

$$\therefore \alpha, 2\alpha$$

$$3\alpha = -(2\alpha - 1) \checkmark$$

$$2\alpha^2 = \alpha^2 + 2$$

$$\frac{9}{2} = \frac{(2\alpha - 1)^2}{\alpha^2 + 2}$$

$$\alpha = ?$$

$$\begin{aligned} \text{Given } 10\alpha^2 &= 3\alpha + 2 \\ 9\alpha^2 &= \alpha^2 \\ \frac{100}{9} &= \frac{(3\alpha + 2)^2}{\alpha^2} \end{aligned}$$

$$\therefore \alpha, \alpha^2$$

$$\alpha + \alpha^2 = \frac{15}{4}$$

$$\begin{aligned} 4\alpha^2 + 4\alpha - 15 &= 0 \\ -6\alpha + 10\alpha & \end{aligned}$$

$$\alpha = ?$$

$$(2\alpha + 5)(2\alpha - 3) = 0$$

$$\alpha = -\frac{5}{2}, \frac{3}{2}$$

$$\alpha = -\frac{125}{8}, \frac{27}{8}$$

1.

$$P^2 - 4(12) = 1$$

4.: $5x_1 + 2x_2 = 1$
 $x_1 + x_2 = -\frac{b}{5}$

$$\frac{8}{2} \cdot x^2 + Px + Q = 0 \quad \begin{matrix} P \\ Q \end{matrix}$$

$$P+Q = -P$$

$$PQ = Q$$

$\textcircled{1} - \textcircled{2} \times 2$

$$3x_1 = 1 + \frac{2b}{5}$$

$x^2 - 2Q(x-1) - 1 = 0$

$\frac{9}{5} \left(\frac{5+2b}{5} \right)^2 + \frac{b}{5} \left(\frac{5+2b}{5} \right) - 28 = 0$

$Q=0 \text{ or } P=1$
 $\text{If } Q=0, P=0 \text{ or } P=1, Q=2$

1. Form a cubic whose roots are cubes of

the roots of the equation

$$x^3 + 3x^2 + 2x + 8 = 0$$

$$x^3 + 3x^2 + 2x + 8 = 0 \quad \text{Let } \beta = x$$

$$y+2 = -3y^{2/3}$$

$$(y+2)^3 = -27y^2$$

$$y^3 + 33y^2 + 12y + 8 = 0$$

$$y = \alpha^3, \beta^3, \gamma^3$$

$$y = x \Rightarrow x = \sqrt[3]{y}$$

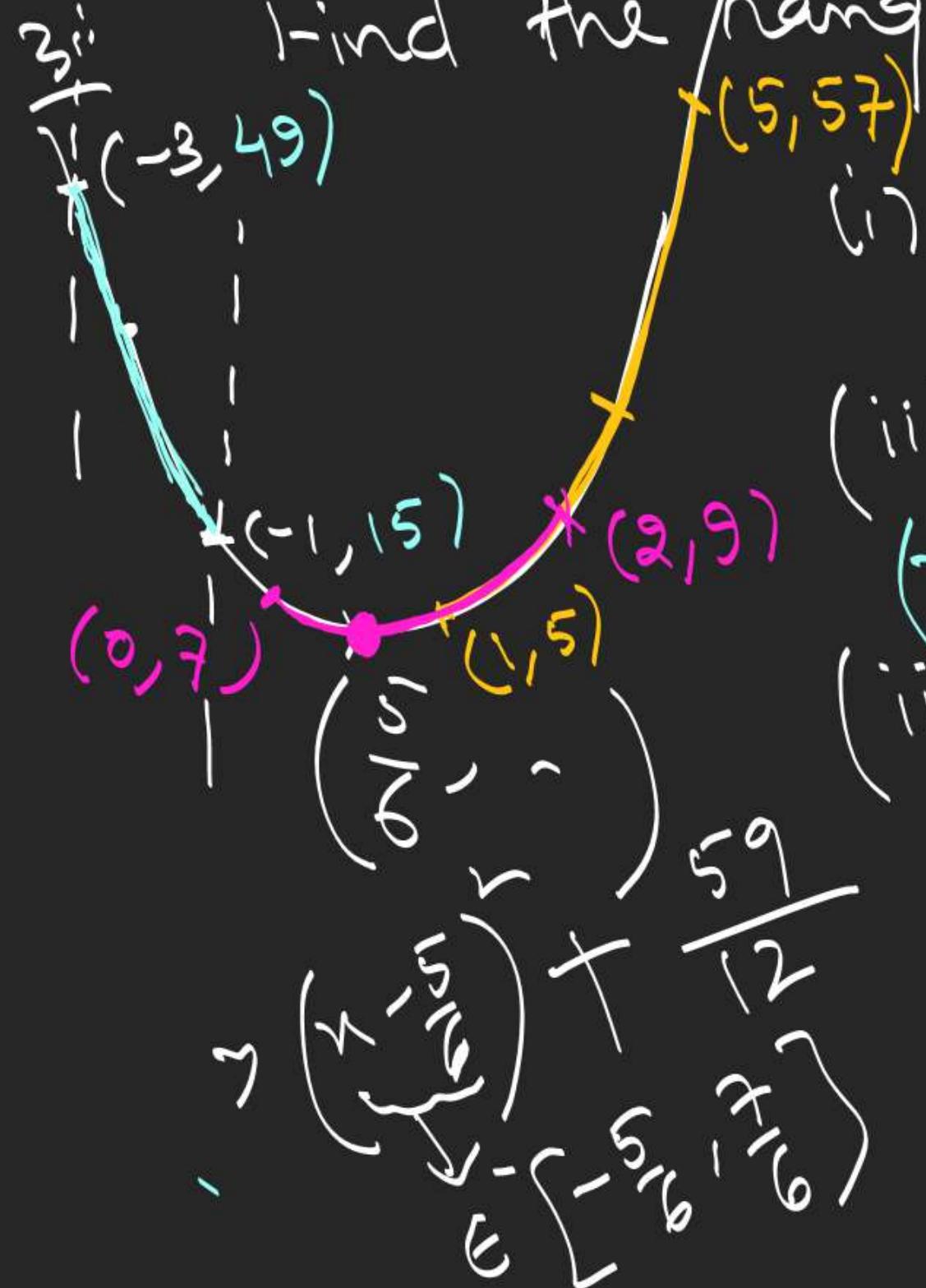
$$2. \quad \sqrt{3(3-a)(3-b)(3-c)} = \sqrt{s(s-a)(s-b)(s-c)} = D = \frac{3}{2}$$

$$4(x-a)(x-b)(x-c) = 4x^3 - 24x^2 + 47x - 30 = 0 \quad \text{Let } a, b, c$$

$$\begin{aligned} 4(27) - 24(9) &= 4(3-a)(3-b)(3-c) \\ + 4(3) - 30 \end{aligned}$$

$$a+b+c = 6 \Rightarrow s = 3$$

Find the range of



$$f(x) = 3x^2 - 5x + 7$$

$$\text{for } x \in [-3, -1] \Rightarrow [15, 49]$$

$$(i) \quad x \in [1, 5] \Rightarrow [5, 57]$$

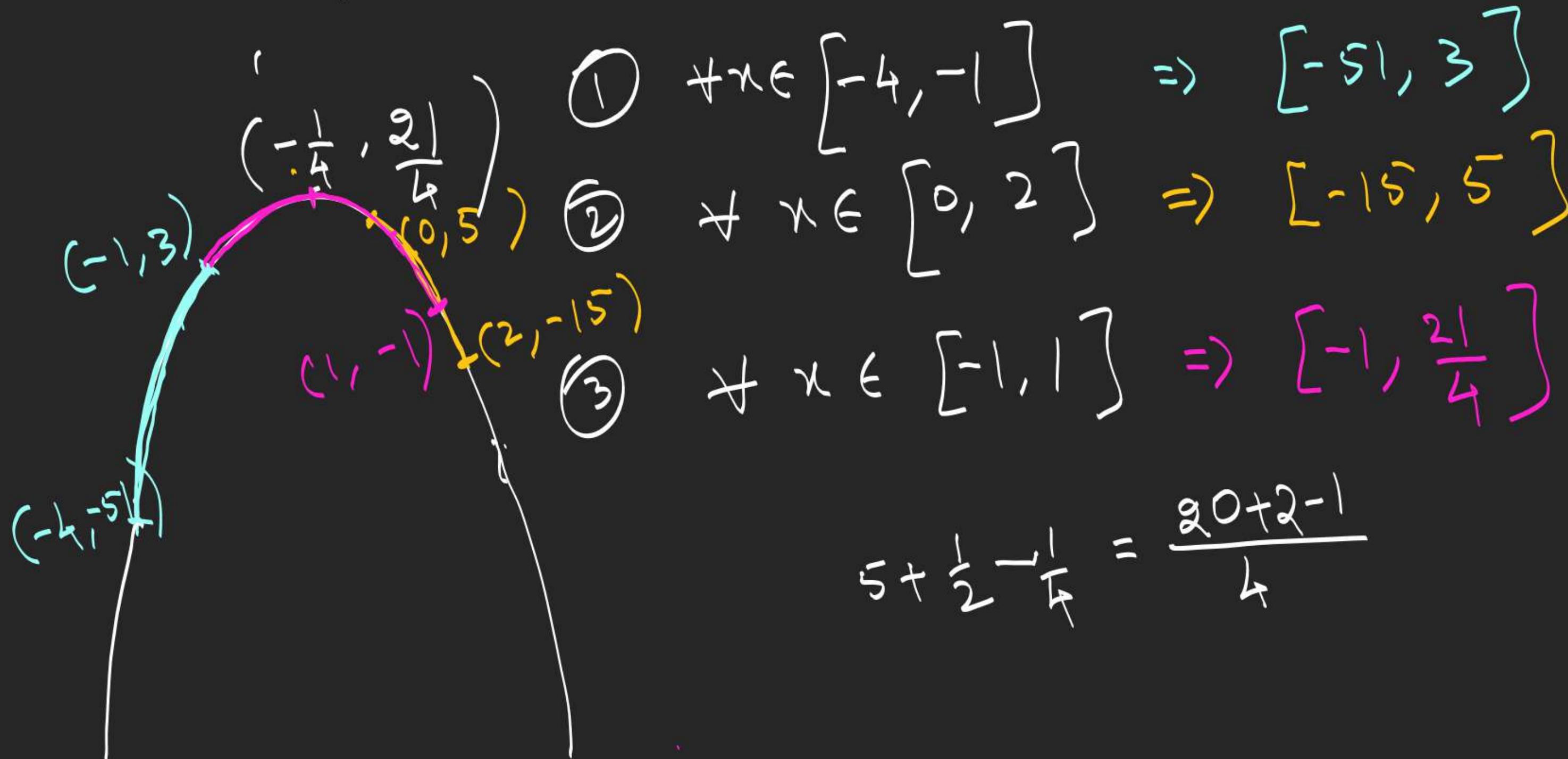
$$(ii) \quad (x - \frac{5}{6})^2 \in [0, \frac{49}{36}]$$

$$(iii) \quad x \in [0, 2]$$

$$[\frac{59}{12}, 9]$$

$$y\left(\frac{5}{6}\right) = 3\left(\frac{25}{36}\right) - \frac{25}{6} + 7 = \frac{75 - 150 + 252}{36} \\ = \frac{177}{36} = \frac{59}{12}$$

Given range of $f(x) = 5 - 2x - 4x^2$



$$5 + \frac{1}{2} - \frac{1}{4} = \frac{20+2-1}{4}$$

5. Find range of function, $f(x) = \frac{x^2 - 3x + 4}{x^2 + 3x + 4}$

$$y = \frac{x^2 - 3x + 4}{x^2 + 3x + 4}$$

$$(y-1)x^2 + (3y+3)x + 4(y-1) = 0$$

$$\text{if } y=1$$

$$6x=0$$

$$x=0$$

OR

$$y = ?$$

$$R_f = \left[\frac{1}{7}, 7 \right]$$

$$y \neq 1$$

$$D \geq 0$$

$$\Rightarrow 9(y+1)^2 - 16(y-1)^2 \geq 0$$

$$(3y+3-4y+4)(3y+3+4y-4) \geq 0$$

$$(7y-1)(y-7) \leq 0$$

$$y \in \left[\frac{1}{7}, 7 \right]$$

$$f = \frac{x^2 - 3x + 4}{x^2 + 3x + 4} = \frac{(x+3x+4)(1) - 6x}{x^2 + 3x + 4} = 1 - \frac{6x}{x^2 + 3x + 4}$$

$$1 - \frac{6x}{x^2 + 3x + 4} \in \left[-\frac{6}{7}, 1+6 \right] = 1 - \frac{6}{x + \frac{4}{x} + 3}$$

$$\frac{x}{x^2 + 3x + 4} \in \left[-1, \frac{1}{2} \right]$$

$$\frac{-6x}{x^2 + 3x + 4} \in \left[-\frac{6}{7}, 6 \right]$$

$$x + \frac{4}{x} + 3 \geq 4 \quad x > 0$$

$$x + \frac{4}{x} + 3 \leq -4 \quad x < 0$$

$$x + \frac{4}{x} + 3 \in (-\infty, -1] \cup [7, \infty)$$

$$\frac{1}{x + \frac{4}{x} + 3} \in (1, 0) \cup (0, \frac{1}{7}]$$

$$f(x) = y = \frac{x^2 - 3x + 4}{x^2 + 3x + 4} = 1 - \frac{6x}{x^2 + 3x + 4}$$

① $D_f = \mathbb{R}$

② $f'(x) = -6 \frac{(x^2 + 3x + 4) - x(2x + 3)}{(x^2 + 3x + 4)^2} = \frac{6(x^2 - 4)}{(x^2 + 3x + 4)^2} = \frac{6(x-2)(x+2)}{(x^2 + 3x + 4)^2}$

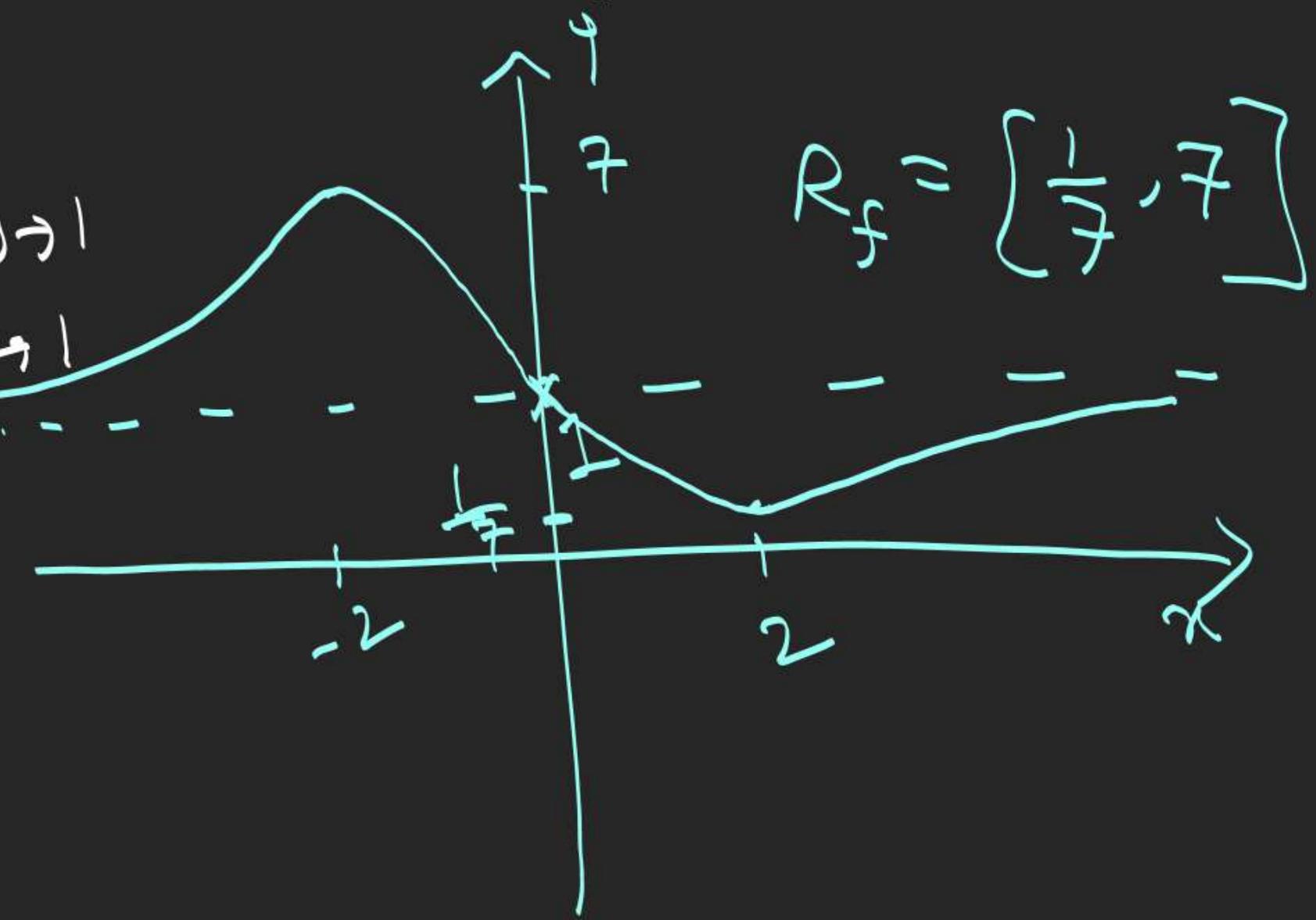
$f \uparrow (-\infty, -2) \cup (2, \infty)$

$y(-2) = \frac{1}{7}$
 $y(2) = 7$

$x \rightarrow -\infty, y \rightarrow 1$
 $x \rightarrow \infty, y \rightarrow 1$

$\downarrow (-2, 2)$

$$y = \frac{\left(-\frac{3}{x} + \frac{4}{x^2}\right)}{\left(1 + \frac{3}{x} + \frac{4}{x^2}\right)}$$



$R_f = \left[\frac{1}{7}, 7\right]$

1. Find the range and sketch the graph of

$$(i) \quad f(x) = \frac{x^2 + 2x - 11}{2(x-3)}$$

$$(ii) \quad f(x) = \frac{(x+1)(x-2)}{x(x+3)}$$

$$(iii) \quad f(x) = \frac{x^2 + 14x + 9}{x^2 + 2x + 3}$$

