Nishant Jindal

Fundamentals of Mathematics

Q. Find value of x in Eqn

$$2\cdot x^{\log_4 3} + 3^{\log_4 x} = 27$$

Q Find value of

$$\frac{2}{\log_4 (2000)^6} + \frac{3}{\log_5 (2000)^6} = ?$$

$$\frac{1}{6} \times 1 = \frac{1}{6}$$

$$2\log_{(2000)^6} 4 + 3\log_{(2000)^6} 5$$

$$\log_{(2000)^6} 16 + \log_{(2000)^6} 125$$

$$\log_{(2000)^6} 16\times 125 = \frac{1}{6}\log_{2000} 2000$$

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Fundamentals of Mathematics

$$a^{m \cdot n} = (a^m)^n$$

Q If x_1 & x_2 are sol. of Eq^n

$$\log_5\left(\log_{64}|x| + 25^x - \frac{1}{2}\right) = 2x \text{ then } x_1 + x_2 = ?$$

$$8 + (-8) = 0$$

$$\log_{64}|x| + 25^x - \frac{1}{2} = 5^{2\cdot x}$$

$$\log_{64}|x| + \cancel{25^x} - \frac{1}{2} = (5^2)^x = \cancel{25^x}$$

$$\log_{64}|x| = \frac{1}{2}$$

$$|x| = (64)^{\frac{1}{2}} = 8$$

$$x = \pm 8 \Rightarrow x = 8 \text{ or } -8$$

x_1 x_2

$$|x| = 2$$

$$x = \pm 2$$

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Q $\log_5\left(\frac{a+b}{3}\right) = \frac{\log_5 a + \log_5 b}{2}$ then $\frac{a^4+b^4}{a^2 \cdot b^2} = ?$

$2\log_5\left(\frac{a+b}{3}\right) = \log_5(a \cdot b)$

$\log_5\left(\frac{a+b}{3}\right)^2 = \log_5(ab)$

$\frac{(a+b)^2}{9} = ab$

$(a+b)^2 = 9ab$

$a^2+b^2+2ab = 9ab$

$a^2+b^2 = 7ab$

Sq^n $(a^2+b^2)^2 = (7ab)^2$

$a^4+b^4+2a^2b^2 = 49a^2b^2$

$a^4+b^4 = 47(a^2b^2)$

$\frac{a^4+b^4}{a^2b^2} = 47$

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Q If $\log_7 \log_7 \sqrt{7\sqrt{7\sqrt{7}}} = 1 - a\log_7 2$

& $\log_{15} \log_{15} \sqrt{15\sqrt{15\sqrt{15\sqrt{15}}}} = 1 - b\log_{15} 2$

then $a+b =$ $a+b = 3+4 = 7$

$\log_7\left\{\log_7 7^{\frac{7}{8}}\right\} = 1 - a\log_7 2$

$\boxed{a=3}$

$\log_7\left\{\frac{7}{8}\right\} = 1 - a\log_7 2$

$\log_7 7 - \log_7 8 = 1 - a\log_7 2$

$1 - 3\log_7 2 = 1 - a\log_7 2$

$\log_{15} \log_{15} \sqrt{15\sqrt{15\sqrt{15\sqrt{15}}}} = 1 - b\log_{15} 2$

$\log_{15}\left(\log_{15} 15^{\frac{15}{16}}\right) = 1 - b\log_{15} 2$

$\log_{15} \frac{15}{16} = 1 - b\log_{15} 2$

$\log_{15} 15 - \log_{15} 16 = 1 - b\log_{15} 2$

$1 - \log_{15} 16 = 1 - b\log_{15} 2$

$1 - 4\log_{15} 2 = 1 - b\log_{15} 2$

$\boxed{b=4}$

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Q Solve

$$\log_a x \cdot \log_a xyz = 48$$

$$\log_a y \; \log_a xyz = 12$$

$$\log_a z \; \log_a xyz = 84$$

find x, y, z?

$\log_a x = A$, $\log_a y = B$, $\log_a z = C$

$$\log_A x\,(\log_A x + \log_A y + \log_A z) = 48 \rightarrow A\cdot(A+B+C) = 48 \rightarrow (1)$$

$$\log_A y\,(\log_A x + \log_A y + \log_A z) = 12 \rightarrow B\cdot(A+B+C) = 12 \rightarrow 2$$

$$\log_A z\,(\log_A x + \log_A y + \log_A z) = 84 \rightarrow C\cdot(A+B+C) = 84$$

Add $(A+B+C)(A+B+C) = 144$

$(A+B+C)^2 = 144 \Rightarrow A+B+C = 12$

$A \cdot 12 = 48 \Rightarrow A = 4 \quad \log_a x = 4 \Rightarrow x = a^4$

$B \cdot 12 = 12 \Rightarrow B = 1 \Rightarrow \log_a y = 1 \Rightarrow y = a^1$

$C \cdot 12 = 84 \Rightarrow C = 7 \Rightarrow \log_a z = 7 \Rightarrow z = a^7$

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Fundamentals of Mathematics

Q $\log_2\left(\frac{1}{\cancel{7^{\log_7}}\cdot 125}\right)$

$\log_2\left(\frac{\overset{8}{\cancel{1000}}}{\cancel{125}}\right)$

$\log_2 2^3 = 3\times 1 = 3$

$\underline{\underline{11^{th}}}$

Q $\log_{\frac{1}{6}} 2 \cdot \log_5 36 \cdot \log_{17} 125 \cdot \log_{\frac{1}{\sqrt{2}}} 17$

$\frac{\log 2}{\log\frac{1}{6}} \times \frac{\log 6^2}{\log 5} \times \frac{\log 5^3}{\cancel{\log 17}} \times \frac{\cancel{\log 17}}{\log\left(\frac{1}{2}\right)^{\frac{1}{2}}}$

$\frac{\cancel{\log 2}}{-\cancel{\log 6}} \times \frac{2\cancel{\log 6}}{\cancel{\log 5}} \times \frac{3\cancel{\log 5}}{-\frac{1}{2}\cancel{\log 2}}$

$\frac{2\times 3}{+\frac{1}{2}} = 12$

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DPP.

Q3 $\log_3(\log_9 x) = \log_9(\log_3 x)$

$\log_{a^\beta} M = \frac{1}{\beta}\log_a M$

$\log_3(\log_{3^2} x) = \log_{3^2}(\log_3 x)$

$\log_3 x = t$

$\log_3\left(\frac{\log_3 x}{2}\right) = \frac{1}{2}\log_3\left(\boxed{\log_3 x}\right)$

$\log_3\left(\frac{t}{2}\right) = \frac{\log_3 t}{2}$

$\log_3 t - \log_3 2 = \frac{\log_3 t}{2}$

$(\log_3 t) - \left(\frac{\log_3 t}{2}\right) = \log_3 2$

$\frac{\log_3 t}{2} = \log_3 2$

$\log_3 t = 2\log_3 2$

$\log_3 t = \log_3 4$

$t = 4$

$\log_3 x = 4$

$x = 3^4 = \boxed{8}\boxed{1}$

$\text{Prod} = 8 \times 1 = 8$

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Q4 $\log_2(\log_3(\log_4 x))=0$, $\log_4(\log_3(\log_2 y))=0$, $\log_3(\log_4(\log_2 z))=0$

$\log_3(\log_4 x) = 2^0 = 1$

$\log_4 x = 3^1 = 3$

$x = 4^3 = 64$

$\log_3(\log_2 y) = 4^0 = 1$

$\log_2 y = 3^1 = 3$

$y = 2^3 = 8$

$\log_4(\log_2 z) = 3^0 = 1$

$\log_2 z = 4^1 = 4$

$z = 2^4 = 16$

$64 > 16 > 8$

$x > z > y$

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$5^{-1} = \frac{1}{5}$

Q5 $\log_3(\log_2 a) + \log_{\frac{1}{3}}(\log_{\frac{1}{2}} b) = 1$ then $ab^3 = ?$

$+ \log_{3^{-1}}(\log_{2^{-1}} b) = 1$

$\log_3 x - \log_3 y$

$- \log_3(-\log_2 b) = 1$

$\log \frac{x}{y}$

$\log_3(\cancel{\log_2 a}) - \log_3(\cancel{\log_2 b^{-1}})$

$\log_3\left(\frac{\log_2 a}{\log_2 b^{-1}}\right) = 1$

$\log_{\frac{1}{b}} a = 3 \Rightarrow a = \left(\frac{1}{b}\right)^3 = \frac{1}{b^3}$

$ab^3 = 1$

$\log A - \log B = \log \frac{A}{B}$

$\log M - \log N = \log \frac{M}{N}$

$\log P - \log R = \log \frac{P}{R}$

$\log T - \log C = \log \frac{T}{C}$

$\log x - \log y = \log$

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Q6 $\log(x+y) = \log 2 + \frac{1}{2}\log x + \frac{1}{2}\log y$

$= \log 2 + \log x^{1/2} + \log y^{1/2}$

$\log(x+y) = \log 2\cdot\sqrt{x}\cdot\sqrt{y}$

$x+y = 2\sqrt{x}\sqrt{y}$

$x+y-2\sqrt{x}\sqrt{y} = 0$

$(\sqrt{x})^2 - 2\sqrt{x}\sqrt{y} + (\sqrt{y})^2 = 0$

$(\sqrt{x}-\sqrt{y})^2 = 0 \Rightarrow \sqrt{x}-\sqrt{y} = 0$

$\sqrt{x} = \sqrt{y}$

$\log t = \log 4$

$t = 4$

$x = y$

$x - y = 0$

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Q7 $\dfrac{1}{\log_{\sqrt{ab}} abc} + \dfrac{1}{\log_{\sqrt{bc}} abc} + \dfrac{1}{\log_{\sqrt{ca}} abc}$

$\log_{abc}\sqrt{ab} + \log_{abc}\sqrt{bc} + \log_{abc}\sqrt{ca}$

$\log_{abc}\left\{\sqrt{ab}\cdot\sqrt{bc}\cdot\sqrt{ca}\right\}$

$\log_{abc}\left\{\sqrt{(abc)^2}\right\} = \log_{abc} abc = 1$