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3.

$$x = \boxed{3\underbrace{\tan^{-1}\frac{1}{2}}_{\theta_1 \in \left(0, \frac{\pi}{6}\right)}} + 2\underbrace{\tan^{-1}\frac{1}{5}}_{\theta_2 \in \left(0, \frac{\pi}{4}\right)} = \tan^{-1}\frac{11}{2} + \tan^{-1}\frac{5}{12}$$

$$3\theta_1 \in \left(0, \pi/2\right)$$

$$\tan(3\theta_1) = \frac{3\left(\frac{1}{2}\right) - \frac{1}{8}}{1 - 3\left(\frac{1}{4}\right)} = \frac{\frac{11}{8}}{\frac{1}{4}} = \frac{11}{2}$$

$$3\theta_1 = \tan^{-1}\tan 3\theta_1 = \tan^{-1}\frac{11}{2}$$

$$\tan 2\theta_2 = \frac{\frac{2}{5}}{1 - \frac{1}{25}} = \frac{5}{12}$$

$$2\theta_2 = \tan^{-1}\tan 2\theta_2 = \tan^{-1}\frac{5}{12}$$

$$\frac{11}{2} \times \frac{5}{12} > 1$$

$$\left(\frac{\pi}{2}, \pi\right)$$

$$= \pi +$$

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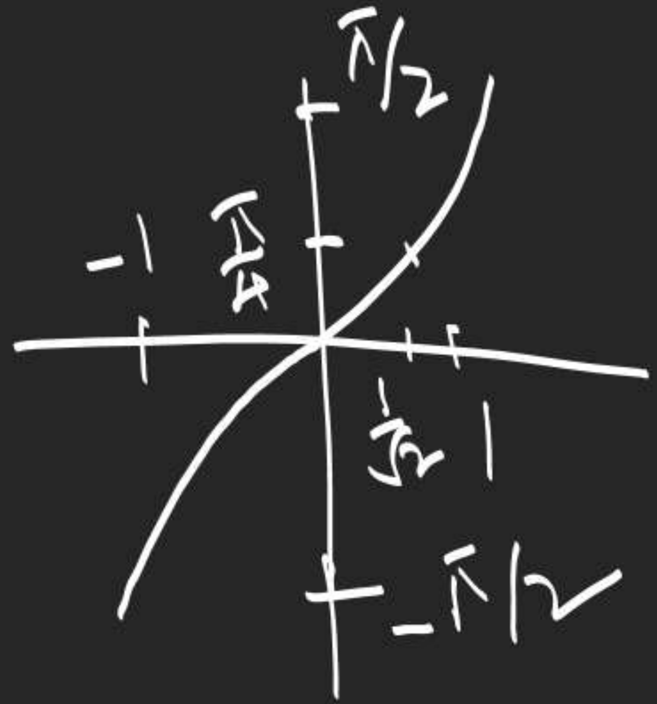
4. $\cos^{-1}\cos\left(\frac{\pi}{4}+\frac{9\pi}{10}\right)$

6. $\rightarrow$ leave.

7. $\frac{\pi}{2}-\sin^{-1}x > \sin^{-1}x$

$$\boxed{\sin^{-1}x < \frac{\pi}{4}}$$

$$x\in\left[-1,\frac{1}{\sqrt{2}}\right)$$



9. $\tan\left(\frac{1}{2}\underbrace{\cos^{-1}\frac{\sqrt{5}}{3}}_{\theta}\right)=\frac{1-\cos\theta}{\sin\theta}$

$\theta\in\left(0,\frac{\pi}{2}\right)$ ✓

$$=\frac{1-\frac{\sqrt{5}}{3}}{\frac{2}{3}}$$

$$\cot\left(\underbrace{\cot^{-1}\alpha}_{\theta_1}+\underbrace{\cot^{-1}\beta}_{\theta_2}\right)$$

$$\frac{\alpha\beta-1}{\alpha+\beta}=\frac{\cot\theta_1\cot\theta_2-1}{\cot\theta_1+\cot\theta_2}$$

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Solve $S_\infty = \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$ $\quad n \to \infty$

1. $\tan^{-1}\left(\frac{2}{2+1^2+1^4}\right)+\tan^{-1}\left(\frac{4}{2+2^2+2^4}\right)+\tan^{-1}\left(\frac{6}{2+3^2+3^4}\right)+\dots$ upto n terms.

$$=\sum_{r=1}^{n} \tan^{-1}\left(\frac{2r}{2+r^2+r^4}\right)=\sum_{r=1}^{n}\tan^{-1}\left(\frac{2r}{1+(1+r^2+r^4)}\right)$$

$$(\tan^{-1}3-\tan^{-1}1)+(\tan^{-1}7-\tan^{-1}3)+(\tan^{-1}13-\tan^{-1}7)+\dots$$

$$=\sum_{r=1}^{n} \tan^{-1}\left(\frac{(1+r+r^2)-(1-r+r^2)}{1+(1+r+r^2)(1-r+r^2)}\right)=\sum_{r=1}^{n}\left(\tan^{-1}(1+r+r^2)-\tan^{-1}(1-r+r^2)\right)$$

$r=n$ $\quad r=1$

$$=\tan^{-1}(1+n+n^2)-\tan^{-1}1$$

$$\tan^{-1}\left(1+(r+1)+(r+1)^2\right)-\tan^{-1}\left(1-(r+1)+(r+1)^2\right)$$

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Q. $\sum_{r=1}^{n} \tan^{-1}\left(\frac{4r}{r^4-2r^2+2}\right) = \sum_{r=1}^{n} \tan^{-1}\left(\frac{(r+1)^2-(r-1)^2}{1+(r+1)^2(r-1)^2}\right)$

$S_\infty = ?$

$= \sum_{r=1}^{n}\left(\tan^{-1}(r+1)^2 - \tan^{-1}(r-1)^2\right)$

$r = n, n-1$ $\qquad$ $r = 1, 2$

$= \tan^{-1}(n+1)^2 + \tan^{-1}n^2 - 0 - \frac{\pi}{4}$

$\left(\cancel{\tan^{-1}2^2} - \tan^{-1}0\right) + \left(\cancel{\tan^{-1}3^2} - \tan^{-1}1^2\right)$
$+ \left(\tan^{-1}4^2 - \cancel{\tan^{-1}2^2}\right) + \left(\tan^{-1}5^2 - \cancel{\tan^{-1}3^2}\right)$
$+ - - -$

$n \to \infty,\ S_\infty = \frac{\pi}{2} + \frac{\pi}{2} - \frac{\pi}{4} = \boxed{\frac{3\pi}{4}}$

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3: $\cot^{-1}\left(\frac{2}{a}+a\right)+\cot^{-1}\left(\frac{2}{a}+3a\right)+\cot^{-1}\left(\frac{2}{a}+6a\right)+\cot^{-1}\left(\frac{2}{a}+10a\right)+ \ldots$ infinite, $a>0$.

$$= \sum_{r=1}^{n} \cot^{-1}\left(\frac{2}{a}+\frac{r(r+1)}{2}a\right) = \sum_{r=1}^{n} \tan^{-1}\left(\frac{2a}{4+r(r+1)a^2}\right)$$

$$= \sum_{r=1}^{n} \tan^{-1}\left(\frac{(r+1)\frac{a}{2}-r\frac{a}{2}}{1+\left(\frac{ra}{2}\right)\left(\frac{(r+1)a}{2}\right)}\right)$$

$$\sum_{r=1}^{n}\left(\tan^{-1}(r+1)\frac{a}{2} - \tan^{-1}\frac{ra}{2}\right)$$

$$= \tan^{-1}(n+1)\frac{a}{2} - \tan^{-1}\frac{a}{2}$$

$$S_\infty = \frac{\pi}{2} - \tan^{-1}\frac{a}{2} = \cot^{-1}\frac{a}{2}$$

$S = 1 + 3 + 6 + 10 + 15 + \ldots T_n$

$\quad\;\; 1 + 3 + 6 + 10 + \ldots T_{n-1} + T_n$

$S =$

$0 = (1+2+3+4+\ldots n\text{ term}) - T_n$

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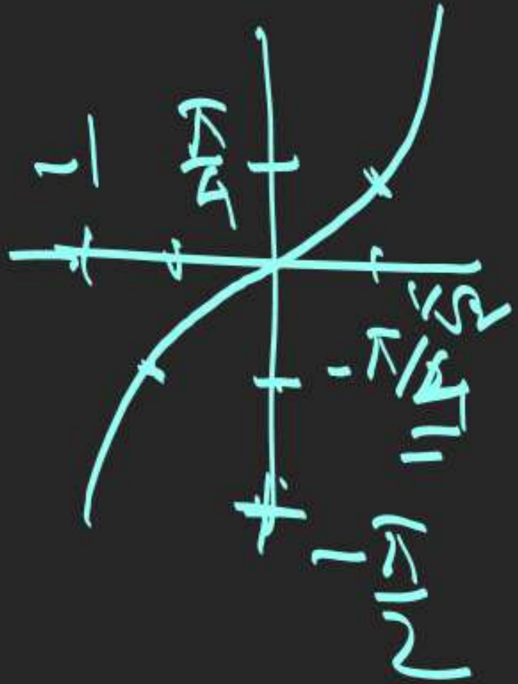
$$\sin^{-1}\left(2x\sqrt{1-x^2}\right) = \sin^{-1}\left(2\sin\theta\cos\theta\right) = \sin^{-1}\sin 2\theta$$

$$\sin^{-1}x = \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

$$2\theta \in [-\pi, \pi]$$

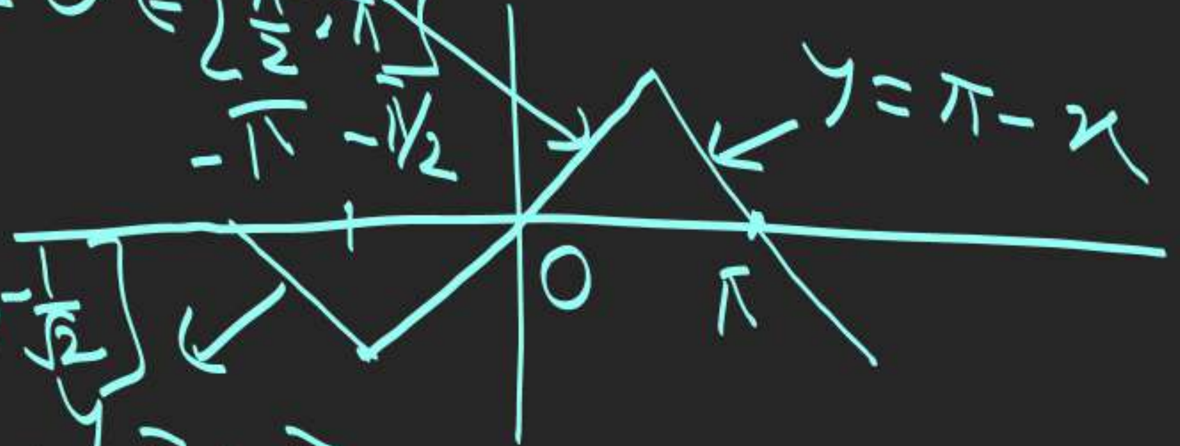
$$1-x^2 = \cos^2\theta$$

$$\sqrt{1-x^2} = |\cos\theta| = \cos\theta$$



$$\begin{cases} -\pi-2\theta & 2\theta \in \left[-\pi, -\frac{\pi}{2}\right] \\ 2\theta & 2\theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \\ \pi-2\theta & 2\theta \in \left[\frac{\pi}{2}, \pi\right] \end{cases}$$

$y = x$



$$= \begin{cases} -\pi - 2\sin^{-1}x & x \in \left[-1, -\frac{1}{\sqrt{2}}\right] \\ 2\sin^{-1}x & x \in \left[-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right] \\ \pi - 2\sin^{-1}x & x \in \left[\frac{1}{\sqrt{2}}, 1\right] \end{cases}$$

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1: , 4(a) , 3(v,) vii, (viii), (ix)

2: (b) iv , 4(d)

1: $f(-3) = f(3)$

$f(-1) = f(1)$

$f(-7) = f(1)$

$f(20) = f(4)$

2: (b) (iv) $\sin\left(\underbrace{\frac{1}{4}\sin^{-1}\left(\frac{\sqrt{63}}{8}\right)}\right)$

$\theta \in (0, \frac{\pi}{2})$

$\cos\frac{\theta}{2} = \sqrt{\frac{1+\cos\theta}{2}}$

$= \sqrt{\frac{1+\frac{1}{8}}{2}} = \frac{3}{4}$

$\sin\frac{\theta}{4} = \sqrt{\frac{1-\cos\theta/2}{2}}$

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$$\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$$

$\tan^{-1}x = \theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) - \{0\}$

$$= \tan^{-1}\left(\frac{\sec\theta - 1}{\tan\theta}\right) = \tan^{-1}\left(\frac{1-\cos\theta}{\sin\theta}\right)$$

$$= \tan^{-1}\tan\frac{\theta}{2} = \frac{\theta}{2}$$

$$= \frac{1}{2}\tan^{-1}x \quad , \quad x \neq 0$$

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Solve for x

1. $\cos^{-1}x - \sin^{-1}x = \cos^{-1}(x\sqrt{3})$

Ans $\boxed{x = 0, \frac{1}{2}, -\frac{1}{2}}$

$\boxed{\text{Check}}$ ✓

$\frac{\pi}{2} - 2\sin^{-1}x = \cos^{-1}(x\sqrt{3})$

$\sin(\underbrace{2\sin^{-1}x}_{\theta}) = \cos\left(\frac{\pi}{2} - 2\sin^{-1}x\right) = \cos(\cos^{-1}x\sqrt{3})$

$\boxed{\theta_1 = \theta_2}$

$\Rightarrow \cos\theta_1 = \cos\theta_2$ ✓

$2x\sqrt{1-x^2} = x\sqrt{3}$

$\boxed{x=0}$, $4 - 4x^2 = 3 \Rightarrow \boxed{x = \pm\frac{1}{2}}$

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HW

$\boxed{\begin{array}{l} \text{Ex I} \quad (\text{complete}) \\ \text{Ex - } \overline{\underline{\text{III}}} \end{array}}$