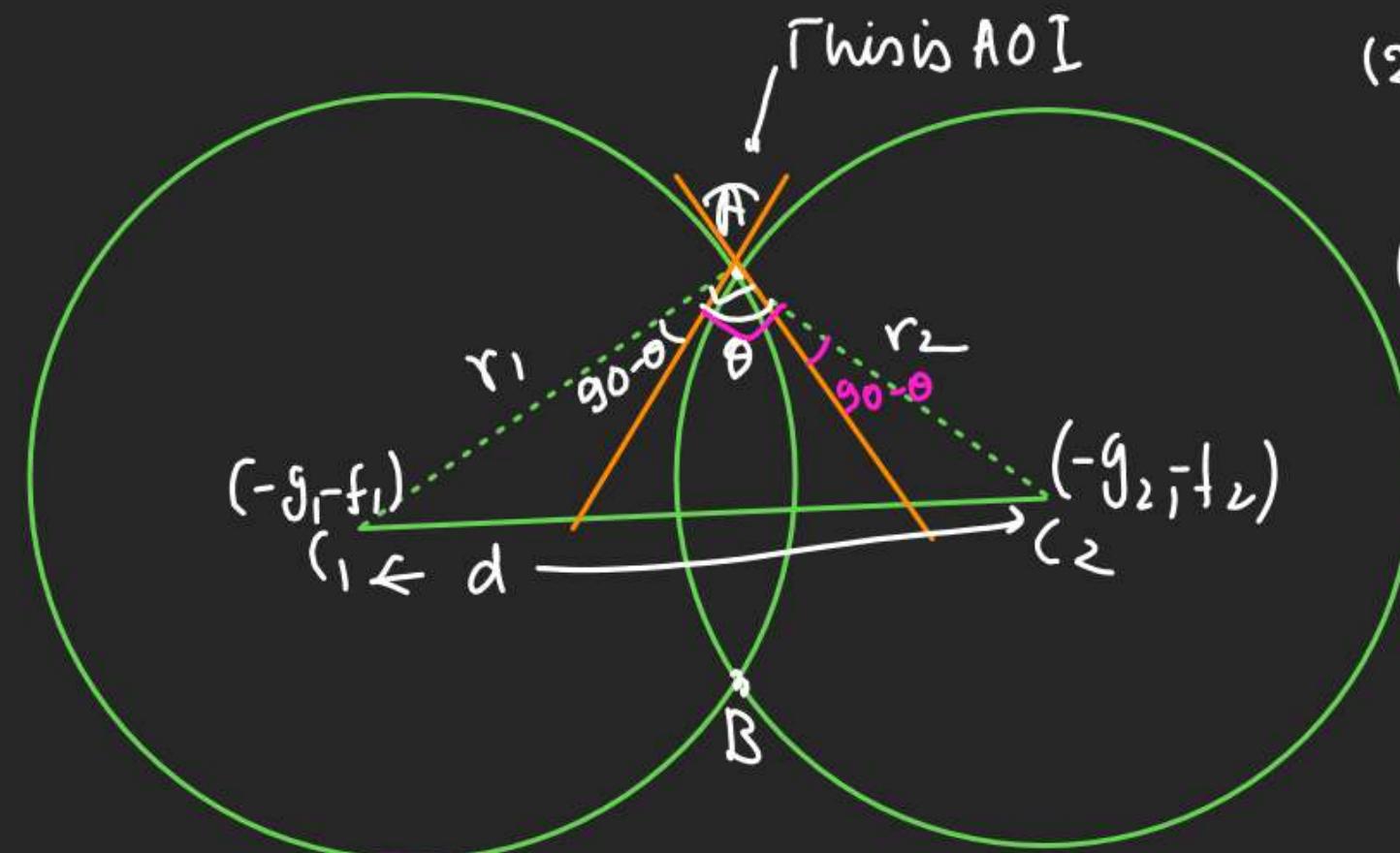


Angle of Intersection of 2 circles



(4) Special Case \rightarrow When $\theta = \frac{\pi}{2}$ then S_1, S_2

are known as Orthogonal Circles.

$$\theta = \frac{\pi}{2} \Rightarrow \text{or } \theta = 0 \Rightarrow \theta = \frac{d^2 - r_1^2 - r_2^2}{2r_1 r_2}$$

(1) AOI Betw 2 Curves is AOI Int. betw tangents at POI

$$(2) \angle C_1 A C_2 = 90 - \theta + \theta + 90 - \theta = 180 - \theta$$

(3) $\triangle C_1 A C_2 \rightarrow r_1, r_2, d$ Available.

$$\text{or } (180 - \theta) = \frac{r_1^2 + r_2^2 - d^2}{2r_1 r_2}$$

This is direct formula

$$\text{or } \theta = \frac{d^2 - r_1^2 - r_2^2}{2r_1 r_2}$$

for AOI Betw 2 circles.

Central form:

$$\left(\sqrt{g_1^2 + f_1^2 - c_1} \right)^2 + \left(\sqrt{g_2^2 + f_2^2 - c_2} \right)^2 = \left(\sqrt{(g_1 - g_2)^2 + (f_1 - f_2)^2} \right)^2$$

$$+ c_1 + c_2 = + 2g_1 g_2 + 2f_1 f_2$$

Cond'n of orthogonality Standard form

Q Circles $x^2 + y^2 + x + y = 0$

& $x^2 + y^2 + x - y = 0$ Intersects

$d =$ at an angle of?

$$\sqrt{0^2+1^2} \left\{ \begin{array}{l} C_1 = (-\frac{1}{2}, -\frac{1}{2}) \\ r_1 = \sqrt{\frac{1}{4} + \frac{1}{4} - 0} = \frac{1}{2} \end{array} \right.$$

$$= 1 \left\{ \begin{array}{l} C_2 = (-\frac{1}{2}, \frac{1}{2}) \\ r_2 = \sqrt{\frac{1}{4} + \frac{1}{4} - 0} = \frac{1}{2} \end{array} \right.$$

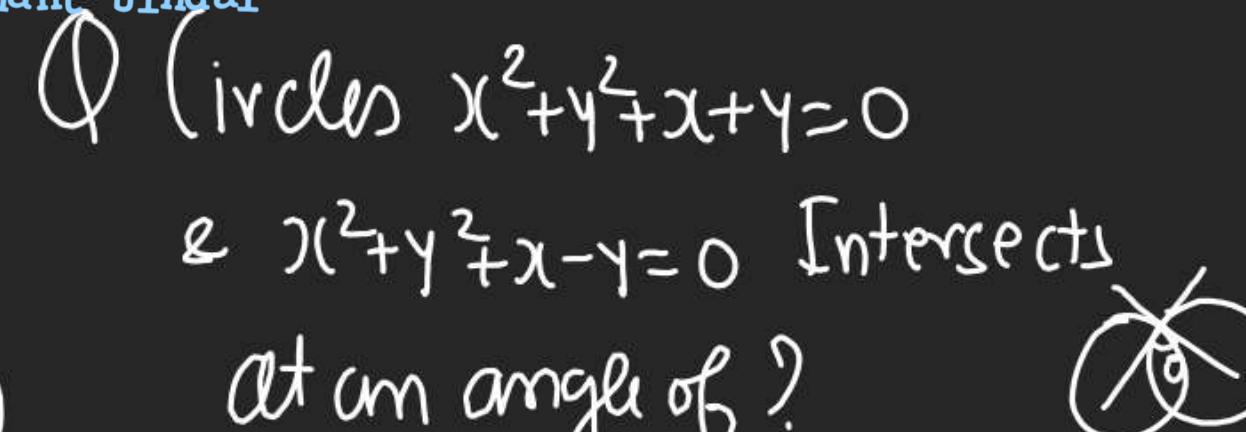
$$\theta = \frac{d^2 - r_1^2 - r_2^2}{2r_1 r_2} = \frac{1^2 - \frac{1}{2} - \frac{1}{2}}{2 \times \frac{1}{2} \times \frac{1}{2}} = 0$$

$\theta = \frac{\pi}{2} \Rightarrow S_1 \& S_2$ are orthogonal.

RK(1) Line & circle are orthogonal

then Line is normal of circle.

(2) If angle b/w tangent is θ then
angle b/w their Normal is θ also



Q A Circle Passes thru origin

& has its centre at $y = x$. If

$$it \ cuts \ x^2 + y^2 - 4x - 6y + 10 = 0$$

Orthogonally then Eqn of circle?

① Let circle $\rightarrow x^2 + y^2 + 2gx + 2fy + c = 0$
(centre $(-g, -f)$) lies on $y = x$
 $-g = -f \Rightarrow g = f$

② Centre $= (-g, -g)$ & (-0)

$$S_1: \text{Circle} \rightarrow x^2 + y^2 + 2gx + 2gy + c = 0$$

$$S_2: \text{Circle} \rightarrow x^2 + y^2 - 4x - 6y + 10 = 0$$

(centre $(2, 3)$, $r = \sqrt{4+9-10} = \sqrt{3}$)

$$(3) S_1 + S_2 \Rightarrow 2g_1 g_2 + 2f_1 f_2 = c_1 + c_2$$

$$2g(-2) + 2g(-3) = 0 + 10$$

$$-5g = 10 \Rightarrow g = -2$$

$$\Rightarrow S_1: x^2 + y^2 - 2x - 2y = 0$$

Q If Line $2x + y = b$ is

orthogonal to $x^2 + y^2 - 2x - 2y = 0$ then $b = ?$

1) As Line is orthogonal to
(circle \Rightarrow Line is Normal)
2) It will pass from centre.

$$2x + y - b = 0$$

(centre $(1, 1)$)

$$2 \times 1 + 1 = b$$

$$\boxed{b = 3}$$

Q 2 given circles

$$x^2 + y^2 + ax + by + c = 0$$

$$\& x^2 + y^2 + dx + ey + f = 0$$

will intersect orthogonally
then find 6nd.

$$(1) (g_1, f_1) = \left(\frac{a}{2}, \frac{b}{2} \right)$$

$$(2) (g_2, f_2) = \left(\frac{d}{2}, \frac{e}{2} \right)$$

Cond of orthogonality

$$2g_1 g_2 + 2f_1 f_2 = a_1 + a_2$$

$$2 \cdot \frac{a}{2} \cdot \frac{d}{2} + 2 \cdot \frac{b}{2} \cdot \frac{e}{2} = (+f)$$

$$ad + be = 2(+2f)$$

Repaired 6nd

Q If (circles) of same radius

a & centres at (2, 3) & (5, 6)
Cut orthogonally then a=?

$$S_1: (x-2)^2 + (y-3)^2 = a^2$$

$$S_2: (x-5)^2 + (y-6)^2 = a^2$$

$$r_1^2 + r_2^2 = d^2 \leftarrow \text{cond of orthogonality}$$

$$a^2 + a^2 = \left(\sqrt{(5-2)^2 + (6-3)^2} \right)^2$$

$$2a^2 = 18$$

$$a = \pm 3$$

Q A circle S passes thru Pt-(0,1) & is

Ans 14 orthogonal to circle $(x+1)^2 + y^2 = 16$ & $x^2 + y^2 = 1$

then A) Rad=8 B) Rad=7 C) centre (-1,1)

D) centre (-8,1)

Q Let circle S is $x^2 + y^2 + 2gx + 2fy + c = 0$ P.I.

$$0+1+0+2f+c=0 \Rightarrow \boxed{2f+1=-1} \quad (0,1)$$

$$(2) S \perp S_1 \Rightarrow S_1: x^2 + y^2 - 2x - 15 = 0$$

$$2(g)(-1) + 2(f)(0) = () + (-15)$$

$$\boxed{-2g = -15}$$

$$(3) S \perp S_2 \quad 2(g)(0) + 2(f)(0) = (-1) + ()$$

$$\boxed{c=1}$$

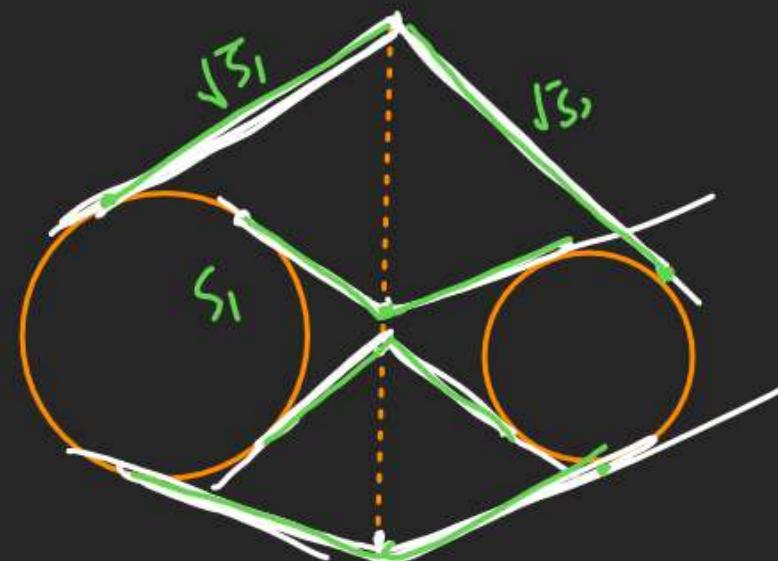
$$\therefore -2g = -15 \Rightarrow g = 7 \quad \left\{ \begin{array}{l} \text{centre } (-g-f) = (-7,1) \\ 2f+1 = -1 \Rightarrow f = -1 \end{array} \right.$$

$$\text{Radius} = \sqrt{7^2 + f^2 - 1} = 7$$

Radical Axis & Radical Centre.

Def 1 Radical Axis of 2 circles is the locus of a pt. whose Power of pt. wrt Both circles is ≤ 0 .

Def 2 R.A. is Locus of pt. from which Length of tangent to Both circles are ≤ 0 .



(2) So for R.A.

$$\sqrt{S_1} = \sqrt{S_2}$$

$$\Rightarrow S_1 = S_2$$

$$\Rightarrow S_1 - S_2 = 0 \text{ in } \\ \text{R.A. for } S_1 \text{ & } S_2$$

If 2 circles are

$$S_1: x^2 + y^2 + 2x - 3y - 11 = 0$$

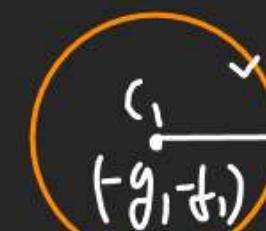
$$S_2: \underline{x^2 + y^2 = 4} \text{ find R.A?}$$

$$\therefore \text{R.A. in } S_1 - S_2 = 0$$

$$2x - 3y - 7 = 0$$

$$2x - 3y - 7 = 0 \text{ in Rad. Axis}$$

(3)* Slope of R.A.

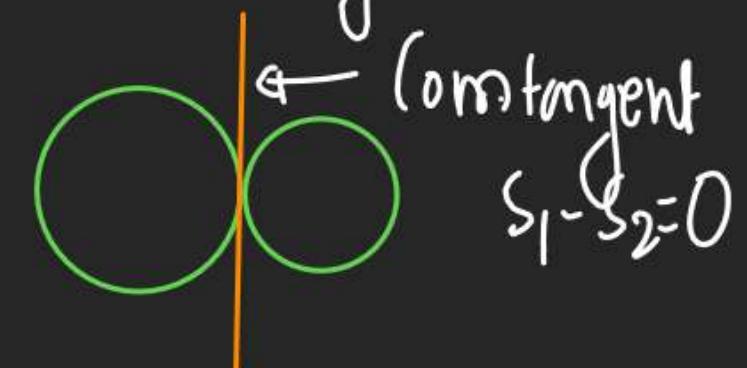


RA is always \perp to Line joining centres.

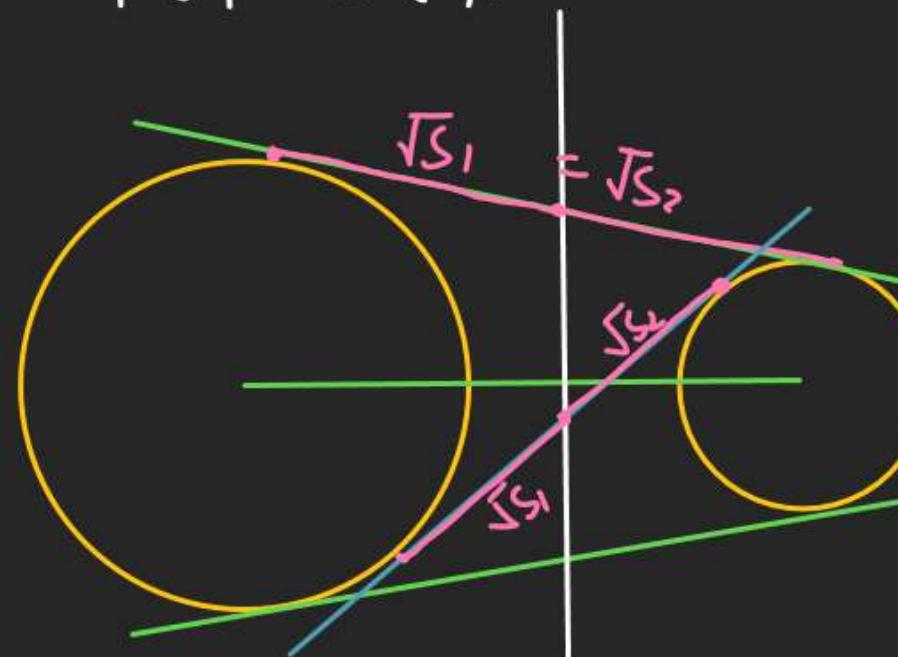
$$(S.l)_{C_1 C_2} = \frac{-f_2 + d_1}{-g_2 + g_1}$$

$$\therefore (S.l)_{\text{RA}} = -\left(\frac{g_1 - g_2}{f_1 - f_2}\right)$$

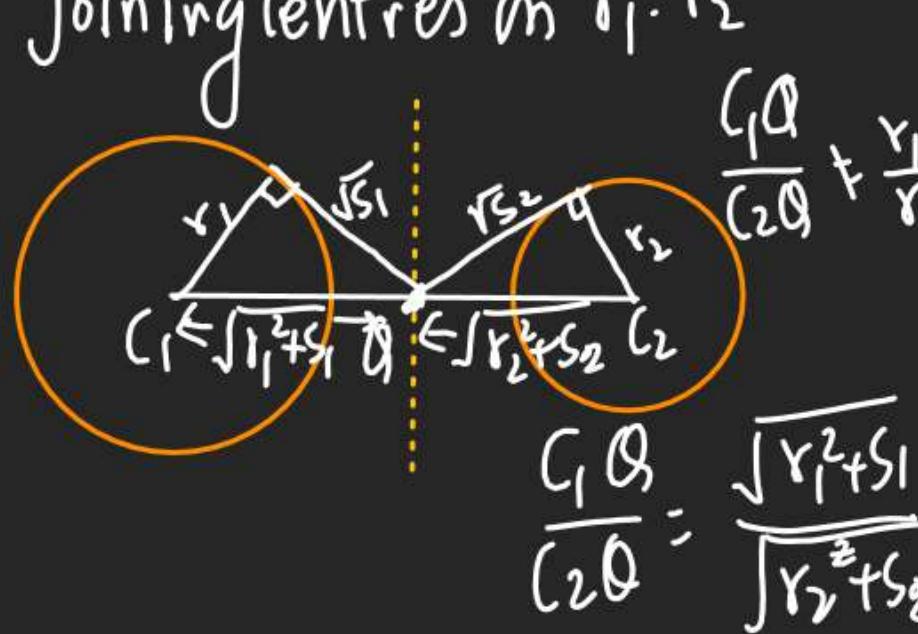
(4)* In then 2 circles touches. RA is com. tangent.



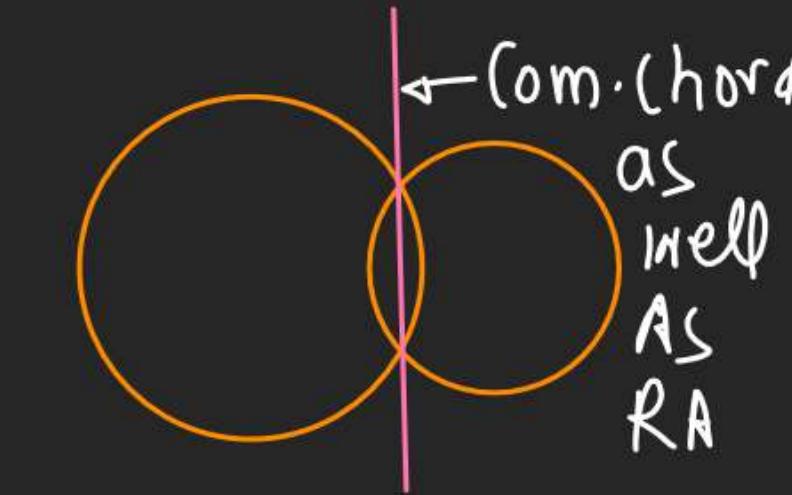
(1) RA always Bisects
TCT & DCT.



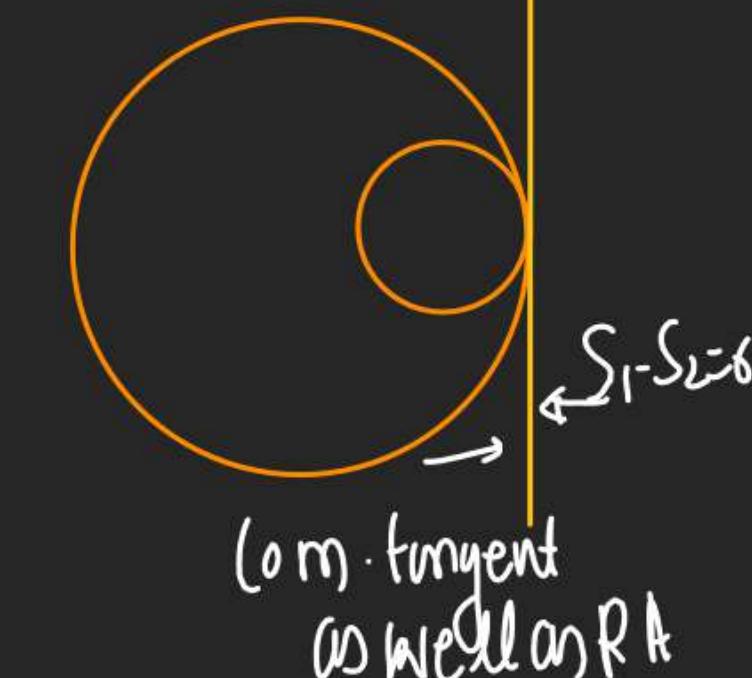
(8) R.A. does not divide line
Joining centres in $r_1 : r_2$



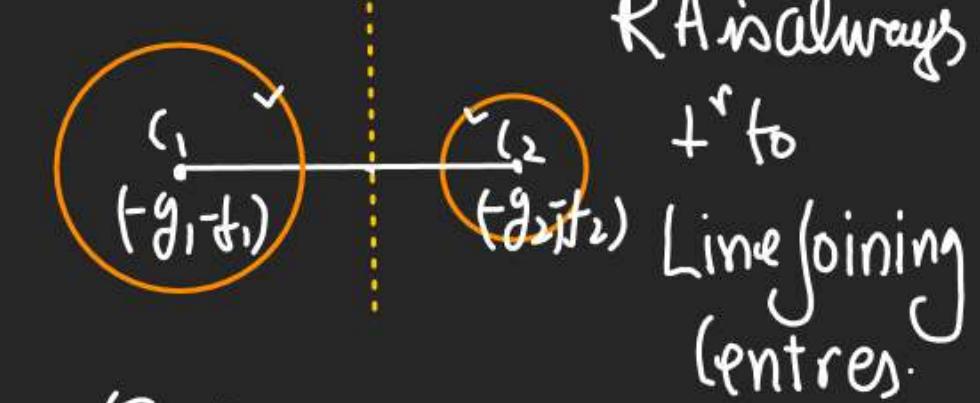
(5) When 2 circles Intersect
them (om. chord is R.A.)



$$(om. chord) \Rightarrow S_1 - S_2 = 0$$



(3)* Slope of RA.

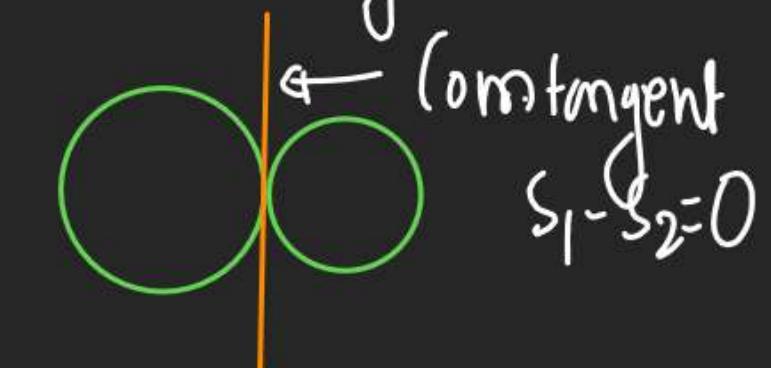


RA is always
perpendicular
to Line joining
centres.

$$(S l)_{C_1 C_2} = -\frac{f_2 + f_1}{g_2 + g_1}$$

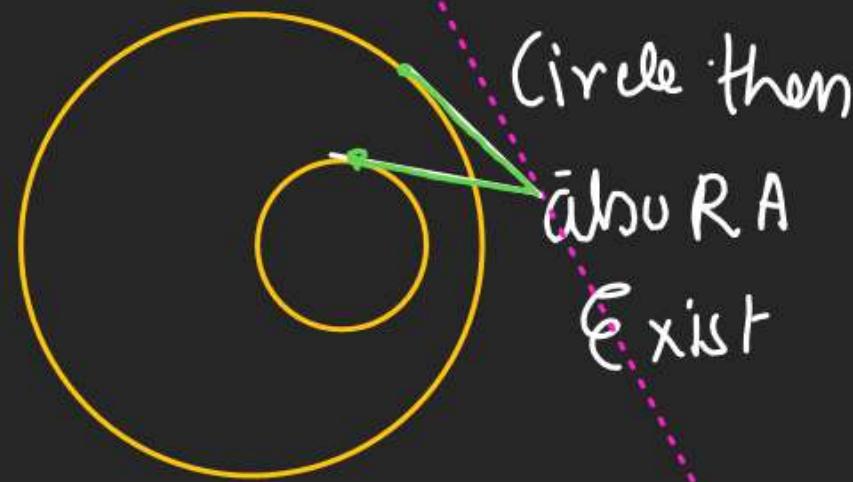
$$\therefore (S l)_{RA} = -\left(\frac{g_1 - g_2}{f_1 - f_2}\right)$$

(4)* In when 2 circles touches
RA is com. tangent.



com tangent
 $S_1 - S_2 = 0$

(g) When a circle is Inside another



Circle then
also RA
exist

Q Tangents are drawn to circle.

$$S_1: x^2 + y^2 = 12 \text{ at } P \text{ to inner}$$

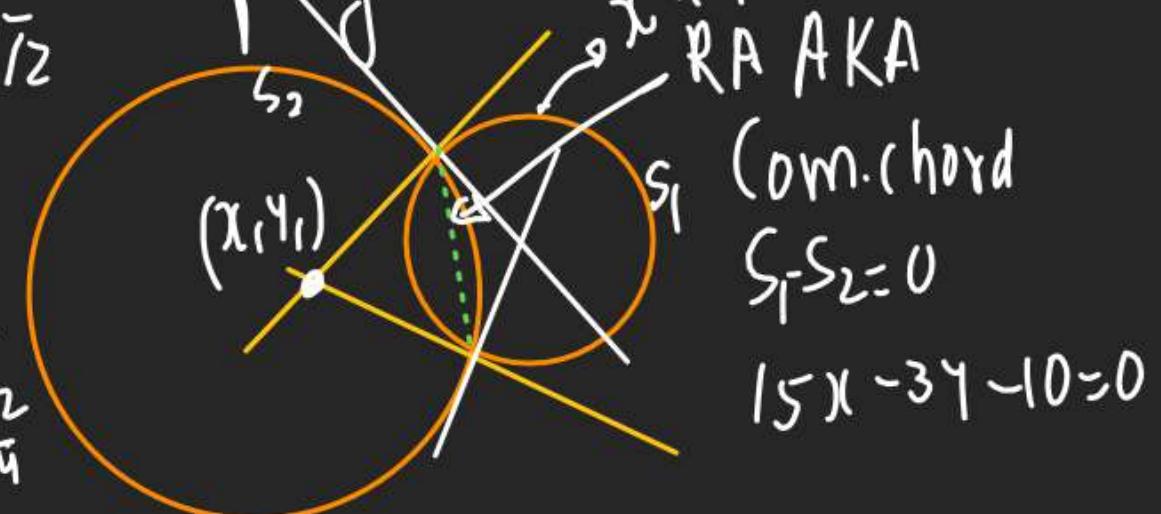
$$\text{meet circle } S_2: x^2 + y^2 - 15x + 3y - 2 = 0$$

find POI of tangents.

$$(0,0) \quad r_1 = \sqrt{12}$$

$$\left(\frac{15}{2}, -\frac{3}{2}\right)$$

$$r = \sqrt{\frac{225}{4} + \frac{9}{4} + 2} \\ = \sqrt{\frac{242}{4}}$$



(2) This com. chord is OC.

$$\text{for } r(x_1, y_1) \rightarrow i=0$$

$$x_1 x_1 + y_1 y_1 - 12 = 0$$

$$15 x_1 - 3 y_1 - 10 = 0$$

$$\frac{x_1}{15} = \frac{y_1}{-3} = \pm \frac{12}{\sqrt{105}}$$

$$x_1 = 18, y_1 = -\frac{18}{5}$$

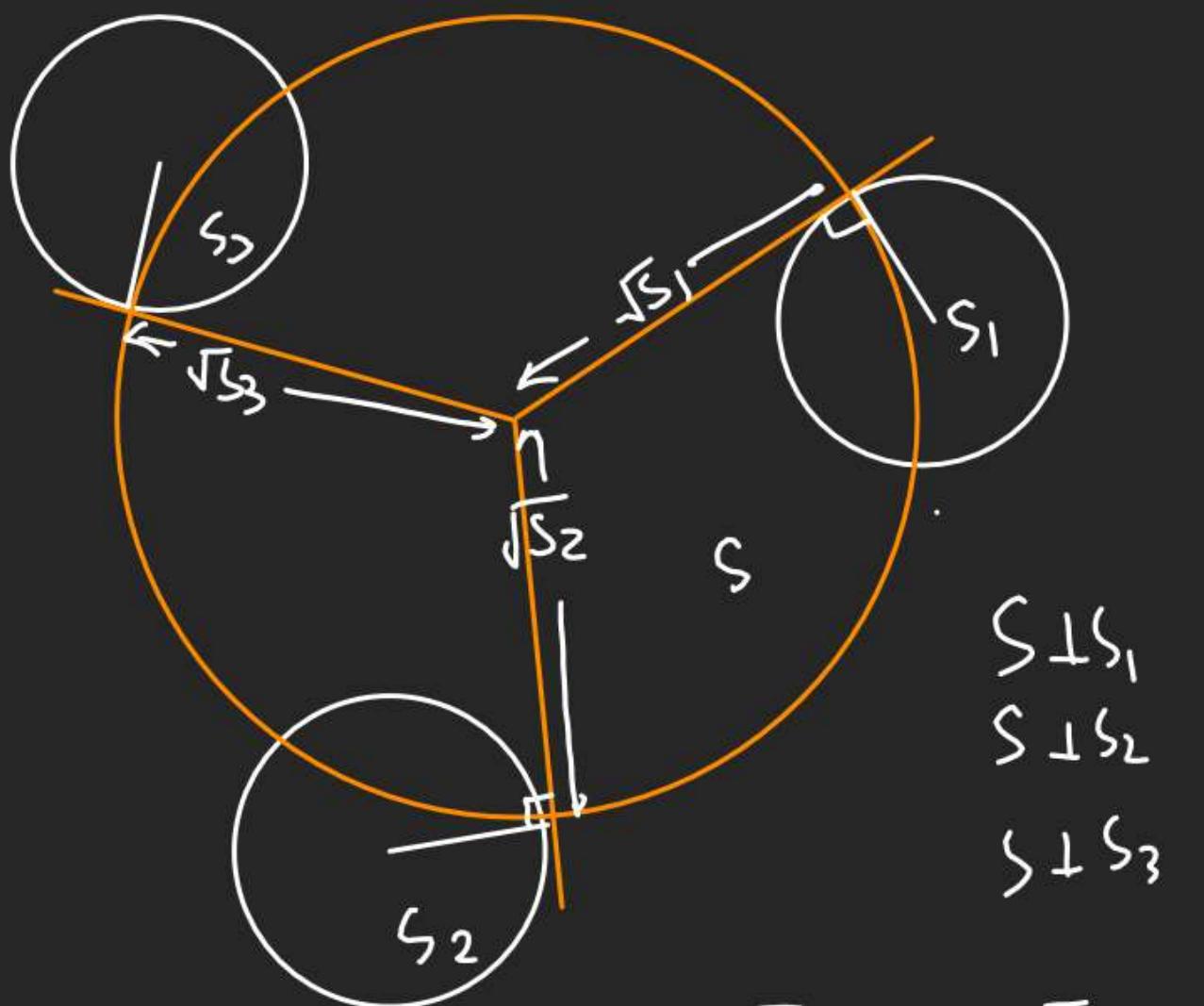
$$\left(18, -\frac{18}{5}\right)$$

$$x^2 + y^2 = 12$$

RA AKA

(0m. chord)
 $S_1 - S_2 = 0$

$$15x - 3y - 10 = 0$$



$$S \perp S_1$$

$$S \perp S_2$$

$$S \perp S_3$$

$$\sqrt{S_1} = \sqrt{S_2} = \sqrt{S_3}$$

$$\left. \begin{array}{l} S_1 - S_2 = 0 \rightarrow ① \\ S_2 - S_3 = 0 \rightarrow ② \end{array} \right\} R.C.$$