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MAXIMA AND MINIMA

GEOMETRICAL PROBLEMS ON MAXIMUM & MINIMUM

1. The radius of a right circular cylinder of greatest curved surface which can be inscribed in a given right circular cone is

(A) one third that of the cone (B) $1/\sqrt{2}$ times that of the cone

(C) 2/3 that of the cone (D) 1/2 that of the cone

2. The dimensions of the rectangle of maximum area that can be inscribed in the ellipse $(x/4)^2 + (y/3)^2 = 1$ are

(A) $\sqrt{8}, \sqrt{2}$ (B) 4,3 (C) $2\sqrt{8}, 3\sqrt{2}$ (D) $\sqrt{2}, \sqrt{6}$

3. The largest area of a rectangle which has one side on the x-axis and the two vertices on the curve $y = e^{-x^2}$ is

(A) $\sqrt{2}e^{-1/2}$ (B) $2e^{-1/2}$ (C) $e^{-1/2}$ (D) None of these

4. Two vertices of a rectangle are on the positive x-axis. The other two vertices lie on the lines $y = 4x$ and $y = -5x + 6$. Then the maximum area of the rectangle is

(A) 4/3 (B) 3/5 (C) 4/5 (D) 3/4

5. In a regular triangular prism the distance from the centre of one base to one of the vertices of the other base is ℓ. The altitude of the prism for which the volume is greatest is

(A) $\frac{\ell}{2}$ (B) $\frac{\ell}{\sqrt{3}}$ (C) $\frac{\ell}{3}$ (D) $\frac{\ell}{4}$

6. The maximum area of the rectangle whose sides pass through the angular points of a given rectangle of sides a and b is

(A) 2 (ab) (B) $\frac{1}{2}(a+b)^2$ (C) $\frac{1}{2}(a^2+b^2)$ (D) None of these

7. The least area of a circle circumscribing any right triangle of area S is

(A) πS (B) $2\pi S$ (C) $\sqrt{2}\pi S$ (D) $4\pi S$

8. The lower corner of a leaf in a book is folded over so as to just reach the inner edge of the page. The fraction of width folded over if the area of the folded part is minimum is

(A) 5/8 (B) 2/3

(C) ¾ (D) 4/5

9. The lateral edge of a regular hexagonal pyramid is 1 cm. If the volume is maximum, then its height must be equal to

(A) $\frac{1}{3}$ (B) $\frac{2}{3}$

(C) $\frac{1}{\sqrt{3}}$ (D) 1

10. A triangular park is enclosed on two sides by a fence and on the third side by a straight river bank. The two sides having fence are of same length x. The maximum area enclosed by the park is -

(A) $\sqrt{\frac{x^3}{8}}$ (B) $\frac{1}{2}x^2$ (C) πx^2 (D) $\frac{3}{2}x^2$

11. The sum of the legs of a triangle is 9 cm. When the triangle rotates about one of the legs, a cone results which has the maximum volume. Then

(A) slant height of such a cone is $3\sqrt{5}$

(B) maximum volume of the cone is 32π

(C) curved surface of the cone is $18\sqrt{5}\pi$

(D) semi vertical angle of cone is $\tan^{-1}\sqrt{2}$

12. If the sum of the lengths of the hypotenuse and another side of a right angled triangle is given, show that the area of the triangle is a maximum when the angle between these sides is $\pi/3$.

13. A wire of length 20 m is to be cut into two pieces. One of these pieces is to be made into a square and other into a circle. What should be the length of the two pieces so that the combined area of square and circle is minimum.

14. Show that the semi vertical angle of a right circular cone of maximum volume and of a given slant height is $\tan^{-1}\sqrt{2}$.

15. The fuel charges for running a train are proportional to the square of the speed generated in m.p.h. & costs Rs. 48/- per hour at 16mph. What is the most economical speed if the fixed charges i.e. salaries etc. amount to Rs. 300/- per hour.

16. A statue 4 meters high sits on a column 5.6 meters high. How far from the column must a man, whose eye level is 1.6 meters from the ground, stand in order to see the statue at the greatest angle?

17. A running track of 440ft. is to be laid out enclosing a football field, the shape of which is a rectangle with semi circle at each end. If the area of the rectangular portion is to be maximum, find the length of its sides.

18. A trapezium ABCD is inscribed into a semicircle of radius l so that the base AD of the trapezium is a diameter and the vertices B & C lie on the circumference. Find the base angle θ of the trapezium ABCD which has the greatest perimeter.

19. A perpendicular is drawn from the centre to a tangent to an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$. Find the greatest value of the intercept between the point of contact and the foot of the perpendicular.

20. A given quantity of metal is to be casted into a half cylinder i.e. with a rectangular base and semicircular ends. Show that in order that total surface area may be minimum, the ratio of the height of the cylinder to the diameter of the semi circular ends is $\pi/(\pi+2)$.

21. A telephone company has 500 subscribers on its list and collects fixed charges of Rs. 300 per subscriber per year. The company proposes to increase the annual subscription and it is believed that for every increase of Rs. 1 in the charge, one subscriber will discontinue find the charge per subscriber that will maximize the income of the company.

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22. A factory D is to be connected by a road to a straight railway line on which a town A is situated. The distance DB of the factory to the railway line is $5\sqrt{3}$ km, the length AB of the railway line is 20 km. Freight charges on the road are twice the freight charges on the railway. At what point P on the railway line should the road DP be connected so as to ensure minimum cost of transporting goods from factory to town.

23. A wire of length 2 units is cut into two parts which are bent respectively to form a square of side = x units and a circle of radius = r units. If the sum of the areas of the square and the circle so formed is minimum, then:
[JEE Main 2016]

(A) $(4-\pi)\pi = \pi r$ (B) $x = 2r$

(C) $2x = r$ (D) $2x = (\pi + 4)r$

24. Twenty meters of wire is available for fencing off a flower-bed in the form of a circular sector. Then the maximum area (in sq. m) of the flower-bed, is : **[Jee Main 2017]**

(A) 12.5 (B) 10 (C) 25 (D) 30

25. A rectangular sheet of fixed perimeter with sides having their lengths in the ratio 8: 15 is converted into an open rectangular box by folding after removing squares of equal area from all four corners. If the total area of removed squares is 100, the resulting box has maximum volume. Then the lengths of the sides of the rectangular sheet are **[JEE 2013]**

(A) 24 (B) 32 (C) 45 (D) 60

26. A cylindrical container is to be made from certain solid material with the following constraints : It has a fixed inner volume of $V mm^3$, has a 2 mm thick solid wall and is open at the top. The bottom of the container is a solid circular dise of thickness 2 mm and is of radius equal to the outer radius of the container.

If the volume of the material used to make the container is minimum when the inner radius of the container is 10 mm, then the value of $\frac{V}{250\pi}$ is. **[JEE Adv. 2015]**

27. Consider all rectangles lying in the region

$$\left\{(x, y) \in R \times R: 0 \leq x \leq \frac{\pi}{2} \text{ and } 0 \leq y \leq 2\sin(2x)\right\}$$

and having one side on the x-axis. The area of the rectangle which has the maximum perimeter among all such rectangles, is **[JEE Adv.2020]**

(A) $\frac{3\pi}{2}$ (B) π (C) $\frac{\pi}{2\sqrt{3}}$ (D) $\frac{\pi\sqrt{3}}{2}$

ALGEBRAIC PROBLEMS ON MAXIMUM & MINIMUM

28. The set of values of p for which the extrema of the function, $f(x) = x^3 - 3px^2 + 3(p^2 - 1)x + 1$ lie in the interval $(-2,4)$ is

(A) $(-3,5)$ (B) $(-3,3)$ (C) $(-1,3)$ (D) $(-1,4)$

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29. $f(x) = 1 + 2x^2 + 4x^4 + 6x^6 + +100x^{100}$ is polynomial in a real variable x, then f(x) has

(A) neither a maximum nor a minimum (B) only one maximum

(C) only one minimum (D) one maximum and one minimum

30. On the interval [0,1] the function $x^{25}(1-x)^{75}$ takes its maximum value at

(A) 0 (B) ½ (C) 1 (D) 1/4

31. The product of minimum value of x^x and maximum value of $\left(\frac{1}{x}\right)^x$ is

(A) e (B) e^{-1} (C) 1 (D) e^2

32. The minimum value of the function defined by $f(x) = \max(x, x+1, 2-x)$ is

(A) 0 (B) ½ (C) 1 (D) 3/2

33. Let $f(x) = \{x\}$, For $f(x), x = 5$ is (where $\{*\}$ denotes the fractional part)

(A) a point of local maxima

(B) a point of local minima

(C) neither a point of local minima nor maxima

(D) None of these

34. The number of values of x where $f(x) = \cos x + \cos \sqrt{2}x$ attains its maximum value is

(A) 1 (B) 0 (C) 2 (D) infinite

35. If x is real, the maximum value of $\frac{3x^2+9x+17}{3x^2+9x+7}$ is

(A) 41 (B) 1 (C) 17/7 (D) 1/4

36. Let $f(x) = \begin{cases} \sin \frac{\pi}{2}x & , 0 \le x < 1 \\ 3-2x & , x \ge 1 \end{cases}$ then

(A) f(x) has local maxima at $x = 1$

(B) f(x) has local minima at $x = 1$

(C) f(x) does not have any local extrema at $x = 1$

(D) f(x) has a global minima at $x = 1$

37. The greatest and the least values of the function, $f(x) = 2 - \sqrt{1+2x+x^2}, x \in [-2,1]$ are

(A) 2,1 (B) 2, −1 (C) 2,0 (D) None of these

38. The difference between the greatest and least values of the function $f(x) = \sin 2x - x$ on $[-\pi/2, \pi/2]$ is

(A) $\frac{\sqrt{3}+\sqrt{2}}{2}$ (B) $\frac{\sqrt{3}+\sqrt{2}}{2} + \frac{\pi}{6}$ (C) $\frac{\pi}{2}$ (D) π

39. A possible ordered pair (a, b) such that all the local extremum values of the function $f(x) = x^3 + ax^2 - 9x + b$ are positive and the local minimum value occurs at point $x = 1$ is

(A) (3,5) (B) (3,6) (C) (3,4) (D) (3,3)

40. The greatest value of $f(x) = (x+1)^{1/3} - (x-1)^{1/3}$ in $[0,1]$ is

(A) 1 (B) 2 (C) 3 (D) $2^{1/3}$

41. Let $f(x) = \begin{cases} x^3 - x^2 + 10x - 5, & x \le 1 \\ -2x + \log_2(b^2 - 2), & x > 1 \end{cases}$ the set of values of b for which f(x) has greatest value at $x = 1$ is given by

(A) $1 \le b \le 2$ (B) $b = \{1,2\}$

(C) $b \in (-\infty, -1)$ (D) $[-\sqrt{130}, -\sqrt{2}) \cup (\sqrt{2}, \sqrt{130}]$

42. The minimum value of $(x-p)^2 + (x-q)^2 + (x-r)^2$ will be at x equals to

(A) pqr (B) $\sqrt[3]{pqr}$ (C) $\frac{p+q+r}{3}$ (D) $p^2 + q^2 + r^2$

43. $f(x) = \begin{cases} \tan^{-1} x, & |x| < \frac{\pi}{2} \\ \frac{\pi}{2} - |x|, & |x| \ge \frac{\pi}{2} \end{cases}$ then

(A) f(x) has no point of local maxima

(B) f(x) has only one point of local maxima

(C) f(x) has exactly two points of local maxima

(D) f(x) has exactly two points of local minima

44. The maximum slope of the curve $y = -x^3 + 3x^2 + 2x - 27$ will be

(A) $-165/8$ (B) -27 (C) 5 (D) None of these

45. If x_1 and x_2 are abscissa of two points on the curve $f(x) = x - x^2$ in the interval $[0,1]$, then maximum value of the expression $(x_1 + x_2) - (x_1^2 + x_2^2)$ is

(A) 1/2 (B) 1/4 (C) 1 (D) 2

46. The maximum value of $f(x) = 2bx^2 - x^4 - 3b$ is g(b), where $b > 0$, if b varies then the minimum value of g(b) is

(A) $\frac{3}{2}$ (B) $\frac{9}{2}$ (C) $-\frac{9}{4}$ (D) $-\frac{9}{2}$

47. The function ' f ' is defined by $f(x) = x^p(1-x)^q$ for all $x \in R$, where p, q are positive integers, has a maximum value, for x equal to

(A) $\frac{pq}{p+q}$ (B) 1 (C) 0 (D) $\frac{p}{p+q}$

48. If $f(x) = \frac{x^2-1}{x^2+1}$, for every real number, then minimum value of f(x)

(A) does not exist

(B) is not attained even though f is bounded

(C) is equal to 1

(D) is equal to -1

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49. If $f(x) = a\ell n|x| + bx^2 + x$ has its extremum values at $x = -1$ and $x = 2$, then

(A) $a = 2, b = -1$
(B) $a = 2, b = -1/2$
(C) $a = -2, b = 1/2$
(D) None of these

50. If $f(x) = \begin{bmatrix} -\sqrt{1-x^2} & , 0 \le x \le 1 \\ -x & , x > 1 \end{bmatrix}$, then

(A) Maximum of f(x) exist at $x = 1$
(B) Maximum of f(x) doesn't exists
(C) Minimum of $f^{-1}(x)$ exist at $x = -1$
(D) Minimum of $f^{-1}(x)$ exist at $x = 1$

51. An extremum value of the function $f(x) = (\arcsin\ x)^3 + (\arccos\ x)^3$ is

(A) $\frac{7\pi^3}{8}$
(B) $\frac{\pi^3}{8}$
(C) $\frac{\pi^3}{32}$
(D) $\frac{\pi^3}{16}$

52. For the function $f(x) = x^{2/3}$, which of the following statement(s) is/are true ?

(A) $\frac{dy}{dx}$ at the origin is non existent
(B) equation of the tangent at the origin is $x = 0$
(C) f(x) has an extremum at $x = 0$
(D) origin is the point of inflection

53. Let $f(x) = \begin{cases} x^3 + x^2 - 10x & -1 \le x < 0 \\ \sin\ x & 0 \le x < \pi/2 \\ 1 + \cos\ x & \pi/2 \le x \le \pi \end{cases}$ then f(x) has

(A) local maximum at $x = \pi/2$
(B) local minima at $x = \pi/2$
(C) absolute minima at $x = 0, \pi$
(D) absolute maxima at $x = \pi/2$

54. If $f(x) = \frac{x}{1+x\tan\ x}, x \in \left(0, \frac{\pi}{2}\right)$, then

(A) f(x) has exactly one point of minima
(B) f(x) has exactly one point of maxima
(C) f(x) is increasing in $\left(0, \frac{\pi}{2}\right)$
(D) maxima occurs at x_0 where $x_0 = \cos\ x_0$

55. If the function $y = f(x)$ is represented as, $x = \phi(t) = t^5 - 5t^3 - 20t + 7$
$y = \psi(t) = 4t^3 - 3t^2 - 18t + 3(|t| < 2)$, then

(A) $y_{max} = 12$
(B) $y_{max} = 14$
(C) $y_{min} = -67/4$
(D) $y_{min} = -69/4$

56. The function $f(x) = \sin\ x - x\cos\ x$ is

(A) maximum or minimum for all integral multiple of π
(B) maximum if x is an odd positive or even negative integral multiple of π
(C) minimum if x is an even positive or odd negative integral multiple of π
(D) None of these

57. Let $f(x) = 40/(3x^4 + 8x^3 - 18x^2 + 60)$, consider the following statement about f(x).

(A) f(x) has local minima at $x = 0$
(B) f(x) has local maxima at $x = 0$
(C) absolute maximum value of f(x) is not defined
(D) f(x) is local maxima at $x = -3, x = 1$

58. Find values of a and b such that $f(x) = \frac{a}{x} + bx$ has a minimum at point (1,6).

59. Find the points of local maxima/minima of following functions
(i) $f(x) = 2x^3 - 21x^2 + 36x - 20$
(ii) $f(x) = -(x-1)^3(x+1)^2$
(iii) $f(x) = x\ell nx$

60. Let $f(x) = \begin{cases} 2\sin x & x > 0 \\ x^2 & x \le 0 \end{cases}$. Investigate the function for maxima/minima at $x = 0$.

61. Find the number of critical points of the following functions.
(i) $f(x) = -\frac{3}{4}x^4 - 8x^3 - \frac{45}{2}x^2 + 105;\ x \in R$
(ii) $f(x) = |x-2| + |x+1|\ ;\ x \in R$
(iii) $f(x) = \min(\tan x, \cot x)\ ;\ x \in (0, \pi)$

62. Let $f(x) = x^2; x \in [-1,2)$. Then show that f(x) has exactly one point of local maxima but global maximum is not defined.

63. Let $f(x) = x + \sqrt{x}$. Find the greatest and least value of f(x) for $x \in (0,4)$.

64. Let $f(x) = \begin{cases} 3 - x & 0 \le x < 1 \\ x^2 + \ln b & x \ge 1 \end{cases}$ find the set of values of b such that f(x) has a local minima at $x = 1$.

65. Find the points of local maxima/minima of following functions
(i) $f(x) = x + \frac{1}{x}$
(ii) $f(x) = \operatorname{cosec} x$
Hence find maxima and minima values of f(x).

66. Show that $\sin^p \theta \cos^q \theta, p, q \in N$ attains maximum value when $\theta = \tan^{-1}\sqrt{\frac{p}{q}}$. Identity if it is a global maxima or not.

67. If $y = \frac{ax+b}{(x-1)(x-4)}$ has a turning value at $(2,-1)$ find a& b and show that the turning value is a maximum.

68. Consider the following graph. In each case identify if $x = a$ is a point of maxima, minima or neither maxima nor minima

69. Find the polynomial f(x) of degree 6, which satisfies $\underset{x\to 0}{\text{Limit}}\left(1 + \frac{f(x)}{x^3}\right)^{1/x} = e^2$ and has local maximum at $x = 1$ and local minimum at $x = 0\&2$.

70.

Column - I	Column – II
(A) If the greatest and least values of the function $f(x) = \begin{cases} 2x^2 + \frac{2}{x^2}, & x \in [-2,2] \sim \{0\} \\ 1, & x = 0 \end{cases}$ are G and L respectively, then	(P) $[G + L] = 1$ where [.] = greatest integer function
(B) If the greatest and	(Q) $[G + L] = 6$ where [.] = greatest integer function

least values of the function

$f(x) = x^3 - 6x^2 + 9x + 1$

on [0,2] are G and L respectively, then

(C) If the greatest and least value of the function $f(x) = \arctan\ x - \frac{1}{2}$ function $\ln\ x$ on $\left[\frac{1}{\sqrt{3}}, \sqrt{3}\right]$ are G and L respectively, then

(R) $[G + L] = 6$ where $[.] =$. greatest integer

(S) $(G + L) = 2$ where $(.) =$ least integer function

(T) $(G + L) = 10(.) =$ least integer function

71. If p and q are positive real numbers such that $p^2 + q^2 = 1$, then the maximum value of $(p + q)$ is- **[AIEEE 2007]**

(A) 2 (B) $\frac{1}{2}$ (C) $\frac{1}{\sqrt{2}}$ (D) $\sqrt{2}$

72. Let $f: R \to R$ be defined by **[AIEEE 2010]**

$f(x) = \begin{cases} k - 2x, & \text{if} \quad x \notin -1 \\ 2x + 3, & \text{if} \quad x > -1 \end{cases}$ If f has a local minimum at $x = -1$, then a possible value of k is

(A) 1 (B) 0 (C) $-\frac{1}{2}$ (D) -1

73. For $x \in \left(0, \frac{5\pi}{2}\right)$, define **[AIEEE 2011]**

$$f(x) = \int_0^x \sqrt{t}\sin\ t dt$$

Then f has:

(A) local maximum at π and 2π

(B) local minimum at π and 2π

(C) local maximum at π and local maximum at 2π

(D) local maximumat π and local minimum at 2π

74. Let $a, b \in R$ be such that the function f given by $f(x) = \ln\ |x| + bx^2 + ax, x \neq 0$ has extreme values at $x = -1$ and $x = 2$. **[AIEEE 2012]**

Statement 1 : f has local maximum at $x = -1$ and at $x = 2$

Statement 2: $a = \frac{1}{2}$ and $b = \frac{-1}{4}$

(A) Statement-1 is true, Statement-2 is true, Statement 2 is not a correct explanation for statement-1.

(B) Statement 1 is true, Statement 2 is false.

(C) Statement 1 ix false, Statement 2 is true.

(D) Statement 1 is true, Statement 2 is true, Statement 2 is a correct explanation for statement 1.

75. The real number k for which the equation, $2x^3 + 3x + k = 0$ has two distinct real roots in [0,1]

[AIEEE - 2013]

(A) lies between -1 and 0 (B) does not exist

(C) lies between 1 and 2 (D) lies between 2 and 3

76. The total number of local maxima and local minima of the function

$f(x) = \begin{cases} (2+x)^3, & -3 < x \le -1 \\ x^{2/3}, & -1 < x < 2 \end{cases}$ is **[JEE 2008]**

(A) 0 (B) 1 (C) 2 (D) 3

77. **(i)** Which of the following is true ?

(A) $(2+a)^2 f''(1) + (2-a)^2 f''(-1) = 0$

(B) $(2-a)^2 f''(1) - (2+a)^2 f''(-1) = 0$

(C) $f'(1)f'(-1) = (2-a)^2$

(D) $f'(1)f'(-1) = -(2+a)^2$

78. Which of the following is true?

(A) f(x) is decreasing on $(-1,1)$ and has a local minimum at $x = 1$

(B) f(x) is increasing on $(-1,1)$ and has a local maximum at $x = 1$

(C) f(x) is increasing on $(-1,1)$ but has neither a local maximum and nor a local minimum at $x = 1$.

(D) f(x) is decreasing on $(-1,1)$ but has neither a local maximum and nor a local minimum at $x = 1$.

79. Let $g(x) = \int_0^{e^x} \frac{f'(t)}{1+t^2} dt$

Which of the following is true ?

(A) $g'(x)$ is positive on $(-\infty, 0)$ and negative on $(0, \infty)$

(B) $g'(x)$ is negative on $(-\infty, 0)$ and positive on $(0, \infty)$

(C) $g'(x)$ changes sign on both $(-\infty, 0)$ and $(0, \infty)$

(D) $g'(x)$ does not change sign on $(-\infty, \infty)$

80. Let p(x) be a polynomial of degree 4 having extremum at $x = 1,2$ and $\lim_{x\to 0}\left(1 + \frac{p(x)}{x^2}\right) = 2$. Then the value of p(2) is **[JEE 2009]**

81. Let f, g and h be real-valued functions defined on the interval $[0,1]$ by $f(x) = e^{x^2} + e^{-x^2}$, $g(x) = xe^{x^2} + e^{-x^2}$ and $h(x) = x^2e^{x^2} + e^{-x^2}$. If a, b and c denote respectively, the absolute maximum of f, g and h on $[0,1]$, then **[JEE 2010]**

(A) $a = b$ and $c \ne b$ (B) $a = c$ and $a \ne b$ (C) $a \ne b$ and $c \ne b$ (D) $a = b = c$

82. Let f be a function defined on R (the set of all real numbers) such that $f'(x) = 2010$

$(x - 2009)(x - 2010)$^2 $(x - 2011)$^3 $(x - 2012)$^4, for all $x \in R$. If g is a function defined on R with values in the interval $(0, \infty)$ such that $f(x) = \ell n\,(g(x))$, for all $x \in R$, then the number of points in R at which g has a local maximum is

83. Let p(x) be a real polynomial of least degree which has a local maximumat $x = 1$ and a local minimumat $x = 3$. If $p(1) = 6$ and $p(3) = 2$, then $p'(0)$ is **[JEE 2012]**

84. et f: $IR \to IR$ be defined as $f(x) = |x| + |x^2 - 1|$. The total number of points at which f attains either a local maximum or a local minimum is **[JEE 2012]**

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85. If $f(x) = \int_0^x e^{t^2}(t-2)(t-3)dt$ for all $x \in (0, \infty)$, then **[JEE 2012]**

(A) f has a local maximum at $x = 2$

(B) f is decreasing on (2,3)

(C) there exists some $c \in (0, \infty)$ such that $f''(c) = 0$

(D) f has a local minimum at $x = 3$

86. The function $f(x) = 2|x| + |x+2| - ||x+2| - 2|x||$ has a local minimum or a local maximum at $x =$ **[JEE 2013]**

(A) -2 (B) $\frac{-2}{3}$

(C) 2 (D) $\frac{2}{3}$

87. The least value of $\alpha \in R$ for which $4\alpha x^2 + \frac{1}{x} \geq 1$ for all $x > 0$, is **[JEE Adv.2016]**

(A) $\frac{1}{64}$ (B) $\frac{1}{32}$ (C) $\frac{1}{27}$ (D) $\frac{1}{25}$

88. Let $f(x) = \frac{\sin \pi x}{x^2}, x > 0$.

Let $x_1 < x_2 < x_3 < \cdots < x_n < \cdots$ be all the points of local maximum of f and $y_1 < y_2 < y_3 < \cdots . < y_n < \cdots$ be all the points of local minimum of f.

Then which of the following options is/are correct ? **[JEE Adv.2019]**

(A) $x_1 < y_1$ (B) $x_n \in \left(2n, 2n + \frac{1}{2}\right)$ for every n

(C) $|x_n - y_n| > 1$ for every n (D) $x_{n+1} - x_n > 2$ for every n

SPECIAL CONCEPT OF GEOMETRICAL DISTANCE

89. The point on the curve $4x^2 + a^2y^2 = 4a^2, 4 < a^2 < 8$, that is farthest from the point $(0, -2)$ is

(A) (2,0) (B) (0,2) (C) $(2, -2)$ (D) $(-2,2)$

90. Which of the following point lying on the line $x + 2y = 5$ is at minimum distance from the origin

(A) (1,2) (B) (3,1) (C) $(-1,3)$ (D) (2,3/2)

91. The maximum distance of the point $(a, 0)$ from the curve $2x^2 + y^2 - 2x = 0$ is-

(A) $\sqrt{(1 - 2a + a^2)}$ (B) $\sqrt{(1 + 2a + 2a^2)}$

(C) $\sqrt{(1 + 2a - a^2)}$ (D) $\sqrt{(1 - 2a + 2a^2)}$

92. The point on the line $y = x$ such that the sum of the squares of its distance from the point $(a, 0), (-a, 0)$ and $(0, b)$ is minimum will be -

(A) $(a/6, a/6)$ (B) (a, a)

(C) (b, b) (D) $(b/6, b/6)$

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2^{nd} ORDER DERIVATIVE, CONCAVITY & POINT OF INFLECTION

93. f(c) is a minimum value of f(x) if -

(A) $f'(c)=0, f''(c)>0$ (B) $f'(c)=0, f''(c)<0$

(C) $f'(c)\neq 0, f''(c)=0$ (D) $f'(c)<0, f''(c)>0$

94. If $f'(c)<0$ and $f'(c)>0$, then at $x=c$, f(x) is-

(A) maximum (B) minimum

(C) neither maximum nor minimum (D) either maximum or minimum

95. Let $f''(x)>0 \forall x\in R$ and $g(x)=f(2-x)+f(4+x)$. Then g(x) is increasing in

(A) $(-\infty,-1)$ (B) $(-\infty,0)$ (C) $(-1,\infty)$ (D) None of these

96. Find the set of value(s) of ' a ' for which the function $f(x)=\frac{ax^3}{3}+(a+2)x^2+(a-1)x+2$ possess a negative point of inflection.

(A) $a\in(-\infty,-2)\cup(0,\infty)$ (B) $a\in(-\infty,-3)\cup(0,\infty)$

(C) $a\in(-\infty,-2)\cup(2,\infty)$ (D) $a\in(-\infty,-1)\cup(3,\infty)$

97. If the point (1,3) serves as the point of inflection of the curve $y=ax^3+bx^2$ then the value of ' a ' and ' b ' are

(A) $a=3/2 \& b=-9/2$ (B) $a=3/2 \& b=9/2$

(C) $a=-3/2 \& b=-9/2$ (D) $a=-3/2 \& b=9/2$

98. The curve $y=\frac{x+1}{x^2+1}$ has

(A) $x=1$, the point of inflection (B) $x=-2+\sqrt{3}$, the point of inflection

(C) $x=-1$, the point of minimum (D) $x=-2-\sqrt{3}$, the point of inflection

99. If $f''(x)>0 \forall x\in R, f'(4)=0$ and $g(x)=f(\cot^2 x-2\cot x+5); 0<x<\frac{\pi}{2}$, then

(A) g(x) is increasing in $\left(0,\frac{\pi}{2}\right)$

(B) g(x) is decreasing in $\left(0,\frac{\pi}{2}\right)$

(C) g(x) is increasing in $\left(\frac{\pi}{4},\frac{\pi}{2}\right)$

(D) g(x) is decreasing in $\left(0,\frac{\pi}{4}\right)$

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ANALYSIS OF CUBIC

100. If $f(x)=x^3-3x^2+3x+7$, then -

(A) $f(x)$ has a maximum at $x=1$

(B) $f(x)$ has a minimum at $x=1$

(C) $f(x)$ has a point of inflexion at $x=1$

(D) None of these

101. If $f(x)=x^3+ax^2+bx+c$ is minimum at $x=3$ and maximum at $x=-1$, then-

(A) $a=-3, b=-9, c=0$ (B) $a=3, b=9, c=0$

(C) $a=-3, b=-9, c\in R$ (D) None of these

102. The set of values of ' a ' for which the function $f(x)=\frac{ax^3}{3}+(a+2)x^2+(a-1)x+2$ possesses a negative point of inflection -

(A) $(-\infty,-2)\cup(0,\infty)$ (B) $\{-4/5\}$

(C) $(-2,0)$ (D) empty set

103. Find the value of a if $x^3-3x+a=0$ has three real and distinct roots -

(A) $a>2$ (B) $a<2$ (C) $-2<a<2$ (D) none

104. The equation $x^3-3x+[a]=0$, will have three real and distinct roots if (where $[*]$ denotes the greatest integer function)

(A) $a\in(-\infty,2)$ (B) $a\in(0,2)$

(C) $a\in(-\infty,2)\cup(0,\infty)$ (D) $a\in[-1,2)$

105. Number of solution(s) satisfying the equation, $3x^2-2x^3=\log_2(x^2+1)-\log_2 x$ is

(A) 1 (B) 2 (C) 3 (D) None of these

106. If the function $f(x)=x^3+3(a-7)x^2+3(a^2-9)x-1$ has a positive point of maximum, then -

(A) $a\in(3,\infty)\cup(-\infty,-3)$

(B) $a\in(-\infty,-3)\cup(3,29/7)$

(C) $(-\infty,7)$

(D) $(-\infty,29/7)$

107. If the function $f(x)=x^3-9x^2+24x+c$ has three real and distinct roots α,β and γ then the possible values of c are

(A) $(-20,-16)$ (B) $(-20,4)$ (C) $(-4,20)$ (D) none of these

108. If the derivative of an odd cubic polynomial vanishes at two different values of ' x ' then (A) coefficient of x^3&x in the polynomial must be same in sign
(B) coefficient of x^3&x in the polynomial must be different in sign
(C) the values of ' x ' where derivative vanishes are closer to origin as compared to the respective roots on either side of origin
(D) the values of ' x ' where derivative vanishes are far from origin as compared to the respective roots on either side of origin

109. A cubic f(x) vanishes at $x=-2$& has relative minimum/maximum at $x=-1$ and $x=1/3$. If $\int_{-1}^{1} f(x)dx=\frac{14}{3}$, find the cubic f(x).

110. Find the values of ' a ' for which the function $f(x)=\frac{a}{3}x^3+(a+2)x^2+(a-1)x+2$ possess a negative point of minimum.

COMPREHENSION

A cubic $f(x)=ax^3+bx^2+cx+d$ vanishes at $x=-2$ and has relative minimum / maximum at $x=-1$ and $x=\frac{1}{3}$ and if $\int_{-1}^{1} f(x)dx=\frac{14}{3}$

111. The function f(x) is
(A) x^3+x^2+x-2 (B) x^3-x^2+x-2 (C) x^3-x^2-x+2 (D) x^3+x^2-x+2

112. f(x) decreases in the interval
(A) $\left(-\frac{1}{3},1\right)$ (B) $\left(-\frac{1}{3},-1\right)$ (C) $\left(-1,\frac{1}{3}\right)$ (D) $\left(1,\frac{3}{2}\right)$

113. The nature of roots of $f(x)=3$ is
(A) one root is real and other two are distinct
(B) all roots real and distinct
(C) all roots are real; two of them are equal
(D) none of the above

114. Suppose the cubic x^3-px+q has three distinct real roots where $p>0$ and $q>0$. Then which one of the following holds? **[AIEEE 2008]**
(A) The cubic has minima at $-\sqrt{\frac{p}{3}}$ and maxima at $\sqrt{\frac{p}{3}}$
(B) The cubic has manima at both $\sqrt{\frac{p}{3}}$ and $-\sqrt{\frac{p}{3}}$
(C) The cubic has maxima at both $\sqrt{\frac{p}{3}}$ and $-\sqrt{\frac{p}{3}}$
(D) The cubic has minima at $\sqrt{\frac{p}{3}}$ and maxima at $-\sqrt{\frac{p}{3}}$

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115. For $x \in \left(0, \frac{5\pi}{2}\right)$, define **[AIEEE 2011]**

$$f(x) = \int_0^x \sqrt{t}\sin\, t dt$$

Then f has:
(A) local maximum at π and 2π
(B) local minimum at π and 2π
(C) local maximum at π and local maximum at 2π
(D) local maximumat π and local minimum at 2π

116. The maximum value of the function $f(x) = 2x^3 - 15x^2 + 36x - 48$ on the set $A = \{x \mid x^2 + 20 \leq 9x\}$ is

117. The number of distinct real roots of $x^4 - 4x^3 + 12x^2 + x - 1 = 0$ is **[JEE 2011]**

118. Let $f: R \to R$ be given by $f(x) = (x-1)(x-2)(x-5)$. Define

$$F(x) = \int_0^x f(t)dt, x > 0$$

Then which of the following options is/are correct? **[JEE Adv.2019]**
(A) F has two local maxima and one local minimum in $(0, \infty)$
(B) F has a local maximum at $x = 2$
(C) $F(x) \neq 0$ for all $x \in (0,5)$
(D) Fhas a local minimum at $x = 1$

MIXED PROBLEMS

119. Minimum value of $\frac{1}{3\sin\,\theta - 4\cos\,\theta + 7}$ is

(A) $\frac{7}{12}$ (B) $\frac{5}{12}$ (C) $\frac{1}{12}$ (D) $\frac{1}{6}$

120. A variable point P is chosen on the straight line $x + y = 4$ and tangents PA and PB are drawn from it to circle $x^2 + y^2 = 1$. Then the position of P for the smallest length of chord of contact AB is

(A) (3,1) (B) (0,4) (C) (2,2) (D) (4,0)

121. A and B are the points (2,0) and (0,2) respectively. The coordinates of the point P on the line $2x + 3y + 1 = 0$ are

(A) $(7, -5)$ if $|PA - PB|$ is maximum (B) $\left(\frac{1}{5}, \frac{1}{5}\right)$ if $|PA - PB|$ is maximum

(C) $(7, -5)$ if $|PA - PB|$ is minimum (D) $\left(\frac{1}{5}, \frac{1}{5}\right)$ if $|PA - PB|$ is minimum

122. Two points A(1,4)& B(3,0) are given on the ellipse $2x^2 + y^2 = 18$. The co-ordinates of a point C on the ellipse such that the area of the triangle ABC is greatest is

(A) $(\sqrt{6}, \sqrt{6})$ (B) $(-\sqrt{6}, \sqrt{6})$ (C) $(\sqrt{6}, -\sqrt{6})$ (D) $(-\sqrt{6}, -\sqrt{6})$

123. Least value of the function, $f(x) = 2^{x^2} - 1 + \frac{2}{2^{x^2}+1}$ is

(A) 0 (B) 3/2 (C) 2/3 (D) 1

124. The co-ordinate of the point for minimum value of $z = 7x - 8y$ subject to the conditions $x + y - 20 \le 0, y \ge 5, x \ge 0, y \ge 0$

(A) (20,0) (B) (15,5) (C) (0,5) (D) (0,20)

125. Maximum and minimum value of $f(x) = \max(\sin t), 0 < t < x, 0 \le x \le 2\pi$ are

(A) 1,0 (B) $1, -1$ (C) $0, -1$ (D) None of these

126. If $a^2x^4 + b^2y^4 = c^6$, then the maximum value of xy is

(A) $\frac{c^3}{2ab}$ (B) $\frac{c^3}{\sqrt{2|ab|}}$ (C) $\frac{c^3}{ab}$ (D) $\frac{c^3}{\sqrt{|ab|}}$

127. Let $f(x) = \sin\frac{\{x\}}{a} + \cos\frac{\{x\}}{a}$. Then the set of values of a for which f can attain its maximum values is (where $a > 0$ and $\{*\}$ denotes the fractional part function)

(A) $\left(0, \frac{4}{\pi}\right)$ (B) $\left(\frac{4}{\pi}, \infty\right)$

(C) $(0, \infty)$ (D) None of these

128. The maximum value of f(x), if $f(x) + f\left(\frac{1}{x}\right) = \frac{1}{x}, x \in$ domain of f

(A) -1 (B) 2 (C) 1 (D) 1/2

129. If $\lim_{x\to a} f(x) = \lim_{x\to a} [f(x)]$ and f(x) is non-constant continuous function, then (where $[*]$ denotes the greatest integer function)

(A) $\lim_{x\to a} f(x)$ is integer (B) $\lim_{x\to a} f(x)$ is non-integer

(C) f(x) has local maximum at $x = a$ (D) f(x) has local minima at $x = a$

130. Let $f(x) = \ln(2x - x^2) + \sin\frac{\pi x}{2}$. Then

(A) graph of f is symmetrical about the line $x = 1$

(B) graph of f is symmetrical about the line $x = 2$

(C) maximum value of f is 1

(D) minimum value of f does not exist

131. A function is defined as $f(x) = ax - b|x|$ where a and b are constants then at $x = 0$ we will have a maxima of f(x) if

(A) $a > 0, b > 0$ (B) $a > 0, b < 0$ (C) $a < 0, b < 0$ (D) $a < 0, b > 0$

132. Draw graph of $f(x) = x|x - 2|$ and hence find points of local maxima/minima.

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133. Let $f(x) = x^2 + \frac{1}{x^2}$ and $g(x) = x - \frac{1}{x}$, $x \in \mathbf{R} - \{-1,0,1\}$. If $h(x) = \frac{f(x)}{g(x)}$, then the local minimum value of h(x) is: **[Jee-MAIN 2018]**

(A) $2\sqrt{2}$ (B) 3 (C) -3 (D) $-2\sqrt{2}$

134. Let $f: R \to R$ and $g: R \to R$ be respectively given by $f(x) = |x| + 1$ and $g(x) = x^2 + 1$.

Define $h: R \to R$ by

$$h(x) = \begin{cases} \max\{f(x), g(x)\} & \text{if } x£0, \\ \min\{f(x), g(x)\} & \text{if } x > 0 \end{cases}$$

The number of points at which h(x) is not differentiable is **[JEE 2014]**

135. Let $a \in R$ and let $f: R \to R$ be given by $f(x) = x^5 - 5x + a$ Then **[JEE Adv. 2014]**

(A) f(x) has three real roots if $a > 4$
(B) f(x) has only one real root if $a > 4$
(C) f(x) has three real roots if $a < -4$
(D) f(x) has three real roots if $-4 < a < 4$

136. Let $f: R \to R$ be given by

$$f(x) = \begin{cases} x^5 + 5x^4 + 10x^3 + 10x^2 + 3x + 1, & x < 0 \\ x^2 - x + 1, & 0 \le x < 1; \\ \frac{2}{3}x^3 - 4x^2 + 7x - \frac{8}{3}, & 1 \le x < 3 \\ (x-2)\log_e(x-2) - x + \frac{10}{3}, & x \ge 3 \end{cases}$$

Then which of the following options is /are correct? **[JEE Adv.2019]**

(A) f is increasing on $(-\infty, 0)$
(B) f is onto
(C) f' has a local maximum at $x = 1$
(D) f' is NOT differentiable at $x = 1$

137. Let PQRS be quadrilateral in a plane, where $QR = 1, \angle PQR = \angle QRS = 70^\circ$ and $\angle PQS = 15^\circ$ and $\angle PRS = 40^\circ$. If $\angle RPS = \theta^\circ, PQ = \theta^\circ, PQ = \alpha$ and $PS = \beta$, then the interval(s) that contains(s) the value of $4\alpha\beta\sin\theta^\circ$ is/are **[JEE Adv.2022]**

(A) $(0, \sqrt{2})$ (B) $(1,2)$ (C) $(\sqrt{2}, 3)$ (D) $(2\sqrt{2}, 3\sqrt{2})$

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ANSWER KEY

1. (D) **2.** (C) **3.** (A) **4.** (C) **5.** (B) **6.** (B) **7.** (A)

8. (B) **9.** (C) **10.** (B) **11.** (A,C) **13.** $\frac{20\pi}{4+\pi}, \frac{80\pi}{4+\pi}$ **15.** 40 m/H

16. $\sqrt[4]{2}$ m **17.** 110 & 170 **18.** $\frac{\pi}{3}$ **19.** |a-b| **21.** 400

22. 5 km from B towards A **23.** (C) **24.** (C) **25.** (A,C) **26.** (4)

27. (C) **28.** (C) **29.** (C) **30.** (D) **31.** (C) **32.** (D) **33.** (B)

34. (A) **35.** (A) **36.** (A) **37.** (C) **38.** π **39.** (B) **40.** (B)

41. (D) **42.** (C) **43.** (B) **44.** (C) **45.** (A) **46.** (C) **47.** (D)

48. (D) **49.** (B) **50.** (A,C) **51.** (A,C) **52.** (A,C) **53.** (A,C) **54.** (BD)

55. (B,D) **56.** (A,B,C) **57.** (A,C,D) **58.** a = 3, b = 3

59. **(i)** x = 1 max. ,x = 6 min. **(ii)** x = -1 min. ,x = -1/5 max. **(iii)** x = 1/e, minima

60. minima at x = 0 **61.** **(i)** 3 **(ii)** infinite **(iii)** 2

63. Greatest and least values are not defined

64. (0,e]

65. **(i)** max at x-1,min at x = 1 **(ii)** minima at x =(2n+1)$\frac{\pi}{2}$, maxima at x = (2n-1)$\frac{\pi}{2}$ $f_{max}1, f_{min} = -1$

67. at =1 & b = 0

68. **(i)** maxima ,**(ii)** minima ,**(iii)** Neither ,**(iv)** Neither , **(v)** Neither,**(vi)** maxima

69. $f(x) = \frac{2}{3}x^6 - \frac{12}{5}x^5 + 2x^4$

70. (A-R,T; B-Q; C-P,S) **71.** (D) **72.** (D) **73.** (D) **74.** (A) **75.** (B)

76. (C) **77.** (A) **78.** (A) **79.** (B) **80.** (0) **81.** (D) **82.** (1)

83. (9) **84.** (5) **85.** (A, B, C, D) **86.** (A, B) **87.** (C) **88.** (A, C, D)

89. (B) **90.** (A) **91.** (D) **92.** (D) **93.** (A) **94.** (C) **95.** (C)

96. (A) **97.** (D) **98.** (A, B, D) **99.** (C, D) **100.** (C) **101.** (C)

102. (A) **103.** (C) **104.** (D) **105.** (A) **106.** (B) **107.** (A) **108.** (B, C)

109. $f(x) = x^3 + x^2 - x + 2$ **110.** (1,∞) **111.** (D) **112.** (C) **113.** (C)

114. (D) **115.** (D) **116.** (1) **117.** (2) **118.** (B, C, D) **119.** (C)

120. (C) **121.** (A) **122.** (D) **123.** (D) **124.** (D) **125.** (A) **126.** (B)

127. (A) **128.** (D) **129.** (A, D) **130.** (A, C, D) **131.** (A, C)

132. Min at x = 2, Max. at x = 1 **133.** (A) **134.** (3) **135.** (B, D) **136.** (B, C, D)

137. (A)