DPP 01

Q.1 The mass density of a planet of radius R varies with the distance r from its centre as $\rho(r) = \rho_0\left(1 - \frac{r^2}{R^2}\right)$. Then the gravitational field is maximum at

(A) $r = \sqrt{\frac{3}{4}}R$ (B) $r = R$ (C) $r = \frac{1}{\sqrt{3}}R$ (D) $r = \sqrt{\frac{5}{9}}R$

Q.2 A test particle is moving in a circular orbit in the gravitational field produced by a mass density $\rho(r) = \frac{K}{r^2}$. Identify the correct relation between the radius R of the particle's orbit and its period T

(A) TR is a constant (B) T/R^2 is a constant

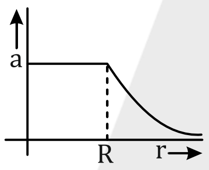
(C) T^2/R^3 is a constant (D) T/R is a constant.

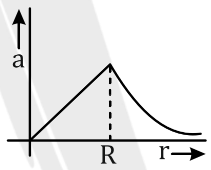
Q.3 A straight rod of length L extends from $x = a$ to $x = L + a$. The gravitational force it exerts on a point mass m at $x = 0$, if the mass per unit length of the rod is $A + Bx^2$, is given by

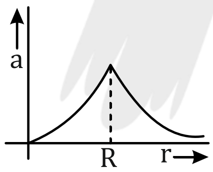
(A) $Gm\left[A\left(\frac{1}{a} - \frac{1}{a+L}\right) - BL\right]$ (B) $Gm\left[A\left(\frac{1}{a+L} - \frac{1}{a}\right) - BL\right]$

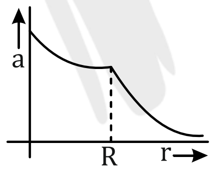
(C) $Gm\left[A\left(\frac{1}{a+L} - \frac{1}{a}\right) + BL\right]$ (D) $Gm\left[A\left(\frac{1}{a} - \frac{1}{a+L}\right) + BL\right]$

Q.4 The mass density of a spherical body is given by $\rho(r) = \frac{k}{r}$ for $r \leq R$ and $\rho(r) = 0$ for $r > R$, where r is the distance from the centre. The correct graph that describes qualitatively the acceleration, a of a test particle as a function of r is

(A) 

(B) 

(C) 

(D) 

Q.5 Suppose the gravitational force varies inversely as the n^{th} power of distance. Then the time period of a planet in circular orbit of radius R around the sun will be proportional to

(A) $R^{\left(\frac{n+1}{2}\right)}$ (B) $R^{\left(\frac{n-1}{2}\right)}$ (C) R^n (D) $R^{\left(\frac{n-2}{2}\right)}$

Q.6 The height ' h ' at which the weight of a body will be the same as that at the same depth ' h ' from the surface of the earth is (Radius of the earth is R and effect of the rotation of the earth is neglected)

(A) $\frac{R}{2}$ (B) $\frac{\sqrt{5}}{2}R - R$ (C) $\frac{\sqrt{3}R-R}{2}$ (D) $\frac{\sqrt{5}R-R}{2}$

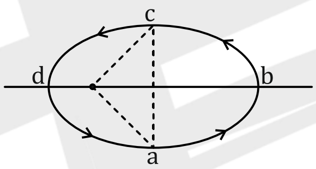
Q.7 The acceleration due to gravity on the earth's surface at the poles is g and angular velocity of the earth about the axis passing through the pole is ω. An object is weighed at the equator and at a height h above the poles by using a spring balance. If the weights are found to be same, then h is ($h << R$, where R is the radius of the earth)

(A) $\frac{R^2\omega^2}{2g}$ (B) $\frac{R^2\omega^2}{g}$ (C) $\frac{R^2\omega^2}{4g}$ (D) $\frac{R^2\omega^2}{8g}$

Q.8 The ratio of the weights of a body on the Earth's surface to that on the surface of a planet is 9: 4. The mass of the planet is $\frac{1}{9}$ th of that of the Earth. If ' R ' is the radius of the Earth, what is the radius of the planet? (Take the planets to have the same mass density)

(A) $\frac{R}{3}$ (B) $\frac{R}{9}$ (C) $\frac{R}{4}$ (D) $\frac{R}{2}$

Q.9 Figure shows elliptical path abcd of a planet around the sun S such that the area of triangle csa is $\frac{1}{4}$ the area of the ellipse (see figure). With db as the semimajor axis, and ca as the semiminor axis. If t_1 is the time taken for planet to go over path abc and t_2 for path taken over cda then



(A) $t_1 = 4t_2$ (B) $t_1 = 2t_2$ (C) $t_1 = 3t_2$ (D) $t_1 = t_2$

Q.10 On the x-axis and at a distance x from the origin, the gravitational field due to a mass distribution is given by $\frac{Ax}{(x^2+a^2)^{3/2}}$ in the x-direction. The magnitude of gravitational potential on the x-axis at a distance x, taking its value to be zero at infinity, is

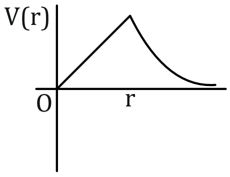
(A) $A(x^2 + a^2)^{3/2}$ (B) $A(x^2 + a^2)^{1/2}$ (C) $\frac{A}{(x^2+a^2)^{3/2}}$ (D) $\frac{A}{(x^2+a^2)^{1/2}}$

Q.11 Two planets have masses M and 16M and their radii are a and 2a, respectively. The separation between the centre of the planets is 10a. A body of mass m is fired from the surface of the larger planet towards the smaller planet along the line joining their centres. For the body to be able to reach at the surface of smaller planet, the minimum firing speed needed is

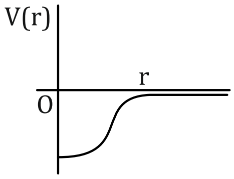
(A) $2\sqrt{\frac{GM}{a}}$ (B) $4\sqrt{\frac{GM}{a}}$ (C) $\sqrt{\frac{GM^2}{ma}}$ (D) $\frac{3}{2}\sqrt{\frac{5GM}{a}}$

Q.12 Which of the following most closely depicts the correct variation of the gravitational potential V(r) due to a large planet of radius R and uniform mass density?
(figures are not drawn to scale)

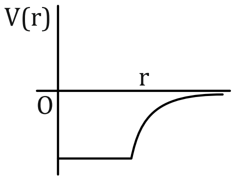
(A)



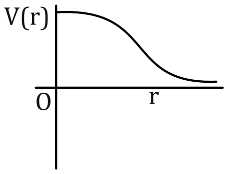
(B)



(C)



(D)



ANSWER KEY

1.	(D)	**2.**	(D)	**3.**	(D)	**4.**	(A)	**5.**	(A)	**6.**	(D)	**7.**	(A)
8.	(D)	**9.**	(C)	**10.**	(D)	**11.**	(D)	**12.**	(B)				