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Indefinite Integration

Antiderivative / Primitive / Integral of a function

reverse process of differentiation.

$\int 1\,dx = x + C$

$$\frac{d}{dx} f(x) = g(x)$$

$$\Rightarrow \quad d(f(x)) = g(x)\,dx$$

$d(\sin x) = \cos x\,dx$

$d(x^4) = 4x^3\,dx$

$\frac{d}{dx}(f(x)+2) = g(x)$

$$\boxed{\int g(x)\,dx = f(x) + C}$$

Integrand.

arbitrary constant

↓

Indefinite integration constant.

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$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C \qquad , n \neq -1$$

$$\int (ax+b)^n\,dx = \frac{(ax+b)^{n+1}}{a\,(n+1)} + C \qquad , n \neq -1$$

$$\frac{d}{dx}\frac{(ax+b)^{n+1}}{a(n+1)} = (ax+b)^n$$

$$\int \frac{dx}{x} = \ln|x| + C$$

$$\int \frac{dx}{ax+b} = \frac{1}{a}\ln|ax+b| + C$$

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$$\int e^x\,dx = e^x + C \quad ; \quad \int e^{px+q}\,dx = \frac{1}{p}e^{px+q} + C$$

$$\int a^x\,dx = \frac{a^x}{\ln a} + C \quad ; \quad \int a^{px+q}\,dx = \frac{a^{px+q}}{p\ln a} + C$$

$$\int \sin x\,dx = -\cos x + C \quad ; \quad \int \sin(ax+b)\,dx = -\frac{1}{a}\cos(ax+b) + C$$

$$\int \cos x\,dx = \sin x + C \quad ; \quad \int \cos(ax+b)\,dx = \frac{1}{a}\sin(ax+b) + C$$

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$$\int \sec^2 x\,dx = \tan x + C \quad ; \quad \int \sec^2(ax+b)\,dx = \frac{1}{a}\tan(ax+b) + C$$

$$\int \operatorname{cosec}^2 x\,dx = -\cot x + C \quad ; \quad \int \operatorname{cosec}^2(ax+b)\,dx = -\frac{1}{a}\cot(ax+b) + C$$

$$\int \sec x \tan x\,dx = \sec x + C \quad ; \quad \int \sec(ax+b)\tan(ax+b)\,dx = \frac{1}{a}\sec(ax+b) + C$$

$$\int \operatorname{cosec} x \cot x\,dx = -\operatorname{cosec} x + C \quad ; \quad \int \operatorname{cosec}(ax+b)\cot(ax+b)\,dx = -\frac{1}{a}\operatorname{cosec}(ax+b) + C.$$

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$$\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1}x + C \quad ; \quad \int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + C$$

$$\int \frac{dx}{1+x^2} = \tan^{-1}x + C \quad ; \quad \int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$$

$$\int \frac{dx}{|x|\sqrt{x^2-1}} = \sec^{-1}x + C \quad ; \quad \int \frac{dx}{|x|\sqrt{x^2-a^2}} = \frac{1}{a}\sec^{-1}\left(\frac{x}{a}\right) + C$$

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1. $\displaystyle\int 2^{\ln x}\,dx = \int x^{\ln 2}\,dx = \frac{x^{\ln 2+1}}{(\ln 2+1)} + C.$

$\displaystyle\int \frac{(x+5)+(x-2)}{(x+5)(x-2)}\,dx = \int\left(\frac{1}{x-2} + \frac{1}{x+5}\right)dx = \ln|x-2| + \ln|x+5| + C.$

2. $\displaystyle\int \frac{\sqrt{x^4+x^{-4}+2}}{x^3}\,dx = \int \frac{x^2+\frac{1}{x^2}}{x^3}\,dx = \int\left(\frac{1}{x} + \frac{1}{x^5}\right)dx$

$\displaystyle= \ln|x| - \frac{1}{4x^4} + C.$

3. $\displaystyle\int \frac{x\,dx}{(x^2+2x+1)} = \int \frac{(x+1-1)\,dx}{(x+1)^2} = \int\left(\frac{1}{x+1} - \frac{1}{(x+1)^2}\right)dx$

$\displaystyle= \ln|x+1| + \frac{1}{(x+1)} + C.$

4. $\displaystyle\int \frac{(2x+3)\,dx}{(x^2+3x-10)}$

5. $$\int \frac{x\,dx}{(x^2-5x+6)} = \int \frac{\left(3(x-2)-2(x-3)\right)}{(x-2)(x-3)}dx = \int\left(\frac{3}{x-3}-\frac{2}{x-2}\right)dx$$

$$= 3\ln|x-3| - 2\ln|x-2| + C.$$

6. $$\int (2^x+3^x)^2\,dx = \int\left(4^x+9^x+2\cdot 6^x\right)dx = \frac{4^x}{\ln 4}+\frac{9^x}{\ln 9}+\frac{2\cdot 6^x}{\ln 6}+C$$

7. $$\int \frac{(e^{3x}+e^{5x})\,dx}{(e^x+e^{-x})} = \int \frac{e^x e^{3x}(1+e^{2x})}{(1+e^{2x})}dx = \int e^{4x}\,dx = \frac{e^{4x}}{4}+C.$$

$$e^{-x} = \frac{1}{e^x}$$

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8.

$$\int \cos 2x \cos 3x \, dx = \int \frac{1}{2}(\cos 5x + \cos x)\, dx = \frac{1}{2}\left(\frac{\sin 5x}{5} + \sin x\right) + C.$$

9.

$$\int \sin^4 x \, dx = \int \left(\frac{1-\cos 2x}{2}\right)^2 dx = \frac{1}{4}\int\left(1+\cos^2 2x - 2\cos 2x\right) dx$$

$$= \frac{1}{4}\int\left(1 + \frac{1+\cos 4x}{2} - 2\cos 2x\right)dx = \frac{1}{4}\int\left(\frac{3}{2} + \frac{\cos 4x}{2} - 2\cos 2x\right)dx$$

$$= \frac{1}{4}\left(\frac{3}{2}x + \frac{\sin 4x}{8} - \sin 2x\right) + C.$$

10.

$$\int \frac{(\cos 5x + \cos 4x)}{(1 - 2\cos 3x)}\, dx = \int \frac{2\cos\frac{9x}{2}\cos\frac{x}{2}\, dx}{1 - 2\left(2\cos^2\frac{3x}{2} - 1\right)} = \int \frac{2\cos\frac{3x}{2}\cancel{\cos\frac{9x}{2}}^{\,-1}\cos\frac{x}{2}\, dx}{\cancel{\left(3\cos\frac{3x}{2} - 4\cos^3\frac{3x}{2}\right)}}$$

$$= 3 - 4\cos^2\frac{3x}{2}$$

$$= \int -2\cos\frac{3x}{2}\cos\frac{x}{2}\, dx$$

$$= -\int(\cos x + \cos 2x)\, dx$$

$$-\sin x - \frac{\sin 2x}{2} + C.$$

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11. $\int \sin(x^\circ)\,dx = \int \sin\left(\frac{\pi}{180}x\right)dx = -\frac{180}{\pi}\cos\frac{\pi x}{180} + C$

$= -\frac{180}{\pi}\cos x^\circ + C$.

$\cot^2 x \cos^2 x = \cot^2 x - \cos^2 x$

$\tan^2 x \sin^2 x = \tan^2 x - \sin^2 x$

12. $\int\left(\frac{1-\cos x}{1+\cos x}\right)dx = \int \tan^2\frac{x}{2}\,dx = \int\left(\sec^2\frac{x}{2} - 1\right)dx = 2\tan\frac{x}{2} - x + C$

13. $\int \sec^2 x \csc^2 x\,dx = \int \frac{\sin^2 x + \cos^2 x}{\sin^2 x \cos^2 x}\,dx = \int(\sec^2 x + \csc^2 x)\,dx$

$\downarrow$

$\int 4\csc^2 2x\,dx$

$= \tan x - \cot x + C$

14. $\int \cot^2 x \cos^2 x\,dx$

$= \int(\cot^2 x - \cos^2 x)\,dx$

$= \int\left(\csc^2 x - 1 - \frac{1+\cos 2x}{2}\right)dx = -\cot x - \frac{3}{2}x - \frac{\sin 2x}{4} + C$

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15. $$\int \frac{\left(\operatorname{cosec}x + \tan^2 x + \sin^2 x\right)dx}{\sin x} = \int \left(\operatorname{cosec}^2 x + \tan x \sec x + \sin x\right)dx$$

$$= -\cot x + \sec x - \cos x + C.$$

16. $$\int \frac{dx}{(1-\sin 3x)} = \int \left(\frac{1+\sin 3x}{\cos^2 3x}\right)dx = \int \left(\sec^2 3x + \overset{\tan x \sec x}{\tan 3x \sec 3x}\right)dx$$

$$= \frac{\tan 3x}{3} + \frac{\sec 3x}{3} + C.$$

17. $$\int \frac{\left(x^2 + \cos^2 x\right)\operatorname{cosec}^2 x\, dx}{\left(x^2+1\right)}$$

$$\int \frac{\left(x^2+1-\sin^2 x\right)\operatorname{cosec}^2 x}{x^2+1}dx = \int \left(\operatorname{cosec}^2 x - \frac{1}{x^2+1}\right)dx$$

$$= -\cot x - \tan^{-1} x + C.$$

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1. $$\int \frac{x^4\,dx}{1+x^2}$$

2. $$\int \frac{dx}{\sqrt{9-4x^2}}$$

3. $$\int \frac{dx}{(x^2-4x+4)(x^2-4x+5)}$$

4. $$\int \frac{dx}{(2x-7)\sqrt{(x-3)(x-4)}}$$