Nishant Jindal

$\times$ Ex 2.2 9, 10, 13, 15, 12, 3, ✓

Ex 1 $\rightarrow$ 3, 4, 5 DPP 1

1) 9(A) $\tan^{-1}\frac{1}{3}+\tan^{-1}\frac{2}{9}+\dots+\tan^{-1}\frac{2^{n-1}}{1+2^{2n-1}}$ Dn.

(B) $\tan^{-1}\left(\frac{1}{x^2+x+1}\right)+\tan^{-1}\left(\frac{1}{x^2+3x+3}\right)+\dots$ DN

2) $\sum\sum\tan^{-1}\left(\frac{m}{n}\right)=K\pi$ Dn

$\sin^{-1}x+\sin^{-1}y$

$x\geq 0,\ y\geq 0$

10) $\underbrace{\sin^{-1}x}_{\alpha}+\underbrace{\sin^{-1}2x}_{\beta}=\boxed{\frac{\pi}{3}}$.

$x=\sin\alpha\ ,\ 2x=\sin\beta$.

$\alpha+\beta=\frac{\pi}{3}$

$\beta=\frac{\pi}{3}-\alpha$

$\sin\beta=\sin\left(\frac{\pi}{3}-\alpha\right)$

$\sin\beta=\sin\frac{\pi}{3}.\cos\alpha-\cos\frac{\pi}{3}.\sin\alpha$

$2x=\frac{\sqrt{3}}{2}\sqrt{1-x^2}-\frac{1}{2}.x$

$4x=\sqrt{3}\sqrt{1-x^2}-x \Rightarrow 5x=\sqrt{3}\sqrt{1-x^2}$

$25x^2=3-3x^2$

$28x^2=3$

$\boxed{x=\pm\sqrt{\frac{3}{28}}}$.

$x=\sqrt{\frac{3}{28}}$

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Q $\tan^{-1}(x-1)+\tan^{-1}x+\tan^{-1}(x+1)=\tan^{-1}3x$

$$\tan^{-1}\left(\frac{(x-\cancel{1})+(x+\cancel{1})}{1-(x^2-1)}\right)+\tan^{-1}x$$

$$\tan^{-1}\left(\frac{2x}{2-x^2}\right)+\tan^{-1}x=$$

$$\tan^{-1}\left(\frac{\frac{2x}{2-x^2}+x}{1-\frac{2x^2}{2-x^2}}\right)=\tan^{-1}(3x)$$

$$\tan^{-1}\left(\frac{\frac{2x+2x-x^3}{\cancel{(2-x^2)}}}{\frac{2-x^2-2x^2}{\cancel{(2-x^2)}}}\right)=\tan^{-1}(3x)$$

$$\frac{4x-x^3}{2-3x^2}=3x$$

$$4x-x^3=6x-9x^3.$$

$$8x^3-2x=0$$

$$2x(4x^2-1)=0$$

$$\boxed{x=0}\quad \tfrac{1}{2},-\tfrac{1}{2}$$

1) $\tan^{-1}(-1)+\tan^{-1}(0)+\tan^{-1}(1)=\tan^{-1}0$

$$-\frac{\pi}{4}+0+\frac{\pi}{4}=0$$

2) $\cancel{\tan^{-1}(-\tfrac{1}{2})}+\cancel{\tan^{-1}(\tfrac{1}{2})}+\tan^{-1}(\tfrac{3}{2})=\tan^{-1}(\tfrac{3}{2})$

$$\tan^{-1}\left(\tfrac{3}{2}\right)=\tan^{-1}\left(\tfrac{3}{2}\right)$$

3) $\tan^{-1}\left(-\tfrac{3}{2}\right)+\tan^{-1}\left(\cancel{-\tfrac{1}{2}}\right)+\tan^{-1}\left(\cancel{\tfrac{1}{2}}\right)=\tan^{-1}\left(-\tfrac{3}{2}\right)$

(C) $\tan^{-1}\left(\frac{x-1}{x+1}\right)+\tan^{-1}\left(\frac{2x-1}{2x+1}\right)=\tan^{-1}\left(\frac{23}{36}\right)$

$\tan^{-1}\left(\frac{x-\tan\frac{\pi}{4}}{x+\tan\frac{\pi}{4}}\right)+\tan^{-1}\left(\frac{2x-\tan\frac{\pi}{4}}{2x+\tan\frac{\pi}{4}}\right)$

$\cot^{-1}(-x)=\pi-\cot^{-1}x$

$\cot^{-1}(-x)-\pi=-\cot^{-1}x$

$\tan^{-1}(x)-\underline{\tan^{-1}(\tan\frac{\pi}{4})}+\tan^{-1}(2x)-\tan^{-1}(\tan\frac{\pi}{4})=\tan^{-1}\left(\frac{23}{36}\right)$

$\tan^{-1}(x)+\tan^{-1}(2x) = \frac{\pi}{2}+\tan^{-1}\left(\frac{23}{36}\right)$

$\tan^{-1}\left(\frac{x+2x}{1-2x^2}\right)=\frac{\pi}{2}+\tan^{-1}\left(\frac{23}{36}\right) = \pi-\cot^{-1}\left(\frac{23}{36}\right)$

$\tan\left(\tan^{-1}\left(\frac{3x}{1-2x^2}\right)\right)=\pi-\tan^{-1}\left(\frac{36}{23}\right)$

**

$\frac{3x}{1-2x^2}=\tan(\pi-\theta)=-\tan\theta=-\tan\left(\tan^{-1}\frac{36}{23}\right)$

$\frac{3x}{1-2x^2}=-\frac{36}{23}$ 288

$69x=-36+72x^2$

$72x^2-69x-36=0$

$24x^2-23x-12=0$

$24x^2-32x+9x-12=0$

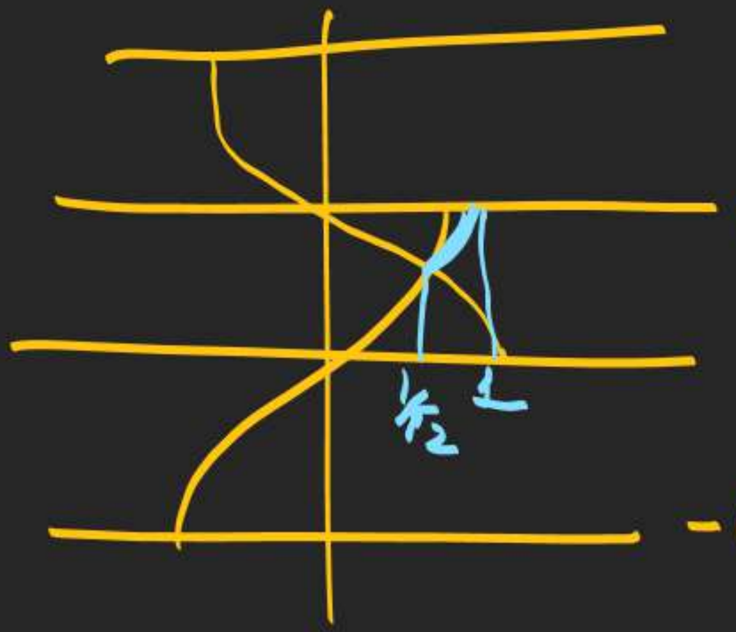
$8x(3x-4)+3(3x-4)=0$

$x=-\frac{3}{8}$ ✗ | $\frac{4}{3}$ ✓

(3) $f(2)+g(2)+h(2) = \cancel{\pi} - \cancel{2\tan^{-1}x} + \cancel{2\tan^{-1}x} - \cancel{\pi} + 2\tan^{-1}x = 2\tan^{-1}x$

$= 2\tan^{-1}2$

13) $(\cos^{-1}x)^2 - 5\cos^{-1} + 6 > 0$



B) $\sin^{-1}x > \cos^{-1}x$

C) $\tan^2(\sin^{-1}x) > 1$

$x \in (-1,0)$ $\quad 2\tan^{-1}x - \cancel{2\tan^{-1}x} + 2\tan^{-1}x = \frac{\pi}{2}$

$\tan^{-1}x = \frac{\pi}{4} \Rightarrow x = 2$

$$f(x) = \sin^{-1}\left(\frac{2x}{1-x^2}\right) = \begin{cases} -\pi - 2\tan^{-1}x & x<-1 \\ \boxed{2\tan^{-1}x} \rightarrow & -1 \le x \le 1 \\ \pi - 2\tan^{-1}x & x>1 \end{cases}$$

$-\frac{\pi}{4} \quad \frac{\pi}{4} = (-1, 1)$

$$g(x) = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) = \begin{cases} -2\tan^{-1}x & x<0 \\ 2\tan^{-1}x & \boxed{x \ge 0} \end{cases}$$

If $x \in (-1,1)$

$x \in (0,1)$ $\quad f(x)+g(x)+h(x) = \frac{\pi}{2}$

$2\tan^{-1}x + 2\tan^{-1}x + 2\tan^{-1}x = \frac{\pi}{2}$

$\tan^{-1}x = \frac{\pi}{12} \Rightarrow x = \tan\frac{\pi}{12} = 2-\sqrt{3}$

$$h(x) = \tan^{-1}\left(\frac{2x}{1-x^2}\right) = \begin{cases} 2\tan^{-1}x + \pi & x<-1 \\ 2\tan^{-1}x & -1 \le x \le 1 \\ -\pi + 2\tan^{-1}x & x>1 \end{cases}$$

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$(\sin^{-1}(-1))^2 = \left(-\frac{\pi}{2}\right)^2 = \frac{\pi^2}{4}$

1) $g: R \to \underline{\left(0, \frac{\pi}{3}\right]}$ $\quad g(x) = \cos^{-1}\left(\frac{x^2-K}{1+x^2}\right)$ $\quad g = \underline{\text{Surjective fxn.}}$

onto

Range = $\underline{\text{Cod}}$

$\sin^{-1}x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$(\sin^{-1}x)^2 \in \left[0, \frac{\pi^2}{4}\right]$

(4) Range $f(x) = \cos^{-1}\left(\frac{1}{e^x+e^{-x}}\right)$

$\infty > e^x + \frac{1}{e^x} \geq 2$

$\infty > e^x + e^{-x} \geq 2$

$0 < \frac{1}{e^x+e^{-x}} \leq \frac{1}{2}$

$\cos^{-1}(0) > \cos^{-1}\left(\frac{1}{e^x+e^{-x}}\right) \geq \cos^{-1}\left(\frac{1}{2}\right)$

$\frac{\pi}{2} > f(x) \geq \frac{\pi}{3}$

$\left[\frac{\pi}{3}, \frac{\pi}{2}\right)$

$0 < \cos^{-1}\left(\frac{x^2-K}{1+x^2}\right) \leq \frac{\pi}{3}$

$1 > \frac{x^2-K}{1+x^2} \geq \frac{1}{2}$

$\frac{x^2-K}{1+x^2} < 1$

$\cancel{x^2} - K < 1 + \cancel{x^2}$

$\underline{K > -1}$

$\frac{x^2-K}{1+x^2} \geq \frac{1}{2}$

$2x^2 - 2K \geq 1 + x^2$

$\boxed{x^2 - 2K - 1} \geq 0$

$D \leq 0$

$0 - 4 \times 1 \times (-2K+1) \leq 0$

$2K + 1 \leq 0$

$K \leq -\frac{1}{2}$

$\left.\begin{aligned}(x,y,z) &= (1,1,0)\\ &= (-1,1,0)\end{aligned}\right\}$ $\underline{\text{2 triplet}}$

(5)

$(\sin^{-1}x)^2 = \frac{\pi^2}{4} + \underbrace{(\sec^{-1}y)^2 + (\tan^{-1}z)^2}_{=\,0}$

$\sec^{-1}y = 0 = \tan^{-1}z$

$y = \sec 0$ | $z = \tan 0$

$y = 1$ | $z = 0$

$(\sin^{-1}x)^2 = \frac{\pi^2}{4}$

$x = 1, -1$

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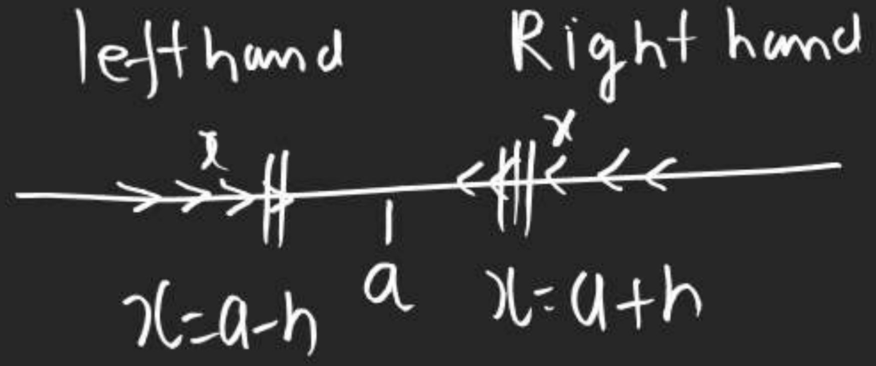
Limit

1) If $\boxed{x \to a}$ gives $\underline{f(x) \to m}$ then $\boxed{\lim_{x \to a} f(x) = m}$. Dhokha

x approaches to a

f(x) approaching to m.

(2) $x \to a \Rightarrow$ Read as x tends to a.



(3) "h" Kya h?

h is infinitely small +ve No

$h \to 0$ $\quad h = .0000000.....1$ (Point me)

$$\boxed{\begin{aligned} \{h\} &= h \\ \{-h\} &= 1-h \end{aligned}}$$

$\{2\} = .2$

$\{.02\} = .02$

$\{.00002\} = .00002$

$[h] = [0+h] = 0$

$[6-h] = [6 \text{ se Kam}] = 5$

$[6+h] = [6 \text{ se Bda}] = 6$

(4) If "Limit Exist" at $x \to a$ is given then we say that LHL = RHL

LHL = Left hand limit

$$= \lim_{x \to a^-} f(x) \qquad (x = a-h)$$

$$= f(a-h)$$

RHL = Rt. hand limit

$$= \lim_{x \to a^+} f(x) = \underline{f(a+h)}$$

(5) We check LHL = RHL for existance of limit only when $x = a$ in in domain of $f(x)$

* If $x = a$ is in Middle of domain then we check LHL = RHL

If yes then limit Exist

(6) If $x = a$ is at Boundary of Domain of $f(x)$ then we donot check LHL = RHL

We consider whoso ever available

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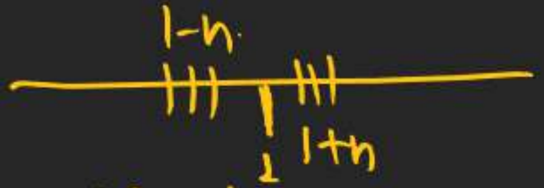
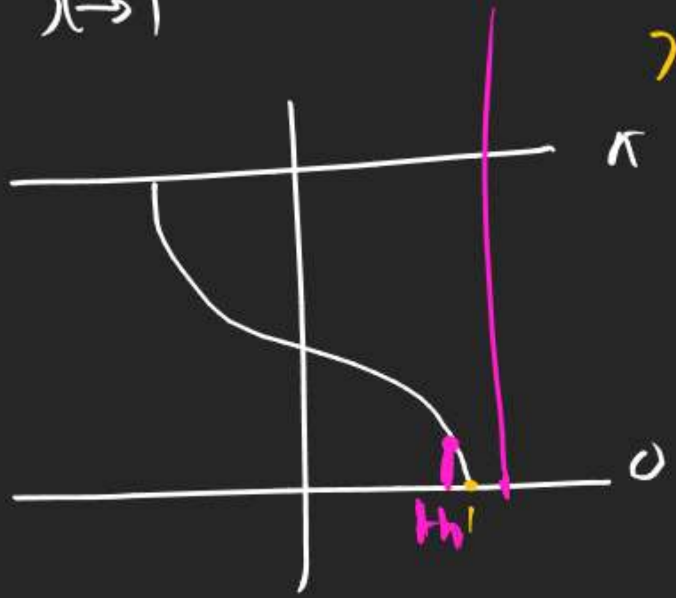
Q $\lim_{x\to 1} \cos^{-1} x$



$x = 1-h$

$x = 1+h$



$x \in [-1, 1]$

at $x = 1+h$
No graph available
on $x = 1+h$ in
not in domain
So we will not
Consider RHL

∴ Answer will LHL
value

$\lim_{h\to 0} \cos^{-1}(1-h)$

$\cos^{-1}(1-0) = 0$

$$\therefore \boxed{\lim_{x\to 1} \cos^{-1} x = 0}$$

(7) $\lim_{\boxed{x\to a}} f(x)$

$\boxed{x \neq a}$

(8) We Use limit in which Case?

A. We Use limit When fxn gives
Indeterminite values.

$$\left\{ \frac{0}{0}, 0^0, \infty - \infty, \frac{\infty}{\infty}, \boxed{1^\infty}, \infty \times 0, \infty^0 \right\}$$

7 Indeterminite fxn

Q $\lim_{x\to 1} \frac{2x+1}{3x+4} = \frac{2+1}{3+4} = \frac{3}{7}$

(9)

$$\lim_{x\to\infty} a^x = \begin{cases} \infty & a>1 \\ \boxed{0 \quad -1<a<1} \\ 1 & a=1 \end{cases}$$

$\left(\frac{2}{3}\right)^\infty = 0$

$\downarrow$

$(\text{Base}<1)^\infty = 0$

$\left(\frac{3}{2}\right)^\infty \to \infty$

$(\text{Exactly } 1)^\infty = 1$

$\left(\frac{3}{4}\right)^\infty = 0$

$\left(\frac{4}{3}\right)^\infty \to \infty$

$\left(\frac{4}{4}\right)^\infty = 1$

Q $\lim_{n\to\infty} \frac{n\cdot 3^n}{n\cdot(x-2)^n + n\cdot 3^{n+1} + 3^n} = \frac{1}{3}$

find Range of x?

$\lim_{n\to\infty} \frac{\cancel{n\cdot 3^n}\cdot 1}{\cancel{n\cdot 3^n}\left(\frac{(x-2)^n}{3^n} + 3 + \frac{1}{n}\right)} = \frac{1}{3}$

Answer will be matched

if $\lim_{n\to\infty}\left(\frac{x-2}{3}\right)^n = 0$

$-1 < \frac{x-2}{3} < 1$

$-3 < x-2 < 3$

$-1 < x < 5 \Rightarrow x \in (-1, 5)$

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Existance of Limit Based Qs.

1) Existance of Limit at $x=a$ is Possible When LHL = RHL

2) LHL = RHL

$$\Rightarrow \lim_{x\to a^-} f(x) = \lim_{x\to a^+} f(x)$$

$$\Rightarrow f(a-h) = f(a+h)$$

$$\Rightarrow f(a-0) = f(a+0)$$

$$\Rightarrow f(a^-) = f(a^+)$$

Ex:→ LHL at $x=1$ for $f(x) = \cos x$

$$\Rightarrow \lim_{x\to 1^-} \cos x$$

$f(x) = \cos x$ & $f(2^-) = ?$

$$f(2^-) = \lim_{x\to 2^-} \cos x$$

Q $\lim\limits_{x\to \frac{\pi}{4}^-} \tan x = ?$

LHL → $\lim\limits_{h\to 0} \tan\left(\frac{\pi}{4} - h\right)$

$$= \tan\left(\frac{\pi}{4} - 0\right) = \tan\left(\frac{\pi}{4}\right) = 1$$

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* We Check LHL = RHL only when following fxn are given.

1) [ ] 2) { } 3) | |

4) Sgn(x) (5) Defined fxn → $\boxed{f(x) = \begin{cases} x^2 & x \geq 1 \\ -x & x < 1 \end{cases}}$

(6) Chor fxn → Whenever fxn has

$\frac{1}{x}, \frac{1}{x^3}, \frac{1}{x^5} \ldots \frac{1}{x^{odd}}$

DPP-1, 2