DDP–5

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1. $\log_{49} 28 = \dfrac{\log_7 28}{\log_7 49} = \dfrac{2\log_7 2 + \log_7 7}{2\log_7 7} = \dfrac{2m+1}{2}$]

2. ${}_x(\log_2 7)^2 +_y (\log_3 5)^2 +_z (\log_5 216)^2 = \left(x^{(\log_2 7)}\right)^{(\log_2 7)} + \left(y^{(\log_3 5)}\right)^{(\log_3 5)} + \left(z^{(\log_5 216)}\right)^{(\log_5 216)}$

$= (8)\log_2 7 + (81)\log_3 5 + (3\sqrt{5})^{\log_5 216} = 2\log_3 7^3 + 3\log_3 5^4 + 5^{\log_5 (216)^{\frac{1}{3}}}$

$$= 7^3 + 5^4 + (216)^{1/3} = 974.$$

3. $a_n = \dfrac{1}{\log_n 2002} = \log_{2002}{}^{n}$

$\therefore \quad b = \log_{2002}(2 \cdot 3 \cdot 4 \cdot 5)$

and $\quad c = \log_{2002}(10 \cdot 11 \cdot 12 \cdot 13 \cdot 14)$

$\therefore \quad b - c = \log_{2002}\left(\dfrac{2\cdot3\cdot4\cdot5}{10\cdot11\cdot12\cdot13\cdot14}\right) = \log_{2002}\left(\dfrac{1}{2002}\right) = -1$ Ans.]

4. Take log both sides

(Here $a^{\sqrt{\log_a b}} = b^{\sqrt{\log_b a}}$) $\log_{10} x \cdot \log_{10} x = 2 + \log_{10} x$

$\Rightarrow \quad t^2 - t - 2 = 0$ Here $t = \log_{10} x$

$\therefore \quad t = 2, t = -1$

$\therefore \quad x = 10^2, x = 10^{-1}$

$\therefore \quad$ product of solution is $10^2 \times \dfrac{1}{10} = 10.$ Ans.]

5. As, $L = \sum_{r=7}^{2400} \log_7\left(\dfrac{r+1}{r}\right) = \log_7\left(\dfrac{8}{7}\right) + \log_7\left(\dfrac{9}{8}\right) + \log_7\left(\dfrac{10}{9}\right) + \cdots \ldots + \log_7\left(\dfrac{2401}{2400}\right)$

$= \log_7\left(\dfrac{8}{7} \times \dfrac{9}{8} \times \dfrac{10}{9} \times \ldots \ldots . . \times \dfrac{2401}{2400}\right) = \log_7\left(\dfrac{2401}{7}\right) = \log_7(343) = 3$

$M = \prod_{r=2}^{1023} \log_r(r+1) = (\log_2 3 \times \log_3 4 \times \log_4 5 \times \ldots \ldots . . \times \log_{1023} 1024)$

$= \log_2(1024) = \log_2 2^{10} = 10$

Also, $N = \sum_{r=2}^{2011}\left(\dfrac{1}{\log_r p}\right)$

$= \sum_{r=2}^{2011} (\log_p r) = \log_p 2 + \log_p 3 + \log_p 4 + \cdots \ldots \ldots + \log_p 2011 = \log_p(2 \cdot 3 \cdot 4.$

$= \log_p r = 1$

Hence, $(L + M + N) = 3 + 10 + 1 = 14.$]

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6. We have, $(\log_3 4)(\log_4 5)(\log_5 6)\ (\log_{25} 26)(\log_{26} 27) = 3^x$

$\Rightarrow 3 = 3^x \Rightarrow x = 1$

7. $5^{-P} = 5^{-\log_5 (\log_5 3)} \Rightarrow 5^{-P} = \log_3 5$

$\Rightarrow 3^{C+\log_3 5} = 405$
$\Rightarrow 3^C \cdot 5 = 405$
$\Rightarrow 3^C = 81 \Rightarrow C = 4$

8. Let $E = z(\log_{xyz} x)(1 + \log_x yz) = \frac{z}{(1+\log_x y+\log_x z)}(1 + \log_x y + \log_x z) = z$

9. $\frac{a+\log_4 3}{a+\log_2 3} = \frac{a+\log_8 3}{a+\log_4 3} = \frac{\log_4 3-\log_8 3}{\log_2 3-\log_4 3} = \frac{1}{3} = b]$

10. (A) In L.H.S. $\log \tan 45^0 = \log 1 = 0 \Rightarrow$ L.H.S. $= 0$
(B) In L.H.S. $\log \sin 90^\circ = \log 1 = 0 \Rightarrow$ L.H.S. $= 0$
(C) $7^{\log_3 5} + 3^{\log_5 7} - 7^{\log_3 5} - 3^{\log_5 7} = 0$
(D) $\log \tan 1^0 \cdot \tan 2^0 \cdot \tan 3^0 \ldots\ldots \cdot \tan 89^\circ = \log \cot 89^0 \cdot \cot 88^0 \ldots . \tan 45^\circ \ldots \tan 89^0 \Rightarrow$
$\log 1 = 0$

11. (A) $\log_x(x^{x^2}) + \log_x(x^{-5x}) = \log_x\left(\frac{1}{x^6}\right) \quad \Rightarrow (x^{x^2})(x^{-5x}) = \left(\frac{1}{x^6}\right)$

$\Rightarrow x^{x^2-5x} = x^{-6} \quad \Rightarrow x^2 - 5x + 6 = 0 \quad \Rightarrow (x-2)(x-3) = 0 \quad \Rightarrow x = 2,3$

hence sum = 2 + 3 = 5

(B) $3^{\tan x} = 27^{\sin x} \quad \Rightarrow \tan x = 3\sin x \Rightarrow \sin x = 3\sin x\cos x \Rightarrow \sin x(1 - 3\cos x) = 0$

$\sin x \neq 0, \cos x = \frac{1}{3} \Rightarrow \sec x = 3$

(C) $\frac{\log_a (x-3)\log_b (x+10)}{\log_b (x-3)} = 2 \Rightarrow \frac{\log (x-3)\log (x+10)\log (x-4)}{\log (x-2)\log (x-4)\log (x-3)} = \log_{(x-2)}(x+10) = 2$

$\Rightarrow \quad x^2 - 4x + 4 = x + 10 \Rightarrow x^2 - 5x - 6 = 0$
$\Rightarrow \quad (x-6)(x+1) = 0;\ x \neq -1,$
$\therefore \quad x = 6$

(D) $\sqrt{2x} = \sqrt{x+7} - 1 \Rightarrow 2x = x + 7 - 2\sqrt{x+7} + 1 \Rightarrow x - 8 = -2\sqrt{x+7}$

$\Rightarrow x^2 - 16x + 64 = 4x + 28 \Rightarrow x^2 - 20x + 36 = 0 \Rightarrow (x-18)(x-2) = 0$

$\Rightarrow x = 2,18$. Only $x = 2$ satisfying.