Nishant Jindal

# Advance level Qs. of Trigo Eqn.

Q $2\sin 11x + \cos 3x + \sqrt{3}\sin 3x = 0$ GS?

$$\sin 11x + \left(\frac{\sqrt{3}}{2}\sin 3x + \frac{1}{2}\cos 3x\right) = 0$$

$$\sin 11x + \sin\left(\frac{\pi}{6} + 3x\right) = 0$$

$$2\sin\left(7x + \frac{\pi}{12}\right)\cos\left(4x - \frac{\pi}{12}\right) = 0$$

$$\sin\left(7x + \frac{\pi}{12}\right) = 0 \qquad \cos\left(4x - \frac{\pi}{12}\right) = 0$$

$7x + \frac{\pi}{12} = n\pi$ | $\left(4x - \frac{\pi}{12}\right) = (2n+1)\frac{\pi}{2}$

$x = \frac{n\pi}{7} - \frac{\pi}{84}$ ; $x = \frac{n\pi}{4} + \frac{7\pi}{48}$

Q2 $\sqrt{13 - 18\tan x} = 6\tan x - 3$ No of sol. in $-2\pi \le x \le 2\pi$?

$$36\tan^2 x - 18\tan x - 4 = 0$$

$$18\tan^2 x - 9\tan x - 2 = 0$$

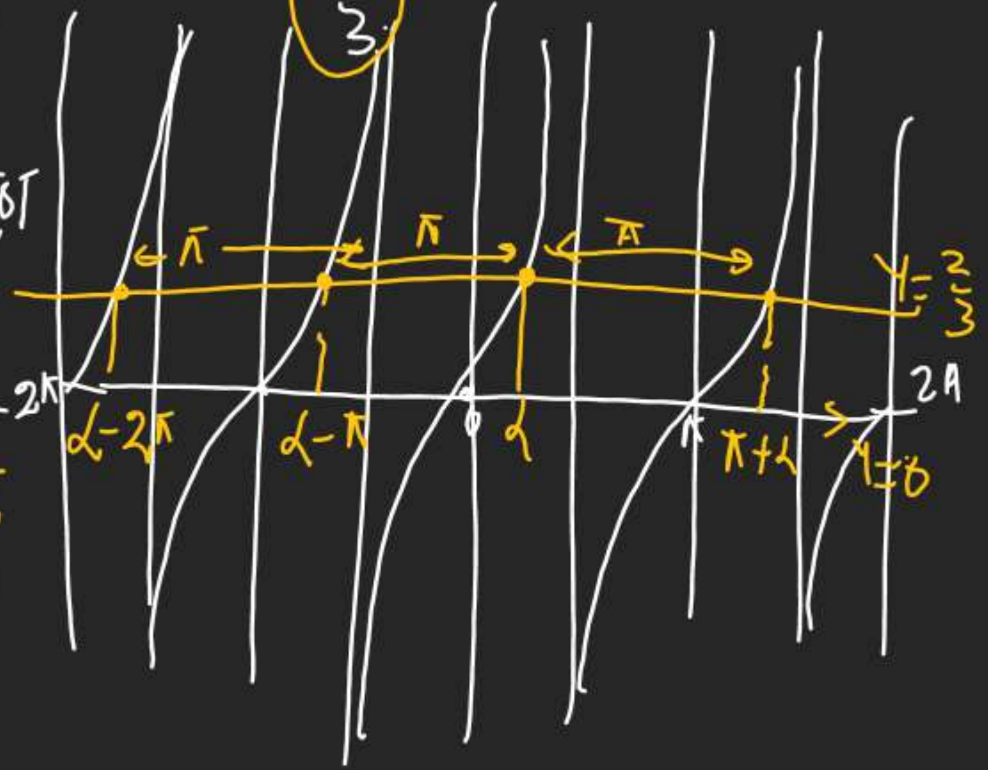
$$(6\tan x + 1)(3\tan - 2) = 0$$

$\tan x = -\frac{1}{6}$ ✗ or $\tan x = \frac{2}{3}$ ; $Y = \frac{2}{3}$

Repeat

good Students

(+)




$x = \alpha - 2\pi, \alpha - \pi, \alpha, \alpha + \pi$

Nishant Jindal

Q3 Find smallest +ve value of $x$ satisfying

$$\sqrt{1+\sin 2x} - \sqrt{2}\cos 3x = 0$$

$$\downarrow$$

$$\sqrt{(\sin x+\cos x)^2} - \sqrt{2}\cos 3x = 0$$

$$\sin x + \cos x = \sqrt{2}\cos 3x$$

$$\frac{1}{\sqrt{2}}\sin x + \frac{1}{\sqrt{2}}\cos x = \cos 3x.$$

$$\cos\left(\frac{\pi}{4}-x\right) = \cos 3x.$$

$$\frac{\pi}{4} - x = 2n\pi \pm 3x$$

$\rightarrow \frac{\pi}{4} - x = 2n\pi + 3x$

$n=0 \rightarrow \frac{\pi}{4} - x = 3x \Rightarrow 4x = \frac{\pi}{4} \Rightarrow \boxed{x = \frac{\pi}{16}}$

$\rightarrow \frac{\pi}{4} - x = 2n\pi - 3x$

$\frac{\pi}{4} - x = -3x \Rightarrow 2x = -\frac{\pi}{4}$ ⊗

Q General value of θ so that Q Eqn.

$$(\sin\theta)x^2 + (2\cos\theta)x + \left(\frac{\cos\theta+\sin\theta}{2}\right)$$ is Sqr of Linear fxn?

$\sin\theta > 0$ $\quad D=0$ (Perfect Sqr $\Rightarrow D$)

$$\cancel{2}4\cos^2\theta = \cancel{4}\sin\theta \times \frac{(\sin\theta+\cos\theta)}{\cancel{2}}$$

$$2(1+\cos 2\theta) = 2\sin^2\theta + 2\sin\theta\cdot\cos\theta$$

$$2 + 2\cos 2\theta = 1 - \cos 2\theta + \sin 2\theta$$

$$1 + 3\cos 2\theta = \sin 2\theta \Rightarrow 1 + 3\cdot\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right) = \frac{2\tan\theta}{(1+\tan^2\theta)}$$

$$\Rightarrow 1+\tan^2\theta + 3 - 3\tan^2\theta = 2\tan\theta$$

$$2\tan^2\theta + 2\tan\theta - 4 = 0$$

$$\tan^2\theta + \tan\theta - 2 = 0$$

$$(\tan\theta+2)(\tan\theta-1) = 0$$

(1st) $\tan\theta = 1$ & $\tan\theta = -2$ (2nd)

Nishant Jindal

Q5 No. of Sol of Eqn.

$\log_{\frac{-x^2-6x}{10}}(\sin 3x+\sin x)=\log_{\frac{-x^2-6x}{10}}(\sin 2x)$ is.

$\log_a f(x)$ की 3 Condn.

Base > 0, Base ≠ 1, f(x) > 0

① $\frac{-x^2-6x}{10}>0$

$-x^2-6x>0$

$x^2+6x<0$

$(x)(x+6)<0$

$-6<x<0$

② $\sin 3x+\sin x>0$

$\sin(\cancel{-6\pi}+\pi)$ ... 0

$+\sin\left(-2\pi+\frac{\pi}{3}\right)$

$\sin\frac{\pi}{3}>0$

③ $\sin 2x>0$

$\sin\left(-4\pi+\frac{2\pi}{3}\right)$

$=\sin\frac{2\pi}{3}$ (+)

$\sin 3x+\sin x=\sin 2x$

$2\sin 2x\cdot\cos(x)=\sin 2x.$

$\sin 2x(2\cos x-1)=0$

$\sin 2x = 0$ OR $\cos x=\frac{1}{2}=\cos\frac{\pi}{3}.$

(X) as $\sin 2x>0$

$x=2n\pi\pm\frac{\pi}{3}.$

$n=-1$: $x=-2\pi-\frac{\pi}{3}$ X, $-2\pi+\frac{\pi}{3}$

$-6.28-1.04$ , $-6.28+1.04$

$n=0$: $x=\frac{\pi}{3},-\frac{\pi}{3}$ (X)

(1.03) (−1.03)

$n=1$: $x=2\pi+\frac{\pi}{3}$ X, $2\pi-\frac{\pi}{3}$ X

$6.28+1.04$ , $6.28-1.04$

$\frac{\pi}{3}\simeq\frac{3.14}{3}\simeq 1.04$

$x=-\frac{5\pi}{3}$ is only answer

**Q6** G.S. of $3-2\cos\theta-4\sin\theta-\cos2\theta+\sin2\theta=0$

$$3-2\cos\theta-4\sin\theta-1+2\sin^2\theta+2\sin\theta\cos\theta=0$$

$$2-2\cos\theta-4\sin\theta+2\sin^2\theta+2\sin\theta\cos\theta=0$$

$$1-\cos\theta-2\sin\theta+\sin^2\theta+\sin\theta\cos\theta=0$$

$$(1-\sin\theta)-\sin\theta+\sin^2\theta-\cos\theta+\sin\theta\cos\theta=0$$

$$(1-\sin\theta)-\sin\theta(1-\sin\theta)-\cos\theta(1-\sin\theta)=0$$

$$(1-\sin\theta)(1-\sin\theta-\cos\theta)=0$$

$1-\sin\theta=0$

$\sin\theta=1$

$\theta=2n\pi+\frac{\pi}{2}$

$1-\sin\theta-\cos\theta=0$

$\sin\theta+\cos\theta=1$

$\cos\left(\theta-\frac{\pi}{4}\right)=\frac{1}{\sqrt{2}}=\cos\frac{\pi}{4}$ ✓

$3-4+1=0$

**Q7** No of sol. of

$\log_{2|\sec x|}(|\cos x|+1)=1$ in $[0,2\pi]$

$$|\cos x|+1=(2|\sec x|)^1$$

$$|\cos x|+1=2|\sec x|$$

$$|\cos x|+1=\frac{2}{|\cos x|}$$

$$t+1=\frac{2}{t}$$

$$t^2+t-2=0$$

$$(t+2)(t-1)=0$$

$t=-2$ OR $t=1$

✗ $|\cos x|=-2$

(+) $\neq$ -ve


$|\cos x|=1$

$\cos x=\pm1$

$\boxed{x=n\pi}$ ✓

Nishant Jindal

Q8 G.S. of

$\tan^2 x \cdot \tan^2 3x \cdot \tan 4x = \tan^2 x - \tan^2 3x + \tan 4x$ is

$\tan^2 x \cdot \tan^2 3x \cdot \tan 4x - \tan 4x = \tan^2 x - \tan^2 3x$

$\tan 4x \{\tan^2 x \cdot \tan^2 3x - 1\} = (\tan x + \tan 3x)(\tan x - \tan 3x)$

$\tan 4x = \boxed{\dfrac{(\tan 3x + \tan x)}{(1 - \tan x \cdot \tan 3x)}} \boxed{\dfrac{(\tan 3x - \tan x)}{(1 + \tan x \cdot \tan 3x)}}$

$= \tan(3x + x) \tan(3x - x)$

$\tan 4x = \tan 4x \cdot \tan 2x \Rightarrow \tan 4x \cdot \tan 2x - \tan 4x = 0$

$\tan 4x (\tan 2x - 1) = 0$

$\tan 2x = 1$

$\tan 4x = 0$

$4x = n\pi$

$x = \dfrac{n\pi}{4}$

$\dfrac{\pi}{4}, \dfrac{\pi}{2}, \dfrac{3\pi}{4}, \dfrac{4\pi}{4}$

$1 \times 1$ $\times$ $\times$ $\pi$

$\boxed{x = n\pi}$

$\tan 2x = \tan \dfrac{\pi}{4} = 1$

$2x = n\pi + \dfrac{\pi}{4}$

$x = \dfrac{n\pi}{2} + \dfrac{\pi}{8}$

Nishant Jindal

Q $\dfrac{\sqrt{5}-1}{\sin x}+\dfrac{\sqrt{10+2\sqrt{5}}}{\cos x}=8$ ; $x\in\left[0,\frac{\pi}{2}\right)$

$$\frac{\sqrt{5}-1}{4}\cdot\frac{1}{\sin x}+\frac{\sqrt{10+2\sqrt{5}}}{4}\cdot\frac{1}{\cos x}=2$$

$$\frac{\sin 18^\circ}{\sin x}+\frac{\cos 18^\circ}{\cos x}=2$$

$$\sin 18^\circ\cdot\cos x+\cos 18^\circ\cdot\sin x=2\sin x\cos x$$

$$\sin\left(\frac{\pi}{10}+x\right)=\sin 2x.$$

Solve Urself

Q 10 Find G.V. of $x, y$ Satisfying

$$5\sin x\cos y=1 \quad \& \quad 4\tan x=\tan y$$

$$\boxed{\sin x\cdot\cos y=\frac{1}{5}} \qquad \left|\quad \frac{4\sin x}{\cos x}=\frac{\sin y}{\cos y}\right.$$

$$4\sin x\cdot\cos y=\sin y\cos x$$

$$\boxed{\frac{4}{5}=\sin y\cdot\cos x}$$

$$\sin(x+y)=\sin x\cdot\cos y+\cos x\cdot\sin y=\frac{1}{5}+\frac{4}{5}=1 \Rightarrow \sin(x+y)=1$$

$$\sin(x-y)=\sin x\cos y-\cos x\cdot\sin y=\frac{1}{5}-\frac{4}{5}=\frac{-3}{5}\Rightarrow \sin(x-y)=-\frac{3}{5}$$

$$\left.\begin{aligned}x+y&=n\pi+(-1)^n\cdot\frac{\pi}{2}\\ x-y&=n\pi+(-1)^n\cdot\sin^{-1}\left(-\frac{3}{5}\right)\end{aligned}\right\}\text{ Solve \& } (x,y)$$

Nishant Jindal

Q $\sin^3 x\cdot\cos 3x+\cos^3 x\cdot\sin 3x+\frac{3}{8}=0$ then $\sin$?

$\sin^3 x(4\cos^3 x-3\cos x)+\cos^3 x\cdot(3\sin x-4\sin^3 x)=-\frac{3}{8}$

$4\sin^3 x\cancel{\cos^3 x}-3\sin^3 x\cos x+3\cos^3 x\cdot\sin x-4\cancel{\sin^3 x\cos^3 x}=-\frac{3}{8}$

$\cancel{3}\sin x\cdot\cos x(\underbrace{\cos^2 x-\sin^2 x}_{\cos 2x})=-\frac{\cancel{3}}{8}$

$\underbrace{2\sin x\cdot\cos x}(\cos 2x)=-\frac{1}{4}$

$2\sin 2x\cdot\cos 2x=-\frac{1}{2}$

$\sin 4x=-\frac{1}{2}$ DY

Q-12 No of Roots of

$|\sin x\cdot\cos x|+\sqrt{2+\tan^2 x+\cot^2 x}=\sqrt{3}$ ; $x\in[0,4\pi]$

$|\sin x\cdot\cos x|+\sqrt{(\tan x+\cot x)^2}=\sqrt{3}$.

$|\sin x\cdot\cos x|+|\tan x+\cot x|=\sqrt{3}$.

$|\sin x\cdot\cos x|+\left|\frac{\sin x}{\cos x}+\frac{\cos x}{\sin x}\right|=\sqrt{3}$.

$\underbrace{|\sin x\cdot\cos x|}+\frac{1}{\underbrace{|\sin x\cdot\cos x|}}=\sqrt{3}$.

$\to$ RHS $=1.72$

$f(x)+\frac{1}{f(x)}\geq 2$ LHS $\geq 2$

Can not be matched

No Roots

Nishant Jindal

Q13 No. of Sol. of

$$\frac{\sin x}{\cos 3x}+\frac{\sin 3x}{\cos 9x}+\frac{\sin 9x}{\cos 27x}=0 \text{ in } \left(0,\frac{\pi}{4}\right)$$

$$\frac{\sin 3x\cdot\cancel{\cos x}}{\cos 3x\cdot\cancel{\cos x}}-\frac{\cancel{\cos 3x}\cdot\sin x}{\cancel{\cos 3x}\cdot\cos x}$$

$$\tan 3x-\tan x$$

$$\frac{2\sin x\cos x}{2\cos 3x\cos x}+\frac{2\sin 3x\cos 3x}{2\cos 9x\cos 3x}+\frac{2\sin 9x\cos 9x}{2\cos 27x\cos 9x}=0$$

$$\frac{\sin(2x)}{2\cos 3x\cdot\cos x}+\frac{\sin(6x)}{2\cos 9x\cdot\cos 3x}+\frac{\sin 18x}{2\cos 27x\cdot\cos 9x}=0$$

open. $\frac{\sin(3x-x)}{\cos 3x\cos x}+\frac{\sin(9x-3x)}{\cos 9x\cdot\cos 3x}+\frac{\sin(27x-9x)}{\cos 27x\cos 9x}=0$

$$(\tan\cancel{3x}-\tan x)+(\tan\cancel{9x}-\cancel{\tan 3x})+(\tan 27x-\cancel{\tan 9x})=0$$

$\tan 27x-\tan x=0 \Rightarrow \tan 27x=\tan x$

$27x=n\pi+x$

$26x=n\pi$

$x=\frac{n\pi}{26}$

$\left(\frac{\pi}{26},\frac{2\pi}{26},\frac{3\pi}{26},\frac{4\pi}{26},\frac{5\pi}{26},\frac{6\pi}{26}\right)$

Nishant Jindal

Q14 No. of Sol. of Eqn.

$8 = (2\sqrt{2})^2$

$(\sqrt{3}+1)^{2x} + (\sqrt{3}-1)^{2x} = 2^{3x}$ is 2

$(\sqrt{3}+1)^{2x} + (\sqrt{3}-1)^{2x} = (2\sqrt{2})^{2x} \quad \div (2\sqrt{2})^{2x}$

$\left(\frac{\sqrt{3}+1}{2\sqrt{2}}\right)^{2x} + \left(\frac{\sqrt{3}-1}{2\sqrt{2}}\right)^{2x} = 1$

$(\cos 15^\circ)^{2x} + (\sin 15^\circ)^{2x} = 1$

$\cos^{2x}\theta + \sin^{2x}\theta = 1$ तभी होगा

जब $x=1$

$\cos^2\theta + \sin^2\theta = 1$

Q No. of Sol. of

$\cos^2\left(x+\frac{\pi}{6}\right) + \cos^2\frac{\pi}{6} - 2\cos\left(x+\frac{\pi}{6}\right)\cdot\cos\frac{\pi}{6} = \sin^2 x$ in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$\left(\cos\left(x+\frac{\pi}{6}\right) - \cos\frac{\pi}{6}\right)^2 = \sin^2 x = \left(2\sin\frac{x}{2}\cdot\cos\frac{x}{2}\right)^2$

$\cancel{4}\sin^2\left(\frac{x}{2}+\frac{\pi}{6}\right)\cdot\sin^2\left(\frac{x}{2}\right) = \cancel{4}\sin^2\frac{x}{2}\cdot\cos^2\frac{x}{2}$

$\sin^2\left(\frac{x}{2}\right)\left\{\sin^2\left(\frac{x}{2}+\frac{\pi}{6}\right) - \cos^2\frac{x}{2}\right\} = 0$

$\parallel$

$0$

$\sin\frac{x}{2} = 0$

$x = 2n\pi$

$\cos\left(x+\frac{\pi}{6}\right)\cdot\cos\left(\frac{\pi}{6}\right) = 0$

$\cos\left(x+\frac{\pi}{6}\right) = 0$

$x+\frac{\pi}{6} = (2n+1)\frac{\pi}{2}$