## HOME WORK –1

## (PROBLEMS BASED ON FUNDAMENTALS)

**Q.** Solve for $\theta$:

**1.** $\sin 3\theta = 0$

**Ans.** $\theta = \frac{n\pi}{3}$

**Sol.** We have, $\sin 3\theta = 0$

$\Rightarrow 3\theta = n\pi$

$\Rightarrow \theta = \frac{n\pi}{3}$, where $n \in I$

**2.** $\cos^2(5\theta) = 0$

**Ans.** $\theta = \frac{1}{5}\left(n\pi \pm \left(\frac{\pi}{2}\right)\right)$

**Sol.** We have, $\cos^2(5\theta) = 0$

$\Rightarrow \cos^2(5\theta) = \cos^2\left(\frac{\pi}{2}\right)$

$\Rightarrow (5\theta) = n\pi \pm \left(\frac{\pi}{2}\right)$

$\Rightarrow \theta = \frac{1}{5}\left(n\pi \pm \left(\frac{\pi}{2}\right)\right)$, where $n \in I$

**3.** $\tan\theta = \sqrt{3}$

**Ans.** $\theta = n\pi + \left(\frac{\pi}{3}\right)$

**Sol.** We have, $\tan \theta = \sqrt{3}$

$\Rightarrow \tan \theta = \tan\left(\frac{\pi}{3}\right)$

$\Rightarrow \theta = n\pi + \left(\frac{\pi}{3}\right)$, where $n \in I$

**4.** $\sin 2\theta = \sin\theta$

**Ans.** $\theta = n\pi$ and $\theta = 2n\pi \pm \frac{\pi}{3}$

**Sol.** We have, $\sin 2\theta = \sin \theta$

$\Rightarrow 2\sin \theta \cos \theta = \sin \theta$

$\Rightarrow \sin \theta(2\cos \theta - 1) = 0$

$\Rightarrow \sin \theta = 0$ and $(2\cos \theta - 1) = 0$

$\Rightarrow \sin \theta = 0$ and $\cos \theta = \frac{1}{2}$

$\Rightarrow \theta = n\pi$ and $\theta = 2n\pi \pm \frac{\pi}{3}$, where $n \in I$

**5.** $\sin(9\theta) = \sin\theta$

**Ans.** $\theta = (2n+1)\frac{\pi}{10}$ and $\theta = \left(\frac{n\pi}{4}\right)$

**Sol.** We have, $\sin(9\theta) = \sin\ \theta$

$\Rightarrow\ \sin(9\theta) - \sin\ \theta = 0$

$\Rightarrow\ 2\cos\left(\frac{9\theta+\theta}{2}\right)\sin\left(\frac{9\theta-\theta}{2}\right) = 0$

$\Rightarrow\ 2\cos(5\theta)\sin(4\theta) = 0$

$\Rightarrow\ \cos(5\theta) = 0$ and $\sin(4\theta) = 0$

$\Rightarrow\ (5\theta) = (2n+1)\frac{\pi}{2}$ and $(4\theta) = n\pi$

$\Rightarrow\ \theta = (2n+1)\frac{\pi}{10}$ and $\theta = \left(\frac{n\pi}{4}\right)$

where $n \in I$

**6.** $5\sin^2\theta + 3\cos^2\theta = 4$

**Ans.** $\theta = n\pi \pm \left(\frac{\pi}{4}\right)$

**Sol.** We have, $5\sin^2\ \theta + 3\cos^2\ \theta = 4$

$\Rightarrow 2\sin^2\ \theta + 3(\sin^2\ \theta + \cos^2\ \theta) = 4$

$\Rightarrow 2\sin^2\ \theta + 3 = 4$

$\Rightarrow 2\sin^2\ \theta = 1$

$\Rightarrow \sin^2\ \theta = \frac{1}{2}$

$\Rightarrow \sin^2\ \theta = \left(\frac{1}{\sqrt{2}}\right)^2 = \sin^2\left(\frac{\pi}{4}\right)$

$\Rightarrow \theta = n\pi \pm \left(\frac{\pi}{4}\right)$, where $n \in I$

**7.** $\tan(\theta - 15°) = 3\tan(\theta + 15°)$

**Ans.** $\theta = (4n+1)\frac{\pi}{4}$

**Sol.** We have, $\tan(\theta - 15°) = \tan(\theta + 15°)$

$\Rightarrow\ \frac{\tan(\theta-15°)}{\tan(\theta+15°)} = \frac{3}{1}$

$\Rightarrow\ \frac{\tan(\theta-15°)+\tan(\theta+15°)}{\tan(\theta-15°)-\tan(\theta+15°)} = \frac{3+1}{3-1}$

$\Rightarrow \frac{\sin(\theta+15°+\theta-15°)}{\sin(\theta+15°-\theta+15°)} = \frac{3+1}{3-1}$

$\Rightarrow 2\sin(2\theta) = 2$

$\Rightarrow \sin(2\theta)° = 1$

$\Rightarrow \theta = (4n+1)\frac{\pi}{4}, n \in I$

**8.** $\tan^2(\theta) + \cot^2(\theta) = 2$

**Ans.** $\theta = n\pi \pm \left(\frac{\pi}{4}\right)$

**Sol.** We have, $\tan^2(\theta) + \cot^2(\theta) = 2$

$\Rightarrow \tan^2(\theta) + \frac{1}{\tan^2(\theta)} = 2$

$\Rightarrow \tan^4(\theta) - 2\tan^2(\theta) + 1 = 0$

$\Rightarrow (\tan^2(\theta) - 1)^2 = 0$

$\Rightarrow (\tan^2(\theta) - 1) = 0$

$\Rightarrow \theta = n\pi \pm \left(\frac{\pi}{4}\right), n \in I$

**9.** $\cos(\theta) + \cos(2\theta) + \cos(3\theta) = 0$

**Ans.** $\theta = (2n+1)\left(\frac{\pi}{4}\right)$ and $\theta = n\pi \pm \left(\frac{2\pi}{3}\right)$

**Sol.** We have, $(\cos(3\theta) + \cos(\theta)) + \cos(2\theta) = 0$

$\Rightarrow (\cos(3\theta) + \cos(\theta)) + \cos(2\theta) = 0$

$\Rightarrow 2\cos(2\theta)\cos(\theta) + \cos(2\theta) = 0$

$\Rightarrow \cos(2\theta)(2\cos(\theta) + 1) = 0$

$\Rightarrow \cos(2\theta) = 0$ and $(2\cos(\theta) + 1) = 0$

$\Rightarrow \cos(2\theta) = 0$ and $\cos(\theta) = -\frac{1}{2}$

$\Rightarrow \theta = (2n+1)\left(\frac{\pi}{4}\right)$ and $\theta = n\pi \pm \left(\frac{2\pi}{3}\right)$

**10.** $\sin(2\theta) + \sin(4\theta) + \sin(6\theta) = 0$

**Ans.** $\theta = \left(\frac{n\pi}{4}\right)$ and $\theta = \left(\frac{n\pi}{2}\right) \pm \left(\frac{\pi}{3}\right)$

**Sol.** We have $\sin(2\theta) + \sin(4\theta) + \sin(6\theta) = 0$

$\sin(6\theta) + \sin(2\theta) + \sin(4\theta) = 0$

$2\sin(4\theta), \cos(2\theta) + \sin(4\theta) = 0$

$\sin(4\theta)(2\cos(2\theta) + 1) = 0$

$\sin(4\theta) = 0$ and $(2\cos(2\theta) + 1) = 0$

$\Rightarrow (4\theta) = n\pi$ and $\cos(2\theta) = -\frac{1}{2}$

$\Rightarrow \theta = \left(\frac{n\pi}{4}\right)$ and $(2\theta) = n\pi \pm \left(\frac{2\pi}{3}\right)$

$\Rightarrow \theta = \left(\frac{n\pi}{4}\right)$ and $\theta = \left(\frac{n\pi}{2}\right) \pm \left(\frac{\pi}{3}\right), n \in I$

**11.** $\tan(\theta) + \tan(2\theta) + \tan(\theta).\tan(2\theta) = 1$

**Ans.** $\theta = \left(\frac{n\pi}{3}\right) + \left(\frac{\pi}{12}\right)$

**Sol.** We have $\tan(\theta) + \tan(2\theta) + \tan(\theta)\tan(2\theta) = 1$

$\Rightarrow \tan(2\theta) + \tan(\theta) = 1 - \tan(\theta)\tan(2\theta)$

$\Rightarrow \frac{\tan(2\theta)+\tan(\theta)}{1-\tan(\theta)\tan(2\theta)} = 1$

$\Rightarrow \tan(3\theta) = 1$

$\Rightarrow \tan(3\theta) = \tan\left(\frac{\pi}{4}\right)$

$\Rightarrow (3\theta) = n\pi + \left(\frac{\pi}{4}\right)$

$\Rightarrow \theta = \left(\frac{n\pi}{3}\right) + \left(\frac{\pi}{12}\right), n \in I$

**12.** $\tan(\theta) + \tan(2\theta) + \tan(3\theta) = \tan(\theta).\tan(2\theta).\tan(3\theta)$

**Ans..** $\theta = \left(\frac{n\pi}{3}\right), n \in I$

**Sol.** We have $\tan(\theta) + \tan(2\theta) + \tan(3\theta)$

$= \tan(\theta), \tan(2\theta), \tan(3\theta)$

$\Rightarrow \tan(\theta) + \tan(2\theta)$

$= -\tan(3\theta) + \tan(\theta)\tan(2\theta)\tan(3\theta)$

$\Rightarrow \tan(q) + \tan(2\theta) = -\tan(3\theta)(1 - \tan(\theta), \tan(2\theta))$

$\Rightarrow \left(\frac{\tan(\theta)+\tan(2\theta)}{(1-\tan(\theta)\cdot\tan(2\theta))}\right) = -\tan\theta$

$\Rightarrow \tan(3\theta) = -\tan(3\theta)$

$\Rightarrow 2\tan(3\theta) = 0$

$\Rightarrow (3\theta) = n\pi$

$\Rightarrow \theta = \left(\frac{n\pi}{3}\right), n \in I$

**13.** $\cot^2\theta + \frac{3}{\sin\theta} + 3 = 0$

**Ans.** $\theta = (4n-1)\frac{\pi}{2}, \theta = n\pi + (-1)^n\left(-\frac{\pi}{6}\right)$

**Sol.** Given equation is $\cot^2\theta + \frac{3}{\sin\theta} + 3 = 0$

$\Rightarrow \cot^2\theta + 3(1 + \operatorname{cosec}\theta) = 0$

$\Rightarrow +3(1 + \operatorname{cosec}\theta) = 0$

$\Rightarrow (\operatorname{cosec}\theta - 1 + 3)(1 + \operatorname{cosec}\theta) = 0$

$\Rightarrow (\operatorname{cosec}\theta + 2)(1 + \operatorname{cosec}\theta) = 0$

$\Rightarrow \operatorname{cosec}\theta = -1, -2$

$\Rightarrow \sin\theta = -1, \sin\theta = -\frac{1}{2}$

$\Rightarrow \theta = (4n-1)\frac{\pi}{2}, \theta = n\pi + (-1)^n\left(-\frac{\pi}{6}\right), n \in I$

**14.** $2\tan\theta - \cot\theta = -1$

**Ans.** $\theta = \left(n\pi - \frac{\pi}{4}\right), \theta = n\pi + \alpha, \alpha = \tan^{-1}\left(\frac{1}{2}\right)$

**Sol.** Given equation is $2\tan\theta - \cot\theta = -1$

$\Rightarrow 2\tan\theta = \cot\theta - 1$

$\Rightarrow 2\tan\theta = \frac{1}{\tan\theta} - 1$

$\Rightarrow 2\tan^2\theta + \tan\theta - 1 = 0$

$\Rightarrow 2\tan^2\theta + 2\tan\theta - \tan\theta - 1 = 0$

$\Rightarrow 2\tan\theta(\tan\theta + 1) - (\tan\theta + 1) = 0$

$\Rightarrow (2\tan\theta - 1)(\tan\theta + 1) = 0$

$\Rightarrow (2\tan\theta - 1) = 0, (\tan\theta + 1) = 0$

$\Rightarrow \tan\theta = -1, \frac{1}{2}$

$\Rightarrow \theta = \left(n\pi - \frac{\pi}{4}\right), \theta = n\pi + \alpha, \alpha = \tan^{-1}\left(\frac{1}{2}\right)$

**15.** $\tan^2\theta + (1 - \sqrt{3})\tan\theta - \sqrt{3} = 0$

**Ans.** $\theta = n\pi + \frac{\pi}{3}, \theta = n\pi - \frac{\pi}{4}$

**Sol.** Given equation is $\tan^2\theta + (1-\sqrt{3})\tan\theta - \sqrt{3} = 0$

$\Rightarrow \tan^2\theta + \tan\theta - \sqrt{3}(\tan\theta + 1) = 0$

$\Rightarrow \tan\theta(\tan\theta + 1) - \sqrt{3}(\tan\theta + 1) = 0$

$\Rightarrow (\tan\theta - \sqrt{3})(\tan\theta + 1) = 0$

$\Rightarrow \tan\theta = \sqrt{3}, \tan\theta = -1$

$\Rightarrow \theta = n\pi + \frac{\pi}{3}, \theta = n\pi - \frac{\pi}{4}, n \in I$

**16.** $\tan\theta + \tan\left(\theta + \frac{\pi}{3}\right) + \tan\left(\theta + \frac{2\pi}{3}\right) = 3$

**Ans.** $\theta = \frac{n\pi}{3} + \frac{\pi}{12}$

**Sol.** Given equation is $\tan\theta + \tan\left(\theta + \frac{\pi}{3}\right) + \tan\left(\theta + \frac{2\pi}{3}\right) = 3$

$\Rightarrow \tan\theta + \tan\left(\frac{\pi}{3} + \theta\right) + \tan\left(\pi - \left(\frac{\pi}{3} - \theta\right)\right) = 3$

$\Rightarrow \tan\theta + \tan\left(\frac{\pi}{3} + \theta\right) - \tan\left(\frac{\pi}{3} - \theta\right) = 3$

$\Rightarrow \tan\theta + \frac{\sqrt{3}+\tan\theta}{1-\sqrt{3}\tan\theta} - \frac{\sqrt{3}-\tan\theta}{1+\sqrt{3}\tan\theta} = 3$

$\Rightarrow \tan\theta + \frac{8\tan\theta}{1-3\tan^2\theta} = 3$

$\Rightarrow \frac{9\tan\theta - 3\tan^3\theta}{1-3\tan^2\theta} = 3$

$\Rightarrow \frac{3\tan\theta - \tan^3\theta}{1-3\tan^2\theta} = 1$

$\Rightarrow \tan(3\theta) = 1$

$\Rightarrow 3\theta = n\pi + \frac{\pi}{4}$

$\Rightarrow \theta = \frac{n\pi}{3} + \frac{\pi}{12}, n \in I$

**17.** $3\tan(\theta - 60^\circ) = \tan(\theta + 60^\circ)$

**Ans.** no solution.

**Sol.** Given equation is $3\tan(\theta - 60^\circ) = \tan(\theta + 60^\circ)$

$\Rightarrow 3 = \frac{\tan(\theta+60^\circ)}{\tan(\theta-60^\circ)}$

$\Rightarrow \frac{\tan(\theta+60^\circ)}{\tan(\theta-60^\circ)} = 3$

$\Rightarrow \frac{\tan(\theta+60^\circ)}{\tan(\theta-60^\circ)} = \frac{3}{1}$

$\Rightarrow \frac{\tan(\theta+60^\circ)+\tan(\theta-60^\circ)}{\tan(\theta+60^\circ)-\tan(\theta-60^\circ)} = \frac{3+1}{3-1}$

$\Rightarrow \frac{\sin(\theta+60^\circ+\theta-60^\circ)}{\sin(\theta+60^\circ-\theta+60^\circ)} = 2$

$\Rightarrow \frac{\sin(2\theta)}{\sin(120^\circ)} = 2$

$\Rightarrow \sin(2\theta) = 2\sin(120^\circ)$

$\Rightarrow \sin(2\theta) = 2 \times \frac{\sqrt{3}}{2} = \sqrt{3}$

It is not possible.
Hence, the equation has no solution.

**18.** $\tan\theta + \tan 2\theta + \tan 3\theta = 0$

**Ans.** $\theta = n\pi \pm \alpha, \alpha = \tan^{-1}\left(\frac{1}{\sqrt{2}}\right)$

**Sol.** Given equation is $\tan\theta + \tan 2\theta + \tan 3\theta = 0$

$\Rightarrow \tan\theta + \tan 2\theta + \tan(2\theta+\theta) = 0$

$\Rightarrow \tan\theta + \tan 2\theta + \frac{\tan(2\theta)+\tan(\theta)}{1-\tan(2\theta)\tan(\theta)} = 0$

$\Rightarrow (\tan\theta + \tan 2\theta)\left(1 + \frac{1}{1-\tan(2\theta)\tan(\theta)}\right) = 0$

$\Rightarrow (\tan\theta + \tan 2\theta) = 0, \left(1 + \frac{1}{1-\tan(2\theta)\tan(\theta)}\right) = 0$

when $(\tan\theta + \tan 2\theta) = 0$

$\Rightarrow \tan\theta + \frac{2\tan\theta}{1-\tan^2\theta} = 0$

$\Rightarrow \tan\theta\left(1 + \frac{2}{1-\tan^2\theta}\right) = 0$

$\Rightarrow \tan\theta = 0, \left(1 + \frac{2}{d-\tan^2}\right) = 0$

$\Rightarrow \tan\theta = 0, \frac{2}{1-\tan^2\theta} = -1$

$\Rightarrow \tan\theta = 0, 1-\tan^2\theta = -2$

$\Rightarrow \tan\theta = 0, \tan^2\theta = 3$

$\Rightarrow \theta = n\pi, \theta = n\pi \pm \frac{\pi}{3}, n \in I$

when $\left(1 + \frac{1}{1-\tan(2\theta)\tan(\theta)}\right) = 0$

$\Rightarrow -\frac{1}{1-\tan\theta\tan 2\theta} - 1$

$\Rightarrow 1 - \tan\theta\tan 2\theta = -1$

$\Rightarrow \tan\theta \tan 2\theta = 2$

$\Rightarrow \tan\theta\left(\frac{2\tan\theta}{1-\tan^2\theta}\right) = 2$

$\Rightarrow \tan 2\theta = 1 - \tan^2\theta$

$\Rightarrow \tan^2\theta = \frac{1}{2} = \tan^2\alpha, \alpha = \tan^{-1}\left(\frac{1}{\sqrt{2}}\right)$

$\Rightarrow \theta = n\pi \pm \alpha, n \in 1$

**19.** $\cos 2\theta \cos 4\theta = \frac{1}{2}$

**Sol.** Given equation is

$\Rightarrow \cos 2\theta \cos 4\theta = \frac{1}{2}$

$\Rightarrow 2\cos(4\theta)\cos(2\theta) = 1$

$\Rightarrow \cos(6\theta) + \cos(2\theta) = 1$

$\Rightarrow \cos(6\theta) = 1 - \cos(2\theta)$

**20.** $\cot\theta - \tan\theta = \cos\theta - \sin\theta$

**Ans.** $\theta = n\pi + \frac{\pi}{4}$

**Sol.** Given equation is $\cot\theta - \tan\theta = \cos\theta - \sin\theta$

$\Rightarrow (\cos\theta - \sin\theta)\left(\frac{(\cos\theta+\sin\theta)}{\sin\theta\cos\theta} - 1\right) = 0$

$\Rightarrow (\cos\theta - \sin\theta) = 0, (\cos\theta + \sin\theta) = \sin\theta\cos\theta$

$\Rightarrow \tan\theta = 1, (\cos\theta + \sin\theta) = \sin q\cos\theta$

when $\tan\theta = 1$

$\Rightarrow \theta = n\pi + \frac{\pi}{4}, n \in I$

when $(\cos\theta + \sin\theta) = \sin\theta\cos\theta$
No real value of $\theta$ satisfies the given equation.

**21.** $(1 - \tan\theta)(1 + \sin 2\theta) = 1 + \tan\theta$

**Ans.** $\theta = n\pi - \frac{\pi}{4}, \theta = n\pi, n \in I$

**Sol.** Given equation is $(1 - \tan\theta)(1 + \sin 2\theta) = 1 + \tan\theta$

$\Rightarrow (\cos\theta - \sin\theta)(\cos\theta + \sin\theta)^2 = (\cos\theta + \sin\theta)$

$\Rightarrow (\cos\theta + \sin\theta)(\cos 2\theta - 1) = 0$

$\Rightarrow \tan(\theta) = -1, \sin^2 \theta = 0$

$\Rightarrow \tan(\theta) = -1, \sin(\theta) = 0$

$\Rightarrow \theta = n\pi - \frac{\pi}{4}, \theta = n\pi, n \in I$

**22.** $2\sin^2\theta + \sin^2 2\theta = 2$

**Ans.** $\theta = (2n+1)\frac{\pi}{2}, \theta = n\pi \pm \frac{\pi}{4}$

**Sol.** Given equation is $2\sin^2 \theta + \sin^2 2\theta = 2$

$\Rightarrow 2\sin^2 \theta + 4\sin^2 \theta\cos^2 \theta = 2$

$\Rightarrow \sin^2 \theta + 2\sin^2 \theta\cos^2 \theta = 1$

$\Rightarrow 2\sin^2 \theta\cos^2 \theta = 1 - \sin^2 \theta$

$\Rightarrow \theta\cos^2 \theta = \cos^2 \theta$

$\Rightarrow (2\sin^2 \theta - 1)\cos^2 \theta = 0$

$\Rightarrow (2\sin^2 \theta - 1) = 0, \cos^2 \theta = 0$

$\Rightarrow \sin^2 \theta = \frac{1}{2}, \cos \theta = 0$

$\Rightarrow \theta = (2n+1)\frac{\pi}{2}, \theta = n\pi \pm \frac{\pi}{4}, n \in I$

**23.** $\sin 3\alpha = 4\sin\theta \sin(\theta+\alpha) \sin(\theta-\alpha), \alpha \neq n\pi, n \in Z$

**Ans.** $\theta = n\pi \pm \frac{\pi}{3}$

**Sol.** Given equation is $\sin(3\alpha) = 4\sin \theta\sin(\theta+\alpha)\sin(\theta+\alpha)$

$\Rightarrow \sin(3\alpha) = 4\sin \theta(\sin^2 \theta - \sin^2 \alpha)$

$\Rightarrow 3\sin \alpha - 4\sin^2 \alpha = 4\sin \alpha(\sin^2 \theta - \sin^2 \alpha)$

It is possible only when $\sin^2 \theta = \frac{3}{4}$

$\Rightarrow \sin^2 \theta = \left(\frac{\sqrt{3}}{2}\right)^2$

$\Rightarrow n \in I$

**24.** $4\sin\theta \sin 2\theta \sin 4\theta = \sin 3\theta$

**Ans.** $\theta = n\pi, \theta = n\pi \pm \frac{\pi}{3}$

**Sol.** Given equation is $4\sin \theta\sin 2\theta\sin 4\theta = \sin 3\theta$

$\Rightarrow 4\sin\ \theta\sin\ (3\theta-\theta)\sin\ (3\theta+\theta)=\sin\ 3\theta$

$\Rightarrow 4\sin\ \theta[\sin^2(3\theta)-\sin^2(\theta)]=\sin\ 3\theta$

$\Rightarrow 4\sin\ \theta[\sin^2(3\theta)-\sin^2(\theta)]=3\sin\ \theta-4\sin^3\ \theta$

$\Rightarrow \sin\ \theta[4\sin^2(3\theta)-4\sin^2(\theta)+4\sin^2\ \theta-3]=0$

$\Rightarrow \sin\ \theta[4\sin^2(3\theta)-3]=0$

$\Rightarrow \sin\ \theta=0,[4\sin^2(3\theta)-3]=0$

$\Rightarrow\ \sin\ \theta=0,\sin^2(3\theta)=\frac{3}{4}$

$\theta=n\pi,\theta=n\pi\pm\frac{\pi}{3},n\in I$

PRINCIPAL VALUE

**Q.** Find the principal value of

**25.** $\sin(\theta)=-\frac{1}{2}.$

**Ans.** $\left(-\frac{\pi}{6}\right)$ 26. $\frac{\pi}{4}$

**Sol.** We have $\sin(\theta)+\cos(\theta)=1$

$$\Rightarrow\ \sqrt{2}\left(\frac{1}{\sqrt{2}}\sin(\theta)+\frac{1}{\sqrt{2}}\cos(\theta)\right)=1$$
$$\Rightarrow\ \left(\frac{1}{\sqrt{2}}\sin(\theta)+\frac{1}{\sqrt{2}}\cos(\theta)\right)=\frac{1}{\sqrt{2}}$$
$$\Rightarrow\ \sin\left(\theta+\frac{\pi}{4}\right)=\frac{1}{\sqrt{2}}$$
$$\Rightarrow\ \sin\left(\theta+\frac{\pi}{4}\right)=\left(\sin\left(\frac{\pi}{4}\right)\right)$$
$$\Rightarrow\ \left(\theta+\frac{\pi}{4}\right)=\left(n\pi+(-1)^n\left(\frac{\pi}{4}\right)\right)$$
$$\Rightarrow\ \theta=\left(n\pi+(-1)^n\left(\frac{\pi}{4}\right)-\frac{\pi}{4}\right),n\in I$$

**26.** $\sin(\theta)=\frac{1}{\sqrt{2}}$

**Ans.** $\frac{\pi}{4}$

**Sol.** We have $\sqrt{3}\sin(\theta)+\cos(\theta)=2$

$$\Rightarrow\ \frac{\sqrt{3}}{2}\sin\ \theta+\frac{1}{2}\cos\ \theta=1$$

$\Rightarrow \sin\left(\theta+\frac{\pi}{6}\right)=1$

$\Rightarrow \sin\left(\theta+\frac{\pi}{6}\right)=\sin\left(\frac{\pi}{2}\right)$

$\Rightarrow \left(\theta+\frac{\pi}{6}\right)=n\pi+(-1)^n\left(\frac{\pi}{2}\right)$

$\Rightarrow \theta=n\pi+(-1)^n\left(\frac{\pi}{2}\right)-\frac{\pi}{6}$

**27.** $\tan(\theta)=-\sqrt{3}$

**Ans.** $-\frac{\pi}{3}$

**Sol.** We have

$\sin(2\theta)+\cos(2\theta)+\sin(\theta)+\cos(\theta)+1=0$

$\Rightarrow (\sin(\theta)+\cos(\theta))+(1+\sin(2\theta))+\cos(2\theta)=0$

$\Rightarrow (\sin(\theta)+\cos(\theta))+(\sin(\theta)+\cos(\theta))^2$
$+(\cos^2\theta-\sin^2\theta)=0.$

$\Rightarrow (\sin(\theta)+\cos(\theta))+(\sin(\theta)+\cos(\theta))^2$
$+(\cos\theta+\sin\theta)(\cos\theta-\sin\theta)=0$

$\Rightarrow (\sin(\theta)+\cos(\theta))$
$\Rightarrow (1+(\sin(\theta)+\cos(\theta))+(\cos\theta-\sin\theta))=0$

$\Rightarrow (\sin(\theta)+\cos(\theta))(1+2\cos\theta)=0$

$\Rightarrow (\sin(\theta)+\cos(\theta))=0$ and $(1+2\cos\theta)=0$

$\Rightarrow \left(\frac{\pi}{4}+\theta\right)=n\Big)=0$ and $\cos\theta=-\frac{1}{2}$

$\Rightarrow \theta=n\pi-\frac{\pi}{4}$ and $\theta=2n\pi\pm\left(\frac{2\pi}{3}\right), n\in I$

**28.** $\tan\theta=-1$

**Ans.** $-\frac{\pi}{4}$

**Sol.** We have $\sin^2\theta+\sin\theta\cos\theta+\cos^2\theta=1$

$\Rightarrow (\sin^3\theta+\cos^3\theta)+\sin\theta\cos\theta=1$

$\Rightarrow (\sin\theta+\cos\theta)(1-\sin\theta\cos\theta)+\sin\theta\cos\theta=1$
$\Rightarrow (\sin\theta+\cos\theta)(1-\sin\theta\cos\theta)=(1-\sin\theta\cos\theta)$

$\Rightarrow (\sin\theta+\cos\theta-1)(1-\sin\theta\cos\theta)=0$
$\Rightarrow (\sin\theta+\cos\theta-1)=0$ and $(1-\sin\theta\cos\theta)=0$

$\Rightarrow (\sin\theta+\cos\theta)=1$ and $\sin(2\theta)=\frac{1}{2}$

$\Rightarrow \left(\sin\left(\theta+\frac{\pi}{4}\right)\right)=\sin\left(\frac{\pi}{2}\right)$

and $\sin(2\theta)=\sin\left(\frac{\pi}{6}\right)$

$\Rightarrow \theta=n\pi+(-1)^n\left(\frac{\pi}{2}\right)-\left(\frac{\pi}{4}\right)$

and $\theta=\frac{1}{2}\left(n\pi+(-1)^n\left(\frac{\pi}{6}\right)\right)$, where $n\in I$

**29.** $\cos\theta = \frac{1}{2}$

**Ans.** $\frac{\pi}{3}$

**Sol.** Given equation is

$$\sin\ \theta + \sqrt{3}\cos\ \theta = \sqrt{2}$$

$$\Rightarrow \quad \frac{1}{2}\sin\ \theta + \frac{\sqrt{3}}{2}\cos\ \theta = \frac{1}{\sqrt{2}}$$

**30.** $\cos\theta = -\frac{1}{2}$

**Ans.** $\frac{2\pi}{3}$

**Sol.** $\Rightarrow\ \sin\left(\theta + \frac{\pi}{3}\right) = \frac{1}{\sqrt{2}}$

$\Rightarrow\ \left(\theta + \frac{\pi}{3}\right) = n\pi + (-1)^n\left(\frac{\pi}{4}\right)$

$\Rightarrow\ \theta = n\pi + (-1)^n\left(\frac{\pi}{4}\right) - \frac{\pi}{3}, n \in I$

**31.** $\tan\theta = -\sqrt{3}$

**Ans.** $-\frac{\pi}{3}$

**Sol.** Do yourself.

**32.** $\sec\theta = \sqrt{2}$.

**Ans.** $\frac{\pi}{4}$

**Sol.** Do yourself.

## SOLUTIONS IN CASE IF TWO EQUATIONS ARE GIVEN:

**33.** If $\sin(\theta) = \frac{1}{\sqrt{2}}$ and $\cos(\theta) = -\frac{1}{\sqrt{2}}$, then find the general values of $\theta$

**Ans.** $\left(2n\pi + \frac{3\pi}{4}\right)$

**Sol.** Do yourself.

**34. If** $\sin(\theta) = \frac{1}{\sqrt{2}}$ and $\tan(\theta) = -1$, then find the general values of $\theta$

**Ans.** $\left(2n\pi + \frac{3\pi}{4}\right)$

**Sol.** Do yourself.

**35.** If $\cos\theta = \frac{1}{\sqrt{2}}$ and $\tan\theta = -1$, then find the general value of $\theta$

**Ans.** $2n\pi + \frac{7\pi}{4}$

**Sol.** Do yourself.

**36.** Find the most general value of $\theta$ which satisfy the equations $\sin\theta = \frac{1}{2}$ and $\tan\theta = \frac{1}{\sqrt{3}}$

**Ans.** $2n\pi+\frac{\pi}{3}$

**Sol.** Do yourself.

## DIFFERENT TYPES IF TRIGNOMETRIC EQUATION

## TYPE–1

**Q.** Solve for x :

**37.** $5\cos 2x+2\cos^2\left(\frac{x}{2}\right)+1=0$

**Ans.** $2n\pi\pm\frac{\pi}{3}$

**Sol.** Given equation is

$$\cos\theta+\sqrt{3}\sin\theta=2\cos 2\theta$$
$$\Rightarrow \frac{1}{2}\cos\theta+\frac{\sqrt{3}}{2}\sin\theta=\cos 2\theta$$
$$\Rightarrow \cos\left(\theta-\frac{\pi}{3}\right)=\cos 2\theta$$
$$\Rightarrow \left(\theta-\frac{\pi}{3}\right)=2n\pi\pm 2\theta$$

Taking positive one, we get

$$\theta=-\left(2n\pi+\frac{\pi}{3}\right)$$

Taking negative one, we get,

$$\Rightarrow \theta=\frac{2n\pi}{3}+\frac{\pi}{9}, n\in I$$

**38.** $4\sin^4 x+\cos^4 x=1$

**Ans.** $n\pi, x=n\pi\pm\alpha, \alpha=\sin^{-1}\left(\sqrt{\frac{2}{5}}\right)$

**Sol.** Given equation is

$$\sqrt{3}(\cos\theta-\sqrt{3}\sin\theta)=4\sin 2\theta\cdot\cos 3\theta$$
$$\Rightarrow \sqrt{3}\cos\theta-3\sin\theta=2(\sin 5\theta-\sin\theta)$$
$$\Rightarrow \sqrt{3}\cos\theta-\sin\theta=2(\sin 5\theta)$$
$$\Rightarrow \frac{\sqrt{3}}{2}\cos\theta-\frac{1}{2}\sin\theta=(\sin 5\theta)$$
$$\Rightarrow \sin\left(\frac{\pi}{3}-\theta\right)=\sin 5\theta$$
$$\Rightarrow 5\theta=n\pi+(-1)^n\left(\frac{\pi}{3}-\theta\right)$$

when n is even

$$5\theta = 2k\pi + \left(\frac{\pi}{3} - \theta\right)$$

$$\Rightarrow\ 6\theta = 2k\pi + \frac{\pi}{3}$$

$$\Rightarrow\ \theta = \frac{k\pi}{3} + \frac{\pi}{18}, k \in I$$

when n is odd

$$5\theta = (2k+1)\pi - \left(\frac{\pi}{3} - \theta\right)$$

$$\Rightarrow\ 4\theta = (2k+1)\pi - \frac{\pi}{3}$$

$$\Rightarrow\ \theta = (2k+1)\frac{\pi}{4} - \frac{\pi}{12}, k \in I$$

**39.** $4\cos^2 x \sin x - 2\sin^2 x = 2\sin x$

**Ans.** $n\pi, x = (4n+1)\frac{\pi}{2}$

**Sol.** We have $\sin(\theta) = -\frac{1}{2}$

$$\Rightarrow\ \theta = -\frac{\pi}{6}$$

Hence, the principal value of $\theta$ is $\left(-\frac{\pi}{6}\right)$

**40.** $\sin 3x + \cos 2x = 1$

**Ans.** $n\pi, x = n\pi + (-1)^n\alpha, \alpha = \sin^{-1}\left(\frac{\sqrt{13}-1}{4}\right)$

**Sol.** We have $\sin(\theta) = \frac{1}{\sqrt{2}}$

$$\Rightarrow\ \theta = \frac{\pi 3\pi}{4,4}$$

Hence the principal value of $\theta$ is $\frac{\pi}{4}$

**41.** $2\cos 2x + \sqrt{2\sin x} = 2$

**Ans.** $n\pi, n = n\pi + (-1)^n\left(\frac{\pi}{4}\right)$

**Sol.** We have $\tan(\theta) = -\sqrt{3}$

$$\Rightarrow\ (\theta) = -\frac{\pi}{3}$$

Hence, the principal value of $\theta$ is $-\frac{\pi}{3}$.

**42.** $1 + \sin^3 x + \cos^3 x = \frac{3}{2}\sin 2x$

**Ans.** $n\pi + (-1)^n\left(-\frac{\pi}{4}\right) - \frac{\pi}{4}$

**Sol.** Given, $\tan\ \theta = -1$

$$\Rightarrow\ \theta = \frac{3\pi}{4}, -\frac{\pi}{4}$$

Hence, the principal value of $\theta$ is $-\frac{\pi}{4}$.

**43.** $\sin^6 x + \cos^6 x = \frac{7}{16}$

**Ans.** $\frac{n\pi}{3} \pm \frac{\pi}{6}$

**Sol.** Given, $\cos\ \dot{\theta} = \frac{1}{2}$

$$\Rightarrow\ \theta = \frac{\pi}{3}, \frac{\pi}{3}$$

Hence, the principal value of $\theta$ is $\frac{\pi}{3}$.

**44.** $\sin^8\ x + \cos^8\ x = \frac{17}{16}\cos^2 2x$

**Ans.** $\frac{n\pi}{2} \pm \frac{\alpha}{2}, \alpha = \sin^{-1}\left(\sqrt{\frac{\sqrt{5}-1}{4}}\right)$

**Sol.** Given, $\cos\ \theta = -\frac{1}{2}$

$$\Rightarrow\ \theta = \frac{2\pi}{3}, -\frac{2\pi}{3}$$

Hence, the principal value of $\theta$ is $\frac{2\pi}{3}$.

**45.** $2\sin^3\ x + 2 = \cos^2 3x$

**Ans.** $(4n-1)\frac{\pi}{2}$,

**Sol.** Given, $\tan\ \theta = -\sqrt{3}$

$$\Rightarrow\ \theta = \frac{2\pi}{3}, -\frac{\pi}{3}$$

Hence, the principal value of $\theta$ is $-\frac{\pi}{3}$.

**46.** $\cos 4x = \cos^2 3x$

**Ans.** $n\pi$

**Sol.** Given, $\sec\ \theta = \sqrt{2}$.

$$\Rightarrow\ \cos\ \theta = \frac{1}{\sqrt{2}}$$

$$\Rightarrow\ \theta = \frac{\pi}{4}, -\frac{\pi}{4}$$

Hence, the principal value of θ is $\frac{\pi}{4}$.

**47.** $\cos 2x = 6\tan^2 x - 2\cos^2 x$

**Ans.** $n\pi \pm \frac{\pi}{6}$

**Sol.** Now $\sin(\theta) = \frac{1}{\sqrt{2}}$

$\Rightarrow \theta = \frac{\pi 3\pi}{4'4}$

and $\cos(\theta) = -\frac{1}{\sqrt{2}}$

$\Rightarrow \theta = \frac{3\pi}{4}$

Thus, the common value of θ is $\frac{3\pi}{4}$.

Hence, the general values of θ is

$$\left(2n\pi + \frac{3\pi}{4}\right)$$

TYPE–2

**Q.** Solve for x :

**48.** $(2\sin x - \cos x)(1 + \cos x) = \sin^2 x$

**Ans.** $x = 2n\pi \pm \pi, x = n\pi + (-1)^n \frac{\pi}{6}, n \in Z$

**Sol.** We have $\sin(\theta) = \frac{1}{\sqrt{2}}$

$\Rightarrow \theta = \frac{\pi 3\pi}{4,4}$

Also, $\tan\theta = -1$

$\Rightarrow \theta = \frac{3\pi 7\pi}{4,4}$

Thus, the common value of θ is $\frac{3\pi}{4}$.

Hence, the general values of θ is $\left(2n\pi + \frac{3\pi}{4}\right)$, where $n \in I$.

**49.** $2\sin^2 x + \sin x - 1 = 0$ where $0 \le x \le 2\pi$

**Ans.** $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}$

**Sol.** Given, $\cos\theta = \frac{1}{\sqrt{2}}$

$\Rightarrow \theta = \frac{\pi 7\pi}{4,4}$

and $\tan\theta = -1$

$\Rightarrow \theta = \frac{3\pi 7\pi}{4,4}$

Hence, the general solution is

$$\theta = 2n\pi + \frac{7\pi}{4}, n \in I$$

**50.** $5\sin^2 x + 7\sin x - 6 = 0$ where $0 \le x \le 2\pi$

**Ans.** $\sin^{-1}\left(\frac{3}{5}\right), \pi - \sin^{-1}\left(\frac{3}{5}\right)$

**Sol.** Given, $\sin\theta = \frac{1}{2}$

$$\Rightarrow \theta = \frac{\pi 2\pi}{3,3}.$$

and $\tan\theta = \frac{1}{\sqrt{3}}$

$$\Rightarrow \theta = \frac{\pi 4\pi}{3,3}.$$

Hence, the general solution is

$$\theta = 2n\pi + \frac{\pi}{3}, n \in I$$

**51.** $\sin^2 x - \cos x = \frac{1}{4}$, where $0 \le x \le 2\pi$

**Ans.** $\frac{\pi}{3}, \frac{5\pi}{3}$

**Sol.** We have, $(1 + \tan A)(1 + \text{tab } B) = 2$

$$\Rightarrow 1 + \tan A + \tan B + \tan A \cdot \tan B = 2$$
$$\Rightarrow \tan A + \tan B = 1 - \tan A \cdot \tan B$$
$$\Rightarrow \left(\frac{\tan A + \tan B}{1 - \tan A \cdot \tan B}\right) = 1$$
$$\Rightarrow \tan(A + B) = 1$$
$$\Rightarrow \tan(A + B) = \tan\left(\frac{\pi}{4}\right)$$
$$\Rightarrow (A + B) = n\pi + \left(\frac{\pi}{4}\right), \text{ where } n \in I$$

**52.** $\tan^2 x - 2\tan x - 3 = 0$

**Ans.** $n\pi - \frac{\pi}{4}, x = n\pi + \alpha, \alpha = \tan^{-1}(3)$

**Sol.** Given, $\tan(A - B) = 1$

$$\Rightarrow (A - B) = \frac{\pi}{4}, \frac{5\pi}{4}$$

Also, $\sec(A + B) = \frac{2}{\sqrt{3}}$

$$\Rightarrow \cos(A + B) = \frac{\sqrt{3}}{2}$$
$$\Rightarrow (A + B) = \frac{\pi \cdot 11\pi}{6,6}$$

Here, we observe that A − B is positive
So, A > B

$$\Rightarrow A+B>A-B$$

$$\begin{cases} A+B=\frac{11\pi}{6} \\ A-B=\frac{\pi}{4} \end{cases} \text{or} \begin{cases} A+B=\frac{11\pi}{6}. \\ A-B=\frac{\sigma}{4} \end{cases}$$

On solving, we get,

$$\begin{cases} A=\frac{25\pi}{24} \\ B=\frac{19\pi}{24} \end{cases} \text{or} \begin{cases} A=\frac{19\pi}{24} \\ B=\frac{7\pi}{24} \end{cases}$$

General values of tan $\tan(A-B)=1$
is $(A-B)=n\pi+\frac{\pi}{4}, n\in I$

General values of sec $(A+B)=\frac{2}{\sqrt{3}}$
is $(A+B)=2n\pi+\frac{\pi}{6}, n\in I$
On solving (i) and (ii), we get

$$\begin{cases} A=(2n+m)\frac{\pi}{2}+\frac{\pi}{24} \\ B=(2n-m)\frac{\pi}{2}-\frac{5\pi}{24} \end{cases}$$

**53.** $2\cos^2 x-\sqrt{3}\sin x+1=0$

**Ans.** $n\pi+(-1)^n\left(\frac{\pi}{3}\right)$

**Sol.** We have $\sin(\pi\cos\theta)=\cos(\pi\sin\theta)$

$$\Rightarrow \sin(\pi\cos\theta)=\sin\left(\frac{\pi}{2}-\pi\sin\theta\right)$$

$$\Rightarrow (\pi\cos\theta)=\left(\frac{\pi}{2}-\pi\sin\theta\right)$$

$$\Rightarrow \cos\theta=\left(\frac{1}{2}-\sin\theta\right)$$

$$\Rightarrow \cos\theta+\sin\theta=\frac{1}{2}$$

$$\Rightarrow \frac{1}{\sqrt{2}}\cos\theta+\frac{1}{\sqrt{2}}\sin\theta=\frac{1}{2\sqrt{2}}$$

$$\Rightarrow \cos\left(\theta+\frac{\pi}{4}\right)=\frac{1}{2\sqrt{2}}$$

Similarly, we can prove that,cos $\left(\theta-\frac{\pi}{4}\right)=\frac{1}{2\sqrt{2}}$

## TYPE–3

**Q.** Solve for x :

**54.** $\sin x + \sin 3x + \sin 5x = 0, 0 \le x \le \frac{\pi}{2}$

**Ans.** $0, \frac{\pi}{3}$

**Sol.** We have $\tan(\pi\cos\theta) = \cot(\pi\sin\theta)$

$\Rightarrow \tan(\pi\cos\theta) = \tan\left(\frac{\pi}{2} - \pi\sin\theta\right)$

$\Rightarrow (\pi\cos\theta) = \left(\frac{\pi}{2} - \pi\sin\theta\right)$

$\Rightarrow \cos(\theta) + \sin(\theta) = \frac{1}{2}$

$\Rightarrow \frac{1}{\sqrt{2}}\cos(\theta) + \frac{1}{\sqrt{2}}\sin(\theta) = \frac{1}{2\sqrt{2}}$

$\Rightarrow \cos\left(\theta - \frac{\pi}{4}\right) = \frac{1}{2\sqrt{2}}$

**55.** $\cos x - \cos 2x = \sin 3x$

**Ans.** $n\pi + \frac{\pi}{4}, 2n\pi - \frac{\pi}{2}$

**Sol.** Given, $\sin A = \sin B$
and $\cos A = \cos B$
Dividing (i) and (ii), we get,

$$\frac{\sin A}{\cos A} = \frac{\sin B}{\cos B}$$

$\Rightarrow \tan A = \tan B$

$\Rightarrow A = n\pi + B$, where $n \in I$

**56.** $\sin 7x + \sin 4x + \sin x = 0, 0 \le x \le \frac{\pi}{2}$

**Ans.** $0, \frac{2\pi}{9}, \frac{\pi}{4}, \frac{\pi}{2}$

**Sol.** Given equations are

$3\sin^2 A + 2\sin^2 B = 1$

and $3\sin 2A - 2\sin 2B = 0$
From (ii), we get,

$$3\sin 2A = 2\sin 2B$$

$$\Rightarrow \frac{\sin 2A}{2} = \frac{\sin 2B}{3}$$

$$\Rightarrow \frac{\sin 2B}{\sin 2A} = \frac{3}{2}$$

From (i), we get

$$\frac{3}{2}(2\sin^2 A) + (2\sin^2 B) = 1$$
$$\Rightarrow \frac{3}{2}(1 - \cos 2A) + (1 - \cos 2B) = 1$$
$$\Rightarrow \frac{3}{2}\cos 2A + \cos 2B = \frac{3}{2}$$
$$\Rightarrow \frac{\sin 2B}{\sin 2A}\cos 2A + \cos 2B = \frac{\sin 2B}{\sin 2A}$$
$$\Rightarrow \sin 2B\cos 2A + \sin 2A\cos 2B = \sin 2B$$
$$\Rightarrow \sin(2A + 2B) = \sin 2B$$
$$\Rightarrow \sin(2A + 2B) = \sin(\pi - 2B)$$
$$\Rightarrow (2A + 2) = (\pi = 2B)$$
$$\Rightarrow (2A + 4B) = \pi$$
$$\Rightarrow (A + 2B) = \frac{\pi}{2}$$

**57.** $\cos 3x + \cos 2x = \sin\left(\frac{3x}{2}\right) + \sin\left(\frac{x}{2}\right), 0 \le x \le 2\pi$

**Ans.** $\frac{\pi}{3}, \frac{\pi}{5}, \frac{3\pi}{5}, \frac{13\pi}{15}, \frac{17\pi}{15}, \frac{7\pi}{5}, \frac{5\pi}{3}, \frac{29\pi}{15}$

**Sol.** Given, $x + y = \frac{\pi}{4}$ and $\tan x + \tan y = 1$

$$\Rightarrow \tan(x + y) = \tan\left(\frac{\pi}{4}\right)$$
$$\frac{\tan x + \tan y}{1 - \tan x \cdot \tan y} = 1$$
$$\Rightarrow 1 - \tan x \cdot \tan y = 1$$
$$\Rightarrow \tan x \cdot \tan y = 0$$
$$\Rightarrow \tan x = 0 \& \tan y = 0$$
$$\Rightarrow x = n\pi = y$$

Thus, no values of x and y satisfy the given equations. Therefore, the given equations have no solutions.

**58.** $\sin x + \sin 2x + \sin 3x = \cos x + \cos 2x + \cos 3x, -\pi \le x \le 2\pi$

**Sol.** Given, $\sin x + \sin y = 1$
and $\cos 2x - \cos 2y = 1$
From (ii), we get, $\cos 2x - \cos 2y = 1$

$$\Rightarrow 1 - 2\sin^2 x - 1 + \sin^2 y = 1$$
$$\Rightarrow \sin^2 x - \sin^2 y = -\frac{1}{2}$$
$$\Rightarrow y = n\pi + (-1)^n \sin^{-1}\left(\frac{3}{4}\right),$$

where $n \in I$

$$\Rightarrow \sin x - \sin y = -\frac{1}{2}$$

Adding (i) and (iii), we get,

$$2\sin\ x = \frac{1}{2}$$

$$\Rightarrow \quad \sin\ x = \frac{1}{4}$$

$$\Rightarrow \quad x = n\pi + (-1)^n \sin^{-1}\left(\frac{1}{4}\right), n \in I,$$

Subtracting (i) and (iii), we get

$$2\sin\ y = \frac{3}{2}$$

$$\Rightarrow \quad \sin\ y = \frac{3}{4}$$

$$\Rightarrow \quad y = n\pi + (-1)^n \sin^{-1}\left(\frac{3}{4}\right), n \in I$$

**59.** $\cos 2x + \cos 4x = 2\cos x$

**Ans.** $\frac{2n\pi}{3} \pm \frac{2\pi}{9}, x = (2n+1)\frac{\pi}{2}$

**Sol.** Given, $\sin\ x = 2\sin\ y$

$$\Rightarrow \sin\ x = 2\sin\left(\frac{2\pi}{3} - x\right)$$

$$\Rightarrow \sin\ x = 2\left(\frac{\sqrt{3}}{2}\cos\ x + \frac{1}{2}\sin\ x\right)$$

$$\Rightarrow \sin\ x = \sqrt{3}\cos\ x + \sin\ x$$

$$\Rightarrow \sqrt{3}\cos\ x = 0$$

$$\Rightarrow \cos\ x = 0$$

$$\Rightarrow x = (2n+1)\frac{\pi}{2}$$

when $x = (2n+1)\frac{\pi}{2}$, then $y = n\pi - \frac{\pi}{6}$ Hence, the solutions are

$$\left\{\begin{array}{l} x = (2n+1)\dfrac{\pi}{2} \\ y = n\pi - \dfrac{\pi}{6} \end{array}\right., n \in I$$

**60.** $\sin 2x + \cos 2x + \sin x + \cos x + 1 = 0$

**Ans.** $n\pi - \frac{\pi}{4}, x = 2n\pi \pm \frac{2\pi}{3}$

**Sol.** Given, $x + y = \frac{2\pi}{3}$ and $\cos\ x + \cos\ y = \frac{3}{2}$

Now $\cos\ x + \cos\ y = \frac{3}{2}$

$$\Rightarrow \cos x + \cos\left(\frac{2\pi}{3} - x\right) = \frac{3}{2}$$

$$\Rightarrow \cos x - \frac{1}{2}\cos x + \frac{\sqrt{3}}{2}\sin x = \frac{3}{2}$$

$$\Rightarrow \frac{1}{2}\cos x + \frac{\sqrt{3}}{2}\sin x = \frac{3}{2}$$

$$\Rightarrow \cos x + \sqrt{3}\sin x = 3$$

It is not possible, since the maximum value of LHS is 2.
So, the given system of equations has no solutions.

**61.** $\tan x + \tan 2x + \tan 3x = 0$

**Sol.** Given equations are

$$r\sin\theta = 3$$

and $r = 4(1 + \sin\theta)$

Eliminating (i) and (ii), we get

$$4(1 + \sin\theta)\sin\theta = 3$$

$$\Rightarrow 4\sin^2\theta + 4\sin\theta - 3 = 0$$

$$\Rightarrow 4\sin^2\theta + 6\sin\theta - 2\sin\theta - 3 = 0$$

$$\Rightarrow 2\sin\theta(2\sin\theta + 3) - 1(2\sin\theta + 3) = 0$$

$$\Rightarrow (2\sin\theta + 3)(2\sin\theta - 1) = 0$$

$$\Rightarrow \sin\theta = -\frac{3}{2}, \frac{1}{2}$$

$$\Rightarrow \sin\theta = \frac{1}{2}$$

$$\Rightarrow \theta = \frac{\pi}{6}, 6$$

**62.** $\tan 3x + \tan x = 2\tan 2x$

**Ans.** $\frac{n\pi}{2}, x = n\pi$

**Sol.** Given equations are

$$\sin x + \sin y = 1$$

and $\cos 2x - \cos 2y = 1$

Now, $\cos 2x - \cos 2y = 1$

$$\Rightarrow 1 - 2\sin^2 x - 1 + 2\sin^2 y = 1$$

$$\Rightarrow -2\sin^2 x - 1 + 2\sin^2 y = 0$$

$$\Rightarrow 2(\sin^2 x - \sin^2 y) = -1$$

$$\Rightarrow (\sin x + \sin y)(\sin x - \sin y) = -\frac{1}{2}$$

$$\Rightarrow (\sin x - \sin y) = -\frac{1}{2}$$

On solving, we get

$$\sin x = 0, \sin y = 1$$

$$\Rightarrow \quad x = n\pi; y = (4n+1)\frac{\pi}{2}, n \in I$$

Hence, the solutions are

$$\begin{cases} x = n\pi \\ y = (4n+1)\frac{\pi}{2}, n \in I \end{cases}$$

**63.** $(1 - \tan x)(1 + \sin 2x) = (1 + \tan x)$

**Ans.** $n\pi, x = n\pi - \frac{\pi}{4}$

**Sol.** Given curves are $y = \cos x$ and $y = \sin 2x$
Thus, $\sin 2x = \cos x$

$\Rightarrow \quad 2\sin x \cos x = \cos x$
$\Rightarrow \quad (2\sin x - 1)\cos x = 0$
$\Rightarrow \quad (2\sin x - 1) = 0, \cos x = 0$
$\Rightarrow \quad \sin x = \frac{1}{2}, \cos x = 0$
$\Rightarrow \quad x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{\pi}{2}, \frac{3\pi}{2}$

then $y = \frac{\sqrt{3}}{2}, -\frac{\sqrt{3}}{2}, 0$

Hence, the solutions are

$$\left(\frac{\pi}{6}, \frac{\sqrt{3}}{2}\right), \left(\frac{5\pi}{6}, -\frac{\sqrt{3}}{2}\right), \left(\frac{\pi}{2}, 0\right), \left(\frac{3\pi}{2}, 0\right)$$

**64.** $\sin x - 3\sin 2x + \sin 3x = \cos x \; 3\cos 2x + \cos 3x$

**Ans.** $\frac{n\pi}{2} + \frac{\pi}{8}$

**Sol.** Given equation is

$\Rightarrow \cos x + \cos y + \cos(x+y) = -\frac{3}{2}$

$\Rightarrow 2(\cos x + \cos y) + 2\cos(x+y) + 2 = 1$

$\Rightarrow 4\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right) + 4\cos^2\left(\frac{x+y}{2}\right) = 1$

$\Rightarrow 4\cos^2\left(\frac{x+y}{2}\right) + 4\cos\left(\frac{x-y}{2}\right)\cos\left(\frac{x+y}{2}\right) + 1 = 0$

For real x and y,

$$16\cos^2\left(\frac{x-y}{2}\right) - 16 \geq 0$$

$$\Rightarrow \quad \cos^2\left(\frac{x-y}{2}\right) \geq 1$$

$$\Rightarrow \quad \cos^2\left(\frac{x-y}{2}\right) = 1$$

$$\Rightarrow \quad \left(\frac{x-y}{2}\right) = 0$$

$$\Rightarrow \quad x = y$$

The given equation

$$4\cos^2\left(\frac{x+y}{2}\right) + 4\cos\left(\frac{x-y}{2}\right)\cos\left(\frac{x+y}{2}\right) + 1 = 0$$

reduces to $4\cos^2(x) + \cos(x) + 1 = 0$

$\Rightarrow \ (2\cos(x) + 1)^2 = 0$

$$\Rightarrow \ \cos(x) = -\frac{1}{2}$$

$$\Rightarrow \ x = \frac{2\pi}{3} = y$$