Solution of DPP - 1

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1. $q_1 = 2 \times 10^{-3}\,C \qquad q_2 = -3 \times 10^{-6}\,C$

$\bullet \longleftarrow x = 10\ cm \longrightarrow \bullet$

Nature:- Attractive [because charges are opposite in nature]

$$|F| = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2} = \frac{9 \times 10^9 \times 2 \times 10^{-3} \times 3 \times 10^{-6}}{(10^{-1})^2}$$

$$r = 10\ cm = 10^{-1}\ m = \frac{54 \times 10^9 \times 10^9}{10^{-2}} = 5400\ N$$

2. $(-1,1,1)m \qquad (3,1,-2)m$

$\bullet \qquad\qquad \bullet$

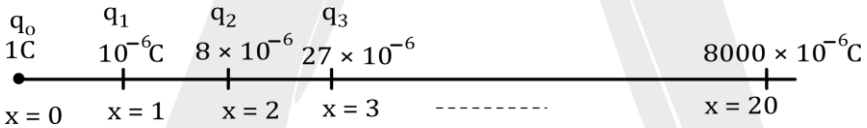
$q_1 = 20C \qquad q_2 = 25\mu C$

$$\vec{F}_{21} = \frac{kq_1q_2}{|\bar{r}_{12}|^3}\vec{r}_{12} \qquad \vec{r}_{12} = \vec{r}_2 - \vec{r}_1$$

$$\hat{r}_{12} = \frac{4i - 3k}{5} \qquad |\bar{r}_{12}| = \sqrt{16+9} = 5\ m$$

$$|\bar{F}_{22}| = \frac{5 \times 10^9 \times 20 \times 10^{-6} \times 25 \times 10^{-6}}{28} = 18 \times 10^{-2} = 0.18\ N$$

3.



Fnet on 1c due to all charges

$$F = \frac{kq_0q_1}{1^2} + \frac{kq_0q_2}{2^2} + \cdots + \frac{kq_0q_{20}}{(20)^2}$$

$$F = Kq_0\left[\frac{10^{-6}}{1} + \frac{8 \times 10^{-6}}{4} + \frac{27 \times 10^{-6}}{9} + \cdots + \frac{8000 \times 10^{-6}}{400}\right]$$

$$F = 9 \times 10^9 \times 10^{-6}[1 + 2 + 3 + \cdots + 20]$$

$$F = 9 \times 10^3[210] = 189 \times 10^4 = 1.89 \times 10^6\ N$$

use sum of AP $\quad a = 1,\ d = 1\ n = 20$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$= \frac{20}{2}[2 \times 1 + 19] = 21 \times 10 = 210$

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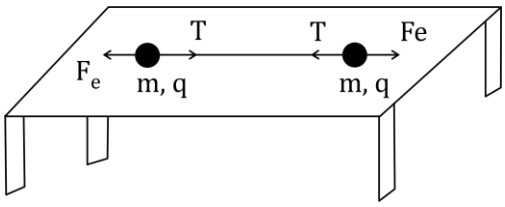
4. (i)



Tension T = Fe (electrostatic force)

$$T = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{l^2} \Rightarrow T = 9 \times 10^9 \times 16 \times 10^{-12}$$

$$T = 144 \times 10^{-3} = .144\ N$$

(ii) After cut the string T = 0 only F_e is act.

$$F_e = ma \Rightarrow a = \frac{F_e}{m} = \frac{.144}{24 \times 10^{-3}}$$

$$a_1 = 6\ m/s^2$$

(iii) In magnitude $a = 6\ m/s^2$ and direction is opposite.

5. Let charges are q_1 & q_2

First $F = \frac{kq_1q_2}{(0.5)^2} = 0.108\ N$ ... (1)

After touching

$$\frac{k(q_1+q_2)^2}{4(0.5)^2} = 0.036N \quad \text{... (ii)}$$

After Solving 1 & 2

$$q_1 = \pm 1 \times 10^{-6}C \quad q_2 = \mp 3 \times 10^{-6}C$$

6. $T\cos\theta = mg$

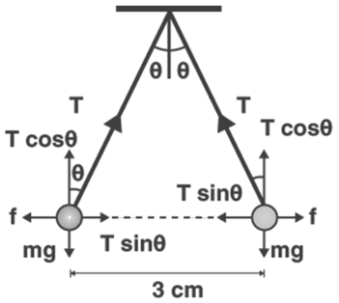
Here, T is the tension force, m is the mass of the sphere, g is the acceleration due to gravity, and θ is the angle.

$$T\cos\theta = 0.1 \times 10^{-3}\ kg \times \frac{10\ m}{s^2} \Rightarrow T\cos\theta = 10^{-3}\ N$$

$$\Rightarrow T\sin\theta = \frac{kq^2}{r^2} \Rightarrow \frac{T\sin\theta}{T\cos\theta} = \frac{\frac{9 \times 10^9 Nm^2/C^2 \times (10^{-9}c)^2}{\left(3cm \times \frac{1m}{100cm}\right)^2}}{10^{-3}N}$$

$$\tan\theta = \frac{1}{100} \Rightarrow \theta = \tan^{-1}\left(\frac{1}{100}\right)$$

$$\theta = 0.6^\circ$$



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7. $F_{qe} = F_{q,4e}$ at equilibrium position

$$\frac{keq}{x^2} = \frac{K4eq}{(\ell + x)^2} \Rightarrow (\ell + x)^2 = 4x^2$$

$$\ell + x = 2x$$

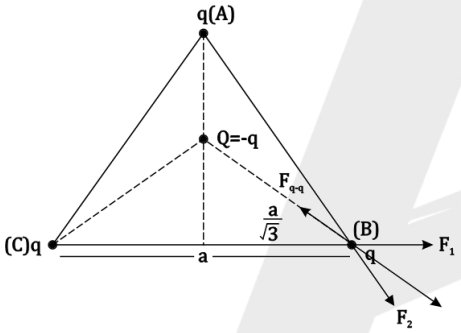
$$\Rightarrow x = \ell \Rightarrow x = \ell$$

$\Rightarrow$ q $\rightarrow$ Placed ℓ from e.

$\Rightarrow$ q = + ive [stable]

q = − ive [unstanbe]

8. (a)



$$F_{q(-q)} = \frac{kq^2 \times 3}{a^2} = \frac{3kq2}{a^2}$$

Force on

$$q(B) = \sqrt{F_1^2 + F_2^2} = \frac{kq^2}{a^2}\sqrt{3}$$

So net force on q(B) towards (−q)

(b) First we find net force on corner charge (q)

$$\Rightarrow \frac{kqQ}{\left(\frac{a}{\sqrt{3}}\right)^2} + \frac{kq^2\sqrt{3}}{a^2} = 0$$

$$\Rightarrow \frac{3kqQ}{a^2} = \frac{-kq^2\sqrt{3}}{a^2}$$

$$\Rightarrow \boxed{Q = -\frac{q}{\sqrt{3}}}$$

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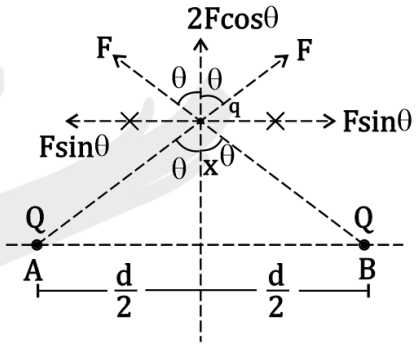
9. F_{net} on q

$= 2F\cos\theta$

$$= \frac{2kqQ}{\left[\left(\frac{d}{2}\right)^2 + x^2\right]} \times \frac{x}{\left(x^2 + \frac{d^2}{2}\right)^{1/2}} = \frac{2kqQx}{\left[x^2 + \left(\frac{d}{2}\right)^2\right]^{3/2}}$$

For F max $\frac{dF}{dx} = 0 \Rightarrow \frac{d}{dx}\frac{2kqQx}{\left[x^2 + \left(\frac{d}{2}\right)^2\right]^{3/2}}$

$$2kqQ\frac{d}{dx}\left(\frac{x}{\left[x^2 + \left(\frac{d}{2}\right)^2\right]^{3/2}}\right) = 0$$



$$x \times \frac{3}{2}\left(x^2 + \frac{d^2}{4}\right)^{1/2} \cdot 2x - \left(x^2 + \frac{d^2}{4}\right)^{3/2} \cdot 1 = 0$$

$$\frac{3x}{2}\left(x^2 + \frac{d^2}{4}\right)^{1/2} \cdot 2x = \left(x^2 + \frac{d^2}{4}\right)^{1}\left(x^2 + \frac{d^2}{4}\right)^{1/2}$$

$$3x^2 = x^2 + \frac{d^2}{4}$$

$$\Rightarrow 2x^2 = \frac{d^2}{4}$$

$$\Rightarrow x^2 = \frac{d^2}{4 \times 2}$$

$$\Rightarrow \boxed{x = \frac{d}{2\sqrt{2}}}$$

10. $q_1(2,-1,3)$ $q_2(0,0,0)$

$$\vec{F}_{21} = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r_{12}^3}(\vec{r}_{12})$$

$$\vec{r}_{12} = (-2i + j - 3\hat{k}) \quad |\bar{r}_{12}| = \sqrt{14}$$

$$\overrightarrow{F_{21}} = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{14\sqrt{14}}(j - 2i - 3k)$$

$$\vec{F}_{21} = \frac{q_1q_2}{56\sqrt{14}\pi\varepsilon_0}(\hat{j} - 2i - 3\hat{k})$$

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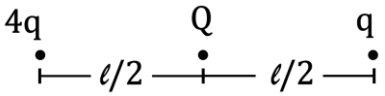
11.

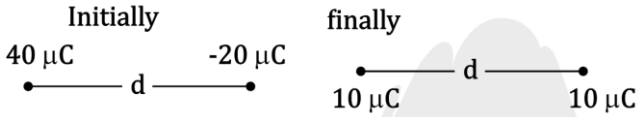


F_{net} on q

$$\Rightarrow \frac{4kqq}{l^2} + \frac{4kqQ}{l^2} = 0$$

$$\boxed{Q = -q}$$

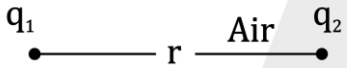
12.



$$F_1 = \frac{K40 \times 20}{d^2} \qquad F_2 = \frac{k \times 10 \times 10}{d2}$$

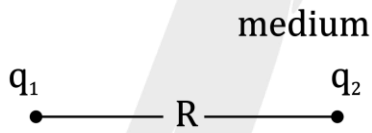
$$\frac{F_1}{F_2} = 8:1$$

13. Initially



$$F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}$$

Finally



$$F' = \frac{1}{4\pi\varepsilon_0 \times 16}\frac{q_1 q_2}{R^2} = 4F$$

$$\frac{1}{4\pi\varepsilon_0 \times 16}\frac{q_1 q_2}{R^2} = 4 \times \frac{1}{4\pi\varepsilon_0}\frac{\varepsilon_0 q_2}{r^2}$$

$$r^2 = 64R^2 \Rightarrow r = 8R$$

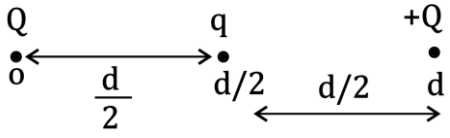
$$R = r/8$$

14. F_{net} on Q (Placed at 0) = 0

$$\frac{kQq}{d^2/4} + \frac{kQ^2}{d^2} = 0$$

$$\frac{4kqQ}{d^2} = \frac{-kQ^2}{d^2}$$

$$q = -Q/4$$



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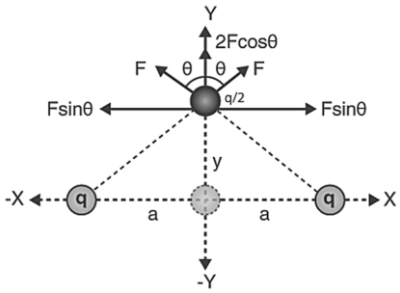
15. $F_{net} = 2F\cos\theta = 2\frac{k\cdot q\cdot q/2}{\left(\sqrt{a^2+y^2}\right)^2}\cdot\frac{y}{\sqrt{a^2+y^2}} = \frac{kq^2y}{(a^2+y^2)^{3/2}}$



As y ≪< a we write

$F_{net} = \frac{kq^2y}{(a^3)} \Rightarrow$ i.e. $F \propto y$

16. $\tan\theta = \frac{Fe}{mg} \Rightarrow \qquad \tan\theta = \frac{kq^2}{x^2mg}$

$\tan\theta \to \sin\theta[x << \ell)]$

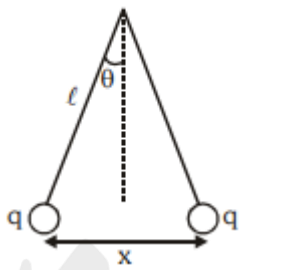
$\frac{x}{2\ell} = \frac{kq^2}{x^2mg} \Rightarrow x^3 \propto q^2$

$q \propto x^{3/2} \Rightarrow$ diff both Side w.r.t t

$\frac{dq}{dt} \propto \frac{d}{dt}\left(x^{3/2}\right) \Rightarrow \frac{dq}{dt} =$ Constant [given]

$\Rightarrow \frac{dq}{dt} \propto \frac{3}{2}x^{1/2}\frac{dx}{dt} \Rightarrow \frac{dq}{dt} = \frac{3}{2}x^{1/2}\cdot v$

$v \propto x^{-1/2}$



17. From fig,

$\tan\theta = Fe/mg \Rightarrow \tan 15° = \frac{kq^2}{d^2mg}$

$\tan 15° = \frac{kq^2}{1.6Vgd^2}$ ..... [V is the volume]

When system is suspended in liquid,

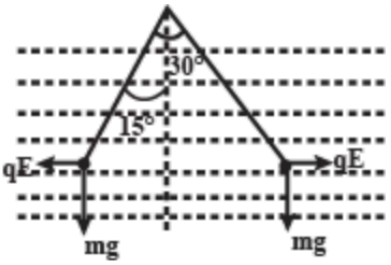
$\tan 15° = \frac{kq^2}{K(mg-\rho Vg)d^2}$

$\tan 15° = \frac{kq^2}{K(1.6-0.8)Vgd^2}$ ..... (2)

from (1) and (2) we get,

$\frac{kq^2}{K(1.6-0.8)Vgd^2} = \frac{kq^2}{1.6Vgd}$

$\therefore K = 2 =$ Dielectric constant of liquid.



18. It is obvious that by charge conservation law, electronic charge must be independent of the acceleration due to gravity. Hence it will remain constant irrespective of where the experiment is performed.

Hence $\frac{\text{electronic charge on moon}}{\text{electronic charge on earth}} = 1$

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19. **Calculation of initial force**

Let A and B have charge Q each initially and are seperated by r distance.

From Coulomb's law, force between them $F = \frac{KQ^2}{r^2}$

Distribution of charges

When an identical uncharged spherical conductor C is brought in contact with charged sphere B. By symmetry, the total charge will be shared equally among them, as both have equal radii.

$\therefore$ Charge on B and C $= \frac{Q}{2}$

Now, when the conductor A is brought in contact with C.

They will also share equal charge among themselves, as both have equal radii.

$\therefore$ Charge on A and C $= \frac{Q+\frac{Q}{2}}{2} = \frac{3Q}{4}$

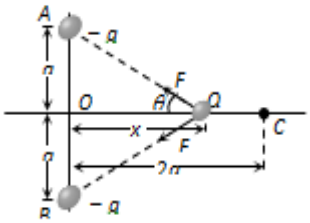
Calculation of new force

New coulomb force between B and C:

$$F' = \frac{K\frac{Q}{2} \times \frac{3Q}{4}}{r^2} = \frac{3}{8}\frac{KQ^2}{r^2}$$

From equation (1), we get: $F' = \frac{3}{8}F$

20. The net force is given as,



$F_{net} = 2F\cos\theta$

$$\vec{F} = 2 \times \frac{1}{4\pi\varepsilon_0}\left(\frac{qQ}{a^2+x^2}\right) \times \left(\frac{-x}{(a^2+x^2)^{\frac{1}{2}}}\right)$$

Thus, the restoring force is not linear, So, the motion will not be simple harmonic motion but the motion will be oscillatory.

21. Charge q is in equilibrium since charges A and B exert equal and opposite forces on it. For equilibrium of charge Q at B;

$F_{BC} + F_{AB} = 0$

$$\Rightarrow \frac{1}{4\pi\epsilon_0}\frac{qQ}{(L/2)^2} + \frac{1}{4\pi\epsilon_0}\frac{Q \cdot Q}{L^2} = 0$$

$$\Rightarrow \frac{1}{4\pi\epsilon_0}\frac{Q}{L^2}(4q+Q) = 0 \Rightarrow q = -\frac{Q}{4}.$$

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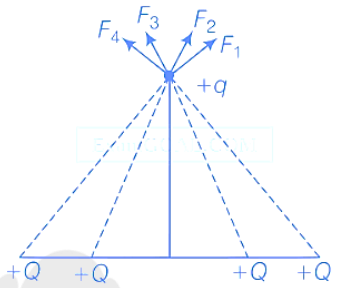
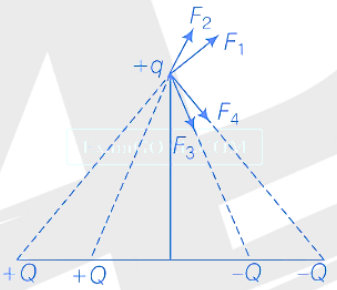
22. Explanation

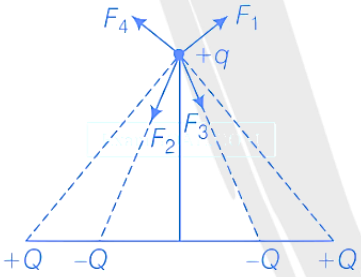
(P) Component of forces along x-axis will vanish. Net force along positive y-axis.



(Q) Component of forces along y-axis will vanish. Net force along positive x-axis



(S) Component of forces along y-axis will vanish. Net force along negative x-axis.



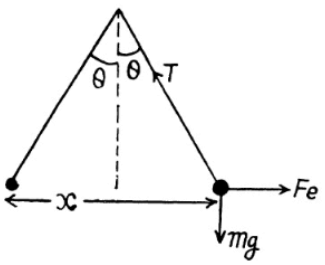
23. Let the balls be deviated by an angle θ, from the vertical when separation between then equals x.

Applying Newton's second law of motion for any one of the sphere, we get, $T\cos\theta = mg$ (1)

and $T\sin\theta = F_e$ (2)

From the Eq. (1) and (2)

$\tan\theta = \frac{F_e}{mg}$ (3)



But from the figure

$\tan\theta = \frac{x}{2\sqrt{\ell^2-\left(\frac{x}{2}\right)^2}} = \frac{x}{2\ell}$ as $x \ll \ell$ From Eq. (3) and (4)

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$$F_e = \frac{mgx}{2\ell} \text{ or } \frac{q^2}{4\pi\epsilon_0 x^2} = \frac{mgx}{2\ell}$$

Thus

$$q^2 = \frac{2\pi\epsilon_0 mgx^3}{\ell} \ldots (5)$$

Differentiating Eqn. (5) with respect to time

$$2q\frac{dq}{dt} = \frac{2\pi\epsilon_0 mg}{\ell} 3x^2 \frac{dx}{dt}$$

According to the problem $\frac{dx}{dt} = v = a\sqrt{x}$ (approach velocity is $\frac{dx}{dt}$)

$$\text{So, } \left(\frac{2\pi\epsilon_0 mg}{\ell} x^3\right)^{1/2} \frac{dq}{dt} = \frac{3\pi\epsilon_0 mg}{\ell} x^2 \frac{a}{\sqrt{x}}$$

$$\text{Hence, } \frac{dq}{dt} = \frac{3}{2} a \sqrt{\frac{2\pi\epsilon_0 mg}{\ell}}.$$