Nishant Jindal

Q $(x+2y^3)\frac{dy}{dx}=y$ Solve. $\boxed{\frac{x}{y}=y^2+C}$

① $\frac{dy}{dx}=\frac{y}{x+2y^3}$

$\frac{dx}{dy}=\frac{x+2y^3}{y}$

$\frac{dx}{dy}=\frac{x}{y}+2y^2$

$\frac{dx}{dx}-\frac{x}{y}=2y^2$

② $P=-\frac{1}{y}$ $Q=2y^2$

③ $IF=e^{-\int\frac{1}{y}dy}=e^{-\ln y}=e^{\ln\frac{1}{y}}=\frac{1}{y}$.

④ $x\cdot IF=\int Q\,IF$

$\Rightarrow \frac{x}{y}=\int 2y^2\cdot\frac{1}{y}dy=y^2+C$

Q.

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Solve. Aaj K Lecture K Liye Kshma (Level Uncha Rakhna Pdega)

Q. $(1+y^2)+(x-e^{\tan^{-1}y})\frac{dy}{dx}=0$

$$\frac{dy}{dx}=\frac{(1+y^2)}{(e^{\tan^{-1}y}-x)}$$

$$\Rightarrow \frac{dx}{dy}=\frac{e^{\tan^{-1}y}-x}{(1+y^2)}$$

$$\frac{dx}{dy}=-\frac{x}{1+y^2}+\frac{e^{\tan^{-1}y}}{1+y^2}$$

$$\frac{dx}{dy}+\frac{x}{1+y^2}=\frac{e^{\tan^{-1}y}}{1+y^2}$$

$P=\frac{1}{1+y^2}$

$Q=\frac{e^{\tan^{-1}y}}{1+y^2}$

① $IF=e^{\int\frac{1}{1+y^2}dy}=e^{\tan^{-1}y}$

② $x\cdot IF=\int Q\cdot IF$

$$x\cdot e^{\tan^{-1}y}=\int\frac{(e^{\tan^{-1}y})^2}{1+y^2}dy$$

$e^{\tan^{-1}y}=t$

$\frac{e^{\tan^{-1}y}}{1+y^2}dy=dt$

$$=\int t\,dt$$

$$xe^{\tan^{-1}y}=\frac{t^2}{2}+C$$

$$x\cdot e^{\tan^{-1}y}=\frac{(e^{\tan^{-1}y})^2}{2}+C$$

Q $\frac{dy}{dx}=\frac{1}{x+y+1}$

$$\frac{dx}{dy}=x+y+1$$

$$\frac{dx}{dy}-x=y+1$$

$P=-1$

$Q=y+1$

① $IF=e^{\int -1\cdot dy}=e^{-y}$

② $x\cdot e^{-y}=\int(y+1)e^{-y}dy$

$$x\cdot e^{-y}=(y+1)\frac{e^{-y}}{-1}-(1)\frac{e^{-y}}{(-1)^2}+C$$

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BDE = Bernoulli's DE

① $\frac{dy}{dx} + Py = Qy^n$ type.

② highest deg of y se divide

Q $x \cdot \frac{dy}{dx} + y = x^3 y^6$

Solve.

① y की deg : 1 & 6

((ज्यादा Jyada deg) Pehchaan)

① $\frac{dy}{dx}$ Akela

$$\frac{dy}{dx} + \frac{y}{x} = x^2 y^6$$

② $\div y^6$ Sath vala Pas vala

$$\frac{1}{y^6}\frac{dy}{dx} + \frac{1}{x} \cdot \frac{1}{y^5} = x^2$$

(3) $\frac{dy}{dx}$ के Pas वाले y की term = t

$$\frac{1}{y^5} = t \Rightarrow \frac{-5}{y^6}\frac{dy}{dx} = \frac{dt}{dx}$$

$$\Rightarrow \frac{1}{y^6}\frac{dy}{dx} = -\frac{1}{5}\frac{dt}{dx}$$

$$-\frac{1}{5}\frac{dt}{dx} + \frac{t}{x} = x^2$$

$$\frac{dt}{dx} - \frac{5t}{x} = -5x^2$$

Normal LDE Start

④ $IF = e^{-\int \frac{5}{x} dx} = e^{-5\ln x} = x^{-5}$

⑤ $t \cdot IF = \int Q \cdot IF$

$$\frac{t}{x^5} = -\int \frac{5\cancel{x^2}}{\cancel{x^5}} x^{-3} = \frac{-5x^{-2}}{-2} + C$$

Q $\frac{dy}{dx} + xy = x^3 y^3$ BDE

$y \to 1, 3$

$\div y^3$

$$\frac{1}{y^3}\frac{dy}{dx} + \frac{x}{y^2} = x^3$$

$$\frac{1}{y^2} = t$$

$$\frac{-2}{y^3}\frac{dy}{dx} = \frac{dt}{dx}$$

$$-\frac{1}{2}\frac{dt}{dx} + tx = x^3$$

$$\frac{dt}{dx} - 2tx = -2x^3$$

① $IF = e^{-\int 2x\,dx} = e^{-x^2}$

$-x^2 = t$

$x\,dx$

② $t \cdot IF = \int Q\, IF$

$$t \cdot e^{-x^2} = -\int 2x^3 e^{-x^2} dx$$

$$= -2\int x \cdot x^2 e^{-x^2} dx$$

$$= -\frac{2}{2}\int t e^{t} dt$$

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Q Gen Sol. of

Imp

$y' + y\cdot\phi'(x) - \phi(x)\cdot\phi'(x) = 0$

$\frac{dy}{dx} + \phi'(x)(y - \phi(x)) = 0$

$dy + \phi'(x)dx(y - \phi(x)) = 0$

$\phi(x) = t$

$\phi'(x)dx = dt$

$dy + (y - t)dt = 0$

$\frac{dy}{dt} + y = t \quad \Big| \begin{matrix} P = 1 \\ Q = t \end{matrix}$

① $IF = e^{\int 1\, dt} = e^t$

② $y\cdot e^t = \int t\cdot e^t\, dt$

$y\cdot e^t = e^t(t-1) + C$

Q. Sol. of

Imp.

$e^x(x+1)dx + (ye^y - xe^x)dy = 0$

& $f(0) = 0$.

let $x\cdot e^x = t$

$x\cdot e^x + e^x\, dx = dt$

$e^x(x+1)dx = dt$

$dt + (ye^y - t)dy = 0$

$\frac{dt}{dy} + (ye^y - t) = 0$

$\frac{dt}{dy} - t = -ye^y$

① $IF = e^{-\int 1.dy} = e^{-y}$

② $t\cdot e^{-y} = -\int y\cdot \cancel{e^y}\cdot \cancel{e^{-y}}\, dy$

$t\cdot e^{-y} = -\frac{y^2}{2} + C$

Q Sol. of DE $\boxed{y = \sin\frac{1}{x} - \cos\frac{1}{x}}$

$x^2\cdot\frac{dy}{dx}\cdot\cos\frac{1}{x} - y\cdot\sin\frac{1}{x} = -1$

(Where $y \to -1$ if $x \to \infty$)

Akela

$\frac{dy}{dx} - \frac{y}{x^2}\cdot\tan\frac{1}{x} = -\frac{\sec\frac{1}{x}}{x^2}$

① $IF = e^{-\int \frac{1}{x^2}\tan\frac{1}{x}\cdot dx}$

$= e^{-\ln(\cos\frac{1}{x})} = \sec(\frac{1}{x})$

② $y\cdot\sec(\frac{1}{x}) = -\int \frac{\sec\frac{1}{x}\times\sec\frac{1}{x}}{x^2}dx$

$y\sec\frac{1}{x} = +\tan\frac{1}{x} + C$

$x \to \infty$, $y \to -1$: $-1\sec(0) = \tan(0) + C \Rightarrow \boxed{C = -1}$

$y\sec\frac{1}{x} = \tan\frac{1}{x} - 1$

Nishant Jindal

Q Let $y(x)$ be a Sol. of DE

2015

$(1+e^x)\cdot y' + y\cdot e^x = 1$. If $y(0)=2$

then $y(-4)=$?

$$\frac{dy}{dx} + \frac{y\cdot e^x}{1+e^x} = \frac{1}{1+e^x}$$

① $IF = e^{\int \frac{e^x}{1+e^x}dx} = e^{\ln(1+e^x)} = (1+e^x)$

② $y\cdot(1+e^x) = \int \frac{1}{\cancel{1+e^x}} \times \cancel{1+e^x}\, dx$

$y=y(x) \rightarrow y(1+e^x) = x + C$

$y(0)=2 \quad 2(1+e^0) = 0 + C \Rightarrow C = 4$

$y(-4) \quad y(1+e^x) = x+4$

$y(1+e^{-4}) = -4+4 \Rightarrow y=0 \Rightarrow \boxed{y(-4)=0}$

Q Consider family of Circles

Ans. 2016 whose centre lies on st. line $y=x$. If this family of Circle rep. by DE

$Py'' + Qy' + 1 = 0$ then find P & Q

① Centre lies on $y=x$

② $\therefore$ Circle $\rightarrow x^2+y^2+2ax+2ay+C=0$

③ $2x + 2y\cdot y' + 2a + 2ay' = 0$

$a(1+y') = -(x+yy')$

$a = -\frac{(x+yy')}{(1+y')}$

④ $1+(y')^2 + y\cdot y'' + ay'' = 0$

$\Rightarrow 1+(y')^2 + y\cdot y'' - \frac{x+yy'}{1+y'}\cdot y'' = 0$

$\Rightarrow 1+y'+(y')^2+(y')^3+y\cdot y''+\cancel{yy'y''}$

$-xy'' - \cancel{yy'y''} = 0$

$y''(y-x) + y'(1+y'+(y')^2) + 1 = 0$

$P = y-x$

$Q = 1+y'+(y')^2$

Nishant Jindal

Q If $f(x) = \int_0^x f(t)\cos t\,dt - \cos(t-x)dt$

If $f(x)$ is a diff^ble fxn find $f(x)$?

$$f(x) = \int_0^x f(t)\cos t\,dt - \int_0^x \cos(t-x)\,dt$$

PR4 $t \to x-t$

$$-\int_0^x \cos(x-t-\cancel{x})\,dt$$

(ab DE 30Qs

ADL 30Qs

$$f(x) = \underbrace{\int_0^x f(t)\cos t\,dt}_{NL} - \int_0^x \cos t\,dt$$

$$f'(x) = f(x)\cos x - \cos x$$

$$\frac{dy}{dx} = y\cos x - \cos x$$

$$\frac{dy}{dx} - y\cos x = -\cos x$$

① $IF = e^{-\int \cos x\,dx} = e^{-\sin x}$

② $y\cdot e^{-\sin x} = -\int \cos x\cdot e^{-\sin x}\,dx$

मुख्य आकर्षण $y\cdot e^{-\sin x} = e^{-\sin x} + C$

$$y = 1 + e^{\sin x}$$

Q Let $f(x)$ exist for $x \ge 2$ & K is fixed +ve Real No. P.T. if $\frac{d}{dx}(x\cdot f(x)) \le -Kf(x)$ then $f(x) \le A\cdot x^{-1-K}$ where A is Ind. of x.

$$\frac{d(x\cdot f(x))}{dx} + Kf(x) \le 0$$

$$x\cdot f'(x) + f(x) + K\cdot f(x) \le 0$$

$$x\cdot\frac{dy}{dx} + y(1+K) \le 0$$

$$\frac{dy}{dx} + y\frac{(1+K)}{x} \le 0$$

① $IF = e^{\int \frac{1+K}{x}dx} = e^{(1+K)\ln x} = x^{1+K}$

② $x^{1+K}\frac{dy}{dx} + \frac{y(1+K)x^{1+K}}{x} \le 0$

$$\int \frac{d}{dx}\left(x^{1+K}\cdot y\right) \le \int 0$$

$$\boxed{x^{1+K}\cdot y \le C} \quad y \le \boxed{C}\,x^{-1-K}$$

H.P.