EXERCISE # 1

1. $\int \frac{1-x^7}{x(1+x^7)}dx$ equals -

(A) $\ln x + \frac{2}{7}\ln(1+x^7) + C$ (B) $\ln x - \frac{2}{7}\ln(1-x^7) + C$

(C) $\ln x - \frac{2}{7}\ln(1+x^7) + C$ (D) $\ln x + \frac{2}{7}\ln(1-x^7) + C$

2. Primitive of $\frac{3x^4-1}{(x^4+x+1)^2}$ w.r.t. x is -

(A) $\frac{x}{x^4+x+1} + C$ (B) $-\frac{x}{x^4+x+1} + C$ (C) $\frac{x+1}{x^4+x+1} + C$ (D) $-\frac{x+1}{x^4+x+1} + C$

3. If $\int \frac{\cos x - \sin x + 1 - x}{e^x + \sin x + x}dx = \ln(f(x)) + g(x) + C$ where C is the constant of integration and f(x) is positive, then f(x) + g(x) has the value equal to

(A) $e^x + \sin x + 2x$ (B) $e^x + \sin x$ (C) $e^x - \sin x$ (D) $e^x + \sin x + x$

4. Integral of $\sqrt{1 + 2\cot x(\cot x + \operatorname{cosec} x)}$ w.r.t. x is

(A) $2\ell n\cos\frac{x}{2} + C$ (B) $2\ell n \sin\frac{x}{2} + C$

(C) $\frac{1}{2}\ln\cos\frac{x}{2} + C$ (D) $\ln\sin x - \ln(\operatorname{cosec} x - \cot x) + C$

5. $\int x \cdot \frac{\ln\left(x+\sqrt{1+x^2}\right)}{\sqrt{1+x^2}}dx$ equals –

(A) $\sqrt{1+x^2}\ln\left(x+\sqrt{1+x^2}\right) - x + C$ (B) $\frac{x}{2}\cdot\ln^2\left(x+\sqrt{1+x^2}\right) - \frac{x}{\sqrt{1+x^2}} + C$

(C) $\frac{x}{2}\cdot\ln^2\left(x+\sqrt{1+x^2}\right) + \frac{x}{\sqrt{1+x^2}} + C$ (D) $\sqrt{1+x^2}\ln\left(x+\sqrt{1+x^2}\right) + x + C$

6. Let g(x) be an antiderivative for f(x). Then $\ell n(1 + (g(x))^2)$ is an antiderivative for

(A) $\frac{2f(x)g(x)}{1+(f(x))^2}$ (B) $\frac{2f(x)g(x)}{1+(g(x))^2}$

(C) $\frac{2f(x)}{1+(f(x))^2}$ (D) none

7. A function $y = f(x)$ satisfies $f''(x) = -\frac{1}{x^2} - \pi^2\sin(\pi x)$; $f'(2) = \pi + \frac{1}{2}$ and $f(1) = 0$. The value of $f\left(\frac{1}{2}\right)$ is

(A) $\ell n2$ (B) 1 (C) $\frac{\pi}{2} - \ln 2$ (D) $1 - \ell n2$

8. Consider $f(x) = \frac{x^2}{1+x^3}$; $g(t) = \int f(t)\,dt$. If $g(1) = 0$ then g(x) equals -

(A) $\frac{1}{3}\ln(1+x^3)$ (B) $\frac{1}{3}\ln\left(\frac{1+x^3}{2}\right)$ (C) $\frac{1}{2}\ln\left(\frac{1+x^3}{3}\right)$ (D) $\frac{1}{3}\ln\left(\frac{1+x^3}{3}\right)$

9. $\int \frac{e^{\sqrt{x}}}{\sqrt{x}}(x+\sqrt{x})dx$

(A) $2e^{\sqrt{x}}[x - \sqrt{x} + 1] + C$ (B) $e^{\sqrt{x}}[x - 2\sqrt{x} + 1] + C$

(C) $e^{\sqrt{x}}(x + \sqrt{x}) + C$ (D) $e^{\sqrt{x}}(x + \sqrt{x} + 1) + C$

10. $\int \frac{dx}{\sqrt[3]{x^{5/2}(x+1)^{7/2}}}$

(A) $-\left(\frac{x+1}{x}\right)^{1/6}+C$ (B) $6\left(\frac{x+1}{x}\right)^{-1/6}+C$ (C) $\left(\frac{x}{x+1}\right)^{5/6}+C$ (D) $-\left(\frac{x}{x+1}\right)^{5/6}+C$

11. Let $f(x)=\frac{2\sin^2 x-1}{\cos x}+\frac{\cos x(2\sin x+1)}{1+\sin x}$ then $\int e^x(f(x)+f'(x))dx$ (where C is the constant of integration)

(A) $e^x\tan x+C$ (B) $e^x\cot x+C$ (C) $e^x\operatorname{cosec}^2 x+C$ (D) $e^x\sec^2 x+C$

12. $\int \frac{x^2(1-\ln x)}{\ln^4 x-x^4}dx$ equals

(A) $\frac{1}{2}\ln\left(\frac{x}{\ln x}\right)-\frac{1}{4}\ln(\ln^2 x-x^2)+C$ (B) $\frac{1}{4}\ln\left(\frac{\ln x-x}{\ln x+x}\right)-\frac{1}{2}\tan^{-1}\left(\frac{\ln x}{x}\right)+C$

(C) $\frac{1}{4}\ln\left(\frac{\ln x+x}{\ln x-x}\right)+\frac{1}{2}\tan^{-1}\left(\frac{\ln x}{x}\right)+C$ (D) $\frac{1}{4}\left(\ln\left(\frac{\ln x-x}{\ln x+x}\right)+\tan^{-1}\left(\frac{\ln x}{x}\right)\right)+C$

13. $\int \frac{(2x+3)}{x(x+1)(x+2)(x+3)+1}=C-\frac{1}{f(x)}$, where f(x) is of the form of ax^2+bx+c then $(a+b+c)$ equals

(A) 4 (B) 5 (C) 6 (D) none

14. $\int e^x\left(\frac{x^2-3}{(x-1)^2}\right)dx$ is equal to-

(A) $e^x\left(\frac{x+3}{x-1}\right)+C$ (B) $e^x\left(\frac{x-3}{x-1}\right)+C$ (C) $e^x\left(\frac{x+1}{x-1}\right)+C$ (D) $e^x\left(\frac{1}{x-1}\right)^2+C$

(where ' C ' is integration constant)

15. $\int \frac{x^3}{(2x^2+1)^3}dx$ is equal to -

(A) $\frac{1}{4}\left(2+\frac{1}{x^2}\right)^{-2}+C$ (B) $-\frac{1}{4}\left(2+\frac{1}{x^2}\right)^{-2}+C$

(C) $\frac{1}{2}\left(2+\frac{1}{x^2}\right)^{-2}+C$ (D) $\frac{1}{4}\left(2+\frac{1}{x^2}\right)^{2}+C$

EXERCISE # 2

1. $\int \frac{(\sqrt{x}+1)(x^2-\sqrt{x})}{x\sqrt{x}+x+\sqrt{x}}dx$

2. A function g defined for all positive real numbers, satisfies $g'(x^2) = x^3$ for all $x > 0$ and $g(1) = 1$. Compute $g(4)$.

3. $\int \left[\sin\ \alpha \sin\ (x-\alpha) + \sin^2\ \left(\frac{x}{2}-\alpha\right)\right]dx$

4. $\int \frac{x^2+3}{x^6(x^2+1)}dx$

5. $\int \frac{dx}{\cot\frac{x}{2}\cdot\cot\frac{x}{3}\cdot\cot\frac{x}{6}}$

6. $\int \sqrt{\frac{\text{cosec } x-\cot x}{\text{cosec } x+\cot x}}\cdot\frac{\sec x}{\sqrt{1+2\sec x}}dx$

7. $\int \frac{\ln\left(\ln\left(\frac{1+x}{1-x}\right)\right)}{1-x^2}dx$

8. $\int \left[\left(\frac{x}{e}\right)^x + \left(\frac{e}{x}\right)^x\right]\ln\ x dx$

9. $\int \frac{x^5+3x^4-x^3+8x^2-x+8}{x^2+1}dx$

10. $\int \frac{(\sqrt{x}+1)dx}{\sqrt{x}(\sqrt[3]{x}+1)}$

11. $\int \sin^{-1}\sqrt{\frac{x}{a+x}}dx$

12. $\int \frac{x\ell nx}{(x^2-1)^{3/2}}\cdot dx$

13. $\int \left[\frac{\sqrt{x^2+1}[\ln(x^2+1)-2\ell nx]}{x^4}\right]dx$

14. $\int \frac{\tan\ 2\theta}{\sqrt{\cos^6\ \theta+\sin^6\ \theta}}d\theta$

15. $\int \frac{3x^2+1}{(x^2-1)^3}dx$

16. $\int \frac{(ax^2-b)dx}{x\sqrt{c^2x^2-(ax^2+b)^2}}$

17. $\int \frac{\left(e^{\sqrt{x}}-e^{-\sqrt{x}}\right)\cos\left(e^{\sqrt{x}}+e^{-\sqrt{x}}+\frac{\pi}{4}\right)+\left(e^{\sqrt{x}}+e^{-\sqrt{x}}\right)\cos\left(e^{\sqrt{x}}-e^{-\sqrt{x}}+\frac{\pi}{4}\right)}{\sqrt{x}}dx$

18. $\int \frac{x^2+x}{(e^x+x+1)^2}dx$

19. $\int \frac{e^{\cos\ x}(x\sin^3\ x+\cos\ x)}{\sin^2\ x}dx$

20. $\int \frac{5x^4+4x^5}{(x^5+x+1)^2}dx$

21. $\int (\sin x)^{-11/3}(\cos x)^{-1/3}dx$

22. $\int \frac{dx}{\sin x+\sec x}$

23. $\int \frac{4x^5-7x^4+8x^3-2x^2+4x-7}{x^2(x^2+1)^2}dx$

24. Let $\int \frac{f'(x)g(x)-g'(x)f(x)}{(f(x)+g(x))\sqrt{f(x)g(x)-g^2(x)}}dx = \sqrt{m}\tan^{-1}\left(\sqrt{\frac{f(x)-g(x)}{ng(x)}}\right)+C.$

Where $m, n \in N$ and ' C ' is constant of integration $(g(x) > 0)$. Find the value of $(m^2 + n^2)$.

25. If the value $\int \frac{1-(\cot x)^{2008}}{\tan x+(\cot x)^{2009}}dx = \frac{1}{k}\ln\left|\sin^k x + \cos^k x\right| + C$, then find k.

26. $\int \frac{dx}{(x-\alpha)\sqrt{(x-\alpha)(x-\beta)}}$

27. Suppose $\int \frac{1-7\cos^2 x}{\sin^7 x\cos^2 x}dx = \frac{g(x)}{\sin^7 x} + C$, where C is arbitrary constant of integration. Then find the value of $g'(0) + g''\left(\frac{\pi}{4}\right)$

ANSWER KEY

EXERCISE # 1

1.	C	**2.**	B	**3.**	B	**4.**	B	**5.**	A	**6.**	B	**7.**	D
8.	B	**9.**	A	**10.**	B	**11.**	A	**12.**	B	**13.**	B	**14.**	C
15.	A												

EXERCISE # 2

1. $\frac{x^2}{2} - x + C$

2. $\frac{67}{5}$

3. $\frac{1}{2}(x - \sin x) + C$

4. $C - \frac{2}{x} + \frac{2}{3x^3} - \frac{3}{5x^5} - 2\tan^{-1} x$

5. $2\ln\left(\sec\frac{x}{2}\right) - 3\ln\left(\sec\frac{x}{3}\right) - 6\ln\left(\sec\frac{x}{6}\right) + C$

6. $\sin^{-1}\left(\frac{1}{2}\sec^2\frac{x}{2}\right) + C$

7. $\frac{1}{2}\left[\ln\left(\frac{1+x}{1-x}\right)\cdot\ln\left(\ln\frac{1+x}{1-x}\right) - \ln\left(\frac{1+x}{1-x}\right)\right] + C$

8. $\left(\frac{x}{e}\right)^x - \left(\frac{e}{x}\right)^x + C$

9. $\frac{x^4}{4} + x^3 - x^2 + 5x + \frac{1}{2}\ln(x^2+1) + 3\tan^{-1} x + C$

10. $6\left[\frac{t^4}{4} - \frac{t^2}{2} + t + \frac{1}{2}\ln(1+t^2) - \tan^{-1} t\right] + C$ where $t = x^{1/6}$

11. $(a+x)\arctan\sqrt{\frac{x}{a}} - \sqrt{ax} + C$

12. $\text{arcsec}\, x - \frac{\ln x}{\sqrt{(x^2-1)}} + C$

13. $\frac{(x^2+1)\sqrt{x^2+1}}{9x^3}\left[2 - 3\ln\left(1+\frac{1}{x^2}\right)\right]$

14. $\ln\left(\frac{1+\sqrt{1+3\cos^2 2\theta}}{\cos 2\theta}\right) + C$

15. $C - \frac{x}{(x^2-1)^2}$

16. $\sin^{-1}\left(\frac{ax^2+b}{cx}\right) + k$

17. $2\sqrt{2}\left(\cos\left(e^{-\sqrt{x}}\right)\right)\left(\sin\left(e^{\sqrt{x}}\right) + \cos\left(e^{\sqrt{x}}\right)\right) + C$

18. $C - \ln(1 + (x+1)e^{-x}) - \frac{1}{1+(x+1)e^{-x}}$

19. $C - e^{\cos x}(x + \operatorname{cosec} x)$

20. $C - \frac{x+1}{x^5+x+1}$ or $C + \frac{x^5}{x^5+x+1}$

21. $-\frac{3(1+4\tan^2 x)}{8(\tan x)^{8/3}} + C$

22. $\frac{1}{2\sqrt{3}} \ln \frac{\sqrt{3}+\sin x-\cos x}{\sqrt{3}-\sin x+\cos x} + \arctan(\sin x + \cos x) + C$

23. $4\ln x + \frac{7}{x} + 6\tan^{-1}(x) + \frac{6x}{1+x^2} + C$

24. 8

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26. $\frac{-2}{\alpha-\beta}\sqrt{\frac{x-\beta}{x-\alpha}} + C$

27. 5