Nishant Jindal

APPLICATION of Derivative = AOD

Chapters.

① tangent & Normal 4 L → 5 L

② Monotonocity. → 3-4

(3) Rolle's & lagrange. - $\boxed{1}$

(4) Rate measurer / Approx ①

(5) Max / Min. ⑤

16 Lec

2Qs Mains

Adv → 2Qs - 3Qs

12th Board

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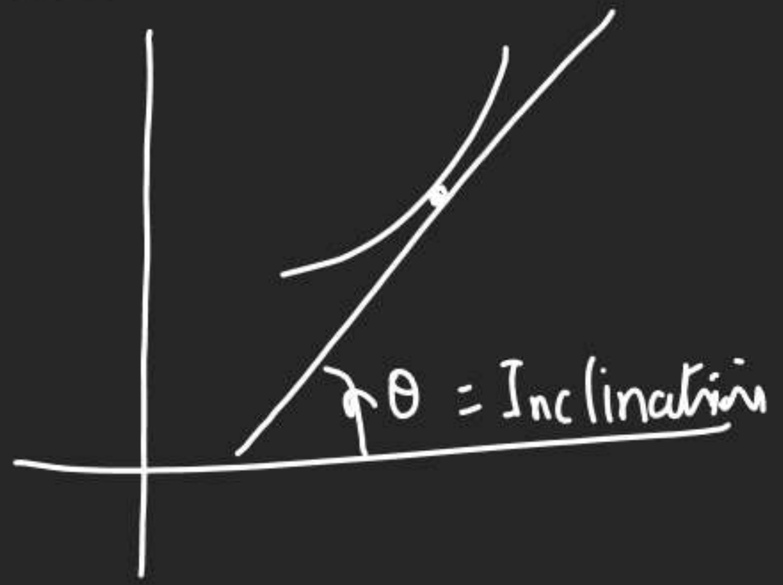
Tangent & Normal

(1) Inclination $= \theta$



(2) Slope $= m = \tan\theta$

Coordinate Geometry

(1) $ax+by+c=0 \rightarrow$ Slope $= -\frac{\text{Coff of } x}{\text{Coff of } y} = \frac{-a}{b}$

(2) $y = \boxed{m}x + c$ Slope $= m$

(3) line joining (x_1,y_1) & (x_2,y_2) $= \frac{y_2-y_1}{x_2-x_1} = m$.

(4) If Curve $y=f(x)$ has a tangent at $x=c$ then Slope of tangent

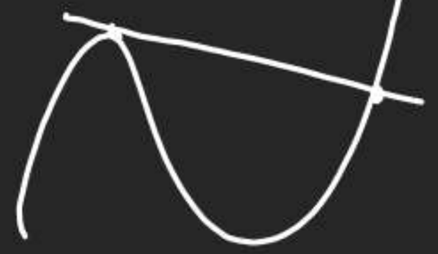
$$\left.\frac{dy}{dx}\right|_{x=c} = f'(x) = f'(c)$$

Q Slope of tangent at $x=0$ for $y=\sin x$?

$$\left.\frac{dy}{dx}\right|_{x=0} = \cos x = \cos 0 = 1$$

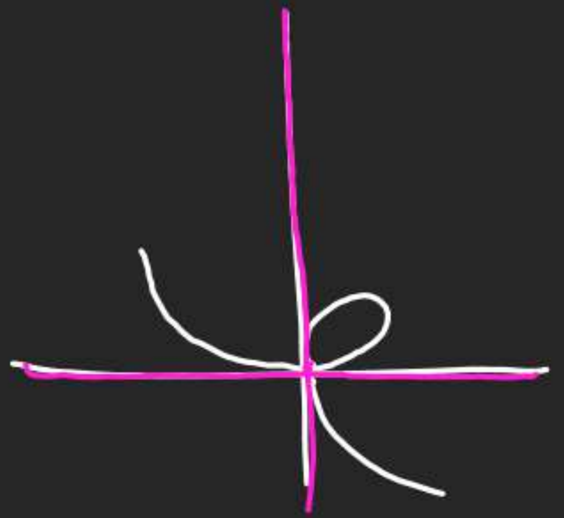
$$(Sl)_T = 1$$

Q A tangent can cross Curve [T/F]



Q A tangent can be a Normal also [T/F]

Yes



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Eqⁿ of tangent

EOT at (x_1, y_1) for $y = f(x)$

① Pt. (x_1, y_1) ② Slope $= \left.\frac{dy}{dx}\right|_{(x_1,y_1)}$

$$\boxed{y - y_1 = \left.\frac{dy}{dx}\right|_{(x_1,y_1)} (x - x_1)}$$

Eqⁿ of Normal : EON

Normal ⊥ʳ tangent

$Sl_N \times Sl_T = -1$

$$Sl_N = \frac{-1}{(Sl)_T} = \frac{-1}{\left.\frac{dy}{dx}\right|_{x_1,y_1}}$$

EON

$$\boxed{(y - y_1) = \frac{-1}{\left.\frac{dy}{dx}\right|_{(x_1,y_1)}} (x - x_1)}$$

Q Find EOT/EON for

C: $x^{2/3} + y^{2/3} = 1$ at $(1,1)$

Pt. Curve पर है या नहीं check कर लेना

$(1)^{2/3} + (1)^{2/3} \neq 1$

No EOT/EON by above formula Possible.

Q. Find EOT/EON for

C: $x^{2/3} + y^{2/3} = 2$ at $(1,1)$, with $x_1 = 1$, $y_1 = 1$

① $\frac{dy}{dx}$

$$\frac{2}{3} x^{-1/3} + \frac{2}{3} y^{-1/3} \cdot \frac{dy}{dx} = 0$$

$$\frac{2}{3} y^{-1/3} \frac{dy}{dx} = -\frac{2}{3} x^{-1/3}$$

$$\left.\frac{dy}{dx}\right|_{(1,1)} = -\left(\frac{y}{x}\right)^{1/3} = -1$$

② EOT $(y-1) = -1(x-1)$

$x + y = 2$

③ EON $(y-1) = \frac{+1}{+1}(x-1)$

$y - 1 = x - 1$

$x - y = 0$

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diffⁿ Using Newton Liebnitz Theorem.

We diffᵗᵉ Integration with variable limit by N L Thm.

$$\frac{d\left(\int_{\psi(x)}^{\phi(x)} f(t)\,dt\right)}{dx} = f(\text{upper Limit})\times(\text{Upper limit})' - f(\text{lower Limit})\times(\text{Lower Limit})'$$

$\sqrt{2}x+y-\sqrt{2}=0$

Q $\frac{d\left(\int_{x}^{x^3}\sin t\,dt\right)}{dx} = \sin(x^3)\times 3x^2 - \sin(x)\times 1$

$= 3x^2\sin x^3 - \sin x$

Q $\frac{d\left(\int_{x^2}^{1}\ln t\,dt\right)}{dx} = \ln(1)\cdot 0 - \ln(x^2)\cdot 2x$

$= -2x\ln x^2$ Ans

Q Find EON to Curve $y = \int_{1}^{x^3}\frac{dt}{\sqrt{1+t^2}}$ at $x=1$?

① $\left.\frac{dy}{dx}\right|_{x=1} = \frac{1}{\sqrt{1+(x^3)^2}}\times 3x^2 - \frac{1}{\sqrt{1+(x^2)^2}}\times 2x = \frac{3}{\sqrt{2}} - \frac{2}{\sqrt{2}} = \frac{1}{\sqrt{2}}$

② EON → pt. → $x=1$ Put Curve $y = \int_1^1\frac{dt}{\sqrt{1+t^2}} = 0$

$\therefore$ pt $(x_1,y_1) = (1,0)$

③ $(y-0) = \frac{-1}{1/\sqrt{2}}(x-1) \Rightarrow y = -\sqrt{2}(x-1)$

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Q EOT for Curve $x = at^2, y = 2at$?

$\boxed{\text{Parametric Curve में Curve का Pt. होता है। } x_1 = at^2,\ y_1 = 2at}$

① $\frac{dx}{dt} = 2at$ ② $\frac{dy}{dx} = 2a$.

③ $\frac{dy}{dx} = \frac{2a}{2at} = \frac{1}{t}$

EOT → $(y - 2at) = \frac{1}{t}(x - at^2)$

$yt - 2at^2 = x - at^2$

$x = yt - 2at^2 + at^2$

$x = ty - at^2$

EOT → $(y - y_1) = \frac{dy}{dx}(x - x_1)$

EON → $(y - y_1) = \frac{-1}{\left(\frac{dy}{dx}\right)}(x - x_1)$

1Q EON for Curve $x = at^2, y = 2at$?

$(y - 2at) = \frac{-1}{1/t}(x - at^2)$

$y - 2at = -tx + at^3$.

$y + tx = 2at + at^3$

Q Slope of tangent at $(2, -1)$ to Curve $x = t^2 + 3t - 8$ & $y = 2t^2 - 2t - 5$ is?

* $2 = t^2 + 3t - 8 \Rightarrow t^2 + 3t - 10 = 0$
$(t + 5)(t - 2) = 0$
$t = 2, -5$

* $-1 = 2t^2 - 2t - 5 \Rightarrow 2t^2 - 2t - 4 = 0$
$t^2 - t - 2 = 0$
$t = 2, -1$

$t = 2$

1) $\frac{dx}{dt} = 2t + 3$

2) $\frac{dy}{dt} = 4t - 2$

3) $\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{4t - 2}{2t + 3}$

यहाँ value of t Chahiye

4) $\left.\frac{dy}{dx}\right|_{t=2} = \frac{4 \times 2 - 2}{2 \times 2 + 3} = \frac{6}{7}$

$(S1)_T$

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Q EOT/EON to:

define fxn & Krantikari formulae & diffte

$$x = \begin{cases} 2t + t^2 \cdot \sin\frac{1}{t} & t \neq 0 \\ 0 & t = 0 \end{cases}$$

$$y = \begin{cases} \frac{1}{t}\sin t^2 & t \neq 0 \\ 0 & t = 0 \end{cases}$$

at $t = 0$

① $t = 0 \to x = 0, y = 0$

Pt. $(0,0)$

② $y'(0) = \lim_{t\to 0} \frac{f(t) - f(0)}{t - 0}$

$\left.\frac{dy}{dt}\right|_{t=0} = \lim_{t\to 0} \frac{\frac{1}{t}\sin t^2 - 0}{t} = \lim_{t\to 0} \frac{\sin t^2}{t^2} = 1$

(3) $x'(0) = \lim_{t\to 0} \frac{f(t) - f(0)}{t - 0}$

$\left.\frac{dx}{dt}\right|_{t=0} = \lim_{t\to 0} \frac{2t + t^2 \cdot \sin 1/t - 0}{t}$

$= \lim_{t\to 0} 2 + t \cdot \sin\frac{1}{t}$

$= 2 + 0 \cdot \sin\infty$

$x'(0) = 2 + 0 = 2$

(4) $\frac{dy}{dx} = \frac{\left.\frac{dy}{dt}\right|_{t=0}}{\left.\frac{dx}{dt}\right|_{t=0}} = \frac{1}{2}$

(5) EOT

$(y - 0) = \frac{1}{2}(x - 0)$

$2y - x = 0$

EON

$(y - 0) = \frac{-1}{1/2}(x - 0)$

$2x + y = 0$

Q Find EOT to curve.

$x = a\sin^3 t,\ y = a\cos^3 t$ at $t = \frac{\pi}{2}$

① $\left.\frac{dx}{dt}\right| = 3a\sin^2 t \cdot \cos t$

② $\frac{dy}{dt} = -3a\cos^2 t \cdot \sin t$

Pt.

$x = a\sin^3\frac{\pi}{2} = a$

$y = a\cos^3\frac{\pi}{2} = 0$

3) $\left.\frac{dy}{dx}\right|_{t=\frac{\pi}{2}} = \frac{-\cancel{3a}\cos^{\cancel{2}} t \cdot \cancel{\sin t}}{\cancel{3a}\sin^{\cancel{2}} t \cdot \cancel{\cos t}} = -\cot t$

$= 0$

④ $(y - 0) = 0(x - a)$

$y = 0$

Q EOT/EON if exist to curve

C: $y = x^{1/3}(1-\cos x)$ at $x=0$?

A) Pt. $x=0 \to y = 0^{1/3}(1-\cos 0) = 0$

$(0,0)$

B) $y = x^{1/3} - x^{1/3}\cos x \to \frac{dy}{dx} = \infty - \infty$ आरहाथा

① $\left.\frac{dy}{dx}\right|_{x=0} = \lim_{x\to 0} \frac{f(x)-f(0)}{x-0}$ ← Limit की Jrurat mahsos hoithi

$= \lim_{x\to 0} \frac{\cancel{x^{1/3}}(1-\cos x) - 0}{x\, x^{2/3}} = \frac{0}{0}$ IDL

$= \lim_{x\to 0} \frac{\sin x \cdot x^{1/3}}{\frac{2}{3}} = \frac{0}{2/3} = 0$

② EOT

$(y-0) = 0(x-0)$

$\underline{y=0}$

(3) EON Solve करने की Jarurat hai Kya??

$(y-0) = \frac{-1}{0}(x-0)$

$0 = -x$

$\boxed{x=0}$

Special Case.

(1) tangent $\|^{rl}$ x axis.

$\frac{dy}{dx} = 0$

(2) tangent $\|^{rl}$ Y Axis

$\frac{dy}{dx} = \frac{1}{0}$

(3) tangent equally Inclined to Both axes

$\frac{dy}{dx} = \pm 1$

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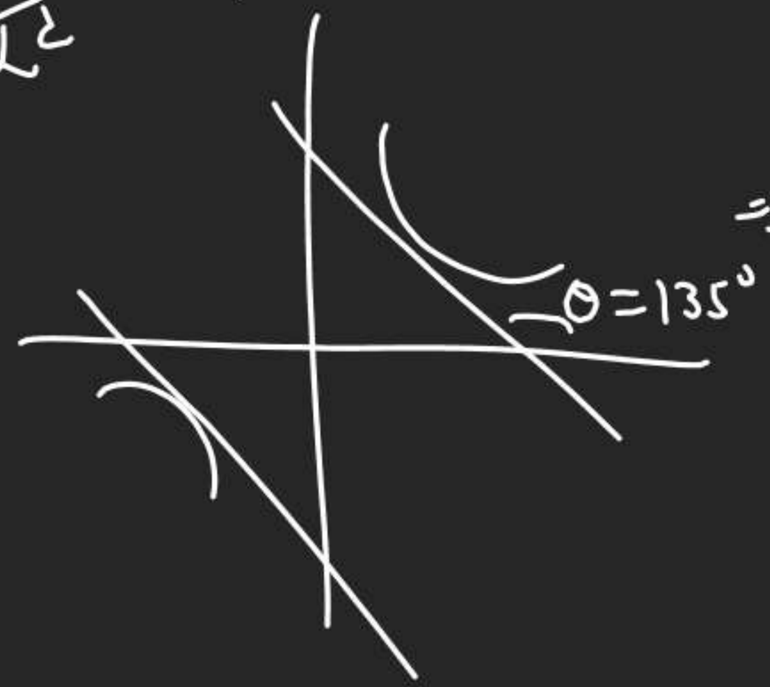
(4) When tangent is making equal Non Zero Intercepts

I^2



$\Rightarrow m = -1$