Nishant Jindal

Q $\displaystyle\int \frac{(2x+5)\,dx}{\sqrt{7-6x-x^2}} = A\sqrt{7-6x-x^2} + B\sin^{-1}\left(\frac{x+3}{4}\right)+C$

(A, B)?

Q $\displaystyle\int \frac{\tan x}{1+\tan x+\tan^2 x}\,dx = x - \frac{K}{\sqrt{A}}\tan^{-1}\left(\frac{K\tan x+1}{\sqrt{A}}\right)+C$

Q $\displaystyle\int \frac{\tan x+\tan\alpha}{\tan x-\tan\alpha}\,dx = A(x)\cos 2x + B(x)\sin 2\alpha + C$

$$\int \frac{d\left(\frac{x}{3}\right)}{\sqrt{1-\left(\frac{x}{3}\right)^2}}$$

$$\int \frac{dy}{\sqrt{1-y^2}} = \sin^{-1}y + 1$$

$$= \sin^{-1}\left(\frac{x}{3}\right)+C$$

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Q8 $\int \frac{dx}{x^2-7x+10}$

$\int \frac{dx}{\left(x-\frac{7}{2}\right)^2-\left(\frac{7}{2}\right)^2+10}$ $\quad 10-\frac{49}{4}$

$\int \frac{dx}{\left(x-\frac{7}{2}\right)^2-\left(\frac{3}{2}\right)^2} \rightarrow \int \frac{dx}{x^2-a^2}$

$\frac{1}{2\times\frac{3}{2}} \ln\left(\frac{\left(x-\frac{7}{2}\right)-\left(\frac{3}{2}\right)}{\left(x+\frac{7}{2}\right)+\frac{3}{2}}\right)$

Q $\int 4\cos\frac{x}{2}\cdot\cos x\cdot\sin\frac{21x}{2}$

$2\cdot\int 2\cos\frac{x}{2}\cdot\sin\frac{21x}{2}\cdot\cos x\cdot dx$

$2\int\left(\sin(11x)+\sin(+10x)\right)\cdot\cos x\,dx$

$\int 2\sin 11x\cdot\cos x+2\sin 10x\cdot\cos x\,dx$

$\int \sin(12x)+\sin(10x) \quad +\int \sin(11x)+\sin(9x)\,dx$

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Q21 $\int \frac{\cos^2 x - \sin^2 x}{\cos^2 x \cdot \sin^2 x}\,dx$

$\int \operatorname{cosec}^2 x - \sec^2 x\,dx$

$-\cot x - \tan x + C$

Q34 $\int \frac{x^6 - 1}{x^2+1}$

$$\begin{array}{r|l} x^2+1 & x^6 - 1 \quad (x^4 + x^2 + 1 \\ & \underline{x^6 + x^4} \\ & -x^4 - 1 \\ & \underline{-x^4 - x^2} \\ & x^2 - 1 \\ & \underline{x^2 + 1} \\ & -2 \end{array}$$

$\int x^4 + x^2 + 1 - \frac{2}{x^2+1}\,dx$

$\frac{x^5}{5} + \frac{x^3}{3} + x - 2\tan^{-1}x + C$

~~Q35~~

Q24 $\int \frac{\cos x - \sin x}{\cos x + \sin x}(2 + 2\sin 2x)\,dx$

$2\int \frac{\cos x - \sin x}{\cos x + \sin x}(1 + \sin 2x)\,dx$

$2\int \frac{\cos x - \sin x}{\cancel{\cos x + \sin x}}(\cos x + \sin x)^{\cancel{2}}$

$2\int \cos 2x\,dx = 2\frac{\sin 2x}{2} + C$

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Q25 $\int 3\sin x\cdot\cos^2 x-\sin^3 x$

$\int 3\sin x(1-\sin^2 x)-\sin^3 x$

$\int 3\sin x-3\sin^3 x-\sin^3 x$

$\int 3\sin x-4\sin^3 x\,dx$

$\int \sin 3x\,dx$

$-\dfrac{\cos 3x}{3}+C$

Q 28 hold

Q37 $\int\sqrt{1-\sin 2x}\,dx$

$\int\sqrt{(\cos x-\sin x)^2}\,dx$

$\int|\cos x-\sin x|\,dx$

$\cos x-\sin x +$: $\int \cos x-\sin x\,dx=(\sin x+\cos x)+C$

$\cos x-\sin x \ominus$: $-\int(\cos x-\sin x)\,dx=-(\sin x+\cos x)+C$

[illegible] $\underbrace{(\sin x+\cos x)\,\mathrm{sgn}(\cos x-\sin x)}+C$

Nishant Jindal

Q $\int x^{51}\cdot(\tan^{-1}x+\cot^{-1}x)\,dx$

$\frac{\pi}{2}\int x^{51}\cdot dx$

$\frac{\pi}{2}\cdot\frac{x^{52}}{52}+C$

Q. $\int\sqrt{x-3}\cdot(\sin^{-1}(\ln x)+\cos^{-1}(\ln x))\,dx$

$x-3\geq 0$

$x\geq 3$

$-1\leq \ln x\leq 1$

$-1\leq \log_e x\leq 1$

$\frac{1}{e}\leq x\leq e^1$

$x\in[.35, 2.7]$

$x=\phi$

fxn DNE

Int = DNE

Q $\int\frac{\sin x}{\sin(x-a)}dx$

$\int\frac{\sin((x-a)+a)}{\sin(x-a)}dx$

$\int\cot x\,dx=\ln|\sin x|$

$\int\frac{\cancel{\sin(x-a)}\cdot\cos a}{\cancel{\sin(x-a)}}+\frac{\cos(x-a)\sin a}{\sin(x-a)}$

$\int\cos a\,dx+\sin a\int\cot(x-a)dx$

$\cos a\int dx+\sin a\times\ln|\sin(x-a)|$

$x\cos a+\sin a\cdot\ln|\sin(x-a)|+C$

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Q $\displaystyle\int \frac{dx}{\sin(x-a)\cdot\sin(x-b)}$ $\qquad$ $\displaystyle\int \frac{dx}{\cos(x-a)\cos(x-b)}$ $\qquad$ $\displaystyle\int \frac{dx}{\sin(x-a)\cos(x-b)}$

Sin(a-b) से Multiply. $\qquad$ Cos(a-b) से Multiply.

$$\int \frac{dx}{\sin(x-a)\sin(x-b)} \times \frac{\sin(a-b)}{\sin(a-b)}$$

ऊपर खोलो ध्यान से

$$\frac{1}{\sin(a-b)}\int \frac{\sin\{(x-b)-(x-a)\}}{\sin(x-a)\cdot\sin(x-b)}\,dx$$

$$\frac{1}{\sin(a-b)}\int \frac{\cancel{\sin(x-b)}\cdot\cos(x-a)}{\sin(x-a)\,\cancel{\sin(x-b)}} - \frac{\cos(x-b)\cancel{\sin(x-a)}}{\cancel{\sin(x-a)}\sin(x-b)}\,dx$$

$$\frac{1}{\sin(a-b)}\left\{\int \cot(x-a)\,dx - \int \cot(x-b)\,dx\right\}$$

$$\frac{1}{\sin(a-b)}\cdot\left\{\ln|\sin(x-a)| - \ln|\sin(x-b)|\right\} + C$$

Nishant Jindal

$1+\cos 2x = 2\cos^2 x$

Q $\int \frac{dx}{1+\cos 5x}$

M1 $\int \frac{dx}{2\cos^2\left(\frac{5x}{2}\right)}$

$\frac{1}{2}\int \sec^2\left(\frac{5x}{2}\right)\cdot dx$

$\frac{1}{2}\times \frac{\tan\left(\frac{5x}{2}\right)}{\frac{5}{2}}+C$

M2 $\int \frac{1}{1+\cos 5x}\times\frac{1-\cos 5x}{1-\cos 5x}\,dx$

$\int \frac{1-\cos 5x}{\sin^2(5x)} = \int (\operatorname{cosec}^2 5x - \operatorname{cosec}(5x)\cdot\cot 5x)\,dx$

$\Rightarrow -\frac{\cot 5x}{5} + \frac{\operatorname{cosec} 5x}{5} + C$

Q $\int \frac{\sin 2x\,dx}{\sin\left(x+\frac{\pi}{6}\right)\cdot\sin\left(x-\frac{\pi}{6}\right)}$

$\Rightarrow \int \frac{\sin\left(\left(x+\frac{\pi}{6}\right)+\left(x-\frac{\pi}{6}\right)\right)}{\sin\left(x+\frac{\pi}{6}\right)\cdot\sin\left(x-\frac{\pi}{6}\right)}dx$

Open Nr & Divide

Q Main $\int \frac{\sin\frac{5x}{2}}{\sin\frac{x}{2}}\cdot dx$

$\int \frac{\sin\frac{5x}{2}-\sin\frac{3x}{2}}{\sin\frac{x}{2}}+\frac{\sin\frac{3x}{2}-\sin\frac{x}{2}}{\sin\frac{x}{2}}+\frac{\sin\frac{x}{2}}{\sin\frac{x}{2}}\,dx$

$\int \frac{2\cos(2x)\cdot\sin\left(\frac{x}{2}\right)}{\sin\frac{x}{2}}+\frac{2\cos(x)\cdot\sin\left(\frac{x}{2}\right)}{\sin\left(\frac{x}{2}\right)}+1\,dx$

DIY

Q Adv $\int \frac{\sin 9x}{\sin x}\cdot dx$

$\int \underbrace{\frac{\sin 9x-\sin 7x}{\sin x}}+\underbrace{\frac{\sin 7x-\sin 5x}{\sin x}}+\underbrace{\frac{\sin 5x-\sin 3x}{\sin x}}+\underbrace{\frac{\sin 3x-\sin x}{\sin x}}+\frac{\cancel{\sin x}}{\cancel{\sin x}}$

$\int \frac{2\cos(8x)\cancel{\sin(x)}}{\cancel{\sin x}}+\frac{2\cos(6x)\cancel{\sin(x)}}{\cancel{\sin x}}+\frac{2\cos(4x)\cancel{\sin(x)}}{\cancel{\sin x}}+\frac{2\cos(2x)\cancel{\sin(x)}}{\cancel{\sin x}}+1\,dx$

$\Rightarrow +2\frac{\sin 8x}{8}+2\frac{\sin 6x}{6}+2\frac{\sin 4x}{4}+2\frac{\sin 2x}{2}+x+C$

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$\frac{x^4}{4}-\frac{2x^2}{2}+\frac{3x^3}{3}+5x+\frac{1}{2}\ln(x^2+1)+3\tan^{-1}x+C$

Q $\int \frac{\cos 6\theta+6\cos 4\theta+15\cos 2\theta+10}{\cos 5\theta+5\cos 3\theta+10\cos\theta}\,d\theta$

$\int \frac{(\cos 6\theta+\cos 4\theta)+5\cos 4\theta+5\cos 2\theta+10\cos 2\theta+10}{\text{"}}$

$\int \frac{2\cos(5\theta)\cos(\theta)+5(\cos(3\theta)+\cos(\theta))+10(2\cos^2\theta)}{\text{"}}$

$\int \frac{2\cos\theta\{\cancel{\cos 5\theta+5\cos 3\theta+10\cos\theta}\}}{\cancel{(\cos 5\theta+5\cos 3\theta+10\cos\theta)}}$

$2\int \cos\theta\,d\theta = 2\sin\theta + C$

Q $\int \frac{x^5+3x^4-x^3+8x^2-x+8}{x^2+1}\,dx$

$\int \frac{x^3(x^2+1)+3x^4+3x^2+5x^2+5+3-2x^3-x}{}$

$=\int \frac{x^3(x^2+1)+3x^2(x^2+1)+5(x^2+1)-x(x^2+1)-x^3+3}{(x^2+1)}$

$\int \frac{\cancel{(x^2+1)}(x^3+3x^2+5-x)}{\cancel{(x^2+1)}}-\frac{(x^3+x)-x+3}{x^2+1}$

$\frac{x^4}{4}+\frac{3x^3}{3}+5x-\frac{x^2}{2}-\int\frac{x\cancel{(x^2+1)}}{\cancel{x^2+1}}+\frac{1}{2}\int\frac{2x}{x^2+1}+\frac{3}{x^2+1}$

$\text{''} \quad , \quad , \quad , \quad \ldots -\frac{x^2}{2} \quad \frac{x^2}{2}+\frac{1}{2}\int\frac{d(x^2+1)}{x^2+1}+3\tan^{-1}x+C$

Nishant Jindal

Q $\int \frac{x^4(1-x)^4}{x^2+1}\,dx$

$(a+b)^4 = {}^4C_0\cdot(a)^0(b)^4 + {}^4C_1(a)^1(b)^3 + {}^4C_2\cdot(a)^2(b)^2 + {}^4C_3(a)^3(b) + {}^4C_4(a)^4(b)^0$

$$\int \frac{x^4\left\{{}^4C_0\cdot 1^0\cdot(-x)^4 + {}^4C_1(1)^1\cdot(-x)^3 + {}^4C_2(1)^2\cdot(-x)^2 + {}^4C_3\cdot(1)^3(-x)^1 + {}^4C_4(1)^4(-x)^0\right\}}{x^2+1}$$

$$\int \frac{x^4(x^4-4x^3+6x^2-4x+1)}{x^2+1}\,dx$$

Nishant Jindal

Tricky

Q $\int \frac{5x+7}{(x+2)(4x+5)}\,dx$

Sum या diff check करो

$(x+2)+(4x+5)$

$=5x+7$

$\int \frac{(x+2)+(4x+5)\,dx}{(x+2)(4x+5)}$

$\int \frac{\cancel{(x+2)}}{\cancel{(x+2)}(4x+5)} + \frac{\cancel{4x+5}}{(x+2)\cancel{(4x+5)}}\,dx$

$\int \frac{dx}{4x+5} + \int \frac{dx}{x+2}$

$\frac{1}{4}\ln|4x+5| + \ln|x+2| + C$

Tricky

Q $\int \frac{(6x+5)\,dx}{(3x+1)(9x+6)}$

$\Rightarrow \int \frac{(9x+6)-(3x+1)\,dx}{(3x+1)(9x+6)}$

$\int \frac{\cancel{(9x+6)}}{(3x+1)\cancel{(9x+6)}} - \frac{\cancel{(3x+1)}}{\cancel{(3x+1)}(9x+6)}\,dx$

$\int \frac{dx}{3x+1} - \int \frac{dx}{9x+6}$

$\frac{\ln|3x+1|}{3} - \frac{\ln|9x+6|}{9} + C$

Trick.

Q $\int \frac{2x^2+3}{(x^2-1)(x^2+4)}\,dx$

$\Rightarrow \int \frac{(x^2-1)+(x^2+4)}{(x^2-1)(x^2+4)}$

$\int \frac{\cancel{x^2-1}}{\cancel{(x^2-1)}(x^2+4)} + \frac{\cancel{(x^2+4)}}{(x^2-1)\cancel{(x^2+4)}}$

$\int \frac{dx}{x^2+2^2} + \int \frac{dx}{x^2-1^2}$

$\frac{1}{2}\tan^{-1}\frac{x}{2} - \frac{1}{2\times 1}\ln\left|\frac{x-1}{x+1}\right| + C$

Nishant Jindal

Q $\int \frac{dx}{x^2-7x+10}$

$\int \frac{dx}{(x-2)(x-5)}$

Bda, chh.

diff=3

$\frac{1}{\text{diff}}\left(\frac{1}{\text{chh}}-\frac{1}{\text{Bda}}\right)$

$\frac{1}{3}\left(\frac{1}{(x-5)}-\frac{1}{(x-2)}\right)$

$\Rightarrow \frac{1}{3}\int \frac{1}{(x-5)}-\frac{1}{(x-2)}\,dx$

$\Rightarrow \frac{1}{3}\{\ln|x-5|-\ln|x-2|\}+C$

Q $\int \frac{dx}{x^2+x-2}$

1	1
-2	1

$\int \frac{dx}{(x+2)(x-1)}$

diff=3.

Sh. Trick

$\boxed{\text{P.F.}}$

$\frac{1}{3}\int \frac{1}{x-1}-\frac{1}{(x+2)}\,dx$

$\frac{1}{3}\left|\ln|x-1|-\ln|x+2|\right|+C$

$\frac{1}{3}\ln\frac{(x-1)}{(x+2)}\,dx$

Q $\int \frac{dx}{x^2-6x+8}$

$\int \frac{dx}{(x-2)(x-4)}$

diff=2

$\frac{1}{2}\ln\left|\frac{x-4}{x-2}\right|+C$

Nishant Jindal

Q $\displaystyle\int \frac{\sin 2x\, dx}{(\sin^2 x-1)(\sin^2 x+1)}$

diff^n ans

diff = 2

$\dfrac{1}{2}\ln\left|\dfrac{\sin^2 x-1}{\sin^2 x+1}\right|+C$

Q $\displaystyle\int \frac{e^x . dx}{(e^x-1)(e^x+5)}$

diff = 6

$\dfrac{1}{6}\ln\left|\dfrac{e^x-1}{e^x+5}\right|+C$

Nr's Deg $\geq$ Dr's Deg $\rightarrow$ Divide

Nr's Deg $<$ Dr's Deg $\rightarrow$ P.F.

Integration By Substitution (Imp.)

Q $\displaystyle\int e^{\sin x}\cdot \cos x \cdot dx$

let
$\sin x = t$
$\cos x \cdot dx = dt$

$\displaystyle\int e^t dt$

$= e^t + C$

$= e^{\sin x} + C$

Q $\displaystyle\int \frac{\sin^{-1}x}{\sqrt{1-x^2}}\, dx$

$\sin^{-1}x = t$

$\dfrac{dx}{\sqrt{1-x^2}} = dt$

$\displaystyle\int t\, dt$

$= \dfrac{t^2}{2} + C$

$= \dfrac{(\sin^{-1}x)^2}{2} + C$