Nishant Jindal

ANGULAR MOMENTUM CONSERVATION (A·M·C)

$$\vec{\tau}_{net} = \frac{d\vec{L}}{dt}$$

if $\vec{\tau}_{net} = 0$ either about point or axis then angular momentum Conservation about that point or that axis.

⇒ Important points regarding (A·M·C)

- [Angular velocity of each body must be w.r.t earth ✓

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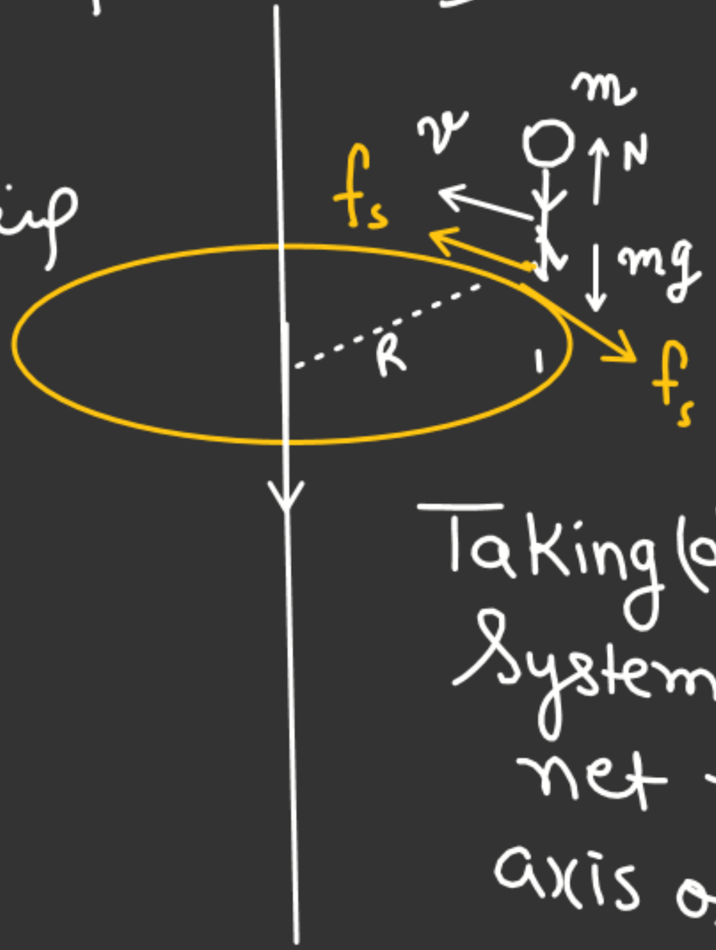
ANGULAR MOMENTUM CONSERVATION

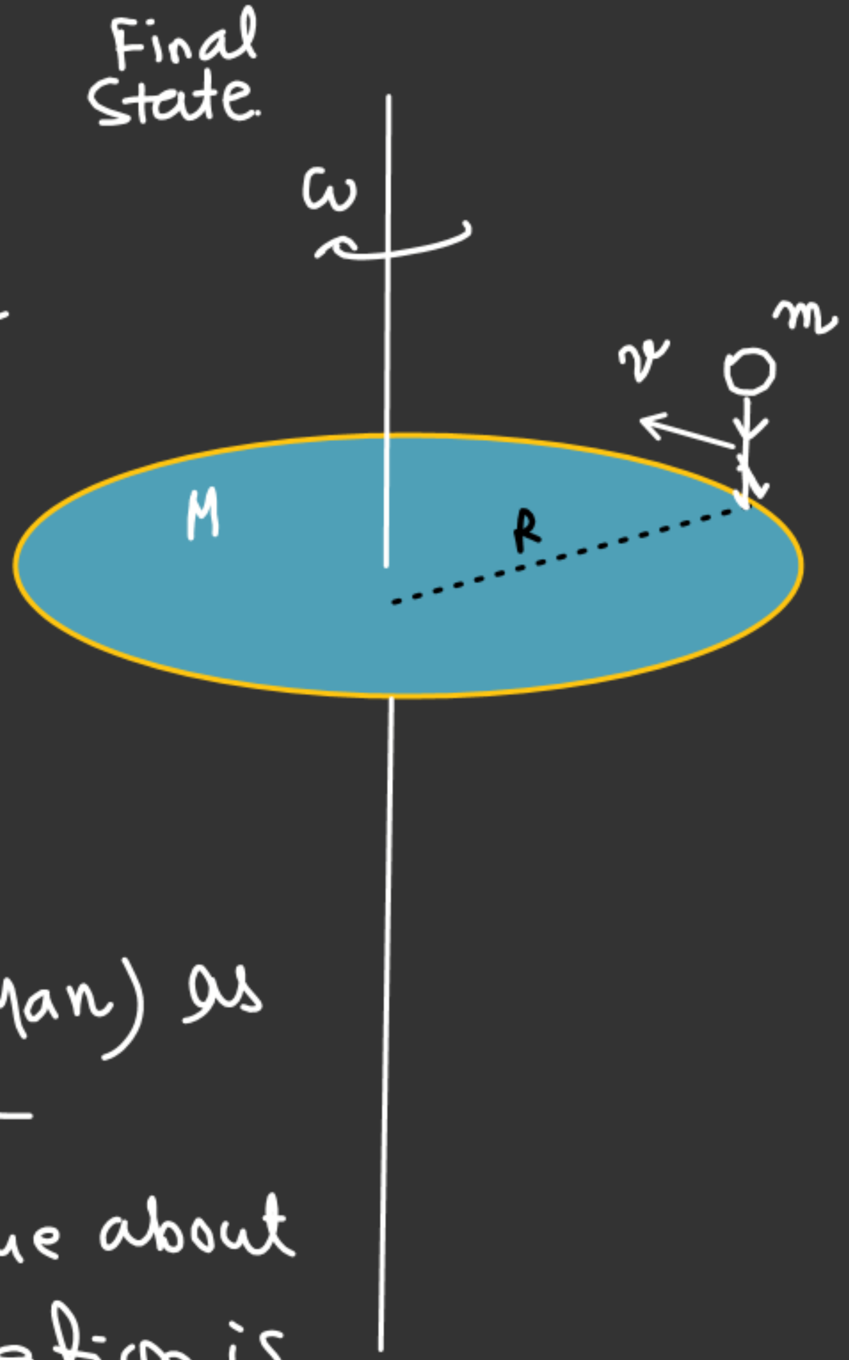
(Man + Disc) System

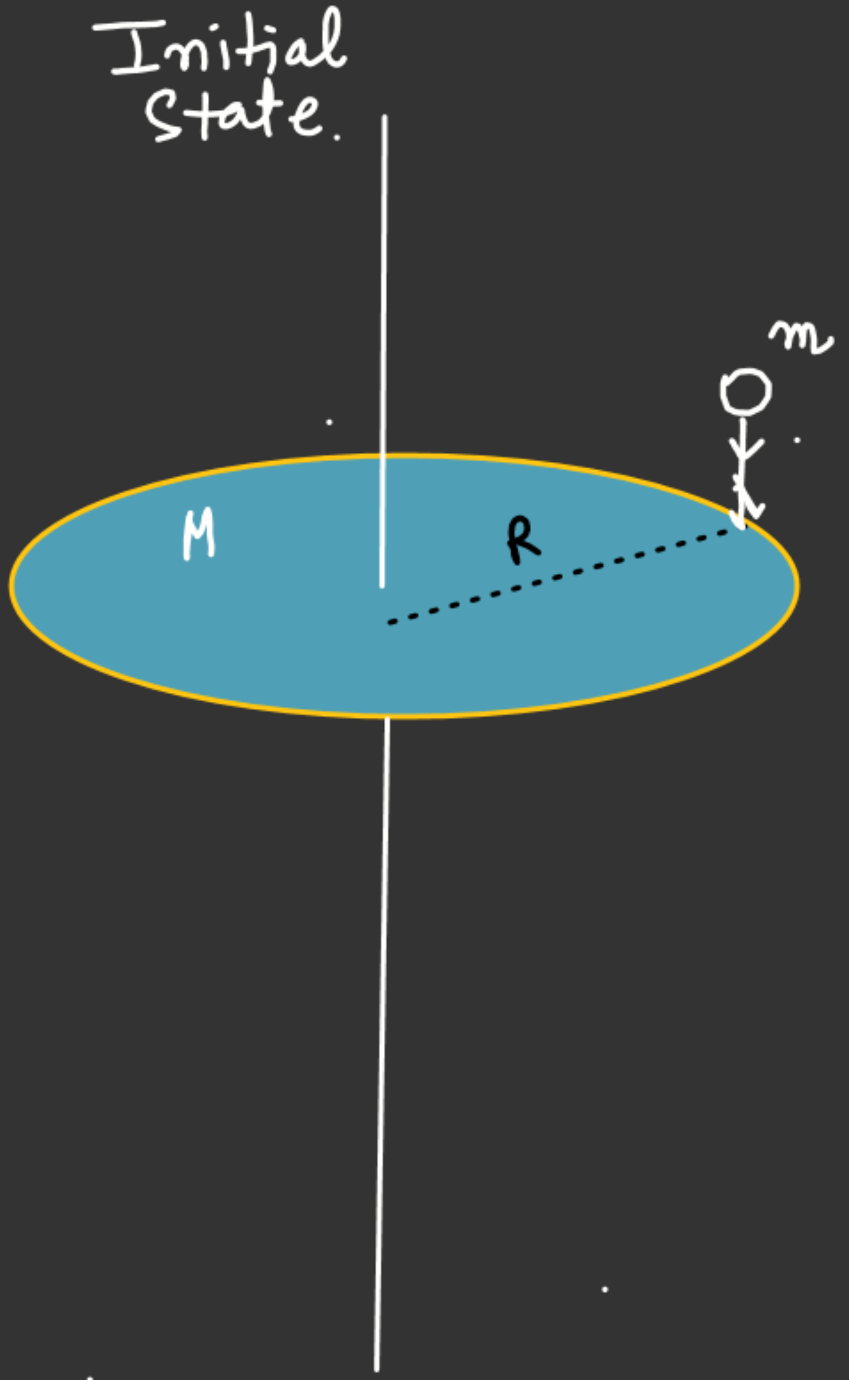
Case-1 (v w.r.t earth or w.r.t present state of the disc)

Note :- f_s providing angular impulse.



Taking (disc + Man) as System net net torque about axis of rotation is zero.





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ANGULAR MOMENTUM CONSERVATION

A.M.C.

$$\vec{L}_i = \vec{L}_f$$

$$0 = -(I_{disc.}\,\omega)\hat{j} + mvR\,\hat{j}$$

$$\frac{MR^2}{2}\omega = mvR$$

$$\omega = \frac{2mvR}{MR^2}$$

$$\boxed{\omega = \frac{2mv}{MR}}$$

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ANGULAR MOMENTUM CONSERVATION

(Man + Disc) System

$$\vec{\omega}_{man/E} = \vec{\omega}_{man/disc} + \vec{\omega}_{disc/E}$$

$$= +\frac{v}{R}\hat{j} - \omega\hat{j} \checkmark$$

$$\vec{v}_{man/E} = R\,\vec{\omega}_{man/E}$$

$$= (v - R\omega)\hat{j} \checkmark$$

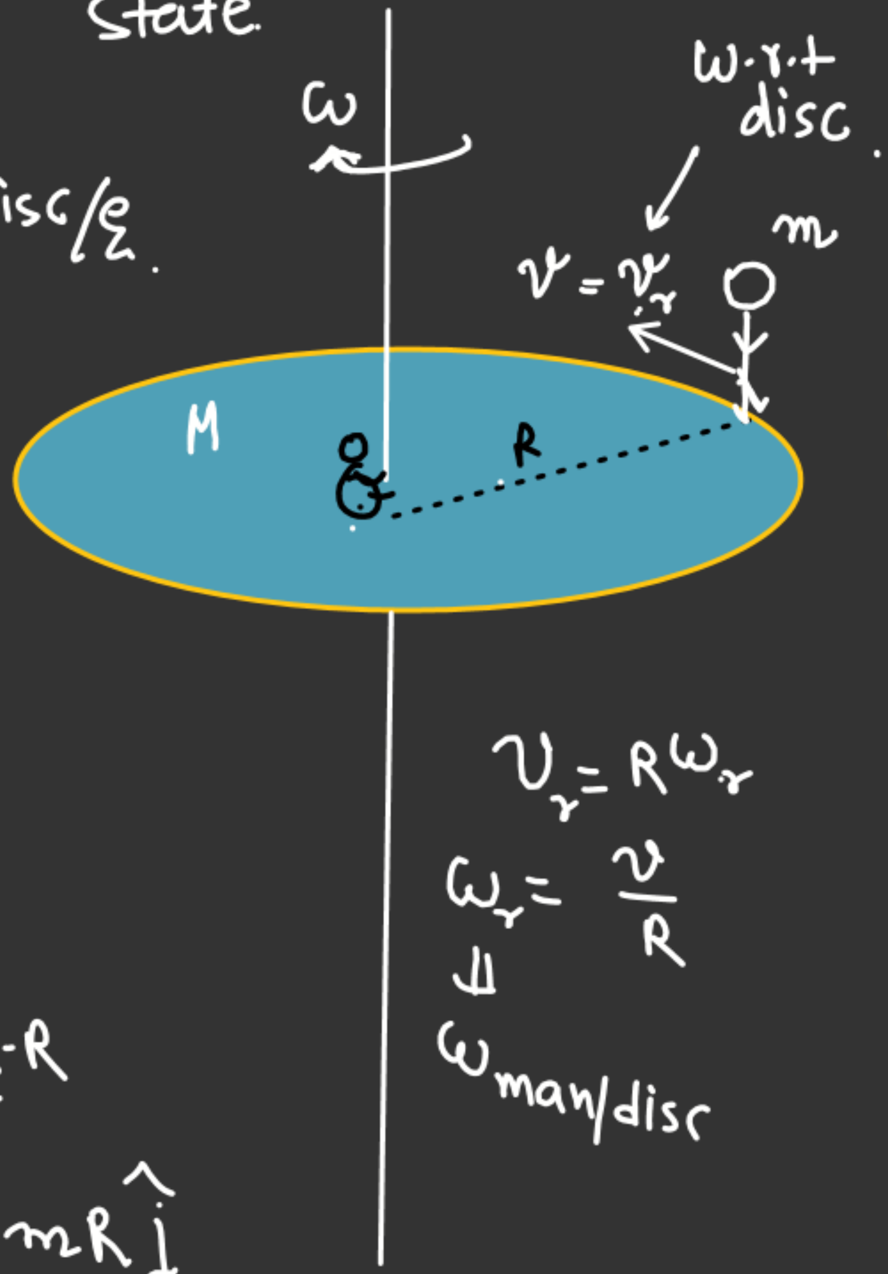
$$\vec{L}_i = \vec{L}_f$$

$$0 = \frac{MR^2}{2}\omega(-\hat{j}) + m\vec{v}_{man/E}\cdot R$$

$$0 = \frac{M\omega R^2}{2}(-\hat{j}) + (v - R\omega)mR\hat{j}$$

$$\left(\frac{M}{2}+m\right)\omega \cancel{R} = mv\cancel{R} \Rightarrow \omega = \frac{mv}{\left(\frac{M}{2}+m\right)R} = \frac{2mv}{(M+2m)R}$$

Final State



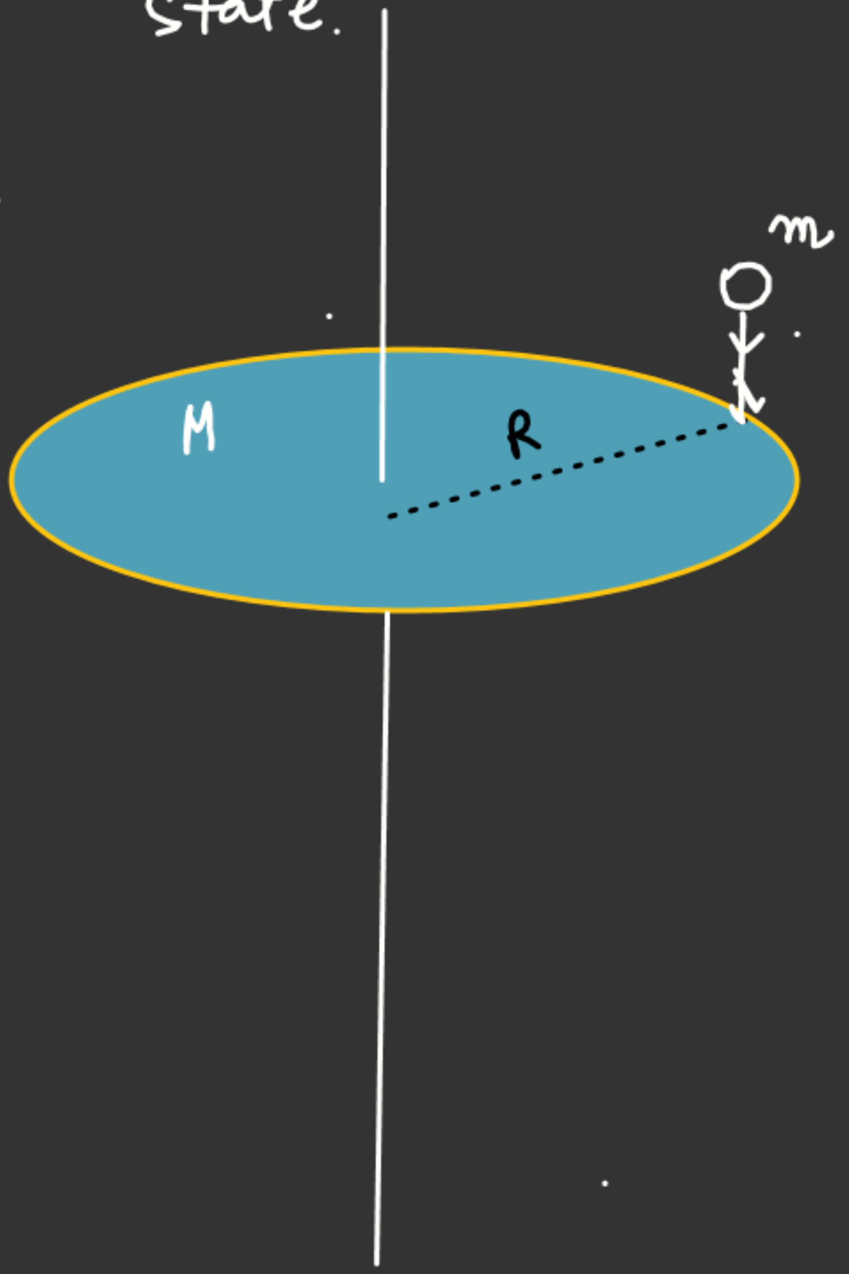
$v_r = R\omega_r$

$\omega_r = \frac{v}{R}$

$\Downarrow$

$\omega_{man/disc}$

Initial State



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ANGULAR MOMENTUM CONSERVATION

$\tau_{net} = 0$

$L = C.$

$L = I\omega.$

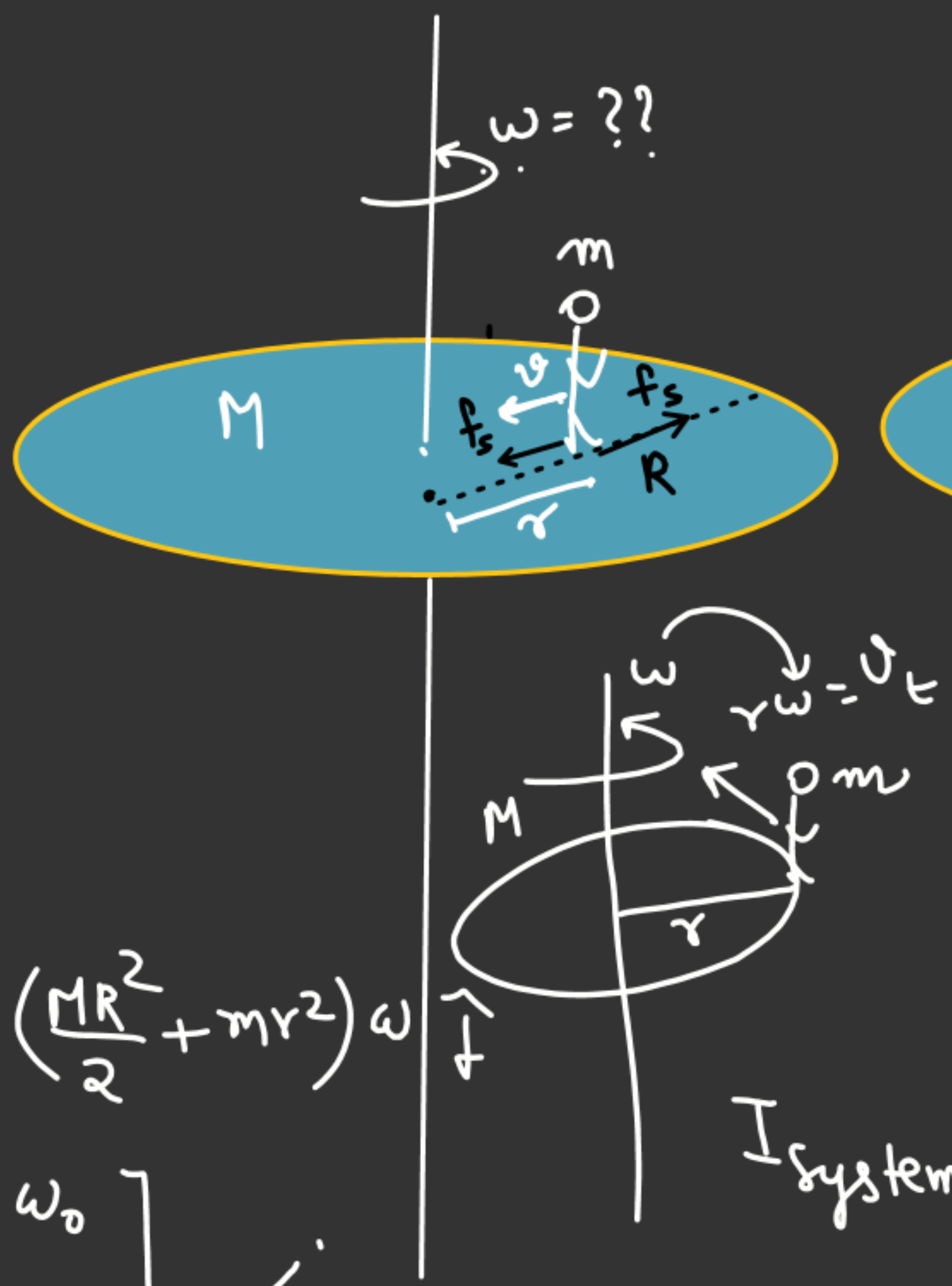
$C = I\downarrow \omega\uparrow$

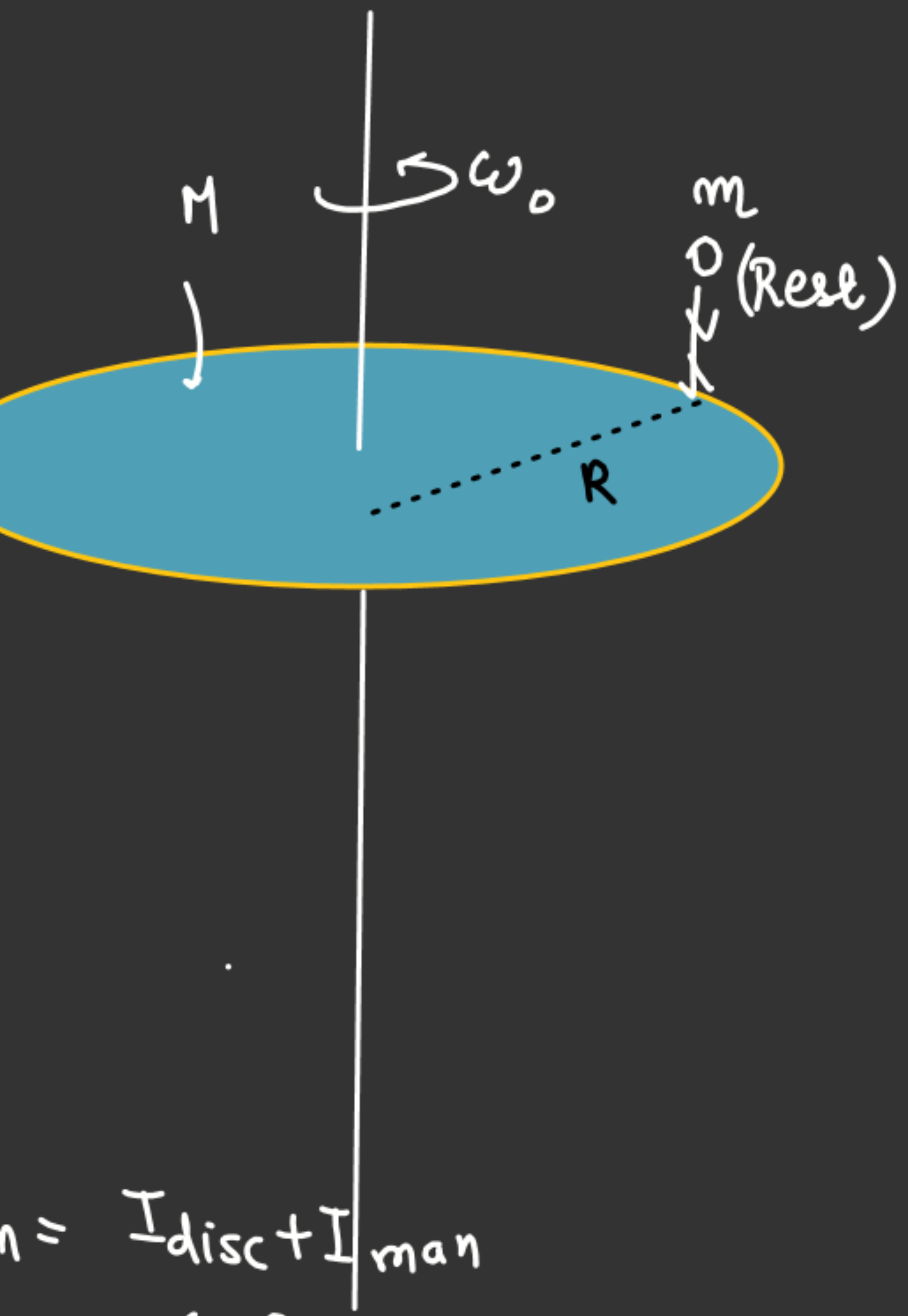
(Constant)

$\vec{L}_i = \vec{L}_f$

$$\hat{j}\left(\frac{MR^2}{2} + mR^2\right)\omega_0 = \left(\frac{MR^2}{2} + mr^2\right)\omega\,\hat{j}$$

$$\omega = \left[\frac{\left(\frac{MR^2}{2} + mR^2\right)\omega_0}{\left(\frac{MR^2}{2} + mr^2\right)}\right] \checkmark$$





$$I_{system} = I_{disc} + I_{man}$$

$$= \left(\frac{MR^2}{2} + mr^2\right)$$

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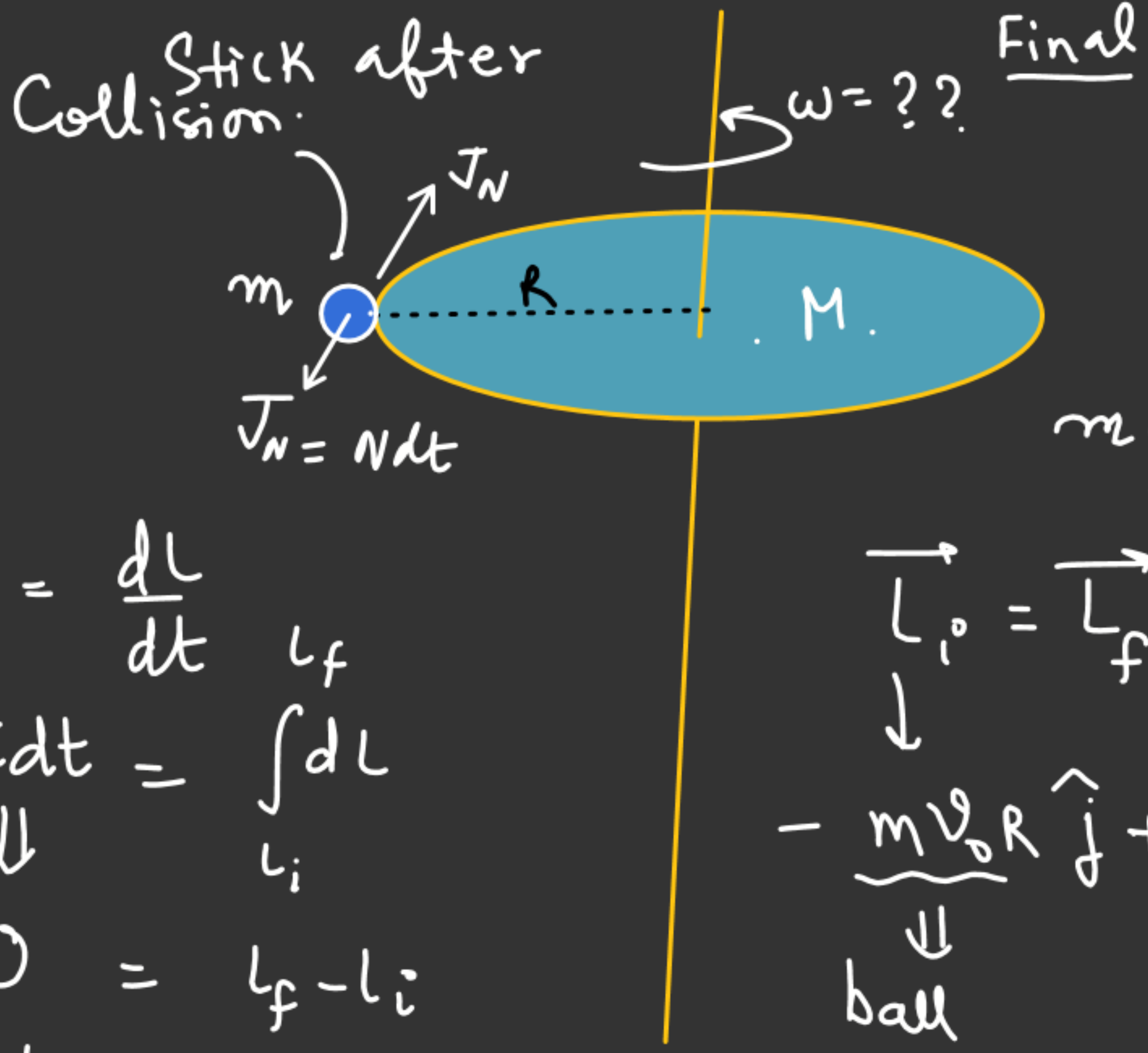
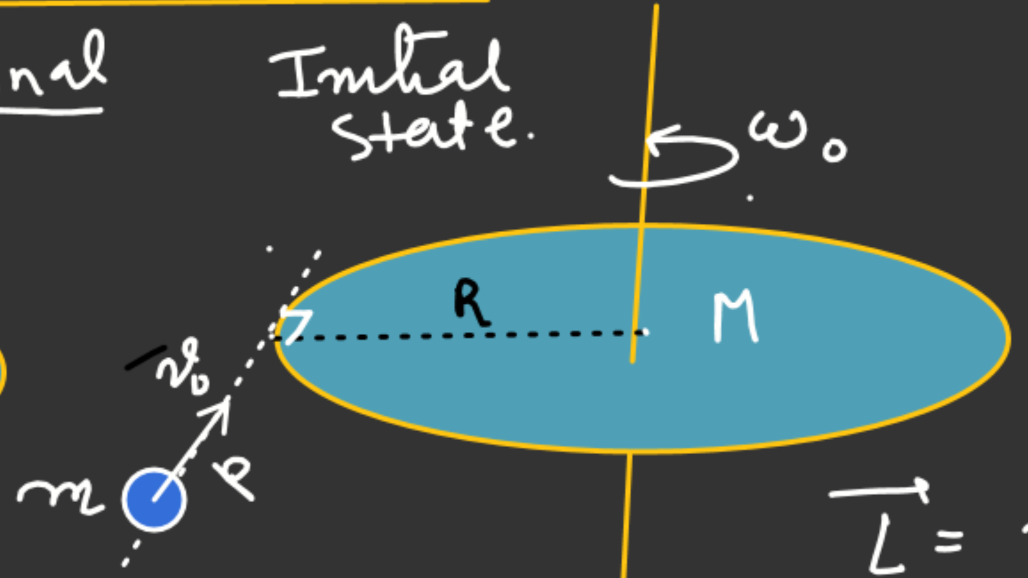
ANGULAR MOMENTUM CONSERVATION





$\vec{L} = m\vec{r} \times \vec{v}$

$|\vec{L}| = \underline{m v r_\perp}$

$$\tau = \frac{dL}{dt}$$

$$\int_0^{\Delta t} \tau \cdot dt = \int_{L_i}^{L_f} dL$$

$$\Downarrow$$

$$0 = L_f - L_i$$

$$L_f = L_i$$

$$\vec{L}_i = \vec{L}_f$$

$$\downarrow$$

$$-\underbrace{m v_0 R}_{\text{ball}}\hat{j} + \underbrace{\left(\frac{MR^2}{2}\right)\omega_0}_{\text{Disc}}\hat{j} = \left(\frac{MR^2}{2} + mR^2\right)\omega\,\hat{j}$$

$$\omega = \frac{\left(\frac{MR^2}{2}\omega_0\right) - m v_0 R}{\left(\frac{MR^2}{2} + mR^2\right)} \checkmark$$

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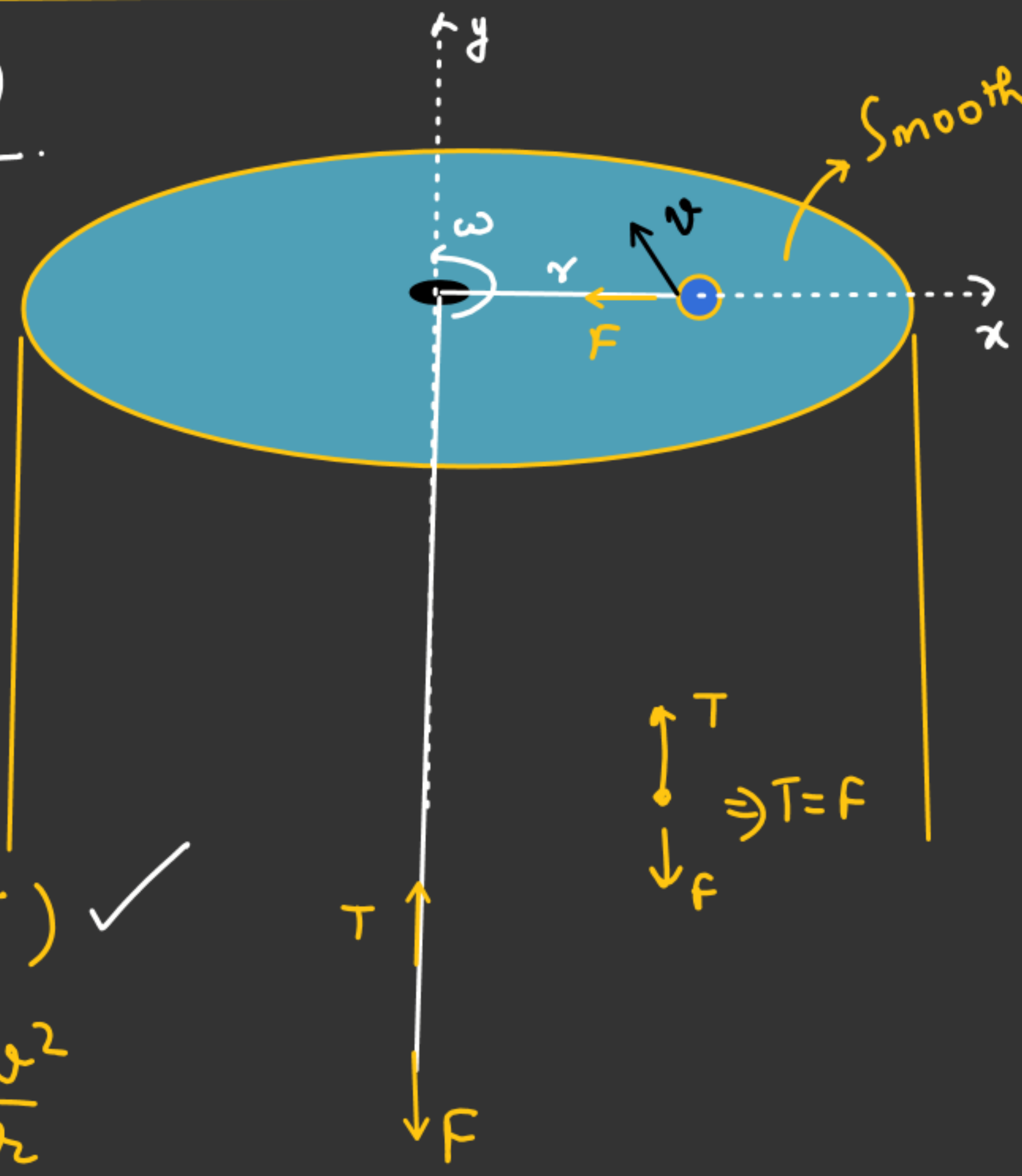
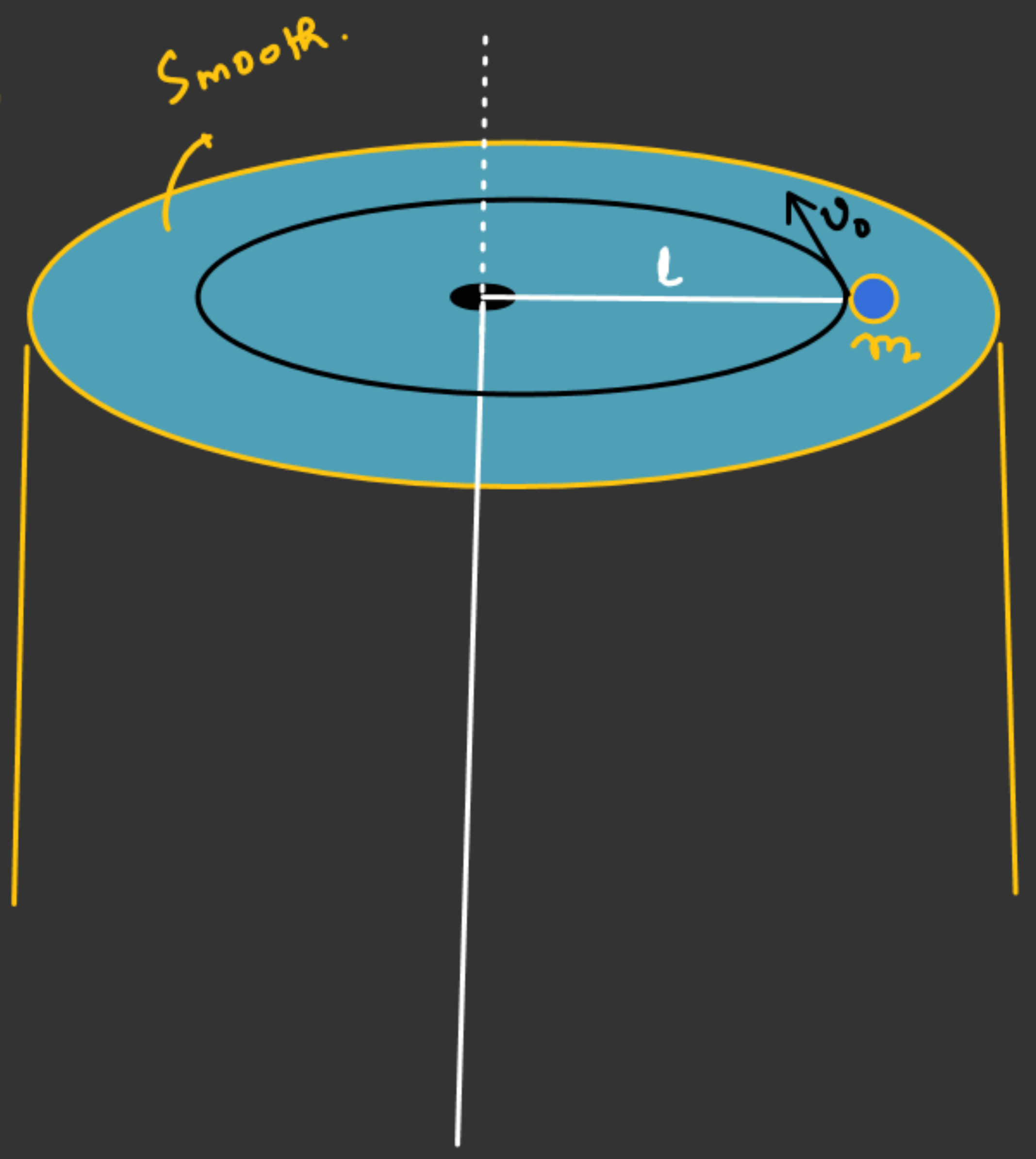
ANGULAR MOMENTUM CONSERVATION

$v = f(r)$





$\tau_F = 0.$

$L_i = L_f$

$m v_0 L = m v r$

$v = \left(\frac{v_0 L}{r}\right)$ ✓

$F = \frac{m v^2}{r}$

$F = \left(\frac{m v_0^2 L^2}{r^3}\right)$

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ANGULAR MOMENTUM CONSERVATION

$$F = -\frac{m v_0^2 l^2}{r^2}$$

Work done by F

$$dW = -F \cdot dr$$

$$W = -m v_0^2 l^2 \int_l^r \frac{dr}{r^3}$$

$$W = -m v_0^2 l^2 \int_l^r r^{-3} \cdot dr$$

$$W = -m v_0^2 l^2 \left[\frac{r^{-2}}{-2}\right]_l^r$$

$$W = +\frac{m v_0^2 l^2}{2}\left[\frac{1}{r^2} - \frac{1}{l^2}\right]$$

$$W =$$

Another Method

$$W_F = \Delta K.E$$

$$= \frac{1}{2} m (v^2 - v_0^2)$$

$$= \frac{1}{2} m \left[\frac{v_0^2 l^2}{r^2} - v_0^2\right]$$

$$= \frac{m v_0^2 l^2}{2}\left[\frac{1}{r^2} - \frac{1}{l^2}\right]$$