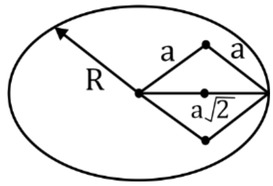
Nishant Jindal

DPP – 02

SOLUTION

1. $R = a\sqrt{2}$



$\rho = \frac{M}{\pi R^2} \qquad = \frac{M}{2\pi a^2}$

$$\rho = \frac{M}{2\pi a^2}$$

position of final centre of mass

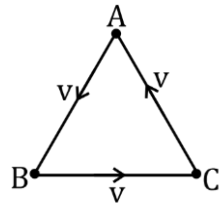
$$x = \frac{M \times 0 - \frac{M}{2\pi a^2} \times a \times a \times \frac{a\sqrt{2}}{2}}{M - \frac{M}{2\pi a^2} \cdot a^2}$$

$$x = \frac{-\frac{M}{2\pi} \cdot \frac{a\sqrt{2}}{2}}{\frac{M}{2\pi}(2\pi - 1)} = \frac{-a\sqrt{2}}{2(2\pi - 1)}$$

$$x = \frac{-a\sqrt{2}}{2(2\pi - 1)} \Rightarrow x = \frac{-R}{2(2\pi - 1)}$$

i.e. $k = 2$

2. $\overline{BC} = vi$

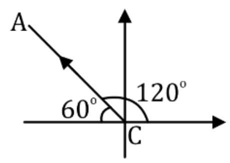


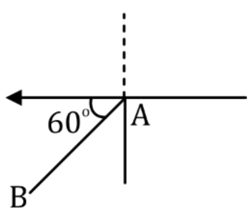
$AC = v(-i)\cos\ 60 + v\sin\ 60(j)$

$$\overline{AC} = \frac{-vi}{2} + \frac{v\sqrt{3}}{2}j$$

$AB = -i\ v\cos 60 - j\ v\sin 60°$

$\overline{AB} = -\frac{vi}{2} - \frac{vj\sqrt{3}}{2}$





velocity of the COM.

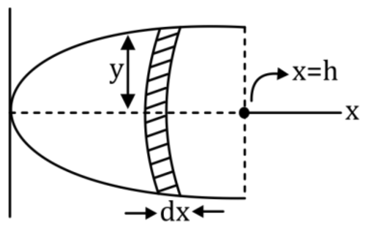
$v_{com} = \frac{m[AB+BC+CA]}{3m}$

$= \frac{\frac{-Vi}{2} - \frac{v\sqrt{3}}{2}j + vi - \frac{vi}{2} + \frac{v\sqrt{3}}{2}j}{3}$

$V_{com} = 0.$

so $\gamma - 1 = 0 \Rightarrow \gamma = 1$ Answer.

3. Volume density = ρ says.



$dm = \pi y^2 \cdot dx \cdot \rho$

$dm = \rho\pi y^2 dx$ _____(1)

$\because x = ky^2$

$y^2 = \frac{x}{k}$ _____(2)

$dm = \frac{\rho\pi x}{k} dx$ _____(3)

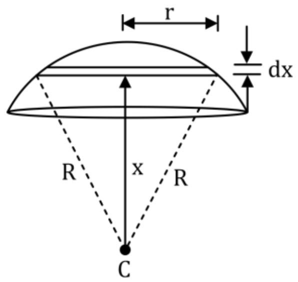
$$x_{com} = \frac{\int x \cdot dm}{\int dm} = \frac{\int_0^h \frac{\rho\pi x^2}{k} dx}{\int_0^h \frac{\rho\pi x}{k} dx}$$

$$x_{com} = \frac{\frac{\rho\pi}{k}\left[\frac{x^3}{3}\right]_0^h}{\frac{\rho\pi}{K}\left[\frac{x^2}{2}\right]_0^h} = \frac{h^3}{3} \times \frac{2}{h^2}$$

$x_{com} = \frac{2h}{3}$ so $\alpha + \beta = 2 + 3 = 5.$

$\alpha + \beta = 5$

4. To find centre of mass of object A, we consider a small elemental disc of width dx at a distance x from the centre C as shown in Figure.



Radius r of this elemental disc is

$$r = \sqrt{R^2 - x^2}$$

If ρ be the density of the sphere, then mass of the elemental disc will be

$dm = \rho dV = \rho(\pi r^2 dx) = \pi\rho(R^2 - x^2)dx$

Mass of object A can be obtained as

$$M = \int_{R/2}^{R} dm = \pi\rho \int_{R/2}^{R} (R^2 - x^2)dx$$

$$\Rightarrow\ M = \pi\rho\left(R^2x - \frac{x^3}{3}\right)\Big|_{R/2}^{R}$$

$$\Rightarrow\ M = \pi\rho\left[\left(R^3 - \frac{R^3}{3}\right) - \left(\frac{R^3}{2} - \frac{R^3}{24}\right)\right] = \frac{5}{24}\pi\rho R^3$$

Since, $y_{cm} = \frac{1}{M}\int_{R/2}^{R} xdm$

$$\Rightarrow\ y_{cm} = \frac{24}{5\rho\pi R^3}\int_{R/2}^{R}\rho\pi(R^2 - x^2)xdx$$

$$\Rightarrow\ y_{cm} = \frac{24}{5R^3}\left(\frac{R^2x^2}{2} - \frac{x^4}{4}\right)\Big|_{R/2}^{R}$$

$$\Rightarrow y_{cm} = \frac{12}{5R^3}\left[\left(R^4 - \frac{R^4}{2}\right) - \left(\frac{R^4}{4} - \frac{R^4}{32}\right)\right]$$

$$\Rightarrow y_{cm} = \frac{27}{40}R$$

5. $a_x = \frac{g}{2}i,\ a_y = -\frac{g}{2}j.$

(a) $r_{com} = -\frac{l}{2}i - \frac{l}{2}j = \bar{r}_i$

(b) V_{com} at $t = 0$ i.e. at initial position.

(c) $v_{cm} = \frac{dr_{cm}}{dt} = \frac{gt}{4}i - \frac{gt}{4}j$

(d) $a_{com} = \frac{\frac{mg}{2}i - \frac{mg}{2}j}{2m} = \frac{g}{4}i - \frac{g}{4}j$

(e) $\because\ s = \bar{r}_f - \bar{r}_i = \frac{1}{2}at^2$

$$\bar{r}_f - \bar{r}_i = \frac{1}{2}\cdot\left(\frac{g}{4}i - \frac{g}{4}j\right)t^2$$

$$r_f - r_i = \frac{gt^2}{8}i - \frac{gt^2}{8}j$$

$$r_f = \left(-\frac{l}{2} + \frac{gt^2}{8}\right)i - \left(\frac{gt^2}{8} + \frac{l}{2}\right)j$$

$$r_f = \left(\frac{gt^2}{8} - \frac{l}{2}\right)i - \left(\frac{gt^2}{8} + \frac{l}{2}\right)j$$

(f) Trajectory

$$x + y = \frac{gt^2}{8} - \frac{l}{2} - \frac{gt^2}{8} - \frac{l}{2}$$

$$x + y = -l$$

$$x + y + l = 0$$

6. (a) If v_{com} and a_{com} are parellel (or) antiparellel then motion will be linear (or) straight line motion

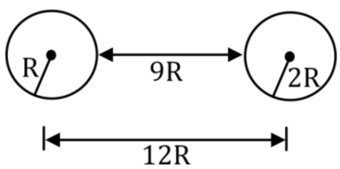
(b) If v_{com} and a_{com} have angle other than 0 (or) 180° then it will be parabolic.

7. say x_1 and x_2 distance covered by both bodies then $x_1 + x_2 = 9R$ ____(I)

$$M_1x_1 = M_2x_2$$

$$Mx_1 = 5Mx_2$$

$$x_2 = \frac{x_1}{5} \qquad \text{____}\mathbf{(II)}$$



from (I) and (II)

$$x + \frac{x_1}{5} = 9R$$

$$6x_1 = 45R$$

$$x_1 = \frac{15}{2}R$$

$$x_1 = 7.5R$$

8. If bigger block moves toward right by distance x then smaller block will move towards left by a distance $(2.2 - x)$ COM of system will be same

i.e. $\sum m_i x_{icom} = 0$

$$M(2.2 - x) = 10Mx$$

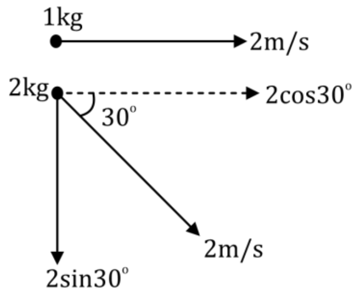
Nishant Jindal

$2.2 - x = 10x$

$x = 0.2$

$10x = 2$ meter

9. $v_{cm\,x} = \left(\frac{m_1 v_{1x} + m_2 v_{2x}}{m_1 + m_2}\right) i$



$$v_{cm\,x} = \left(\frac{2}{3} + \frac{2\sqrt{3}}{3}\right) i$$

$$v_{cmy} = \frac{m_1 v_{1y} + m_2 v_{2y}}{m_1 + m_2} = \frac{1 \times 0 + 2 \times 2\sin\ 30(-j)}{1 + 2}$$

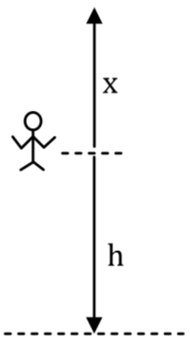
$$v_{cm\,y} = \frac{-2j}{3}$$

$V_{cm} = v_{cmx} + v_{cmy}$

$$v_{cm} = \left(\frac{2 + 2\sqrt{3}}{3}\right) i - \frac{2}{3} j$$

10. No external force is present.

$Mx + m(-h) = 0.$



$x = \frac{mh}{M}$

find distance $= h + x$

$$= h + \frac{mh}{M}$$

$= h\left(1 + \frac{m}{M}\right)$ Answer