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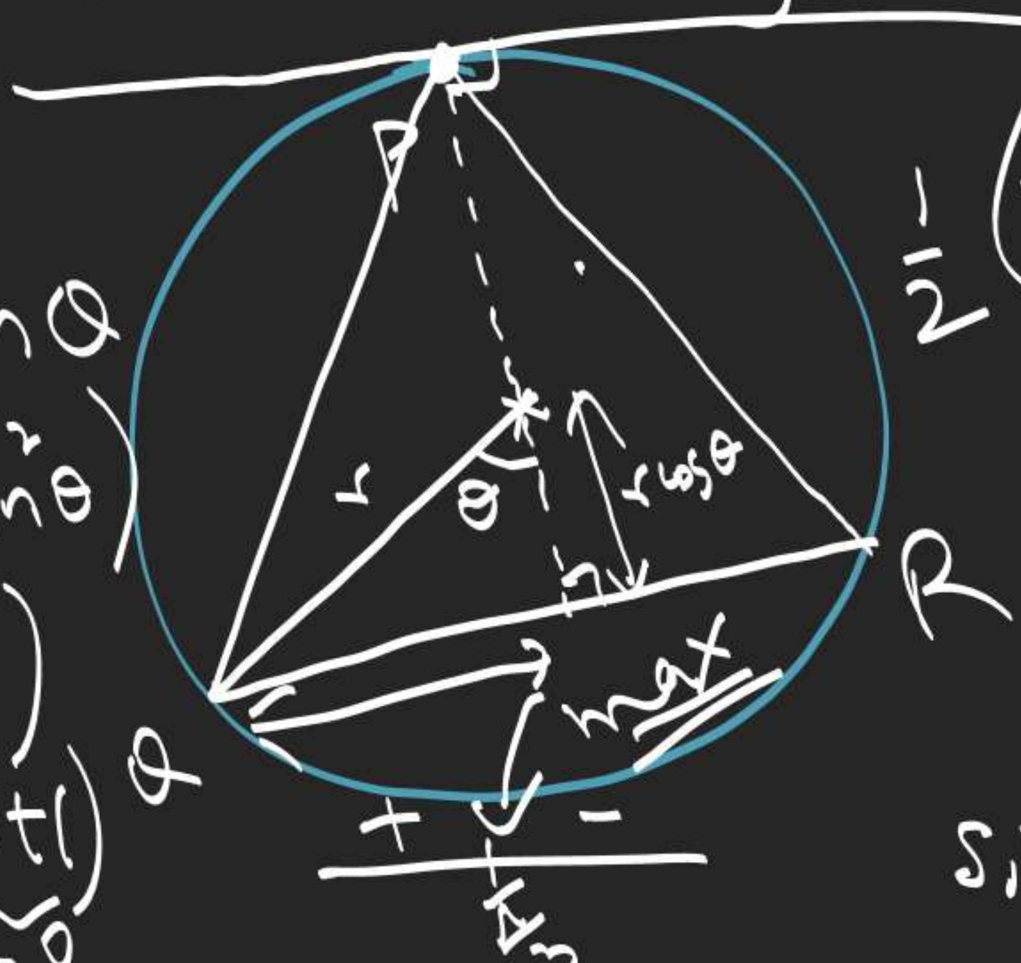
Q: A point 'P' is given on the circumference of a circle of radius 'r'. Chords QR are parallel to tangent at 'P'. Determine the maximum possible area of ΔPQR.

$A = \frac{3\sqrt{3}r^2}{4}$ if $\frac{1}{3}\cos^2\frac{\theta}{2} = \sin^2\frac{\theta}{2}$.



$$A = r^2(1+\cos\theta)\sin\theta$$

$$\frac{dA}{d\theta} = r^2(\cos\theta+\cos^2\theta-\sin^2\theta)$$

$$= r^2(2\cos^2\theta+\cos\theta-1)$$

$$= r^2(2\cos\theta-1)\underbrace{(\cos\theta+1)}_{>0}$$

$$\frac{1}{2}(r+r\cos\theta)\times(2r\sin\theta)$$

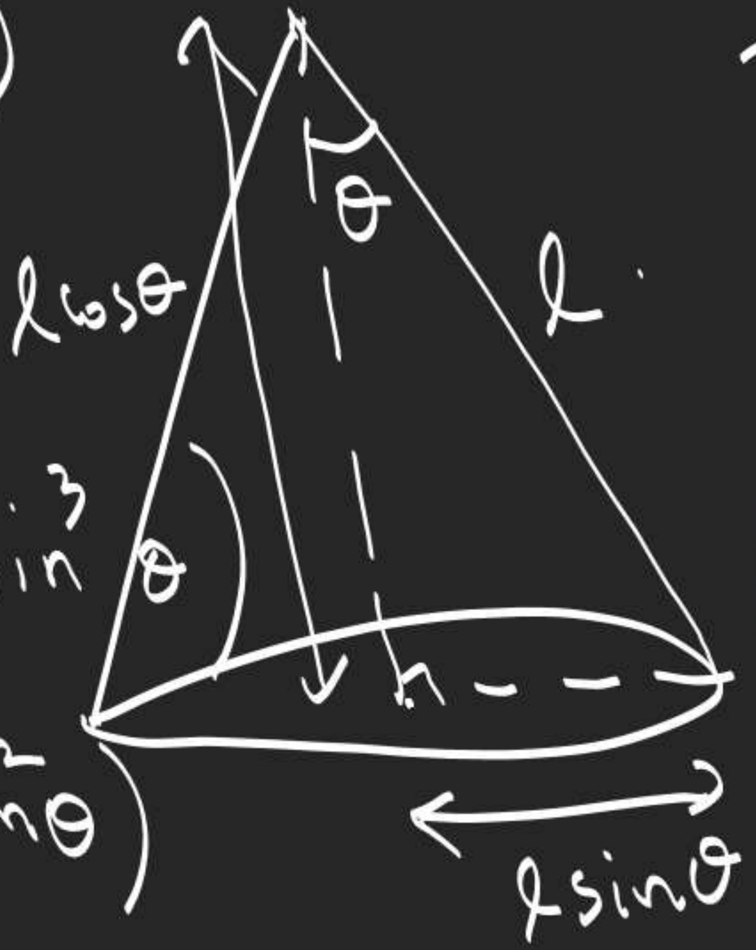
$$= 4r^2\sin\frac{\theta}{2}\cos^3\frac{\theta}{2} \le \frac{3\sqrt{3}r^2}{4}$$

$$\sin^2\frac{\theta}{2}\cos^6\frac{\theta}{2} \le 27\left(\frac{\frac{1}{3}\cos^2\frac{\theta}{2}\cdot 3+\sin^2\frac{\theta}{2}}{4}\right)^4$$

$$\sin\frac{\theta}{2}\cos^3\left(\frac{\theta}{2}\right) \le \frac{3\sqrt{3}}{16}$$

$$V = \frac{1}{3}\pi (l\sin\theta)^2 (l\cos\theta)$$

$$= \frac{\pi l^3}{3} \boxed{\sin^2\theta\cos\theta}$$



$\theta \in \left(0, \frac{\pi}{2}\right)$

$$\frac{dV}{d\theta} = 0 = \frac{\pi l^3}{3}\left(2\sin\theta\cos^2\theta - \sin^3\theta\right)$$

$$= \frac{\pi l^3}{3}\sin\theta\cos^2\theta\left(2 - \tan^2\theta\right)$$

$$\left(\frac{\sin^4\theta}{4}\cos^2\theta\right)^{1/3} \leq \frac{\frac{\sin^2\theta}{2} + \frac{\sin^2\theta}{2} + \cos^2\theta}{3}$$

$$\sin^2\theta + \cos^2\theta = 1$$

$$\sin^4\theta\cos^2\theta \leq \frac{4}{27}$$

$$\sin^2\theta\cos\theta \leq \frac{2}{3\sqrt{3}}$$

max

$f'(\theta)$ + ↓ −

$\tan^{-1}\sqrt{2}$ $\quad \theta$

V is max

$$\theta = \tan^{-1}\sqrt{2}$$

$$\sin^2\theta\cos\theta = \frac{2}{3\sqrt{3}}$$

if $\frac{\sin^2\theta}{2} = \cos^2\theta$

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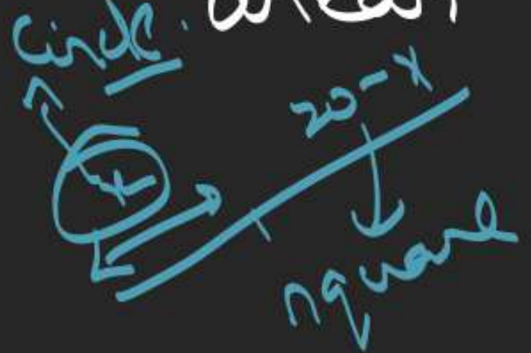
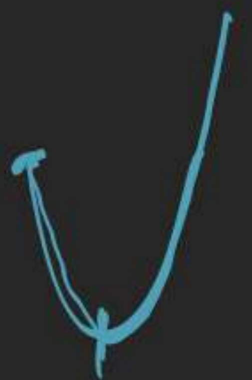
3. A wire of length 20 cm is cut into two pieces. One piece converted into a circle and the other into a square. Find where the wire is to be cut from so that sum of areas of two plane figures is (i) minimum (ii) maximum.



$$A = \pi\left(\frac{x}{2\pi}\right)^2 + \left(\frac{20-x}{4}\right)^2 = \frac{x^2}{4\pi} + \frac{400+x^2-40x}{16}$$



$$= \frac{(4+\pi)x^2 - 40\pi x + 400\pi}{16\pi} \qquad x \in [0,20]$$

(i) A min. at $x = \frac{20\pi}{4+\pi} < 10$ (ii) A max at $x = 20$.

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4. Find the eqn. of line thru (1, 8) cutting +ive coordinate axes at A & B, if

(i) area of ΔOAB, (O = origin) is minimum.

$y-8=m(x-1)$

$\frac{1}{2}\times\left(1-\frac{8}{m}\right)(8-m)$

$= -\frac{(m-8)^2}{2m}$

(ii) the intercept between coordinate axes is minimum.

$= \frac{1}{2}\left(-m-\frac{64}{m}+16\right)$

$m =$

(iii) sum of intercept on coordinate axes is minimum.

$\boxed{y-8=-8(x-1)}$

$ab = ?$

$\Rightarrow \frac{1}{a}+\frac{8}{b}=1$

$\boxed{\text{Ex-I (T/N)}}$

$\frac{\frac{1}{a}+\frac{8}{b}}{2} \geq \sqrt{\frac{8}{ab}}$

$\frac{1}{2} \geq \sqrt{\frac{8}{ab}}$

$ab \geq 32$

$\Delta \geq 16$

$\Delta = 16$ if $\frac{1}{a}=\frac{8}{b}=\frac{1}{2}$

$\frac{b}{a}=8$

$\frac{(-m)+\left(-\frac{64}{m}\right)}{2} \geq$