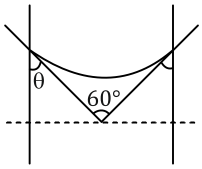
DPP 03

SOLUTION

1. $r = 0.15\ \text{mm}, T = 0.05\ \text{N m}^{-1}$

$\rho = 667\ \text{kg/m}^3, g = 10\ \text{m/s}^2$

Given situation is shown in the figure.



Angle of contact,

$\theta = 90^\circ - 60^\circ = 30^\circ$

$$h = \frac{2T\cos\,\theta}{r\rho g} = \frac{2 \times 0.05 \times \frac{\sqrt{3}}{2}}{(0.15 \times 10^{-3}) \times 667 \times 10} = 0.08656$$

Hence, $h \approx 0.087\ \text{m}$

2. $P_{\text{excess }1} = 1.01 - 1 = 0.01\ \text{atm}$

$P_{\text{excess }2} = 2.02 - 1 = 0.02\ \text{atm}$

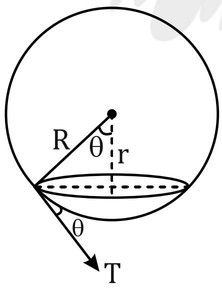
Excess pressure inside the bubble, $P = \frac{4T}{R}$

$$\therefore \frac{P_1}{P_2} = \frac{R_2}{R_1}; \frac{R_2}{R_1} = \frac{0.01}{0.02} = \frac{1}{2}$$

$$\frac{V_1}{V_2} = \left(\frac{R_1}{R_2}\right)^3 = \left(\frac{2}{1}\right)^3$$

Therefore, $V_1 : V_2 = 8 : 1$

3. Force due to surface tension



$$= \int T dl \sin\,\theta$$

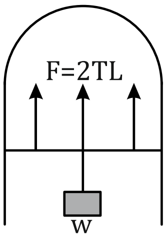
$$= (T\sin\,\theta)\int dl = T\left(\frac{r}{R}\right)(2\pi r)$$

This force will balance the force of buoyancy.

So, $T(2\pi r)\left(\frac{r}{R}\right) = \rho_W\left(\frac{4}{3}\pi R^3\right)g$

$\Rightarrow r^2 = \frac{2}{3}\frac{\rho_W g}{T}R^4 \Rightarrow r = R^2\sqrt{\frac{2\rho_W g}{3T}}.$

4. The force due to the surface tension will balance the weight.



$F = w$

$2TL = w \Rightarrow T = \frac{w}{2L}$

Substituting the given values, we get

$T = \frac{1.5 \times 10^{-2}\ N}{2 \times 30 \times 10^{-2}\ m} = 0.025 Nm^{-1}$

5. Here, surface tension, $S = 0.03\ N\ m^{-1}$

$r_1 = 3\ cm = 3 \times 10^{-2}\ m, r_2 = 5\ cm = 5 \times 10^{-2}\ m$

Since bubble has two surfaces,

Initial surface area of the bubble

$= 2 \times 4\pi r_1^2 = 2 \times 4\pi \times (3 \times 10^{-2})^2 = 72\pi \times 10^{-4}\ m^2$

Final surface area of the bubble

$= 2 \times 4\pi r_2^2 = 2 \times 4\pi(5 \times 10^{-2})^2 = 200\pi \times 10^{-4}\ m^2$

Increase in surface energy

$= 200\pi \times 10^{-4} - 72\pi \times 10^{-4} = 128\pi \times 10^{-4}$

$\therefore$ Work done $= S \times$ increase in surface energy

$= 0.03 \times 128 \times \pi \times 10^{-4} = 3.84\pi \times 10^{-4} = 4\pi \times 10^{-4}\ J = 0.4\pi mJ$

6. Let V be the volume of the droplet.

In equilibrium, $Vdg = \rho\frac{V}{2}g + T(2\pi r)$

or $\frac{4}{3}\pi r^3\frac{g}{2}(2d - \rho) = T(2\pi r)$

or $r = \sqrt{\frac{3T}{(2d - \rho)g}}$

7. For cylindrical shape, excess pressure is given by

$$\Delta P = \frac{T}{R}$$

8. $h = \frac{2\sigma}{r\rho g}\cos\theta$

Since σ, g and ρ are constant, $h \propto \frac{1}{r}$

If $r' = 2r, h' = h/2$

Now, $M = \rho \times V = \rho \times \pi r^2 h$

$\therefore\ M' = \rho \times \pi r'^2 h' = \rho \times \pi(2r)^2 h/2 = 2M$

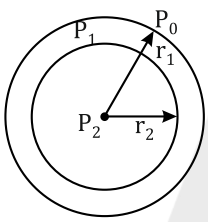
9. From ascent formula, $h = \frac{2S\cos\theta}{r\rho g}$

So, $h_1 = \frac{2S_1\cos\theta_1}{r_1\rho_1 g}$ and $h_2 = \frac{2S_2\cos\theta_2}{r_2\rho_2 g}$

$\because h_1 = h_2$ then $\frac{2S_1\cos\theta_1}{r_1\rho_1 g} = \frac{2S_2\cos\theta_2}{r_2\rho_2 g}$

$\Rightarrow \frac{r_1}{r_2} = \frac{S_1\rho_2\cos\theta_1}{S_2\rho_1\cos\theta_2} = \frac{7.5\cos 135^\circ}{13.6\cos 0^\circ} \simeq \frac{2}{5}$

10. Excess pressure inside the inner bubble,



$P_2 - P_1 = \frac{4T}{r_2}$ ... (i)

Excess pressure inside the outer bubble,

$P_1 - P_0 = \frac{4T}{r_1}$ ... (ii)

From eqn (i) and (ii), $P_2 - P_0 = 4T\left(\frac{1}{r_2} + \frac{1}{r_1}\right) = \frac{4T}{r}$

Here r is required radius of a soap bubble,

$\therefore\ r = \frac{r_2 r_1}{r_1 + r_2} = \frac{4 \times 6}{4 + 6} = \frac{24}{10} = 2.4$ cm