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$a = \alpha\beta$

$6 = \alpha\gamma$

$\frac{a}{6} = \frac{\beta}{\gamma} = \frac{4}{3}$

$c^2 < 4ab$

$-c^2 > -4ab$

$f(x) \geq -c^2 > -4ab$

$a = 8$

$(a-5)(a-1)f(0) < 0$

$(a+2)a \geq 0$

$6\alpha^9$

$6\beta^9$

$\left(-\frac{c}{b}, -\frac{c^2}{a}\right)$

$-6\alpha + 2\alpha^8 = 0$

$-\left(3c^2 - 6c^2 + 2c^2\right)$

$2(\alpha^9 - \beta^9)$

$= 3$

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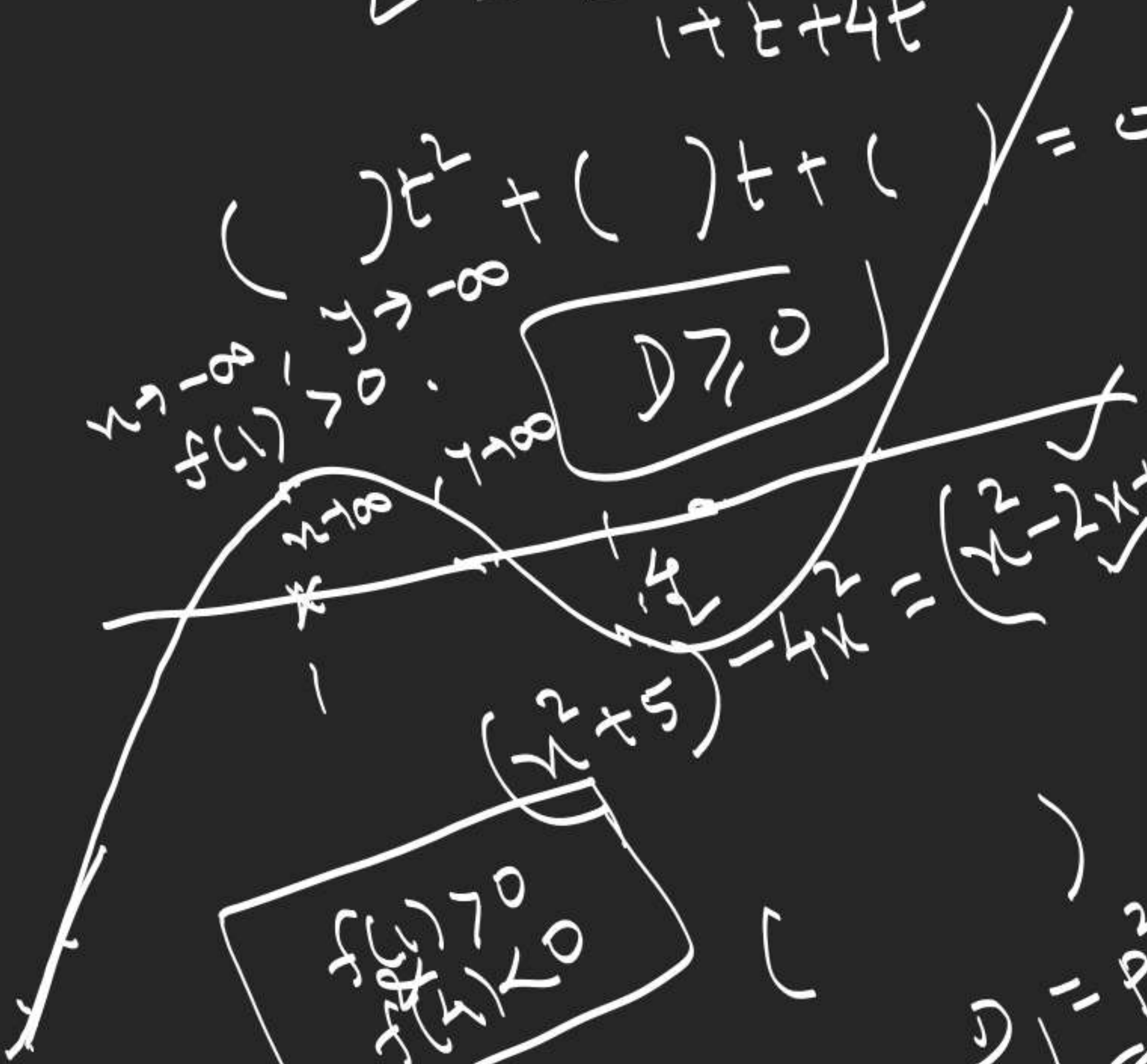
$$E = \frac{1+t^2}{1+t+4t^2}$$

$$(\quad)t^2 + (\quad)t + (\quad) = 0$$

$$\boxed{D \geq 0}$$



$$\boxed{\begin{matrix} f(1) > 0 \\ f(4) < 0 \end{matrix}}$$

$$(x^2+5)^2 - 4x^2 = (x^2-2x+5)(x^2+5+2x)$$

$$(x,y) = (r\cos\theta, r\sin\theta)$$

$$E = \frac{1}{\cos^2\theta + \cos\theta\sin\theta + 4\sin^2\theta}$$

$$1 + 3\sin^2\theta + \sin\theta\cos\theta$$

$$= \frac{2}{2 + 3(1-\cos 2\theta) + \sin 2\theta}$$

$$① \times 3 - ②$$

$$14x^2 - 28x + 70 = 0$$

$$x^2 - 2x + 5 = 0$$

$$D_1 = p^2 - 4q$$

$$D_2 = r^2 - 4s$$

$$D_1 + D_2 = p^2 + r^2 - 2pr = (p-r)^2 \geq 0$$

$$\frac{(x-a)(x-c)}{(x-b)(x-d)} = -\frac{\lambda}{\mu}$$

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$$P(x) - 7 = (x-\alpha)(x-\beta)(x-\gamma)(x-\delta)\,\underline{g(x)}$$

Let $P(k) = 14$

$k \in I$.

Put $x = k$.

$$\underset{\downarrow}{7} = (\underline{k-\alpha})(\underline{k-\beta})(\underline{k-\gamma})(\underline{k-\delta})\,\underline{g(k)}$$

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1. The sum of first 3 consecutive terms of a G.P. is 19 and their product is 216. Find sum of first 'n' terms, also find sum upto infinite terms if exist.

$\frac{a}{r}, a, ar$

$a^3 = 216 \Rightarrow \boxed{a = 6}$

$6\left(\frac{1}{r} + 1 + r\right) = 19$

$r = \frac{3}{2}, \frac{2}{3}$

$r = \frac{3}{2}$

$T_1 = \frac{6}{3/2} = 4$

$S_n = \frac{4\left(\left(\frac{3}{2}\right)^n - 1\right)}{\left(\frac{3}{2} - 1\right)}$

S_∞ not exist

If $r = \frac{2}{3}$, $T_1 = \frac{6}{2/3} = 9$

$S_n = \frac{9\left(1 - \left(\frac{2}{3}\right)^n\right)}{\left(1 - \frac{2}{3}\right)}$

$S_\infty = \frac{9}{1 - \frac{2}{3}} = \boxed{27}$

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2. $a_1, a_2, a_3, a_4, \ldots, a_n$ are in G.P.

$a_1+a_2+a_3 = 13 = a_1(1+r+r^2)$ ✓ ①

$a_1^2+a_2^2+a_3^2 = 91 = a_1^2(1+r^2+r^4)$ ②

$$\frac{①^2}{②} \Rightarrow \frac{(1+r+r^2)^2}{(1+r^2+r^4)} = \frac{(13)^2}{91}$$

$$\Rightarrow \frac{(1+r+r^2)^2}{(1-r+r^2)(1+r+r^2)}$$

$$= \frac{13}{7} = \frac{1+r+r^2}{1-r+r^2}$$

$$\boxed{1+x^2+x^4 = (1+x+x^2)(1-x+x^2)}$$

$6r^2-20r+6=0$

$3r^2-10r+3=0$

$-9r-r$

$(3r-1)(r-3)=0$

$$\boxed{r=3, \frac{1}{3}}$$

If $r=\frac{1}{3}$, $a_1=9$

$$\boxed{S_n = \frac{9\left(1-\left(\frac{1}{3}\right)^n\right)}{\left(1-\frac{1}{3}\right)}}$$

If $r=3$, $a_1=1$

$$\boxed{S_n = \frac{1(3^n-1)}{(3-1)}}$$

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3. The sum of infinite terms of G.P. is 15 and sum of their squares is 45. Find the series.

$a, ar, ar^2, ar^3, \ldots$

$a^2, a^2r^2, a^2r^4, a^2r^6, \ldots$

$$\frac{a}{1-r} = 15 \quad -(1)$$

$$\frac{a^2}{1-r^2} = 45 \quad -(2)$$

$$\frac{(1)^2}{(2)} \Rightarrow \frac{1-r^2}{(1-r)^2} = \frac{15^2}{45} = \boxed{5 = \frac{1+r}{1-r}}$$

$$6r^2 - 10r + 4 = 0$$

$$3r^2 - 5r + 2 = 0$$

$$-3r - 2r$$

$$(3r-2)(r-1) = 0$$

$$\boxed{r = 2/3}$$

$$\boxed{a = 5}$$

$5, \frac{10}{3}, \frac{20}{9}, \frac{40}{27}, \ldots$

4. Find the sum, (i) $9 + 99 + 999 + \ldots + \underbrace{999\ldots9}_{n \text{ times}}$

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$$9+99+999+9999+\cdots+\underbrace{999\cdots9}_{n\text{ times}}$$

$$=(10-1)+(10^2-1)+(10^3-1)+\cdots+(10^n-1)$$

$$=(10+10^2+10^3+\cdots+10^n)-n$$

$$=\frac{10(10^n-1)}{(10-1)}-n$$

$0.6+0.66+0.666+0.6666$

$+\cdots$ + upto n terms

$$=\frac{6}{9}\left(0.9+0.99+0.999+\cdots+0.999\cdots9\right)$$

$$=\frac{6}{9}\left((1-10^{-1})+(1-10^{-2})+(1-10^{-3})+\cdots(1-10^{-n})\right)$$

$$\frac{6}{9}\left[n-\frac{\frac{1}{10}\left(1-\left(\frac{1}{10}\right)^n\right)}{1-\frac{1}{10}}\right]$$

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5: If $S_1, S_2, S_3, S_4, \ldots, S_p$ are sums of infinite G.P. whose first terms are $1, 2, 3, 4, \ldots, p$ and whose common ratios are $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \ldots, \frac{1}{p+1}$ respectively.

P.T. $S_1 + S_2 + S_3 + S_4 + \ldots S_p = \frac{p}{2}(p+3)$

$$\frac{1}{1-\frac{1}{2}} + \frac{2}{1-\frac{1}{3}} + \frac{3}{1-\frac{1}{4}} + \frac{4}{1-\frac{1}{5}} + \ldots + \frac{p}{1-\frac{1}{p+1}}$$

$$= 2 + 3 + 4 + 5 + \ldots + (p+1) = \frac{p}{2}(2+p+1) = \frac{p}{2}(p+3)$$

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6. Insert 4 G.M.s between 5 & 160

$5, G_1, G_2, G_3, G_4, 160$

$160 = 5 \cdot r^5 \Rightarrow r^5 = 32 \Rightarrow \boxed{r=2}$

$10, 20, 40, 80$

7. If AM between a & b is 15 and GM between a & b is 9. Find a, b.

$\frac{a+b}{2} = 15$

$\boxed{a+b=30}$

$\sqrt{ab} = 9$

$\boxed{ab = 81}$

$x^2 - 30x + 81 = 0 \begin{matrix} a \\ b \end{matrix}$

$27, 3$

$(a,b) = (3,27)$ or $(27,3)$

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8. If a, b, c are in G.P. and x, y are the A.M.s between a, b and b, c respectively.

P.T. (i) $\frac{1}{x}+\frac{1}{y}=\frac{2}{b}$ (ii) $\frac{a}{x}+\frac{c}{y}=2$.

$x=\frac{a+b}{2}$, $y=\frac{b+c}{2}$, $b^2=ac$

(i) $\frac{2}{a+b}+\frac{2}{b+c}=\frac{2(b+c+a+b)}{(a+b)(b+c)}=\frac{2(2b+a+c)}{ab+ac+b^2+bc}$

$ac \to b^2$

$=\frac{2(2b+a+c)}{ab+2b^2+bc}=\frac{2\cancel{(2b+a+c)}}{b\cancel{(a+2b+c)}}=\frac{2}{b}$

JEE Adv.

Ex-II (19-20)

Ex-IV → (1-13)