DPP-02 (Solution)

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1. $\int \tan x dx.$

Ans. $c - \ln|\cos x|.$

Sol. $\int \frac{\sin x}{\cos x} \cdot dx$

Let
$\cos x = t$
$-\sin x \cdot dx = dt$
$\sin x dx = -dt$

$= \int \frac{-1dt}{t}$

$= -\int \frac{1}{t} \cdot dt \because \int \frac{1}{x} \cdot dx = \ln|x| + c$

$= -\ln|t| + c$

$= -\ln|\cos x| + c = \ln|\sec x| + c$

2. $\int \cot x\, dx.$

Ans. $\ln|\sin x| + c$

Sol. $\int \frac{\cos x}{\sin x} \cdot dx$

let $\sin x = t \Rightarrow \cos x \cdot dx = dt$

$\int \frac{dt}{t} = \ln|t| + c$

$= \ln|\sin x| + c \quad = -\ln|\operatorname{cosec} x| + c$

3. $\int \tan 3x dx$

Ans. $c - \frac{1}{3}\ln|\cos 3x|$

Sol. $= \frac{-\ln(\cos 3x| + c}{\frac{d}{dx}(3x)} \quad = -\frac{1}{3}\ln|\cos 3x| + c$

4. $\int \cot(2x+1)dx.$

Ans. $\frac{1}{2}\ln|\sin(2x+1)| + c$

Sol. $= \frac{\ln|\sin(2x+1)| + c}{\frac{d}{dx}(2x+1)} = \frac{1}{2}\ln|\sin(2x+1)| + c$

5. $\int \frac{d\left(\frac{x}{3}\right)}{\sqrt{1-\left(\frac{x}{3}\right)^2}}.$

Ans. $\arcsin\frac{x}{3} + c$

Sol. let $\frac{x}{3} = t$

$\int \frac{dt}{\sqrt{1-t^2}} \quad \left(\because \frac{1}{\sqrt{1-x^2}} \cdot dx = \sin^{-1}(x) + c\right)$

$= \sin^{-1}(t) + c \quad = \sin^{-1}\left(\frac{x}{3}\right) + c$

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6. $\int \frac{dx}{\sqrt{1-25x^2}}$.

Ans. $\frac{1}{5}\arcsin 5x + c$

Sol. $\left(\therefore \frac{1}{\sqrt{a^2-x^2}} \cdot dx = \sin^{-1}\left(\frac{x}{a}\right) + c\right)$

$\int \frac{dx}{5\sqrt{\frac{1}{25}-x^2}} = = \frac{1}{5}\int \frac{dx}{\sqrt{\left(\frac{1}{5}\right)^2-x^2}} = \frac{1}{5}\sin^{-1}\left(\frac{x}{1/5}\right) + c = \frac{1}{5}\sin^{-1}(5x) + c$

Alter →

$\int \frac{dx}{\sqrt{1-(5x)^2}}$

$\frac{\sin^{-1}(5x)}{\frac{d}{dx}(5x)} + c$

$\frac{1}{5}\sin^{-1}(5x) + c$

7. $\int \frac{dx}{1+9x^2}$.

Ans. $\frac{1}{3}\arctan 3x + c$

Sol. $\left(\because \frac{1}{1+x^2} \cdot dx = \tan^{-1}(x) + c\right)$

$\int \frac{dx}{1+(3x)^2} = \frac{\tan^{-1}(3x)}{\frac{d}{dx}(3x)} + c = \frac{1}{3}\tan^{-1}(3x) + c$

8. $\int \frac{dx}{x^2-7x+10}$

Ans. $\frac{1}{3}\ln\left|\frac{x-5}{x-2}\right| + c$

Sol. $\left(\because \frac{1}{x^2-a^2} dx = \frac{1}{2a}\ln\left|\frac{x-a}{x+a} + c\right|\right)$

$\int \frac{dx}{\left(x-\frac{7}{2}\right)^2-\frac{49}{4}+10} = \int \frac{dx}{\left(x-\frac{7}{2}\right)^2-\frac{9}{4}}$

$\int \frac{dx}{\left(x-\frac{7}{2}\right)^2-\left(\frac{3}{2}\right)^2} = \frac{1}{2\left(\frac{3}{2}\right)}\ln\left|\frac{x-\frac{7}{2}-\frac{3}{2}}{x-\frac{7}{2}+\frac{3}{2}}\right| + c = \frac{1}{3}\ln\left|\frac{x-5}{x-2}\right| + c$

9. $\int \frac{dx}{x^2+3x-10}$

Ans. $\frac{1}{7}\ln\left|\frac{x-2}{x+5}\right| + c$

Sol. $\int \frac{dx}{\left(x+\frac{3}{2}\right)^2-\frac{9}{4}-10}$

$= \int \frac{dx}{\left(x+\frac{3}{2}\right)^2-\left(\frac{7}{2}\right)^2} = \frac{1}{2\left(\frac{7}{2}\right)}\ln\left|\frac{x+\frac{3}{2}-\frac{7}{2}}{x+\frac{3}{2}+\frac{7}{2}}\right| + c = \frac{1}{7}\ln\left|\frac{x-2}{x+5}\right| + c$

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10. $\int \frac{dx}{4x^2-9}$

Ans. $\frac{1}{12}\ln\left|\frac{2x-3}{2x+3}\right| + c$

Sol. $\int \frac{dx}{(2x)^2-(3)^2} = \frac{\frac{1}{2(3)}\ln\left|\frac{2x-3}{2x+3}\right|}{\frac{d}{dx}(2x)} + c$

$= \frac{1}{12}\ln\left|\frac{2x-3}{2x+3}\right| + c$

11. $\int \frac{dx}{2-3x^2}$

Ans. $\frac{1}{2\sqrt{6}}\ln\left|\frac{\sqrt{2}+x\sqrt{3}}{\sqrt{2}-x\sqrt{3}}\right| + c$

Sol. $\because \int \frac{1}{a^2-x^2}\cdot dx = \frac{1}{2a}\ln\left|\frac{a+x}{a-x}\right| + c$

$\int \frac{dx}{(\sqrt{2})^2-(\sqrt{3}x)^2} = \frac{\frac{1}{2\sqrt{2}}\ln\left|\frac{\sqrt{2}+\sqrt{3}x}{\sqrt{2}-\sqrt{3}x}\right|}{\frac{d}{dx}(\sqrt{3}x)} + c$

$= \frac{1}{2\sqrt{6}}\ln\left|\frac{\sqrt{2}+\sqrt{3}x}{\sqrt{2}-\sqrt{3}x}\right| + c$

12. $\int \frac{dx}{(x-1)^2+4}$

Ans. $\frac{1}{2}\arctan\frac{x-1}{2} + c$

Sol. $= \int \frac{dx}{(x-1)^2+2^2}$

$\because \int \frac{1}{x^2+a^2}dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + c$

$= \frac{\frac{1}{2}\tan^{-1}\left(\frac{x-1}{2}\right)}{\frac{d}{dx}(x-1)} + c = \frac{1}{2}\tan^{-1}\left(\frac{x-1}{2}\right) + c$

13. $\int \frac{dx}{\sqrt{1-(2x+3)^2}}$.

Ans. $\frac{1}{2}\arcsin(2x+3) + c$

Ans. $= \frac{\sin^{-1}(2x+3)}{\frac{d}{dx}(2x+3)} + c = \frac{1}{2}\sin^{-1}(2x+3) + c$

14. $\int \frac{dx}{\sqrt{4x-3-x^2}}$

Ans. $\arcsin(x-2) + c$

Sol. $= \int \frac{dx}{\sqrt{-(x^2-4x+3)}}$

$= \int \frac{dx}{\sqrt{-\{(x-2)^2-4+3\}}}$

$= \int \frac{dx}{\sqrt{1-(x-2)^2}}$

$= \frac{\sin^{-1}(x-2)}{\frac{d}{dx}(x-2)} + c = \sin^{-1}(x-2) + c$

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15. $\int 2^x \cdot e^x dx$

Ans. $\frac{2^x \cdot e^x}{1+\ln 2} + c$

Sol. $\because \int a^x \cdot dx = \frac{a^x}{\log_e a} + c$

$= \int (2e)^x \cdot dx$

$= \frac{(2e)^x}{\log(2e)} + c = \frac{2^x \cdot e^x}{\log_e 2 + \log_e e} + c = \frac{2^x \cdot e^x}{1+\log_e 2} + c$

16. $\int \frac{1+\cos^2 x}{1+\cos 2x} dx$

Ans. $\frac{1}{2}(\tan x + x) + c$

Sol. $\int \frac{1+\cos^2 x}{2\cos^2 x} \cdot dx (\because 1 + \cos 2x = 2\cos^2 x)$

$\frac{1}{2}\int \frac{1+\cos^2 x}{\cos^2 x} \cdot dx$

$\frac{1}{2}\int (\sec^2 x + 1) \cdot dx$

$\frac{1}{2}(\tan x + x) + c$

17. $\int \frac{1-\tan^2 x}{1+\tan^2 x} dx$

Ans. $\frac{1}{2}\sin 2x + c$

Sol. $= \int (\cos 2x) \cdot dx$

$= \frac{\sin 2x}{2} + c$

18. $\int \frac{1+\tan^2 x}{1+\cot^2 x} dx$

Ans. $\tan x - x + c$

Sol. $\int \frac{1+\tan^2 x}{1+\frac{1}{\tan^2 x}} \cdot dx = \int \frac{1+\tan^2 x}{\frac{\tan^2 x + 1}{\tan^2 x}} = \int \frac{\tan^2 x(1+\tan^2 x)}{(1+\tan^2 x)} \cdot dx = \int (\sec^2 x - 1) \cdot dx = (\tan x - x) + c$

19. $\int \frac{e^{5\ln x} - e^{4\ln x}}{e^{3\ln x} - e^{2\ln x}} dx$

Ans. $\frac{x^3}{3} + c$

Sol. $\int \frac{e^{\ln x^5} - e^{\ln x^4}}{e^{\ln x^3} - e^{\ln x^2}} \cdot d(\because e^{\ln a} = a)$

$\int \frac{x^5 - x^4}{x^3 - x^2} \cdot dx = \int \frac{x^4(x-1)}{x^2(x-1)} \cdot dx = \int x^2 \cdot dx = \frac{x^3}{3} + c$

20. $\int \left(e^{a\ln x} + e^{x\ln a}\right) dx \ (a > 0)$

Ans. $\frac{x^{a+1}}{a+1} + \frac{a^x}{\ell n a} + c$

Sol. $= \int e^{\ln x^a} + e^{\ln a^x} \cdot dx = \int (x^a + a^x) \cdot dx = \frac{x^{a+1}}{a+1} + \frac{a^x}{\log_e a} + c$

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21. $\int \frac{\cos 2x}{\cos^2 x \sin^2 x} dx$

Ans. $-(\cot x + \tan x) + c$

Sol. $\int \frac{\cos^2 x}{\cos^2 x \cdot \sin^2 x} - \frac{\sin^2 x}{\cos^2 x \cdot \sin^2 x} \cdot dx = \int (\operatorname{cosec}^2 x - \sec^2 x) \cdot dr = -\cot x - \tan x + c$

$= -(\tan x + \cot x) + c$

22. $\int \frac{1+2x^2}{x^2(1+x^2)} dx$

Ans. $-\frac{1}{x} + \tan^{-1} x + c$

Sol. $\int \frac{(1+x^2)+x^2}{x^2(1+x^2)} \cdot dx = \int \frac{(1+x^2)}{x^2(1+x^2)} \cdot dx + \int \frac{x^2}{x^2(1+x^2)} \cdot dx$

$= \int \frac{1}{x^2} \cdot dx + \int \frac{1}{1+x^2} \cdot dx = \left(-\frac{1}{x}\right) + \tan^{-1} x + c$

23. $\int 4\cos \frac{x}{2} \cdot \cos x \cdot \sin \frac{21}{2} x dx$

Ans. $-\left[\frac{1}{9}\cos 9x + \frac{1}{10}\cos 10x + \frac{1}{11}\cos 11x + \frac{1}{12}\cos 12x\right] + C$

Sol. $= \int 2\left(2\cos \frac{x}{2} \sin \frac{21x}{2}\right) \cos x \cdot dx$

$= \int 2\left\{\sin\left(\frac{21x}{2} + \frac{x}{2}\right) + \sin\left(\frac{21x}{2} - \frac{x}{2}\right)\right\} \cos x \cdot dx$

$= \int 2(\sin 11x + \sin 10x)\cos x dx$

$= \int 2\sin 11x \cos x + 2\sin 10x \cdot \cos x \cdot dx$

$= \int (\sin 12x + \sin 10x + \sin 11x + \sin 9x) \cdot dx$

$= -\frac{\cos 12x}{12} - \frac{\cos 11x}{11} - \frac{\cos 10x}{10} - \frac{\cos 9x}{9} + c$

24. $\int \frac{\cos x - \sin x}{\cos x + \sin x}(2 + 2\sin 2x) dx$

Ans. $\sin 2x + c$

Sol. $2\int \frac{\cos x - \sin x}{\cos x + \sin x} \cdot (1 + \sin 2x) \cdot dx$

$= \int \frac{\cos x - \sin x}{(\cos x + \sin x)} \cdot (\cos x + \sin x)^2 = 2\int (\cos^2 x - \sin^2 x) \cdot dx$

$= 2\int \cos 2x \cdot dx = 2\int \cos 2x \cdot dx = 2\left(\frac{\sin 2x}{2}\right) + C = \sin 2x + C$

25. $\int (3\sin x \cos^2 x - \sin^3 x) dx$

Ans. $-\frac{\cos 3x}{3} + c$

Ans. $\int [3\sin x(1 - \sin^2 x) - \sin^3 x] \cdot dx = \int (3\sin x - 3\sin^3 x - \sin^3 x) \cdot dx$

$\int (3\sin x - 4\sin^3 x) dx \ (\because 3\sin\theta - 4\sin^3\theta = \sin 3\theta)$

$\int \sin 3x = -\frac{\cos 3x}{3} + c$

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26. $\int \cos x^\circ dx$

Ans. $\frac{180}{\pi}\sin x^\circ + c$

Sol. $\int \cos\left(\frac{\pi x}{180}\right) dx = \frac{\sin\left(\frac{\pi x}{180}\right)}{\frac{d}{dx}\left(\frac{\pi}{180}\right)} + c = \frac{\sin(x^\circ)}{(\pi/180)} + c = \frac{180}{\pi}\sin(x^\circ) + c$

27. $\int \frac{(1+x)^2}{x(1+x^2)} dx$

Ans. $\ln x + 2\tan^{-1}x + c$

Sol. $\int \frac{(1+x^2)+2x}{x(1+x^2)} \cdot dx = \int \frac{(1+x^2)}{x(1+x^2)} + \frac{2x}{x(1+x^2)} \cdot dx = \int \frac{1}{x} \cdot dx + 2\int \frac{1}{1+x^2} dx$

$\ln|x| + 2 \cdot \tan^{-1}(x) + c$

28. $\int \frac{x}{2x+1} dx$

Ans. $\frac{1}{2}\left[x - \frac{\ln(2x+1)}{2}\right] + c]$

Sol. $= \frac{1}{2}\int \frac{2x}{2x+1} \cdot dx = \frac{1}{2}\int \frac{(2x+1)-1}{(2x+1)} \cdot dx = \frac{1}{2}\int \left[1 - \frac{1}{(2x+1)}\right] \cdot dx$

$= \frac{1}{2}\left\{x - \frac{\ln(2x+1)}{\frac{d}{dx}(2x+1)}\right\} + c = \frac{x}{2} - \frac{1}{4}\ln(2x+1) + c$

29. $\int \frac{\sec 2x - 1}{\sec 2x + 1} dx$

Ans. $\tan x - x + c$

Sol. $\int \frac{\frac{1}{\cos 2x} - 1}{\frac{1}{\cos 2x} + 1} \cdot dx = \int \frac{1-\cos 2x}{1+\cos 2x} \cdot dx = \int \tan^2 x \cdot dx = \int (\sec^2 x - 1) \cdot dx = (\tan x - x) + c$

30. $\int \frac{2x-1}{x-2} dx$

Ans. $2x + 3\ln(x-2) + c$

Sol. $\int \frac{(2x-1)-3+3}{x-2} \cdot dx = \int \frac{(2x-4)+3}{(x-2)} \cdot dx = \int \left[\frac{2(x-2)}{x-2} + \frac{3}{(x-2)}\right] \cdot dx$

$2\int 1 \cdot dx + 3\int \frac{1}{x-2} \cdot dx = 2(x) + 3\ln(x-2) + c$

31. $\int \frac{e^{2x}-1}{e^x} dx$

Ans. $e^x + e^{-x} + c$

Sol. $\int \left\{\frac{e^{2x}}{e^x} - \frac{1}{e^x}\right\} \cdot dx = \int (e^x - e^{-x}) \cdot dx = e^x - (-e^{-x}) + c = e^x + e^{-x} + c$

32. $\int \frac{\sin x + \cos x}{\sqrt{1+\sin 2x}} dx (\cos x + \sin x > 0)$

Ans. $x + C$

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Sol. $\int \frac{\sin x+\cos x}{\sqrt{(\sin x+\cos x)^2}} \cdot dx$

$\int \frac{(\sin x+\cos x)}{|\sin x+\cos x|} \cdot dx \left(\because \sqrt{x^2}=|x|\right)$

$\int \frac{(\sin x+\cos x)}{(\sin x+\cos x)} \cdot dx$

$\int 1 \cdot dx = x + c$

33. $\int \frac{\cos 2x-\cos 2\alpha}{\cos x-\cos\alpha} dx$

Ans. $2(\sin x + x\cos\alpha) + c$

Sol. $\cos 2\theta = 2\cos^2\theta - 1$

$\int \frac{(2\cos^2 x-x)-(2\cos^2\alpha-x)}{\cos x-\cos\alpha} \cdot dx = 2\int \frac{\cos^2 x-\cos^2\alpha}{\cos x-\cos\alpha} \cdot dx$

$= 2\int \frac{(\cos x+\cos\alpha)(\cos x-\cos\alpha)}{\cos x-\cos\alpha} dx = 2\int \cos x \cdot dx + 2\cos\alpha\int 1 \cdot dx$

$= 2\sin x + 2x\cos\alpha + c = 2(\sin x + x\cos\alpha) + c$

34. $\int \frac{x^6-1}{x^2+1} dx$

Ans. $\frac{x^5}{5} - \frac{x^3}{3} + x - 2\tan^{-1}x + c$

Sol. $\text{Degree}(N^r) > \text{Degree}(D^r)$

" Using Long division method "

$\therefore \int \frac{x^6-1}{x^2+1} dx \int\int x^4 - x^2 + 1 + \frac{(-2)}{x^2+1}\Big] \cdot dx$

$= \frac{x^5}{5} - \frac{x^3}{3} + x - 2\tan^{-1}(x) + c$

35. $\int \frac{\sin^3 x+\cos^3 x}{\sin^2 x\cos^2 x} dx$

Ans. $\sec x - \operatorname{cosec} x + c$

Sol. $\int \frac{\sin^3 x}{\sin^2 x\cos^2 x} \cdot dx + \int \frac{\cos^2 x}{\sin^2 x\cos^2 x} \cdot dx$

$\int \sec x \cdot \tan x \cdot dx + \int \operatorname{cosec} x \cdot \cot x \cdot dx = \sec x - \operatorname{cosec} x + c$

36. $\int \frac{x^4+x^2+1}{2(1+x^2)} dx$

Ans. $\frac{1}{2}\left(\frac{x^3}{3} + \tan^{-1}x\right) + c$

Sol. $\int \left\{\frac{x^2(x^2+1)}{2(1+x^2)} + \frac{1}{2(1+x^2)}\right\} \cdot dx = \frac{1}{2}\int \left(x^2 + \frac{1}{1+x^2}\right) \cdot dx = \frac{1}{2}\left(\frac{x^3}{3} + \tan^{-1}x\right) + c$

37. $\int \sqrt{1-\sin 2x} dx$

Ans. $(\sin x + \cos x)\text{sgn}(\cos x - \sin x) + c$

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Sol. $\therefore 1 - \sin 2x = \begin{matrix} (\sin x - \cos x)^2 \\ (\cos x - \sin x)^2 \end{matrix}$

$\int \sqrt{(\sin x - \cos x)^2} \cdot dx$

$\int |\sin x - \cos x| \cdot dx$

case(i) If $\sin x - \cos x > 0$

$\therefore \int (\sin x - \cos x) \cdot dx$

$= \cos x + \sin x$

case(ii) If $\sin x - \cos x < 0$

$\therefore -\int (\sin x - \cos x) \cdot dx$

$= -(\cos x + \sin x)$

$(\cos x + \sin x) \cdot \text{sgn}(\sin x - \cos x) + c$

38. $\int \frac{\sin^6 x + \cos^6 x}{\sin^2 x \cdot \cos^2 x} dx$

Ans. $\tan x - \cot x - 3x + c$

Sol. $\because a^3 + b^3 = (a+b)^3 - 3ab(a+b)$

$\int \frac{(\sin^2 x)^3 + (\cos^2 x)^3}{\sin^2 x \cdot \cos^2 x} \cdot dx$

$\int \frac{(\sin^2 x + \cos^2 x)^3 - 3\sin^2 x \cdot \cos^2 x(\sin^2 x + \cos^2 x)}{\sin^2 x \cdot \cos^2 x} \cdot dx$

$\int \frac{1 - 3\sin^2 x \cos^2 x}{\sin^2 x \cdot \cos^2 x} \cdot dx$

$\int \frac{\sin^2 x + \cos^2 x}{\sin^2 x \cdot \cos^2 x} - \frac{3\sin^2 x \cos^2 x}{\sin^2 x \cdot \cos^2 x}$

$\int \left(\frac{1}{\cos^2 x} + \frac{1}{\sin^2 x} - 3\right) \cdot dx$

$(\sec^2 x + \text{cosec}^2 x - 3) \cdot dx$

$(\tan x - \cot x - 3x + c)$

39. $\int \frac{(\sqrt{x}+1)(x^2 - \sqrt{x})}{x\sqrt{x} + x + \sqrt{x}} dx$

Ans. $\frac{x^2}{2} - x + c$

Sol. $\int \frac{(\sqrt{x}+1) \cdot \sqrt{x} \cdot \left(x^{\frac{3}{2}} - 1\right)}{\sqrt{x} \cdot (x + \sqrt{x} + 1)} \cdot dx$

$\int \frac{(\sqrt{x}+1)[(\sqrt{x})^3 - (1)^3] \cdot dx}{(x + \sqrt{x} + 1)} =$

$\int \frac{(\sqrt{x}+1)(\sqrt{x}-1)\{(\sqrt{x})^2 + (\sqrt{x})(1) + (1)^2\}}{(x + \sqrt{x} + 1)} dx$

$\because (a^3 - b^3) = (a-b)(a^2 + ab + b^2)$

$= \int (\sqrt{x})^2 - (1)^2 \cdot dx = \int (x-1) dx = \frac{x^2}{2} - x + c$