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Q1. If $\vec{a}$ & $\vec{b}$ $\boxed{\text{L.I.}}$ vectors

& $(\sqrt{3}\tan\theta-1)\vec{a}+(\sqrt{3}\sec\theta-2)\vec{b}=0$

find general values of θ.

Concept: $\lambda_1\vec{a_1}+\lambda_2\vec{a_2}=0$ & $\vec{a}_1$, & $\vec{a_2}$ are L.I

then $\lambda_1=\lambda_2=0$

here

$\sqrt{3}\tan\theta-1=0$ & $\sqrt{3}\sec\theta-2=0$

$\tan\theta=\frac{1}{\sqrt{3}}$ $\sec\theta=\frac{2}{\sqrt{3}}$

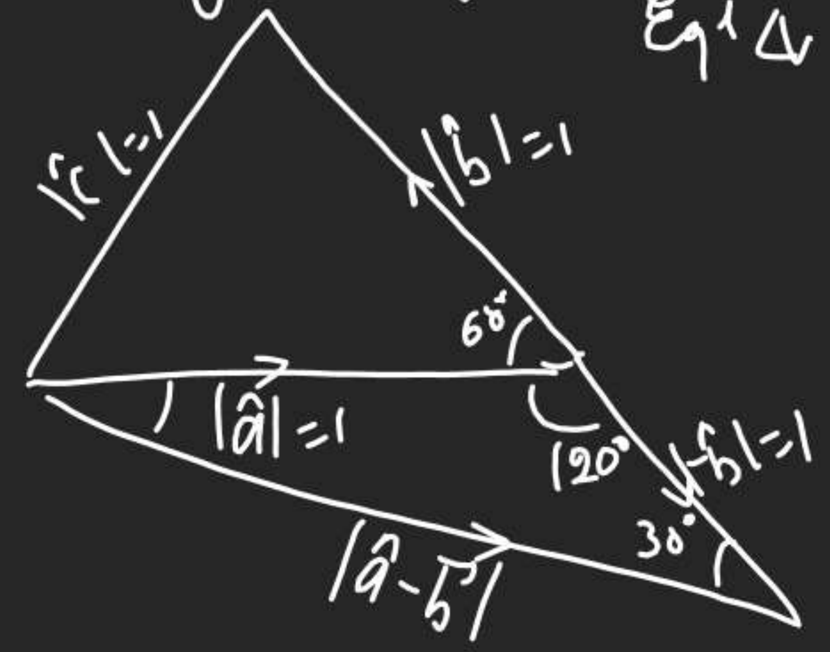
$\tan\theta=\tan\frac{\pi}{6}$ $\cos\theta=\frac{\sqrt{3}}{2}=\cos\frac{\pi}{6}$

$\therefore$ General value $=2n\pi+\frac{\pi}{6}$

Q If $\vec{a},\vec{b},\vec{c}$ are 3 vectors such that every pair is non collinear & $\vec{a}$ is (hold.

Q Sum of 2 unit vectors is a unit vector. find Magnitude of their difference?

Eq^l Δ



$\frac{|\hat{a}-\hat{b}|}{\sin 120°}=\frac{|\hat{a}|}{\sin 30°}$

$|\hat{a}-\hat{b}|=\frac{1}{\frac{1}{2}}\times\frac{\sqrt{3}}{2}=\sqrt{3}$.

Q 3 (Main)

$\vec a, \vec b, \vec c$ are 3 Non Collinear Vectors.
& $\vec a + 2\vec b$ is Collinear with $\vec c$
& $\vec b + 3\vec c$ is Collinear with $\vec a$
find $\vec a + 2\vec b + 6\vec c$?

$\vec a + 2\vec b$ is Collinear with $\vec c$

$$\vec a + 2\vec b = \lambda \vec c$$
$$\vec b + 3\vec c = \mu \vec a \quad \times 2$$

$$\vec a + \cancel{2\vec b} = \lambda \vec c$$
$$\cancel{2\vec b} + 6\vec c = 2\mu \vec a$$
$$\vec a - 6\vec c = \lambda \vec c - 2\mu \vec a$$

$-2\mu = 1$ | $\lambda = -6$

$$a + 2b = -6\vec c$$
$$\vec a + 2\vec b + 6\vec c = 0$$

Q 4 (Adv.)

find λ for which $-\lambda^2\hat i + \hat j + \hat k$, $\hat i - \lambda^2\hat j + \hat k$, $\hat i + \hat j - \lambda^2\hat k$ are L.D. vectors

Coplanar

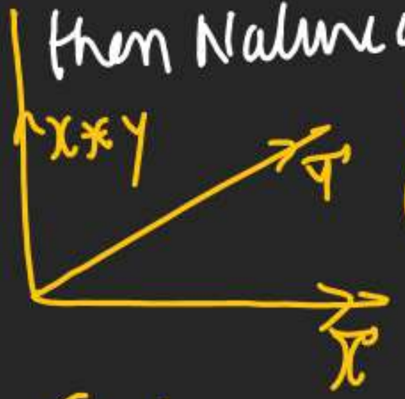
$$\begin{vmatrix} -\lambda^2 & 1 & 1 \\ 1 & -\lambda^2 & 1 \\ 1 & 1 & -\lambda^2 \end{vmatrix} = 0$$

$$(-\lambda^6 + 1 + 1) - (-\lambda^2 - \lambda^2 - \lambda^2) = 0$$
$$-\lambda^6 + 3\lambda^2 + 2 = 0$$
$$\lambda^6 - 3\lambda^2 - 2 = 0$$
$$(\lambda^2 - 2)(\lambda^4 + 2\lambda^2 + 1) = 0$$
$$(\lambda^2 - 2)(\lambda^2 + 1)^2 = 0 \Rightarrow \boxed{\lambda = \pm\sqrt{2}}$$

Q (Main) If x, y 2 Non Collinear Vectors
& Side length of ΔABC is a, b, c
& this Satisfies

$$(20a - 15b)\vec x + (15b - 12c)\vec y + (12c - 20a)\vec x \times \vec y = 0$$

then Nature of ΔABC



given $\lambda_1 \vec x + \lambda_2 \vec y + \lambda_3 \vec x \times \vec y = 0$
& $\vec x, \vec y$ & $\vec x \times \vec y$ Non L.I ← Copl.

$\Rightarrow$ Scalar Part $= 0$

$20a - 15b = 0$ & $15b - 12c = 0$ & $12c - 20a = 0$

$\Rightarrow 20a = 15b = 12c \quad \div 60$

$\frac{a}{3} = \frac{b}{4} = \frac{c}{5} = K \Rightarrow a = 3K, b = 4K, c = 5K$

Rt. angle Δ.

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Q If vectors

$a\hat{i}+\hat{j}+\hat{k},\ \hat{i}+b\hat{j}+\hat{k},\ \hat{i}+\hat{j}+c\hat{k}$

are L.D. vectors find value

of $\frac{1}{1-a}+\frac{1}{1-b}+\frac{1}{1-c}=?$

$$\Delta=\begin{vmatrix} a & 1 & 1 \\ 1 & b & 1 \\ 1 & 1 & c \end{vmatrix}=0$$

$C_1\to C_1-C_3$ & $C_2\to C_2-C_3$

$$\begin{vmatrix} a-1 & 0 & 1 \\ 0 & b-1 & 1 \\ 1-c & 1-c & c \end{vmatrix}=0$$

$(a-1)((b-1)c-(1-c))-1(1-c)(b-1)=0$

$(a-1)(b-1)c-(a-1)(1-c)-(1-c)(b-1)=0$

$(1-a)(1-b)c+(1-a)(1-c)+(1-c)(1-b)=0\ \div(1-a)(1-b)(1-c)$

$\frac{1-(1-c)}{1-c}+\frac{1}{1-b}+\frac{1}{1-a}=0$

$\frac{1}{1-c}-1+\frac{1}{1-b}+\frac{1}{1-a}=0$

$\frac{1}{1-a}+\frac{1}{1-b}+\frac{1}{1-c}=1$

Vector Eq^n of a Line

In 2D We need 2 things ① Slope ② Pt.

here in vectors Same 2 things ① Slope ② fix pt.

A) When a line is P.T. $\vec{a}$ & $\parallel^{rt}$ to $\vec{b}$

$\vec{AR}\parallel\vec{b}$

$\vec{AR}=\lambda\vec{b}$

$\vec{r}-\vec{a}=\lambda\vec{b}$

$\boxed{\vec{r}=\lambda\vec{b}+\vec{a}}$ is

Vector Eq^n of a Line P.T. $\vec{a}$ & $\parallel^{rt}$ to $\vec{b}$

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(B) Vector EOL when 2 fixed Pts. are given

$\overrightarrow{AR} \parallel \overrightarrow{AB}$

$\overrightarrow{AR} = \lambda \overrightarrow{AB}$

$\vec{r} - \vec{a} = \lambda(\vec{b} - \vec{a})$

$$\boxed{\vec{r} = \vec{a} + \lambda(\vec{b} - \vec{a})}$$

Q EOL P.T. A$(\hat{i} - \hat{j} - \hat{k})$ & $\parallel^{rt}$ to $\hat{i} + \hat{j} + \hat{k}$?

$\vec{r} = \vec{a} + \lambda \vec{b}$

$\vec{a}$: F.P

$\vec{b}$: Jiske $\parallel^{l}$ hai

Kota style $\vec{r} = \langle 1, -1, -1 \rangle + \lambda \langle 1, 1, 1 \rangle$

$\vec{r} = (\hat{i} - \hat{j} - \hat{k}) + \lambda(\hat{i} + \hat{j} + \hat{k})$

x, y, z H Eqn Booby

Q Find Cartesian form of

$\vec{r} = \langle 1, -1, -1 \rangle + \lambda \langle 1, 1, 1 \rangle$

$x\hat{i} + y\hat{j} + z\hat{k} = \hat{i} - \hat{j} - \hat{k} + \lambda(\hat{i} + \hat{j} + \hat{k})$

$x = 1 + \lambda \mid y = -1 + \lambda \mid z = -1 + \lambda$

$\lambda = \boxed{\frac{x-1}{1} = \frac{y+1}{1} = \frac{z+1}{1}}$ → Cart form of Line

Q $\vec{r} = (\hat{i} - 2\hat{j}) + \lambda(-3\hat{i} + \hat{j} - 2\hat{k})$

Cart. form

$= \langle 1, -2, 0 \rangle + \lambda \langle -3, 1, -2 \rangle$

$\frac{x-1}{-3} = \frac{y+2}{1} = \frac{z-0}{-2}$

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POI of 2 Lines.

Q $L_1: \vec{r} = \hat{i} - \hat{j} - \hat{k}) + \lambda(\hat{i} - 2\hat{j} - \hat{k})$ → दो Part Same नहीं है

$L_2: \vec{r} = (\hat{i} + \hat{j} + \hat{k}) + \mu(\hat{i} + 2\hat{j} + 3\hat{k})$

find PoI ?

⇒ No ∥ line or No Coincident.

A) Find Gen Pt. on Both the Lines

$L_1 = \langle 1+\lambda, -1-2\lambda, -1-\lambda \rangle$

$\parallel$

$L_2 = \langle 1+\mu, 1+2\mu, 1+3\mu \rangle$

$1+\lambda = 1+\mu$

$\lambda = \mu$

$-1-2\lambda = 1+2\mu$

$2\lambda + 2\mu = -2$

$\lambda + \mu = -1$

$2\lambda = -1$

$\lambda = -\frac{1}{2}, \mu = -\frac{1}{2}$

$-1-\lambda = 1+3\mu$

$-1+\frac{1}{2} = 1-\frac{3}{2}$

$-\frac{1}{2} = -\frac{1}{2}$ ✓

(B) PoI is

$\left(1-\frac{1}{2}, -1+\cancel{2}\times\frac{1}{\cancel{2}}, -1-\left(-\frac{1}{2}\right)\right)$

$\left(\frac{1}{2}, 0, -\frac{1}{2}\right)$ is PoI

Q $L_1: \vec{r} = 2\hat{k} + \lambda(3\hat{i} + 2\hat{j} + \hat{k})$

$L_2: \vec{r} = 3\hat{i} + 2\hat{j} + 3\hat{k} + \mu(6\hat{i} + 4\hat{j} + 2\hat{k})$

find PoI ?

$L_1: \vec{r} = \langle 0,0,2 \rangle + \lambda \langle 3,2,1 \rangle$ → ∥ या Coincident

$L_2: \vec{r} = \langle 3,2,3 \rangle + \mu \langle 6,4,2 \rangle$

$L_1 = \langle 3\lambda, 2\lambda, 2+\lambda \rangle$

$L_2 = \langle 3+6\mu, 2+4\mu, 3+2\mu \rangle$

$3+6\mu = 3\lambda$

$\lambda - 2\mu = 1$

$2+4\mu = 2\lambda$

$\lambda - 2\mu = 1$

$3+2\mu = 2+\lambda$

$\lambda - 2\mu = 1$

∞ Solution.

Coincident Lines.

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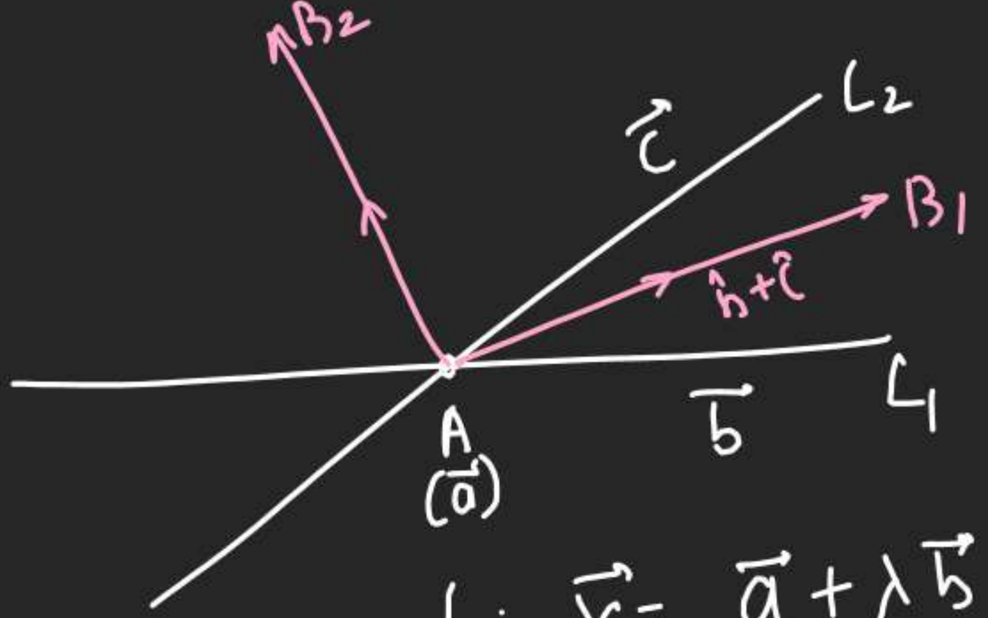
Vector Eqⁿ of Angle Bisectors



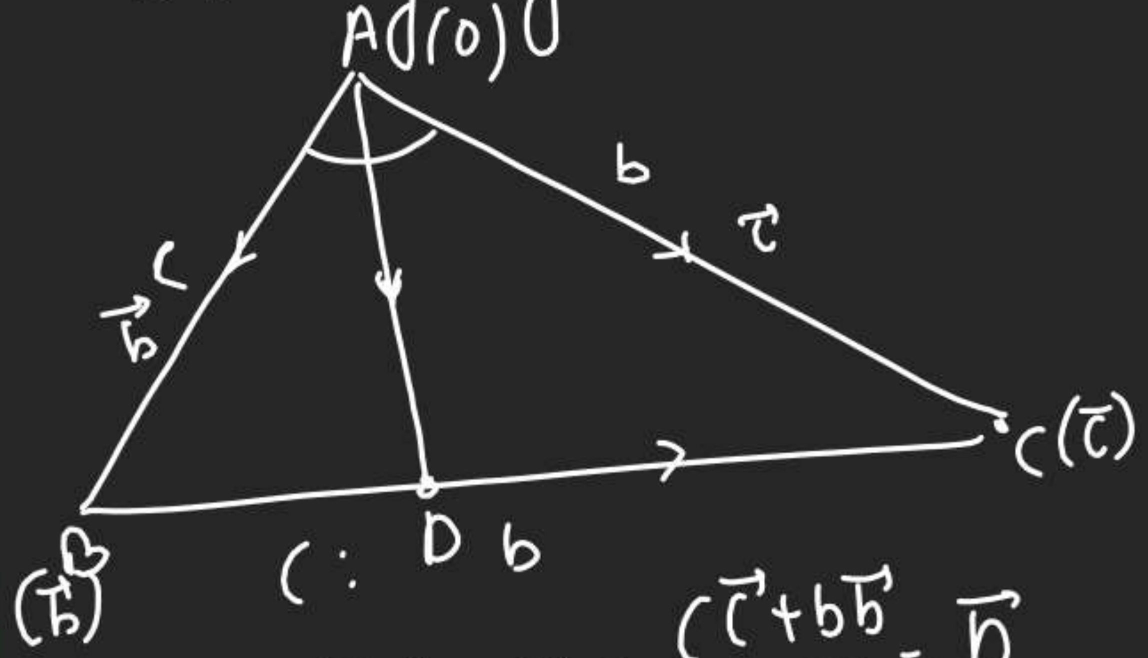
$L_1: \vec{r} = \vec{a} + \lambda\vec{b}$

$L_2: \vec{r} = \vec{a} + \mu\vec{c}$

Int. A.B. $B_1 \to \vec{r} = \vec{a} + t_1(\hat{b}+\hat{c})$

Ext. A.B. B_2 $\vec{r} = \vec{a} + t_2(\hat{b}-\hat{c})$

Q Use Vector Method to Prove that Internal Bisector of a $\triangle$ divide the opp side in Ratio of Sides containing angle.



To prove $\to \dfrac{c\vec{c}+b\vec{b}}{b+c} = \vec{D}$

Thought Process D is PoI of BC & AD

$L_{BC} \Rightarrow \vec{r} = \vec{b} + \lambda(\vec{c}-\vec{b})$

Gen. pt $= \langle 1-\lambda, \lambda \rangle$

$L_{AD} \Rightarrow \vec{r} = 0 + \mu(\hat{b}+\hat{c})$

$\vec{r} = \mu\left(\dfrac{\vec{b}}{|\vec{b}|} + \dfrac{\vec{c}}{|\vec{c}|}\right)$

$= \mu\left(\dfrac{\vec{b}}{c} + \dfrac{\vec{c}}{b}\right)$

$\left\langle \dfrac{\mu}{c}, \dfrac{\mu}{b} \right\rangle$

$1-\lambda = \dfrac{\mu}{c}$ | $\lambda = \dfrac{\mu}{b}$

$1 - \dfrac{\mu}{b} = \dfrac{\mu}{c} \Rightarrow \mu\left(\dfrac{1}{b}+\dfrac{1}{c}\right) = 1$

$\mu = \left(\dfrac{bc}{b+c}\right)$

$\vec{r} = b\cdot$

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Product of 2 Vectors.

1) Product of 2 Vectors → Scalar Product = Dot Product ; Vector Prod = Cross Prod.

(2) Dot Product me Product is Scalar.

Cross Product me Product is Vector.

$\vec{a} \cdot \vec{b}$ = Ans is Scalar

$\vec{a} \times \vec{b}$ = Ans. is a vector

(3) any kind of Prod Scalar or Vector need 2 vectors.

$\underbrace{(\vec{a} \cdot \vec{b})}_{\text{Scalar}} \times \underbrace{\vec{b}}_{\text{vector}}$ meaningles

Cross