Nishant Jindal

Basic Maths (Physics)

→ **Differentiation:-**

1. $\frac{d}{dx}(x^n) = nx^{n-1}$ ✓
2. $\frac{d}{dx}(\sin x) = \cos x$
3. $\frac{d}{dx}(\cos x) = -\sin x$
4. $\frac{d}{dx}(\tan x) = \sec^2 x$
5. $\frac{d}{dx}(\cot x) = -\mathrm{cosec}^2 x$
6. $\frac{d}{dx}(\mathrm{cosec}\,x) = -\mathrm{cosec}\,x \cdot \cot x$
7. $\frac{d}{dx}(\sec x) = \sec x \cdot \tan x$

$$y = \tan x$$

$$y = \frac{\sin x \rightarrow N}{\cos x \rightarrow D}$$

$$\frac{dy}{dx} = \frac{\cos x\left[\frac{d}{dx}(\sin x)\right] - \sin x \frac{d}{dx}(\cos x)}{\cos^2 x}$$

$$\frac{dy}{dx} = \frac{\cos x(\cos x) - \sin x(-\sin x)}{\cos^2 x}$$

$$\frac{d}{dx}(\tan x) = \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x} = \sec^2 x.$$

$$y = \frac{f(x) \rightarrow N}{g(x) \rightarrow D}$$

$$\frac{dy}{dx} = \frac{D\frac{d}{dx}(N) - N\frac{d}{dx}(D)}{D^2}$$

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$y = \text{Cosec}\,x$

$\left(\text{Cosec}\,x = \frac{1}{\sin x}\right)$

$\frac{d}{dx}(1) = 0$

$\frac{d}{dx}(\text{Cosec}\,x)$

$\frac{d}{dx}\left(\frac{1}{\sin x}\right)$ N D

$$= \frac{\sin x \frac{d}{dx}(1) - 1 \cdot \frac{d}{dx}(\sin x)}{(\sin x)^2}$$

$$= \frac{-\cos x}{\sin^2 x} = -\left(\frac{\cos x}{\sin x}\right) \times \frac{1}{\sin x}$$

$$= -\text{Cosec}\,x \cdot \text{Cot}\,x$$

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Basic Maths (Physics)

$\Rightarrow$ $\boxed{y = e^x}$ $\leftarrow$ (exponential function)

$$\boxed{\frac{d}{dx}(e^x) = e^x}$$

$\Rightarrow$ $y = \log_e x = \ln x$.

$$\boxed{\frac{d}{dx}(\ln x) = \frac{1}{x}}$$

$$\boxed{\begin{aligned} y &= \log_{10} x = \log(x) \\ y &= \log_e x = \ln x \end{aligned}}$$

$y = 2e^x + 3\sin x + 1$.

$$\frac{dy}{dx} = \frac{d}{dx}(2e^x) + \frac{d}{dx}(3\sin x) + \frac{d}{dx}(1)$$

$$= 2\frac{d}{dx}(e^x) + 3\frac{d}{dx}(\sin x) + 0$$

$$\frac{dy}{dx} = 2e^x + 3\cos x. \quad \text{Ans}$$

Basic Maths (Physics)

$y = e^x \ln x$

Find $\left(\frac{dy}{dx}\right)_{x=2} = ?$

$$\left[\frac{d}{dx}[(\mathrm{I})(\mathrm{II})] = \mathrm{I}\frac{d}{dx}(\mathrm{II}) + \mathrm{II}\frac{d}{dx}(\mathrm{I})\right]$$

Sol^n

$$\frac{d}{dx}(\underset{\mathrm{I}}{e^x}\,\underset{\mathrm{II}}{\ln x}) = e^x \frac{d}{dx}(\ln x) + \ln x \frac{d}{dx}(e^x)$$

$$= e^x \times \frac{1}{x} + \ln x\, e^x$$

$$= \frac{e^x}{x} + \ln x\, e^x$$

$$\frac{dy}{dx} = e^x\left[\frac{1}{x} + \ln x\right]$$

$$\left(\frac{dy}{dx}\right)_{x=2} = (e^2)\left[\frac{1}{2} + \ln 2\right]$$

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Basic Maths (Physics)

→ Chain rule :→

$y = f(x)$ | $x \to f(t)$

$$\left[\frac{dy}{dt} = ??\right]$$

$\left(\frac{dy}{dx}\right)$ multiply $\left(\frac{dx}{dt}\right)$ ✓

$$\boxed{\frac{dy}{dt} = \frac{dy}{\cancel{dx}} \times \frac{\cancel{dx}}{dt}}$$

\# $y = 2x^2$, $x = (2t+1)$

Find $\left(\frac{dy}{dt}\right)_{t=2} = ??$

$y \to f(x)$, $x \to f(t)$

$$\frac{dy}{dt} = \underbrace{\left(\frac{dy}{dx}\right)}_{4x} \times \underbrace{\left(\frac{dx}{dt}\right)}_{2}$$

$y = 2x^2$

$\frac{dy}{dx} = 2\frac{d}{dx}(x^2) = (2 \times 2)x^{2-1} = 4x$

$x = (2t+1)$

$\frac{dx}{dt} = 2\frac{d}{dt}(t) + \frac{d}{dt}(1) = 2$

$\frac{d}{dt}(t^1) = 1t^{1-1}$
$= 1$

$\frac{dy}{dt} = 4x \times 2$
$= 8x$

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Basic Maths (Physics)

$$\frac{dy}{dx} = 4x \qquad \frac{dx}{dt} = 2 \qquad x = (2t+1)$$

$$Y = \cos(x^2) = ??$$

$$\frac{dy}{dt} = \left(\frac{dy}{dx}\right) \times \frac{dx}{dt}$$

$$= 4x \times 2$$

$$\boxed{\frac{dy}{dt} = 8x}$$

$$\frac{dy}{dt} = 8(2t+1)$$

$$\frac{dy}{dt} = (16t+8)$$

$$\left(\frac{dy}{dt}\right)_{t=2} = (16 \times 2) + 8 = 40$$

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Basic Maths (Physics)

$$\frac{dy}{dx} = 4x \qquad \frac{dx}{dt} = 2$$

$$x = (2t+1)$$

$$Y = \cos(x^2) = ??$$

$$\frac{dy}{dt} = \left(\frac{dy}{dx}\right) \times \frac{dx}{dt}$$

$$= 4x \times 2$$

$$\boxed{\frac{dy}{dt} = 8x}$$

$$\frac{dy}{dt} = 8(2t+1)$$

$$\frac{dy}{dt} = (16t+8)$$

$$\left(\frac{dy}{dt}\right)_{t=2} = (16\times2)+8 = 40$$

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$y = \sqrt{\sin x}$

$\frac{dy}{dx} = ??$

put $\sin x = t$

$y = \sqrt{t}$

$t = \sin x$

$\left(\frac{dy}{dt}\right) = \frac{d}{dt}(t^{1/2}) = \frac{1}{2\sqrt{t}}$

$\frac{dt}{dx} = (\cos x)$

$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx} = \frac{1}{2\sqrt{t}} \times \cos x$

$$\boxed{\frac{dy}{dx} = \frac{\cos x}{2\sqrt{\sin x}}}$$

$y = \cos(x^2)$

put $x^2 = t$.

$y = \cos t$ | $t = x^2$

$\frac{dy}{dt} = -\sin t$

$\frac{dt}{dx} = 2x$

$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$

$= -(\sin t)(2x)$

$= -2x \sin x^2$ ✓

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$y = \sin kx$ $K = \text{Constant}$

$$\frac{d}{dx}(\sin kx) = k\cos kx$$

$$\frac{d}{dx}(\cos 5x) = -5\sin 5x$$

$y = \sin kx \rightarrow y = \sin t,\ t = kx.$

put $kx = t$

$$\frac{dy}{dt} = \cos t, \quad \frac{dt}{dx} = k\frac{d}{dx}(x) = k.$$

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx} = k\cos t = k\cos kx$$

$$\frac{d}{dx}[2\sin x] = 2\frac{d}{dx}(\sin x) = 2\cos x$$

$$\frac{d}{dx}\sin(2x) = 2\cos 2x$$

$$\frac{d}{dx}(\cos x) = -\sin x$$

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$y = \ln(3x+2)$

$$\frac{dy}{dx} = ?? \quad \frac{1}{(3x+2)} \times 3 = \frac{3}{(3x+2)} \text{ Ans.}$$

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$y = e^{\underbrace{ax+b}_{t}}$ (where a & b are Constant)

$$\frac{dy}{dx} = ??$$

$$y = e^t$$

$$\frac{dy}{dt} = e^t$$

$$t = (\underline{ax+b})$$

$$\frac{dt}{dx} = a\frac{d}{dx}(x) + \underbrace{\frac{d}{dx}(b)}_{0}$$

$$= \underline{\underline{a}}$$

$$\frac{d}{dx}(e^x) = e^x.$$

$$\frac{dy}{dx} = \left(\frac{dy}{dt}\right) \times \left(\frac{dt}{dx}\right) = ae^t = \underline{ae^{(ax+b)}}. \checkmark$$

$\boxed{\frac{d}{dx} e^{(\underline{ax+b})} = \underline{\underline{a}}e^{(ax+b)}}$

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$y = \dfrac{1}{\sqrt{x^2+2}}$

find $\left(\dfrac{dy}{dx}\right) = ??$

put $x^2+2 = t$

$y = \dfrac{1}{\sqrt{t}}$, $t = x^2+2$

$\dfrac{dy}{dt} = \dfrac{d}{dt}\left(t^{-1/2}\right)$ → n.

$\dfrac{dt}{dx} = (2x)$

$= \left(-\dfrac{1}{2}\right) t^{\left(-\frac{1}{2}-1\right)}$

$= -\dfrac{1}{2}\left(t^{-3/2}\right) = \dfrac{-1}{2\left(t^{3/2}\right)}$

$=$

$\dfrac{dy}{dx} = \dfrac{dy}{dt} \times \dfrac{dt}{dx}$

$= \dfrac{-1}{\cancel{2}\left(t^{3/2}\right)} \times \cancel{2}x$

$= \left[\dfrac{-x}{(x^2+2)^{3/2}}\right]$ Ans

$y = \ln(ax+b)$ → x

[a & b are Constants.

$\dfrac{dy}{dx} = \dfrac{1}{(ax+b)} \times a = \left(\dfrac{a}{ax+b}\right)$

put $ax+b = t$

$y = \ln t$, $t = (ax+b)$

$\dfrac{dy}{dt} = \dfrac{1}{t}$ $\dfrac{dt}{dx} = a$

$\dfrac{dy}{dx} = \dfrac{dy}{dt} \times \dfrac{dt}{dx}$

$= \left(\dfrac{1}{ax+b}\right) \times a = \left(\dfrac{a}{ax+b}\right)$

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\# $y = \frac{1}{3}\sin(18x)$ $\to K$

Find $\frac{dy}{dx} = ??$

$$\frac{dy}{dx} = \frac{1}{3} \times 18 \cos 18x$$

$$= 6 \sin 18x$$

\# $y = e^{7x}$

Fin $\frac{dy}{dx} = ??$

$$\frac{dy}{dx} = 7e^{7x}$$

✓