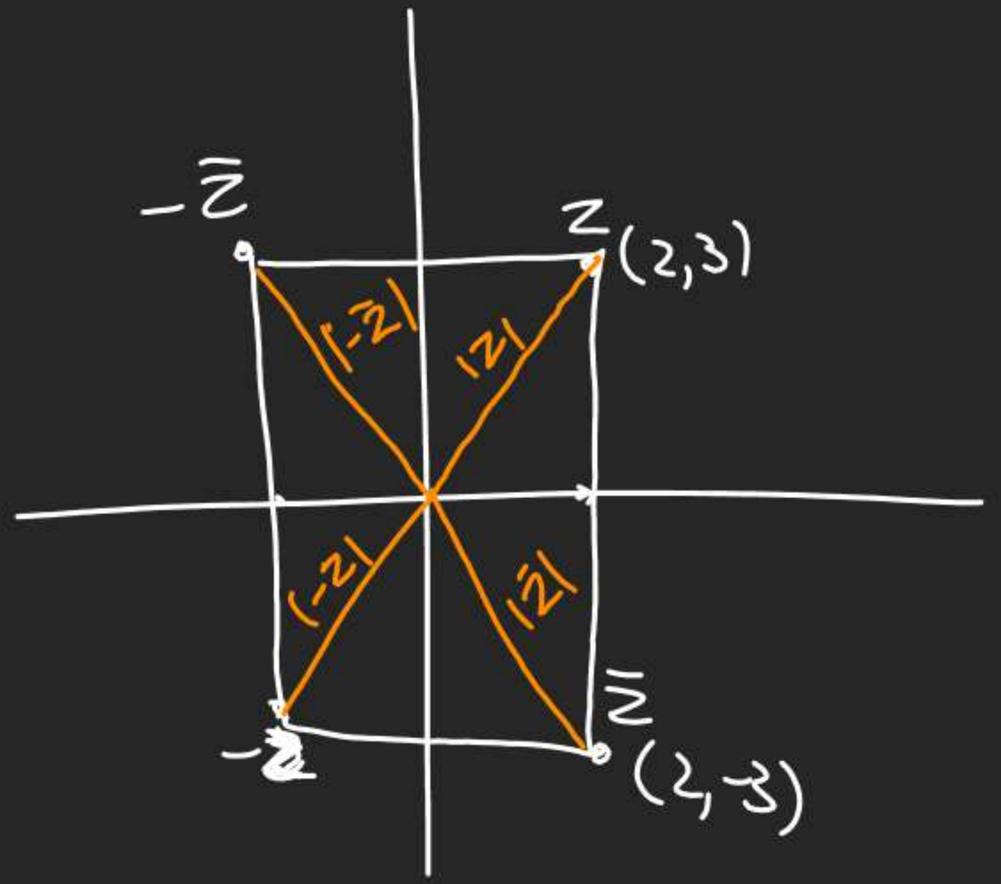
Nishant Jindal

Ex. If $z = 2+3i = (2,3)$

then $\bar{z} = 2-3i = (2,-3)$

$-z = -2-3i = (-2,-3)$

$-\bar{z} = -2+3i = (-2,3)$



1) $|z|$ = distance of z from origin

2) If $|z| = 0 \Rightarrow z = (0,0)$

3) If $|z| > 0 \Rightarrow z \neq (0,0)$

C.N. → 1-20QS

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Q 1. If $z = 3+4i$ then $\bar{z} = 3-4i$

2. If $z = i-5$ then $\bar{z} = -i-5$

3. If $z = 5$ then $\bar{z} = 5$

4. If $z = -2i$ then $\bar{z} = 2i$

5. If $z = \frac{3+4i}{5-i}$ then $\bar{z} = \frac{11-23i}{26}$

$$z = \frac{3+4i}{5-i} \times \frac{5+i}{5+i} = \frac{15+3i+20i+4i^2}{(5)^2-(i)^2}$$

$$= \frac{11+23i}{26}$$

Properties of $\bar{z}$

This must be remembered. Otherwise Qs Solving forget.

(1) $\overline{(\bar{z})} = z$

(2) $z + \bar{z} = 2Re(z)$

$a+ib+a-ib = 2a$

(3) $z - \bar{z} = 2i Im(z)$

$(a+ib)-(a-ib) = 2ib$

(4) $\overline{(z_1+z_2)} = \bar{z}_1 + \bar{z}_2$

$\overline{(a+ib+c+id)} = \overline{(a+c)+i(b+d)}$

$(a+c) - i(b+d)$

$a-ib + c-id$

$= \bar{z}_1 + \bar{z}_2$

(5) $\overline{(z_1 \cdot z_2)} = \bar{z}_1 \cdot \bar{z}_2$

(6) $\overline{\left(\frac{z_1}{z_2}\right)} = \frac{\bar{z}_1}{\bar{z}_2}$

(7) $\overline{(z^n)} = (\bar{z})^n$

(8) If $z = \bar{z}$ then

$Im(z) = 0$

$\Rightarrow z$ is a Real No.

(9) $z = -\bar{z}$ then

$Re(z) = 0$

$\Rightarrow z$ is an Img No.

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If $Z = \bar{Z}$

$a + ib = a - ib$

$2ib = 0$

$b = 0$

$\text{Im } Z = 0$

$Z = a + i \cdot 0$

$Z = a \Rightarrow Z$ is a Real No.

Q If $Z = -\bar{Z}$ then Nature of Z.

$a + ib = -(a - ib)$

$a + \cancel{ib} = -a + \cancel{ib}$

$2a = 0$

$a = 0$

$\text{Re}(Z) = 0$

$Z = 0 + ib$

Z is an Imaginary No.

Rem:-

If Z is Purely Real $\Rightarrow Z = \bar{Z}$

If Z is Purely Img $\Rightarrow Z = -\bar{Z}$

Q If $Z = x + iy$ then find (x, y) in terms of Z.

$\frac{1}{i} = -i$

$Z = x + iy$

$\bar{Z} = x - iy$

$Z + \bar{Z} = 2x$

$\boxed{x = \frac{Z + \bar{Z}}{2}}$

$Z - \bar{Z} = 2iy$

$\boxed{y = \frac{Z - \bar{Z}}{2i}} = -i\left(\frac{Z - \bar{Z}}{2}\right)$

$y = \frac{(\bar{Z} - Z)i}{2}$

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Q Convert St. line $x+2y=3$ in C.N. form?

$$x+2y=3$$

$$\frac{z+\bar{z}}{2}+\frac{2(z-\bar{z})}{2i}=3$$

$$\frac{z+\bar{z}}{2}-\frac{2i(z-\bar{z})}{2}=3$$

$$z+\bar{z}-2iz+2i\bar{z}=6$$

$$(z+\bar{z})-2i(z-\bar{z})=6$$

is St. Line.

Q If $(a+bi)^5=p+iq$

then P.T. $(b+ia)^5=\cancel{p+iq}\ q+ip$

$$\overline{(z^n)}=(\bar{z})^n$$

① $\overline{(a+bi)^5}=\overline{p+iq}$

$$(\overline{a+ib})^5=\overline{p+iq}$$

$$(\overline{a+ib})^5=p-iq$$

$$(a-ib)^5=p-iq$$

$$(-i)^5\left(b+\frac{a}{-i}\right)^5=p-iq$$

$$-i\left(b-a(-i)\right)^5=p-iq$$

$$(b+ai)^5=\frac{p}{-i}+\frac{iq}{\cancel{i}}=q+pi$$

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Q If $(x+iy)^5 = 4+5i$ then

$(y+ix)^5 = ?$

$= 5+4i$

Q $\sin x + i\cos 2x$ & $\cos x - i\sin 2x$

are conjugate for $x \in$?

(A) $x = n\pi$ (B) $x = (2n+1)\frac{\pi}{4}$

(C) $x = 0$ (D) No value of x.

(A) $\overline{\sin x + i\cos 2x} = \cos x - i\sin 2x$

$\sin x - i\cos 2x = \cos x - i\sin 2x$

$\sin x = \cos x$ & $\cos 2x = \sin 2x$

$\sin x = \cos x$ | & $\sin 2x = \cos 2x$

$\tan x = 1$ | $\tan 2x = 1$

$x = n\pi + \frac{\pi}{4}$ & $2x = n\pi + \frac{\pi}{4}$

$x = n\pi + \frac{\pi}{4} \cap x = \frac{n\pi}{2} + \frac{\pi}{8}$

$\frac{\pi}{4}$	$\frac{\pi}{8}$
$\pi + \frac{\pi}{4} = \frac{5\pi}{4}$	$\frac{\pi}{2} + \frac{\pi}{8} = \frac{5\pi}{8}$
$-\frac{3\pi}{4}$	$-\frac{\pi}{2} + \frac{\pi}{8} = -\frac{3\pi}{8}$
$2\pi + \frac{\pi}{4} = \frac{9\pi}{4}$	$\pi + \frac{\pi}{8} = \frac{9\pi}{8}$
$-\frac{7\pi}{4}$	$-\pi + \frac{\pi}{8} = -\frac{7\pi}{8}$

$x = \phi$ (No value of x)

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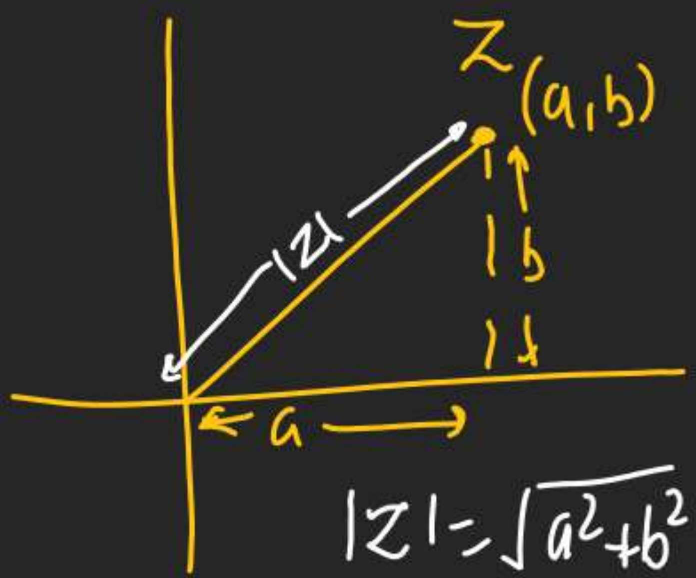
Modulus of a C.N.

(1) $|z|$ is Modulus of z.

(2) $|z|$ Rep. distance of z from origin.

(3) $Z = a+ib$



$$|z| = \sqrt{a^2+b^2}$$

4) z is C.N S.T. $Z = a+ib$ then $\sqrt{a^2+b^2}$ is $|z|$

Q $Z = 1-5i$ then $|z|$

$$|z| = \sqrt{1^2+(-5)^2}$$
$$= \sqrt{26}$$

Q $Z = 1+\sqrt{2}i$ then $|z|$

$$|z| = \sqrt{1^2+\sqrt{2}^2}$$
$$= \sqrt{3}$$

Q $Z = \dfrac{1+\sqrt{3}i}{2}$ then $|z| = ?$

$$|z| = \sqrt{\left(\tfrac{1}{2}\right)^2+\left(\tfrac{\sqrt{3}}{2}\right)^2} = \sqrt{\tfrac{1}{4}+\tfrac{3}{4}} = \sqrt{1} = 1$$

Q If z is C.N. Satisfying $(z^3+3)^2 = -16$ find $|z| = ?$

$$(z^3+3) = \pm\sqrt{-16}$$
$$z^3+3 = \pm 4i$$

$z^3+3 = 4i$	$z^3+3 = -4i$
$z^3 = -3+4i$	$z^3 = -3-4i$
Hold $z = (-3+4i)^{1/3}$	$z = (-3-4i)^{1/3}$
$\|z\| = \|(-3+4i)^{1/3}\|$	$\|z\| = \|(-3-4i)^{1/3}\|$
$= \|-3+4i\|^{1/3}$	$= \|-3-4i\|^{1/3}$
$= 5^{1/3}$	$\|z\| = 5^{1/3}$

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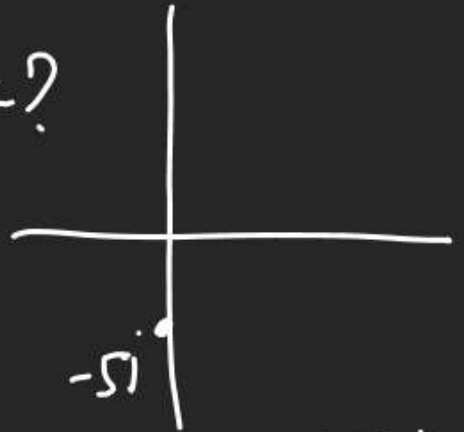
Q $Z = -5i$ then $|Z| = ?$

$|Z| = \sqrt{0 + (-5)^2} = 5$



Q P.T. $\left(\frac{\sqrt{3}}{2} + \frac{i}{2}\right)^5 + \left(\frac{\sqrt{3}}{2} - \frac{i}{2}\right)^5$ is Purely Real?

$\rightarrow a+ib$ $\quad \rightarrow a-ib$

ये पास वाले का Conjugate है

$\left(\frac{\sqrt{3}}{2} + \frac{i}{2}\right)^5 + \left(\overline{\frac{\sqrt{3}}{2} + \frac{i}{2}}\right)^5 \qquad \overline{(z^n)} = (\bar{z})^n$

$\left(\frac{\sqrt{3}}{2} + \frac{i}{2}\right)^5 + \overline{\left(\frac{\sqrt{3}}{2} + \frac{i}{2}\right)^5}$

ये $Z + \bar{Z}$ हो जाएगा

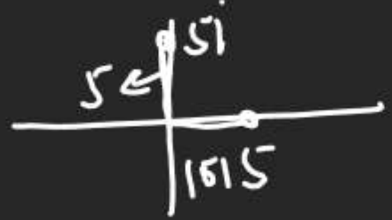
$= 2\,Re(Z) \Rightarrow$ it is a Real No

H.P.

Q Make diagram if

$|Z - \bar{Z}| + |Z + \bar{Z}| = 4$

$Z = 5 \qquad Z = 5i$



Rem :$\rightarrow$ $Z + \bar{Z} = 2x$

$Z - \bar{Z} = 2iy$

$|2iy| + |2x| = 4$

$|iy| + |x| = 2$

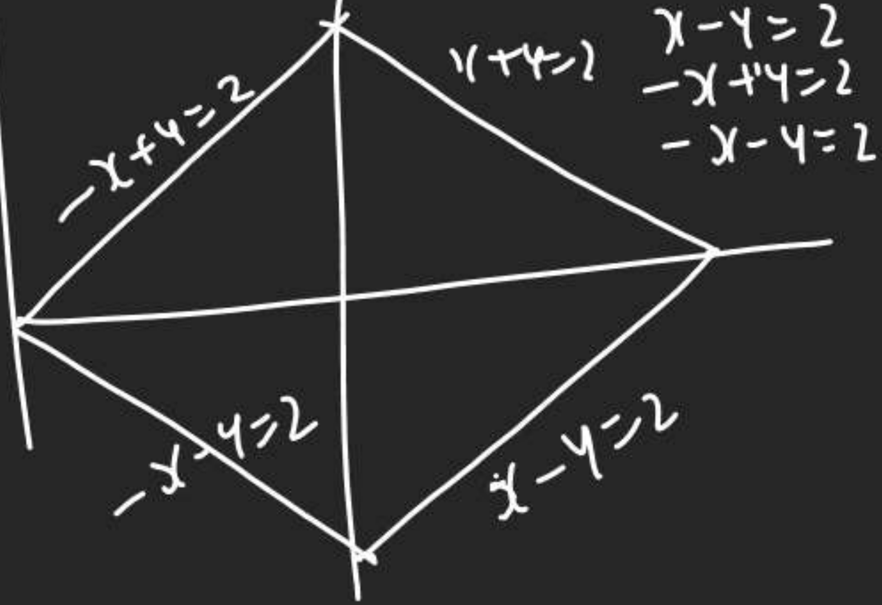
$|x| + |y| = 2$

$x + y = 2$

$x - y = 2$

$-x + y = 2$

$-x - y = 2$



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Properties of Modulus.

(1) $|z| = |-z| = |\bar{z}| = |-\bar{z}|$

(2) $|z| = 1$ then $z\bar{z} = 1$.

(3) $\boxed{\text{If } \underbrace{|z| = 1}_{} \Rightarrow z \cdot \bar{z} = 1 \quad \bar{z} = \frac{1}{z} \quad \bar{z} = z^{-1}}$

z is Unimodular.

(4) If $|z| = 1$ then $z^{-1} = \frac{\bar{z}}{|z|^2}$

(5) $|z_1 \cdot z_2| = |z_1||z_2|$

$|z_1 \cdot z_2 \cdot z_3 \cdots z_n| = |z_1||z_2||z_3| \cdots |z_n|$

(6) $\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}$

(7) $|z^n| = |z|^n$

Q If $|z| = 1$ then $z\bar{z} = 1$ [T | F]

$\downarrow$

$\sqrt{a^2+b^2} = 1$

$a^2+b^2 = 1$

$(a+ib)(a-ib) = 1$

$a^2 - (ib)^2 = 1$

$a^2+b^2 = 1$

Q If $|z| = 1$ then $z^{-1} = \frac{\bar{z}}{|z|^2}$

$\downarrow$

$|z|^2 = 1$

& $z^{-1} = \bar{z}$ (We know)

$z^{-1} = \frac{\bar{z}}{1}$

$z^{-1} = \frac{\bar{z}}{|z|^2}$

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Q If $Re\left(\frac{1}{z}\right) < \frac{1}{2}$ then Locus of z

$Re(z^{-1}) < \frac{1}{2}$

$Re\left(\frac{\bar{z}}{|z|^2}\right) < \frac{1}{2}$

$Re\left(\frac{x-iy}{x^2+y^2}\right) < \frac{1}{2}$

$Re\left(\frac{x}{x^2+y^2} - i\frac{y}{x^2+y^2}\right) < \frac{1}{2}$

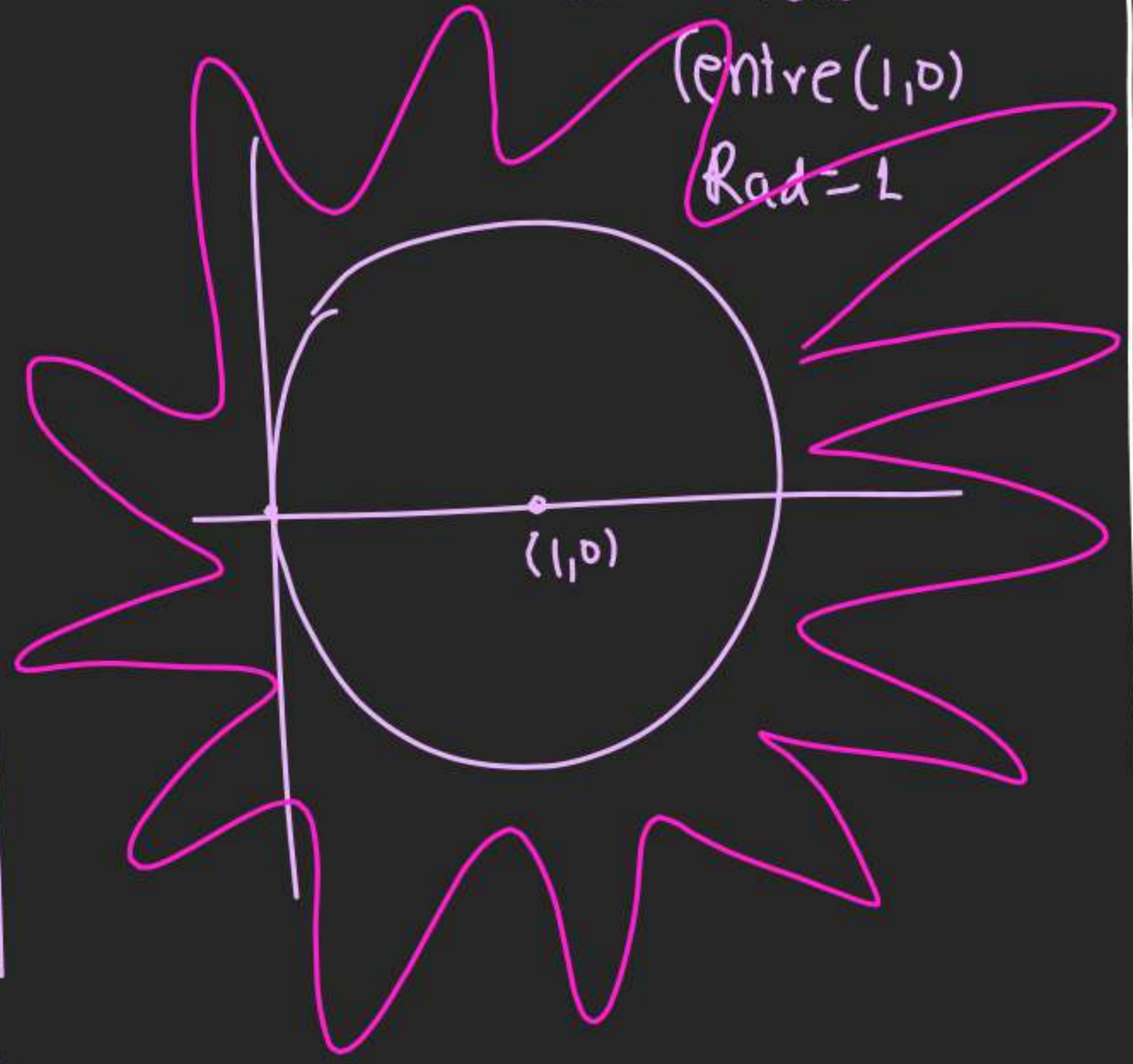
$\Rightarrow \frac{x}{x^2+y^2} < \frac{1}{2} \Rightarrow 2x < x^2+y^2$

$x^2+y^2-2x > 0$

$(x^2-2x+1) + y^2 > 1$ → Circle [illegible]

$(x-1)^2+(y)^2 > 1$

$(x-1)^2+(y)^2 = 1 \Rightarrow$ Circle



Q Is $z \cdot \bar{z} = |z|^2$?

$(a+ib)(a-ib) = a^2+b^2$

$a^2-(ib)^2 = a^2+b^2$

$a^2+b^2 = a^2+b^2$

Are ye to true

from Now onwards

if $|z|^2$ is given

We can write $z \cdot \bar{z}$

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Q If $\left|\dfrac{\bar{z}_1 - 2\bar{z}_2}{2 - z_1\bar{z}_2}\right| = 1$, $|z_2| \neq 1$ then S.T. $|z_1| = 2$

$\overline{(z_1+z_2)} = \bar{z}_1 + \bar{z}_2$

$|z|^2 = z \cdot \bar{z}$

$|\bar{z}_1 - 2\bar{z}_2| = |2 - z_1 \cdot \bar{z}_2|$ C.M

$\Rightarrow |\bar{z}_1 - 2\bar{z}_2|^2 = |2 - z_1 \cdot \bar{z}_2|^2$

$\Rightarrow (\bar{z}_1 - 2\bar{z}_2)\overline{(\bar{z}_1 - 2\bar{z}_2)} = (2 - z_1\bar{z}_2)\cdot\overline{(2 - z_1\bar{z}_2)}$

$(\bar{z}_1 - 2\bar{z}_2)(\bar{\bar{z}}_1 - 2\bar{\bar{z}}_2) = (2 - z_1\bar{z}_2)(\bar{2} - \bar{z}_1\bar{\bar{z}}_2)$

$(\bar{z}_1 - 2\bar{z}_2)(z_1 - 2z_2) = (2 - z_1\cdot\bar{z}_2)(2 - \bar{z}_1\cdot z_2)$

$z_1\bar{z}_1 - 2\cancel{z_1\bar{z}_2} - 2\cancel{\bar{z}_1 z_2} + 4z_2\bar{z}_2 = 4 - 2\cancel{z_1\bar{z}_2} - 2\cancel{\bar{z}_1 z_2} + z_1\bar{z}_1\cdot z_2\bar{z}_2$

$|z_1|^2 \quad + 4|z_2|^2 = 4 \quad + |z_1|^2|z_2|^2$

$|z_1|^2 + 4|z_2|^2 - 4 - |z_1|^2|z_2|^2 = 0$

$|z_1|^2(1 - |z_2|^2) - 4(1 - |z_2|^2) = 0 \Rightarrow (1 - |z_2|^2)(|z_1|^2 - 4) = 0 \Rightarrow |z_1|^2 = 4 \Rightarrow \boxed{|z_1| = 2}$

Q If $|z_1 + z_2|^2 = |z_1|^2 + |z_2|^2$ then $\dfrac{z_1}{z_2}$ is

Purely Imag = ?