Nishant Jindal

QUADRATIC EQUATION

Ex 3 (Trigo Phi)

Q1

$$\cos x + \sin x = \frac{1}{2} \qquad \tan x$$

$$\cos^2 x + \sin^2 x + 2\sin x\cos x = \frac{1}{4}$$

$$2\sin x \cdot \cos x = -\frac{3}{4}$$

$$\sin 2x = -\frac{3}{4}$$

$$\frac{2\tan x}{1+\tan^2 x} = -\frac{3}{4}$$

$$8\tan x = -3 - 3\tan^2 x$$

$$3\tan^2 x + 8\tan x + 3 = 0$$

$$\tan x = \frac{-8 \pm \sqrt{64-36}}{6}$$

$$= \frac{-8 \pm 2\sqrt{7}}{6}$$

$$= -\frac{4+\sqrt{7}}{3} \;\Big|\; \frac{-4-\sqrt{7}}{3}$$

1) $\sin 2x = \dfrac{2\tan x}{1+\tan^2 x}$

2) $\cos 2x = \dfrac{1-\tan^2 x}{1+\tan^2 x}$

3) $\tan 2x = \dfrac{2\tan x}{1-\tan^2 x}$

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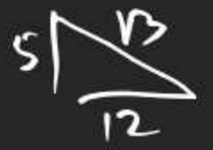
QUADRATIC EQUATION

2) $\cos(\alpha+\beta) = \frac{4}{5}\frac{B}{H}$ $\quad \sin(\alpha-\beta) = \frac{5}{13}$



$\sin(\alpha+\beta) = \frac{3}{5}\frac{P}{H}$ $\quad \overline{\cos(\alpha-\beta)} = \frac{12}{13}.$

$= \tan(\alpha+\beta) = \frac{3}{4}$ $\quad \tan(\alpha-\beta) = \frac{5}{12}$

$$\tan 2\alpha = \tan((\alpha+\beta)+(\alpha-\beta)) = \frac{\tan(\alpha+\beta)+\tan(\alpha-\beta)}{1-\tan(\alpha+\beta)\cdot\tan(\alpha-\beta)}$$

$$= \frac{\frac{3}{4}+\frac{5}{12}}{1-\frac{3}{4}\times\frac{5}{12}}$$

Q

$A = \sin^2 x + \cos^4 x$ $\quad \boxed{\frac{3}{4} \le A \le 1}$

$= \sin^2 x + (1-\sin^2 x)^2$

$= \sin^2 x + \sin^4 x - 2\sin^2 x + 1$

$= \sin^4 x - \sin^2 x + 1$ $\quad \frac{1}{2}$

$= (\sin^2 x - \frac{1}{2})^2 - (\frac{1}{2})^2 + 1$

$= (\boxed{\sin^2 x} - \frac{1}{2})^2 + \frac{3}{4}$

$0 + \frac{3}{4}$ $\quad (0-\frac{1}{2})^2 + \frac{3}{4}$ $\quad (1-\frac{1}{2})^2 + \frac{3}{4}$

$\frac{3}{4}$ Min $\quad 1$ M. $\quad 1$

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QUADRATIC EQUATION

Q3 $3\sin P + 4\cos Q = 6$
$4\sin Q + 3\cos P = 1$ } Angle R?

$9\sin^2 P + 16\cos^2 Q + 24\sin P\cos Q = 36$

$9\cos^2 P + 16\sin^2 Q + 24\sin Q\cos P = 1$

$9 + 16 + 24(\sin P\cos Q + \cos P\sin Q) = 37$

$24\sin(P+Q) = 12$

$P+Q+R = \pi$ $\quad \sin(P+Q) = \frac{1}{2}$

$\sin(\pi - R) = \frac{1}{2}$

$\sin R = \frac{1}{2} \Rightarrow \boxed{R = \frac{\pi}{6}}$

Q5 $\frac{\tan A}{1-\cot A} + \frac{\cot A}{1-\tan A} = ?$

$$\frac{\frac{\sin A}{\cos A}}{1 - \frac{\cos A}{\sin A}} + \frac{\frac{\cos A}{\sin A}}{1 - \frac{\sin A}{\cos A}}$$

$\frac{1}{\sin A\cos A} + 1$

$1 + \sec A \cdot \operatorname{cosec} A$ $\boxed{B}$

$$\frac{\sin A}{\cos A} \times \frac{\sin A}{\sin A - \cos A} + \frac{\cos A}{\sin A} \times \frac{\cos A}{(\cos A - \sin A)}$$

$$\frac{1}{\sin A - \cos A}\left[\frac{\sin^2 A}{\cos A} - \frac{\cos^2 A}{\sin A}\right]$$

$$\frac{1}{\sin A - \cos A} \times \frac{\sin^3 A - \cos^3 A}{\sin A\cos A}$$

$$\frac{\cancel{(\sin A - \cos A)}(\sin^2 A + \cos^2 A + \sin A\cos A)}{\cancel{(\sin A - \cos A)}(\sin A\cos A)}$$

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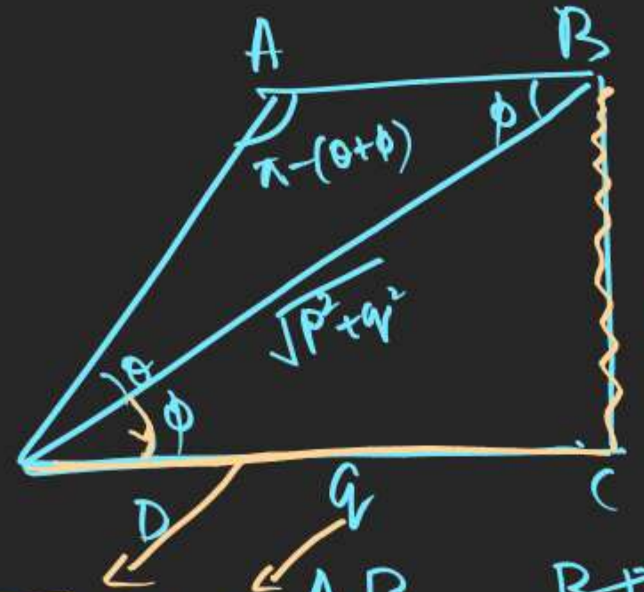
QUADRATIC EQUATION



$p = \sqrt{p^2+q^2}\cdot\sin\phi$

$\sqrt{p^2+q^2}\cos\phi = q$

$\cos\phi = \frac{q}{\sqrt{p^2+q^2}}$

$\frac{AB}{\sin\theta} = \frac{\cancel{BD}\ \sqrt{p^2+q^2}}{\sin(\pi-(\theta+\phi))}$

$AB = \frac{\sqrt{p^2+q^2}\sin\theta}{\sin(\theta+\phi)} = \frac{\sqrt{p^2+q^2}\sin\theta}{\sin\theta\cdot\boxed{\cos\phi}+\cos\theta\sin\phi}$

$= \frac{\sqrt{p^2+q^2}\sin\theta}{\sin\theta\cdot\frac{q}{\sqrt{p^2+q^2}}+\cos\theta\cdot\frac{p}{\sqrt{p^2+q^2}}}$

Q8 $f_K(x) = \frac{1}{K}(\sin^K x+\cos^K x)$ $f_4(x)-f_6(x)$

$f_4(x)-f_6(x) = \frac{\sin^4 x+\cos^4 x}{4} - \frac{\sin^6 x+\cos^6 x}{6}$

$= \frac{1-2\sin^2 x\cdot\cos^2 x}{4} - \frac{1-3\sin^2 x\cdot\cos^2 x}{6}$

$= \frac{3-6\sin^2 x\cdot\cancel{\cos^2 x}-2+6\sin^2 x\,\cancel{\cos^2 x}}{12}$

$= \frac{1}{12}$

Ex5

1 – 6	8 – 12

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QUADRATIC EQUATION

$\sqrt{6} = \boxed{\sqrt{2}} \times \sqrt{3}$

$\sqrt{Pr.No}$ $\sqrt{Pr.No}$

Quad

$\alpha + \beta$, $\alpha\beta$, $\sqrt{D}/a$

50

Nature of Roots. (Quad Eqⁿ) $= \boxed{a}x^2 + \boxed{b}x + \boxed{c} = 0$

$\boxed{a, b, c \in Q}$

Discriminant → $D = b^2 - 4ac$.

$D = 0$

1) $x = \frac{-b \pm \sqrt{0}}{2a} \rightarrow x = \frac{-b}{2a}$

2) Identical Roots (α, α)

3) Q Eqⁿ = Per sqʳ.

4) $\boxed{ax^2 + bx + c = a\left(x + \frac{b}{2a}\right)^2}$

5) $f(x) \geq 0$

$D \neq 0$

$D > 0$

$D = 25$ (Per sqʳ)

$x = \frac{-b \pm \sqrt{25}}{2a}$

$= \frac{-b+5}{2a}, \frac{-b-5}{2a}$

Roots Rational

Distinct

$D = 24$ (Normal No.)

$x = \frac{-b \pm \sqrt{24}}{2a}$

Irr.

$x = \frac{-b \pm 2\sqrt{6}}{2a}$

Roots Irr.

Bogo Offer.

Conj: $\alpha + \sqrt{\beta}$ then $\alpha - \sqrt{\beta}$

$D < 0$

$D = -ve$

$x = \frac{-b \pm \sqrt{-ve}}{2a}$ → iota.

Imaginary Roots

BOGO Offer

$\alpha + i\beta$ then other will be $\alpha - i\beta$. (Conjugate Roots)

2π

QUADRATIC EQUATION

Q Form a Quad Eqn with Rational (off) whose One Root is $\boxed{\tan\frac{\pi}{8}}$?

① Dara $\alpha = \sqrt{2}-1$, $\beta = -\sqrt{2}+1$ } गलती!

② $\tan\frac{\pi}{8}$ नहीं आता था)

$\alpha + \sqrt{\beta}$

$\alpha = \tan\frac{\pi}{8} = \sqrt{2}-1 = -1+\sqrt{2}$

then other Root will be $\beta = -1-\sqrt{2}$

$x^2-(\alpha+\beta)x+\alpha\beta=0$

$x^2-(-1+\cancel{\sqrt{2}}-1-\cancel{\sqrt{2}})x+(-1+\sqrt{2})(-1-\sqrt{2})=0$

$x^2+2x+(-1)^2-(\sqrt{2})^2=0$

$x^2+2x-1=0$

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QUADRATIC EQUATION

Q Form a Quad. polynomial $F(x)$ with rational coff whose one Zero is $\sqrt{3}+1$ & $\boxed{f(1)=4}$

$F(x)=ax^2+bx+c$

$F(x)=a(x^2-(\alpha+\beta)x+\alpha\beta)$

$F(x)=a(x^2-2x-2)$

$x \to 1$

$F(1)=a(1^2-2-2)$

$4=-3a \Rightarrow a=-\frac{4}{3}$

$\therefore F(x)=-\frac{4}{3}(x^2-2x-2)$

$\alpha+\sqrt{\beta}$ then $\alpha-\sqrt{\beta}$.

$\alpha=\sqrt{3}+1$ Dang Se Likho.

$\alpha=1+\sqrt{3}$ then other root

$\beta=1-\sqrt{3}$

$\alpha+\beta=2$

$\alpha\cdot\beta=(1+\sqrt{3})(1-\sqrt{3})$

$=1^2-\sqrt{3}^2$

$=1-3=-2$

$ax^2+bx+c=0 \quad \alpha, \beta$

$a(x-\alpha)(x-\beta)=0$

$a(x^2-\alpha x-\beta x+\alpha\beta)=0$

$a(x^2-x(\alpha+\beta)+\alpha\beta)=0$

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QUADRATIC EQUATION

Learning :-

1) Q $\boxed{Eq^n}$ ← Use Kra.

$ax^2+bx+c=0$ → α, β } Roots

$a(x-\alpha)(x-\beta)=0$.

2) Q $\boxed{Poly}$ ← Use Kra

Zeroes. α, β.

$ax^2+bx+c = a(x-\alpha)(x-\beta)$

$$ax^2+bx+c = a(x-\alpha)(x-\beta)$$

$$= a\left(x^2-(\alpha+\beta)x+\alpha\beta\right)$$

$$= a\left(x^2-\left(-\frac{b}{a}\right)x+\frac{c}{a}\right)$$

$$= a\left(x^2+\frac{bx}{a}+\frac{c}{a}\right)$$

$$= ax^2+bx+c$$

QUADRATIC EQUATION

Q Find value of m if eqⁿ

$x^2-(m+2)x+(m^2-4m+4)=0$

has Coincident Roots

||

Identical Root $\boxed{D=0}$

$b^2-4ac=0$

$b^2=4ac$

$[-(m+2)]^2=4\times1\times(m^2-4m+4)$

$m^2+4m+4=4m^2-16m+16$

$3m^2-20m+12=0$

$(3m-2)(m-6)=0 \Rightarrow m=\frac{2}{3},6$

Same

Q Find value of m if Eqⁿ $x^2-(m+2)x+(m^2-4m+4)=0$

is a Perfect Sqⁿ?

Q Find value of K for which a Quad Poly

$F(x)=(4-K)x^2+(2K+4)x+(8K+1)$ is a Perfect Sqⁿ. $D=0$

$(2K+4)^2=4\times(4-K)(8K+1)$

$\cancel{4}(K+2)^2=\cancel{4}(4-K)(8K+1)$

$K^2+4K+\cancel{4}=-8K^2+31K+\cancel{4}$

$9K^2-27K=0$

$9K(K-3)=0$

$K=0,3$

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QUADRATIC EQUATION

V. imp.

Q If eqn $\boxed{p(q-r)}x^2 + \boxed{q(r-p)}x + \boxed{r(p-q)} = 0$ has equal Roots then P.T. $\frac{2}{q} = \frac{1}{p} + \frac{1}{r}$

1) Coffnt → $p(q-r), q(r-p), r(p-q)$

2) Coffnt are in Cyclic Order then check if Q fxn can be satisfied by $x=1$

3) $F(x) = p(q-r)x^2 + q(r-p)x + r(p-q)$

$F(1) = p(q-r)\times 1^2 + q(r-p)\times 1 + r(p-q)$

$= \cancel{pq} - \cancel{pr} + \cancel{qr} - \cancel{pq} + \cancel{pr} - \cancel{qr} = 0$

★ Onwords → whenever Coff will be in Cyclic Order we will take Root $\alpha = 1$

here in this QS.

$\alpha = 1$ then $\beta = 1$ (Equal Root)

Prod $= \alpha \cdot \beta = 1 \times 1 = 1$

$\Rightarrow \frac{c}{a} = 1$

$\frac{r(p-q)}{p(q-r)} = 1$

$pr - qr = pq - pr$

$2pr = qr + pq \quad \div pqr$

$\boxed{\frac{2}{q} = \frac{1}{p} + \frac{1}{r}}$