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IRODOVE

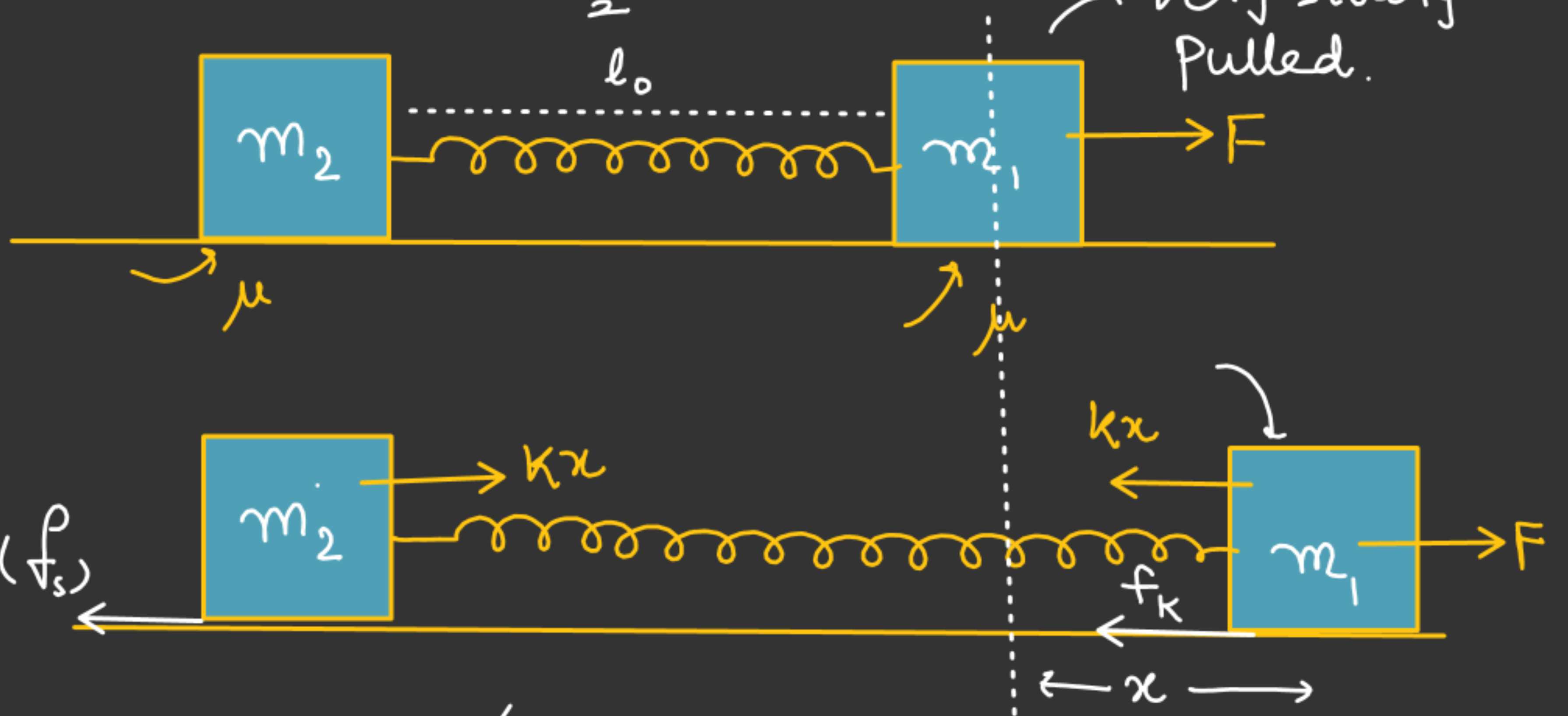
F is gradually increases from 0 to F.

Find F_{min} for block m_2 just to move.

$W_{spring} = -\Delta U$

$= U_i - U_f$

$= 0 - \frac{1}{2}Kx^2$



Sol^n: when block m_2 about to move.

$Kx = (f_s)_{max} = \mu m_2 g$

$x = \left(\frac{\mu m_2 g}{K}\right)$ —(1)

By work-Energy theorem,

$W_F + W_{spring\,force} + W_{f_K} = \cancel{(\Delta K E)} \to 0$

$Fx - \frac{1}{2}Kx^2 - \mu m_1 g x = 0$

$F = \left(\mu m_1 g + \frac{1}{2}Kx\right)$ —(2)

$F = \left(\mu m_1 g + \frac{1}{2}\mu m_2 g\right)$

$F = \mu g\left(m_1 + \frac{m_2}{2}\right)$ Ans

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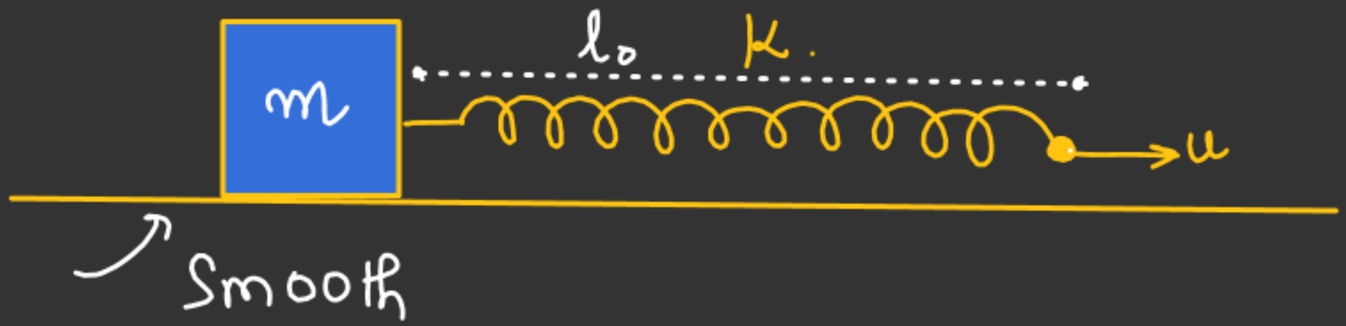
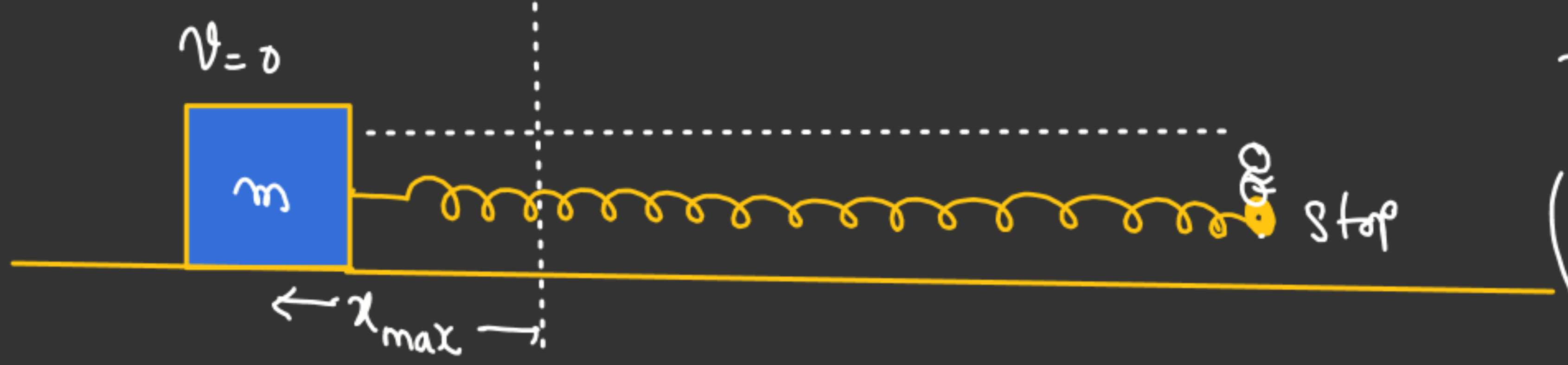
Maximum elongation in the Spring



When Spring at its Natural length then the free end has velocity u m/s.

Note

For maximum elongation or Compression the relative velocity of the two ends of the Spring should be zero.



$$\frac{1}{2}mu^2 = \frac{1}{2}Kx_{max}^2$$

$$x_{max} = \sqrt{\frac{mu^2}{K}}$$

$$\left(x_{max} = u\sqrt{\frac{m}{K}}\right)$$

w.r.t earth

At the time of maximum elongation

$v_{rel} = 0$ $\left[v_{A/B} \text{ or } v_{B/A} \right.$

$\left[S_{rel} = \text{Constant} \right.$

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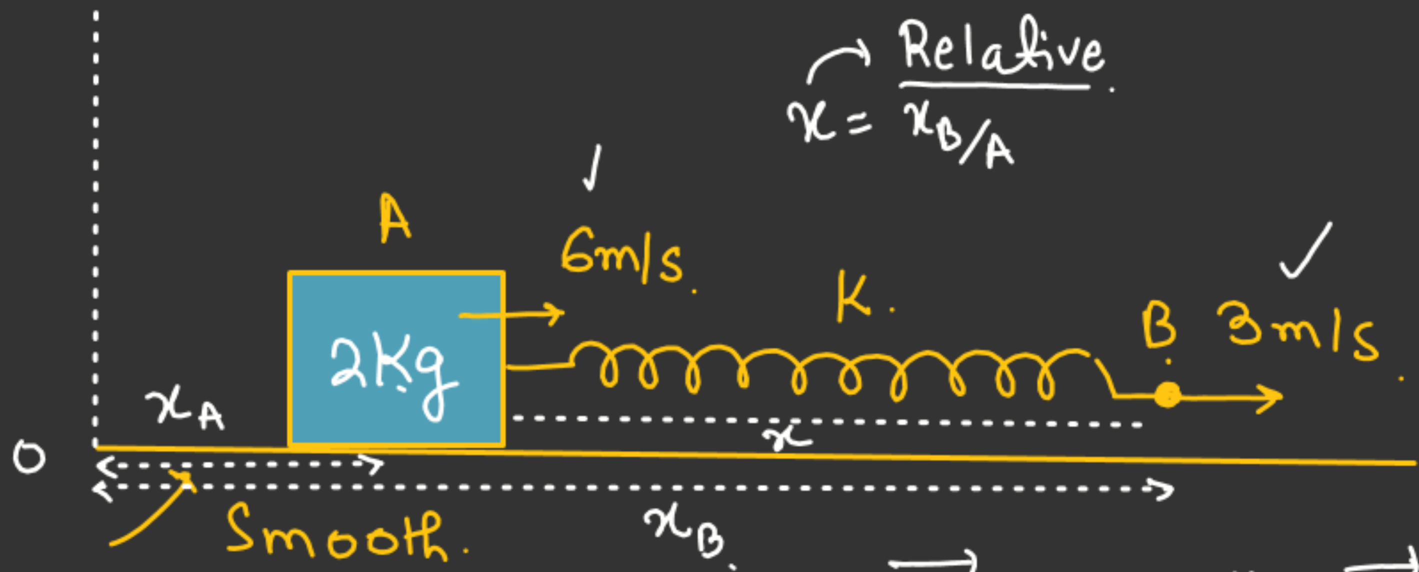
At this instant the rate of Change of Potential energy of Spring is (15 J/s) ✓ Find accleration of block.



Relative

$x = x_{B/A}$

x = Relative distance b/w A and B.

$x_{B/A} = x_B - x_A$

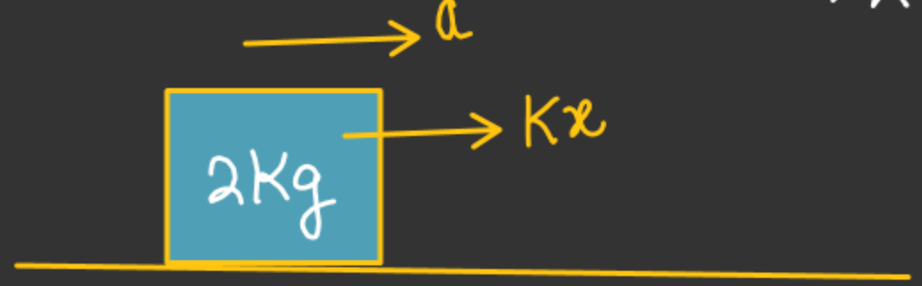
$\frac{dx_{B/A}}{dt} = \frac{d(x_B)}{dt} - \frac{d(x_A)}{dt}$

$v_{B/A} = \frac{dx_{B/A}}{dt} = (3-6) = -3$ m/s ✓

$\vec{v}_{B/A} = \vec{v}_B - \vec{v}_A$

$= 3\hat{i} - 6\hat{i}$

$= -3\hat{i}$ ✓



$a = \left(\frac{Kx}{2}\right)$

$U = \frac{1}{2}Kx^2$

$\Downarrow$

$\frac{dU}{dt} = \frac{1}{2}K\frac{d}{dt}(x^2)$

$15 = \frac{1}{\cancel{2}}K(\cancel{2}x)\left(\frac{dx}{dt}\right)$

$\left|\frac{dx}{dt}\right| = 3$ m/s.

$Kx = \frac{15}{3} = 5$ N.

$a = \frac{5}{2} = 2.5\,ms^{-2}$ ✓

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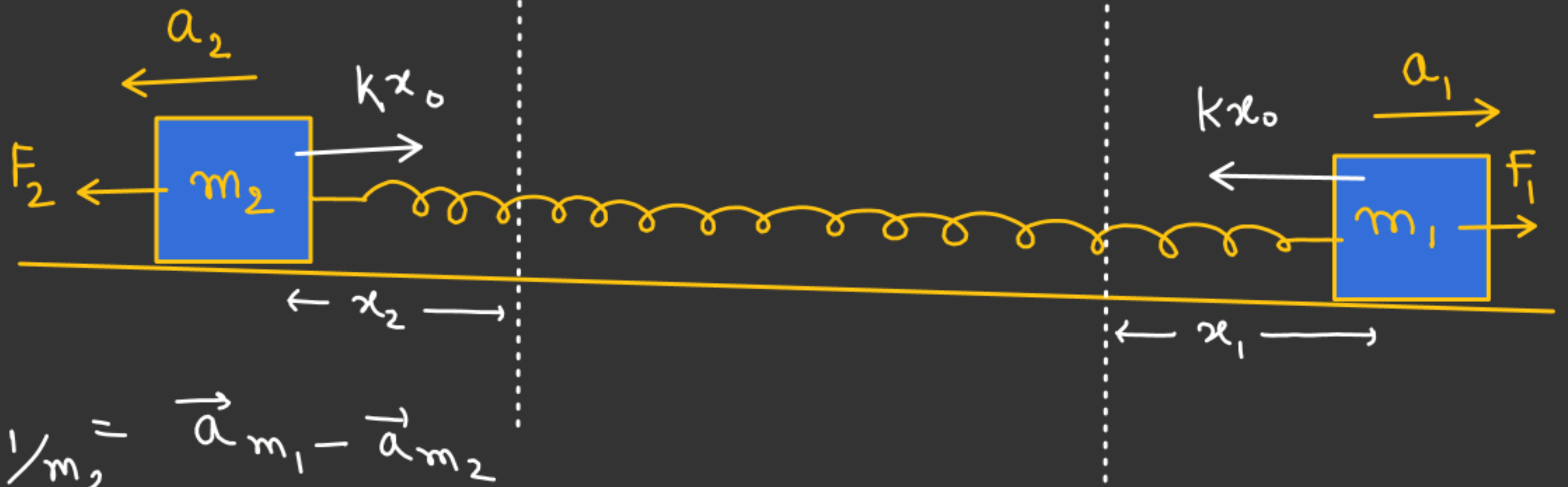
Maximum Elongation in the Spring

F_1 & F_2 Constant force.



$$x = (x_1 + x_2)$$

Elongation at any instant

From Netwon's 2^{nd} Law on both the blocks.

$$F_1 - Kx = m_1 a_1$$

$$F_2 - Kx = m_2 a_2$$

$$a_1 = \left(\frac{F_1}{m_1} - \frac{Kx}{m_1}\right)$$

$$a_2 = \left(\frac{F_2}{m_2} - \frac{Kx}{m_2}\right)$$

$$\vec{a}_{m_1/m_2} = \vec{a}_{m_1} - \vec{a}_{m_2}$$

$$\Downarrow$$

$$= \left(\frac{F_1}{m_1} - \frac{Kx}{m_1}\right)\hat{i} - \left(\frac{F_2}{m_2} - \frac{Kx}{m_2}\right)(-\hat{i})$$

$$\vec{a}_{m_1/m_2} = \left[\left(\frac{F_1}{m_1} + \frac{F_2}{m_2}\right) - \left(\frac{K}{m_1} + \frac{K}{m_2}\right)x\right]\hat{i}$$

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$$a_{m_1/m_2} = \left(\frac{F_1}{m_1} + \frac{F_2}{m_2}\right) - \left(\frac{K}{m_1} + \frac{K}{m_2}\right)x$$

$$\Downarrow$$

$$v_r \frac{dv_r}{dx} = \left(\frac{F_1}{m_1} + \frac{F_2}{m_2}\right) - \left(\frac{K}{m_1} + \frac{K}{m_2}\right)x$$

$$\int v_r\, dv_r = \int_0^{x_{max}} \left(\frac{F_1}{m_1} + \frac{F_2}{m_2}\right) dx - \int_0^{x_{max}} \left(\frac{K}{m_1} + \frac{K}{m_2}\right) x\, dx$$

$$0 = \left(\frac{F_1}{m_1} + \frac{F_2}{m_2}\right) x_{max} - K\left(\frac{1}{m_1} + \frac{1}{m_2}\right) \frac{x_{max}^2}{2}$$

For Maximum elongation.

$(v_{rel}) = 0$.

$$\boxed{x_{max} = 2\left(\frac{m_2F_1 + m_1F_2}{m_1 + m_2}\right)}$$

M-2 [COM]

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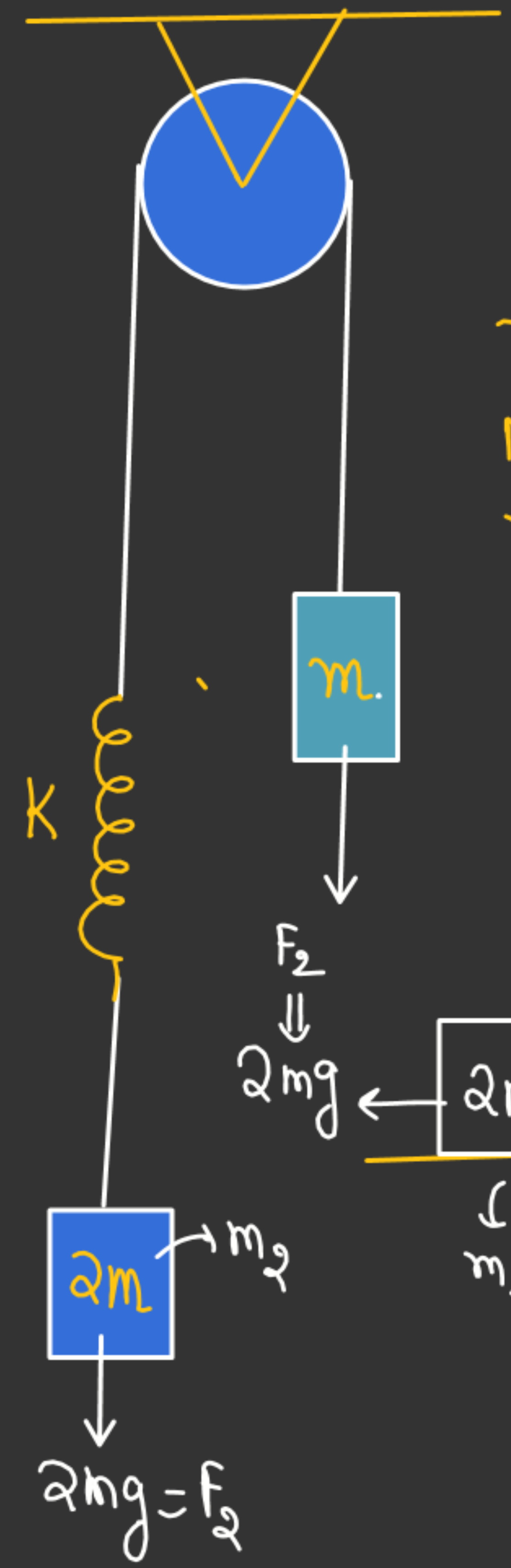


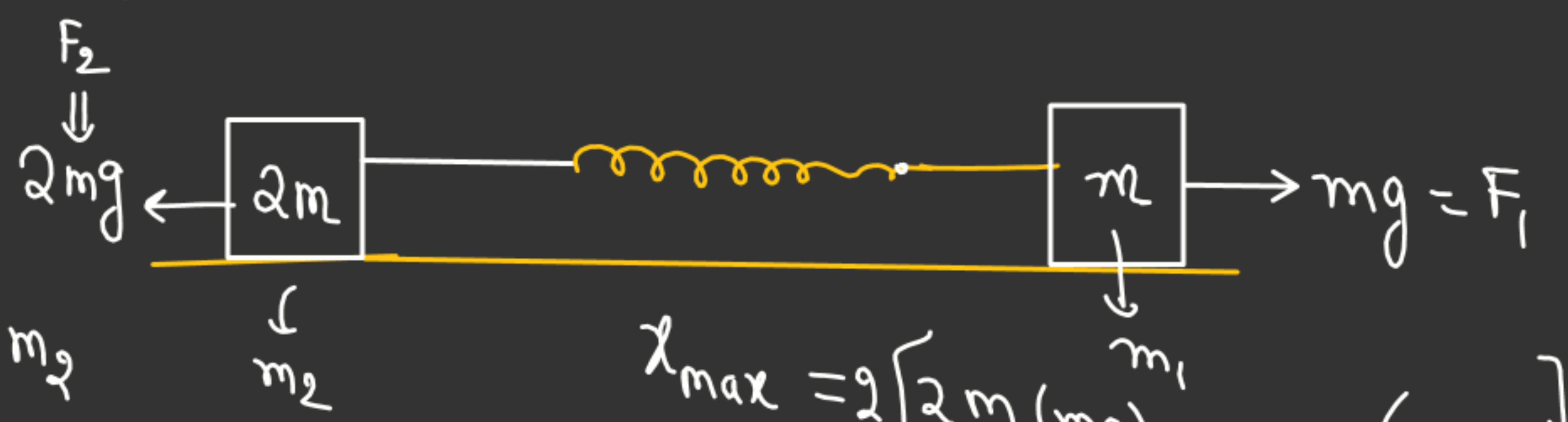
System is released from rest.

Maximum elongation in the spring.

When forces in same direction.

$$x_{max} = \frac{2}{K}\left[\frac{m_2F_1 - m_1F_2}{m_1+m_2}\right]$$

$$x_{max} = \left(\frac{m_2F_1 + m_1F_2}{m_1+m_2}\right)$$



$$x_{max} = \frac{2}{K}\left[\frac{2m(mg) + m(2mg)}{3m}\right] = \frac{2}{K}\left(\frac{4m^2g}{3m}\right) = \frac{4}{3}mg\times\frac{2}{K} = \frac{8mg}{3K}$$ Ans

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System is released from rest.

Find $x_{max} = ??$