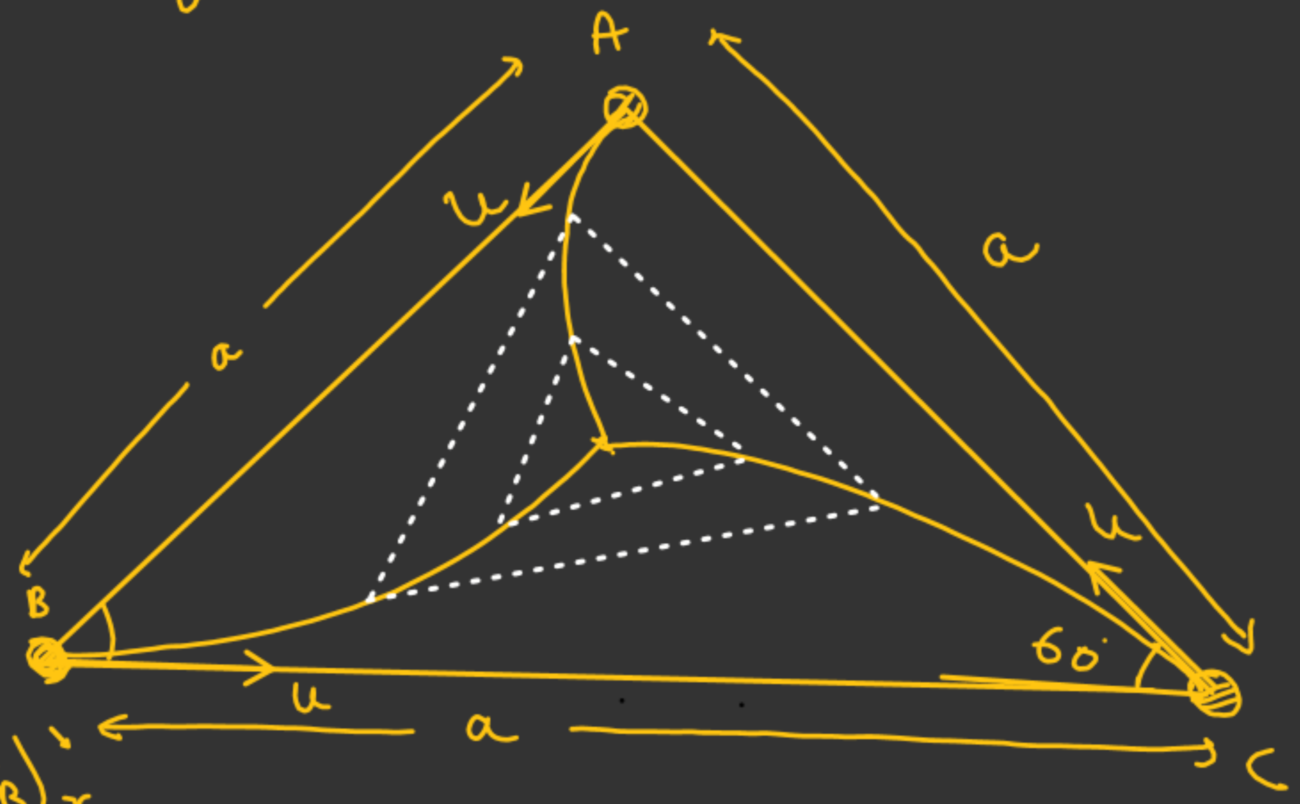
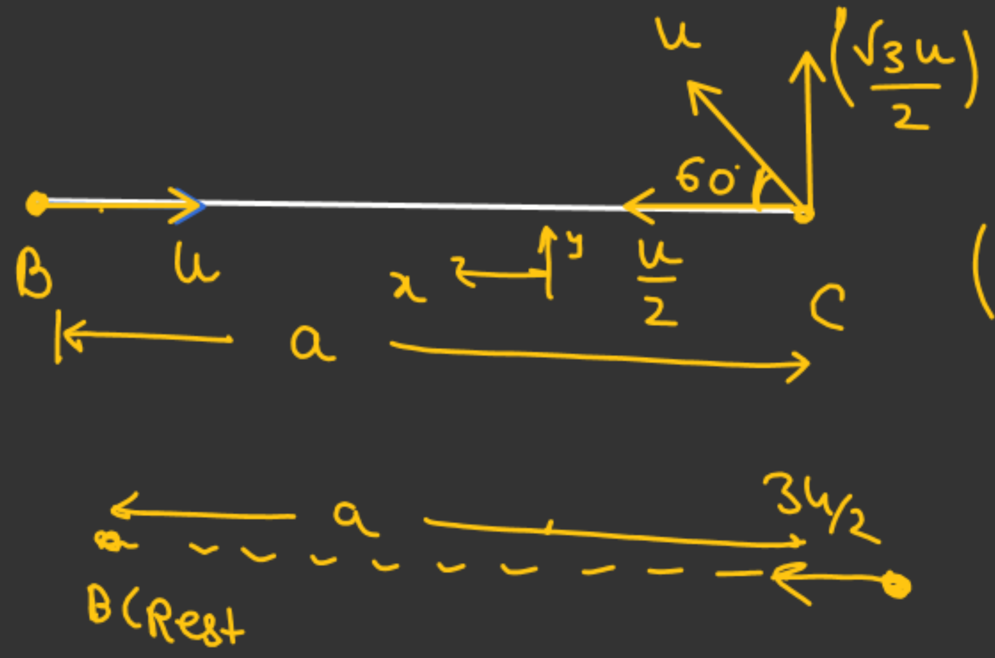
Nishant Jindal

Three particles are moving along the Side of an equilateral triangle in Such a way that particles are directed towards each other.

If velocity of each particle is u m/s. find time when all the three particle meet.

Sol^n :-





$$\left(\vec{v}_{C/B}\right)_x = \left(\vec{v}_C\right)_x - \left(\vec{v}_B\right)_x$$

$$= -\frac{u}{2}\hat{i} - u\hat{i}$$

$$= \left(-\frac{3u}{2}\hat{i}\right)$$

$$T = \frac{a}{\left(\frac{3u}{2}\right)} = \left(\frac{2a}{3u}\right)$$

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(4) Cat & dog problem :→

V = velocity of dog.

u = velocity of Cat.

dog always directed towards Cat.

$$-\left(\frac{dl}{dt}\right) = (v - u\cos\theta)$$

Relative distance decreasing w.r.t time

$$-\int_l^0 dl = \int_0^T (v-u\cos\theta)\,dt$$

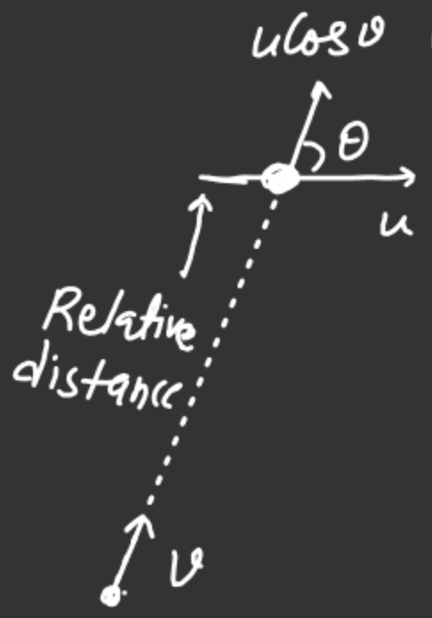
$$l = v\int_0^T dt - u\int_0^T \cos\theta\,dt$$

$$l = VT - u\int_0^T \cos\theta\,dt \quad \text{—} \ (1)$$

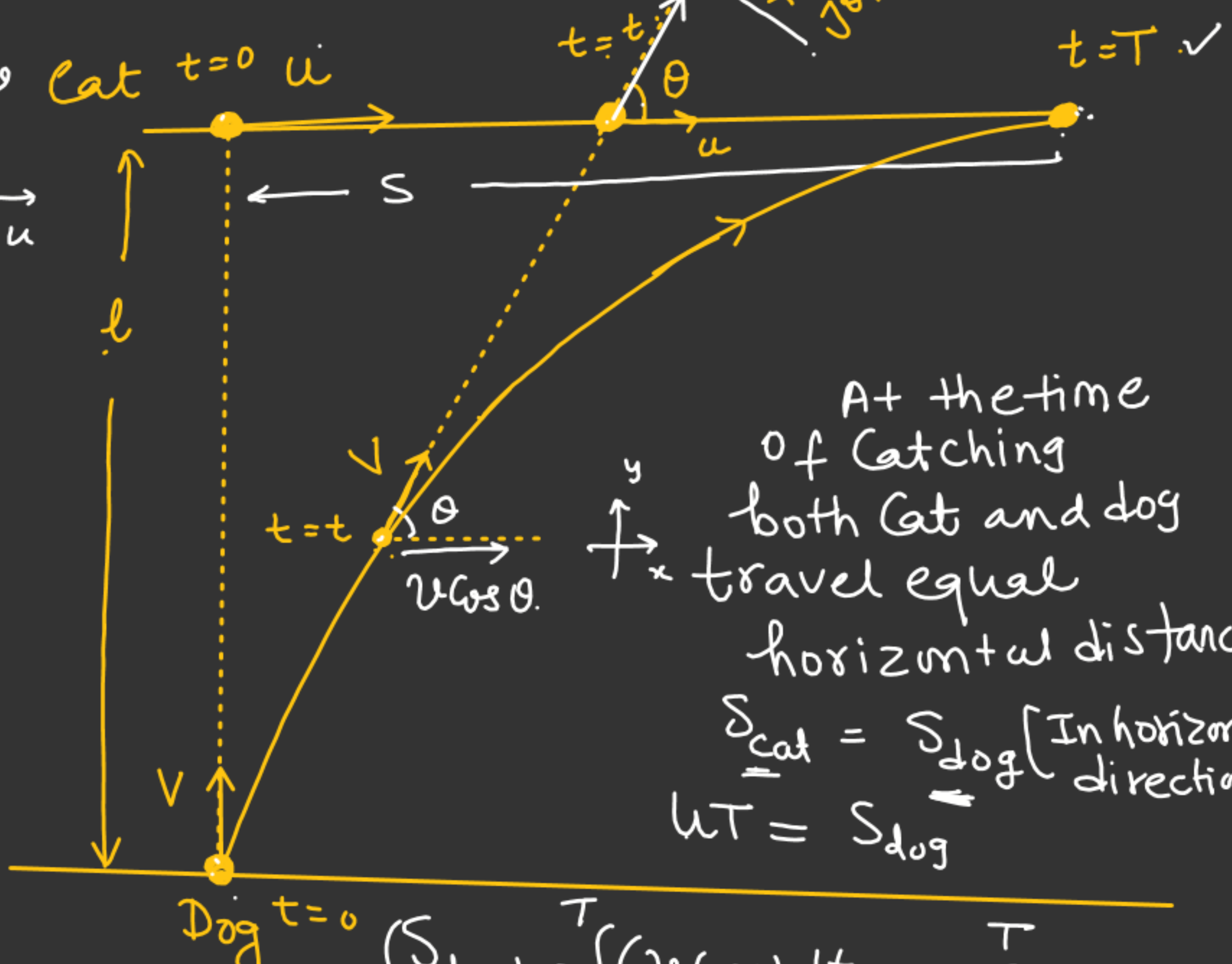
$$l = VT - u\left(\frac{uT}{V}\right)$$

$$l = \left(\frac{V^2-u^2}{V}\right)T$$

$$\boxed{T = \frac{Vl}{V^2-u^2}}$$



$$ds_x = \int v_x\,dt$$



At the time of catching both Cat and dog travel equal horizontal distance

$$S_{cat} = S_{dog} \ [\text{In horizontal direction}]$$

$$uT = S_{dog}$$

$$(S_{dog}) = \int_0^T (v\cos\theta)\,dt = v\int_0^T \cos\theta\,dt$$

$$v\int_0^T \cos\theta\,dt = uT$$

$$\int_0^T \cos\theta\,dt = \left(\frac{uT}{v}\right) \quad \text{—} \ (2)$$

Nishant Jindal

Relative velocity

Relative velocity of aeroplane w.r.t air

w.r.t air

Q.1 An aircraft flies at 400 km/hr in still air. A wind of $200\sqrt{2}$ km/hr is blowing from the south. The pilot wishes to travel from A to a point B north east of A. Find the direction he must steer and time of his journey if AB = $\boxed{1000 \text{ km.}}$

Sol^n.

$$\frac{\sin\alpha}{200\sqrt{2}} = \frac{\sin 45^\circ}{400} = \frac{\sin[180-(45+\alpha)]}{V_{A/E}}$$

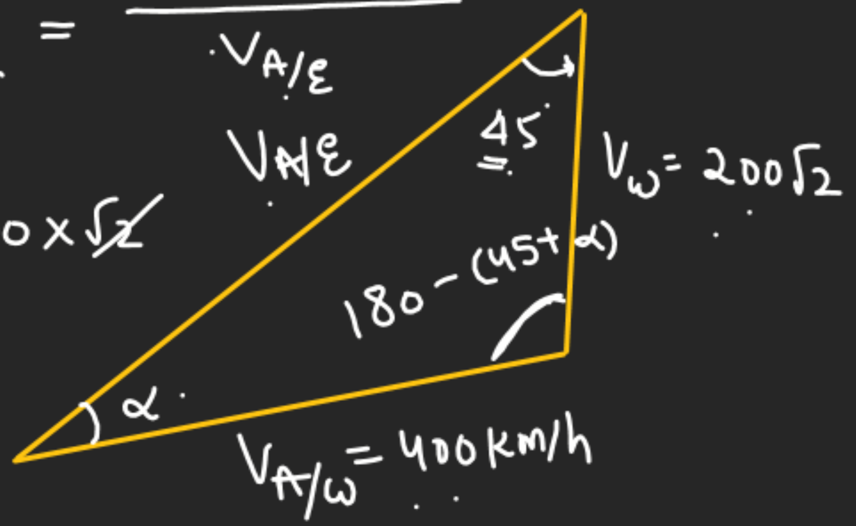
$$\sin\alpha = \frac{1}{\cancel{\sqrt{2}}} \times \frac{1}{400} \times 200 \times \cancel{\sqrt{2}}$$

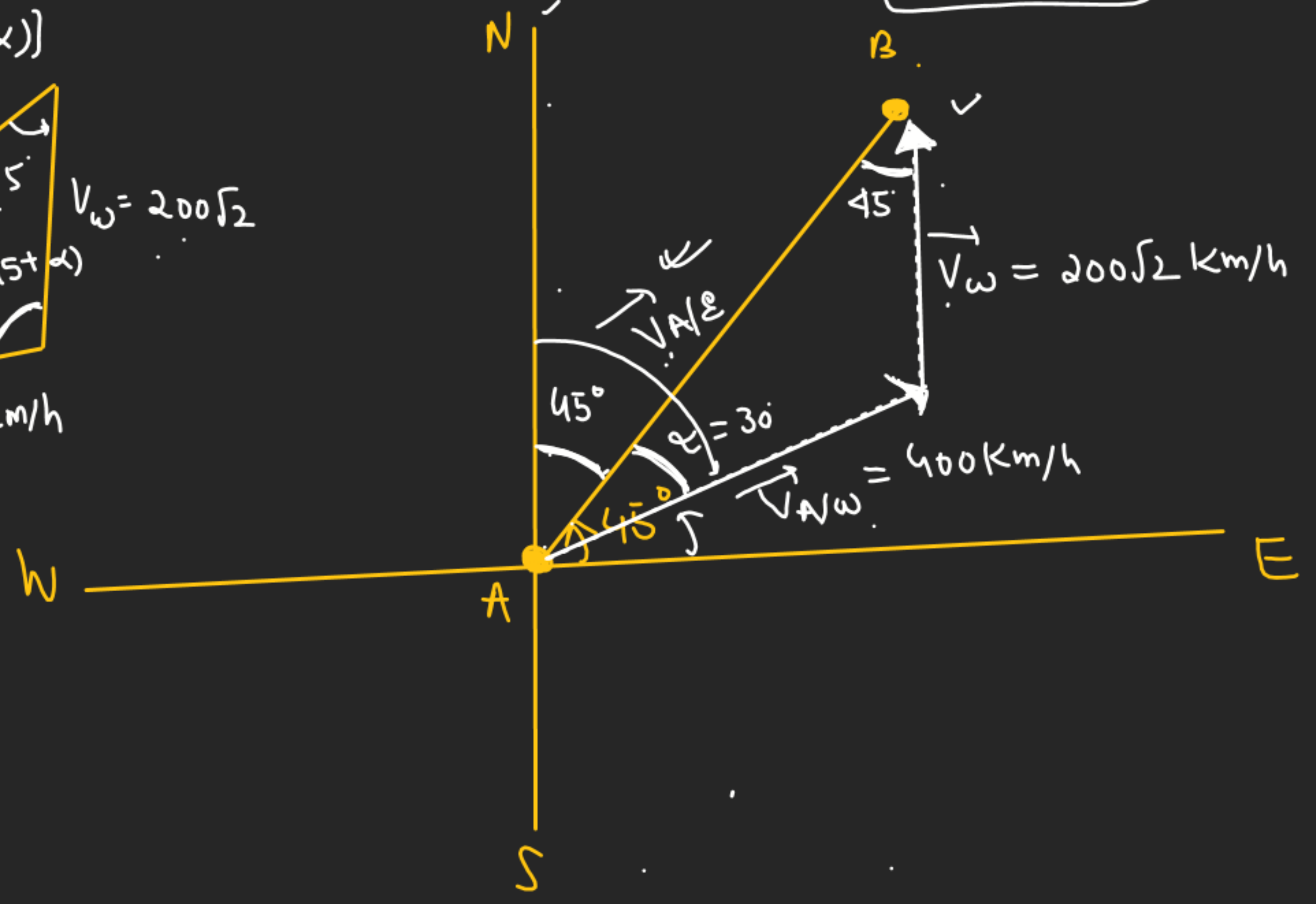
$$\sin\alpha = \frac{1}{2}$$

$$\boxed{\alpha = 30^\circ}$$

From vertical → 75°

From horizonal → 90−75° = 15°





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Relative velocity

$$\frac{\sin 45^\circ}{400} = \frac{\sin[180 - (45+\alpha)]}{v_{A/E}}$$

$$V_{A/E} = \sin(75^\circ) \times \frac{400}{(\sin 45^\circ)}$$

$$V_{A/E} = (400\sqrt{2})[\sin(75^\circ)]$$

$$T = \frac{1000}{|\vec{V}_{A/E}|} =$$

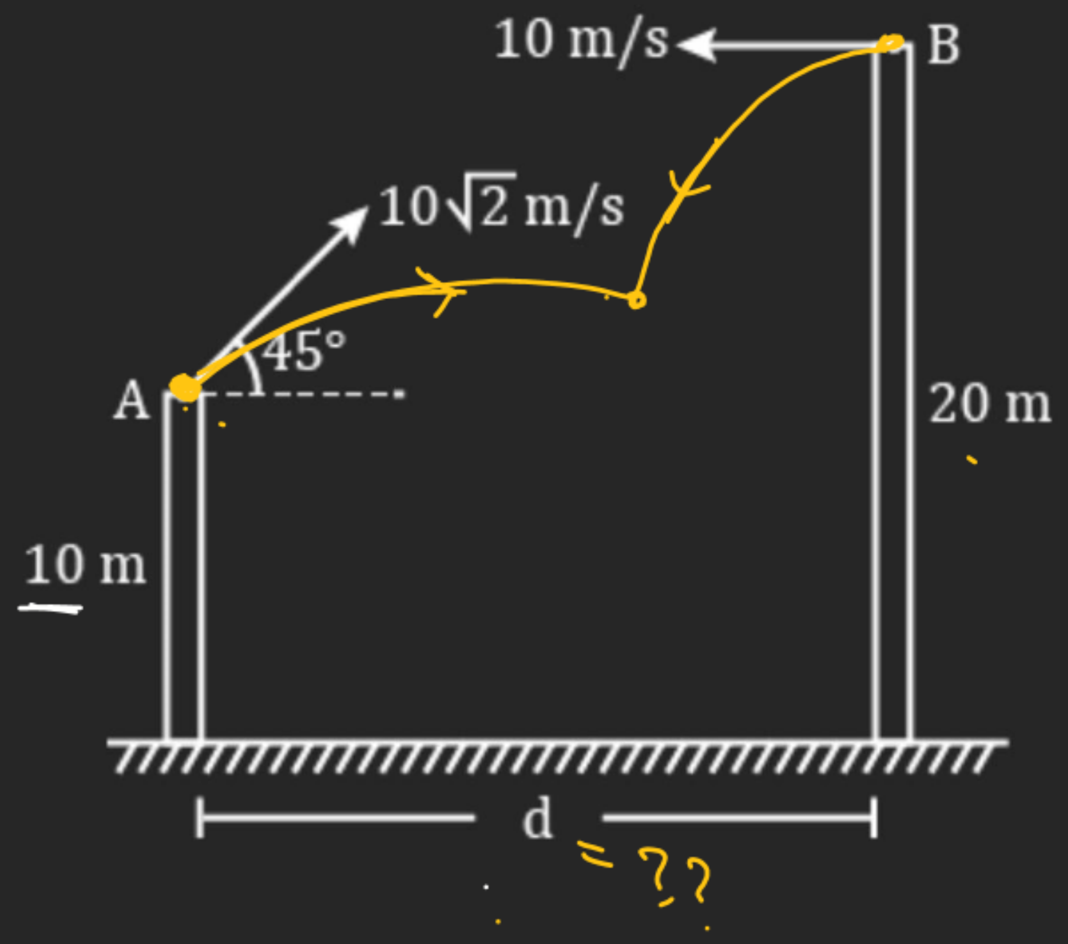
$$\sin(A+B) = [\sin A \cdot \cos B + \cos A \cdot \sin B]$$

$$\sin(75^\circ) = \sin(45^\circ + 30^\circ) = \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \times \frac{1}{2}$$

$$= \left(\frac{\sqrt{3}+1}{2\sqrt{2}}\right) \qquad = \frac{(\sqrt{3}+1)}{2\sqrt{2}}$$

Relative velocity

Q.2 Two particles A and B are projected from the two towers of height 10 m and 20 m respectively. Particle A is projected with an initial speed of $10\sqrt{2}\text{ m/s}$ at an angle of 45° with horizontal, while particle B is projected horizontally with speed 10 m/s. If they collide in air, what is the distance ' d ' between the towers?



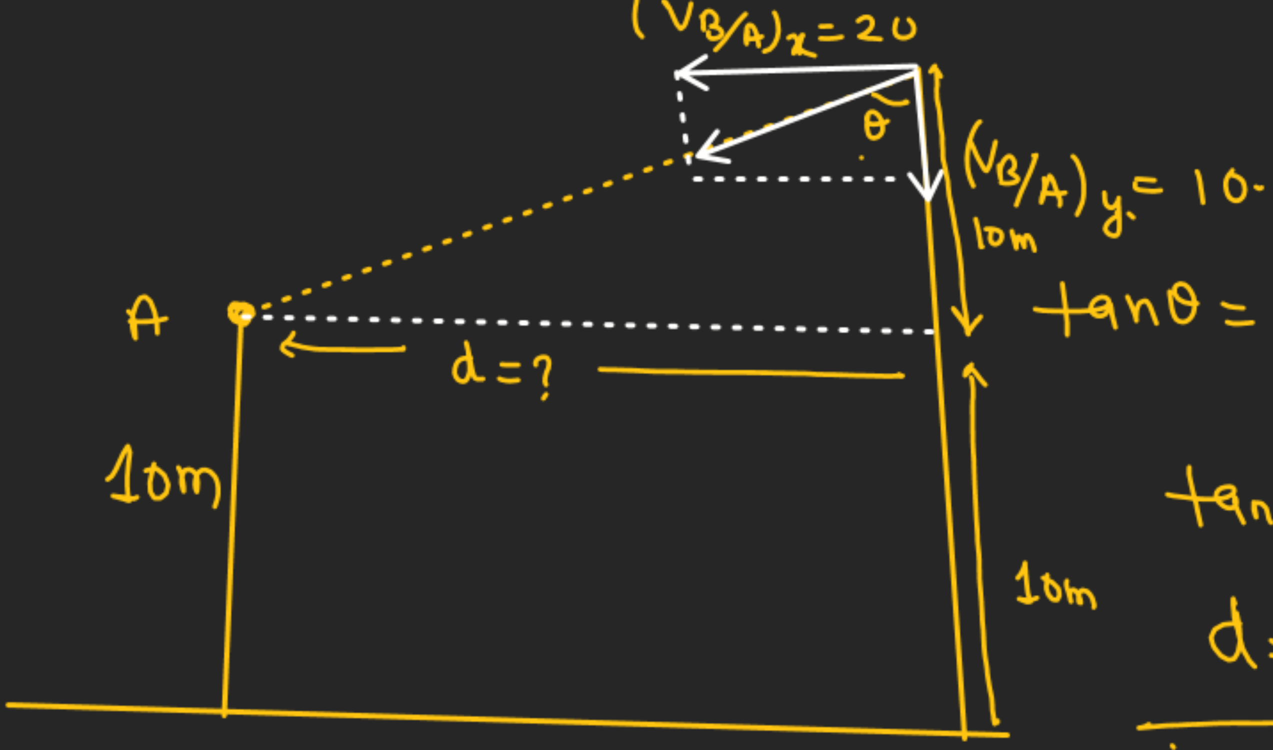
Nishant Jindal

Relative velocity

$$\vec{V}_{A/E} = 10\sqrt{2}\cos 45^\circ \hat{i} + 10\sqrt{2}\sin 45^\circ \hat{j}$$
$$= (10\hat{i} + 10\hat{j})$$

$$\vec{V}_{B/E} = -10\hat{i}$$

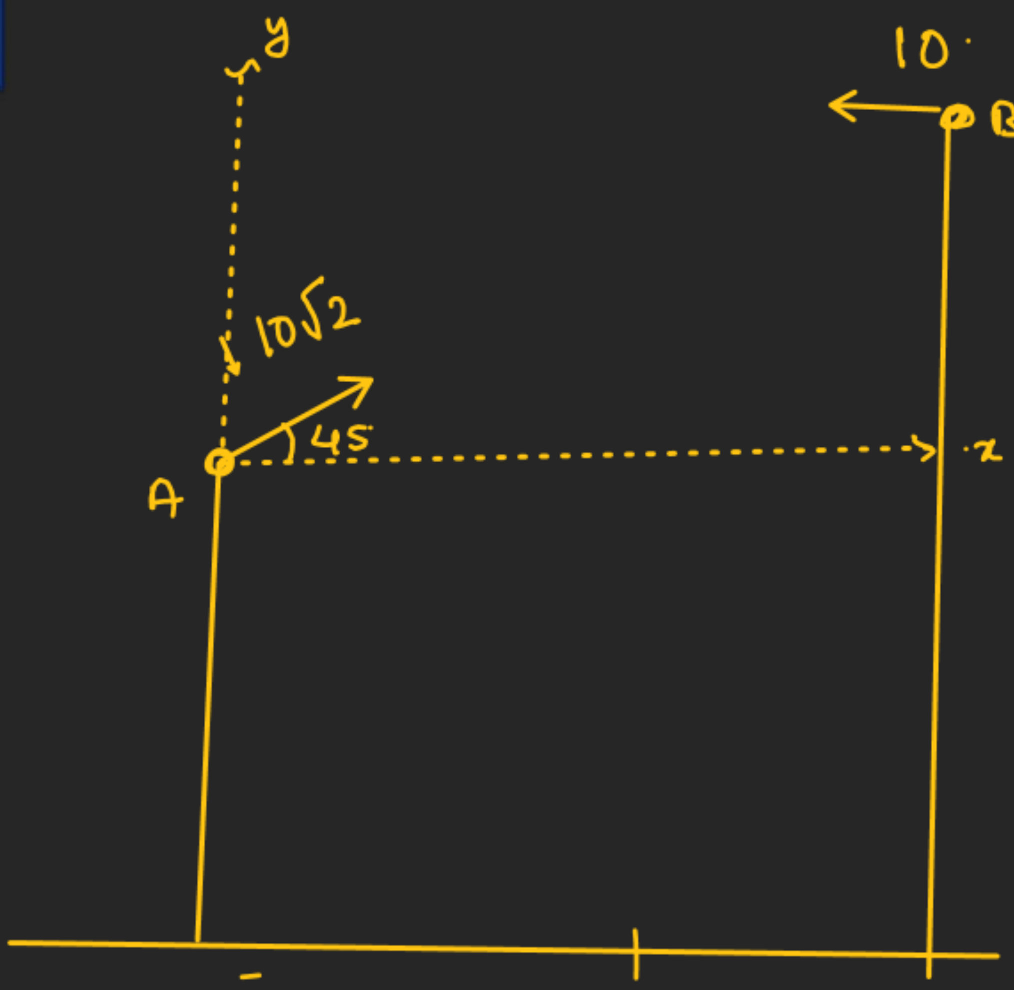
$$\vec{V}_{B/A} = \vec{V}_{B/E} - \vec{V}_{A/E}$$
$$= -10\hat{i} - (10\hat{i} + 10\hat{j})$$
$$= -20\hat{i} - 10\hat{j}$$



$$\tan\theta = \frac{(V_{B/A})_x}{(V_{B/A})_y} = \frac{20}{10} = 2$$

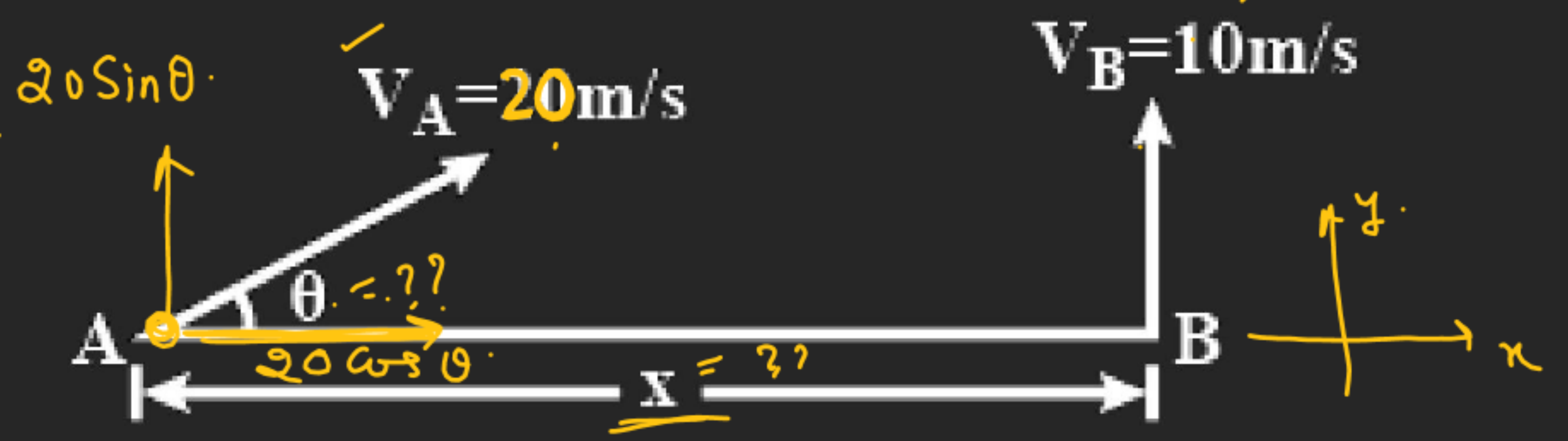
$$\tan\theta = \frac{d}{10} = 2$$

$$d = 20\,m$$



Relative velocity

Q.3 Two particles A and B are projected simultaneously in the directions shown in figure with velocities $v_A = 20\ m/s$ and $v_B = 10\ m/s$ respectively. They collide in air after $\frac{1}{2}$ s. Find (a) the angle θ (b) the distance x.



Sol^n :-

$\vec{a}_{A/E} = -g\hat{j}$

$\vec{a}_{B/E} = -g\hat{j}$

$\boxed{\vec{a}_{A/B} = 0}$

$\vec{V}_{A/E} = 20\cos\theta\,\hat{i} + 20\sin\theta\,\hat{j}$

$\vec{V}_{B/E} = 10\hat{j}$

$\vec{V}_{B/A} = \vec{V}_{B/E} - \vec{V}_{A/E}$

$= 10\hat{j} - 20\cos\theta\,\hat{i} - 20\sin\theta\,\hat{j}$

$= -20\cos\theta\,\hat{i} + (10 - 20\sin\theta)\hat{j}$

$(\vec{v}_{B/A}) = -\left(20 \times \frac{\sqrt{3}}{2}\right)\hat{i}$

$= -10\sqrt{3}\,\hat{i}$

$x = 10\sqrt{3} \times \frac{1}{2}$

$\boxed{x = 5\sqrt{3}}$

$(v_{B/A})_y = 0$

For Collision

For Collision

$10 - 20\sin\theta = 0$

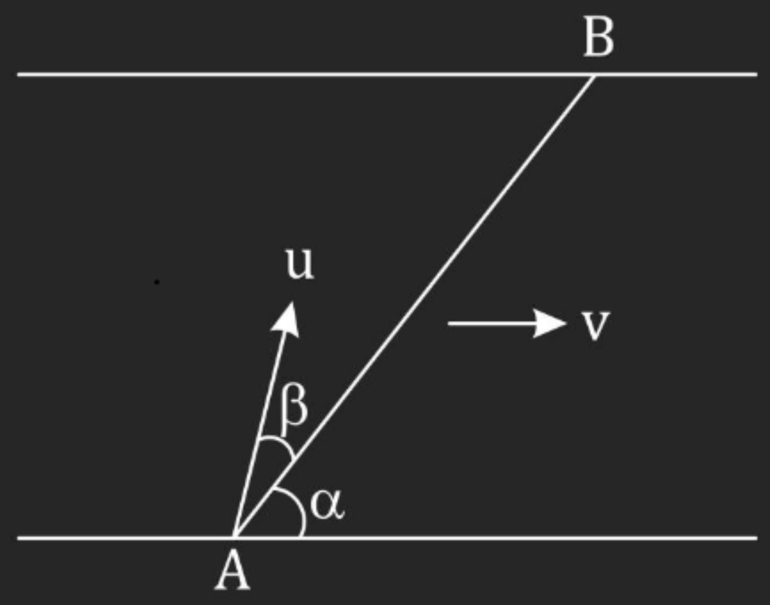
$\sin\theta = \frac{1}{2} \Rightarrow \theta = 30$

Nishant Jindal

Relative velocity

H.W.

Q.4 A launch plies between two points A and B on the opposite banks of a river always following the line AB. The distance S between points A and B is $1,200\ \mathrm{m}$. The velocity of the river current $v = 1.9\ \mathrm{m/s}$ is constant over the entire width of the river. The line AB makes an angle $\alpha = 60^\circ$ with the direction of the current. With what velocity u and at what angle β to the line AB should the launch move to cover the distance AB and back in a time $t = 5\ \mathrm{min}$? The angle β remains the same during the passage from A to B and from B to A.



Nishant Jindal

Relative velocity

H.W

Q.7 The slopes of wind screen of two cars are $\alpha_1 = 30^\circ$ and $\alpha_2 = 15^\circ$ respectively. At what ratio $\frac{v_1}{v_2}$ of the velocities of the cars will their drivers see the hail stones bounced back by the wind screen on their cars in vertical direction? Assume hail stones fall vertically downwards and collisions to be elastic.

Nishant Jindal

Relative velocity

H.W

Q.8 A river of width ' a ' with straight parallel banks flows due north with speed u. The points O and A are on opposite banks and A is due east of O. Coordinate axes Ox and Oy are taken in the east and north directions respectively. A boat, whose speed is v relative to water, starts from O and crosses the river. If the boat is steered due east and u varies with x as: $u = x(a-x)\frac{v}{a^2}$. Find.

(A) equation of trajectory of the boat

(B) time taken to cross the river

(C) absolute velocity of boatman when he reaches the opposite bank

(D) the displacement of boatman when he reaches the opposite bank from the initial position.