

THERMAL EXPANSION

★★

Case of bimetallic Strip

$$\alpha_1 < \alpha_2$$



$$\alpha_1 > \alpha_2$$



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Case of bimetallic Strip

$$\alpha_1 > \alpha_2$$



$$\Delta T = (T - T_0)$$

γ = Mean radius

$$L_1 = (\gamma + t) \theta$$

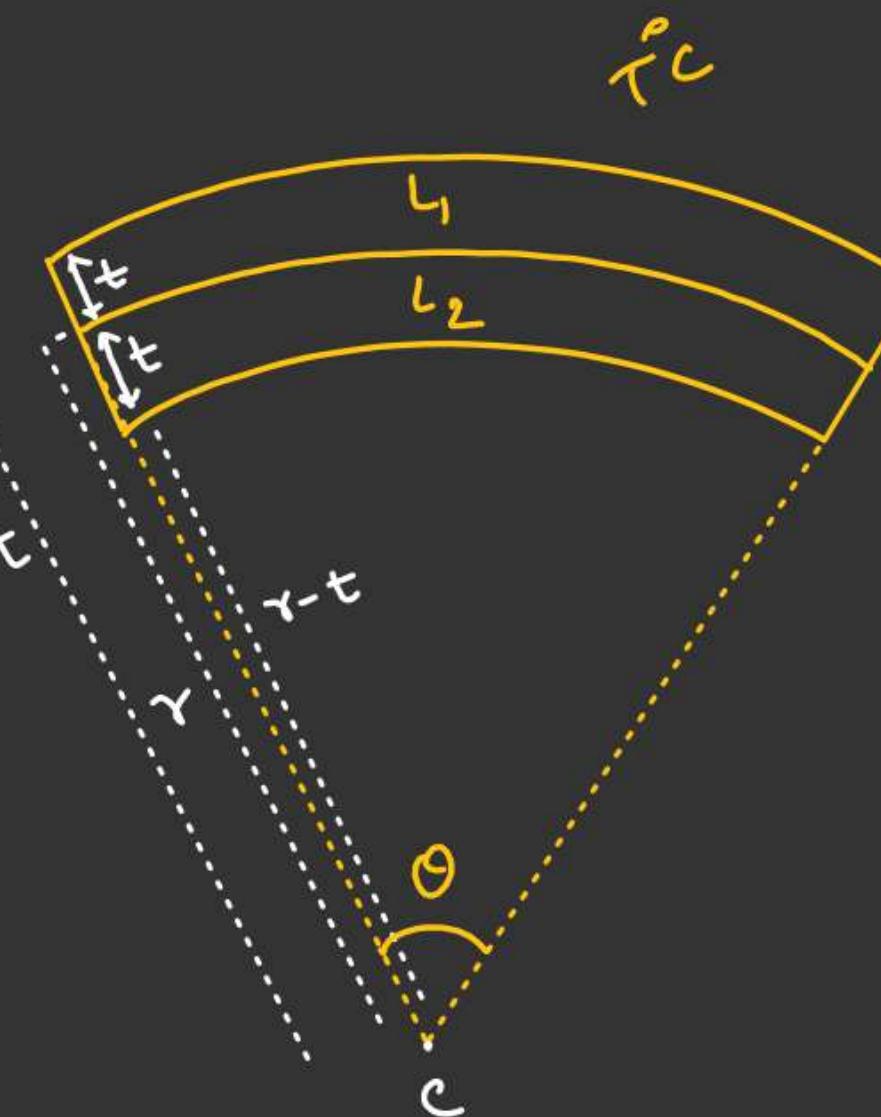
$$L_2 = (\gamma - t) \theta$$

$$L_1 = L (1 + \alpha_1 \Delta T)$$

$$L_2 = L (1 + \alpha_2 \Delta T)$$

$$\frac{L_1}{L_2} = \left(\frac{\gamma + t}{\gamma - t} \right) \Rightarrow \frac{L (1 + \alpha_1 \Delta T)}{L (1 + \alpha_2 \Delta T)} = \left(\frac{\gamma + t}{\gamma - t} \right)$$

=)



$$\frac{(1+\alpha_1 \Delta T)}{(1+\alpha_2 \Delta T)} = \frac{\gamma+t}{\gamma-t}$$

$$\frac{(1+\alpha_1 \Delta T) + (1+\alpha_2 \Delta T)}{(1+\alpha_1 \Delta T) - (1+\alpha_2 \Delta T)} = \frac{(\gamma+t) + (\gamma-t)}{(\gamma+t) - (\gamma-t)}$$

$$\Rightarrow \frac{2 + (\alpha_1 + \alpha_2) \Delta T}{(\alpha_1 - \alpha_2) \Delta T} = \frac{2\gamma}{2t}$$

$$\Rightarrow \boxed{\gamma = \frac{t [2 + (\alpha_1 + \alpha_2) \Delta T]}{(\alpha_1 - \alpha_2) \Delta T}}$$

$$\rightarrow \gamma = \frac{2t \left[1 + \frac{(\alpha_1 + \alpha_2) \Delta T}{2} \right]}{(\alpha_1 - \alpha_2) \Delta T}$$

|>> $(\alpha_1 + \alpha_2) \Delta T$

$$\boxed{\gamma = \frac{2t}{(\alpha_1 - \alpha_2) \Delta T}}$$

ΔABC equilateral triangle form by

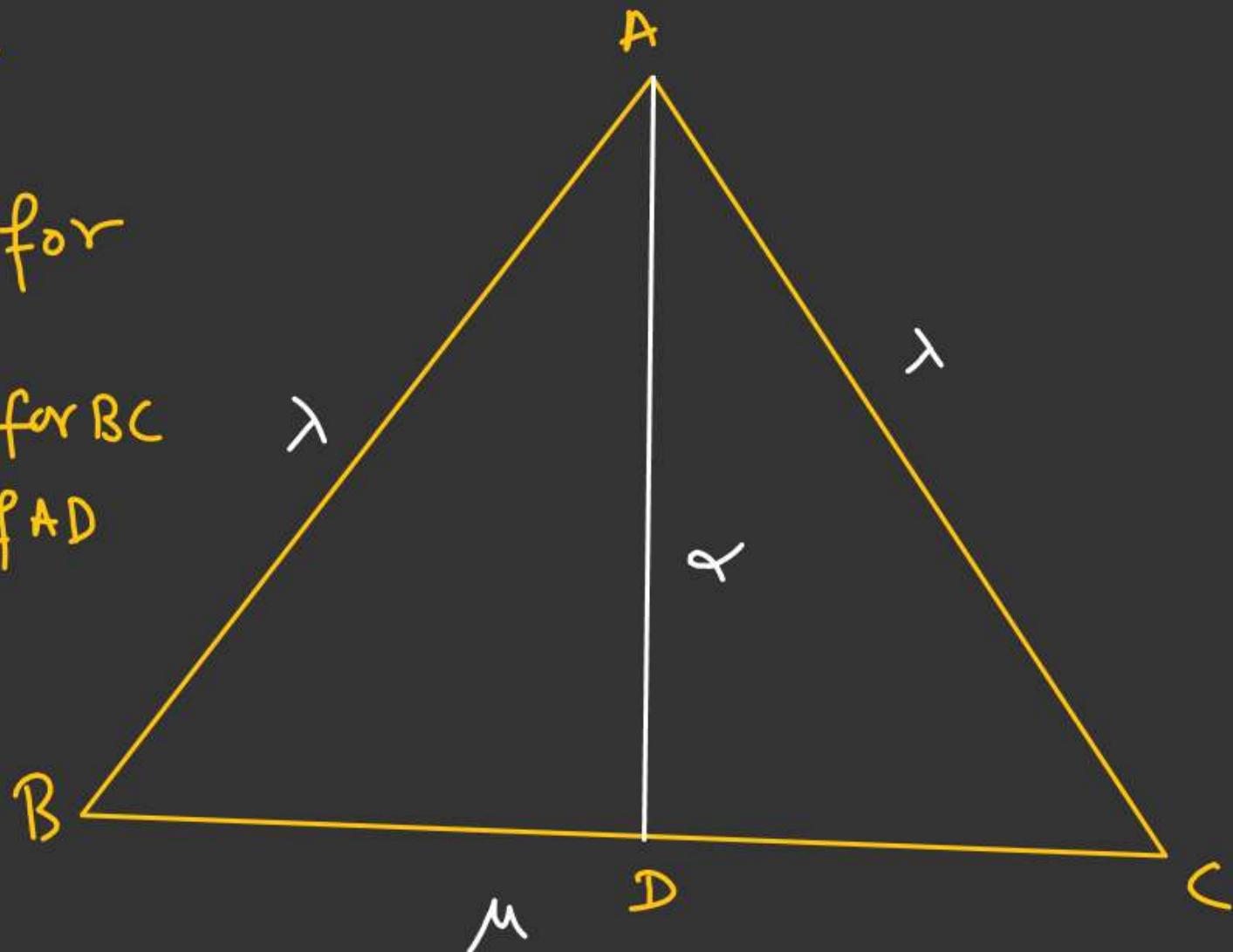
3-rods AB , BC & CA

λ is the coeffⁿ of linear expansion for
 AB and AC

μ be the coeffⁿ of linear expansion for BC

α be the coeffⁿ of linear expansion of AD

Find α so that the frame will
not deformed if heated from
 $T_0^{\circ}\text{C}$ to $T^{\circ}\text{C}$



For \overline{AB} .

$$L' = L(1 + \lambda \Delta T)$$

$| \gg \alpha \Delta T \text{ or } \mu \Delta T \text{ or } \lambda \Delta T$

For \overline{BD} .

$$\frac{L'}{2} = \frac{L}{2}(1 + \mu \Delta T)$$

$| \gg \alpha \Delta T$

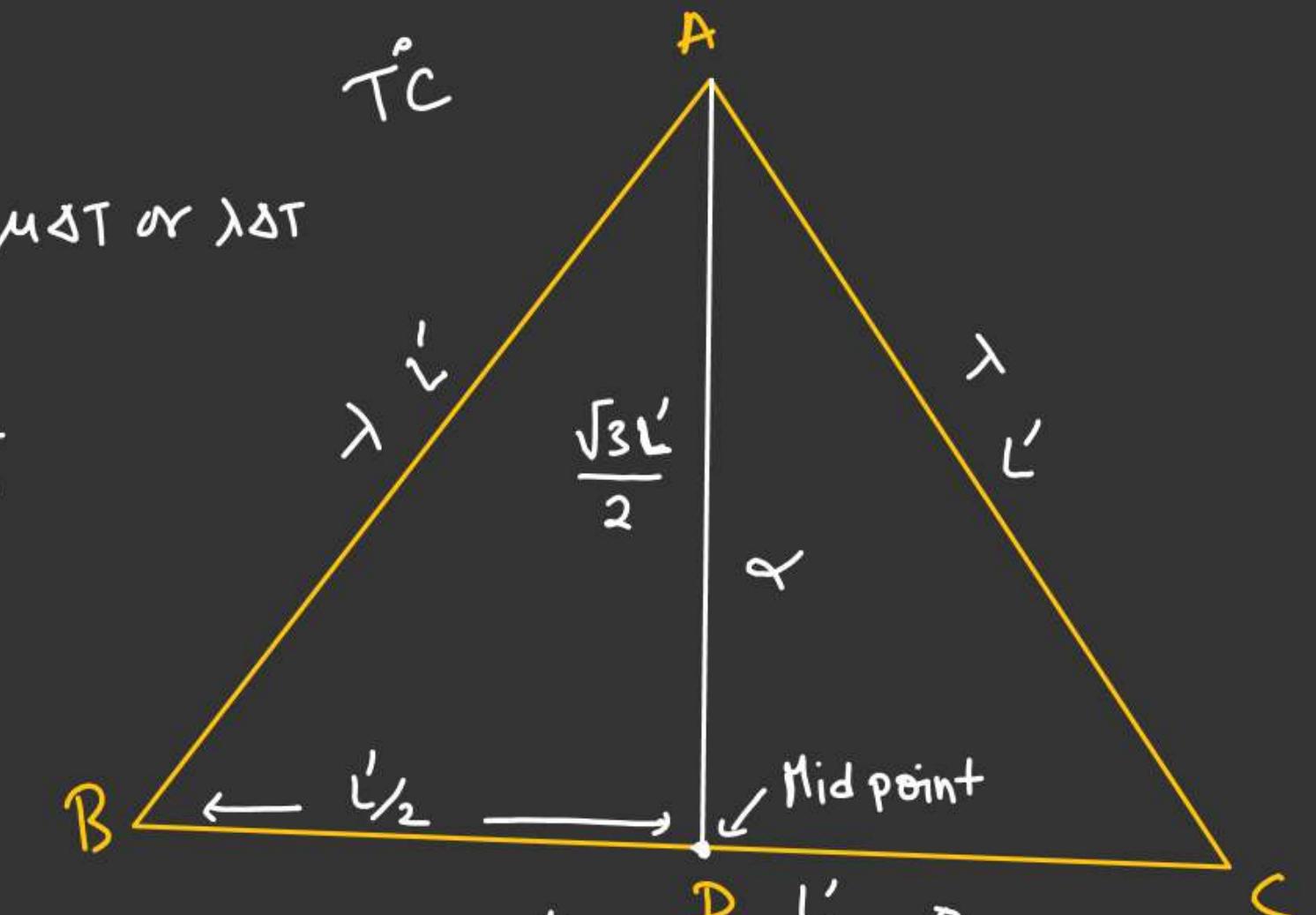
For \overline{AD}

$$\frac{\sqrt{3} L'}{2} = \frac{\sqrt{3} L}{2}(1 + \alpha \Delta T)$$

$$L'^2 = \left(\frac{L'}{2}\right)^2 + \left(\frac{\sqrt{3} L'}{2}\right)^2$$

$$(1 + \lambda \Delta T)^2 = \frac{1}{4} (1 + \mu \Delta T)^2 + \frac{3}{4} L'^2 (1 + \alpha \Delta T)^2$$

$$(1 + \lambda \Delta T)^2 = \frac{1}{4} (1 + \mu \Delta T)^2 + \frac{3}{4} (1 + \alpha \Delta T)^2$$



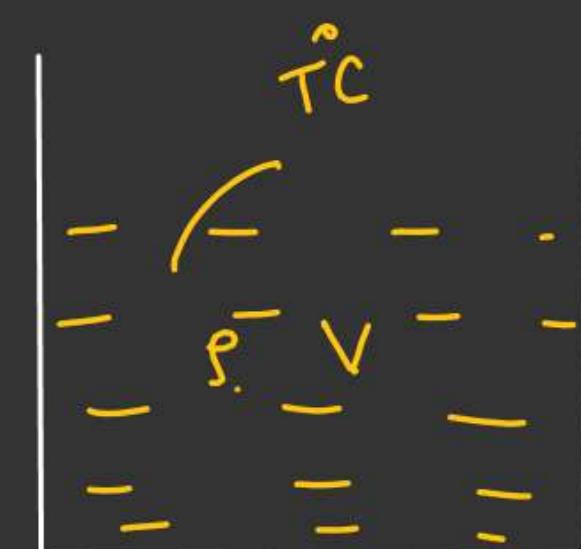
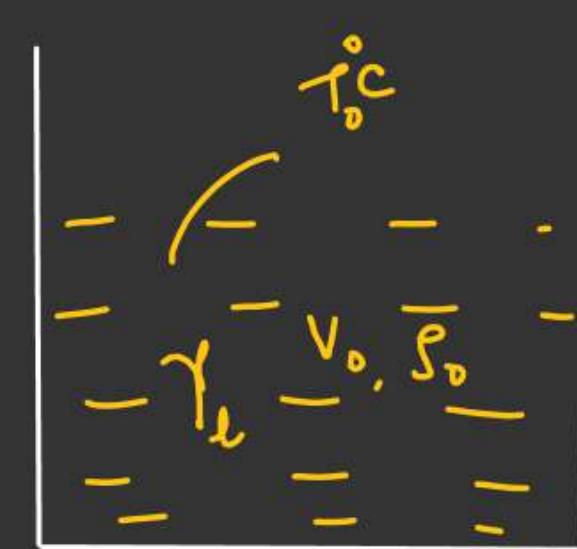
$$\begin{aligned}
 1 + \frac{\lambda^2 \Delta T^2}{4} + 2\lambda \Delta T &= \frac{1}{4} (1 + \mu^2 \Delta T^2 + 2\mu \Delta T) \\
 + \frac{3}{4} (1 + \alpha^2 \Delta T^2 + 2\alpha \Delta T) \\
 1 + 2\lambda \Delta T &= 1 + \frac{\mu}{2} \Delta T + \frac{3}{2} \alpha \Delta T \\
 2\lambda - \frac{\mu}{2} &= \frac{3}{2} \alpha \Rightarrow \alpha = \frac{4\mu - 1}{3} \lambda
 \end{aligned}$$

$\Delta \rho$ Thermal Expansion in liquid.Case when only expansion of liquid not vessel

$$\rho = \frac{m}{V}$$

$$(\rho \propto \frac{1}{V})$$

$$\frac{m = C}{\text{---}}$$



$$1 >> \gamma_L \Delta T$$

$$\rho = \rho_0 (1 + \gamma_L \Delta T)^{-1}$$

$$\rho = \rho_0 (1 - \gamma_L \Delta T)$$

✓

$\rho_0 \rightarrow$ Density of liquid at $T_0^{\circ}\text{C}$

$$V = V_0 (1 + \gamma_L \Delta T)$$

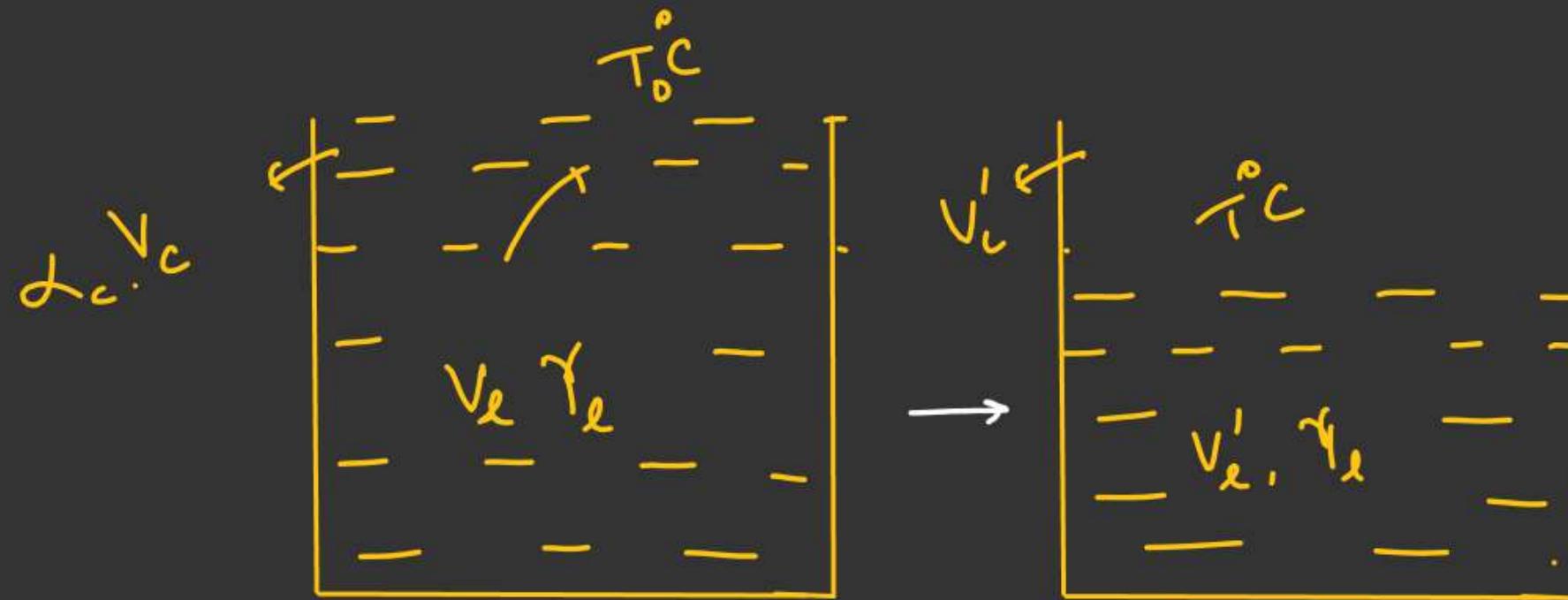
$\rho \rightarrow$ Density of liquid at $T_C^{\circ}\text{C}$

$$\frac{\rho}{\rho_0} = \frac{V_0}{V}$$

$$\boxed{\rho = \frac{\rho_0}{(1 + \gamma_L \Delta T)}}$$

 T_C°

$$\frac{\rho}{\rho_0} = \frac{V_0}{V_0(1 + \gamma_L \Delta T)}$$

At $T^{\circ}\text{C}$ At $T_0^{\circ}\text{C}$ ✓ V_l = Volume of liquid✓ V_c = Volume of container

$\gamma_l > 3\alpha_c \Rightarrow$ level of liquid increase
 $\gamma_l < 3\alpha_c \Rightarrow$ level of liquid decrease.

At $T^{\circ}\text{C}$ V'_l = Volume of liquid V'_c = Volume of vessel

For liquid

$$V'_l = V_l(1 + \gamma_l \Delta T)$$

$$V'_c = V_c(1 + \gamma_c \Delta T)$$

$$\gamma_c = 3\alpha_c$$

$$V'_c = V_c(1 + 3\alpha_c \Delta T)$$

$$(\gamma = 3\alpha)$$

$$(V_l = V_c = V_0)$$

$$\Delta V_l = V'_l - V'_c$$

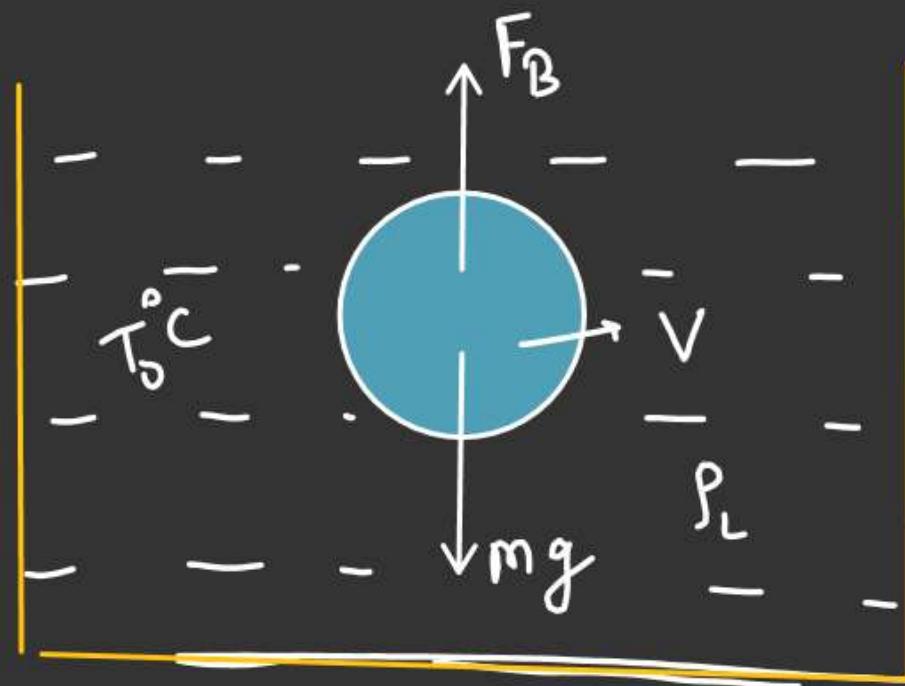
$$= V_l(1 + \gamma_l \Delta T) - V_c(1 + 3\alpha_c \Delta T)$$

$$\Delta V_l = V_0(\gamma_l - 3\alpha_c)\Delta T$$

~~A/A:~~

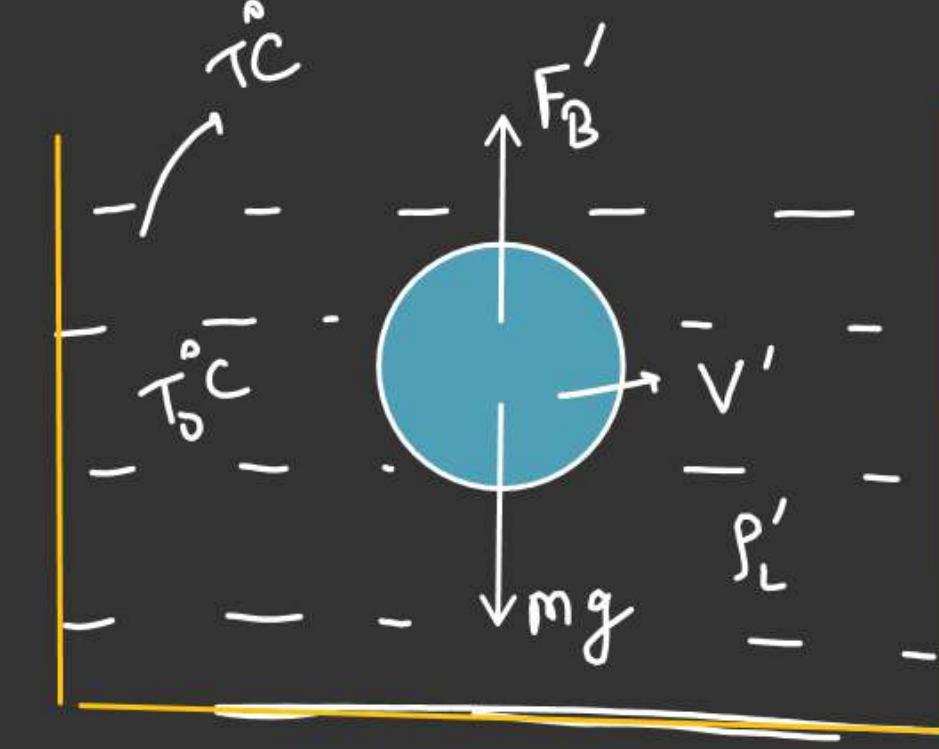
Effect of Temperature on the Apparent Weight of the body fully submerged

No expansion of vessel



$$W_{app} = (mg - F_B)$$

$$W_{app} = mg - V \rho_L g$$



$$W'_{app} = mg - F'_B = mg - V' \rho'_L g$$

$$W'_{app} - W_{app} = V \rho_L g - V' \rho'_L g$$

F_B = Weight of displaced liquid
 $= V_e \rho_L g$
 $V_e = V_s$
 V_s = Volume of submerged part of body

sof gss

$$\frac{W'_{app} - W_{app}}{V'} = V\rho_L g - V'\rho'_L g$$

$$V' = V(1 + \gamma_s \Delta T)$$

$$\rho'_L = \frac{\rho_L}{(1 + \gamma_e \Delta T)} = \rho_L (1 - \gamma_e \Delta T)$$

$$\Delta W_{app} = V\rho_L g - [V(1 + \gamma_s \Delta T) \rho_L (1 - \gamma_e \Delta T) g]$$

$$\begin{aligned}\Delta W_{app} &= V\rho_L g - V\rho_L g [(1 + \gamma_s \Delta T)(1 - \gamma_e \Delta T)] \\ &= V\rho_L g - V\rho_L g [1 - \gamma_e \Delta T + \gamma_s \Delta T - \gamma_s \gamma_e \Delta T^2] \\ &= V\rho_L g [1 - (1 + (\gamma_s - \gamma_e) \Delta T)]\end{aligned}$$

$$\boxed{\Delta W_{app} = V\rho_L g [\gamma_e - \gamma_s] \Delta T}$$

① $\Delta W_{app} > 0 \Rightarrow \gamma_e > \gamma_s$

② $\Delta W_{app} < 0 \Rightarrow (W_{app})_f < (W_{app})_i \Rightarrow \gamma_s > \gamma_e$

③ if $\gamma_s = \gamma_e$

$$3\alpha_s = \gamma_e$$

No change in
apparent weight.

SCALE ERROR

$$l_a = l_0 [1 + \alpha(T - T_0)]$$

l_a = Actual length

l_0 = Observed or Measured length

T_0 = Temperature at which Scale gives Correct reading

T = Temperature at which Observation is made