DPP-02

SLOPE OF LINE & ANGLE BETWEEN TWO LINES

SOLUTION

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1. The line joining the points (x, 2x) and (3,5) makes an obtuse angle with the positive direction of the x-axis. Then find the values of x.

Ans. $x \in (5/2, 3)$

Sol. The slope joining points A(x, 2x) and B(3,5) is $(2x-5)/(x-3)$ If AB makes an angle θ with the positive direction of the x-axis, then

$$\tan\theta = \frac{2x-5}{x-3}$$

Since θ is obtuse, $\tan\theta < 0$

$$\text{or } \frac{2x-5}{x-3} < 0$$

$$\text{or } x \in \left(\frac{5}{2}, 3\right)$$

2. If the line passing through (4,3) and (2, k) is parallel to the line $y = 2x + 3$, then find the value of k.

Ans. -1

Sol. The slope of line passing through (4,3) and (2, k)

$$m_1 = \frac{k-3}{2-4} = \frac{3-k}{2}$$

The slope of the given line $y = 2x + 3$,

$$m_2 = 2$$

The lines are parallel. Therefore,

$$\frac{3-k}{2} = 2$$

or $k = -1$

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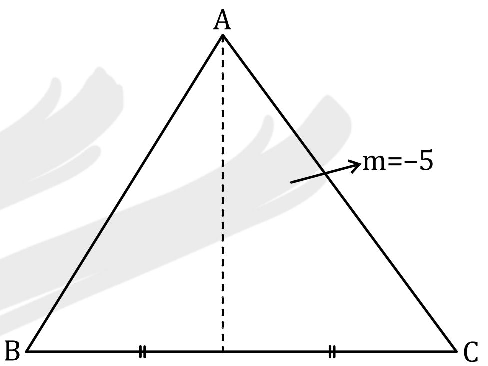
3. Triangle ABC lies in the cartesian plane and has an area of 70 sq. units. The coordinates of B and C are (12,19) and (23,20), respectively. The line containing the median to the side BC has slope -5 . Find the possible coordinates of point A.

Ans. (15,32) or (20,7)

Sol. $m = \frac{\left(\frac{39}{2}-k\right)}{\left(\frac{35}{2}-h\right)} = -5$

$$\frac{39}{2} - k = \frac{-175}{2} + 5h$$
$$\frac{39+175}{2} = 5h + k$$

$5h + k = 107$ (1)



$$\frac{1}{2}\left\|\begin{matrix} h & k & 1 \\ 12 & 19 & 1 \\ 23 & 20 & 1 \end{matrix}\right\| = 70$$
$$\left|\begin{matrix} h & k & 1 \\ 12 & 19 & 1 \\ 23 & 20 & 1 \end{matrix}\right| = \pm 140$$

$11k - h = 337$ (2)

&

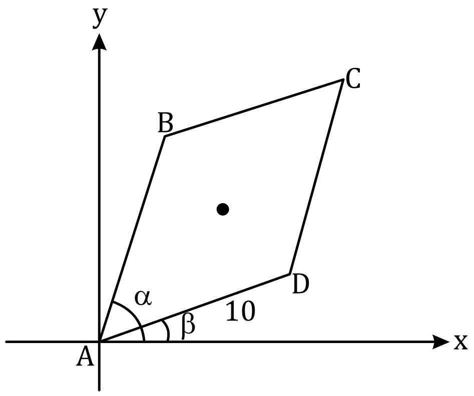
$11k - h = 57$ (3)

from (1) & (2) $\Rightarrow A(15,32)$
& from (1) & (3) $\Rightarrow A(20,7)$

4. ABCD is a rhombus of side 10 units where slope of AB is 4/3 and slope of AD is 3/4. If coordinates of A are (0,0), then find the coordinates of B, C and D.

Ans. B(6,8), C(14,14), D(8,6)

Sol. In the figure, $\tan\,\alpha = \frac{4}{3}$. $\therefore$ Coordinates of point B



$\equiv (10\cos\,\alpha, 10\sin\,\alpha)$

$\equiv \left(10 \times \frac{3}{5}, 10 \times \frac{4}{5}\right) \equiv (6,8)$

Also, $\tan\,\beta = \frac{3}{4}$.

$\therefore$ Coordinates of point D

$\equiv (10\cos\,\beta, 10\sin\,\beta)$

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$\equiv \left(10 \times \frac{4}{5}, 10 \times \frac{3}{5}\right) \equiv (8,6)$

Diagonals of rhombus bisect.
So, midpoint of BD is (7,7), which is midpoint of AC. Therefore, coordinates of C are (14,14).

5. The line joining the points A(2,1), and B(3,2) is perpendicular to the line $(a^2)x + (a+2)y + 2 = 0$. Find the values of a.

Ans. $a = 2, -1$

Sol. The slope of the line joining A(2,1) and B(3,2) is

$$\frac{2-1}{3-2} = 1$$

The slope of the line $(a^2)x + (a+2)y + 2 = 0$ is

$$-\frac{a^2}{a+2}$$

The lines are perpendicular. Therefore,

$$\left(-\frac{a^2}{a+2}\right)(1) = -1$$

or $a^2 = a + 2$

or $a^2 - a - 2 = 0$

or $(a-2)(a+1) = 0$

or $a = 2, -1$

6. Find the angle between the line joining the points $(1,-2)$, $(3,2)$ and the line $x + 2y - 7 = 0$.

Ans. $\pi/2$

Sol. $x + 2y - 7 = 0$
Slope of line (i) $= m_1 = -\frac{1}{2}$
Slope of line PQ $= m_2 = \frac{2-(-2)}{3-1} = 2$
where $P \equiv (1,-2)$ and $Q \equiv (3,2)$.
Since $m_1 \cdot m_2 = -1$ the angle between line (1) and line PQ is $\pi/2$.

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7. The orthocenter of $\triangle$ ABC with vertices $B(1,-2)$ and $C(-2,0)$ is $H(3,-1)$. Find the vertex A.

Ans. $(3/7,-34/7)$

Sol. Let the coordinates of vertex A be (x, y).

$AH \perp BC$

or $\quad$ (Slope of AH) $\times$ (Slope of BC) $=-1$

or $\quad \frac{y+1}{x-3} \times \frac{-2-0}{1-(-2)}=-1$

or $\quad -2y-2=9-3x$

or $\quad 3x-2y=11$

Also, $BH \perp AC$

or (Slope of BH) $\times$ (Slope of AC) $=-1$

or $\quad \dfrac{-2-(-1)}{1-3} \times \dfrac{y-0}{x-(-2)}=-1$

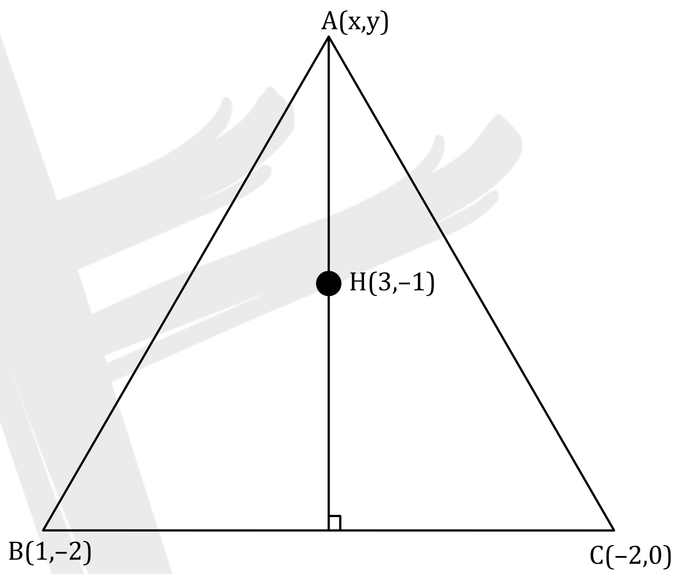
or $\quad -y=2x+4$

or $\quad 2x+y=-4$

Solving (1) and (2), we get

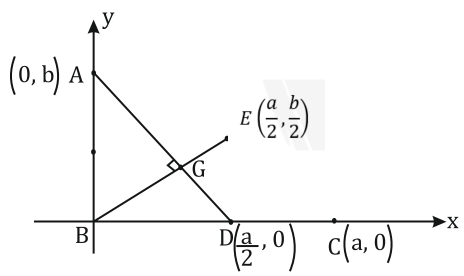
$x=\dfrac{3}{7}, y=-\dfrac{34}{7}$

Hence, the orthocenter is $(3/7,-34/7)$.



8. The medians AD and BE of the triangle with vertices $A(0, b), B(0,0)$ and $C(a, 0)$ are mutually perpendicular. Prove that $a^2=2b^2$.

Sol.



From the figure,

Slope of BE, $m_{BE}=\frac{b}{a}$

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Slope of AD, $m_{AD} = -\frac{2b}{a}$

Given that AD and BE are perpendicular.

$\therefore\ m_{BE} \times m_{AD} = -\frac{2b^2}{a^2} = -1$

$\Rightarrow\ a^2 = 2b^2$

LOCUS

9. Find the locus of a point whose distance from (a, 0) is equal to its distance from the y-axis.

Ans. $y^2 - 2ax + a^2 = 0$

Sol. Let the point be (h, k). Therefore,

$$\text{or}\quad \begin{aligned} (h-a)^2 + (k-0)^2 &= h^2 \\ h^2 + a^2 - 2ah + k^2 &= h^2 \end{aligned}$$

Hence, the locus is $y^2 - 2ax + a^2 = 0$.

10. The coordinates of the points A and B are (a, 0) and (−a, 0), respectively. If a point P moves so that $PA^2 - PB^2 = 2k^2$, when k is constant, then find the equation to the locus of the point P.

Ans. $2ax + k^2 = 0$

Sol. Let the point be (x, y). Then,

$(x-a)^2 + y^2 - (x+a)^2 - y^2 = 2k^2$

or $\quad -4ax - 2k^2 = 0$

or $\quad 2ax + k^2 = 0$

This is the required equation to the locus of point P.

11. Let A(2, −3) and B(−2,1) be the vertices of △ ABC. If the centroid of the triangle moves on the line 2x + 3y = 1. then find the locus of the vertex C.

Ans. $2x + 3y = 9$

Sol. Let C be (α, β).

The centroid is

$$\left(\frac{2-2+\alpha}{3}, \frac{-3+1+\beta}{3}\right), \text{ i.e., } \left(\frac{\alpha}{3}, \frac{\beta-2}{3}\right)$$

This lies on 2x + 3y = 1. Therefore, we get

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$$2\left(\frac{\alpha}{3}\right)+3\left(\frac{\beta-2}{3}\right)=1$$

or $2\alpha+3\beta=9$
Hence, the locus of (α,β) is $2x+3y=9$.

12. Q is a variable point whose locus is $2x-3y-4=0$; corresponding to a particular position of Q, P is the point of section of OQ, O being the origin, such that $OP:PQ=3:1$. Find the locus of P.

Ans. $2x-3y-3=0$

Sol. $(h,k)=\left(\frac{3\alpha}{4},\frac{3\beta}{4}\right)$

$\therefore h=\frac{3\alpha}{4}$ & $k=\frac{3\beta}{4}$
$\therefore \alpha=\frac{4h}{3}$ & $\beta=\frac{4k}{3}$

$2\alpha-3\beta-4=0$

$$2\left(\frac{4h}{3}\right)-3\left(\frac{1k}{3}\right)=4$$

$$\frac{2h}{3}-k=1$$

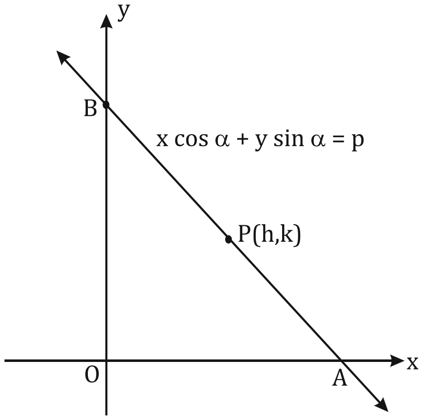
$2h-3k=3$

or $2x-3y-3=0$

13. Find the locus of the middle point of the portion of the line $x\cos\alpha+y\sin\alpha=p$ which is intercepted between the axes, given that p remains constant.

Ans. $\frac{1}{x^2}+\frac{1}{y^2}=\frac{4}{p^2}$

Sol.



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The equation of the variable line is

$$x\cos\alpha + y\sin\alpha = p$$

Here p is a constant and α is the parameter (variable).
Let the line in (1) cuts x - and y-axes at A and B, respectively.
Putting $y = 0$ in (1), we get $A \equiv (p\sec\alpha, 0)$.
Putting $x = 0$ in (1), we get $B \equiv (0, p\operatorname{cosec}\alpha)$.
AB is the portion of (1) intercepted between the axes.
Let $P(h, k)$ be the midpoint of AB. We have to find the locus of point $P(h, k)$, For this, we will have to eliminate α and find a relation in h and k. Therefore,

$$h = \frac{p\sec\alpha + 0}{2} = \frac{p}{2}\sec\alpha$$

and $k = \frac{0+p\operatorname{cosec}\alpha}{2} = \frac{p}{2}\operatorname{cosec}\alpha$
From (2), we get

$$\cos\alpha = \frac{p}{2h}$$

From (3), we get

$$\sin\alpha = \frac{p}{2k}$$

Squaring and adding (4) and (5), we get

$$\cos^2\alpha + \sin^2\alpha = \frac{p^2}{4h^2} + \frac{p^2}{4k^2}$$

or

$$\frac{1}{h^2} + \frac{1}{k^2} = \frac{4}{p^2}$$

Hence, the locus of point $P(h, k)$ is

$$\frac{1}{x^2} + \frac{1}{y^2} = \frac{4}{p^2}$$

14. Find the locus of the point of intersection of lines $x\cos\alpha + y\sin\alpha = a$ and $x\sin\alpha - y\cos\alpha = b$ (a is a variable).

Ans. $x^2 + y^2 = a^2 + b^2$

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Sol. Let (h, k) be the point of intersection of $x \cos\alpha + y \sin\alpha = a$ and $x \sin\alpha - y \cos\alpha = b$. Then,

$h\cos\ \alpha + k\sin\ \alpha = a$

$h\sin\ \alpha - k\cos\ \alpha = b$

Squaring and adding (1) and (2), we get

$(h\cos\ \alpha + k\sin\ \alpha)^2 + (h\sin\ \alpha - k\cos\ \alpha)^2 = a^2 + b^2$

or $h^2 + k^2 = a^2 + b^2$

Hence, the locus of (h, k) is

$x^2 + y^2 = a^2 + b^2$

15. A point moves such that the area of the triangle formed by it with the points $(1,5)$ and $(3, -7)$ is 21 sq. units. Then, find the locus of the point.

Ans. $6x + y = 32$ or $6x + y = -10$

Sol. Let (x, y) be the required point. Therefore,

$$\frac{1}{2}\begin{vmatrix} x & y \\ 1 & 5 \\ 3 & -7 \\ x & y \end{vmatrix} = \pm 21$$

or $\quad 5x - y - 7 - 15 + 3y + 7x = \pm 42$

i.e., $\quad 12x + 2y = 64$ or $12x + 2y = -20$

i.e., $\quad 6x + y = 32$ or $6x + y = -10$

16. A variable line through point $P(2,1)$ meets the axes at A and B. Find the locus of the circumcenter of triangle OAB (where O is the origin).

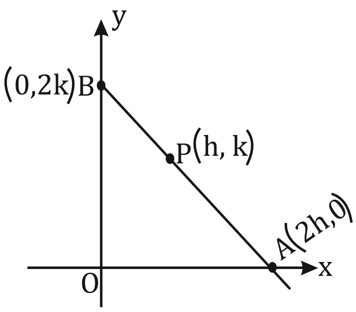
Ans. $x + 2y - 2xy = 0$

Sol. Since triangle OAB is right-angled, its circumcenter is the midpoint of hypotenuse AB.

So, let the midpoint of AB be $Q(h, k)$.

Then the coordinates of A and B are $(2h, 0)$ and $(0,2k)$, respectively. Now, points A, B, and P are collinear. Therefore,

$$\begin{vmatrix} 2h & 0 \\ 2 & 1 \\ 0 & 2k \\ 2h & 0 \end{vmatrix} = 0$$



or $2h + 4k - 4hk = 0$
(0. or $x + 2y - 2y = 0$

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17. A straight line is drawn through P(3,4) to meet the axis of x and y at A and B, respectively. If the rectangle OACB is completed, then find the locus of C.

Ans. $\frac{3}{x}+\frac{4}{y}=1$

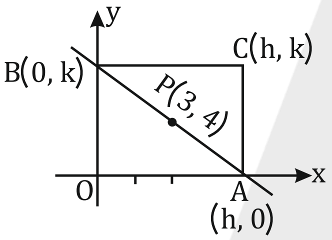
Sol. Let the point C be (h, k).
Then the coordinates of A and B are (h, 0) and (0, k), respectively.
Since points A. P, and B are collinear, we have

$$\begin{vmatrix} h & 0 \\ 3 & 4 \\ 0 & k \\ h & 0 \end{vmatrix}=0$$

or $4h+3k-hk=0$

or $4x+3y-xy=0$

Therefore, the required locus is $\frac{3}{x}+\frac{4}{y}=1$



ANSWER KEY

1. $x \in (5/2,3)$

2. -1

3. (15,32) or (20,7)

4. B(6,8), C(14,14), D(8,6)

5. $a=2,-1$

6. $\pi/2$

7. $(3/7,-34/7)$

9. $y^2-2ax+a^2=0$

10. $2ax+k^2=0$

11. $2x+3y=9$

12. $2x+3y+3=0$

13. $\frac{1}{x^2}+\frac{1}{y^2}=\frac{4}{p^2}$

14. $x^2+y^2=a^2+b^2$

15. $6x+y=32$ or $6x+y=-10$

16. $x+2y-2xy=0$

17. $\frac{3}{x}+\frac{4}{y}=1$