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Q There are n Urns each containing $(n+1)$ balls such that i^{th} Urn contain i White balls & $(n+1-i)$ Red balls

Let U_i be the event of selecting i^{th} Urn, $i = 1, 2, 3 \ldots n$ & W denotes the event of getting a white ball

A) if $P(U_i) \propto i$, $i = 1, \ldots n$ then $\lim\limits_{n\to\infty} P(W) = ?$

$$P(U_i) = Ki$$

$$\sum Ki = 1 \Rightarrow K\sum i = 1$$

$$K\{1+2+3+\cdots+n\} = 1 \Rightarrow K = \frac{2}{(n)(n+1)}$$

$$\lim_{n\to\infty} P(W) = \sum P(U_i) \times P\left(\frac{W}{U_i}\right)$$

$$= \sum \frac{2i}{(n)(n+1)} \times \frac{i}{(n+1)}$$

$$= \frac{2}{(n)(n+1)^2} \sum i^2$$

$$= \frac{2}{(\cancel{n})(n+1)} \times \frac{(\cancel{n})(\cancel{n+1})(2n+1)}{6}$$

$$= \lim_{n\to\infty} \frac{2(2n+1)}{6(n+1)} \quad \frac{\infty}{\infty} = \frac{4}{6} = \frac{2}{3}$$

$$P\left(\frac{W}{U_5}\right) = \frac{5}{(n+1)}$$

$$P\left(\frac{W}{U_6}\right) = \frac{6}{n+1}$$

$$P\left(\frac{W}{U_9}\right) = \frac{9}{n+1}$$

hold

(B) $P(U_i) = C$ then $P\left(\frac{U_n}{W}\right) = ?$ → White ball आ गया है।

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Total Prob. Theorem.

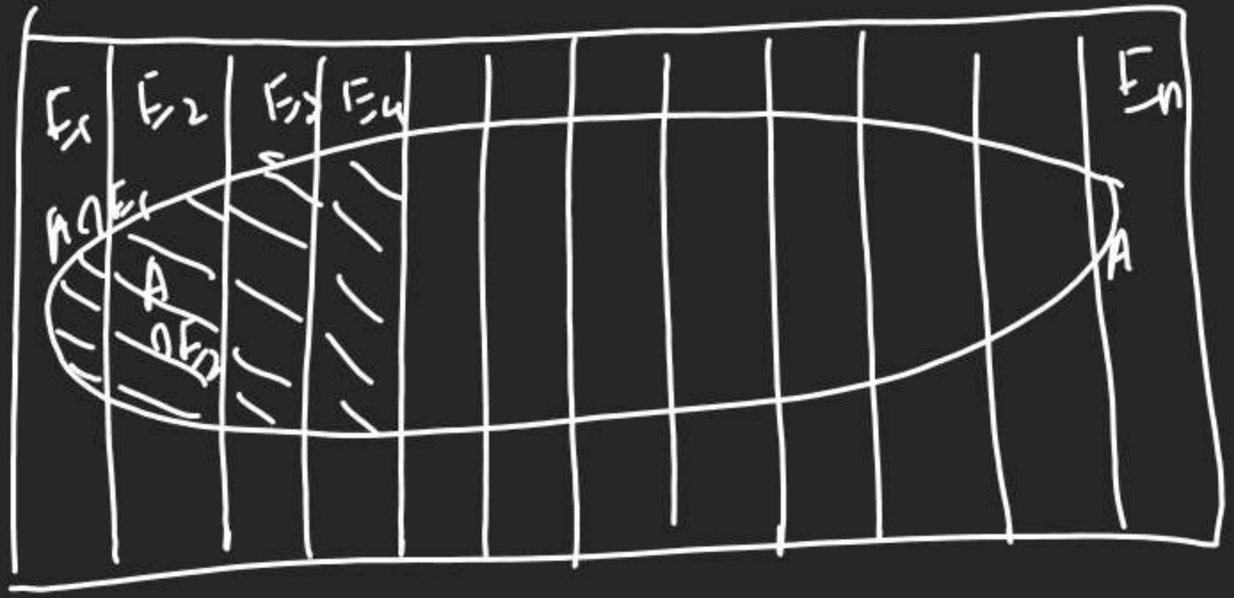
Let there are $E_1 E_2 E_3 \dots E_n$

n M.E. & exhaustive events

& Another Event A is happenning

among all $E_1, E_2, E_3 \dots E_n$

then Prob. of happonning of A = ?



$$P(E_1)+P(E_2)+P(E_3) - - - + P(E_n) = 1$$

$$P(A) = P(A \cap E_1) + P(E_2 \cap A) + P(E_3 \cap A) + - - - + P(E_n \cap A)$$

$$P(A) = P(E_1) \cdot P\left(\frac{A}{E_1}\right) + P(E_2) \cdot P\left(\frac{A}{E_2}\right) + P(E_3) P\left(\frac{A}{E_3}\right) + - - + P(E_n) P\left(\frac{A}{E_n}\right)$$

$$\boxed{P(A) = \sum_{i=1}^{n} P(E_i) \cdot P\left(\frac{A}{E_i}\right)}$$

Q Prob. of 53 Sunday in a selected year is?

$$P(\text{leap year}) = \frac{1}{4}, \quad P(53 \text{ Sunday}) = \frac{2}{7}$$

$$P(53 \text{ Sunday}) = P(LY) \cdot P\left(\frac{53S}{LY}\right) + P(NY) \cdot P\left(\frac{53S}{NY}\right)$$

$$= \frac{1}{4} \times \frac{2}{7} + \frac{3}{4} \times \frac{1}{7}$$

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Q.

B_1	B_2	B_3
3W, 4R	4W, 5R	2R, 7B

Prob. of getting a Red Ball from a Selected Bag.

$$P(\text{Red Ball}) = P(B_1)\cdot P\left(\frac{R}{B_1}\right) + P(B_2)\cdot P\left(\frac{R}{B_2}\right) + P(B_3)\cdot P\left(\frac{R}{B_3}\right)$$

$$= \frac{1}{3}\times\frac{4}{7} + \frac{1}{3}\times\frac{5}{9} + \frac{1}{3}\times\frac{2}{9}$$

$$\sum_{i=1}^{3} P(B_i)\cdot P\left(\frac{R}{B_i}\right)$$

Bay's Theorem.

Q If

B_1	B_2	B_3
3W, 4R	4W, 5R	2R, 7B

& a Red Ball has been Selected What is the Prob. that it was selected from B_2

Now Qs in $P\left(\frac{B_2}{R}\right) = \frac{P(R\cap B_2)}{P(R)}$

$$= \frac{P(B_2)\cdot P\left(\frac{R}{B_2}\right)}{P(B_1)\cdot P\left(\frac{R}{B_1}\right) + P(B_2)\cdot P\left(\frac{R}{B_2}\right) + P(B_3)\cdot P\left(\frac{R}{B_3}\right)}$$

$$= \frac{\frac{1}{3}\times\frac{5}{9}}{\frac{1}{3}\times\frac{4}{7} + \left(\frac{1}{3}\times\frac{5}{9}\right) + \frac{1}{3}\times\frac{2}{9}}$$

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Baye's Thm.

If an Event A can occur only with one of n ME & Exhaustive Events $B_1\, B_2\, B_3 \ldots B_n$ & Prob. $P\left(\frac{A}{B_1}\right), P\left(\frac{A}{B_2}\right) \ldots P\left(\frac{A}{B_n}\right)$ are known. then

$$\boxed{P\left(\frac{B_i}{A}\right) = \frac{P(B_i)\cdot P\left(\frac{A}{B_i}\right)}{\sum P(B_i)\, P\left(\frac{A}{B_i}\right)}}$$

Q ABox has 4 Dice in it. Three of them are fair dice but 4th one has No. 5 on all of its faces. A die is chosen at Random from the box and is rolled 3 times & shows up the face 5 on all 3 occassions.

The chance that die chosen was a Rigged die, is?

$$P\left(\frac{\text{Rigged Dice}}{\text{5 Aa Chuka hai}}\right) = \frac{\text{आने से Rigged के chance}}{\text{5 के आने के Total Chance}}$$

$$= \frac{\frac{1}{4}\times 1\times 1\times 1}{\frac{3}{4}\times\frac{1}{6}\times\frac{1}{6}\times\frac{1}{6} + \frac{1}{4}\times 1\times 1\times 1} = \frac{216}{219}$$

Q. A Card from Set of 52 Cards is lost. If one Card is Selected from Rest of the Cards then find Prob. of this Card be a King?

P(King) = P(lost = King) या P(lost = Non King)

$$= \frac{4}{52}\times\frac{3}{51} + \frac{48}{52}\times\frac{4}{51}$$

Q

B_1	B_2
7W	8W
8R	7R

If one ball is drawn from B_1 & mixed it in B_2 & another ball is drawn from B_2 find Prob of this ball being Red.

P(Red ball) = B_1 से Red निकली या B_1 से Red नहीं निकली

$$= \frac{8}{15} \times \frac{8}{16} + \frac{7}{15} \times \frac{7}{16}$$

Q If drawn Card is a King find Prob that lost Card was also King.

$$\Rightarrow P\left(\frac{\text{lost = King}}{\text{drawn = King}}\right) = \frac{\frac{4}{52} \times \frac{3}{51}}{\frac{4}{52} \times \frac{3}{51} + \frac{48}{52} \times \frac{4}{51}}$$

Q. A Card from Set of 52 Cards is lost. If one Card is Selected from Rest of the Cards then find Prob. of this Card be a King?

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$$= \frac{4}{52} \times \frac{3}{51} + \frac{48}{52} \times \frac{4}{51}$$

B_1

18 Fair
2 Biased → Both Head.

20 Coin

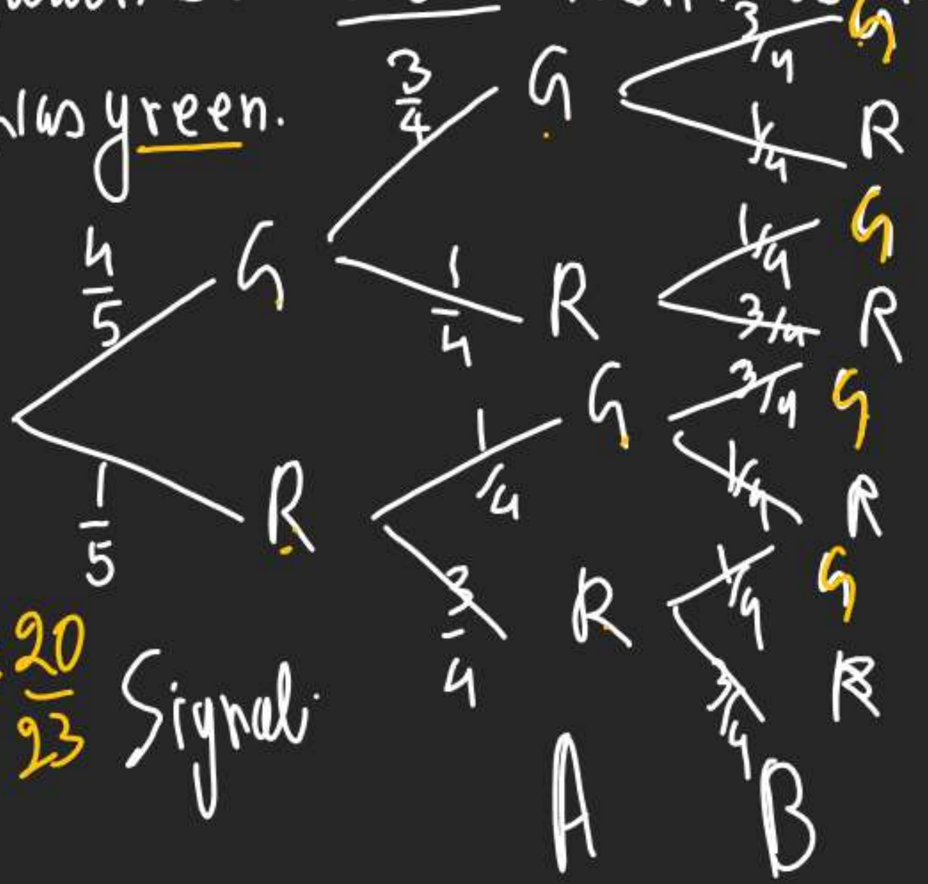
One Coin is selected & tossed then H comes then find Prob that it was a Fair coin?

$$P\left(\frac{\text{Fair coin th}}{\text{Head Aya}}\right) = \frac{\text{Fair aur head aya}}{P(\text{Head Aya})}$$

$$= \frac{\frac{18}{20} \times \frac{1}{2}}{\left\{\underbrace{\frac{18}{20} \times \frac{1}{2}}_{\text{Fair \& Head}} + \underbrace{\frac{2}{20} \times 1}_{\text{Bias \& Head}}\right\}}$$

TREE diagram.

Q A signal which can be green or Red with Prob. $\frac{4}{5}$ & $\frac{1}{5}$, is recieved by Station A then transmitted to Station B. The Prob. that each station recieving signal correctly is $\frac{3}{4}$, if the signal recieved at Station B is Green then Prob. that original Signal was green.



$$= \frac{\frac{4}{5} \times \left\{\frac{3}{4} \times \frac{3}{4} + \frac{1}{4} \times \frac{1}{4}\right\}}{\left\{\frac{4}{5} \times \frac{3}{4} \times \frac{3}{4} + \frac{4}{5} \times \frac{1}{4} \times \frac{1}{4}\right\} + \frac{1}{5} \times \frac{1}{4} \times \frac{3}{4} + \frac{1}{5} \times \frac{3}{4} \times \frac{1}{4}} = \frac{20}{23}$$

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Q With Respect to a Particular Qs of MCQ a Student knows the Answer therefore can eliminate 3 out of 4 Choices with Prob $= \frac{2}{3}$, Can eliminate 2 out of 4 choices With Prob $= 1/6$, Can eliminate 1 out of 4 choices with Prob $= 1/9$, Can eliminate None with Prob $= \frac{1}{18}$. If Answer given by Student is Correct, then the Prob. that he knew Answer $= \frac{a}{b}$ find $a+b = ?$

$$\frac{1}{1} \times \frac{2}{3} + \frac{1}{2} \times \frac{1}{6} + \frac{1}{3} \times \frac{1}{9} + \frac{1}{4} \times \frac{1}{18}$$

Binomial Prob.