

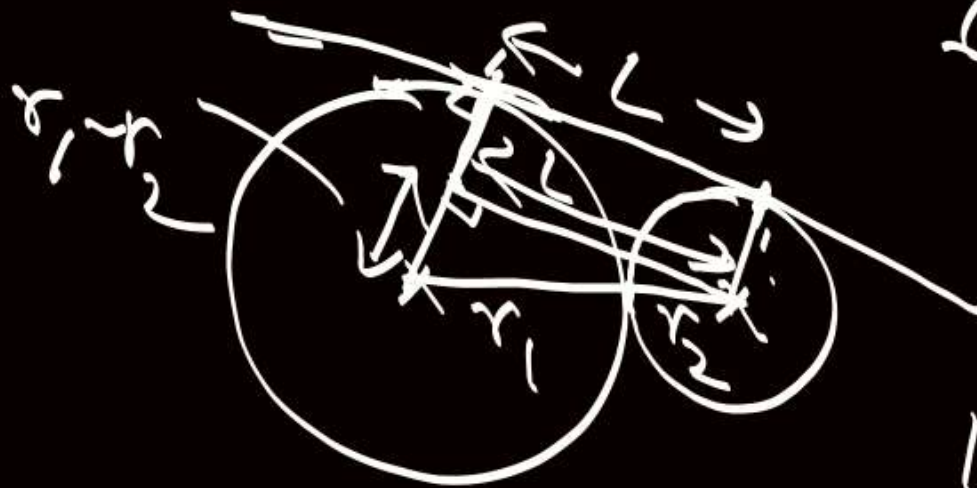
$$\sin \frac{A}{2} = \frac{r-r_1}{r+r_1}$$

$$\tan \frac{A}{2} = \frac{r-r_1}{2\sqrt{r_1 r}}$$

$$\sum \tan \frac{A}{2} \tan \frac{B}{2} = 1$$



$$r = ?$$



$$L = \sqrt{(r_1+r_2)^2 - (r_1-r_2)^2} = 2\sqrt{r_1 r_2}$$

1. Simplify  $\begin{vmatrix} a^2+x^2 & ab & ca \\ ab & b^2+x^2 & bc \\ ca & cb & c^2+x^2 \end{vmatrix} = x^4(a^2+b^2+c^2+x^2)$

$$= \frac{1}{abc} \begin{vmatrix} a(a^2+x^2) & a^2b & ca^2 \\ ab^2 & b(b^2+x^2) & b^2c \\ c^2a & c^2b & c(c^2+x^2) \end{vmatrix} = \begin{vmatrix} a^2+x^2 & a^2 & a^2 \\ b^2 & b^2+x^2 & b^2 \\ c^2 & c^2 & c^2+x^2 \end{vmatrix}$$

$\downarrow R_1 \rightarrow R_1 + R_2 + R_3$

$$\begin{pmatrix} a^2+b^2+c^2+x^2 \end{pmatrix} \begin{vmatrix} 1 & 0 & 0 \\ b^2 & x^2 & 0 \\ c^2 & 0 & x^2 \end{vmatrix}$$

$\leftarrow \begin{pmatrix} a^2+b^2+c^2+x^2 \end{pmatrix}$

$C_2 \rightarrow C_2 - C_1, C_3 \rightarrow C_3 - C_1$

$$\begin{vmatrix} 1 & 1 & 1 \\ b^2 & b^2+x^2 & b^2 \\ c^2 & c^2 & c^2+x^2 \end{vmatrix}$$



2. Simplify

$$C_1 \rightarrow xC_1 + yC_2 + zC_3$$

$$= \frac{(x^2+y^2+z^2)}{x} \begin{vmatrix} a & ay+bx & cx+az \\ b & by-cz-ax & bz+cy \\ c & bz+cy & cz-ax-by \end{vmatrix}$$

$$= \frac{(x^2+y^2+z^2)(a^2+b^2+c^2)}{ax}$$

$$\begin{vmatrix} 1 & y & z \\ b & by-cz-ax & bz+cy \\ c & bz+cy & cz-ax-by \end{vmatrix}$$

$$\begin{vmatrix} 1 & y & z \\ b & by-cz-ax & bz+cy \\ c & bz+cy & cz-ax-by \end{vmatrix}$$

$$\begin{vmatrix} ay+bx & cx+az \\ by-cz-ax & bz+cy \\ bz+cy & cz-ax-by \end{vmatrix}$$

$$(x^2+y^2+z^2)(ax+by+cz)(a^2+b^2+c^2)$$

$$R_1 \rightarrow aR_1 + bR_2 + cR_3$$

$$R_2 \rightarrow R_2 - bR_1$$

$$R_3 \rightarrow R_3 - cR_1$$

$$\begin{vmatrix} 1 & y & z \\ 0 & -cz-ax & cy \\ 0 & bz & -ax-by \end{vmatrix}$$

$$\begin{vmatrix} 1 & y & z \\ 0 & -cz-ax & cy \\ 0 & bz & -ax-by \end{vmatrix}$$

3. Solve 
$$\begin{vmatrix} u+a^2x & l+abx & m+acx \\ l+abx & v+b^2x & n+bcx \\ m+acx & n+bcx & w+c^2x \end{vmatrix} = 0 \text{ for } x$$

expressing result in terms of determinant.

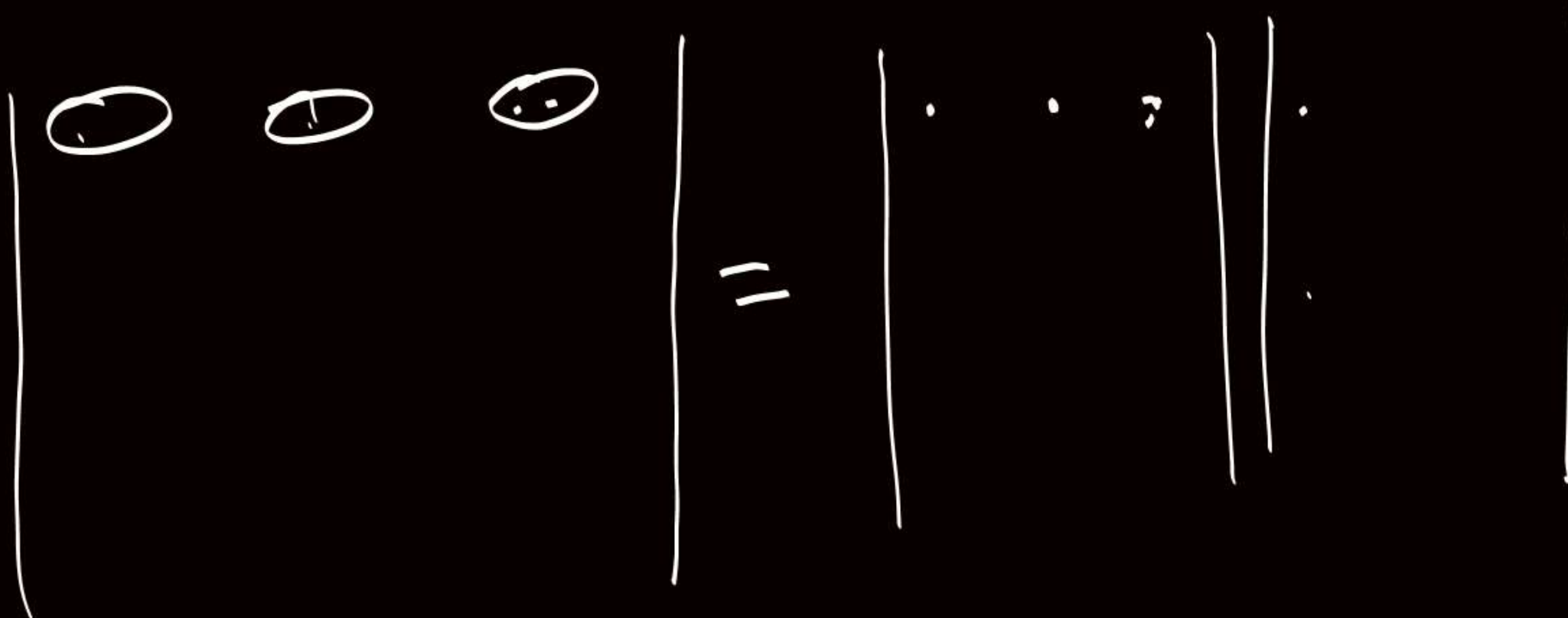
# Product of determinants

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} \begin{vmatrix} d_1 & d_2 & d_3 \\ e_1 & e_2 & e_3 \\ f_1 & f_2 & f_3 \end{vmatrix} = \begin{vmatrix} a_1 d_1 + a_2 e_1 + a_3 f_1 & a_1 d_2 + a_2 e_2 + a_3 f_2 & a_1 d_3 + a_2 e_3 + a_3 f_3 \\ b_1 d_1 + b_2 e_1 + b_3 f_1 & b_1 d_2 + b_2 e_2 + b_3 f_2 & b_1 d_3 + b_2 e_3 + b_3 f_3 \\ c_1 d_1 + c_2 e_1 + c_3 f_1 & c_1 d_2 + c_2 e_2 + c_3 f_2 & c_1 d_3 + c_2 e_3 + c_3 f_3 \end{vmatrix}$$

Row, Column ✓

Row  
Column  
Column

Row  
Row  
Column





$$\therefore \text{Simplify } \begin{vmatrix} a_1 l_1 + b_1 m_1 & a_1 l_2 + b_1 m_2 & a_1 l_3 + b_1 m_3 \\ a_2 l_1 + b_2 m_1 & a_2 l_2 + b_2 m_2 & a_2 l_3 + b_2 m_3 \\ a_3 l_1 + b_3 m_1 & a_3 l_2 + b_3 m_2 & a_3 l_3 + b_3 m_3 \end{vmatrix}$$

$$= \begin{vmatrix} a_1 & b_1 & 0 \\ a_2 & b_2 & 0 \\ a_3 & b_3 & 0 \end{vmatrix} \begin{vmatrix} l_1 & l_2 & l_3 \\ m_1 & m_2 & m_3 \\ 0 & 0 & 0 \end{vmatrix} = 0$$

2.

$$a_1^2 - 2a_1b_1 + b_1^2 \quad \begin{vmatrix} (a_1-b_1)^2 & (a_1-b_2)^2 & (a_1-b_3)^2 \\ (a_2-b_1)^2 & (a_2-b_2)^2 & (a_2-b_3)^2 \\ (a_3-b_1)^2 & (a_3-b_2)^2 & (a_3-b_3)^2 \end{vmatrix} = \begin{vmatrix} a_1^2 & -2a_1 & 1 \\ a_2^2 & -2a_2 & 1 \\ a_3^2 & -2a_3 & 1 \end{vmatrix} \begin{vmatrix} 1 & 1 & 1 \\ b_1 & b_2 & b_3 \\ b_1^2 & b_2^2 & b_3^2 \end{vmatrix}$$

$$2 \begin{vmatrix} 1 & a_1 & a_1^2 \\ 1 & a_2 & a_2^2 \\ 1 & a_3 & a_3^2 \end{vmatrix} \begin{vmatrix} 1 & b_1 & b_1^2 \\ 1 & b_2 & b_2^2 \\ 1 & b_3 & b_3^2 \end{vmatrix}$$

$$\begin{array}{l} \Sigma x - \text{III} \\ \Sigma x - \text{IV} \end{array}$$

$$\underline{\underline{a a.5 +}}$$

$$= 2(a_1 - a_2)(a_2 - a_3)(a_3 - a_1) \\ (b_1 - b_2)(b_2 - b_3)(b_3 - b_1)$$