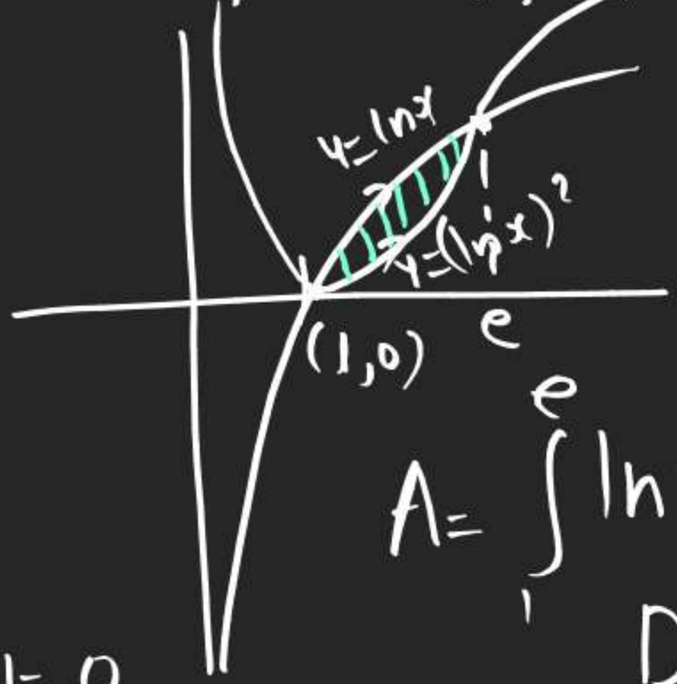
Nishant Jindal

Q.1 ABB $y = \ln x,\ y = (\ln x)^2$



$A = \int_1^e \ln x - (\ln x)^2 \, dx$

DY

$\ln 1 = 0$

$\ln e = 1$

$x \in (1, e)$

$y \in (\ln 1, \ln e)$

$y \in (0, 1)$

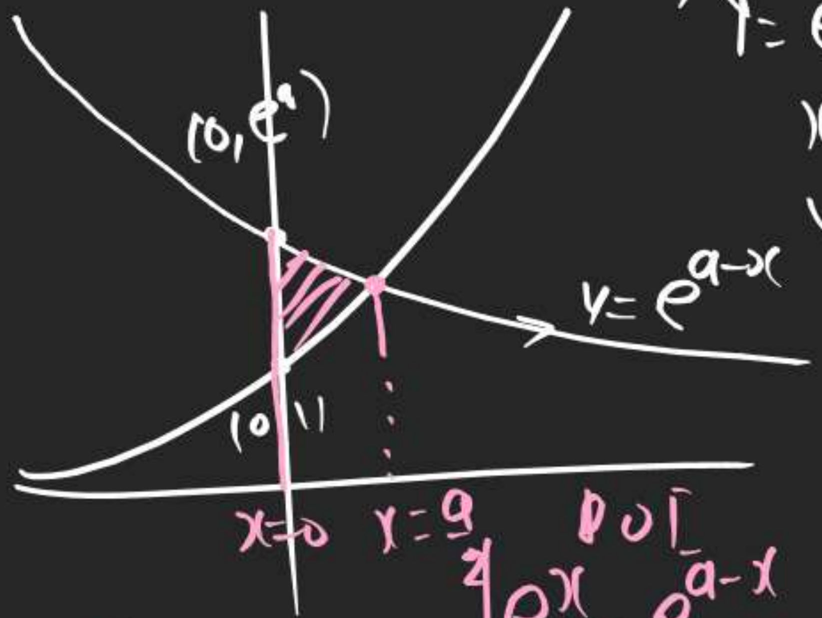
Q2 $y = e^x,\ y = e^{a-x}$ & y Axis   ABB $a > 0$

$y = e^{-x}$

$\to y = e^a \cdot e^{-x}$

$x = 0$ पर

$y = e^a$



$A = \int_0^{a/2} e^{a-x} - e^x \, dx$

DY

$e^x = e^{a-x}$

$e^x = \frac{e^a}{e^x}$

$e^{2x} = e^a$

$2x = a$

$x = \frac{a}{2}$

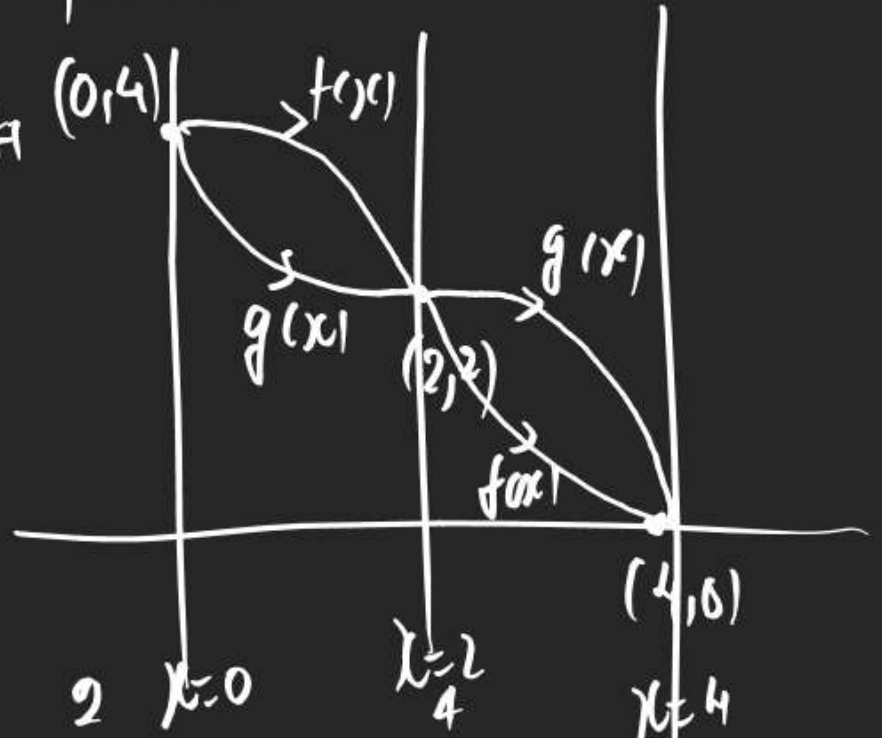
Q3 $f(x), g(x)$ Intersect at 3 pts $(0,4), (2,2), (4,0)$

With $\boxed{f(x) > g(x)}$ for $0 < x < 2$

& $f(x) < g(x)$ for $2 < x < 4$

$f(x), g(x)$ find Area.

$x \in (0,2)$ में



$\text{Area} = \int_0^2 f(x) - g(x) + \int_2^4 g(x) - f(x)$

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Q A.B.B. $y^2+x^4=x^2$

$y^2=x^2-x^4$

$y^2=(x)^2(1-x^2)$ → $y=x\sqrt{1-x^2}$, $y=-x\sqrt{1-x^2}$

$y=|x|\sqrt{1-x^2}$ → Dom

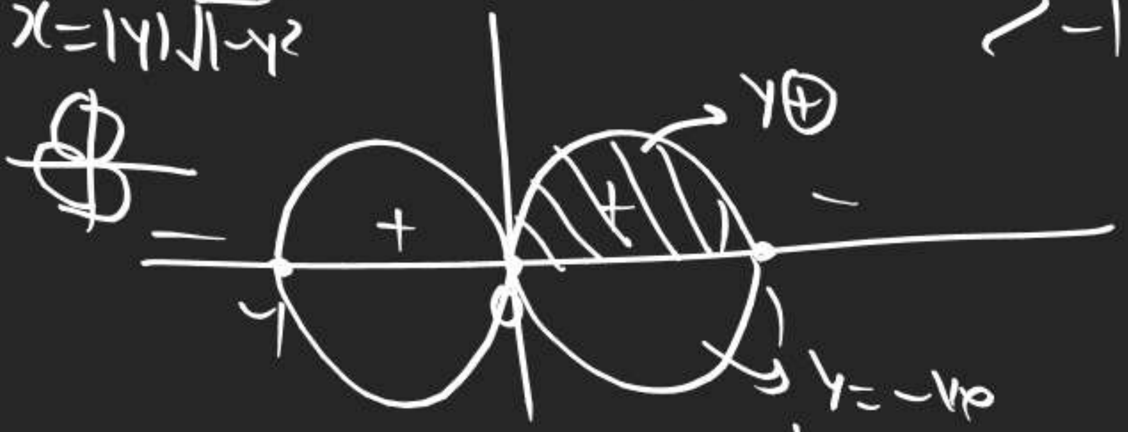
$1-x^2\geq 0$

$y^2=-(x)^2(x^2-1)$ → $x^2-1\leq 0$

$y^2=-(x)^2(x+1)(x-1)$ → $(x-1)(x+1)\leq 0$

$-1\leq x\leq 1$

$x=|y|\sqrt{1-y^2}$



$A=4\int_0^1 x\sqrt{1-x^2}\,dx$

$x\in(0,1)$
$y=+ve$

Symmetricity

$y^2=4ax$

y ka deg even

⇒ Symm about x Axis

$y=x^2$

deg of x even

⇒ Symm about Y Axis } 2 times

$y^2=x^2-x^4$

deg of x, y Even

⇒ Symm about x & y Axis

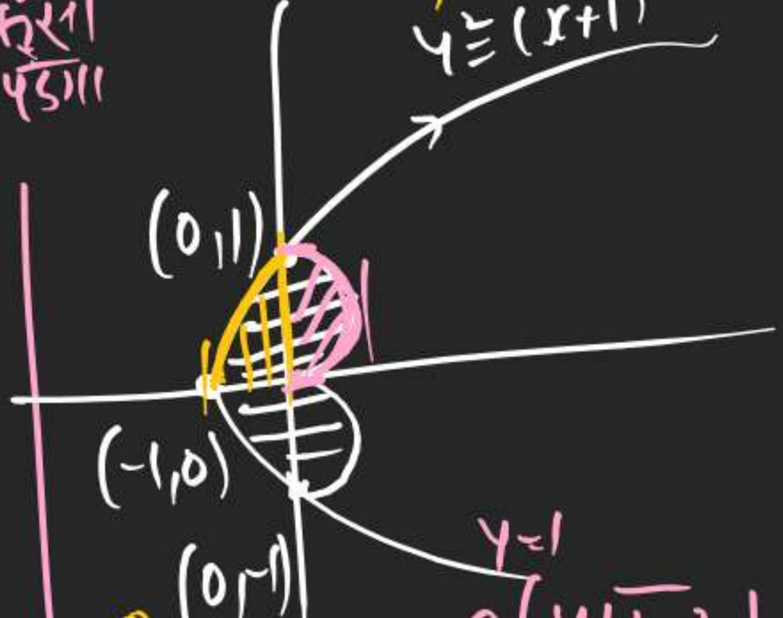
Both | 4 times

Q5 ABB $x=y^2-1$

$x=|y|\sqrt{1-y^2}$

$(y)^2=(x+1)$ ← $y^2=4ax$

vertex of Par. at $(-1,0)$



$A=2\int_{-1}^{0}\sqrt{x+1}\,dx+2\int_{y=0}^{y=1} y\sqrt{1-y^2}\,dy$

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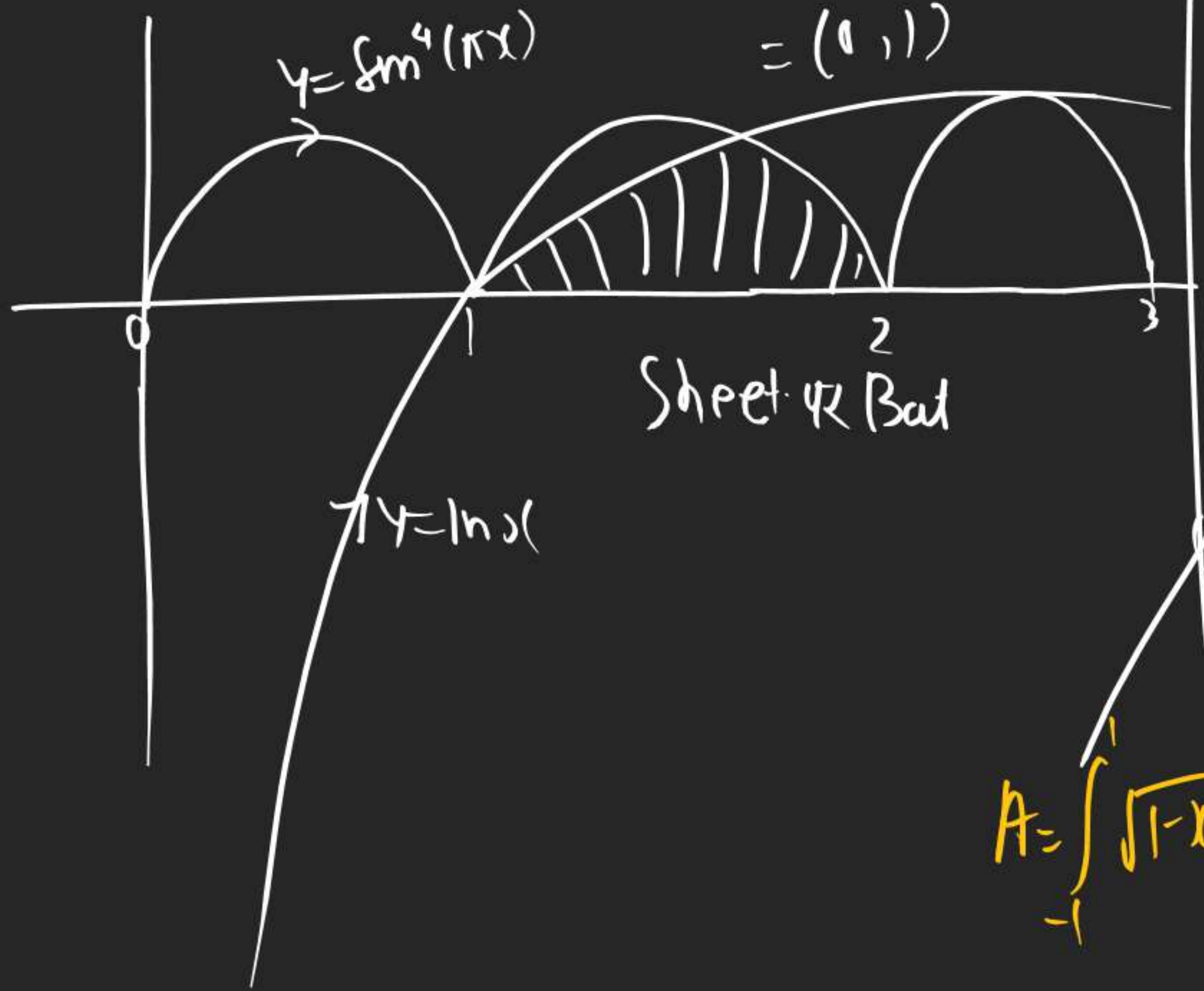
Q ABB $y = \ln x$ & $y = \sin^4(\pi x)$ & x Axis

$y = \sin(\pi x)$

$\sin x \to (0, \pi)$

$\sin \pi x \to \left(\frac{0}{\pi}, \frac{\pi}{\pi}\right)$

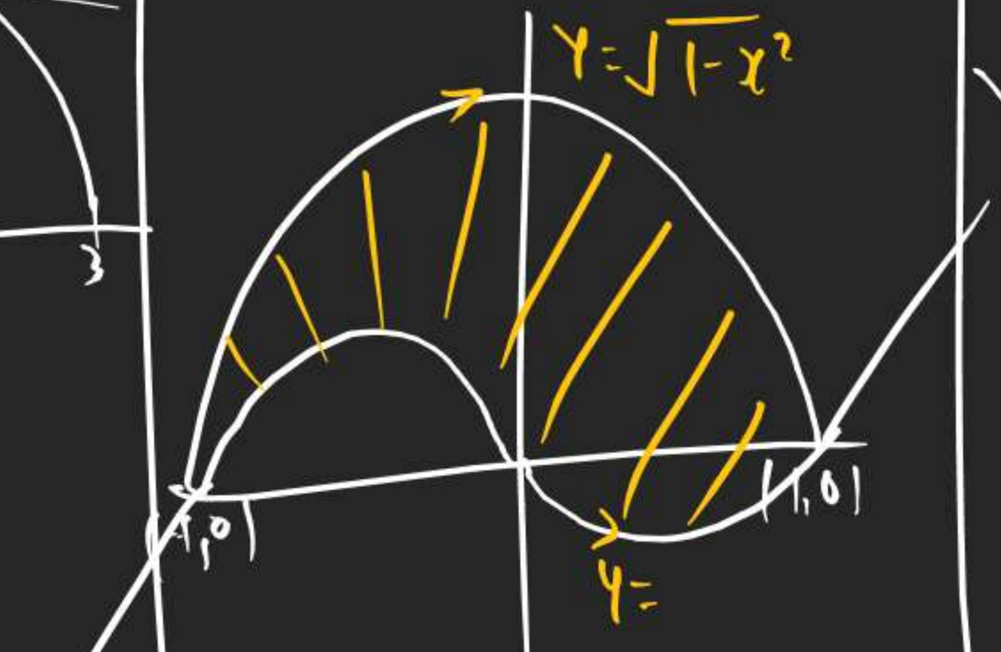
$= (0, 1)$



Q ABB $y = \sqrt{1-x^2} \to$ Circle

& $y = x^3 - x$

$y^2 = 1 - x^2 \Rightarrow x^2 + y^2 = 1$



$$A = \int_{-1}^{1} \sqrt{1-x^2} - (x^3 - x)\,dx$$

$y = x^3 - x$

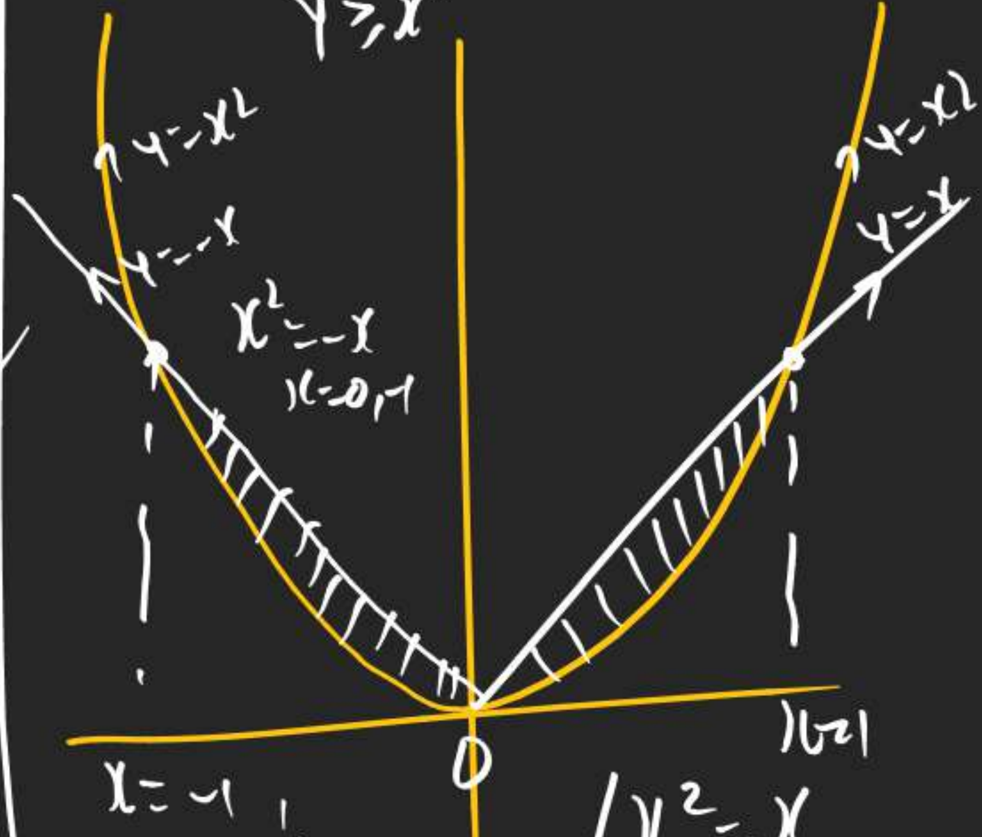
$= (x)(x-1)(x+1)$

Q $\{(x,y) : x^2 \le y \le |x|\}$ ABB?

NCERT

$y = x^2$ & $y = |x|$

$y \ge x^2$



$$A = 2\int_0^1 x - x^2\,dx$$

D Y

$x^2 = x$

$x(x-1) = 0$

$x = 0, x = 1$

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Q ABB $y^2 \le 4x$ & $4x^2+4y^2 \le 9$

NCERT.

$y^2 = 4x$ → के अन्दर → $y = 2\sqrt{x}$

$4x^2+4y^2 = 9$ → Circle → के अन्दर

↓ $(0,0)$, $\frac{3}{2}$, $(0, \frac{3}{2})$

$4y^2 = 9-4x^2$

$2y = \sqrt{9-x^2}$

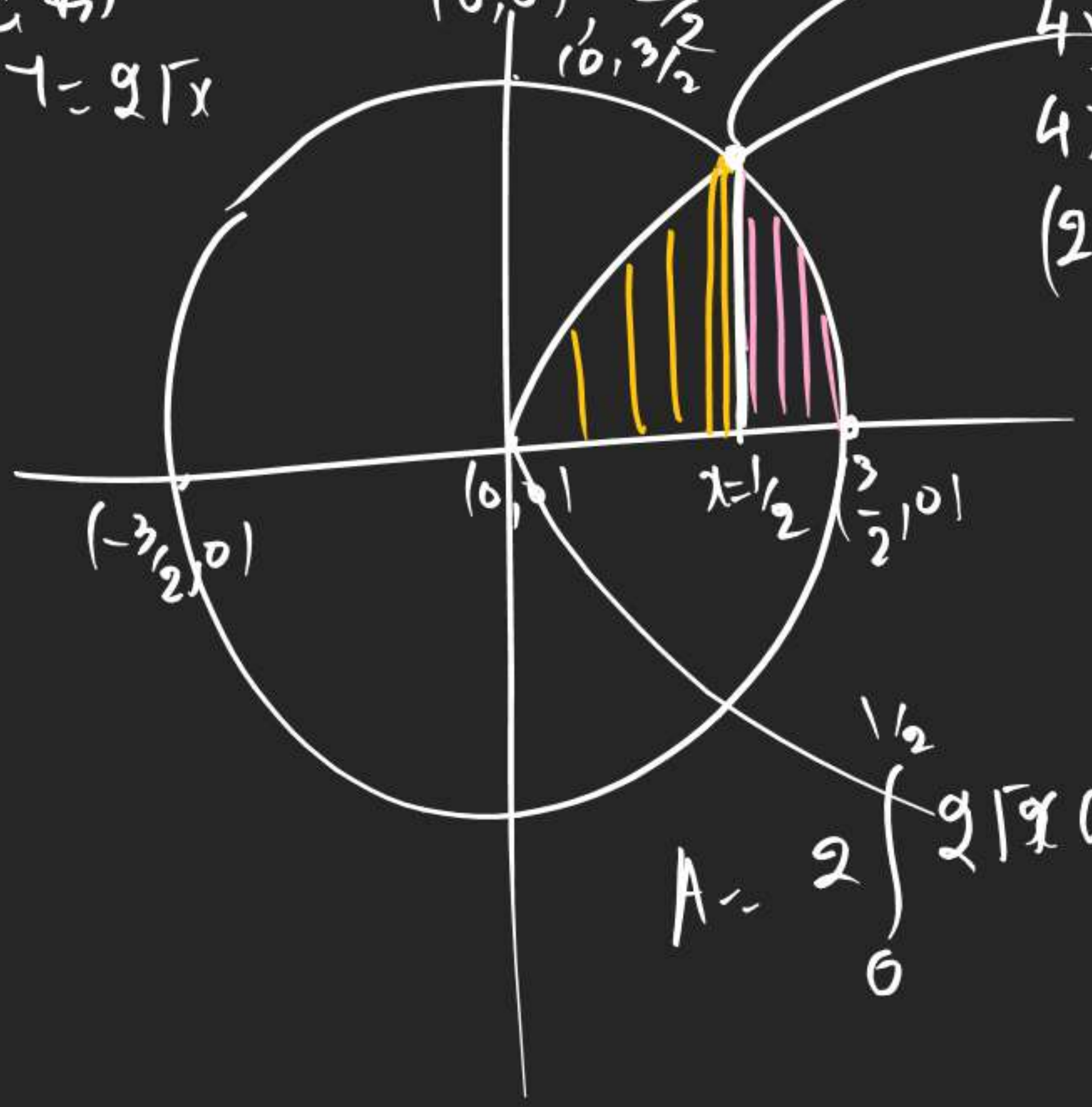
$4x^2+4(4x)=9$

$4x^2+16x-9=0$

$4x^2+18x-2x-9=0$

$(2x-1)(2x+9)=0$

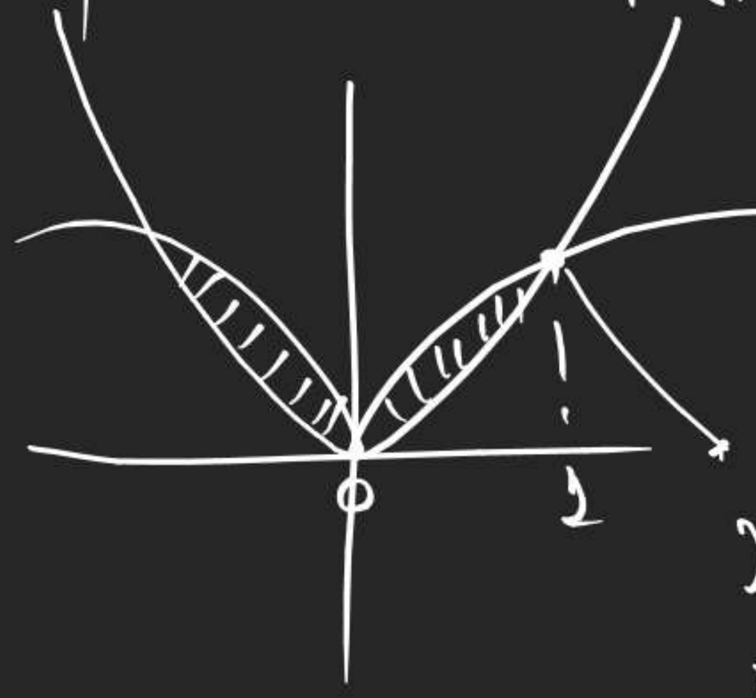
$x = \frac{1}{2}, -\frac{9}{2}$



$$A = 2\int_0^{1/2} 2\sqrt{x}\,dx + \frac{2}{2}\int_{1/2}^{3/2} \sqrt{3^2-(2x)^2}\,dx$$

→ $\sqrt{a^2-x^2}\,dx$

Q ABB $y = x^2$ & $y = \sqrt{|x|}$ → $y=\sqrt{x}$, $y=\sqrt{-x}$



$x^2 = \sqrt{x}$

$x^4 = x$

$x(x^3-1)=0$

$x = 0, 1$

$$A = 2\int_0^1 \sqrt{x} - x^2\,dx$$

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Q ABB $y = -\sqrt{-x}$ & $x = -\sqrt{-y}$

$y = \sqrt{-x}$

$y = -\sqrt{-x}$

$x = \sqrt{-y}$

$x = -\sqrt{-y}$

Q $0 < y < 3 - 2x - x^2$; $x > 0$

$y = 0$ के ऊपर

$y = -x^2 - 2x + 3$ के downward नीचे Parab

$x^2 + 2x - 3 = 0$

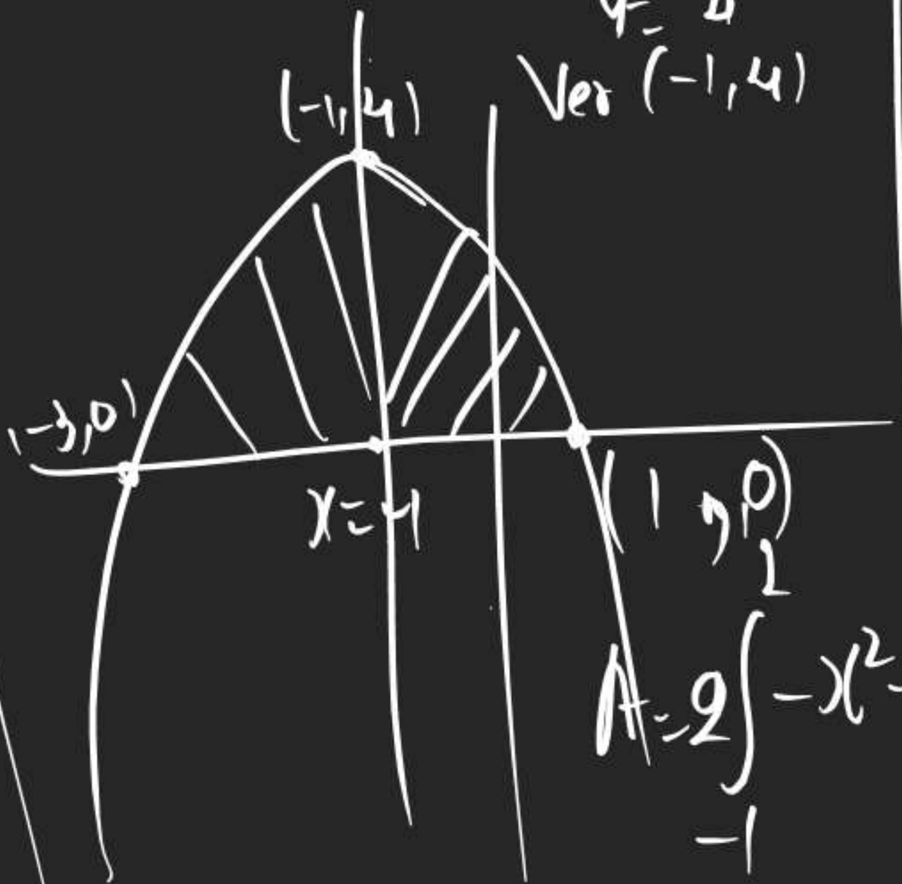
$(x+3)(x-1) = 0$

$\frac{dy}{dx} = -2x - 2 = 0$

$x = -1$

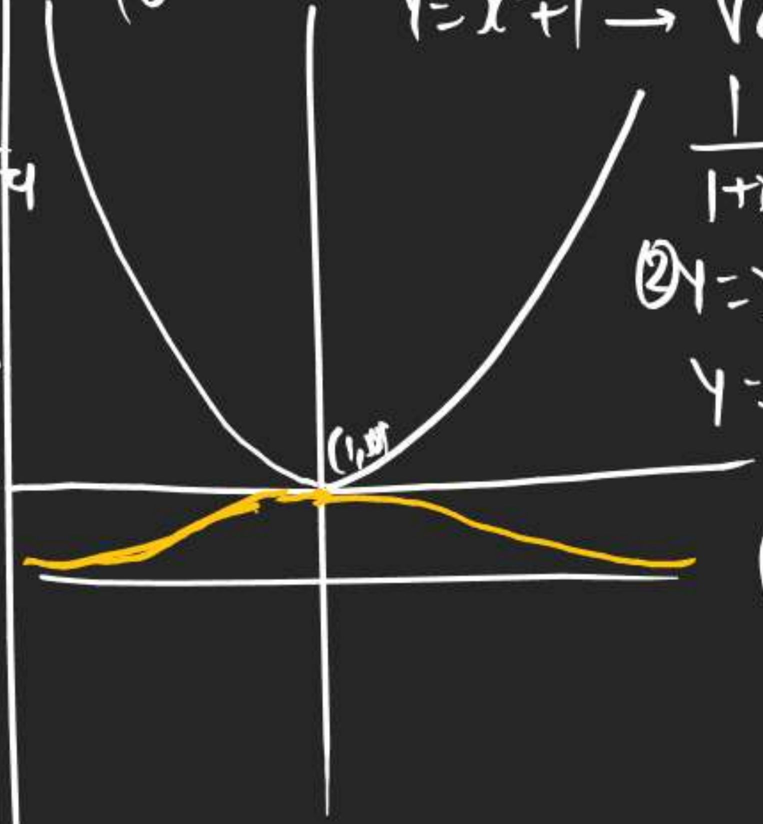
$y = 4$

Ver $(-1, 4)$



$A = 2\int_{-1}^{1} -x^2 - 2x + 3\,dx$

Q 10 $y = \frac{1}{1+x^2}$



① $y = x^2 + 1 \rightarrow$ (Min value 1 होगी

$\frac{1}{1+x^2}$ की value भी 1 होगी Max

② $y = x^2 + 1$ की value ∞ है

$y = \frac{1}{x^2+1}$ — 0

③ $y = x^2 + 1$, $x > 0$ पर ↑ing है

∴ $y = \frac{1}{x^2+1}$ ↓ होगा

④ $y = \frac{1}{x^2+1}$ Even

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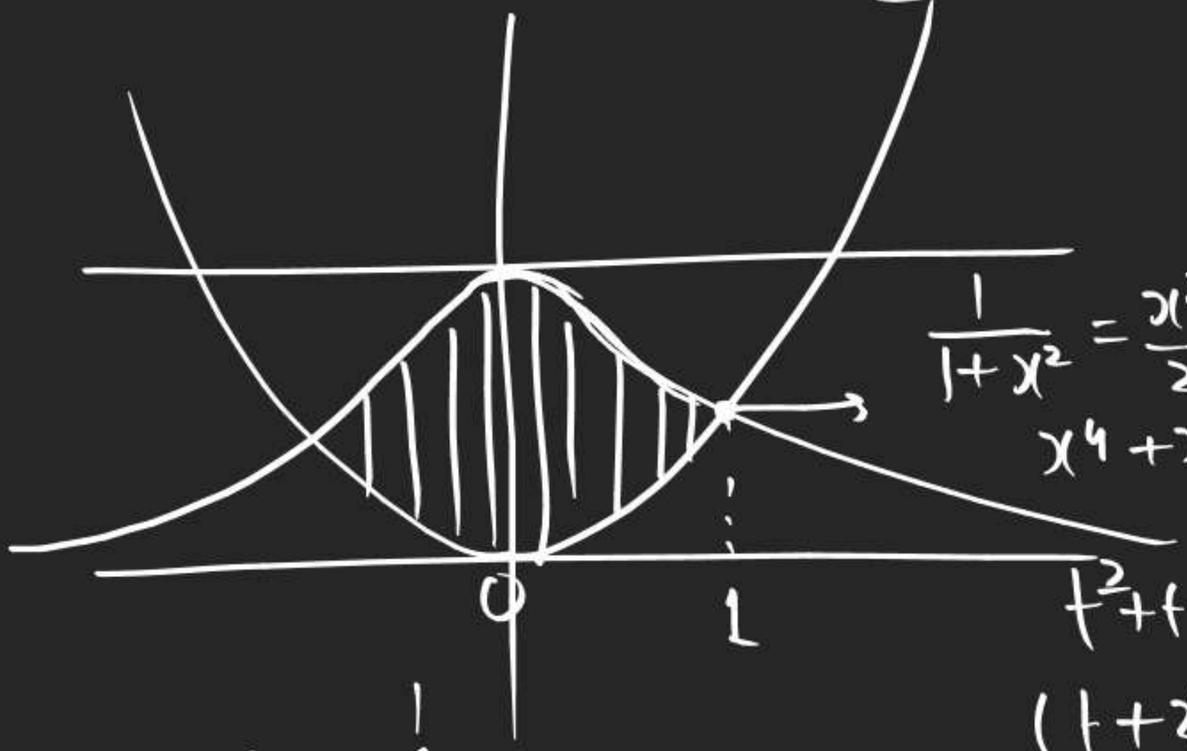
Q A.B.B $y = \frac{1}{1+x^2}$ & $y = \frac{x^2}{2}$



$\frac{1}{1+x^2} = \frac{x^2}{2}$

$x^4 + x^2 = 2$

$t^2 + t - 2 = 0$

$(t+2)(t-1) = 0$

$t = -2,\ t = 1$

$x^2 = -2$ ⊗ | $x^2 = 1$

$$A = 2\int_0^1 \frac{1}{1+x^2} - \frac{x^2}{2}\, dx$$

DY

Q $y = mx$ Bisects ABB.

12

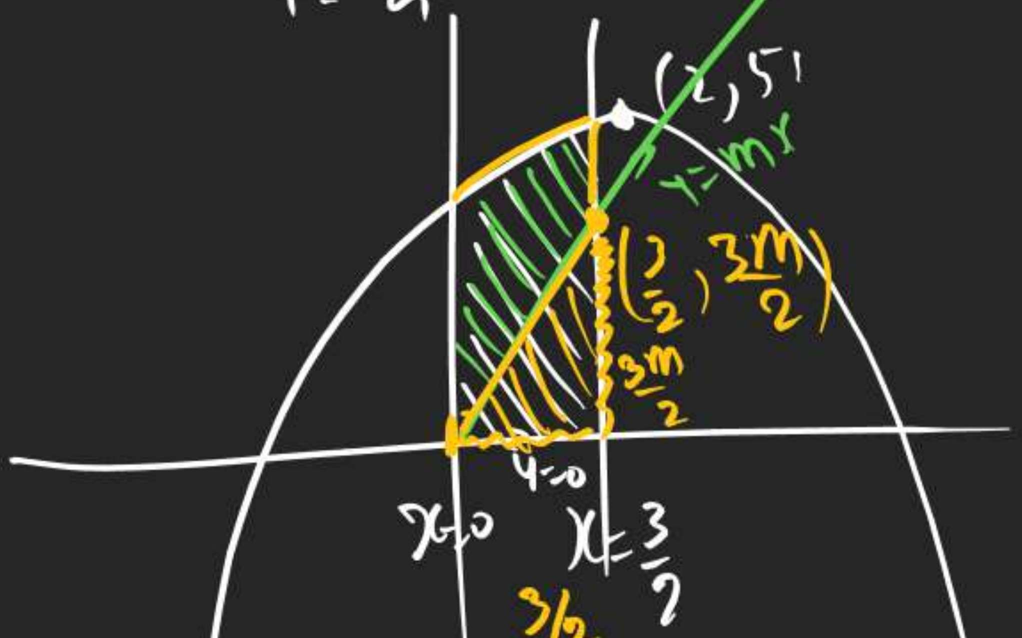
$y = 1 + 4x - x^2$, line $x = 0$, $x = \frac{3}{2}$

& $y = 0$ find m.

Ver → $\frac{dy}{dx} = 4 - 2x = 0$

$x = 2$ $(2, 5)$

$y = 5$



$$\int_0^{3/2} (1 + 4x - x^2 - mx)\, dx = \frac{1}{2} \times \frac{3}{2} \times \frac{3m}{2}$$

Solve & get m.

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Q $C_1 : y = \frac{1}{x}$, $C_2 : y = \ln x$

D_1 denote region surrounded by C_1, C_2 & line $x=1$ & D_2 denotes region surrounded by C_1, C_2 & line $x=a$. If $D_1 = D_2$ then $a = ?$

POI

$\frac{1}{x} = \ln x$

होगा नहीं

Solve.

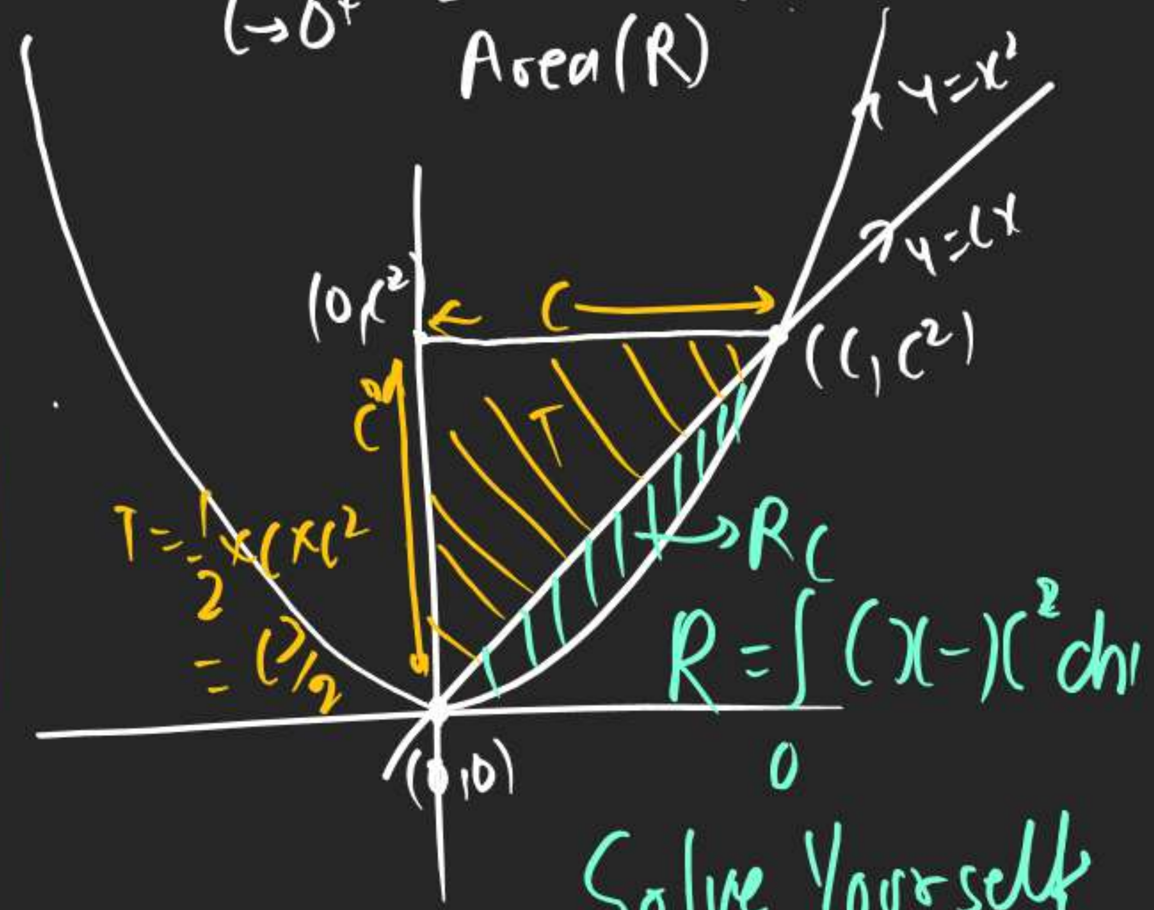
$\therefore x = b$ पर Intersected maanलो।

$$\int_1^b \left(\frac{1}{x} - \ln x\right)dx = \int_b^a \left(\ln x - \frac{1}{x}\right)dx$$

Solve & see Chamatkaar ·

b will get dissolved.

Q T is a △ with vertices $(0,0)$, $(0,c^2)$, (c,c^2) & R be the region betⁿ $\boxed{y=cx}$ & $y = x^2$; $c>0$ then $\lim\limits_{c \to 0^+} \frac{\text{Area}(T)}{\text{Area}(R)} = ?$



$T = \frac{1}{2} \times c \times c^2$

$= c^3/2$

$R = \int_0 (x - x^2)\,dx$

Solve Yourself

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Q) $f(x)=x^2+6x+1$ & R denotes Set of Pt. (x,y) in Coord Plane Such that $f(x)+f(y)\le 0$ & $f(x)-f(y)\le 0$ then R = ?

$f(y)=y^2+6y+1$

① $f(x)+f(y)=x^2+y^2+6x+6y+2\le 0$

Circle → $(-3,-3)$, $r=\sqrt{9+9-2}=4$

के अन्दर का

② $f(x)-f(y)=x^2-y^2+6x-6y$

$=(x-y)\{x+y+6\}\le 0$

$(x-y)(x+y+6)\le 0$

↓ 2 का Prod = −ve

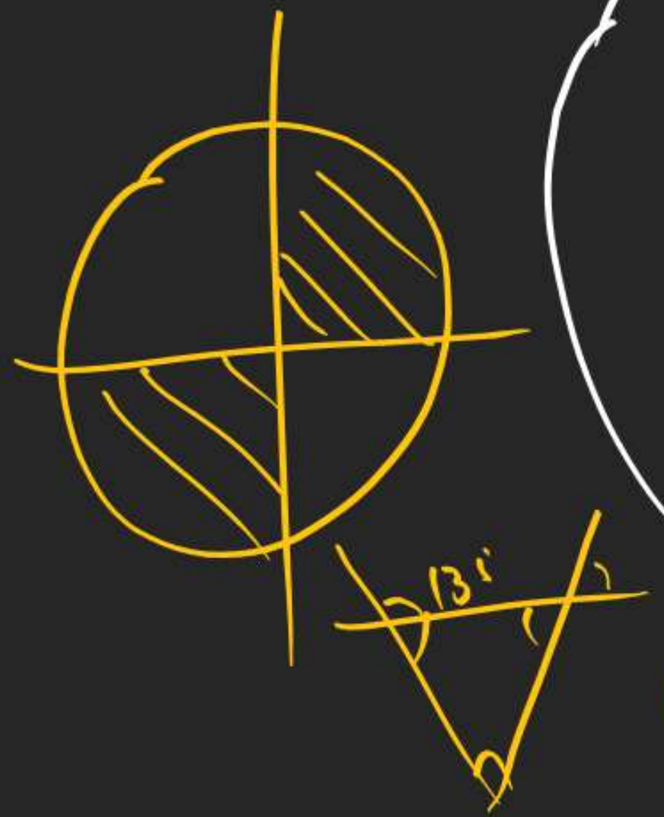
$x-y\le 0$ & $x+y+6\ge 0$ | या | $x-y\ge 0$ & $x+y+6\le 0$

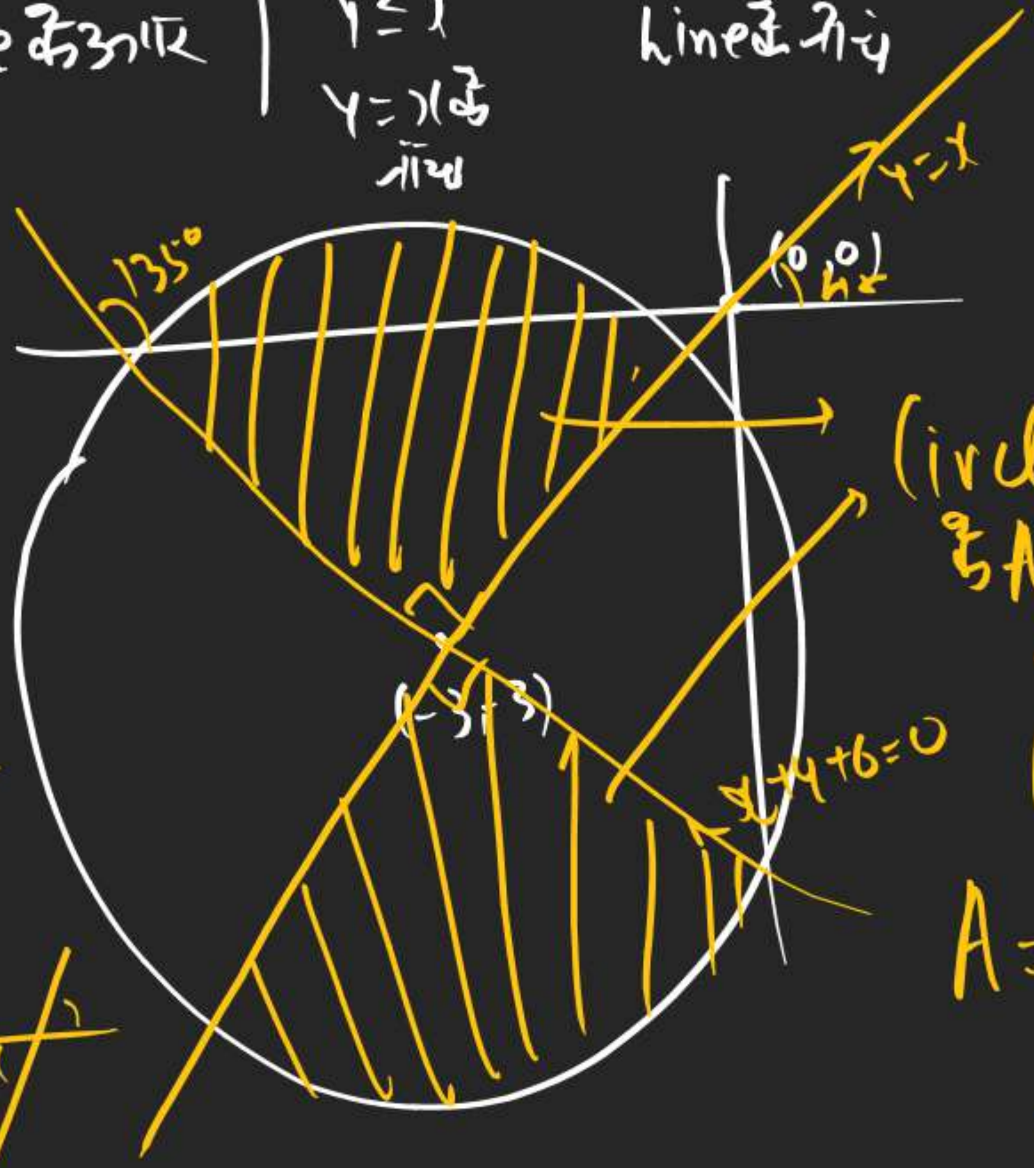
$y\ge x$, $y=x$ के ऊपर | line के ऊपर | $y\le x$, $y=x$ के नीचे | line के नीचे

$x+y+6=0$ को $(-3,-3)$ Satisfy

$y=-x-6$

$m=-1$





Circle के Area का half

$A=\frac{\pi\times 4^2}{2}$

$=8\pi$