

$$f(x) < f(2x) < f(3x)$$

$$\frac{f(x)}{f(x)} < \frac{f(2x)}{f(x)} < \frac{f(3x)}{f(x)}$$

$$\lim_{x \rightarrow \infty} \frac{f(2x)}{f(x)} = 1$$

$$\lim_{x \rightarrow \infty} \frac{f(cx)}{f(x)} = ? \quad c > 0$$

$$0 < c < 1$$

$$2^{\frac{1}{n}} < c < \frac{1}{2^{\frac{1}{n}-1}}$$

$$\lim_{x \rightarrow \infty} \frac{f(2^{\frac{1}{n}} x)}{f(x)} = 1$$

Let $c > 1 \Rightarrow 2^{n-1} \leq c \leq 2^n$

$$f(2^{n-1} x) < f(cx) < f(2^n x)$$

$$\lim_{x \rightarrow \infty} \frac{f(2^2 x)}{f(2x)} = 1$$

$$\lim_{x \rightarrow \infty} \frac{f(2^3 x)}{f(2^2 x)} = 1$$

$$\lim_{x \rightarrow \infty} \frac{f(2^n x)}{f(2^{n-1} x)} = 1$$

$$\lim_{n \rightarrow 0} \left[n \times \frac{k}{n} - n \left\{ \frac{k}{n} \right\} \right] \stackrel{k}{=} 0$$

$\sum_{k=1}^k k$

$$\frac{\cot x (1 - \sin x)}{\tan\left(\frac{\pi}{2} - x\right) \left(1 - \cos\left(\frac{\pi}{2} - x\right)\right)} = \frac{2^0 \left(\frac{\pi}{2} - x\right) \left(\frac{\pi}{2} - x\right)^2}{\left(\frac{\pi}{2} - x\right) \left(\frac{\pi}{2} - x\right)^2}$$

1.

1

1.

$\lim_{x \rightarrow 0}$

$$\frac{e^x(e^x + e^{-x}) - (x+1)(e^x + e^{-x})}{x^2 \left(\frac{e^x - 1}{x} \right)}$$

2.

$$[-2, 0)$$

$$D \leq 0$$

$$k^2 - 4 \leq 0$$

$$k \in [-2, 2]$$

3.

$$\phi$$

$$\geq 0 \quad \forall x \in \mathbb{R}$$

$$\frac{(e^x + e^{-x})(e^x - x - 1)}{x^2 \left(\frac{e^x - 1}{x} \right)} = 1$$

$$f(x) =$$

$$\sqrt{x^2 + kx + 1}$$

$$x^2 - k$$

$$\neq 0$$

$$k < 0$$

cont.
 $\forall x \in \mathbb{R}$
 $k = ?$

$\lim_{x \rightarrow 0}$

$$\frac{3}{3} \cdot q(x) = \boxed{\operatorname{cosec} 2x + \operatorname{cosec} 2^2 x + \operatorname{cosec} 2^3 x + \dots + \operatorname{cosec} 2^{n-1} x + \boxed{\operatorname{cosec} 2^n x + \cot 2^n x}} \\ = \cot x$$

$$\text{RHL} = \lim_{x \rightarrow 0^+} \left((\cos x)^{\cot x} + (\sec x)^{\operatorname{cosec} x} + 1 \right) = 2 + 1 = 3$$

$$\begin{aligned} & \frac{(\cos x - 1)x}{x^2 \frac{\tan x}{x}} + \frac{(1 - \cos x)x}{\cos x x^2 \frac{\sin x}{x}} \\ \text{LHL} = \lim_{x \rightarrow 0^-} & \frac{e^x + e^{-x} - 2 + 2(1 - \cos x)}{x^2 \frac{\sin x}{x}} = 2 \end{aligned}$$

$$p = \phi$$

1. Let $f(x) = \lim_{n \rightarrow \infty} \frac{x^{2n-1} + ax^2 + bx}{x^{2n} + 1} = \begin{cases} , n \in \mathbb{N} \\ ax^2 + bx, |x| < 1 \\ \frac{1}{x}, |x| > 1 \end{cases}$
 is continuous $\forall x \in \mathbb{R}$, find a, b .

Cont. At $x=1$ ✓

$$\text{LHL} = \lim_{x \rightarrow 1^-} \frac{ax^2 + bx}{1} = a + b$$

$$\text{RHL} = \lim_{x \rightarrow 1^+} \frac{1}{x} = 1$$

$$f(1) = \frac{1 + a + b}{2}$$

$$a = 0, b = 1$$

$$a + b = 1$$

$$a - b = -1$$

At $x = -1$

$$\text{LHL} = \lim_{x \rightarrow -1^-}$$

at

Cont. $x = -1$

$$\text{LHL} = -1$$

$$\text{RHL} = a - b$$

$$f(-1) = \frac{a - b - 1}{2}$$

$$\frac{1 + a + b}{2} \quad x = 1$$

$$\frac{-1 + a - b}{2} \quad x = -1$$

2. Let $f(x) = \left(\frac{\frac{a^x - 1}{x} \left(\frac{b(\sin x - x) + (bx - \sin bx)}{x^3} \right) + \frac{\sin x}{x} \left(\frac{(\cos x - 1) + (1 - \cos bx)b^2}{x^2} \right)}{b - a} \right)^n$

$\frac{(a^x - 1)}{x} \left(\frac{b \sin x - \sin bx}{x(\cos x - \cos bx)} \right)^n, x > 0$

$\frac{a^x \sin(bx) - b^x \sin(ax)}{bx \tan(bx) - \tan(ax) ax}, x < 0$

LHL = $\frac{b - a}{b - a} = 1$

is continuous at $x = 0$. Obtain $f(0)$ and a relation between a, b & n .

$$\ln a \left(\frac{-\frac{b}{6} + \frac{b^3}{6}}{-\frac{1}{2} + \frac{b^2}{2}} \right)^n$$

$$f(0) = 1 = \ln a \left(\frac{b}{3} \right)^n$$

At 5:45 pm

∴ Let $f(x+y) = f(x)f(y) \quad \forall x, y \in \mathbb{R}$ and

$f(x) = 1 + g(x)G(x)$, where $\lim_{x \rightarrow 0} g(x) = 0$ &

$\lim_{x \rightarrow 0} G(x)$ exists. P.T. $f(x)$ is continuous

$\forall x \in \mathbb{R}$.

cont. at $x=a$

$a \in \mathbb{R}$

$LHL = RHL = f(a)$
cont. at $x=a$.

$$LHL = \lim_{h \rightarrow 0} f(a-h) = f(a) \lim_{h \rightarrow 0} f(-h) = f(a) \lim_{h \rightarrow 0} \left(1 + \underbrace{g(-h)G(-h)}_{0 \times \text{finite}} \right)$$

$$RHL = \lim_{h \rightarrow 0} f(a+h) = f(a) \lim_{h \rightarrow 0} f(h) = f(a) \lim_{h \rightarrow 0} \left(1 + \underbrace{g(h)G(h)}_{= f(a)} \right) = f(a)$$