Nishant Jindal

$\sqrt{\ }$ type Qs.

Q $\int \frac{1}{x}\sqrt{\frac{x-1}{x+1}}\cdot dx \times \frac{x-1}{x-1}$

$\int \frac{(x-1)\,dx}{x\sqrt{x^2-1}}$ Split.

$\int \frac{dx}{\sqrt{x^2-a^2}}$

$\int \frac{x\,dx}{x\sqrt{x^2-1}} - \int \frac{1\,dx}{x\sqrt{x^2-1}}$

$\ln|x+\sqrt{x^2-1}| - \sec^{-1}x + C$

Q $\int\sqrt{\frac{a-x}{a+x}}\cdot dx$

Rat.

$\int\sqrt{\frac{a-x}{a+x}\times\frac{a-x}{a-x}}$

$\int\frac{(a-x)\,dx}{\sqrt{a^2-x^2}}$

$\int\frac{a\cdot dx}{\sqrt{a^2-x^2}} - \int\frac{x\cdot dx}{\sqrt{a^2-x^2}}$

$a\left(\sin^{-1}\frac{x}{a}\right) + \sqrt{a^2-x^2} + C$

Q $\int\frac{(x+1)\,dx}{\sqrt{x^2+1}}$

$\int\frac{x\,dx}{\sqrt{x^2+1^2}} + \int\frac{1\,dx}{\sqrt{x^2+1^2}}$

$\sqrt{x^2+1} + \ln|x+\sqrt{x^2+1}|$

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Q $\int (x+1)\sqrt{x^2+1}\,dx$

$x^2+1=t$ $\quad \int x\sqrt{x^2+1}\cdot dx + \int \sqrt{x^2+1^2}\,dx \rightarrow \int \sqrt{x^2+a^2}\cdot dx$

$x\,dx=\frac{dt}{2}$ $\quad \frac{1}{2}\int \sqrt{t}\,dt$

$\frac{1}{2}\times\frac{2}{3}(t)^{3/2} + \quad \text{''} \quad + \frac{x}{2}\left(\sqrt{x^2+1}\right) + \frac{1^2}{2}\left(\ln\left|x+\sqrt{x^2+1}\right|\right)+c$

Q $\int x^2\sqrt{a^2-x^6}\cdot dx$

$\int x^2\sqrt{(a)^2-(x^3)^2}\,dx \quad x^3=t$

$x^2dx=\frac{dt}{3}$

$\frac{1}{3}\int \sqrt{(a)^2-(t)^2}\,dt$

$\frac{1}{3}\left[\frac{t}{2}\sqrt{a^2-t^2}+\frac{a^2}{2}\sin^{-1}\frac{t}{a}\right]+c$

Q $\int \sqrt{\frac{a-x}{x}}\cdot dx \rightarrow$ M1 $\quad x = a\sin^2\theta$

M2 Dr ko Bethe fxn ko $\sqrt{\ }$ se free kar do.

$x=t^2$

$dx=2t\,dt$

$\int \sqrt{\frac{a-t^2}{\cancel{t^2}}}\times 2\cancel{t}\,dt$

$2\int \sqrt{(\sqrt{a})^2-(t)^2}\,dt \rightarrow \int\sqrt{a^2-x^2}\cdot dx$

$2\left[\frac{t}{2}\sqrt{a-t^2}+\frac{a}{2}\sin^{-1}\frac{t}{\sqrt{a}}\right]+c$

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Substitution

1) $1 - \sin^2\theta = \cos^2\theta$

2) $\sec^2\theta - 1 = \tan^2\theta$

3) $1 + \tan^2\theta = \sec^2\theta$

4) $\dfrac{1-\cos 2\theta}{1+\cos 2\theta} = \tan^2\theta$

Q $\displaystyle\int \sqrt{\frac{x}{x+a}}\, dx$

Dr $\sqrt{\ }$ Se free.

$x + a = t^2$

$dx = 2t\,dt$

$\displaystyle\int \sqrt{\frac{t^2 - a}{\cancel{t^2}}} \cdot \times 2t\,dt$

for:

$\displaystyle 2\int \sqrt{(t)^2 - (\sqrt{a})^2}\, dt$

$\displaystyle 2\left[\frac{t}{2}\sqrt{} - \frac{a}{2}\ln\left|\right|\right] + c$

Q $\displaystyle\int \frac{dx}{x\sqrt{1 - x^{2016}}}$

$1 - x^{2016} = t^2$

$-2016\, x^{2015}\, dx = 2t\,dt$

$dx = \dfrac{2t\,dt}{-2016\, x^{2015}}$

$\displaystyle -\int \frac{2\cancel{t}\,dt}{2016\, x^{2015} \cdot x\sqrt{\cancel{t^2}}}$

$\displaystyle -\int \frac{2\,dt}{2016 \cdot x^{2016}}$

$\displaystyle -\int \frac{2\,dt}{2016 \cdot (1 - t^2)}$

$\displaystyle \frac{2}{2016}\int \frac{dt}{t^2 - 1^2}$

$\displaystyle \frac{2}{2016} \times \frac{1}{2\times 1} \ln\left|\right| + c$

Q $\displaystyle\int \frac{(x^2+1)\,dx}{\sqrt[3]{x^3 + 3x + 6}}$

$x^3 + 3x + 6 = t^3$

$(3x^2 + 3)\,dx = 3t^2\,dt$

$(x^2 + 1)\,dx = t^2\,dt$

$\displaystyle\int \frac{t^2\,dt}{(t^3)^{1/3}}$

$\displaystyle = \int t\,dt = \frac{t^2}{2} + c$

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Q $\displaystyle\int \frac{\sqrt{1-x^2}-x}{\sqrt{1-x^2}\,(1+x\sqrt{1-x^2})}\,dx$

$x = 1\cdot\sin\theta$

$x = \sin\theta \Rightarrow dx = \cos\theta\cdot d\theta$

$$\int \frac{(\cos\theta-\sin\theta)\cdot\cancel{\cos\theta}\cdot d\theta}{\cancel{\cos\theta}\,(1+\sin\theta\cdot\cos\theta)}$$

$$\Rightarrow 2\int \frac{\cos\theta-\sin\theta}{2+\sin 2\theta}\cdot d\theta$$

$$\Rightarrow 2\int \frac{(\cos\theta-\sin\theta)\,d\theta}{2+(1+\sin 2\theta)-1}$$

$$= 2\int \frac{(\cos\theta-\sin\theta)\,d\theta}{1+(\sin\theta+\cos\theta)^2}$$

$\sin\theta+\cos\theta = z$

$(\cos\theta-\sin\theta)\,d\theta = dz$

$$\Rightarrow 2\int \frac{dz}{1+z^2} = 2\tan^{-1}z + C$$

Q Prove that $\displaystyle\int \frac{dx}{\sqrt{x^2+a^2}} = \ln|x+\sqrt{x^2+a^2}| + C$

$x = a\tan\theta$

$\tan\theta = \frac{x}{a}$

$dx = a\sec^2\theta\cdot d\theta$

$$\int \frac{a\sec^2\theta\cdot d\theta}{\sqrt{a^2+a^2\tan^2\theta}}$$

$$\int \frac{\sec^{\cancel{2}}\theta\,d\theta}{\cancel{\sec\theta}}$$

$$\Rightarrow \ln|\sec\theta+\tan\theta| + C$$

$$\Rightarrow \ln|\tan\theta+\sqrt{1+\tan^2\theta}| + C$$

$$\Rightarrow \ln\left|\frac{x}{a}+\sqrt{1+\frac{x^2}{a^2}}\right| + C$$

$$\Rightarrow \ln|x+\sqrt{a^2+x^2}| + C$$

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Q $\int \frac{dx}{\sqrt{a^2-x^2}}$

$x = a\sin\theta \Rightarrow \sin\theta = \frac{x}{a}$

$dx = a\cos\theta \cdot d\theta$

$\theta = \sin^{-1}\frac{x}{a}$

$\int \frac{a\cos\theta \cdot d\theta}{\sqrt{a^2 - a^2\sin^2\theta}}$

$\int \frac{\cancel{\cos\theta} \cdot d\theta}{\cancel{\cos\theta}} = \int d\theta = \theta + c$

$= \sin^{-1}\frac{x}{a} + c$

Q $\int \frac{x\,dx}{2-x^2+\sqrt{2-x^2}}$

$x = \sqrt{2}\sin\theta$

$dx = \sqrt{2}\cos\theta \cdot d\theta$

$\int \frac{\sqrt{2}\sin\theta \cdot \sqrt{2}\cos\theta \cdot d\theta}{(2-2\sin^2\theta) + \sqrt{2-2\sin^2\theta}}$

$\int \frac{2\sin\theta \cdot \cos\theta \cdot d\theta}{2\cos^2\theta + \sqrt{2}\cos\theta}$

$\int \frac{\sqrt{2}\sin\theta \cdot d\theta}{(\sqrt{2}\cos\theta + 1)}$

$\sqrt{2}\cos\theta + 1 = t$

$\sqrt{2}\sin\theta \cdot d\theta = -dt$

$\Rightarrow -\int \frac{dt}{t} = -\ln|t| + c$

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Q $\int \frac{x+\sqrt{x+1}}{x+2}\,dx$

$x+1=t^2$

$dx=2t\,dt$

$\int \frac{(t^2-1+t)\,2t\,dt}{t^2+1}$

$2\int \left(\frac{(t^2+1)+t-2}{t^2+1}\right) t\,dt$

$2\int \frac{\cancel{(t^2+1)}}{\cancel{t^2+1}} t\,dt + 2\int \frac{(t^2+1)-1}{t^2+1}\,dt - 2\int \frac{2t}{t^2+1}\,dt$ $\longrightarrow t^2+1=z$

$2t\,dt=dz$

$\cancel{2}\frac{t^2}{\cancel{2}} + 2t - 2\int \frac{dt}{t^2+1} - 2\int \frac{dz}{z} \Rightarrow t^2+2t-2\tan^{-1}t-2\ln|z|+c$

Q $\int \cos\left(2\cot^{-1}\sqrt{\frac{1-x}{1+x}}\right)dx$

$x=\cos\theta \Rightarrow dx=-\sin\theta\,d\theta$

$-\int \cos\left(2\cot^{-1}\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}\right)\sin\theta\cdot d\theta$

$-\int \cos\left(2\cot^{-1}\sqrt{\tan^2\frac{\theta}{2}}\right)\sin\theta\cdot d\theta$

$\Rightarrow -\int \cos\left(2\cot^{-1}\tan\frac{\theta}{2}\right)\sin\theta\cdot d\theta$

$\Rightarrow -\int \cos\left(2\left(\frac{\pi}{2}-\tan^{-1}\tan\frac{\theta}{2}\right)\right)\sin\theta\cdot d\theta$

$\Rightarrow -\int \cos(\pi-\theta)\cdot\sin\theta\cdot d\theta$

$+\frac{1}{2}\int 2\cos\theta\cdot\sin\theta\,d\theta$

$\frac{1}{2}\int \sin 2\theta\cdot d\theta =$

$-\frac{\cos 2\theta}{4}+c$

$-\frac{(2\cos^2\theta-1)}{4}+c$

$-\frac{x^2}{2}+c$

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Q $\int \frac{\sqrt{x^2-a^2}}{x}\cdot dx$

M1: $x = a\sec\theta$

M2:

$x^2-a^2=t^2$

$2x\,dx = 2t\,dt$

$dx = \frac{t\,dt}{x}$

$\int \frac{\sqrt{t^2}\cdot t\cdot dt}{x^2}$

$\int \frac{t^2\cdot dt}{t^2+a^2}$

$\int \frac{(t^2+a^2)-a^2}{t^2+a^2}\cdot dt$

$\int dt - a^2\int\frac{dt}{a^2+t^2}$

$t - \frac{a^2}{a}\tan^{-1}\frac{t}{a}+C$

Q $\int \sqrt{\frac{a}{x+a}}\cdot dx$

$\Rightarrow \sqrt{a}\int\frac{dx}{\sqrt{x+a}} \to \int\frac{dx}{\sqrt{x}}$

$\to$ Linear fxn x की तरह behave.

$\sqrt{a}\times 2\sqrt{x+a}+C$.

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Q $\int \sqrt{\dfrac{\cos x-\cos^3 x}{1^3-\cos^3 x}}\,dx \quad \xleftrightarrow{\;dr.\ \exists\ a^3-b^3\;} \quad$ Q $\int \sqrt{\dfrac{x}{a^3-x^3}}\,dx$

$$\int \frac{\sin x\sqrt{\cos x}\cdot dx}{\sqrt{(1)^2-(\cos^{3/2}x)^2}}$$

$\cos^{3/2}x=t$

$-\frac{3}{2}\cos^{1/2}x\sin x\cdot dx=dt$

$\sin x\sqrt{\cos x}\cdot dx=-\frac{2}{3}dt$

$$-\frac{2}{3}\int\frac{dt}{\sqrt{1^2-t^2}}$$

$$-\frac{2}{3}\sin^{-1}t+c$$

$$\int\frac{\sqrt{x}\,dx}{\sqrt{(a^{3/2})^2-(x^{3/2})^2}}$$

$x^{3/2}=t$

$\frac{3}{2}x^{1/2}dx=dt$

$\sqrt{x}\,dx=\frac{2}{3}dt$

$$\frac{2}{3}\int\frac{dt}{\sqrt{(\ )^2-t^2}}$$

$$\frac{2}{3}\sin^{-1}\frac{t}{a^{3/2}}+c$$

Nishant Jindal

"different fraction degrees are given."

↓

take LCM of Dr. of all degrees.

Q $\int \frac{x+x^{2/3}+x^{1/6}}{x(1+x^{1/3})}\cdot dx$

$LCM\left(\frac{1}{1},\frac{1}{3},\frac{1}{6}\right)=\frac{1}{6}$

$x^{1/6}=t$

$x=t^6$

$dx=6t^5\cdot dt$

$\Rightarrow \int \frac{t^6+(t^6)^{2/3}+(t^6)^{1/6}}{t^6(1+(t^6)^{1/3})}\cdot 6t^5 dt$

$\Rightarrow 6\int \frac{(t^6+t^4+t)}{\cancel{t^6}(1+t^2)}\cancel{t^5}\cdot dt$

$\Rightarrow 6\int \frac{\cancel{t}(t^5+t^3+1)}{\cancel{t}(1+t^2)}dt$

$\Rightarrow 6\int \frac{t^3(\cancel{t^2+1})}{(\cancel{t^2+1})}dt+6\int\frac{1}{1+t^2}dt$

Q $\int \frac{dx}{x^{1/2}+x^{1/3}}$

$LCM\left(\frac{1}{2},\frac{1}{3}\right)=\frac{1}{6} \Rightarrow x^{1/6}=t$

$x=t^6$

$\int \frac{6t^5\cdot dt}{t^3+t^2}$ divide.

"Odd degree Brk"

$\int \sin^{odd}x\cdot dx$ OR $\int \cos^{odd}x\cdot dx$

Q $\int \sin^3 x\cdot dx$ M2

$\int \sin^2 x\cdot \sin x\cdot dx$

$\Rightarrow \int (1-\cos^2 x)\cdot \sin x\cdot dx$

$\cos x=t$

$\sin x\, dx=-dt$

$-\int (1-t^2)dt$ DY

Nishant Jindal

1) $\int e^x(f(x)+f'(x))\,dx$ type　　2) $\int x\cdot f'(x)+f(x)\cdot dx$ type.

$$\int \underbrace{e^x}_{U'}\cdot\underbrace{f(x)}_{V}+\underbrace{e^x}_{U}\cdot\underbrace{f'(x)}_{V'}\cdot dx$$

$$\int (U\cdot V'+U'\cdot V)$$

$$=\int (U\cdot V)'$$

$$=U\cdot V+C$$

$$\boxed{\int e^x(f(x)+f'(x))dx=e^x\cdot f(x)+C}$$

$$\int \underbrace{x}_{U}\cdot\underbrace{f'(x)}_{V'}+\underbrace{f(x)}_{V}\cdot\underbrace{1}_{U'}$$

$$\int (U\cdot V'+U'\cdot V)$$

$$\int (U\cdot V)'$$

$$=U\cdot V+C$$

$$=x\cdot f(x)+C$$

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$$\int e^x\left(f(x)+f'(x)\right)\cdot dx$$

$$= e^x\cdot f(x)+C$$

Q $$\int e^x(\underset{f}{\sin x}+\underset{f'}{\cos x})dx$$

$$= e^x\cdot \sin x + C$$

Q $$\int e^x(\underset{f'}{\sin x}-\underset{f}{\cos x})dx$$

$$e^x(-\cos x)+C$$

Q $$\int e^x\left(\frac{x-1}{x^2}\right)\cdot dx$$

$$\int e^x\left(\underset{f}{\frac{1}{x}}-\underset{f'}{\frac{1}{x^2}}\right)\cdot dx$$

$$\Rightarrow \frac{e^x}{x}+C$$

Q. $$\int e^x\left(\frac{2x+1}{2\sqrt{x}}\right)\cdot dx$$

$$\int e^x\left(\underset{f}{\sqrt{x}}+\underset{f'}{\frac{1}{2\sqrt{x}}}\right)\cdot dx$$

$$\Rightarrow e^x(\sqrt{x})+C$$

Q $$\int \frac{x^2\cdot e^x\cdot dx}{(x+2)^2}$$

$$\Rightarrow \int e^x\cdot\left(\frac{(x^2-4)+4}{(x+2)^2}\right)dx$$

$$\Rightarrow \int e^x\left(\underset{f}{\frac{x-2}{x+2}}+\underset{f'}{\frac{4}{(x+2)^2}}\right)\cdot dx$$

$$\Rightarrow e^x\left(\frac{x-2}{x+2}\right)+C$$

$$y=\frac{x-2}{x+2}$$

$$y'=\frac{(x+2)-(x-2)}{(x+2)^2}=\frac{4}{(x+2)^2}$$

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Q $\displaystyle\int e^x\left\{\frac{x^2+5x+7}{(x+3)^2}\right\}dx$ $\quad x^2+5x+6$

$$\int e^x\left\{\frac{x^2+5x+6+1}{(x+3)^2}\right\}dx$$

$$\int e^x\left\{\frac{(x+2)\cancel{(x+3)}}{(x+3)^2}+\frac{1}{(x+3)^2}\right\}dx$$

$$\int e^x\left\{\underbrace{\frac{x+2}{x+3}}_{f}+\underbrace{\frac{1}{(x+3)^2}}_{f'}\right\}$$

$$\Rightarrow e^x\left(\frac{x+2}{x+3}\right)+C$$

When Qs has too much log

Q

$$\int\frac{\log x}{(1+\log x)^2}dx$$

$\log_e x=t$

$x=e^t$

$dx=e^t\,dt$

$$\int\frac{e^t\cdot t}{(1+t)^2}\,dt$$

$$\int e^t\left(\frac{(t+1)-1}{(t+1)^2}\right)dt$$

$\left(\frac{1}{x}\right)'=\frac{-1}{x^2}$

$$\int e^t\left(\underbrace{\frac{1}{(t+1)}}_{f}-\underbrace{\frac{1}{(t+1)^2}}_{f'}\right)dt$$

$$=\frac{e^t}{t+1}+C=\frac{x}{1+\ln x}+C$$