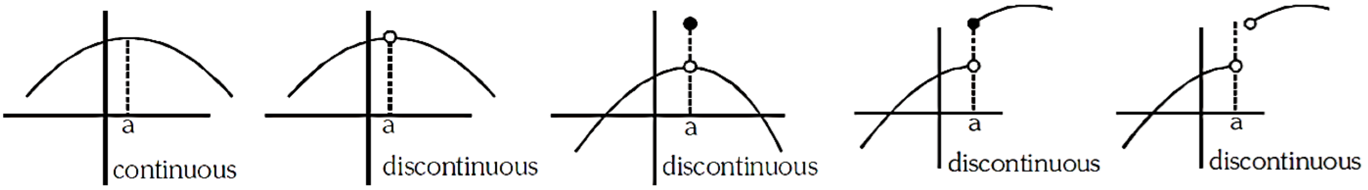
1. CONTINUOUS FUNCTIONS :

A function for which a small change in the independent variable causes only a small change and not a sudden jump in the dependent variable are called continuous functions. Naively, we may that a function is continuous at fixed point if we can draw the graph of the function around the point without lifting the pen from the plane of the paper.



Continuity of a function at a point :

A function f(x) is said to be continuous at $x = a$, if $\text{Lim}_{x\to a} f(x) = f(a)$. Symbolically f is continuous at $x = a$ if $\text{Lim}_{h\to 0} f(a-h) = \text{Lim}_{h\to 0} f(a+h) = f(a), h > 0$

i. e. $(\text{LHL}_{x=a} = \text{RHL})$ equals value of $f'f'$ at $x = a$.

Illustration 1: If $f(x) = \begin{cases} \sin\frac{\pi x}{2}, & x < 1 \\ [x] & x \geq 1 \end{cases}$ then find whether f(x) is continuous or not at $x = 1$,

where [ ] denotes greatest integer function.

Solution: $f(x) = \begin{cases} \sin\frac{\pi x}{2}, & x < 1 \\ [x] & x \geq 1 \end{cases}$

For continuity at $x = 1$, we determine, $f(1), \lim_{x\to 1^-} f(x)$ and $\lim_{x\to 1^+} f(x)$.

Now, $f(1) = [1] = 1$

$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} \sin\frac{\pi x}{2} = \sin\frac{\pi}{2} = 1$ and $\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} [x] = 1$

So $f(1) = \lim_{x\to 1^-} f(x) = \lim_{x\to 1^+} f(x)$

$\therefore$ f(x) is continuous at $x = 1$

Illustration 2: Let $f(x) = \begin{cases} \frac{a(1-x\sin x)+b\cos x+5}{x^2} & x < 0 \\ 3 & x = 0 \\ \left(01+\left(\frac{cx+dx^3}{x^2}\right)\right)^{\frac{1}{x}} & x > 0 \end{cases}$

If f is continuous at $x = 0$, then find out the values of a, b, c and d.

Solution: Since f(x) is continuous at $x = 0$, so at $x = 0$, both left and right limits must exist and both must be equal to 3 .

Now $\text{Lim}_{x\to 0^-} \frac{a(1-x\sin x)+b\cos x+5}{x^2} = \text{Lim}_{x\to 0^-} \frac{(a+b+5)+\left(-a-\frac{b}{2}\right)x^2+\cdots}{x^2} = 3$ (By the expansion of sin x and cos x)

If $\lim_{x\to 0^-} f(x)$ exists then $a + b + 5 = 0$ and $-a - \frac{b}{2} = 3 \Rightarrow a = -1$ and $b = -4$ since

$\lim_{x\to 0^+} \left(1+\left(\frac{cx+dx^3}{x^2}\right)\right)^{\frac{1}{x}}$ exists $\Rightarrow \lim_{x\to 0^+} \frac{cx+dx^3}{x^2} = 0 \Rightarrow c = 0$

Now $\lim_{x\to 0^+}(1+dx)^{\frac{1}{x}} = \lim_{x\to 0^+}\left[(1+dx)^{\frac{1}{dx}}\right]^d = e^d$

So $e^d = 3 \Rightarrow d = \ln\ 3$,

Hence $a = -1, b = -4, c = 0$ and $d = \ln\ 3$.

Do yourself - 1 :

(i) If $f(x) = \begin{cases} \cos\ x; x \geq 0 \\ x + k; x < 0 \end{cases}$ find the value of k if f(x) is continuous at $x = 0$.

(ii) If $f(x) = \begin{cases} \frac{|x+2|}{\tan^{-1}(x+2)} & ; x \neq -2 \\ 2 & ; x = -2 \end{cases}$ then discuss the continuity of f(x) at $x = -2$

CONTINUITY OF THE FUNCTION IN AN INTERVAL :

(a) A function is said to be continuous in (a, b) if f is continuous at each and every point belonging to (a, b).

(b) A function is said to be continuous in a closed interval [a, b] if :

(i) f is continuous in the open interval (a, b)

(ii) f is right continuous at 'a' i.e. $\text{Lim}_{x\to a^+}\ f(x) = f(a) =$ a finite quantity

(iii) f is left continuous at 'b' i.e. $\text{Lim}_{x\to b^-}\ f(x) = f(\ b) =$ a finite quantity

Note :

(i) All polynomials, trigonometrical functions, exponential and logarithmic functions are continuous in their domains.

(ii) If f(x) and g(x) are two functions that are continuous at $x = c$ then the function defined by : $F_1(x) = f(x) \pm g(x)$; $F_2(x) = Kf(x)$, where K is any real number ; $F_3(x) = f(x).g(x)$ are also continuous at $x = c$.

Further, if g(c) is not zero, then $F_4(x) = \frac{f(x)}{g(x)}$ is also continuous at $x = c$.

Illustration 3: Discuss the continuity of $f(x) = \begin{cases} |x+1| & , & x < -2 \\ 2x+3 & , & -2 \leq x < 0 \\ x^2+3 & , & 0 \leq x < 3 \\ x^3 - 15 & , & x \geq 3 \end{cases}$

Solution : We write f(x) as $f(x) = \begin{cases} -x-1 & , & x < -2 \\ 2x+3 & , & -2 \leq x < 0 \\ x^2+3 & , & 0 \leq x < 3 \\ x^3 - 15, & & x \geq 3 \end{cases}$

As we can see, f(x) is defined as a polynomial function in each of intervals $(-\infty, -2)$, $(-2,0)$, $(0,3)$ and $(3, \infty)$. Therefore, it is continuous in each of these four open intervals. Thus we check the continuity at $x = -2,0,3$.

At the point $x = -2$

$$\lim_{x\to -2^-} f(x) = \lim_{x\to -2^-}(-x-1) = +2 - 1 = 1$$

$$\lim_{x\to -2^+} f(x) = \lim_{x\to -2^+}(2x+3) = 2.(-2) + 3 = -1$$

Therefore, $\lim_{x\to -2} f(x)$ does not exist and hence f(x) is discontinuous at $x = -2$.
At the point $x = 0$

$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} (2x+3) = 3$$

$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} (x^2+3) = 3$$

$$f(0) = 0^2 + 3 = 3$$

Therefore f(x) is continuous at $x = 0$.
At the point $x = 3$

$$\lim_{x\to 3^-} f(x) = \lim_{x\to 3^-} (x^2+3) = 3^2 + 3 = 12$$

$$\lim_{x\to 3^+} f(x) = \lim_{x\to 3^+} (x^3-15) = 3^3 - 15 = 12$$

$$f(3) = 3^3 - 15 = 12$$

Therefore, f(x) is continuous at $x = 3$.
We find that f(x) is continuous at all points in R except at $x = -2$

Do yourself - 2:

(i) If $f(x) = \begin{cases} \dfrac{x^2}{a} & ; \quad 0 \le x < 1 \\ -1 & ; \quad 1 \le x < \sqrt{2} \\ \dfrac{2b^2 - 4b}{x^2} & ; \quad \sqrt{2} \le x < \infty \end{cases}$ then find the value of a and b if $f(x)$ is continuous in $[0, \infty)$

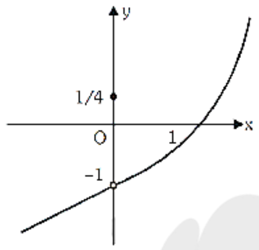
(ii) Discuss the continuity of $f(x) = \begin{cases} |x-3| & ; \quad 0 \le x < 1 \\ \sin x & ; \quad 1 \le x \le \dfrac{\pi}{2} \\ \log_{\frac{\pi}{2}} x & ; \quad \dfrac{\pi}{2} < x < 3 \end{cases}$ in [0, 3)

3.TYPES OF DISCONTINUITIES :

Type-1 :(Removable type of discontinuities) : - In case $\lim_{x\to a} f(x)$ exists but is not equal to f (a) then the function is said to have a removable discontinuity or discontinuity of the first kind. In this case we can redefine the function such that $\lim_{x\to a} f(x) = f(a)$ and make it continuous at $x = a$. Removable type of discontinuity can be further classified as :
(a) Missing point discontinuity :
Where $\lim_{x\to a} f(x)$ exists but f(a) is not defined.

(b) Isolated point discontinuity :
Where $\text{Lim}_{x\to a}$ f(x) exists and f(a) also exists but ; $\text{Lim}_{x\to a}$ $f(x) \neq f(a)$.

Illustration 4 : Examine the function, $f(x) = \begin{cases} x-1, & x<0 \\ 1/4, & x=0 \\ x^2-1, & x>0 \end{cases}$. Discuss the continuity, and if discontinuous remove the discontinuity by redefining the function (if possible).



$y = f(x)$ before redefining

Graph of f(x) is shown, from graph it is seen that
$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x) = -1$, but $f(0) = 1/4$
Thus, f(x) has removable discontinuity and f(x) could be made continuous by taking

$f(0) = -1$

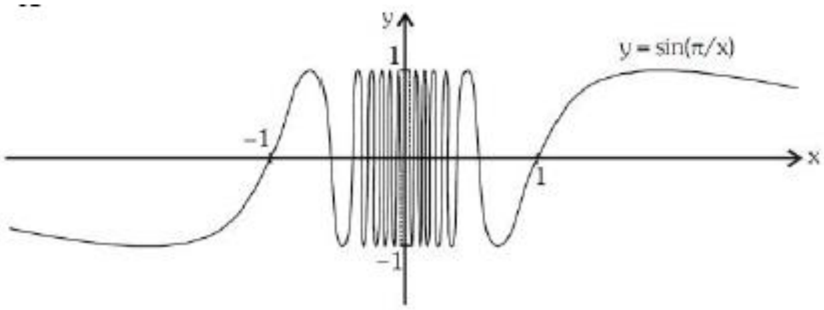
$$\Rightarrow f(x) = \begin{cases} x-1 & , \quad x<0 \\ -1 & , \quad x=0 \\ x^2-1 & , \quad x>0 \end{cases}$$

Do yourself - 3:

(i) If $f(x) = \begin{cases} \dfrac{1}{x-1} & ; \quad 0 \le x < 2 \\ x^2-3 & ; \quad 2 \le x < 4 \\ 5 & ; \quad x = 4 \\ 14 - \dfrac{x^{1/2}}{2} & ; \quad x > 4 \end{cases}$ then discuss the types of discontinuity for the function.

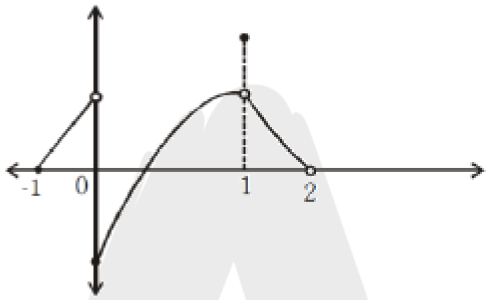
Type - 2: (Non-Removable type of discontinuities) :
In case $\text{Lim}_{x\to a}$ f(x) does not exist then it is not possible to make the function continuous by redefining it. Such a discontinuity is known as non- removable discontinuity or discontinuity of the 2nd kind. Non-removable type of discontinuity can be further classified as :

(i) Finite type discontinuity : In such type of discontinuity left hand limit and right hand limit at a point exists but are not equal.
(ii) Infinite type discontinuity : In such type of discontinuity at least one of the limit viz. LHL and RHL is tending to infinity.
(iii) Oscillatory type discontinuity :
e.g. $f(x) = \sin\frac{\pi}{x}$ at $x = 0$



f(x) has non removable oscillatory type discontinuity at $x = 0$

Example :



From the adjacent graph note that
(i) f is continuous at $x = -1$
(ii) f has isolated discontinuity at $x = 1$
(iii) f has missing point discontinuity at $x = 2$
(iv) f has non removable (finite type) discontinuity at the origin.
Note : In case of non-removable (finite type) discontinuity the non-negative difference between the value of the RHL at $x = a$ and LHL at $x = a$ is called the jump of discontinuity. A function having a finite number of jumps in a given interval I is called a piece wise continuous or sectionally continuous function in this interval.

Illustration 5:

Show that the function, $f(x) = \begin{cases} \frac{e^{1/x}-1}{e^{1/x}+1} & ;\ \text{when } x \neq 0 \\ 0, & ;\ \text{when } x = 0 \end{cases}$ has non-removable discontinuity at $x = 0$.

Solution :

We have, $f(x) = \begin{cases} \frac{e^{1/x}-1}{e^{1/x}+1} & ;\ \text{when } x \neq 0 \\ 0, & ;\ \text{when } x = 0 \end{cases}$

$$\Rightarrow \lim_{x\to 0^+} f(x) = \lim_{h\to 0} f(0+h) = \lim_{h\to 0} \frac{e^{\frac{1}{h}}-1}{e^{\frac{1}{h}}+1} = \lim_{h\to 0} \frac{1-\frac{1}{e^{1/h}}}{1+\frac{1}{e^{1/h}}} = 1[\because e^{1/h} \to \infty]$$

$$\Rightarrow \lim_{x\to 0^-} f(x) = \lim_{h\to 0} \frac{e^{-1/h}-1}{e^{-1/h}+1} = \frac{0-1}{0+1} = -1\ [\because h \to 0; e^{-1/h} \to 0]$$

$\lim_{x\to 0^-} f(x) = -1$

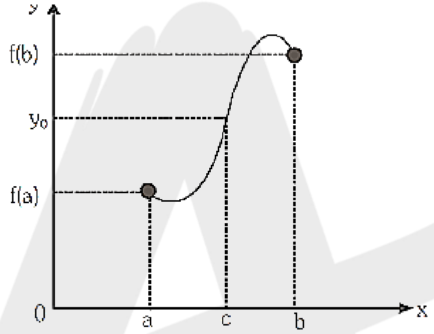
$\Rightarrow \lim_{x\to 0^+} f(x) \neq \lim_{x\to 0^-} f(x)$. Thus f(x) has non-removable discontinuity.

Do yourself - 4:

(i) Discuss the type of discontinuity for $f(x) = \begin{cases} -1 & ; \quad x \le -1 \\ |x| & ; \quad -1 < x < 1 \\ (x+1) & ; \quad x \ge 1 \end{cases}$

4.THE INTERMEDIATE VALUE THEOREM :

Suppose f(x) is continuous on an interval I, and a and b are any two points of I. Then if y_0 is a number between f(a) and f(b), there exists a number c between a and b such that $f(c) = y_0$



The function f, being continuous on [a,b] takes on every value between f(a) and f(b)

Note that a function f which is continuous in [a, b] possesses the following properties:
(i) If f(a) and f(b) posses opposite signs, then there exists at least one root of the equation $f(x) = 0$ in the open interval (a, b).
(ii) If K is any real number between f(a) and f(b), then there exists at least one root of the equation $f(x) = K$ in the open interval (a, b).

Note: In above cases the number of roots is always odd.

Illustration 6:

Show that the function, $f(x) = (x-a)^2(x-b)^2 + x$, takes the value $\frac{a+b}{2}$ for some $x_0 \in (a, b)$

Solution :

$f(x) = (x-a)^2(x-b)^2 + x$

$f(b) = b$

$f(a) = a$

and $\dfrac{a+b}{2} \in (f(a), f(b))$

$\therefore$ By intermediate value theorem, there is at least one $x_0 \in (a, b)$ such that $f(x_0) = \frac{a+b}{2}$.

Illustration 7:

Let $f: [0,1] \xrightarrow{\text{onto}} [0,1]$ be a continuous function, then prove that $f(x) = x$ for atleast one $x \in [0,1]$

Solution :

Consider $g(x) = f(x) - x$

$g(0) = f(0) - 0 = f(0) \geq 0 \{\because 0 \leq f(x) \leq 1\}$

$g(1) = f(1) - 1 \leq 0$

$\Rightarrow g(0) \cdot g(1) \leq 0$

$\Rightarrow g(x) = 0$ has at least one root in $[0,1]$

$\Rightarrow f(x) = x$ for at least one $x \in [0,1]$

Do yourself - 5:

(i) If $f(x)$ is continuous in $[a, b]$ such that $f(c) = \frac{2f(a)+3f(b)}{5}$, then prove that $c \in (a, b)$

5. SOME IMPORTANT POINTS :

(a) If $f(x)$ continuous and $g(x)$ is discontinuous at $x = a$ then the product function $\phi(x) = f(x) \cdot g(x)$ will not necessarily be discontinuous at $x = a$, e.g.

$$f(x) = x \text{ and } g(x) = \begin{cases} \sin\frac{\pi}{x} & x \neq 0 \\ 0 & x = 0 \end{cases}$$

$f(x)$ is continuous at $x = 0$ and $g(x)$ is discontinuous at $x = 0$. but $f(x) \cdot g(x)$ is continuous at $x = 0$.

(b) If $f(x)$ and $g(x)$ both are discontinuous at $x = a$ then the product function $\phi(x) = f(x) \cdot g(x)$ is not necessarily be discontinuous at $x = a$, e..

$$f(x) = -g(x) = \begin{cases} 1 & x \geq 0 \\ -1 & x < 0 \end{cases}$$

$f(x)$ and $g(x)$ both are discontinuous at $x = 0$ but the product function $f(x) \cdot g(x)$ is still continuous at $x = 0$

(c) If $f(x)$ and $g(x)$ both are discontinuous at $x = a$ then $f(x) \pm g(x)$ is not necessarily be discontinuous at $x = a$

(d) A continuous function whose domain is closed must have a range also in closed interval.

(e) If f is continuous at $x = a$ and g is continuous at $x = f(a)$ then the composite $g[f(x)]$ is continuous at $x = a$. eg. $f(x) = \frac{x\sin x}{x^2+2}$ and $g(x) = |x|$ are continuous at $x = 0$, hence the composite $(g \circ f)(x) = \left|\frac{x\sin x}{x^2+2}\right|$ will also be continuous at $x = 0$

Illustration 8:

If $f(x) = \frac{x+1}{x-1}$ and $g(x) = \frac{1}{x-2}$, then discuss the continuity of $f(x), g(x)$ and $f \circ g(x)$ in R.

Solution : $f(x) = \frac{x+1}{x-1}$

$f(x)$ is a rational function it must be continuous in its domain and f is not defined at $x = 1$.

$\therefore$ f is discontinuous at $x = 1$

$$g(x) = \frac{1}{x-2}$$

$g(x)$ is also a rational function. It must be continuous in its domain and g is not defined at $x = 2$.

$\therefore$ g is discontinuous at $x = 2$

Now $f \circ g(x)$ will be discontinuous at $x = 2$ (point of discontinuity of $g(x)$)

Consider $g(x) = 1$ (when $g(x) =$ point of discontinuity of $f(x)$)

$$\frac{1}{x-2} = 1 \Rightarrow x = 3$$

$\therefore f \circ g(x)$ is discontinuous at $x = 2$ and $x = 3$.

Do yourself - 6 :

(i) Let $f(x) = [x]$ and $g(x) = \text{sgn}(x)$ (where [.] denotes greatest integer function), then discuss the continuity of $f(x) \pm g(x), f(x) \cdot g(x)$ and $\frac{f(x)}{g(x)}$ at $x = 0$.

(ii) If $f(x) = \sin|x|$ and $g(x) = \tan|x|$ then discuss the continuity of $f(x) \pm g(x); \frac{f(x)}{g(x)}$ and $f(x)g(x)$

6. SINGLE POINT CONTINUITY :

Functions which are continuous only at one point are said to exhibit single point continuity

Illustration 9

: If $f(x) = \begin{cases} x & \text{if } x \in Q \\ -x & \text{if } x \notin Q \end{cases}$, find the points where $f(x)$ is continuous

Solution : Let $x = a$ be the point at which $f(x)$ is continuous.

$$\Rightarrow \lim_{\substack{x\to a \\ \text{through rational}}} f(x) = \lim_{\substack{x\to a \\ \text{through irrational}}} f(x)$$

$$\Rightarrow a = -a \Rightarrow a = 0 \Rightarrow \text{ function is continuous at } x = 0.$$

Do yourself - 7:

(i) If $g(x) = \begin{cases} x & \text{if } x \in Q \\ 0 & \text{if } x \notin Q \end{cases}$, then find the points where function is continuous.

(ii) If $f(x) = \begin{cases} x^2 & ; \quad x \in Q \\ 1 - x^2 & ; \quad x \notin Q \end{cases}$, then find the points where function is continuous.

ANSWERS FOR DO YOURSELF

1. (i) 1 (ii) discontinuous at $x = -2$

2. (i) $a = -1$ and $b = 1$ (ii) Discontinuous at $x = 1$ and continuous at $x = \frac{\pi}{2}$

3. (i) Non-removable infinite type discontinuity at $x = 1$, isolated point removable discontinuity at $x = 4$.

4. (i) Finite type non-removable discontinuity at $x = -1, 1$

6. (i) All are discontinuous at $x = 0$.

(ii) $f(x)g(x)$ and $f(x) \pm g(x)$ are discontinuous at

$x = (2n+1)\frac{\pi}{2}; n \in I$ $\frac{f(x)}{g(x)}$ is discontinuous at $x = \frac{n\pi}{2}; n \in I$

7. (i) $x = 0$

(ii) $x = \pm\frac{1}{\sqrt{2}}$

CONTINUITY
EXERCISE # 1
[SINGLE CORRECT CHOICE TYPE]

1. Let $f(x) = \begin{cases} ax+1 & \text{if } x<1 \\ 3 & \text{if } x=1 \\ bx^2+1 & \text{if } x>1 \end{cases}$ If f(x) is continuous at x = 1 then (a − b) is equal to-

(A) 0 (B) 1 (C) 2 (D) 4

2. For the function $f(x) = \frac{1}{x+2^{\left(\frac{1}{x-2}\right)}}$, $x \neq 2$ which of the following holds ?

(A) $f(2) = 1/2$ and f is continuous at $x = 2$ (B) $f(2) \neq 0, 1/2$ and f is continuous at $x = 2$

(C) f can not be continuous at $x = 2$ (D) $f(2) = 0$ and f is continuous at $x = 2$.

3. The function $f(x) = \frac{4-x^2}{4x-x^3}$, is-

(A) Discontinuous at only one point in its domain.

(B) Discontinuous at two points in its domain.

(C) Discontinuous at three points in its domain.

(D) Continuous everywhere in its domain.

4. If $f(x) = \frac{x^2-bx+25}{x^2-7x+10}$ for $x \neq 5$ and f is continuous at $x = 5$, then f(5) has the value equal to-

(A) 0 (B) 5 (C) 10 (D) 25

5. If $f(x) = \frac{x-e^x+\cos 2x}{x^2}$, $x \neq 0$ is continuous at $x = 0$, then -

(A) $f(0) = \frac{5}{2}$ (B) $[f(0)] = -2$ (C) $\{f(0)\} = -0.5$ (D) $[f(0)].\{f(0)\} = -1.5$

where [.]and{.}denotesgreatestintegerandfractionalpartfunction

6. $y = f(x)$ is a continuous function such that its graph passes through $(a, 0)$.

Then $\lim_{x\to a} \frac{\log_e(1+3f(x))}{2f(x)}$ is-

(A) 1 (B) 0 (C) $\frac{3}{2}$ (D) $\frac{2}{3}$

7. In [1,3], the function $[x^2+1]$, [.] denoting the greatest integer function, is continuous -

(A) For all x (B) For all x except at nine points

(C) For all x except at seven points (D) For all x except at eight points

8. Number of points of discontinuity of $f(x) = [2x^3 - 5]$ in [1,2), is equal to(Where [x] denotes greatest integer less than or equal to x)

(A) 14 (B) 13 (C) 10 (D) 8

9. Given $f(x) = \begin{cases} |x+1| & \text{if} \quad x<-2 \\ 2x+3 & \text{if} \quad -2\le x<0 \\ x^2+3 & \text{if} \quad 0\le x<3 \\ x^3-15 & \text{if} \quad x\ge 3 \end{cases}$. Then number of point(s) of discontinuity of f(x) is-

(A) 0 (B) 1 (C) 2 (D) 3

10. If f(x) is continuous and $f\left(\frac{9}{2}\right) = \frac{2}{9}$, then the value of $\lim_{x\to 0} f\left(\frac{1-\cos 3x}{x^2}\right)$ is-

(A) $\frac{2}{9}$ (B) $\frac{9}{2}$ (C) 0 (D) data insufficient

11. f is a continuous function on the real line. Given that $x^2 + (f(x) - 2)x - \sqrt{3} \cdot f(x) + 2\sqrt{3} - 3 = 0$. Then the value of $f(\sqrt{3})$

(A) cannot be determined
(B) is $2(1-\sqrt{3})$
(C) is zero
(D) is $\frac{2(\sqrt{3}-2)}{\sqrt{3}}$

12. The function $f(x) = [x]^2 - [x^2]$ (where [y] is the greatest integer less than or equal to y), is discontinuous at :

(A) all integers
(B) all integers except 0&1
(C) all integers except 0
(D) all integers except 1

13. Let $f: R \to R$ be a continuous function $\forall x \in R$ and $f(x) = 5 \forall x \in$ irrational. Then the value of f(3) is -

(A) 1
(B) 2
(C) 5
(D) cannot determine

14. If $f(x) = \frac{1}{(x-1)(x-2)}$ and $g(x) = \frac{1}{x^2}$, then points of discontinuity of $f\{g(x)\}$ are -

(A) $\left\{-1, 0, 1, \frac{1}{\sqrt{2}}\right\}$
(B) $\left\{-\frac{1}{\sqrt{2}}, -1, 0, 1, \frac{1}{\sqrt{2}}\right\}$
(C) $\{0,1\}$
(D) $\left\{0, 1, \frac{1}{\sqrt{2}}\right\}$

15. Consider the function $f(x) = \begin{cases} \frac{x}{[x]} & \text{if} \quad 1 \le x < 2 \\ 1 & \text{if} \quad x = 2 \\ \sqrt{6-x} & \text{if} \quad 2 < x \le 3 \end{cases}$

Where [x] denotes step up function then at x = 2 function -

(A) has missing point removable discontinuity
(B) has isolated point removable discontinuity
(C) has non removable discontinuity finite type
(D) is continuous

16. Consider $f(x) = \begin{cases} x[x]^2 \log_{(1+x)} 2 & \\ \frac{\ln\left(e^{x^2} + 2\sqrt{\{x\}}\right)}{\tan \sqrt{x}}, & \text{for } -1 < x < 0 \\ & \text{for } 0 < x < 1 \end{cases}$

where $[*]\&\{\,^*\}$ are the greatest integer function & fractional part function respectively, then :-

(A) $f(0) = \ln\ 2 \Rightarrow$ f is continuous at $x = 0$
(B) $f(0) = 2 \Rightarrow$ f is continuous at $x = 0$
(C) $f(0) = e^2 \Rightarrow$ f is continuous at $x = 0$
(D) f has an irremovable discontinuity at $x = 0$

17. The function $f(x) = [x] \cdot \cos \frac{2x-1}{2}\pi$, where $[\cdot]$ denotes the greatest integer function, is discontinuous at :-

(A) all x
(B) all integer points
(C) no x
(D) x which is not an integer

18. Consider the function defined on $[0,1] \to R$, $f(x) = \frac{\sin x - x\cos x}{x^2}$ if $x \ne 0$ and $f(0) = 0$, then the function f(x) :-

(A) has a removable discontinuity at $x = 0$
(B) has a non removable finite discontinuity at $x = 0$
(C) has a non removable infinite discontinuity at $x = 0$
(D) is continuous at $x = 0$

19. Which one of the following function is discontinuous for atleast one real value of x ?

(A) $f(x)=\sqrt{1+\operatorname{sgn}\ x}$
(B) $g(x)=\frac{e^x+1}{e^x+3}$
(C) $h(x)=\left(\frac{2^{2x}+1}{2^{3x}+5}\right)^{\frac{5}{7}}$
(D) $k(x)=\sqrt{3+2\sin\ x}$

[Note : sgn x denotes signum function of x.]

20. Let $f(x)=\frac{\sqrt{x^2+px+1}}{x^2-p}$. If f(x) is discontinuous at exactly 2 values of x and continuous for all remaining value of $x\in R$, then number of integers in the range of p is

(A) 1 (B) 2 (C) 3 (D) 4

21. Let $f(x)=\begin{cases}\frac{(1+\tan\ x)^{\frac{1}{x}}-e}{x}, & \text{if } x\neq 0\\ k, & \text{if } x=0\end{cases}$

If f(x) is continuous at $x=0$, then the value of k is

(A) e (B) $\frac{-e}{2}$ (C) $\frac{-e}{4}$ (D) None

22. Let $f(x)=\begin{cases} pxx^2-px+q, & x<1\\ x-1, & 1\leq x\leq 3\\ lx^2+mx+2, & x>3\end{cases}$

If f(x) is continuous $\forall x\in R$, then the value of $\frac{ql-m}{l}$ is equal to

(A) 1 (B) 2 (C) 3 (D) 4

23. Let f(x) and g(x) are continuous function on [a, b] such that $f(a)\geq g(a)$ and $f(b)\leq g(b)$. Then

(A) $f(x_0)=g(x_0)$ for exactly one $x_0\in[a,b]$.
(B) $f(x_0)=g(x_0)$ for atleast one $x_0\in[a,b]$.
(C) $f(x_0)=g(x_0)$ for no values of $x_0\in[a,b]$.
(D) $f(x_0)=g(x_0)$ for infinitely many values of $x_0\in[a,b]$.

24. If f(x) is continuous at $x=0$, then α can be

(A) 1 (B) 2 (C) 3 (D) 4

25. Number of points of discontinuity of the function $f(x)=[\cos^{-1}(\cos\ x)-\sin^{-1}(\sin\ x)]$ in $x\in\left[\frac{-\pi}{2},\frac{\pi}{2}\right]$, is

[Note : [k] denotes the largest integer less than or equal to k.]

(A) 1 (B) 2 (C) 3 (D) 4

26. Let f: [0,1] → R be a continuous function and assumes only rational values. If $f(0)=2$ then the value of $\tan^{-1}\left(f\left(\frac{1}{2}\right)\right)+\tan^{-1}\left(\frac{3}{2}f\left(\frac{1}{2}\right)\right)$ is

(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{6}$ (C) $\frac{3\pi}{4}$ (D) π

27. $f(x)=\begin{cases}x+1; & x<0\\ \cos\ x; & x\geq 0\end{cases}$ at $x=0$, f(x) is

(A) continuous
(B) having removable discontinuity
(C) discontinuous
(D) none

EXERCISE - 2

1. If the function $f(x)=\frac{3x^2+ax+a+3}{x^2+x-2}$ is continuous at $x=-2$. Find $f(-2)$.

2. Find all possible values of a and b so that f(x) is continuous for all $x \in R$ if

$$f(x)=\begin{cases} |ax+3| & \text{if } \quad x\le -1 \\ |3x+a| & \text{if } \quad -1<x\le 0 \\ \dfrac{b\sin\ 2x}{x}-2b & \text{if } \quad 0<x<\pi \\ \cos^2\ x-3 & \text{if } \quad x\ge\pi \end{cases}$$

3. Determine the values of ' a ' & ' b ', if f is continuous at $x=\pi/2$.

4. Suppose that $f(x)=x^3-3x^2-4x+12$ and $h(x)=\left|\begin{array}{ll} \frac{f(x)}{x-3} & ,x\neq 3 \\ K & ,x=3 \end{array}\right.$ then

(a) find all zeroes of f(x).

(b) find the value of K that makes h continuous at $x=3$.

(c) using the value of K found in (b) determine whether h is an even function.

5. Let $f(x)=\left|\begin{array}{ll} \frac{1-\sin\ \pi x}{1+\cos\ 2\pi x}, & x<\frac{1}{2} \\ p, & x=\frac{1}{2} \\ \frac{\sqrt{2x-1}}{\sqrt{4+\sqrt{2x-1}-2}}, & x>\frac{1}{2} \end{array}\right.$. Determine the value of p, if possible, so that the function is continuous at $x=1/2$.

6. Given the function $g(x)=\sqrt{6-2x}$ and $h(x)=2x^2-3x+a$. Then

(a) evaluate $h(g(2))$

(b) If $f(x)=\left[\begin{array}{ll} g(x), & x\le 1 \\ h(x), & x>1 \end{array}\right.$, find 'a' so that f is continuous

7. Let $f(x)=\left|\begin{array}{ll} 1+x, & 0\le x\le 2 \\ 3-x, & 2<x\le 3 \end{array}\right.$. Determine the form of $g(x)=f[f(x)]$& hence find the point of discontinuity of g, if any.

8. Let $f(x)=\left[\begin{array}{ll} \frac{\ell\text{los}\ x}{\sqrt[4]{1+x^2}-1} & \text{if } x>0 \\ \frac{e^{\sin\ 4x}-1}{\ell\text{n}\ (1+\tan\ 2x)} & \text{if } x<0 \end{array}\right.$

Is it possible to define f(0) to make the function continuous at $x=0$. If yes what is the value of f(0), if not then indicate the nature of discontinuity.

9. Determine a& b so that f is continuous at $x=\frac{\pi}{2}$ where $f(x)=\left|\begin{array}{lll} \frac{1-\sin^3\ x}{3\cos^2\ x} & & \text{if } x<\frac{\pi}{2} \\ a & & \text{if } \ x=\frac{\pi}{2} \\ \frac{b(1-\sin\ x)}{(\pi-2x)^2} & \text{if} & x>\frac{\pi}{2} \end{array}\right.$

10. Determine the values of a, b & c for which the function

$$f(x)=\left|\begin{array}{lll} \frac{\sin\ (a+1)x+\sin\ x}{x} & \text{for} & x<0 \\ c & \text{for} & x=0 \\ \frac{(x+bx^2)^{1/2}-x^{1/2}}{bx^{3/2}} & \text{for} & x>0 \end{array}\right.$$

is continuous at $x=0$.

EXERCISE - 3 (JM)

1. The function $f: R/\{0\} \rightarrow R$ given by $f(x) = \frac{1}{x} - \frac{2}{e^{2x}-1}$ can be made continuous at $x = 0$ by defining $f(0)$ as- **[AIEEE 2007]**

(A) 2 (B) -1 (C) 0 (D) 1

2. The values of p and q for which the function $f(x) = \begin{cases} \frac{\sin(p+1)x + \sin x}{x}, & x < 0 \\ q & , x = 0 \\ \frac{\sqrt{x+x^2}-\sqrt{x}}{x^{\frac{3}{2}}} & , x > 0 \end{cases}$ is continuous for all x in R are :- **[AIEEE 2011]**

(A) $p = -\frac{3}{2}, q = \frac{1}{2}$ (B) $p = \frac{1}{2}, q = \frac{3}{2}$ (C) $p = \frac{1}{2}, q = -\frac{3}{2}$ (D) $p = \frac{5}{2}, q = \frac{1}{2}$

3. Define $F(x)$ as the product of two real functions $f_1(x) = x, x \in \mathbb{R}$, and $f_2(x) = \begin{cases} \sin\frac{1}{x}, & \text{if} \quad x \neq 0 \\ 0, & \text{if} \quad x = 0 \end{cases}$ as follows : $F(x) = \begin{cases} f_1(x) \cdot f_2(x) & \text{if} \quad x \neq 0 \\ 0, & \text{if} \quad x = 0 \end{cases}$ **[AIEEE 2011]**

Statement-1: $F(x)$ is continuous on $\mathbb{R}$.

Statement-2 : $f_1(x)$ and $f_2(x)$ are continuous on $\mathbb{R}$.

(A) Statemen-1 is false, statement- 2 is true.

(B) Statemen-1 is true, statement-2 is true; Statement-2 is correct explanation for statement.

(C) Statement-1 is true, statement-2 is true, statement- 2 is not a correct explanation for statement 1

(D) Statement-1 is true, statement-2 is false

4. If $f(x)$ is continuous and $f(9/2) = 2/9$, then $\lim_{x\to 0} f\left(\frac{1-\cos 3x}{x^2}\right)$ is equal to: **[JEE Mains 2014]**

(A) 9/2 (B) 0 (C) 2/9 (D) 8/9

5. If the function $f(x) = \begin{cases} \frac{\sqrt{2+\cos x}-1}{(\pi-x)^2}, & x \neq \pi \\ k, & x = \pi \end{cases}$ is continuous at $x = \pi$, then k equals:-

(A) $\frac{1}{4}$ (B) $\frac{1}{2}$ (C) 2 (D) 0 **[JEE Mains 2014]**

6. Let $f: R \rightarrow R$ be a function defined as : $f(x) = \begin{cases} 5, & \text{if} \quad x \leq 1 \\ a + bx, & \text{if} \quad 1 < x < 3 \\ b + 5x, & \text{if} \quad 3 \leq x < 5 \\ 30, & \text{if} \quad x \geq 5 \end{cases}$ Then, f is :

(A) continuous if $a = 5$ and $b = 5$ (B) continuous if $a = -5$ and $b = 10$ **[JEE Mains 2019]**

(C) continuous if $a = 0$ and $b = 5$ (D) not continuous for any values of a and b

7. If a function f(x) defined by $f(x)=\begin{cases} ae^x+be^{-x}, & -1\le x<1 \\ cx^2 & 1\le x\le 3 \\ ax^2+2cx, & 3<x\le 4\end{cases}$ be continuous for some $a, b, c \in R$ and $f'(0)+f'(2)=e$, then the value of a is : **(JEE Main 2020)**

(A) $\frac{e}{e^2+3e+13}$ (B) $\frac{e}{e^2-3e+13}$ (C) $\frac{e}{e^2-3e-13}$ (D) $\frac{1}{e^2-3e+13}$

8. If the function $f(x)=\begin{cases} \frac{1}{x}\log_e\left(\frac{1+\frac{x}{a}}{1-\frac{x}{b}}\right), & ,x<0 \\ k & ,x=0 \\ \frac{\cos^2 x-\sin^2 x-1}{\sqrt{x^2+1}-1}, & x>0\end{cases}$ is continuous at $x=0$, then $\frac{1}{a}+\frac{1}{b}+\frac{4}{k}$ is equal to :

(A) -5 (B) 5 (C) -4 (D) 4 **(JEE Main 2021)**

9. Let $f:\mathbb{R}\to\mathbb{R}$ be defined as $f(x)=\begin{cases} [e^x], & x<0 \\ ae^x+[x-1], & 0\le x<1 \\ b+[\sin(\pi x)], & 1\le x<2 \\ [e^{-x}]-c, & x\ge 2\end{cases}$

where $a, b, c \in \mathbb{R}$ and [t] denotes greatest integer less than or equal to t. Then, which of the following statements is true ? **(JEE Main 2022)**

(A) There exists $a, b, c \in \mathbb{R}$ such that f is continuous of $\mathbb{R}$.
(B) If f is discontinuous at exactly one point, then $a+b+c=1$.
(C) If f is discontinuous at exactly one point, then $a+b+c\neq 1$.
(D) f is discontinuous at atleast two points, for any values of a, b and c.

10. Let $f(x)=\begin{cases} x^2\sin\left(\frac{1}{x}\right) & ,x\neq 0 \\ 0 & ,x=0\end{cases}$; Then at $x=0$ **(JEE Main 2023)**

(A) f is continuous but not differentiable
(B) f is continuous but f' is not continuous
(C) f and f' both are continuous
(D) f' is continuous but not differentiable

11. If the function $f(x)=\begin{cases} (1+|\cos x|)^{\frac{\lambda}{|\cos x|}}, & 0<x<\frac{\pi}{2} \\ \mu & ,x=\frac{\pi}{2} \\ e^{\frac{\cot 6x}{\cot 4x}} & ,\frac{\pi}{2}<x<\pi\end{cases}$

is continuous at $x=\frac{\pi}{2}$, then $9\lambda+6\log_e\mu+\mu^6-e^{6\lambda}$ is equal to **(JEE Main 2023)**

(A) 11 (B) 8 (C) $2e^4+8$ (D) 10

EXERCISE - 4 (JA)

SECTION-1

1. For every integer n, let a_n and b_n be real numbers. Let function $f: \mathbb{R} \to \mathbb{R}$ be given by

$f(x) = \begin{cases} a_n + \sin \pi x, & \text{for } x \in [2n, 2n+1] \\ b_n + \cos \pi x, & \text{for } x \in (2n-1, 2n) \end{cases}$, for all integers n If f is continuous, then which of the following holds(s) for all n ? **[JEE 2012]**

(A) $a_{n-1} - b_{n-1} = 0$
(B) $a_n - b_n = 1$
(C) $a_n - b_{n+1} = 1$
(D) $a_{n-1} - b_n = -1$

2. For every pair of continuous function $f, g: [0,1] \to \mathbb{R}$ such that

$\max\{f(x): x \in [0,1]\} = \max\{g(x): x \in [0,1]\}$ the correct statement(s) is(are) :

(A) $(f(c))^2 + 3f(c) = (g(c))^2 + 3g(c)$ for some $c \in [0,1]$ **[JEE(Advanced)-2014]**

(B) $(f(c))^2 + f(c) = (g(c))^2 + 3g(c)$ for some $c \in [0,1]$
(C) $(f(c))^2 + 3f(c) = (g(c))^2 + g(c)$ for some $c \in [0,1]$
(D) $(f(c))^2 = (g(c))^2$ for some $c \in [0,1]$

3. Let [x] be the greatest integer less than or equals to x. Then, at which of the following point(s) the function $f(x) = x\cos(\pi(x + [x]))$ is discontinuous ? **[JEE(Advanced)-2017]**

(A) $x = -1$ (B) $x = 0$ (C) $x = 2$ (D) $x = 1$

SECTION-2

4. Discuss the continuity of the function $f(x) = \begin{cases} \frac{e^{1/(x-1)} - 2}{e^{1/(x-1)} + 2}, & x \neq 1 \\ 1, & x = 1 \end{cases}$ at $x = 1$.

[REE 2001 (Mains), 3 out of 100]

EXERCISE - 5
[MULTIPLE CORRECT CHOICE TYPE]

1. Which of the following function(s) is/are discontinuous at $x=0$?
(A) $f(x)=\sin\frac{\pi}{2x}, x\neq 0$ and $f(0)=1$
(B) $g(x)=x\sin\left(\frac{\pi}{x}\right), x\neq 0$ and $g(0)=\pi$
(C) $h(x)=\frac{|x|}{x}, x\neq 0$ and $h(0)=1$
(D) $k(x)=\frac{1}{1+e^{\cot x}}, x\neq 0$ and $k(0)=0$.

2. A function $f(x)$ is defined as $f(x)=\frac{A\sin x+\sin 2x}{x^3}, (x\neq 0)$. If the function is continuous at $x=0$, then -
(A) $A=-2$
(B) $f(0)=-1$
(C) $A=1$
(D) $f(0)=1$

3. Which of the following function(s) not defined at $x=0$ has/have non-removable discontinuity at the point $x=0$?
(A) $f(x)=\frac{1}{1+2^{\frac{1}{x}}}$
(B) $f(x)=\arctan\frac{1}{x}$
(C) $f(x)=\frac{e^{\frac{1}{x}}-1}{e^{\frac{1}{x}}+1}$
(D) $f(x)=\frac{1}{\ln|x|}$

4. Which of the following function(s) not defined at $x=0$ has/have removable discontinuity at $x=0$?
(A) $f(x)=\frac{1}{1+2^{\cot x}}$
(B) $f(x)=\cos\left(\frac{|\sin x|}{x}\right)$
(C) $f(x)=x\sin\frac{\pi}{x}$
(D) $f(x)=\frac{1}{\ell n|x|}$

5. Let $f(x)=\begin{cases}\frac{e^x-1+ax}{x^2}, & x>0\\ b, & x=0\\ \frac{\sin\frac{x}{2}}{x}, & x<0\end{cases}$, then -
(A) $f(x)$ is continuous at $x=0$ if $a=-1, b=\frac{1}{2}$.
(B) $f(x)$ is discontinuous at $x=0$, if $b\neq\frac{1}{2}$.
(C) $f(x)$ has non-removable discontinuity at $x=0$ if $a\neq -1$.
(D) $f(x)$ has removable discontinuity at $x=0$ if $a=-1, b\neq\frac{1}{2}$

6. Which of the following function(s) has removable type of discontinuity at $x=0$?
(A) $f(x)=\frac{1-\sec^2 2x}{4x^2}$
(B) $g(x)=\frac{\csc x-1}{x\csc x}$ (where $\csc x=\operatorname{cosec} x$)
(C) $h(x)=\frac{\sin 5x}{x}$
(D) $\ell(x)=(1+2x^2)^{\frac{1}{x^2}}$

7. If f is defined on an interval $[a,b]$. Which of the following statement(s) is/are INCORRECT ?
(A) If $f(a)$ and $f(b)$, have opposite sign, then there must be a point $c\in(a,b)$ such that $f(c)=0$.
(B) If f is continuous on $[a,b]$, $f(a)<0$ and $f(b)>0$, then there must be a point $c\in(a,b)$ such that $f(c)=0$.
(C) If f is continuous on $[a,b]$ and there is a point c in (a,b) such that $f(c)=0$, then $f(a)$ and $f(b)$ have opposite sign.
(D) If f has no zeroes on $[a,b]$, then $f(a)$ and $f(b)$ have the same sign.

8. Which of the following functions f has/have a removable discontinuity at the indicated point?
(A) $f(x)=\frac{x^2-2x-8}{x+2}$ at $x=-2$
(B) $f(x)=\frac{x-7}{|x-7|}$ at $x=7$
(C) $f(x)=\frac{x^3+64}{x+4}$ at $x=-4$
(D) $f(x)=\frac{3-\sqrt{x}}{9-x}$ at $x=9$

9. In which of the following cases the given equations has atleast one root in the indicated interval?
(A) $x-\cos x=0$ in $(0,\pi/2)$
(B) $x+\sin x=1$ in $(0,\pi/6)$
(C) $\frac{a}{x-1}+\frac{b}{x-3}=0, a, b>0$ in $(1,3)$
(D) $f(x)-g(x)=0$ in (a, b) where f and g are continuous on $[a, b]$ and $f(a)>g(a)$ and $f(b)<g(b)$.

10. Indicate all correct alternatives if, $f(x)=\frac{x}{2}-1$, then on the interval $[0, \pi]$
(A) $\tan (f(x))$ & $\frac{1}{f(x)}$ are both continuous
(B) $\tan (f(x))$ & $\frac{1}{f(x)}$ are both discontinuous
(C) $\tan (f(x))$ & $f^{-1}(x)$ are both continuous
(D) $\tan (f(x))$ is continuous but $\frac{1}{f(x)}$ is not

[MATRIX TYPE]

11.

Column-I	**Column-II**
(A) $\lim_{x\to 1}\frac{x^3-1}{\ell n x}$ is	(P) 2
(B) $\lim_{x\to 0}\frac{x(\cos x-\cos 2x)}{2\sin x-\sin 2x}$ is	(Q) 3
(C) $\lim_{x\to 0}\frac{\tan x\sqrt{\tan x}-\sin x\sqrt{\sin x}}{x^3\sqrt{x}}$ is	(R) $\frac{3}{2}$
(D) If $f(x)=\cos\left(x\cos\frac{1}{x}\right)$ and $g(x)=\frac{\ell n(\sec^2 x)}{x\sin x}$ are	(S) $\frac{3}{4}$

both continuous at $x=0$ then $f(0)+g(0)$ equals

12. Match the function in Column-I with its behaviour at $x=0$ in column-II, where [.] denotes greatest integer function & sgn (x) denotes signum function.

Column-I	**Column-II**
(A) $f(x)=[x][1+x]$	(P) LHL exist at $x=0$
(B) $f(x)=[-x][1+x]$	(Q) RHL exist at $x=0$
(C) $f(x)=(\operatorname{sgn}(x))[2-x][1+\lvert x\rvert]$	(R) Continuous at $x=0$
(D) $f(x)=[\cos x]$	(S) $\lim_{x\to 0} f(x)$ exists but function is discontinuous at $x=0$

EXERCISE - 6

1. If $f(x)=\frac{\sin 3x+A\sin 2x+B\sin x}{x^5}$ $(x\neq 0)$ is cont. at $x=0$. Find A and B. Also find $f(0)$.

2. Let $f(x)=\begin{cases}\frac{\left(\frac{\pi}{2}-\sin^{-1}(1-\{x\}^2)\right)\sin^{-1}(1-\{x\})}{\sqrt{2}(\{x\}-\{x\}^3)} & \text{for } x\neq 0 \text{ where } \{x\} \text{ is the fractional part of } x.\\ \frac{\pi}{2} & \text{for } x=0\end{cases}$ Consider another function g(x); such that

$g(x)=\begin{bmatrix} f(x) & \text{for} & x\geq 0\\ 2\sqrt{2}f(x) & \text{for} & x<0\end{bmatrix}$ Discuss the continuity of the functions $f(x)\&g(x)$ at $x=0$.

3. If $f(x)=x+\{-x\}+[x]$, where $[x]$ is the integral part & $\{x\}$ is the fractional part of x. Discuss the continuity of f in $[-2,2]$.

4. Find the locus of (a, b) for which the function $f(x)=\begin{cases} ax-b & \text{for} & x\leq 1\\ 3x & \text{for} & 1<x<2\\ bx^2-a & \text{for} & x\geq 2\end{cases}$ is continuous at $x=1$ but discontinuous at $x=2$.

5. A function $f: R\to R$ is defined as $f(x)=\lim_{n\to\infty}\frac{ax^2+bx+c+e^{nx}}{1+c\cdot e^{nx}}$ where f is continuous on R. Find the values of a, b and c.

6. Let $f(x)=\begin{cases}(\sin x+\cos x)^{\operatorname{cosec} x} & ; \frac{-\pi}{2}<x<0\\ a & ; x=0\\ \frac{e^{1/x}+e^{2/x}+e^{3/|x|}}{ae^{2/x}+be^{3/|x|}} & ; 0<x<\frac{\pi}{2}\end{cases}$

If f(x) is continuous at $x=0$, find the value of (a^2+b^2).

7. Given $f(x)=\sum_{r=1}^{n}\tan\left(\frac{x}{2^x}\right)\sec\left(\frac{x}{2^{r-1}}\right)$; $r,n\in N$

$$g(x)=\operatorname{limit}_{n\to\infty}\frac{\ln\left(f(x)+\tan\frac{x}{2^n}\right)-\left(f(x)+\tan\frac{x}{2^n}\right)^n\cdot\left[\sin\left(\tan\frac{x}{2}\right)\right]}{1+\left(f(x)+\tan\frac{x}{2^n}\right)^n}$$

$=k$ for $x=\frac{\pi}{4}$ and the domain of g(x) is $(0,\pi/2)$. where [] denotes the greatest integer function. Find the value of k, if possible, so that g(x) is continuous at $x=\pi/4$. Also state the points of discontinuity of g(x) in $(0,\pi/4)$, if any.

8. Let $f(x)=x^3-x^2-3x-1$ and $h(x)=\frac{f(x)}{g(x)}$, where h is a rational function such that (a) it is continuous every where except when $x=-1$,

(b) $\lim_{x\to\infty}h(x)=\infty$ and (c) $\lim_{x\to -1}h(x)=\frac{1}{2}$

Find $\lim_{x\to 0}(3h(x)+f(x)-2g(x))$

9. (a) Let f be a real valued continuous function on R and satisfying $f(-x)-f(x)=0\forall\in R$. If $f(-5)=5, f(-2)=4, f(3)=-2$ and $f(0)=0$ then find the minimum number of zero's of the equation $f(x)=0$

(b) Find the number of points of discontinuity of the function $f(x)=[5x]+\{3x\}$ in $[0,5]$ where $[y]$ and $\{y\}$ denote largest integer less than or equal to y and fractional part of y respectively.

10. (a) If $g:[a,b]\to[a,b]$ is continuous & onto function, then show that there is some $c\in[a,b]$ such that $g(c)=c$.

(b) Let f be continuous on the interval $[0,1]$ to R such that $f(0)=f(1)$. Prove that there exists a point c in $\left[0,\frac{1}{2}\right]$ such that $f(c)=f\left(c+\frac{1}{2}\right)$

ANSWER KEY

CONTINUITY
EXERCISE-1

1.	A	**2.**	C	**3.**	D	**4.**	A	**5.**	D	**6**	C	**7.**	D
8.	B	**9.**	B	**10.**	A	**11.**	B	**12.**	D	**13.**	C	**14.**	B
15.	B	**16.**	D	**17.**	C	**18.**	D	**19.**	A	**20.**	B	**21.**	B
22.	C	**23.**	B	**24.**	A	**25.**	C	**26.**	C	**27.**	A		

EXERCISE-2

1. -1

2. $a = 0, b = 1$

3. $a = 0; b = -1$

4. (a) $-2,2,3$; (b) $K = 5$;(c) even

5. P not possible.

6. (a) $4 - 3\sqrt{2} + a$,(b) $a = 3$

7. $g(x) = 2 + x$ for $0 \le x \le 1, 2 - x$ for $1 < x \le 2, 4 - x$ for $2 < x \le 3$, g is discontinuous at $x = 1$& $x = 2$

8. $f(0^+) = -2; f(0^-) = 2$ hence $f(0)$ not possible to define

9. $a = 1/2, b = 410. a = -3/2, b \neq 0, c = 1/2$

EXERCISE-3 (JM)

1.	D	**2.**	A	**3.**	D	**4.**	C	**5.**	A	**6.**	D
7.	B	**8.**	A	**9.**	C	**10.**	B	**11.**	D		

EXERCISE-4 (JA)
SECTION-1

1.	BD	**2.**	AD	**3.**	ACD

SECTION-2

4. Discontinuous at $x = 1$: $f(1^+) = 1$ and $f(1) = -1$

EXERCISE-5 (JA)

1.	ABCD	**2.**	AB	**3.**	ABC	**4.**	BCD	**5.**	ABCD	**6.**	ACD
7.	ACD	**8.**	ACD	**9.**	ABCD	**10.**	CD				

11. $(A) \to (Q); (B) \to (R); (C) \to (S); (D) \to (P)$

12. $(A) \to (P, Q, R); (B) \to (P, Q, T); (C) \to (P, Q, T); (D) \to (P, Q, S)$

EXERCISE-6 (JA)

1. $A=-4, B=5, f(0)=1$

2. $f(0^+)=\frac{\pi}{2}; f(0^-)=\frac{\pi}{4\sqrt{2}} \Rightarrow$ f is discount. at $x=0$; $g(0^+)=g(0^-)=g(0)=\pi/2 \Rightarrow$ g is cont. at $x=0$

3. discontinuous at all integral values in $[-2,2]$

4. locus $(a,b)\rightarrow x,y$ is $y=x-3$ excluding the points where $y=3$ intersects it.

5. $c=1, a, b\in R$

6. e^2+e^{-2}

7. $k=0; g(x)=\begin{cases} \ell n(\tan x) \text{ if } 0<x<\frac{\pi}{4} \\ 0 \text{ if } \frac{\pi}{4}\le x<\frac{\pi}{2}\end{cases}$. Hence g(x) is continuous everywhere.

8. $g(x)=4(x+1)$ and limit $=-\frac{39}{4}$

9. (a) 5 (b) 30