Nishant Jindal

$$(6\sqrt{6}+14)^{2n+1}-(6\sqrt{6}-14)^{2n+1}=2\left[{}^{2n+1}C_1(6\sqrt{6})^{2n}(14)+{}^{2n+1}C_3(6\sqrt{6})^{2n-2}(14)^3+\cdots\right]$$

$$[x]+\underbrace{(\{x\}-N)}_{\in I}=2K\quad, K\in I.$$

$$0\le\{x\}<1$$
$$0<N<1$$
$$0<-N<0$$ (sic)
$$-1<-N<0$$
$$-1<\{x\}-N<1$$

$$\boxed{\{x\}-N=0}$$

$$x\{x\}=xN=(20)^{2n+1}$$

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$$(\sqrt{3}+1)^{2n}+(\sqrt{3}-1)^{2n} = 2^n\left((2+\sqrt{3})^n+(2-\sqrt{3})^n\right)$$

$$[x]+\left(\{x\}+N\right) = 2^{n+1}\left[{}^nC_0\,2^n+{}^nC_2\,2^{n-2}(\sqrt{3})^2+{}^nC_4\,2^{n-4}(\sqrt{3})^4+\cdots\right]$$

$$= [x]+1$$

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1. Find the remainder when

(i) 2^{2005} is divided by $\underline{17}$

$2(17-1)^{501} = 2(17\lambda - 1)$

$= 17\lambda - 17 + 15 = 17K + \boxed{15}$

(ii) 5^{99} is divided by $13 \Rightarrow 5(26-1)^{49} = 13\lambda - 5 = 13\lambda - 13 + 8$

$= 13K + \boxed{8}$

(iii) 7^{103} is divided by $25 \to 7(50-1)^{51} = 25\lambda - 7$

$= 25K + \boxed{18}$

(iv) $2^{(32)^{32}}$ is divided by 7

(v) $(32)^{(32)^{32}}$ is divided by 7

$(32)^{32} = 2^{160} = (3-1)^{160} = 3K+1$

$2^{5(3K+1)} = 2^{3\lambda+2}$

$= 4(1+7)^{\lambda} = 7\mu + \boxed{4}$

$2^{3K+1} = 2(1+7)^{K} = 7\lambda + \boxed{2}$

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$$\boxed{r\,{}^{n}C_r} = n\,{}^{n-1}C_{r-1}$$

div. by r.

$$\frac{{}^{n}C_r}{r+1} = \frac{{}^{n+1}C_{r+1}}{n+1}$$

Note → ① If r, n are coprime, then ${}^{n}C_r$ is divisible by 'n'.

② m, n are coprime, then $m, 2m, 3m, \ldots, (n-1)m$ give all remainders from 1 to $n-1$ if divided by n.

$nQ_1 + R = K_1 m$, $K_2 m = nQ_2 + R$

$(K_1 - K_2)m = n(Q_1 - Q_2)$ impossible

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$$(1+x)^n = {}^nC_0 + {}^nC_1 x + {}^nC_2 x^2 + {}^nC_3 x^3 + \cdots + {}^nC_r x^r + \cdots + {}^nC_n x^n$$

$x=1$, $\quad 2^n = {}^nC_0 + {}^nC_1 + {}^nC_2 + {}^nC_3 + \cdots + {}^nC_n \Rightarrow \boxed{\sum_{r=0}^{n} {}^nC_r = 2^n}$

$x=-1$, $\quad 0 = {}^nC_0 - {}^nC_1 + {}^nC_2 - {}^nC_3 + \cdots (-1)^n\,{}^nC_n \Rightarrow \boxed{\sum_{r=0}^{n} {}^nC_r (-1)^r = 0}$

$$2^{n-1} = {}^nC_0 + {}^nC_2 + {}^nC_4 + {}^nC_6 + \cdots$$

$$2^{n-1} = {}^nC_1 + {}^nC_3 + {}^nC_5 + {}^nC_7 + \cdots$$

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1. $ {}^nC_1 + {}^nC_3 2 + {}^nC_5 2^2 + {}^nC_7 2^3 + \dots = ?$

$= \frac{1}{\sqrt{2}} \left[{}^nC_1(\sqrt{2}) + {}^nC_3(\sqrt{2})^3 + {}^nC_5(\sqrt{2})^5 + {}^nC_7(\sqrt{2})^7 + \dots \right]$

$(1+x)^n$

$$= \frac{(1+\sqrt{2})^n - (1-\sqrt{2})^n}{2\sqrt{2}}$$

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$$(1+x)^n = {}^nC_0 + {}^nC_1 x + {}^nC_2 x^2 + {}^nC_3 x^3 + \cdots + {}^nC_r x^r + \cdots + {}^nC_n x^n$$

$$\sum_{r=0}^{n} r(r-1)(r-2)\,(-1)^r\,{}^nC_r = 0$$

$$n(1+x)^{n-1} = 1\cdot{}^nC_1 x^0 + 2\cdot{}^nC_2 x + 3\,{}^nC_3 x^2 + 4\,{}^nC_4 x^3 + \cdots + r\,{}^nC_r x^{r-1} + \cdots + n\,{}^nC_n x^{n-1}$$

$$\sum_{r=0}^{n} (-1)^r r^3\,{}^nC_r = 0$$

$x=1$, $$n\cdot 2^{n-1} = 1\cdot{}^nC_1 + 2\cdot{}^nC_2 + 3\cdot{}^nC_3 + 4\cdot{}^nC_4 + \cdots + n\cdot{}^nC_n = \sum_{r=1}^{n} r\,{}^nC_r$$

$$\boxed{\sum_{r=0}^{n} (-1)^r r^m\,{}^nC_r = 0, \quad m \in W}$$

$x=-1$, $$0 = 1\cdot{}^nC_1 - 2\,{}^nC_2 + 3\,{}^nC_3 - 4\,{}^nC_4 + \cdots + (-1)^{n-1}\,{}^nC_n$$

$$\sum_{r=0}^{n} (-1)^r r\,{}^nC_r = 0$$

$\underline{x=-1}$ $$0 = \sum_{r=0}^{n} r(r-1)\,{}^nC_r(-1)^r$$

$$n(n-1)(1+x)^{n-2} = 2\cdot 1\cdot{}^nC_2 x^0 + 3\cdot 2\,{}^nC_3 x + 4\cdot 3\,{}^nC_4 x^2 + \cdots + n(n-1)\,{}^nC_n x^{n-2}$$

$x=1$, $$n(n-1)\,2^{n-2} = \sum_{r=2} r(r-1)\,{}^nC_r$$

$$\sum_{r=0}^{n} (-1)^r r^2\,{}^nC_r = 0$$

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1. ${}^nC_0 + 5\,{}^nC_1 + 9\,{}^nC_2 + 13\,{}^nC_3 + \cdots$ + upto $(n+1)$ terms = ?

$$= \sum_{r=0}^{n} (1+4r)\,{}^nC_r = \sum_{r=0}^{n} {}^nC_r + 4\sum_{r=1}^{n} r\,{}^nC_r$$

$$= \sum_{r=0}^{n} {}^nC_r + 4n\sum_{r=1}^{n} {}^{n-1}C_{r-1}$$

$$= 2^n + 4n\,2^{n-1}$$

$$= (2n+1)2^n$$

$\boxed{\text{Ex-II (Complete)}}$

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$$(1+x)^n = \sum_{r=0}^{n} {}^nC_r\, x^r \qquad\qquad \sum_{r=0}^{n} (4r+1)\, {}^nC_r = ?$$

$$(1+x^4)^n = \sum_{r=0}^{n} {}^nC_r\, x^{4r}$$

$$x(1+x^4)^n = \sum_{r=0}^{n} {}^nC_r\, x^{4r+1}$$

$$(1+x^4)^n + x\, n(1+x^4)^{n-1}(4x^3) = \sum_{r=0}^{n} {}^nC_r\, (4r+1)\, x^{4r}$$

$x=1$ $$2^n + n\,2^{n-1}(4) = \sum_{r=0}^{n} {}^nC_r(4r+1)$$