Nishant Jindal

LIMIT

1 Ex1

2

3

4

5

$$\lim_{x\to 2} \frac{1+\sqrt{2+x}-3}{(x-2)\left(\sqrt{1+\sqrt{2+x}}+\sqrt{3}\right)} = \frac{\sqrt{2+x}-2}{(x-2)\left(\sqrt{3}+\sqrt{3}\right)}$$

Rat

$$= \frac{1}{2\sqrt{3}} \lim_{x\to 2} \frac{\cancel{2+x-4}}{\cancel{(x-2)}\left(\sqrt{2+x}+2\right)}$$

$$= \frac{1}{2\sqrt{3}} \times \frac{1}{2+2} = \frac{1}{8\sqrt{3}}$$

Q4 $\lim_{x\to 1} \frac{\sqrt[n]{x}-1}{\sqrt[m]{x}-1}$ DL

$x = 1+h$

$$\frac{(1+h)^{\frac{1}{n}}-1}{(1+h)^{\frac{1}{m}}-1} = \frac{\cancel{1}+\frac{h}{n}-\cancel{1}}{\cancel{1}+\frac{h}{m}-\cancel{1}} = \frac{m}{n}$$

$$\frac{3}{2} = \frac{1}{2\sqrt{a}} \Rightarrow a =$$

Q5 $\lim_{x\to a}$

$$\frac{2-\frac{1}{2\sqrt{x^2+3a^2}} \times 2x}{\frac{1}{2\sqrt{x+a}}\times 1 - 0}$$

$$\frac{2-\frac{a}{2a}}{\frac{1}{2\sqrt{2a}}} = \sqrt{2}$$

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LIMIT

Q 6 $\lim\limits_{x\to 0} \dfrac{\ln(\sin 3x)}{\ln(\sin x)}$ $\frac{\infty}{\infty}$ DL

$y = \ln(\sin x)$

$\dfrac{dy}{dx} = \dfrac{1}{\sin x} \times \cos x$

x^n

$\lim\limits_{x\to 0} \dfrac{\frac{3\cos 3x}{\sin 3x}}{\frac{\cos x}{\sin x}} = \dfrac{3\cot 3x}{\cot x} = 3\lim\limits_{x\to 0} \dfrac{\tan x}{\tan 3x}$ $\left(\frac{0}{0}\right)$ DL

$= 3\lim\limits_{x\to 0} \dfrac{\sec^2(x)}{3\sec^2(3x)} = 3\times\dfrac{\sec^2 0}{3\sec^2 0} = \dfrac{3\times 1}{3\times 1} = 1$

(8) $\lim\limits_{x\to 1} \dfrac{(7+x)^{\frac{1}{3}} - (3+x^2)^{\frac{1}{2}}}{x-1}$ $\frac{2-2}{1-1} = \frac{0}{0}$

$x = 1+h$

$\lim\limits_{x\to 1} \dfrac{\frac{1}{3}(7+x)^{-\frac{2}{3}} - \frac{1}{2}(3+x)^{-\frac{1}{2}}}{1-0}$ DL

$\frac{1}{3}(2^3)^{-\frac{2}{3}} - \frac{1}{2}(2^2)^{-\frac{1}{2}}$

Q10 $\lim\limits_{x\to\infty} \dfrac{(\boxed{x}+1)^{10} + (\boxed{x}+2)^{10} + \cdots + (\boxed{x}+100)^{10}}{x^{10}+10^{10}}$

$\lim\limits_{x\to\infty} \dfrac{\cancel{x^{10}}\left\{\left(1+\frac{1}{x}\right)^{10} + \left(1+\frac{2}{x}\right)^{10} + \cdots + \left(1+\frac{100}{x}\right)^{10}\right\}}{\cancel{x^{10}}\left\{1+\frac{10^{10}}{x^{10}}\right\}} = \dfrac{\overbrace{1+1+1+\cdots+1}^{\leftarrow 100 \text{ terms} \rightarrow}}{1+0} = 100$

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LIMIT

$$\lim_{x\to 1} \frac{(7+x^3)^{\frac{1}{3}}-(3+x^2)^{\frac{1}{2}}}{x-1}$$

$$\lim_{x\to 1} \frac{\frac{1}{\cancel{3}}(7+x^3)^{-\frac{2}{3}}\times(\cancel{3}x^2)-\frac{1}{\cancel{2}}(3+x^2)^{-\frac{1}{2}}\times(0+\cancel{2}x)}{1-0}$$

$$(2^{\cancel{3}})^{-\frac{2}{\cancel{3}}}\times 1^2-(2^{\cancel{2}})^{-\frac{1}{\cancel{2}}}\times 1$$

$$\frac{1}{4}-\frac{1}{2}$$

13 $$U_n=\frac{n!}{(n+2)!}=\frac{\cancel{n!}}{(n+2)(n+1)\cancel{\lfloor n}}$$

$$S_n=\sum \frac{1}{\underbrace{(n+1)(n+2)}_{\text{diff}=1}}$$

$$S_n=\sum_{n=1}^{n}\left(\frac{1}{n+1}-\frac{1}{n+2}\right)$$

$$=\left(\frac{1}{2}-\cancel{\frac{1}{3}}\right)+\left(\cancel{\frac{1}{3}}-\cancel{\frac{1}{4}}\right)+\left(\cancel{\frac{1}{4}}-\frac{1}{5}\right)$$

$$---+\left(\frac{1}{n+1}-\frac{1}{n+2}\right)$$

$$\lim_{n\to\infty} S_n=\lim_{n\to\infty}\frac{1}{2}-\cancel{\frac{1}{n+2}}=\frac{1}{2}-0=\frac{1}{2}$$

LIMIT

14) $a_n = \sum_{K=1}^{n} 2K$

$= 2\sum_{K=1}^{n} K$

$= 2[1+2+3+ \cdots +n]$

$= \cancel{2} \times \frac{(n)(n+1)}{\cancel{2}}$

$a_n = n^2+n$

$b_n = \sum_{K=1}^{n}(2K-1)$

$= 1+3+5+ \cdots (2n-1)$

$\leftarrow$ n odd no. Sum $\rightarrow$

$b_n = n^2$

Q $\lim_{n\to\infty} \sqrt{a_n} - \sqrt{b_n}$

$\lim_{n\to\infty} \sqrt{n^2+n} - n$ Rat

$\lim_{n\to\infty} \frac{\cancel{n^2}+n - \cancel{n^2}}{\sqrt{n^2+n}+n} = \frac{E}{E} = \frac{1}{1+1} = \frac{1}{2}$

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LIMIT

Q 15

$$P_n = \prod_{K=2}^{n} 1 - \frac{1}{{}^{K+1}C_2} = \prod 1 - \frac{2}{(K)(K+1)}$$

$${}^{n}C_2 = \frac{(n)(n-1)}{1\cdot 2} \qquad {}^{K+1}C_2 = \frac{(K+1)(K)}{1\cdot 2}$$

$$= \prod\left(\frac{K^2+K-2}{(K)(K+1)}\right)$$

$$= \prod_{K=2}^{n}\left(\frac{(K+2)(K-1)}{(K)(K+1)}\right)$$

$$= \prod_{K=2}^{n}\frac{K+2}{K+1} \times \prod_{K=2}^{n}\frac{K-1}{K}$$

$$= \left\{\frac{\cancel{4}}{3}\times\frac{\cancel{5}}{\cancel{4}}\times\frac{\cancel{6}}{\cancel{5}}\times\frac{7}{\cancel{6}}\times\frac{8}{\cancel{7}} \quad \frac{n+2}{\cancel{n+1}}\right\}\times\left\{\frac{1}{2}\times\frac{\cancel{2}}{\cancel{3}}\times\frac{\cancel{3}}{\cancel{4}}\times\frac{\cancel{4}}{5}\times\cdots\times\frac{n-1}{n}\right\}$$

$$\lim_{n\to\infty}\frac{(n+2)}{3}\times\frac{1}{n} = \frac{n+2}{3n} \;\frac{\infty}{\infty} = \frac{1}{3}$$

$$a+b=4$$

LIMIT

Q2 $\lim_{x\to 1} \dfrac{\sum_{k=1}^{100} x^k - 100}{x-1}$

$\lim_{x\to 1} \dfrac{x^1+x^2+x^3+\cdots+x^{100}-100}{x-1}$ $\dfrac{(\overbrace{1+1+1\cdots 1}^{100})}{}$ $\dfrac{100-100}{1-1}=\dfrac{0}{0}$

DL.

$\lim_{x\to 1} \dfrac{1+2x+3x^2+4x^3+\cdots+100x^{99}-0}{1}$

$1+2+3+\cdots+100 = \dfrac{(\cancel{100}^{50})(101)}{\cancel{2}} = 5050$

LIMIT

Q5 $\lim_{\theta\to\frac{\pi}{4}} \dfrac{\sqrt{2}-\cos\theta-\sin\theta}{(4\theta-\pi)^2} = \dfrac{\sqrt{2}-\left(\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}\right)}{\left(4\times\frac{\pi}{4}-\pi\right)^2} = \dfrac{\sqrt{2}-\sqrt{2}}{0} = \dfrac{0}{0} = DL$

$\lim_{\theta\to\frac{\pi}{4}} \dfrac{0+\sin\theta-\cos\theta}{\boxed{2}(4\theta-\pi)\times\boxed{(4-0)}} = \dfrac{1}{8}\lim_{\theta\to\frac{\pi}{4}} \dfrac{\sin\theta-\cos\theta}{(4\theta-\pi)} = \dfrac{\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{2}}}{0} = \dfrac{0}{0}$

DL

$= \dfrac{1}{8}\lim_{\theta\to\frac{\pi}{4}} \dfrac{\cos\theta+\sin\theta}{(4-0)} = \dfrac{1}{32}\times\left(\dfrac{1}{\sqrt{2}}+\dfrac{1}{\sqrt{2}}\right)$

$= \dfrac{\sqrt{2}}{32}$

LIMIT

Q 6 $\lim_{h\to 0} \frac{\sin\left(\frac{\pi}{3}+4h\right)-4\sin\left(\frac{\pi}{3}+3h\right)+6\sin\left(\frac{\pi}{3}+2h\right)-4\sin\left(\frac{\pi}{3}+h\right)+\sin\frac{\pi}{3}}{h^4}$ $\frac{0}{0}$

DL

$\lim_{h\to 0} \frac{4\cos\left(\frac{\pi}{3}+4h\right)-12\cos\left(\frac{\pi}{3}+3h\right)+12\cos\left(\frac{\pi}{3}+2h\right)-4\cos\left(\frac{\pi}{3}+h\right)+0}{4h^3}$ $\frac{0}{0}$

$\lim_{h\to 0} \frac{-4^2\sin\left(\frac{\pi}{3}+4h\right)+36\sin\left(\frac{\pi}{3}+3h\right)-24\sin\left(\frac{\pi}{3}+2h\right)+4\sin\left(\frac{\pi}{3}+h\right)}{12h^2}$

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Q If $\lim_{x\to 0} \frac{f(x)}{x^2} = 2$ find $\lim_{x\to 0} \frac{f(x)}{x} = ?$

$$\lim_{x\to 0} \frac{f(x)}{x} = \lim_{x\to 0} \boxed{\frac{f(x)}{x^2}} \times x$$

$$2 \times 0 = 0$$

Q $\lim_{x\to a} f(x)+g(x) = 2$ & $\lim_{x\to a} f(x)-g(x) = 1$ then (1) find $\lim_{x\to a} f(x) = ?$ $3/2$

(2) find $\lim_{x\to a} 4 f(x)\cdot g(x) = ?$

$$4 \times \frac{3}{2} \times \frac{1}{2} = 3$$

$$\lim_{x\to a} f(x)+g(x) = 2$$

$$\lim_{x\to a} f(x)-g(x) = 1$$

$$- \quad + \quad -$$

add $\lim_{x\to a} 2f(x) = 3 \Rightarrow \lim_{x\to a} f(x) = \frac{3}{2}$

Subtract

$\lim_{x\to a} 2g(x) = 1$

$\Rightarrow \lim_{x\to a} g(x) = \frac{1}{2}$

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LIMIT

Q Find value of $\prod_{n=3}^{\infty}\left(1-\frac{4}{n^2}\right)=?$

$$\prod_{n=3}^{\infty}\left(\frac{n^2-4}{n^2}\right)=\prod_{n=3}^{\infty}\frac{(n-2)}{n}\frac{(n+2)}{n}$$

$$\prod_{n=3}\left(\frac{n-2}{n}\right)\times\prod_{n=3}\frac{n+2}{n}$$

$$\left\{\frac{1}{\cancel{3}}\times\frac{2}{\cancel{4}}\times\frac{\cancel{3}}{\cancel{5}}\times\frac{\cancel{4}}{\cancel{6}}\cdots\times\frac{\cancel{n-3}}{n-1}\times\frac{\cancel{n-2}}{n}\right\}\times\left\{\frac{\cancel{5}}{3}\times\frac{\cancel{6}}{4}\times\frac{\cancel{7}}{\cancel{5}}\times\frac{\cancel{8}}{\cancel{6}}\times\cdots\frac{(n+1)}{\cancel{n-1}}\times\frac{n+2}{\cancel{n}}\right\}$$

$$\frac{1\times 2}{(n)(n-1)}\times\frac{(n+1)(n+2)}{3\times 4_2}$$

$$\frac{1}{6}\times\lim_{n\to\infty}\frac{(n+1)(n+2)}{(n)(n-1)}\frac{\varepsilon}{\varepsilon}$$

$$\frac{1}{6}\times 1=\frac{1}{6}$$

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LIMIT

Q $\lim_{n\to\infty} \sqrt{n^2+n+1} - [\sqrt{n^2+n+1}]$ $\quad n \in I$ $\qquad \{x+n\} = \{x\}$

$x - [x] = \{x\}$

$\lim_{n\to\infty} \{\sqrt{n^2+n+1}\}$

$\{x\} = x - [x]$

$\lim_{n\to\infty} \{(n^2+n+1)^{1/2}\}$

$\lim_{n\to\infty} \left\{n\left(1+\left(\frac{1}{n}+\frac{1}{n^2}\right)\right)^{1/2}\right\} \to BT$

$\lim_{n\to\infty} \left\{n\left(1+\frac{1}{2n}+\frac{1}{2n^2}\right)\right\}$

$\frac{1}{2} - 0 = \frac{1}{2}$

1/2 Km

$\lim_{n\to\infty} \left\{n+\frac{1}{2}+\frac{1}{2n}\right\} = \lim_{n\to\infty}\left\{\frac{1}{2}+\frac{1}{2n}\right\} = \lim_{n\to\infty}\left(\frac{1}{2}+\cancel{\frac{1}{2n}}\right) - \left[\frac{1}{2}+\frac{1}{2n}\right]$

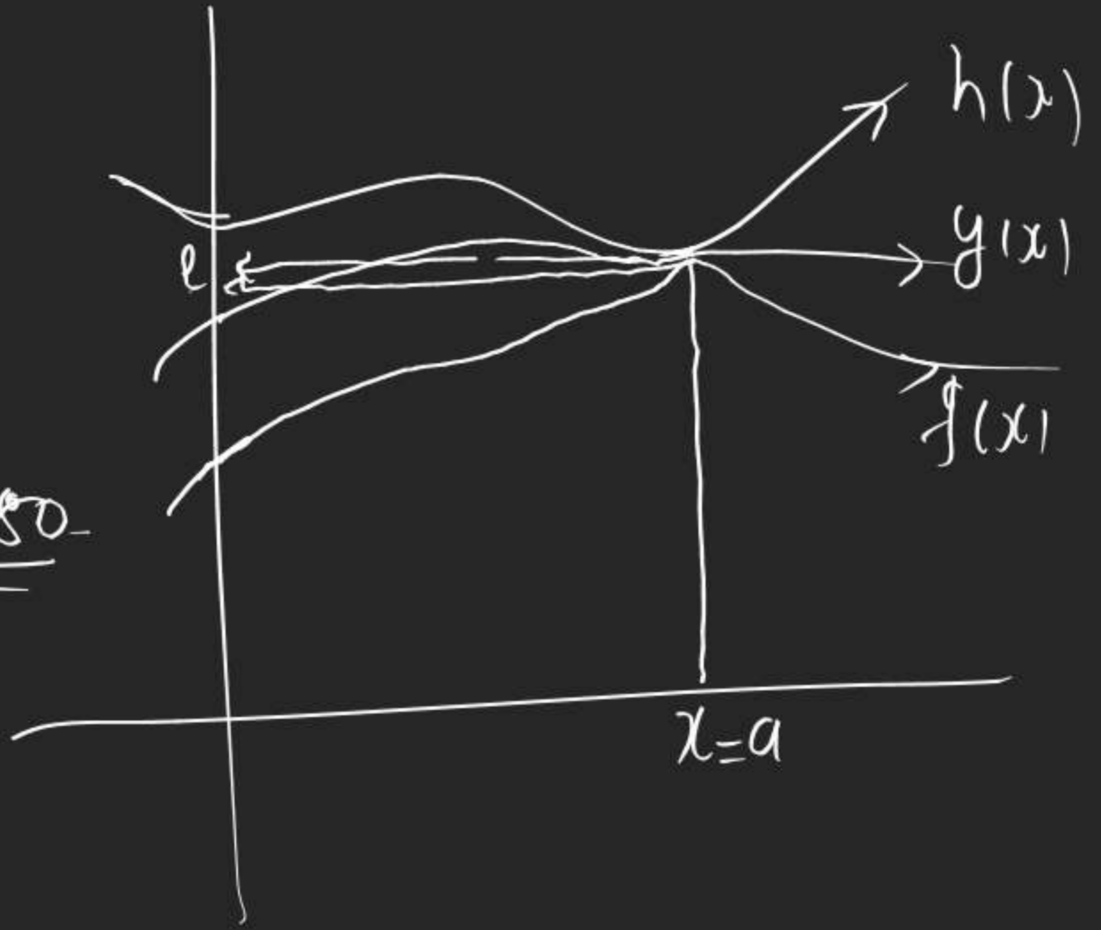
Sandwich Theorem [Must need for all Qs where Inequality works]

Let $f(x) \le g(x) \le h(x)$ $\forall x$ belonging to common domain

Then for some real No. a

if $\lim_{x \to a} f(x) = l$ & $\lim_{x \to a} h(x) = l$

Then by Sandwich Theorem $\boxed{\lim_{x \to a} g(x) = l}$ also



$$\boxed{f(x)} < g(x) \le h(x)$$

$$l < \lim_{x \to a} g(x) \le l$$

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LIMIT

Q $\lim_{n\to\infty} \dfrac{[x]+[2x]+[3x]+\cdots+[nx]}{n^2}$

$x-1 < [x] \le x$

$2x-1 < [2x] \le 2x$

$3x-1 < [3x] \le 3x$

$\vdots$

$nx-1 < [nx] \le nx$

$\lim_{n\to\infty} \dfrac{(x+2x\cdots+nx)-n}{n^2} < \lim_{n\to\infty} \dfrac{[x]+\cdots+[nx]}{n^2} \le \lim_{n\to\infty} \dfrac{x+2x\cdots+nx}{n^2}$

$\lim_{n\to\infty} \dfrac{x(1+2+3+\cdots+n)}{n^2} - \dfrac{n}{n^2} < \lim_{n\to\infty} \cdots \le \lim_{n\to\infty} \dfrac{x(1+2+\cdots+n)}{n^2}$

diff = 1

$\dfrac{x}{2} - 0 < \lim_{n\to\infty} \cdots \le \dfrac{x}{2}$

$\therefore \lim_{n\to\infty} \dfrac{[x]+[2x]\cdots+[nx]}{n^2} = \dfrac{x}{2}$

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Q

$$\lim_{n\to\infty} \boxed{\frac{n}{n^2+1}}^{\text{Bdi}} + \frac{n}{n^2+2} + \frac{n}{n^2+3} + \cdots + \boxed{\frac{n}{n^2+n}}^{\text{Chh.}} = L \text{ (assume)}$$

n term

Dr Bda Vo Sabse Chhota

$$\frac{n}{n^2+n} + \frac{n}{n^2+n} + \frac{n}{n^2+n} \cdots \le L \le \frac{n}{n^2+1} + \frac{n}{n^2+1} + \frac{n}{n^2+1} \cdots$$

← n times → ← n times →

$$\lim_{n\to\infty} \frac{n \times n}{n^2+n} \le L \le \lim_{n\to\infty} \frac{n \times n}{n^2+1}$$

$$1 \le L \le 1$$

$$\therefore \boxed{L = 1}$$

(1) When Qs is of $\lim_{n\to\infty}$ with a Series having + + + + Sign

(2) Process.

Always try to find Min^m & Max^n of Series & Repeat it times equal. to No of terms

(3) See Ex.

(3) Com. Sense

$$1+1+1+1 < 1+2+3+4 < 4+4+4+4$$

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Q $\lim_{n\to\infty} \dfrac{1+(1+2)+(1+2+3)\dots n\text{ term}}{n^3}$ → "Upr Misc. Series (S&P) Ki hai

2) Ye AP, GP, HP, AGP Kuchh nh hai

3) find $S_n = \sum T_n$ ← then ter.

$= \sum 1+2+3+\dots+n$

$S_n = \sum \dfrac{(n)(n+1)}{2}$

$S_n = \dfrac{1}{2}\sum n^2+n$

$= \dfrac{1}{2}\left\{\sum n^2 + \sum n\right\}$

$= \dfrac{1}{2}\left\{\dfrac{(n)(n+1)(2n+1)}{6} + \dfrac{(n)(n+1)}{2}\right\}$

$\dfrac{1}{2}\lim_{n\to\infty} \dfrac{\left\{\dfrac{(n)(n+1)(2n+1)}{6} + \dfrac{(n)(n+1)}{2}\right\}}{n^3}$

Tob=3, → 2, $n^3 \to \boxed{3}$

$\dfrac{1}{2} \times \dfrac{\dfrac{1\times1\times2}{6}}{1} = \dfrac{1}{6}$

Ex1 36, 39, 44

Ex2 → 20

Ex3 → 1, 8, 9, 10, 11

Dscuss Ex4 (2, 3.)

LIMIT

Q $\lim_{n\to\infty} \frac{[x]+[2x]+\cdots+[nx]}{n^2} = \frac{x}{\text{deg}} = \frac{x}{2}$

diff = 1

$[1^2x+1^2] = [1^2x+1]$

Q $\lim_{n\to\infty} \frac{[1^2x]+[2^2x]+[3^2x]+\cdots+[n^2x]}{n^3} = \frac{x}{3}$

diff = 1

Q $\lim_{n\to\infty} \frac{[1^2x+1^2]+[2^2x+2^2]+\cdots+[n^2x+n^2]}{n^3}$

$\left\{\frac{[1^2x]+[2^2x]\cdots[n^2x]}{n^3}\right\} + \left(\frac{1^2+2\cdots n^2}{n^3}\right)$

diff = 1

$\frac{x}{3} + \frac{1}{3} = \frac{x+1}{3}$