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$$\vec{x} \times \vec{a} + (\vec{x} \cdot \vec{b})\vec{a} = \vec{c}$$

$$(\cancel{\vec{x} \cdot \vec{b}})(\vec{a} + \vec{a} \times \vec{b}) - (\vec{a} \cdot \vec{b})\vec{x} = \vec{c} \times \vec{b}$$

$$\frac{\vec{c} \cdot \vec{a}}{|\vec{a}|^2}$$

$$\vec{v} = \frac{1}{|\vec{b}|^2}\vec{b} + \frac{1}{|\vec{a}|^2 |\vec{b}|^2}(\vec{a} \times \vec{b})$$

$$\vec{v} = x\vec{a} + y\vec{b} + z(\vec{a} \times \vec{b})$$

$$0 = \vec{v} \cdot \vec{a} = x|\vec{a}|^2 + 0 + 0 \Rightarrow \boxed{x = 0}$$

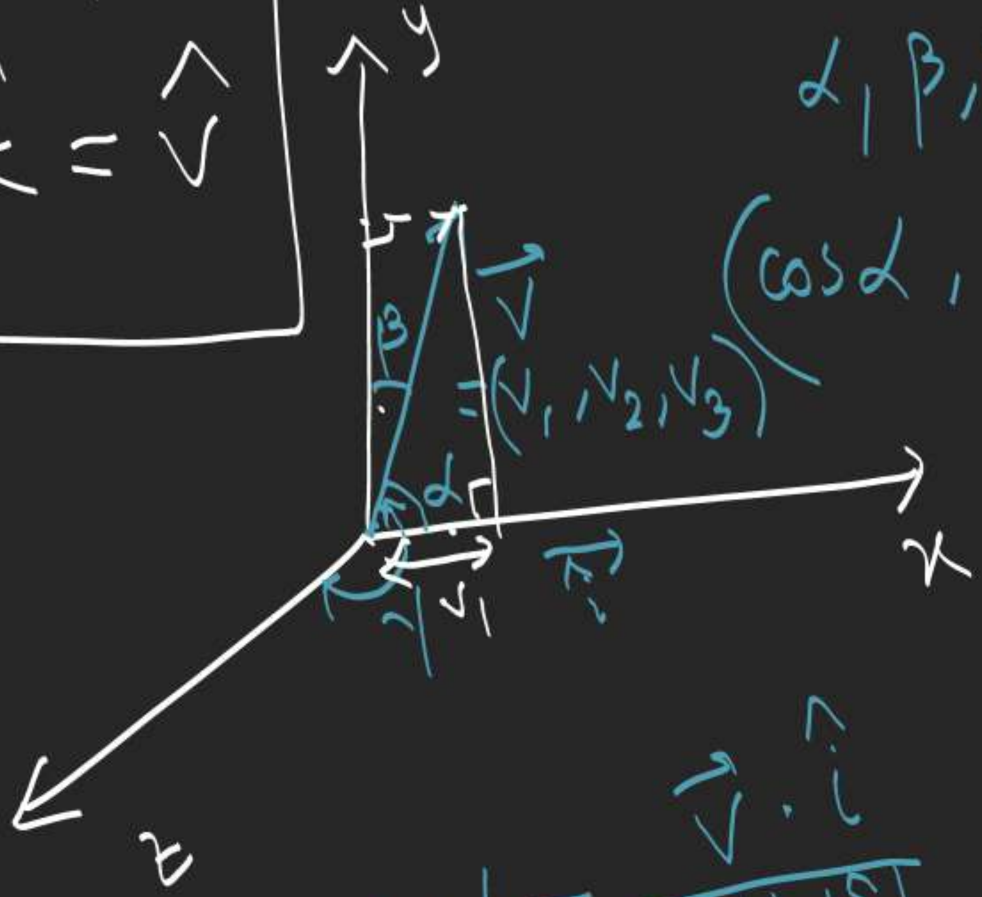
$$1 = \vec{v} \cdot \vec{b} = 0 + y|\vec{b}|^2 + 0 \Rightarrow y = \frac{1}{|\vec{b}|^2}$$

$$1 = \vec{v} \cdot (\vec{a} \times \vec{b}) = 0 + 0 + z|\vec{a} \times \vec{b}|^2 = z|\vec{a}|^2 |\vec{b}|^2$$

Direction Angle / Direction Cosine of a vector

$$\boxed{l\hat{i}+m\hat{j}+n\hat{k}=\hat{v}}$$

$$l^2+m^2+n^2=1$$



$\alpha, \beta, \gamma \rightarrow$ direction angles

$(\cos\alpha, \cos\beta, \cos\gamma) \rightarrow$ direction cosines of vector.

$\quad\ \ \|$

(l, m, n)

$$l = \cos\alpha = \frac{\vec{v}\cdot\hat{i}}{|\vec{v}||\hat{i}|} = \frac{v_1}{\sqrt{v_1^2+v_2^2+v_3^2}}$$

$$m = \cos\beta = \frac{\vec{v}\cdot\hat{j}}{|\vec{v}||\hat{j}|} = \frac{v_2}{\sqrt{v_1^2+v_2^2+v_3^2}}$$

$$n = \frac{\vec{v}\cdot\hat{k}}{|\vec{v}||\hat{k}|} = \frac{v_3}{\sqrt{v_1^2+v_2^2+v_3^2}}$$

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Direction Cosine & Direction Ratios for a line

$\frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{2}{\sqrt{6}} \rightarrow$ d.r.

$\vec{r} = \vec{a} + \lambda \vec{b}$

$l\hat{i} + m\hat{j} + n\hat{k} = \hat{b}$

1, 1, 2

−2, −2, −4

Direction Ratios

$\alpha\hat{i} + \beta\hat{j} + \gamma\hat{k} = \lambda(l\hat{i} + m\hat{j} + n\hat{k})$ d.r. $= (\alpha, \beta, \gamma)$

$= \lambda \hat{b}$

$\frac{\alpha}{l} = \frac{\beta}{m} = \frac{\gamma}{n}$

$\alpha\hat{i} + \beta\hat{j} + \gamma\hat{k}$ = vector ∥ to line

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Angle b/n two Lines L_1 & L_2 whose d.c.s are (l_i, m_i, n_i) $i=1,2$

$$\cos\theta = \left|\frac{\vec{m}_1 \cdot \vec{m}_2}{|\vec{m}_1||\vec{m}_2|}\right| = \left|l_1 l_2 + m_1 m_2 + n_1 n_2\right|$$

$L_1 \perp L_2$ $\quad l_1 l_2 + m_1 m_2 + n_1 n_2 = 0$

$L_1 \parallel L_2$ $\quad \frac{l_1}{l_2} = \frac{m_1}{m_2} = \frac{n_1}{n_2}$

D.r.s are (a_1, b_1, c_1) & (a_2, b_2, c_2)

$$\cos\theta = \frac{|a_1 a_2 + b_1 b_2 + c_1 c_2|}{\sqrt{a_1^2+b_1^2+c_1^2}\,\sqrt{a_2^2+b_2^2+c_2^2}}$$

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1. Find the direction cosines of a line perpendicular to two lines whose direction ratios are 1, 2, 3 and −2, 1, 4.

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ -2 & 1 & 4 \end{vmatrix} = 5\hat{i} - 10\hat{j} + 5\hat{k}$$

$1, -2, 1$

$$(l, m, n) = \left(\frac{1}{\sqrt{6}}, \frac{-2}{\sqrt{6}}, \frac{1}{\sqrt{6}}\right)$$

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2. The direction cosines l, m, n of two lines are connected by relations $l+m+n=0$ & $2lm+2ln-mn=0$

Find them and the angle between the lines.

$$\cos\theta = \left| \frac{1}{\sqrt6}\frac{1}{\sqrt6} + \left(-\frac{2}{\sqrt6}\right)\frac{1}{\sqrt6} + \left(\frac{1}{\sqrt6}\right)\left(\frac{-2}{\sqrt6}\right) \right| = \frac{1}{2}$$

$$2lm - (2l-m)(l+m) = 0 \Rightarrow m^2 + lm - 2l^2 = 0$$

$$(m+2l)(m-l) = 0$$

$$\boxed{\theta = \frac{\pi}{3}}$$

Case II $l=m$, $n=-2l$

$l:m:n = 1:1:-2$

$$\boxed{(l,m,n) = \left(\frac{1}{\sqrt6}, \frac{1}{\sqrt6}, \frac{-2}{\sqrt6}\right)}$$

Case I $m=-2l$, $n=l$

$l:m:n = 1:-2:1$

$l^2+m^2+n^2=1$

$$\boxed{(l,m,n) = \left(\frac{1}{\sqrt6}, \frac{-2}{\sqrt6}, \frac{1}{\sqrt6}\right)}$$

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3. A line makes angles $\alpha, \beta, \gamma, \delta$ with four diagonals of a cube. Find the value of

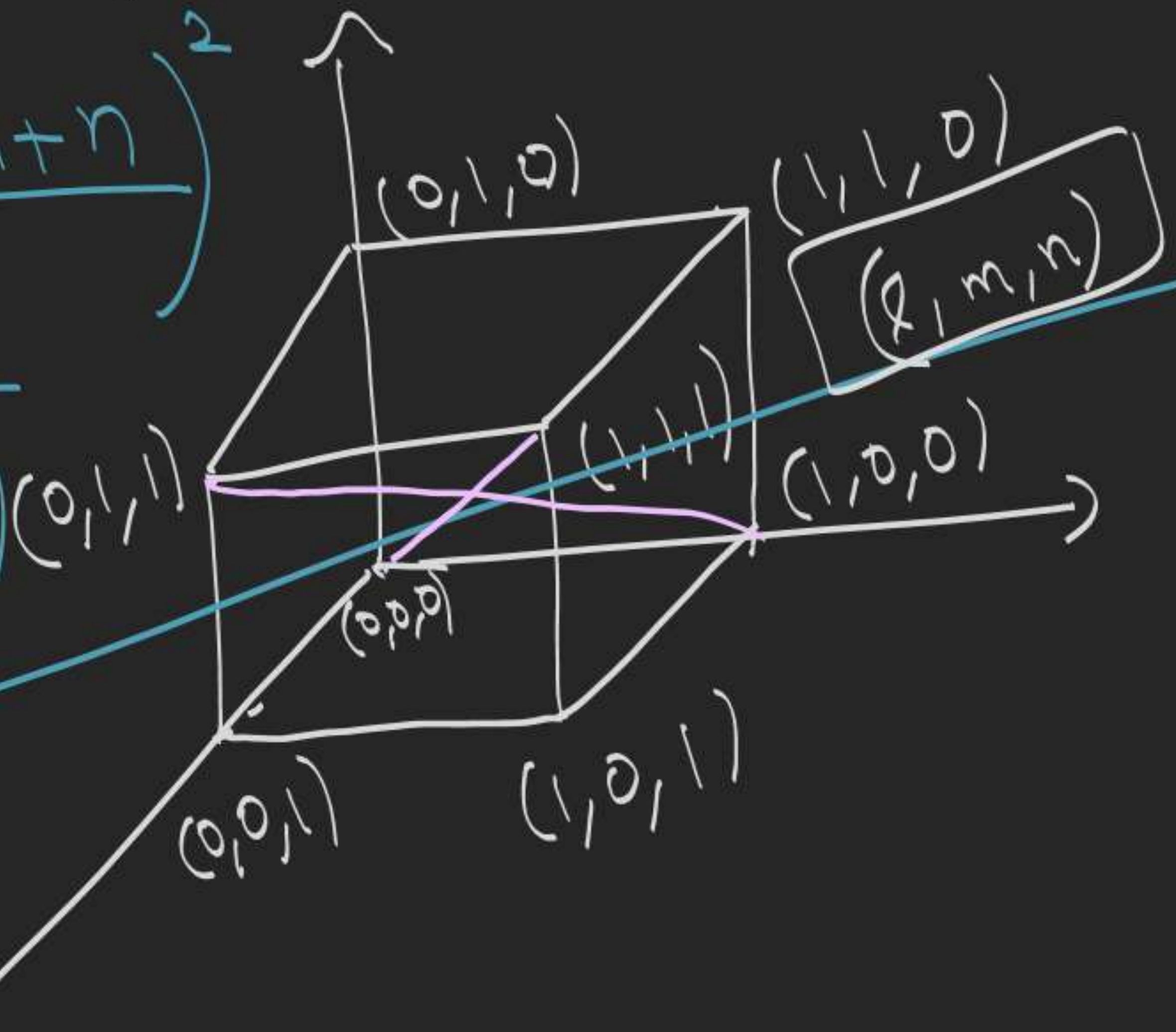
$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma + \cos^2\delta.$$

$$\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$$

$$\left(-\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$$

$$\left(\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$$

$$\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}\right)$$



$$= \left(\frac{l+m+n}{\sqrt{3}}\right)^2 \left(\frac{-l+m+n}{\sqrt{3}}\right)^2 + \left(\frac{l-m+n}{\sqrt{3}}\right)^2 + \left(\frac{l+m-n}{\sqrt{3}}\right)^2$$

$$= \frac{4(l^2+m^2+n^2)}{3}$$

$$= \boxed{\frac{4}{3}}$$

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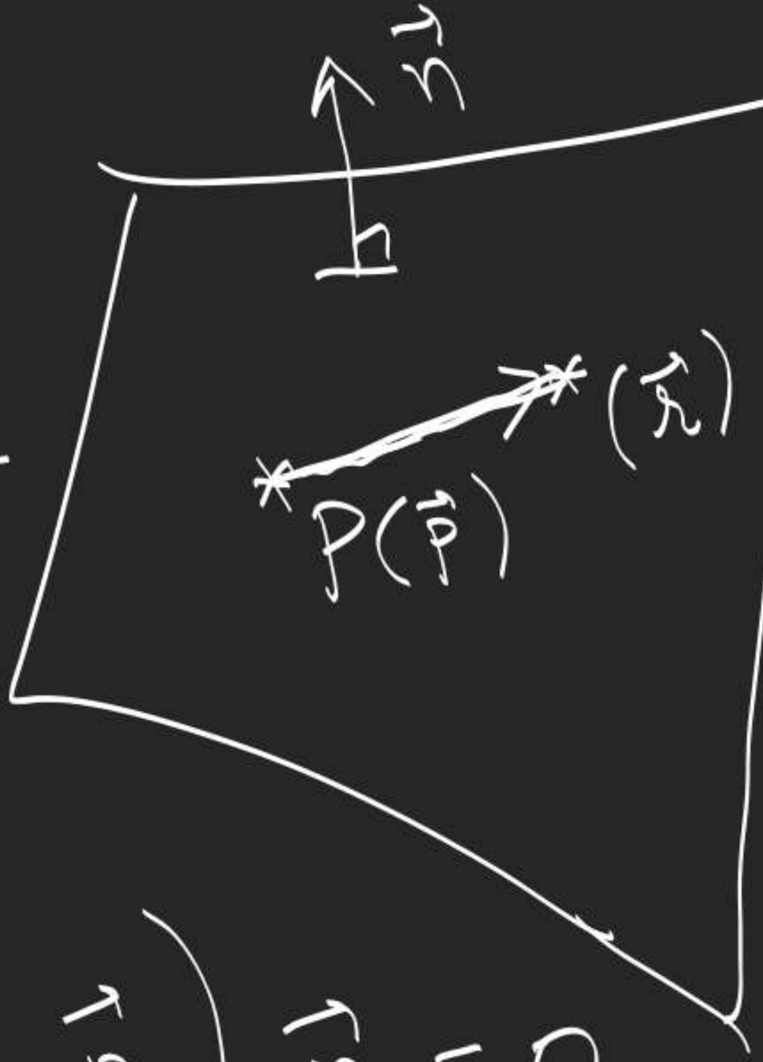
Plane

$\vec{n}$ $\perp$ar to plane.

&

$\vec{p}$ = p.v of point lying on plane



$$\boxed{\vec{r}\cdot(\hat{i}+\hat{j}-3\hat{k}) = 8}$$

$\hat{i}+\hat{j}-3\hat{k}$ = vector $\perp$ to plane.

$$(\vec{r}-\vec{p})\cdot\vec{n} = 0$$

$$\vec{r}\cdot\vec{n} = \vec{p}\cdot\vec{n}$$

$$\boxed{\vec{r}\cdot\vec{n} = d}$$

Vector form.

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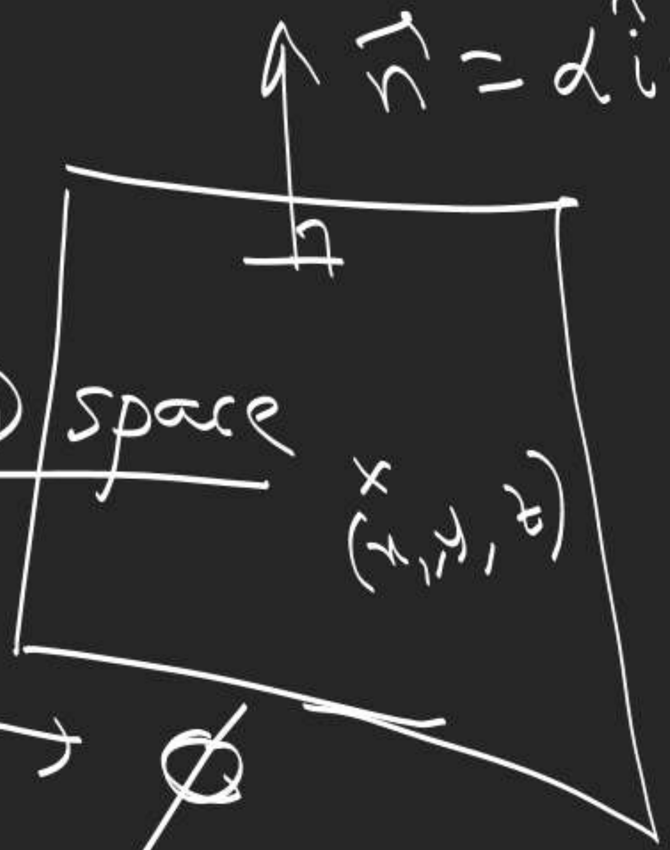
$\vec{n} = \alpha\hat{i} + \beta\hat{j} + \gamma\hat{k}$.



$\alpha = \beta = \gamma = 0 = d \rightarrow$ Complete 3D space

$\vec{r}\cdot\vec{n} = d$

$\alpha = \beta = \gamma = 0,\ d \neq 0 \rightarrow \phi$

$$\boxed{\alpha x + \beta y + \gamma z = d}$$

atleast one of α, β, γ is nonzero.

Plane in General form

$$\boxed{\vec{n} = 2\hat{i} - \hat{j} + 3\hat{k}}$$

$\perp$ar to plane.

$2x - y + 3z = 13$

$\vec{r}\cdot(2\hat{i} - \hat{j} + 3\hat{k}) = 13$

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$x + 2y = 3 \longrightarrow$ plane.

xy plane $\longrightarrow z = 0$

$x +$

$\{x - 4$

$1 - 13$

$- \{12\}$.