Mass of Nucleus & electron Compariable

Taking Nucleus and electron as System.
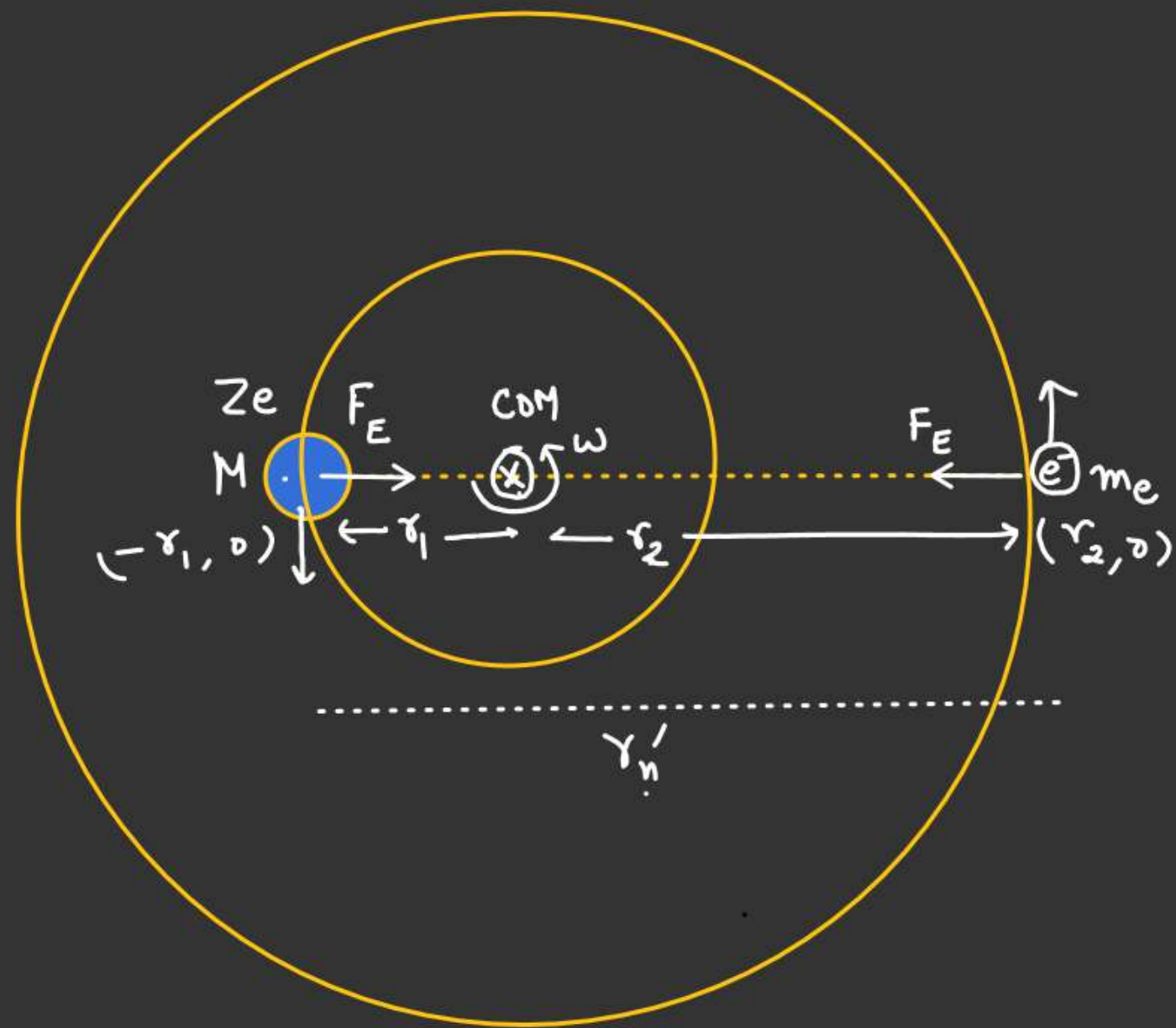
$$R_{com} = \frac{m_e r_2 - M r_1}{m_e + M}$$

$$\Downarrow$$

$$0 = m_e r_2 - M r_1 \quad - \textcircled{1}$$

$$r_1 + r_2 = r_n' \quad - \textcircled{2}$$

$$r_1 = \left(\frac{m_e r_n'}{m_e + M}\right), \quad r_2 = \left(\frac{M r_n'}{m_e + M}\right)$$

For electron

$$r_2 = \left(\frac{M r_n'}{M + m_e}\right)$$

$$L = \frac{nh}{2\pi}$$

$$F_E = m_e \omega^2 r_2$$

$\Downarrow$

$$\frac{1}{4\pi\varepsilon_0} \frac{z e^2}{(r_n')^2} = m_e \omega^2 \left(\frac{M r_n'}{M + m_e}\right)$$

$$L = I_{system} \cdot \omega$$

$$L = \left(M r_1^2 + m_e r_2^2\right)\omega$$

$$L = \left[M\left(\frac{m_e r_n'}{M + m_e}\right)^2 + m_e\left(\frac{M r_n'}{M + m_e}\right)^2\right]\omega$$

$$\frac{1}{4\pi\varepsilon_0} \frac{z e^2}{(r_n')^2} = \left(\frac{M m_e}{M + m_e}\right)\omega^2 r_n'$$

$\Downarrow$
μ

$$L = \left(\frac{M m_e}{M + m_e}\right) r_n'^2 \cdot \omega$$

$\Downarrow$
μ

$$\frac{z e^2}{4\pi\varepsilon_0} = \mu \omega^2 (r_n')^3 - \text{(III)} \checkmark$$

$$\frac{nh}{2\pi} = L = \mu \omega r_n'^2$$

From (III) & (IV) $\checkmark$

$$\boxed{r_n' = r_n\left(\frac{m_e}{\mu}\right)}$$

$$\overset{\text{जहाँ}}{=\!=}\left(r_n = 0.529 Å \frac{n^2}{z}\right)$$

$$\frac{nh}{2\pi} = \mu \omega r_n'^2 - \text{(IV)} \checkmark$$

$$v_n = \left(\frac{ze^2}{2\varepsilon_0 n h} \right)$$

$\downarrow$

Independent of mass.

$$\boxed{v_n' = v_n} \quad \overset{\&\&}{=}$$

ENERGY

$$U = -\frac{ze^2}{4\pi\varepsilon_0 r_n'}$$

$$K.E = \frac{1}{2}(I_{system})\omega^2$$

$$= \frac{1}{2}\left[Mr_1^2 + m_e r_2^2 \right]\omega^2$$

$$E_n' = (U + K.E)$$

$$E_n' = -\frac{13.6 z^2}{n^2} \times \left(\frac{\mu}{m_e} \right)$$

$$\overset{\&\&}{=}$$

$$\boxed{E_n' = E_n \times \frac{\mu}{m_e}} \checkmark$$

$$\left[r_1 = \left(\frac{m_e r_n'}{m_e + M} \right) \quad \omega = \frac{nh}{2\pi M (r_n')^2} \right.$$

$$\left. r_2 = \left(\frac{M r_n'}{m_e + M} \right) \right]$$

$$\mu = \frac{M m_e}{M + m}$$

Reduced
mass

Q.9 Consider a hydrogen-like ionized atom with atomic number Z with a single electron. In the emission spectrum of this atom, the photon emitted in the $n = 2$ to $n = 1$ transition has energy $74.8 \, eV$ higher than the photon emitted in the $n = 3$ to $n = 2$ transition. The ionization energy of the hydrogen atom is $13.6 \, eV$. The value of Z is

(2018)

Q.10   An electron in a hydrogen atom undergoes a transition from an orbit with quantum number n_i to another with quantum number n_f. V_i and V_f are respectively the initial and final potential energies of the electron. If $\dfrac{V_i}{V_f} = 6.25$, then the smallest possible n_f is   (2017)

Q.11   A hydrogen atom in its ground state is irradiated by light of wavelength 970Å. Taking $hc/e = 1.237 \times 10^{-6}$ V m and the ground state energy of hydrogen atom as -13.6eV, the number of lines present in the emission spectrum is   **(2016)**

Nishant Jindal

Q.12 Consider a hydrogen atom with its electron in the nth orbital. An electromagnetic radiation of wavelength 90 nm is used to ionize the atom. If the kinetic energy of the ejected electron is 10.4eV, then the value of n is $(hc = 1242\text{eVnm})$ **(2015)**

ATOMIC STRUCTURE

Q.23 A photon collides with a stationary hydrogen atom in ground state inelastically. Energy of the colliding photon is $10.2\,eV$. After a time interval of the order of micro second another photon collides with same hydrogen atom inelastically with an energy of $15\,eV$. What will be observed by the detector? **(2005)**

(A) One photon of energy $10.2\,eV$ and an electron of energy $1.4\,eV$

(B) Two photons of energy $1.4\,eV$

(C) Two photons of energy $10.2\,eV$

(D) One photon of energy $10.2\,eV$ and another photon of $1.4\,eV$.

ATOMIC STRUCTURE

Q.13 In hydrogen-like atom $(Z = 11)$, n^{th} line of Lyman series has wavelength λ. The de-Broglie's wavelength of electron in the level from which it originated is also λ. Find the value of n.

(2006)

Q.15 The potential energy of a particle of mass m is given by

$$V(x) = \begin{cases} E_0; & 0 \le x \le 1 \\ 0; & x > 1 \end{cases}$$

λ_1 and λ_2 are the de-Broglie wavelengths of the particle, when $0 \le x \le 1$ and $x > 1$

respectively. If the total energy of particle is $2E_0$, find $\lambda_1/\lambda_2 = ??$ **(2005)**

$\lambda_1 \qquad \lambda_2$

Sol^n $\quad |E_T| = P.E + K.E$

$\qquad\qquad\qquad K.E = E_T - P.E$

$0 \le x \le 1 \qquad\qquad = 2E_0 - E_0$

$K.E_1 = E_T = 2E_0 \qquad = E_0$

$\lambda_1 = \dfrac{h}{\sqrt{2m(K.E)_1}} = \dfrac{h}{\sqrt{2mE_0}}$

$x > 1$

$P.E = 0 \checkmark$

$E_T = (K.E)_2 + P.E^0$

$K.E_2 = E_T = 2E_0$

$\lambda_2 = \dfrac{h}{\sqrt{2m(K.E)_2}} = \dfrac{h}{\sqrt{4mE_0}}$
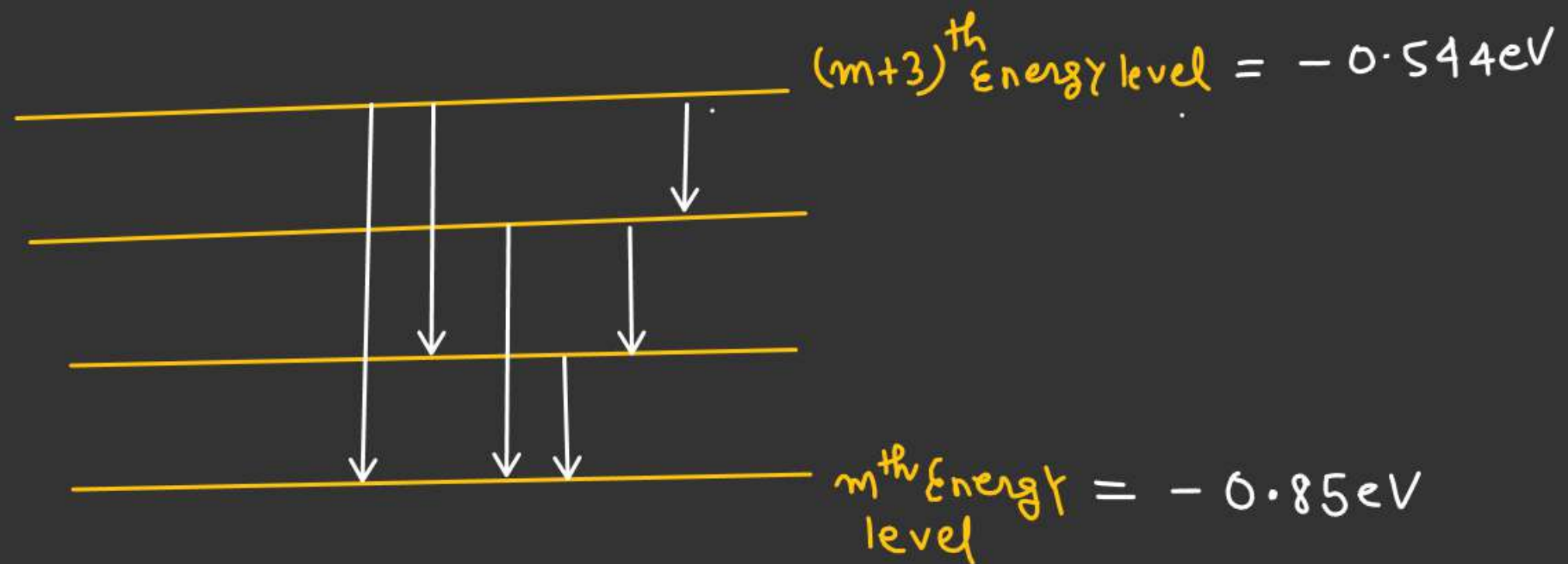
$\dfrac{\lambda_1}{\lambda_2} = \sqrt{2}$ A

ATOMIC STRUCTURE

Q.16 A hydrogen-like atom (described by the Bohr model) is observed to emit six wavelengths, originating from all possible transitions between a group of levels. These levels have energies between -0.85eV and -0.544eV (including both these values).

(a) Find the atomic number of the atom.

(b) Calculate the smallest wavelength emitted in these transitions.

(Take $hc = 1240\text{eV-nm}$, ground state energy of hydrogen atom $= -13.6\text{eV}$)

(2002)

$\longrightarrow 6 \text{ Wavelength}$

$n = \text{No of Energy level}$

$n(n-4) + 3(n-4) = 0$

$^n C_2 = 6$

$\dfrac{n!}{2!\,(n-2)!} = 6$

$\dfrac{n(n-1)}{2} = 6$

$n^2 - n - 12 = 0$

$n^2 - 4n + 3n - 12 = 0$

$\boxed{n = -3} \quad n = 4 \checkmark$

$\times$

$\Updownarrow$

No of energy level

$(m+3)^{th}$ Energy level $= -0.544 eV$

m^{th} Energy level $= -0.85 eV$

$$Z = 3$$
$$m = 12$$

Ans

$-0.544 = \dfrac{-13.6 z^2}{(m+3)^2} \Rightarrow \left(\dfrac{z}{m+3}\right)^2 = \left(\dfrac{0.544}{13.6}\right) = \dfrac{1}{25} \Rightarrow \left(\dfrac{z}{m+3} = \dfrac{1}{5}\right)$

$-0.85 = \dfrac{-13.6 z^2}{m^2} \Rightarrow \left(\dfrac{z}{m}\right)^2 = \left(\dfrac{0.85}{13.6}\right) = \dfrac{1}{16} \Rightarrow \left(\dfrac{z}{m} = \dfrac{1}{4}\right)$

ATOMIC STRUCTURE

Q.17 A hydrogen-like atom of atomic number Z is in an excited state of quantum number $2n$. It can emit a maximum energy photon of $204eV$. If it makes a transition to quantum state n, a photon of energy $40.8eV$ is emitted. Find n, Z and the ground state energy (in eV) for this atom. Also calculate the minimum energy (in eV) that can be emitted by this atom during de-excitation. Ground state energy of hydrogen atom is $-13.6eV$.
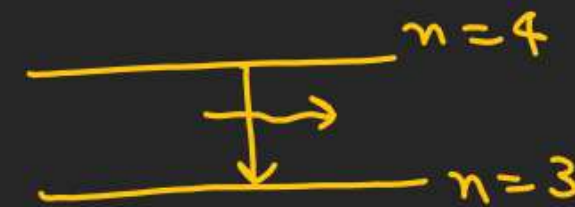
(2000)

$$204 = 13.6\, z^2 \left[1 - \frac{1}{4n^2} \right]$$

$$204 = 13.6\, z^2 \left[\frac{4n^2-1}{4n^2} \right] \quad -\text{①}$$

$$E = h\nu = 204\, eV$$
$$\text{max}$$

$$40.8 = 13.6\, z^2 \left[\frac{1}{n^2} - \frac{1}{4n^2} \right]$$

$$40.8 = 13.6\, z^2 \left[\frac{3}{4n^2} \right] \quad -\text{②}$$

$$\begin{array}{c} n = 2 \\ z = 4 \end{array} \quad \text{Ans}$$

$$E_{min} = 13.6 (4)^2 \left[\frac{1}{3^2} - \frac{1}{4^2} \right]$$
$$\hookrightarrow = \checkmark$$

$2n$

$40.8eV$

$n = n$

$n = 1$

$n = 4$

$n = 3$

ATOMIC STRUCTURE

Q.18 A hydrogen like atom (atomic number Z) is in a higher excited state of quantum number n. The excited atom can make a transition to the first excited state by successively emitting two photons of energy 10.2eV and 17.0eV respectively. Alternately, the atom from the same excited state can make a transition to the second excited state by successively emitting two photons of energies 4.25eV and 5.95eV respectively. Determine the values of n and Z. (Ionization energy of H-atom 13.6eV)

$n = 2$

$n = 3$

(1994)

$$(17 + 10.2) = 13.6 Z^2 \left[\frac{1}{4} - \frac{1}{n^2} \right] - ①$$

$$(4.25 + 5.95) = 13.6 Z^2 \left[\frac{1}{9} - \frac{1}{n^2} \right] - ②$$

$$① \div ②$$

$$n = 6$$

Put $n = 6$ in ①

$$Z = 3$$

n'

17

$10.2\,eV$

$n = 2$

Q.19 An electron, in a hydrogen-like atom, is in an excited state. It has a total energy of $-3.4eV$. Calculate (i) the kinetic energy and (ii) the de Broglie wavelength of the electron. **(1996)**

$$K \cdot E = |E_T| = 3.4 \, e \cdot V.$$

$$1 \, e \cdot V = 1.6 \times 10^{-19} \, J$$

$$\lambda = \frac{h}{mv} = \frac{h}{p} = \frac{h}{\sqrt{2m(K \cdot E)}}$$

$$\lambda = \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times \left(3.4 \times 1.6 \times 10^{-19}\right)}}$$

$$\lambda = 6.63 \, \overset{\circ}{A} \quad \checkmark$$

ATOMIC STRUCTURE

Q.20 A free hydrogen atom after absorbing a photon of wavelength λ_a gets excited from the state $n = 1$ to the state $n = 4$. Immediately after that the electron jumps to $n = m$ state by emitting a photon of wavelength λ_e. Let the change in momentum of atom due to the absorption and the emission are Δp_a and Δp_e respectively. If $\lambda_a/\lambda_e = 1/5$, ✓ which of the option(s) is/are correct?

[Use $hc = 1242 \, eVnm$; $1 \, nm = 10^{-9} \, m$, h and c are Planck's constant and speed of light, respectively]

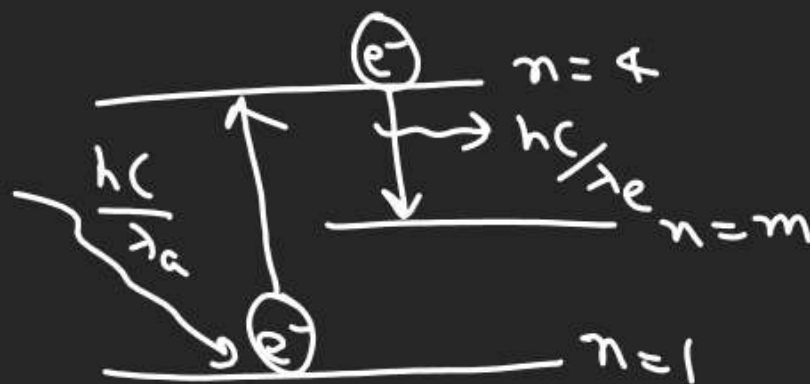(a) The ratio of kinetic energy of the electron in the state $n = m$ to the state $n = 1$ is 1/4 ✓

(b) $m = 2$ ✓

(c) $\Delta p_a/\Delta p_e = 1/2$

(d) $\lambda_e = 418 \, nm$

(2019)

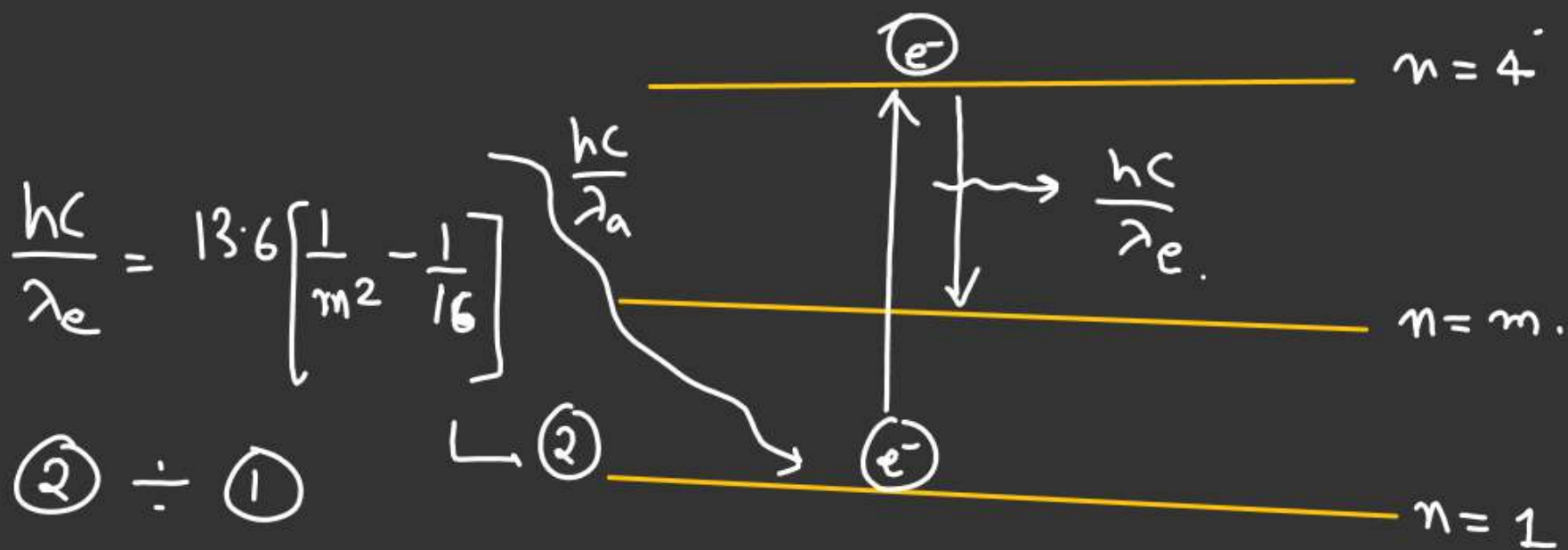$$|E_7| = K.E = \frac{|P.E|}{2}$$

$$E_4 - E_1 = \frac{hc}{\lambda_a}$$

$$-\frac{13.6}{4^2} - \left(\frac{-13.6}{1^2}\right) = \frac{hc}{\lambda_a}$$

$$13.6 \left[1 - \frac{1}{16}\right] = \frac{hc}{\lambda_a}$$

$$13.6 \left[\frac{15}{16}\right] = \frac{hc}{\lambda_a} \quad -①$$

$$\frac{hc}{\lambda_e} = 13.6 \left[\frac{1}{m^2} - \frac{1}{16}\right]$$

$$② ÷ ①$$

$$\frac{\lambda_a}{\lambda_e} = \left(\frac{\frac{1}{m^2} - \frac{1}{16}}{\frac{15}{16}}\right)$$

$$\frac{1}{5} \times \frac{15}{16} = \frac{1}{m^2} - \frac{1}{16}$$

$$\frac{3}{16} + \frac{1}{16} = \frac{1}{m^2}$$

$$\frac{1}{m^2} = \frac{4}{16} = \frac{1}{4}$$

$$\underline{m = 2} \checkmark$$

$\underline{n=1}$

$$K \cdot E_1 = |E_T| = \frac{13.6}{(1)^2}$$

$\underline{n = m = 2}$

De excitation $K \cdot E_2 = |E_T| = \frac{13.6}{(2)^2}$

$\underline{n = m}$

$$\frac{K \cdot E_1}{K \cdot E_2} = 4$$

$$\frac{1}{\lambda_a} = R\left[1 - \frac{1}{(4)^2}\right]$$

$$\lambda_a = - \checkmark$$

$RhC = 13-6$

$R = \left(\frac{13-6}{hC}\right)$

By De broglie Equation

$$\lambda = \frac{h}{mv} = \frac{h}{P}$$

$$P_{\lambda_a} = \sqrt{2m (K \cdot E)_1}$$

$$P_{\lambda_e} = \sqrt{2m (K \cdot E_2)}$$

$$\frac{P_{\lambda_a}}{\lambda_{\lambda_e}} = \sqrt{\frac{(K \cdot E)_1}{(K \cdot E)_2}} = \sqrt{\frac{4}{1}} = 2$$

$$K \cdot E = \frac{P^2}{2m}$$

$$P = \sqrt{2m(K \cdot E)}$$

$hC = \underline{1242 \; eV \cdot nm}$