Nishant Jindal

Energy density of transverse & Longitudinal wave

$$y = A\sin(\omega t - Kx)$$

$$\boxed{K.E \text{ per Unit Volume} = \frac{1}{2}\rho\left(\frac{\partial y}{\partial t}\right)^2} \checkmark$$

$$\boxed{P.E \text{ per Unit Volume} = \frac{1}{2}\rho v^2\left(\frac{\partial y}{\partial s}\right)^2} \checkmark$$

$$\frac{\partial y}{\partial s} = -\frac{1}{v}\left(\frac{\partial y}{\partial t}\right)$$

$$\frac{\partial y}{\partial t} = -v\left(\frac{\partial y}{\partial s}\right)$$

(K.E per Unit Volume = P.E per Unit Volume)

Nishant Jindal

Avg. K.E per unit volume

$$y = A \sin(\omega t - Kx)$$

$$K.E \text{ per Unit Volume} = \frac{1}{2}\rho\left(\frac{\partial y}{\partial t}\right)^2$$

$$\frac{\partial y}{\partial t} = A\omega \cos(\omega t - Kx)$$

K.E per Unit Volume

$$\frac{1}{2}\rho A^2\omega^2 \cos^2(\omega t - Kx)$$

Avg K.E per Unit Volume b/w $x = 0$ to $x = \lambda$

$$(K.E_{avg})_{\text{per Unit Volume}} = \frac{\frac{1}{2}\rho A^2\omega^2 \int_0^{\lambda} \underbrace{\cos^2(\omega t - Kx)\, dx}_{\frac{\lambda}{2}}}{\int_0^{\lambda} dx}$$

$$(K.E_{avg})_{\text{per Unit Volume}} = \frac{\frac{1}{2}\rho A^2\omega^2 \cdot (\cancel{\lambda}/2)}{\cancel{\lambda}}$$

$$\boxed{(K.E)_{\text{avg per Unit Volume}} = \frac{1}{4}\rho A^2\omega^2}$$

A = Amplitude

ρ = density of Medium

$\omega = 2\pi f$

$$\boxed{\text{Avg. P.E per Unit Volume.} = \frac{1}{4}\rho A^2\omega^2}$$

Total Energy density

$$E_T = (K\cdot E)_{\substack{\text{per unit}\\\text{Volume}}} + (P\cdot E)_{\substack{\text{per Unit}\\\text{Volume}}}$$

$$\boxed{E_T = \frac{1}{2}\rho\left(\frac{\partial y}{\partial t}\right)^2 + \frac{1}{2}\rho\left(v\frac{\partial y}{\partial x}\right)^2}$$

$$(E_T)_{avg} = \frac{1}{4}\rho A^2\omega^2 + \frac{1}{4}\rho A^2\omega^2$$

$$\boxed{(E_T)_{avg} = \frac{1}{2}\rho A^2\omega^2}$$

Nishant Jindal

$\Delta\Delta$:

$$\text{Intensity} = \frac{\text{Energy}}{(\text{time}).(\text{Area})}$$

$$I = \left(\frac{\text{Energy}}{(\text{Volume})}\right) \times \frac{\text{Area} \times \text{time}}{(\text{Volume})}$$

$dV = A\,dx$

$$I = \frac{\text{Energy density}}{\frac{\cancel{\text{Area}} \times \text{time}}{\cancel{\text{Area}} \times \text{displacement}}}$$

$$\boxed{I = \frac{1}{2}\rho A^2 \omega^2 \cdot v}$$

$$\boxed{I \propto A^2}$$

For transverse Wave $v = \sqrt{\frac{T}{\mu}}$

For Longitudinal $v = \sqrt{\frac{B}{\rho}}$.

$$\boxed{I = \text{Energy density} \times \text{Velocity of wave propagation}}$$

Nishant Jindal

For Longitudinal wave

$$B K S_0 = P_0$$

$\Downarrow$ displacement amplitude

$\Downarrow$ Excess pressure amplitude

$$I = \left(\frac{1}{2}\rho\omega^2 S_0^2\right)\cdot v$$

$$B\frac{\omega}{v}\cdot S_0 = P_0$$

$$\omega S_0 = \left(\frac{P_0 v}{B}\right)$$

$$v = \sqrt{\frac{B}{\rho}}$$

$$v^2 = \frac{B}{\rho}$$

$$I = \frac{1}{2}\rho\frac{P_0^2 v^2}{B^2}\times v$$

$$\boxed{I = \frac{P_0^2}{2\rho v}}$$

$$\boxed{I \propto P_0^2}$$

Nishant Jindal

STANDING WAVE

- **Condition**

Two wave pulse of same amplitude travelling in opposite direction intefere to give standing wave.

Properties of Standing wave

- Energy Confined b/w two points.
- Points which are at rest are called Nodes.
- Points which are at it's maximum displacement (Amplitude) are called AntiNodes.
- Particles b/w any two nodes Vibrate in Same phase.
- Distance b/w two nodes is $\frac{\lambda}{2}$.

Nishant Jindal

#

$$y_1 = A\sin(kx - \omega t)$$

$$y_2 = A\sin(kx + \omega t)$$

$$y_R = y_1 + y_2$$

$$y_R = A[\sin(kx - \omega t) + \sin(kx + \omega t)]$$

$$y_R = A[2\sin kx \cdot \cos\omega t]$$

$$\boxed{y_A = \underbrace{2A\sin kx}_{\text{Amplitude of Standing wave}} \cdot \cos\omega t}$$

Amplitude of Standingwave $= (2A\sin kx)$

For Nodes

$$2A\sin kx = 0$$

$$\sin kx = 0$$

$$kx = n\pi$$

$$\frac{2\cancel{\pi}}{\lambda}x = n\cancel{\pi}$$

$$\boxed{x = \frac{n\lambda}{2}}$$

$n = 0, 1, 2, 3, 4 \ldots$

Nodes $= 0, \frac{\lambda}{2}, \lambda, \frac{3\lambda}{2}, 2\lambda \ldots$

$\frac{\lambda}{2}$ $\lambda/2$ $\lambda/2$

For AntiNodes

Amplitude maximum

$$\sin kx = \pm 1$$

$$kx = (2n+1)\frac{\pi}{2} \;/\; (2n-1)\frac{\pi}{2}$$

$k = \frac{2\pi}{\lambda}$

$n = 0, 1, 2.$ $\quad$ $n = 1, 2, 3$

$$\boxed{x = (2n+1)\frac{\lambda}{4} \text{ or } (2n-1)\frac{\lambda}{4}}$$

$n = 0, 1, 2, 3.$ $\quad$ $n = 1, 2, 3$

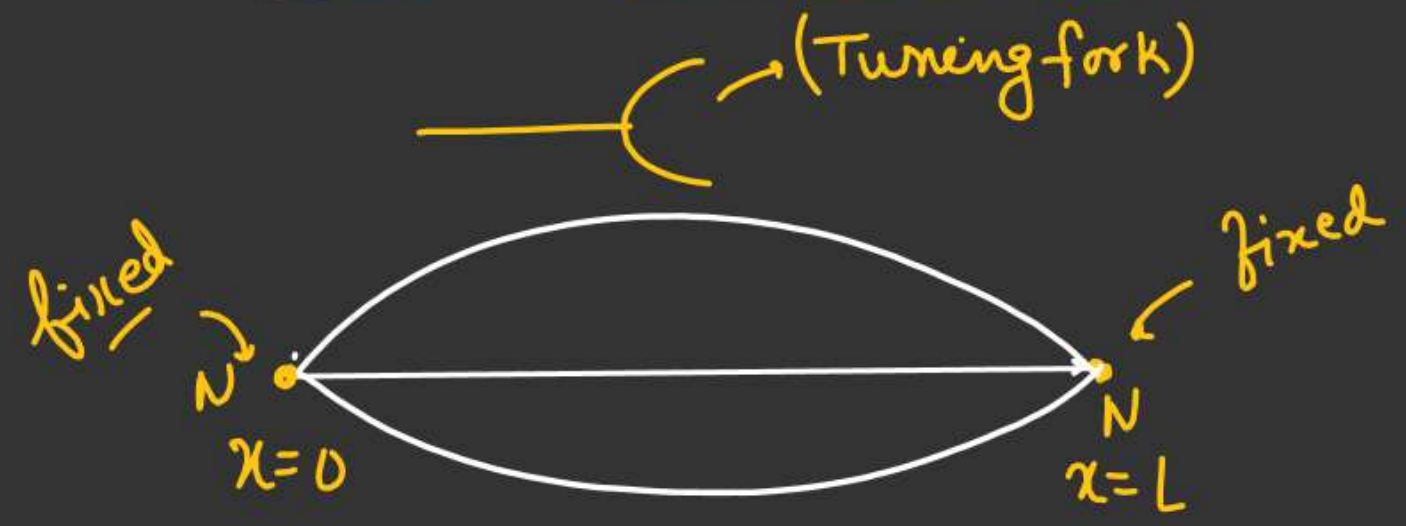
Nishant Jindal

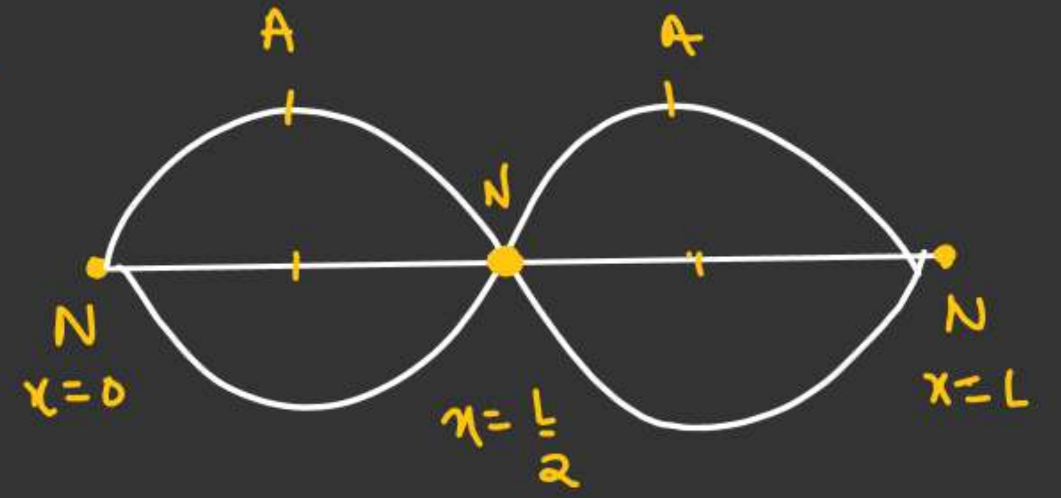
Standing wave in a String

In Resonating condition

$$\boxed{\left(f_{string} = f_{\substack{tuning\\fork}}\right)}$$

Case-1 :- String fixed at both ends





$$L = \frac{\lambda}{2}$$

$$v = f_0\lambda$$

$$\lambda = \frac{v}{f_0}$$

$$L = \frac{v}{2f_0}$$

$$\boxed{f_0 = \frac{v}{2L}}$$

Fundamental frequency
1st harmonic

$$L = \lambda$$

$$L = \frac{v}{f_1}$$

$$f_1 = \frac{v}{L} = 2f_0$$

↳ 2nd harmonic
or
1st overtone

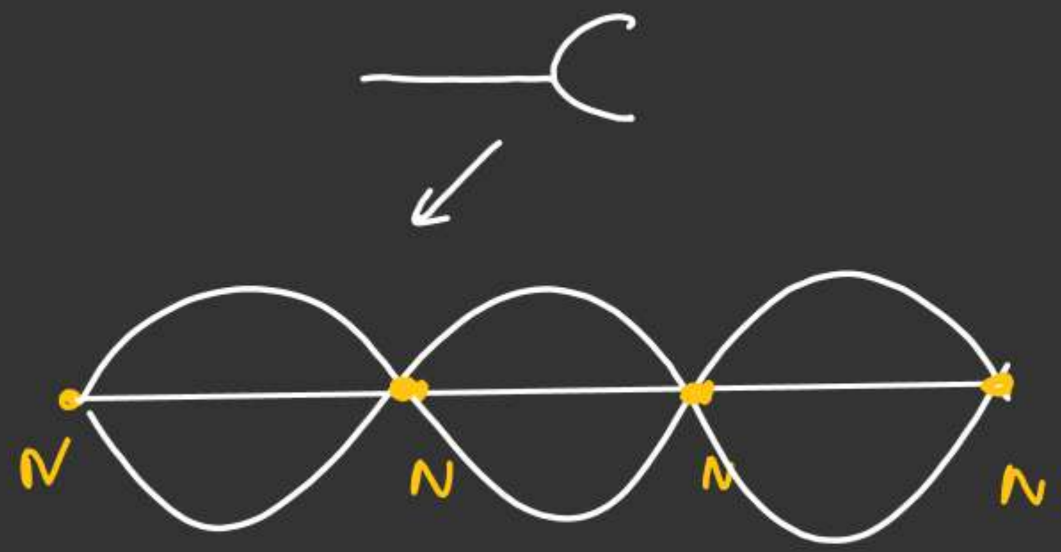
Antinodes

$$x = \frac{\lambda}{4}$$

$$x = \frac{L}{4},$$

$$x = \left(\frac{L}{2}+\frac{L}{4}\right) = \frac{3L}{4}.$$

Nishant Jindal



$$L = \frac{3\lambda}{2}$$

$$L = \frac{3}{2}\frac{v}{f}$$

$$f = \frac{3v}{2L}$$

$$f = 3f_0$$

$\Downarrow$

3rd harmonic or
2nd overtone.

$\Rightarrow$ In string fixed at both ends of the integral multiple of fundamental frequency are the overtone.

$$\rho = \frac{m}{AL}$$

$$f = \frac{nv}{2L} = nf_0$$

$$\mu = \frac{m}{L} = \rho A$$

$$\boxed{f = \frac{n}{2L}\sqrt{\frac{T}{\mu}} = \frac{n}{2L}\sqrt{\frac{T}{\rho A}}}$$

Nishant Jindal

Case-2 :- String fixed at one end

$x = (2n-1)\dfrac{\lambda}{4}$ (Free to oscillate)

$n = 1, 2, 3 \ldots$

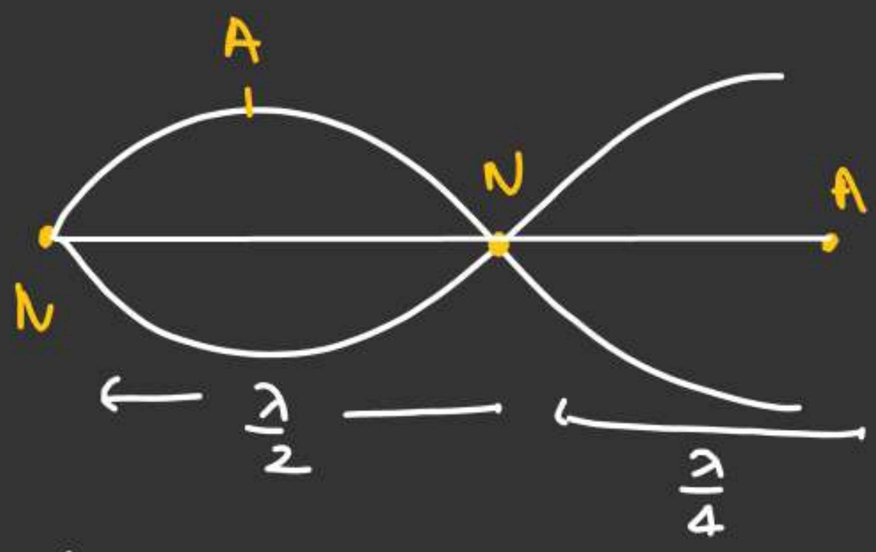
$\underline{n=1}$

$L = \dfrac{\lambda}{4} = \dfrac{v}{4f_0}$

$\boxed{f_0 = \dfrac{v}{4L}}$ → Fundamental frequency or 1st harmonic

$\underline{n=2}$

$L = \dfrac{3\lambda}{4} = \underline{\dfrac{\lambda}{2} + \dfrac{\lambda}{4}}$



$L = \dfrac{3\lambda}{4} = \dfrac{3}{4}\dfrac{v}{f}$

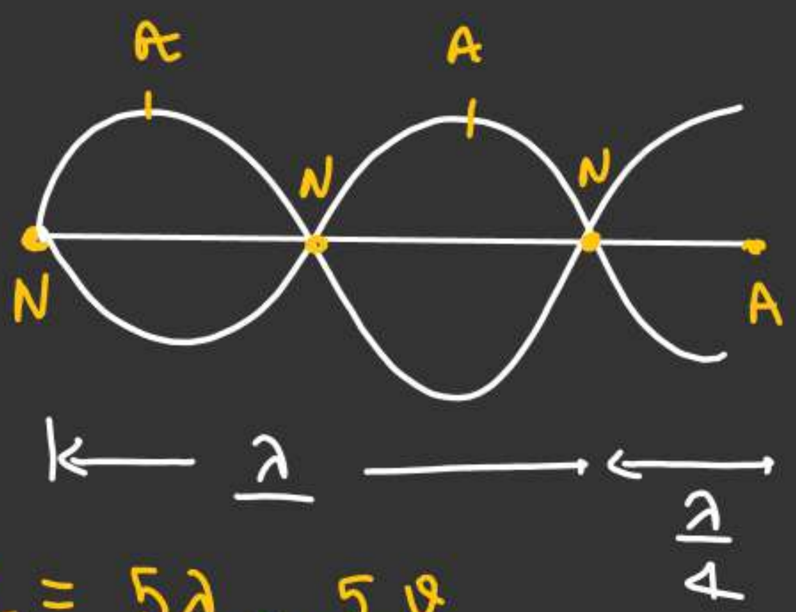
$f = \dfrac{3v}{4L} = 3f_0$

⇓

2nd harmonic or 1st overtone.

$\underline{n=3}$

$L = \dfrac{5\lambda}{4} = \lambda + \dfrac{\lambda}{4}$



$L = \dfrac{5\lambda}{4} = \dfrac{5}{4}\dfrac{v}{f}$

$f = \dfrac{5v}{4L} = \underline{5f_0}$

⇓

3rd harmonic or 2nd overtone

⋮

Nishant Jindal

In string fixed at one end only odd harmonics are the allowed overtone.

In general, string fixed at one end

$$\boxed{f = (2n-1)\frac{v}{4L}} \quad n = 1, 2, 3 \ldots$$

or

$$\boxed{f = (2n+1)\frac{v}{4L}} \quad n = 0, 1, 2, 3 \ldots$$

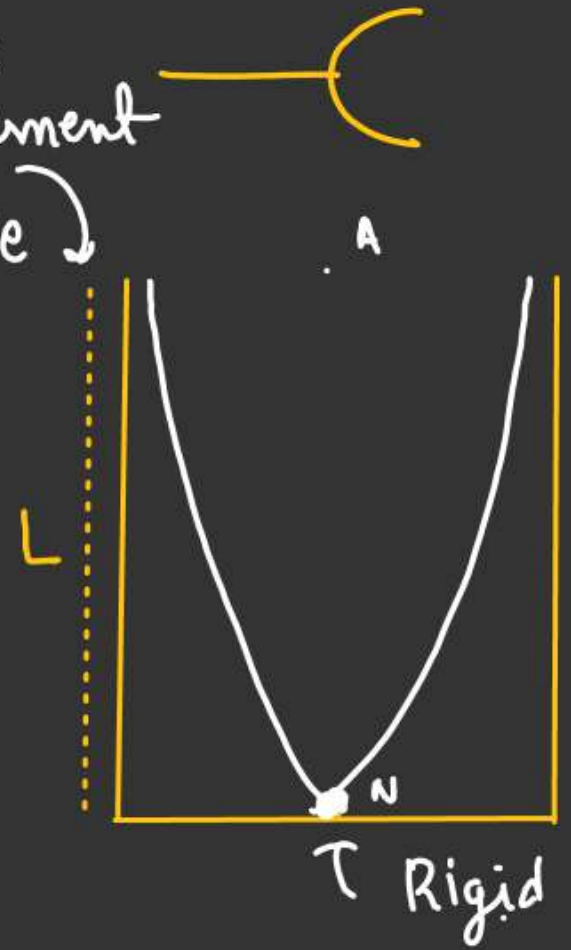
$$\left(v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{T}{\rho A}} \right).$$

Nishant Jindal

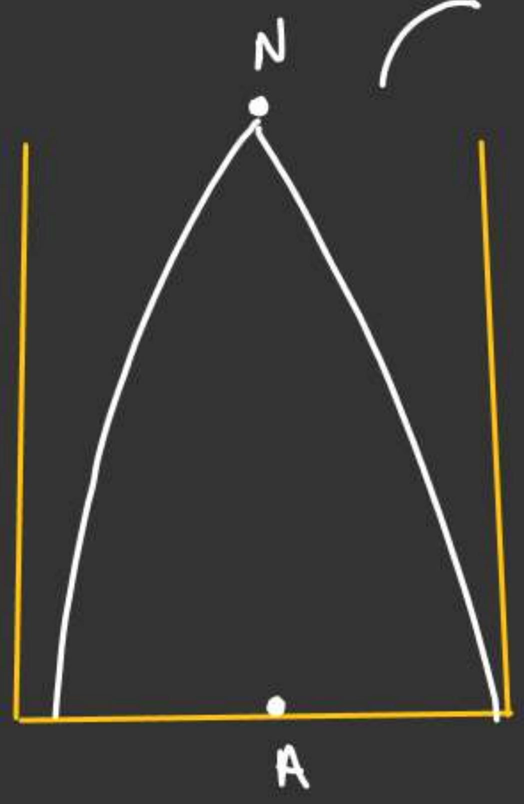
Standing wave in Organ pipe

Case-1 :- Close organ pipe (one end closed)

In terms of displacement of particle



In terms of pressure amplitude



Displacement Amplitude = 0 (N)

Pressure amplitude = maxium (A)

$$L = \frac{\lambda}{4}$$

In general

$$\boxed{f = (2n-1)\frac{v}{4L}}$$

Same as string fixed at one end

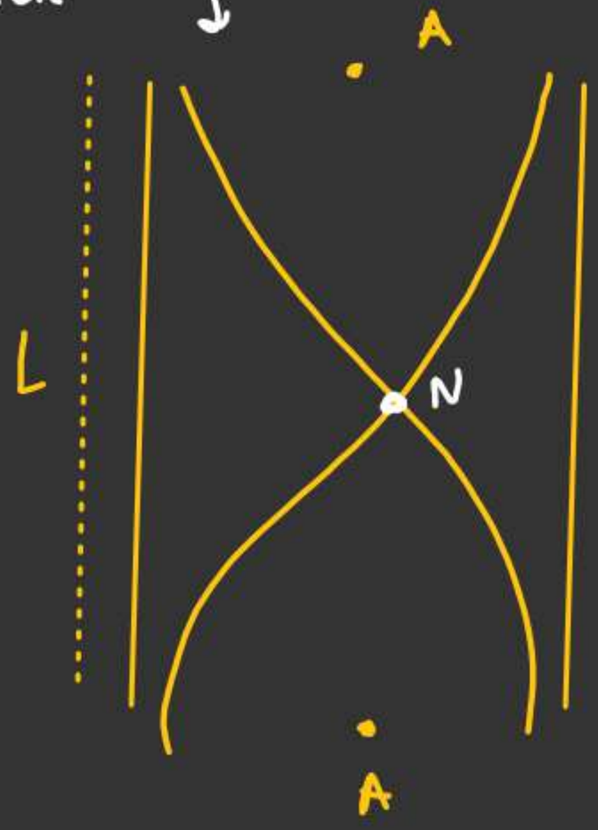
$$v = \sqrt{\frac{\gamma RT}{M}} = \sqrt{\frac{P}{\rho}}$$

$f = 3f_0, 5f_0,$

Nishant Jindal

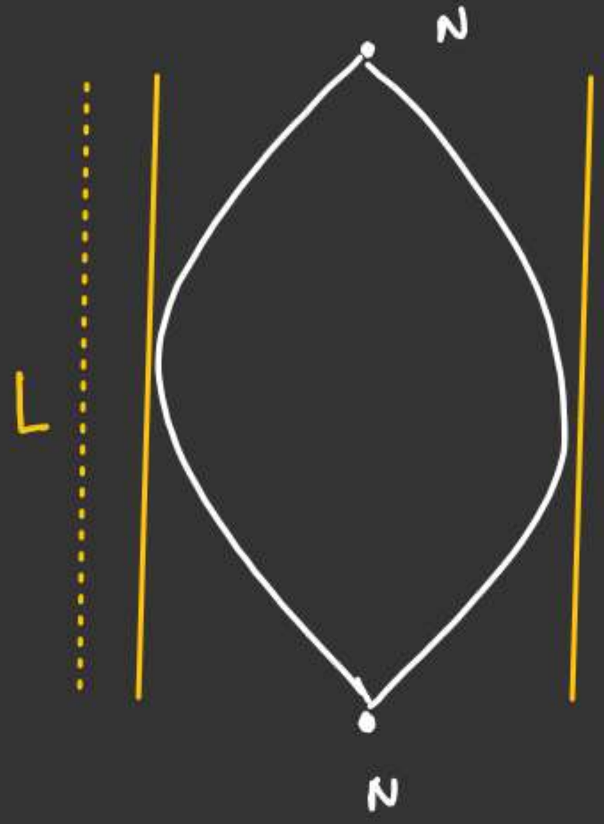
1. Open organ pipe (open at both end)

In terms of displacement of particle



$L = \lambda$

In terms of pressure amplitude.



In general

$$\boxed{f = \frac{nv}{2L}}$$

Same as string fixed at both end.

$$v = \sqrt{\frac{\gamma RT}{M}} = \sqrt{\frac{P}{\rho}}$$

Nishant Jindal

Beats

$y_1 = A\sin(\omega_1 t - \underline{Kx})$

$y_2 = A\sin(\omega_2 t - \underline{Kx})$

Interferene at $x=0$

$y_1 = A\sin\omega_1 t$

$y_2 = A\sin\omega_2 t$

$y_R = y_1 + y_2$

$= A(\sin\omega_1 t + \sin\omega_2 t)$

$= 2A\sin\left(\frac{\omega_1+\omega_2}{2}\right)t\ \cos\left(\frac{\omega_1-\omega_2}{2}\right)t$

$y_R = 2A\cos\left(\frac{\omega_1-\omega_2}{2}\right)t \cdot \sin\left(\frac{\omega_1+\omega_2}{2}\right)t$

Amplitude

$\underline{f_1 > f_2}$. $\quad \omega_1 = 2\pi f_1 t, \quad \omega_2 = 2\pi f_2 t$

For Amplitude to be maximum.

$\cos\frac{2\pi(f_1-f_2)t}{2} = 1$

$\underline{\underline{t = 0}}$.

$\cos\frac{2\pi(f_1-f_2)t}{2} = -1$

$\frac{\cancel{2}\cancel{\pi}(f_1-f_2).t}{\cancel{2}} = \cancel{\pi}$

$t = \left(\frac{1}{f_1-f_2}\right)$

Time interval b/w two Consecutive maxima

$\Delta t = t - 0 = \frac{1}{f_1-f_2}$

Beat frequency $= \frac{1}{\Delta t}$

$\Downarrow$

(No of beat per second)

$= |f_1 - f_2|$