PROBABILITY

2. One ticket is selected at random from 50 tickets numbered $00, 01, 02, \ldots\ldots, 49$. Then the probability that the sum of the digits on the selected ticket is 8, given that the product of these digits is zero, equals [JEE MAIN 2009]

(A) $\frac{1}{7}$ (B) $\frac{5}{14}$ (C) $\frac{1}{50}$ (D) $\frac{1}{14}$

00, 01, 02, 03, 04, 05, 06, 07, 08

09, 10, 20, 30, 40

→ 14 ticket → Prod = 0

PROBABILITY

3. An urn contains nine balls of which three are red, four are blue and two are green. Three balls are drawn at random without replacement from the urn. The probability that the three balls have different colours is- [JEE MAIN 2010]

(A) $\frac{1}{3}$ (B) $\frac{2}{7}$ (C) $\frac{1}{21}$ (D) $\frac{2}{23}$

3R 4B 2G

$$\frac{{}^{4}C_1 \times {}^{3}C_1 \times {}^{2}C_1}{{}^{9}C_3} = \frac{24 \times 6}{9 \cdot 8 \cdot 7}$$

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4. Four numbers are chosen at random (without replacement) from the set $\{1, 2, 3, \ldots., 20\}$.

Statement -1: The probability that the chosen numbers when arranged in some order will form an AP is $\frac{1}{85}$.

20 ; 14, 8, 2

Statement-2: If the four chosen numbers form an AP, then the set of all possible values of common diference is $\{\pm 1, \pm 2, \pm 3, \pm 4, \pm 5\}$ [JEE MAIN 2010]

(A) Statement (1) is true and statement (2) is true and statement (2) is correct explanation for Statement (1)

(B) Statement (1) is true and statement (2) is true and statement (2) is NOT a correct explanation for Statement (1)

(C) Statement (1) is true but (2) is false ✓

(D) Statement (1) is false but (2) is true

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5. Consider 5 independent Bernoulli's trials each with probability of success p. If the probability of at least one failure is greater than or equal to $\frac{31}{32}$, then p lies in the interval: [JEE MAIN 2011]

(A) $\left(\frac{1}{2}, \frac{3}{4}\right]$ (B) $\left(\frac{3}{4}, \frac{11}{12}\right]$ (C) $\left[0, \frac{1}{2}\right]$ (D) $\left(\frac{11}{12}, 1\right]$

$1 - p^5 \geq \frac{31}{32}$

$\frac{1}{32} \geq p^5$

$p \leq \frac{1}{2}$

PROBABILITY

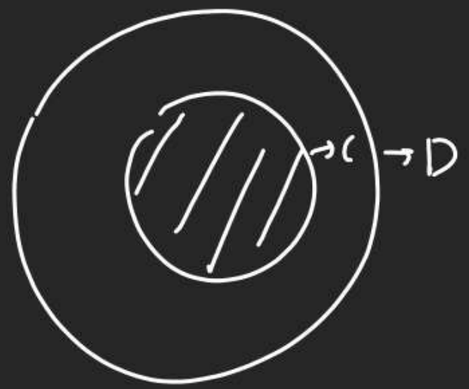
6. If C and D are two events such that $C \subset D$ and $P(D) \neq 0$, then the correct statement among the following is: [JEE MAIN 2011]

(A) $P(C \mid D) = P(C)$

(B) $P(C \mid D) \geq P(C)$

(C) $P(C \mid D) < P(C)$

(D) $P(C \mid D) = \frac{P(D)}{P(C)}$



$$P\left(\frac{C}{D}\right) = \frac{P(C \cap D)}{P(D)} = \frac{P(C)}{P(D)} \leq 1$$

$$P\left(\frac{C}{D}\right) < P(C)$$

PROBABILITY

7. Let A, B and C be pairwise independent events with $P(C) > 0$ and $P(A \cap B \cap C) = 0$.

Then, $P(A^C \cap B^C / C)$ is equal to **[JEE MAIN 2012]**

(A) $P(A^C) - P(B)$ (B) $P(A) - P(B^C)$ (C) $P(A^C) + P(B^C)$ (D) $P(A^C) - P(B^C)$

$$\frac{P(C) - P(A \cap B) - P(B \cap C) + P(A \cap B \cap C)}{P(C)}$$

Q Supp 2 dices are Rolled.

$A = \{1^{st}$ Roll is $1,2,3\}$

$B = \{1^{st}$ Roll is $3,4,5\}$

$C = \{$Sum of 2 Rolls is $9\}$

(Check Independance

$P(A \cap B) = \frac{1}{6} \neq P(A) \cdot P(B) = \frac{1}{2} \times \frac{1}{2}$

$P(B \cap C) = \frac{1+1+1}{36} = \frac{1}{12} \neq \frac{1}{2} \times \frac{4^2}{36} P(B) \cdot P(C)$

$P(A \cap C) = \frac{1}{36} \neq \frac{1}{2} \times \frac{4}{36} = P(A) \cdot P(C)$

} No Pair Wise Ind.

Mutually

$P(A \cap B \cap C) = \frac{1}{36} = \frac{1}{2} \times \frac{1}{2} \times \frac{4}{36} P(A) \cdot P(B) \cdot P(C)$

PROBABILITY

9. A multiple choice examination has 5 questions. Each question has three alternative answers of which exactly one is correct. The probability that a student will get 4 or more correct answers just by guessing is [JEE-MAIN 2013]

(A) $\frac{11}{3^5}$ (B) $\frac{10}{3^5}$ (C) $\frac{17}{3^5}$ (D) $\frac{13}{3^5}$

$$P(X \geq 4) = P(X=4) + P(X=5)$$

$$= {}^5C_4 \left(\frac{1}{3}\right)^4 \left(\frac{2}{3}\right) + {}^5C_5 \left(\frac{1}{3}\right)^5 \left(\frac{2}{3}\right)^0$$

PROBABILITY

10. Let A and B be two events such that $P(\overline{A \cup B}) = \frac{1}{6}, P(A \cap B) = \frac{1}{4}$ and $P(\bar{A}) = \frac{1}{4}$, where $\bar{A}$ stands for the complement of the event A. Then the events A and B are:

[JEE-MAIN 2014]

(A) mutually exclusive and independent.
(B) equally likely but not independent
(C) independent but not equally likely
(D) independent and equally likely

$1 - P(A \cup B) = \frac{1}{6}$

$P(A \cup B) = \frac{5}{6}$

$P(A \cap B) = \frac{1}{4}$

$P(\bar{A}) = \frac{1}{4}$

$P(A) = \frac{3}{4}$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\frac{5}{6} = \frac{3}{4} + P(B) - \frac{1}{4}$

$P(B) = \frac{1}{3}$

$P(A \cap B) = P(A) \cdot P(B)$

$\frac{1}{4} = \frac{3}{4} \times \frac{1}{3} = \frac{1}{4}$

$P(A) \neq P(B)$

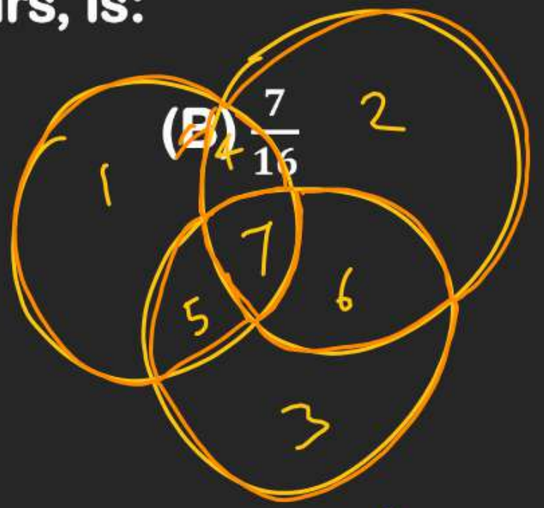
PROBABILITY

11. For there events A,B and C,P(Exactly one of A or B occurs) = P (Exctly one of B or C occures) = P (Exactly one of C or A occurs) $= \frac{1}{4}$ and P(All the three events occur simultaneously) $= \frac{1}{16}$. Then the probability that at least one of the events occurs, is:

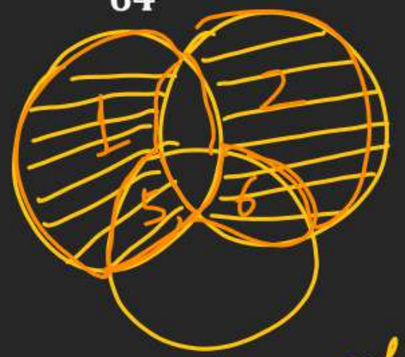
[JEE-MAIN 2017]

(A) $\frac{7}{32}$

(B) $\frac{7}{16}$



(C) $\frac{7}{64}$



(D) $\frac{3}{16}$

$1+5+2+6=\frac{1}{4}$

$2+3+4+5=\frac{1}{4}$

$1+4+6+3=\frac{1}{4}$

$2(1+2+3+4+5+6)=\frac{3}{4}$

$(1+2+3+4+5+6)=\frac{3}{8}=\frac{6}{16}$

$7=\frac{1}{16}$

$A \cup B \cup C = 1+2+3+4+5+6+7 = \frac{7}{16}$

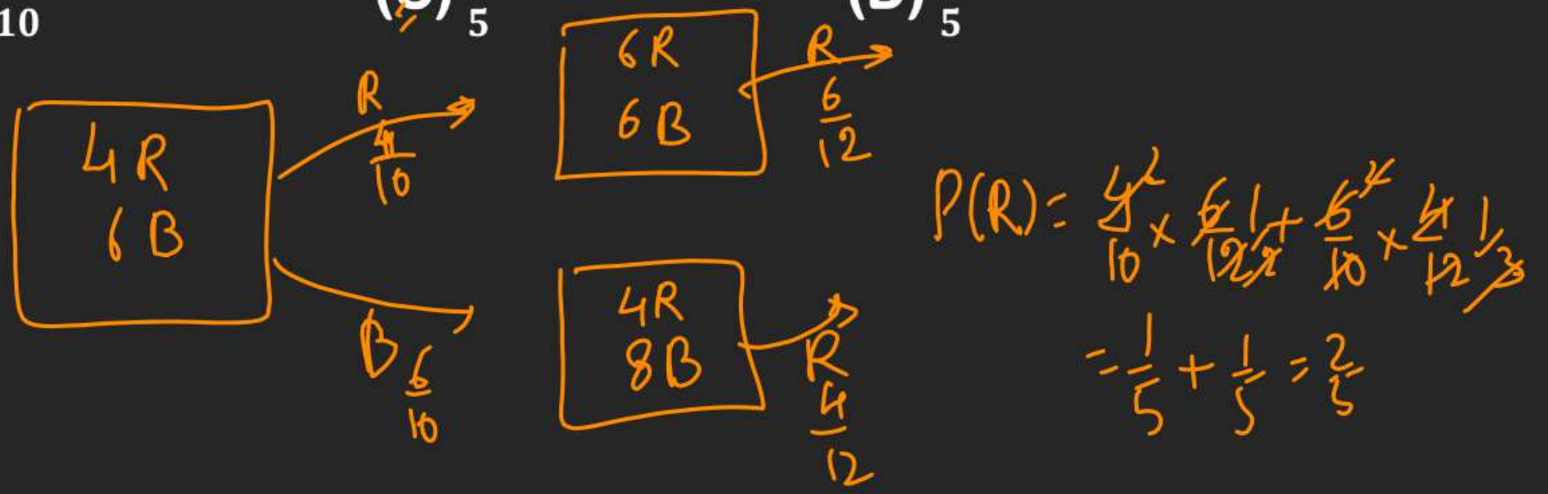
Exactly one of A or B occurs

Exactly one of B or C occurs

PROBABILITY

12. A bag contains 4 red and 6 black balls. A ball is drawn at random from the bag, its colour is observed and this ball along with two additional balls of the same colour are returned to the bag. If now a ball is drawn at random from the bag, then the probability that this drawn ball is red, is: [JEE-MAIN 2018]

(A) $\frac{3}{4}$ (B) $\frac{3}{10}$ (C) $\frac{2}{5}$ (D) $\frac{1}{5}$



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PROBABILITY

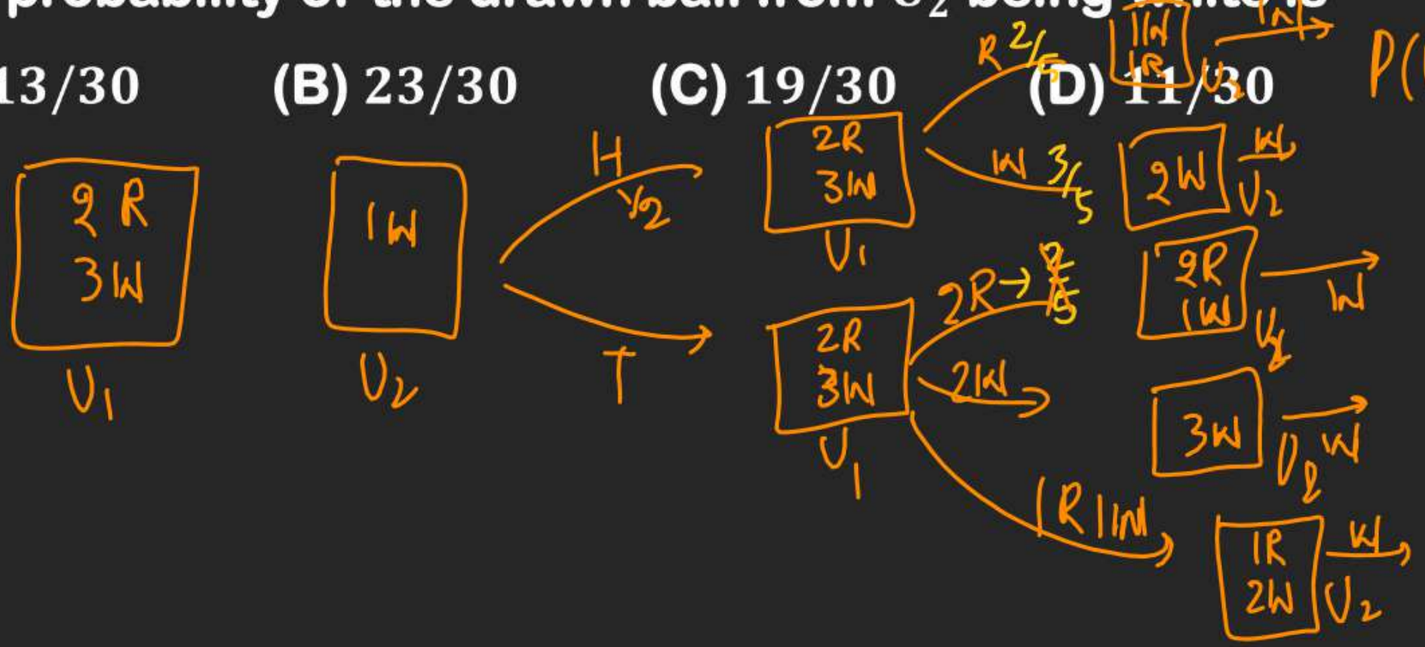
Paragraph for Question No. 2 to 3

Let U_1 and U_2 be two urns such that U_1 contains 3 white and 2 red balls, and U_2 contains only 1 white ball. A fair coin is tossed. If head appears then 1 ball is drawn at random from U_1 and put into U_2. However, if tail appears then 2 balls are drawn at random from U_1 and put into U_2. Now 1 ball is drawn at random from U_2.

[JEE ADVANCED 2011]

2. The probability of the drawn ball from U_2 being white is

(A) $13/30$ (B) $23/30$ (C) $19/30$ (D) $11/30$



$$P(W) = \left(\frac{1}{2}\times\frac{2}{5}\times\frac{1}{2} + \frac{1}{2}\times\frac{3}{5}\times 1\right) + \left(\frac{1}{2}\times\frac{2}{5}\times\frac{1}{3} + \frac{1}{2}\times\frac{3}{5}\times 1 + \frac{1}{2}\times\frac{3}{5}\times\frac{2}{3}\right)$$

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Paragraph for Question No. 2 to 3

Let U_1 and U_2 be two urns such that U_1 contains 3 white and 2 red balls, and U_2 contains only 1 white ball. A fair coin is tossed. If head appears then 1 ball is drawn at random from U_1 and put into U_2. However, if tail appears then 2 balls are drawn at random from U_1 and put into U_2. Now 1 ball is drawn at random from U_2.

[JEE ADVANCED 2011]

3. Given that the drawn ball from U_2 is white, the probability that head appeared on the coin is

(A) $17/23$ (B) $11/23$ (C) $15/23$ (D) $12/23$

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4. Let E and F be two independent events. The probability that exactly one of them occurs is $\frac{11}{25}$ and the probability of none of them occurring is $\frac{2}{25}$. If $\mathrm{P(T)}$ denotes the probability of occurrence of the event T, then

[JEE ADVANCED 2011]

(A) $\mathrm{P(E)} = \frac{4}{5}, \mathrm{P(F)} = \frac{3}{5}$

(B) $\mathrm{P(E)} = \frac{1}{5}, \mathrm{P(F)} = \frac{2}{5}$

Done.

(C) $\mathrm{P(E)} = \frac{2}{5}, \mathrm{P(F)} = \frac{1}{5}$

(D) $\mathrm{P(E)} = \frac{3}{5}, \mathrm{P(F)} = \frac{4}{5}$

PROBABILITY

5. A ship is fitted with three engines E_1, E_2 and E_3. The engines function independently of each other with respective probabilities $\frac{1}{2}, \frac{1}{4}$ and $\frac{1}{4}$. For the ship to be operational at least two of its engines must function. Let X denote the event that the ship is operational and let X_1, X_2 and X_3 denote respectively the events that the engines E_1, E_2 and E_3 are functioning. Which of the following is (are) true?

[JEE ADVANCED 2012]

(A) $P[X_1^c \mid X] = \frac{3}{16}$

(B) P [Exactly two engines of the ship are functioning $\mid X] = \frac{7}{8}$

(C) $P[X \mid X_2] = \frac{5}{16}$

(D) $P[X \mid X_1] = \frac{7}{16}$

$P(E_1) = 1/2 \quad P(E_2) = 1/4 \quad P(E_3) = 1/4$

$P(E_1 \cap E_2) = P(E_1) \cdot P(E_2) = 1/2 \times 1/4 = 1/8$

(2) $P(X) = \frac{1}{2} \times \frac{1}{4} \times \frac{3}{4} + \frac{1}{2} \times \frac{1}{4} \times \frac{1}{4} + \frac{1}{2} \times \frac{3}{4} \times \frac{1}{4} + \frac{1}{2} \times \frac{1}{4} \times \frac{1}{4} = \frac{3+1+3+1}{32} = \frac{8}{32} = 1/4$

(1) $P\left(\frac{\bar{X}_1}{X}\right) = \frac{\bar{E}_1 \cap X}{P(X)} = \frac{\frac{1}{2} \times \frac{1}{4} \times \frac{1}{4}}{1/4} = 1/8$

(3) $P\left(\frac{X}{X_2}\right) = \frac{X_2 \cap X}{P(X_2)} = \frac{\frac{3}{32} + \frac{1}{32} + \frac{1}{32}}{1/4} = \frac{5}{32} \times \frac{4}{1} = 5/8$

$P\left(\frac{E \cap 2}{X}\right) = \frac{\frac{1}{2} \times \frac{1}{4} \times \frac{3}{4} + \frac{1}{2} \times \frac{1}{4} \times \frac{1}{4} + \frac{1}{2} \times \frac{3}{4} \times \frac{1}{4}}{\left(\frac{1}{2} \times \frac{3}{4} \times \frac{1}{4} + \frac{1}{2} \times \frac{1}{4} \times \frac{1}{4} + \frac{1}{2} \times \frac{3}{4} \times \frac{1}{4} + \frac{1}{2} \times \frac{1}{4} \times \frac{1}{4}\right)}$

PROBABILITY

Paragraph for Question 10 and 11

A box B_1 contains 1 white ball, 3 red balls and 2 black ballls, Another box B_2 contains 2 whilte balls, 3 red balls and 4 black balls. A third box B_3 contains 3 while balls, 4 red balls and 5 black balls. [JEE ADVANCED 2013]

12. Three boys and two girls stand in a queue. The probability, that the number of boys ahead of every girl is at least one more than the number of girls ahead of her, is [JEE ADVANCED 2014]

(A) $\frac{1}{2}$ (B) $\frac{1}{3}$ (C) $\frac{2}{3}$ Best (D) $\frac{3}{4}$

B_2 B_3
B_3 B_2
B_1 G_2
G_2 B_1
G_1 G_1

PROBABILITY

PARAGRAPH Question No. 16 to 17

Let n_1 and n_2 be the number of red and black balls, respectively, in box I. Let n_3 and n_4 be the number of red and black balls, respectively, in box II.

[JEE ADVANCED 2013]

16. One of the two boxes, box I and box II, was selected at random and a ball was drawn randomly out of this box. The ball was found to be red. If the probability that this red ball was drawn from box II is $\frac{1}{3}$, then the correct option(s) with the possible values of n_1, n_2, n_3 and n_4 is(are)

(A) $n_1 = 3, n_2 = 3, n_3 = 5, n_4 = 15$ (B) $n_1 = 3, n_2 = 6, n_3 = 10, n_4 = 50$

(C) $n_1 = 8, n_2 = 6, n_3 = 5, n_4 = 20$ (D) $n_1 = 6, n_2 = 12, n_3 = 5, n_4 = 20$

20 mins