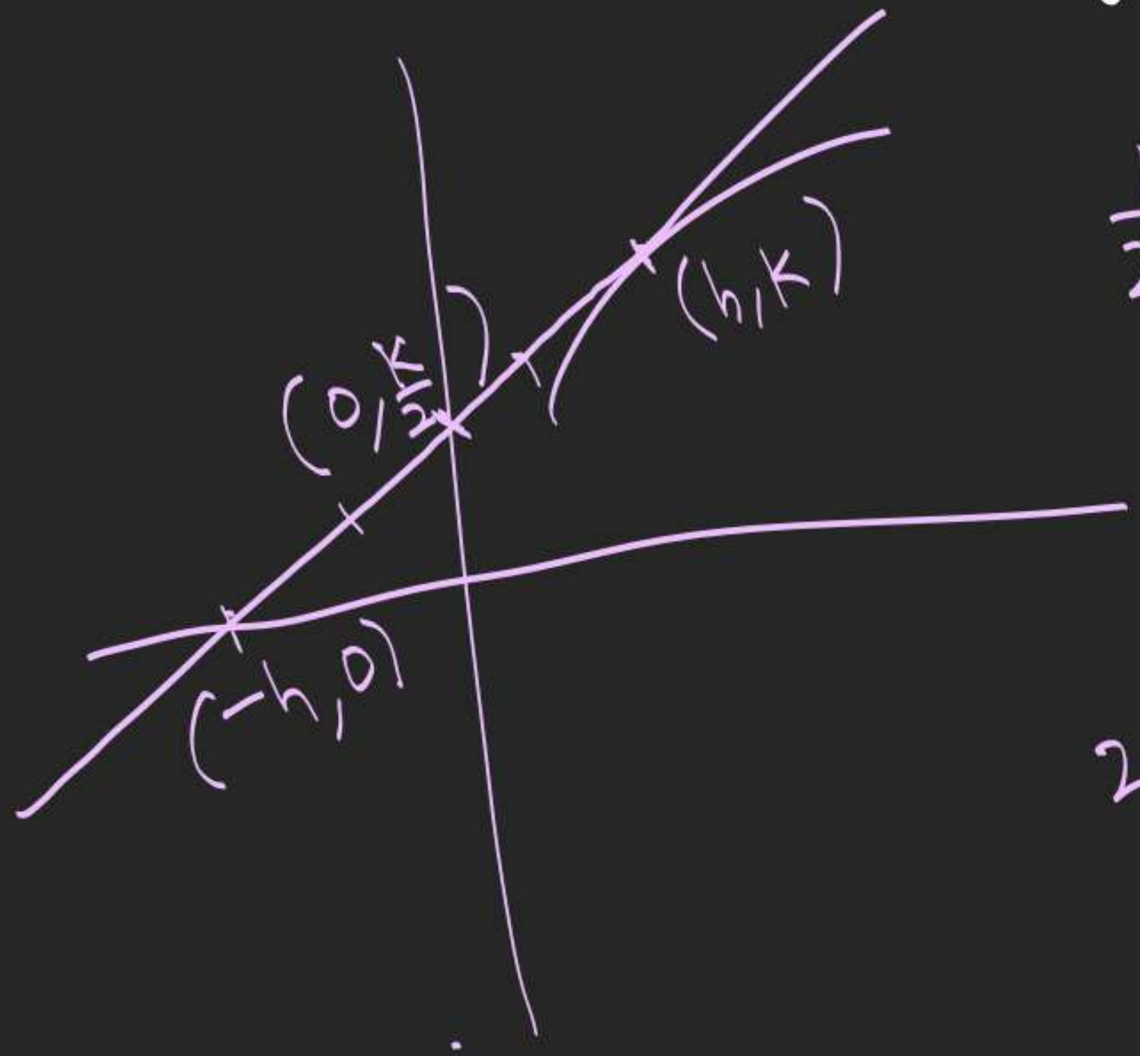
Nishant Jindal

1. Find the curve passing thru $(1,2)$ for which the segment of tangent between point of contact on curve and x-axis is bisected by y-axis.



$$\frac{k}{2h} = \left(\frac{dy}{dx}\right)_{(h,k)}$$

$$\frac{dy}{dx} = \frac{y}{2x}$$

$$2\int\frac{dy}{y} = \frac{dx}{x}$$

$$2\ln y = \ln x + C$$

$(1,2)$ $\quad 2\ln 2 = C$

$$\boxed{y^2 = 4x}$$

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2. $$\frac{x\,dx-y\,dy}{x\,dy-y\,dx}=\sqrt{\frac{1+x^2-y^2}{x^2-y^2}}$$

$x=r\sec\theta,\ y=r\tan\theta$

$$\frac{r\,dr}{r^2\sec\theta\,d\theta}=\frac{\sqrt{1+r^2}}{r}$$

$$\int\frac{dr}{\sqrt{1+r^2}}=\int\sec\theta\,d\theta \Rightarrow \ln\left(r+\sqrt{1+r^2}\right)=\ln(\sec\theta+\tan\theta)+\ln c$$

$$\sqrt{x^2-y^2}+\sqrt{1+x^2-y^2}=\frac{c(x+y)}{\sqrt{x^2-y^2}}$$

$$\int\frac{\frac{1}{2}d(x^2-y^2)}{\sqrt{(x^2-y^2)}\sqrt{1+x^2-y^2}}=\int\frac{d\left(\frac{y}{x}\right)}{1-\left(\frac{y}{x}\right)^2}$$

$$\boxed{x^2-y^2=t^2}$$

$$\int\frac{dt}{\sqrt{1+t^2}}$$

$$=\frac{1}{2}\ln\left|\frac{1+\frac{y}{x}}{1-\frac{y}{x}}\right|+\ln c$$

3. $$\frac{x\,dx+y\,dy}{\sqrt{x^2+y^2}}=\frac{y\,dx-x\,dy}{x^2}$$

$x=r\cos\theta,\ y=r\sin\theta$

$$\frac{d(x^2+y^2)}{2\sqrt{x^2+y^2}}=-d\left(\frac{y}{x}\right)$$

$$\sqrt{x^2+y^2}=-\frac{y}{x}+c$$

$x\,dy+y\,dx$

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4. Find the curve, $y=f(x)$, $f(x)>0$ for which area bounded by curve, coordinate axes and a variable ordinate is equal to length of corresponding arc. Given that curve passes thru $(0,1)$.

$$\int_0^h f(t)\,dt = \int_0^h \sqrt{(dx)^2+(dy)^2} = \int_0^h \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$$

$$f(h) = \sqrt{1+\left(\frac{dy}{dx}\right)^2}_{(h,k)}$$

$$y = \sqrt{1+\left(\frac{dy}{dx}\right)^2}$$

$$\pm\sqrt{y^2-1} = \frac{dy}{dx}$$

$$\boxed{y=1}$$

$$\int \frac{dy}{\sqrt{y^2-1}} = \pm\int dx$$

$$\ln\left(y+\sqrt{y^2-1}\right) = \pm x + \cancel{c} \quad 0$$

$$\boxed{(0,1)}$$

$$y+\sqrt{y^2-1} = e^{x} \text{ or } e^{-x}$$

$$y-\sqrt{y^2-1} = e^{-x} \qquad e^{x}$$

$$\boxed{y = \frac{e^{x}+e^{-x}}{2}}$$

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Homogeneous DE

$$\boxed{\begin{gathered}\frac{dy}{dx} = f\left(\frac{y}{x}\right) \\ \text{Put } \frac{y}{x} = t\end{gathered}}$$

$y = tx$

$$\frac{dy}{dx} = t + x\frac{dt}{dx}$$

$$t + x\frac{dt}{dx} = f(t)$$

$$x\frac{dt}{dx} = f(t) - t$$

$$\int \frac{dt}{f(t)-t} = \int \frac{dx}{x}$$

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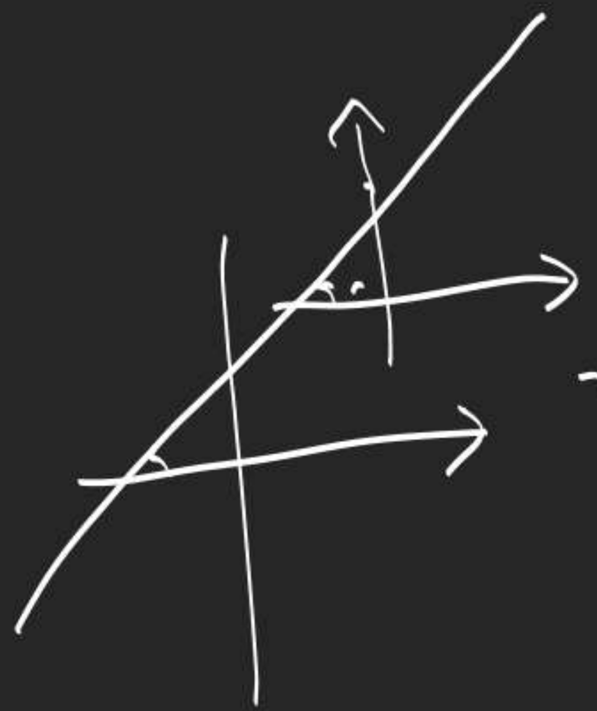
$$\frac{dy}{dx} = \frac{a_1x+b_1y+c_1}{a_2x+b_2y+c_2} \quad , \quad \frac{a_1}{a_2} \neq \frac{b_1}{b_2} \ \& \ b_1+a_2 \neq 0$$

$$\left.\begin{array}{l} a_1x+b_1y+c_1=0 \\ a_2x+b_2y+c_2=0 \end{array}\right\rangle \text{Intersection point} = (\alpha, \beta)$$



$$x-\alpha = X \Rightarrow dX = dx$$

$$y-\beta = Y$$

$$\frac{dy}{dx} = \frac{dY}{dX}$$

$$\frac{dY}{dX} = \frac{a_1(X+\alpha)+b_1(Y+\beta)+c_1}{a_2(X+\alpha)+b_2(Y+\beta)+c_2}$$

$$\frac{dY}{dX} = \frac{a_1X+b_1Y}{a_2X+b_2Y}$$

$$Y = tX$$

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1. $\left(\frac{dy}{dx} - \frac{y}{x}\right)\tan^{-1}\left(\frac{y}{x}\right) = 1$, $y\Big|_{x=1} = 0$.

$y = tx$ $\qquad$ $t + x\frac{dt}{dx} = \frac{dy}{dx}$

$\tan^{-1}t \; x\frac{dt}{dx} = 1$

$\int \tan^{-1}t \, dt = \int \frac{dx}{x}$

$t\tan^{-1}t - \frac{1}{2}\ln(1+t^2) = \ln x + C$

$\frac{y}{x}\tan^{-1}\frac{y}{x} - \frac{1}{2}\ln\left(1+\frac{y^2}{x^2}\right) = \ln x + C$

put $(1,0) \Rightarrow \boxed{C=0}$

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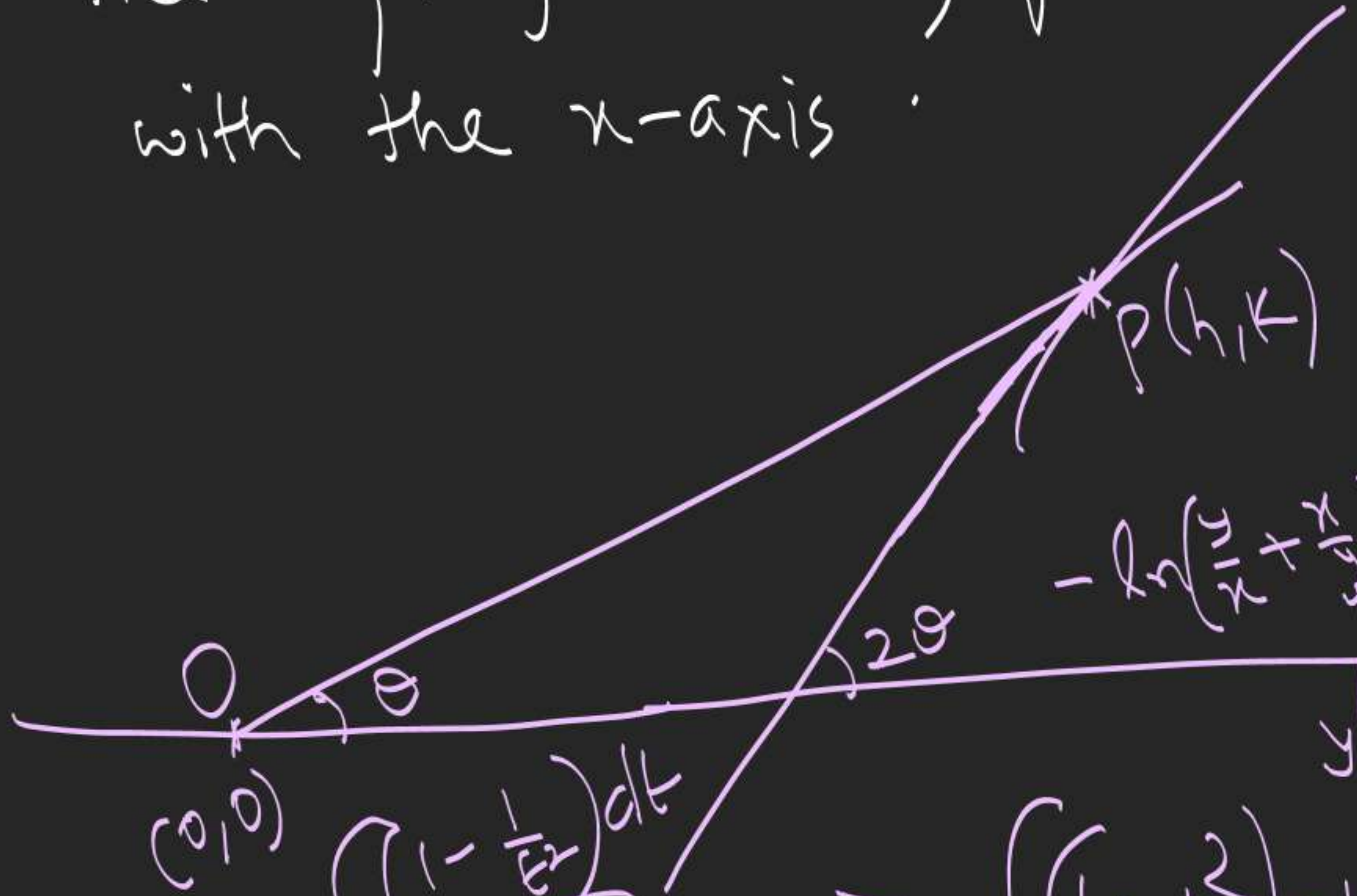
2. Find the curve s.t. angle formed with x-axis by the tangent to curve at any of its points, is twice the angle formed by polar radius of the point of tangency with the x-axis.



$$\tan 2\theta = \left(\frac{dy}{dx}\right)_{(h,k)} = \frac{2\left(\frac{k}{h}\right)}{1-\left(\frac{k}{h}\right)^2}$$

$$\frac{dy}{dx} = \frac{2\frac{y}{x}}{1-\left(\frac{y}{x}\right)^2}$$

$$y = tx$$

$$x\frac{dt}{dx} = \frac{2t}{1-t^2} - t = \frac{t+t^3}{1-t^2}$$

$$-\int \frac{\left(1-\frac{1}{t^2}\right)dt}{t+\frac{1}{t}} = \int \frac{(1-t^2)\,dt}{t+t^3} = \int \frac{dx}{x}$$

$$-\ln\left(\frac{y}{x}+\frac{x}{y}\right) = \ln x + \ln c$$

$$\boxed{x^2+y^2 = c'y}$$

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3.

$$y^3 \frac{dy}{dx} = x^2\left(\frac{3x^3+y^4-7}{x^3-2y^4+8}\right)$$

$$\boxed{\frac{3}{4}\frac{dY}{dX} = \frac{3X+Y}{X-2Y}}$$

$$x^3 - \frac{6}{7} = X$$

$$y^4 - \frac{31}{7} = Y$$

$$\frac{4y^3}{3x^2}\frac{dy}{dx} = \frac{dY}{dX}$$

$$\cancel{6}x + 2y = 1\cancel{4}$$

$$x - 2y = -8$$

$$7x = 6$$

$$x = \frac{6}{7}$$

$$y = \frac{\frac{6}{7}+8}{2}$$

$$y = \frac{31}{7}$$

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Linear DE of 'n' order

$$\boxed{\frac{d^n y}{dx^n} + a_{n-1}(x)\frac{d^{n-1} y}{dx^{n-1}} + a_{n-2}(x)\frac{d^{n-2} y}{dx^{n-2}} + \cdots + a_1(x)\frac{dy}{dx} + a_0(x)\,y = P(x)}$$

$$y'\,y'' + y^2 + x = 3$$

Linear DE

$\left(\frac{dy}{dx}\right)^3$ ✗

$\frac{dy}{dx}\frac{d^2y}{dx^2}$

y^2

$y\frac{dy}{dx}$

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First Order Linear D E

$$\frac{dy}{dx} + P(x)\,y = Q(x)$$

Integrating Factor = I.F. $= e^{\int P(x)\,dx}$

$$\boxed{y\,e^{\int P(x)dx} = \int Q(x)\, e^{\int P(x)\,dx}\, dx}$$

$$e^{\phi(x)}\frac{dy}{dx} + e^{\phi(x)} P(x)\,y = Q(x)\,e^{\phi(x)}$$

$$\frac{d}{dx}\left(e^{\phi(x)}y\right) = Q(x)\,e^{\phi(x)}$$

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Bernoulli's form

$$\frac{dy}{dx} + P(x)\,y = Q(x)\,y^n$$

$$\frac{1}{y^n}\frac{dy}{dx} + \frac{P(x)}{y^{n-1}} = Q(x)$$

Put $\frac{1}{y^{n-1}} = t$

$$\frac{(1-n)}{y^n}\frac{dy}{dx} = \frac{dt}{dx}$$

$$\frac{1}{(1-n)}\frac{dt}{dx} + P(x)\,t = Q(x)$$

$$\frac{dt}{dx} + (1-n)P(x)\,t = (1-n)Q(x)$$

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1. $$x(x^2+1)\frac{dy}{dx} = y(1-x^2) + x^2\ln x$$

2. $$(x^2-1)\sin x\frac{dy}{dx} + \left(2x\sin x + (x^2-1)\cos x\right)y - (x^2-1)\cos x = 0$$