Nishant Jindal

REFLECTION (CURVE SURFACE)

Reflection from a parabolic Surface

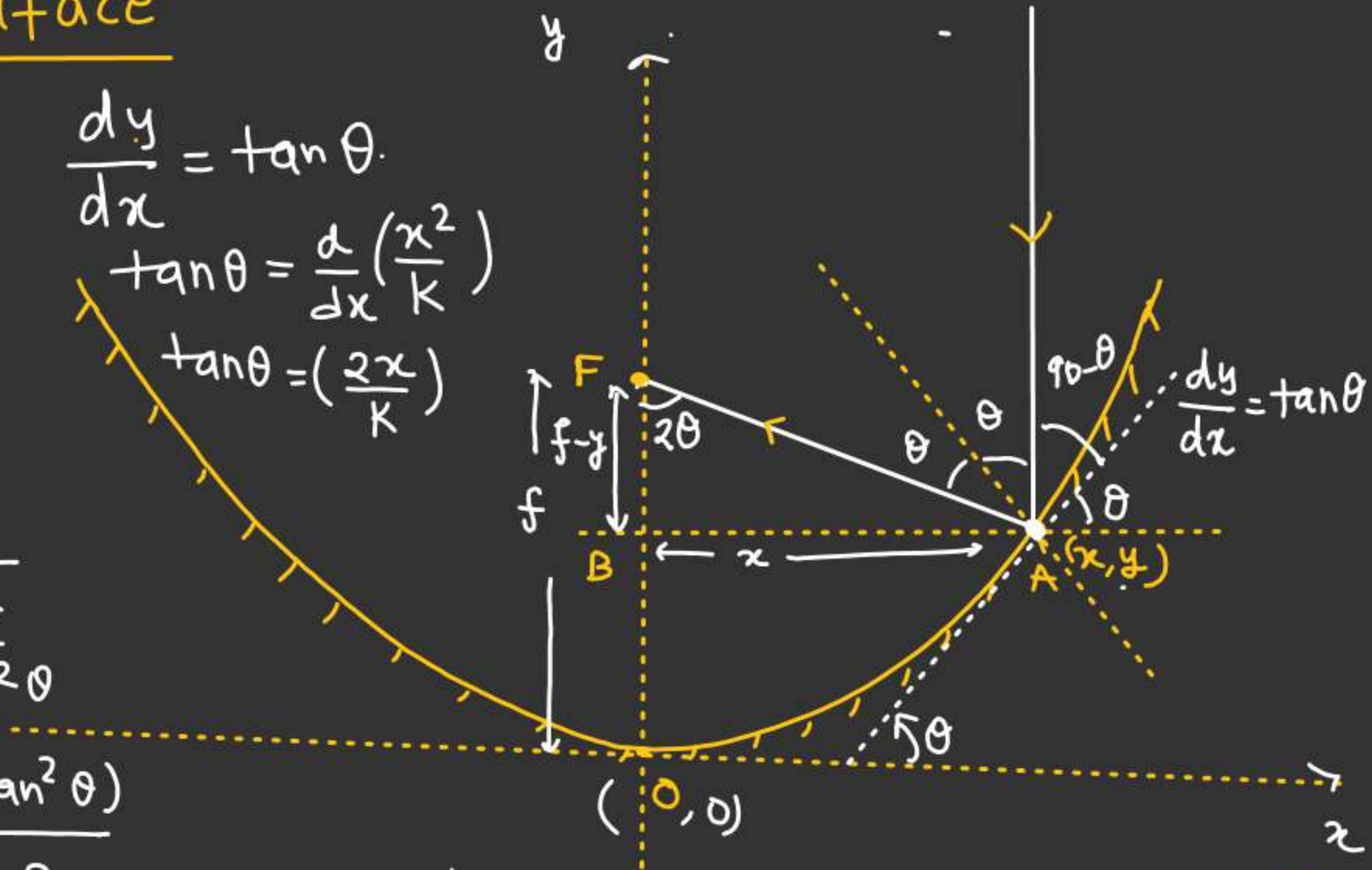
$x^2 = ky$.

Prove that any ray parallel to y axis intersect at focus.

$$\frac{dy}{dx} = \tan\theta.$$

$$\tan\theta = \frac{d}{dx}\left(\frac{x^2}{k}\right)$$

$$\tan\theta = \left(\frac{2x}{k}\right)$$



In Δ FBA

$$\tan 2\theta = \frac{x}{f-y}$$

$$f-y = \frac{x}{\tan 2\theta}$$

$$f = \left(y + \frac{x}{\tan 2\theta}\right)$$

$$f = \frac{x^2}{K} + \frac{x}{\frac{2\tan\theta}{1-\tan^2\theta}}$$

$$f = \frac{x^2}{K} + \frac{x(1-\tan^2\theta)}{2\tan\theta}$$

$$f = \frac{x^2}{k} + \frac{x\left(1 - \frac{4x^2}{k^2}\right)}{\left(\frac{4x}{k}\right)} \Rightarrow f = \frac{\cancel{\frac{4x^3}{k^2}} + x - \cancel{\frac{4x^3}{k^2}}}{\frac{4x}{k}} = \boxed{f = \frac{k}{4}}$$

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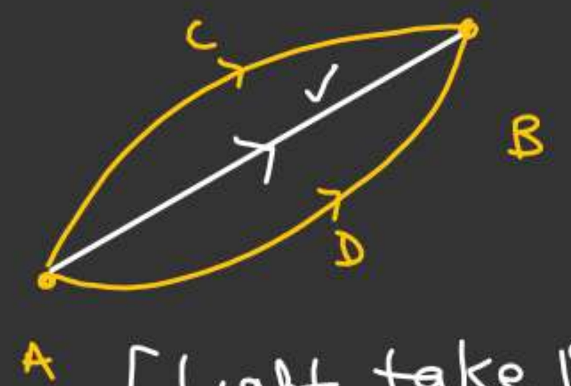
⇒ FERMET PRINCIPLE

Acutual path followed by a light ray b/w any two points is the path in which light ray take min. time b/w the two points.



[Light take path AB Instead of ACB or ADB

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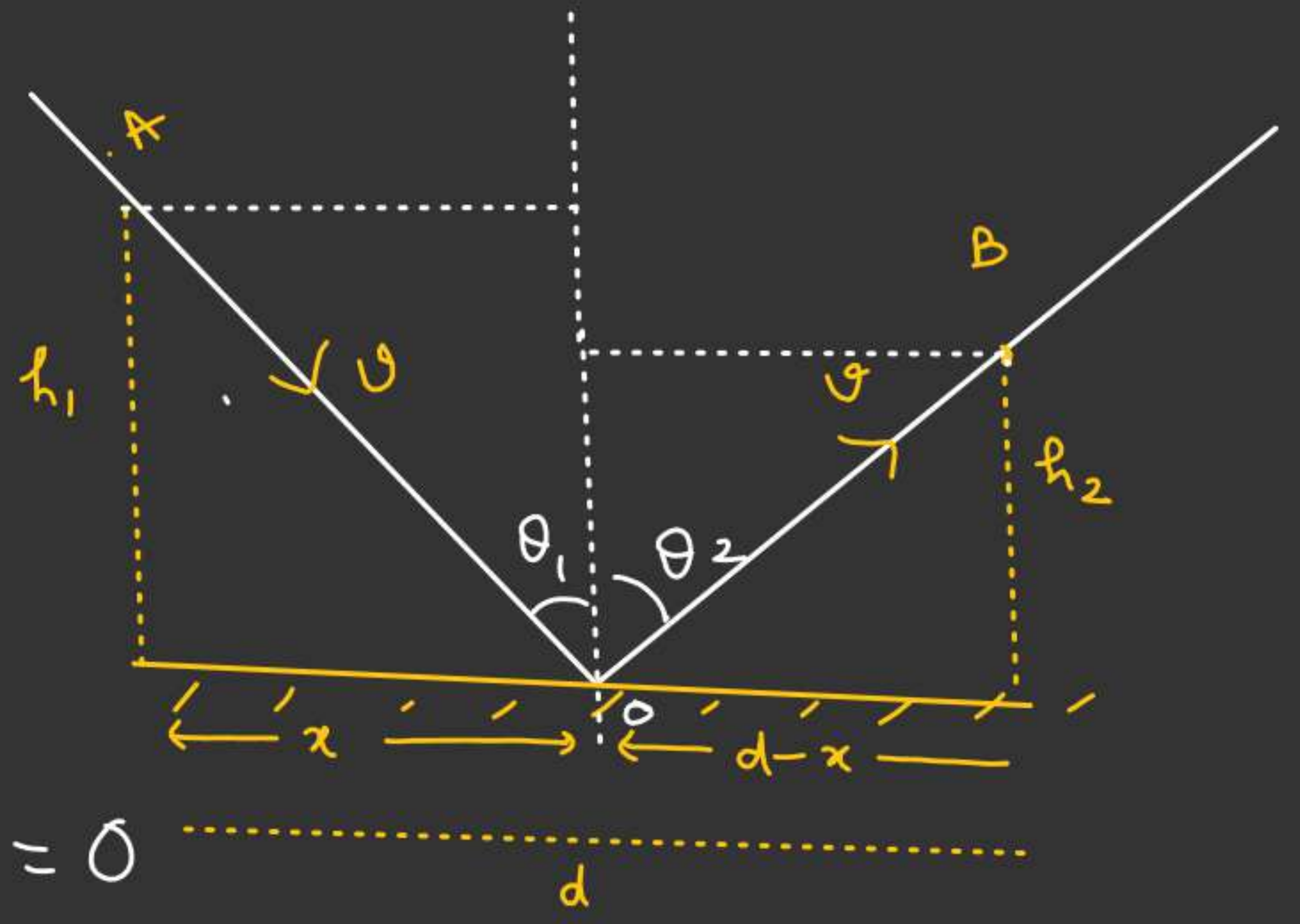
REFLECTION (CURVE SURFACE)

$$t_{AOB} = t_{OA} + t_{OB}.$$

$$t_{AOB} = \frac{\sqrt{h_1^2 + x^2}}{v} + \frac{\sqrt{h_2^2 + (d-x)^2}}{v}.$$



For t_{AOB} to be Min.

$$\frac{d}{dx}(t_{AOB}) = 0$$

$$\frac{1}{v}\left[\frac{1}{2\sqrt{h_1^2+x^2}} \times 2x + \frac{1}{2\sqrt{h_2^2+(d-x)^2}} \times 2(d-x)(-1)\right] = 0$$

$$\frac{x}{\sqrt{h_1^2+x^2}} = \frac{d-x}{\sqrt{h_2^2+(d-x)^2}}.$$

$\Downarrow$

$$\sin\theta_1 = \sin\theta_2$$

$\theta_1 = \theta_2 \Rightarrow$ <u>Law of Reflection</u>.

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REFLECTION (CURVE SURFACE)

For distance SQF to be minimum.

$SQF = (SQ) + (QF)$ [$PF = PP'$]

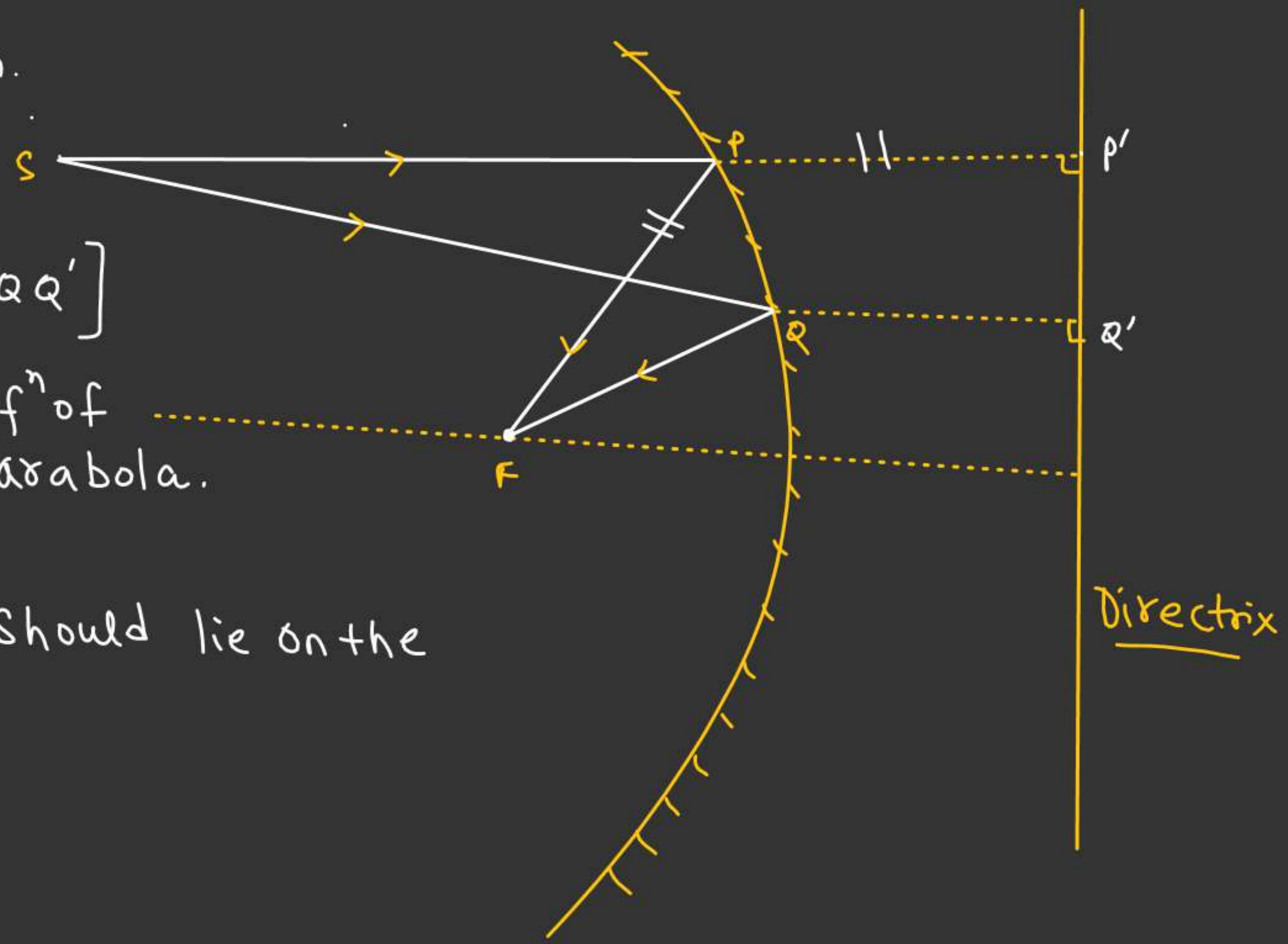
[$QF = QQ'$]

⇓

By def^n of Parabola.

$\underset{\text{Min.}}{\underline{SQF}} = \underset{\text{Min}}{\underline{SQ}} + \underset{\text{Min.}}{\underline{QQ'}}$

For SQF min, Q should lie on the line SPP'



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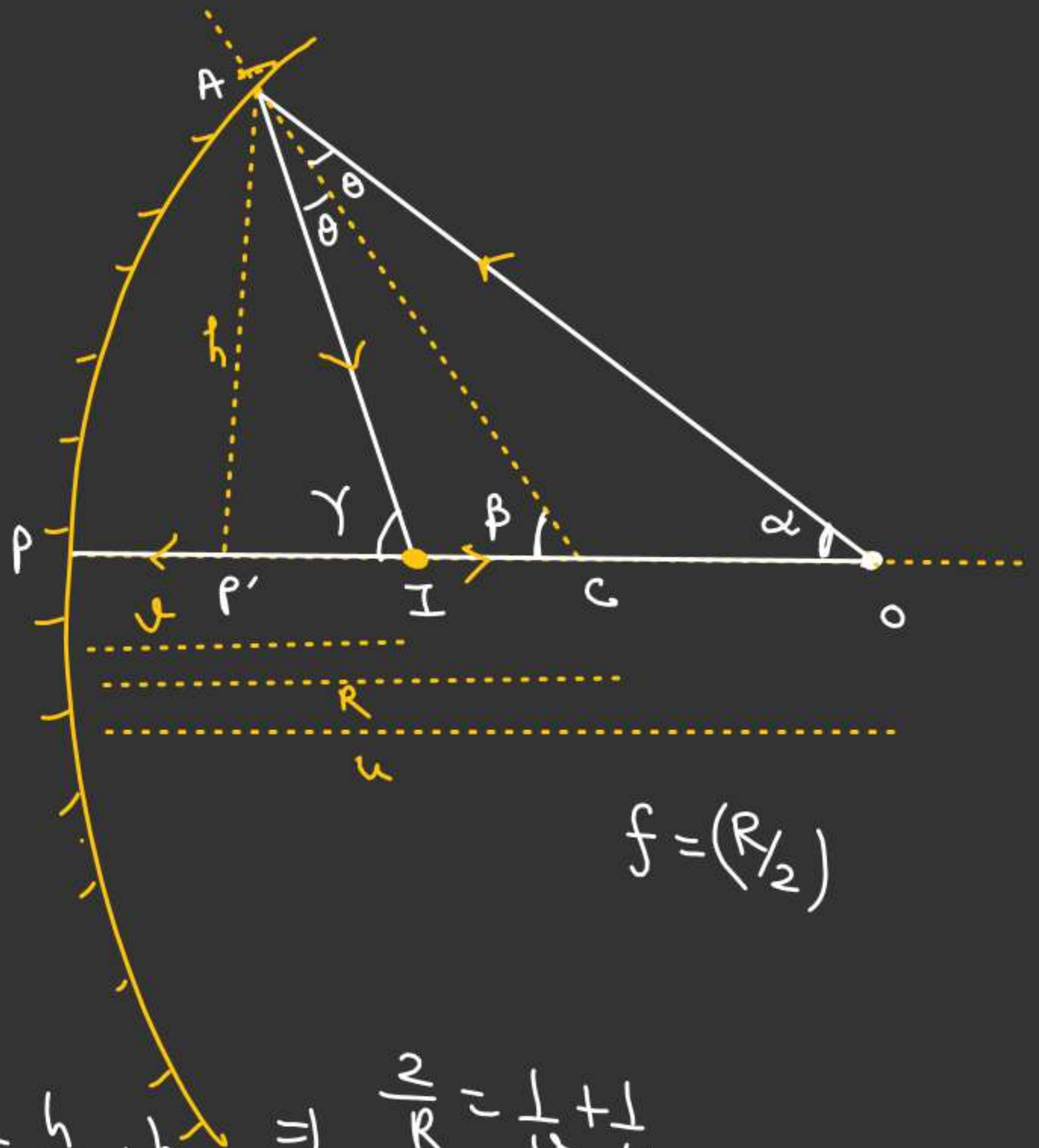
REFLECTION (CURVE SURFACE)

Mirror Formula

Assumption :- • For paraxial rays
• angle of incidence very small.



Distance from P′ is same as from P.

$\alpha \simeq \tan\alpha = \frac{h}{OP'} = \frac{h}{OP} = \frac{h}{u}$

$\beta \simeq \tan\beta = \frac{h}{CP'} = \frac{h}{CP} = \frac{h}{R}$

$\gamma \simeq \tan\gamma = \frac{h}{P'I} = \frac{h}{PI} = \frac{h}{v}$

$f = \left(\frac{R}{2}\right)$

In $\triangle ACO$

$\beta = \theta + \alpha$

In $\triangle AIC$

$\gamma = \theta + \beta$

$\beta = (\gamma - \beta) + \alpha$

$2\beta = \gamma + \alpha \Rightarrow \frac{2h}{R} = \frac{h}{v} + \frac{h}{u} \Rightarrow \frac{2}{R} = \frac{1}{v} + \frac{1}{u}$

$\Rightarrow \boxed{\frac{1}{v} + \frac{1}{u} = \frac{1}{f}}$

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REFLECTION (CURVE SURFACE)

Mirror Formula

$$\boxed{\frac{1}{v} + \frac{1}{u} = \frac{1}{f}}$$

$\Downarrow$

Put Known quantites with Sign
Unknown quantity Come with Sign

Sign Convention

- All the distances measured from pole.
- Distance measured along the incident ray taken as +ve and distances measured opposite to incident ray taken as −ve.
- Distances measured above the principal axis taken as +ve and below the principal axis taken as −ve

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REFLECTION (CURVE SURFACE)

MAGNIFICATION

Linear Magnification
OR Transverse Magnification
OR Lateral Magnification

In general

$$\boxed{m = \frac{\text{height of image}}{\text{height of object}} = \frac{h_I}{h_o}}$$

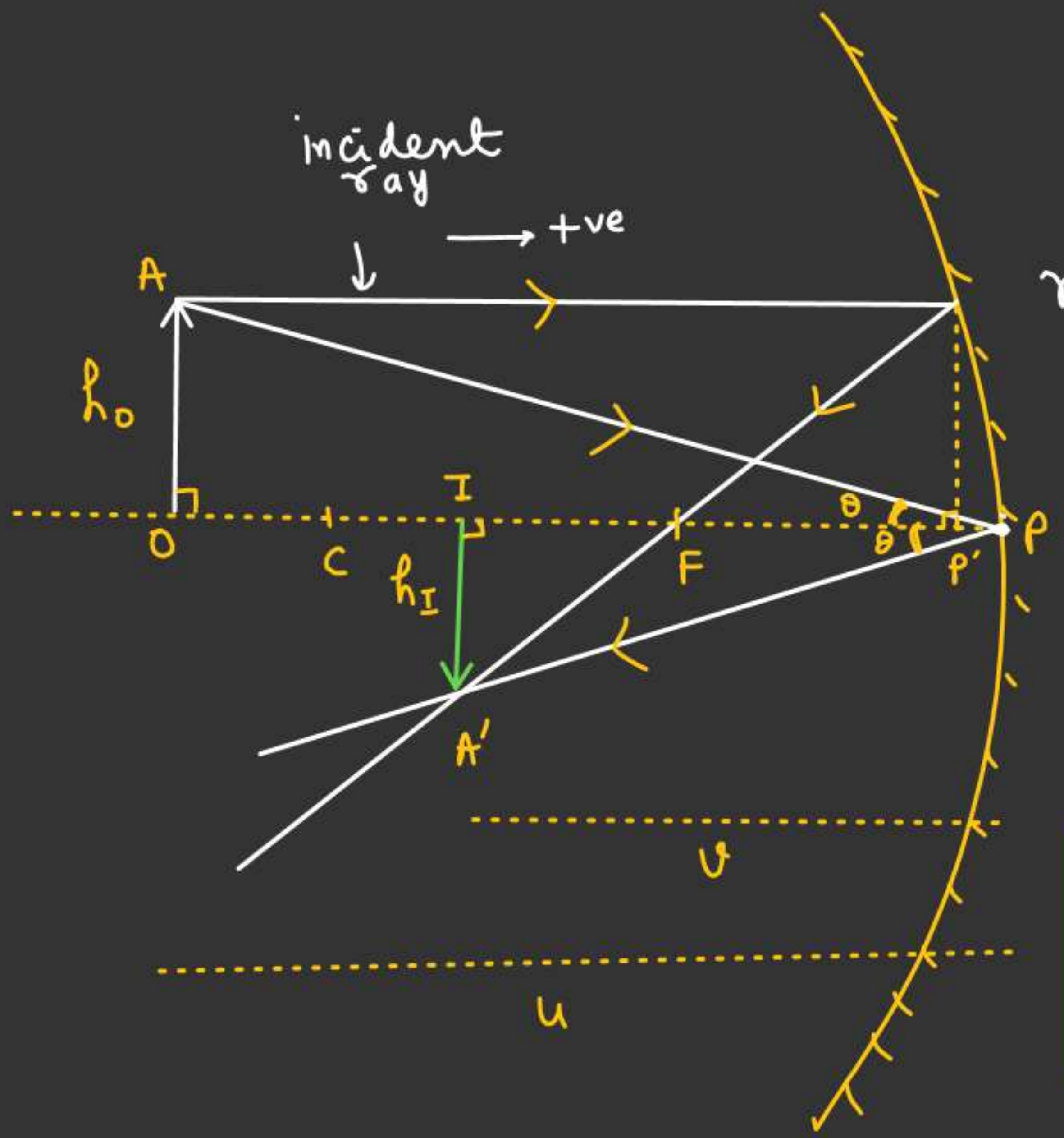
For Mirror.

$$\boxed{\frac{h_I}{h_o} = \frac{-v}{u}}$$

Longitudinal Magnification.

$$\hookrightarrow \quad m_l = \left(\frac{\text{Length of image}}{\text{Length of object}}\right)$$

REFLECTION (CURVE SURFACE)



In ΔAOP and $A'P'I$

$$m = \frac{h_I}{h_o} = \frac{v}{u}$$

Apply Sign Convention

$$m = \frac{-h_I}{+h_o} = \left(\frac{-v}{-u}\right)$$

$$\boxed{m = \frac{h_I}{h_o} = \frac{-v}{u}}$$

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{v} = \frac{1}{f} - \frac{1}{u}$$

$$v = \left(\frac{uf}{u-f}\right)$$

$$m = \left(\frac{-f}{u-f}\right)$$

$$\boxed{m = \frac{f}{f-u}}$$

$$\frac{1}{u} = \frac{1}{f} - \frac{1}{v}$$

$$u = \frac{vf}{v-f}$$

$$m = \frac{-v(v-f)}{vf}$$

$$\boxed{m = \frac{f-v}{f}}$$

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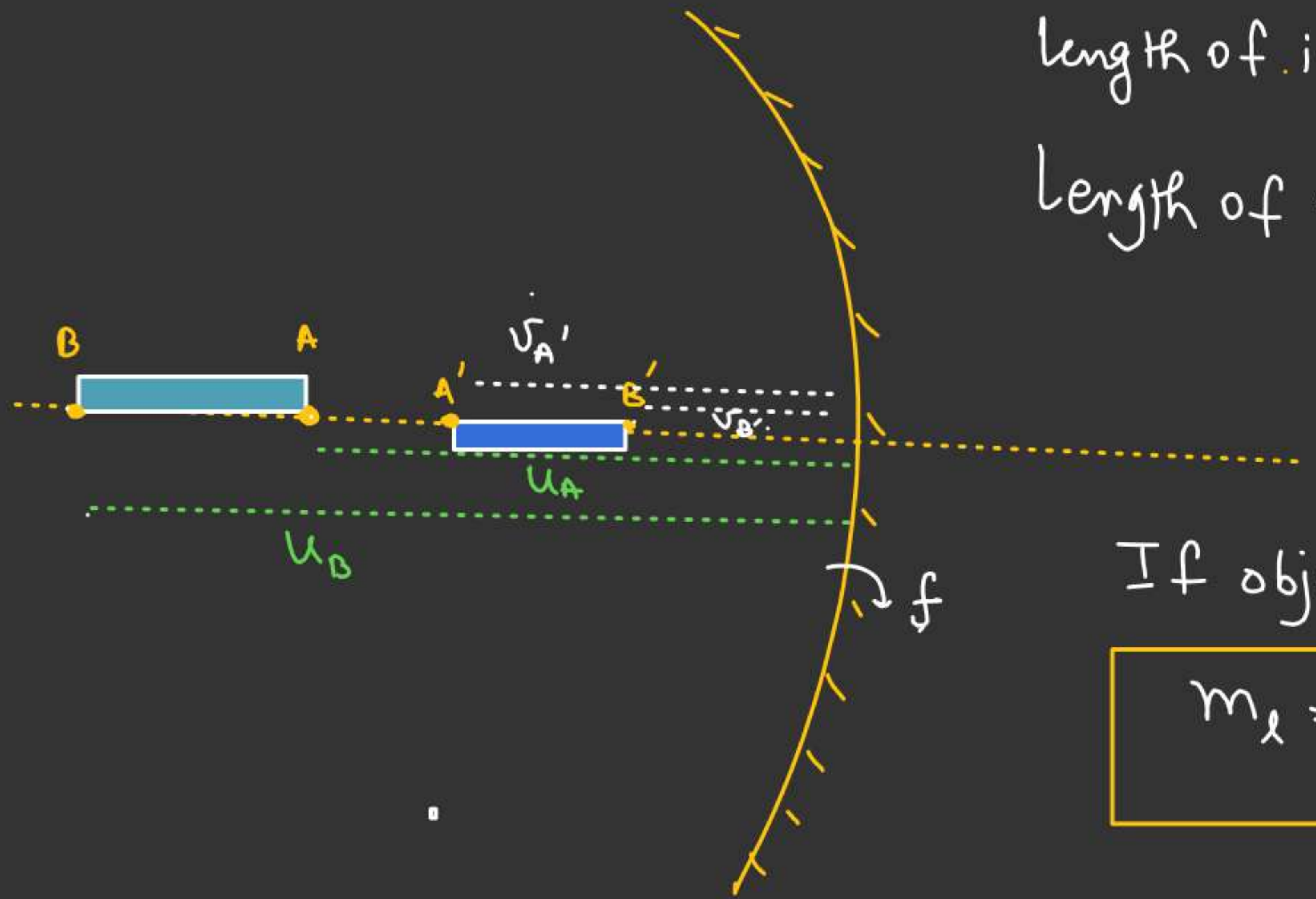
REFLECTION (CURVE SURFACE)

Longitudinal Magnification



length of image $= |v_{A'} - v_{B'}|$

Length of object $= |u_B - u_A|$

$$m_l = \frac{|v_{A'} - v_{B'}|}{|u_B - u_A|}$$

If object length is small.

$$\boxed{m_l = \frac{dv}{du}}$$

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REFLECTION (CURVE SURFACE)

Relation b/w Longitudinal & Transverse (For Mirror)

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}. \qquad m_\ell = \frac{dv}{du}$$

Differentiating both side w.r.t u.

$$-\frac{1}{v^2}\left(\frac{dv}{du}\right) - \frac{1}{u^2}\frac{d}{du}(u) = 0 \qquad \frac{-v}{u} = m$$

$$\frac{dv}{du} = -\frac{v^2}{u^2} \qquad \frac{v}{u} = (-m)$$

$$m_\ell = -(-m)^2$$

$$\boxed{m_\ell = -m^2}$$