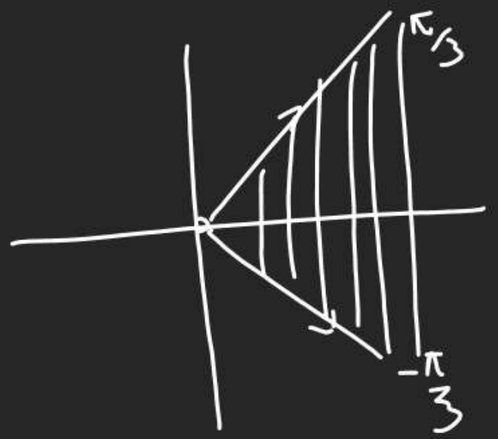
Nishant Jindal

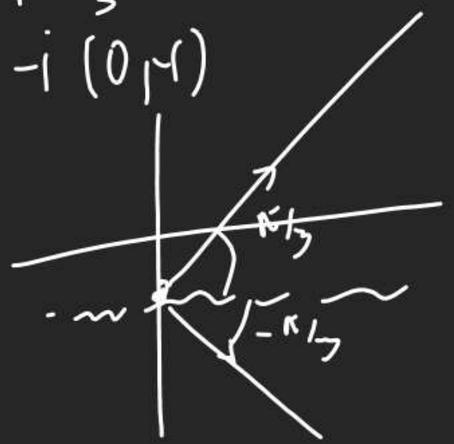
Q $|\text{Arg}(z)| \le \frac{\pi}{3}$ Locus.

$-\frac{\pi}{3} \le \text{Arg}(z) \le \frac{\pi}{3}$



Q $|\text{Arg}(z+i)| \le \frac{\pi}{3}$ ($z - z_1$)

Ray starts P.I. $\to -i$ $(0,-1)$



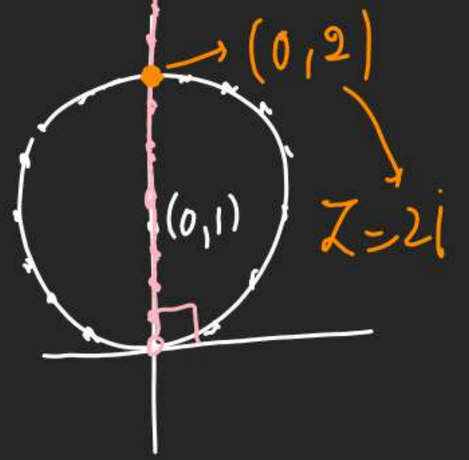
Q Find Complex No.

Satisfying

$|z-i| = 1$ ← Rad. & $\text{Arg}\, z = \frac{\pi}{2}$ $(z \to 0)$

$|z - z_1| = K$ — Rep. Circle. | Ray.

(entre $-i = (0,1)$



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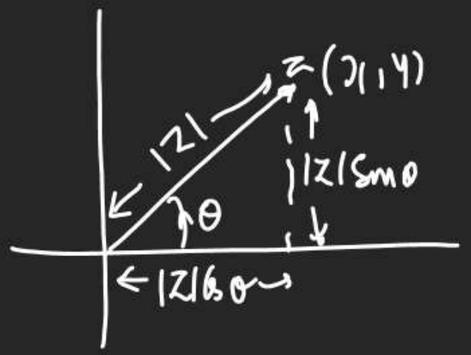
Different forms of C.N.

(1) Cart. form → $Z = x+iy$; (x,y) x,y Real.

(2) Polar form → $|Z|, \theta$



$Z = x+iy$

$= |Z|\cos\theta + i|Z|\sin\theta$

$= |Z|(\cos\theta + i\sin\theta)$; $\theta = \text{Arg } Z$

$Z = |Z|\text{cis}\,\theta = |Z|\text{cis}(\text{Arg } Z)$

$1+\sqrt{3}i = (1,\sqrt{3}) = 1^{st}\,Q$. $\Rightarrow \text{Arg } Z = \tan^{-1}(\sqrt{3}/1) = \frac{\pi}{3}$ & $|Z| = 2$

$Z = 2\,\text{cis}\left(\frac{\pi}{3}\right)$

$1-\sqrt{3}i = (1,-\sqrt{3}) = 4^{th}$ $\Rightarrow \text{Arg } Z = -\tan^{-1}\left|\frac{-\sqrt{3}}{1}\right| = -\frac{\pi}{3}$ & $|Z| = 2$

$Z = 2\,\text{cis}\left(-\frac{\pi}{3}\right)$

$-1-\sqrt{3}i = (-1,-\sqrt{3}) = 3^{rd} \Rightarrow \text{Arg } Z = -\pi + \tan^{-1}\left|\frac{-\sqrt{3}}{-1}\right| = -\pi + \frac{\pi}{3} = -\frac{2\pi}{3}$.

$Z = 2\,\text{cis}\left(-\frac{2\pi}{3}\right)$

$-1+\sqrt{3}i = (-1,\sqrt{3}) = 2^{nd} \rightarrow \text{Arg } Z = \pi - \tan^{-1}\left|\frac{\sqrt{3}}{-1}\right| = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$

$Z = 2\,\text{cis}\left(\frac{2\pi}{3}\right)$

$5i = (0,5) = +\text{ve Im}$ $\quad |Z| = 5$ $\quad Z = 5\,\text{cis}\,\frac{\pi}{2}$

$-5 = (-5,0) = -\text{ve Real}$ $\quad Z = 5\,\text{cis}(\pi)$

$1+\sqrt{2} = (1+\sqrt{2},0) = +\text{ve Real}$. $\quad Z = 1+\sqrt{2}\,\text{cis}\,0$

$\text{Arg } Z = 0$

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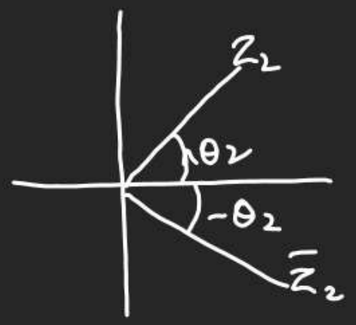
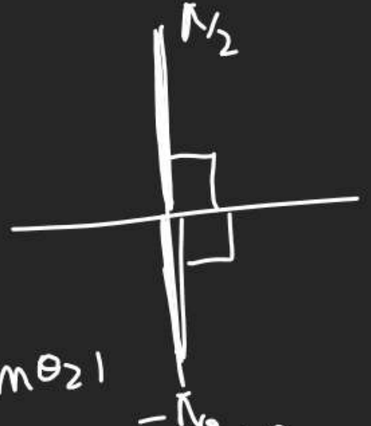
Result

$|z_1+z_2|^2 = |z_1|^2+|z_2|^2+2Re(z_1\bar{z}_2)$

$|z_1-z_2|^2 = |z_1|^2+|z_2|^2-2Re(z_1\bar{z}_2)$



Progress:



$Re(z_1\cdot z_2) = |z_1|\cdot cis\theta_1 \cdot |z_2|\cdot cis\theta_2$

$= |z_1||z_2|(\cos\theta_1+i\sin\theta_1)\cdot(\cos\theta_2+i\sin\theta_2)$

$= |z_1||z_2|\cdot(\cos\theta_1\cos\theta_2-\sin\theta_1\sin\theta+i\sin\theta_1\cos\theta+i\sin\theta_2\cos\theta_1)$

$Re(z_1\cdot z_2) = |z_1||z_2|(\cos\theta_1\cdot\cos\theta_2-\sin\theta_1\cdot\sin\theta_2)$ $\quad z=-\bar{z}$

$= |z_1||z_1|\cos(\theta_1+\theta_2)$

$\therefore Re(z_1\bar{z}_2) = |z_1||\bar{z}_2|\cos(\theta_1-\theta_2) = |z_1||z_2|\cos(\theta_1-\theta_2)$

New Results

$|z_1+z_2|^2 = |z_1|^2+|z_2|^2+2|z_1||z_2|\cos(\theta_1-\theta_2)$

$|z_1-z_2|^2 = |z_1|^2+|z_2|^2-2|z_1||z_2|\cos(\theta_1-\theta_2)$

Q P.T. $|z_1+z_2|^2+|z_1-z_2|^2 = 2(|z_1|^2+|z_2|^2)$

Prove करो।

Q If $|z_1+z_2|^2 = |z_1|^2+|z_2|^2$ then P.T. $z_1\bar{z}_2 = -z_2\bar{z}_1$

here $2|z_1||z_2|\cos(\theta_1-\theta_2)=0$

$\Rightarrow \cos(\theta_1-\theta_2)=0 \Rightarrow \theta_1-\theta_2 = \pm\frac{\pi}{2}$

$\Rightarrow \arg z_1-\arg z_2 = \pm\frac{\pi}{2} \Rightarrow \arg\left(\frac{z_1}{z_2}\right) = \pm\frac{\pi}{2}$

$\Rightarrow \frac{z_1}{z_2}$ is Img. No $\Rightarrow \frac{z_1}{z_2} = -\left(\frac{\bar{z}_1}{\bar{z}_2}\right)$

$z_1\bar{z}_2 = -z_2\bar{z}_1$ H.P.

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Properties of $(\bar{z})$

(1) $(\bar{\bar{z}}) = z$

(2) $z + \bar{z} = 2\mathrm{Re}(z)$ $\quad z+\bar{z} = x+iy+x-iy = 2x$

(3) $x = \frac{z+\bar{z}}{2}$

(4) $z - \bar{z} = 2i\,\mathrm{Im}(z)$

$y = \frac{z-\bar{z}}{2i}$

(5) $\overline{(z_1+z_2)} = \bar{z}_1 + \bar{z}_2$

(6) $\overline{(z_1 - z)} = \bar{z}_1 - \bar{z}_2$

(7) $\left(\frac{\bar{z}_1}{z_2}\right) = \frac{\bar{z}_1}{\bar{z}_2}$ (8) $\overline{(z_1 \cdot z_2)} = \bar{z}_1 \cdot \bar{z}_2$

(9) $(\bar{z}^n) = (\bar{z})^n$

Properties of Arg(z)

(1) $\mathrm{Arg}(\bar{z}) = -\mathrm{Arg}\,z$

(2) $\mathrm{Arg}\,z_1 + \mathrm{Arg}\,z_2 = \mathrm{Arg}(z_1 \cdot z_2)$

(3) $\mathrm{Arg}\left(\frac{z_1}{z_2}\right) = \mathrm{Arg}\,z_1 - \mathrm{Arg}\,z_2 + 2k\pi$

(4) $\mathrm{Arg}(z^n) = n\,\mathrm{Arg}\,z$

(5) $\mathrm{Arg}(iz) = \mathrm{Arg}\,i + \mathrm{Arg}\,z = \frac{\pi}{2} + \mathrm{Arg}\,z$

(6) $\mathrm{Arg}(\omega z) = \mathrm{Arg}\,\omega + \mathrm{Arg}\,z = \frac{2\pi}{3} + \mathrm{Arg}\,z$

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Q If $|z_1+z_2|=|z_1|+|z_2|$ then

$\text{Arg}\frac{z_1}{z_2}=?$

$|z_1+z_2|^2=(|z_1|+|z_2|)^2$

$|z_1|^2+|z_2|^2+2|z_1||z_2|\cos(\theta_1-\theta_2)=|z_1|^2+|z_2|^2+2|z_1||z_2|$

$\Rightarrow \cos(\theta_1-\theta_2)=1$

$\theta_1-\theta_2=0$

$\text{Arg}\,z_1-\text{Arg}\,z_2=0$

$\text{Arg}\frac{z_1}{z_2}=0$

Q If $|z_1+z_2|=|z_1|-|z_2|$

then $\text{Arg}\frac{z_1}{z_2}=?$

After Sqr

$2|z_1||z_2|\cos(\theta_1-\theta_2)=-2|z_1||z_2|$

$\cos(\theta_1-\theta_2)=-1$

$\theta_1-\theta_2=\pi$

$\text{Arg}\left(\frac{z_1}{z_2}\right)=\pi$

Q If $|z_1+z_2|=|z_1-z_2|$ then

$\text{Arg}\frac{z_1}{z_2}=?$

Sqr

$2|z_1||z_2|\cos(\theta_1-\theta_2)=-2|z_1||z_2|\cos(\theta_1-\theta_2)$

$2\cos(\theta_1-\theta_2)=0$

$\theta_1-\theta_2=\pm\frac{\pi}{2}$

$\frac{z_1}{z_2}$ = Purely I. nry

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Q If $|z| = |w|$ & $\text{Arg}(z \cdot w) = \pi$

then $z = ?$

$w \quad \bar{w} \quad -w \quad -\bar{w}$

$\text{Arg}(zw) = \pi$

$\text{Arg}\, z + \text{Arg}\, w = \pi$

$\text{Arg}\, z = \pi - \theta$

$z = |z|(\cos\theta + i\sin\theta)$ — Hota hai

$z = |z|(\cos(\pi-\theta) + i\sin(\pi-\theta))$

$= (-\cos\theta + i\sin\theta)$ → (is form ko Convert)

$= -|z|(\cos\theta - i\sin\theta)$

$= -|z|(\cos(-\theta) + i\sin(-\theta))$

$= -|w|(\cos(-\text{Arg}\, w) + i\sin(-\text{Arg}\, w))$

$= -|\bar{w}|(\cos(\text{Arg}\,\bar{w}) + i\sin(\text{Arg}\,\bar{w}))$ — $\bar{w}$ का Polarform

$= -\bar{w}$

Q If $|zw| = 1$ & $\text{Arg}\, z - \text{Arg}(w) = \frac{\pi}{2}$

then $\bar{z}w = ?$

$\bar{z}w = |\bar{z}w|(\cos(\text{Arg}\,\bar{z}w) + i\sin(\text{Arg}\,\bar{z}w))$

1) $|z||w| = 1$
$|\bar{z}||w| = 1$
$|\bar{z}w| = 1$

$= 1\left(\cos\left(-\frac{\pi}{2}\right) + i\sin\left(-\frac{\pi}{2}\right)\right)$

$= -i$

2) $\text{Arg}\, z - \text{Arg}\, w = \frac{\pi}{2}$

$\text{Arg}\, z + \text{Arg}\,\bar{w} = \frac{\pi}{2}$

$\text{Arg}(z \cdot \bar{w}) = \frac{\pi}{2}$

$\text{Arg}(\bar{z}w) = \text{Arg}(\overline{z\bar{w}})$

$= -\frac{\pi}{2}$

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① Correct it

$$Z = 6(\sin 310^\circ - i\cos 310^\circ)$$

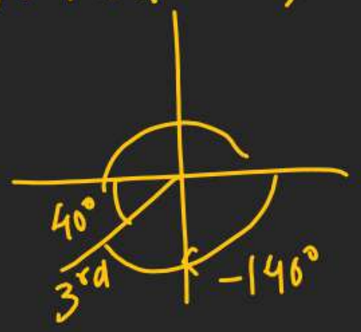
$\rightarrow i$ com ⊗

$$= 6(\cos(90^\circ - 310^\circ) - i\sin(90 - 310^\circ))$$

$$= 6(\cos(-220^\circ) - i\sin(-220^\circ))$$

$$= 6(\cos 220^\circ + i\sin 220^\circ)$$

$$= 6(\cos(-140^\circ) + i\sin(-140^\circ))$$



② Correct

$$-5(\cos 40^\circ - i\sin 40^\circ)$$

$$Z = -5\cos 40^\circ + 5i\sin 40^\circ$$

(-ve, +ve) = 2nd Q.

$$|Z| = 5$$

$$\text{Arg}(Z) = \pi - \tan^{-1}\left|\frac{\cancel{5}\sin 40^\circ}{-\cancel{5}\cos 40^\circ}\right|$$

$$= \pi - 40^\circ$$

$$Z = 5(\cos(\pi - 40^\circ) + i\sin(\pi - 40^\circ))$$

$$= 5(\cos 140^\circ + i\sin 140^\circ)$$

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Exponantial form / Euler form.

$z = x+iy$

$= |z|(\cos\theta + i\sin\theta)$

$\boxed{z = |z| \cdot e^{i\theta}}$ $\theta = \text{Arg } z$

$e^x = 1 + \frac{x}{1} + \frac{x^2}{2} + \frac{x^3}{3}$, $\sin x = x - \frac{x^3}{3}$, $\cos x = 1 - \frac{x^2}{2}$

$e^{i\theta} = 1 + i\theta - \frac{\theta^2}{2} - \frac{i\theta^3}{3} + \frac{\theta^4}{4}$

$= \left(1 - \frac{\theta^2}{2} + \frac{\theta^4}{4} \cdots\right) + i\left(\theta - \frac{\theta^3}{3} + \frac{\theta^5}{5}\right)$

$= \cos\theta + i\sin\theta$

Q If $(\cos\theta + i\sin\theta)\cdot(\cos 2\theta + i\sin 2\theta)\cdot(\cos 3\theta + i\sin 3\theta)\cdots(\cos n\theta + i\sin n\theta) = 1$

find θ = ?

$e^{i\theta} \times e^{i2\theta} \times e^{i3\theta} \cdots e^{in\theta} = e^{i(0+2K\pi)}$

$e^{i\theta(1+2+3\cdots n)} = e^{i(2K\pi)} \Rightarrow \boxed{\theta = \frac{4K\pi}{(n)(n+1)}}$

Q $\left[\frac{1+i\tan\alpha}{1-i\tan\alpha}\right]^{2n} - \left[\frac{1+i\tan 2n\alpha}{1-i\tan 2n\alpha}\right] = ?$

$n \in I$

$\left[\frac{\cos\alpha + i\sin\alpha}{\cos\alpha - i\sin\alpha}\right]^{2n} - \left[\frac{\cos 2n\alpha + i\sin 2n\alpha}{\cos 2n\alpha - i\sin 2n\alpha}\right]$

$\left[\frac{\cos\alpha + i\sin\alpha}{\cos(-\alpha) + i\sin(-\alpha)}\right]^{2n} - \left[\frac{\cos 2n\alpha + i\sin 2n\alpha}{\cos(-2n\alpha) + i\sin(-2n\alpha)}\right]$

$\left[\frac{e^{i\alpha}}{e^{-i\alpha}}\right]^{2n} - \left[\frac{e^{i2n\alpha}}{e^{-i2n\alpha}}\right]$

$\left[e^{2i\alpha}\right]^{2n} - \left[e^{i4n\alpha}\right]$

$e^{i4n\alpha} - e^{i4n\alpha} = 0$

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Q $z_1 = 1+\sqrt{3}i \to (1,\sqrt{3})$ ①

$z_2 = 1-\sqrt{3}i \to (1,-\sqrt{3})$ ④

then $\dfrac{z_1^{100}+z_2^{100}}{z_1+z_2}$

$|z_1| = 2 = |z_2|$

$z_1 = 2(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}) = 2e^{i\frac{\pi}{3}}$

$z_2 = 2(\cos(-\frac{\pi}{3}) + i\sin(-\frac{\pi}{3})) = 2e^{-i\frac{\pi}{3}}$

$z_1^{100} = 2^{100}\cdot e^{i\frac{100\pi}{3}} = 2^{100}(\cos\frac{100\pi}{3} + i\sin\frac{100\pi}{3})$

$z_2^{100} = 2^{100}\cdot e^{-i\frac{100\pi}{3}} = 2^{100}(\cos(-\frac{100\pi}{3}) + i\sin(-\frac{100\pi}{3}))$

$\dfrac{z_1^{100}+z_2^{100}}{z_1+z_2} = \dfrac{2^{100}\times 2\cos\frac{100\pi}{3}}{2(2\cos\frac{\pi}{3})} = 2^{99}(\quad)$

Q $\dfrac{z_1}{z_2} = \dfrac{|z_1|}{|z_1|}\cdot\dfrac{(\cos\theta_1)}{(\cos\theta_2} = \dfrac{|z_1|}{|z_2|}\cdot\dfrac{e^{i\theta_1}}{e^{i\theta_2}} = \dfrac{|z_1|}{|z_2|}\cdot e^{i(\theta_1-\theta_2)}$

$\dfrac{z_1}{z_2} = \dfrac{|z_1|}{|z_2|}\cdot(\cos(\theta_1-\theta_2)$

Q. $z_1\cdot z_2 = |z_1||z_2|\cdot(\cos(\theta_1+\theta_2)$

Q. $z_1\cdot\bar{z}_2 = |z_1||z_2|(\cos(\theta_1-\theta_2)$

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Q If $x+\frac{1}{x}=2\cos\theta$ then $x^n+\frac{1}{x^n}=2\cos n\theta$ (P.T.)

↳ (is को yad

$x=1\cdot(\cos\theta+i\sin\theta) \Rightarrow x=e^{i\theta}$

$\frac{1}{x}=1(\cos(-\theta)+i\sin(-\theta)) \Rightarrow \frac{1}{x}=e^{-i\theta}$

$x+\frac{1}{x}=2\cos\theta$

$x^n+\frac{1}{x^n}=(e^{i\theta})^n+(e^{-i\theta})^n$

$=e^{in\theta}+e^{-in\theta}$

$=\cos(n\theta)+i\sin\theta+\cos(-n\theta)+i\sin(-n\theta)$

$=2\cos n\theta_{RHS}.$