Nishant Jindal

Q Evaluate

$$\begin{vmatrix} 3 & -1 & -2 \\ 0 & 0 & -1 \\ 3 & -5 & 0 \end{vmatrix}$$

$$(0+0+3)-(0+15+0)$$

$$=-12$$

(B) $$\begin{vmatrix} 3 & -4 & 5 \\ 1 & 1 & -2 \\ 2 & 3 & 1 \end{vmatrix}$$

$$(3+16+15)-(10+-18+-4)$$

$$34+12=46$$

Q. $A = \begin{vmatrix} 2 & b & 1 \\ b & b^2+1 & b \\ 1 & b & 2 \end{vmatrix}$; $b>0$ then

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min value of $\dfrac{\det(A)}{b} = ?$

$$\begin{vmatrix} 2 & b & 1 \\ b & b^2+1 & b \\ 1 & b & 2 \end{vmatrix} \qquad \begin{vmatrix} 2 & b & 1 \\ b & b^2+1 & b \\ 1 & b & 2 \end{vmatrix}$$

$$\left(4(b^2+1)+b^2+b^2\right) - \left(b^2+1+2b^2+2b^2\right)$$

$$\det(A) = b^2+3$$

Now $\dfrac{\det(A)}{b} = \dfrac{b^2+3}{b} = b+\dfrac{3}{b}$

AM $\geq$ GM

$$\frac{b+\frac{3}{b}}{2} \geq \sqrt{b \times \frac{3}{\cancel{b}}}$$

$$b+\frac{3}{b} \geq 2\sqrt{3}$$

$\therefore$ Min $= 2\sqrt{3}$.

$$Z = b+\frac{3}{b}$$

$$\frac{dZ}{db} = 1-\frac{3}{b^2} = 0$$

$$\frac{3}{b^2} = 1 \Rightarrow b = \pm\sqrt{3}$$

$$Z = \sqrt{3} + \frac{\cancel{3}\,\sqrt{3}}{\cancel{\sqrt{3}}} = \underline{\underline{2\sqrt{3}}}$$

Q In $\triangle ABC$ if $\begin{vmatrix} 1 & a & b \\ 1 & c & a \\ 1 & b & c \end{vmatrix} = 0$ then

$\sin^2 A + \sin^2 B + \sin^2 C = ?$

$\sin^2 60 + \sin^2 60 + \sin^2 60$

$= \frac{3}{4} + \frac{3}{4} + \frac{3}{4} = \frac{9}{4}$

$$\begin{vmatrix} 1 & a & b \\ 1 & c & a \\ 1 & b & c \end{vmatrix} \qquad \begin{vmatrix} 1 & a & b \\ 1 & c & a \\ 1 & b & c \end{vmatrix}$$

$$\Delta = (c^2 + a^2 + b^2) - (bc + ab + ac) = 0$$

$$\frac{1}{2}\left\{(a^2 - 2ab + b^2) + (a^2 - 2ac + c^2) + b^2 - 2bc + c^2\right\} = 0$$

$$\frac{1}{2}\left\{(a-b)^2 + (a-c)^2 + (b-c)^2\right\} = 0$$

$a - b = 0$ & $a - c = 0$ & $b - c = 0 \Rightarrow a = b = c \Rightarrow A = B = C = 60^\circ$

Q

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$$\Delta_1 = \begin{vmatrix} x & \sin\theta & \cos\theta \\ -\sin\theta & -x & 1 \\ \cos\theta & 1 & x \end{vmatrix} \qquad \Delta_2 = \begin{vmatrix} x & \sin 2\theta & \cos 2\theta \\ -\sin 2\theta & -x & 1 \\ \cos 2\theta & 1 & x \end{vmatrix}$$

$x \neq 0$ then for all $\theta \in (0, \frac{\pi}{2})$

$\Delta_1 - \Delta_2 = x(\cos 2\theta - \cos 4\theta)$

$\Delta_1 + \Delta_2 = -2x^3$

$\Delta_1 + \Delta_2 = -2(x^3 + x - 1)$

$\Delta_1 - \Delta_2 = -2x^3$

$\Delta_1 = (-x^3 + \cancel{\sin\theta\cos\theta} + \cancel{-\sin\theta\cos\theta}) - (-x\cancel{\cos^2\theta} + \cancel{x} - \cancel{x\sin^2\theta})$

$= -x^3$

$\Delta_2 = (-x^3 + \cancel{\sin 2\theta\cos 2\theta} + \cancel{-\sin 2\theta\cos 2\theta}) - (-x\cos^2 2\theta + x + -x\sin^2 2\theta)$

$- (-x(\underbrace{\cos^2 2\theta + \sin^2 2\theta}) + x)$

$= (-\cancel{x} + \cancel{x})$

$= -x^3$

$\Delta_1 + \Delta_2 = -2x^3$

$$\Delta = \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} \quad \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}$$

$$(aei + gfb + cdh) - (gec + ahf + bdi)$$

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Value of Determinant in terms of Minor & Cofactors.

$$\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

① Sum of Prod of elements of a Row / Col. and Corresponding Cofactors of elements of Same Row / Col give value of Δ.

② Sum of Prod of elements of a Row / Col. and Corresponding Cofactors of elements of any other Row give value $= 0$

$\Delta = a_{11}M_{11} - a_{12}M_{12} + a_{13}M_{13}$ (Using 1st Row)

$= a_{31}M_{31} - a_{32}M_{32} + a_{33}M_{33}$ (Using 3rd Row)

$= -a_{12}M_{12} + a_{22}M_{22} - a_{32}M_{32}$ (Using 2nd Col.)

$$\boxed{\Delta = a_{11}C_{11} + a_{12}C_{12} + a_{13}C_{13}}$$

$\Delta = a_{31}C_{31} + a_{32}C_{32} + a_{33}C_{33}$.

$$\boxed{a_{11}C_{21} + a_{12}C_{22} + a_{13}C_{23} = 0}$$

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Q $\Delta = \begin{vmatrix} +p & q & r \\ -x & y & z \\ a & b & c \end{vmatrix}$

$-x M_{21} + y M_{22} - z M_{23} = \Delta$.

× A) $x M_{21} - y M_{22} + z M_{23} = \Delta$.

B) $a \boxed{C_{11}} + b \boxed{C_{12}} + c \boxed{C_{13}} = 0$ ✓

C) $x C_{21} - y C_{22} + z C_{23} = \Delta$ ⓧ

D) $+p M_{11} - q M_{12} + r M_{13} = \Delta$. ✓

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Q By Using elements 1 & -1 all Possible det. of 3^{rd} Order are formed. Find Max^m Value of Determinant?

$$\Delta = \left|\begin{matrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{matrix}\right| \begin{matrix} a_1 & a_2 \\ b_1 & b_2 \\ c_1 & c_2 \end{matrix}$$

$$\Delta = \underbrace{(a_1 b_2 c_3 + a_2 b_3 c_1 + a_3 b_1 c_2)}_{a_1 a_2 a_3\, b_1 b_2 b_3\, c_1 c_2 c_3} - \underbrace{(a_3 b_2 c_1 + a_1 b_3 c_2 + a_2 b_1 c_3)}_{a_1 a_2 a_3\, b_1 b_2 b_3\, c_1 c_2 c_3}$$

T.P.

$$\Delta = \underbrace{(1 + 1 + 1)}_{Prod = 1} - \underbrace{(-1 - 1 - 1)}_{Prod = -1} = 6$$

$$\Delta = \underbrace{(1 + 1 + 1)}_{Pr = 1} - \underbrace{(-1 - 1 + 1)}_{1} = 4 \text{ (Max)}$$

$Min^m = ?$

$$\Delta = \underbrace{(-1 - 1 - 1)}_{Pr = -1} - \underbrace{(1 + 1 + 1)}_{Pr = 1} \quad \boxed{\times}$$

$$\Delta = \underbrace{(-1 - 1 - 1)}_{Pr = -1} - \underbrace{(1 + 1 - 1)}_{Pr = -1} = -4 \text{ (Min)}$$

Properties of det.

① Determinant value of Upper Δ^r or Lower Δ^r matrix is equal to prod of diag. elements.

$$\Delta = \begin{vmatrix} a & b & c \\ 0 & d & e \\ 0 & 0 & f \end{vmatrix} \Rightarrow \Delta = adf$$

Upper Δ^r

(2) $|A^T| = |A|$

(3) If corresponding elements of 2 Rows / col. are same or Proportional then $\Delta = 0$

(4) all elements of any Row / Col $= 0$ then $\Delta = 0$

(5) If any 2 Rows are interchanged then value of det get multiplied to -1

(6) If elements of any Row / Col. are multiplied by a Constant then Δ is also multiplied to that constant.

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Proof of 6

$$\Delta_1 = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

$$\Delta = a_{11}M_{11} - a_{12}M_{12} + a_{13}M_{13}.$$

$$\Delta' = \begin{vmatrix} Ka_{11} & Ka_{12} & Ka_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

$$\Delta' = Ka_{11}M_{11} - Ka_{12}M_{12} + Ka_{13}M_{13}.$$

$$= K(a_{11}M_{11} - a_{12}M_{12} + a_{13}M_{13})$$

$$\Delta' = K\Delta$$

Q Value of $\Delta = \begin{vmatrix} 4 & 8 & 20 \\ 3 & 4 & 9 \\ 6 & 12 & 30 \end{vmatrix} = ?$

$\underbrace{4 \quad 8 \quad 20}$, $\underbrace{6 \quad 12 \quad 30} \rightarrow$ 6 Com

$$= 4 \times 6 \begin{vmatrix} 1 & 2 & 5 \\ \\ 1 & 2 & 5 \end{vmatrix}$$ Identical

$$= 0$$

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Q $\begin{vmatrix} ax & by & cz \\ x^2 & y^2 & z^2 \\ 1 & 1 & 1 \end{vmatrix} = \begin{vmatrix} a & b & c \\ x & y & z \\ yz & xz & xy \end{vmatrix}$

$\downarrow$ x Com $\quad\downarrow$ y Com $\quad\downarrow$ z Com.

$(xyz)\begin{vmatrix} a & b & c \\ x & y & z \\ 1/x & 1/y & 1/z \end{vmatrix}$

$\begin{vmatrix} a & b & c \\ x & y & z \\ yz & xz & xy \end{vmatrix} = \text{RHS}.$

Q $x, y, z > 0$

P.T.? $\begin{vmatrix} 1 & \log_x y & \log_x z \\ \log_y x & 1 & \log_y z \\ \log_z x & \log_z y & 1 \end{vmatrix} = ?$

$\Rightarrow \begin{vmatrix} \log_x x & \log_x y & \log_x z \\ \log_y x & \log_y y & \log_y z \\ \log_z x & \log_z y & \log_z z \end{vmatrix}$

$\log_a b = \dfrac{\log b}{\log a}$

$\Rightarrow \begin{vmatrix} \frac{\log x}{\log x} & \frac{\log y}{\log x} & \frac{\log z}{\log x} \\ \frac{\log x}{\log y} & \frac{\log y}{\log y} & \frac{\log z}{\log y} \\ \frac{\log x}{\log z} & \frac{\log y}{\log z} & \frac{\log z}{\log z} \end{vmatrix} = \dfrac{1}{\log x \log y \log z} \begin{vmatrix} \log x & \log y & \log z \\ \log x & \log y & \log z \\ \end{vmatrix}$

0

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$$\Delta_1 = \begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix} \quad \Delta_2 = \begin{vmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{vmatrix}$$

$$\frac{\Delta_1}{2\Delta_2} = ?$$

$$\Delta_2 = \begin{array}{c} \begin{vmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{vmatrix} \end{array} \begin{array}{l} \leftarrow a \\ \leftarrow b \\ \leftarrow c \end{array}$$

$$= \frac{1}{abc} \begin{vmatrix} a & a^2 & abc \\ b & b^2 & abc \\ c & c^2 & abc \end{vmatrix}$$

$$= \frac{\cancel{abc}}{\cancel{abc}} \begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ c & c^2 & 1 \end{vmatrix} = - \begin{vmatrix} a & 1 & a^2 \\ b & 1 & b^2 \\ c & 1 & c^2 \end{vmatrix} = \cancel{-} \times \cancel{-} \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$$

$$= \Delta_1$$

$\Delta_1 = \Delta_2$

$\frac{\Delta_1}{\Delta_2} = 1$

Ans $= \frac{1}{2}$

Nishant Jindal

Q Let $A=[a_{ij}]$ & $B=[b_{ij}]$ be two 3×3 Real Matrices such that $\boxed{b_{ij}=(3)^{i+j-2}a_{ji}}$; $i,j=1,2,3$. If determinant of B is 81 then $\det(A)=?$

Main 2020

$b_{11}=(3)^{1+1-2}a_{11}$

$b_{12}=(3)^{1+2-2}a_{21}$

$b_{13}=(3)^{1+3-2}a_{31}$

$b_{21}=(3)^{2+1-2}a_{12}$

$$B=\begin{vmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{vmatrix}$$

$$B=\begin{vmatrix} a_{11} & 3a_{21} & 9a_{31} \\ 3a_{12} & 9a_{22} & 27a_{32} \\ 9a_{13} & 27a_{23} & 81a_{33} \end{vmatrix} = 3\times9\begin{vmatrix} a_{11} & a_{21} & a_{31} \\ 3a_{12} & 3a_{22} & 3a_{32} \\ 9a_{13} & 9a_{23} & 9a_{33} \end{vmatrix} = 27\times27\begin{vmatrix} a_{11} & a_{21} & a_{31} \\ a_{12} & a_{22} & a_{32} \\ a_{13} & a_{23} & a_{33} \end{vmatrix}$$

$$=(27)^2\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

$(\cancel{27})^{\cancel{2}}\ 9\ \mathrm{Det}(A) = |B| = \cancel{81}\ 3$

$9|A| = 1 \Rightarrow \boxed{|A|=\frac{1}{9}}$

Nishant Jindal

Q let a, b, c be such that $b(c+a) \neq 0$

Mains

if $\begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix} + \begin{vmatrix} a+1 & b+1 & c-1 \\ a-1 & b-1 & c+1 \\ (-1)^{n+2}a & (-1)^{n+1}b & (-1)^{n}c \end{vmatrix} = 0$

then $n=$ A) Zero B) Even Int ~~C)~~ odd Int D) any Int

$$\begin{vmatrix} & & \\ & & \\ & & \end{vmatrix} + \begin{vmatrix} a+1 & a-1 & (-1)^{n+2}a \\ b+1 & b-1 & (-1)^{n+1}b \\ c-1 & c+1 & (-1)^{n}c \end{vmatrix}$$

$$\begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix} + \begin{vmatrix} (-1)^{n+2}a & a+1 & a-1 \\ (-1)^{n+1}b & b+1 & b-1 \\ (-1)^{n}c & c-1 & c+1 \end{vmatrix} = 0 \Rightarrow \begin{vmatrix} a+(-1)^{n+2}a & a+1 & a-1 \\ -b+(-1)^{n+1}b & b+1 & b-1 \\ c+(-1)^{n}c & c-1 & c+1 \end{vmatrix} = 0$$

$(-1)^{n+2}a \to -a$, $(-1)^{n+1}b \to +b$, $(-1)^{n}c \to -c$

Nishant Jindal

(7)* If each element of any Row / Col. can be expressed as a sum of 2 terms then det can also be expressed as sum of 2 det.

$$\begin{vmatrix} a+x & b+y & c+z \\ 2 & 3 & 4 \\ 5 & 6 & 7 \end{vmatrix} = \begin{vmatrix} a & b & c \\ 2 & 3 & 4 \\ 5 & 6 & 7 \end{vmatrix} + \begin{vmatrix} x & y & z \\ 2 & 3 & 4 \\ 5 & 6 & 7 \end{vmatrix}$$

Q $\begin{vmatrix} \sqrt{13}+\sqrt{3} & 2\sqrt{5} & \sqrt{5} \\ \sqrt{15}+\sqrt{26} & 5 & \sqrt{10} \\ 3+\sqrt{65} & \sqrt{15} & 5 \end{vmatrix} = 5\sqrt{3}\begin{vmatrix} 1 & 2 & 1 \\ \sqrt{5} & \sqrt{5} & \sqrt{2} \\ \sqrt{3} & \sqrt{3} & \sqrt{5} \end{vmatrix}$ (T/F)

$$= \begin{vmatrix} \sqrt{13} & 2\sqrt{5} & \sqrt{5} \\ \sqrt{26} & 5 & \sqrt{10} \\ \sqrt{65} & \sqrt{15} & 5 \end{vmatrix} + \begin{vmatrix} \sqrt{3} & 2\sqrt{5} & \sqrt{5} \\ \sqrt{15} & 5 & \sqrt{10} \\ 3 & \sqrt{15} & 5 \end{vmatrix}$$

$$\sqrt{13}\times\sqrt{5}\times\sqrt{5}\,\cancel{\begin{vmatrix} 1 & 2 & 1 \\ \sqrt{2} & \sqrt{5} & \sqrt{2} \\ \sqrt{5} & \sqrt{3} & \sqrt{5} \end{vmatrix}} + \sqrt{3}\sqrt{5}\sqrt{5}\begin{vmatrix} 1 & 2 & 1 \\ \sqrt{5} & \sqrt{5} & \sqrt{2} \\ \sqrt{3} & \sqrt{3} & \sqrt{5} \end{vmatrix} = T$$

0

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(8) Value of Determinant is unaltered by adding to elements of any Row (Col) with a constant multiple of corresponding elements of any other Row (Col.)

$$\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

$\boxed{\text{Matrix} \to \text{JA, JM}}$

$$R_1 \to R_1 + pR_3$$

$$\Delta' = \begin{vmatrix} a_{11}+pa_{31} & a_{12}+pa_{32} & a_{13}+pa_{33} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} + \begin{vmatrix} pa_{31} & pa_{32} & pa_{33} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

Prop.

$$\Delta + 0$$

(9) $|KA| = K^n |A|$

(10) $|A \cdot B| = |A| \cdot |B|$

(11) $|A^2| = |A|^2$

$|A^3| = |A|^3$