

$$\underline{3.} \quad B = \frac{\frac{\log 23}{\log 41} \cdot \frac{\log 86}{\log 21} \cdot \frac{\log 91}{\log 23} \cdot \frac{\log 41}{\log 86}}{\frac{\log 91}{\log 11} \cdot \frac{\log 11}{\log 21}} = 1$$

$$A = \left| \log_2 |\cos 24^\circ| + \log_2 |\cos 48^\circ| + \log_2 |\cos 96^\circ| + \log_2 |\cos 192^\circ| \right|$$

$$\left| \log_2 \left| \frac{\sin 384^\circ}{16 \sin 24^\circ} \right| \right| = 4.$$

$$\tan \theta = \frac{1 - \cos 2\theta}{\sin 2\theta} = \frac{1 - \left(-\frac{7}{25}\right)}{-\frac{24}{25}}$$

$$\frac{4 \left( \cancel{\sin \frac{\pi}{7}} + \sin \frac{3\pi}{7} - \cancel{\sin \frac{\pi}{7}} \right)}{\sin \frac{4\pi}{7}} \quad 2\theta \in \left( \pi, \frac{3\pi}{2} \right)$$

$\theta \in \left( \frac{\pi}{2}, \frac{3\pi}{4} \right)$

II

$$\frac{2\cos 2\alpha + 1}{\cos \alpha \cos 2\alpha}$$

$$= \frac{1 + 2\cos \frac{2\pi}{7}}{\cos \frac{\pi}{7} \cos \frac{4\pi}{7}}$$

$$= \frac{\cancel{\cos \frac{\pi}{7}} \cos \frac{4\pi}{7} + \cancel{\sin \frac{\pi}{7}} + 8 \sin \frac{\pi}{7} \cos \frac{4\pi}{7}}{\sin \frac{4\pi}{7}}$$

$$\sqrt{\sin^4 + 4 - 4\sin^2} = 2 - \sin^2$$

$$\frac{\cos(\alpha - \beta)}{\cos(\alpha + \beta)} = \frac{-\cos(\gamma + \delta)}{\cos(\gamma - \delta)} \Rightarrow \frac{\cos(\alpha - \beta) - \cos(\alpha + \beta)}{\cos(\alpha - \beta) + \cos(\alpha + \beta)} =$$

$$\sqrt{x} = 141$$

$$\text{antilog} \left( \frac{2}{3} \right)_{27} = (27)^{2/3} = \frac{-\cos(\gamma + \delta) - \cos(\gamma - \delta)}{-\cos(\gamma + \delta) + \cos(\gamma - \delta)}$$

$$x = 0.6666 \dots$$

$$\log x = 6.6666 \dots$$



$$\log_{2011} 2012 + \log_{2012} 2011 > 2$$

$$a_1 = 2010 > 0 \quad \frac{\cos^3 9^\circ - (4\cos^3 9^\circ - 3\cos 9^\circ)}{\cos 9^\circ} + \boxed{x \in [\frac{1}{3}, 3] \cup [3, \infty)}$$

$$\frac{\tan 5^\circ - \tan 2^\circ - \tan 3^\circ}{\tan 2^\circ \tan 3^\circ \tan 5^\circ}$$

$$\boxed{x = 20 - 3 \times 2x}$$

$$\frac{3\cos 9^\circ - 3\cos 9^\circ}{\cos 9^\circ}$$

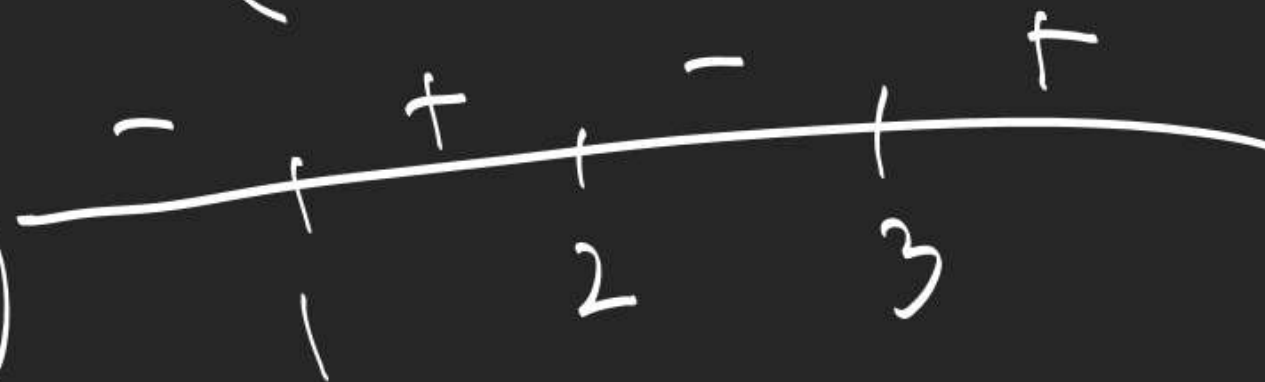
$$x \in \mathbb{Q}$$

$$\log_3 x \in [1, 2] \cup [3, \infty)$$

$$\log_3^3 x - 6 \log_2^2 x + 11 \log_3 x - 6 \geq 0$$

$$(t-1)(t^2-5t+6) \geq 0$$

$$(t-1)(t-2)(t-3) \geq 0$$



Note  $\rightarrow f(x) = ax^2 + bx + c$ ,  $a, b, c \in \mathbb{R}$   
 $a \neq 0$   $> 0$  or  $= 0$

①  $f(x) > 0 \quad \forall x \in \mathbb{R}$   
 $\Rightarrow a > 0, D < 0$

②  $f(x) < 0 \quad \forall x \in \mathbb{R}$   
 $\Rightarrow a < 0, D < 0$

③  $f(x) \geq 0 \quad \forall x \in \mathbb{R}$   
 $\Rightarrow a > 0 \& D \leq 0$

④  $f(x) \leq 0 \quad \forall x \in \mathbb{R}$   
 $\Rightarrow a < 0 \& D \leq 0$





# Roots

$$ax^2 + bx + c = 0 \quad \begin{matrix} \alpha \\ \beta \end{matrix}$$

$$\alpha + \beta = -\frac{b}{a}$$

$$\alpha\beta = \frac{c}{a}$$

$$\alpha, \beta = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\frac{3 \pm \sqrt{9+40}}{4}$$

coeff of  $x$   $b = -a(\alpha + \beta)$   
 $ax^2 + bx + c = a(x^2 - (\alpha + \beta)x + \alpha\beta)$

$a \neq 0, a, b, c \in \mathbb{R}$   
Ident  $c = a\alpha\beta$   
 Identity

$$ax^2 + bx + c = a(x - \alpha)(x - \beta)$$

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$x^2 - 3x + 2 = (x-1)(x-2)$$

$$2x^2 - 3x - 5 = 2\left(x - \frac{5}{2}\right)(x+1)$$

Form a quad. having  
roots  $\alpha, \beta$

$$(x - \alpha)(x - \beta) = 0$$

$$x^2 - (\alpha + \beta)x + \alpha\beta = 0$$



Form a cubic having roots  $\alpha, \beta, \gamma$ .

$$\begin{aligned}\alpha + \beta + \gamma &= -\frac{b}{a} \\ \alpha\beta + \beta\gamma + \gamma\alpha &= \frac{c}{a} \\ \alpha\beta\gamma &= -\frac{d}{a}\end{aligned}$$

$$(x - \alpha)(x - \beta)(x - \gamma) = 0$$

Coefficient

$$x^3 - (\alpha + \beta + \gamma)x^2 + (\alpha\beta + \beta\gamma + \gamma\alpha)x - \alpha\beta\gamma = 0$$

Identity

$$ax^3 + bx^2 + cx + d = 0 \quad \begin{matrix} \alpha \\ \beta \\ \gamma \end{matrix}$$

$$\begin{aligned}ax^3 + bx^2 + cx + d &= a(x - \alpha)(x - \beta)(x - \gamma) \\ &= a(x^3 - (\alpha + \beta + \gamma)x^2 + (\alpha\beta + \beta\gamma + \gamma\alpha)x - \alpha\beta\gamma)\end{aligned}$$

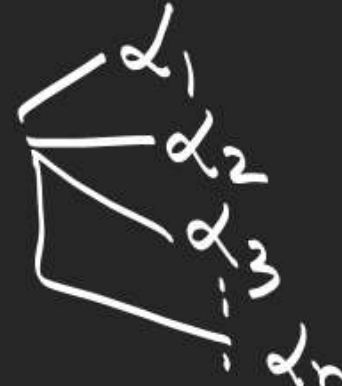
Coeff. of  $x^2$

$$b = -a(\alpha + \beta + \gamma)$$

Coeff. of  $x$ ,  $c = a(\alpha\beta + \beta\gamma + \gamma\alpha)$

Const.  $d = -a\alpha\beta\gamma$ .



$$a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \dots + a_2 x^2 + a_1 x + a_0 = 0$$


$$a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \dots + a_1 x + a_0 = a_n (x-d_1)(x-d_2)(x-d_3) \dots (x-d_n)$$

Coeff. of  $x^{n-1}$ ,  $a_{n-1} = a_n(-d_1 - d_2 - \dots - d_n) \Rightarrow \sum d_i = -\frac{a_{n-1}}{a_n}$

Coeff. of  $x^{n-2}$ ,  $a_{n-2} = a_n(d_1 d_2 + d_1 d_3 + d_1 d_4 + \dots + d_1 d_n + d_2 d_3 + \dots + d_2 d_n + \dots + d_{n-1} d_n)$

$$\Rightarrow \sum d_i d_j = \frac{a_{n-2}}{a_n}$$

Coeff. of  $x^{n-3}$ ,  $a_{n-3} = -a_n(d_1 d_2 d_3 + d_1 d_2 d_4 + \dots + d_{n-2} d_{n-1} d_n)$

$$\sum d_i d_j d_k = -\frac{a_{n-3}}{a_n}$$

$\sum d_i d_j d_k d_l = \frac{a_{n-4}}{a_n}$

$\vdots$

$$a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \dots + a_1 x + a_0 = 0 \quad \begin{matrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_n \end{matrix}$$

## Vieta's Theorem

$$\sum \alpha_i = -\frac{a_{n-1}}{a_n} = -\frac{\text{Coeff. of } x^{n-1}}{\text{Coeff. of } x^n}$$

$$\sum \alpha_i \alpha_j = \frac{a_{n-2}}{a_n} = \frac{\text{Coeff. of } x^{n-2}}{\text{Coeff. of } x^n}$$

$$\sum \alpha_i \alpha_j \alpha_k = -\frac{a_{n-3}}{a_n} = -\frac{\text{Coeff. of } x^{n-3}}{\text{Coeff. of } x^n}$$

$$\sum \alpha_i \alpha_j \alpha_k \alpha_l = \frac{a_{n-4}}{a_n} = \frac{\text{Coeff. of } x^{n-4}}{\text{Coeff. of } x^n}$$

$$\alpha_1 \alpha_2 \alpha_3 \dots \alpha_n = (-1)^n \frac{a_0}{a_n}$$