ELECTROSTATICS

CHARGE AND ITS PROPERTIES

- Study of characteristics of electric charges at rest is known as electrostatics.
- Electric charge is the property associated with a body or a particle due to which it is able to produce as well as experience the electric and magnetic effects.
- Charge is a fundamental property of matter and is never found free.
- The excess or deficiency of electrons in a body gives the concept of charge.
- There are two types of charges namely positive and negative charges.
- The deficiency of electrons in a body is known as positively charged body.
- The excess of electrons in a body is known as negatively charged body.
- Charge is relativistically invariant, i.e. it does not change with motion of charged particle and no change in it is possible, whatsover may be the circumstances. i.e.
- Charge is a derived physical quantity with dimensions [AT].

  Quantization of Charge : The electric charge is discrete. It has been verified by Millikan's oil drop experiment.
- Charge is quantized. The charge on any body is an integral multiple of the minimum charge or electron charge, i.e if q is the charge then $q = \pm ne$ when n is an integer, and e is the charge of electron $= 1.6 \times 10^{-19}\,C$.
- The minimum charge possible is $1.6 \times 10^{-19}\,C$.
- If a body possesses n_1 protons and n_2 electrons, then net charge on it will be $(n_1 - n_2)e$, i.e. $n_1(e) + n_2(-e) = (n_1 - n_2)e$

Law of conservation of charge

- The total net charge of an isolated physical system always remains constant,

  i.e. $q = q_+ + q_- =$ constant.
- In every chemical or nuclear reaction, the total charge before and after the reaction remains constant.
- This law is applicable to all types of processes like nuclear, atomic, molecular and the like.
- Charge is conserved. It can neither be created nor destroyed. It can only be transferred from one object to the other.
- Like charges repel each other and unlike charges attract each other.
- Charge always resides on the outer surface of a charged body. It accumulates more at sharp points.
- The total charge on a body is algebric sum of the charges located at different points on the body.

- **Electrification :** A body can be charged by friction, conduction and induction.
- **By Friction :** When two bodies are rubbed together, equal and opposite charges are produced on both the bodies.
- **By Conduction :** An uncharged body acquiring charge when kept in contact with a charged body is called conduction. Conduction preceeds repulsion.
- **By Induction** : If a charged body is brought near a neutral body, the charged body will attract opposite charge and repel like charge present in the neutral body. Opposite charge is induced at the near end and like charge at the farther end. Inducing body neither gains nor loses charge. Induction always preceeds attraction.
- Repulsion is the sure test of electrification.
- Induced charge $q^1 = -q\left[1-\frac{1}{K}\right]$ where K is Dielectric constant

  Coulomb's Law : The force of attraction or repulsion between two stationary electic charges is directly proportional to the product of magnitude of the two charges and is inversely proportional to the square of the distance between them and this force acts along the line joining those two charges
- $F = \frac{1}{4\pi\epsilon_0 \epsilon_r}\frac{q_1 q_2}{r^2}$

  ϵ_0 - permittivity of free space or vacuum or air.

  ϵ_r - Relative permittivity or dielectric constant of the medium in which the charges are situated.
- $\epsilon_0 = 8.857\times10^{-12}\frac{C^2}{Nm^2}$ or $\frac{\text{farad}}{\text{metre}}$, and $\frac{1}{4\pi\epsilon_0} = 9\times10^9\,Nm^2/C^2$

  Permittivity of Medium: Permittivity is the measure of the degree of the medium which resists the flow of charges

  In SI. for medium other than free space, the constant $K = \frac{1}{4\pi\epsilon}$ so that we can write the equation for the force between the charges as $F = \frac{1}{4\pi\epsilon}\frac{q_1 q_2}{r^2}$

  $\therefore \frac{F_0}{F} = \frac{\epsilon}{\epsilon_0} = \epsilon_r$

  ϵ_r is known as the relative permittivity of the medium. It is a constant for a given medium and the force between the charges, separated by a medium, decreases compared with the force between the same charges in free space separated by the same distance.

Relative permittivity $\in_r$ is also known as dielectric constant K of the medium or specific inductive capacity.

Relative permittivity of a medium is defined as the ratio of permittivity of the medium to permittivity of free space (or) air (or) Relative permittivity of a medium is defined as the electrostatic force (F_0) between two charges in air to the force (F) between the same two charges kept in the medium at same distance. Dielectric constant (or) Relative permittivity

$$K = \frac{\text{Permittivity the medium}}{\text{Permittivity of free space}}$$

It has no units and no dimensions

Hence, the mathematical form of inverse square law is given as $F = \frac{1}{4\pi\in}\frac{q_1q_2}{r^2} = \frac{1}{K}\frac{1}{4\pi\in_0}\frac{q_1q_2}{r^2}$

For force in vacuum or air K = 1 and for a good conductor like metals, $K = \infty$

Conclusion :

1) The introduction of a glass slab between two charges will decrease the magnitude of force between them.

2) The introduction of a metallic slab between two charges will decrease the magnitude of force to zero.

Note : 1 When some charges are separated by the same distance in two different media,

$$F_1 = \frac{1}{K_1}\frac{1}{4\pi\in_0}\frac{q_1q_2}{r^2} \qquad (1)$$

and $$F_2 = \frac{1}{K_2}\frac{1}{4\pi\in_0}\frac{q_1q_2}{r^2} \qquad (2)$$

from (1) and (2) $\Rightarrow$ $F_1K_1 = F_2K_2$

Note : 2 When the same charges are separated by different distance in the same medium Fd^2 = constant (or) $F_1d_1^2 = F_2d_2^2$

Note: 3 If different charges are at the same separation in a given medium $\frac{F^1}{F} = \frac{q_1^1q_2^1}{q_1q_2}$

Note : 4 If the force between two charges in two different media is the same for different separations.

$$F = \frac{1}{K}\frac{1}{4\pi\in_0}\frac{q_1q_2}{r^2} = \text{constant}$$

Kr^2 = constant or $K_1r_1^2 = K_2r_2^2$

If the force between two charges separated by a distance 'r_0' in vacuum or air is same as the force between the same charges separated by a distance 'r' in a medium.

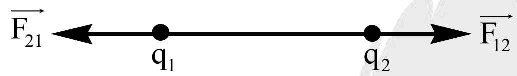
$$Kr^2 = r_0^2 \Rightarrow r = \frac{r_0}{\sqrt{K}}$$

Here K is dielectric constant of the medium. The effective distance 'r' in medium for a distance r_0 in vacuum $= \frac{r_0}{\sqrt{K}}$.

Similarly, the effective distance in vacuum for a dielectric slab of thickness $'x'$ and dielectric constant K is $x_{eff} = x\sqrt{K}$

Coulomb's Law in Vector Form

- $\overrightarrow{F_{12}} = \frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r_2^2}\hat{r}_{12}$ and $\overrightarrow{F_{21}} = -\overrightarrow{F_{12}}$



Here F_{12} is force exerted by q_1 on q_2 and F_{21} is force exerted by q_2 on q_1

- Suppose the position vector of two charges q_1 and q_2 are $\vec{r_1}$ and $\vec{r_2}$, then electric force on charge q_1 due to q_2 is, $\vec{F_1} = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{|\vec{r_1}-\vec{r_2}|^3}(\vec{r_1}-\vec{r_2})$

  Similarly, electric force on q_2 due to charge q_1 is $\vec{F_2} = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{|\vec{r_2}-\vec{r_1}|^3}(\vec{r_2}-\vec{r_1})$

  Here q_1 and q_2 are to be substituted with sign.

  $\vec{r_1} = x_1i + y_1j + z_1k$ and $\vec{r_2} = x_2i + y_2j + z_2k$ where (x_1, y_1, z_1) and (x_2, y_2, z_2) are the co-ordinates of charges q_1 and q_2.

Limitations of Coulomb's Law

- Coulomb's law holds for stationary charges only which are point sized.

  This law is valid for all types of charge distributions.

  This law is valid at distances greater than 10^{-15} m.

  This law obeys Newton's third law.

  This law represents central forces.

  This law is analogous to Newton law of gravitation in mechanics.

- The electric force is an action reaction pair, i.e the two charges exert equal and opposite forces on each other.
- The electric force is conservative in nature.
- Coulomb force is central.

- Coulomb force is much stronger than gravitational force. $\left(10^{36}F_g = F_E\right)$

Forces between multiple charges :

- Force on a charged particle due to a number of point charges is the resultant of forces due to individual point charges $\vec{F} = \vec{F_1} + \vec{F_2} + \vec{F_3} + ...$

Linear charge density (λ) is defined as the charge per unit length.

$$\lambda = \frac{dq}{d\ell}$$

Where dq is the charge on an infinitesimal length dl.

Units of λ are Coulomb/meter (C/m)

Examples:- Charged straight wire, circular charged ring

Surface charge density (σ) is defined as the charge per unit area.

$$\sigma = \frac{dq}{ds}$$

Where dq is the charge on an infinitesimal surface area ds. Units of σ are coulomb/meter^2 $\left(C/m^2\right)$.

Example :- Plane sheet of charge, conducting sphere.

Volume charge density (ρ) is defined as charge per unit volume.

$$\rho = \frac{dq}{dv}$$

Where dq is the charge on an infinitesimal volume element dv. Units of (ρ) are coulomb/meter^3 $\left(C/m^3\right)$

Electric Field : The space around electric charge upto which its influence is felt is known as electric field.

- Electric field is a conservative field.

Intensity of Electric Field : The intensity of electric field or electric field strength E at a point in space is defined as the force experienced by unit positive test charge placed at that point.

The intensity of electric field is also often called as electric field strength.

Consider an electric field in a given region. Bring a charge q_0 to a given point in that field without disturbing any other charge that has produced the field.

Let $\vec{F}$ be the electric force experienced by q_0 and it is found to be proportional to q_0

$$\vec{F} \propto q_0 \Rightarrow \vec{F} = \vec{E}q_0 .$$

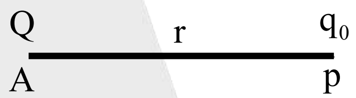
Here $\vec{E}$ is proportionality constant called electric field strength $\vec{E} = \frac{\vec{F}}{q_0}$

Electric field strength is a vector quantity. Its direction is the direction along which a free positive charge experiences the force in the electric field.

The S.I unit of electric field strength is newton per coulomb $\left(NC^{-1}\right)$. It can also be expressed in volt per meter $\left(Vm^{-1}\right)$.

Electric field internsity due to an isolated point charge :

Consider a point charge 'Q' placed at point A as shown. Let us find the electric field $\vec{E}$ at a point P at a distance 'r' from charge Q. Imagine a positive test charge q_0 at P. The charge Q produces a field $\vec{E}$ at P.



The force applied by Q on q_0 is given by $F = \frac{1}{4\pi \in_0} \frac{Qq_0}{r^2}$. This acts along AP.

According to definition $\vec{E} = \frac{\vec{F}}{q_0} \Rightarrow \vec{E} = \frac{1}{4\pi \in_0} \frac{Q}{r^2} \hat{r}$

If 'q' is positive, E is along $\overrightarrow{AP}$ and if 'q' is negative E will be along $\overrightarrow{PA}$.

If the charge 'q' is in medium of permittivity ε, and dielectric constant K, $\left(K = \frac{\varepsilon}{\varepsilon_0}\right)$ the intensity of electric field in a medium $\left(E_{med}\right)$ is given by

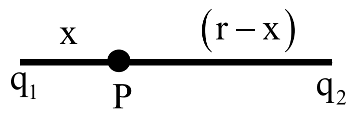
$$E_{med} = \frac{1}{4\pi\varepsilon} \frac{Q}{r^2} \therefore E_{med} = \frac{E_{\text{free space}}}{K}$$

NULL POINT OR NEUTRAL POINT

In the case of a system of charges if the net electric field is zero at a point, it is known as null point.

Application : Two point (like) charge q_1 and q_2 are separated by a distance 'r' and fixed, we can locate the point on the line joining those charges where resultant or net field is zero.

Case 1 : If the charges are like, the neutral point will be between the charges.



Let P be the null point where $\vec{E}_{net} = 0$

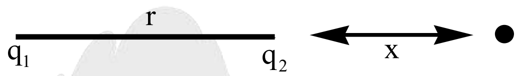
$\Rightarrow$ $\vec{E}_1 + \vec{E}_2 = 0$ (due to those charges) or $\vec{E}_1 = -\vec{E}_2$ and $E_1 = E_2$

$$\Rightarrow \frac{1}{4\pi\epsilon_0}\frac{q_1}{x^2} = \frac{1}{4\pi\epsilon_0}\frac{q_2}{(r-x)^2}$$

or $\dfrac{q_1}{x^2} = \dfrac{q_2}{(r-x)^2}$

on solving we get $x = \dfrac{r}{\sqrt{\dfrac{q_2}{q_1}}+1}$

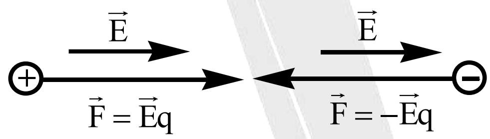
Case 2 : If the charges are unlike, the neutral point will be outside the charge on the line joining them.



In this case $\dfrac{q_1}{x^2} = \dfrac{q_2}{(r+x)^2}$

On solving we get $x = \dfrac{r}{\sqrt{\dfrac{q_2}{q_1}}-1}$

- If q_0 is positive charge then the force acting on it is in the direction of the field.
- If q_0 is negative then the direction of this force is opposite of the field direction.



Motion of a charge particle in a uniform electric field :

(a) A charged body of mass 'm' and charge 'q' is initially at rest in a uniform electric field of intensity E. The force acting on it, F = Eq.

- Here the direction of F is in the direction of field if 'q' is +ve and opposite to the field if 'q' is $-$ve.
- The body travels in a straight line path with uniform acceleration, $a = \dfrac{F}{m} = \dfrac{Eq}{m}$, initial velocity, u = 0.

  At an instant of time t.

  Its final velocity, $v = u + at = \left(\dfrac{Eq}{m}\right)t$

Displacement $s = ut + \frac{1}{2}at^2 = \frac{1}{2}\left(\frac{Eq}{m}\right)t^2$

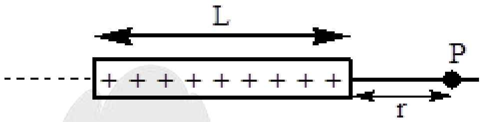
Momentum, $P = mv = (Eq)t$

Kinetic energy,

$K \cdot E = \frac{1}{2}mv^2 = \frac{1}{2}\left(\frac{E^2q^2}{m}\right)t^2$

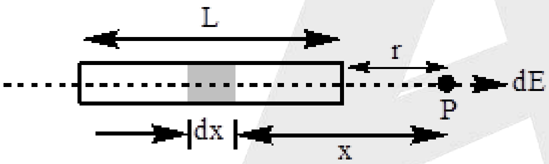
Electric field strength due to a uniformly charged rod

At an axial point:



Consider a rod of length L, uniformly charged with a charge Q.

To calculate the electric field strength at a point P situated at a distance 'r' from one end of the rod, consider an element of length dx on the rod as shown in the figure.



Charge on the elemental length dx is $dq = \frac{Q}{L}dx$

$$dE = \frac{dq}{4\pi\varepsilon_0 x^2} = \frac{Qdx}{4\pi\varepsilon_0 Lx^2}$$

The net electric field at point P can be given by integrating this expression over the length of the rod.

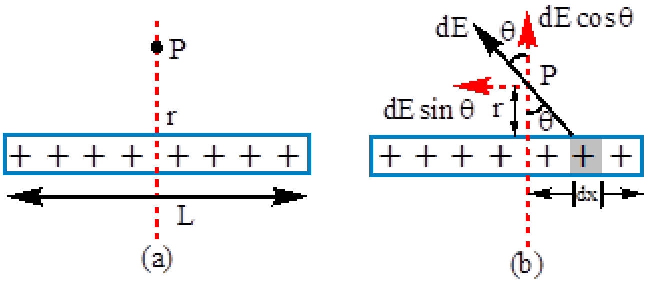
$$E_P = \int dE = \int_r^{r+L} \frac{Q}{Lx^2 4\pi\varepsilon_0}dx = \frac{Q}{4\pi\varepsilon_0 L}\int_r^{r+L}\frac{1}{x^2}dx$$

$$E_P = \frac{Q}{4\pi\varepsilon_0 L}\left[\frac{-1}{x}\right]_r^{r+L}$$

$$E_P = \frac{Q}{4\pi\varepsilon_0 L}\left[\frac{1}{r} - \frac{1}{r+L}\right] = \frac{Q}{4\pi\varepsilon_0 r(r+L)}$$

At an equatorial point:

To find the electric field due to a rod at a point P situated at a distance 'r' from its centre on its equatorial line



Consider an element of length dx at a distance 'x' from centre of rod as in figure (b). Charge on the element is $dq = \frac{Q}{L}dx$

The strength of electric field at P due to this point charge dq is dE.

$$\Rightarrow dE = \frac{dq}{4\pi\varepsilon_0\left(r^2+x^2\right)}$$

The component $dE\sin\theta$ will get cancelled and net electric field at point P will be due to integration of $dE\cos\theta$ only

Net electric field strength at point P can be given as

$$E_P = \int dE\cos\theta = \int_{-\frac{L}{2}}^{+\frac{L}{2}} \frac{Qdx}{L\left(r^2+x^2\right)} \times \frac{r}{\sqrt{r^2+x^2}} \times \frac{1}{4\pi\varepsilon_0}$$

$$E_P = \frac{Qr}{4\pi\varepsilon_0 L}\int_{-\frac{L}{2}}^{+\frac{L}{2}} \frac{dx}{\left(r^2+x^2\right)^{3/2}}$$

From the diagram $\tan\theta = \frac{x}{r}$

$x = r\tan\theta$; On differentiation; $dx = r\sec^2\theta d\theta$

$$E_P = \frac{Qr}{4\pi\varepsilon_0 L}\int \frac{r\sec^2\theta d\theta}{r^3\sec^3\theta}; = \frac{Q}{4\pi\varepsilon_0 Lr}\int \frac{r\sec^2 d\theta}{r^3\sec^3\theta}$$

$$= \frac{Q}{4\pi\varepsilon_0 Lr}\int \cos\theta d\theta = \frac{Q}{4\pi\varepsilon_0 Lr}[\sin\theta]$$

Substituting $\theta = \tan^{-1}\frac{x}{r} = \sin^{-1}\frac{x}{\sqrt{x^2+r^2}}$

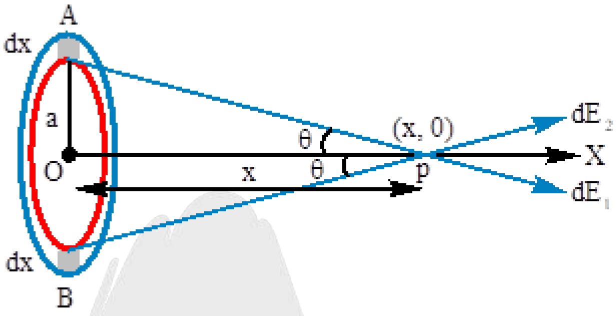
$$E_P = \frac{Q}{4\pi\varepsilon_0 L}\left[\frac{x}{\sqrt{x^2+r^2}}\right]_{-\frac{L}{2}}^{\frac{L}{2}} = \frac{Q}{4\pi\varepsilon_0 r}\left(\frac{1}{\sqrt{\frac{L^2}{4}+r^2}}\right)$$

$$E_P = \frac{Q}{4\pi\varepsilon_0 r}\left\{\frac{2}{\sqrt{L^2+4r^2}}\right\}$$

Electric field due to a uniformly charged ring:

The intensity of electric field at a distance 'x' meter from the Centre along the axis:

Consider a circular ring of radius 'a' having a charge 'q' uniformly distributed over it as shown in figure. Let 'O' be the centre of the ring.



Consider an element dx of the ring at point A. The charge on this element is given by

$$dq = dx \times \text{ charge density}$$

$$dq = dx\frac{q}{2\pi a} = \frac{qdx}{2\pi a}$$

(a) The intensity of electric field dE_1 at point P due to the element dx at A is given by

$$dE_1 = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2}$$

The direction of dE_1 is as shown in figure. The component of intensity along x-axis will be

$$\frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2}\cos\theta = dE_1\cos\theta$$

The component of intensity along y-axis will be

$$\frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2}\sin\theta = dE_1\sin\theta$$

Similarly, if we consider an element dx of the ring opposite to A which lies at B, the component of intensity perpendicular to the axis will be equal and opposite to the component of intensity perpendicular to the axis due to element at A. Hence, they cancel each other. Due to symmetry of ring the component of intensity due to all elements of the ring perpendicular to the axis will cancel.

So the resultant intensity is only along the axis of the ring. The resultant intensity is given by

$$E = \int\frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2}\cos\theta$$

$$E = \frac{1}{4\pi\varepsilon_0}\int\frac{qdx}{2\pi ar^2}\times\frac{x}{r}\ (\text{where } \cos\theta = x/r)$$

$$E=\frac{1}{4\pi\varepsilon_0}\times\frac{qx}{(2\pi a)}\times\frac{1}{\left(a^2+x^2\right)^{\frac{3}{2}}}\int dx$$

$$\left[\therefore r^3=\left(a^2+x^2\right)^{3/2}\right]$$

$$E=\frac{1}{4\pi\varepsilon_0}\frac{qx}{2\pi a}\frac{1}{\left(a^2+x^2\right)^{3/2}}\times 2\pi a$$

$$E=\frac{1}{4\pi\varepsilon_0}\frac{qx}{\left(a^2+x^2\right)^{3/2}}$$

Electric field at centre is zero.

By symmetry we can say that electric field strength at centre due to every small segment on ring is cancelled by the electric field at centre due to the element exactly opposite to it. As in the figure the electric field at centre due to segment A is cancelled by that due to segment B. Thus net electric field strength at the centre of a uniformly charged ring is $E_{centre}=0$

Electric field strength due to a uniformly charged disc

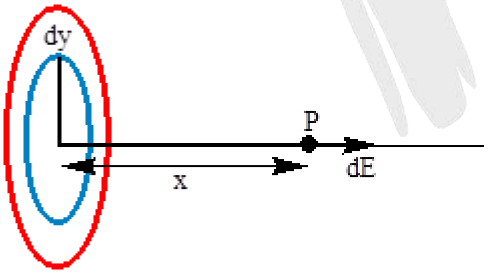
Consider a disc of radius R, charged on its surface with a charge density σ.

Let us find electric field strength due to this disc at a distance 'x' from the center of disc on its axis at point P as shown in figure.

Consider an elemental ring of radius 'y' and width dy in the disc as shown in figure. The charge on this elemental ring dq can be given as

$$dq=\sigma 2\pi\, y dy$$

{Area of elemental ring $ds = dy = 2\pi\, y dy$}



Electric field strength due to a ring of radius y, charge Q at a distance x from its centre on its axis can be given as

$$E=\frac{Qx}{4\pi\varepsilon_0\left(x^2+y^2\right)^{3/2}}$$

Due to the elemental ring electric field strength dE at point P can be given as

$$dE = \frac{xdq}{4\pi\varepsilon_0\left(x^2+y^2\right)^{3/2}} = \frac{\sigma 2\pi ydyx}{4\pi\varepsilon_0\left(x^2+y^2\right)^{3/2}}$$

Net electric field at point P due to whole disc is given by integrating above expression within the limits from 0 to R

$$E = \int dE = \int_0^R \frac{\sigma 2\pi xydy}{4\pi\varepsilon_0\left(x^2+y^2\right)^{3/2}}$$

$$= \frac{\sigma\pi x}{4\pi\varepsilon_0}\int_0^R \frac{2ydy}{\left(x^2+y^2\right)^{3/2}} = \frac{2\sigma\pi x}{4\pi\varepsilon_0}\left[\frac{-1}{\sqrt{x^2+y^2}}\right]_0^R$$

$$E = \frac{\sigma}{2\varepsilon_0}\left[1-\frac{x}{\sqrt{x^2+R^2}}\right]$$

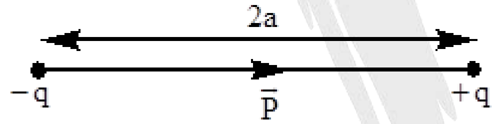
Electric field strength due to uniformly charged disc at a distance x from its surface is given as

$$E = \frac{\sigma}{2\varepsilon_0}\left[1-\frac{x}{\sqrt{x^2+R^2}}\right]$$

If we put $x=0$ we get $E=\frac{\sigma}{2\varepsilon_0}$

Electric dipole:

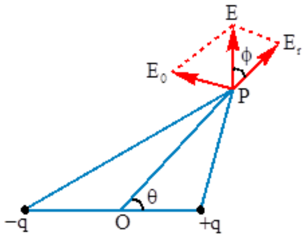
A system of two equal and opposite point charges fixed at a small distance constitutes an electric dipole. Electric dipole is analogous to bar magnet or magnetic dipole in magnetism. Every dipole has a characteristic property called dipole moment, which is similar to magnetic moment of a bar magnet. If 2a is the distance between the charges +q and -q, then electric dipole moment is $p = q.2a$



Dipole moment is a vector quantity and its direction is from negative charge to positive charge as shown.

Electric field at any point due to a dipole:

We know that the electric field is the negative gradient of potential. In polar form if V is the potential at (r,θ) the electric field will have two components radial and transverse components which are represented by E_r & E_θ respectively.



Then
$$E_r = -\left(\frac{\partial V}{\partial r}\right) = -\frac{p\cos\theta}{4\pi\varepsilon_0}\frac{\partial}{\partial r}\left(\frac{1}{r^2}\right)$$

$$E_r = \frac{2p\cos\theta}{4\pi\varepsilon_0 r^3}$$

The transverse component of electric field

$$E_\theta = -\frac{1}{r}\frac{\partial V}{\partial \theta} = -\frac{1}{r}\left(-\frac{p\sin\theta}{4\pi\varepsilon_0 r^2}\right)$$

$$E_\theta = \frac{p\sin\theta}{4\pi\varepsilon_0 r^3}$$

$$E = \sqrt{E_\theta^2 + E_R^2}$$

$$E = \sqrt{\frac{p^2\sin^2\theta}{\left(4\pi\varepsilon_0 r^3\right)^2} + \frac{4p^2\cos^2\theta}{\left(4\pi\varepsilon_0 r^3\right)^2}}$$

$$E = \frac{p}{4\pi\varepsilon_0 r^3}\sqrt{4\cos^2\theta + \sin^2\theta}$$

$$\Rightarrow E = \frac{p}{4\pi\varepsilon_0 r^3}\sqrt{\left[1+3\cos^2\theta\right]}$$

Field at a point on the axial line: $(\theta = 0°)$

$$\vec{E}_{\text{axial}} = \frac{2\vec{p}}{4\pi\varepsilon_0 r^3}$$

Field at a point on the equatorial line $(\theta = 90°)$

$$\vec{E}_{\text{equitorial}} = \frac{-\vec{p}}{4\pi\varepsilon_0 r^3}$$

The direction of E at any point is given by

$$\tan\phi = \frac{E_\theta}{E_r} = \frac{\dfrac{p\sin\theta}{4\pi\varepsilon_0 r^3}}{\dfrac{2p\cos\theta}{4\pi\varepsilon_0 r^3}} \Rightarrow \tan\phi = \frac{1}{2}\tan\theta$$

$$\phi = \tan^{-1}\left[1/2\tan\theta\right]$$

Oscillatory motion of dipole in an electric field

When dipole is displaced from its position of equilibrium. The dipole will then experience a torque given by

$\tau = -pE\sin\theta$

For small value of θ,

$\tau = -pE\theta$ .....(1)

Where negative sign shows that torque is acting against increasing value of θ

Also, $\tau = I\alpha$

Where, I = moment of inertia and

α = angular acceleration

$\tau = I\dfrac{d^2\theta}{dt^2}$ ....(2)

Hence, from equations (1) and (2), we have

$I\dfrac{d^2\theta}{dt^2} = -pE\theta$ or $\dfrac{d^2\theta}{dt^2} = \dfrac{-pE}{I}\theta$ .....(3)

$$\frac{d^2\theta}{dt^2} \propto -\theta$$

This equation represents simple harmonic motion (SHM). When dipole is displaced from its mean position by small angle, then it will have SHM.

Equation (3) can be written as $\dfrac{d^2\theta}{dt^2} + \dfrac{pE}{I}\theta = 0$

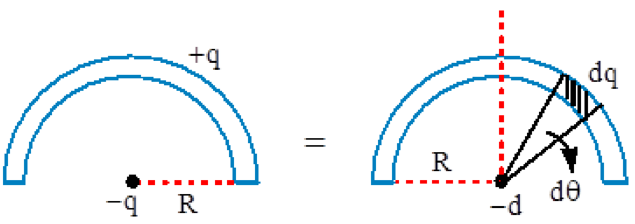
On comparing above equation with standard equation of SHM.

$\dfrac{d^2\theta}{dt^2} + \omega^2 y = 0$, we have; $\omega^2 = \dfrac{pE}{I} \Rightarrow \omega = \sqrt{\dfrac{pE}{I}}$

$T = 2\pi\sqrt{\dfrac{I}{pE}}$, where T is the time period of oscillations.

Distributed dipole:

Consider a half ring with a charge +q uniformly distributed and another equal negative charge -q placed at its centre. Here -q is point charge while +q is distributed on the ring. Such a system is called distributed dipole.



The net dipole moment is $p_{net} = \dfrac{2qR}{\pi}$

If $\theta = \phi$ $\quad p_{net} = 2\int_0^{\phi/2} dp\cos\theta; \; = \dfrac{2qR}{\pi}\sin\phi/2$

If the arrangement is a complete circle,

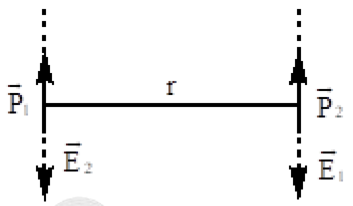
$$\frac{\phi}{2} = \pi \Rightarrow p_{net} = 0$$

Force between two short dipoles

Consider two short dipoles separated by a distance r.

There are two possibilities.

(a) If the dipoles are parallel to each other.

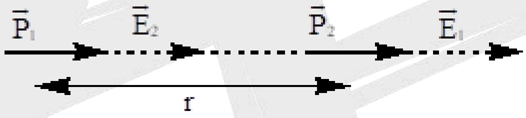


$$F = \frac{1}{4\pi\varepsilon_0}\frac{3p_1p_2}{r^4}$$

As the force is positive, it is repulsive.

Similarly if the dipoles are antiparallel to each other, the force is attractive

(b) If the dipoles are on the same axis



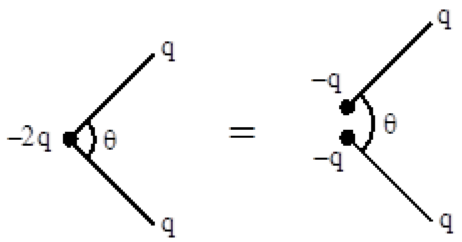
$$F = -\frac{1}{4\pi\varepsilon_0}\frac{6p_1p_2}{r^4}$$

As the force is negative, it is attractive

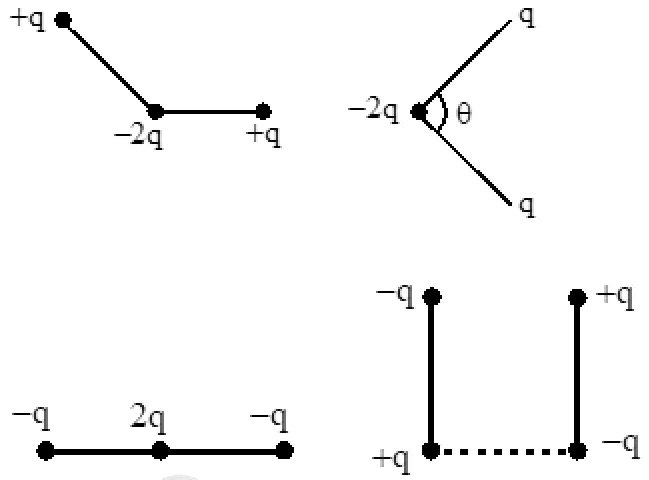
Quadrapole:

We have discussed about electric dipole with two equal and unlike point charges separated by a small distance. But in some cases, the two charges are not concentrated at its ends. Consider a situation as shown in the figure (Like in water molecule). Here three charges -2q, q and q are arranged as shown. It can be visualized as the combination of two dipoles each of dipole moment p = qd at an angle θ between them. The arrangement of two electric dipoles is called a quadrapole. As dipole moment is a vector the resultant dipole moment of the system is

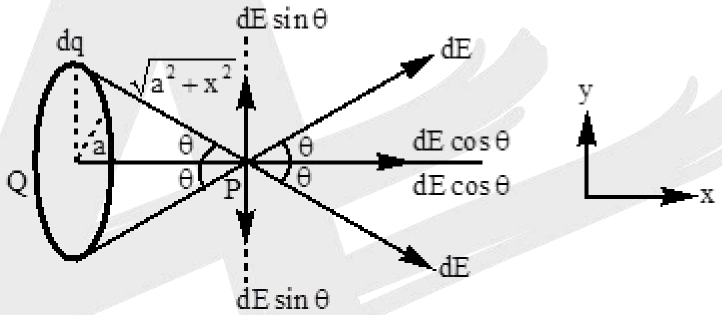
$$p' = 2p\cos\theta/2$$



Few other quadrapoles are also as shown in the following figures.



Electric field at the axis of a circular uniformly charged ring



Intensity of electric field at a point P that lies on the axis of the ring at a distance x from its centre is

$$E = \frac{1}{4\pi\varepsilon_0}\frac{qx}{\left(x^2+a^2\right)^{3/2}}$$

Where $\left\{\cos\theta = \frac{x}{\sqrt{a^2+x^2}}\right\}$

Where a is the radius of the ring.

From the above, expression E = 0 at the centre of the ring

E will be maximum when $\frac{dE}{dx} = 0$

Differentiating E w.r.t. x and putting it equal to zero we get $x = \frac{a}{\sqrt{2}}$ and $E_{max} = \frac{2}{3\sqrt{3}}\left(\frac{1}{4\pi\varepsilon_0}\frac{q}{a^2}\right)$

Electric field due to a charged spherical conductor (Spherical shell)

Let 'q' amount of charge be uniformly distributed over a spherical shell of radius 'R'

σ = Surface charge density, $\sigma = \frac{q}{4\pi R^2}$

When point 'P' lies outside the shell:

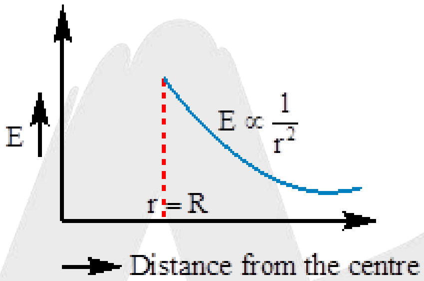
$$E = \frac{1}{4\pi\varepsilon_0} \times \frac{q}{r^2}$$

This is the same expression as obtained for electric field at a point due to a point charge. Hence a charged spherical shell behaves as a point charge concentrated at the centre of it.

$$E = \frac{1}{4\pi\varepsilon_0}\frac{\sigma.4\pi R^2}{r^2} \quad \because \sigma = \frac{q}{4\pi r^2};\ E = \frac{\sigma.R^2}{\varepsilon_0 r^2}$$

When point 'P' lies on the shell: $E = \frac{\sigma}{\varepsilon_0}$

When point 'P' lies inside the shell: $E = 0$



Electric field due to a uniformly charged non – conducting sphere

Electric field intensity due to a uniformly charged non – conducting sphere of charge Q, of radius R at a distance r from the centre of the sphere.

Let q amount of charge be uniformly distributed over a solid sphere of radius R

$$\rho \text{ = Volume charge density } \rho = \frac{q}{\frac{4}{3}\pi R^3}$$

When point 'P' lies inside sphere:

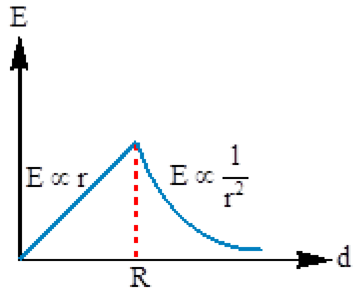
$$E = \frac{1}{4\pi\varepsilon_0}\frac{Qr}{R^3} \text{ for } r < R,\ E = \frac{\rho.r}{3\varepsilon_0}$$

When point 'P' lies on the sphere:

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{R^2}; \qquad E = \frac{\rho.R}{3\varepsilon_0}$$

When point 'P' lies outside the sphere:

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}; \qquad E = \frac{\rho.R^3}{3\varepsilon_0 r^2}$$



Electric potential:

Work done to bring a unit positive charge from infinite distance to a point in the electric field is called electric potential at that point

It is given by $V = \frac{W}{q}$

It represents the electrical condition or state of the body and it is similar to temperature

Potential at a point due to a point charge $= \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$

Potential due to a group of charges is the algebraic sum of their individual potentials

$$\text{i.e., } V = V_1 + V_2 + V_3 + \ldots..$$

when a charged particle is accelerated from rest through a potential difference 'V', work done,

$$W = Vq = \frac{1}{2}mv^2 \text{ (or) } v = \sqrt{\frac{2Vq}{m}}$$

The work done in moving a charge of q coulomb between two points separated by potential difference $V_2 - V_1$ is $q(V_2 - V_1)$

Relation among E, V and d in a uniform electric field is $E = \frac{V}{d}$ (or) $E = -\frac{dV}{dx}$

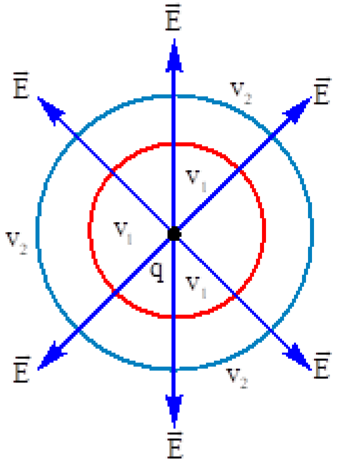
Electric field is always in the direction of decreasing potential

The component of electric field in any direction is equal to the negative of potential gradient in that direction.

$$\vec{E} = -\left(\frac{\partial V}{\partial x}\hat{i} + \frac{\partial V}{\partial y}\hat{j} + \frac{\partial V}{\partial z}\hat{k}\right)$$

An equatorial surface has a constant value of potential at all points on the surface.

For single charge q



Electric field at every point is normal to the equipotential surface passing through that point.

No work is required to move a test charge on an equipotential surface.

Zero Potential Point:

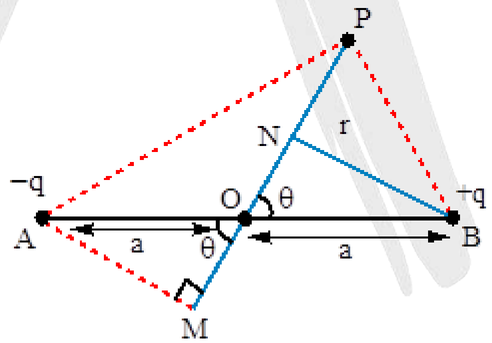
Two unlike charges Q_1 and $-Q_2$ are separated by a distance 'd'. The net potential is zero at two points on the line joining them, one (x) in between them and the other (y) outside them

$$\frac{Q_1}{x}=\frac{Q_2}{d-x} \text{ and } \frac{Q_1}{y}=\frac{Q_2}{d+y}$$

Potential due to a dipole:

An electric dipole consists of two equal and opposite charges separated by a very small distance.

If 'q' is the charge and 2a the length of the dipole then electric dipole moment will be given by $p=(2a)q$



Let AB be a dipole whose centre is at 'O' and 'P' be the point where the potential due to dipole is to be determined. Let r, θ be the position coordinates of 'P' w.r.t the dipole as shown in figure. Let BN & AM be the perpendiculars drawn on to OP and the line produced along PO. From geometry ON $=a\cos\theta=$ OM.

Hence the distance, BP from +q charge is

$$r-a\cos\theta$$

[because PB = PN as AB is very small in comparison with r]

For similar reason

$$AP = r + a\cos\theta \ [\because AP = PM]$$

Hence potential at P due to charge +q situated at B is $V_1 = \frac{1}{4\pi\varepsilon_0}\frac{q}{(r - a\cos\theta)}$

Similarly potential at P due to charge -q at A is $V_2 = \frac{1}{4\pi\varepsilon_0}\frac{-q}{(r + a\cos\theta)}$

Hence the total potential at P is $V = V_1 + V_2$

$$V = \frac{q}{4\pi\varepsilon_0(r - a\cos\theta)} - \frac{q}{4\pi\varepsilon_0(r + a\cos\theta)}$$

$$V = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{(r - a\cos\theta)} - \frac{1}{(r + a\cos\theta)}\right]$$

$$V = \frac{q(2a\cos\theta)}{4\pi\varepsilon_0(r^2 - a^2\cos^2\theta)}$$

But $r >> a \quad \therefore r^2 - a^2\cos^2\theta \approx r^2$

$$\therefore V = \frac{p\cos\theta}{4\pi\varepsilon_0 r^2}$$

Hence, potential varies inversely as the square of the distance from the dipole.

SPECIAL CASSES:

1. On the axial line: For a point on the axial line $\theta = 0°$

$\therefore V_{axial} = p/4\pi\varepsilon_0 r^2$ volts for a dipole.

2. Point on the equatorial line: For a point on the equatorial line $\theta = 90° \quad \therefore V_{equitorial} = 0$ volts.

Equatorial line is a line where the potential is zero at any point

1. Equipotential surfaces:

Equipotential surface in an electric field is a surface on which the potential is same at every point. In other words, the locus of all points which have the same electric potential is called equipotential surface. An equipotential surface may be the surface of a material body or a surface drawn in an electric field. The important properties of equipotential surfaces are as given below.

(a) As the potential difference between any two points on the equipotential surface is zero, no work is done in taking a charge from one point to another.

(b) The electric field is always perpendicular to an equipotential surface.
In other words electric field or lines of force are perpendicular to the equipotential surface

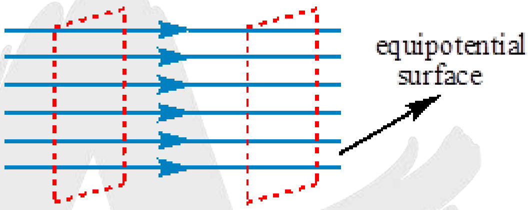
(c) No two equipotential surfaces intersect. If they intersect like that, at the point of intersection field will have two different directions or at the same point there will be two different potentials which is impossible.

(d) The spacing between equipotential surfaces enables to identify regions of strong and weak fields

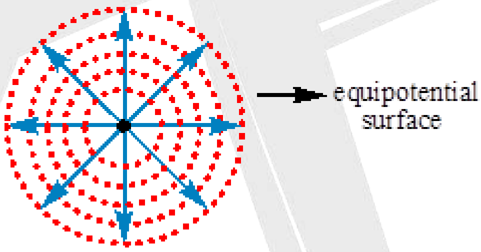
$$E=-\frac{dV}{dr}.\ \text{So}\ E\propto\frac{1}{dr}\ \text{(if dV is constant).}$$

(e) At any point on the equipotential surface component of electric field parallel to the surface is zero.

In uniform field, the lines of force are straight and parallel and equipotential surfaces are planes perpendicular to the lines of force as shown in figure.



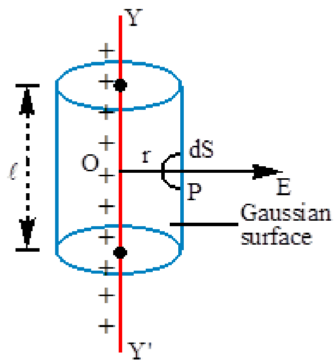
The equipotential surfaces are a family of concentric spheres for a uniformly charged sphere or for a point charge as shown in figure.



Equipotential surfaces in electrostatics are similar to wave fronts in optics. The wave fronts in optics are the locus of all points which are in the same phase. Light rays are normal to the wave fronts. On the other hand the equipotential surfaces are perpendicular to the lines of force.

Electric Potential due to a Linear Charge Distribution

Consider a thin infinitely long line charge having a uniform linear charge density λ placed along YY'. Let P is a point at distance 'r' from the line charge then magnitude of electric field at point P is given by $E=\frac{\lambda}{2\pi\varepsilon_0 r}$



We know that $V(r)=-\int \bar{E}.d\bar{r}$

Here $E=\frac{\lambda}{2\pi\varepsilon_0 r}$ and $\bar{E}.d\bar{r}=Edr$

So $V(r) = -\int E dr = -\int \frac{\lambda}{2\pi\varepsilon_0 r} dr$

$$\therefore V(r) = \left(\frac{-\lambda}{2\pi\varepsilon_0}\log_e r\right) + C$$

Where C is constant of integration and V(r) gives electric potential at a distance 'r' from the linear charge distribution

Electric Potential due to Infinite Plane Sheet of charge (Non conducting)

Consider an infinite thin plane sheet of positive charge having a uniform surface charge density σ on both sides of the sheet. By symmetry, it follows that the electric field is perpendicular to the plane sheet of charge and directed in outward direction.

The electric field intensity is $E = \frac{\sigma}{2\varepsilon_0}$

Electrostatic potential due to an infinite plane sheet of charge at a perpendicular distance r from the sheet given by

$$V(r) = -\int \bar{E}.d\bar{r} = -\int E dr$$

$$V(r) = -\int \frac{\sigma}{2\varepsilon_0} dr = \left(\frac{-\sigma}{2\varepsilon_0} r\right) + C$$

Where C is constant of integration.

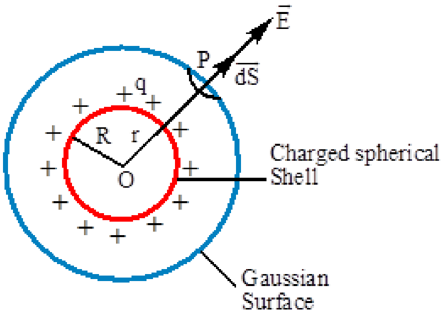
Similarly, the electric potential due to an infinite plane conducting plate at a perpendicular distance r from the plate is given by

$$V(r) = -\int \bar{E}.d\bar{r} = -\int E dr$$

$$V(r) = -\int \frac{\sigma}{2\varepsilon_0} dr = \left(\frac{-\sigma}{2\varepsilon_0} r\right) + C$$

Where C is constant of integration

Electric Potential due to a Charged Spherical Shell (or Conducting Sphere):



Consider a thin spherical shell of radius R and having charge +q on the spherical shell.

Case (i): When point P lies outside the spherical shell. The electric field at the point is

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} \text{(for } r > R\text{)}$$

The potential $V(r) = -\int \bar{E}.d\bar{r} = -\int Edr$

$$= -\int \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}dr = \frac{1}{4\pi\varepsilon_0}\frac{q}{r} + C$$

Where C is constant of integration

If $r \to \infty$, $V(\infty) \to 0$ and $C = 0$

$$V(r) = \frac{1}{4\pi\varepsilon_0}\frac{q}{r} \quad (r > R)$$

Case (ii): When point P lies on the surface of spherical shell then r = R

Electrostatic potential at P on the surface is

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{R}$$

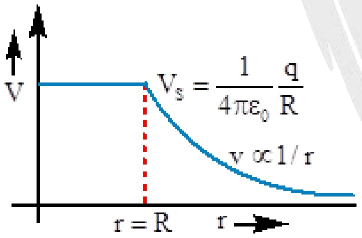
Case (iii): For points inside the charged spherical shell $(r < R)$, the electric field $E = 0$

So we can write $-\frac{dV}{dr} = 0$

$\Rightarrow$ V is constant and is equal to that on the surface

So, $V = \frac{1}{4\pi\varepsilon_0}\frac{q}{R}$ for $r \le R$

The variation of V with distance 'r' from centre is as shown in the graph.



Electric Potential due to a Uniformly Charged Non-conducting solid Sphere:

Consider a charged sphere of radius R with total charge q uniformly distributed on it.

Case (i): For points outside the sphere $(r > R)$ the electric field at any point is

$$E = \frac{1}{4\pi\varepsilon_0}, \frac{q}{r^2} \qquad \text{(for } r > R\text{)}$$

The potential at any point outside the shell is

$$V(r) = -\int \bar{E}.d\bar{r} = -\int Edr$$

$$= -\int \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}dr = \frac{1}{4\pi\varepsilon_0}\frac{q}{r} + C$$

Where C is constant of integration

If $r \to \infty, V(\infty) \to 0$ and $C = 0$

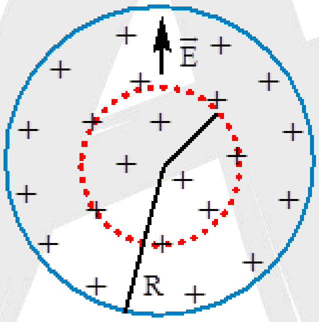
$$V(r) = \frac{1}{4\pi\varepsilon_0}\frac{q}{r} \qquad (r > R)$$

Case (ii): When point P lies on the surface of spherical shell then $r = R$ The electrostatic potential at P on the surface is

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{R}$$

Case (iii): For points inside the sphere $(r < R)$

The electric field is $E = \dfrac{1}{4\pi\varepsilon_0}\dfrac{qr}{R^3}$



$$dV = \bar{E}.d\bar{r} = -Edr$$

$$\int_{V_s}^{V} dV = -\int_R^r E\,dr = -\int_R^r \frac{1}{4\pi\varepsilon_0}\frac{qr}{R^3}dr$$

$$V - V_s = -\frac{1}{4\pi\varepsilon_0}\frac{q}{R^3}\left(\frac{r^2}{2}\right)_R^r$$

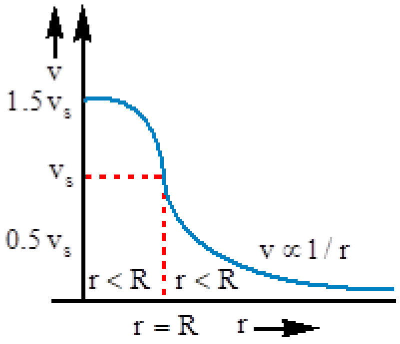
$$V - \frac{1}{4\pi\varepsilon_0}\frac{q}{R} = -\frac{1}{4\pi\varepsilon_0}\frac{q}{R^3}\left[\frac{r^2}{2} - \frac{R^2}{2}\right]$$

$$\Rightarrow V = \frac{1}{4\pi\varepsilon_0}\frac{q}{R} = -\frac{1}{4\pi\varepsilon_0}\frac{q}{R^3}\left[\frac{r^2}{2} - \frac{R^2}{2}\right]$$

$$\Rightarrow V = \frac{1}{4\pi\varepsilon_0}\frac{q}{R} \times \left[\frac{3}{2} - \frac{r^2}{2R^2}\right]$$

At the centre $r = 0$ then potential at centre $V_C = \dfrac{1}{4\pi\varepsilon_0}\dfrac{3q}{2R} = \dfrac{3}{2}\dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{R}$

The variation of V with distance 'r' from centre is as shown in the graph.



Potential of a charged ring:

A charge q is distributed over the circumference of ring (either uniformly or non-uniformly), then electric potential at the centre of the ring is

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{R}$$

At distance 'r' from the centre of ring on its axis would be $V = \frac{1}{4\pi\varepsilon_0}.\frac{q}{\sqrt{R^2+r^2}}$

Electric potential of a uniformly charged disc

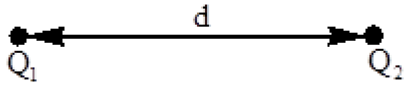
Consider a uniformly charged circular disc having surface charge density σ.

- Potential at a point on its axial line at distance x from the centre is $V = \frac{\sigma}{2\varepsilon_0}\left[\sqrt{R^2+x^2}-x\right]$
- At the center of disc $x = 0$, $V = \frac{\sigma R}{2\varepsilon_0}$
- For $x >> R$, $V = \frac{q}{4\pi\varepsilon_0 x}$
- Potential on the edge of the disc is $V = \frac{\sigma R}{\pi\varepsilon_0}$

Potential Energy of System of Charges

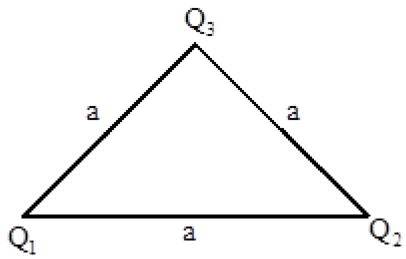
- Two charges Q_1 and Q_2 are separated by a distance 'd'. The P.E. of the system of charges is

$$U = \frac{1}{4\pi\varepsilon_0}.\frac{Q_1Q_2}{d} \text{ from } U = W = Vq$$



- Three charges Q_1, Q_2, Q_3 are placed at the three vertices of an equilateral triangle of side 'a'.

The P.E. of the system of charges is $U = \frac{1}{4\pi\varepsilon_0}\left[\frac{Q_1Q_2}{a} + \frac{Q_2Q_3}{a} + \frac{Q_3Q_1}{a}\right]$ or $U = \frac{1}{4\pi\varepsilon_0}\frac{\Sigma Q_1Q_2}{a}$



- If two unlike charges are brought closer, P.E. of the system decreases.

  For an attractive system U is always NEGATIVE

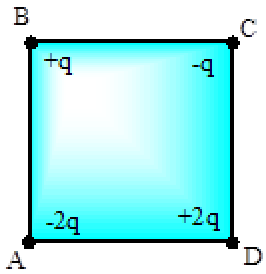
  For a repulsive system U s always POSITIVE

  For a stable system U is MINIMUM

  i.e., $F = -\frac{dU}{dx} = 0$ (for stable system)

EXERCISE-1

1. Four charges are arranged at the corners of a square ABCD, as shown. The force on a +ve charge kept at the centre of the square is



(A) Zero (B) Along diagonal AC

(C) Along diagonal BD (D) Perpendicular to the side AB

2. Which of the following is a volt?

(A) Erg per cm (B) Joule per coulomb

(C) Erg per ampere (D) Newton/ (coulomb x m^2)

3. When a negative charge is released and moves in electric field, it moves toward a position of

(A) lower electric potential and lower potential energy

(B) lower electric potential and higher potential energy

(C) higher electric potential and lower potential energy

(D) higher electric potential and higher potential energy

4. Two particles X and Y, of equal mass and with unequal positive charges, are free to move and are initially far away from each other. With Y at rest, X begins to move towards it with initial velocity u. After a long time, finally

(A) X will stop, Y will move with velocity u

(B) X and Y will both move with velocities $\frac{u}{2}$ each

(C) X will stop, Y will move with velocity < u

(D) Both will move with velocities < $\frac{u}{2}$

5. In a uniform electric field, the potential is 10V at the origin of coordinates, and 8V at each of the points (1, 0, 0), (0, 1, 0) and (0, 0, 1). The potential (in volt) at the point (1, 1, 1) will be

6. Point P lies on the axis of a dipole. If the dipole is rotated by 90° anticlockwise, the electric field vector a at P will rotate by

(A) 90° clockwise (B) 180° clockwise

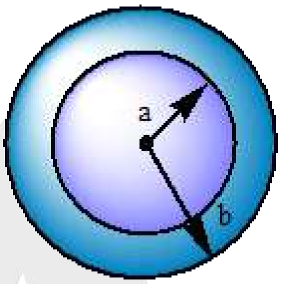
(C) 90° anticlockwise (D) none of these

7. Electric flux through a surface of area $100\,m^2$ lying in the xy plane is (in V-m) if $\vec{E}=\hat{i}+\sqrt{2}\hat{j}+\sqrt{3}\hat{k}$

(A) 100 (B) 141.4 (C) 173.2 (D) 200

8. A hollow metal sphere of radius 5cm is charged such that the potential on its surface is 10V. The potential (in V) at the centre of the sphere is

9. If the electric potential of the inner metal sphere is 10volt and that of the outer shell is 5 volt, then the potential (in volt) at the centre will be



10. An isolated system consists of two charged particles of equal mass. Initially the particles are far apart, have zero potential energy and one of the particles has non – zero speed. If the radiation is neglected, which of the following is true for the total energy of the system at any later time?

(A) It is negative and constant

(B) It is positive and constant

(C) It is constant, but the sign cannot be determined unless the initial velocities of both particles are known

(D) It cannot be constant of the motion because the particles exert force on each other

11. A point charge Q is located at the centre of a hollow spherical conductor of inner radius R_1 and outer radius R_2, the conductor being uncharged initially. The potential at the inner surface will be

(A) $KQ\left[\frac{1}{R_1}+\frac{1}{R_2}\right]$ (B) $KQ\left[\frac{1}{R_1}-\frac{1}{R_2}\right]$

(C) $KQ\left[\frac{1}{R_2}-\frac{1}{R_1}\right]$ (D) None of the above

12. A number of spherical conductors of different radii are charged to some potential. The surface charge density of each conductor is related with its radius as

(A) $\sigma\propto\frac{1}{R^2}$ (B) $\sigma\propto\frac{1}{R}$ (C) $\sigma\propto R$ (D) None of these

13. A dipole of $2\mu C$ charges each consists of the positive charge at the point P (1, -1) and the negative charge is placed at the point Q (-1, 1). The work done in displacing a charge of $+1\mu C$ from point $A(-3,-3)$ to B (4, 4) is

(A) 1.6×10^{-19} J (B) 3.2×10^{-19} J (C) zero (D) 4.8 eV

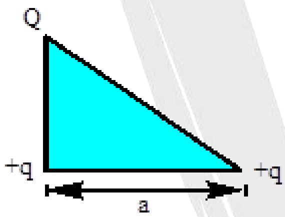
14. There are two concentric metal shells of radii r_1 and $r_2\left(>r_1\right)$. If the outer shell has a charge q and the inner shell is grounded, the charge on the inner shell is

(A) zero (B) $-\left(\frac{r_1}{r_2}\right)q$ (C) r_1r_2q (D) $\left(\frac{r_2}{r_1}\right)q$

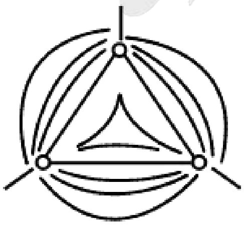
15. If the intensity of electric field at a distance x from the centre in axial position of small electric dipole is equal to intensity at distance y in equatorial position, then

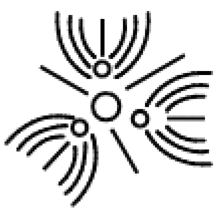
(A) $x=y$ (B) $x=\frac{y}{2}$ (C) $y=\frac{x}{2^{2/3}}$ (D) $y=\frac{x}{2^{1/3}}$

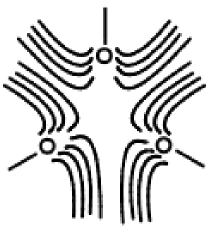
16. Three charges Q, +q and +q are placed at the vertices of a right- angled isosceles triangle as shown in the figure. The net electrostatic energy of the configuration will be zero if Q is equal to $\frac{-\alpha q}{\beta+\sqrt{2}}$. Find $\alpha+\beta$?

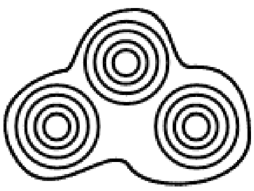


17. Three positive charges of equal value q are placed at the vertices of an equilateral triangle. The resulting lines of force should be sketched as in

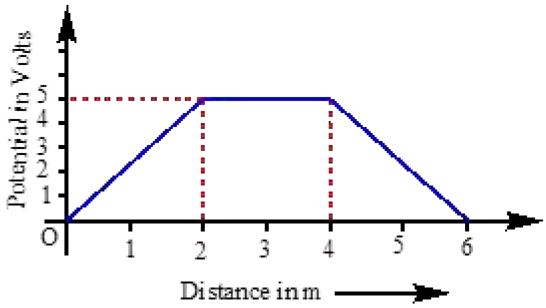

(A)


(B)


(C)

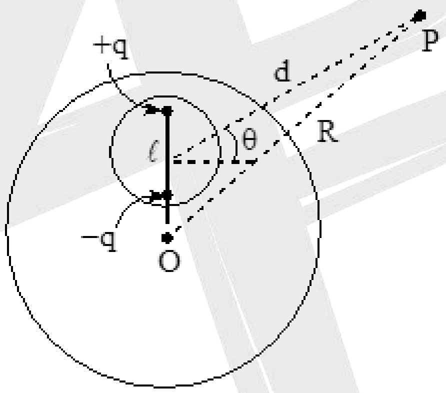

(D)

18. The variation of potential with distance R from fixed point is shown in the figure. The electric field at R = 5m is



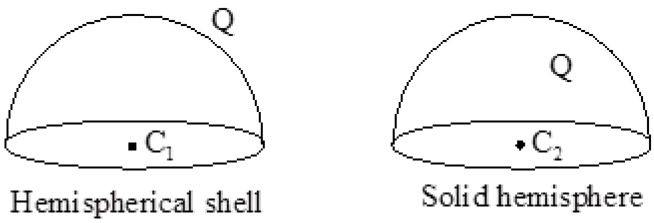
(A) 2.5 V/m (B) -2.5 V/m (C) $\left(\frac{2}{5}\right)V/m$ (D) $\left(\frac{-2}{5}\right)V/m$

19. A spherical cavity is created in a neutral solid conducting sphere. Inside the cavity, a dipole is placed as shown in the figure. Electrostatic potential at point P only due to charge induced on the inner surface of the cavity is (assume that $\ell << d$)



(A) $\frac{q\ell}{4\pi\varepsilon_0}\frac{\sin\theta}{d^2}$ (B) $\frac{-q\ell\sin\theta}{4\pi\varepsilon_0 d^2}$ (C) $-\frac{q\ell}{4\pi\varepsilon_0}\frac{\sin\theta}{d}$ (D) $\frac{q\ell\sin\theta}{4\pi\varepsilon_0 d}$

20. Charge Q each are distributed uniformly on a non-conducting hemispherical shell (on the surface) and in the volume of a non-conducting solid hemisphere. A point charge q is brought slowly at the centre C_1 of shell and at the centre C_2 of solid hemisphere from infinity.



If the work done in these two processes are W_1 and W_2 then, the ratio $\left(\frac{W_1}{W_2}\right)$ is

(A) $\frac{3}{4}$ (B) $\frac{3}{2}$ (C) $\frac{4}{3}$ (D) $\frac{2}{3}$

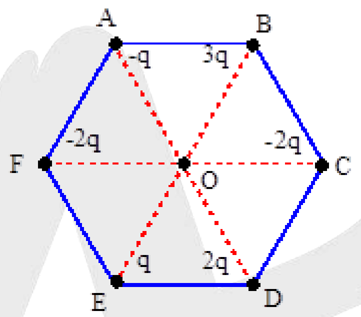
EXERCISE-2

1. Two free positive charges 4q and q are a distance ℓ apart. What charge Q is needed to achieve equilibrium for the entire system and where should it be placed from charge q?

(A) $Q=\frac{4}{9}q$ (negative) at $\frac{\ell}{3}$ (B) $Q=\frac{4}{9}q$ (positive) at $\frac{\ell}{3}$

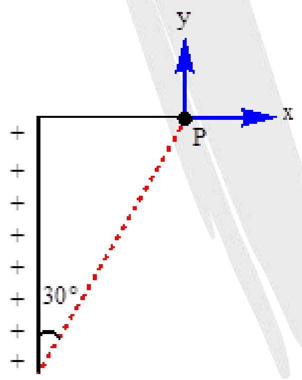
(C) $Q=q$ (positive) at $\frac{\ell}{3}$ (D) $Q=q$ (negative) at $\frac{\ell}{3}$

2. Six charges are placed at the corner of a regular hexagon as shown. if an electron is placed at its centre O, force on it will be



(A) Zero (B) along OF (C) along OC (D) none of the above

3. The direction (θ) of $\vec{E}$ at point P due to uniformly charged finite rod will be



(A) at angle 30° from x – axis (B) 45° from x – axis

(C) 60° from x – axis (D) None of the above

4. Two equal negative charges are fixed at the points [0, a] and [0, -a] on the y – axis. A positive charge Q is released from rest at the points [2a, 0] on the x – axis. The charge Q will

(A) execute simple harmonic motion about the origin

(B) move to the origin and remain at rest

(C) move to infinity

(D) execute oscillatory but not simple harmonic motion

5. An infinite non – conducting sheet of charge has a surface charge density of $10^{-7}C/m^2$. The separation between two equipotential surfaces near the sheet whose potential differ by 5V is $\frac{88}{n}mm$. Find n?

6. Two positively charged particles X and Y are initially far away from each other and at rest. X begins to move towards Y with some initial velocity. The total momentum and energy of the system are p and E

(A) If Y is fixed, both p and E are conserved

(B) If Y is fixed, E is conserved, but not p

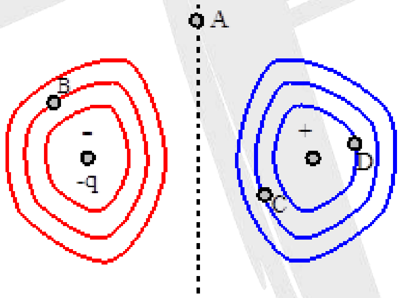
(C) If both are free to move, p is conserved but not E

(D) If both are free, E is conserved, but not p

7. The equation of an equipotential line in an electric field is $y = 2x$, then the electric field strength vector at (1, 2) may be

(A) $4\hat{i}+3\hat{j}$ (B) $4\hat{i}+8\hat{j}$ (C) $8\hat{i}+4\hat{j}$ (D) $-8\hat{i}+4\hat{j}$

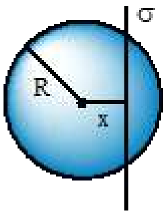
8. Figure shows equi – potential surfaces for a two charges system. At which of the labelled points will an electron have the highest potential energy?



(A) Point A (B) Point B (C) Point C (D) Point D

9. In a certain region of space, the potential is given by $V = k\left[2x^2 - y^2 + z^2\right]$. The electric field at the point (1, 1, 1) has magnitude is $2k\sqrt{\alpha}$. Find α ?

10. An infinite uniformly charged sheet with surface charge density σ cuts through a spherical Gaussian surface of radius R at a distance x from its centre, as shown in the figure. The electric flux ϕ through the Gaussian surface is



(A) $\frac{\pi R^2\sigma}{\varepsilon_0}$ (B) $\frac{2\pi\left(R^2-x^2\right)\sigma}{\varepsilon_0}$ (C) $\frac{\pi\left(R-x\right)^2\sigma}{\varepsilon_0}$ (D) $\frac{\pi\left(R^2-x^2\right)\sigma}{\varepsilon_0}$

11. A positive charge q is placed in a spherical cavity made in a positively charged sphere. The centres of sphere and cavity are separated by a small distance $\vec{\ell}$. Force on charge q is

(A) in the direction parallel to vector $\vec{\ell}$

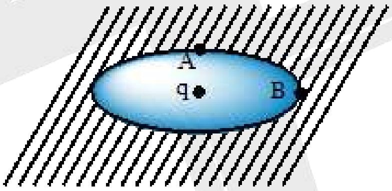
(B) in radial direction

(C) in a direction which depends on the magnitude of charge density in sphere

(D) direction cannot be determined

12. A non – conducting ring of radius 0.5m carries a total charge of 1.11×10^{-10}C distributed non – uniformly on its circumference producing an electric field $\vec{E}$ everywhere in space. The value of the line integral $\int_{\ell=\infty}^{\ell=0} -\vec{E}.d\vec{\ell}$ ($\ell=0$ being centre of the ring) in volts is

13. An ellipsoidal cavity is carved within a perfect conductor. A positive charge q is placed at the centre of the cavity. The points A and B are on the cavity surface as shown in the figure. Then



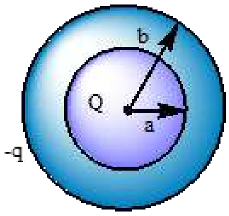
(A) electric field near A in the cavity = electric field near B in the cavity

(B) charge density at A = charge density at B

(C) potential at A = potential at B

(D) total electric field flux through the surface of the cavity is $\frac{q}{\varepsilon_0}$

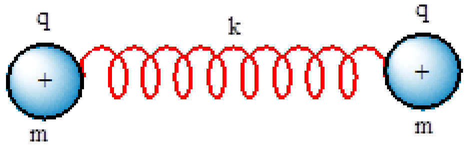
14. The system of charges as shown in the figure. A thick spherical shell with an inner radius a and an outer radius b is made of conducting material. A point charge +Q is placed at the centre of the spherical shell and a total charge –q is placed on the shell. Assume that the electrostatic potential is zero at an infinite distance from the spherical shell. The electrostatic potential at a distance R $(a<R<b)$ from the centre of the shell is



(A) 0 (B) $\frac{KQ}{a}$ (C) $K\frac{Q-q}{R}$ (D) $K\frac{Q-q}{b}$

(where $K=\frac{1}{4\pi\varepsilon_0}$)

15. The ratio of the time periods of small oscillations of the insulated spring and mass system before and after charging the masses is



16. A sphere of radius R is having charge Q uniformly distributed over it. The energy density of the electric field in the air, at a distance r is $\frac{Q^{\alpha}}{\beta\pi^{\gamma}\varepsilon_0 r^{P}}$. Find $\alpha+\beta+\gamma+P$? (r > R) is given by (in J/m^3)

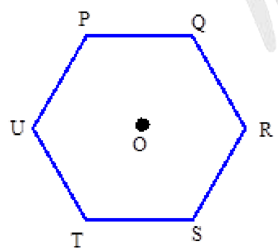
17. If V_o be the potential at origin in an electric field $\vec{E}=E_x\hat{i}+E_y\hat{j}$, then the potential at point P(x, y) is

(A) $V_o+xE_x+yE_y$ (B) $V_o+xE_x-yE_y$

(C) $V_o-xE_x-yE_y$ (D) $\sqrt{(x^2+y^2)}\sqrt{E_x^2+E_y^2}-V_o$

18. Electric charges q, q and -2q are placed at the corners of an equilateral triangle ABC of side L. The magnitude of electric dipole moment of the system is $\sqrt{\alpha}\,qL$. Find α ?

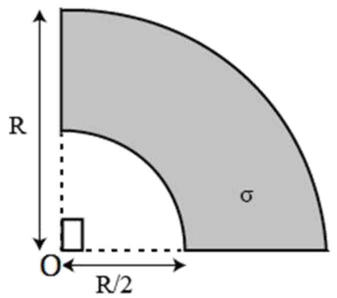
19. Six charges of equal magnitude, 3 positive and 3 negative are to be placed on PQRSTU corners of a regular hexagon, such that field at the centre is double that of what it would have been if only one +ve charge is placed at R. identify the correct sequence of charge.



(A) $+,+,+,-,-,-$ (B) $-,+,+,+,-,-$ (C) $-,+,+,-,+,-$ (D) $+,-,+,-,+,-$

20. A point charge q is placed at a point on the axis of a non – conducting circular plate of radius r at a distance $R(>>r)$ from its centre. The electric flux associated with the plate is $\frac{qr^{\alpha}}{4\pi\varepsilon_0 R^{\beta}}$. Find $\alpha+\beta$?

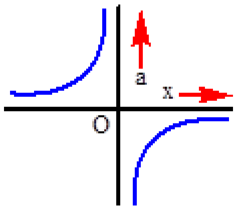
21. The diagram shows part of a disc of radius R carrying uniformly distributed charge from $\frac{R}{2}$ to R of density σ.Electric potential at the center C is

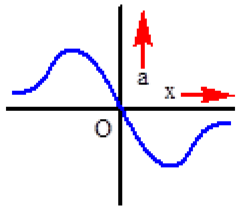


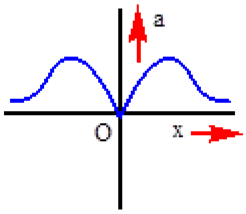
(A) $\frac{\sigma R}{2\in_0}$ (B) $\frac{\sigma R}{16\in_0}$ (C) $\frac{\sigma R}{24\in_0}$ (D) $\frac{\sigma R}{32\in_0}$

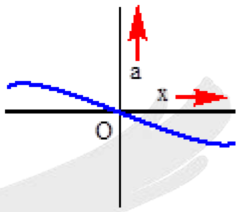
EXERCISE-3

1. Two identical positive charges are fixed on the y – axis, at equal distances from the origin O. A particle with a negative charge starts on the x – axis at a large distance from O, moves along the +x – axis, passes through O and moves far away from O. Its acceleration a is taken as positive along its direction of motion. The particle's acceleration a is plotted against its x – coordinate. Which of the following best represents the plot?

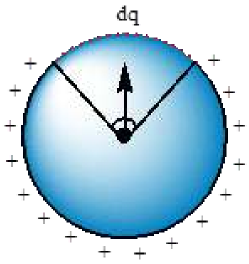
(A) 

(B) 

(C) 

(D) 

2. A non-conducting ring of radius R has uniformly distributed positive charge Q. A small part of the ring, of length d, is removed (d<R). The electric field at the centre of the ring will now be



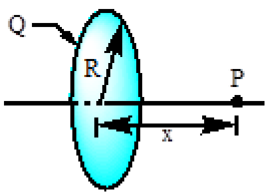
(A) directed towards the gap, inversely proportional to R^3

(B) directed towards the gap, inversely proportional to R^2

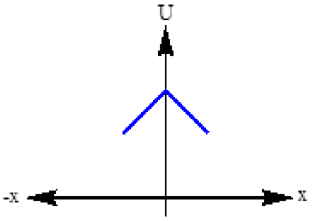
(C) directed away from the gap, inversely proportional to R^3

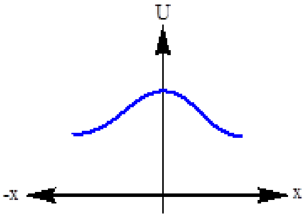
(D) directed away from the gap, inversely proportional to R^2

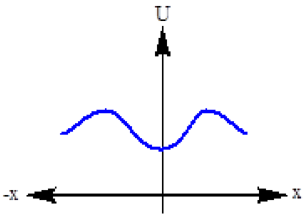
3. A small particle of mass m and charge –q is placed at point P on the axis of uniformly charged ring and released. If R > x, the particle will undergo oscillations along the axis of symmetry with an angular frequency that is equal to $\sqrt{\dfrac{\alpha Qq}{\beta\pi\varepsilon_0 R^{\gamma} m}}$. Find $\alpha+\beta+\gamma$

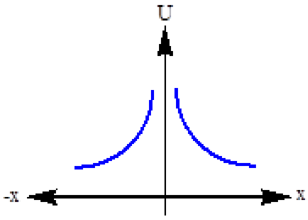


4. Four equal charges +q are placed at four corners of a square with its centre at origin and lying in yz plane. The electrostatic potential energy of a fifth charge +q varies on x – axis as

(A) 

(B) 

(C) 

(D) 

5. Two identical thin rings, each of radius R meter are coaxially placed at distance R meter apart. If Q_1 and Q_2 coulomb are respectively the charges uniformly spread on the two rings, the minimum work done in moving a charge q from the centre of one ring to that of the other is

(A) zero

(B) $\frac{q(Q_1-Q_2)(\sqrt{2}-1)}{(\sqrt{2}.4\pi\varepsilon_0 R)}$

(C) $\frac{q\sqrt{2}(Q_1+Q_2)}{4\pi\varepsilon_0 R}$

(D) $\frac{q(Q_1-Q_2)(\sqrt{2}+1)}{(\sqrt{2}.4\pi\varepsilon_0 R)}$

6. In a regular polygon of n sides, each corner is at a distance r from the centre. Identical charges are placed at (n – 1) corners. At the centre, the intensity is E and the potential is V. the ratio V/E has magnitude

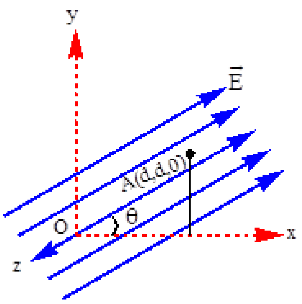
(A) r n

(B) r (n – 1)

(C) $\frac{(n-1)}{r}$

(D) $\frac{r(n-1)}{n}$

7. A uniform electric field having strength $\vec{E}$ is existing in x – y plane as shown in figure. Find the p.d. between origin O and A (d, d, 0)



(A) $Ed(\cos\theta+\sin\theta)$

(B) $-Ed(\sin\theta-\cos\theta)$

(C) $\sqrt{2}Ed$

(D) None of these

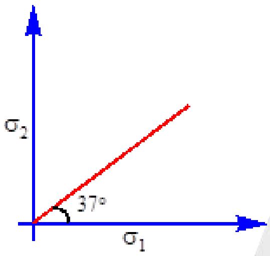
8. Uniform electric field of magnitude 100V/m in space is directed along the line $y = 3 + x$. Find the potential difference between point A(3, 1) and B(1, 3)

(A) 100V (B) $200\sqrt{2}$V (C) 200V (D) 0

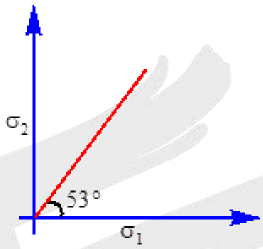
9. The dipole moment of a system of charge +q distributed uniformly on an arc of radius R subtending an angle $\pi/2$ at its centre where another charge –q is placed at centre

(A) $\frac{2\sqrt{2}qR}{\pi}$ (B) $\frac{\sqrt{2}qR}{\pi}$ (C) $\frac{qR}{\pi}$ (D) $\frac{2qR}{\pi}$

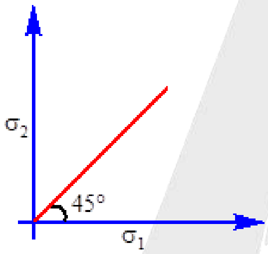
10. Two sphere of radii R_1 and R_2, joined by a fine long wire, are raised to a potential V. if σ_1 and σ_2 represent the respective surface charge densities of the spheres, then for $\frac{R_1}{R_2} = \frac{3}{4}$ we have



(A)



(B)



(C)

(D) None of these

11. Two point charges (Q each) are placed at (0, y) and (0, -y). A point charge q of the same polarity can move along X – axis. Then

(A) the force on q is maximum at $x = \pm\frac{y}{\sqrt{2}}$

(B) the charge q is in equilibrium at the origin

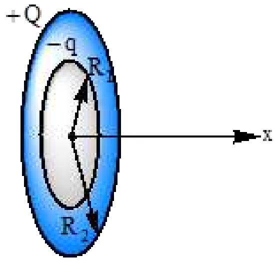
(C) the charge q performs an oscillatory motion about the origin

(D) for any position of q other then origin the force is directed away from origin

12. Three equal point changes (Q) are kept at the three corners of an equilateral triangle ABC of side a. P is a point having equal distance a from A, B and C. if E is the magnitude of electric field and V is the potential at point P, then

(A) $E = \frac{3Q}{4\pi\varepsilon_0 a^2}$ (B) $E = \frac{\sqrt{6}Q}{4\pi\varepsilon_0 a^2}$ (C) $E = \frac{3Q}{4\pi\varepsilon_0 a}$ (D) $E = \frac{3\sqrt{6}Q}{4\pi\varepsilon_0 a^2}$

13. Two concentric rings of radii $R_1 = \sqrt{6}$m and $R_2 = 4$m are placed in y – z plane with their centres at origin. They have uniform charge –q and $+Q = 2\sqrt{2}q$ on the inner and outer rings respectively. Consider the electrostatic potential to be zero at infinity. Then



(A) The electic potential is zero at origin

(B) The electric field intensity is zero at r = 2m

(C) A positive charged particle disturbed from origin along the x – axis will restore back to origin

(D) Where potential is maximum on the x – axis, field intensity is zero

14. Two particles A and B, each carrying a charge Q, are held fixed with a separation d between them. A particle C having a mass m and charge –q, in kept at the middle point of line AB. It displaced through a distance x perpendicular to AB. Assume x << d. Then :

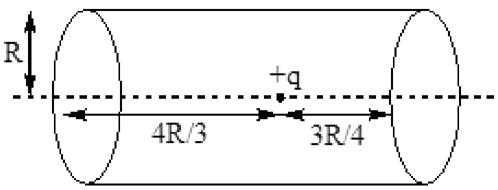
(A) Force experienced by C is proportional to x

(B) Force experienced by C is proportional to d

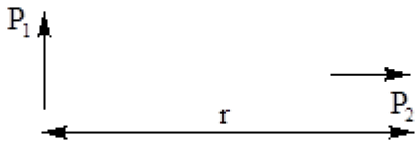
(C) Particle C may executes SHM with time period $\left(\frac{m\pi^3\varepsilon_0 d^3}{qQ}\right)^{1/2}$

(D) Particle C may executes SHM with time period $\left(\frac{m\pi^2\varepsilon_0 d^2}{qQ}\right)^{1/2}$

15. A point charge +q is placed on the axis of a closed cylinder of radius R and height 25R/12 as shown. If electric flux coming out from the curved surface of cylinder is $\frac{xq}{10\epsilon_0}$, then calculate x.

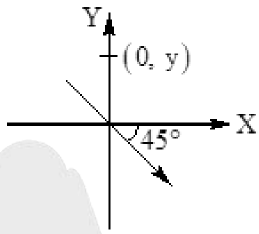


16. Two point electric dipoles with dipole moments 'P_1' and 'P_2' are separated by a distance 'r' with their dipole axes mutually perpendicular as shown. The force of interaction between the dipoles $\left(\text{where, } k = \frac{1}{4\pi\varepsilon_0}\right)$



(A) $\frac{2kP_1P_2}{r^4}$ (B) $\frac{3kP_1P_2}{r^4}$ (C) $\frac{4kP_1P_2}{r^4}$ (D) $\frac{6kP_1P_2}{r^4}$

17. A dipole is placed at origin of co-ordinate system as shown in figure. Electric field at point P (0, y) is given as



(A) $\frac{K\rho}{\sqrt{3}y3}\left(-\hat{i}-\hat{j}\right)$ (B) $\frac{K\rho}{y^3}\left(-\hat{i}+\sqrt{2}\,\hat{j}\right)$ (C) $\frac{K\rho}{y^3}\left(-\hat{i}-2\hat{j}\right)$ (D) $\frac{K\rho}{\sqrt{2}y^3}\left(-\hat{i}-2\hat{j}\right)$

18. Two fixed charges $-2Q$ and Q are located at the point of co-ordinates $(-3a,\,0)$ and $(3a,\,0)$ respectively in (x – y) plane. Then all the points in x – y plane where potential is zero lies on a

(A) straight line parallel to x-axis
(B) straight line parallel to y-axis
(C) a circle of radius 4a
(D) circle of radius 2a

19. Two free charges +Q and +4Q are placed at a separation L. The system to stay in equilibrium.

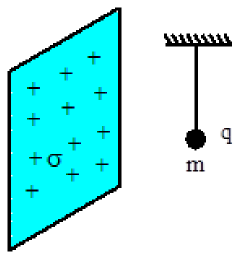
(A) The location of 3rd charge is at $\frac{L}{3}$ from Q are $\frac{2L}{3}$ for 4Q

(B) The location of 3rd charge is at $\frac{2L}{3}$ from Q are $\frac{L}{3}$ for 4Q

(C) $q=-\frac{4Q}{q}$

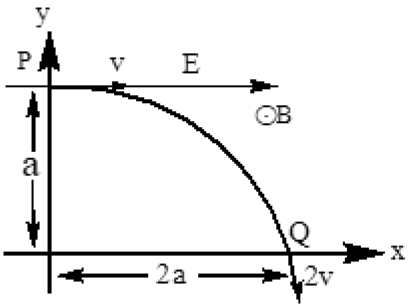
(D) $q=+\frac{4Q}{q}$

20. A small charged particle of mass m and charge q is suspended by an insulated thread in front of a very large sheet of charge density σ. The angle made by the thread with the vertical, in equilibrium is



(A) $\tan^{-1}\left(\frac{\sigma q}{2\varepsilon_0 mg}\right)$
(B) $\tan^{-1}\left(\frac{\sigma q}{\varepsilon_0 mg}\right)$
(C) $\tan^{-1}\left(\frac{2\sigma q}{\varepsilon_0 mg}\right)$
(D) zero

21. A particle of charge $+q$ and mass 'm' moving under the influence of a uniform electric field $E\hat{i}$ and uniform magnetic field $B\hat{k}$ follows a trajectory from P to Q as shown in figure. The velocities at P and Q are $v\hat{i}$ and $-2\hat{j}$. Which of the following statement(s) is/are correct?



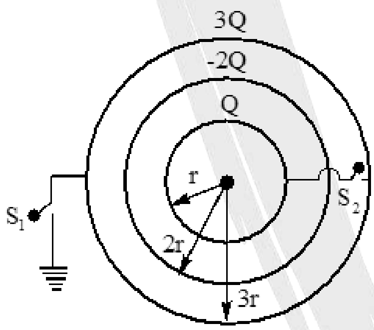
(A) $E=\frac{3}{4}\left[\frac{mv^2}{qa}\right]$

(B) Rate of work done by the electric field at P is $\frac{3}{4}\left[\frac{mv^3}{a}\right]$

(C) Rate of work done by the electric field at P is zero

(D) Rate of work done by both the fields at Q is zero

22. In the given diagram three concentric conducting charged spherical shells are indicated. Initially both the switches are open. Select the correct alternatives (s)



(A) If only switch S_2 is closed then the charge transferred through this switch will be $\frac{Q}{2}$

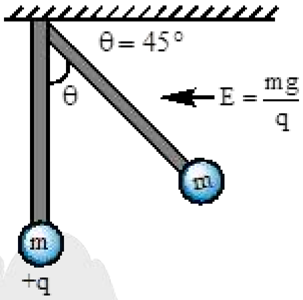
(B) If only switch S_2 is closed then the charge transferred through this switch will be $\frac{Q}{3}$

(C) If only switch S_1 is closed then the charge transferred through this switch will be $2Q$

(D)If only switch S_1 is closed then the charge transferred through this switch will be $\frac{Q}{2}$

EXERCISE-4

1. In space of horizontal EF (E = (mg)/q) exist as shown in figure and a mass m attached at the end of a light rod. If mass m is released from the position shown in figure, the angular velocity of the rod when it passes through the bottom most position is $\sqrt{\frac{\alpha g}{\beta \ell}}$. Find $\alpha+\beta$

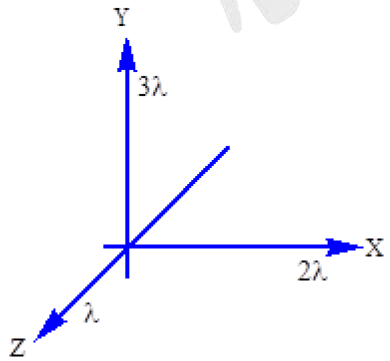


2. Two identical particles of mass m carry a charge Q each. Initially one is at rest on a smooth horizontal plane and the other is projected along the plane directly towards first particle from a large distance with speed ν. The closest distance of approach is

(A) $\frac{1}{4\pi\varepsilon_0}\frac{Q^2}{m\nu}$ (B) $\frac{1}{4\pi\varepsilon_0}\frac{4Q^2}{m\nu^2}$ (C) $\frac{1}{4\pi\varepsilon_0}\frac{2Q^2}{m\nu^2}$ (D) $\frac{1}{4\pi\varepsilon_0}\frac{3Q^2}{m\nu^2}$

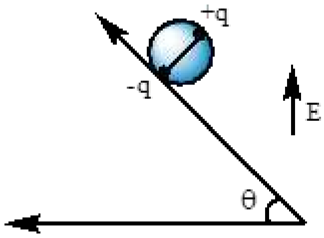
3. A charged particle of charge Q is held fixed and another charged particle of mass m and charge q (of the same sign) is released from a distance r. The impulse of the force exerted by the external agent on the fixed charge by the time distance between Q and q becomes 2r is $\sqrt{\frac{\alpha mQq}{\beta\pi\varepsilon_0 r}}$. Find $\alpha+\beta$

4. The diagram shows three infinitely long uniform line charges placed on the X, Y and Z axis. The work done in moving a unit positive charge from (1, 1, 1) to (0, 1, 1) is equal to

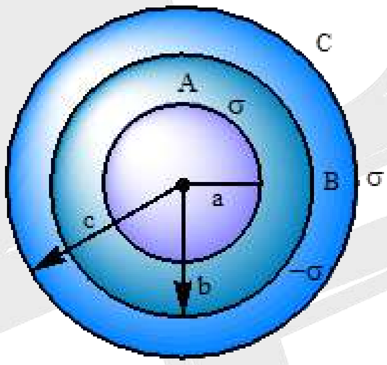


(A) $\frac{(\lambda \ln 2)}{2\pi\varepsilon_0}$ (B) $\frac{(\lambda \ln 2)}{\pi\varepsilon_0}$ (C) $\frac{(3\lambda \ln 2)}{2\pi\varepsilon_0}$ (D) None

5. A wheel having mass m has charges +q and –q on diametrically opposite points. It remains in equilibrium on a rough inclined plane in the presence of uniform vertical electric field E = $\frac{mg}{nq}$. Find n?



6. The figure shows charged spherical shells A, B and C having charge densities σ, $-\sigma$, σ and radii a, b , c respectively. If $V_A = V_C$, then b equal to



(A) $a+c$ (B) $\sqrt{a^2+c^2}$ (C) $\sqrt{ac}$ (D) c – a

7. A charged particle moves with a speed ν in a circular path of radius r around a long uniformly charged conductor. Then

(A) $\nu \propto r$ (B) $\nu \propto \frac{1}{r}$ (C) $\nu \propto \frac{1}{\sqrt{r}}$ (D) ν is independent of r

8. A fixed point charge Q is at origin. At t = 0, a charge q with mass m is at x = a with leftward velocity V_0 which satisfies $\frac{kQq}{a} = 3mV_0^2$. The particle turns around and starts to move rightward at the position $b < a$.

(A) The ratio $\frac{b}{a}$ is $\frac{6}{7}$ (B) The ratio $\frac{b}{a}$ is $\frac{3}{7}$

(C) The velocity of the particle at a large distance from the origin is $\sqrt{7}V_0$

(D) The velocity of the particle at a large distance from the origin is zero

9. You are moving a negative charge $q < 0$ at a small constant speed away from a uniformly charged non – conducting spherical shell on which resides a negative charge Q. the electrostatic field of Q is E. let U be the total energy of the system, W_a the work done by the force F_a you exert on q and W_E the work done by the electrostatic force F_E on q. then, as q is being moved

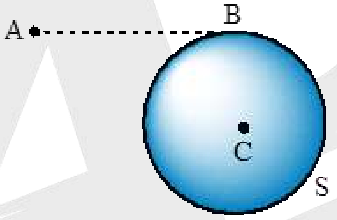
(A) $W_a = -W_E$, therefore U remains constant

(B) $F_a = -F_E$

(C) U increases

(D) U decreases

10. S is a solid neutral conducting sphere. A point charge q of $1\times10^{-6}C$ is placed at point A. C is the centre of sphere and AB is a tangent. BC = 3m and AB = 4m.



(A) the electric potential of the conductor is 1.8 kV

(B) the electric potential of the conductor is 2.25 kV

(C) the electric potential at B due to induced charges on the spheres is -0.45 kV

(D) the electric potential at B due to induced charges on the spheres is 0.45 kV

11. Two infinite, parallel, non – conducting sheets carry equal positive charge denstiy σ. One is placed in the yz plane and the other at distance x = a. take potential V = 0 at x = 0

(A) For $0 \le x \le a$, potential $V_x = 0$

(B) For $x \ge a$, potential $V_x = -\frac{\sigma}{\varepsilon_0}(x-a)$

(C) For $x \ge a$, potential $V_x = \frac{\sigma}{\varepsilon_0}(x-a)$

(D) For $x \le 0$, potential $V_x = \frac{\sigma}{\varepsilon_0}x$

12. Small identical balls with equal charges are fixed at the vertices of a regular polygon of N sides, each of length d. at a certain instant, one of the ball is released. After a long time interval, the adjacent ball to the previous one is released. The difference in kinetic energies of the two released balls is K at a sufficiently long distance from the polygon

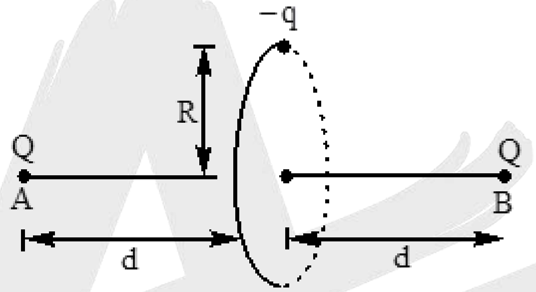
(A) Final kinetic energy of the first ball is greater than of the second ball

(B) Final kinetic energy of the second ball is greater than that of the first ball

(C) Charge on each ball is $\sqrt{2\pi\varepsilon_0 dK}$

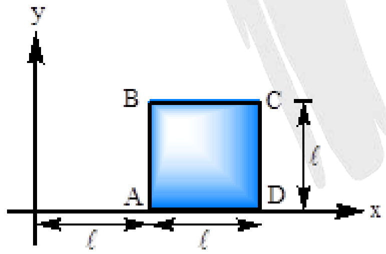
(D) Charge on each ball is $\sqrt{4\pi\varepsilon_0 dK}$

13. Two positive point charges each of magnitude 10C are fixed at positions A and B at a separation $2d = 6m$. A negatively charged particle of mass m = 90 gm and charge of magnitude $10\times10^{-6}C$ is revolving in a circular path of radius 4m in the plane perpendicular to the line AB and bisecting the line AB. Neglect the effect of gravity. Find the angular velocity of the particle. If gravity is also considered will it still move in the circular path assuming AB to be horizontal.



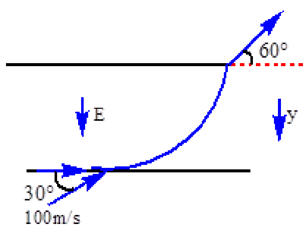
(A) 100 rad/s (B) 200 rad/s (C) 300 rad/s (D) 400 rad/s

14. A square loop of side 'ℓ' having uniform linear charge density 'λ' is placed in 'xy' plane as shown in figure. There is a non – uniform electric field $\vec{E} = \frac{a}{\ell}(x+\ell)\hat{i}$ where a is a constant. Find the resultant electric force on the loop if $\ell = 10cm$, $a = 2N/C$ and charge density $\lambda = 2\mu C/m$



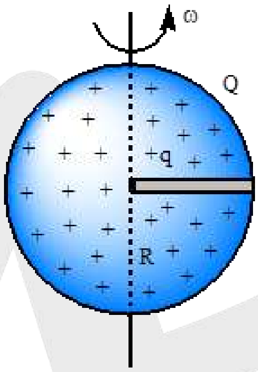
(A) $2\times10^{-6}N$ (B) $4\times10^{-6}N$ (C) $6\times10^{-6}N$ (D) $8\times10^{-6}N$

15. Find the magnitude of uniform electric field E in N/C (direction shown in figure) if an electron entering with velocity 100m/s making $30°$ comes out making $60°$ (see figure), after a time numerically equal to $\frac{m}{e}$ of electron.



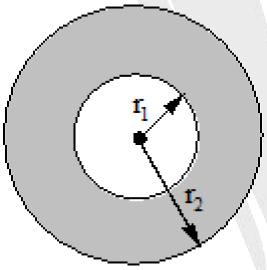
(A) 25 (B) 50 (C) 75 (D) 100

16. A small ball of mass 1kg and charge $\frac{2}{3}\mu C$ is placed at the centre of a uniformly charged sphere of radius 1m and charge $\frac{1}{3}mC$. A narrow smooth groove is made in the sphere from centre to surface as shown in figure. The sphere is made to rotate about its vertical diameter at a constant rate of $\frac{1}{2\pi}$ revolutions per second. Find the speed w.r.t. ground with which the ball slides out from the groove. Neglect any magnetic force acting on ball.



(A) 1 m/sec (B) 2 m/sec (C) 3 m/sec (D) 4 m/sec

17. In given diagram, there is a conducting sphere of radius r_1 which is surrounded by dielectric $(\in_r)$. If conducting sphere is given charge 'q' then surface density of polarization charges on outer surface of dielectric layer is



(A) $\frac{\in_r q}{4\pi r_2^2}$ (B) $\frac{q}{4\pi \in_r r_2^2}$ (C) $\frac{(\in_r - 1)q}{4\pi r_2^2}$ (D) $\frac{(\in_r - 1)q}{4\pi \in_r r_2^2}$

18. Three particles, each of mass 1g and carrying a charge q are suspended from a common point by three insulated massless strings, each 100 cm long. If the particles are in equilibrium and are located at the corners of an equilateral triangle of side length 3 cm, calculate the charge q on each particle. $(\text{Take } g = 10 \text{ ms}^{-2})$.

(A) $1.17\times10^{-9}C$ (B) $2.17\times10^{-9}C$ (C) $3.17\times10^{-9}C$ (D) $4.17\times10^{-9}C$

19. Two identical positive point charges, each having a charge Q are fixed at a separation 2a. A point charge q lies midway between the fixed charges. For small

(1) displacement (compared to a) along the line joining the fixed charges, the charge q executes SHM, if it is positive in nature.

(2) lateral displacement, the charge q executes SHM, if it is negative in nature. The ratio of periods of oscillations in the above two cases.

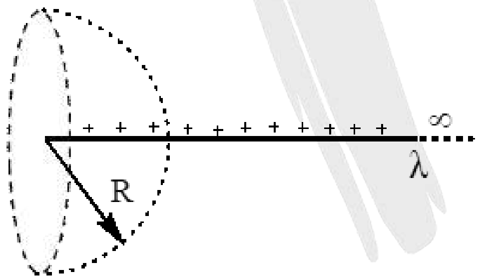
(A) $\frac{1}{\sqrt{2}}$ (B) $\frac{1}{\sqrt{3}}$ (C) $\frac{1}{2}$ (D) $\frac{1}{3}$

20. A charge +q is fixed at each of the points $x = x_0$, $x = 3x_0$, $x = 5x_0$,......., ∞ on the x – axis and a charge –q is fixed at each of the points $x = 2x_0$, $x = 4x_0$, $x = 6x_0$,......., ∞. Here, x_0 is a positive quantity. Take the electric potential at a point due to charge Q at a distance r from it to be $\frac{Q}{4\pi\varepsilon_0 r}$.

Then, the potential at origin due to the above system of charges is

(A) 0 (B) ∞ (C) $\frac{q}{8\pi\varepsilon_0 x_0 \ln 2}$ (D) $\frac{q \ln 2}{4\pi\varepsilon_0 x_0}$

21. Consider an imaginary hemispherical surface. A semi–infinite wire of charge density λ is kept with one of its end coinciding with center of hemisphere and wire is kept along the symmetric axis of the hemisphere as shown in the figure. The electric flux through the spherical surface (Curved surface) of the hemisphere is



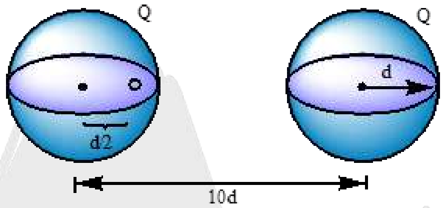
(A) $\frac{\lambda R}{3\epsilon_0}$ (B) $\frac{\lambda R}{4\epsilon_0}$ (C) $\frac{3\lambda R}{4\epsilon_0}$ (D) $\frac{\lambda R}{2\epsilon_0}$

EXERCISE-5

1. A small electric dipole is placed at origin with its axis being directed along the positive x – axis. The direction of electric field due to the dipole at a point $\left(1\text{m}, \sqrt{2}\text{m}, 0\right)$ is along the

(A) z – axis (B) y – axis (C) x – axis (D) line y = x

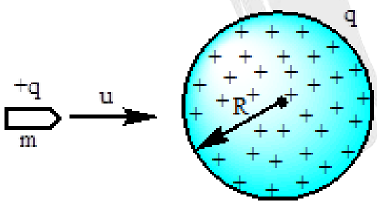
2. Two spherical, non – conducting, and very thin shells of uniformly distributed positive charge Q and radius d are located a distance 10d from each other. A positive point charge q is placed inside one of the shells at a distance d/2 from the centre, on the line connecting the centres of the two shells, as shown in the figure. What is the net force on the charge q?



(A) $\frac{qQ}{361\pi\varepsilon_0 d^2}$ to the left (B) $\frac{qQ}{361\pi\varepsilon_0 d^2}$ to the right

(C) $\frac{362qQ}{361\pi\varepsilon_0 d^2}$ to the left (D) $\frac{360qQ}{361\pi\varepsilon_0 d^2}$ to the right

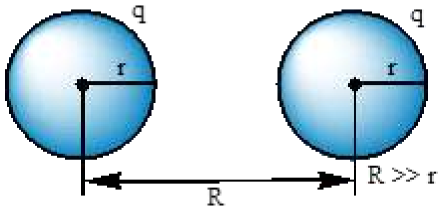
3. A bullet of mass m and charge q is fired towards a solid uniformly charged sphere of radius R and total charge +q. If it strikes the surface of sphere with speed u, the minimum speed u so that it can penetrate through the sphere is $\frac{\alpha q}{\sqrt{\beta\pi\varepsilon_0 mR}}$. Find $\alpha+\beta$?



(Neglect all resistance forces or friction acting on bullet except electrostatic forces)

4. There are four concentric shells A, B, C and D of radii a, 2a, 3a and 4a respectively. Shells B and D are given charges +q and –q respectively. Shell C is now earthed. The potential difference $V_A - V_C$ is $\frac{Kq}{na}$. Find n?

5. The potential energy of a system of two identically charged hollow spheres (as shown in the figure) is equal to $\frac{q^\alpha}{\beta\pi\varepsilon_0}\left[\frac{1}{R}-\frac{1}{r}\right]$. find $\alpha+\beta$?(Assume the charge distribution to be uniform)

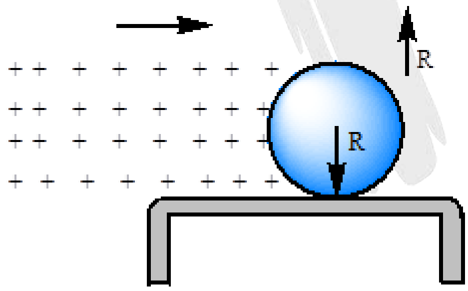


6. If charges q/2 and 2q are placed at the centre of face and at the corner of a cube, then the total flux through the cube is $\frac{q}{\alpha\varepsilon_0}$. Find α

7. A large insulating thick sheet of thickness 2d carries a uniform charge per unit volume ρ. A particle of mass m, carrying a charge q having a sign opposite to that of the sheet, is released from the surface of the sheet. The sheet does not offer any mechanical resistance to the motion of the particle. Find the oscillation frequency ν of the particle inside the sheet.

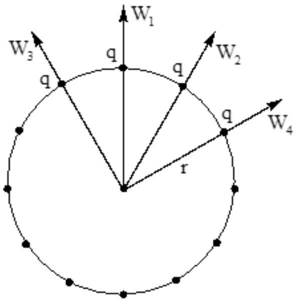
(A) $\nu = \frac{1}{2\pi}\sqrt{\frac{q\rho}{m\varepsilon_0}}$ (B) $\nu = \frac{1}{2\pi}\sqrt{\frac{2q\rho}{m\varepsilon_0}}$ (C) $\nu = \frac{1}{4\pi}\sqrt{\frac{q\rho}{m\varepsilon_0}}$ (D) $\nu = \frac{1}{4\pi}\sqrt{\frac{2q\rho}{m\varepsilon_0}}$

8. Figure shows a metal ball of mass 50kg and radius $\frac{2}{\sqrt{\pi}}$ m is placed on an insulating uncharged stand. In space an upward electric field $5\times10^5 N/C$ is switched on. A stream of light ions is incident on the ball from left side at a speed $2\times10^6 m/s$ as shown in figure. If charge on ball at t = 0 was zero, find the time at which ball will be lifted from the stand. The charge density of ion beam is 5×10^{-12} coulomb m^3. Assume that all charge incident on the ball is absorbed.



(A) 5sec (B) 15 sec (C) 25 sec (D) 35 sec

9. N identical charges each of charge 'q' are symmetrically placed on the circumference of circle of radius 'r'. Four adjacent charges are slowly taken to infinity in the sequence a shown in figure. If W_1, W_2, W_3 and W_4 is the respective work done in slowly displacing q_1, q_2, q_3 and q_4. If $(W_3 - W_4)$ is $x\left[\frac{-q^2}{4\pi t_0 r \sin\frac{3\pi}{n}}\right]$ then ___

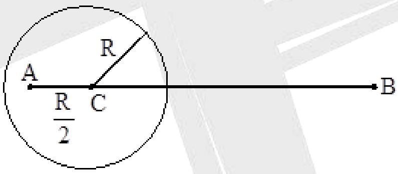


(A) 0.20 (B) 0.50 (C) 0.71 (D) 0.82

10. Two equal charges Q are at opposite corners of a square of side 'a' and an electric dipole of moment p is at a third corner, pointing towards one of the charges. If $p = 2\sqrt{2}a$ Q; The field strength at the fourth corner of the square is :-

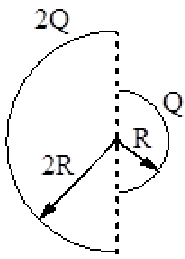
(A) $\dfrac{\sqrt{17/2}}{4\pi\in_0 a^2}Q$ (B) $\dfrac{\sqrt{17/4}}{4\pi\in_0 a^2}Q$ (C) $\dfrac{Q}{4\sqrt{2}\pi\in_0 a^2}$ (D) $\dfrac{\sqrt{3}}{4\pi\in_0 a^2}Q$

11. Consider a thin conducting shell of radius R carrying total charge Q. Two point charges Q and 2Q are placed on points A and B, which are at a distances of $\frac{R}{2}$ and 2R from the center of the cell respectively as shown in the figure. If the cell is earthed how much charge will flow to the earth?



(A) 3Q (B) Q (C) 2Q (D) $\dfrac{Q}{2}$

12. Two hemispherical thin shells made of insulating materials are concentrically arranged in a free space as shown. The radii of the smaller and larger hemispheres are 'R' and '2R' and they carry positive charges 'Q' and '2Q' respectively. The charges are uniformly distributed over the surfaces of the shells. Then choose the correct option(s).



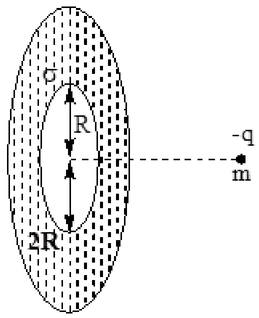
(A) The electrostatic force between the hemispherical shells is $\dfrac{Q^2}{8\pi\varepsilon_0 R^2}$

(B) The electrostatic force between the hemispherical shells is $\dfrac{Q^2}{16\pi\varepsilon_0 R^2}$

(C) The net electric potential at the centre 'O' of the shells is $\dfrac{Q}{2\pi\varepsilon_0 R}$

(D) The net electric potential at the centre 'O' of the shells is $\dfrac{3Q}{2\pi\varepsilon_0 R}$

13. A charged fixed annular disc of uniform positive charge density σ has inner and outer radii R and 2R respectively. A negatively charged particle of mass m and charge –q is released on the axis of the disc from a distance x from the centre. Then



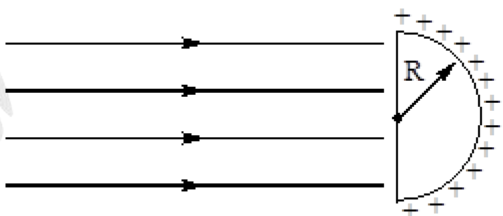
(A) The speed of the particle when reaches the centre of ring, $v=\sqrt{\frac{q\sigma}{m\varepsilon_0}\left(R+\sqrt{R^2+x^2}-\sqrt{4R^2+x^2}\right)}$

(B) The speed of the particle when it reaches the centre of ring, $v=\sqrt{\frac{q\sigma}{m\varepsilon_0}\left(R+\sqrt{4R^2+x^2}-\sqrt{R^2+x^2}\right)}$

(C) Work done by electrostatic force till the particle reaches the centre of ring $=\frac{q\sigma}{2\varepsilon_0}\left(R+\sqrt{R^2+x^2}-\sqrt{4R^2+x^2}\right)$

(D) If x << R, the frequency of oscillation of the particle is $f=\frac{1}{2\pi}\sqrt{\frac{q\sigma}{4\varepsilon_0 mR}}$

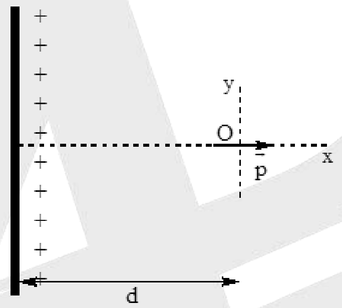
14. A semi-circular disc of mass M and radius R with linear charge density λ on its curved circumference is hinged at its centre and placed in a uniform electric field as shown in the figure. Match the statements from List I with those in List II and select the correct answer using the code given below the lists.



	List – I		List – II
(A)	The net force acting on the ring is	(P)	$\pi\sqrt{\frac{M}{\lambda E}}$
(B)	If the ring is slightly rotated about O and released, find its time period of oscillation	(Q)	$\pi R\lambda E$
(C)	The work done by an external agency to rotate it through an angle α is	(R)	$E-\frac{2K\lambda}{R}$
(D)	Magnitude of electric field at 'O' will be	(S)	$4\lambda ER^2\sin^2\left(\frac{\alpha}{2}\right)$

	A	B	C	D
(A)	P	Q	R	S
(B)	Q	S	P	R
(C)	Q	P	S	R
(D)	R	S	P	Q

15. An infinite non-conducting plate with uniform charge density σ is kept parallel to yz plane and at a distance 'd' from a dipole $\vec{p}$ which itself is located at the origin. An equipotential surface for this system is spherical, centred at origin, having radius R(<d). Given that $R=\left(\frac{p}{n\pi\sigma}\right)^{1/3}$; find the integer n.



16. A small ball of mass 1 kg and charge $8\ \mu C$ is placed at the centre of a uniformly charged sphere of radius 2m and charge $\frac{1}{6}$mC. A narrow smooth groove is made in the sphere from centre to the surface of sphere and the sphere is made to rotate about its vertical diameter at a constant rate of $\frac{1}{4\pi}$ revolution per second. Find the speed (in m/s) with respect to ground with which the ball slides out from the groove. Ignore any magnetic force acting on the ball. (Take $\sqrt{2}=1.41$)

(A) 2.82 (B) 1.82 (C) 0.82 (D) 3.82

17. An infinite non-conducting uniformly charged sheet has a hole of radius 'R' in it (change density = σ). An electron is placed on an axis passing through the centre of hole and perpendicular to the plane of sheet at a distance 'R' from the centre. The speed with which the electron reaches the centre is ....

$(\sigma > 0)$ Given : electric field on the axis of a uniformly charged disc of radius 'r'

$$E(x)=\frac{\sigma}{2\in}\left[1-\frac{x}{\sqrt{x^2+r^2}}\right]$$

(A) $\sqrt{\frac{\sigma eR}{m\varepsilon_0}}$ (B) $\sqrt{\frac{2\sigma eR}{m\varepsilon_0}}$ (C) $\sqrt{\frac{\left(\sqrt{2}-1\right)\sigma eR}{m\varepsilon_0}}$ (D) $\sqrt{\frac{\sqrt{2}\sigma eR}{m\varepsilon_0}}$

18. If electric force $\left(\vec{F}\right)$ on a point charge 'q' due to another charge 'Q' obeys following law

$$\vec{F}=\frac{Qq\left(1-\sqrt{\alpha r}\right)\vec{r}}{4\pi\epsilon_0 r^3}$$

Where α; positive constant, $\vec{r}$ is position vector of charge 'a' relative to 'Q'.

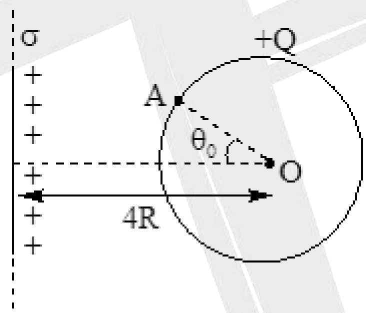
(A) Electric field to point charge Q is $\vec{E}=\frac{Q\left(1-\sqrt{\alpha r}\right)}{4\pi\epsilon_0 r^3}\vec{r}$

(B) $\oint\vec{E}\cdot\vec{d\ell}$ over a closed path will be equal to zero

(C) Gauss's law $\oint\vec{E}\cdot\vec{ds}=\frac{q_{enclosed}}{\epsilon_0}$ holds true

(D) All statements are correct

19. A conducting sphere of radius R and charge Q is placed near a uniformly charged non-conducting infinitely large thin plate having surface charge density σ. Then find the potential at point A (on the surface of sphere) due to charge on sphere $\left(\text{here } K=\frac{1}{4\pi\epsilon_0},\ \theta_0=\frac{\pi}{3}\right)$



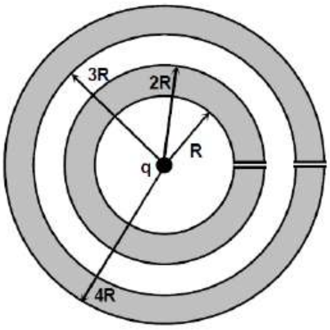
(A) $K\frac{Q}{R}-\frac{\sigma}{4\epsilon_0}R$ (B) $K\frac{Q}{R}-\frac{\sigma}{2\epsilon_0}$ (C) $K\frac{Q}{R}$ (D) None of these

20. A segment of a charged wire of length ℓ, charge density λ_1, and an infinitely long charged wire, charge density λ_1, lie in a place at right angles to each other. The separation between the wires is r_0. Determine the force of interaction between the wires.

(A) $F=\frac{\lambda_1\lambda_2}{2\pi\varepsilon_0}\log_e\left[1+\frac{\ell}{r_0}\right]$ (B) $F=\frac{\lambda_1\lambda_2}{4\pi\varepsilon_0}\log_e\left[1+\frac{\ell}{r_0}\right]$

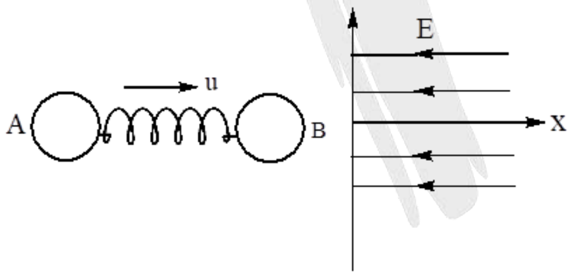
(C) $F=\frac{2\lambda_1\lambda_2}{\pi\varepsilon_0}\log_e\left[1+\frac{\ell}{r_0}\right]$ (D) $F=\frac{4\lambda_1\lambda_2}{\pi\varepsilon_0}\log_e\left[1+\frac{\ell}{r_0}\right]$

21. A charge particle ‘q’ lies at the center of two concentric hollow spheres of inner radii R and 3R and outer radii 2R and 4R respectively. What amount of work has to be performed to slowly transfer the charge ‘q’ from center through the orifice to infinity.



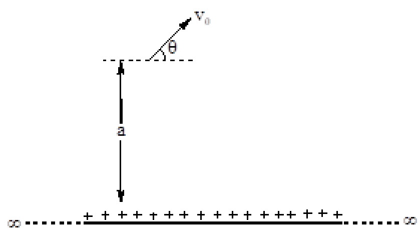
(A) $\frac{5}{96}\frac{q^2}{\pi\varepsilon_0 R}$ (B) $\frac{q^2}{16\pi\varepsilon_0 R}$ (C) $\frac{7}{96}\frac{q^2}{\pi\varepsilon_0 R}$ (D) $\frac{q^2}{96\pi\varepsilon_0 R}$

22. A dumbbell like structure is made by affixing two particles A and B at the ends of a light non conducting spring. Both the particles have equal mass m and the particle B carries a positive charge q. A uniform electric field of intensity E pointing in the negative x – direction is established in the region x > 0 and gravity is absent everywhere. The dumbbell is initially at rest on the x – axis in the region x < 0. It is projected with a velocity u in the positive x – direction as shown in the figure. After a while, the dumbbell is observed moving in the negative x – direction with the same speed u. During this interval, particle A never enters the region of electric field and the spring length becomes minimum only once. How much time the particle B spends in the electric field? (spring is in its natural length before entering and after coming of the electric field)



(A) $\frac{mu}{2qE}$ (B) $\frac{2mu}{qE}$ (C) $\frac{mu}{4qE}$ (D) $\frac{4mu}{qE}$

23. There is an infinite line of uniform linear density of charge $+\lambda$. A particle of charge ‘-q’ and mass ‘m’ is projected with initial velocity v_0 at an angle θ with the line of charge from a distance ‘a’ from it. The speed of the particle is found to be minimum when its distance from the line of charge is $ae^{(Km\varepsilon_0 v_0^2\sin^2\theta/q\lambda)}$.Then the Value of K is

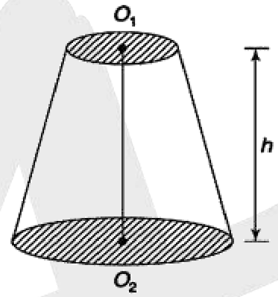


(A) π (B) 2π (C) 3π (D) 4π

24. Two long wires have uniform charge density λ per unit length each. The wires are non-coplanar and mutually perpendicular. Shortest distance between them is d. The interaction force between them is:

(A) $\frac{\lambda^2}{2\varepsilon_0}$ (B) $\frac{\lambda^2}{2\pi\varepsilon_0}$ (C) $\frac{2\lambda^2}{\varepsilon_0}$ (D) $\frac{\lambda^2}{4\varepsilon_0}$

25. A frustum is cut from a right circular cone. The two circular faces have radii R and 2R and their centers are at O_1 and O_2 respectively. Height of the frustum is h = 3R. When a point charge Q is placed at O_1, the flux of electric field through the circular face of radius 2R is ϕ_1 and when the charge Q is placed at O_2, the flux through the other circular face is ϕ_2. Then



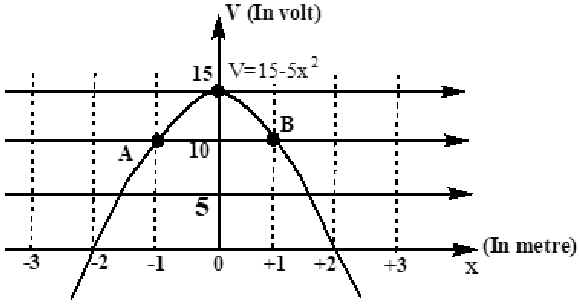
(A) $\frac{\phi_1}{\phi_2}=\left(\frac{\sqrt{13}-3}{\sqrt{10}-3}\right)\sqrt{\frac{10}{13}}$ (B) $\frac{\phi_1}{\phi_2}=\left(\frac{\sqrt{13}-3}{\sqrt{10}-3}\right)\sqrt{\frac{13}{10}}$

(C) $\frac{\phi_1}{\phi_2}=\left(\frac{\sqrt{10}-3}{\sqrt{13}-3}\right)\sqrt{\frac{10}{13}}$ (D) $\frac{\phi_1}{\phi_2}=\left(\frac{\sqrt{13}-6}{\sqrt{10}-6}\right)\sqrt{\frac{10}{13}}$

26. A non-conducting sphere of radius 'a' and unit permittivity has a surface charge density 'σ' varying with polar angle 'θ' as $\sigma=\sigma_0\cos\theta$ [σ_0 is a positive constant]. Intensity of electric field at (r,θ) where $r<a$ is given by

(A) $\frac{\sigma_0\cos\theta}{3\epsilon_0}$ (B) $\frac{\sigma_0 r}{3\epsilon_0 a}$ (C) $\frac{\sigma_0}{3\epsilon_0}$ (D) $\frac{\sigma_0 r\cos\theta}{3\epsilon_0 a}$

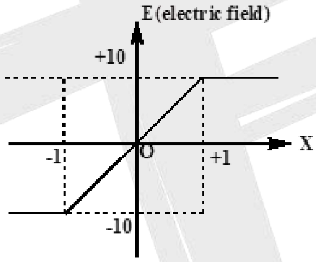
27. The figure shows the variation of electrostatic potential V in volt with the distance of position of point along x – axis from origin due to continuous volume charge distribution. In the region x = -1 m to X = + 1m, the graph is parabolic $\left(V=15-5x^2\right)$ and rest portion of graph is straight line. Choose the correct option(s) ($\varepsilon_0 \Rightarrow$ permittivity of free space = $8.85\times10^{-12} N^{-1}m^{-2}C^2$). The direction of $\vec{E}$ along Positive x – axis is considered as positive.



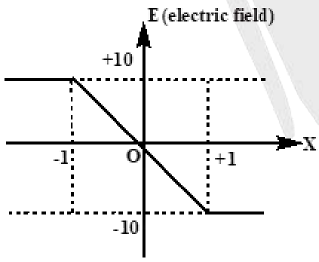
(A) This graph of potential may be due to a thick sheet of infinite dimension $(-1m \le x \le 1m, -\infty < y < \infty and -\infty < z < \infty)$ with constant volume charge density $1.77 \times 10^{-10} C/m^3$

(B) This graph of potential may be due to a thick sheet of infinite dimension $(-1m \le x \le 1m, -\infty < y < \infty and -\infty < z < \infty)$ with constant volume charge density $0.885 \times 10^{-10} C/m^3$

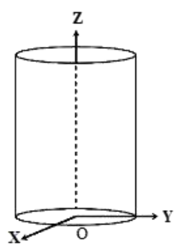
(C) The variation of electric field with the distance due to a thick sheet of infinite dimension $(-1m \le x \le 1m, -\infty < y < \infty and -\infty < z < \infty)$ may be



(D) The variation of electric field with the distance due to a thick sheet of infinite dimension $(-1m \le x \le 1m, -\infty < y < \infty and -\infty < z < \infty)$ may be



28. The electric field in the space is given by $\vec{E} = E_0\left(x\hat{i} + yj + zk\right)$. Consider a right circular cylindrical surface whose radius is 'a' and height 'h'. Now choose the correct option(s).



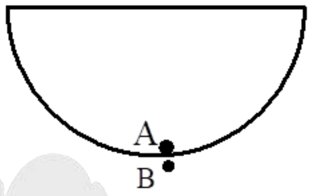
(A) The electric flux through lower circular base is zero

(B) The electric flux through upper circular top is $\pi a^2 h E_0$

(C) The electric flux through lateral surface is $2\pi a^2 h E_0$

(D) The total electric flux through cylindrical surface is $4\pi a^2 h E_0$

29. A charged hemispherical shell has a uniform charge density σ. Calculate the ratio of electric field due to shell at A to that at B



Proficiency Test-1

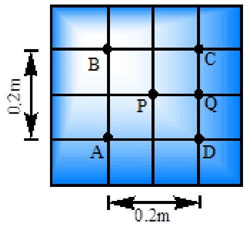
1. A, B, C, D, P and Q are points in a uniform electric field. The potentials at these points are

V(A) = 2 volt

V(P) = V(B) = V(D) = 5 volt

V(C) = 8 volt

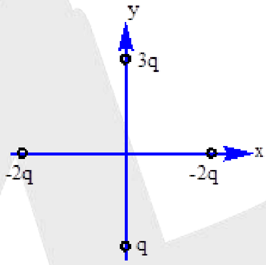
The electric field at P is



(A) $10\,Vm^{-1}$ along PQ (B) $15\sqrt{2}\,Vm^{-1}$ along PA

(C) $5\,Vm^{-1}$ along PC (D) $5\,Vm^{-1}$ along PA

2. 4 charges are placed each at a distance a from origin. The dipole moment of configuration is



(A) $2qa\hat{j}$ (B) $3qa\hat{j}$ (C) $2aq\left[\hat{i}+\hat{j}\right]$ (D) none of these

3. Three concentric metallic spherical shell A, B and C or radii a, b and c (a < b < c) have surface charge densities $-\sigma$, $+\sigma$, and $-\sigma$ respectively. The potential of shell A is

(A) $\left(\frac{\sigma}{\varepsilon_0}\right)[a+b-c]$ (B) $\left(\frac{\sigma}{\varepsilon_0}\right)[a-b+c]$

(C) $\left(\frac{\sigma}{\varepsilon_0}\right)[b-a-c]$ (D) None of these

4. Two identical small conducting spheres, having charges of opposite sign, attract each other with a force of 0.108N when separated by 0.5m. The spheres are connected by a conducting wire, which is then removed, and thereafter, they repel each other with a force of 0.036N. the initial charges on the spheres are

(A) $\pm 5\times10^{-6}C$ and $\mp 15\times10^{-6}C$ (B) $\pm 1.0\times10^{-6}C$ and $\mp 3.0\times10^{-6}C$

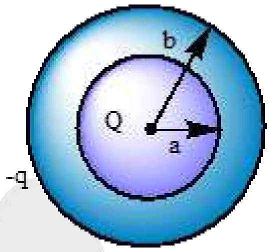
(C) $\pm 2.0\times10^{-6}C$ and $\mp 6.0\times10^{-6}C$ (D) $\pm 0.5\times10^{-6}C$ and $\mp 1.5\times10^{-6}C$

5. A solid sphere of radius R is charged uniformly. At what distance from its surface is the electrostatic potential half of the potential at the centre?

(A) R (B) $\frac{R}{2}$ (C) $\frac{R}{3}$ (D) 2R

6. n small conducting drops of same size are charged to V volts each. If they coalesce to form a single large drop, then its potential is $Vn^{\alpha/\beta}$. Find $\alpha+\beta$?

7. The system of charges as shown in the figure. A thick spherical shell with an inner radius a and an outer radius b is made of conducting material. A point charge +Q is placed at the centre of the spherical shell and a total charge –q is placed on the shell. Charge –q is distributed on the surfaces as



(A) –Q on the inner surface, -q on outer surface

(B) –Q on the inner surface, -q + Q on outer surface

(C) +Q on the inner surface, -q - Q on outer surface

(D) The charge –q is spread uniformly between the inner and outer surface

8. A charge –Q is uniformly distributed over a non – conducting semicircular rod of radius R. the potential at the centre is

(A) 0 (B) $\frac{1}{4\pi\varepsilon_0}\cdot\frac{(-Q)}{R}$ (C) $\frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{2R}$ (D) $\frac{1}{4\pi\varepsilon_0}\cdot\frac{2Q}{R}$

9. Two point charges +q and –q are held fixed at points (-d, 0) and (d, 0), respectively, of a (x, y) coordinate system. Then

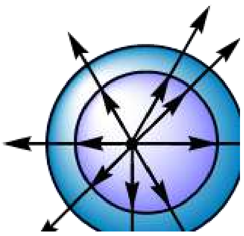
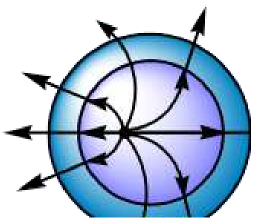
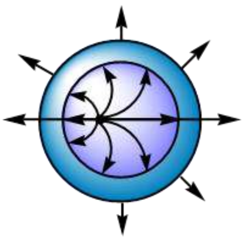
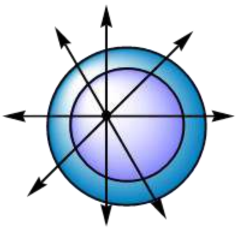
(A) the electric field $\vec{E}$ at all points on the x – axis has same direction

(B) $\vec{E}$ at all points on the Y – axis is along $\hat{j}$

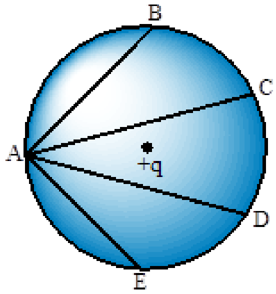
(C) No work is done in bringing a test charge from infinity to the origin

(D) The dipole moment is 2qd directed along $\hat{i}$

10. A positive point charge is placed inside a spherical metallic shell. The electric field lines are represented by

(A)  (B)  (C)  (D) 

11. In the electric field due to a point charge q, a test charge is carried from A to the points B, C, D and E lying on the same circle around q. The work done is



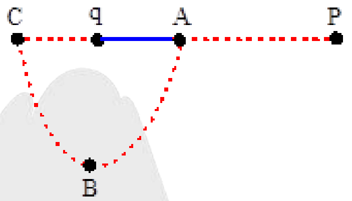
(A) the least along AB

(B) the least along AD

(C) zero along any one of the paths AB, AD, AC and AE

(D) the least along AE

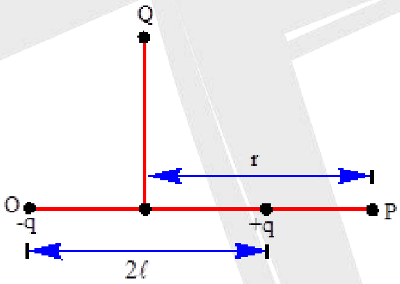
12. Consider the situation depicted in the adjacent figure. The work done in taking a point charge from P to A is W_A, from P to B is W_B and from P to C is W_C. Therefore



(A) $W_A < W_B < W_C$ (B) $W_A > W_B > W_C$

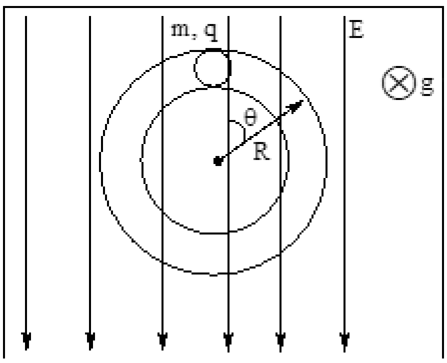
(C) $W_A = W_B = W_C$ (D) $W_A > W_B < W_C$

13. In the figure, the ratio of electic field at point P to that at point Q will be $(r = 2\ell)$.



(A) 2:1 (B) $5:\sqrt{\frac{5}{3}}$ (C) $20\sqrt{5}:9$ (D) $5\sqrt{5}:3$

14. The smooth circular pipe is kept fixed in a horizontal plane. If mass m with charge +q is slightly displaced from the position shown, then distance travelled by the ball when the magnitude of normal becomes minimum would be

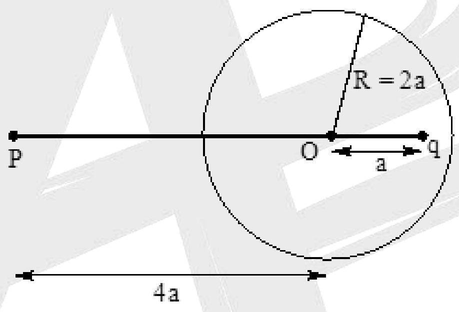


(A) $R\cos^{-1}\left(\frac{2}{3}\right)$ (B) $R\cos^{-1}1/3$ (C) $R\tan^{-1}\frac{\sqrt{5}}{2}$ (D) πR

15. A uniformly charged square plate having side L carries a uniform surface charge density σ. The plate lies in the y-z plane with its center at the origin. A point charge q lies on the x-axis. The flux of the electric field of q through the plate is ϕ_0; while the force on the point charge q due to the plate is F_0, along the x-axis. Then,

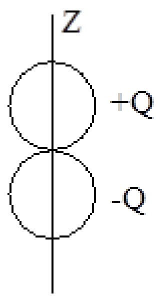
(A) $\sigma=\frac{F_0}{\phi_0 L}$ (B) $\sigma=\frac{F_0}{\phi_0}$ (C) $\sigma=\frac{F_0 L}{\phi_0}$ (D) $\sigma=\frac{\phi_0}{F_0}$

16. A point charge 'q' is placed at distance 'a' from the centre of an unchanged thin spherical conducting shell of radius R = 2a. A point 'P' is located at a distance '4a' from the centre of the conducting shell as shown. The electric potential due to induced charge on the inner surface of the conducting shell at point 'P' is



(A) $-\frac{kq}{a}$ (B) $-\frac{5kq}{a}$ (C) $+\frac{kq}{5a}$ (D) $-\frac{kq}{5a}$

17. An electric dipole is constructed by fixing two circular charged rings, each of radius a, with an insulating contact, one of these rings has a total charge +Q and the other has total charge –Q. If the charge is distributed uniformly along each ring, the dipole moment about the point of contact will be $nQa\hat{Z}/4$, n = ________.

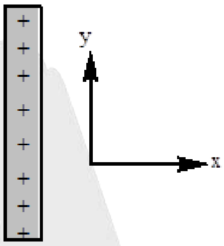


Proficiency Test-2

1. A charge Q has to be divided between two solid spheres of radii R which are at distance d from each other $(d >> R)$. What should be the value of charge, which we should be placed on the spheres, so that the force of attraction between them is maximum?

(A) $\frac{Q}{4}, \frac{3Q}{4}$ (B) $\frac{Q}{3}, \frac{2Q}{3}$ (C) $\frac{Q}{2}, \frac{Q}{2}$ (D) $\frac{Q}{5}, \frac{4Q}{5}$

2. An insulating rod of uniform linear charge density λ and uniform linear mass density μ lies on a smooth table whose surface is xy – plane. A uniform electric field E is switched on



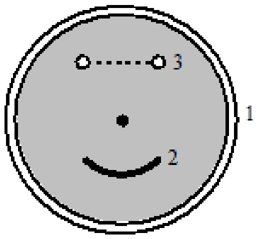
(A) If electric field is along x – axis, the speed of the rod when it has travelled a distance d is $\sqrt{\frac{2\lambda Ed}{\mu}}$

(B) If electric field E is at an angle $\theta(< 90^\circ)$ with x – axis, the speed of the rod when it has travelled a distance d is $\sqrt{\frac{2\lambda Ed\cos\theta}{\mu}}$

(C) Torque on the rod due to the field about centre of mass in case B is into the plane of paper

(D) Torque on the rod due to the field about centre of mass in case B is zero

3. The arrangement shown consists of three elements



(a) a thin rod of charge $-3.0\mu C$ that forms a full circle of radius 6.0 cm

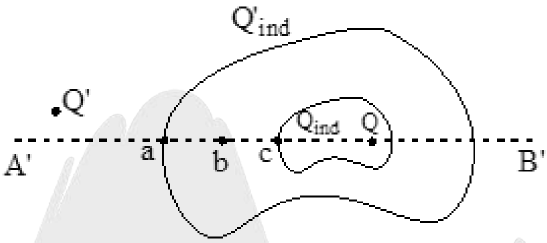
(b) a second thin rod of charge $2.0\mu C$ that forms a circular arc of radius 4.0 cm and concentric with the full circle, subtending an angle of 90° at the centre of the full circle

(c) an electric dipole with a dipole moment that is perpendicular to a radial line and has magnitude $1.28\times10^{-21} C-m$.

Find the net electric potential in volts at the centre.

(A) 0V (B) 1V (C) 2V (D) 3V

4. Figure shows a irregular space conductor with irregular cavity inside it. A charge Q is placed inside cavity and a charge Q' is placed outside conductor. Let Q'_{ind} be charge induced at outside surface of conductor and Q_{ind} be the charge induced at inside surface of cavity. A 'B' is an arbitrary line passing through charge Q and a, b, c be the points on line as shown in figure. Let $\vec{E}, \vec{E}', \vec{E}_{ind}$ and $\vec{E}'_{ind}$ represent electric field at different points due to charge $Q, Q'\ Q_{ind}, \&\ Q'_{ind}$ respectively. If V_a, V_b and V_c represent potential at point a, b, and c. Choose the incorrect statement



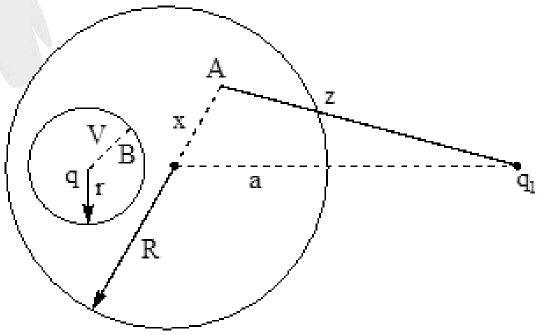
(A) $V_a = V_b = V_c$

(B) $V_b = \int_{-\infty}^{\vec{r}_0} \left(\vec{E} + \vec{E}' + \vec{E}_{ind} + \vec{E}'_{ind}\right) \cdot \vec{dr}$, where $\vec{r_b}$ = position vector of point b

(C) $V_b = \int_{-\infty}^{\vec{r}_b} \left(\vec{E}' + \vec{E}'_{ind}\right) \cdot \vec{dr}$

(D) At point 'b', $\vec{E}' = 0$

5. A cavity is made inside a solid conducting sphere and a charge q is placed inside the cavity at the centre. A charge q_1 is placed outside the sphere as shown in the figure. Point A is inside the sphere and point B is inside the cavity. Then



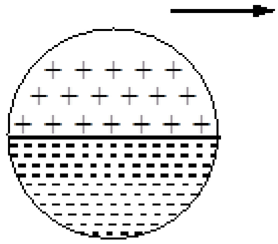
(A) Electric field at point A is zero

(B) Electric field at point B is $\frac{1}{4\pi\varepsilon_0}\frac{q}{y^2}$

(C) Potential at point A is $\frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{a} + \frac{q}{R}\right]$

(D) Potential at point B is $\frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{a} + \frac{q}{R} + \frac{q}{y} - \frac{q}{r}\right]$

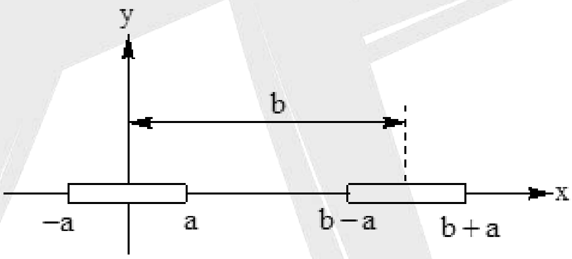
6. A disc having uniform surface charge density in upper part $+\rho$ and $-\rho$ in lower half placed on rough horizontal surface as shown in figure. A uniform electric field is set up as shown if mass of disc is M and its radius is R as well sufficient friction is present to prevent slipping. The acceleration of disc is



(A) $\frac{8}{9}\frac{\rho R^2 E_0}{M}$ (B) $\frac{9}{8}\frac{\rho R^2 E_0}{M}$ (C) $\frac{4}{3}\frac{\rho R^2 E_0}{M}$ (D) $\frac{3}{4}\frac{\rho R^2 E_0}{M}$

7. Two identical thin rods of length 2a carry equal charges +Q that is distributed uniformly along their lengths. The rods lie along the x-axis with their centers separated by a distance b > 2a. The magnitude of the force exerted by the left rod on the right one is $F = \frac{Q^2}{\alpha\pi\varepsilon_0 a^2}\log_e\left[\frac{b^2}{b^2-\beta a^2}\right]$.

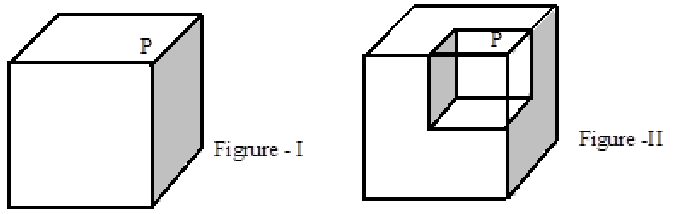
Find $\alpha+\beta$?



8. Two spherical shells of inner radius R and 3R are placed far from each other. They are made of some thin conducting material, the width of their wall d is thin: d≪R. At the centres of the spheres there are charges of 2Q and Q respectively. What is the minimum work which should be done in order to interchange the charges? (There are small holes on the walls.)

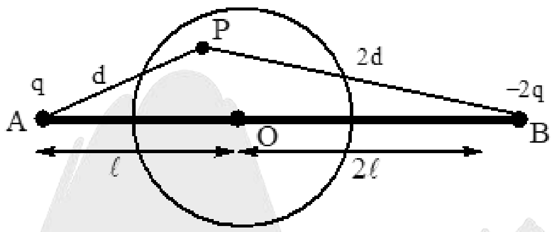
(A) $\frac{5d}{6\pi\varepsilon_0}\frac{Q^2}{R^2}$ (B) $\frac{d}{4\pi\varepsilon_0}\frac{Q^2}{R^2}$ (C) $\frac{d}{8\pi\varepsilon_0}\frac{Q^2}{R^2}$ (D) $\frac{d}{3\pi\varepsilon_0}\frac{Q^2}{R^2}$

9. Consider a cube as shown in the figure-I: with uniformly distributed charge in its entire volume. Intensity of electrical field and potential at one of its vertex P are E_0 and V_0 respectively. A portion of half the size (half edge length) of the original cube is cut and removed as shown in the figure –II. Find modulus of electric field and potential at the point P in the new structure



(A) $\frac{E_0}{2}$ and $\frac{3V_0}{4}$ (B) $\frac{3E_0}{4}$ and $\frac{V_0}{2}$ (C) $\frac{3E_0}{4}$ and $\frac{7V_0}{8}$ (D) $\frac{7E_0}{8}$ and $\frac{7V_0}{8}$

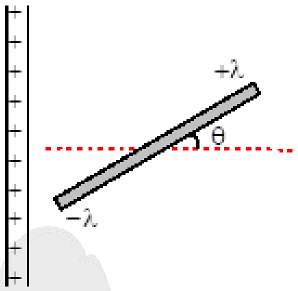
10. Two-point charges are kept as shown in figure from a neutral conducting shell and P is a point inside the shell. Find potential at P due to induced charges.



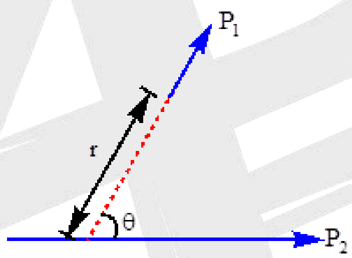
(A) $\frac{kq}{d}-\frac{kq}{2\ell}$ (B) $\frac{kq}{2d}-\frac{kq}{\ell}$ (C) $\frac{kq}{d}-\frac{kq}{\ell}$ (D) 0

Proficiency Test-3

1. A large sheet carries uniform surface charge density σ. A rod of length 2ℓ has a linear charge density λ on one half and $-\lambda$ on the second half. The rod is hinged at mid point O and makes an angle θ with the normal to the sheet. The torque experienced by the rod is $\frac{\sigma\lambda\ell^{\alpha}}{\beta\varepsilon_0}$. Find $\alpha+\beta$?



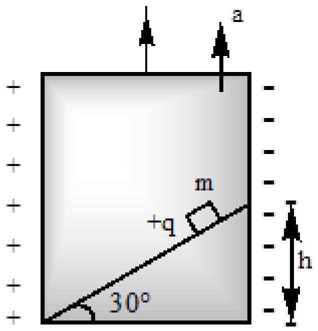
2. Two short electric dipoles are placed as shown. The energy of electric interaction between these dipoles will be



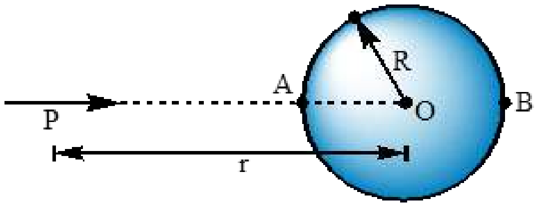
(A) $\frac{2kP_1P_2\cos\theta}{r^3}$ (B) $\frac{-2kP_1P_2\cos\theta}{r^3}$ (C) $\frac{-2kP_1P_2c\sin\theta}{r^3}$ (D) $\frac{-4kP_1P_2\cos\theta}{r^3}$

3. A small block of mass m is kept on a smooth inclined plane of angle 30° placed in an elevator going upward with acceleration a. electric field E exists between the vertical sides of the wall of the Elevator. The charge on the block is +q. the time taken by the block to come to the lowest point of the inclined plane is $2\sqrt{\frac{\alpha h}{\left(g+a\right)-\frac{gE}{m}\sqrt{\beta}}}$. Find $\alpha+\beta$?

(Take the surface to be smooth)



4. An ideal dipole of dipole moment $\vec{P}$ is placed in front of an uncharged conducting sphere of radius R as shown.



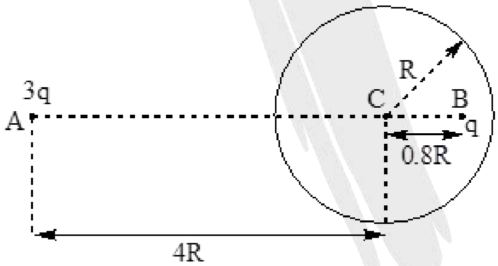
(A) the potential at point A is $\dfrac{KP}{(r-R)^2}$

(B) The potential at point A is $\dfrac{KP}{r^2}$

(C) the potential due to dipole at point B is $\dfrac{KP}{(r+R)^2}$

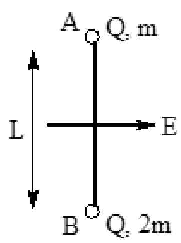
(D) The potential due to dipole at point B is $\dfrac{KP}{r^2}$

5. A thin conducting spherical shell of radius 'R' is given charge '2q'. Two point charges 'q' and '3q' are placed at distances 0.8R and 4R from the centre 'C' of the shell respectively as shown. If the shell is earthed, find the charge (in μC) that will flow from conducting shell to the earth. (Given $q = 2\mu C$)



(A) 6.50 (B) 7.50 (C) 5.50 (D) 8.50

6. Two small balls A and B of positive charge Q each and masses m and 2m respectively are connected by a non-conducting light rigid rod of length L. This system is released in a uniform electric field of strength E as shown. Just after the release (assume no other force acts on the system).



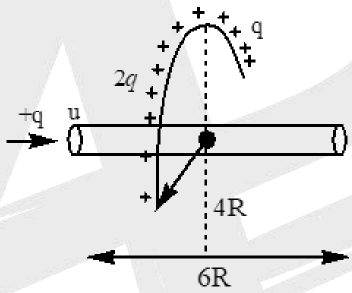
(A) rod has zero angular acceleration

(B) rod has angular acceleration $\frac{QE}{2mL}$ in anticlockwise direction

(C) acceleration of point A is $\frac{2QE}{m}$ towards right

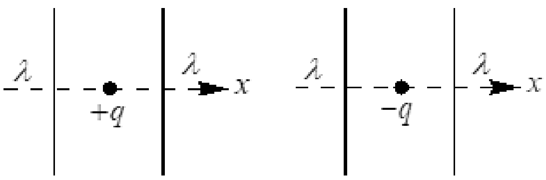
(D) acceleration of point A is $\frac{QE}{m}$ towards right

6. On a semicircular ring of radius 4R, charge $+3q$ is distributed in such a way that on one quarter $+q$ is uniformly distributed and on another quarter $+2q$ is uniformly distributed. Along its axis a smooth non-conducting and uncharged pipe of length 6R is fixed axially as shown. A small ball of mass m and charge $+q$ is thrown from the other end of pipe. The ball can come out of the pipe if (Neglect the effect of induction and also consider pipe to be friction less)



(A) $u<\sqrt{\frac{3q^2}{40\pi\varepsilon_0 Rm}}$ (B) $u>\sqrt{\frac{3q^2}{40\pi\varepsilon_0 Rm}}$ (C) $u\geq\sqrt{\frac{3q^2}{80\pi\varepsilon_0 Rm}}$ (D) $u<\sqrt{\frac{q^2}{40\pi\varepsilon_0 Rm}}$

7. The figures below depict two situations in which two infinitely long static line charges of constant positive line charge density λ are kept parallel to each other. In their resulting electric field, point, charges, q and –q are kept in equilibrium between them. The point charges are confined to move in the x direction only. If they are given a small displacement about their equilibrium positions, then the correct statement(s) is(are)



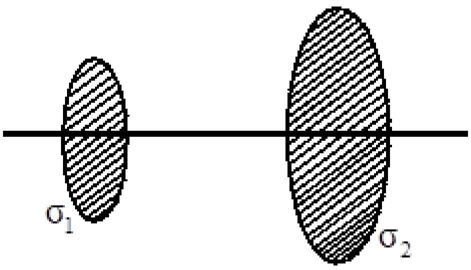
(A) Both charges execute simple harmonic motion.

(B) Both charges will continue moving in the direction of their displacement.

(C) Charge +q executes simple harmonic motion while charge –q continues moving in the direction of its displacement.

(D) Charge –q executes simple harmonic moving in the direction of its displacement.

8. Two discs having uniform charge densities σ_1 and σ_2 respectively are placed parallel to each other with their axis coinciding as shown. Flux of electric field linked with disc 1 is ϕ_1 and flux of electric field linked with disc 2 is ϕ_2. Then



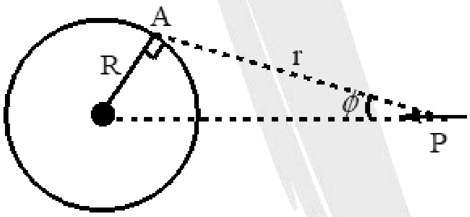
(A) $\phi_1 = \phi_2$

(B) $\dfrac{\phi_1}{\phi_2} = \dfrac{\sigma_2}{\sigma_1}$

(C) $\dfrac{\phi_1}{\phi_2} = \dfrac{\sigma_1}{\sigma_2}$

(D) For any set of σ_1 and σ_2, the relation $\sigma_1\phi_1 = \sigma_2\phi_2$ is always true

9. A small dipole having dipole moment p is placed in front of a fixed solid uncharged conducting sphere as shown in the diagram.



(A) The net potential at point A lying on the surface of the sphere is $\dfrac{kp\cos\phi}{r^2}$

(B) The net potential at point A lying on the surface of the sphere is $\dfrac{kp\cos^2\phi}{r^2}$

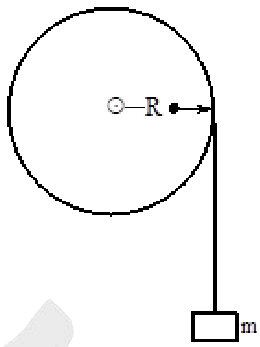
(C) The potential at point A lying on the surface due to induced charge will be

$$\frac{kp\cos\phi(\cos\phi - 1)}{r^2}$$

(D) The potential at point A lying on the surface due to induced charge will be

$$\frac{kp\cos\phi(1 - \cos\phi)}{r^2}$$

10. A uniform dielectric hollow cylinder of mass M, radius R, length l carrying uniform charge of surface charge density σ can rotate without friction about a fixed horizontal axle that coincides with the axis of the cylinder. Several turns of a light thin insulating cord are wrapped on the cylinder and a block of mass m is suspended from the free end of the cord. Initially the block is held at rest as shown in the figure. Find acceleration of the block after it is released. Neglect charge transferred to the cord and fringing of magnetic field at the ends of the cylinder. Acceleration due to gravity is g and permeability of the medium inside the cylinder is μ_0 choose correct options.



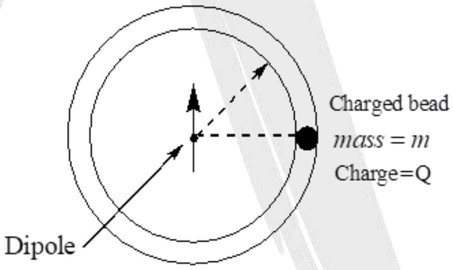
(A) Acceleration of the block just after release is $\dfrac{mg}{m+\mu_0\sigma^2\pi R^2 l}$.

(B) Acceleration of the block just after release is $\dfrac{mg}{m+M+\mu_0\sigma^2\pi R^2 l}$

(C) Tension in the string just after release is zero

(D) Tension in the string just after release is non-zero

11. A small charged bead can slide on a circular frictionless, insulating wire frame. A point like dipole is fixed at the center of circle; dipole moment is $\vec{p}$. Initially the bead is on the plane of symmetry of the dipole. Bead is released from rest. Ignore the effect of gravity. Select the correct options.



(A) Magnitude of velocity of bead as function of its angular position θ (wrt the axis of the dipole) is $\sqrt{\dfrac{Qp(-\cos\theta)}{2\pi\varepsilon_0 mr^2}}$.

(B) Normal force exerted by the wire frame on bead is zero.

(C) If the wire frame were not present bead executes circular motion and returns to initial point after tracing a complete circle.

(D) Bead would move along a circle path until it reaches the opposite its starting position and executes periodic motion.

EXERCISE-1

1	2	3	4	5	6	7	8	9	10
D	B	B	A	4	A	C	10	10	B
11	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
D	B	C	B	D	4	C	A	B	D

EXERCISE-2

1	2	3	4	5	6	7	8	9	10
A	D	A	D	100	B	D	B	6	D
11	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
A	2	C	D	1	40	C	3	C	4

EXERCISE-3

1	2	3	4	5	6	7	8	9	10
B	A	8	B	B	B	A	D	A	A
11	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
ABD	BC	BCD	AC	7	B	D	C	AC	B
21	**22**								
ABD	AC								

EXERCISE-4

1	2	3	4	5	6	7	8	9	10
3	B	5	B	2	D	D	AC	BD	AC
11	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
ABD	AD	D	B	D	B	D	C	A	D
21									
D									

EXERCISE-5

1	2	3	4	5	6	7	8	9	10
B	A	5	6	6	2	A	C	B	A
11	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
A	BC	ACD	C	2	A	C	AB	A	A
21	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
C	D	A	A	A	C	AC	ABC	00.17	

Proficiency Test-1

1	2	3	4	5	6	7	8	9	10
B	A	C	B	C	5	B	B	C	C
11	**12**	**13**	**14**	**15**	**16**	**17**			
C	C	C	A	B	D	8			

Proficiency Test-2

1	2	3	4	5	6	7
C	AD	A	ABCD	ABCD	A	20
8	**9**	**10**				
D	A	D				

Proficiency Test-3

1	2	3	4	5	6
4	B	5	BC	B	D
7	**8**	**9**	**10**	**11**	**12**
C	BD	BC	BD	ABD	