DPP – 02
(LOCATION OF ROOTS)

SOLUTION

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1. Find the values of m for which both roots of equation $x^2 - mx + 1 = 0$ are less than unity.

Ans. $(-\infty, -2)$

Sol. Let $f(x) = x^2 - mx + 1$

Apply

(i) $D \geq 0$

$\Rightarrow m^2 - 4 \geq 0 \Rightarrow (m+2)(m-2) \geq 0$

$\Rightarrow\ m \leq -2$ and $m \geq 2$

(ii) $a \,.\, f(1) > 0$

$\Rightarrow\ (1 - m + 1) < 0 \Rightarrow\ m < 2$

(iii) $\alpha + \beta < 2 \Rightarrow m < 2$

From (i), (ii) and (iii), we get

$m < -2$

2. For what real values of m both roots of the equation $x^2 - 6mx + 9m^2 - 2m + 2 = 0$ exceed 3?

Ans. $\left(\frac{11}{9}, \infty\right)$

Sol. Let $f(x) = x^2 - 6mx + 9m^2 - 2m + 2$

Apply

(i) $D \geq 0$

$36m^2 - 36m^2 + 8m - 8 \geq 0$

$m \geq 1$

(ii) $a \cdot f(3) > 0$

$1 \cdot f(3) > 0$

$f(3) > 0$

$9 - 18m + 9m^2 - 2m + 2 > 0$

$9m^2 - 20m + 11 > 0$

$(m-1)(9m-11) > 0$

$m < 1$ and $m > \frac{11}{9}$

(iii) $\alpha + \beta > 6$

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$6m > 6$

$m > 6$

From (i), (ii) and (iii), we get, $m \in \left(\frac{11}{9}, \infty\right)$

3. Find the values of m for which exactly one root of the equation $x^2 - 2mx + m^2 - 1 = 0$ lies in the interval $(-2, 4)$.

Ans. $(-3, -1) \cup (3,5)$

Sol. Let $f(x) = x^2 - 2mx + m^2 - 1$

Apply

(i) $D \geq 0$

$4m^2 - 4(m^2 - 1) \geq 0$

$4 > 0$

It is true all values of m

(ii) $f(-2)f(4) < 0$

$(m^2 + 4m + 3)(m^2 - 8m + 15) < 0$

$(m + 1)(m + 3)(m - 3)(m - 5) < 0$

$m \in (-3, -1) \cup (3,5)$

From (i) and (ii), we get

$m \in (-3, -1) \cup (3,5)$

4. If the equation $ax^2 + bx + c = 0 (a > 0)$ has two roots α and β such that $\alpha < -2$ and $\beta > 2$ then

(A) $b^2 - 4ac > 0$

(B) $4a + 2|b| + c < 0$

(C) $a + |b| + c = 0$

(D) $c < 0$

Ans. (A, B)

Sol. Let $f(x) = ax^2 + bx + c$

Apply

(i) $D > 0$

$\Rightarrow b^2 - 4ac > 0$

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(ii) $af(-2) < 0$

$\Rightarrow 4a - 2b + c < 0$

(iii) $af(2) < 0$

$\Rightarrow 4a + 2b + c < 0$

5. Find the value of ' λ ' for which $2x^2 - 2(2\lambda + 1)x + \lambda(\lambda + 1) = 0$ may have one root less than λ and another root greater than λ.

Ans. $(-\infty, -1) \cup (0, \infty)$

Sol. If $f(x) = (x - \alpha)(x - \beta)$,

then $f(\lambda) = (\lambda - \alpha)(\lambda - \beta) =$ ive as the two factors are of opposite signs by given conditions. Also roots must be real

$\therefore \Delta \geq 0 \Rightarrow 8\left(\lambda^2 + \lambda + \frac{1}{2}\right)$

$= 8\left[\left(\lambda + \frac{1}{2}\right)^2 + \frac{1}{4}\right]$ is +ive, which is true.

Now $f(\lambda) = -$ ive

$\Rightarrow 2\lambda^2 - 2(2\lambda + 1)\lambda + \lambda(\lambda + 1) < 0$

or $-\lambda^2 - \lambda < 0$ or $\lambda(\lambda + 1) > 0$

or $\lambda < -1$ or $\lambda > 0$

6. Find the value of ' a ' for which the equation $2x^2 - 2(2a + 1)x + a(a - 1) = 0$ has roots α and β such that $\alpha < a < \beta$.

Ans. $(-\infty, -1) \cup (0, \infty)$

Sol. Let $f(x) = 2x^2 - 2(2a + 1)x + a(a + 1)$

The condition for a to lie in between the roots is

$af(a) < 0$

$\Rightarrow 2a^2 - 2a(2a + 1) + a(a + 1) < 0$ ($\because A = 2 > 0$, so $f(a) < 0$)

$\Rightarrow -a^2 - a < 0 \Rightarrow a^2 + a > 0 \Rightarrow a > 0$ or $a < -1$.

7. Find all values of b for which $x^2 - x + b - 3 < 0$ for atleast one negative x.

Ans. $(-\infty, 3)$

Sol. Given $x^2 - x + b - 3 < 0$

For at least one negative value of x.

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The coefficient of x^2 is positive. The expression on the left-hand side must be positive at infinitely many points. It can be negative at least one point if the smaller root were negative.

$\frac{1-\sqrt{13-4b}}{2} < 0$

$\Rightarrow \sqrt{13-4b} > 1 \Rightarrow 13-4b > 1$

$\Rightarrow 4b > 12 \Rightarrow b < 3$

$\Rightarrow b \in (-\infty, 3)$

8. Find all possible parameters ' a ' for which $f(x) = (a^2+a-2)x^2 - (a+5)x - 2$ is non +ve for every $x \in [0,1]$.

Ans. $[-3,3]$

Sol. $f(x) = (a^2+a-2)x^2 - (a+5)x - 2$

Case I: If $a^2+a-2 > 0$

$(a+2)(a-1) > 0$

$a \in (-\infty, -2) \cup (1, \infty)$ ... (i)

$\therefore f(0) \leq 0 \Rightarrow -2 \leq 0$ (always)

$f(1) \leq 0 \Rightarrow a \in [-3,3]$ ... (ii)

$\therefore$ from (i) (ii) $a \in [-3,-2) \cup (1,3]$

Case II: If $a^2+a-2 \leq 0$

$a \in [-2,1]$ ... (iii)

(A) If $D \geqslant 0$

$(a+5)^2 - 4(a^2+a-2)(-2) \geqslant 0$

$(a+1)^2 \geqslant 0$

$a \in R$ ... (iv)

and $f(1) < 0$ & $f(0) < 0$

$a \in (-3,3)$ ... (v)

from (iii), (iv) & (v)

$a \in [-2,1]$

(B) If $D < 0$

$(a+1)^2 < 0$

$a = \phi$

$\therefore$ from case (I) & case (II) $\Rightarrow a \in [-3,3]$