CIRCLE

KEY CONCEPTS

STANDARD RESULTS:

1. EQUATION OF A CIRCLE IN VARIOUS FORM:

(a) The circle with center (h, k) & radius 'r' has the equation; $(x-h)^2+(y-k)^2=r^2$

(b) The general equation of a circle is $x^2+y^2+2gx+2fy+c=0$

with center as: $(-g,-f)$ & radius $=\sqrt{g^2+f^2-c}$.

Remember that every 2degree equation in x & y in which coefficient of x^2 = coefficient of y^2 & there is no xy term always represents a circle.

if $g^2+f^2-c>0 \Rightarrow$ real circle.

$g^2+f^2-c=0 \Rightarrow$ point circle.

$g^2+f^2-c<0 \Rightarrow$ imaginary circle.

Note: That the general equation of a circle contains three arbitrary constants, g, f & c which corresponds to the fact that a unique circle passes through three non collinear points.

(c) The equation of circle with (x_1, y_1) & (x_2, y_2) as its diameter is:

$(x-x_1)(x-x_2)+(y-y_1)(y-y_2)=0$

Note: That this will be the circle of least radius passing through (x_1, y_1) & (x_2, y_2).

2. INTERCEPTS MADE BY A CIRCLE ON THE AXES:

The intercepts made by the circle $x^2+y^2+2gx+2fy+c=0$ on the co-ordinate axes are $2\sqrt{g^2-c}$ & $2\sqrt{f^2-c}$ respectively.

Note: If $g^2-c>0$ $\Rightarrow$ circle cuts the x axis at two distinct points.

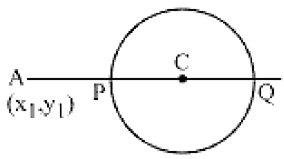
if $g^2=c$ $\Rightarrow$ circle touches the x-axis.

If $g^2<c$ $\Rightarrow$ circle lies completely above or below the x-axis.

3. POSITION OF A POINT w.r.t. A CIRCLE:

The point (x_1, y_1) is inside, on or outside the circle $x^2+y^2+2gx+2fy+c=0$. According as $x_1^2+y_1^2+2gx_1+2fy_1+c \Leftrightarrow 0$.

Note: The greatest & the least distance of a point A from a circle with center C & radius r is $AC+r$ & $AC-r$ respectively.



4. LINE & A CIRCLE:

Let $L=0$ be a line & $S=0$ be a circle. If r is the radius of the circle & p is the length of the perpendicular from the centre on the line, then:

(i) $p>r \Leftrightarrow$ the line does not meet the circle i. e. passes out side the circle.

(ii) $p=r \Leftrightarrow$ the line touches the circle.

(iii) $p<r \Leftrightarrow$ the line is a secant of the circle.

(iv) $p=0 \Rightarrow$ the line is a diameter of the circle.

5. PARAMETRIC EQUATIONS OF A CIRCLE:

The parametric equations of $(x-h)^2+(y-k)^2=r^2$ are: $x=h+r\cos\theta$;

$y=k+r\sin\theta$; $-\pi<\theta\le\pi$ where (h,k) is the center, r is the radius & θ is a parameter.

Note: That equation of a straight line joining two point α & β on the circle $x^2+y^2=a^2$ is

$$x\cos\frac{\alpha+\beta}{2}+y\sin\frac{\alpha+\beta}{2}=a\cos\frac{\alpha-\beta}{2}$$

6. TANGENT & NORMAL:

(a) The equation of the tangent to the circle $x^2+y^2=a^2$ at its point (x_1,y_1) is, $xx_1+y_1=a^2$.

Hence equation of a tangent at $(a\cos\alpha, a\sin\alpha)$ is; $x\cos\alpha+y\sin\alpha=a$.

The point of intersection of the tangents at the points

$P(\alpha)$ and $Q(\beta)$ is $\dfrac{a\cos\frac{\alpha+\beta}{2}}{\cos\frac{\alpha-\beta}{2}}, \dfrac{a\sin\frac{\alpha+\beta}{2}}{\cos\frac{\alpha-\beta}{2}}$.

(b) The equation of the tangent to the circle $x^2+y^2+2gx+2fy+c=0$ at its point (x_1,y_1) is $xx_1+yy_1+g(x+x_1)+f(y+y_1)+c=0$

(c) $y=mx+c$ is always a tangent to the circle $x^2+y^2=a^2$ if $c^2=a^2(1+m^2)$ and the point of contact is $\left(-\frac{a^2m}{c},\frac{a^2}{c}\right)$.

(d) If a line is normal/orthogonal to a circle then it must pass through the center of the circle. Using this fact normal to the circle $x^2+y^2+2gx+2fy+c=0$ at (x_1,y_1) is

$$y-y_1=\frac{y_1+f}{x_1+g}(x-x_1)$$

7. AFAMILY OF CIRCLES:

(a) The equation of the family of circles passing through the points of intersection of two circles $S_1=0$ & $S_2=0$ is: $S_1+KS_2=0$ ($K\ne-1$).

(b) The equation of the family of circles passing through the point of intersection of a circle $S=0$ & a line $L=0$ is given by $S+KL=0$.

(c) The equation of a family of circles passing through two given points (x_1,y_1) & (x_2,y_2) can be written in the form: $(x-x_1)(x-x_2)+(y-y_1)(y-y_2)+K\begin{vmatrix}x & y & 1\\ x_1 & y_1 & 1\\ x_2 & y_2 & 1\end{vmatrix}=0$

where K is a parameter.

(d) The equation of a family of circles touching a fixed line $y-y_1=m(x-x_1)$ at the fixed point (x_1,y_1) is $(x-x_1)^2+(y-y_1)^2+K[y-y_1-m(x-x_1)]=0$, where K is a parameter. In case the line through (x_1,y_1) is parallel to y - axis the equation of the family of circles touching it at (x_1,y_1) becomes $(x-x_1)^2+(y-y_1)^2+K(x-x_1)=0$

Also, if line is parallel to x - axis the equation of the family of circles touching it at (x_1, y_1) becomes $(x - x_1)^2 + (y - y_1)^2 + K(y - y_1) = 0$

(e) Equation of circle circumscribing a triangle whose sides are given by $L_1 = 0; L_2 = 0 \ \& \ L_3 = 0$ is given by; $L_1 L_2 + \lambda L_2 L_3 + \mu L_3 L_1 = 0$ provided co-efficient of $xy = 0$ & co-efficient of $x^2 =$ co-efficient of y^2.

(f) Equation of circle circumscribing a quadrilateral whose side in order are represented by the lines $L_1 = 0,\ L_2 = 0,\ L_3 = 0 \ \& \ L_4 = 0$ is $L_1 L_3 + \lambda L_2 L_4 = 0$ provided co-efficient of $x^2 =$ co-efficient of y^2 and co-efficient of $xy = 0$.

8. LENGTH OF A TANGENT AND POWER OF A POINT :

The length of a tangent from an external point (x_1, y_1) to the circle

$S \equiv x^2 + y^2 + 2gx + 2fy + c = 0$ is given by $L = \sqrt{{x_1}^2 + {y_1}^2 + 2gx_1 + 2f_1y + c} = \sqrt{S_1}$.

Square of length of the tangent from the point P is also called THE POWER OF POINT w.r.t. a circle. Power of a point remains constant w.r.t. a circle.

Note: That Power of a point P is positive, negative or zero according as the point ' P ' is outside, inside or on the circle respectively.

9. DIRECTOR CIRCLE:

The locus of the point of intersection of two perpendicular tangents is called the DIRECTOR CIRCLE of the given circle. The director circle of a circle is the concentric circle having radius equal to $\sqrt{2}$ times the original circle.

10. EQUATION OF THE CHORD WITH AGIVEN MIDDLE POINT:

The equation of the chord of the circle $S \equiv x^2 + y^2 + 2gx + 2fy + c = 0$ in terms of its mid point $M(x_1, y_1)$ is $y - y_1 = -\frac{x_1 + g}{y_1 + f}(x - x_1)$.

This on simplication can be put in the form

$x_1 + y_1 + g(x + x_1) + f(y + y_1) + c = x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c$ which is designated by $T = S_1$.

Note: That: the shortest chord of a circle passing through a point ' M ' inside the circle, is one chord whose middle point is M.

11. CHORD OF CONTACT:

If two tangents PT_1 & PT_2 are drawn from the point $P(x_1, y_1)$ to the circle

$S \equiv x^2 + y^2 + 2gx + 2fy + c = 0$,

then the equation of the chord of contact T_1T_2 is:

$xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c = 0$.

REMEMBER:

(a) Chord of contact exists only if the point ' P ' is not inside.

(b) Length of chord of contact $T_1T_2 = \frac{2LR}{\sqrt{R^2 + L^2}}$.

(c) Area of the triangle formed by the pair of the tangents & its chord of contact $= \frac{RL^3}{R^2 + L^2}$

Where R is the radius of the circle &L is the length of the tangent from (x_1, y_1) on $S = 0$.

(d) Angle between the pair of tangents from $(x_1, y_1) = \tan^{-1}\left(\frac{2RL}{L^2 - R^2}\right)$

where R = radius ; L = length of tangent.

(e) Equation of the circle circumscribing the triangle $PT_1\ T_2$ is:

$(x - x_1)(x + g) + (y - y_1)(y + f) = 0$

(f) The joint equation of a pair of tangents drawn from the point $A(x_1, y_1)$ to the circle

$x^2 + y^2 + 2gx + 2fy + c = 0$ is: $SS_1 = T^2$. Where $S \equiv x^2 + y^2 + 2gx + 2fy + c$;

$S_1 \equiv {x_1}^2 + {y_1}^2 + 2gx_1 + 2fy_1 + c$, $T \equiv x_1 + y_1 + g(x + x_1) + f(y + y_1) + c$

12. POLE & POLAR:

(i) If through a point P in the plane of the circle, there be drawn any straight line to meet the circle in Q and R, the locus of the point of intersection of the tangents at Q & R is called the Polar Of The Point P; also P is called the Pole Of The Polar.

(ii) The equation to the polar of a point $P(x_1,\ y_1)$ w.r.t. the circle $x^2 + y^2 = a^2$ is given by $xx_1 + yy_1 = a^2$, & if the circle is general then the equation of the polar becomes $xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c = 0$.

Note: That if the point (x_1, y_1) be on the circle then the chord of contact, tangent & polar will be represented by the same equation.

(iii) Pole of a given line $Ax + By + C = 0$ w.r.t. any circle $x^2 + y^2 = a^2$ is $\left(-\frac{Aa^2}{C}, -\frac{Ba^2}{C}\right)$.

(iv) If the polar of a point P pass through a point Q, then the polar of Q passes through P.

(v) Two lines L_1& L_2 are conjugate of each other if Pole of L_1 lies on L_2 & vice versa Similarly two points P & Q are said to be conjugate of each other if the polar of P passes through

Q & vice-versa.

13. COMMON TANGENTS TO TWO CIRCLES:

(i) Where the two circles neither intersect nor touch each other, there are FOUR common tangents, two of them are transverse & the others are direct common tangents.

(ii) When they intersect there are two common tangents, both of them being direct.

(iii) When they touch each other:

(a) **Externally:** there are three common tangents, two direct and one is the tangent at the point of contact.

(b) **INTERNALLY:** only one common tangent possible at their point of contact.

(iv) Length of an external common tangent & internal common tangent to the two circles is given by: $L_{ext} = \sqrt{d^2 - (r_1 - r_2)^2}$ & $L_{int} = \sqrt{d^2 - (r_1 + r_2)^2}$. Where d = distance between the centers of the two circles r_1 & r_2 are the radii of the two circles.

(v) The direct common tangents meet at a point which divides the line joining center of circles externally in the ratio of their radii. Transverse common tangents meet at a point which divides the line joining center of circles internally in the ratio of their radii.

14. RADICAL AXIS & RADICAL CENTRE:

The radical axis of two circles is the locus of points whose powers w.r.t. the two circles are equal. The equation of radical axis of the two circles $S_1 = 0$ & $S_2 = 0$ is given; $S_1 - S_2 = 0$ i.e. $2(g_1 - g_2)x + 2(f_1 - f_2)y + (c_1 - c_2) = 0$.

NOTE: That

(a) If two circles intersect, then the radical axis is the common chord of the two circles.

(b) If two circles touch each other than the radical axis is the common tangent of the circles at the common point of contact.

(c) Radical axis is always perpendicular to the line joining the centers of the two circles.

(d) Radical axis need not always pass through the midpoint of the line joining the centers of the two circles.

(e) Radical axis bisects a common tangent between the two circles.

(f) The common point of intersection of the radical axes of three circles taken two at a time is called the radical center of three circles.

(g) A system of circles, every two which have the same radical axis, is called a coaxal system.

(h) Pairs of circles which do not have radical axis are concentric.

15. ORTHOGONALITY OF TWO CIRCLES:

Two circles $S_1 = 0$ & $S_2 = 0$ are said to be orthogonal or said to intersect orthogonally if the tangents at their point of intersection include a right angle. The condition for two circles to be orthogonal is: $2g_1g_2 + 2f_1f_2 = c_1 + c_2$.

Note: **(a)** Locus of the center of a variable circle orthogonal to two fixed circles is the radical axis between the two fixed circles.

(b) If two circles are orthogonal, then the polar of a point 'P' on first circle w.r.t. the second circle passes through the point Q which is the other end of the diameter through P. Hence locus of a point which moves such that its polars w. r. t. the circles $S_1 = 0, S_2 = 0$ & $S_3 = 0$ are concurrent in a circle which is orthogonal to all the three circles.

EXERCISE-I

1. Determine the nature of the quadrilateral formed by four lines $3x + 4y - 5 = 0$; $4x - 3y - 5 = 0$; $3x + 4y + 5 = 0$ and $4x - 3y + 5 = 0$. Find the equation of the circle inscribed and circumscribing this quadrilateral.

2. Suppose the equation of the circle which touches both the coordinate axes and passes through the point with abscissa -2 and ordinate 1 has the equation $x^2 + y^2 + Ax + By + C = 0$, find all the possible ordered triplet (A, B, C).

3. A circle $S = 0$ is drawn with its center at $(-1, 1)$ so as to touch the circle $x^2 + y^2 - 4x + 6y - 3 = 0$ externally. Find the intercept made by the circle $S = 0$ on the coordinate axes.

4. The line $lx + my + n = 0$ intersects the curve $ax^2 + 2hxy + by^2 = 1$ at the point P and Q. The circle on PQ as diameter passes through the origin. Prove that $n^2(a + b) = l^2 + m^2$.

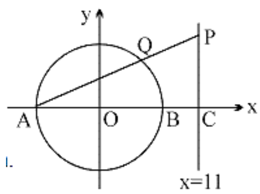
5. One of the diameters of the circle circumscribing the rectangle ABCD is $4y = x + 7$. If A & B are the points $(-3, 4)$ & $(5, 4)$ respectively, then find the area of the rectangle.

6. Find the equation to the circle which is such that the length of the tangents to it from the points $(1, 0), (2, 0)$ and $(3, 2)$ are $1, \sqrt{7}, \sqrt{2}$ respectively.

7. A circle passes through the points $(-1, 1), (0, 6)$ and $(5, 5)$. Find the points on the circle the tangents at which are parallel to the straight line joining origin to the centre.

8. Find the equations of straight lines which pass through the intersection of the lines $x - 2y - 5 = 0$, $7x + y = 50$ & divide the circumference of the circle $x^2 + y^2 = 100$ into two arcs whose lengths are in the ratio 2: 1

9. In the given figure, the circle $x^2 + y^2 = 25$ intersects the x-axis at the point A and B. The line $x = 11$ intersects the x-axis at the point C. Point P moves along the line $x = 11$ above the x-axis and AP intersects the circle at Q. Find



(i) The coordinates of the point P if the triangle AQB has the maximum area.

(ii) The coordinates of the point P if Q is the middle point of AP.

(iii) The coordinates of P if the area of the triangle AQB is $\left(\frac{1}{4}\right)^{th}$ of the area of the triangle APC.

10. A circle is drawn with its center on the line $x + y = 2$ to touch the line $4x - 3y + 4 = 0$ and pass through the point $(0, 1)$. Find its equation.

11. **(a)** Find the area of an equilateral triangle inscribed in the circle $x^2 + y^2 + 2gx + 2fy + c = 0$.

(b) If the line $x \sin\alpha - y + a \sec\alpha = 0$ touches the circle with radius 'a' and centre at the origin then find the most general values of ' α ' and sum of the values of 'α' lying in $[0, 100\pi]$.

12. A point moving around circle $(x+4)^2+(y+2)^2=25$ with centre C broke away from it either at the point A or point B on the circle and moved along a tangent to the circle passing through the point $D(3,-3)$. Find the following.

(i) Equation of the tangents at A and B. **(ii)** Coordinates of the points A and B.

(iii) Angle ADB and the maximum and minimum distances of the point D from the circle.

(iv) Area of quadrilateral ADBC and the $\triangle$ DAB.

(v) Equation of the circle circumscribing the $\triangle$ DAB and also the intercepts made by this circle on the coordinate axes.

13. Find the locus of the midpoint of the chord of a circle $x^2+y^2=4$ such that the segment intercepted by the chord on the curve $x^2-2x-2y=0$ subtends a right angle at the origin.

14. Find the equation of a line with gradient 1 such that the two circles $x^2+y^2=4$ and $x^2+y^2-10x-14y+65=0$ intercept equal length on it.

15. Find the locus of the middle points of portions of the tangents to the circle $x^2+y^2=a^2$ terminated by the coordinate axes.

16. Tangents are drawn to the concentric circles $x^2+y^2=a^2$ and $x^2+y^2=b^2$ at right angle to one another. Show that the locus of their point of intersection is a 3^{rd} concentric circle. Find its radius.

17. Find the equation of the circle passing through the three points $(4,7),(5,6)$ and $(1,8)$. Also find the coordinates of the point of intersection of the tangents to the circle at the points where it is cut by the straight line $5x+y+17=0$

18. Consider a circle S with centre at the origin and radius 4. Four circles A, B, C and D each with radius unity and centers $(-3,0),(-1,0),(1,0)$ and $(3,0)$ respectively are drawn. A chord PQ of the circle S touches the circle B and passes through the centre of the circle C. If the length of this chord can be expressed as $\sqrt{x}$, find x

19. Obtain the equations of the straight lines passing through the point $A(2,0)$ & making $45°$ angle with the tangent at A to the circle $(x+2)^2+(y-3)^2=25$. Find the equations of the circles each of radius 3 whose centers are on these straight lines at a distance of $5\sqrt{2}$ from A.

20. Consider a curve $ax^2+2hxy+by^2=1$ and a point P not on the curve. A line is drawn from the point P intersects the curve at points Q & R. If the product PQ. PR is independent of the slope of the line, then show that the curve is a circle.

21. The line $2x-3y+1=0$ is tangent to a circle $S=0$ at (1,1).

If the radius of the circle is $\sqrt{13}$. Find the equation of the circle S.

22. Find the equation of the circle which passes through the point $(1, 1)$ & which touches the circle $x^2 + y^2 + 4x - 6y - 3 = 0$ at the point $(2, 3)$ on it.

23. Let K denotes the square of the diameter of the circle whose diameter is the common chord of the two circles $x^2 + y^2 + 2x + 3y + 1 = 0$ and $x^2 + y^2 + 4x + 3y + 2 = 0$ and W denotes the sum of the abscissa and ordinates of a point P where all variable chords of the curve $y^2 = 8x$ subtending right angles at the origin, are concurrent. and H denotes the square of the length of the tangent from the point $(3, 0)$ on the circle $2x^2 + 2y^2 + 5y - 16 = 0$. Find the value of KWH.

24. Show that the equation of a straight line meeting the circle $x^2 + y^2 = a^2$ in two points at equal distances 'd' from a point (x_1, y_1) on its circumference is $xx_1 + yy_1 - a^2 + (d^2/2) = 0$.

25. The radical axis of the circles $x^2 + y^2 + 2gx + 2fy + c = 0$ and $2x^2 + 2y^2 + 3x + 8y + 2c = 0$ touches the circle $x^2 + y^2 + 2x - 2y + 1 = 0$. Show that either $g = 3/4$ or $f = 2$.

26. Find the equation of the circle through the points of intersection of circles $x^2 + y^2 - 4x - 6y - 12 = 0$ and $x^2 + y^2 + 6x + 4y - 12 = 0$& cutting the circle $x^2 + y^2 - 2x - 4 = 0$ orthogonally.

27. The centre of the circle $S = 0$ lie on the line $2x - 2y + 9 = 0$ & $S = 0$ cuts orthogonally the circle $x^2 + y^2 = 4$. Show that circle $S = 0$ passes through two fixed points & find their coordinates.

28. **(a)** Find the equation of a circle passing through the origin if the line pair, $xy - 3x + 2y - 6 = 0$ is orthogonal to it. If this circle is orthogonal to the circle $x^2 + y^2 - kx + 2ky - 8 = 0$ then find the value of k.

(b) Find the equation of the circle which cuts the circle $x^2 + y^2 - 14x - 8y + 64 = 0$ and the coordinate axes orthogonally.

29. Find the equation of the circle whose radius is 3 and which touches the circle $x^2 + y^2 - 4x - 6y - 12 = 0$ internally at the point $(-1, -1)$.

30. Show that the locus of the centers of a circle which cuts two given circles orthogonally is a straight line & hence deduce the locus of the centers of the circles which cut the circles $x^2 + y^2 + 4x - 6y + 9 = 0$ & $x^2 + y^2 - 5x + 4y + 2 = 0$ orthogonally. Interpret the locus.

EXERCISE-II

1. A variable circle passes through the point $A(a, b)$ & touches the x-axis; show that the locus of the other end of the diameter through A is $(x-a)^2 = 4by$.

2. Find the equation of the circle passing through the point $(-6, 0)$ if the power of the point $(1, 1)$ w.r.t. the circle is 5 and it cuts the circle $x^2 + y^2 - 4x - 6y - 3 = 0$ orthogonally.

3. Consider a family of circles passing through two fixed points $A(3, 7)$ & $B(6, 5)$.
The the chords in which the circle $x^2 + y^2 - 4x - 6y - 3 = 0$ cuts the members of the family are concurrent at a point. Find the coordinates of this point.

4. Find the equation of circle passing through $(1, 1)$ belonging to the system of co-axal circles that are tangent at $(2, 2)$ to the locus of the point of intersection of mutually perpendicular tangent to the circle $x^2 + y^2 = 4$

5. Find the locus of the mid point of all chords of the circle $x^2 + y^2 - 2x - 2y = 0$ such that the pair of lines joining $(0, 0)$ & the point of intersection of the chords with the circles make equal angle with axis of x.

6. The circle $C: x^2 + y^2 + kx + (1 + k)y - (k + 1) = 0$
passes through the same two points for every real number k. Find
(i) The coordinates of these two points.
(ii) The minimum value of the radius of a circle C.

7. Find the equation of a circle which is co-axial with circles $2x^2 + 2y^2 - 2x + 6y - 3 = 0$ & $x^2 + y^2 + 4x + 2y + 1 = 0$. It is given that the centre of the circle to be determined lies on the radical axis of these two circles.

8. Show that the locus of the point of intersection of the tangents from which to the circle $x^2 + y^2 - a^2 = 0$ include a constant angle α is $(x^2 + y^2 - 2a^2)^2 \tan^2\alpha = 4a^2(x^2 + y^2 - a^2)$.

9. A circle with center in the first quadrant is tangent to $y = x + 10, y = x - 6$, and the y-axis. Let (h, k) be the center of the circle. If the value of $(h + k) = a + b\sqrt{a}$ where $\sqrt{a}$ is a surd, find the value of $a + b$.

10. A circle is described to pass through the origin and to touch the lines $x = 1, x + y = 2$.
Prove that the radius of the circle is a root of the equation $(3 - 2\sqrt{2})t^2 - 2\sqrt{2}t + 2 = 0$.

11. Find the condition such that the four points in which the circle $x^2 + y^2 + ax + by + c = 0$ and $x^2 + y^2 + a'x + b'y + c' = 0$ are intercepted by the straight lines
$Ax + By + C = 0$
& $A'x + B'y + C' = 0$
respectively, lie on another circle.

12. A circle C is tangent to the x and y axis in the first quadrant at the points P and Q respectively. BC and AD are parallel tangents to the circle with slope −1. If the points A and B are on the y-axis while C and D are on the x-axis and the area of the figure ABCD is $900\sqrt{2}$ sq. units then find the radius of the circle.

13. The circle $x^2 + y^2 - 4x - 4y + 4 = 0$ is inscribed in a triangle which has two of its sides along the coordinate axes. The locus of the circumcenter of the triangle is $x + y - xy + K\sqrt{x^2 + y^2} = 0$. Find K.

14. Let A, B, C be real numbers such that

(i) $(\sin A, \cos B)$ lies on a unit circle centered at origin.

(ii) $\tan C$ and $\cot C$ are defined.

If the minimum value of $(\tan C - \sin A)^2 + (\cot C - \cos B)^2$ is $a + b\sqrt{2}$ where $a, b \in I$, find the value of $a^3 + b^3$

15. An isosceles right angled triangle whose sides are $1, 1, \sqrt{2}$ lies entirely in the first quadrant with the ends of the hypotenuse on the coordinate axes. If it slides prove that the locus of its centroid is $(3x - y)^2 + (x - 3y)^2 = \frac{32}{9}$.

16. A rhombus ABCD has sides of length 10. A circle with centre 'A' passes through C (the opposite vertex) likewise, a circle with centre B passes through D.

If the two circles are tangent to each other, find the area of the rhombus.

17. Find the equation of a circle which touches the lines $7x^2 - 18xy + 7y^2 = 0$ and the circle $x^2 + y^2 - 8x - 8y = 0$ and is contained in the given circle.

18. Let W_1 & W_2 denote the circles $x^2 + y^2 + 10x - 24y - 87 = 0$ & $x^2 + y^2 - 10x - 24y + 153 = 0$ respectively. Let m be the smallest positive value of 'a' for which the line $y = ax$ contains the centre of a circle that is externally tangent to W_2 and internally tangent to W_1.

Given that $m^2 = p/q$ where p and q are relatively prime integers, find $(p + q)$.

19. Find the equation of the circle which passes through the origin, meets the x-axis orthogonally & cuts the circle $x^2 + y^2 = a^2$ at an angle of 45°.

20. Circles C_1 and C_2 are externally tangent and they are both internally tangent to the circle C_3. The radii of C_1 and C_2 are 4 and 10, respectively and the centres of the three circles are collinear. A chord of C_3 is also a common internal tangent of C_1 and C_2.

Given that the length of the chord is $\frac{m\sqrt{n}}{p}$ where m, n and p are positive integers, m and p are relatively prime and n is not divisible by the square of any prime, find the value of $(m + n + p)$.

EXERCISE-III

1. **(a)** If two distinct chords, drawn from the point (p, q) on the circle $x^2 + y^2 = px + qy$ (Where $pq \neq q$) are bisected by the x-axis, then:

(A) $p^2 = q^2$ (B) $p^2 = 8q^2$ (C) $p^2 < 8q^2$ (D) $p^2 > 8q^2$

(b) Let L_1 be a straight line through the origin and L_2 be the straight line $x + y = 1$. If the intercepts made by the circle $x^2 + y^2 - x + 3y = 0$ on L_1 & L_2 are equal, then which of the following equations can represent L_1 ?

(A) $x + y = 0$ (B) $x - y = 0$ (C) $x + 7y = 0$ (D) $x - 7y = 0$

(c) Let T_1, T_2 be two tangents drawn from $(-2,0)$ onto the circle $C: x^2 + y^2 = 1$.

Determine the circles touching C and having T_1, T_2 as their pair of tangents. Further, find the equations of all possible common tangents to these circles, when taken two at a time.

[JEE ' 99, 2 + 3 + 10 (out of 200)]

2. **(a)** The triangle PQR is inscribed in the circle, $x^2 + y^2 = 25$. If Q and R have co-ordinates $(3, 4)$ & $(-4,3)$ respectively, then $\angle QPR$ is equal to:

(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{4}$ (D) $\frac{\pi}{6}$

(b) If the circles, $x^2 + y^2 + 2x + 2ky + 6 = 0$ & $x^2 + y^2 + 2ky + k = 0$ intersect orthogonally, then ' k ' is: **[JEE ' 2000 (Screening) 1 + 1]**

(A) 2 or $-\frac{3}{2}$ (B) -2 or $-\frac{3}{2}$ (C) 2 or $\frac{3}{2}$ (D) -2 or $\frac{3}{2}$

3. **(a)** Extremities of a diagonal of a rectangle are (0,0) & (4,3). Find the equation of the tangents to the circumcircle of a rectangle which are parallel to this diagonal.

(b) Find the point on the straight line, $y = 2x + 11$ which is nearest to the circle, $16(x^2 + y^2) + 32x - 8y - 50 = 0$

(c) A circle of radius 2 units rolls on the outer side of the circle, $x^2 + y^2 + 4x = 0$, touching it externally. Find the locus of the centre of this outer circle. Also find the equations of the common tangents of the two circles when the line joining the centres of the two circles is inclined at an angle of 60° with x-axis. **[REE '2000 (Mains) +3 + 5]**

4. **(a)** Let PQ and RS be tangents at the extremities of the diameter PR of a circle of radius r. If PS and RQ intersect at a point X on the circumference of the circle then 2r equals

(A) $\sqrt{PQ \cdot RS}$ (B) $\frac{PQ + RS}{2}$ (C) $\frac{2PQ \cdot RS}{PQ + RS}$ (D) $\sqrt{\frac{(PQ)^2 + (RS)^2}{2}}$

[JEE '2001 (Screening) 1 out of 35]

(b) Let $2x^2 + y^2 - 3xy = 0$ be the equation of a pair of tangents drawn from the origin 'O' to a circle of radius 3 with centre in the first quadrant. If A is one of the points of contact, find the length of OA. **[JEE '2001 (Mains) 5 out of 100]**

5. **(a)** Find the equation of the circle which passes through the points of intersection of circles $x^2 + y^2 - 2x - 6y + 6 = 0$ and $x^2 + y^2 + 2x - 6y + 6 = 0$ and intersects the circle $x^2 + y^2 + 4x + 6y + 4 = 0$ orthogonally. **[REE '2001 (Mains) 3 out of 100]**

(b) Tangents TP and TQ are drawn from a point T to the circle $x^2 + y^2 = a^2$. If the point T lies on the line $px + qy = r$, find the locus of centre of the circumcircle of triangle TPQ.

[REE '2001 (Mains) 5 out of 100]

6. **(a)** If the tangent at the point P on the circle $x^2 + y^2 + 6x + 6y = 2$ meets the straight line $5x - 2y + 6 = 0$ at a point Q on the y-axis, then the length of PQ is

(A) 4 (B) $2\sqrt{5}$ (C) 5 (D) $3\sqrt{5}$

(b) If $a > 2b > 0$ then the positive value of m for which $y = mx - b\sqrt{1 + m^2}$ is a common tangent to $x^2 + y^2 = b^2$ and $(x - a)^2 + y^2 = b^2$ is **[JEE'2002 (Scr) 3 + 3 out of 270]**

(A) $\frac{2b}{\sqrt{a^2-4b^2}}$ (B) $\frac{\sqrt{a^2-4b^2}}{2b}$ (C) $\frac{2b}{a-2b}$ (D) $\frac{b}{a-2b}$

7. The radius of the circle, having centre at (2,1), whose one of the chord is a diameter of the circle $x^2 + y^2 - 2x - 6y + 6 = 0$

(A) 1 (B) 2 (C) 3 (D) $\sqrt{3}$ **[JEE '2004 (Scr)]**

8. Line $2x + 3y + 1 = 0$ is a tangent to a circle at $(1, -1)$. This circle is orthogonal to a circle which is drawn having diameter as a line segment with end points $(0, -1)$ and $(-2, 3)$. Find equation of circle. **[JEE '2004, 4 out of 60]**

9. A circle is given by $x^2 + (y - 1)^2 = 1$, another circle C touches it externally and also the x-axis, then the locus of its centre is **[JEE ' 2005 (Scr)]**

(A) $\{(x, y): x^2 = 4y\} \cup \{(x, y): y \le 0\}$ (B) $\{(x, y): x^2 + (y - 1)^2 = 4\} \cup \{x, y): y \le 0\}$

(C) $\{(x, y): x^2 = y\} \cup \{(0, y): y \le 0\}$ (D) $\{(x, y): x^2 = 4y\} \cup \{(0, y): y \le 0\}$

10. **(a)** Let ABCD be a quadrilateral with area 18, with side AB parallel to the side CD and AB = 2CD. Let AD be perpendicular to AB and CD. If a circle is drawn inside the quadrilateral ABCD touching all the sides, then its radius is

(A) 3 (B) 2 (C) 3/2 (D) 1

(b) Tangents are drawn from the point (17,7) to the circle $x^2 + y^2 = 169$. **[JEE 2007, 3+3]**

STATEMENT-1: The tangents are mutually perpendicular.

because

STATEMENT-2: The locus of the points from which mutually perpendicular tangents can be drawn to the given circle is $x^2 + y^2 = 338$.

(A) Statement-1 is true, statement- 2 is true; statement- 2 is correct explanation for statement-1.

(B) Statement-1 is true, statement-2 is true; statement-2 is NOT a correct explanation for statement- 1.

(C) Statement- 1 is true, statement-2 is false.

(D) Statement- 1 is false, statement-2 is true

11. **(a)** Consider the two curves $C_1: y^2 = 4x; C_2: x^2 + y^2 - 6x + 1 = 0$. Then,

(A) C_1 and C_2 touch each other only at one point

(B) C_1 and C_2 touch each other exactly at two points

(C) $C_1 \& C_2$ intersect (but do not touch) at exactly two points

(D) C_1 and C_2 neither intersect nor touch each other

(b) Consider, $L_1: 2x + 3y + P - 3 = 0;\ L_2: 2x + 3y + P + 3 = 0$, where p is a real number, and $C: x^2 + y^2 + 6x - 10y + 30 = 0$.

STATEMENT-1: If line L_1 is a diameter of circle C, then line L_2 is not always a diameter of circle C.

because

STATEMENT-2: If line L_1 is a diameter of circle C, then line L_2 is not a chord of circle C.

(A) Statement-1 is True, Statement-2 is True; statement-2 is a correct explanation for statement-1

(B) Statement-1 is True, Statement- 2 is True; statement-2 is NOT a correct explanation for statement-1

(C) Statement- 1 is True, Statement-2 is False

(D) Statement- 1 is False, Statement- 2 is True

(c) Comprehension (3 questions together):

A circle C of radius 1 is inscribed in an equilateral triangle PQR. The points of contact of C with the sides PQ, QR, RP are D, E, F respectively. The line PQ is given by the equation $\sqrt{3}x + y - 6 = 0$ and the point D is $\left(\frac{3\sqrt{3}}{2}, \frac{3}{2}\right)$. Further, it is given that the origin and the centre of C are on the same side of the line PQ. **[JEE 2008, 3+3+4+4+4]**

(i) The equation of circle C is

(A) $(x - 2\sqrt{3})^2 + (y - 1)^2 = 1$

(B) $(x - 2\sqrt{3})^2 + \left(y + \frac{1}{2}\right)^2 = 1$

(C) $(x - \sqrt{3})^2 + (y + 1)^2 = 1$

(D) $(x - \sqrt{3})^2 + (y - 1)^2 = 1$

(ii) Points E and F are given by

(A) $\left(\frac{\sqrt{3}}{2}, \frac{3}{2}\right), (\sqrt{3}, 0)$ (B) $\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right), (\sqrt{3}, 0)$

(C) $\left(\frac{\sqrt{3}}{2}, \frac{3}{2}\right), \left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$ (D) $\left(\frac{3}{2}, \frac{\sqrt{3}}{2}\right), \left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

(iii) Equations of the sides RP, RQ are

(A) $y = \frac{2}{\sqrt{3}}x + 1, y = -\frac{2}{\sqrt{3}}x - 1$ (B) $y = \frac{1}{\sqrt{3}}x, y = 0$

(C) $y = \frac{\sqrt{3}}{2}x + 1, y = -\frac{\sqrt{3}}{2}x - 1$ (D) $y = \sqrt{3}x, y = 0$

12. Tangents drawn from the point P(1,8) to the circle $x^2 + y^2 - 6x - 4y - 11 = 0$ touch the circle at the points A and B. The equation of the circumcircle of the triangle PAB is **[IIT 2009]**

(A) $x^2 + y^2 + 4x - 6x + 19 = 0$ (B) $x^2 + y^2 - 4x - 10y + 19 = 0$

(C) $x^2 + y^2 - 2x + 6y - 29 = 0$ (D) $x^2 + y^2 - 6x - 4y + 19 = 0$

13. The centers of two circles C_1 and C_2 each of unit radius are at a distance of 6 units from each other. Let P be the mid point of the line segment joining the centres of C_1 and C_2 and C be a circle touching circles C_1 and C_2 externally. If a common tangent to C_1 and C passing through P is also a common tangent to C_2 and C, then the radius of the circle C is. **[IIT 2009]**

14. The circle passing through the point $(-1, 0)$ and touching the y-axis at $(0, 2)$ also passes through the point **[IIT 2011]**

(A) $\left(-\frac{3}{2}, 0\right)$ (B) $\left(-\frac{5}{2}, 2\right)$ (C) $\left(-\frac{3}{2}, \frac{5}{2}\right)$ (D) $(-4, 0)$

15. The straight line $2x - 3y = 1$ divides the circular region $x^2 + y^2 \leq 6$ into two parts.

If $S = \left\{\left(2, \frac{3}{4}\right), \left(\frac{5}{4}, \frac{3}{4}\right), \left(\frac{1}{4}, -\frac{1}{4}\right), \left(\frac{1}{8}, \frac{1}{4}\right)\right\}$, then the number of point(s) in S lying inside the smaller part is **[IIT 2011]**

16. The locus of the mid-point of the chord of contact of tangents drawn from points lying on the straight line $4x - 5y = 20$ to the circle $x^2 + y^2 = 9$ is **[IIT 2012]**

(A) $20(x^2 + y^2) - 36x + 45y = 0$ (B) $20(x^2 + y^2) + 36x - 45y = 0$

(C) $36(x^2 + y^2) - 20x + 45y = 0$ (D) $36(x^2 + y^2) + 20x - 45y = 0$

Paragraph for Question Nos. 17 to 18

A tangent PT is drawn to the circle $x^2 + y^2 = 4$ at the point $P(\sqrt{3}, 1)$. A straight line L, perpendicular to PT is a tangent to the circle $(x - 3)^2 + y^2 = 1$. **[IIT 2012]**

17. A common tangent of the two circles is

(A) $x = 4$ (B) $y = 2$ (C) $x + \sqrt{3}y = -1$ (D) $x + 2\sqrt{2}y = 6$

18. A possible equation of L is

(A) $x - \sqrt{3}y = +1$ (B) $x + \sqrt{3}y = 1$ (C) $x - \sqrt{3}y = -1$ (D) $x + \sqrt{3}y = 5$

19. The circle passing through $(1, -2)$ and touching the axis of x at $(3, 0)$ also passes through the point: **[IIT JEE Main 2013]**

(A) $(-2, 5)$ (B) $(-5, 2)$

(C) $(2, -5)$ (D) $(5, -2)$

20. Circle(s) touching x-axis at a distance 3 from the origin and having an intercept of length $2\sqrt{7}$ on y-axis is (are) **[IIT JEE Advance 2013]**

(A) $x^2 + y^2 - 6x + 8y + 9 = 0$ (B) $x^2 + y^2 - 6x + 7y + 9 = 0$

(C) $x^2 + y^2 - 6x - 8y + 9 = 0$ (D) $x^2 + y^2 - 6x - 7y + 9 = 0$

21. Let C be the circle with centre at (1,1) and radius = 1. If T is the circle centered at $(0, y)$, passing through origin and touching the circle C externally, then the radius of T is equal to

(A) $\frac{1}{4}$ (B) $\frac{\sqrt{3}}{\sqrt{2}}$ (C) $\frac{\sqrt{3}}{2}$ (D) $\frac{1}{2}$ **[IIT JEE Main 2014]**

22. A circle S passes through the point (0,1) and is orthogonal to the circles $(x - 1)^2 + y^2 = 16$ and $x^2 + y^2 = 1$. Then **[IIT JEE Advance 2014]**

(A) radius of S is 8 (B) radius of S is 7

(C) center of S is $(-7,1)$ (D) center of S is $(-8, 1)$

23. The number of common tangents to the circles $x^2 + y^2 - 4x - 6y - 12 = 0$ and $x^2 + y^2 + 6x + 18y + 26 = 0$, is: **[IIT JEE Main 2015]**

(A) 4 (B) 1 (C) 2 (D) 3

24. Locus of the image of the point (2,3) in the line $(2x - 3y + 4) + k(x - 2y + 3) = 0, k \in R$, is a:

(A) Circle of radius $\sqrt{3}$ (B) straight line parallel to x-axis

(C) straight line parallel to y-axis (D) circle of radius $\sqrt{2}$ **[IIT JEE Main 2015]**

25. The centers of those circles which touch the circle, $x^2 - y^2 - 8x - 8y - 4 = 0$, externally and also touch the x-axis, lie on: **[IIT JEE Main 2016]**

(A) a circle (B) a hyperbola

(C) an ellipse which is not a circle (D) a parabola

26. If one of the diameters of the circle, given by the equation, $x^2 + y^2 - 4x + 6y - 12 = 0$, is a chord of a circle S, whose centre is at $(-3, 2)$, then the radius of S is: **[IIT JEE Main 2016]**

(A) $5\sqrt{2}$ (B) $5\sqrt{3}$ (C) 5 (D) 10

27. Let P be the point on the parabola, $y^2 = 8x$ which is at a minimum distance from the centre C of the circle, $x^2 + (y + 6)^2 = 1$. Then the equation of the circle, passing through C and having its center at P is: **[IIT JEE Main 2016]**

(A) $x^2 + y^2 - 4x + 8y + 12 = 0$ (B) $x^2 + y^2 - x + 4y - 12 = 0$

(C) $x^2 + y^2 - \frac{x}{4} + 2y - 24 = 0$ (D) $x^2 + y^2 - 4x + 9y + 18 = 0$

28. The circle $C_1: x^2+y^2=3$, with centre at O, intersects the parabola $x^2=2y$ at the point P in the first quadrant. Let the tangent to the circle C_1 at P touches other two circles C_2 and C_3 at R_2 and R_3, respectively. Suppose C_2 and C_3 have equal radii $2\sqrt{3}$ and centres Q_2 and Q_3, respectively. If Q_2 and Q_3 lie on the y-axis, then **[JEE Advance 2016]**

(A) $Q_2Q_3=12$ (B) $R_2R_3=4\sqrt{6}$

(C) area of the triangle OR_2R_3 is $6\sqrt{2}$ (D) area of the triangle PQ_2Q_3 is $4\sqrt{2}$

29. Let RS be the diameter of the circle $x^2+y^2=1$, where S is the point $(1,0)$. Let P be a variable point (other than R and S) on the circle and tangents to the circle at S and P meet at the point Q. The normal to the circle at P intersects a line drawn through Q parallel to RS at point E. Then the locus of E passes through the point(s). **[JEE Advance 2016]**

(A) $\left(\frac{1}{3},\frac{1}{\sqrt{3}}\right)$ (B) $\left(\frac{1}{4},\frac{1}{2}\right)$ (C) $\left(\frac{1}{3},-\frac{1}{\sqrt{3}}\right)$ (D) $\left(\frac{1}{4},-\frac{1}{2}\right)$

30. Let $\alpha,\lambda,\mu\in\mathbb{R}$. Consider the system of linear equations **[JEE Advance 2016]**

$\alpha x+2y=\lambda,\ 3x-2y=\mu$ Which of the following statement(s) is(are) correct?

(A) If $\alpha=-3$, then the system has infinitely many solutions for all values of λ and μ.
(B) If $\alpha\neq-3$, then the system has a unique solution for all values of λ and μ.
(C) If $\lambda+\mu=0$, then the system has infinitely many solutions for $\alpha=-3$
(D) If $\lambda+\mu\neq0$, then the system has no solution for $\alpha=-3$

31. The radius of a circle, having minimum area, which touches the curve $y=4-x^2$ and the lines, $y=|x|$ is: **[JEE Main 2017]**

(A) $4(\sqrt{2}-1)$ (B) $4(\sqrt{2}+1)$ (C) $2(\sqrt{2}+1)$ (D) $2(\sqrt{2}-1)$

32. For how many values of p, the circle $x^2+y^2+2x+4y-p=0$ and the coordinate axes have exactly three common points? **[JEE Advance 2017]**

Comprehension (Q.33 to Q.34):

Let S be the circle in the xy-plane defined by the equation $x^2+y^2=4$.

33. Let E_1E_2 and F_1F_2 be the chords of S passing through the point $P_0(1,1)$ and parallel to the x-axis and the y axis, respectively. Let G_1G_2 be the chord of S passing through P_0 and having slope -1. Let the tangents to S at E_1 and E_2 meet at E_3, the tangents to S at F_1 and F_2 meet at F_3, and the tangents to S at G_1 and G_2 meet at G_3. Then, the points E_3F_3, and G_3 lie on the curve

(A) $x+y=4$ (B) $(x-4)^2+(y-4)^2=16$ **[JEE Advanced 2018]**

(C) $(x-4)(y-4)=4$ (D) $xy=4$

34. Let P be a point on the circle S with both coordinates being positive. Let the tangent to S at P intersect the coordinate axes at the points M and N. Then, the mid-point of the line segment MN must lie on the curve **[JEE Advanced 2018]**

(A) $(x+y)^2=3xy$ (B) $x^{2/3}+y^{2/3}=2^{4/3}$ (C) $x^2+y^2=2xy$ (D) $x^2+y^2=x^2y^2$

35. Let T be the line passing through the points $P(-2,7)$ and $Q(2,-5)$. Let F_1 be the set of all pairs of circles (S_1, S_2) such that T is tangent to S_1 at P and tangent to S_2 at Q, and also such that S_1 and S_2 touch each other at a point, say, M. Let E_1 be the set representing the locus of M as the pair (S_1, S_2) varies in F_1. Let the set of all straight line segments joining a pair of distinct points of E_1 and passing through the point $R(1,1)$ be F_2. Let E_2 be the set of the mid-points of the line segments in the set F_2. Then, which of the following statement(s) is (are) TRUE?

[JEE Advanced 2018]

(A) The point $(-2,7)$ lies in E_1

(B) The point $\left(\frac{4}{5}, \frac{7}{5}\right)$ does NOT lie in E_2

(C) The point $\left(\frac{1}{2}, 1\right)$ lies in E_2

(D) The point $\left(0, \frac{3}{2}\right)$ does NOT lie in E_1

36. A line $y = mx + 1$ intersects the circle $(x-3)^2 + (y+2)^2 = 25$ at the points P and Q. If the midpoint of the line segment PQ has x-coordinate $-\frac{3}{5}$, then which one of the following options is correct? **[JEE Advanced 2019]**

(A) $2 \le m < 4$

(B) $4 \le m < 6$

(C) $6 \le m < 8$

(D) $-3 \le m < -1$

37. Let the point B be the reflection of the point $A(2,3)$ with respect to the line $8x - 6y - 23 = 0$. Let Γ_A and Γ_B be circles of radii 2 and 1 with centres A and B respectively. Let T be a common tangent to the circles Γ_A and Γ_B such that both the circles are on the same side of T. If C is the point of intersection of T and the line passing through A and B, then the length of the line segment AC is……. **[JEE Advanced 2019]**

Answer the following by appropriately matching the lists based on the information given in the paragraph (Q.38 to Q.39) **[JEE Advanced 2019]**

Let the circles $C_1: x^2 + y^2 = 9$ and $C_2: (x-3)^2 + (y-4)^2 = 16$, intersect at the points X and Y. Suppose that another circle $C_3: (x-h)^2 + (y-k)^2 = r^2$ satisfies the following conditions:

(i) Centre of C_3 is collinear with the centres of C_1 and C_2.

(ii) C_1 and C_2 both lie inside C_3 and

(iii) C_3 touches C_1 at M and C_2 at N.

Let the line through X and Y intersect C_3 at Z and W, and let a common tangent of C_1 and C_3 be a tangent to the parabola $x^2 = 8\alpha y$. There are some expressions given in the List-I whose values are given in List-II below:

List-I		List-II	
(I)	$2h + k$	(P)	6
(II)	$\frac{\text{Length of ZW}}{\text{Length of XY}}$	(Q)	$\sqrt{6}$
(III)	$\frac{\text{Area of triangle MZN}}{\text{Area of triangle ZMW}}$	(R)	$\frac{5}{4}$
(IV)	α	(S)	$\frac{21}{5}$
		(T)	$2\sqrt{6}$
		(U)	$\frac{10}{3}$

38. Which of the following is the only INCORRECT combination?

(A) (IV), (S) (B) (III), (R) (C) (I), (P) (D) (IV), (U)

39. Which of the following is the only CORRECT combination?

(A) (II), (Q) (B) (I), (U) (C) (II), (T) (D) (I), (S)

40. Let O be the centre of the circle $x^2 + y^2 = r^2$, where $r > \frac{\sqrt{5}}{2}$. Suppose PQ is a chord of this circle and the equation of the line passing through P and Q is $2x + 4y = 5$. If the centre of the circumcircle of the triangle OPQ lies on the line $x + 2y = 4$, then the value of r is_______

[JEE Advanced 2020]

41. Consider a triangle Δ whose two sides lie on the x-axis and the line $x + y + 1 = 0$. If the orthocenter of Δ is $(1, 1)$, then the equation of the circle passing through the vertices of the triangle Δ is: **[JEE Advanced 2021]**

(A) $x^2 + y^2 - 3x + y = 0$ (B) $x^2 + y^2 + x + 3y = 0$

(C) $x^2 + y^2 + 2y - 1 = 0$ (D) $x^2 + y^2 + x + y = 0$

Paragraph for Question Nos. 42 to 43 **[JEE Advanced 2021]**

Let $M = \{(x, y) \in R \times R : x^2 + y^2 \le r^2\}$

where $r > 0$. Consider the geometric progression $a_n = \frac{1}{2^{n-1}}, n = 1, 2, 3, \ldots$. Let $S_0 = 0$ and, for $n \ge 1$, let S_n denote the sum of the first n terms of this progression. For $n \ge 1$, let C_n denote the circle with center $(S_{n-1}, 0)$ and radius a_n, and D_n denote the circle with center (S_{n-1}, S_{n-1}) and radius a_n.

42. Consider M with $r = \frac{1025}{513}$. Let k be the number of all those circles C_n that are inside M. Let ℓ be the maximum possible number of circles among these k circles such that no two circles intersect. Then

(A) $k + 2\ell = 22$ (B) $2k + \ell = 26$ (C) $2k + 3\ell = 34$ (D) $3k + 2\ell = 40$

43. Consider M with $r = \frac{(2^{199} - 1)\sqrt{2}}{2^{198}}$. The number of all those circles D_n that are inside M is

(A) 198 (B) 199 (C) 200 (D) 201

ANSWER-SHEET

EXERCISE-I

1. square of side 2; $x^2+y^2=1; x^2+y^2=2$

2. $x^2+y^2+10x-10y+25=0$ **OR** $x^2+y^2+2x-2y+1=0, (10,-10,25)(2,-2,1)$

3. zero, zero

5. 32 Sq. Unit

6. $2(x^2+y^2)+6x-17y-6=0]$

7. $(5,1)$ & $(-1,5)$

8. $4x-3y-25=0$ OR $3x+4y-25=0$

9. **(i)** $(11,16)$, **(ii)** $(11,8)$, **(iii)** $(11,12)$

10. $x^2+y^2-2x-2y+1=0$ **OR** $x^2+y^2-42x+38y-39=0$

11. **(a)** $\frac{3\sqrt{3}}{4}(g^2+f^2-c)$; **(b)** $\alpha=n\pi$; 5050π

12. **(i)** $3x-4y=21; 4x+3y=3$; **(ii)** $A(0,1)$ and $B(-1,-6)$; **(iii)** $90°, 5(\sqrt{2}\pm 1)$ units
(iv) 25 sq. units, 12.5 sq. units; **(v)** $x^2+y^2+x+5y-6$, x intercept 5; y intercept 7

13. $x^2+y^2-2x-2y=0$

14. $2x-2y-3=0$

15. $a^2(x^2+y^2)=4x^2y^2$

16. $x^2+y^2=a^2+b^2; r=\sqrt{a^2+b^2}$

17. $(-4,2), x^2+y^2-2x-6y-15=0$

18. 63

19. $x-7y=2, 7x+y=14; (x-1)^2+(y-7)^2=3^2; (x-3)^2+(y+7)^2=3^2$
$(x-9)^2+(y-1)^2=3^2;\ (x+5)^2+(y+1)^2=3^2$

21. $x^2+y^2-6x+4y=0$ OR $x^2+y^2+2x-8y+4=0$

22. $x^2+y^2+x-6y+3=0$

23. 64

26. $x^2+y^2+16x+14y-12=0$

27. $(-4,4); (-1/2,1/2)$

28. **(a)** $x^2+y^2+4x-6y=0; k=1$; **(b)** $x^2+y^2=64$

29. $5x^2+5y^2-8x-14y-32=0$

30. $9x-10y+7=0$; radical axis

EXERCISE-II

2. $x^2+y^2+6x-3y=0$

3. $\left(2,\frac{23}{3}\right)$

4. $x^2+y^2-3x-3y+4=0$

5. $x+y=2$

6. $(1,0)$ & $\left(\frac{\frac{1}{2,1}}{2}\right)$; $r=\frac{1}{2\sqrt{2}}$

7. $4x^2+4y^2+6x+10y-1=0$

9. 10

11. $\begin{vmatrix} a-a' & b-b' & c-c' \\ A & B & C \\ A' & B' & C' \end{vmatrix}=0$

12. $r=15$

13. $K=1$

14. 19

16. 75 sq. unit

17. $x^2+y^2-12x-12y+64=0$

18. 169

19. $x^2+y^2\pm a\sqrt{2}x=0$

20. 19

EXERCISE-III

1. **(a)** (D) **(b)** (B, C)

(c) $c_1: (x-4)^2 + y^2 = 9;\ c_2: \left(x+\frac{4}{3}\right) + y^2 = \frac{1}{9}$
common tangent between c & c_1: $T_1 = 0; T_2 = 0$ and $x - 1 = 0$;
common tangent between c & c_2: $T_1 = 0; T_2 = 0$ and $x + 1 = 0$;
common tangent between c_1 & c_2: $T_1 = 0; T_2 = 0$ and $y = \pm\frac{5}{\sqrt{39}}\left(x+\frac{4}{5}\right)$
where $T_1: x - \sqrt{3}y + 2 = 0$ and $T_2: x + \sqrt{3}y + 2 = 0$

2. **(a)** (C) **(b)** (A)

3. **(a)** $6x - 8y + 25 = 0$ & $6x - 8y - 25 = 0$

(b) $\left(-\frac{9}{2,2}\right)$

(c) $x^2 + y^2 + 4x - 12 = 0,$

$T_1: \sqrt{3}x - y + 2\sqrt{3} + 4 = 0,$

$T_2: \sqrt{3}x - y + 2\sqrt{3} - 4 = 0$ (D.C.T.)

$T_3: x + \sqrt{3}y - 2 = 0, T_4: x + \sqrt{3}y + 6 = 0$ (T.C.T.)

4. **(a)** (A) **(b)** $OA = 3(3+\sqrt{10})$

5. **(a)** $x^2 + y^2 + 14x - 6y + 6 = 0;$ **(b)** $2px + 2qy = r$

6. **(a)** (C) **(b)** (A)

7. (C) **8.** $2x^2 + 2y^2 - 10x - 5y + 1 = 0$ **9.** (D)

10. **(a)** (B) **(b)** (A)

11. **(a)** (B) **(b)** (C) **(c)** **(i)** (D) **(ii)** (A) **(iii)** (D)

12. (B) **13.** (8) **14.** (D) **15.** (2) **16.** (A) **17.** (D) **18.** (A)

19. (D) **20.** (A, C) **21.** (A) **22.** (B, C) **23.** (D) **24.** (D) **25.** (D)

26. (B) **27.** (A) **28.** (A, B, C) **29.** (A, C) **30.** (B, C, D) **31.** (A) **32.** (2)

33. (A) **34.** (D) **35.** (BD) **36.** (A) **37.** (10.00) **38.** (A) **39.** (A)

40. (2) **41** (B) **42.** (D) **43.** (B)