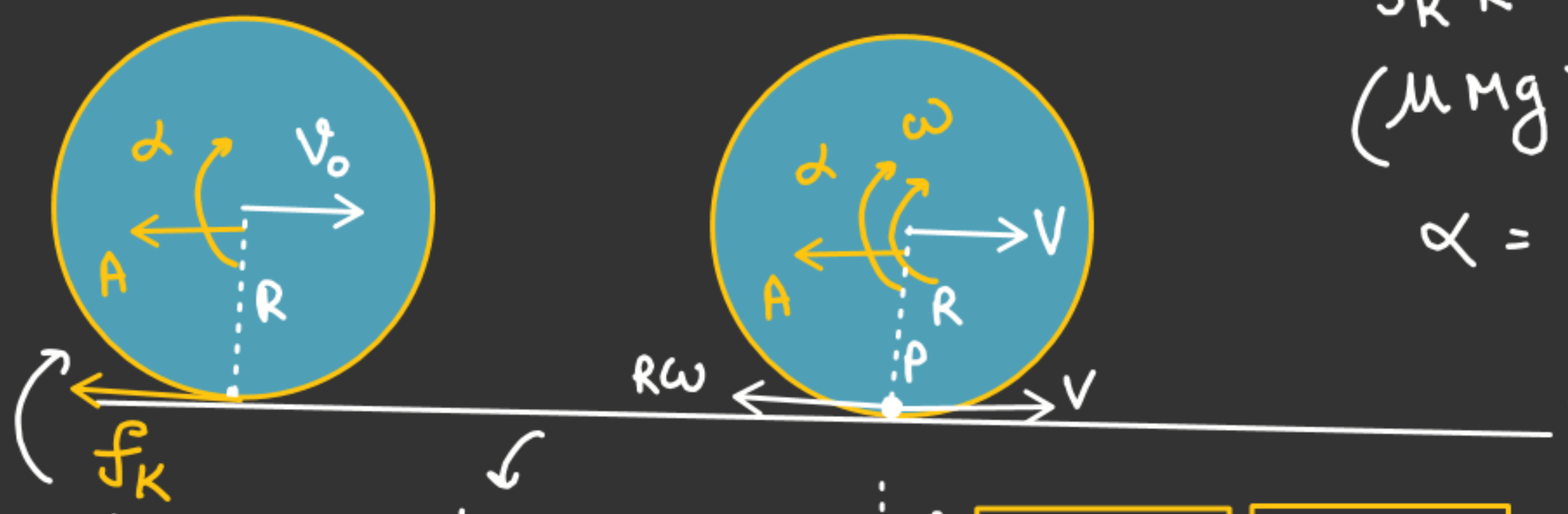
Nishant Jindal

Role of kinetic friction in pure rolling

$t = 0$

(M, I) Disc.

$t = t$



At $t = 0$ (Rough)

$\mu =$ coff of friction b/w body & ground

If $\boxed{V = R\omega}$, $\boxed{A = R\alpha}$

then body starts pure rolling.

$f_K = 0$ After pure rolling start

$$A = \frac{f_K}{M} = \frac{\mu M g}{M} = \mu g$$

$$f_K \cdot R = I\alpha$$

$$(\mu M g) R = I\alpha$$

$$\alpha = \left(\frac{\mu M g R}{I}\right)$$

$$\alpha = \frac{\mu M g R}{\frac{MR^2}{2}} = \left(\frac{2\mu g}{R}\right)$$

$v_0 \rightarrow v = R\omega \leftarrow 0$

$V = R\omega \rightarrow$ At the time of pure rolling.

$$v_0 - \mu g t = R(\alpha t)$$

$$v_0 - \mu g t = R \times \frac{2\mu g}{R} \times t$$

$$v_0 = 3\mu g t$$

$$t = \frac{v_0}{3\mu g}$$

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At the time of pure rolling.

$$v = v_0 - At$$

$$v = v_0 - \mu g\left(\frac{v_0}{3\mu g}\right)$$

$$v = v_0 - \frac{v_0}{3} = \left(\frac{2v_0}{3}\right) \rightarrow \omega = \frac{v}{R}$$

$$\omega = \alpha t = \frac{2\mu g}{R} \times \frac{v_0}{3\mu g}$$

$$\omega = \left(\frac{2v_0}{3R}\right)$$

work done by friction force

$$(W_{f_k})_{translational} = (\Delta K.E)_{translational}$$

$$(W_{f_k})_{translational} = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2$$

$$= \frac{1}{2}m\left[\left(\frac{2v_0}{3}\right)^2 - v_0^2\right]$$

$$= \frac{1}{2}m\left[\frac{4v_0^2}{9} - v_0^2\right] = -\frac{5Mv_0^2}{18} J$$

$$(W_{f_k})_{Rotational} = (\Delta K.E)_{Rotational}$$

$$= \frac{1}{2}I(\omega^2 - 0^2)$$

$$= \frac{1}{2}\left(\frac{MR^2}{2}\right) \times \left(\frac{2v_0}{3R}\right)^2$$

$$= +\frac{Mv_0^2}{9} J$$

$$(W_{f_k})_{net} = (W_{f_k})_{translational} + (W_{f_k})_{Rotational}$$

$$= \left(-\frac{5Mv_0^2}{18} + \frac{Mv_0^2}{9}\right) = -\frac{3Mv_0^2}{18} = -\frac{Mv_0^2}{6}$$

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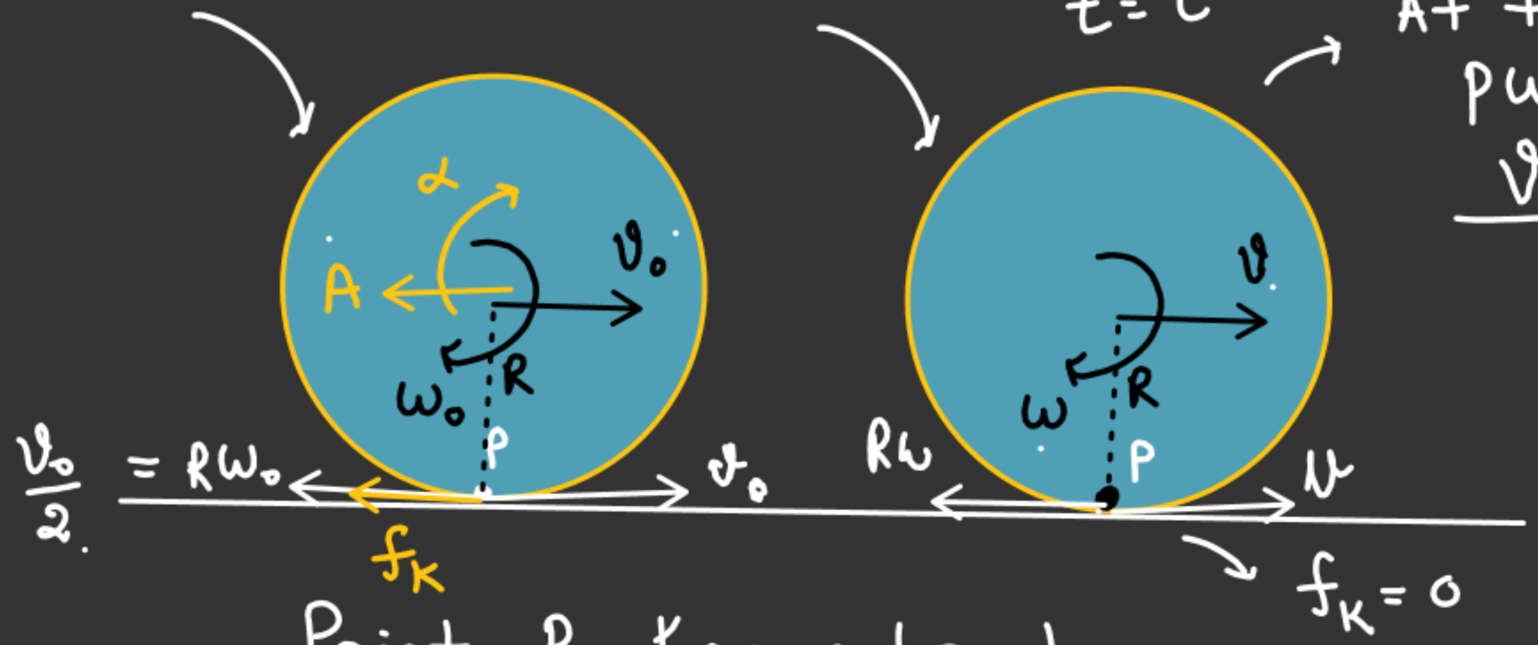
Disc.

At $t=0$ $\left(\omega_0 = \frac{v_0}{2R}\right)$ (given)

$t = t$.

At the time of pure rolling
$v = R\omega$.



Point P has a tendency of forward slipping.
So, f_K acts backward.

$$A = \frac{f_K}{M} = \mu g. \checkmark$$

$$f_K \cdot R = \frac{MR^2}{2} \cdot \alpha$$

$$\mu m g R = \frac{MR^2}{2} \alpha$$

$$\alpha = \left(\frac{2\mu g}{R}\right) \text{ — } \checkmark$$

$$v = R\omega$$

$$\downarrow$$

$$v_0 - At = R(\omega_0 + \alpha t)$$

$$v_0 - \mu g t = R\omega_0 + R\alpha t$$

$$v_0 - \mu g t = R\left(\frac{v_0}{2R}\right) + R\left(\frac{2\mu g}{R}\right)t$$

$$\frac{v_0}{2} = \mu g t + 2\mu g t$$

$$\frac{v_0}{2} = 3\mu g t \Rightarrow t = \left(\frac{v_0}{6\mu g}\right)$$

Time of pure rolling

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At the time of pure rolling.

$$v = v_0 - \mu g t$$

$$v = v_0 - \mu g \frac{v_0}{6\mu g}$$

$$v = \left(v_0 - \frac{v_0}{6}\right)$$

$$v = \left(\frac{5v_0}{6}\right) \text{ m/s}$$

$$\omega = (\omega_0 + \alpha t)$$

$$\omega = \frac{v}{R} = \left(\frac{5v_0}{6R}\right) \text{ rad/sec}$$

Work done by friction force.

$$W_{f_k} = (W_{f_k})_T + (W_{f_k})_R$$

$$= \frac{1}{2}m(v^2 - v_0^2) + \frac{1}{2}\left(\frac{MR^2}{2}\right)(\omega^2 - \omega_0^2)$$

$$= -\left(\frac{3Mv_0^2}{72}\right)$$

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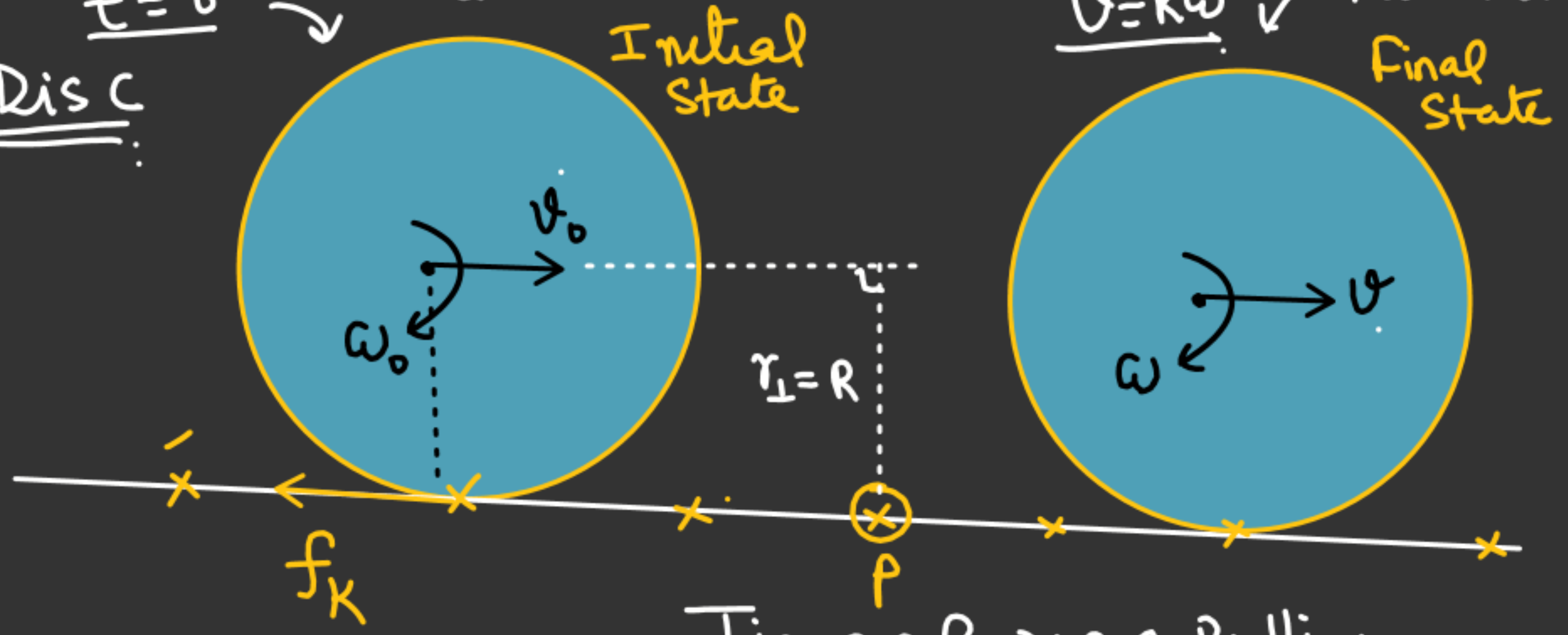
A*: Another Method

$t = 0$ Disc: $\omega_0 = \frac{v_0}{2R}$ (given)

At $t = t$

$v = R\omega$ ✓ Pure Rolling



Time of pure Rolling.

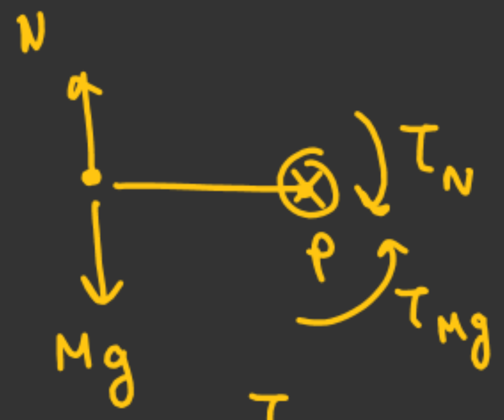
$$A = \mu g.$$

$$v = v_0 - \mu g t$$

$$t = \left(\frac{v_0 - v}{\mu g}\right) = \left(v_0 - \frac{5v_0}{6}\right)\frac{1}{\mu g}$$

$$t = \frac{v_0}{6\mu g} \checkmark$$

$N = Mg$



$\tau_{Mg} = -\tau_N$

$$\vec{L} = I_{com}\vec{\omega} + M(\vec{r} \times \vec{v}_{com})$$

$$|\vec{L}| = (I_{com}\omega + M v r_\perp)$$

A.M.C about any point P on the ground:-

$$-\frac{MR^2}{2}\omega_0 - Mv_0R = -\frac{MR^2}{2}\omega - MvR$$

$$\left(\frac{MR^2}{2} \times \frac{v_0}{2R}\right) + Mv_0R = \frac{MR^2}{2} \times \frac{v}{R} + MvR$$

$$\frac{Mv_0R}{4} + Mv_0R = \frac{MvR}{2} + MvR$$

$$\frac{5}{4}\cancel{M}v_0\cancel{R} = \frac{3}{2}\cancel{M}v\cancel{R}$$

$$\left[\frac{5}{6}v_0 = v\right] \quad \omega = \frac{v}{R} = \left(\frac{5v_0}{6R}\right)$$