

# Fundamentals of Mathematics

U hv to Improve

$$\textcircled{1} \quad \frac{2x-3}{3x-7} > 0$$

$$\textcircled{2} \quad \frac{x+5}{x-x^2-1} < 0$$

$$\textcircled{3} \quad \frac{x^2-3x+2}{x^2+x+1} < 0$$

$$\textcircled{4} \quad \frac{(x-1)(x+2)^2}{-1-x} < 0$$

## I. Algebra, Trigonometry, and Elementary Functions

tion  $(x - x_1)^{k_1}$  does not change sign when  $x$  passes through the point  $x_1$  and, consequently, the function  $F(x)$  does not change sign. If  $k_p$  is an even number, then the function  $(x - x_p)^{k_p}$  changes sign when  $x$  passes through the point  $x_p$  and, consequently, the function  $F(x)$  also changes sign.

Example 4. Solve the inequality  $(x - 1)^3 (x + 1)^2 (x - 4) < 0$ .

Solution. The function  $F(x) = (x - 1)^3 (x + 1)^2 (x + 4)$  changes sign only when  $x$  passes through the points  $x_1 = -1$ ,  $x_2 = 4$ . We have  $F(x) > 0$  on the interval  $(4, +\infty)$ ,  $F(x) < 0$  on the next interval  $(-1, 4)$ , excluding the point  $x = 1$  at which  $F(x) = 0$ , and  $F(x) > 0$  on the last interval  $(-\infty, -1)$ .

Answer:  $(-\infty, -1) \cup (-1, 1) \cup (1, 4)$ .

Example 5. Solve the inequality  $\frac{(x-1)^3 (x+1)^2}{x^4 (x-2)^3} \leq 0$ .

Solution. The function  $F(x) = \frac{(x-1)^3 (x+1)^2}{x^4 (x-2)^3}$  changes sign only when the variable  $x$  passes through the points  $x_1 = -1$ ,  $x_2 = 2$ . When  $x$  passes through the points  $x_3 = 0$  and  $x_4 = 1$ , the function  $F(x)$  does not change sign. We have  $F(x) > 0$  on the interval  $(2, +\infty)$ ,  $F(x) < 0$  on the next intervals  $(1, 2)$ ,  $(0, 1)$ ,  $(-1, 0)$ , and  $F(x) > 0$  on the interval  $(-\infty, -1)$ . At the point  $x_4 = 1$  the inequality is satisfied and at the point  $x_3 = 0$  the function  $F(x)$  is not defined.

Answer:  $(-\infty, -1) \cup (0, 2)$ .

Solve the following inequalities (27-135).

27.  $(x-1)(3-x)(x-2)^2 > 0$ .
28.  $\frac{6x-5}{4x+1} < 0$ .
29.  $\frac{2x-3}{3x-7} > 0$ .
30.  $\frac{0.5}{x-x^2-1} < 0$ .
31.  $\frac{x^2-5x+6}{x^2+x+1} < 0$ .
32.  $\frac{x^2+2x-3}{x^2+1} < 0$ .
33.  $\frac{(x-1)(x+2)^2}{-1-x} < 0$ .
34.  $\frac{x^2+4x+4}{2x^2-x-1} > 0$ .
35.  $x^4 - x^2 + 4 < 0$ .
36.  $x^4 - 2x^2 - 63 \leq 0$ .

## I.5. Equations of Higher Degrees. Rational Inequalities

37.  $\frac{3}{x-2} < 1$ .
38.  $\frac{1}{x-1} \leq 2$ .
39.  $\frac{4x+3}{2x-5} < 6$ .
40.  $\frac{5x-6}{x+6} < 1$ .
41.  $\frac{5x+8}{4-x} < 2$ .
42.  $\frac{x-1}{x+3} > 2$ .
43.  $\frac{7x-5}{8x+3} > 4$ .
44.  $\frac{x}{x-5} > \frac{1}{2}$ .
45.  $\frac{5x-1}{x^2+3} < 1$ .
46.  $\frac{x-2}{x^2+1} < -\frac{1}{2}$ .
47.  $\frac{x+1}{(x-1)^3} < 1$ .
48.  $\frac{x^2-7x+12}{2x^2+4x+5} > 0$ .
49.  $\frac{x^2+6x-7}{x^2+1} \leq 2$ .
50.  $\frac{x^4+x^2+1}{x^3-4x-5} < 0$ .
51.  $\frac{1+3x^2}{2x^3-21x+40} < 0$ .
52.  $\frac{1+x^4}{x^2-5x+6} < 0$ .
53.  $\frac{x^4+x^2+1}{x^2-4x-5} > 0$ .
54.  $\frac{1-2x-3x^4}{3x-x^4-5} > 0$ .
55.  $\frac{x^4-5x+7}{-2x^4+3x+2} > 0$ .
56.  $\frac{2x^3-3x-459}{x^3+1} > 1$ .
57.  $\frac{x^3-1}{x^3+x+1} < 1$ .

$$\textcircled{5} \quad \frac{x^2+4x+4}{2x^2-x-1} > 0$$

$$\textcircled{6} \quad \frac{1+x^2}{x^2-5x+6} < 0$$

$$\textcircled{7} \quad \frac{1-2x-3x^2}{3x-x^2-5} > 0$$

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$$Q_2 \frac{x^2 - 5}{x - x^2 - 1} < 0$$

$$\frac{+}{-} = 0$$

D<sub>r</sub> must be -ve

$$a=1, b=-1, c=-1$$

$$x - x^2 - 1 < 0$$

$$\Rightarrow x^2 - x + 1 > 0 \quad D = (-1)^2 - 4 \times 1 \times 1$$

$$x \in \mathbb{R}$$

$$= -3 \text{ -ve}$$

$$x^2 - x + 1 > 0$$

$$\text{for } a \in \mathbb{R}$$

$$Q \frac{x^2 - 3x + 2}{x^2 + x + 1} < 0$$

$$\frac{(x-1)(x-2)}{(x^2+x+1)} < 0$$

$$a=1, b=1, c=1$$

$$D = 1^2 - 4 \times 1 \times 1 \\ = -3.$$

$$(x-1)(x-2) < 0$$

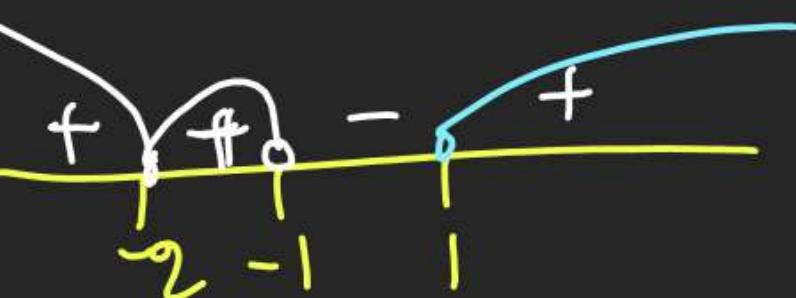


$$x \in (-1, 2)$$

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$$4) \frac{(-1)(x+2)^2}{-(1-x)} < 0$$

$$\frac{(-1)(x+2)^2}{(x+1)^2} > 0$$



Even Odd Odd

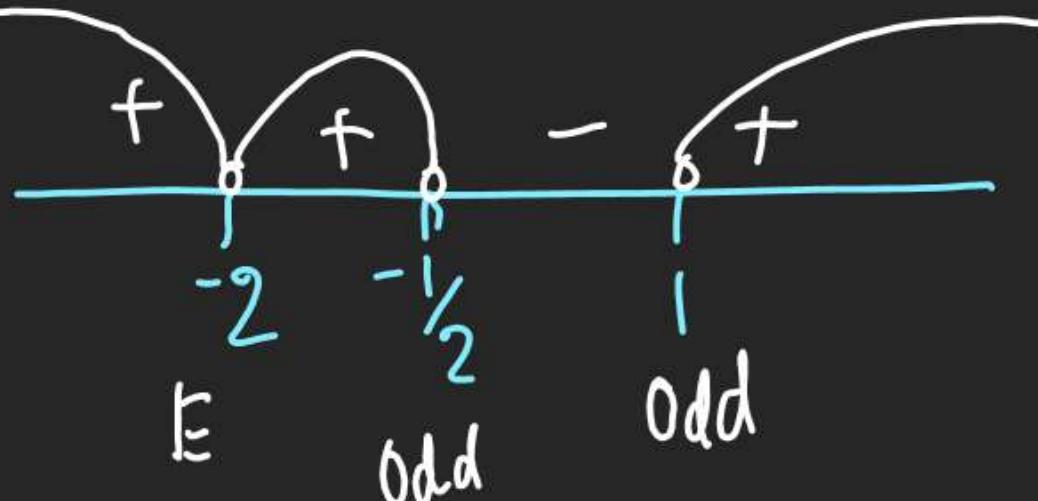
$$x \in (-\infty, -2) \cup (-2, -1) \cup (1, \infty)$$

एवेन ऑड

Sign change } Last  
video

$$Q5: \frac{x^2+4x+4}{2x^2-x-1} > 0$$

$$\Rightarrow \frac{(x+2)^2}{(2x+1)(x-1)} > 0$$



Even Odd Odd

$$x \in (-\infty, -2) \cup (-2, -\frac{1}{2}) \cup (1, \infty)$$

# Fundamentals of Mathematics

Q 6  $\frac{(1+x^2)}{x^2 - 5x + 6} < 0$

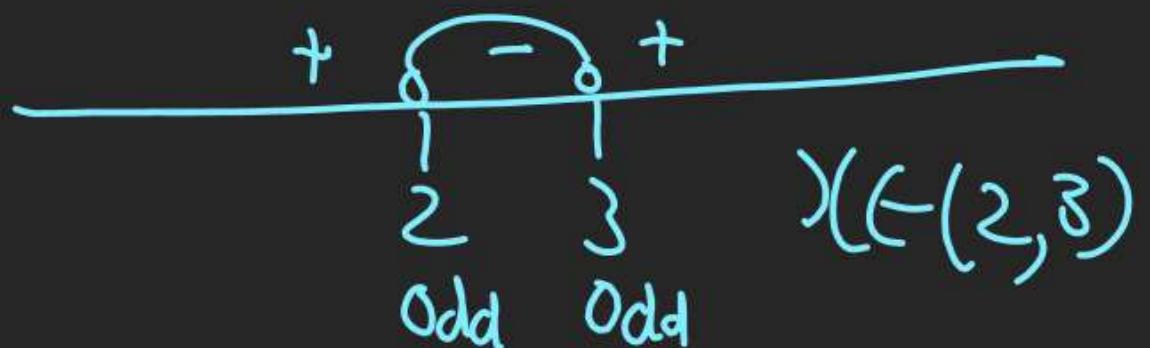
$$\begin{aligned}a &= 1, b = 0, c = 1 \\D &= 0^2 - 4 \times 1 \times 1 \\&= -4 (-\text{ve})\end{aligned}$$

$$\frac{(1+x^2)^{\oplus}}{(x-2)(x-3)} < 0 \Rightarrow \frac{\oplus}{(-)} = -\text{ve}$$

$$1+x^2 > 0$$

Dr - ve Hona Padega.

$$(x-2)(x-3) < 0$$



factors Practice  
क्षेत्रफलों

# Fundamentals of Mathematics

$$\emptyset \quad x^4 - 5x^2 + 4 \leq 0$$

$\downarrow x^2 = t$

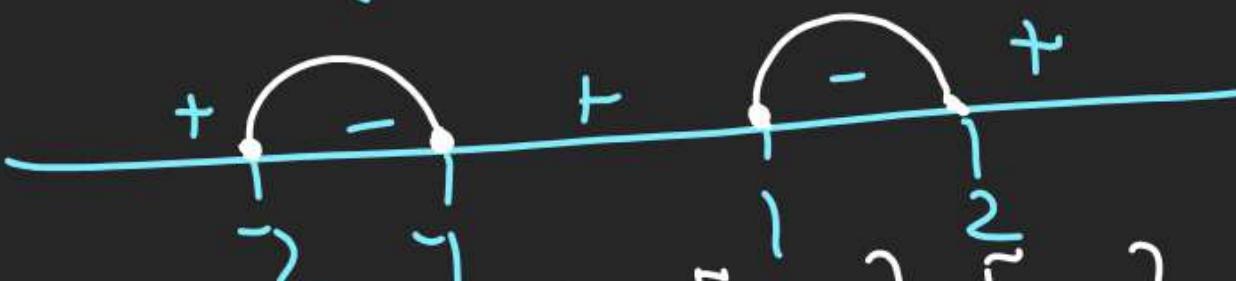
[lose = अमीर  
Equal = बराबर  
Darrk = कम्ते]

$$t^2 - 5t + 4 \leq 0$$

$$(t-1)(t-4) \leq 0$$

$$(x^2-1)(x^2-4) \leq 0$$

$$(x-1)(x+1)(x-2)(x+2) \leq 0$$



$$x \in [-2, -1] \cup [1, 2]$$

$$\emptyset \quad x^4 - x^2 - 6 \leq 0$$

$\downarrow x^2 = t$

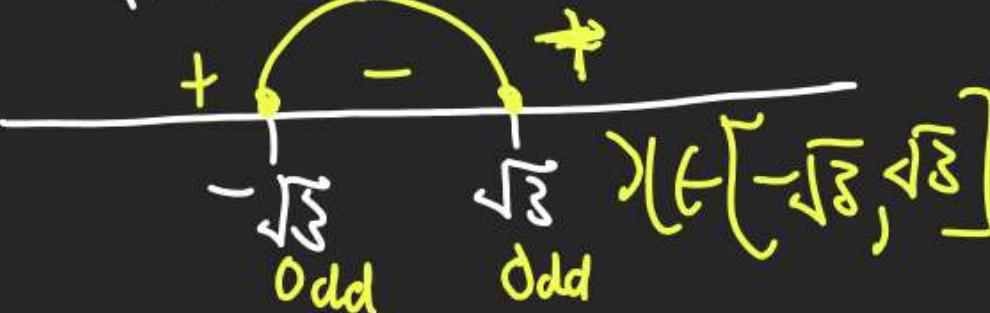
$$t^2 - t - 6 \leq 0$$

$$a^2 - b^2 \quad (t-3)(t+2) \leq 0 \quad \text{factor 48}$$

$$\leftrightarrow (x^2-3)(x^2+2) \leq 0 \quad | u=1, b=0, (-2)$$

$$(x-\sqrt{3})(x+\sqrt{3})(x^2+2) \leq 0$$

$$(x-\sqrt{3})(x+\sqrt{3}) \leq 0$$



## Fundamentals of Mathematics

$$\cap (-\infty, 1) \cup \left(\frac{5}{2}, \infty\right)$$

When RHS don't have  $<0$  or  $>0$

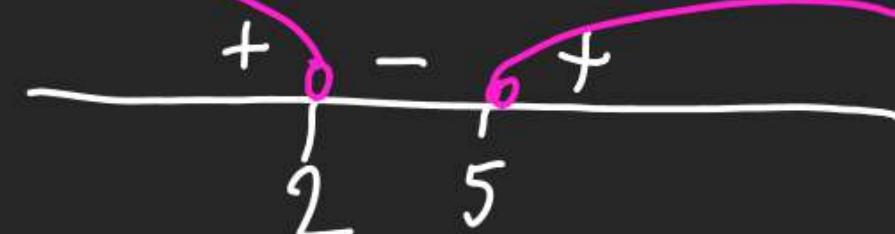
$$Q. \frac{3}{x-2} < 1$$

< 0 Nh hai !!

LHS me hao.

$$\frac{3}{x-2} - 1 < 0 \Rightarrow \frac{3 - (x-2)}{(x-2)} < 0$$

$$\frac{(5-x)}{(x-2)} < 0 \Rightarrow \frac{(x-5)}{(x-2)} > 0$$



$$x \in (-\infty, 2) \cup (5, \infty)$$

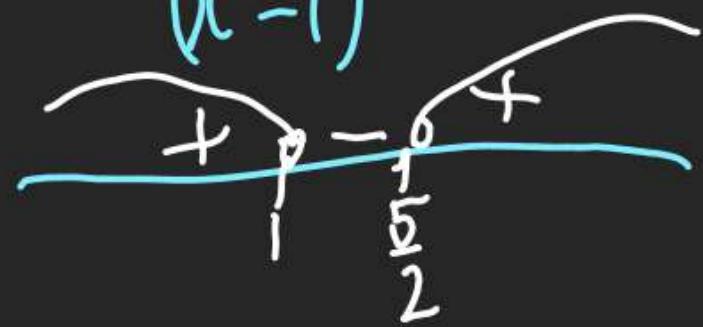
$$Q. \frac{3}{(x-1)} < 2$$

$$\Rightarrow \frac{3}{(x-1)} - 2 < 0$$

$$\Rightarrow \frac{3 - 2(x-1)}{(x-1)} < 0$$

$$\Rightarrow \frac{(5-2x)}{(x-1)} < 0$$

$$\frac{(2x-5)}{(x-1)} > 0$$



# Fundamentals of Mathematics

$$\text{H.W. } \textcircled{1} \frac{1}{x-1} \leq 2$$

$$\textcircled{2} \quad \frac{x+3}{2x-5} < 6$$

$$\textcircled{3} \quad \frac{5x-6}{x+6} < 1$$

$$\textcircled{4} \quad \frac{7x-3}{5x+3} > 4$$

Q

$$\frac{x+1}{(x-1)^2} < 1$$

Solve.

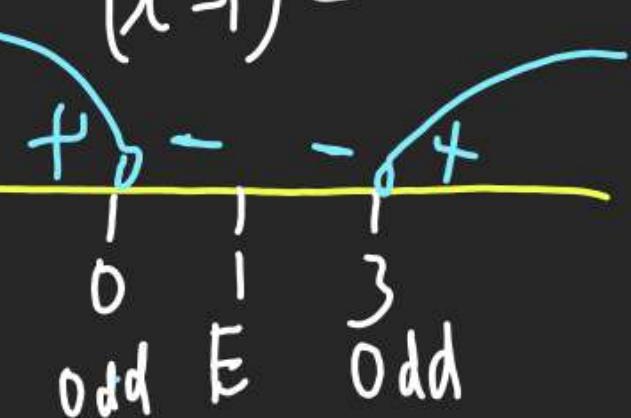
$$x \in (-\infty, 0) \cup (3, \infty)$$

$$\frac{x+1}{(x-1)^2} - 1 < 0 \Rightarrow \frac{(x+1) - (x-1)^2}{(x-1)^2} < 0$$

$$\Rightarrow \frac{x+1 - x^2 + 2x - 1}{(x-1)^2} < 0$$

$$\Rightarrow \frac{-x^2 + 3x}{(x-1)^2} < 0 \Rightarrow \frac{-x(x-3)}{(x-1)^2} < 0$$

$$\frac{(x)(x-3)}{(x-1)^2} > 0$$



# Fundamentals of Mathematics

Q

$$\frac{2x^2 - 3x - 459}{x^2 + 1} > 1$$

$$\frac{2x^2 - 3x - 459}{x^2 + 1} - 1 > 0$$

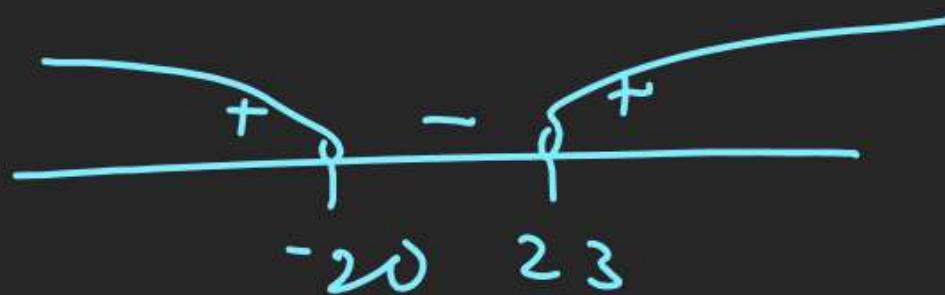
$$\frac{2x^2 - 3x - 459 - (x^2 + 1)}{x^2 + 1} > 0$$

$$\frac{x^2 - 3x - 460}{(x^2 + 1)_{+ve}} > 0$$

a +ve  
D - ve

$$x^2 - 3x - 460 > 0$$

$$(x - 23)(x + 20) > 0$$



$$x \in (-\infty, -20) \cup (23, \infty)$$

# Fundamentals of Mathematics

Logarithm.

John Napier (1700)

36 is  $6^2$

$$\log_3 9 = 2$$

$$\log_2 8 = 3$$

$$2^3 = 8$$

Exp.

$N(36)$

↓ Exponantial  
form

$$6^2 = 36$$

↓ log form.

$$\log_6 36 = 2$$

Antilog form.

Antilog of  $\frac{3}{4}$  to Base 16

$$= (16)^{\frac{3}{4}} = (2^4)^{\frac{3}{4}} = 2^3 = 8$$

# Fundamentals of Mathematics

\*  $\log_a N = x$  is given

$$\frac{\sqrt{2} \cdot \sqrt{2} \cdot \sqrt{2}}{2\sqrt{2}}$$

$\Rightarrow N$ , a Ki Kitni degree hai  
& Ansiner is  $x$

$$11^3 = 1331$$

$$\Rightarrow N = a^x$$

Solve

$$Q1) \log_3 9 = 2$$

$$2) \log_{13} 169 = 2$$

$$3^2 = 9$$

$$3) \log_{11} 1331 = 3$$

$$4) \log_{\sqrt{5}} 5 = 2$$

$$(\sqrt{5})^2 = 5$$

$$5) \log_{\sqrt{2}} 2\sqrt{2} = 3$$

$$(\sqrt{2})^3 = 2\sqrt{2}$$

$$(6) \log_9 9 = 1 \quad 9^1 = 9$$

$$(7) \log_3 \frac{1}{3} = -1 \quad (3)^{-1} = \frac{1}{3}$$

$$(8) \log_{\frac{1}{3}} 3 = -1 \quad \left(\frac{1}{3}\right)^{-1} = 3$$

$$(9) \log_{\frac{1}{3}} 9 = -2 \quad \left(\frac{1}{3}\right)^{-2} = 3^2 = 9$$

# Fundamentals of Mathematics

$$\text{Q) } \log_{27} 1 = ?$$

$= 0$

L, 27 कि कितनी देग हाँ

$$(27)^0 = 1$$

$$\text{Q) } \log_9 1 = 0$$

$(9)^0 = 1$

Results

$$1) \log_N N = 1$$

$$N^1 = N$$

$$2) \log_{\frac{1}{N}} N = -1$$

$$\left(\frac{1}{N}\right)^{-1} = N$$

$$3) \log_N 1 = 0 \quad (N)^0 = 1$$

$$(N)^0 = 1$$

# Fundamentals of Mathematics

(4)  $\log_a N$  is defined

3 Shrt Puri ho

1) Base  $a > 0$

2) Base  $a \neq 1$

3)  $N > 0$

Otherwise  
 $\log_a N$  is  
 Undefined

$\log_2 \boxed{-4}$  = Not defined

$N = -4 \not> 0$

$\log_{100} 0 \rightarrow$  Undefined

$\log_{0.5} 5 \rightarrow$  Undefined

$\log_{1} 1 \rightarrow$  Not defined

# Fundamentals of Mathematics

Q  $\log_4(x-3)$  is defined?

(1)  $4 > 0$  ✓

(2)  $4 \neq 1$  ✓

(3)  $(x-3) > 0$

$$x > 3$$

$$x \in (3, \infty)$$

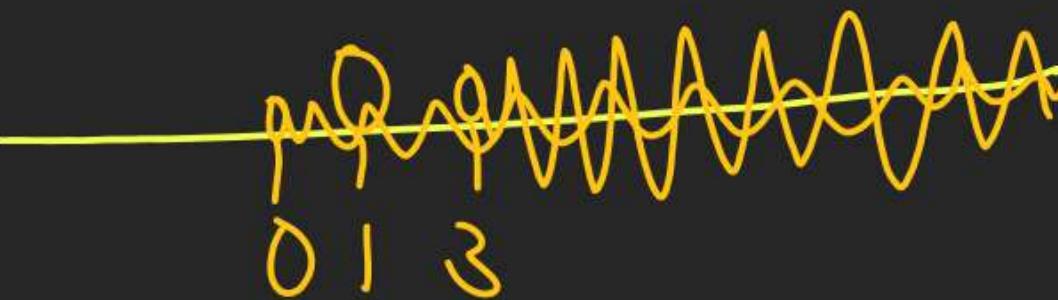
Q  $y = \log_x(x-3)$  is defined?

1)  $x > 0$

2)  $x \neq 1$

3)  $x-3 > 0$

$$x > 3$$



$$x \in (3, \infty)$$

# Fundamentals of Mathematics

Q)  $y = \log_{\frac{1}{3}}(8x-33)$  is defined?

1)  $\frac{1}{3} > 0 \quad \checkmark$

2)  $\frac{1}{3} \neq 1 \quad \checkmark$

3)  $8x - 33 > 0$

$$x > \frac{33}{8}$$

$$x \in \left(\frac{33}{8}, \infty\right)$$

Q)  $\log_N 1 = ?$

$$N > 0, N \neq 1$$

$$= 0$$

Q)  $\log_{\cot 45^\circ} \tan 45^\circ = ?$

$$\log_1 1 = ND$$

→ Base +