

$$\begin{aligned}
 10: & \cos(A+B) + \sin(A-B) = \frac{2(\sin 2\theta \cos 2\theta) \cos 4\theta}{\sin 2\theta} \\
 &= \sin\left(\frac{\pi}{2} + A+B\right) + \sin(A-B) \\
 &= 2 \sin\left(\frac{\pi}{2} + 2A\right) \cos\left(\frac{\pi}{2} + 2B\right)
 \end{aligned}$$

$$\begin{aligned}
 13: & \frac{\tan 5\theta + \tan 3\theta}{\tan 5\theta - \tan 3\theta} = \frac{\frac{\sin 5\theta}{\cos 5\theta} + \frac{\sin 3\theta}{\cos 3\theta}}{\frac{\sin 5\theta}{\cos 5\theta} - \frac{\sin 3\theta}{\cos 3\theta}} = \frac{\sin 8\theta}{\cos 8\theta \cos 2\theta} = \frac{\sin 2\theta}{\cos 2\theta \cos 2\theta} \\
 &= \frac{\sin 8\theta}{\sin 2\theta} = \frac{(\sin 4\theta + \cos 4\theta)}{\sin 4\theta \cos 4\theta + \cos 4\theta \sin 4\theta} \\
 &= \frac{2 \sin 4\theta \cos 4\theta}{\sin 2\theta} = \frac{2 \sin(2\theta+2\theta) \cos 4\theta}{\sin 2\theta}
 \end{aligned}$$

$$\text{L.H.S.} = \frac{-2 \sin A \sin 2A}{2 \sin A \cos 2A} + \frac{2 \sin A \sin 3A}{2 \sin A \cos 3A}$$

$$= \frac{\sin 3A}{\cos 3A} - \frac{\sin 2A}{\cos 2A}$$

$$= \frac{\sin 3A \cos 2A - \sin 2A \cos 3A}{\cos 2A \cos 3A} = \frac{\sin A}{\cos 2A \cos 3A}$$

$$\begin{aligned}
 & \frac{1}{b} \cdot \frac{(\sin(A-C) + \sin(A+C)) + 2\sin A}{(\sin(B-C) + \sin(B+C)) + 2\sin B} \\
 &= \frac{2\sin A \cos C + 2\sin A}{2\sin B \cos C + 2\sin B} = \frac{2\sin A (\cancel{\cos C + 1})}{2\sin B (\cancel{\cos C + 1})} \\
 &= \frac{\sin A}{\sin B}
 \end{aligned}$$

$$\tan A + \tan B = \frac{\sin A}{\cos A} + \frac{\sin B}{\cos B} = \frac{\sin A \cos B + \sin B \cos A}{\cos A \cos B}$$

$$\tan A + \tan B = \frac{\sin(A+B)}{\cos A \cos B}$$

$$\tan A - \tan B = \frac{\sin(A-B)}{\cos A \cos B}$$

$$\cot A + \cot B = \frac{\sin(A+B)}{\sin A \sin B}$$

$$\cot A - \cot B = \frac{\sin(B-A)}{\sin A \sin B}$$

$$\frac{19}{1569} \frac{(\sin A - \sin 5A) + (\sin 9A - \sin 13A)}{(\cos A - \cos 5A) - (\cos 9A - \cos 13A)}$$

$$\frac{1155}{1569} = \frac{(-2 \sin 2A \cos 3A) + (-2 \sin 2A \cos 11A)}{(2 \sin 2A \sin 3A) - (2 \sin 2A \sin 11A)}$$

$$\frac{\min(1155, 1569)}{1569} \frac{\cos 3A + \cos 11A}{\sin 11A - \sin 3A} = \frac{2 \cos 7A \cos 4A}{2 \sin 4A \cos 7A}$$

$$\text{rank } \langle \min(1155, 1569) \rangle = \max(2, 3) = 3 \\ \min(2, 3) = 2 \\ \gamma = \cot 4A$$

$$\underline{24} : \frac{\cos(A+B+C) + \cos(-A+B+C) + \cos(A-B+C) + \cos(A+B-C)}{(\sin(A+B+C) + \sin(-A+B+C)) - (\sin(A-B+C) - \sin(A+B-C))}$$

$$= \frac{2\cos(B+C)\cos A + 2\cos A \cos(B-C)}{2\sin(B+C)\cos A - 2\sin(C-B)\cos A}$$

$$= \frac{\cos(B+C) + \cos(B-C)}{\sin(B+C) + \sin(B-C)} = \frac{2\cos B \cos C}{2\sin B \cos C} = \cot B.$$

$$\begin{aligned}
 30: & \quad \cos\left(\theta + \left(n - \frac{3}{2}\right)\phi\right) - \cos\left(\theta + \left(n + \frac{3}{2}\right)\phi\right) \\
 & = 2 \sin \frac{3\phi}{2} \sin(\theta + n\phi) \quad 28:
 \end{aligned}$$

$$\begin{aligned}
 28: & \quad 27: (\sin 50^\circ - \sin 70^\circ) + \sin 10^\circ \\
 & = -2 \sin 10^\circ \cancel{\cos 60^\circ} + \sin 10^\circ \\
 & = -\sin 10^\circ + \sin 10^\circ = 0
 \end{aligned}$$

$$\begin{aligned}28. \quad & \frac{\sin 10^\circ + \sin 20^\circ + \sin 40^\circ + \sin 50^\circ}{(\sin 10^\circ + \sin 50^\circ) + (\sin 20^\circ + \sin 40^\circ)} \\&= 2 \sin 30^\circ \cos 20^\circ + 2 \sin 30^\circ \cos 10^\circ \\&= \cos 20^\circ + \cos 10^\circ \\&= \sin 70^\circ + \sin 80^\circ\end{aligned}$$

$$\begin{aligned}\tan(A+B) &= \frac{\sin(A+B)}{\cos(A+B)} = \frac{\sin A \cos B + \cos A \sin B}{\cos A \cos B - \sin A \sin B} \\ &= \frac{\left(\frac{\sin A \cos B + \cos A \sin B}{\cos A \cos B} \right)}{\left(\frac{\cos A \cos B - \sin A \sin B}{\cos A \cos B} \right)} \\ &= \frac{\tan A + \tan B}{1 - \tan A \tan B}\end{aligned}$$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\cot(A+B) = \frac{(\tan A \tan B) / (\tan A + \tan B)}{(\tan A + \tan B) / (\tan A \tan B)} = \frac{\tan A \tan B}{1 + \tan A \tan B}$$

$$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

$$\cot(A+B) = \frac{\cot A \cot B - 1}{\cot A + \cot B}$$

$$\cot(A-B) = \frac{\cot A \cot B + 1}{\cot B - \cot A}$$

$$\tan(A+B) - \tan A - \tan B = \tan(A+B)\tan A \tan B$$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\tan(A+B) - \tan(A+B)\tan A \tan B = \tan A + \tan B$$

$$\tan(A+B) - \tan A - \tan B = \frac{\tan(A+B)\tan A}{\tan B}$$

$$\begin{aligned}
 & \text{L.H.S.} \\
 & \cot 16^\circ \cot 44^\circ + \cot 44^\circ \cot 76^\circ - \cot 76^\circ \cot 16^\circ = ? \\
 & = (\underbrace{\cot 16^\circ \cot 44^\circ - 1}_{\cot(A+B) = \frac{\cot A \cot B - 1}{\cot A + \cot B}}) + (\cot 44^\circ \cot 76^\circ - 1) - (\cot 76^\circ \cot 16^\circ + 1) + 3 \\
 & = \cot(16^\circ + 44^\circ)(\cot 16^\circ + \cot 44^\circ) + \cot 120^\circ (\cot 44^\circ + \cot 76^\circ) \\
 & \quad - \cot(76^\circ - 16^\circ)(\cot 16^\circ - \cot 76^\circ) + 3 \\
 & \left. \begin{array}{l} \cot(A+B) = \frac{\cot A \cot B - 1}{\cot A + \cot B} \\ \cot(A-B) = \frac{\cot A \cot B + 1}{\cot B - \cot A} \end{array} \right\} = \frac{1}{\sqrt{3}} (\cot 16^\circ + \cot 44^\circ) - \frac{1}{\sqrt{3}} (\cot 44^\circ + \cot 76^\circ) \\
 & \quad - \frac{1}{\sqrt{3}} (\cot 16^\circ - \cot 76^\circ) + 3 \\
 & = \frac{1}{\sqrt{3}} [\cot 16^\circ + \cot 44^\circ - \cot 44^\circ - \cot 76^\circ - \cot 16^\circ - \cot 76^\circ] + 3
 \end{aligned}$$

Q. P.T. $\tan 54^\circ = \frac{\cos 9^\circ + \sin 9^\circ}{\cos 9^\circ - \sin 9^\circ}$

$$\tan(45^\circ + 9^\circ) = \frac{\tan 45^\circ + \tan 9^\circ}{1 - \tan 45^\circ \tan 9^\circ} = \frac{1 + \tan 9^\circ}{1 - \tan 9^\circ}$$

$$= \frac{1 + \frac{\sin 9^\circ}{\cos 9^\circ}}{1 - \frac{\sin 9^\circ}{\cos 9^\circ}}$$

$$= \frac{\cos 9^\circ + \sin 9^\circ}{\cos 9^\circ - \sin 9^\circ}$$

H.W
Ex-15 (Leave Q. 15)
Ex-16 (Q 5, 6, 7)