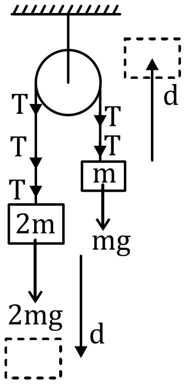
DPP – 5

SOLUTION

CONSTRAINED MOTION & SPRING

1. System $(m + 2m + \text{ string })$

$w_g + w_T = k_f - k_i$

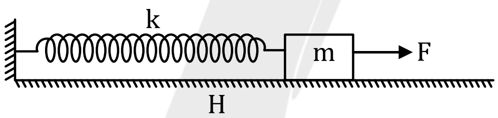


$$2mgd - mgd + 0 = \frac{1}{2}(2m)v^2 + \frac{1}{2}mv^2 - 0$$

$$mgd = \frac{1}{2}3mv^2$$

$$v = \sqrt{\frac{2gd}{3}}$$

2. $W_g + w_n + W_F + W_S = K_f - K_L$



$$o + o + fx_0 - \frac{k}{2}(x_0^2 - o^2) = \frac{1}{2}mv_{max}^2$$

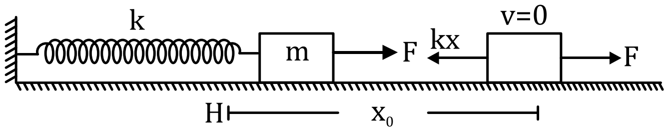
$$F \times \frac{F}{k} - \frac{K}{2}\frac{F^2}{k^2} = \frac{1}{2}mv_{max}^2$$

$$\frac{F^2}{k} - \frac{F^2}{2k} = \frac{1}{2}mv_{max}^2$$

$$v\frac{F}{\sqrt{km}_{max}}$$

3. v is maximum $\Rightarrow a = 0 \Rightarrow$ Fret $= 8$ Extension is maxima $= v = 0 \Rightarrow$

max extension $\Rightarrow V = 0 \Rightarrow K \cdot E = 0$



$$W_g + W_F + W_S + w_{Fri} + W_H = k_S - k_1$$

$$0 + F \cdot x_0 + \frac{k}{2}[x_0^2 - 0] + \left[(-\mu m_g \times x_0)\right] + 0 = 0$$

$$Fx_0 - \frac{k}{2}x_0^2 - 4mgx_0 = 0$$

$$F - \frac{k}{2}x_0 - Hmg = 0$$

$$F - Hmg = \frac{k}{2}x_0$$

$$x_0 = \frac{2F - 2Hmg}{k}$$

$$x_0 = \frac{2[F - Hmg]}{k}$$

4. As the block of mass M, descends down, it loses potential energy, which appears in the form of kinetic energy.

Now, if the block of mass 2M moves by a distance s, and its velocity be v. then the distance descended by block of mass M would be s/2 and so its velocity would be v/2.

Now, loss in potential energy of the system $= Mg\left(\frac{s}{2}\right)$ .... (i)

Work done against friction $= 2M\mu gs$ .... (ii)

Gain in kinetic energy of the two blocks

$= \frac{1}{2}(2M)v^2 + \frac{1}{2}M\left(\frac{v}{2}\right)^2$ .... (iii)

By conservation of energy:

$$Mg\left(\frac{S}{2}\right) = 2M\mu gs + Mv^2 + \frac{Mv^2}{8}$$

$$\frac{gs}{2} = 2\mu gs + \frac{9}{8}v^2$$

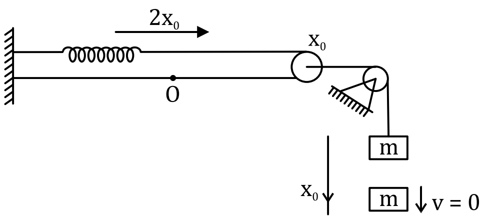
$$\frac{9}{8}v^2 = \frac{gs}{2}(1 - 4\mu) \Rightarrow v = \sqrt{\frac{4}{9}gs(1 - 4\mu)}$$

Velocity of mass, $M = \frac{v}{2}$

Required velocity $= \frac{1}{3}\sqrt{gs(1 - 4\mu)}$

5. System

(spring +block+ string + pulley)



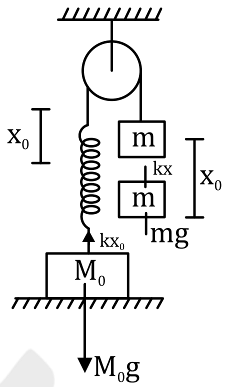
$$w_g + w_s + w_T = k_f - k_i$$

$$mgx_0 + \left(-\frac{k}{2}((2x_0)^2 - 0)\right) + 0 = 0 - 0$$

$$mgx_0 = \frac{2k}{2}x_0^2$$

$$x_0 = \frac{mg}{2k}$$

6. $kx \geq M_0 g.$



$$Wg + w_s = k_f - k_i.$$

$$mg\,x_0 - \frac{k}{2}(x^2 - 0^2) = 0 - 0.$$

$$\frac{kx}{2} = mg$$

$$x = \frac{2mg}{k}$$

$$k\frac{2mg}{k} \geq M_0 g$$

$$m \geq \frac{M_0}{m}$$

$$\text{minimum} = \frac{M_O}{2}$$

7. $mg\sin\theta = \frac{3}{5}mg$

$$f_{max} = \mu mg\cos\ \theta = \left(\frac{3}{4}\right)\left(\frac{4}{5}\right)mg = \frac{3}{5}mg$$

$$f_{max} = mg\sin\ \theta = \frac{3}{5}mg$$

Block will move upwards when

$$kx_0 = f_{max} + mg\sin\ \theta = \frac{6}{5}mg$$

From conservation of mechanical energy

$$Mgx_0 = \frac{1}{2}kx_0^2$$

$$M = \frac{kx_0}{2g} = \frac{\frac{6}{5}mg}{2g} = \frac{3}{5}m$$

8. At maximum compression reduced mass of system equals $\left(\frac{mM}{M+m}\right)$ and the initial relative velocity of approach is $(v_1 - v_2)$

$$\frac{1}{2}kx_{max}^2 = \frac{1}{2}\left(\frac{mM}{M+m}\right)(v_1 - v_2)^2$$

$$x_{max} = \sqrt{\frac{mM}{k(M+m)}}(v_1 - v_2)^2$$

9. Using energy conservation

$mgh = mgh' \Rightarrow h = h'$

For AO path: $v = 0 + at_{AO}$; where $a = g\sin\theta$

$\Rightarrow \quad t_{AO} = \frac{v}{g\sin c}$

For OB path: $v = 0 - at_{AO}$; where $a = g\sin\ 2\theta$

$t_{OB} = \frac{v}{g\sin 2\theta} \Rightarrow t_{AO} > t_{OB}$