Nishant Jindal

Capacitor

(*) Correction :→

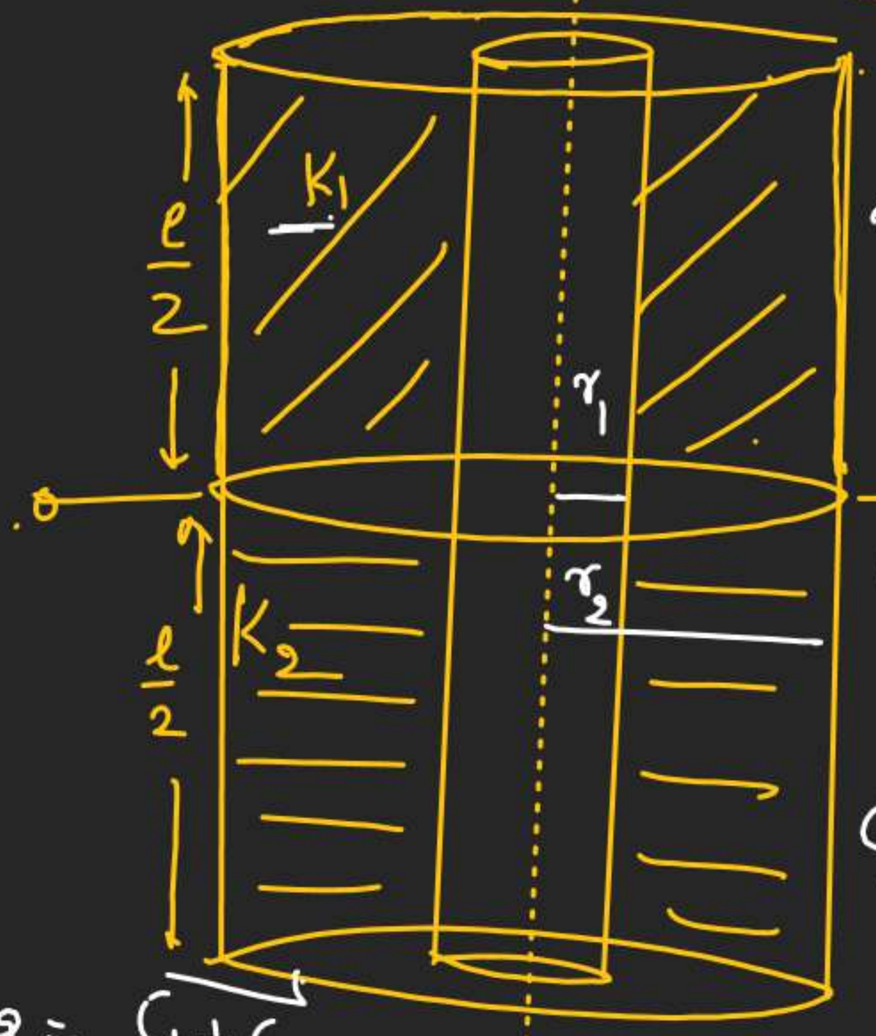
$$E = \frac{\lambda}{2\pi\varepsilon_0 r}$$

$$\lambda = \frac{Q}{l}.$$

**

$$\boxed{C = \frac{2\pi\varepsilon_0 l}{\ln\left(\frac{r_2}{r_1}\right)}}$$

⇒ $C = ??$



⇒

$$C_1 = \frac{2\pi K_1\varepsilon_0 (l/2)}{\ln\left(\frac{r_2}{r_1}\right)}$$

$$C_2 = \frac{2\pi K_2\varepsilon_0 (l/2)}{\ln\left(\frac{r_2}{r_1}\right)}$$

$$C_{eq} = C_1 + C_2 = \frac{\pi\varepsilon_0 l}{\ln\left(\frac{r_2}{r_1}\right)}(K_1+K_2)$$ ✓

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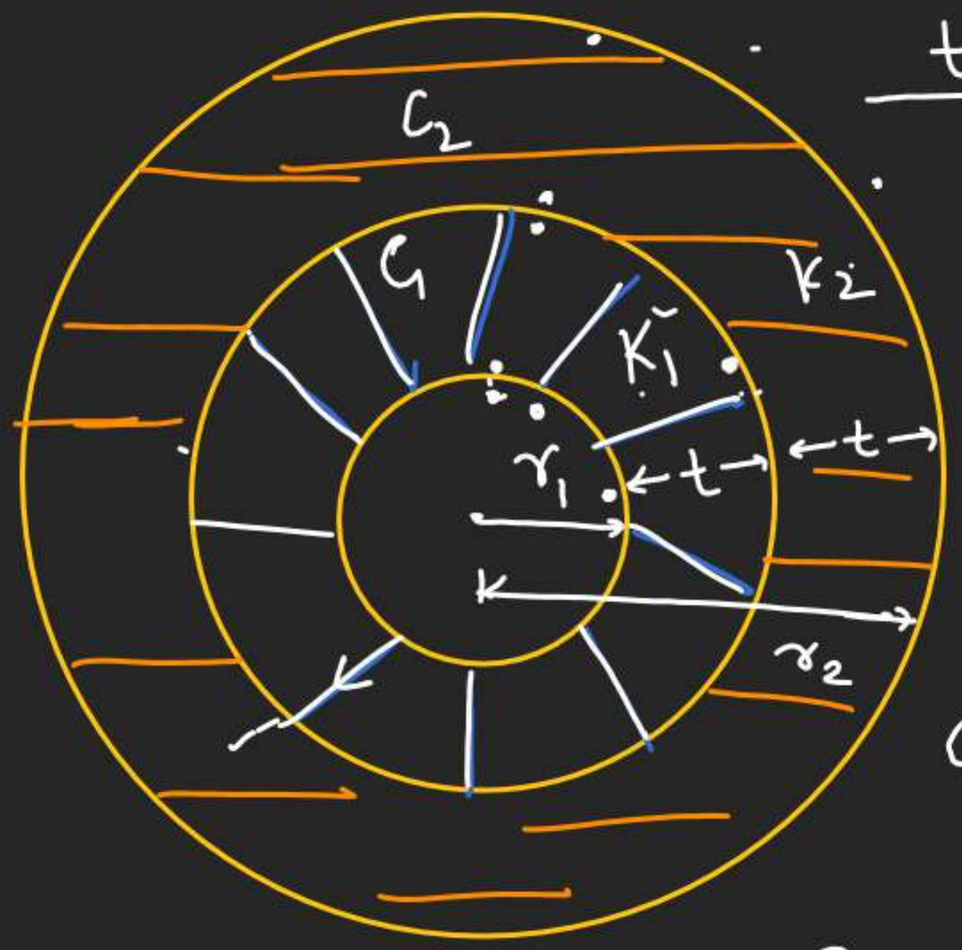
$$C = \frac{4\pi\varepsilon_0 r_1 r_2}{r_2 - r_1}$$



t = thickness.

Find Capacitance of the System

$r_2 = r_1 + 2t$

$2t = (r_2 - r_1)$

$t = \left(\frac{r_2 - r_1}{2}\right)$

$r_1 + t = \frac{r_2 - r_1}{2} + r_1$

$= \frac{(r_2 + r_1)}{2}$

$$C_1 = \frac{4\pi K_1 \varepsilon_0 r_1 (r_1 + t)}{[(r_1 + t) - r_1]}$$

$$C_1 = \frac{4\pi K_1 \varepsilon_0 r_1}{\frac{(r_2 - r_1)}{2}} \times \frac{(r_2 + r_1)}{2}$$

$$\boxed{C_1 = K_1 4\pi \varepsilon_0 r_1 \frac{(r_1 + r_2)}{(r_2 - r_1)}}$$

$$C_2 = \left[\frac{4\pi K_2 \varepsilon_0 (r_1 + t) r_2}{r_2 - (r_1 + t)}\right]$$

$$C_2 = \frac{4\pi K_2 \varepsilon_0 r_2 (r_1 + r_2)}{2\left[r_2 - \frac{(r_2 + r_1)}{2}\right]}$$

$$\boxed{C_2 = \frac{4\pi \varepsilon_0 K_2 r_2 (r_1 + r_2)}{(r_2 - r_1)}}$$

$C_{eq} = \frac{C_1 C_2}{C_1 + C_2}$, $\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$

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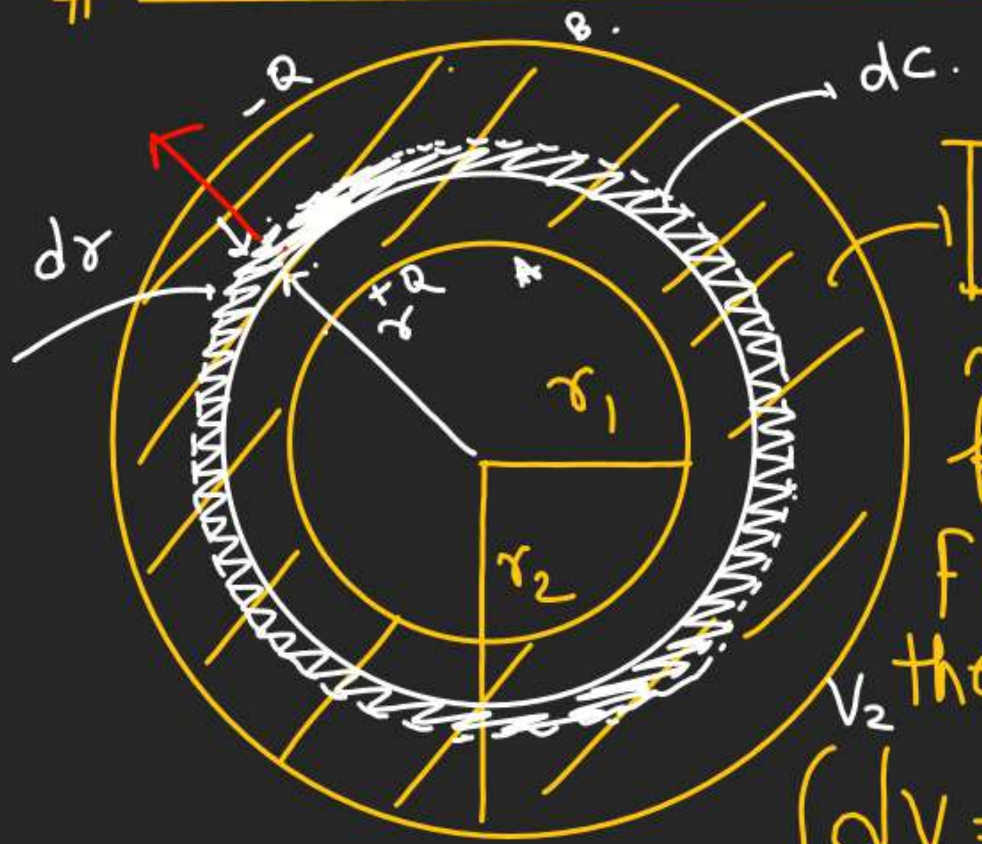
Case of Variable dielectric

Capacitor



$$\boxed{K = \frac{K_0}{r}}$$

$r \rightarrow$ radial distance from the Center.

Find Capacitance of the System.

$$E_{net} = \frac{E}{K_r} = \left(\frac{Q}{4\pi\varepsilon_0 r^2} \times \frac{1}{K_r}\right)$$

$$K_r = K_0 r$$

$$K_{r+dr} = K_0(r+dr)$$

$$\boxed{K_r \simeq K_{r+dr}}$$

As dr is very small.

$$\int_{V_A}^{V_2} dV = -\int E_r \cdot dr = -\int_{r_1}^{r_2} \frac{Q}{4\pi K_r \varepsilon_0} \times \frac{1}{r^2} dr$$

$$(V_2 - V_1) = \frac{-Q}{4\pi\varepsilon_0} \int_{r_1}^{r_2} \frac{dr}{K_r r^2}$$

$$= \frac{-Q}{4\pi\varepsilon_0} \int_{r_1}^{r_2} \frac{dr}{\frac{K_0}{r} \times r^2}$$

$$V_1 - V_2 = \frac{Q}{4\pi\varepsilon_0 K_0} \int_{r_1}^{r_2} \frac{dr}{r}$$

$$V_1 - V_2 = \frac{Q}{4\pi\varepsilon_0 K_0} \ln\left(\frac{r_2}{r_1}\right) \Rightarrow C = \frac{4\pi\varepsilon_0 K_0}{\ln(r_2/r_1)}$$

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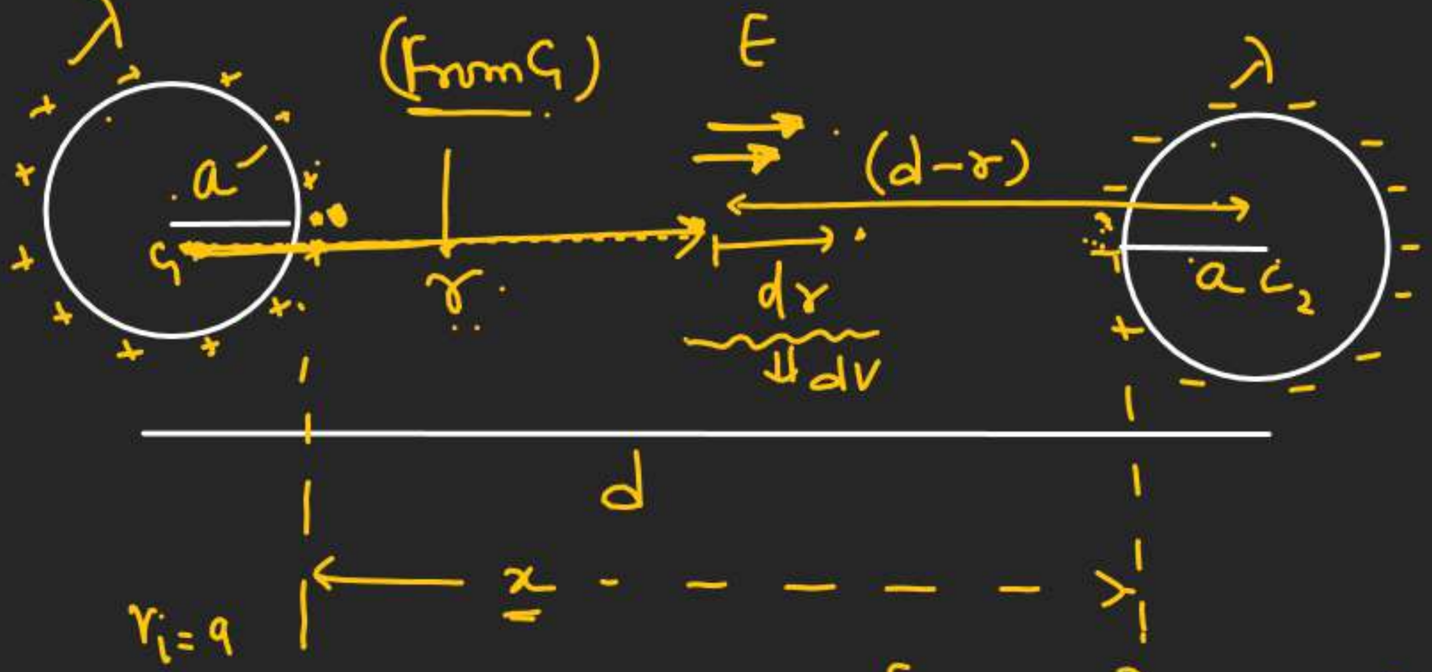
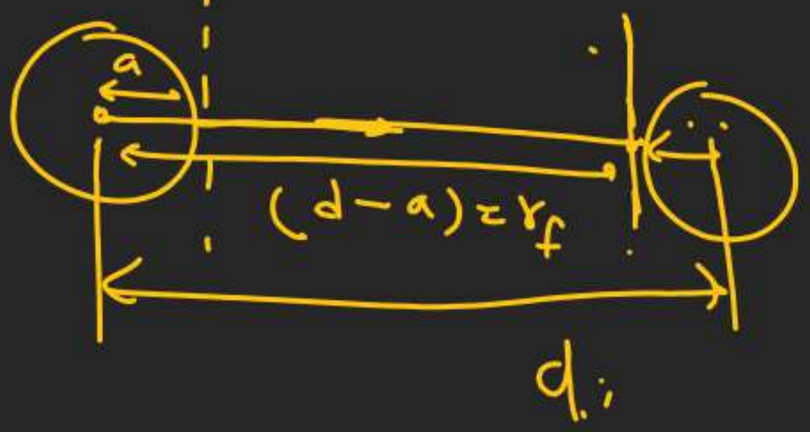
Capacitor

Capacitance b/w two parallel very long thin wire per unit length :→

$\boxed{d>>a}$





$$E_r = \frac{\lambda}{2\pi\varepsilon_0 r} + \frac{\lambda}{2\pi\varepsilon_0 (d-r)}$$

$$= \frac{\lambda}{2\pi\varepsilon_0}\left[\frac{1}{r} + \frac{1}{(d-r)}\right]$$

$$\int \frac{dx}{a+bx} = \frac{\ln(a+bx)}{(b)}$$

$$\int_{V_1}^{V_2} dV = -\int_{a}^{(d-a)} E_r \cdot dr$$

$$(V_2 - V_1) = \frac{-\lambda}{2\pi\varepsilon_0}\left[\int_{a}^{(d-a)} \frac{1}{r}\,dr + \int_{a}^{(d-a)} \frac{dr}{d-r}\right]$$

$$(V_1 - V_2) = \frac{\lambda}{2\pi\varepsilon_0}\left[\ln[r]_a^{d-a} - \ln(d-r]_a^{d-a}\right]$$

$$= \frac{\lambda}{2\pi\varepsilon_0}\left[\ln\left(\frac{d-a}{a}\right) - \ln\left(\frac{a}{d-a}\right)\right]$$

$$= \frac{\lambda}{2\pi\varepsilon_0}\ln\left(\frac{d-a}{a}\right)^2 = \frac{\lambda}{\pi\varepsilon_0}\ln\left(\frac{d-a}{a}\right)$$

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$$V_1 - V_2 = \frac{\lambda}{\pi\varepsilon_0}\ln\left(\frac{d-a}{a}\right)$$

$q = CV.$

$$\lambda = \left[\frac{\pi\varepsilon_0}{\ln\left(\frac{d-a}{a}\right)}\right](V_1 - V_2)$$

$\lambda \Downarrow$ Charge per Unit length

$\Downarrow$ Capacitace per Unit length.

$q = \frac{\lambda}{l} \rightarrow 1\text{unit}$

$d >> a$

$d - a \simeq d.$

$$\boxed{C_{\substack{\text{per}\\ \text{unit length}}} = \frac{\pi\varepsilon_0}{\ln\left(\frac{d-a}{a}\right)}} \Rightarrow C_{\substack{\text{per}\\ \text{Unit length}}} = \frac{\pi\varepsilon_0}{\ln(d/a)}$$

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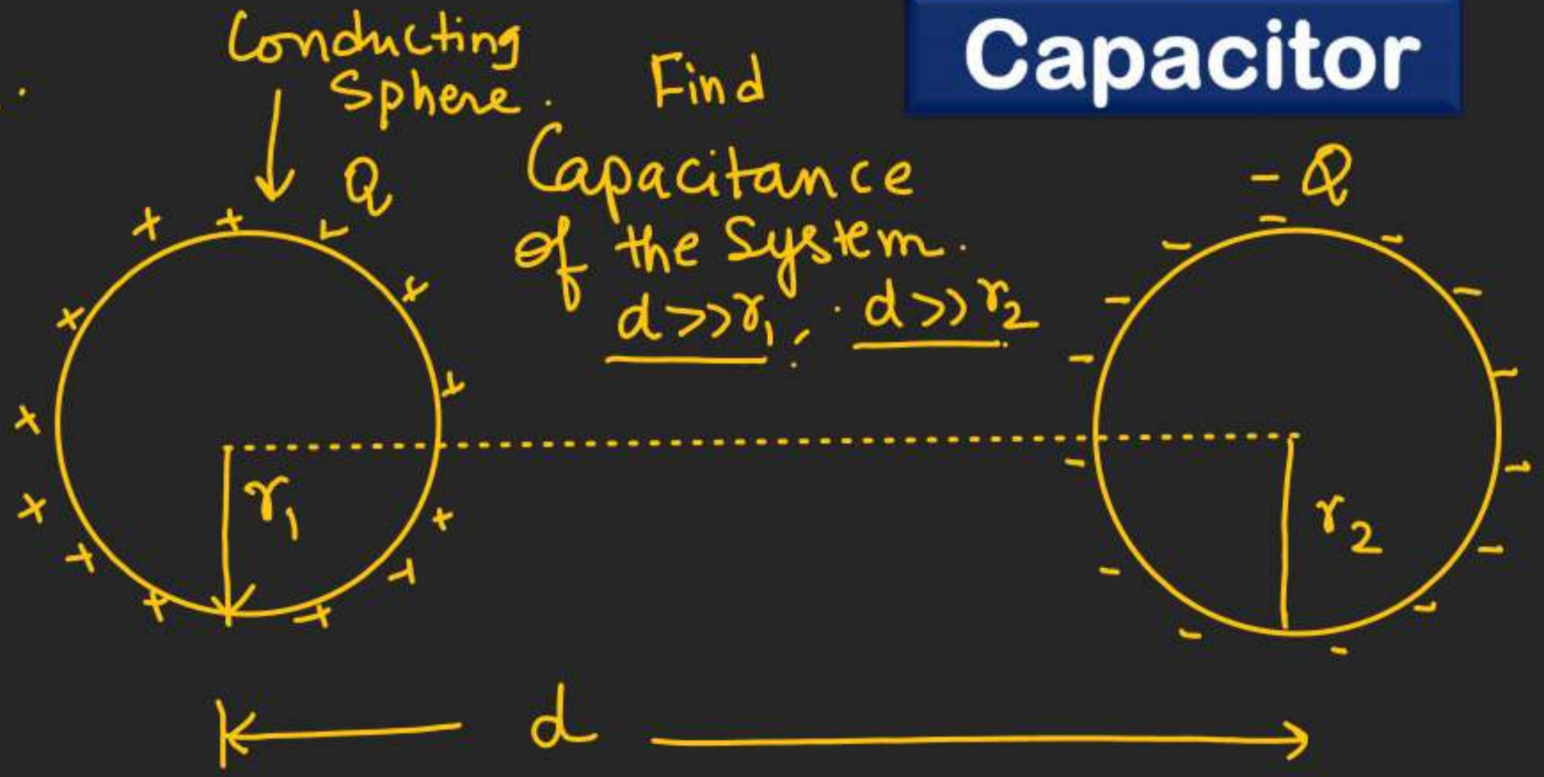
Capacitor

H.W.

Find Capacitance of the System.

$d \gg r_1$; $d \gg r_2$



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Capacitor

(4)

Energy Stored in a Conductor/ Capacitor: →

$\left[\frac{U}{q} = V\right]$

$\left(\frac{dU}{dq} = dV\right)$

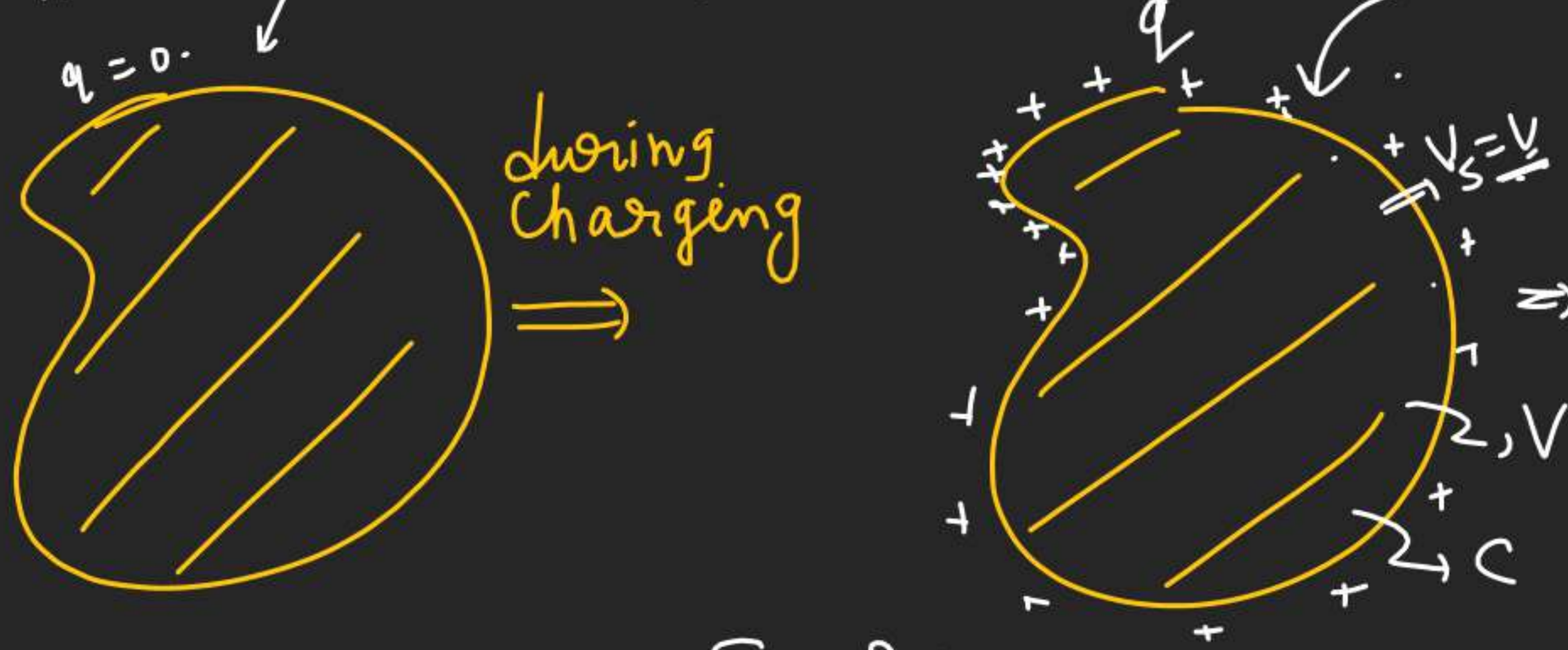
$+dq$ $V_\infty = 0$

At $t = 0$

$q = 0$

Conductor

during Charging ⟹



$\Rightarrow \boxed{q = CV}$

$dq = C\,dV$

$dU = dq \cdot V$

$$\int_0^U dU = C\int_0^V V\,dV$$

$$\boxed{U = \frac{CV^2}{2}}$$

$**$

[P.E for Conductor or Capacitor]

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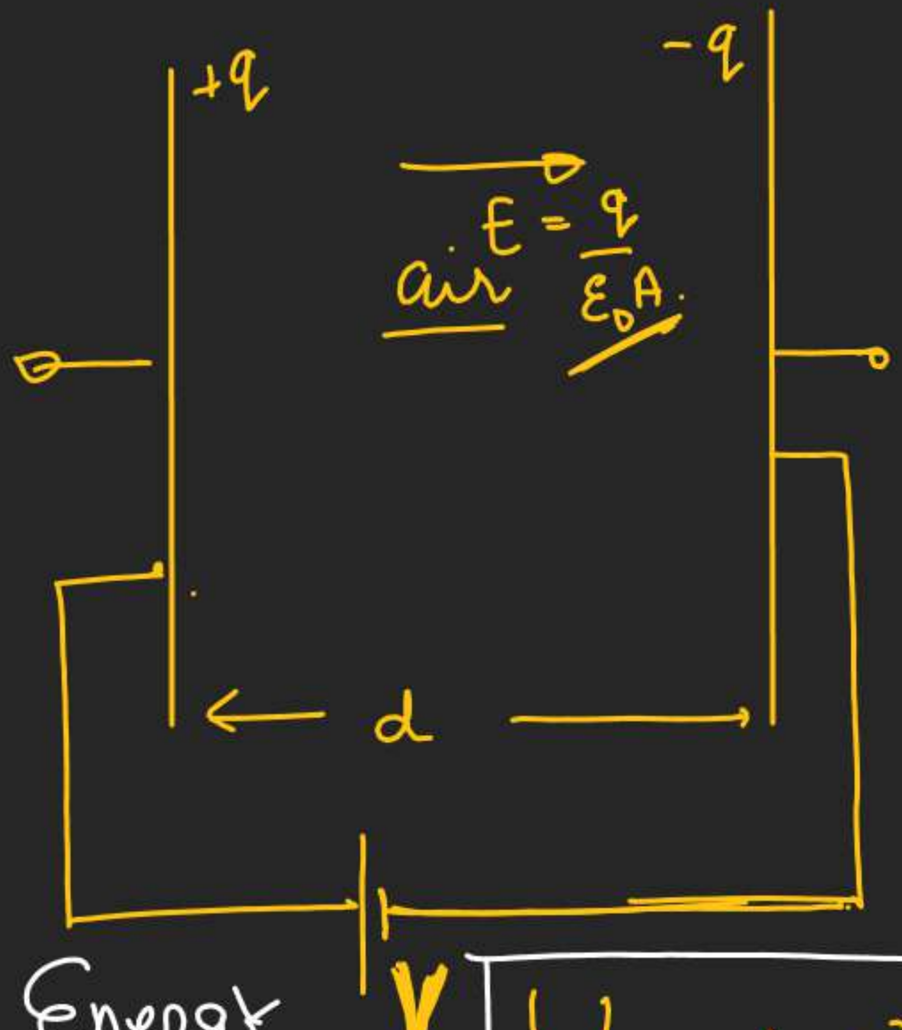
Capacitor



V → Potential difference b/w two plates

$$U = \frac{1}{2} C V^2 \rightarrow V = \bar{C}$$

$$q = CV \Rightarrow V = \left(\frac{q}{C}\right)$$

$$U = \frac{1}{2}\left(\frac{\varepsilon_0 A}{d}\right) V^2$$

$$U = \frac{1}{2} \times C \times \left(q/C\right)^2$$

$$\boxed{U = \frac{q^2}{2C}} \; (*) \rightarrow q = C$$

$$U = \frac{q^2}{2\left(\frac{\varepsilon_0 A}{d}\right)} = \frac{d}{2}\left(\frac{q}{\varepsilon_0 A}\right)^2 \times \varepsilon_0 A$$

$$\frac{q}{\varepsilon_0 A} = E$$

$$U = \frac{1}{2}\varepsilon_0 (Ad) E^2$$

$$U = \left(\frac{1}{2}\varepsilon_0 E^2\right)(\text{Volume})$$

Volume = Ad

Energy density $\Leftarrow \boxed{\frac{U}{\text{Volume}} = \frac{1}{2}\varepsilon_0 E^2}$

↳ True for all Capacitors and Conductor.

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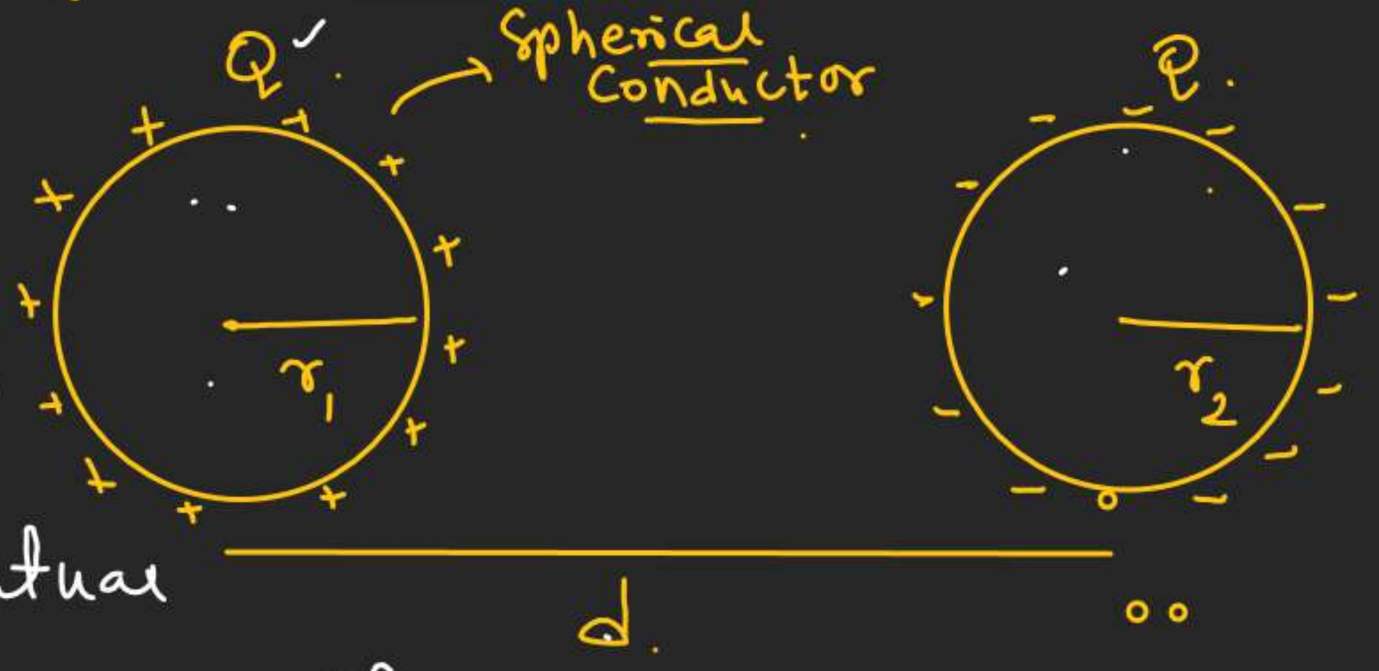
Capacitor

(8) **Calculating Capacitance by the help of P.E :-**

$$\underbrace{U}_{\text{System (P.E)}} = \frac{1}{2}CV^2 = \boxed{\frac{q^2}{2C}}$$



$$U_T = U_{self} + U_{mutual}$$

$$U_T = \frac{Q^2}{8\pi\varepsilon_0 r_1} + \frac{Q^2}{8\pi\varepsilon_0 r_2} - \frac{1}{4\pi\varepsilon_0}\frac{Q^2}{d}$$

$$U_T = \frac{Q^2}{8\pi\varepsilon_0}\left[\frac{1}{r_1}+\frac{1}{r_2}-\frac{2}{d}\right] \rightarrow \text{Compare with } \frac{Q^2}{2C}$$

$$** \quad \frac{1}{C} = \frac{1}{4\pi\varepsilon_0}\left[\frac{1}{r_1}+\frac{1}{r_2}-\frac{2}{d}\right]$$

$$\boxed{C = \left(\frac{4\pi\varepsilon_0}{\frac{1}{r_1}+\frac{1}{r_2}-\frac{2}{d}}\right)}$$