Nishant Jindal

Q1 $f(x) = \begin{cases} e^{2x} & x \le 0 \\ 2\sin x & x > 0 \end{cases}$

Check diffy at $x = 0$.

$e^{2\times 0} = 2\sin 0$

$1 = 0$

D.C. $\Rightarrow$ ND.

Q. 2 $f(x) = \begin{cases} x + [2x] & x < 1 \\ \{x\} + 1 & x \ge 1 \end{cases}$

Check diffy?

$f(1^-) = 1 + [2(1-h)] = 1 + [2 - 2h)$

$= 1 + 1 = 2$

$f(1^+) = \{1+h\} + 1 = \{h\} + 1$

$= h + 1 = 1$

$f(1^+) \ne f(1^-) \Rightarrow$ D.C

$\Rightarrow$ ND.

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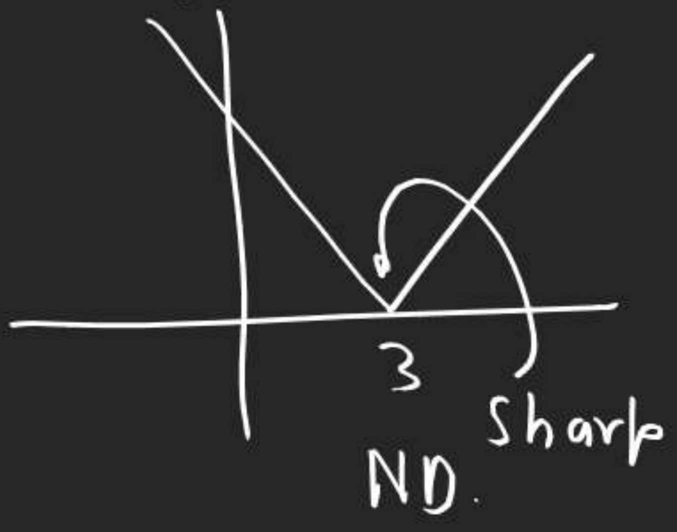
★ Modulus fxn are N.D. at turning pt (Cr.) pt.

$Y = \boxed{|x-3|}$ is N.D. at



$Y = \frac{x+1}{|x|}$ is N.D. at

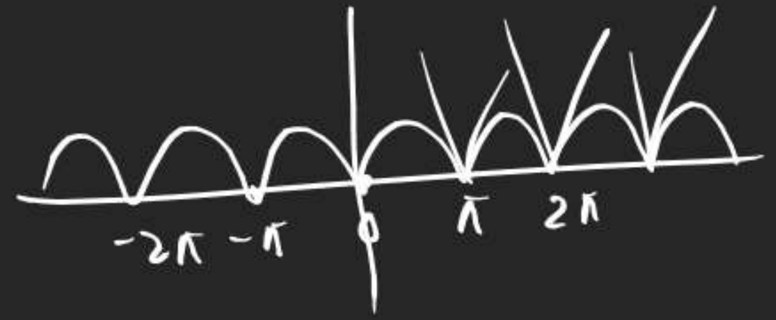
$x = 0$ is Cr. pt. of $|x|$

but Domain of

$f(x)$ doesn't contain

$x = 0$

Q $Y = |\sin x|$ is N.D. at



$\sin x = 0 \Rightarrow x = n\pi$

Q $Y = |\ln x|$ is N.D. at

N.D. where $\ln x = 0$

$x = e^0 = 1$

Q $y = e^{|x|}$ is N.D. at

Doubt $\rightarrow x = 0$

$$y = e^{|x|} = \begin{cases} e^{x} & x \geq 0 \\ e^{-x} & x < 0 \end{cases}$$

Cont^y

$$e^{0} = e^{-0} \Rightarrow 1 = 1 \checkmark$$

$$f'(x) = \begin{cases} e^{x} & x \geq 0 \\ -e^{-x} & x < 0 \end{cases}$$

$$\left.\begin{aligned} LHD &= -e^{-0} = -1 \\ RHD &= e^{0} = +1 \end{aligned}\right\} \text{N.D.}$$

Q $f(x) = |x^3|$ is N.D. at

Doubt $\rightarrow x^3 = 0 \Rightarrow x = 0$

$$y = |x^3| = \begin{cases} x^3 & x \geq 0 \\ -x^3 & x < 0 \end{cases}$$

Cont^y

$$0^3 = -0^3 \Rightarrow 0 = 0 \checkmark$$

$$f'(x) = \begin{cases} 3x^2 & x \geq 0 \\ -3x^2 & x < 0 \end{cases}$$

$$\left.\begin{aligned} LHD &= -3(0)^2 = 0 \\ RHD &= 3(0)^2 = 0 \end{aligned}\right\} \text{Cont}^s \text{ \& Diff}$$

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Q $f(x) = \dfrac{x}{1+|x|}$ is N.D. at ?

Doubt $\to x = 0$

$$Y = \frac{x}{1+|x|} = \begin{cases} \dfrac{x}{1+x} & x \geq 0 \\ \dfrac{x}{1-x} & x < 0 \end{cases}$$

Cnt^y $\dfrac{0}{1+0} = \dfrac{0}{1-0} \Rightarrow 0 = 0$

$$f'(x) = \begin{cases} \dfrac{(1+x)\cdot 1 - x\cdot(1)}{(1+x)^2} = \dfrac{1}{(1+x)^2} & x \geq 0 \\ \dfrac{(1-x)\cdot 1 - x(-1)}{(1-x)^2} = \dfrac{1}{(1-x)^2} & x < 0 \end{cases}$$

$$\left.\begin{aligned} LHD &= \frac{1}{(1-0)^2} = 1 \\ RHD &= \frac{1}{(1+0)^2} = 1 \end{aligned}\right\} \text{Diff at everywhere}$$

Q $f(x) = \begin{cases} \dfrac{x}{1+|x|} & |x| \geq 1 \\ \dfrac{x}{1-|x|} & |x| < 1 \end{cases}$

Where check Cnt^y & diff^y.

$$f(x) = \begin{cases} \dfrac{x}{1+|x|} & -\infty < x \leq -1 \cup 1 \leq x < \infty \\ \dfrac{x}{1-|x|} & -1 < x < 1 \end{cases}$$

$x = -ve$, $x = +ve$

-1 0 1

$$f(x) = \begin{cases} \dfrac{x}{1-x} & -\infty < x \leq -1 \\ \dfrac{x}{1-x} & 0 \leq x < 1 \\ \dfrac{x}{1+x} & 1 \leq x < \infty \\ \dfrac{x}{1+x} & -1 < x < 0,\ x = -ve \end{cases}$$

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$-\infty \qquad -1 \qquad 0 \qquad 1 \qquad \infty$

$$f(x) = \begin{cases} \dfrac{x}{1-x} & -\infty < x \le -1 \cup 0 \le x < 1 \\ \dfrac{x}{1+x} & 1 \le x < \infty \cup -1 < x < 0 \end{cases}$$

$$f(x) = \begin{cases} \dfrac{x}{1-x} & 0 \le x < 1 \\ \dfrac{x}{1+x} & -1 < x < 0 \end{cases}$$

Critical pt $\rightarrow -1, 0, 1$.

-1: $\dfrac{-1}{1-(-1)} \ne \dfrac{-1}{1+(-1)}$ D.C. $\rightarrow$ N.D

0: $\dfrac{0}{1-0} = \dfrac{0}{1+0} \Rightarrow 0 = 0 \Rightarrow$ Cont^s

$x = 1$: $\dfrac{1}{1-1} \ne \dfrac{1}{1+1}$ D.C ND

$$f'(x) = \begin{cases} \dfrac{(1-x)\cdot 1 - x(-1)}{(1-x)^2} = \dfrac{1}{(1-x)^2} & 0 \le x < 1 \\ \dfrac{(1+x) - x}{(1+x)^2} = \dfrac{1}{(1+x)^2} & -1 < x < 0 \end{cases}$$

LHD $= \dfrac{1}{(1+0)^2} = 1$

RHD $= \dfrac{1}{(1-0)^2} = 1$ } Diff.

Q $f(x) = \begin{cases} \dfrac{x}{1+|x|} & |x| \ge 1 \\ \dfrac{x}{1-|x|} & |x| < 1 \end{cases}$

Atleast 2 time self

Where check Cont^y & diff^y.

$$f(x) = \begin{cases} \dfrac{x}{1+|x|} & -\infty < x \le -1 \cup 1 \le x < \infty \\ \dfrac{x}{1-|x|} & -1 < x < 1 \end{cases}$$

$x = -ve$ $\qquad$ $x = +ve$

$-1 \qquad 0 \qquad 1$

$$f(x) = \begin{cases} \dfrac{x}{1-x} & -\infty < x \le -1 \\ \dfrac{x}{1-x} & 0 \le x < 1 \\ \dfrac{x}{1+x} & 1 \le x < \infty \\ \dfrac{x}{1+x} & -1 < x < 0,\ x = -ve \end{cases}$$

Nishant Jindal

Jahan Drd Vohan humdard

then No Drd

Q $y = |x-1|$ is N.D. at

$x = 1$ N.D

Q $y = (x-1)|x-1|$ is Diff or N.D.?

↳ Doubt $\Rightarrow x = 1$

$$y = (x-1)\underbrace{|x-1|}_{x=1} = \begin{cases} (x-1)(x-1) & x \geq 1 \\ -(x-1)(x-1) & x < 1 \end{cases}$$

$$= \begin{cases} (x-1)^2 & x \geq 1 \\ -(x-1)^2 & x < 1 \end{cases}$$

Cont & diff

$y = (x-1)^2$

$y = -(x-1)^2$

$y = |x|$ is N.D. at $x = 0$

$$y = x|x| = \begin{cases} x^2 & x \geq 0 \\ -x^2 & x < 0 \end{cases}$$

Diff

$y = x|x|$ is diff at $x = 0$

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Q $y = (x-2)\,|x^2-3x+2|$ is N.D. at

$y = (x-2)\,|(x-1)(x-2)|$ is N.D. at $x=1$

$x=2$ ✗

Drd $\Rightarrow x = 1, 2$ ✗

Q $y = (x^2-3x+2)\,|x^3-6x^2+11x-6|$ is N.D. at

$y = (x-1)(x-2)\,|(x-1)(x-2)(x-3)|$ is N.D at $x=3$

1 ✗ 2 ✗

N.D at $x = 1, 2, 3$

Q $f(x) = (x-1)\,|\ln x|$ is N.D/Diff

$x=1$

$\ln x = 0$ N.D

$x = 1$ ✗

diff'ble at $x=1$

$f : (0,1) \to R$ $\quad f(x) = [4x]\,(x-1/4)^2(x-1/2)$

$x = \frac{1}{4}, \frac{1}{2}, \frac{3}{4}$

$\frac{1}{4}$ ✗ $\quad \frac{1}{2}$ ✗

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Q $f(x) = \begin{cases} x \cdot \sin\frac{1}{x} & x \neq 0 \\ 0 & x = 0 \end{cases}$

Check cont^y & diff^y of $f(x)$ at $x=0$.

Cont^y

$LL = \lim_{x \to 0} x \cdot \sin\frac{1}{x} = 0 \cdot \sin\infty$

$= 0 \times (-1 \text{ to } +1)$

$= 0$

fxn is cont^s

$f(0) = 0$

diff^y

$$f'(0+h) = \lim_{h \to 0} \frac{f(0+h) - f(0)}{h}$$

$$= \lim_{h \to 0} \frac{h \cdot \sin\frac{1}{h} - 0}{h} = \sin\infty = \underbrace{(-1 \text{ to } +1)}_{\infty \text{ value}}$$

RHD DNE

Diff^y ×

$f(x)$ is cont^s but not diff

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Q $f(x) = \begin{cases} x^2 \sin\frac{1}{x} & x \neq 0 \\ 0 & x = 0 \end{cases}$

Check Cont^y & diff^y at $x = 0$

$\boxed{\text{Cont}^y}$ $\lim_{x \to 0} x^2 \cdot \sin\frac{1}{x} = 0^2 \times \sin\infty$

$= 0 \times (-1 \text{ to } +1)$

$= 0 = f(0)$

Cont^s

diff^y

$RHD = f'(0+h) = \lim_{h \to 0} \frac{f(0+h) - f(0)}{h}$

$= \lim_{h \to 0} \frac{h^{\cancel{2}} \cdot \sin\frac{1}{h} - 0}{\cancel{h}} = 0 \cdot \sin\infty = 0 \times (-1 \text{ to } 1)$

$= 0$

$f'(0-h) = \lim_{h \to 0} \frac{f(0-h) - f(0)}{-h} = \lim_{h \to 0} \frac{(-h)^2 \cdot \sin\left(\frac{1}{-h}\right) - 0}{-h}$

$= \lim_{h \to 0} \frac{+h^{\cancel{2}} \cdot \sin\frac{1}{h}}{+\cancel{h}} = 0 \cdot \sin\infty = 0$

Cont^s & diff both

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① $f(x) = \begin{cases} x \cdot \sin\frac{1}{x} & x \neq 0 \\ 0 & x = 0 \end{cases}$ $\cos\frac{1}{x}$

Cont^s But not diff

(2) $f(x) = \begin{cases} x^2 \cdot \sin\frac{1}{x} & x \neq 0 \\ 0 & x = 0 \end{cases}$ $\cos\frac{1}{x}$

Cont^s & diff both.

$f(x) = \begin{cases} x^1 \cdot \underbrace{\boxed{\dfrac{e^{\frac{1}{x}} - e^{-1/x}}{e^{1/x} + e^{-1/x}}}}_{\text{LDNE}} & x \neq 0 \\ 0 & x = 0 \end{cases}$ $\to$ LHL $= -1$, $\to$ RHL $= +1$

$n = 1 \Rightarrow$ Cont^s But ND.

$f(x) = \begin{cases} x^n \cdot \overset{\text{LDNE}}{\boxed{\sin\frac{1}{x}}} & \boxed{x \neq 0} \\ 0 & x = 0 \end{cases}$

$n \leq 0$	Niether Cont^s Nor diff
$0 < n \leq 1$	Cont^s But ND.
$n > 1$	Cont^s & Diff.

Q $f(x) = \begin{cases} x^1 \cdot \underbrace{\boxed{e^{-\left\{\frac{1}{|x|} + \frac{1}{x}\right\}}}}_{\text{LDNE}} & x \neq 0 \\ 0 & x = 0 \end{cases}$ $\to 0$, $x > 0$, $x < 0$

$n = 1$ Cont^s But ND