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Binomial Prob. Distribution

① When an experiment is repeated n times.

(2) Everytime we do it is known as trial.

(3) Every trial has 2 outcomes → Success & failure

(4) Success = p & failure = q & p+q = 1 always

(5) Prob. of getting "r" Success = $P(\overset{\nearrow \text{No of Success}}{X}=r)$

$$= {}^{n}C_{r}\,(p)^{r}(q)^{n-r}$$

(6) Prob. of getting atleast 3 Success.

$$P(X\geq 3) = P(x=3)+P(x=4)+P(x=5) - -$$

(7) Prob of getting atmost 3 success.

$$P(0\leq X\leq 3)$$

$$= P(X=0)+P(X=1)+P(X=2)+P(X=3)$$

Q A Coin is tossed $\overset{\text{B.P.D.}}{\boxed{10 \text{ times}}}$. What is the Prob. of getting exactly 6 heads.

n = 10, Success = heads का आना

$p = 1/2$, $q = 1/2$

$$P(x=6) = {}^{10}C_{6}\left(\tfrac{1}{2}\right)^{6}\left(\tfrac{1}{2}\right)^{4}$$

Q An ordinary 6 Sided die is Rolled 16 times →BPD. The Prob. that on exactly 5 of the Rolls either a 3 or 4 comes up, is?

n = 16, S = {1, 2, 3, 4, 5, 6}

3, 4 का आना = Success.

$p = \frac{2}{6} = 1/3$, $q = \frac{2}{3}$

$$P(X=5) = {}^{16}C_{5}\left(\tfrac{1}{3}\right)^{5}\left(\tfrac{2}{3}\right)^{11}$$

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Q A Pair of dice is thrown

6 times getting a doublet $\xrightarrow{\quad} p=\frac{6}{36}=\frac{1}{6}$

BPT ←

is Considered as a Success. $q=\frac{5}{6}$

Find Prob of

A) No Success

$$P(X=0)={}^6C_0\left(\frac{1}{6}\right)^0\left(\frac{5}{6}\right)^6.$$

(B) Exactly 1 success.

$$P(X=1)={}^6C_1\cdot\left(\frac{1}{6}\right)^1\left(\frac{5}{6}\right)^5$$

(C) Atleast one Success.

$$P(X\geq 1)=P(X=1)+P(X=2)+P(X=3)+P(X=4)+P(X=5)+P(X=6)$$

un

$$=1-P(\text{No success})$$

$$=1-{}^6C_0\left(\frac{1}{6}\right)^0\left(\frac{5}{6}\right)^6\ \underline{\underline{A}}$$

(D) Atmost 1 Success.

$$P(X\leq 1)=P(X=0)+P(X=1)$$

$$={}^6C_0\cdot\left(\frac{1}{6}\right)^0\cdot\left(\frac{5}{6}\right)^6+{}^6C_1\cdot\left(\frac{1}{6}\right)^1\left(\frac{5}{6}\right)^5$$

Q A drinker takes one step forward or backward. The probability that he takes step forward is .4. Find Prob. that end of 11th Step, he is one step away from starting pt.

$p=.4,\ q=.6\quad n=11$

$${}^{11}C_6(.4)^6\cdot(.6)^5+{}^{11}C_5(.4)^5(.6)^6$$

Q In a Binomial distribution $B(n, p = \frac{1}{4})$ If the Prob. of atleast one success is gr. than or equal to $\frac{9}{10}$ then $n \geq$?

$$P(\text{atleast one Success}) \geq \frac{9}{10}$$

$$1 - P(\text{No succ.}) \geq \frac{9}{10}$$

$$1 - {}^{n}C_{0} \cdot \left(\frac{1}{4}\right)^{0} \cdot \left(\frac{3}{4}\right)^{n} \geq \frac{9}{10}$$

$$\frac{1}{10} \geq \left(\frac{3}{4}\right)^{n}$$

$$\log_{10}\left(\frac{1}{10}\right) \geq n\left(\log_{10} 3 - \log_{10} 4\right)$$

$$-1 \geq n\left(\log_{10} 3 - \log_{10} 4\right)$$

$$1 \leq n\left(\log_{10} 4 - \log_{10} 3\right)$$

$$n \geq \frac{1}{\log_{10} 4 - \log_{10} 3}$$

Q For an Initial screening of admission test, a candidate is given 50 problems to solve. If Prob. that candidate solve any Prob is $\frac{4}{5}$, then Prob. that he is unable to solve less than 2 problems, is?

$$n = 50, \quad p = \frac{1}{5}, \quad q = \frac{4}{5}$$

Solve nhi hona is success.

= P(unable to solve Zero Prob) + P(unable to solve 1 prob).

$$= {}^{50}C_{0} \cdot \left(\frac{1}{5}\right)^{0} \cdot \left(\frac{4}{5}\right)^{50} + {}^{50}C_{1} \cdot \left(\frac{1}{5}\right)^{1} \cdot \left(\frac{4}{5}\right)^{49}$$

$$= \frac{(4)^{50}}{(5)^{50}} + 50 \times \frac{(4)^{49}}{(5)^{50}}$$

$$= \frac{(4)^{50} + (4)^{49} \cdot 50}{(5)^{50}}$$

$$= \frac{(4)^{49}(54)}{(5)^{50}}$$

$$= \frac{54}{5} \cdot \left(\frac{4}{5}\right)^{49} \text{ Ans}$$

Q In a bombing attack, there is 50% chance that bomb will hit the target. At least 2 Independent hits are Required to destroy the target Then Min No of bombs, that must be dropped to ensure that at least 99% chance of completely destroying the target, is

target destroy करने के लिए 12 bombardment is Required

target will be destroyed $= 1 - P(X=0) - P(X=1) \geq 99\%$

$$1 - \left({}^{n}C_{0} \cdot \left(\tfrac{1}{2}\right)^{0} \cdot \left(\tfrac{1}{2}\right)^{n} + {}^{n}C_{1} \left(\tfrac{1}{2}\right)^{1} \cdot \left(\tfrac{1}{2}\right)^{n-1} \right) \geq \frac{99}{100}$$

$$1 - \left(\frac{1}{2}\right)^{n} - n \cdot \left(\frac{1}{2}\right)^{n} \geq \frac{99}{100}$$

$$\frac{1}{100} \geq \left(\frac{1}{2}\right)^{n} (1+n)$$

$$\frac{1}{100} \geq \frac{n+1}{2^{n}}$$

$$\frac{2^{n}}{n+1} \geq 100$$

$$\frac{2^{11}}{12} \geq 100$$

$$n \geq 11$$

$n = 11$ (Min Bombs) Required

Probability Distributions

1) It is a mathematical fxn that gives the Prob. of occurence of different outcomes of an Experiments.

(2) Prob. dist. of a random Variable is given by.

X	x_1	x_2	x_3	
P(x)	$P(x_1)$	$P(x_2)$	$P(x_3)$	

x_1, x_2, x_3 -- are values of Random Var. X.

$P(x_1), P(x_2) P(x_3)$ are Respective Prob. of these Variables.

(3) $P(X=x_i) = p_i$

$P(x=x_3) = P(x_3) = p_3$

(4) $p_i > 0$ & $\sum p_i = 1$

Q:- A pair of dice is thrown 3 times. If getting a Sum of 9 on them, is Considered as Success. Write the Prob. distribution of No. of Successes.

$p = \frac{4}{36} = \frac{1}{9}$, $q = \frac{8}{9}$

$P(x=0) = {}^3C_0 \left(\frac{1}{9}\right)^0 \cdot \left(\frac{8}{9}\right)^3$

$P(x=1) = {}^3C_1 \left(\frac{1}{9}\right)^1 \left(\frac{8}{9}\right)^2$

$P(x=2) = {}^3C_2 \left(\frac{1}{9}\right)^2 \left(\frac{8}{9}\right)^1$

$P(X=3) = {}^3C_3 \left(\frac{1}{9}\right)^3 \left(\frac{8}{9}\right)^0$

X	0	1	2	3
$P(X_i)$	$\left(\frac{8}{9}\right)^3$	${}^3C_1 \frac{(8)^2}{9^3}$	${}^3C_2 \cdot \frac{8}{9^3}$	$\cdot {}^3C_3 \cdot \left(\frac{1}{9}\right)^3$

Q 2 Cards are drawn Successively with Replacement from a deck of 52 Cards Find Prob. distribution o No. of Aces.

X	0 Ace	1 Ace	2 Ace.
$P(x=x_i)$ $P(x_i)$	$\frac{(48)^2}{(52)^2}$	$\frac{8\times48}{(52)^2}$	$\left(\frac{4}{52}\right)^2$

Prob of 0 Ace = $\frac{48}{52}\cdot\frac{48}{52}$

Prob. of 1 Ace = $\frac{4}{52}\times\frac{48}{52}\times2$

Prob of 2 Aces = $\frac{4}{52}\times\frac{4}{52}$

Q A Random Variable X has following Prob. distribution

X	1	2	3	4	5
P(X)	K^2	$2K$	K	$2K$	$5K^2$

then $P(X>2)$

$= P(X=3)+P(X=4)+P(X=5)$

$= K+2K+5K^2$

$= 5K^2+3K$

$= 5\times\frac{1}{36}+\frac{3}{6}$

$= \frac{23}{36}$

$K^2+2K+K+2K+5K^2=1$

$6K^2+5K-1=0$

$6K^2+6K-K-1=0$

$(6K-1)(K+1)=0$

$K=1/6,\ K=-1$ $\otimes$

Mean & Variance

1) Mean = $\bar{x}$

We take Mean = μ also.

2) $$\bar{x} = \frac{\sum p_i x_i}{\sum p_i} = \frac{\sum p_i x_i}{1}$$

$$\downarrow$$

$$\mu = \sum p_i x_i$$

(3) Variance = σ^2

$$\sigma^2 = \sum p_i x_i^2 - \mu^2$$

$$\boxed{\sigma^2 = \sum p_i \cdot x_i^2 - \left(\sum p_i x_i\right)^2}$$

Q A Random Variable X is specified by following distribution

X	2 (x_1)	3 (x_2)	4 (x_3)
P(X)	.3 p_1	.4 p_2	.3 p_3

Then variance of distribution is?

① $\mu = p_1 \cdot x_1 + p_2 x_2 + p_3 x_3$

$= 2 \times .3 + 3 \times .4 + 4 \times .3$

$= .6 + 1.2 + 1.2 = 3$

(2) $\sigma^2 = p_1 x_1^2 + p_2 \cdot x_2^2 + p_3 \cdot x_3^2 - (3)^2$

$= .3 \times 4 + .4 \times 9 + .3 \times 16 - 9$

$= 1.2 + 3.6 + 4.8 - 9 = .6$

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Q Prob. dist. of Random variable X is given by

x	1	2	3	4	5
P(x)	K	2K	2K	3K	K

let $p = P(1<X<4/X<3)$

If $5p = \lambda K$ then $\lambda = ?$

Jee mains Qs

13+24

① $p = P\left(\frac{1<x<4}{x<3}\right) = \frac{P(1<x<4 \cap x<3)}{P(x<3)}$

$= \frac{P(X=2)}{P(X<3)} = \frac{P(X=2)}{P(X=1)+P(X=2)}$

$= \frac{2K}{K+2K} = \frac{2K}{3K} = \frac{2}{3}$

② $K+2K+2K+3K+K=1$

$9K=1$

$K=1/9$

③ $5p = \lambda K$

$5\times\frac{2}{\cancel{3}} = \lambda\cdot\frac{1}{\cancel{9}_3}$

$\boxed{\lambda=30}$

Mean & Variance of B.P.D.

→ n trials, Succ = p

failure = q.

$\boxed{\begin{array}{l}\text{mean} = np \\ \text{Variance} = npq\end{array}}$

Q If X follows a Binomial Distribution with mean = 3 & variance = $\frac{3}{2}$ Find

① $P(x\geq 1) = 1 - P(x=0)$

$= 1 - {}^6C_0\cdot\left(\frac{1}{2}\right)^0\cdot\left(\frac{1}{2}\right)^6$

(2) $P(X\leq 5) = 1 - P(X=6)$

$= 1 - {}^6C_6\cdot\left(\frac{1}{2}\right)^6\cdot\left(\frac{1}{2}\right)^0$

$np = 3$

$npq = \frac{3}{2}$

$\Rightarrow \frac{npq}{np} = \frac{3/2}{3}$

$\Rightarrow q = 1/2$

$\Rightarrow p = 1/2$

$n = 6$