Nishant Jindal

Q 15 $7^{103} \div 25$ find Rem.

Q. $\left(3\sqrt{\frac{a}{\sqrt{b}}}+\sqrt{\frac{b}{\sqrt[3]{a}}}\right)^{21}$

$T_{r+1} = {}^{21}C_r \left(\left(\frac{a}{b^{1/2}}\right)^{1/3}\right)^{21-r} \cdot \left(\left(\frac{b}{a^{1/3}}\right)^{1/2}\right)^{r}$

$= {}^{21}C_r \left(\frac{a}{b^{1/2}}\right)^{\frac{21-r}{3}} \cdot \left(\frac{b}{a^{1/3}}\right)^{\frac{r}{2}}$

$= {}^{21}C_r \cdot (a)^{\frac{21-r}{3}-\frac{r}{6}} (b)^{\frac{r}{2}-\frac{21-r}{6}}$

$\frac{21-r}{3}-\frac{r}{6} = \frac{r}{2}-\frac{21-r}{6}$

$\frac{42-2r-r}{\cancel{6}} = \frac{3r-21+r}{\cancel{6}}$

$63 = 7r$

$r = 9$

Q 5 $9^7+7^9 \div 2^n$ gr. value of $n \in N$.

$(8+1)^7+(8-1)^9$

72
56
128

$\left({}^7C_0 \cdot 8^7 + {}^7C_1 \cdot 8^6 + {}^7C_2 \cdot 8^5 + {}^7C_3 \cdot 8^4 + {}^7C_4\, 8^3 + {}^7C_5 \cdot 8^2 + \underbrace{{}^7C_6 \cdot 8}_{7\times 8} + \cancel{1}\right)$

$+\left({}^9C_0 \cdot 8^9 - {}^9C_1 8^8 + {}^9C_2 \cdot 8^7 - {}^9C_3 \cdot 8^6 + {}^9C_4 \cdot 8^5 - {}^9C_5 8^4 + {}^9C_6\, 8^3 - {}^9C_7 8^2 + \underbrace{{}^9C_8}_{9\times 8} - \cancel{1}\right)$

$8^2\{8^5 + 7\cdot 8^5 - - + 21 + (2)\}$

8^2 Min Com ले सकते है

$\therefore 2^6$

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I

9) hold (Notes)

10) ✓

11) $9^{3n} - 7x - 1 \div 49.$

12) ✓

13) ✓

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Rational/Irrational terms in $(a^{1/p}+b^{1/q})^n$

$\sqrt{\sqrt{2}}$ Irr. $\sqrt{\sqrt{\sqrt{3}}}$ = Irr.

Q Find No of rational terms in Exp of $\left(2^{\frac{1}{4}}+3^{\frac{1}{8}}\right)^{256}$?

$T_{r+1} = {}^{256}C_r \cdot \left(2^{\frac{1}{4}}\right)^{256-r} \left(3^{\frac{1}{8}}\right)^{r}$

$= {}^{256}C_r (2)^{\frac{256-r}{4}} \cdot (3)^{\frac{r}{8}}$

$(2)^{\frac{256-r}{4}}$ Kha-Khu Integer Deg a?? / $3^{r/8}$ Kha-Kha Integer Dega??

$\frac{256-r}{4} \rightarrow 0, 4, 8, 12, 16 \ldots\ldots 256 \rightarrow d_1 = 4$

$\frac{r}{8} \rightarrow 0, 8, 16, 24 \ldots\ldots 256 \rightarrow d_2 = 8$

Com. A.P ⇒ Com. Com. diff. $LCM(4,8) = 8$

∴ Com AP → $0, 8, 16, 24 \ldots\ldots 256$ ⇒ total term $n = \frac{l-a}{d}+1 = \frac{256-0}{8}+1 = 32+1 = 33$ term

M2 $\left(2^{\frac{1}{4}}+3^{\frac{1}{8}}\right)^{256}$

$2^{\frac{256}{4}}\left(1+\frac{3^{1/8}}{2^{1/4}}\right)^{256} = 2^{64}\left(1+\frac{3^{\frac{1}{8}}}{(2^2)^{1/8}}\right)^{256}$

$= 2^{64}\left(1+\left(\frac{3}{4}\right)^{\frac{1}{8}}\right)^{256}$

${}^{256}C_r\left(\frac{3}{4}\right)^{\frac{r}{8}}$

r = 0, 8, 16, 24, …, 256 → 33 term

∴ 1) No of Rational terms = 33 terms.

2) No. of Irrational terms = 257 − 33 = 224 terms.

M3 rational term = $\left[\frac{256}{LCM(4,8)}\right]+1 = \left[\frac{256}{8}\right]+1$

$= 32+1 = 33$

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Q Find No. of Rational terms in Exp of $(5^{1/2}+7^{1/8})^{1024}$.

$$T_{r+1} = {}^{1024}C_r \cdot (5^{1/2})^{1024-r} (7^{1/8})^r$$

$$= {}^{1024}C_r \cdot (5)^{\frac{1024-r}{2}} \cdot (7)^{\frac{r}{8}}$$

$\frac{1024-r}{2} \to 0, 2, 4, 6, 8, 10, 12, 14, 16, 18 ——— 1024 \quad d_1 = 2$

$\frac{r}{8} \to 0, 8, 16, 24 - - - \quad 1024 \quad d_2 = 8$

Com AP $\Rightarrow 0, 8, 16, \ldots 1024$

$$n = \frac{1024-0}{8} + 1 = 128+1 = 129$$

M_2

$$= \left[\frac{1024}{LCM(2,8)}\right] + 1$$

$$= \left[\frac{1024}{8}\right] + 1 = 129$$

Q. In Exp of $(2^{1/2}+5^{1/5})^{10}$ $\quad {}^{10}C_r \cdot (2)^{\frac{10-r}{2}} \cdot (5)^{\frac{r}{5}}$

(1) No. of Rat. terms

$\frac{10-r}{2} \to 0, 2, 4, 6, 8, 10$ $\quad \left[\frac{10}{LCM(2,5)}\right] + 1 = \left[\frac{10}{10}\right] + 1$

$\frac{r}{5} \to 0, 5, 10$ $\quad = 1+1$

$= 2$

(2) No of Irr. terms

$= 11 - 2 = 9$

(3) Sum of Rational term - (2 terms Rational हैं)

2 Rational term $\to r=0 \quad {}^{10}C_0 \, 2^5 \cdot 5^0$

$\to r=10 \quad {}^{10}C_{10} \, 2^0 \cdot 5^2$

$32 + 25$

$= 57$

(4) Value of Middle Terms.

$n = 10$ (Even) 1 M.

$$T_{\frac{10+2}{2}} = T_6 = {}^{10}C_5 \cdot (2)^{5/2} \cdot (5)^1$$

Q No. of term free from $\boxed{\text{Radical Sign.}}$ in $\left(1+3^{1/3}+7^{1/7}\right)^{10}$.

कोई fractional degree नहीं चाहिए

deg	1	$3^{1/3}$	$7^{1/7}$
	10	0	0
	7	3	0
	3	0	7
	0	3	7
	4	6	0
	1	9	0

No of term without Radical Sign $= 6$

$$(1)^{10}\cdot(3^{1/3})^{0}\cdot(7^{1/7})^{0} = 1\times 3^{0}\times 7^{0} = 1\times1\times1 = 1$$

$$(1)^{7}\cdot(3^{1/3})^{3}\cdot(7^{1/7})^{0} = 1\times 3^{1}\times 7^{0} = 1\times3\times1 = 3$$

$$(1)^{3}\cdot(3^{1/3})^{0}\cdot(7^{1/7})^{7} = 1\times 3^{0}\times 7^{1} = 1\times1\times7 = 7$$

$$(1)^{0}\cdot(3^{1/3})^{3}\cdot(7^{1/7})^{7} = 1\times 3^{1}\times 7^{1} = 63$$

$$(1)^{4}(3^{1/3})^{6}\cdot(7^{1/7})^{0} = 1\times 3^{2}\times 7^{0} = 9$$

$$(1)^{1}(3^{1/3})^{9}\times(7^{1/7})^{0} = 1\times 3^{3}\times 7^{0} = 27$$

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Digit at Unit Place / Last 2 digit / Last 3 digits → Anyhow make No. as a Multiple of 10.

$697 = 6\times10^2 + 9\times10^1 + 7\times1$

Q Which of the following is true for 17^{10}?

~~A)~~ last digit is 9.

~~B)~~ last 2 digits are 49.

~~C)~~ last 3 digits are 449

D). ______ are 749.

$17^{10} = (289)^5 = (290-1)^5$ → 6 terms.

$= [{}^5C_0(290)^5 - {}^5C_1(290)^4 + {}^5C_2(290)^3 - {}^5C_3(290)^2] + {}^5C_4(290) - {}^5C_5(290)^0$

$(29)^5\times100000 - 5\times(29)^4\times10000 + (29)^3\,10000 - (29)^2\times1000$

$= 1000[(29)^5\times100 - 29^4\cdot50 + 29^3\cdot10 - 29^2] + 1450 - 1$

$1000K + 1449$

47000	158000	1023000
1449	1449	1449
48$\boxed{449}$	159$\boxed{449}$	1024$\boxed{449}$

Last 3 digit = 449

Last 2 digit = 49

Last digit = 9

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Q Last 3 digits of 11^{50} ?

$$11^{50} = (1+10)^{50}$$

Main Sheet $= {}^{50}C_0\,10^0 + {}^{50}C_1 \cdot 10^1 + {}^{50}C_2 \cdot 10^2 + {}^{50}C_3 \cdot 10^3$

$$= 1 + 50\times 10 + \frac{\overset{25}{\cancel{50}}\cdot 49}{\cancel{2}} \cdot 100 + 1000K$$

$$= 501 + 122500 + 1000K$$

$$= 1000K + 123\boxed{001}$$

Last 3 digits $= 001$.

56
58
61
62

Extra:
Q9, 12

Q. Last 3 digit of 9^{50}

$$(10-1)^{50}$$

$${}^{50}C_2$$

$$-{}^{50}C_{47}10^3 + {}^{50}C_{48}10^2 - {}^{50}C_{49}10 + {}^{50}C_{50}10^0(-1)^{50}$$

$$1000K + \frac{\overset{25}{\cancel{50}}\cdot 49}{1\cdot\cancel{2}} \cdot 100 - 500 + 1$$

$$1000K + 122500 - 499$$

$$1000K + 122001$$