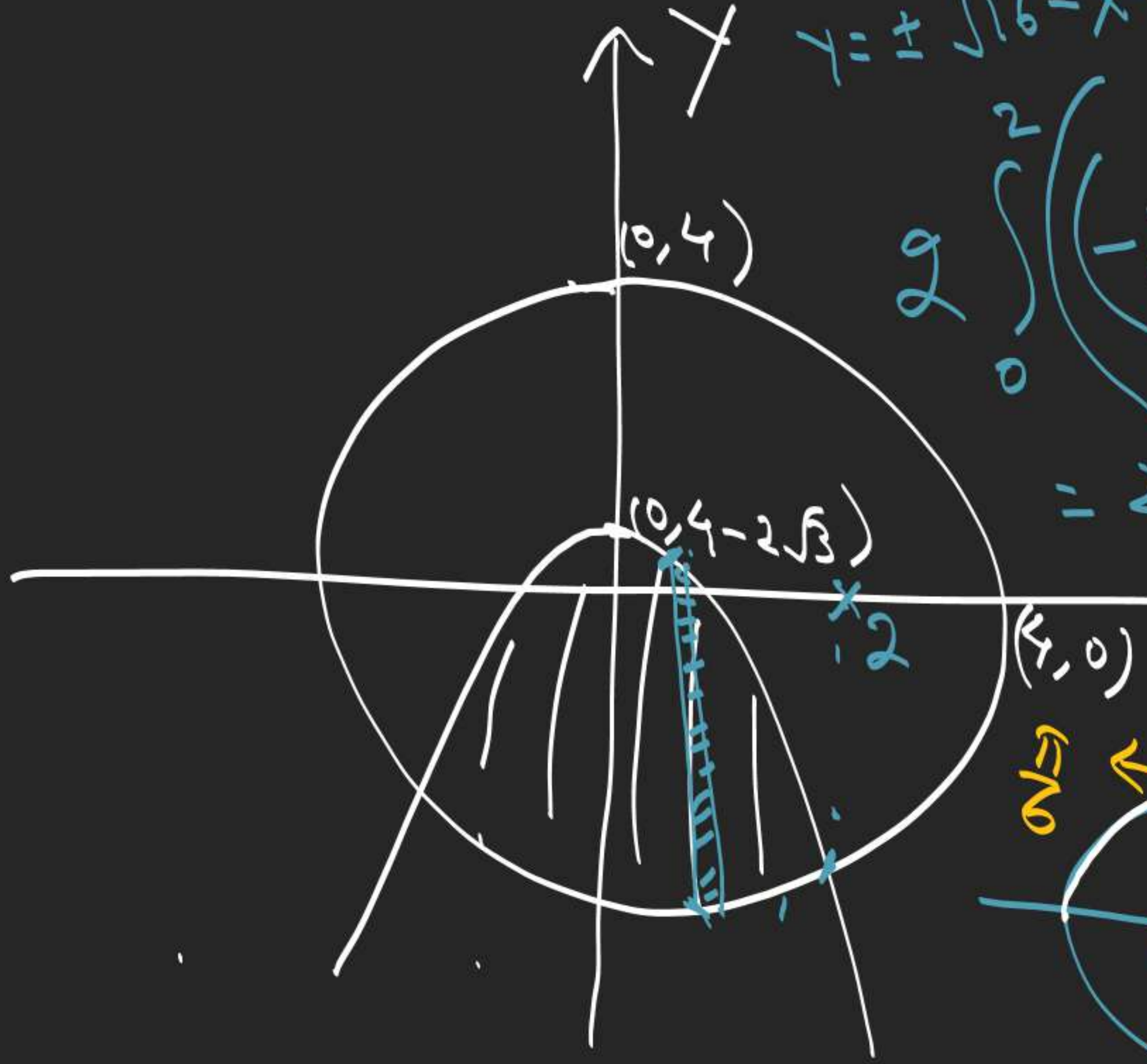
Nishant Jindal

$$x^2+y^2+4y-2x-11=0 = (x-1)^2+(y+2)^2-16 \Rightarrow X^2+Y^2=16$$

$$y=-x^2+2x+1-2\sqrt{3} \Rightarrow y+2=-(x-1)^2+4-2\sqrt{3} \Rightarrow Y=-X^2+4-2\sqrt{3}.$$

$y=\pm\sqrt{16-x^2}$



$$2\int_0^2 \left(\left(-X^2+(4-2\sqrt{3})\right)-\left(-\sqrt{16-X^2}\right)\right)dX$$

$$=2\left[-\frac{8}{3}+(4-2\sqrt{3})2 + \int_0^2 \sqrt{16-X^2}\,dX\right]$$

$$=\frac{1}{2}\times 16\times\frac{\pi}{6}+\frac{1}{2}\times 2\times 2\sqrt{3}$$

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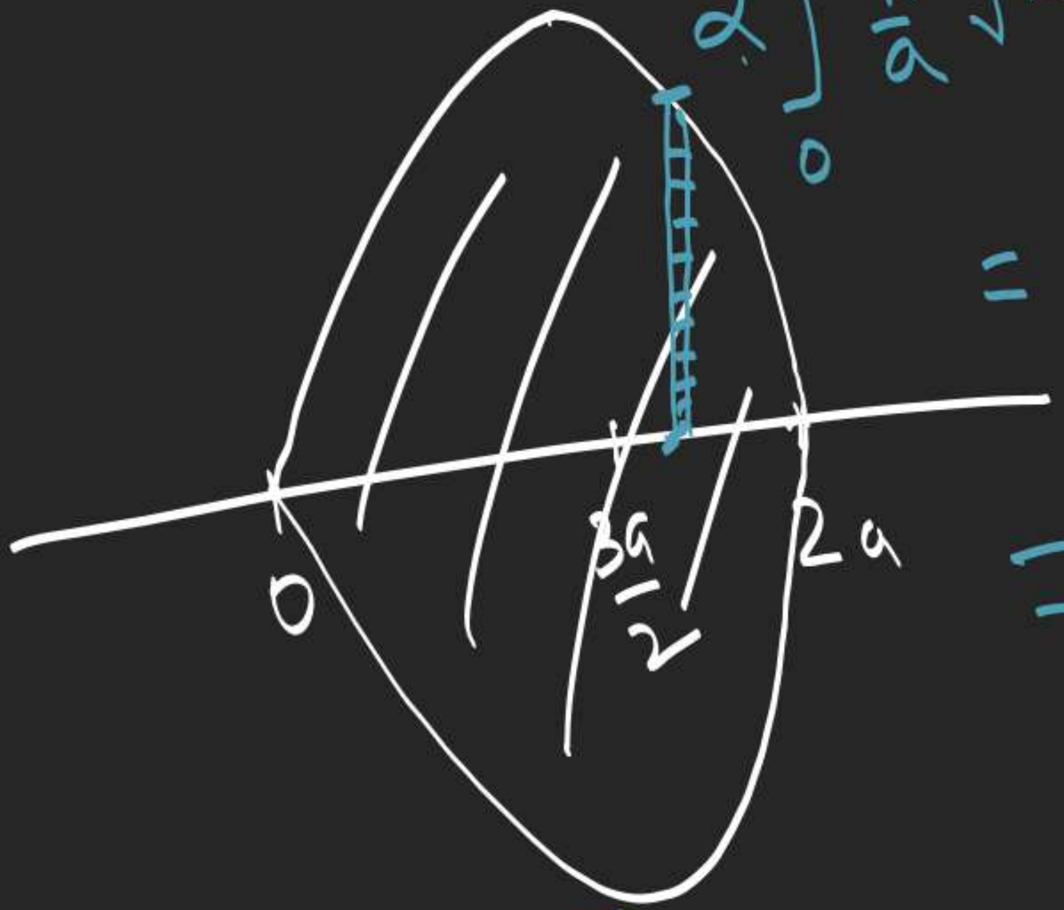
1. Find the area enclosed by the curve

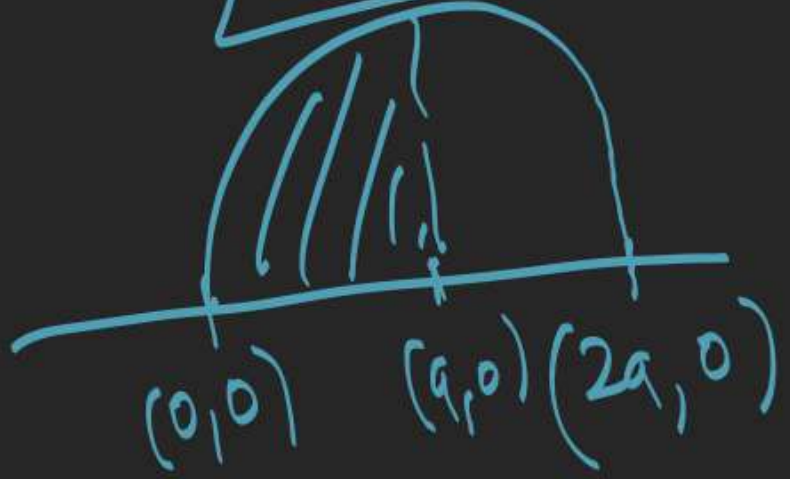
$$a^2y^2 = x^3(2a-x)$$



$$2\int_0^{2a} \frac{x}{a}\sqrt{x(2a-x)}\,dx$$

$$= \frac{2\times 2\cancel{a}}{\cancel{a}}\int_0^{a} \sqrt{x(2a-x)}\,dx$$

$$= \boxed{\pi a^2}$$

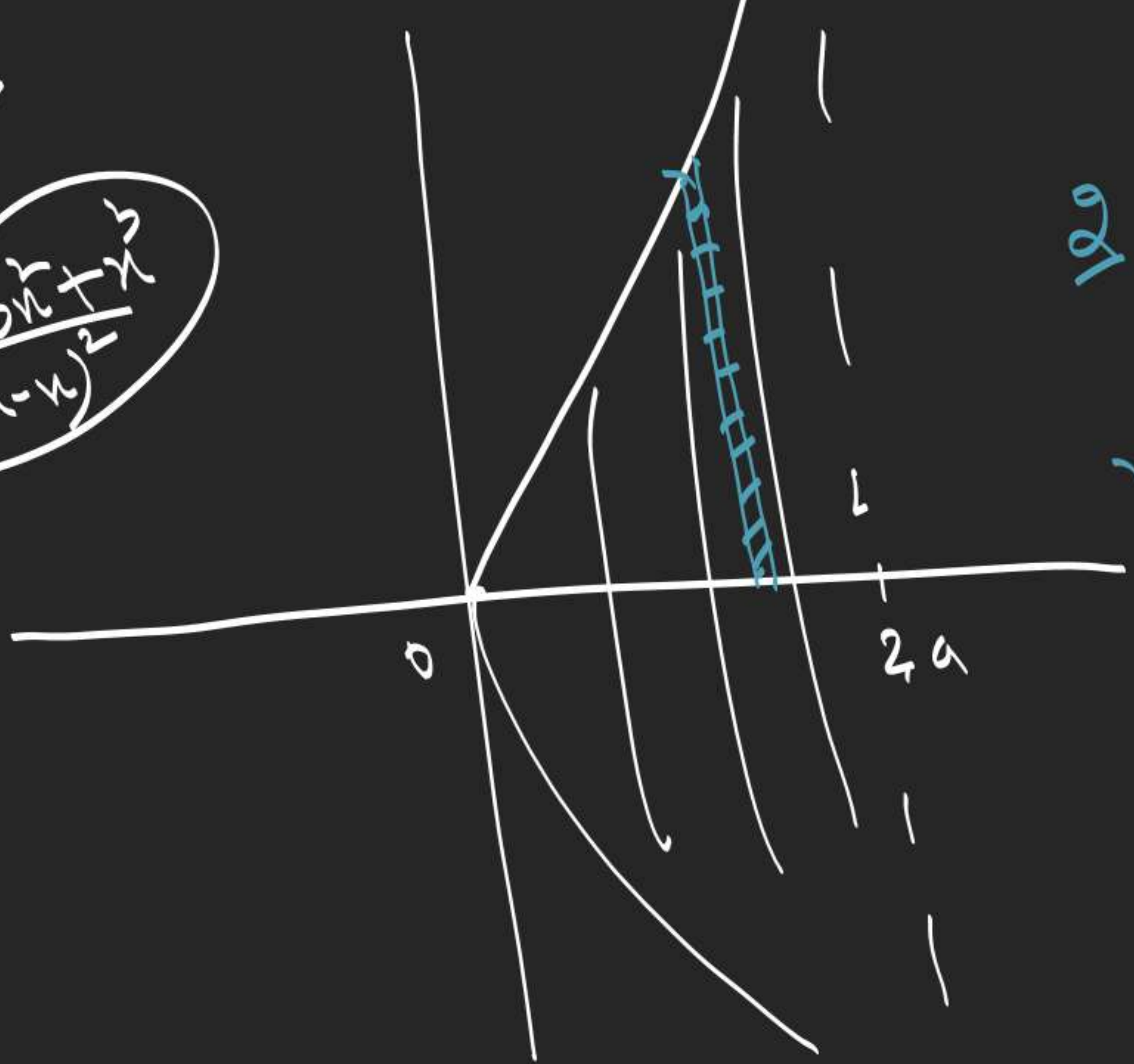


$\rightarrow = \sqrt{x(2a-x)}$

2. Find the area bounded by curve $y^2(2a-x) = x^3$ and its asymptote

$$y^2 = \frac{x^3}{2a-x}$$

$$2yy' = \frac{(2a-x)3x^2 + x^3}{(2a-x)^2}$$



$$2\int_0^{2a} x\sqrt{\frac{x}{2a-x}}\,dx$$

$$x = 2a\sin^2\theta$$

$$= 2\int_0^{\pi/2} 2a\sin^2\theta \,\frac{\sin\theta}{\cos\theta}\, 4a\sin\theta\cos\theta\,d\theta$$

$$= 16a^2\int_0^{\pi/2} \sin^4\theta\,d\theta$$

$$= \boxed{3\pi a^2}$$

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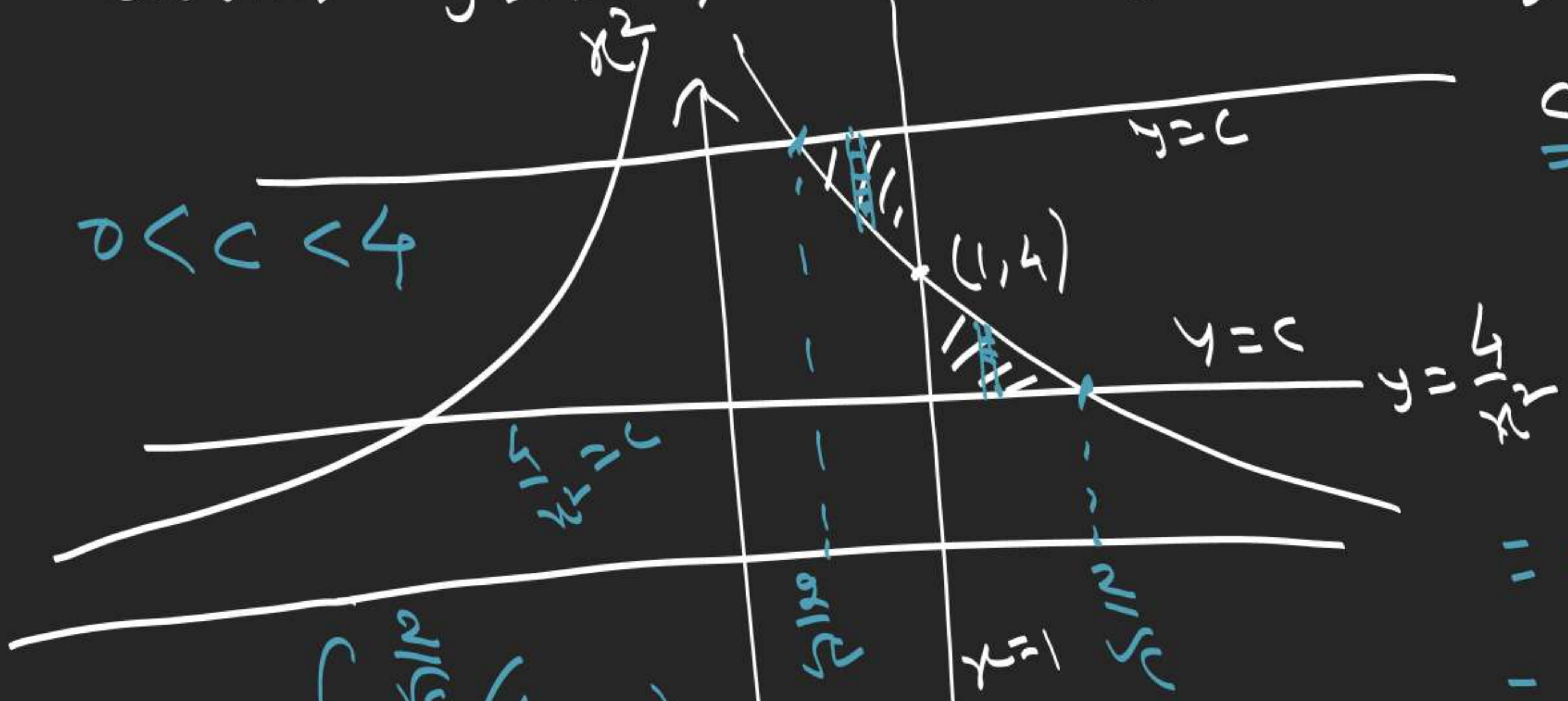
3. Find 'c' for which area of figure bounded by curves $y=\frac{4}{x^2}$, $x=1$ and $y=c$ is equal to $\frac{9}{4}$

$\boxed{c=\frac{1}{4},\frac{49}{4}}$

$0<c<4$



$$\int_1^{\frac{2}{\sqrt{c}}}\left(\frac{4}{x^2}-c\right)dx=\frac{9}{4}$$

$\boxed{c=\frac{1}{4}}$

$\boxed{c=\frac{49}{4}}$

$c>4$

$$\int_{\frac{2}{\sqrt{c}}}^{1}\left(c-\frac{4}{x^2}\right)dx$$

$$=c\left(1-\frac{2}{\sqrt{c}}\right)+4\left(1-\frac{\sqrt{c}}{2}\right)$$

$$=c+4-4\sqrt{c}$$

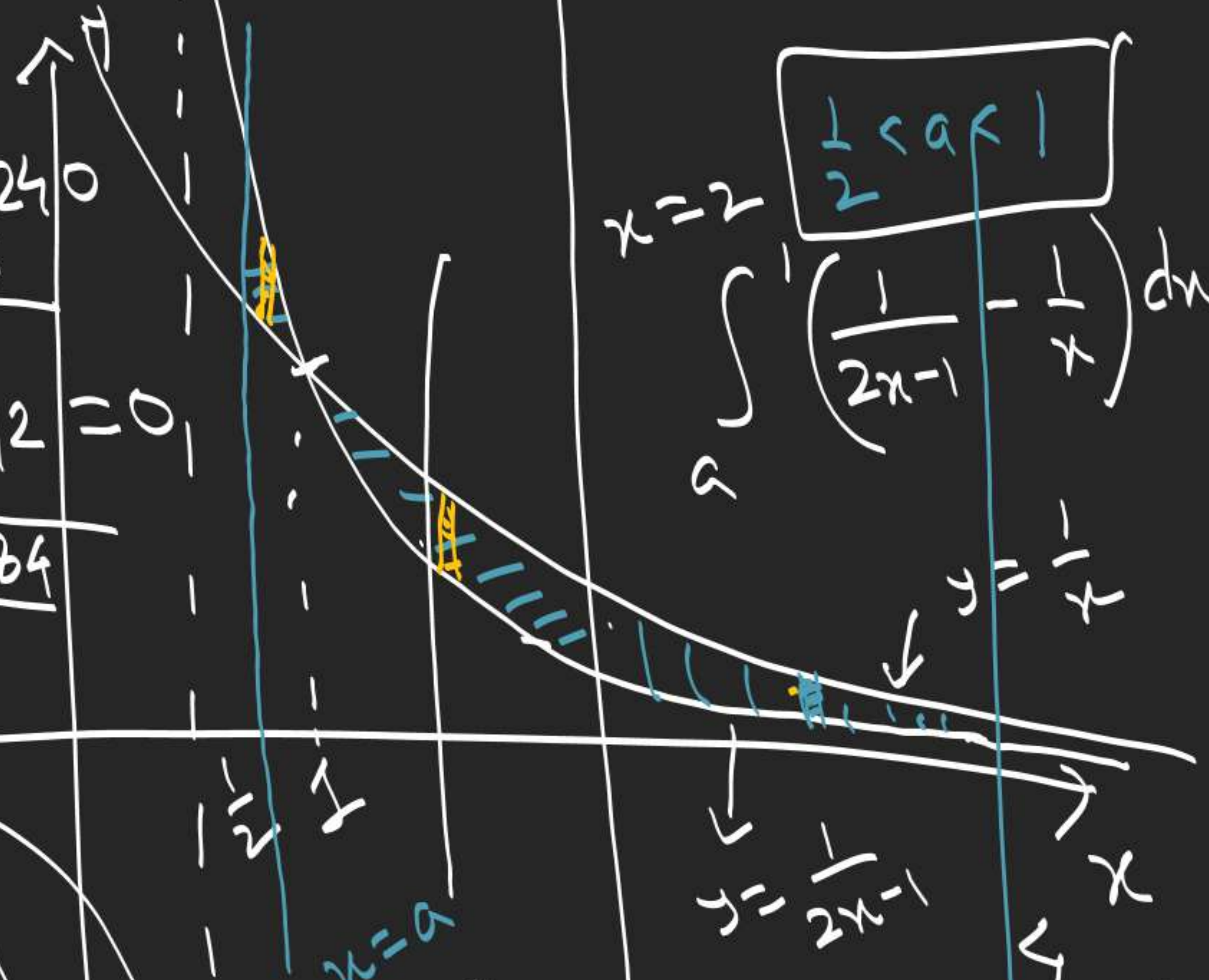
$$\frac{9}{4}=\left(\sqrt{c}-2\right)^2$$

$$\sqrt{c}-2=\pm\frac{3}{2}\Rightarrow\sqrt{c}=\frac{7}{2},\frac{1}{2}$$

4. Find 'a' for which area bounded by $y=\frac{1}{x}$, $y=\frac{1}{2x-1}$, $x=2$ and $x=a$ is equal to $\ln\left(\frac{4}{\sqrt{5}}\right)$

$a>2$

$$\int_2^a\left(\frac{1}{x}-\frac{1}{2x-1}\right)dx$$



$$\boxed{\frac{1}{2}<a<1}$$

$x=2$

$$\int_a^1\left(\frac{1}{2x-1}-\frac{1}{x}\right)dx+\int_1^2\left(\frac{1}{x}-\frac{1}{2x-1}\right)dx$$

$$-\frac{1}{2}\ln(2a-1)+\ln a+\ln\frac{2}{\sqrt{3}}$$

$$\ln\frac{a}{\sqrt{2a-1}}+\ln\frac{2}{\sqrt{3}}$$

$$\frac{4}{\sqrt{5}}=\frac{2a}{\sqrt{6a-3}}$$

$$5a^2=24a-12$$

$$5a^2-24a+12=0$$

$$a=\frac{12\pm\sqrt{84}}{5}$$

$$\frac{24\pm\sqrt{576-240}}{10}$$

336

$$a=\boxed{\frac{12-2\sqrt{21}}{5}}$$

$$\boxed{a=8}$$

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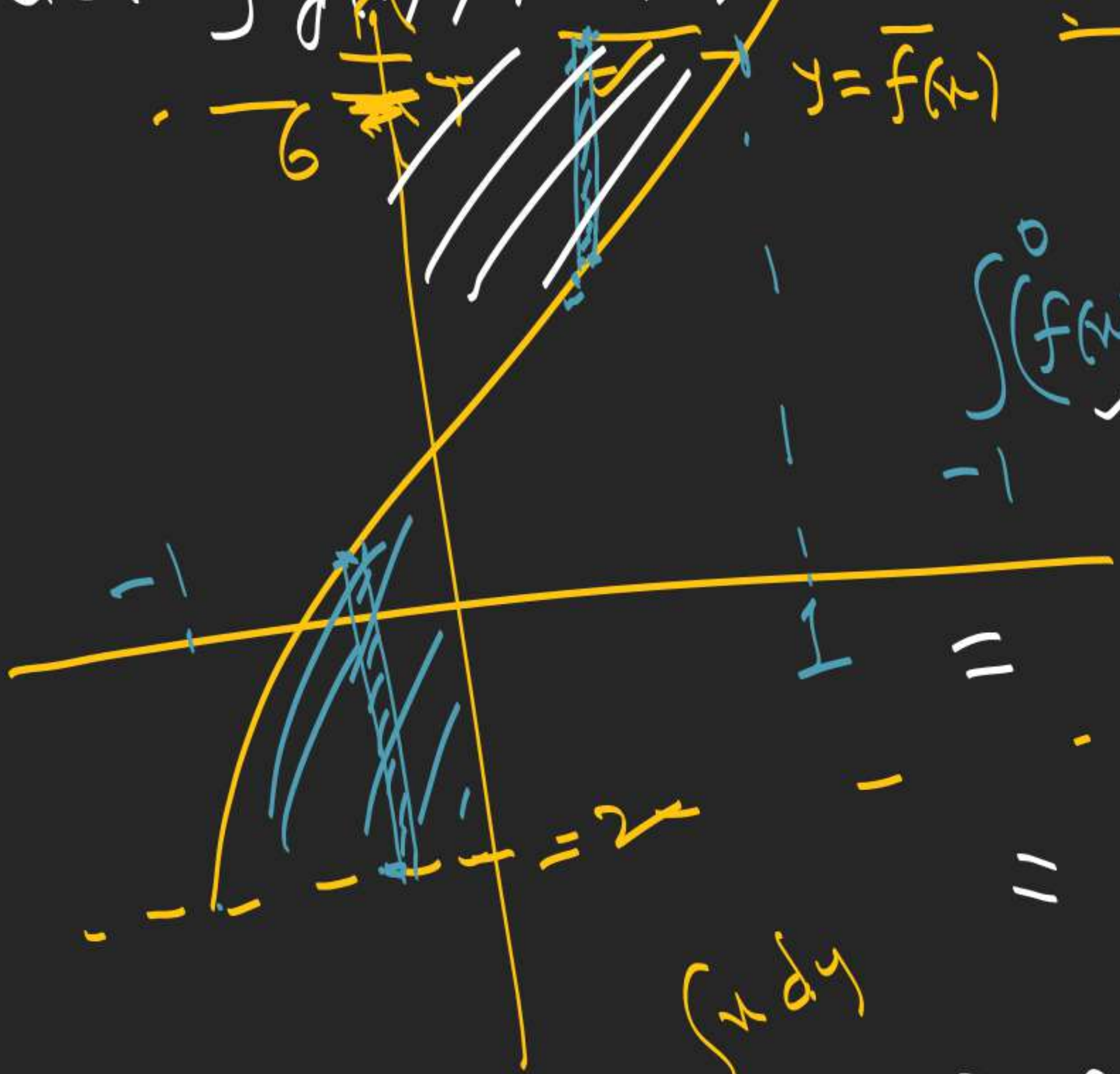
5. Let $f(x) = x^3 + 3x + 2$ and $g(x)$ is inverse of it. Find the area bounded by $g(x)$, x-axis and ordinates $x = -2$, $x = 6$.



$\int x\,dy$

$$\int_{-1}^{0} (f(x)+2)\,dx + \int_{0}^{1} (6 - f(x))\,dx$$

$$= \int_{0}^{1} (f(-x)+2+6-f(x))\,dx$$

$$= \int_{0}^{1} (8 - 2x^3 - 6x)\,dx$$

$$= 8 - \frac{1}{2} - 3 = \boxed{\frac{9}{2}}$$

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6. Find 'k', if the area bounded by $y = x^2 + 2x - 3$ and the line $y = kx + 1$ is least. Also find the least area.

1-5 (Ex-5)

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$$\frac{1}{\sqrt{r}+\sqrt{r+1}} < \frac{1}{2\sqrt{r}} < \frac{1}{\sqrt{r}+\sqrt{r-1}}$$

1-5

$$\sqrt{r+1}-\sqrt{r} < \frac{1}{2\sqrt{r}} < \sqrt{r}-\sqrt{r-1}$$

$$\sqrt{10^6}-\sqrt{1}$$

$$<\sqrt{10^6+1}-\sqrt{1} < \frac{1}{2}\sum_{r=1}^{10^6}\frac{1}{\sqrt{r}} < \sqrt{10^6}-\sqrt{0}$$

$$2\times 999 < \sum \frac{1}{\sqrt{r}} < 2000$$

$$\frac{1}{\sqrt{1}} + \left(\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}+\cdots\frac{1}{\sqrt{10^6}}\right) < \frac{1}{\sqrt{1}}+2\left(\sqrt{10^6}-\sqrt{1}\right)$$