

Case I $x \leq -8$

$$7 - x + 3x - 6 - 4x - 32 - x = 21$$

$$x = -\frac{52}{3}$$

III $x \in [0, 2]$

$$7 - x + 3x - 6 + 4x + 32 + x = 21$$

$$7x = -12 \quad \times$$

II $-8 \leq x \leq 0$

$$7 - x + 3x - 6 + 4x + 32 - x = 21$$

$$5x = -12$$

$$x = -\frac{12}{5}$$

IV $2 \leq x \leq 7$

$$7 - x - 3x + 6 + 4x + 32 + x = 21$$

$$x = -24 \quad \times$$

V $x \geq 7$

$$x - 7 - 3x + 6 + 4x + 32 + x = 21$$

$$x = -\frac{10}{2} \quad \times$$

$8x$

2.

$$x \geq -2$$

OR

$$x < -2$$

$$\frac{x+2-x}{x} < 2$$

$$\frac{-x-2-x}{x} < 2$$

$$x \in (-\infty, 0) \cup (1, \infty)$$

$$\frac{2}{x} - 2 < 0$$

$$\frac{2(1-x)}{x} < 0$$

$$\frac{x-1}{x} \geq 0$$

$$x \in (-\infty, 0) \cup (1, \infty)$$

$$x \in [-2, 0) \cup (1, \infty)$$

$$\frac{2+2x}{x} + 2 > 0$$

$$\frac{2(1+2x)}{x} > 0$$

$$x \in (-\infty, -\frac{1}{2}) \cup (0, \infty)$$

$$x \in (-\infty, -2)$$

3. $x \geq 0$

$$\frac{x^2 - 7x + 10}{x^2 - 6x + 9} < 0$$

$$\frac{(x-5)(x-2)}{(x-3)^2} < 0$$

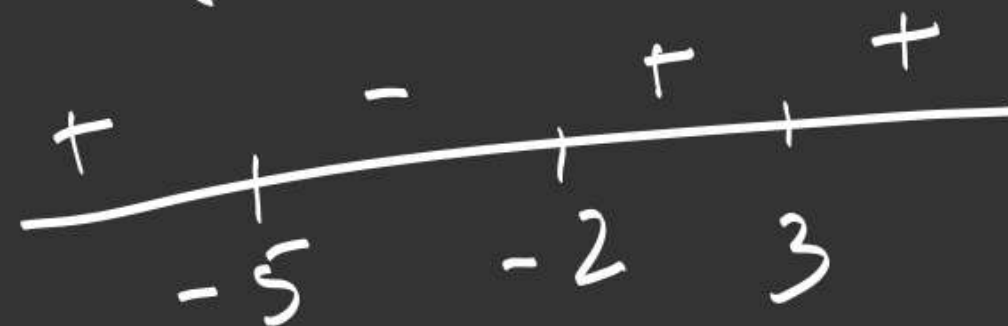


$$x \in (2, 3) \cup (3, 5) \checkmark$$

OR

$x < 0 \checkmark$

$$\frac{(x+2)(x+5)}{(x-3)^2} = \frac{x^2 + 7x + 10}{(x-3)^2} < 0$$



$$x \in (-5, -2)$$

4.

$$(-7, -4) \cup (-4, 1)$$

$$x \in (-5, -2) \cup (2, 3) \cup (3, 5)$$

$$\underline{7.} \quad x \geq 3$$

OR

$$x < 3 \quad \checkmark$$

$$\frac{x-3}{(x-3)(x-2)} \geq 2$$

$$\frac{1}{x-2} - 2 \geq 0$$

$$\frac{1-2x+4}{x-2} \geq 0$$

$$\frac{2x-5}{x-2} \leq 0$$

$$(2, \frac{5}{2}]$$

$$x \neq 3$$

$$x \in \left[\frac{3}{2}, 2 \right)$$

$$\frac{3-x}{(x-3)(x-2)} \geq 2$$

$$2 + \frac{1}{x-2} \leq 0 \quad x \neq 3$$

$$\frac{2x-3}{x-2} \leq 0$$

$$\left[\frac{3}{2}, 2 \right)$$

$$\underline{6.} \quad x \geq 1$$

$$\frac{x-1}{x+2} - 1 < 0$$

$$\frac{-3}{x+2} > 0$$

$$(-2, \infty)$$

$$x \in [1, \infty)$$

$$x < 1 \quad \checkmark$$

$$1 + \frac{x-1}{x+2} > 0$$

$$\frac{2x+1}{x+2} > 0$$

$$(-\infty, -2) \cup \left(-\frac{1}{2}, \infty\right)$$

$$x \in (-\infty, -2) \cup \left(-\frac{1}{2}, 1\right)$$

$$x \in (-\infty, -2) \cup \left(-\frac{1}{2}, \infty\right)$$

5.

$$\frac{x^2 - 5x + 6}{|x| + 7} < 0$$

$$|x| + 7 > 0$$

$$(x-2)(x-3) < 0$$

$$x \in (2, 3)$$

S.L. Loney
Trigonometry

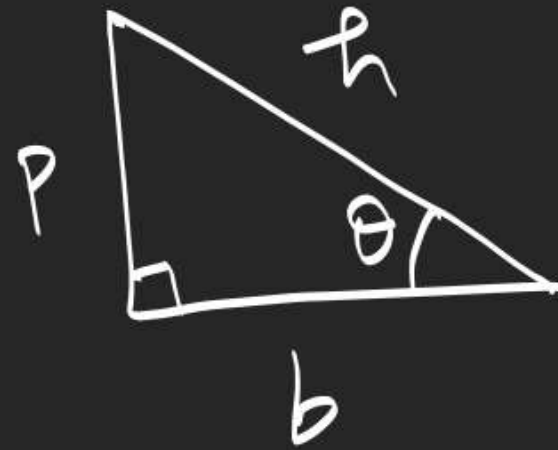
11.

Trigonometry → measurement.

3 sides 2 sides

$$\begin{aligned}\csc \theta &= \frac{h}{p} \\ \sec \theta &= \frac{h}{b} \\ \cot \theta &= \frac{b}{p}\end{aligned}$$

$$\begin{aligned}\sin \theta &= \frac{p}{h} \\ \cos \theta &= \frac{b}{h} \\ \tan \theta &= \frac{p}{b}\end{aligned}$$



Identity

Equation which holds true for all values of parameters wherever defined.

$$a^2 + b^2 = c^2 \quad x = x \rightarrow \text{Identity}$$

not identity

$$x^2 - 3x + 2 = (x-1)(x-2) \rightarrow \text{Identity}$$

$$(a+b)^2 = a^2 + b^2 + 2ab \rightarrow \text{Identity}$$

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\sec^2 \theta - \tan^2 \theta = 1$$

$$\operatorname{cosec}^2 \theta - \cot^2 \theta = 1$$

P.T.

$$1. \quad \text{P.T.} \quad (\sec \theta + \operatorname{cosec} \theta)(\sin \theta + \cos \theta) = \sec \theta \operatorname{cosec} \theta + 2$$

$$\left(\frac{1}{\cos \theta} + \frac{1}{\sin \theta} \right) (\sin \theta + \cos \theta) = \frac{(\sin \theta + \cos \theta)^2}{\sin \theta \cos \theta}$$

$$\sec \theta \operatorname{cosec} \theta + 2 = \frac{1 + 2 \sin \theta \cos \theta}{\sin \theta \cos \theta} = \frac{(\sin^2 \theta + \cos^2 \theta) + 2 \sin \theta \cos \theta}{\sin \theta \cos \theta}$$

$$2. \quad \left(\sec^2 \theta + \tan^2 \theta \right) (\operatorname{cosec}^2 \theta + \cot^2 \theta) = 1 + 2 \sec^2 \theta \operatorname{cosec}^2 \theta$$

$$\frac{(1 + \sin^2 \theta)}{\cos^2 \theta} \cdot \frac{(1 + \cos^2 \theta)}{\sin^2 \theta} = \frac{(1 + \sin^2 \theta + \cos^2 \theta) + \sin^2 \theta \cos^2 \theta}{\sin^2 \theta \cos^2 \theta}$$

$$3. \quad \tan^2 \theta \sin^2 \theta = \tan^2 \theta - \sin^2 \theta$$

$$2 \operatorname{cosec}^2 \theta \sec^2 \theta + 1 = \frac{2 + \sin^2 \theta \cos^2 \theta}{\sin^2 \theta \cos^2 \theta}$$

$$\tan^2 \theta \sin^2 \theta = \tan^2 \theta - \sin^2 \theta$$

$$\cot^2 \theta \cos^2 \theta = \cot^2 \theta - \cos^2 \theta$$

$$\tan^2 \theta \sin^2 \theta$$

$$= \tan^2 \theta (1 - \cos^2 \theta)$$

$$= \tan^2 \theta - \sin^2 \theta$$

$$\begin{aligned} \cot^2 \theta - \cos^2 \theta &= \cot^2 \theta (1 - \sin^2 \theta) \\ &= \cot^2 \theta \cos^2 \theta \end{aligned}$$

4.

$$\frac{\cot \theta + \operatorname{cosec} \theta - 1}{\cot \theta - \operatorname{cosec} \theta + 1} = \cot \theta + \operatorname{cosec} \theta$$

$$\frac{\cot \theta + \operatorname{cosec} \theta - (\operatorname{cosec}^2 \theta - \cot^2 \theta)}{\cot \theta - \operatorname{cosec} \theta + 1} = \frac{(\cot \theta + \operatorname{cosec} \theta)(1 - \cancel{\operatorname{cosec} \theta - \cot \theta})}{\cot \theta - \cancel{\operatorname{cosec} \theta + 1}}$$

$$= \cot \theta + \operatorname{cosec} \theta$$

5.

$$\left(\frac{1 + \sin \alpha}{1 + \cos \alpha} \right) \left(\frac{1 + \sec \alpha}{1 + \operatorname{cosec} \alpha} \right) = \tan \alpha$$

$$\left(\frac{1 + \sin \alpha}{1 + \cos \alpha} \right) \left(\frac{1 + \frac{1}{\cos \alpha}}{1 + \frac{1}{\sin \alpha}} \right) = \left(\frac{1 + \sin \alpha}{1 + \cos \alpha} \right) \left(\frac{1 + \cos \alpha}{1 + \sin \alpha} \right) \frac{\sin \alpha}{\cos \alpha} = \tan \alpha$$

$$\underline{1.} \text{ P.T. } \frac{\sin x + \cos x}{\cos^3 x} = \tan^3 x + \tan^2 x + \tan x + 1$$

$$\frac{\sin x}{\cos x} \frac{1}{\cos^2 x} + \frac{1}{\cos^2 x} = \tan x \sec^2 x + \sec^2 x$$

$$= \tan x (1 + \tan^2 x) + 1 + \tan^2 x$$

$$= \tan^3 x + \tan x + \tan^2 x + 1$$

2. Simplify

$$\operatorname{cosec}^2 A \cot^2 A - \sec^2 A \tan^2 A - (\cot^2 A - \tan^2 A)$$

$$(\sec^2 A \operatorname{cosec}^2 A - 1)$$

P.T.

3.

$$\frac{\sin A}{1 + \cos A} + \frac{1 + \cos A}{\sin A} = 2 \operatorname{cosec} A$$

4.

$$\cos^6 A + \sin^6 A = 1 - 3 \sin^2 A \cos^2 A$$

5.

$$\sin^8 A - \cos^8 A = (\sin^2 A - \cos^2 A)(1 - 2 \sin^2 A \cos^2 A)$$

$$2. \operatorname{cosec}^2 A \cot^2 A - \sec^2 A \tan^2 A - (\cot^2 A - \tan^2 A)(\sec^2 A \operatorname{cosec}^2 A - 1)$$

$$\frac{\cos^2 A}{\sin^4 A} - \frac{\sin^2 A}{\cos^4 A} - \frac{(\cos^4 A - \sin^4 A)}{\sin^2 A \cos^2 A} \frac{(1 - \sin^2 A \cos^2 A)}{\sin^2 A \cos^2 A}$$

$$= \frac{\cos^6 A - \sin^6 A - (\cos^4 A - \sin^4 A)(1 - \sin^2 A \cos^2 A)}{\sin^4 A \cos^4 A}$$

$$\cos^6 A - \sin^6 A - (\cos^2 A - \sin^2 A)(1 - \sin^2 A \cos^2 A) \frac{(\sin^2 A + \cos^2 A)^2}{\sin^4 A \cos^4 A} \frac{\cos^6 A - \sin^6 A}{\sin^4 A \cos^4 A}$$

$$\frac{(\cos^6 A - \sin^6 A) - (\cos^2 A - \sin^2 A)(\sin^4 A + \cos^4 A + \sin^2 A \cos^2 A)}{\sin^4 A \cos^4 A} = 0$$

Ineq.

sec 1.5

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