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2.

$$\underbrace{3\tan^{-1}\frac{1}{2}}_{\theta\in(0,\frac{\pi}{6})} + 2\underbrace{\tan^{-1}\frac{1}{5}}_{\phi\in(0,\frac{\pi}{4})} + \sin^{-1}\frac{142}{65\sqrt{5}}$$

$$= \tan^{-1}\frac{11}{2} + \tan^{-1}\frac{5}{12} + \tan^{-1}\left(\frac{142}{31}\right)$$

$$\pi + \tan^{-1}\left(\quad\right)$$

$$\tan^{-1}\tan 3\theta = \tan^{-1}\frac{\frac{3}{2}-\frac{1}{8}}{1-\frac{3}{4}} = \tan^{-1}\frac{11}{2}$$

$$= 3\theta$$

$$\tan 2\phi = \frac{\frac{2}{5}}{1-\frac{1}{25}} = \frac{10}{24} = \frac{5}{12}$$

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3. $\frac{ax}{c}\sqrt{1-\frac{b^2x^2}{c^2}}+\frac{bx}{c}\sqrt{1-\frac{a^2x^2}{c^2}}=x$, $x=0$

$$\frac{a^2}{c^2}\left(1-\frac{b^2x^2}{c^2}\right)=1-\frac{2b}{c}\sqrt{1-\frac{a^2x^2}{c^2}}+\frac{b^2}{c^2}\left(1-\frac{a^2x^2}{c^2}\right)$$

a^2+b^2

$$-\frac{2b^2}{c^2}=\frac{a^2-c^2-b^2}{c^2}=\frac{a^2}{c^2}-1-\frac{b^2}{c^2}=-\frac{2b}{c}\sqrt{1-\frac{a^2x^2}{c^2}}$$

$$\frac{b^2}{c^2}=1-\frac{a^2x^2}{c^2}\Rightarrow\frac{a^2x^2}{c^2}=1-\frac{b^2}{c^2}=\frac{a^2}{c^2}$$

$$x^2=1$$

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4.

$$\cos^{-1}\sqrt{6}x = \sin^{-1}3\sqrt{3}x^2$$

$$\sqrt{1-6x^2} = 3\sqrt{3}\,x^2$$

$$1-6x^2 = 27x^4$$

Check.

9.

$$\cos^{-1}y + \cos^{-1}bxy = \cos^{-1}ax$$

$$bxy^2 - \sqrt{1-y^2}\sqrt{1-b^2x^2y^2} = ax$$

$$x^2\left(by^2-a\right)^2 = \left(1-y^2\right)\left(1-b^2x^2y^2\right)$$

$$x^2\left(\cancel{b^2y^4}+a^2-2aby^2\right) = 1-y^2-b^2x^2y^2+\cancel{b^2x^2y^4}$$

$$a^2x^2-2abx^2y^2 = 1-y^2-b^2x^2y^2$$

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12. (P)

$$\frac{1}{y^2}\left(\frac{\frac{1}{\sqrt{1+y^2}}+\frac{y^2}{\sqrt{1+y^2}}}{\frac{\sqrt{1-y^2}}{y}+\frac{y}{\sqrt{1-y^2}}}\right)^{2}+y^4$$

$$\frac{\sqrt{1+y^2}}{\frac{1}{y\sqrt{1-y^2}}}=y(1-y^2)^{\frac{1}{2}}(1+y^2)^{\frac{1}{2}}$$

$$1-y^4+y^4=1$$

① $\cos x+\cos y=-\cos z$

② $\sin x+\sin y=-\sin z$

$①^2+②^2$ $\quad 2+2\cos(x-y)=1$

Q $\cos x+\cos y+\cos z$
$=0=\sin x+\sin y+\sin z.$

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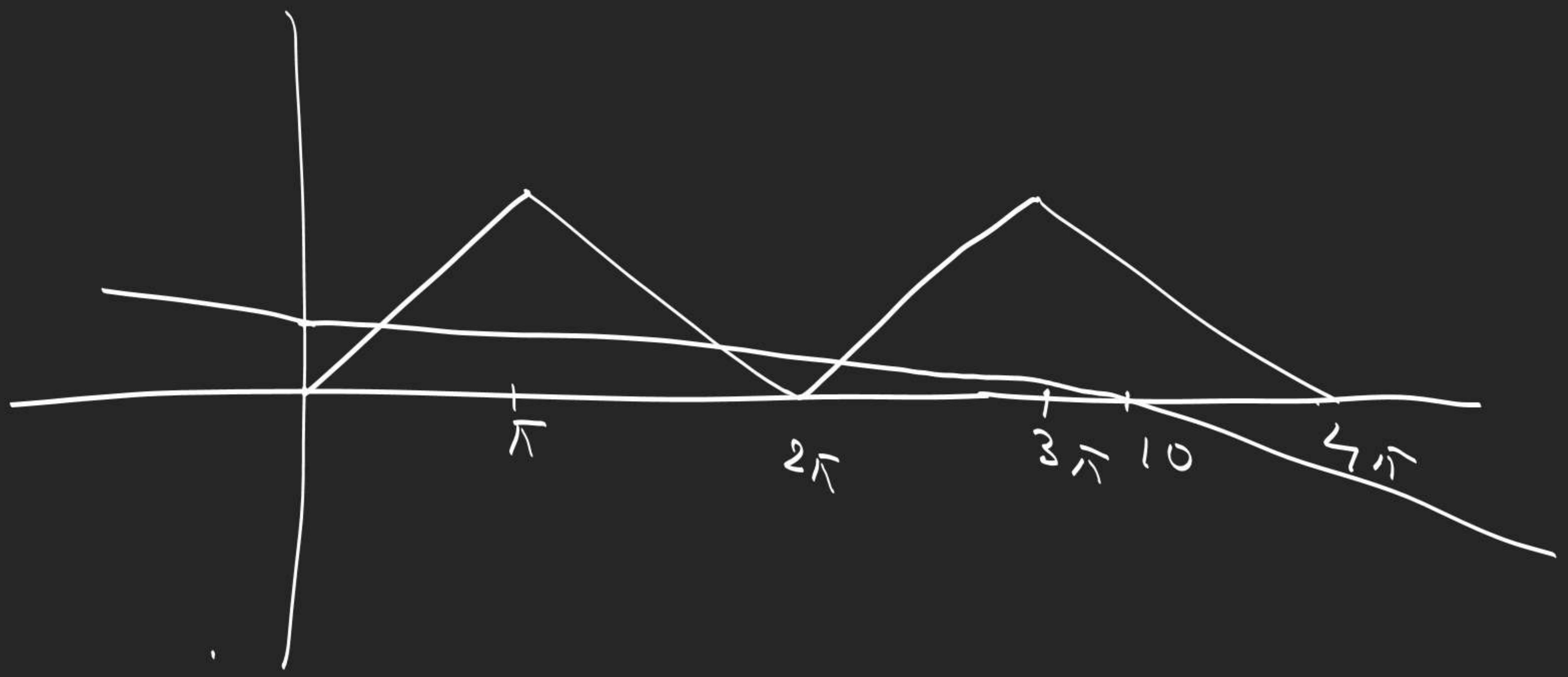

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15. $-1 \le \ln\left(\frac{x}{x-1}\right) \le 1$

$x = 0$

$\frac{1}{e} \le \frac{x-}{x-1} \le e$

$$\underbrace{\left(\frac{x^2}{1-x} - x\,\frac{\frac{x}{2}}{1-\frac{x}{2}}\right)}_{\in[-1,1]} = \frac{-\frac{x}{2}}{1+\frac{x}{2}} - \frac{-x}{1+x}$$

16.

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Trigonometric Limits

$$\lim_{\theta \to 0} \frac{\theta}{\sin\theta}$$

$$\lim_{x \to 0} \left(\frac{\sin x}{x}\right) = 1 \qquad \lim_{x \to 0} \frac{\sin^{-1} x}{x} = 1$$

$$\lim_{x \to 0} \left(\frac{\tan x}{x}\right) = 1 \qquad \lim_{x \to 0} \frac{\tan^{-1} x}{x} = 1$$

$$\lim_{x \to 0} \frac{1-\cos x}{x^2} = \frac{1}{2}$$

$$\lim_{x \to 0} \frac{2\sin^2 \frac{x}{2}}{x^2} = \lim_{x \to 0} \frac{1}{2} \left(\frac{\sin \frac{x}{2}}{\frac{x}{2}}\right)^2 = \frac{1}{2}$$

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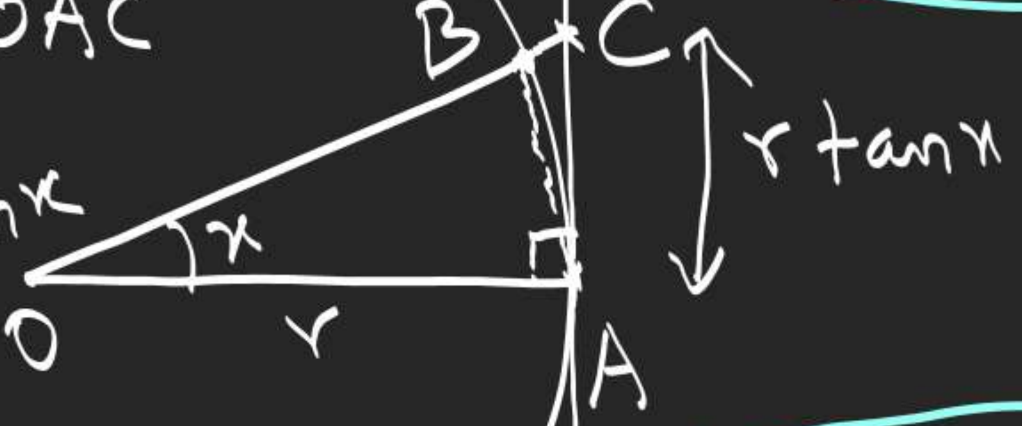
$x > 0 \rightarrow \cos x < \dfrac{x}{\tan x} < 1 \Rightarrow 1 < \dfrac{\tan x}{x} < \sec x$

$x \to 0, \ \dfrac{\tan x}{x} \to 1^+$

$\Delta OAB < \text{sector } OAB < \Delta OAC$



$\lim\limits_{x \to 0} \dfrac{\sin x}{x}$

$\frac{1}{2} r^2 \sin x < \frac{1}{2} r^2 x < \frac{1}{2} r^2 \tan x$

RHL $= \lim\limits_{h \to 0} \dfrac{\sin h}{h}$

LHL $= \lim\limits_{h \to 0} \dfrac{\sin(-h)}{-h}$

$\sin x < x < \tan x$

$x \to 0^+$

$1 < \dfrac{x}{\sin x} < \dfrac{1}{\cos x}$

$\cos x < \dfrac{\sin x}{x} < 1$

$x \to 0, \ \dfrac{\sin x}{x} \to 1^-$

$\lim\limits_{x \to 0} \cos x = 1$

$\lim\limits_{x \to 0} 1 = 1$

$\Rightarrow \lim\limits_{x \to 0} \dfrac{\sin x}{x} = 1$

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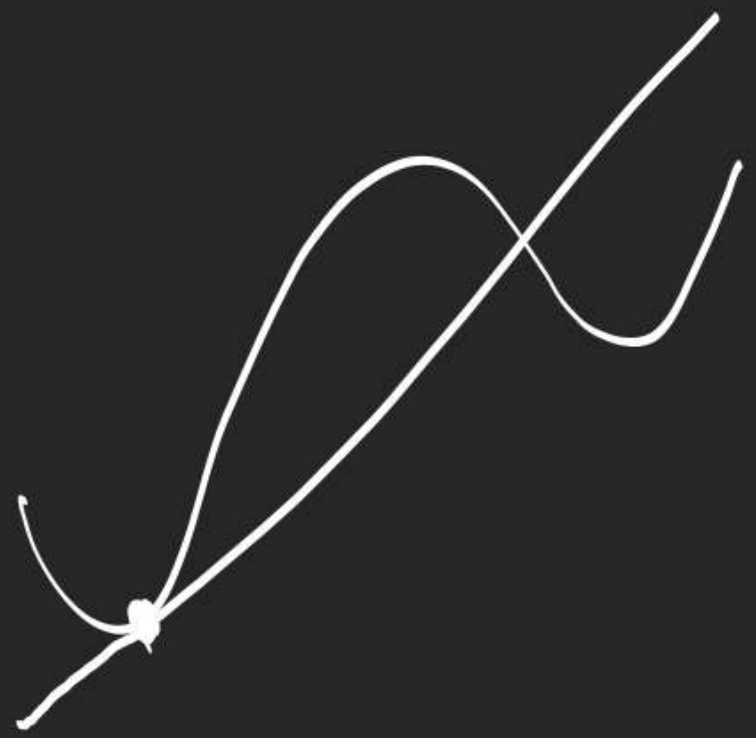

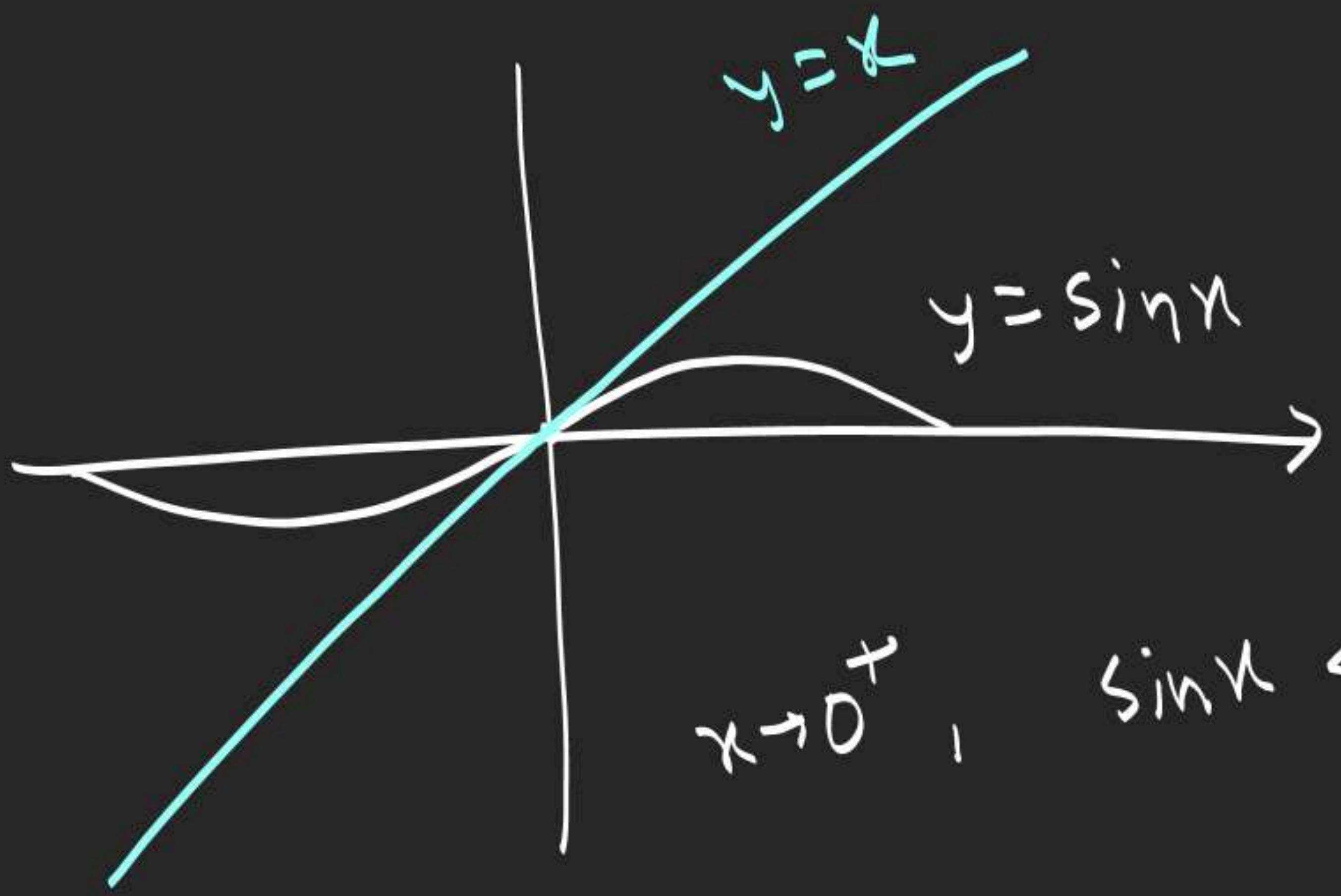

$x \to 0^+, \quad \sin x < x \Rightarrow \frac{\sin x}{x} < 1$

$x \to 0^-, \quad \sin x > x \Rightarrow \frac{\sin x}{x} < 1$

Nishant Jindal

1. $\lim_{x\to0}\left(\frac{1-\cos5x}{3x^2}\right) = \lim_{x\to0}\frac{1-\cos5x}{(5x)^2}\ \frac{25}{3} = \frac{1}{2}\times\frac{25}{3}$

$= \frac{25}{6}.$

2. $\lim_{x\to0}\left(\frac{1-\cos(1-\cos x)}{x^4}\right) = \lim_{x\to0}\left(\frac{1-\cos(1-\cos x)}{(1-\cos x)^2}\right)\left(\frac{1-\cos x}{x^2}\right)^2$

$= \frac{1}{2}\times\left(\frac{1}{2}\right)^2 = \frac{1}{8}.$

3. $\lim_{x\to0}\left(\frac{\sqrt{1+\tan x}-\sqrt{1+\sin x}}{x^3}\right) = \lim_{x\to0}\frac{\tan x-\sin x}{\left(\sqrt{1+\tan x}+\sqrt{1+\sin x}\right)x^3}$

$\lim_{x\to0}\frac{1}{\left(\sqrt{1+\tan x}+\sqrt{1+\sin x}\right)}\cdot\frac{\tan x}{x}\left(\frac{1-\cos x}{x^2}\right)$

$\boxed{\frac{1}{4}}$

Nishant Jindal

4. $$\lim_{x\to 0}\left(\frac{1-(\cos x)\sqrt{\cos 2x}}{x^2}\right)$$

$\sin^2 x+2\sin^2 x\cos^2 x$

$$=\lim_{x\to 0}\frac{1-\cos^2 x(1-2\sin^2 x)}{\left(1+\cos x\sqrt{\cos 2x}\right)x^2}=\lim_{x\to 0}\frac{\sin^2 x\left(1+2\cos^2 x\right)}{x^2\left(1+\cos x\sqrt{\cos 2x}\right)}$$

$$=\frac{3}{2}$$

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5. $$\lim_{x\to 0}\left(\frac{\tan(a+2x)-2\tan(a+x)+\tan a}{x^2}\right)$$

$$\lim_{x\to 0}\frac{\big(\tan(a+2x)-\tan(a+x)\big)-\big(\tan(a+x)-\tan a\big)}{x^2}$$

$$\tan(A-B)=\frac{\tan A-\tan B}{1+\tan A\tan B}$$

$$\lim_{x\to 0}\frac{\tan x\left[\big(\cancel{1}+\tan(a+2x)\tan(a+x)\big)-\big(\cancel{1}+\tan(a+x)\tan a\big)\right]}{x^2}$$

$$\boxed{2\tan a\sec^2 a}=\lim_{x\to 0}2\left(\frac{\tan x}{x}\right)\tan(a+x)\left(\frac{\tan 2x}{2x}\right)\big(1+\tan a\tan(a+2x)\big)$$

Nishant Jindal

6. $\lim_{x\to 0}\left(\left\{1000\underbrace{\frac{\sin x}{x}}_{1^-}\right\}+\left[98\underbrace{\frac{\tan^{-1}x}{x}}_{1^-}\right]\right)$

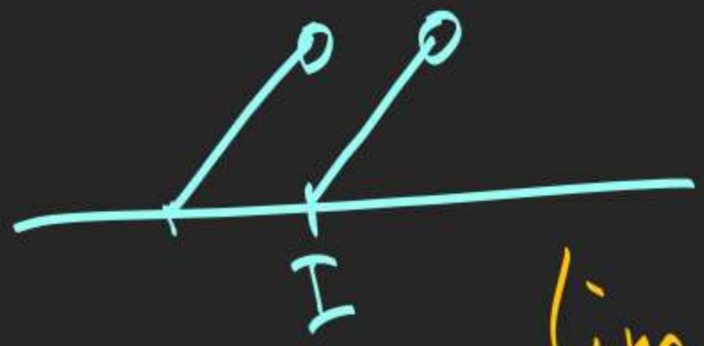
$[\cdot] = G.I.F$

$\{\cdot\} = FPF$

$1 + 97 = \boxed{98}$

$\frac{1000\sin x}{x} \to 1000^-$

$98\frac{\tan^{-1}x}{x} \to 98^-$



$\lim_{x\to 0}\left(\frac{1000\sin x}{x}-\left[\frac{1000\sin x}{x}\right]+\left[\frac{98\tan^{-1}x}{x}\right]\right)$

$1000 - 999 + 97$

7: $\lim_{x\to 0^+}\left(\frac{\cos^{-1}(1-x)}{\sqrt{x}}\right)$

$\cos^{-1}(1-x)=\theta$

$1-x=\cos\theta$

$x=1-\cos\theta$

$$\lim_{\theta\to 0^+}\left(\frac{\theta}{\sqrt{1-\cos\theta}}\right)=\lim_{\theta\to 0^+}\frac{|\theta|=\sqrt{\theta^2}}{\sqrt{1-\cos\theta}}=\lim_{\theta\to 0^+}\frac{1}{\sqrt{\frac{1-\cos\theta}{\theta^2}}}=\sqrt{2}.$$

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8. $$\lim_{x\to\frac{\pi}{6}} \frac{\sin\left(x-\frac{\pi}{6}\right)}{\frac{\sqrt{3}}{2}-\cos x}$$

$\cos\frac{\pi}{6}$

$$=\lim_{x\to\frac{\pi}{6}} \frac{\cancel{2}\cancel{\sin\left(\frac{x-\frac{\pi}{6}}{2}\right)}\cos\left(\frac{x-\frac{\pi}{6}}{2}\right)}{\cancel{2}\cancel{\sin\left(\frac{x-\frac{\pi}{6}}{2}\right)}\sin\left(\frac{x+\frac{\pi}{6}}{2}\right)}$$

$$=\frac{1}{\sin\frac{\pi}{6}}=2$$

$$\lim_{x\to\frac{\pi}{6}} \frac{\dfrac{\sin\left(x-\frac{\pi}{6}\right)}{x-\frac{\pi}{6}}}{-\left(\dfrac{\cos x-\cos\frac{\pi}{6}}{x-\frac{\pi}{6}}\right)}$$

$$=\frac{1}{\sin\frac{\pi}{6}}$$

$=2$.

$x=\frac{\pi}{6}+h$

$$\lim_{h\to 0} \frac{\sin h}{\frac{\sqrt{3}}{2}-\cos\left(\frac{\pi}{6}+h\right)}=\lim_{h\to 0}\frac{\sin h}{\frac{\sqrt{3}}{2}-\frac{\sqrt{3}}{2}\cos h+\frac{1}{2}\sin h}$$

$$=\lim_{h\to 0}\frac{\dfrac{\sin h}{h}}{\dfrac{\sqrt{3}}{2}\left(\dfrac{1-\cos h}{h^2}\right)h+\dfrac{1}{2}\dfrac{\sin h}{h}}=\frac{1}{0+\frac{1}{2}}$$

$=2$.

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$\boxed{306 - 350}$ Berman.