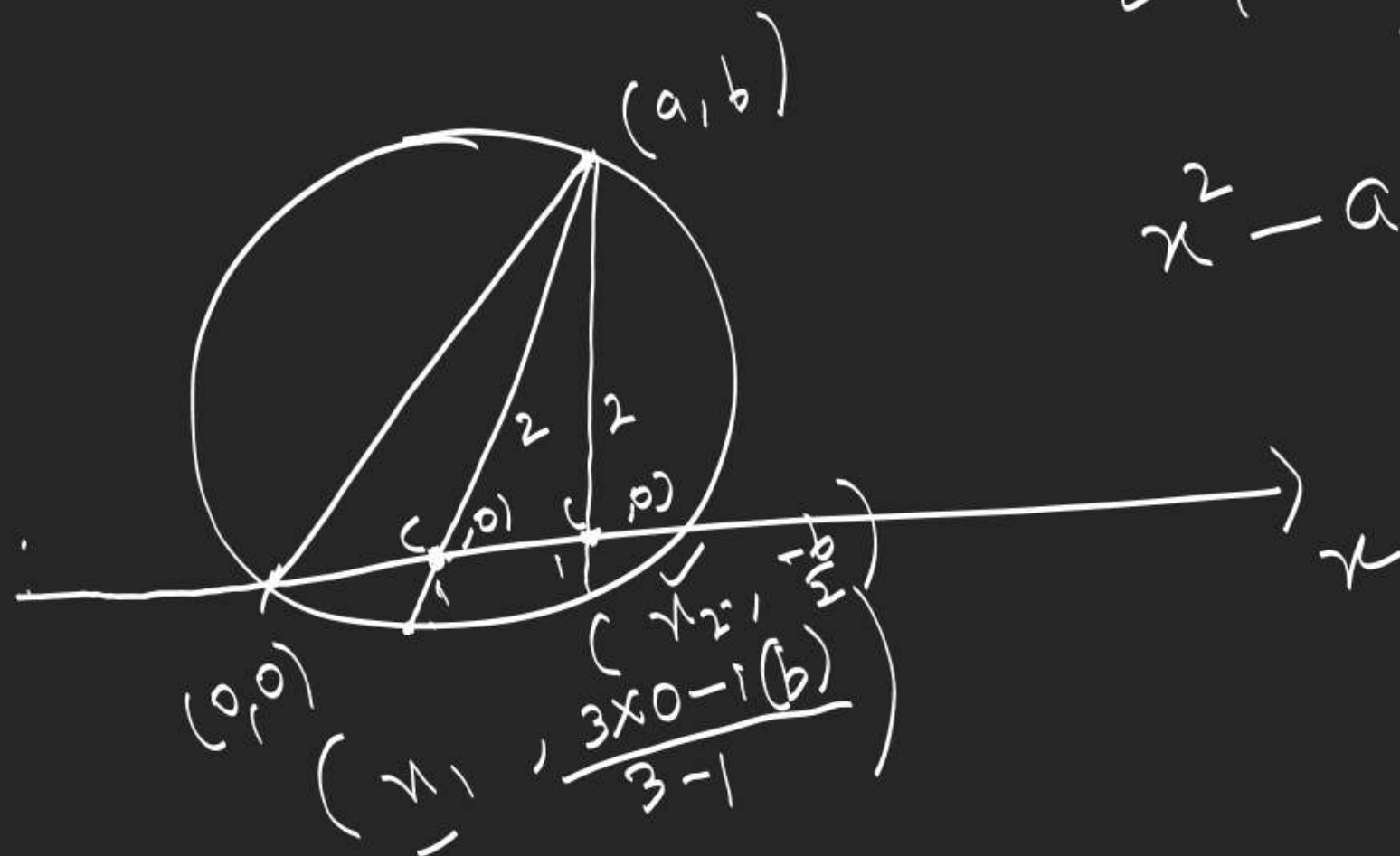$$\rightarrow hx + ky = h^2 + k^2$$

$$x^2 + y^2 + (2gx + 2fy)\left(\frac{hx + ky}{h^2 + k^2}\right) + c\left(\frac{hx + ky}{h^2 + k^2}\right)^2 = 0$$

$$2 + \frac{2gh + 2fk}{h^2 + k^2} + \frac{c(h^2 + k^2)}{(h^2 + k^2)^2} = 0$$

Diagram labels: T, (h,k), O $(0,0)$, B

1. If 2 distinct chords of circle $x^2 + y^2 - ax - by = 0$ drawn from point (a, b) is divided by x-axis in the ratio $2:1$, then P.T. $a^2 > 3b^2$.
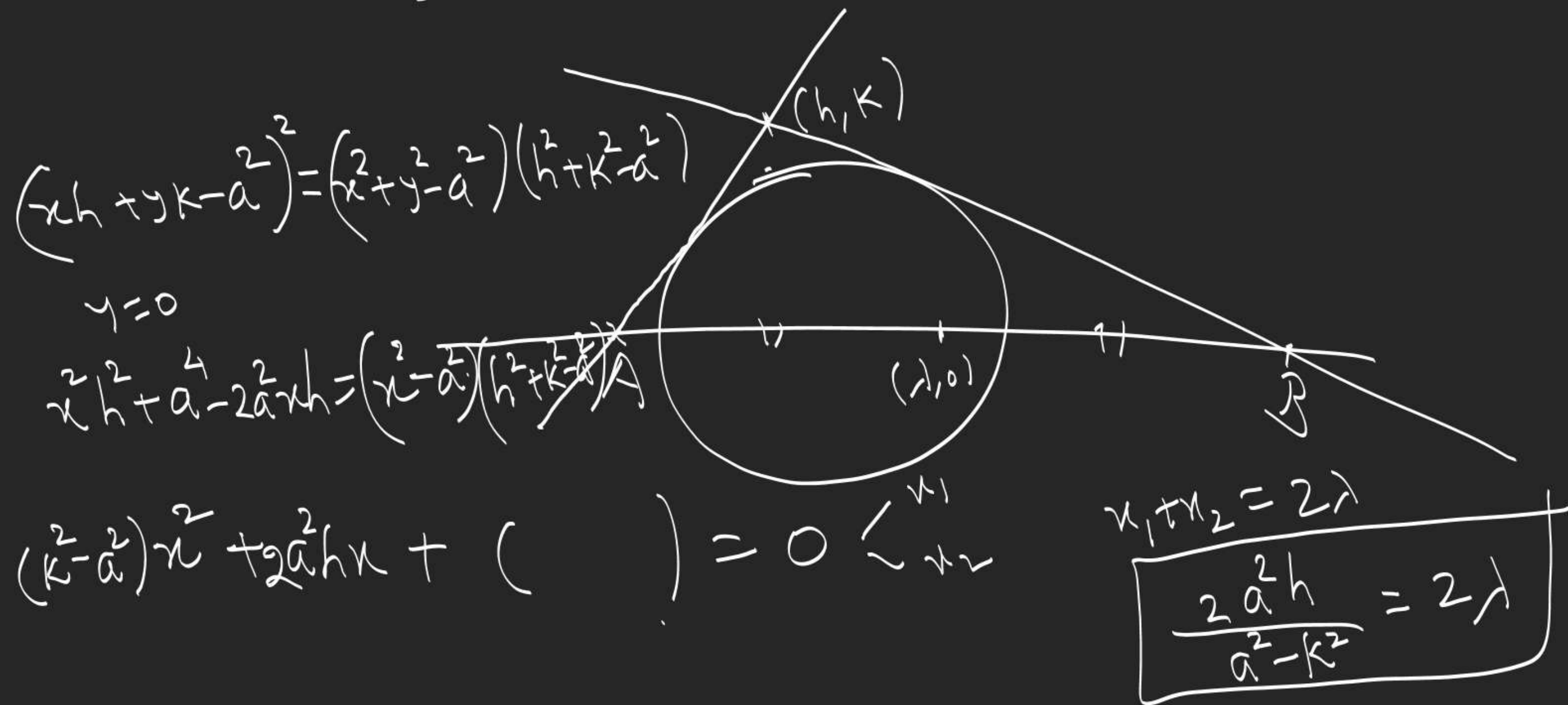
$y = \dfrac{-b}{2}$

(a, b)

$(0, 0)$

$\left(\dfrac{a}{3}, 0\right)$ $\left(\dfrac{a}{3}, 0\right)$

2 2

1 1

$\left(x_2, \dfrac{-b}{2}\right)$

$\left(x_1, \dfrac{3 \times 0 - 1(b)}{3 - 1}\right)$

$x^2 - ax + \dfrac{b^2}{4} + \dfrac{b^2}{2} = 0$

$x^2 - ax + \dfrac{3b^2}{4} = 0$

$\boxed{D > 0}$

2. Tangents are drawn to circle $x^2+y^2=a^2$ from two points on x-axis equidistant from point $(\lambda,0)$. Show that locus of their intersection point is $\lambda y^2 = a^2(\lambda-x)$.

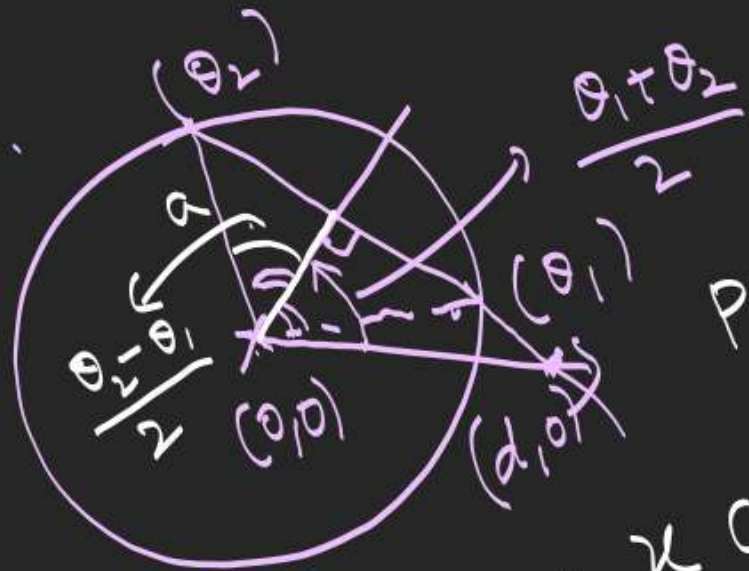$$\left(xh+yk-a^2\right)^2 = \left(x^2+y^2-a^2\right)\left(h^2+k^2-a^2\right)$$

$$y=0$$

$$x^2h^2+a^4-2a^2xh = \left(x^2-a^2\right)\left(h^2+k^2-a^2\right)$$

$$\left(k^2-a^2\right)x^2 + 2a^2hx + \left(\quad\right) = 0 \begin{cases} x_1 \\ x_2 \end{cases}$$

(h,k)

$(\lambda,0)$

B

$x_1+x_2 = 2\lambda$

$$\boxed{\dfrac{2a^2h}{a^2-k^2} = 2\lambda}$$

3. Let $A = (a\cos\theta_1, a\sin\theta_1)$, $B(a\cos\theta_2, a\sin\theta_2)$, $C(\theta_3)$

on circle $x^2 + y^2 = a^2$. Then. $D(\theta_4)$

(i) If chord AB passes thru $(d, 0)$, then find $\left(\tan\dfrac{\theta_1}{2} \tan\dfrac{\theta_2}{2}\right)$

$(d, 0) \checkmark \longrightarrow (-d, 0)$

$\left(\tan\dfrac{\theta_1}{2}\tan\dfrac{\theta_2}{2}\right)\left(\tan\dfrac{\theta_3}{2}\tan\dfrac{\theta_4}{2}\right)$

(ii) If chords AB & CD intersect x-axis at 2 distinct points

$= \left(\dfrac{d-a}{d+a}\right)\left(\dfrac{-d-a}{-d+a}\right) = 1$

equidistant from origin, then find $\dfrac{1}{4}$

$\dfrac{d-a}{d+a} = \tan\dfrac{\theta_1}{2}\tan\dfrac{\theta_2}{2}$

$\displaystyle\prod_{i=1}^{4}\tan\dfrac{\theta_i}{2}$

But $(d, 0)$

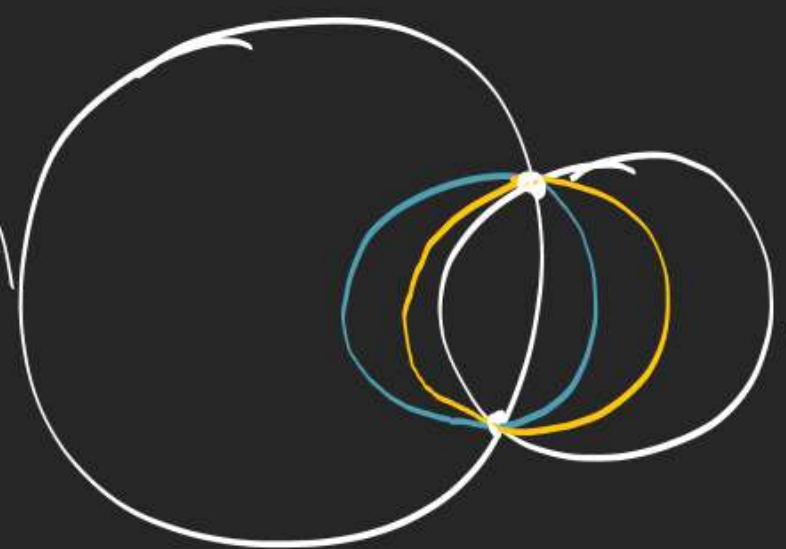$\dfrac{d}{a} = \dfrac{\cos\left(\dfrac{\theta_1 - \theta_2}{2}\right)}{\cos\left(\dfrac{\theta_1 + \theta_2}{2}\right)}$

$p = a\cos\left(\dfrac{\theta_2 - \theta_1}{2}\right)$

$x\cos\left(\dfrac{\theta_1 + \theta_2}{2}\right) + y\sin\left(\dfrac{\theta_1 + \theta_2}{2}\right) = a\cos\left(\dfrac{\theta_1 - \theta_2}{2}\right)$

Family of Circles passing thru intersection point of Two given circles

$$x^2 + y^2 + 2g_1 x + 2f_1 y + c_1 + \lambda \left(x^2 + y^2 + 2g_2 x + 2f_2 y + c_2 \right) = 0 \qquad \lambda \neq -1$$

$$S_1 + \lambda S_2 = 0 , \lambda \neq -1$$

$$S_2 = x^2 + y^2 + 2g_2 x + 2f_2 y + c_2 = 0$$

$$\boxed{S_1 - S_2 = 0} \rightarrow \text{Line } \ell$$

$$S_1 = x^2 + y^2 + 2g_1 x + 2f_1 y + c_1 = 0$$

Common chord

$$\boxed{\text{St. Line} \quad (\text{Subj} - 1 - 15}$$