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MECHANICAL PROPERTIES OF SOLIDS

ELASTICITY

Property of a body by virtue of which the body opposes any change in its shape and size when deforming force is applied and recover its original shape as soon as deforming force is removed.

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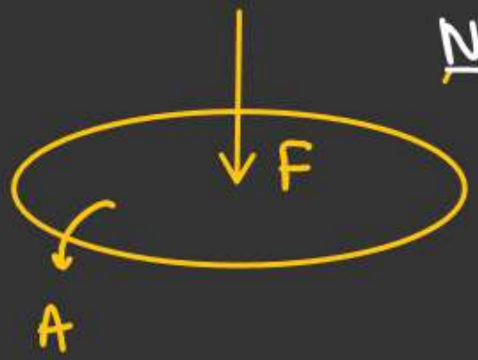
MECHANICAL PROPERTIES OF SOLIDS

STRESS & STRAIN

LONGITUDINAL STRESS
OR
NORMAL STRESS



$$\boxed{\frac{F}{A}}$$

Force always perpendicular to area

$S \cdot I = N/m^2$

LONGITUDINAL STRAIN

$$\frac{\text{Change in length}}{\text{Initial length}} = \left(\frac{L_f - L_i}{L_i}\right) = \left(\frac{\Delta L}{L_i}\right)$$

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MECHANICAL PROPERTIES OF SOLIDS

YOUNG'S MODULUS OF ELASTICITY

For Solid

Ratio of Longitudinal Stress to Longitudinal strain is called Young's Modulus of Elasticity

$$Y = \left(\frac{\text{Longitudinal stress}}{\text{Longitudinal strain}}\right)$$

$$\boxed{Y = \frac{F/A}{\frac{\Delta L}{L}}}$$

HOOK'S LAW

↳ For small deformation Stress is directly proportional to Strain.

Stress $\propto$ Strain

$$\boxed{\left(\frac{\text{Stress}}{\text{Strain}}\right) = \text{Constant}}$$

(Moduli of Elasticity)

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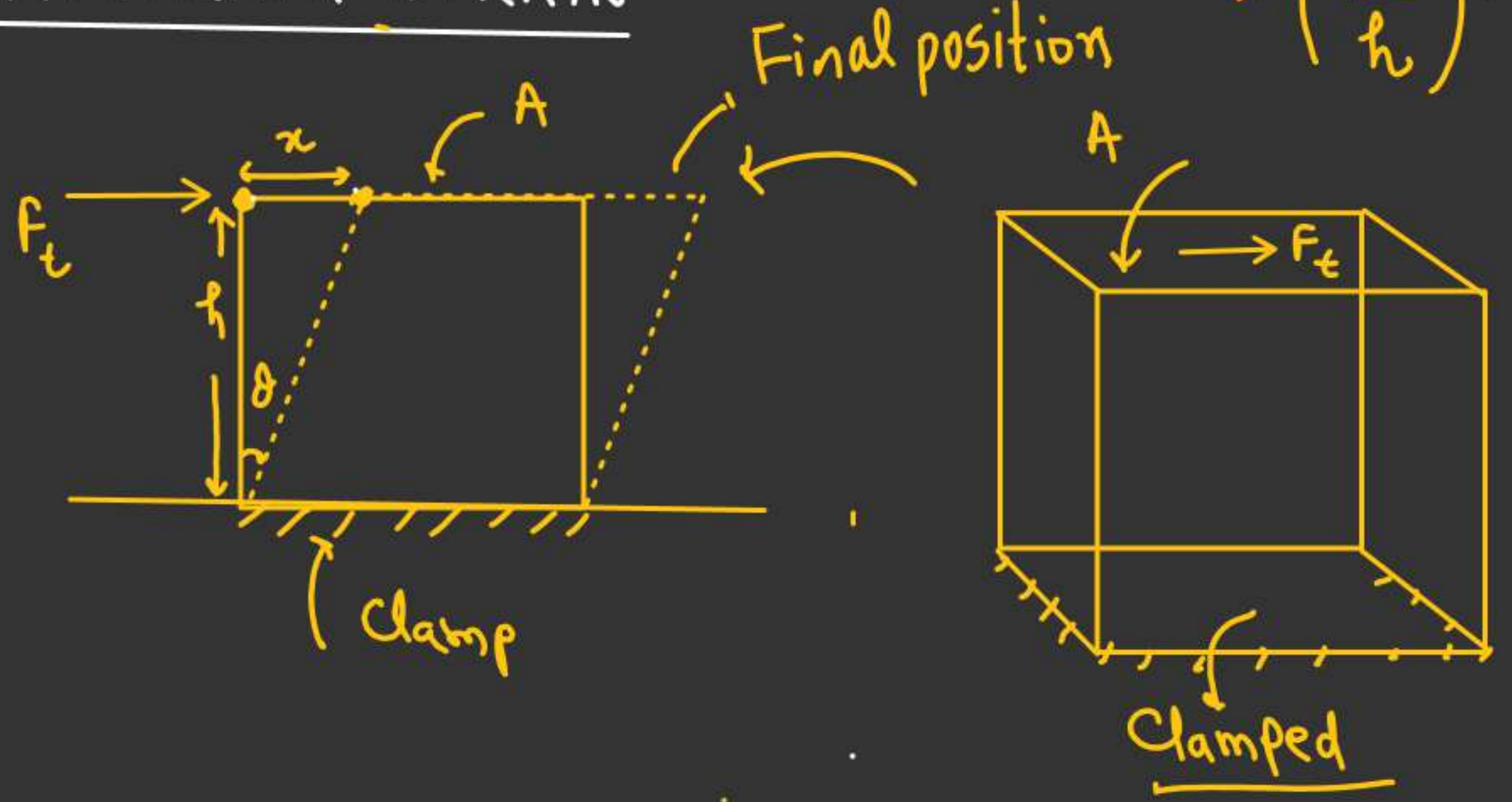
MECHANICAL PROPERTIES OF SOLIDS

SHEARING STRESS & SHEARING STRAIN

✓ SHEARING STRESS = $\dfrac{\text{Tangential Force}}{\text{Area}}$

$= \left(\dfrac{F_t}{A}\right)$

✓ SHEARING STRAIN $\longrightarrow \left(\dfrac{x}{h}\right)$



Modulii of Rigidity / Shear Modulus

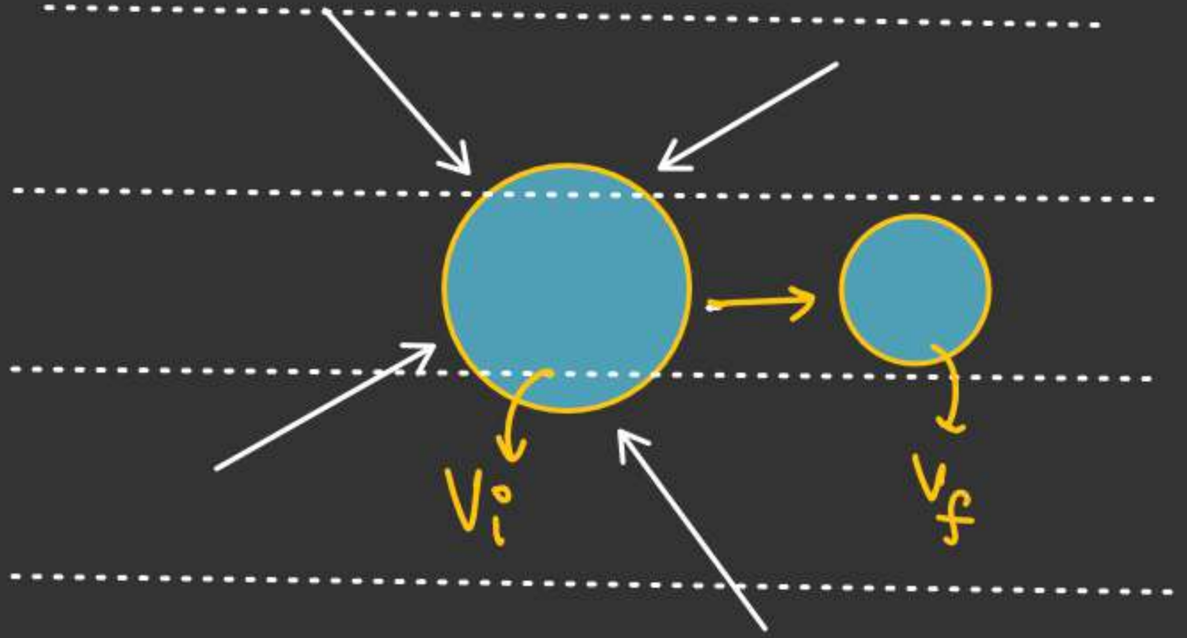
$$\eta = \frac{\text{Shearing Stress}}{\text{Shearing strain}}$$

$$\eta = \frac{(F_t/A)}{(x/h)}$$

MECHANICAL PROPERTIES OF SOLIDS

Volumetric Stress (For Fluid)

⟶ (Pressure)



Volumetric Strain (For fluid)

$$= \frac{\text{Change in volume}}{\text{Initial Volume}}$$

$$= \left(\frac{V_f - V_i}{V_i}\right) \qquad (V_i > V_f)$$

$$= \left(-\frac{\Delta V}{V}\right)$$

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★★: Bulk Modulus (liquid or Gas)

↳ It is ratio of Volumetric Stress to Volumetric Strain

$$\boxed{B = \frac{-\Delta P}{\left(\frac{\Delta V}{V}\right)}}$$

$\Delta P \rightarrow$ Excess pressure or P

$$\boxed{B = \frac{-dP}{\left(\frac{dV}{V}\right)}}$$

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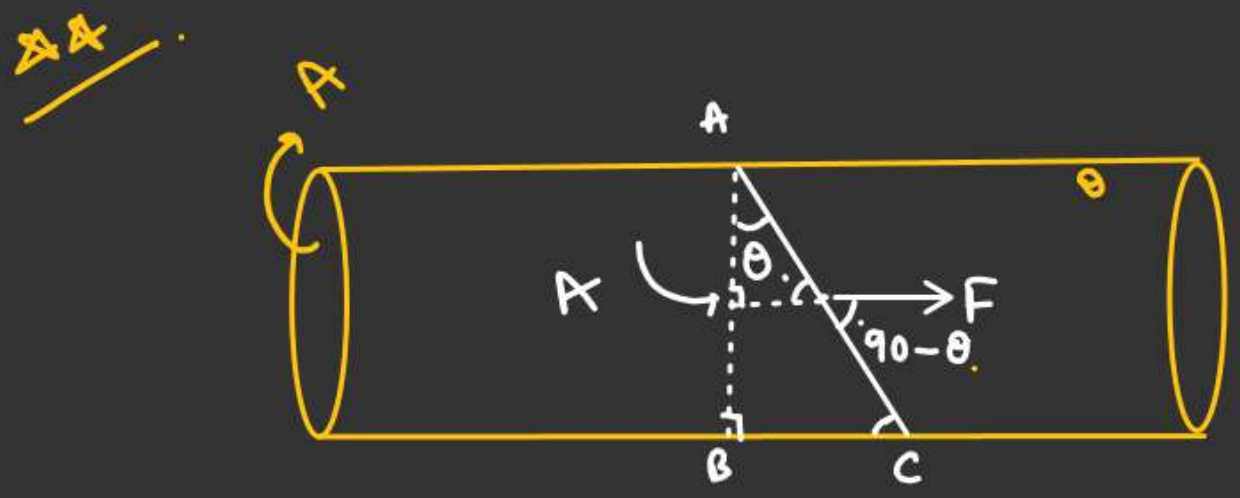
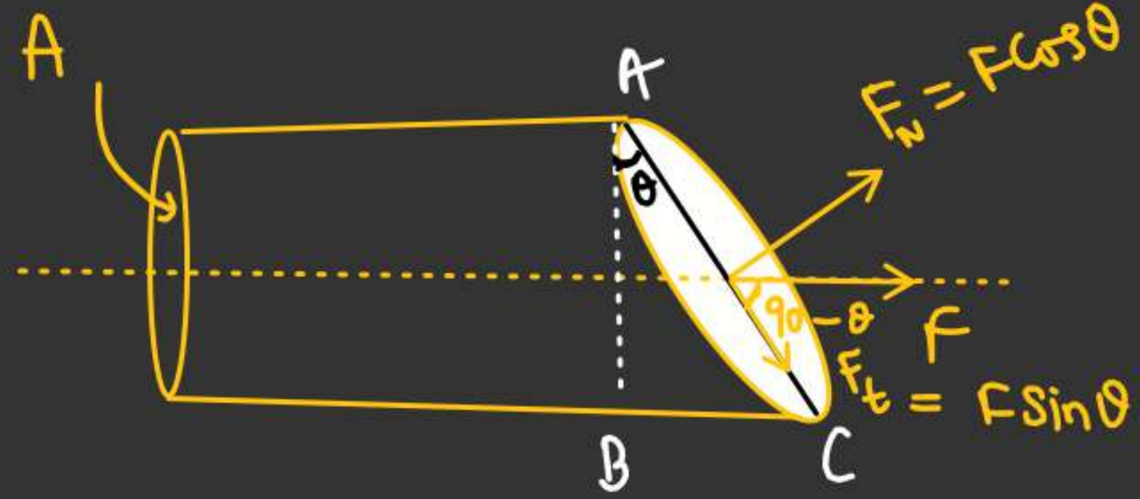
MECHANICAL PROPERTIES OF SOLIDS



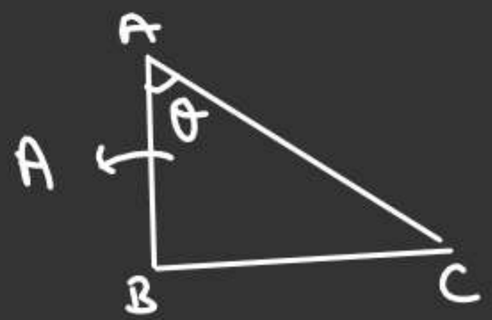


Find Stress due to F on AC.

A = Crosectional area of Cylinder.

Longitudinal Stress $= \dfrac{F\cos\theta}{A_{AC}} = \dfrac{F\cos\theta}{A/\cos\theta} = \left(\dfrac{F\cos^2\theta}{A}\right)$

A_{AC} = Crosectional area of AC.



$\cos\theta = \dfrac{AB}{AC}$

$AC = \left(\dfrac{AB}{\cos\theta}\right)$

$\left(A_{AC} = \dfrac{A}{\cos\theta}\right)$

Tangential Stress

$= \dfrac{F_t}{A_{AC}} = \dfrac{F\sin\theta}{A/\cos\theta}$

$= \dfrac{F}{A}\sin\theta\cdot\cos\theta$

$= \dfrac{F}{2A}\sin 2\theta$ ✓

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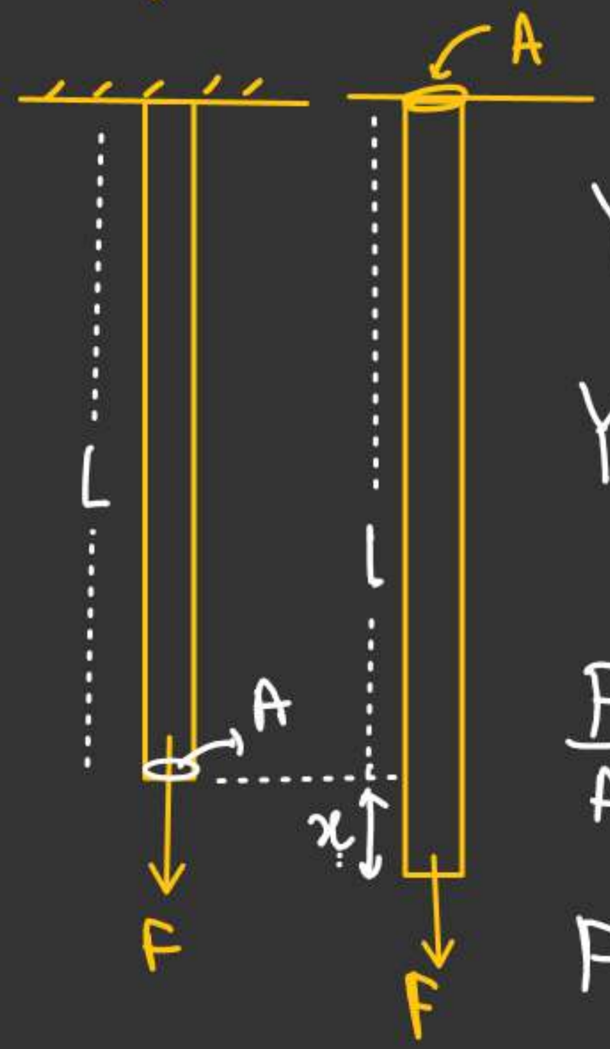
MECHANICAL PROPERTIES OF SOLIDS

Equivalent Spring Constant of a Rod



$$Y = \frac{\text{Stress}}{\text{Strain}}$$

$$Y = \frac{F/A}{\frac{x}{L}}$$

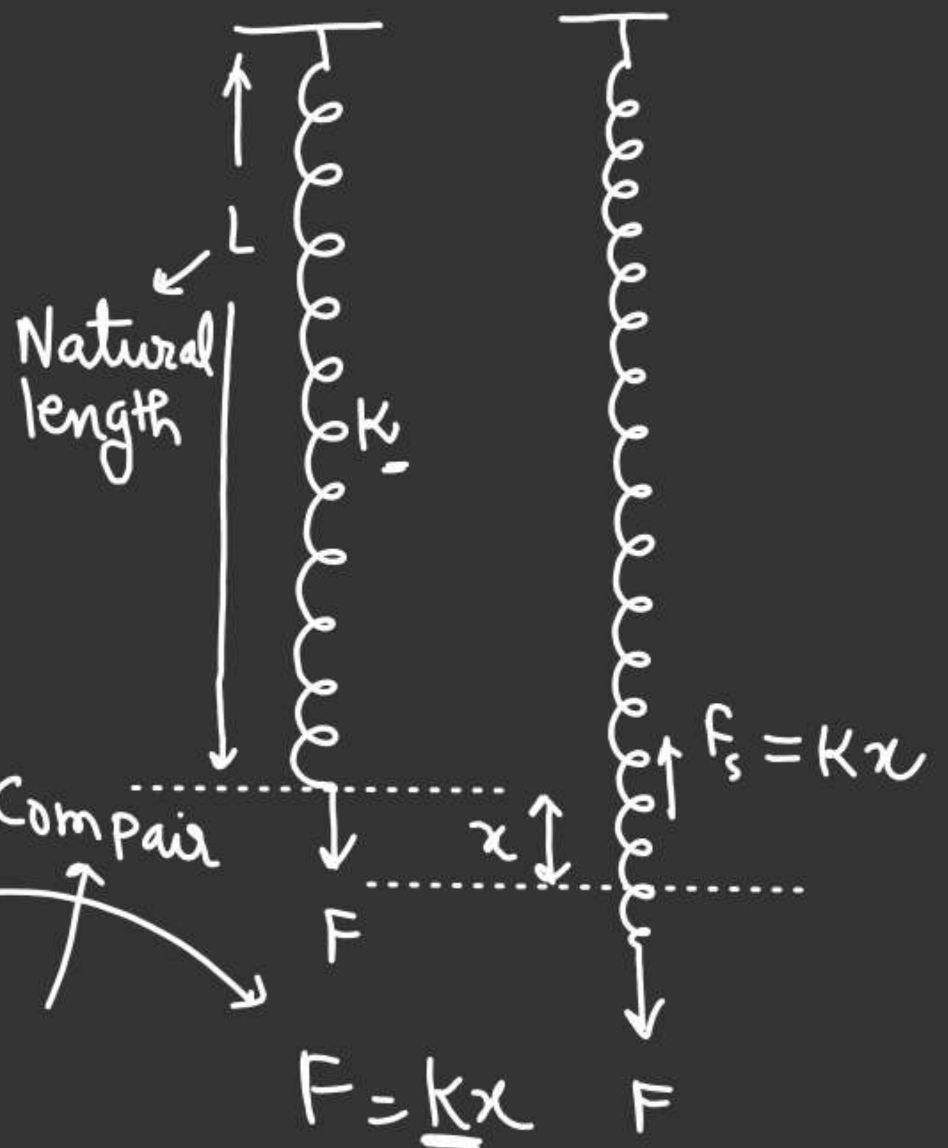
$$\frac{F}{A} = \frac{Y}{L}x$$

$$F = \left(\frac{YA}{L}\right)x$$

$$\Downarrow$$

$$K = \left(\frac{YA}{L}\right)$$

Compair



$F = \underline{K}x$

$\downarrow$

Spring Constant

$K \rightarrow$ Spring Constant of Rod $= \left(\frac{YA}{L}\right)$

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Energy stored

Stress = Y Strain

Strain = $\frac{x}{L}$

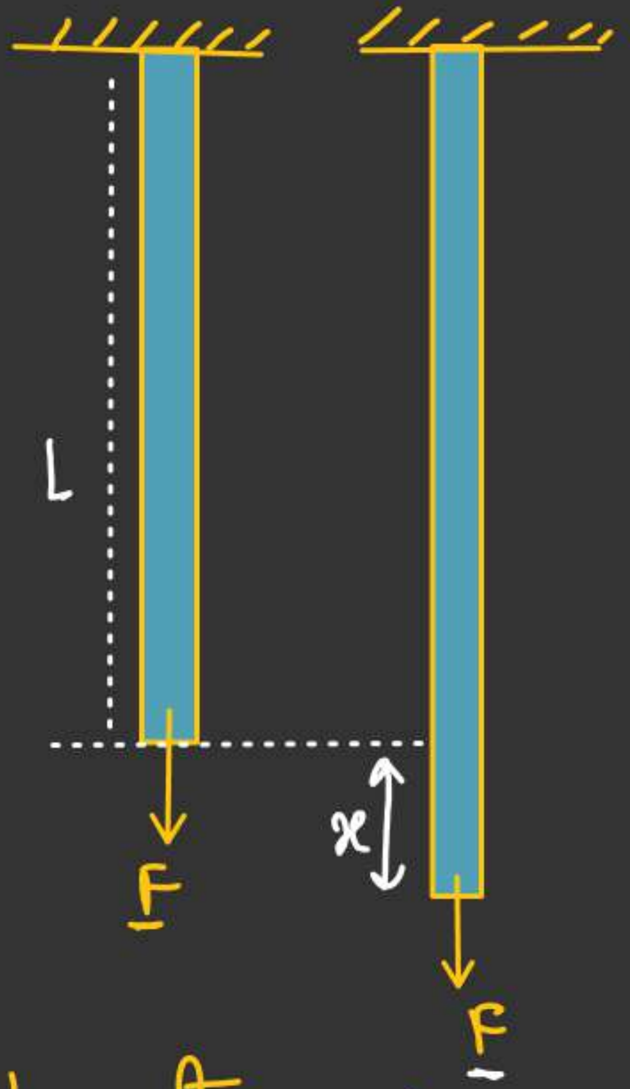
$K = \frac{YA}{L}$

$U = \frac{1}{2} K x^2$

$U = \frac{1}{2}\left(\frac{YA}{L}\right) x^2$

$U = \frac{1}{2}\left(\frac{Yx}{L}\right)\left(\frac{Ax}{L}\right) \times L$

$U = \frac{1}{2}\underbrace{\left(\frac{Yx}{L}\right)}_{\text{Stress}} \times \underbrace{\left(\frac{x}{L}\right)}_{\text{Strain}} \times \underbrace{AL}_{\text{Volume of Rod}}$



$\frac{U}{AL} = \frac{1}{2} \times \text{Stress} \times \text{Strain}$

$\boxed{\text{P.E per Unit Volume} = \frac{1}{2} \times \text{Stress} \times \text{Strain}}$

- → Elongation due to self weight neglected
- → Change in radius neglected

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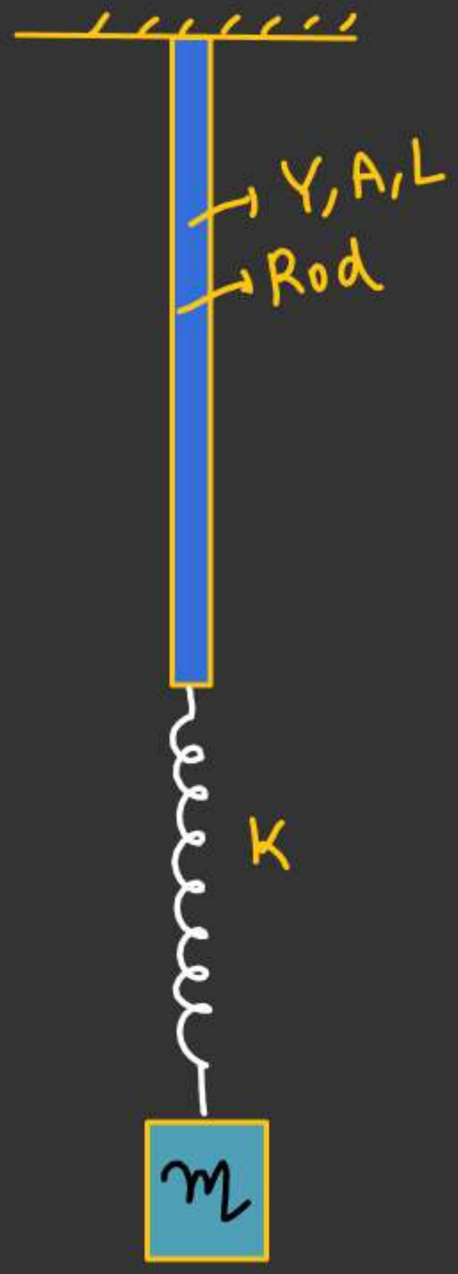
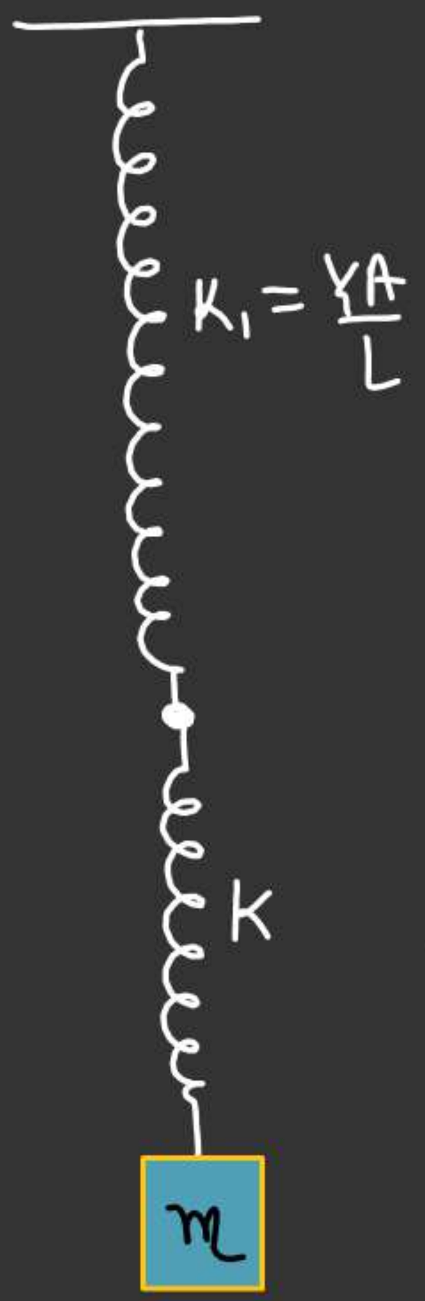
Find time period of the block. (Rod is Massless.)





$$K_{eq} = \frac{K_1 \cdot K}{K_1 + K}$$

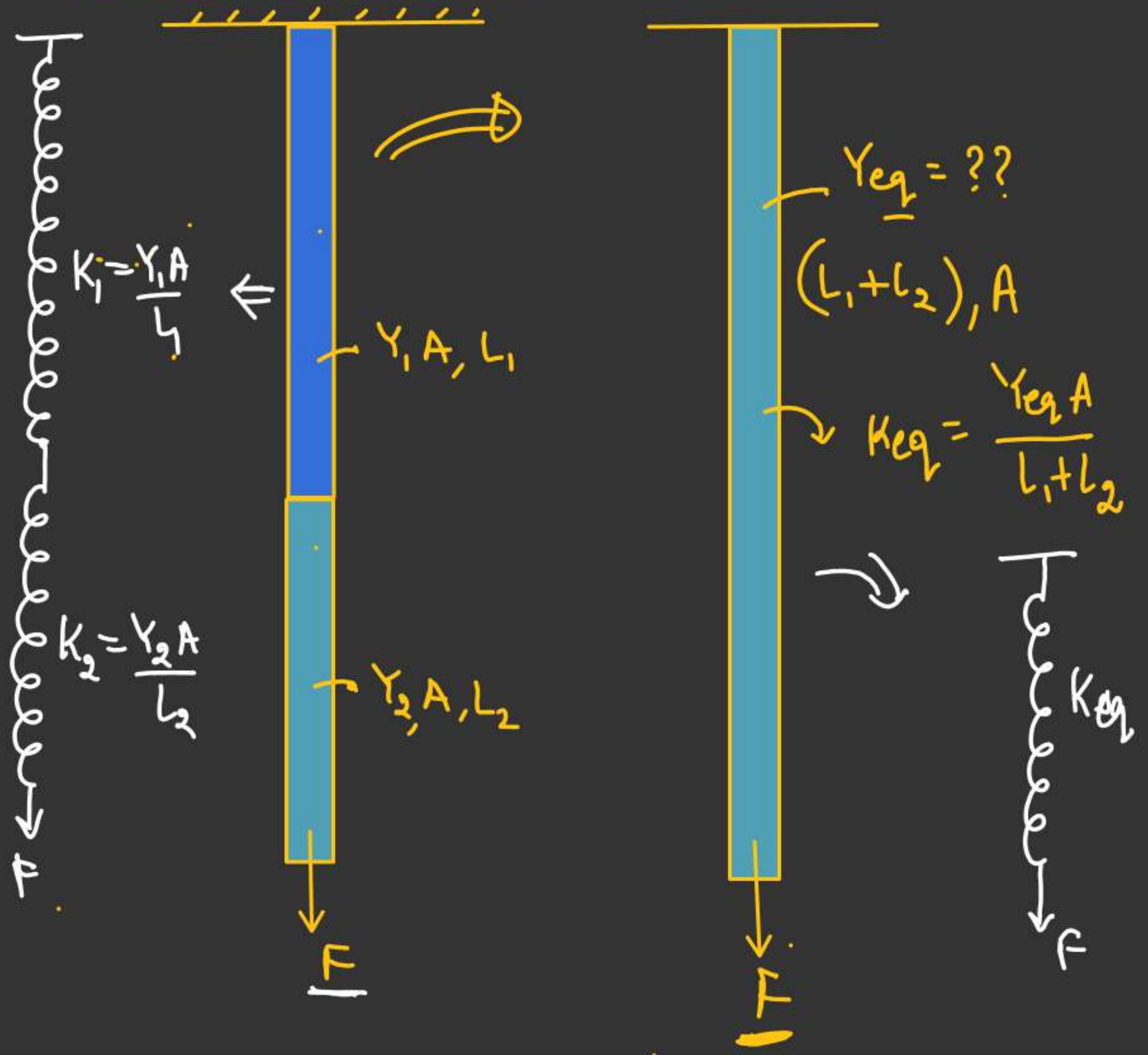
$$K_{eq} = \left(\frac{\frac{YAK}{L}}{K + YA/L}\right) = \left(\frac{YAK}{KL + YA}\right)$$

$$T = 2\pi\sqrt{\frac{m}{K_{eq}}}$$

$$T = 2\pi\sqrt{\frac{m(KL + YA)}{YAK}}.$$

MECHANICAL PROPERTIES OF SOLIDS

Composite Rod System (Elongation due to self weight neglected)



$$\frac{1}{K_{eq}} = \left(\frac{1}{K_1} + \frac{1}{K_2}\right)$$

$$\frac{L_1+L_2}{Y_{eq}\cancel{A}} = \frac{L_1}{Y_1\cancel{A}} + \frac{L_2}{Y_2\cancel{A}}$$

$$Y_{eq} = \left(\frac{L_1+L_2}{\frac{L_1}{Y} + \frac{L_2}{Y_2}}\right)$$

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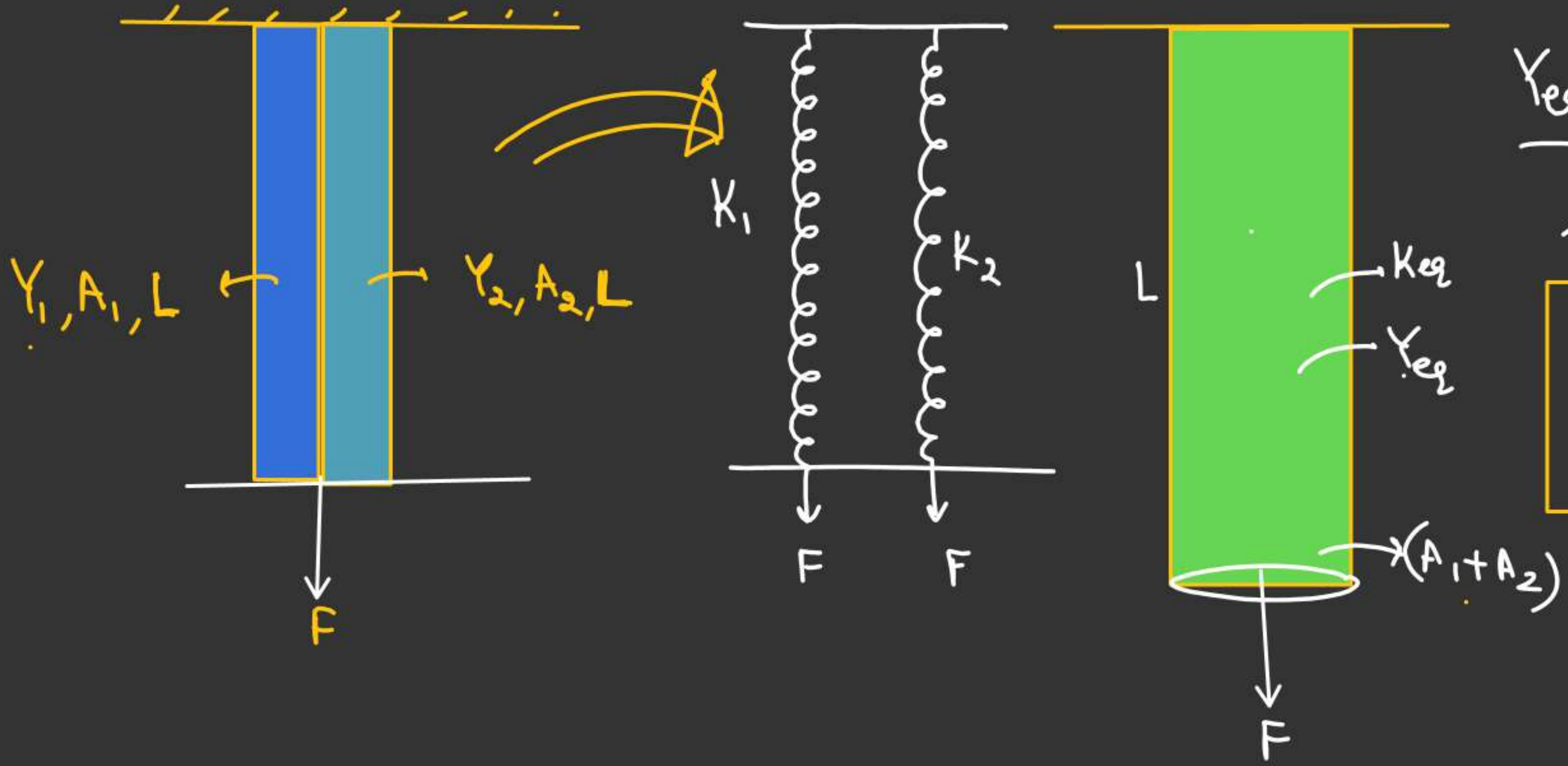
MECHANICAL PROPERTIES OF SOLIDS

Parallel Combination (Y_{eq} = ??)



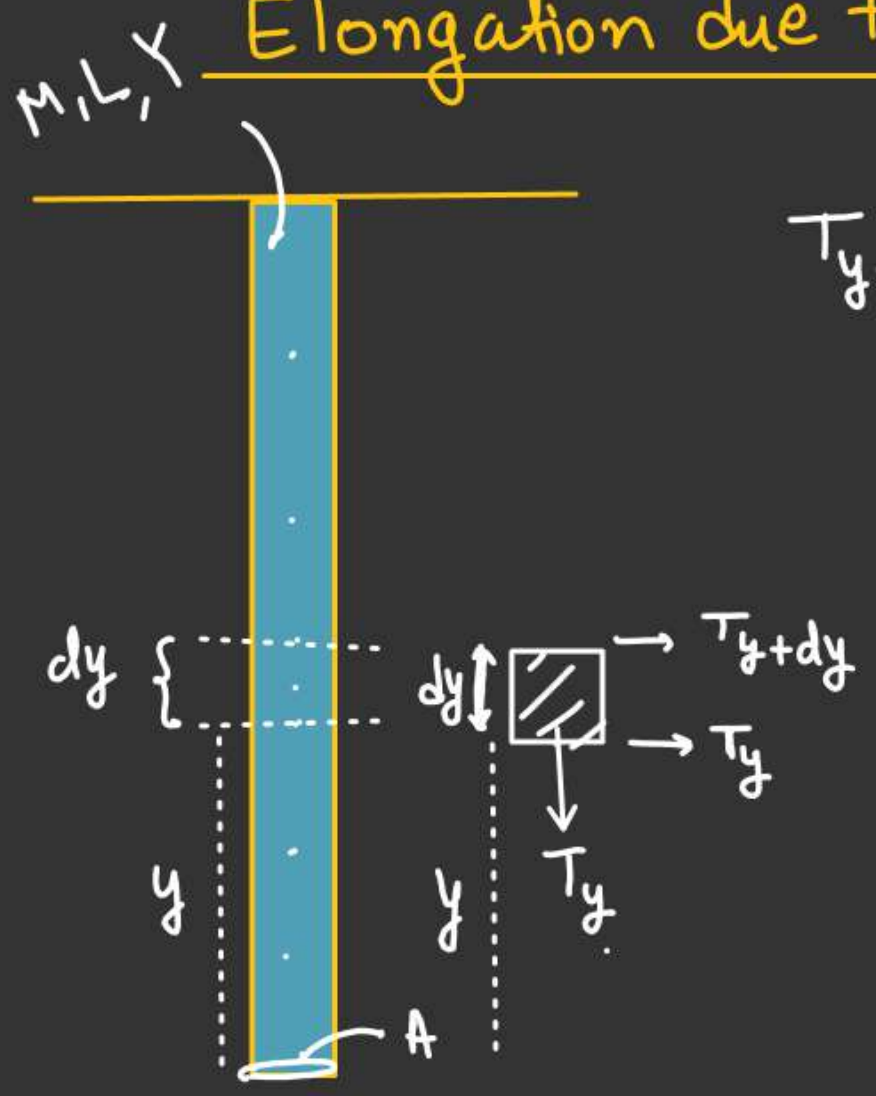
$$K_{eq} = K_1 + K_2$$

$$\frac{Y_{eq}(A_1+A_2)}{\cancel{L}} = \frac{Y_1A_1}{\cancel{L}} + \frac{Y_2A_2}{\cancel{L}}$$

$$\boxed{Y_{eq} = \frac{Y_1A_1+Y_2A_2}{A_1+A_2}}$$

MECHANICAL PROPERTIES OF SOLIDS

Elongation due to Self weight (Rod is uniform) (A = Crossectional area)



$T_{y+dy} \simeq T_y$.

T_y = Resposible for producing stress in dy length of Rod.

T_y = Weight of y length of Rod

$$T_y = \left(\frac{M}{L}y\right)g$$

$$\text{Stress} = \frac{T_y}{A} = \left(\frac{Mgy}{LA}\right)$$

let, dy' be change in length in dy length of rod

$$\text{Strain} = \left(\frac{dy'}{dy}\right)$$

$$Y = \frac{\text{Stress}}{\text{strain}}$$

$$Y \text{ strain} = \text{stress}$$

$$Y \frac{dy'}{dy} = \frac{Mg}{LA}y.$$

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MECHANICAL PROPERTIES OF SOLIDS

$$Y\frac{dy'}{dy} = \frac{Mg}{LA}y.$$

$$\int_0^{\Delta L} dy' = \frac{Mg}{YAL}\int_0^{L} y\,dy$$

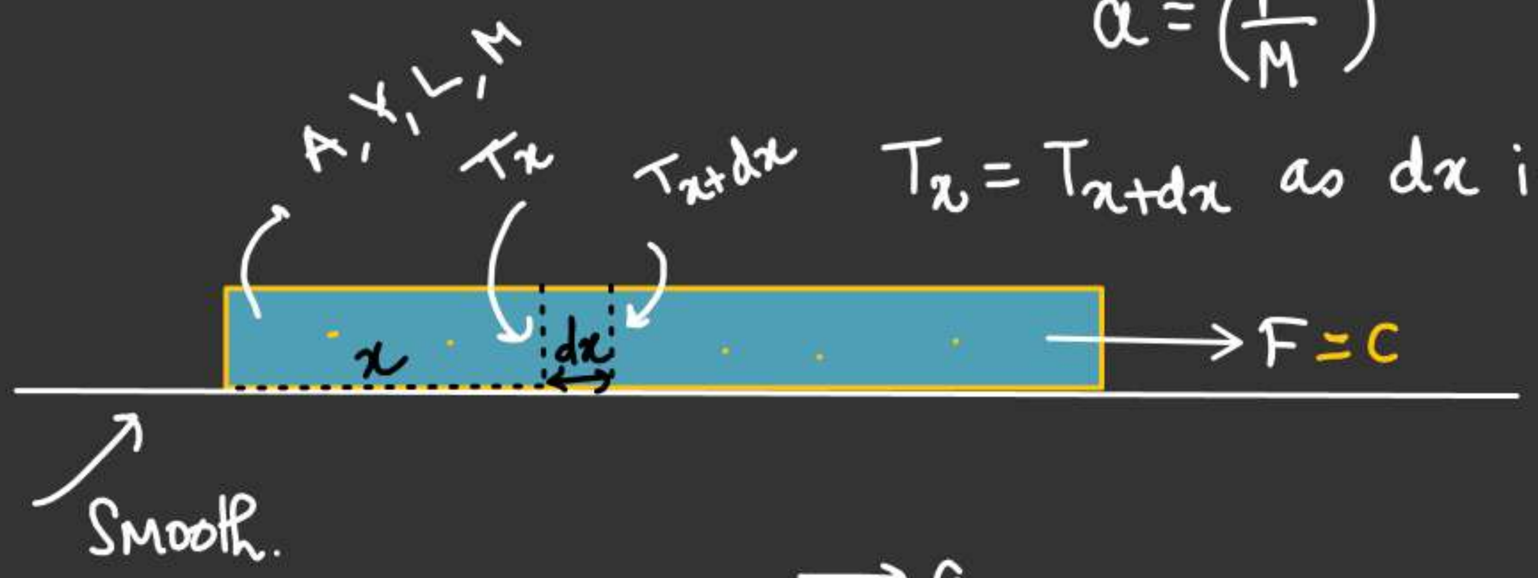
$$\Delta L = \frac{Mg}{YAL} \times \frac{L^2}{2}$$

$$\boxed{\Delta L = \frac{MgL}{2YA}}$$

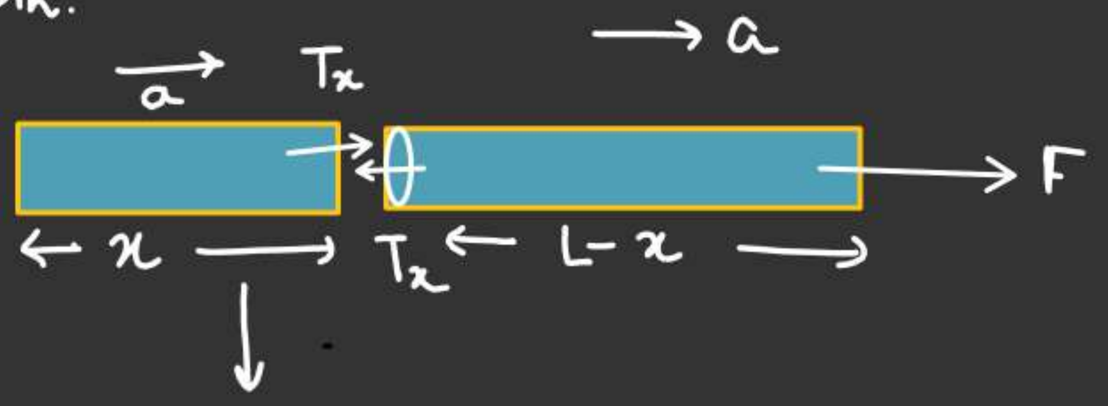
MECHANICAL PROPERTIES OF SOLIDS

Elongation in a uniform acclerated rod.

$$a = \left(\frac{F}{M}\right)$$



$T_x = T_{x+dx}$ as dx is very small.



$$T_x = m_x a$$

$$T_x = \left(\frac{M}{L}x\right)a$$

$$T_x = \frac{\cancel{M}}{L} x \times \frac{F}{\cancel{M}} = \left(\frac{F}{L}x\right)$$

Stress = Y (strain)

let dx' be change in length in dx length

$$\frac{T_x}{A} = Y\frac{dx'}{dx}$$

$$\frac{F x}{YAL} = \frac{dx'}{dx}$$

$$\int_{\Delta L}^{0} dx' = -\frac{F}{YAL}\int_{L}^{0} x\,dx$$

$$-\Delta L = -\frac{F}{YAL} \times \frac{L^2}{2}$$

$$\boxed{\Delta L = \frac{F.L}{2YA}}$$

when Rod is hanging $F \to Mg$.

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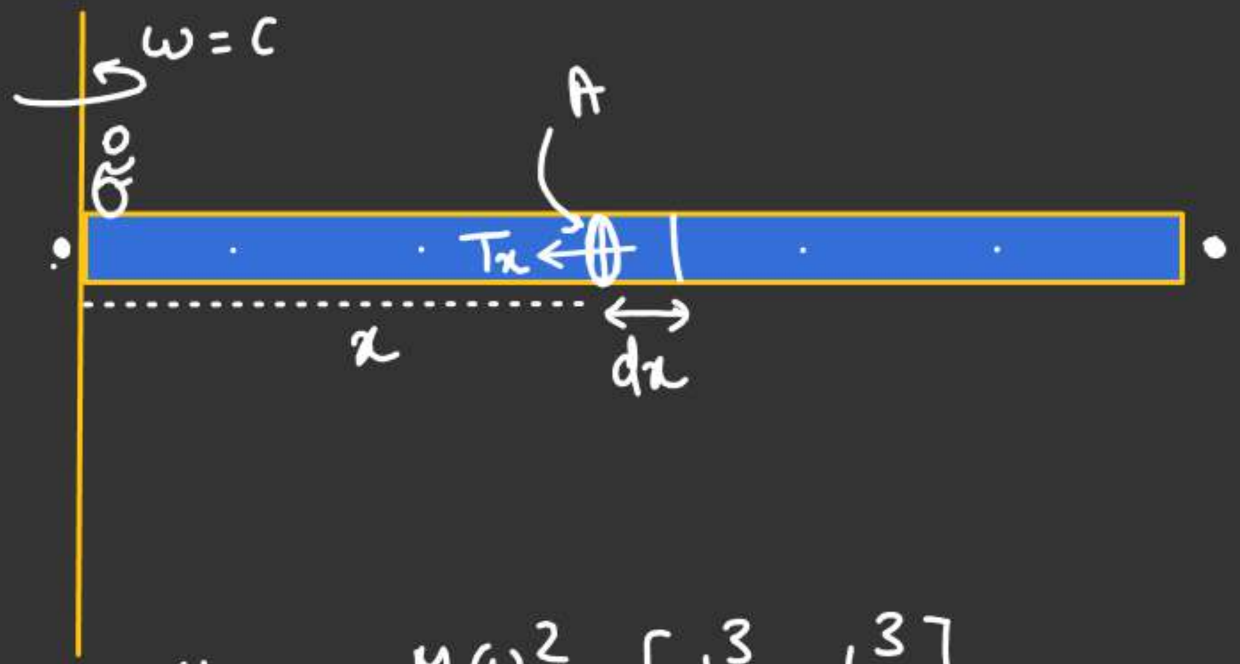
MECHANICAL PROPERTIES OF SOLIDS

✓ Total Elongation in rotating rod

$$\boxed{T_x = \frac{M\omega^2}{2L}(L^2 - x^2)}$$



let, dx' be elongation in dx length.

$$\text{stress} = Y\,\text{strain}$$

$$\frac{T_x}{A} = Y\frac{dx'}{dx}$$

$$\frac{M\omega^2}{A\,2YL}\int_0^L (L^2 - x^2)\,dx = \int_0^{\Delta L} dx'$$

$$\frac{M\omega^2}{2YLA}\left[L^2\int_0^L dx - \int_0^L x^2 dx\right] = \Delta L$$

$$\Delta L = \frac{M\omega^2}{2YLA}\left[L^3 - \frac{L^3}{3}\right]$$

$$\boxed{\Delta L = \left(\frac{M\omega^2 L^2}{3YA}\right)}$$

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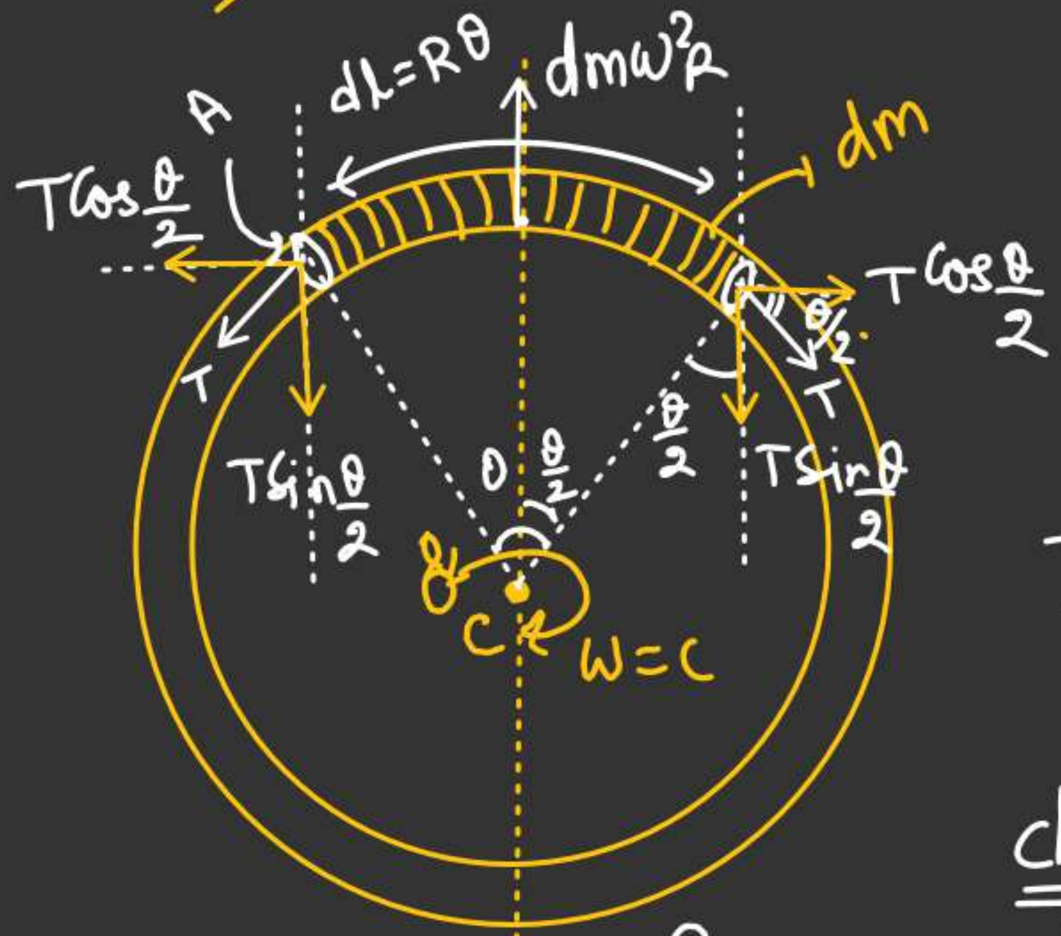
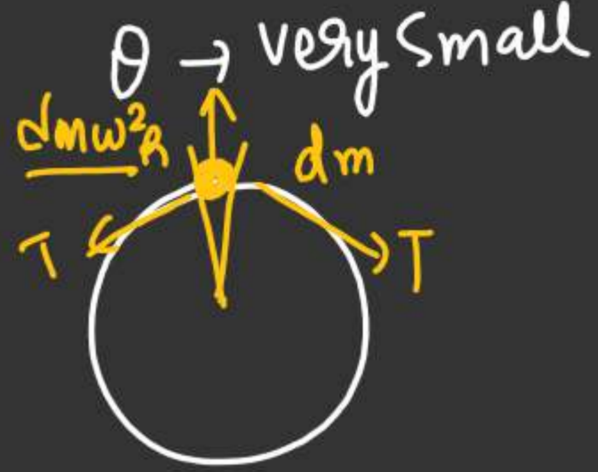
Case of Rotating ring (Elongation = ??)



A = Cross sectional area of ring



$$dm = \frac{M}{2\pi R} \times R\theta = \frac{M\theta}{2\pi}, \quad L = 2\pi R$$

In Rotating frame force balance on dm.

$$dm\,\omega^2 R = 2T\sin\frac{\theta}{2}$$

$$\sin\frac{\theta}{2} \simeq \frac{\theta}{2}$$

$$\frac{M\cancel{\theta}}{2\pi}\omega^2 R = \cancel{2}T\left(\frac{\cancel{\theta}}{\cancel{2}}\right)$$

$$\boxed{T = \left(\frac{M\omega^2 R}{2\pi}\right)}$$

$$\text{Stress} = Y(\text{Strain})$$

$$\frac{T}{A} = Y\left(\frac{\Delta L}{L}\right)$$

$$\frac{M\omega^2 R}{2\pi A} = \frac{Y\,\Delta L}{L}$$

$$\frac{M\omega^2 R}{\cancel{2\pi} A} = \frac{Y\Delta L}{\cancel{2\pi} R}$$

$$\left(\frac{M\omega^2 R^2}{YA} = \Delta L\right)$$

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$$\Delta L = \frac{M\omega^2 R^2}{YA}$$

Change in radius.

$$l_f = 2\pi(R+\Delta R)$$

$$l_i = 2\pi R$$

$$\Delta L = l_f - l_i = 2\pi(R+\Delta R) - 2\pi R$$

$$= (2\pi\,\Delta R)$$

$$2\pi\Delta R = \frac{M\omega^2 R^2}{YA}$$

$$\boxed{\Delta R = \frac{M\omega^2 R^2}{2\pi YA}}$$

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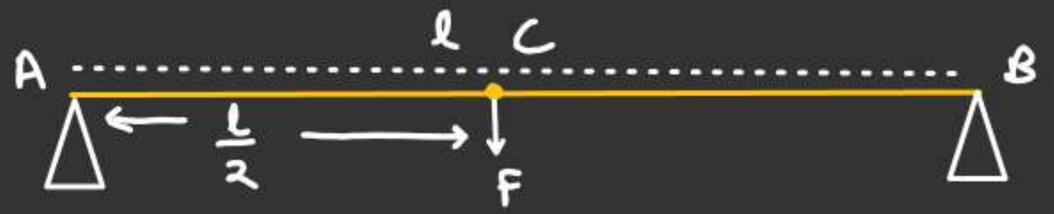
Rod fixed on two rigid Support at its end. A Constant force F applied at its mid-point. due to this mid point displaced vertically by δ. Find stress = ??

($\delta << l$) (given)



$$L_{AC} = L_{CB} = \sqrt{\frac{l^2}{4} + \delta^2}$$

$$\Delta L = (L_{AC} + L_{CB}) - l$$

$$\Delta L = 2\sqrt{\frac{l^2}{4} + \delta^2} - l$$

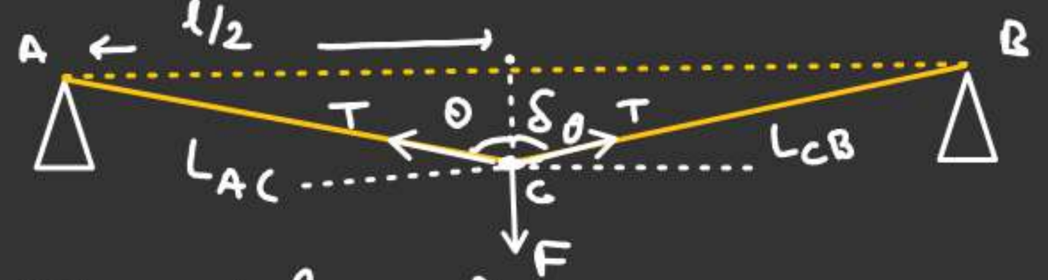
$$= \cancel{2}\frac{l}{\cancel{2}}\left(1 + \frac{4\delta^2}{l^2}\right)^{\frac{1}{2}} - l$$

$$= l\left(1 + \frac{4\delta^2}{l^2} \times \frac{1}{2}\right) - l$$

$$= \left(\frac{2\delta^2}{l^2}\right)$$

$$\text{Stress} = Y(\text{strain})$$

$$= Y\left(\frac{2\delta^2}{l^2}\right)$$



Binomial approximation.