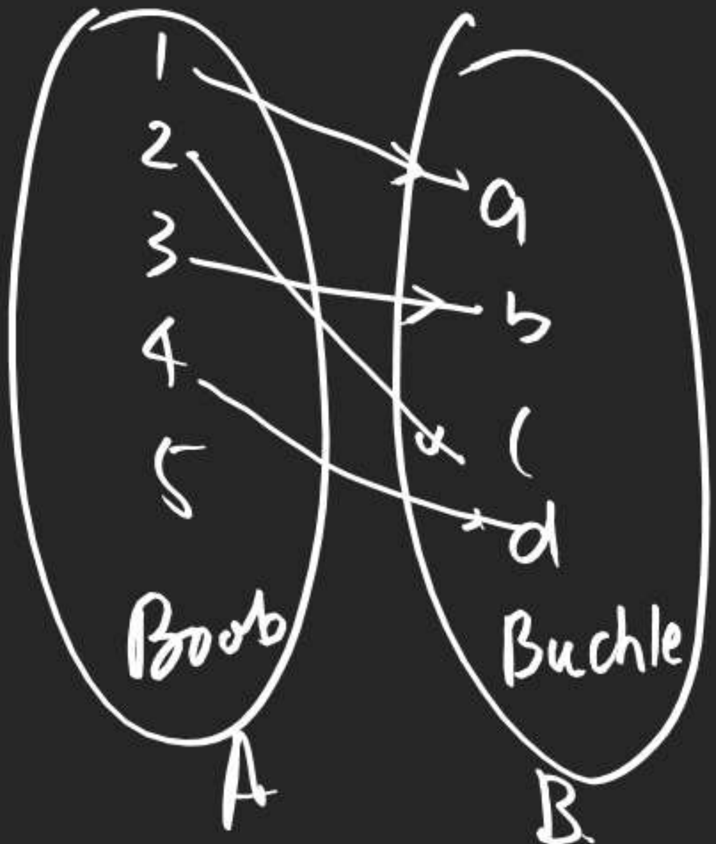
Nishant Jindal

RELATION FUNCTION

(2) $A = \{1,2,3,4,5\}$

$B = \{a,b,c,d\}$

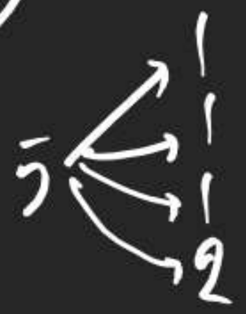


A) No of total fxn $= 4\times4\times4\times4\times4 = 4^5 = 1024$

B) No of 1-2-1 fxn $= 4\times3\times2\times1\times0 = 0$ $(n(A)>n(B))$

C) No of M21 fxn $= 1024-0 = 1024$

D) No of Onto fxn = No of ways of distributing 5 Books in 4 students



$$= \underbrace{\left\{\frac{\cancel{\lfloor 5}^{\,60}}{\lfloor 1\lfloor 1\lfloor 1\cancel{\lfloor 2}} \times \frac{1}{\cancel{\lfloor 3}}\right\}}_{\text{No of division}} \times \underbrace{\cancel{\lfloor 4}^{\,4}}_{\text{Dist}} = 240$$

No of Into $= 1024-240$

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20 Lec

Value Based fxn.

Q. $f(x) = \log_e x$

then ① $f\left(\frac{x}{y}\right) = ?$ ② $f(x \cdot y) = ?$

① $f\left(\frac{x}{y}\right) = \log \frac{x}{y} = \log_e x - \log_e y$

$f\left(\frac{x}{y}\right) = f(x) - f(y)$

② $f(x \cdot y) = \log(x \cdot y) = \log x + \log y$

$f(x \cdot y) = f(x) + f(y)$

③ $\dfrac{f\left(\frac{x}{y}\right) + f(x \cdot y)}{2f(x)} = ?$

$\dfrac{f(x) - \cancel{f(y)} + f(x) + \cancel{f(y)}}{2f(x)} = \dfrac{2f(x)}{2f(x)} = 1$

Q2 $f(x) = \dfrac{x}{x+1}$ ① $\dfrac{f\left(\frac{a}{b}\right)}{f\left(\frac{b}{a}\right)}$

A) $f\left(\frac{a}{b}\right) = \dfrac{a/b}{a/b + 1} = \dfrac{a/\cancel{b}}{\frac{a+b}{\cancel{b}}} = \dfrac{a}{a+b}$

B) $f(b/a) = \dfrac{b/a}{\frac{b}{a} + 1} = \dfrac{b/\cancel{a}}{\frac{a+b}{\cancel{a}}} = \dfrac{b}{a+b}$

C) $\dfrac{f(a/b)}{f(b/a)} = \dfrac{\frac{a}{\cancel{a+b}}}{\frac{b}{\cancel{a+b}}} = \dfrac{a}{b}$

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Q3 $f(x) = \frac{b(x-a)}{(b-a)} + \frac{a(x-b)}{(a-b)}$

$\frac{f(a+b)}{f(a)+f(b)} = ?$

$f(x) = \frac{b(x-a)}{(b-a)} - \frac{a(x-b)}{(b-a)}$

$= \frac{bx - \cancel{ab} - ax + \cancel{ab}}{(b-a)}$

$f(x) = \frac{x\cancel{(b-a)}}{\cancel{(b-a)}} \Rightarrow f(x) = x$

$\frac{f(a+b)}{f(a)+f(b)} = \frac{a+b}{a+b} = 1$

Q4 $f(x+\frac{1}{x}) = x^2 + \frac{1}{x^2}$ then $f(x)$ is → Q fxn / → linear fxn?

$f(x+\frac{1}{x}) = (x^2 + \frac{1}{x^2} + 2) - 2$

$f(x+\frac{1}{x}) = (x+\frac{1}{x})^2 - 2$

$f(t) = t^2 - 2$

$f(x) = x^2 - 2$ Quad

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Q $f(x) = bx^2 + cx + d$

& $f(x+1) - f(x) = 8x + 3$ find b, c, d

$f(x+1) = b(x+1)^2 + c(x+1) + d$

$f(x+1) = bx^2 + 2bx + cx + b + c + d$

$f(x) = bx^2 \quad + cx \quad + d$

$f(x+1) - f(x) = 2bx + (b+c) = 8x + 3$

$2b = 8$ | $b + c = 3$ | $d =$ Kuchbhi

$b = 4$ | $c = -1$ | $d \in R$

fxnl Eqn

Q6 If $\boxed{3f(x) + 2f\left(\frac{x+59}{x-1}\right) = 10x + 30}$ find $\boxed{f(7)}$ =?

$x = 7$

$3f(7) + 2f(11) = 100 \quad \times 3$

$x = 11$

$3f(11) + 2f(7) = 140 \quad \times 2$

$9f(7) + 6f(11) = 300$

$\underline{-\,4f(7) \mathbin{\underline{+}} 6f(11) \mathbin{=} \underline{-}\,280}$

$5f(7) = 20$

$f(7) = 4$

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Q7 $f(x)+2f\left(\frac{2002}{x}\right)=3x$ then $f(2)$

$x=2$ $\quad f(2)+2f(1001)=6$

$x=1001$ $\quad f(1001)+2f(2)=3003 \times 2$

$4f(2)+2\cancel{f(1001)}=6006$

$f(2)+2\cancel{f(1001)}=6$

$3f(2)=6000$

$f(2)=2000$

Q8 If $f(x-y, x+y)=x\cdot y$

find AM of $f(x,y)$ & $f(y,x)$?

① $x-y=p$

$x+y=q$

$x=\frac{p+q}{2}$, sub → $y=\frac{q-p}{2}$

② $f(x-y, x+y)=x\cdot y$

$f(p,q)=\left(\frac{q+p}{2}\right)\left(\frac{q-p}{2}\right)=\frac{q^2-p^2}{4}$

$f(x,y)=\frac{y^2-x^2}{4}$

$f(y,x)=\frac{x^2-y^2}{4}$

$AM=\frac{f(x,y)+f(y,x)}{2}=\frac{\frac{y^2-x^2}{4}+\frac{x^2-y^2}{4}}{2}=0$

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Q_9 for $f(x) \rightarrow \boxed{f(f(x))\cdot(1+f(x)) = -f(x)}$

find $f(3) = ?$

$$f(f(x))\cdot(1+f(x)) = -f(x)$$

$$f(f(x)) = \frac{-f(x)}{1+f(x)}$$

$$f(t) = \frac{-t}{1+t}$$

$$f(3) = \frac{-3}{1+3} = -\frac{3}{4}$$

Q_{10} for a fxn f $\boxed{f(3)=1}$

$\underline{f(3x) = x + f(3x-3)}$ find $f(300) = ?$

$x=2$ $\quad f(6) = 2 + f(3) = 2+1$

$x=3$ $\quad f(9) = 3 + f(6) = 3+2+1$

$x=4$ $\quad f(12) = 4 + f(9) = 4+3+2+1$

$f(3\times4)$

$$f(300) = f(3\times100) = 100+99+98+\dots+2+1$$

$$= 5050$$

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Q4 If $f(1) = 2005$ & $f(1) + f(2) + \cdots + f(n) = n^2 f(n)$

find $f(2004) = ?$

$n=2$ $\quad f(1) + f(2) = 4 \cdot f(2) \Rightarrow 3f(2) = f(1) \Rightarrow f(2) = \frac{f(1)}{3}$

$n=3$ $\quad f(1) + f(2) + f(3) = 9f(3) \Rightarrow f(1) + \frac{f(1)}{3} = 8f(3) \Rightarrow 2\cancel{8}f(3) = \frac{\cancel{4}f(1)}{3} \Rightarrow f(3) = \frac{f(1)}{6}$

$n=4$ $\quad f(1) + f(2) + f(3) + f(4) = 16f(4) \Rightarrow f(1) + \frac{f(1)}{3} + \frac{f(1)}{6} = 15f(4) \Rightarrow \cancel{15}^{5}f(4) = \cancel{9}^{3}\frac{f(1)}{\cancel{6}2}$

$f(4) = \frac{f(1)}{10}$

$f(2) = \frac{f(1)}{3} = \frac{f(1)}{1+2}$

$f(3) = \frac{f(1)}{6} = \frac{f(1)}{1+2+3}$

$f(4) = \frac{f(1)}{10} = \frac{f(1)}{1+2+3+4}$

$f(2004) = \frac{f(1)}{1+2+3+\cdots+2004}$

$= \frac{\cancel{2005} \times 2}{(2004)(\cancel{2005})} = \frac{1}{1002}$

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Q 12 If $\boxed{2f(x\cdot y)=(f(x))^y+(f(y))^x}$ $\forall x, y \in R$

$f(1)=2$ find $\sum_{r=1}^{100} f(r)=?$

$\boxed{\begin{matrix} x=x \\ y\to 1 \end{matrix}}$

$2f(x\cdot 1)=(f(x))^1+(f(1))^x$

$2f(x)=f(x)+2^x$

$f(x)=2^x$

$a\cdot\frac{(r^n-1)}{r-1}$

$\sum_{r=1}^{100} f(r)=\sum_{r=1}^{100} 2^r=2^1+2^2+2^3+\ldots+2^{100}$

$\leftarrow$ 100 term GP $\rightarrow$

$=\frac{2(2^{100}-1)}{(2-1)}=2\cdot(2^{100}-1)$

Q 13 If $f(x)=\frac{4^x}{4^x+2}$ then $\sum_{r=1}^{2001} f\left(\frac{r}{2002}\right)=?$

first step

1) $f(x)+f(1-x)=\frac{4^x}{4^x+2}+\frac{4^{1-x}}{4^{1-x}+2}$

$f(x)+f(1-x)=\frac{4^x}{4^x+2}+\frac{\frac{4}{4^x}}{\frac{4}{4^x}+2}=\frac{4^x}{(4^x+2)}+\frac{2}{(2+4^x)}$

$f(x)+f(1-x)=\frac{\cancel{4^x+2}}{\cancel{4^x+2}}=1$

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$f(x)+f(1-x)=1$

2) Demand $= \sum_{r=1}^{2001} f\left(\frac{r}{2002}\right) = f\left(\frac{1}{2002}\right)+f\left(\frac{2}{2002}\right)+f\left(\frac{3}{2002}\right)+ - - - - + f\left(\frac{1999}{2002}\right)+f\left(\frac{2000}{2002}\right)+f\left(\frac{2001}{2002}\right)$

$\rightarrow 1000.5$

$= f\left(\frac{1}{2002}\right)+f\left(\frac{2}{2002}\right) \quad f\left(1-\frac{3}{2002}\right)+f\left(1-\frac{2}{2002}\right)+f\left(1-\frac{1}{2002}\right)$

$x \qquad x \qquad 1-x \qquad 1-x$

$1 \qquad 1$

$f(x)=\frac{4^x}{4^x+2}$

$f(x)=\frac{9^x}{9^x+3}$

$f(x)=\frac{a^x}{a^x+\sqrt{a}}$

$= 1000 \text{ Pairs} + f\left(\frac{1001}{2002}\right)$

$= 1000 \times 1 + f(1/2)$

$= 1000 + .5 = 1000.5$

$f(x)=\frac{4^x}{4^x+2}$

$f(1/2)=\frac{4^{1/2}}{4^{1/2}+2}$

$=\frac{\sqrt{4}}{\sqrt{4}+2}=\frac{2}{2+2}$

$=\frac{2}{4}=.5$

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Q find $f(x)$ if $f(x)+2f(1-x)=3x$

$$f(x)+2f(1-x)=3x$$

$$x\to 1-x$$

$$f(1-x)+2f(1-(1-x))=3(1-x)$$

$$f(1-x)+2f(x)=3-3x \quad \times 2$$

$$\cancel{2f(1-x)}+4f(x)=6-6x$$

$$\underset{-}{\cancel{2f(1-x)}}+\underset{-}{}f(x)=\underset{-}{}3x$$

$$3f(x)=6-9x$$

$$f(x)=2-3x$$

$x-1=X$

$x=X+1$

Q find $f(x)$ if

15

$$f(x-1)+4f\left(\frac{1-x}{x}\right)=3x\ ?$$

$$f(x-1)+4f\left(\frac{1}{x}-1\right)=3x$$

$$x\to\frac{1}{x}$$

$$f\left(\frac{1}{x}-1\right)+4f(x-1)=3\left(\frac{1}{x}\right) \quad \times 4$$

$$\cancel{4f\left(\frac{1}{x}-1\right)}+16f(x-1)=\frac{12}{x}$$

$$\underset{-}{\cancel{4f\left(\frac{1}{x}-1\right)}}+\underset{-}{}f(x-1)=\underset{-}{}3x$$

$$15f(x-1)=\frac{12}{x}-3x$$

$$15f(X)=\frac{12}{X+1}-3(X+1)$$

$$f(x)=\frac{1}{15}\left\{\frac{12}{x+1}-3(x+1)\right\}$$

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Q16 $f(\sin x) + 3f(\cos x) = \tan^2 x$

then $f(x) = ?$

Hint $x \to \frac{\pi}{2} - x$

$\sin x \to t$

$$f(x) = \frac{1}{8}\left\{\frac{3(1-x^2)}{x^2} - \frac{x^2}{1-x^2}\right\}$$

Q $f(x+y) = f(x) + y(y+2x+1)$; $f(5) = 32$

$x = 5$

$$f(x) = (x-5)^2 + 11(x-5) + 32$$

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Even & Odd fxn.

① Even fxn → $f(-x) = f(x)$

Ex: → $f(x) = |x|$

$f(-x) = |-x| = |x|$

$f(-x) = f(x)$ Even

Ex: $f(x) = \cos x$

$f(-x) = \cos(-x)$

$= \cos x$

$f(-x) = f(x)$ Even

② Odd fxn → $\boxed{f(-x) = -f(x)}$

Ex: → $f(x) = x^3$

$f(-x) = (-x)^3$

$= -(x)^3$

$f(-x) = -f(x)$ odd

Ex: $f(x) = e^x - e^{-x}$

$f(-x) = e^{-x} - e^x$

$= -(e^x - e^{-x})$

$f(-x) = -f(x)$

odd

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(3) Niether Even Nor Odd fxn.

(NENO)

$f(-x) \neq f(x)$

$\neq -f(x)$

Ex:- $f(x) = e^{x} \to \frac{1}{e^{x}}$

$f(-x) = e^{-x} \neq e^{x}$

$\neq -e^{x}$

$f(-x) \neq f(x)$

$\neq -f(x)$

NENO

(B) Graph of Even/odd fxn.

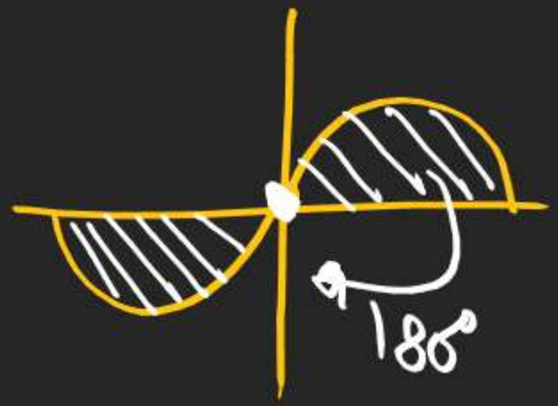
A) Even fxn → Symmetric about y Axis $(x=0)$

$y = x^2$

$y = |x|$

$y = \cos x$

B) Odd fxn → Symmetrical About Origin



$y = \sin x$

$y = x^3$

$y = \mathrm{sgn}\, x$

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Q $f(x) = \sqrt{1+x+x^2} - \sqrt{1-x+x^2}$ E/O?

$f(-x) = \sqrt{1-x+x^2} - \sqrt{1+x+x^2}$

$= -\left(\sqrt{1+x+x^2} - \sqrt{1-x+x^2}\right)$

$f(-x) = -f(x)$ odd

Q $f(x) = \ln\left(\frac{1-x}{1+x}\right)$ E/O?

$f(-x) = \ln\left(\frac{1+x}{1-x}\right)$

$= \ln\left(\frac{1-x}{1+x}\right)^{-1} = -\ln\left(\frac{1-x}{1+x}\right)$

$= -f(x)$ odd

Q $h(x) = f(x) + f(-x)$ $h(x)$ E/O?

$h(-x) = f(-x) + f(x)$

$h(-x) = h(x)$ Even.

Q $h(x) = f(x) - f(-x)$ E/O?

$h(-x) = f(-x) - f(x)$

$= -(f(x) - f(-x))$

$h(-x) = -h(x)$ odd

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Result

$$\boxed{\begin{array}{l} y = f(x) + f(-x) \to \text{Even} \\ y\ \ f(x) - f(-x) \to \text{odd.} \end{array}}$$

But $\boxed{f(x) + f(-x) = 0}$

$\Rightarrow \underline{f(-x)} = \underline{-f(x)}$

$\boxed{\text{odd}}$.

Q $f(x) = a^x + a^{-x}$ E/O?

$g(x) + g(-x)$ E

Q $f(x) = \dfrac{2^x + 2^{-x}}{2^x - 2^{-x}}$

$= \dfrac{g(x) + g(-x)}{g(x) - g(-x)} = \dfrac{\text{Even}}{\text{odd}}$

$= \text{odd}$

Q $f(x) = \dfrac{2^{2x}+1}{2^{2x}-1}$ odd

$f(x) = \dfrac{a^x + a^{-x}}{a^x - a^{-x}}$ odd

Q $f(x) = \left(\dfrac{a^x + a^{-x}}{a^x - a^{-x}}\right) \times x$

odd $\times$ odd

$=$ Even

Q $f(x) = \left(\dfrac{a^x + a^{-x}}{a^x - a^{-x}}\right) \cdot x^2$

$=$ odd Even

$=$ Odd

Q $f(x) = \left(\dfrac{a^x + a^{-x}}{a^x - a^{-x}}\right) \cdot x^n$

$=$ Even

$n =$ odd deg.