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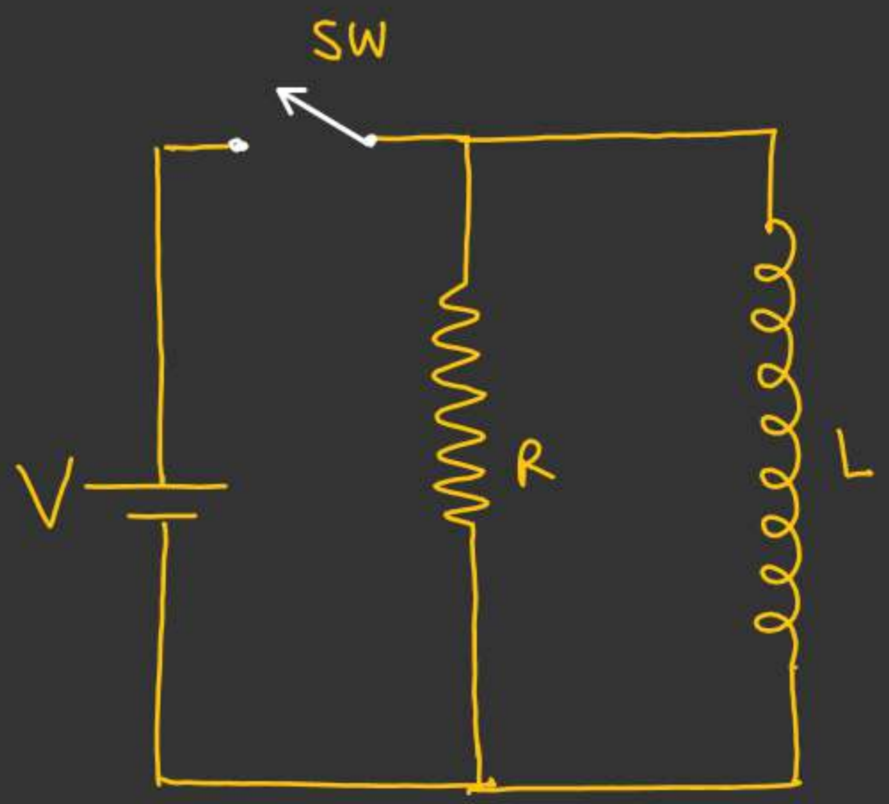
Case when L & R parallel to each other

At $t=0$, Switch is Closed.

Find a) Current as a function of time in the inductor.

b) Find time when Current in the inductor & resistor become equal



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$I = i_1 + i_2$

$I = \text{Constant}$

$$0 = \frac{dI}{dt} = \frac{di_1}{dt} + \frac{di_2}{dt}$$

$$\frac{di_2}{dt} = -\frac{di_1}{dt} \quad -(1)$$

KVL in loop cdefc

$$-L\frac{di_2}{dt} + i_1 R = 0$$

$$i_1 R = L\frac{di_2}{dt}$$

$$i_1 R = L\left(\frac{-di_1}{dt}\right)$$

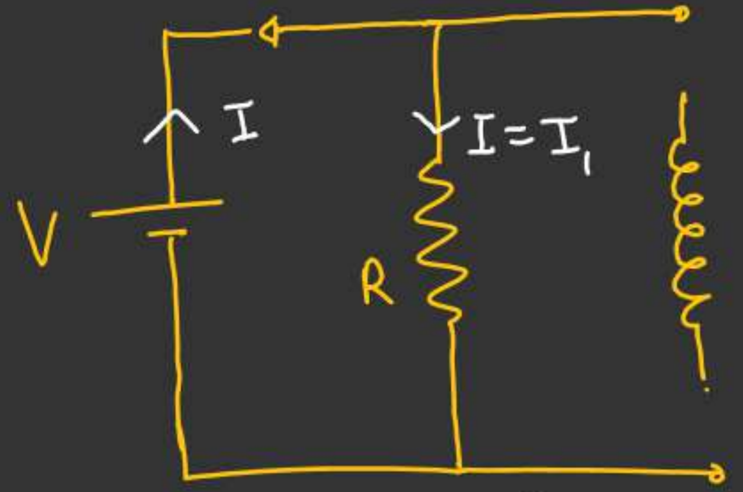
$$\int_I^{i_1} \frac{di_1}{i_1} = -\frac{R}{L}\int_0^t dt$$

$$\ln\left(\frac{i_1}{i}\right) = -\frac{R}{L}t$$

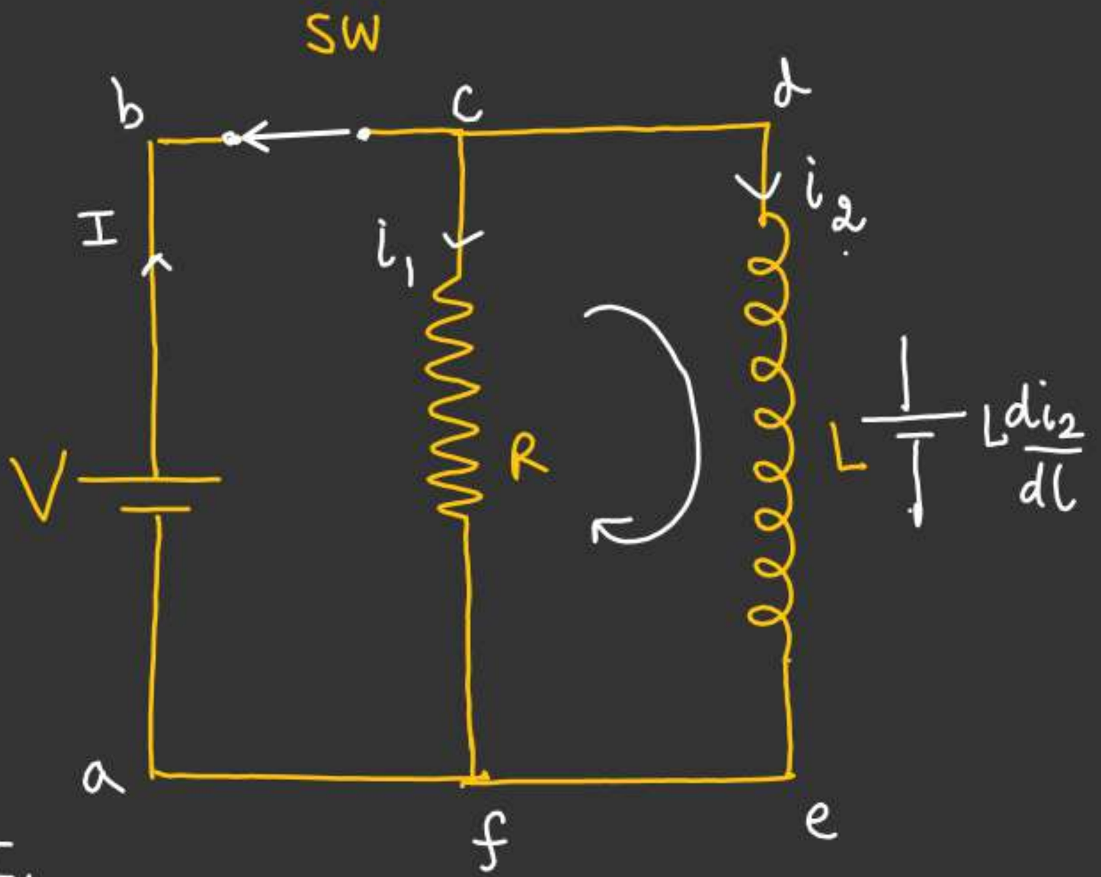
$$i_1 = i e^{-\frac{R}{L}t}$$

$$\boxed{i_1 = \frac{V}{R} e^{-\frac{R}{L}t}}$$

At $t = 0$



I_1 at $t = 0 = (V/R)$



$$i_2 = I - I_1$$

$$i_2 = i - i e^{-R/L\, t}$$

$$i_2 = i(1 - e^{-R/L\, t})$$

$$\boxed{i_2 = \frac{V}{R}(1 - e^{-R/L\, t})}$$

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Time when Current in the inductor and resistor equal.

$$i_1 = i_2$$

$$\frac{V}{R} e^{-\frac{R}{L}t} = \frac{V}{R}\left(1 - e^{-\frac{R}{L}t}\right)$$

$$2e^{-\frac{R}{L}t} = 1$$

$$e^{-\frac{R}{L}t} = \frac{1}{2}$$

$$-\frac{R}{L}t = \ln\left(\frac{1}{2}\right)$$

$$\cancel{-}\frac{R}{L}t = \cancel{-}\ln 2$$

$$t = \ln 2\left(\frac{L}{R}\right)$$

$$t = (0.693)\frac{L}{R}$$

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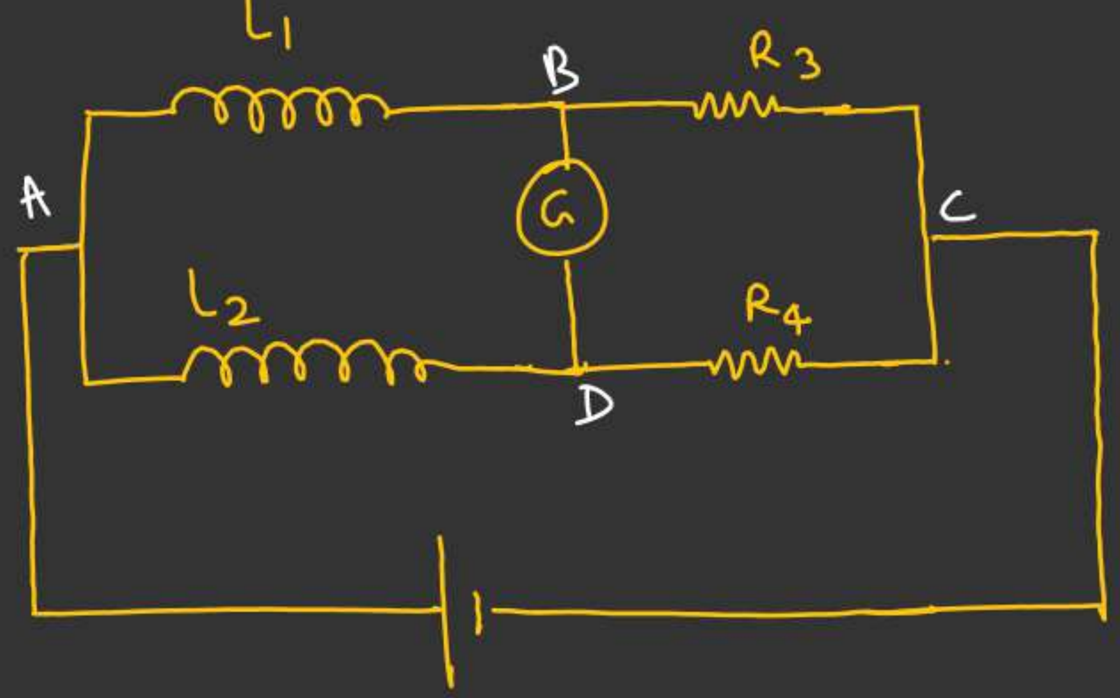
Q. L_1 and L_2 have resistance R_1 & R_2 respectively.

Find condition for Null deflection.



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For Null deflection

$V_B = V_D$

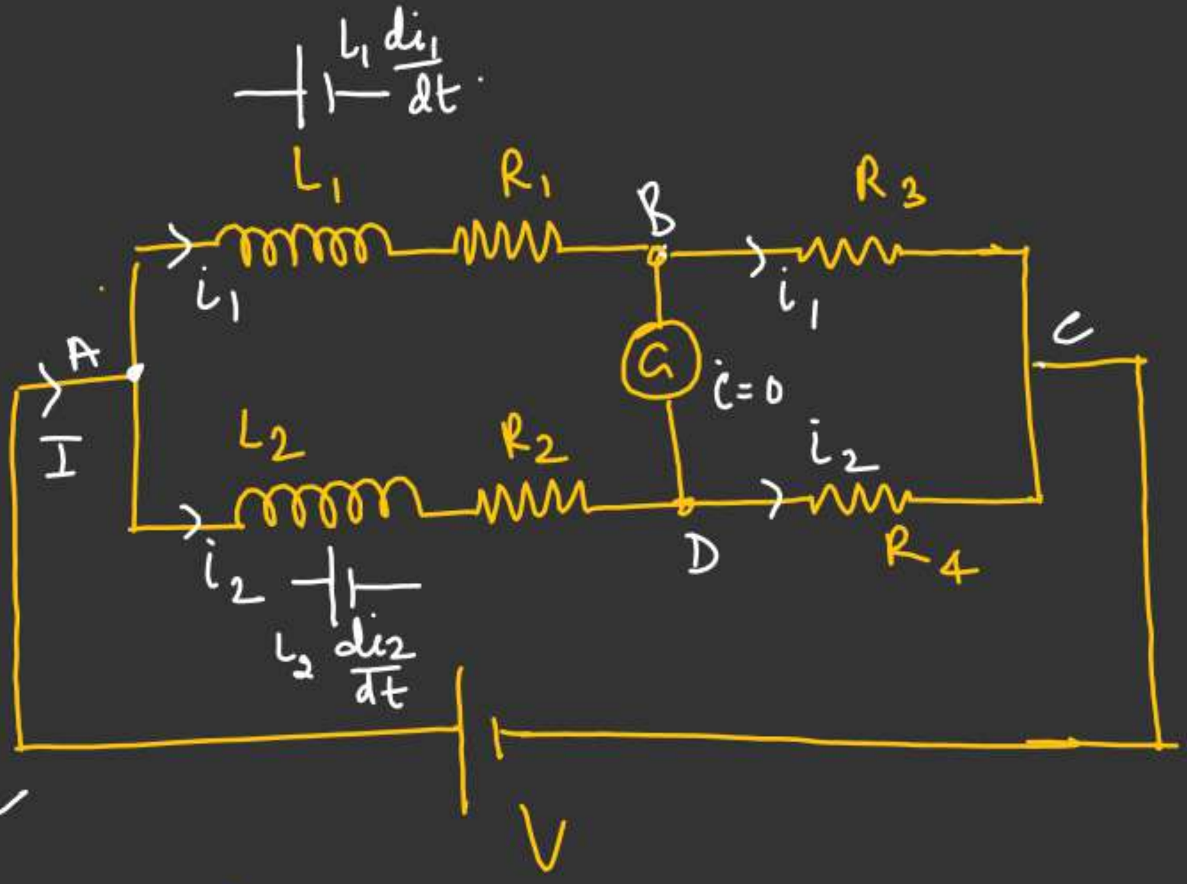
$$\underline{V_{AB} = V_{AD}}$$

$$L_1\left(\frac{di_1}{dt}\right) + i_1R_1 = L_2\frac{di_2}{dt} + i_2R_2$$

$$V_{BC} = V_{DC}$$

$$\underline{i_1R_3 = i_2R_4}$$

$$R_3\frac{di_1}{dt} = R_4\left(\frac{di_2}{dt}\right)$$



$$\frac{di_1}{dt} = \frac{R_4}{R_3}\left(\frac{di_2}{dt}\right) \checkmark$$

$$i_1 = \left(\frac{i_2R_4}{R_3}\right) \checkmark$$

$$L_1\frac{R_4}{R_3}\left(\frac{di_2}{dt}\right) - L_2\frac{di_2}{dt} = \left[i_2R_2 - \left(\frac{i_2R_4}{R_3}\right)R_1\right]$$

$$\boxed{\left(\frac{R_4L_1}{R_3} - L_2\right)\left(\frac{di_2}{dt}\right) = \left(R_2 - \frac{R_4R_1}{R_3}\right)i_2}$$

At $t = 0$, $i_2 = 0$, $\frac{di_2}{dt} \neq 0$.

$$\frac{R_4}{R_3}L_1 - L_2 = 0.$$

$$\frac{L_2}{L_1} = \frac{R_4}{R_3} \quad — (1)$$

$$\underline{\text{At } t \to \infty}$$

$\hookrightarrow \frac{di_2}{dt} = 0$, $i_2 = \text{Constant}$.

$$i_2 = \frac{V}{R_2 + R_4}$$

$$\Rightarrow R_2 - \frac{R_4R_1}{R_3} = 0.$$

$$\Rightarrow \left(\frac{R_2}{R_1} = \frac{R_4}{R_3}\right) — (2)$$

$$\boxed{\begin{array}{c}\frac{L_2}{L_1} = \frac{R_2}{R_1} = \frac{R_4}{R_3} \\ \frac{L_1}{L_2} = \frac{R_1}{R_2} = \frac{R_3}{R_4}\end{array}}$$

$\Rightarrow$ Condition for Null deflection.

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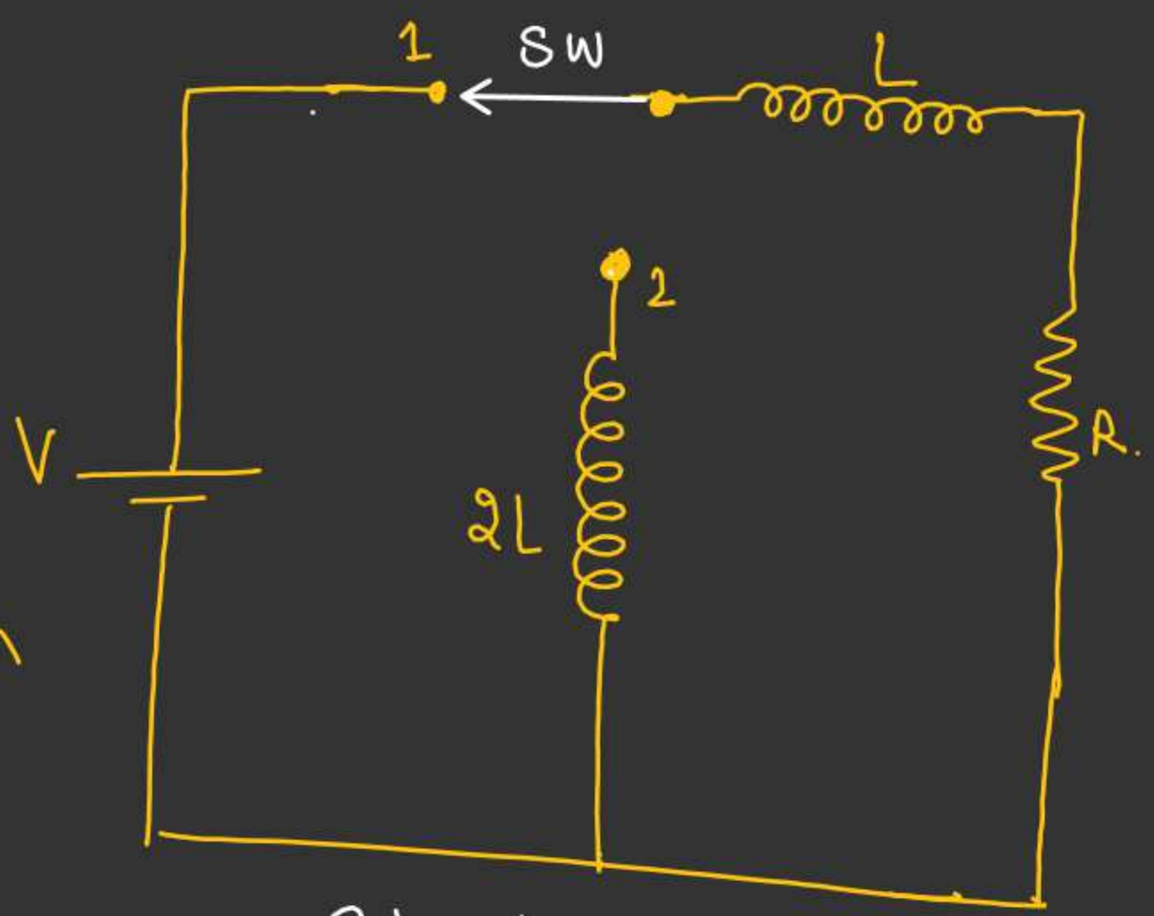
SW is closed for a very long time.

At $t=0$, Switch is shifted from position 1 to 2.

Find Current as a function 2.



Note :- When SW is shifted from position 1 to 2, Current in the Ckt changes at $t=0$.

flux just before shifting of switch will be same as just after shifting.

Steady state current.

$i_0 = \left(\frac{V}{R}\right)$.

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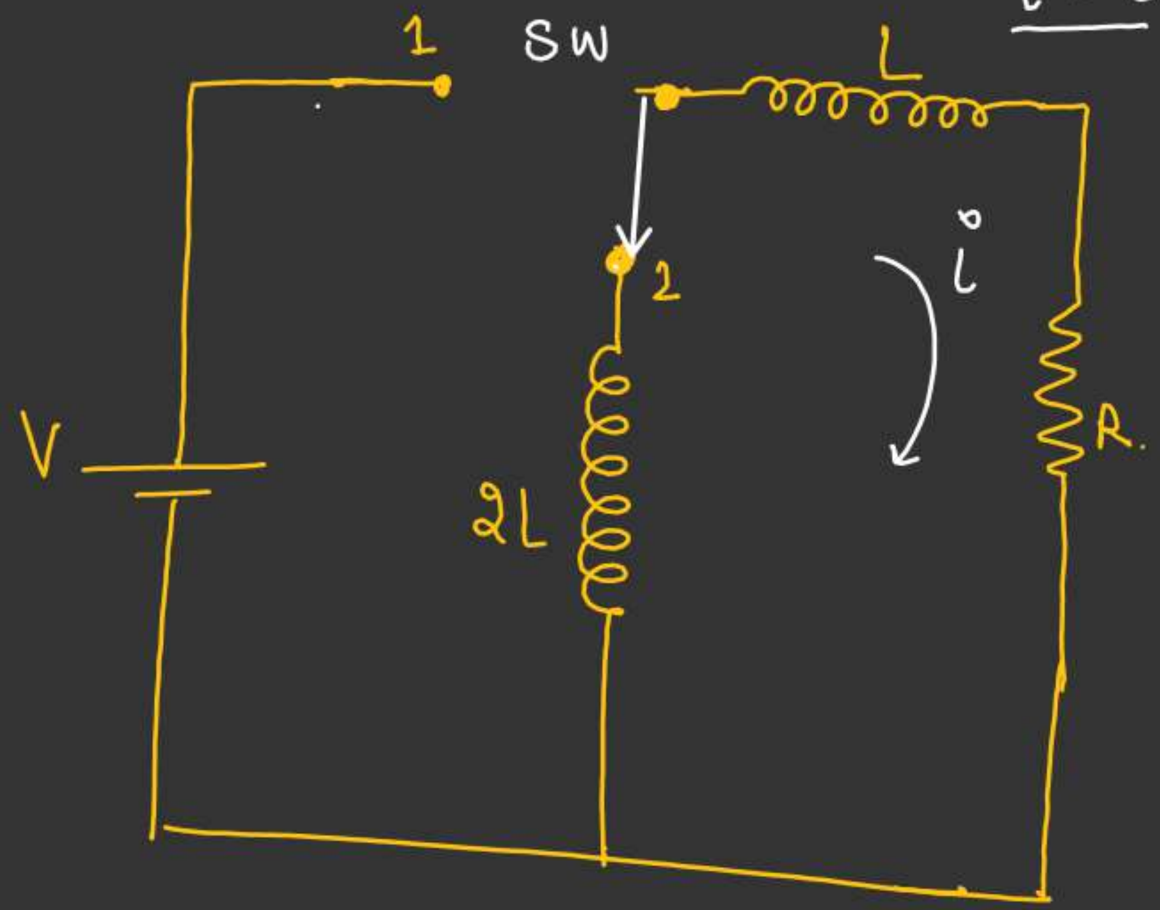
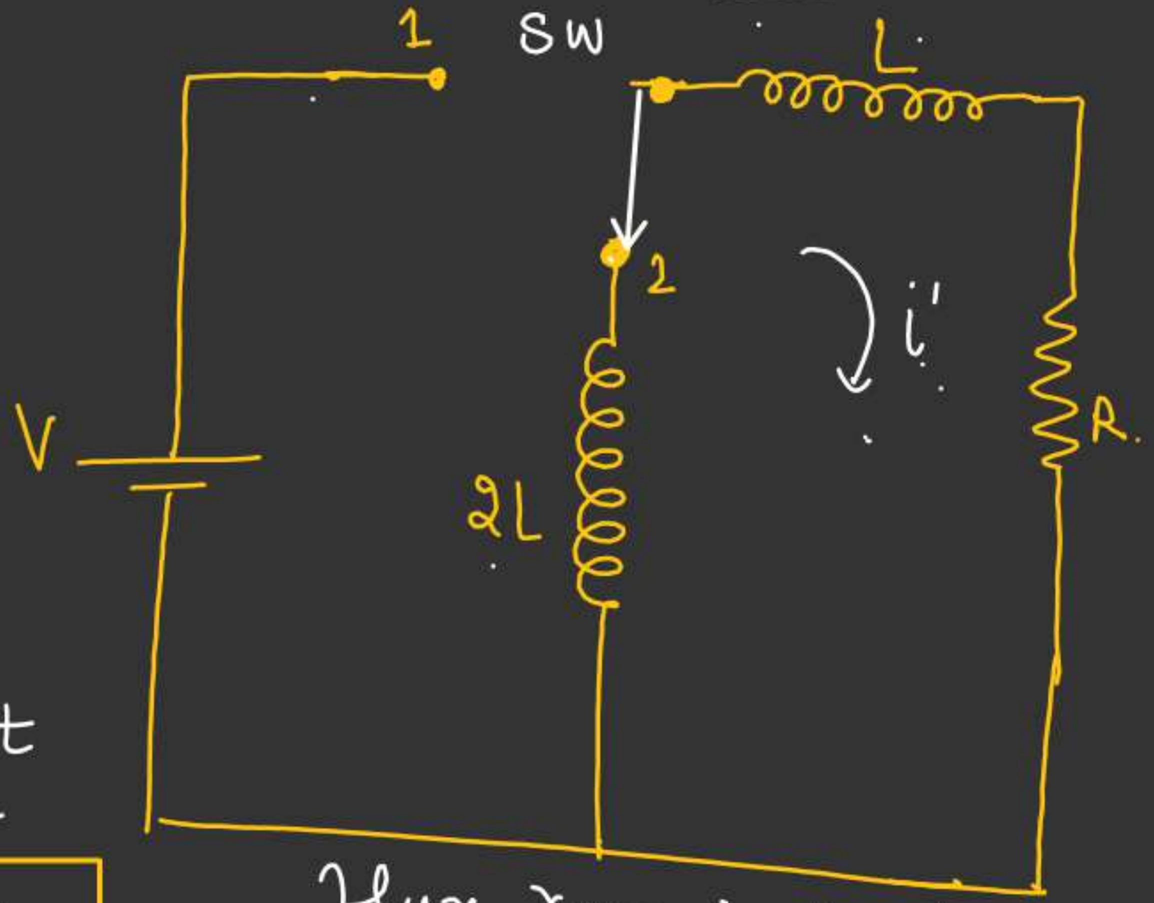
$t = t$.



$(\phi = Li)$

At $t = 0$



$$-3L\frac{di}{dt} = iR$$

$$\int_{i_0/3}^{i}\frac{di}{i} = \frac{-R}{3L}\int_0^t dt$$

$$\ln\left(\frac{i}{i_0/3}\right) = -\frac{R}{3L}t$$

$$\boxed{i = \frac{i_0}{3}e^{-\frac{R}{3L}t}}$$

Flux remain constant just before shifting of switch to just after shifting

$$Li_0 = 3Li' \Rightarrow i' = \left(\frac{i_0}{3}\right)$$

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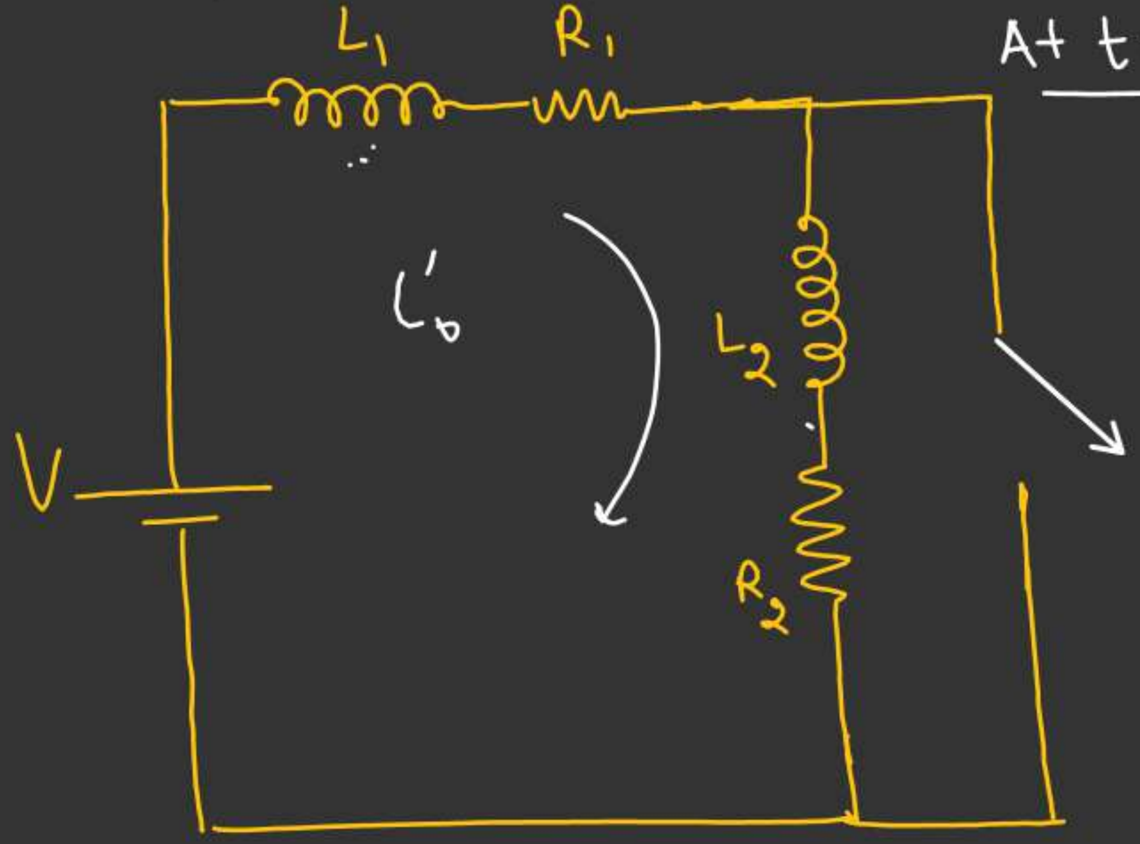
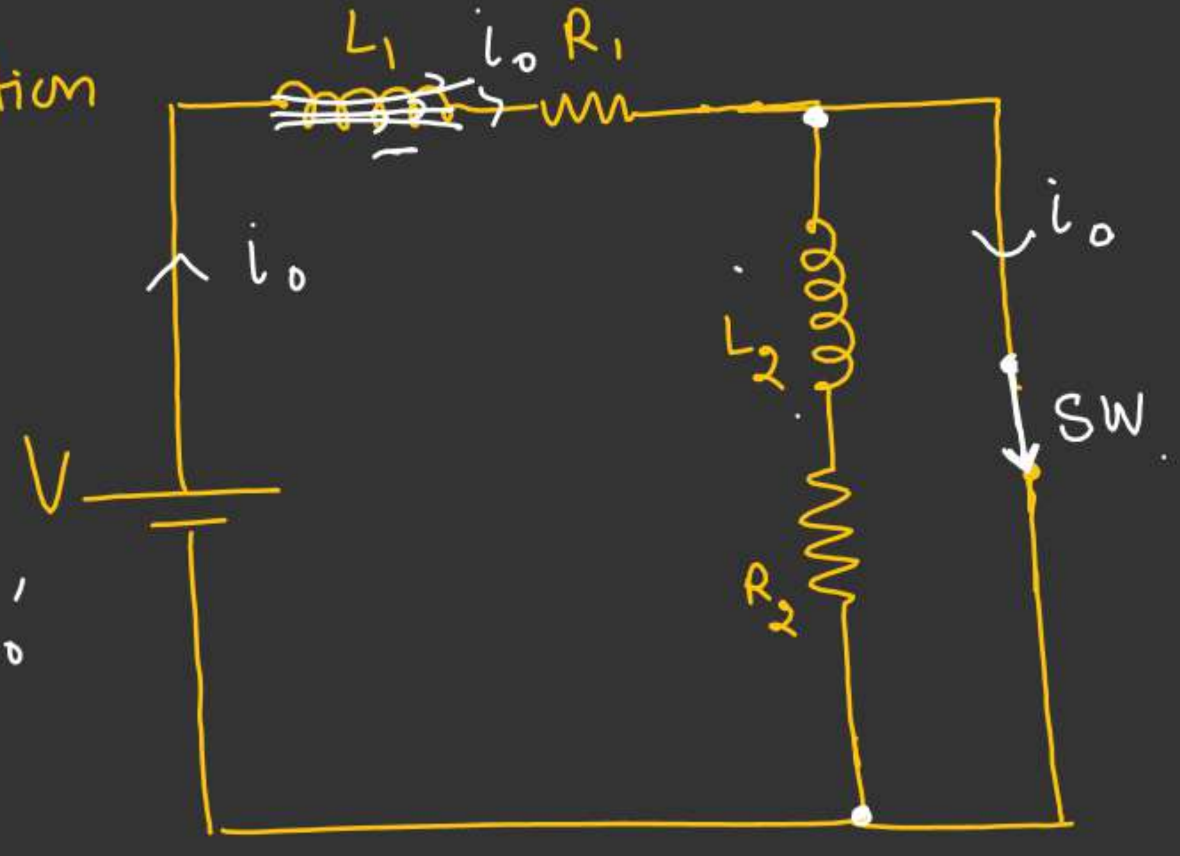
★★ SW Closed for a very long time. Find Current in the ckt as a function of time. At $t=0$ Switch is opened.



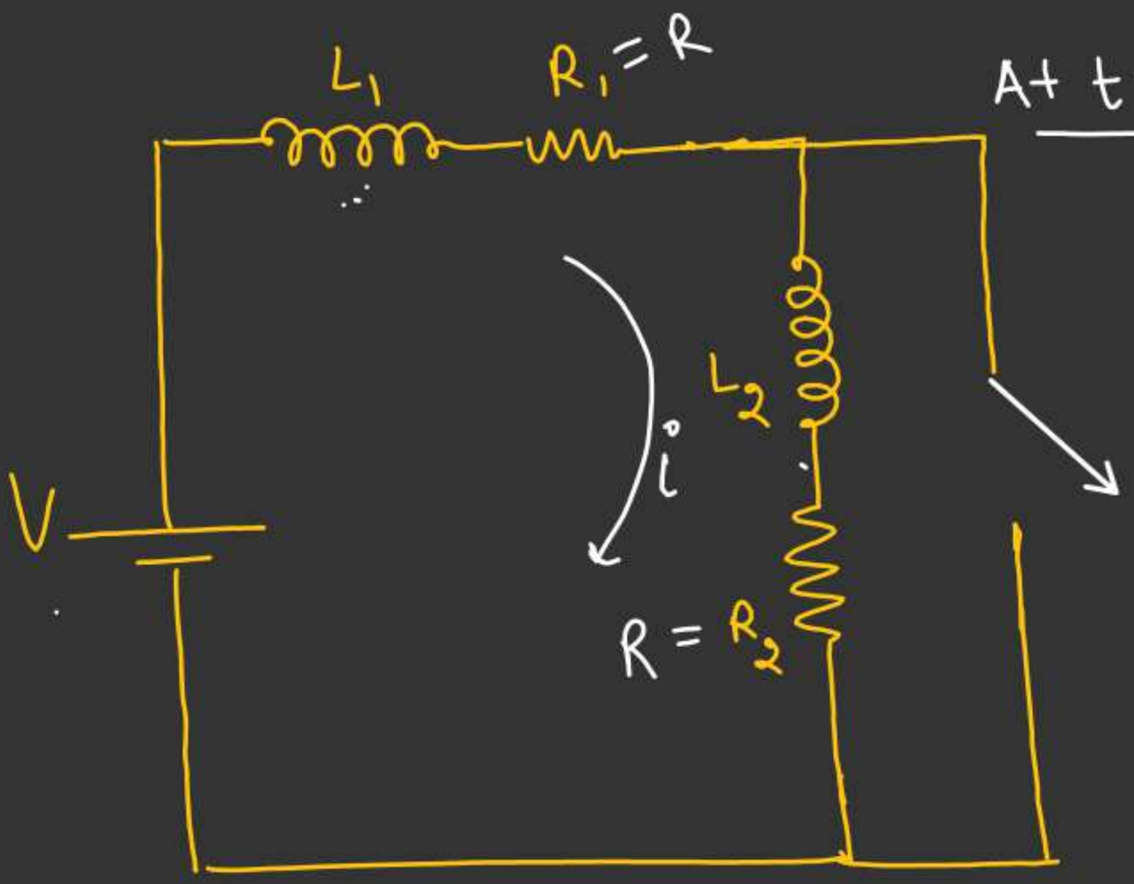
At $t=0$

$$L_1 i_0 = (L_1+L_2)\, i_0'$$

$$i_0' = \left(\frac{L_1}{L_1+L_2}\right) i_0$$



$$i_0 = \left(\frac{V}{R_1}\right).$$



At $t = t$

$$V - (L_1+L_2)\frac{di}{dt} - i2R = 0$$

$$V - 2Ri = (L_1+L_2)\frac{di}{dt}$$

$$\int_{i_0'}^{i} \left(\frac{di}{V-2Ri}\right) = \frac{1}{L_1+L_2}\int_0^t dt$$

$$\ln\left(\frac{V-2Ri}{V-2Ri_0'}\right) = \frac{-2R}{L_1+L_2}t$$

$$V-2Ri = (V-2Ri_0')\, e^{-\frac{2R}{L_1+L_2}t}$$

$$\Downarrow$$

$$i = \frac{V}{2R}\left[1 - \frac{L_2-L_1}{L_1+L_2}\, e^{-\frac{2R}{L_1+L_2}t}\right]$$

Put $i_0' = \dfrac{i_0 L_1}{L_1+L_2}$

$$i_0' = \frac{V}{R}\left(\frac{L_1}{L_1+L_2}\right)$$

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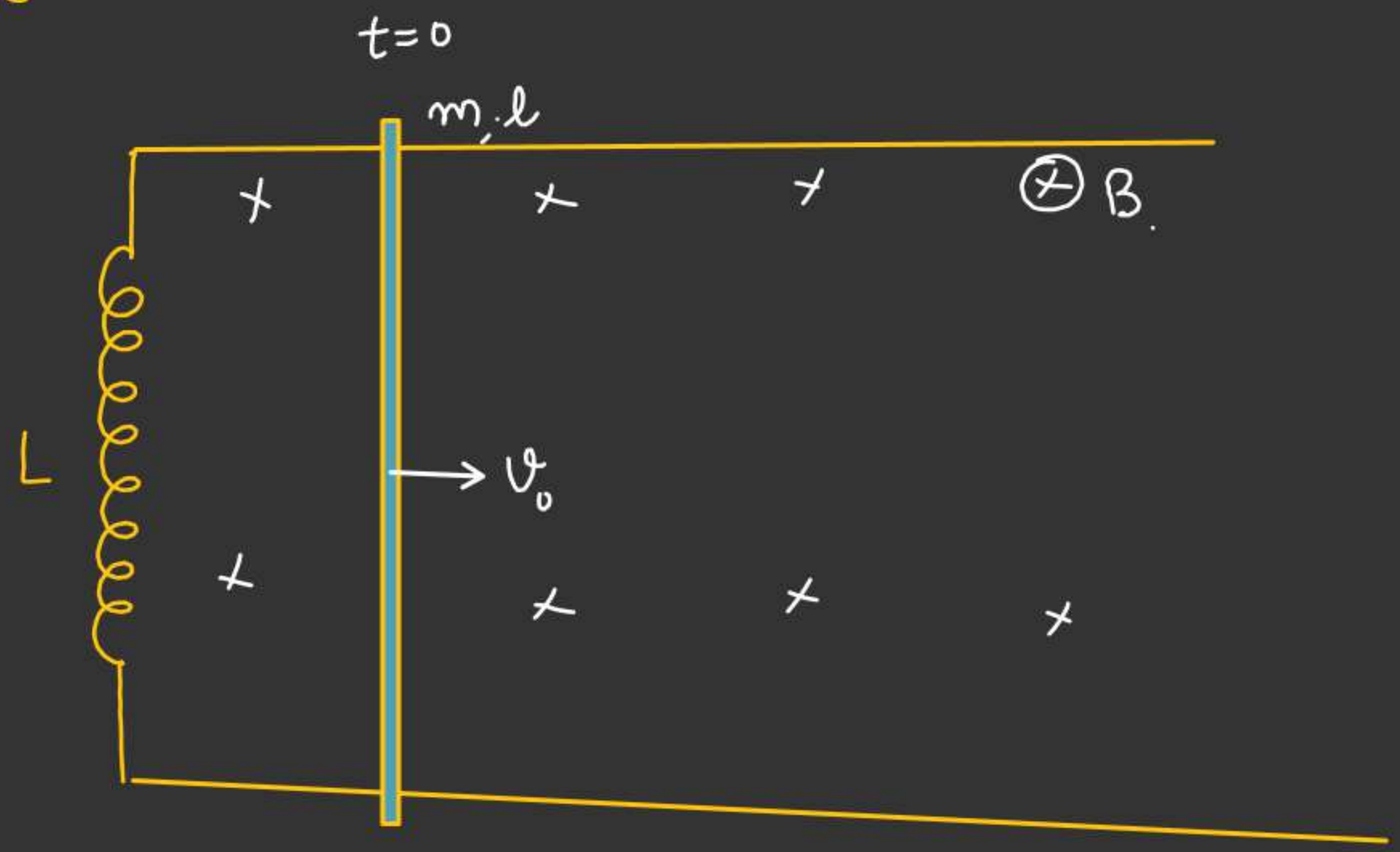
No Electrical resistance in the Ckt.

Slider projected on Conducting frictionless parallel rails with velocity v_0 at $t=0$.

Find a) Displacement of Slider as a function of time.



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$$Bl v = L\frac{dI}{dt}$$

$$\Downarrow$$

$$\frac{dI}{dt} = \left(\frac{Blv}{L}\right)$$

$$F_B = IlB$$

$$\Downarrow$$

$$a = \frac{F_B}{m}$$

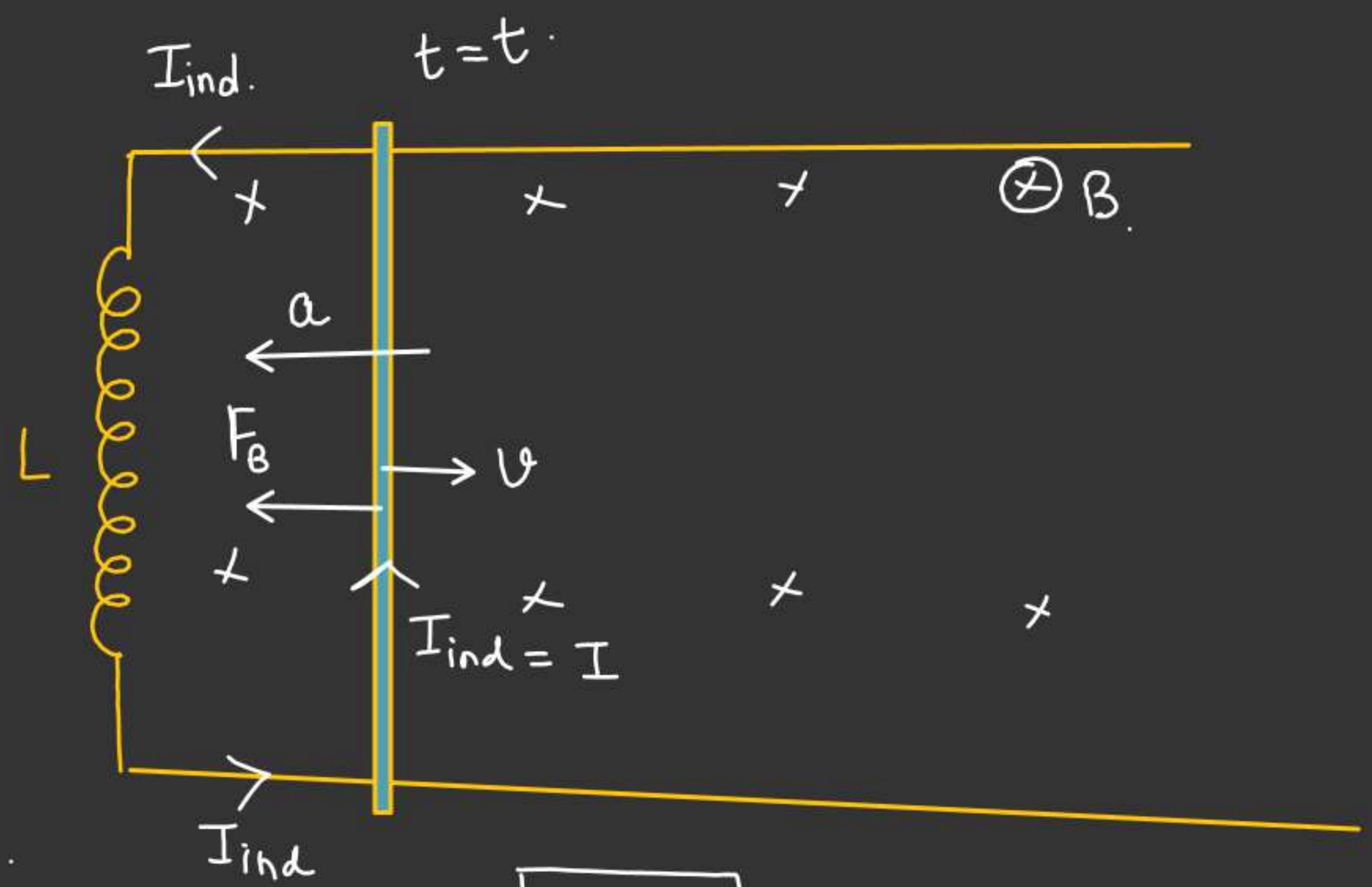
$$a = \frac{Bl}{m} I$$

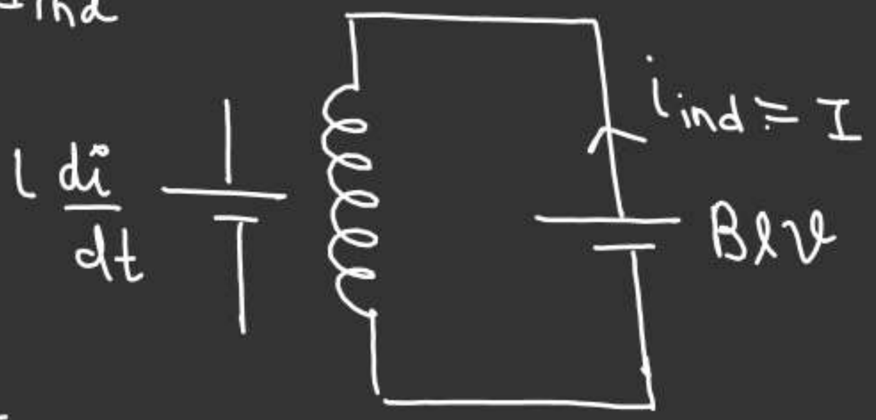
$$-\frac{dv}{dt} = \frac{Bl}{m}(I)$$

Again differentiating w.r.t time.

$$\frac{d^2v}{dt^2} = -\frac{Bl}{m}\left(\frac{dI}{dt}\right)$$

$$\frac{d^2v}{dt^2} = -\frac{Bl}{m}\left(\frac{Blv}{L}\right) = -\left(\frac{B^2l^2}{mL}\right)v$$





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$$\frac{d^2v}{dt^2} = -\frac{B^2l^2}{mL}v$$

$$\frac{B^2l^2}{mL} \Downarrow \omega^2$$

$$\omega^2 = \frac{B^2l^2}{mL}$$

$$\omega = \frac{Bl}{\sqrt{mL}}$$

$$v_{max} = v_0$$

$$v_0 = A\omega$$

$$A = \frac{v_0}{\omega} = \frac{v_0}{Bl}\sqrt{mL}$$

$\Downarrow$

Amplitude

$$v = v_0 \sin(\omega t + \phi)$$

At $t=0$, $v = v_0$.

$$\sin\phi = 1 \quad \phi = \pi/2$$

$$v = v_0 \sin(\omega t + \pi/2)$$

$$\boxed{v = v_0 \cos\omega t}$$

$\Downarrow$

$$\frac{dx}{dt} = v_0 \cos\omega t$$

$$\int_0^x dx = v_0 \int_0^t \cos\omega t \cdot dt$$

$$x = \frac{v_0}{\omega}\left[\sin\omega t\right]_0^t$$

$$a = -\omega^2 x$$

$$\frac{d^2x}{dt^2} = -\omega^2 x$$

$\Downarrow$

$$x = A\sin(\omega t + \phi) \checkmark$$

$$(v_{max} = A\omega)$$

$$x = \frac{v_0}{\omega}\sin\omega t$$

$\Downarrow$

$$\boxed{x = A\sin\omega t}$$

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H.W.: Slider is released from the frictionless vertical rails.
No Electrical resistance.

Find $X \to f(t)$