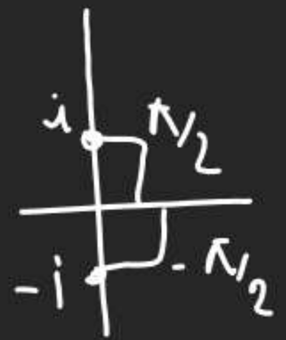
Nishant Jindal

Q If $Arg(-2+3i)=\theta$ then $Arg(3+2i)=?$

hold

Q If $Arg(z\cdot w)=\pi$ & $\bar{z}+i\bar{w}=0$

then $Arg(z)=?$



$Arg(z\cdot w)=\pi$

$Arg\,z+Arg\,w=\pi \rightarrow (A)$

$\bar{z}=-i\bar{w}$

$Arg(\bar{z})=Arg(-i\bar{w})$

$\Rightarrow -Arg\,z=Arg(-i)+Arg(\bar{w})$

$-Arg\,z=-\frac{\pi}{2}-Arg\,w$

$\frac{\pi}{2}=Arg\,z-Arg\,w \rightarrow (B)$

(A)+(B)

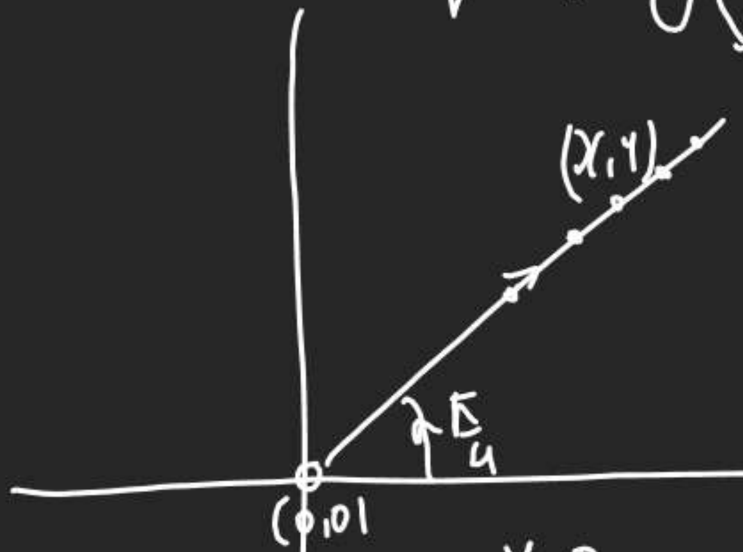
$2Arg\,z=\pi+\frac{\pi}{2}=\frac{3\pi}{2} \Rightarrow \boxed{Arg\,z=\frac{3\pi}{4}}$

Geometrical Prop. of C.N.

A) Locus of z if $Arg\,z=\theta$

then z Rep. a Ray starting from Origin

Q Find locus of z if $Arg\,z=\frac{\pi}{4}$



$\frac{y-0}{x-0}$

$\tan^{-1}\left|\frac{y}{x}\right|=\frac{\pi}{4}$

$\frac{y}{x}=\tan\frac{\pi}{4}$

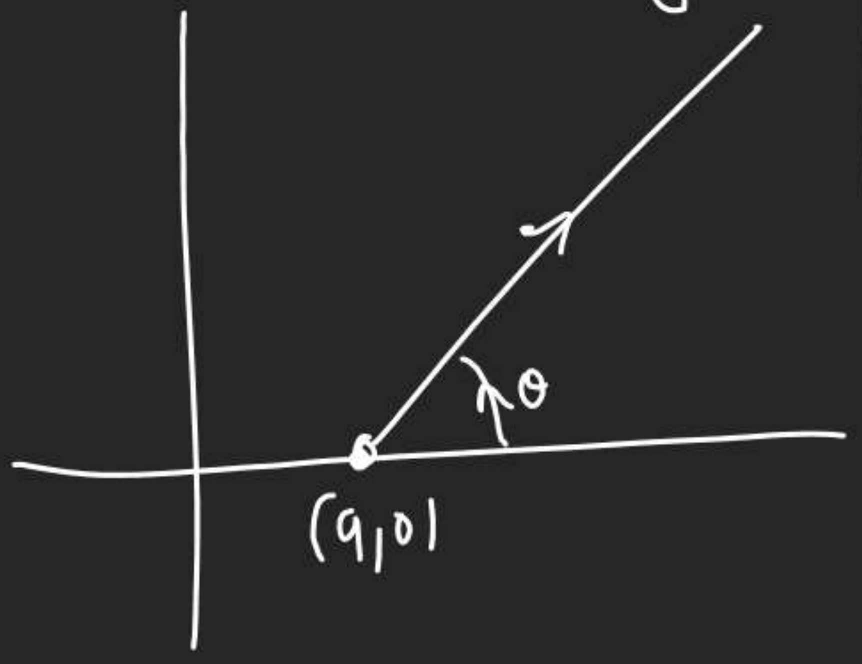
$\frac{y}{x}=1$

$\boxed{y=x}$

(2) Locus of z if

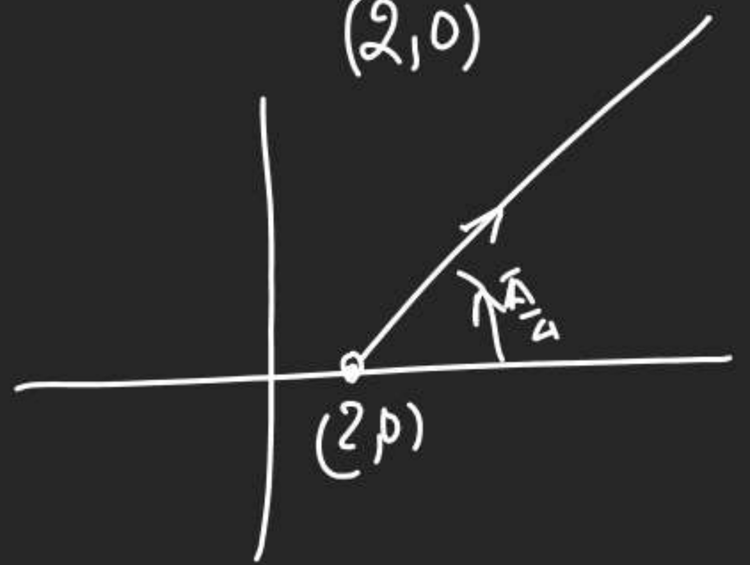
$$\text{Arg}(z-a)=\theta$$

then it Rep. Ray starting from $(a,0)$ at angle θ



Q $\text{Arg}(z-2)=\frac{\pi}{4}$ find locus

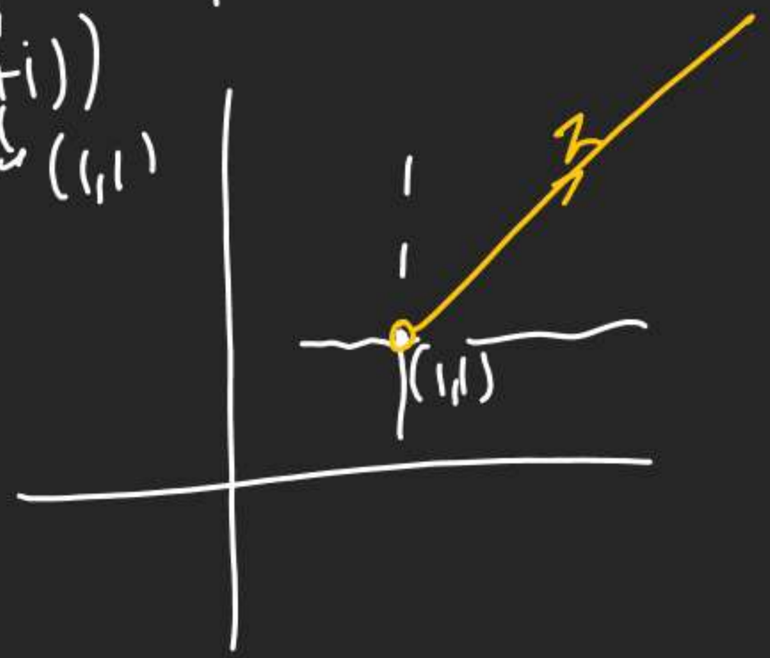
$(2,0)$



Q $\text{Arg}(z-1-i)=\frac{\pi}{4}$ locus of z.

$(z-(1+i))$

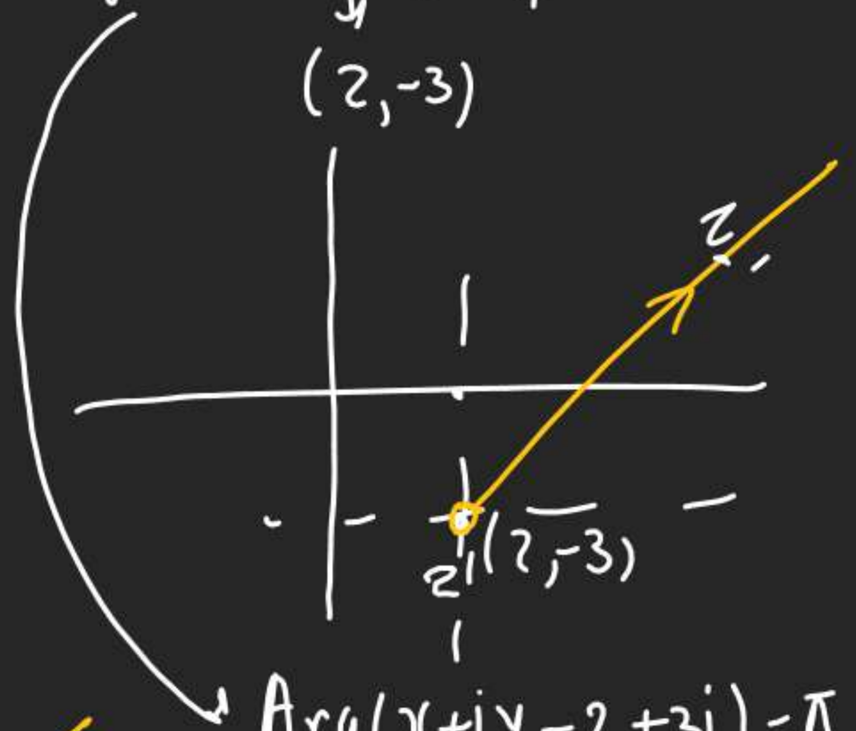
$\downarrow (1,1)$



Q $\text{Arg}(z-2+3i)=\frac{\pi}{4}$ find locus?

$\text{Arg}(z-(2-3i))=\frac{\pi}{4}$

$(2,-3)$



$$\text{Arg}(x+iy-2+3i)=\frac{\pi}{4}$$

$$\text{Arg}((x-2)+i(y+3))=\frac{\pi}{4}$$

$$\tan^{-1}\left(\frac{y+3}{x-2}\right)=\frac{\pi}{4}$$

$$(y+3)=\tan\frac{\pi}{4}(x-2)$$

$$(y+3)=1\cdot(x-2) \rightarrow \text{St. line}$$

$$(y-y_1)=m(x-x_1)$$

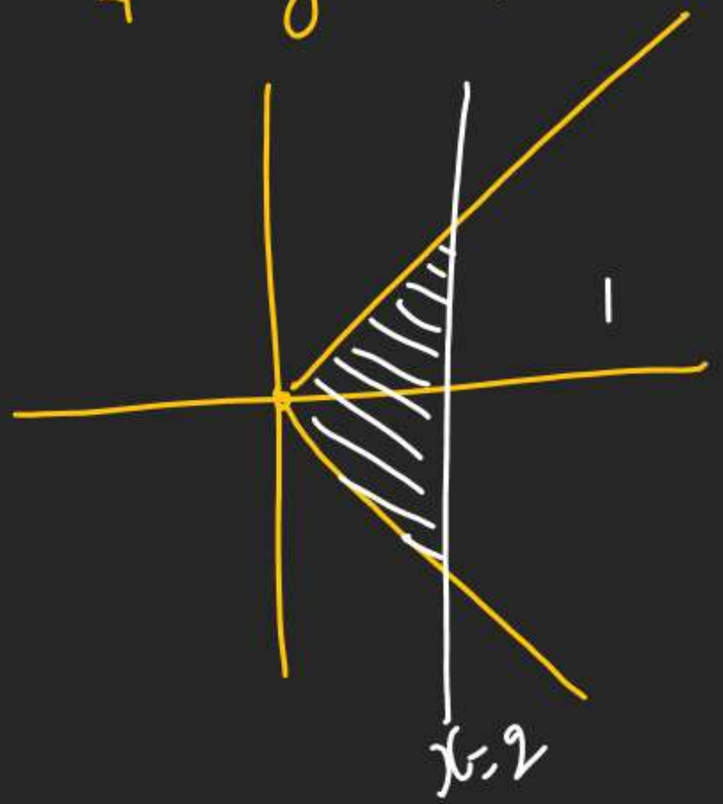
Q Show Area bounded by.

$|Arg\, z| < \frac{\pi}{4}$ &

$|z-1| < |z-3|$

① $|Arg\, z| < \frac{\pi}{4}$

$-\frac{\pi}{4} < Arg(z) < \frac{\pi}{4}$



② $|z-1| < |z-3|$

$|x+iy-1| < |x+iy-3|$

$|(x-1)+iy| < |(x-3)+iy|$

$\sqrt{(x-1)^2+y^2} < \sqrt{(x-3)^2+y^2}$

$(x-1)^2+y^2 < (x-3)^2+\cancel{y^2}$

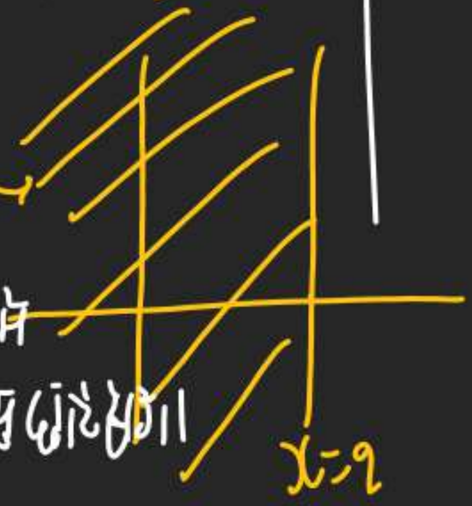
$\cancel{x^2}+1-2x < \cancel{x^2}-6x+9$

$4x < 8$

$x < 2 \rightarrow$

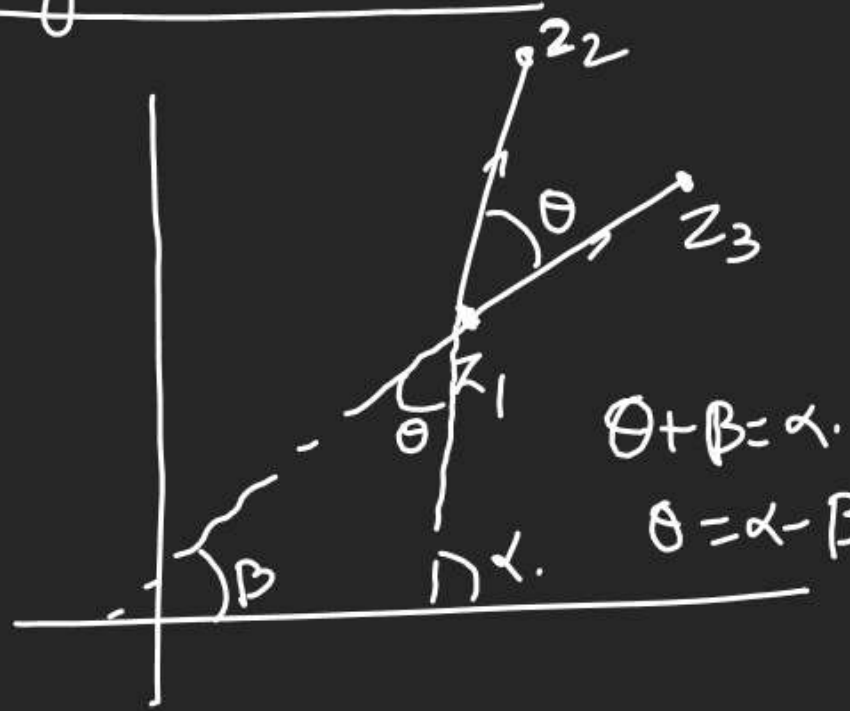
x=2 से left में

यानि x, 2 से छोटा होगा



★ Angle Betn 2 Lines.

A)



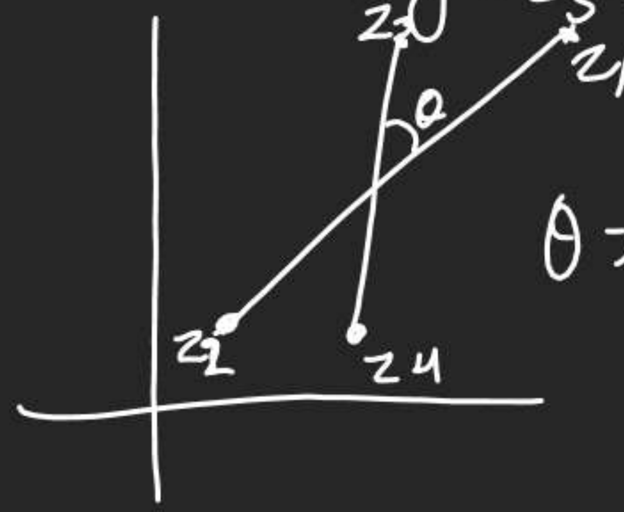
$\theta + \beta = \alpha$

$\theta = \alpha - \beta$

$\theta = Arg(z_2 - z_1) - Arg(z_3 - z_1)$

$\theta = Arg\left(\frac{z_2 - z_1}{z_3 - z_1}\right)$

(B)



$\theta = Arg\left(\frac{z_3 - z_4}{z_1 - z_2}\right)$

Nishant Jindal

Q If $|z-i|=1$ & $\operatorname{Arg} z=\frac{\pi}{2}$

then find $z=?$

$|z-i|=1$

$|x+iy-i|=1$

$|x+i(y-1)|=1$

$\sqrt{x^2+(y-1)^2}=1$

$x^2+(y-1)^2=1$

↳ Circle

Centre $(0,1)$

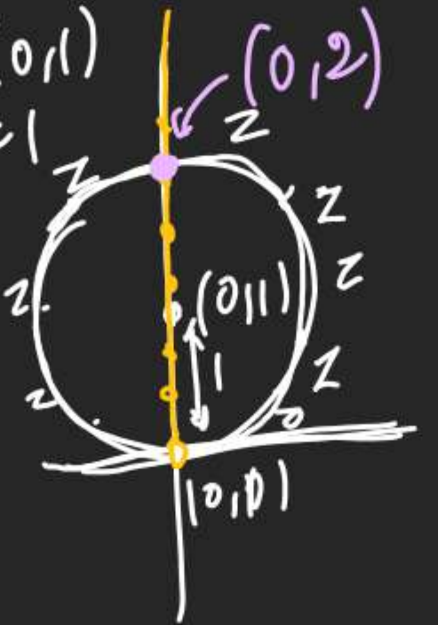
Rad $=1$

$\operatorname{Arg} z=\frac{\pi}{2}$

$\operatorname{Arg}(z-0)=\frac{\pi}{2}$

Such $z=0+2i$

$=2i$



Q find z if

$|z+3-i|=4$ & $\operatorname{Arg}(z)=\frac{\pi}{4}$

$|x+iy+3-i|=4$

$|(x+3)+i(y-1)|=4$

$\sqrt{(x+3)^2+(y-1)^2}=4$

$(x+3)^2+(y-1)^2=4^2 \longleftrightarrow y=x$

Circle

Centre $=(-3,1)$

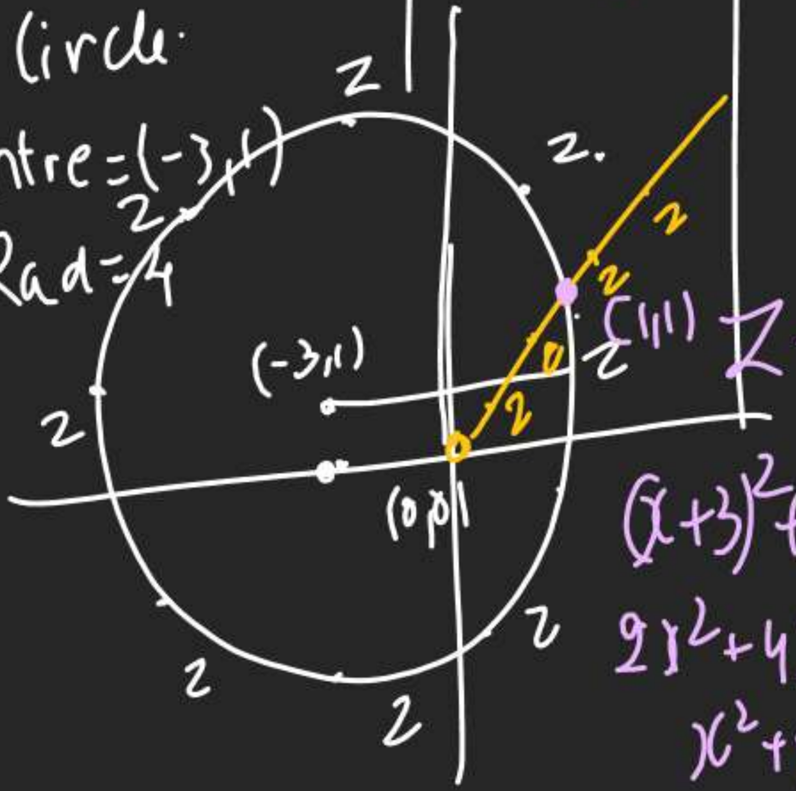
Rad $=4$

$\operatorname{Arg}(z-0)=\frac{\pi}{4}$

$\tan^{-1}\left(\frac{y}{x}\right)=\frac{\pi}{4}$

$\frac{y}{x}=\tan\frac{\pi}{4} \Rightarrow 1$

$y=x$



$z=1+i$ Ans

$(x+3)^2+(x-1)^2=4^2$

$2x^2+4x-6=0$

$x^2+2x-3=0 \Rightarrow (x+3)(x-1)=0$

$x=1, \cancel{-3}$

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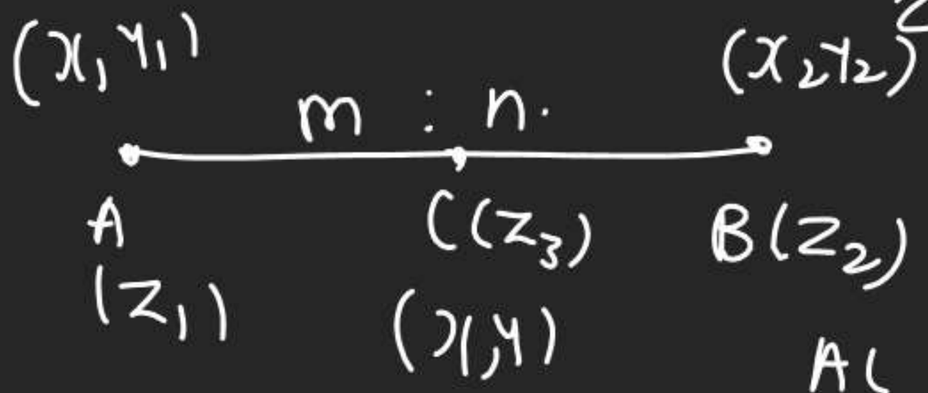
Section Formula in C.N.

$z_1 = x_1 + iy_1$

$z_2 = x_2 + iy_2$



$$\frac{AC}{BC} = \frac{m}{n}$$

$$C \equiv z_3 = \left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right)$$

$$z_3 = x + iy = \left(\frac{mx_2 + nx_1}{m+n}\right) + i\left(\frac{my_2 + ny_1}{m+n}\right)$$

$$= \frac{n(x_1 + iy_1)}{m+n} + \frac{m(x_2 + iy_2)}{m+n}$$

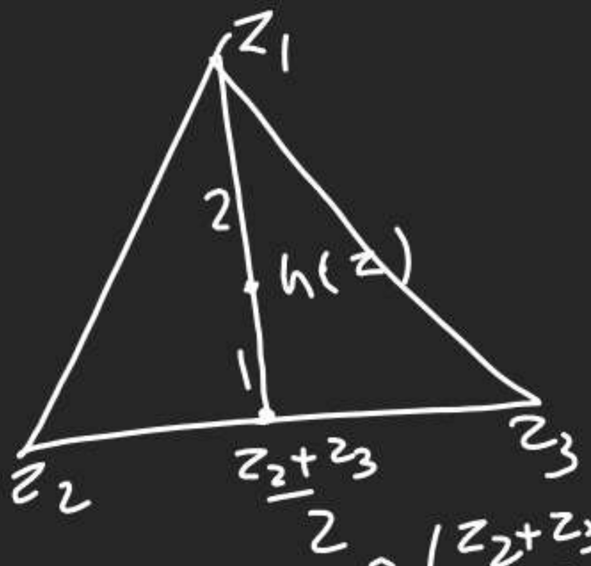
$$z_3 = \frac{mz_2 + nz_1}{m+n}$$

* MidPt.

z_3 is MidPt. z_1 & z_2

$$z_3 = \frac{z_1 + z_2}{2}$$

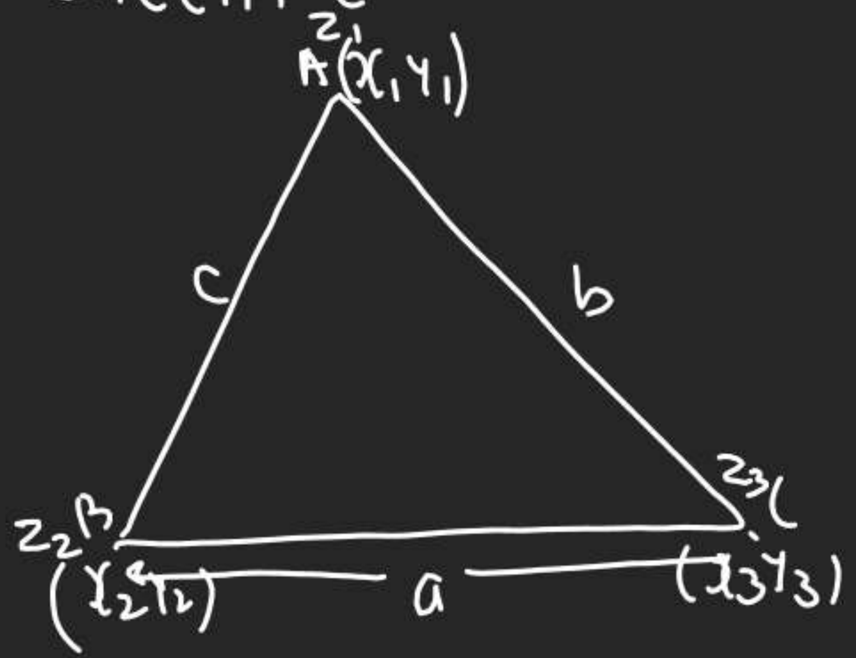
** Centroid



$$z = \frac{2\left(\frac{z_2+z_3}{2}\right) + 1\, z_1}{2+1}$$

$$\boxed{z = \frac{z_1 + z_2 + z_3}{3}}$$

Incentre.



$$(x, y) = \left\{\frac{ax_1 + bx_2 + cx_3}{a+b+c}, \frac{ay_1 + by_2 + cy_3}{a+b+c}\right\}.$$

In C.N.

$$z = \frac{az_1 + bz_2 + cz_3}{a+b+c}$$

$a =$ z_2 & z_3 के बीच dist.

$a = |z_2 - z_3|$ $\quad c = |z_2 - z_1|$

$b = |z_1 - z_3|$

Nishant Jindal

Sq. Root of a C.N.

2 Method → orthodox.
→ By formula.

Q Find value of $\sqrt{8-15i}$

* $\sqrt{C.N}$ also a C.N.

(1) ⇒ $\sqrt{8-15i} = a+ib$

⇒ $8-15i = a^2-b^2+2iab$

Compare.

$a^2-b^2=8$ | $2ab=-15$,

(2) $(a^2+b^2)^2 = (a^2-b^2)^2+4a^2b^2$

$= 64+225=289$

$a^2+b^2 = \pm 17$

$a^2+b^2=17$

$a^2-b^2=8$

$2a^2=25$

$a=\pm\frac{5}{\sqrt{2}}$ | $2b^2=9$

$b=\pm\frac{3}{\sqrt{2}}$

$a^2+b^2=-17$ ✗

3) $\sqrt{8-15i} = \pm\left(\frac{5}{\sqrt{2}}-\frac{3}{\sqrt{2}}i\right)$

$= \pm\frac{(5-3i)}{\sqrt{2}}$

$$\boxed{\sqrt{x \pm iy} = \pm\left\{\sqrt{\frac{|z|+x}{2}} \pm i\sqrt{\frac{|z|-x}{2}}\right\}}$$

$\sqrt{8-15i} = \pm\left\{\sqrt{\frac{17+8}{2}} - i\sqrt{\frac{17-8}{2}}\right\}$

$|z| = \sqrt{8^2+(-15)^2} = 17$

$= \pm\left\{\frac{5}{\sqrt{2}} - i\frac{3}{\sqrt{2}}\right\}$

Q $\sqrt{7+24i} = \pm\left\{\sqrt{\frac{25+7}{2}} + i\sqrt{\frac{25-7}{2}}\right\}$

$|z|=25$

$= \pm\{4+i3\}$

Q $\sqrt{-7+24i} = \pm\left\{\sqrt{\frac{25-7}{2}} + i\sqrt{\frac{25+7}{2}}\right\}$

$x=-7$ $|z|=25$

$= \pm\{3+4i\}$

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Q Value of $\sqrt{i}+\sqrt{-i}$

$\sqrt{0+1\cdot i}+\sqrt{0-1\cdot i}$

$x=0$ $\quad x=0$

$|z|=1$ $\quad |z|=1$

$\pm\left\{\sqrt{\frac{1+0}{2}}+i\sqrt{\frac{1-0}{2}}\right\}\pm\left\{\sqrt{\frac{1+0}{2}}-i\sqrt{\frac{1-0}{2}}\right\}$

$\pm\left\{\frac{1}{\sqrt2}+\frac{i}{\sqrt2}\right\}\pm\left\{\frac{1}{\sqrt2}-\frac{i}{\sqrt2}\right\}$

$=\pm\frac{1}{\sqrt2}(1+i)\pm\frac{1}{\sqrt2}(1-i)$

$(++)$ $\quad (+,-)$

$\frac{1}{\sqrt2}(1+\cancel{i}+1-\cancel{i})$, $\frac{1}{\sqrt2}(\cancel{1}+i-\cancel{1}+i)$

$(-,+)$ $\quad (-,-)$

$\frac{1}{\sqrt2}(\cancel{1}-i+\cancel{1}-i)$, $\frac{1}{\sqrt2}(-1\cancel{-i}-1+\cancel{i})$

$\pm\sqrt2 \quad \pm\sqrt2 i$

$$\boxed{\begin{matrix}\sqrt2, & \sqrt2 i\\ -\sqrt2, & -\sqrt2 i\end{matrix}}$$

Q Find Roots of

$z^2+2(1+2i)z-(11+2i)=0$

$z=\dfrac{-2(1+2i)\pm\sqrt{4(1+2i)^2+4(11+2i)}}{2}$

$=\dfrac{-2-4i\pm\sqrt{4(1-4+4i)+44+8i}}{2}$

$=\dfrac{-2-4i\pm\sqrt{-12+16i+44+8i}}{2}$

$=\dfrac{-2-4i\pm\sqrt{32+24i}}{2}$

$=-1-2i\pm\sqrt{8+6i}$

$=-1-2i\pm\left\{\sqrt{\frac{10+8}{2}}+i\sqrt{\frac{10-8}{2}}\right\}$

$=-1-2i\pm(3+i)$

$z=-1-2i+3+i$ $\quad \Big|\quad -1-2i-3-i$

$=2-i$ $\quad \Big|\quad -4-3i$

$SOR=-2-4i$

$=-2(1+2i)$

Nishant Jindal

Q If Eq^n $2z^2+2(i-1)=z-10$ has a Purely Img Root find other Root.

$z=$ Purely Img Root $=iy$

$2(iy)^2+2(i-1)=iy-10$

$-2y^2+2i-2=iy-10$

$-2y^2-2=-10$ | $\boxed{y=2}$

$-8-2=10$

Other Root $-2i$

different forms

5 Lectures 12th