Nishant Jindal

VECTOR

31. Let $\vec{a} = \hat{i} + \hat{j}$ and $\vec{b} = 2\hat{i} - \hat{k}$. The point of intersection of the lines $\vec{r} \times \vec{a} = \vec{b} \times \vec{a}$ and $\vec{r} \times \vec{b} = \vec{a} \times \vec{b}$ is

(A) $-\hat{i} + \hat{j} + 2\hat{k}$ (B) $3\hat{i} - \hat{j} + \hat{k}$ (C) $3\hat{i} + \hat{j} - \hat{k}$ (D) $\hat{i} - \hat{j} - \hat{k}$

1) $\vec{r} \times \vec{a} = \vec{b} \times \vec{a} \Rightarrow \vec{r} \times \vec{a} - \vec{b} \times \vec{a} = 0$

$(\vec{r} - \vec{b}) \times \vec{a} = 0 \Rightarrow \vec{r} - \vec{b} \parallel \vec{a} \Rightarrow \vec{r} - \vec{b} = \lambda \vec{a}$

$\vec{r} = \vec{b} + \lambda \vec{a}$

2) $\vec{r} \times \vec{b} = \vec{a} \times \vec{b} \Rightarrow \vec{r} \times \vec{b} - \vec{a} \times \vec{b} = 0 \Rightarrow (\vec{r} - \vec{a}) \times \vec{b} = 0 \Rightarrow \vec{r} - \vec{a} \parallel \vec{b}$

$\Rightarrow \vec{r} - \vec{a} = \mu \vec{b}$

$\Rightarrow \vec{r} = \vec{a} + \mu \vec{b}$

$\vec{r} = \langle 2, 0, -1 \rangle + \lambda \langle 1, 1, 0 \rangle$

$\langle 2 + \lambda, 0 + \lambda, -1 \rangle$

PoI $= \langle 3, 1, -1 \rangle$ ✓

$\vec{r} = \langle 1, 1, 0 \rangle + \mu \langle 2, 0, -1 \rangle$

$\vec{r} = \langle 1 + 2\mu, 1 + 0, -\mu \rangle$

$2 + \lambda = 1 + 2\mu$ | $\lambda = 2$ | $-1 = -\mu$

$\mu = 1$

32. Vector $\vec{a}$ and $\vec{b}$ make an angle $\theta = \frac{2\pi}{3}$, if $|\vec{a}| = 1, |\vec{b}| = 2$, then $\{(\vec{a} + 3\vec{b}) \times (3\vec{a} - \vec{b})\}^2$ is equal to

(A) 225 (B) 250 (C) 275 (D) 300

$0 - a \times b + 9\vec{b} \times \vec{a} + 0$

$= \{10 (b \times a)\}^2 = 100 |b \times a|^2$

$= 100 |a|^2 |b|^2 \sin^2\theta \quad \frac{2\pi}{3}$

$= 100 \times 1 \times 4 \times \frac{3}{4}$

$= 300$

Nishant Jindal

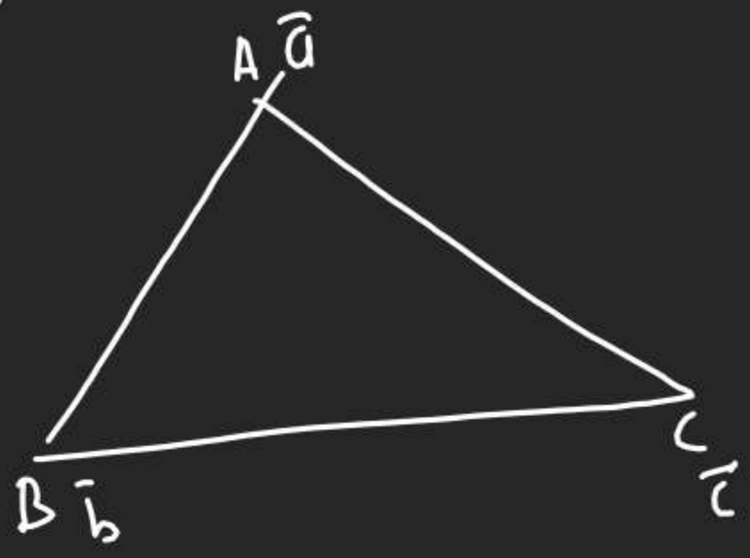
33. Unit vector perpendicular to the plane of the triangle ABC with position vectors $\vec{a}, \vec{b}, \vec{c}$ of the vertices A, B, C is

(A) $\dfrac{(\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a})}{\Delta}$

(B) $\dfrac{(\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a})}{2\Delta}$

(C) $\dfrac{(\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a})}{4\Delta}$

(D) none of these



$$\hat{v} = \frac{\frac{1}{2}(\vec{BC}\times\vec{BA})}{\frac{1}{2}|\vec{BC}\times\vec{BA}|} = \frac{(\vec{BC}\times\vec{BA})}{2\Delta}$$

Nishant Jindal

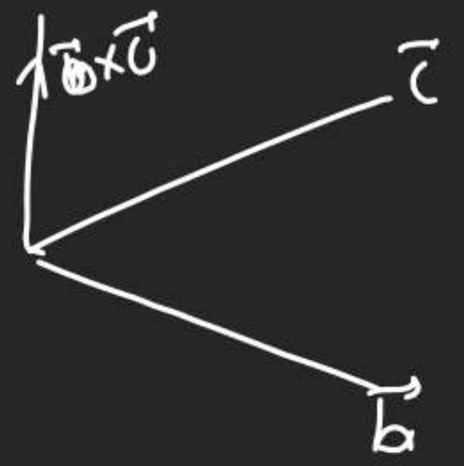
34. If $\vec{b}$ and $\vec{c}$ are two non-collinear vectors such that $\vec{a} \parallel (\vec{b} \times \vec{c})$, then $(\vec{a} \times \vec{b}) \cdot (\vec{a} \times \vec{c})$ is equal to

(A) $\vec{a}^2(\vec{b} \cdot \vec{c})$ ✓

(B) $\vec{b}^2(\vec{a} \cdot \vec{c})$

(C) $\vec{e}^2(\vec{a}, \vec{b})$

(D) none of these



$\vec{a}\cdot(\vec{b}\times(\vec{a}\times\vec{c}))$

$\vec{a}\,((\vec{b}\cdot\vec{c})\vec{a}-(\vec{b}\cdot\vec{a})\vec{c})$

$|a|^2(\vec{b}\cdot\vec{c})-\underbrace{(\vec{a}\cdot\vec{c})}_{0}\times\underbrace{(\vec{b}\cdot\vec{a})}_{0}$

$\vec{a}^2(\vec{b}\cdot\vec{c})$

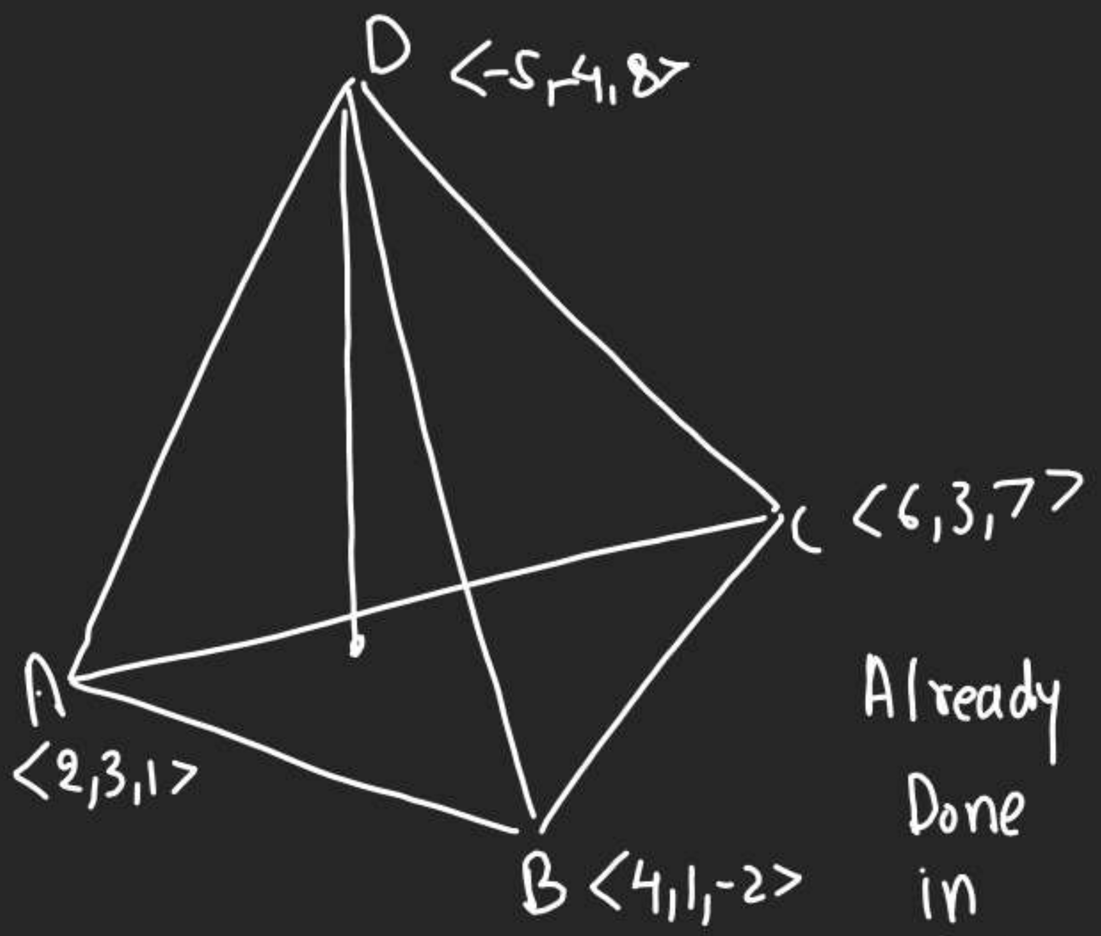
36. Given the vertices $A(2,3,1), B(4,1,-2), C(6,3,7) \& D(-5,-4,8)$ of a tetrahedron. The length of the altitude drawn from the vertex D is

(A) 7

(B) 9

(C) 11

(D) none of these



Already Done in Master Problem.

Nishant Jindal

VECTOR

37. For a non zero vector $\vec{A}$ If the equations $\vec{A} \cdot \vec{B} = \vec{A} \cdot \vec{C}$ and $\vec{A} \times \vec{B} = \vec{A} \times \vec{C}$ hold simultaneously, then

(A) $\vec{A}$ is perpendicular to $\vec{B} - \vec{C}$

(B) $\vec{A} = \vec{B}$

(C) $\vec{B} = \vec{C}$

(D) $\vec{C} = \vec{A}$

$A \cdot (\vec{B} - \vec{C}) = 0 \Rightarrow \vec{A} \perp (\vec{B} - \vec{C})$

$A \times (\vec{B} - \vec{C}) = 0 \Rightarrow \vec{A} \parallel (\vec{B} - \vec{C})$

$\vec{B} = \vec{C}$

Nishant Jindal

38. If $\mathbf{u}$ and $\mathbf{v}$ are unit vectors and θ is the acute angle between them, then $2\mathbf{u} \times 3\mathbf{v}$ is a unit vector for

(A) Exactly two values of θ

(B) More than two values of θ

(C) No value of θ

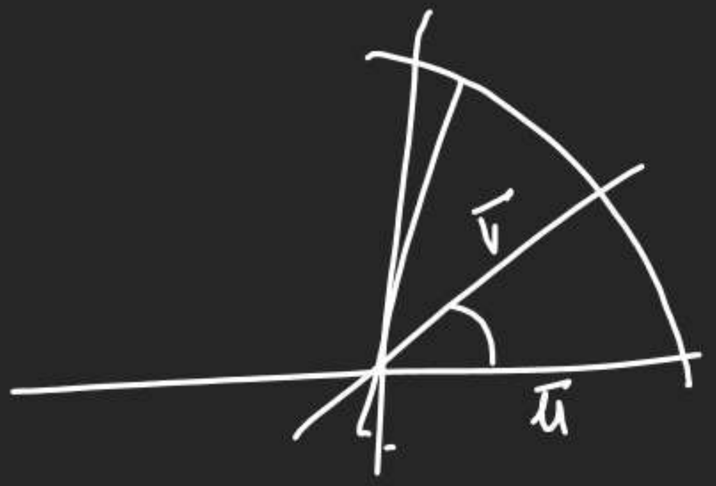
(D) Exactly one value of θ ✓✓

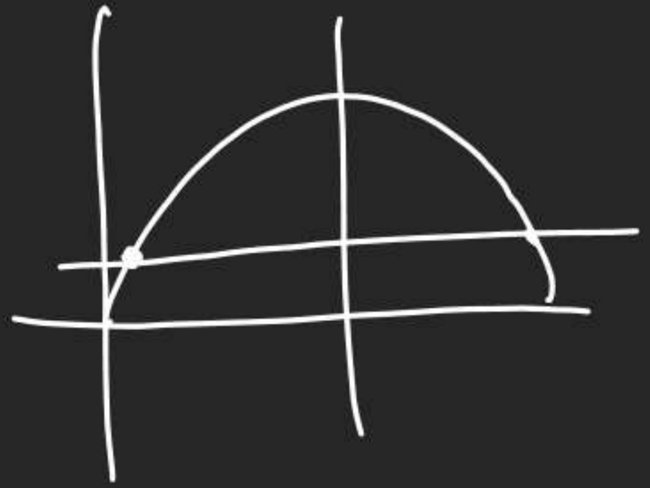
$|6(u \times v)| = 1$

$|u \times v| = \frac{1}{6}$

$|u||v| \cdot \sin\theta = \frac{1}{6}$.

$\sin\theta = \frac{1}{6}$.





VECTOR

39. If $\vec{u} = \vec{a} - \vec{b}, \vec{v} = \vec{a} + \vec{b}$ and $|\vec{a}| = |\vec{b}| = 2$, then $|\vec{u} \times \vec{v}|$ is equal to

Imp.

(A) $\sqrt{2(16 - (\vec{a} \cdot \vec{b})^2)}$

(B) $2\sqrt{(16 - (\vec{a} \cdot \vec{b})^2)}$

(C) $2\sqrt{(4 - (\vec{a} \cdot \vec{b})^2)}$

(D) $\sqrt{2(4 - (\vec{a} \cdot \vec{b})^2)}$

$|u \times v| = |(a-b) \times (a+b)|$

$= |0 + a \times b - b \times a - 0|$

$= 2|a \times b|$

$= 2\sqrt{|a|^2|b|^2 - (a \cdot b)^2}$

$= 2\sqrt{4 \times 4 - (a \cdot b)^2}$

$= 2\sqrt{16 - (a \cdot b)^2}$

$(a \times b)^2 = |a|^2|b|^2 - (a \cdot b)^2$

41. If $\vec{a} = \vec{b} + \vec{c}, \vec{b} \times \vec{d} = 0$ and $\vec{c} \cdot \vec{d} = 0$ then $\frac{\vec{d} \times (\vec{a} \times \vec{d})}{\vec{d}^2}$ is equal to

(A) a (B) $\vec{b}$ (C) c (D) $\vec{d}$

$(\vec{a} = \vec{b} + \vec{c}) \times \vec{d}$

$\vec{a} \times \vec{d} = \cancel{b \times d} + c \times d$

$\vec{a} \times \vec{d} = \vec{c} \times \vec{d}$

$\left.\begin{array}{l} \vec{d} \parallel \vec{b} \\ \vec{d} \perp \vec{c} \end{array}\right\} \vec{b} \perp \vec{c}$

$\frac{\vec{d} \times (\vec{a} \times \vec{d})}{d^2} = \frac{\vec{d} \times (c \times d)}{d^2}$

$= \frac{(d \cdot d)\vec{c} - \cancel{(d \cdot c)\vec{d}}}{d^2}$

$= \vec{c}$

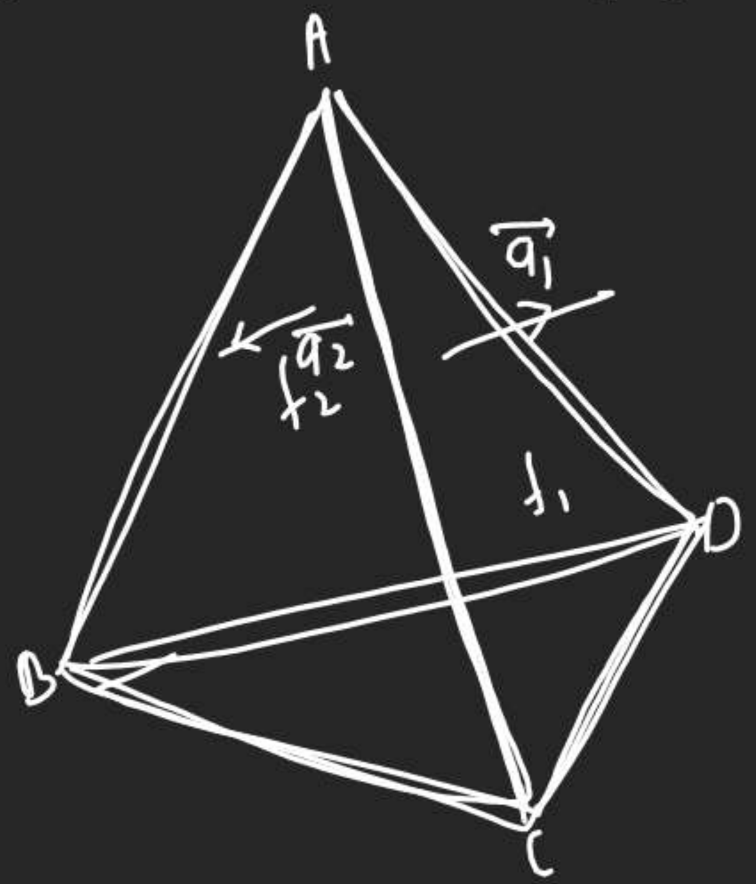
Nishant Jindal

42. Consider a tetrahedron with faces f_1, f_2, f_3, f_4 Let $\vec{a}_1, \vec{a}_2, \vec{a}_3, \vec{a}_4$ be the vectors whose magnitudes are respectively equal to the areas of f_1, f_2, f_3, f_4 and whose directions are perpendicular to these faces in the outward direction. Then

(A) $\vec{a}_1 + \vec{a}_2 + \vec{a}_3 + \vec{a}_4 = 0$ (B) $\vec{a}_1 + \vec{a}_3 = \vec{a}_2 + \vec{a}_4$

(C) $\vec{a}_1 + \vec{a}_2 = \vec{a}_3 + \vec{a}_4$ (D) none of these

$\vec{AB} + \vec{BC} = \vec{AC}$



$\vec{a_1} = \vec{CD} \times \vec{CA}$

$\vec{a_4} = \vec{AB} \times \vec{AB}$

$\vec{a_2} = \vec{AB} \times \vec{AC}$

$\vec{a_3} = \vec{CB} \times \vec{CD}$

$\vec{AB} \times (\vec{AC} - \vec{AD}) = \vec{AB} \times (\vec{CA} + \vec{AD})$

$= -\vec{AB} \times \vec{CD}$

$\vec{CD} \times (\vec{CA} - \vec{CB}) = -\vec{CD} \times (\vec{AC} + \vec{CB})$

$= -\vec{CD} \times \vec{AB}$

43. Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}, \vec{b} = \hat{i} - \hat{j} + 2\hat{k}$ and $\vec{c} = x\hat{i} + (x-2)\hat{j} - \hat{k}$. If the vector $\vec{c}$ lies in the plane of $\vec{a}$ and $\vec{b}$, then x equals

(A) 0 (B) 1 (C) -4 (D) -2

C, A, B Coplanar

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & -1 & 2 \\ x & x-2 & -1 \end{vmatrix} = 0$$

Nishant Jindal

VECTOR

$[a\ b\ c] = a\cdot(b\times c)$

46. The value of $[(\vec{a}+2\vec{b}-\vec{c}),(\vec{a}-\vec{b}),(\vec{a}-\vec{b}-\vec{c})]$ is equal to the box product

(A) $[\vec{a}\vec{b}\vec{c}]$ (B) $2[\vec{a}\vec{b}\vec{c}]$ (C) $3[\vec{a}\vec{b}\vec{c}]$ (D) $4[\vec{a}\vec{b}\vec{c}]$

$(a+2b-c)\cdot\{(a-b)\times(a-b-c)\}$

$(a+2b-c)\cdot\{0-\cancel{a\times b}-a\times c-\cancel{b\times a}+0+b\times c\}$

$(a+2b-c)\cdot\{b\times c-a\times c\}$

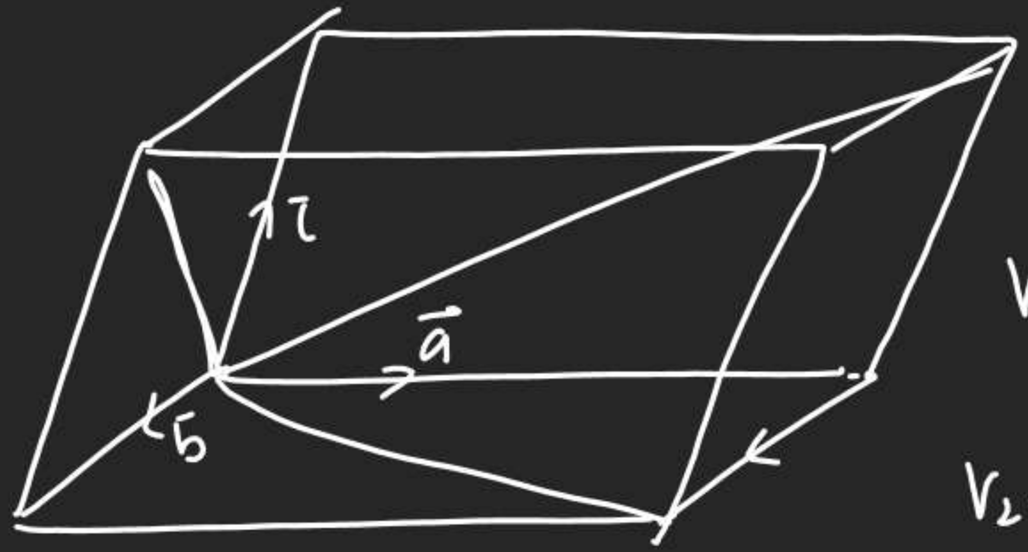
$[a\ b\ c]-0+2\times 0-2[b\ a\ c]+0-0$

$[a\ b\ c]\qquad +2[a\ b\ c]$

$=3[a\ b\ c]$

47. The volume of the parallelopiped constructed on the diagonals of the faces of the given rectangular parallelopiped is m times the volume of the given parallelopiped. Then m is equal to

(A) 2 (B) 3 (C) 4 (D) none of these



$V_1 = [\vec{a}\ \vec{b}\ \vec{c}]$

$V_2 = [\vec{a}+\vec{b}\quad b+c\quad c+a]$

$= 2[\vec{a}\ b\ c]$

48. If $\vec{u}, \vec{v}$ and $\vec{w}$ are three non-coplanar vectors, then $(\vec{u}+\vec{v}-\vec{w}) \cdot [(\vec{u}-\vec{v}) \times (\vec{v}-\vec{w})]$ equals

(A) 0 (B) $\vec{u} \cdot \vec{v} \times \vec{w}$ (C) $\vec{u} \cdot \vec{w} \times \vec{v}$ (D) $3\vec{u} \cdot \vec{v} \times \vec{w}$

$(U + v - W) \cdot \{$

49. Let $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}, \vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ and $\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$ be three non-zero vectors such that $\vec{c}$ is a unit vector perpendiuclar to both $\vec{a}$ and $\vec{b}$. If the angle between $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{6}$, then $\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}^2$ is equal to

$\vec{c}\cdot\vec{b} = 0$
$\vec{c}\cdot\vec{a} = 0$

(A) 0

(B) 1

(C) $\frac{1}{4}(a_1^2 + a_2^2 + a_3^2)(b_1^2 + b_2^2 + b_3^2)$

(D) $\frac{3}{4}(a_1^2 + a_2^2 + a_3^2)(b_1^2 + b_2^2 + b_3^2)(c_1^2 + c_2^2 + c_3^2)$

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} \cdot \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = \begin{vmatrix} \vec{a}\cdot\vec{a} & \vec{a}\cdot\vec{b} & \cancel{a\cdot c} \\ b\cdot a & b\cdot b & \cancel{b\cdot c} \\ \cancel{c\cdot a} & \cancel{c\cdot b} & c\cdot c \end{vmatrix}$$

$$= \begin{vmatrix} |a|^2 & a\cdot b & 0 \\ b\cdot a & |b|^2 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

$$= |a|^2|b|^2 - (a\cdot b)^2$$

$$= |a|^2|b|^2 - (|a|^2|b|^2\cos^2 30^\circ)$$

$$= \frac{|a|^2|b|^2}{4}$$

Nishant Jindal

PROBABILITY

(1) Experiment.

Deterministic

[Chemistry Lab]

Same Result

X

Random.

[Chapter]

2 Coins

3 Coins

2 Dices.

(2) Sample Space → Set of all outcomes.

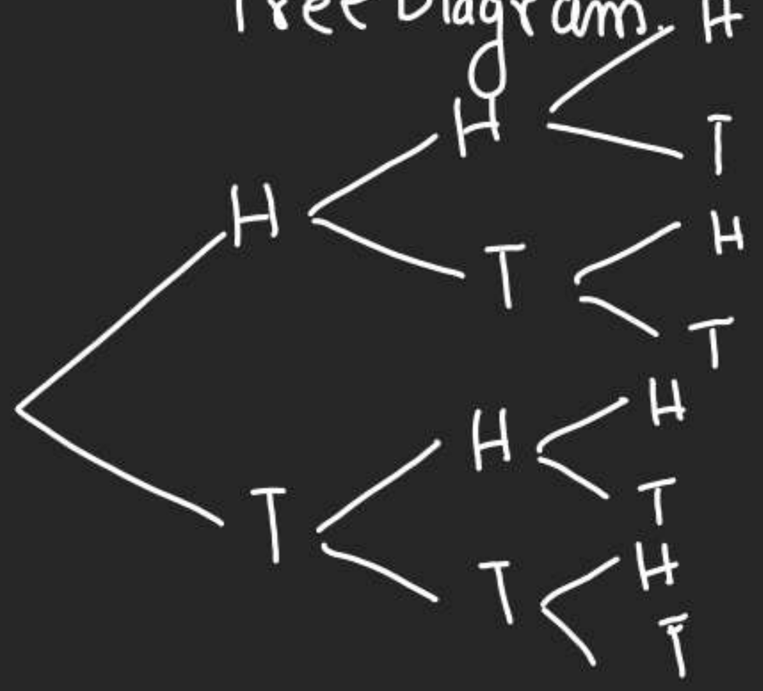
Q. Sample space of 2 fair coins

$\{HH, HT, TH, TT\}$

4 Outcomes $= 2^2$

Q Sample Space of 3 fair Coins?

Tree Diagram.



$S = \{HHH, HHT, HTH, HTT$
$THH, THT, TTH, TTT\}$

8 outcomes $= 2^3$

Q No of Sample space when n Coins are tossed?

No of S.S $= n(S)$
$= 2^n$

Nishant Jindal

Q Sample Space of 1 Dice?

$S.S = \{1, 2, 3, 4, 5, 6\}$.

6 Outcomes.

$n(S) = 6$

Q No of Sample Space when 3 fair dices are tossed

$\Rightarrow n(S) = 6^3 = 216$.

Q Sample Space of 2 Dices?

(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

$n(S) = 36 = 6^2$

Q Sample Space of Cards?

J, K, Q = Face Cards (12)

1) Total = 52 Cards Joker is not a Card.

2) 2 Colors → Red & Black.

Honour Cards = 16

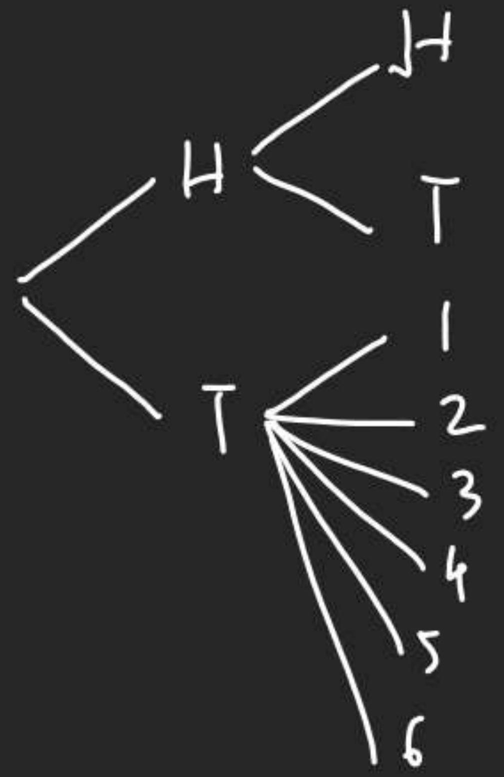
Ace + J + K + Q

13 Denomination

	Red		Black	
	Heart ♡	Diamond ◇	Club ♣	Spade ♠
Ace				
2				
3				
4				
5				
6				
7				
8				
9				
10				
J				
K				
Q				

Nishant Jindal

Q A Fair Coin is tossed if it shows head then again a Coin is tossed & if it shows tail then a dice is Rolled find Sample Spaces?



$S.S = \{HH, HT, T1, T2, T3, T4, T5, T6\}$

$n(S) = 8$