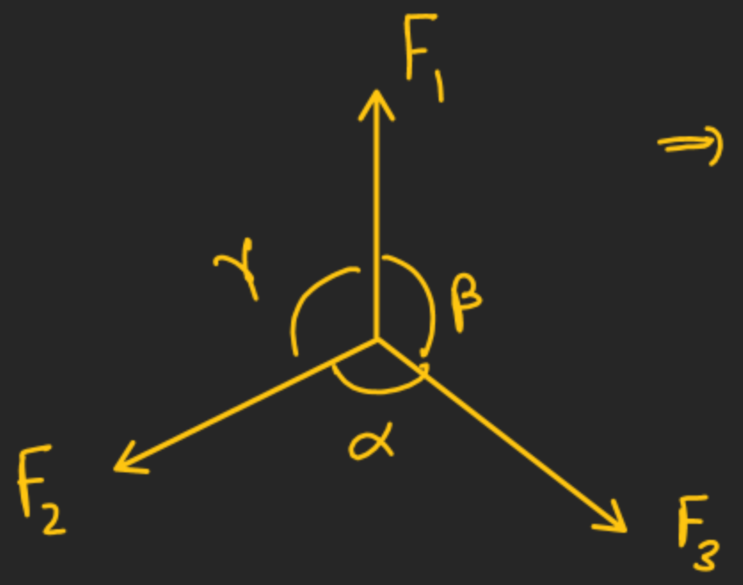
Nishant Jindal

Law of Motion

LAMI'S THEOREM



$$\Rightarrow \boxed{\frac{F_1}{\sin\alpha} = \frac{F_2}{\sin\beta} = \frac{F_3}{\sin\gamma}}$$

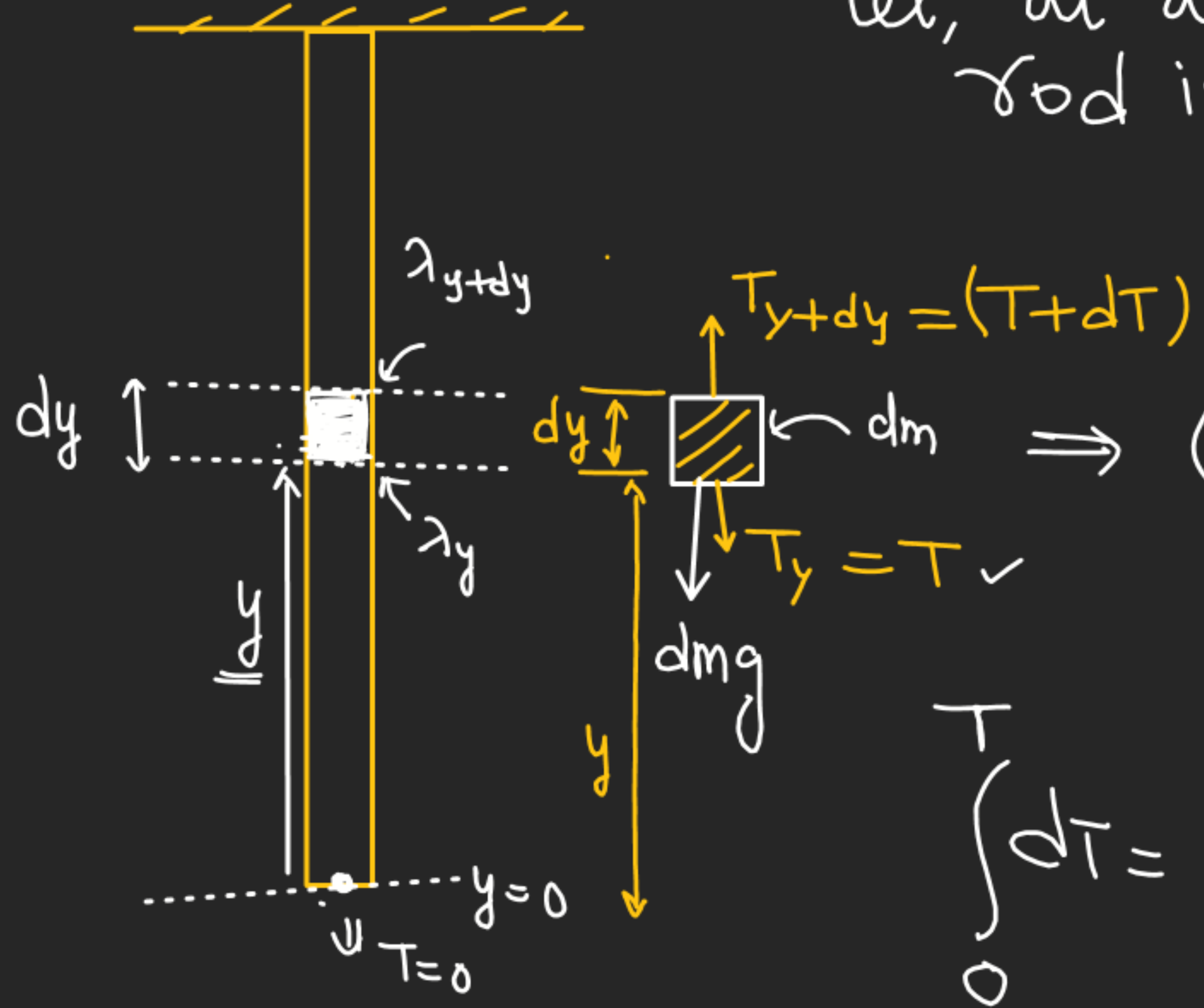
Law of Motion

Tension in a non-uniform rope or rod :→

[$\lambda = \lambda_0 y$] (where y is distance from bottom).

let, at a distance 'y', 'dy' length of the rod is cut whose mass is dm.

let, dT be the increment in the tension



$$\Rightarrow (T+dT) = (dm)g + T$$

$$dT = dm\,g.$$

$$dm = \lambda_y\,dy = (\lambda_0 y\,dy)$$

$$\int_0^T dT = \lambda_0 g \int_0^y y\,dy \Rightarrow T = \frac{\lambda_0 g}{2}\left[y^2\right]_0^y$$

$$T = \frac{\lambda_0 g}{2} y^2 \quad \text{Ans}$$

Since 'dy' is very small so, λ at y is same as λ at y+dy

$\lambda_{y+dy} \simeq \lambda_y$

i.e 'dy' length λ is assumed to be constant

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Law of Motion

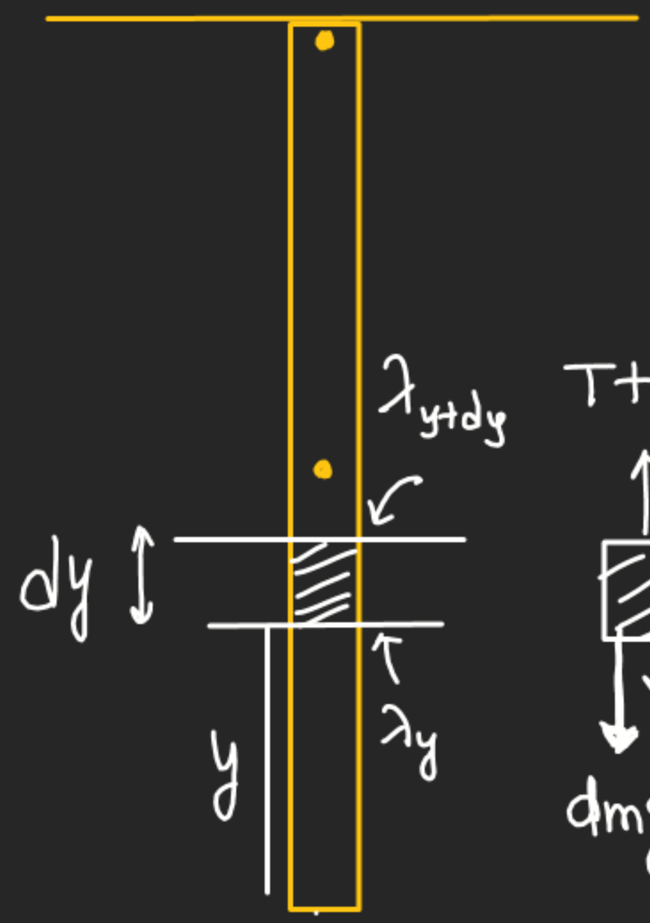
\# $\lambda = (a + by)$ [a & b are +ve constant]

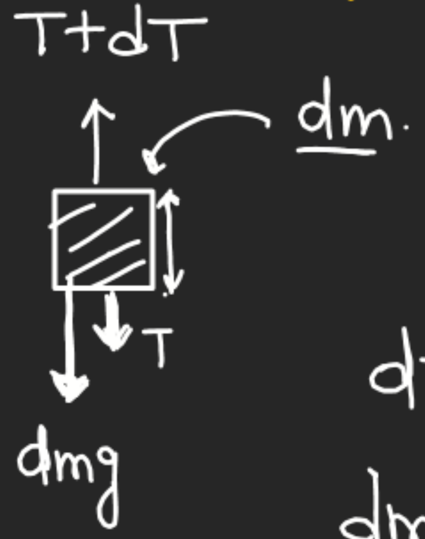
y from bottom.

[$\lambda \to$ linear mass density.]

$\left(\lambda = \frac{M}{L}\right) \leftarrow$

Find ratio of tension at mid point of the rod and at the top of the rod.





$T + dT - T - dmg = 0$

$dT = dm\,g.$

$dm = \lambda_y dy.$

[$\lambda_{y+dy} \simeq \lambda_y$] as dy is Very Small.

$dT = \lambda_y g\, dy$

$\int_0^T dT = \int_0^y (a + by) g\, dy.$

$T = g\left[a\int_0^y dy + b\int_0^y y\, dy\right]$

$T = g\left[ay + \frac{by^2}{2}\right]$

$T = \left[(ag)y + \frac{bg\,y^2}{2}\right]$

$T_{y=\frac{L}{2}} = \left[ag\frac{L}{2} + \frac{bg}{2}\left(\frac{L^2}{4}\right)\right]$

$= \frac{agL}{2} + \frac{bgL^2}{8}$

$= \left(\frac{4agL + bgL^2}{8}\right)$

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Law of Motion

$$T = g\left[ay + \frac{by^2}{2}\right]$$

$$T_{y=L} = agL + \frac{bgL^2}{2}$$

(maximum tension) $= \left[\frac{2agL + bgL^2}{2}\right]$ ✓

$$T_{y=\frac{L}{2}} = \left[\frac{4agL + bgL^2}{8}\right]$$ ✓

$$\frac{T_{y=L/2}}{T_{y=L}} = \frac{4agL + bgL^2}{\cancel{8}} \times \frac{\cancel{2}}{(2agL + bgL^2)}$$

$$= \frac{\cancel{gL}}{4}\frac{(4a+bL)}{\cancel{gL}(2a+bL)} = \frac{(4a+bL)}{4(2a+bL)}.$$

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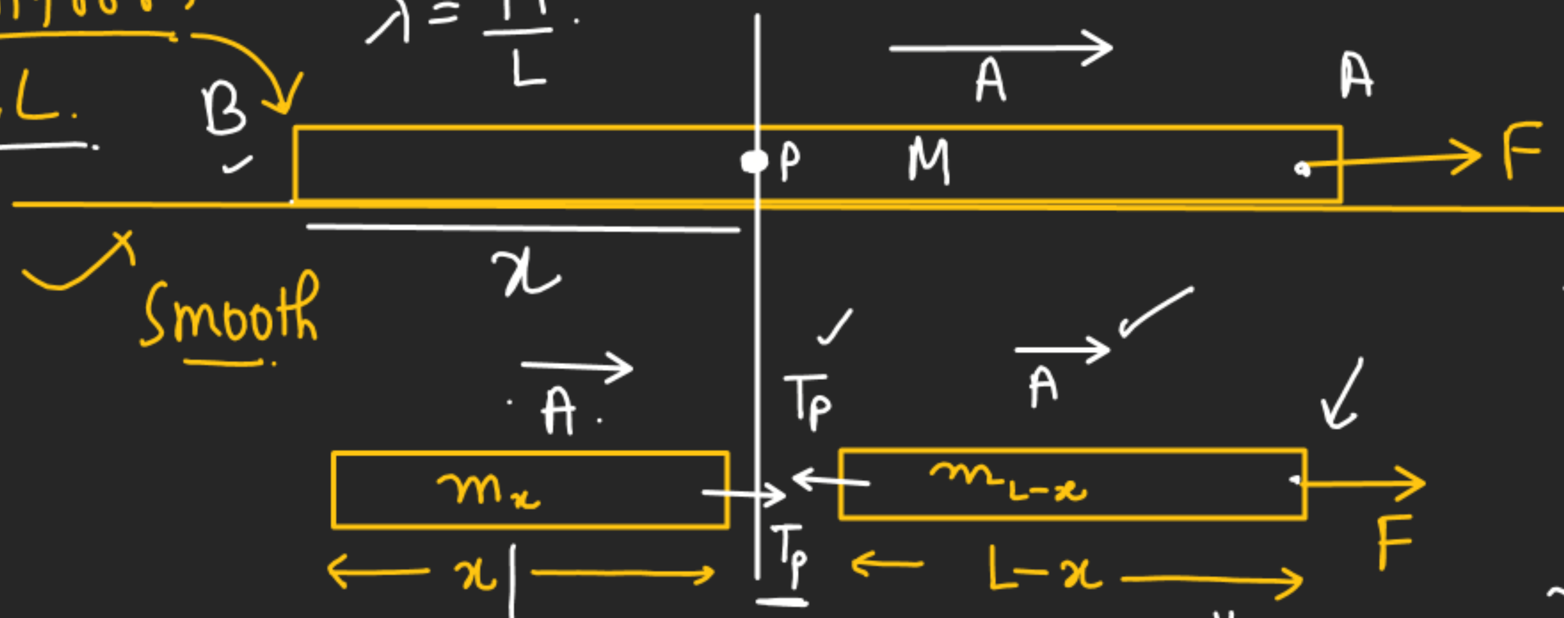
Law of Motion

Tension in a acclerated rope or rod

Uniform

M, L.

$\lambda = \frac{M}{L}$



Newton's 2nd Law

$T_p = m_x A$

$m_x = \left(\frac{M}{L} x\right) \Rightarrow T_p = \left(\frac{\cancel{M}}{L} x\right) \times \frac{F}{\cancel{M}} \Rightarrow \boxed{\frac{F}{L} x}$

Newton's 2nd Law

$\left[F - T_p = m_{L-x} A \right]$

$F = MA$

$A = \left(\frac{F}{M}\right)$

$\Downarrow$

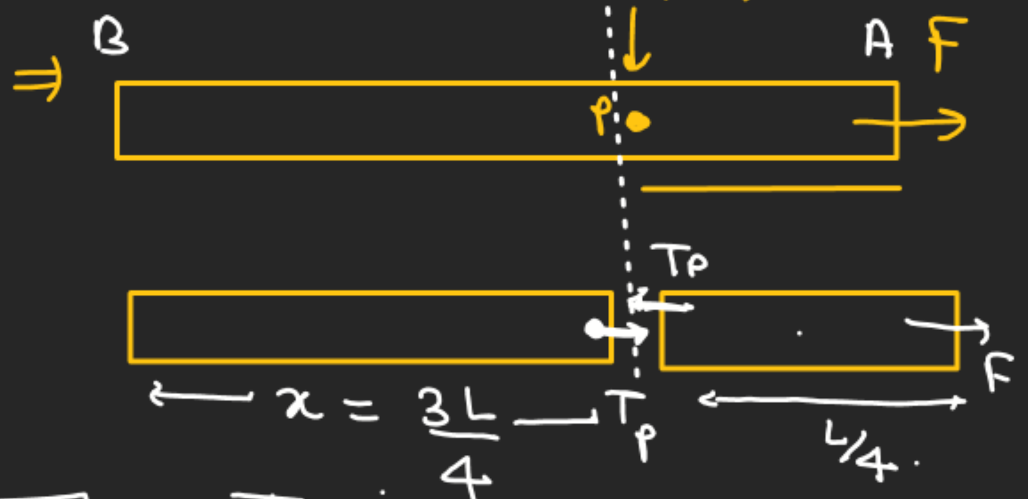
Accleration of the rod

$(\vec{F}_{net})_{ext} = M\vec{A}$

↳ Always net force in accleration direction

$T_p = ??$

$\Rightarrow$



$x = \frac{3L}{4}$ T_p $L/4$

$T_p = \frac{F}{L} \times \frac{3L}{4} = \left(\frac{3F}{4}\right)$

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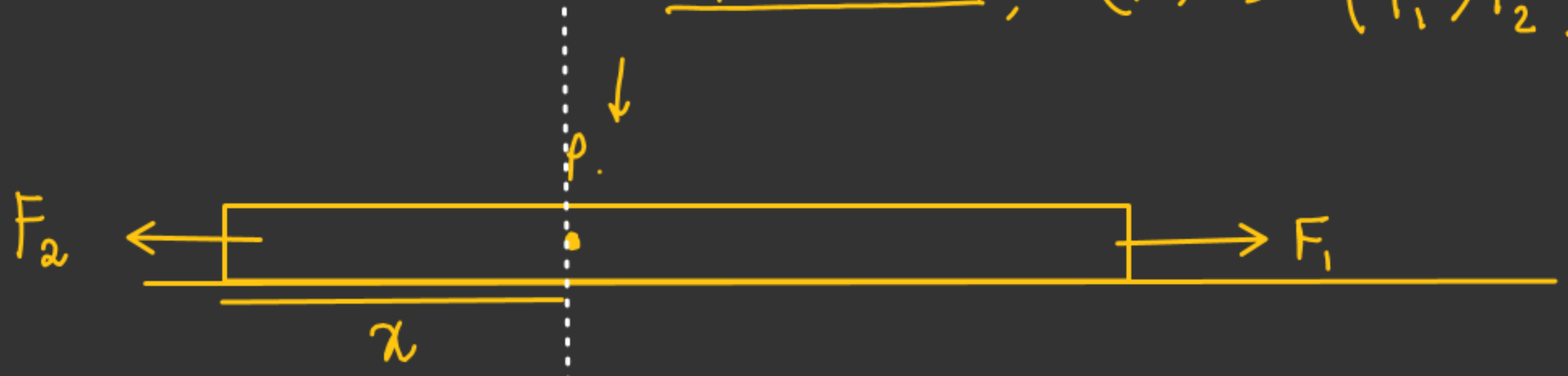
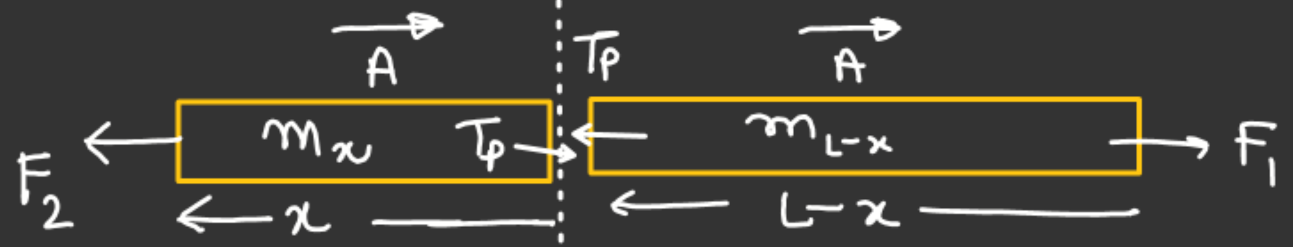
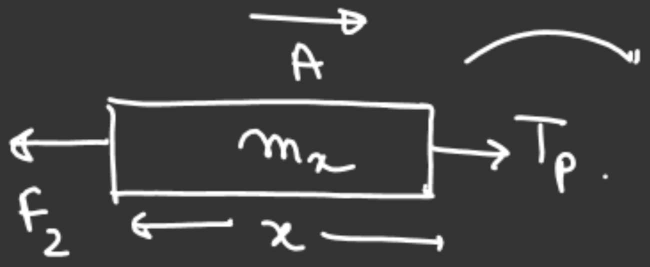
Uniform Rod, (M, L) $(F_1 > F_2)$





$\Downarrow$



$$m_x = \left(\frac{M}{L}x\right)$$

Newton's 2nd Law

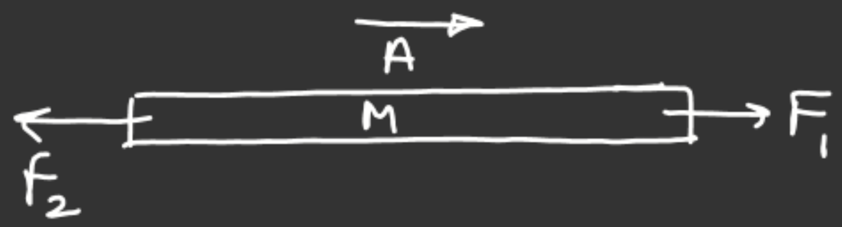
$$T_p - F_2 = m_x A$$

$$T_p = F_2 + m_x A$$

$$T_p = F_2 + \left(\frac{\cancel{M}}{L}x\right)\left(\frac{F_1 - F_2}{\cancel{M}}\right)$$

$$\boxed{T_p = F_2 + \left(\frac{F_1 - F_2}{L}\right)x}$$ **

Accleration of Rod



$$F_1 - F_2 = MA$$

$$A = \left(\frac{F_1 - F_2}{M}\right)$$

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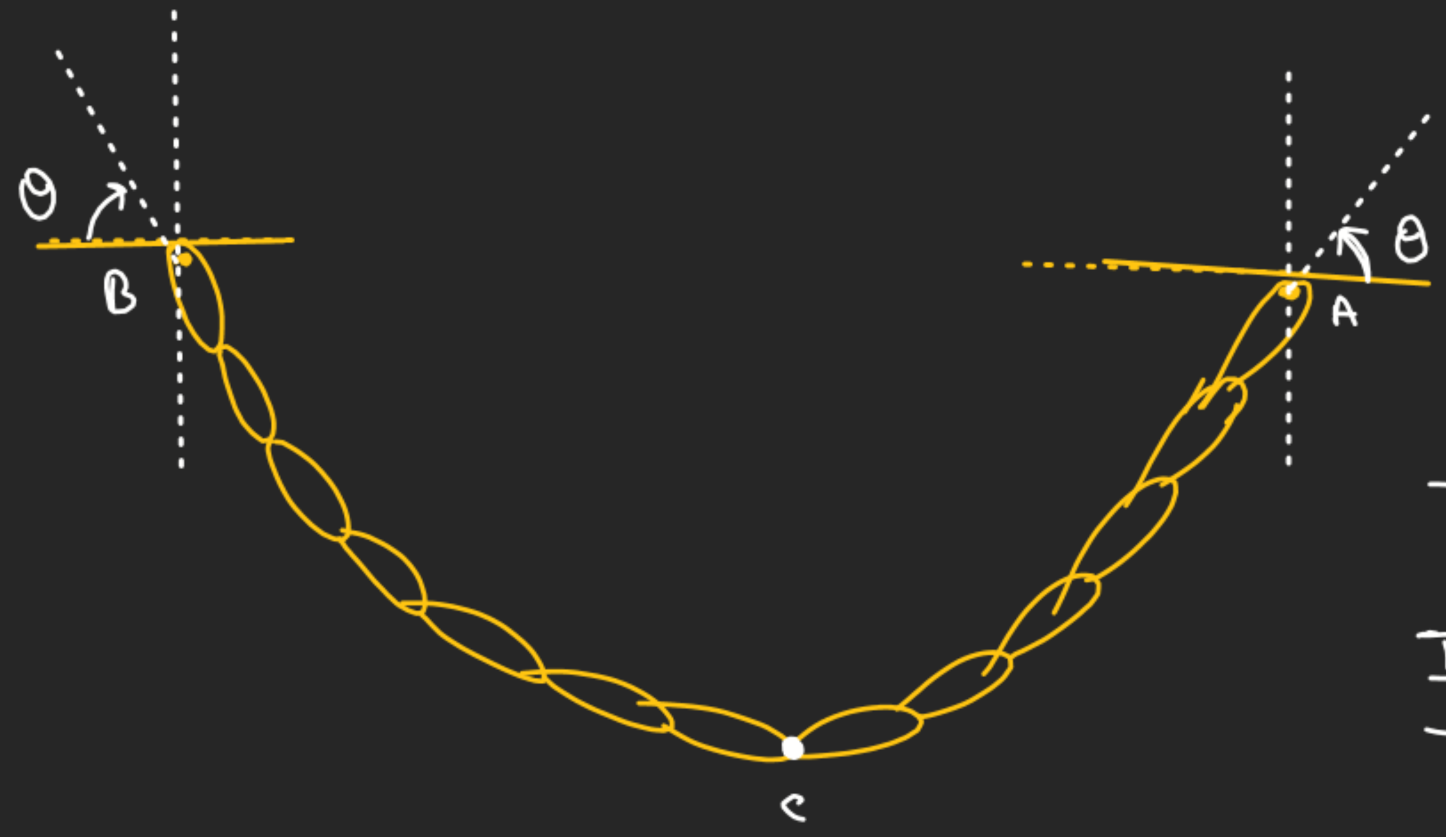
Law of Motion

Q8: Tension in a Uniform Chain :-

⇒ Uniform Chain, (Semi Circular Shape)

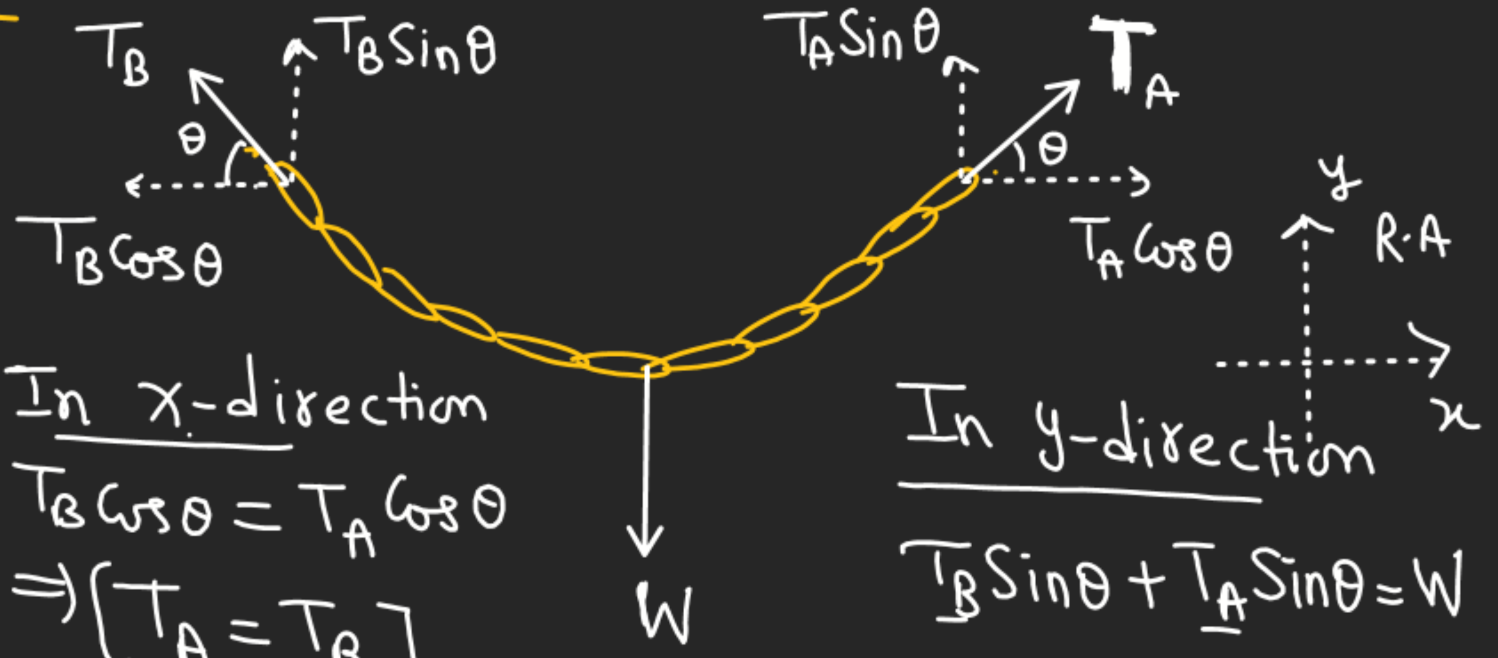


Find tension

$T_A = ?$, $T_B = ?$, $T_c = ?$

Total weight of the Chain $= W$ (N)

F.B.D of whole Chain



In x-direction

$T_B \cos\theta = T_A \cos\theta$

$\Rightarrow \boxed{T_A = T_B}$

let, $T_A = T_B = T$

In y-direction

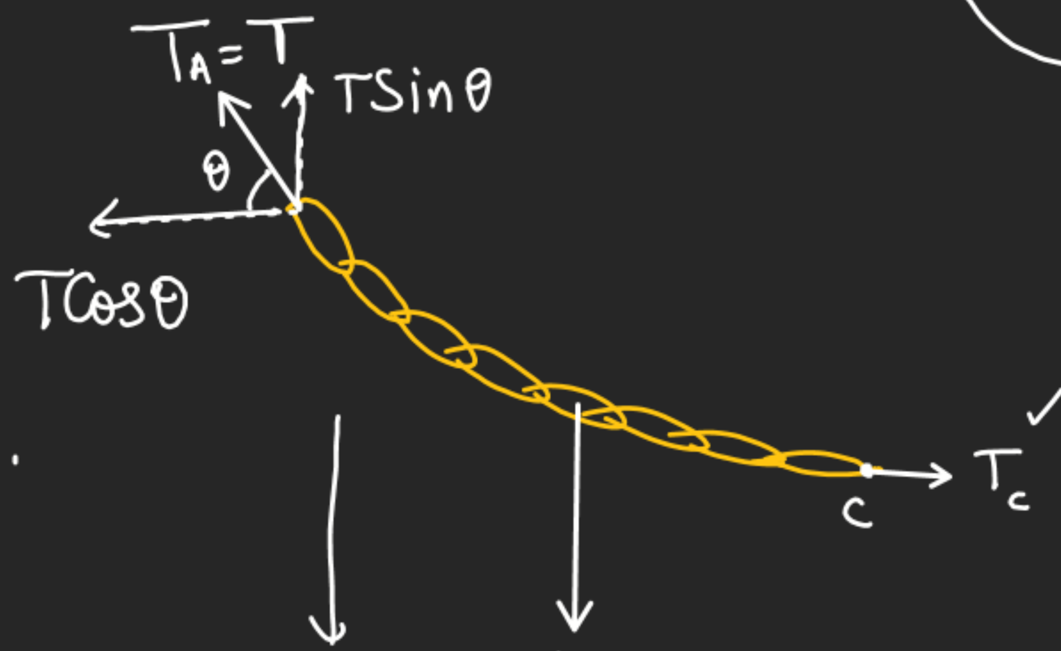
$T_B \sin\theta + T_A \sin\theta = W$

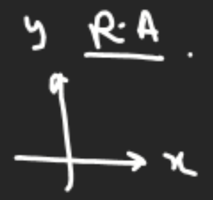
$2T\sin\theta = W$

$$\boxed{T_B = T_A = T = \left(\frac{W}{2\sin\theta}\right)}$$

Law of Motion

For $T_c = ??$





$\frac{W}{2}$

Newton's 1st law:-

$$T\sin\theta = \frac{W}{2}$$

$$T = \frac{W}{2\sin\theta}$$

$$T_c = T\cos\theta$$

$$T_c = \frac{W}{2\sin\theta} \times \cos\theta \Rightarrow \boxed{T_c = \frac{W}{2}\cot\theta}$$ Ans.