Nishant Jindal

308.

$$\lim_{x\to-\infty}\left(\sqrt{x^2+1}-x\right) \to \infty$$

$\downarrow$

∞

$$\frac{\frac{\tan\alpha}{\alpha}}{\alpha^{1/3}\left(\frac{1-\cos\alpha}{\alpha^2}\right)^{2/3}} \to \infty$$

318.

$$\frac{\sin(\alpha^n)}{\alpha^n}\cdot\frac{\alpha^m}{(\sin\alpha)^m}$$

$$\alpha^{n-m} = \begin{cases} \text{not exist} & n-m<0 \\ 0 & n-m>0 \\ 1 & n-m=0 \end{cases}$$

$$\frac{1}{4}\times 3 = \frac{(1-\cos x)(1+\cos x+\cos^2 x)}{2x^2\left(\frac{\sin 2x}{2x}\right)}$$

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329.

$x = \frac{\pi}{2} + h$

$$\lim_{h \to 0} \frac{-\frac{\sin h}{h}}{\left(\frac{1-\cos h}{h^2}\right)^{2/3} h^{1/3}}$$

$-\cot \frac{\pi h}{2a}$

334. $a+h$

$$\lim_{h \to 0} \sin \frac{h}{2} \tan \frac{\pi}{2a}(a+h)$$

$$= -\frac{\sin h/2}{\tan \frac{\pi h}{2a}}$$

$$\sin\left(\frac{y-a}{2}\right) \cot\left(\frac{\pi}{2} - \frac{\pi y}{2a}\right)$$

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338. $$\lim_{h\to 0}\left(2\left(\frac{\pi}{2}+h\right)\tan\left(\frac{\pi}{2}+h\right)-\frac{\pi}{\cos\left(\frac{\pi}{2}+h\right)}\right)$$

$$=\lim_{h\to 0}\left(-(\pi+2h)\cot h+\frac{\pi}{\sin h}\right)$$

$$=\lim_{h\to 0}\left(-\frac{2h}{\tan h}+\pi\left(\frac{\frac{1-\cos h}{h^2}\,h}{\frac{\sin h}{h}}\right)\right)$$

$$=\boxed{-2}$$

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$$\lim_{x\to 0}\frac{1+x\sin x-\cos 2x}{\left(\sqrt{1+x\sin x}+\sqrt{\cos 2x}\right)\tan^2\frac{x}{2}} = \frac{\left(\frac{2\sin^2 x}{x^2}+\frac{x\sin x}{x^2}\right)}{\left(\sqrt{\ }+\sqrt{\ }\right)\frac{\tan^2\frac{x}{2}}{4\left(\frac{x}{2}\right)^2}}$$

50.

$$\lim_{x\to -1^+}\frac{\pi-\cos^{-1}x}{\left(\sqrt{\pi}+\sqrt{\cos^{-1}x}\right)\sqrt{x+1}}$$

$\cos^{-1}x \to \theta$

$$\lim_{\theta\to\pi^-}\frac{\pi-\theta}{\left(\sqrt{\pi}+\sqrt{\cos^{-1}x}\right)\sqrt{1+\cos\theta}}$$

$\theta=\pi-h$

$$\lim_{h\to 0^+}\frac{h}{\left(\sqrt{\pi}+\sqrt{\cos^{-1}h}\right)\sqrt{1-\cos h}}$$

$h\to 0$

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49.
$$\frac{3\left(\underbrace{\tan^{-1}3x}_{3x}+\underbrace{\sin^{-1}3x}_{3x}\right)\left(\sqrt{\quad}+\sqrt{\quad}\right)}{\left((\quad)^{2/3}+(\quad)^{2/3}+(\quad)^{1/3}(\quad)^{1/3}\right)\left(-\frac{\sin^{-1}2x}{2x}-\frac{\tan^{-1}2x}{2x}\right)\times 2}$$

$$\frac{3(2)(2)}{3(-2)2}=-1$$

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1. $\lim_{x\to 8} \dfrac{\sin\{x-10\}}{\{10-x\}} \to$ not exist $\quad \{\cdot\} = \text{FPF}$.

$\lim_{x\to 8} \dfrac{\sin\{x\}}{\{-x\}}$

$\text{LHL} = \dfrac{\to \sin 1}{\to 0} \to \infty$.

$\text{RHL} = \dfrac{\to \sin 0}{\to 1} = 0$.

$f(8-h)$

$\text{LHL} = \lim_{h\to 0} \left(\dfrac{\sin(8-h-[8-h])}{-8+h-[-8+h]} \right) = \lim_{h\to 0} \dfrac{\sin(1-h)}{h} \to \infty$.

$[8-h] \to 7$, $\quad [-8+h] = -8$

$\text{RHL} = \lim_{h\to 0} \dfrac{\sin(8+h-[8+h])}{-8-h-[-8-h]} = \lim_{h\to 0} \dfrac{\sin h}{-8-h-(-9)} = \lim_{h\to 0} \dfrac{\sin h}{1-h} = 0$

Nishant Jindal

Exponential & logarithmic limits

$$\lim_{x\to 0}\left(\frac{e^x-1}{x}\right)=1$$

$$\lim_{x\to 0}\left(\frac{a^x-1}{x}\right)=\ln a$$

$$\lim_{x\to 0}\left(\frac{e^{x\ln a}-1}{x\ln a}\right)\ln a=\boxed{\ln a}$$

$$\lim_{x\to 0}\left(\frac{\ln(1+x)}{x}\right)=1$$

$$\ln(1+x)=t$$

$$x+1=e^t$$

$$\lim_{t\to 0}\frac{t}{e^t-1}=1$$

$$\lim_{x\to 0}\frac{x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+\dots}{x}=1$$

$$\lim_{x\to 0}(1+x)^{\frac{1}{x}}=e$$

$$\lim_{x\to 0}e^{\frac{1}{x}\ln(1+x)}=e^1$$

$$\lim_{x\to 0}\frac{e^x-e^0}{x-0}$$

$$\lim_{x\to 0}\frac{\cancel{1}+\frac{x}{1!}+\frac{x^2}{2!}+\frac{x^3}{3!}+\dots\infty-\cancel{1}}{x}$$

$$=\lim_{x\to 0}\left(\frac{1}{1!}+\frac{x}{2!}+\frac{x^2}{3!}+\dots\right)=1$$

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1. $$\lim_{x\to 0}\left(\frac{e^{x^2}-\cos x}{x^2}\right)$$

$$\lim_{x\to 0}\left(\frac{e^{x^2}-1}{x^2}+\frac{1-\cos x}{x^2}\right)=1+\frac{1}{2}=\frac{3}{2}$$

2. $$\lim_{x\to\infty} x\left(e^{\frac{1}{x}}-1\right)$$

$$\lim_{x\to\infty}\frac{e^{\frac{1}{x}}-1}{\frac{1}{x}}=1$$

$\frac{1}{x}=t$

$$\lim_{t\to 0}\frac{e^{t}-1}{t}=1$$

3. $$\lim_{h\to 0}\left(\frac{a^{x+h}+a^{x-h}-2a^{x}}{h^2}\right)$$

$$a^{x}\lim_{h\to 0}\frac{a^{h}+a^{-h}-2}{h^2}=a^{x}\lim_{h\to 0}\frac{a^{2h}-2a^{h}+1}{h^2a^{h}}=a^{x}\lim_{h\to 0}\left(\left(\frac{a^{h}-1}{h}\right)^2\frac{1}{a^{h}}\right)$$

$$=a^{x}\ln^2 a$$

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4.
$$\lim_{x\to\infty}\left(\frac{e^{\frac{1}{x^2}}-1}{2\tan^{-1}(x^2)-\pi}\right)$$

$$=\lim_{x\to\infty}\frac{e^{\frac{1}{x^2}}-1}{2\left(\tan^{-1}x^2-\frac{\pi}{2}\right)}=\lim_{x\to\infty}\frac{\left(e^{\frac{1}{x^2}}-1\right)}{-2\cot^{-1}(x^2)}$$

$$=\lim_{x\to\infty}\frac{\left(e^{\frac{1}{x^2}}-1\right)}{-2\tan^{-1}\left(\frac{1}{x^2}\right)}$$

$$\lim_{x\to\infty}\frac{\frac{\left(e^{1/x^2}-1\right)}{\frac{1}{x^2}}}{-2\left(\frac{\tan^{-1}\frac{1}{x^2}}{\frac{1}{x^2}}\right)}=\boxed{-\frac{1}{2}}$$

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5. $$\lim_{x\to 0}\left(\frac{\cos(xe^{x})-\cos(xe^{-x})}{x^3}\right)$$

$f_1 g_1 + f_2 g_2$

$$\lim_{x\to 0}\frac{2\sin\frac{x(e^{-x}-e^{x})}{2}}{\frac{x}{2}(e^{-x}-e^{x})}\cdot\frac{\sin\frac{x(e^{x}+e^{-x})}{2}}{\frac{x}{2}(e^{x}+e^{-x})}\cdot\frac{\frac{x^2}{4}\left(e^{-2x}-e^{2x}\right)}{x^3}$$

$$\frac{e^{-2x}-e^{2x}}{4x}$$

$$-2=\lim_{x\to 0} 2\left(\frac{\sin(\quad)}{(\quad)}\right)\left(\frac{\sin(\quad)}{(\quad)}\right)\left(\frac{(1-e^{4x})}{e^{2x}\,4x}\right)$$

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6. $\lim_{x\to 0} \dfrac{\ln(1+x)}{(3^x-1)}$ $\lim_{x\to 0} \dfrac{\frac{\ln(1+x)}{x}}{\frac{3^x-1}{x}} = \dfrac{1}{\ln 3}$

7. $\lim_{x\to 0} (1+2x)^{\frac{5}{x}} = \lim_{x\to 0}\left((1+2x)^{\frac{1}{2x}}\right)^{10} = e^{10}$

$a^{mn} = (a^m)^n$

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8. $\lim_{x\to e}\left(\frac{(\ln x)-1}{x-e}\right)$ $\xrightarrow{} \ln e$ $= \lim_{x\to e}\frac{\ln\left(\frac{x}{e}\right)}{x-e} = \lim_{x\to e}\frac{\ln\left(1+\frac{x-e}{e}\right)}{e\cdot\frac{x-e}{e}}$

$= \frac{1}{e}$.

$\lim_{x\to e}\frac{\ln x-\ln e}{x-e} = \frac{1}{e}$.

9. $\lim_{x\to 1}(1-x)\log_x 2 = \lim_{x\to 1}(1-x)\frac{\ln 2}{\ln x} = \lim_{x\to 1}\frac{-\ln 2\cdot(x-1)}{\ln(1+(x-1))}$

$= -\ln 2$

$-\ln 2 \cdot \frac{(x-1)}{\ln x-\ln 1}$

$= -\ln 2$

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10. $\boxed{f(x)=x^x} = e^{x\ln x}$

$$\lim_{x\to a}\left(\frac{x^x-a^a}{x-a}\right) = a^a(1+\ln a)$$

$a>0$

$$\lim_{x\to a}\frac{f(x)-f(a)}{x-a} = f'(a)$$

$$f'(x) = x^x\left(x\frac{1}{x}+\ln x\right)$$

$$= x^x(1+\ln x)$$

$$= \lim_{x\to a}\frac{e^{x\ln x}-e^{a\ln a}}{x-a} = a^a\lim_{x\to a}\frac{\left(e^{x\ln x-a\ln a}-1\right)}{(x\ln x-a\ln a)}\frac{(x\ln x-a\ln a)}{(x-a)}$$

$$= a^a\lim_{x\to a}\left(\left(\quad\right)\frac{x\ln x-x\ln a+x\ln a-a\ln a}{x-a}\right)$$

$$= a^a\lim_{x\to a}\left(\left(\quad\right)\left(\frac{x\ln\left(1+\frac{x-a}{a}\right)}{a\left(\frac{x-a}{a}\right)}+\ln a\right)\right) = a^a\times1\times(1+\ln a)$$

$$= a^a(1+\ln a)$$