QUESTIONS OF DOMAIN

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1. The domain of the function $f(x) = \frac{1}{\sqrt{{}^{10}C_{x-1} - 3 \times {}^{10}C_x}}$ is

(A) $\{9, 10, 11\}$ (B) $\{9, 10, 12\}$

(C) all natural numbers (D) $\{9,10\}$

Ans. **(D)**

Sol. Given function is defined if ${}^{10}C_{x-1} > 3\,{}^{10}C_x$ or $\frac{1}{11-x} > \frac{3}{x}$ or $4x > 33$ or $x \geq 9$

But $x \leq 10$

$\therefore\ x = 9,10$

2. The domain of the function $f(x) = \frac{\sin^{-1}(3-x)}{\ln(|x|-2)}$ is

(A) $[2, 4]$ (B) $(2, 3) \cup (3, 4]$ (C) $[2, \infty)$ (D) $(-\infty, -3) \cup [2, \infty)$

Ans. **(B)**

Sol. $f(x) = \frac{\sin^{-1}(3-x)}{\log(|x|-2)}$

Let $g(x) = \sin^{-1}(3-x)$
or $-1 \leq 3 - x \leq 1$
The domain of $g(x)$ is $[2,4]$.
Let $h(x) = \log(|x| - 2)$
i.e., $|x| - 2 > 0$ or $|x| > 2$
i.e., $x < -2$ or $x > 2$
$\therefore$ Domain $(-\infty, -2) \cup (2, \infty)$
We know that

$$(fg)(x) = \frac{f(x)}{g(x)} \forall x \in D_1 \cap D_2 - \{x \in R: g(x) = 0\}$$

Therefore, the domain of $f(x)$ is $(2,4] - \{3\} = (2,3) \cup (3,4]$.

3. The domain of $f(x) = \frac{\log_2(x+3)}{x^2+3x+2}$ is

(A) $R - \{-1, -2\}$ (B) $(-2, \infty)$

(C) $R - \{-1, -2, -3\}$ (D) $(-3, \infty) - \{-1, -2\}$

Ans. **(D)**

Sol. Here, $x + 3 > 0$ and $x^2 + 3x + 2 \neq 0$.
Therefore, $x > -3$ and $(x + 1)(x + 2) \neq 0$, i.e., $x \neq -1, -2$.
Therefore, the domain is $(-3, \infty) - \{-1, -2\}$.

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4. The domain of the function $f(x) = \sqrt{x^2 - [x]^2}$, where $[x]$ is the greatest integer less than or equal to x, is

(A) R (B) $[0, +\infty)$ (C) $(-\infty, 0]$ (D) none of these

Ans. **(D)**

Sol. $x^2 - [x]^2 \geq 0$ or $x^2 \geq [x]^2$

This is true for all non-negative values of x and all negative integers x.

5. The domain of the function $f(x) = \log_{3+x}(x^2 - 1)$ is

(A) $(-3, -1) \cup (1, \infty)$ (B) $[-3, -1) \cup [1, \infty)$

(C) $(-3, -2) \cup (-2, -1) \cup (1, \infty)$ (D) $[-3, -2) \cup (-2, -1) \cup [1, \infty)$

Ans. **(C)**

Sol. f(x) is to be defined when $x^2 - 1 > 0$ and $3 + x > 0$ and $3 + x \neq 1$, i.e.,
$x^2 > 1$ and $x > -3$ and $x \neq -2$,
i.e., $x < -1$ or $x > 1$ and $x > -3$ and $x \neq -2$

$\therefore D_f = (-3, -2) \cup (-2, -1) \cup (1, \infty)$

6. The domain of the function $f(x) = \left[\log_{10}\left(\frac{5x-x^2}{4}\right)\right]^{1/2}$ is

(A) $-\infty < x < \infty$ (B) $1 \leq x \leq 4$ (C) $4 \leq x \leq 16$ (D) $-1 \leq x \leq 1$

Ans. **(B)**

Sol. We have $f(x) = \left[\log_{10}\left(\frac{5x-x^2}{4}\right)\right]^{1/2}$
From (1), clearly, f(x) is defined for those values of x for which

$$\log_{10}\left[\frac{5x - x^2}{4}\right] \geq 0$$

or $\left(\frac{5x-x^2}{4}\right) \geq 10^0$
or $\left(\frac{5x-x^2}{4}\right) \geq 1$
or $x^2 - 5x + 4 \leq 0$
or $(x - 1)(x - 4) \leq 0$
Hence, the domain of the function is [1,4].

7. The domain of $f(x) = \log|\log x|$ is

(A) $(0, \infty)$ (B) $(1, \infty)$ (C) $(0,1) \cup (1, \infty)$ (D) $(-\infty, 1)$

Ans. **(C)**

Sol. $f(x) = \log\log x\,|\cdot$ f(x) is defined if $|\log x| > 0$ and $x > 0$, i.e., if $x > 0$ and $x \neq 1$

$(\because |\log x| > 0$ if $x \neq 1)$ or $x \in (0,1) \cup (1, \infty)$

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8. If $x^3 f(x) = \sqrt{1+\cos 2x} + |f(x)|, \frac{-3\pi}{4} < x < \frac{-\pi}{2}$ and $f(x) = \frac{\alpha \cos x}{1+x^3}$, then the value of α is

(A) 2 (B) $-\sqrt{2}$ (C) $\sqrt{2}$ (D) 1

Ans. **(B)**

Sol. $x^3 f(x) = \sqrt{1+\cos 2x} + |f(x)| = \sqrt{2}|\cos x| + |f(x)|$

Since R.H.S is positive, L.H.S must be positive.

So, $x^3 f(x) > 0$

$\Rightarrow f(x) < 0$

$\Rightarrow f(x) < 0 \left(\because \frac{-3\pi}{4} < x < \frac{-\pi}{2} \Rightarrow x^3 < 0\right)$

$\therefore x^3 f(x) = -\sqrt{2}\cos x - f(x)$

$\Rightarrow f(x) = \frac{-\sqrt{2}\cos x}{1+x^3}$

9. The domain of definition of the function f(x) given by the equation $2^x + 2^y = 2$ is

(A) $0 < x \le 1$ (B) $0 \le x \le 1$ (C) $-\infty < x \le 0$ (D) $-\infty < x < 1$

Ans. **(D)**

Sol. It is given that $2^x + 2^y = 2 \forall x, y \in R$
or $2^y = 2 - 2^x$
or $y = \log_2 (2 - 2^x)$
Therefore, function is defined only when $2 - 2^x > 0$ or $2^x < 2$ or $x < 1$.

10. The domain of $f(x) = \cos^{-1}\left(\frac{2-|x|}{4}\right) + [\log(3-x)]^{-1}$ is

(A) $[-2,6]$ (B) $[-6,2) \cup (2,3)$ (C) $[-6,2]$ (D) $[-2,2] \cup (2,3)$

Ans. **(B)**

Sol. $\cos^{-1}\left(\frac{2-|x|}{4}\right)$ exists if

$-1 \le \frac{2-|x|}{4} \le 1$

or $-6 \le -|x| \le 2$

or $-2 \le |x| \le 6$

or $|x| \le 6$

or $-6 \le x \le 6$

The function $[\log(3-x)]^{-1} = \frac{1}{\log(3-x)}$ is defined if $3 - x > 0$ and $x \ne 2$, i.e., if $x \ne 2$ and $x < 3$.
Thus, the domain of the given function is

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$\{x \mid -6 \leq x \leq 6\} \cap \{x \mid x \neq 2, x < 3\} = [-6,2) \cup (2,3)$

11. The domain of the function $f(x) = \sqrt{\log\left(\frac{1}{|\sin x|}\right)}$

(A) $R - \{-\pi, \pi\}$ (B) $R - \{n\pi \mid n \in Z\}$ (C) $R - \{2n\pi \mid n \in z\}$ (D) $(-\infty, \infty)$

Ans. **(B)**

Sol. f(x) is defined for

$$\log\left(\frac{1}{|\sin x|}\right) \geq 0$$

or $\frac{1}{|\sin x|} \geq 1$ and $|\sin x| \neq 0$

or $|\sin x| \neq 0$

or $x \neq n\pi, n \in Z$ $\left[\because \frac{1}{|\sin x|} \geq 1 \text{ for all } x\right]$

Hence, the domain of f(x) is $R - \{n\pi, n \in Z\}$.

12. The domain of the following function is $f(x) = \log_2\left(-\log_{1/2}\left(1 + \frac{1}{x^{1/4}}\right) - 1\right)$

(A) $(0,1)$ (B) $(0,1]$ (C) $[1, \infty)$ (D) $(1, \infty)$

Ans. **(A)**

Sol. f(x) is defined if

$$-\log_{1/2}\left(1 + \frac{1}{x^{1/4}}\right) - 1 > 0$$

or $\log_{1/2}\left(1 + \frac{1}{x^{1/4}}\right) < -1$

or $1 + \frac{1}{x^{1/4}} > \left(\frac{1}{2}\right)^{-1}$

or $\frac{1}{x^{1/4}} > 1$ or $0 < x < 1$

13. The number of real solutions of the $\log_{0.5}|x| = 2|x|$ is

(A) 1 (B) 2 (C) 0 (D) none of these

Ans. **(B)**

Sol. Draw the graph of $y = \log_{0.5}|x|$ and $y = 2|x|$.

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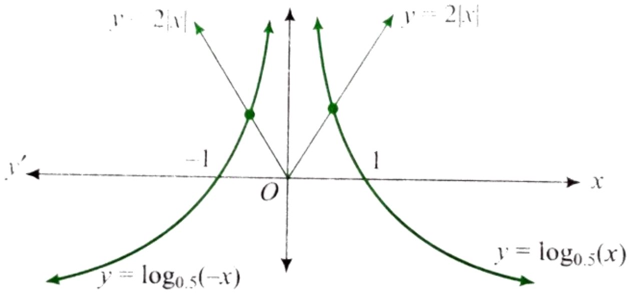

Clearly, from the graph, there are two solutions.

14. The domain of the function $f(x) = \sqrt{\ln_{(|x|-1)}(x^2+4x+4)}$ is

(A) $[-3,-1] \cup [1,2]$ (B) $(-2,-1) \cup [2,\infty)$

(C) $(-\infty,-3] \cup (-2,-1) \cup (2,\infty)$ (D) none of these

Ans. **(C)**

Sol. Case I:

$0 < |x| - 1 < 1$ or $1 < |x| < 2$

Then $x^2 + 4x + 4 \le 1$
or $x^2 + 4x + 3 \le 0$
or $-3 \le x \le -1$
So, $x \in (-2,-1)$
Case II:

$|x| - 1 > 1$ or $|x| > 2$

Then $x^2 + 4x + 4 \ge 1$
or $x^2 + 4x + 3 \ge 0$
or $x \ge -1$ or $x \le -3$
So, $x \in (-\infty,-3] \cup (2,\infty)$
From (1) and (2), $x \in (-\infty,-3] \cup (-2,-1) \cup (2,\infty)$.

15. The domain of $f(x) = \ln(ax^3 + (a+b)x^2 + (b+c)x + c)$, where $a > 0, b^2 - 4ac = 0$, is (where $[\cdot]$represents greatest integer function)

(A) $(-1,\infty) \sim \left\{-\frac{b}{2a}\right\}$ (B) $(1,\infty) \sim \left\{-\frac{b}{2a}\right\}$

(C) $(-1,1) \sim \left\{-\frac{b}{2a}\right\}$ (D) none of these

Ans. **(A)**

Sol. We must have $ax^3 + (a+b)x^2 + (b+c)x + c > 0$

or $ax^2(x+1) + bx(x+1) + c(x+1) > 0$

or $(x+1)(ax^2 + bx + c) > 0$

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or $a(x+1)\left(x+\frac{b}{2a}\right)^2 > 0$ as $b^2 = 4ac$

or $x > -1$ and $x \neq -\frac{b}{2a}$

16. The domain of the function $f(x) = \frac{1}{\sqrt{4x-|x^2-10x+9|}}$ is

(A) $(7-\sqrt{40}, 7+\sqrt{40})$ (B) $(0, 7+\sqrt{40})$ (C) $(7-\sqrt{40}, \infty)$ (D) none of these

Ans. **(D)**

Sol. $f(x) = \frac{1}{\sqrt{4x-|x^2-10x+9|}}$

For $f(x)$ to be defined, $|x^2-10x+9| < 4x$

or $x^2-10x+9 < 4x$ and $x^2-10x+9 > -4x$

or $x^2-14x+9 < 0$ and $x^2-6x+9 > 0$

or $x \in (7-\sqrt{40}, 7+\sqrt{40})$ and $x \in R-\{3\}$

or$x \in (7-\sqrt{40}, 3) \cup (3, 7+\sqrt{40})$

17. The exhaustive domain of the following function is $f(x) = \sqrt{x^{12}-x^9+x^4-x+1}$

(A) $[0,1]$ (B) $[1,\infty)$ (C) $(-\infty, 1]$ (D) R

Ans. **(D)**

Sol. $f(x) = \sqrt{x^{12}-x^9+x^4-x+1}$
We must have $x^{12}-x^9+x^4-x+1 \geq 0$
Obviously, (1) is satisfied by $x \in (-\infty, 0]$.
Also, $x^9(x^3-1)+x(x^3-1)+1 \geq 0 \forall x \in [1,\infty)$.
Further, $x^{12}-x^9+x^4-x+1 = (1-x)+x^4(1-x^5)+x^{12}$ is also satisfied by $x \in (0,1)$.
Hence, the domain is R.

18. The domain of the function $f(x) = \sqrt{\sin x + \cos x} + \sqrt{7x-x^2-6}$ is

(A) $[1,6]$ (B) $\left[1, \frac{3\pi}{4}\right] \cup \left[\frac{7\pi}{4}, 6\right]$ (C) $[1,\pi] \cup \left[\frac{7\pi}{4}, 6\right]$ (D) none of these

Ans. **(B)**

Sol. We have $f(x) = \sqrt{\sin x + \cos x} + \sqrt{7x-x^2-6}$

$= \sqrt{\sqrt{2}\sin\left(\frac{\pi}{4}+x\right)} + \sqrt{7x-x^2-6}$

$f(x)$ is defined if
(i) $7x-x^2-6 \geq 0$

$\Rightarrow 1 \leq x \leq 6$

(ii) $\sin\left(\frac{\pi}{4}+x\right) \geq 0$

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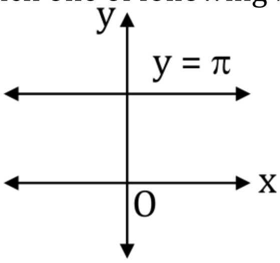
$$x+\frac{\pi}{4}\in\cdots[-2\pi,-\pi]\cup[0,\pi]\cup[2\pi,3\pi]\ldots$$

$$\Rightarrow\ x\in\cdots\left[-\frac{\pi}{4},\frac{3\pi}{4}\right]\cup\left[\frac{7\pi}{4},\frac{11\pi}{4}\right]\ldots$$

From (1) and (2), we get

$$x\in\left[1,\frac{3\pi}{4}\right]\cup\left[\frac{7\pi}{4},6\right]$$

19. Which one of following best represent the graph of $y=x^{\log_x \pi}$?

(A)


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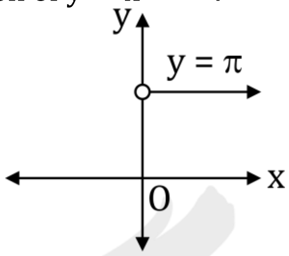

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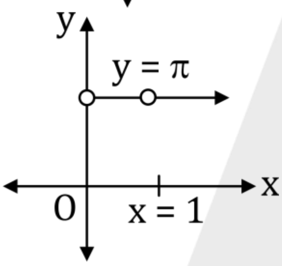

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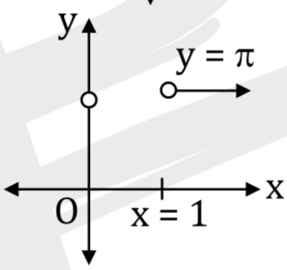

Ans. **(C)**

Sol. Given, $y=x^{\log_x \pi}=\pi$

Domain is $x\in(0,1)\cup(1,\infty)$

Range is $\{\pi\}$