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$$\int \frac{dx}{x} = \ln|x| + c$$

$$\int \frac{dx}{\sqrt{x}} = 2\sqrt{x}$$

$$\int \sqrt{x}\, dx = \frac{2}{3} x^{3/2}$$

$$\int \sqrt{a^2 - x^2}\, dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1}\frac{x}{a}$$

$$\int \sqrt{x^2 - a^2}\, dx = \frac{x}{2}\sqrt{x^2 - a^2} - \frac{a^2}{2} \ln|x + \sqrt{x^2 - a^2}| + c$$

$$\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\frac{x}{a} + c$$

$$\int \frac{dx}{\sqrt{x^2 - a^2}} = \ln|x + \sqrt{x^2 - a^2}|$$

$$\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln\left|\frac{x-a}{x+a}\right|$$

$$\int \sec x = \ln\left|\tan\left(\frac{\pi}{4} + \frac{x}{2}\right)\right| + c$$

$$\int \operatorname{cosec} x \cot x\, dx = -\operatorname{cosec} x + c$$

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DPP2

Q $\int \frac{x^4+1}{x^6+1}\,dx$

$\Rightarrow \int \frac{x^4+1-x^2}{(x^2)^3+(1)^3} + \frac{1}{3}\int \frac{3x^2\,dx}{(x^3)^2+1}$

$\int \frac{\cancel{x^4+1-x^2}\,dx}{(x^2+1)\cancel{(x^4-x^2+1)}} + \frac{1}{3}\int \frac{d(x^3)}{(x^3)^2+1}$

$+\frac{1}{3}\int \frac{dy}{y^2+1}$

$+\frac{1}{3}\tan^{-1}(y)$

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Jee Main

Q If $\int \frac{dx}{(x^2+x+1)^2} = A\tan^{-1}\left(\frac{2x+1}{3}\right) + B\left(\frac{2x+1}{(x^2+x+1)^2}\right) + C$ find A & B?

$\frac{2}{3}x + \frac{1}{3}$.

$\left(\frac{1}{x^2}\right)' \to \frac{-2}{x^3}$

When A, B are hidden. Try to Diff^{te} BTS.

diff^n w

$$\frac{1}{(x^2+x+1)^2} = A \cdot \frac{1}{1+\left(\frac{2x+1}{3}\right)^2} \times \frac{2}{3} + B \times \left((2x+1) \times \frac{-2 \times (2x+1)}{(x^2+x+1)^3} + \frac{1}{(x^2+x+1)^2} \times 2\right)$$

$x=0$

$$1 = \frac{\cancel{2}}{\cancel{3}} \frac{A \cdot \cancel{9}^{3}}{10\cancel{5}} + B\left(\frac{-2}{(1)^3} + 2\right)$$

$$\boxed{A = \frac{5}{3}}$$

$$\boxed{A = \frac{5}{3},\ B = 1}$$

$x=1$

$$\frac{1}{9} = \frac{5}{3} \times \frac{\cancel{2}}{3} \times \frac{1}{\cancel{2}} + B\left(-\frac{\cancel{18}\,2}{\cancel{27}\,3} + \frac{2}{9}\right)$$

$$-\frac{4}{9} = B \times \left(-\frac{4}{9}\right) \Rightarrow B = 1$$

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Q $\int \sin x\,dx$

$= -\cos x + C$

Q $\int \cos x \cdot dx$

$= \sin x$

Q $\int \sin^2 x\,dx$

$= \int \frac{1}{2} - \frac{\cos 2x}{2}\,dx$

$= \frac{x}{2} - \frac{\sin 2x}{2\times 2} + C$

Q $\int \cos^2 x\,dx$

$= \int \frac{1}{2} + \frac{\cos 2x}{2}\,dx$

$= \frac{x}{2} + \frac{\sin 2x}{2\times 2} + C$

Q $\int \sin^3 x\,dx = \frac{3}{4}\int \sin x - \frac{1}{4}\int \sin 3x\,dx$

$\sin 3x = 3\sin x - 4\sin^3 x$

$4\sin^3 x = 3\sin x - \sin 3x$

$\sin^3 x = \frac{3}{4}\sin x - \frac{1}{4}\sin 3x$

$= -\frac{3}{4}\cos x + \frac{1}{4}\frac{\cos 3x}{3} + C$

Q $\int \cos^3 x\,dx = \frac{1}{4}\int \cos 3x\,dx + \frac{3}{4}\int \cos x\,dx$

$= \frac{1}{4}\frac{\sin 3x}{3} + \frac{3}{4}\sin x + C$

$\cos 3x = 4\cos^3 x - 3\cos x$

$\cos 3x + 3\cos x = 4\cos^3 x$

$\frac{1}{4}\cos 3x + \frac{3}{4}\cos x = \cos^3 x$

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Reduction Formula

$$\boxed{\int \tan^n x\,dx = \frac{\tan^{n-1}x}{n-1} + \int \tan^{n-2}x\,dx}$$

Q $\int \sin^4 x\,dx$

$\Rightarrow \int (\sin^2 x)^2\,dx$

$\int \sin x,\ \int \sin^2 x,\ \int \sin^3 x,\ \int \sin^4 x$

$\int \cos x,\ \int \cos^2 x,\ \int \cos^3 x,\ \int \cos^4 x$

$\int \tan x,\ \int \tan^2 x,\ \int \tan^3 x,\ \int \tan^4 x$

$\Rightarrow \int \left(\frac{1}{2} - \frac{\cos 2x}{2}\right)^2 dx$

$\Rightarrow \int \frac{1}{4} + \frac{\cos^2 2x}{4} - 2\times\frac{1}{2}\times\frac{\cos 2x}{2}\,dx$

$\Rightarrow \frac{x}{4} - \frac{1}{2}\frac{\sin 2x}{2} + \frac{1}{4}\int \cos^2(2x)\,dx$

$+ \frac{1}{4}\int \left(\frac{1}{2} + \frac{\cos 4x}{2}\right) dx$

$\frac{x}{4} - \frac{\sin 2x}{4} + \frac{1}{4}\times\frac{x}{2} + \frac{1}{4}\times\frac{\sin 4x}{4\times 2} + C$

Q $\int \cos^4 x\,dx$

$\Rightarrow \int \left(\frac{1}{2} + \frac{\cos 2x}{2}\right)^2 dx$

DY

Q $\int \tan^2 x\,dx$

$\int \sec^2 x - 1\,dx$

$\Rightarrow \tan x - x + C$

Q $\int \cot^2 x\,dx$

$= \int \csc^2 x - 1\,dx$

Q $\int \tan^3 x\,dx$

$= \frac{\tan^2 x}{2} + \int \tan x\,dx$

$= \frac{\tan^2 x}{2} + \ln|\sec x| + C$

Q $\int \tan^4 x\,dx = \frac{\tan^3 x}{3} + \int \tan^2 x\,dx$

$= \frac{\tan^3 x}{3} + \tan x - x + C$

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H/W

Q $\int \tan^6 x \cdot dx$

Q $\int \sin 3x \cdot \cos 4x \, dx$

S.C, C.S, S.S, C.C } ① 1st Multiply by 2 ② Change Pro into Sum / diff 3) Integrate

$\sin C + \sin D = 2 \sin\left(\frac{C+D}{2}\right) \cos\left(\frac{C-D}{2}\right)$

$\sin C - \sin D = 2 \cos(\;,,\;) \sin(\;,,\;)$

$\cos C + \cos D = 2 \cos(\;,,\;) \cos(\;,,\;)$

$\cos C - \cos D = -2 \sin(\;,,\;) \sin(\;,,\;)$

$\frac{1}{2}\int 2 \sin 3x \cos 4x \, dx$

$\frac{1}{2}\int \sin(7x) + \sin(-x) \, dx$

$\frac{1}{2}\int \sin 7x - \sin x \, dx$

$\frac{1}{2}\left[-\frac{\cos 7x}{7} + \cos x\right] + C$

Q $\int \cos 2x \cdot \cos 3x \cdot dx$

$\frac{1}{2}\int 2\cos 2x \cdot \cos 3x \, dx$

$\frac{1}{2}\int \cos(5x) + \cos(+x) \, dx$

$\frac{1}{2}\left[\frac{\sin 5x}{5} + \sin x\right] + C$

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Q $\int \tan 3x \cdot \tan 2x \cdot \tan x \cdot dx$

$\int \tan 3x - \tan 2x - \tan x \, dx$

$\Rightarrow \frac{\ln|\sec 3x|}{3} - \frac{\ln|\sec 2x|}{2} - \ln|\sec x| + c$

$\tan 3x = \tan(2x + x)$

$\tan 3x = \frac{\tan 2x + \tan x}{1 - \tan 2x \cdot \tan x}$

$\tan 3x - \tan 3x \cdot \tan 2x \cdot \tan x = \tan 2x + \tan x$

$\tan 3x - \tan 2x - \tan x = \int \tan 3x \cdot \tan 2x \cdot \tan x$

Q $\int \tan 8\theta \cdot \tan 6\theta \cdot \tan 2\theta \cdot d\theta$

sum

$\int \tan 8\theta - \tan 6\theta - \tan 2\theta \, d\theta$

$\frac{\ln|\sec 8\theta|}{8} - \frac{\ln|\sec 6\theta|}{6} - \frac{\ln|\sec 2\theta|}{2} + c$

Q $\int \frac{dx}{\cot\frac{x}{2} \cdot \cot\frac{x}{3} \cdot \cot\frac{x}{6}}$

sum

$\int \tan\frac{x}{2} \cdot \tan\frac{x}{3} \cdot \tan\frac{x}{6} \, dx$

$\int \tan\frac{x}{2} - \tan\frac{x}{3} - \tan\frac{x}{6} \, dx$

$\frac{\ln|\sec\frac{x}{2}|}{\left(\frac{1}{2}\right)} - \frac{\ln|\sec\frac{x}{3}|}{\frac{1}{3}} - \frac{\ln|\sec\frac{x}{6}|}{\frac{1}{6}} + c$

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Q $\int \sin x \cdot \cos x \cdot \cos 2x \cdot \cos 4x \cdot \cos 8x \, dx \longrightarrow \frac{1}{2^{5-1}} \int \sin 2^{5-1} x \, dx$

$\frac{1}{2} \int (2 \sin x \cdot \cos x) \cdot \cos 2x \cos 4x \cdot \cos 8x \, dx \Rightarrow \frac{1}{16} \int \sin 16x \, dx$

$\frac{1}{2 \times 2} \int (2 \sin 2x \cdot \cos 2x) \cdot \cos 4x \cdot \cos 8x \, dx \Rightarrow \frac{1}{16} x^{-\frac{\cos 16x}{16}} + c$

$\frac{1}{2 \times 4} \int (2 \sin 4x \cdot \cos 4x) \cdot \cos 8x \, dx$

$\frac{1}{2 \times 8} \int 2 \sin 8x \cdot \cos 8x \, dx$

$\frac{1}{16} \int \sin 16x = \frac{1}{16} x^{-\frac{\cos 16x}{16}} + c$

Q NCERT. $\int \frac{1 \, dx}{\sin^2 x \cdot \cos^2 x}$

$\int \frac{\sin^2 x + \cos^2 x}{\sin^2 x \cdot \cos^2 x} dx$

div

$\int \frac{\cancel{\sin^2 x}}{\cancel{\sin^2 x} \cdot \cos^2 x} + \frac{\cancel{\cos^2 x}}{\sin^2 x \cdot \cancel{\cos^2 x}} dx$

$\int \sec^2 x + \operatorname{cosec}^2 x \, dx$

$\tan x - \cot x + c$

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Q $\int \frac{\sin^3 x + \cos^3 x}{\sin^2 x \cdot \cos^2 x} dx$

$\int \frac{\cancel{\sin^3 x}^{\sin x}}{\cancel{\sin^2 x} \cdot \cos^2 x} + \frac{\cancel{\cos^3 x}^{\cos x}}{\sin^2 x \cdot \cancel{\cos^2 x}} dx$

$\int \sec x \cdot \tan x + \operatorname{cosec} x \cot x \, dx$

$\sec x - \operatorname{cosec} x + c$

Q $\int \frac{\sin^4 x + \cos^4 x}{\sin^2 x \cdot \cos^2 x} \cdot dx$

$\int \frac{\cancel{\sin^4 x}^{\sin^2 x}}{\cancel{\sin^2 x} \cdot \cos^2 x} + \frac{\cancel{\cos^4 x}^{\cos^2 x}}{\sin^2 x \cdot \cancel{\cos^2 x}} dx$

$\int \tan^2 x + \cot^2 x$

$\int (\sec^2 x - 1 + \operatorname{cosec}^2 x - 1) dx$

$\therefore \tan x - \cot x - 2x + c$

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Q $\displaystyle\int \frac{\sin^6 x + \cos^6 x}{\sin^2 x \cdot \cos^2 x}\,dx$

$\sin^4 x + \cos^4 x = 1 - 2\sin^2 x \cdot \cos^2 x$

$\sin^6 x + \cos^6 x = 1 - 3\sin^2 x \cdot \cos^2 x$

$\displaystyle\int \frac{1 - 3\sin^2 x \cdot \cos^2 x}{\sin^2 x \cdot \cos^2 x}$

$\displaystyle\int \frac{1}{\sin^2 x \cdot \cos^2 x} - 3\,dx \qquad 1 = S^2 + C^2$

$\Rightarrow \tan x - \cot x - 3x + C$

$\boxed{6B}$

Q $\displaystyle\int \frac{\sin^8 x - \cos^8 x}{1 - 2\sin^2 x \cdot \cos^2 x}\,dx$

$\displaystyle\int \frac{(\sin^4 x - \cos^4 x)\cancel{(\sin^4 x + \cos^4 x)}}{\cancel{\sin^4 x + \cos^4 x}}$

$\displaystyle\int (\sin^2 x - \cos^2 x)\underbrace{(\sin^2 x + \cos^2 x)}_{1}\,dx$

$-\displaystyle\int (\cos^2 x - \sin^2 x)\,dx = -\int \cos 2x$

$= -\dfrac{\sin 2x}{2}$

$\cos 2x = C^2 - S^2$

$= 1 - 2S^2$

$= 2C^2 - 1$

$= \dfrac{1 - T^2}{1 + T^2}$

$\sin 2x = 2SC$

$= \dfrac{2T}{1 + T^2}$

$\tan 2x = \dfrac{2T}{1 - T^2}$

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Q $\displaystyle\int \frac{1+\cos 4x}{\cot x-\tan x}\cdot dx$

$$\boxed{\begin{aligned}1+\cos 2x&=2\cos^2 x\\1-\cos 2x&=2\sin^2 x\end{aligned}}$$

$$\int \frac{2\cos^2(2x)}{\frac{c}{s}-\frac{s}{c}}$$

$$\int \frac{2\cos^2(2x)\,dx}{\frac{(\cos^2 x-\sin^2 x)}{\sin x\cdot\cos x}}$$

$$\int \frac{2\sin x\cdot\cos x\cdot\cos^2(2x)}{\cancel{(\cos 2x)}}\quad 1$$

$$\frac{1}{2}\int 2\sin 2x\cdot\cos 2x$$

$$\frac{1}{2}\int \sin 4x=-\frac{\cos 4x}{8}+c$$

Q good $\displaystyle\int \frac{\cos 2x-\cos 2\theta}{\cos x-\cos\theta}\cdot dx$

$$\int \frac{(2\cos^2 x-\cancel{1})-(2\cos^2\theta-\cancel{1})}{\cos x-\cos\theta}\cdot dx$$

$$2\int \frac{\overset{\cos x+\cos\theta}{\cancel{\cos^2 x-\cos^2\theta}}}{\cancel{\cos x-\cos\theta}}\cdot d\theta$$

$$2\int \cos x\cdot dx+2\cos\theta\int dx$$

$$2\sin x+2\cos\theta\cdot x+C$$

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Q $\int \frac{1-\tan^2 x}{1+\tan^2 x} \cdot dx$

$\Rightarrow \int \cos 2x \, dx$

$\Rightarrow \frac{\sin 2x}{2} + C$

Q $\int \frac{1+\tan x}{1-\tan x} \cdot dx$

$\int \tan\left(\frac{\pi}{4}+x\right) \cdot dx$

$\frac{\ln\left|\sec\left(\frac{\pi}{4}+x\right)\right|}{2} + C$