DPP-03

SOLUTION

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1. The value of $\tan\left[\sin^{-1}\left(\frac{3}{5}\right)+\tan^{-1}\left(\frac{2}{3}\right)\right]$ is

(A) $\frac{6}{17}$ (B) $\frac{7}{16}$ (C) $\frac{5}{7}$ (D) $\frac{17}{6}$

Sol. Given that $\tan\left(\sin^{-1}\frac{3}{5}+\cot^{-1}\frac{3}{2}\right)$

$\therefore \tan\left(\sin^{-1}\frac{3}{5}+\cot^{-1}\frac{3}{2}\right)=\tan\left(\tan^{-1}\frac{3}{\sqrt{5^2-3^2}}+\tan^{-1}\frac{2}{3}\right)$

[as $\sin^{-1}\frac{a}{b}=\tan^{-1}\frac{a}{\sqrt{b^2-a^2}}$ and $\cot^{-1}\frac{a}{b}=\tan^{-1}\frac{b}{a}$]

$=\tan\left(\tan^{-1}\frac{3}{4}+\tan^{-1}\frac{2}{3}\right)$

$=\tan\left[\tan^{-1}\left(\frac{\frac{3}{4}+\frac{2}{3}}{1-\frac{3}{4}\times\frac{2}{3}}\right)\right]$

$=\tan\left[\tan^{-1}(9+8/4\times 3)/(4\times 3-3\times 2)/(4\times 3)\right]$

$=\tan\left(\tan^{-1}17/6\right)=17/6$

2. $\tan\left(\frac{\pi}{4}+\frac{1}{2}\cos^{-1}x\right)+\tan\left(\frac{\pi}{4}-\frac{1}{2}\cos^{-1}x\right)$, $x\neq 0$, is equal to

(A) x (B) 2x (C) $\frac{2}{x}$ (D) $\frac{x}{2}$

Sol. Let $\frac{1}{2}\cos^{-1}(x)=A$

Therefore simplifying the above expression, we get

$$\frac{1-\tan A}{1+\tan A}+\frac{1+\tan A}{1-\tan A}$$

$$=\frac{2(1+\tan^2 A)}{1-\tan^2 A}$$

$$=\frac{2}{\cos 2A}$$

$$=\frac{2}{\cos(\cos^{-1}(x))}$$

$$=\frac{2}{x}$$

3. The value of $\sin^{-1}[\cos\{\cos^{-1}(\cos x)+\sin^{-1}(\sin x)\}]$, where $x\in\left(\frac{\pi}{2},\pi\right)$ is

(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{4}$ (C) $\frac{-\pi}{4}$ (D) $\frac{-\pi}{2}$

Sol. The value of $\sin^{-1}[\cos\{\cos^{-1}(\cos x)+\sin^{-1}(\sin x)\}]$

$=\sin^{-1}[\cos\{x+\pi-x\}]$

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$= \sin^{-1}[\cos(\pi)] = \sin^{-1}(-1)$

$= -\frac{\pi}{2}$

4. If $x < 0$ then value of $\tan^{-1}x + \tan^{-1}\left(\frac{1}{x}\right)$ is equal to

(A) $\frac{\pi}{2}$ (B) $\frac{-\pi}{2}$ (C) 0 (D) None of these

Sol. We know

$$\tan^{-1}\frac{1}{x} = \begin{cases} \cot^{-1}x, & \text{for } x > 0 \\ -\pi + \cot^{-1}x, & \text{for } x < 0 \end{cases}$$

$\therefore \tan^{-1}x + \tan^{-1}\frac{1}{x}$

$= \tan^{-1}x + \cot^{-1}x - \pi (x < 0)$

$= \frac{\pi}{2} - \pi \left(\tan^{-1}x + \cot^{-1}x = \frac{\pi}{2}, \forall x \in R\right)$

$= -\frac{\pi}{2}$

If $x < 0$, then $\tan^{-1}x + \tan^{-1}\frac{1}{x}$ is equal to $-\frac{\pi}{2}$.

5. $\tan^{-1}a + \tan^{-1}b$, where $a > 0, b > 0, ab > 1$ is equal to

(A) $\tan^{-1}\left(\frac{a+b}{1-ab}\right)$ (B) $\tan^{-1}\left(\frac{a+b}{1-ab}\right) - \pi$

(C) $\pi + \tan^{-1}\left(\frac{a+b}{1-ab}\right)$ (D) $\pi - \tan^{-1}\left(\frac{a+b}{1-ab}\right)$

Sol. Given, $\tan^{-1}(a) + \tan^{-1}(b)$

$$= \tan^{-1}\left(\frac{a+b}{1-ab}\right)$$

However, $ab > 1$

Therefore,

$$\pi + \tan^{-1}\left(\frac{a+b}{1-ab}\right)$$

6. The number of solutions of the equation $\mathrm{Tan}^{-1}(1+x) + \mathrm{Tan}^{-1}(1-x) = \frac{\pi}{2}$ is

(A) 3 (B) 2 (C) 1 (D) 4

Sol. Since we know that $\tan^{-1} x + \tan^{-1} y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$

$\Rightarrow \tan^{-1}(1+x) + \tan^{-1}(1-x) = \tan^{-1}\left(\frac{(1+x)+(1-x)}{1-(1+x)(1-x)}\right)$

$= \tan^{-1}\left(\frac{2}{1-(1-x^2)}\right)$ $= \tan^{-1}\left(\frac{2}{x^2}\right)$

Here since $\tan^{-1}(1+x) + \tan^{-1}(1-x) = \frac{\pi}{2}$

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$\Rightarrow \tan^{-1}\left(\frac{2}{x^2}\right) = \frac{\pi}{2}$

$\Rightarrow \tan^{-1}\left(\frac{2}{x^2}\right) = \tan^{-1}(\infty)\left(\because \tan\frac{\pi}{2} = \infty\right)$

$\Rightarrow \frac{2}{x^2} = \infty$ $\qquad\qquad$ $\Rightarrow x^2 = \frac{2}{\infty}$

$\Rightarrow \mathrm{x} = 0$

7. The number of real solutions of $\mathrm{Tan}^{-1}\sqrt{\mathrm{x}(\mathrm{x}+1)} + \sin^{-1}\sqrt{\mathrm{x}^2+\mathrm{x}+1} = \frac{\pi}{2}$ is

(A) Zero (B) one (C) Two (D) Infinite

Sol. From the above expression

$-1 \le \sqrt{\mathrm{x}^2+\mathrm{x}+1} \le 1$ and $\mathrm{x}(\mathrm{x}+1) \ge 0$

$\Rightarrow 0 \le \mathrm{x}^2+\mathrm{x}+1 \le 1$ and $\mathrm{x}^2+\mathrm{x}+1 \ge 1$

$\Rightarrow \mathrm{x}^2+\mathrm{x}+1 = 1$

$\Rightarrow \mathrm{x}^2+\mathrm{x} = 0$

$\Rightarrow \mathrm{x}(\mathrm{x}+1) = 0$

Hence there will be two solutions. One at $\mathrm{x} = -1$ and another at$\mathrm{x} = 0$.

8. If $\cot^{-1}\frac{\mathrm{n}}{\pi} > \frac{\pi}{6}$, $\mathrm{n} \in \mathrm{N}$, then the maximum Value of ' n ' is

(A) 1 (B) 5 (C) 9 (D) None of these

Sol. $\cot^{-1}\frac{n}{\pi} > \frac{\pi}{6}$

$\mathrm{n} < \pi \times \sqrt{3}$

$\mathrm{n} < \sqrt{3}\pi$

$\mathrm{n} < 5.16$

so max value of n is 5

9. Which of the following is correct?

(A) $\mathrm{Tan}1 > \mathrm{Tan}^{-1}1$ (B) $\mathrm{Tan}1 < \mathrm{Tan}^{-1}1$

(c) $\mathrm{Tan}1 = \mathrm{Tan}^{-1}1$ (D) None of these

Sol. We know, $\tan^{-1} 1 = \frac{\pi}{4}$ and $1 > \frac{\pi}{4}$

Now,

$1 > \frac{\pi}{4}$

$\tan 1 > \tan\frac{\pi}{4}$

$\tan 1 > 1$

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$\tan\ 1 > 1 > \frac{\pi}{4}$

$\tan\ 1 > \tan^{-1}\ 1$

10. If $\sum_{i=1}^{n} \cos^{-1}\alpha_i = 0$ then $\sum_{i=1}^{n} \alpha_i =$

(A) n (B) $-n$ (C) 0 (D) None of these

Sol. $\sum_{i=1}^{n} \cos^{-1}\alpha_1 = 0\ \because \cos^{-1}\alpha_k \in [0, \pi]$

$\cos^{-1}(\alpha_1) = 0 \forall i = 1, 2, \ldots, n$

$\alpha_i = 1 \forall i = 1, 2, \ldots, n$

$\sum_{i=1}^{n}\ \alpha_i = \sum_{i=1}^{n}\ 1 = n$

11. If $\sum_{i=1}^{2n} \sin^{-1}x_i = n\pi$, then $\sum_{i=1}^{2n} x_i$ is equal to

(A) n (B) 2n (C) $\frac{n(n+1)}{2}$ (D) None of these

Sol. If $\sum_{i=1}^{2n} \sin^{-1}x_i = n\pi$

then $\sin^{-1}x_i = \frac{\pi}{2}$ for all i

So, $x_i = 1$ for all i from 1 to 2n

Therefore, $\sum_{i=1}^{2n} x_i = \sum_{i=1}^{2n} 1 = 2n$

Hence Proved.

12. The number of solution (s) of the equation $\sin^{-1}x + \cos^{-1}(1-x) = \sin^{-1}(-x)$ is/are

(A) 0 (B) 1 (C) 2 (D) More than 2

Sol. $\sin(x) + \cos^{-1}(1-x) = \sin^{-1}(-x)$

$\cos^{-1}(1-x) = -\sin^{-1} x - \sin^{-1}(x)$ $\quad \because \sin^{-1}(-x) = -\sin^{-1}(x)$

$\cos^{-1}(1-x) = -2\sin^{-1}(x)$

$(1-x) = \cos(-2\sin^{-1}(x))$ $\quad$ let $2\sin^{-1}(x) = t$

$x = \sin\frac{t}{2}$

$1 - x = \cos(-t)$

$1 - x = \cos\ t$ $\quad \because \cos(-t) = \cos\ t$ $\quad \therefore \cos\ t = 1 - 2\sin^2\frac{t}{2}$

$x = 2x^2$ $\quad = 1 - 2x^2$

$2x^2 - x = 0$

$x(2x - 1) = 0$

$x = 0, x = \frac{1}{2}$

Verification: $\sin^{-1}(x) + \cos^{-1}(1-x) = \sin^{-1}(-x)$

for $x = 0$ $\quad \sin^{-1}(0) + \cos^{-1}(1-0) = \sin^{-1}(-0)$

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$0 + 0 = 0$

$x = 1$ $\frac{\pi}{6} + \frac{\pi}{3}$

$= \frac{3\pi}{6}$

$= \frac{\pi}{2} \Rightarrow \sin^{-1}(-1/2)$

$\sin^{-1}(-1/2) = -\frac{\pi}{6} \neq \frac{\pi}{2}$

$\therefore \frac{1}{2}$ is not the solution & only 0 is the solution

No. of solution = 1

The correct answer is option (B)

13. The value of $\text{Tan}^{-1}\left(\frac{a}{b+c}\right) + \text{Tan}^{-1}\left(\frac{b}{c+a}\right)$, if $\angle C = 90^\circ$ in triangle ABC, is

(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{2}$ (D) π

Sol. $\tan^{-1}\left(\frac{a}{b+c}\right) + \tan^{-1}\left(\frac{b}{c+a}\right)$

We know that $\tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$

Replace, x by $\frac{a}{b+c}$ and y by $\frac{b}{c+a}$

$$= \tan^{-1}\left(\frac{\frac{a}{b+c} + \frac{b}{c+a}}{1 - \frac{ab}{(b+c)(c+a)}}\right)$$

$$= \tan^{-1}\left(\frac{\frac{ac + a^2 + b^2 + bc}{(b+c)(c+a)}}{\frac{(b+c)(c+a) - ab}{(b+c)(c+a)}}\right)$$

$$= \tan^{-1}\left(\frac{\frac{a^2 + b^2 + bc + ac}{(b+c)(c+a)}}{\frac{bc + c^2 + ca}{(b+c)(c+a)}}\right)$$

$$= \tan^{-1}\frac{a^2 + b^2 + bc + ac}{bc + c^2 + ca}$$

Given: $\angle C = 90^\circ \Rightarrow a^2 + b^2 = c^2$

$$= \tan^{-1}\frac{c^2 + bc + ac}{bc + c^2 + ca}$$

$$= \tan^{-1}1 = \frac{\pi}{4}$$

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14. If x > 0, $\cos^{-1}\left(\frac{12}{x}\right) = \frac{\pi}{2} - \cos^{-1}\left(\frac{16}{x}\right)$ then x equals

(A) 12 (B) 16 (C) 20 (D) None of these

Sol. Given for $x > 0, \cos^{-1}\left(\frac{12}{x}\right) = \frac{\pi}{2} - \cos^{-1}\left(\frac{16}{x}\right) \Rightarrow \cos^{-1}\left(\frac{12}{x}\right) + \cos^{-1}\left(\frac{16}{x}\right) = \frac{\pi}{2}$

$$\Rightarrow \cos^{-1}\frac{12}{x}\cdot\frac{16}{x} - \sqrt{1-\left(\frac{12}{x}\right)^{-2}}\sqrt{1-\left(\frac{16}{x}\right)^{2}} = \frac{\pi}{2}$$

using $\cos^{-1}x + \cos^{-1}y = \cos^{-1} = \cos^{-1}\left(xy - \sqrt{(1-x^2)(1-y^2)}\right)$

$$\Rightarrow \frac{12}{x}\cdot\frac{16}{x} - \sqrt{1-\left(\frac{12}{x}\right)^{2}}\sqrt{1-\left(\frac{16}{x}\right)^{2}} = \cos\frac{\pi}{2}$$

$$\Rightarrow \left[1-\frac{12}{x}\right]^2\left[1-\frac{16}{x}\right]^2 = \left(\frac{12}{x}\right)^2\left(\frac{16}{x}\right)^2$$

$\Rightarrow x = 20$

15. Number of integral ordered pairs (a, b) for which

$\sin^{-1}(1 + b + b^2 + \cdots\infty) + \cos^{-1}\left(a - \frac{a^2}{3} + \frac{a^2}{9} - \cdots\infty\right) = \frac{\pi}{2}$ is

(A) 0 (B) 4 (c) 9 (D) infinitely many

Sol. We have,

$$\sin^{-1}(1 + b + b^2 + \cdots.\infty) + \cos^{-1}\left(a - \frac{a^2}{3} + \frac{a^2}{9} - \cdots..\infty\right) = \frac{\pi}{2}$$

$$(1 + b + b^2 + \cdots.\infty) = \left(a - \frac{a^2}{3} + \frac{a^2}{9} - \cdots..\infty\right)$$

$$\left(\frac{1}{1-b}\right) = \left(\frac{a}{+\frac{a}{3}}\right)$$

$$\left(\frac{1}{1-b}\right) = \left(\frac{3a}{3+a}\right)$$

Zero ordered pairs of (a, b) are possible.

16. If $\cos^{-1}(2x^2 - 1) = 2\pi - 2\cos^{-1}x$, then

(A) $x \in [-1,0]$ (B) $x \in [0,1]$ (C) $x \in \left[0, \frac{1}{\sqrt{2}}\right]$ (D) $x \in \left[\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right]$

Sol. Now, $\cos^{-1}(2x^2 - 1) = 2\pi - 2\cos^{-1}x$

Put $x = \cos\theta$ in (1), we get

$\cos^{-1}(2\cos^2\theta - 1) = 2\pi - 2\cos^{-1}(\cos\theta)$

$\Rightarrow \cos^{-1}(\cos 2\theta) = 2\pi - 2\theta$

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$\Rightarrow\ \cos 2\theta = \cos(2\pi - 2\theta)$

This is possible when $\pi < 2\theta \leqslant 2\pi$

i.e, $\frac{\pi}{2} < \theta \leqslant \pi$

i.e, $\frac{\pi}{2} < \cos^{-1}x \leqslant \pi$

if $\begin{array}{c} -1 \leqslant x < 0 \\ x \in [1,0) \end{array}$

17. Evaluate each of the following

(i) $\sin^{-1}\left(\sin\frac{7\pi}{6}\right)$

Sol. The value of $\sin\frac{7\pi}{6}$ is $\frac{-1}{2}$

$\therefore$ The question becomes $\sin^{-1}\left(\frac{-1}{2}\right)$

Let $\sin^{-1}\left(\frac{-1}{2}\right) = y$

$\Rightarrow -\sin\ y = \frac{1}{2}$

$= -\sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$

As, $-\sin(\theta)$ is $\sin(-\theta)$.

$\Rightarrow -\sin\left(\frac{\pi}{6}\right) = \sin\left(\frac{-\pi}{6}\right)$

The range of principal value of $\sin^{-1}$ is $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$ and $\sin\left(\frac{\pi}{6}\right) = \frac{-1}{2}$

Therefore, the value of $\sin^{-1}\left(\sin\frac{7\pi}{6}\right)$ is $\frac{-\pi}{6}$.

(ii) $\tan^{-1}\left(\tan\frac{2\pi}{3}\right)$

Sol. $\tan^{-1}(\tan\ \theta) = \theta;\ \theta \in \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$

$\tan^{-1}\left(\tan\left(\frac{2\pi}{3}\right)\right)$

$= \tan^{-1}\left(\tan\left(\pi - \frac{\pi}{3}\right)\right)$

$= \tan^{-1}\left(-\tan\left(\frac{\pi}{3}\right)\right)$

$= \tan^{-1}\left(\tan\left(-\frac{\pi}{3}\right)\right)$

$= -\frac{\pi}{3}$

(iii) $\cos^{-1}\left(\cos\frac{5\pi}{4}\right)$

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Sol. Now,

$\therefore$ The question becomes $\cos^{-1}\left(\frac{-1}{\sqrt{2}}\right)$

Let $\cos^{-1}\left(\frac{-1}{\sqrt{2}}\right) = y$

$\Rightarrow \cos\ y = \frac{-1}{\sqrt{2}}$

$= -\cos\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$

$= \cos\left(\pi - \frac{\pi}{4}\right) = \frac{-1}{\sqrt{2}}$

$\Rightarrow \cos\left(\frac{3\pi}{4}\right) = \frac{-1}{\sqrt{2}}$

The range of principal value of $\cos^{-1}$ is $[0, \pi]$ and $\cos\left(\frac{3\pi}{4}\right) = \frac{-1}{\sqrt{2}}$

Therefore, the value of $\cos^{-1}\left(\cos\left(\frac{5\pi}{4}\right)\right)$ is $\frac{3\pi}{4}$.

(iv) $\sec^{-1}\left(\sec\frac{7\pi}{4}\right)$

Sol. We know that

$\sec^{-1}(\sec\ \theta) = \theta,\ [0, \pi/2) \cup (\pi/2, \pi]$

We have

$\sec^{-1}\left(\sec\ \frac{7\pi}{3}\right) = \sec^{-1}\left[\sec\left(2\pi + \frac{\pi}{3}\right)\right]$

$= \sec^{-1}\left[\sec\left(\frac{\pi}{3}\right)\right]$

$= \frac{\pi}{3}$

18. Find the value of the following

(i) $\sin^{-1}(\sin 5)$

Sol. $\sin^{-1}(\sin\ \theta) = \theta\ \ -\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$

Hence, first we write the expression such that $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$

$\sin^{-1}(\sin\ 5) = \sin^{-1}\ \sin(5 - 2\pi)$ (Since $5 - 2\pi > -\pi/2$)

$\sin^{-1}\ \sin\ 5 = 5 - 2\pi$

(ii) $\cos^{-1}(\cos 10)$

Sol. $\cos - 1(\cos\ 10) = \cos - 1(\cos\ (4\pi - 10)) = 4\pi - 10$

(iii) $\tan^{-1}(\tan(-6))$

Sol. $\therefore \tan - 1\{\tan - 6\}\ = \tan - 1\{\tan(\tan(2\pi - 6))\} = 2\pi - 6$

(iv) $\cot^{-1}(\cot(-10))$

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Sol. $\cot^{-1}(cot(-10))$

$\cot^{-1}(\cot x) = x \; x \in (0, \pi)$

$\cot^{-1}(-10+10)$

$\cot^{-1}(10+(4\pi-10))$

$= 4\pi - 10$

(v) $\cos^{-1}\left(\frac{1}{\sqrt{2}}\left(\cos\frac{9\pi}{10} - \sin\frac{9\pi}{10}\right)\right)$

Sol. $\cos^{-1}\left\{\left(\frac{1}{\sqrt{2}}\right)\left[\cos\left(\frac{9\pi}{10}\right) - \sin\left(\frac{9\pi}{10}\right)\right]\right\}$

$= \cos^{-1}\left\{\left(\frac{1}{\sqrt{2}}\right)\left[\cos\left(\pi - \frac{\pi}{10}\right) - \sin\left(\pi - \frac{\pi}{10}\right)\right]\right\}$ {**Domain of** $\cos^{-1} x$ is $[-1,1]$ }

$= \cos^{-1}\left\{\left(\frac{1}{\sqrt{2}}\right)\left[-\cos\left(\frac{\pi}{10}\right) - \sin\left(\frac{\pi}{10}\right)\right]\right\}$

$= \cos^{-1}\left\{(-1)\left[\frac{1}{\sqrt{2}}\cos\left(\frac{\pi}{10}\right) + \frac{1}{\sqrt{2}}\sin\left(\frac{\pi}{10}\right)\right]\right\}$

$= \cos^{-1}\left\{(-1)\left[\cos\frac{\pi}{4}\cos\left(\frac{\pi}{10}\right) + \sin\frac{\pi}{4}\sin\left(\frac{\pi}{10}\right)\right]\right\}$

$= \cos^{-1}\left\{(-1)\left[\cos\left(\frac{\pi}{4} - \frac{\pi}{10}\right)\right]\right\}$

$[\because \cos A \sin B + \sin A \cos B = \cos(A-B)]$

$= \pi - \cos^{-1}\left(\cos\frac{3\pi}{20}\right) = \pi - \frac{3\pi}{20} = \frac{17\pi}{20}$ $[\because \cos^{-1}[\cos(-\mathbf{x})] = \pi - \mathbf{x}]$

19. Find $\sin^{-1}(\sin\theta), \cos^{-1}(\cos\theta), \tan^{-1}(\tan\theta)$ and $\cot^{-1}(\cot\theta)$ for $\theta \in \left[\frac{3\pi}{2}, 3\pi\right]$

20. Prove each of the following:

(i) $\tan^{-1}x = -\pi + \cot^{-1}\frac{1}{x} = \sin^{-1}\frac{x}{\sqrt{1+x^2}} = -\cos^{-1}\frac{1}{\sqrt{1+x^2}}$ when $x < 0$.

21. Prove that $\sin^{-1}\cos(\sin^{-1}x) + \cos^{-1}\sin(\cos^{-1}x) = \frac{\pi}{2}, |\pi| \leq 1$

Sol. $\cos^{-1} x = \sin^{-1}\sqrt{1-x^2}$

$\therefore \sin(\cos^{-1} x) = \sin\left(\sin^{-1}\sqrt{1-x^2}\right) = \sqrt{1-x^2}$

Similarly, $\cos(\sin^{-1} x) = \cos\left(\cos^{-1}\sqrt{1-x^2}\right) = \sqrt{1-x^2}$

$\therefore \sin^{-1}\left(\cos(\sin^{-1} x)\right) + \cos^{-1}\left(\sin(\cos^{-1} x)\right)$

$= \sin^{-1}\left(\sqrt{1-x^2}\right) + \cos^{-1}\left(\sqrt{1-x^2}\right)$

$= \sin^{-1}\left(\sqrt{1-x^2}\right) + \cos^{-1}\left(\sqrt{1-x^2}\right)$

$= \frac{\pi}{2} \; \ldots \left[\sin^{-1}(\theta) + \cos^{-1}(\theta) = \frac{\pi}{2}\right]$

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22. $\tan(\tan^{-1}x + \tan^{-1}y + \tan^{-1}z) = \cot(\cot^{-1}x + \cot^{-1}y + \cot^{-1}z)$

Sol. $\tan(\tan^{-1}x + \tan^{-1}y + \tan^{-1}z)$

$$= \tan\left(\tan^{-1}\left(\frac{x+y+z-xyz}{1-xy-yz-zx}\right)\right)$$

$$= \left(\frac{x+y+z-xyz}{1-xy-yz-zx}\right)$$

Also, $\cot(\cot^{-1}x\cot^{-1}y + \cot^{-1}z)$

$$= \cot\left(\tan^{-1}\left(\frac{1}{x}\right) + \tan^{-1}\left(\frac{1}{y}\right) + \tan^{-1}\left(\frac{1}{z}\right)\right)$$

$$= \cot\left(\tan^{-1}\left(\frac{\frac{1}{x}+\frac{1}{y}+\frac{1}{z}-\frac{1}{xyz}}{1-\left(\frac{1}{xy}+\frac{1}{xz}+\frac{1}{yz}\right)}\right)\right)$$

$$= \cot\left(\tan^{-1}\left(\frac{xy+yz+zx-1}{xyz-(x+y+z)}\right)\right)$$

$$= \cot\left(\cot^{-1}\left(\frac{xyz-(x+y+z)}{xy+yz+zx-1}\right)\right)$$

$$= \cot\left(\cot^{-1}\left(\frac{(x+y+z)-xyz}{1-(xy+yz+zx)}\right)\right)$$

$$= \left(\frac{(x+y+z)-xyz}{1-(xy+yz+zx)}\right)$$

23. Prove that $\cos^{-1}\left(\sqrt{\frac{1}{3}}\right) - \cos^{-1}\left(\sqrt{\frac{1}{6}}\right) + \cos^{-1}\left(\frac{\sqrt{10}-1}{3\sqrt{2}}\right) = \cos^{-1}\left(\frac{2}{3}\right)$

Sol. $\cos^{-1}x - \cos^{-1}y = \cos^{-1}\left(xy + \sqrt{1-x^2}\sqrt{1-y^2}\right)$

$$\text{L.H.S} = \cos^{-1}\left(\frac{1}{\sqrt{18}} + \sqrt{1-\frac{1}{3}}\sqrt{1-\frac{1}{6}}\right) + \cos^{-1}\cdot\left(\frac{\sqrt{10}-1}{32}\right)$$

$$= \text{Cos}^{-1}\left(\frac{1+\sqrt{2}\sqrt{5}}{\sqrt{18}}\right) + \text{Cos}^{-1}\left(\frac{\sqrt{10}-1}{3\sqrt{2}}\right)$$

$$= \text{Cos}^{-1}\cdot\left(\frac{\sqrt{10}+1}{3\sqrt{2}}\right) + \text{Cos}^{-1}\left(\frac{\sqrt{10}-1}{3\sqrt{2}}\right)$$

$\because \cos^{-1}x + \cos^{-1}y = \cos^{-1}\left(xy - \sqrt{1-x^2}\sqrt{1-y^2}\right)$

$$= \cos^{-1}\left(\frac{10-1}{18} - \sqrt{1-\frac{(10+1+2\sqrt{10}}{18}}\sqrt{\frac{1-(10+1-2\sqrt{10}}{18}}\right)$$

$$= \text{Cos}^{-1}\left(\frac{9}{18} - \left(\sqrt{\frac{7-250}{18}\times\frac{7+2\sqrt{10}}{18}}\right)\right)$$

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$= \cos^{-1}\left(\frac{1}{2} - \frac{\sqrt{49-40}}{18^2}\right)$

$= \cos^{-1}\left(\frac{1}{2} - \frac{3}{18}\right) = \cos^{-1}\left(\frac{1}{2} = \frac{1}{6}\right) \quad = \cos^{-1}\left(\frac{2}{3}\right) =$ R.H.S

24. Prove that $2\tan^{-1}(\text{cosec} \tan^{-1}x - tan \cot^{-1}x) = \tan^{-1}x$

Sol. $2\tan^{-1}(\text{cosec}\ \tan^{-1} x - \tan^{-1}\ \cot^{-1})$

$= 2\tan^{-1}\left[\text{cosec}\left\{\text{cosec}^{-1}\frac{\sqrt{1+x^2}}{x}\right\} - \tan\left\{\tan^{-1}\frac{1}{x}\right\}\right]$

$= 2\tan^{-1}\left[\frac{\sqrt{1+x^2}}{x} - \frac{1}{x}\right]$

$= 2\tan^{-1}\left\{\frac{\sqrt{1+x^2}-1}{x}\right\}$

$= 2\tan^{-1}\left\{\frac{\sec\theta-1}{\tan\theta}\right\} (putx = \tan\theta)$

$= 2\tan^{-1}\left\{\frac{1-\cos\theta}{\sin\theta}\right\}$

$= 2\tan^{-1}\left\{\frac{2\sin^2\frac{\theta}{2}}{2\sin\frac{\theta}{2}\cdot\cos\frac{\theta}{2}}\right\}$

$= 2\tan^{-1}\ \tan\frac{\theta}{2}$

$= 2\cdot\frac{\theta}{2} = \theta = \tan - 1x$

25. Prove that $\cos^{-1}\left(\frac{63}{65}\right) + 2\tan^{-1}\left(\frac{1}{5}\right) = \sin^{-1}\left(\frac{3}{5}\right)$

Sol. $\cos^{-1}\frac{63}{65} + 2\tan^{-1}\frac{1}{5} = \sin^{-1}\frac{3}{5}$

Taking L.H.S., $\sin^{-1}\sqrt{1-\left(\frac{63}{65}\right)^2} + \sin^{-1}\frac{2\cdot\frac{1}{5}}{1+\left(\frac{1}{5}\right)^2}$ ...... $\left[\begin{array}{c} U\ sing, \\ \cos^{-1}x = \sin^{-1}\sqrt{1-x^2} \\ 2\tan^{-1}x = \sin^{-1}\frac{2x}{1+x^2}\end{array}\right]$

$= \sin^{-1}\sqrt{1-\frac{3969}{4225}} + \sin^{-1}\frac{2/5}{26/25}$

$= \sin^{-1}\sqrt{\frac{4225-3969}{4225}} + \sin^{-1}\frac{5}{13}$

$= \sin^{-1}\sqrt{\frac{256}{4225}} + \sin^{-1}\frac{5}{13}$

$= \sin^{-1}\frac{16}{65} + \sin^{-1}\frac{5}{13}$

[Using $\sin^{-1}x + \sin^{-1}y = \sin^{-1}\left(x\{\sqrt{1-x^2}\} + y\{\sqrt{1-x^2}\}\right)$]

$= \sin^{-1}\left\{\frac{16}{65}\sqrt{1-\left(\frac{5}{13}\right)^2} + \frac{5}{13}\sqrt{1-\left(\frac{16}{65}\right)^2}\right\}$

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$= \sin^{-1}\left(\frac{16}{65}\times\frac{12}{13}+\frac{5}{13}\times\frac{63}{65}\right)$

$= \sin^{-1}\left(\frac{16\times12+315}{13\times65}\right)$

$= \sin^{-1}\left(\frac{507}{65\times13}\right) = \sin^{-1}\frac{3}{5}$

26. Prove that

(i) $2\cos^{-1}\frac{3}{\sqrt{13}}+\cot^{-1}\frac{16}{63}+\frac{1}{2}\cos^{-1}\frac{7}{25}=\pi$

Sol. $= 2\cos^{-1}\frac{3}{\sqrt{13}}+\cot^{-1}\frac{16}{63}+\frac{1}{2}\cos^{-1}\frac{7}{25}$

$= 2\tan^{-1}(2/3)+\tan^{-1}(63/16)+\frac{1}{2}\tan^{-1}(24/7)$

$= \tan^{-1}(12/5)+\tan^{-1}(63/16)+\frac{1}{2}(2\tan^{-1}(3/4))$

$= \tan^{-1}(12/5)+\tan^{-1}(63/16)+\tan^{-1}(3/4)$

$= \tan^{-1}(12/5)+\tan^{-1}(3/4)+\tan^{-1}(63/16)$

$= \pi-\tan^{-1}63/16+\tan^{-1}(63/16)$

$= \pi$

(ii) $\cos^{1}\left(\frac{5}{13}\right)+\cos^{-1}\left(-\frac{7}{25}\right)+\sin^{-1}\frac{36}{325}=\pi$

Sol. $= \cos^{-1}\left(\frac{5}{13}\right)+\cos^{-1}\left(\frac{223}{225}\right)+\cos^{-1}\left(-\frac{7}{25}\right)$

$= \cos^{-1}\left(\frac{5}{13}\times\frac{323}{325}-\frac{12}{13}\times\cdot\frac{36}{325}\right)+\cos^{-1}\left(\frac{7}{25}\right)$

$= \cos^{-1}\left(\frac{1183}{13\times325}\right)+\cos^{-1}\left(-\frac{7}{25}\right)$

$= \cos^{-1}\left(\frac{91}{325}\right)+\cos^{-1}\left(\frac{-7}{25}\right)$

$= \cos^{-1}\left(\frac{7}{25}\right)+\cos^{-1}\left(\frac{-7}{25}\right)$

27. Show that:

$\sin^{-1}\left(sin\frac{33\pi}{7}\right)+\cos\left(\frac{46\pi}{7}\right)+\tan^{-1}(-\tan^{3\pi})+\sin\left(\cos\left(\frac{19\pi}{8}\right)\right)=\frac{13\pi}{7}$

Sol. $\sin^{-1}\left(\sin\frac{33\pi}{7}\right)+\cos^{-1}\left(\cos\frac{46\pi}{7}\right)+\tan^{-1}\left(-\tan\frac{13\pi}{8}\right)+\cot^{-1}\left(\cot\left(-\frac{19}{8}\right)\right.$

$= \sin^{-1}\left(\sin\left(4\pi+\frac{5\pi}{7}\right)\right)+\cos^{-1}\left(\cos\left(6\pi+\frac{4\pi}{7}\right)\right)+\tan^{-1}\left(-\tan\left(2\pi-\frac{3\pi}{8}\right)\right)$

$+\cot^{-1}\left(-\cot\left(2\pi+\frac{3\pi}{8}\right)\right)$

$= \sin^{-1}\left(\sin\left(\frac{5\pi}{7}\right)\right)+\cos^{-1}\left(\cos\frac{4\pi}{7}\right)+\tan^{-1}\left(\tan\frac{3\pi}{8}\right)+\cot^{-1}\left(-\cot\frac{3\pi}{8}\right)$

$= \sin^{-1}\left(\sin\left(\pi-\frac{2\pi}{7}\right)\right)+\frac{4\pi}{5}+\frac{3\pi}{8}+\frac{5\pi}{8}$

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$= \frac{13\pi}{7}$

28. Solve for x: $\sin^{-1}\left(\sin\left(\frac{2x^2+4}{1+x^2}\right)\right) < \pi - 3$

29. If the sum $\sum_{n=1}^{10}\sum_{m=1}^{10}\tan^{-1}\left(\frac{m}{n}\right) = k\pi$, find the value of k.

$S = \sum_{b=1}^{10}\sum_{a=1}^{10}\tan^{-1}\left(\frac{a}{b}\right)$

$= \sum_{b=1}^{10}\left[\tan^{-1}\left(\frac{1}{b}\right) + \tan^{-1}\left(\frac{2}{b}\right) + \tan^{-1}\left(\frac{3}{b}\right) + \cdots. + \tan^{-1}\left(\frac{10}{b}\right)\right]$

$= \tan^{-1}\left(\frac{1}{1}\right) + \tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{3}\right) + \cdots. + \tan^{-1}\left(\frac{1}{10}\right)$

$+\tan^{-1}\left(\frac{2}{1}\right) + \tan^{-1}\left(\frac{2}{2}\right) + \tan^{-1}\left(\frac{2}{3}\right) + \cdots. + \tan^{-1}\left(\frac{2}{10}\right)$

$+\tan^{-1}\left(\frac{3}{1}\right) + \tan^{-1}\left(\frac{3}{2}\right) + \tan^{-1}\left(\frac{3}{3}\right) + \cdots. + \tan^{-1}\left(\frac{3}{10}\right)$

$+\tan^{-1}\left(\frac{10}{1}\right) + \tan^{-1}\left(\frac{10}{2}\right) + \tan^{-1}\left(\frac{10}{3}\right) + \cdots. + \tan^{-1}\left(\frac{10}{10}\right)$

$= 10 \times \frac{\pi}{4} + 45 \times \frac{\pi}{2}$

$= 25\pi$

30. Let $y = \sin^{-1}(\sin 8) - \text{tant}^{-1}(\tan 10) + \cos^{-1}(\cos 12)$

$-\sec^{-1}(\sec 9) + \cot^{-1}(\cot 6) - \text{cosec}^{-1}(\text{cosec} 7)$. If y simplifies to aπ + b then find (a − b).

Sol. Step 1: Changing angles based on range of inverse trigonometric function

$y = \sin^{-1}(\sin\ 8) - \tan^{-1}(\tan\ 10) + \cos^{-1}(\cos\ 12)$

$-\sec^{-1}(\sec\ 9) - \cot^{-1}(\cot\ 6) - \csc^{-1}(\csc\ 7)$

$= \sin^{-1}(\sin(3\pi - 8)) - \tan^{-1}(\tan(10 - 3\pi)) + \cos^{-1}(\cos(4\pi - 12)) - \sec^{-1}(\sec(9 - 2\pi))$

$-\cot^{-1}(\cot(6 - \pi)) - \csc^{-1}(\csc(7 - 2\pi))$

Step 2: Simplifying

$y = 3\pi - 8 - 10 + 3\pi + 4\pi - 12 - 9 + 2\pi - 6 + \pi - 7 + 2\pi$

$\Rightarrow y = 13\pi - 40 = a\pi + b$

$\Rightarrow a = 13, b = -40$

$\Rightarrow a - b = 13 + 40 = 53$

Hence value of a - b is 53.

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31. Prove that $\tan\left[\frac{\pi}{4}+\frac{1}{2}\cos^{-1}\frac{a}{b}\right]+\tan\left[\frac{\pi}{4}-\frac{1}{2}\cos^{-1}\frac{a}{b}\right]=\frac{2b}{a}$

32. Solve the following inequalities

(i) $\sin^{-1}x > -1$

(ii) $\cos^{-1}x < 2$

(iii) $\cot^{-1}x < -\sqrt{3}$

(iv) Solve the inequality:

$(\text{arcsecx})^2 - 6(\text{arcsecx}) + 8 > 0$