Nishant Jindal

Q A Line $y=mx+1$ intersects

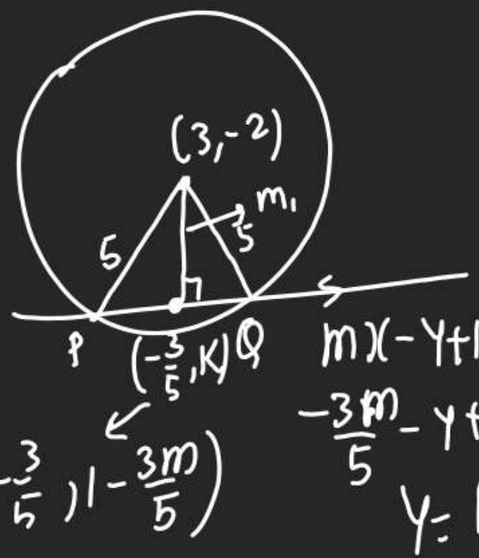
Adv $\equiv$ circle $(x-3)^2+(y+2)^2=25$ at

Pt. P & Q. If Mid Pt. of Line

Segment PQ has x coord $=-\frac{3}{5}$

then m $\in$

Centre $=(3,-2)$, $R=5$



$\left(-\frac{3}{5}, 1-\frac{3m}{5}\right)$

$mx-y+1=0$

$-\frac{3m}{5}-y+1=0$

$y=1-\frac{3m}{5}$

$m_1 \times m_2 = -1$

$m \times \left(\frac{1-\frac{3m}{5}+2}{-\frac{3}{5}-3}\right) = -1$

$m \times \left(3-\frac{3m}{5}\right) = \frac{18}{5}$

$m(15-3m) = 18$

$3m^2-15m+18=$

$3m^2-6m-9m+18=0$

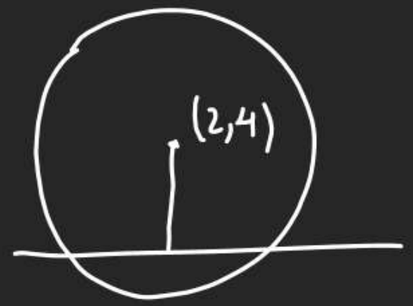
$3m(m-2)-9(m-2)=0$

$m=2, 3$

Q Circle

Mains $x^2+y^2=4x+8y+5$ intersects Line

$3x-4y=m$ at 2 distinct Pt find m $\in$?



$b<r$

$\frac{|6-16-m|}{\sqrt{3^2+-4^2}} < 5$

$|m+10| < 25$

$-25 < m+10 < 25$

$-35 < m < 15$

$-85 < m < -35$

$\cup -35 < m < 15$

$15 < m < 65$

NOT.

$x^2+y^2-4x-8y-5=0$

$(2,4)$

$R=\sqrt{4+16+5}=5$

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Line & Circle [Part 2]

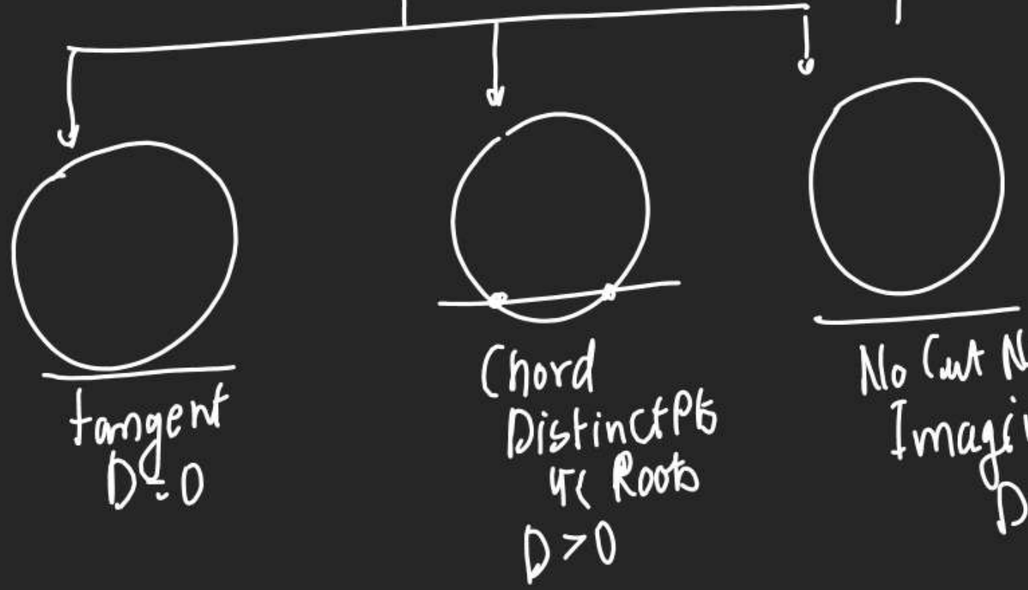
given Line :- $y = mx + c$

Circle → $x^2 + y^2 = a^2$

Combine Eqn करने पर

$x^2 + (mx + c)^2 = a^2$

$(m^2+1)x^2 + 2mcx + c^2 - a^2 = 0$



(A) $D = 0$ → Cond^n of tngency, E.O.T.

↓

Result 1 $B^2 - 4AC = 0$

$4m^2c^2 - 4 \cdot (m^2+1) \cdot (c^2 - a^2) = 0$

$4m^2c^2 - 4m^2c^2 + 4m^2a^2 - 4c^2 + 4a^2 = 0$

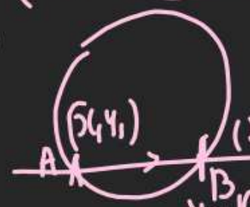
$c^2 = a^2(1+m^2)$

$\boxed{c = \pm a\sqrt{1+m^2}}$ → Cond^n of tngency

Result 2 → Eqn of tngent (Slope form)

$y = mx \pm a\sqrt{1+m^2}$

(B) Chord जो कि Q से गुजरता है



Length of chord = AB

$= \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

(x_1, y_1) & (x_2, y_2) Line

$y = mx + c$ (यही है)

$y_1 = mx_1 + c$

$y_2 = mx_2 + c$

$y_1 - y_2 = m(x_1 - x_2)$

$AB = \sqrt{(x_1 - x_2)^2 + m^2(x_1 - x_2)^2}$

$= |x_1 - x_2|\sqrt{1+m^2}$

$\boxed{AB = \sqrt{(x_1+x_2)^2 - 4x_1x_2}\sqrt{1+m^2}}$

आपको ये याद रखना है ① EOT (Slope form)

1) $y=mx+c$ touches $x^2+y^2=a^2$

then EOT → $y=mx\pm a\sqrt{1+m^2}$

2) $y=mx+c$ touched $(x-h)^2+(y-k)^2=a^2$

$y-k=m(x-h)\pm a\sqrt{1+m^2}$

3) $y=mx+c$ touches $x^2+y^2+2gx+2fy+c=0$

$(x+g)^2+(y+f)^2=(\sqrt{g^2+f^2-c})^2$

EOT → $y+f=m(x+g)\pm\sqrt{g^2+f^2-c}\cdot\sqrt{1+m^2}$

A) Circle given & a pt. on Circle given. & EOT is asked??

then Make changes

(B) $x^2\to xx_1$, $y^2\to yy_1$, $2x\to x+x_1$, $2y\to y+y_1$

$xy=\frac{xy_1+x_1y}{2}$

this will give EOT.

(C) EOT by making such changes is coded as T=0

② EOT. (Cartesian form)

When Line $y=mx+c$ is not given in Qs of tangent instead of that a Pt. (x_1,y_1) is given on circle.

(3) EOT. (Parametric form)

Just Replace (x_1,y_1) By $(a\cos\theta, a\sin\theta)$

Q Tangent of Circle.

$x^2+y^2=5$ at $(1,-2)$ = ?

$(1,-2) \to 1^2+(-2)^2=5$

$\Rightarrow (x_1, y_1) = (1,-2)$ lying on circle.

(cartesian formation)

EOT $\Rightarrow x\cdot 1 + y\cdot(-2) = 5$

$x - 2y = 5$

Q Find EOT at $(1,3)$ for

Circle $x^2+y^2=10$

$(1,3) \to 1^2+3^2=10$ ✓

$x\cdot 1 + y\cdot 3 = 10$

$x+3y=10$ EOT

Q. Find EOT at $(0,0)$ for $x^2+y^2-6x-4y=0$ ($6x \to 3\times 2x$, $4y \to 2\times 2y$)

$(0,0) \to 0^2+0^2-6\times 0-4\times 0=0$

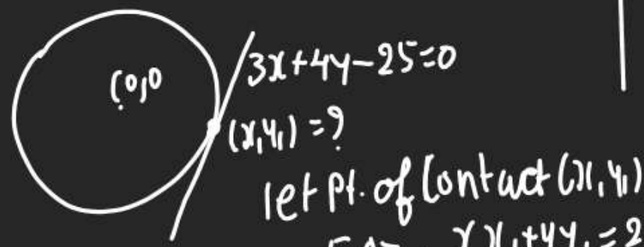
$0=0$ ✓

$x\cdot 0 + y\cdot 0 - 3(x+0) - 2(y+0) = 0$

$-3x-2y=0$

EOT $\equiv 3x+2y=0$

Q. If Line $3x+4y-25=0$ touches $x^2+y^2=25$ find Pt. of Contact.



let Pt. of Contact (x_1, y_1)

EOT $\to xx_1+yy_1=25$

$x_1+3y_1=20 \to (1)$

$8x_1-y_1=-15 \to (2)$

$24x_1-3y_1=-45$

$25x_1=-25 \Rightarrow x_1=-1$

$\frac{x_1-4}{1} = \frac{y_1+3}{-2} = \frac{-4x_1+3y_1}{-5}$

$-5x_1+20 = -4x_1+3y_1$

$xx_1+yy_1-25=0$

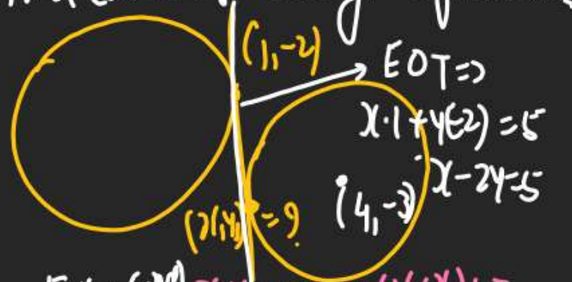
$3x+4y-25=0$

$\frac{x_1}{3} = \frac{y_1}{4} = \frac{-25}{-25} = 1$

$x_1=3, y_1=4 \therefore$ Pt. of Cont $=(3,4)$

Q Tangent to Circle $x^2+y^2=5$ at $(1,-2)$ also touches $x^2+y^2-8x+6y=0$ ($8x \to 4\times 2x$, $6y \to 3\times 2y$)

Find Coorresponding Pt. of Contact



EOT. (2^{nd}) $xx_1+yy_1-4(x+x_1)+3(y+y_1)=0$

$x(x_1-4)+y(y_1+3)-4x_1+3y_1=0$

$-5y_1-15 = 8x_1-6y_1$

$3y_1 = 20+1$

$y_1=7$

$(-1,7)$

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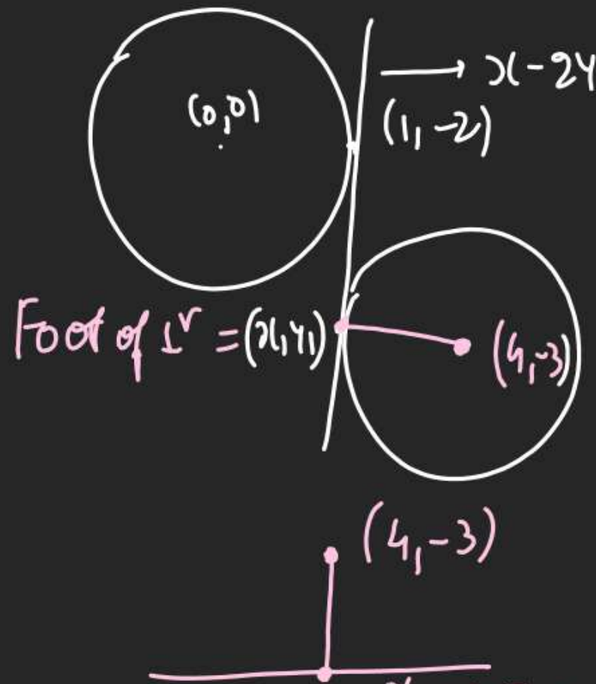
Q. Tangent to circle $x^2+y^2=5$ at $(1,-2)$ also touches Circle. $x^2+y^2-8x+6y+20=0$ Find Pt of contact.



(M3) Combine Eqn of Line

$x-2y-5=0$ & $x^2+y^2-8x+6y+20=0$

$(2y+5)^2+y^2-8(2y+5)+6y+20=0$

$5y^2+10y+5=0$

$y^2+2y+1=0 \Rightarrow (y+1)^2=0$

$y=-1$

$x=2y+5$

$=-2+5=3$

$(3,-1)$

Pt of (cnt =) $(3,-1)$

$\frac{\alpha-4}{1}=\frac{\beta+3}{-2}=\frac{-(4+6-5)}{1^2+-2^2}=-1$

$\alpha=-1+4=3$ | $\beta=2-3=-1$

When Pt. is outside of Circle

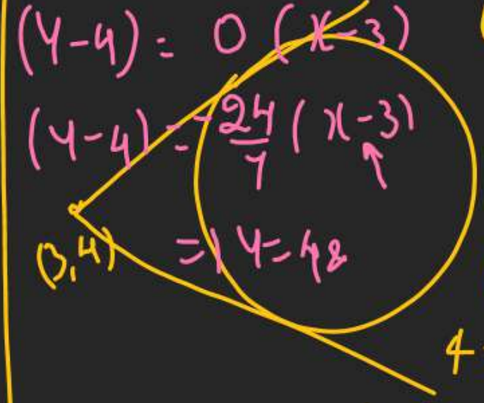
1) If Pt is outside of Circle Never Use Cart form.

2) Always Use Slope form: 1-16, 20-21-22-31

Q Find EOT. to Circle $x^2+y^2=16$ Passing thru $(3,4)$

$S(3,4) \rightarrow 3^2+4^2-16>0$

$(3,4)$ Circle के बाहर



(1) let EOT.

$y=mx\pm4\sqrt{1+m^2}$

(2) Both are P.T. (3,4)

$4=3m\pm4\sqrt{1+m^2}$

$4-3m=\pm4\sqrt{1+m^2}$

$16+9m^2-24m=16+16m^2$

$\Rightarrow 7m^2+24m=0 \Rightarrow m=0,-\frac{24}{7}$