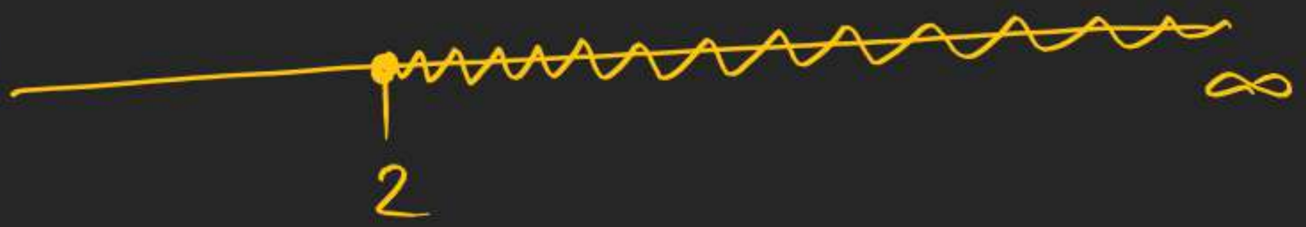
Nishant Jindal

Rk

( ) Open

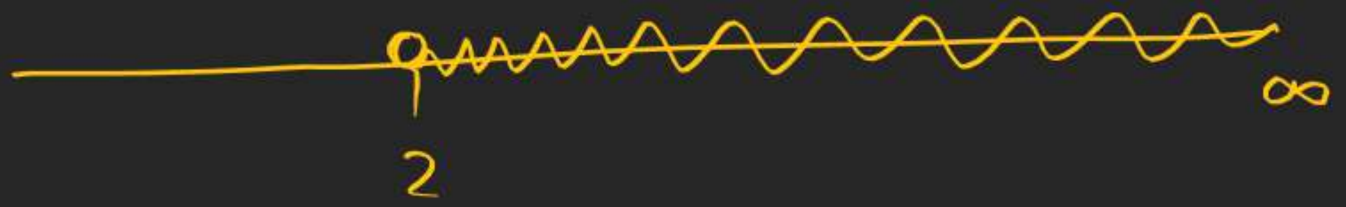
[ ] Close

$x \geq 2 \rightarrow$



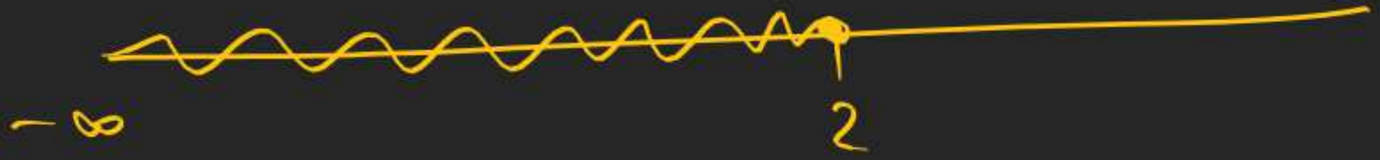
$\Rightarrow x \in [2, \infty)$

$x > 2 \rightarrow$



$\Rightarrow x \in (2, \infty)$

$x \leq 2$



$\Rightarrow x \in (-\infty, 2]$

$x < -5$

$\Rightarrow x \in (-\infty, -5)$

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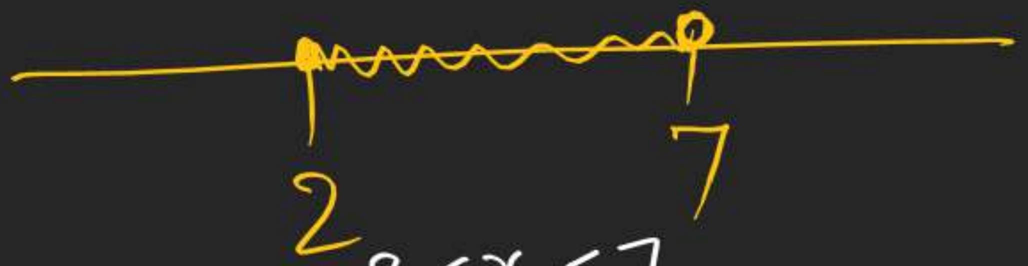
Fundamentals of Mathematics



$2 \le x \le 7$ $\Rightarrow x \in [2, 7)$

[lose]

( ) pen

$-3 < x < 4$ $\Rightarrow x \in (-3, 4)$

$\Rightarrow x \in (-\infty, \infty) \Rightarrow \boxed{x \in R}$

$\Rightarrow x \in (-\infty, \infty) - \{-13, 4\}$

$x \in R - \{-13, 4\}$

Fundamentals of Mathematics

Q. Simplify :

13. $\sqrt{a^{\frac{-2}{3}}b^{4}c^{\frac{-1}{3}}} \div \sqrt{a^{2}b^{4}c^{-1}}$

$$\left(a^{-\frac{2}{3}}b^{4}c^{-\frac{1}{3}}\right)^{\frac{1}{2}} \times \frac{1}{\left(a^{2}b^{4}c^{-1}\right)^{1/2}} = a^{-\frac{1}{3}}b^{2}c^{-\frac{1}{6}} \times \frac{1}{a^{1}b^{2}c^{-\frac{1}{2}}}$$

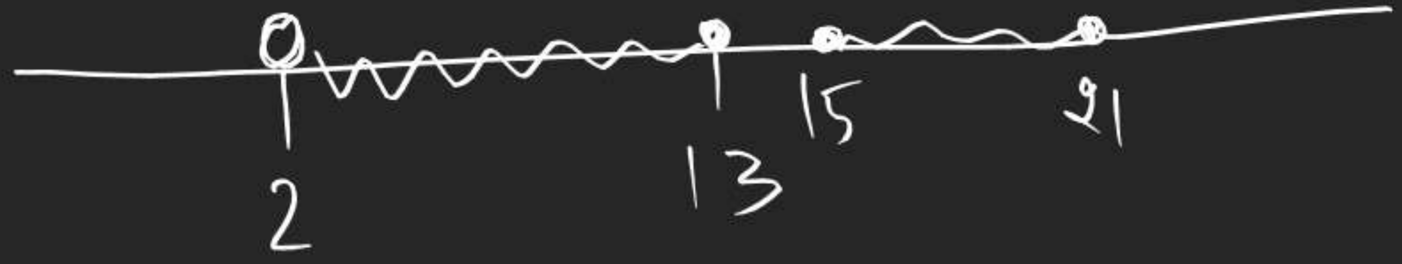
$$= a^{-\frac{1}{3}-1}b^{2-2}c^{-\frac{1}{6}+\frac{1}{2}}$$

$$= a^{-\frac{4}{3}}b^{0}c^{\frac{1}{3}}$$

14. $\sqrt{ab^{-2}c^{3}} \div \left(\sqrt[3]{a^{3}b^{2}c^{-3}}\right)^{-1}$

15. $\left(\frac{a^{-1}b^{2}}{a^{2}b^{-4}}\right)^{7} \div \left(\frac{a^{3}b^{-5}}{a^{-2}b^{3}}\right)^{-5}$

$x \in [2, 13)$



$\Rightarrow x \in (2, 13] \cup [15, 21]$

$x \in (-\infty, 2) \cup [7, \infty)$

Fundamentals of Mathematics

58. $\frac{x}{x^2-3x-4}>0.$

59. $\frac{x^2+7x+10}{x+2/3}>0.$

60. $\frac{3x^2-4x-6}{2x-5}<0.$

61. $\frac{17-15x-2x^2}{x+3}<0.$

62. $\frac{x^2-9}{3x-x^2-24}<0.$

63. $\frac{x+7}{x-5}+\frac{3x+1}{2}\geqslant 0.$

64. $2x^2+\frac{1}{x}>0.$

65. $\frac{x^2-x-6}{x^2+6x}\geqslant 0.$

66. $\frac{x^2-5x+6}{x^2-11x+30}<0.$

67. $\frac{x^2-8x+7}{4x^2-4x+1}<0.$

68. $\frac{x^2-36}{x^2-9x+18}<0.$

69. $\frac{x^2-6x+9}{5-4x-x^2}\geqslant 0.$

70. $\frac{x-1}{x+1}<x.$

71. $\frac{1}{x+2}<\frac{3}{x-3}.$

72. $\frac{14x}{x+1}-\frac{9x-30}{x-4}<0.$

73. $\frac{5x^2-2}{4x^2-x+3}<1.$

74. $\frac{x^2-5x+12}{x^2-4x+5}>3.$

75. $\frac{x^2-3x+24}{x^2-3x+3}<4.$

76. $\frac{x^2-1}{2x+5}<3.$

77. $\frac{x^2+1}{4x-3}>2.$

78. $\frac{x^2+2}{x^2-1}<-2.$

79. $\frac{3x-5}{x^2+4x-5}>\frac{1}{2}.$

80. $\frac{2x+3}{x^2+x-12}\leqslant\frac{1}{2}.$

81. $\frac{5-2x}{3x^2-2x-16}<1.$

82. $\frac{15-4x}{x^2-x-12}<4.$

83. $\frac{1}{x^2-5x+6}\geqslant\frac{1}{2}.$

84. $\frac{(2-x^2)(x-3)^3}{(x+1)(x^2-3x-4)}\geqslant 0.$

85. $\frac{5-4x}{3x^2-x-4}<4.$

86. $\frac{19-33x}{7x^2-11x+4}>2.$

87. $\frac{0.5x+49}{10x^2+x-2}<\frac{1}{2}.$

88. $\frac{(x+2)(x^2-2x+1)}{4+3x-x^2}\geqslant 0.$

89. $\frac{4}{1+x}+\frac{2}{1-x}<1.$

90. $2+\frac{3}{x+1}>\frac{2}{x}.$

91. $1+\frac{2}{x-1}>\frac{6}{x}.$

92. $\frac{x^4-3x^3+2x^2}{x^2-x-30}>0.$

93. $\frac{x-1}{x}-\frac{x+1}{x-1}<2.$

94. $\frac{2(x-3)}{x(x-6)}\leqslant\frac{1}{x-1}.$

95. $\frac{2(x-4)}{(x-1)(x-7)}\geqslant\frac{1}{x-2}.$

96. $\frac{2x}{x^2-9}\leqslant\frac{1}{x+2}.$

97. $\frac{1}{x-2}+\frac{1}{x-1}>\frac{1}{x}.$

98. $\frac{7}{(x-2)(x-3)}+\frac{9}{x-3}+1<0.$

99. $\frac{20}{(x-3)(x-4)}+\frac{10}{x-4}+1>0.$

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Fundamentals of Mathematics

U hv to Improve

(1) $\dfrac{2x-3}{3x-7} > 0$

(2) $\dfrac{.5}{x-x^2-1} < 0$

(3) $\dfrac{x^2-3x+2}{x^2+x+1} < 0$

(4) $\dfrac{(x-1)(x+2)^2}{-1-x} < 0$

44 *1. Algebra, Trigonometry, and Elementary Functions*

tion $(x-x_j)^{k_j}$ does not change sign when x passes through the point x_j and, consequently, the function $F(x)$ does not change sign. If k_p is an even number, then the function $(x-x_p)^{k_p}$ changes sign when x passes through the point x_p and, consequently, the function $F(x)$ also changes sign.

Example 4. Solve the inequality $(x-1)^2(x+1)^3(x-4) < 0$.

Solution. The function $F(x) = (x-1)^2(x+1)^3(x+4)$ changes sign only when x passes through the points $x_1 = -1$, $x_2 = 4$. We have $F(x) > 0$ on the interval $(4, +\infty)$, $F(x) < 0$ on the next interval $(-1, 4)$, excluding the point $x = 1$ at which $F(x) = 0$, and $F(x) > 0$ on the last interval $(-\infty, -1)$.

Answer: $(-1, 1) \cup (1, 4)$.

Example 5. Solve the inequality $\dfrac{(x-1)^2(x+1)^3}{x^4(x-2)} \leqslant 0$.

Solution. The function $F(x) = \dfrac{(x-1)^2(x+1)^3}{x^4(x-2)^3}$ changes sign only when the variable x passes through the points $x_1 = -1$, $x_2 = 2$. When x passes through the points $x_3 = 0$ and $x_4 = 1$, the function $F(x)$ does not change sign. We have $F(x) > 0$ on the interval $(2, +\infty)$, $F(x) < 0$ on the next intervals $(1, 2)$, $(0, 1)$, $(-1, 0)$, and $F(x) > 0$ on the interval $(-\infty, -1)$. At the point $x_4 = 1$ the inequality is satisfied and at the point $x_3 = 0$ the function $F(x)$ is not defined.

Answer: $[-1, 0) \cup (0, 2)$.

Solve the following inequalities (27-135).

27. $(x-1)(3-x)(x-2)^2 > 0$.
28. $\dfrac{6x-5}{4x+1} < 0$.
29. $\dfrac{2x-3}{3x-7} > 0$.
30. $\dfrac{0.5}{x-x^2-1} < 0$.
31. $\dfrac{x^2-5x+6}{x^2+x+1} < 0$.
32. $\dfrac{x^2+2x-3}{x^2+1} < 0$.
33. $\dfrac{(x-1)(x+2)^2}{-1-x} < 0$.
34. $\dfrac{x^2+4x+4}{2x^2-x-1} > 0$.
35. $x^4-5x^2+4 < 0$.
36. $x^4-2x^2-63 \leqslant 0$.

1.5. Equations of Higher Degrees. Rational Inequalities 45

37. $\dfrac{3}{x-2} < 1$.
38. $\dfrac{1}{x-1} \leqslant 2$.
39. $\dfrac{4x+3}{2x-5} < 6$.
40. $\dfrac{5x-6}{x+6} < 1$.
41. $\dfrac{5x+8}{4-x} < 2$.
42. $\dfrac{x-1}{x+3} > 2$.
43. $\dfrac{7x-5}{8x+3} > 4$.
44. $\dfrac{x}{x-5} > \dfrac{1}{2}$.
45. $\dfrac{5x-1}{x^2+3} < 1$.
46. $\dfrac{x-2}{x^2+1} < -\dfrac{1}{2}$.
47. $\dfrac{x+1}{(x-1)^2} < 1$.
48. $\dfrac{x^2-7x+12}{2x^2+4x+5} > 0$.
49. $\dfrac{x^2+6x-7}{x^2+1} \leqslant 2$.
50. $\dfrac{x^4+x^2+1}{x^2-4x-5} < 0$.
51. $\dfrac{1+3x^2}{2x^2-21x+40} < 0$.
52. $\dfrac{1+x^2}{x^2-5x+6} < 0$.
53. $\dfrac{x^4+x^2+1}{x^2-4x-5} > 0$.
54. $\dfrac{1-2x-3x^2}{3x-x^2-5} > 0$.
55. $\dfrac{x^2-5x+7}{-2x^2+3x+2} > 0$.
56. $\dfrac{2x^2-3x-459}{x^2+1} > 1$.
57. $\dfrac{x^2-1}{x^2+x+1} < 1$.

5) $\dfrac{x^2+4x+4}{2x^2-x-1} > 0$

6) $\dfrac{1+x^2}{x^2-5x+6} < 0$

7) $\dfrac{1-2x-3x^2}{3x-x^2-5} > 0$

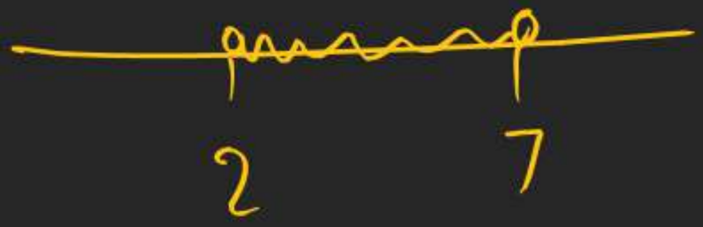
Fundamentals of Mathematics

Interval.

A) Close Interval :- $2 \le x \le 7 \Rightarrow$

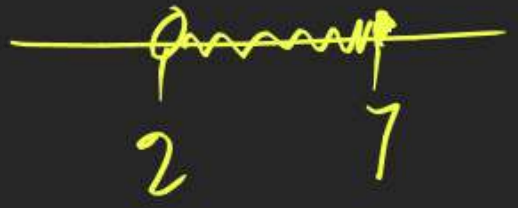
$x \in [2, 7]$

B) Open Interval $2 < x < 7$



$x \in (2, 7)$

C) Semiopen/Close Interval :- $2 < x \le 7$



$x \in (2, 7]$

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Fundamentals of Mathematics

* Discreate Interval → $(x-1)(x+2)=0$

$x-1=0$ or $x+2=0$

$x=1$ or $x=-2$

$x \in \{1, -2\}$

Q $|x| = 2$ find x?

$x = \pm 2 \rightarrow x \in \{2, -2\}$

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Fundamentals of Mathematics

Solving Inequality

$x-1=0$

$x=1$

Q $x^3 - x \le 0$ find $x \in$?

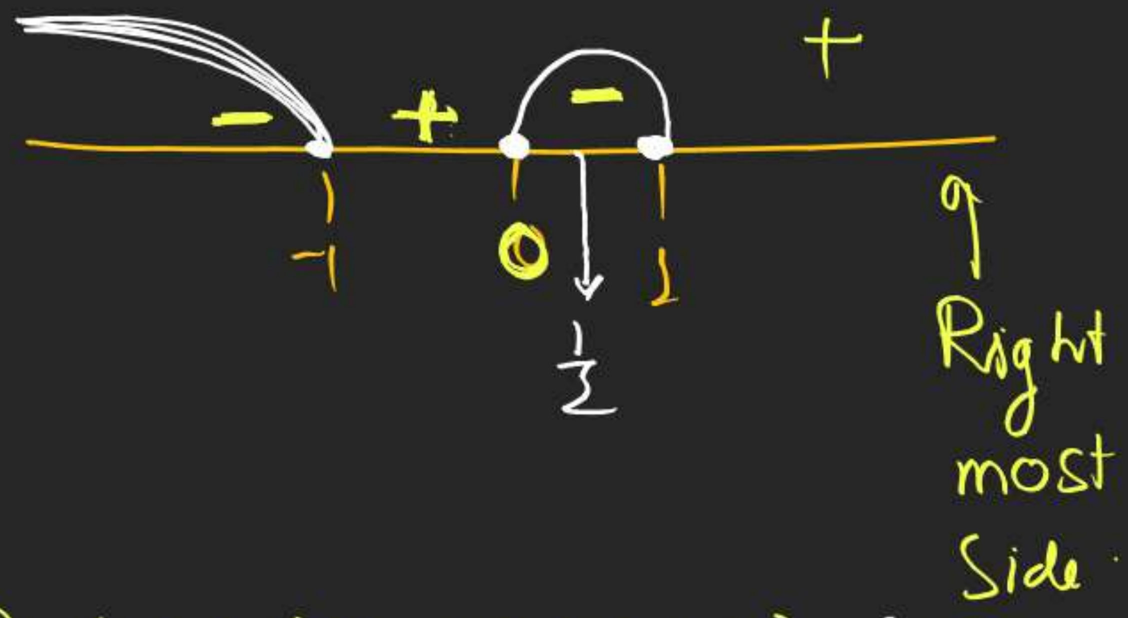
① factorise & find value of x

$(x)(x^2-1) \le 0$

deg of all 3 Br. is odd $\{$ $(x)^1(x-1)^1(x+1)^1 \le 0$

(2) Put values of x on No Line

(3)



(4) $(x)(x-1)(x+1) \le 0$

$\Rightarrow x \in (-\infty,-1] \cup [0,1]$

$\left(\frac{1}{2}\right)\left(\frac{1}{2}-1\right)\left(\frac{1}{2}+1\right)$

$\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)\left(\frac{3}{2}\right) = -\frac{3}{8}$

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Fundamentals of Mathematics

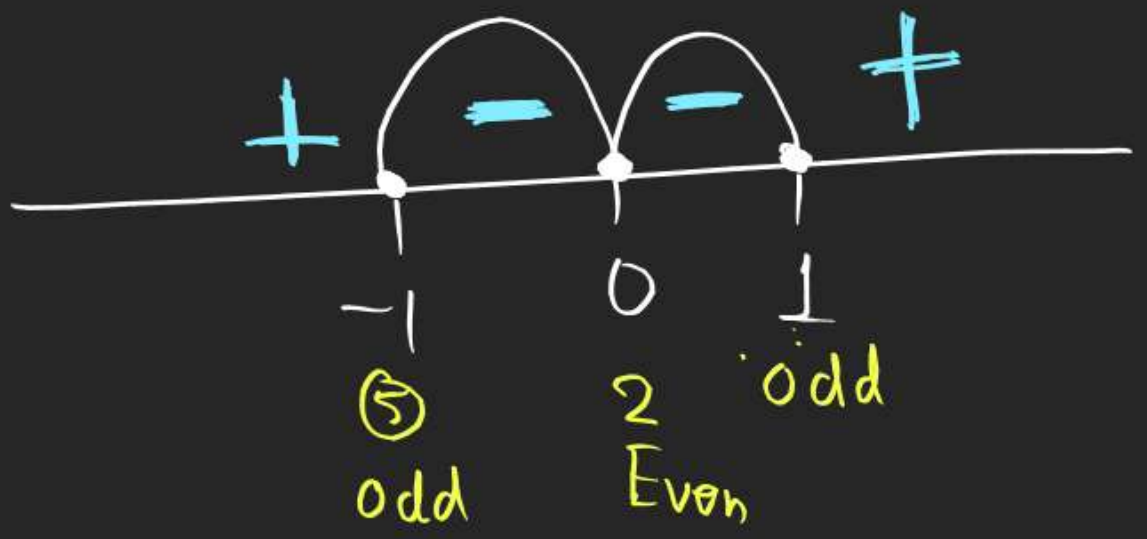
Q. $(x)^2(x-1)^3(x+1)^5 \le 0$

$\quad 0 \qquad 1 \qquad -1$

demand = -ve

(1) Factors already available

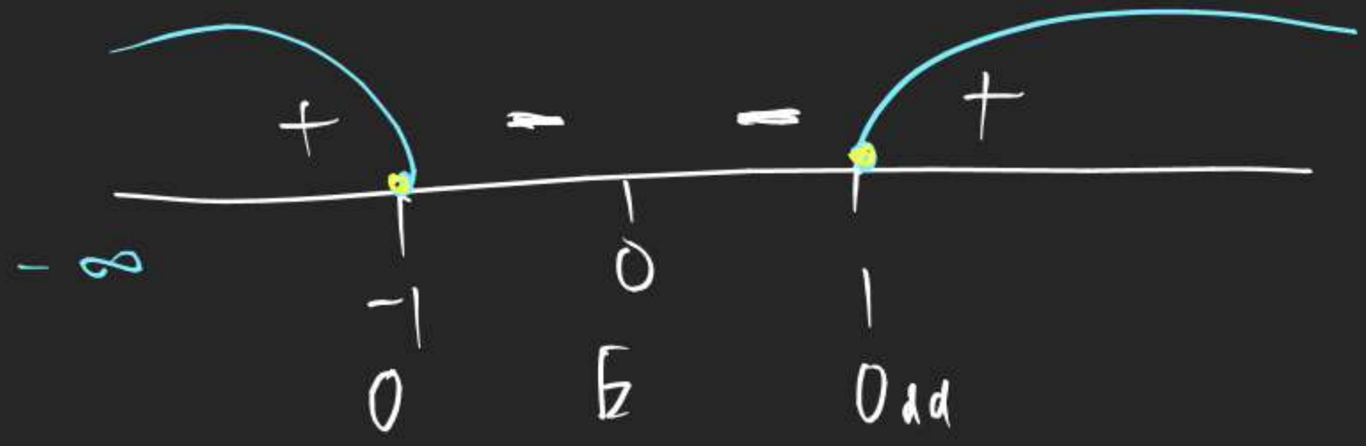
(2)



$x \in [-1, 1]$ ✓

Q. $(x)^2(x-1)^3(x+1)^5 \geq 0$

demand +ve



$x \in (-\infty, -1] \cup [1, \infty) \cup \{0\}$

x = 0 ur check

$(0)^2(0-1)^3(0+1)^5 \geq 0$

$0 \geq 0$ corr

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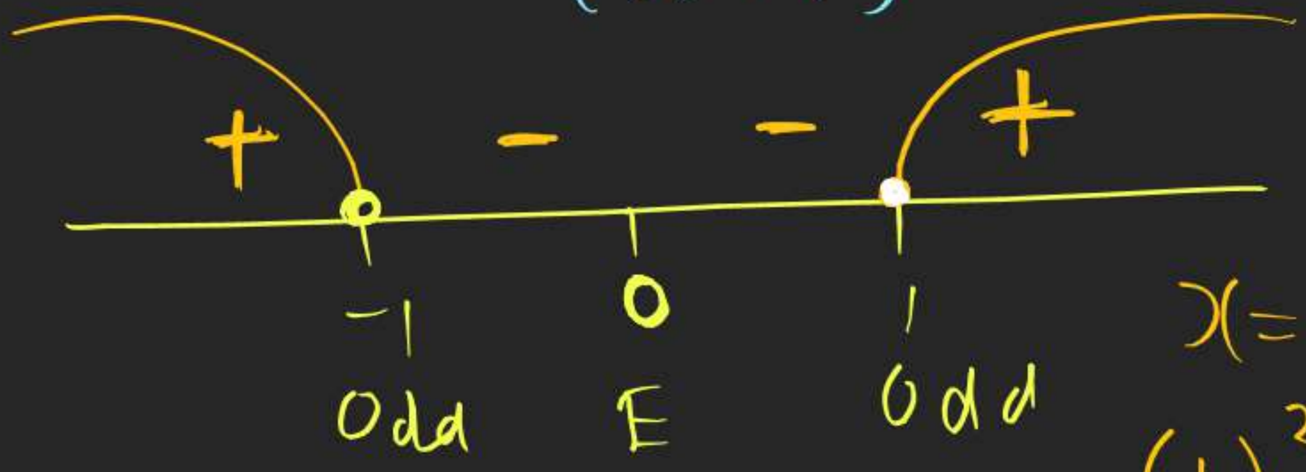
Fundamentals of Mathematics

Q $\dfrac{(x)^2(x-1)^3}{(x+1)^5} \geq 0$ +ve

$x \in (-\infty,-1) \cup [1,\infty) \cup \{0\}$



$x=-1$ पर Check

$\dfrac{(-1)^2(-1-1)^3}{(\cancel{-1+1})^5} \geq 0$

$\dfrac{-8}{0} \to \infty$ Not Defined.

$x=1$ पर think.

$\dfrac{(1)^2(\cancel{1-1})^3}{(1+1)^5} \geq 0$

$0 \geq 0$

Right Statement

We will include 1

Classic.

$x=0$ पर Check.

$\dfrac{(0)^2(0-1)^3}{(0+1)^5} \geq 0$

$0 \geq 0$ Correct

Fundamentals of Mathematics

$(3-1)\cancel{(3-3)}(3-2)^2 \geq 0$

$0 \geq 0$

Q $(x-1)(3-x)(x-2)^2 > 0$

* Factorise given Eqn & make all x +ve

$(x-1)^1(x-3)^1(x-2)^2 < 0$ (-ve)

$x \in (-\infty, 1] \cup \{2\} \cup [3, \infty)$

$x \in (1,2) \cup (2,3)$

Q $(x-1)(3-x)(x-2)^2 \leq 0$

$(x-1)(x-3)(x-2)^2 \geq 0$

$x=1$

$\cancel{(1-1)}(1-3)(1-2)^2 \geq 0$

$0 \geq 0$ ✓

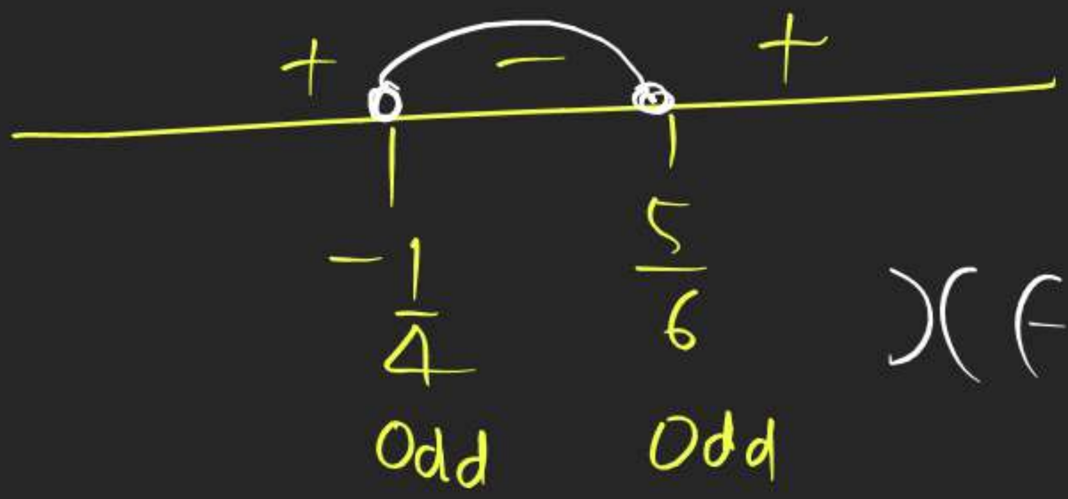
$x=2$

$(2-1)(2-3)\cancel{(2-2)}^2 \geq 0$

$0 \geq 0$

Fundamentals of Mathematics

Q $\dfrac{(6x-5)^1}{(4x+1)^1} < 0$ $\rightarrow \frac{5}{6}$, $\rightarrow -\frac{1}{4}$ $-ve$



$x \in \left(-\frac{1}{4}, \frac{5}{6}\right)$

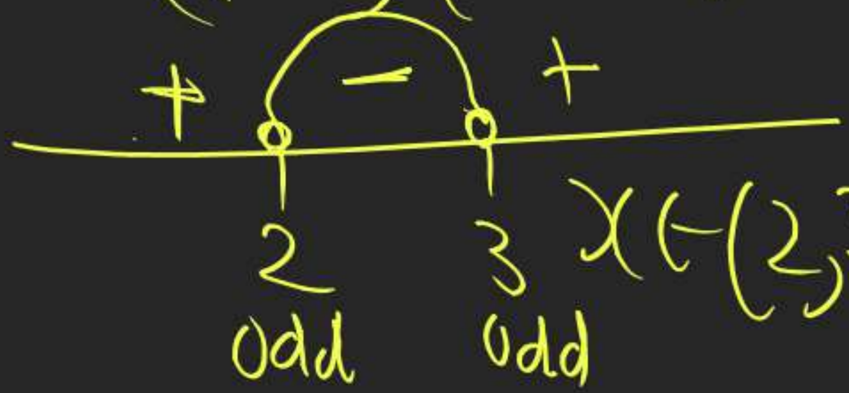
Q $\dfrac{x^2-5x+6}{\boxed{x^2+x+1}} < 0$ $\rightarrow$ Factors Nhi ho Rahe.

$a=1, b=1, c=1$

$\dfrac{(x-2)(x-3)}{(x^2+x+1)} < 0$

$+ve$ है

$(x-2)^1(x-3)^1 < 0$



$x \in (2,3)$

"D" Check करो

$= b^2-4ac$

$= 1^2-4\times1\times1$

$= 1-4 = -3 = -ve$

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Fundamentals of Mathematics

$\boxed{a=1}$, $b=-1$, $c=1$

Q $\dfrac{.5}{(x^2-x+1)} < 0$

+ve

D check

b^2-4ac

$\Rightarrow (-1)^2-4\times1\times1$

$= -ve$

$.5 < 0$

$\frac{1}{2} < 0$

X

$x \in \phi$

Q $\dfrac{.5}{(x^2-x+1)} > 0$

+ve

D check

$D=-3$

-ve

$a=1$ (+)

$.5 > 0$

$\frac{1}{2} > 0$

True always for all x

$x \in (-\infty, \infty) \Rightarrow x \in R$

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Fundamentals of Mathematics

Q $\dfrac{x^2-1}{x^2+1} \le 0$

$a=1,\ b=0,\ c=1$

$(+)$

x^2+1 → factor nahi ho Rahe

D check

$b^2-4ac = 0^2-4\times1\times1$

$= -4\ (-ve)$

$\dfrac{(x-1)(x+1)}{(x^2+1)} \le 0$

(x^2+1): $(+)ve$

$(x-1)(x+1) \le 0$

$-ve$

$x \in [-1, 1]$

$x=-1$: $(-1-1)(-1+1) \le 0$

$0 \le 0$ corr.

$x=1$: $(1-1)(1+1) \le 0$

$0 \le 0$ corr

Ahr D +ve
आए तो factor
ऐसा Shri
Dharacharya
method से
कर लेना।

Fundamentals of Mathematics

Q $\dfrac{(x^2-1)}{-x^2+x-3} \geq 0$ $\longrightarrow$ factor नही होते है

$\dfrac{(x-1)(x+1)}{(x^2-x-3)} \geq 0$

-ve

$(x-1)(x+1) \leq 0$

$x \in [-1, 1]$

check D.

$\boxed{a=-1}$, $b=1$, $c=-3$.

b^2-4ac

$(1)^2-4\times(-1)(-3)$

$-11 = -ve$

a = -ve & D = -ve

Result

① If Quad Eq^n is not factorising check D.

② If a +ve D -ve
Q Eq +ve

③ If a = -ve, D = -ve
Q Eq^n = -ve

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Fundamentals of Mathematics

Q $\dfrac{(-x^2+x-3)}{x^2-1} \geq 0$ → factorise Nahi hua

D = Check

D = −11 −ve

a = −1 −ve

−ve है

$\dfrac{(-x^2+x-3)}{\{(x-1)(x+1)\}} \geq 0$

Ans +ve चाहिए

$\dfrac{\ominus \text{ है}}{\bigcirc}$ = +ve चाहिए

−ve लाना पड़ेगा

It must be −ve

$(x-1)(x+1) < 0$

$x \in (-1, 1)$

Fundamentals of Mathematics

Q. $\dfrac{.5}{x^2-9} \geq 0$

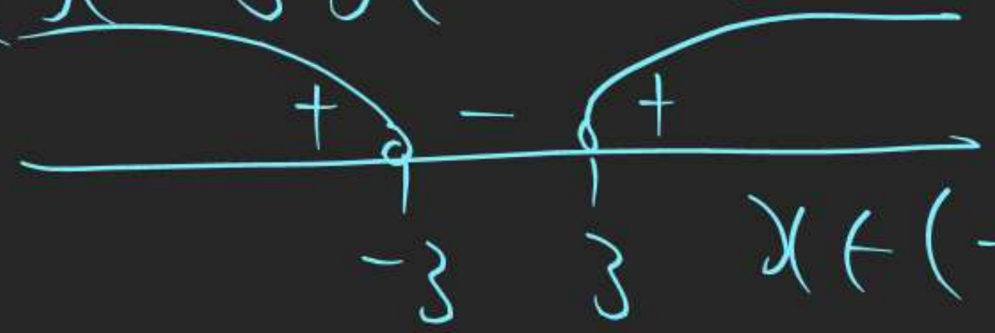
$\dfrac{.5 \; (+)}{(x-3)(x+3)} \geq 0$ (+) Chahiye

+ve

$\dfrac{(+)}{\bigcirc} = (+)$

$\bigcirc$ → (+) hona Padega.

$(x-3)(x+3) > 0$



$x \in (-\infty, -3) \cup (3, \infty)$

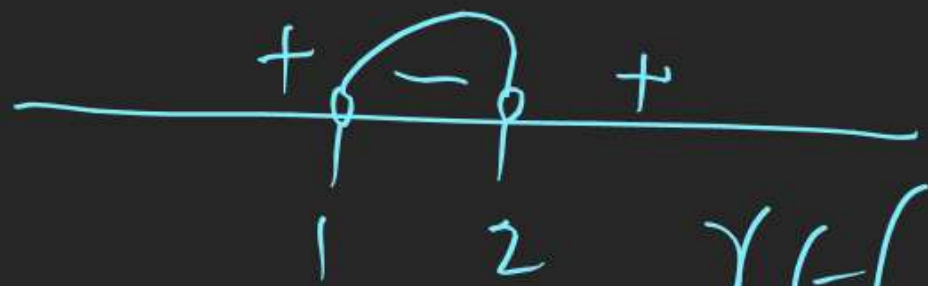
1. $$\frac{2^{+}}{(x^2-3x+2)} \le 0^{-}$$

$$\frac{+}{-} = -\text{ve}$$

$\to$ -ve

$$x^2-3x+2<0$$

$$(x-1)(x-2)<0$$



$$x \in (1,2)$$