Nishant Jindal

Q e^{π} or π^{e} Which one is greater?

Assume $e^{\pi} > \pi^{e}$

$(e^{\pi})^{\frac{1}{\pi e}} > (\pi^{e})^{\frac{1}{\pi e}}$

$e^{\frac{1}{e}} > \pi^{\frac{1}{\pi}}$ (T/F)

① let $f(x) = x^{1/x}$

making graph

(A) Domain $x > 0$

$x \in (0, \infty)$

(B) $\frac{dy}{dx} = x^{1/x}\left\{\frac{d}{dx}\frac{1}{x}\ln x\right\}$

$= x^{1/x}\left\{\frac{1}{x^2} - \frac{\ln x}{x^2}\right\}$

$= \underset{+}{\oplus}\, x^{\frac{1}{x}-2}(1-\ln x) = 0 \Rightarrow x = e \rightarrow f(e) = e^{1/e}$

$f(\infty) = \lim_{x\to\infty} x^{1/x} = \infty^0$

$y = \lim_{x\to\infty} x^{1/x}$

$\ln y = \lim_{x\to\infty} \frac{1}{x}\ln x = \frac{\infty}{\infty} = 0$

$y = e^0 = 1$

$\lim_{x\to 0} x^{1/x} = 0^{\infty}$

$= 0$

$f(e) > f(\pi)$

$e^{\frac{1}{e}} > \pi^{1/\pi} \Rightarrow e^{\pi} > \pi^{e}$ [True]

$f(2016) > f(2017)$

$(2016)^{\frac{1}{2016}} > (2017)^{\frac{1}{2017}}$

$(2016)^{2017} > (2017)^{2016}$

Q Which is greater

Board level

$e^{\frac{2x_1+x_2}{3}}$ or $\frac{2e^{x_1}+e^{x_2}}{3}$.

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Q $f(x)$ is concave downward & $f'(x)>0$

$x_1 \neq x_2$ Which is grtr

$$f'\left(\frac{x_1+x_2}{2}\right) \text{ or } \frac{f'(x_1)+f'(x_2)}{2}$$

Rolle's Thm. [Mean Value Thm]

If a fxn $y=f(x)$ in Interval $[a,b]$

Satisfies

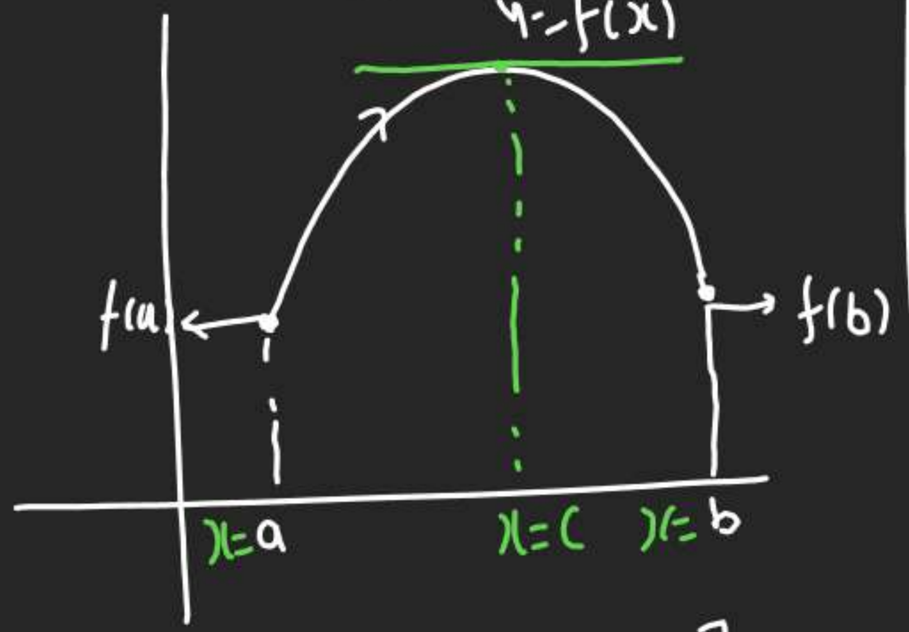
A) $f(x)$ is cont^s in $[a,b]$ ✓

B) $f(x)$ is diff in (a,b) ✓

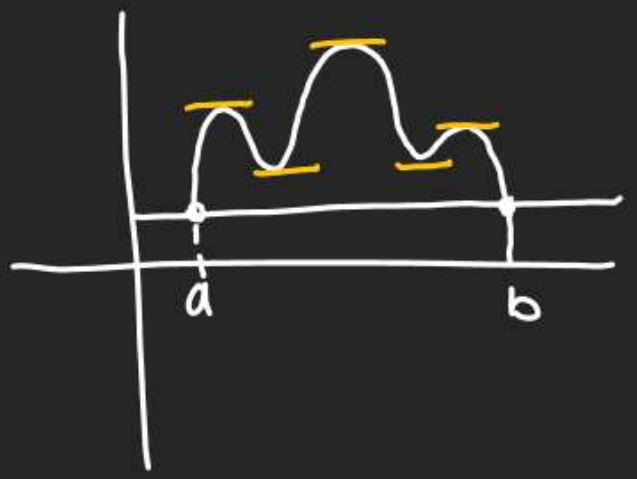
C) $f(a)=f(b)$

then acc. to RMVT $\exists$ at least at pt $x=c$ such that $f'(c)=0$

(B) Geometrical Meaning of RMVT.



Bet $x=a$ & $x=b$ $\exists$ at least one pt $x=c$ where tangent is $||^{r}$ to X Axis



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Varify RMVT fo $f(x) = x^3 - 4x + 3$.

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(1) Varify RMVT for $f(x)=x^2-4x+3$ in $[0,4]$

(1) $f(x)=$ Poly Cont ✓

(2) $f'(x)=2x-4$ in $[0,4]$ Everywhere Existing

diff

(3) $f(0)=3$

$f(4)=16-16+3=3$

$f(0)=f(4)$

all 3 cond^n Satisfied

then acc to RMVT

$\exists$ atleast one pt. $x=c$ Where

$f'(c)=2c-4=0$

$c=2\in(0,4)$

Q If Rolle's Thm is applicable to fxn

BL

f defined by

$$f(x)=\begin{cases} ax^2+b & |x|<1 \\ 1 & |x|=1 \\ \frac{c}{|x|} & |x|>1 \end{cases}$$

for $x\in[-2,2]$ then find $b^2+c^2-a^2$?

as RMVT applicable $\Rightarrow$ fxn is Cont^s

fxn is diff^{ble}

$$f(x)=\begin{cases} ax^2+b & -1<x<1 \\ 1 & x=1,-1 \\ \frac{c}{x} & x>1 \\ \frac{c}{-x} & x<-1 \end{cases}$$

(+1)

$a(1)^2+b=1=\frac{c}{1}$

$a+b=1=c$

(−1)

$\frac{c}{-(-1)}=1=a(-1)^2+b$

$c=\boxed{1=a+b}$

(c=1)

$$f'(x)=\begin{cases} 2ax & -1<x<1 \\ 0 & x=1,-1 \\ -\frac{c}{x^2} & x>1 \\ \frac{c}{x^2} & x<-1 \end{cases}$$

(x=1) LHD=RHD

$2a(1)=-\frac{1}{(1)^2}$

$a=-\frac{1}{2}$

$a+b=1$

$b=1+\frac{1}{2}$

$=\frac{3}{2}$

$b^2+c^2-a^2$

$\frac{9}{4}+1-\frac{1}{4}=3$

Root theorey of RMVT.

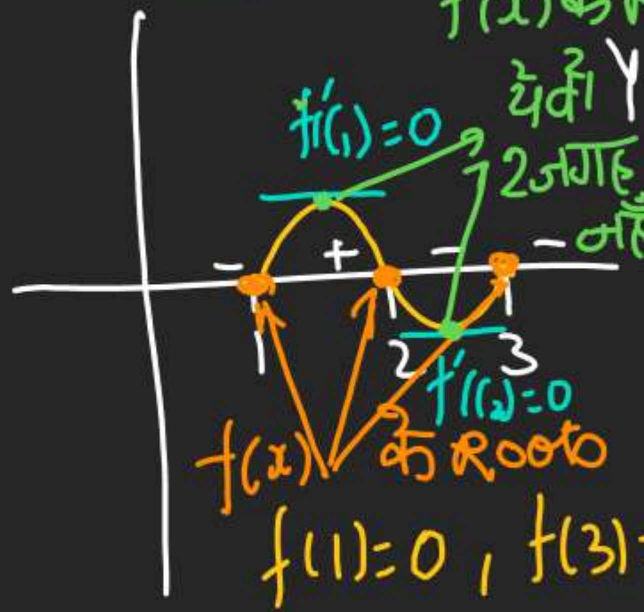
$f'(x)$ के Root

जैसे $Y = x^3 - 6x^2 + 11x - 6$ in $[1,3]$

Poly (at^3) / Deff.

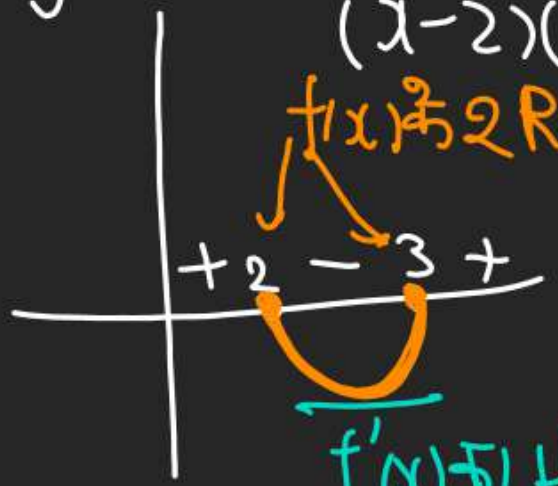
$= (x-1)(x-2)(x-3)$

Roots $= 1, 2, 3$



$f(x) = x^2 - 5x + 6 \quad x \in [2,3]$

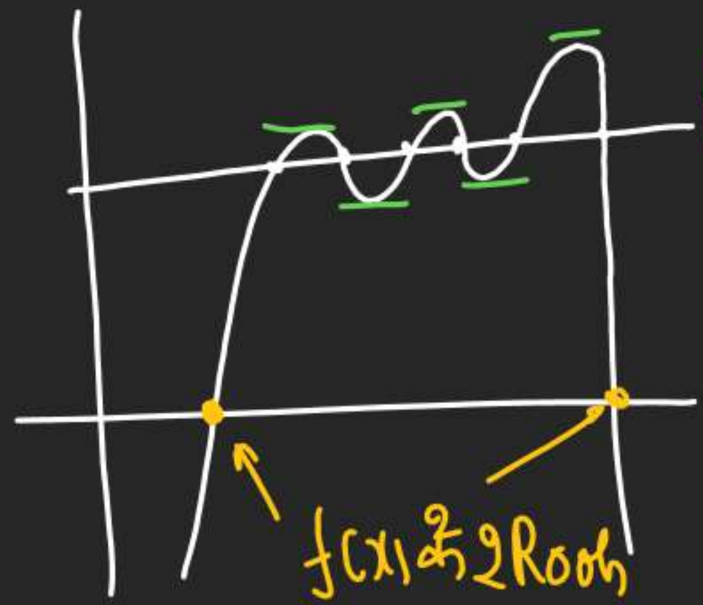
$(x-2)(x-3)$



If $f(x)$ has n Roots

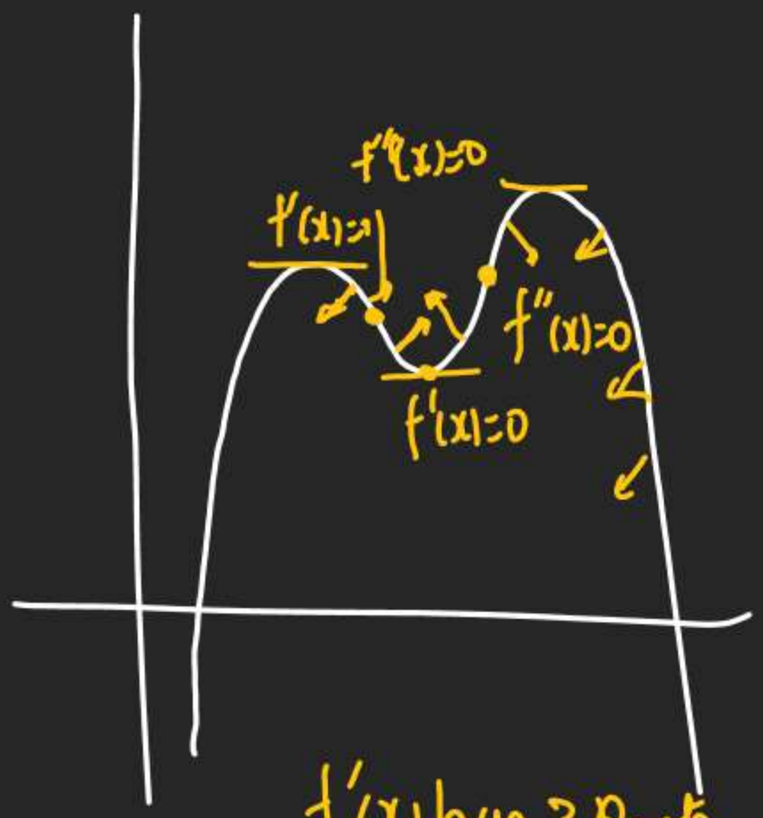
then $f'(x)$ must have $(n-1)$ Root

if $f(x)$ Satisfies RMVT.



$f'(x)$ has n Roots then

$f''(x)$ must have $(n-1)$ Roots



$f'(x)$ has 3 Roots

$f''(x)$ has 2 Roots

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Q If $f(x)$ is twice diff^ble^ fxn

S.T. $f(a)=0, f(b)=2, f(c)=-1$, $f(d)=2, f(e)=0$ where $a<b<c<d<e$ then Min. No. of $\boxed{\text{Zeroes}}$ of $g(x) = (f'(x))^2 + f''(x) \cdot f(x)$ in Interval $[a, e]$ is?

$g(x)$ Poly

$f(x) = x^3$

$f'(x) = 3x^2$

$f(x) \times f'(x) = x^3 \times 3x^2$

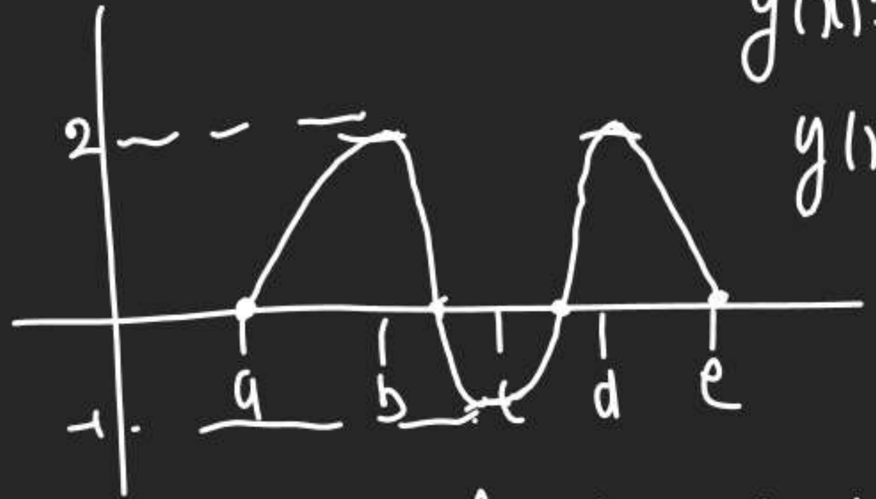
$= 3x^5$

$g(x) = f'(x) \cdot f'(x) + f''(x) \cdot f(x)$

$g(x) = \frac{d}{dx}\left(f(x) \times f'(x)\right)$

$f(x) \times f'(x)$ has 7 Roots

$\to g(x)$ is der. of it, having 6 Roots Zeroes



$f(x)$ has 4 Roots

$f'(x)$ has 3 Roots

Q If $\boxed{a, b \text{ are 2 Roots of Poly. } P(x)=0}$ then S.T. $\exists$ atleast 1 Root of $c \in (a,b)$ Where $P'(c) + 100\,P(c) = 0$

Self

$e^{100x} \times \quad P'(x) + 100\,P(x) = 0$

$e^{100x} \cdot P'(x) + 100\,e^{100x}\,P(x) = 0$

$\frac{d}{dx}\left(P(x) \cdot e^{100x}\right) = 0$

Jab b Strike na Kare Ki Kiska der. hai think about $e^{\text{Kuch Kuch}}$

$f(x) = \boxed{P(x)} \cdot e^{100x} \to$ Conts + diff

$f(a) = 0, f(b) = 0 \Rightarrow f(a) = f(b)$

Bet^n^ $x \in (a,b)$ atleast one hb.

$x = c$ Where $f'(c) = 0$

$e^{100c} \cdot P'(c) + 100\,e^{100c} \cdot P(c) = 0$

$\Rightarrow P'(c) + 100\,P(c) = 0$ [H.P]

Nishant Jindal

Q Betⁿ any 2 Real Root of $e^x \cos x = 1$ $f(x)$

∃ atleast one Real Root of $\tan x = 1$ $f'(x)$

$f(x)$ के 2 Real Roots के बीच atleast one Root of its derivative Exist.

① $e^x \cdot \cos x = 1$ → r_1, r_2 Real

$f(x) = e^x \cdot \cos x - 1$

Ex, Trig

C ✓, D ✓ } RMVT

$f(r_1) = 0$

$f(r_2) = 0$

$f'(x) = e^x(-\sin x) + \cos x \cdot e^x = 0$

$e^x(\cos x - \sin x) = 0$

$\cos x = \sin x$

$\tan x = 1$

Q Betⁿ 2 Real Root of $e^x \cos x = 1$ ∃ atleast one Real Root of $e^x \cdot \sin x = 1$

Make Sure that $e^x \cos x = 1$ is $f(x)$

$e^x \cdot \sin x = 1$ is $f'(x)$

① $e^x \cdot \cos x = 1$ } $\times e^{-x}$

$\cos x = e^{-x}$ $f(x)$

diff

② $+\sin x = +e^{-x}$

$\sin x = \frac{1}{e^x}$

$e^x \cdot \sin x = 1$ → $f'(x)$