

Q. Integrate following functions

1. $\frac{2x}{1+x^2}$

Ans. $\log(1+x^2) + C$

Sol. $\int \frac{2x}{1+x^2} dx$

Put $1+x^2 = t \Rightarrow 2x dx = dt$

$= \log |t| + C$

$\Rightarrow \int \frac{2x}{1+x^2} dx = \int \frac{1}{t} dt$

$= \log(1+x^2) + C$

2. $\frac{(\log x)^2}{x}$

Ans. $\frac{1}{3}(\log|x|)^3 + C$

Sol. $\int \frac{\log^2 x}{x} dx$

put $\log x = t$ (i)

Therefore, $\frac{1}{x} \cdot dx = dt$

$\Rightarrow \int t^2 dt. \quad \Rightarrow \frac{t^3}{3} + C$

Put value of t from eq. (i). we get,

$\Rightarrow \frac{(\log x)^3}{3} + C$

3. $\frac{1}{x+x \log x}$

Ans. $\log|1+\log x| + C$

Sol. $\frac{1}{x+x \log x} = \frac{1}{x(1+\log x)}$

Let $1+\log x = t$

$\therefore \frac{1}{x} dx = dt \Rightarrow \int \frac{1}{x(1+\log x)} dx = \int \frac{1}{t} dt$

$= \log |t| + C = \log |1+\log x| + C$

4. $\sin x \sin(\cos x)$

Ans. $\cos(\cos x) + C$

Sol. $\sin x \cdot \sin(\cos x)$

Let $\cos x = t$

$\therefore -\sin x dx = dt$

$\Rightarrow \int \sin x \cdot \sin(\cos x) dx = - \int \sin t dt$

$= -[-\cos t] + C = \cos t + C = \cos(\cos x) + C$

5. $\sin(ax+b)\cos(ax+b)$

Ans. $-\frac{1}{4a} \cos 2(ax+b) + C$

Sol. $\sin(ax+b)\cos(ax+b) = \frac{2\sin(ax+b)\cos(ax+b)}{2} = \frac{\sin 2(ax+b)}{2}$

Let $2(ax+b) = t$

$\therefore 2a dx = dt$

$= \frac{1}{4a} [-\cos t] + C$

$\Rightarrow \int \frac{\sin 2(ax+b)}{2} dx = \frac{1}{2} \int \frac{\sin t dt}{2a}$

$= \frac{-1}{4a} \cos 2(ax+b) + C$

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6. $\sqrt{ax + b}$

Ans. $\frac{2}{3a}(ax + b)^{\frac{3}{2}} + C$

Sol. Let $ax + b = t$

$\Rightarrow adx = dt \quad \therefore dx = \frac{1}{a}dt$

$\Rightarrow \int (ax + b)^{\frac{1}{2}} dx = \frac{1}{a} \int t^{\frac{1}{2}} dt = \frac{1}{a} \left(\frac{t^{\frac{3}{2}}}{\frac{3}{2}} \right) + C$

$= \frac{2}{3a}(ax + b)^{\frac{3}{2}} + C$

7. $x\sqrt{x + 2}$

Ans. $\frac{2}{5}(x + 2)^{\frac{5}{2}} - \frac{4}{3}(x + 2)^{\frac{3}{2}} + C$

Sol. Let $(x + 2) = t$

$\therefore dx = dt \quad \Rightarrow \int x\sqrt{x + 2} dx = \int (t - 2)\sqrt{t} dt$

$= \int \left(t^{\frac{3}{2}} - 2t^{\frac{1}{2}} \right) dt = \int t^{\frac{3}{2}} dt - 2 \int t^{\frac{1}{2}} dt$

$= \frac{t^{\frac{5}{2}}}{\frac{5}{2}} - 2 \left(\frac{t^{\frac{3}{2}}}{\frac{3}{2}} \right) + C = \frac{2}{5}t^{\frac{5}{2}} - \frac{4}{3}t^{\frac{3}{2}} + C$

$= \frac{2}{5}(x + 2)^{\frac{5}{2}} - \frac{4}{3}(x + 2)^{\frac{3}{2}} + C$

8. $x\sqrt{1 + 2x^2}$

Ans. $\frac{1}{6}(1 + 2x^2)^{\frac{3}{2}} + C$

Sol. Let $1 + 2x^2 = t$

$\therefore 4xdx = dt$

$\Rightarrow \int x\sqrt{1 + 2x^2} dx = \int \frac{\sqrt{t} dt}{4}$

$= \frac{1}{4} \int t^{\frac{1}{2}} dt = \frac{1}{4} \left(\frac{t^{\frac{3}{2}}}{\frac{3}{2}} \right) + C$

$= \frac{1}{6}(1 + 2x^2)^{\frac{3}{2}} + C$

9. $(4x + 2)\sqrt{x^2 + x + 1}$

Ans. $\frac{4}{3}(x^2 + x + 1)^{\frac{3}{2}} + C$

Sol. Let $x^2 + x + 1 = t$

$\therefore (2x + 1)dx = dt$

$\int (4x + 2)\sqrt{x^2 + x + 1} dx$

$= \int 2\sqrt{t} dt = 2 \int \sqrt{t} dt$

$= 2 \left(\frac{t^{\frac{3}{2}}}{\frac{3}{2}} \right) + C = \frac{4}{3}(x^2 + x + 1)^{\frac{3}{2}} + C$

10. $\frac{1}{x - \sqrt{x}}$

Ans. $2\log|\sqrt{x} - 1| + C$

Sol. $\frac{1}{x - \sqrt{x}} = \frac{1}{\sqrt{x}(\sqrt{x} - 1)}$

Let $(\sqrt{x} - 1) = t$

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$$\frac{1}{2\sqrt{x}} dx = dt$$

$$\Rightarrow \int \frac{1}{\sqrt{x}(\sqrt{x}-1)} dx = \int \frac{2}{t} dt$$

$$= 2 \log |\sqrt{x} - 1| + C$$

$$\therefore \frac{1}{2\sqrt{x}} dx = dt$$

$$= 2 \log |t| + C$$

11. $\frac{x}{\sqrt{x+4}}, x > 0$

Ans. $\frac{2}{3}\sqrt{x+4}(x-8) + C$

Sol. Let $x+4 = t$

$$\therefore dx = dt$$

$$\int \frac{x}{\sqrt{x+4}} dx = \int \frac{(t-4)}{\sqrt{t}} dt$$

$$= \int \left(\sqrt{t} - \frac{4}{\sqrt{t}} \right) dt = \frac{t^{\frac{3}{2}}}{\frac{3}{2}} - 4 \left(\frac{t^{\frac{1}{2}}}{\frac{1}{2}} \right) + C$$

$$= \frac{2}{3}(t)^{\frac{3}{2}} - 8(t)^{\frac{1}{2}} + C = \frac{2}{3}t \cdot t^{\frac{1}{2}} - 8t^{\frac{1}{2}} + C$$

$$= \frac{2}{3}t^{\frac{1}{2}}(t-12) + C = \frac{2}{3}(x+4)^{\frac{1}{2}}(x+4-12) + C$$

$$= \frac{2}{3}\sqrt{x+4}(x-8) + C$$

12. $(x^3 - 1)^{\frac{1}{3}} x^5$

Ans. $\frac{1}{7}(x^3 - 1)^{\frac{7}{3}} + \frac{1}{4}(x^3 - 1)^{\frac{4}{3}} + C$

Sol. Let $x^3 - 1 = t$

$$\therefore 3x^2 dx = dt$$

$$\Rightarrow \int (x^3 - 1)^{\frac{1}{3}} x^5 dx = \int (x^3 - 1)^{\frac{1}{3}} x^3 \cdot x^2 dx$$

$$= \int t^{\frac{1}{3}}(t+1) \frac{dt}{3} = \frac{1}{3} \int \left(t^{\frac{4}{3}} + t^{\frac{1}{3}} \right) dt$$

$$= \frac{1}{3} \left[\frac{t^{\frac{7}{3}}}{\frac{7}{3}} + \frac{t^{\frac{4}{3}}}{\frac{4}{3}} \right] + C = \frac{1}{3} \left[\frac{3}{7} t^{\frac{7}{3}} + \frac{3}{4} t^{\frac{4}{3}} \right] + C$$

$$= \frac{1}{7}(x^3 - 1)^{\frac{7}{3}} + \frac{1}{4}(x^3 - 1)^{\frac{4}{3}} + C$$

13. $\frac{x^2}{(2+3x^3)^3}$

Ans. $-\frac{1}{18(2+3x^3)^2} + C$

Sol. Let $2+3x^3 = t$

$$\therefore 9x^2 dx = dt$$

$$\Rightarrow \int \frac{x^2}{(2+3x^3)^3} dx = \frac{1}{9} \int \frac{dt}{(t)^3}$$

$$= \frac{1}{9} \left[\frac{t^{-2}}{-2} \right] + C = \frac{-1}{18} \left(\frac{1}{t^2} \right) + C$$

$$= \frac{-1}{18(2+3x^3)^2} + C$$

14. $\frac{1}{x(\log x)^m}, x > 0, m \neq 1$

Ans. $\frac{(\log x)^{1-m}}{1-m} + C$

Sol. Let $\log x = t$ $\therefore \frac{1}{x} dx = dt$

$$\Rightarrow \int \frac{1}{x(\log x)^m} dx = \int \frac{dt}{(t)^m}$$

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$$= \left(\frac{t^{-m+1}}{1-m} \right) + C$$

$$= \frac{(\log x)^{1-m}}{(1-m)} + C$$

15. $\frac{x}{9-4x^2}$

Ans. $-\frac{1}{8} \log |9 - 4x^2| + C$

Sol. Let $9 - 4x^2 = t$

$$\therefore -8x dx = dt \Rightarrow \int \frac{x}{9-4x^2} dx = \frac{-1}{8} \int \frac{1}{t} dt$$

$$= \frac{-1}{8} \log |t| + C = \frac{-1}{8} \log |9 - 4x^2| + C$$

16. e^{2x+3}

Ans. $\frac{1}{2} e^{2x+3} + C$

Sol. Let $2x + 3 = t$

$$\therefore 2dx = dt$$

$$\Rightarrow \int e^{2x+3} dx$$

$$= \frac{1}{2} \int e^t dt$$

$$= \frac{1}{2} (e^t) + C$$

$$= \frac{1}{2} e^{(2x+3)} + C$$

17. $\frac{x}{e^{x^2}}$

Ans. $-\frac{1}{2e^{x^2}} + C$

Sol. Let $x^2 = t$

$$\therefore 2x dx = dt$$

$$\Rightarrow \int \frac{x}{e^{x^2}} dx = \frac{1}{2} \int \frac{1}{e^t} dt$$

$$= \frac{1}{2} \int e^{-t} dt$$

$$= \frac{1}{2} \left(\frac{e^{-t}}{-1} \right) + C$$

$$= -\frac{1}{2} e^{-x^2} + C$$

$$= \frac{-1}{2e^{x^2}} + C$$

18. $\frac{e^{\tan^{-1}x}}{1+x^2}$

Ans. $e^{\tan^{-1}x} + C$

Sol. Let $\tan^{-1} x = t$

$$\therefore \frac{1}{1+x^2} dx = dt$$

$$\Rightarrow \int \frac{e^{\tan^{-1}x}}{1+x^2} dx = \int e^t dt$$

$$= e^t + C$$

$$= e^{\tan^{-1}x} + C$$

19. $\frac{e^{2x}-1}{e^{2x}+1}$

Ans. $\log(e^x + e^{-x}) + C$

Sol. $\frac{e^{2x}-1}{e^{2x}+1}$

Dividing numerator and denominator by e^x , we obtain

$$\frac{\frac{(e^{2x}-1)}{e^x}}{\frac{(e^{2x}+1)}{e^x}} = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

$$\text{Let } e^x + e^{-x} = t \quad \therefore (e^x - e^{-x})dx = dt$$

$$\Rightarrow \int \frac{e^{2x}-1}{e^{2x}+1} dx = \int \frac{e^x - e^{-x}}{e^x + e^{-x}} dx$$

$$= \int \frac{dt}{t} = \log |t| + C$$

$$= \log |e^x + e^{-x}| + C$$

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20. $\frac{e^{2x}-e^{-2x}}{e^{2x}+e^{-2x}}$

Ans. $\frac{1}{2} \log(e^{2x} + e^{-2x}) + C$

Sol. Let $e^{2x} + e^{-2x} = t \quad \therefore (2e^{2x} - 2e^{-2x})dx = dt$
 $\Rightarrow 2(e^{2x} - e^{-2x})dx = dt$
 $\Rightarrow \int \left(\frac{e^{2x}-e^{-2x}}{e^{2x}+e^{-2x}} \right) dx = \int \frac{dt}{2t} = \frac{1}{2} \int \frac{1}{t} dt$
 $= \frac{1}{2} \log |t| + C = \frac{1}{2} \log |e^{2x} + e^{-2x}| + C$

21. $\tan^2(2x - 3)$

Ans. $\frac{1}{2} \tan(2x - 3) - x + C$

Sol. $\tan^2(2x - 3) = \sec^2(2x - 3) - 1$
 Let $2x - 3 = t \quad \therefore 2dx = dt$
 $\Rightarrow \int \tan^2(2x - 3)dx = \int [(\sec^2(2x - 3)) - 1]dx$
 $= \frac{1}{2} \int (\sec^2 t)dt - \int 1dx = \frac{1}{2} \int \sec^2 t dt - \int 1dx$
 $= \frac{1}{2} \tan t - x + C = \frac{1}{2} \tan(2x - 3) - x + C$

22. $\sec^2(7 - 4x)$

Ans. $-\frac{1}{4} \tan(7 - 4x) + C$

Sol. Let $7 - 4x = t$
 $\therefore -4dx = dt$
 $\therefore \int \sec^2(7 - 4x)dx = \frac{-1}{4} \int \sec^2 t dt$
 $= \frac{-1}{4} (\tan t) + C = \frac{-1}{4} \tan(7 - 4x) + C$

23. $\frac{\sin^{-1}x}{\sqrt{1-x^2}}$

Ans. $\frac{1}{2} (\sin^{-1}x)^2 + C$

Sol. Let $\sin^{-1} x = t$
 $\therefore \frac{1}{\sqrt{1-x^2}} dx = dt \Rightarrow \int \frac{\sin^{-1} x}{\sqrt{1-x^2}} dx = \int t dt$
 $= \frac{t^2}{2} + C = \frac{(\sin^{-1} x)^2}{2} + C$

24. $\frac{2\cos x - 3\sin x}{6\cos x + 4\sin x}$

Ans. $\frac{1}{2} \log|2\sin x + 3\cos x| + C$

Sol. $\frac{2\cos x - 3\sin x}{6\cos x + 4\sin x} = \frac{2\cos x - 3\sin x}{2(3\cos x + 2\sin x)}$
 Let $3\cos x + 2\sin x = t$
 $\therefore (-3\sin x + 2\cos x)dx = dt$
 $\int \frac{2\cos x - 3\sin x}{6\cos x + 4\sin x} dx = \int \frac{dt}{2t}$
 $= \frac{1}{2} \int \frac{1}{t} dt = \frac{1}{2} \log |t| + C$
 $= \frac{1}{2} \log |2\sin x + 3\cos x| + C$

25. $\frac{1}{\cos^2 x (1 - \tan x)^2}$

Ans. $\frac{1}{(1 - \tan x)} + C$

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Sol. $\frac{1}{\cos^2 x(1-\tan x)^2} = \frac{\sec^2 x}{(1-\tan x)^2}$
 Let $(1 - \tan x) = t$
 $\therefore -\sec^2 x dx = dt$
 $\Rightarrow \int \frac{\sec^2 x}{(1-\tan x)^2} dx = \int \frac{-dt}{t^2}$
 $= -\int t^{-2} dt = +\frac{1}{t} + C$
 $= \frac{1}{(1-\tan x)} + C$

26. $\frac{\cos\sqrt{x}}{\sqrt{x}}$

Ans. $2\sin\sqrt{x} + C$

Sol. Let $\sqrt{x} = t$ $\therefore \frac{1}{2\sqrt{x}} dx = dt$
 $\Rightarrow \int \frac{\cos\sqrt{x}}{\sqrt{x}} dx = 2 \int \cos t dt$
 $= 2\sin t + C = 2\sin\sqrt{x} + C$

27. $\sqrt{\sin 2x} \cos 2x$

Ans. $\frac{1}{3}(\sin 2x)^{\frac{3}{2}} + C$

Sol. Let $\sin 2x = t$
 $\therefore 2\cos 2x dx = dt$
 $\Rightarrow \int \sqrt{\sin 2x} \cos 2x dx = \frac{1}{2} \int \sqrt{t} dt$
 $= \frac{1}{2} \left(\frac{t^{\frac{3}{2}}}{\frac{3}{2}} \right) + C = \frac{1}{3} t^{\frac{3}{2}} + C$
 $= \frac{1}{3} (\sin 2x)^{\frac{3}{2}} + C$

28. $\frac{\cos x}{\sqrt{1+\sin x}}$

Ans. $2\sqrt{1+\sin x} + C$

Sol. Let $1 + \sin x = t$
 $\therefore \cos x dx = dt$
 $\Rightarrow \int \frac{\cos x}{\sqrt{1+\sin x}} dx = \int \frac{dt}{\sqrt{t}}$
 $= \frac{t^{\frac{1}{2}}}{\frac{1}{2}} + C = 2\sqrt{t} + C$
 $= 2\sqrt{1+\sin x} + C$

29. $\cot x \log \sin x$

Ans. $\frac{1}{2}(\log \sin x)^2 + C$

Sol. Let $\log \sin x = t$
 $\Rightarrow \frac{1}{\sin x} \cdot \cos x dx = dt \quad \therefore \cot x dx = dt$
 $\Rightarrow \int \cot x \log \sin x dx = \int t dt$
 $\therefore = \frac{t^2}{2} + C = \frac{1}{2}(\log \sin x)^2 + C$

30. $\frac{\sin x}{1+\cos x}$

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Ans. $-\log|1 + \cos x| + C$

Sol. Let $1 + \cos x = t$

$$\therefore -\sin x dx = dt$$

$$\Rightarrow \int \frac{\sin x}{1 + \cos x} dx = \int -\frac{dt}{t}$$

$$= -\log |t| + C = -\log |1 + \cos x| + C$$

31. $\frac{\sin x}{(1 + \cos x)^2}$

Ans. $\frac{1}{1 + \cos x} + C$

Sol. Let $1 + \cos x = t$

$$\therefore -\sin x dx = dt$$

$$\Rightarrow \int \frac{\sin x}{(1 + \cos x)^2} dx = \int -\frac{dt}{t^2}$$

$$= -\int t^{-2} dt = \frac{1}{t} + C$$

$$= \frac{1}{1 + \cos x} + C$$

32. $\frac{1}{1 + \cot x}$

Ans. $\frac{x}{2} - \frac{1}{2} \log |\cos x + \sin x| + C$

Sol. $I = \int \frac{1}{1 + \cot x} dx$

$$= \int \frac{1}{1 + \frac{\cos x}{\sin x}} dx = \int \frac{\sin x}{\sin x + \cos x} dx$$

$$= \frac{1}{2} \int \frac{2 \sin x}{\sin x + \cos x} dx$$

$$\text{Let } = \frac{1}{2} \int \frac{(\sin x + \cos x) + (\sin x - \cos x)}{(\sin x + \cos x)} dx$$

$$= \frac{1}{2} \int 1 dx + \frac{1}{2} \int \frac{\sin x - \cos x}{\sin x + \cos x} dx$$

$$= \frac{1}{2} (x) + \frac{1}{2} \int \frac{\sin x - \cos x}{\sin x + \cos x} dx$$

$$\text{Let } \sin x + \cos x = t \Rightarrow (\cos x - \sin x) dx = dt$$

$$\therefore I = \frac{x}{2} + \frac{1}{2} \int \frac{-(dt)}{t} = \frac{x}{2} - \frac{1}{2} \log |t| + C$$

$$= \frac{x}{2} - \frac{1}{2} \log |\sin x + \cos x| + C$$

33. $\frac{1}{1 - \tan x}$

Ans. $\frac{x}{2} - \frac{1}{2} \log |\cos x - \sin x| + C$

Sol. Let $I = \int \frac{1}{1 - \tan x} dx$

$$= \int \frac{1}{1 - \frac{\sin x}{\cos x}} dx = \int \frac{\cos x}{\cos x - \sin x} dx$$

$$= \frac{1}{2} \int \frac{2 \cos x}{\cos x - \sin x} dx$$

$$= \frac{1}{2} \int \frac{(\cos x - \sin x) + (\cos x + \sin x)}{(\cos x - \sin x)} dx$$

$$= \frac{1}{2} \int 1 dx + \frac{1}{2} \int \frac{\cos x + \sin x}{\cos x - \sin x} dx$$

$$= \frac{x}{2} + \frac{1}{2} \int \frac{\cos x + \sin x}{\cos x - \sin x} dx$$

$$\text{Put } \cos x - \sin x = t \Rightarrow (-\sin x - \cos x) dx = dt$$

$$\therefore I = \frac{x}{2} + \frac{1}{2} \int \frac{-(dt)}{t}$$

$$= \frac{x}{2} - \frac{1}{2} \log |t| + C = \frac{x}{2} - \frac{1}{2} \log |\cos x - \sin x| + C$$

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34. $\frac{\sqrt{\tan x}}{\sin x \cos x}$

Ans. $2\sqrt{\tan x} + C$

Sol. Let $I = \int \frac{\sqrt{\tan x}}{\sin x \cos x} dx$
 $= \int \frac{\sqrt{\tan x} \times \cos x}{\sin x \cos x \times \cos x} dx = \int \frac{\sqrt{\tan x}}{\tan x \cos^2 x} dx$
 $= \int \frac{\sec^2 x dx}{\sqrt{\tan x}}$

Let $\tan x = t \Rightarrow \sec^2 x dx = dt$

$\therefore I = \int \frac{dt}{\sqrt{t}}$
 $= 2\sqrt{t} + C = 2\sqrt{\tan x} + C$

35. $\frac{(1+\log x)^2}{x}$

Ans. $\frac{1}{3}(1 + \log x)^3 + C$

Sol. Let $1 + \log x = t$
 $\therefore \frac{1}{x} dx = dt \Rightarrow \int \frac{(1+\log x)^2}{x} dx = \int t^2 dt$
 $= \frac{t^3}{3} + C = \frac{(1+\log x)^3}{3} + C$

36. $\frac{(x+1)(x+\log x)^2}{x}$

Ans. $\frac{1}{3}(x + \log x)^3 + C$

Sol. $\frac{(x+1)(x+\log x)^2}{x} = \left(\frac{x+1}{x}\right)(x + \log x)^2 = \left(1 + \frac{1}{x}\right)(x + \log x)^2$
 Let $(x + \log x) = t$
 $\therefore \left(1 + \frac{1}{x}\right) dx = dt$
 $\Rightarrow \int \left(1 + \frac{1}{x}\right)(x + \log x)^2 dx = \int t^2 dt$
 $= \frac{t^3}{3} + C = \frac{1}{3}(x + \log x)^3 + C$

37. $\frac{x^3 \sin(\tan^{-1} x^4)}{1+x^8}$

Ans. $-\frac{1}{4} \cos(\tan^{-1} x^4) + C$

Sol. Let $x^4 = t \Rightarrow 4x^3 dx = dt$
 $\Rightarrow \int \frac{x^3 \sin(\tan^{-1} x^4)}{1+x^8} dx = \frac{1}{4} \int \frac{\sin(\tan^{-1} t)}{1+t^2} dt \dots (i)$
 Let $\tan^{-1} t = u \Rightarrow \frac{1}{1+t^2} dt = du$

From (i), we obtain

$\frac{x^3 \sin(\tan^{-1} x^4) dx}{1+x^8} = \frac{1}{4} \int \sin u du$
 $= \frac{1}{4} (-\cos u) + C = \frac{-1}{4} \cos(\tan^{-1} t) + C$
 $= \frac{-1}{4} \cos(\tan^{-1} x^4) + C$

38. $\int \frac{10x^9 + 10^x \log_e 10 dx}{x^{10} + 10^x}$ equals

(A) $10^x - x^{10} + C$

(B) $10^x + x^{10} + C$

(C) $(10^x - x^{10})^{-1} + C$

(D) $\log(10^x + x^{10}) + C$

Ans. D

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Sol. Let $x^{10} + 10^x = t$ $\therefore (10x^9 + 10^x \log_e 10)dx = dt$
 $\Rightarrow \int \frac{10x^9 + 10^x \log_e 10}{x^{10} + 10^x} dx = \int \frac{dt}{t} = \log t + C$
 $= \log (10^x + x^{10}) + C$
Hence, the correct Answer is D.

39. $\int \frac{dx}{\sin^2 x \cos^2 x}$ equals
(A) $\tan x + \cot x + C$ (B) $\tan x - \cot x + C$
(C) $\tan x \cot x + C$ (D) $\tan x - \cot 2x + C$

Ans. B

Sol. $I = \int \frac{dx}{\sin^2 x \cos^2 x} = \int \frac{1}{\sin^2 x \cos^2 x} dx$
 $= \int \frac{\sin^2 x + \cos^2 x}{\sin^2 x \cos^2 x} dx$
Let $= \int \frac{\sin^2 x}{\sin^2 x \cos^2 x} dx + \int \frac{\cos^2 x}{\sin^2 x \cos^2 x} dx$
 $= \int \sec^2 x dx + \int \operatorname{cosec}^2 x dx$
 $= \tan x - \cot x + C$
Hence, the correct Answer is B.

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Answer Key

1. $\log(1 + x^2) + C$
2. $\frac{1}{3}(\log|x|)^3 + C$
3. $\log|1 + \log x| + C$
4. $\cos(\cos x) + C$
5. $-\frac{1}{4a}\cos 2(ax + b) + C$
6. $\frac{2}{3a}(ax + b)^{\frac{3}{2}} + C$
7. $\frac{2}{5}(x + 2)^{\frac{5}{2}} - \frac{4}{3}(x + 2)^{\frac{3}{2}} + C$
8. $\frac{1}{6}(1 + 2x^2)^{\frac{3}{2}} + C$
9. $\frac{4}{3}(x^2 + x + 1)^{\frac{3}{2}} + C$
10. $2\log|\sqrt{x} - 1| + C$
11. $\frac{2}{3}\sqrt{x + 4}(x - 8) + C$
12. $\frac{1}{7}(x^3 - 1)^{\frac{7}{3}} + \frac{1}{4}(x^3 - 1)^{\frac{4}{3}} + C$
13. $-\frac{1}{18(2 + 3x^3)^2} + C$
14. $\frac{(\log x)^{1-m}}{1-m} + C$
15. $-\frac{1}{8}\log|9 - 4x^2| + C$
16. $\frac{1}{2}e^{2x+3} + C$
17. $-\frac{1}{2e^{x^2}} + C$
18. $e^{\tan^{-1}x} + C$
19. $\log(e^x + e^{-x}) + C$
20. $\frac{1}{2}\log(e^{2x} + e^{-2x}) + C$
21. $\frac{1}{2}\tan(2x - 3) - x + C$
22. $-\frac{1}{4}\tan(7 - 4x) + C$
23. $\frac{1}{2}(\sin^{-1}x)^2 + C$
24. $\frac{1}{2}\log|2\sin x + 3\cos x| + C$
25. $\frac{1}{(1 - \tan x)} + C$
26. $2\sin\sqrt{x} + C$
27. $\frac{1}{3}(\sin 2x)^{\frac{3}{2}} + C$
28. $2\sqrt{1 + \sin x} + C$
29. $\frac{1}{2}(\log \sin x)^2 + C$
30. $-\log|1 + \cos x| + C$
31. $\frac{1}{1 + \cos x} + C$
32. $\frac{x}{2} - \frac{1}{2}\log|\cos x + \sin x| + C$
33. $\frac{x}{2} - \frac{1}{2}\log|\cos x - \sin x| + C$
34. $2\sqrt{\tan x} + C$
35. $\frac{1}{3}(1 + \log x)^3 + C$
36. $\frac{1}{3}(x + \log x)^3 + C$
37. $-\frac{1}{4}\cos(\tan^{-1}x^4) + C$
38. D
39. B