

$$Q \quad \operatorname{Im} \frac{\pi}{11} + \operatorname{Im} \frac{2\pi}{11} + \operatorname{Im} \frac{4\pi}{11} + \operatorname{Im} \frac{7\pi}{11} + \operatorname{Im} \frac{9\pi}{11} + \operatorname{Im} \frac{10\pi}{11} = ?$$

① Series जैसा कुछ हा।

② total 6 terms hai

③ Last 3 terms change karong

$$\operatorname{Im} \left(\frac{(11\pi - 4\pi)}{11} \right) + \operatorname{Im} \left(\frac{(11\pi - 2\pi)}{11} \right) + \operatorname{Im} \left(\frac{(11\pi - \pi)}{11} \right)$$

$$\textcircled{4} \quad \frac{10\pi}{11} \rightarrow D_r = 11 \text{का half} = 5.5 \\ N_r = 10 > 5.5$$

$$+ \operatorname{Im} \left(\pi - \frac{4\pi}{11} \right) + \operatorname{Im} \left(\pi - \frac{2\pi}{11} \right) + \operatorname{Im} \left(\pi - \frac{\pi}{11} \right) \text{ (5) as } N_r \text{ ring than half}$$

if $D_r \Rightarrow (\pi - \theta)$ will be used

$$\operatorname{Im} \cancel{\frac{\pi}{11}} + \operatorname{Im} \cancel{\frac{9\pi}{11}} + \operatorname{Im} \cancel{\frac{4\pi}{11}} - \operatorname{Im} \cancel{\frac{5\pi}{11}} - \operatorname{Im} \cancel{\frac{2\pi}{11}} - \operatorname{Im} \cancel{\frac{\pi}{11}}$$

= 0

$$\textcircled{6} \quad \operatorname{Im} \frac{10\pi}{11} = \operatorname{Im} \left(\frac{11\pi - \pi}{11} \right)$$

$$\textcircled{7} \quad \operatorname{Im} \left(\pi - \frac{\pi}{11} \right) = - \operatorname{Im} \frac{\pi}{11}$$



$$\begin{aligned}
 & \textcircled{1} \quad G_s^2\left(\frac{\pi}{16}\right) + G_s^2\left(\frac{3\pi}{16}\right) + G_s^2\left(\frac{5\pi}{16}\right) + G_s^2\left(\frac{7\pi}{16}\right) \\
 & G_s^2\left(\frac{\pi}{16}\right) + G_s^2\left(\frac{3\pi}{16}\right) + G_s^2\left(\frac{8\pi-3\pi}{16}\right) + G_s^2\left(\frac{8\pi-\pi}{16}\right) \\
 & G_s^2\left(\frac{\pi}{16}\right) + G_s^2\left(\frac{3\pi}{16}\right) + G_s^2\left(\frac{\pi}{2} - \frac{3\pi}{16}\right) + G_s^2\left(\frac{\pi}{2} - \frac{\pi}{16}\right) \\
 & G_s^2\left(\frac{\pi}{16}\right) + G_s^2\left(\frac{3\pi}{16}\right) \underbrace{\delta m^2\left(\frac{3\pi}{16}\right)}_{1} + \delta m^2\left(\frac{\pi}{16}\right) \\
 & = 1 + 1 = 2
 \end{aligned}$$

- ① Series की फैल
 ② total terms = 4
 ③ Last का 2 Ko Balance
 ④ Nr = 7
 Dr = 16 \rightarrow 16 का half = 8
 Nr = 7 < 8 = Dr का half
 (5) as Nr < half of dr.
 Now change in $(\frac{\pi}{2} - \theta)$
 (6) $(\frac{\pi}{2} - \theta) = (\frac{8\pi}{16} - \theta)$
 (7) $G_s^2\left(\frac{2\pi}{16}\right) = G_s^2\left(\frac{8\pi-\pi}{16}\right)$
 $= G_s^2\left(\frac{\pi}{2} - \frac{\pi}{16}\right) = \delta m^2\left(\frac{\pi}{16}\right)$

Sahi
Likha

$$\{ \delta m^2 \left(\frac{\pi}{18} \right) + \delta m^2 \left(\frac{\pi}{9} \right) + \delta m^2 \left(\frac{7\pi}{18} \right) + \delta m^2 \left(\frac{4\pi}{9} \right) \}$$

$$\delta m^2 \left(\frac{\pi}{18} \right) + \delta m^2 \left(\frac{2\pi}{18} \right) + \delta m^2 \left(\frac{7\pi}{18} \right) + \delta m^2 \left(\frac{8\pi}{18} \right)$$

$$+ \delta m^2 \left(\frac{9\pi - 2\pi}{18} \right) + \delta m^2 \left(\frac{9\pi - \pi}{18} \right)$$

$$\delta m^2 \left(\frac{\pi}{18} \right) + \delta m^2 \left(\frac{2\pi}{18} \right) + \delta m^2 \left(\left[\frac{\pi}{2} \right] - \frac{2\pi}{18} \right) + \delta m^2 \left(\frac{\pi}{2} - \frac{\pi}{18} \right)$$

$$\underbrace{\delta m^2 \left(\frac{\pi}{18} \right) + \delta m^2 \left(\frac{2\pi}{18} \right)}_1 + \underbrace{\delta m^2 \left(\frac{2\pi}{18} \right) + \delta m^2 \left(\frac{\pi}{18} \right)}_1$$

$$= 1 + 1 = 2$$

① Last 2 term change

$$\textcircled{2} \delta m^2 \left(\frac{8\pi}{18} \right)$$

$$\textcircled{3} Nr = 8$$

$$\textcircled{4} Dr = \boxed{18} \text{ half} = 9$$

$$\textcircled{5} Nr = 8 < 9$$

Nr < Dr half

$$\textcircled{6} \left(\frac{\pi}{2} - \theta \right) \text{ me change}$$

$$\textcircled{7} \left(\frac{\pi}{2} - \theta \right) = \left(\frac{9\pi}{18} - \theta \right)$$

Q $\sin\left(\frac{24\pi}{3}\right)$ then value?

$$\sin(8\pi) = 0$$

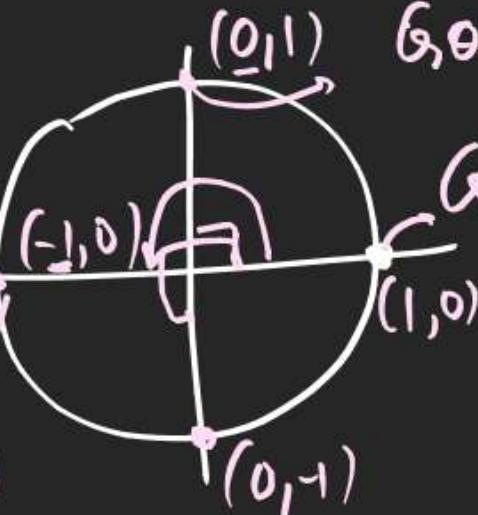


$$\text{at } 180^\circ = -1$$

$$\sin 180^\circ = 0$$

$$\text{at } \theta = 0, \sin \theta = 1$$

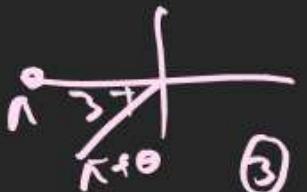
$$\text{at } \theta = 1, \sin \theta$$



$$\text{at } 270^\circ = 0$$

$$\sin 270^\circ = -1$$

Q Value of $\sin \frac{7\pi}{4}$



$$\sin\left(\frac{4\pi+3\pi}{4}\right) = \sin\left(\pi + \frac{3\pi}{4}\right)$$

$$\therefore -\sin\frac{3\pi}{4}$$

$$\therefore -\frac{1}{\sqrt{2}}$$

$$Q \left(\cos\left(\frac{28\pi}{3}\right) \right) = ?$$

$$\cos\left(9\pi + \frac{\pi}{3}\right)$$

$$-\cos\frac{\pi}{3} = -\cos 60^\circ = -\frac{1}{2}$$

$$Q \tan\left(\frac{11\pi}{6}\right) = ?$$

$$\tan\left(\frac{12\pi - \pi}{6}\right) = \tan\left(2\pi - \frac{\pi}{6}\right)$$

$$= -\tan\frac{\pi}{6} = -\tan 30^\circ$$

$$= -\frac{1}{\sqrt{3}}$$

$$Q \tan\left(\frac{17\pi}{4}\right)$$

$$\tan\left(\frac{16\pi + \pi}{4}\right)$$

$$\tan\left(4\pi + \frac{\pi}{4}\right)$$

$$+ \tan\frac{\pi}{4}$$

$$= 1$$

$$\sec\left(\frac{4\pi}{3}\right)$$

$$\begin{aligned} & \sec\left(\frac{3\pi+\pi}{3}\right) \\ & \sec\left(\pi+\frac{\pi}{3}\right) \quad \text{③} \\ & -\sec\frac{\pi}{3} \\ & = -\sec 60^\circ \\ & = -2 \end{aligned}$$

$$\begin{aligned} & \sec\left(\frac{7\pi}{6}\right) \\ & \sec\left(6\pi+\frac{\pi}{6}\right) \\ & \sec\left(\pi+\frac{\pi}{6}\right) \quad \text{③} \\ & -\sec\frac{\pi}{6} \\ & = -\sec 30^\circ \\ & = -2 \end{aligned}$$

$$\begin{aligned} & \tan\left(\frac{35\pi}{4}\right) \\ & \tan\left(\frac{36\pi-\pi}{4}\right) \\ & \tan\left(9\pi-\frac{\pi}{4}\right) \quad \text{②} \\ & -\tan\frac{\pi}{4} \\ & = -1 \end{aligned}$$

$$\text{Q } \tan\left(\frac{43\pi}{11}\right)$$

$$= \tan\left(\frac{44\pi - \pi}{11}\right)$$

~~$$= \tan\left(4\pi - \frac{\pi}{11}\right)$$~~

~~$$= \tan\left(\frac{\pi}{11}\right)$$~~

Standard formulae

1) $\sin(A+B) = \sin A \cos B + \cos A \sin B$

2) $\sin(A-B) = \sin A \cos B - \cos A \sin B$

3) $\cos(A+B) = \cos A \cos B - \sin A \sin B$

4) $\cos(A-B) = \cos A \cos B + \sin A \sin B$

$$\text{Q Value of } \sin(45^\circ + \theta) \cdot \cos(15^\circ + \theta) - \cos(45^\circ + \theta) \cdot \sin(15^\circ + \theta) = ?$$

$$\sin A \quad \cos B \quad - \cos A \cdot \sin B' = \sin(A-B)$$

$$\sin\{(45^\circ + \theta) - (15^\circ + \theta)\} = \sin(45^\circ - 15^\circ) = \sin 30^\circ = \frac{1}{2}$$

Q Find value of $\cos(45^\circ + \theta) \cdot \cos(45^\circ - \theta) - \sin(45^\circ + \theta) \cdot \sin(45^\circ - \theta)$?

$$G_A \cdot G_B - \delta_m(A) \cdot \delta_m(B) = G_{(A \# B)}$$

$$\sqrt{\frac{41^2 - 9^2}{41^2}} = \sqrt{\frac{40^2}{41^2}}$$

$$\sin \beta = \frac{40}{41}$$

$$\sin 0^\circ = 0$$

$$G_0 = 1$$

On 90° = L

690-10

$$\text{so } 180^\circ = D$$

$$6, 180' = -1$$

$\sin 270^\circ = -$

$$6, 270^\circ$$

$$= 90 \left((45^\circ + \theta) + (45^\circ - \theta) \right) = 90 \cdot 90^\circ = 0$$

$$Q \text{ Tim } G_1(\underline{A+B}) \cdot G_1(\underline{A-B}) + S_m(\underline{A+B}) \cdot S_m(\underline{A-B}) =$$

$$G_m M \cdot G_N + G_m M \cdot G_N = G(M-N)$$

$$= G((\alpha + \beta) - (\alpha - \beta))$$

- 6 (2B)

$$Q_{\text{form}} = \frac{3}{5} \quad \text{&} \quad G_B = \frac{9}{41} \quad d, B \in \text{I}^{\text{st}} \text{ Quad}$$

$$\text{find } \theta \text{ in } (\alpha + \beta) \text{ } \textcircled{2} \text{ } \theta(\alpha - \beta) = 9$$

$$\textcircled{1} \quad \sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta = \frac{\frac{3}{5} \cdot \frac{9}{51} + \frac{4}{5} \cdot \frac{40}{51}}{5 \times 51} = \frac{133}{255}$$

$$\textcircled{1} \sin \alpha = \frac{3}{5}$$

$$G\lambda = \sqrt{1 - \delta m^2} \lambda$$

$$= \sqrt{1 - \frac{9}{25}} = \sqrt{\frac{16}{25}}$$

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$$\textcircled{2} \quad \sin(\alpha - \beta) = \sin \alpha \cdot \cos \beta + \cos \alpha \cdot \sin \beta$$

$$= \frac{4}{5} \cdot \frac{9}{41} + \frac{3}{5} \cdot \frac{40}{41}$$

$$= \frac{36 + 120}{205} = \frac{156}{205}$$



$$\sin \alpha = \frac{3}{5} \Rightarrow \cos \alpha = \frac{4}{5}$$

$$\cos \beta = \frac{9}{41} \Rightarrow \sin \beta = \frac{40}{41}$$

$$\sin \beta = \sqrt{1 - \cos^2 \beta}$$

$$= \sqrt{1 - \frac{81}{1681}}$$

Q Value of $\sin 31^\circ + \sin 89^\circ + \sin 151^\circ = ?$ $\xrightarrow{31^\circ \text{ Kina Zayr.}}$

$$\begin{aligned} \sin 120^\circ \\ = \underline{\sin} \left(\frac{\pi}{2} + 30^\circ \right) \end{aligned}$$

$$\begin{aligned} = -\sin 30^\circ \\ = -\frac{1}{2} \end{aligned}$$

$$\begin{aligned} & \sin 31^\circ + \sin \underbrace{(120^\circ - 31^\circ)}_{A - B} + \sin (120^\circ + 31^\circ) \\ &= \sin 31^\circ + \left\{ \sin 120^\circ \sin 31^\circ + \cos 120^\circ \cos 31^\circ \right\} \\ & \quad + \left\{ \sin 120^\circ \sin 31^\circ - \cos 120^\circ \cos 31^\circ \right\} \\ &= \sin 31^\circ + 2 \sin 120^\circ \sin 31^\circ \end{aligned}$$

$$= \sin 31^\circ + 2 \times -\frac{1}{2} \sin 31^\circ = \sin 31^\circ - \sin 31^\circ = 0$$

$\sin(A - B)$	$= \sin A \cos B + \cos A \sin B$
$\sin(A + B)$	$= \sin A \cos B - \cos A \sin B$

$$\text{Q If } \sin \alpha \cdot \sin \beta - \cos \alpha \cdot \cos \beta + 1 = 0$$

then value of $1 + \underline{\cos} \alpha \cdot \sin \beta = 2$

$$\sin \alpha \cdot \sin \beta - \cos \alpha \cdot \cos \beta = -1$$

$$\Rightarrow \cos \alpha \cdot \cos \beta - \sin \alpha \cdot \sin \beta = 1$$

$$\stackrel{\text{पर्याप्ति}}{\Rightarrow} \cos(\alpha + \beta) = 1$$

$\sin(\alpha + \beta) = 0$ hota hai

$$\text{Demand} = 1 + \underline{\cos} \alpha \cdot \sin \beta$$

$$= 1 + \frac{\cos \alpha}{\sin \alpha} \cdot \frac{\sin \beta}{\cos \beta} = \frac{\sin \alpha \cdot \cos \beta + \cos \alpha \cdot \sin \beta}{\sin \alpha \cos \beta} = \frac{\sin(\alpha + \beta)}{\sin \alpha \cos \beta}$$

Correct $\sin(90-0)$

$$\frac{\sin 24^\circ \cos 6^\circ - \sin 6^\circ \cos 24^\circ = 0}{\sin 21^\circ \cos 39^\circ - \cos 21^\circ \sin 39^\circ = 0} \quad \text{Q } \underline{\cos}(\underline{\frac{\pi}{2}-0})$$

$$= \sin 0$$

$$\frac{\sin 24^\circ \cos 6^\circ - \sin 6^\circ \sin(90-24^\circ)}{\sin 21^\circ \cos 39^\circ - \cos(90-39^\circ) \sin(90-21^\circ)}$$

$$\frac{\sin 24^\circ \cos 6^\circ - \sin 6^\circ \cos(24^\circ-6^\circ)}{\sin 21^\circ \cos 39^\circ - \sin 39^\circ \cos(21^\circ-39^\circ)} = \frac{\sin(24^\circ-6^\circ)}{\sin(21^\circ-39^\circ)}$$

$$= \frac{\sin 18^\circ}{\sin(-18^\circ)} = \frac{\sin 18^\circ}{-\sin 18^\circ} = -1$$

$$\text{Q} \quad \sum_{K=1}^{13} \frac{1}{\sin\left(\frac{\pi}{4} + \frac{(K-1)\pi}{6}\right) \cdot \sin\left(\frac{\pi}{4} + \frac{K\pi}{6}\right)}$$

difference = $\frac{\pi}{6}$

$$G\left(\frac{58\pi}{24}\right) = G\left(\frac{48\pi}{24} + \frac{10\pi}{24}\right) = G\left(2\pi + \frac{5\pi}{12}\right)$$

$$-? = G\left(\frac{5\pi}{12}\right)$$

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$$2 \left(G\left(\frac{\pi}{4} + \frac{(K-1)\pi}{6}\right) - G\left(\frac{\pi}{4} + \frac{K\pi}{6}\right) \right)$$

K=1

Multiply & divide
by $\sin\frac{\pi}{6}$.

Change $\sin\frac{\pi}{6}$
acc to Dr.

$$\sum \frac{\sin\frac{\pi}{6}}{\sin\left(\frac{\pi}{4} + \frac{K\pi}{6}\right) \cdot \sin\left(\frac{\pi}{4} + \frac{(K-1)\pi}{6}\right) \cdot \sin\frac{\pi}{6}}$$

$$\frac{1}{\sin\frac{\pi}{6}} \left(\sum \frac{\sin\left\{ \left(\frac{\pi}{4} + \frac{K\pi}{6}\right) - \left(\frac{\pi}{4} + \frac{(K-1)\pi}{6}\right) \right\}}{\sin\left(\frac{\pi}{4} + \frac{K\pi}{6}\right) \cdot \sin\left(\frac{\pi}{4} + \frac{(K-1)\pi}{6}\right)} \right)$$

$$\frac{1}{\pi} \left(\sum \frac{\sin\left(\frac{\pi}{4} + \frac{K\pi}{6}\right) \cdot \left(G\left(\frac{\pi}{4} + \frac{(K-1)\pi}{6}\right) - G\left(\frac{\pi}{4} + \frac{K\pi}{6}\right)\right)}{\sin\left(\frac{\pi}{4} + \frac{K\pi}{6}\right) \cdot \sin\left(\frac{\pi}{4} + \frac{(K-1)\pi}{6}\right)} \right)$$

$$\frac{1}{\pi} \left(\sum \frac{G\left(\frac{\pi}{4} + \frac{K\pi}{6}\right) \cdot \sin\left(\frac{\pi}{4} + \frac{(K-1)\pi}{6}\right)}{\sin\left(\frac{\pi}{4} + \frac{K\pi}{6}\right) \cdot \sin\left(\frac{\pi}{4} + \frac{(K-1)\pi}{6}\right)} \right)$$

$$= 2 \left[G\left(\frac{\pi}{4} + 0\right) - G\left(\frac{\pi}{4} + \frac{\pi}{6}\right) \right]$$

$$+ G\left(\frac{\pi}{4} + \frac{\pi}{6}\right) - G\left(\frac{\pi}{4} + \frac{2\pi}{6}\right)$$

$$+ G\left(\frac{\pi}{4} + \frac{2\pi}{6}\right) - G\left(\frac{\pi}{4} + \frac{3\pi}{6}\right)$$

$$+ G\left(\frac{\pi}{4} + \frac{12\pi}{6}\right) - G\left(\frac{\pi}{4} + \frac{13\pi}{6}\right)$$

$$= 2 \left(G\left(\frac{\pi}{4}\right) - G\left(\frac{\pi}{4} + \frac{13\pi}{6}\right) \right)$$

$$2 \left(1 - G\left(\frac{6\pi + 52\pi}{24}\right) \right)$$

$$2 \left(1 - G\left(\frac{5\pi}{2}\right) \right)$$

$$2 \left(1 - \frac{m}{16} \right)$$