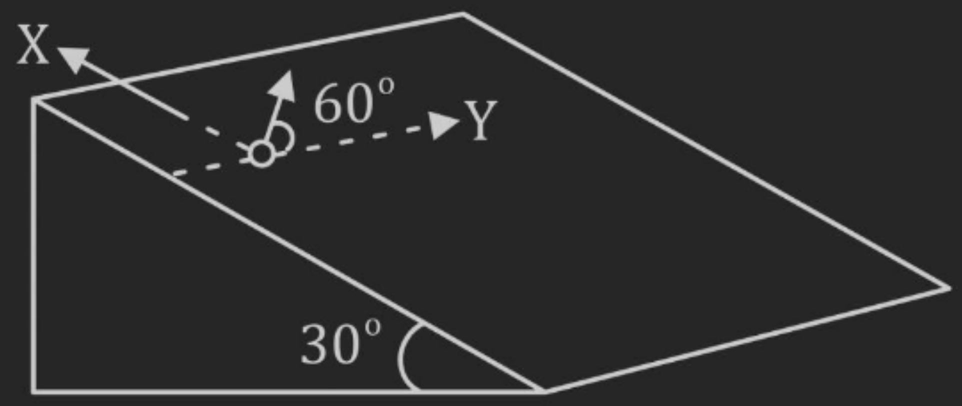
Nishant Jindal

Projectile Motion

H.W.

Q. A small sphere is projected with a velocity of 3 ms^{-1} in a direction 60° from the horizontal y-axis, on the smooth inclined plane (Fig.) The motion of sphere takes place in the x – y plane. Calculate the magnitude v of its velocity after 2 s.



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H.W I.I.T Adv

Projectile Motion

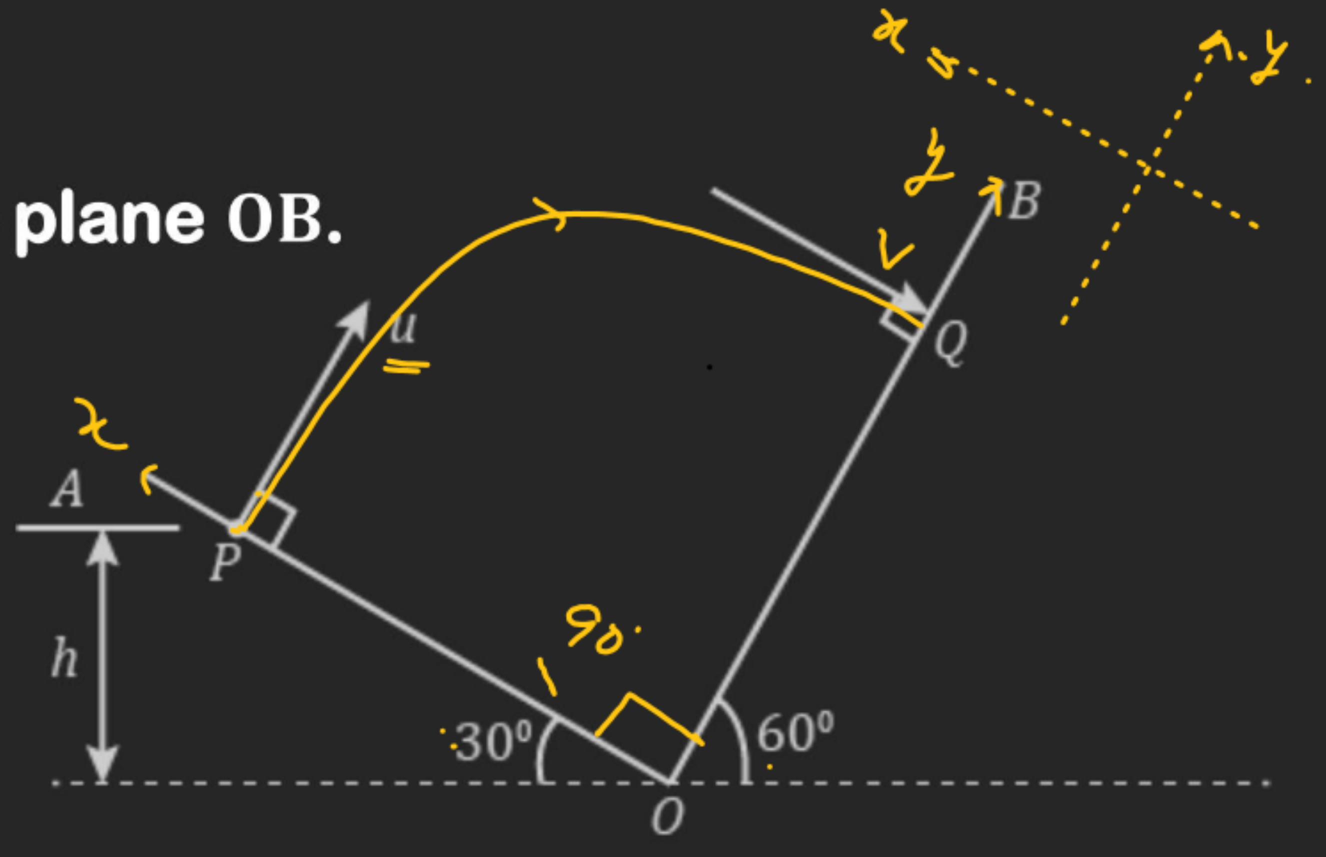
Q. Two inclined planes OA and OB having inclinations 30° and 60° with the horizontal respectively intersect each other at O, as shown in figure. A particle is projected from point P with velocity $u = 10\sqrt{3}\,\frac{m}{s}$ along a direction perpendicular to plane OA. If the particle strikes plane OB perpendicular at Q. Calculate

(A) time of flight

(B) velocity with which the particle strikes the plane OB.

(C) height h of point P from point O.

(D) distance PQ. (Take $g = 10\ m/s^2$)



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In x-direction

$v_x = \cancel{u_x}^{\,0} + a_x t$

$\Downarrow$

$v = (g\cos 60^\circ)t$

$v = 10 \times \frac{1}{2} \times t$

$v = 5t$ — ①

In - y-direction

$v_y = u_y - a_y t$

$\Downarrow$

$0 = u - (g\sin 60^\circ)t$

$0 = u - 10 \times \frac{\sqrt{3}}{2} t$

$0 = u - 5\sqrt{3}t$

$t = \left(\frac{u}{5\sqrt{3}}\sec\right) = \frac{10\sqrt{3}}{5\sqrt{3}} = 2\sec$

$v = 5 \times 2 = 10\,m/s$

$\Delta POQ \rightarrow PQ^2 = OP^2 + OQ^2$

$OQ = Y$.

$Y = ut - \frac{1}{2} \times g\sin 60^\circ \times t^2$

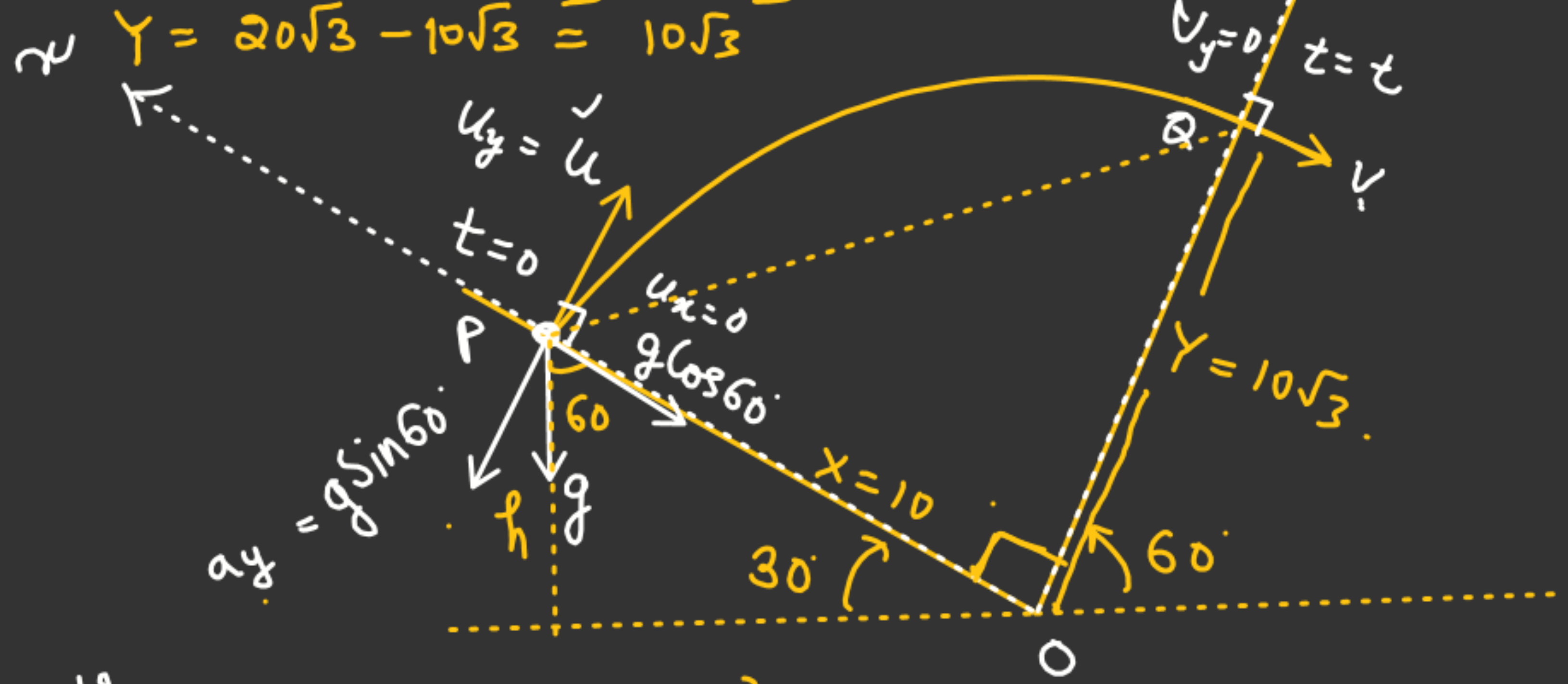
$Y = (10\sqrt{3}) \times 2 - \frac{1}{2} \times 10 \times \frac{\sqrt{3}}{2} \times (2)^2$

$Y = 20\sqrt{3} - 10\sqrt{3} = 10\sqrt{3}$

$PQ^2 = (10)^2 + (10\sqrt{3})^2$

$PQ = \sqrt{(10)^2 + (10\sqrt{3})^2}$

$= 10 \times 2 = 20\,m$



$OP = X$

$X = \frac{1}{2} a_x t^2 = \frac{1}{2} \times 10 \times \frac{1}{2} \times (2)^2$

$X = 10\,m$.

$\sin 30^\circ = \frac{h}{OP} \Rightarrow h = OP\sin 30^\circ = x\sin 30^\circ$

$h = 10 \times \frac{1}{2} = 5\,m$.

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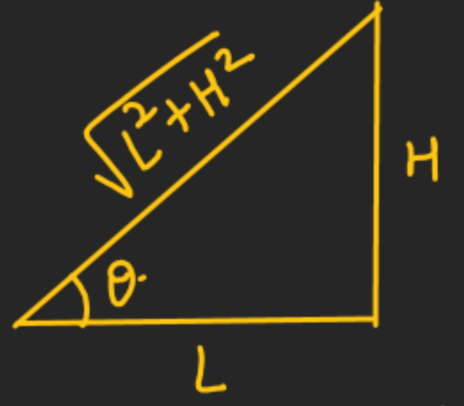
this problem. Also ignore the size of the cannons relative to L and H. The two groups of gunners aim the cannons directly at each other. They fire at each other simultaneously, with equal muzzle speed v_0. What is the value of v_0 for which the two cannon balls collide just as they hit the ground?

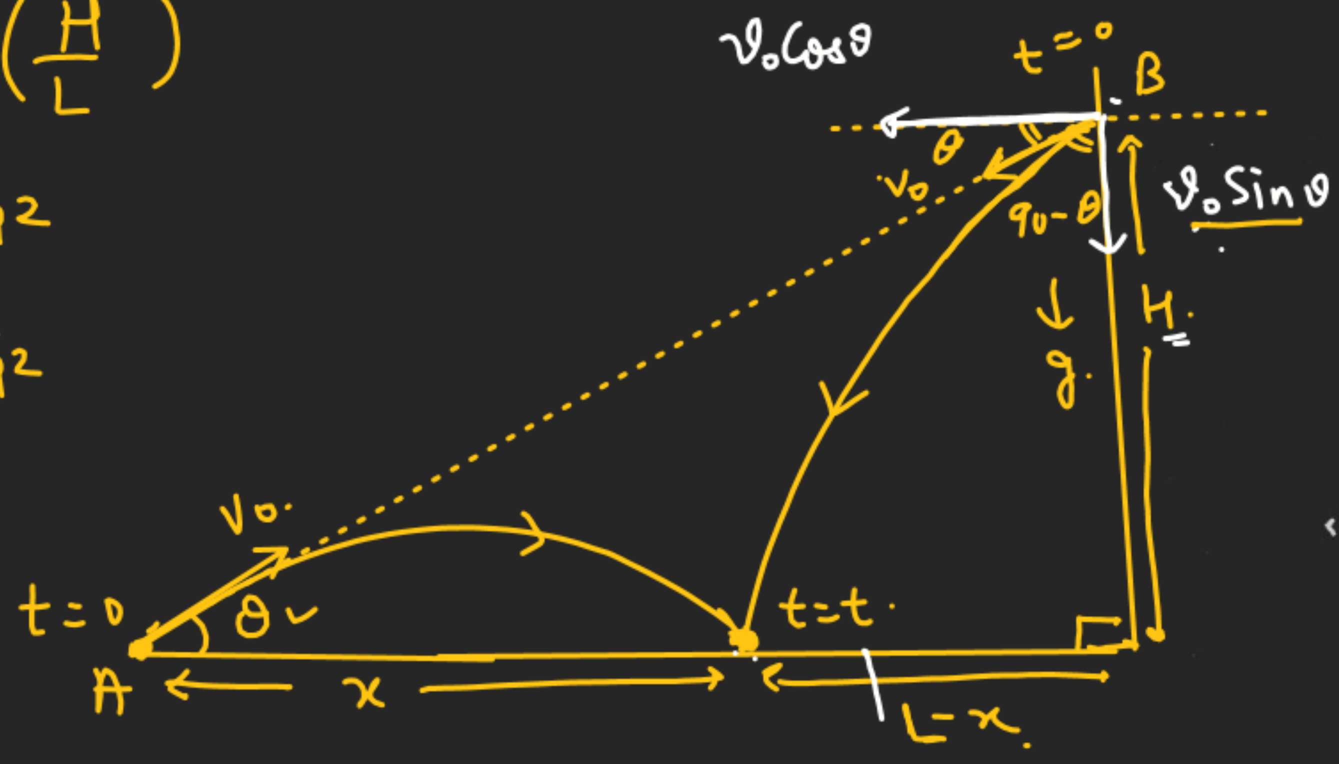
HW

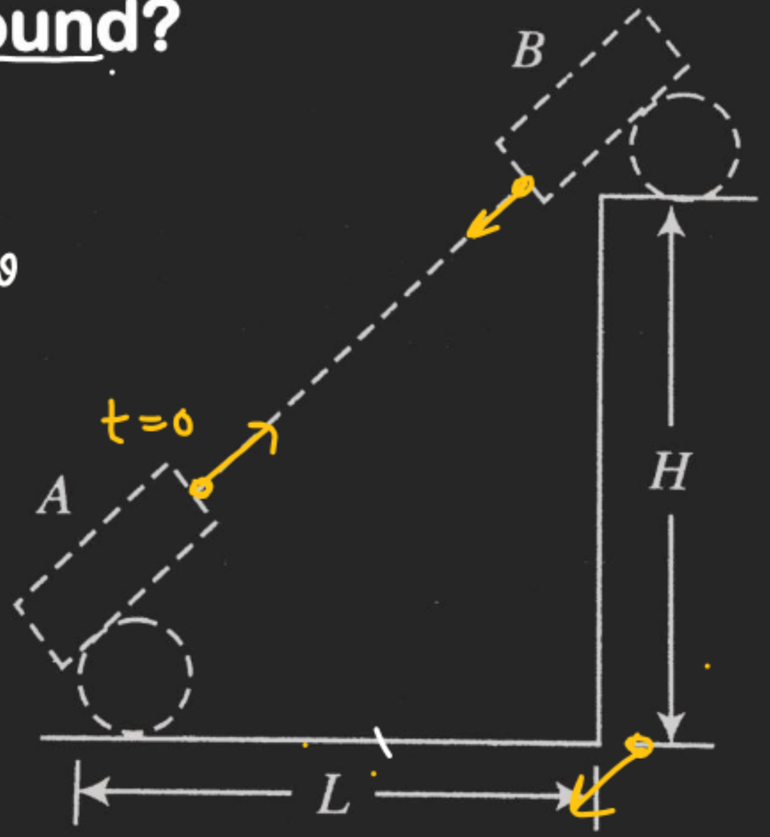


$$\tan\theta = \left(\frac{H}{L}\right)$$

$$\sin\theta = \frac{H}{\sqrt{L^2+H^2}}$$

$$\cos\theta = \frac{L}{\sqrt{L^2+H^2}}$$





For projectile A.

$$(v_0\cos\theta)\,t = x.$$

For B

$$(v_0\cos\theta)\times t = L - x. \Rightarrow L - x = x \Rightarrow x = \frac{L}{2}.$$

$$t = \left(\frac{2v_0\sin\theta}{g}\right)$$

For particle B

In y direction →

$$H = (v_0\sin\theta)\,t + \frac{1}{2} g t^2$$

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$$H = (V_0 \sin\theta)\left(\frac{2V_0 \sin\theta}{g}\right) + \frac{1}{2} g \left(\frac{2V_0 \sin\theta}{g}\right)^2.$$

$$H = \frac{2V_0^2 \sin^2\theta}{g} + \frac{2V_0^2 \sin^2\theta}{g}.$$

$$H = \frac{4V_0^2 \sin^2\theta}{g} \qquad \sin\theta = \left(\frac{H}{\sqrt{H^2+L^2}}\right)$$

$$V_0^2 = \frac{gH}{4\sin^2\theta}$$

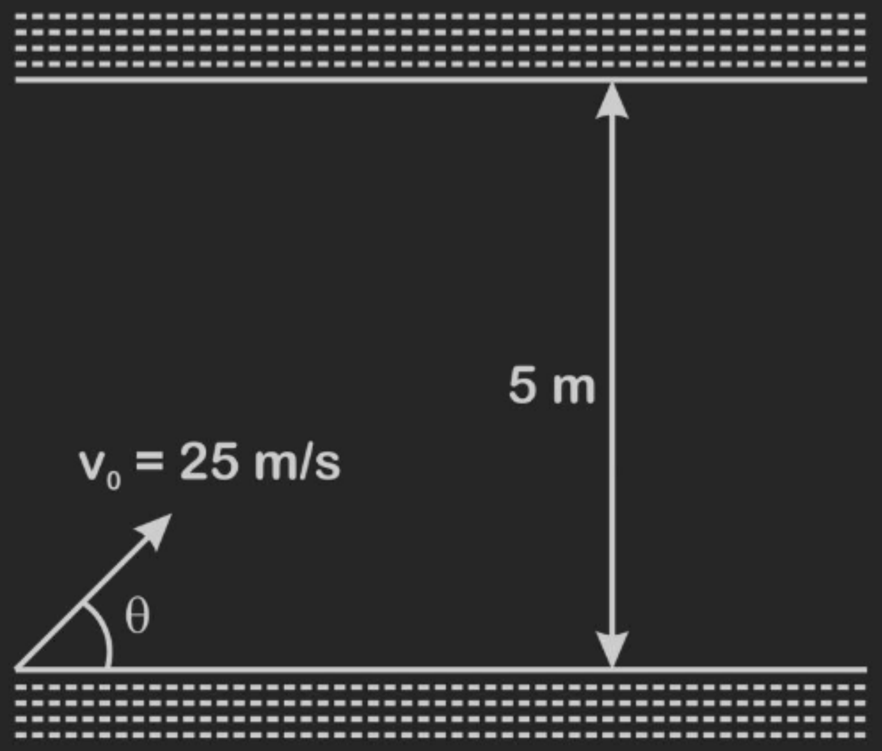
$$V_0^2 = \frac{g\cancel{H}(H^2+L^2)}{4\,\cancel{H^2}} = \left(\frac{g(H^2+L^2)}{4H}\right)$$

$$V_0 = \sqrt{\frac{g(H^2+L^2)}{4H}}.$$

Projectile motion

H.W.

Q. A projectile is launched with a speed $v_B = 25\ \mathrm{m/s}$ from the floor of a $5\ \mathrm{m}$ high tunnel as shown in figure. Determine the maximum horizontal range R of the projectile and the corresponding launch angle θ.



Projectile motion

Q. Two particles are projected simultaneously from the level ground as shown in figure. They may collide after a time :

(a) $\dfrac{x\sin\theta_2}{u_1}$

(b) $\dfrac{x\cos\theta_2}{u_2}$

(c) $\dfrac{x\sin\theta_2}{u_1\sin(\theta_2-\theta_1)}$

(d) $\cancel{\dfrac{x\sin\theta_1}{u_2\sin(\theta_2-\theta_1)}}$

Sol^n

For particle ①

$$h = (u_1\sin\theta_1)t - \frac{1}{2}gt^2$$

For particle-②

$$h = (u_2\sin\theta_2)t - \frac{1}{2}gt^2$$

$$\boxed{u_1\sin\theta_1 = u_2\sin\theta_2}$$

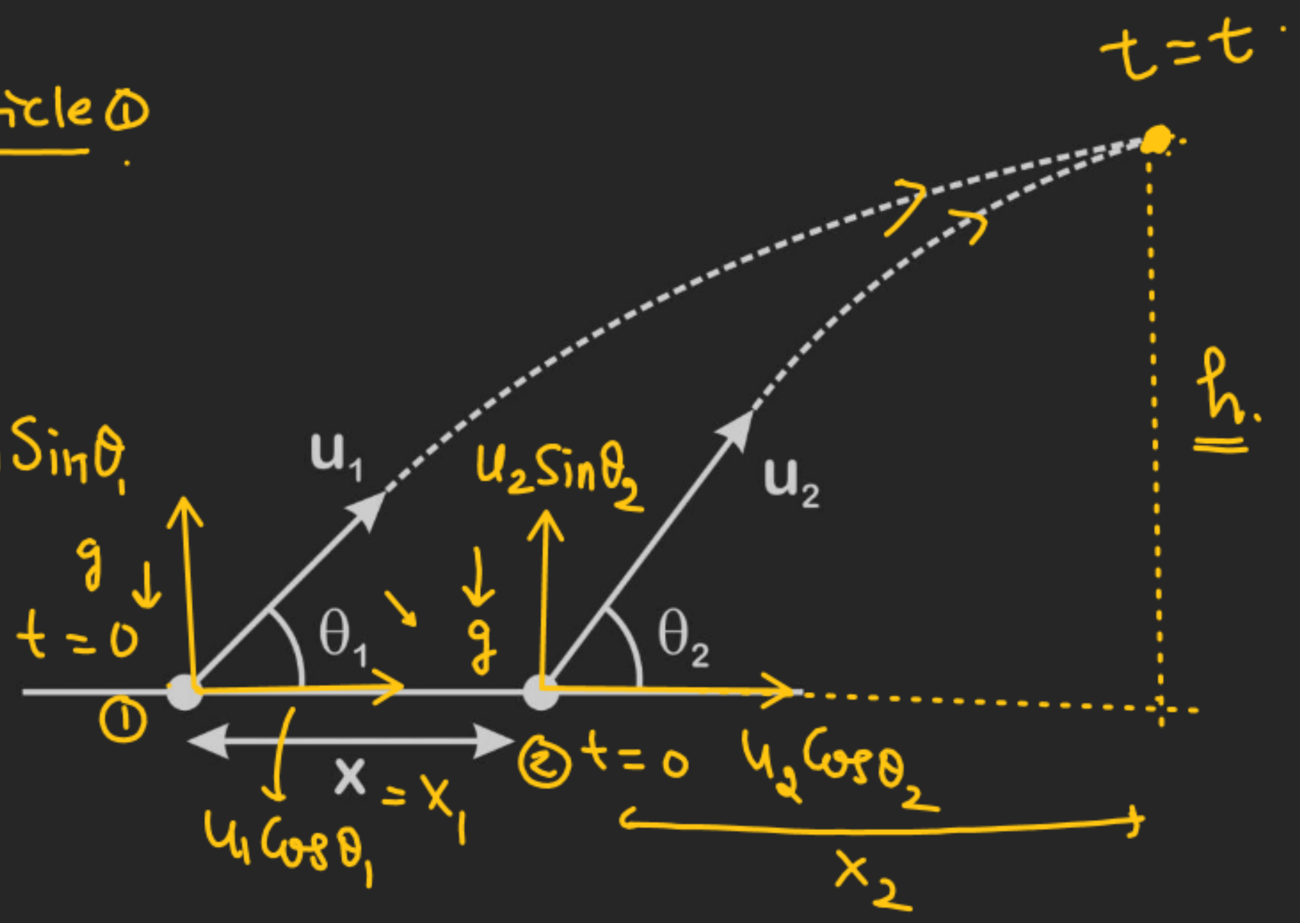
In horizontal direction

① $\to x + x_2 = (u_1\cos\theta_1)t$

② $\to x_2 = (u_2\cos\theta_2)t$

$\Rightarrow x + (u_2\cos\theta_2)t = (u_1\cos\theta_1)t$

$$t = \left(\frac{x}{u_1\cos\theta_1 - u_2\cos\theta_2}\right)$$



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$$(u_1 \sin\theta_1 = u_2 \sin\theta_2) \Rightarrow u_1 = \frac{u_2 \sin\theta_2}{\sin\theta_1}$$

$$t = \frac{x}{u_1\cos\theta_1 - u_2\cos\theta_2}$$

$$t = \frac{x}{\frac{u_2 \sin\theta_2 \times \cos\theta_1}{\sin\theta_1} - u_2\cos\theta_2}$$

$$t = \frac{x\sin\theta_1}{u_2[\sin\theta_2 \cdot \cos\theta_1 - \cos\theta_2 \cdot \sin\theta_1]}$$

$$t = \frac{x\sin\theta_1}{u_2 \sin(\theta_2 - \theta_1)}$$

$$u_2 = \frac{u_1 \sin\theta_1}{\sin\theta_2}$$

~~H.W~~

Sheet :- Ex-1 :- Complete ✓ → Discussion on Sat
Ex-2 → Complete

Projectile motion

??

Q. A particle is projected from the ground. If the equation of the trajectory is $\left(y = x - \frac{x^2}{2}\right)$, then the time of flight is:

(a) $\frac{2}{\sqrt{g}}$

(b) $\frac{3}{\sqrt{g}}$

(c) $\frac{9}{\sqrt{g}}$

(d) $\sqrt{\frac{2}{g}}$

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Projectile motion

$\alpha \to \alpha$

Q. A projectile moves from the ground such that its horizontal displacement is $x = Kt$ and vertical displacement is $y = Kt(1 - \alpha t)$, where K and α are constants and t is time. Find out total time of flight (T) and maximum height attained (Y_{max}) its

(a) $T = \alpha, Y_{max} = \frac{K}{2\alpha}$

(b) $T = \frac{1}{\alpha}, Y_{max} = \frac{2K}{\alpha}$

(c) $T = \frac{1}{\alpha}, Y_{max} = \frac{K}{6\alpha}$

(d) $T = \frac{1}{\alpha}, Y_{max} = \frac{K}{4\alpha}$

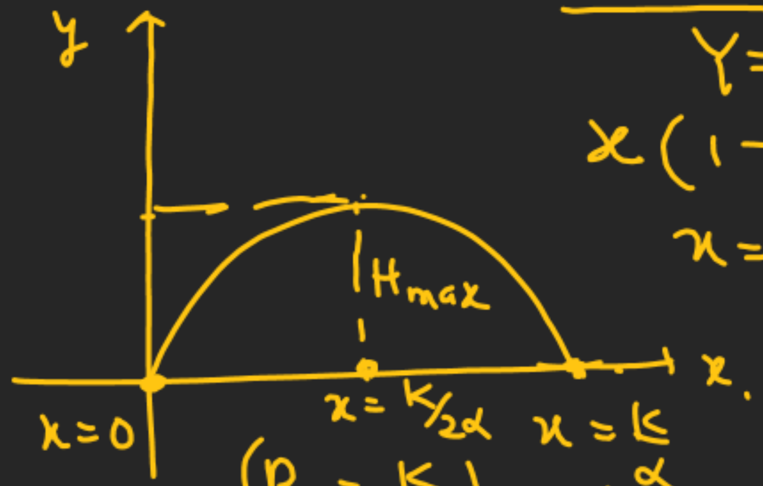
$X = Kt, \quad Y = Kt\cdot(1 - \alpha t)$

$t = \left(\frac{X}{K}\right)$

$Y = K\left(\frac{X}{K}\right)\left(1 - \alpha\left(\frac{X}{K}\right)\right)$

$Y = X\left(1 - \frac{\alpha}{K}X\right)$

$\boxed{Y = X - \frac{\alpha}{K}X^2}$



$\left(R = \frac{K}{\alpha}\right)$

$x = R, \quad t = T$

$T = \frac{R}{K} = \frac{K}{\alpha} \times \frac{1}{K} = \frac{1}{\alpha}$

Time of flight

$Y = 0.$

$x\left(1 - \frac{\alpha}{K}x\right) = 0.$

$x = 0, \quad x = \left(\frac{K}{\alpha}\right)$

At $x = \frac{K}{2\alpha}$

$y = H_{max}.$

$H_{max} = \frac{K}{2\alpha} - \frac{\alpha}{K}\left(\frac{K^2}{4\alpha^2}\right)$

$H_{max} = \frac{K}{2\alpha} - \frac{K}{4\alpha}$

$= \frac{K}{4\alpha}$

Projectile motion

Q. A particle is ejected from the tube at A with a velocity v at an angle θ with the vertical y-axis. A strong horizontal wind gives the particle a constant horizontal acceleration a in the x-direction. If the particle strikes the ground at a point directly under its released position and the downward y-acceleration is taken as g then

(a) $h = \dfrac{2v^2 \sin\theta \cos\theta}{a}$

(b) $h = \dfrac{2v^2 \sin\theta \cos\theta}{g}$

(c) $h = \dfrac{2v^2}{g} \sin\theta \left(\cos\theta + \dfrac{a}{g}\sin\theta\right)$

(d) $h = \dfrac{2v^2}{a} \sin\theta \left(\cos\theta + \dfrac{g}{a}\sin\theta\right)$ ✓

In x-direction

$0 = (v\sin\theta)t - \frac{1}{2}at^2$ — ①

In y direction

$h = (v\cos\theta)t + \frac{1}{2}gt^2$ — ②

From ①

$t\left(v\sin\theta - \frac{1}{2}at\right) = 0$

$t = 0$ $\quad t = \left(\dfrac{2v\sin\theta}{a}\right)$ ✓

$h = (v\cos\theta)\left(\dfrac{2v\sin\theta}{a}\right) + \frac{1}{2}g\left(\dfrac{2v\sin\theta}{a}\right)^2$

$h = \dfrac{2v^2\sin\theta}{a}\left(\cos\theta + \dfrac{g}{\cancel{2}} \dfrac{\cancel{2}\sin\theta}{a}\right)$