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Remaining Part of diff^{y}.

digest

Q let $f(x)$ be a Poly. fxn Satisfying

$$f(x)+f(y)=f(x)\cdot f(y)-f(x\cdot y)+f(1) \quad \forall \boxed{x,y\in R}$$

& $f'(1)=5$. If No. of divisors of $f'(1)\cdot f''(1)\cdot f'''(1)$ is N then $\frac{N}{8}=?$

$$f(x)+f(y)=f(x)\cdot f(y)-f(x\cdot y)+f(1) \quad \forall x,y\in R$$

let $y=\frac{1}{x}$

$$f(x)+f\left(\tfrac{1}{x}\right)=f(x)\cdot f\left(\tfrac{1}{x}\right)-f\left(x\times\tfrac{1}{x}\right)+f(1)$$

$$f(x)+f\left(\tfrac{1}{x}\right)=f(x)\cdot f\left(\tfrac{1}{x}\right)-\cancel{f(1)}+\cancel{f(1)}$$

$$\boxed{f(x)+f\left(\tfrac{1}{x}\right)=f(x)\cdot f\left(\tfrac{1}{x}\right)} \Rightarrow f(x)=1+x^n$$

$$f(x)=1+x^n$$

$$f'(x)=nx^{n-1}$$

$x=1$ $\quad f'(1)=n\cdot(1)^{n-1}=5 \Rightarrow n=5$

$\therefore f(x)=1+x^5$

$f'(x)=5x^4 \xrightarrow{x=1} f'(1)=5$

$f''(x)=20x^3 \rightarrow f''(1)=20$

$f'''(x)=60x^2 \rightarrow f'''(1)=60$

$$f'(1)\cdot f''(1)\cdot f'''(1)=5\times 2^2\times 5\times 3\times 2^2\times 5$$

$$=2^4\times 3^1\times 5^3 \quad [\text{Prime factorise}]$$

No of divisor $=(4+1)(1+1)(3+1)$

$=\underline{\underline{40}}=N \quad \therefore \frac{N}{8}=\frac{40}{8}=5$

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Rem

Q If $f(x+y) = f(x) + f(y)$ & $\boxed{f(x) = x^2 \cdot g(x)}$; $g(x) =$ Cont^s fxn then $f'(x) = ?$

$f(x+h)$

① $f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$

$= \lim_{h\to 0} \frac{\cancel{f(x)} + f(h) - \cancel{f(x)}}{h}$

$= \lim_{h\to 0} \frac{h^{\cancel{2}} \cdot g(h)}{\cancel{h}}$

$f'(x) = 0 \times g(0) = 0$

Q If $f(x+y) = f(x) \cdot f(y)\ \forall x, y \in R$; $\boxed{f(x) = 1 + x \cdot g(x) \cdot h(x)}$

Where $\lim_{x\to 0} g(x) = a$ & $\lim_{x\to 0} h(x) = b$ find $f'(x)$?

$g(0) = a$ $h(0) = b$

$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$

$= \lim_{h\to 0} \frac{f(x) \cdot f(h) - f(x)}{h} = \lim_{h\to 0} \frac{f(x)(f(h) - 1)}{h}$

$= f(x) \cdot \lim_{h\to 0} \frac{f(h) - 1}{h}$

$= f(x) \cdot \lim_{h\to 0} \frac{\cancel{1} + \cancel{h} \cdot g(h) \cdot h(h) - \cancel{1}}{\cancel{h}}$

h is moving
h is variable
x is constant
$f(x)$ "

$= f(x) \{ g(0) \cdot h(0) \}$

$f'(x) = a \cdot b f(x)$

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Q If $f(x+y) = f(x)\cdot f(y) \ \forall \ x, y \in R$

$f(3) = 5$, $f'(0) = 11$ then $f'(3) = ?$

(1) $$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{f(x)\cdot f(h) - f(x)}{h}$$

$$= f(x) \lim_{h\to 0} \frac{f(h) - 1}{h} = \frac{0}{0} \text{ DL}$$

$$= f(x) \lim_{h\to 0} \frac{f'(h) - 0}{1}$$

$$f'(x) = f(x)\cdot f'(0)$$

$x = 3$

$$f'(3) = f(3)\cdot f'(0) = 5 \times 11 = 55$$

$f(x+y) = f(x)\cdot f(y)$

$x = y = 0$

$f(0) = f(0)\cdot f(0)$

$f^2(0) - f(0) = 0$

$f(0)(f(0) - 1) = 0$

$f(0) = 0$ or $f(0) = 1$

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Min & Max^m f(t) type

Q $f(x) = \cos x$.

$$g(x) = \begin{cases} \text{Min } f(t) : 0 \le t \le x \,; & 0 \le x \le \frac{\pi}{2} \\ 3 - x & \frac{\pi}{2} < x < \pi \end{cases}$$

Check diff^y at $x = \frac{\pi}{2}$?

$f(x) = \cos x \Rightarrow f(t) = \cos t$