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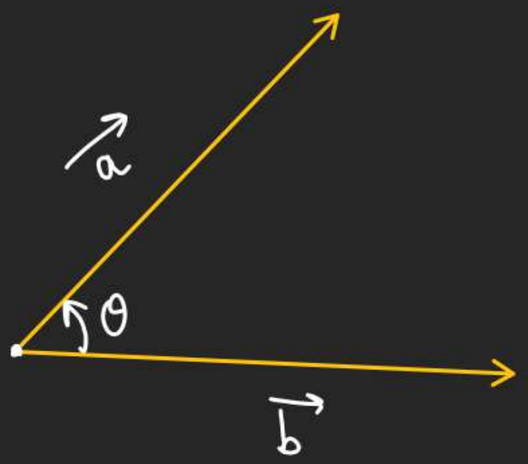
DOT PRODUCT / Scalar product

$\Rightarrow$ If $\vec{a}$ & $\vec{b}$ be two vectors then

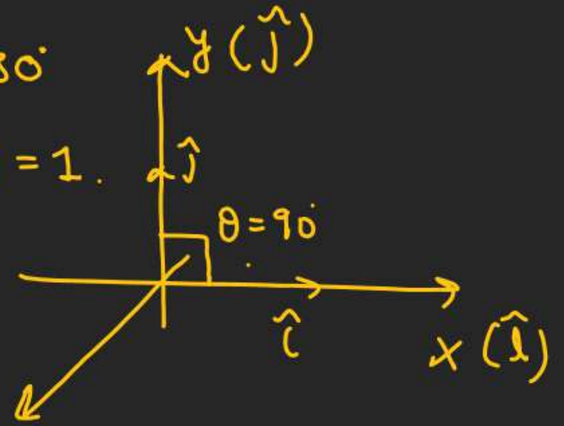
(*) $\boxed{\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}|\cos\theta}$



(*) Dot product of Unit Vector:-

$\hat{i} \cdot \hat{i} = |\hat{i}||\hat{i}|\cos 0$

- $\hat{i} \cdot \hat{i} = (1)(1)(1) = 1$.
- $\hat{j} \cdot \hat{j} = 1$.
- $\hat{k} \cdot \hat{k} = 1$



- $\hat{i} \cdot \hat{j} = |\hat{i}||\hat{j}|\cos 90 = 1 \cdot 1 \cdot (0) = 0$.
- $\hat{j} \cdot \hat{k} = 0$
- $\hat{k} \cdot \hat{i} = 0$

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Dot product of general position vector:→

$$\vec{r_1} = a_1\hat{i} + b_1\hat{j} + c_1\hat{k}$$

$$\vec{r_2} = a_2\hat{i} + b_2\hat{j} + c_2\hat{k}$$

$$\vec{r_1} \cdot \vec{r_2} = (a_1\hat{i} + b_1\hat{j} + c_1\hat{k}) \cdot (a_2\hat{i} + b_2\hat{j} + c_2\hat{k})$$

$$= a_1\hat{i} \cdot (a_2\hat{i} + b_2\hat{j} + c_2\hat{k}) + b_1\hat{j} \cdot (a_2\hat{i} + b_2\hat{j} + c_2\hat{k}) + c_1\hat{k} \cdot (a_2\hat{i} + b_2\hat{j} + c_2\hat{k})$$

$$= (a_1a_2)(1) + (a_1b_2)(0) + a_1c_2(0) + (b_1b_2) + c_1c_2$$

$$\vec{r_1} \cdot \vec{r_2} = a_1a_2 + b_1b_2 + c_1c_2$$

⇓

Magnitude

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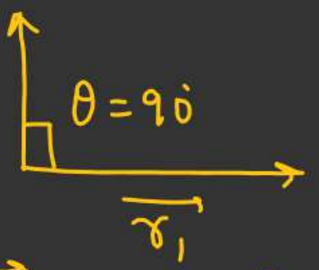
(*) If $\vec{r}_1$ and $\vec{r}_2$ are perpendicular to each other.

$$\vec{r}_1 \cdot \vec{r}_2 = |\vec{r}_1||\vec{r}_2|\cos 90^\circ$$

$$\boxed{\vec{r}_1 \cdot \vec{r}_2 = 0}$$



If $\vec{r}_1 = (3-K)\hat{i} + \hat{j} + 2\hat{k}$ and $\vec{r}_2 = 3\hat{i} - \hat{j} + 2\hat{k}$ are perpendicular to each other then find the value of Constant $K = ??$

Sol^n:- If two vectors are perpendicular $\vec{r}_1 \cdot \vec{r}_2 = 0$

$$[(3-K)\hat{i} + 1\hat{j} + 2\hat{k}] \cdot (3\hat{i} - \hat{j} + 2\hat{k}) = 0$$

$$3(3-k) - 1 + 4 = 0$$

$$9 - 3K + 3 = 0 \Rightarrow 12 = 3K$$

$$\Rightarrow \boxed{K = 4} \leftarrow$$

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(★) Angle b/w the two vectors:-

$\vec{r_1} \cdot \vec{r_2} = r_1 r_2 \cos\theta$

$|\vec{r_1}| = r_1$

$|\vec{r_2}| = r_2$

$$\cos\theta = \frac{\vec{r_1} \cdot \vec{r_2}}{|\vec{r_1}||\vec{r_2}|}$$

$\cos\theta = k$

$\theta = \cos^{-1}(k)$

Ex:- Find angle b/w $\vec{r_1} = -\hat{i} + \hat{j} + \hat{k}$ and $\vec{r_2} = (\sqrt{3}\hat{i} + \hat{j})$

Sol^n.

$$\vec{r_1} \cdot \vec{r_2} = (-\hat{i} + \hat{j} + \hat{k}) \cdot (\sqrt{3}\hat{i} + \hat{j} + 0\hat{k})$$

$$= (-\sqrt{3} + 1)$$

$$|\vec{r_1}| = \sqrt{(-1)^2 + (1)^2 + (1)^2} = \sqrt{3}$$

$$|\vec{r_2}| = \sqrt{(\sqrt{3})^2 + (1)^2} = 2$$

$$\cos\theta = \frac{(-\sqrt{3}+1)}{2\sqrt{3}}$$

$$\theta = \cos^{-1}\left(\frac{-\sqrt{3}+1}{2\sqrt{3}}\right)$$

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(☆)

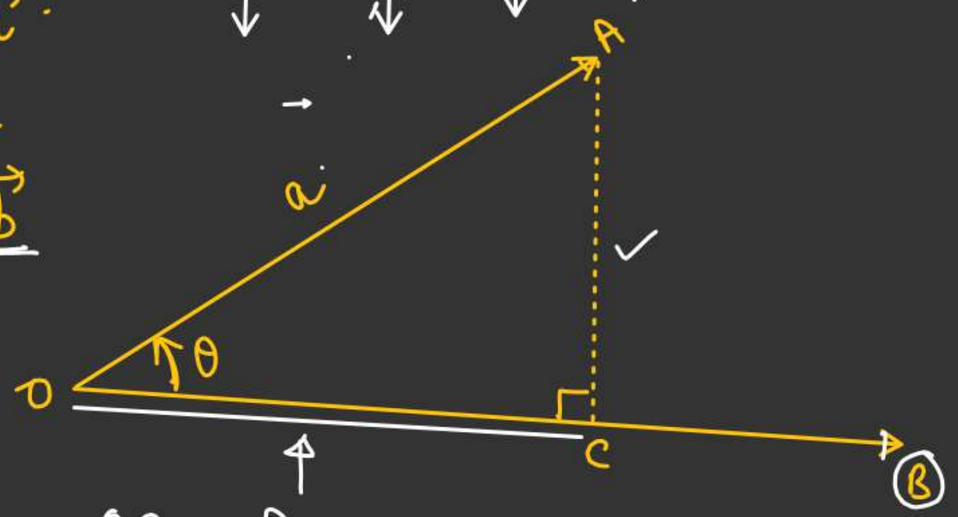
Geometrical meaning of dot production

$\vec{OA} = \vec{a}$

$|\vec{OA}| = a$

$\vec{OB} = \vec{b}$

$|\vec{OB}| = b$



OC = Projection or Component of $\vec{a}$ along $\vec{b}$

$$OC = a\cos\theta$$

$$\boxed{\vec{a}\cdot\vec{b} = ab\cos\theta}$$

In $\triangle OAC$

$$\cos\theta = \frac{OC}{OA}$$

$$OC = OA\cos\theta$$

$$\boxed{OC = a\cos\theta}$$

$$[a\cos\theta] = \left[\frac{\vec{a}\cdot\vec{b}}{b}\right]$$

It is projection or Component of $\vec{a}$ along $\vec{b}$.

⇒ General formula for component of one vector along another vector

$$= \left(\frac{\vec{a}\cdot\vec{b}}{b}\right)$$

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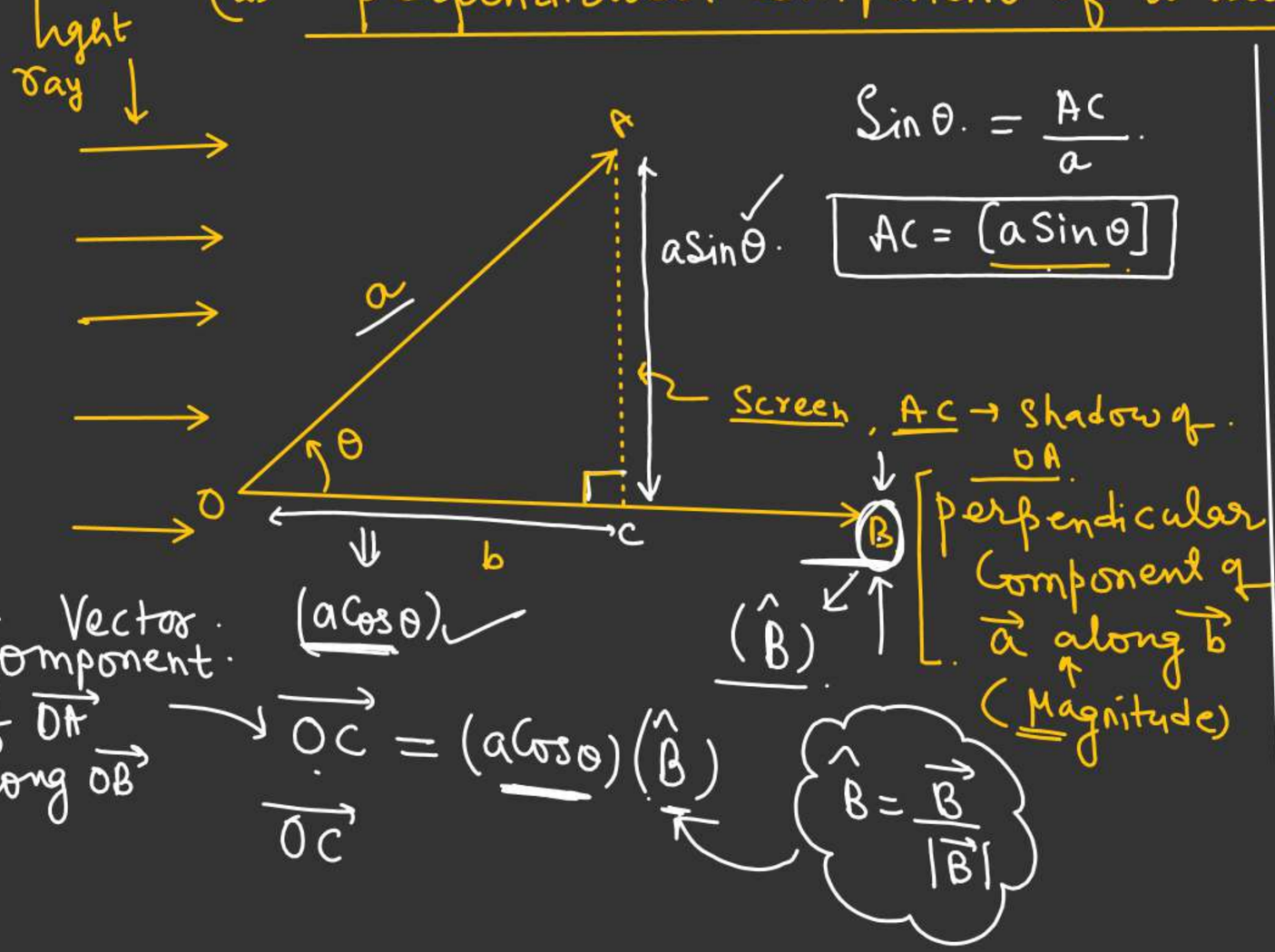
(*) Perpendicular Component of $\vec{a}$ along $\vec{b}$.



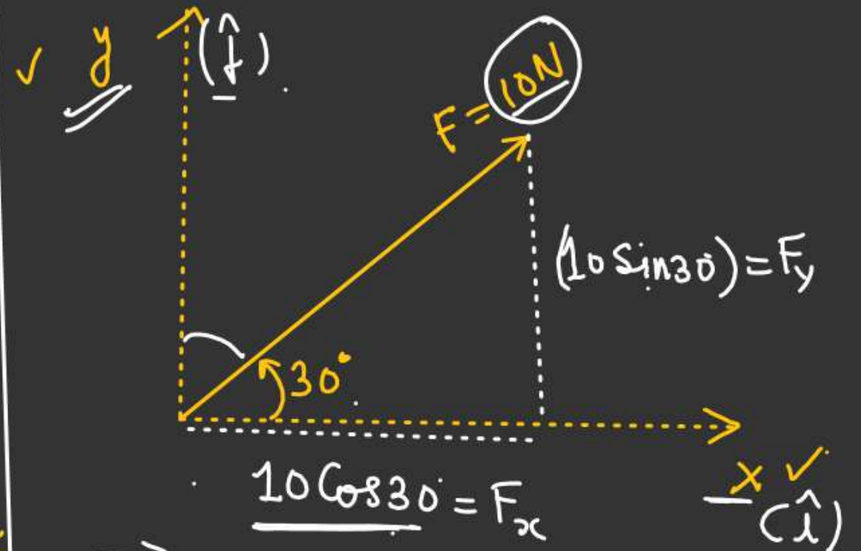
$$\sin\theta = \frac{AC}{a}$$

$$\boxed{AC = [a\sin\theta]}$$

Vector Component of $\overrightarrow{OA}$ along $\overrightarrow{OB}$ → $\overrightarrow{OC}$

$$\overrightarrow{OC} = (a\cos\theta)(\hat{B})$$

$$\hat{B} = \frac{\vec{B}}{|\vec{B}|}$$



$$\vec{F} = F_x\hat{i} + F_y\hat{j}$$

$$\vec{F} = (5\sqrt{3})\hat{i} + 5\hat{j}$$

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. Find the Component of $\vec{a} = 3\hat{i} - \hat{j}$ along vector $\vec{b} = (\hat{i} + \hat{j} - \hat{k})$

Perpendicular Component

Sol^n.

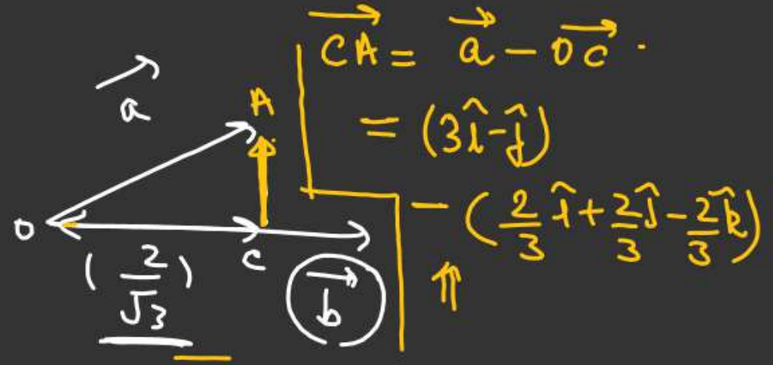
Component of $\vec{a}$ along $\vec{b}$ $= \dfrac{\vec{a} \cdot \vec{b}}{|\vec{b}|}$

$|\vec{b}| = \sqrt{(1)^2 + (1)^2 + (-1)^2}$

$= \sqrt{3}$

$= \dfrac{(3\hat{i} - \hat{j}) \cdot ((1)\hat{i} + \hat{j} - \hat{k})}{(\sqrt{3})}$

$= \left(\dfrac{3-1}{\sqrt{3}}\right) = \left[\dfrac{2}{\sqrt{3}}\right]$ Ans.



[Vector Component of $\vec{a}$ along $\vec{b}$] $= \left(\dfrac{2}{\sqrt{3}}\right)(\hat{b}) = \dfrac{2}{\sqrt{3}}\left(\dfrac{\vec{b}}{|\vec{b}|}\right) = \dfrac{2}{\sqrt{3}}\left(\dfrac{\hat{i} + \hat{j} - \hat{k}}{\sqrt{3}}\right) = \dfrac{2}{3}(\hat{i} + \hat{j} - \hat{k})$

$\Downarrow$

$\vec{OC}$

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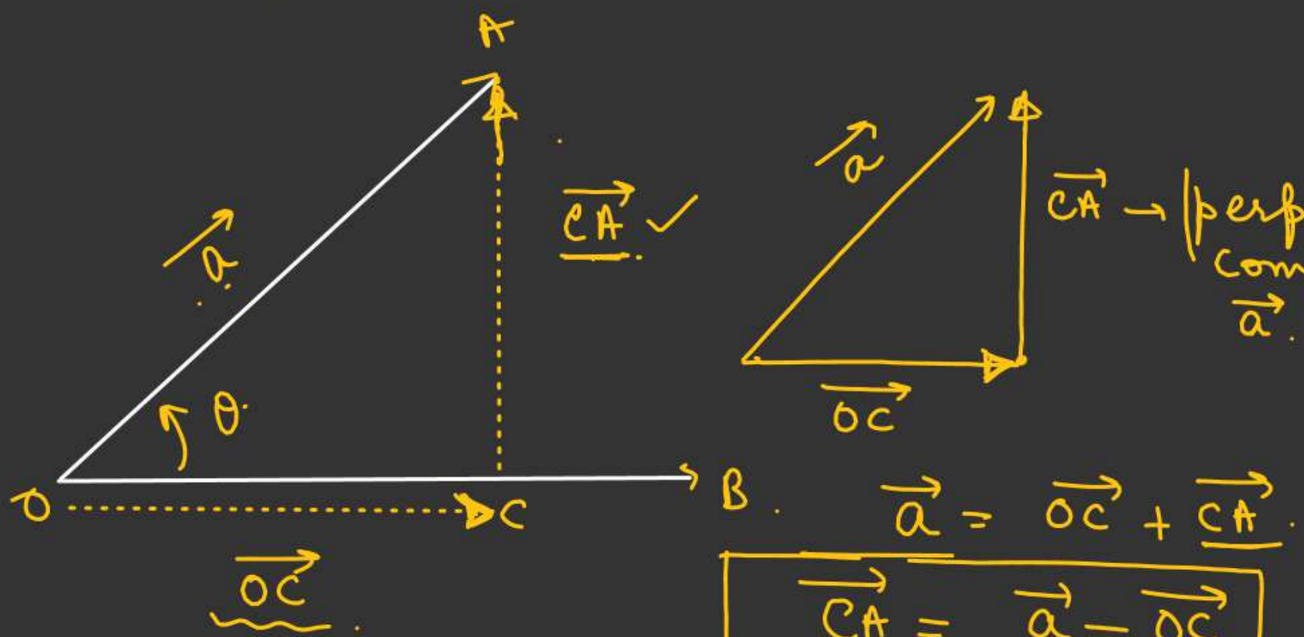
#. Perpendicular Component.



$\vec{CA}$ ✓

$\vec{CA}$ → perpendicular component of $\vec{a}$.

$\vec{a} = \vec{OC} + \vec{CA}$.

$$\vec{CA} = \vec{a} - \vec{OC}$$

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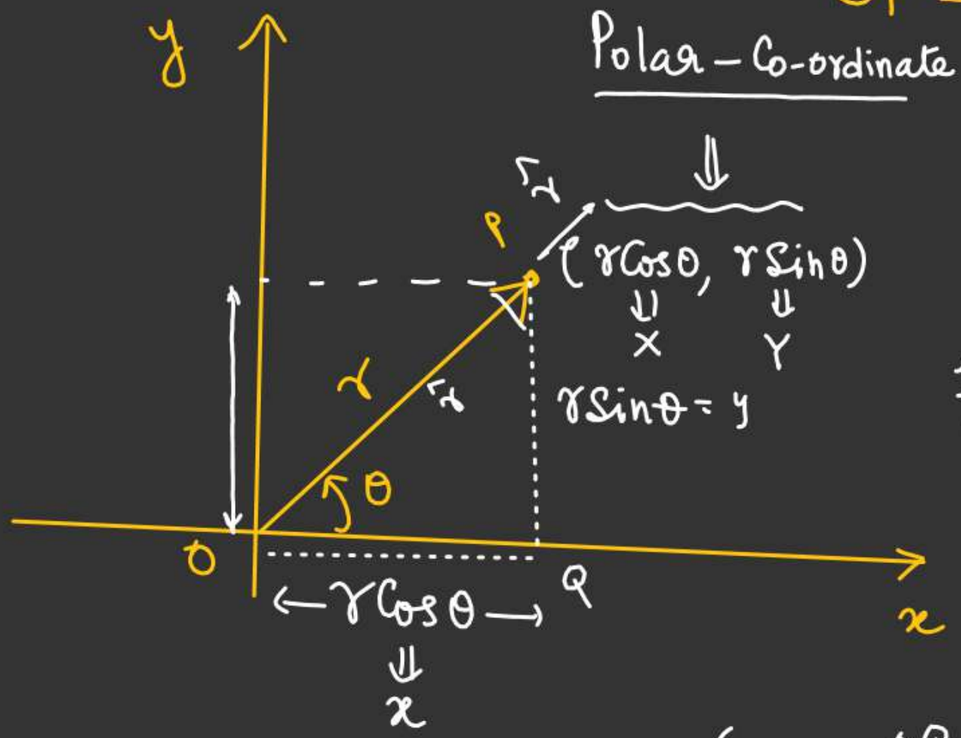
Polar form :-

$$\overrightarrow{OP} = \vec{r} = x\hat{i} + y\hat{j}$$



Polar – Co-ordinate

$$\vec{r} = (r\cos\theta)\hat{i} + (r\sin\theta)\hat{j}$$

$$\vec{r} = r(\cos\theta\hat{i} + \sin\theta\hat{j})$$

$$\hat{r} = \frac{\vec{r}}{|\vec{r}|} = \frac{\vec{r}}{r} = [\cos\theta\hat{i} + \sin\theta\hat{j}]$$

$$\cos\theta = \frac{OQ}{OP}$$

$$OQ = OP\cos\theta$$

$$= r\cos\theta$$

Nishant Jindal

DOT PRODUCT

H.W.

Q.1 Find the dot product of two vectors $\vec{A} = 3\hat{i} + 2\hat{j} - 4\hat{k}$ and $\vec{B} = 2\hat{i} - 3\hat{j} - 6\hat{k}$

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DOT PRODUCT

H.W.

Q.2 Find the value of m so that the vector $3\hat{i} - 2\hat{j} + \hat{k}$ may be perpendicular to the vector $2\hat{i} + 6\hat{j} + m\hat{k}$

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DOT PRODUCT

H.W.

Q.3 What is the angle between the following pair of vectors ?

$\vec{A} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{B} = -2\hat{i} - 2\hat{j} - 2\hat{k}$

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DOT PRODUCT

H.W.

Q.4 If the sum of two unit vectors is a unit vector, then find the magnitude of their difference.

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DOT PRODUCT

H.W.

Q.5 Prove that $(\vec{A} + 2\vec{B}) \cdot (2\vec{A} - 3\vec{B}) = 2A^2 + AB\cos\theta - 6B^2$

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DOT PRODUCT

Do not attempt.

Q.6 A body constrained to move along the z-axis of a co-ordinate system is subjected to a constant force $\vec{F}$ given by $\vec{F} = -\hat{i} + 2\hat{j} + 3\hat{k}$ newton where $\hat{i}, \hat{j}$, and $\hat{k}$ represent unit vectors along $x-, y$, and z-axes of the system, respectively. Calculate the work done by this force in displacing the body through a distance of $4\ m$ along the z-axis.

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DOT PRODUCT

H.W.

Q.7 By vector method, prove that if the diagonals of a parallelogram intersect perpendicularly, then the parallelogram is a rhombus.

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DOT PRODUCT

H.W.

Q.8 $\hat{i}$ and $\hat{j}$ are unit vectors along x - and y-axes respectively. What is the magnitude and direction of the vectors $\hat{i}+\hat{j}$ and $\hat{i}-\hat{j}$? What are the components of a vector $\vec{A}=2\hat{i}+3\hat{j}$ along the direction $\hat{i}+\hat{j}$ and $\hat{i}-\hat{j}$?

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DOT PRODUCT

H.W.

Q.9 If $\vec{A} = \vec{B} + \vec{C}$, and the magnitudes of $\vec{A}, \vec{B}, \vec{C}$ are 5,4 , and 3 units, then the angle between $\vec{A}$ and $\vec{C}$ is

(A) $\cos^{-1}\left(\frac{3}{5}\right)$

(B) $\cos^{-1}\left(\frac{4}{5}\right)$

(C) $\sin^{-1}\left(\frac{3}{4}\right)$

(D) $\frac{\pi}{2}$

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DOT PRODUCT

H-W.

Q.10 Given: $\vec{A} = A\cos\theta\hat{i} + A\sin\theta\hat{j}$. A vector $\vec{B}$, which is perpendicular to $\vec{A}$, is given by

(A) $B\cos\theta\hat{i} - B\sin\theta\hat{j}$

(B) $B\sin\theta\hat{i} - B\cos\theta\hat{j}$

(C) $B\cos\theta\hat{i} + B\sin\theta\hat{j}$

(D) $B\sin\theta\hat{i} + B\cos\theta\hat{j}$

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DOT PRODUCT

H-W.

Q.11 The projection of a vector $\vec{r} = 3\hat{i} + \hat{j} + 2\hat{k}$ on the $x - y$ plane has magnitude

(A) 3

(B) 4

(C) $\sqrt{14}$

(D) $\sqrt{10}$

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DOT PRODUCT

H-W.

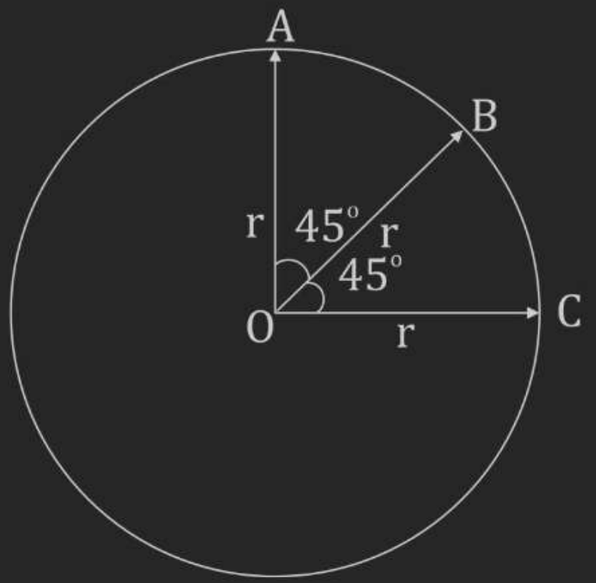
Q.12 The resultant of the three vectors $\overrightarrow{OA}, \overrightarrow{OB}$, and $\overrightarrow{OC}$ shown in Fig. is

(A) r

(B) $2r$

(C) $r(1+\sqrt{2})$

(D) $r(\sqrt{2}-1)$



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DOT PRODUCT

Q.13 Given two vectors $\vec{A} == 3\hat{i} + 4\hat{j}$ and $\vec{B} == \hat{i} + \hat{j} \cdot \theta$ is the angle between $\vec{A}$ and $\vec{B}$. Which of the following statements is/are correct?

(A) $|\vec{A}|\cos\theta\left(\frac{\hat{i}+\hat{j}}{\sqrt{2}}\right)$ is the component of $\vec{A}$ along $\vec{B}$

(B) $|\vec{A}|\sin\theta\left(\frac{\hat{i}-\hat{j}}{\sqrt{2}}\right)$ is the component of $\vec{A}$ perpendicular to $\vec{B}$.

(C) $|\vec{A}|\cos\theta\left(\frac{\hat{i}-\hat{j}}{\sqrt{2}}\right)$ is the component of $\vec{A}$ along $\vec{B}$.

(D) $|\vec{A}|\sin\theta\left(\frac{\hat{i}+\hat{j}}{\sqrt{2}}\right)$ is the component of $\vec{A}$ perpendicular to $\vec{B}$.

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DOT PRODUCT

H.W.

Q.14 A plane is inclined at an angle 30° with horizontal. The component of a vector $\vec{A} = -10\hat{k}$ perpendicular to this plane is: (here z-direction is vertically upwards)

(A) $5\sqrt{2}$

(B) $5\sqrt{3}$

(C) 5

(D) 2.5

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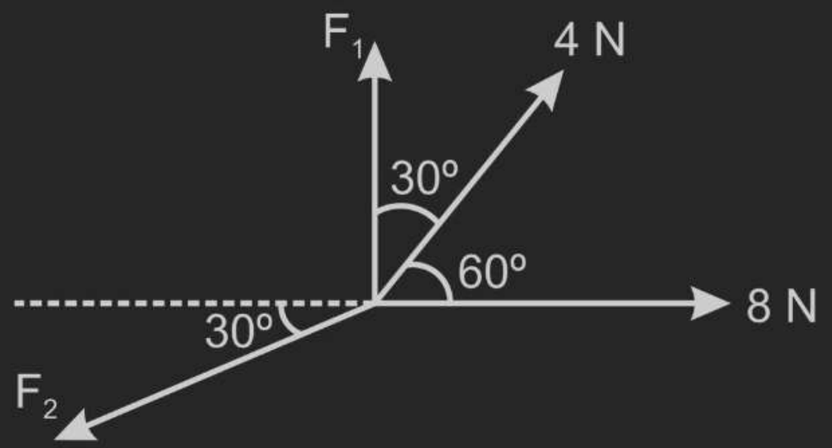
DOT PRODUCT

Q.15 An object is in equilibrium under four concurrent forces in the directions shown in figure. Find the magnitude of $\vec{F_1}$ and $\vec{F_2}$.



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DOT PRODUCT

Q.15 One end of a string $0.5\ \mathrm{m}$ long is fixed to a point A and the other end is fastened to a small object of weight $8\ \mathrm{N}$. The object is pulled aside by a horizontal force F, until it is $0.3\ \mathrm{m}$ from the vertical through A. Find the magnitudes of the tension T in the string and the force F.