


DPP-01

SOLUTIONS

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1. Find the range of

$$f(x) = \sin^{-1} x + \cos^{-1} x + \tan^{-1} x$$

**Sol.** We have,  $f(x) = \sin^{-1} x + \cos^{-1} x + \tan^{-1} x$  is defined when  $-1 \leq x \leq 1$  Now,

$$f(1) = \sin^{-1}(1) + \cos^{-1}(1) + \tan^{-1}(1)$$

$$= \frac{\pi}{2} + 0 + \frac{\pi}{4} = \frac{3\pi}{4}$$

$$\text{and } f(-1) = \sin^{-1}(-1) + \cos^{-1}(-1) + \tan^{-1}(-1)$$

$$= -\frac{\pi}{2} + \pi - \frac{\pi}{4} = \pi - \frac{3\pi}{4} = \frac{\pi}{4}$$

$$\text{Thus, } R_f = \left[ \frac{\pi}{4}, \frac{3\pi}{4} \right]$$

2. Solve for x:  $4\sin^{-1}(x-2) + \cos^{-1}(x-2) = \pi$

**Sol.** We have  $4\sin^{-1}(x-2) + \cos^{-1}(x-2) = \pi$

$$\Rightarrow 3\sin^{-1}(x-2) + \frac{\pi}{2} = \pi$$

$$\Rightarrow 3\sin^{-1}(x-2) = \frac{\pi}{2}$$

$$\Rightarrow \sin^{-1}(x-2) = \frac{\pi}{6}$$

$$\Rightarrow (x-2) = \sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$$

$$\Rightarrow x = 2 + \frac{1}{2} = \frac{5}{2}$$

Hence, the solution is  $x = \frac{5}{2}$

3. Solve for x :

$$\sin^{-1}(x^2 - 2x + 1) + \cos^{-1}(x^2 - x) = \frac{\pi}{2}$$

**Sol.** As we know that, if  $\sin^{-1}(f(x)) + \cos^{-1}(g(x)) = \frac{\pi}{2}$ , then


$$f(x) = g(x)$$

$$\Rightarrow (x^2 - 2x + 1) = (x^2 - x)$$

$$\Rightarrow 2x - x = 1$$

$$\Rightarrow x = 1$$

Hence, the solution is  $x = 1$

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4. Find the number of real solutions of

$$\tan^{-1} \sqrt{x(x+1)} + \sin^{-1} \sqrt{x^2 + x + 1} = \frac{\pi}{2}$$

**Sol.** We have

$$\tan^{-1} \sqrt{x(x+1)} + \sin^{-1} \sqrt{x^2 + x + 1} = \frac{\pi}{2}$$

$$\Rightarrow \cos^{-1} \left( \frac{1}{\sqrt{x^2 + x + 1}} \right) + \sin^{-1} \sqrt{x^2 + x + 1} = \frac{\pi}{2}$$

$$\Rightarrow \left( \frac{1}{\sqrt{x^2 + x + 1}} \right) = \sqrt{x^2 + x + 1}$$

$$\Rightarrow x^2 + x + 1 = 1$$

$$\Rightarrow x^2 + x = 0$$

$$\Rightarrow x(x+1) = 0$$

$$\Rightarrow x = 0 \text{ and } -1$$

Hence, the number of solutions is 2

5. If  $\sin^{-1} \left( x - \frac{x^2}{2} + \frac{x^3}{4} - \dots \right) + \cos^{-1} \left( x^2 - \frac{x^4}{2} + \frac{x^6}{4} - \dots \right) = \frac{\pi}{2}$ , for  $0 < |x| < \sqrt{2}$ , then find x.

**Sol.** As we know that, if  $\sin^{-1} (f(x)) + \cos^{-1} (g(x)) = \frac{\pi}{2}$ , then

$$f(x) = g(x)$$

$$\Rightarrow \left( x - \frac{x^2}{2} + \frac{x^3}{4} - \dots \right) = \left( x^2 - \frac{x^4}{2} + \frac{x^6}{4} - \dots \right)$$

$$\Rightarrow x \left( 1 - \frac{x}{2} + \frac{x^2}{4} - \dots \right) = x^2 \left( 1 - \frac{x^2}{2} + \frac{x^4}{4} - \dots \right)$$

$$\Rightarrow x \left( \frac{1}{1+\frac{x}{2}} \right) = x^2 \left( \frac{1}{1+\frac{x^2}{2}} \right)$$

$$\Rightarrow \left( \frac{2x}{x+2} \right) = \left( \frac{2x^2}{x^2+2} \right)$$

$$\Rightarrow x \left\{ \left( \frac{1}{x+2} \right) - \left( \frac{x}{x^2+2} \right) \right\} = 0$$

$$\Rightarrow x = 0 \text{ and } \left( \frac{1}{x+2} \right) = \left( \frac{x}{x^2+2} \right)$$

$$\Rightarrow x = 0 \text{ and } x = 1$$


6. Solve for x :  $\sin^{-1} x > \cos^{-1} x$

**Sol.** We have  $\sin^{-1} x > \cos^{-1} x$

$$\Rightarrow 2\sin^{-1} x > \sin^{-1} x + \cos^{-1} x$$

$$\Rightarrow 2\sin^{-1} x > \frac{\pi}{2}$$

$$\Rightarrow \sin^{-1} x > \frac{\pi}{4}$$

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$$\Rightarrow x > \sin\left(\frac{\pi}{4}\right)$$

$$\Rightarrow x > \frac{1}{\sqrt{2}}$$

$$\Rightarrow x \in \left(\frac{1}{\sqrt{2}}, 1\right]$$

7.  $(\sin^{-1} x)^2 - 3\sin^{-1} x + 2 = 0$

Sol.  $(\sin^{-1} x)^2 - 3\sin^{-1} x + 2 = 0$

$$\Rightarrow (\sin^{-1} x - 1)(\sin^{-1} x - 2) = 0$$

$$\Rightarrow (\sin^{-1} x - 1) = 0, (\sin^{-1} x - 2) = 0$$

$$\Rightarrow \sin^{-1} x = 1, 2$$

$$\Rightarrow \sin^{-1} x = 1$$

$$\Rightarrow x = \sin(1)$$

8.  $\sin^{-1} x + \sin^{-1} 2y = \pi$

Sol. Given equation is  $\sin^{-1} x + \sin^{-1} 2y = \pi$ . It is possible only when

$$\Rightarrow \sin^{-1} x = \frac{\pi}{2}, \sin^{-1} (2y) = \frac{\pi}{2}$$

$$\Rightarrow x = 1, 2y = 1$$

$$\Rightarrow x = 1, y = \frac{1}{2}$$

9.  $\cos^{-1} x + \cos^{-1} x^2 = 2\pi$

Sol. Given equation is  $\cos^{-1} x + \cos^{-1} x^2 = 2\pi$ . It is possible only when

$$\Rightarrow \cos^{-1} x = \pi, \cos^{-1} (x^2) = \pi$$

$$\Rightarrow x = -1, x^2 = -1$$

$$\Rightarrow x = \varphi$$

10.  $\cos^{-1} x + \cos^{-1} x^2 = 0$

Sol. Given equation is  $\cos^{-1} x + \cos^{-1} x^2 = 0$  It is possible only when

$$\Rightarrow \cos^{-1} x = 0, \cos^{-1} (x^2) = 0$$

$$\Rightarrow x = 1 \text{ and } x^2 = 1$$


$$\Rightarrow x = 1$$

11.  $4\sin^{-1} (x - 1) + \cos^{-1} (x - 1) = \pi$

Sol. Given equation is

$$4\sin^{-1} (x - 1) + \cos^{-1} (x - 1) = \pi$$

$$\Rightarrow 3\sin^{-1} (x - 1) + \frac{\pi}{2} = \pi$$

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$$\Rightarrow 3\sin^{-1}(x-1) = \frac{\pi}{2}$$

$$\Rightarrow \sin^{-1}(x-1) = \frac{\pi}{6}$$

$$\Rightarrow (x-1) = \sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$$

$$\Rightarrow x = \frac{3}{2}$$

Hence, the solution is  $x = \frac{3}{2}$

12.  $\cot^{-1}\left(\frac{1}{x^2-1}\right) + \tan^{-1}(x^2-1) = \frac{\pi}{2}$

Sol. Given equation is

$$\cot^{-1}\left(\frac{1}{x^2-1}\right) + \tan^{-1}(x^2-1) = \frac{\pi}{2}$$

It is possible only when

$$\Rightarrow \frac{1}{x^2-1} = x^2-1$$

$$\Rightarrow (x^2-1)^2 = 1$$

$$\Rightarrow (x^2-1) = \pm 1$$

$$\Rightarrow x^2 = 1 \pm 1 = 2, 0$$

$$\Rightarrow x = \{-\sqrt{2}, 0, \sqrt{2}\}$$

13.  $\cot^{-1}\left(\frac{x^2-1}{2x}\right) + \tan^{-1}\left(\frac{2x}{x^2-1}\right) = \frac{2\pi}{3}$

Sol. Given equation is

$$\cot^{-1}\left(\frac{x^2-1}{2x}\right) + \tan^{-1}\left(\frac{2x}{x^2-1}\right) = \frac{2\pi}{3}$$

$$\Rightarrow \tan^{-1}\left(\frac{2x}{x^2-1}\right) + \tan^{-1}\left(\frac{2x}{x^2-1}\right) = \frac{2\pi}{3}$$

$$\Rightarrow 2\tan^{-1}\left(\frac{2x}{x^2-1}\right) = \frac{2\pi}{3}$$

$$\Rightarrow \tan^{-1}\left(\frac{2x}{x^2-1}\right) = \frac{\pi}{3}$$


$$\Rightarrow \tan^{-1}\left(\frac{2x}{1-x^2}\right) = -\frac{\pi}{3}$$

$$\Rightarrow 2\tan^{-1}x = -\frac{\pi}{3}$$

$$\Rightarrow \tan^{-1}x = -\frac{\pi}{6}$$

$$\Rightarrow x = \tan\left(-\frac{\pi}{6}\right) = -\frac{1}{\sqrt{3}}$$

Hence, the solution is  $x = -\frac{1}{\sqrt{3}}$

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14.  $4\sin^{-1} x + \cos^{-1} x = \frac{3\pi}{4}$

Sol. Given equation is

$$4\sin^{-1} x + \cos^{-1} x = \frac{3\pi}{4}$$

$$\Rightarrow 3\sin^{-1} x + \frac{\pi}{2} = \frac{3\pi}{4}$$

$$\Rightarrow 3\sin^{-1} x = \frac{\pi}{4}$$

$$\Rightarrow \sin^{-1} x = \frac{\pi}{12}$$

$$\Rightarrow x = \sin\left(\frac{\pi}{12}\right) = \frac{\sqrt{3}-1}{2\sqrt{2}}$$

15.  $5\tan^{-1} x + 3\cot^{-1} x = \frac{7\pi}{4}$

Sol. Given equation is

$$5\tan^{-1} x + 3\cot^{-1} x = \frac{7\pi}{4}$$

$$\Rightarrow 2\tan^{-1} x + \frac{3\pi}{2} = \frac{7\pi}{4}$$

$$\Rightarrow 2\tan^{-1} x = \frac{\pi}{4}$$

$$\Rightarrow \tan^{-1} x = \frac{\pi}{8}$$

$$\Rightarrow x = \tan\left(\frac{\pi}{8}\right) = (\sqrt{2} - 1)$$

16.  $5\tan^{-1} x + 4\cot^{-1} x = 2\pi$

Sol. Given equation is

$$5\tan^{-1} x + 4\cot^{-1} x = 2\pi$$

$$\Rightarrow \tan^{-1} x + 2\pi = 2\pi$$

$$\Rightarrow \tan^{-1} x = 0$$

$$\Rightarrow x = \tan(0) = 0$$

Hence, the solution is  $x = 0$ .

17.  $\cot^{-1} x - \cot^{-1} (x+1) = \frac{\pi}{2}$


Sol. Given equation is

$$\cot^{-1} x - \cot^{-1} (x+1) = \frac{\pi}{2}$$

$$\Rightarrow \tan^{-1}\left(\frac{1}{x}\right) - \tan^{-1}\left(\frac{1}{x+1}\right) = \frac{\pi}{2}$$

$$\Rightarrow \tan^{-1}\left(\frac{\frac{1}{x} - \frac{1}{x+1}}{1 + \frac{1}{x} \times \frac{1}{x+1}}\right) = \frac{\pi}{2}$$

$$\Rightarrow \tan^{-1}\left(\frac{1}{x^2+x+1}\right) = \frac{\pi}{2}$$

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$$\Rightarrow \left( \frac{1}{x^2+x+1} \right) = \tan \left( \frac{\pi}{2} \right) = \infty$$

$$\Rightarrow x^2 + x + 1 = \frac{1}{\infty} = 0 \quad \Rightarrow x^2 + x + 1 = 0$$

So, no real values of  $x$  satisfies the above equation. Hence, the solution is  $x = \varphi$

**18.**  $[\sin^{-1} x] + [\cos^{-1} x] = 0$

**Sol.** Given equation is

$$[\sin^{-1} x] + [\cos^{-1} x] = 0$$

It is possible only when

$$[\sin^{-1} x] = 0, [\cos^{-1} x] = 0$$

$$\Rightarrow 0 \leq \sin^{-1} x < 1 \text{ and } 0 \leq \cos^{-1} x < 1 \Rightarrow 0 \leq x < \sin(1) \text{ and } \cos(1) < x \leq 1$$

$$\Rightarrow x \in [\cos(1), \sin(1)]$$

**19.**  $[\tan^{-1} x] + [\cot^{-1} x] = 0$

**Sol.** Given equation is

$$[\tan^{-1} x] + [\cot^{-1} x] = 0$$

It is possible only when

$$[\tan^{-1} x] = 0 \text{ and } [\cot^{-1} x] = 0$$

$$\Rightarrow 0 \leq \tan^{-1} x < 1 \text{ and } 0 \leq \cot^{-1} x < 1$$

$$\Rightarrow \cot(1) < x < \tan(1)$$

$$\text{Hence, } x \in (\cot(1), \tan(1))$$

**20.**  $[\sin^{-1} \cos^{-1} \sin^{-1} \tan^{-1} x] = 0$

**Sol.** Given equation is

$$[\sin^{-1} \cos^{-1} \sin^{-1} \tan^{-1} x] = 0$$

$$\Rightarrow 0 \leq \sin^{-1} \left( \cos^{-1} \left( \sin^{-1} \left( \tan^{-1} x \right) \right) \right) < 1$$


$$\Rightarrow 0 \leq \left( \cos^{-1} \left( \sin^{-1} \left( \tan^{-1} x \right) \right) \right) < \sin(1)$$

$$\Rightarrow \cos(\sin(1)) < \left( \sin^{-1} \left( \tan^{-1} x \right) \right) \leq 1$$

$$\Rightarrow \sin(\cos(\sin(1))) < \left( \tan^{-1} x \right) \leq \sin(1)$$

$$\Rightarrow \tan(\sin(\cos(\sin(1)))) < x \leq \tan(\sin(1))$$

**21.**  $[\sin^{-1} \cos^{-1} \sin^{-1} \tan^{-1} x] = 1$

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22.  $(\tan^{-1} x)^2 + (\cot^{-1} x)^2 = \frac{5\pi^2}{8}$

Sol. Given equation is

$$(\tan^{-1} x)^2 + (\cot^{-1} x)^2 = \frac{5\pi^2}{8}$$

$$\Rightarrow (\tan^{-1} x + \cot^{-1} x)^2 - 2\tan^{-1} x \cdot \cot^{-1} x = \frac{5\pi^2}{8}$$

$$\Rightarrow \frac{\pi^2}{4} - 2a\left(\frac{\pi}{2} - a\right) = \frac{5\pi^2}{8}, a = \tan^{-1} x$$

$$\Rightarrow 2a\left(\frac{\pi}{2} - a\right) + \frac{3\pi^2}{8} = 0$$

$$\Rightarrow a\pi - 2a^2 + \frac{3\pi^2}{8} = 0$$

$$\Rightarrow 8a\pi - 16a^2 + 3\pi^2 = 0$$

$$\Rightarrow 16a^2 - 8a\pi - 3\pi^2 = 0$$

$$\Rightarrow 16a^2 - 12a\pi + 4a\pi - 3\pi^2 = 0 \Rightarrow 4a(4a - 3\pi) + \pi(4a - 3\pi) = 0$$

$$\Rightarrow (4a + \pi)(4a - 3\pi) = 0 \Rightarrow a = \frac{3\pi}{4}, -\frac{\pi}{4} \Rightarrow \tan^{-1} x = \frac{3\pi}{4}, -\frac{\pi}{4}$$

$$\Rightarrow x = \tan\left(\frac{3\pi}{4}\right), \tan\left(-\frac{\pi}{4}\right) \Rightarrow x = -1$$

23. Find the value of  $\cos\left(\frac{1}{2}\cos^{-1}\left(\frac{3}{5}\right)\right)$ .

Sol. Let  $\frac{1}{2}\cos^{-1}\left(\frac{3}{5}\right) = \theta$

$$\Rightarrow \cos^{-1}\left(\frac{3}{5}\right) = 2\theta$$

$$\Rightarrow \cos(2\theta) = \frac{3}{5}$$

$$\Rightarrow 2\cos^2 \theta - 1 = \frac{3}{5}$$

$$\Rightarrow 2\cos^2 \theta = 1 + \frac{3}{5} = \frac{8}{5}$$

$$\Rightarrow \cos^2 \theta = \frac{4}{5} \Rightarrow \cos \theta = \frac{2}{\sqrt{5}}$$


24. Find the value of  $\sin\left(\frac{\pi}{4} + \sin^{-1}\left(\frac{1}{2}\right)\right)$ .

Sol. We have

$$\sin\left(\frac{\pi}{4} + \sin^{-1}\left(\frac{1}{2}\right)\right)$$

$$= \sin\left(\frac{\pi}{4} + \theta\right), \theta = \sin^{-1}\left(\frac{1}{2}\right)$$

$$= \sin\left(\frac{\pi}{4} + \theta\right), \sin \theta = \frac{1}{2}$$

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$$\begin{aligned}
 &= \sin\left(\frac{\pi}{4}\right) \cos(\theta) + \cos\left(\frac{\pi}{4}\right) \sin(\theta) \\
 &= \frac{1}{\sqrt{2}} \cos(\theta) + \frac{1}{\sqrt{2}} \sin(\theta) \\
 &= \frac{\sqrt{3}+1}{2\sqrt{2}}
 \end{aligned}$$

25. If  $m$  is a root of  $x^2 + 3x + 1 = 0$ , then find the value of  $\tan^{-1}(m) + \tan^{-1}\left(\frac{1}{m}\right)$ .

Sol. Let  $m_1$  and  $m_2$  be the roots of  $x^2 + 3x + 1 = 0$

$$\text{Thus, } m_1 + m_2 = -3 < 0$$

$$\text{and } m_1 \cdot m_2 = 1$$

It is possible only when both are negative.

$$\begin{aligned}
 \text{Thus, } \tan^{-1}(m) + \tan^{-1}\left(\frac{1}{m}\right) \\
 &= \tan^{-1}(m) - \pi + \cot^{-1}(m) = \tan^{-1}(m) + \cot^{-1}(m) - \pi \\
 &= \frac{\pi}{2} - \pi \\
 &= -\frac{\pi}{2}
 \end{aligned}$$

26. Prove that

$$\cos\left(\tan^{-1}\left(\sin\left(\cot^{-1} x\right)\right)\right) = \sqrt{\frac{x^2 + 1}{x^2 + 2}}$$

Sol. We have  $\cos\left(\tan^{-1}\left(\sin\left(\cot^{-1} x\right)\right)\right)$

$$= \cos\left(\tan^{-1}(\sin \theta)\right), \cot \theta = x$$

$$= \cos\left(\tan^{-1}\left(\frac{1}{\sqrt{1+x^2}}\right)\right)$$

$$= \cos \varphi, \tan \varphi = \left(\frac{1}{\sqrt{1+x^2}}\right)$$

$$= \sqrt{\frac{x^2+1}{x^2+2}}$$

Questions Solve for  $x$ :


27.  $6(\sin^{-1} x)^2 - \pi \sin^{-1} x \leq 0$

Sol. Given,  $6(\sin^{-1} x)^2 - \pi \sin^{-1} x \leq 0$

$$\Rightarrow \sin^{-1} x (6 \sin^{-1} x - \pi) \leq 0$$

$$\Rightarrow 0 \leq \sin^{-1} x \leq \frac{\pi}{6}$$

$$\Rightarrow 0 \leq x \leq \frac{1}{2}$$

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28.  $\frac{2\tan^{-1} x + \pi}{4\tan^{-1} x - \pi} \leq 0$

**Sol.** Given, in-equation is

$$\frac{2\tan^{-1} x + \pi}{4\tan^{-1} x - \pi} \leq 0$$

$$\Rightarrow -\frac{\pi}{2} \leq \tan^{-1} x \leq \frac{\pi}{4}$$

$$\Rightarrow -\infty < x < 1$$

29.  $\sin^{-1} x < \sin^{-1} x^2$

**Sol.** Given inequation is

$$\Rightarrow \sin^{-1} x < \sin^{-1} x^2$$

$$\Rightarrow x^2 > x$$

$$\Rightarrow x(x-1) > 0$$

$$\Rightarrow x > 1 \text{ and } x < 0$$

$$\Rightarrow x \in [-1, 0)$$

30.  $\cos^{-1} x > \cos^{-1} x^2$

**Sol.** Given in-equation is

$$\Rightarrow \cos^{-1} x > \cos^{-1} x^2$$

$$\Rightarrow x^2 > x$$

$$\Rightarrow x^2 - x > 0$$

$$\Rightarrow x(x-1) > 0$$

$$\Rightarrow -1 \leq x < 0$$

31.  $\log^2 (\tan^{-1} x) > 1$

**Sol.** Given in-equation is

$$\log_2 (\tan^{-1} x) > 1$$

$$\Rightarrow \tan^{-1} x > 2$$

$$\Rightarrow x > \tan (2)$$

Hence, the solution is

$$(\tan 2, \infty)$$


32.  $(\cot^{-1} x)^2 - 5\cot^{-1} x + 6 > 0$

**Sol.** Given in-equation is

$$(\cot^{-1} x)^2 - 5\cot^{-1} x + 6 > 0$$

$$\Rightarrow (\cot x - 2)(\cot^{-1} x - 3) > 0$$

$$\Rightarrow (\cot^{-1} x - 2) < 0, (\cot^{-1} x - 3) > 0$$

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$$\Rightarrow x > \cot(2), x < \cot(3)$$

$$\Rightarrow x \in (\cot 2, \cot 3)$$

33.  $\sin^{-1} x < \cos^{-1} x$

Sol. Given in-equation is

$$\sin^{-1} x < \cos^{-1} x$$

$$\Rightarrow 2\sin^{-1} x < \frac{\pi}{2}$$

$$\Rightarrow \sin^{-1} x < \frac{\pi}{4}$$

$$\Rightarrow x < \frac{1}{\sqrt{2}}$$

$$\Rightarrow x \in \left[-1, \frac{1}{\sqrt{2}}\right)$$

34.  $\sin^{-1} x > \sin^{-1} (1 - x)$

Sol. Given in-equation is  $\sin^{-1} x > \sin^{-1} (1 - x)$

$$x > (1 - x)$$

$$2x > 1$$

$$x > \frac{1}{2}$$

$$\text{Hence, the solution is } x \in \left(\frac{1}{2}, 1\right]$$

35.  $\sin^{-1} 2x > \operatorname{cosec}^{-1} x$

Sol. Given in-equation is

$$\sin^{-1} 2x > \operatorname{cosec}^{-1} x \Rightarrow \sin^{-1} (2x) > \sin^{-1} \left(\frac{1}{x}\right) \Rightarrow 2x > \frac{1}{x} \Rightarrow 2x - \frac{1}{x} > 0$$

$$\Rightarrow \frac{2x^2 - 1}{x} > 0 \Rightarrow \frac{(\sqrt{2}x + 1)(\sqrt{2}x - 1)}{x} > 0 \Rightarrow x \in \left(-\frac{1}{\sqrt{2}}, 0\right) \cup \left(\frac{1}{\sqrt{2}}, 1\right]$$