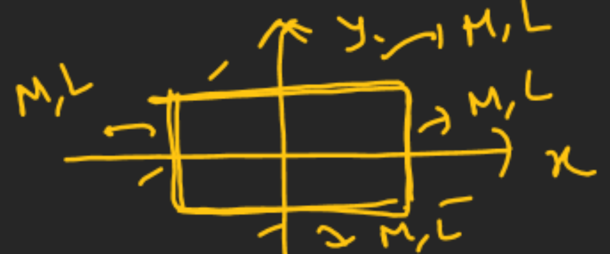
MAGNETIC FIELD



Magnetic Moment and Torque

Q.7 A uniform, constant magnetic field $\vec{B}$ is directed at an angle of 45° to the x-axis in the xy-plane. PQRS is a rigid, square wire frame carrying a steady current I_0, with its centre at the origin O. At time $t = 0$, the frame is at rest in the position as shown in figure, with its sides parallel to the x and y-axis. Each side of the frame is of mass M and length L. **(1998)**

(a) What is the torque τ about O acting on the frame due to the magnetic field?

(b) Find the angle by which the frame rotates under the action of this torque in a short interval of time Δt, and the axis about which rotation occurs. (Δt is so short that any variation in the torque during this interval may be neglected). Given: the moment of inertia of the frame about an axis through its centre perpendicular to its plane is $\frac{4}{3}ML^2 = (I_z)_{loop}$

- Take $\tau \to$ constant in $\Delta t \to$ interval

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MAGNETIC FIELD

Magnetic Moment and Torque

$$\vec{\tau} = \vec{M} \times \vec{B}.$$

$$\vec{M} = (l^2 I)\hat{k}$$

$$\vec{B} = B\cos 45^\circ\,\hat{i} + B\sin 45^\circ\,\hat{j}$$

$$= \left(\frac{B}{\sqrt{2}}\hat{i} + \frac{B}{\sqrt{2}}\hat{j}\right)$$

$$\vec{\tau} = (\vec{M} \times \vec{B})$$

$$= (I l^2)\hat{k} \times \frac{B}{\sqrt{2}}(\hat{i} + \hat{j})$$

$$= \frac{IBl^2}{\sqrt{2}}\left[\hat{k} \times (\hat{i} + \hat{j})\right]$$

$$\vec{\tau} = \frac{I l^2 B}{\sqrt{2}}\left[\hat{j} - \hat{i}\right]$$

$$|\vec{\tau}| = IBl^2,$$

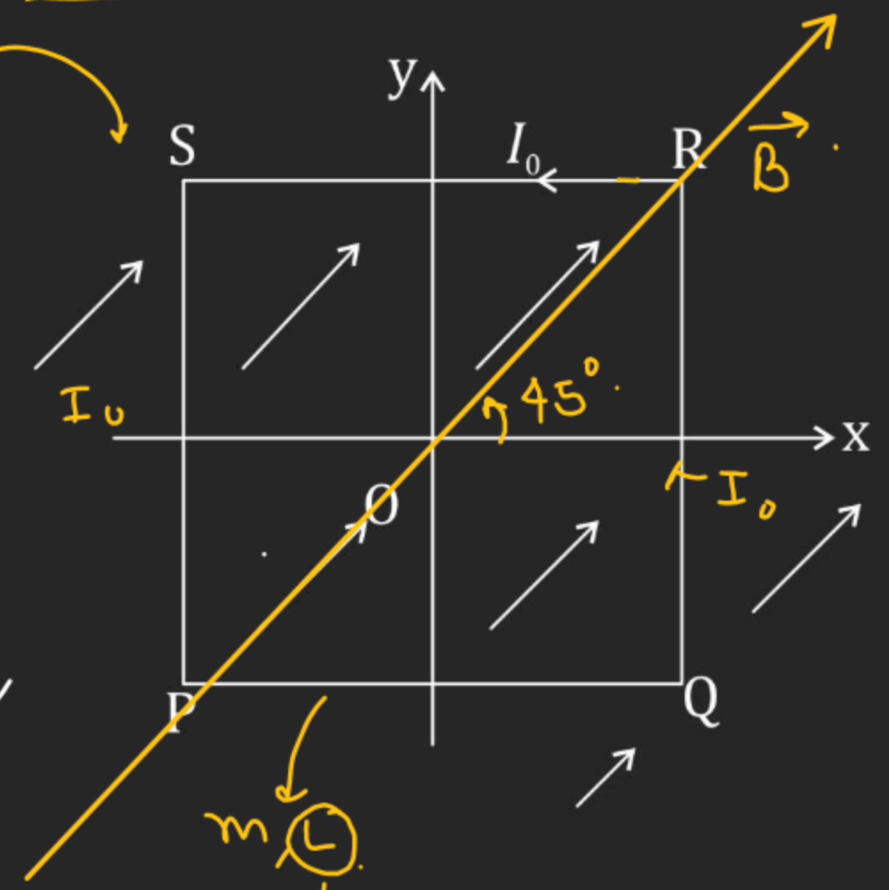
(b) $\tau = I\alpha.$

$$\underset{\text{Constant}}{\alpha} = \left(\frac{\tau}{I}\right)$$

$$\Delta\theta = \underset{0}{\omega_0}\Delta t + \frac{1}{2}\alpha\Delta t^2$$

$$\boxed{\Delta\theta = \frac{1}{2}\alpha\Delta t^2}$$

Square frame



m(L) → PQ

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(4) [The direction of torque gives the natural axis of Rotation]

$\vec{\tau} = \frac{I l^2 B}{\sqrt{2}}(\hat{j} - \hat{i})$

By perpendicular axis theorem

$I_x + I_y = I_z$

By Symmetry

$\boxed{I_x = I_y}$

$2I_x = 2I_y = I_z$

$I_x = I_y = \frac{I_z}{2} = \frac{\frac{4}{3}Ml^2}{2} = \frac{2}{3}Ml^2$ → About axis of Rotation

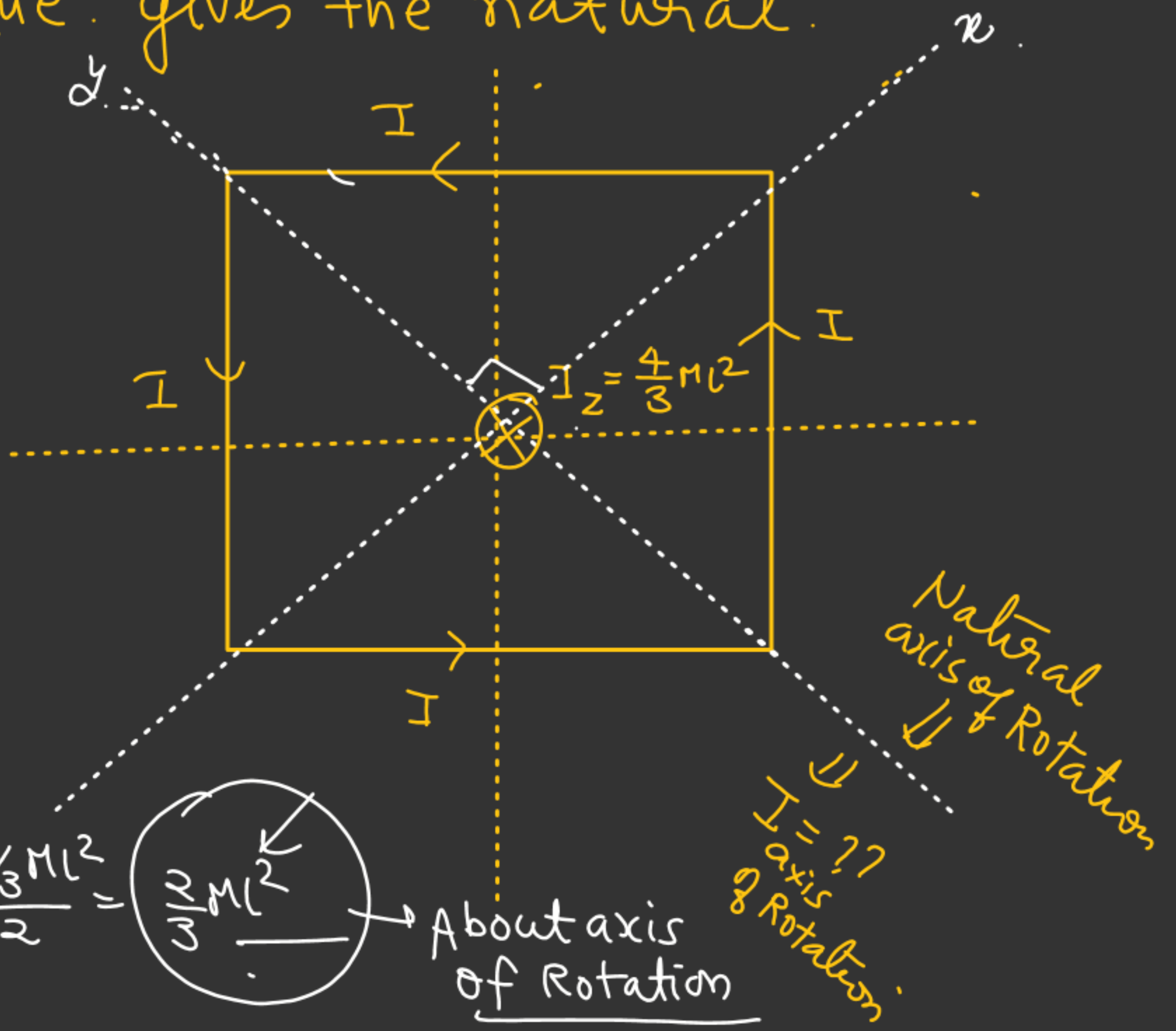
$\alpha = \frac{\tau}{I_{\text{axis of Rotation}}}$

$\alpha = \frac{Bl^2 I}{\frac{2}{3}Ml^2}$

$\alpha = \left(\frac{3BI}{2M}\right)$

$\Delta\theta = \frac{1}{2}\left(\frac{3BI}{2M}\right)(\Delta t)^2$

$= \left(\frac{3BI}{4M}\right)(\Delta t)^2$ Ans



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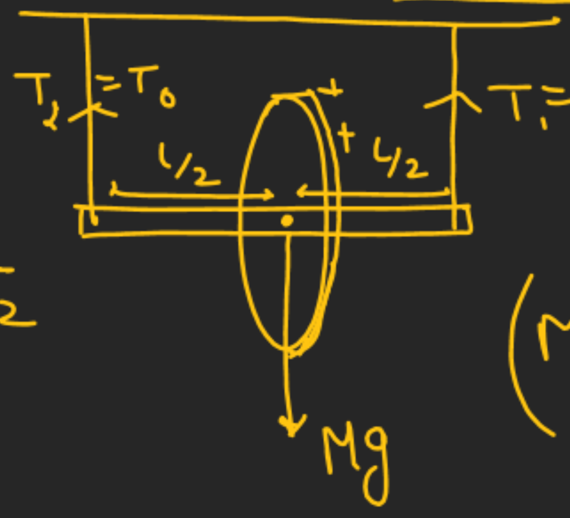
MAGNETIC FIELD

Magnetic Moment and Torque

Q.8 A ring of radius R having uniformly distributed charge Q is mounted on a rod suspended by two identical strings. The tension in strings in equilibrium is T_0. Now a vertical magnetic field is switched on and the ring is rotated at constant angular velocity ω. Find the maximum ω with which the ring can be rotated if the strings can withstand a $\boxed{\text{maximum tension}}$ of $3T_0/2$.

(2004)

Before B Switch on ← At Equilibrium



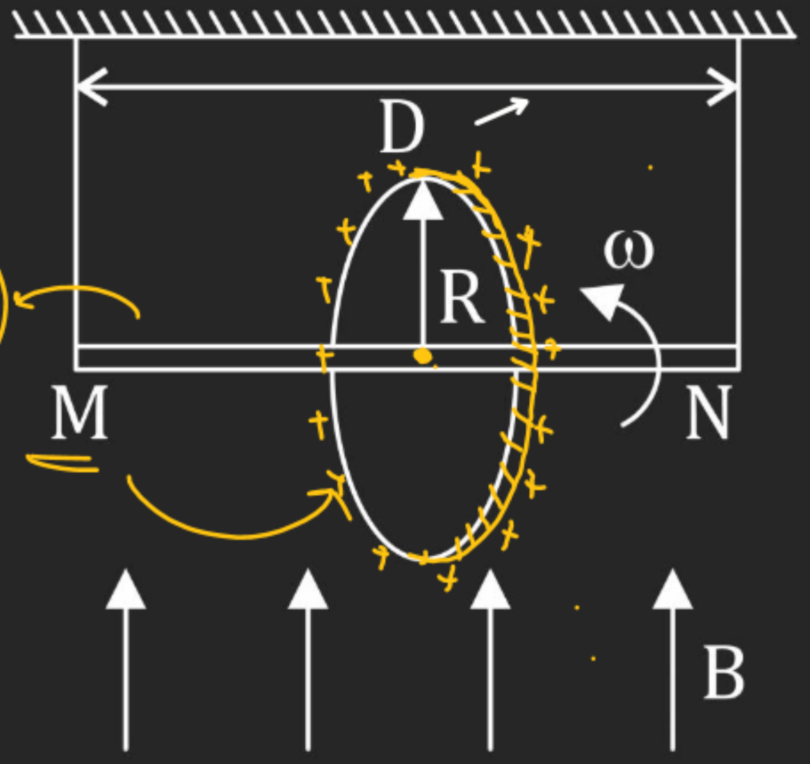
$T_1 = T_2 = T_0.$

$T_1 \cdot \frac{L}{2} = T_2 \cdot \frac{L}{2} \Rightarrow T_1 = T_2$

$2T_0 = Mg.$

$T_0 = \left(Mg/2\right) \Rightarrow Mg = 2T_0$

(M → Mass of system)



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Magnetic Moment and Torque

$$I = \frac{Q}{T} = \left(\frac{Q\omega}{2\pi}\right)$$

$$\vec{\tau}_B \rightarrow (-\hat{i} \times \hat{j})$$

$$\vec{\tau}_B \rightarrow -\hat{k}$$

$$\vec{\tau}_B = \vec{M} \times \vec{B} = \left(\frac{Q\omega R^2 B}{2}\right)(-\hat{k})$$

$$\vec{M} = \left(\frac{Q\omega}{2\pi} \times \pi R^2\right)(-\hat{i})$$

$$\vec{M} = \left(\frac{Q\omega R^2}{2}\right)(-\hat{i})$$

$$\vec{B} = B\hat{j}$$

$$\vec{\tau}_{T_1} = \left(T_1 \frac{D}{2}\right)(-\hat{k})$$

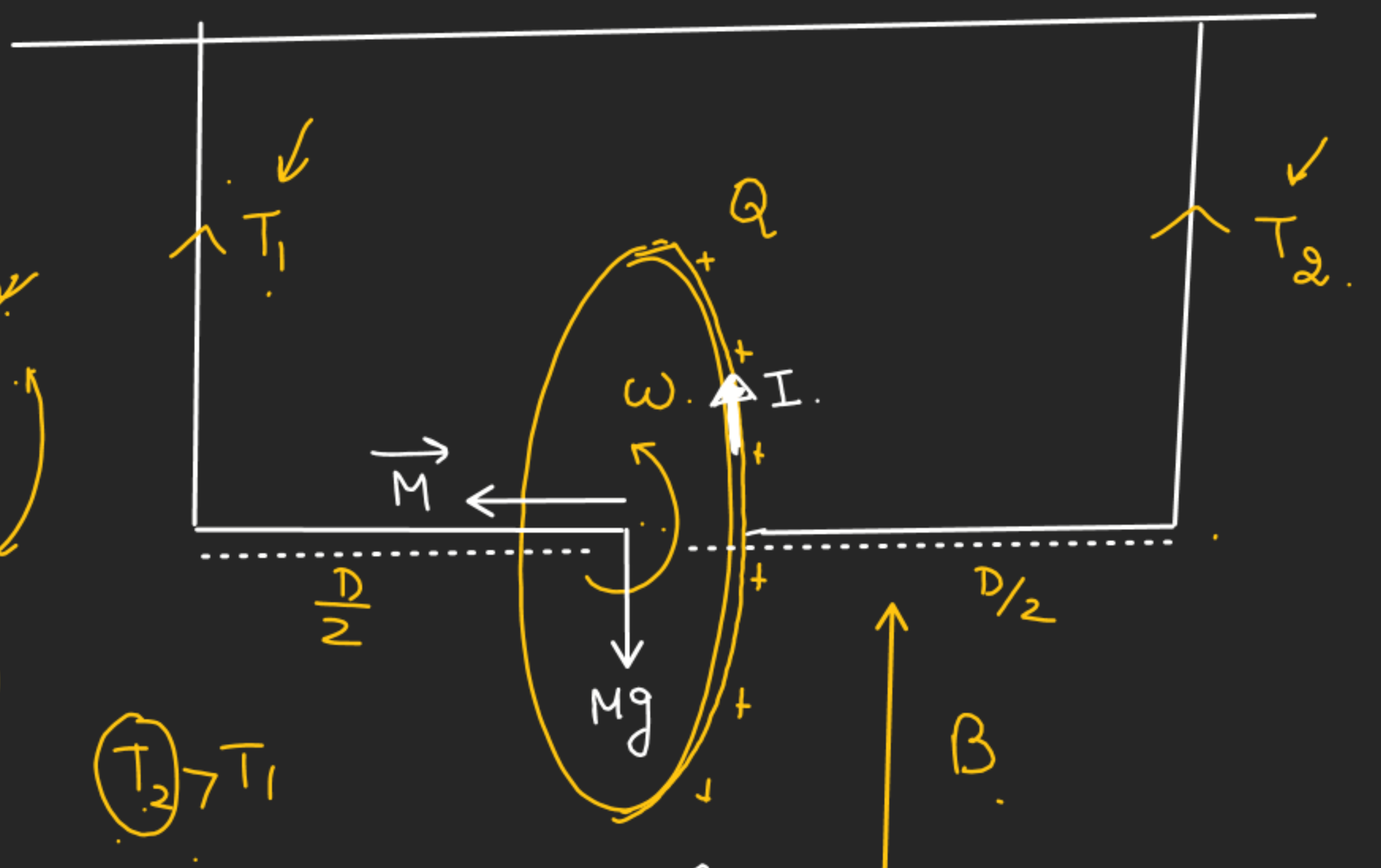
$$\vec{\tau}_{T_2} = T_2 \cdot \frac{D}{2}(+\hat{k})$$

$$T_1 + T_2 = Mg \quad \text{—(1)}$$

$$\frac{T_1 D}{2} + \frac{QB\omega R^2}{2} = T_2 \cdot \frac{D}{2}$$

$$T_2 - T_1 = \frac{QB\omega R^2}{D} \quad \text{—(2)}$$

$$T_1 = Mg - (T_2)_{max} = 2T_0 - \frac{3T_0}{2} = \left(\frac{T_0}{2}\right)$$



$$T_2 > T_1$$

$$(T_2)_{max} = \left(\frac{3T_0}{2}\right), \quad \left(T_1 = \frac{T_0}{2}\right)$$

$$T_2 - T_1 = \frac{QB\omega_{max} R^2}{D}$$

$$\frac{3T_0}{2} - \frac{T_0}{2} = \frac{QBR^2}{D}\omega_{max}$$

$$\omega_{max} = \left(\frac{DT_0}{QBR^2}\right) \text{ Ans}$$

MAGNETIC FIELD

Magnetic Moment and Torque

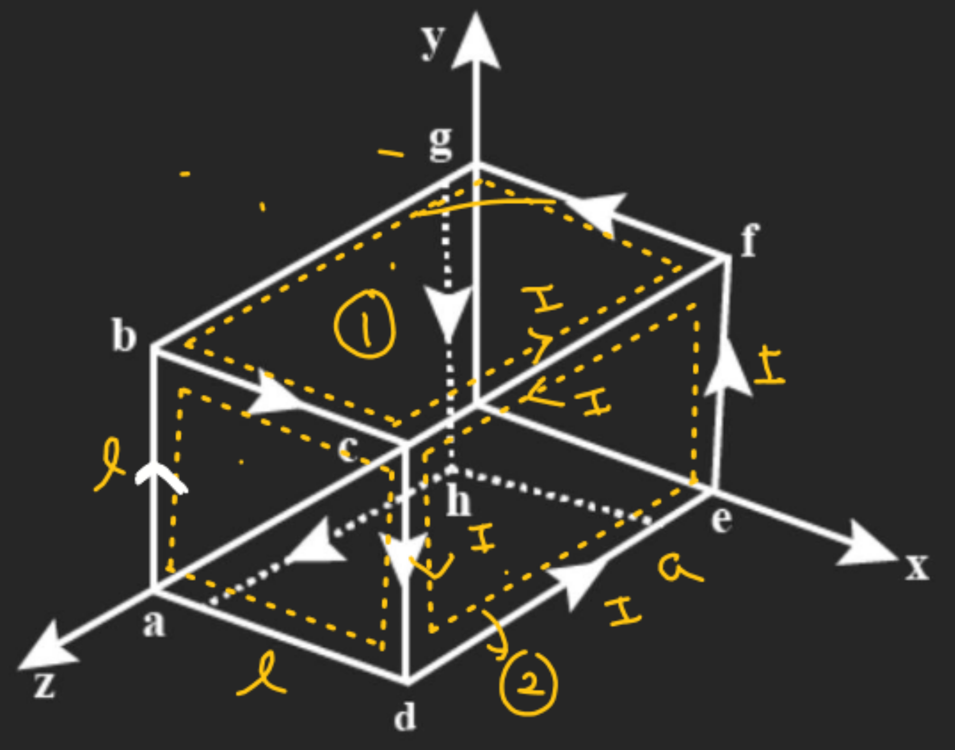
Q.9 A conductor carries a constant current I along the closed path abcdefgha involving 8 of the 12 edges of length l. Find the magnetic dipole moment of the closed path.

$$\vec{M}_{net} = [2 I l^2]\,\hat{j}$$

⇓

only due to face.
① & ②

a



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MAGNETIC FIELD

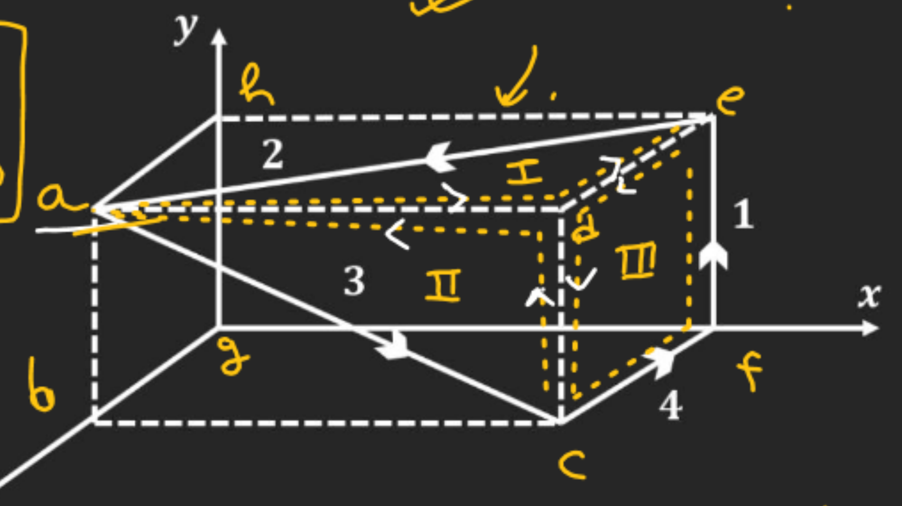
Magnetic Moment and Torque

Q.10 A wire carrying a 10 A current is bent to pass through various sides of a cube of side 10 cm as shown in Fig(a). A magnetic field $\vec{B} = (2\hat{i} - 3\hat{j} + \hat{k})$T is present in the region. Then find :

(a) the net force on the loop shown. $= 0$ [Net force on closed loop is zero. $B = C$]

(b) the magnetic moment vector of the loop.

(c) the net torque on the loop.



[$2 \rightarrow$ triangular loop + 1 square loop]

$$\vec{M} = \vec{M}_{I} + \vec{M}_{II} + \vec{M}_{III}$$

$$\vec{M}_{I} = I\left(\frac{1}{2}\times a\times a\right)\hat{j}$$

$$\vec{M}_{II} = I\left(\frac{1}{2}\times a\times a\right)(+\hat{k})$$

$$\vec{M}_{III} = Ia^2\,\hat{i}$$

$$= \frac{Ia^2}{2}\hat{i} + \frac{Ia^2}{2}\hat{j} + Ia^2\hat{k}$$

$$\vec{\tau} = \vec{M}\times\vec{B}$$

$$= Ia^2(\hat{i}+\hat{j}+\hat{k})\times(2\hat{i}-3\hat{j}+\hat{k})$$

$$\vec{\tau} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{Ia^2}{2} & \frac{Ia^2}{2} & Ia^2 \\ 2 & -3 & 1 \end{vmatrix} = \_\_\_\_$$

MAGNETIC FIELD

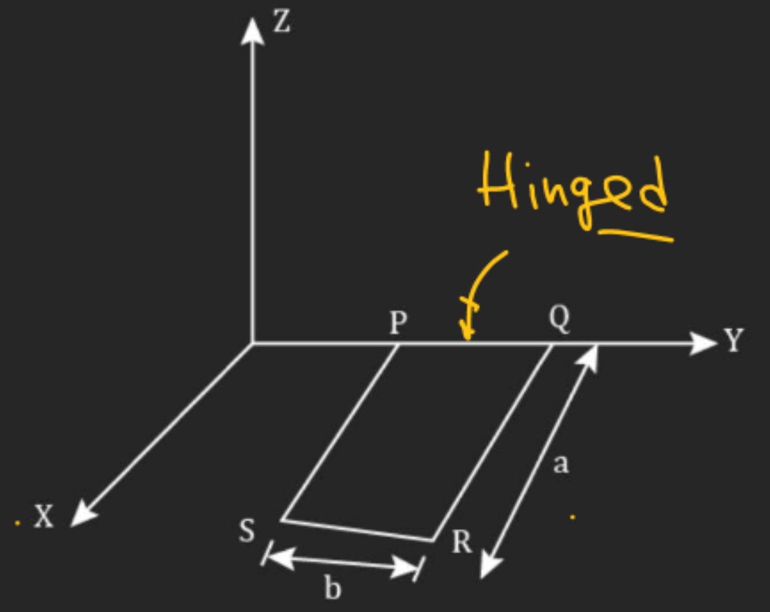
Magnetic Moment and Torque

Q.11 A rectangular loop $PQRS$ made from a uniform wire has length a, width b and mass m. It is free to rotate about the aim PQ which remains hinged along a horizontal line taken as the y-axis (see Fig). Take the vertically upward direction as the z-axis. A uniform magnetic field $\vec{B} = (3\hat{i} + 4\hat{k})B_0$ exists in the region. The loop is now released and is found to stay in the horizontal position in equilibrium.

(a) What is the direction of the current I in PQ ?

(b) Find the magnetic force on the arm RS.

(c) Find the expression for I in terms of B_0, a, b and m.



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$\vec{B} = (3\hat{i} + 4\hat{k})B_0$

$\vec{\tau} = \vec{r} \times \vec{F}$

$\vec{\tau}_{Mg} = \left(Mg\frac{a}{2}\right)(\hat{i} \times -\hat{k})$ $\quad \vec{r} = \frac{a}{2}\hat{i}$

$\vec{F} = Mg(-\hat{k})$

$\vec{\tau}_{Mg} = \left(Mg\frac{a}{2}\right)(+\hat{j})$

$\vec{\tau}_B = -\vec{\tau}_{Mg}$ (For Rotational Equlibrium.

$\Rightarrow \vec{\tau}_B \rightarrow (-\hat{j})$

$I \rightarrow (P \text{ to } Q)$

$\vec{M} = ab(-\hat{k})$

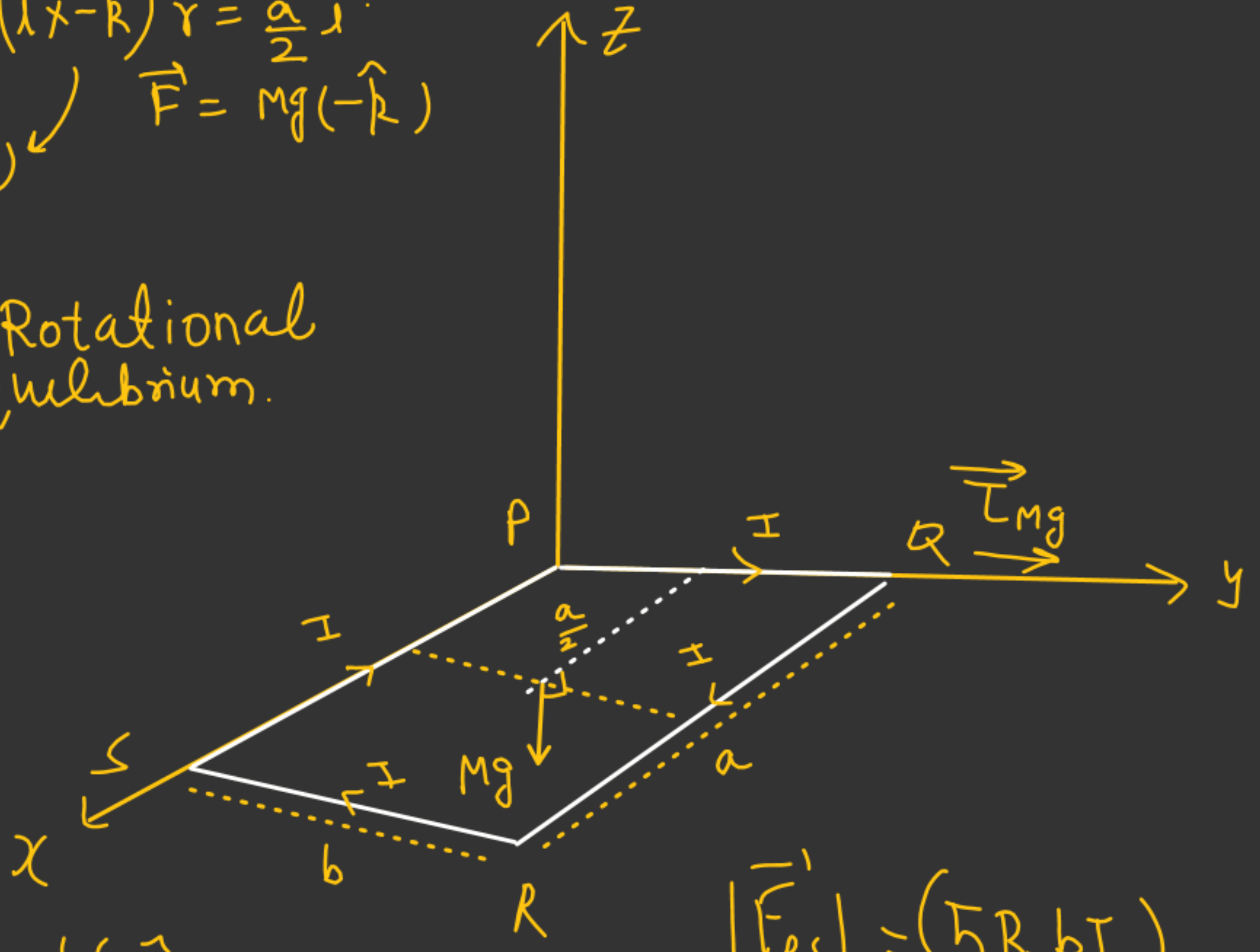
$\vec{\tau} = (\vec{M} \times \vec{B}) = (3ab)(-\hat{j})$

$\vec{F}_{RS} = I(\vec{L}_{RS} \times \vec{B})$ $\quad \vec{L}_{RS} = b(-\hat{j})$

$= B_0 I[b(-\hat{j}) \times (3\hat{i} + 4\hat{k})]$ $\quad \vec{B} = (3\hat{i} + 4\hat{k})B_0$

$\vec{F}_{RS} = B_0 I[3b(+\hat{k}) - 4b\hat{i}] \Rightarrow \vec{F}_{RS} = B_0 b I[3\hat{k} - 4\hat{i}]$

$|\vec{F}_{RS}| = (5B_0 bI)$



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H.W.:

Find I so that Cylinder is in equilibrium.

Hint