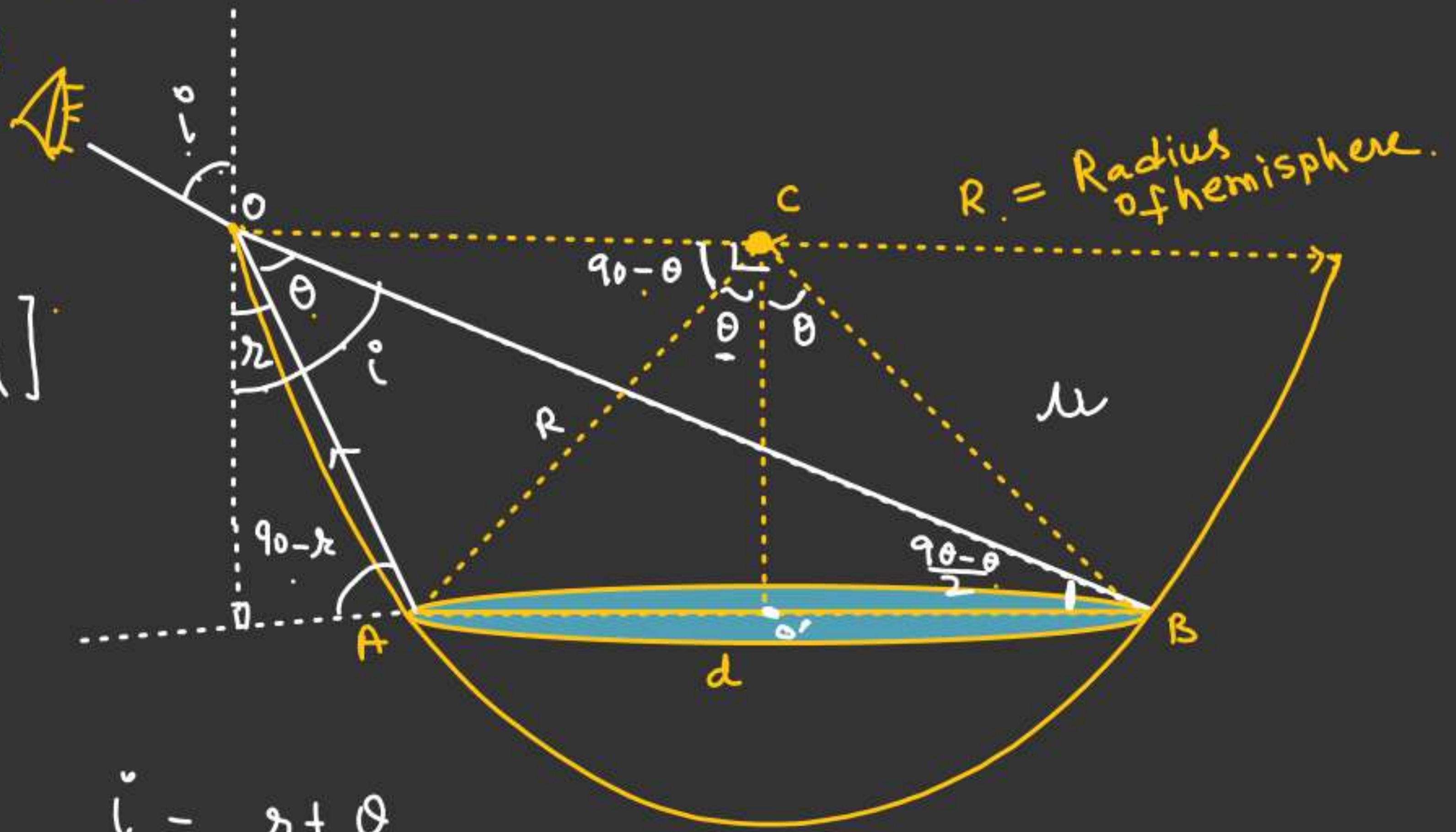
Nishant Jindal

Viewer only see the point B.

a liquid having refractive Index μ is poured in the hemispherical Vessel so that Viewer Can see the Complete disc. Find d=??



$\angle AOB = \frac{1}{2}\angle ACB = \theta$ [For AB Chord]

For OA Chord

$\angle OCA = 90-\theta$

$\angle OBA = \left(\frac{90-\theta}{2}\right)$

In $\underline{\Delta OAB}$

$90-r = \theta + \frac{90-\theta}{2} = \frac{90+\theta}{2}$

$r = 90-\left(\frac{90+\theta}{2}\right)$

$r = \left(\frac{90-\theta}{2}\right)$ ✓

$i = r+\theta$

$i = \frac{90-\theta}{2}+\theta$

$i = \left(\frac{90+\theta}{2}\right)$ ✓

In $\Delta ACO'$

$\sin\theta = \frac{d}{2R}$

Nishant Jindal

By Snell's law.

$$1 \cdot \sin i = \mu \cdot \sin r$$

$$\sin\left(\frac{90+\theta}{2}\right) = \mu \cdot \sin\left(\frac{90-\theta}{2}\right)$$

$$\frac{\sin\left(45^\circ+\frac{\theta}{2}\right)}{\sin\left(45^\circ-\frac{\theta}{2}\right)} = \mu$$

$$\frac{\sin 45^\circ \cdot \cos\theta/2 + \cos 45 \cdot \sin\theta/2}{\sin 45 \cos\frac{\theta}{2} - \cos 45 \cdot \sin\frac{\theta}{2}} = \mu.$$

$$\frac{\cos\theta/2 + \sin\theta/2}{\cos\frac{\theta}{2} - \sin\frac{\theta}{2}} = \mu$$

Squaring on both side.

$$\frac{\cos^2\frac{\theta}{2} + \sin^2\frac{\theta}{2} + 2\sin\frac{\theta}{2}\cdot\cos\frac{\theta}{2}}{\cos^2\frac{\theta}{2} + \sin^2\frac{\theta}{2} - 2\sin\frac{\theta}{2}\cdot\cos\frac{\theta}{2}} = \mu^2$$

$$\frac{1+\sin\theta}{1-\sin\theta} = \mu^2$$

$$1+\sin\theta = \mu^2 - \mu^2\sin\theta$$

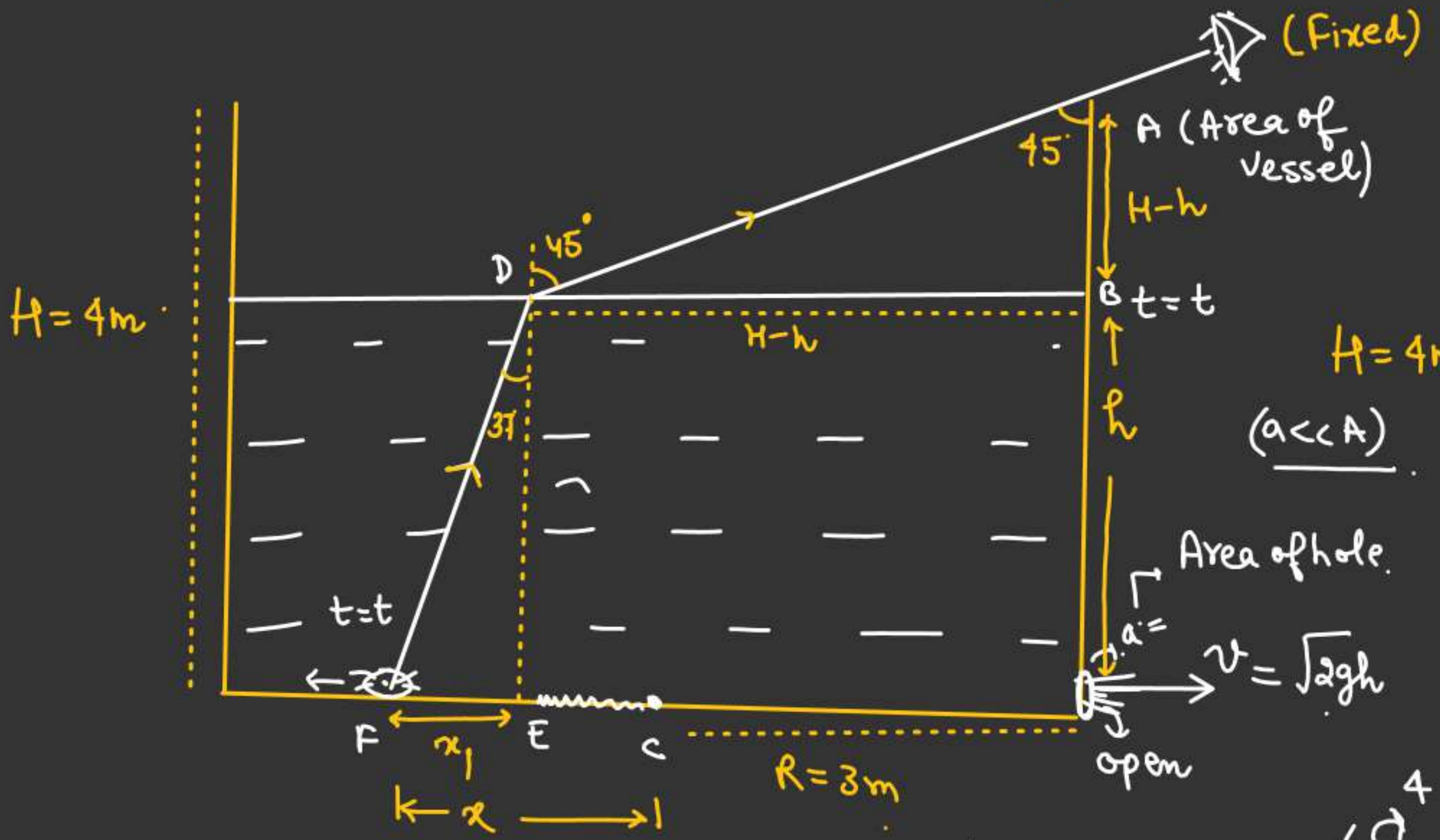
$$(1+\mu^2)\sin\theta = \mu^2 - 1$$

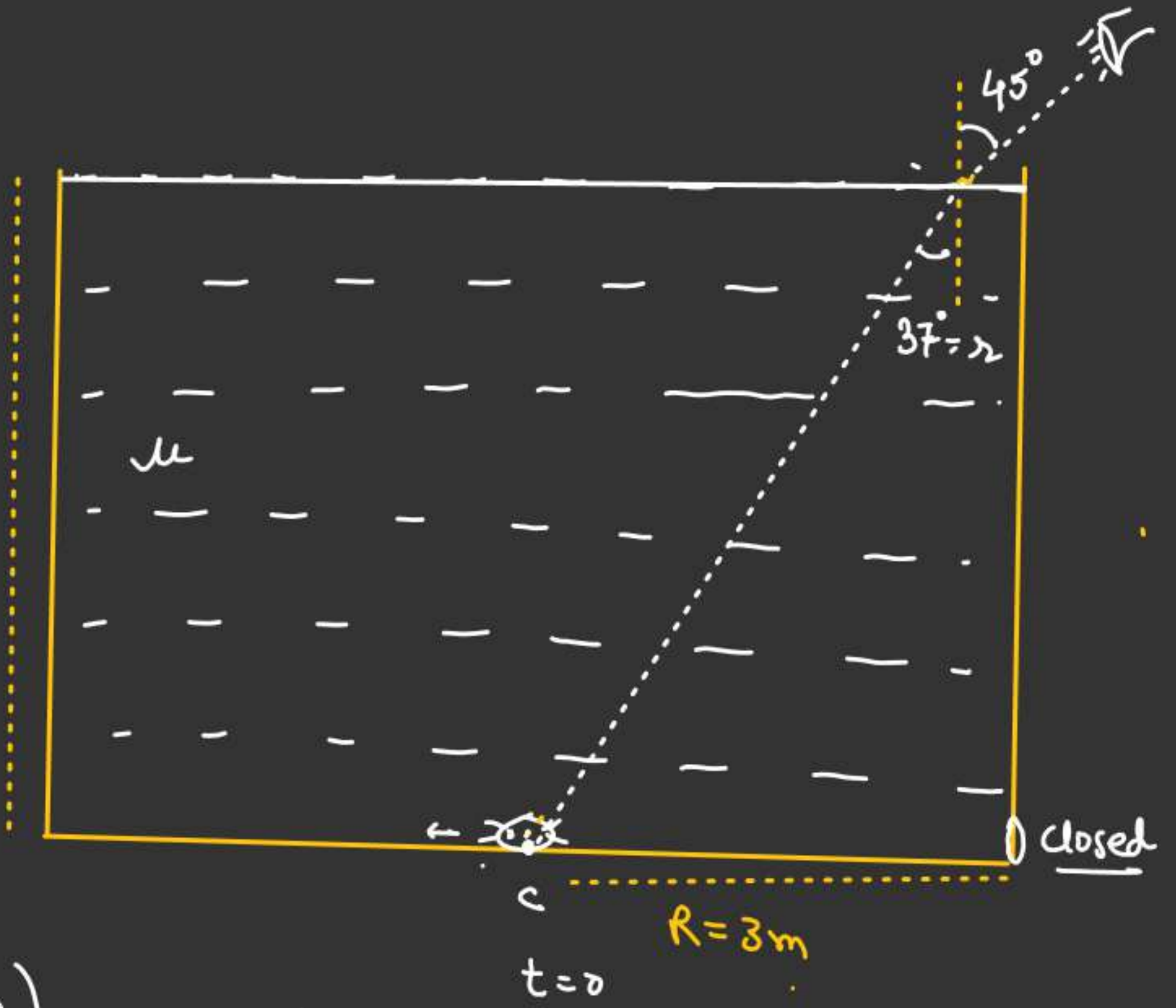
$$\sin\theta = \left(\frac{\mu^2-1}{1+\mu^2}\right)$$

$$\Downarrow$$

$$\frac{d}{2R} = \frac{(\mu^2-1)}{(1+\mu^2)} \Rightarrow d = \frac{2R(\mu^2-1)}{(1+\mu^2)} \text{ Ans}$$

Insect at the Center of the vessel. at $t=0$, valve opened. and insect starts moving. Find the velocity of insect as a function of time so that observer always see the insect.





In $\triangle$ DEF

$\tan 37^\circ = \dfrac{x_1}{h}$

$x_1 = h\tan 37^\circ$

$x_1 = \left(\dfrac{3h}{4}\right)$

$EC = (H-h-3)$

$x = EC + EF$

$x = H-h-3+x_1$

$x = H-h-3+\dfrac{3h}{4}$

$x = \left(H - 3 - \dfrac{h}{4}\right)$ with $H = 4$

$x = (4-3) - \dfrac{h}{4}$

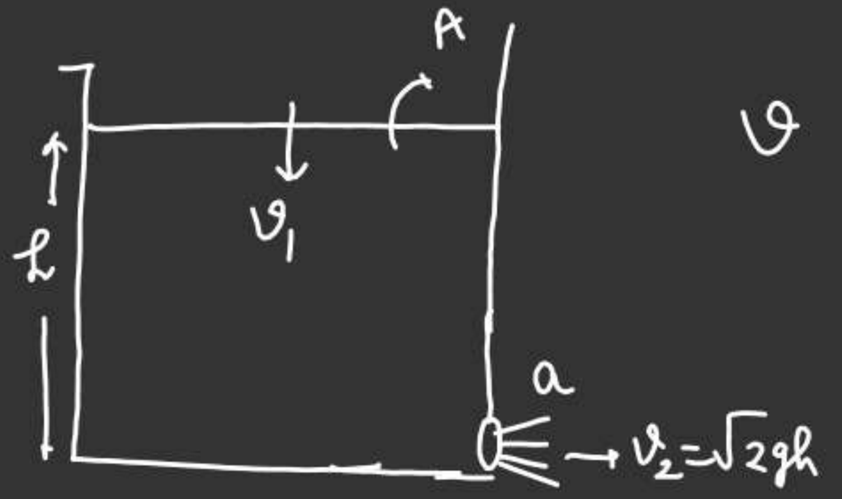
$\left(x = 1 - \dfrac{h}{4}\right)$ ✓

$\tan r = \dfrac{3}{4} \Rightarrow r = 37^\circ$

Nishant Jindal

$$x = 1 - \frac{h}{4}$$

$$\boxed{\left(\frac{dx}{dt}\right) = -\frac{1}{4}\left(\frac{dh}{dt}\right)}$$



$$A_1 v_1 = A_2 v_2$$

$$A\left(-\frac{dh}{dt}\right) = a\sqrt{2gh}$$

$$\int_H^h \frac{dh}{\sqrt{h}} = -\frac{a\sqrt{2g}}{A}\int_0^t dt$$

$$2\left[\sqrt{h}\right]_H^h = -\frac{a}{A}\sqrt{2g}\,t$$

$$2\left(\sqrt{h} - \sqrt{H}\right) = -\frac{a}{A}\sqrt{2g}\,t$$

$$\sqrt{H} - \sqrt{h} = \frac{a}{A}\sqrt{\frac{g}{2}}\,t$$

$$\sqrt{H} - \frac{a}{A}\sqrt{\frac{g}{2}}\,t = \sqrt{h}$$

$$\left(\sqrt{H} - \frac{a}{A}\sqrt{\frac{g}{2}}\,t\right)^2 = h$$

$$\frac{dh}{dt} = 2\left(\sqrt{H} - \frac{a}{A}\sqrt{\frac{g}{2}}\,t\right)\left(-\frac{a}{A}\sqrt{\frac{g}{2}}\right)$$

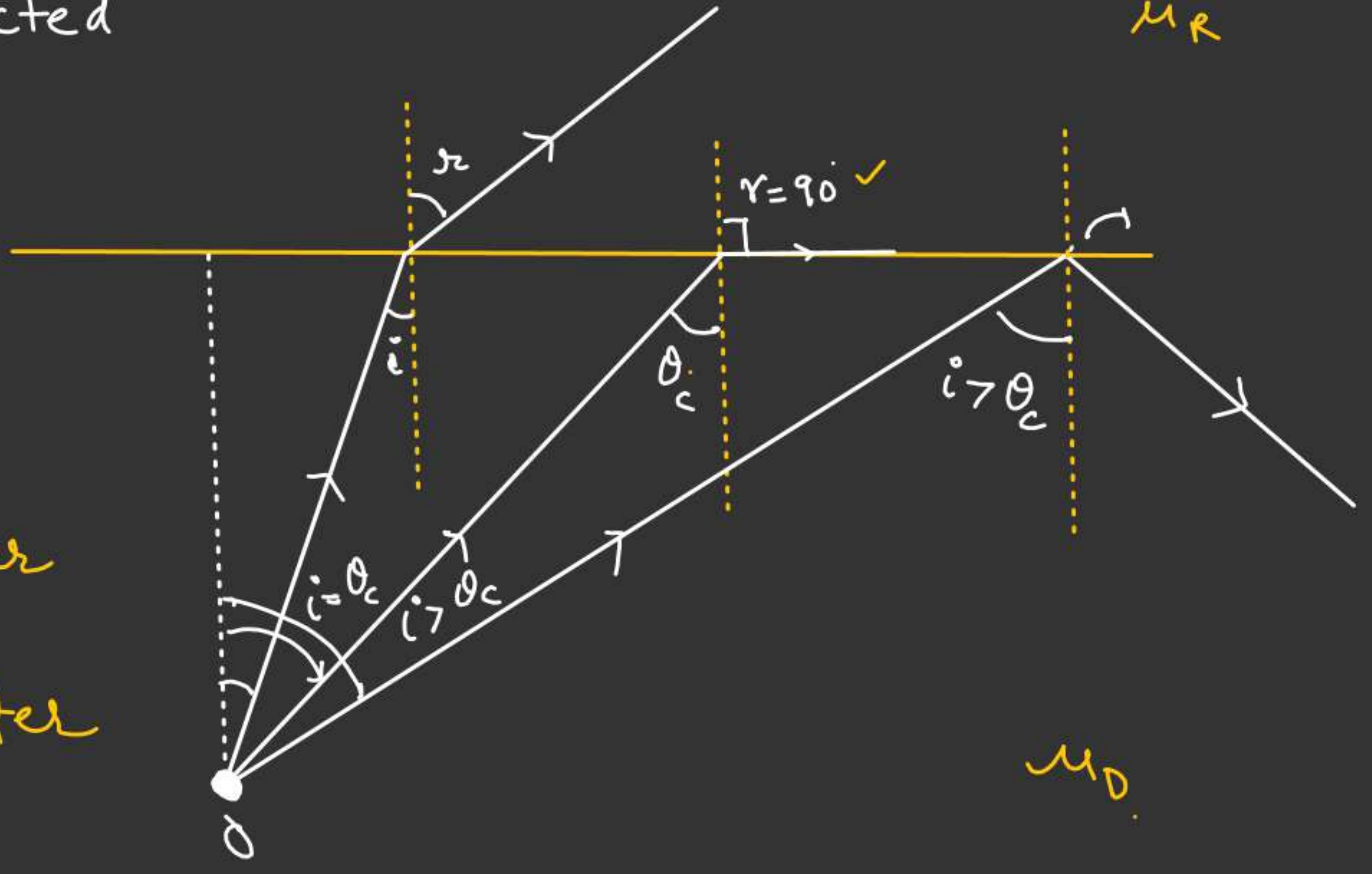
$$\left(-\frac{dh}{dt}\right) = \frac{a}{A}\sqrt{2g}\left(\sqrt{H} - \frac{a}{A}\sqrt{\frac{g}{2}}\,t\right)$$

$$v = \frac{dx}{dt} = \left[\frac{a}{4A}\sqrt{2g}\left[\sqrt{H} - \frac{a}{A}\sqrt{\frac{g}{2}}\,t\right]\right]$$

Nishant Jindal

TIR (Total Internal Reflection).

"Phenomena by Virtue of which when light ray travel from denser to rarer medium get reflected back to denser medium when angle of incidence is greater than Critical angle"



Condition of TIR

- light always travel from denser to rarer.
- Angle of incidence must be grater the Critical angle.

Nishant Jindal

Critical Angle

Angle of incidence Corresponding to which angle of refraction is 90°

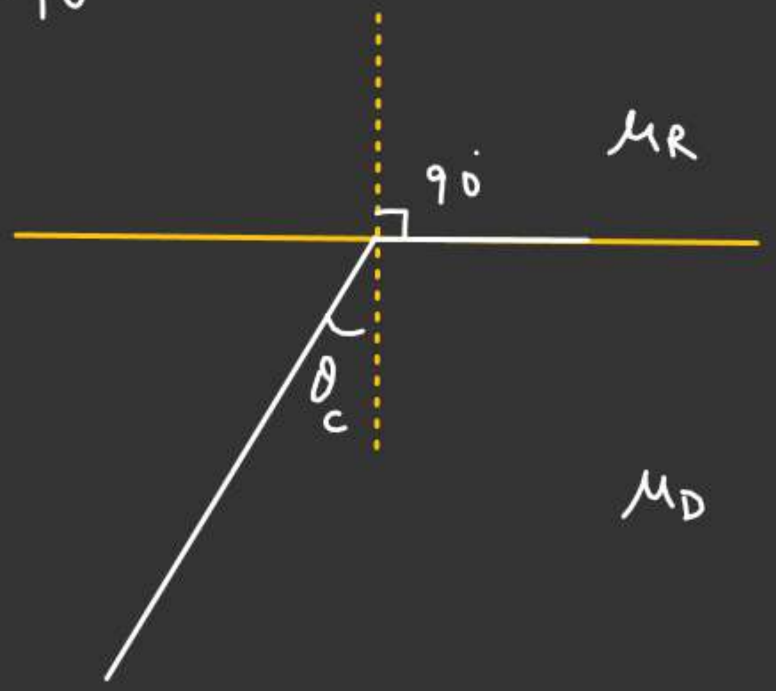
By Snell's Law.

$$\mu_D \sin\theta_c = \mu_R \sin 90°$$

$$\sin\theta_c = \frac{\mu_R}{\mu_D}$$

$$\sin\theta_c = \left(\frac{1}{{}_R\mu_D}\right)$$

$$\theta_c = \sin^{-1}\left(\frac{1}{{}_R\mu_D}\right)$$



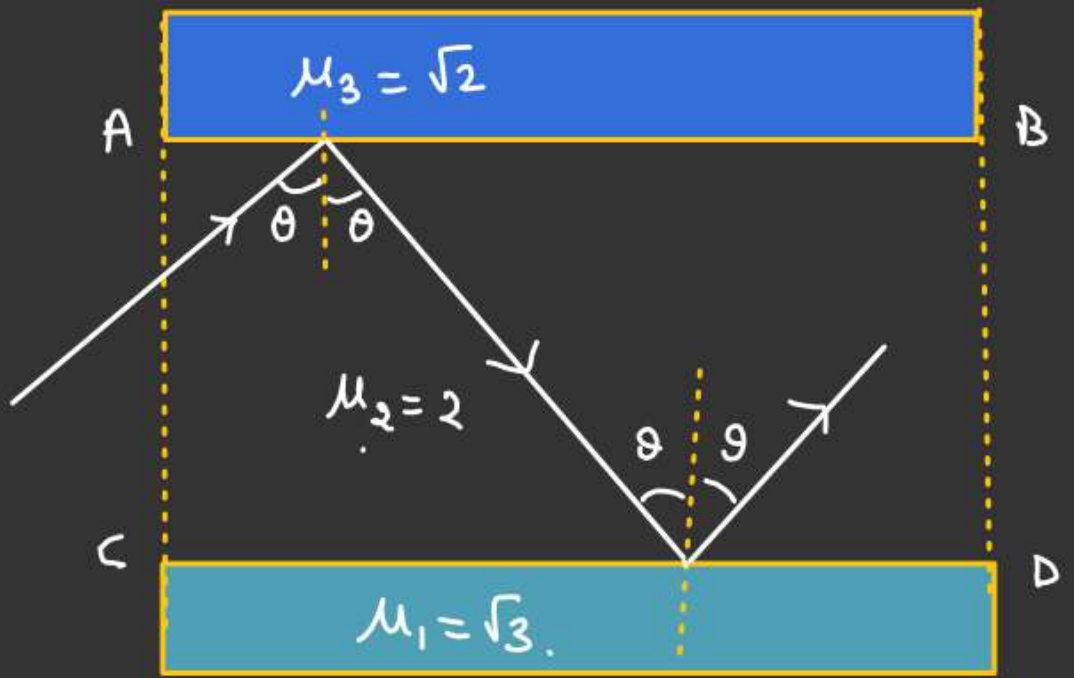
If $\mu_R = 1$. $\mu_D = \mu$.

$$\sin\theta_c = \frac{1}{\mu}$$

$$\boxed{\theta_c = \sin^{-1}\left(\frac{1}{\mu}\right)}$$

Nishant Jindal

\# Fin min value of θ so that TIR takes place at AB as well as CD.



θ_c for interface AB.

$$\mu_2 \sin(\theta_c)_{AB} = \mu_3 \sin 90^\circ$$

$$\sin(\theta_c)_{AB} = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$$

$$(\theta_c)_{AB} = 45^\circ \checkmark$$

$$\sin(\theta_c)_{CD} = \frac{\sqrt{3}}{2}$$

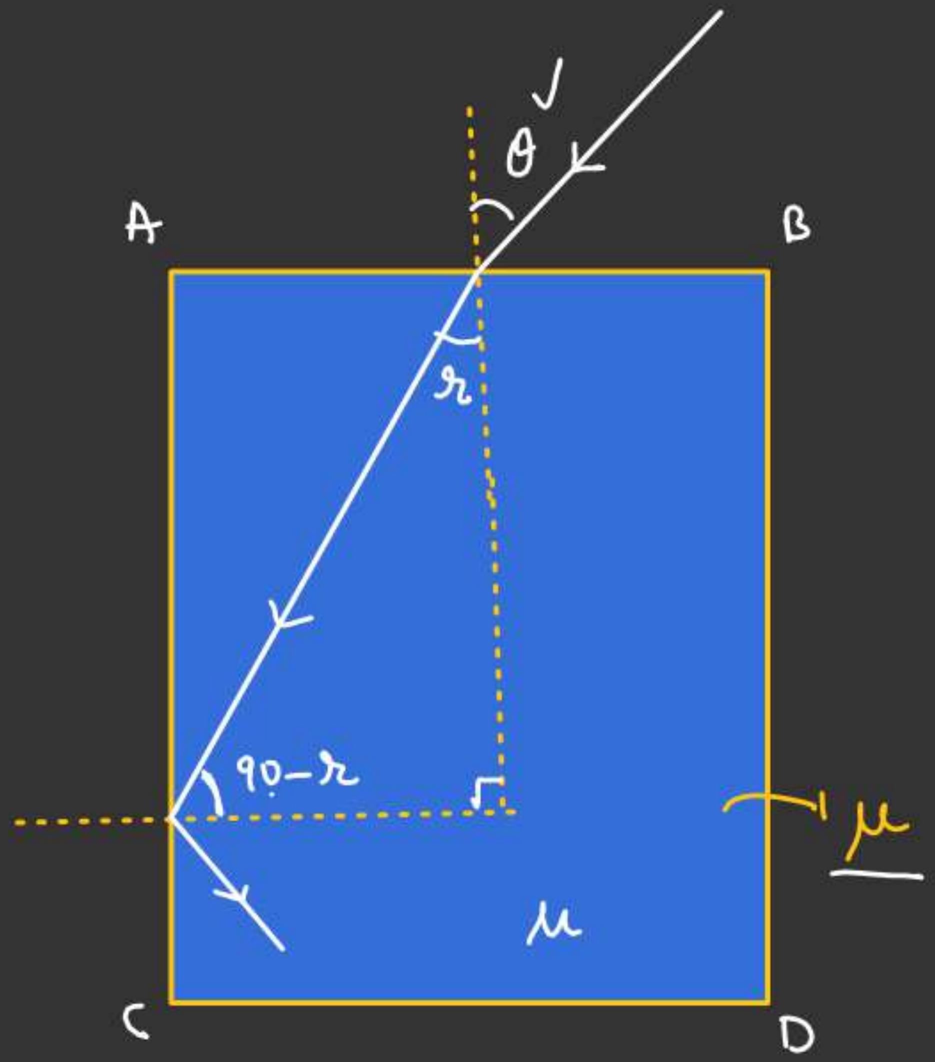
$$(\theta_c)_{CD} = 60^\circ \checkmark$$

$$\theta \geq 60^\circ$$

$$\underline{\theta_{min} = 60^\circ} \checkmark$$

Nishant Jindal

Find max θ so that light doesn't come out from the vertical face AC.



For TIR to take's place

$$(90-r) \geq \theta_c$$

$$\sin(90^\circ - r) \geq \sin\theta_c$$

$$\cos r \geq \sin\theta_c$$

$$\cos r \geq \frac{1}{\mu} \quad -(1)$$

Snell's law at AB interface

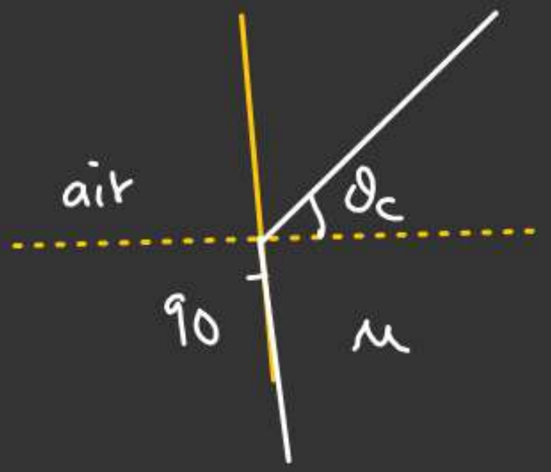
$$1 \cdot \sin\theta = \mu \cdot \sin r$$

$$\sin r = \left(\frac{\sin\theta}{\mu}\right) \quad -(2)$$

From (1)

$$\sqrt{1-\sin^2 r} \geq \frac{1}{\mu}$$

$$\sqrt{1-\frac{\sin^2\theta}{\mu^2}} \geq \frac{1}{\mu}$$



$$\sin\theta_c = \frac{1}{\mu}$$

Nishant Jindal

$$\sqrt{1-\frac{\sin^2\theta}{\mu^2}} \geq \frac{1}{\mu}$$

if ($\mu = 1.25$) given

$$1-\frac{\sin^2\theta}{\mu^2} \geq \frac{1}{\mu^2}$$

$$1-\frac{1}{\mu^2} \geq \frac{\sin^2\theta}{\mu^2}$$

$$\mu^2-1 \geq \sin^2\theta$$

$$\sin\theta \leq \sqrt{\mu^2-1}$$

$$\boxed{\theta \leq \sin^{-1}\left(\sqrt{\mu^2-1}\right)}$$

$$\theta_{max} = \sin^{-1}\left(\sqrt{\mu^2-1}\right)$$