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Parametric Differentiation

$x = f(t)$

$y = g(t)$

$$\frac{dy}{dx} = \frac{dy}{dt}\frac{dt}{dx} = \left(\frac{\frac{dy}{dt}}{\frac{dx}{dt}}\right)$$

$$\frac{dt}{dx} = \frac{1}{\left(\frac{dx}{dt}\right)}$$

$$\frac{dt}{dx} = \lim_{\Delta x \to 0} \frac{\Delta t}{\Delta x} = \left(\lim_{\Delta t \to 0} \frac{1}{\left(\frac{\Delta x}{\Delta t}\right)}\right)$$

$$= \frac{1}{\left(\frac{dx}{dt}\right)}$$

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Derivative of $f(x)$ w.r.t $g(x)$

$$\frac{d}{d\,g(x)}\left(f(x)\right) = \frac{f'(x)}{g'(x)}$$

$x,\ x+\Delta x$

$$\boxed{\begin{aligned} y &= \sin x \\ dy &= \cos x \, dx \end{aligned}}$$

$$\frac{d\,f(x)}{d\,g(x)} \quad \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{g(x+\Delta x) - g(x)} = \lim_{\Delta x \to 0} \frac{\left(\dfrac{f(x+\Delta x) - f(x)}{\Delta x}\right)}{\left(\dfrac{g(x+\Delta x) - g(x)}{\Delta x}\right)}$$

$$= \frac{f'(x)}{g'(x)}$$

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1. If $x = a\sqrt{\cos 2t}\cos t$ and $y = a\sqrt{\cos 2t}\sin t$

find $\frac{dy}{dx}$ at $t = \frac{\pi}{6}$

$$\frac{dy}{dx} = \frac{a\left(\cos t\sqrt{\cos 2t} + \frac{\sin t(-2\sin 2t)}{2\sqrt{\cos 2t}}\right)}{a\left(-\sin t\sqrt{\cos 2t} + \frac{-2\sin 2t\cos t}{2\sqrt{\cos 2t}}\right)}$$

2. If $x = \sec\theta - \cos\theta$

$y = \sec^n\theta - \cos^n\theta$, then P.T. $(x^2+4)\left(\frac{dy}{dx}\right)^2 = n^2(y^2+4)$

$$\frac{dy}{dx} = \frac{n\sec^n\theta\tan\theta + n\cos^{n-1}\theta\sin\theta}{\sec\theta\tan\theta + \sin\theta}$$

$$= \frac{n\cancel{\tan\theta}(\sec^n\theta + \cos^n\theta)}{\cancel{\tan\theta}(\sec\theta + \cos\theta)}$$

$$\Rightarrow \left(\frac{dy}{dx}\right)^2 = \frac{n^2\left[(\sec^n\theta - \cos^n\theta)^2 + 4\right]}{(\sec\theta - \cos\theta)^2 + 4}$$

$$= \frac{n^2(y^2+4)}{(x^2+4)}$$

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3: Find derivative of

(i) $(\ln x)^{\tan x}$ w.r.t. x^x

(ii) $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$ w.r.t. $\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$

$$\frac{(\ln x)^{\tan x}\left(\sec^2 x \ln(\ln x) + \frac{\tan x}{x \ln x}\right)}{x^x\left(1 + \ln x\right)}$$

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$$\sin^{-1}\left(\frac{2x}{1+x^2}\right) = \begin{cases} -\pi-2\tan^{-1}x & x\in(-\infty,-1] \\ 2\tan^{-1}x & x\in[-1,1] \\ \pi-2\tan^{-1}x & x\in[1,\infty) \end{cases}$$

$$\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) = \begin{cases} -2\tan^{-1}x & x\in(-\infty,0] \\ 2\tan^{-1}x & x\in[0,\infty) \end{cases}$$

$$\tan^{-1}x=\theta\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$$

$$2\theta\in(-\pi,\pi)$$

$$\frac{d\,f(x)}{d\,g(x)} = \begin{cases} 1 & x\in(-\infty,-1) \\ -1 & x\in(-1,0) \\ 1 & x\in(0,1) \\ -1 & x\in(1,\infty) \end{cases}$$

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$\sin^{-1}\left(\frac{2x}{1+x^2}\right)$

$\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$

$\downarrow$

$2-1-x^2$

$\frac{2}{1+x^2}-1$

$$\frac{\dfrac{1}{\sqrt{1-\dfrac{4x^2}{(1+x^2)^2}}} \times \dfrac{2\left(1+x^2-x(2x)\right)}{\cancel{(1+x^2)^2}}}{\dfrac{-1}{\sqrt{1-\dfrac{(1-x^2)^2}{(1+x^2)^2}}} \times \dfrac{-4x}{\cancel{(1+x^2)^2}}}$$

$$= \frac{\dfrac{1}{|1-x^2|}\, 2\left(1-x^2\right)}{\dfrac{1}{2|x|}\, 4x} = \frac{(1-x^2)\,|x|}{|1-x^2|\; x}$$

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4. $y = \tan^{-1}\left(\dfrac{\sqrt{1+x^2}-1}{x}\right)$, $\dfrac{dy}{dx} = ?$

$= \frac{1}{2}\tan^{-1}x$, $x \in R - \{0\}$

$y' = \dfrac{1}{2(1+x^2)}$

5. If $f(x) = x^3 + x^5$ and $g(x) = f^{-1}(x)$,

find $g'(2)$.

$f'(x) = 3x^2 + 5x^4$

$\boxed{f(1) = 2}$

$g(f(x)) = x \Rightarrow g'(f(x))\,f'(x) = 1$

Put $x = 1$

$g'(2)\,f'(1) = 1$

$\Rightarrow \boxed{g'(2) = \frac{1}{8}}$

6. Let $f(x) = e^{x^3+x^2+x}$, $g(x) = f^{-1}(x)$

find $g'(e^3) = \dfrac{1}{6e^3}$

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6. If $x^y = e^{x-y}$, then P.T. $\frac{dy}{dx} = \frac{\ln x}{(1+\ln x)^2}$

$y \ln x = x - y \Rightarrow y = \frac{x}{1+\ln x}$

7. If $\boxed{\sin y = x \sin(a+y)}$, then P.T. $\frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}$.

Also find $\frac{dy}{dx}$ explicitly in terms of x.

$$y' = \frac{(-1)\times\left(\frac{-1}{x^2 \sin a}\right)}{1+\left(\frac{\frac{1}{x}-\cos a}{\sin a}\right)^2}$$

$$\frac{\sin y}{\sin(a+y)} \Rightarrow \frac{dx}{dy} = \frac{\sin(a+y)\cos y - \sin y \cos(a+y)}{\sin^2(a+y)} = \frac{\sin a}{\sin^2(a+y)}$$

$$\frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}$$

$$\frac{1}{x} = \frac{\sin(a+y)}{\sin y} = \sin a \cot y + \cos a$$

$$y + \boxed{n\pi} = \cot^{-1}\cot y = \cot^{-1}\left(\frac{\frac{1}{x}-\cos a}{\sin a}\right) \cancel{=} \quad \cot y = \frac{\frac{1}{x}-\cos a}{\sin a}.$$

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9. If $\sqrt{1-x^2}+\sqrt{1-y^2}=a(x-y)$, then P.T.

$$\frac{dy}{dx}=\sqrt{\frac{1-y^2}{1-x^2}}$$