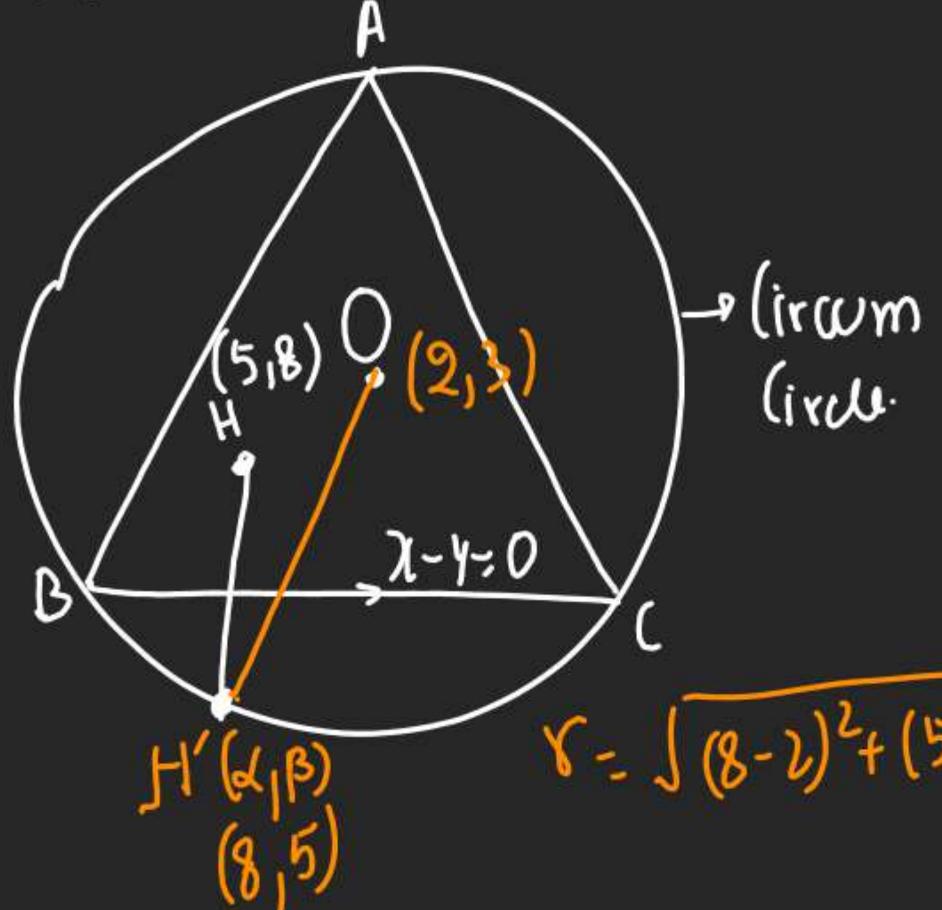
Nishant Jindal

Q In $\triangle ABC$, Line BC is $y=x$

If O (circ.) $(2,3)$ & $H \equiv (5,8)$

find Eqn of Circumcircle.

Concept:- Image of H in Base always lie on Circumcircle.



$$\frac{\alpha-5}{1} = \frac{\beta-8}{-1} = \frac{-2(5-8)}{1^2+(-1)^2}$$

$$\frac{\alpha-5}{1} = \frac{\beta-8}{-1} = 3$$

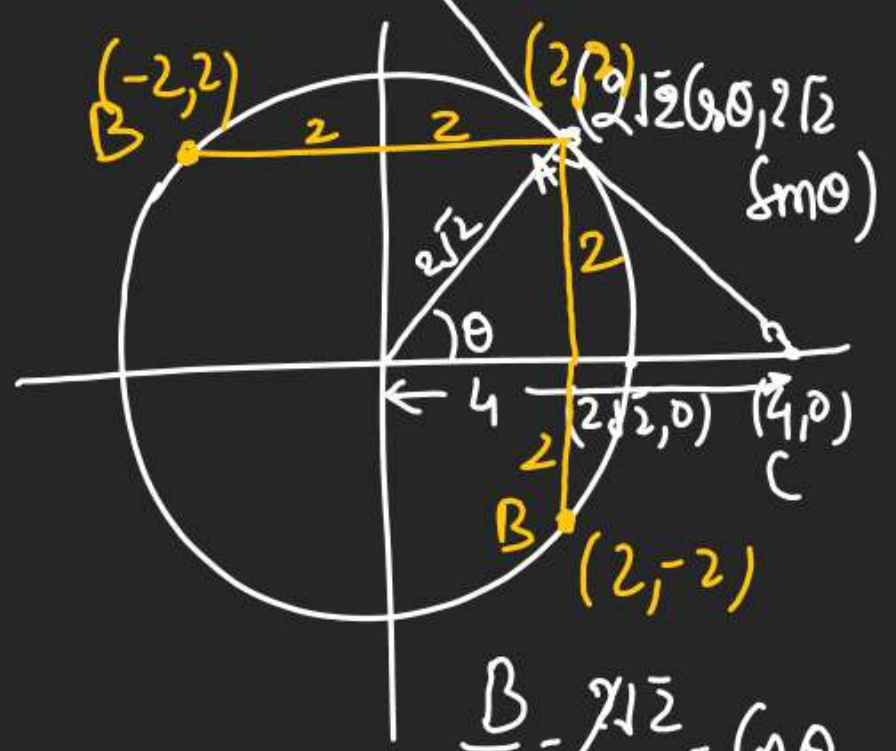
$$\alpha = 8,\ \beta = 5$$

$$(x-2)^2+(y-3)^2 = 40$$

$$r = \sqrt{(8-2)^2+(5-3)^2} = \sqrt{40}$$

Q2 Tangent drawn from $(4,0)$ to circle $x^2+y^2=8 \rightarrow$ touches at Pt. A in 1^{st} Quad. Find Coord of B on circle such that $AB = 4$.

$C \equiv (0,0)$, $R = 2\sqrt{2}$



$$\frac{B}{H} = \frac{2\sqrt{2}}{4} = \cos\theta$$

$$\theta = \frac{\pi}{4}$$

$$\therefore A = \left(2\sqrt{2}\cos\frac{\pi}{4},\ 2\sqrt{2}\sin\frac{\pi}{4}\right)$$

$$A = (2,2)$$

So 4 units down we have B on circle $= (2,-2)$ & $(-2,2)$

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Q Find Eq^n of tangent from $(2,3)$ to Circle $(x^2+y^2=4)$ $\rightarrow a=2$

Position $(2,3) \rightarrow 2^2+3^2-4>0$ outside.

then $y=mx \pm a\sqrt{1+m^2}$ Use

$\therefore$ EOT $\Rightarrow y = mx \pm 2\sqrt{1+m^2}$ P.T. $(2,3)$

$3 = 2m \pm 2\sqrt{1+m^2}$

$3-2m = \pm 2\sqrt{1+m^2}$

$9+4m^2-12m = 4+4m^2$

$(4-4)m^2 - 12m + 5 = 0$ m_1, m_2

So one Root $m \rightarrow \infty$ ✓

$-12m = -5$

$m = \frac{5}{12}$ ✓

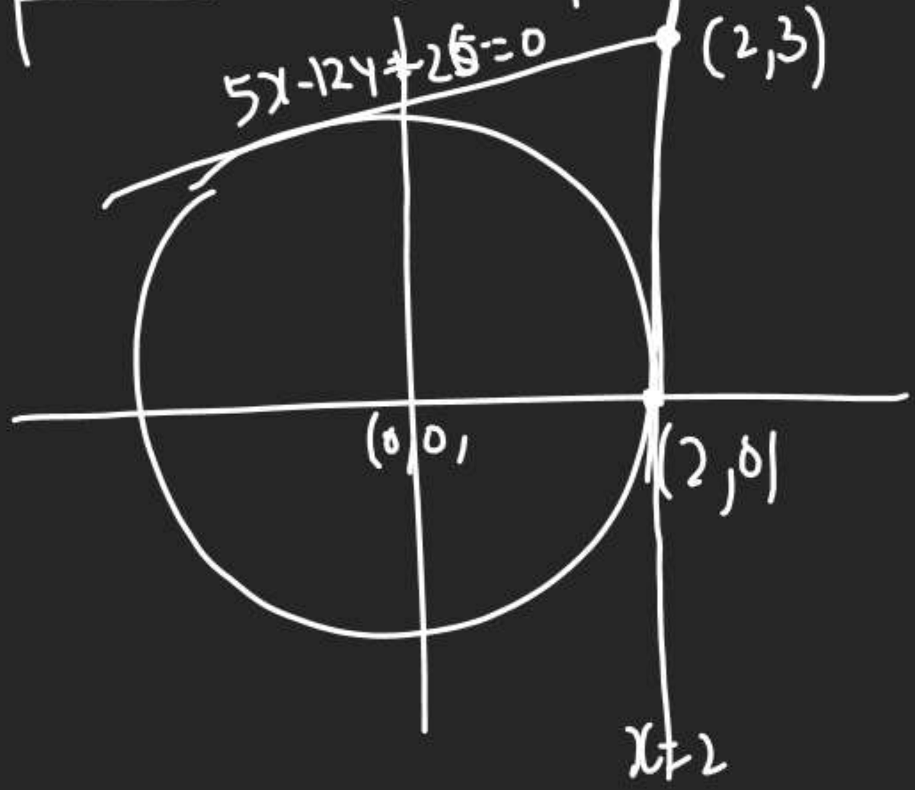
EOT.

$(y-3) = m(x-2)$

$y-3 = \frac{1}{0}(x-2)$ | $y-3 = \frac{5}{12}(x-2)$

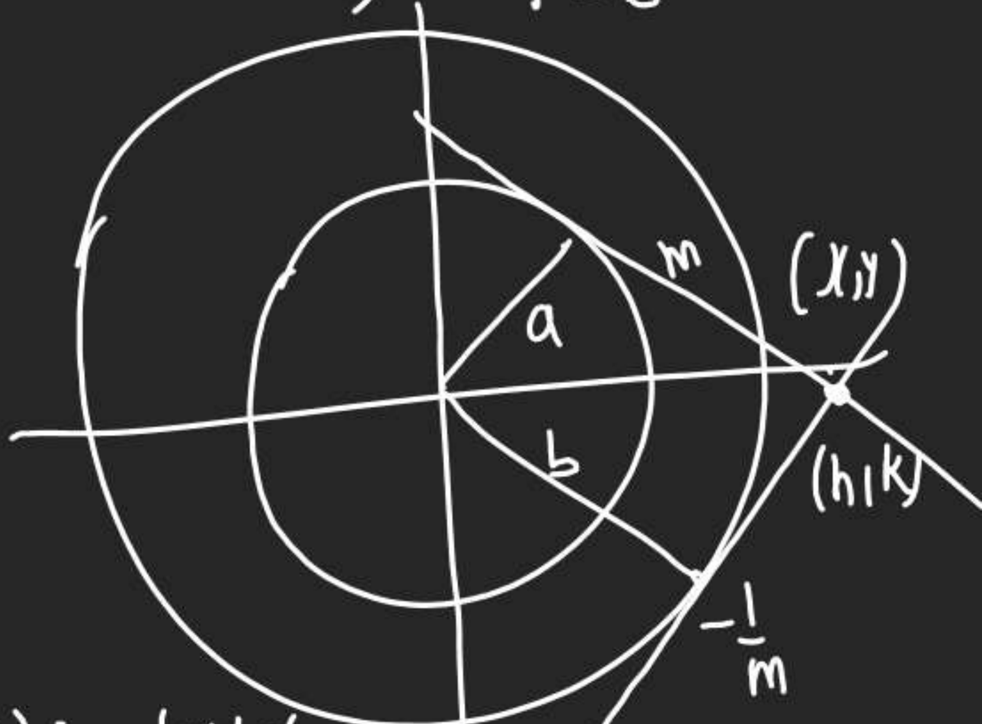
$\boxed{x=2}$

$5x-12y+26=0$



Q4 2 Concentric Circles $x^2+y^2=a^2$, $x^2+y^2=b^2$ tangents are drawn $\perp^r$ find locus of their POI.

$x^2+y^2=a^2$, $x^2+y^2=b^2$



(x,y) is outside from both Circles clearly

$y = mx \pm a\sqrt{1+m^2}$

$y = -\frac{1}{m}x \pm b\sqrt{1+(-1/m)^2}$

$y - mx = \pm a\sqrt{1+m^2} \Rightarrow$

$my + x = \pm b\sqrt{m^2+1}$

$y^2+m^2x^2-2mxy = a^2+a^2m^2$

$m^2y^2+x^2+2mxy = b^2+b^2m^2$

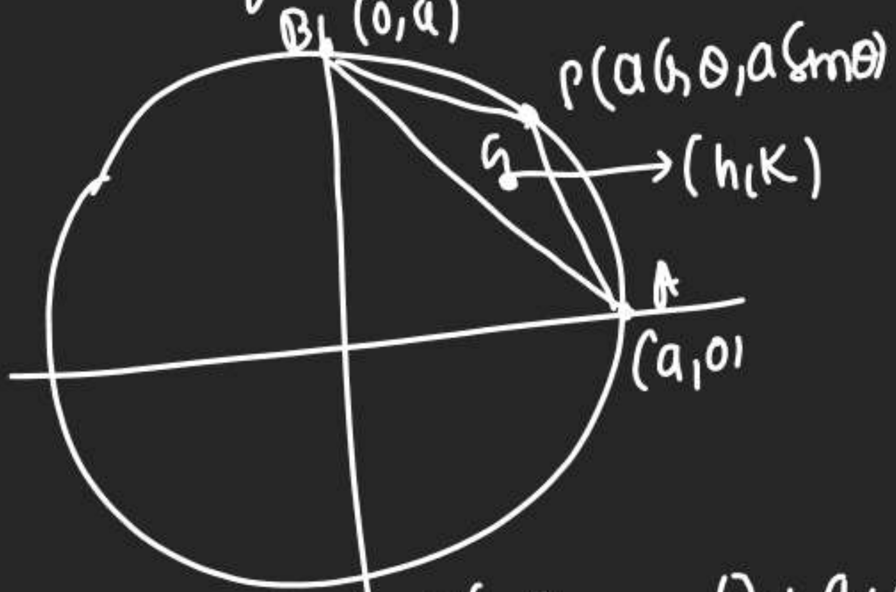
$y^2(1+m^2)+x^2(1+m^2) = (a^2+b^2)+m^2(a^2+b^2)$

$\cancel{(1+m^2)}(x^2+y^2)$

$= (a^2+b^2)\cancel{(1+m^2)}$

$x^2+y^2 = (\sqrt{a^2+b^2})^2$

Q5 If Circle $x^2+y^2=a^2$ Intersect x Axis & y Axis at A & B & P is a Random Pt. on Circle find Locus of Centroid of ΔPAB.



$$h=\frac{a+0+a\cos\theta}{3} \qquad K=\frac{0+a+a\sin\theta}{3}$$

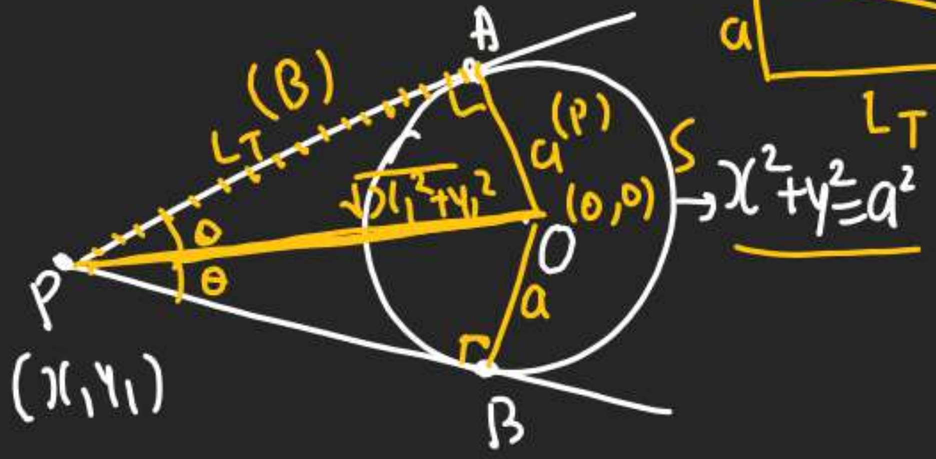
$$3h=a(1+\cos\theta) \qquad 3K=a(1+\sin\theta)$$

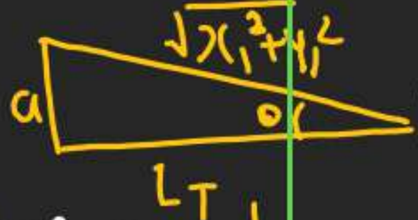
$$\cos\theta=\frac{3h}{a}-1 \qquad \sin\theta=\frac{3K}{a}-1$$

$$s^2+c^2=1 \Rightarrow \left(\frac{3x-a}{a}\right)^2+\left(\frac{3y-a}{a}\right)^2=1$$

$$(3x-a)^2+(3y-a)^2=a^2$$

Length of Tangent





① $PA=PB=L_T$

$$=\sqrt{\left(\sqrt{x_1^2+y_1^2}\right)^2-a^2}$$

$$=\sqrt{x_1^2+y_1^2-a^2}$$

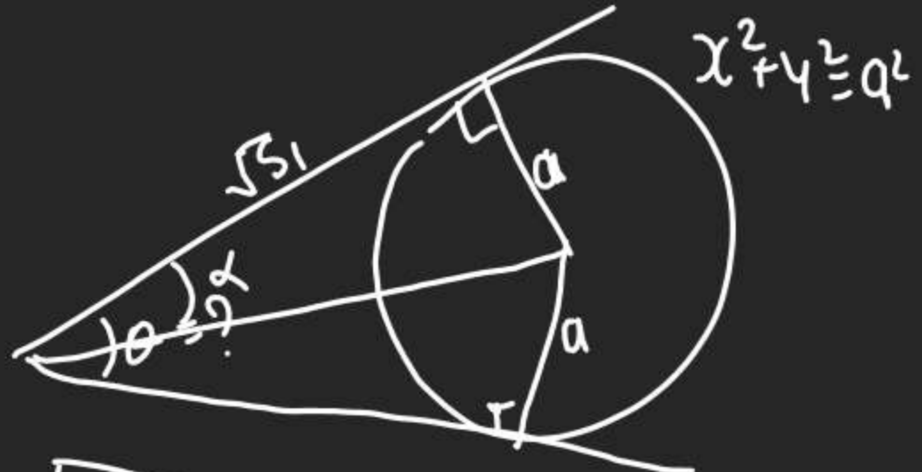
$$=\sqrt{\text{Circle at }(x_1,y_1)}$$

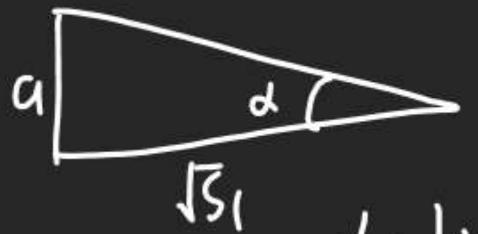
$$L_T=\sqrt{S_1}$$

(2) Rem: → If Circle is $S: x^2+y^2+2gx+2fy+c=0$ & Pt (x_1,y_1) then $L_T=\sqrt{S_1}$

$$=\sqrt{x_1^2+y_1^2+2gx_1+2fy_1+c}$$

Angle Betn tangents



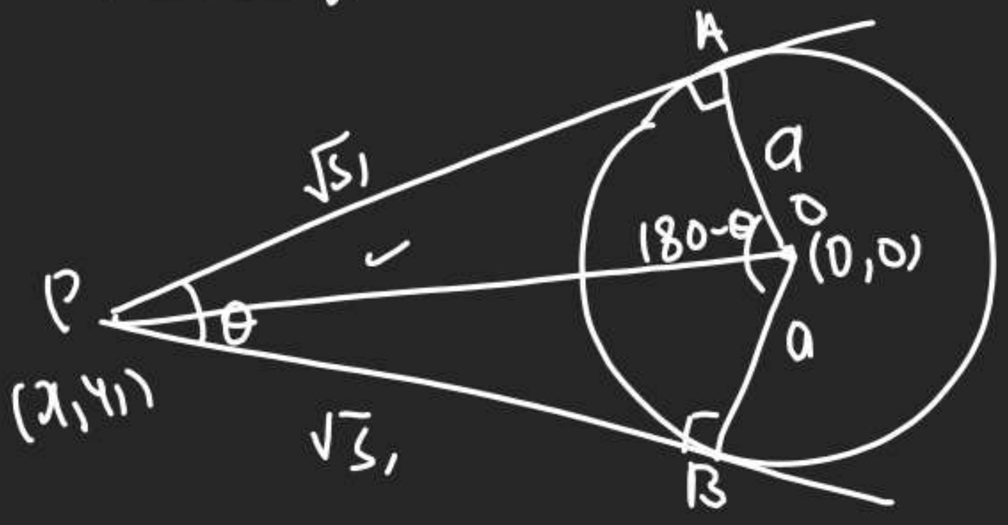


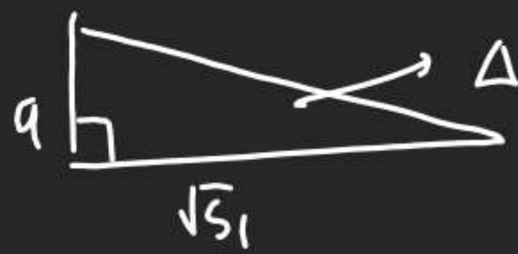
$$\tan\alpha=\frac{a}{\sqrt{S_1}}$$

$$\alpha=\tan^{-1}\frac{a}{\sqrt{S_1}}$$

$$\boxed{\text{angle bet}^n\text{ tangents}=2\tan^{-1}\frac{a}{\sqrt{S_1}}}$$

Area of Quadrilateral POAB



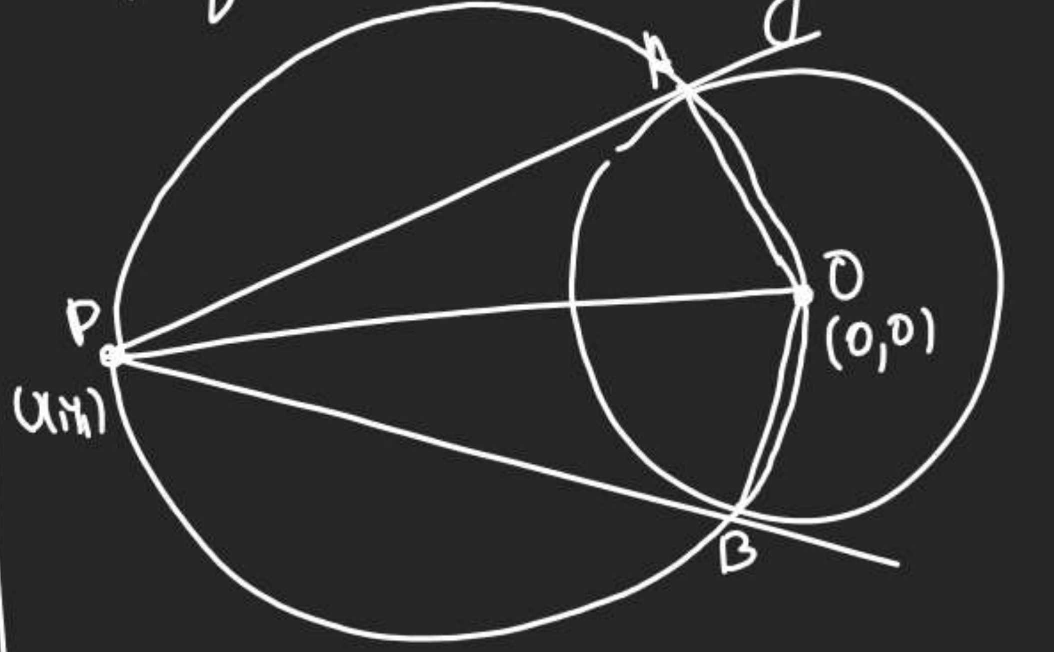


$\Delta = \frac{1}{2}\sqrt{S_1}\, a$

$\square POAB = 2\Delta = 2\times\frac{1}{2}a\sqrt{S_1}$
$= a\sqrt{S_1}$

Rk $\square$ POAB is cyclic Quad. ✓

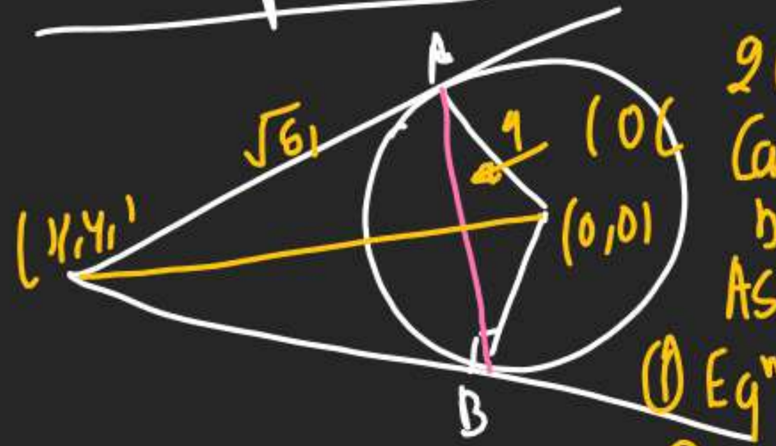
Eqn of Circle Circumscribing $\square$POAB



OP is diameter by Diameter.

$$\boxed{(x-x_1)(x-0)+(y-y_1)(y-0)=0}$$

Chord of Contact = COC



2 Qs can be Asked
(1) Eqn of COC
(2) Length of COC

(1) Eqn of COC is $\boxed{T=0}$ Change.

$x^2 \to xx_1,\ y^2 \to yy_1$ in T=0
$2x \to x+x_1,\ 2y \to y+y_1$

T=0

- If (x_1,y_1) Lying on Circle then T=0 is EOT

- If (x_1,y_1) is outside circle then T=0 is COC

- If (x_1,y_1) inside Circle then T=0 is Eqn of Polar [out of syllabus]

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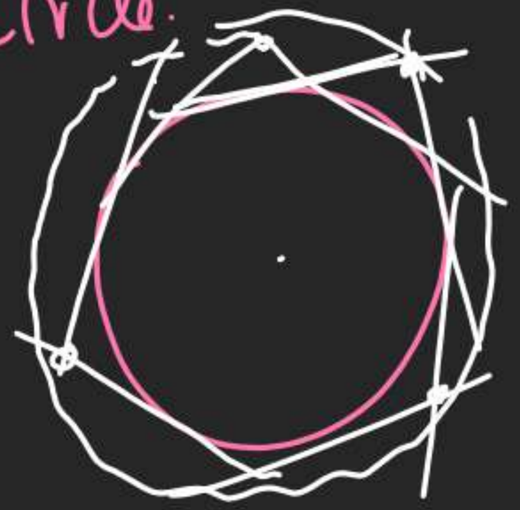
Special Case

When Both tangent to Circle are $\perp^r$ to eachother then Locus of $P(x_1, y_1)$ is known as Director Circle.



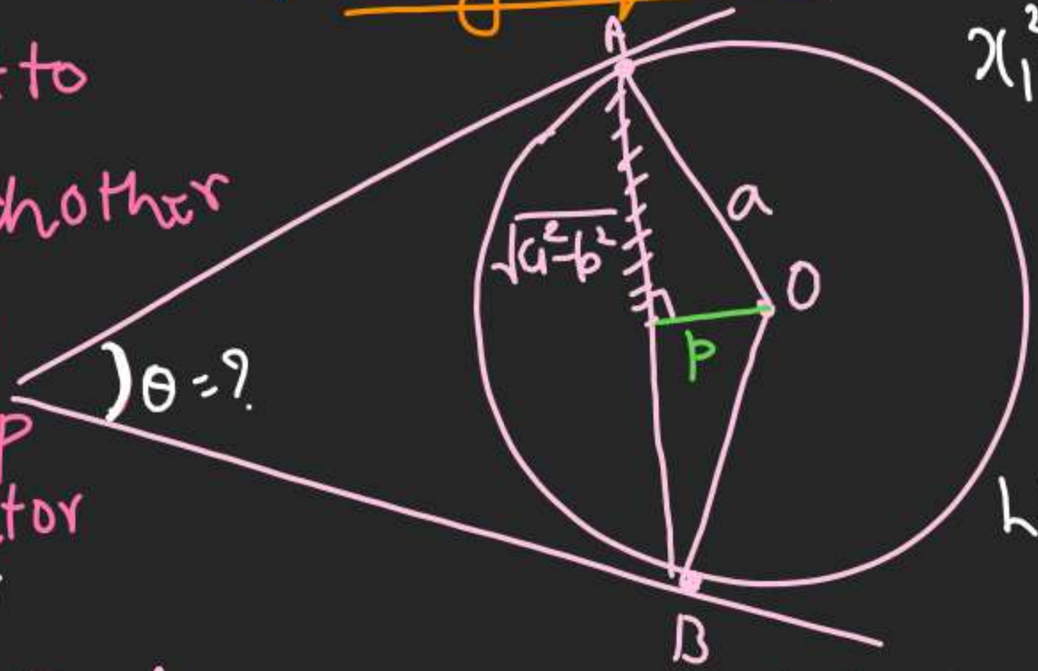
(2) Dir. circle is Locus of 2 $\perp^r$ tangent to Curve.

(3) $2\tan^{-1}\frac{a}{\sqrt{S_1}} = \frac{\pi}{2} \Rightarrow \tan^{-1}\frac{a}{\sqrt{S_1}} = \frac{\pi}{4}$

$\frac{a}{\sqrt{S_1}} = \tan\frac{\pi}{4} = 1$

$\Rightarrow 1 \quad \sqrt{S_1} = a$

(2) Length of COC

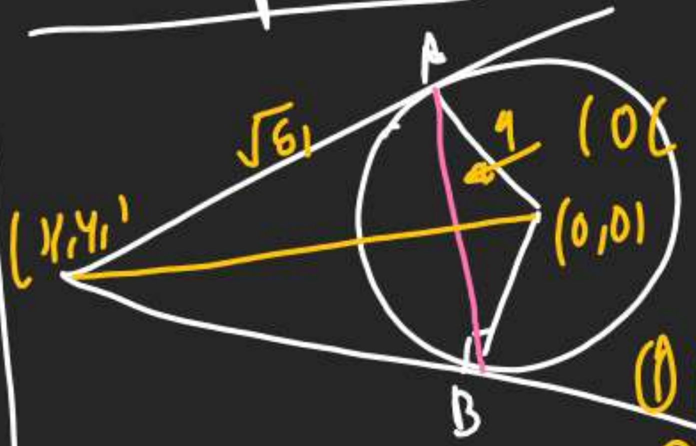


① find distance p of Centre & COC

(2) Use pythagorus

(3) $L_{COC} = 2\sqrt{a^2-b^2}$

Chord of Contact ≡ COC



$S_1 = a^2$

$x_1^2 + y_1^2 - a^2 = a^2$

$x_1^2 + y_1^2 = 2a^2$

$x_1^2 + y_1^2 = (\sqrt{2}a)^2$

Locus → $\boxed{x^2 + y^2 = (\sqrt{2}a)^2}$

Dir. Circle.

2 Qs Can be Asked

① Eqn of COC

② Length of COC

① Eqn of COC is $\boxed{T = 0}$

Change.

$x^2 \to x x_1, \; y^2 \to y y_1$

$2x \to x + x_1, \quad 2y \to y + y_1$ in $T = 0$

$T = 0$

If (x_1, y_1) Lying on Circle

then $T = 0$ is EOT

If (x_1, y_1) is outside circle then $T = 0$ is COC

If (x_1, y_1) inside Circle then $T = 0$ in Eqn of Polar [out of syllabus]

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4) Circle $\rightarrow x^2+y^2=a^2$

then D.C $\Rightarrow x^2+y^2=2a^2$

$$\boxed{\text{Radius of D.C} = \sqrt{2} \times \text{Rad of Circle}}$$

(5) If Circle is

$x^2+y^2+2gx+2fy+c=0$

$(x+g)^2+(y+f)^2=\left(\sqrt{g^2+f^2-c}\right)^2$

So D.C $(x+g)^2+(y+f)^2=2\left(\sqrt{g^2+f^2-c}\right)^2$

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Q From Pt. P(3,4) tangents are drawn to circle $x^2+y^2+6x+8y=0$ meets at A & B. If centre of circle is O then

(1) Length of Tangent

(2) Angle betw^n Tangents

(3) Eq^n of COC

(4) Length of COC

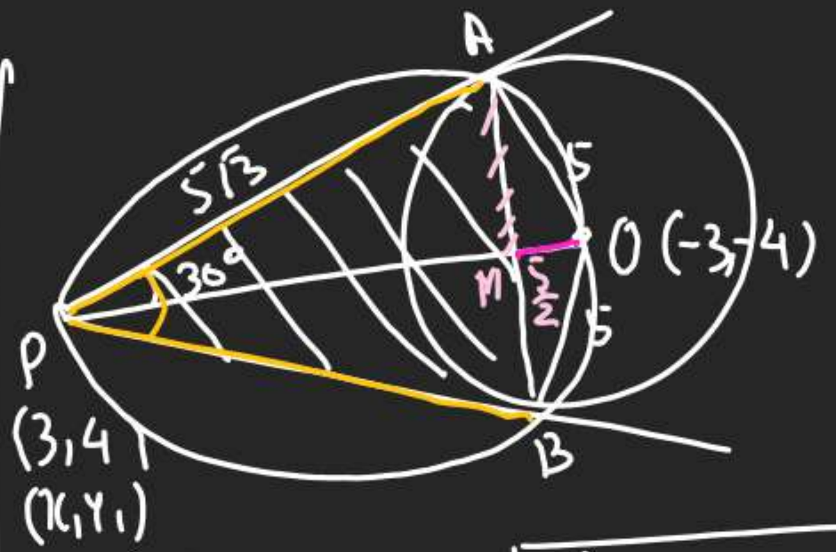
(5) Area of $\triangle$ POAB

(6) Area of $\triangle$ PAB

(7) Area circumscribing $\triangle$ PAB

(8) Director Circle of Circle

(9) Eq^n of Tangents.



(1) $L_T=\sqrt{S_1}=\sqrt{3^2+4^2+6\times3+8\times4}$

$=5\sqrt{3}$

(2) $\theta=2\tan^{-1}\frac{a}{\sqrt{S_1}}=2\tan^{-1}\frac{5}{5\sqrt{3}}$

$=2\times\frac{\pi}{6}=\frac{\pi}{3}$

(3) Eq^n of COC $\to T=0$ (Change)

$x\cdot3+y\cdot4+3(x+3)+4(y+4)=0$

$6x+8y+25=0$

(4) A) $p=$ distance of $(-3,-4)$ from $6x+8y+25=0$

$=\frac{|-18-32+25|}{\sqrt{6^2+8^2}}=\frac{5}{2}$

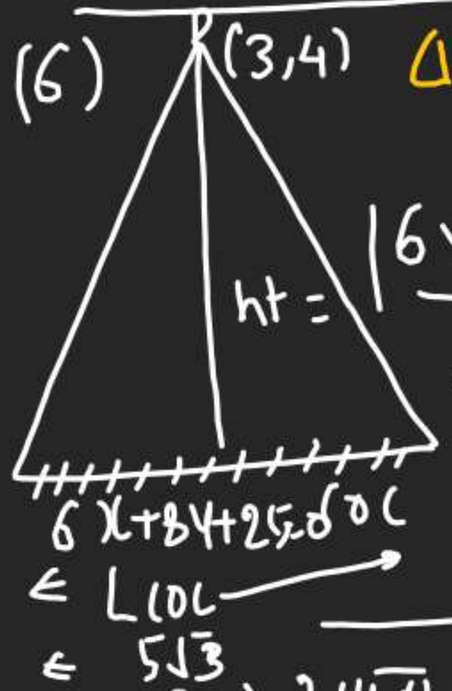
(B) $AM=\sqrt{5^2-\left(\frac{5}{2}\right)^2}=\frac{5\sqrt{3}}{2}$

$L_{COC}=2AM=5\sqrt{3}$

Q 29 $\to$ 70 $\to$ 18

(5) $\triangle POAB=a\sqrt{S_1}=5\times5\sqrt{3}=25\sqrt{3}$

(6)



$\Delta=\frac{1}{2}5\sqrt{3}\times5\sqrt{3}\times\sin60°=\frac{75}{2}\times\frac{\sqrt{3}}{2}$

$ht=\frac{|6\times3+8\times4+25|}{\sqrt{6^2+8^2}}=\frac{75}{10}=\frac{15}{2}$

$\Delta=\frac{1}{2}\times5\sqrt{3}\times\frac{15}{2}=\frac{75\sqrt{3}}{4}$

(7) अपना वाला circle $\to$ $\boxed{Area=\pi\times25}$

$(x-3)(x+3)+(y-4)(y+4)=0\Rightarrow x^2+y^2=25$

(8) D.C $\to (x+3)^2+(y+4)^2=2\times5^2$

(9) $(y+4)=m(x+3)\pm5\sqrt{1+m^2}$ P.T. (3,4)

$8-6m=\pm5\sqrt{1+m^2}$

$36m^2+64-96m=25+25m^2$