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$az^2+bz+c=0 \begin{cases} z_1 \\ z_2 \end{cases}$ $|a|=|b|=|c|$, $|z_1|=1$

$\frac{b^2}{ac} = 1 = \frac{\left(-\frac{b}{a}\right)^2}{\frac{c}{a}} = \frac{(z_1+z_2)^2}{z_1z_2}$

$z_1+z_2 = -\frac{b}{a}$ ✓

$z_1z_2 = \frac{c}{a}$ $|z_1||z_2| = 1 \Rightarrow |z_2| = 1$

$|z_1+z_2| = 1$

$(z_1+z_2)(\bar{z}_1+\bar{z}_2) = 1$

$(z_1+z_2)\left(\frac{1}{z_1}+\frac{1}{z_2}\right) = 1$

(z_1+z_2)

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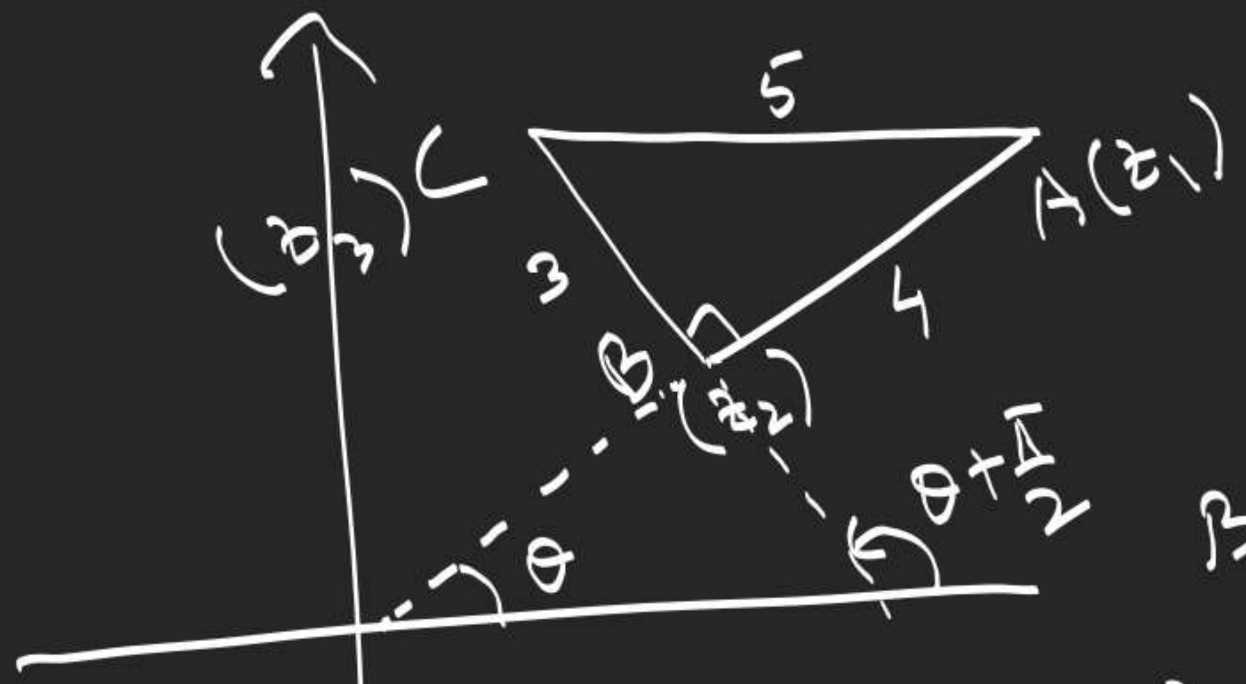
Rotation



Express z_3 in terms of z_1, z_2

BC BA

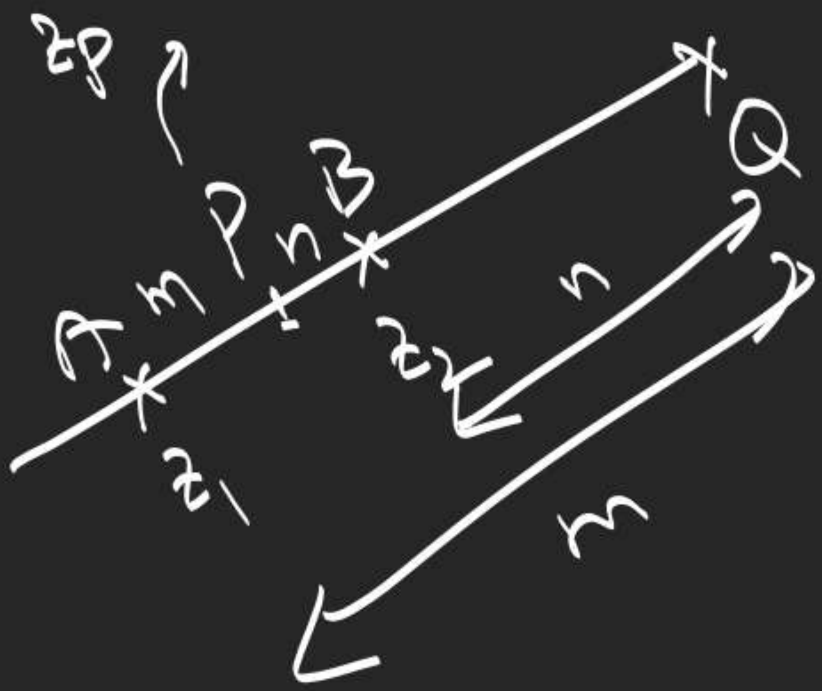
$z_3 - z_2 \;?\; z_1 - z_2$

$\frac{z_3-z_2}{3} \;?\; \frac{z_1-z_2}{4}$

$$\frac{z_3-z_2}{3} = \frac{z_1-z_2}{4} e^{i\frac{\pi}{2}}$$

$$z_1 - z_2 = 4e^{i\theta}$$

$$\left(\frac{z_1-z_2}{4}\right) e^{i\pi/2} = e^{i\left(\theta+\frac{\pi}{2}\right)}$$



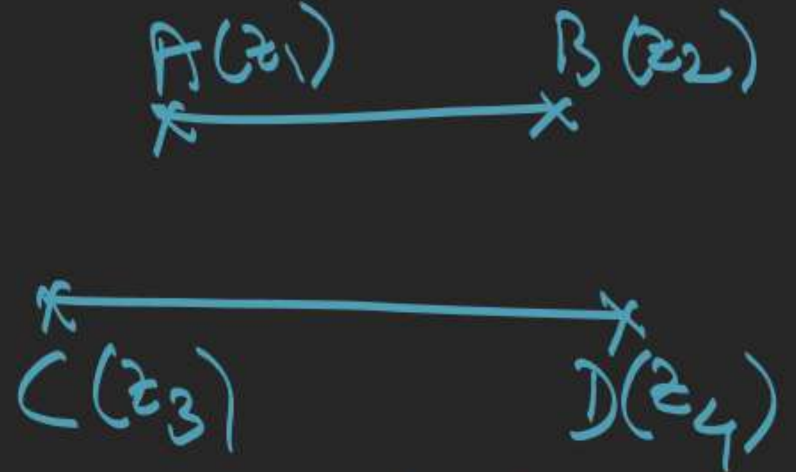
$$\frac{z_p - z_1}{AP} = \frac{z_2 - z_p}{PB}$$

$$z_p - z_1 = \frac{m}{n}(z_2 - z_p)$$

$$\boxed{z_p = \frac{m z_2 + n z_1}{m+n}}$$

$$\frac{z_Q - z_1}{AQ} = \frac{z_Q - z_2}{BQ} \Rightarrow z_Q - z_1 = \frac{m}{n}(z_Q - z_2)$$

$$\boxed{z_Q = \frac{m z_2 - n z_1}{m-n}}$$



AB ∥ CD

AB ⊥ CD

$$\frac{z_1-z_2}{|z_1-z_2|}e^{i\frac{\pi}{2}} = \frac{z_3-z_4}{|z_3-z_4|}$$

$$\frac{z_1-z_2}{|z_1-z_2|} = \frac{z_3-z_4}{|z_3-z_4|}$$

$$\boxed{\frac{z_1-z_2}{z_3-z_4} \in R}$$

$$\Rightarrow \frac{z_1-z_2}{z_3-z_4} = \frac{\bar{z}_1-\bar{z}_2}{\bar{z}_3-\bar{z}_4}$$

$$\frac{z_1-z_2}{z_3-z_4} = -i\,\frac{|z_1-z_2|}{|z_3-z_4|}$$

$$\boxed{\frac{z_1-z_2}{z_3-z_4} \text{ is purely imaginary}}$$

$$\Rightarrow \frac{z_1-z_2}{z_3-z_4} + \frac{\bar{z}_1-\bar{z}_2}{\bar{z}_3-\bar{z}_4} = 0$$

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Parabola → Ex - 3 $\boxed{(1-15)}$

1. If z_1, z_2, z_3 are vertices of equilateral triangle then P.T. $\sum z_1^2 = \sum z_1 z_2$. And if z_0 is its circumcentre, then P.T. $3z_0^2 = z_1^2 + z_2^2 + z_3^2$.

2. If z_r $(r = 1, 2, \ldots, 6)$ are vertices of a regular hexagon then P.T $\sum_{r=1}^{6} z_r^2 = 6z_0^2$, where z_0 is its circumcentre.

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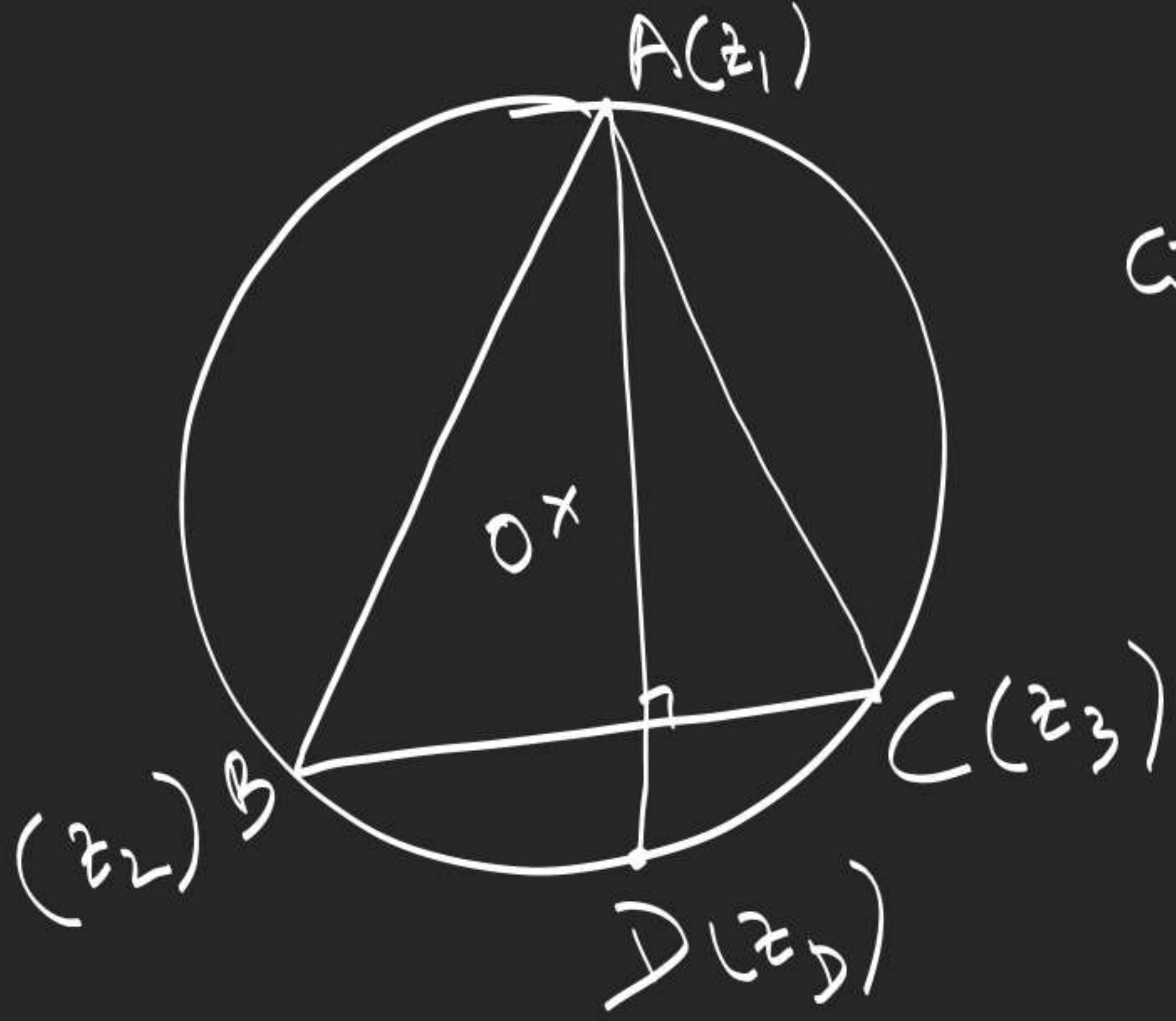
3.



Circumcentre is origin.

P.T. $z_D = -\dfrac{z_2 z_3}{z_1}$

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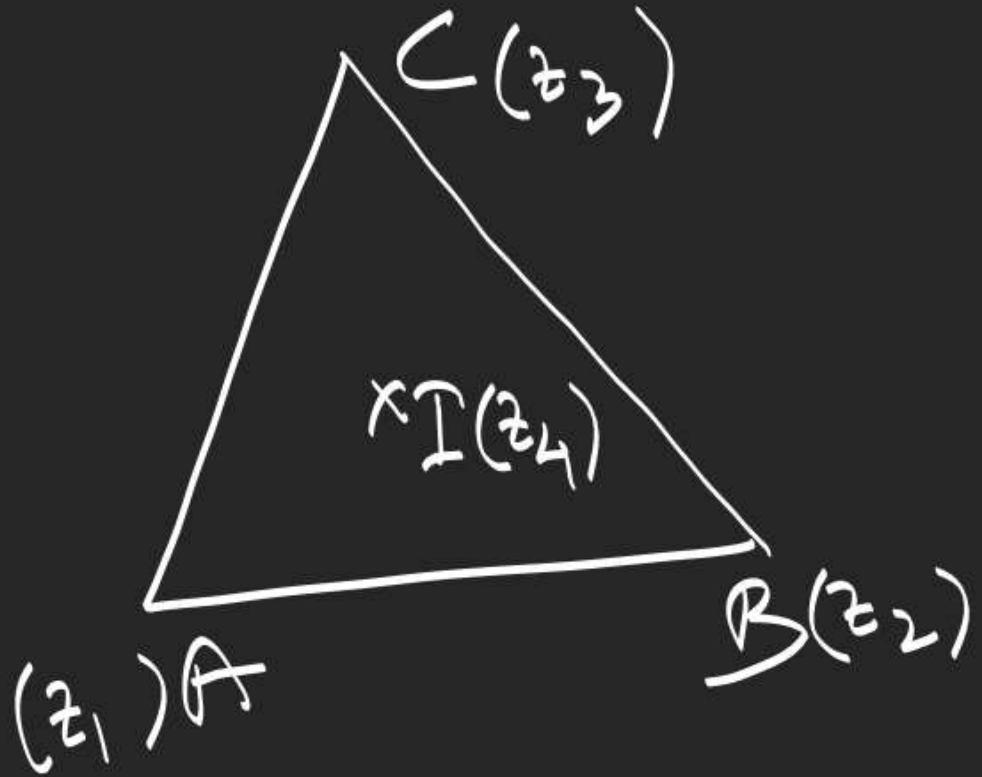
4.

$AC = BC$

$\angle CAB = \theta$

$I = \text{Incentre}$



P.T. $(z_2 - z_1)(z_3 - z_1) = (1 + \sec\theta)(z_4 - z_1)^2$

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5. Let z_1 & z_2 be roots of eqn. $z^2+pz+q=0$, where the coefficients p, q may be complex numbers. Let A and B represent z_1 & z_2 in complex plane. If $\angle AOB = \alpha \neq 0$ and $OA = OB$, where 'O' is origin, then P.T. $p^2 = 4q\cos^2\frac{\alpha}{2}$

6. Let z_1, z_2, z_3 be non zero complex numbers s.t. $|z_1| = |z_2| = |z_3| = R$ and $z_2 \neq z_3$, then P.T.

$$\min_{a\in R}\left|az_2+(1-a)z_3-z_1\right| = \frac{1}{2R}|z_1-z_2||z_1-z_3|$$