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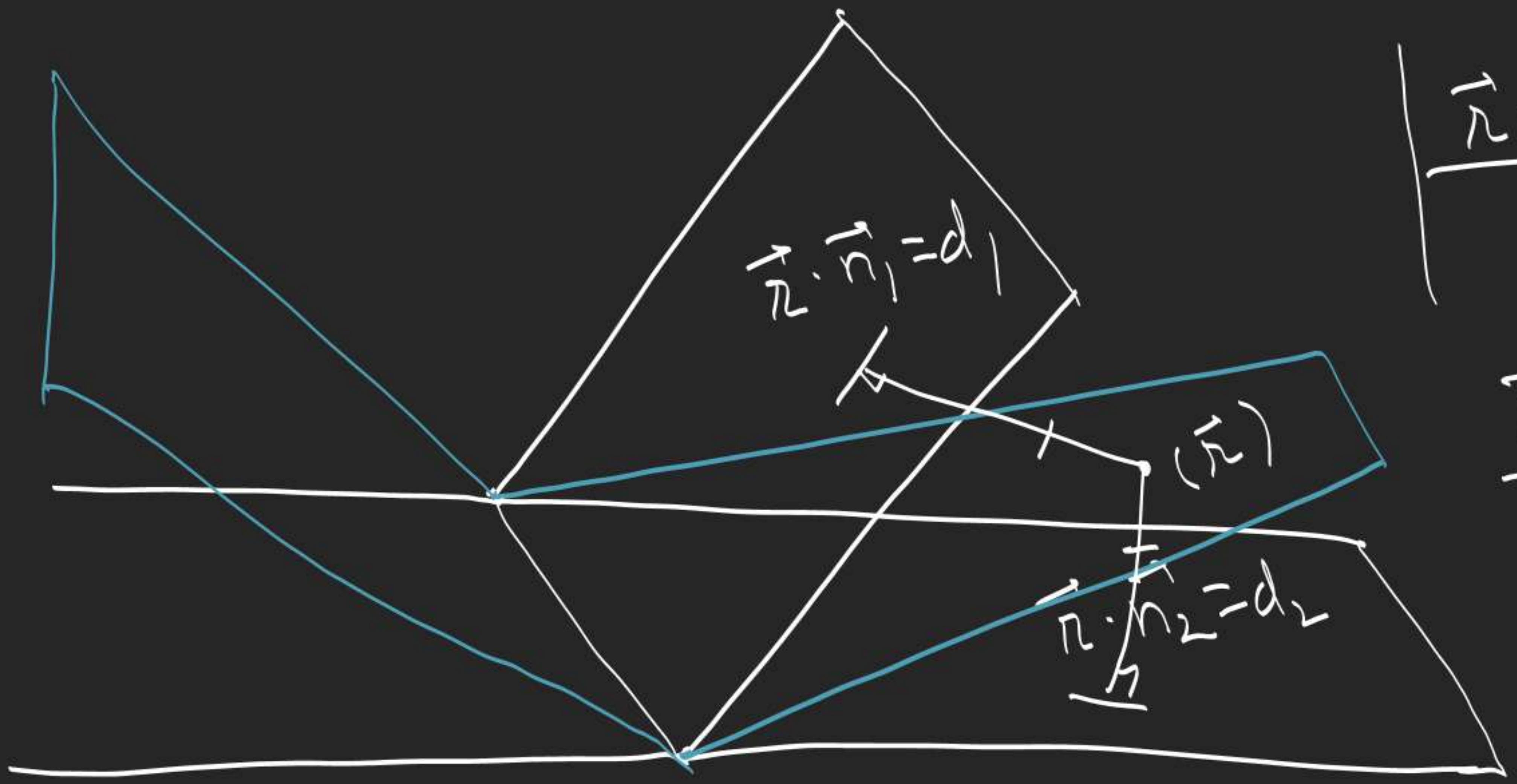
Angle Bisector of Two Planes



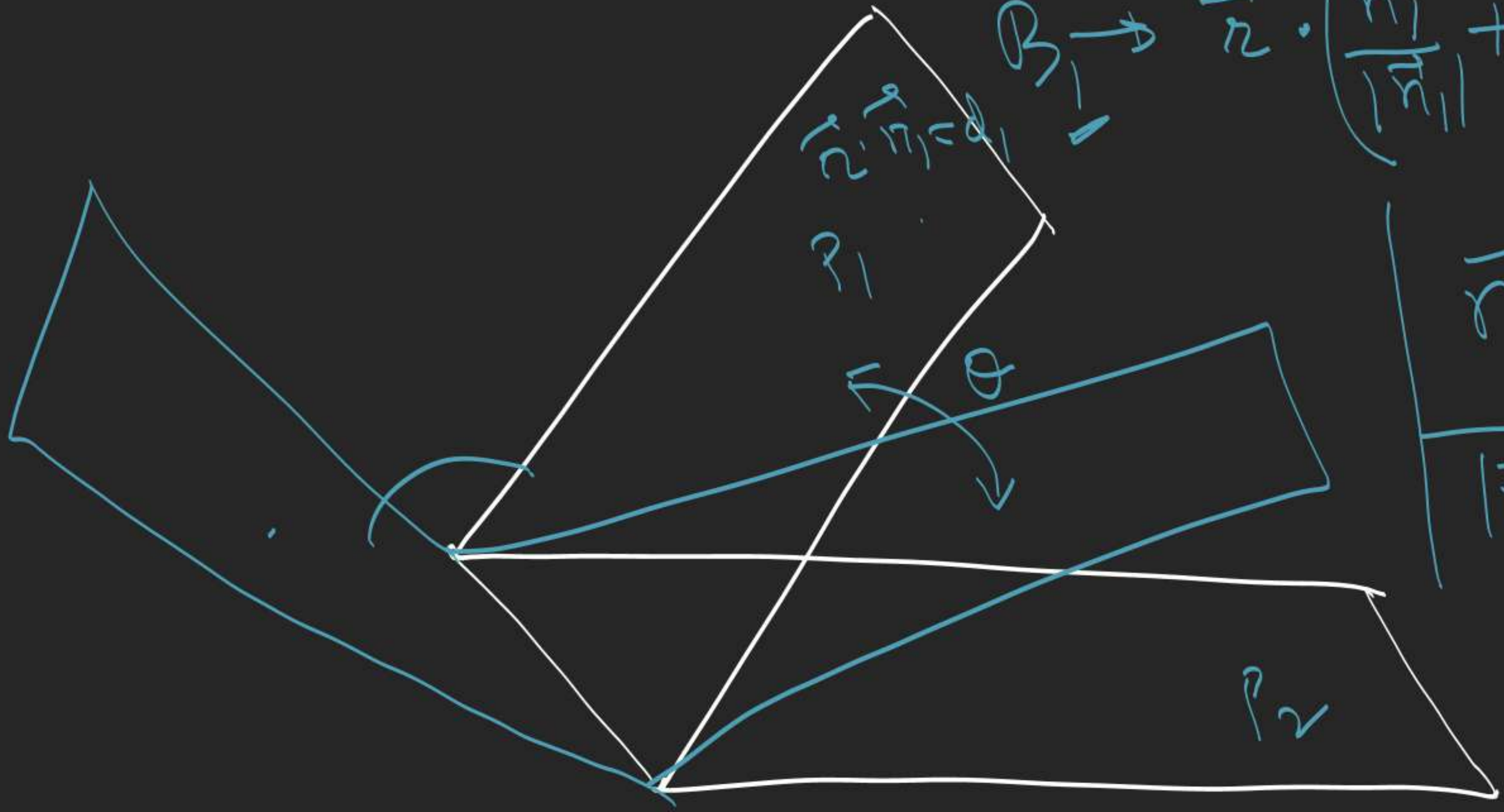
$$\left|\frac{\vec{r}\cdot\vec{n}_1-d_1}{|\vec{n}_1|}\right| = \left|\frac{\vec{r}\cdot\vec{n}_2-d_2}{|\vec{n}_2|}\right|$$

$$\frac{\vec{r}\cdot\vec{n}_1-d_1}{|\vec{n}_1|} = \pm\frac{\vec{r}\cdot\vec{n}_2-d_2}{|\vec{n}_2|}$$

$$\vec{r}\cdot\left(\frac{\vec{n}_1}{|\vec{n}_1|}-\frac{\vec{n}_2}{|\vec{n}_2|}\right) = \frac{d_1}{|\vec{n}_1|}-\frac{d_2}{|\vec{n}_2|}$$

$$\vec{r}\cdot\left(\frac{\vec{n}_1}{|\vec{n}_1|}+\frac{\vec{n}_2}{|\vec{n}_2|}\right) = \frac{d_1}{|\vec{n}_1|}+\frac{d_2}{|\vec{n}_2|}$$

Acute & Obtuse Bisector



$B_1 \rightarrow \vec r\cdot\left(\frac{\vec n_1}{|\vec n_1|}+\frac{\vec n_2}{|\vec n_2|}\right)=\frac{d_1}{|\vec n_1|}+\frac{d_2}{|\vec n_2|}$

$$\left|\frac{\vec n_{B_1}\cdot \vec n_1}{|\vec n_B|\,|\vec n_1|}\right| = \cos\theta$$

$\cos\theta > \frac{1}{\sqrt2} \Rightarrow B_1$ is acute $\angle$ bisector

$\cos\theta < \frac{1}{\sqrt2} \Rightarrow B_1$ is obtuse $\angle$ bisector.

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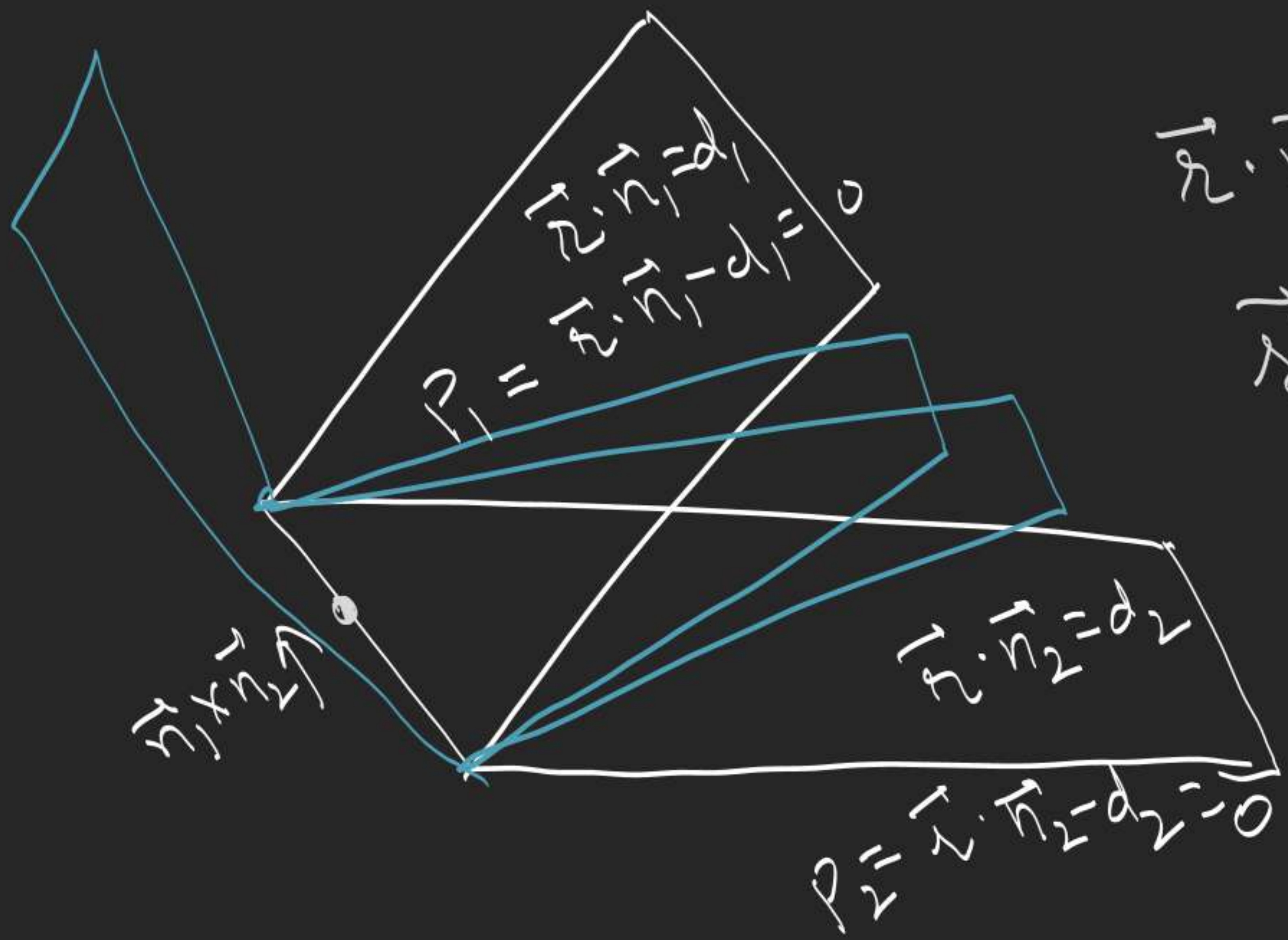
Family of Planes thru intersection of two given planes



$$\vec{r}\cdot\vec{n}_1-d_1+\lambda\left(\vec{r}\cdot\vec{n}_2-d_2\right)=0,\ \lambda\in R$$

$$\vec{r}\cdot\left(\vec{n}_1+\lambda\vec{n}_2\right)=d_1+\lambda d_2$$

$$\boxed{P_1+\lambda P_2=0\ ,\ \lambda\in R}$$

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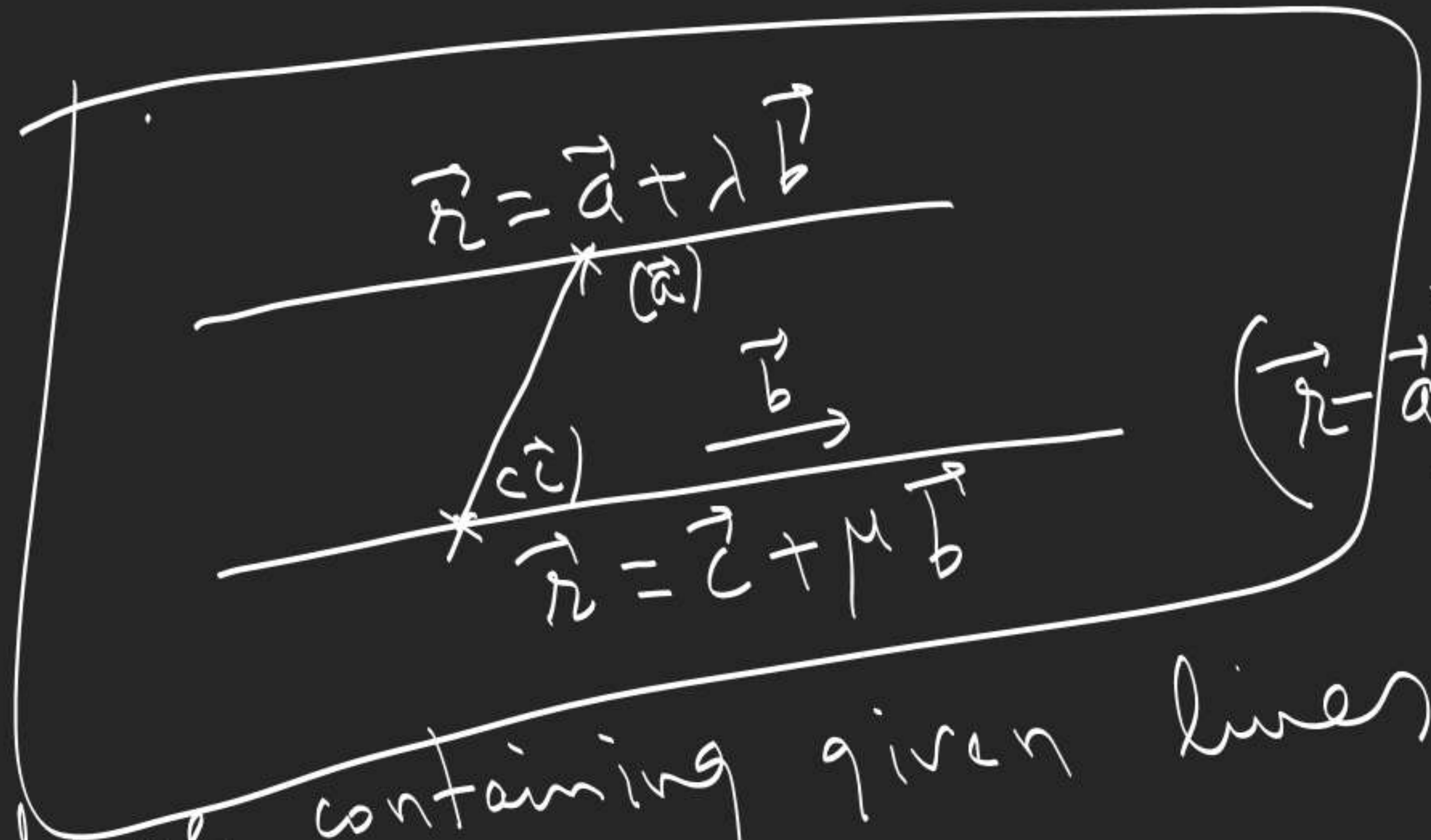
1.



$$(\vec{r} - \vec{a}) \cdot ((\vec{a} - \vec{c}) \times \vec{b}) = 0$$

Plane containing given lines

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Q. Find the eqn. of plane passing thru points A (2, 2, 1) and B (1, −2, 3) and ⊥ar to plane

$x - 2y + 3z + 4 = 0$

$\vec{n}_1$

$\vec{n} = \vec{AB} \times \vec{n}_1 = (-\hat{i} - 4\hat{j} + 2\hat{k}) \times (\hat{i} - 2\hat{j} + 3\hat{k})$

$$\left(\vec{r} - (2\hat{i} + 2\hat{j} + \hat{k})\right) \cdot (8\hat{i} - 5\hat{j} - 6\hat{k}) = 0$$

$$\vec{r} \cdot (8\hat{i} - 5\hat{j} - 6\hat{k}) = 0$$

$$\boxed{8x - 5y - 6z = 0}$$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & -4 & 2 \\ 1 & -2 & 3 \end{vmatrix} = -8\hat{i} + 5\hat{j} + 6\hat{k}$$

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3. Find the eqn. of plane which is parallel to plane $x+5y-4z+5=0$ and the sum of whose intercepts on the coordinate axes is 19. Also find the ⊥ar distance b/n these planes.

$$x+5y-4z-20=0$$

$$d=\frac{25}{\sqrt{42}}$$

$$x+5y-4z=\lambda$$

$$\lambda+\frac{\lambda}{5}+\left(-\frac{\lambda}{4}\right)=19 \Rightarrow \boxed{\lambda=20}$$

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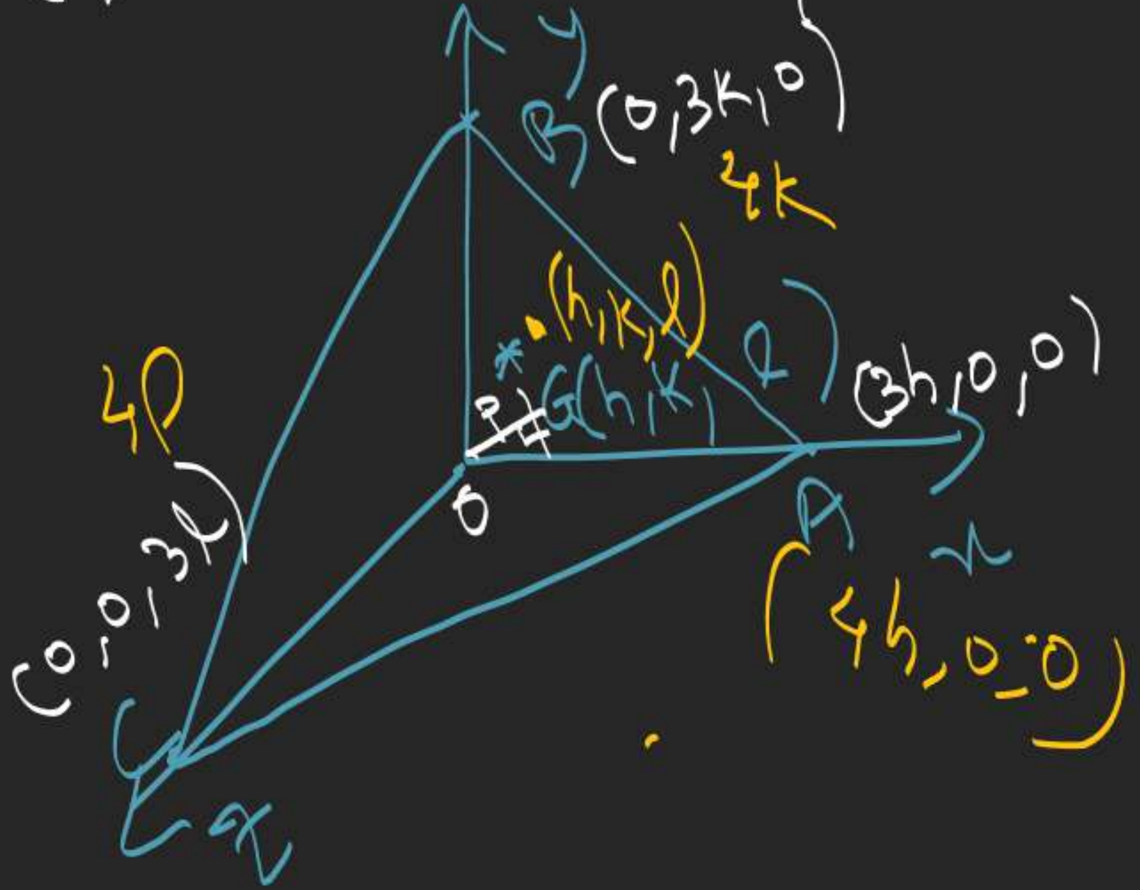
4. A plane which always remain at a constant distance 'p' from origin cuts coordinate axes at A, B, C. Find the locus of

(i) Centroid of triangle ABC

(ii) Centroid of tetrahedron OABC, 'O' is origin.



$$\frac{x}{3h}+\frac{y}{3k}+\frac{z}{3l}=1$$

$4h \quad 4k \quad 4l$

$$\frac{|-1|}{\sqrt{\frac{1}{9h^2}+\frac{1}{9k^2}+\frac{1}{9l^2}}}=p \Rightarrow \boxed{\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=\frac{9}{p^2}}$$

$16 \quad 16 \quad 16$

$$\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=\frac{16}{p^2}$$

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5. Find the eqn. of plane containing the line of intersection of the planes $\vec{r}\cdot\vec{n}_1 = q_1$ & $\vec{r}\cdot\vec{n}_2 = q_2$ and is parallel to line of intersection of planes $\vec{r}\cdot\vec{n}_3 = q_3$ & $\vec{r}\cdot\vec{n}_4 = q_4$.

$\vec{n}_3 \times \vec{n}_4$

$$\vec{r}\cdot\vec{n}_1 - q_1 + \lambda(\vec{r}\cdot\vec{n}_2 - q_2) = 0$$

$$\vec{r}\cdot(\vec{n}_1 + \lambda\vec{n}_2) = q_1 + \lambda q_2$$

$$(\vec{n}_1 + \lambda\vec{n}_2)\cdot(\vec{n}_3 \times \vec{n}_4) = 0 \Rightarrow \lambda = -\frac{[\vec{n}_1\ \vec{n}_3\ \vec{n}_4]}{[\vec{n}_2\ \vec{n}_3\ \vec{n}_4]}$$

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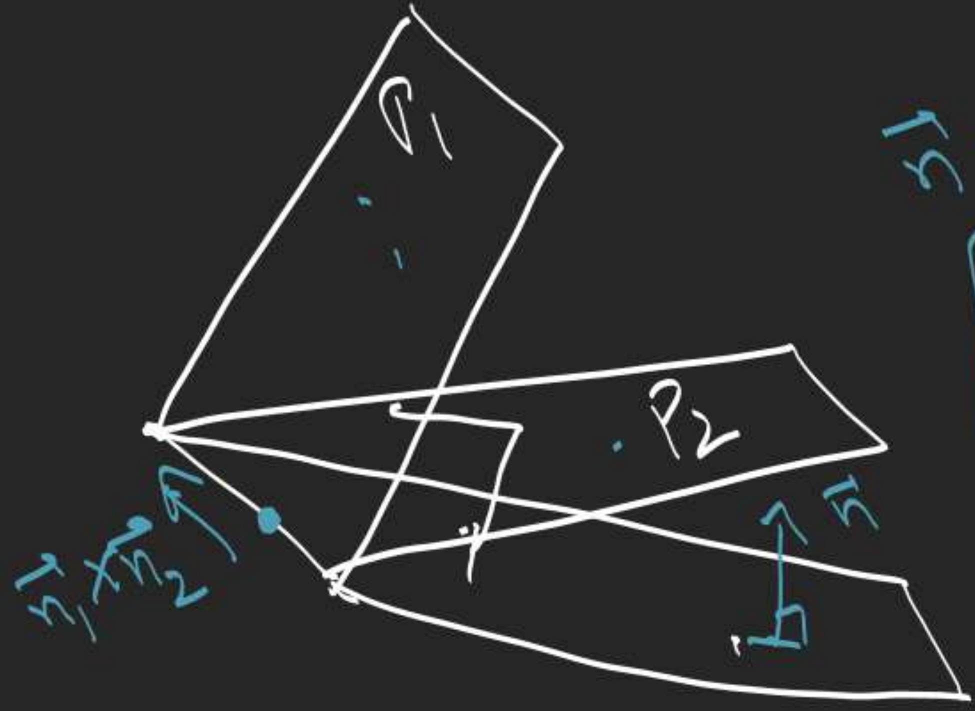
6. The plane $\underset{P_1}{x-y-z=4}$ is rotated thru 90° about its line of intersection with the plane $\underset{P_2}{x+y+2z=4}$. Find its eqn. in new position.

$z=0$

$x-y=4$

$x+y=4$

$\boxed{(4,0,0)}$



$$\vec{n} = \vec{n}_1\times(\vec{n}_1\times\vec{n}_2) = (\vec{n}_1\cdot\vec{n}_2)\vec{n}_1 - |\vec{n}_1|^2\vec{n}_2$$

$$= -2(\hat{i}-\hat{j}-\hat{k}) - 3(\hat{i}+\hat{j}+2\hat{k})$$

$$-5\hat{i}-\hat{j}-4\hat{k}$$

$$\boxed{5x+y+4z=20}$$

$$P_1' \equiv x-y-z-4+\lambda(x+y+2z-4)=0$$

$$(\vec{r}-4\hat{i})\cdot(5\hat{i}+\hat{j}+4\hat{k})=0.$$

$$1(\lambda+1)-1(\lambda-1)-1(2\lambda-1)=0$$

$$\lambda=\frac{3}{2}$$

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7. Find the reflection of plane $P_1: 2x-3y+6z+1=0$ in the plane $P_2: 14x-2y-5z+3=0$

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Straight Line

$\vec{m} = (\alpha\hat{i} + \beta\hat{j} + \gamma\hat{k})$

$(\vec{r}) = (x\hat{i} + y\hat{j} + z\hat{k})$

$(\vec{a}) = (x_1\hat{i} + y_1\hat{j} + z_1\hat{k})$

$$\boxed{\vec{r} = \vec{a} + \lambda\vec{m}}$$

$\boxed{x\text{ axis}}$

$$\frac{x}{1} = \frac{y}{0} = \frac{z}{0}.$$

vector ∥ to line $= \alpha\hat{i} + \beta\hat{j} + \gamma\hat{k}$

Symmetric Form

$$\frac{x - x_1}{\alpha} = \frac{y - y_1}{\beta} = \frac{z - z_1}{\gamma}$$

$$(x, y, z) = \left(\frac{1}{2}\lambda + \frac{1}{2}, \frac{3}{4} - \frac{5}{4}\lambda, -7\right)$$

$$2x - 1 = \frac{3 - 4y}{5} \quad ; \quad z = -7$$

$$\boxed{\frac{x - \frac{1}{2}}{\frac{1}{2}} = \frac{y - \frac{3}{4}}{-\frac{5}{4}} = \frac{z + 7}{0} = \lambda}$$

$\vec{m} = \frac{1}{2}\hat{i} - \frac{5}{4}\hat{j}$

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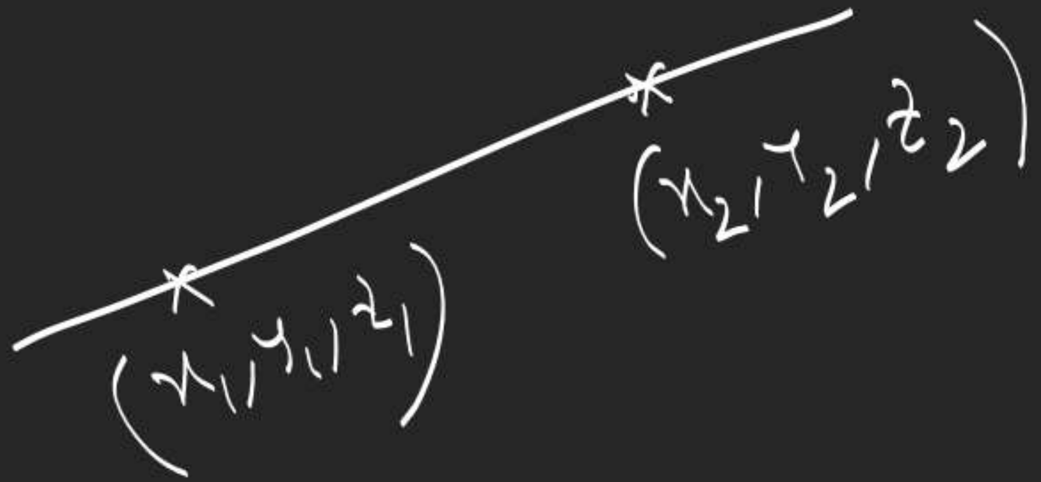

$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1}$$

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Unsymmetric Form

2 points (common) $z = z_1$

$$a_1x + b_1y + c_1z + d_1 = 0$$

&

$$a_2x + b_2y + c_2z + d_2 = 0$$

represent line.

intersection point

(x_1, y_1, z_1)

$$a_1x + b_1y + c_1z + d_1 = 0 = a_2x + b_2y + c_2z + d_2$$ is a line

$$\vec{m} = \vec{n}_1 \times \vec{n}_2$$

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1. Find the eqn of line which passes thru point $(2,-1,-1)$, is parallel to plane $4x+y+z+2=0$ and is $\perp$ar to line of intersection of the planes $2x+y=0=x-y+z$.

$\vec{n}_1$ $\quad$ $\vec{n}_2$ $\quad$ $\vec{n}_3$

$\vec{r} = 2\hat{i}-\hat{j}-\hat{k}+\lambda(\hat{i}-13\hat{j}+9\hat{k})$

$$\boxed{\frac{x-2}{1} = \frac{y+1}{-13} = \frac{z+1}{9}}$$

$\vec{n}_2 \times \vec{n}_3$

$\vec{m} = \vec{n}_1 \times (\vec{n}_2 \times \vec{n}_3)$

$= (\vec{n}_1 \cdot \vec{n}_3)\vec{n}_2 - (\vec{n}_1 \cdot \vec{n}_2)\vec{n}_3$

$= 4(2\hat{i}+\hat{j}) - 9(\hat{i}-\hat{j}+\hat{k}) = -\hat{i}+13\hat{j}-9\hat{k}$

$(1-30)$

$\{x-4$

$\{x-3 \rightarrow (1-5)$