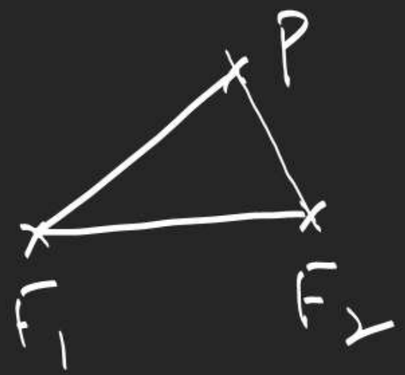
Hyperbola

$|PF_1 - PF_2| = \text{constant} = 2a$

F_1, F_2 are fixed points



locus of P

Hyperbola $\leftarrow 2a < F_1F_2$ ✓

P lies on line joining F_1F_2 excluding interior points of segment $\bar{F}_1\bar{F}_2$ $\leftarrow 2a = F_1F_2$

$2a > F_1F_2 \rightarrow$ no locus

$2a < 2c$

$$\left(\sqrt{(x+c)^2+y^2} - \sqrt{(x-c)^2+y^2}\right)^2 = (2a)^2$$

$P(x,y)$

$(-c,0)\ F_1 \qquad F_2\ (c,0)$

$$2(x^2+c^2+y^2) - 2\sqrt{x^2+c^2+y^2+2cx}\ \sqrt{x^2+c^2+y^2-2cx} = 4a^2$$

$$\left(x^2+c^2+y^2-2a^2\right)^2 = \left(x^2+c^2+y^2\right)^2 - 4c^2x^2$$

$$-4a^2(x^2+c^2+y^2) + 4a^4 = -4c^2x^2$$

$$(c^2-a^2)x^2 - a^2y^2 = a^2(c^2-a^2)$$

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

$$b^2 = c^2 - a^2$$

$$\frac{x^2}{a^2} - \frac{y^2}{c^2-a^2} = 1$$

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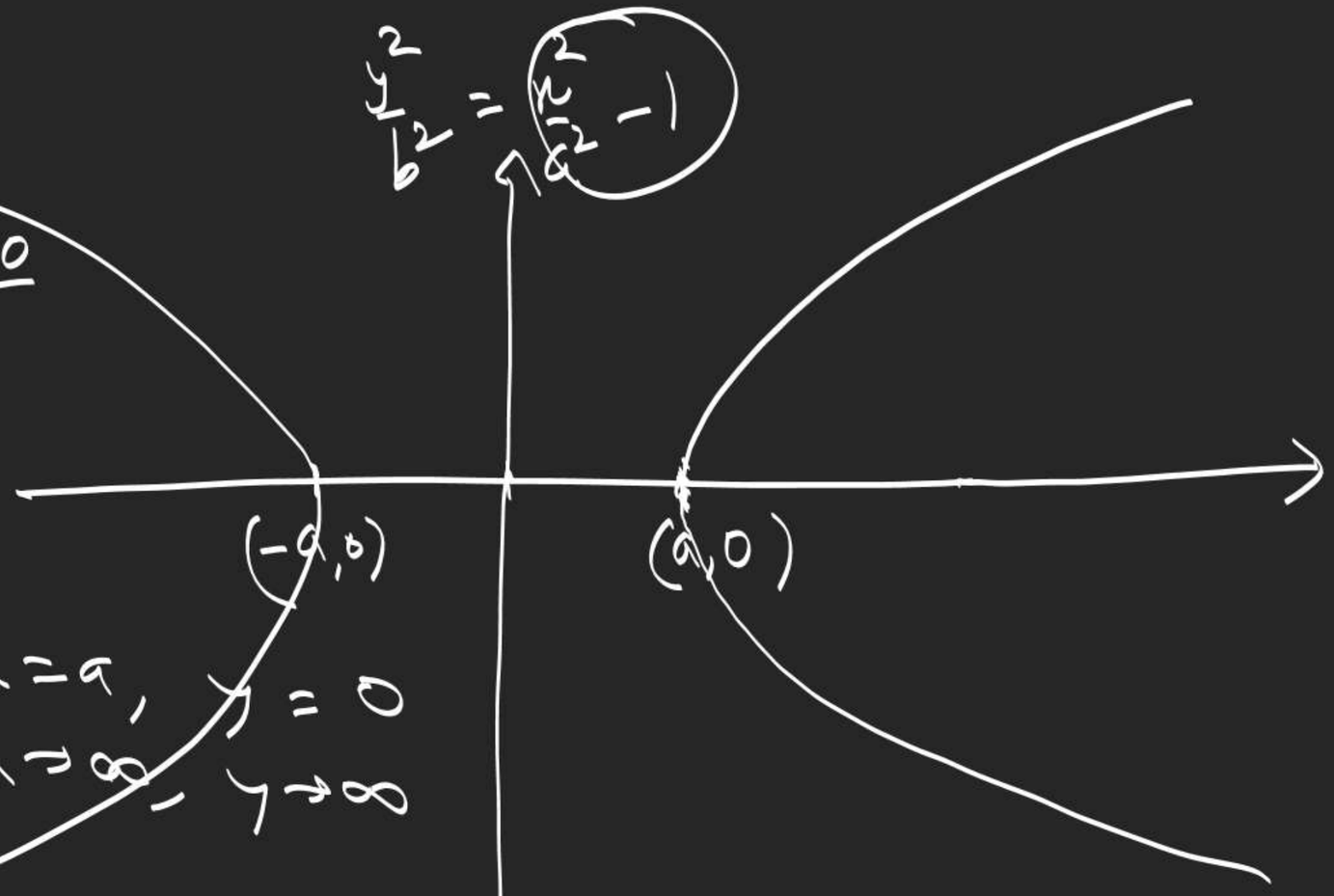
$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

$$\frac{y^2}{b^2} = \frac{x^2}{a^2} - 1$$

$y \geq 0$



$x = a, \quad y = 0$

$x \to \infty, \quad y \to \infty$

$$\frac{2yy'}{b^2} = \frac{2x}{a^2}$$

$y' < 0, \quad x < 0$

$> 0, \quad x > 0$

$$\frac{yy''}{b^2} + \frac{(y')^2}{b^2} = \frac{1}{a^2}$$

$$\frac{yy''}{b^2} = \frac{1}{a^2} - \frac{1}{b^2}\left(\frac{x}{a^2}\frac{b^2}{y}\right)^2 = \frac{1}{a^2} - \frac{x^2}{y^2}\frac{b^2}{a^4} = \frac{b^2}{a^2 y^2}\left(\frac{y^2}{b^2} - \frac{x^2}{a^2}\right) = -\frac{b^2}{a^2 y^2}$$

$< 0.$

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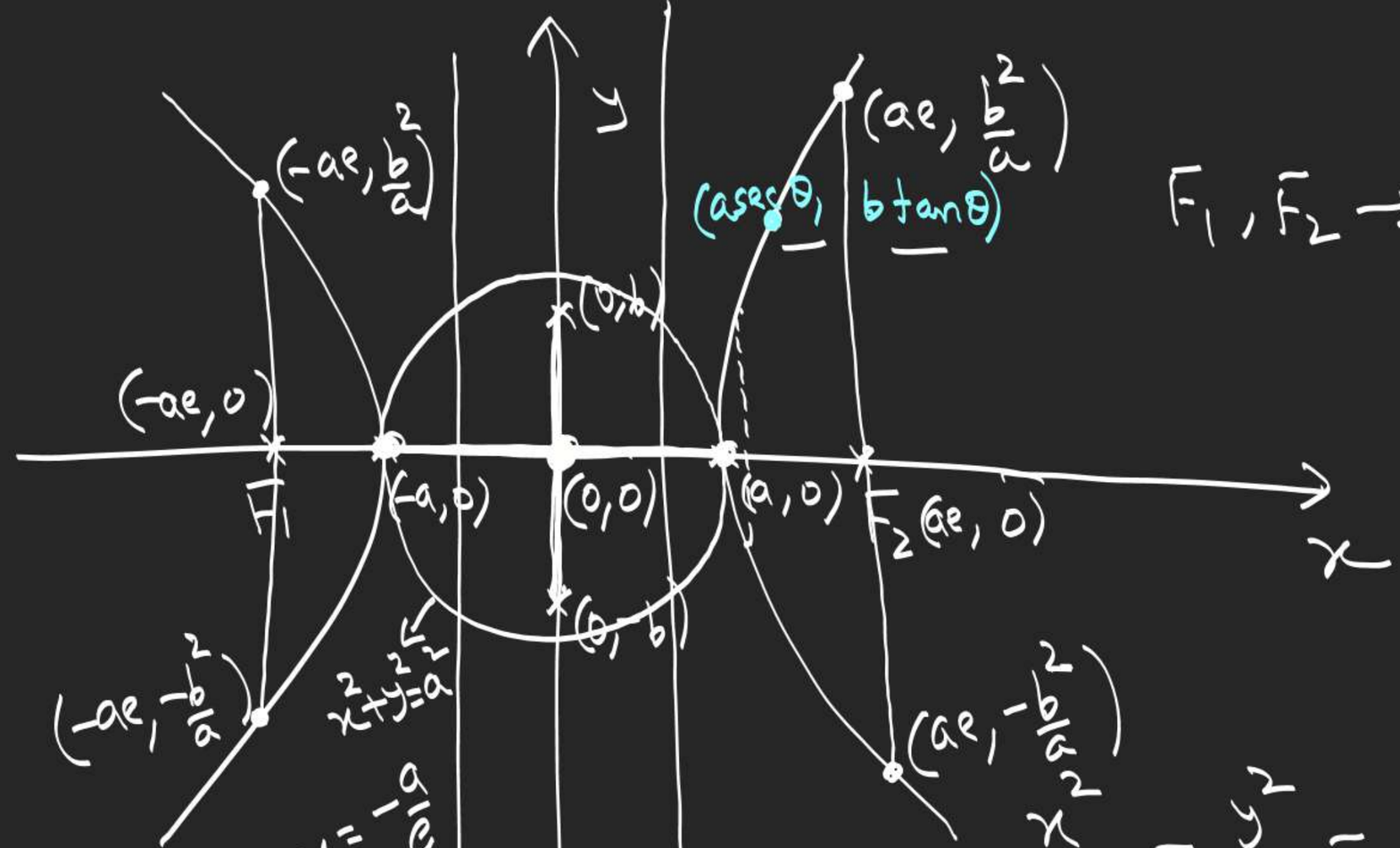
$b^2 = c^2 - a^2$

$2a$ = Length of TA

$2b$ = Length of CA



$F_1, F_2 \rightarrow$ foci

$b^2 = a^2e^2 - a^2$

$\boxed{b^2 = a^2(e^2-1)}$

$b^2\frac{x^2}{a^2} = b^2(e^2-1) = y^2$

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

$b^2 = a^2(e^2-1)$

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Transverse axis
Centre
Conjugate axis
} Principle axis

Vertices

Double Ordinate

Latus Rectum

$$\text{eccentricity} = \frac{\text{Distance b/n centre \& focus}}{\text{Distance b/n centre \& vertex}}$$

$$e = \frac{c}{a}$$

$$c = ae$$

Auxiliary Circle

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Equation of Hyperbola

$$\frac{(\text{Ian Distance of any point 'P' on Hyperbola from conjugate axis})^2}{(\text{Semi TA})^2} - \frac{(\text{Ian distance of 'P' from TA})^2}{(\text{Semi CA})^2} = 1$$

$$(\text{Semi CA})^2 = (\text{Semi TA})^2(e^2 - 1)$$

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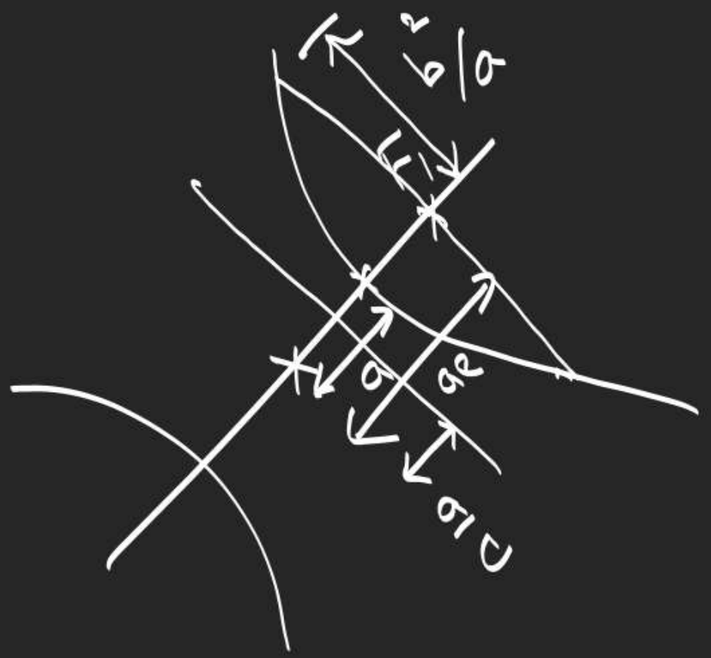

a = semi TA

b = semi CA

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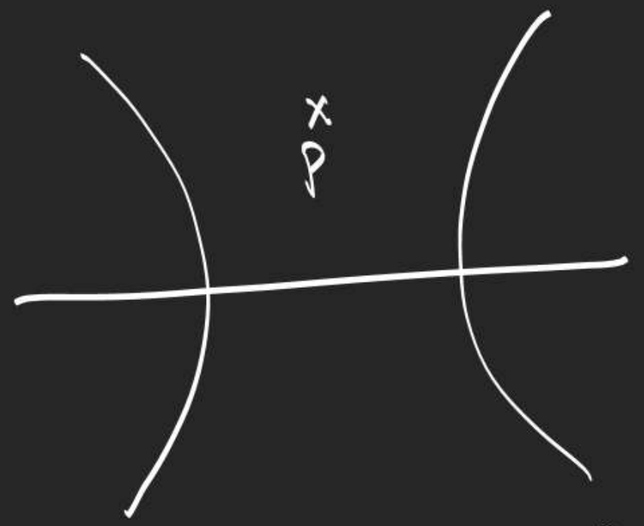
Position of point w.r.t. Hyperbola

$S_1 < 0 \Rightarrow P$ lies outside

$S_1 > 0 \Rightarrow P$ lies inside



$$\frac{x^2}{a^2} - \frac{y^2}{b^2} - 1 = 0$$

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Conjugate Hyperbola

Square

e_1, e_2 ?

$$\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$$

$$b^2 = a^2(e_1^2 - 1) \Rightarrow a^2 + b^2 = a^2e_1^2$$

$$a^2 = b^2(e_2^2 - 1) \Rightarrow a^2 + b^2 = b^2e_2^2$$

$$\frac{1}{e_1^2} + \frac{1}{e_2^2} = \frac{a^2}{a^2+b^2} + \frac{b^2}{a^2+b^2} = 1$$

$$\boxed{\frac{1}{e_1^2} + \frac{1}{e_2^2} = 1}$$

$$A = \frac{1}{2} \times \left(2\sqrt{a^2+b^2}\right)^2$$

$$\boxed{A = 2(a^2+b^2)}$$

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

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Rectangular Hyperbola

$$TA = CA$$

$$x^2 - y^2 = a^2$$

$$e = \sqrt{2}$$

$$a^2 = a^2(e^2 - 1)$$

Suggested Problems
of Ellipse
(SL Loney)