Nishant Jindal

WAVE

Which of the following represent travelling wave Equation

$$y = e^{\left(t - x/v\right)^2} \checkmark$$

$$y = e^{\left(t^2 + x^2/v^2\right)} \times$$

$$y = A \sin(x + vt) \checkmark$$

$$y = A \sin^2(x - vt) \checkmark$$

$$y = A \cos(x^2 + v^2t^2) \times$$

$$\left(\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}\right)$$

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A4 :

$$y = f(t - x/v)$$

$y = A\sin\omega t$ → For particle at $x = 0$

$$\boxed{y = A\sin\omega\left(t - \frac{x}{v}\right)}$$

↳ Travelling in +x-direction

$$\boxed{y = A\sin\omega\left(t + \frac{x}{v}\right)}$$

↳ Travelling in −x-direction

$$y = A\sin\left(\omega t - \underbrace{\frac{\omega}{v}}_{K}x\right)$$

$$\boxed{y = A\sin(\omega t - Kx)}$$

↳ +x-direction

$$\boxed{K = \frac{\omega}{v}}$$

Wave No.

$$K = \frac{2\pi}{\cancel{T}} \times \frac{\cancel{T}}{\lambda} \qquad \left(v = \frac{\lambda}{T}\right)$$

$$\boxed{K = \frac{2\pi}{\lambda}}$$

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① – $y = A\sin(\omega t - kx)$

② – $y = A\sin(kx - \omega t)$

$y = A\sin[(\omega t - kx) + \pi]$

Both represent travelling wave Equation having phase difference π

Put $x = 0$ in ① & ②

$y = A\sin\omega t$ from ①

$y = -A\sin\omega t$ from ②

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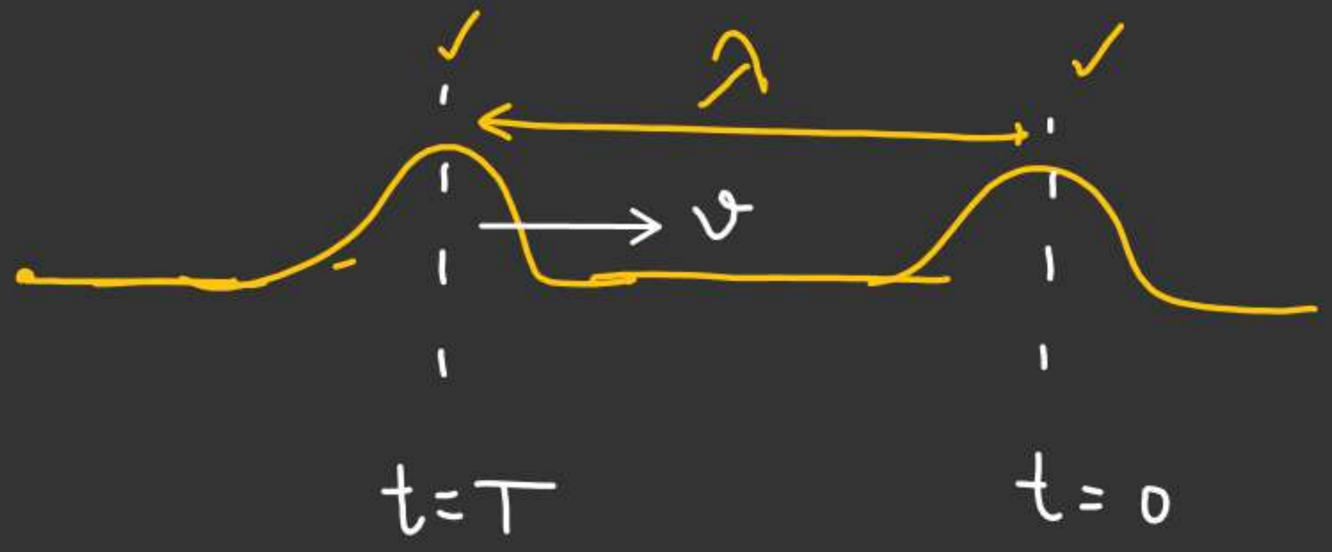
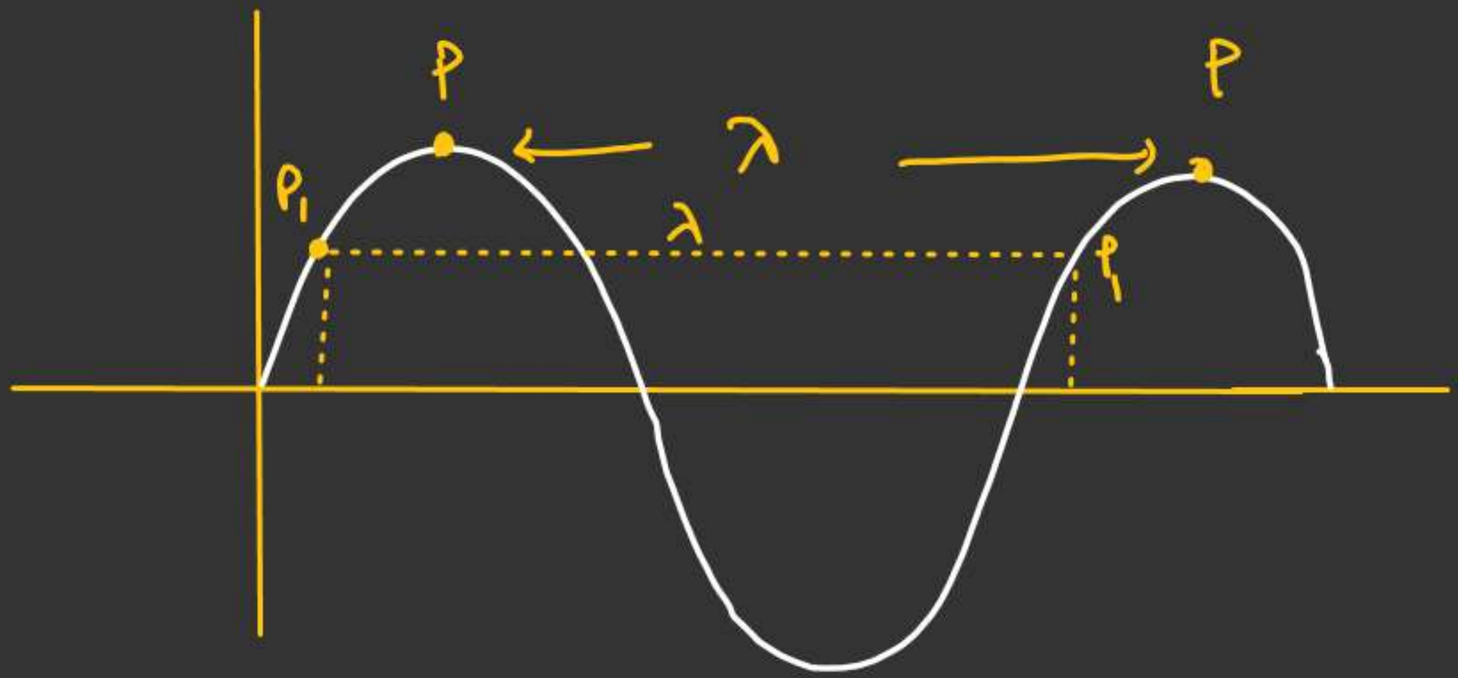
4. Wave length :- Min. distance b/w two particles vibrating in Same phase.





$$\boxed{\lambda = VT}$$

$$T = \frac{2\pi}{\omega}.$$

V = Wave velocity.

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$(x = C)$

$$y = A\sin(\omega t - Kx)$$

$$v_p = \frac{\partial y}{\partial t} = A\cos(\omega t - Kx)\frac{\partial}{\partial t}(\omega t - Kx)$$

$$\boxed{v_p = \frac{\partial y}{\partial t} = A\omega\cos(\omega t - Kx)}$$

$$a_p = \frac{\partial v_p}{\partial t} = \frac{\partial^2 y}{\partial t^2} = -\underbrace{A\omega^2 \sin(\omega t - Kx)}_{y}$$

$$a_p = -\omega^2 y \Rightarrow \text{S.H.M of particles.}$$

$$K = \frac{\omega}{v}$$

$$v = \frac{\omega}{K}$$

$\Downarrow$

Wave velocity

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Wave velocity of Transverse wave

$$\boxed{v = \sqrt{\frac{T}{\mu}}}$$

T → Represent Elastic property of String

μ → Represent inertial property of String.

T = Tension in String

$\mu = \frac{M}{l}$ = linear mass density Of String.

(Neighbourhood Particle responds the disturbance faster)

Neighbourhood particle responds the disturbance slower

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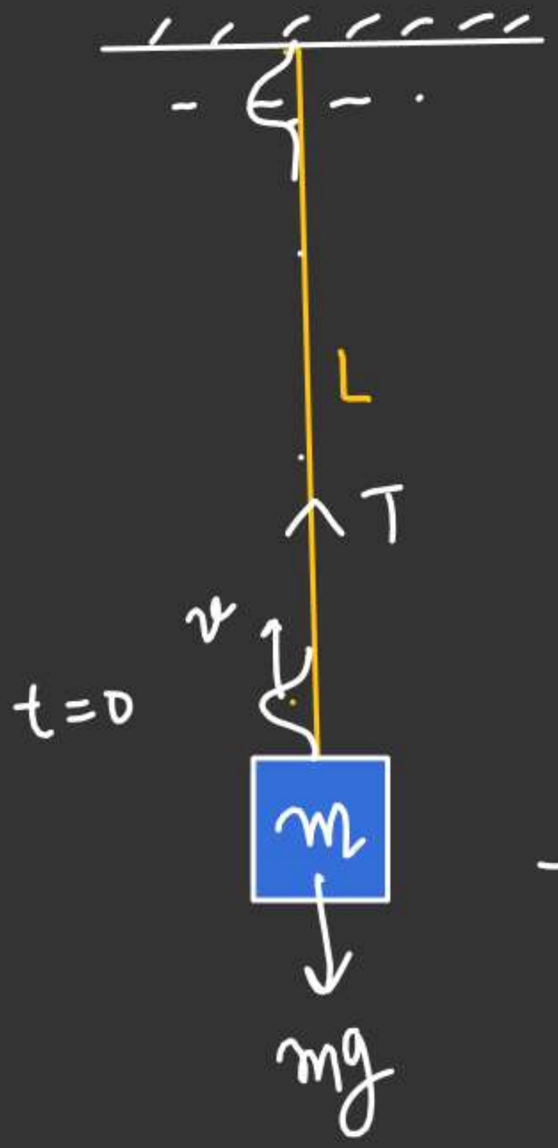
String ⇒ Tension due to self weight neglected i.e tension is uniform
($\mu \neq 0$)



$t = ??$

$v =$ uniform.

$$v = \sqrt{\frac{T}{\mu}}$$

$T = mg$

$$v = \sqrt{\frac{mg}{\mu}}$$

$$t = \frac{L}{v} = L\sqrt{\frac{\mu}{mg}} \checkmark$$

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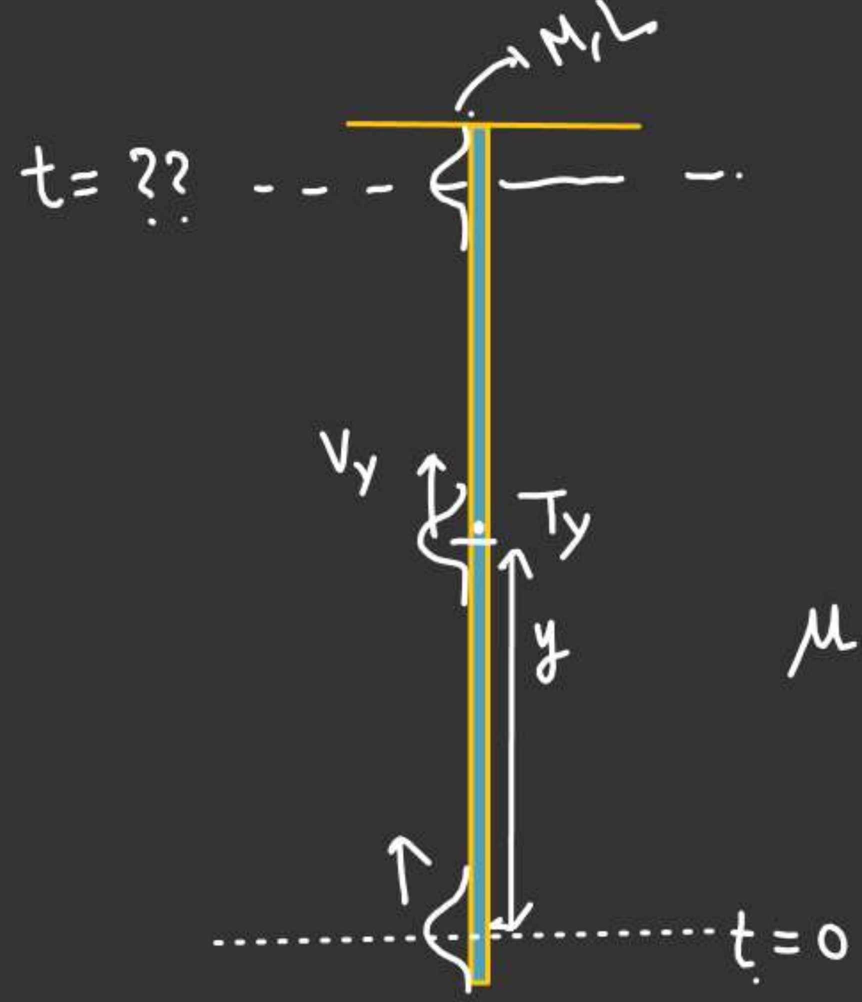
velocity of transverse rope in thick rope



$$T_y = \left(\frac{M}{L} y g\right) = \mu y g.$$

$$V_y = \sqrt{\frac{T_y}{\mu}}$$

$$\mu = \frac{M}{L}, \quad V_y = \sqrt{\frac{\cancel{\mu} y g}{\cancel{\mu}}}$$

$$\boxed{V_y = \sqrt{yg}}$$

$$\frac{dy}{dt} = \sqrt{yg}$$

$$\int_0^y \frac{dy}{\sqrt{y}} = \sqrt{g} \cdot \int_0^t dt$$

$$\int_0^y y^{-1/2}\, dy = \sqrt{g}\, t$$

$$2\sqrt{y} = \sqrt{g}\, t$$

$$\boxed{t = 2\sqrt{y/g}}$$ ✓

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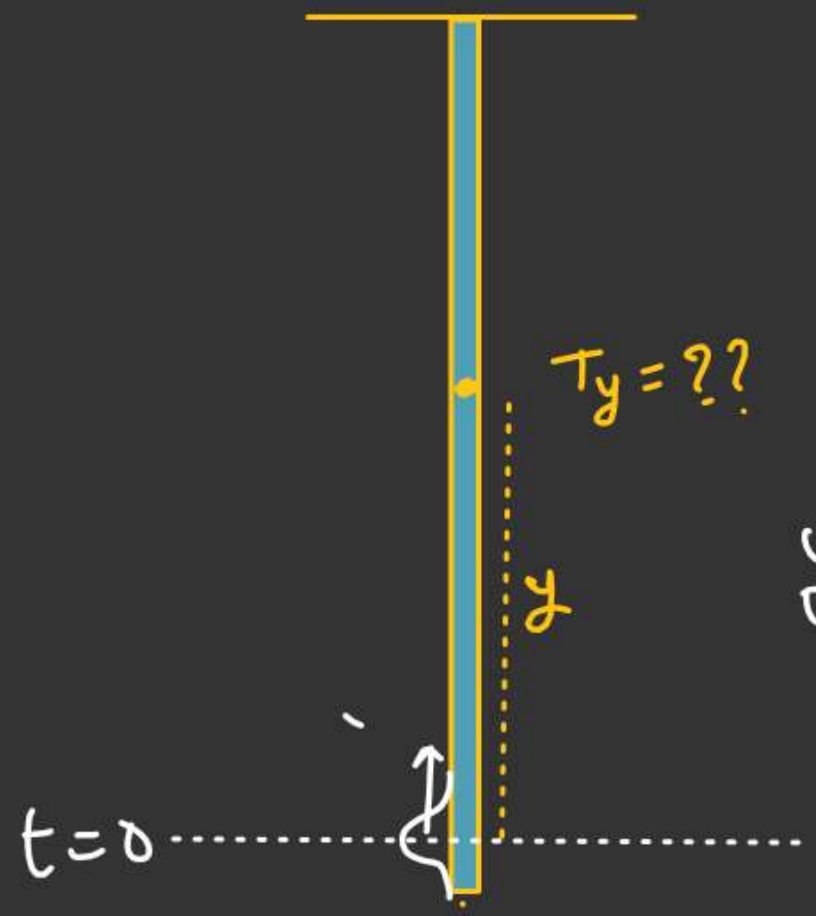
Case-2

If rope is non uniform

$\mu = \mu_0 y$.

T_y = weight of y length of the rope.

$= m_y g$.



$dm_y = \mu_y dy$

$\int_0^{m_y} dm_y = \mu_0 \int_0^{y} y\,dy$

$m_y = \left(\dfrac{\mu_0 y^2}{2}\right)$

$y = L$

$M = \dfrac{\mu_0 L^2}{2}$

$\checkmark$

$T_y = \dfrac{\mu_0 g}{2} y^2$.

$\mu = \left(\dfrac{M}{L}\right)$

$M = \int_0^L dm_y = \int_0^L \mu_y dy$

$M = \mu_0 \int_0^L y\,dy$

$M = \left(\dfrac{\mu_0 L^2}{2}\right)$

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Case-2

$$T_y = \frac{\mu_0 g}{2} y^2.$$

$$M = \left(\frac{\mu_0 L^2}{2}\right)$$

$$\mu = \frac{M}{L} = \left(\frac{\mu_0 L}{2}\right)$$

$$V_y = \sqrt{\frac{\frac{\mu_0 g y^2}{2}}{\frac{\mu_0 L}{2}}}$$

$$V_y = \sqrt{\frac{g}{L}} \cdot y.$$

H.W (Repeat the question)

Find $t = ??$

if $\mu = \mu_0(1+y)$