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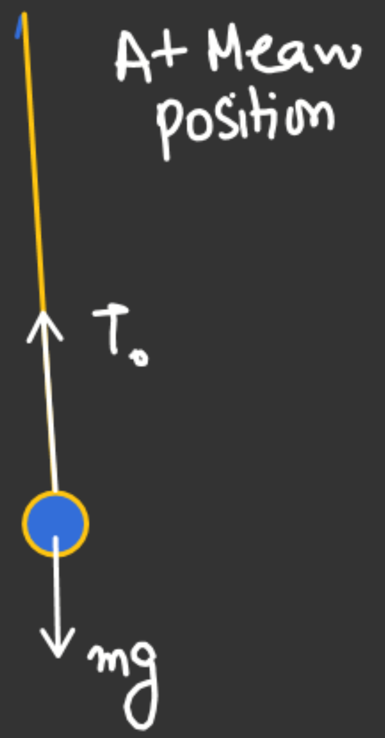
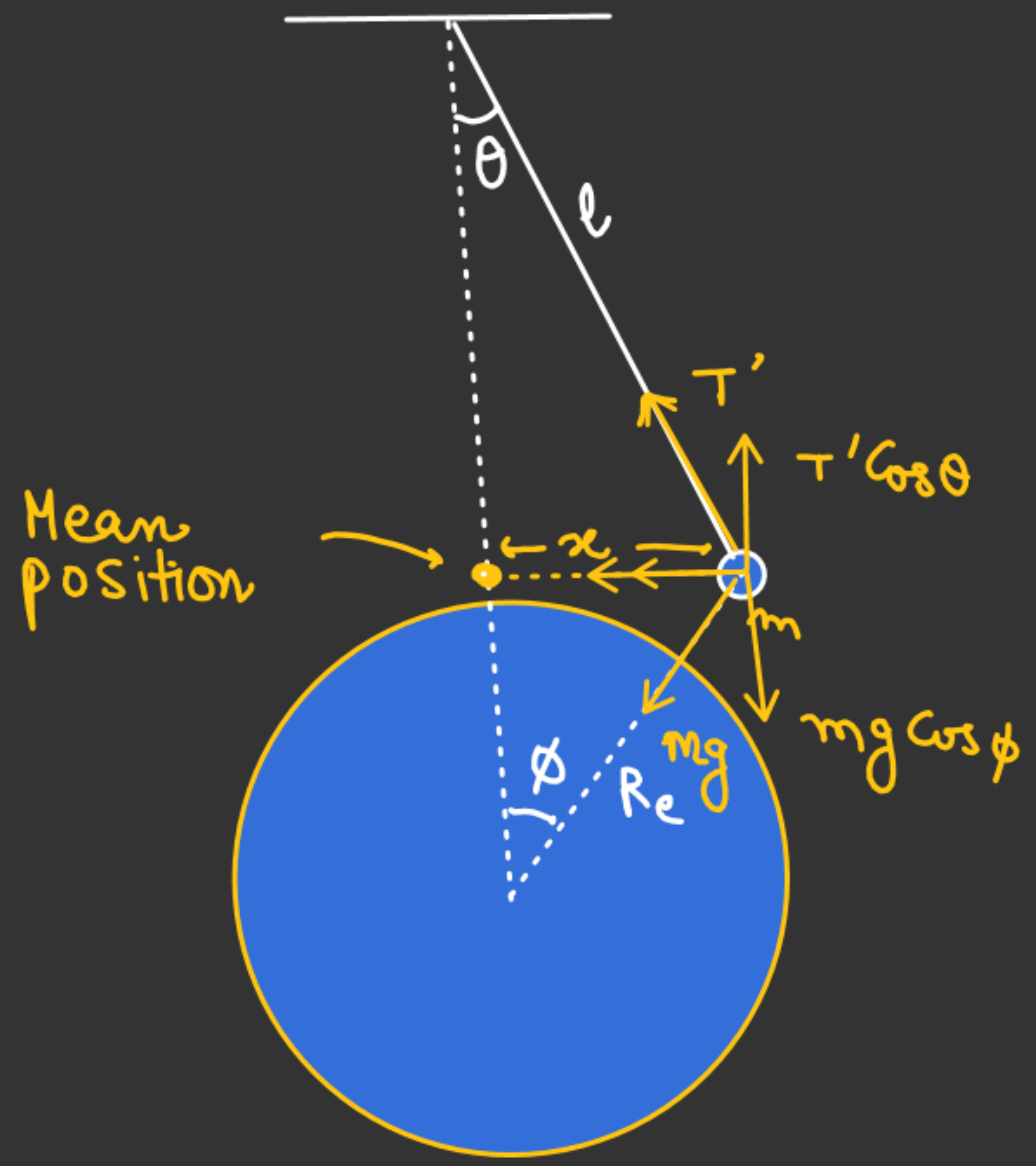
# Case of Simple pendulum when length of the String is Comparable w.r.t radius of earth.

$$F_r = -\left[T'\sin\theta + mg\sin\phi\right]$$

$\theta$ & $\phi$ are very small

$$F_r = -\left(T'\theta + mg\phi\right)$$




For vertical equilibrium

$$T'\cos\theta = mg\cos\phi$$

$$\theta \to 0 \quad \phi \to 0$$

$$T' = mg$$

$$\left[\theta = \frac{x}{l} \qquad \phi = \frac{x}{R_e}\right.$$

$$F_r = -mg(\theta+\phi) = -mg\left(\frac{1}{l}+\frac{1}{R_e}\right)x$$

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$$F_r = -mg(\theta+\phi) = -mg\left(\frac{1}{l}+\frac{1}{R_e}\right)x$$

$$a = -g\left(\frac{1}{l}+\frac{1}{R_e}\right)x$$

$$a = -\omega^2 x$$

$$\omega = \sqrt{g\left(\frac{1}{l}+\frac{1}{R_e}\right)} =$$

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{1}{g\left(\frac{1}{l}+\frac{1}{R_e}\right)}}$$

$$T = 2\pi\sqrt{\frac{l R_e}{g(l+R_e)}}$$

$$T = 2\pi\sqrt{\frac{\cancel{R_e}\cdot l}{g \cancel{R_e}(1+l/R_e)}}$$

$$\boxed{T = 2\pi\sqrt{\frac{l}{g(1+l/R_e)}}}$$

If $l << R_e$

$$T = 2\pi\sqrt{\frac{l}{g}} \checkmark$$

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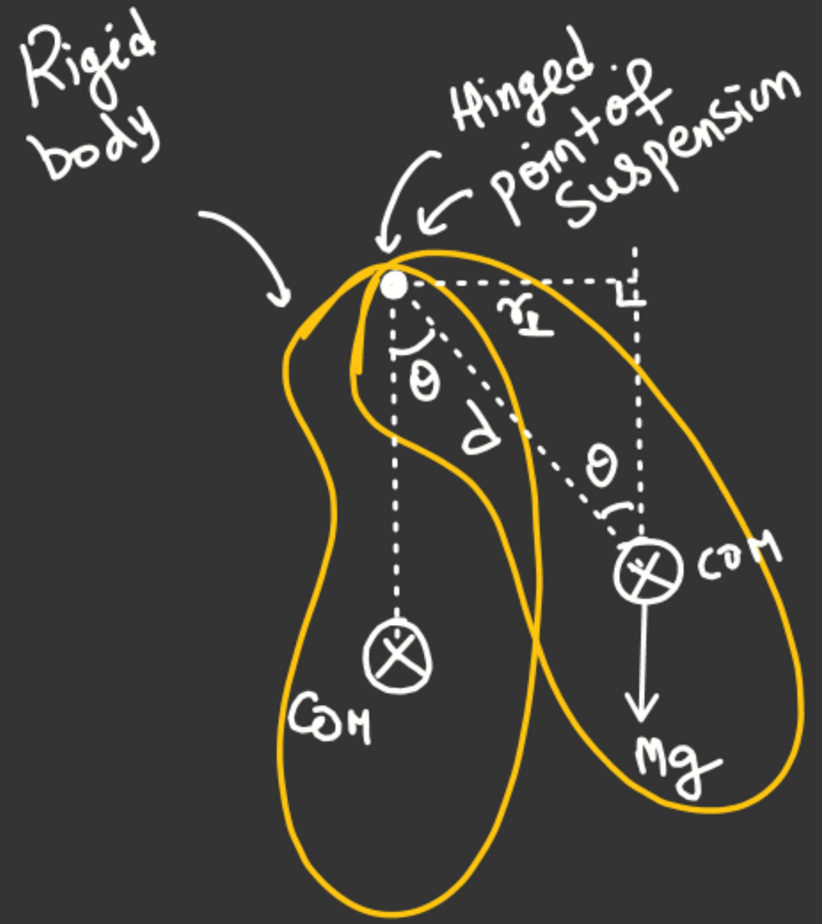
# physical pendulum

$\sin\theta \simeq \theta.$

1




$r_\perp = d\sin\theta.$

$\tau_r = -mg\,r_\perp$

$\tau_r = -mgd\sin\theta$

$\tau_r = -(mgd)\,\theta$

$\alpha = -\left(\dfrac{mgd}{I}\right)\theta$

$\alpha = -\omega^2\theta$

$I = I_{\text{body about}}$ axis passing through point of suspension

$\omega = \sqrt{\dfrac{mgd}{I}}$

$T = \dfrac{2\pi}{\omega} = 2\pi\sqrt{\dfrac{I}{mgd}}$

$$\boxed{T = 2\pi\sqrt{\dfrac{I}{mgd}}}$$

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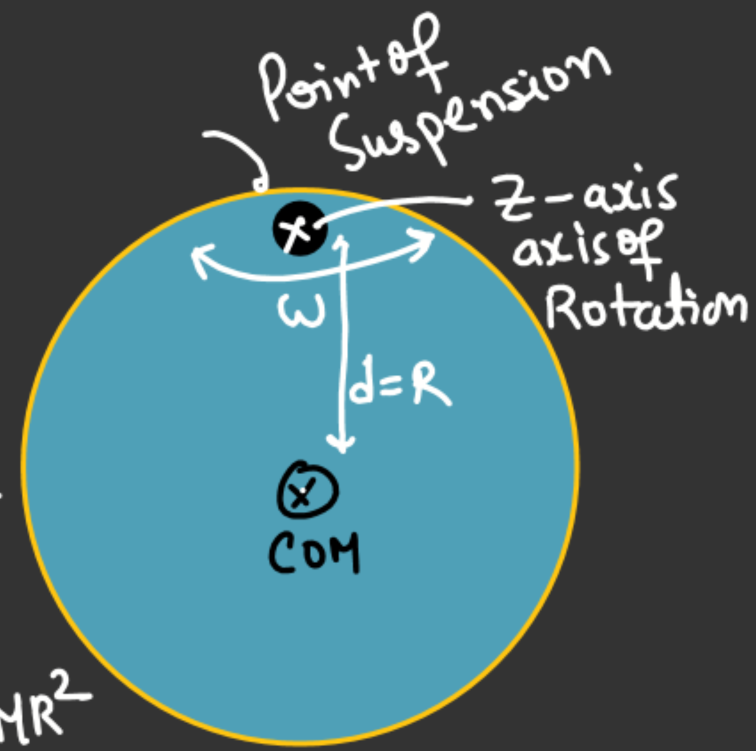
Find the ratio of time period of the uniform disc.

$T_1$ be the time period when disc oscillate in the plane of disc.

$T_2$ be the time period when disc oscillate perpendicular to the time period.

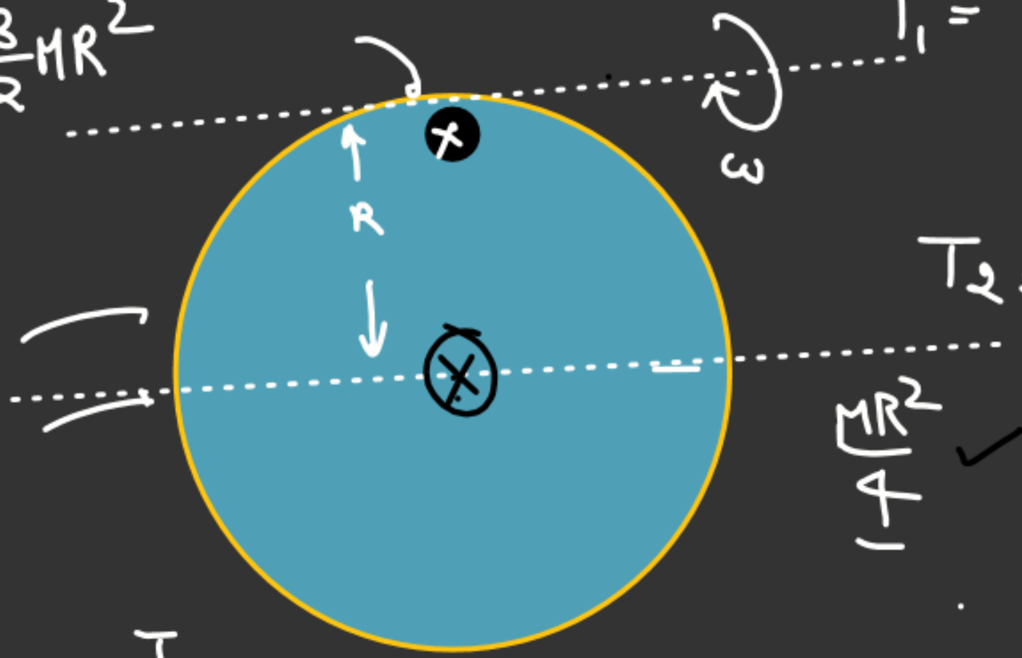
Uniform disc

M,R




$I_1 = \frac{MR^2}{2} + MR^2$

$I_1 = \frac{3}{2}MR^2$




$\frac{MR^2}{4}$ ✓

$I_2 = \frac{MR^2}{4} + MR^2 = \frac{5MR^2}{4}$

$\frac{T_1}{T_2} = ?$

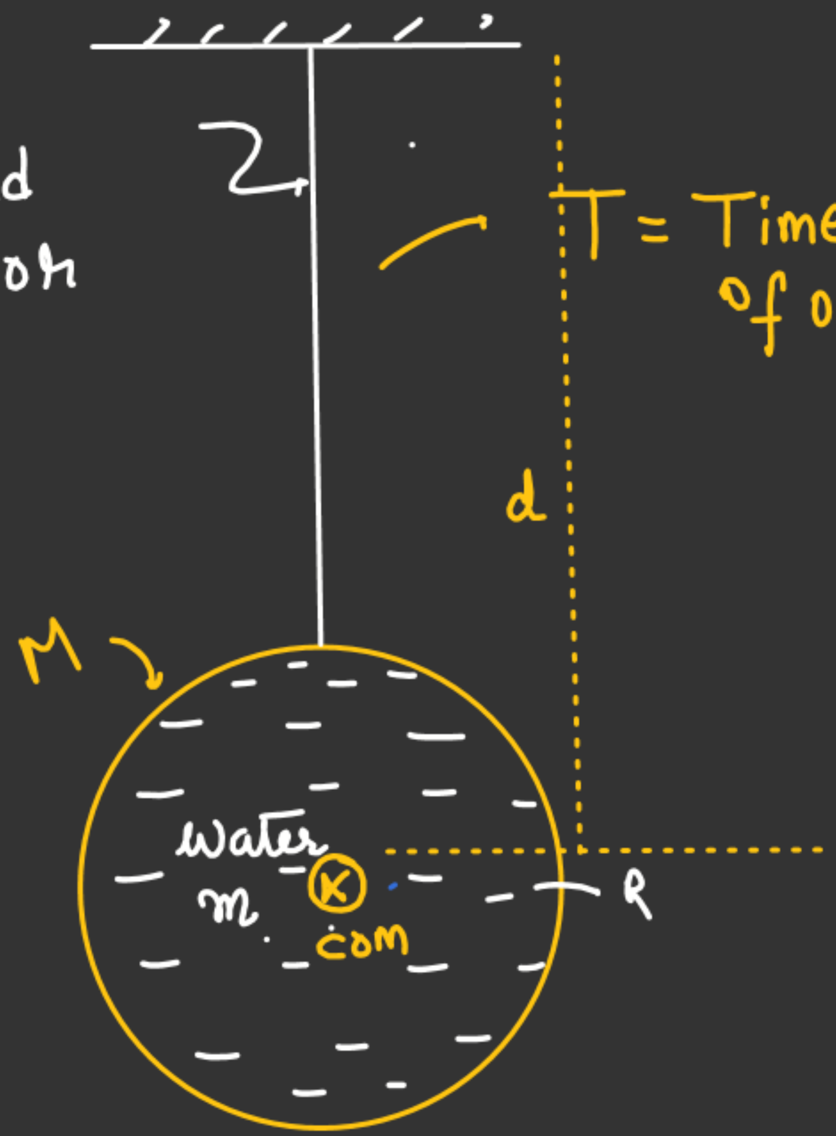
$T_1 = 2\pi\sqrt{\frac{I_1}{mgR}}$,

$T_2 = 2\pi\sqrt{\frac{I_2}{mgR}}$

$\frac{T_1}{T_2} = \sqrt{\frac{I_1}{I_2}}$

$\frac{T_1}{T_2} = \sqrt{\frac{3}{2} \times \frac{4}{5}} = \sqrt{\frac{6}{5}}$

Rigid Wire or Rod.




$T$ = Time period of oscillation.


Com of sphere

COM of water

$T' = ??$

increase, decrease or remain same ??

$T = 2\pi\sqrt{l/g}$. $l \rightarrow l_{eff}$

$l_{eff} \uparrow$ increase. when water leaks

after water completely drain out COM on the center of sphere i.e $d \rightarrow$ decreases.

[First increases then decreases]

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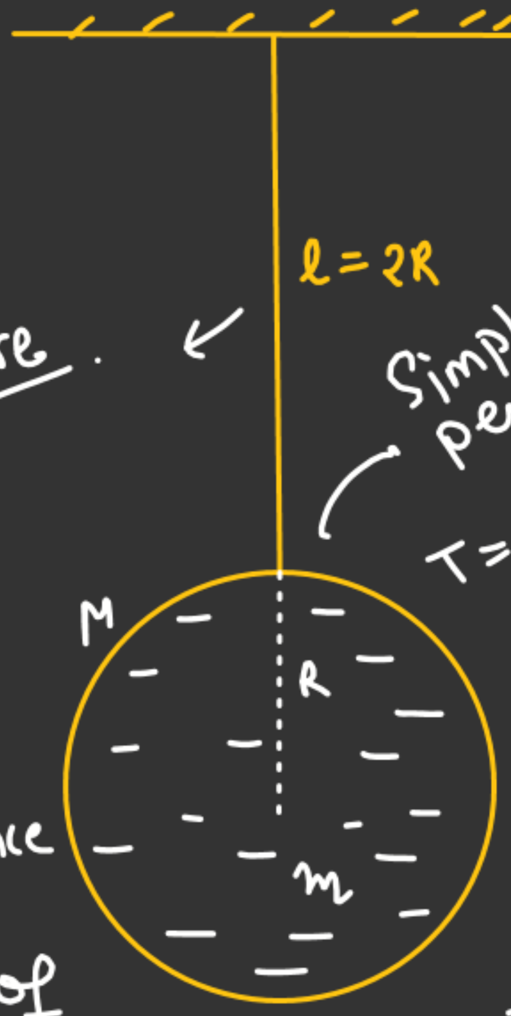


Wire

Simple pendulum

$$T = 2\pi\sqrt{\frac{l_{eff}}{g}}$$

$-T^{\circ}C$

$l_{eff}$ = distance of COM of Sphere and Water Sphere w.r.t point of suspension


Physical pendulum

$$T = 2\pi\sqrt{\frac{I}{mgd}}$$

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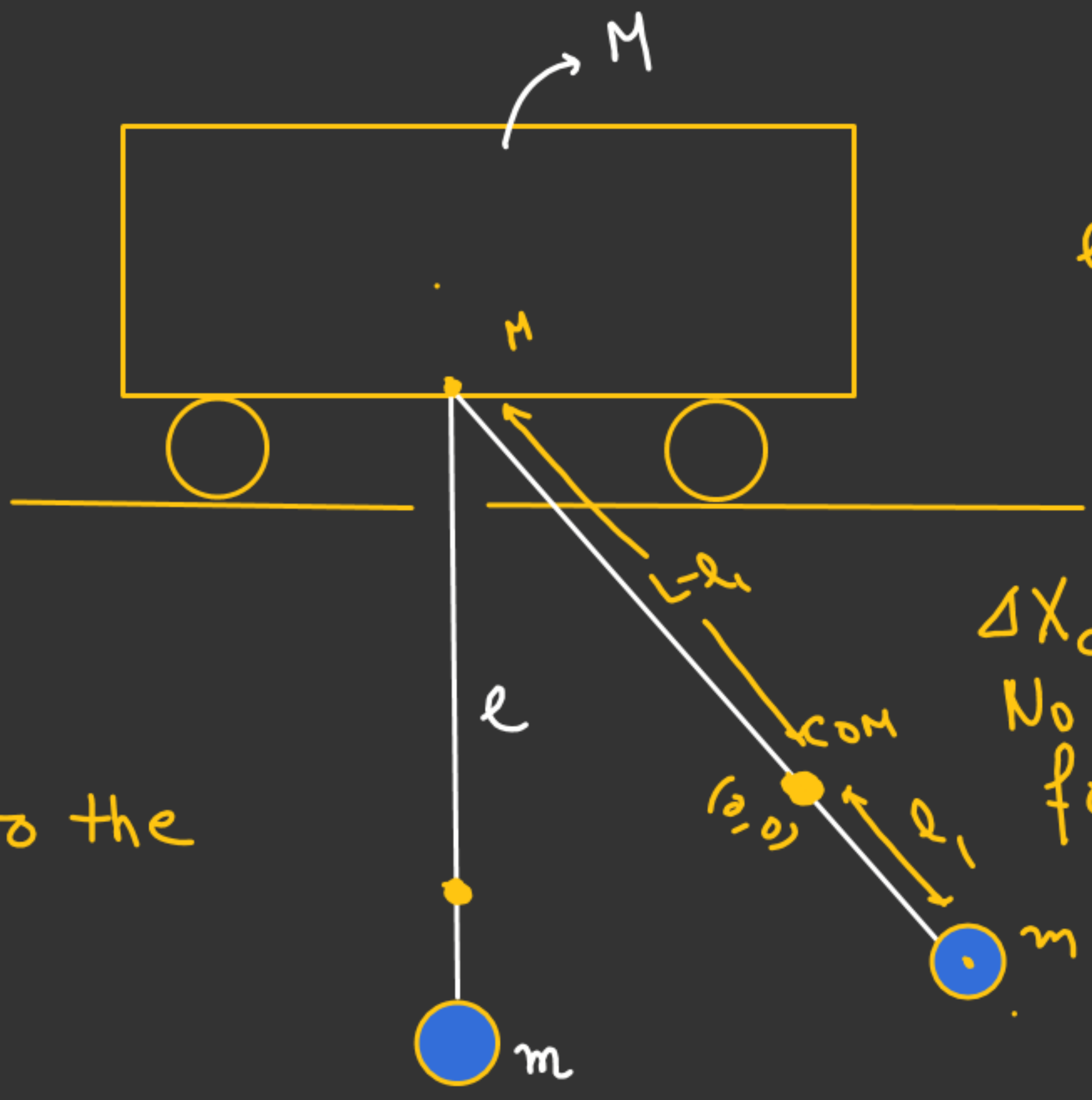
Trolley can move horizontally on the parallel track

Find time period of string-bob system if

1) Bob oscillate along the plane of trolley.

2) Bob oscillate perpendicular to the plane of trolley.

$\hookrightarrow \quad T = 2\pi\sqrt{L/g}$





$\Delta X_{com} = 0$.

No external force in x-direction

$T = 2\pi\sqrt{\dfrac{ML}{(M+m)g}}$

$0 = \dfrac{ml_1 - M(L-l_1)}{M+m}$

$ml_1 = M(L-l_1)$

$l_1 = \left(\dfrac{ML}{M+m}\right)$

$\Downarrow$

$l_{eff}$

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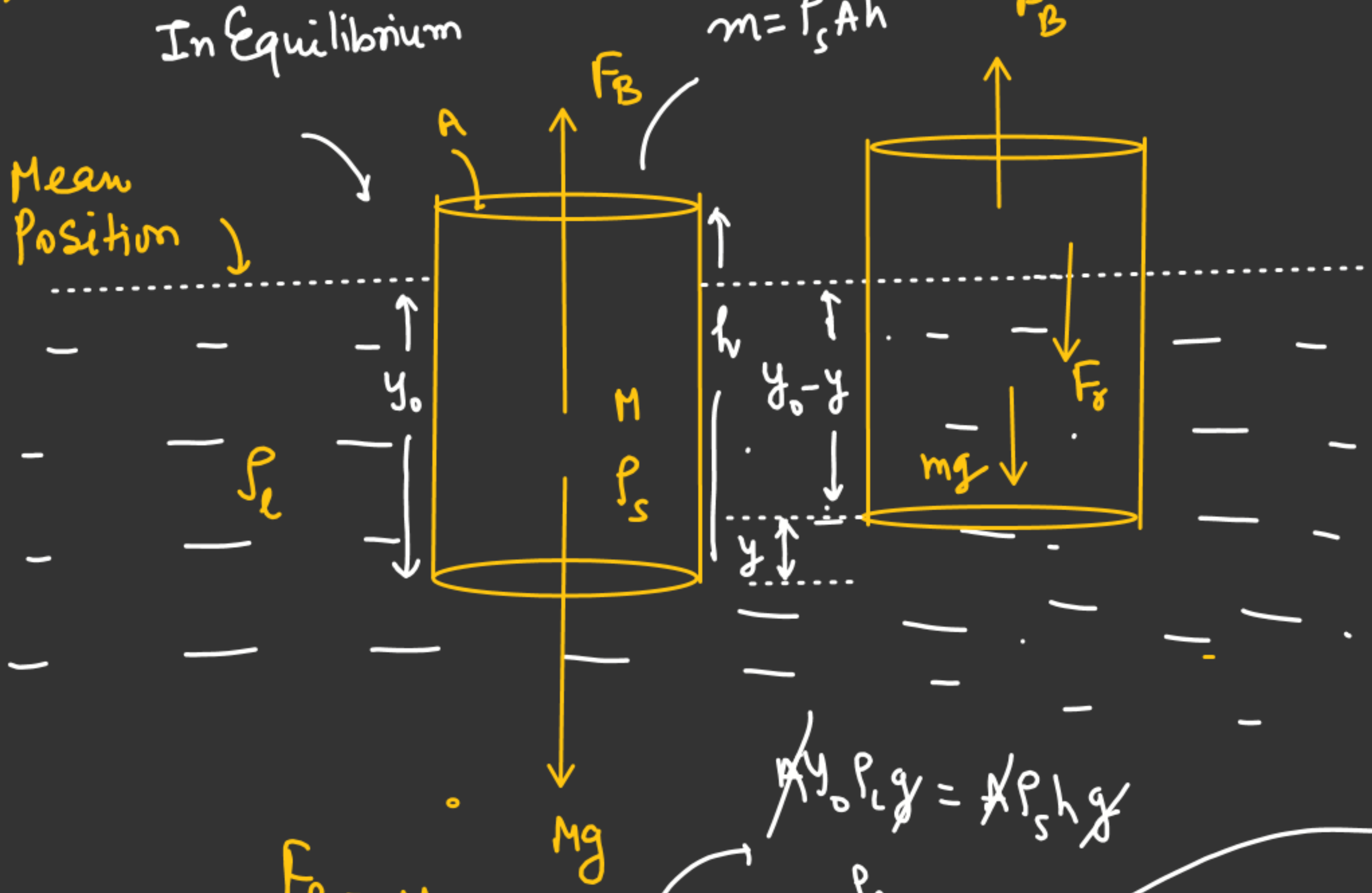
# S.H.M in Fluid.




$$F_B = Mg$$

$$A y_0 \rho_L g = Mg \quad \text{—(1)}$$

$$\cancel{A} y_0 \rho_L \cancel{g} = \cancel{A} \rho_s h \cancel{g}$$

$$\frac{\rho_\ell}{\rho_s} = \frac{h}{y_0}$$

$$F_r = -(mg - F_B')$$

$$F_r = -\left(mg - [\rho_L A (y_0 - y) g]\right)$$

$$F_r = -\left(\cancel{mg - \rho_L A g y_0} + \rho_\ell A y g\right)$$

$$F_r = -\rho_\ell A g y, \quad a = \frac{-F_r}{m}$$

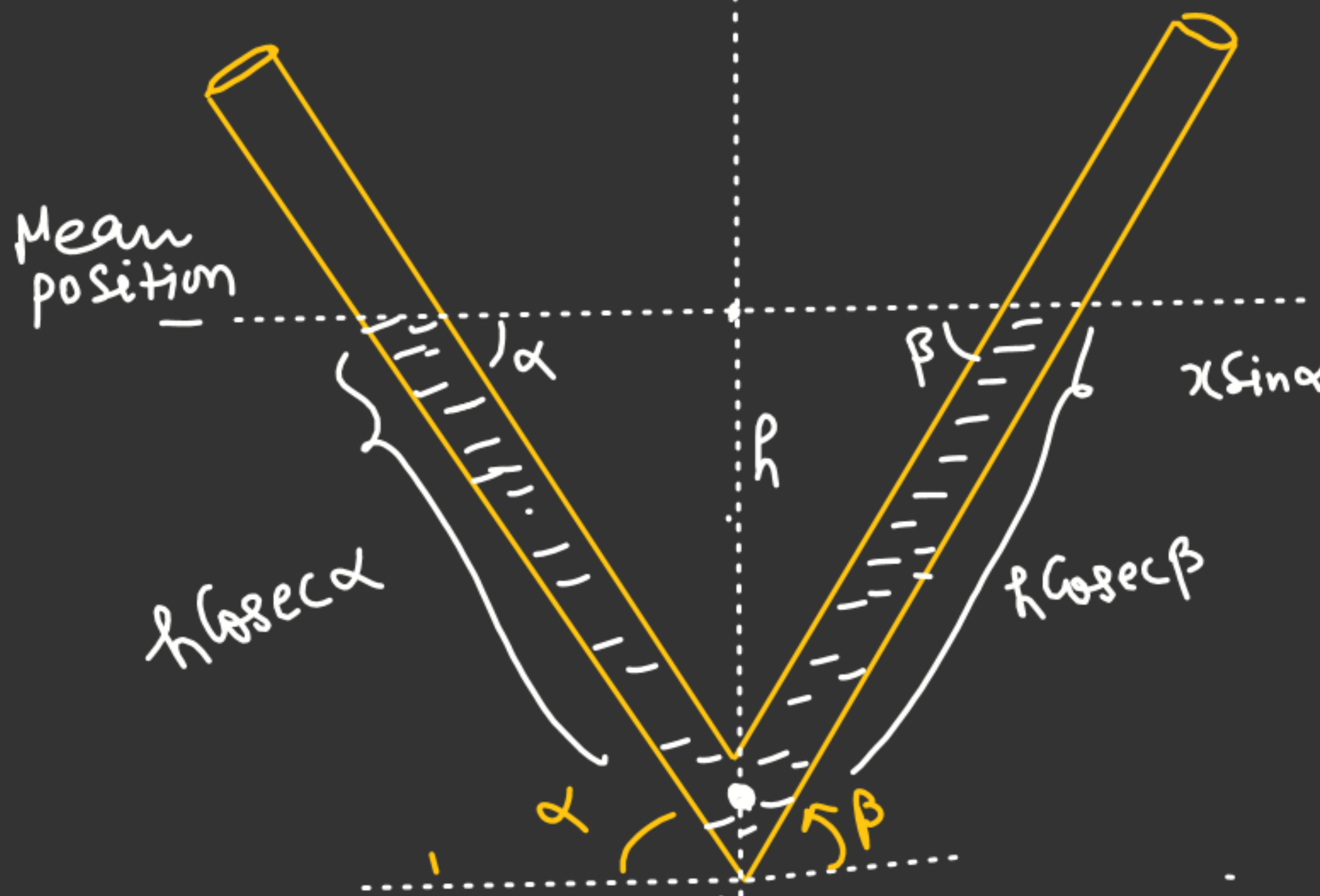
$$a = -\frac{\rho_L A g y}{(\rho_s A h)}$$

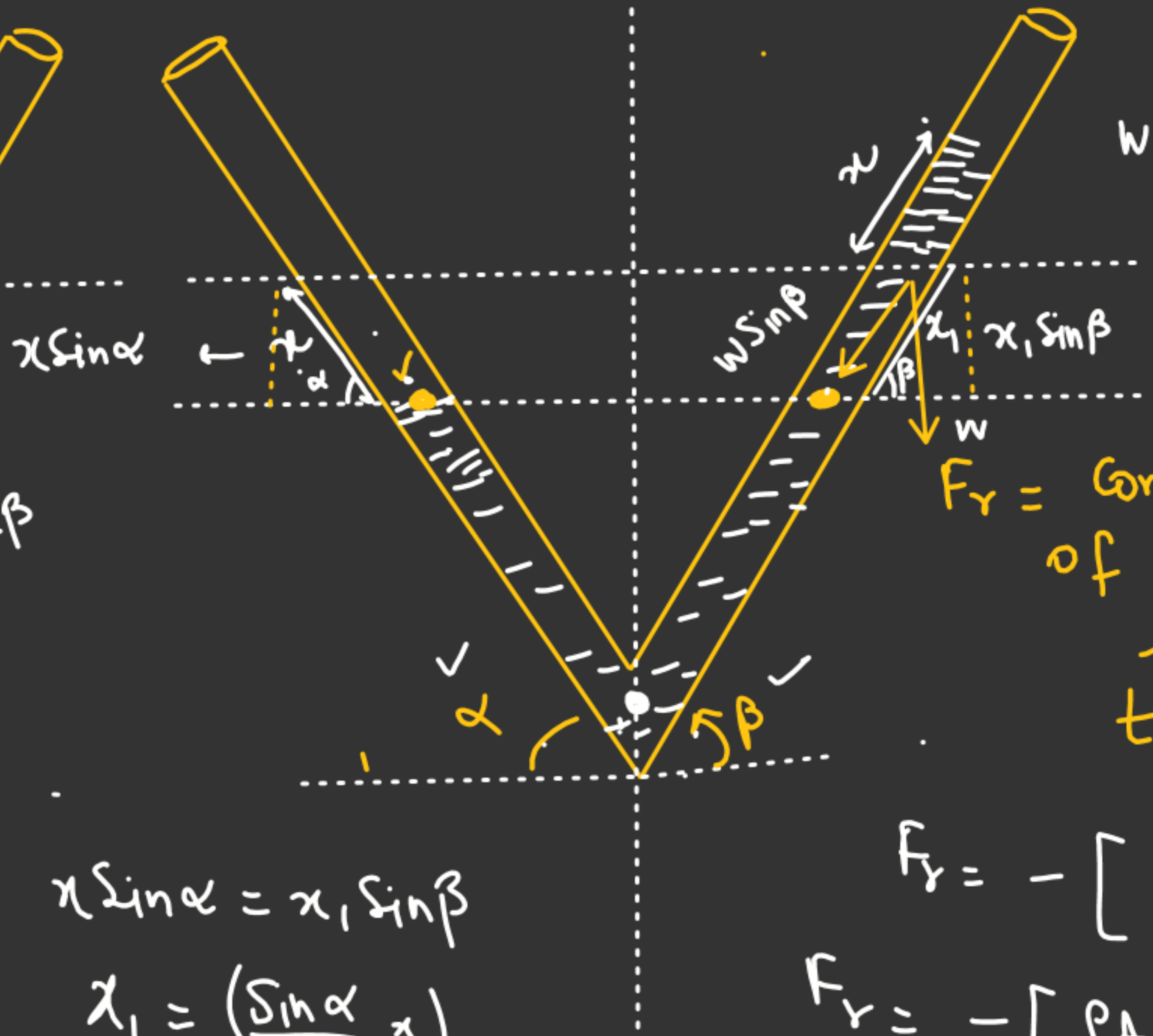
$$a = -\left(\frac{\rho_\ell}{\rho_s}\frac{g}{h}\right) y$$

$$a = -\left(\frac{g}{y_0}\right) y$$

$$a = -\omega^2 y$$

$$\boxed{T = 2\pi\sqrt{\frac{y_0}{g}}}$$




$W$ = weight of $(x+x_1)$ length of liquid.

$F_r$ = Component of weight of $(x+x_1)$ length of liquid along the tube.

$$x\sin\alpha = x_1\sin\beta$$

$$x_1 = \left(\frac{\sin\alpha}{\sin\beta}x\right)$$

$$F_r = -\left[\rho A(x+x_1)g\sin\beta\right]$$

$$F_r = -\left[\rho A g\sin\beta\left(x + x\frac{\sin\alpha}{\sin\beta}\right)\right]$$

$$F_r = -\underline{\rho A g(\sin\alpha+\sin\beta)}x$$

$m$ = mass of liquid

$$= \underline{\rho A h(\operatorname{cosec}\alpha + \operatorname{cosec}\beta)}$$

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$$F_r = -\rho A g(\sin\alpha + \sin\beta)x$$

$m$ = mass of liquid

$$= \rho A h(\operatorname{cosec}\alpha + \operatorname{cosec}\beta)$$

$$a = -\frac{F_r}{m}$$

$$a = -\frac{g(\sin\alpha \cdot \sin\beta)}{h}x$$

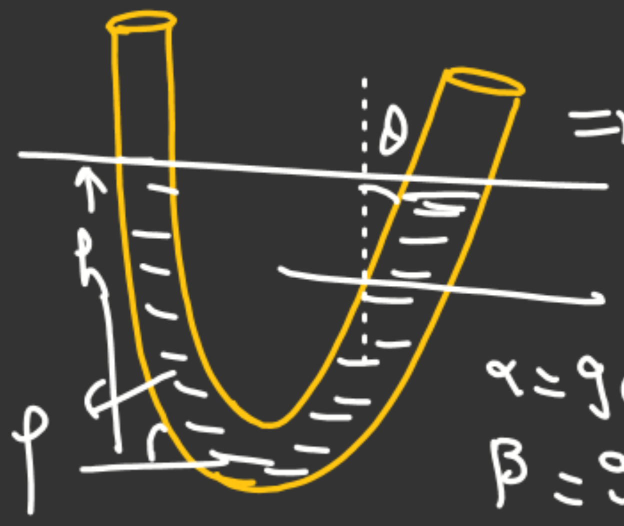
$$a = -\omega^2 x$$

$$\omega = \sqrt{\frac{g\sin\alpha \cdot \sin\beta}{h}}$$

$$\boxed{T = 2\pi\sqrt{\frac{h}{g\sin\alpha \cdot \sin\beta}}}$$

$\alpha$ & $\beta$ inclination of tube from horizontal.

$h$ = distance b/w mean position & Vertex of tube at equilibrium.

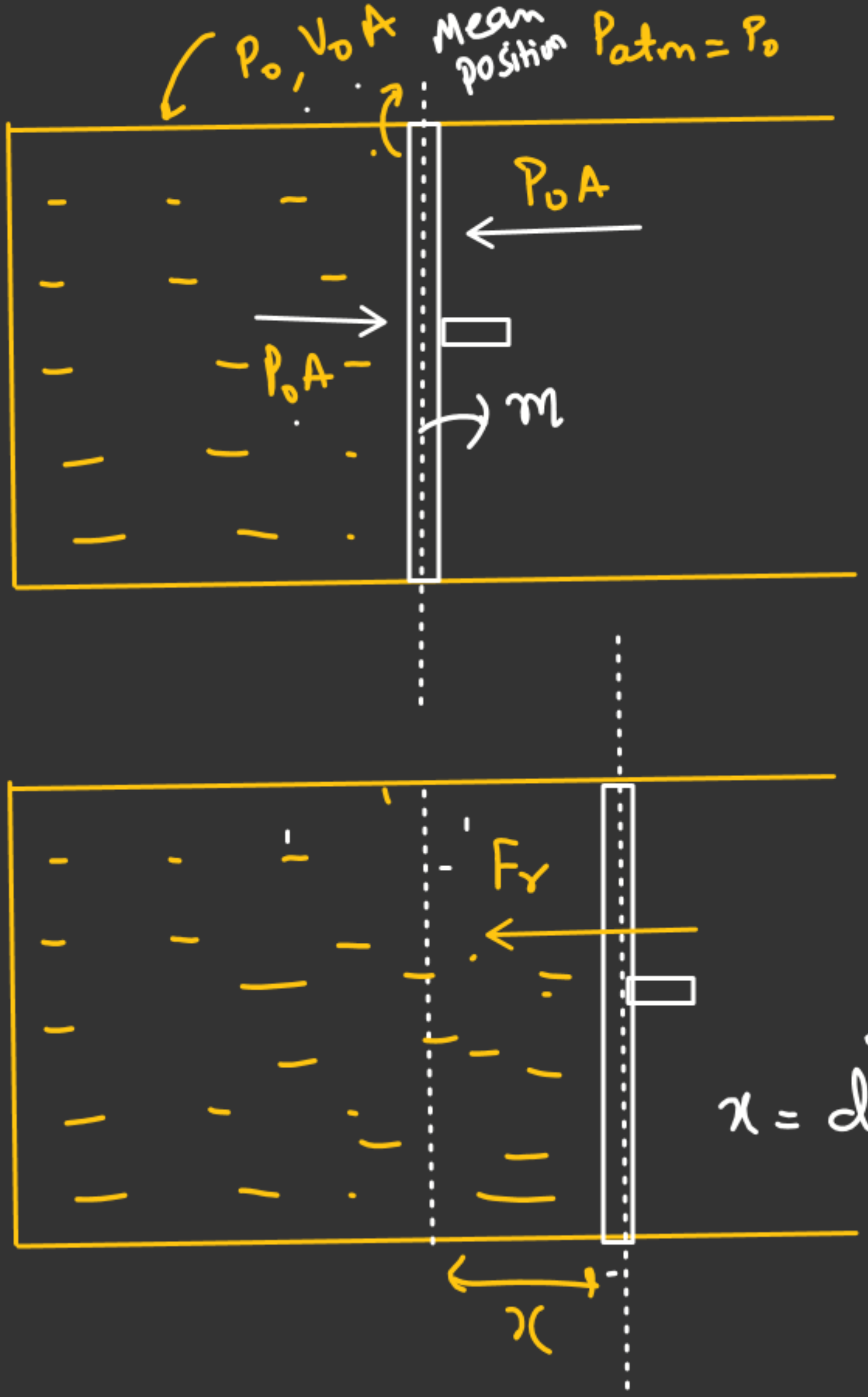



$\alpha = 90$

$\beta = 90 - \theta$

$$\Rightarrow T = 2\pi\sqrt{\frac{h}{g\sin 90 \cdot \sin(90-\theta)}}$$

$$T = 2\pi\sqrt{\frac{h}{g\cos\theta}}$$




Piston either compressed or expand adibatically form mean position. then. T = ??

$$PV^{\gamma} = C$$

$$\ln P + \gamma \ln V = \ln C$$

$$\frac{1}{P}\frac{dP}{dV} + \frac{\gamma}{V} = 0$$

$$\frac{dP}{dV} = -\frac{P\gamma}{V}$$

$$dV = Ax$$

$x$ = displacement from mean position

$P \to P_0$

$V \to V_0$

$$dP = -\frac{P_0\gamma}{V_0}\,dV$$

$$dV = Ax$$

$$dP = -\frac{P_0\gamma A}{V_0}x.$$

$$F_r = -(dP)A$$

$$F_r = \frac{-P_0\gamma A^2}{V_0}x$$

$$a = \frac{-P_0\gamma A^2}{mV_0}x$$

$$a = -\omega^2 x$$

$$T = 2\pi\sqrt{\frac{mV_0}{P_0\gamma A^2}}$$

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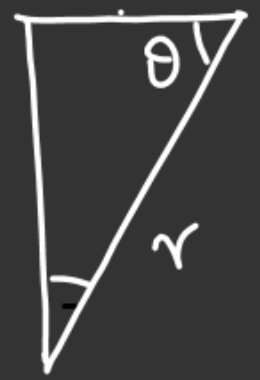
$$F_r = -mE\cos\theta.$$

$E$ = Gravitational field of earth at a radial distance $r$.

$$E_r = \left(\frac{GM}{R^3}r\right)$$

$x = r\cos\theta$

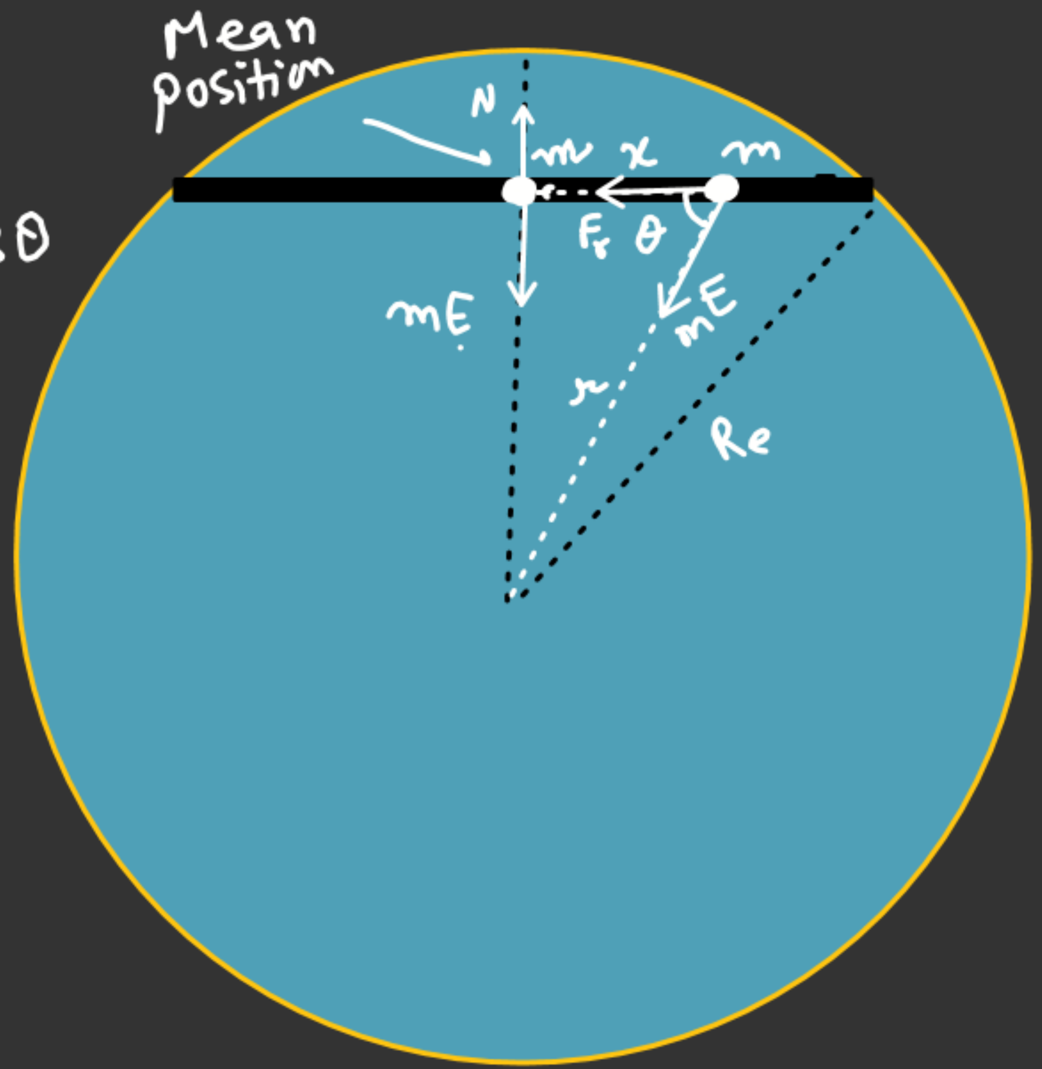



$$F_r = -m\frac{GM}{R^3}(r\cos\theta)$$

$$a = \frac{F_r}{m} = -\left(\frac{GM}{R^3}x\right) \qquad g = \frac{GM}{R^2}$$

$$\boxed{T = 2\pi\sqrt{\frac{R^3}{GM}}}$$

$$T = 2\pi\sqrt{\frac{R}{(GM/R^2)}} \Rightarrow \boxed{T = 2\pi\sqrt{R/g}}$$



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$$T = 2\pi\sqrt{\frac{R}{g}}$$

$$T_1 = T_2 = T_3$$

$$= 2\pi\sqrt{R/g}$$

R = Radius of earth.