Fundamentals of Mathematics

Q31 $\log_2\left(\frac{1}{7^{\log_7 .125}}\right) = ?$

$\log_2\left(\frac{\cancel{1000}^{8}}{\cancel{125}}\right) = \log_2 8$

$= \log_2 2^3 = 3 \times 1 = 3.$

सारे Qs नहीं बने तो घबराना नहीं है।

Q $\left(\frac{1}{4}\right)^{-\log_2 3}$ → 2, $\frac{1}{4}$ me Koi Relation h Kya?

$\left(\frac{1}{4}\right)^{-\frac{1}{2}} = (4)^{\frac{1}{2}} = \sqrt{4} = 2$

$(2^{+2})^{+\log_2 3}$

$2^{2\log_2 3}$

$= 2^{\log_2 3^2}$

$= 3^2 = 9$

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Fundamentals of Mathematics

Q $e^{\ln 85} = ?$

$e^{\log_e 85}$

$= 85$

Q_3

$e^{x \log_e a}$

$e^{\log_e a^x}$

$= a^x$

Q_4 $2^{\log_2 x^2} - 3x - 4 = 0$ find x?

$x^2 - 3x - 4 = 0$

$(x-4)(x+1) = 0$

$x = 4, -1$

Check

Fundamentals of Mathematics

Q5 $2^{2\log_2 x} - 3x - 4 = 0$ find x

$$2^{\log_2 x^2} - 3x - 4 = 0$$

$$x^2 - 3x - 4 = 0$$

$$(x-4)(x+1) = 0$$

$$x = 4 \;\&\; x = -1 \;\times$$

Check

$2^{2\log_2(-1)} - 3\times -1 - 4 = 0$

ND

Check

$2^{2\log_2 4} - 3\times 4 - 4 = 0$

$$\boxed{x = 4}$$

Q6: If $K^{\log_2 5} = 16$ find value of $K^{(\log_2 5)^2} = ?$

Demand: $K^{(\log_2 5)^2} = K^{\log_2 5 \times \log_2 5}$

$$\boxed{\text{Sona hai} \rightarrow a^{m\times n} = (a^m)^n}$$

$$= \left(K^{\log_2 5}\right)^{\log_2 5} = (16)^{\log_2 5}$$

$$= 2^{4\log_2 5} = 2^{\log_2 5^4} = 5^4$$

$$= 625$$

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Fundamentals of Mathematics

$7^3 + 11^2 = 343 + 121 = 464$

Q. If $a^{\log_3 7} = 27$, $b^{\log_7 11} = 49$

then value of $a^{(\log_3 7)^2} + b^{(\log_7 11)^2} = ?$

$\left(a^{\log_3 7}\right)^{\log_3 7} + \left(b^{\log_7 11}\right)^{\log_7 11}$

$(27)^{\log_3 7} + (49)^{\log_7 11}$

$3^{3\log_3 7} + 7^{2\log_7 11}$

$3^{\log_3 7^3} + 7^{\log_7 11^2}$

Fundamental Identity 2nd

① $a^{\log_a N} = N$

② We can write any exponential fxn in $e^{\ln \dots}$ form

$\boxed{a^x = e^{x \ln a}}$

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Fundamentals of Mathematics

Ex 1) $2^{x} = e^{x \ln 2}$

2) $3^{4} = e^{4 \ln 3}$

3) $2^{-2} = e^{-2 \ln 2}$

4) $2^{\ln z} = ?$

$e^{\ln z \cdot \ln 2}$

Q8 $x^{\ln y - \ln z} \cdot y^{\ln z - \ln x} \cdot z^{\ln x - \ln y} = ?$

$e^{(\ln y - \ln z)\ln x} \cdot e^{(\ln z - \ln x)\ln y} \cdot e^{(\ln x - \ln y)\ln z}$

$e^{\ln x \cancel{\ln y} - \ln x \cancel{\ln z} + \ln \cancel{z} \cdot \ln y - \ln \cancel{x} \cdot \ln y + \ln \cancel{x} \ln z - \ln \cancel{y} \ln z}$

e^{0}

$e^{0} = 1$

Fundamentals of Mathematics

$\log x^{1/2} = \log\sqrt{x}$

Q9 $\log(x+y) = \log 2 + \frac{1}{2}\log x + \frac{1}{2}\log y$ then $x-y=0$

[T/F]

$\log(x+y) = \log 2 + \log\sqrt{x} + \log\sqrt{y}$ $\{\log A + \log B + \log C$

$= \log A\cdot B\cdot C\}$

$\cancel{\log}(x+y) = \cancel{\log}(2\sqrt{xy})$

$x+y = 2\sqrt{xy}$

$x+y-2\sqrt{x}\sqrt{y} = 0$

$(\sqrt{x}-\sqrt{y})^2 = 0$

$\sqrt{x}-\sqrt{y} = 0$

$\sqrt{x} = \sqrt{y}$

$x = y$

$\boxed{x-y=0}$

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Fundamentals of Mathematics

Q10 If $\log_2(\log_3(\log_4 x)) = 0$ & $\log_4(\log_3(\log_2 y)) = 0$ & $\log_3(\log_4(\log_2 z)) = 0$

then $x > y > z$ [T/F]

$\log_2(\log_3(\log_4 x)) = 0$

$\log_3(\log_4 x) = 2^0 = 1$

$\log_4 x = 3^1 = 3$

$\boxed{x = 4^3 = 64}$

$\log_4(\log_3(\log_2 y)) = 0$

$\log_3(\log_2 y) = 4^0 = 1$

$(\log_2 y) = 3^1 = 3$

$\boxed{y = 2^3 = 8}$

$\boxed{x > z > y}$

$\log_3(\log_4(\log_2 z)) = 0$

$\log_4(\log_2 z) = 3^0 = 1$

$(\log_2 z) = 4^1 = 4$

$\boxed{z = 2^4 = 16}$

Fundamentals of Mathematics

Q11 $\log_{\frac{4}{3}}\left(\dfrac{56+\sqrt{56+\sqrt{56+\sqrt{56+\cdots\infty}}}}{\sqrt{64\sqrt{64\sqrt{64\cdots\infty}}}}\right) = ?$

$\log_{\frac{4}{3}}\left(\dfrac{\cancel{64}^{\,1}}{\cancel{64}}\right)$

$\log_{\frac{4}{3}} 1 = 0$

Dhyaan se

$S = 56+\sqrt{56+\sqrt{56+\sqrt{56\cdots\infty}}}$

$S = 56+\sqrt{S}$

$S - \sqrt{S} - 56 = 0$

$t^2 - t - 56 = 0$

$(t-8)(t+7) = 0$

$t = 8$ or $t = -7$

$\sqrt{S} = 8$ or $\sqrt{S} = -7$

$+ \neq -ve$

$\boxed{S = 64}$

let $N = \sqrt{64\sqrt{64\sqrt{64\cdots\infty}}}$

$N = \sqrt{64N}$

$\sqrt{N} = \sqrt{64}$

$\boxed{N = 64}$

Fundamentals of Mathematics

Q12 Let A denotes real value of x satisfying eqⁿ $x^3+3x^2+3x+4 = \log_{12}(1728)$

& $B = \sqrt{132+\sqrt{132+\sqrt{132+\dots\infty}}}$ then $A-B = ?$ $= -1-12$
$= -13$

$B = \sqrt{132+\boxed{\sqrt{132+\sqrt{132+\dots\infty}}}}$

$B = \sqrt{132+B}$

$B^2 = 132+B$

$B^2-B-132=0$

$(B-12)(B+11)=0$

$B=12,\ B=-11$

$x^3+3x^2+3x+4 = \log_{12}1728$

$= \log_{12}(12^3)$

$= 3\boxed{\log_{12}12} = 3\times1 = 3$

$x^3+3x^2+3x+4 = 3 \Rightarrow x^3+3x^2+3x+1=0$

$\Rightarrow (x+1)^3=0 \Rightarrow x+1=0$

$\boxed{x=-1}$ $A=-1$

I seqⁿ को Solve करके
जो x आएगा
that is A

IIT Concept

Fundamentals of Mathematics

Rk:- 1) $\boxed{\begin{aligned} A^3+B^3 &= (A+B)(A^2+B^2-AB) \\ A^3-B^3 &= (A-B)(A^2+B^2+AB) \end{aligned}}$

2) $A^3+B^3+C^3-3ABC = (A+B+C)(A^2+B^2+C^2-AB-BC-CA)$

$= (A+B+C)\frac{1}{2}(2A^2+2B^2+2C^2-2AB-2BC-2AC)$

$= (A+B+C)\frac{1}{2}\{(A^2-2AB+B^2)+(A^2-2AC+C^2)+(B^2-2BC+C^2)\}$

$$\boxed{A^3+B^3+C^3-3ABC = (A+B+C)\{(A-B)^2+(C-A)^2+(B-C)^2\}}$$

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Fundamentals of Mathematics

$$A^3+B^3+C^3-3ABC = \frac{(A+B+C)}{2}\left\{(A-B)^2+(B-C)^2+(C-A)^2\right\}$$

$(A-B)^2 = 0$, $(B-C)^2 = 0$, $(C-A)^2 = 0$

$\boxed{A+B+C=0}$ $\qquad$ $\boxed{A=B=C}$

$\Rightarrow A^3+B^3+C^3=3ABC$ $\qquad$ $A^3+B^3+C^3=3ABC$

BASE CHANGE Theorem.

Lalch Theorem.

RIP'N"

Agr Pasndida Base nahi dia ho Use BCT

$$\boxed{\log_N M = \frac{\log_P M}{\log_P N}}$$

AWAJ

Agr 2 log Multiply me diye ho
Use BCT.

Niche Aao Log Denge
Tumko

Fundamentals of Mathematics

Q12 $\log_a b \times \log_b a = ?$ find value?

$$\frac{\cancel{\log_e b}}{\cancel{\log_e a}} \times \frac{\cancel{\log_e a}}{\cancel{\log_e b}} = 1$$

Q13 $\log_b a \times \log_c b \times \log_d c = ?$

$$\frac{\log a}{\cancel{\log b}} \times \frac{\cancel{\log b}}{\cancel{\log c}} \times \frac{\cancel{\log c}}{\log d} = \frac{\log a}{\cancel{\log} d} = \log_d a$$

Q14 $\log_{\boxed{N^\alpha}} M^\beta = ?$

BCT

$$\Rightarrow \frac{\log M^\beta}{\log N^\alpha}$$

$$\Rightarrow \frac{\beta}{\alpha} \cdot \frac{\log M}{\cancel{\log} N}$$

$$\frac{\beta}{\alpha} \cdot \log_N M$$

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Result

$$\boxed{\begin{aligned} &1)\ \log_N M^{\beta} = \beta \log_N M \\ &2)\ \log_{N^{\alpha}} M^{\beta} = \frac{\beta}{\alpha} \cdot \log_N M \end{aligned}}$$

3) $\dfrac{1}{\log_a b} = \log_b a$

Q14 $\dfrac{1}{\log_{xy} xyz} + \dfrac{1}{\log_{yz} xyz} + \dfrac{1}{\log_{zx} xyz} = ?$

logo Ko niche rhna Psnd nhi

$\log_{xyz} xy + \log_{xyz} yz + \log_{xyz} zx$

$= \log_{xyz}(xy \cdot yz \cdot zx)$

$= \log_{xyz}(xyz)^2 = 2$

Fundamentals of Mathematics

Q15 If $\log_a b + \log_b c + \log_c a$ $\boxed{\text{vanishes}}$ when a, b, c are +ve & real different than Unity then value of $(\log_a b)^3 + (\log_b c)^3 + (\log_c a)^3 = ?$

$\log_a b + \log_b c + \log_c a = 0$ given.

$$\boxed{\begin{array}{l} A+B+C=0 \\ A^3+B^3+C^3=3ABC \end{array}}$$

So $(\log_a b)^3 + (\log_b c)^3 + (\log_c a)^3 = 3 \log_a b \times \log_b c \times \log_c a$

$$= 3 \frac{\cancel{\log b}}{\cancel{\log a}} \times \frac{\cancel{\log c}}{\cancel{\log b}} \times \frac{\cancel{\log a}}{\cancel{\log c}} = 3$$

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Fundamentals of Mathematics

16) $81^{\frac{1}{\log_5 3}} + 27^{\log_9 36} + 3^{\frac{4}{\log_7 9}}$

$(81)^{\log_3 5} + 3^{3\log_{3^2} 36} + 3^{4\log_9 7}$

$3^{4\log_3 5} + 3^{\frac{3}{2}\log_3 36} + 3^{4\log_{3^2} 7}$

$3^{\log_3 5^4} + 3^{\log_3 (36)^{3/2}} + 3^{\frac{2 4}{2}\log_3 7}$

$5^4 + (36)^{3/2} + 7^2$

$625 + 216 + 49$

DY

$a^{3/2} = a\sqrt{a}$

$36^{3/2} = 36\sqrt{36}$

$= 36 \times 6$

$= 216$