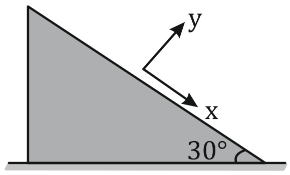
DPP – 06

SOLUTION

1. $t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{4+\left(\frac{R}{\sqrt{3}}\right)}{g}}$

Since, $R = \upsilon_A t$

$\Rightarrow R = 2\sqrt{\frac{4+\left(\frac{R}{\sqrt{3}}\right)}{g}}$



Squaring and solving, we get

$R \approx 2$ m and $t \approx 1$ s

Now, $u_x = 2\cos(30°) = \sqrt{3}\ ms^{-1}$ and

$u_y = 2\sin(30°) = 1\ ms^{-2}$

$a_x = g\sin(30°) = 4.9\ ms^{-2}$ and

$a_y = -g\cos\ 30° = -4.9\sqrt{3}\ ms^{-2}$

Speed after bouncing is

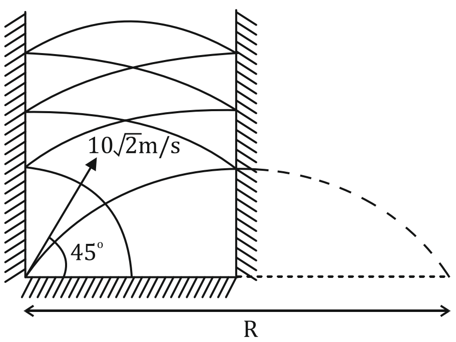
$\upsilon = \sqrt{v_x^2 + (ev_y)^2}$

$\Rightarrow \upsilon = \sqrt{(\sqrt{3} + 4.9 \times 1)^2 + (0.6(1 - 4.9\sqrt{3} \times 1))^2}$

$\Rightarrow \upsilon \cong 8\ ms^{-1}$

2. $R = \frac{v^2}{g} = \frac{200}{10} = 20$ m

Number of collision



$= \dfrac{20}{3} = 6.667$

$\Rightarrow n = 6$

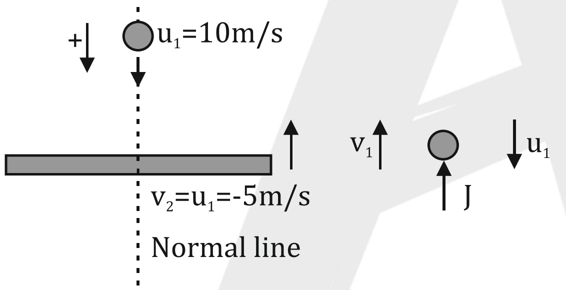
3. $t_1 = \frac{l}{3v}$

$t_2 = \dfrac{l - 2v(t_1)}{5v} = \dfrac{l}{15v}$

$t_3 = \dfrac{l - 2v(t_1 + t_2)}{7v} = \dfrac{l}{5 \times 7v}$

$\Rightarrow t_1 + t_2 + t_3 = \dfrac{45}{v} = 5\text{ s} \Rightarrow v = 9\text{ m/s}$

4. As we know $e = \left[\frac{(v_2 - v_1)}{(u_1 - u_2)}\right]_n$



$1 = \frac{(-5)-(v_1)}{(10)-(-5)} \Rightarrow v_1 = -20\text{ m/s}$

Hence, ball will start moving towards upward direction with velocity 20 m/s.

Let impulse imparted by plate force on the ball is J in upward direction.

$mu_1 - J = -mv_1$

$\Rightarrow J = m(u_1 + v_1) = 1(10 + 20) = 30\text{ kg m/s}$

5. $e = 1 = \frac{\text{Velocity of separation}}{2V-(-V)}$

Velocity of separation $= 3\text{ V}$

Required time $= \frac{2\pi r}{3V}$

6. $m_1V = m_2V_2 - m_1\frac{V}{10}$ ____(i)

$e = 1 = \frac{V_2-(-V/10)}{V-0}$

or $V_2 + \frac{V}{10} = V$ ____(ii)

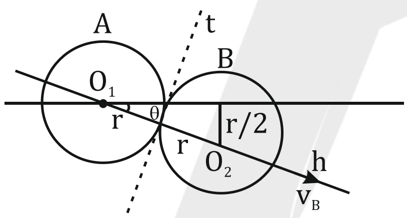
From eqn. (i) $\frac{m_1}{m_2}V = V_2 - \frac{V}{10}\frac{m_1}{m_2}$

From eqn. (ii), $\frac{m_1}{m_2}V = \left(V - \frac{V}{10}\right) - \frac{V}{10}\frac{m_1}{m_2}$

or $11m_1 = 9m_2 \Rightarrow m_2 > m_1$

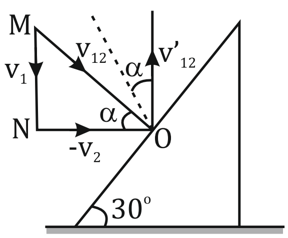
7. M_1 is very large as compared to M_2. Hence for collision between M_1 and M_2, M_1 can be considered equivalent to a wall and M_2 as a small block. Thus the velocity of M_2 will be $2v_0$ after collision with M_1. Similarly after collision between M_2 and M_3, the velocity of M_3 will be $2(2v_0)$. In sequence, the velocity of M_4 shall be $2(2(2v_0)) = 8v_0$ after collision with M_3.

8. $\sin\theta = \frac{r/2}{2r} = \frac{1}{4}$



As collision is elastic velocity along common normal, it will be interchanged. Hence $v_B = v\cos\theta = \frac{v\cdot\sqrt{15}}{4}$

9.



In the figure $\vec{v}_{12}$ = velocity of ball w.r.t. wedge before the collision and $\vec{v}'_{12}$ = velocity of ball w.r.t. wedge after the collision, which must be in vertically upward direction as shown. In elastic collision, $\vec{v}_{12}$ and $\vec{v}'_{12}$ will make equal angle (say a) with the normal to the plane. We can show that $\alpha = 30°$ $\therefore \angle MON = 30°$

Now $\frac{v_1}{v_2} = \tan\ 30^\circ = \frac{1}{\sqrt{3}}$

10. During each collision with wall A, velocity of the ball increases by $2v_1$ but velocity of wall remains unchanged.

$$\frac{2x}{v} = \frac{(-\Delta x)}{v_1} \text{ but } \Delta v = 2v_1$$

$$\Rightarrow \frac{-\Delta x}{\Delta v} = \frac{x}{v}; \text{ or } k - \int_{\ell}^{\ell/2} \frac{dx}{x} = \int_{v_0}^{v} \frac{dv}{v} \rightarrow v = 2v_0$$