Nishant Jindal

$$\cos^2(\alpha+\beta)+\cos(\alpha-\beta)\cos(\alpha+\beta)+\frac{1}{4}=0$$

$$\left(\cos(\alpha+\beta)+\frac{1}{2}\cos(\alpha-\beta)\right)^2+\frac{1}{4}\sin^2(\alpha-\beta)=0$$

$= 0$ $\qquad = 0$

$$\boxed{\alpha=\beta}$$

$0<\alpha<\frac{\pi}{2}$

$-\frac{\pi}{2}<-\beta<0$

$\alpha-\beta\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$

$\left(-\infty,\frac{1-\sqrt{5}}{2}\right]\cup\left[\frac{1+\sqrt{5}}{2},\infty\right)$

$2\sin t\in\left[-2,\frac{1-\sqrt{5}}{2}\right]\cup\left[\frac{1+\sqrt{5}}{2},2\right]$

$(a^2+b^2-2ab)-4+4a+4b>0 \quad \forall b\in R$

$b^2+(4-2a)b+(a^2+4a-4)>0 \quad \forall b\in R$

$\Downarrow$

$D<0$

$y=1$

a $\quad$ b

$a=?$

$1=\boxed{(x-a)(x-b)}$

$\cos 2\alpha=-\frac{1}{2}$

$\left[-\frac{\pi}{2},-\frac{\pi}{10}\right]\cup\left[\frac{3\pi}{10},\frac{\pi}{2}\right]$

$-\frac{\pi}{2}$, 0, $\frac{\pi}{2}$

$\alpha+\alpha^2=-\frac{p}{3}$

$\alpha^3=1 \Rightarrow \underbrace{(\alpha^2+\alpha+1)}_{\neq 0}(\alpha-1)=0$

$1-\frac{p}{3}=0$

Nishant Jindal

$$a^2+b^2+c^2+2\sum ab-3\lambda\sum ab \geq 0.$$

$$2\sum ab > \sum a^2 \geq (3\lambda-2)\sum ab$$

$$\sum a^2 \geq \sum ab$$

$$|a-b| < c \Rightarrow \quad a^2+b^2-2ab < c^2$$

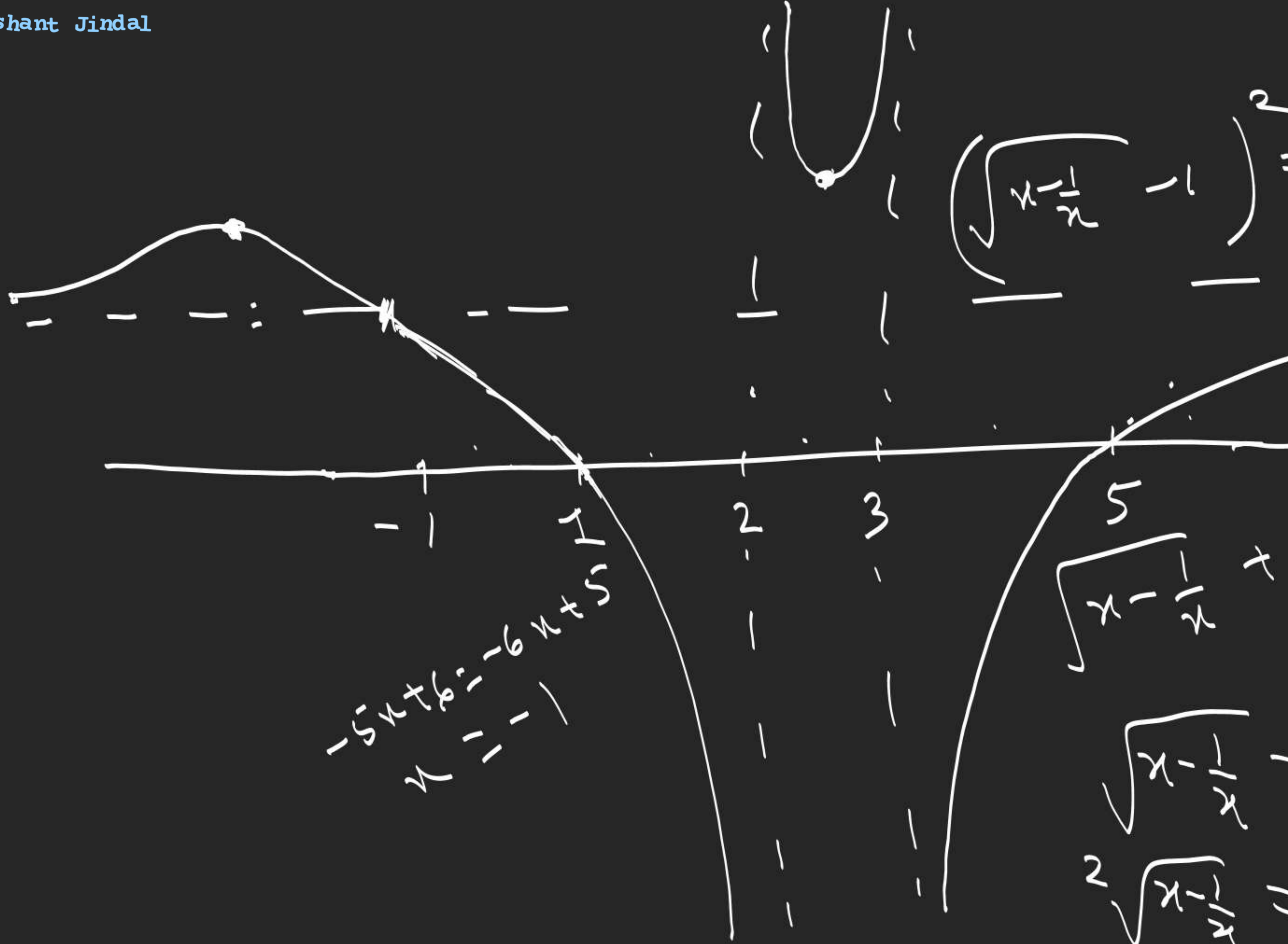
$$b^2+c^2-2bc < a^2$$

$$c^2+a^2-2ca < b^2$$

$$2\underbrace{\sum ab}_{>0} > (3\lambda-2)\underbrace{\sum ab}_{>0}$$

$$\sum a^2 < 2\sum ab$$

$$2 > 3\lambda-2.$$



$-5x+6=-6x+5$

$x=-1$

$$\left(\sqrt{x-\frac{1}{x}}-1\right)^2=0$$

$$x-\frac{1}{x}=1$$

$x>0$

$$\sqrt{x-\frac{1}{x}}+\sqrt{1-\frac{1}{x}}=x$$

$$\sqrt{x-\frac{1}{x}}-\sqrt{1-\frac{1}{x}}=\frac{x-1}{x}$$

$$2\sqrt{x-\frac{1}{x}}=x+1-\frac{1}{x}$$

Nishant Jindal

$$\left(-\frac{2}{3}, -\frac{1}{3}\right) \cup (1, 5)$$

$$(x+5)(x-2a) \le 0$$

$2a$

$$\boxed{5 \le 2a}$$

Nishant Jindal

1. If sum of two numbers a, b $(a>b)$ is n $(n>2)$ times their G.M., then show that

$$a:b = n+\sqrt{n^2-4} : n-\sqrt{n^2-4}.$$

$$a+b = n\sqrt{ab}$$

$$\sqrt{\frac{a}{b}} + \sqrt{\frac{b}{a}} = n$$

$$t + \frac{1}{t} = n$$

$$t^2 - nt + 1 = 0$$

$$t = \frac{n \pm \sqrt{n^2-4}}{2}$$

$$\Rightarrow \frac{a}{b} = \frac{\left(n \pm \sqrt{n^2-4}\right)^2}{4} = \frac{\left(n \pm \sqrt{n^2-4}\right)^2}{\left(n+\sqrt{n^2-4}\right)\left(n-\sqrt{n^2-4}\right)}$$

reject

$$\frac{n-\sqrt{n^2-4}}{n+\sqrt{n^2-4}} \text{ or } \boxed{\frac{n+\sqrt{n^2-4}}{n-\sqrt{n^2-4}}}$$

Nishant Jindal

Inequality

If $x_1, x_2, x_3, \ldots, x_n > 0$, then

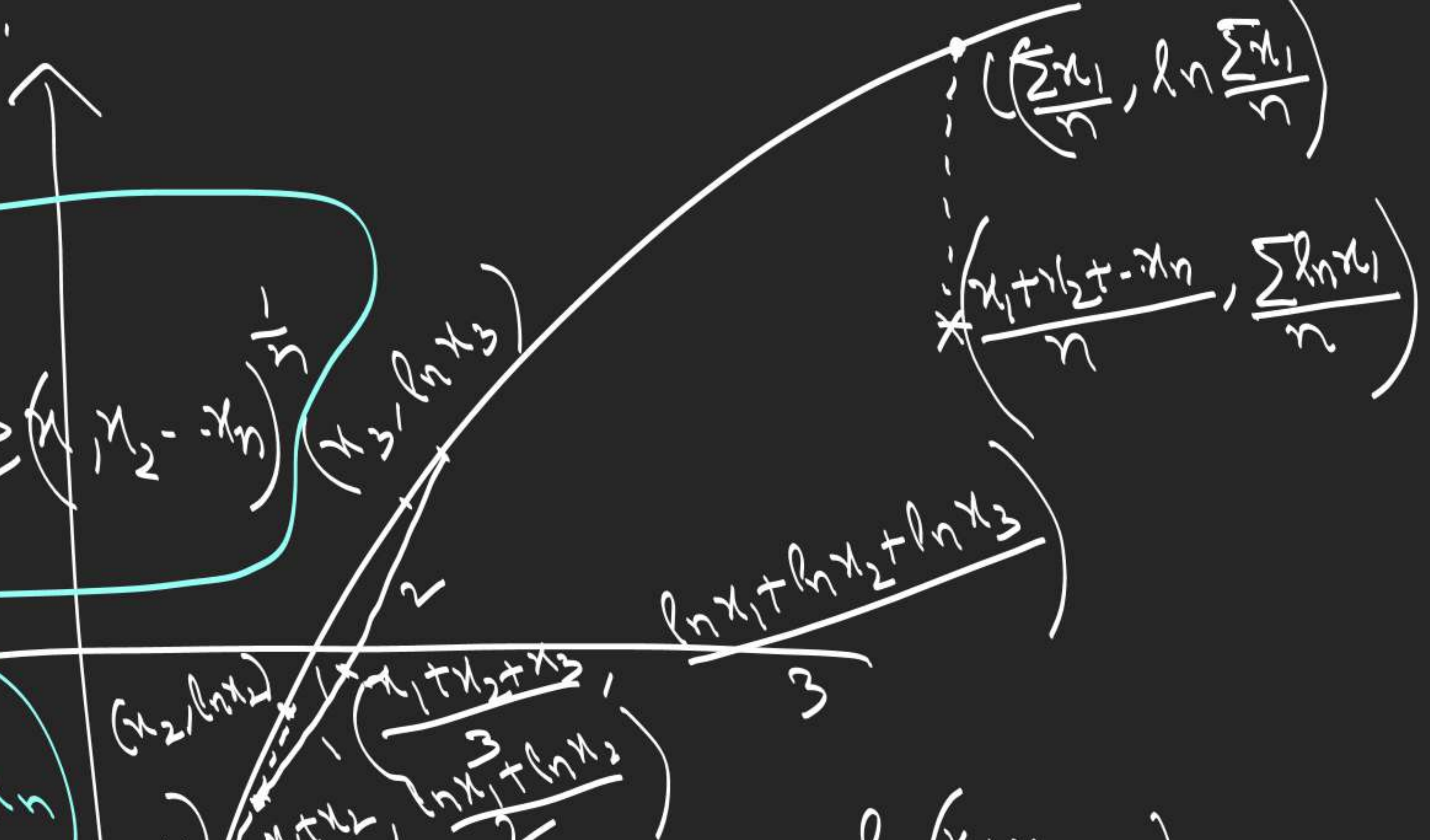
$$\frac{x_1+x_2+\cdots+x_n}{n} \geq \left(x_1 x_2 x_3 \cdots x_n\right)^{\frac{1}{n}}$$

& equality holds if $x_1 = x_2 = x_3 = \ldots = x_n$

$A.M \geq G.M.$

Nishant Jindal

$f(x) = \ln x$



$$\boxed{\frac{x_1+x_2+\ldots+x_n}{n} \geq (x_1 x_2 \ldots x_n)^{\frac{1}{n}}}$$

$$\boxed{AM = GM \text{ if } x_1 = x_2 = \ldots = x_n}$$

$$\ln\left(\frac{x_1+x_2+\ldots x_n}{n}\right) \geq \frac{1}{n}\ln(x_1 x_2 \ldots x_n)$$

$$\Rightarrow \ln\left(\frac{x_1+x_2+\ldots x_n}{n}\right) \geq \ln(x_1 x_2 \ldots x_n)^{\frac{1}{n}}$$

Nishant Jindal

1. If $x>0, y>0, z>0$, then P.T.

$$(x+y)(y+z)(z+x) \geq 8xyz.$$

AM = GM if $\frac{a}{2} = \frac{b}{3} = \frac{c}{2}$

$6 > 2$

$5 > 1$

$6\times5 > 2\times1$

$$\frac{x+y}{2} \geq \sqrt{xy}$$
$$\frac{y+z}{2} \geq \sqrt{yz}$$
$$\frac{z+x}{2} \geq \sqrt{zx}$$

$$\Rightarrow \frac{(x+y)(y+z)(z+x)}{8} \geq xyz.$$

2. If $a+b+c=3$ and $a, b, c > 0$, then P.T. max value of $a^2b^3c^2 = ?$

$$a^2 b^3 c^2 \leq \left(\frac{3}{7}\right)^7 \times 2^2 \times 3^3 \times 2^2$$

$$\frac{a}{3} = \frac{2a}{3} = \frac{b}{3} = \frac{c}{2}$$

$$\frac{3^{10}\,2^4}{7^7}$$

$$\frac{\frac{a}{3}+\frac{2a}{3}+\frac{b}{3}+\frac{c}{2}}{7} \geq a^2 b^3 c^2 \leq \frac{3^{10}\,2^4}{7^7}$$

$$a^2b^3c^2 \leq$$

$$\frac{3}{7} = \frac{a+b+c}{7} = \frac{\frac{a}{2}+\frac{a}{2}+\frac{b}{3}+\frac{b}{3}+\frac{b}{3}+\frac{c}{2}+\frac{c}{2}}{7} \geq \left(\frac{a}{2}\right)^2\left(\frac{b}{3}\right)^3\left(\frac{c}{2}\right)^2$$

$\frac{1}{7}$

Nishant Jindal

$\boxed{\text{rem. Ex-4}}$

Hall & Knight

Ex-IV(a)

19, 21, 20, 22, 23, 24

IV(b) → 11, 12, 13, 14, 11-24

$\sin\theta \le 2$

max value of $\sin\theta = 1$

min of $x^2 + \frac{4}{x^2}$

$$\frac{x^2 + \frac{4}{x^2}}{2} \geq \sqrt{(x^2)\frac{4}{x^2}} = 2$$

$$x^2 + \frac{4}{x^2} \geq 4.$$

min of $x^2 + \frac{4}{x^2} = 4$ if $x^2 = \frac{4}{x^2} \Rightarrow x^2 = 2$

$\boxed{x = \pm\sqrt{2}}$

$x + \frac{1}{x}$, $x > 0$,

$$\frac{x + \frac{1}{x}}{2} \geq \sqrt{x \times \frac{1}{x}} = 1$$

$$\boxed{x + \frac{1}{x} \geq 2}$$

$\left(x + \frac{1}{x}\right)_{min} = 2$

if $AM = GM$

$x = \frac{1}{x} \Rightarrow x = 1$