$$(a+x)^n = \overset{T_1}{{}^nC_0 \cdot a^n} + \overset{T_2}{{}^nC_1 a^{n-2}x} + \overset{T_3}{{}^nC_2 \cdot a^{n-2}x^2} + \overset{T_4}{{}^nC_3 a^{n-3}x^3} + \cdots = \underbrace{(T_1+T_3+T_5+\cdots)}_{P} + \underbrace{(T_2+T_4+T_6+\cdots)}_{Q} = P+Q.$$

$$(a-x)^n = \overset{T_1}{{}^nC_0 a^n} - \overset{T_2}{{}^nC_1 a^{n-1}x} + \overset{T_3}{{}^nC_2 \cdot a^{n-2}x^2} - \overset{T_4}{{}^nC_n \cdot a^{n-3}x^3}. = (T_1+T_3+T_5 \cdots) - (T_2+T_4+\cdots) = P-Q$$

P = Sum of odd term

Q = Sum of Even terms.

① $(a+x)^n + (a-x)^n = (P+Q) + (P-Q) = 2P = 2[T_1 + T_5 + \cdots]$

② $(a+x)^n - (a-x)^n = (P+Q) - (P-Q) = 2Q = 2[T_2 + T_4 + \cdots]$

(3) $(a-x)^{2n} + (a+x)^{2n} = (P \pm Q)^2 + (P-Q)^2 = 2(P^2+Q^2)$

(4) $(a+x)^{2n} - (a-x)^{2n} = (P+Q)^2 - (P-Q)^2 = 4PQ.$

half.

1) No of Rational term in

A) in $(x+a)^n + (x-a)^n$ are $\left(\frac{n}{2}+1\right)$ terms $\{n = \text{Even}\}$

B) in $(x+a)^n - (x-a)^n$ are $\frac{n}{2}$ terms $\{n =$ ~~odd~~ Even$\}$

Q $(\sqrt{2}+1)^6 + (\sqrt{2}-1)^6 = ?$

$(P+Q) + (P-Q) = 2P = 2[T_1 + T_3 + T_5 + T_7]$

$2\left[{}^6C_0 \sqrt{2}^6 + {}^6C_2 (\sqrt{2})^4 (1)^2 + {}^6C_4 (\sqrt{2})^2 (1)^4 + {}^6C_6 (\sqrt{2})^0 (1)^6\right] = 2\left[2^3 + 15 \cdot 2^2 + 15 \cdot 2 + 1\right] = 198$

all terms are without under root ∴ they all all Rational No. Now. } No of Rational terms = 4

Q $(\tan\frac{3\pi}{8})^7 + (-\cot\frac{3\pi}{8})^7 = ?$

$(\tan\frac{3\pi}{8})^7 - ((\cot\frac{3\pi}{8})^7$

$(\sqrt{2}+1)^7 - (\sqrt{2}-1)^7$

$(a+x)^n - (a-x)^n = (P+Q) - (P-Q) = 2Q$

$2[T_2 + T_4 + T_6 + T_8]$

$2[{}^7C_1(\sqrt{2})^6 + {}^7C_3(\sqrt{2})^4 + {}^7C_5(\sqrt{2})^2 + {}^7C_7(\sqrt{2})^0]$

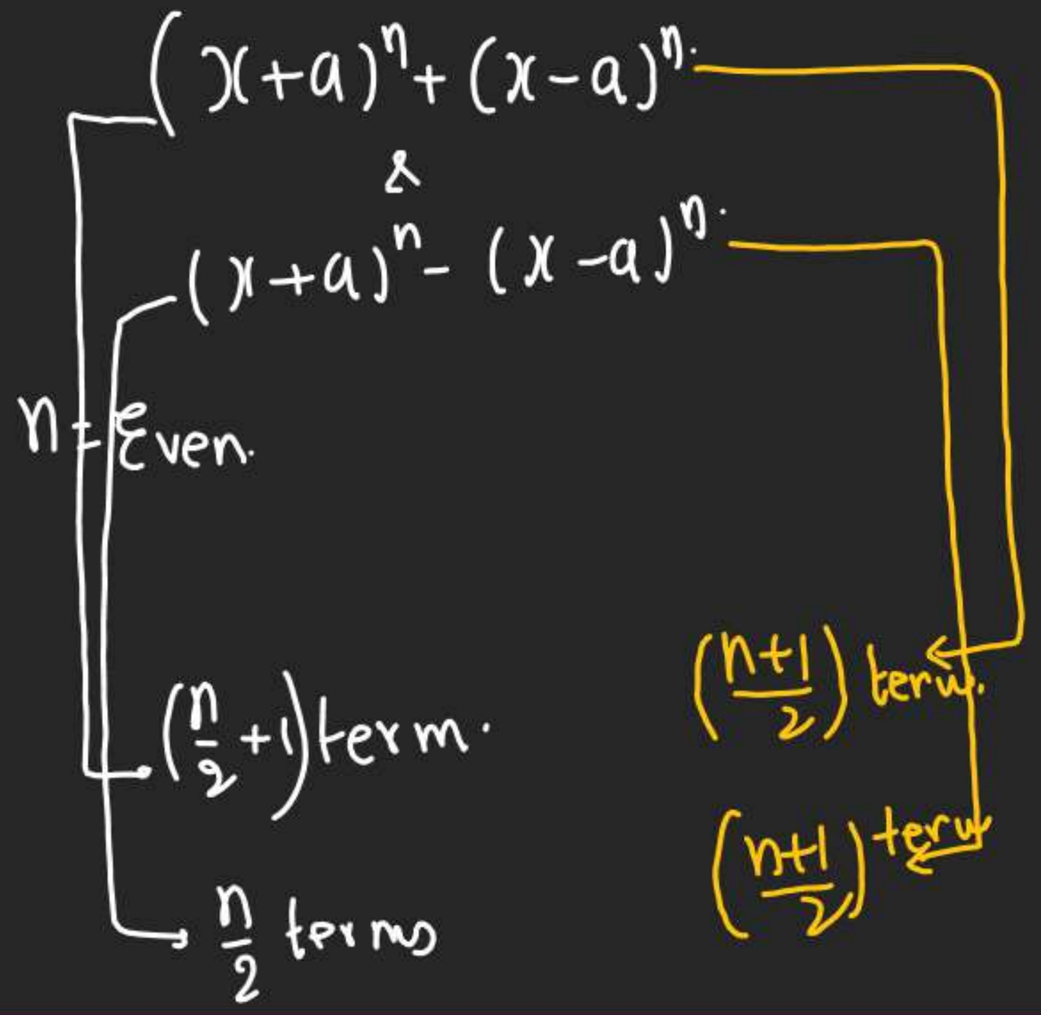
$= 2[7\times 8 + 35\cdot 4 + 21\cdot 2 + 1\times 1]$

$= 2[56 + 140 + 42 + 1]$

196
43
39

$= 2\times 239 = 478.$

Imp. No of terms in (Formula)



$(x+a)^n + (x-a)^n$

&

$(x+a)^n - (x-a)^n$

n = Even

$(\frac{n}{2}+1)$ term.

$\frac{n}{2}$ terms

$(\frac{n+1}{2})$ term.

$(\frac{n+1}{2})$ term

Q Total No. of terms in $(x+a)^{50} + (x-a)^{50}$?

(A) Formula → $n = 50$ (Even) ⇒ $(\frac{50}{2}+1) = 26$ terms.

(B) $P+Q+P-Q = 2P = 2[\underbrace{T_1 + T_3 + T_5 \sim T_{51}}_{26 \text{ terms}}]$

Q Total No of terms in

$(x+a)^{101} - (x-a)^{101}$?

$n = 101 = $ odd.

total No. of terms $= \left(\frac{n+1}{2}\right) = \frac{101+1}{2}$

$= \underline{51 \text{ terms}}$

Q total No of terms in

$(x+a)^{18} - (x-a)^{18}$?

$n = 18$ (Even)

total terms $= \frac{n}{2} = \frac{18}{2} = 9$ ter.

$(5+2\sqrt{6})^n$ $\quad 2\sqrt{6} \simeq 2.4 \times 2$

$(5-2\sqrt{6})^n$ $\quad \simeq 4.8$

$(5-4.8)^n = (.2)^n =$ fraction (Decimal

Integral & Fractional Part of $(a+b\sqrt{c})^n$

(3, 4 Qs are Necessary for frndshp)

Purpose :- to find Integral & fractional Part of $(a+b\sqrt{c})^n$; $a, b, c \in N$.

Algorithm. → (1) Write Expression $= I + f$

(2) Now Replace +ve Sign to −ve Sign & denote it by f'

(3) add or Subtract acc. to Qs.

(4) always Care $0 < f + f' < 2$ & $0 < f - f' < 1$

Q If n is a +ve Integer then Integral Part of $(3+\sqrt{7})^n$ is an odd Int?

1) let $(3+\sqrt{7})^n = I + f$ $\quad 0 < f < 1$

2) $(3-\sqrt{7})^n = f'$ $\quad 0 < f' < 1$

(3) $(3+\sqrt{7})^n + (3-\sqrt{7})^n = I + (f + f')$ $\quad \boxed{0 < f + f' < 2}$

$= 2P = 2[T_1 + T_3 + T_5 + \cdots] =$ Even Integer $= \underline{I} + (f + f')$

Int ∈ $\quad$ Integer ∈ + Kuchh तो Integer होगा यही

Even $= I + 1$

$I =$ Even Integer − 1

$I =$ odd Integer

Q Show that Integral Part of each of the following is odd.

$(5+2\sqrt{6})^n$.

① let $(5+2\sqrt{6})^n = I+f$ $\quad 0<f<1$

2) $(5-2\sqrt{6})^n = f'$ $\quad 0<f'<1$

$0<f+f'<2$

3) $(5+2\sqrt{6})^n + (5-2\sqrt{6})^n = I+f+f'$

$2P = 2[T_1+T_3+T_5+ \dots] = I+(f+f')$

Even Int. $=$ I + Kuch (Int.)

Even $= I+1$

Even $-1 = I$

Odd Int. $= I$

odd. $=$ Integral Part

(2) $(8+3\sqrt{7})^n$

() $(6+\sqrt{35})^n$

Q Find Gr. Integer Value of $(\sqrt{2}+1)^7$.

1) let $(\sqrt{2}+1)^7 = I+f$ | $0<f<1$

2) $(\sqrt{2}-1)^7 = f'$ | $0<f'<1$

3) $(\sqrt{2}+1)^7 - (\sqrt{2}-1)^7 = I+f-f'$

$$0<f<1$$
$$-1<-f'<0$$
$$-1<f-f'<1$$

$2Q =$ Even Integer $=$ Integer $+$ Kuch $\xrightarrow{} 0$

$2[T_2+T_4+T_6+T_8] =$ Integer

$2[7\cdot 8+35\times 4+21\times 2+1] =$ Integral Part

$2[56+140+42+1] =$ '' ''

$478 =$ Integral. an

$[(\sqrt{2}+1)^7] = 478$

When 3 Consecutive Terms are in AP.

Q If Coff. of T_r, T_{r+1}, T_{r+2} terms of $(1+x)^{14}$ are in AP then $r = ?$ 6, 7, 8, 9 ??

${}^nC_{r-1}, {}^nC_r, {}^nC_{r+1}$ are in AP

$$2\times {}^nC_r = {}^nC_{r-1} + {}^nC_{r+1}$$

$$2 = \frac{{}^nC_{r-1}}{{}^nC_r} + \frac{{}^nC_{r+1}}{{}^nC_r} = 2 = \frac{1}{\left(\frac{{}^nC_r}{{}^nC_{r-1}}\right)} + \frac{n-r}{r+1}$$

$$2 = \frac{1}{\left(\frac{n-r+1}{r}\right)} + \frac{n-r}{r+1} \Rightarrow 2 = \frac{r}{14-r+1} + \frac{14-r}{r+1}$$

$$2(15-r)(r+1) = r^2+r+(14-r)(15-r)$$

$$2(15r-r^2+15-r) = r^2+r+210-29r+r^2$$

$$4r^2-56r+180=0 \to r=5,9$$

$r = \frac{n\pm\sqrt{n+2}}{2} = \frac{14\pm\sqrt{16}}{2}$

$\frac{14+4}{2}, \frac{14-4}{2}$

$r = 9, 5$

Most Imp.

$$\frac{{}^nC_r}{{}^nC_{r-1}} = \frac{n-r+1}{r}$$

✓ $$\frac{{}^nC_{r+1}}{{}^nC_r} = \frac{n-(r+\cancel{1})+\cancel{1}}{r+1} = \frac{n-r}{r+1}$$

Q. If Coff of r^{th}, $(r+1)^{th}$, $(r+2)^{th}$ term in Bin. Exp. of $(1+y)^m$ are in AP, then m & r satisfy Eq^n

A) $m^2 - m(4r-1)+4r^2-2=0$

B) $m^2 - m(4r+1)+4r^2+2=0$

C) $m^2 - m(4r+1)+4r^2-2=0$ ✓

D) $m^2 - m(4r-1)+4r^2+2=0$

$$r = \frac{m \pm \sqrt{m+2}}{2}$$

$$2r = m \pm \sqrt{m+2}$$

$$(2r-m)^2 = m+2$$

$$m^2+4r^2-4mr = m+2$$

$$m^2-4mr-m+4r^2-2=0$$

$$m^2-m(4r+1)+4r^2-2=0$$

$\boxed{C}$

6, 10-18

19-28, 31, 32

58, 61, 67, 91*

Q. If Coff of 4 consecutive terms in Exp. of $(1+x)^n$ are a_1, a_2, a_3, a_4 then $\frac{a_1}{a_1+a_2}, \frac{a_2}{a_2+a_3}, \frac{a_3}{a_3+a_4}$ are in?

$n=3.$

$$(1+x)^3 = 1+3x+3x^2+x^3$$

$$= a_1=1, a_2=3, a_3=3, a_4=1$$

$$\frac{a_1}{a_1+a_2} = \frac{1}{1+3} = \frac{1}{4}$$

$$\frac{a_2}{a_2+a_3} = \frac{3}{3+3} = \frac{1}{2}$$

$$\frac{a_3}{a_3+a_4} = \frac{3}{3+1} = \frac{3}{4}$$

$\frac{1}{4}, \frac{1}{2}, \frac{3}{4}$ AP