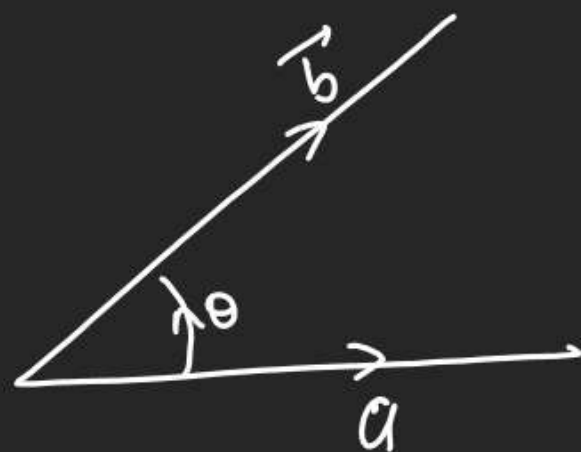


① Dot Product

$$1) \vec{a} \cdot \vec{b} = |\vec{a}| |\vec{b}| \cos \theta$$

$$0 \leq \theta < \pi$$

$$2) \vec{a} \cdot \vec{b} = |\vec{a}| |\vec{b}| \cos \theta = \text{Scalar}$$

\downarrow \downarrow \downarrow
 Sc. Sc. Sc.

$$(3) \text{ Angle bet}^n \vec{a} \text{ \& } \vec{b} = \cos \theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| |\vec{b}|}$$

$$(4) \vec{a} = a_1 \hat{i} + a_2 \hat{j} + a_3 \hat{k}$$

$$\vec{b} = b_1 \hat{i} + b_2 \hat{j} + b_3 \hat{k}$$

$$\vec{a} \cdot \vec{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$$

$$(5) \cos \theta = \frac{a_1 b_1 + a_2 b_2 + a_3 b_3}{\sqrt{a_1^2 + a_2^2 + a_3^2} \sqrt{b_1^2 + b_2^2 + b_3^2}}$$

$$\cos \theta = \frac{\sum a_i b_i}{\sqrt{\sum a_i^2} \sqrt{\sum b_i^2}}$$

$$(6) \vec{a}^2 = ?$$

$$\vec{a}^2 = \vec{a} \cdot \vec{a}$$

$$= |\vec{a}| |\vec{a}| \cos 0^\circ$$

$$(\vec{a})^2 = (a_1^2 + a_2^2 + a_3^2) = |\vec{a}|^2$$

Scalar

$$(7) (\vec{a} + \vec{b})^2 = ?$$

$$(\vec{a} + \vec{b}) \cdot (\vec{a} + \vec{b})$$

$$|\vec{a}|^2 + |\vec{b}|^2 + \vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{a}$$

$$|\vec{a}|^2 + |\vec{b}|^2 + 2\vec{a} \cdot \vec{b}$$

$$(8) \vec{a} \cdot \vec{b} = |\vec{a}| |\vec{b}| \cos \theta$$

$\theta = 0$	$\theta = \pi$	$\theta = \frac{\pi}{2}$
$\vec{a} \cdot \vec{b} = \vec{a} \vec{b} $	$\vec{a} \cdot \vec{b} = \vec{a} \vec{b} \cos \pi$	$\vec{a} \perp \vec{b}$
Max.	$\vec{a} \cdot \vec{b} = - \vec{a} \vec{b} $	$\vec{a} \cdot \vec{b} = 0$
	Min.	

$$-|\vec{a}| |\vec{b}| \leq \vec{a} \cdot \vec{b} \leq |\vec{a}| |\vec{b}|$$

$$(9) (\vec{a} \cdot \vec{b})^2 \in ?$$

$$\vec{a} \cdot \vec{b} = |\vec{a}| |\vec{b}| \cos \theta$$

$$(\vec{a} \cdot \vec{b})^2 = |\vec{a}|^2 |\vec{b}|^2 \cos^2 \theta$$

$$0 \leq \cos^2 \theta \leq 1$$

$$0 \leq |\vec{a}|^2 |\vec{b}|^2 \cos^2 \theta \leq |\vec{a}|^2 |\vec{b}|^2$$

$$0 \leq (\vec{a} \cdot \vec{b})^2 \leq |\vec{a}|^2 |\vec{b}|^2$$

Range.

(10) $\theta = \text{acute angle.}$

$$\vec{a} \cdot \vec{b} > 0$$

obtuse angle $\rightarrow \cos \theta < 0$

$$\vec{a} \cdot \vec{b} < 0$$

$$11) \vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}$$

$$\vec{a} \cdot (\vec{b} + \vec{c}) = \vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c}$$

$$\vec{a} \perp \vec{b} \Rightarrow \vec{a} \cdot \vec{b} = 0$$

But opposite is not true.

$$\vec{a} \cdot \vec{b} = 0$$

$$\begin{array}{c} \downarrow \quad \downarrow \quad \downarrow \\ \vec{a} = 0 \quad \vec{b} = 0 \quad \vec{a} \perp \vec{b} \end{array}$$

$$Q. \vec{a} \cdot \hat{i} = ?$$

$$(a_1 \hat{i} + a_2 \hat{j} + a_3 \hat{k}) \cdot \hat{i}$$

$$a_1 + 0 + 0 = a_1$$

$$Q. \vec{a} \parallel \vec{b} \text{ then.}$$

$$\frac{a_1}{b_1} = \frac{a_2}{b_2} = \frac{a_3}{b_3}$$

$$Q$$

$$(\vec{a} \cdot \hat{i}) \hat{i} + (\vec{a} \cdot \hat{j}) \hat{j} + (\vec{a} \cdot \hat{k}) \hat{k} = ?$$

$$\downarrow \quad \quad \quad \downarrow \quad \quad \quad \downarrow$$

$$a_1 \hat{i} + a_2 \hat{j} + a_3 \hat{k} = \vec{a}$$

$$Q$$

$$(\vec{a} \cdot \hat{i})^2 + (\vec{a} \cdot \hat{k})^2 + (\vec{a} \cdot \hat{j})^2 = ?$$

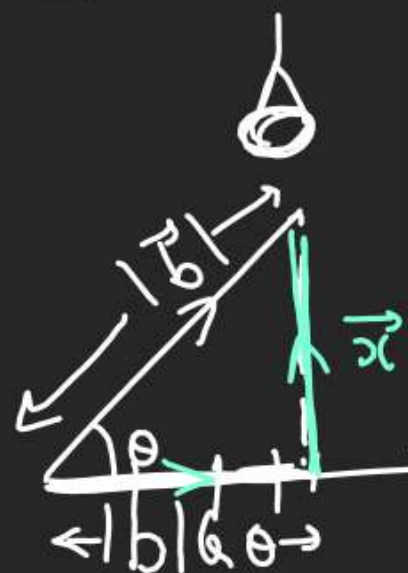
$$\downarrow \quad \quad \quad \downarrow \quad \quad \quad \downarrow$$

$$a_1^2 + a_2^2 + a_3^2 = |\vec{a}|^2$$

$$\text{as } |\vec{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$$

Projection of \vec{b} on \vec{a}

Scalar value & Vector value can be asked



$$\vec{a} \cdot \vec{b} = |\vec{a}| |\vec{b}| \cos \theta$$

$$|\vec{b}| \cos \theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}|}$$

$$|\vec{b}| \cos \theta \hat{a} + \vec{x} = \vec{b}$$

$$\vec{x} = \vec{b} - |\vec{b}| \cos \theta \hat{a} \quad \text{or } \vec{x} = \vec{b} - \frac{\vec{a} \cdot \vec{b}}{|\vec{a}|^2} \vec{a}$$

$$\text{Proj of } \vec{b} \text{ on } \vec{a} = |\vec{b}| \cos \theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}|}$$

$$\text{Proj vector of } \vec{b} \text{ on } \vec{a} = |\vec{b}| \cos \theta \cdot \hat{a} = \left(\frac{\vec{a} \cdot \vec{b}}{|\vec{a}|} \right) \hat{a} = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}|^2} \vec{a}$$

$$\text{Component of } \vec{b} \text{ along } \vec{a} = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}|} = \left(\frac{\vec{a} \cdot \vec{b}}{|\vec{a}|^2} \right) \vec{a}$$

$$\text{Component of } \vec{b} \perp \text{ to } \vec{a} = \vec{b} - \left(\frac{\vec{a} \cdot \vec{b}}{|\vec{a}|^2} \right) \vec{a}$$

$$\text{Proj. vector of } \vec{P} \text{ on } \vec{m} = \frac{\vec{P} \cdot \vec{m}}{|\vec{m}|} \hat{m}$$

Q If \vec{m} is decomposed into \parallel^r & \perp^r

vector of \vec{n} find vectors.

$$(1) \parallel^r = \left(\frac{\vec{m} \cdot \vec{n}}{|\vec{n}|} \right) \hat{n}$$

$$(2) \perp^r = \vec{m} - \left(\frac{\vec{m} \cdot \vec{n}}{|\vec{n}|} \right) \hat{n}$$

Q Angle betⁿ $\underline{1-2\hat{j}+\hat{k}}$ & $3\hat{i}-2\hat{j}+\hat{k}$

$$|\vec{a}| = \sqrt{1+4+1} = \sqrt{6}$$

$$|\vec{b}| = \sqrt{9+4+1} = \sqrt{14}$$

$$\cos \theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| |\vec{b}|} = \frac{3 \times 1 + -2 \times -2 + 1 \times 1}{\sqrt{6} \sqrt{14}}$$

$$\cos \theta = \frac{8}{2\sqrt{21}}$$

$$\theta = \cos^{-1}\left(\frac{4}{\sqrt{21}}\right)$$

Q $|\vec{a}| = |\vec{b}| = |\vec{a} + \vec{b}| = 1$ & find $|\vec{a} - \vec{b}| = ?$

Qs already \rightarrow Lemni Thm (Sine Rule)

$$|\vec{a} + \vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 + 2\vec{a} \cdot \vec{b}$$

$$|\vec{a} - \vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 - 2\vec{a} \cdot \vec{b}$$

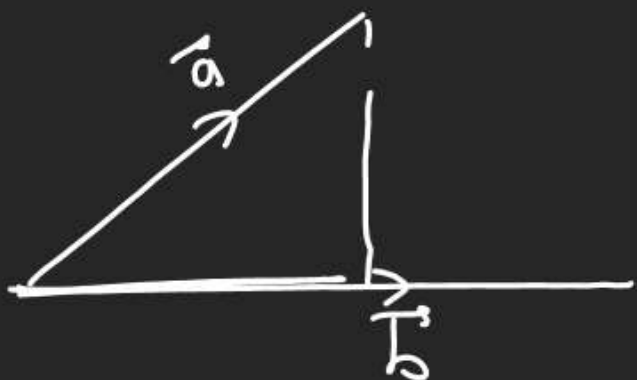
$$\frac{|\vec{a} + \vec{b}|^2 + |\vec{a} - \vec{b}|^2}{2} = |\vec{a}|^2 + |\vec{b}|^2$$

$$1^2 + |\vec{a} - \vec{b}|^2 = 2(1^2 + 1^2)$$

$$|\vec{a} - \vec{b}|^2 = 3$$

$$|\vec{a} - \vec{b}| = \sqrt{3}$$

Q $\vec{a} = 4\hat{i} + 6\hat{j}$, $\vec{b} = 3\hat{j} + 4\hat{k}$ find.
Component of \vec{a} along \vec{b}



$$\left(\frac{\vec{a} \cdot \vec{b}}{|\vec{b}|^2} \right) \cdot \vec{b}$$

$$\vec{a} \cdot \vec{b} = 4 \times 0 + 6 \times 3 + 0 \times 4 = 18$$

$$|\vec{b}| = \sqrt{0 + 9 + 16} = 5$$

$$= \left(\frac{18}{25} \right) \cdot (3\hat{j} + 4\hat{k})$$

$$\vec{a} \cdot \vec{b} > 0$$

Q. If $\vec{r} = ((a^2 - 4)\hat{i} + 2\hat{j} - (a^2 - 9)\hat{k})\hat{k}$
has acute angle with o axes
then Interval of a ?

\vec{r} & Axis \hat{i}
Acute
 $\vec{r} \cdot \hat{i} > 0$

$$(a^2 - 4) > 0$$

$$(a - 2)(a + 2) > 0$$

$$a < -2 \vee a > 2$$

\vec{r} & Axis \hat{j}
Acute
 $\vec{r} \cdot \hat{j} > 0$

$$2 > 0$$

\mathbb{R}

$\vec{r} \cdot \hat{k} > 0$

$$-(a^2 - 9) > 0$$

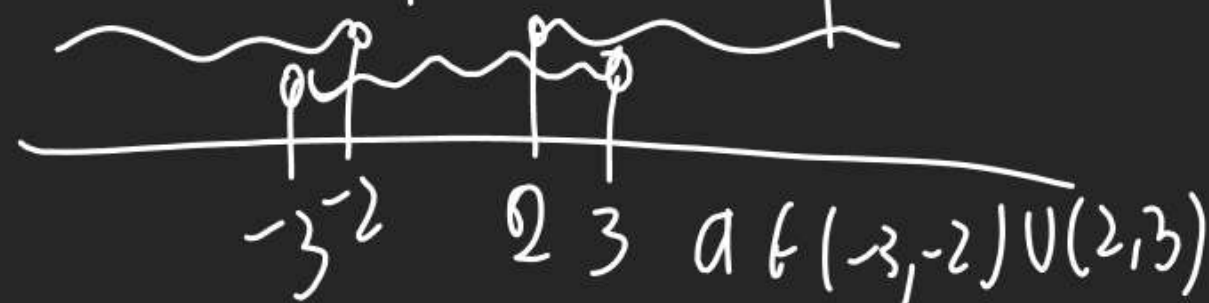
$$a^2 - 9 < 0$$

$$(a - 3)(a + 3) < 0$$

$$-3 < a < 3$$

$$\begin{aligned} \hat{i} \cdot \hat{i} &= 1 \\ \hat{i} \cdot \hat{j} &= 0 \\ \hat{i} \cdot \hat{k} &= 0 \end{aligned}$$

	\hat{i}	\hat{j}	\hat{k}
\hat{i}	1	0	0
\hat{j}	0	1	0
\hat{k}	0	0	1



Q4 If $\vec{a}, \vec{b}, \vec{c}$ are non zero vectors.

Such that $\vec{a} + \vec{b} + \vec{c} = 0$

then if $m = \vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$ then

$m < 0$ $m > 0$ $m = 0$ NOT.

$$(\vec{a} + \vec{b} + \vec{c})^2 = 0$$

$$|\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) = 0$$

$$2m = -(|\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2)$$

$$m = -\frac{(|\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2)}{2}$$

$$m = -\frac{10}{2} \Rightarrow m < 0$$

Q $\vec{a}, \vec{b}, \vec{c}$ are vectors such that

$$|\vec{a}| = 3, |\vec{b}| = 4, |\vec{c}| = 5 \text{ \&}$$

$$(\vec{a} + \vec{b}) \text{ is } \perp \text{ to } \vec{c}, (\vec{b} + \vec{c}) \text{ is } \perp \text{ to } \vec{a}$$

$$(\vec{c} + \vec{a}) \text{ is } \perp \text{ to } \vec{b} \text{ find } |\vec{a} + \vec{b} + \vec{c}|$$

$$|\vec{a} + \vec{b} + \vec{c}|^2 = (\vec{a} + \vec{b} + \vec{c})^2$$

$$= |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) \quad \vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{c} = \vec{c} \cdot \vec{a} = 0$$

$$|\vec{a} + \vec{b} + \vec{c}|^2 = 9 + 16 + 25$$

$$|\vec{a} + \vec{b} + \vec{c}| = \sqrt{50} = 5\sqrt{2}$$

$$(\vec{a} + \vec{b}) \cdot \vec{c} = 0 \Rightarrow \vec{a} \cdot \vec{c} + \vec{b} \cdot \vec{c} = 0$$

$$(\vec{b} + \vec{c}) \cdot \vec{a} = 0 \Rightarrow \vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c} = 0$$

$$(\vec{c} + \vec{a}) \cdot \vec{b} = 0 \Rightarrow \vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} = 0$$

$$2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) = 0$$

Q4 If $\vec{r} \cdot \hat{i} = \vec{r} \cdot \hat{j} = \vec{r} \cdot \hat{k} \wedge |\vec{r}| = 3$

find \vec{r} ?

$$\text{let } \vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

$$\vec{r} \cdot \hat{i} = x$$

$$\vec{r} \cdot \hat{j} = y, \vec{r} \cdot \hat{k} = z$$

$$x = y = z = k$$

$$|\vec{r}| = \sqrt{x^2 + y^2 + z^2}$$

$$= \sqrt{k^2 + k^2 + k^2} = k\sqrt{3}$$

$$k\sqrt{3} = 3 \Rightarrow k = \sqrt{3}$$

$$\therefore \vec{r} = \sqrt{3}\hat{i} + \sqrt{3}\hat{j} + \sqrt{3}\hat{k}$$

$$= \sqrt{3}(\hat{i} + \hat{j} + \hat{k})$$

Q $\vec{a}, \vec{b}, \vec{c}$ are unit vectors.

2 $\vec{a} + \vec{b} + \vec{c} = 0$ find value of
 $\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a} = ?$

$$(\vec{a} + \vec{b} + \vec{c})^2 = 0$$

$$|\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) = 0$$

$$1^2 + 1^2 + 1^2 + 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) = 0$$

$$\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a} = -\frac{3}{2}$$

Shortcut
 use $\hat{i}, \hat{j}, \hat{k}$

Q $\vec{a}, \vec{b}, \vec{c}$ are mutually \perp
vectors of equal Magnitudes.
 find angle betⁿ \vec{a} & $\vec{a} + \vec{b} + \vec{c}$?

$$(1) \vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{c} = \vec{c} \cdot \vec{a} = 0$$

$$(2) |\vec{a}| = |\vec{b}| = |\vec{c}| = K.$$

$$(3) |\vec{a} + \vec{b} + \vec{c}|^2 = |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\quad)$$

$$|\vec{a} + \vec{b} + \vec{c}|^2 = K^2 + K^2 + K^2$$

$$|\vec{a} + \vec{b} + \vec{c}|^2 = 3K^2$$

$$|\vec{a} + \vec{b} + \vec{c}| = \sqrt{3}K$$

$$\vec{a} \cdot \vec{b}$$

$$\cos \theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| |\vec{b}|}$$

$$\cos \theta = \frac{\vec{a} \cdot (\vec{a} + \vec{b} + \vec{c})}{|\vec{a}| |\vec{a} + \vec{b} + \vec{c}|}$$

$$= \frac{|\vec{a}|^2 + 0 + 0}{|\vec{a}| |\vec{a} + \vec{b} + \vec{c}|}$$

$$\cos \theta = \frac{K^2}{K \times \sqrt{3}K}$$

$$\theta = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

$$\vec{a} + \vec{b} - \vec{c} = 0$$

$$Q \text{ If } |\vec{a}| = |\vec{b}| = |\vec{c}| = k$$

& $\vec{a} + \vec{b} = \vec{c}$ find angle betⁿ \vec{a} & \vec{b} .

Solⁿ.

$$(\vec{a} + \vec{b})^2 = (\vec{c})^2$$

$$\cos \theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| |\vec{b}|}$$

$$|\vec{a}|^2 + |\vec{b}|^2 + 2(\vec{a} \cdot \vec{b}) = |\vec{c}|^2$$

$$k^2 + k^2 + 2(\vec{a} \cdot \vec{b}) = k^2$$

$$\vec{a} \cdot \vec{b} = -\frac{k^2}{2}$$

$$\cos \theta = \frac{-\frac{k^2}{2}}{k \times k}$$

$$\theta = 120^\circ = \frac{2\pi}{3}$$

Q θ is angle betⁿ unit vectors

\vec{a} & \vec{b} then find $\frac{1}{2} |\vec{a} + \vec{b}|$ & $\frac{1}{2} |\vec{a} - \vec{b}|$?

$$① |\vec{a} + \vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 + 2|\vec{a}||\vec{b}|\cos \theta$$

$$|\vec{a} + \vec{b}|^2 = 1 + 1 + 2 \times 1 \times 1 \times \cos \theta$$

$$= 2 + 2 \cos \theta$$

$$= 2(1 + \cos \theta)$$

$$|\vec{a} + \vec{b}|^2 = 2 \times 2 \cos^2 \frac{\theta}{2}$$

$$\frac{1}{2} |\vec{a} + \vec{b}|^2 = \cos^2 \frac{\theta}{2}$$

$$\cos \frac{\theta}{2} = \frac{1}{2} |\vec{a} + \vec{b}|$$

$$② |\vec{a} - \vec{b}|^2 =$$

$$|\vec{a} - \vec{b}|^2 = 1 + 1 - 2 \times 1 \times 1 \times \cos \theta$$

$$= 2 - 2 \cos \theta$$

$$= 2(1 - \cos \theta)$$

$$|\vec{a} - \vec{b}|^2 = 2 \times 2 \sin^2 \frac{\theta}{2}$$

$$\sin \frac{\theta}{2} = \frac{1}{2} |\vec{a} - \vec{b}|$$

Q Find gr. value of

 $|\vec{a} + \vec{b}| + |\vec{a} - \vec{b}|$ if \vec{a}, \vec{b} are Unit Vectors?

$$= 2\cos\frac{\theta}{2} + 2\sin\frac{\theta}{2}$$

Result of R.K. Verma

$$\frac{1}{2}|\vec{a} + \vec{b}| = \cos\frac{\theta}{2}$$

$$\frac{1}{2}|\vec{a} - \vec{b}| = \sin\frac{\theta}{2}$$

$$-\sqrt{2^2 + 2^2} \leq 2\sin\frac{\theta}{2} + 2\cos\frac{\theta}{2} \leq \sqrt{2^2 + 2^2}$$

$$-2\sqrt{2} \leq$$

$$\leq 2\sqrt{2}$$

$$\therefore \text{gr. value} = 2\sqrt{2}$$

Saturday Vector 5:45 PM

Q $\vec{a}, \vec{b}, \vec{c}$ are Unit vector

$$\text{then } |\vec{a} - \vec{b}|^2 + |\vec{b} - \vec{c}|^2 + |\vec{c} - \vec{a}|^2$$

does not exceed?

$$|\vec{a} - \vec{b}|^2 + |\vec{b} - \vec{c}|^2 + |\vec{c} - \vec{a}|^2$$

$$= 2(|\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2) - 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a})$$

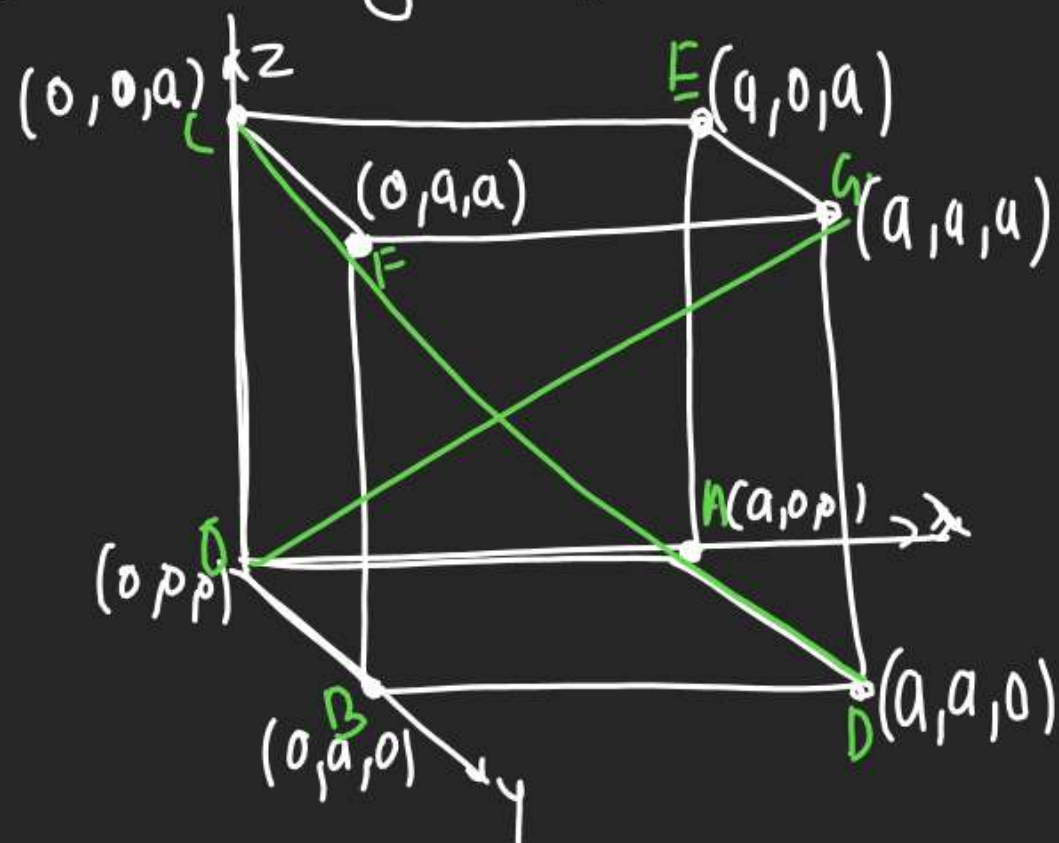
$$= 3(|\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2) - \{|\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a})\}$$

$$\text{Exp.} = 3(1+1+1) - |\vec{a} + \vec{b} + \vec{c}|^2$$

$$\text{Exp. gr.} = 9 - |\vec{a} + \vec{b} + \vec{c}|^2_{\text{Min.}}$$

$$= 9 - 0 = 9$$

Q Find angle betⁿ diagonal of cube?



$$\vec{OH} = \langle a, a, a \rangle - \langle 0, 0, 0 \rangle = a\hat{i} + a\hat{j} + a\hat{k}$$

$$\vec{OD} = \langle a, a, 0 \rangle - \langle 0, 0, a \rangle = a\hat{i} + a\hat{j} - a\hat{k}$$

$$\cos \theta = \frac{\vec{OH} \cdot \vec{OD}}{|\vec{OH}| |\vec{OD}|} = \frac{a^2 + a^2 - a^2}{\sqrt{3a^2} \sqrt{3a^2}} = \frac{a^2}{3a^2}$$

$$\theta = \cos^{-1} \frac{1}{3}$$

Q $\vec{U}, \vec{V}, \vec{W}$ are 3 vectors s.t. Proj. of \vec{V} on \vec{U} is equal to Proj. of \vec{W} on \vec{U} & \vec{V}, \vec{W} are \perp to each other
find $|\vec{U} - \vec{V} + \vec{W}| = ?$

$$① |\vec{U} - \vec{V} + \vec{W}|^2 = |\vec{U}|^2 + |\vec{V}|^2 + |\vec{W}|^2 - 2\vec{U} \cdot \vec{V} - 2\vec{V} \cdot \vec{W} + 2\vec{U} \cdot \vec{W}$$

$$\therefore |\vec{U} - \vec{V} + \vec{W}| = \sqrt{|\vec{U}|^2 + |\vec{V}|^2 + |\vec{W}|^2}$$

$$② |\vec{V}| \cos \theta = |\vec{W}| \cos \theta$$

$$\frac{\vec{U} \cdot \vec{V}}{|\vec{U}|} = \frac{\vec{U} \cdot \vec{W}}{|\vec{U}|}$$

$$③ \vec{V} \perp \vec{W}$$

$$\vec{V} \cdot \vec{W} = 0$$

$$\vec{V} \cdot \vec{V} = |\vec{V}| |\vec{V}| \cos 0$$

$$|\vec{W} \cdot \vec{V}| = |\vec{W}| |\vec{V}| \cos \theta$$

Q 1-15, 18-19, 21-22