Nishant Jindal

$$x = \frac{1-\cos y}{\sin y} - \ln \frac{2(1+\sin y)}{\sin y}$$

$$\frac{dx}{dy} = \left(\frac{1}{\sin^2 y} - \frac{\cos y}{\sin^2 y}\right) - \frac{\cos y}{1+\sin y} + \frac{\cos y}{\sin y}$$

$$= \frac{1}{1+\cos y} - \frac{\cos y}{1+\sin y} + \frac{\cos y}{\sin y}$$

$$\frac{dy}{dx} = \frac{\cancel{(1+\sin y)}\cancel{(1+\cos y)}\sin y}{(1+\sin y+\cos y)^2}$$

$$\sin y + \sin^2 y - \cos y \sin y - \cos^2 y \sin y + \cos y(1+\sin y)(\cos y + \sin y + \cos y)$$

$$\cancel{2+2\sin y+2\cos y+2\sin y\cos y}$$

$$(1+\cos y)(1+\sin y)\sin y$$

$$= \frac{2}{(1+\sin y+\cos y)\sin y} \cdot 2 = \frac{1+\sin y+\cos y}{(1+\sin y)(1+\cos y)\sin y}$$

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$$\frac{2}{\sqrt{a^2-b^2}} \qquad \frac{\frac{1}{2}\sqrt{\frac{a-b}{a+b}}\sec^2\frac{x}{2}}{1+\left(\frac{a-b}{a+b}\right)\tan^2\frac{x}{2}} = \frac{}{\frac{a\left(1+\tan^2\frac{x}{2}\right)+b\left(1-\tan^2\frac{x}{2}\right)}{1+\tan^2\frac{x}{2}}}$$

$$\frac{2\ln(1+x)\sin\frac{1}{x}}{(1+x)} - \frac{\ln^2(1+x)\cos\frac{1}{x}}{x^2}$$

$x > 0$

$x \leq 0$

$$y' = \frac{1}{a+b\cos x}$$

0

$$n = \frac{n-1+n-2}{\boxed{n=3}}$$

$$f(x) = x^3 + ax^2 + bx + c$$

$a = 3 + 2a + b$ $\quad c = 6$

$b = 12 + 2a$

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$$\frac{-1}{\sqrt{1-\frac{\cos 3x}{\cos^3 x}}}\cdot\frac{1}{2}\cdot\frac{1}{\sqrt{\frac{\cos 3x}{\cos^3 x}}}\cdot\frac{\cos^3 x(-3\sin 3x)+3\cos^2 x\sin x\cos 3x}{\cos^6 x}$$

$$\frac{-1}{\sqrt{\frac{3\cos x-3\cos^3 x}{3\cos x\sin^2 x}}\sqrt{\cos 3x}}\cdot\frac{3(-\sin 2x)}{2\cos x}$$

$$=\frac{3}{\sqrt{3\cos x\cos 3x}}=\sqrt{\frac{3}{\cos x\cos 3x}}$$

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$$a_1 \sin x + a_2 \sin 2x + \cdots + a_n \sin nx = f(x)$$

$$|f(x)| \le |\sin x|$$

$$f(0) = 0$$

$$\left|\sum r a_r\right| \le 1$$

$$|f'(0)| \le 1$$

$$|a_1 \sin x + a_2 \sin 2x + \cdots + a_n \sin nx| \le |\sin x|$$

$$|f'(0)| = \left|\lim_{x\to 0} \frac{f(x)}{x}\right| = \left|\lim_{x\to 0} \sum_{r=1}^{n} r a_r \frac{\sin rx}{rx}\right| \le \lim_{x\to 0} \frac{|\sin x|}{|x|}$$

$$f > g$$

$$\lim_{x\to a} f > \lim_{x\to a} g$$

$x = a$

$$|f(x)| \le |\sin x|$$

$$x \ne 0$$

$$\frac{|f(x)|}{|x|} \le \frac{|\sin x|}{|x|}$$

$$\boxed{\lim_{x\to 0} \left|\frac{f(x)}{x}\right|} \le \lim_{x\to 0} \left|\frac{\sin x}{x}\right|$$

$$|f'(0)| \le 1$$

1. $$\int \underset{\text{II}}{x}\,\underset{\text{I}}{\tan^{-1}x}\,dx = \frac{x^2}{2}\tan^{-1}x - \frac{1}{2}\int \frac{x^2+1-1}{1+x^2}\,dx$$

$$= \frac{x^2}{2}\tan^{-1}x - \frac{1}{2}\left(x - \tan^{-1}x\right) + C \qquad 1 - \frac{1}{1+x^2}$$

2. $$\int \frac{\sin^{-1}x\,dx}{(1-x^2)^{3/2}} \checkmark$$

$\tan^{-1}x = \theta \Rightarrow x = \tan\theta$, $dx = \sec^2\theta\,d\theta$

$$\int \underset{\text{I}}{\theta}\,\underbrace{\tan\theta\sec^2\theta}_{\text{II}}\,d\theta = \theta\frac{\sec^2\theta}{2} - \frac{1}{2}\int 1\cdot\sec^2\theta\,d\theta$$

$$\frac{\tan^2\theta}{2} = \frac{\sec^2\theta}{2} - \frac{1}{2}$$

$$= \frac{\theta\sec^2\theta}{2} - \frac{1}{2}\tan\theta + C.$$

$$\frac{\theta\tan^2\theta}{2} - \int\frac{\tan^2\theta}{2}\,d\theta \qquad \tan^2\theta \to \sec^2\theta - 1$$

$$= \frac{\theta\tan^2\theta}{2} - \frac{\tan\theta}{2} + \frac{\theta}{2} + C.$$

$\sec\theta(\sec\theta\tan\theta)$

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$$\sin^{-1}x = \theta \Rightarrow x = \sin\theta \,,\quad dx = \cos\theta\, d\theta$$

$$\int \frac{\sin^{-1}x\, dx}{(1-x^2)^{3/2}} = \int \frac{\theta\, \cos\theta\, d\theta}{\cos^3\theta} = \int \underset{I}{\theta}\, \underset{II}{\sec^2\theta}\, d\theta = \theta\tan\theta - \int \tan\theta\, d\theta$$

$$= \theta\tan\theta - \ln|\sec\theta| + C$$

$$\int \overset{I}{\sin^{-1}x}\, \frac{dx}{(1-x^2)^{3/2}} = \frac{x\sin^{-1}x}{\sqrt{1-x^2}} + \frac{1}{2}\int \frac{-2x\quad dx}{1-x^2}$$

$$= \frac{x\sin^{-1}x}{\sqrt{1-x^2}} + \frac{1}{2}\ln|1-x^2| + C.$$

$$\int \frac{dx}{(1-x^2)^{3/2}} = -\frac{1}{2}\int \frac{-2\,dx}{x^3\left(\frac{1}{x^2}-1\right)^{3/2}}$$

$$= \frac{1}{\sqrt{\frac{1}{x^2}-1}}$$

$$= \frac{x}{\sqrt{1-x^2}}$$

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3.

$$\int \underset{\text{I}}{x^2}\,\underset{\text{II}}{e^{3x}}\,dx = x^2\frac{e^{3x}}{3} - \frac{2}{3}\int \underset{\text{I}}{x}\,\underset{\text{II}}{e^{3x}}\,dx$$

$$= \frac{x^2}{3}e^{3x} - \frac{2}{3}x\frac{e^{3x}}{3} + \frac{2}{9}\int e^{3x}\,dx$$

$\int \sin x\,dx = -\cos x$

$\frac{d}{dx}(-\cos x) = \sin x$

$$= \left(\frac{x^2}{3} - \frac{2}{9}x + \frac{2}{27}\right)e^{3x} + C$$

$$\boxed{\int (a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0)e^{\alpha x}\,dx = e^{\alpha x}(B_n x^n + B_{n-1}x^{n-1} + \cdots + B_1 x + B_0) + C}$$

$\int f(x)\,dx = g(x)$ $\quad$ $g'(x) = f(x)$

$$\cancel{e^{\alpha x}}\left(\alpha(B_n x^n + \cdots + B_1 x + B_0) + (nB_n x^{n-1} + \cdots + B_1)\right) = (a_n x^n + \cdots + a_1 x + a_0)\cancel{e^{\alpha x}}$$

Equate coeff.

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$$\int (x^3-3x^2+6x-4)e^{2x}\,dx = (Ax^3+Bx^2+Cx+D)e^{2x} + E$$

$$(x^3-3x^2+6x-4)\cancel{e^{2x}} = \left(2(Ax^3+Bx^2+Cx+D)+3Ax^2+2Bx+C\right)\cancel{e^{2x}}$$

$$1 = 2A \Rightarrow A = \frac{1}{2}$$

$$-3 = 2B+3A \Rightarrow B = -\frac{9}{4}$$

$$6 = 2C+2B \Rightarrow C = \frac{21}{4}$$

$$-4 = 2D+C \Rightarrow D = -\frac{37}{8}$$

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4. $\int \sec^{-1}x\,dx \cdot = \int \underset{\mathrm{I}}{\theta}\,\underbrace{\sec\theta\tan\theta\,d\theta}_{\mathrm{II}}$ $\quad \sec^{-1}x=\theta \Rightarrow x=\sec\theta$

$= \theta\sec\theta - \int \sec\theta\,d\theta$

$= \theta\sec\theta - \ln|\sec\theta+\tan\theta| + C.$

$\boxed{1832-1855}$

$\boxed{1910-1950}$

$$\int \overset{\mathrm{II}}{1}\cdot\overset{\mathrm{I}}{\sec^{-1}x}\,dx = x\sec^{-1}x - \int \frac{\cancel{x}\,dx}{\cancel{x}\sqrt{x^2-1}}$$

$$= x\sec^{-1}x - \ln\left|x+\sqrt{x^2-1}\right| + C.$$

$$\int \overset{\mathrm{II}}{1}\cdot\overset{\mathrm{I}}{f(x)}\,dx = x\,f(x) - \int x\,f'(x)\,dx.$$