KEY CONCEPTS (STRAIGHT LINE)

1. DISTANCE FORMULA:

The distance between the points $A(x_1, y_1)$ and $B(x_2, y_2)$ is $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$.

2. SECTION FORMULA:

If $P(x, y)$ divides the line joining $A(x_1, y_1)$ & $B(x_2, y_2)$ in the ratio $m : n$, then;

$$x = \frac{mx_2 + nx_1}{m + n}; \quad y = \frac{my_2 + ny_1}{m + n}$$

If $\frac{m}{n}$ is positive, the division is internal, but if $\frac{m}{n}$ is negative, the division is external.

Note: If P divides AB internally in the ratio $m : n$ & Q divides AB externally in the ratio $m : n$ then P & Q are said to be harmonic conjugate of each other w. r. t. AB.

Mathematically; $\frac{2}{AB} = \frac{1}{AP} + \frac{1}{AQ}$ i.e. AP, AB & AQ are in H.P.

3. CENTROID AND INCENTRE :

If $A(x_1, y_1), B(x_2, y_2), C(x_3, y_3)$ are the vertices of triangle ABC, whose sides BC, CA, AB are of lengths a, b, c respectively, then the coordinates of the centroid are : $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$

& the coordinates of the incentre are : $\left(\frac{ax_1 + bx_2 + cx_3}{a + b + c}, \frac{ay_1 + by_2 + cy_3}{a + b + c}\right)$

Note: That incentre divides the angle bisectors in the ratio $(b + c) : a$; $(c + a) : b$ & $(a + b) : c$.

REMEMBER:

(i) Orthocenter, Centroid & circumcenter are always collinear & centroid divides the line joining orthocenter & circumcenter in the ratio 2: 1.

(ii) In an isosceles triangle G, O, I & C lie on the same line.

4. SLOPE FORMULA:

If θ is the angle at which a straight line is inclined to the positive direction of x-axis, & $0° \le \theta < 180°, \theta \ne 90°$, then the slope of the line, denoted by m, is defined by $m = \tan\theta$. If θ is 90°, m does not exist, but the line is parallel to the y-axis. If $\theta = 0$, then $m = 0$ & the line is parallel to the x-axis. If $A(x_1, y_1)$ & $B(x_2, y_2), x_1 \ne x_2$, are points on a straight line, then the slope m of the line is given by: $m = \left(\frac{y_1 - y_2}{x_1 - x_2}\right)$

5. CONDITION OF COLLINEARITY OF THREE POINTS - (SLOPE FORM):

Points A (x_1, y_1), B (x_2, y_2), C (x_3, y_3) are collinear if $\left(\frac{y_1 - y_2}{x_1 - x_2}\right) = \left(\frac{y_2 - y_3}{x_2 - x_3}\right)$.

6. EQUATION OF A STRAIGHT LINE IN VARIOUS FORMS:

(i) **Slope - intercept form:** $y = mx + c$ is the equation of a straight line whose slope is m & which makes an intercept c on the y-axis.

(ii) **Slope one point form:** $y - y_1 = m(x - x_1)$ is the equation of a straight line whose slope is m & which passes through the point (x_1, y_1).

(iii) **Parametric form:** The equation of the line in parametric form is given by $\frac{x-x_1}{\cos\theta}=\frac{y-y_1}{\sin\theta}=r$ (say). Where 'r' is the distance of any point (x, y) on the line from the fixed point (x_1, y_1) on the line. r is positive if the point (x, y) is on the right of (x_1, y_1) and negative if (x, y) lies on the left of (x_1, y_1).

(iv) **Two point form:** $y-y_1=\frac{y_2-y_1}{x_2-x_1}(x-x_1)$ is the equation of a straight line which passes through the points (x_1, y_1) & (x_2, y_2).

(v) **Intercept form:** $\frac{x}{a}+\frac{y}{b}=1$ is the equation of a straight line which makes intercepts a & b on OX & OY respectively.

(vi) **Perpendicular form:** $x\cos\alpha+y\sin\alpha=p$ is the equation of the straight line where the length of the perpendicular from the origin O on the line is p and this perpendicular makes angle α with positive side of x-axis.

(vii) **General Form:** $ax+by+c=0$ is the equation of a straight line in the general form

7. POSITION OF THE POINT (x_1, y_1) RELATIVE TO THE LINE $ax+by+c=0$:

If ax_1+b_1+c is of the same sign as c, then the point (x_1, y_1) lie on the origin side of $ax+by+c=0$. But if the sign of ax_1+by_1+c is opposite to that of c, the point (x_1, y_1) will lie on the non-origin side of $ax+by+c=0$.

8. THE RATIO IN WHICH A GIVEN LINE DIVIDES THE LINE SEGMENT JOINING TWO POINTS:

Let the given line $ax+by+c=0$ divide the line segment joining $A(x_1, y_1)$ & $B(x_2, y_2)$ in the ratio m: n, then $\frac{m}{n}=-\frac{ax_1+by_1+c}{ax_2+by_2+c}$

If A & B are on the same side of the given line then $\frac{m}{n}$ is negative but if A & B are on opposite sides of the given line, then $\frac{m}{n}$ is positive

9. LENGTH OF PERPENDICULAR FROM A POINT ON A LINE:

The length of perpendicular from $P(x_1, y_1)$ on $ax+by+c=0$ is $\left|\frac{ax_1+by_1+c}{\sqrt{a^2+b^2}}\right|$.

10. ANGLE BETWEEN TWO STRAIGHT LINES IN TERMS OF THEIR SLOPES:

If m_1 & m_2 are the slopes of two intersecting straight lines $(m_1m_2\neq-1)$ & θ is the acute angle between them, then $\tan\theta=\left|\frac{m_1-m_2}{1+m_1m_2}\right|$.

Note: Let m_1, m_2, m_3 are the slopes of three lines $L_1=0$; $L_2=0$; $L_3=0$ where $m_1>m_2>m_3$ then the interior angles of the $\triangle$ ABC found by these lines are given by,

$$\tan A=\frac{m_1-m_2}{1+m_1m_2};\ \tan B=\frac{m_2-m_3}{1+m_2m_3}\ \&\ \tan C=\frac{m_3-m_1}{1+m_3m_1}$$

11. PARALLEL LINES:

(i) When two straight lines are parallel their slopes are equal. Thus any line parallel to $ax + by + c = 0$ is of the type $ax + by + k = 0$. Where k is a parameter.

(ii) The distance between two parallel lines with equations $ax + by + c_1 = 0$ & $ax + by + c_2 = 0$ is $\left|\frac{c_1 - c_2}{\sqrt{a^2 + b^2}}\right|$.

Note: That the coefficients of x & y in both the equations must be same.

(iii) The area of the parallelogram $= \frac{p_1 p_2}{\sin\theta}$, where p_1 & p_2 are distances between two pairs of opposite sides & θ is the angle between any two adjacent sides.

Note: That area of the parallelogram bounded by the lines $y = m_1x + c_1, y = m_1x + c_2$ and $y = m_2x + d_1, y = m_2x + d_2$ is given by $\left|\frac{(c_1-c_2)(d_1-d_2)}{m_1-m_2}\right|$.

12. PERPENDICULAR LINES:

(i) When two lines of slopes m_1&m_2 are at right angles, the product of their slopes is -1, i.e. $m_1m_2 = -1$. Thus any line perpendicular to $ax + by + c = 0$ is of the form $bx - ay + k = 0$, where k is any parameter.

(ii) Straight lines $ax + by + c = 0$ & $a'x + b'y + c' = 0$ are at right angles if & only if $aa' + bb' = 0$.

13. Equations of straight lines through (x_1, y_1) making angle α with $y = mx + c$ are:
$(y - y_1) = \tan(\theta - \alpha)\,(x - x_1)$ & $(y - y_1) = \tan(\theta + \alpha)(x - x_1)$, where $\tan\theta = m$.

14. CONDITION OF CONCURRENCY:

Three lines $a_1x + b_1y + c_1 = 0, a_2x + b_2y + c_2 = 0$ & $a_3x + b_3y + c_3 = 0$ are concurrent if $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$. Alternatively: If three constants A, B & C can be found such that $A(a_1x + b_1y + c_1) + B(a_2x + b_2y + c_2) + C(a_3x + b_3y + c_3) \equiv 0$, then the three straight lines are concurrent.

15. AREA OF A TRIANGLE:

If $(x_i, y_i), i = 1, 2, 3$ are the vertices of a triangle, then its area is equal to $\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$, provided the vertices are considered in the counter clockwise sense. The above formula will give a (-) ve area if the vertices $(x_i, y_i), i = 1, 2, 3$ are placed in the clockwise sense.

16. CONDITION OF COLLINEARITY OF THREE POINTS-(AREAFORM):

The points $(x_i, y_i), i = 1, 2, 3$ are collinear if $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$.

17. THE EQUATION OF A FAMILY OF STRAIGHT LINES PASSING THROUGH THE POINTS OF INTERSECTION OF TWO GIVEN LINES:

The equation of a family of lines passing through the point of intersection of $a_1x + b_1y + c_1 = 0$ & $a_2x + b_2y + c_2 = 0$ is given by $(a_1x + b_1y + c_1) + k(a_2x + b_2y + c_2) = 0$, where k is an arbitrarv real number.

Note: If $u_1 = ax + by + c, u_2 = a'x + b'y + d, u_3 = ax + by + c', u_4 = a'x + b'y + d'$ then, $u_1 = 0;\ u_2 = 0;\ u_3 = 0; u_4 = 0$ form a parallelogram. $u_2u_3 - u_1u_4 = 0$ represents the diagonal BD.

Proof: Since it is the first degree equation in x & y it is a straight line. Secondly point B satisfies the equation because the co-ordinates of B satisfy $u_2 = 0$ and $u_1 = 0$. Similarly for the point D. Hence the result. On the similar lines $u_1u_2 - u_3u_4 = 0$ represents the diagonal AC.

Note: The diagonal AC is also given by $u_1 + \lambda u_4 = 0$ and $u_2 + \mu u_3 = 0$, if the two equations are identical for some λ and μ.

[For getting the values of λ & μ compare the coefficients of x, y & the constant terms].

18. BISECTORS OF THE ANGLES BETWEEN TWO LINES:

(i) Equations of the bisectors of angles between the lines $ax + by + c = 0$ & $a'x + b'y + c' = 0$ $(ab' \neq a'b)$ are : $\frac{ax + by + c}{\sqrt{a^2 + b^2}} = \pm \frac{a'x + b'y + c'}{\sqrt{a'^2 + b'^2}}$

(ii) To discriminate between the acute angle bisector & the obtuse angle bisector

If θ be the angle between one of the lines & one of the bisectors, find $\tan\theta$.

If $|\tan\theta| < 1$, then $2\theta < 90°$ so that this bisector is the acute angle bisector.

If $|\tan\theta| > 1$, then we get the bisector to be the obtuse angle bisector.

(iii) To discriminate between the bisector of the angle containing the origin & that of the angle not containing the origin. Rewrite the equations, $ax + by + c = 0$ & $a'x + b'y + c' = 0$ such that the constant terms c, c′ are positive. Then; $\frac{ax + by + c}{\sqrt{a^2 + b^2}} = + \frac{a'x + b'y + c'}{\sqrt{a'^2 + b'^2}}$ gives the equation of the bisector of the angle containing the origin & $\frac{ax + by + c}{\sqrt{a^2 + b^2}} = - \frac{a'x + b'y + c'}{\sqrt{a'^2 + b'^2}}$ gives the equation of the bisector of the angle not containing the origin.

(iv) To discriminate between acute angle bisector & obtuse angle bisector proceed as follows

Write $ax + by + c = 0$ & $a'x + b'y + c' = 0$ such that constant terms are positive.

If $aa' + bb' < 0$, then the angle between the lines that contains the origin is acute and the equation of the bisector of this acute angle is $\frac{ax + by + c}{\sqrt{a^2 + b^2}} = + \frac{a'x + b'y + c'}{\sqrt{a'^2 + b'^2}}$

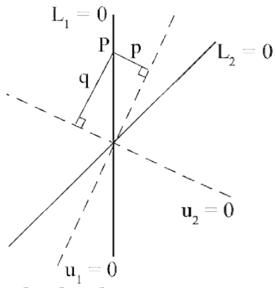
therefore $\frac{ax + by + c}{\sqrt{a^2 + b^2}} = - \frac{a'x + b'y + c'}{\sqrt{a'^2 + b'^2}}$ is the equation of other bisector.

If, however , $aa' + bb' > 0$, then the angle between the lines that contains the origin is obtuse & the equation of the bisector of this obtuse angle is:

$\frac{ax + by + c}{\sqrt{a^2 + b^2}} = +\frac{a'x + b'y + c'}{\sqrt{a'^2 + b'^2}}$; therefore $\frac{ax + by + c}{\sqrt{a^2 + b^2}} = -\frac{a'x + b'y + c'}{\sqrt{a'^2 + b'^2}}$ is the equation of other bisector.

(v) Another way of identifying an acute and obtuse angle bisector is as follows:

Let $L_1 = 0$ & $L_2 = 0$ are the given lines & $u_1 = 0$ and $u_2 = 0$ are the bisectors between $L_1 = 0$ & $L_2 = 0$. Take a point P on any one of the lines $L_1 = 0$ or $L_2 = 0$ and drop perpendicular on $u_1 = 0$ & $u_2 = 0$ as shown.



If, $|p| < |q| \Rightarrow u_1$ is the acute angle bisector.

$|p| > |q| \Rightarrow u_1$ is the obtuse angle bisector.

$|p| = |q| \Rightarrow$ the lines L_1 & L_2 are perpendicular.

Note: Equation of straight lines passing through $P(x_1, y_1)$ & equally inclined with the lines $a_1x + b_1y + c_1 = 0$ & $a_2x + b_2y + c_2 = 0$ are those which are parallel to the bisectors between these two lines & passing through the point P.

19. A PAIR OF STRAIGHT LINES THROUGH ORIGIN:

(i) A homogeneous equation of degree two of the type $ax^2 + 2hxy + by^2 = 0$ always represents a pair of straight lines passing through the origin & if:

(a) $h^2 > ab \Rightarrow$ lines are real & distinct.

(b) $h^2 = ab \Rightarrow$ lines are coincident.

(c) $h^2 < ab \Rightarrow$ lines are imaginary with real point of intersection i.e. $(0, 0)$

(ii) If $y = m_1x$ & $y = m_2x$ be the two equations represented by $ax^2 + 2hxy + by^2 = 0$, then; $m_1 + m_2 = -\frac{2h}{b}$ & $m_1m_2 = \frac{a}{b}$.

(iii) If θ is the acute angle between the pair of straight lines represented by,

$ax^2 + 2hxy + by^2 = 0$, then; $\tan\theta = \left|\frac{2\sqrt{h^2 - ab}}{a + b}\right|$

The condition that these lines are:

(a) At right angles to each other is $a + b = 0$.
i.e. co-efficient of x^2 + coefficient of $y^2 = 0$.

(b) Coincident is $h^2 = ab$.

(c) Equally inclined to the axis of x is $h = 0$.
i.e. coeff. of $xy = 0$.

Note: A homogeneous equation of degree n represents n straight lines passing through origin.

20. **GENERAL EQUATION OF SECOND DEGREE REPRESENTING APAIR OF STRAIGHT LINES:**

(i) $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represents a pair of straight lines if:

$abc + 2fgh - af^2 - bg^2 - ch^2 = 0$, i.e. if $\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = 0$

(ii) The angle θ between the two lines representing by a general equation is the same as that between the two lines represented by its homogeneous part only.

21. The joint equation of a pair of straight lines joining origin to the points of intersection of the line given by

$1x + my + n = 0$ ... ... **(i) &**

the 2nd degree curve : $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ ... ... **(ii)**

is $ax^2 + 2hxy + by^2 + 2gx\left(\frac{lx+my}{-n}\right) + 2fy\left(\frac{lx+my}{-n}\right) + c\left(\frac{lx+my}{-n}\right)^2 = 0$ ... ... **(iii)**

(iii) is obtained by homogenizing (ii) with the help of (i), by writing (i) in the form: $\left(\frac{lx + my}{-n}\right) = 1$.

22. The equation to the straight lines bisecting the angle between the straight lines, $ax^2 + 2hxy + by^2 = 0$ is $\frac{x^2-y^2}{a-b} = \frac{xy}{h}$.

23. The product of the perpendiculars, dropped from (x_1, y_1) to the pair of lines represented by the equation, $ax^2 + 2hxy + by^2 = 0$ is $\frac{ax_1{}^2 + 2hx_1y_1 + by_1{}^2}{\sqrt{(a-b)^2 + 4h^2}}$.

24. Any second degree curve through the four point of intersection of $f(xy) = 0$ & $xy = 0$ is given by $f(xy) + \lambda xy = 0$ where $f(xy) = 0$ is also a second degree curve.

EXERCISE–I

1. Line $\frac{x}{6}+\frac{y}{8}=1$ intersects the x and y axes at M and N respectively. If the coordinates of the point P lying inside the triangle OMN (where ' O ' is origin) are (a, b) such that the areas of the triangle POM, PON and PMN are equal. Find

(a) the coordinates of the point P and

(b) the radius of the circle escribed opposite to the angle N.

2. Find the co-ordinates of the orthocenter of the triangle, the equations of whose sides are $x+y=1, 2x+3y=6, 4x-y+4=0$, without finding the co-ordinates of its vertices.

3. Two vertices of a triangle are $(4,-3)$ & $(-2,5)$. If the orthocenter of the triangle is at $(1,2)$, find the coordinates of the third vertex.

4. The point A divides the join of $P(-5,1)$ & $Q(3,5)$ in the ratio $K:1$. Find the two values of K for which the area of triangle ABC, where B is $(1,5)$ & C is $(7,-2)$, is equal to 2 units in magnitude.

5. Determine the ratio in which the point $P(3,5)$ divides the join of $A(1,3)$ & $B(7,9)$.
Find the harmonic conjugate of P w.r.t. A & B.

6. A line is such that its segment between the straight lines $5x-y-4=0$ and $3x+4y-4=0$ is bisected at the point $(1,5)$. Obtain the equation.

7. A line through the point $P(2,-3)$ meets the lines $x-2y+7=0$ and $x+3y-3=0$ at the points A and B respectively . If P divides AB externally in the ratio $3:2$
then find the equation of the line AB.

8. The area of a triangle is 5. Two of its vertices are $(2,1)$ & $(3,-2)$. The third vertex lies on $y=x+3$. Find the third vertex.

9. A variable line, drawn through the point of intersection of the straight lines $\frac{x}{a}+\frac{y}{b}=1$ & $\frac{x}{b}+\frac{y}{a}=1$, meets the coordinate axes in A & B. Show that the locus of the mid point of AB is the curve $2xy(a+b)=ab(x+y)$.

10. Two consecutive sides of a parallelogram are $4x+5y=0$ & $7x+2y=0$.
If the equation to one diagonal is $11x+7y=9$, find the equation to the other diagonal.

11. The line $3x+2y=24$ meets the y-axis at A & the x-axis at B. The perpendicular bisector of AB meets the line through $(0,-1)$ parallel to x-axis at C. Find the area of the triangle ABC.

12. If the straight line drawn through the point $P(\sqrt{3},2)$ & inclined at an angle $\frac{\pi}{6}$ with the x-axis, meets the line $\sqrt{3}x-4y+8=0$ at Q. Find the length PQ.

13. Find the condition that the diagonals of the parallelogram formed by the lines $ax+by+c=0$; $ax+by+c'=0$; $a'x+b'y+c=0$ & $a'x+b'y+c'=0$ are at right angles.
Also find the equation to the diagonals of the parallelogram.

14. A triangle has side lengths 18, 24 and 30. Find the area of the triangle whose vertices are the incentre, circumcenter and centroid of the triangle.

15. The points $(1,3)$ & $(5,1)$ are two opposite vertices of a rectangle. The other two vertices lie on the line $y = 2x + c$. Find c & the remaining vertices.

16. A straight line L is perpendicular to the line $5x - y = 1$. The area of the triangle formed by the line L & the coordinate axes is 5. Find the equation of the line.

17. Two equal sides of an isosceles triangle are given by the equations $7x - y + 3 = 0$ and $x + y - 3 = 0$ & its third side passes through the point $(1, -10)$. Determine the equation of the third side.

18. The vertices of a triangle OBC are $O(0,0), B(-3,-1), C(-1,-3)$. Find the equation of the line parallel to BC & intersecting the sides OB & OC, whose perpendicular distance from the point $(0,0)$ is half.

19. Starting at the origin, a beam of light hits a mirror (in the form of a line) at the point $A(4,8)$ and is reflected at the point $B(8,12)$. Compute the slope of the mirror.

20. Given vertices $A(1,1), B(4,-2)$ & $C(5,5)$ of a triangle, find the equation of the perpendicular dropped from C to the interior bisector of the angle A.

21. Triangle ABC lies in the Cartesian plane and has an area of 70 sq. units. The coordinates of B and C are $(12,19)$ and $(23,20)$ respectively and the coordinates of A are (p,q). The line containing the median to the side BC has slope -5. Find the largest possible value of $(p + q)$.

22. A straight line is drawn from the point $(1,0)$ to the curve $x^2 + y^2 + 6x - 10y + 1 = 0$, such that the intercept made on it by the curve subtends a right angle at the origin. Find the equations of the line.

23. Determine the range of values of $\theta \in [0, 2\pi]$ for which the point $(\cos\theta, \sin\theta)$ lies inside the triangle formed by the lines $x + y = 2$; $x - y = 1$ & $6x + 2y - \sqrt{10} = 0$.

24. The points $(-6,1), (6,10), (9,6)$ and $(-3,-3)$ are the vertices of a rectangle. If the area of the portion of this rectangle that lies above the x axis is $\frac{a}{b}$, find the value of $(a + b)$, given a and b are coprime.

25. The two line pairs $y^2 - 4y + 3 = 0$ and $x^2 + 4xy + 4y^2 - 5x - 10y + 4 = 0$ enclose a 4 sided convex polygon find

(i) area of the polygon;

(ii) length of its diagonals.

EXERCISE–II

1. The equations of perpendiculars of the sides AB & AC of triangle ABC are $x - y - 4 = 0$ and $2x - y - 5 = 0$ respectively. If the vertex A is $(-2, 3)$ and point of intersection of perpendiculars bisectors is $\left(\frac{3}{2}, \frac{5}{2}\right)$, find the equation of medians to the sides AB & AC respectively.

2. The interior angle bisector of angle A for the triangle ABC whose coordinates of the vertices are $A(-8, 5)$; $B(-15, -19)$ and $C(1, -7)$ has the equation $ax + 2y + c = 0$. Find 'a' and 'c'.

3. Show that all the chords of the curve $3x^2 - y^2 - 2x + 4y = 0$ which subtend a right angle at the origin are concurrent. Does this result also hold for the curve, $3x^2 + 3y^2 - 2x + 4y = 0$?
If yes, what is the point of concurrency & if not, give reasons.

4. The coordinates of the vertices of a quadrilateral are $A(0, 0)$; $B(16, 0)$, $C(8, 8)$, $D(0, 8)$.
Find the equation of the line parallel to AC that halves the area of the quadrilateral in the form of $y = mx + c$.

5. Find the equation of the straight lines passing through $(-2, -7)$ & having an intercept of length 3 between the straight lines $4x + 3y = 12$, $4x + 3y = 3$

6. Without finding the vertices or angles of the triangle, show that the three straight lines $au + bv = 0$; $au - bv = 2ab$ and $u + b = 0$ from an isosceles triangle where $u \equiv x + y - b$ & $v \equiv x - y - a$ & $a, b \neq 0$.

7. Two sides of a rhombus ABCD are parallel to the lines $y = x + 2$ & $y = 7x + 3$. If the diagonals of the rhombous intersect at the point (1,2) & the vertex A is on the y-axis, find the possible coordinates of A.

8. The equations of the perpendicular bisectors of the sides AB & AC of a triangle ABC are $x - y + 5 = 0$ & $x + 2y = 0$, respectively. If the point A is $(1, -2)$ find the equation of the line BC.

9. A triangle is formed by the lines whose equations are
AB: $x + y - 5 = 0$,
BC: $x + 7y - 7 = 0$ and
CA: $7x + y + 14 = 0$.
Find the bisector of the interior angle at B and the exterior angle at C. Determine the nature of the interior angle at A and find the equaion of the bisector.

10. A point P is such that its perpendicular distance from the line $y - 2x + 1 = 0$ is equal to its distance from the origin. Find the equation of the locus of the point P. Prove that the line $y = 2x$ meets the locus in two points Q & R, such that the origin is the mid point of QR.

11. A triangle has two sides $y = m_1x$ and $y = m_2x$ where m_1 and m_2 are the roots of the equation $b\alpha^2 + 2h\alpha + a = 0$. If (a, b) be the orthocentre of the triangle, then find the equation of the third side in terms of a, b and h.

12. Find the area of the triangle formed by the straight lines whose equations are $x + 2y - 5 = 0$; $2x + y - 7 = 0$ and $x - y + 1 = 0$ without determining the coordinates of the vertices of the triangle. Also compute the tangent of the interior angles of the triangle and hence comment upon the nature of triangle.

13. Find the equation of the two straight lines which together with those given by the equation $6x^2 - xy - y^2 + x + 12y - 35 = 0$ will make a parallelogram whose diagonals intersect in the origin.

14. Find the equations of the sides of a triangle having $(4, -1)$ as a vertex, if the lines $x - 1 = 0$ and $x - y - 1 = 0$ are the equations of two internal bisectors of its angles.

15. Equation of a line is given by $y + 2at = t(x - at^2)$, t being the parameter. Find the locus of the point of intersection of the lines which are at right angles.

16. The ends A, B of a straight line segment of a constant length 'c' slide upon the fixed rectangular axes OX & OY respectively. If the rectangle OAPB be completed then show that the locus of the foot of the perpendicular drawn from P to AB is $x^{2/3} + y^{2/3} = c^{2/3}$.

17. The sides of a triangle are $U_r \equiv x\cos\alpha_r + y\sin\alpha_r - p_r = 0, (r = 1, 2, 3)$.
Show that the orthocenter is given by $U_1\cos(\alpha_2 - \alpha_3) = U_2\cos(\alpha_3 - \alpha_1) = U_3\cos(\alpha_1 - \alpha_2)$.

18. P is the point $(-1, 2)$, a variable line through P cuts the x & y axes at A & B respectively Q is the point on AB such that PA, PQ, PB are H.P. Show that the locus of Q is the line $y = 2x$.

19. The equations of the altitudes AD, BE, CF of a triangle ABC are $x + y = 0, x - 4y = 0$ and $2x - y = 0$ respectively. The coordinates of A are $(t, -t)$. Find coordinates of B & C.
Prove that if t varies the locus of the centroid of the triangle ABC is $x + 5y = 0$.

20. The distance of a point (x_1, y_1) from each of two straight lines which passes through the origin of co-ordinates is δ; find the combined equation of these straight lines.

EXERCISE–III

1. Let PQR be a right angled isosceles triangle, right angled at P(2,1). If the equation of the line QR is $2x + y = 3$, then the equation representing the pair of lines PQ and PR is **[JEE'99,]**

(A) $3x^2 - 3y^2 + 8xy + 20x + 10y + 25 = 0$ (B) $3x^2 - 3y^2 + 8xy - 20x - 10y + 25 = 0$

(C) $3x^2 - 3y^2 + 8xy + 10x + 15y + 20 = 0$ (D) $3x^2 - 3y^2 - 8xy - 10x - 15y - 20 = 0$

2. The equation of two equal sides AB and AC of an isosceles triangle ABC are $x + y = 5$ & $7x - y = 3$ respectively. Find the equations of the side BC if the area of the triangle of ABC is 5 units.

[REE '99, 6]

3. **(a)** The incentre of the triangle with vertices $(1, \sqrt{3})$, $(0, 0)$ and $(2, 0)$ is:

(A) $\left(1, \frac{\sqrt{3}}{2}\right)$ (B) $\left(\frac{2}{3}, \frac{1}{\sqrt{3}}\right)$ (C) $\left(\frac{2}{3}, \frac{\sqrt{3}}{2}\right)$ (D) $\left(1, \frac{1}{\sqrt{3}}\right)$

(b) Let PS be the median of the triangle with vertices, $P(2, 2)$, $Q(6, -1)$ and $R(7, 3)$.

The equation of the line passing through $(1, -1)$ and parallel to PS is

(A) $2x - 9y - 7 = 0$ (B) $2x - 9y - 11 = 0$

(C) $2x + 9y - 11 = 0$ (D) $2x + 9y + 7 = 0$ **[JEE 2000]**

(c) For points $P = (x_1, y_1)$ and $Q = (x_2, y_2)$ of the co-ordinate plane, a new distance $d(P, Q)$ is defined by $d(P, Q) = |x_1 - x_2| + |y_1 - y_2|$. Let $O = (0, 0)$ and $A = (3, 2)$.

Prove that the set of points in the first quadrant which are equidistant (with respect to the new distance) from O and A consists of the union of a line segment of finite length and an infinite ray. Sketch this set in a labelled diagram. **[JEE 2000 (Mains) 10 out of 100]**

4. Find the position of point (4,1) after it undergoes the following transformations successively.

(i) Reflection about the line, $y = x - 1$ **[REE 2000 (Mains)]**

(ii) Translation by one unit along x-axis in the positive direction.

(iii) Rotation through an angle $\pi/4$ about the origin in the anti-clockwise direction.

5. **(a)** Area of the parallelogram formed by the lines $y = mx$, $y = mx + 1$, $y = nx$ and $y = nx + 1$ equals

(A) $\frac{|m + n|}{(m - n)^2}$ (B) $\frac{2}{|m + n|}$ (C) $\frac{1}{|m + n|}$ (D) $\frac{1}{|m - n|}$

(b) The number of integer values of m, for which the x co-ordinate of the point of intersection of the lines $3x + 4y = 9$ and $y = mx + 1$ is also an integer, is **[JEE 2001 (Screening)]**

(A) 2 (B) 0 (C) 4 (D) 1

6. **(a)** Let $P = (-1, 0)$, $Q = (0, 0)$ and $R = (3, 3\sqrt{3})$ be three points. Then the equation of the bisector of the angle PQR is

(A) $\frac{\sqrt{3}}{2}x + y = 0$ (B) $x + \sqrt{3}y = 0$ (C) $\sqrt{3}x + y = 0$ (D) $x + \frac{\sqrt{3}}{2}y = 0$

(b) A straight line through the origin O meets the parallel lines $4x + 2y = 9$ and $2x + y + 6 = 0$ at points P and Q respectively. Then the point O divides the segment PQ in the ratio

(A) 1: 2 (B) 3: 4 (C) 2: 1 (D) 4: 3

(c) The area bounded by the curves $y = |x| - 1$ and $y = -|x| + 1$ is **[JEE 2002 (Screening)]**

(A) 1 (B) 2 (C) $2\sqrt{2}$ (D) 4

(d) A straight line L through the origin meets the line $x + y = 1$ and $x + y = 3$ at P and Q respectively. Through P and Q two straight lines L_1 and L_2 are drawn, parallel to $2x - y = 5$ and $3x + y = 5$ respectively. Lines L_1 and L_2 intersect at R. Show that the locus of R, as L varies, is a straight line. **[JEE 2002 (Mains)]**

(e) A straight line L with negative slope passes through the point (8,2) and cuts the positive coordinates axes at points P and Q. Find the absolute minimum value of OP + OQ, as L varies, where O is the origin. **[JEE 2002 Mains,]**

7. The area bounded by the angle bisectors of the lines $x^2 - y^2 + 2y = 1$ and the line $x + y = 3$, is

(A) 2 (B) 3 (C) 4 (D) 6 **[JEE 2004 (Screening)]**

8. The area of the triangle formed by the intersection of a line parallel to x-axis and passing through P(h, k) with the lines $y = x$ and $x + y = 2$ is $4h^2$.

Find the locus of the point P. **[JEE 2005, Mains, 2]**

9. **(a)** Let O(0, 0), P(3, 4), Q(6, 0) be the vertices of the triangle OPQ. The point R inside the triangle OPQ is such that the triangles OPR, PQR, OQR are of equal area. The coordinates of R are

(A) $\left(\frac{4}{3}, 3\right)$ (B) $\left(3, \frac{2}{3}\right)$ (C) $\left(3, \frac{4}{3}\right)$ (D) $\left(\frac{4}{3}, \frac{2}{3}\right)$

(b) Lines $L_1: y - x = 0$ and $L_2: 2x + y = 0$ intersect the line $L_3: y + 2 = 0$ at P and Q, respectively. The bisector of the acute angle between L_1 and L_2 intersects L_3 at R.

Statement-1: The ratio PR: RQ equals $2\sqrt{2}: \sqrt{5}$

because

Statement-2: In any triangle, bisector of an angle divides the triangle into two similar triangles.

(A) Statement-1 is true, statement-2 is true; statement-2 is a correct explanation for statement-1.

(B) Statement-1 is true, statement-2 is true; statement-2 is NOT a correct explanation for statement-1.

(C) Statement-1 is true, statement-2 is false.

(D) Statement- 1 is false, statement- 2 is true. **[JEE 2007, 3+3]**

10. Consider the lines given by $L_1 = x + 3y - 5 = 0$; $L_2 = 3x - ky - 1 = 0$; $L_3 = 5x + 2y - 12 = 0$

Match the statements/Expression in **Column-I** with the statements/Expressions in **Column-II** and indicate your answer by darkening the appropriate bubbles in the 4×4 matrix given in OMR.

[JEE 2008, 6]

Column-I	Column-II
(A) L_1, L_2, L_3 are concurrent, if	(P) $k = -9$
(B) One of L_1, L_2, L_3 is parallel to at least one of the other two, if	(Q) $k = -\frac{6}{5}$
(C) L_1, L_2, L_3 form a triangle, if	(R) $k = \frac{5}{6}$
(D) L_1, L_2, L_3 do not form a triangle, if	(S) $k = 5$

11. The locus of the orthocenter of the triangle formed by the lines $(1+p)x - py + p(1+p) = 0, (1+q)x - qy + q(1+q) = 0$, and $y = 0$, where $p \neq q$, is **[IIT 2009]**

(A) a hyperbola (B) a parabola

(C) an ellipse (D) a straight line

12. A straight line L through the point $(3, -2)$ is inclined at an angle $60°$ to the line $\sqrt{3}x + y = 1$. If L also intersects the x-axis, then the equation of L is: **[IIT 2011]**

(A) $y + \sqrt{3}x + 2 - 3\sqrt{3} = 0$ (B) $y - \sqrt{3}x + 2 + 3\sqrt{3} = 0$

(C) $\sqrt{3}y - x + 3 + 2\sqrt{3} = 0$ (D) $\sqrt{3}y + x - 3 + 2\sqrt{3} = 0$

13. The x-coordinate of the incentre of the triangle that has the coordinates of mid points of its sides as (0,1) (1,1) and (1,0) is: **[IIT JEE Main - 2013]**

(A) $1 - \sqrt{2}$ (B) $2 + \sqrt{2}$ (C) $2 - \sqrt{2}$ (D) $1 + \sqrt{2}$

14. A ray of light along $x + \sqrt{3}y = \sqrt{3}$ gets reflected upon reaching x-axis, the equation of the reflected ray is: **[IIT JEE Main - 2013]**

(A) $\sqrt{3}y = x - 1$ (B) $y = x + \sqrt{3}$ (C) $\sqrt{3}y = x - \sqrt{3}$ (D) $y = \sqrt{3}x - \sqrt{3}$

15. For $a > b > c > 0$, the distance between (1,1) and the point of intersection of the lines $ax + by + c = 0$ and $bx + ay + c = 0$ is less than $2\sqrt{2}$. Then **[IIT JEE Advance - 2013]**

(A) $a + b - c > 0$ (B) $a - b + c < 0$ (C) $a - b + c > 0$ (D) $a + b - c < 0$

16. Let PS be the median of the triangle with vertices P(2,2), Q(6,−1) and R(7,3). The equation of the line passing through (1,−1) and parallel to PS is **[IIT JEE Main - 2014]**

(A) $2x - 9y - 11 = 0$ (B) $4x - 7y - 11 = 0$ (C) $2x + 9y + 7 = 0$ (D) $4x + 7y + 3 = 0$

17. For a point P in the plane, let $d_1(P)$ and $d_2(P)$ be the distances of the point P from the lines $x - y = 0$ and $x + y = 0$ respectively. The area of the region R consisting of all points P lying in the first quadrant of the plane and satisfying $2 \leq d_1(P) + d_2(P) \leq 4$, is

[IIT JEE Advance - 2014]

18. The normal to the curve, $x^2 + 2xy - 3y^2 = 0$, at (1,1) : **[IIT JEE Main - 2015]**

(A) meets the curve again in the fourth quadrant

(B) does not meet the curve again

(C) meets the curve again in the second quadrant

(D) meets the curve again in the third quadrant

19. The number of points, having both co-ordinates as integers, that lie in the interior of the triangle with vertices $(0,0)$, $(0,41)$ and $(41,0)$, is: **[IIT JEE Main - 2015]**

(A) 780 (B) 901 (C) 861 (D) 820

20. Two sides of a rhombus are along the lines, $x-y+1=0$ and $7x-y-5=0$. If its diagonals intersect at $(-1,-2)$, then which one of the following is a vertex of this rhombus?

(A) $(-3,-9)$ (B) $(-3,-8)$ (C) $\left(\frac{1}{3},-\frac{8}{3}\right)$ (D) $\left(-\frac{10}{3},-\frac{7}{3}\right)$ **[IIT JEE Main - 2016]**

21. Let $\alpha, \lambda, \mu \in R$ Consider the system of linear equations **[JEE Advanced - 2016]**

$\alpha x + 2y = \lambda$

$3x - 2y = \mu$

Which of the following statement(s) is(are) correct?

(A) If $\alpha = -3$, then the system has infinitely many solutions for all values of λ and μ.

(B) If $\alpha \neq -3$, then the system has a unique solution for all values of λ and μ.

(C) If $\lambda + \mu = 0$, then the system has infinitely many solution for $\alpha = -3$

(D) If $\lambda + \mu \neq 0$, then the system has no solution for $\alpha = -3$

22. Let k be an integer such the triangle with vertices $(k,-3k)$, $(5,k)$ and $(-k,2)$ has area 28 Sq. Units. Then the orthocentre of this triangle is at the point: **[IIT JEE Main - 2017]**

(A) $\left(1,-\frac{3}{4}\right)$ (B) $\left(2,\frac{1}{2}\right)$ (C) $\left(2,-\frac{1}{2}\right)$ (D) $\left(1,\frac{3}{4}\right)$

23. A straight line through a fixed point $(2,3)$ intersects the coordinate axes at distinct points P and Q. If O is the origin and the rectangle OPRQ is completed, then the locus of R is **[JEE Main 2018]**

(A) $3x+2y=6xy$ (B) $3x+2y=6$ (C) $2x+3y=xy$ (D) $3x+2y=xy$

Question Stem for Question Nos. 24 and 25

Question Stem **[JEE Advanced - 2021]**

Consider the lines L_1 and L_2 defined by

$L_1: x\sqrt{2}+y-1=0$ and $L_2: x\sqrt{2}-y+1=0$

For a fixed constant λ, let C be the locus of a point P such that the product of the distance of P from L_1 and the distance of P from L_2 is λ_2. The line $y=2x+1$ meets C at two points R and S, where the distance between R and 5 is $\sqrt{270}$.

Let the perpendicular bisector of RS meet C at two distinct points R′ and S′.

Let D be the square of the distance between R′ and S′.

24. The value of λ^2 is___________.

25. The value of D is__________.

Answer Key

EXERCISE–I

Q. 1 **(a)** $\left(2, \frac{8}{3}\right)$; **(b)** (4) **Q. 2** $\left(\frac{3}{7}, \frac{22}{7}\right)$ **Q. 3** $(33, 26)$ **Q. 4** $K = 7$ or $\frac{31}{9}$

Q. 5 $1:2; Q(-5,-3)$ **Q. 6** $83x - 35y + 92 = 0$ **Q. 7** $2x + y - 1 = 0$

Q. 8 $\left(\frac{7}{2}, \frac{13}{2}\right)$ or $\left(-\frac{3}{2}, \frac{3}{2}\right)$ **Q. 10** $x - y = 0$ **Q. 11** 91 Sq. Units **Q. 12** 6 Units

Q. 13 $a^2 + b^2 = a'^2 + b'^2; (a + a')x + (b + b')y + (c + c') = 0; (a - a')x + (b - b')y = 0$

Q. 14 3 Units **Q. 15** $c = -4; B(2, 0); D(4, 4)$

Q. 16 $x + 5y + 5\sqrt{2} = 0$ or $x + 5y - 5\sqrt{2} = 0$

Q. 17 $x - 3y - 31 = 0$ or $3x + y + 7 = 0$ **Q. 18** $2x + 2y + \sqrt{2} = 0$ **Q. 19** $\frac{1 + \sqrt{10}}{3}$

Q. 20 $x - 5 = 0$ **Q. 21** 47

Q. 22 $x + y = 1; x + 9y = 1$ **Q. 23** $0 < \theta < \frac{5\pi}{6} - \tan^{-1} 3$

Q. 24 533 **Q. 25** (i) area = 6 Sq. Units, (ii) diagonals are $\sqrt{5}$ & $\sqrt{53}$

EXERCISE-II

Q. 1 $x + 4y = 4; 5x + 2y = 8$ **Q. 2** $a = 11, c = 78$ **Q. 3** $(1, -2)$, yes $\left(\frac{1}{3}, -\frac{2}{3}\right)$

Q. 4 $y = x + 8\sqrt{3} - 16$ **Q. 5** $7x + 24y + 182 = 0$ or $x = -2$

Q. 7 $(0, 0)$ or $\left(0, \frac{5}{2}\right)$ **Q. 8** $14x + 23y = 40$

Q. 9 $3x + 6y - 16 = 0; 8x + 8y + 7 = 0; 12x + 6y - 11 = 0$

Q. 10 $x^2 + 4y^2 + 4xy + 4x - 2y - 1 = 0$

Q. 11 $(a + b)(ax + by) = ab(a + b - 2h)$

Q. 12 $\frac{3}{2}$ sq. units, $\left(3, 3, \frac{3}{4}\right)$, isosceles **Q. 13** $6x^2 - xy - y^2 - x - 12y - 35 = 0$

Q. 14 $2x - y + 3 = 0, 2x + y - 7 = 0, x - 2y - 6 = 0$ **Q. 15** $y^2 = a(x - 3a)$

Q. 19 $B\left(-\frac{2t}{3}, -\frac{t}{6}\right), C\left(\frac{t}{2}, t\right)$ **Q. 20** $(y_1{}^2 - \delta^2)x^2 - 2x_1y_1xy + (x_1{}^2 - \delta^2)y^2 = 0$

EXERCISE–III

Q. 1 (B)

Q. 2 $x - 3y + 21 = 0, x - 3y + 1 = 0, 3x + y = 12, 3x + y = 2$

Q. 3 **(a)** (D); **(b)** (D) **Q. 4** $(4,1) \to (2, 3) \to (3, 3) \to (0, 3\sqrt{2})$ **Q. 5** **(a)** D; **(b)** A

Q. 6 (a) C;(b) B;(c) B;(d) $x - 3y + 5 = 0$;(e) 18 **Q. 7** (A)

Q. 8 $y = 2x + 1, y = -2x + 1$ **Q. 9** (a) C; (b) C **Q. 10** (A)S; (B) P, Q; (C) R;(D) P, Q, S

Q. 11 (D) **Q. 12** (B) **Q. 13** (C) **Q. 14** (C) **Q. 15** (A/C or A, C) **Q. 16** (C)

Q. 17 (6) **Q. 18** (A) **Q. 19** (A) **Q. 20** (C) **Q. 21** (B, C, D) **Q.22** (B)

Q. 23 (D) **Q. 24** (9.00) **Q. 25** (77.14)