DPP-5

1. Prove that $\cot\theta - \tan\theta = 2\cot 2\theta$.

2. Prove that $\frac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta} = \sec 2\theta - \tan 2\theta$.

3. Prove that $\tan\left(\frac{\pi}{4} + \theta\right) - \tan\left(\frac{\pi}{4} - \theta\right) = 2\tan 2\theta$.

4. Prove that $1 + \tan\theta \tan 2\theta = \sec 2\theta$

5. Prove that $\frac{1 + \sin 2A - \cos 2A}{1 + \sin 2A + \cos 2A} = \tan A$

6. Show that $\frac{1}{\sin 10^\circ} - \frac{\sqrt{3}}{\cos 10^\circ} = 4$

7. Prove that $\operatorname{cosec} A - 2\cot 2A \cos A = 2\sin A$.

8. Prove that $\frac{1 + \sin 2A}{\cos 2A} = \frac{\cos A + \sin A}{\cos A - \sin A} = \tan\left(\frac{\pi}{4} + A\right)$.

9. Prove that $\cos^3\theta \sin 3\theta + \sin^3\theta \cos 3\theta = \frac{3}{4}\sin 4\theta$

10. Prove that $\tan\theta + \tan(60^\circ + \theta) + \tan(120^\circ + \theta) = 3\tan 3\theta$

11. If α and β are the two different roots of equation $a\cos\theta + b\sin\theta = c$, prove that

a) $\tan(\alpha + \beta) = \frac{2ab}{a^2 - b^2}$ b) $\cos(\alpha + \beta) = \frac{a^2 - b^2}{a^2 + b^2}$

12. If $\cos\theta = \frac{\cos\alpha - \cos\beta}{1 - \cos\alpha\cos\beta}$, prove that one of the values of $\tan\frac{\theta}{2}$ is $\tan\frac{\alpha}{2}\cot\frac{\beta}{2}$.

13. If $\tan\theta \tan\phi = \sqrt{\frac{(a-b)}{(a+b)}}$, prove that $(a - b\cos 2\theta)(a - b\cos 2\phi)$ is independent of θ and ϕ.

14. If θ is an acute angle and $\sin\frac{\theta}{2} = \sqrt{\frac{x-1}{2x}}$, find $\tan\theta$ in terms of x.

15. Prove that $(1 + \sec 2\theta)(1 + \sec 4\theta)(1 + \sec 8\theta) = \frac{\tan 8\theta}{\tan\theta}$

16. Prove that $\frac{\sin^2 3A}{\sin^2 A} - \frac{\cos^2 3A}{\cos^2 A} = 8\cos 2A$.

17. If $A = 110^\circ$, then prove that $\frac{1 + \sqrt{1 + \tan^2 2A}}{\tan 2A} = -\tan A$.

18. In triangle ABC, $a = 3, b = 4$ and $c = 5$. Then find the value of $\sin A + \sin 2B + \sin 3C$.

Answer Key

14. $\sqrt{x^2 - 1}$ **18.** $\frac{14}{25}$