Nishant Jindal

Energy Method for Calculating elongation in Spring

- Body moved very slowly.
- System is released from rest when spring at its Natural length.

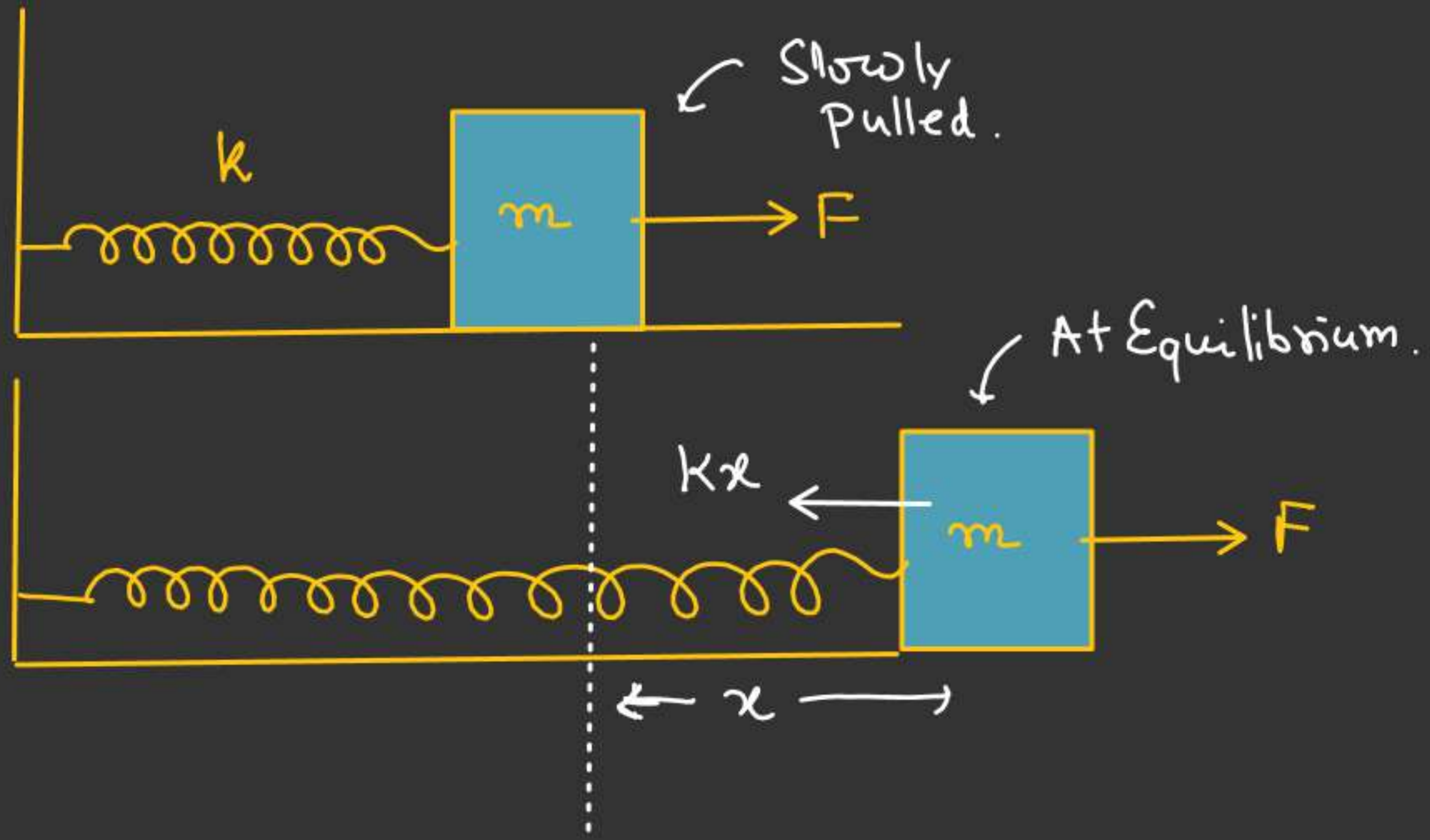
At Equilibrium

$$F = Kx$$

$$x = \left(\frac{F}{K}\right)$$

$$U = \frac{1}{2}Kx^2 = \frac{1}{2}K\left(\frac{F}{K}\right)^2$$

$$\boxed{U = \frac{F^2}{2K}}$$



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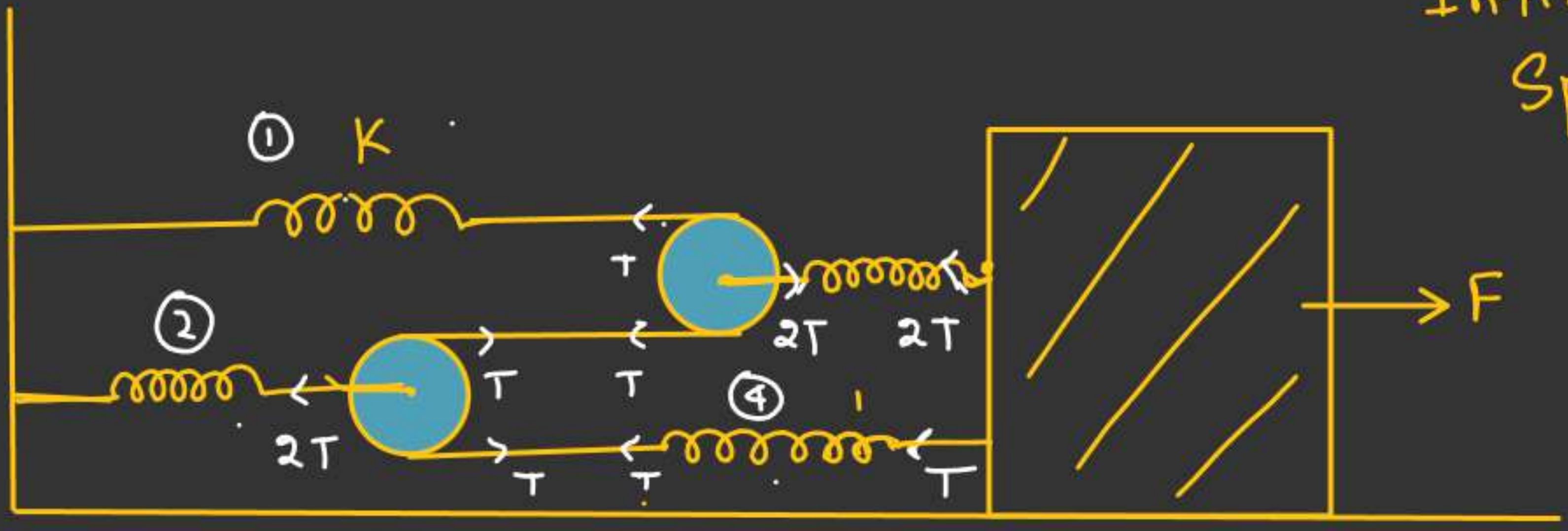
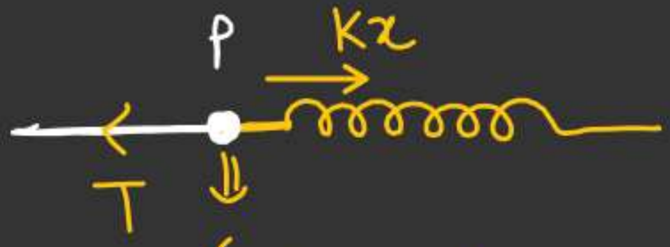


Intially all the Springs at its natural length. Block pulled by Constant force F very Slowly.

string, Spring & pully massless.

Find displaced of block when it is in equilibrium position.

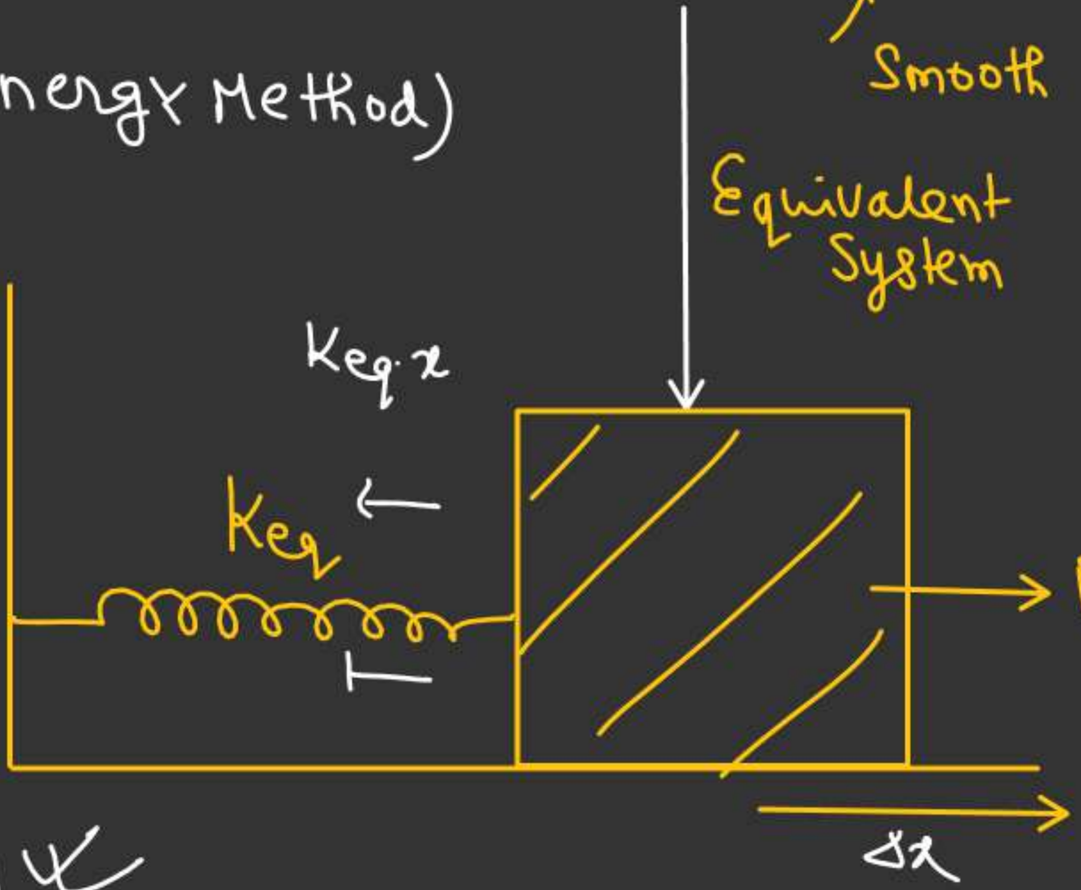
Unique Approach (Energy Method)



$(T = kx)$

$\left(x = \frac{T}{k}\right)$

$$\left(\begin{matrix} x_1 = T/K & x_3 = \frac{2T}{K} \\ x_2 = \frac{2T}{K} & x_4 = \frac{T}{K} \end{matrix}\right)$$



$$\left[\begin{matrix} F = K_{eq}\Delta x \\ \Delta x = \left(\frac{F}{K_{eq}}\right) \end{matrix}\right]$$

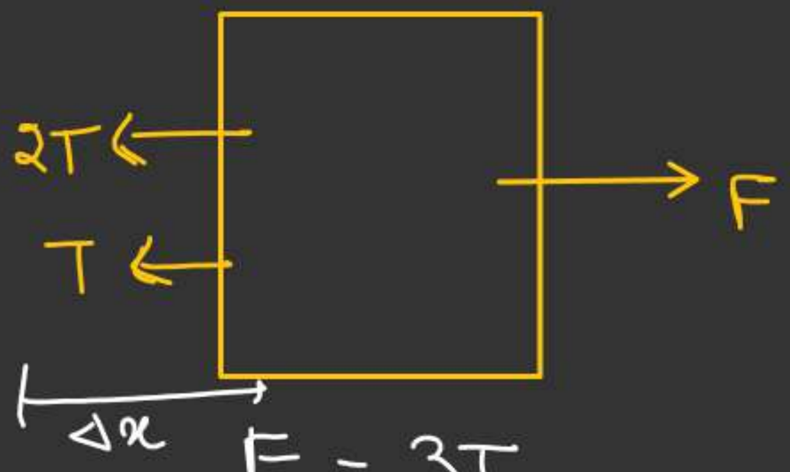
$$U_T = U_1 + U_2 + U_3 + U_4$$

$$\Downarrow$$

$$\frac{F^2}{2K_{eq}} = \frac{1}{2}kx_1^2 + \frac{1}{2}kx_2^2 + \frac{1}{2}kx_3^2 + \frac{1}{2}kx_4^2$$

$$\frac{F^2}{2K_{eq}} = \frac{1}{2}k\left[\left(\frac{T}{k}\right)^2 + \left(\frac{2T}{k}\right)^2 + \left(\frac{2T}{k}\right)^2 + \left(\frac{T}{k}\right)^2\right]$$

At Equilibrium.



$$F = 3T$$

$$T = \left(\frac{F}{3}\right)$$

$$\frac{F^2}{2K_{eq}} = \frac{10T^2}{2K} = \left(\frac{5T^2}{k}\right)$$

$$\frac{F^2}{2K_{eq}} = \frac{5}{k}\left(\frac{F}{3}\right)^2 \Rightarrow \frac{1}{2K_{eq}} = \frac{5}{9k} \Rightarrow k_{eq} = \left(\frac{9k}{10}\right)$$

$$\Delta x = \left(\frac{F}{K_{eq}}\right) = \left(\frac{10F}{9k}\right) \text{ Ans}$$

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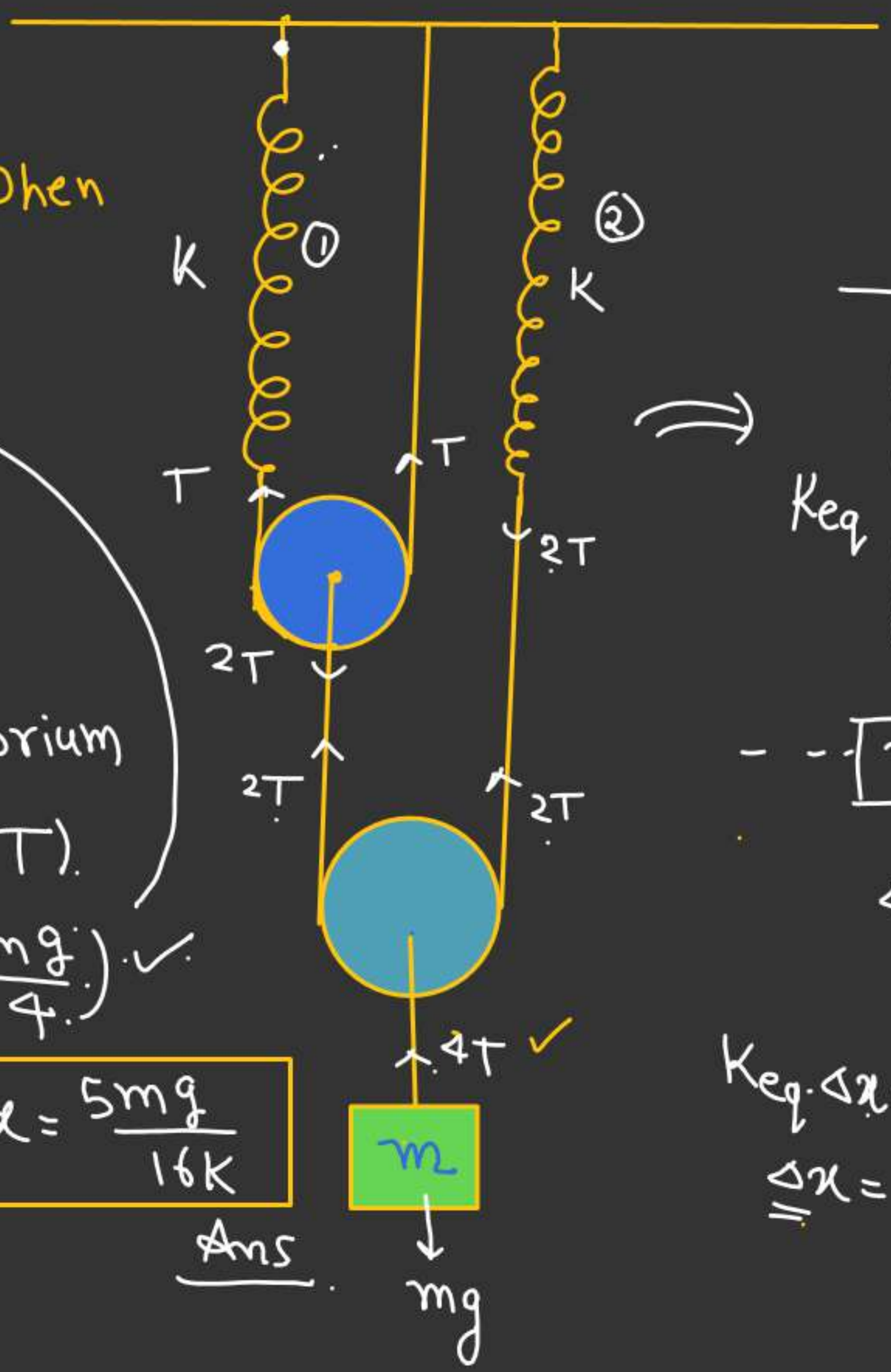
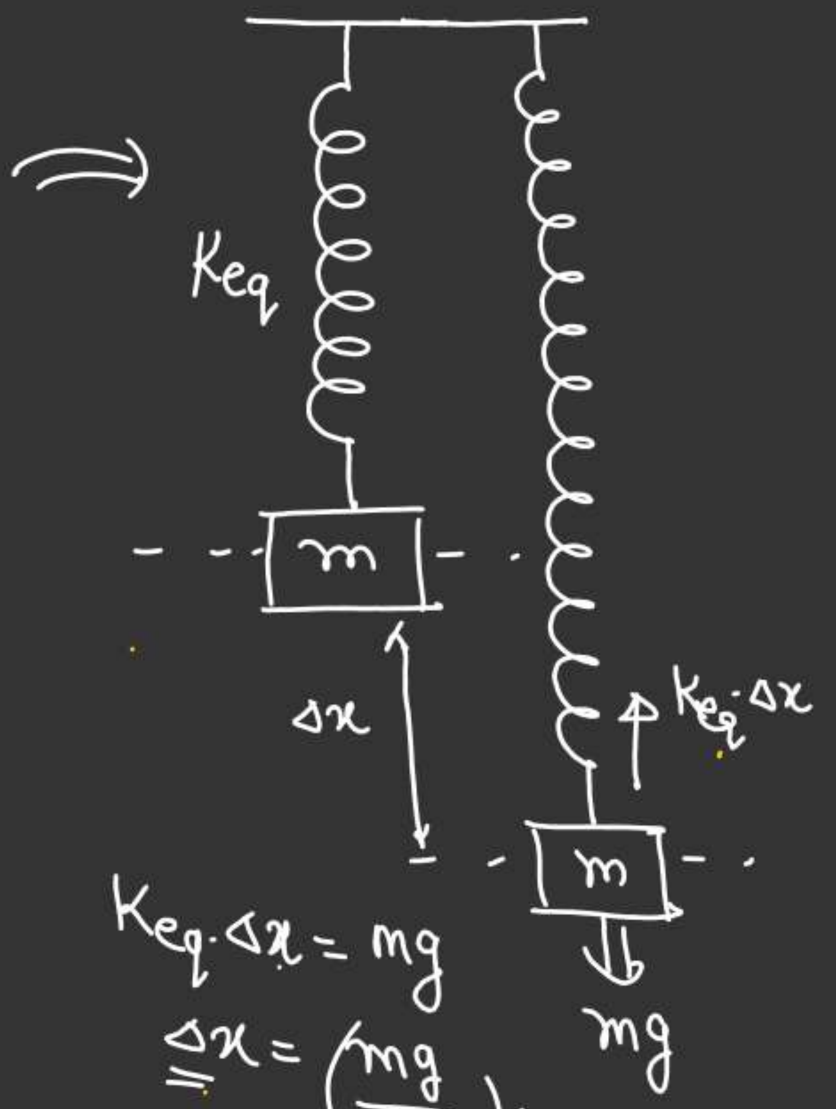
Block is released when Spring at its natural length. Find the displacement of block when it is in equilibrium position.





$$\text{Keq}\cdot\Delta x = mg$$

$$\Delta x = \left(\frac{mg}{K_{eq}}\right)$$

$\Rightarrow$ Elongation in Spring $\left(\frac{T^2}{2k}\right)$

$$x_1 = \frac{T}{K}$$

$$x_2 = \frac{(2T)}{K}$$

$$(U_T) = U_1 + U_2$$

$\Downarrow$

Spring P.E

At Equilibrium

$$mg = (4T)$$

$$T = \left(\frac{mg}{4}\right)$$

$$\frac{1}{2}K_{eq}\left(\frac{mg}{K_{eq}}\right)^2 = \frac{1}{2}K\left(\frac{T}{K}\right)^2 + \frac{1}{2}K\left(\frac{2T}{K}\right)^2$$

$$\frac{m^2g^2}{2K_{eq}} = \frac{T^2}{2K} + \frac{4T^2}{2K} = \frac{5T^2}{2K}$$

$$\frac{\cancel{m^2g^2}}{\cancel{2}K_{eq}} = \frac{5}{\cancel{2}K} \times \frac{\cancel{m^2g^2}}{16} \Rightarrow K_{eq} = \left(\frac{16K}{5}\right)$$

$$\boxed{\Delta x = \frac{5mg}{16K}}$$

Ans

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Case of Uniform Chain

$\frac{\ell}{3}$ part of the chain is hanging. System is released from the position shown in the fig. Find the K.E of the chain when it just about to leave the table.

Energy Conservation

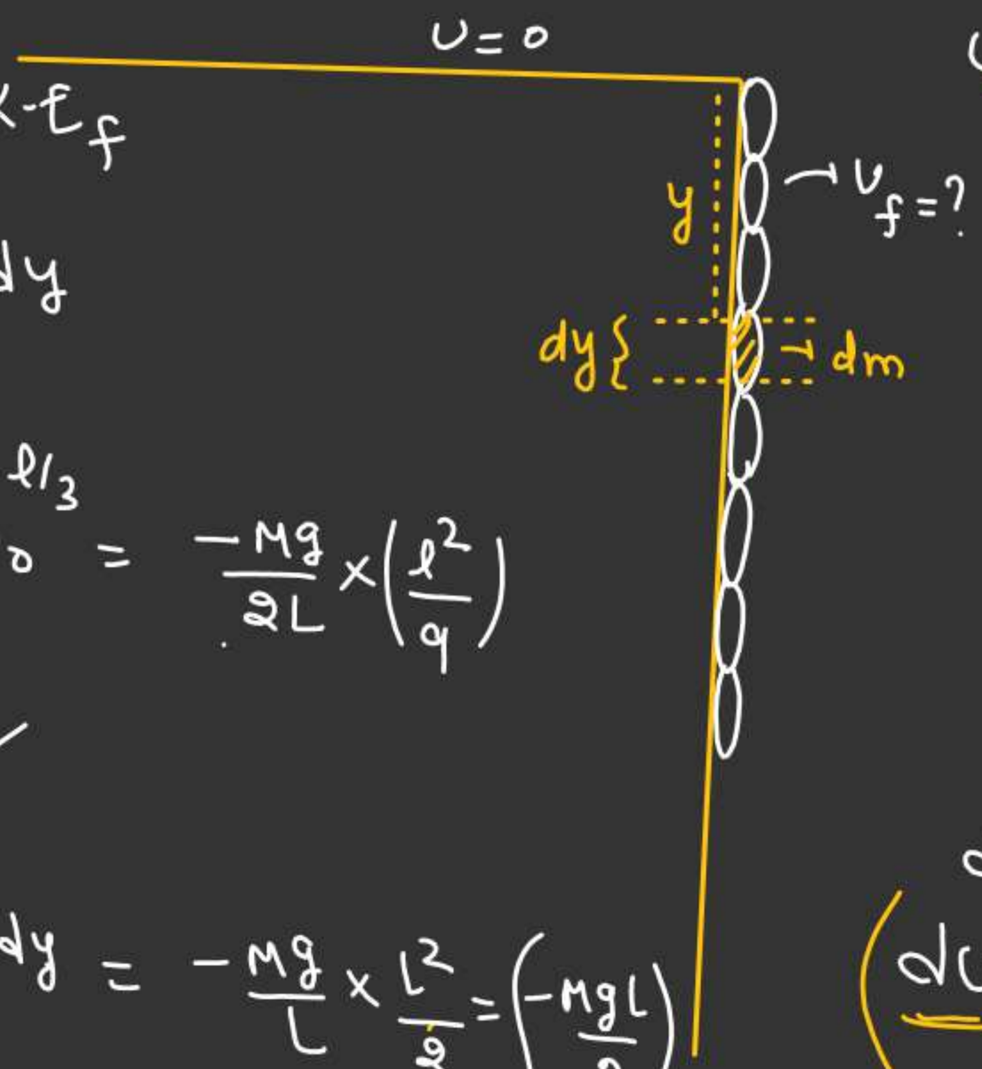
$$U_i + \cancel{K.E_i}^{\,0} = U_f + K.E_f$$

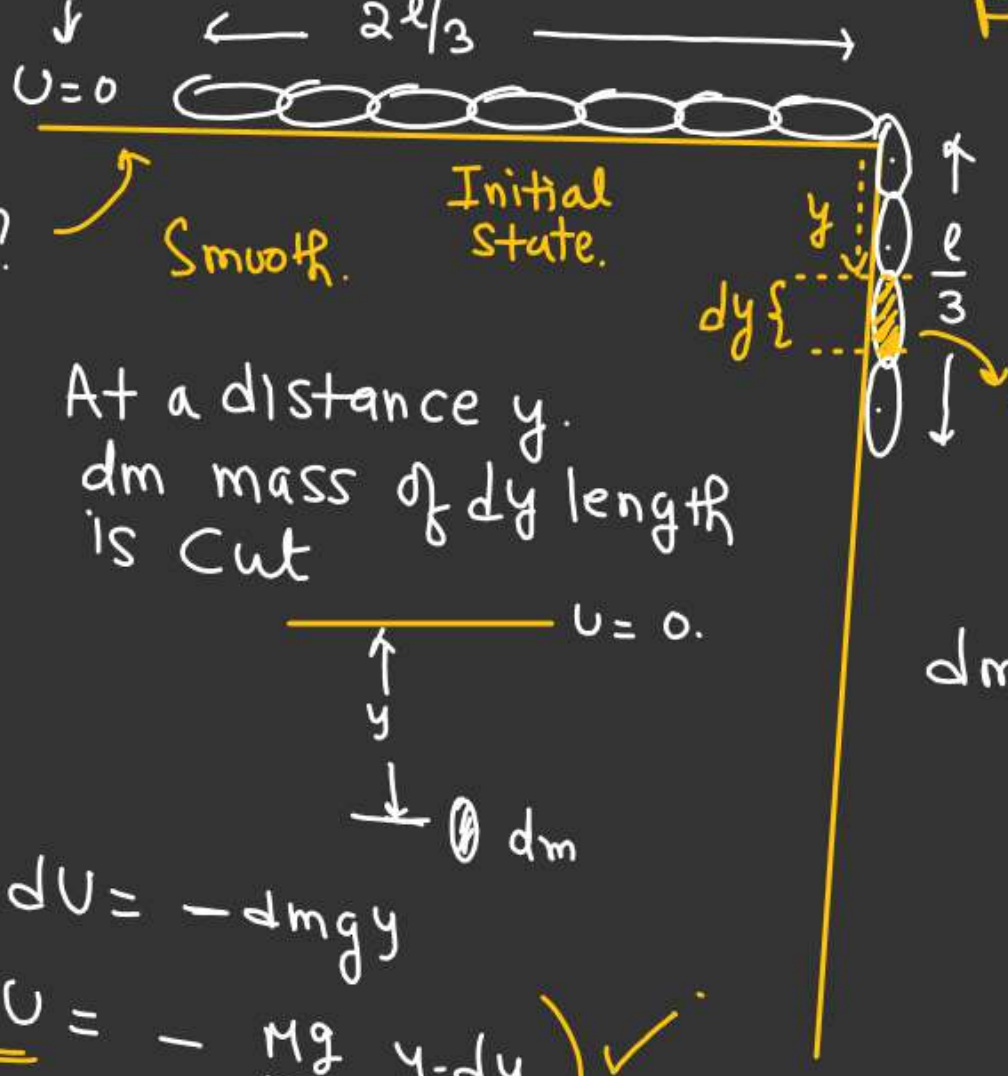
$$U_i = -\frac{Mg}{L}\int_0^{\ell/3} y\,dy$$

$$U_i = -\frac{Mg}{L}\left[\frac{y^2}{2}\right]_0^{\ell/3} = -\frac{Mg}{2L}\times\left(\frac{\ell^2}{9}\right)$$

$$U_i = -\frac{Mg\ell}{18} \checkmark$$

$$U_f = -\frac{Mg}{L}\int_0^{L} y\,dy = -\frac{Mg}{L}\times\frac{L^2}{2} = \left(-\frac{MgL}{2}\right)$$





At a distance y. dm mass of dy length is Cut

$$dm = \left(\frac{M}{L}dy\right)$$

$$dU = -dm\,g\,y$$

$$\left(dU = -\frac{Mg}{L}\,y\,dy\right) \checkmark$$

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Energy Conservation.

$$U_i + \cancel{K.E_i}^{\,0} = U_f + K.E_f$$

$$K.E_f = U_i - U_f$$

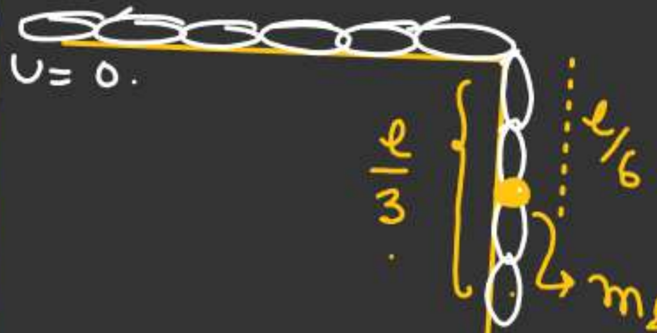
$$= -\frac{Mgl}{18} - \left(-\frac{MgL}{2}\right)$$

$$= -\frac{MgL}{18} + \frac{MgL}{2}$$

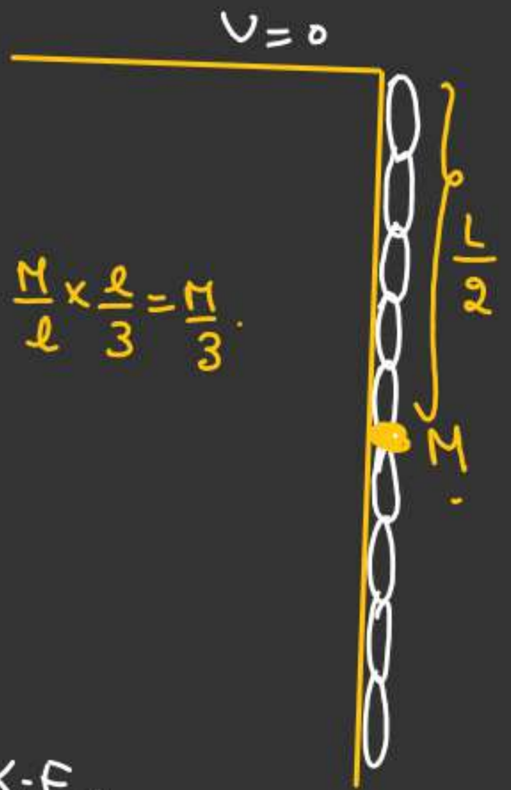
$$K.E_f = \frac{8MgL}{18} = \left(\frac{4MgL}{9}\right) \checkmark$$

TRICK :→ (By COM)

↳ For a uniform Rod or Chain the whole mass is assumed to be concentrated at its Mid-point.



$m_{l/3} = \frac{M}{l} \times \frac{l}{3} = \frac{M}{3}$.



$$U_i + \cancel{K.E_i}^{\,0} = U_f + K.E_f$$

$$\Downarrow$$

$$-\frac{M}{3}g\frac{l}{6} = \left(-\frac{MgL}{2}\right) + K.E_f$$

$$K.E_f = \left(\frac{4MgL}{9}\right) \checkmark$$

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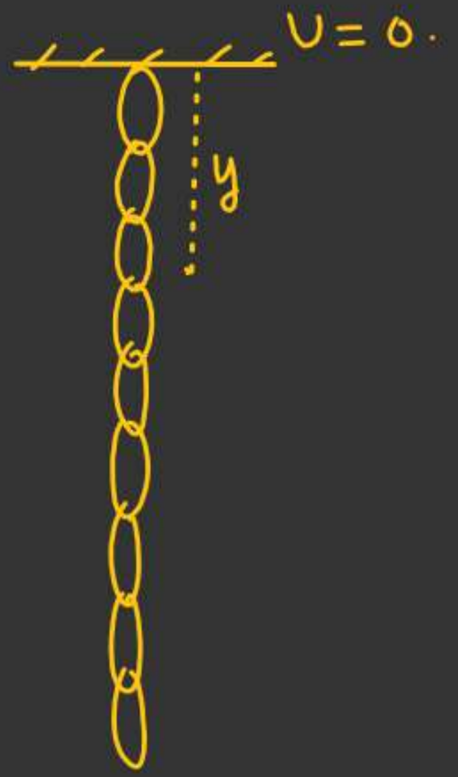
H.W

If Chain is non-Uniform its linear mass density is $\lambda = \lambda_0 y$. Find it P.E.



H.W

P.E of Chain = ??