Q EOP P.T. (1,1,1) $\perp^r$ to planes $2x+y-2z=5$ & $3x-6y-2z=7$ is?

Adv 2017

1) Normal of asked Plane can be found out using $n_1 \times n_2$

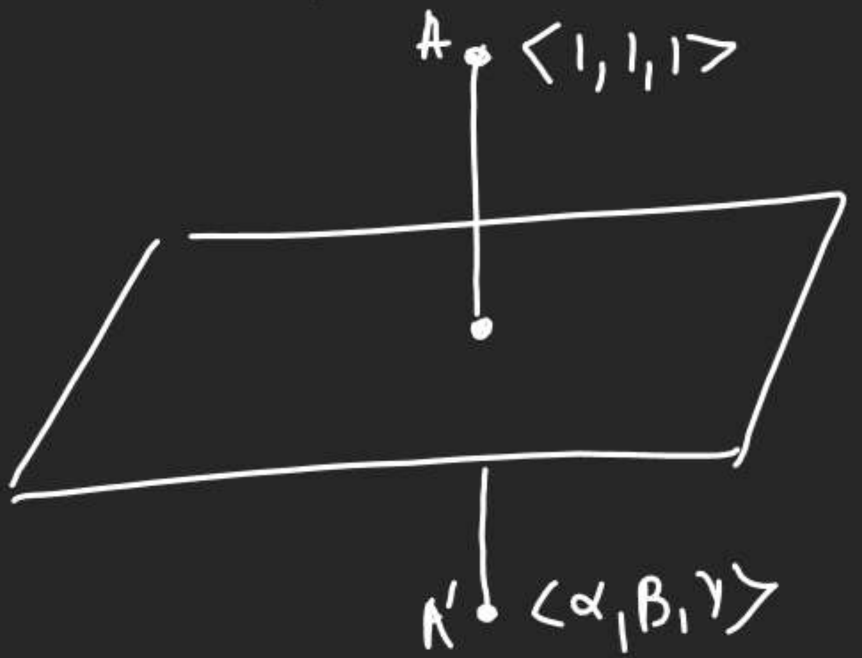
$$\vec{n} = \begin{vmatrix} i & j & k \\ 2 & 1 & -2 \\ 3 & -6 & -2 \end{vmatrix} = \langle -14, -2, -15 \rangle$$

2) $-14(x-1) - 2(y-1) - 15(z-1) = 0$

$-14x - 2y - 15z = -31$

$14x + 2y + 15z = 31$

Q Find Image of $\langle 1,1,1 \rangle$ in $x - y + z = 2$



$$\frac{\alpha-1}{1} = \frac{\beta-1}{-1} = \frac{\gamma-1}{1} = \frac{-2(1-1+1-2)}{1^2+(-1)^2+1^2}$$

$$\frac{\alpha-1}{1} = \frac{\beta-1}{-1} = \frac{\gamma-1}{1} = \frac{2}{3}$$

$\alpha = \frac{5}{3}$, $\beta = \frac{1}{3}$, $\gamma = \frac{5}{3}$

$\left\langle \frac{5}{3}, \frac{1}{3}, \frac{5}{3} \right\rangle$

Q Let P is Image of $\langle 3,1,7 \rangle$ in Plane $x - y + z = 3$ Then EOP P.T. P & containing Line $\frac{x}{1} = \frac{y}{2} = \frac{z}{1}$

ADV 2016

A) P: $\frac{\alpha-3}{1} = \frac{\beta-1}{-1} = \frac{\gamma-7}{1} = \frac{-2(3-1+7-3)}{1^2+1^2+1^2}$

$\frac{\alpha-3}{1} = \frac{\beta-1}{-1} = \frac{\gamma-7}{1} = -4$

$\langle \alpha, \beta, \gamma \rangle = \langle -1, 5, 3 \rangle$

Line's DR $= \langle 1,2,1 \rangle$

(B) Plane's DR $= \begin{vmatrix} i & j & k \\ 1 & 2 & 1 \\ -1 & 5 & 3 \end{vmatrix} = \langle 1, -4, 7 \rangle$

EOP

$1(x-0) - 4(y-0) + 7(z-0) = 0$

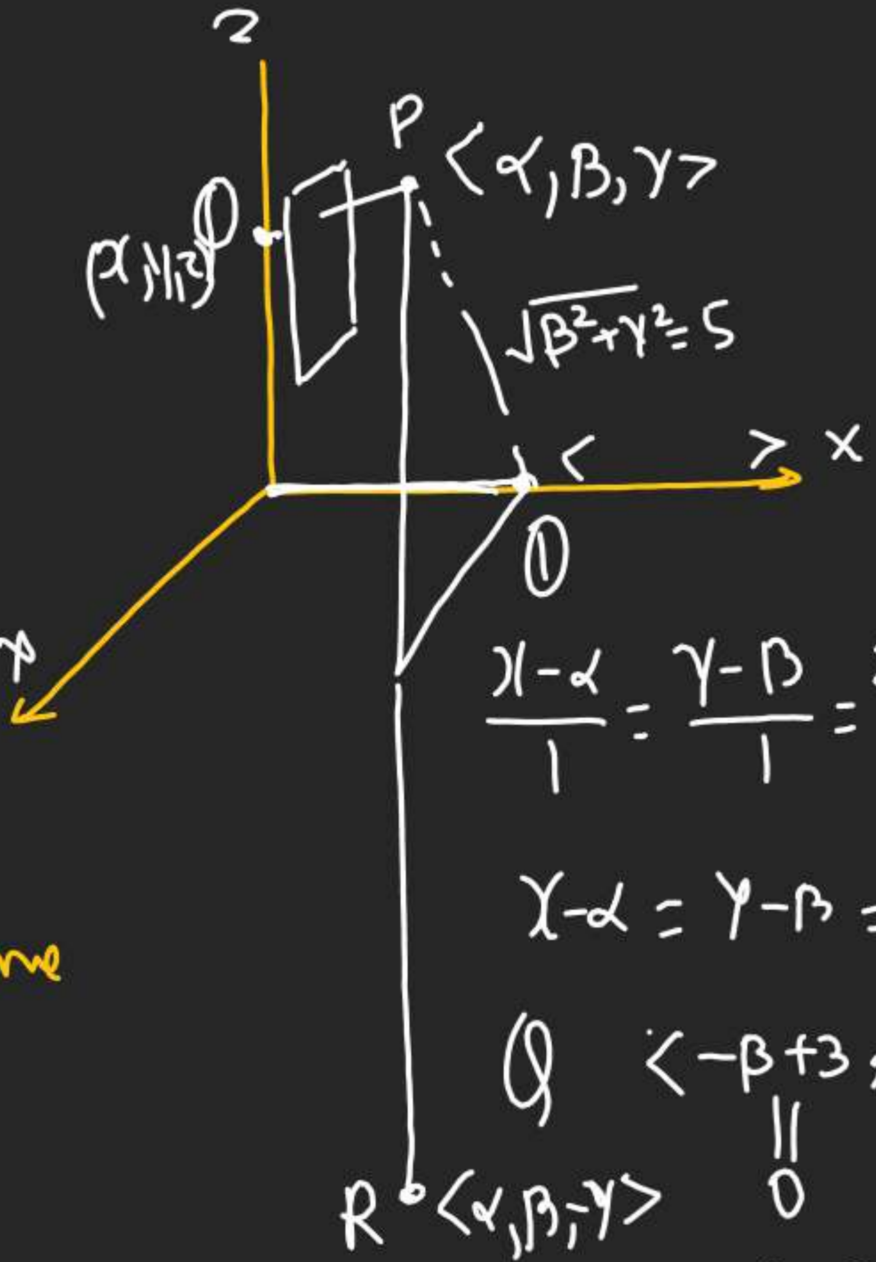
$x - 4y + 7z = 0$

Nishant Jindal

Q Let P be a pt. in 1^{st} Octant

Adv

Whose Image Q in Plane is $x+y=3$ {that is, the Line Segment PQ is $\perp^r$ to Plane $x+y=3$ & Mid Pt. of PQ lies in Plane $x+y=3$} lies on Z Axis. Let the distance of P from x Axis be 5. If R is image of P in xy Plane then length of PR is?



$L_{PR} = 2\gamma$

$$\frac{x-\alpha}{1} = \frac{y-\beta}{1} = \frac{z-\gamma}{0} = \frac{-2(\alpha+\beta-3)}{1^2+1^2+0^2}$$

$$x-\alpha = y-\beta = z-\gamma = -\alpha-\beta+3$$

Q $\langle -\beta+3, -\alpha+3, \gamma \rangle$ lies on Z Axis

$-\beta+3 = 0$, $-\alpha+3 = 0$

$\beta = 3$, $\alpha = 3$

So $P = \langle 3, 3, \gamma \rangle$

(2) Hint 2 → $\sqrt{\beta^2+\gamma^2} = 5$

$\sqrt{9+\gamma^2} = 5 \Rightarrow \gamma = 4$

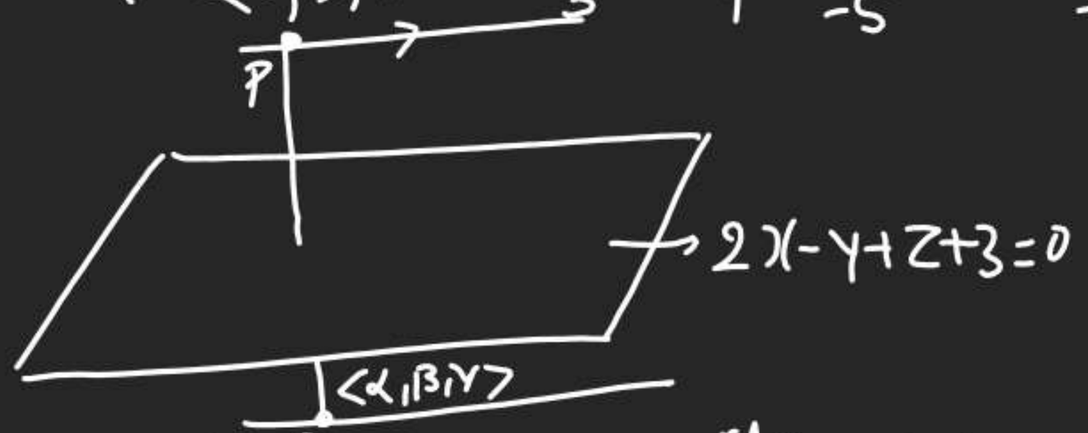
(3) Hint 3

$PR = 2\gamma = 8$

Nishant Jindal

Q Image of Line $\frac{x-1}{3}=\frac{y-3}{1}=\frac{z-4}{-5}$ in Plane $2x-y+z+3=0$ is line = ?

Adv Board



(1) Cond^n of Line Check करो ll^{rl} है या Intersecting है?

$2\times3+-1\times1++1\cdot-5 = 6-1-5=0$

(2) Fix pt on Image वाली Line is Image of <1,3,4> in Plane.

$\frac{\alpha-1}{2}=\frac{\beta-3}{-1}=\frac{\gamma-4}{1}=\frac{-2\cancel{(2-3+4+3)}}{\cancel{2^2+1^2+1^2}}$

$\frac{\alpha-1}{2}=\frac{\beta-3}{-1}=\frac{\gamma-4}{1}=-2$

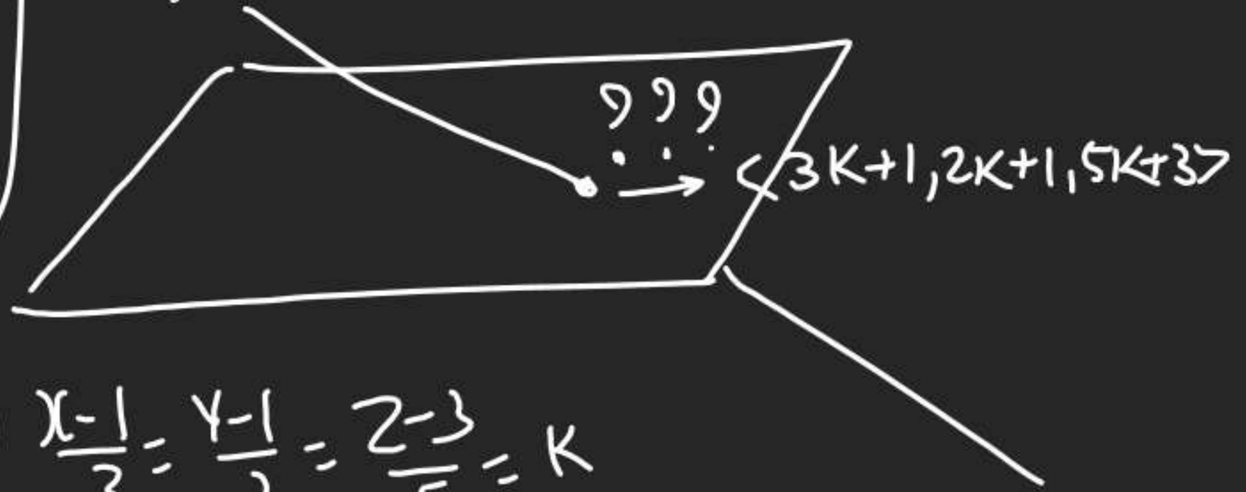
$Q = \langle -3, 5, 2\rangle$

(3) Image वाली Line's DR = Main Line's DR as Lines are ll^{rl}.

$\Rightarrow$ ∴ so Image's Line

$\frac{x+3}{3}=\frac{y-5}{1}=\frac{z-2}{-5}$

Q If Line $\frac{x-1}{3}=\frac{y-1}{2}=\frac{z-3}{5}$ Intersect Plane $x-y+z=2$ find PoI.



(1) $\frac{x-1}{3}=\frac{y-1}{2}=\frac{z-3}{5}=K$

Gen. Pt. $\langle 3K+1, 2K+1, 5K+3\rangle$

(2) as Pt lies on Plane so It will satisfy.

$(3K+\cancel{1})-(2K+\cancel{1})+(5K+3)=2$

$6K=-1 \Rightarrow K=-\frac{1}{6}$

(3) ∴ Pt of Int $=\langle -3\times\frac{1}{6}+1, -2\times\frac{1}{6}+1, -5\times\frac{1}{6}+3\rangle$

$=\langle \frac{1}{2}, \frac{2}{3}, \frac{13}{6}\rangle$

Nishant Jindal

① 3 Lines are given by

Adv Board

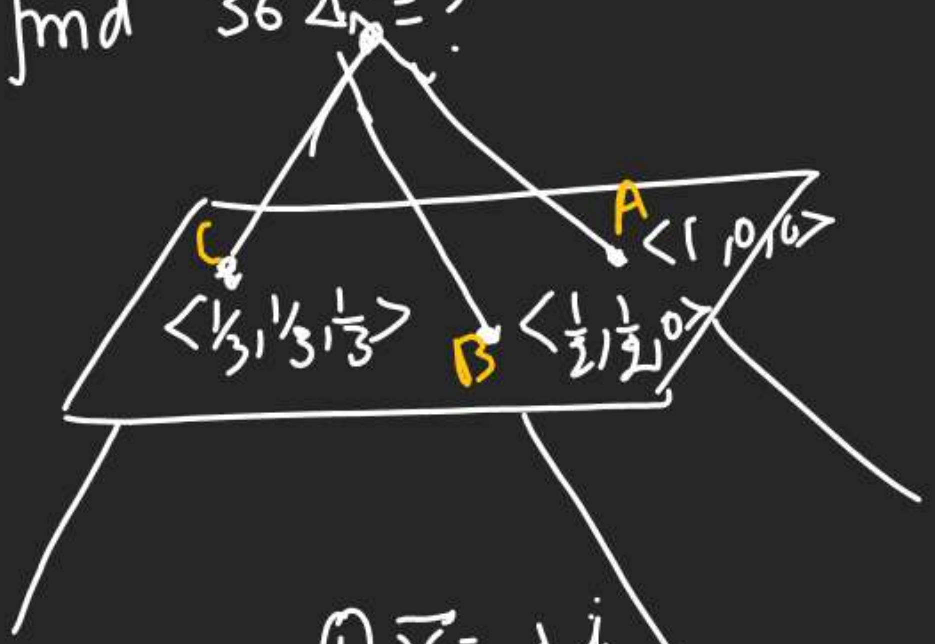
$\vec{r} = \lambda\hat{i}$, $\vec{r} = \mu(\hat{i}+\hat{j})$

$\vec{r} = \nu(\hat{i}+\hat{j}+\hat{k})$

Let Lines cut Plane $x+y+z=1$

at A, B, C & Area of Δ ABC is

Δ find $36\Delta^2 = ?$



① $\vec{r} = \lambda i$

$\vec{r} = \langle 0,0,0\rangle + \lambda\langle 1,0,0\rangle$

$\frac{x-0}{1} = \frac{y-0}{0} = \frac{z-0}{0} = K \Rightarrow \langle K,0,0\rangle \Rightarrow K+0+0=1 \Rightarrow K=1$

(2) $\vec{r} = \mu\langle \hat{i}+\hat{j}\rangle$

$\vec{r} = \langle 0,0,0\rangle + \mu\langle 1,1,0\rangle$

G.P.

$\frac{x-0}{1} = \frac{y-0}{1} = \frac{z-0}{0} = m \Rightarrow \langle m,m,0\rangle$ lies $x+y+z=1$

$m+m+0=1 \Rightarrow m=\frac{1}{2}$

(3) $\vec{r} = \nu(\hat{i}+\hat{j}+\hat{k})$

$\Rightarrow \vec{r} = \langle 0,0,0\rangle + \nu\langle 1,1,1\rangle$

$\Rightarrow \frac{x}{1} = \frac{y}{1} = \frac{z}{1} = n$

G.P $\Rightarrow \langle n,n,n\rangle$

$n+n+n=1 \Rightarrow n=\frac{1}{3}$.

$36 \times \frac{3}{144} = .75$

(4) $\Delta = \frac{1}{2}\left|a\times b + b\times c + c\times a\right|$

$= \frac{1}{2}\left|\vec{AB}\times\vec{AC}\right|$

$= \frac{1}{2}\begin{vmatrix} i & j & k \\ -\frac{1}{2} & \frac{1}{2} & 0 \\ -\frac{2}{3} & \frac{1}{3} & \frac{1}{3} \end{vmatrix} = \frac{1}{12}\begin{vmatrix} i & j & k \\ -1 & 1 & 0 \\ -2 & 1 & 1 \end{vmatrix} = \frac{1}{12}\langle 1,1,1\rangle = \frac{\sqrt{3}}{12}$

$\vec{AB} = \langle -\frac{1}{2}, \frac{1}{2}, 0\rangle$

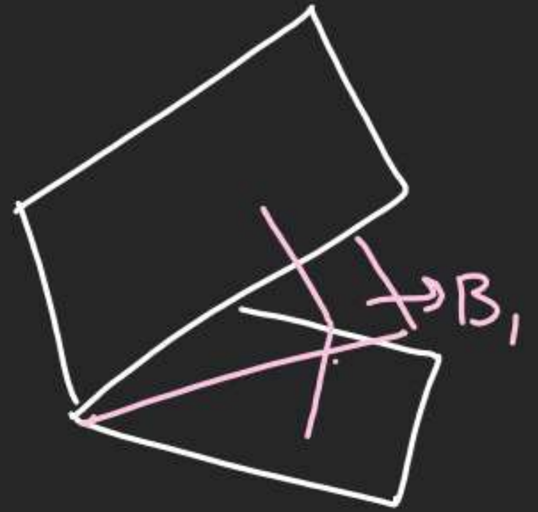
$\vec{AC} = \langle -\frac{2}{3}, \frac{1}{3}, \frac{1}{3}\rangle$

Nishant Jindal

Eqⁿ of Bisector Plane.

$P_1 : a_1x+b_1y_1+c_1z+d_1=0$

$P_2 \Rightarrow a_2x+b_2y+c_2z+d_2=0$



Bisector Plane.

$$\frac{a_1x+b_1y+c_1z+d_1}{\sqrt{a_1^2+b_1^2+c_1^2}} = \pm\frac{(a_2x+b_2y+c_2z+d_2)}{\sqrt{a_2^2+b_2^2+c_2^2}}$$

Family of Plane.

1) If P_1 & P_2 are 2 Planes then all the Plane P.T. Joint of P_1 & P_2 are their Family of Planes.

2) New plane P.T. Joint of P_1 & P_2 will be $P : P_1+\lambda P_2=0$

Q In R^3, Consider Plane
Adv Board $P_1 : y=0$ & $P_2 : x+z=1$
Let P_3 be a plane different from P_1 & P_2 which passes Intersection of P_1 & P_2. If distance of pt. $\langle 0,1,0\rangle$ from P_3 is 1 & distance of $\langle \alpha,\beta,\gamma\rangle$ from P_3 is 2 find Relation betⁿ α,β,γ?

1) It is obvious that P_3 is Family of P_1 & $P_2 \Rightarrow P_3 : x+z-1+\lambda y=0$

(2) dist of P_3 from $\langle 0,1,0\rangle = 1$

$$\frac{|0+\lambda+0-1|}{\sqrt{1^2+1^2+\lambda^2}} = 1 \Rightarrow |\lambda-1| = \sqrt{2+\lambda^2}$$

$$\cancel{\lambda^2}-2\lambda+1 = 2+\cancel{\lambda^2} \Rightarrow -2\lambda=1 \Rightarrow \lambda=-\tfrac{1}{2}$$

(3) $\therefore P_3 : x-\frac{y}{2}+z=1$

(4) $$\frac{|\alpha-\frac{\beta}{2}+\gamma+1|^2}{\sqrt{1+1/4+1}} = 9 \Rightarrow |\alpha-\tfrac{\beta}{2}+\gamma+1| = 3$$

$\alpha-\frac{\beta}{2}+\gamma = 2$