

① Series जैसा Kuchh hai

② total 6 terms hai

③ Last 3 terms change karenge

④  $\frac{10\pi}{11} \rightarrow Dr = 11 \div 2 \text{ half} = 5.5$   
 $Nr = 10 > 5.5$ ⑤ as  $Nr$  is gr than half  
 of  $Dr \Rightarrow (\pi - 0)$  will be used

⑥  $\tan \frac{10\pi}{11} = \tan \left( \frac{11\pi - \pi}{11} \right)$

⑦  $\tan \left( \pi - \frac{\pi}{11} \right) = -\tan \frac{\pi}{11}$

Q  $\tan \frac{\pi}{11} + \tan \frac{2\pi}{11} + \tan \frac{4\pi}{11} + \tan \frac{7\pi}{11} + \tan \frac{9\pi}{11} + \tan \left( \frac{10\pi}{11} \right) = ?$

$$+ \tan \left( \frac{11\pi - 4\pi}{11} \right) + \tan \left( \frac{11\pi - 2\pi}{11} \right) + \tan \left( \frac{11\pi - \pi}{11} \right)$$

$$+ \tan \left( \pi - \frac{4\pi}{11} \right) + \tan \left( \pi - \frac{2\pi}{11} \right) + \tan \left( \pi - \frac{\pi}{11} \right)$$

$$\cancel{\tan \frac{\pi}{11}} + \cancel{\tan \frac{9\pi}{11}} + \cancel{\tan \frac{4\pi}{11}} - \cancel{\tan \frac{4\pi}{11}} - \cancel{\tan \frac{2\pi}{11}} - \cancel{\tan \frac{\pi}{11}}$$

$$= 0$$

$$\textcircled{1} \cos^2\left(\frac{\pi}{16}\right) + \cos^2\left(\frac{3\pi}{16}\right) + \cos^2\left(\frac{5\pi}{16}\right) + \cos^2\left(\frac{7\pi}{16}\right)$$

$$\cos^2\left(\frac{\pi}{16}\right) + \cos^2\left(\frac{3\pi}{16}\right) + \cos^2\left(\frac{8\pi-3\pi}{16}\right) + \cos^2\left(\frac{8\pi-\pi}{16}\right)$$

$$\cos^2\left(\frac{\pi}{16}\right) + \cos^2\left(\frac{3\pi}{16}\right) + \cos^2\left(\frac{\pi}{2} - \frac{3\pi}{16}\right) + \cos^2\left(\frac{\pi}{2} - \frac{\pi}{16}\right)$$

$$\cos^2\frac{\pi}{16} + \cos^2\left(\frac{3\pi}{16}\right) \quad \cos^2\left(\frac{3\pi}{16}\right) + \cos^2\frac{\pi}{16}$$

$$= 1 + 1 = 2$$

① Series of term

② total terms = 4

③ Last 2 ko Balenge

④  $Nr = 7$

$Dr = 16 \rightarrow 16 \text{ Kahalf} = 8$

$Nr = 7 < 8 = Dr \cdot \text{Kahalf}$

⑤ as  $Nr < \text{half of } dr$ .

Now change in  $\left(\frac{\pi}{2} - \theta\right)$

$$\textcircled{6} \left(\frac{\pi}{2} - \theta\right) = \left(\frac{8\pi}{16} - \theta\right)$$

$$\textcircled{7} \cos^2\left(\frac{7\pi}{16}\right) = \cos^2\left(\frac{8\pi-\pi}{16}\right)$$

$$= \cos^2\left(\frac{\pi}{2} - \frac{\pi}{16}\right) = \cos^2\frac{\pi}{16}$$



Q  $\sin^2\left(\frac{\pi}{18}\right) + \sin^2\left(\frac{\pi}{9}\right) + \sin^2\left(\frac{7\pi}{18}\right) + \sin^2\left(\frac{4\pi}{9}\right)$

Sahi  
Likha

$$\sin^2\left(\frac{\pi}{18}\right) + \sin^2\left(\frac{2\pi}{18}\right) + \sin^2\left(\frac{7\pi}{18}\right) + \sin^2\left(\frac{8\pi}{18}\right)$$

$$+ \sin^2\left(\frac{9\pi-2\pi}{18}\right) + \sin^2\left(\frac{9\pi-\pi}{18}\right)$$

$$\sin^2\left(\frac{\pi}{18}\right) + \sin^2\frac{2\pi}{18} + \sin^2\left(\frac{\pi}{2} - \frac{2\pi}{18}\right) + \sin^2\left(\frac{\pi}{2} - \frac{\pi}{18}\right)$$

$$\sin^2\left(\frac{\pi}{18}\right) + \sin^2\left(\frac{2\pi}{18}\right) + \sin^2\left(\frac{2\pi}{18}\right) + \sin^2\left(\frac{\pi}{18}\right)$$

$$\underbrace{\hspace{10em}}_{= 1+1=2}$$

① Last 2 term change

②  $\sin^2\left(\frac{8\pi}{18}\right)$

3)  $Nr = 8$

4)  $Dr = \lfloor 18 \rfloor \div 2 = 9$

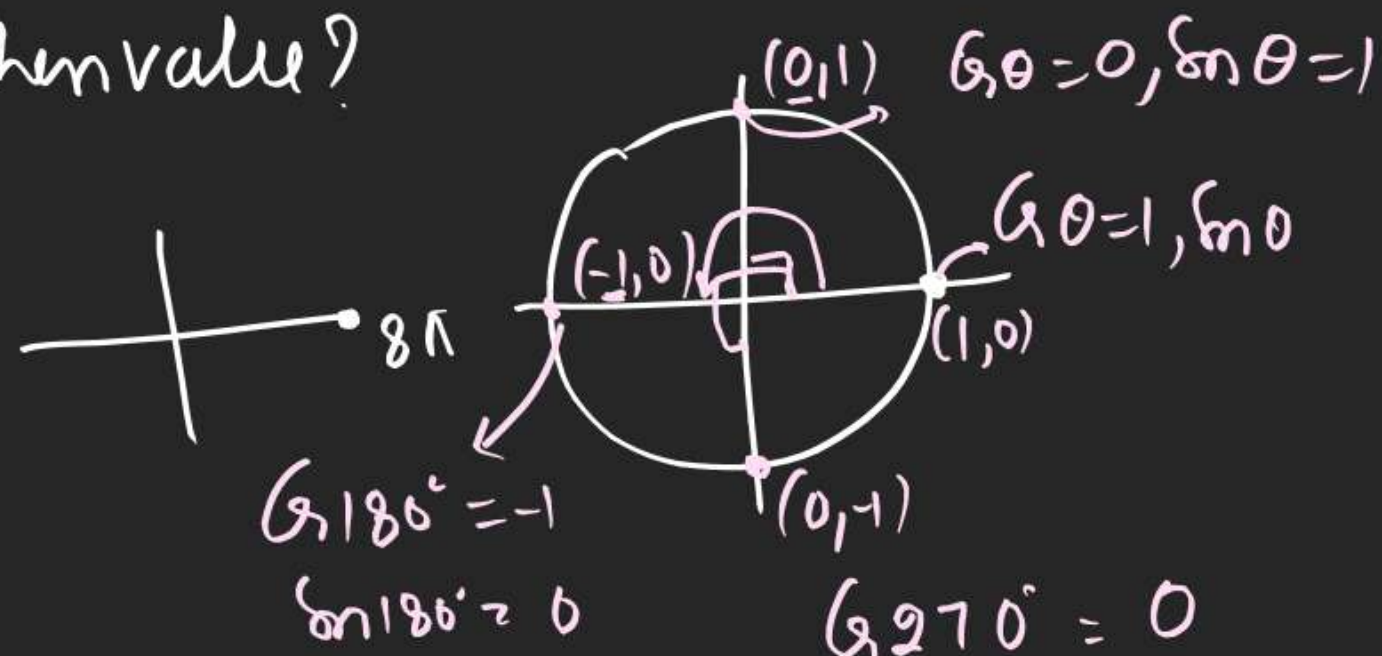
5)  $Nr = 8 < 9$   
 $Nr < Dr \div 2$

6)  $\left(\frac{\pi}{2} - \theta\right)$  me change

7)  $\left(\frac{\pi}{2} - \theta\right) = \left(\frac{9\pi}{18} - \theta\right)$

Q  $\sin\left(\frac{24\pi}{3}\right)$  then value?

$$\sin(8\pi) = 0$$



Q Value of  $\sin \frac{7\pi}{4}$



$$\sin\left(\frac{4\pi + 3\pi}{4}\right) = \sin\left(\pi + \frac{3\pi}{4}\right)$$

$$= -\sin \frac{3\pi}{4}$$

$$= -\frac{1}{\sqrt{2}}$$

$$G(\underline{9\bar{1}} + \frac{\bar{1}}{3})$$

$$-G \frac{\pi}{3} = -G 60^\circ = -\frac{1}{2}$$

Q  $\tan\left(\frac{11\pi}{6}\right) = ?$

$$\tan\left(\frac{12\pi - \pi}{6}\right) = \tan\left(2\pi - \frac{\pi}{6}\right)$$

$$= -\tan \frac{\pi}{6} = -\tan 30^\circ$$

$$= -\frac{1}{\sqrt{3}}$$

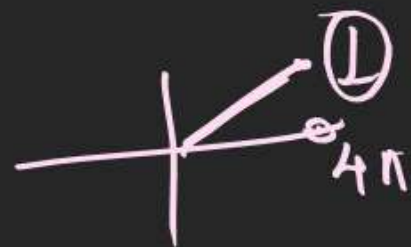
$$\phi \tan\left(\frac{17.1^\circ}{4}\right)$$

$$\tan\left(\frac{16\pi + \pi}{4}\right)$$

$$\tan\left(4\pi + \frac{\pi}{4}\right)$$

$$+ \ln \frac{\pi}{4}$$

1





$$Q \sec\left(\frac{4\pi}{3}\right)$$

$$\sec\left(\frac{3\pi+\pi}{3}\right)$$

$$\sec\left(\pi+\frac{\pi}{3}\right)$$

$$-\sec\frac{\pi}{3}$$

$$= -\sec 60^\circ$$

$$= -2$$

$$Q \sec\left(\frac{7\pi}{6}\right)$$

$$\sec\left(\frac{6\pi+\pi}{6}\right)$$

$$\sec\left(\pi+\frac{\pi}{6}\right)$$

$$-\sec\frac{\pi}{6}$$

$$= -\sec 30^\circ$$

$$= -2$$

$$Q \tan\left(\frac{35\pi}{4}\right)$$

$$\tan\left(\frac{36\pi-\pi}{4}\right)$$

$$\tan\left(9\pi-\frac{\pi}{4}\right)$$

$$= \tan\frac{\pi}{4}$$

$$= 1$$

$$Q \sin\left(\frac{43\pi}{11}\right)$$

$$= \sin\left(\frac{44\pi - \pi}{11}\right)$$

$$\text{---} \sin\left(4\pi - \frac{\pi}{11}\right)$$

$$= \sin\left(\frac{\pi}{11}\right)$$

## Standard formulae

$$① \sin(A+B) = \sin A \cos B + \cos A \sin B$$

$$2) \sin(A-B) = \sin A \cos B - \cos A \sin B$$

$$3) \cos(A+B) = \cos A \cos B - \sin A \sin B$$

$$4) \cos(A-B) = \cos A \cos B + \sin A \sin B$$

$$Q \text{ Value of } \sin(45^\circ + \theta) \cdot \cos(15^\circ + \theta) - \cos(45^\circ + \theta) \cdot \sin(15^\circ + \theta) = ?$$

$$\sin A \cos B - \cos A \sin B = \sin(A-B)$$

$$\sin\{(45^\circ + \theta) - (15^\circ + \theta)\} = \sin(45^\circ - 15^\circ) = \sin 30^\circ = \frac{1}{2}$$



Q Find value of  $\cos(45^\circ + \theta) \cdot \cos(45^\circ - \theta) - \sin(45^\circ + \theta) \cdot \sin(45^\circ - \theta) = ?$

$$\cos A \cdot \cos B - \sin A \cdot \sin B = \cos(A + B)$$

$$= \cos((45^\circ + \theta) + (45^\circ - \theta)) = \cos 90^\circ = 0$$

$$\sqrt{\frac{41^2 - 9^2}{41^2}} = \sqrt{\frac{40^2}{41^2}}$$

$$\sin B = \frac{40}{41}$$

$$\sin 0^\circ = 0$$

$$\cos 0 = 1$$

$$\sin 90^\circ = 1$$

$$\cos 90 = 0$$

$$\sin 180^\circ = 0$$

$$\cos 180^\circ = -1$$

$$\sin 270^\circ = -1$$

$$\cos 270^\circ = 0$$

Q Find  $\cos(A+B) \cdot \cos(A-B) + \sin(A+B) \cdot \sin(A-B) = ?$

$$\cos M \cdot \cos N + \sin M \cdot \sin N = \cos(M - N)$$

S.L. Loney

$$= \cos((A+B) - (A-B))$$

$$= \cos(2B)$$

Q If  $\sin \alpha = \frac{3}{5}$  &  $\cos \beta = \frac{9}{41}$   $\alpha, \beta \in I^{\text{st}} \text{ Quad}$

find ①  $\sin(\alpha + \beta)$  ②  $\cos(\alpha - \beta) = ?$

$$\begin{aligned} \text{① } \sin(\alpha + \beta) &= \sin \alpha \cos \beta + \cos \alpha \sin \beta = \frac{27}{160} - \frac{133}{805} \\ &= \frac{\frac{3}{5} \cdot \frac{9}{41} - \frac{4}{5} \cdot \frac{40}{41}}{5 \times 41} = \frac{-133}{805} \end{aligned}$$

2) When  $\cos \beta = \frac{9}{41}$

$$\begin{aligned} \sin \beta &= \sqrt{1 - \cos^2 \beta} \\ &= \sqrt{1 - \frac{9^2}{41^2}} \end{aligned}$$

$$\text{① } \sin \alpha = \frac{3}{5}$$

$$\begin{aligned} \cos \alpha &= \sqrt{1 - \sin^2 \alpha} \\ &= \sqrt{1 - \frac{9}{25}} = \sqrt{\frac{16}{25}} \end{aligned}$$

$$\cos \alpha = \frac{4}{5}$$



$$(2) \cos(\alpha - \beta) = \cos \alpha \cdot \cos \beta + \sin \alpha \cdot \sin \beta$$

$$= \frac{4}{5} \cdot \frac{9}{41} + \frac{3}{5} \cdot \frac{40}{41}$$

$$= \frac{36 + 120}{205} = \frac{156}{205}$$

$$\sin \alpha = \frac{3}{5} \Rightarrow \cos \alpha = \frac{4}{5}$$

$$\cos \beta = \frac{9}{41} \Rightarrow \sin \beta = \frac{40}{41}$$

$$\sin \beta = \sqrt{1 - \cos^2 \beta} = \sqrt{1 - \frac{81}{1681}}$$



Q Value of  $\cos 31^\circ + \cos 89^\circ + \cos 151^\circ = ?$   $\rightarrow 31^\circ$  Kina zar.

$$\cos 31^\circ + \cos(\overset{A}{120^\circ} - \overset{B}{31^\circ}) + \cos(\overset{A}{120^\circ} + \overset{B}{31^\circ})$$

$$= \cos 31^\circ + \left\{ \cos 120^\circ \cos 31^\circ + \sin 120^\circ \sin 31^\circ \right\} + \left\{ \cos 120^\circ \cos 31^\circ - \sin 120^\circ \sin 31^\circ \right\}$$

$$= \cos 31^\circ + 2 \cos 120^\circ \cos 31^\circ$$

$$= \cos 31^\circ + 2 \times -\frac{1}{2} \cos 31^\circ = \cos 31^\circ - \cos 31^\circ = 0$$

$$\cos 120^\circ = \cos\left(\frac{\pi}{2} + 30^\circ\right) = -\sin 30^\circ = -\frac{1}{2}$$

$$\cos(A - B) = \cos A \cos B + \sin A \sin B$$

$$\cos(A + B) = \cos A \cos B - \sin A \sin B$$



Q If  $\sin \alpha \cdot \sin \beta - \cos \alpha \cdot \cos \beta + 1 = 0$   
then value of  $1 + \cos \alpha \cdot \tan \beta = ?$

$$\sin \alpha \sin \beta - \cos \alpha \cos \beta = -1$$

$$\Rightarrow \cos \alpha \cos \beta - \sin \alpha \sin \beta = 1$$

$$\Rightarrow \cos(\alpha + \beta) = 1$$

$\sin(\alpha + \beta) = 0$  होता है

$$\text{Demand} = 1 + \cos \alpha \cdot \tan \beta$$

$$= 1 + \frac{\cos \alpha}{\sin \alpha} \cdot \frac{\sin \beta}{\cos \beta} = \frac{\sin \alpha \cdot \cos \beta + \cos \alpha \cdot \sin \beta}{\sin \alpha \cos \beta} = \frac{\sin(\alpha + \beta)}{\sin \alpha \cos \beta} = \frac{0}{\sin \alpha \cos \beta} = 0$$

(correct  $\sin(90-0)$ )  
Q  $\frac{\sin 24^\circ \cdot \cos 6^\circ - \sin 6^\circ \cdot \sin 66^\circ}{\sin 21^\circ \cdot \cos 39^\circ - \cos 51^\circ \cdot \sin 69^\circ} = ?$  ①

$$\frac{\sin 24^\circ \cdot \cos 6^\circ - \sin 6^\circ \cdot \sin(90^\circ - 24^\circ)}{\sin 21^\circ \cdot \cos 39^\circ - \cos(90^\circ - 39^\circ) \cdot \sin(90^\circ - 21^\circ)} = \sin 0$$

$$\sin 24^\circ \cdot \cos 6^\circ - \sin 6^\circ \cdot \cos 24^\circ = \sin(24^\circ - 6^\circ)$$

$$\frac{\sin 21^\circ \cdot \cos 39^\circ - \sin 39^\circ \cdot \cos 21^\circ}{\sin \alpha \cos \beta - \cos \alpha \sin \beta} = \frac{\sin(21^\circ - 39^\circ)}{\sin(\alpha - \beta)} = \frac{\sin 18^\circ}{\sin(-18^\circ)}$$

$$= \frac{\sin 18^\circ}{-\sin 18^\circ} = -1$$



Q

$$\sum_{K=1}^{13} \frac{1}{\sin\left(\frac{\pi}{4} + \frac{(K-1)\pi}{6}\right) \cdot \sin\left(\frac{\pi}{4} + \frac{K\pi}{6}\right)} = ? = \cot\frac{5\pi}{12}$$

Multiply & divide

$$= 2 \sum_{K=1}^{13} \left( \cot\left(\frac{\pi}{4} + \frac{(K-1)\pi}{6}\right) - \cot\left(\frac{\pi}{4} + \frac{K\pi}{6}\right) \right)$$
$$= 2 \left[ \cot\left(\frac{\pi}{4} + 0\right) - \cancel{\cot\left(\frac{\pi}{4} + \frac{\pi}{6}\right)} \right]$$

$$\ln\left(\frac{\pi}{6}\right)$$


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$$\ln\left(\frac{\pi}{4} + \frac{k\pi}{6}\right) \cdot \ln\left(\frac{\pi}{4} + \frac{(k-1)\pi}{6}\right) \cdot \ln\frac{\pi}{6}$$

$$\frac{1}{\sin \frac{\pi}{6}} \left\{ \frac{\sin \left( \frac{\pi}{4} + \frac{k\pi}{6} \right) - \left( \frac{\pi}{4} + \frac{(k-1)\pi}{6} \right)}{\sin \left( \frac{\pi}{4} + \frac{k\pi}{6} \right) \cdot \sin \left( \frac{\pi}{4} + \frac{(k-1)\pi}{6} \right)} \right\}$$

$$\frac{1}{k} \left( \frac{\cancel{\sin\left(\frac{\pi}{4} + \frac{k\pi}{2}\right)} \cdot \cancel{\cos\left(\frac{\pi}{4} + \frac{(k-1)\pi}{2}\right)}}{\cancel{\sin\left(\frac{\pi}{4} + \frac{k\pi}{2}\right)} \cdot \cancel{\sin\left(\frac{\pi}{4} + \frac{(k-1)\pi}{2}\right)}} - \frac{\cancel{\cos\left(\frac{\pi}{4} + \frac{k\pi}{2}\right)} \cdot \cancel{\sin\left(\frac{\pi}{4} + \frac{(k-1)\pi}{2}\right)}}{\cancel{\sin\left(\frac{\pi}{4} + \frac{k\pi}{2}\right)} \cdot \cancel{\sin\left(\frac{\pi}{4} + \frac{(k-1)\pi}{2}\right)}} \right)$$

Multiply & divide  
by  $\ln \frac{\pi}{6}$ .

Change from  $\frac{\pi}{6}$  all to Dr.

$$= 2 \left( \begin{aligned} & \cancel{\Gamma\left(\frac{\pi}{4} + 0\right)} - \cancel{\Gamma\left(\frac{\pi}{4} + \frac{\pi}{6}\right)} \\ & + \cancel{\Gamma\left(\frac{\pi}{4} + \frac{\pi}{6}\right)} - \cancel{\Gamma\left(\frac{\pi}{4} + \frac{2\pi}{6}\right)} \\ & + \cancel{\Gamma\left(\frac{\pi}{4} + \frac{2\pi}{6}\right)} - \cancel{\Gamma\left(\frac{\pi}{4} + \frac{3\pi}{6}\right)} \\ & \vdots \\ & + \cancel{\Gamma\left(\frac{\pi}{4} + \frac{12\pi}{6}\right)} - \Gamma\left(\frac{\pi}{4} + \frac{13\pi}{6}\right) \end{aligned} \right)$$

$$2 \left( 1 - 64^{\left( \frac{6\pi + 52\pi}{24} \right)} \right)$$

$$2\left(1 - 64 \frac{5T}{12}\right)$$

pm.  $g(1 - \ln 16)$