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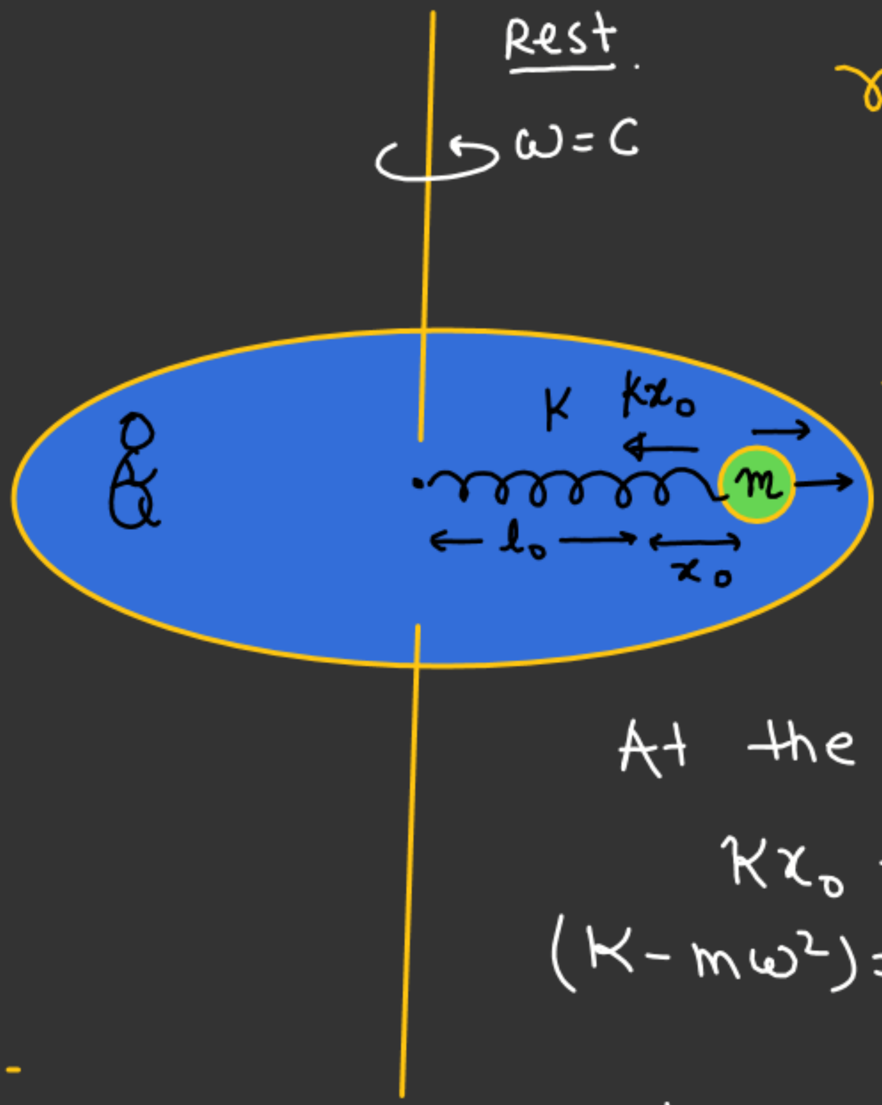
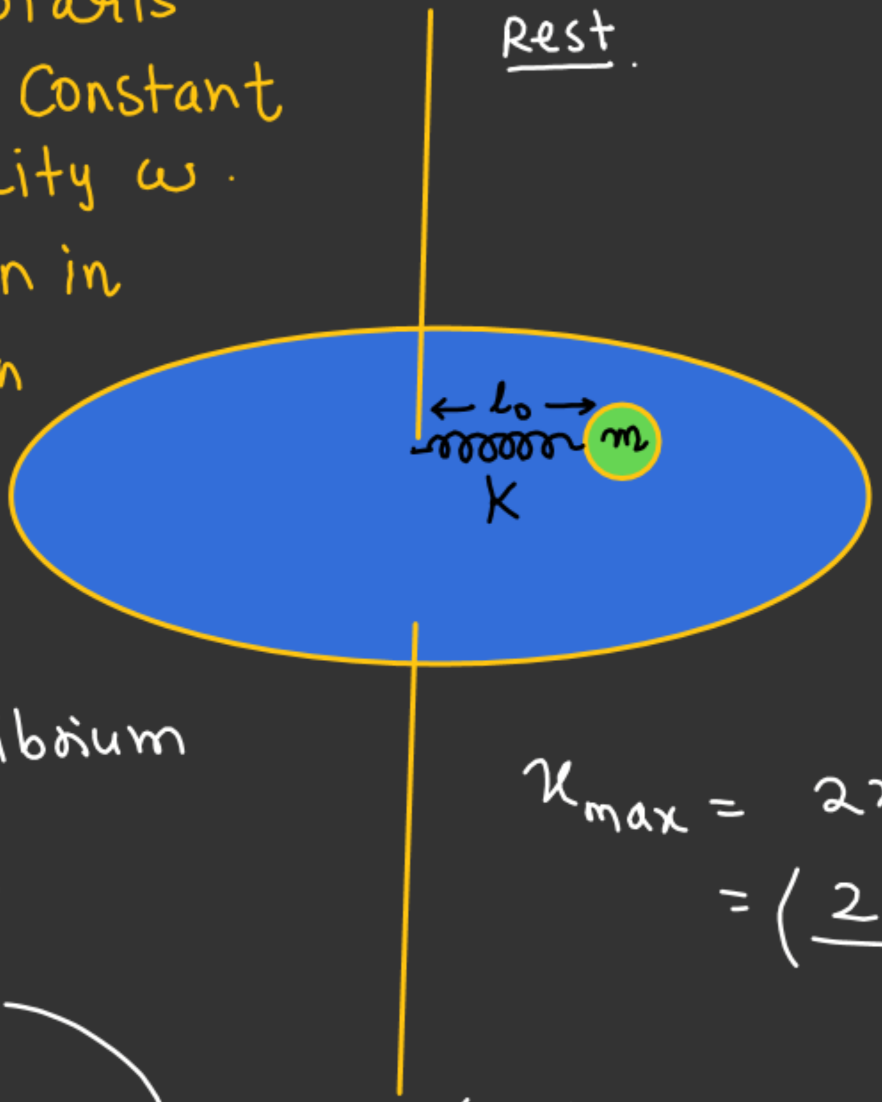


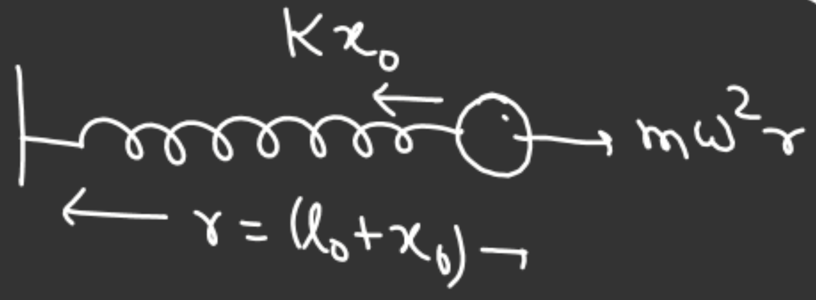
Turn table Starts rotating with Constant angular velocity ω. find elongation in the Spring when ball is in equilibrium



At the time of Equilibrium

$$Kx_0 = m\omega^2(l_0+x_0)$$

$$(K-m\omega^2)x_0 = m\omega^2 l_0$$



$$x_0 = \left(\frac{m\omega^2 l_0}{K-m\omega^2}\right)$$ Ans.

$$x_{max} = 2x_0$$

$$= \left(\frac{2m\omega^2 l_0}{K-m\omega^2}\right)$$ ✓

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\# Find ω so that ball doesn't slip.

$$r = R\sin\theta$$

In Rotating frame.

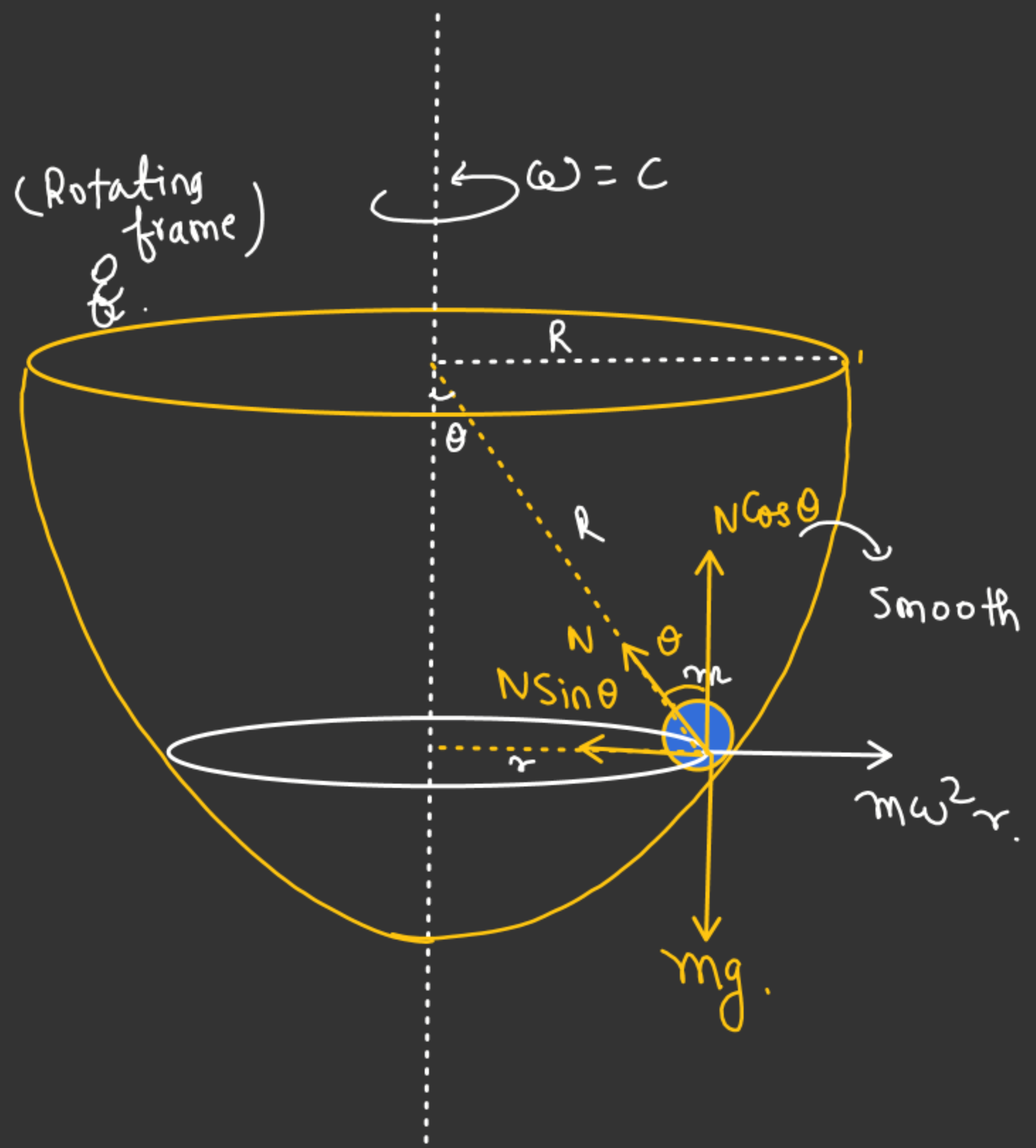
$$N\cancel{\sin\theta} = m\omega^2 r = m\omega^2 R\cancel{\sin\theta}$$

$$N = m\omega^2 R$$

$$N\cos\theta = mg$$

$$\cancel{m}\omega^2 R\cos\theta = \cancel{m}g$$

$$\omega = \sqrt{\frac{g}{R\cos\theta}}$$



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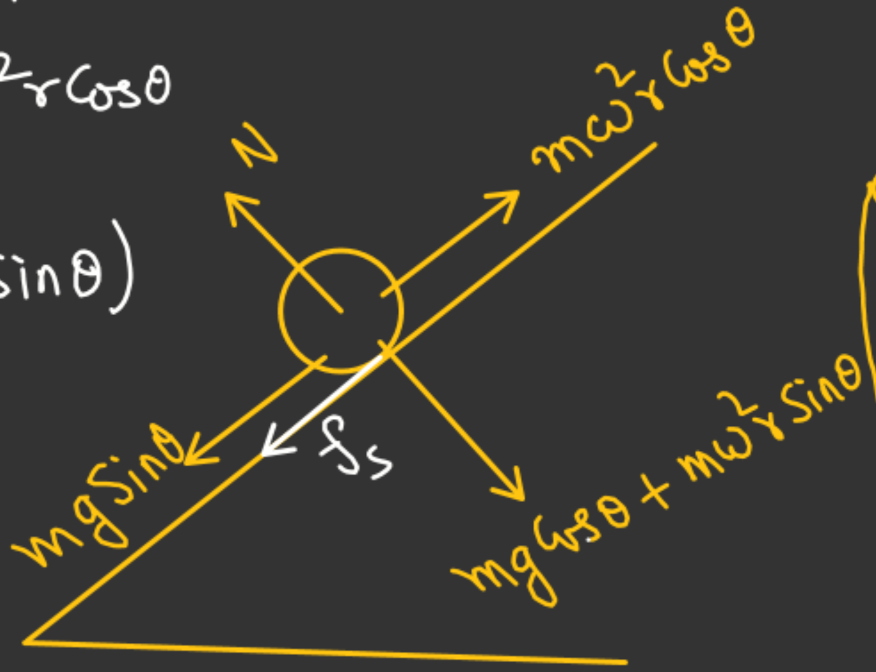
Case-1 :- If $m\omega^2 r\cos\theta > mg\sin\theta$.

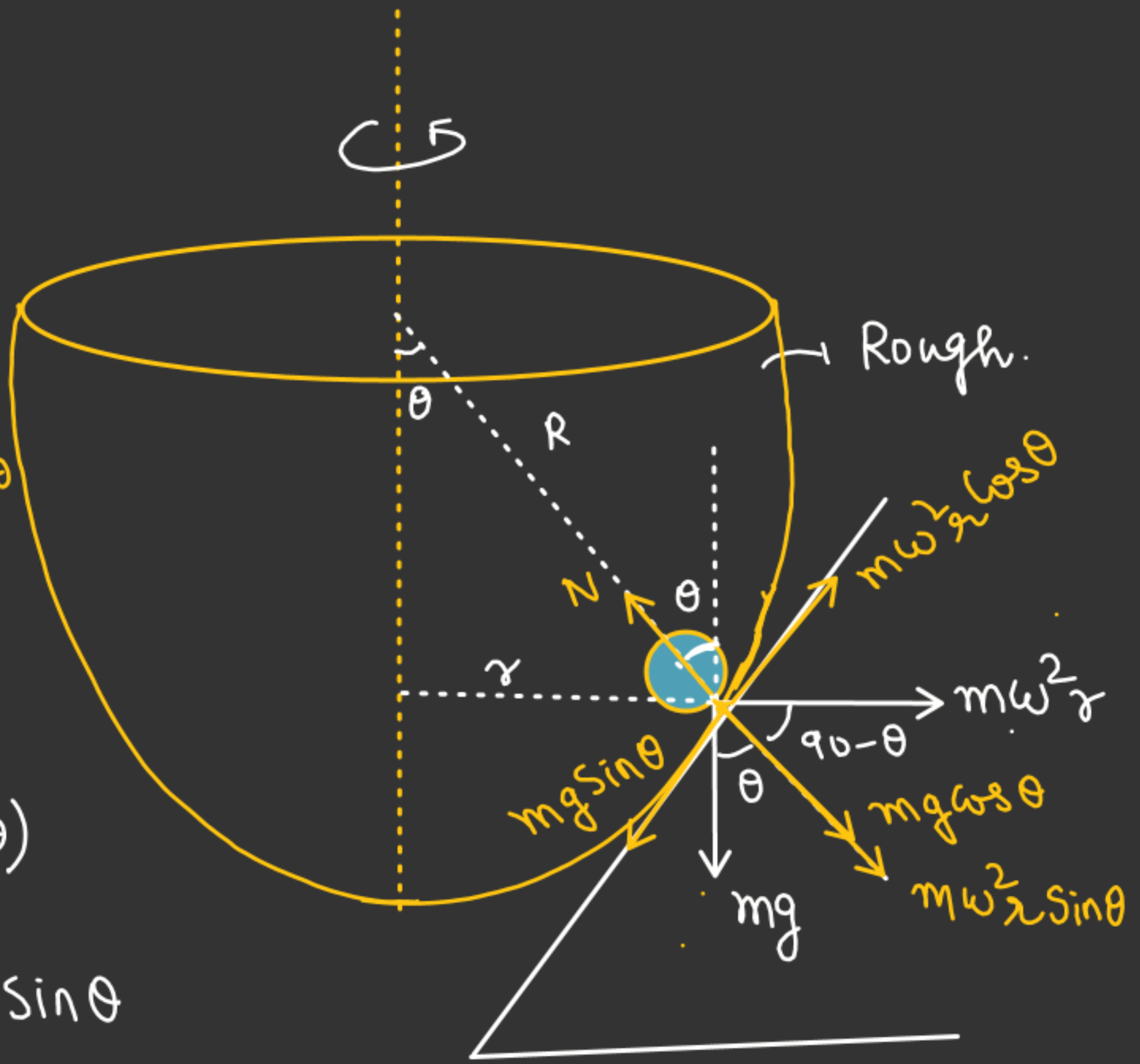
For block not to slip.

$$mg\sin\theta + f_s = m\omega^2 r\cos\theta$$

$$f_s = (m\omega^2 r\cos\theta - mg\sin\theta)$$

$$f_s \le (f_s)_{max}$$





$$m\omega^2 r\cos\theta - mg\sin\theta \le \mu(mg\cos\theta + m\omega^2 r\sin\theta)$$

$$m\omega^2 r\cos\theta - \mu m\omega^2 r\sin\theta \le \mu mg\cos\theta + mg\sin\theta$$

$$r\omega^2(\cos\theta - \mu\sin\theta) \le g(\mu\cos\theta + \sin\theta)$$

$$\omega \le \sqrt{\frac{g}{r}\left(\frac{\mu\cos\theta + \sin\theta}{\cos\theta - \mu\sin\theta}\right)} \Rightarrow \omega_{max} = \frac{g}{r}\left(\frac{\mu\cos\theta + \sin\theta}{\cos\theta - \mu\sin\theta}\right)$$

$$\omega \le \sqrt{\frac{g}{r}\left(\frac{\tan\theta + \mu}{1 - \mu\tan\theta}\right)}$$

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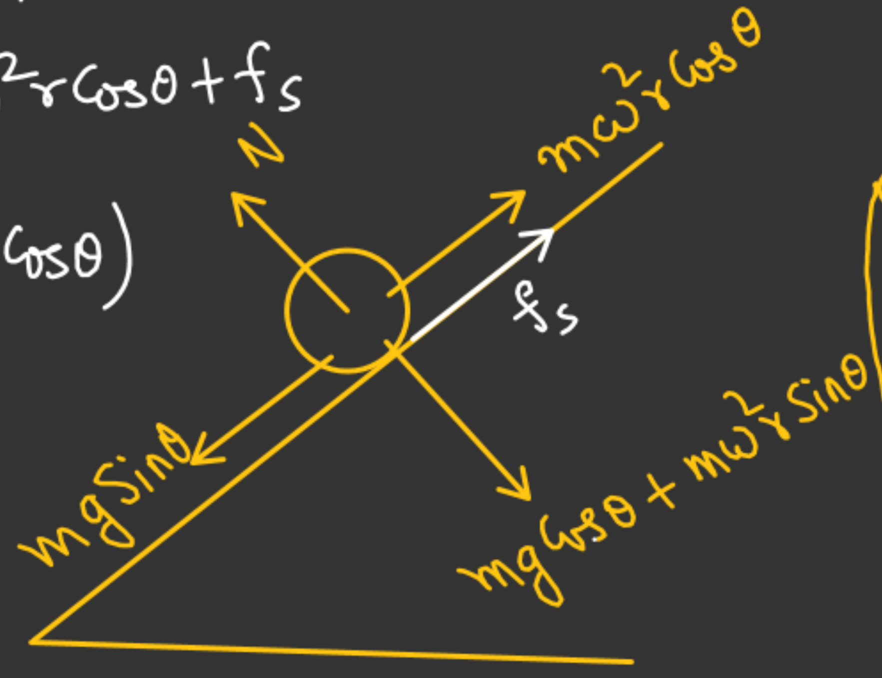
Case-2 :- If $m\omega^2 r\cos\theta < mg\sin\theta$.

For block not to Slip.

$$mg\sin\theta = m\omega^2 r\cos\theta + f_s$$

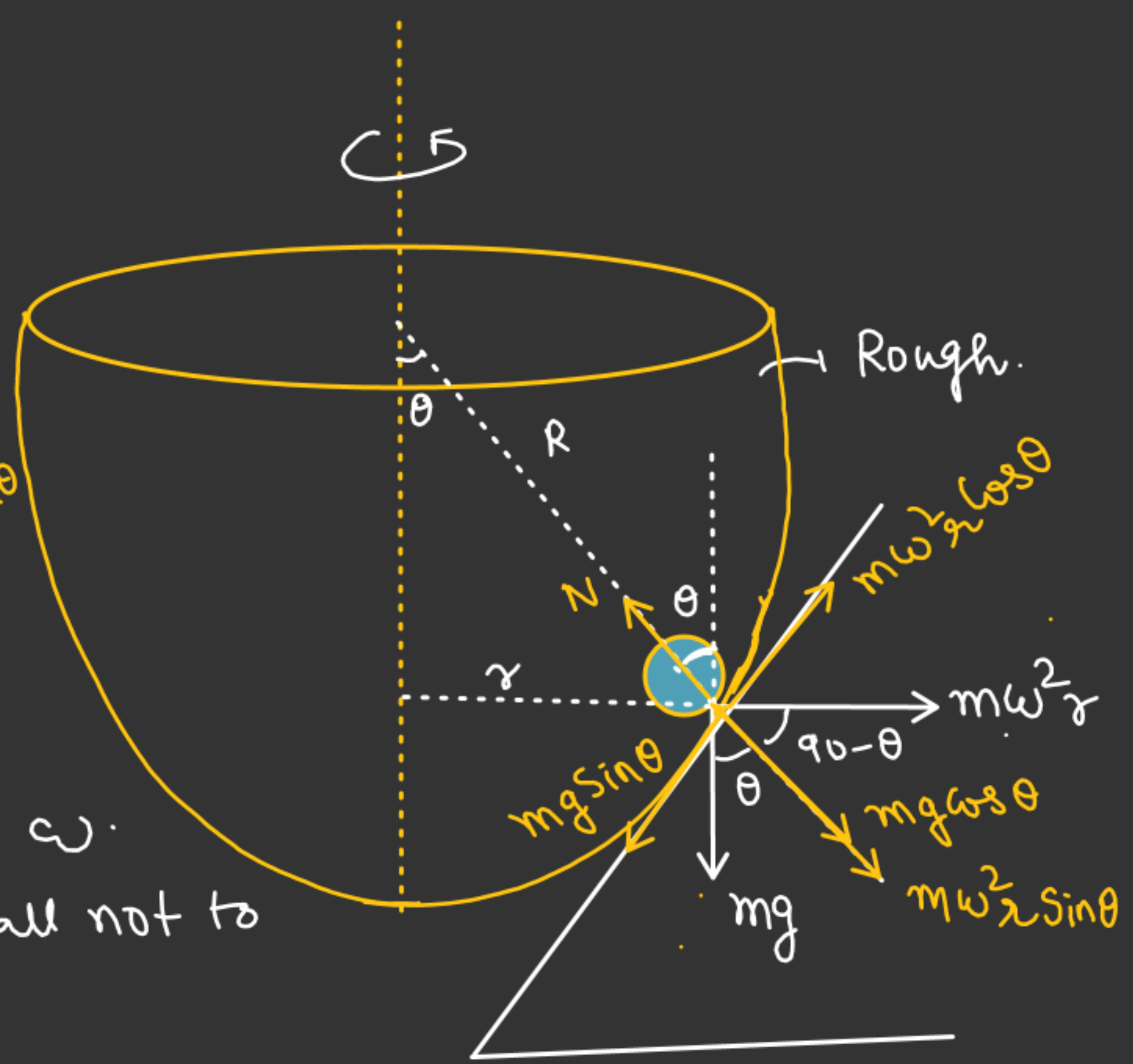
$$f_s = (mg\sin\theta - m\omega^2 r\cos\theta)$$

$$f_s \le (f_s)_{max}$$



$$\omega \geq \sqrt{\frac{g}{r}\left(\frac{\mu\cos\theta - \sin\theta}{\cos\theta + \mu\sin\theta}\right)}$$

$$\omega_{min} = \sqrt{\frac{g}{r}\frac{\mu\cos\theta - \sin\theta}{(\cos\theta + \mu\sin\theta)}}$$



Range ω.

For ball not to Slip

$$\boxed{\omega_{min} \le \omega \le \omega_{max}}$$

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v_{max} for Safe turn

Static friction force providing necessary Centripetal force for turning.

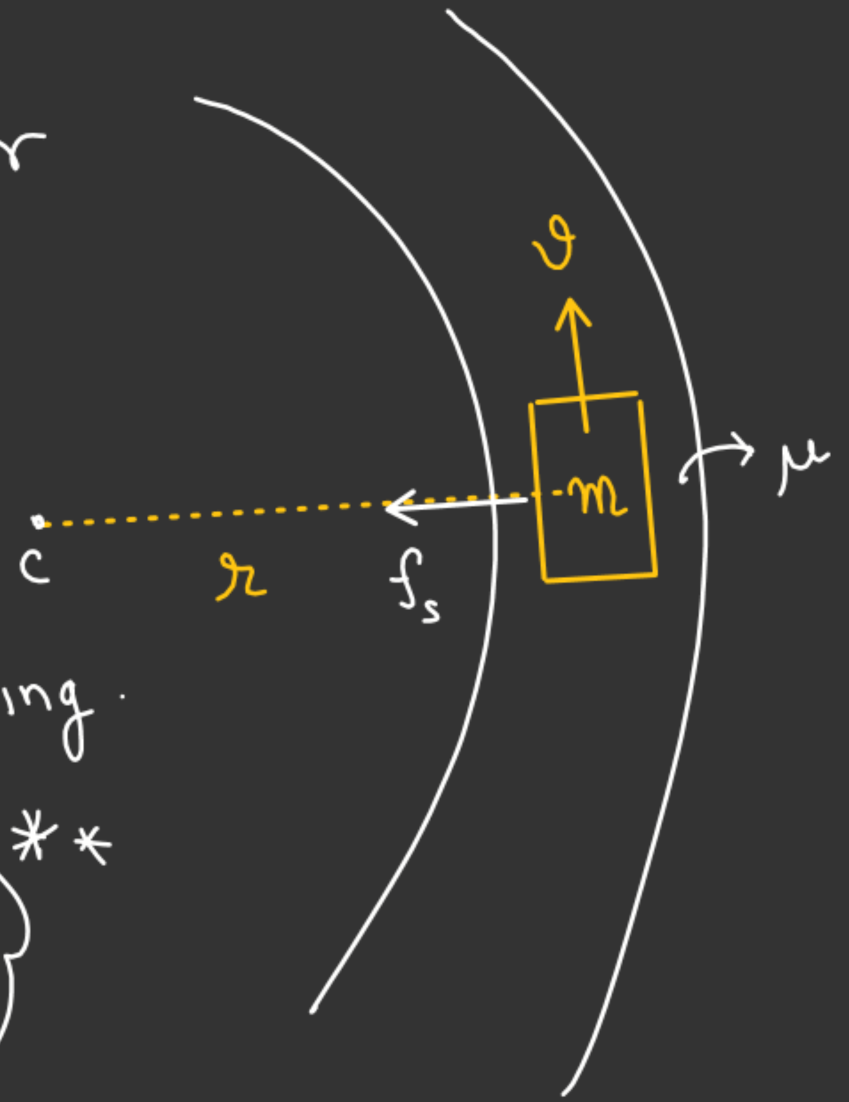
$$f_s = \frac{mv^2}{r}$$

$$f_s \le (f_s)_{max}$$

$$\frac{\cancel{m}v^2}{r} \le \mu \cancel{m} g$$

$$v^2 \le \mu g r$$

$$\boxed{v \le \sqrt{\mu g r}}$$



For Safe turning.

$$v_{max} = \sqrt{\mu g r}$$ **

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Concept of Banking of road

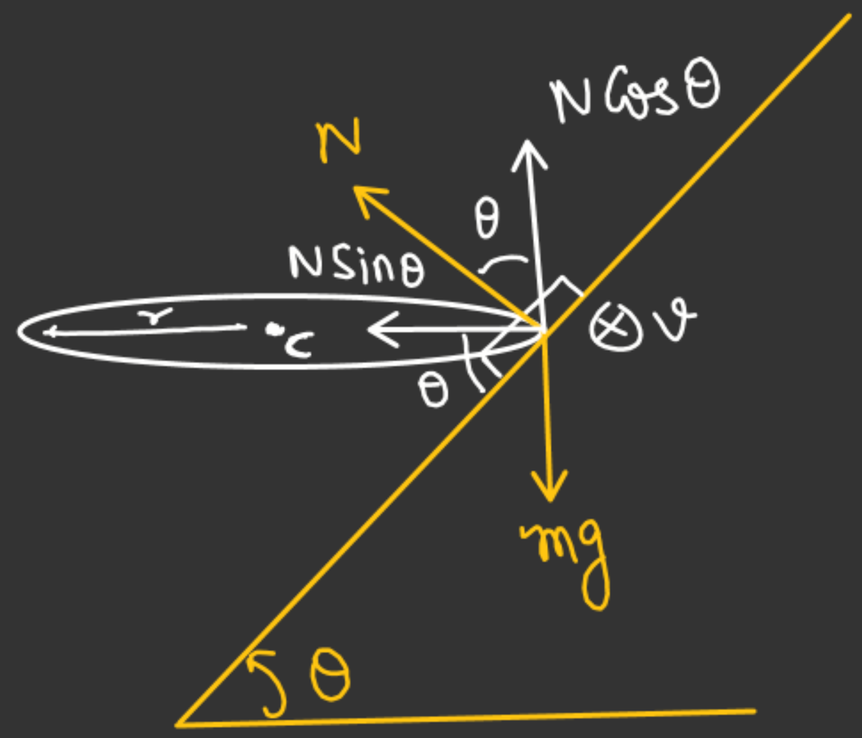
Case-1 :- Banked road without friction

$$N\sin\theta = \frac{mv^2}{r}$$

$$N\cos\theta = mg$$

$$\tan\theta = \left(\frac{v^2}{rg}\right)$$

$$\boxed{\theta = \tan^{-1}\left(\frac{v^2}{rg}\right)}$$



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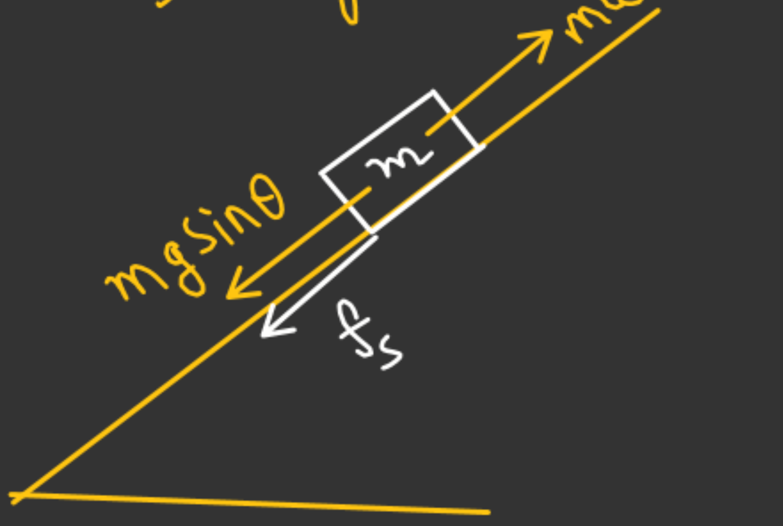
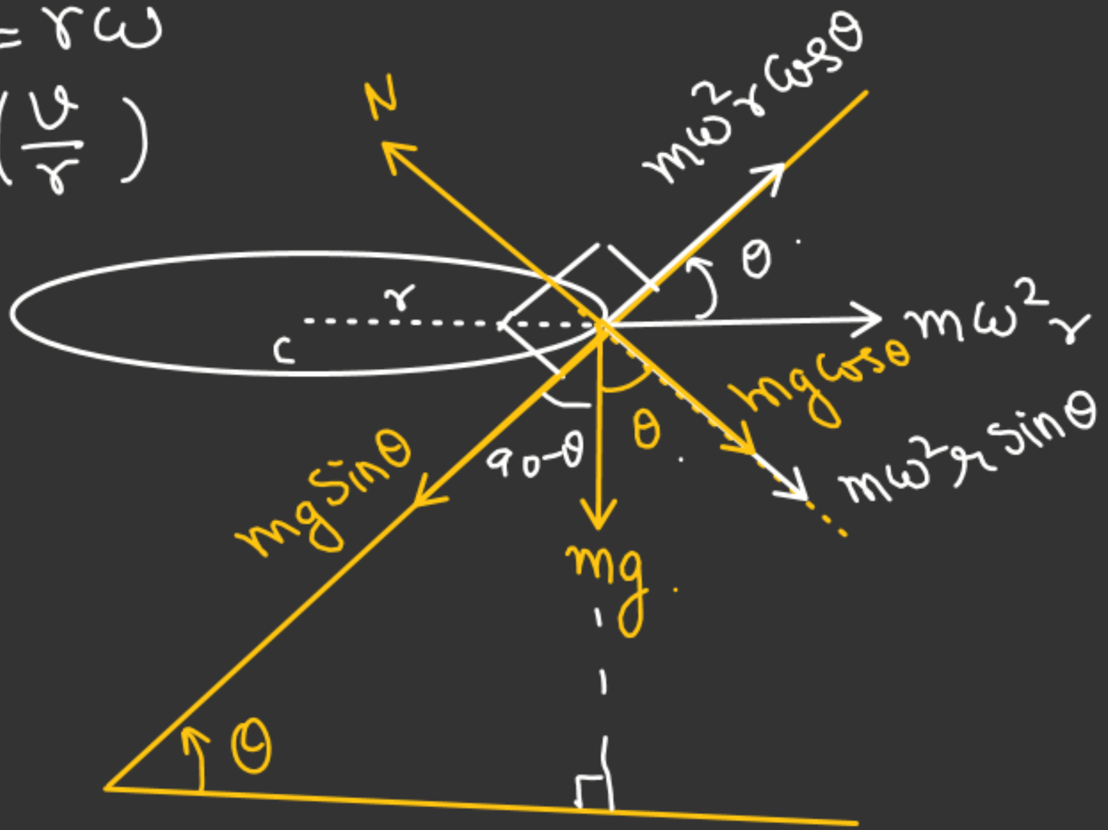
**: Case-2 :- Banked road with friction :-

Case-1 $m\omega^2 r\cos\theta > mg\sin\theta$



$v = r\omega$

$\omega = \left(\frac{v}{r}\right)$



$$N = (mg\cos\theta + m\omega^2 r\sin\theta)$$

For No Slipping

$$mg\sin\theta + f_s = m\omega^2 r\cos\theta$$

$$f_s = (m\omega^2 r\cos\theta - mg\sin\theta)$$

$$f_s \le (f_s)_{max}$$

$$m\omega^2 r\cos\theta - mg\sin\theta \le \mu(mg\cos\theta + m\omega^2 r\sin\theta)$$

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$$f_s \le (f_s)_{max}$$

$$v = r\omega$$

$$\omega = \frac{v}{r}$$

$$m\omega^2 r\cos\theta - mg\sin\theta \le \mu(mg\cos\theta + m\omega^2 r\sin\theta)$$

$$m\omega^2 r\cos\theta - \mu m\omega^2 r\sin\theta \le \mu mg\cos\theta + mg\sin\theta$$

$$\cancel{m}\omega^2 r(\cos\theta - \mu\sin\theta) \le \cancel{m}g(\mu\cos\theta + \sin\theta)$$

$$\frac{v^2}{r} \le g\left(\frac{\sin\theta + \mu\cos\theta}{\cos\theta - \mu\sin\theta}\right)$$

$$v \le \sqrt{rg\left(\frac{\sin\theta + \mu\cos\theta}{\cos\theta - \mu\sin\theta}\right)}$$

Range

$$\boxed{\sqrt{rg\left(\frac{\sin\theta - \mu\cos\theta}{\cos\theta + \mu\sin\theta}\right)} \le v \le \sqrt{rg\left(\frac{\sin\theta + \mu\cos\theta}{\cos\theta - \mu\sin\theta}\right)}}$$

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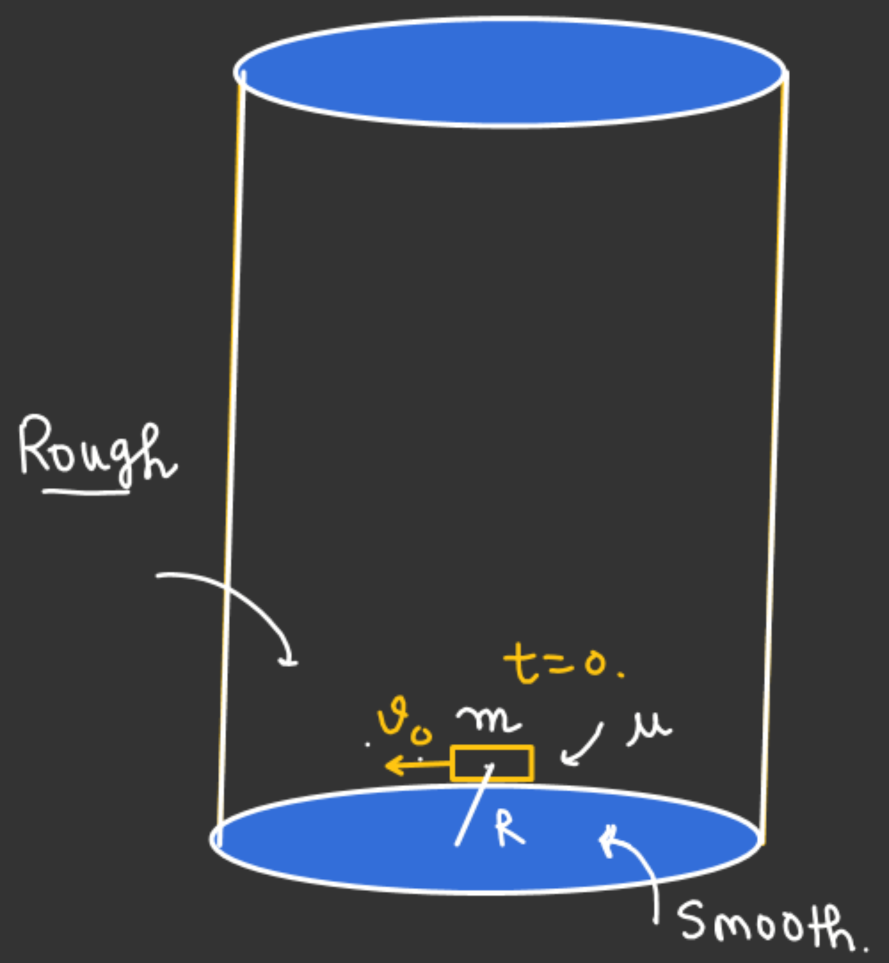
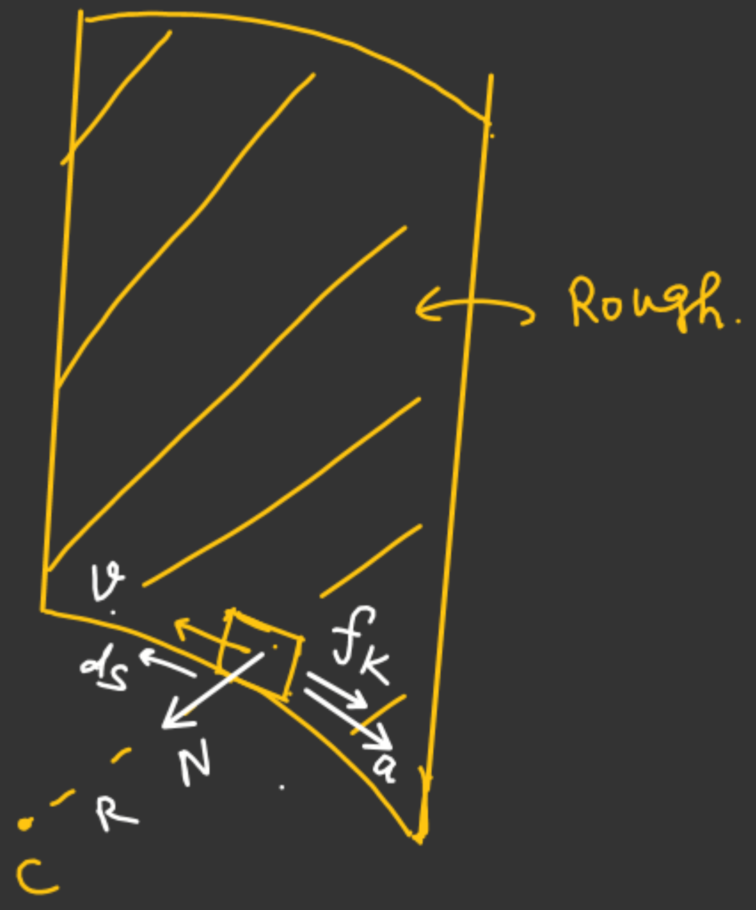
Block projected with v_0 tangentially.



∴ At any time $t = t$.
let, velocity of block be v.



$$N = \frac{mv^2}{R}.$$

$$f_K = \mu N = \left(\frac{\mu m v^2}{R}\right)$$

Speed after one Complete rotation = ??

$$a_t = \frac{-f_K}{m} = \frac{-\mu v^2}{R}$$

$$\Downarrow$$

$$v\frac{dv}{ds} = -\frac{\mu v^2}{R} \Rightarrow \int_{v_0}^{v}\frac{dv}{v} = -\frac{\mu}{R}\int_0^{2\pi R} ds \Rightarrow \ln\frac{v}{v_0} = -\frac{\mu}{\cancel{R}} \times 2\pi \cancel{R}$$

$$\ln\frac{v}{v_0} = -2\pi\mu \Rightarrow \boxed{v = v_0 e^{-2\pi\mu}}$$

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H.W

H.C.V (Complete Chapter. Circular Motion)