Nishant Jindal

MOD discussion HW2

$$\sqrt{1-x^6}+\sqrt{1-y^6}=a(x^3-y^3)$$

Q5

$$\sqrt{1-x^2}+\sqrt{1-y^2}=a(x-y) \text{ then } \frac{dy}{dx}.$$

$$x=\sin\theta,\ y=\sin\phi\ ;\quad \theta=\sin^{-1}x,\ \phi=\sin^{-1}y.$$

$$\cos\theta+\cos\phi=a(\sin\theta-\sin\phi)$$

$$\cancel{2}\cancel{\cos\left(\frac{\theta+\phi}{2}\right)}\cdot\cos\left(\frac{\theta-\phi}{2}\right)=a\left(\cancel{2}\cancel{\cos\left(\frac{\theta+\phi}{2}\right)}\cdot\sin\left(\frac{\theta-\phi}{2}\right)\right)$$

$$\cot\left(\frac{\theta-\phi}{2}\right)=a$$

$$\frac{\theta-\phi}{2}=\cot^{-1}a \Rightarrow \theta-\phi=2\cot^{-1}a$$

$$\Rightarrow \sin^{-1}x-\sin^{-1}y=2\cot^{-1}a$$

$$\Rightarrow \frac{1}{\sqrt{1-x^2}}-\frac{1}{\sqrt{1-y^2}}\cdot\frac{dy}{dx}=0 \Rightarrow \frac{dy}{dx}=\frac{\sqrt{1-y^2}}{\sqrt{1-x^2}}$$

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10) $y = (1+x)(1+x^2)(1+x^4)\cdots(1+x^{2n})$ $\left.\dfrac{dy}{dx}\right|_{x=0}$.

$(1-x^2)$ $(1-x^4)$ $(1-x^8)$ $\rightarrow (1-x^{4n})$

$$y = \frac{(1-x)(1+x)(1+x^2)(1+x^4)\cdots(1+x^{2n})}{(1-x)}$$

$$y = \frac{(1-x^{4n})}{1-x}$$

(16) Copy done

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Diffⁿ of Determinants.

Let $\Delta(x) = \begin{vmatrix} f(x) & g(x) & h(x) \\ p(x) & q(x) & r(x) \\ m(x), & n(x) & t(x) \end{vmatrix}$

diffⁿ Row wise / Col. wise

$$\Delta'(x) = \begin{vmatrix} f' & g' & h' \\ p & q & r \\ m & n & t \end{vmatrix} + \begin{vmatrix} f & g & h \\ p' & q' & r' \\ m & n & t \end{vmatrix} + \begin{vmatrix} f & g & h \\ p & q & r \\ m' & n' & t' \end{vmatrix}$$

Q $Y = \begin{vmatrix} 3 & 6x^2 & 1 \\ 2 & 2x^3 & a \\ 1 & x^4 & a^2 \end{vmatrix}$ then $f''(a) = ?$

Col. wise

$$\frac{dy}{dx} = \begin{vmatrix} 0 & & \\ 0 & & \\ 0 & & \end{vmatrix} + \begin{vmatrix} 3 & 12x & 1 \\ 2 & 6x^2 & a \\ 1 & 4x^3 & a^2 \end{vmatrix} + \begin{vmatrix} & & 0 \\ & & 0 \\ & & 0 \end{vmatrix}$$

$$\frac{d^2y}{dx^2} = \begin{vmatrix} 0 & & \\ 0 & & \\ 0 & & \end{vmatrix} + \begin{vmatrix} 3 & 12 & 1 \\ 2 & 12x & a \\ 1 & 12x^2 & a^2 \end{vmatrix} + \begin{vmatrix} & & 0 \\ & & 0 \\ & & 0 \end{vmatrix}$$

$$\left.\frac{d^2y}{dx^2}\right|_{x=a} = 12 \begin{vmatrix} 3 & 1 & 1 \\ 2 & a & a \\ 1 & a^2 & a^2 \end{vmatrix} = 0$$

Q If $f(x) = \begin{vmatrix} \cos(x+x^2) & \sin(x+x^2) & -\cos(x+x^2) \\ \sin(x-x^2) & \cos(x-x^2) & \sin(x-x^2) \\ \sin 2x & 0 & \sin 2x^2 \end{vmatrix}$

$f'(x) = ?$

$x + x^2 = A$

$x - x^2 = B$

$2x = A + B$

$2x^2 = A - B$

$$f(x) = \begin{vmatrix} \cos A & \sin A & -\cos A \\ \sin B & \cos B & \sin B \\ \sin(A+B) & 0 & \sin(A-B) \end{vmatrix}$$

= opening it from 3rd Row

$$= \sin(\underbrace{A+B}_{X}) \cos(\underbrace{A-B}_{Y}) - 0 + \sin(\underbrace{A-B}_{Y}) \cos(\underbrace{A+B}_{X})$$

$$= \sin(X+Y) = \sin(A+\cancel{B}+A-\cancel{B}) = \sin 2A$$

$$f(x) = \sin(2x+2x^2) \Rightarrow f'(x) = \cos(2x+2x^2) \times (2+4x)$$

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Q 17, 18

Q23 $Y = \sin^{-1}\left(\frac{2x}{1+x^2}\right) = 2\tan^{-1}x$ for $-1 < x < 1$ (Normally)

$\left.\frac{dy}{dx}\right|_{x=-2}$

$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$\left(-\frac{\pi}{4}, \frac{\pi}{4}\right)$

$x \in \left(\tan -\frac{\pi}{4}, \tan\frac{\pi}{4}\right)$

$x \in (-1, 1)$

$Y = \sin^{-1}\left(\frac{2x}{1+x^2}\right) = \begin{cases} -\pi - 2\tan^{-1}x & x \le -1 \\ 2\tan^{-1}x & -1 < x < 1 \\ \pi - 2\tan^{-1}x & x \ge 1 \end{cases}$ $(x-2)$

at $x = -2$ $\sin(\sin^{-1}x) = x$

$Y = \sin^{-1}\left(\frac{2x}{1+x^2}\right) = -\pi - 2\tan^{-1}x$

$\frac{dy}{dx} = -\frac{2}{1+x^2}$

25) $\frac{d}{dx}\left(\sin\left(\cot^{-1}\frac{1}{\sqrt{\frac{1+x}{1-x}}}\right)\right)^2$

$\left(\sin\left(\cot^{-1}\sqrt{\frac{1-x}{1+x}}\right)\right)^2 \to \frac{B}{P}$

$\frac{P}{H}$

$\frac{d}{dx}\left(\sin\left(\sin^{-1}\left(\sqrt{\frac{1+x}{2}}\right)\right)\right)^2$

$\frac{d}{dx}\left(\frac{1}{2} + \frac{x}{2}\right) = \frac{1}{2}$

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24) $$\frac{d}{dx}\left[\tan^{-1}\left(\frac{\sqrt{x^2+a^2}+x}{\sqrt{x^2+a^2}-x}\right)^{1/2}\right]$$

$$x = a\tan\theta \Rightarrow \tan\theta = \frac{x}{a} \Rightarrow \theta = \tan^{-1}\left(\frac{x}{a}\right)$$

$$\tan^{-1}\left(\frac{\sqrt{a^2\tan^2\theta+a^2}+a\tan\theta}{\sqrt{a^2\tan^2\theta+a^2}-a\tan\theta}\right)^{1/2}$$

Best Q 26

$$\tan^{-1}\left(\frac{\sec\theta+\tan\theta}{\sec\theta-\tan\theta}\right)^{1/2} = \tan^{-1}\left(\frac{1+\sin\theta}{1-\sin\theta}\right)^{1/2} = \tan^{-1}\left(\frac{(\cos\theta/2+\sin\theta/2)^{\cancel{2}}}{(\cos\frac{\theta}{2}-\sin\frac{\theta}{2})^{\cancel{2}}}\right)^{1/2}$$

Bnana hai

$$= \tan^{-1}\left(\frac{1+\tan\frac{\theta}{2}}{1-\tan\frac{\theta}{2}}\right) = \tan^{-1}\left(\tan\left(\frac{\pi}{4}+\frac{\theta}{2}\right)\right) = \frac{d}{dx}\left(\frac{\pi}{4}+\frac{\theta}{2}\right) = \frac{d}{dx}\left(\frac{\pi}{4}+\frac{1}{2}\tan^{-1}\frac{x}{a}\right)$$

$$= 0 + \frac{1}{2}\times\frac{1}{1+\left(\frac{x}{a}\right)^2}\times\frac{1}{a}$$

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30) $f(\theta) = \sin\left(\tan^{-1}\left(\frac{\sin\theta}{\sqrt{\cos 2\theta}}\right)\right) \begin{matrix}\to P \\ \to B\end{matrix}$ $\quad \frac{d\, f(\theta)}{d(\tan\theta)}$?

Sinθ  $\sqrt{\sin^2\theta + \cos 2\theta}$

$\sqrt{\cos 2\theta}$ $= \sqrt{\sin^2\theta + 1 - 2\sin^2\theta}$

$= \sqrt{1 - \sin^2\theta}$

$= \cos\theta$

$$\frac{d\left(\sin\left(\sin^{-1}\frac{\sin\theta}{\cos\theta}\right)\right)}{d(\tan\theta)} = \frac{d(\tan\theta)}{d(\tan\theta)} = 1$$

HW3 (Pending)

Q3) of $\sin^{-1}x$ WRT $3\cdot\sin^{-1}(3x-4x^3)$

$\sin^{-1}(3x-4x^3) = 3\sin^{-1}x \to -\frac{1}{2} \le x \le \frac{1}{2}$

$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$\left[-\frac{\pi}{6}, \frac{\pi}{6}\right]$

$\left[\sin\left(-\frac{\pi}{6}\right), \sin\frac{\pi}{6}\right]$

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Diff^n of Inverse fxn.

Q If $g(x)$ is Inverse of $f(x)$ then $g'(x)$?

$$\boxed{g(x) = f^{-1}(x)} \longrightarrow \boxed{f(x) = g^{-1}(x)}$$

$f(g(x)) = x$

$f'(g(x)) \times g'(x) = 1$

$$\boxed{f'(g(x)) = \frac{1}{g'(x)}}$$

$g(f(x)) = x$

$g'(f(x)) \times f'(x) = 1$

$$\boxed{g'(f(x)) = \frac{1}{f'(x)}}$$

Q If $f(x) = e^{x^3+x^2+x}$ & $g(x)$ is Inverse fxn of $f(x)$ then $g'(e^3) = ?$

① $f(x)$ me x ki jgh kya Rakhne Par e^3 aayega?

$f(x) = e^{x^3+x^2+x}$

$f(1) = e^{1+1+1} = e^3$

② Demand

$= g'(e^3)$

$= g'(f(1))$ manga

$f'(x) = e^{x^3+x^2+x} \times (3x^2+2x+1)$

$f'(1) = e^3 \times (3+2+1)$

$= 6e^3$

$$\boxed{\frac{1}{f'(1)} = \frac{1}{6e^3}}$$

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Q If $f(x) = e^{\frac{x}{2} + x^3}$ & $g(x) = f^{-1}(x)$ then $g'(1) = ?$

Demand $= g'(1)$

$= g'(f(0))$

manga

$= \frac{1}{f'(0)} = \frac{1}{1/2} = 2$

(f(x) में x Ki Jgh Kya Rakhe Par 1 Aayega)

$f(x) = e^{\frac{x}{2} + x^3}$

$f(0) = e^{0 + 0^3} = e^0 = 1$

$f'(x) = e^{\left(\frac{x}{2} + x^3\right)} \times \left(\frac{1}{2} + 3x^2\right)$

$f'(0) = e^0 * \left(\frac{1}{2} + 3 \times 0\right) = \frac{1}{2}$

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Q If $f(x)=\sin^{-1}\{\cancel{[3x+2]}-\{3x+(x-\{2x\})\}\}$; $x\in(0,\frac{\pi}{12})$; $g\circ f(x)=x\ \forall x\in(0,\frac{\pi}{12})$

Integer Degn.

(HW3) find $g'(\frac{\pi}{6})=?$

1) $2x\in(0,\frac{\pi}{6})$

$\in(0,\frac{3.14}{6})\simeq(0,.5)$ → fractional No.

$\therefore \{2x\}=2x$

Kab.

$=\sin^{-1}\{-\{3x+(x-\{2x\}\}\}$

$=\sin^{-1}\{-\{3x+(x-2x)\}\}$

$=\sin^{-1}\{-\{2x\}\}$

$=\sin^{-1}\{-2x\}=\sin^{-1}(1-2x)$

$f(x)=\sin^{-1}(1-2x)$

$f'(x)=\frac{1}{\sqrt{1-(1-2x)^2}}\times(-2)$

$f'(\frac{1}{4})=\frac{-2}{\sqrt{1-(\frac{1}{2})^2}}=\frac{-4}{\sqrt{3}}$

② Demand $=g'(\frac{\pi}{6})$

$=g'(f(\frac{1}{4}))$

$=\frac{1}{f'(\frac{1}{4})}=\frac{-\sqrt{3}}{4}$

$\sin^{-1}(1-2x)=\frac{\pi}{6}$

$1-2x=\frac{1}{2}$

$\frac{1}{2}=2x$

$\boxed{x=\frac{1}{4}}$

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Back to Determinant Lecture.

Q4b $\begin{vmatrix} x^2 & x & x+1 \\ x & 1 & 1 \\ 2 & 1 & x \end{vmatrix} = \begin{vmatrix} x & 2 & x^2 \\ 1 & 1 & x \\ 1 & x & 4 \end{vmatrix}$ then $x = ?$

$\downarrow$ Transpose

$$\begin{vmatrix} x^2 & x & x+1 \\ x & 1 & 1 \\ 2 & 1 & x \end{vmatrix} = \begin{vmatrix} x & 1 & 1 \\ 2 & 1 & x \\ x^2 & x & 4 \end{vmatrix}$$

$$\begin{vmatrix} x^2 & x & x+1 \\ x & 1 & 1 \\ 2 & 1 & x \end{vmatrix} = \begin{vmatrix} x^2 & x & 4 \\ x & 1 & 1 \\ 2 & 1 & x \end{vmatrix} \Rightarrow$$

$$\Rightarrow \begin{vmatrix} x^2 & x & x+1 \\ x & 1 & 1 \\ 2 & 1 & x \end{vmatrix} - \begin{vmatrix} x^2 & x & 4 \\ x & 1 & 1 \\ 2 & 1 & x \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} 0 & 0 & (x-3) \\ x & 1 & 1 \\ 2 & 1 & x \end{vmatrix} = 0$$

$$(x-3)(x-2) = 0$$

$$\boxed{x = 2, 3}$$

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Q $a, b, c \in R_0$ then
$$\Delta = \begin{array}{c} \begin{vmatrix} b^2c^2 & bc & b+c \\ c^2a^2 & ca & c+a \\ a^2b^2 & ab & a+b \end{vmatrix} \end{array} \begin{array}{c} \leftarrow a \\ \leftarrow b \\ \leftarrow c. \end{array}$$

$$\frac{1}{abc}\begin{vmatrix} (abc)bc & abc & ab+ac \\ (abc)ca & abc & bc+ab \\ (abc)(ab) & abc & ac+bc \end{vmatrix}$$

$$\frac{(abc)^2}{abc}\begin{vmatrix} bc & 1 & ab+ac \\ ca & 1 & bc+ab \\ ab & 1 & ac+bc \end{vmatrix} = (abc)\begin{vmatrix} bc & 1 & ab+bc+ca \\ ca & 1 & ab+bc+ca \\ ab & 1 & ab+bc+ca \end{vmatrix}$$

$C_3 \to C_3 + C_1$
$$= (abc)(ab+bc+ca)\begin{vmatrix} bc & 1 & 1 \\ ca & 1 & 1 \\ ab & 1 & 1 \end{vmatrix} = 0$$

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Q $\begin{vmatrix} x^2+x & x+1 & x-2 \\ 2x^2+3x-1 & 3x & 3x-3 \\ x^2+2x+3 & 2x-1 & 2x-1 \end{vmatrix} = Px-12$ thus $P=?$

let $x=1$

$\begin{vmatrix} 2 & 2 & -1 \\ 4 & 3 & 0 \\ 6 & 1 & 1 \end{vmatrix} = P-12$

$(6+0+-4)-(-18+0+8)=P-12$

$2+10=P-12$

$P=24$

Q $\begin{vmatrix} a^2 & b^2 & c^2 \\ (a+\lambda)^2 & (b+\lambda)^2 & (c+\lambda)^2 \\ (a-\lambda)^2 & (b-\lambda)^2 & (c-\lambda)^2 \end{vmatrix} = K\lambda \begin{vmatrix} a^2 & b^2 & c^2 \\ a & b & c \\ 1 & 1 & 1 \end{vmatrix}$

$\lambda \neq 0,\ K=?$

$a=1,\ b=2,\ c=3,\ \lambda=1$

$\begin{vmatrix} 1 & 4 & 9 \\ 4 & 9 & 16 \\ 0 & 1 & 4 \end{vmatrix} = K \begin{vmatrix} 1 & 4 & 9 \\ 1 & 2 & 3 \\ 1 & 1 & 1 \end{vmatrix}$

$\boxed{K=4}$

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Some Special Determinants.

1) Symmetric Determinants

$$a_{ij} = a_{ji}$$

2) Skew. $\rightarrow$ $a_{ji} = -a_{ij}$

3) Skew symm determinant of odd order has value = 0

4) Symm det. of even order det has always sq. of same quantity.

Q If

$$\underbrace{ax^4 + bx^3 + cx^2 + dx}_{\downarrow} + e = \begin{vmatrix} x^3+3x & x-1 & x+3 \\ x+1 & -2x & x-4 \\ x-3 & x+4 & 3x \end{vmatrix}$$

then e = ?

Put $x = 0$

$$e = \begin{vmatrix} 0 & -1 & 3 \\ 1 & 0 & -4 \\ -3 & 4 & 0 \end{vmatrix} \rightarrow \text{Skew of order 3}$$

$$e = 0$$

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Summation Property

Use of Σ.

While applying Σ, all other elements except the row applied Σ, should remains constant

$$\Delta = \begin{vmatrix} f(r) & m & n \\ g(r) & p & q \\ h(r) & r & s \end{vmatrix} \quad \text{find } \sum_{r=1}^{n} \Delta(r) = \begin{vmatrix} \sum f(r) & m & n \\ \sum g(r) & p & q \\ \sum h(r) & r & s \end{vmatrix}$$

$$\sum_{r=0}^{n} 1 = 1+1+1+ - - - +1 = n+1 \quad \leftarrow (n+1) \text{ बार}$$

$$\sum_{r=0}^{n} r = 0+\underline{1+2+3 - - n} = \frac{(n)(n+1)}{2}$$

$$\sum_{r=0}^{n} (2r-1) = -1 + \underbrace{1+3+5+7 - - - (2n-1)} = n^2-1$$

Q $\Delta(r) = \begin{vmatrix} 1 & x & n+1 \\ r & y & \frac{(n)(n+1)}{2} \\ 2r-1 & z & n^2-1 \end{vmatrix}$ then $\sum_{r=0}^{n} \Delta(r) = ?$

$$\Delta(r) = \begin{vmatrix} \sum_{r=0}^{n} 1 & x & n+1 \\ \sum_{r=0}^{n} r & y & \frac{(n)(n+1)}{2} \\ \sum_{r=0}^{n} (2r-1) & z & n^2-1 \end{vmatrix}$$

$$= \begin{vmatrix} n+1 & x & n+1 \\ \frac{(n)(n+1)}{2} & y & \frac{(n)(n+1)}{2} \\ n^2-1 & z & n^2-1 \end{vmatrix} = 0$$

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① $\Delta_r = \begin{vmatrix} 2^{r-1} & 2\cdot 3^{r-1} & 4\cdot 5^{r-1} \\ \alpha & \beta & \gamma \\ 2^n-1 & 3^n-1 & 5^n-1 \end{vmatrix}$

$\sum_{r=1}^{n} \Delta_r = ?$

$\sum_{r=1}^{n} 2^{r-1} = \underbrace{2^0+2^1+2^2+\dots+2^{n-1}}_{\leftarrow n\text{ term} \rightarrow} = \frac{1\cdot(2^n-1)}{(2-1)} = 2^n-1$

$2\sum 3^{r-1} = 2\left[3^0+3^1+3^2+\dots+3^n\right] = \cancel{2}\left[\frac{1\cdot(3^n-1)}{\cancel{(3-1)}}\right] = 3^n-1$

$4\sum 5^{r-1} = 4\left[5^0 - - - \right] = \cancel{4}\left[\frac{5^n-1}{\cancel{(5-1)}}\right] = 5^n-1$

$\Delta = \begin{vmatrix} \sum 2^{r-1} & 2\sum 3^{r-1} & 4\sum 5^{r-1} \\ \alpha & \beta & \gamma \\ 2^n-1 & 3^n-1 & 5^n-1 \end{vmatrix}$

$= \begin{vmatrix} 2^n-1 & 3^n-1 & 5^n-1 \\ 2^n-1 & 3^n-1 & 5^n-1 \end{vmatrix} = 0$

Q $D_r = \begin{vmatrix} r & r-1 \\ r-1 & r \end{vmatrix}$ $\sum_{r=0}^{100} D_r = ?$

Q $\begin{vmatrix} r & \frac{(n)(n+1)}{2} & r-1 \\ 2r-1 & n^2 & 2r+1 \\ r^3 & \left(\frac{(n)(n+1)}{2}\right)^2 & r^3-1 \end{vmatrix}$ $\sum_{r=1}^{n} D_r$

CRammer Rule (Max^m Qs)