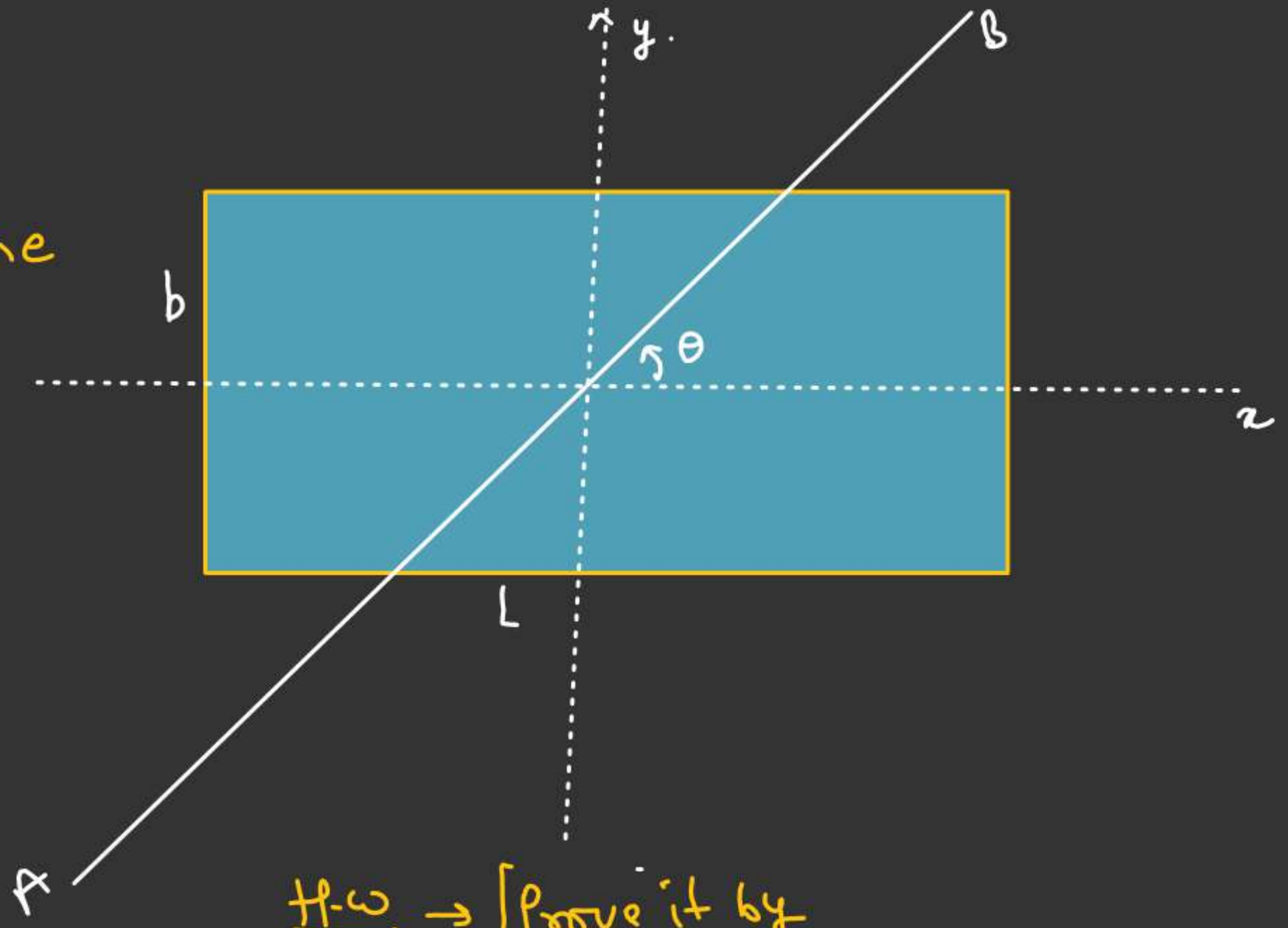
Nishant Jindal

AB → Axis in the plane of rectangular lamina and passing through the center.



$$I_{AB} = I_x \cos^2\theta + I_y \sin^2\theta$$

$$I_x = \frac{Mb^2}{12}, \quad I_y = \frac{ML^2}{12}$$

$$\boxed{I_{AB} = \frac{Mb^2}{12}\cos^2\theta + \frac{ML^2}{12}\sin^2\theta}$$

H.W → [Prove it by integration

→

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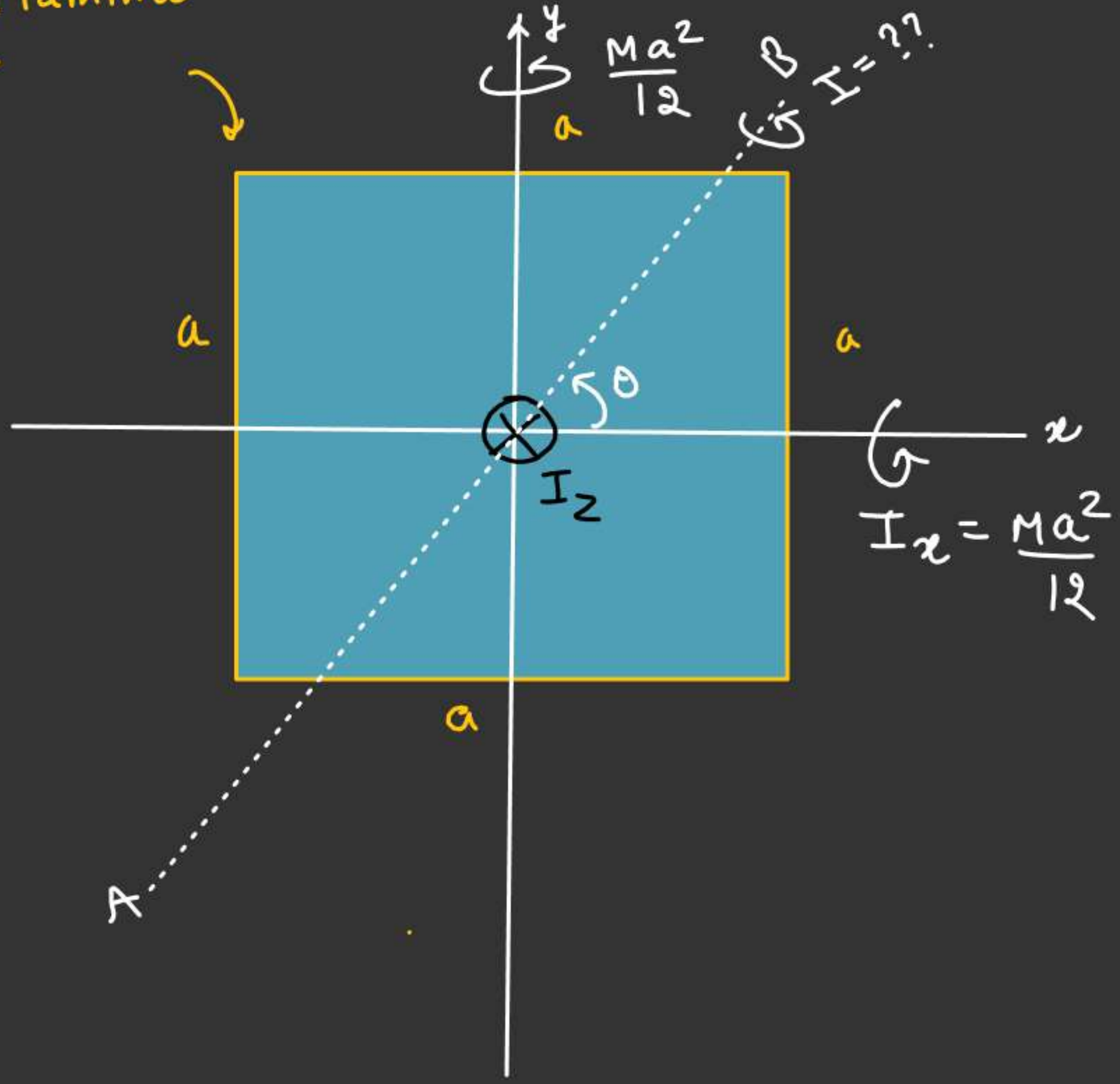
$$I_z = I_x + I_y$$

$$= \frac{Ma^2}{12} + \frac{Ma^2}{12}$$

$$\boxed{I_z = \frac{Ma^2}{6}}$$

Squar lamina of side a

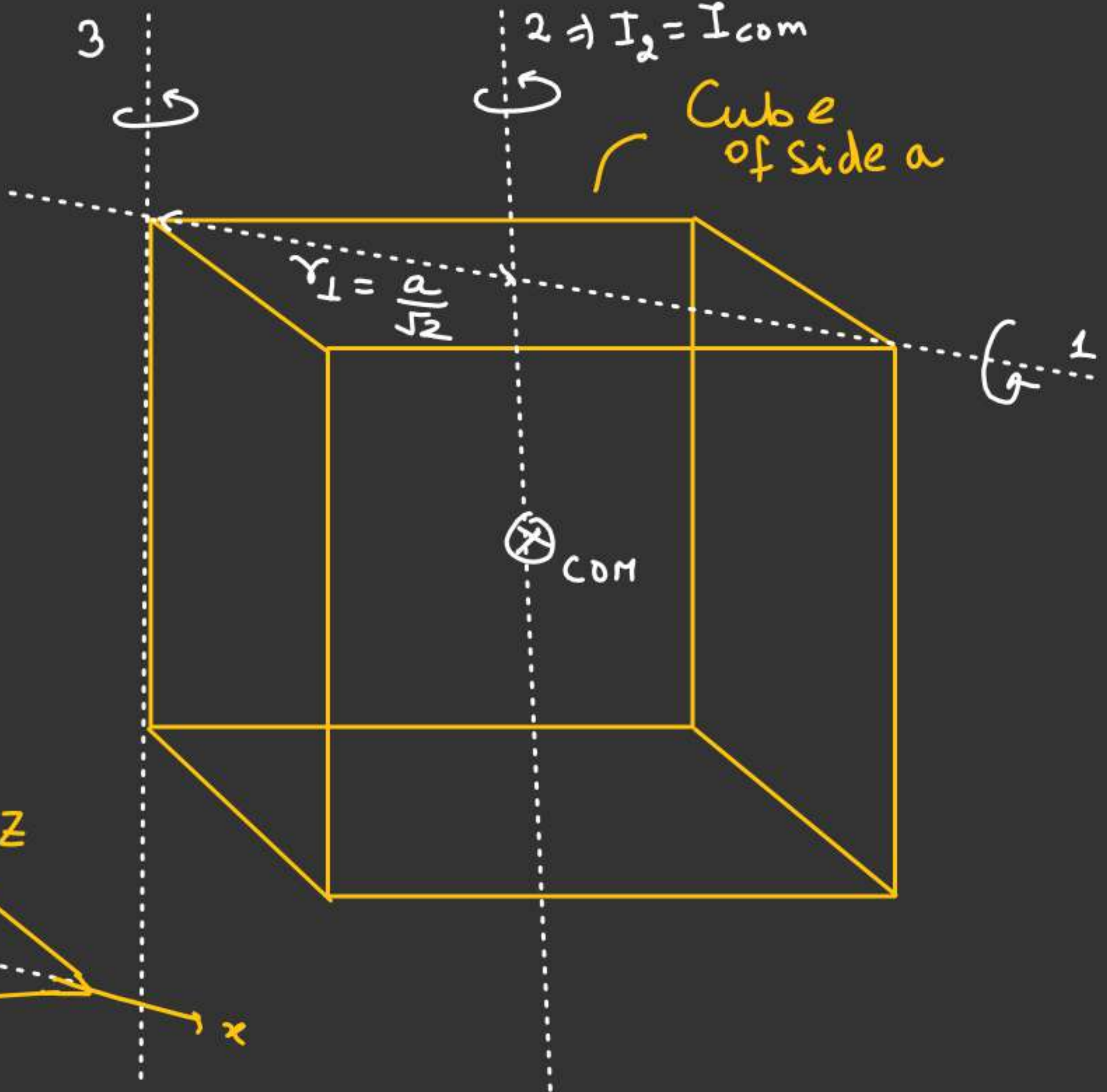


$$I_{AB} = \frac{Ma^2}{12}\sin^2\theta + \frac{Ma^2}{12}\cos^2\theta$$

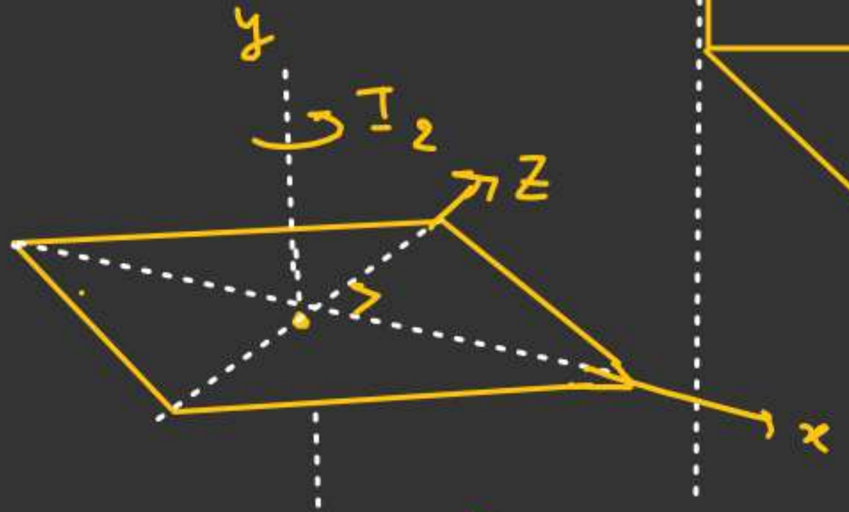
$$\boxed{I_{AB} = \frac{Ma^2}{12}}$$

$$I_1 = \frac{Ma^2}{12}$$
$$I_2 = \frac{Ma^2}{6}$$

Just like Square lamina as mass distribution is same is Square and Cube w.r.t axis 1 & 2.



$$I_3 = I_{com} + M\left(\frac{a}{\sqrt{2}}\right)^2$$
$$= \left(\frac{Ma^2}{6} + \frac{Ma^2}{2}\right)$$
$$= \frac{Ma^2 + 3Ma^2}{6}$$
$$= \frac{4Ma^2}{6}$$
$$= \frac{2Ma^2}{3} \checkmark$$



$$I_x + I_z = I_y$$
$$\frac{Ma^2}{12} + \frac{Ma^2}{12} = I_y$$
$$I_y = \frac{Ma^2}{6}$$

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$$I_2 = \frac{M}{12}\left(L^2+\omega^2\right)$$

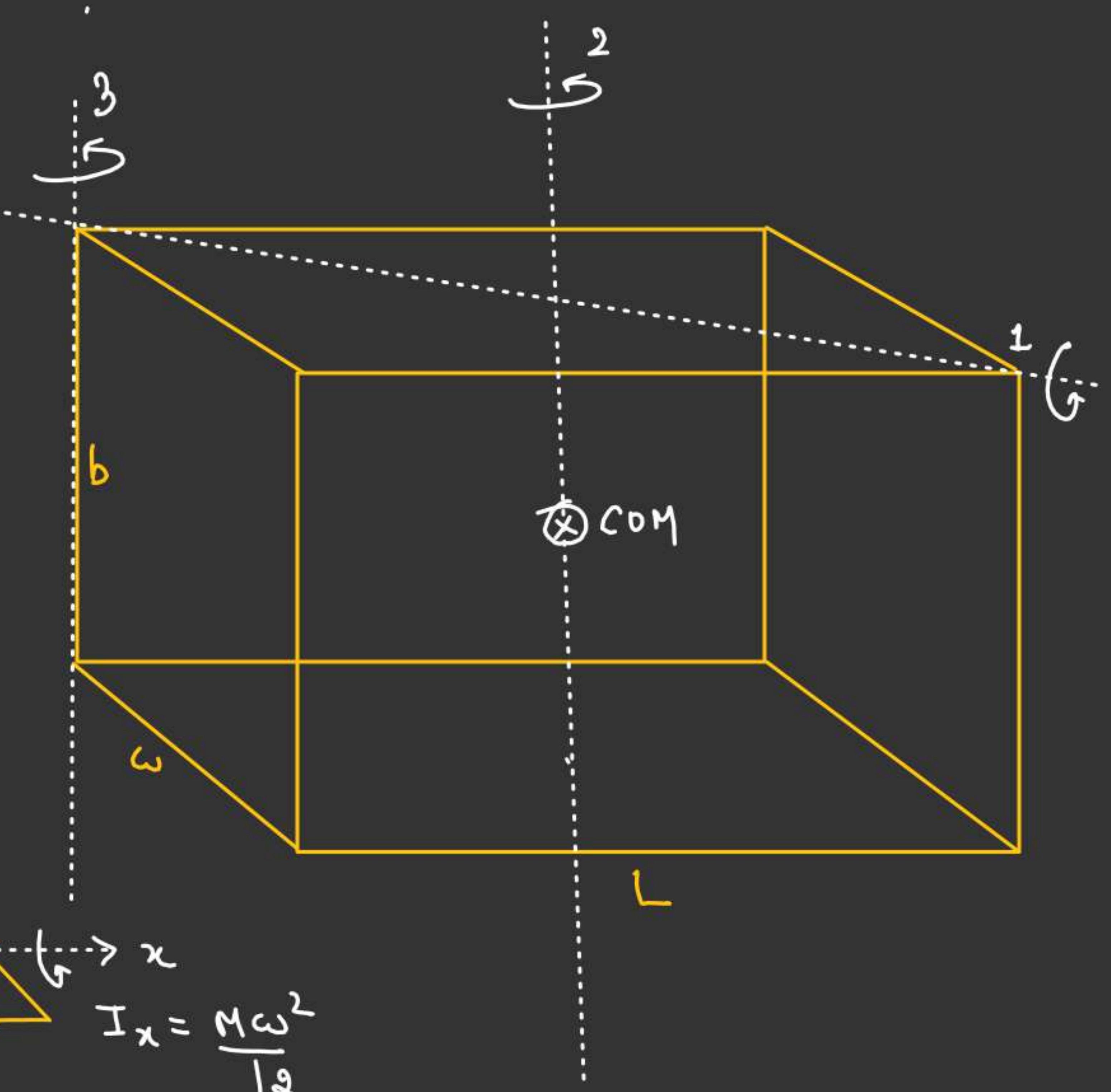
$$I_{com} = I_2$$

$$I_3 = I_{com} + M r_1^2$$

$$r_1 = \frac{\sqrt{L^2+\omega^2}}{2}$$

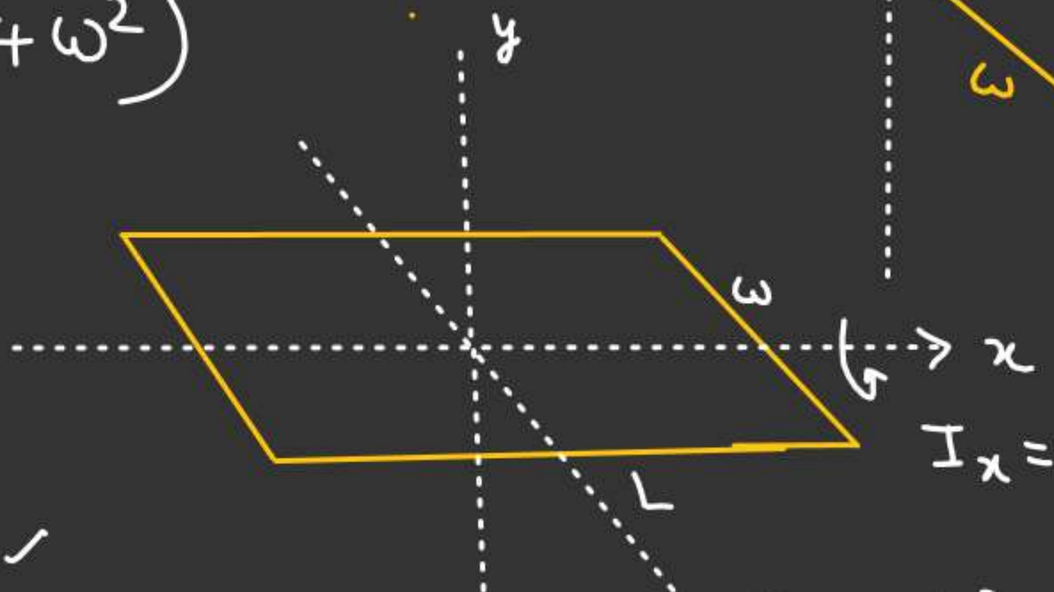
$$I_3 = \frac{M}{12}\left(L^2+\omega^2\right) + M\frac{\left(L^2+\omega^2\right)}{4}$$

$$= \frac{M}{3}\left(L^2+\omega^2\right)$$



$$I_y = I_x + I_z$$

$$= \frac{M}{12}\left(L^2+\omega^2\right)$$



$$I_x = \frac{M\omega^2}{12}$$

$$I_z = \frac{ML^2}{12}$$

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44:

$I_{AB} = ??$

CD parallel to AB and passing through its Center

$$OP = \frac{a}{\sqrt{2}}$$

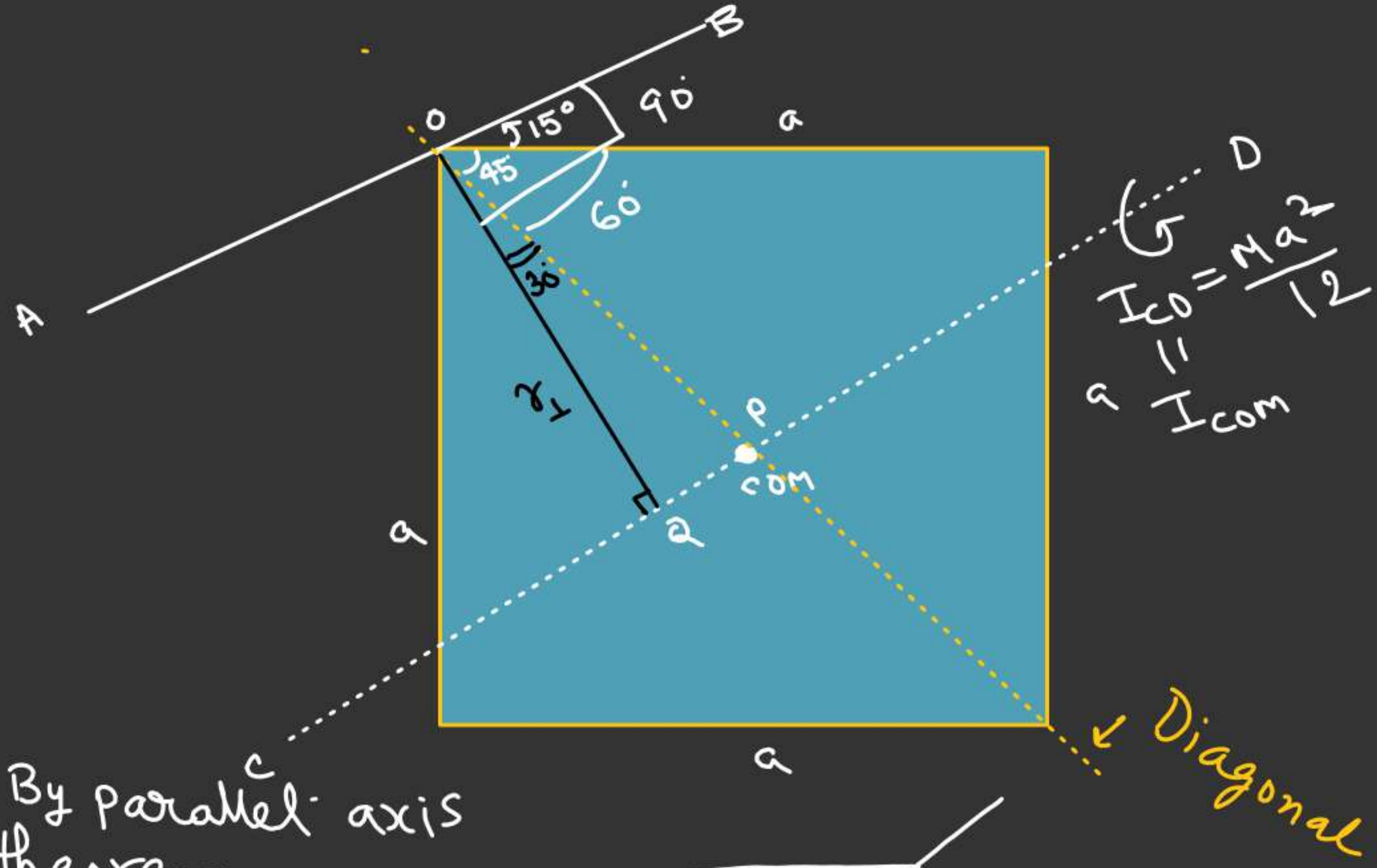
In ΔOPQ

$$\cos 30^\circ = \frac{r_\perp}{OP}$$

$$r_\perp = OP\cos 30^\circ$$

$$= \frac{a}{\sqrt{2}} \times \frac{\sqrt{3}}{2}$$

$$= \frac{\sqrt{3}a}{2\sqrt{2}}$$

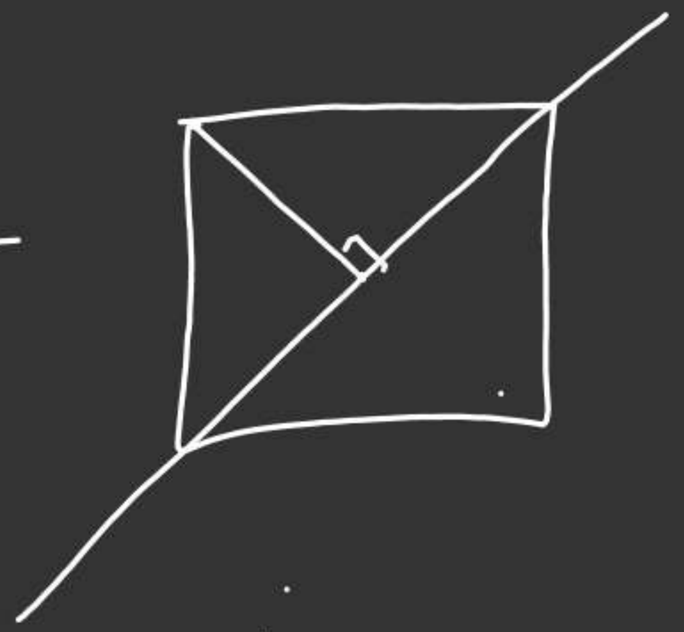


By parallel axis theorem

$$I_{AB} = I_{com} + Mr_\perp^2$$

$$= \left(\frac{Ma^2}{12} + \frac{M3a^2}{8}\right)$$

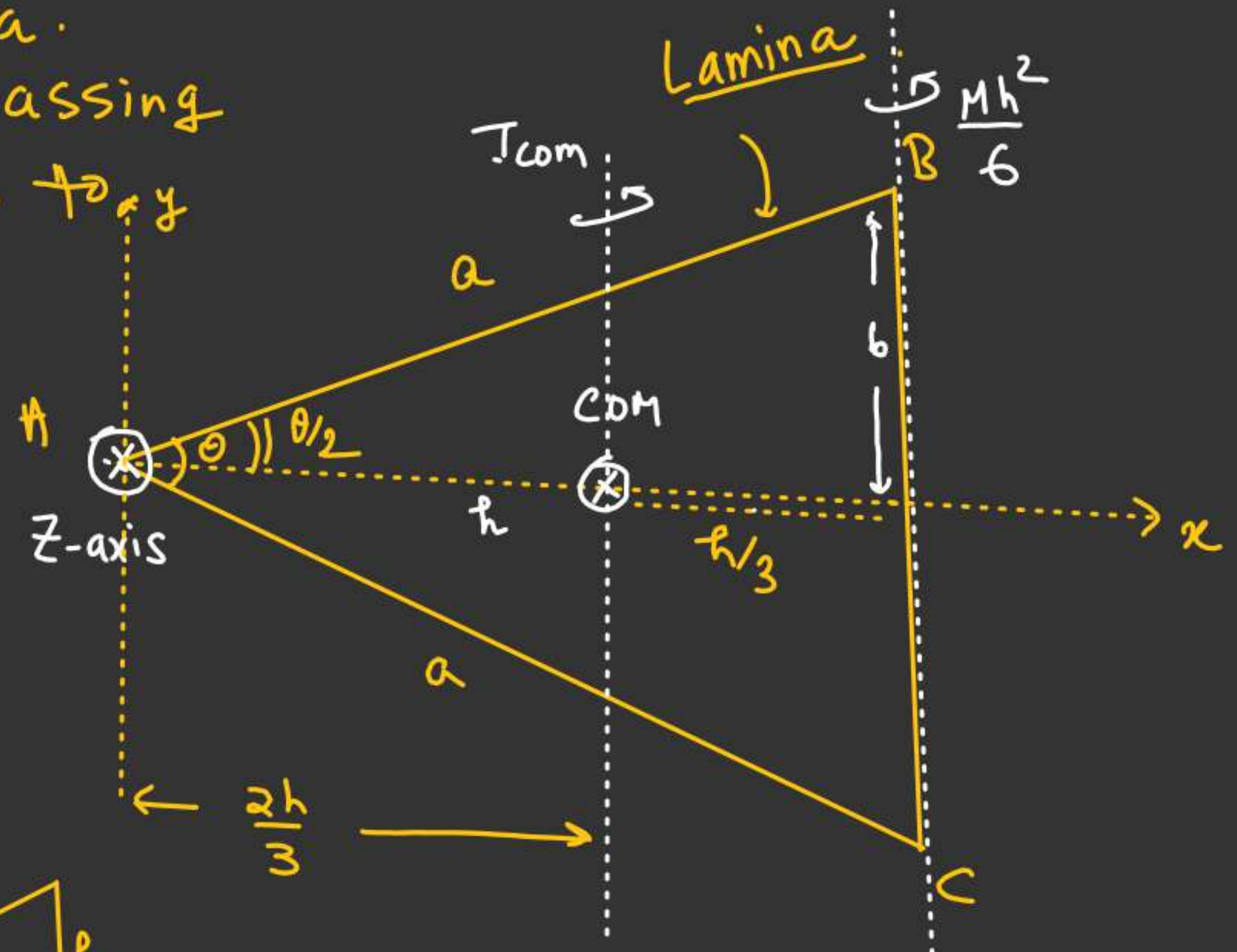
$$= \frac{11ma^2}{24}$$



Nishant Jindal

Isosceles triangular lamina.

Find M.I about an axis passing through A and perpendicular to plane of ABC.



$$I_{BC} = I_{com} + M\left(\frac{h}{3}\right)^2$$

$$\frac{Mh^2}{6} = I_{com} + \frac{Mh^2}{9}$$

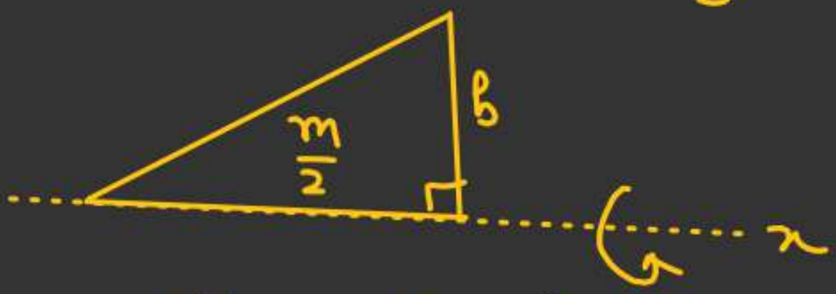
$$I_{com} = \left(\frac{Mh^2}{6} - \frac{Mh^2}{9}\right)$$

$$I_{com} = \left(\frac{Mh^2}{18}\right)$$

$$I_y = I_{com} + m\left(\frac{2h}{3}\right)^2$$

$$= \frac{Mh^2}{18} + \frac{4Mh^2}{9}$$

$$= \left(\frac{Mh^2}{2}\right).$$



$$I_x = \left(\frac{m}{2}\right)\frac{b^2}{6} \times 2 = \left(\frac{Mb^2}{6}\right)$$

$$\begin{cases} b = a\sin\frac{\theta}{2} \\ h = a\cos\frac{\theta}{2} \end{cases}$$

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By perpendicular axis theorem at A

$$I_z = I_x + I_y$$

$$= \left(\frac{Mb^2}{6} + \frac{Mh^2}{2}\right)$$

$$= \frac{M}{6}\left(a\sin\frac{\theta}{2}\right)^2 + \frac{M}{2}\left(a\cos\frac{\theta}{2}\right)^2$$

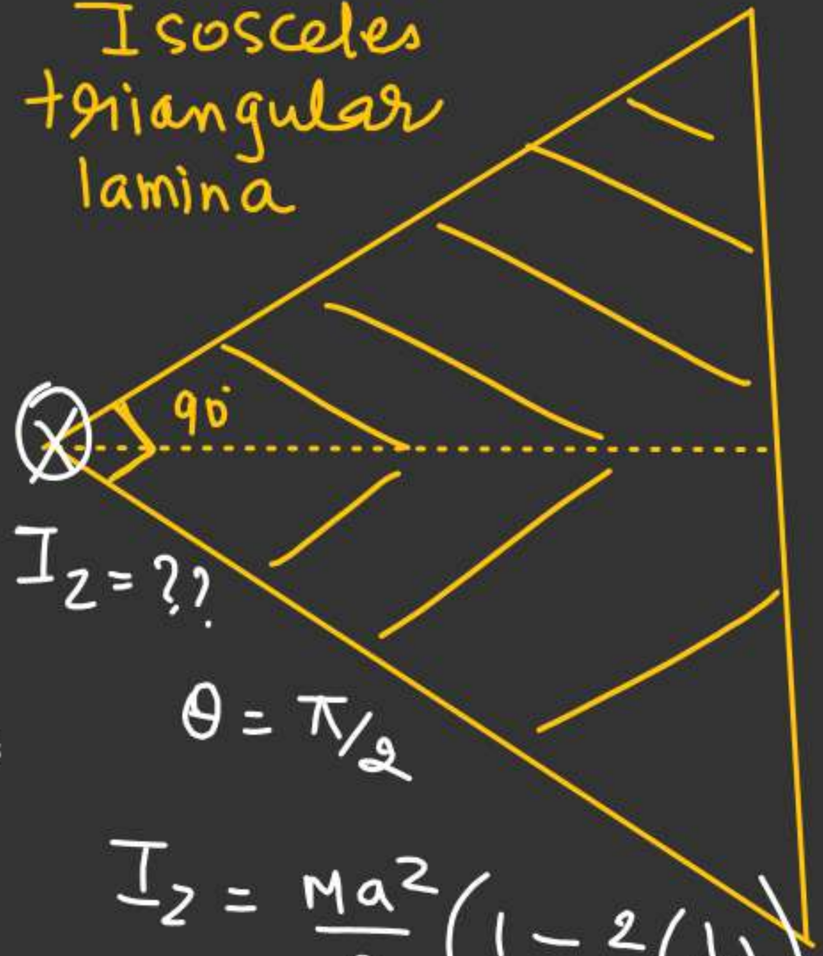
$$I_z = \frac{Ma^2}{6}\sin^2\frac{\theta}{2} + \frac{Ma^2}{2}\cos^2\frac{\theta}{2}$$

$$= \frac{Ma^2}{2}\left(\cos^2\frac{\theta}{2} + \frac{1}{3}\sin^2\frac{\theta}{2}\right)$$

$$= \frac{Ma^2}{2}\left(1 - \sin^2\frac{\theta}{2} + \frac{1}{3}\sin^2\frac{\theta}{2}\right)$$

$$I_z = \frac{Ma^2}{2}\left(1 - \frac{2}{3}\sin^2\frac{\theta}{2}\right)$$

Right angle Isosceles triangular lamina



$I_z = ??$

$\theta = \pi/2$

$$I_z = \frac{Ma^2}{2}\left(1 - \frac{2}{3}\left(\frac{1}{2}\right)\right)$$

$$\boxed{I_z = \frac{Ma^2}{3}}$$

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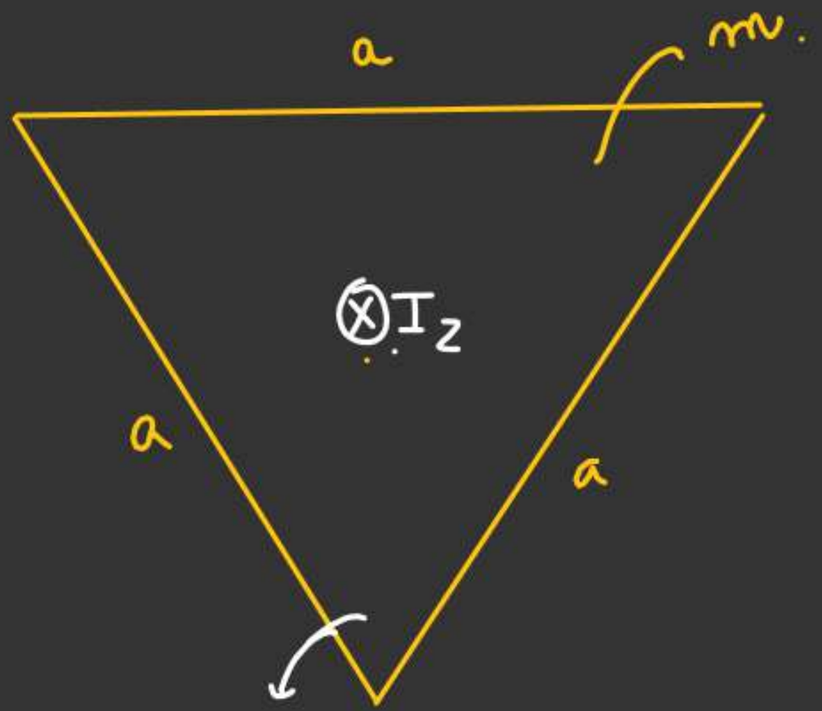
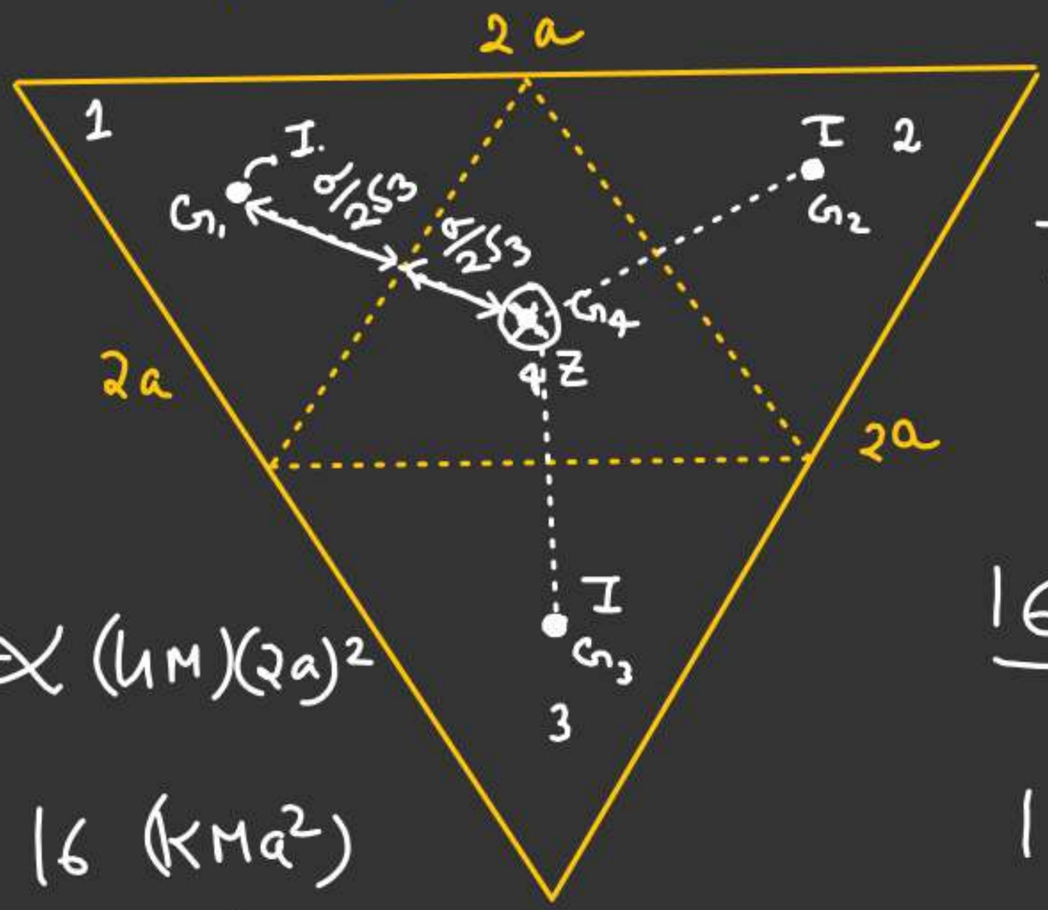
$I_z \rightarrow$ M.I about the axis passing through the Centeriod and perpendicular to the plane.



$I \propto Ma^2$

About z-axis

$I = kMa^2$



$I_1 \propto (4M)(2a)^2$

$I_1 = 16\,(kMa^2)$

$$\boxed{I_1 = 16I}$$

$$I_1 = \left[I + m\left(\frac{a}{\sqrt{3}}\right)^2\right]\times 3 + I$$

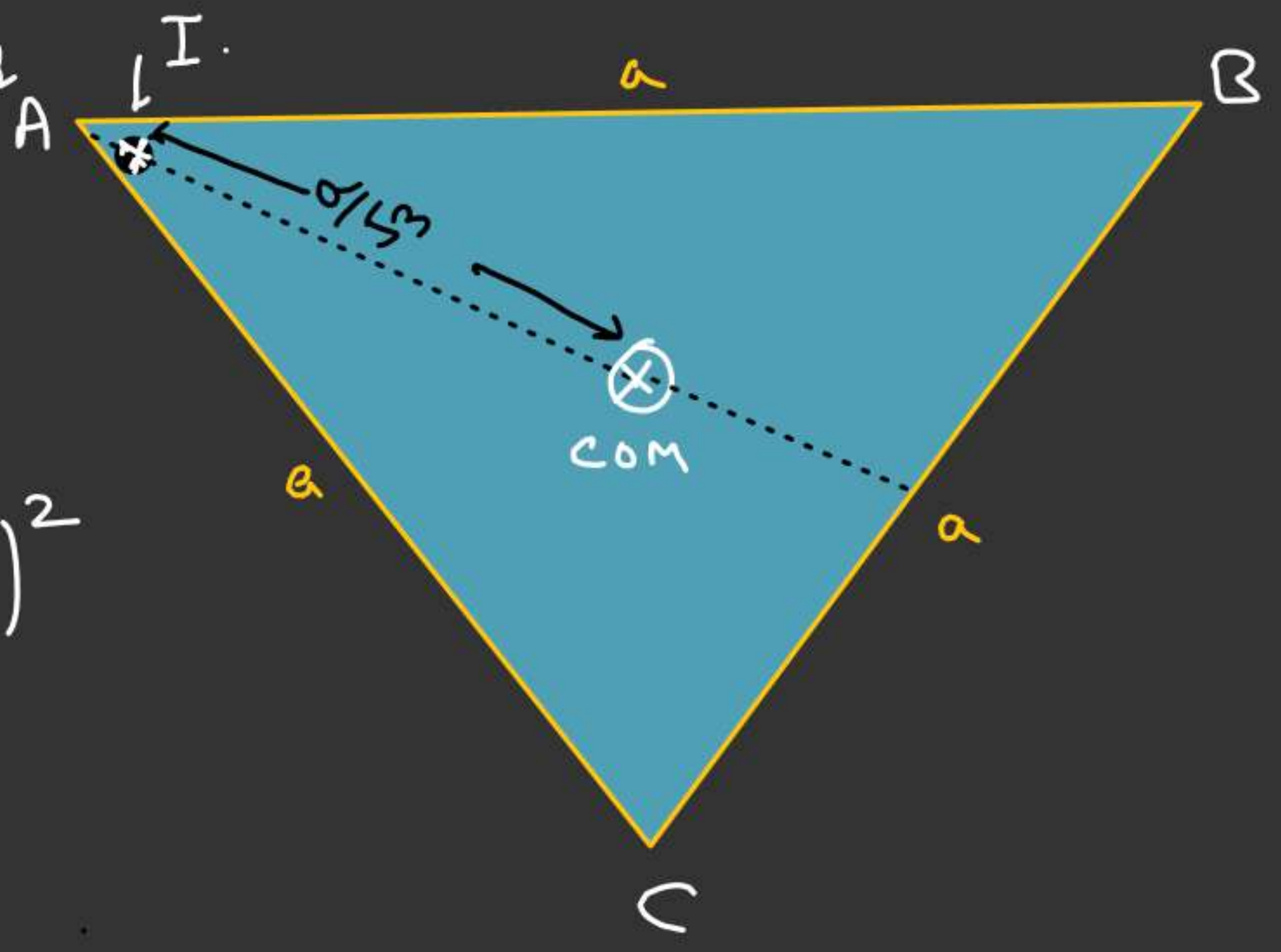
$$16I = (3I + I) + Ma^2$$

$$12I = Ma^2$$

$$\boxed{I = \frac{Ma^2}{12}}$$

Nishant Jindal

I ⇒ Axis passing through A and perpendicular to plane of ABC.



$$I_{com} = \frac{Ma^2}{12}$$

$$I = I_{com} + M\left(\frac{a}{\sqrt{3}}\right)^2$$

$$= \frac{Ma^2}{12} + \frac{Ma^2}{3}$$

$$= \frac{Ma^2 + 4Ma^2}{12}$$

$$= \left(\frac{5Ma^2}{12}\right)$$