

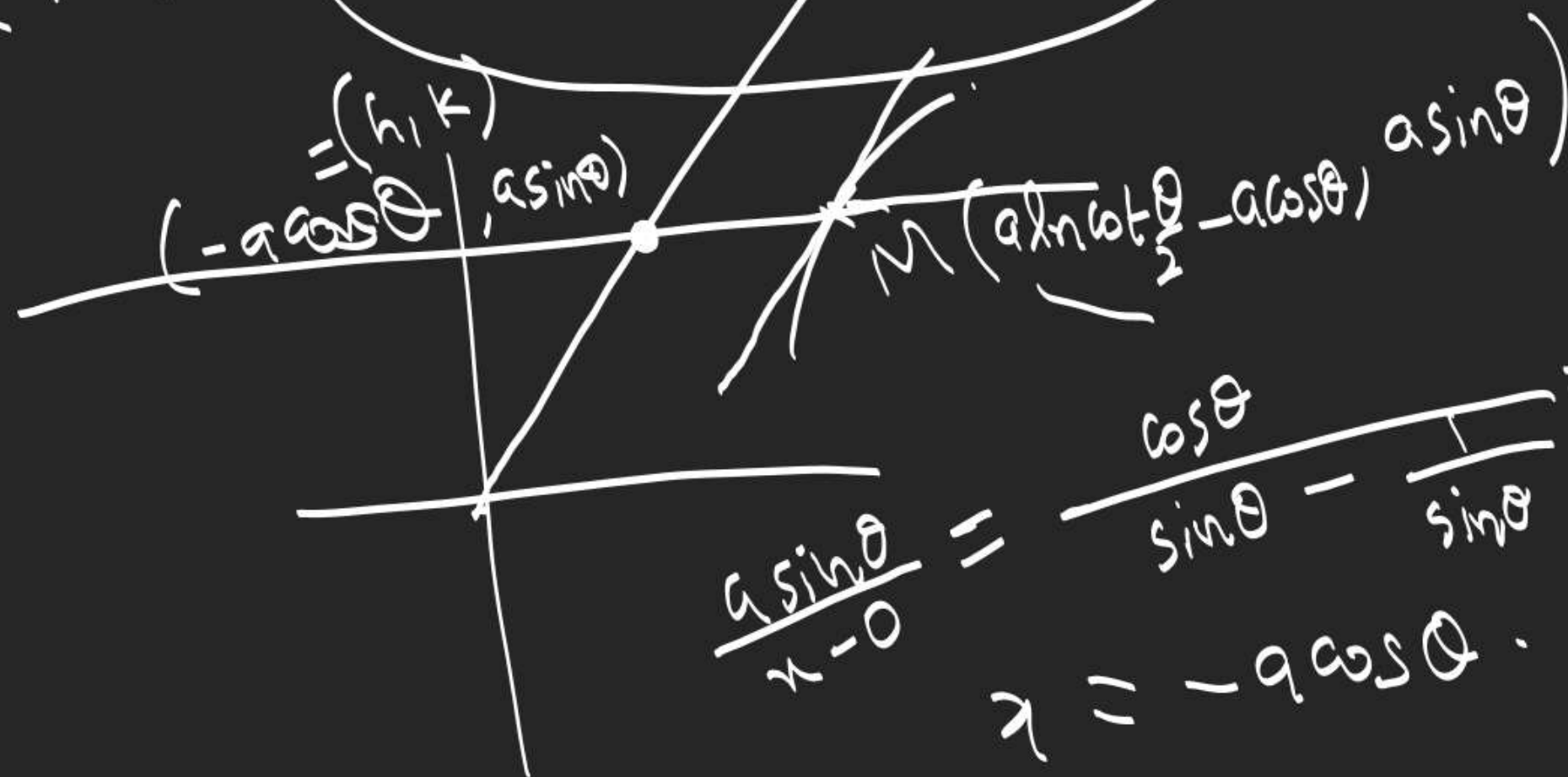
$$2yy' - 6x^2 - 4y' = 0$$

$$y' = \frac{3x^2}{y-2}$$

$$\frac{y_1-2}{x_1-1} = \frac{3x_1^2}{y_1-2}$$

$$(y_1-2)^2 = 3x_1^2(x_1-1) = 2x_1^3 - 4$$

$$(y_1-2)^2 = 2x_1^3 - 4$$



$$\frac{a \sin \theta}{x-0} = \frac{\cos \theta}{\sin \theta} - \frac{1}{\sin \theta} = \frac{\sin \theta}{\cos \theta}$$

$$x = -a \cos \theta$$

$$L'(\theta) = -\frac{8 \cos \theta}{\sin^2 \theta} + \frac{\sin \theta}{\cos^2 \theta}$$

$$= \frac{\sin^3 \theta - 8 \cos^3 \theta}{\sin^2 \theta \cos^2 \theta}$$

$$\cos^3 \theta (\tan^3 \theta - 8)$$

$$\frac{-1 + \sqrt{1 - 4 \tan^2 2}}{2}$$

$$\frac{8 + \tan \theta}{1 + \frac{8 \cot \theta}{2}}$$

$$\frac{7}{9 + 4\sqrt{2}} - 25$$

$$y - 9 = -2.52(x - 1) \quad \boxed{\text{min}}$$

$$y - 8 = -8(x - 1)$$

$$\Delta = \frac{1}{2} (8 + \tan \theta) (1 + 8 \cot \theta) - \frac{1}{2} [16 + (\tan \theta + 64 \cot \theta)] \geq 16$$

$$y - 8 = -2(x - 1)$$





# Cauchy's Inequality

$$(a_1^2 + a_2^2 + a_3^2 + \dots + a_n^2)(b_1^2 + b_2^2 + b_3^2 + \dots + b_n^2) \geq (a_1 b_1 + a_2 b_2 + \dots + a_n b_n)^2$$

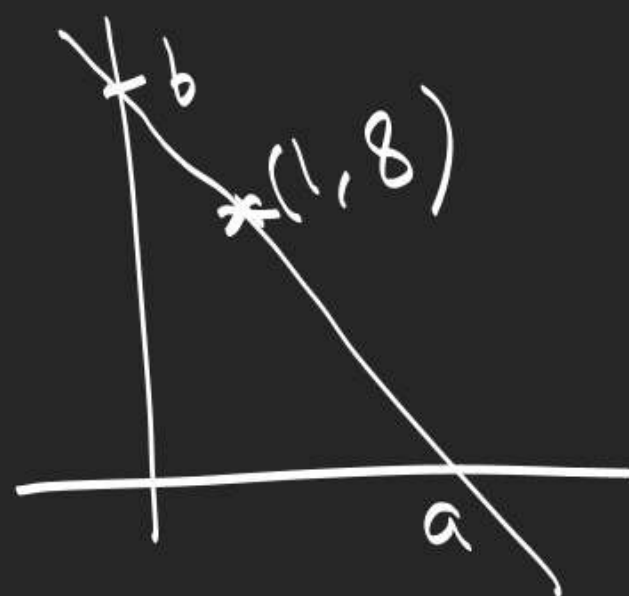
Equality holds if  $\frac{a_1}{b_1} = \frac{a_2}{b_2} = \frac{a_3}{b_3} = \dots = \frac{a_n}{b_n}$ .

$$(a_1 + \lambda b_1)^2 + (a_2 + \lambda b_2)^2 + \dots + (a_n + \lambda b_n)^2 \geq 0$$

$$(b_1^2 + b_2^2 + \dots + b_n^2) \lambda^2 + 2(a_1 b_1 + a_2 b_2 + \dots + a_n b_n) \lambda + (a_1^2 + a_2^2 + \dots + a_n^2) \geq 0$$

$\Delta \leq 0 \Rightarrow (\sum a_i b_i)^2 - (\sum a_i^2)(\sum b_i^2) \leq 0 \quad \forall \lambda \in \mathbb{R}$ .

$\Delta = 0$



$$\frac{x}{a} + \frac{y}{b} = 1 \quad \checkmark$$

$$\boxed{\frac{1}{a} + \frac{8}{b} = 1}$$

$$m = -\frac{b}{a}$$

$$(a+b)_{\min} = ?$$

$$(a+b) \left( \frac{1}{a} + \frac{8}{b} \right) \geq (1 + 2\sqrt{2})^2$$

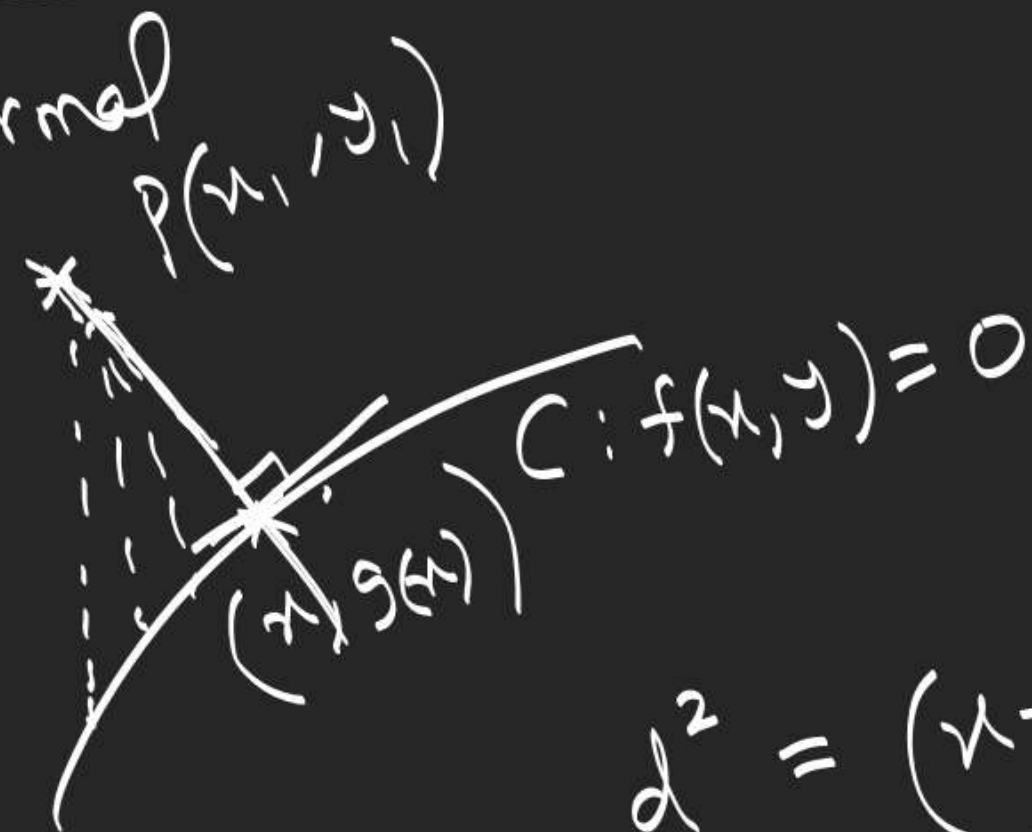
$$a+b \geq \frac{(1+2\sqrt{2})^2}{\frac{b}{a}}$$

$$\Rightarrow \frac{b}{a} = 2\sqrt{2} \quad \checkmark$$

# Max / Min distance of a point

from a curve

occur along normal  
from  $P$  to  $C$ .



$$y = g(x)$$

$$d^2 = (x - x_1)^2 + (g(x) - y_1)^2$$

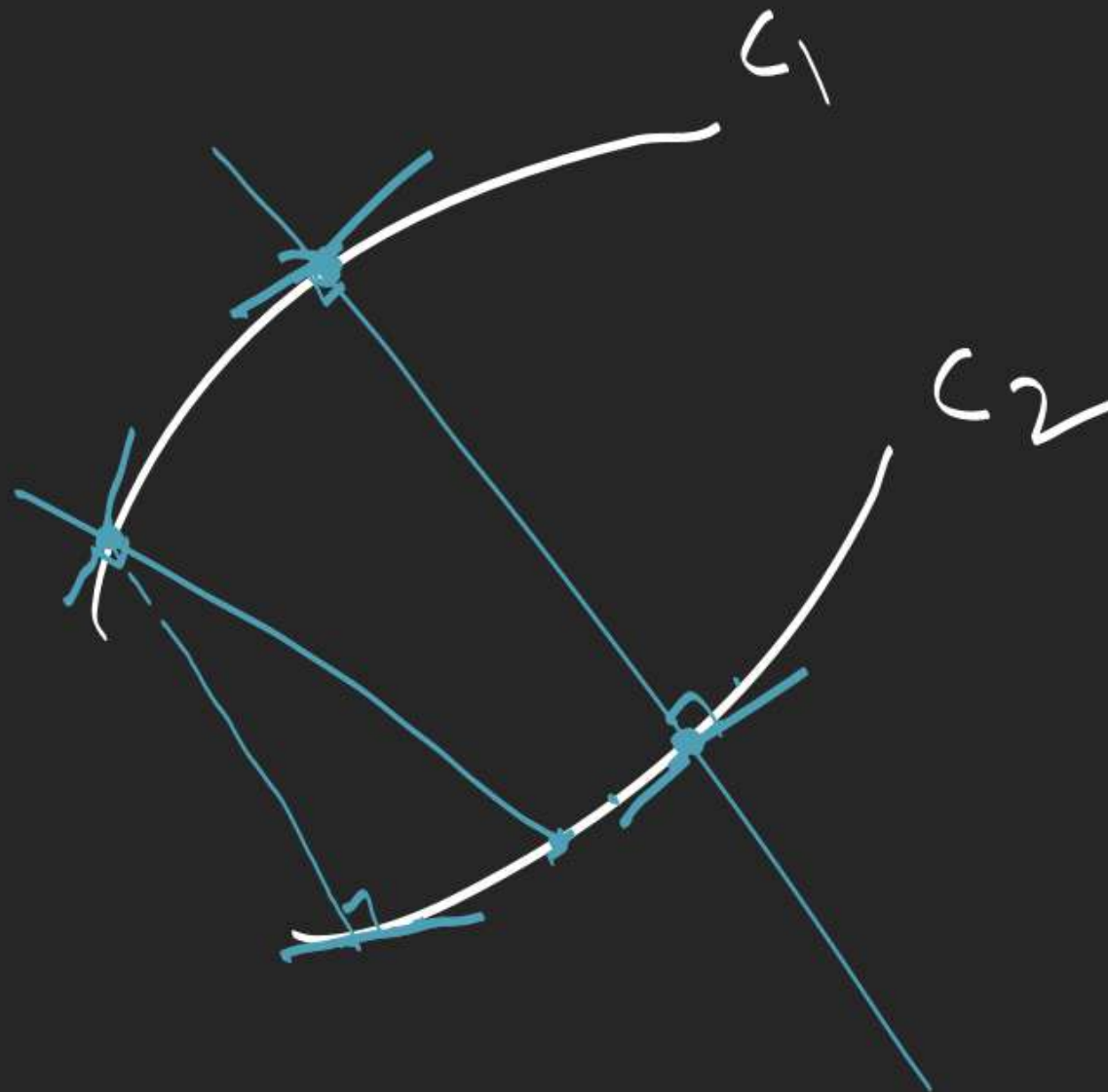
$$\frac{d(d^2)}{dx} = 0 = 2(x - x_1) + 2(g(x) - y_1)g'(x)$$

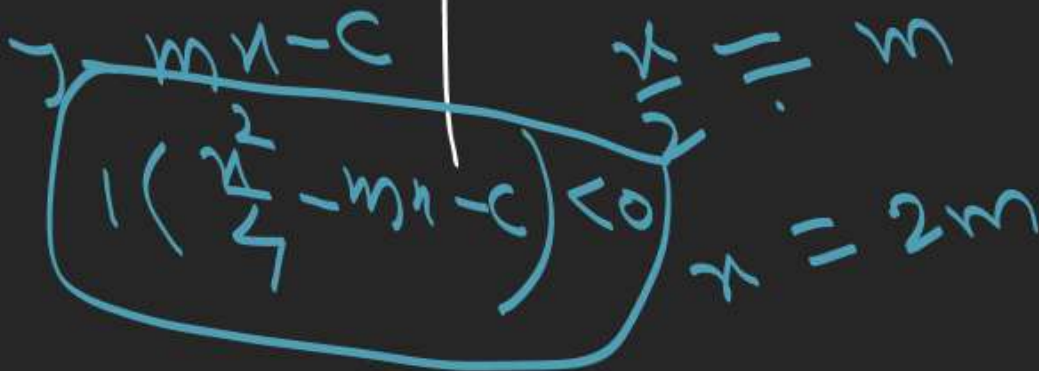
$$g'(x) = -\frac{x - x_1}{g(x) - y_1}$$



# Max/Min distance between two Curves

⇓  
occur along common normal to curves



$$y = mx + c$$


$ax + by + c = 0$   
 $(x_1, y_1)$  lies  $\frac{\text{above}}{\text{below}}$   $(ax_1 + by_1 + c) > 0$   
 $< 0$

2. Four points  $A, B, C, D$ , with  $A = (-2, 3), B = (-1, 1)$   
 $D = (2, 7)$  lie in order on parabola  $y = ax^2 + bx + c$ .  
Find 'C' for which area of quadrilateral  
ABCD is the greatest.

$$\sum x - \frac{II}{IV} \left( \frac{I}{N} \right)$$
$$\sum x - \frac{IV}{IV} \left( \frac{I}{N} \right)$$