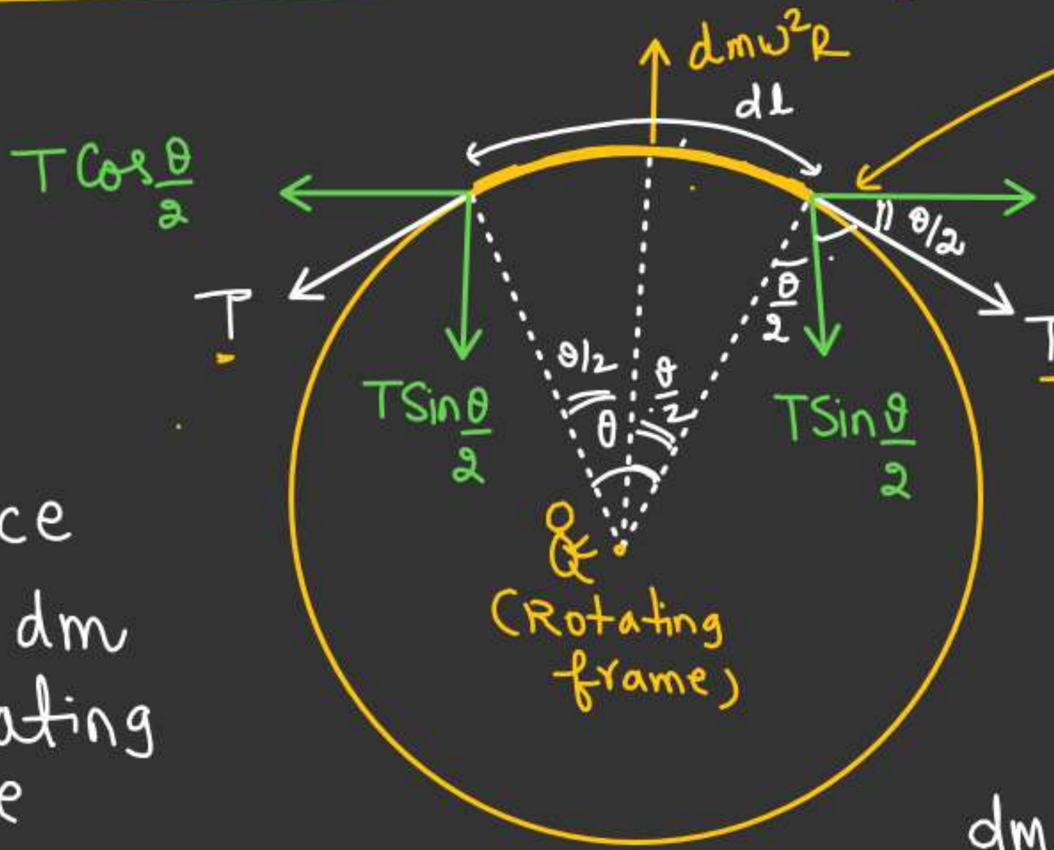
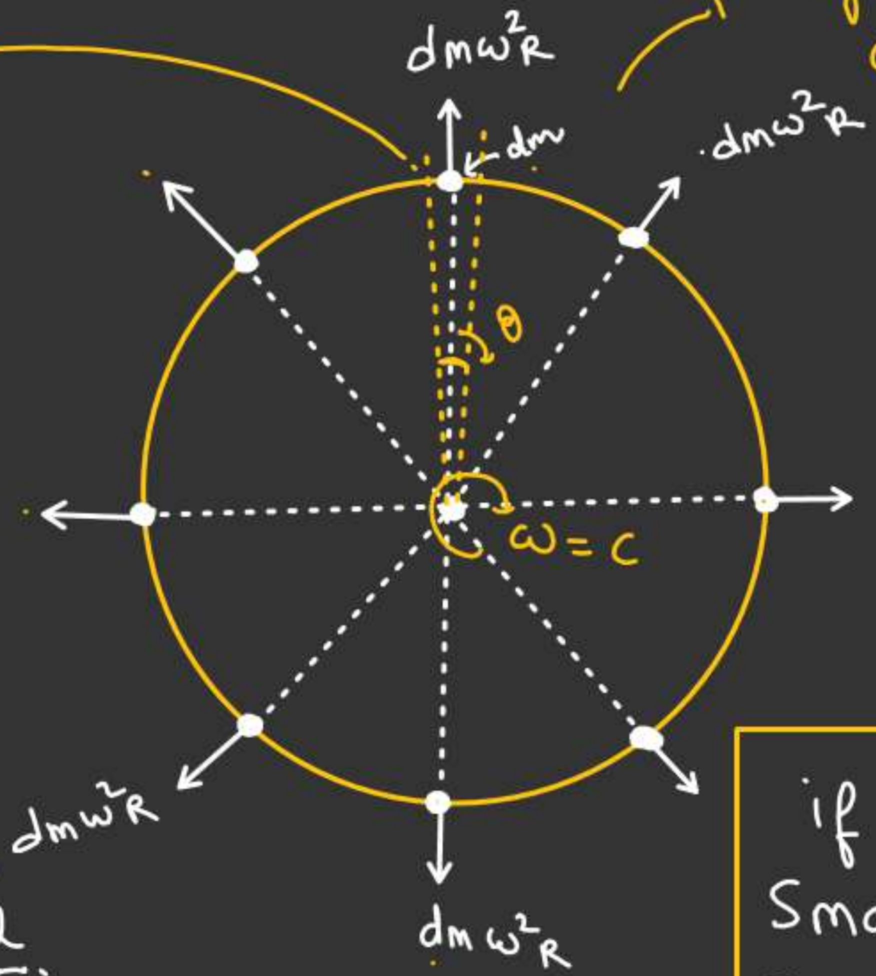
Nishant Jindal

CIRCULAR MOTION

Tension in a rotating ring



$d\ell = R\theta$



System on a smooth horizontal table

Net force zero on dm in rotating frame

$$dm\omega^2 R = 2T\sin\frac{\theta}{2}$$

$$\left(\frac{M}{2\pi}\right)\cancel{\theta}\,\omega^2 R = \cancel{2}T\left(\frac{\cancel{\theta}}{\cancel{2}}\right)$$

$$\boxed{T = \frac{M\omega^2 R}{2\pi}}$$

$$dm = \left(\frac{M}{L}\right) \times d\ell$$

$$dm = \frac{M}{2\pi R} \times R\theta = \left(\frac{M}{2\pi}\theta\right)$$

θ is very small

$\sin\frac{\theta}{2} \simeq \frac{\theta}{2}$

if θ is very small

$\sin\theta \simeq \theta$

$\tan\theta \simeq \theta$.

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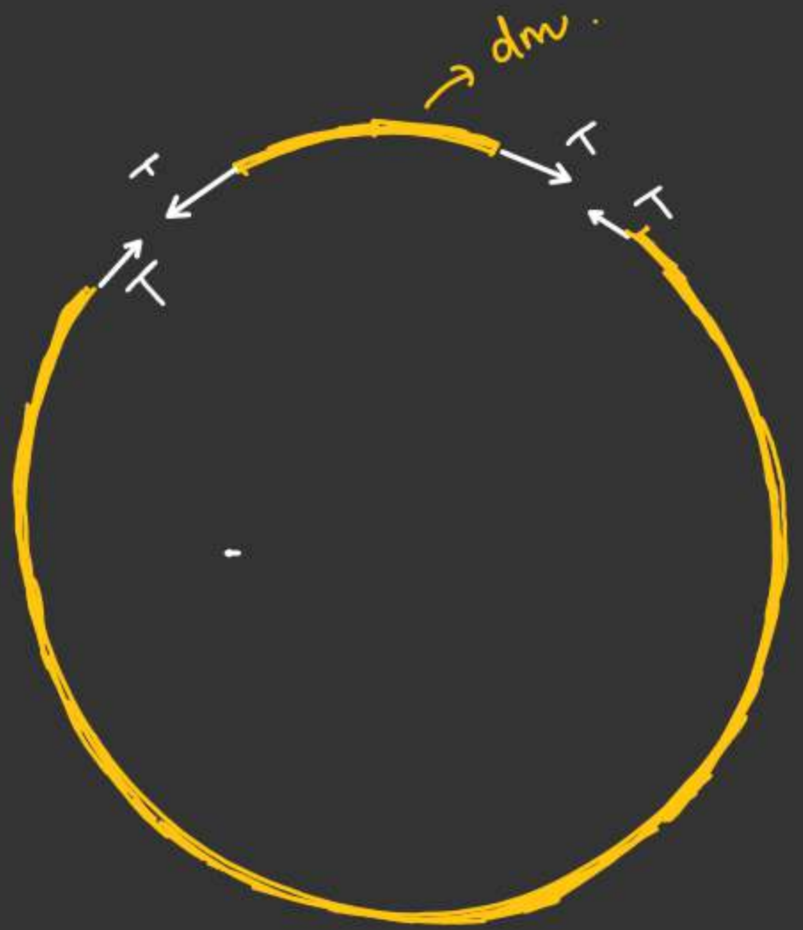
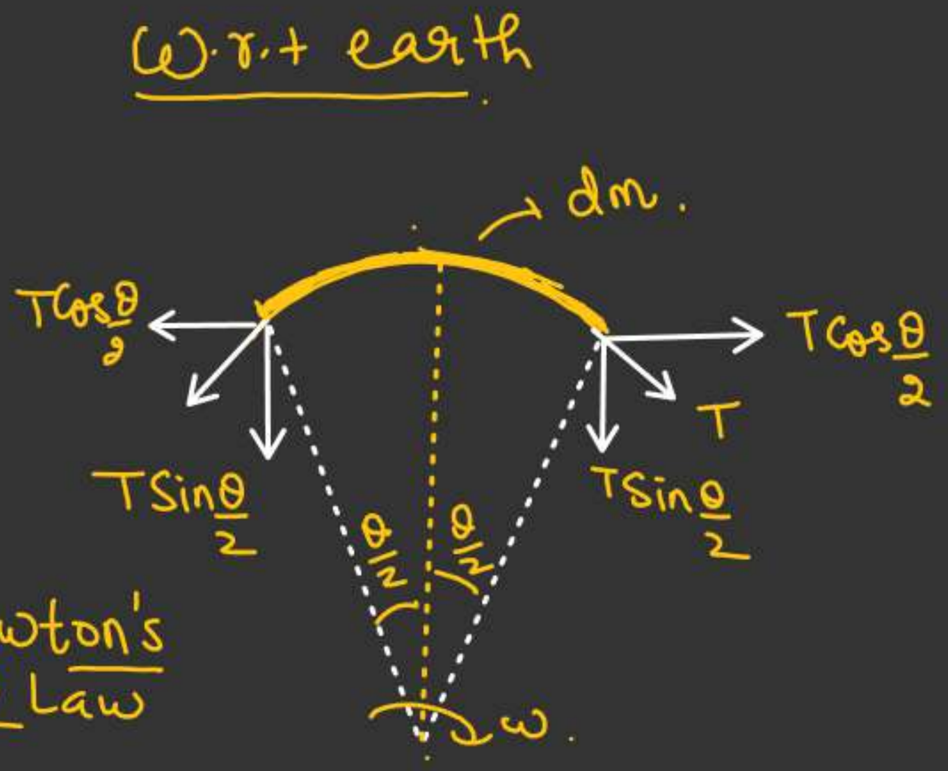


$$T = \frac{m\omega^2 R}{2\pi}$$

w.r.t earth



Newton's 2nd Law

$$\left[2T\sin\frac{\theta}{2} = dm\,\omega^2 R\right]$$

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Q:

Find $\theta = ??$

$r = (L + l\sin\theta)$

w.r.t earth

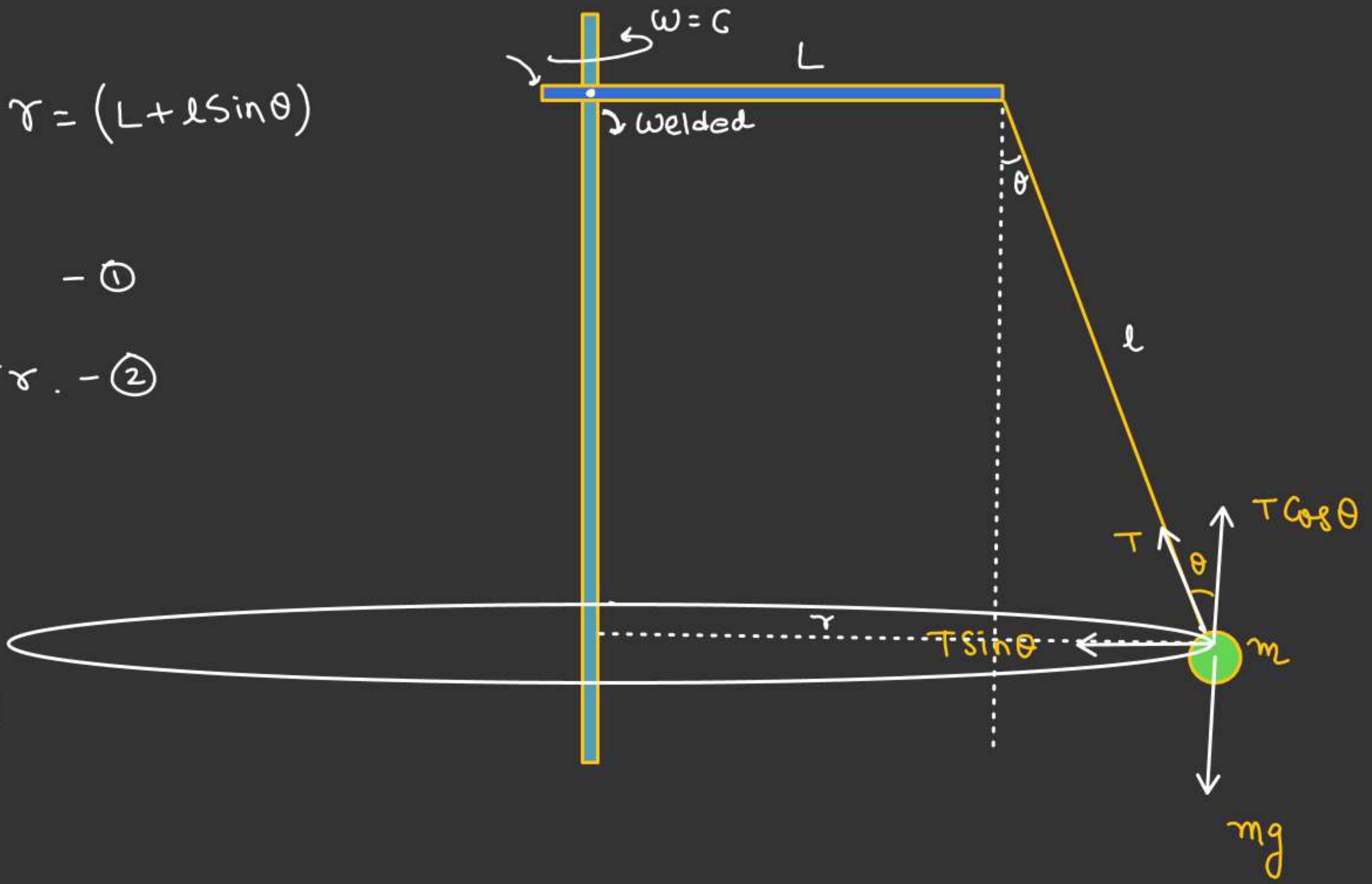
$T\cos\theta = mg \quad -①$

$T\sin\theta = m\omega^2 r. \quad -②$

② ÷ ①

$\tan\theta = \left(\frac{\omega^2 r}{g}\right)$

$\tan\theta = \frac{\omega^2}{g}(L + l\sin\theta)$



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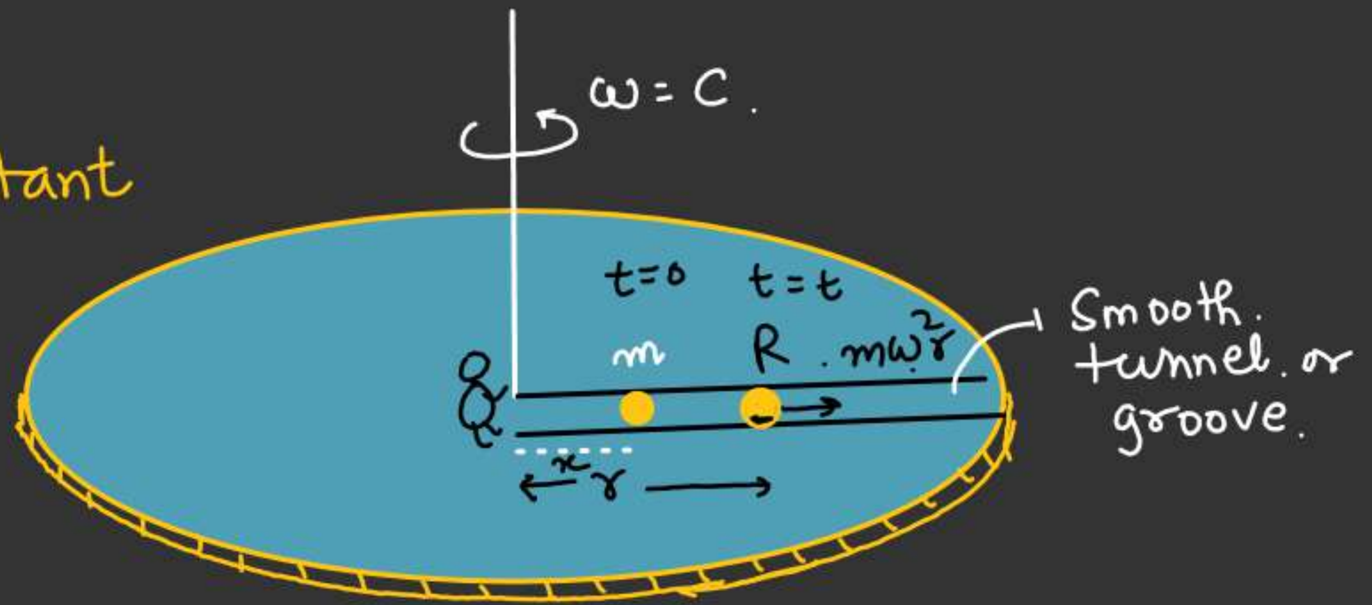
A Groove on a rotating disc is made. Disc rotate with constant angular velocity ω. A particle of mass m gently released in the groove. find velocity of particle as a function of radial distance r. w.r.t disc



At $t = t$, let, ball is at a radial distance r from the center of the disc.

In rotating frame, Along the tunnel.

$$\cancel{m}\omega^2 r = \cancel{m} a$$

$$a = \omega^2 r$$

$$v\frac{dv}{dr} = \omega^2 r$$

$$\int_0^v v\,dv = \omega^2 \int_x^r r\,dr$$

$$\frac{v^2}{\cancel{2}} = \omega^2 \frac{(r^2 - x^2)}{\cancel{2}}$$

$$v = \omega\sqrt{r^2 - x^2} \checkmark$$

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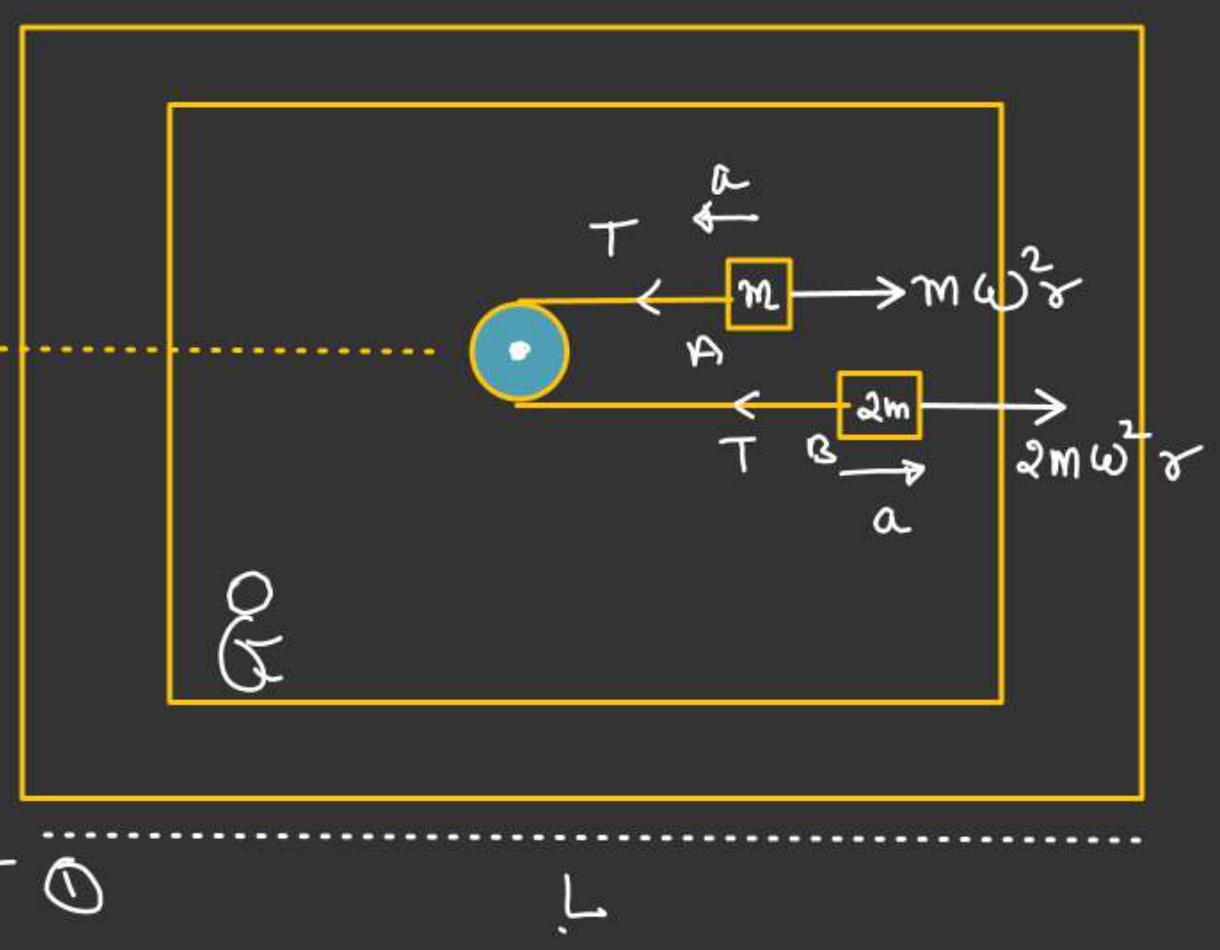
The whole table is rotating in circle of radius r which is very large as compared to dimension of the table. Pully is fixed on the table with two blocks attached with a string.

$[r >> L]$

Top View.



a) Find accleration of the blocks w.r.t table.

b) Tension in the String

w.r.t Rotating frame :-

For block A

$$T - m\omega^2 r = ma \quad -(1)$$

For block B

$$2m\omega^2 r - T = 2ma \quad -(2)$$

(1) + (2)

$$m\omega^2 r = 3ma$$

$$a = \left(\frac{\omega^2 r}{3}\right)$$

From (1)

$$T = ma + m\omega^2 r = \frac{m\omega^2 r}{3} + m\omega^2 r = \frac{4m\omega^2 r}{3}\ \text{N.}$$

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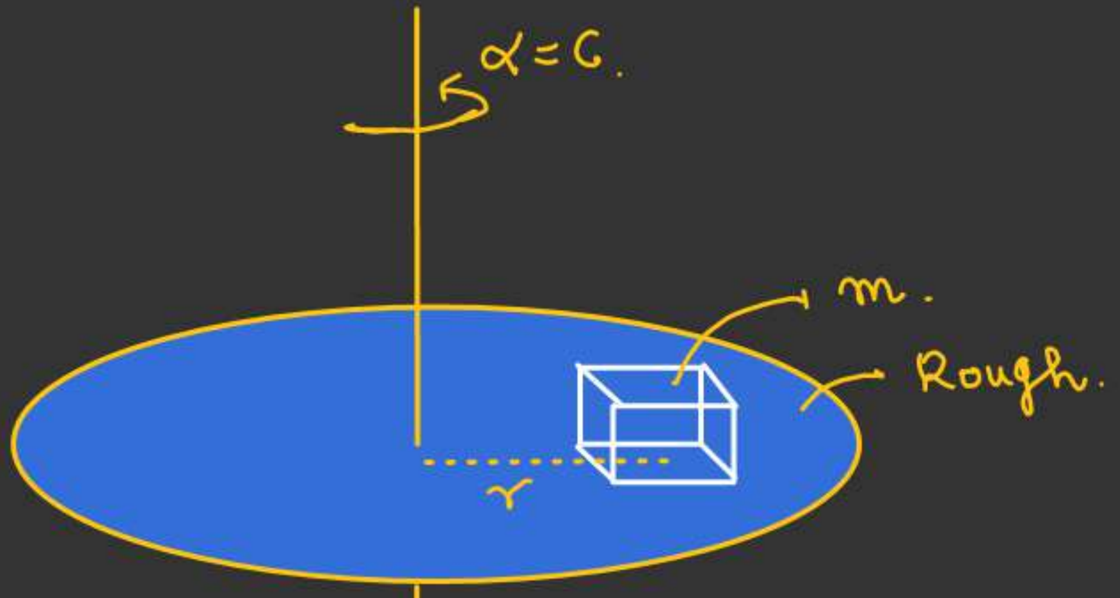
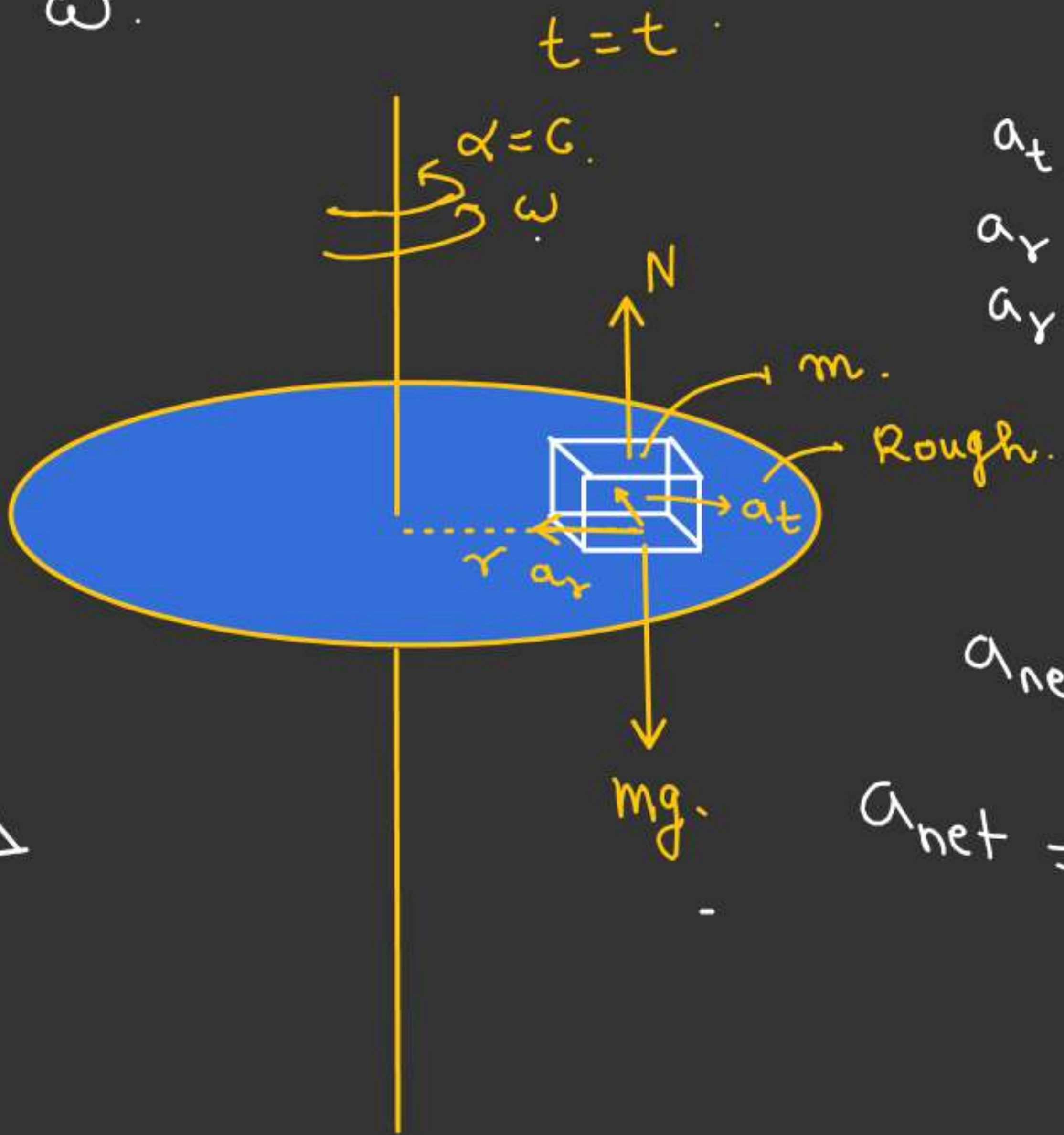
Disc rotating with constant angular accleration α.

If block is at rest w.r.t disc ie no slipping of disc w.r.t disc.

Find a) Accleration of block at $t=t$.

Sol^n At $t=t$, let angular velocity of disc be ω.





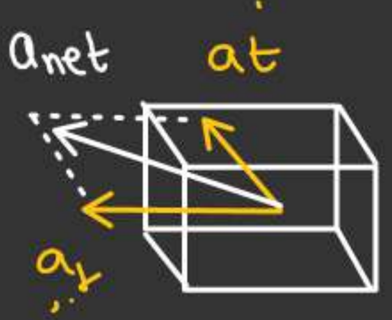
$a_t = r\alpha$ ✓

$\omega = \cancel{\omega_0}^{\,0} + \alpha t$

$a_r = \omega^2 r$

$a_r = \alpha^2 r \cdot t^2$ ✓

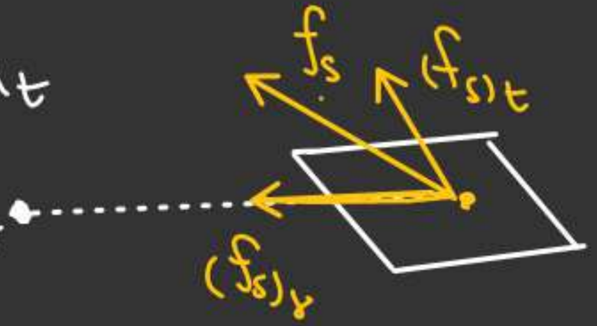
$\omega = \alpha t$



$f_s = m a_{net}$

$(f_s)_t = m a_t$

$(f_s)_r = m a_r$



$$\sqrt{a_t^2 + a_r^2} = a_{net}$$

$$a_{net} = \sqrt{r^2\alpha^2 + (\alpha^2 t^2 r)^2}$$

$$a_{net} = \sqrt{\alpha^2 r^2 + r^2\alpha^4 t^4} = \underline{\alpha r\sqrt{1+\alpha^2 t^4}}$$ ✓

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$$f_s = m a_{net}.$$

$$f_s = m \alpha r \sqrt{1+\alpha^2 t^4}.$$

At the time of Slipping

$$f_s = (f_s)_{max} = \mu m g$$

$\Downarrow$

$t = ??$ $\Rightarrow$ [Time when block starts Slipping.]