Nishant Jindal

** Energy in Case of S.H.M

$$E_T = P.E + K.E$$

$$v_x = \omega\sqrt{A^2 - x^2}$$

$$K.E = \frac{1}{2}mv_x^2$$

$$\boxed{K.E = \frac{1}{2}m\omega^2(A^2 - x^2)}$$

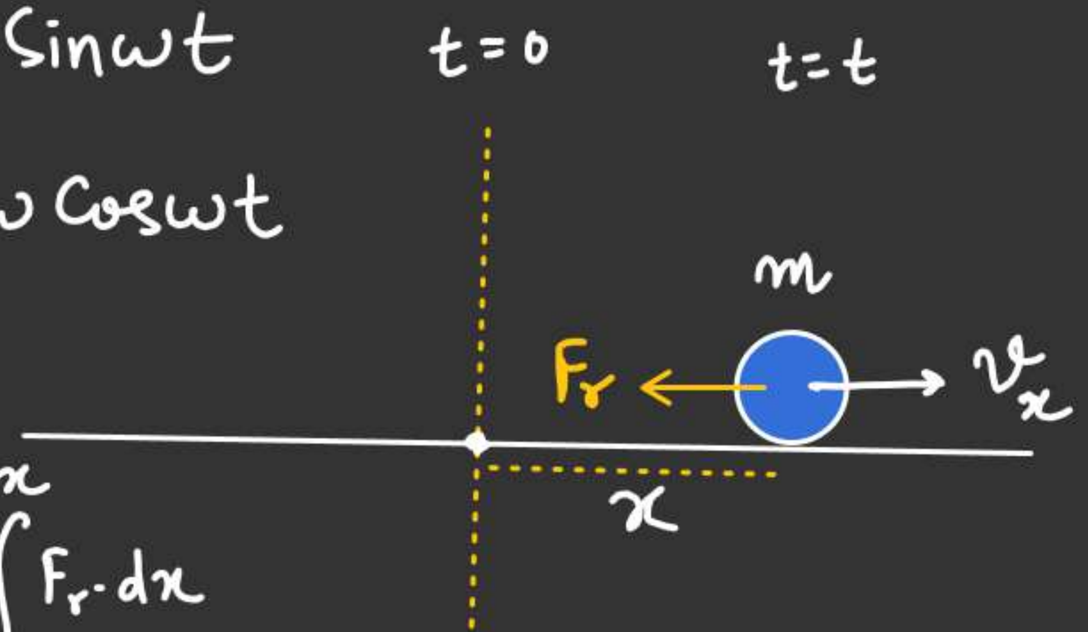
$$K.E = \frac{1}{2}m\omega^2(A^2 - A^2\sin^2\omega t)$$

$$K.E = \frac{1}{2}m\omega^2A^2(1 - \sin^2\omega t)$$

$$\boxed{K.E = \frac{1}{2}m\omega^2A^2\cos^2\omega t} \quad \text{— (1)}$$

$$x = A\sin\omega t$$

$$v = \frac{dx}{dt} = A\omega\cos\omega t$$



$$W_{F_r} = \int_0^x F_r \cdot dx$$

$$W_r = -\int_0^x kx\,dx$$

$$-W_r = \int_0^x kx\,dx$$

$$\Downarrow$$

$$U_x - \cancel{U_{x=0}}^{\,0} = \frac{kx^2}{2}$$

$$U_x = \frac{1}{2}kx^2$$

$$F_r = -kx \qquad \omega^2 = \frac{k}{m}$$

$$k = m\omega^2$$

$$U_x = \frac{1}{2}kA^2\sin^2\omega t$$

$$\boxed{U_x = \frac{1}{2}m\omega^2A^2\sin^2\omega t} \quad \text{— (2)}$$

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S·H·M

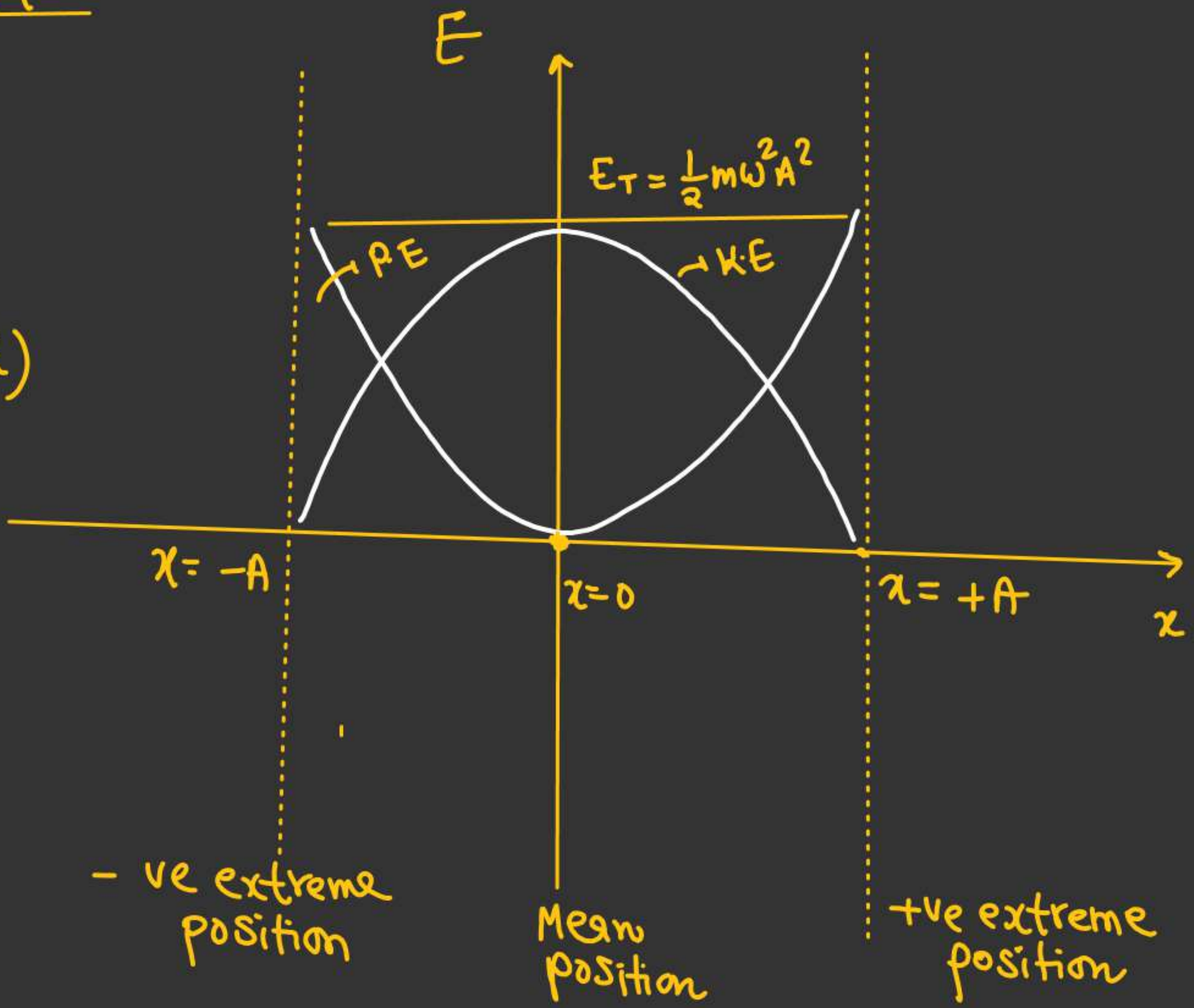
Total Energy

$$E_T = P.E + K.E$$

$$= \frac{1}{2}m\omega^2A^2(\sin^2\omega t + \cos^2\omega t)$$

$$\boxed{E_T = \frac{1}{2}m\omega^2A^2}$$

$$\boxed{\frac{dE_T}{dt} = 0}$$



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S·H·M

Avg P·E & K·E in S·H·M

$$P.E = \frac{1}{2} m\omega^2 A^2 \sin^2\omega t$$

$$P.E_{avg} = \frac{\int_0^{2\pi/\omega} \frac{1}{2} m\omega^2 A^2 \sin^2\omega t \cdot dt}{\int_0^{2\pi/\omega} dt}$$

$$P.E_{avg} = \frac{1}{2} m\omega^2 A^2 \left[\frac{\int_0^{2\pi/\omega = T} \sin^2\omega t \cdot dt}{\int_0^{T = \frac{2\pi}{\omega}} dt} \right] \Downarrow \frac{1}{2}$$

$$\boxed{P.E_{avg} = \frac{1}{4} m\omega^2 A^2}$$

$$K.E_{avg} = \frac{1}{2} m\omega^2 A^2 \left(\frac{\int_0^T \cos^2\omega t \cdot dt}{\int_0^T dt} \right) \Downarrow \frac{1}{2}$$

$$\boxed{K.E_{avg} = \frac{1}{4} m\omega^2 A^2}$$

Energy Oscillates 2-times in One-time period

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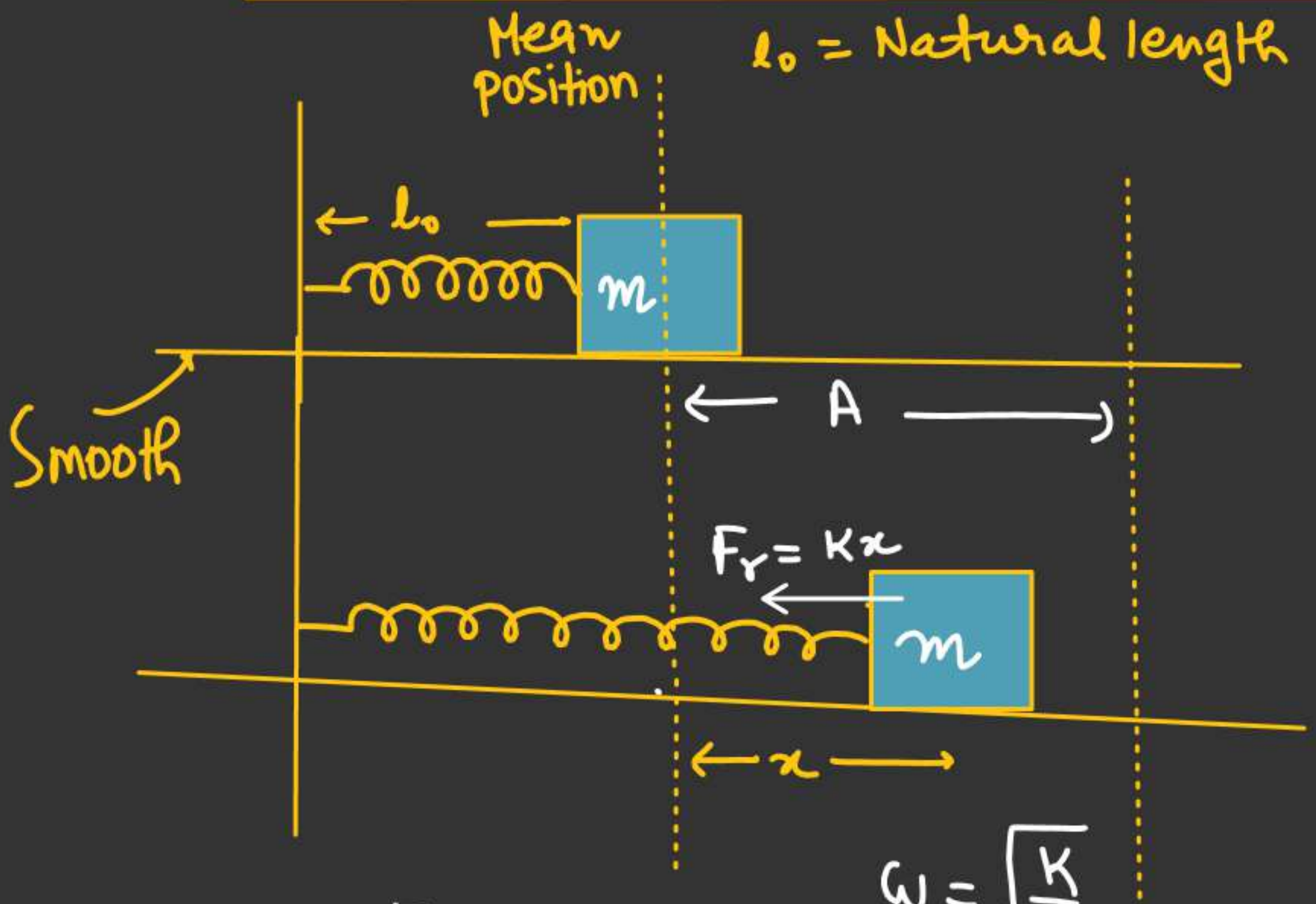
S.H.M

Time period of Spring-block System



$$v_{max} = A\omega .$$

$$\left[\frac{1}{2}KA^2 = \frac{1}{2}mv_0^2\right]$$

$$F_r = -Kx$$

$$a = -\frac{K}{m}x$$

$$a = -\omega^2 x$$

$$\omega = \sqrt{\frac{K}{m}}$$

$$\boxed{T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{K}}}$$

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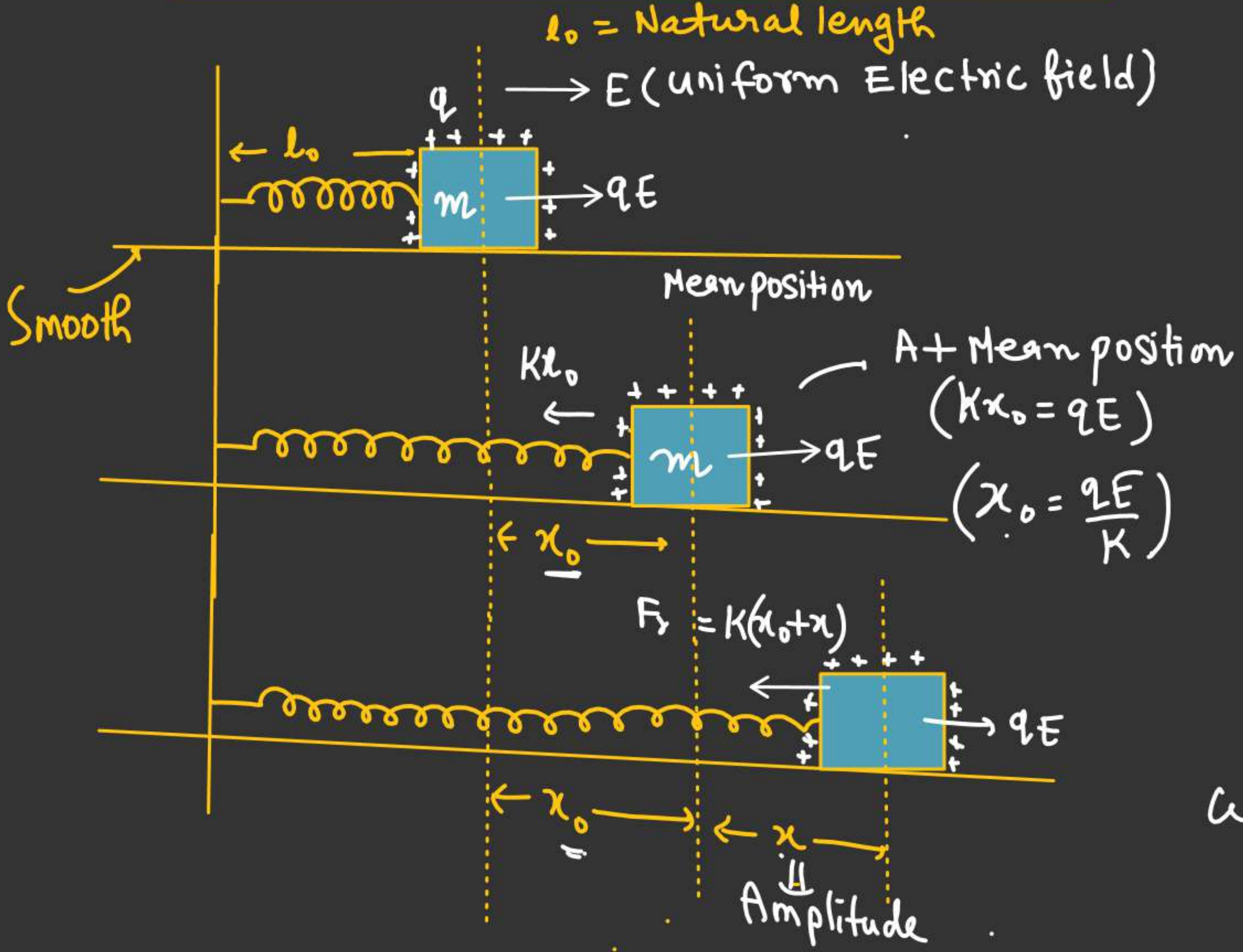
S.H.M

Time period of Spring-block System



$$F_r = -\left[K(x_0+x) - qE\right]$$

$$F_r = -\left[\underbrace{\cancel{Kx_0} - \cancel{qE}} + \underbrace{Kx}_{\Downarrow}\right]$$

Extra Spring force after mean position

$$F_r = -Kx$$

$$a = -\frac{K}{m}x$$

$$a = -\omega^2 x$$

$$\omega = \sqrt{\frac{K}{m}}, \left(T = 2\pi\sqrt{\frac{m}{K}}\right)$$

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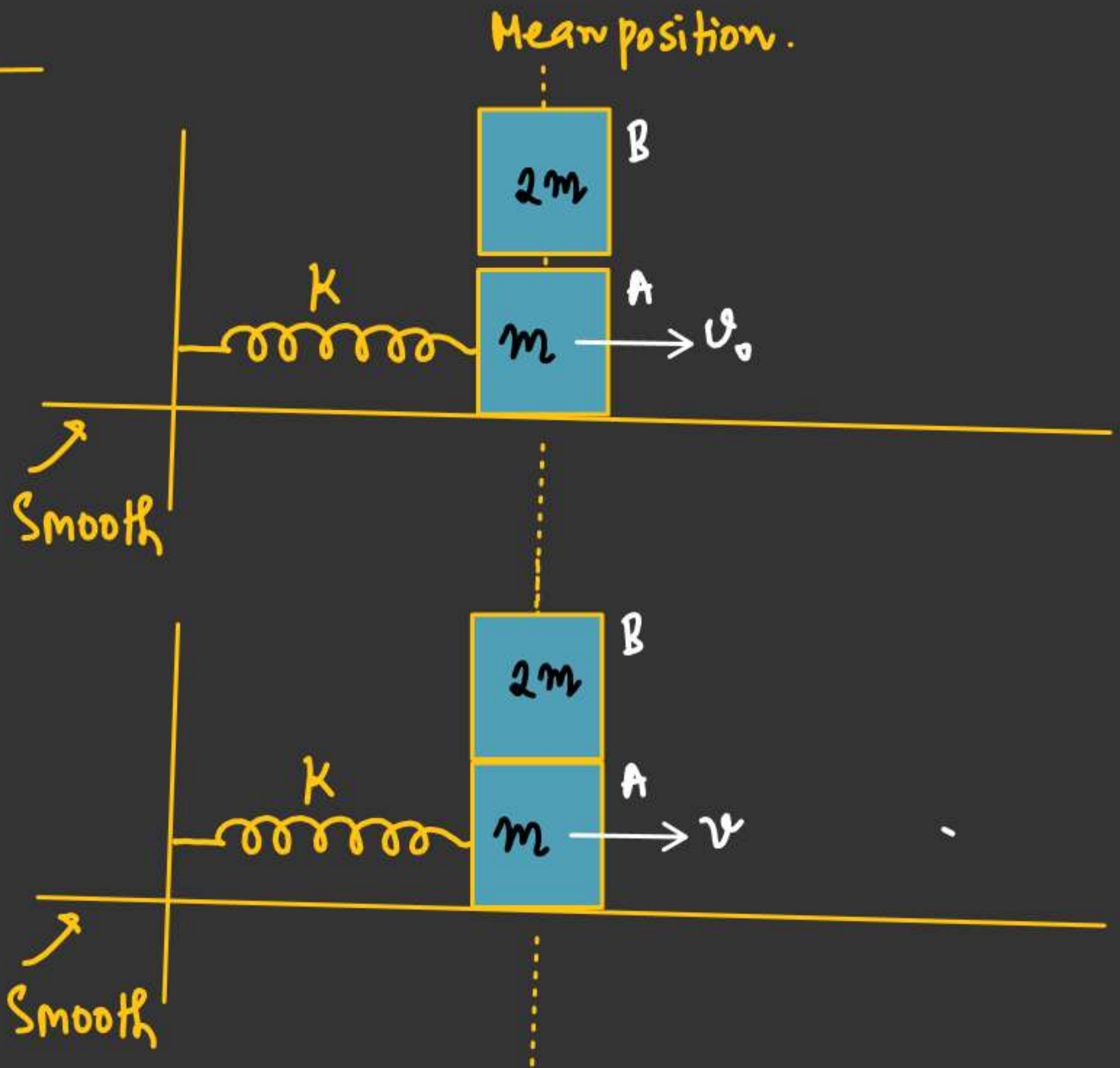
S.H.M

\# When block A passing through it's mean position another block B gently placed on A.
No relative slipping b/w A and B
Find
1) New Amplitude
11) New time period.



$$T_0 = 2\pi\sqrt{\frac{m}{K}}, \quad \omega_0 = \sqrt{\frac{K}{m}}, \quad v_0 = A_0\omega_0$$

No external force in x-direction during loading at mean position

$$m v_0 = (3m)v$$

$$\boxed{v = \frac{v_0}{3}}$$

Mean position Velocity after loading

$$\omega = \sqrt{\frac{K}{3m}},$$

$$v = A\omega$$

$$A = \frac{v}{\omega} = \sqrt{\frac{3m}{K}} \times \frac{v_0}{3} \Rightarrow A = \frac{1}{\sqrt{3}} \times \sqrt{\frac{m}{K}} \times v_0$$

$$A = \frac{1}{\sqrt{3}}\frac{v_0}{\omega_0} = \frac{1}{\sqrt{3}}A_0$$

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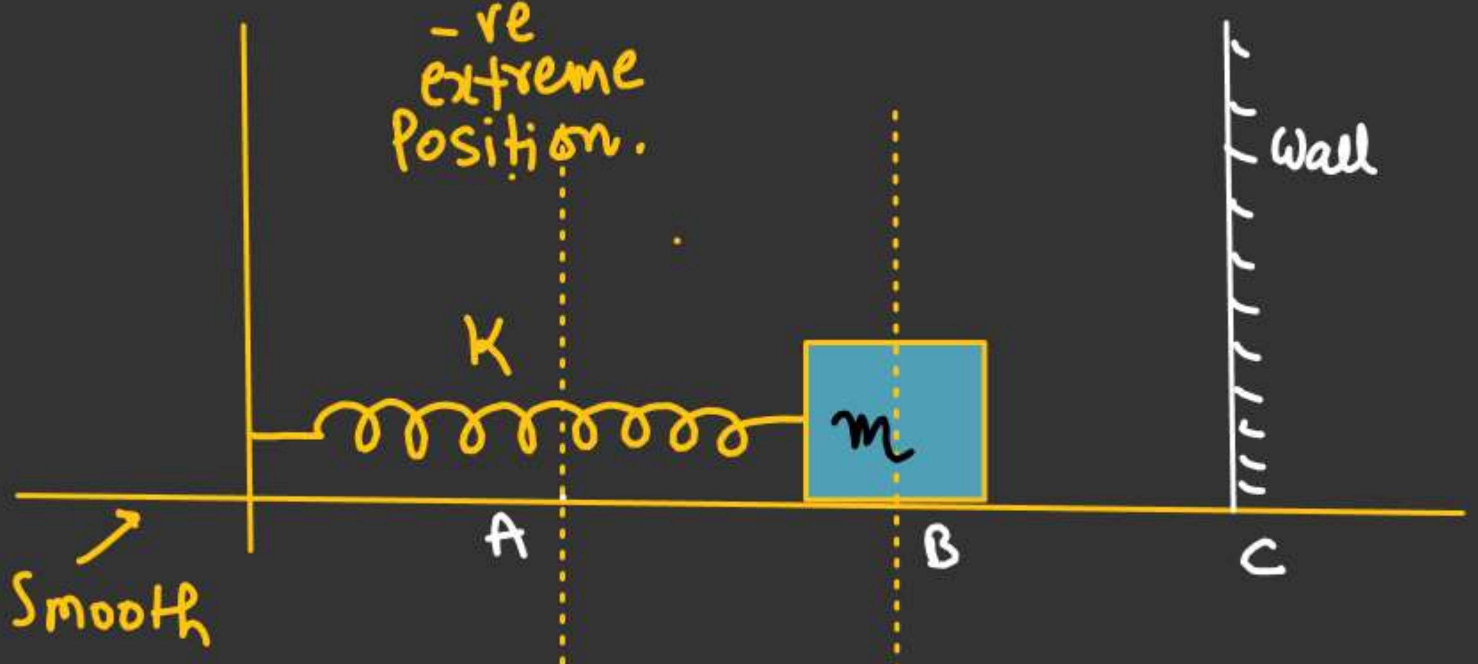
S.H.M

\# Collision b/w block & wall is perfectly elastic. Block compressed by A distance and released. Find the time period of block

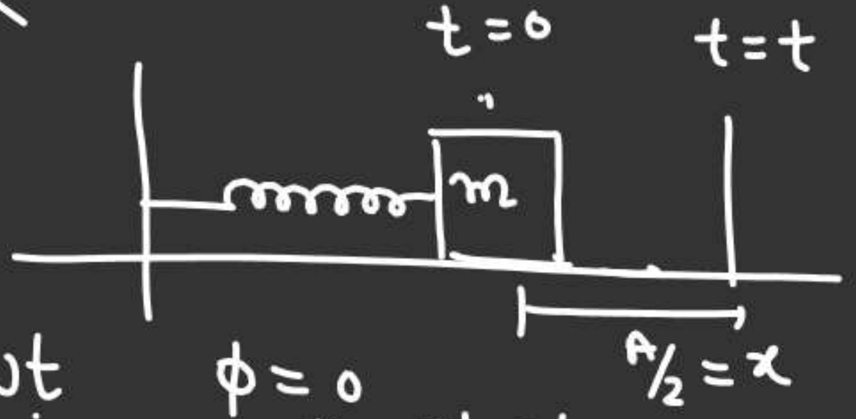


$$T = 2\pi\sqrt{\frac{m}{K}}$$

$$t_{AB} = \frac{T}{4} \checkmark$$

$$= \frac{\pi}{2}\sqrt{\frac{m}{K}}$$

$$\underline{t_{BC}} = ??$$



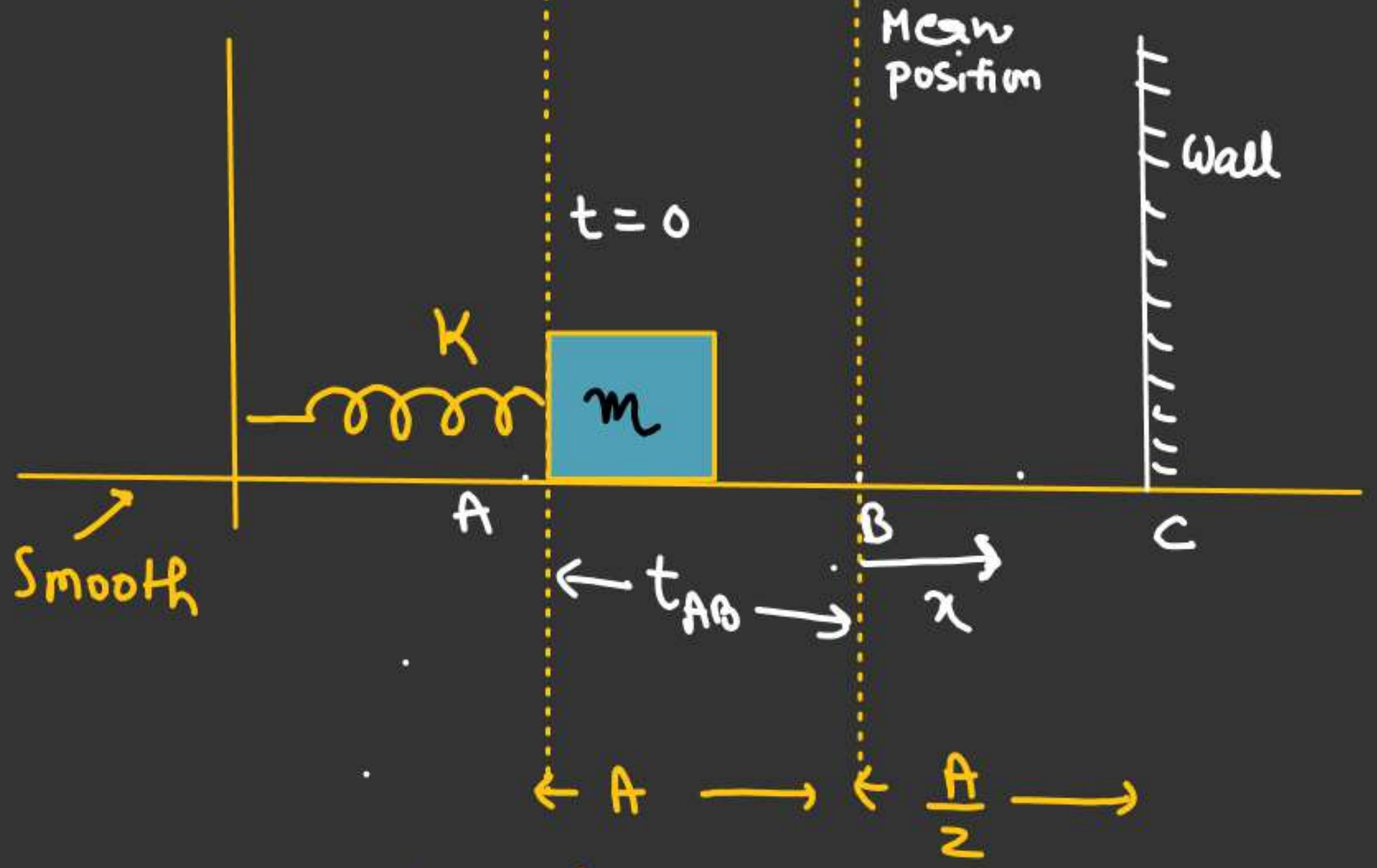
$$X = A\sin\omega t$$

$\phi = 0$
Motion starts from mean position

$$\frac{A}{2} = A\sin\omega t_{BC}$$

$$\sin\omega t_{BC} = \frac{1}{2}$$

$$\omega t_{BC} = \sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}.$$



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S.H.M

$$\omega t_{BC} = \frac{\pi}{6}$$

$$\underline{t_{BC}} = \frac{\pi}{6\omega} = \frac{\cancel{\pi}\, T}{6\times 2\cancel{\pi}} = \frac{T}{12}$$

$$T' = 2\left(t_{AB} + t_{BC}\right)$$

$$T' = 2\left(\frac{T}{4} + \frac{T}{12}\right)$$

$$T' = 2\left(\frac{4T}{12}\right)$$

$$T' = \left(\frac{2T}{3}\right) = \frac{2}{3}\left(2\pi\sqrt{\frac{m}{K}}\right)$$

$$= \frac{4\pi}{3}\sqrt{\frac{m}{K}}$$

S.H.M

** Time period of two blocks & Spring System

Both the block's stretched from their mean position and released Simultaneously

x_0 = Total elongation in the Spring

No external force in x-direction.

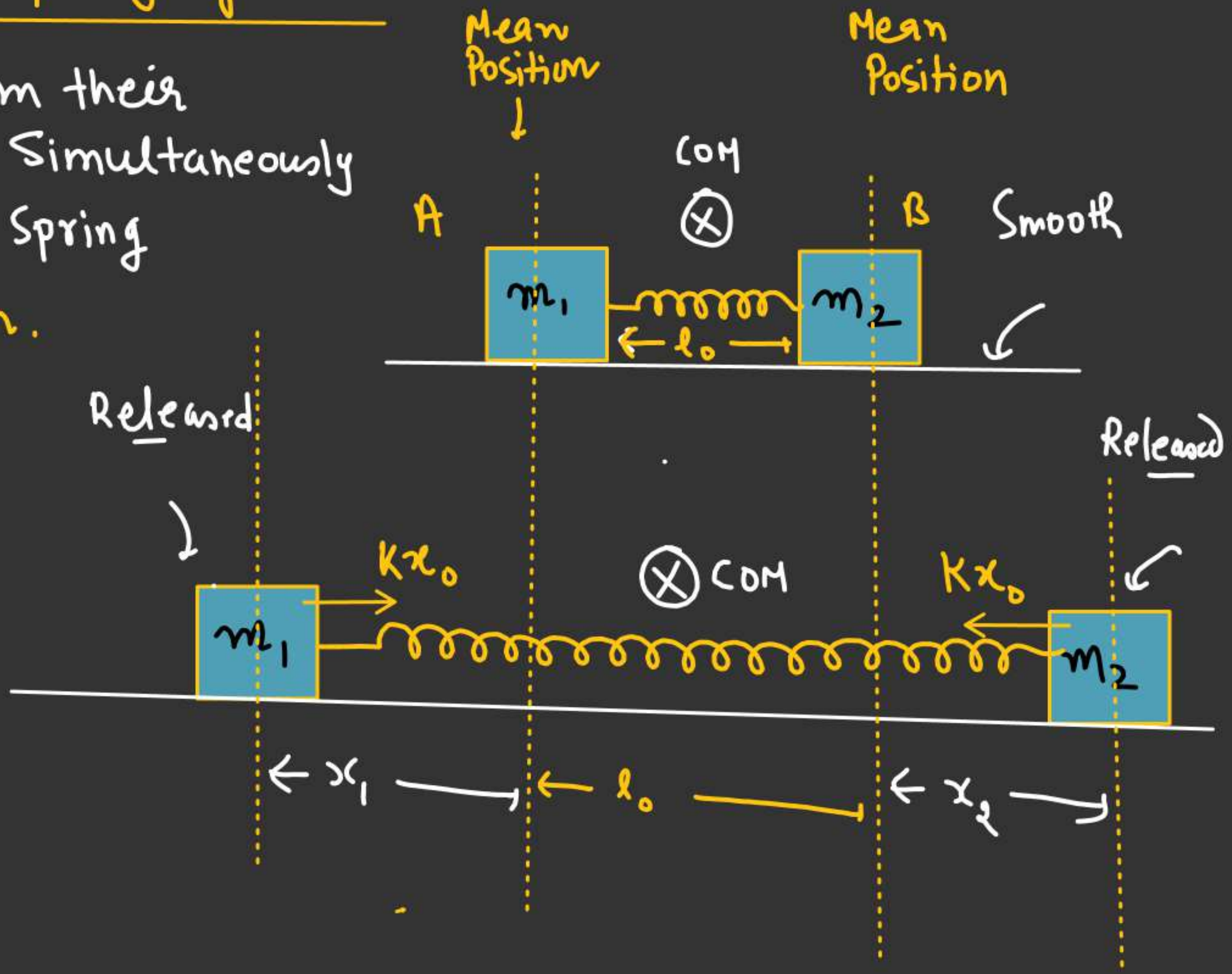
$$\Delta X_{com} = 0.$$

$$\frac{-m_1 x_1 + m_2 x_2}{m_1 + m_2} = 0.$$

$$m_1 x_1 = m_2 x_2 \quad -(1)$$

$$x_0 = x_1 + x_2 \quad -(2)$$

$$x_1 = \left(\frac{m_2 x_0}{m_1 + m_2}\right), \quad x_2 = \left(\frac{m_1 x_0}{m_1 + m_2}\right)$$



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S.H.M

$$F_r = -Kx_0 \qquad x_1 = \frac{m_2 x_0}{m_1+m_2} \Rightarrow x_2 = \frac{m_1 x_0}{m_1+m_2}$$

For block A

$$F_r = -Kx_0$$

$$F_r = -K\left(\frac{m_1+m_2}{m_2}\right)x_1$$

$$a = -K\left(\frac{m_1+m_2}{m_1 m_2}\right)x_1$$

$$a = -\omega^2 x_1$$

$$\omega = \sqrt{\frac{K(m_1+m_2)}{m_1 m_2}}$$

$$T_A = 2\pi\sqrt{\frac{m_1 m_2}{K(m_1+m_2)}}$$

$$\frac{m_1 m_2}{m_1+m_2} = \mu$$

Reduced Mass

For block B

$$F_r = -Kx_0$$

$$F_r = -K\left(\frac{m_1+m_2}{m_1}\right)x_2$$

$$a = -K\left(\frac{m_1+m_2}{m_1 m_2}\right)x_2$$

$$a = -\omega^2 x_2$$

$$T_B = 2\pi\sqrt{\frac{m_1 m_2}{K(m_1+m_2)}}$$

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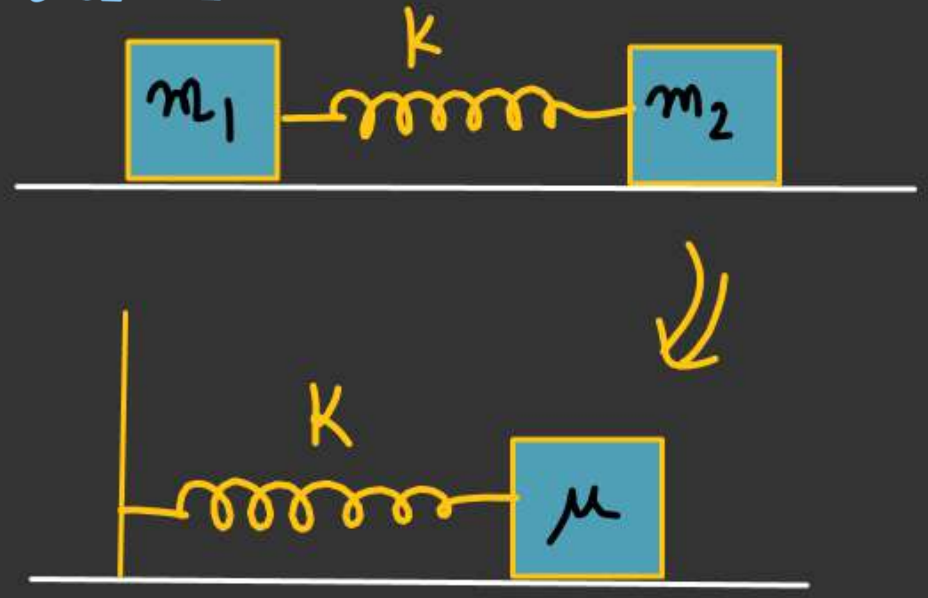
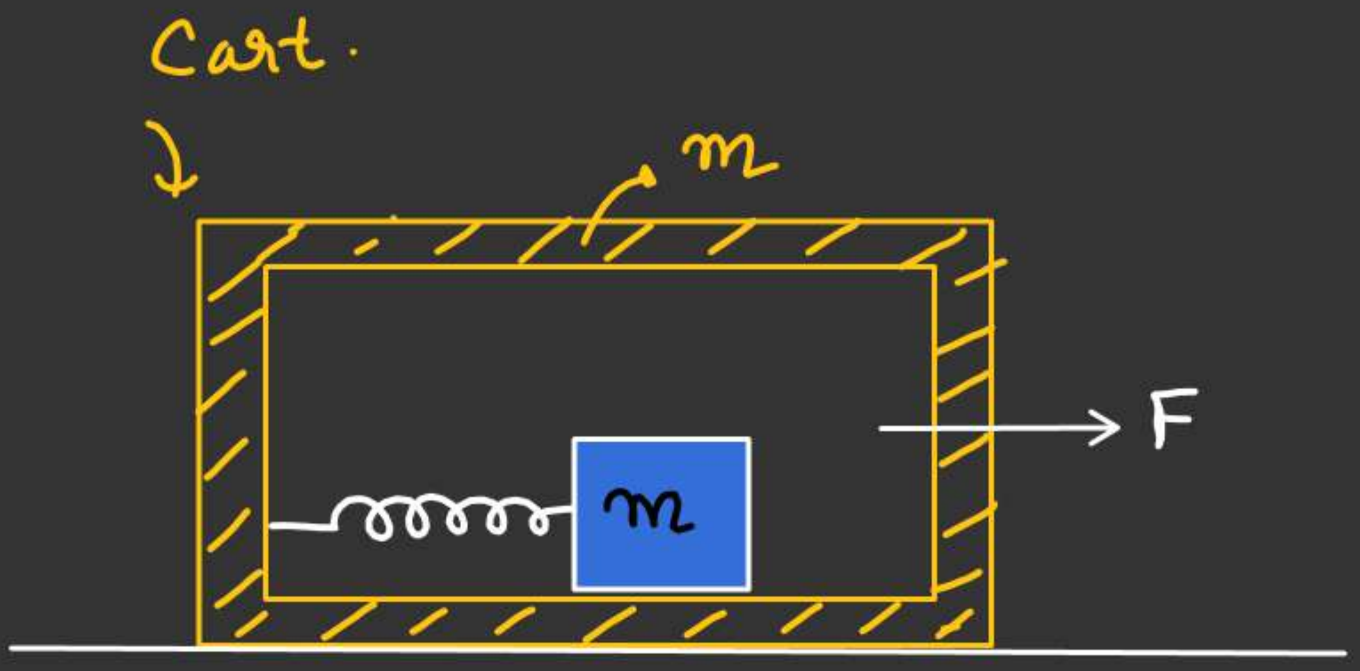


$$T = \left(2\pi\sqrt{\frac{\mu}{K}}\right)$$

$$\mu = \frac{m_1 m_2}{m_1 + m_2}$$

S.H.M



Cart pulled by constant force F.

a) Find Time Period of Cart

2) velocity of the Cart at the instant 'B' when compression in the Spring is maximum

$$T = 2\pi\sqrt{\frac{\mu}{K}}$$

$$\mu = \frac{m}{2}$$

$$T = 2\pi\sqrt{\frac{m}{2K}}$$

$$V_{com} = A_{com} \cdot t$$

$$V_{com} = \left(\frac{F}{2m} t\right)$$

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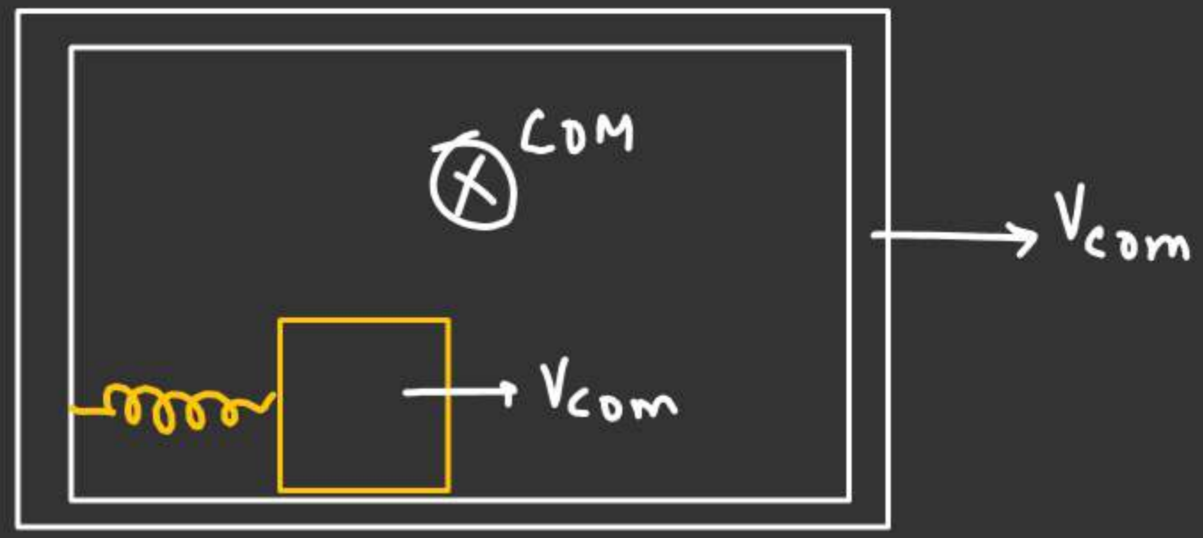
At the time when block at it's maximum compression



In COM fram block & Cart perform S.H.M.

In earth frame (S.H.M + translation)

$$t = \frac{T}{4} = \frac{1}{4} 2\pi\sqrt{\frac{m}{2K}}$$

$$= \frac{\pi}{2}\sqrt{\frac{m}{2K}} \checkmark$$

$$\vec{v}_{cart/E} = \vec{v}_{cart/COM} + \vec{V}_{COM/E}$$

$$= 0 + V_{com}$$

$$(v_{car}) = V_{com} = A_{com} t = \frac{F}{2m} \times \frac{\pi}{2}\sqrt{\frac{m}{2k}} = \frac{F\pi}{4\sqrt{2mk}}$$

At the time of Maximum Compression