY

Q $2 \sin 14^\circ \cos 6^\circ$

$= \sin(14^\circ+6^\circ) + \sin(14^\circ-6^\circ)$

$= \sin 20^\circ + \sin 8^\circ$

Q $2 \cos 20^\circ \cos 10^\circ$ चार चार 3

$\cos(20+10) + \cos(20-10)$

$\cos 30 + \cos(10^\circ)$

Q $2 \cos 71^\circ \cdot \sin 29^\circ$ चार बक्स 2

$\sin(71^\circ+29^\circ) - \sin(71^\circ-29^\circ)$

$\sin(100^\circ) - \sin(42^\circ)$

Q $2 \sin 3A \cdot \sin 5A$ साल सबसे 4

$\cos(3A-5A) - \cos(3A+5A)$

$\cos(-2A) - \cos(8A)$

$\cos 2A - \cos 8A$

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# Trigonometry

Prod. Prod. Prod $\longrightarrow$ Sum या diff.

Q $\cos A \cdot \sin(B-C) + \cos B \sin(C-A) + \cos C \sin(A-B) = ?$

चार मत्र. (2)

$$\frac{1}{2}\left(2\cos A\sin(B-C) + 2\cos B\sin(C-A) + 2\cos C\sin(A-B)\right)$$

$$\frac{1}{2}\left(\sin(A+B-C) - \sin(A-(B-C)) + \sin(B+C-A) - \sin(B-(C-A)) + \sin(C+A-B) - \sin(C-(A-B))\right)$$

$$\frac{1}{2}\left(\sin(\cancel{A+B-C}) - \sin(A-\cancel{B+C}) + \sin(B+\cancel{C-A}) - \sin(\cancel{B-C+A}) + \sin(C+\cancel{A-B}) - \sin(\cancel{C}-A+B)\right)$$

$= 0$