

LOGARITHMS

1. If  $\log_7(x+1) + \log_7(x-5) = 1$ , then  $x$  is equal to

- (A) 6 (B) -2 (C) -6 (D) 2

Ans. (A)

Sol.  $\log_7(x+1) + \log_7(x-5) = 1$

$$\Rightarrow \log_7(x+1)(x-5) = 1 \Rightarrow (x+1)(x-5) = 7^1 = 7 \Rightarrow x^2 - 4x - 12 = 0$$

$$\Rightarrow (x-6)(x+2) = 0 \Rightarrow x = 6, -2$$

But  $x \neq -2$ , because then in the given equation we shall have logarithms of negative numbers which are not defined.

Hence  $x = 6$ .

2. If  $A = \log_2 \log_2 \log_4 256 + 2 \log_{\sqrt{2}} 2$ , then  $A$  is equal to

- (A) 2 (B) 3 (C) 5 (D) 7

Ans. (C)

Sol.  $A = \log_2 \log_2 \log_4 256 + 2 \log_{\sqrt{2}} 2 = \log_2 \log_2 \log_4 4^4 + 2 \cdot \frac{1}{(1/2)} \log_2 2$

$$[\because \log_{a^\beta} n = (1/\beta) \log_a n]$$

$$= \log_2 \log_2 4 + 4 = \log_2 (2 \log_2 2) + 4 = \log_2 2 + 4 = 1 + 4 = 5.$$

3.  $\left[ (0.16)^{\log_{2.5} \left( \frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} + \dots \right)} \right]^{1/2}$  is equal to

- (A) 0 (B) 1 (C) -1 (D) 2

Ans. (D)

Sol. Let the given expression be  $A$ , then

$$A = \left[ (0.16)^{\log_{2.5} \left( \frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} + \dots \right)} \right]^{1/2} = \left[ \left( \frac{16}{100} \right)^{\log_{2.5} \left( \frac{1}{1-1/3} \right)} \right]^{1/3} = \left[ \left( \frac{16}{100} \right)^{\log_{2.5} 2} \left( \frac{1}{2} \right) \right]^{1/2}$$

$$\therefore \log A = \frac{1}{2} \log_{2.5} \left( \frac{1}{2} \right) \cdot \log \left( \frac{16}{100} \right) = -\frac{1}{2} \log_{2.5} 2 \cdot \log \left( \frac{4}{25} \right) = -\frac{1}{2} \frac{\log 2}{\log 2.5} (\log 4 - \log 25)$$

$$= -\frac{\log 2 (2 \log 2 - \log 5)}{2(\log 25 - \log 10)} = \frac{\log 2 (\log 5 - \log 2)}{2 \log 5 - (\log 5 + \log 2)} = \frac{\log 2 (\log 5 - \log 2)}{(\log 5 - \log 2)} = \log 2$$

$$\therefore A = 2.$$

4. If  $\log_7 2 = m$ , then  $\log_{49} 28$  is equal to

- (A)  $1 + m$  (B)  $2(1 + 2m)$  (C)  $\frac{1}{2}(1 + 2m)$  (D)  $\frac{1}{2}(1 + m)$

Ans. (C)

Sol.  $\log_{49} 28 = \frac{\log_7 28}{\log_7 49} = \frac{\log_7 7 + \log_7 4}{2 \log_7 7} = \frac{1 + 2 \log_7 2}{2} = \frac{1}{2}(1 + 2m).$

(MATHEMATICS)

DIWALI ASSIGNMENT

5. If  $3^x = 4^{x-1}$ , then x is equal to

- (A)  $\frac{2\log_3 2}{2\log_3 2 - 1}$  (B)  $\frac{2}{2 - \log_2 3}$  (C)  $\frac{1}{1 - \log_4 3}$  (D)  $\frac{2\log_2 3}{2\log_2 3 - 1}$

Ans. (A,B,C)

Sol. Taking log to base 4, we have

$$x \log_4 3 = x - 1$$

$$\Rightarrow x(1 - \log_4 3) = 1$$

$$\Rightarrow x = \frac{1}{1 - \log_4 3} \quad \dots(i)$$

$$= \frac{1}{1 - \frac{1}{2} \log_2 3}$$

$$= \frac{2}{2 - \log_2 3} \quad \dots(ii)$$

$$= \frac{2}{2 - \frac{1}{\log_3 2}}$$

$$= \frac{2\log_3 2}{2\log_3 2 - 1} \quad \dots(iii)$$

(i), (ii), (iii) show that (1), (2), (3) are correct answers. [1,2,3]

COMPLEX NUMBERS

6. The smallest positive integer n such that  $\left(\frac{1+i}{1-i}\right)^n = 1$  is

- (A) 16 (B) 12 (C) 8 (D) 4

Ans. (D)

Sol.  $\left(\frac{1+i}{1-i}\right)^n = i^n$

$$\Rightarrow \text{Also } i^2 = -1, i^3 = -i, i^4 = 1, \therefore n = 4.$$

$$\text{Note: } (1+i)^n = (1-i)^n \Rightarrow n = 4k, k \in \mathbb{Z}$$

$$\therefore (1+i)^n = (1-i)^n \Rightarrow \left(\frac{1+i}{1-i}\right)^n = 1$$

$$\Rightarrow i^n = 1 \Rightarrow n = 4k, k \in \mathbb{Z}$$

7. If  $z_1, z_2$  are two complex numbers such that  $|z_1 + z_2| = |z_1| + |z_2|$ , then  $\arg(z_1) - \arg(z_2)$  equals

- (A)  $\pi/2$  (B)  $-\pi/2$  (C) 0 (D)  $\pi$

Ans. (C)

(MATHEMATICS)

DIWALI ASSIGNMENT

**Sol.** Let  $z_1 = (r_1, \theta_1)$  and  $z_2 = (r_2, \theta_2)$ , then

$$z_1 + z_2 = (r_1 \cos \theta_1 + r_2 \cos \theta_2) + i(r_1 \sin \theta_1 + r_2 \sin \theta_2)$$

$$\Rightarrow |z_1 + z_2| = [(r_1 \cos \theta_1 + r_2 \cos \theta_2)^2 + (r_1 \sin \theta_1 + r_2 \sin \theta_2)^2]^{1/2}$$

$$= [r_1^2 + r_2^2 + 2r_1 r_2 \cos(\theta_1 - \theta_2)]^{1/2}$$

$$\therefore |z_1 + z_2| = |z_1| + |z_2|$$

$$\Rightarrow r_1^2 + r_2^2 + 2r_1 r_2 \cos(\theta_1 - \theta_2) = (r_1 + r_2)^2 \Rightarrow \cos(\theta_1 - \theta_2) = 1 \Rightarrow \theta_1 - \theta_2 = 0$$

$$\Rightarrow \text{amp}(z_1) - \text{amp}(z_2) = 0. \quad [3]$$

Aliter. If  $z_1 = \overrightarrow{OP}$ ,  $z_2 = \overrightarrow{OQ}$  and  $z_1 + z_2 = \overrightarrow{OA}$ , then

$$|z_1 + z_2| = OA, |z_1| = OP \text{ and } |z_2| = OQ = PA.$$

$$\text{Now } |z_1 + z_2| = |z_1| + |z_2|$$

$$\Rightarrow OA = OP + PA \Rightarrow O, P, A \text{ are collinear} \Rightarrow O, P, Q \text{ are collinear} \Rightarrow \text{amp}(z_1) - \text{amp}(z_2) = 0.$$

**8.** If  $z_0$  is the circumcentre of an equilateral triangle with vertices  $z_1, z_2, z_3$ , then  $z_1^2 + z_2^2 + z_3^2$  is equal to

- (A)  $z_0^2$  (B)  $z_0^2/3$  (C)  $3z_0^2$  (D)  $2z_0^2/3$

**Ans. (C)**

**Sol.** Since  $z_1, z_2, z_3$  are vertices of an equilateral triangle, so

$$z_1^2 + z_2^2 + z_3^2 = z_1 z_2 + z_2 z_3 + z_3 z_1$$

Further the circumcenter of an equilateral triangle is same as its centroid, so

$$z_0 = (z_1 + z_2 + z_3)/3 \Rightarrow 9z_0^2 = z_1^2 + z_2^2 + z_3^2 + 2(z_1 z_2 + z_2 z_3 + z_3 z_1)$$

$$= z_1^2 + z_2^2 + z_3^2 + 2(z_1^2 + z_2^2 + z_3^2) \therefore z_1^2 + z_2^2 + z_3^2 = 3z_0^2.$$

**9.** If  $\alpha, \beta$  are two different complex numbers and  $|\beta| = 1$ , then  $\left| \frac{\beta - \alpha}{1 - \bar{\alpha}\beta} \right|$  is equal to

- (A) 0 (B)  $1/2$  (C) 1 (D) 2

**Ans. (C)**

(MATHEMATICS)

DIWALI ASSIGNMENT

**Sol.**  $\because |\beta| = 1 \Rightarrow \beta\bar{\beta} = 1 \therefore \left| \frac{\beta-\alpha}{1-\bar{\alpha}\beta} \right| = \left| \frac{\beta-\alpha}{\beta\bar{\beta}-\bar{\alpha}\beta} \right| = \frac{|\beta-\alpha|}{|\beta||\bar{\beta}-\alpha|} = 1$

**10.** If two complex numbers  $z_1, z_2$  are such that  $|z_1| = 12$  and  $|z_2 - 3 - 4i| = 5$ , then the least value of  $|z_1 - z_2|$  is

- (A) 0 (B) 2 (C) 7 (D) 17

**Ans. (B)**

**Sol.**  $|z_2 - 3 - 4i| = 5 \Rightarrow |z_2 - (3 + 4i)| = 5 \Rightarrow |z_2| - |3 + 4i| \leq 15$  [ $\because |z_1| - |z_2| \leq |z_1 - z_2|$ ]

$\Rightarrow |z_2| - 5 \leq 15 \Rightarrow |z_2| \leq 10$  .....(1)

Now  $|z_1 - z_2| \geq |z_1| - |z_2| \Rightarrow |z_1 - z_2| \geq 12 - 10$  [ from (1)]

$\Rightarrow |z_1 - z_2| \geq 2.$

Aliter:-  $z_1$  and  $z_2$  lie on circles of radii 12 and 5. Also second circle passes through the centre (0,0) of first circle. Hence required minimum distance =  $12 - 2(5) = 2$ .

**11.** If  $1, \omega, \omega^2, \dots, \omega^{n-1}$  are  $n$ th roots of unity, then  $(1 - \omega)(1 - \omega^2) \dots (1 - \omega^{n-1})$  is equal to  
(A) 0 (B) 1 (C)  $n$  (D)  $n^2$

**Ans. (C)**

**Sol.**  $\because 1, \omega, \omega^2, \dots, \omega^{n-1}$  are  $n$ th roots of unity

$\Rightarrow x^n - 1 = (x - 1)(x - \omega)(x - \omega^2) \dots (x - \omega^{n-1}) \Rightarrow \frac{x^n - 1}{x - 1} = (x - \omega)(x - \omega^2) \dots (x - \omega^{n-1})$

$\Rightarrow \lim_{x \rightarrow 1} \frac{x^n - 1}{x - 1} = \lim_{x \rightarrow 1} (x - \omega)(x - \omega^2) \dots (x - \omega^{n-1}) \Rightarrow n = (1 - \omega)(1 - \omega^2) \dots (1 - \omega^{n-1})$

**12.** If  $z$  is a complex number such that  $|z| \geq 2$ , then the minimum value of  $|z + 1/2|$

(A) is strictly greater than  $5/2$ .

(B) is strictly greater than  $3/2$  but less than  $5/2$ .

(C) is equal to  $5/2$ .

(D) lies in interval  $(1, 2)$ .

**Ans. (D)**

(MATHEMATICS)

DIWALI ASSIGNMENT

**Sol.**  $|z| \geq 2 \Rightarrow z$  lies on or outside the circle with centre 0 and radius = 2.

Now  $|z + 1/2|$  = distance of  $z$  from the point  $(-1/2, 0)$

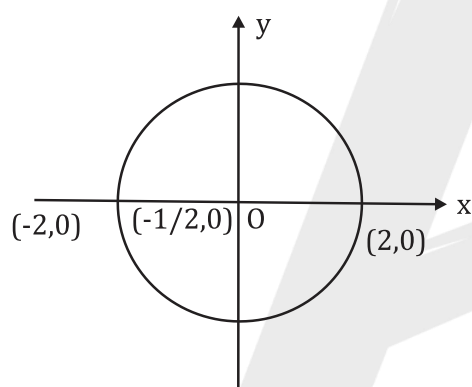
$\therefore \min. |z + 1/2|$  = distance between  $(-1/2, 0)$  and  $(-2, 0)$  [ $\because (-2, 0)$  is the nearest end of the diameter passing through  $(-1/2, 0)$ ]

$$= \sqrt{(-1/2 + 2)^2 + 0} = 3/2 \in (1, 2)$$

Aliter.

$$|z + 1/2| \geq ||z| - |1/2|| \geq |2 - 1/2| \quad (\because |z| \geq 2)$$

$$\geq 3/2 \in (1, 2).$$



QUADRATIC EQUATIONS

**13.** The number of real roots of the equation  $e^{\sin x} - e^{-\sin x} - 4 = 0$  is

- (A) 2 (B) 1 (C) infinite (D) none

**Ans.** (D)

**Sol.** Let  $e^{\sin x} = y$ , then given equation reduces to

$$y - 1/y - 4 = 0 \Rightarrow y^2 - 4y - 1 = 0 \Rightarrow y = \frac{4 \pm \sqrt{20}}{2} = 2 \pm \sqrt{5} = 4.23, -0.23$$

But  $y = e^{\sin x}$  is never negative. So  $y = e^{\sin x} = 4.23$

$\Rightarrow \sin x = \log 4.23 > 1$ , which is not possible.

Hence the equation has no real root.

(MATHEMATICS)

DIWALI ASSIGNMENT

14. If  $x$  be real then the maximum and minimum values of the expression  $\frac{x^2-x+1}{x^2+x+1}$  are  
 (A)  $3, -1/3$  (B)  $-3, 1/3$  (C)  $3, 1/3$  (D)  $-3, -1/3$

Ans. (C)

Sol. Let  $\frac{x^2-x+1}{x^2+x+1} = y \Rightarrow x^2(y-1) + (y+1)x + (y-1) = 0$

$x$  is real, so  $B^2 - 4AC \geq 0 \Rightarrow (y+1)^2 - 4(y-1) \geq 0 \Rightarrow (y-3)(3y-1) \leq 0$

$$\Rightarrow \begin{cases} y-3 \leq 0 & \text{and } 3y-1 \geq 0 \\ y-3 \geq 0 & \text{and } 3y-1 \leq 0 \end{cases} \Rightarrow 1/3 \leq y \leq 3.$$

So maximum and minimum values are  $3, 1/3$ .

15. If  $a < b < c < d$ , then roots of the equation  $(x-a)(x-c) + 2(x-b)(x-d) = 0$  are  
 (A) real and equal (B) real and different  
 (C) imaginary (D) rational

Ans. (B)

Sol. Given equation is

$$3x^2 - (a+c+2b+2d)x + (ac+2bd) = 0$$

$$\therefore B^2 - 4AC = (a+c+2b+2d)^2 - 12(ac+2bd)$$

$$= [(a+2d) - (c+2b)]^2 + 4(a+2d)(c+2b) - 12(ac+2bd)$$

$$= [(a+2d) - (c+2b)]^2 + 8(c-b)(d-a) > 0 \Rightarrow \text{roots are real and different.}$$

16. If roots of the equation  $x^2 - 2ax + a^2 + a - 3 = 0$  are real and less than 3, then  
 (A)  $a < 2$  (B)  $2 \leq a \leq 3$  (C)  $3 < a \leq 4$  (D)  $a > 4$

Ans. (A)

Sol. Roots are real and less than 3  $\Rightarrow b^2 - 4ac \geq 0, f(3) > 0, -b/2a < 3$

$$\Rightarrow 4a^2 - 4(a^2 + a - 3) \geq 0; a^2 - 5a + 6 > 0; 2a/2 < 3$$

$$\Rightarrow -a + 3 \geq 0; (a-2)(a-3) > 0; a < 3 \Rightarrow a \leq 3, a < 2 \text{ or } a > 3; a < 3 \Rightarrow a < 2.$$

(MATHEMATICS)

DIWALI ASSIGNMENT

17. If  $(1 - p)$  is a root of the equation  $x^2 + px + (1 - p) = 0$ , then its roots are

- (A) 0,1 (B) 0, -1 (C) -1,1 (D) -1,2

Ans. (B)

Sol.  $\because (1 - p)$  is a root of the given equation, so

$$(1 - p)^2 + p(1 - p) + (1 - p) = 0 \Rightarrow (1 - p)(1 - p + p + 1) = 0 \Rightarrow p = 1$$

So given root is 0. If other root be  $\alpha$ , then  $\alpha + 0 = -p \Rightarrow \alpha = -1$ . Hence roots are 0, -1.

18. If  $\tan P/2$  and  $\tan Q/2$  are roots of the equation  $ax^2 + bx + c = 0$  ( $a \neq 0$ ) and in

$\triangle PQR$ ,  $\angle R = \pi/2$ ; then

- (A)  $a + b = c$  (B)  $b + c = a$  (C)  $c + a = b$  (D)  $b = c$

Ans. (A)

Sol. As given

$$\frac{\tan P}{2} + \frac{\tan Q}{2} = -\frac{b}{a} \quad \dots\dots(1)$$

$$\tan P/2 \tan Q/2 = c/a \quad \dots\dots(2)$$

Also

$$\angle R = \pi/2 \Rightarrow P + Q = \pi/2 \Rightarrow P/2 + Q/2 = \pi/4 \Rightarrow \tan(P/2 + Q/2) = 1 \Rightarrow \frac{\tan P/2 + \tan Q/2}{1 - \tan P/2 \tan Q/2} = 1$$

$$\Rightarrow \frac{-b/a}{1 - c/a} = 1 \quad [\text{from (1), (2)}] \Rightarrow b = c - a \Rightarrow a + b = c.$$

19. If  $\alpha, \beta$  are roots of the equation  $x^2 - px + r = 0$  and  $\alpha/2, \beta/2$  are roots of the equation

$x^2 - qx + r = 0$ , then  $r$  is equal to

- (A)  $\frac{2}{9}(p - q)(2q - p)$  (B)  $\frac{2}{9}(q - p)(2p - q)$   
(C)  $\frac{2}{9}(q - 2p)(2q - p)$  (D)  $\frac{2}{9}(2p - q)(2q - p)$

(MATHEMATICS)

DIWALI ASSIGNMENT

Ans. (D)

Sol. As given  $\alpha + \beta = p$  .....(1)

$$\alpha\beta = r \quad \text{.....(2)}$$

$$\alpha/2 + 2\beta = q \Rightarrow \alpha + 4\beta = 2q \quad \text{.....(3)}$$

$$(1), (3) \Rightarrow \alpha = \frac{2}{3}(2p - q), \beta = \frac{1}{3}(2q - p)$$

$$\therefore \text{from (2),} \quad r = \frac{2}{9}(2p - q)(2q - p).$$

PROGRESSIONS

20. If the sum of  $p$  terms of an AP is  $q$  and the sum of its  $q$  terms is  $p$ , then the sum of its  $(p + q)$  terms will be

- (A) 0                      (B)  $p - q$                       (C)  $p + q$                       (D)  $-(p + q)$

Ans. (D)

Sol. Let  $a$  be the first term and  $d$  be the common difference of the given AP, then  
and  $q = \frac{p}{2}[2a + (p - 1)d]$  and

$$(1) - (2), \Rightarrow = \frac{q}{2}[2a + (q - 1)d] \quad q - p = \frac{1}{2}[2a(p - q) + d(p^2 - p - q^2 + q)]$$

$$= \frac{1}{2}(p - q)[2a + (p + q - 1)d] \quad \Rightarrow -1 = \frac{1}{2}[2a + (p + q - 1)d]$$

$$\Rightarrow -(p + q) = \frac{p+q}{2}[2a + (p + q - 1)d] \quad \Rightarrow -(p + q) = \text{Sum of } (p + q) \text{ terms.}$$

21. If  $\frac{1}{q+r}, \frac{1}{r+p}, \frac{1}{p+q}$  are in AP, then correct statement is

- (A)  $p, q, r$  are in AP                      (B)  $p^2, q^2, r^2$  are in AP  
(C)  $1/p, 1/q, 1/r$  are in AP                      (D)  $1/p^2, 1/q^2, 1/r^2$  are in AP

Ans. (B)

Sol. We have  $\frac{1}{r+p} - \frac{1}{q+r} = \frac{1}{p+q} - \frac{1}{r+p} \Rightarrow \frac{q+r-r-p}{(r+p)(q+r)} = \frac{r+p-p-q}{(p+q)(r+p)} \Rightarrow q^2 - p^2 = r^2 - q^2$

$$\Rightarrow p^2, q^2, r^2 \text{ are in AP.}$$

(MATHEMATICS)

DIWALI ASSIGNMENT

22.  $2^{1/4} \cdot 4^{1/8} \cdot 8^{1/16} \cdot 16^{1/32} \cdot \dots$  is equal to

- (A) 1 (B) 2 (C)  $3/2$  (D)  $5/2$

Ans. (B)

Sol. The given product =  $2^{1/4} \cdot 2^{2/8} \cdot 2^{3/16} \cdot 2^{4/32} \cdot \dots$

$$= 2^{\frac{1}{4} + \frac{2}{8} + \frac{3}{16} + \frac{4}{32} + \dots} = 2^S \quad [\text{say}]$$

$$\text{Now } S = \frac{1}{4} + \frac{2}{8} + \frac{3}{16} + \frac{4}{32} + \dots \quad \dots\dots(1)$$

$$\Rightarrow \frac{1}{2}S = \frac{1}{8} + \frac{2}{16} + \frac{3}{32} + \dots \quad \dots\dots(2)$$

$$(1) - (2) \Rightarrow \frac{1}{2}S = \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \dots = \frac{1/4}{1-1/2} = \frac{1}{2} \Rightarrow S = 1 \quad \text{Product} = 2^1 = 2.$$

23. If  $x > 1, y > 1, z > 1$  are in GP, then  $\frac{1}{1+\ln x}, \frac{1}{1+\ln y}, \frac{1}{1+\ln z}$  are in

- (A) AP (B) GP (C) HP (D) none of these

Ans. (C)

Sol.  $x, y, z$  are in GP  $\Rightarrow y^2 = xz \Rightarrow 2\log y = \log x + \log z \Rightarrow 2(1 + \log y) = (1 + \log x) + (1 + \log z)$

$$\Rightarrow 1 + \ln x, 1 + \ln y, 1 + \ln z \text{ are in AP} \Rightarrow \frac{1}{1+\ln x}, \frac{1}{1+\ln y}, \frac{1}{1+\ln z} \text{ are in HP.}$$

24. If  $A_1, A_2; G_1, G_2$  and  $H_1, H_2$  are respectively two AM's, two GM's and two HM's between two numbers, then  $\frac{A_1+A_2}{H_1+H_2}$  equals

- (A)  $\frac{H_1H_2}{G_1G_2}$  (B)  $\frac{G_1G_2}{H_1H_2}$  (C)  $\frac{H_1H_2}{A_1A_2}$  (D)  $\frac{G_1G_2}{A_1A_2}$

Ans. (B)

Sol. Let given two numbers be  $a$  and  $b$ . Then

$$A_1 + A_2 = a + b, G_1G_2 = ab \quad \dots\dots(1)$$

$$\frac{1}{H_1} + \frac{1}{H_2} = \frac{1}{a} + \frac{1}{b} \Rightarrow \frac{H_1+H_2}{H_1H_2} = \frac{a+b}{ab} \Rightarrow \frac{H_1+H_2}{H_1H_2} = \frac{A_1+A_2}{G_1G_2} \quad [\text{from (1)}]$$

$$\therefore \frac{A_1+A_2}{H_1+H_2} = \frac{G_1G_2}{H_1H_2}$$

(MATHEMATICS)

DIWALI ASSIGNMENT

25. In a  $\triangle PQR$  if  $\sin P, \sin Q, \sin R$  are in AP, then its

- (A) altitudes are in AP (B) altitudes are in HP  
(C) medians are in GP (D) medians are in AP

Ans. (B)

Sol. Length of the altitudes from vertices are :

$q \sin R, r \sin P, p \sin Q$  or  $k \sin Q \sin R, k \sin R \sin P, k \sin P \sin Q$  or

$\frac{k \sin P \sin Q \sin R}{\sin P}, \frac{k \sin P \sin Q \sin R}{\sin Q}, \frac{k \sin P \sin Q \sin R}{\sin R}$  But  $\sin P, \sin Q, \sin R$  are in AP, so above lengths are in HP.

26. Let  $a_1, a_2, a_3, \dots$  be terms of an AP. If  $\frac{a_1 + a_2 + \dots + a_p}{a_1 + a_2 + \dots + a_q} = \frac{p^2}{q^2}, p \neq q$ , then  $\frac{a_6}{a_{21}}$  equals

- (A)  $2/7$  (B)  $7/2$  (C)  $11/41$  (D)  $41/11$

Ans. (C)

Sol. Let  $d$  be the common difference of given AP. Then  $\frac{\frac{p}{2}[2a_1 + (p-1)d]}{\frac{q}{2}[2a_1 + (q-1)d]} = \frac{p^2}{q^2} \Rightarrow \frac{a_1 + \left(\frac{p-1}{2}\right)d}{a_1 + \left(\frac{q-1}{2}\right)d} = \frac{p}{q} \dots \dots (1)$

Now when  $\frac{p-1}{2} = 5$ , i.e.,  $p = 11$  and  $\frac{q-1}{2} = 20$ , i.e.,  $q = 41$ , we shall have

$$\frac{a_1 + 5d}{a_1 + 20d} = \frac{11}{41} \Rightarrow \frac{a_6}{a_{21}} = \frac{11}{41}$$

PERMUTATIONS & COMBINATIONS

27.  ${}^{47}C_4 + \sum_{r=1}^5 {}^{52-r}C_3$  is equal to

- (A)  ${}^{51}C_4$  (B)  ${}^{52}C_4$  (C)  ${}^{53}C_4$  (D) none of these

Ans. (B)

Sol. The given expression can be written as

$$\begin{aligned} \sum_{r=1}^5 {}^{52-r}C_3 + {}^{47}C_4 &= {}^{51}C_3 + {}^{50}C_3 + {}^{49}C_3 + {}^{48}C_3 + {}^{47}C_3 + {}^{47}C_4 \\ &= {}^{51}C_3 + {}^{50}C_3 + {}^{49}C_3 + {}^{48}C_3 + {}^{48}C_4 = {}^{51}C_3 + {}^{50}C_3 + {}^{49}C_3 + {}^{49}C_4 \\ &= {}^{51}C_3 + {}^{50}C_3 + {}^{50}C_4 = {}^{51}C_3 + {}^{51}C_4 = {}^{52}C_4 \end{aligned}$$

(MATHEMATICS)

DIWALI ASSIGNMENT

28. The number of words from the letters of the word 'BHARAT' in which B and H will never come together, is
- (A) 360 (B) 240 (C) 120 (D) none of these

Ans. (B)

Sol. There are 6 letters in the word 'BHARAT', 2 of them are identical. Hence total number of words with these letters =  $6!/2 = 360$ .

Also, the number of words in which B and H come together =  $15 \cdot 12/2 = 120$

$\therefore$  the required number of words =  $360 - 120 = 240$ .

29. The number of divisors of 9600 is
- (A) 46 (B) 48 (C) 58 (D) 60

Ans. (B)

Sol.  $\because 9600 = 2^7 \times 3 \times 5^2$ .

Now by taking one or more of these prime factors of 9600, we shall get a divisor of 9600. So, number of such divisors =  $(7 + 1)(1 + 1)(2 + 1) - 1 = 47$ .

But 1 is also a divisor of 9600 which is not included in these divisors, so required number =  $47 + 1 = 48$ .

30. The sides AB, BC, CA of a triangle ABC have 3, 4 and 5 interior points respectively on them. The number of triangles that can be constructed using these points as vertices, is
- (A) 220 (B) 210 (C) 205 (D) 200

Ans. (C)

Sol. Total points are  $3 + 4 + 5 = 12$ . The number of all possible triangles formed by joining these points taking 3 at a time =  ${}^{12}C_3 = 220$ . But the number of triangles by joining 3 points of AB =  ${}^3C_3 = 1$ , then number of triangles by joining 3 points of BC =  ${}^4C_3 = 4$ , and the number of triangles by joining 3 points of

CA =  ${}^5C_3 = 10$ . Actually, we shall have no such triangles.

$\therefore$  the required number =  $220 - (1 + 4 + 10) = 205$ .

(MATHEMATICS)

DIWALI ASSIGNMENT

31. A student is to answer 10 out of 13 questions in an examination such that he must choose at least 4 from the first 5 questions. The number of choices available to him is

- (A) 346 (B) 140 (C) 196 (D) 280

Ans. (C)

Sol. He can select 4 questions from first 5 questions and 6 from remaining 8 questions or he can select first 5 questions and 5 from remaining 8 questions. So total number of choices

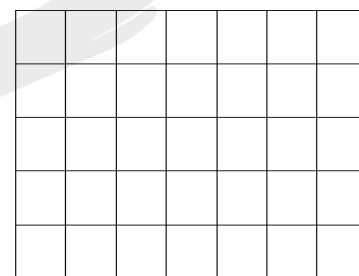
$$= ({}^5C_4 \times {}^8C_6) + ({}^5C_5 \times {}^8C_5) = (5 \times 28) + (1 + 56) = 196$$

32. A rectangle with sides  $(2m - 1)$  and  $(2n - 1)$  units is divided into squares of unit length by drawing parallel lines as shown in the diagram. The number of rectangles possible with odd side lengths is

- (A)  $m^2n^2$  (B)  $4^{m+n-1}$  (C)  $mn(m + 1)(n + 1)$  (D)  $(m + n + 1)^2$

Ans. (A)

Sol. In the given diagram, there are  $2m$  vertical (say numbered  $1, 2, 3, \dots, 2m$ ) and  $2n$  horizontal (say numbered  $1, 2, 3, \dots, 2n$ ) lines. To have a rectangle with odd side lengths, we should select two vertical lines one even numbered, other odd numbered and two horizontal lines in the same way. The number of such selections is  $({}^mC_1 {}^mC_1)({}^nC_1 {}^nC_1) = m^2n^2$



33. The total number of ways in which 5 balls of different colours can be distributed among 3 persons so that each person gets at least one ball is

- (A) 75 (B) 150 (C) 210 (D) 243

Ans. (B)

Sol. Five different balls can be distributed among three persons in the following ways :

Case I: 1 1 3 (balls)  
Case II: 2 2 1 (balls)

So total ways of distribution of 5 balls in case I

$$= \frac{|5|}{|1|1|3|} \cdot \frac{1}{|2|} = 10 \text{ total ways of distribution of 5 balls in case II} = \frac{|5|}{|2|2|1|} \cdot \frac{1}{|2|} = 15$$

$$\therefore \text{Total ways of distributions of 5 balls to 3 persons} = (10 + 15) \times |3| = 150.$$

(MATHEMATICS)

DIWALI ASSIGNMENT

BINOMIAL THEOREM

34. Given  $r > 1, n > 2$  and the coefficients of  $(3r)$ th and  $(r + 2)$  th terms in the expansion of  $(1 + x)^{2n}$  are equal, then

- (A)  $n = 2r$  (B)  $n = 2r - 1$  (C)  $n = 2r + 1$  (D) none of these

Ans. (A)

Sol. Coef. of  $T_{3r} = \text{Coef. of } T_{r+2} \Rightarrow {}^{2n}C_{3r-1} = {}^{2n}C_{r+1} \Rightarrow (3r - 1) + (r + 1) = 2n \Rightarrow n = 2r$ .

35. In the expansion of  $\left(\frac{x+1}{x^{2/3}-x^{1/3}+1} - \frac{x-1}{x-x^{1/2}}\right)^{10}$ , the term independent of  $x$  is

- (A)  ${}^{10}C_7$  (B)  ${}^{10}C_4$  (C)  ${}^{10}C_5$  (D) does not exist

Ans. (B)

Sol. Exp.  $= \left(\frac{(x^{1/3}+1)(x^{2/3}+1-x^{1/3})}{x^2-x^{1/3}+1} - \frac{(\sqrt{x}-1)(\sqrt{x}+1)}{\sqrt{x}(\sqrt{x}-1)}\right)^{10} = \left(x^{1/3} + 1 - \frac{\sqrt{x}+1}{\sqrt{x}}\right)^{10}$   
 $= \left(x^{1/3} + 1 - 1 - \frac{1}{x^{1/2}}\right)^{10} = \left(x^{1/3} - \frac{1}{x^{1/2}}\right)^{10}$  Now if  $T_{r+1}$  is the term independent of  $x$ ,  
 then  $r = \frac{10(1/3)}{1/3+1/2} = 4. \therefore$  required term  $= T_{4+1} = {}^{10}C_4(-1)^4 = {}^{10}C_4$

36. If the sum of the coefficients in the expansion of  $(x + y)^n$  is 4096, then in this expansion the greatest binomial coefficient is

- (A) 930 (B) 925 (C) 924 (D) none of these

Ans. (C)

Sol. If we take  $x = 1 = y$ ; then the sum of the coefficients in the expansion of  $(x + y)^n$  will be

$$(1 + 1)^n = 4096 \Rightarrow 2^n = 4096 = 2^{12} \Rightarrow n = 12.$$

Now the term with greatest binomial coefficient is the middle term which is  $(12/2) + 1 = 7$  th term. Hence required coefficient  $= {}^{12}C_6 = 924$ .

37. If  $(1 + x)^n = C_0 + C_1x + C_2x^2 + \dots + C_nx^n$ , then  $C_0 + \frac{C_1}{2} + \frac{C_2}{3} + \frac{C_3}{4} + \dots + \frac{C_n}{n+1}$  is equal to

- (A)  $\frac{2^{n-1}-1}{n+1}$  (B)  $\frac{2^{n+1}-1}{n+1}$  (C)  $\frac{2^{n-1}}{n-1}$  (D)  $\frac{2^{n-1}}{n+1}$

(MATHEMATICS)

DIWALI ASSIGNMENT

Ans. (B)

Sol. Integrating both sides of given expansion with respect x in  $[0,1]$ , we get

$$\Rightarrow \left[ \frac{(1+x)^{n+1}}{n+1} \right]_0^1 = \left[ C_0 x + C_1 \frac{x^2}{2} + C_2 \frac{x^3}{3} + \dots + C_n \frac{x^{n+1}}{n+1} \right]_0^1 \Rightarrow \frac{2^{n+1}-1}{n+1} = C_0 + \frac{C_1}{2} + \frac{C_2}{3} + \dots + \frac{C_n}{n+1}$$

Aliter. If  $n = 1$ , then

$$\text{Exp.} = C_0 + \frac{C_1}{2} = 1 + \frac{1}{2} = \frac{3}{2}, \text{ which is true only in (2) alternative.}$$

38. If  $n$  is a positive integer, then integral part of  $(3 + \sqrt{7})^n$  is

- (A) an even number (B) an odd number  
(C) a prime number (D) none of these

Ans. (B)

Sol. Let  $(3 + \sqrt{7})^n = I + F$ , where  $I$  is integral part and  $F$  is fractional part. Further let

$$(3 - \sqrt{7})^n = f \text{ Obviously, } 0 < f < 1 [\because 0 < (3 - \sqrt{7}) < 1]$$

$$\text{Now } I + F = 3^n + {}^nC_1 3^{n-1} \sqrt{7} + {}^nC_2 3^{n-2} (\sqrt{7})^2 + \dots (1)$$

$$f = 3^n - {}^nC_1 3^{n-1} \sqrt{7} + {}^nC_2 3^{n-2} (\sqrt{7})^2 - \dots (2)$$

$$(1) + (2) \Rightarrow I + F + f = 2[3^n + {}^nC_2 3^{n-2} (\sqrt{7})^2 + \dots] \Rightarrow I + F + f = \text{an even number} \dots (3)$$

Further  $0 < F < 1$  and  $0 < f < 1$ .

$$\Rightarrow 0 < F + f < 2 \dots (4)$$

Now (3) and (4) show that

$$F + f = 1 \dots (5)$$

Also (3), (5)  $\Rightarrow I + 1 = \text{an even integer} \Rightarrow I = \text{an odd integer.}$

39. The greatest integer which divides  $101^{100} - 1$  is

- (A) 100 (B) 1000 (C) 10,000 (D) 100,000

Ans. (C)

(MATHEMATICS)

DIWALI ASSIGNMENT

**Sol.**  $101^{100} - 1 = (100 + 1)^{100} - 1 = 100^{100} + {}^{100}C_1 100^{99} + {}^{100}C_2 100^{98} + \dots + 1 - 1$

$$= 100^{100} + {}^{100}C_1 100^{99} + {}^{100}C_2 100^{98} + \dots + {}^{100}C_{99} 100^1$$

$$= 100(100^{99} + {}^{100}C_1 100^{98} + \dots + {}^{100}C_{99})$$

$$= 100(100^{99} + {}^{100}C_1 100^{98} + \dots + {}^{100}C_{98} 100 + {}^{100}C_{99})$$

$$= 100(100^{99} + {}^{100}C_1 100^{98} + \dots + {}^{100}C_{98} 100 + 100)$$

$$= 100^2(100^{98} + {}^{100}C_1 100^{97} + \dots + {}^{100}C_2 + 1)$$

$\therefore$  the greatest integer which divides given number  $= 100^2 = 10,000$ .

- 40.** The number of integral terms in the expansion of  $(\sqrt{3} + \sqrt[8]{5})^{256}$  is  
 (A) 35 (B) 34 (C) 33 (D) 32

**Ans. (C)**

**Sol.** Exp.  $= (3^{1/2} + 5^{1/8})^{256}$

$$T_{r+1} = {}^{256}C_r (3^{1/2})^{256-r} (5^{1/8})^r = {}^{256}C_r \cdot 3^{(256-r)/2} \cdot 5^{r/8}, r = 0, 1, 2, \dots, 256$$

Now  $T_{r+1}$  is an integral term when  $(256 - r)/2$  and  $r/8$  both are integers. This is possible for  $r = 0, 8, 16, 24, \dots, 256$  (33 values). Hence required number of terms = 33.

TRIGONOMETRICAL FUNCTIONS

- 41.** If  $\operatorname{cosec} A + \cot A = 11/2$ , then  $\tan A$  is  
 (A) 21/12 (B) 15/16 (C) 44/17 (D) 117/43

**Ans. (C)**

**Sol.**  $\operatorname{cosec} A + \cot A = 11/2 \Rightarrow \frac{1}{\operatorname{cosec} A + \cot A} = \frac{2}{11} \Rightarrow \operatorname{cosec} A - \cot A = \frac{2}{11}$

$$(1) - (2) \Rightarrow 2\cot A = \frac{11}{2} - \frac{2}{11} = \frac{117}{22} \Rightarrow \tan A = \frac{44}{117}$$

- 42.**  $\sin 47^\circ + \sin 61^\circ - \sin 11^\circ - \sin 25^\circ$  equals  
 (A)  $\sin 36^\circ$  (B)  $\sin 7^\circ$  (C)  $\cos 36^\circ$  (D)  $\cos 7^\circ$

**Ans. (D)**

(MATHEMATICS)

DIWALI ASSIGNMENT

**Sol.**  $\text{Exp.} = (\sin 47^\circ - \sin 11^\circ) + (\sin 61^\circ - \sin 25^\circ)$

$$= 2\cos 29^\circ \sin 18^\circ + 2\cos 43^\circ \sin 18^\circ = 2\sin 18^\circ (2\cos 36^\circ \cos 7^\circ) = 4 \left( \frac{\sqrt{5}-1}{4} \right) \left( \frac{\sqrt{5}+1}{4} \right) \cos 7^\circ = \cos 7^\circ$$

**43.** If  $\sqrt{3}\cos\theta + \sin\theta = \sqrt{2}$ , then general solution of  $\theta$  is

(A)  $n\pi + (-1)^n \frac{\pi}{4}$  (B)  $(-1)^n \frac{\pi}{4} - \frac{\pi}{3}$  (C)  $n\pi + \frac{\pi}{4} - \frac{\pi}{3}$  (D)  $n\pi + (-1)^n \frac{\pi}{4} - \frac{\pi}{3}$

**Ans. (D)**

**Sol.**  $\sqrt{3}\cos\theta + \sin\theta = \sqrt{2} \Rightarrow \frac{\sqrt{3}}{2}\cos\theta + \frac{1}{2}\sin\theta = \frac{\sqrt{2}}{2} \Rightarrow \sin\left(\theta + \frac{\pi}{3}\right) = \frac{1}{\sqrt{2}} = \sin\frac{\pi}{4} \therefore \theta + \frac{\pi}{3} = n\pi + (-1)^n \frac{\pi}{4} \Rightarrow \theta = n\pi - \frac{\pi}{3} + (-1)^n \frac{\pi}{4}, n \in \mathbb{Z}$

**44.** If  $x\cos\theta = y\cos(\theta + 2\pi/3) = z\cos(\theta + 4\pi/3)$ , then  $\frac{1}{x} + \frac{1}{y} + \frac{1}{z}$  is equal to

(A) 1 (B) 2 (C) 0 (D)  $3\cos\theta$

**Ans. (C)**

**Sol.** Let  $x\cos\theta = y\cos\left(\theta + \frac{2\pi}{3}\right) = z\cos\left(\theta + \frac{4\pi}{3}\right) = \lambda \Rightarrow \frac{\lambda}{x} + \frac{\lambda}{y} + \frac{\lambda}{z} = \cos\theta + \cos\left(\theta + \frac{2\pi}{3}\right) + \cos\left(\theta + \frac{4\pi}{3}\right)$

$$= \cos\theta + \cos\left(\pi - \frac{\pi}{3} + \theta\right) + \cos\left(\pi + \frac{\pi}{3} + \theta\right)$$

$$= \cos\theta - \cos(\pi/3 - \theta) - \cos(\pi/3 + \theta) = \cos\theta - 2\cos\pi/3\cos\theta = 0 \quad \frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 0$$

**45.**  $\sin\frac{\pi}{14} \cdot \sin\frac{3\pi}{14} \cdot \sin\frac{5\pi}{14} \cdot \sin\frac{7\pi}{14} \cdot \sin\frac{9\pi}{14} \cdot \sin\frac{11\pi}{14} \cdot \sin\frac{13\pi}{14}$  is equal to

(A)  $1/64$  (B)  $1/32$  (C)  $1/16$  (D)  $1/8$

**Ans. (A)**

**Sol.**  $\text{Exp.} = \sin\frac{\pi}{14} \cdot \sin\frac{3\pi}{14} \cdot \sin\frac{5\pi}{14} \cdot 1 \cdot \sin\left(\pi - \frac{5\pi}{14}\right) \cdot \sin\left(\pi - \frac{3\pi}{14}\right) \cdot \sin\left(\pi - \frac{\pi}{14}\right)$

$$= \left(\sin\frac{\pi}{14} \cdot \sin\frac{3\pi}{14} \cdot \sin\frac{5\pi}{14}\right)^2 \cdot 1 = \left\{\cos\left(\frac{\pi}{2} - \frac{\pi}{14}\right) \cdot \cos\left(\frac{\pi}{2} - \frac{3\pi}{14}\right) \cdot \cos\left(\frac{\pi}{2} - \frac{5\pi}{14}\right) \cdot \sin\left(\pi - \frac{\pi}{14}\right)\right\}^2$$

$$= \left(\cos\frac{3\pi}{7} \cdot \cos\frac{2\pi}{7} \cdot \cos\frac{\pi}{7}\right)^2 = \left(\cos\frac{\pi}{7} \cdot \cos\frac{2\pi}{7} \cdot \cos\frac{3\pi}{7}\right)^2 = \left(-\cos\frac{\pi}{7} \cdot \cos\frac{2\pi}{7} \cdot \cos\frac{4\pi}{7}\right)^2 = \left[-\frac{\sin 2^3\pi/7}{2^3\sin\pi/7}\right]$$

$$\left[\because \cos\theta \cdot \cos 2\theta \cdot \cos 2^2\theta \cdot \dots \cdot \cos 2^{n-1}\theta = \frac{\sin 2^n\theta}{2^n\sin\theta}\right] = \frac{1}{64} \cdot \left(\frac{\sin 8\pi/7}{\sin\pi/7}\right)^2 = \frac{1}{64}.$$

(MATHEMATICS)

DIWALI ASSIGNMENT

46. If  $\cos\left(\frac{\pi}{4} - x\right) \cos 2x + \sin x \sin 2x \sec x = \cos x \sin 2x \sec x + \cos(\pi/4 + x) \cos 2x$ , then

possible value(s) of  $\sec x$  is(are)

- (A) 1 (B) 2 (C)  $\sqrt{2}$  (D)  $\sqrt{3}$

Ans. (A, C)

Sol. From the given relation, we have

$$\cos 2x [\cos(\pi/4 - x) - \cos(\pi/4 + x)] + 2\sin^2 x = 2\sin x \cos x$$

$$\Rightarrow \cos 2x (\sqrt{2} \sin x) + 2\sin^2 x = 2\sin x \cos x \Rightarrow \sqrt{2} \sin x \{\cos 2x + \sqrt{2} \sin x - \sqrt{2} \cos x\} = 0$$

$$\Rightarrow \sin x \{\cos^2 x - \sin^2 x + \sqrt{2}(\sin x - \cos x)\} = 0 \Rightarrow \sin x (\cos x - \sin x)(\sin x + \cos x - \sqrt{2}) = 0$$

$$\Rightarrow \sin x = 0 \text{ or } \cos x - \sin x = 0 \text{ or } \sin x + \cos x - \sqrt{2} = 0$$

$$\Rightarrow \sin x = 0 \text{ or } \tan x = 1 \text{ or } \sin(x + \pi/4) = 1 \quad \therefore x = 0 \text{ or } x = \pi/4 \text{ or } x = \pi/4$$

Hence  $\sec x = 1$  or  $\sqrt{2}$ .

PROPERTIES AND SOLUTIONS OF A TRIANGLE

47. In triangle ABC, if  $\frac{\cos A}{a} = \frac{\cos B}{b} = \frac{\cos C}{c}$  and  $a = 2$ , then area of this triangle is

- (A) 1 (B) 2 (C)  $\sqrt{3}/2$  (D)  $\sqrt{3}$

Ans. (D)

Sol. Given  $\frac{\cos A}{a} = \frac{\cos B}{b} = \frac{\cos C}{c} \dots (1)$   $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \dots (2)$

$$(1), (2) \Rightarrow \frac{\cos A}{\sin A} = \frac{\cos B}{\sin B} = \frac{\cos C}{\sin C} \Rightarrow \angle A = \angle B = \angle C \Rightarrow \text{triangle is equilateral.}$$

$$\text{Hence, its area} = \frac{\sqrt{3}}{4} (\text{side})^2 = \frac{\sqrt{3}}{4} (2)^2 = \sqrt{3}.$$

48. In triangle ABC,  $\cos A + \cos B + \cos C$  is equal to

- (A)  $1 + R/r$  (B)  $1 + r/R$  (C)  $1 - R/r$  (D)  $1 - r/R$

Ans. (B)

Sol.  $\cos A + \cos B + \cos C$

$$= 1 + 4 \sin \frac{A}{2} \sin \frac{B}{2} \sin \frac{C}{2} \text{ [identity relation]} = 1 + \frac{r}{R}. \left[ \because r = 4R \sin \frac{A}{2} \sin \frac{B}{2} \sin \frac{C}{2} \right]$$

(MATHEMATICS)

DIWALI ASSIGNMENT

49. In a triangle ABC,  $\frac{b-c}{r_1} + \frac{c-a}{r_2} + \frac{a-b}{r_3}$  is equal to

- (A) 1 (B) 0 (C) abc (D)  $r_1 r_2 r_3$

Ans. (B)

Sol. Exp.  $= \frac{b-c}{\frac{\Delta}{s-a}} + \frac{c-a}{\frac{\Delta}{s-b}} + \frac{a-b}{\frac{\Delta}{s-c}} = \frac{1}{\Delta} [(b-c)(s-a) + (c-a)(s-b) + (a-b)(s-c)]$   
 $= \frac{1}{\Delta} [s(b-c+c-a+a-b) - a(b-c) - b(c-a) - c(a-b)] = 0$

50. In a triangle ABC, if  $3a = b + c$ , then  $\cot \frac{B}{2} \cot \frac{C}{2}$  is equal to

- (A)  $\sqrt{2}$  (B)  $\sqrt{3}$  (C) 1 (D) 2

Ans. (D)

Sol. Exp.  $= \sqrt{\frac{s(s-b)}{(s-c)(s-a)}} \cdot \sqrt{\frac{s(s-c)}{(s-a)(s-b)}} = \frac{s}{s-a} = \frac{2a}{2a-a} [\because 3a = b + c \Rightarrow 4a = 2s] = 2$

51. In a triangle ABC, if  $\angle A = 45^\circ, \angle B = 75^\circ$ , then  $a + c\sqrt{2}$  is equal to

- (A) 1 (B) 0 (C) b (D) 2b

Ans. (D)

Sol.  $\because \angle C = 180^\circ - (45^\circ + 75^\circ) = 60^\circ$ , so, using sine formula, we have

$$\frac{a}{\sin 45^\circ} = \frac{b}{\sin 75^\circ} = \frac{c}{\sin 60^\circ} \Rightarrow \frac{a}{1/\sqrt{2}} = \frac{b}{(\sqrt{3}+1)/2\sqrt{2}} = \frac{c}{\sqrt{3}/2}$$

$$\Rightarrow a = \frac{2b}{\sqrt{3}+1} \text{ and } c = \frac{\sqrt{6}b}{\sqrt{3}+1} \Rightarrow a + c\sqrt{2} = \frac{2b}{\sqrt{3}+1} + \frac{2\sqrt{3}b}{\sqrt{3}+1} = 2b.$$

52. In a triangle ABC, if  $a \cos^2 C/2 + c \cos^2 A/2 = 3b/2$ , then a, b, c are in

- (A) AP (B) GP (C) HP (D) none of these

Ans. (A)

Sol. We have  $a \frac{s(s-c)}{ab} + c \frac{s(s-a)}{bc} = \frac{3b}{2} \Rightarrow 2s(s-c+s-a) = 3b^2 \Rightarrow 2s(b) = 3b^2$   
 $\Rightarrow a + b + c = 3b \Rightarrow a + c = 2b \Rightarrow a, b, c \text{ are in AP.}$

(MATHEMATICS)

DIWALI ASSIGNMENT

53. In a triangle ABC, if  $\Delta = a^2 - (b - c)^2$ , then  $\tan A$  is equal to.

- (A)  $1/2$  (B)  $8/15$  (C)  $15/16$  (D)  $8/17$

Ans. (B)

Sol.  $\Delta = a^2 - (b - c)^2 = (a - b + c)(a + b - c) \Rightarrow \sqrt{s(s - a)(s - b)(s - c)} = (2s - 2b)(2s - 2c)$

$$\Rightarrow \sqrt{s(s - a)} = 4\sqrt{(s - b)(s - c)} \Rightarrow \sqrt{\frac{(s - b)(s - c)}{s(s - a)}} = \frac{1}{4} \Rightarrow \tan \frac{A}{2} = \frac{1}{4}$$

$$\therefore \tan A = \frac{2 \tan \frac{A}{2}}{1 - \tan^2 \frac{A}{2}} = \frac{2(1/4)}{1 - (1/16)} = \frac{8}{15}$$

HEIGHT & DISTANCE

54. A man from the top of a 100m high tower sees a car moving towards the tower at an angle of depression of  $30^\circ$ . After some time, the angle of depression becomes  $60^\circ$ . The distance (in meters) travelled by the car during this time is

- (A)  $100\sqrt{3}$ m (B)  $(200\sqrt{3})/3$ m  
(C)  $(100\sqrt{3})/3$ m (D)  $200\sqrt{3}$ m

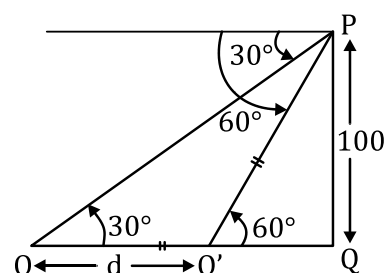
Ans. (B)

Sol. Let  $d$  be the distance travelled by car. Now from adjoining diagram in  $\Delta PQO'$

$$\frac{100}{PO'} = \sin 60^\circ \Rightarrow PO' = \frac{200}{\sqrt{3}}. \text{ Also, in } \Delta PQO \angle OPO' =$$

$$= 60^\circ - 30^\circ = 30^\circ = \angle POO'$$

$$\Rightarrow OO' = PO' \Rightarrow d = \frac{200}{\sqrt{3}} = \frac{(200\sqrt{3})}{3} \text{ m.}$$



55. The angle of elevation of the top of a tower from the top and bottom of a building of height  $a$  are  $30^\circ$  and  $45^\circ$  respectively. If the tower and the building stand at the same level, the height of the tower is.

- (A)  $a(\sqrt{3} + 1)$  (B)  $\left(\frac{a}{2}\right)(3 + \sqrt{3})$   
(C)  $a(\sqrt{3} - 1)$  (D)  $a\sqrt{3}$

(MATHEMATICS)

DIWALI ASSIGNMENT

Ans. (B)

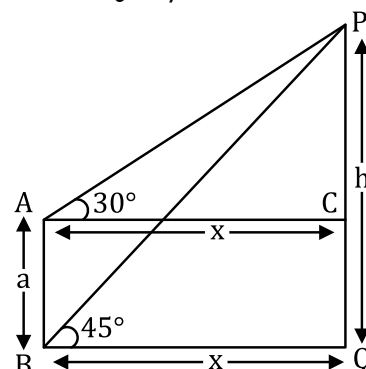
**Sol.** Let PQ be given tower of height  $h$  and AB be given building of height  $a$ . From the adjoining diagram  $\angle PAC = 30^\circ$  and  $\angle PBQ = 45^\circ$ . If  $BQ = AC = x$ , then in  $\triangle PQB$   $h/x = \tan 45^\circ$

$$\Rightarrow x = h \dots (1)$$

Also, in  $\triangle PCA$

$$\frac{h-a}{x} = \tan 30^\circ \Rightarrow h - a = h \left( \frac{1}{\sqrt{3}} \right) [\because x = h]$$

$$\Rightarrow h(1 - 1/\sqrt{3}) = a \therefore h = \frac{\sqrt{3}}{\sqrt{3}-1} a = \frac{3+\sqrt{3}}{2}.$$



**56.** The angles of elevation of the top of a tower at two points which are at distances  $a$  and  $b$  from the foot in the same horizontal line and on the same sides of the tower, are complementary. The height of the tower is

- (A)  $ab$  (B)  $\sqrt{ab}$  (C)  $\sqrt{a/b}$  (D)  $\sqrt{b/a}$

Ans. (B)

**Sol.** Let PQ be given tower of height  $h$ . If A, B be given points then suppose  $\angle PAQ = \alpha$  and  $\angle PBQ = \beta$ . Then  $\alpha + \beta = \pi/2$ .

Now in  $\triangle PAQ$ ,

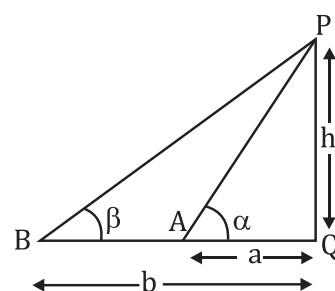
$$\tan \alpha = h/a \dots (1)$$

in  $\triangle PBQ$ ,

$$\tan \beta = h/b \dots (2)$$

$$(1), (2) \Rightarrow \tan \alpha \tan \beta = (h/a) \cdot (h/b)$$

$$\Rightarrow \tan \alpha \tan(\pi/2 - \alpha) = h^2/ab \Rightarrow h = \sqrt{ab}.$$



**57.** ABC is a triangular area where  $AB = AC = 100$  m. A TV tower is standing at the mid-point of BC. If angles of elevation of the top of the tower with respect to A, B, C are  $45^\circ, 60^\circ, 60^\circ$ , then the height of the tower is

- (A) 50 m (B)  $50\sqrt{3}$  m (C)  $50/\sqrt{3}$  m (D) none of these

Ans. (B)

(MATHEMATICS)

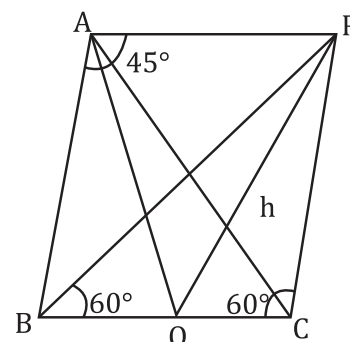
DIWALI ASSIGNMENT

**Sol.** Let PO be the tower with height h. Then from diagram

$$AO = h \cot 45^\circ = h$$

$$OB = OC = h \cot 60^\circ = \frac{h}{\sqrt{3}}$$

$$\text{Now } AB^2 = AO^2 + OB^2 \Rightarrow 100^2 = h^2 + \frac{h^2}{3} \Rightarrow h = 50\sqrt{3} \text{ m.}$$



THE POINT

**58.** Points  $(0, 8/3)$ ,  $(1, 3)$  and  $(82, 30)$  are vertices of a

- (A) right angled triangle (C) obtuse angled triangle  
(B) acute angled triangle (D) none of these

**Ans.** (D)

**Sol.** Let given points be A, B, C. Then

$$AB = \sqrt{10}/3, BC = \sqrt{(81)^2 + (27)^2} = 27\sqrt{10}, AC = \sqrt{(82)^2 + (82/3)^2} = 82\sqrt{10}/3.$$

$$\therefore AB + BC = 82\sqrt{10}/3 = AC. \Rightarrow A, B, C \text{ are collinear.}$$

**59.** If the points  $(x_1 + t(x_2 - x_1), y_1 + t(y_2 - y_1))$  divides the line joining  $(x_1, y_1)$  and  $(x_2, y_2)$  internally, then

- (A)  $t < 0$  (B)  $0 < t < 1$  (C)  $t > 1$  (D)  $t = 1$

**Ans.** (B)

**Sol.** Coordinates of the dividing point can be written as

$$\left( \frac{tx_2 + (1-t)x_1}{t + (1-t)}, \frac{ty_2 + (1-t)y_1}{t + (1-t)} \right) \text{ This shows that the division 'ratio' } = \frac{t}{1-t}. \text{ But this division is internal, so}$$

$$\frac{t}{1-t} > 0 \Rightarrow \frac{t}{t-1} < 0 \Rightarrow t \text{ and } (t-1) \text{ are in opposite signs. } \Rightarrow 0 < t < 1.$$

**60.** The point  $(4, 1)$  undergoes the following three transformations successively :

- (i) Reflection about the line  $y = x$ .  
(ii) Transformation through a distance 2 units along the positive direction of x-axis.  
(iii) Rotation through an angle  $\pi/4$  about the origin in the counter clockwise direction.

The final position of the point is given by the coordinates :

- (A)  $(7/\sqrt{2}, -1/\sqrt{2})$  (B)  $(7/\sqrt{2}, 1/\sqrt{2})$  (C)  $(-1/\sqrt{2}, 7/\sqrt{2})$  (D) none of these

(MATHEMATICS)

DIWALI ASSIGNMENT

Ans. (B)

**Sol.** After first transformation the coordinates of the point become (1,4) which after second transformation will become (1 + 2,4), i.e., (3,4). If (x', y') be the coordinates of the point after third transformation, then  $x' = 3\cos\pi/4 + 4\sin\pi/4 = 7\sqrt{2}$

$$y' = -3\sin\pi/4 + 4\cos\pi/4 = 1/\sqrt{2} \therefore \text{final coordinates are } (7/\sqrt{2}, 1/\sqrt{2}).$$

**61.** If a vertex of a triangle is (1,1) and the midpoints of two sides through this vertex are (-1,2) and (3,2), then the centroid of the triangle is

- (A) (1/3, 7/3) (B) (1, 7/3) (C) (-1/3, 7/3) (D) (-1, 7/3)

Ans. (B)

**Sol.** Let ABC be the given triangle and A be the given vertex (1,1). Let B  $\equiv$  (a, b) and C  $\equiv$  (c, d). Then  $a + 1 = -2, b + 1 = 4 \Rightarrow B \equiv (-3, 3)$  and  $c + 1 = 6, d + 1 = 5 \Rightarrow C \equiv (5, 3)$ .

$$\therefore \text{Centroid} \equiv \left( \frac{1-3+5}{3}, \frac{1+3+3}{3} \right) = \left( 1, \frac{7}{3} \right).$$

**62.** Let A(h, k), B(1,1) and C(2,1) be vertices of a right-angled triangle with AC as the hypotenuse. If the area of the triangle is 1, then the set of values which k can take is given by

- (A) {1,3} (B) {0,2} (C) {-1,3} (D) {-3,-2}

Ans. (C)

**Sol.**  $\therefore \text{Area of } \triangle ABC = 1 \Rightarrow \frac{1}{2}(AB)(BC) = 1 \Rightarrow [(h-1)^2 + (k-1)^2] \cdot 1 = 4$

$$\Rightarrow (h-1)^2 + (k-1)^2 = 4 \quad \dots(1) \text{ Again } AB \perp BC \Rightarrow \frac{k-1}{h-1} = \infty \quad [\because \text{slope of } BC = 0]$$

$$\Rightarrow h-1 = 0 \Rightarrow h = 1 \quad \dots(2) \quad (1), (2) \Rightarrow (k-1)^2 = 4 \Rightarrow k = -1, 3.$$

STRAIGHT LINE

**63.** The system of lines  $ax + by + c = 0$  where  $3a + 2b + 4c = 0$ , passes through the point

- (A) (0,0) (B) (1/2, 3/4) (C) (3/4, 1/2) (D) none of these

Ans. (C)

(MATHEMATICS)

DIWALI ASSIGNMENT

**Sol.** From the given relation  $4c = -3a - 2b$ . Putting this value of  $c$  in the given equation, we have

$$4ax + 4by - 3a - 2b = 0 \Rightarrow a(4x - 3) + b(4y - 2) = 0 \Rightarrow (4x - 3) + (b/a)(4y - 2) = 0$$

which is of the form  $L + \lambda L' = 0$ . Hence the system passes through the point of intersection of  $4x - 3 = 0$  and  $4y - 2 = 0$  which is  $(3/4, 1/2)$ .

**64.** A straight-line segment of length  $l$  moves with its ends on the coordinate axes. The locus of the point which divides the line in the ratio 1: 2 is

(A)  $9x^2 + 36y^2 = l^2$

(B)  $36x^2 + 9y^2 = l^2$

(C)  $36x^2 + 9y^2 = 4l^2$

(D)  $9x^2 + 36y^2 = 4l^2$

**Ans.** (D)

**Sol.** Let the equation of the line be  $x/a + y/b = 1$  and its ends on coordinate axes be A and B. Then  $A \equiv (a, 0), B \equiv (0, b)$ . Now  $AB = l \Rightarrow a^2 + b^2 = l^2$ . .....(1)

Let the division point be  $(h, k)$ . Then  $h = 2a/3, k = b/3$ . .....(2)

From (1) and (2) eliminating  $a, b$ , we have  $9h^2/4 + 9k^2 = l^2$

$\therefore$  required locus is  $9x^2 + 36y^2 = 4l^2$ .

**65.** If the sum of the distances of a point from two perpendicular coplanar lines is 1, then the locus of this point is a

(A) line

(B) pair of lines

(C) square

(D) circle

**Ans.** (C)

**Sol.** If we take given perpendicular lines as coordinate axes, then their equations will be  $x = 0$ ,

$y = 0$ . Let  $P(x, y)$  be the given point, then as given  $|x| + |y| = 1$ , which represents following four lines :  $x + y = 1, -x - y = 1, x - y = 1, -x + y = 1$ . Obviously, they are the sides of a square.

**66.** Two consecutive sides of a parallelogram are  $4x + 5y = 0$  and  $7x + 2y = 0$ . If the equation to one diagonal is  $11x + 7y = 9$ , then equation of the other diagonal is

(A)  $x - y = 0$

(B)  $x + 2y = 0$

(C)  $2x + y = 0$

(D) none of these

**Ans.** (A)

(MATHEMATICS)

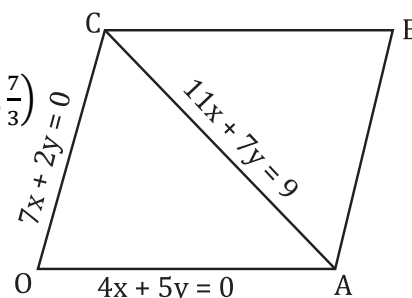
DIWALI ASSIGNMENT

**Sol.** Since given two consecutive sides say OA and OC pass through (0,0) and given diagonal does not pass through (0,0), so it is diagonal AC. Now

$$\begin{cases} 4x + 5y = 0 \\ 11x + 7y = 9 \end{cases} \Rightarrow A \equiv \left(\frac{5}{3}, -\frac{4}{3}\right) \text{ and } \begin{cases} 7x + 2y = 0 \\ 11x + 7y = 9 \end{cases} \Rightarrow C \equiv \left(-\frac{2}{3}, \frac{7}{3}\right)$$

Now mid-point of AC = (1/2, 1/2).

So equation of other diagonal OB is  $y = x$ .



**67.** Orthocenter of the triangle with vertices (0,0); (3,4) and (4,0) is

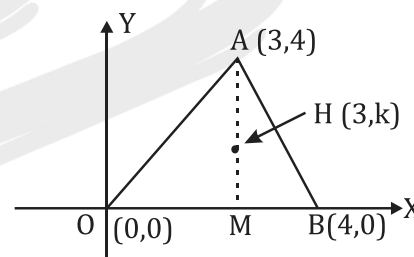
- (A) (3,5/4)      (B) (3,12)      (C) (3,3/4)      (D) (3,9)

**Ans.** (C)

**Sol.** Let given points be O, A, B respectively. Obviously as shown in the diagram  $\triangle OAB$  is acute angled and lies in first quadrant. If AM is altitude from A, then  $M \equiv (3,0)$ ; So let orthocentre  $H \equiv (3, k)$ . Now

$$OH \perp AB \Rightarrow (k/3)(-4/1) = -1$$

$$\Rightarrow k = -3/4. \text{ Orthocenter} = (3, 3/4).$$



**68.** The locus of the mid-point of the intercept of the variable line  $x \cos \alpha + y \sin \alpha = p$  ( $p$  constant) between coordinate axes is

- (A)  $x^{-2} + y^{-2} = p^{-2}$       (B)  $x^{-2} + y^{-2} = 2p^{-2}$   
(C)  $x^{-2} + y^{-2} = 4p^{-2}$       (D) none of these

**Ans.** (C)

**Sol.** If AB be the given intercept, then  $A \equiv \left(\frac{p}{\cos \alpha}, 0\right)$  and  $B \equiv \left(0, \frac{p}{\sin \alpha}\right)$ .

Let (x,y) be the mid-point of AB, then  $2x = \frac{p}{\cos \alpha}$ ,  $2y = \frac{p}{\sin \alpha}$ . Eliminating  $\alpha$ , we get the required locus as  $x^{-2} + y^{-2} = 4p^{-2}$ .

**69.** For  $a > b > c > 0$ , the distance between (1,1) and the point of intersection of the lines

$ax + by + c = 0$  and  $bx + ay + c = 0$  is less than  $2\sqrt{2}$ . Then

- (A)  $a + b - c > 0$       (B)  $a - b + c < 0$       (C)  $a - b + c > 0$       (D)  $a + b - c < 0$

**Ans.** (A,C)

(MATHEMATICS)

DIWALI ASSIGNMENT

**Sol.** Solving given equations, we get their point of intersection as

$$\left(-\frac{c}{a+b}, -\frac{c}{a+b}\right) \text{ From given condition, we have } \sqrt{2\left(1 + \frac{c}{a+b}\right)^2} < 2\sqrt{2}$$

$\Rightarrow \left|1 + \frac{c}{a+b}\right| < 2 \quad [\because a, b, c > 0] \Rightarrow 1 + \frac{c}{a+b} < 2 \Rightarrow \frac{c}{a+b} < 1 \Rightarrow a + b - c > 0$ . Further, we may note that  $a > b > c > 0 \Rightarrow a - b + c > 0$  (always). So (1) and (3) are correct answers.

CIRCLE

**70.** The equation of the circle passing through the origin and cutting intercepts  $a, b$  from coordinate axes, is

(A)  $x^2 + y^2 + ax + by = 0$  (B)  $x^2 + y^2 - ax - by = 0$

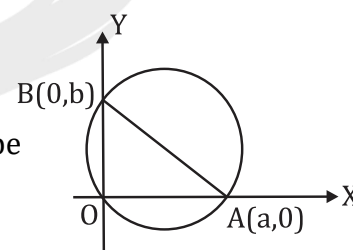
(C)  $x^2 + y^2 + bx + ay = 0$  (D) none of these

**Ans.** (B)

**Sol.** If the circle meets the axes at  $A$  and  $B$ , then  $A \equiv (a, 0), B \equiv (0, b)$

$\angle AOB = 90^\circ$ , so  $AB$  is a diameter of the circle. Hence its equation will be

$$(x - a)(x - 0) + (y - 0)(y - b) = 0 \Rightarrow x^2 + y^2 - ax - by = 0$$



**71.** A tangent to the circle  $x^2 + y^2 = a^2$  meets the axes at point  $A$  and  $B$ . The locus of the midpoint of  $AB$  is

(A)  $\frac{1}{x^2} + \frac{1}{y^2} = \frac{1}{a^2}$  (B)  $\frac{1}{x^2} + \frac{1}{y^2} = \frac{4}{a^2}$  (C)  $\frac{1}{x^2} + \frac{1}{y^2} = 4a^2$  (D)  $\frac{1}{x^2} + \frac{1}{y^2} = \frac{a^2}{4}$

**Ans.** (B)

**Sol.** Any tangent to the given circle is  $x \cos \theta + y \sin \theta = a \Rightarrow \frac{x}{a \sec \theta} + \frac{y}{a \csc \theta} = 1$

$\therefore A \equiv (a \sec \theta, 0)$  and  $B \equiv (0, a \csc \theta)$ . If  $(x, y)$  be the midpoint of  $AB$ , then  $2x = a \sec \theta, 2y$

$$= a \csc \theta \Rightarrow \cos \theta = a/2x, \sin \theta = a/2y \Rightarrow \frac{1}{x^2} + \frac{1}{y^2} = \frac{4}{a^2}, \text{ which is the required locus.}$$

**72.** If a circle with centre  $(-1, 1)$  touches the line  $x + 2y + 12 = 0$ , then its point of contact is

(A)  $(-7/2, -4)$  (B)  $(-18/5, -21/5)$

(C)  $(2, -7)$  (D)  $(-2, -5)$

**Ans.** (B)

(MATHEMATICS)

DIWALI ASSIGNMENT

**Sol.** Radius of the circle  $= \frac{-1+2+12}{\sqrt{1+4}} = \frac{13}{\sqrt{5}}$  Let point of contact  $\equiv (x_1, y_1)$ . Then

$x_1 + 2y_1 + 12 = 0$  Also, line joining  $(x_1, y_1)$  and  $(-1, 1)$  is perpendicular to given line, so

$$\left(\frac{y_1-1}{x_1+1}\right)\left(-\frac{1}{2}\right) = -1 \Rightarrow 2x_1 - y_1 + 3 = 0 \text{ From (1) and (2) } x_1 = -18/5, y_1 = -21/5$$

**73.** If the angle between tangents drawn from a point P on the circle

$x^2 + y^2 + 4x - 6y + 9\sin^2\alpha + 13\cos^2\alpha = 0$  is  $2\alpha$ , then locus of P is

(A)  $x^2 + y^2 + 4x - 6y - 4 = 0$

(C)  $x^2 + y^2 + 4x - 6y - 9 = 0$

(B)  $x^2 + y^2 + 4x - 6y + 4 = 0$

(D)  $x^2 + y^2 + 4x - 6y + 9 = 0$

**Ans. (D)**

**Sol.** Centre of the circle  $C \equiv (-2, 3)$

$$(\text{radius})^2 = r^2 = 4 + 9 - 9\sin^2\alpha - 13\cos^2\alpha = 4 - 4\cos^2\alpha = 4\sin^2\alpha$$

If tangents meet the circle at  $T_1$  and  $T_2$ , then  $\frac{CT_1}{CP} = \sin\alpha \Rightarrow \frac{2\sin\alpha}{CP} = \sin\alpha$

$\Rightarrow CP = 2$  (constant) Thus P is always at a constant distance 2 from the centre  $C(-2, 3)$ . Hence its locus is  $(x + 2)^2 + (y - 3)^2 = 4 \Rightarrow x^2 + y^2 + 4x - 6y + 9 = 0$ .

**74.** If the tangent at a point P of the circle  $x^2 + y^2 + 6x + 6y - 2 = 0$  meets the line

$5x - 2y + 6 = 0$  at point Q which lies on y-axis, then length PQ is equal to

(A) 4

(B)  $2\sqrt{5}$

(C) 5

(D)  $3\sqrt{5}$

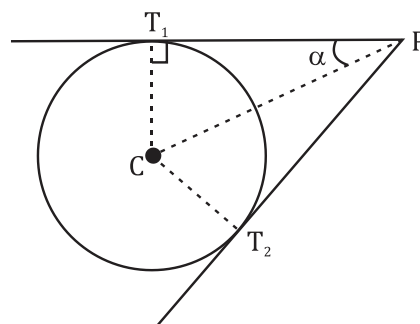
**Ans. (C)**

**Sol.** Let  $Q = (0, k)$ . Since it also lies on  $5x - 2y + 6 = 0$ .

$$\text{So } 0 - 2k + 6 = 0 \Rightarrow k = 3 \therefore Q = (0, 3).$$

Now  $PQ =$  length of the tangent from Q on given

$$\text{circle} = \sqrt{S_1} = \sqrt{0 + 9 + 0 + 18 - 2} = 5.$$



(MATHEMATICS)

DIWALI ASSIGNMENT

75. The centre of the circle inscribed in the square formed by lines  $x^2 - 8x + 12 = 0$  and

$$y^2 - 14y + 45 = 0 \text{ is}$$

- (A) (4,7) (B) (7,4) (C) (9,4) (D) (4,9)

Ans. (A)

Sol.  $\because x^2 - 8x + 12 = 0 \Rightarrow x - 2 = 0, x - 6 = 0$  and  $y^2 - 14y + 45 = 0 \Rightarrow y - 5 = 0, y - 9 = 0$   
So equations of the sides of the given square are  $x - 2 = 0, y - 5 = 0, x - 6 = 0, y - 9 = 0$ .

Vertices of its one diagonal are (2,5) and (6,9). The centre of the circle inscribed in the square

$$= \text{the midpoint of the diagonals} = \left( \frac{2+6}{2}, \frac{5+9}{2} \right) = (4,7).$$

76. A circle passes through (a, b) and cuts the circle  $x^2 + y^2 = k^2$  orthogonally. The locus of its centre is

- (A)  $2ax + 2by - (a^2 - b^2 + k^2) = 0$   
(B)  $2ax + 2by - (a^2 + b^2 + k^2) = 0$   
(C)  $x^2 + y^2 - 3ax - 4by + (a^2 + b^2 - k^2) = 0$   
(D)  $x^2 + y^2 - 2ax - 3by + (a^2 - b^2 - k^2) = 0$

Ans. (B)

Sol. Let equation of the given circle be  $x^2 + y^2 + 2gx + 2fy + c = 0$  since it passes through (a, b), so

$$a^2 + b^2 + 2ga + 2by + c = 0 \quad \dots(1) \text{ Its cuts circle } x^2 + y^2 = k^2 \text{ orthogonally, so we have}$$

$$2g(0) + 2f(0) = c - k^2 \quad \Rightarrow c = k^2 \quad \dots(2)$$

$$(1), (2) \Rightarrow a^2 + b^2 + 2ga + 2bf + k^2 = 0 \Rightarrow a^2 + b^2 - 2(-g)a - 2(-f)b + k^2 = 0$$

Hence locus of the centre  $(-g, -f)$  of the given circle will be

$$a^2 + b^2 - 2ax - 2by + k^2 = 0 \quad 2ax + 2by - (a^2 + b^2 + k^2) = 0.$$

PARABOLA

77. The axis of the parabola  $9y^2 - 16x - 12y - 57 = 0$  is

- (A)  $3y = 2$  (B)  $x + 3y = 3$  (C)  $2x = 3$  (D)  $y = 3$

Ans. (A)

(MATHEMATICS)

DIWALI ASSIGNMENT

**Sol.**  $9y^2 - 12y = 16x + 57 \Rightarrow (3y - 2)^2 = 16x + 61 \Rightarrow \left(y - \frac{2}{3}\right)^2 = \frac{16}{9}\left(x + \frac{61}{16}\right)$ .

which shows that equation of the axis is  $y - \frac{2}{3} = 0$  or  $3y = 2$

**78.** If  $lx + my + n = 0$  is a tangent to the parabola  $x^2 = y$ , then

(A)  $l = 4m^2n^2$

(B)  $P^2 = 4mn$

(C)  $l^2 = 2mn$

(D) none of these

**Ans. (B)**

**Sol.** Given line is  $y = -\frac{l}{m}x - \frac{n}{m}$ . Using condition of tangency  $c = -am^2$ , we have  $-\frac{n}{m} = -\left(\frac{1}{4}\right)\left(-\frac{l}{m}\right)^2$   
 $\Rightarrow l^2 = 4mn$ .

**79.** The length of a focal chord of the parabola  $y^2 = 4ax$  which makes  $\alpha$  angle with x-axis is

(A)  $4a\sin^2\alpha$

(B)  $4a\operatorname{cosec}^2\alpha$

(C)  $4a\cos\alpha\operatorname{cosec}^2\alpha$

(D)  $4a\cos\alpha\operatorname{cosec}\alpha$

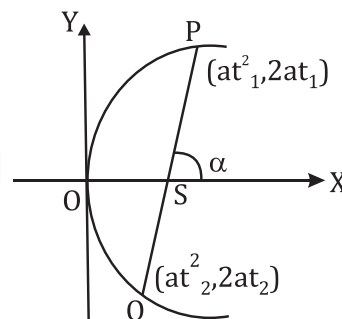
**Ans. (B)**

**Sol.** Let PQ be a focal chord which makes  $\alpha$  angle with x-axis and  $P \equiv (at_1^2, 2at_1)$ ;  $Q \equiv (at_2^2, 2at_2)$

Then  $= a\sqrt{(t_1 + t_2)^2 - 4t_1t_2}\sqrt{(t_1 + t_2)^2 + 4} = a[(t_1 + t_2)^2 + 4]$

Now  $\tan\alpha = \frac{2a(t_2 - t_1)}{a(t_2^2 - t_1^2)} = \frac{2}{t_1 + t_2} \Rightarrow t_1 + t_2 = 2\cot\alpha \therefore PQ = a[4\cot^2\alpha + 4]$

$[\because t_1t_2 = -1] = 4a\operatorname{cosec}^2\alpha$ .



**80.** Two tangents of the parabola  $y^2 = 8x$  meet its tangent at vertex at points P and Q. If  $PQ = 4$ , then the locus of the point of intersection of these two tangents is

(A)  $y^2 = 8(x - 2)$

(B)  $y^2 = 8(x + 2)$

(C)  $x^2 = 8(y - 2)$

(D)  $x^2 = 8(y + 2)$

**Ans. (B)**

**Sol.** Let given tangents be  $t_1y = x + 2t_1^2$  and  $t_2y = x + 2t_2^2$  Now tangent at the vertex is  $x = 0$ , so

$P = (0, 2t_1)$  and  $Q = (0, 2t_2)$  Also, then

$PQ = 4 \Rightarrow t_1 - t_2 = 2 \dots\dots(1)$

(MATHEMATICS)

DIWALI ASSIGNMENT

If point of intersection be  $(x, y)$ , then

$$x = 2t_1t_2 \text{ and } y = 2(t_1 + t_2) \quad \dots(2)$$

Now

$$x = 2t_1t_2 \text{ and } y = 2(t_1 + t_2) \quad (t_1 - t_2)^2 = 4 \Rightarrow (t_1 + t_2)^2 - 4t_1t_2 = 4$$

$$\Rightarrow \frac{y^2}{4} - 2x = 4 \quad [\text{by (1)}] \quad \Rightarrow y^2 = 8(x + 2) \text{ which is the required locus.}$$

81. If  $x - 1 = 0$  is the directrix of the parabola  $y^2 - kx + 8 = 0$ , then one value of  $k$  is

- (A)  $1/8$  (B)  $8$  (C)  $4$  (D)  $1/4$

Ans. (C)

Sol. Parabola is  $y^2 = k\left(x - \frac{8}{k}\right)$  Its directrix is  $x - \frac{8}{k} = -\frac{k}{4}$ , i.e.,  $x - \left(\frac{8}{k} - \frac{k}{4}\right) = 0$

But it is given to be  $x - 1 = 0$ . So, we have  $\frac{8}{k} - \frac{k}{4} = 1 \Rightarrow k^2 + 4k - 32 = 0$

$$\Rightarrow (k - 4)(k + 8) = 0 \Rightarrow k = 4, -8 \text{ Hence from given options } k = 4.$$

82. The locus of the midpoint of the line segment joining a variable point on the parabola  $y^2 = 4ax$  to its focus is a parabola. The directrix of this second parabola is

- (A)  $x = -a$  (B)  $x = -a/2$  (C)  $x = 0$  (D)  $x = a/2$

Ans. (C)

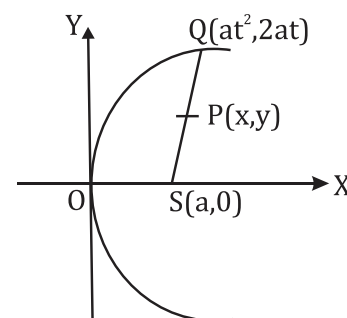
Sol. Let  $Q(at^2, 2at)$  be a variable point on the parabola  $y^2 = 4ax$ , and  $P(x, y)$  be the midpoint of the line segment  $QS$ . Then  $2x = at^2 + a \quad \dots\dots\dots(1)$

$$2y = 2at \quad \dots\dots\dots(2)$$

$$(1), (2) \Rightarrow y^2 = a^2 \left(\frac{2x-a}{a}\right) \Rightarrow y^2 = 2a(x - a/2)$$

which is the locus of  $P$  (parabola). Its directrix is

$$x - \frac{a}{2} = -\frac{a}{2} \Rightarrow x = 0.$$



(MATHEMATICS)

DIWALI ASSIGNMENT

83. A focal chord of  $y^2 = 16x$  is a tangent to  $(x - 6)^2 + y^2 = 2$ . Then the possible values of the slope of this chord are  
 (A)  $\{-1, 1\}$  (B)  $\{-2, 2\}$  (C)  $\{-2, 1/2\}$  (D)  $\{2, -1/2\}$

Ans. (A)

Sol. Any tangent to the circle  $(x - 6)^2 + y^2 = 2$  is  $y = m(x - 6) + \sqrt{2}\sqrt{1 + m^2}$

If it is a focal chord of  $y^2 = 16x$ , then it passes through its focus  $(4, 0)$ . So

$$0 = -2m + \sqrt{2}\sqrt{1 + m^2} \Rightarrow 2m^2 = 1 + m^2 \Rightarrow m = \pm 1$$

ELLIPSE & HYPERBOLA

84. Equation of the ellipse whose eccentricity is  $1/2$  and foci are  $(\pm 1, 0)$  will be

- (A)  $\frac{x^2}{3} + \frac{y^2}{4} = 1$  (B)  $\frac{x^2}{4} + \frac{y^2}{3} = 1$   
 (C)  $\frac{x^2}{4} + \frac{y^2}{3} = \frac{4}{3}$  (D) none of these

Ans. (B)

Sol. Foci are on x-axis and distance between them = 2  $\therefore 2ae = 2 \Rightarrow a(1/2) = 1 \Rightarrow a = 2$

$$\text{Further } b^2 = a^2(1 - e^2) \Rightarrow b^2 = 4(1 - 1/4) \Rightarrow b^2 = 3$$

Obviously, centre of the ellipse (midpoint of  $SS'$ ) is the origin and its axes are along coordinate axes. Hence its equation will be  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \Rightarrow \frac{x^2}{4} + \frac{y^2}{3} = 1$

85. If LR of an ellipse is half of its minor axis, then its eccentricity is

- (A)  $3/2$  (B)  $2/3$  (C)  $\sqrt{3}/2$  (D)  $\sqrt{2}/3$

Ans. (C)

Sol. As given  $2b^2/a = b \Rightarrow 2b = a \Rightarrow 4b^2 = a^2 \Rightarrow 4a^2(1 - e^2) = a^2 \Rightarrow 1 - e^2 = 1/4$

$$\therefore e = \sqrt{3}/2$$

86. Area of the greatest rectangle that can be inscribed in the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  is

- (A)  $a/b$  (B)  $\sqrt{ab}$  (C)  $ab$  (D)  $2ab$

(MATHEMATICS)

DIWALI ASSIGNMENT

Ans. (D)

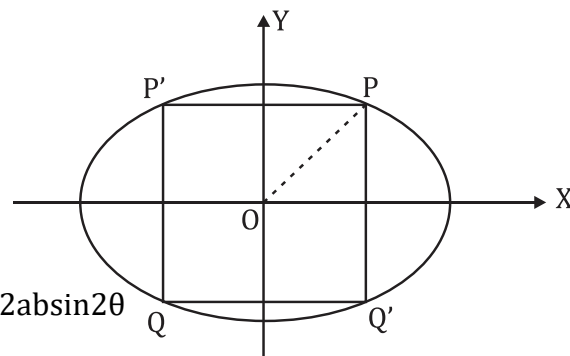
**Sol.** Inscribed rectangle will have greatest area when its sides are parallel to the axes and centre coincides with the centre of the ellipse.

Let  $PP'QQ'$  be such a rectangle and  $P = (a\cos\theta, b\sin\theta)$ .

Then obviously lengths of two sides of this rectangle

will be  $2a\cos\theta$  and  $2b\sin\theta$   $A = 4ab\cos\theta\sin\theta = 2ab\sin 2\theta$

Its maximum value =  $2ab$ .



**87.** The eccentricity of the hyperbola  $4x^2 - 9y^2 - 8x = 32$  is

- (A)  $\sqrt{5}/3$  (B)  $\sqrt{13}/3$  (C)  $\sqrt{13}/2$  (D)  $3/2$

Ans. (B)

**Sol.**  $4x^2 - 9y^2 - 8x = 32 \Rightarrow 4(x-1)^2 - 9y^2 = 36 \Rightarrow \frac{(x-1)^2}{9} - \frac{y^2}{4} = 1$

Here  $a^2 = 9, b^2 = 4 \therefore$  eccentricity  $e = \sqrt{1 + b^2/a^2} = \sqrt{1 + 4/9} = \sqrt{13}/3$

**88.** Slopes of the common tangent to the hyperbola  $\frac{x^2}{9} - \frac{y^2}{16} = 1$  and  $\frac{y^2}{9} - \frac{x^2}{16} = 1$  are

- (A)  $2, -2$  (B)  $1, -1$  (C)  $1, 2$  (D)  $-1, -2$

Ans. (B)

**Sol.** Any tangent to the hyperbola  $\frac{x^2}{9} - \frac{y^2}{16} = 1$  is

$$y = mx + \sqrt{9m^2 - 16}$$

Any tangent to the hyperbola  $\frac{x^2}{(-16)} - \frac{y^2}{(-9)} = 1$  parallel to

$$(1) \text{ is } y = mx + \sqrt{-16m^2 + 9}$$

If (1) and (2) are identical, then  $9m^2 - 16 = -16m^2 + 9 \Rightarrow m^2 = 1 \Rightarrow m = \pm 1$

(MATHEMATICS)

DIWALI ASSIGNMENT

89. If  $\alpha, \beta$  are eccentric angles of end points of a focal chord of the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ , then

$\tan \frac{\alpha}{2} \cdot \tan \frac{\beta}{2}$  is equal to

- (A)  $\frac{e-1}{e+1}$  (B)  $\frac{1-e}{1+e}$  (C)  $\frac{e+1}{e-1}$  (D)  $e$

Ans. (A)

Sol. Equation of the line joining points ' $\alpha$ ' and ' $\beta$ ' is

$$\frac{x}{a} \cos \frac{\alpha + \beta}{2} + \frac{y}{b} \sin \frac{\alpha + \beta}{2} = \cos \frac{\alpha - \beta}{2}$$

If it is a focal chord, then it passes through focus  $(ae, 0)$ , so

$$\cos \frac{\alpha + \beta}{2} = \cos \frac{\alpha - \beta}{2}$$

$$\frac{\cos \frac{\alpha - \beta}{2}}{\cos \frac{\alpha + \beta}{2}} = \frac{e}{1} \Rightarrow \frac{\cos \frac{\alpha - \beta}{2} - \cos \frac{\alpha + \beta}{2}}{\cos \frac{\alpha - \beta}{2} + \cos \frac{\alpha + \beta}{2}} = \frac{e-1}{e+1} \Rightarrow \frac{2 \sin \frac{\alpha}{2} \sin \frac{\beta}{2}}{2 \cos \frac{\alpha}{2} \cos \frac{\beta}{2}} = \frac{e-1}{e+1} \Rightarrow \tan \frac{\alpha}{2} \cdot \tan \frac{\beta}{2} = \frac{e-1}{e+1}$$

90. If the sum of the intercepts made by the tangent to the ellipse  $x^2/27 + y^2 = 1$  at its point  $(3\sqrt{3}\cos\theta, \sin\theta)$ ,  $\theta \in (0, \pi/2)$  is minimum, then  $\theta$  is equal to

- (A)  $\pi/3$  (B)  $\pi/6$  (C)  $\pi/8$  (D)  $\pi/4$

Ans. (B)

Sol. Equation of the tangent at given point is  $\frac{x \cos \theta}{3\sqrt{3}} + y \sin \theta = 1$  If  $\lambda$  be the sum of intercepts made by

it on axes, then  $\lambda = 3\sqrt{3} \sec \theta + \csc \theta = f(\theta) \Rightarrow \frac{d\lambda}{d\theta} = 3\sqrt{3} \sec \theta \tan \theta - \csc \theta \cot \theta =$

$\frac{3\sqrt{3} \sin^3 \theta - \cos^3 \theta}{\sin^2 \theta \cos^2 \theta}$  Now  $\frac{d\lambda}{d\theta} = 0 \Rightarrow \tan \theta = \frac{1}{\sqrt{3}} \Rightarrow \theta = \frac{\pi}{6}$  where  $\frac{d^2\lambda}{d\theta^2} > 0$ . Hence at  $\theta = \pi/6$ ,  $\lambda$  is minimum.