Nishant Jindal

$$g(x) = \left(|x| + |4x-7|\right)\left(\underbrace{[x^2] - 3}\right)$$

$$\left(-\frac{1}{2}, 2\right)$$

$$\boxed{0, 1, \sqrt{2}, \sqrt{3}}$$

$x \in \left(-\frac{1}{2}, 1\right)$ → -3

$x \in [1, \sqrt{2})$ → -2

$x \in [\sqrt{2}, \sqrt{3})$ → -1

$x \in [\sqrt{3}, 2)$ → 0

$$\lim_{x \to 0} \frac{-3\left(|x| + \cancel{7} - 4x\right) + \cancel{21}}{x}$$

$$3\left(4 - \frac{|x|}{x}\right)$$

$$\lim_{x \to \frac{7}{4}} \frac{0}{x - 7/4} = 0$$

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$y=0 \qquad f(x) = f(x)f'(0) + f'(x)$

$\frac{1}{2} f(x) = f'(x) \qquad \frac{dy}{dx} = \frac{1}{2} y$

(B) $x\,g(x) = e^x - 1$

$$g(x) = \begin{cases} \frac{e^x-1}{x} & x \neq 0 \\ 1 & x=0 \end{cases}$$

$$\lim_{x\to 0} \frac{\frac{e^x-1}{x} - 1}{x-0}$$

$x=y=0$

$f(0) = 2f(0)f'(0)$

$f'(0) = \frac{1}{2}$

$$\int \frac{dy}{y} = \frac{1}{2}\int dx$$

$\ln y = \frac{1}{2}x + \cancel{c}$, $c \to 0$

$\ln(y+1) = x + \cancel{c}$, $c \to 0$ $\Leftarrow \int \frac{dy}{y+1} = \int dx$

$$\boxed{f(x) = e^x - 1}$$

$$\lim_{x\to 1} \frac{\cancel{(x-1)}\left(x^2+\sin x\right) g(x) - 0}{\cancel{x-1}} = 0$$

$$\lim_{x\to 0} \frac{0}{x}$$

$$f'(x) = (f(x)+1)\lim_{h\to 0}\frac{f(h)}{h}$$

$$f'(x) = (f(x)+1) f'(0) = f(x)+1$$

$$f'(0) = \lim_{x\to 0} \frac{\cancel{x}g(x) - \cancel{0}}{\cancel{x}} = 1$$

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$$y = a\cos(\ln x) + b\sin(\ln x)$$

$$x^2 y_3 + 3xy_2 + 2y_1 = 0$$

$$xy_1 = -a\sin(\ln x) + b\cos(\ln x)$$

$$x(xy_2 + y_1) = -a\cos(\ln x) - b\sin(\ln x) = -y$$

$$x^2 y_2 + xy_1 = -y$$

$$2xy_2 + x^2 y_3 + xy_2 + y_1 = -y_1$$

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$$y^{\frac{1}{m}} + y^{-\frac{1}{m}} = 2x \qquad (x^2-1)y_3 + 3xy_2 + (1-m^2)y_1 = 0$$

$$t + \frac{1}{t} = 2x \Rightarrow t^2 - 2xt + 1 = 0, \quad t = x \pm \sqrt{x^2-1} = y^{\frac{1}{m}}$$

$$y = \left(x - \sqrt{x^2-1}\right)^m$$

$$y_1 = m\left(x - \sqrt{x^2-1}\right)^{m-1}\left(1 - \frac{x}{\sqrt{x^2-1}}\right) = \frac{-my}{\sqrt{x^2-1}}$$

$$(x^2-1)y_1^2 = m^2y^2 \Rightarrow 2(x^2-1)y_1y_2 + 2xy_1^2 = 2m^2yy_1$$

$$(x^2-1)y_2 + xy_1 = m^2y \Rightarrow (x^2-1)y_3 + 2xy_2 + xy_2 + y_1$$

$$= m^2y_1$$

Nishant Jindal

1. Use the substitution $\boxed{x = \tan\theta}$ to show that the equation $\frac{d^2y}{dx^2} + \frac{2x}{1+x^2}\frac{dy}{dx} + \frac{y}{(1+x^2)^2} = 0$

$$-\sin 2\theta \cancel{\cos^2\theta} \frac{dy}{d\theta} + \cos^4\theta \frac{d^2y}{d\theta^2} + \sin 2\theta \cancel{\cos^2\theta} \frac{dy}{d\theta} + y\cos^4\theta = 0$$

$$\frac{dx}{d\theta} = \sec^2\theta$$

$$\frac{dy}{dx} = \frac{dy}{d\theta}\frac{d\theta}{dx} = \cos^2\theta \frac{dy}{d\theta}$$

$$\cos^4\theta\left(\frac{d^2y}{d\theta^2} + y\right) = 0$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\cos^2\theta \frac{dy}{d\theta}\right) = \frac{d}{d\theta}\left(\cos^2\theta \frac{dy}{d\theta}\right)\frac{d\theta}{dx} = \cos^2\theta\left(-\sin 2\theta \frac{dy}{d\theta} + \cos^2\theta \frac{d^2y}{d\theta^2}\right)$$

$$\frac{d^2y}{dx^2} = -\sin 2\theta \cos^2\theta \frac{dy}{d\theta} + \cos^4\theta \frac{d^2y}{d\theta^2}$$

Nishant Jindal

Q. P.T. $\dfrac{d^2x}{dy^2} = -\dfrac{\frac{d^2y}{dx^2}}{\left(\frac{dy}{dx}\right)^3} \Rightarrow \dfrac{d^3x}{dy^3} = -\dfrac{d}{dx}\left(\dfrac{\frac{d^2y}{dx^2}}{\left(\frac{dy}{dx}\right)^3}\right)\dfrac{dx}{dy}$

$$\frac{3(y'')^2 - y'y'''}{(y')^5} = -\left(\frac{(y')^3y''' - y''\,3(y')^2y''}{(y')^6}\right)\frac{1}{y'}$$

$$\frac{dx}{dy} = \frac{1}{\left(\frac{dy}{dx}\right)} \qquad -\frac{1}{\left(\frac{dy}{dx}\right)^2}\frac{d^2y}{dx^2}\frac{1}{\left(\frac{dy}{dx}\right)} = \frac{-\frac{d^2y}{dx^2}}{\left(\frac{dy}{dx}\right)^3}$$

$$\frac{d^2x}{dy^2} = \frac{d}{dy}\left(\frac{1}{\frac{dy}{dx}}\right) = \frac{d}{dx}\left(\frac{1}{\left(\frac{dy}{dx}\right)}\right)\frac{dx}{dy}$$

$$= \frac{\left(\frac{dy}{dx}\right)(0) - 1\left(\frac{d^2y}{dx^2}\right)}{\left(\frac{dy}{dx}\right)^2}\frac{1}{\left(\frac{dy}{dx}\right)}$$

Nishant Jindal

Derivative of determinant

$$D(x) = \begin{vmatrix} f_1(x) & f_2(x) & f_3(x) \\ g_1(x) & g_2(x) & g_3(x) \\ h_1(x) & h_2(x) & h_3(x) \end{vmatrix}$$

$D'(x) =$

$$D'(x) = \begin{vmatrix} f_1'(x) & f_2'(x) & f_3'(x) \\ g_1(x) & g_2(x) & g_3(x) \\ h_1(x) & h_2(x) & h_3(x) \end{vmatrix} + \begin{vmatrix} f_1(x) & f_2(x) & f_3(x) \\ g_1'(x) & g_2'(x) & g_3'(x) \\ h_1(x) & h_2(x) & h_3(x) \end{vmatrix} + \begin{vmatrix} f_1(x) & f_2(x) & f_3(x) \\ g_1(x) & g_2(x) & g_3(x) \\ h_1'(x) & h_2'(x) & h_3'(x) \end{vmatrix}$$

Nishant Jindal

$$\lim_{\Delta x \to 0} \frac{D(x+\Delta x) - D(x)}{\Delta x} = \begin{vmatrix} \lim \frac{f_1(x+\Delta x) - f_1(x)}{\Delta x} & \lim \frac{f_2(x+\Delta x) - f_2(x)}{\Delta x} & \lim \frac{f_3(x+\Delta x) - f_3(x)}{\Delta x} \\ \lim g_1(x+\Delta x) & \lim g_2(x+\Delta x) & \lim g_3(x+\Delta x) \\ \lim h_1(x+\Delta x) & \lim h_2(x+\Delta x) & \lim h_3(x+\Delta x) \end{vmatrix} + \begin{vmatrix} f_1(x) & f_2(x) & f_3(x) \\ \frac{g_1(x+\Delta x) - g_1(x)}{\Delta x} & \frac{g_2(x+\Delta x) - g_2(x)}{\Delta x} & \frac{g_3(x+\Delta x) - g_3(x)}{\Delta x} \\ h_1(x+\Delta x) & h_2(x+\Delta x) & h_3(x+\Delta x) \end{vmatrix}$$

$$+ \begin{vmatrix} f_1(x) & f_2(x) & f_3(x) \\ g_1(x) & g_2(x) & g_3(x) \\ \frac{h_1(x+\Delta x) - h_1(x)}{\Delta x} & \frac{h_2(x+\Delta x) - h_2(x)}{\Delta x} & \frac{h_3(x+\Delta x) - h_3(x)}{\Delta x} \end{vmatrix} = \begin{vmatrix} f_1' & f_2' & f_3' \\ g_1 & g_2 & g_3 \\ h_1 & h_2 & h_3 \end{vmatrix} +$$

$$\boxed{PT-1 \; \& \; PT-2}$$

$$\lim_{\Delta x \to 0} \underline{f_1}\,\underline{g_2}\,\underline{h_3} - \ldots - -$$