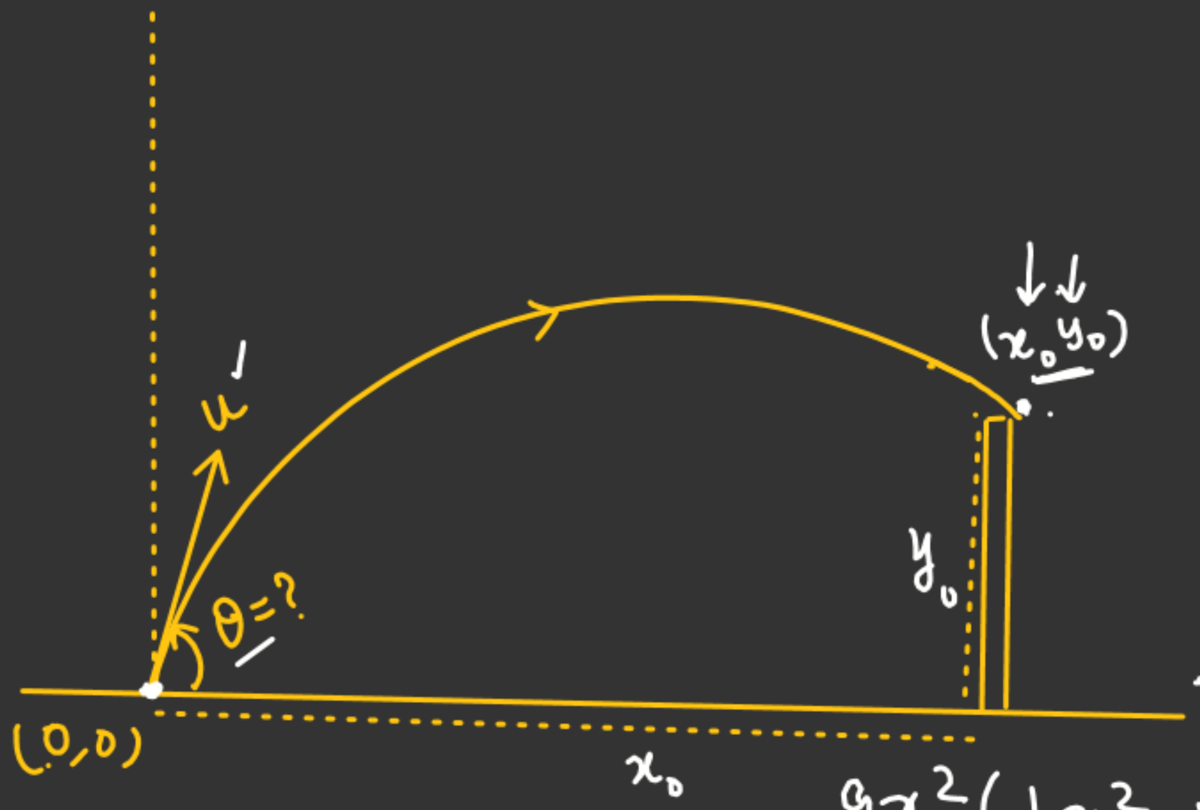
Nishant Jindal

For projectile to graze the tower:-

$[\sec^2\theta - \tan^2\theta = 1]$



Trajectory Equation

$$y = x\tan\theta - \frac{g}{2u^2\cos^2\theta}x^2$$

$$y = x\tan\theta - \frac{g}{2u^2}\sec^2\theta \cdot x^2$$

$$y = x\tan\theta - \frac{gx^2}{2u^2}(1+\tan^2\theta)$$

put ($x = x_0$ and $y = y_0$)

$$\underset{\downarrow(y_0)}{y} = x\tan\theta - \frac{gx^2}{2u^2} - \frac{gx^2\tan^2\theta}{2u^2}$$

$$\frac{gx_0^2}{2u^2}(\tan^2\theta) - x_0\tan\theta + \left(y_0 + \frac{gx_0^2}{2u^2}\right) = 0$$

$$\underbrace{(gx_0^2)}_{a}\tan^2\theta - \overset{b}{(2u^2x_0)}\tan\theta + \underbrace{(2u^2y_0 + gx_0^2)}_{c} = 0 \Rightarrow \tan\theta = \left[\frac{-b \pm \sqrt{b^2-4ac}}{2a}\right]$$

$$\boxed{ax^2 - bx + c = 0}$$

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Projectile Motion

For real 'θ'

$$D \geq 0$$

$$b^2 - 4ac \geq 0$$

$$(-2u^2x_0)^2 - 4gx_0^2(2u^2y_0 + gx_0^2) \geq 0$$

$$4u^4x_0^2 - 4gx_0^2(2u^2y_0 + gx_0^2) \geq 0$$

$$4x_0^2(u^4 - g(2u^2y_0 + gx_0^2)) \geq 0$$

$$u^4 - 2u^2gy_0 - g^2x_0^2 \geq 0$$

$$[(u^2)^2 - 2(u^2)(gy_0) + g^2y_0^2] - g^2y_0^2 - g^2x_0^2 \geq 0$$

$$(u^2 - gy_0)^2 \geq g^2(y_0^2 + x_0^2)$$

$$(u^2 - gy_0) \geq g\sqrt{y_0^2 + x_0^2}$$

$$u^2 \geq gy_0 + g\sqrt{y_0^2 + x_0^2}$$

$$\boxed{u \geq \sqrt{g\left[y_0 + \sqrt{y_0^2 + x_0^2}\right]}} \quad **$$

$$\rightarrow u_{min} = \sqrt{g\left[y_0 + \sqrt{y_0^2 + x_0^2}\right]}$$

$$\boxed{\tan\theta = \frac{u^2}{2x_0}} \quad **$$

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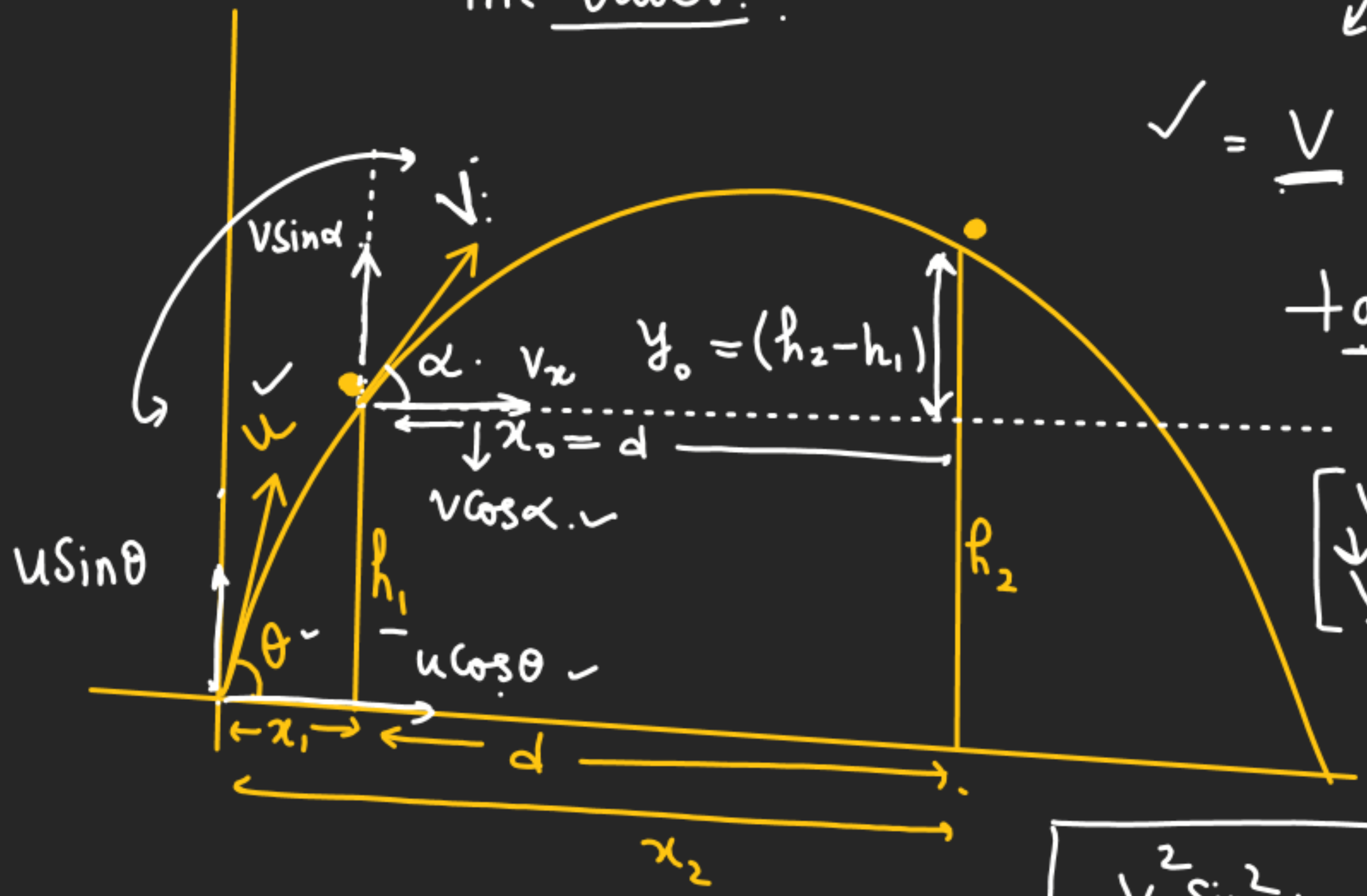
Projectile Motion

Application :- Find 'u'$_{min}$ So that it grazes both the tower.



$$u = \sqrt{g\left[y_0 + \sqrt{y_0^2 + x_0^2}\right]}$$

$$V = \sqrt{g\left[(h_2 - h_1) + \sqrt{(h_2-h_1)^2 + d^2}\right]}$$

$$\tan\alpha = \left(\frac{V^2}{2d}\right)$$

$$\left[\begin{array}{l} V_x = u\cos\theta \\ V\cos\alpha = u\cos\theta \end{array}\right]$$

$$u = \left[\frac{V\cos\alpha}{\cos\theta}\right]$$

$$\boxed{V^2\sin^2\alpha = u^2\sin^2\theta - 2gh_1}$$

3rd Equation

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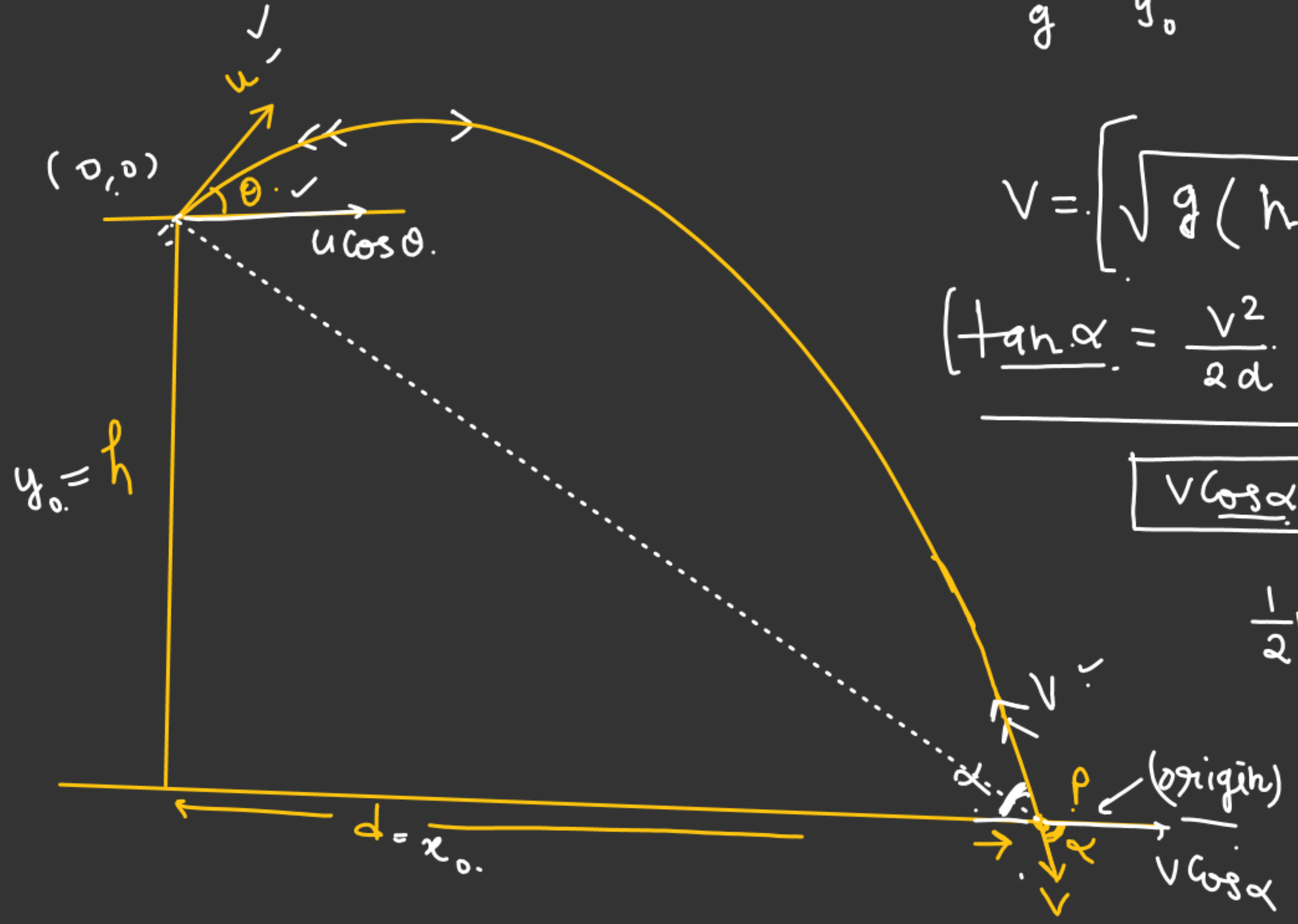

$g \quad y_0$

$$\boxed{V = \sqrt{g\left(h + \sqrt{h^2+d^2}\right)}}$$

$$\left[\tan\alpha = \frac{V^2}{2d}\right]$$

$$\boxed{V\cos\alpha = u\cos\theta} \quad -(1)$$

$$\frac{1}{2}mv^2 = mgh + \frac{1}{2}mu^2$$

$$v^2 = 2gh + u^2$$

$$u = \sqrt{v^2 - 2gh}$$

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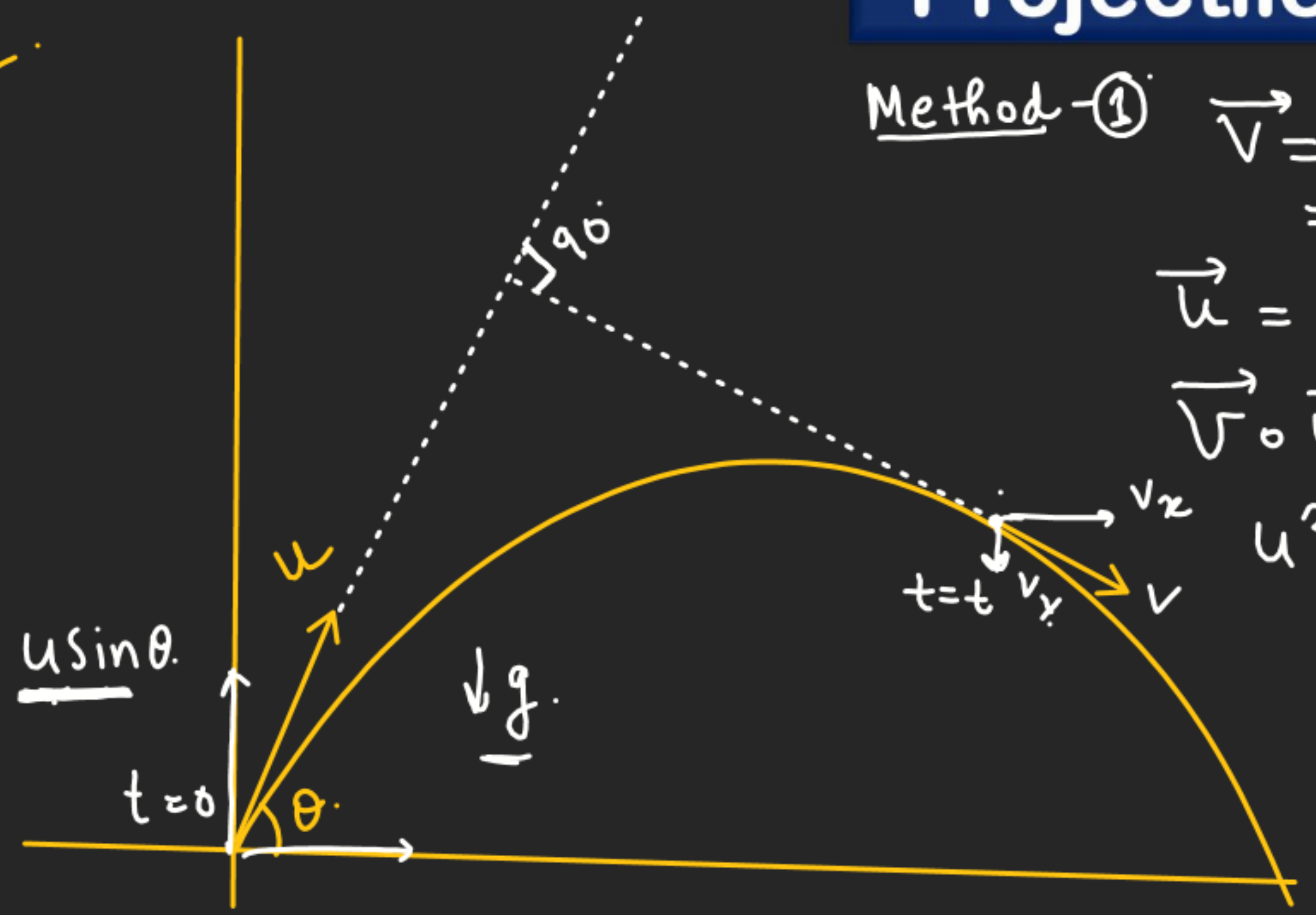
Projectile Motion

(A).



Method-①

$\vec{V} = V_x\hat{i} + V_y\hat{j}$

$= (u\cos\theta)\hat{i} + (u\sin\theta - gt)\hat{j}$

$\vec{u} = u\cos\theta\,\hat{i} + u\sin\theta\,\hat{j}$

$\vec{V}\cdot\vec{u} = 0$

$u^2\cos^2\theta + u^2\sin^2\theta - (u\sin\theta)gt = 0$

$u^2(\cos^2\theta + \sin^2\theta) = (u\sin\theta)gt$

$\left(t = \dfrac{u}{g\sin\theta}\right)$

Find the time when the velocity of the projectile is perpendicular to initial velocity of projection

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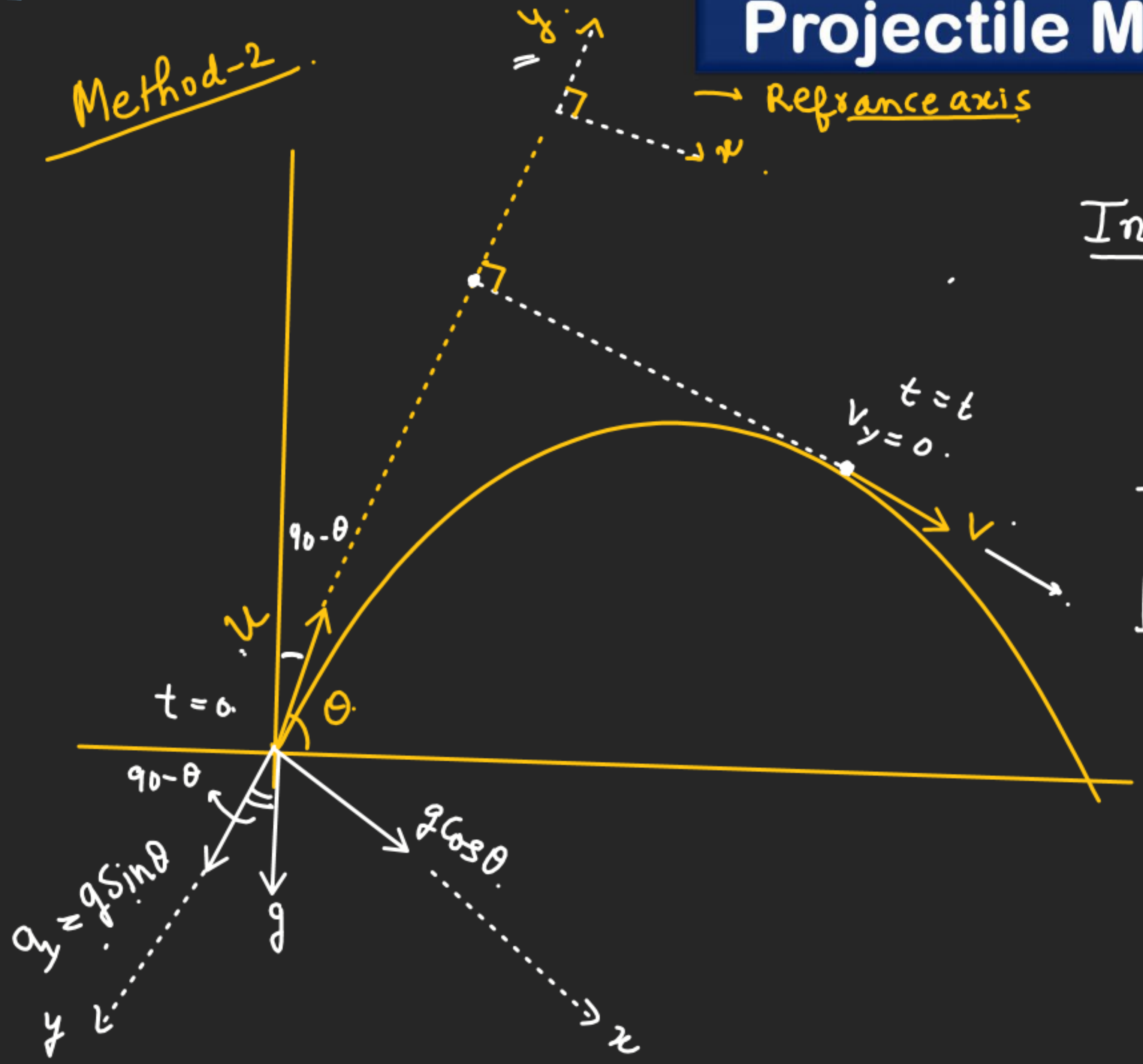
Projectile Motion

Method-2



→ Refrance axis

In - y direction

$$v_y = u_y - a_y t$$

$$\Downarrow$$

$$0 = u - g\sin\theta . t$$

$$\boxed{t = \frac{u}{g\sin\theta}}$$ ✓

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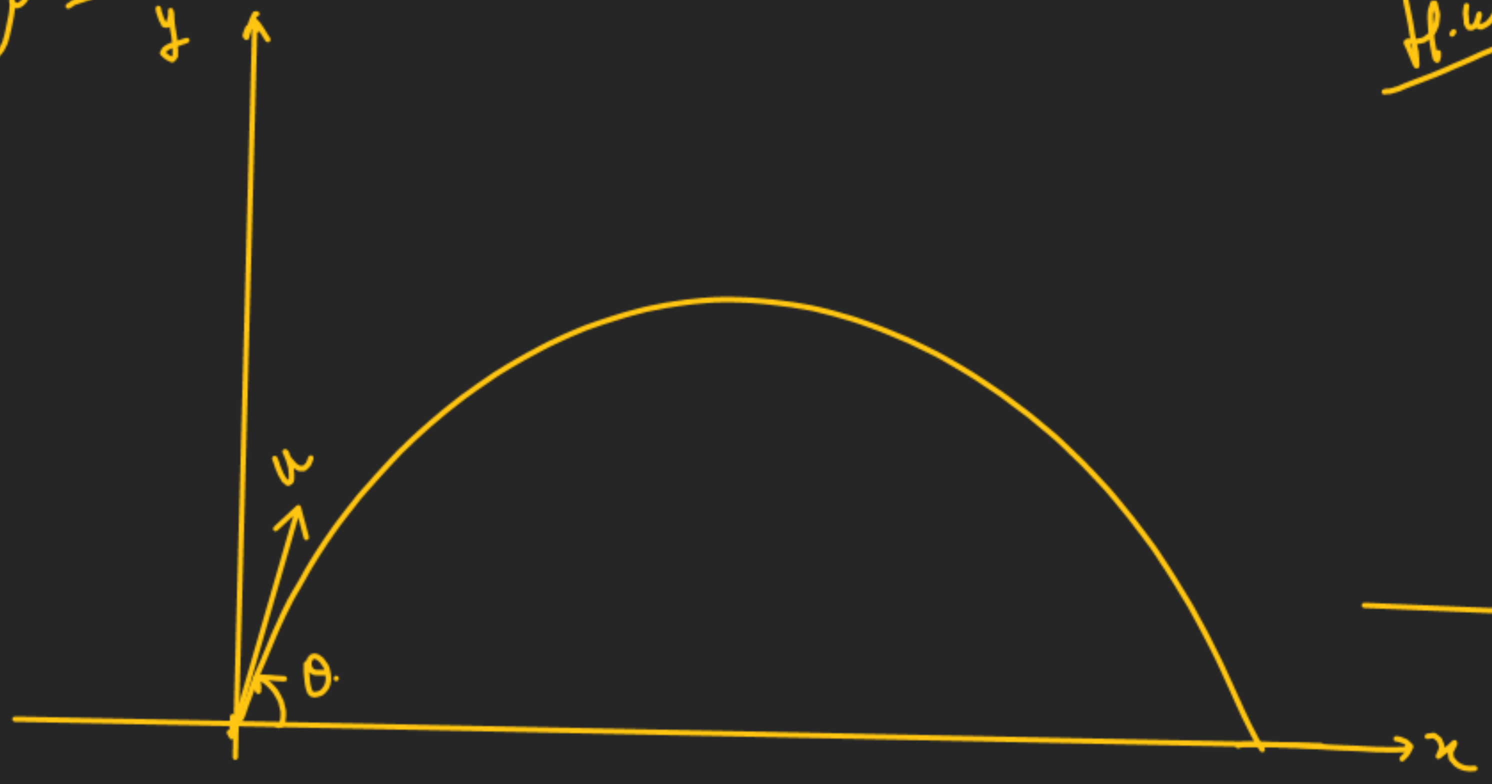
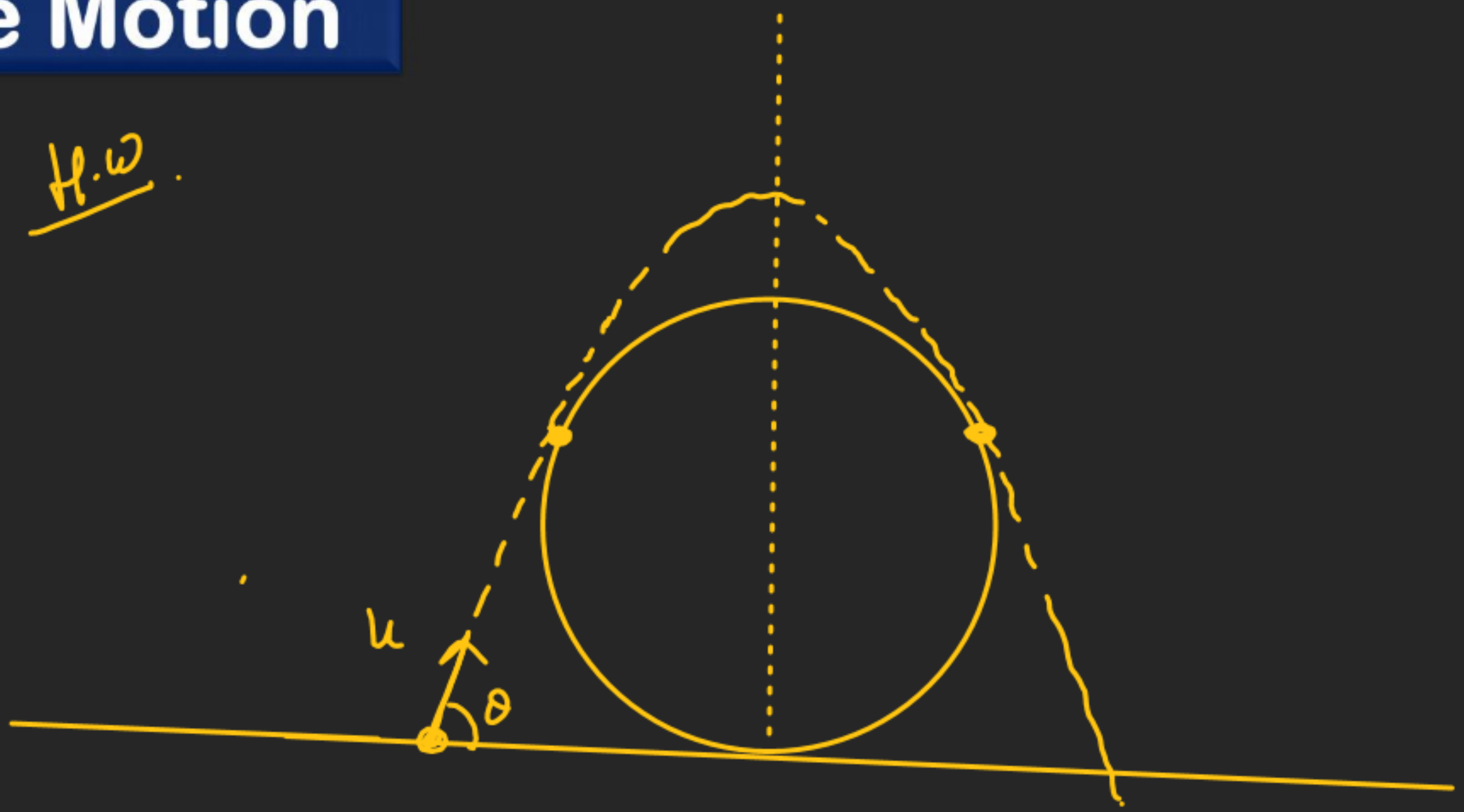
Projectile Motion

H.W

Method-3



H.W



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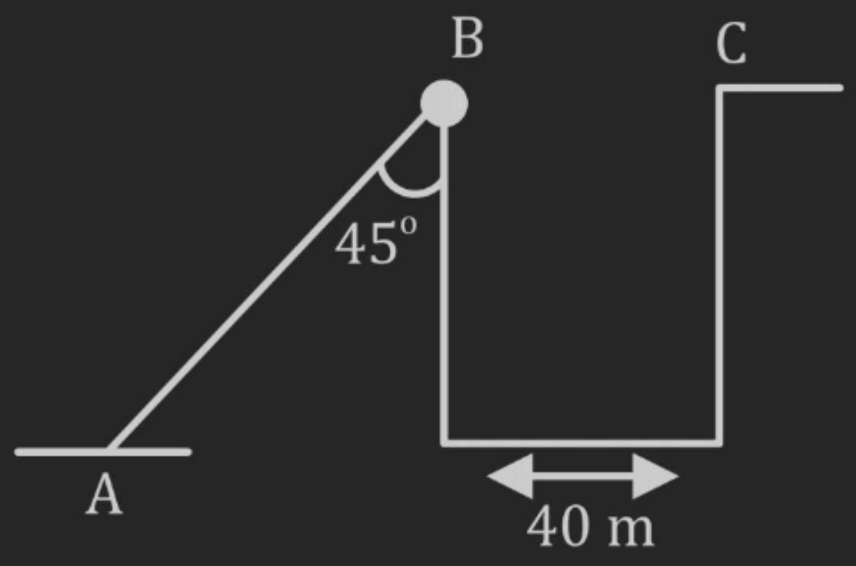
Projectile Motion

H.W

Q. A body is projected up along a smooth inclined plane with velocity u from the point A as shown in Fig. The angle of inclination is 45° and the top is connected to a well of diameter $40\ m$. If the body just manages to cross the well, what is the value of u ? The length of inclined plane is $20\sqrt{2}\ m$.



(A) $40\ ms^{-1}$

(B) $40\sqrt{2}\ ms^{-1}$

(C) $20\ m\ s^{-1}$

(D) $20\sqrt{2}\ ms^{-1}$