Nishant Jindal

Properties

(1) row/column

$$\begin{vmatrix} & & \\ 0 & 0 & 0 \\ & & \end{vmatrix}$$

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = 0$$

(2) rows/columns

(3)

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

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4. Scalar Multiplication

$$\Delta = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

$$k\Delta = \begin{vmatrix} a_1 & ka_2 & a_3 \\ b_1 & kb_2 & b_3 \\ c_1 & kc_2 & c_3 \end{vmatrix} = ka_2 C_{12} + kb_2 C_{22} + kc_2 C_{32}$$

$$= k$$

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5 Sum.

$$\begin{vmatrix} a_1+d_1 & a_2+d_2 & a_3+d_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} + \begin{vmatrix} d_1 & d_2 & d_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

$\downarrow$

$$(a_1+d_1)C_{11} + (a_2+d_2)C_{12} + (a_3+d_3)C_{13} = (a_1C_{11} + a_2C_{12} + a_3C_{13}) + (d_1C_{11} + d_2C_{12} + d_3C_{13})$$

$$\begin{vmatrix} a_1+d_1 & a_2+d_2 & a_3+d_3 \\ b_1+e_1 & b_2+e_2 & b_3+e_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1+e_1 & b_2+e_2 & b_3+e_3 \\ c_1 & c_2 & c_3 \end{vmatrix} + \begin{vmatrix} d_1 & d_2 & d_3 \\ b_1+e_1 & b_2+e_2 & b_3+e_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

$$= \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} + \begin{vmatrix} a_1 & a_2 & a_3 \\ e_1 & e_2 & e_3 \\ c_1 & c_2 & c_3 \end{vmatrix} + \begin{vmatrix} d_1 & d_2 & d_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} + \begin{vmatrix} d_1 & d_2 & d_3 \\ e_1 & e_2 & e_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

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6.

rows / columns

$$\Delta = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

$$\Delta' = \begin{vmatrix} c_1 & c_2 & c_3 \\ b_1 & b_2 & b_3 \\ a_1 & a_2 & a_3 \end{vmatrix} = -\Delta$$

$$\Delta'' = \begin{vmatrix} c_1 & c_2 & c_3 \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = \Delta.$$

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7.

Row/Column Transformation

$$\Delta = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} \qquad R_1 \to R_1 + k_2R_2 + k_3R_3$$

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} + k_2\begin{vmatrix} b_1 & b_2 & b_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} + k_3\begin{vmatrix} c_1 & c_2 & c_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ \_\_ & & \\ \_\_ & & \end{vmatrix} + \begin{vmatrix} k_2b_1+k_3c_1 & - & - \\ \_\_ & & \\ \_\_ & & \end{vmatrix} = \begin{vmatrix} a_1+k_2b_1+k_3c_1 & a_2+k_2b_2+k_3c_2 & a_3+k_2b_3+k_3c_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

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$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} \xrightarrow[R_3 \to R_3 - R_2]{R_2 \to R_2 - R_1} \frac{1}{K_1}\begin{vmatrix} K_1 a_1 & K_1 a_2 & K_1 a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

$\downarrow R_1 \to R_1 + K_2 R_2 + K_3 R_3$

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} \xrightarrow{R_1 \to K_1 R_1 + K_2 R_2 + K_3 R_3}$$

$$= \frac{1}{K_1}\begin{vmatrix} K_1 a_1 + K_2 b_1 + K_3 c_1 & K_1 a_2 + K_2 b_2 + K_3 c_2 & K_1 a_3 + K_2 b_3 + K_3 c_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

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1.

$$\Delta = \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} \xrightarrow[R_3 \to R_3 - R_1]{R_2 \to R_2 - R_1} \begin{vmatrix} 1 & x & x^2 \\ 0 & y-x & y^2-x^2 \\ 0 & z-x & z^2-x^2 \end{vmatrix}$$

$$= (x-y)(y-z)(z-x)$$

$$= (y-x)(z-x)\begin{vmatrix} 1 & x & x^2 \\ 0 & 1 & y+x \\ 0 & 1 & z+x \end{vmatrix}$$

But $x = y$ $\Rightarrow \Delta = 0$

$y = z$, $\Delta = 0$

$z = x$, $\Delta = 0$

$$= (y-x)(z-x)(z-y)$$

$$\Delta = k(y-x)(z-y)(z-x)$$

$$= (x-y)(y-z)(z-x)$$

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 1 & 1 \end{vmatrix} = -2$$

$x = 1, y = 0, z = -1$

$$\Delta = k(0-1)(-1-0)(-1-1)$$

$$= -2k$$

$$-2k = -2 \Rightarrow \boxed{k = 1}$$

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$$\begin{vmatrix} 1 & x & x^3 \\ 1 & y & y^3 \\ 1 & z & z^3 \end{vmatrix} = (x-y)(y-z)(z-x)(x+y+z)$$

2 z 7

$$\begin{vmatrix} 1 & x^2 & x^3 \\ 1 & y^2 & y^3 \\ 1 & z^2 & z^3 \end{vmatrix} = (x-y)(y-z)(z-x)(xy+yz+zx)$$

$$\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} \xrightarrow{R_1 \to R_1+R_2+R_3} (a+b+c)\begin{vmatrix} 1 & 1 & 1 \\ b & c & a \\ c & a & b \end{vmatrix} \xrightarrow[C_3 \to C_3 - C_1]{C_2 \to C_2 - C_1} = (a+b+c)\begin{vmatrix} 1 & 0 & 0 \\ b & c-b & a-b \\ c & a-c & b-c \end{vmatrix}$$

$$= (a+b+c)\left(\sum ab - \sum a^2\right)$$

$$= 3abc - a^3 - b^3 - c^3$$

$$a(bc-a^2) - b(b^2-ac) + c(ab-c^2)$$

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$$\boxed{\begin{array}{l} Ex\text{-}\underline{I}\ (21-25) \\ Ex\text{-}\underline{II}\ (1-15) \end{array}}$$