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3. (ix) / (viii), vii

$0 < x^2 - 5x + 13 < 7$ $\boxed{(2,3)}$

$1 - \log_7 (x^2 - 5x + 13) > 0$

$\log_7 (\quad) < 1$

(vii) $-1 \le \left(\dfrac{3}{2 + \sin\left(\frac{9\pi x}{2}\right)} \le 1 \right)$

$3 \le 2 + \sin\frac{9\pi x}{2}$

$\sin\frac{9\pi x}{2} \geq 1$

$\frac{9\pi x}{2} = 2n\pi + \frac{\pi}{2}, \ n \in I$

$\boxed{x = \frac{4n+1}{9}}$ $n \in I$.

$\boxed{x = \left\{ \cancel{\frac{21}{9}}\ \frac{7}{3}, \frac{25}{9} \right\}}$

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(viii)

$$\ln\sqrt{\underbrace{x-[x]}_{\{x\}}}$$

$$-1 \le \frac{x}{2} \le 1$$

$$x \notin I$$

$$-2 \le x \le 2$$

$$D_f = (-2,-1) \cup (-1,0) \cup (0,1) \cup (1,2)$$

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(ix) $f(x) = e^{\cos^{-1}\left(\frac{2\sin x+1}{2\sqrt{2\sin x}}\right)}$

$\boxed{T = 2\pi}$

$\boxed{D_f = \left\{2n\pi + \frac{\pi}{6}\right\}_{n \in I}}$

$-1 \le \boxed{\frac{2\sin x+1}{2\sqrt{2\sin x}} \le 1}$

$2\sin x + 1 - 2\sqrt{2\sin x} \le 0$

$\left(\sqrt{2\sin x} - 1\right)^2 \le 0$

$=$

$\boxed{\sin x = \frac{1}{2}}$

$\boxed{\frac{\pi}{6}, \cancel{\frac{5\pi}{6}}}$

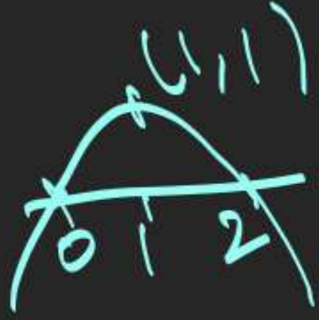
$\sqrt{\sin \underbrace{\cos \frac{5\pi}{6}}}$

$\sin\left(-\frac{\sqrt{3}}{2}\right) < 0$

4. (iii) $f(x) = \sin(\underbrace{\tan^{-1} x}_{\theta})$ $\qquad g(x) = \frac{x}{\sqrt{1+x^2}}$

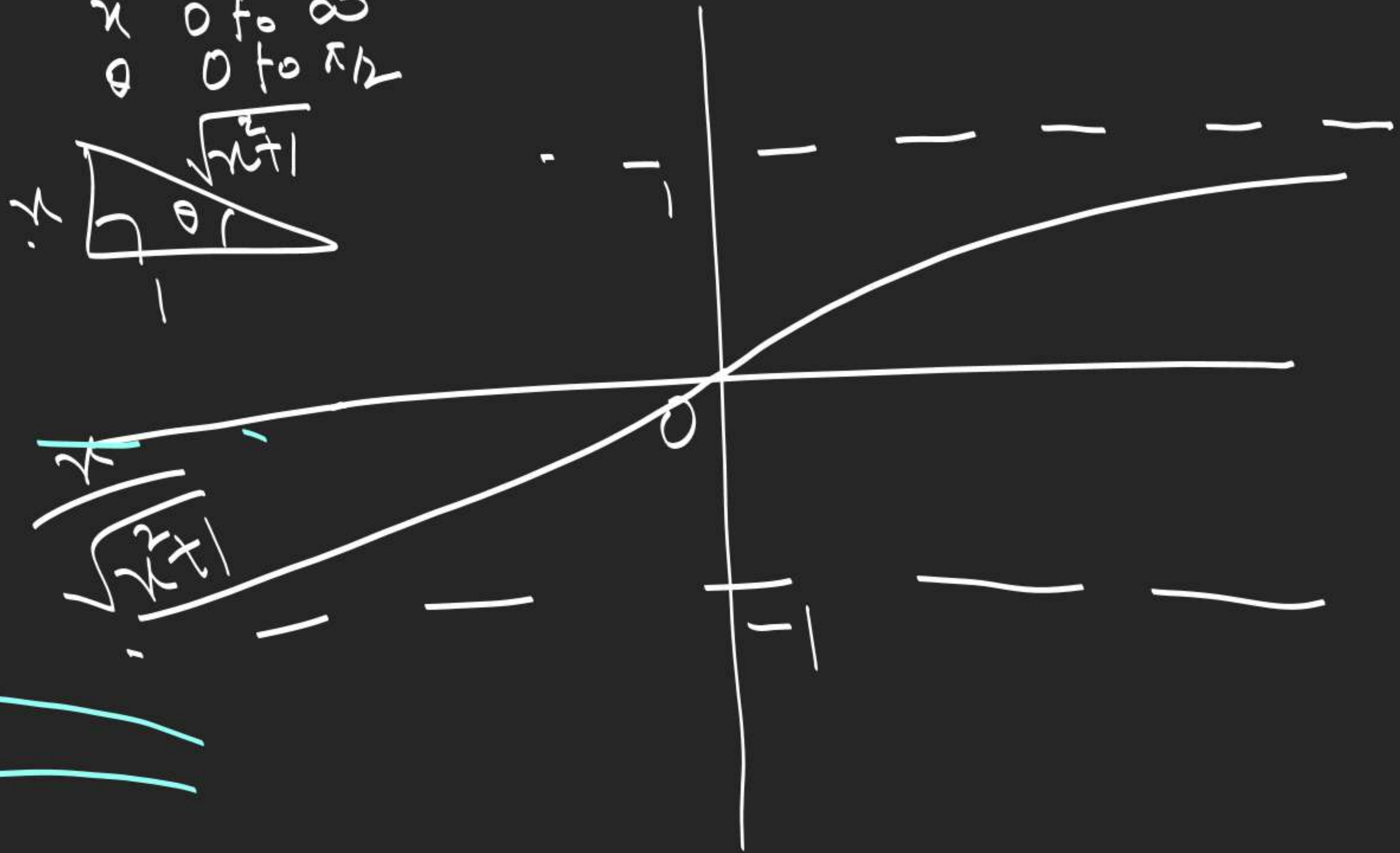
x 0 to ∞

θ 0 to $\pi/2$



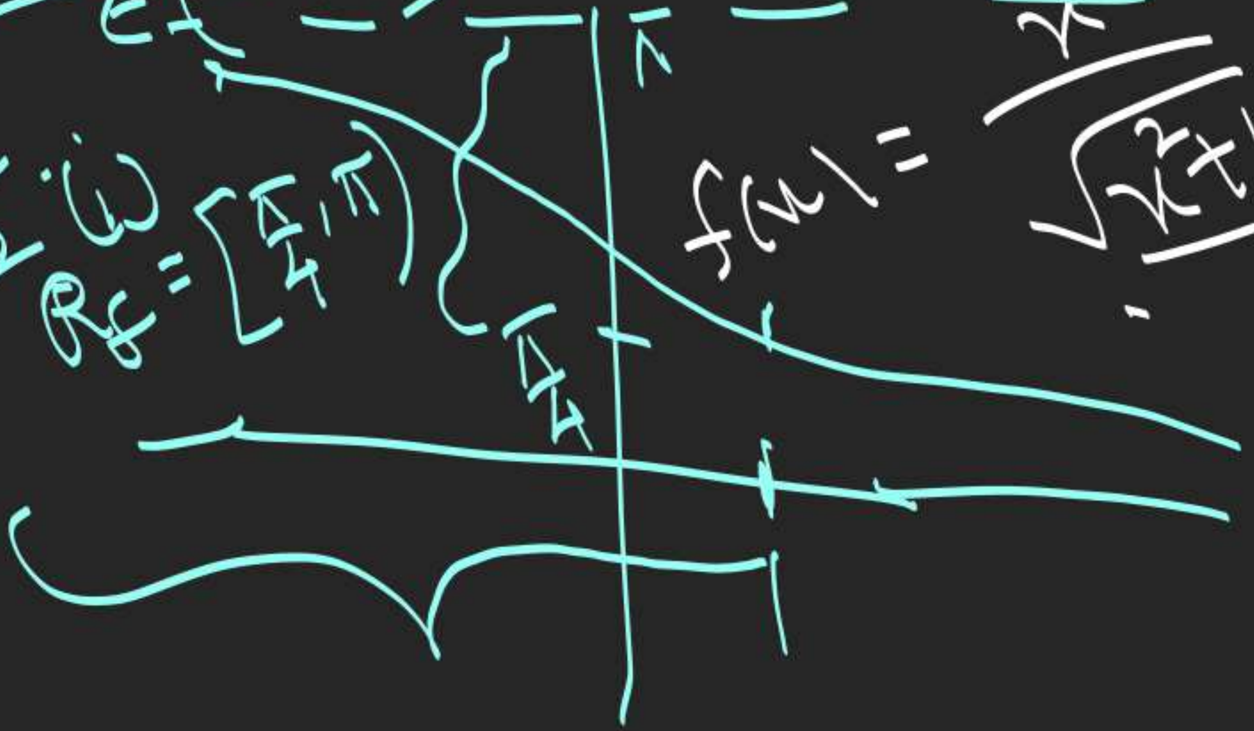
$\boxed{2x - x^2} \in (-\infty, 1]$

$f(x) = \frac{x}{\sqrt{x^2+1}}$



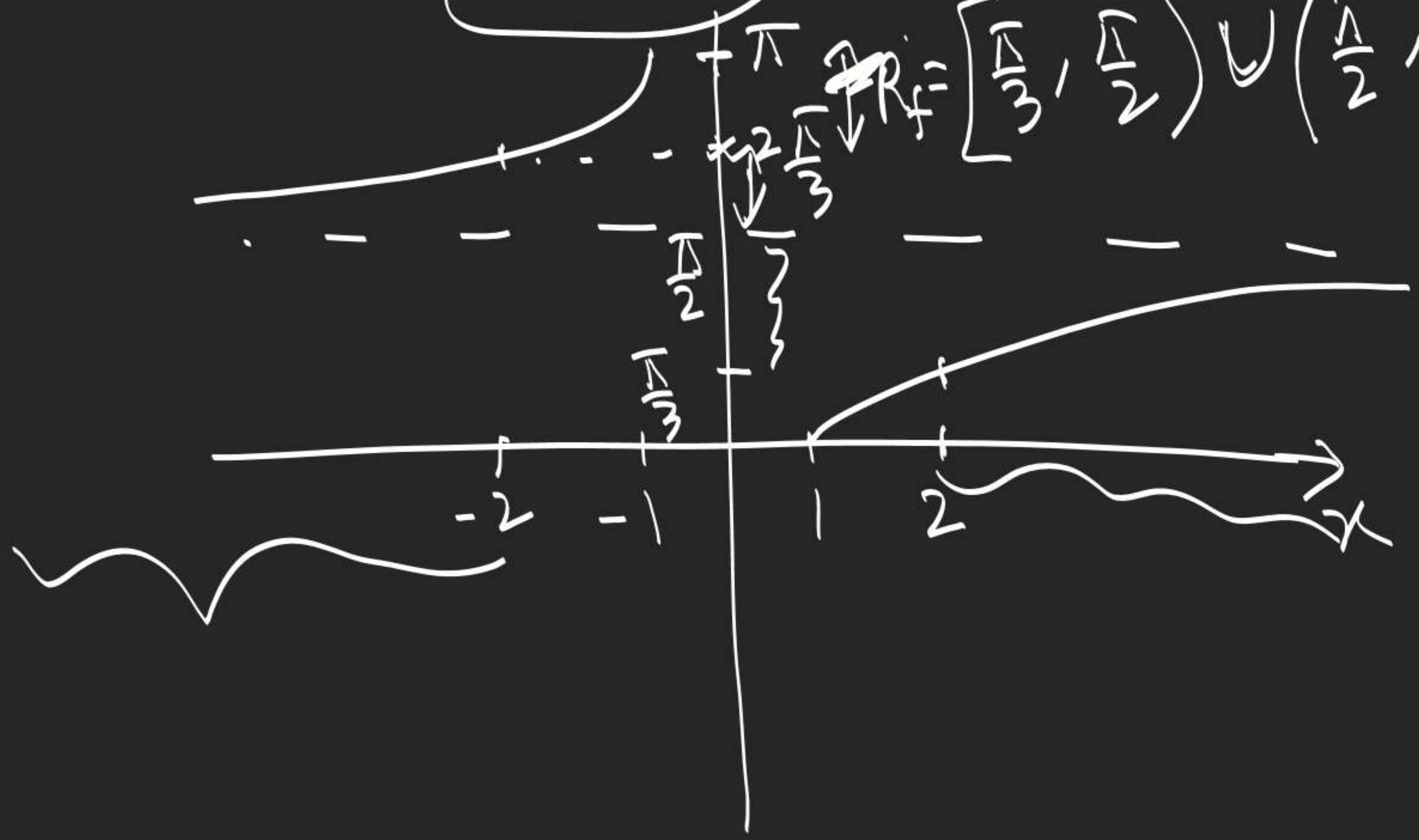
5. (ii)

$R_f = [\frac{\pi}{4}, \pi]$



(11) $\sec^{-1}\left(\underbrace{t+\frac{1}{t}}_{=x}\right)$ $(-\infty,-2] \cup [2,\infty)$

$R_f = \left[\frac{\pi}{3}, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \frac{2\pi}{3}\right]$



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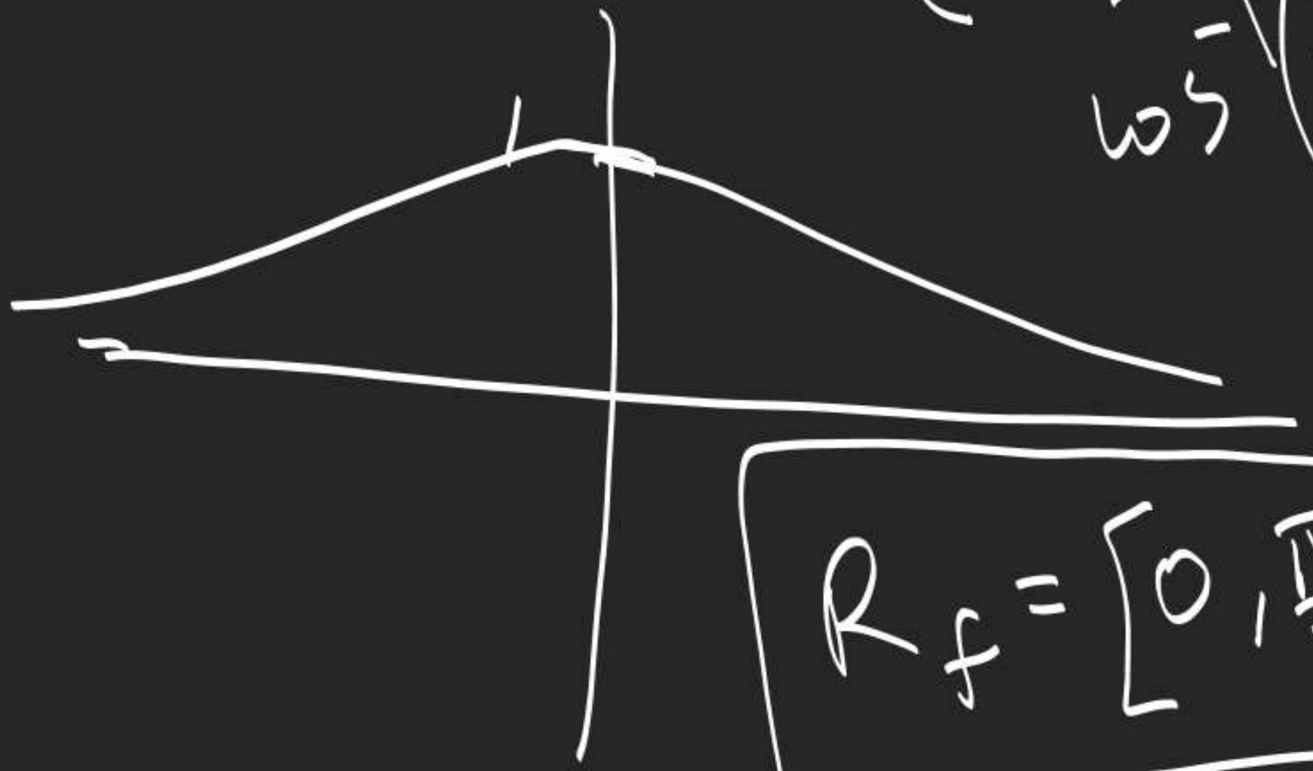
(iii) $g(x) = \dfrac{\sqrt{2x^2+1}}{x^2+1}$ $\qquad \dfrac{|x|\sqrt{2+\frac{1}{x^2}}}{x^2\left(1+\frac{1}{x^2}\right)}$ $\qquad x\to\infty,\ g(x)\to 0$

$$g'(x) = \frac{\dfrac{(x^2+1)\cancel{4x}^{\,2}}{\cancel{2}\sqrt{2x^2+1}} - \sqrt{2x^2+1}\;2x}{(x^2+1)^2} = \frac{2x\left(x^2+1-2x^2-1\right)}{\sqrt{2x^2+1}\,(x^2+1)^2}$$

$$= \frac{-2x^3}{\sqrt{2x^2+1}\,(x^2+1)^2}$$

$\downarrow \quad x>0.$



$\cos^{-1}\left(\underbrace{\dfrac{\sqrt{2x^2+1}}{x^2+1}}_{(0,1]}\right)$

$$\boxed{R_f = \left[0, \frac{\pi}{2}\right)}$$

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8. $\alpha = \tan^{-1}\left(\frac{36}{\quad}\right)$

$\beta = \tan^{-1}\frac{3}{4}$

$\gamma = \tan^{-1}\frac{8}{15}$

$\tan(\alpha+\beta+\gamma) = \frac{S_1 - S_3}{1 - S_2}$

$1 - S_2 = 0$

$\alpha+\beta+\gamma = \frac{\pi}{2}$

$\alpha+\beta+\gamma \in \left(0, \frac{3\pi}{2}\right)$

sin

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10. (a) $\underbrace{2\cos^{-1}\frac{3}{\sqrt{13}}}_{\theta_1 \in (0,\frac{\pi}{2})} + \cot^{-1}\frac{16}{63} + \frac{1}{2}\underbrace{\cos^{-1}\frac{7}{25}}_{\theta_2 \in (0,\pi/2)}$

$2\theta_1 \in (0,\pi)$

$(0,\pi/2)$

$\tan(2\theta_1) = \dfrac{2\left(\frac{2}{3}\right)}{1-\frac{4}{9}} = \dfrac{12}{5}$

$2\theta_1 = \tan^{-1}\tan 2\theta_1 = \tan^{-1}\dfrac{12}{5}$

$\tan\dfrac{\theta_2}{2} = \dfrac{1-\cos\theta_2}{\sin\theta_2}$

$= \dfrac{1-\frac{7}{25}}{\frac{24}{25}} = \dfrac{3}{4}$

$\pi + \tan^{-1}\left(\dfrac{\frac{12}{5}+\frac{3}{4}}{1-\frac{12}{5}\times\frac{3}{4}}\right)$

$\boxed{\tan^{-1}\frac{12}{5} + \tan^{-1}\frac{3}{4}} + \tan^{-1}\frac{63}{16}$

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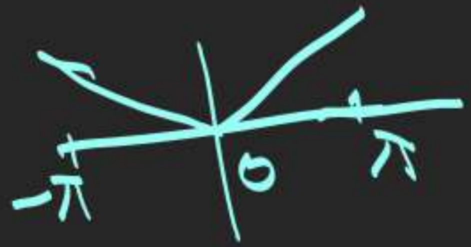
$$\cos^{-1}\frac{5}{13} + \pi - \cos^{-1}\frac{7}{25} + \sin^{-1}\frac{36}{325}$$



$$\tan^{-1}\left(\frac{a-b}{1+ab}\right) + \tan^{-1}\left(\frac{b-c}{1+bc}\right) + \pi + \tan^{-1}\left(\frac{c-a}{1+ac}\right)$$

$$\tan^{-1}a - \tan^{-1}b + \tan^{-1}b - \tan^{-1}c + \pi + \tan^{-1}c - \tan^{-1}a$$

12. (a) $\cos^{-1}x = \theta \in \left(0, \frac{\pi}{3}\right)$ $\quad \cos^{-1}\left(\frac{x}{2} + \frac{\sqrt{3}}{2}\sqrt{1-x^2}\right) = \cos^{-1}\left(\cos\left(\theta - \frac{\pi}{3}\right)\right)$

$\boxed{\theta - \frac{\pi}{3} \in \left(-\frac{\pi}{3}, 0\right)}$ $\quad \frac{1}{2}\cos\theta + \frac{\sqrt{3}}{2}\sin\theta = \frac{\pi}{3} - \theta$

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13.
$$\tan^{-1}\left(\frac{6\tan\alpha}{5(1+\tan^2\alpha)+3(1-\tan^2\alpha)}\right)$$

$$= \tan^{-1}\left(\frac{3\tan\alpha}{4+\tan^2\alpha}\right) = \tan^{-1}\left(\frac{\frac{3}{4}\tan\alpha}{1+\frac{1}{4}\tan^2\alpha}\right)$$

$$= \tan^{-1}\left(\frac{\left(\tan\alpha-\frac{1}{4}\tan\alpha\right)}{1+(\tan\alpha)\left(\frac{\tan\alpha}{4}\right)}\right) = \tan^{-1}\tan\alpha - \tan^{-1}\left(\frac{1}{4}\tan\alpha\right)$$

$x, y < 0$

$$\tan^{-1}x - \tan^{-1}y = -\tan^{-1}(-x) + \tan^{-1}(-y) = \tan^{-1}\left(\frac{-y+x}{1+xy}\right)$$

$$= \tan^{-1}\left(\frac{x-y}{1+xy}\right)$$

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15. (c) $\tan^{-1}\left(\frac{2\frac{n}{m}}{1-\left(\frac{n}{m}\right)^2}\right)$

$-1 < \frac{n}{m} < 1$

$\tan^{-1}\frac{n}{m} = \theta_1 \in \left(-\frac{\pi}{4}, \frac{\pi}{4}\right)$

$2\theta_1 \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$= \tan^{-1}\tan 2\theta_1$

$= 2\theta_1$

$= 2\tan^{-1}\frac{n}{m}$

16. $\frac{1}{2} + \frac{1}{2} + K + \frac{1}{2} + 2K$

$= \frac{1}{2}\left(\frac{1}{2}+K\right)\left(\frac{1}{2}+2K\right)$

$K = ?$ → Check.

$\cancel{2}\tan^{-1}\frac{n}{m} + \cancel{2}\tan^{-1}\frac{q}{p} = \cancel{2}\tan^{-1}\frac{N}{M}$

17(a)

$\sec^{-1}x \in \left[0,\frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, 2\right)$

$\sec^{-1}x = 2$

$x = \sec 2$

$x \in (-\infty, \sec 2] \cup [1, \infty)$

(b) $\frac{\tan^2 x}{1+\tan^2 x} + \frac{\tan^2 y}{1+\tan^2 y} < 1$

$\tan^2 x \tan^2 y < 1$

$\boxed{-1 < \tan x \tan y < 1}$

20- $(\sin^{-1}x)^3+(\cos^{-1}x)^3 = (s^{-1}+c^{-1})^3 - 3s^{-1}c^{-1}\underline{(s^{-1}+c^{-1})}$

$$= \frac{\pi^3}{8} - 3\frac{\pi}{2}\, t\left(\frac{\pi}{2}-t\right)$$

$$= \frac{\pi^3}{8} + \frac{3\pi}{2}\boxed{t\left(t-\frac{\pi}{2}\right)} \to \in \left[-\frac{\pi^2}{16}, \frac{\pi^2}{2}\right]$$

$\sin^{-1}x = t \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$\alpha\,\underline{\underline{\pi^3}}$ $\left(-\frac{\pi}{2}, \frac{\pi^2}{2}\right)$ $\left(\frac{\pi}{2},\right.$ $\left(\frac{\pi}{4}, -\frac{\pi^2}{16}\right)$

$$R_f = \left[\frac{\pi^3}{8}+\frac{3\pi}{2}\left(-\frac{\pi^2}{16}\right), \frac{\pi^3}{8}+\frac{3\pi}{2}\left(\frac{\pi^2}{2}\right)\right]$$

$$= \left[\frac{\pi^3}{32}, \frac{7\pi^3}{8}\right]$$

$$\alpha \in \left[\frac{1}{32}, \frac{7}{8}\right]$$

18.

$R \to \left(0, \frac{\pi}{2}\right]$

$y = x^2 + 4x + \alpha^2 - \alpha$

$\boxed{D = 0}$

×

✓ $\boxed{[0, \infty)}$

×

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1. $\sin^{-1}x + \sin^{-1}(2x) = \frac{\pi}{3}$

$$\sin^{-1}2x = \frac{\pi}{3} - \sin^{-1}x$$

$$\sin(\sin^{-1}2x) = \sin\left(\frac{\pi}{3} - \sin^{-1}x\right)$$

$$2x = \frac{\sqrt{3}}{2}\sqrt{1-x^2} - \frac{1}{2}x$$

$$\frac{5}{\cancel{2}}x = \frac{\sqrt{3}}{\cancel{2}}\sqrt{1-x^2}$$

$$25x^2 = 3 - 3x^2$$

$$x = \pm\sqrt{\frac{3}{28}}$$

If $x = -\sqrt{\frac{3}{28}}$ (rejected)

$$\underbrace{\sin^{-1}x}_{\left(-\frac{\pi}{2},0\right)} + \underbrace{\sin^{-1}2x}_{\left(-\frac{\pi}{2},0\right)}$$

$$\boxed{x = \sqrt{\frac{3}{28}}}$$

Ans

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2. $\tan^{-1}\left(\frac{x+1}{x-1}\right)+\tan^{-1}\left(\frac{x-1}{x}\right)=\tan^{-1}(-7)$

$0, \pi/2$ $\quad \hookrightarrow (0, \pi/2)$ $\quad (-\pi/2, 0)$

$\underline{x=2}$

$$\tan\left(\tan^{-1}\frac{x+1}{x-1}+\tan^{-1}\frac{x-1}{x}\right)=\tan(\tan^{-1}(-7))$$

$$\frac{\frac{x+1}{x-1}+\frac{x-1}{x}}{1-\frac{(x+1)}{(x-1)}\left(\frac{x-1}{x}\right)}=-7$$

$x=2$

$\underline{\text{check}}$

$\boxed{x\in\phi} \Rightarrow \underline{\text{Ans}}$

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$\boxed{\text{Ex-2, 3}}$ → leave Q15

3: Find x satisfying

$$(\tan^{-1}x)^2 - 3\tan^{-1}x + 2 > 0 \;\&\; [\sin^{-1}x] > [\cos^{-1}x], \quad [\cdot] = \text{G.I.F}$$

$\tan^{-1}x \in (-\infty, 1) \cup (2, \infty)$

$\tan^{-1}x \in \left(-\frac{\pi}{2}, 1\right)$

$$\boxed{x \in (-\infty, \tan 1)}$$

$\tan\theta > 1$

$\theta \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

$\tan^{-1}x = 1$

$x = \tan 1$

0, 1, 2, 3

−2, −1, 0, 1

0

1

$\sin^{-1}x \in \left[1, \frac{\pi}{2}\right]$ & $\cos^{-1}x \in [0, 1)$

$x \in [\sin 1, 1]$

Ans

$x \in [\sin 1, 1]$ $\qquad$ $x \in (\cos 1, 1]$

$$\boxed{[\sin 1, 1]}$$

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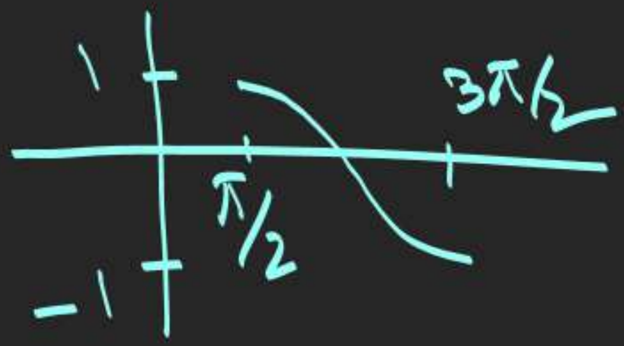
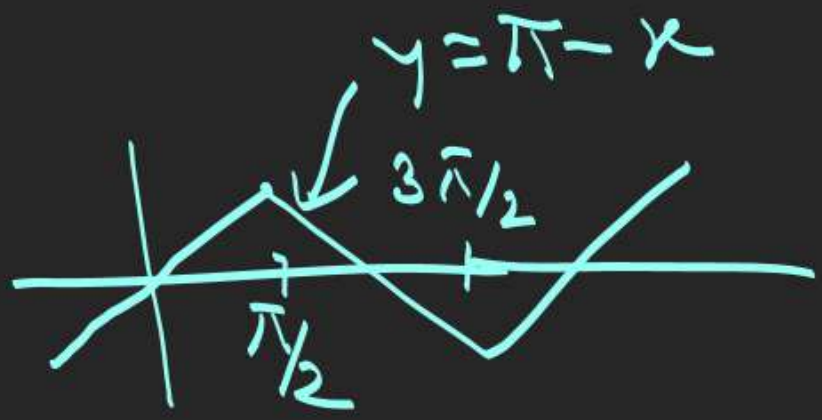
$f : \left[\frac{\pi}{2}, \frac{3\pi}{2}\right] \to [-1, 1]$, $f(x) = \sin x$, find $f^{-1}(x)$.





$f^{-1} : [-1, 1] \to \left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$

$f(f^{-1}(x)) = x \Rightarrow \sin(f^{-1}(x)) = x$.

$\pi - f^{-1}(x) = \sin^{-1}\sin(f^{-1}(x)) = \sin^{-1}x$

$f^{-1}(x) = \pi - \sin^{-1}x$.