Nishant Jindal

$z = x^3, \; y = 6x^2 + 15x + 15$

$\frac{dz}{dx} < \frac{dy}{dx}$

$\frac{dy}{dz} > 1$

(2) $f(x) = \frac{|x-1|}{x^2}$

$f(x) = \begin{cases} \frac{x-1}{x^2} = \frac{1}{x} - \frac{1}{x^2} & x \geq 1 \\ -\frac{(x-1)}{x^2} = -\frac{1}{x} + \frac{1}{x^2} & x < 1 \end{cases}$

$f'(x) = \begin{cases} -\frac{1}{x^2} + \frac{2}{x^3} < 0 \\ \frac{1}{x^2} - \frac{2}{x^3} < 0 \end{cases}$

$\frac{-x+2}{x^3} < 0 \Rightarrow \frac{(x-2)}{(x)^3} > 0$

$\frac{x-2}{x^3} < 0$

$x \in [0,1) \cup (2, \infty)$

(3) hold.

(4) "

(5) "

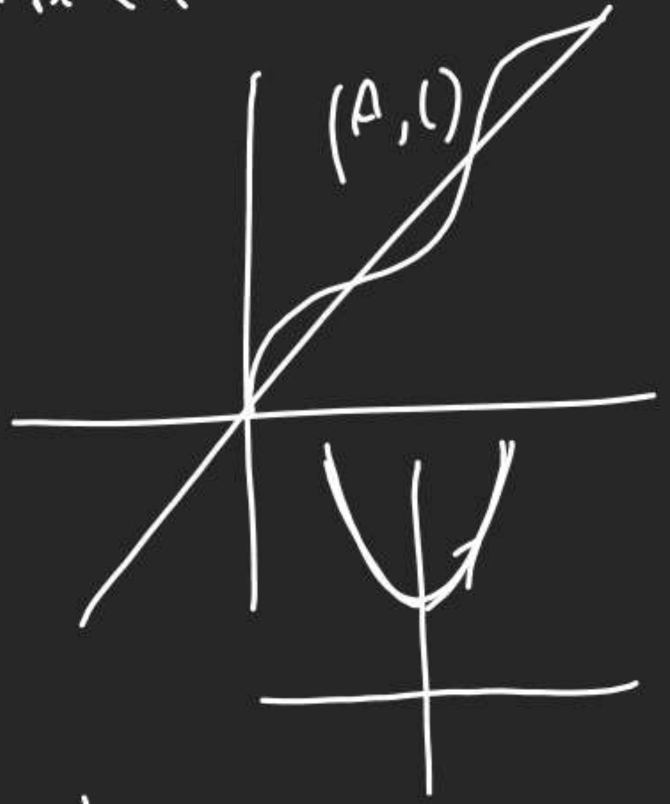
Q 6 $f(x) = x^2 - x \cdot \sin x$

$f'(x) = 2x - x \cdot \cos x - \sin x$ (1) $x \in [0, \frac{\pi}{2}]$

$= x(2 - \cos x) - \sin x > 0$ $\sin x < x$

(7) (8) main

(10)



Nishant Jindal

46) ✓

59) $y = -\sqrt{x} + 2$

$y = x$

$x = -\sqrt{x} + 2$

$x - 2 = -\sqrt{x}$

$x^2 - 4x + 4 = x$

55) Copy

57) $r = 9m,\ \Delta r = .03$

$S = 4\pi r^2$

$\frac{dS}{dr} = 8\pi r$

$dS = 8\pi r \cdot \Delta r = 8 \times \pi \times 9(.03) = 2.16\pi m^2$

(60) $y = x - x^2 \Rightarrow y^2 = (x - x^2)x$

$$\frac{dy^2}{dx^2} = \frac{\cancel{2}(x-x^2)x(1-2x)}{\cancel{2}x}$$

$= (1-x)x(1-2x)$

$= 2x^2 - 3x + 1$

(62) $y^2 = x^3 + x^2$ at $(0,0)$

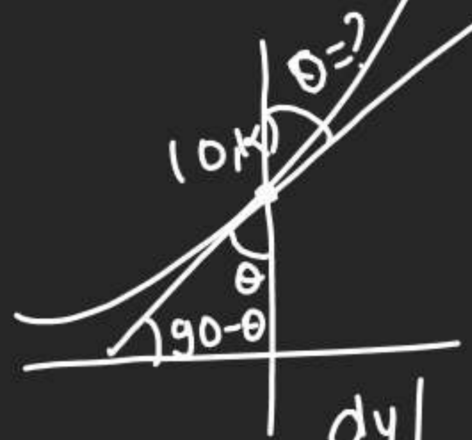
origin से Pass होने वाली cur.

EOT → lowest deg term $= 0$

$y^2 - x^2 = 0$

$y = x,\ y = -x$

63 $y = K \cdot e^{Kx}$ at y Axis Ang.



$\left.\frac{dy}{dx}\right|_{(0,K)} = \tan\theta = K^2 e^{Kx}$

$= K^2 e^0 = K^2$

$\theta = \tan^{-1} K^2$

71

θ is same as diagram में

$90 - \theta = \tan^{-1} K^2$

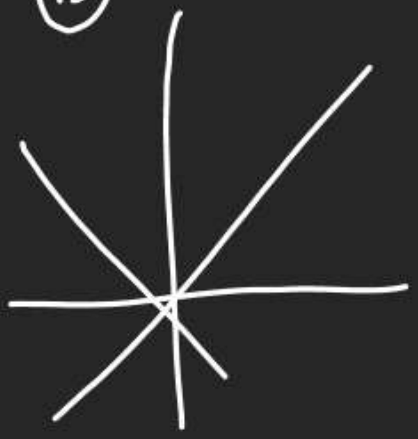
$90 - \tan^{-1} K^2 = \theta$

$\theta = \cot^{-1} \frac{K^2}{1}$

$\theta = \sec^{-1}\left(\frac{\sqrt{1+K^4}}{K^2}\right)$

$\theta = \tan^{-1} \frac{1}{\sqrt{1+K^4}}$

73



Nishant Jindal

Q Check fxn $f(x)=\int_{x^2}^{x^3}\frac{dt}{\ln t}$ in

↑ or ↓. in which Interval?

$$f'(x)=\frac{1}{\ln(x^3)}\times 3x^2-\frac{1}{\ln(x^2)}\cdot 2x$$

$$f'(x)=\frac{\cancel{3}x^2}{\cancel{3}\ln x}-\frac{\cancel{2}x}{\cancel{2}\ln x}$$

① ↑ in? $f'(x)=\frac{x^2}{\ln x}-\frac{x}{\ln x}>0$

$$=\frac{x(x-1)}{\ln x}>0$$

$-$ $+$ $+$

0 1

↑ in $x\in(0,1)\cup(1,\infty)$

fxn is always ↑.

Q $f(x)=\int_{1}^{e^x}\left(\sin^2(2\ln t)-3\sin(2\ln t)+2\right)dt$

in ↓ in?

$$f'(x)=\left(\sin^2(2\ln e^x)-3\sin(2\ln e^x)+2\right)e^x-0$$

$$=e^x\left(\sin^2 2x-3\sin 2x+2\right)$$

$$=\underset{\oplus}{e^x}\left((\sin 2x-1)(\sin 2x-2)\right)\le 0 \ \downarrow$$

$$(\sin 2x-1)(\sin 2x-2)\le 0$$

$$1\le \sin 2x\le 2$$

only $\sin 2x=1$ there $f'(x)=0$

No where ↓ ⇐ $f'(x)<0$ Nhi

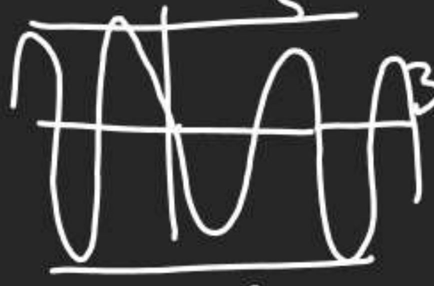
Nishant Jindal

Q If $f(x)$ $f: R \to R$.

$$f(x) = x^3 + ax^2 + bx + 5\sin^2 x$$

is an ↑ing fxn find Relation betn a & b?

$$f'(x) = 3x^2 + 2ax + b + 5\sin 2x \geq 0 \quad \uparrow$$



$$3x^2 + 2ax + b \geq (-5\sin 2x)_{Max}$$

$$3x^2 + 2ax + b \geq 5$$

$$3x^2 + 2ax + b - 5 \geq 0 \longrightarrow QE \geq 0$$

$$D \leq 0$$

$$4a^2 - 4 \times 3 \times (b-5) \leq 0$$

$$a^2 - 3b + 15 \leq 0$$

↳ Req Cond^n for fxn to ↑

Q If $f(x) = \sin x - a\sin 2x - \frac{1}{3}\sin 3x + 2ax$ is ↑ing in $x \in R$ then find a? ↑

$$f'(x) = \cos x - 2a\cos 2x - \cos 3x + 2a \geq 0$$

$$= \cos x + 2a(1 - \cos 2x) - 4\cos^3 x + 3\cos x \geq 0$$

$$= 4\cos x + 2a \times 2\sin^2 x - 4\cos^3 x \geq 0$$

$$= 4\cos x(1 - \cos^2 x) + 4a\sin^2 x \geq 0$$

$$= 4\sin^2 x\,(a + \cos x) \geq 0$$

⊕ ⊕

fxn will be ↑ing $a + \cos x \geq 0$

$$a \geq (-\cos x)_{Max}$$

$$a \geq 1 \qquad a \in [1, \infty)$$

Q Find a for which $f(x) = a\cos 2x - \sin x$ has exactly one c.r. pt in $(0, \pi)$

exactly one pt where $\frac{dy}{dx} = 0$

$$f'(x) = -2a\sin 2x - \cos x = 0$$

$$= 4a\sin x\cos x + \cos x = 0$$

$$= \cos x\,(4a\sin x + 1) = 0$$

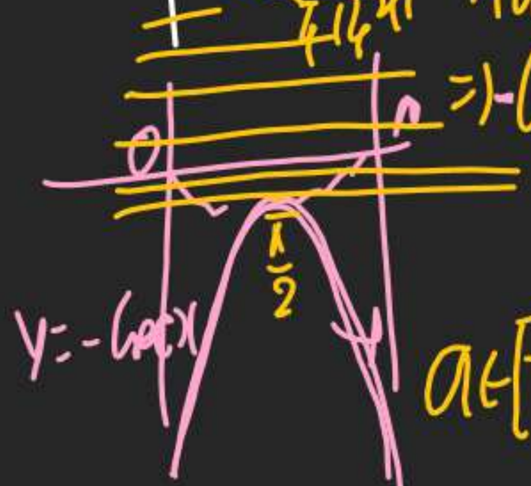
$\cos x = 0$, $x = \frac{\pi}{2}$ — 1 c.r. pt मिल गया

Now इसे Zero नहीं चाहिए

$$4a\sin x + 1 \neq 0$$

$$4a\sin x \neq -1$$

$$\Rightarrow -\operatorname{cosec} x \neq 4a$$



$$4a \geq -1$$

$$a \geq -\frac{1}{4}$$

$$a \in \left[-\frac{1}{4}, \infty\right)$$

Nishant Jindal

Q6 $f:(0,\infty)\to R$ $f(x)=\int_{1/x}^{x} e^{-\left(t+\frac{1}{t}\right)}\frac{dt}{t}$

1) $f(x)$ is Mon. ↑ in $[1,\infty)$ ✓✓

2) $f(x)$ is Mon. ↓ in $(0,1)$ ⊗

3) $f(x)+f\left(\frac{1}{x}\right)=0$ for al $x\in(0,\infty)$ ✓

4) $f(2^x)$ is an odd fxn in $x\in R$. ✓

$$f'(x)=\frac{e^{-\left(x+\frac{1}{x}\right)}}{x}\times 1+\frac{e^{-\left(\frac{1}{x}+x\right)}}{\frac{1}{x}}\times\frac{1}{x^2}$$

$$=\frac{e^{-\left(x+\frac{1}{x}\right)}}{x}+\frac{e^{-\left(x+\frac{1}{x}\right)}}{x}$$

$$=\frac{2e^{-\left(x+\frac{1}{x}\right)}}{x}\geq 0 \text{ hoga } \forall x\in(0,\infty)$$

op (3)

$$f(x)+f\left(\frac{1}{x}\right)=\int_{1/x}^{x} e^{-\left(t+\frac{1}{t}\right)}\frac{dt}{t}+\int_{x}^{1/x}\frac{e^{-\left(t+\frac{1}{t}\right)}}{t}dt$$

$$=\int_{x}^{1/x}\cancel{\frac{e^{-\left(t+\frac{1}{t}\right)}}{t}dt}-\int_{x}^{1/x}\cancel{\frac{e^{-\left(t+\frac{1}{t}\right)}}{t}}dt$$

$$=0$$

op (4)

$f(2^x)+f(2^{-x})=0$ A Jayega

Correct

Nishant Jindal

Q1 $f(x) = \frac{2}{\sqrt{3}} \tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right) - \ln(x^2+x+1) + (b^2-5b+3)x$

is ↓ for $x \in R$ find b.

$$f'(x) = \frac{2}{\sqrt{3}} \times \frac{1}{1+\left(\frac{2x+1}{\sqrt{3}}\right)^2} \times \frac{2}{\sqrt{3}} - \frac{2x+1}{x^2+x+1} + (b^2-5b+3) \le 0$$

$$= \frac{4}{\cancel{3}} \times \frac{\cancel{3}}{4x^2+4x+4} - \frac{2x+1}{x^2+x+1} + (b^2-5b+3) \le 0$$

$$= \frac{\cancel{1} - 2x - \cancel{1}}{x^2+x+1} + (b^2-5b+3) \le 0$$

$$\Rightarrow b^2-5b+3 \le \left(\frac{2x}{x^2+x+1}\right)_{Min}$$

$$\le \left(\frac{2}{x+\frac{1}{x}+1}\right)_{Min}$$

$$b^2-5b+3 \le -2 \Rightarrow b^2-5b+5 \le 0$$

$$\left(b-\left(\frac{5+\sqrt{5}}{2}\right)\right)\left(b-\left(\frac{5-\sqrt{5}}{2}\right)\right) \le 0$$

$$b \in \left(\frac{5-\sqrt{5}}{2}, \frac{5+\sqrt{5}}{2}\right)$$

$x < 0$

$x + \frac{1}{x} \le -2$

$x + \frac{1}{x} + 1 \le -1$

$-\infty < x + \frac{1}{x} + 1 \le -1$

$0 > \frac{1}{x+\frac{1}{x}+1} \ge \frac{1}{-1}$

$0 > \frac{2}{x+\frac{1}{x}+1} \ge -2$

$x > 0$

$x + \frac{1}{x} \ge 2$

$x + \frac{1}{x} + 1 \ge 3$

$3 \le x + \frac{1}{x} + 1 < \infty$

$\frac{1}{3} \ge \frac{1}{x+\frac{1}{x}+1} > 0$

$\frac{2}{3} \ge \frac{2}{x+\frac{1}{x}+1} > 0$

$$\frac{2}{x+\frac{1}{x}+1} \in [-2, 0) \cup \left(0, \frac{2}{3}\right]$$

$$\left(\frac{2}{x+\frac{1}{x}+1}\right)_{Min} = -2$$

$$b = \frac{5 \pm \sqrt{25-20}}{2}$$

$$= \frac{5 \pm \sqrt{5}}{2}$$

Nishant Jindal

Q8 If $f(x) = \sin^3 x - a\sin^2 x$, then find a s.t. $f(x)$ has no cr. pt. in $\left(\frac{\pi}{6}, \frac{\pi}{3}\right)$

$$\frac{dy}{dx} \neq 0 \text{ in } \left(\frac{\pi}{6}, \frac{\pi}{3}\right)$$

$$f'(x) = 3\sin^2 x \cdot \cos x - 2a\sin x\cos x$$

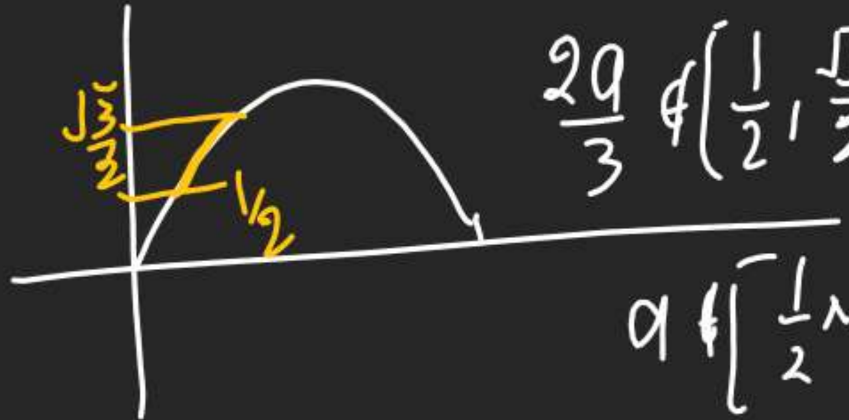
$$= \sin x \cdot \cos x\,(3\sin x - 2a) \neq 0$$

$x \in (30°, 60°)$ $\quad \sin x \neq 0 \quad \cos x \neq 0$

$$3\sin x - 2a \neq 0$$

$$\sin x \neq \frac{2a}{3}$$

$$a \notin \left[\frac{3}{4}, \frac{3\sqrt{3}}{4}\right]$$



$$\frac{2a}{3} \notin \left[\frac{1}{2}, \frac{\sqrt{3}}{2}\right]$$

$$a \notin \left[\frac{1}{2} \times \frac{3}{2}, \frac{\sqrt{3}}{2} \times \frac{3}{2}\right]$$

Q9 $f(x) = (b^2 + (a-1)b + 2)x + \int (\sin^2 x + \cos^4 x)\,dx$

is Mon ↑ in $x \in R$, $b \in R$ find a?

$$f'(x) = (b^2 + (a-1)b + 2) + \sin^2 x + \cos^4 x \geq 0$$

$$b^2 + (a-1)b + 2 \geq \left(-(\sin^2 x + \cos^4 x)\right)_{\text{Max.}}$$

$$b^2 + (a-1)b + 2 \geq -\frac{3}{4}$$

QE ≥ 0

$$4b^2 + 4(a-1)b + 11 \leq 0 \qquad D \leq 0$$

$$16(a-1)^2 - 16 \times 11 \leq 0$$

$$(a-1)^2 - (\sqrt{11})^2 \leq 0$$

$$(a-1-\sqrt{11})(a-1+\sqrt{11}) \leq 0$$

$$1 - \sqrt{11} \leq a \leq 1 + \sqrt{11}$$ Ans

Q46 Monot.

$$\sin^2 x + \cos^4 x \in \left[\frac{3}{4}, 1\right]$$

$$-(\sin^2 x + \cos^4 x) \in \left[-1, -\frac{3}{4}\right]$$

$$y = \sin^2 x + \cos^4 x$$

$$= \sin^2 x + (1 - \sin^2 x)^2$$

$$= \sin^4 x - \sin^2 x + 1$$

$$= \left(\sin^2 x - \frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2 + 1$$

$$y = \left(\sin^2 x - \frac{1}{2}\right)^2 + \frac{3}{4}$$

$\frac{1}{2}$: $y = \frac{3}{4}$ Min

0: $\left(0 - \frac{1}{2}\right)^2 + \frac{3}{4} = 1$ Max

1: $\left(1 - \frac{1}{2}\right)^2 + \frac{3}{4} = 1$ Max

$$\therefore \sin^2 x + \cos^4 x \in \left[\frac{3}{4}, 1\right]$$