IOTA

Express each of the complex number given in the Exercises 1 to 10 in the form a + ib.

1. $(5i)\left(-\frac{3}{5}i\right)$

2. $i^9 + i^{19}$

3. i^{-39}

4. $3(7+i7)+i(7+i7)$

5. $(1-i)-(-1+i6)$

6. $\left(\frac{1}{5}+i\frac{2}{5}\right)-\left(4+i\frac{5}{2}\right)$

7. $\left[\left(\frac{1}{3}+i\frac{7}{3}\right)+\left(4+i\frac{1}{3}\right)\right]-\left(-\frac{4}{3}+i\right)$

8. $(1-i)^4$

9. $\left(\frac{1}{3}+3i\right)^3$

10. $\left(-2-\frac{1}{3}i\right)^3$

11. Evaluate $\sum_{n=1}^{13}(i^n + i^{n+1})$, where $n \in \mathbf{N}$.

12. Evaluate: $\left[i^{18}+\left(\frac{1}{i}\right)^{25}\right]^3$.

ALGEBRA OF COMPLEX NUMBER

Find the multiplicative inverse of each of the complex numbers given in the Exercises 13 to 15.

13. $4-3i$

14. $\sqrt{5}+3i$

15. $-i$

16. Let $x, y \in \mathbf{R}$, then $x + iy$ is a non real complex number if:

(A) $x = 0$
(B) $y = 0$
(C) $x \neq 0$
(D) $y \neq 0$

17. If $a + ib = c + id$, then

(A) $a^2 + c^2 = 0$
(B) $b^2 + c^2 = 0$
(C) $b^2 + d^2 = 0$
(D) $a^2 + b^2 = c^2 + d^2$

18. Express the following expression in the form of a + ib :

$$\frac{(3+i\sqrt{5})(3-i\sqrt{5})}{(\sqrt{3}+\sqrt{2}i)-(\sqrt{3}-i\sqrt{2})}$$

19. Reduce $\left(\frac{1}{1-4i}-\frac{2}{1+i}\right)\left(\frac{3-4i}{5+i}\right)$ to the standard form.

20. If $\left(\frac{1+i}{1-i}\right)^m = 1$, then find the least positive integral value of m,

21. For a positive integer n, find the value of $(1-i)^n\left(1-\frac{1}{i}\right)^n$

22. If $x + iy = \frac{a+ib}{a-ib}$, prove that $x^2 + y^2 = 1$

22. For any two complex numbers z_1 and z_2, prove that Re $(z_1z_2) = \text{Re}z_1\text{Re}z_2 - \text{Im}z_1\text{Im}z_2$

24. Let $z_1 = 2 - i, z_2 = -2 + i$. Find

(i) Re $\left(\frac{z_1z_2}{\bar{z}_1}\right)$,**(ii)** Im $\left(\frac{1}{z_1\bar{z}_1}\right)$.

25. Find the real numbers x and y if $(x-iy)(3+5i)$ is the conjugate of $-6-24i$.

26. If $(x+iy)^3 = u+iv$, then show that $\frac{u}{x}+\frac{v}{y} = 4(x^2-y^2)$.

27. If $\left(\frac{1+i}{1-i}\right)^3 - \left(\frac{1-i}{1+i}\right)^3 = x+iy$, then find (x,y).

28. If $\frac{(1+i)^2}{2-i} = x+iy$, then find the value of $x+y$.

29. If $\left(\frac{1-i}{1+i}\right)^{100} = a+ib$, then find (a,b).

30. If $a = \cos\theta + i\sin\theta$, find the value of $\frac{1+a}{1-a}$.

31. If $\frac{(a^2+1)^2}{2a-i} = x+iy$, what is the value of x^2+y^2 ?

32. The real value of θ for which the expression $\frac{1+i\cos\theta}{1-2i\cos\theta}$ is a real number is:

(A) $n\pi+\frac{\pi}{4}$

(B) $n\pi+(-1)^n\frac{\pi}{4}$

(C) $2n\pi\pm\frac{\pi}{2}$

(D) none of these.

33. If $(a+ib)^5 = \alpha+i\beta$ then $(b+ia)^5$ is equal to

(A) $\beta+i\alpha$ (B) $\alpha-i\beta$ (C) $\beta-i\alpha$ (D) $-\alpha-i\beta$

34. If $z(\neq -1)$ is a complex number such that $\frac{z-1}{z+1}$ is purely imaginary, then $|z|$ is equal to

(A) 1 (B) 2 (C) 3 (D) 5

35. A value of θ for which $\frac{2+3i\sin\theta}{1-2i\sin\theta}$ is purely imaginary, is : **[JEE - MAIN 2016]**

(A) $\frac{\pi}{6}$ (B) $\sin^{-1}\left(\frac{\sqrt{3}}{4}\right)$ (C) $\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$ (D) $\frac{\pi}{3}$

CONJUGATE

36. Find the conjugate of $\frac{(3-2i)(2+3i)}{(1+2i)(2-i)}$.

37. What is the conjugate of $\frac{2-i}{(1-2i)^2}$?

38. The conjugate of a complex number is $\frac{1}{i-1}$. Then that complex number is- **[AIEEE - 2008]**

(A) $\frac{1}{i+1}$ (B) $\frac{-1}{i+1}$ (C) $\frac{1}{i-1}$ (D) $\frac{-1}{i-1}$

39. $\sin x + i\cos 2x$ and $\cos x - i\sin 2x$ are conjugate to each other for:

(A) $x = n\pi$

(B) $x = \left(n+\frac{1}{2}\right)\frac{\pi}{2}$

(C) $x = 0$

(D) No value of x

40. If $z = x+iy$ lies in the third quadrant, then $\frac{\bar{z}}{z}$ also lies in the third quadrant if

(A) $x>y>0$

(B) $x<y<0$

(C) $y<x<0$

(D) $y>x>0$

41. If $x - iy = \sqrt{\frac{a-ib}{c-id}}$ prove that $(x^2+y^2)^2 = \frac{a^2+b^2}{c^2+d^2}$.

42. If $(1+i)z = (1-i)\bar{z}$, then show that $z = -i\bar{z}$.

43. The value of $(z+3)(\bar{z}+3)$ is equivalent to

(A) $|z+3|^2$ (B) $|z-3|$

(C) z^2+3 (D) None of these

44. A real value of x satisfies the equation $\left(\frac{3-4ix}{3+4ix}\right) = \alpha - i\beta$ $(\alpha, \beta \in \mathbf{R})$ if $\alpha^2+\beta^2 =$

(A) 1 (B) -1 (C) 2 (D) -2

MODULUS

45. Find the modulus of $\frac{1+i}{1-i} - \frac{1-i}{1+i}$.

46. Find the number of non-zero integral solutions of the equation $|1-i|^x = 2^x$.

47. If $(a+ib)(c+id)(e+if)(g+ih) = A + i\,B$, then show that

$(a^2+b^2)(c^2+d^2)(e^2+f^2)(g^2+h^2) = A^2+B^2$

48. If $|z_1| = |z_2|$, is it necessary that $z_1 = z_2$?

49. Find $\left|(1+i)\frac{(2+i)}{(3+i)}\right|$

50. If z is a complex number, then

(A) $|z^2| > |z|^2$ (B) $|z^2| = |z|^2$ (C) $|z^2| < |z|^2$ (D) $|z^2| \geq |z|^2$

51. If $z_1 = 2-i, z_2 = 1+i$, find $\left|\frac{z_1+z_2+1}{z_1-z_2+1}\right|$.

52. If $a+ib = \frac{(x+i)^2}{2x^2+1}$, prove that $a^2+b^2 = \frac{(x^2+1)^2}{(2x^2+1)^2}$.

53. If α and β are different complex numbers with $|\beta| = 1$, then find $\left|\frac{\beta-\alpha}{1-\bar{\alpha}\beta}\right|$

54. If $\frac{z-1}{z+1}$ is a purely imaginary number $(z \neq -1)$, then find the value of $|z|$.

55. If $|z_1| = 1 (z_1 \neq -1)$ and $z_2 = \frac{z_1-1}{z_1+1}$, then show that the real part of z_2 is zero.

56. If $|z_1| = |z_2| = \cdots = |z_n| = 1$, then show that $|z_1+z_2+z_3+\cdots+z_n| = \left|\frac{1}{z_1}+\frac{1}{z_2}+\frac{1}{z_3}+\cdots+\frac{1}{z_n}\right|$.

57. Where does z lie, if $\left|\frac{z-5i}{z+5i}\right| = 1$.

58. The complex number z which satisfies the condition $\left|\frac{i+z}{i-z}\right| = 1$ lies on

(A) circle $x^2+y^2 = 1$ (B) the x-axis

(C) the y-axis (D) the line $x+y = 1$.

59. If $f(z) = \frac{7-z}{1-z^2}$, where $z = 1 + 2i$, then $|f(z)|$ is

(A) $\frac{|z|}{2}$ (B) $|z|$ (C) $2|z|$ (D) none of these.

60. If $|z-2| \geq |z-4|$ then correct statement is-

(A) $R(z) \geq 3$ (B) $R(z) \leq 3$ (C) $R(z) \geq 2$ (D) $R(z) \leq 2$

61. If $|z_1 - 1| < 1, |z_2 - 2| < 2, |z_3 - 3| < 3$ then $|z_1 + z_2 + z_3|$

(A) is less than 6 (B) is more than 3

(C) is less than 12 (D) lies between 6 and 12

62. If $iz^3 + z^2 - z + i = 0$, then $|z|$ equals

(A) 4 (B) 3 (C) 2 (D) 1

63. If $|z_1| = 2, |z_2| = 3, |z_3| = 4$ and $|2z_1 + 3z_2 + 4z_3| = 4$ then absolute value of $8z_2z_3 + 27z_3z_1 + 64z_1z_2$ equals

(A) 24 (B) 48 (C) 72 (D) 96

64. If z is a complex number such that $|z| \geq 2$, then the minimum value of $\left|z + \frac{1}{2}\right|$:

[JEE-MAIN 2014]

(A) is equal to $\frac{5}{2}$ (B) lies in the interval $(1,2)$

(C) is strictly greater than $\frac{5}{2}$ (D) is strictly greater than $\frac{3}{2}$ but less than $\frac{5}{2}$

ARGUMENT

65. Find principal argument of $(1 + i\sqrt{3})^2$.

66. The value of arg (x) when $x < 0$ is:

(A) 0 (B) $\frac{\pi}{2}$ (C) π (D) none of these

67. The argument of the complex number $\sin\frac{6\pi}{5} + i\left(1 + \cos\frac{6\pi}{5}\right)$ is.

(A) $\frac{6\pi}{5}$ (B) $\frac{5\pi}{6}$ (C) $\frac{9\pi}{10}$ (D) $\frac{2\pi}{5}$

68. z_1 and z_2 are two complex numbers such that $|z_1| = |z_2|$ and $\arg(z_1) + \arg(z_2) = \pi$, then show that $z_1 = -\bar{z}_2$.

69. If z_1, z_2 and z_3, z_4 are two pairs of conjugate complex numbers, then find $\arg\left(\frac{z_1}{z_4}\right) + \arg\left(\frac{z_2}{z_3}\right)$.

70. If for complex numbers z_1 and z_2, $\arg(z_1) - \arg(z_2) = 0$, then show that $|z_1 - z_2| = |z_1| - |z_2|$

71. Fill in the blanks of the following

(i) For any two complex numbers z_1, z_2 and any real numbers a, b,

$|az_1 - bz_2|^2 + |bz_1 + az_2|^2 = \cdots$

(ii) The value of $\sqrt{-25} \times \sqrt{-9}$ is

(iii) The number $\frac{(1-i)^3}{1-i^3}$ is equal to

(iv) The sum of the series $i + i^2 + i^3 + \cdots$ upto 1000 terms is

(v) Multiplicative inverse of $1 + i$ is

(vi) If z_1 and z_2 are complex numbers such that $z_1 + z_2$ is a real number, then $z_2 = \cdots$.

(vii) $\arg(z) + \arg \bar{z} (\bar{z} \neq 0)$ is

(viii) If $|z + 4| \leq 3$, then the greatest and least values of $|z + 1|$ are ..... and .....

72. Find z if $|z| = 4$ and $\arg(z) = \frac{5\pi}{6}$.

73. Which of the following is correct for any two complex numbers z_1 and z_2 ?

(A) $|z_1 z_2| = |z_1||z_2|$
(B) $\arg(z_1 z_2) = \arg(z_1) \cdot \arg(z_2)$
(C) $|z_1 + z_2| = |z_1| + |z_2|$
(D) $|z_1 + z_2| \geq |z_1| - |z_2|$

74. $|z_1 + z_2| = |z_1| + |z_2|$ is possible if

(A) $z_2 = \bar{z}_1$
(B) $z_2 = \frac{1}{z_1}$
(C) $\arg(z_1) = \arg(z_2)$
(D) $|z_1| = |z_2|$

75. Let z and w are two non zero complex number such that $|z| = |w|$, and $\text{Arg}(z) + \text{Arg}(w) = \pi$ then-

(A) $z = w$
(B) $z = \bar{w}$
(C) $\bar{z} = \bar{w}$
(D) $z = -\bar{w}$

76. Let z, w be complex numbers such that $\bar{z} + i\bar{w} = 0$ and $\arg zw = \pi$. Then $\arg z$ equals-

(A) $\pi/4$
(B) $\pi/2$
(C) $3\pi/4$
(D) $5\pi/4$

77. If $z_1 = -3 + 5i$; $z_2 = -5 - 3i$ and z is a complex number lying on the line segment joining $z_1 \& z_2$, then $\arg(z)$ can be

(A) $-\frac{3\pi}{4}$
(B) $-\frac{\pi}{4}$
(C) $\frac{\pi}{6}$
(D) $\frac{5\pi}{6}$

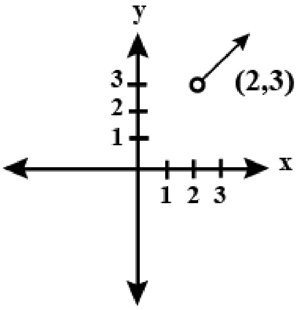
78. If z and ω are two non- zero comlex numbers such that $|z\omega| = 1$, and $\text{Arg}(z) - \text{Arg}(\omega) = \frac{\pi}{2}$, then $\bar{z}\omega$ is equal to-

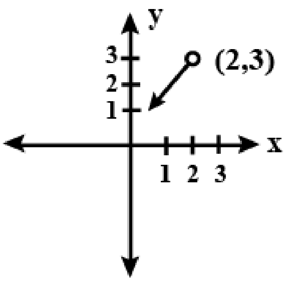
(A) $-I$
(B) 1
(C) -1
(D) i

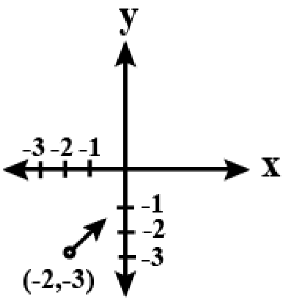
79. If z_1 and z_2 are two non-zero complex numbers such that $|z_1 + z_2| = |z_1| + |z_2|$, then $\arg z_1$ - $\arg z_2$ is equal to –

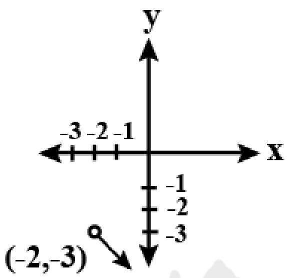
(A) $\frac{\pi}{2}$
(B) $-\pi$
(C) 0
(D) $\frac{-\pi}{2}$

80. If Arg $(z-2-3i)=\frac{\pi}{4}$, then the locus of z is

(A)


(B)


(C)


(D)


81. If z is a complex number of unit modulus and argument θ, then arg $\left(\frac{1+z}{1+\bar{z}}\right)$ equals

[JEE-MAIN 2013]

(A) θ (B) $\pi-\theta$ (C) $-\theta$ (D) $\frac{\pi}{2}-\theta$

COMPLEX NUMBER EQUATION

Short Answer Type

82. Solve the equation $|z| = z + 1 + 2i$.

83. The number of solutions of the system of equations Re $(z^2) = 0, |z| = 2$ is

(A) 4 (B) 3 (C) 2 (D) 1

84. Let $z(\neq 2)$ be a complex number such that $\log_{1/2}|z-2| > \log_{1/2}|z|$, then

(A) Re $(z) > 1$ (B) Im $(z) > 1$ (C) Re $(z) = 1$ (D) Im $(z) = 1$

SQUARE ROOT OF A COMPLEX NUMBER

85. In one root of the quadratic equation $(1+i)x^2-(7+3i)x+(6+8i)=0$ is $4-3i$, then the other root must be

(A) $1 + I$ (B) $4 + 3i$ (C) $1 - I$ (D) $4i + 3$

ANSWER KEY

1.	$3+i0$	2.	$0+i0$	3.	$0+i1$
4.	$14+28i$	5.	$2-7i$	6.	$-\frac{19}{5}-\frac{21i}{10}$
7.	$\frac{17}{3}+i\frac{5}{3}$	8.	$-4+i0$	9.	$-\frac{242}{27}-26i$
10.	$\frac{-22}{3}-i\frac{107}{27}$	11.	$-1+i$	12.	$2-2i$
13.	$\frac{4}{25}+i\frac{3}{25}$	14.	$\frac{\sqrt{5}}{14}-i\frac{3}{14}$	15.	$0+i1$
16.	(D)	17.	(D)	18.	$0-i\frac{7\sqrt{2}}{2}$
19.	$\frac{307+599i}{442}$	20.	(4)	21.	(2^n)
22.	(1)	24.	(i) $\frac{-2}{5}$, (ii) 0	25.	$x=3, y=-3$
27.	$(0,-2)$	28.	$\frac{2}{5}$	29.	$(1,0)$
30.	$i\cot\frac{\theta}{2}$	31.	$\frac{(a^2+1)^4}{4a^2+1}$	32.	(C)
33.	(A)	34.	(A)	35.	(C)
37.	$\frac{-2}{25}-i\frac{11}{25}$	38.	(B)	39.	(D)
40.	(B)	42.	$\frac{3}{2}-2i$	43.	(A)
44.	(A)	45.	(2)	46.	(0)
48.	(No)	49.	(1)	50.	(B)
51.	$(\sqrt{2})$	53.	(1)	54.	(1)
57.	(Real axis)	58.	(B)	59.	(A)
60.	(A)	61.	(C)	62.	(D)
63.	(D)	64.	(B)	65.	$\left(\frac{2\pi}{3}\right)$
66.	(C)	67.	(C)	69.	(0)
71.	(i) $(a^2+b^2)(\lvert z_1\rvert^2+\lvert z_2\rvert^2)$		(ii) -15		
	(iii) -2 (iv) 0 (v) $\frac{1}{2}-\frac{i}{2}$		(vi) $\bar{z}_1$ (vii) 0		
	(viii) 6 and 0		(ix) a circle		
72.	$-2\sqrt{3}+2i$	73.	(A)	74.	(C)
75.	(D)	76.	(C)	77.	(D)
78.	(A)	79.	(C)	80.	(A)
81.	(A)	82.	$\frac{3}{2}-2i$	83.	(A)
84.	(A)	85.	(A)		