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7.

$$(a + 2hm + bm^2) = 0$$

$$bm^2 + 2hm + a = 0$$

$$(a'm^2 - 2h'm + b') = 0$$

Put $x = -my$

$$\frac{m^2}{2hb' + 2h'a} = \frac{m}{aa' - bb'} = \frac{1}{-2h'b - 2ha'}$$

$$\frac{2hb' + 2h'a}{aa' - bb'} = -\frac{aa' - bb'}{2h'b + 2ha'}$$

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10.

$$x = \frac{-(by+b') \pm \sqrt{b^2y^2+b'^2+2bb'y-4(acy^2+(ac'+a'c)y+a'c')}}{2(ay+a')}$$

$$(ay+a')x^2+(by+b')x+cy+c' = 0$$

$$(b^2-4ac)y^2+(2bb'-4ac'-4a'c)y+b'^2-4a'c'$$

$$(\quad)y+(\quad)=0$$

$$(b^2-4ac)(y-\alpha)^2$$

$$\boxed{D=0}$$

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both roots in $[k_1, k_2]$

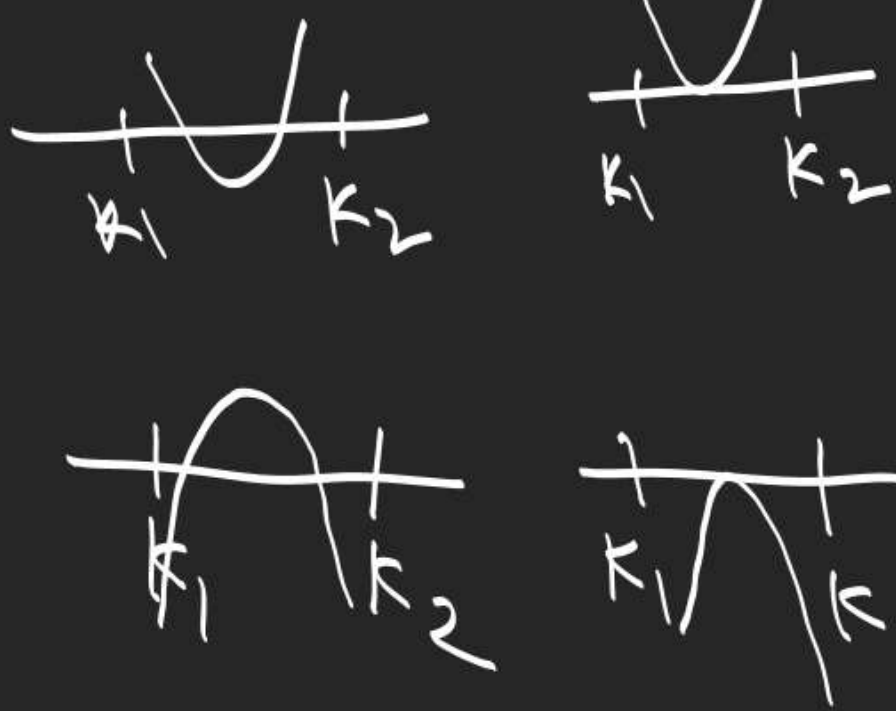
$$f(x) = ax^2 + bx + c$$

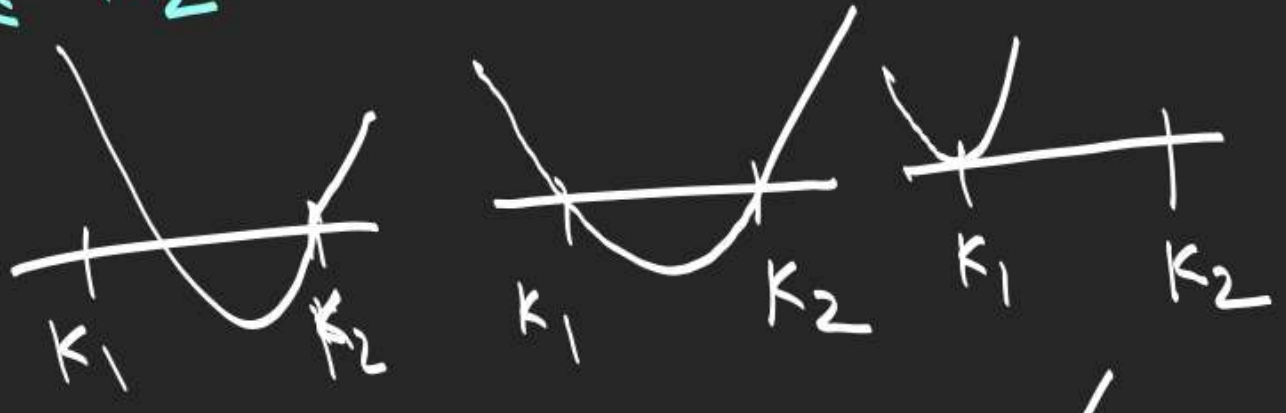
$$D \geq 0$$

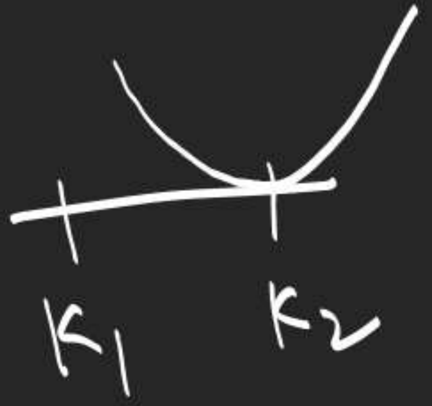
$$k_1 \leq -\frac{b}{2a} \leq k_2$$

$$a f(k_1) \geq 0$$

$$a f(k_2) \geq 0$$







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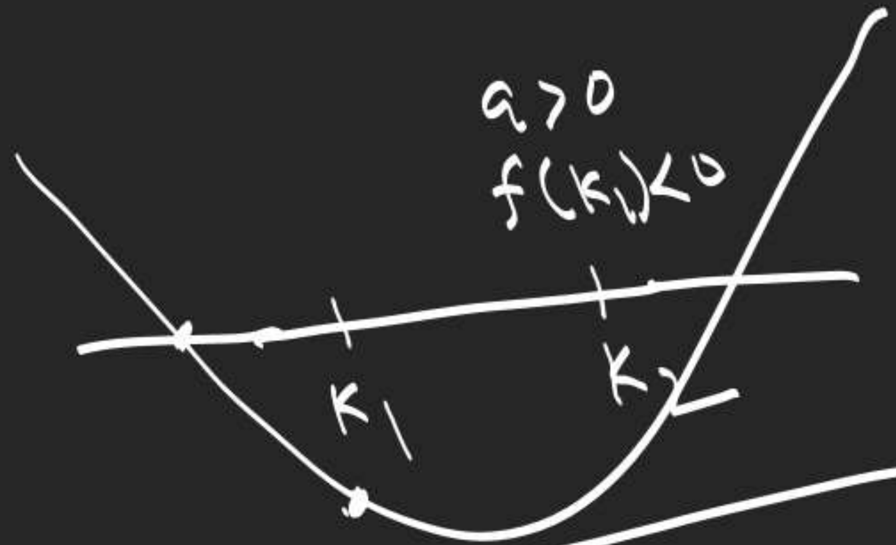
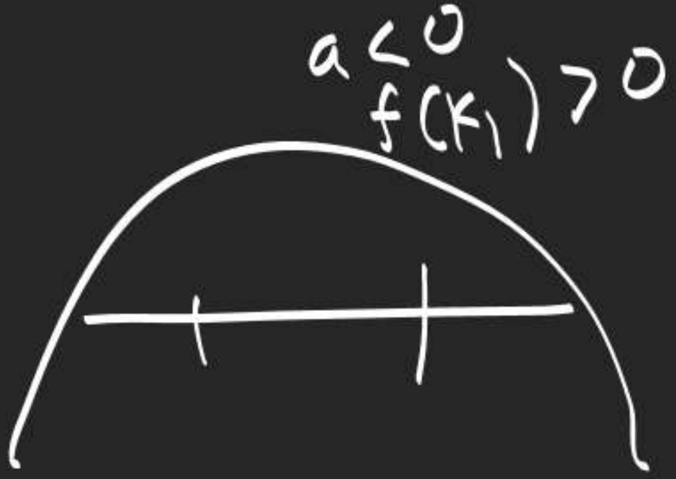
One root $< K_1$, other $> K_2$, $K_1 < K_2$



$a\, f(K_1) < 0$ ✓

$a\, f(K_2) < 0$ ✓

$D > 0$ ✓



$x \to -\infty$ $f(x) \to \infty$

✓ $\boxed{f(K_1) < 0}$ ✓

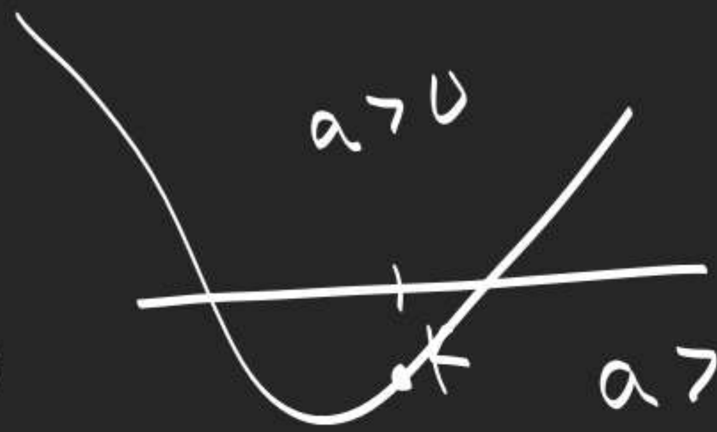
$x \to \infty$, $f(x) \to \infty$

Condition for $f(x) = ax^2 + bx + c = 0$, $a, b, c \in R$, $a \neq 0$

to have roots lying on either side of k.

one root $< K$ & other $> K$.

$a f(K) < 0$



$a > 0$
$f(K) < 0$

$f(K) > 0$
$a < 0$

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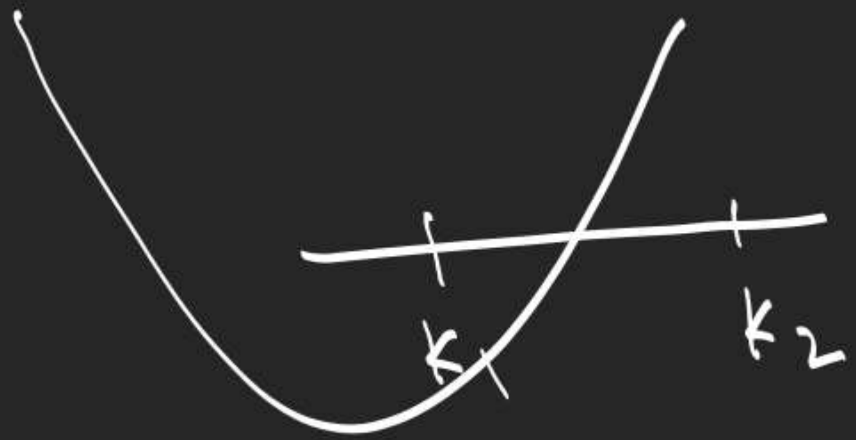
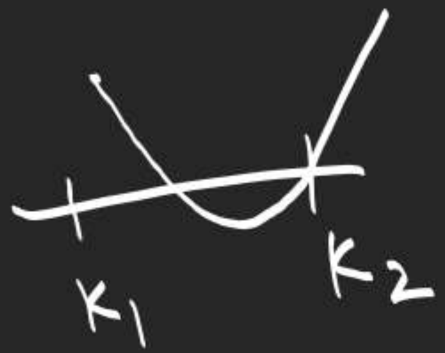
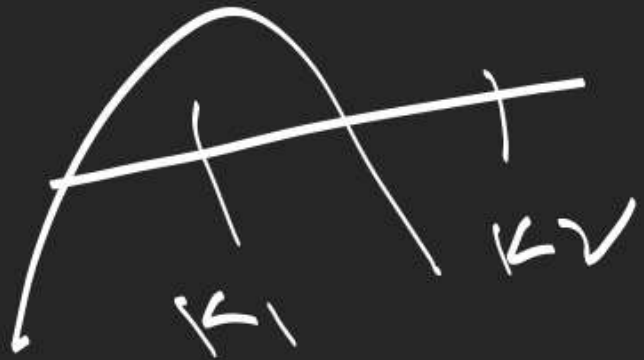
Exactly one root in (k_1, k_2)







$$\boxed{f(k_1)f(k_2) < 0}$$

OR

$$\boxed{\begin{array}{c}\text{Check} \\ f(k_1)\,f(k_2) = 0\end{array}}$$

Nishant Jindal

1. Find 'd' for which both the roots of the equation $f(x) = x^2 - 6dx + (2-2d+9d^2) = 0$ exceed 3.

$\boxed{d \in \left(\frac{11}{9}, \infty\right)} \Rightarrow$ Ans

1. $f(3) > 0 \Rightarrow \boxed{d \in (-\infty, 1) \cup \left(\frac{11}{9}, \infty\right)}$

$9 - 18d + 2 - 2d + 9d^2 > 0$

$9d^2 - 20d + 11 > 0$

$-9d - 11d$

$(9d-11)(d-1)$

$D \geq 0 \Rightarrow 36d^2 - 4(2-2d+9d^2) \geq 0$

$d - 1 \geq 0$

$\boxed{d \geq 1}$

$3d > 3$

$\boxed{d \in (1, \infty)}$

Q. Find 'a' for which zeroes of quadratic polynomial $f(x) = (a^2+a+1)x^2 + (a-1)x + a^2$ are located on either side of 3.

$$(a^2+a+1)f(3) < 0$$

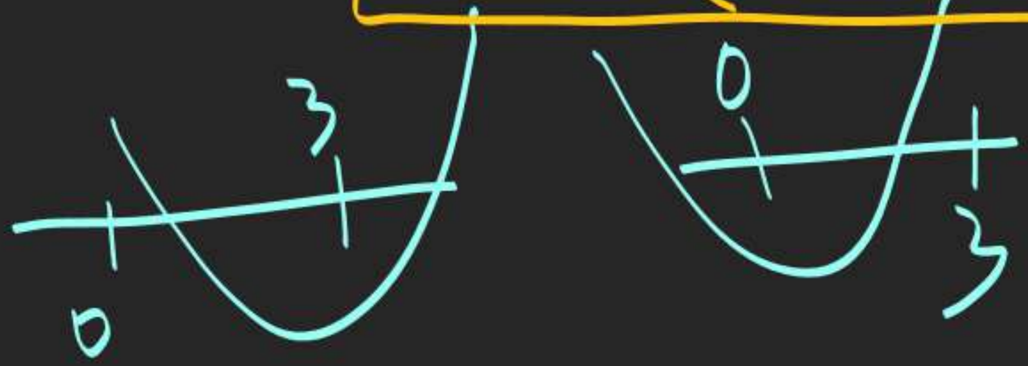
$$(a^2+a+1)(9a^2+9a+9+3a-3+a^2) < 0$$

$$\underbrace{(a^2+a+1)}_{>0}\underbrace{(10a^2+12a+6)}_{>0} < 0$$

$$\boxed{a \in \phi}$$

3: Find 'a' for which exactly one root of eqn: $f(x) = x^2-(a+1)x+2a=0$ lie in interval $(0,3)$.

$$\boxed{a\in(-\infty,0]\cup(6,\infty)}$$



$f(0)f(3)<0$ OR

Check

$f(0)f(3)=0$

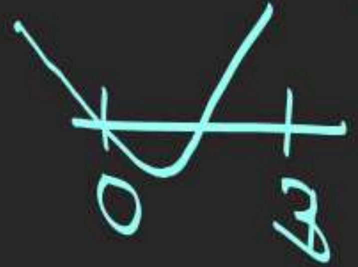
$\Rightarrow a=0,6$

$2a(-a+6)<0$

$a(a-6)>0$

$a\in(-\infty,0)\cup(6,\infty)$

If $a=0$ ✓ $x^2-x=0$, $\boxed{x=0, x=1}$



If $a=6$ × $x^2-7x+12=0$

$x=4,3$

Nishant Jindal

1: If α, β are roots of equation

$$x^2+2(k-3)x+9=0, \quad \alpha \neq \beta.$$

$\boxed{\text{PT-1 \& PT-2}}$

Find k if $\alpha, \beta \in (-6, 1)$.

2: Find k for which one root of quadratic equation $(k-5)x^2-2kx+k-4=0$ is smaller than 1 and the other root exceed 2.