Nishant Jindal

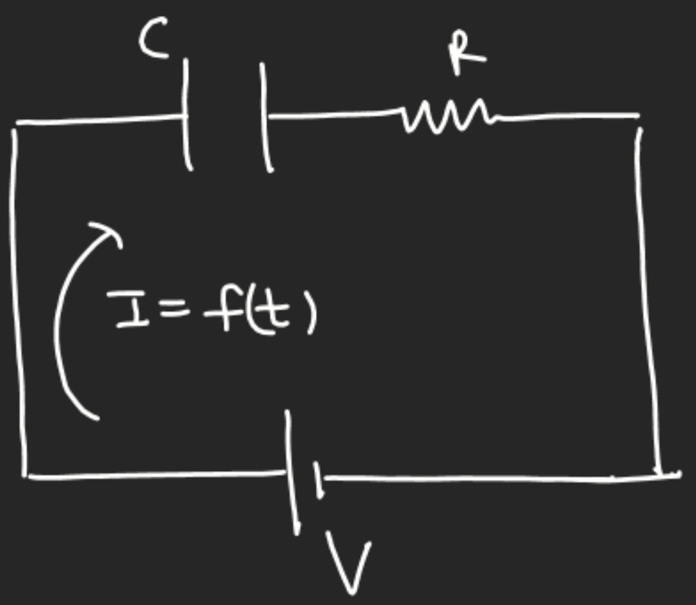
RC ckt

CURRENT ELECTRICITY

Power dissipiated in R-C Ckt :→

$$I = I_0 e^{-t/\tau}$$

$$I_0 = \left(\frac{V}{R}\right)$$



$$P = i^2 R$$

$$P = \left(I_0 e^{-t/\tau}\right)^2 \cdot R$$

$$\boxed{P = (I_0^2 R)\, e^{-2t/\tau}}$$

↳ $P = f(t)$

(*) Total heat energy dissipiated

$$P = \frac{dH}{dt} \rightarrow \int_0^H dH = \int_0^\infty P \cdot dt$$

$$H = I_0^2 R \int_0^\infty e^{-2t/\tau} \cdot dt$$

$$H = I_0^2 R \int_0^\infty e^{\left(-\frac{2}{\tau}\right)t} \cdot dt$$

$$H = I_0^2 R \frac{\left[e^{-\frac{2}{\tau}t}\right]_0^\infty}{\left(-\frac{2}{\tau}\right)}$$

$$H = I_0^2 R \times \left(\frac{-\tau}{2}\right)\left[e^{-\infty} - e^0\right]$$

$$H = \frac{I_0^2 R \tau}{2}$$

$I_0 = \frac{V}{R}$

$$\to H = \frac{V^2}{R^2} \times R \times \frac{RC}{2}$$

$$H = \frac{1}{2} C V^2$$

$$\boxed{H = U}$$

**

$$\boxed{\int e^{-\alpha x} \cdot dx = \frac{e^{-\alpha x}}{-\alpha}}$$

$$\int e^x \cdot dx = e^x$$

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$$\int e^{-\alpha x}\,dx = \left[\frac{e^{-\alpha x}}{-\alpha}\right].$$

44. Find avg power in the interval $t=0$ to $t=\tau$. in R-C Ckt during discharging of Capacitor:-

$$y = f(x)$$

$$y_{avg} = \left[\frac{\int_{x_i}^{x_f} y\cdot dx}{\int_{x_i}^{x_f} dx}\right]$$

$$I_0 = \frac{V}{R}$$

$$P = i^2 R$$

$$I = I_0 e^{-t/\tau}$$

$$P_{inst} = \left(I_0^2 R\, e^{-2t/\tau}\right)$$

$$P_{avg} = \frac{\int_0^\tau P\cdot dt}{\int_0^\tau dt} = \frac{I_0^2 R}{\tau}\int_0^\tau e^{-\frac{2t}{\tau}}\cdot dt$$

$$P_{avg} = \frac{I_0^2 R}{\tau}\frac{\left[e^{-\frac{2t}{\tau}}\right]_0^\tau}{\left(-\frac{2}{\tau}\right)} = -\frac{I_0^2 R}{2}\left[e^{-2} - e^0\right]$$

$$P_{avg} = -\frac{I_0^2 R}{2}\left(\frac{1}{e^2}-1\right) = \frac{I_0^2 R}{2}\left(1-\frac{1}{e^2}\right) = \underline{\frac{V^2}{2R}\left(1-\frac{1}{e^2}\right)}$$ ✓

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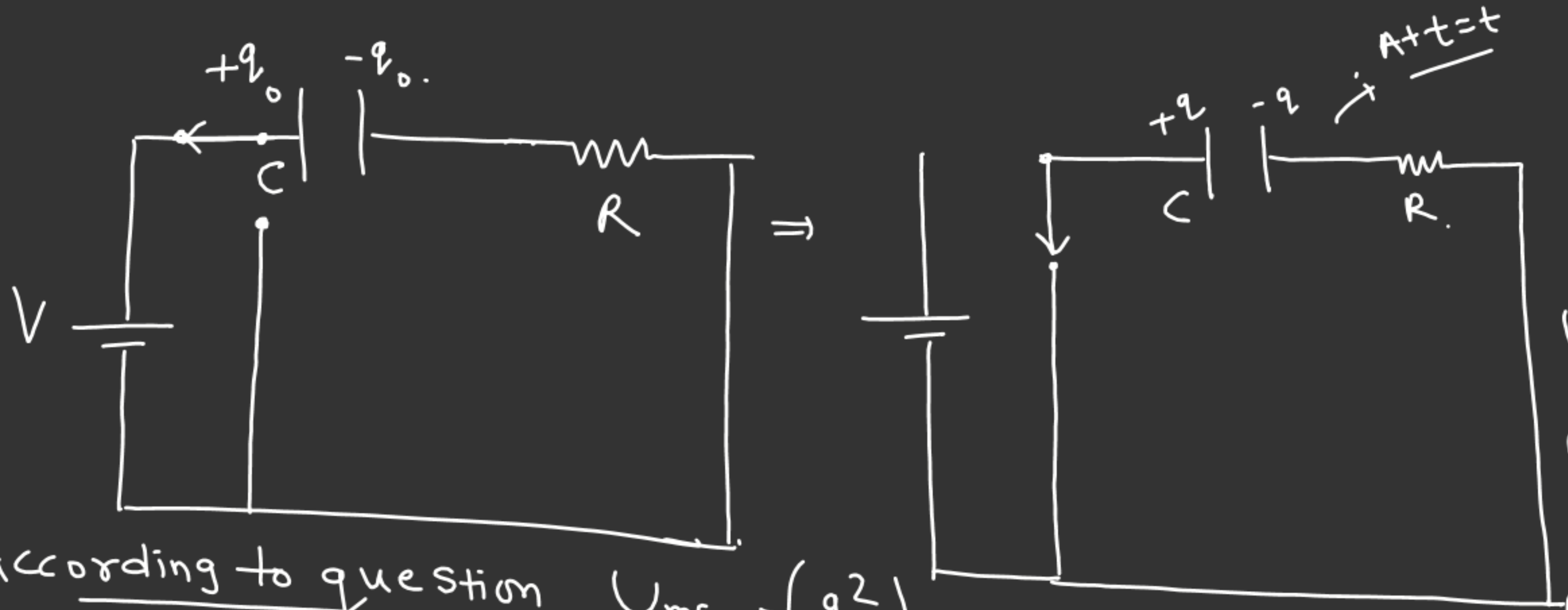
Find the time when energy stored in the Capacitor become half of its maximum value during discharging of Capacitor ??

Sol^n.



During discharging.

$q \to f(t)$

$$\left(q = q_0 e^{-t/\tau}\right)$$

$$U = \frac{q^2}{2C}$$

$$U = \frac{q_0^2}{2C}\left(e^{-2t/\tau}\right)$$

$\Downarrow$

Energy at any instant in the Capacitor during discharing

According to question $\quad U_{max} = \left(\frac{q_0^2}{2C}\right)$

$$U = \frac{U_{max}}{2}$$

$$\frac{\cancel{q_0^2}}{\cancel{2C}} e^{\frac{-2t}{\tau}} = \frac{1}{2}\left(\frac{\cancel{q_0^2}}{\cancel{2C}}\right) \Rightarrow e^{\frac{-2t}{\tau}} = \frac{1}{2} \Rightarrow \frac{-2t}{\tau} = -\ln 2$$

$$t = \left(\frac{\tau \ln 2}{2}\right) = \tau\left(\frac{0.693}{2}\right) = 0.34\tau$$

RC ckt

CURRENT ELECTRICITY

$$\boxed{q = q_0(1 - e^{-t/\tau})}$$

Q. During discharging of R-C Ckt find time when

a) Current become half of its maximum value

b) Charge become half of its maximum value

Sol^n

$$\boxed{I = I_0 e^{-t/\tau}}$$

$I = I_0/2$

$I_0 e^{-t/\tau} = (I_0/2)$

$e^{-t/\tau} = -\ln(2)$

$t = \tau \ln 2$

$q = q_0 e^{-t/\tau}$

$q = (q_0/2)$

$\frac{q_0}{2} = q_0 e^{-t/\tau}$

$\frac{1}{2} = e^{-t/\tau}$

$(\cancel{-} t/\tau) = \ln(\frac{1}{2}) = \cancel{-} \ln(2)$

$$\boxed{t = \tau \ln 2}$$ ✓

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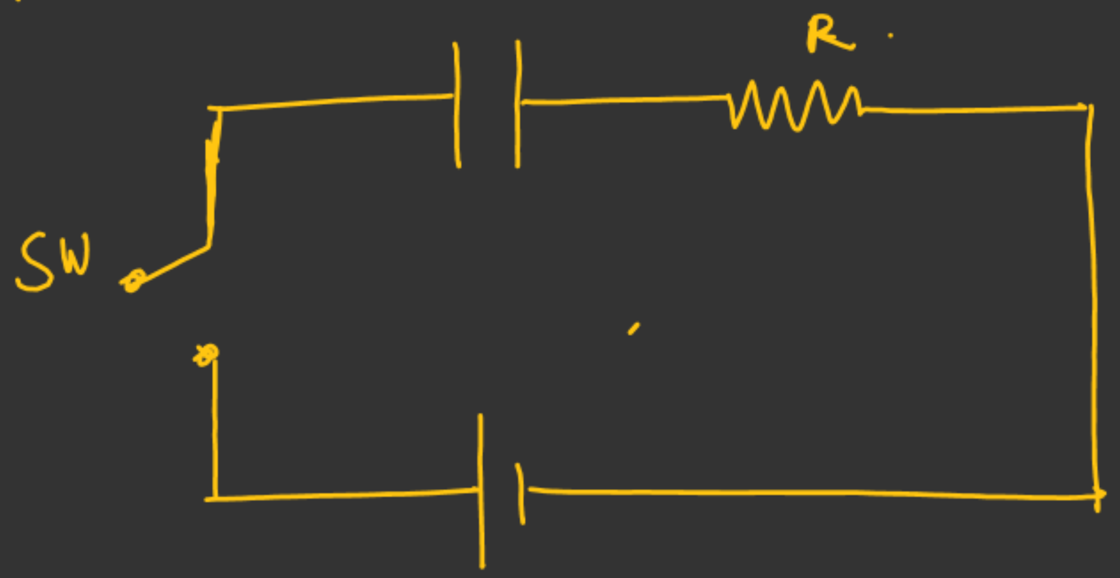
Initially Capacitor is uncharged. At $t=0$ SW is closed. Find charge on the Capacitor as a function of time if $R=(R_0+\alpha t)$ (where R_0 & α are constant)



K.V.L

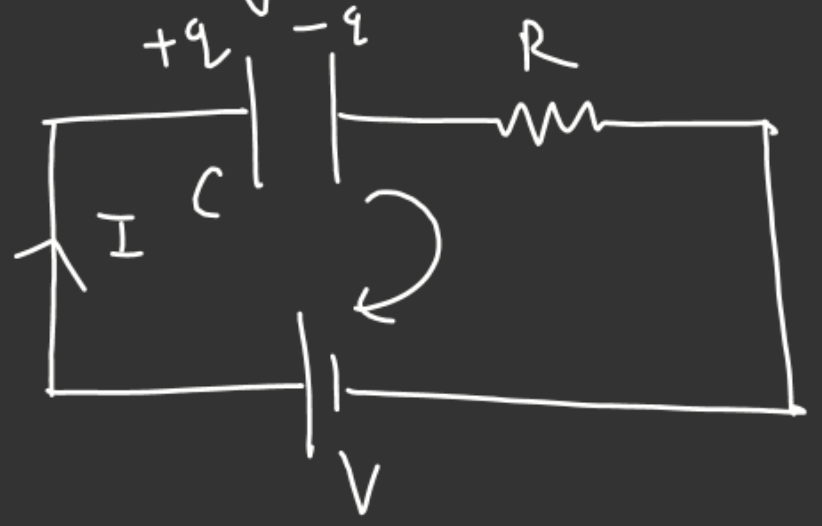
$$V - \frac{q}{C} - iR = 0$$

$$i = +(dq/dt)$$

$$V - \frac{q}{C} - R\frac{dq}{dt} = 0$$

$$V - \frac{q}{C} = R\frac{dq}{dt}$$

let, at $t=0$, Switch is closed. at $t=t$, let Charge on the Capacitor be q.



$$\frac{CV-q}{C} = R\frac{dq}{dt}$$

$$CV - q = CR\frac{dq}{dt}$$

$$\int_0^q \frac{dq}{CV-q} = \frac{1}{C}\int_0^t \frac{dt}{R_0+\alpha t}$$

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$$\int_0^q \frac{dq}{CV-q} = \frac{1}{C}\int_0^t \frac{dt}{R_0+\alpha t}$$

$$\frac{\ln[CV-q]_0^q}{(-1)} = \frac{1}{C}\,\frac{\ln[(R_0+\alpha t)]_0^t}{\alpha}$$

$$\ln\left[\frac{CV-q}{CV}\right] = \frac{-1}{C\alpha}\ln\left[\frac{R_0+\alpha t}{R_0}\right] \checkmark$$

$$\ln\left[\frac{CV-q}{CV}\right] = \ln\left[\frac{R_0+\alpha t}{R_0}\right]^{\frac{-1}{C\alpha}}$$

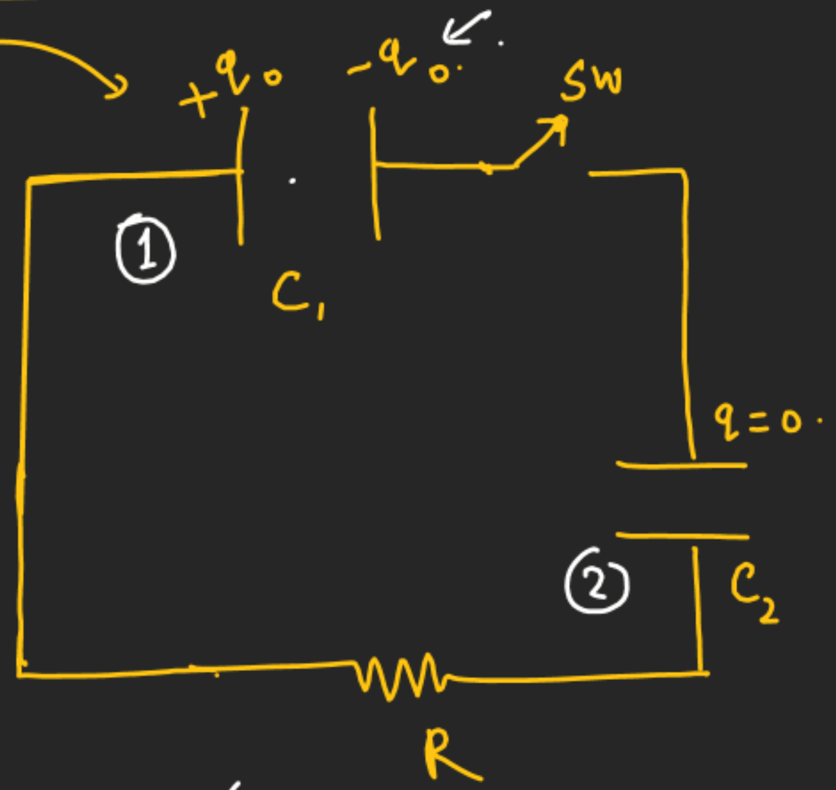
$$\frac{CV-q}{CV} = \left(\frac{R_0+\alpha t}{R_0}\right)^{\frac{-1}{C\alpha}}$$

$$CV-q = CV\left[\frac{R_0+\alpha t}{R_0}\right]^{\frac{-1}{C\alpha}}$$

$$q = CV - CV\left[\frac{R_0+\alpha t}{R_0}\right]^{\frac{-1}{C\alpha}}$$

$$q = CV\left[1-\left[1+\frac{\alpha}{R_0}t\right]^{\frac{-1}{C\alpha}}\right]$$

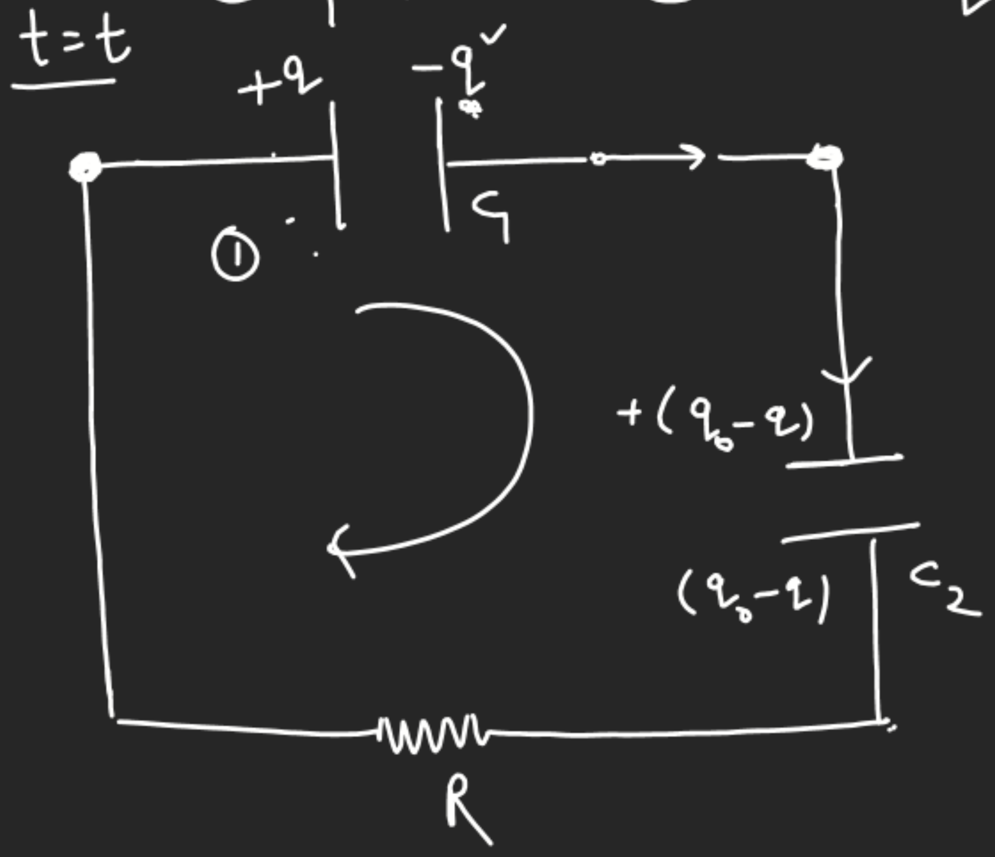
Case of 2-Capacitors in R-C Ckt (one discharging and other charging)



(flow of charge in the ckt $\rightarrow (q_0 - q)$)

Switch is closed at $t=0$. find $I = f(t)$.

At any time $t=t$; let charge on the Capacitor ① be q.



$$I = \frac{d}{dt}(q_0 - q)$$

charge flow in the ckt

$$I = \left(-\frac{dq}{dt}\right)$$

$$\left[V_{C_1} = V_{C_2} + V_R\right]$$

$$\frac{q}{C_1} = \frac{q_0 - q}{C_2} + IR$$

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CURRENT ELECTRICITY

RC ckt

$$\frac{1}{C_1}+\frac{1}{C_2}=\frac{1}{C}$$

$$\frac{q}{C_1}=\left(\frac{q_0-q}{C_2}\right)+IR \qquad I=\left(-dq/dt\right)$$

$$q\left(\frac{1}{C_1}+\frac{1}{C_2}\right)=\frac{q_0}{C_2}-R\frac{dq}{dt}$$

$$R\frac{dq}{dt}=\frac{q_0}{C_2}-q\left(\frac{1}{C}\right)$$

$$\frac{dq}{dt}=\frac{Cq_0-C_2q}{RC_2C}$$

$$\int_{q_0}^{q}\frac{dq}{Cq_0-C_2q}=\frac{1}{RC_2C}\int_0^t dt$$

$$\frac{\ln\left[Cq_0-C_2q\right]_{q_0}^{q}}{-C_2}=\frac{1}{RC_2C}t$$

$$\ln\left[Cq_0-C_2q\right]-\ln\left(Cq_0-C_2q_0\right)=-\frac{1}{RC}t$$

$$\ln\left[\frac{Cq_0-C_2q}{Cq_0-C_2q_0}\right]=-\frac{1}{RC}t$$

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$$\ln\left[\frac{Cq_0 - C_2 q}{Cq_0 - C_2 q_0}\right] = -\frac{1}{RC}t$$

$$C = \frac{1}{C_1} + \frac{1}{C_2}$$

$$C = \left(\frac{C_1 C_2}{C_1 + C_2}\right)$$

$$Cq_0 - C_2 \underline{q} = \left(Cq_0 - C_2 q_0\right)e^{-\frac{t}{RC}}$$

$$C_2 q = Cq_0 - \left(Cq_0 - C_2 q_0\right)e^{-t/RC}.$$

$$\boxed{q = \frac{Cq_0}{C_2} - \left[\frac{Cq_0}{C_2} - q_0\right]e^{-t/RC}}$$

$$q = \frac{C_1 q_0}{C_1 + C_2} - \left[\frac{C_1 q_0}{C_1 + C_2} - q_0\right]e^{-t/RC}$$

$$q = \left(\frac{C_1 q_0}{C_1 + C_2}\right) - \left[\frac{-C_2 q_0}{C_1 + C_2}\right]\underline{\underline{e}}^{-t/RC}$$

$$\underline{q = \frac{q_0}{C_1 + C_2}\left[C_1 + C_2 e^{-t/RC}\right]} \checkmark$$