Nishant Jindal

$e^{y \ln x}$

$2\tan^{-1}x$

$-\pi$

$+\pi$

$\log x = \log_e x = \ln x$

$y = \frac{1-\cos x}{\sin x} = \tan\frac{x}{2}$

$\boxed{\frac{\frac{2}{1+x^2}}{2x}}$

$1 + y' = x^y\left(\frac{y}{x} + y'\ln x\right)$

$-a\sin(2x+2y)(1+y') = 0$

$\pi^c = 180^\circ$

$y = \sqrt{x+y}$

$y^2 - y = x$

$(2y-1)y' = 1$

$y' = \frac{1}{2y-1}$

$y^2 - y = \ln x$

$(2y-1)y' = \frac{1}{x}$

$y''(0)$

$y' = a e^{ax+b}$

$y'' = a^2 e^{ax+b}$

$\boxed{y''(0) = a^2 e^b}$

$\frac{d}{dx}\sec\left(\frac{\pi x}{180}\right) = \frac{\pi}{180}\sec\frac{\pi x}{180}\tan\frac{\pi x}{180}$

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1. Let $f(x) = \begin{vmatrix} x^3 & \sin x & \cos x \\ 6 & -1 & 0 \\ p & p^2 & p^3 \end{vmatrix}$ where p is constant.

find $\frac{d^3 f(x)}{dx^3}$ at $x=0$.

$\boxed{\underline{\text{Ans}} \quad 0}$

$$f'(x) = \begin{vmatrix} 3x^2 & \cos x & -\sin x \\ 6 & -1 & 0 \\ p & p^2 & p^3 \end{vmatrix}$$

$$f''(x) = \begin{vmatrix} 6x & -\sin x & -\cos x \\ 6 & -1 & 0 \\ p & p^2 & p^3 \end{vmatrix}$$

$$f'''(x) = \begin{vmatrix} 6 & -\cos x & \sin x \\ 6 & -1 & 0 \\ p & p^2 & p^3 \end{vmatrix}$$

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Q. If $D = \begin{vmatrix} f & g & h \\ (xf)' & (xg)' & (xh)' \\ (x^2f)'' & (x^2g)'' & (x^2h)'' \end{vmatrix}$ P.T. $D' = \begin{vmatrix} f & g & h \\ f' & g' & h' \\ (x^3f'')' & (x^3g'')' & (x^3h'')' \end{vmatrix}$

$$= \begin{vmatrix} f & g & h \\ f+xf' & g+xg' & h+xh' \\ 2f+4xf'+x^2f'' & 2g+4xg'+x^2g'' & 2h+4xh'+x^2h'' \end{vmatrix}$$

$R_2 \to R_2 - R_1$, $R_3 \to R_3 - 4R_2 + 2R_1$

$$= \begin{vmatrix} f & g & h \\ xf' & xg' & xh' \\ x^2f'' & x^2g'' & x^2h'' \end{vmatrix} = \begin{vmatrix} f & g & h \\ f' & g' & h' \\ x^3f'' & x^3g'' & x^3h'' \end{vmatrix}$$

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Polynomial

$f(x) = \underline{(x-\alpha)^2} g(x) \Rightarrow f(\alpha) = 0 = f'(\alpha)$

$f'(x) = 2(x-\alpha)g(x) + (x-\alpha)^2 g'(x)$

$f(x) = \underline{(x-\alpha)^3} g(x) \Rightarrow \boxed{f(\alpha) = f'(\alpha) = f''(\alpha) = 0}$

$f'(x) = 3(x-\alpha)^2 g(x) + (x-\alpha)^3 g'(x)$

$\vdots$

$\boxed{\text{If } f(x) = (x-\alpha)^n g(x) \Rightarrow f(\alpha) = f'(\alpha) = f''(\alpha) = \dots = f^{(n-1)}(\alpha) = 0}$

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L'Hospital's rule

If $f'(x)$ and $g'(x)$ are continuous at $x=a$ and

$\lim_{x\to a} f(x)=0$, $\lim_{x\to a} g(x)=0$ or $\lim_{x\to a} f(x)=\pm\infty$ & $\lim_{x\to a} g(x)=\pm\infty$

then $\lim_{x\to a}\left(\frac{f(x)}{g(x)}\right) = \lim_{x\to a}\frac{f'(x)}{g'(x)}$

$\frac{0}{0}$ or $\frac{\pm\infty}{\pm\infty}$

$$\lim_{x\to a}\frac{\frac{g'(x)}{g^2(x)}}{\frac{f'(x)}{f^2(x)}} = l = \lim_{x\to a}\frac{\frac{1}{g(x)}}{\frac{1}{f(x)}} =$$

$$\lim_{x\to a}\frac{f(x)-f(a)}{g(x)-g(a)} = \lim_{x\to a}\frac{(f(x)-f(a))/(x-a)}{(g(x)-g(a))/(x-a)} = \frac{f'(a)}{g'(a)}$$

$$\lim_{x\to a}\frac{f'(x)}{g'(x)}$$

Limit

- Standard limits ✓
- Series
- L'Hospital

$\frac{0}{0}$ $\quad$ $\frac{\infty}{\infty}$

1. $\lim\limits_{x\to 0^+} \left(x^x\right)$ $\qquad 0^0$

$x \ln x$

$$= \lim_{x\to 0^+} e^{\frac{\ln x}{1/x}} \qquad \frac{\infty}{\infty}$$

$$= \lim_{x\to 0^+} e^{\frac{1/x}{-\frac{1}{x^2}}} = \lim_{x\to 0^+} e^{-x}$$

$$= \boxed{1}$$

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2. $\lim_{x\to 0^+} (\operatorname{cosec} x)^{\frac{1}{\ln x}}$ $\quad \infty^0$ $= \lim_{x\to 0^+} e^{\frac{\ln \operatorname{cosec} x}{\ln x}}$

$$- \frac{\ln \sin x}{\ln x}$$

3. $\lim_{x\to 1} \left(\frac{x^x - x}{x - 1 - \ln x} \right)$

$$\lim_{x\to 0^+} e^{-\left(\frac{\ln\left(\frac{\sin x}{x}\right) + \ln x}{\ln x} \right)}$$

$$e^{-\frac{\cot x}{1/x}}$$

$$= \lim_{x\to 0^+} e^{-\frac{x}{\tan x}}$$

$$\lim_{x\to 0^+} e^{-\left(1 + \frac{\ln \frac{\sin x}{x}}{\ln x}\right)}$$

$$e^{-1} = \lim_{x\to 0^+} e^{-\left(1 + \frac{\ln \frac{\sin x}{x}}{\ln x}\right)}$$

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$$\lim_{x\to 1} \frac{x^x - x}{x-1-\ln x} = \lim_{x\to 1} \frac{x\left(x^{x-1}-1\right)}{x-1-\ln x}$$

$$2 = \frac{x\left(e^{(x-1)\ln x}-1\right)}{(x-1)\ln x} \cdot \frac{(x-1)\ln(1+(x-1))}{\left((x-1)-\ln(1+(x-1))\right)}$$

$$\frac{(x-1)^2 \cdot \frac{\ln(1+(x-1))}{(x-1)}}{(x-1)-\ln(1+(x-1))}$$

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$$\lim_{x\to 0}\left(\frac{\log_{\sec\frac{x}{2}}(\cos x)}{\log_{\sec x}\left(\cos\frac{x}{2}\right)}\right) = \lim_{x\to 0}\left(\frac{\ln\cos x}{\ln\cos\frac{x}{2}}\right)^2$$

$$\boxed{PT-3}$$

$$16 = \left(\frac{\frac{\ln(1+(\cos x-1))}{(\cos x-1)}\cdot\frac{(\cos x-1)}{x^2}}{\frac{\ln\left(1+\cos\frac{x}{2}-1\right)}{\cos\frac{x}{2}-1}\cdot\frac{\left(\cos\frac{x}{2}-1\right)}{\left(\frac{x}{2}\right)^2}\,\frac{1}{4}}\right)^2$$