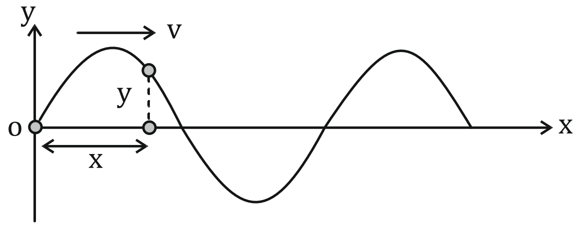
Equation of Progressive Wave

1. A progressive wave is considered a harmonic progressive wave when the particles of the medium undergo simple harmonic motion (SHM) around their mean position during wave propagation.

2. In the given figure, a plane simple harmonic wave propagates from the origin along the positive x-axis in the left-to-right direction.



3. The displacement (y) of a particle O from its mean position at any given time (t) can be expressed as follows:

$y = A\sin\omega t$ ...(1)

In general along x-axis, $y = A\sin(\omega t \pm kx)$ + sign for a wave travelling along –ve X direction

Where y is displacement of the particle after a time t from mean position, x is displacement of the wave, A is Amplitude.

ω is angular frequency or angular velocity $\omega = \frac{2\pi}{T} = 2\pi n$

k is propagation constant & $k = \frac{2\pi}{\lambda}$

➢ For a given time 't', y-x graph gives the shape of pulse on string

Various forms of progressive wave function:

(i) $y = A\sin(\omega t \pm kx)$ (or) $y = A\sin(kx \pm \omega t)$

(ii) $y = A\cos(\omega t \pm kx)$ (or) $y = A\cos(kx \pm \omega t)$

(iii) $y = A\sin\left(\omega t \pm \frac{2\pi}{\lambda}x\right)$

(iv) $y = A\sin 2\pi\left[\frac{t}{T} \pm \frac{x}{\lambda}\right]$

(v) $y = A\sin\frac{2\pi}{T}\left[t \pm x\frac{T}{\lambda}\right]$

(vi) $y = A\sin\frac{2\pi}{\lambda}(vt \pm x)$

(vii) $y = A\sin\omega\left(t \pm \frac{x}{v}\right)$

(viii) $y = A\sin 2\pi\left(\frac{t}{T} \pm \frac{x}{\lambda}\right)$

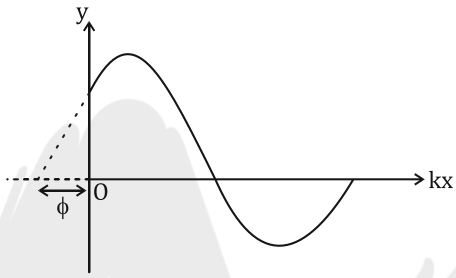
General Expression for a Sinusoidal wave

$$Y = A\sin(kx - \omega t + \phi) \text{ (or) } Y = A\sin(\omega t - kx + \phi)$$

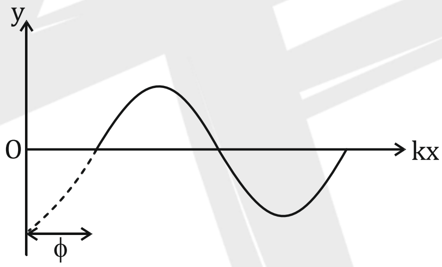
The phase constant ϕ plays a significant role, as we have encountered in our study of periodic motion. This constant is determined based on the initial conditions of the system.

Positive and Negative Initial Phase Constants

(i) Positive initial phase constant $y = A\sin(kx - \omega t + \phi)$. The sine curve starts from the left of the origin.

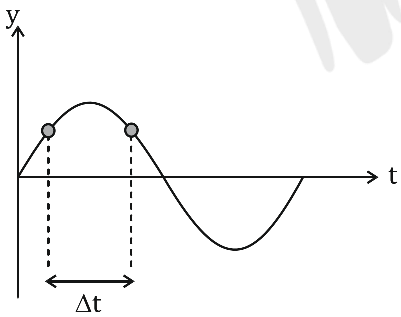


(ii) Negative initial phase constant $y = A\sin(kx - \omega t - \phi)$. The sine curve starts from the right of the origin.



Change in Phase with time for a constant x, i.e., at a fixed point in the medium

$$[\phi]_{t_1} = 2\pi\left(\frac{t_1}{T} - \frac{x}{\lambda}\right) + \phi; \; [\phi]_{t_2} = 2\pi\left(\frac{t_2}{T} - \frac{x}{\lambda}\right) + \phi$$



(For the wave travelling in positive x-direction)

$$\Delta\phi = [\phi]_{t_2} - [\phi]_{t_1} = \frac{2\pi}{T} \times (t_2 - t_1) = \frac{2\pi}{T} \times \Delta t \Rightarrow \Delta\phi = \frac{2\pi \times \Delta t}{T}$$

Phase difference $= \dfrac{2\pi}{T} \times$ Time difference

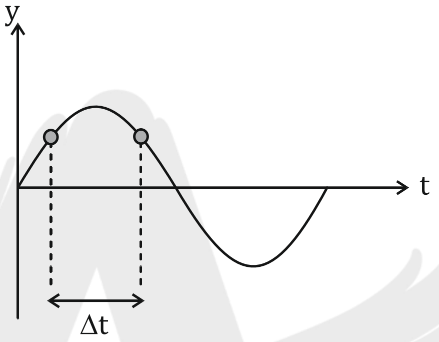
Variation of Phase with Distance

At a given instant of time t = t, Phase at $x = x_1$

$$[\phi]_{x_1} = 2\pi\left(\frac{t}{T} - \frac{x_1}{\lambda}\right) + \phi$$

(For the wave travelling in positive x-direction and phase at $x = x_2$),

$$[\phi]_{x_2} = 2\pi\left(\frac{t}{T} - \frac{x_2}{\lambda}\right) + \phi$$



$$\Rightarrow \Delta\phi = [\phi]_{x_2} - [\phi]_{x_1} = \frac{2\pi}{\lambda}(x_2 - x_1) = \frac{2\pi}{\lambda}\Delta x$$

$$\Delta\phi = \frac{2\pi}{\lambda}\Delta x$$

i.e., Phase difference $= \frac{2\pi}{\lambda} \times$ Path difference

Particle Velocity: The rate of change of displacement (y) with respect to time (t) is commonly referred to as particle velocity. Therefore, from the given expression $y = A\sin(\omega t - kx)$

Particle velocity, $v_P = \frac{\partial y}{\partial t} = A\omega\cos(\omega t - kx)$

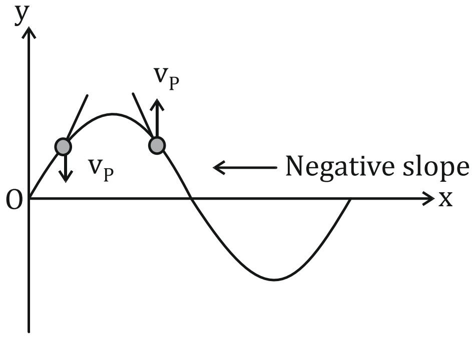
Maximum particle velocity $(v_P)_{max} = A\omega$

Also, $\frac{\partial y}{\partial t} = -\frac{\omega}{k} \times \frac{\partial y}{\partial x}$

- The particle velocity at a specific position and time is equal to the negative product of the wave velocity and the slope of the wave at that particular point, expressed as follows:

$$v_{particle} = -v_{Wave}\left(\frac{\partial y}{\partial x}\right)$$

Particle veloicty = -(Wave velocity) × slope of wave curve



Energy, Power And Intensity of a Wave:

If ρ is the density of the medium, kinetic energy of the wave per unit volume will be

$$=\frac{1}{2}\rho\left[\frac{\partial y}{\partial t}\right]^2=\frac{1}{2}\rho\omega^2A^2\cos^2(\omega t-kx)$$

and its maximum value will be equal to energy per unit volume i.e., energy density U.

$$U=\frac{1}{2}\rho A^2\omega^2$$

Intensity is defined as power per unit area. $I=\frac{\Delta E}{S\Delta t}=\frac{P}{S}=\frac{1}{2}\rho v\omega^2A^2=2\pi^2f^2A^2\rho v$

If frequency f is constant then $I\propto A^2$

Reflection and Refraction of Waves:

When waves encounter a boundary between two different media, a portion of the incident waves reflects back into the initial medium (reflection), while the remaining waves are divided, with part of them being absorbed and the other part transmitted into the second medium (refraction).

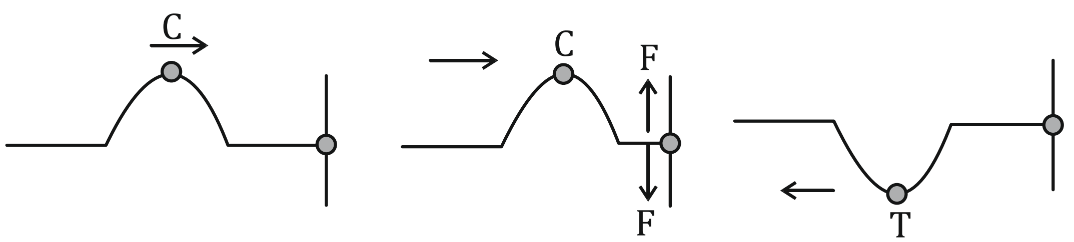
Boundary conditions:

The reflections of a wave pulse from a boundary depend on the characteristics and properties of that boundary.

Rigid end:

When an incident wave reaches a fixed end, it applies an upward pull on that end. In accordance with Newton's third law, the fixed end exerts an equal and opposite downward force on the string. This interaction leads to the inversion of the pulse or a phase change of π.

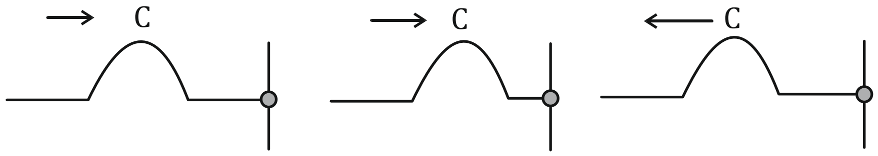
Crest (C) reflects as trough (T) and vice-versa. Time changes by $\frac{T}{2}$ and path changes by $\frac{\lambda}{2}$



Free end:

When a wave or pulse is reflected from a free end, then ther is no change of phase (as there is no reaction force).

Crest (C) reflects as crest (C) ang trough (T) reflects as trough (T), time changes by zero and path changes by zero.

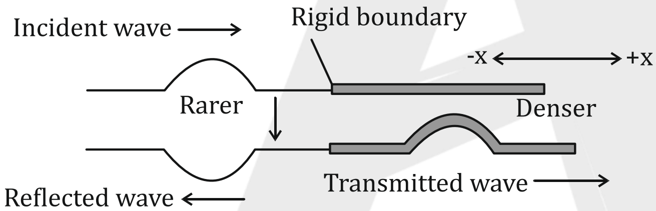


Note: Exception:

Longitudinal pressure waves suffer no change in phase from rigid end. i.e., compression pulse reflects as compression pulse. On the other hand if longitudinal pressure wave reflects from free end, it suffer a phase change of π, i.e., compression reflects as rarefaction and vice-versa.

Wave in a combination of string

(i) Wave goes from thin to thick string



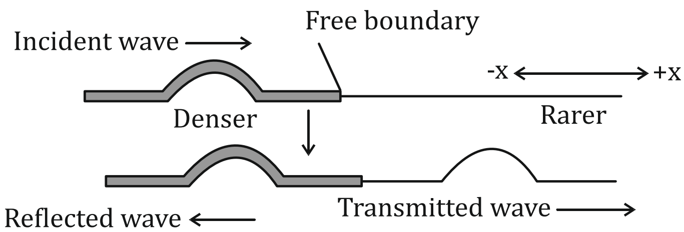
Incident wave $y_i = a_i \sin(\omega t - k_1 x)$

Reflected wave $y_r = a_r \sin[\omega t - k_1(-x) + \pi]$

$= -a_r \sin(\omega t + k_1 x)$

Transmitted wave, $y_t = a_t \sin(\omega t - k_2 x)$

(ii) Wave goes from thick to thin string



Incident wave $y_i = a_t \sin(\omega t - k_1 x)$

Reflected wave $y_r = a_r \sin[\omega t - k_1(-x) + 0]$

$= a_r \sin(\omega t + k_1 x)$

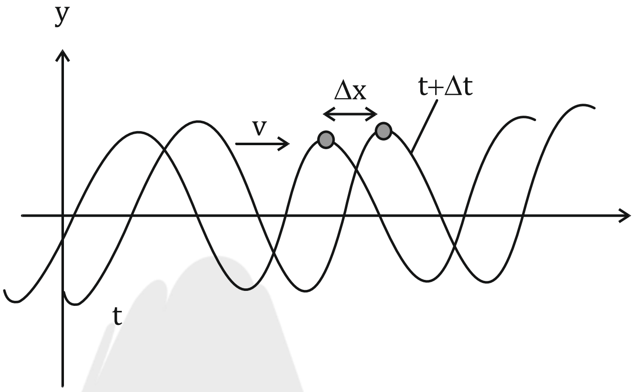
Transmitted wave $y_t = a_t \sin(\omega t - k_2 x)$

Note: Ratio of amplitudes: It is given as follows

$$\frac{a_r}{a_i} = \frac{k_1 - k_2}{k_1 + k_2} = \frac{v_2 - v_1}{v_2 + v_1} \text{ and } \frac{a_t}{a_i} = \frac{2k_1}{k_1 + k_2} = \frac{2v_2}{v_1 + v_2}$$

The speed of A Travelling Wave

(i) Let a wave move along the +ve x-axis with velocity 'v' as shown in figure.



For the same particle displacement 'y' at two different positions, $kx - \omega t$ = constant ....(1)

$$\Rightarrow k\Delta x - \omega \Delta t = 0$$

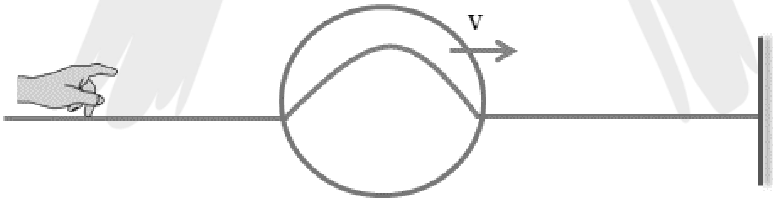
$$\Rightarrow \frac{\Delta x}{\Delta t} = \frac{\omega}{k} \Rightarrow v = \frac{\Delta x}{\Delta t} = \frac{\omega}{k}$$

$$\Rightarrow v = \frac{2\pi n}{2\pi / \lambda} = n\lambda$$

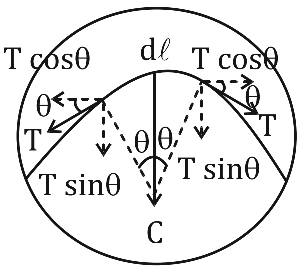
$$\left(\because \omega = 2\pi n \text{ and } k = \frac{2\pi}{\lambda}\right)$$

Speed of transverse wave in a string

(i) Let a transverse pulse is travelling on a stretched string as shown in figure (a)



(a)

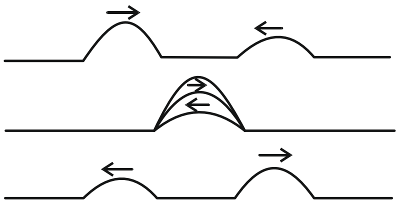


(b)

$$v = \sqrt{\frac{T}{\mu}} \quad .....(3)$$

Principle of Superposition:

If $\bar{y}_1, \bar{y}_2, \bar{y}_3 \ldots\ldots$ are the displacements at a particular time at a particular position, due to individual waves, then the resultant displacement. $\bar{y} = \bar{y}_1 + \bar{y}_2 + \bar{y}_3 + \ldots\ldots$



Interference of Sound waves

When two waves with the same frequency, wavelength, and nearly equal amplitudes move in the same direction, their combination results in interference.

Since Intensity (I) $\propto$ $(\text{Amplitude A})^2 \Rightarrow \frac{I_1}{I_2} = \left(\frac{a_1}{a_2}\right)^2$

Therefore, the resultant intensity is given by $I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$

Table : Constructive and destructive interference	
When the waves meet a point with same phase, constructive interference is obtained at that point (i.e., maximum sound)	When the waves meet a point with opposite phase, destructive interference is obtained at that point (i.e., minimum sound)
Phase difference between the waves at the point of observation $\phi = 0°$ (or) $2n\pi$	Phase difference $\phi = 180°$ (or) $(2n-1)\pi$; $n = 1, 2, \ldots\ldots$
Phase differnce between the waves at the point of observation $\Delta = n\lambda$ (i.e., even multiple of $\lambda/2$)	Phase difference $\Delta = (2n-1)\frac{\lambda}{2}$ (i.e., odd multiple of $\lambda/2$)
Resultant amplitude at the point of observation will be maximum $A_{max} = a_1 + a_2$ If $a_1 = a_2 = a_0 \Rightarrow A_{max} = 2a_0$	Resultant amplitude at the point of observation will be minimum $A_{max} = a_1 - a_2$ If $a_1 = a_2 \Rightarrow A_{max} = 0$
Resultant intensity at the point of observation will be maximum $I_{max} = I_1 + I_2 + 2\sqrt{I_1 I_2} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2$ If $I_1 = I_2 = I_0 \Rightarrow I_{min} = 0$	Resultant intensity at the point of observation will be minimum $I_{min} = I_1 + I_2 + 2\sqrt{I_1 I_2} = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2$ If $I_1 = I_2 = I_0 \Rightarrow I_{min} = 4I_0$

7. $\frac{I_{max}}{I_{min}} = \left(\frac{\sqrt{I_1}+\sqrt{I_2}}{\sqrt{I_1}-\sqrt{I_2}}\right)^2 = \left(\frac{a_1+a_2}{a_1-a_2}\right)^2 = \left(\frac{\frac{a_1}{a_2}+1}{\frac{a_1}{a_2}-1}\right)^2$

Standing Waves or Stationary Waves:

When two sets of progressive wave trains of the same type (either both longitudinal or both transverse), having identical amplitudes, time periods/frequencies/wavelengths, and traveling at the same speed along the same straight line but in opposite directions, they superimpose and give rise to a new set of waves. These waves are known as stationary waves or standing waves.

1. Suppose that two super imposing waves are incident wave $y_1 = a\sin(\omega t - kx)$ and reflected wave $y_2 = a\sin(\omega t + kx)$

(If reflection takes place from rigid end, then equation of stationary wave will be $y = \pm 2a\, sin\, k\, x\, cos\, \omega\, t$)

2. As this equation satisfies the wave equation. $\frac{\partial^2 y}{\partial t^2} = v^2 \frac{\partial^2 y}{\partial x^2}$. It represents a wave

7. Amplitude of standing waves in two different cases:

Table : Amplitude in two different cases

Reflection at opend end or free boundary	Reflection at closed end or rigid boundary
$A_{SW} = 2a\cos kx$	$A_{SW} = 2a\sin kx$
Amplitude is maximum when $\cos kx = \pm 1$ $\Rightarrow kx = 0, 2\pi, ...n\pi \Rightarrow x = 0, \frac{\lambda}{2}, \lambda.....\frac{n\lambda}{2}$ Where $k = \frac{2\pi}{\lambda}$ and $n = 0, 1, 2, 3.....$	Amplitude is maximum when $\sin kx = \pm 1$ $\Rightarrow kx = \frac{\pi}{2}, \frac{3\pi}{2}.....\frac{(2n-1)\pi}{2}$ $\Rightarrow x = \frac{\lambda}{4}, \frac{3\lambda}{4}......$ Where $k = \frac{2\pi}{\lambda}$ and $n = 1, 2, 3, ...$
Amplitude is minimum when $\cos kx = 0$ $\Rightarrow kx = \frac{\pi}{2}, \frac{3\pi}{2}.....\frac{(2n-1)\pi}{2}$ $\Rightarrow x = \frac{\lambda}{4}, \frac{3\lambda}{4}......$	Amplitude is minimum when $\sin kx = 0$ $\Rightarrow kx = \frac{\pi}{2}, \frac{3\pi}{2}.....\frac{(2n-1)\pi}{2}$ $\Rightarrow x = 0, \frac{\lambda}{2}, \lambda......\frac{n\lambda}{2}$

Terms related to the Application of Stationary wave

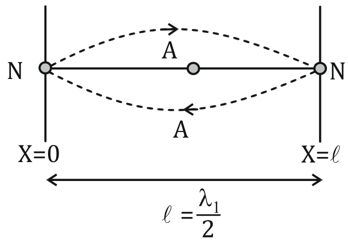
1. Harmonics: The frequency which are the integral multiple of the fundamental frequency are known as harmonics e.g. if n be the fundamental frequency, then the frequencies n, 2n, 3n.... are termed as first, second, third..... harmonics.

2. Overtone: The harmonics other than the first (fundamental note) which are actually produced by the instrument are called overtones. e.g. the tone with frequency immediately higher than the fundamental is defined as first overtone.

3. Octave: The tone whose frequency is doubled the fundamental frequency is defined as Octave.

Standing Waves on a String:

Fundamental mode of vibration



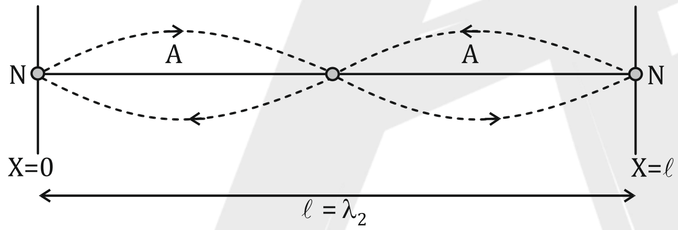
(i) Number of loops p = 1

(ii) Plucking at $\frac{\ell}{2}$ (from one fixed end)

(iii) $\ell = \frac{\lambda_1}{2} \Rightarrow \lambda_1 = 2\ell$

(iv) Fundamental frequency or first harmonic $n_1 = \frac{1}{\lambda_1}\sqrt{\frac{T}{\mu}} = \frac{1}{2\ell}\sqrt{\frac{T}{\mu}}$

Second mode of vibration:



(i) Number of loops p = 2

(ii) Plucking at $\frac{\ell}{2\times 2} = \frac{\ell}{4}$ (from one fixed end)

(iii) $\ell = \lambda_2$

(iv) Second harmonic or first over tone $n_2 = \frac{1}{\lambda_2}\sqrt{\frac{T}{\mu}} = \frac{1}{\ell}\sqrt{\frac{T}{\mu}} = 2n_1$

Third mode of vibration:

(i) Number of loops p = 3

(ii) Plucking at $\frac{\ell}{2\times 3} = \frac{1}{6}$ (from one fixed one)

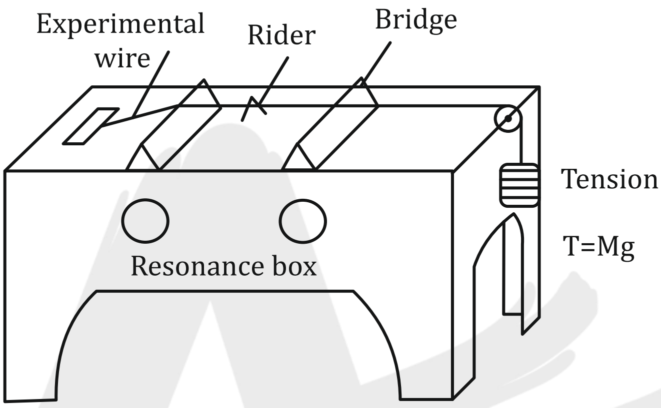
(iii) $\ell = \frac{3\lambda_3}{2} \Rightarrow \lambda_3 = \frac{2\ell}{3}$

(iv) Third harmonic or second over tone

$$n_3 = \frac{1}{\lambda_3}\sqrt{\frac{T}{\mu}} = \frac{3}{2\ell}\sqrt{\frac{T}{\mu}} = 3n_1$$

Sonometer

1. This apparatus is designed to create resonance, matching the frequency of a tuning fork (or any sound source), with a stretched vibrating string.
2. The apparatus comprises a hollow rectangular box made of lightweight wood. The experimental setup attached to the box is depicted below.



3. The box serves the function of amplifying the sound produced by the vibrating wire.
4. If the length of the wire between the two bridges is represented by 'L', then the frequency of vibration is given by.

$$n = \frac{1}{2\ell}\sqrt{\frac{T}{\mu}} = \sqrt{\frac{T}{\pi r^2 \rho}}$$

Laws of string

(i) Law of length: If T and μ are constant then $n \propto \frac{1}{\ell} \Rightarrow n\ell$ = constant $\Rightarrow n_1\ell_1 = n_2\ell_2$

If % change is less than 5% then $\frac{\Delta n}{n} = -\frac{\Delta \ell}{\ell}$ or $\frac{\Delta n}{n} \times 100\% = -\frac{\Delta \ell}{\ell} \times 100\%$

(ii) Law of mass: If T and ℓ are constant then

$$n \propto \frac{1}{\sqrt{\mu}} \Rightarrow \frac{n_1}{n_2} = \sqrt{\frac{\mu_2}{\mu_1}}$$

If % change is less than 5% then $\frac{\Delta n}{n} = -\frac{1}{2}\frac{\Delta \mu}{\mu}$ or $\frac{\Delta n}{n} \times 100\% = -\frac{\Delta \mu}{\mu} \times 100\%$

(iii) Law of density: If T, ℓ and r are constant

then $n \propto \frac{1}{\sqrt{\rho}} \Rightarrow n\sqrt{\rho}$ = constant $\Rightarrow \frac{n_1}{n_2} = \sqrt{\frac{\rho_2}{\rho_1}}$

If % change is less than 5% then

$$\frac{\Delta n}{n} = -\frac{1}{2}\frac{\Delta\rho}{\rho} \text{ or } \frac{\Delta n}{n}\times 100\% = -\frac{\Delta\rho}{\rho}\times 100\%$$

(iv) Law of tension: If ℓ and μ are constant then $n \propto \sqrt{T}$

$$\Rightarrow \frac{n}{\sqrt{T}} = \text{constant} \Rightarrow \frac{n_1}{n_2} = \sqrt{\frac{T_1}{T_2}} = \sqrt{\frac{M_1}{M_2}}$$

If % change is less than 5% then

$$\frac{\Delta n}{n} = \frac{1}{2}\frac{\Delta T}{T} \text{ or } \frac{\Delta n}{n}\times 100\% = -\frac{\Delta T}{T}\times 100\%$$

Sound Waves :

Sound is a type of energy that travels through longitudinal waves, propagating vibrations that can be perceived as auditory sensations when they reach the ear. Any object capable of vibrating can act as a source of sound. These longitudinal mechanical waves can travel through solids, liquids, and gases.

The intensity of sound, which determines its loudness, is directly related to the amplitude of vibration. A higher amplitude results in a greater intensity, producing a louder sound. However, the perception of loudness is subjective and relies on the sensitivity of the listener's ear. Therefore, the loudness of a sound with a particular intensity may vary among different individuals.

The average energy transmitted by a wave per unit normal area per second is called intensity of a wave. $I = \frac{E}{At}$. Its SI Unit : W/m^2

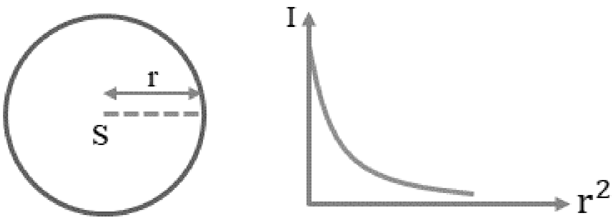
- It is the average power transmitted by a wave through the given area.

$$I = \frac{P_{avg}}{area};\ I = 2\pi^2 n^2 A^2 \rho v$$

Where ρ - density of medium, v – velocity of wave, A – Amplitude, n – Frequency

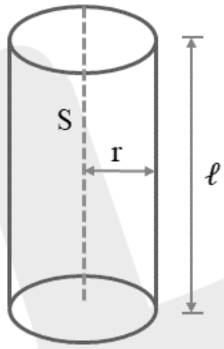
- Human ear responds to sound intensities over a wide range from $10^{-12}\ W/m^2$ to $1\ W/m^2$.
- In a spherical wave front, which originates from a point source, the amplitude of the wave changes in an inverse relationship with the distance from the source's position.

i.e., $A \alpha \frac{1}{r} \Rightarrow I \propto \frac{1}{r^2}$



- In a cylindrical wave front, which originates from a linear source, the amplitude of the wave decreases inversely proportional to the square root of the distance from the axis of the source.

i.e., $A \propto \frac{1}{\sqrt{r}} \Rightarrow I \propto \frac{1}{r}$



Sound level in decibles is given by $\beta = 10\log\left(\frac{I}{I_0}\right)$

If β_1 and β_2 be the sound levels corresponding to sound intensities I_1 and I_2 respectively. Then,

$(\text{or})\ \ \beta_2 - \beta_1 = 10\log\left(\frac{I_2}{I_1}\right)$

Velocity of Sound

- The equation for velocity of sound through a medium is given by $v = \sqrt{\frac{E}{\rho}}$ where E = modulus of elasticity; ρ = density

- In case of fluids (liquids and gases) $v = \sqrt{\frac{B}{\rho}}$ where B is the Bulk modulus

Velocity of sound in Gases :

- **Newton's formula :**

  Newton's formula is based on the assumption that sound propagates through a gas under isothermal conditions.

- Isothermal Bulk modulus, B = P

  $\therefore \quad v_s = \sqrt{\frac{P}{\rho}}$

- At S.T.P. $v = \sqrt{\frac{1.013\times10^5}{1.29}} \approx 280\text{ms}^{-1}$

  Which is less than the experimental vlaue $(332\ \text{m/s})$

- Laplace's correction : Laplace's assumption was that sound propagates through a gas under adiabatic conditions.

- Adiabatic Bulk modulus, $B = \gamma P$ $\quad\therefore\ v = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\frac{\gamma PV}{m}} = \sqrt{\frac{\gamma nRT}{m}} = \sqrt{\frac{\gamma RT}{M}}$

  where V = volume, m is mass, M = molecular weight. T is absolute temperature

- For air $\gamma = 1.4$. Therefore

  At STP $v_0 = 280\sqrt{1.4} \approx 330\ \text{m/s}^{-1}$, which agrees with the experimentally calculated value.

- The velocity of sound in a gas varies directly with the square root of the absolute temperature.

$$\frac{v_t}{v_0} = \sqrt{\frac{T}{T_0}} = \left(\frac{t+273}{273}\right)^{1/2} \quad \left(\because\ v \propto \sqrt{T}\right)$$

$$\Rightarrow\ v_t = v_0\left(1 + \frac{t}{546}\right)$$

$$\Rightarrow\ v_t = v_0 + \frac{v_0 t}{546} = v_0 + 0.61t°C$$

(1) Pressure Wave :

If the displacement wave is represented by $y = A\sin(\omega t - kx)$ then the corresponding pressure wave will be represented by $\Delta P = -B\frac{dy}{dx}$ (B = Bulk modulus of elasticity of medium)

$\therefore\ \Delta P = BAk\cos(\omega t - kx) = \Delta P_0\cos(\omega t - kx)$

Where ΔP_0 = pressure amplitude = BAk

According to definition of Bulk's modulus

$$B = -\Delta P\left(\frac{v}{\Delta v}\right) = \Delta P\left(\frac{\rho_0}{\Delta\rho}\right)$$

$$\Rightarrow\ \Delta\rho = \frac{\rho_0}{B}\cdot\Delta p$$

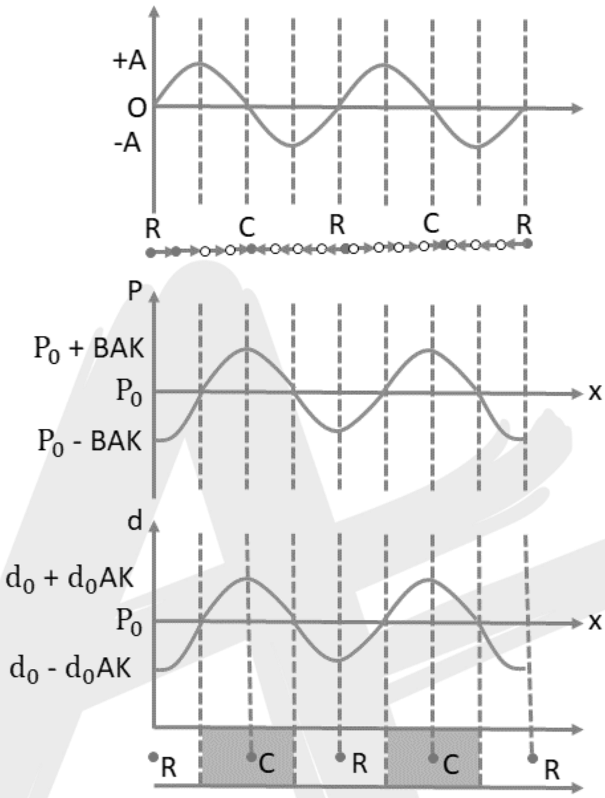
$$\Rightarrow\ \Delta\rho = \frac{\rho_0}{B}(\Delta p)_{max}\cos(kx - \omega t)$$

$$\Rightarrow\ \Delta\rho = \rho_0 Ak\cos(kx - \omega t)$$

$$\left(\because\ (\Delta p)_{max} = BAk\right)$$

$$\Rightarrow \quad \Delta\rho = (\Delta\rho)_{max}\cos(kx - \omega t),$$

where $(\Delta\rho)_{max} = \rho_0 Ak$ is called density amplitude. Thus the density wave is in phase with the pressure wave and this is 90° out of phase (lags) with the displacement wave as shown in the figure.



Note 1 : The relation between density amplitude and pressure amplitude is $(\Delta\rho)_{max} = (\Delta p)_{max}\left(\frac{\rho}{B}\right)$

Note 2 : Average Intensity $I = \frac{P}{S} = \frac{1}{2}\rho\omega^2 A^2 v$

In terms of pressure amplitude, sound intensity

$$I = \frac{1}{2}\rho\omega^2\left(\frac{\Delta p_{max}}{Bk}\right)^2 v = \frac{1}{2}\frac{(\Delta p_{max})^2}{\rho v}$$

$$\left[\because (\Delta P)_{max} = BAk,\ k = \frac{\omega}{v} \text{ and } B = \rho v^2\right]$$

Hence, the intensity of a wave is directly proportional to the square of the pressure amplitude, displacement amplitude, or density amplitude, and it remains unaffected by the frequency of the wave.

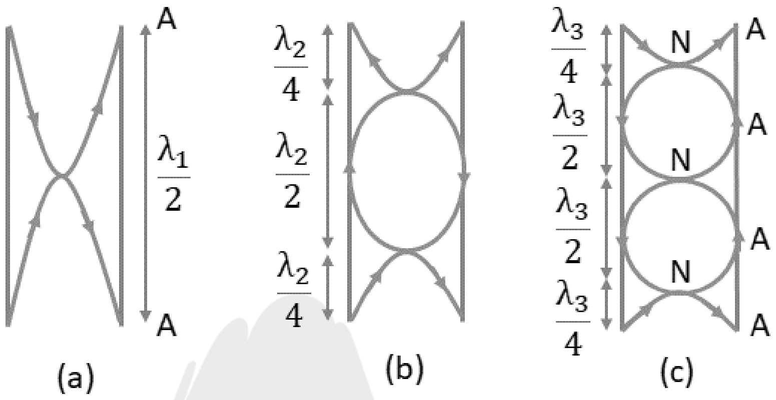
Organ pipes

Organ pipe:

An organ pipe refers to a cylindrical tube with a consistent cross-sectional area, where a column of gas is confined or trapped.

Open pipe :

When a pipe has both ends open and a flow of air is directed against one end, it can establish standing longitudinal waves within the tube. At the open ends, there occurs a displacement antinode, where the amplitude of the wave is at its maximum.



- Figure (a) For fundamental mode of vibrations or I harmonic

$$L = \frac{\lambda_1}{2}; \quad \therefore \quad \lambda_1 = 2L$$

$$V = \lambda_1 n_1; \quad \therefore \quad V = 2Ln_1 \Rightarrow n_1 = \frac{V}{2L} \qquad ....(1)$$

- Figure (b) For the second harmonic or first overtone,

$$L = \lambda_2$$

$$V = \lambda_2 n_2 \quad \therefore \quad V = Ln_2 \Rightarrow n_2 = \frac{2V}{2L} \qquad ....(2)$$

- Figure (c) For the third harmonic or second overtone,

$$L = 3\times\frac{\lambda_3}{2} \quad \therefore \lambda_3 = \frac{2}{3}L$$

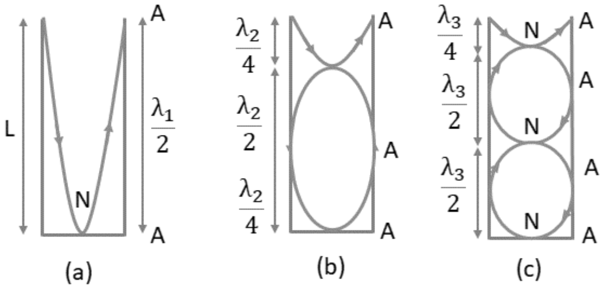
$$V = \lambda_3 n_3 \quad \therefore V = \frac{2}{3}Ln_3 \quad \Rightarrow \quad n_3 = \frac{3V}{2L} \qquad ....(3)$$

- From (1), (2) and (3) we get,

$$n_1 : n_2 : n_3 ..... = 1 : 2 : 3 : .....$$

Closed pipe :

If one end of a pipe is closed, then reflected wave is $180°$ out of phase with the wave. Consequently, the closed end of the pipe must always have zero displacement for the small volume elements. Therefore, the closed end serves as a displacement node, where the amplitude of the wave is minimal or non-existent.



- Figure (a) for the fundamental mode of vibration or I harmonic :

  $L = \frac{\lambda_1}{4} \quad \therefore \lambda_1 = 4L$

  If n_1 is the fundamental frequency, then the velocity of sound waves is given as

  $V = \lambda_1 n_1 \quad \therefore \; V = 4Ln_1 \; \Rightarrow \; n_1 = \frac{V}{4L}$ ....(1)

- Figure (b) for third harmonic or first overtone.

  $L = 3 \times \frac{\lambda_2}{4}, \quad \therefore \lambda_2 = \frac{4}{3}L$

  $V = \lambda_2 n_2, \; \therefore \; V = \frac{4}{3}Ln_2 \; \Rightarrow \; n_2 = \frac{3V}{4L}$ ....(2)

- Figure (c) for fifth harmonic or second overtone.

  $L = 5 \times \frac{\lambda_3}{4}, \quad \lambda_3 = \frac{4}{5}L$

  $V = \lambda_3 n_3, \; V = \frac{4}{5}Ln_3 \; \Rightarrow \; n_3 = \frac{5V}{4L}$ ....(3)

  From (1), (2) and (3) we get,

  $n_1 : n_2 : n_3 ...... = 1 : 3 : 5 : .....$

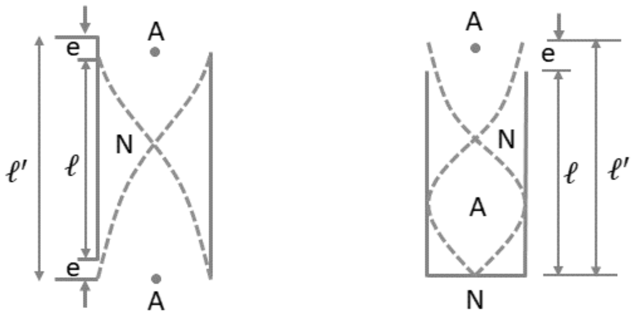
- In the general case, $\lambda = \frac{4L}{(2p+1)}$, where $p = 0, 1, 2, ...$

- p^{th} harmonic frequency $= \frac{(2p-1)V}{4L}$, where $p = 1, 2, ...$

End Correction

The distance of the antinode from the open end is referred to as the "end correction" (e).

It is given by $e = 0.6r$, where r = radius of pipe.



(A) Open pipe (B) Close pipe

Effective length in open organ pipe $\ell' = (\ell + 2e)$

Effective length in closed organ pipe $\ell' = (\ell + e)$

Note : When the end correction is considered, then

(i) the fundamental frequency of open pipe

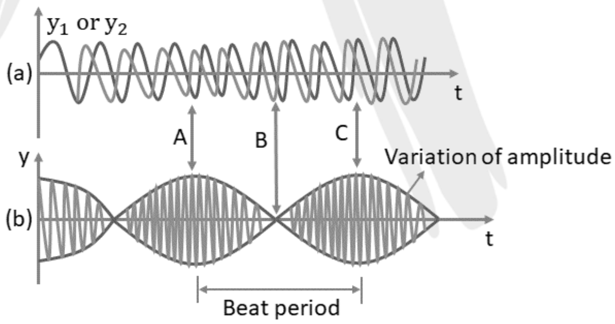
$$n = \frac{V}{2(\ell + 2e)} \Rightarrow n = \frac{V}{2(\ell + 1.2r)}$$

(ii) The fundamental frequency of closed pipe

$$n = \frac{V}{4(\ell + e)} \Rightarrow n = \frac{V}{4(\ell + 0.6r)}$$

BEATS

It is the phenomenon of periodic fluctuations in the intensity of sound when two waves of slightly different frequencies, traveling in the same direction, superimpose or overlap with each other.



Analytical treatment of Beats :

- Equations of waves producing beats are given as $y_1 = a\sin\omega_1 t$ and $y_2 = a\sin\omega_2 t$ let $\omega_1 > \omega_2$
- Resultant wave equation is $y = y_1 + y_2 = 2a\cos\left(\frac{\omega_1 - \omega_2}{2}\right)t\sin\left(\frac{\omega_1 + \omega_2}{2}\right)t$

$$y = A(t)\cos\left(\frac{\omega_1 - \omega_2}{2}\right)t$$

Here $A(t) = 2a\sin\frac{(\omega_1+\omega_2)}{2}t$

Beat frequency = $n_1 \sim n_2$

- The time period of one beat (or) the time interval between two successive maxima or minima is $\frac{1}{n_1 \sim n_2}$
- The time interval between a minima and the immediate maxima is $\frac{1}{2(n_1 \sim n_2)}$
- As the persistence of human hearing is about 0.1 sec, beats will be detected by the ear only if beat period is $\Delta t \geq 0.1\text{sec}$ or beat frequency $\Delta n = n_1 \sim n_2 \leq 10\,\text{Hz}$
- If a_1, a_2 are amplitudes of two sound waves that interfere to produce beats then the ratio of maximum and minimum intensity of sound is, $\frac{I_{max}}{I_{min}} = \left(\frac{a_1+a_2}{a_1-a_2}\right)^2$

Doppler's effect : Whenever there is relative motion between a sound source and an observer (listener), the frequency of the sound perceived by the observer differs from the actual frequency emitted by the source. This perceived frequency is known as the apparent frequency, and it can either be lower or higher than the actual emitted frequency, depending on the relative motion between the source and the observer. The amount of difference in frequency is determined by their relative motion.

General expression for apparent frequency :

If v, v_0, v_s are the velocities of sound, observer, source respectively and velocity of medium is v_m then apparent frequency observed by observer when wind blows in the direction of v (from the source to observer) is given by

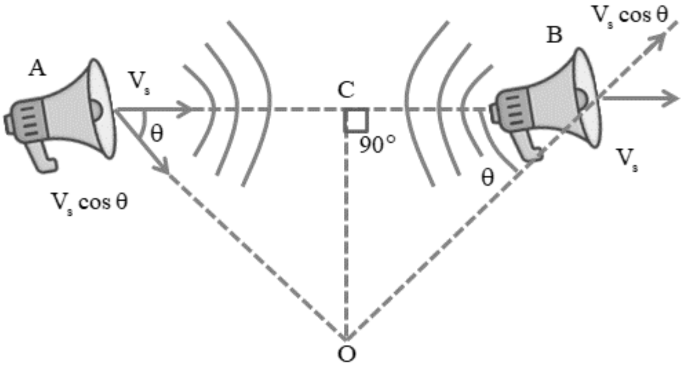
$n' = \left[\frac{(v+v_m) \pm v_0}{(v+v_m) \pm v_s}\right] n$ and in opposite direction of v

(from observer to source) $n' = \left[\frac{(v-v_m) \pm v_0}{(v-v_m) \pm v_s}\right] n$

If medium is stationary i.e., $v_m = 0$ then $n' = \left(\frac{v \pm v_0}{v \pm v_s}\right) n$

Transverse Doppler's Effect

(i) If a source is moving in a direction making an angle θ w.r.t. the observer.



The apparent frequency herd by observer O at rest

At point A : $n' = \dfrac{nv}{v - v_s \cos\theta}$

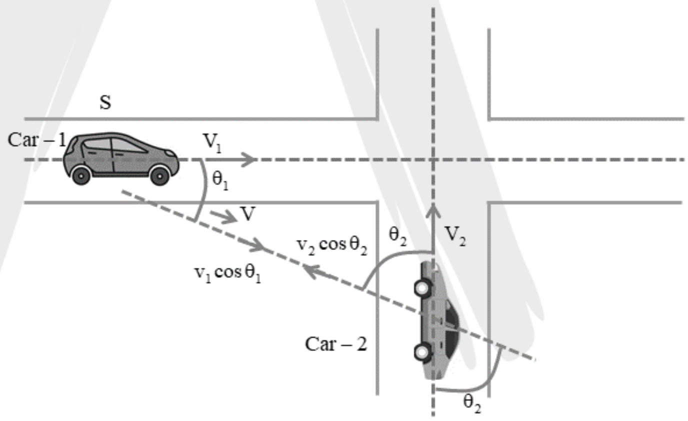
As source moves along AB, value of θ increases, $\cos\theta$ decreases, n' goes on decreasing.

At point C :

$\theta = 90°,\ \cos\theta = \cos 90° = 0,\ n' = n$

At point B : The apparent frequency of sound becomes $n'' = \dfrac{nv}{v + v_s \cos\theta}$

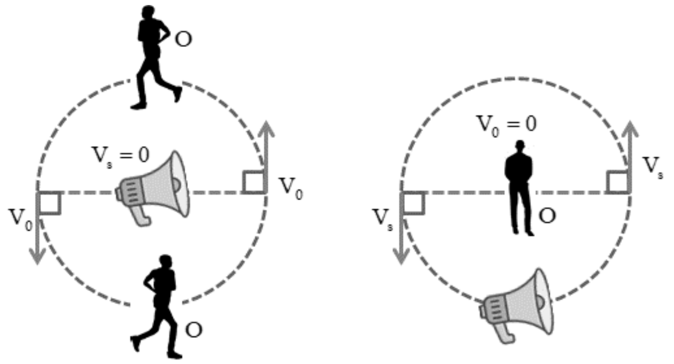
(ii) When two cars are moving on perpendicular roads: When car-1 sounds a horn of frequency n, the apparent frequency of sound heard by car-2 can be given as $n' = n\left[\dfrac{v + v_2 \cos\theta_2}{v - v_1 \cos\theta_1}\right]$



Rotating source/observer :

Suppose that a source of sound/observer is rotating in a circle of radius r with angular velocity ω (Linear velocity $v_s = r\omega$)

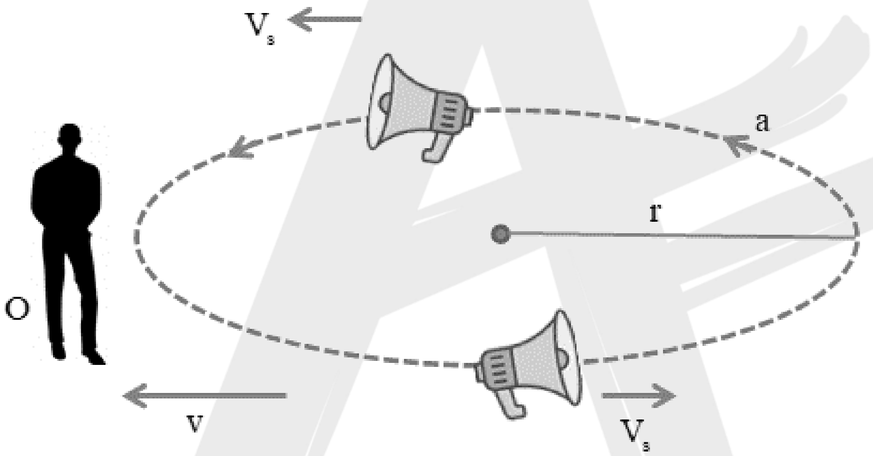
(i) When both the source and the observer are at rest at the center of a circle, and one of them is rotating in a circular path around the other, the line of sight between them remains perpendicular to the direction of motion. As a result, there is no Doppler effect observed in this scenario.$\therefore\ n' = n$



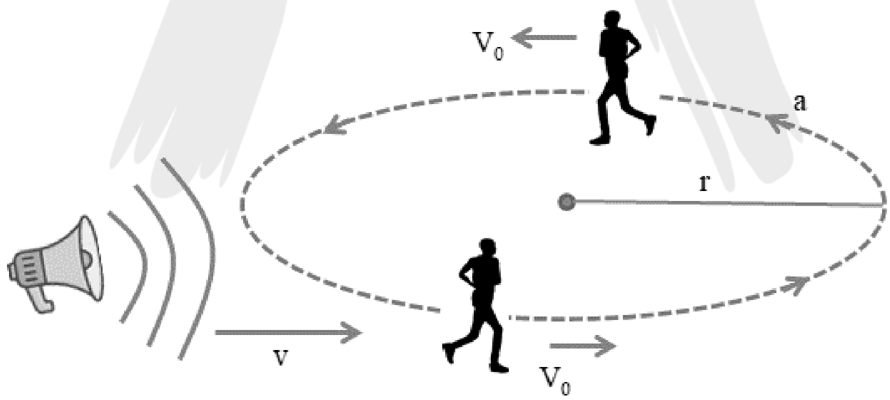
(ii) When source is rotating

(a) Towards the observer heard frequency will be maximum i.e., $n_{max} = \dfrac{nv}{v - v_s}$

(b) Away from the observer heard frequency will be minimum and $n_{min} = \dfrac{nv}{v + v_s}$



(iii) When oberver is rotating



(a) Towards the source heard frequency will be maximum

i.e., $n_{max} = n\left(\dfrac{v + v_0}{v}\right)$

(b) Away from the source heard frequency will be minimum and $n_{min} = n\left(\dfrac{v - v_0}{v}\right)$

(c) Ratio of maximum and minimum frequency $\frac{n_{max}}{n_{min}} = \frac{v+v_0}{v-v_0}$

Doppler shift in RADAR : A microwave beam is directed towards the aeroplane and is received back after reflection from it. It 'v' is the speed of the plane and 'n' is the actual frequency of the microwave beam then the frequency of the microwave beam then the frequency received by moving plane $n' = \left(\frac{c+v}{c}\right)n$

Now the plane act as a moving source, the frequency of the wave from it is $n'' = \left(\frac{c+v}{c-v}\right)n$

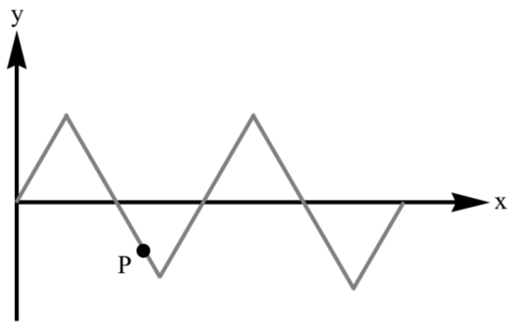
(c is velocity of microwave)

Change in frequency $\Delta n \approx \frac{2nv}{c}$

By measuring Δn, the speed 'v' can be obtained.

EXCERCISE-I

1. A triangular transverse wave is propagating in the positive X – direction. Velocity of P at this instant will be



(A) Vertically upward
(B) Vertically downward
(C) At rest
(D) cannot be determined

2. If at t = 0 a travelling wave pulse on a string is described by the function $y=\frac{10}{\left(5+x^{2}\right)}$, the wave function representing the pulse at time t, if the pulse is travelling along positive x – axis with speed $2\,ms^{-1}$ will be

(A) $\frac{10}{5+(x+2t)^{2}}$
(B) $\frac{10+2t}{5+x^{2}}$
(C) $\frac{10}{5+(x-2t)^{2}}$
(D) $\frac{10}{5+(4x+t)^{2}}$

3. The amplitude of a wave represented by displacement equation $y=\frac{1}{\sqrt{a}}\sin\omega t\pm\frac{1}{\sqrt{b}}\cos\omega t$ will be

(A) $\frac{a+b}{ab}$
(B) $\frac{\sqrt{a}+\sqrt{b}}{ab}$
(C) $\frac{\sqrt{a}\pm\sqrt{b}}{ab}$
(D) $\sqrt{\frac{a+b}{ab}}$

4. A transverse sinusoidal wave moves along a string in the positive x direction. In figure (I) displacement of particle at P as a function of time is given and in figure (II) at a particular time t – the snap shot of wave is shown. The wave velocity (cm/s) and velocity of particle at P (cm/s) will be

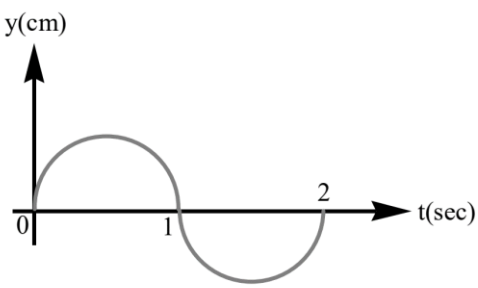


Figure - I

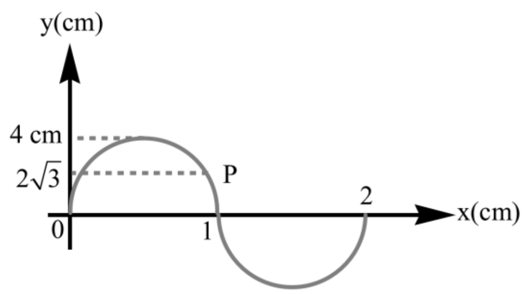
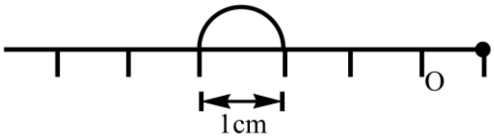
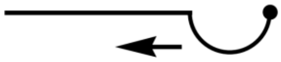


Figure - II

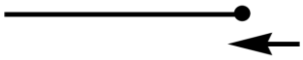
(A) $1\hat{i},-3\pi\hat{j}$
(B) $1\hat{i},3\pi\hat{j}$
(C) $1\hat{i},2\pi\hat{j}$
(D) $-1\hat{i},-2\pi\hat{j}$

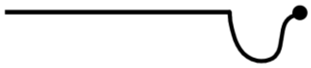
5. A wave pulse on a string has the shape at a particular instant as shown in the figure. The wave speed is $v = 1cm/s$. The point O is a fixed end. The total wave on the string at t = 4.5 seconds is represented by



(A) 

(B) 

(C) 

(D) 

6. What is the percentage change in the tension necessary in a sonometer of fixed length to produce a note one octave lower (half of original frequency) than before

7. A wave equation is given as $y = \cos(500t - 70x)$, where y is in mm, x is in m and t is in sec.

(A) The wave must be a transverse propagating wave

(B) The speed of the wave is $\frac{100}{7} m/s$

(C) The frequency of oscillations is $100\pi Hz$

(D) Two closest points which are in same phase have separation $\frac{20\pi}{7} cm$

8. Which of the following actions would make a pulse travel faster along a stretched string?

(A) Move your hand up and down more quickly as you generate the pulse

(B) Use a heavier string of the same length, under the same tension

(C) Use a lighter string of the same length, under the same tension

(D) Stretch the string tighter to increase the tension

9. The wave function for the wave pulse is $y(x,t) = \frac{0.1C^3}{C^2+(x-vt)^2}$ with $C = 4cm$ at x = 0 the displacement $y = (x,t)$ is observed to decrease from its maximum value of half of its maximum value in time $t = 2 \times 10^{-3}s$.

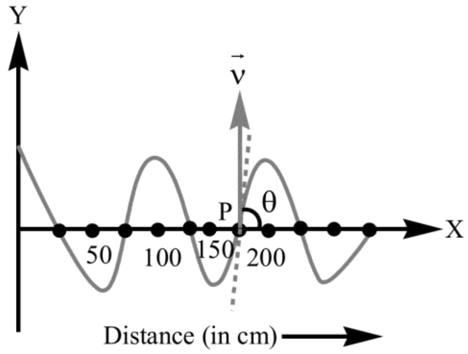
(A) $y(x,t)$ represents the motion of travelling pulse moving along positive X direction

(B) $y(x,t)$ does not represent the motion of a travelling pulse

(C) Speed of wave pulse is 20m/s

(D) Speed of wave pulse is 10m/s

10. Shape of string transmitting wave along x – axis at some instant is as shown. velocity of point P is $v = 4\pi cm/s$ and angle $\theta = \tan^{-1}(0.004\pi)$.



(A) Amplitude of wave is 2mm

(B) Velocity of wave is 5m/s

(C) Maximum acceleration of particle is $80\pi^2 \, cm/sec^2$

(D) Wave is travelling in –ve x – direction

11. A sinusoidal wave of amplitude A and wavelength λ is incident from heavier string on a joint between two strings of which one is heavy and another is light. Choose correct option(s)

(A) The amplitude of transmitted wave is more than that of the incident wave

(B) The wavelength of the transmitted wave is less than that of the incident wave

(C) The amplitude of the reflected wave is less than the amplitude of incident wave

(D) The wavelength of the reflected wave is same as that of the incident wave

12. Two mechanical waves, $y_1 = 2\sin 2\pi(50t - 2x)$ and $y_2 = 4\sin 2\pi(ax + 100t)$ propagates in a medium with same speed

(A) The ratio of their intensities is 1:16 (B) The ratio of their intensities is 4:24

(C) The value of 'a' is 4 units (D) The value of 'a' is 2 units

13. Under similar conditions of temperature and pressure, in which of the following gases the velocity of sound will be largest?

(A) H_2 (B) N_2 (C) He (D) CO_2

14. A sound wave of frequency 500 Hz covers a distance of 1000m in 5 second between the points X and Y. Then the number of waves between X and Y is

15. Propagation of a sound wave in a gas is quite close to

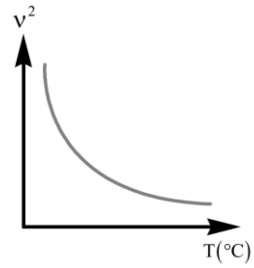
(A) An isothermal process

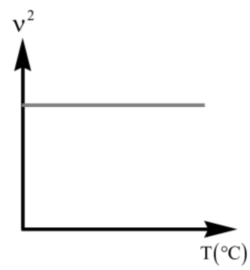
(B) An adiabatic process

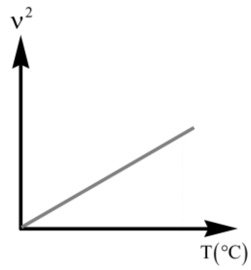
(C) An isochoric process

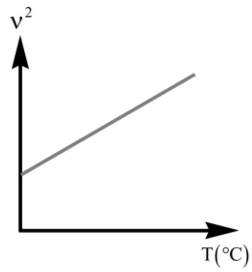
(D) A process that does not exhibit properties close to any of the three given in (A), (B), (C)

16. The graph between the $(\text{Velocity})^2$ and temperature T of a gas is

(A) 

(B) 

(C) 

(D) 

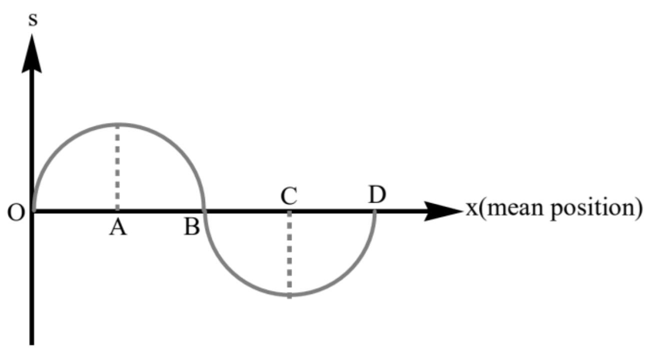
17. The velocity of sound in a gas at temperature $27°C$ is V then in the same gas its velocity will be 2V at temperature (in $°C$)

18. When a sound wave is reflected from a wall, the phase difference between the reflected and incident pressure wave is

(A) 0 (B) π (C) $\frac{\pi}{2}$ (D) $\frac{\pi}{4}$

19. In an experiment three tuning fork are used one of frequency f_1 and other two of frequency f_2. When f_1 and f_2 are struck together a beat of frequency 5 Hz is observed. When wax is applied on f_2 and struck along with f_1 beat frequency increases to 6 Hz and when f_1 filled and struck along with f_2 it produced beats of frequency 7 Hz. What will be the beat frequency (in Hz) when filled f_1 and waxed f_2 are struck together (Assume $f_1 > f_2$)?

20. A sound wave is travelling in air along positive x – direction. Displacement(s) of particles from their mean positions at a particular time t is shown in the figure. Choose the correct option(s) for that instant only



(A) Particle located at C has zero velocity

(B) Particle located at D has its velocity in the negative direction

(C) The pressure at C is equal to normal atmospheric pressure

(D) Particles located near B are under Refraction

21. The displacement amplitude of 5000 Hz sound wave is 1.20×10^{-10} m. At $t=0.00$ s and $x=0.00$ m the displacement is zero. (Speed of sound in air is 330m/s)

(A) Displacement wave function for this sound is

$$y = (1.2\times10^{-10}\text{m})\sin[(95.15\text{m}^{-1})x-(10^4\pi\text{rad/s})t]$$

(B) Displacement wave function for this sound is

$$y = (1.2\times10^{-10}\text{m})\sin\left[(95.15\text{m}^{-1})x-(10^4\pi\text{rad/s})t+\frac{\pi}{2}\right]$$

(C) Velocity of the particle at t = 0sec and x = 0 is $1.2\pi\times10^{-6}$ m / s

(D) Velocity of the particle at t = 0 sec and x = 0 is zero

EXCERCISE-II

1. A sinusoidal wave travelling in the positive direction of x on a stretched string has amplitude 2.0cm, wavelength 1m and wave velocity 5.0m/s. At x = 0 and t = 0, it is given that displacement y = 0 and $\frac{\partial y}{\partial t} < 0$. Express the wave function correctly in the form $y = f(x, t)$

(A) $y = (0.02\,m)\sin 2\pi(x - 5t)$ (B) $y = (0.02\,cm)\cos 2\pi(x - 5t)$

(C) $y = (0.02\,m)\sin 2\pi\left(x - 5t + \frac{1}{4}\right)$ (D) $y = (0.02\,cm)\cos 2\pi\left(x - 5t + \frac{1}{4}\right)$

2. A man generates a symmetrical pulse in a string by moving his hand up and down. At t = 0 the point in his hand moves downward. The pulse travels with speed of 3m/s on the string and his hands passes 6times in each second from the mean position. Then the point on the string at a distance 3m will reach its upper extreme first time at time t

(A) 1.25sec (B) 1 sec (C) $\frac{13}{12}$sec (D) 2.25sec

3. Two small boats are 10m apart on a lake. Each pops up and down with a period of 4.0 seconds due to wave motion on the surface of water. When one boat is at its highest point, the other boat is at its lowest point. Both boats are always within a single cycle of the waves. The speed of the waves (in m/s) is

4. A violin string oscillating in its fundamental mode, generates a sound wave with wavelength λ. To generate a sound wave with wavelength $\frac{\lambda}{2}$ by the string, still oscillating in its fundament mode, tension must be changed by the multiple

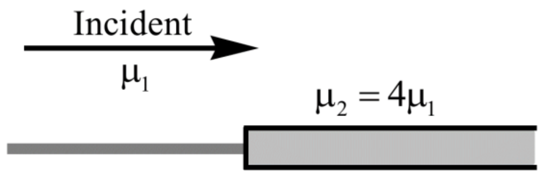
5. The displacement from the position of equilibrium of a point 4 cm from a source of sinusoidal oscillations is half the amplitude at the moment $t = \frac{T}{6}$ (T is the time period). Assume that the source was at mean position at t = 0. The wavelength of the running wave is

(A) 0.96 m (B) 0.48 m

(C) 0.24 m (D) 0.12 m

6. A wire under tension vibrates with a fundamental frequency of 600Hz. If the length of the wire is doubled, the radius is halved and the wire is made to vibrate under one – ninth the tension. Then the fundamental frequency (in Hz) will become

7. String #1 is connected with string #2. The mass per unit length in string #1 is μ_1 and the mass per unit length in string #2 is $4\mu_1$.The tension in the strings is T. A travelling wave is coming from the left. What fraction of energy in the incident wave goes into string #2?



(A) $\frac{1}{8}$ (B) $\frac{4}{9}$ (C) $\frac{2}{3}$ (D) $\frac{8}{9}$

8. In a standing wave formed as a result of reflection from a surface, the ratio of the amplitude at an antinode to that at node is x. The fraction of energy that is reflected is

(A) $\left[\frac{(x-1)}{x}\right]^2$ (B) $\left[\frac{x}{(x+1)}\right]^2$

(C) $\left[\frac{(x-1)}{(x+1)}\right]^2$ (D) $\left[\frac{1}{x}\right]^2$

9. In case of a transverse wave in a string

(A) If $\frac{\lambda}{2\pi} > A_0$, maximum particle speed is more than the wave speed

(B) if $\frac{\lambda}{2\pi} = A_0$, maximum particle speed is equal to the wave speed

(C) In harmonic wave, wave velocity is constant

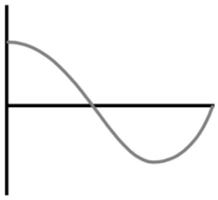
(D) In harmonic wave, particle speed is constant

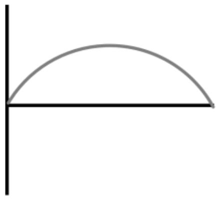
10. A girl stands 130m away from a high wall. She claps her hangs together at a steady rate such that 37claps are made in 30seconds, starting at time t = 0 and ending at t = 30s. Each clap, except the first, coincides with the echo of the one before. Calculate the speed of sound (in m/s)

11. Four sources of sound each of sound level 10 dB are sounded together, the resultant intensity level (in dB) will be

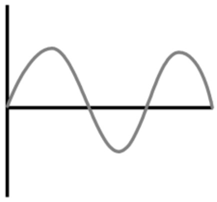
12. Two sound waves of slightly different frequencies have amplitude ratio $\frac{11}{9}$. What is the difference of sound levels in decibels of maximum and minimum intensities heard at a point?

13. Which of the figures, shows the pressure difference from regular atmospheric pressure for an organ pipe of length L closed at one end, corresponds to the 1st overtone for the pipe?

(A) 

(B) 

(C) 

(D) 

14. An open pipe is suddenly closed at one end with the result that the frequency of third harmonic of the closed pipe is found to be higher by 100 Hz than the fundamental frequency of the open pipe. The fundamental frequency (in Hz) of the open pipe is

15. Two organ pipes, a pipe of fundamental frequency 440 Hz, closed at one end, and a pipe of fundamental frequency 660 Hz, open at both ends, produce overtones. Which choice below correctly describes overtones present in both pipes?

(A) After the first overtone of each pipe, every second overtone of the first pipe matches every third overtone of the second pipe.

(B) After the first overtone of each pipe, every third overtone of the first pipe matches every second overtone of the second pipe.

(C) After the first overtone of each pipe, every second overtone of the first pipe matches every fourth overtone of the second pipe.

(D) After the first overtone of each pipe, every third overtone of the first pipe matches every fourth overtone of the second pipe.

16. A closed organ pipe of length L is vibrating in its first overtone. There is a point Q inside the pipe at a distance $\frac{7L}{9}$ from the open end. The ratio of pressure amplitude at Q to the maximum pressure amplitude in the pipe is

(A) 1:2 (B) 2:1 (C) 1:1 (D) 2:3

17. A policemen buzz a whistle of frequency 400 Hz. A car driver is approaching the policemen. The speed of car is $54 kmh^{-1}$. The change in frequency experienced by the driver, when driver approaches the policemen and after he crosses the policemen, is (Velocity of sound is 350m/s)

(A) 42.8 Hz (B) 34.2 Hz (C) 38.6 Hz (D) 27.6 Hz

18. The difference between the apparent frequencies of a sound source as perceived by a stationary observer during its approach and recession is 2% of the actual frequency of the source. The speed of source (in m/s) is (Speed of sound = 300m/s)

19. A man standing in front of a vertical wall at a certain distance beats a drum at regular intervals. The drumming rate is gradually increased and he finds that the echo is not heard distinctly when the drumming rate becomes 2 beats in seconds. He then moves nearer to the wall by 90 m and finds the echo is again not heard when the drumming rate becomes 1 per second. From this data, the velocity of sound (in m/s) must be

20. You are provided with three similar, but slightly different tuning forks. When A and B are both struck, a beat frequency of f_{AB} is heard. When A and C are both struck, a beat frequency of f_{AC} is heard. If B and C are simultaneously struck, what will be the observed beat frequency?

(A) $|f_{AB}+f_{AC}|$

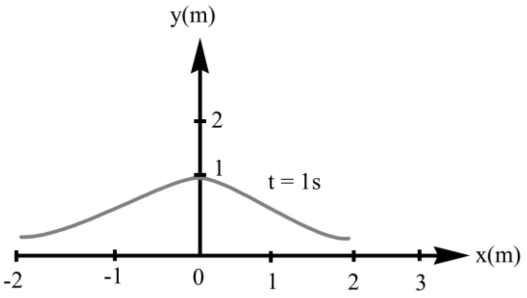
(B) $|f_{AB}-f_{AC}|$

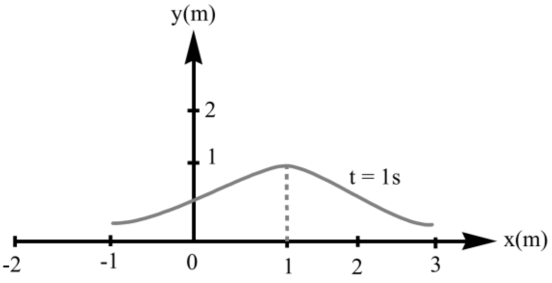
(C) Either $|f_{AB}+f_{AC}|$ or $|f_{AB}-f_{AC}|$ will be heard

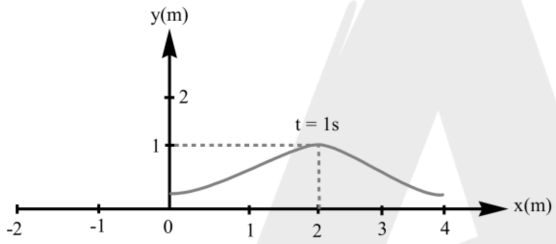
(D) $|f_{AB}+f_{AC}|$ and $|f_{AB}-f_{AC}|$ will simultaneously be heard

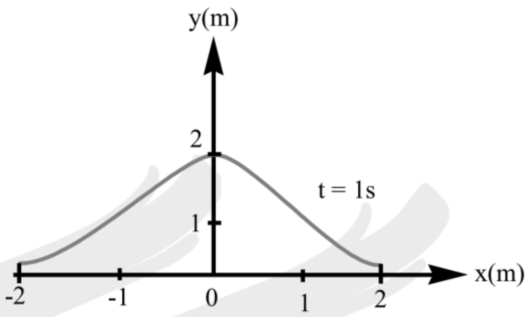
EXCERCISE-III

1. A pulse is given by the equation $y = f(x,t) = A\exp[-B(x-vt)^2]$. Given $A = 1.0m$, $B = 1.0m^{-2}$ and $v = +2.0m/s$, which of the following graph shows the correct wave profile at the instant t = 1s?

(A)


(B)


(C)


(D)


2. Two particles of medium disturbed by the wave propagation are at $x_1 = 0$ and $x_2 = 1cm$. The respective displacements (in cm) of the particles can be given by the equations

$y_1 = 2\sin 3\pi t$ $y_2 = 2\sin\left(3\pi t - \frac{\pi}{8}\right)$

Find the wave velocity (in cm/sec) is

3. In the above question, the displacement (in cm) of particle at $t = 1sec$ and $x = 4cm$ is

4. A loop of a string of mass per unit length μ and radius R is rotated about an axis passing through centre perpendicular to the plane with an angular velocity ω. A small disturbance is created in the loop having the same sense of rotation. The linear speed of the disturbance for a stationary observer is $\alpha\omega R$. Find α ?

5. A transverse wave is propagating along +x direction. At t = 2sec, the particle at x = 4m is at y = 2mm. With the passage of time its y coordinate increases and reaches to a maximum of 4mm. The wave equation is (using ω and k with their usual meanings)

(A) $y = 4\sin\left[\omega(t+2) + k(x-2) + \frac{\pi}{6}\right]$ (B) $y = 4\sin\left[\omega(t+2) + k(x) + \frac{\pi}{6}\right]$

(C) $y = 4\sin\left[\omega(t-2) - k(x-4) + \frac{5\pi}{6}\right]$ (D) $y = 4\sin\left[\omega(t-2) - k(x-4) + \frac{\pi}{6}\right]$

6. Sinusoidal waves 5.00cm in amplitude are to be transmitted along a string having a linear mass density equal to $4.00\times10^{-2}\,kg/m$. If the source can deliver a maximum power of 90W and the string is under a tension of 100N, then the highest frequency (in Hz) at which the source can operate is (take $\pi^2=10$)

7. If $y_1=5(mm)\sin\pi t$ is equation of oscillation of source S_1 and $y_2=5(mm)\sin\left(\pi t+\frac{\pi}{6}\right)$ be that of S_2 and it takes 1 sec and $\frac{1}{2}$ sec for the transverse waves to reach point A from sources S_1 and S_2 respectively then the resulting amplitude (in mm) at point A, is

8. A string with a mass density of $4\times10^{-3}\,kg/m$ is under tension of 360N and is fixed at both ends. One of its resonance frequencies is 375 Hz. The next higher resonance frequency is 450 Hz. The mass of the string is

(A) $2\times10^{-3}kg$ (B) $3\times10^{-3}kg$

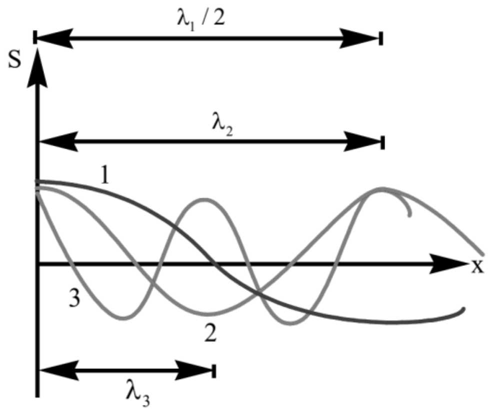
(C) $4\times10^{-3}kg$ (D) $8\times10^{-3}kg$

9. Two vibrating strings of the same material but lengths L and 2L have radii 2r and r respectively. They are stretched under the same tension. Both the strings vibrate in their fundamental modes, the one of length L with frequency f_1 and the other with frequency f_2. The ratio $\frac{f_1}{f_2}$ is given by

10. A string of length 'ℓ' is fixed at both ends. It is vibrating in its 3rd overtone with maximum amplitude 'a'. The amplitude at a distance $\frac{\ell}{3}$ from one end is $\frac{\sqrt{\alpha}a}{\beta}$. Find $\alpha+\beta$?

11. A wire having a linear density 0.1 kg/m is kept under a tension of 490N. It is observed that it resonates at a frequency of 400Hz and the next higher frequency 450 Hz. The length of the wire is $\frac{7}{n}m$. Find n?

12. Figure shown is a graph, at a certain time t, of the displacement function S(x, t) of three sound waves 1, 2 and 3 as marked on the curves that travel along x – axis through air. If P_1, P_2 and P_3 represent their pressure amplitudes respectively, then correct relation between them is



(A) $P_1 > P_2 > P_3$ (B) $P_3 > P_2 > P_1$ (C) $P_1 = P_2 = P_3$ (D) $P_2 > P_3 > P_1$

13. A person is standing at a distance D from an isotropic point source of sound. He walks 50.0 m towards the source and observes that the intensity of the sound has doubled. His initial distance D from the source is

(A) $50\sqrt{2}$m

(B) $\dfrac{50\sqrt{2}}{\sqrt{2}-1}$m

(C) $\dfrac{50}{\sqrt{2}-1}$m

(D) $100\sqrt{2}$ m

14. A bird is singing on a tree and a man is hearing at a distance 'r' from the bird. Calculate the displacement of the man towards the bird so that the loudness heard by man increases by 20dB. [Assume that the motion of man is along the line joining the bird and the man]

(A) $\dfrac{9r}{10}$ (B) $\dfrac{r}{10}$ (C) $\dfrac{3r}{5}$ (D) $\dfrac{4r}{5}$

15. Two radio frequency point sources S_1 and S_2, separated by distance 2.5m are emitting in phase waves of wavelength 1m. A detector moves in a large circular path around the two sources in a plane containing them. The number of maxima that will be detected by it over the complete circular path, are

16. In an organ pipe whose one end is at x = 0, the pressure is expressed by $P = P_0 \cos\frac{3\pi x}{2} \sin 3\ 0\pi t$, where x is in meter and t is in sec. The organ pipe can be

(A) Closed at one end, open at another with length = 0.5m

(B) Open at both ends, length = 1m

(C) Closed at both ends, length = 2m

(D) Closed at one end, open at another with length $= \dfrac{2}{3}$m

17. Air columns in an organ pipe P_1 closed at one end vibrates in its first harmonic and in another pipe P_2 open at both ends vibrates in its third harmonic are in resonance with a given tuning fork. The ratio of the length of P_1 to that of P_2 is

(A) $\frac{1}{6}$ (B) $\frac{1}{3}$ (C) $\frac{1}{2}$ (D) $\frac{8}{3}$

18. A longitudinal sound wave given by $P = 25 \sin\frac{\pi}{2}(x - 660t)$ (P is in N/m^2 and x is in m and t in s) is sent down a closed organ pipe. If the pipe vibrates is second overtone then the length (in m) of the pipe is

19. In Quincke's tube a detector detects minimum intensity. Now one of the tube is displaced by 5cm. During displacement detector detects maximum intensity 10 times, then finally a minimum intensity (When displacement is complete). The wavelength of sound (in cm) is

20. Two tuning forks A and B produce 8beats/s when sounded together. A gas column 37.5cm long in a pipe closed at one end resonate to its fundamental mode with fork A whereas a column of length 38.5cm of the same gas in a similar pipe is required for a similar resonance with fork B. The frequencies of these two tuning forks are

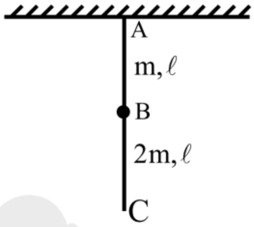
(A) 308 Hz, 300 Hz (B) 208 Hz, 200 Hz

(C) 300 Hz, 400 Hz (D) 350 Hz, 500 Hz

21. An ultrasonic burglar alarm in still air transmits a signal at a frequency of 4.5×10^4 Hz, part of which is reflected by the burglar to receiver alongside the transmitter. The alarm is triggered by any beat frequency greater than 5 Hz. Velocity of sound in air is 340 m/s. The minimum velocity (in mm/s) of approach of the burglar to activate the alarm, will be

EXCERCISE-IV

1. In the figure shown strings AB and BC have masses m and 2m respectively. Both are of same length ℓ. Mass of each string is uniformly distributed on its length. The string is suspended vertically from the ceiling of a room. A small jerk wave pulse is given at the end 'C'. it goes up to upper end 'A' in time 't'. if $m = 2kg$, $\ell = \frac{9610}{1681}$m, $g = 10 m/s^2$, $\sqrt{2} = 1.4$, $\sqrt{3} = 1.7$ then 't' is equal to



(A) $\frac{620}{697}$s (B) $\frac{434}{205}$s (C) 2s (C) None of these

2. The equation of a travelling wave in a uniform string of mass per unit length μ is given as $y = A\sin(\omega t - kx)$. Find the total energy transferred through the origin in time interval from t = 0 to t = $\frac{\pi}{12\omega}$.

(A) $\frac{(\pi+3)}{12}\frac{\mu\omega^2A^2}{K}$

(B) $\frac{(\pi+3)}{8}\frac{\mu\omega^2A^2}{K}$

(C) $\frac{(\pi+3)}{18}\frac{\mu\omega^2A^2}{K}$

(D) $\frac{(\pi+3)}{24}\frac{\mu\omega^2A^2}{K}$

3. The fundamental frequency of a sonometer wire increases by 6 Hz if its tension is increased by 44% keeping the length constant. The change in the fundamental frequency of the sonometer wire in Hz when the length of the wire is increased by 20%, keeping the original tension in the wire will be

4. A string fixed at one end is vibrating in its second overtone. The length of the string is 10cm and maximum amplitude of vibration of particles of the string is 2mm. Then the amplitude of the particle at 9cm from the open end is

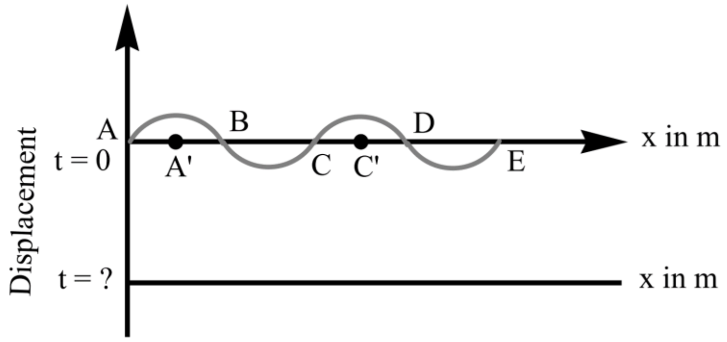
(A) $\sqrt{3}$mm

(B) $\sqrt{2}$mm

(C) $\frac{\sqrt{3}}{2}$mm

(D) $\sqrt{5}$mm

5. The pattern of standing waves formed on a stretched string at two instants of time (extreme, mean) are shown in figure. The velocity of two waves superimposing to form stationary waves is $360 ms^{-1}$ and their frequencies are 256 Hz. Which is not possible value of t (in sec)?



(A) 9.8×10^{-4} (B) 10^{-3} (C) 2.94×10^{-3} (D) 4.9×10^{-3}

6. A 40cm long wire having a mass 3.2 gm and area of cross section $1mm^2$ is stretched between the support 40.05 cm apart. In its fundamental mode, it vibrates with a frequency $\frac{1000}{64}$ Hz. Find the Young's modulus of the wire

(A) $1\times10^6 Nm^2$ (B) $1\times10^9 Nm^2$ (C) $2\times10^6 Nm^2$ (D) $4\times10^9 Nm^2$

7. A string of length 3L is fixed at both ends. It resonates with a tuning fork in third harmonic with amplitude at antinode equal to A_0. At time t = 0, a string element at position of antinode is at half its positive amplitude and moving towards mean position. Displacement of a string element at $\frac{L}{2}$ is given by

(A) $\frac{A_0}{2}\sin\left(\omega t+\frac{11\pi}{6}\right)$ (B) $\frac{\sqrt{3}A_0}{2}\sin\left(\omega t+\frac{5\pi}{6}\right)$

(C) $A_0\sin\left(\omega t+\frac{5\pi}{6}\right)$ (D) $\frac{A_0}{2}\sin\left(\omega t+\frac{5\pi}{6}\right)$

8. A string of length L fixed at the two ends is oscillating such that the point at $\frac{3L}{8}$ is an antinode. What is the possible number of loops in the string?

9. A steel wire of length L, cross – sectional area A and density ρ is fixed between two rigid supports, with the wire just taut. Y = Young's modulus and α = coefficient of thermal expansion of steel. If the wire temperature is now lowered by ΔT, then the frequency of the fundamental note produced by plucking the wire in the middle is

(A) $\frac{1}{L}\left[\frac{\alpha YL\Delta T}{\rho}\right]^{1/2}$ (B) $\frac{1}{2L}\left[\frac{\alpha Y\Delta T}{\rho}\right]^{1/2}$

(C) $\frac{1}{2L}\left[\frac{\alpha Y\Delta T}{\rho L}\right]^{1/2}$ (D) $\frac{1}{L}\left[\frac{\alpha Y\Delta T}{\rho}\right]^{1/2}$

10. Equation of a travelling wave is $y = 5e^{(-ax^2-bt^2-2\sqrt{ab}xt)}$ where x and y are in meter and t is in sec. $a = 25m^{-2}$ and $b = 9sec^{-1}$

(A) Travelling wave propagates along (+) x direction

(B) Travelling wave propagates along (-) x direction

(C) Speed of wave is $\frac{3}{5}$ m/sec

(D) Maximum displacement of particle is 10m

11. 200 students are taking an examination in a room, and the sounds of pens scratching on paper, sighs, groans and muttered imprecations has created a more or less continuous sound level of this noise of 60dB. Assuming each student contributes equally to this noise and nothing else changes or adds to it, what will the sound (in dB) level in the room be when only 50 students are left? $(\log_{10} 2 = 0.3)$

12. In a city sound intensity level increases uniformly by about 1 dB annually. How many years will it take for intensity of sound to become double?

13. A sound wave propagating along x-axis, in medium I of density $\rho_1 = 1.5kg/m^3$ is transmitted to a medium II of density $\rho_2 = 3kg/m^3$. The equation of excess pressure developed by wave in medium I and that in medium II respectively are

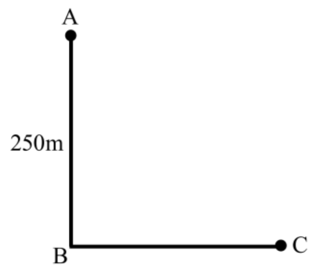
$p_1 = 4 \times 10^{-2} \cos\omega\left(t - \frac{x}{400}\right)$ (in SI units)

$p_2 = 3 \times 10^{-2} \cos\omega\left(t - \frac{x}{1200}\right)$ (in SI units)

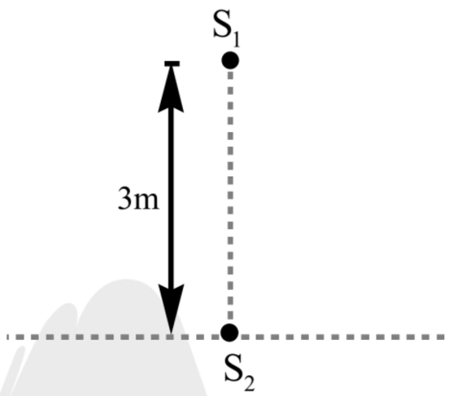
Then the ratio of intensity of transmitted wave I_2 (wave in medium II) to the intensity of incident wave I_1 (wave in medium I), that is, $\frac{I_2}{I_1}$ is

(A) $\frac{3}{4}$ (B) $\frac{9}{16}$ (C) $\frac{3}{32}$ (D) $\frac{32}{3}$

14. Two radio transmitters radiating in phase are located at points A and B, 250 m apart. The radio wave have frequency of 3 MHz. A radio receiver is moved out from point 'B' along a line BC (perpendicular to AB). The distance (in m) from B beyond which the detector does not detect any minima



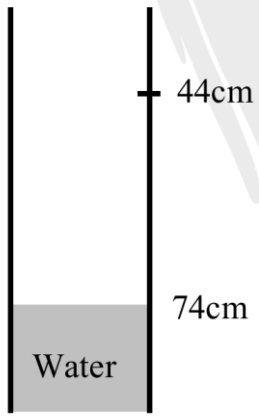
15. S_1 and S_2 are two coherent sources of sound having no initial phase difference. The velocity of sound is 330m/s. No minima will be formed on the line passing through S_2 and perpendicular to the line joining S_1 and S_2, if the frequency of both the sources is



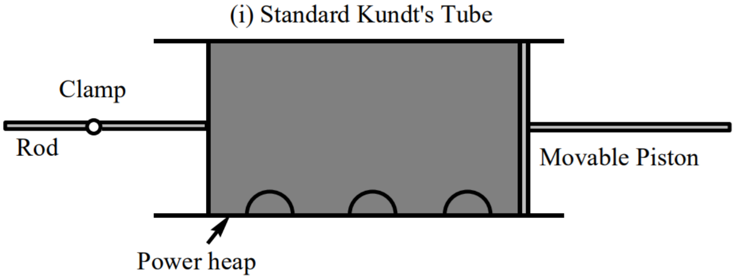
(A) 50 Hz (B) 60 Hz (C) 70 Hz (D) 80 Hz

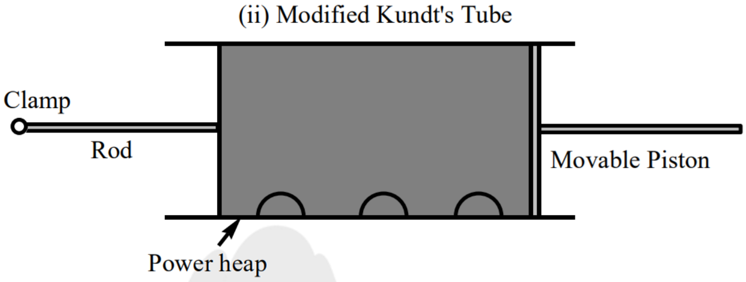
16. A vertical tube is completely filled with water. A small sound source of constant frequency is held a little above the open upper end and water is run out from the lower end. A number of resonance positions are detected. The first of these occurs when the water surfaces is 7cm below the top of the tube and another occurs at 39cm. Find the distance where should resonance detected?

17. Sound waves of frequency 320 Hz are sent into the top of a vertical tube containing water at a level than can be adjusted. Standing waves are produced at two successive water levels 44cm and 74cm from open end. The distance of nearest displacement antinode from open end (in cm) is

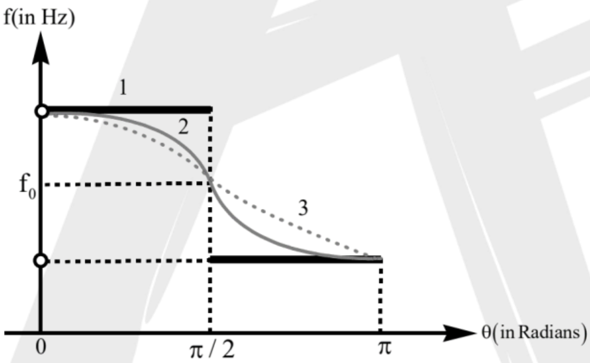


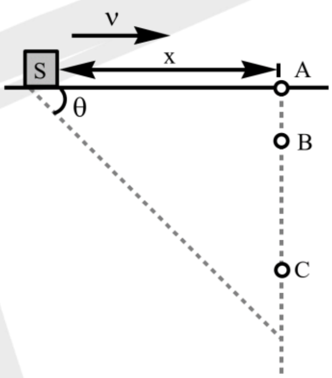
18. In the Kundt's tube experiment (shown in figure (i)), the rod is clamped at the end instead of clamping it at the centre as shown in figure (ii). It is known that speed of sound in air is 330m/s, powder piles up at successive distance of 0.6m and length of rod used is 1m, speed of sound in rod is 100n m/s. Find n?





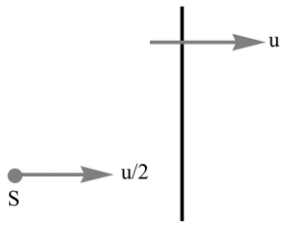
19. A source is moving with a constant speed u on a straight line, emitting a sound of frequency f_0. There are three observers A, B and C. A on track, B at a perpendicular distance of d from the track and C at a perpendicular distance of 2d from the track as shown in the figure. The variation of the observed frequency with respect to the position x





(A) A – 3, B – 2, C – 1
(B) A – 2, B – 3, C – 1
(C) A – 1, B – 2, C – 3
(D) A – 1, B – 3, C – 2

20. A wall is moving with velocity u and a source of sound moves with velocity $\frac{u}{2}$ in the same direction as shown in figure. Assuming that the sound travels with velocity 10u. The ratio of incident sound wavelength on the wall to the reflected sound wavelength by the wall, is equal to (assume observer for reflected sound is at rest)

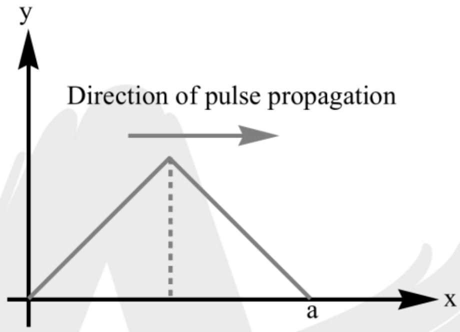


(A) 9:11
(B) 11:19
(C) 4:5
(D) 5:4

EXCERCISE-V

1. The wave function of a triangular wave pulse is defined by the relation below at time t = 0 sec

$$y=\begin{cases} mx & \text{for } 0\le x\le \frac{a}{2} \\ -m(x-a) & \text{for } \frac{a}{2}\le x\le a \\ 0 & \text{everywhere else, where } m<<1 \end{cases}$$



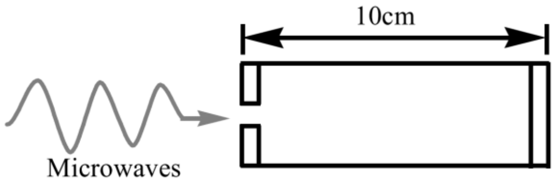
The wave pulse in moving in the +x direction in a string having tension T and mass per unit length μ. The total energy present with the wave pulse is

(A) $\frac{m^2Ta}{2}$ (B) m^2Ta (C) mTa (D) $\frac{mTa}{2}$

2. Two wires of same material and radii r and 2r respectively are welded together end to end and the combination is used in the sonometer and is kept under tension T. The welded point is midway between the bridges in the sonometer. What should be the ratio of number of loops formed in wires such that the joint is a node?

(A) 2:3 (B) 1:4 (C) 1:2 (D) 1:3

3. You have a micro – wave generator that can produce micro – waves at any frequency between 1GHz and 10GHz. The microwave radiation enters a 10cm long cylinder with reflective end caps, as shown in the figure. What frequency of the microwave generator, in GHz, will produce the lowest – order standing wave with an antinode (maximum) in the center of the cavity? Note that with reflectors at both ends, the electromagnetic standing wave acts just like the standing wave on a string that is tied at both ends. The velocity of waves as $3\times10^8\ m/s$



(A) 4.5 (B) 3.0 (C) 1.5 (D) 7.5

4. Transverse waves are being produced at the one end (x = 0) of a wire stretched along x – axis by a tuning fork oscillating along y – axis. The frequency of the fork is 400 Hz and linear mass density of wire is 0.05 kg/m. It is observed that at a certain moment of time two consecutive particles at extreme are located at x = 100cm and x = 200cm. Now choose the correct option(s)

(A) Tension in the string is 32×10^3N

(B) Speed of the wave is 800m/s

(C) Wavelength of the wave is 4m

(D) If maximum velocity of particle of the wire is $80\pi m/s$, then displacement amplitude of particle is 10cm

5. Two particles of medium disturbed by the wave propagation are at $x_1 = 0$ and $x_2 = 1\text{cm}$. The respective displacements (in cm) of the particles can be given by the equations:

$y_1 = 2\sin 3\pi t$ $\quad$ $y_2 = 2\sin\left(3\pi t - \frac{\pi}{8}\right)$

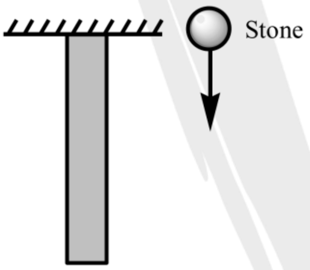
(A) The wave velocity is 16cm/sec

(B) The wave velocity is 24cm/sec

(C) The displacement of particle at t = 1sec and x = 4cm is zero

(D) The displacement of particle at t = 1sec and x = 4cm is 2cm

6. A uniform wire of length ℓ and mass m is hanging from a fixed support as shown. A transverse wave pulse is produced at the top of rod, and a stone is dropped simultaneously from top. Then



(A) Stone will definitely overtake the pulse, irrespective of length of wire

(B) Location of overtake will be independent of mass of wire, provided length remains same

(C) As the wave pulse travels downward its length of pulse decreases

(D) As the wave pulse travels downward its length of pulse increases

7. Consider a travelling simple harmonic wave on a string of mass per unit length μ and tension T. Kinetic energy per unit length is given by $\mu_k = \frac{1}{2}\mu\left(\frac{\partial y}{\partial t}\right)^2$ and potential energy per unit length is given by $\mu_p = \frac{1}{2}T\left(\frac{\partial y}{\partial x}\right)^2$. Mark the correct option(s).

(A) Power transmitted by wave equals $\left(u_k + u_p\right)\sqrt{\frac{T}{\mu}}$

(B) u_k and u_p simultaneously attain their maximum and minimum values

(C) Total energy per unit length of a string is constant when a harmonic wave travels on it

(D) A small part of string has maximum potential energy when it is at its equilibrium position

8. A long string of mass per unit length $0.2\,kg\,m^{-1}$ is stretched to a tension of 500N. Choose the correct option(s)

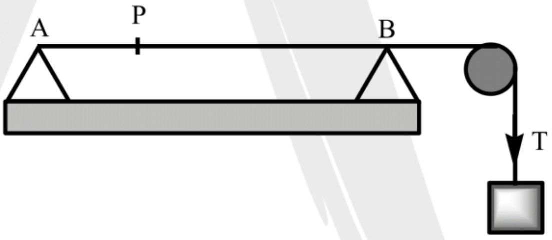
(A) Speed of transverse waves on string is $50\,ms^{-1}$

(B) Speed of transverse waves on string is $25\,ms^{-1}$

(C) Mean power required to maintain a travelling wave of amplitude 10mm and wavelength 0.5m is 197W

(D) Mean power required to maintain a travelling wave of amplitude 10mm and wavelength 0.5m is 150W

9. A sonometer string AB of length 1m is stretched by a load and the tension T is adjusted so that the string resonates to a frequency of 1 kHz. Any point P of the wire may be held fixed by use of a movable bridge that can slide along the base of sonometer.



(A) If point P is fixed so that AP:PB::1:4, then the smallest frequency for which the sonometer wire resonates is 5 kHz

(B) If P taken at midpoint of AB and fixed, then, when the wire vibrates in the third harmonic of its fundamental, the number of nodes in the wire (including A and B) will be totally seven

(C) If the fixed point P divides AB in the ratio 1:2, then the tension needed to make the string vibrate at 1 kHz will be 3T. (neglecting the terminal effects)

(D) The fundamental frequency of the sonometer wire when P divides AB in the ratio a:b will be the same as the fundamental frequency when P divides AB in the ratio b:a

10. A standing wave of time period T is set up in a string clamped between two rigid supports. At t = 0 antinode is at its maximum displacement 2A

(A) The energy density of a node is equal to energy density of an antinode for the first time at $t = \frac{T}{4}$

(B) The energy density of node and antinode becomes equal after $\frac{T}{2}$ second

(C) The displacement of the particle at antinode at $t = \frac{T}{8}$ is $\sqrt{2}A$

(D) The displacement of the particle at node is zero

11. A wave equation is represented as $\psi = A\sin\left[a\left(\frac{x-y}{2}\right)\right]\cos\left[\omega t - \alpha\left(\frac{x+y}{2}\right)\right]$, where x and y are in meter and t is in second then

(A) The wave is stationary wave

(B) The wave is a progressive wave propagating along +x – axis

(C) The wave is progressive wave propagating at right angle to the +x – axis

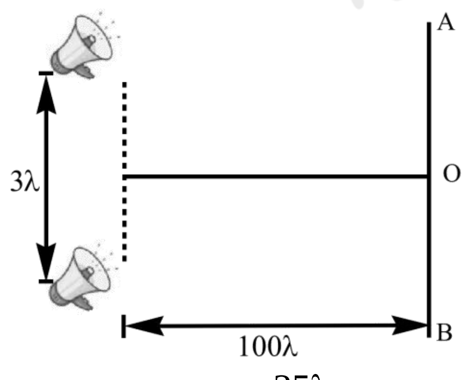
(D) All point lying on line $y = x + \frac{4\pi}{\alpha}$ are always at rest

12. A sound wave is travelling in a uniform pipe with gas of adiabatic exponent γ. If u is the particle velocity at any point in medium and c is the wave velocity, then relative change in pressure $\frac{dP}{P}$ while wave passes through this point is

(A) $\frac{u}{\gamma c}$ (B) $\gamma\sqrt{\frac{u}{c}}$ (C) $\gamma\frac{u}{c}$ (D) $\frac{u^2}{\gamma c^2}$

13. A plane progressive sound wave of frequency 160 Hz is moving along vector $3\hat{i} + 4\hat{j}$. The phase difference between point A(1, 0, 2) and point B(3, 1, 4) is nπ. Find n? (Given velocity of sound = 320 m/s)

14. Two loudspeakers are emitting sound waves of wavelength λ with an initial phase difference of $\frac{\pi}{2}$. At what minimum distance from O on line AB will one hear a maxima?



(A) 25λ (B) $\frac{100\lambda}{\sqrt{15}}$ (C) $\frac{25\lambda}{3}$ (D) 50λ

15. Two radio station that are 250m apart emit radio waves of wavelength 100m. Point A is 400m from both stations. Point B is 450m from both stations. Point C is 400m from one station and 450m from the other station. The radio station emits radio waves in phase. Which of the following statement is true?

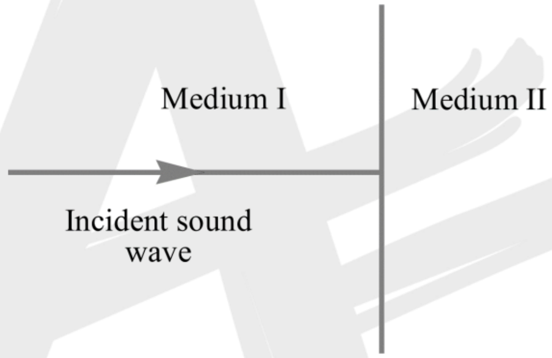
(A) There will constructive interference at A and B and destructive interference at C

(B) There will be destructive interference at A and B and constructive interference at C

(C) There will be constructive interference at B and C and destructive interference at A

(D) There will be destructive interference at A, B and C

16. A sound wave propagating along x – axis, in medium I of density $\rho_1 = 1.5 kg/m^3$ is transmitted to a medium II of density $\rho_2 = 3 kg/m^3$ as shown.



The equation of excess pressure developed by wave in medium I and that in medium II respectively are

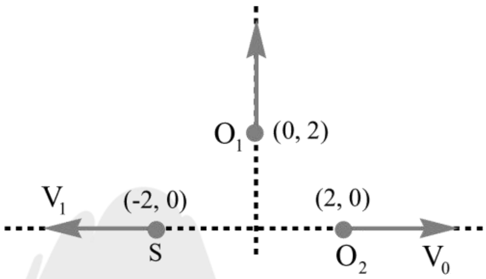
$$p_1 = 4\times10^{-2}\cos\omega\left(t-\frac{x}{400}\right) \quad \text{(in SI units)}$$

$$p_2 = 3\times10^{-2}\cos\omega\left(t-\frac{x}{1200}\right) \quad \text{(in SI units)}$$

Then the ratio of intensity of transmitted wave I_2 (wave in medium II) to the intensity of incident wave I_1 (wave in medium I), that is, $\frac{I_2}{I_1}$ is $\frac{3}{\alpha 16}$. Find α ?

17. Standing wave produced in a metal rod of length 1m fixed at the left end is represented by the equation $y = 10^{-6}\sin\frac{\pi x}{2}\sin 2\,00\pi t$, where x is in meter and t is in seconds. The maximum tensile stress at the midpoint of the rod is $\frac{\pi}{\alpha\sqrt{2}}\times10^{\beta} N/m^2$. Find $\alpha+\beta$? (Young's modulus of material of rod $= 10^{12} N/m^2$)

18. At t = 0 a sound source S and observers O_1 and O_2 are placed as shown. S is moving with speed V_1 and O_1 and O_2 are moving with speed V_0. At any later instant, let f_1 and f_2 represent apparent frequencies of sound observed by O_1 and O_2 respectively. The ratio $\frac{f_1}{f_2}$ is (Note: V_1 and V_0 are less than speed of sound in this medium)



(A) Zero
(B) 1
(C) Between 0 and 1
(D) Greater than 1

19. A man while sitting on an oscillating swing whistle at a frequency 1000 Hz. The swing is oscillating with an amplitude $\pm\frac{\pi}{6}$ radian about the mean position in a vertical plane. Man is sitting over the swing which is at a distance of 2m from the point of suspension of the swing. Another man standing just in front of the mean position of the swing hears the whistle. The maximum frequency (in Hz) heard by the man standing in front of the swing is (Speed of sound in air = 332 m/s)

20. Two vibrating tuning forks produce progressive waves given by $y_1 = 4\sin(500\pi t)$ and $y_2 = 2\sin(506\pi t)$. These tuning forks are held near the ear of a person. The person will hear

(A) 3beats/s with intensity ratio between maxima and minima equal to 4
(B) 3beats/s with intensity ratio between maxima and minima equal to 9
(C) 6beats/s with intensity ratio between maxima and minima equal to 4
(D) 6beats/s with intensity ratio between maxima and minima equal to 9

21. Four tuning forks of frequencies 200, 210, 204 and 206 Hz are sounded together. The beat frequency will be

PROFICIENCY TEST-I

1. At t = 0, transverse wave pulse travelling in the positive x direction with a speed of 2 m/s in a wire is described by the function $y = \frac{6}{x^2}$, given that $x \neq 0$. Transverse velocity of a particle at $x = 2\text{m}$ and $t = 2$ seconds is

(A) 3 m/s (B) -3 m/s (C) 8 m/s (D) -8 m/s

2. Which of the following function correctly represent the travelling wave equation for finite values of x and t

(A) $y = x^2 - t^2$ (B) $y = \cos x^2 \sin t$

(C) $y = \log(x^2 - t^2) - \log(x - t)$ (D) $y = e^{2x} \sin t$

3. The shape of a wave propagating in the positive x or negative x – direction is given $y = \frac{1}{\sqrt{1+x^2}}$ at $t = 0$ and $y = \frac{1}{\sqrt{2-2x+x^2}}$ at t = 1s where x and y are in meter. The shape of the wave disturbance does not change during propagation, then the velocity of the wave is

(A) 1m/s in positive x direction (B) 1m/s in negative x direction

(C) $\frac{1}{2}$ m/s in positive x direction (D) $\frac{1}{2}$ m/s in negative x direction

4. A point mass is subjected to two simultaneous sinusoidal displacements in x – direction, $x_1(t) = A \sin \omega t$ and $x_2(t) = A \sin\left(\omega t + \frac{2\pi}{3}\right)$. Adding a third sinusoidal displacement $x_3(t) = B \sin(\omega t + \phi)$ brings the mass to a complete rest. the values of B and ϕ are

(A) $\sqrt{2}A, \frac{3\pi}{4}$ (B) $A, \frac{4\pi}{3}$ (C) $\sqrt{3}A, \frac{5\pi}{6}$ (D) $A, \frac{\pi}{3}$

5. A standing wave pattern is formed on a string. One of the waves is given by equation $y_1 = a\cos\left(\omega t - kx + \frac{\pi}{3}\right)$ then the equation of the other wave such that at x = 0 a node is formed

(A) $y_2 = a \sin\left(\omega t + kx + \frac{\pi}{3}\right)$ (B) $y_2 = a \cos\left(\omega t + kx + \frac{\pi}{3}\right)$

(C) $y_2 = a \cos\left(\omega t + kx + \frac{2\pi}{3}\right)$ (D) $y_2 = a \cos\left(\omega t + kx + \frac{4\pi}{3}\right)$

6. A wave is propagating along x-axis. The displacement of particles of the medium in z-direction at $t = 0$ is given by, $z = \exp[-(x+2)^2]$, where 'x' is in meters. At $t = 1\text{s}$, the same wave disturbance is given by, $z = \exp[-(2-x)^2]$. Then, the wave propagation velocity is

(A) 4 m/s in +x direction (B) 4 m/s in –x direction

(C) 2 m/s in +x direction (D) 2 m/s in –x direction

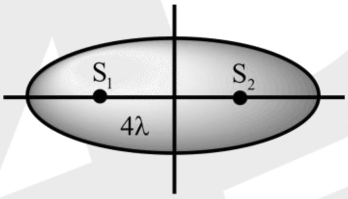
7. The equation of displacement due to a sound wave is $s = s_0 \sin^2(\omega t - kx)$. If the bulk modulus of the medium is B, then the equation of pressure variation due to that sound is

(A) $Bks_0 \sin(2\omega t - 2kx)$ (B) $-Bks_0 \sin(2\omega t - 2kx)$

(C) $Bks_0 \cos^2(\omega t - kx)$ (D) $-Bks_0 \cos^2(\omega t - kx)$

8. A person is talking in a small room and the sound intensity level is 60dB everywhere within the room. If there are eight people talking simultaneously in the room, what is the sound intensity (in dB) level?

9. S_1, S_2 are two coherent sources of sound located along x – axis separated by 4λ where λ is wavelength of sound emitted by them. Number of maxima located on the elliptical boundary around it will be

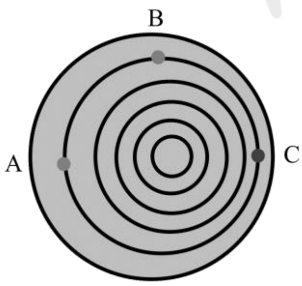


(A) 16 (B) 12 (C) 8 (D) 4

10. A closed organ pipe of length ℓ is sounded together with another closed organ pipe of length $\ell + x(x < \ell)$ both in fundamental mode. If ν = speed of sound, the beat frequency heard is

(A) $\frac{\nu x}{2\ell^2}$ (B) $\frac{\nu x}{4\ell^2}$ (C) $\frac{\nu x^2}{4\ell}$ (D) $\frac{\nu x}{\ell^2}$

11. Three observers A, B and C are listening to a moving source of sound. The diagram below shows the location of the wave crests of the moving source with respect to the three stationary observers. Which of the following is true?



(A) The wavefronts move faster at C than at A and B

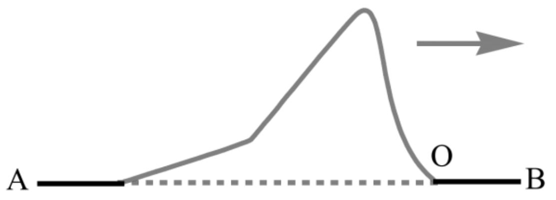
(B) The frequency of the sound is highest at A

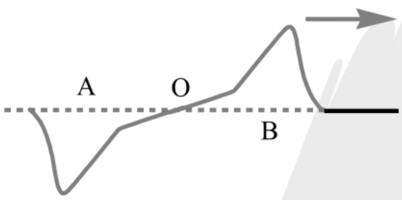
(C) The frequency of the sound is highest at B

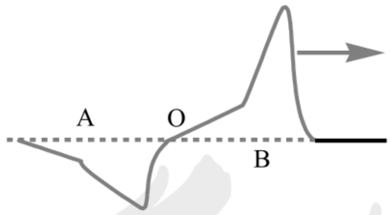
(D) The frequency of the sound is highest at C

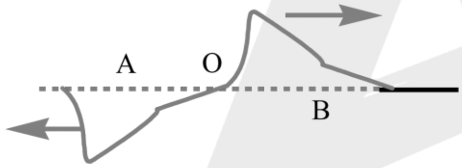
PROFICIENCY TEST-II

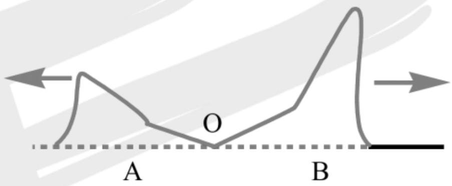
1. Two strings A and B, of lengths 4L and L respectively and same mass M each, are tied together to form a knot 'O' and stretched under the same tension. A transverse wave pulse it sent along the composite string from the side A, as shown to the right. Which of the following diagrams correctly shows the reflected and transmitted wave pulses near the knot 'O'?



(A) 

(B) 

(C) 

(D) 

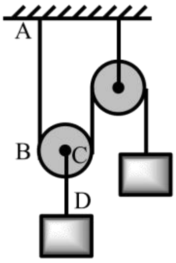
2. A sonometer wire resonates with a given tuning fork forming a standing wave with five antinodes between the two bridges when a mass of 9kg is suspended from the wire. When this mass is replaced by a mass 'M' kg, the wire resonates with the same tuning fork forming three antinodes for the same positions of the bridges, then the value of M is

3. A standing wave given by the equation $y = 2A\sin\left(\frac{\pi x}{L}\right)\sin\omega t$ is formed between the points $x = -L$ and $x = L$. What is the minimum separation between two particles whose maximum velocity is half the maximum velocity of the antinodes?

(A) $\frac{L}{12}$ (B) $\frac{L}{6}$ (C) $\frac{L}{3}$ (D) $\frac{2L}{3}$

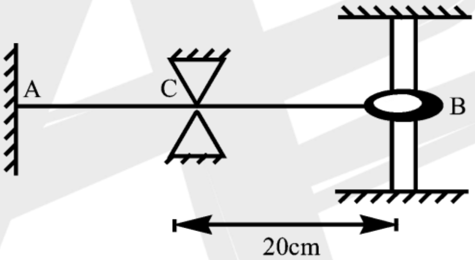
4. The vibrations of a string of length 600cm fixed at both ends are represented by the equation $y = 4\sin\left(\frac{\pi x}{15}\right)\cos(96\pi t)$, where x and y are in cm and t in second. What is the maximum displacement at a point at x = 5cm is $\alpha\sqrt{3}$cm. Find α ?

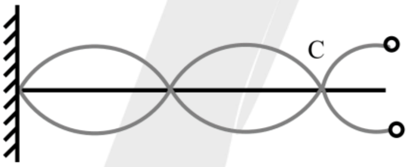
5. Both the strings, shown in figure are made of same material and diameter of CD is double that of AB. The pulleys are light. The speed of a transverse wave in the string AB is v_1 and in CD it is v_2, then $\frac{v_1}{v_2}$ is



(A) $\sqrt{5}$ (B) $\sqrt{3}$ (C) $\sqrt{2}$ (D) $\frac{1}{\sqrt{2}}$

6. A 1m long wire having tension T is fixed at A and free at B. The point C, 20cm from B is constrained to be stationary. What is shape of string for fundamental mode?





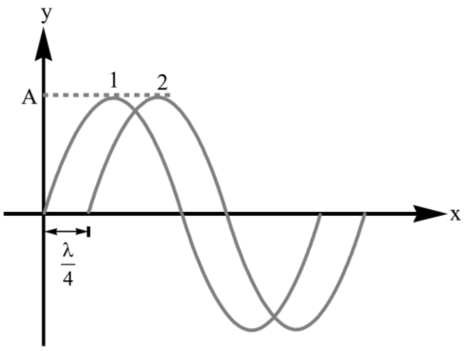
(A)

(B)

(C)

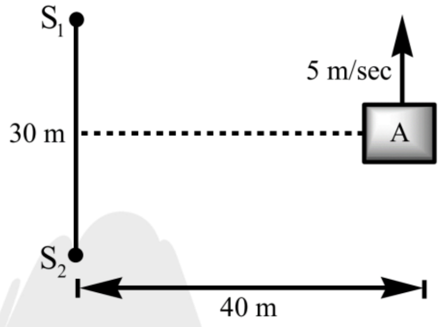
(D)

7. In the given figure two identical waves each of intensity I_0, 1 and 2 are superimposed. The resulting intensity is



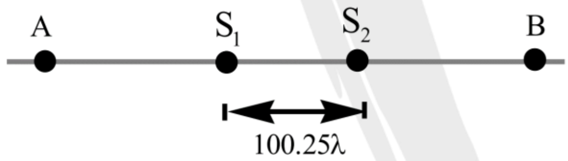
(A) I_0 (B) $2\sqrt{2}I_0$ (C) $4I_0$ (D) $2I_0$

8. As shown in figure, there are two sources S_1 and S_2 producing progressive waves $y_1 = 10\sin 2\pi(t-2x)$ and $y_2 = -5\cos\pi\left(2t-\frac{x}{5}\right)$ respectively and A is moving with constant velocity 5 m/s. Then find the resultant amplitude of the wave where A would be at t = 3s.



(A) 5 (B) $5\sqrt{5}$ (C) $5\sqrt{3}$ (D) 10

9. S_1 and S_2 are two coherent sources of radiations separated by distance 100.25λ, where λ is the wavelength of radiation. S_1 leads S_2 in phase by $\frac{\lambda}{2}$. A and B are two points on the line joining S_1 and S_2 as shown in figure. The ratio of amplitudes of source S_1 and S_2 are in ratio 1:2. The ratio of intensity at A to that of B $\left(\frac{I_A}{I_B}\right)$ is



(A) ∞ (B) $\frac{1}{9}$ (C) 0 (D) 9

10. An open organ pipe containing air resonates in fundamental mode due to a tuning fork. The measured values of length ℓ (in cm) of the pipe and radius r (in cm) of the pipe are $\ell = 94 \pm 0.1$, $r = 5 \pm 0.05$. The velocity of the sound in air is accurately known. The maximum percentage error in the measurement of the frequency of that tuning fork by this experiment will be

(A) 0.16 (B) 0.64 (C) 1.2 (D) 1.6

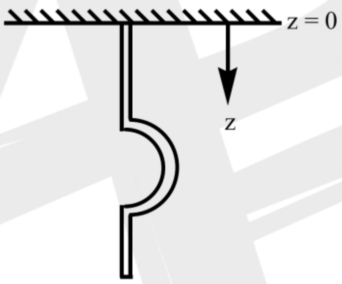
PROFICIENCY TEST-III

1. A composite string is made up by joining two strings of different masses per unit length 1m and 4m. The composite string is under the same tension. A transverse wave pulse $Y=(6mm)\sin(5t+40x)$, where t is in seconds and x in meters, is sent along the lighter string towards the joint. The joint is at x = 0. The equation of the wave pulse reflected from the joint is

(A) $(2mm)\sin(5t-40x)$ (B) $(4mm)\sin(40x-5t)$

(C) $-(2mm)\sin(5t-40x)$ (D) $(2mm)\sin(5t-10x)$

2. A rope hangs from a rigid support. A pulse is set by jiggling the bottom end. We want to design a rope in which velocity v of pulse is independent of z, the distance of the pulse from fixed end of the rope. If the rope is very long the desired function for mass per unit length $\mu(z)$ in terms of μ_0 mass per unit length of the rope at the top $(z=0)$, g, v and z is



(A) $\mu(z)=\mu_0 e^{-[g/v^2]z}$ (B) $\mu(z)=\mu_0 e^{+[g/v^2]z}$

(C) $\mu(z)=\mu_0 \log_e\left(\frac{g}{v^2}\right)z$ (D) $\mu(z)=\mu_0 e+\left(\frac{v^2}{g}\right)z$

3. Two heavy bodies of mass 'm' and '3m' tied together with a light string of mass density μ are dropped from a helicopter as shown. A constant air friction force 'F' acts on both of them. If the length of the string is ℓ. Determine the time taken by a pulse to go from top to bottom of the string.

(A) $\ell\sqrt{\frac{\mu}{F}}$ (B) $\ell\sqrt{\frac{2\mu}{F}}$ (C) $\ell\sqrt{\frac{4\mu}{F}}$ (D) $\ell\sqrt{\frac{\mu}{2F}}$

4. A pulse is started at a time t = 0 along the +x direction on a long, taut string. The shape of the pulse at t = 0 is given by function f(x) with

$$f(x)=\begin{cases}\frac{x}{4}+1 & \text{for} \quad -4<x\le 0\\ -x+1 & \text{for} \quad 0<x<1\\ 0 & \text{for} \quad \text{otherwise}\end{cases}$$

Here f and x are in centimetres. The linear mass density of the string is 50g/m and it is under a tension of 5N

(A) The shape of the string is drawn at t = 0 then the area of the pulse enclosed by the string and the x-axis is $2.5cm^2$

(B) The shape of the string is drawn at t = 0 then the area of the pulse enclosed by the string and the x-axis is $5cm^2$

(C) The transverse velocity of the particle at x = 13cm and t = 0.015s will be -250cm/s

(D) The transverse velocity of the particle at x = 13cm and t = 0.015s will be 250cm/s

5. How long will it take sound waves to travel a distance ℓ between points A and B if the air temperature between them varies linearly form T_1 to T_2? (The velocity of sound in air at temperature T is given by $v=\alpha\sqrt{T}$, where α is a constant)

(A) $\frac{2\ell}{\alpha\sqrt{T_1T_2}}$ (B) $\alpha\ell\sqrt{\frac{T_1}{T_2}}$ (C) $\sqrt{T_1+T_2}.\alpha\ell$ (D) $\frac{2\ell}{\alpha\left(\sqrt{T_2}+\sqrt{T_1}\right)}$

6. Sound waves are emitted uniformly in all directions from a point source. The dependence of sound level β in decibels on the distance r can be expressed as (a and b are positive constants)

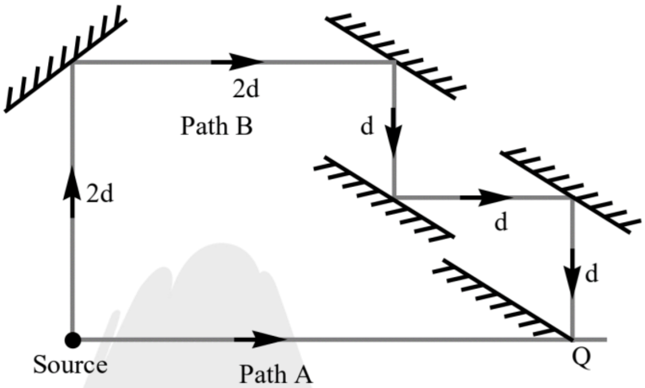
(A) $\beta=-b\log r^a$ (B) $\beta=a-b(\log r)^2$

(C) $\beta=a-b\log r$ (D) $\beta=a-\frac{b}{r^2}$

7. Three coherent sonic sources emitting sound of single wavelength λ are placed on the x – axis at points $\left(\frac{-\lambda\sqrt{11}}{6},0\right)$, (0, 0), $\left(\frac{\lambda\sqrt{11}}{6},0\right)$. The intensity reaching a point $\left(0,\frac{5\lambda}{6}\right)$ from each source has the same value I_0. Then, the resultant intensity at this point due to the interference of the three waves will be

(A) $6I_0$ (B) $7I_0$ (C) $9I_0$ (D) $5I_0$

8. A sound source emits two sinusoidal sound waves, both of wavelength λ, along paths A and B as shown in figure. The sound travelling along path B is reflected from five surfaces as shown and then merges at point Q, producing minimum intensity at that point. The minimum value of d in terms of λ is $\frac{\lambda}{n}$. Find n



9. An open pipe 0.400m in length is placed vertically in a cylindrical bucket and nearly touches the bottom of the bucket which has an area of 0.100m^2. Water is slowly poured into the bucket until a sounding tuning fork of frequency 440 Hz, held over the pipe, produces resonance. Find the mass of water in the bucket at this moment. Take speed of sound in air as 330m/s.

(A) 18.75kg (B) 21.25kg (C) 40.00kg (D) 26.75kg

10. The fundamental frequency of a sonometer wire of length ℓ is n_0. A bridge is now introduced at a distance of $\Delta\ell(<\ell)$ from the centre of the wire. The lengths of wire on the two sides of the bridge are now vibrated in their fundamental modes. Then, the beat frequency nearly is $\frac{\alpha n_0 \Delta\ell}{\ell}$. Find α ?

ANSWER KEY

EXERCISE-I_KEY

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
A	C	D	C	D	75	AD	CD	AC	ACD
11	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
ACD	AC	A	2500	B	D	927	A	8	ABC
21									
AC									

EXERCISE-II_KEY

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
A	A	5	4	B	200	D	C	BC	313
11	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
16	20	A	200	D	A	B	3	360	C

EXERCISE-III_KEY

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
C	24	2	2	D	30	5	D	1	5
11	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
10	B	B	A	10	C	A	5	1	A
21									
20									

EXERCISE-IV_KEY

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
C	D	5	B	B	B	C	4	B	BC
11	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
54	3	C	600	A	23	1	11	C	A

EXERCISE-V_KEY

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
B	C	C	ABD	BD	ABC	ABD	AC	ABD	CD
11	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
D	C	2	C	A	2	8	D	1007	B
21									
12									

PROFICIENCY TEST-I_KEY

1	2	3	4	5	6	7	8	9	10
B	C	A	B	D	A	A	69	A	B
11									
D									

PROFICIENCY TEST-II_KEY

1	2	3	4	5	6	7	8	9	10
A	25	C	2	C	A	D	B	B	A

PROFICIENCY TEST-III_KEY

1	2	3	4	5	6	7	8	9	10
C	A	B	AC	D	C	B	8	B	8