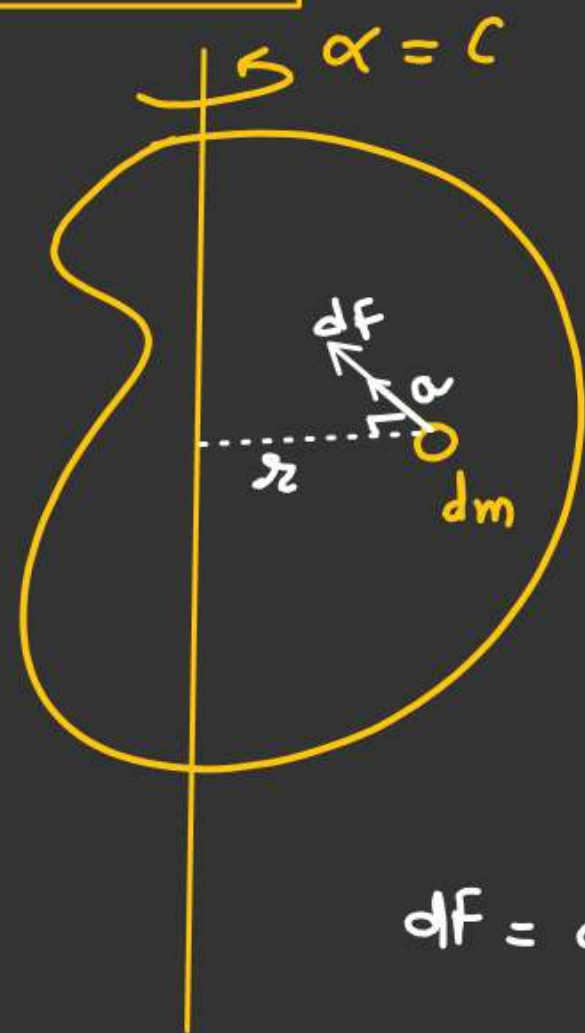


$$\vec{\tau} = I \vec{\alpha}$$



$$a = r\alpha$$

$$dF = dm a$$

$$d\tau = dF \cdot r$$

$$d\tau = dm a r$$

$$\int d\tau = \int dm r^2 \alpha$$

$$\tau_{\text{net}} = (\underbrace{\int dm r^2}_{I}) \alpha$$

\Downarrow
 I_{body}
 about axis
 of rotation

$$\tau_{\text{net}} = I \alpha$$

\hookrightarrow Newton's 2nd Law

K.E of a hinged body

$$v = r\omega$$

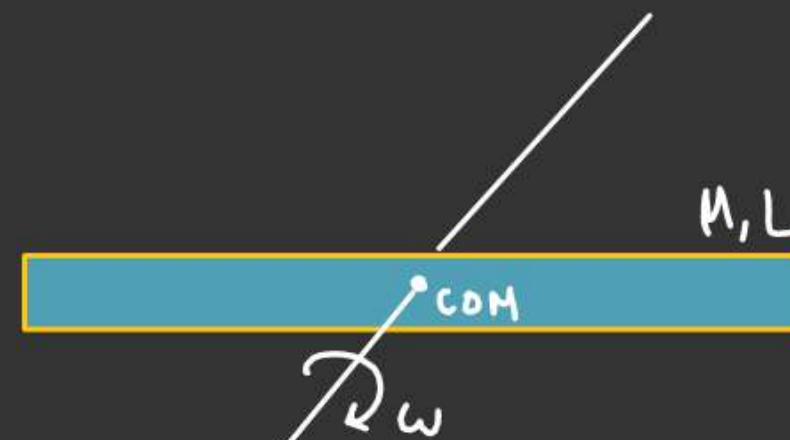
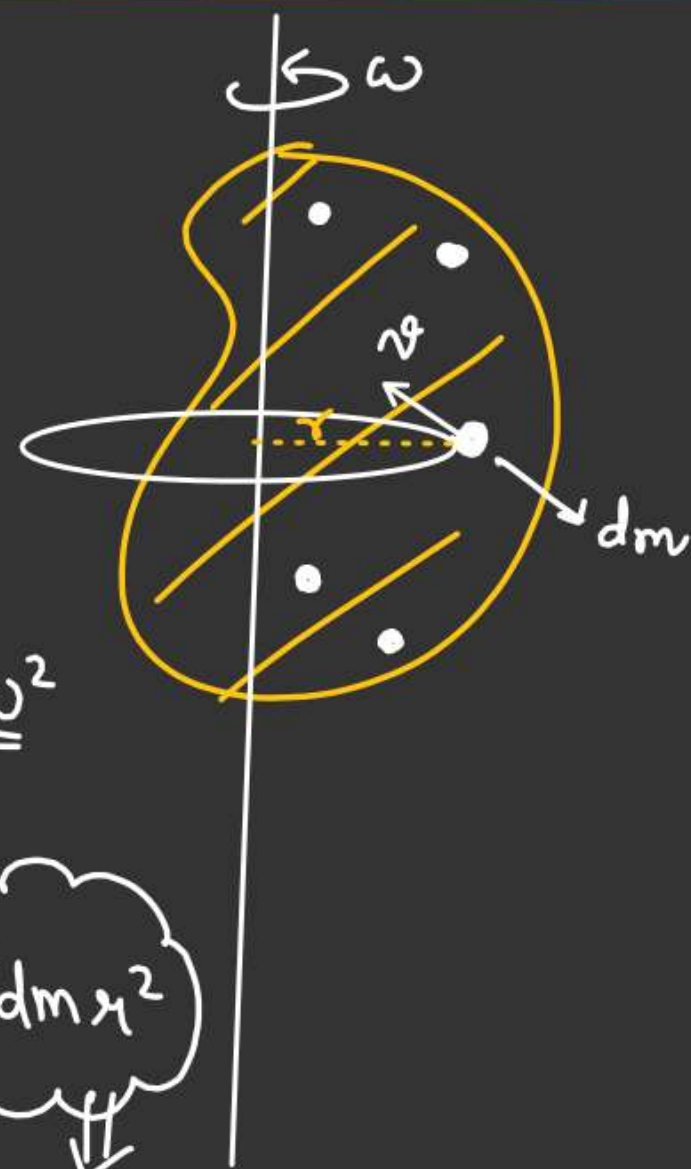
$$d(K.E) = \frac{1}{2}(dm)v^2$$

$$\int d(K.E) = \int \frac{1}{2}(dm)r^2 \omega^2$$

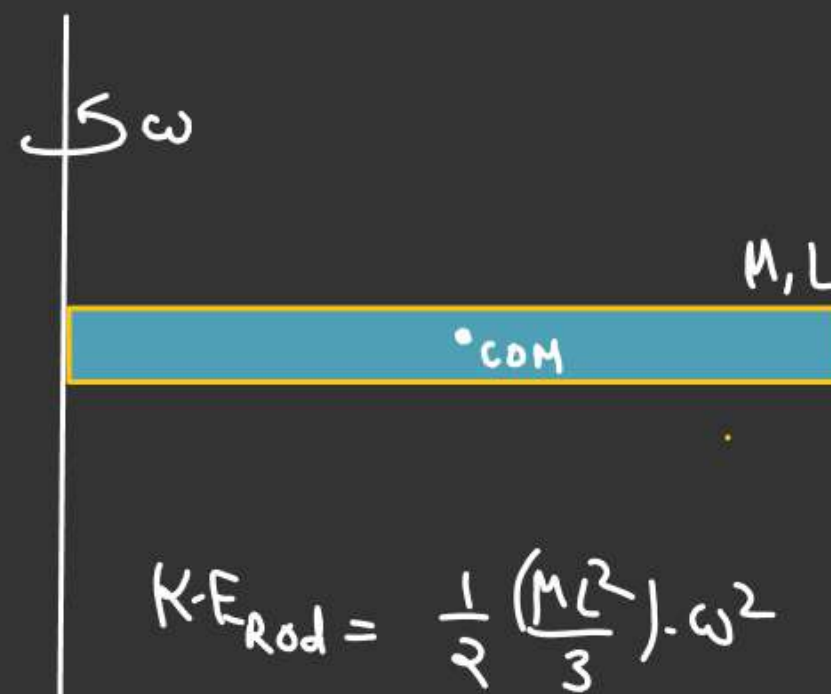
$$K.E_{body} = \frac{1}{2}\omega^2 \left\{ \int dm r^2 \right\}$$

$$K.E_{body} = \frac{1}{2} I_{axis} \cdot \omega^2$$

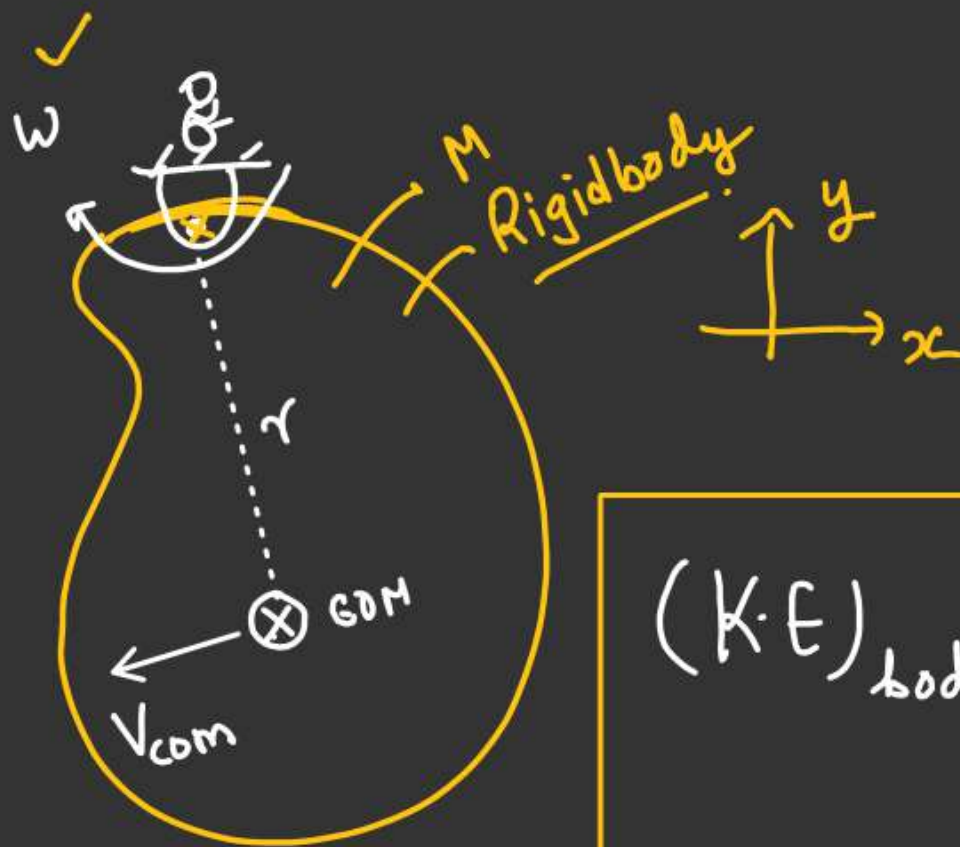
(I_{body}) about axis of Rotation



$$K.E_{Rod} = \frac{1}{2} I_{COM} \cdot \omega^2 = \frac{1}{2} \left(\frac{ML^2}{12} \right) \omega^2 = \frac{ML^2 \omega^2}{24}$$



$$K.E_{Rod} = \frac{1}{2} \left(\frac{ML^2}{3} \right) \cdot \omega^2 = \frac{ML^2 \omega^2}{6}$$



$$\underline{v_{com} = r\omega}$$

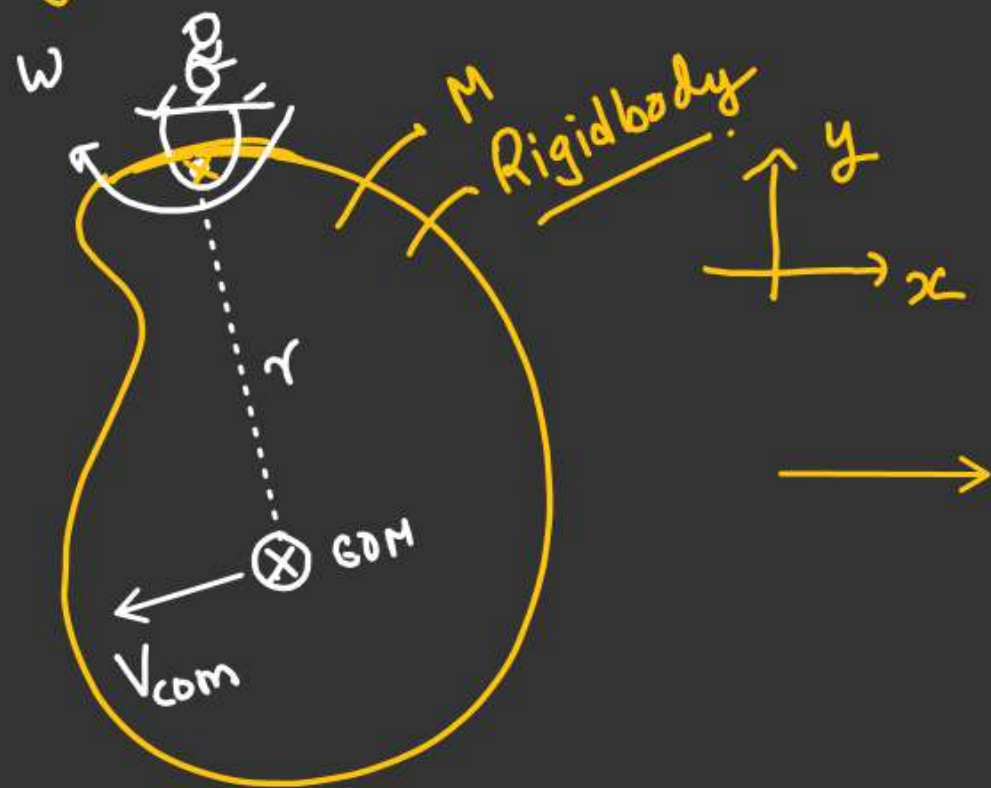
$$(K.E)_{body} = \frac{1}{2} \left(I_{body \text{ about axis of Rotation}} \right) \omega^2$$

$$I_{body \text{ about axis of rotation}} = [I_{com} + mr^2]$$

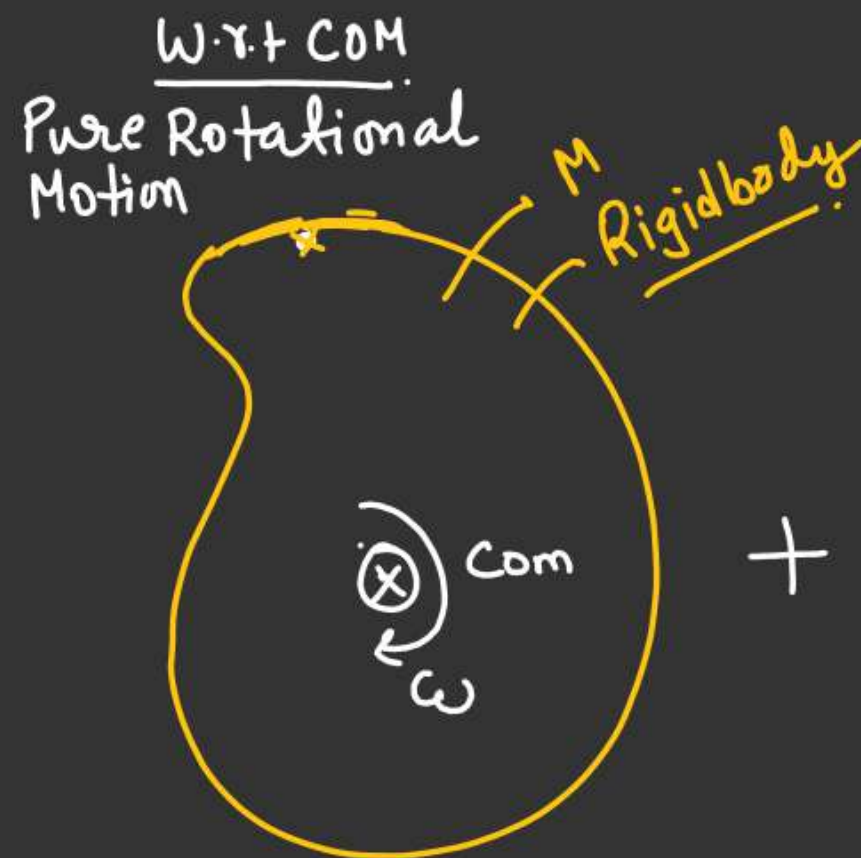
$$K.E_{body} = \frac{1}{2} [I_{com} + mr^2] \omega^2$$

$$K.E_{body} = \frac{1}{2} I_{com} \omega^2 + \frac{1}{2} M v_{com}^2$$

$$K.E_{body} = \frac{1}{2} (I_{com}) \omega^2 + \frac{1}{2} m \underline{r^2 \omega^2}$$

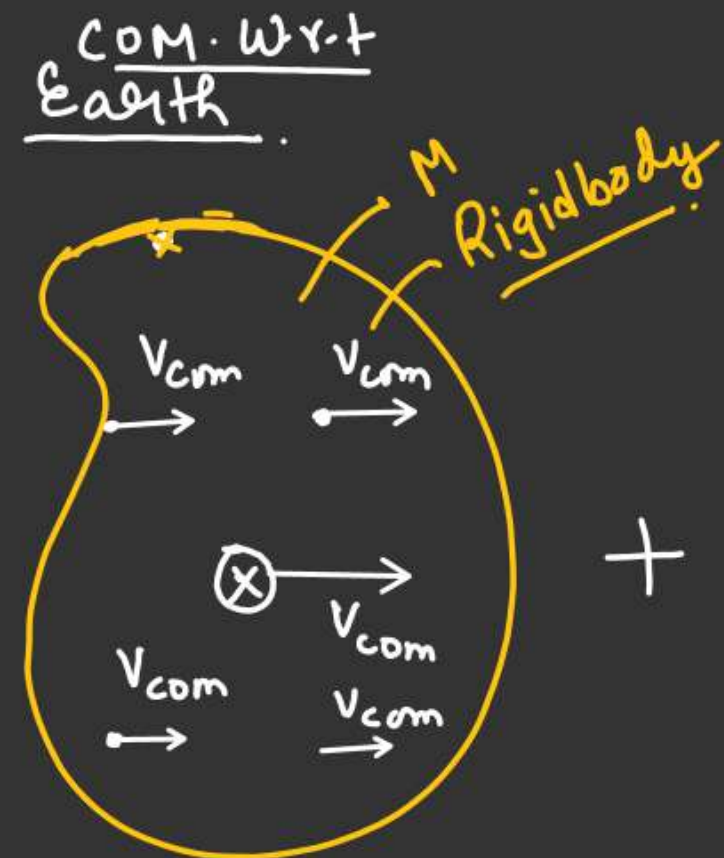


$$\underline{\underline{K.E_{body} = \frac{1}{2} I_{com} \omega^2 + \frac{1}{2} M v_{com}^2}}$$



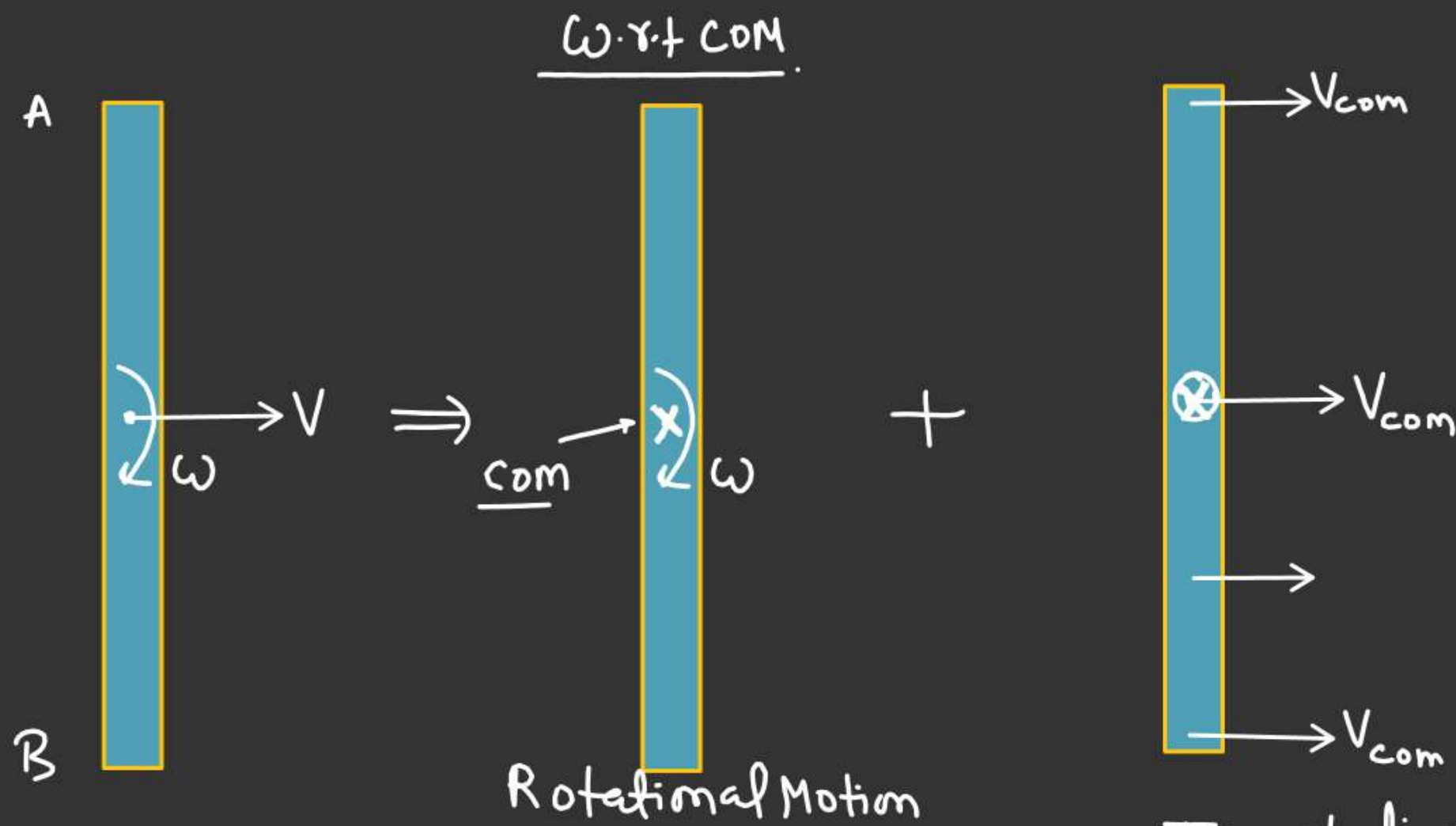
$$\Downarrow$$

$$\Leftarrow K.E_{Rotational} = \frac{1}{2} I_{com} \omega^2$$



$$\Downarrow$$

$$K.E_{translational} = \frac{1}{2} M v_{com}^2$$



Note:- When a body is free to rotate it always rotate about COM

$\begin{array}{c} \bigcirc \rightarrow V_{\text{com}} \\ \bigotimes \rightarrow V_{\text{com}} \end{array}$

Earth

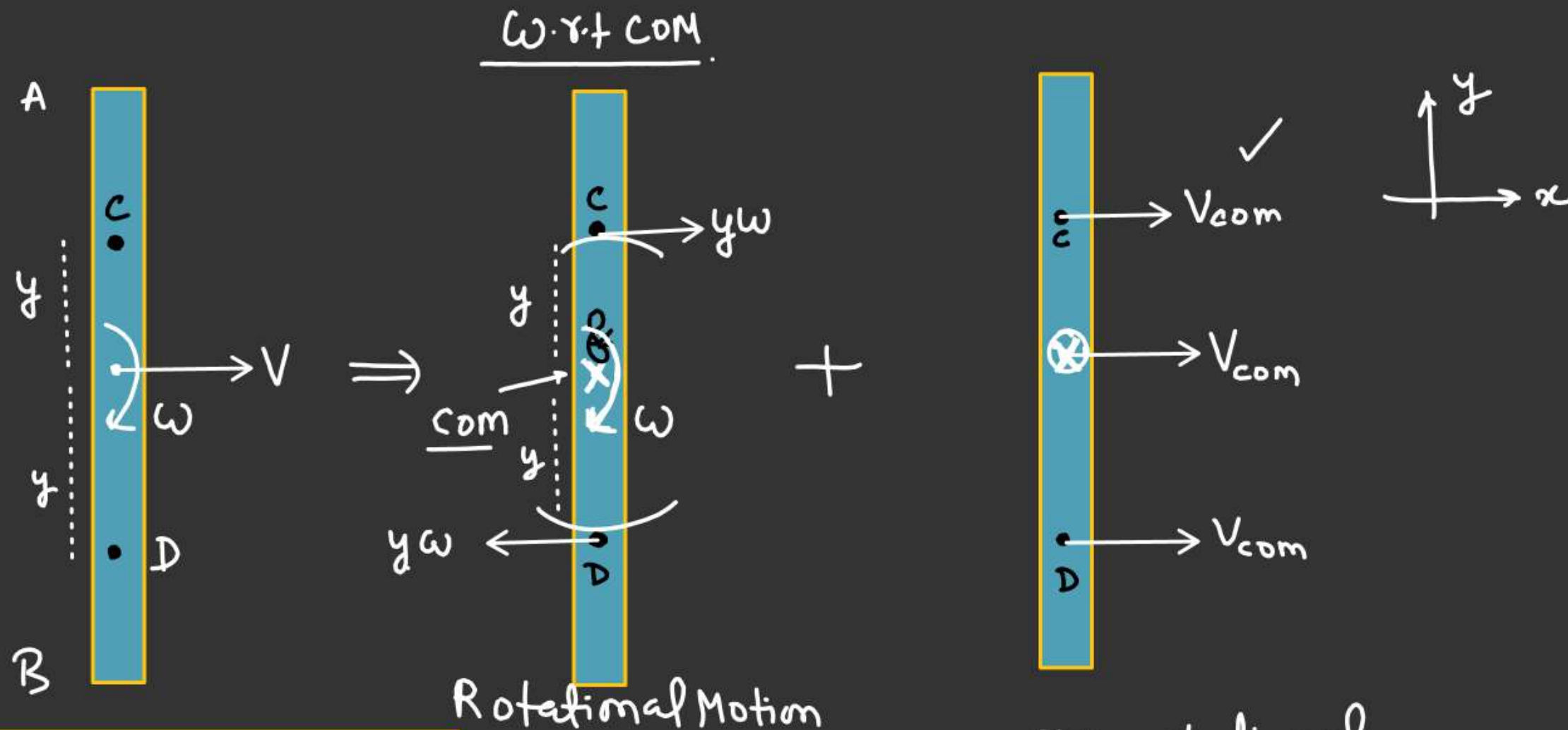
$$\begin{aligned} \text{K.E}_{\text{Rotational}} &= \frac{1}{2} I_{\text{com}} \omega^2 \\ &= \frac{1}{2} \left(\frac{ML^2}{12} \right) \omega^2 \end{aligned}$$

Translational Motion.

$$\text{K.E}_{\text{Translational}} = \frac{1}{2} M V_{\text{com}}^2$$

$$|\vec{V}_c| = ??$$

$$|\vec{a}_c| = ??$$



$$\vec{V}_{c/\varepsilon} = \vec{V}_{c/\text{com}} + \vec{V}_{\text{com}/\varepsilon}$$

$$\begin{aligned} \vec{V}_{c/\varepsilon} &= y\omega \hat{i} + V_{\text{com}} \hat{i} \\ &= (V_{\text{com}} + y\omega) \hat{i} \end{aligned}$$

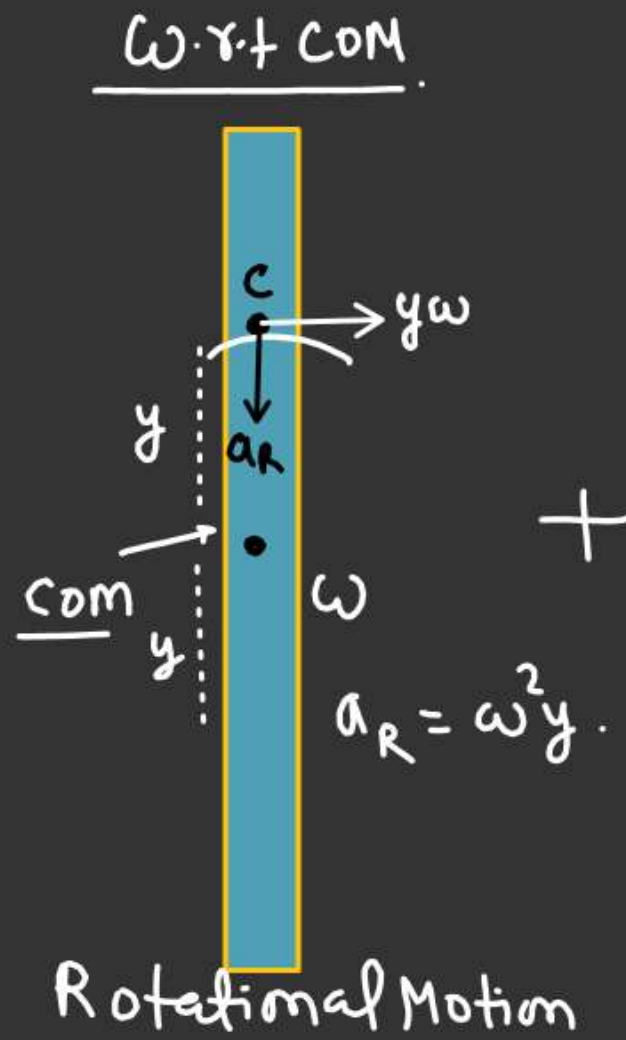
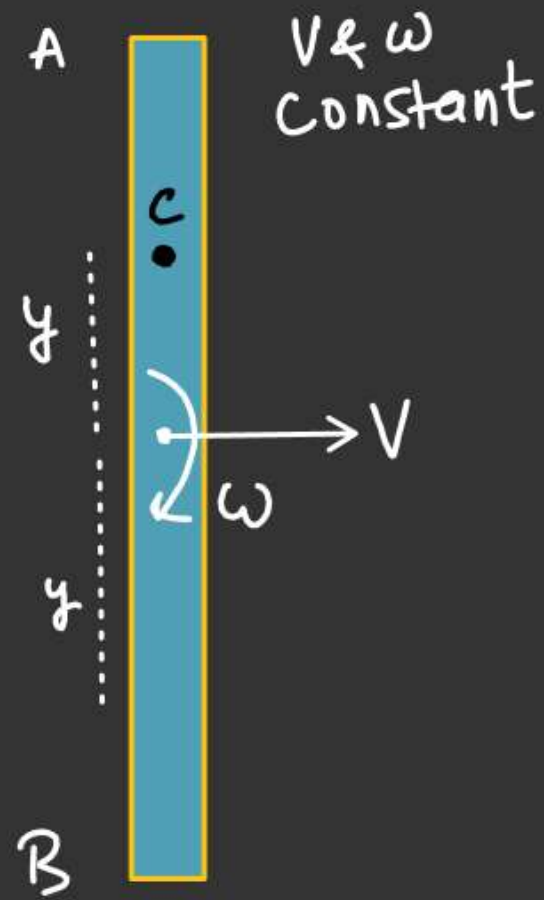
$$\begin{aligned} \text{O} &\rightarrow V_{\text{com}} \\ \text{X} &\rightarrow V_{\text{com}} \end{aligned}$$

$$\text{Earth}$$

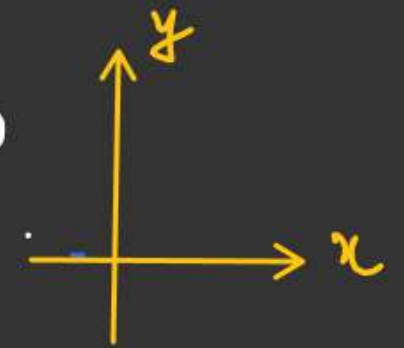
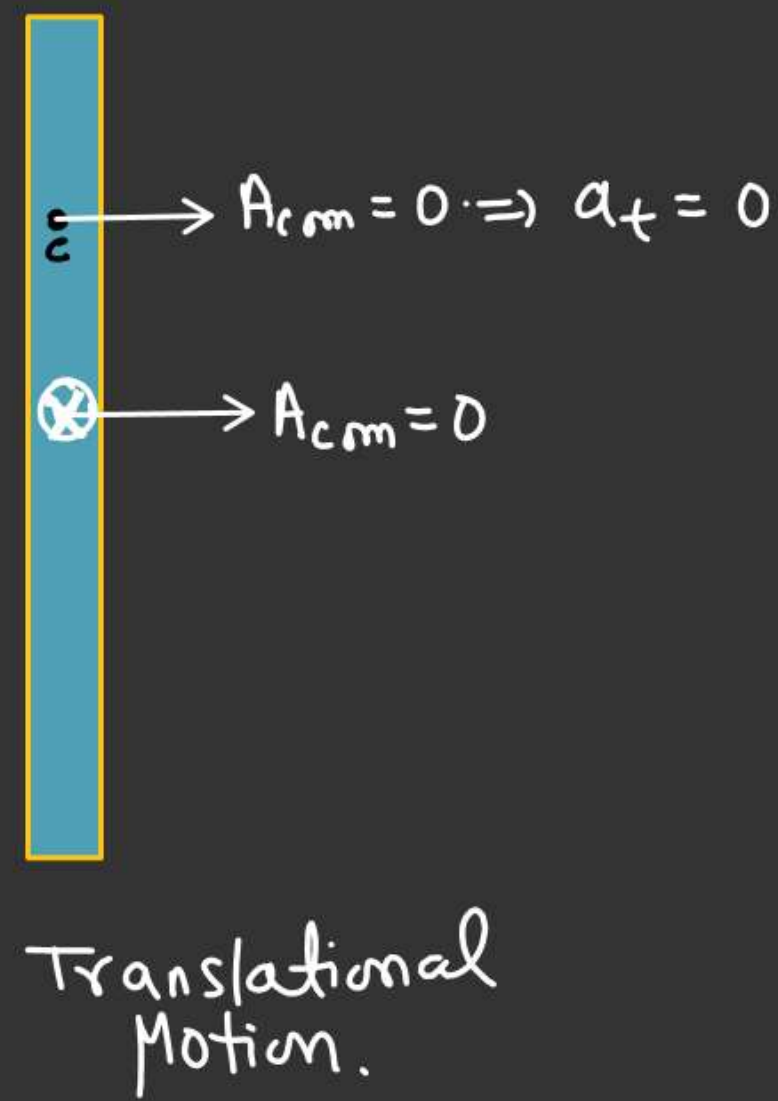
Translational Motion.

$$\begin{aligned} \vec{V}_{D/\varepsilon} &= \vec{V}_{D/\text{com}} + \vec{V}_{\text{com}/\varepsilon} \\ &= -y\omega \hat{i} + V_{\text{com}} \hat{i} \\ &= (V_{\text{com}} - y\omega) \hat{i} \end{aligned}$$

$$|\vec{a}_c| = ??$$



$\frac{0}{\text{Earl}}$

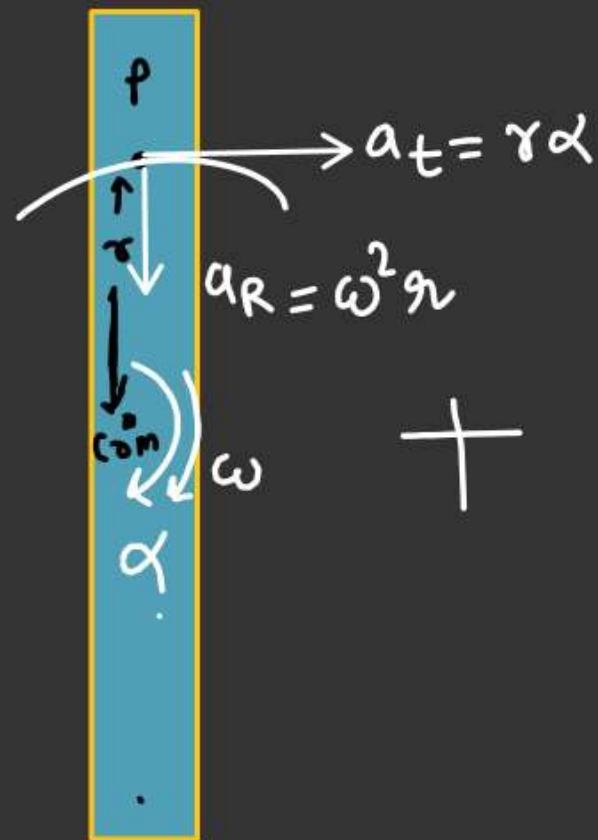


$$\vec{a}_{c/\varepsilon} = \vec{a}_{c/\text{COM}} + \vec{a}_{\text{COM}/\varepsilon}$$

$$\vec{a}_{c/\varepsilon} = -\omega^2 y \hat{j} + 0$$

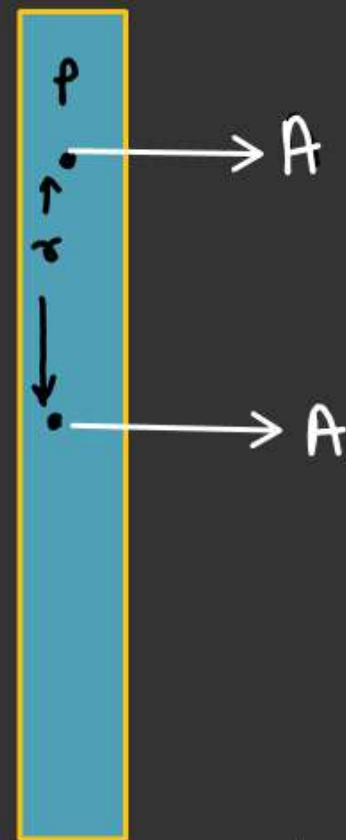


A & $\alpha \rightarrow \text{Constant.}$
 $a_p = ?$ $a_q = ?$



Rotational Motion

$$\vec{a}_{p/\text{com}} = \left(r\alpha \hat{i} - \omega^2 r \hat{j} \right) \quad (\text{com frame})$$

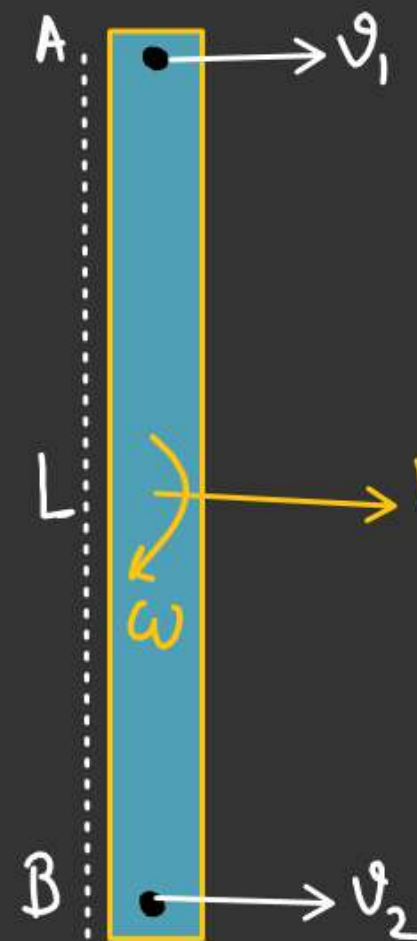


Translational Motion

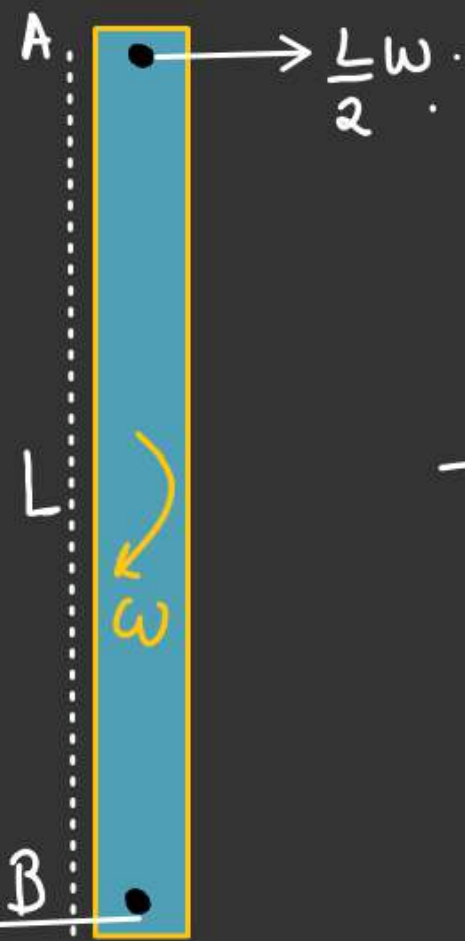
$$\vec{a}_{\text{com}/\mathcal{E}} = A \hat{i}$$

$$\begin{aligned} \vec{a}_{p/\mathcal{E}} &= \vec{a}_{p/\text{com}} + \vec{a}_{\text{com}/\mathcal{E}} \\ &= (r\alpha \hat{i} - \omega^2 r \hat{j}) + A \hat{i} \\ &= \underline{(A + r\alpha) \hat{i} - \omega^2 r \hat{j}} \end{aligned}$$

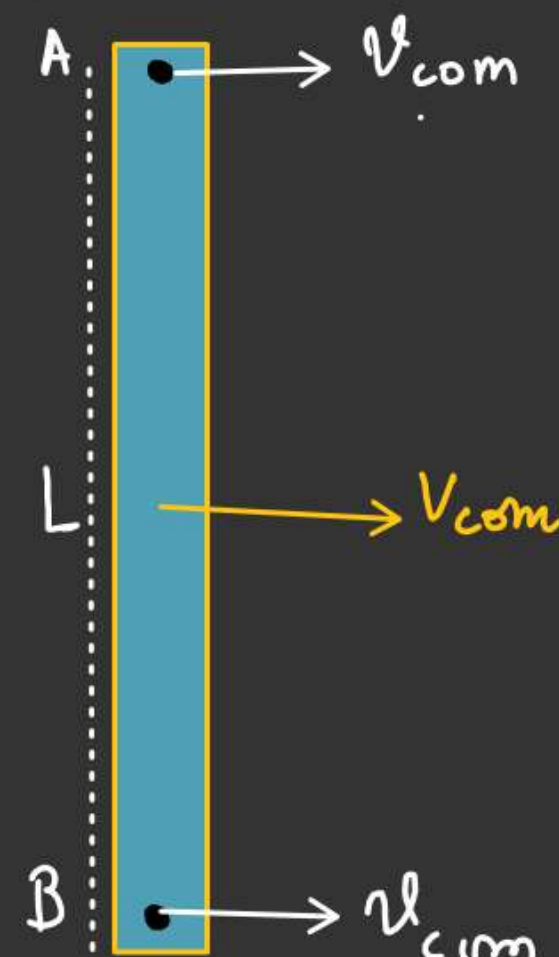
v_1, v_2



$V_{com} = ??, \omega = ??$



+



$$v_1 = V_{com} + \frac{L}{2}\omega$$

$$v_2 = V_{com} - \frac{L}{2}\omega$$

$$v_1 + v_2 = 2V_{com}$$

$$V_{com} = \frac{v_1 + v_2}{2}$$

Rotational Motion
(COM frame)

$$\begin{aligned} \frac{L}{2}\omega &= v_1 - V_{com} \\ &= v_1 - \left(\frac{v_1 + v_2}{2}\right) \end{aligned}$$

$$\frac{L}{2}\omega = \frac{v_1 - v_2}{2}$$

$$\omega = \frac{v_1 - v_2}{L}$$