Nishant Jindal

$$C_1 \to C_1 - C_2$$
$$C_2 \to C_2 - C_3$$

$$(b+c+a)^2 \begin{vmatrix} b+c-a & 0 & a^2 \\ b-c-a & c+a-b & b^2 \\ 0 & c-a-b & (a+b)^2 \end{vmatrix}$$

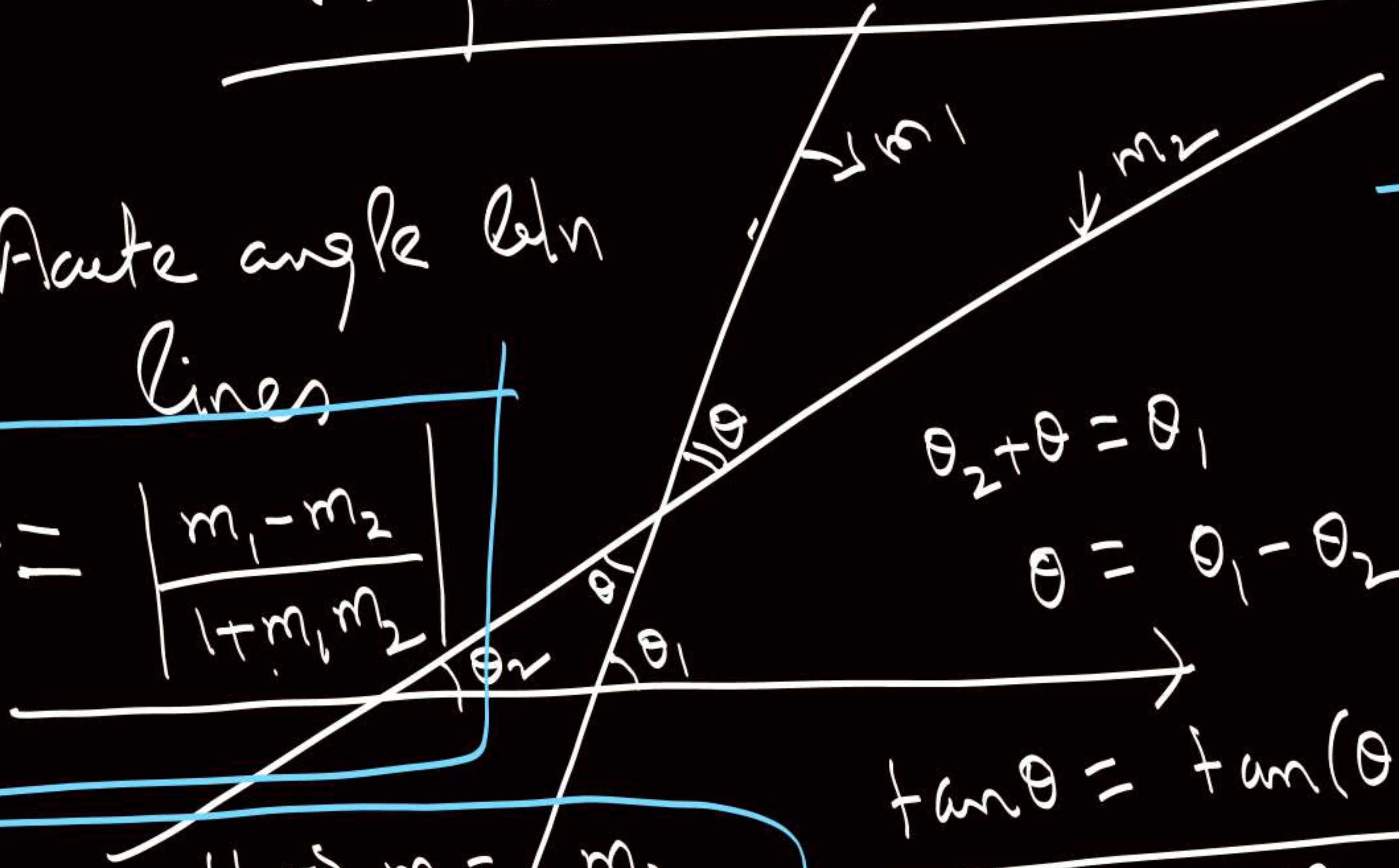
$$C_1 \to \frac{1}{a}C_3 + C_1$$
$$C_2 \to \frac{1}{b}C_3 + C_2$$

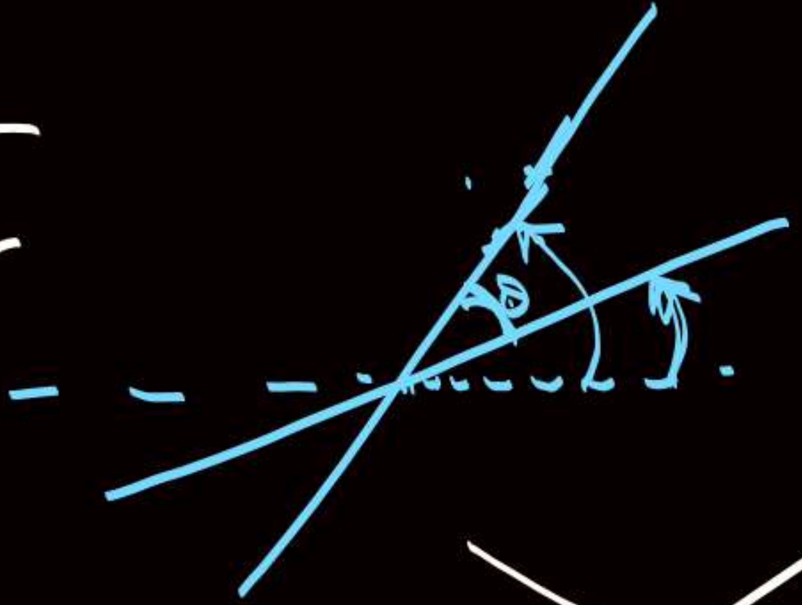
$$\downarrow R_3 \to R_3 - R_2 - R_1$$

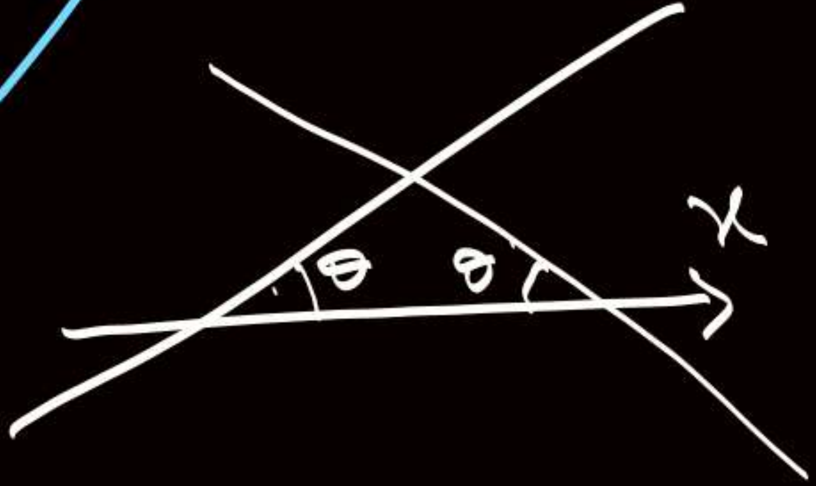
$$(b+c+a)^2 \begin{vmatrix} b+c-a & 0 & a^2 \\ b-c-a & c+a-b & b^2 \\ 2a-2b & -2a & 2ab \end{vmatrix} \to (b+c+a)^2 \begin{vmatrix} b+c-a & 0 & a^2 \\ 0 & c+a-b & b^2 \\ -2b & -2a & 2ab \end{vmatrix}$$

Nishant Jindal

Angle between 2 lines







Acute angle btn lines

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

$\theta_2 + \theta = \theta_1$

$\theta = \theta_1 - \theta_2$

$\tan\theta = \tan(\theta_1 - \theta_2)$

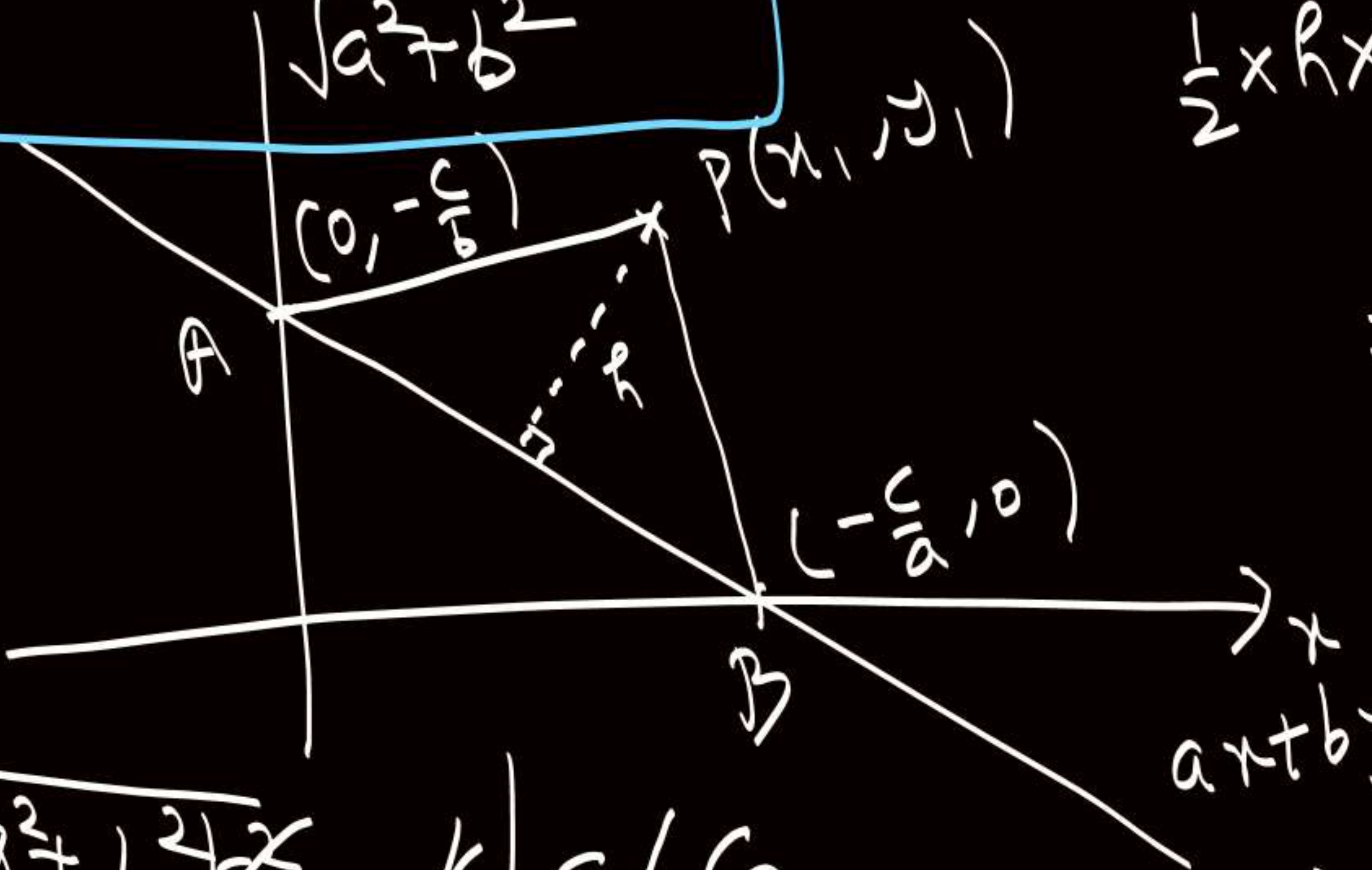
$$\boxed{\tan\theta = \frac{m_1 - m_2}{1 + m_1 m_2}}$$

- Lines are $\parallel \Rightarrow m_1 = m_2$
- Lines are $\perp$ar $\Rightarrow m_1 m_2 = -1$
- Lines (non $\parallel$) are equally inclined with x-axis $\Rightarrow m_1 + m_2 = 0$

Nishant Jindal

Perpendicular distance of a point from a line

$$\boxed{h = \frac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}}}$$



$$\frac{1}{2}\times h\times\sqrt{\frac{c^2}{a^2}+\frac{c^2}{b^2}} = \Delta PAB$$

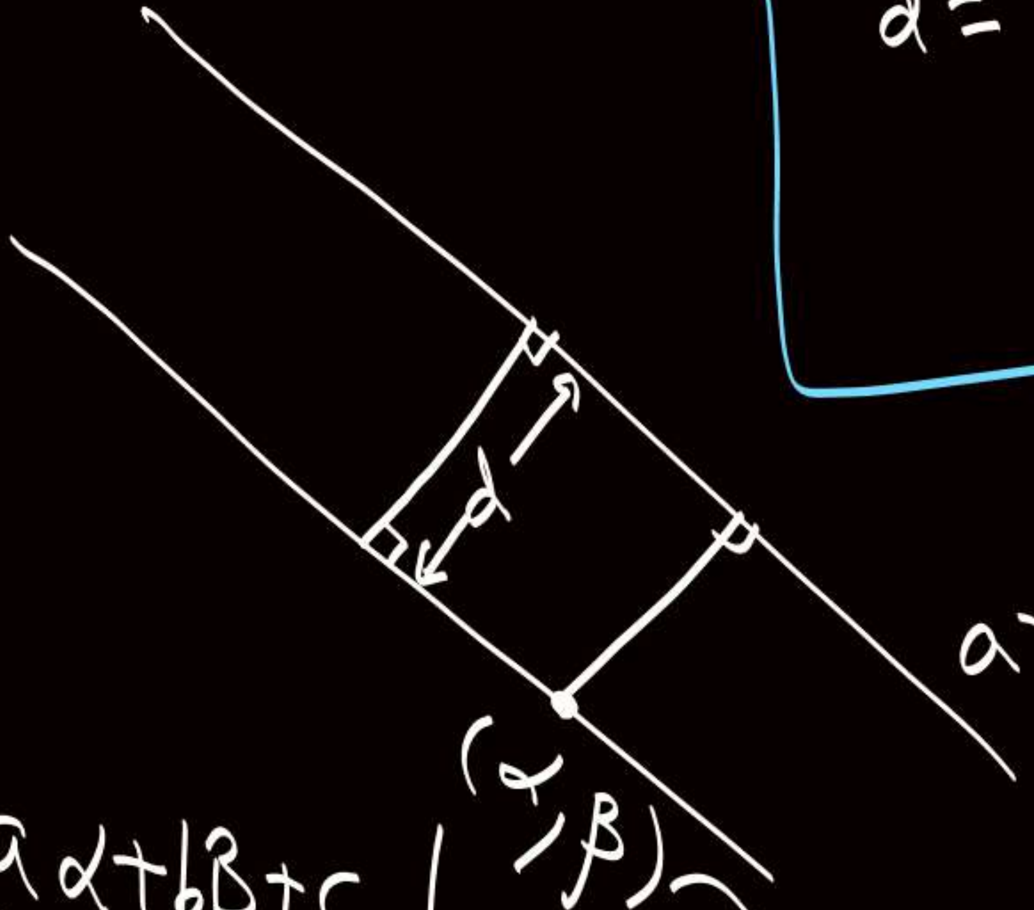
$$= \frac{1}{2}\text{ modulus of }\begin{vmatrix} x_1 & y_1 & 1 \\ 0 & -\frac{c}{b} & 1 \\ -\frac{c}{a} & 0 & 1 \end{vmatrix}$$

$$= \frac{1}{2}\left| x_1\left(-\frac{c}{b}\right) - y_1\left(\frac{c}{a}\right) + \left(-\frac{c^2}{ab}\right)\right|$$

$$\cancel{\frac{1}{2}}h\times\sqrt{\frac{(a^2+b^2)\cancel{c^2}}{\cancel{a^2b^2}}} = \cancel{\frac{1}{2}}\left|\frac{\cancel{c}}{\cancel{ab}}(ax_1+by_1+c)\right|$$

Perpendicular distance b/n two parallel lines

$$\boxed{d = \frac{|c_1 - c_2|}{\sqrt{a^2+b^2}}}$$



$$d = \frac{|a\alpha + b\beta + c_2|}{\sqrt{a^2+b^2}}$$

$$= \frac{|c_2 - c_1|}{\sqrt{a^2+b^2}}$$

$$a\alpha + b\beta + c_1 = 0$$

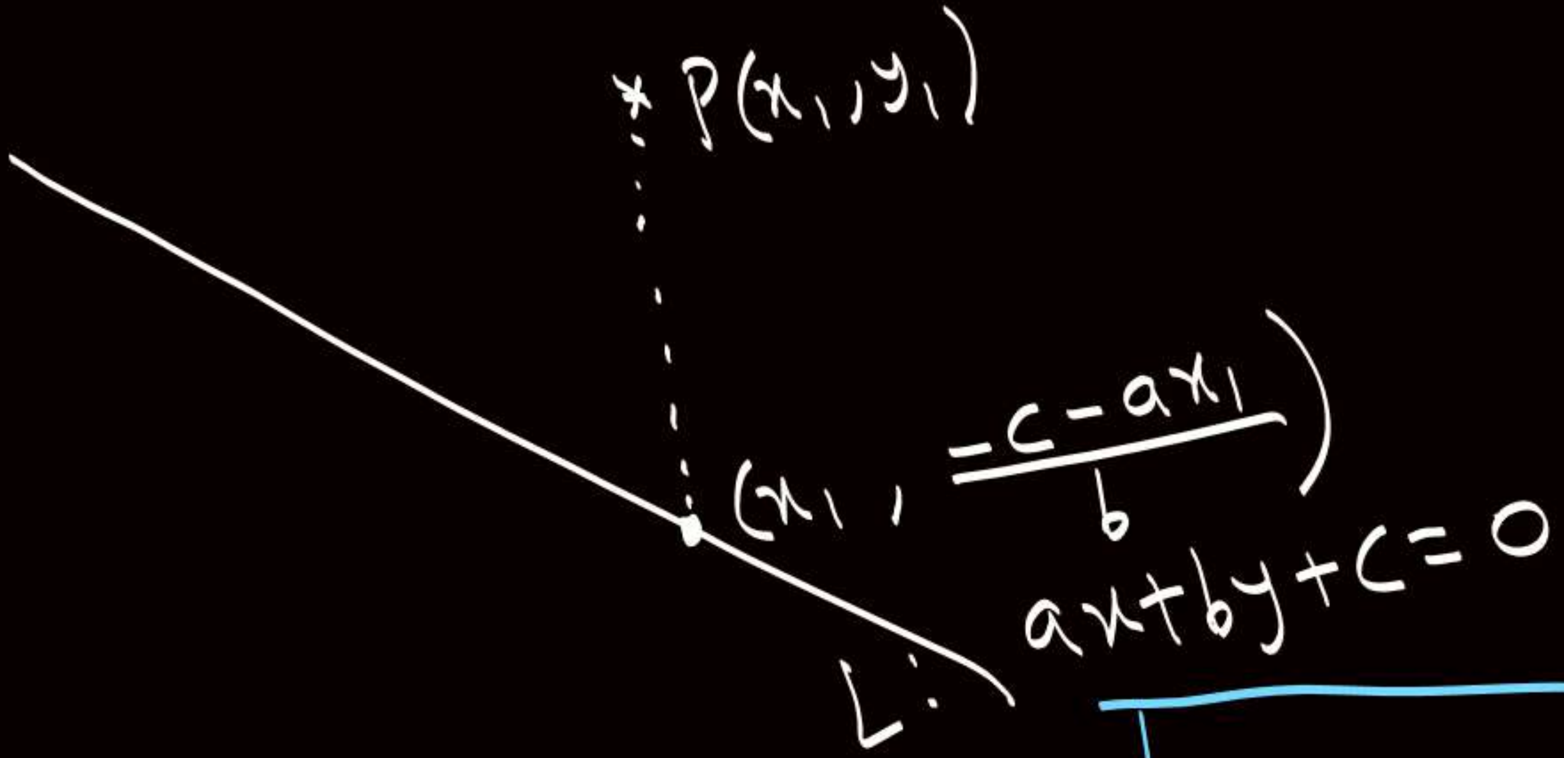
$$x + 2y = 3 \Rightarrow x + 2y - 3 = 0$$

$$2x + 4y = 7 \Rightarrow x + 2y - \frac{7}{2} = 0$$

$$d = \frac{\left|-3 + \frac{7}{2}\right|}{\sqrt{1^2+2^2}}$$

Nishant Jindal

Position of Point w.r.t. Line



$$y_1 > \frac{-c-ax_1}{b}$$

$$y_1 + \frac{c+ax_1}{b} > 0$$

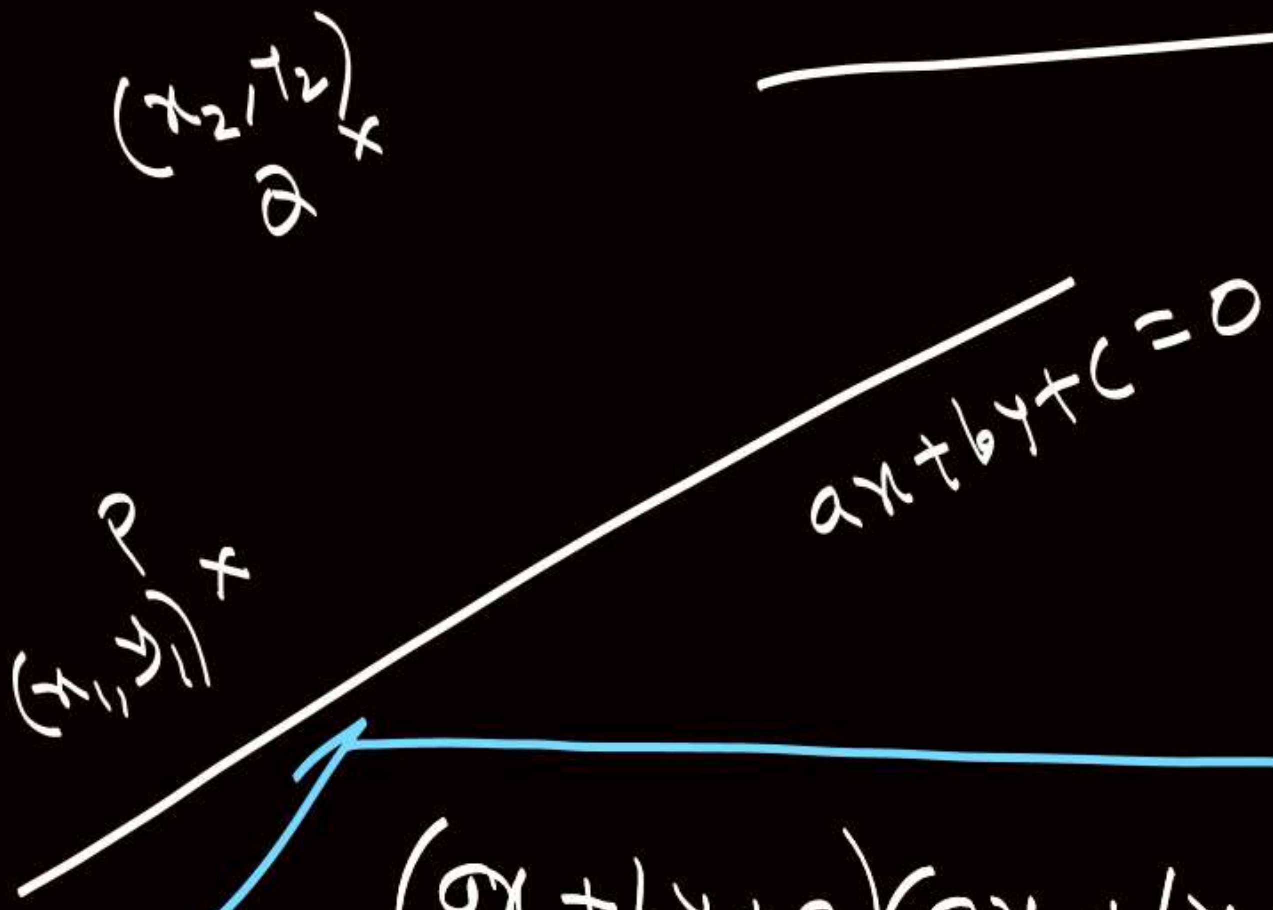
$$\frac{ax_1+by_1+c}{b} > 0$$

$b(ax_1+by_1+c) > 0 \Rightarrow P$ lies above line 'L'.

$b(ax_1+by_1+c) < 0 \Rightarrow P$ lies below 'L'.

Nishant Jindal

Relative position of 2 points (x_1, y_1) & (x_2, y_2) w.r.t. a line $L: ax+by+c=0$.



$b(ax_1+by_1+c) > 0$ & $b(ax_2+by_2+c) > 0$

or

$b(ax_1+by_1+c) < 0$ & $b(ax_2+by_2+c) < 0$

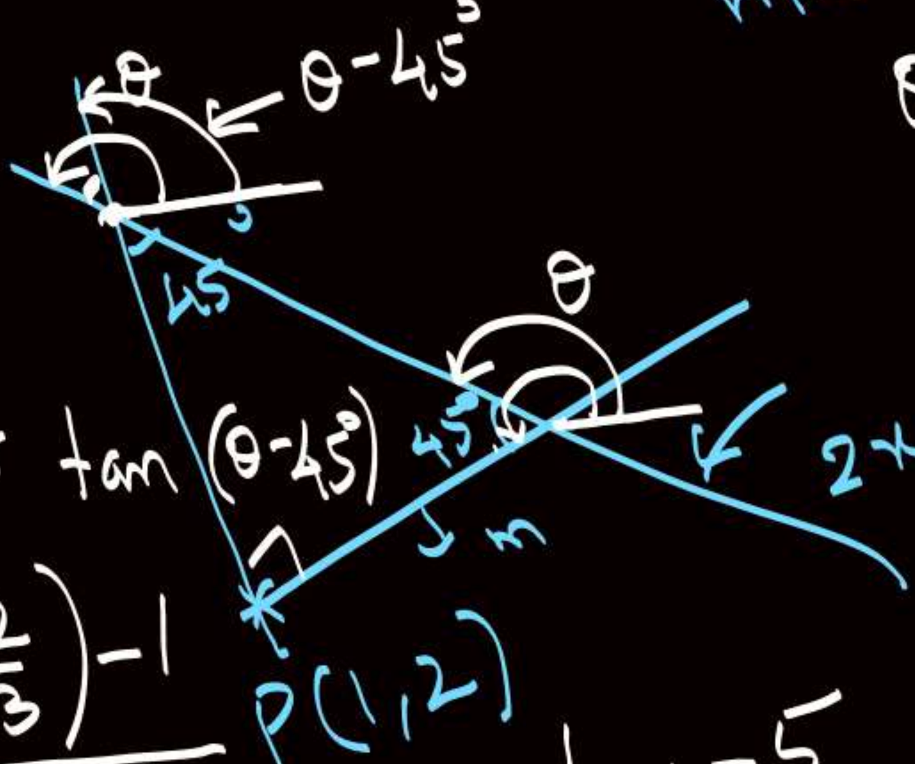
$(ax_1+by_1+c)(ax_2+by_2+c) > 0 \Rightarrow$ P, Q lie on same side of L.

$(ax_1+by_1+c)(ax_2+by_2+c) < 0 \Rightarrow$ P, Q lie on opposite side of L.

Nishant Jindal

1. Find the equation of line passing thru $(1,2)$ making an angle of 45° with the line $2x+3y=10$.

$\tan\theta = -\frac{2}{3}$



$m = \tan(\theta+45^\circ)$ or $\tan(\theta-45^\circ)$

$$= \frac{-\frac{2}{3}+1}{1-\left(-\frac{2}{3}\right)(1)}, \frac{\left(-\frac{2}{3}\right)-1}{1+\left(-\frac{2}{3}\right)(1)} = \frac{1}{5}, -5$$

$m = -5, \frac{1}{5}$

$\theta+45^\circ$

$$\tan 45^\circ = \left|\frac{m-\left(-\frac{2}{3}\right)}{1+m\left(-\frac{2}{3}\right)}\right|$$

$$\pm 1 = \frac{3m+2}{3-2m}$$

$$y-2 = \frac{1}{5}(x-1)$$

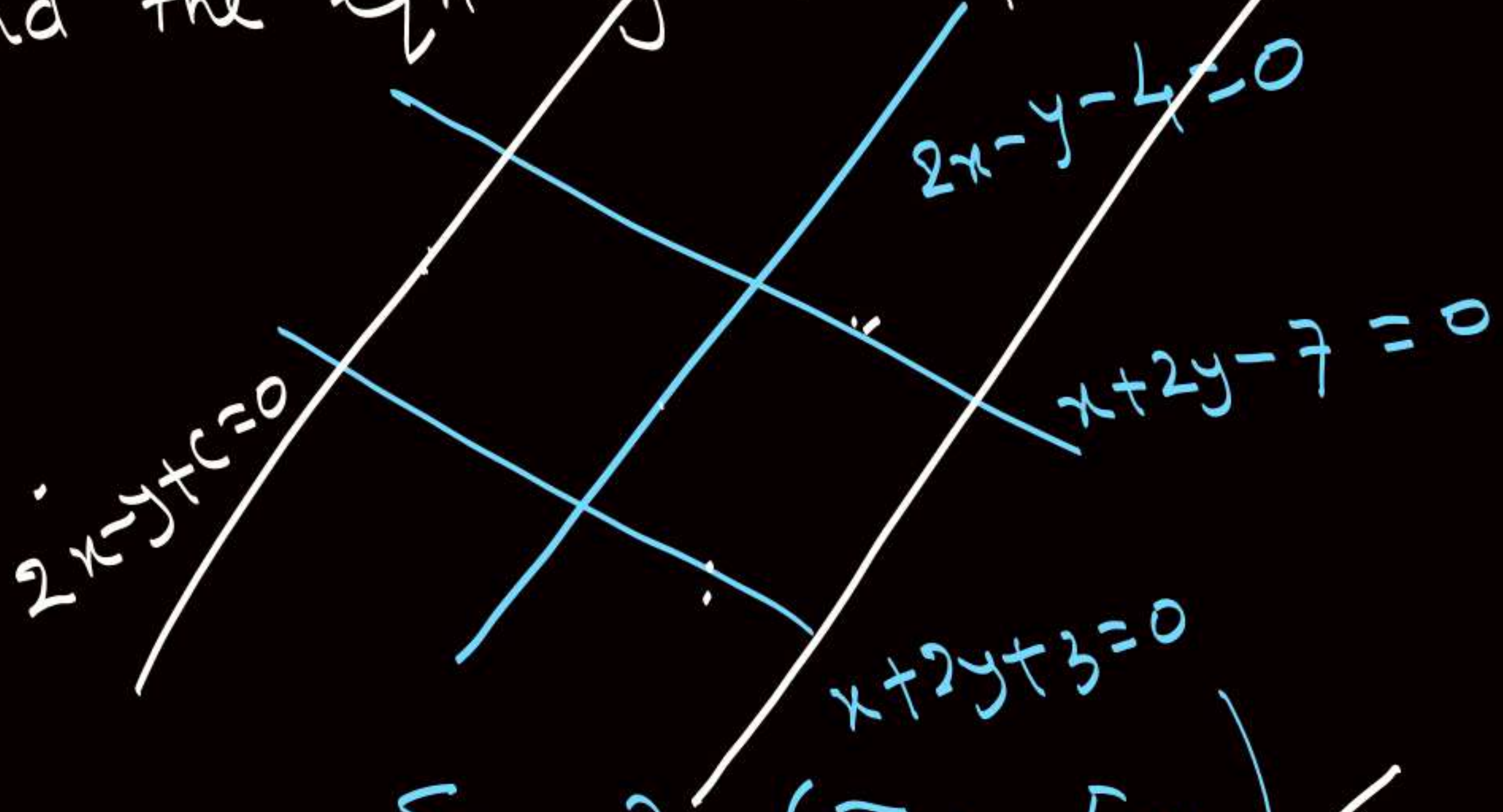
$$y-2 = -5(x-1)$$

$2x+3y-10=0$

$2x+3y-10$

$3(2+6-10) < 0$

Q: The 3 lines $x+2y+3=0$, $x+2y-7=0$ and $2x-y-4=0$ form the 3 sides of a square. Find the eqn. of its 4th side.



$$\boxed{\begin{array}{c} 2x-y+6=0 \\ 2x-y-14=0 \end{array}}$$

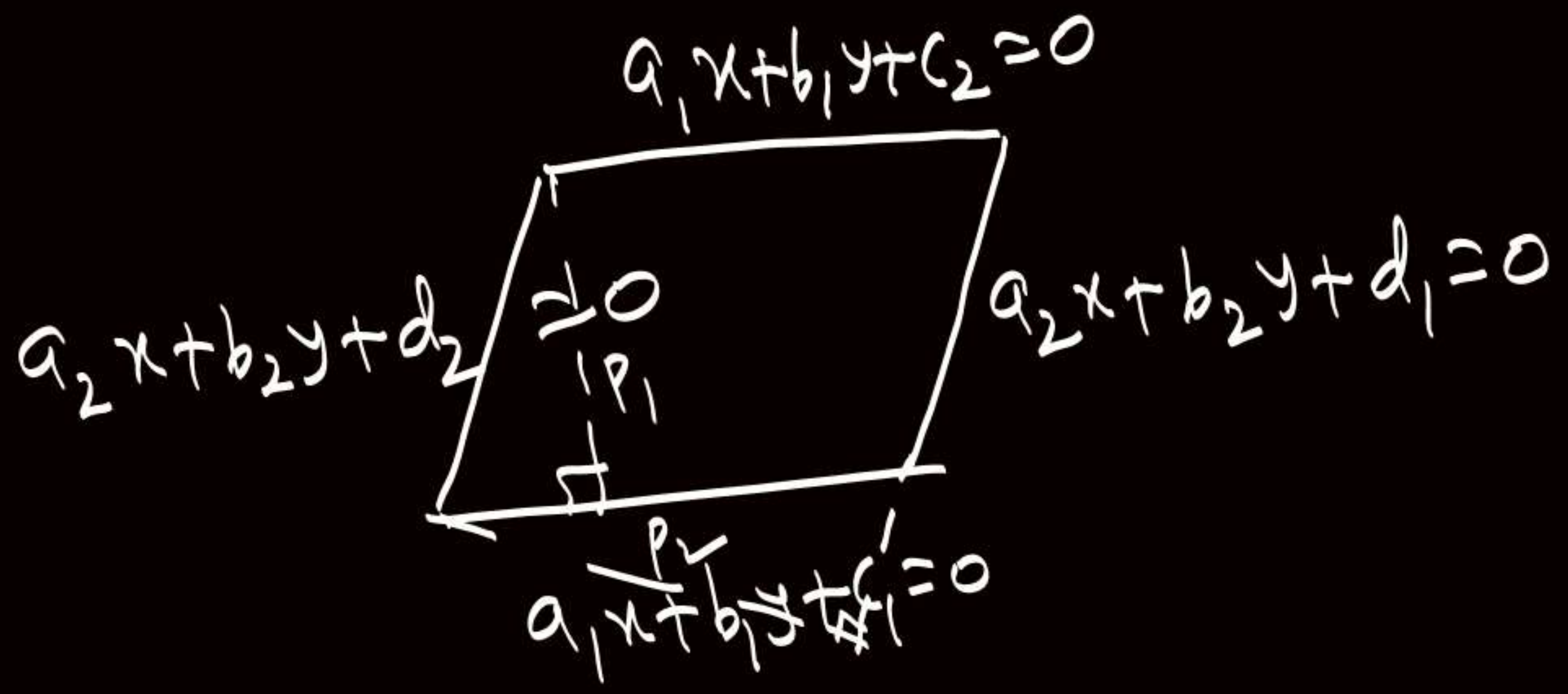
$$\left|\frac{3-(-7)}{\sqrt{1^2+2^2}}\right| = \left|\frac{c+4}{\sqrt{2^2+1^2}}\right|$$

$$c+4=\pm 10$$

$$c=6,\ -14.$$

Ex-2 (Trig. Eqn.)

3.



Find the condition for parallelogram as shown to become rhombus.

$$\frac{|c_1-c_2|}{\sqrt{a_1^2+b_1^2}} = \frac{|d_1-d_2|}{\sqrt{a_2^2+b_2^2}}$$