1. STANDARD EQUATION & DEFINITIONS :

Standard equation of an ellipse referred to its principal axes along the co-ordinate axes is $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.

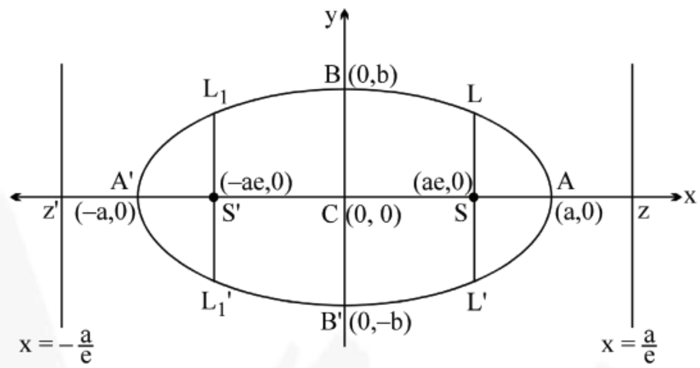
Where $a > b$ & $b^2 = a^2(1 - e^2)$ $\Rightarrow$ $a^2 - b^2 = a^2 e^2$.

Where e = eccentricity ($0 < e < 1$).

FOCI : $S \equiv (a e, 0)$ & $S' \equiv (-a e, 0)$.

EQUATIONS OF DIRECTRICES :

$x = \frac{a}{e}$ & $x = -\frac{a}{e}$.



VERTICES :

$A' \equiv (- a, 0)$ & $A \equiv (a, 0)$.

MAJOR AXIS :

The line segment A′ A in which the foci S′ & S lie is of length 2a & is called the **major axis** ($a > b$) of the ellipse. Point of intersection of major axis with directrix is called **the foot of the directrix (z).**

MINOR AXIS :

The y–axis intersects the ellipse in the points $B' \equiv (0, - b)$ & $B \equiv (0, b)$. The line segment B′B of length 2b ($b < a$) is called the **Minor Axis** of the ellipse.

PRINCIPAL AXIS :

The major & minor axis together are called **Principal Axis** of the ellipse.

CENTRE :

The point which bisects every chord of the conic drawn through it is called the **centre** of the conic. $C \equiv (0, 0)$ the origin is the centre of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.

DIAMETER :

A chord of the conic which passes through the centre is called a **diameter** of the conic.

FOCAL CHORD : A chord which passes through a focus is called a **focal chord**.

DOUBLE ORDINATE :

A chord perpendicular to the major axis is called a **double ordinate.**

LATUS RECTUM :

The focal chord perpendicular to the major axis is called the **latus rectum**. Length of latus rectum

(LL′) = $\frac{2b^2}{a}=\frac{(minor\ \ axis)^2}{major\ \ axis}=2a(1-e^2)$ = 2e (distance from focus to the corresponding directrix)

NOTE :

(i) The sum of the focal distances of any point on the ellipse is equal to the major Axis. Hence distance of focus from the extremity of a minor axis is equal to semi major axis. **i.e. BS = CA.**

(ii) If the equation of the ellipse is given as $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ & nothing is mentioned, then the rule is to assume that a > b.

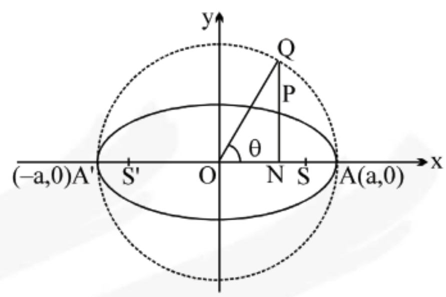
2. POSITION OF A POINT w.r.t. AN ELLIPSE :

The point $P(x_1, y_1)$ lies outside, inside or on the ellipse according as ; $\frac{x_1^2}{a^2}+\frac{y_1^2}{b^2}-1$ > < or = 0.

3. AUXILIARY CIRCLE / ECCENTRIC ANGLE :

A circle described on major axis as diameter is called the **auxiliary circle**.

Let Q be a point on the auxiliary circle $x^2 + y^2 = a^2$ such that QP produced is perpendicular to the x-axis then P & Q are called as the **Corresponding Points** on the ellipse & the auxiliary circle respectively 'θ' is called the **Eccentric Angle** of the point P on the ellipse ($0 \le \theta < 2\pi$).



Note that $\frac{\ell(PN)}{\ell(QN)}=\frac{b}{a}=\frac{\textit{Semi minor axis}}{\textit{Semi major axis}}$

Hence " If from each point of a circle perpendiculars are drawn upon a fixed diameter then the locus of the points dividing these perpendiculars in a given ratio is an ellipse of which the given circle is the auxiliary circle".

4. PARAMETRIC REPRESENTATION :

The equations $x = a\cos\theta$ & $y = b\sin\theta$ together represent the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.

Where θ is a parameter. Note that if $P(\theta) \equiv (a\cos\theta, b\sin\theta)$ is on the ellipse then ; $Q(\theta) \equiv (a\cos\theta, a\sin\theta)$ is on the auxiliary circle.

5. LINE AND AN ELLIPSE :

The line $y = mx + c$ meets the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ in two points real, coincident or imaginary according as c^2 is < = or > $a^2m^2 + b^2$.

Hence $y = mx + c$ is tangent to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ if $c^2 = a^2m^2 + b^2$.

The equation to the chord of the ellipse joining two points with eccentric angles α & β is given by

$$\frac{x}{a}\cos\frac{\alpha+\beta}{2}+\frac{y}{b}\sin\frac{\alpha+\beta}{2}=\cos\frac{\alpha-\beta}{2}.$$

6. TANGENTS :

(i) $\frac{xx_1}{a^2}+\frac{yy_1}{b^2}=1$ is tangent to the ellipse at (x_1, y_1).

Note : The figure formed by the tangents at the extremities of latus rectum is rhoubus of area $\frac{2a^2}{e}$

(ii) $y = mx \pm \sqrt{a^2m^2+b^2}$ is tangent to the ellipse for all values of m.

Note that there are two tangents to the ellipse having the same m, i.e. there are two tangents parallel to any given direction.

(iii) $\frac{x\cos\theta}{a}+\frac{y\sin\theta}{b}=1$ is tangent to the ellipse at the point $(a\cos\theta, b\sin\theta)$.

(iv) The eccentric angles of point of contact of two parallel tangents differ by π. Conversely if the difference between the eccentric angles of two points is p then the tangents at these points are parallel.

(v) Point of intersection of the tangents at the point α & β is $a\frac{\cos\frac{\alpha+\beta}{2}}{\cos\frac{\alpha-\beta}{2}}, b\frac{\sin\frac{\alpha+\beta}{2}}{\cos\frac{\alpha-\beta}{2}}$.

7. NORMALS :

(i) Equation of the normal at (x_1, y_1) is $\frac{a^2x}{x_1}-\frac{b^2y}{y_1}=a^2-b^2=a^2e^2$.

(ii) Equation of the normal at the point $(a\cos\theta, b\sin\theta)$ is ; $ax\sec\theta - by\,\text{cosec}\,\theta = (a^2-b^2)$.

(iii) Equation of a normal in terms of its slope 'm' is $y = mx - \frac{(a^2-b^2)m}{\sqrt{a^2+b^2m^2}}$.

8. DIRECTOR CIRCLE :

Locus of the point of intersection of the tangents which meet at right angles is called the **Director Circle**. The equation to this locus is $x^2+y^2=a^2+b^2$ i.e. a circle whose centre is the centre of the ellipse & whose radius is the length of the line joining the ends of the major & minor axis.

9. Chord of contact, pair of tangents, chord with a given middle point, pole & polar are to be interpreted as they are in parabola.

10. DIAMETER :

The locus of the middle points of a system of parallel chords with slope 'm' of an ellipse is a straight line passing through the centre of the ellipse, called its diameter and has the equation $y = -\frac{b^2}{a^2m}x$.

11. IMPORTANT HIGHLIGHTS : Refering to an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.

H – 1 If P be any point on the ellipse with S & S′ as its foci then $\ell(SP)+\ell(S'P)=2a$.

H – 2 The product of the length's of the perpendicular segments from the foci on any tangent to the ellipse is b^2 and the feet of these perpendiculars Y,Y′ lie on its auxiliary circle. The tangents at these feet to the auxiliary circle meet on the ordinate of P and that the locus of their point of intersection is a similiar ellipse as that of the original one. Also the lines joining centre to the feet of the perpendicular Y and focus to the point of contact of tangent are parallel.

H – 3 If the normal at any point P on the ellipse with centre C meet the major & minor axes in G & g respectively, & if CF be perpendicular upon this normal, then

(i) PF. PG = b^2 **(ii)** PF. Pg = a^2 **(iii)** PG. Pg = SP. S′ P **(iv)** CG. CT = CS^2

(v) locus of the mid point of Gg is another ellipse having the same eccentricity as that of the original ellipse.

[where S and S′ are the focii of the ellipse and T is the point where tangent at P meet the major axis]

H – 4 The tangent & normal at a point P on the ellipse bisect the external & internal angles between the focal distances of P. This refers to the well known reflection property of the ellipse which states that rays from one focus are reflected through other focus & vice–versa. Hence we can deduce that the straight lines joining each focus to the foot of the perpendicular from the other focus upon the tangent at any point P meet on the normal PG and bisects it where G is the point where normal at P meets the major axis.

H – 5 The portion of the tangent to an ellipse between the point of contact & the directrix subtends a right angle at the corresponding focus.

H – 6 The circle on any focal distance as diameter touches the auxiliary circle.

H – 7 Perpendiculars from the centre upon all chords which join the ends of any perpendicular diameters of the ellipse are of constant length.

H – 8 If the tangent at the point P of a standard ellipse meets the axis in T and t and CY is the perpendicular on it from the centre then,

(i) T t. PY = $a^2 - b^2$ and (ii) least value of Tt is a + b.

Suggested problems from Loney: **Exercise-32** (Q.2 to 7, 11, 12, 14, 16, 24), **Exercise-33 (Important)** (Q.3, 5, 15, 18, 24, 25, 26), **Exercise-35** (Q.4, 6, 7, 8, 11, 12, 15)

EXERCISE-I

1. **(a)** Find the equation of the ellipse with its centre (1, 2), focus at (6, 2) and passing through the point (4, 6).

(b) An ellipse passes through the points (– 3, 1) & (2, –2) & its principal axis are along the coordinate axes in order. Find its equation.

2. The tangent at any point P of a circle $x^2 + y^2 = a^2$ meets the tangent at a fixed point A (a, 0) in T and T is joined to B, the other end of the diameter through A, prove that the locus of the intersection of AP and BT is an ellipse whose ettentricity is $1/\sqrt{2}$.

3. The tangent at the point α on a standard ellipse meets the auxiliary circle in two points which subtends a right angle at the centre. Show that the eccentricity of the ellipse is $(1 + \sin^2\alpha)^{-1/2}$.

4. If any two chords be drawn through two points on the major axis of an ellipse equidistant from the centre, show that $\tan\frac{\alpha}{2}\cdot\tan\frac{\beta}{2}\cdot\tan\frac{\gamma}{2}\cdot\tan\frac{\delta}{2}=1$, where α, β, γ, δ are the eccentric angles of the extremities of the chords.

5. If the normal at the point P(θ) to the ellipse $\frac{x^2}{14}+\frac{y^2}{5}=1$, intersects it again at the point Q(2θ), show that $\cos\theta = -(2/3)$.

6. If the normals at the points P, Q, R with eccentric angles α, β, γ on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ are concurrent, then show that, $\begin{vmatrix} \sin\alpha & \cos\alpha & \sin 2\alpha \\ \sin\beta & \cos\beta & \sin 2\beta \\ \sin\gamma & \cos\gamma & \sin 2\gamma \end{vmatrix} = 0$.

7. Prove that the equation to the circle, having double contact with the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ (having eccentricity e) at the ends of a latus rectum, is $x^2 + y^2 - 2ae^3x = a^2(1 - e^2 - e^4)$.

8. Find the equations of the lines with equal intercepts on the axes & which touch the ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$.

9. The tangent at $P\left(4\cos\theta, \frac{16}{\sqrt{11}}\sin\theta\right)$ to the ellipse $16x^2 + 11y^2 = 256$ is also a tangent to the circle $x^2 + y^2 - 2x - 15 = 0$. Find θ. Find also the equation to the common tangent.

10. A tangent having slope $-\frac{4}{3}$ to the ellipse $\frac{x^2}{18}+\frac{y^2}{32}=1$, intersects the axis of x & y in points A & B respectively. If O is the origin, find the area of triangle OAB.

11. 'O' is the origin & also the centre of two concentric circles having radii of the inner & the outer circle as 'a' & 'b' respectively. A line OPQ is drawn to cut the inner circle in P & the outer circle in Q. PR is drawn parallel to the y-axis & QR is drawn parallel to the x-axis. Prove that the locus of R is an ellipse touching the two circles. If the focii of this ellipse lie on the inner circle, find the ratio of inner : outer radii & find also the eccentricity of the ellipse.

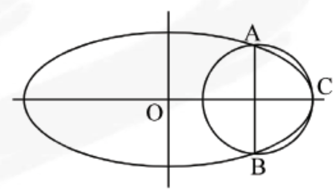
12. ABC is an isosceles triangle with its base BC twice its altitude. A point P moves within the triangle such that the square of its distance from BC is half the rectangle contained by its distances from the two sides. Show that the locus of P is an ellipse with eccentricity $\sqrt{2/3}$ passing through B & C.

13. Let d be the perpendicular distance from the centre of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ to the tangent drawn at a point P on the ellipse. If F_1 & F_2 are the two foci of the ellipse, then show that $(PF_1 - PF_2)^2 = 4a^2\left[1-\frac{b^2}{d^2}\right]$.

14. Common tangents are drawn to the parabola $y^2 = 4x$ & the ellipse $3x^2 + 8y^2 = 48$ touching the parabola at A & B and the ellipse at C & D. Find the area of the quadrilateral.

15. If the normal at a point P on the ellipse of semi axes a, b & centre C cuts the major & minor axes at G & g, show that $a^2.(CG)^2 + b^2.(Cg)^2 = (a^2 - b^2)^2$. Also prove that $CG = e^2CN$, where PN is the ordinate of P.

16. A circle intersects an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}$ = 1 precisely at three points A, B, C as shown in the figure. AB is a diameter of the circle and is perpendicular to the major axis of the ellipse. If the eccentricity of the ellipse is 4/5, find the length of the diameter AB in terms of a.



17. The tangent at a point P on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ intersects the major axis in T & N is the foot of the perpendicular from P to the same axis. Show that the circle on NT as diameter intersects the auxiliary circle orthogonally.

18. The tangents from (x_1, y_1) to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ intersect at right angles. Show that the normals at the points of contact meet on the line $\frac{y}{y_1}=\frac{x}{x_1}$.

19. If the tangent at any point of an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ makes an angle α with the major axis and an angle β with the focal radius of the point of contact then show that the eccentricity 'e' of the ellipse is given by the absolute value of $\frac{\cos\beta}{\cos\alpha}$.

20. An ellipse has foci at $F_1(9, 20)$ and $F_2(49, 55)$ in the xy-plane and is tangent to the x-axis. Find the length of its major axis.

EXERCISE-II

1. PG is the normal to a standard ellipse at P, G being on the major axis. GP is produced outwards to Q so that PQ = GP. Show that the locus of Q is an ellipse whose eccentricity is $\frac{a^2-b^2}{a^2+b^2}$ & find the equation of the locus of the intersection of the tangents at P & Q.

2. P & Q are the corresponding points on a standard ellipse & its auxiliary circle. The tangent at P to the ellipse meets the major axis in T. Prove that QT touches the auxiliary circle.

3. The point P on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is joined to the ends A, A′ of the major axis. If the lines through P perpendicular to PA, PA′ meet the major axis in Q and R then prove that
l(QR) = length of latus rectum.

4. Given the equation of the ellipse $\frac{(x-3)^2}{16}+\frac{(y+4)^2}{49}=1$, a parabola is such that its vertex is the lowest point of the ellipse and it passes through the ends of the minor axis of the ellipse. The equation of the parabola is in the form $16y = a(x-h)^2 - k$. Determine the value of $(a + h + k)$.

5. A tangent to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ touches at the point P on it in the first quadrant & meets the coordinate axes in A & B respectively. If P divides AB in the ratio 3 : 1 reckoning from the x-axis find the equation of the tangent.

6. Prove that the length of the focal chord of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ which is inclined to the major axis at angle θ is $\frac{2ab^2}{a^2\sin^2\theta+b^2\cos^2\theta}$.

7. A tangent to the ellipse $x^2 + 4y^2 = 4$ meets the ellipse $x^2 + 2y^2 = 6$ at P & Q. Prove that the tangents at P & Q of the ellipse $x^2 + 2y^2 = 6$ are at right angles.

8. Rectangle ABCD has area 200. An ellipse with area 200π passes through A and C and has foci at B and D. Find the perimeter of the rectangle.

9. Consider the parabola $y^2 = 4x$ and the ellipse $2x^2 + y^2 = 6$, intersecting at P and Q.

(a) Prove that the two curves are orthogonal.

(b) Find the area enclosed by the parabola and the common chord of the ellipse and parabola.

(c) If tangent and normal at the point P on the ellipse intersect the x-axis at T and G respectively then find the area of the triangle PTG.

10. A normal inclined at 45° to the axis of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is drawn. It meets the x-axis & the y-axis in P & Q respectively. If C is the centre of the ellipse, show that the area of triangle CPQ is $\frac{(a^2-b^2)^2}{2(a^2+b^2)}$ sq. units.

11. Tangents are drawn to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ from the point $\left(\frac{a^2}{\sqrt{a^2-b^2}}, \sqrt{a^2+b^2}\right)$. Prove that they intercept on the ordinate through the nearer focus a distance equal to the major axis.

12. A straight line AB touches the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ & the circle $x^2 + y^2 = r^2$; where $a > r > b$. A focal chord of the ellipse, parallel to AB intersects the circle in P & Q, find the length of the perpendicular drawn from the centre of the ellipse to PQ. Hence show that PQ = 2b.

13. A ray emanating from the point (– 4, 0) is incident on the ellipse $9x^2 + 25y^2 = 225$ at the point P with abscissa 3. Find the equation of the reflected ray after first reflection.

14. If p is the length of the perpendicular from the focus 'S' of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ on any tangent at 'P', then show that $\frac{b^2}{p^2}=\frac{2a}{\ell(SP)}-1$.

15. Variable pairs of chords at right angles and drawn through any point P (with eccentric angle $\pi/4$) on the ellipse $\frac{x^2}{4}+y^2=1$, to meet the ellipse at two points say A and B. If the line joining A and B passes through a fixed point Q(a, b) such that $a^2 + b^2$ has the value equal to $\frac{m}{n}$, where m, n are relatively prime positive integers, find (m + n).

EXERCISE-III

1. (a) If x_1, x_2, x_3 as well as y_1, y_2, y_3 are in G.P. with the same common ratio, then the points (x_1, y_1), (x_2, y_2) & (x_3, y_3) :
(A) lie on a straight line (B) lie on on ellipse (C) lie on a circle (D) are vertices of a triangle.

(b) On the ellipse, $4x^2 + 9y^2 = 1$, the points at which the tangents are parallel to the line $8x = 9y$ are :
(A) $\left(\frac{2}{5}, \frac{1}{5}\right)$ (B) $\left(-\frac{2}{5}, \frac{1}{5}\right)$ (C) $\left(-\frac{2}{5}, -\frac{1}{5}\right)$ (D) $\left(\frac{2}{5}, -\frac{1}{5}\right)$

(c) Consider the family of circles, $x^2 + y^2 = r^2$, $2 < r < 5$. If in the first quadrant, the common tangent to a circle of the family and the ellipse $4x^2 + 25y^2 = 100$ meets the co–ordinate axes at A & B, then find the equation of the locus of the mid–point of AB. [JEE '99, 2 + 3 + 10 (out of 200)]

2. Find the equation of the largest circle with centre (1, 0) that can be inscribed in the ellipse $x^2 + 4y^2 = 16$. [REE '99, 6]

3. Let ABC be an equilateral triangle inscribed in the circle $x^2 + y^2 = a^2$. Suppose perpendiculars from A, B, C to the major axis of the ellipse, $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, (a > b) meet the ellipse respectively at P, Q, R so that P, Q, R lie on the same side of the major axis as A, B, C respectively. Prove that the normals to the ellipse drawn at the points P, Q and R are concurrent. [JEE '2000, 7]

4. Let C_1 and C_2 be two circles with C_2 lying inside C_1. A circle C lying inside C_1 touches C_1 internally and C_2 externally. Identify the locus of the centre of C. [JEE '2001, 5]

5. Find the condition so that the line px + qy = r intersects the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}$ = 1 in points whose eccentric angles differ by $\frac{\pi}{4}$. [REE '2001, 3]

6. Prove that, in an ellipse, the perpendicular from a focus upon any tangent and the line joining the centre of the ellipse to the point of contact meet on the corresponding directrix. [JEE ' 2002, 5]

7. (a) The area of the quadrilateral formed by the tangents at the ends of the latus rectum of the ellipse $\frac{x^2}{9}+\frac{y^2}{5}=1$ is

(A) $9\sqrt{3}$ sq. units (B) $27\sqrt{3}$ sq. units (C) 27 sq. units (D) none

(b) The value of θ for which the sum of intercept on the axis by the tangent at the point $\left(3\sqrt{3}\cos\theta, \sin\theta\right)$, $0 < \theta < \pi/2$ on the ellipse $\frac{x^2}{27}+y^2=1$ is least, is : [JEE ' 2003 (Screening)]

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{8}$

8. The locus of the middle point of the intercept of the tangents drawn from an external point to the ellipse $x^2 + 2y^2 = 2$, between the coordinates axes, is [JEE 2004 (Screening)]

(A) $\frac{1}{x^2}+\frac{1}{2y^2}=1$ (B) $\frac{1}{4x^2}+\frac{1}{2y^2}=1$ (C) $\frac{1}{2x^2}+\frac{1}{4y^2}=1$ (D) $\frac{1}{2x^2}+\frac{1}{y^2}=1$

9. (a) The minimum area of triangle formed by the tangent to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}$ = 1 and coordinate axes is

(A) ab sq. units (B) $\frac{a^2+b^2}{2}$ sq. units (C) $\frac{(a+b)^2}{2}$ sq. units (D) $\frac{a^2+ab+b^2}{3}$ sq. units

[JEE 2005 (Screening)]

(b) Find the equation of the common tangent in 1st quadrant to the circle $x^2 + y^2 = 16$ and the ellipse $\frac{x^2}{25}+\frac{y^2}{4}$ = 1. Also find the length of the intercept of the tangent between the coordinate axes.

[JEE 2005 (Mains), 4]

10. The line passing through the extremity A of the major axis and extremity B of the minor axis of the ellipse $x^2 + 9y^2 = 9$ meets its auxiliary circle at the point M. Then the area of the triangle with vertices at A, M and the origin O is **[JEE 2009]**

(A) $\frac{31}{10}$ (B) $\frac{29}{10}$ (C) $\frac{21}{10}$ (D) $\frac{27}{10}$

11. The normal at la point P on the ellipse $x^2 + 4y^2 = 16$ meets the x-axis at Q. If M is the mid point of the line segment PQ, then the locus of M intersects the latus rectums of the given ellipse at the points **[JEE 2009]**

(A) $\left(\pm\frac{3\sqrt{5}}{2},\pm\frac{2}{7}\right)$ (B) $\left(\pm\frac{3\sqrt{5}}{2},\pm\frac{\sqrt{19}}{4}\right)$ (C) $\left(\pm 2\sqrt{3},\pm\frac{1}{7}\right)$ (D) $\left(\pm 2\sqrt{3},\pm\frac{4\sqrt{3}}{7}\right)$

Paragraph for questions 12 to 14 **[JEE 2010]**

Tangents are drawn from the point P(3, 4) to the ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$ touching the ellipse at points A and B.

12. The coordinates of A and B are

(A) (3, 0) and (0, 2)

(B) $\left(-\frac{8}{5},\frac{2\sqrt{161}}{15}\right)$ and $\left(-\frac{9}{5},\frac{8}{5}\right)$

(C) $\left(-\frac{8}{5},\frac{2\sqrt{161}}{15}\right)$ and (0, 2)

(D) (3, 0) and $\left(-\frac{9}{5},\frac{8}{5}\right)$

13. The orthocentre of the triangle PAB is

(A) $\left(5,\frac{8}{7}\right)$ (B) $\left(\frac{7}{5},\frac{25}{8}\right)$ (C) $\left(\frac{11}{5},\frac{8}{5}\right)$ (D) $\left(\frac{8}{25},\frac{7}{25}\right)$

14. The equation of the locus of the point whose distances from the point P and the line AB are equal, is

(A) $9x^2 + y^2 - 6xy - 54x - 62y + 241 = 0$

(B) $x^2 + 9y^2 + 6xy - 54x + 62y - 241 = 0$

(C) $9x^2 + 9y^2 - 6xy - 54x - 62y - 241 = 0$

(D) $x^2 + y^2 - 2xy + 27x + 31y - 120 = 0$

15. The ellipse $E_1 : \frac{x^2}{9}+\frac{y^2}{4}=1$ is inscribed in a reactangle R whose sides are parallel to the coordinate axes. Another ellipse E_2 passing through the point (0, 4) circumscribes the reactangle R. The eccentricity of the ellipse E_2 is **[JEE 2012]**

(A) $\frac{\sqrt{2}}{2}$ (B) $\frac{\sqrt{3}}{2}$ (C) $\frac{1}{2}$ (D) $\frac{3}{4}$

16. The equation of the circle passing through the foci of the ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$, and having centre at (0, 3) is:

[IIT JEE Main - 2013]

(A) $x^2 + y^2 - 6y + 5 = 0$

(B) $x^2 + y^2 - 6y - 7 = 0$

(C) $x^2 + y^2 - 6y + 7 = 0$

(D) $x^2 + y^2 - 6y - 5 = 0$

17. A vertical line passing through the point (h, 0) inersects the ellipse $\frac{x^2}{4}+\frac{y^2}{3}=1$ at the points P and Q. Let the tangents to the ellipse at P and Q meet at the point R. If $\Delta(h)$ = area of the triangle PQR, $\Delta_1 = \max\limits_{1/2\le h\le 1} \Delta(h)$ and $\Delta_2 = \min\limits_{1/2\le h\le 1} \Delta(h)$, then $\frac{8}{\sqrt{5}}\Delta_1 - 8\Delta_2 =$ **[IIT JEE Advance - 2013]**

18. The locus of the foot of perpendicular drawn from the centre of the ellipse $x^2 + 3y^2 = 6$ on any tangent to it is **[IIT Main - 2014]**

(A) $(x^2 + y^2)^2 = 6x^2 - 2y^2$

(B) $(x^2 - y^2)^2 = 6x^2 + 2y^2$

(C) $(x^2 - y^2)^2 = 6x^2 - 2y^2$

(D) $(x^2 + y^2)^2 = 6x^2 + 2y^2$

19. **List - I** **List - II** **[IIT JEE Advance - 2014]**

(P) Let $y(x) = \cos(3\cos^{-1}x)$, $x \in [-1, 1]$, $x \neq \pm\frac{\sqrt{3}}{2}$. **(1)** 1

Then $\frac{1}{y(x)}\left\{(x^2-1)\frac{d^2y(x)}{dx^2}+x\frac{dy(x)}{dx}\right\}$ equals

(Q) Let $A_1, A_2, \ldots, A_n$ $(n > 2)$ be the vertices of a regular polygon of n sides with its centre at the origin. Let $\vec{a}_k$ be the position vector of the point A_k, k = 1,2,...., n. If $\left|\sum_{k=1}^{n-1}(\vec{a}_k \times \vec{a}_{k+1})\right| = \left|\sum_{k=1}^{n-1}(\vec{a}_k \cdot \vec{a}_{k+1})\right|$, then the minimum value of n is **(2)** 2

(R) If the normal from the point P(h, 1) on the ellipse $\frac{x^2}{6}+\frac{y^2}{3}=1$ is perpendicular to the line x + y = 8, then the value of h is **(3)** 8

(S) Number of positive solutions satisfying the equation $\tan^{-1}\left(\frac{1}{2x+1}\right)+\tan^{-1}\left(\frac{1}{4x+1}\right)=\tan^{-1}\left(\frac{2}{x^2}\right)$ is **(4)** 9

Code :

	P	Q	R	S
(A)	4	3	2	1
(B)	2	4	3	1
(C)	4	3	1	2
(D)	2	4	1	3

20. The area (in sq. units) of the quadrilateral formed by the tangents at the end points of the latera recta to the ellipse $\frac{x^2}{9}+\frac{y^2}{5}=1$, is : **[JEE Main - 2015]**

(A) 27 (B) $\frac{27}{4}$ (C) 18 (D) $\frac{27}{2}$

21. Let E_1 and E_2 be two ellipses whose centers are at the origin. The major axes of E_1 and E_2 lie along the x-axis and the y-axis, respectively. Let S be the circle $x^2 + (y-1)^2 = 2$. The straight line x + y = 3 touches the curves S, E_1 and E_2 at P, Q and R, respectively. Suppose that $PQ = PR = \frac{2\sqrt{2}}{3}$. If e_1 and e_2 are the eccentricities of E_1 and E_2, respectively, then the correct expression(s) is(are) : **[IIT JEE Advance - 2015]**

(A) $e_1^2+e_2^2=\frac{43}{40}$ (B) $e_1e_2=\frac{\sqrt{7}}{2\sqrt{10}}$ (C) $\left|e_1^2-e_2^2\right|=\frac{5}{8}$ (D) $e_1e_2=\frac{\sqrt{3}}{4}$

22. Suppose that the foci of the ellipse $\frac{x^2}{9}+\frac{y^2}{5}=1$ are $(f_1, 0)$ and $(f_2, 0)$ where $f_1 > 0$ and $f_2 < 0$. Let P_1 and P_2 be two parabolas with a common vertex at (0, 0) and with foci at $(f_1, 0)$ and $(2f_2, 0)$, respectively. Let T_1 be a tangent to P_1 which passes through $(2f_2, 0)$ and T_2 be a tangent to P_2 which passes through $(f_1, 0)$. If m_1 is the slope of T_1 and m_2 is the slope of T_2, then the value of $\left(\frac{1}{m_1^2}+m_2^2\right)$ is : **[IIT JEE Advance - 2015]**

Paragraph-(Q.23 to Q.24)

Let $F_1(x_1, 0)$ and $F_2(x_2, 0)$, for $x_1 < 0$ and $x_2 > 0$, be the foci of the ellipse $\frac{x^2}{9}+\frac{y^2}{8}=1$. Suppose a parabola having vertex at the origin and focus at F_2 intersects the ellipse at point M in the first quadrant and at point N in the fourth quadrant. **[IIT JEE Advance - 2016]**

23. The orthocentre of the triangle F_1MN is

(A) $\left(-\frac{9}{10}, 0\right)$ (B) $\left(\frac{2}{3}, 0\right)$ (C) $\left(\frac{9}{10}, 0\right)$ (D) $\left(\frac{2}{3}, \sqrt{6}\right)$

24. If the tangents to the ellipse at M and N meet at R and the nomal to the parabola at M meets the x-axis at Q, then the ratio of area of the triangle MQR to area of the quadrilateral MF_1NF_2 is

(A) 3 : 4 (B) 4 : 5 (C) 5 : 8 (D) 2 : 3

25. The eccentricity of an ellipse whose centre is at the origin is $\frac{1}{2}$. If one of its directrices is x = – 4, then the equation of the normal to it at $\left(1, \frac{3}{2}\right)$ is : **[JEE Main - 2017]**

(A) 4x + 2y = 7 (B) x + 2y = 4 (C) 2y – x = 2 (D) 4x – 2y = 1

26. Consider two straight lines, each of which is tangent to both the circle $x^2 + y^2 = \frac{1}{2}$ and the parabola $y^2 = 4x$. Let these lines intersect at the point Q. Consider the ellipse whose center is at the origin O(0, 0) and whose semi-major axis is OQ. If the length of the minor axis of this ellipse is $\sqrt{2}$, then which of the following statement(s) is (are) TRUE? **[JEE Advanced 2018]**

(A) For the ellipse, the eccentricity is $\frac{1}{\sqrt{2}}$ and the length of the latus rectum is 1

(B) For the ellipse, the eccentricity is $\frac{1}{2}$ and the length of the latus rectum is $\frac{1}{2}$

(C) The area of the region bounded by the ellipse between the lines $x=\frac{1}{\sqrt{2}}$ and x = 1 is $\frac{1}{4\sqrt{2}}(\pi-2)$

(D) The area of the region bounded by the ellipse between the lines $x=\frac{1}{\sqrt{2}}$ and x = 1 is $\frac{1}{16}(\pi-2)$

27. Define the collections $\{E_1, E_2, E_3, ...\}$ of ellipses and $\{R_1, R_2, R_3, ...\}$ of rectangles as follows :

$E_1 : \frac{x^2}{9} + \frac{y^2}{4} = 1$;

R_1 : rectangle of largest area, with sides parallel to the axes, inscribed in E_1;

E_n : ellipse $\frac{x^2}{a_n^2} + \frac{y^2}{b_n^2} = 1$ of largest area inscribed in R_{n-1}, $n > 1$;

R_n : rectangle of largest area, with sides parallel to the axes, inscribed in E_n, $n > 1$.
Then which of the following options is/are correct? **[JEE Advanced - 2019]**

(A) The distance of a focus from the centre in E_9 is $\frac{\sqrt{5}}{32}$

(B) $\sum_{n=1}^{N} (\text{area of } R_n) < 24$, for each positive integer N

(C) The eccentricities of E_{18} and E_{19} are NOT equal

(D) The length of latus rectum of E_9 is $\frac{1}{6}$

28. Let a, b and λ be positive real numbers. Suppose P is an end point of the latus rectum of the parabola $y^2 = 4\lambda x$, and suppose the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ passes through the point P. If the tangents to the parabola and the ellipse at the point P are perpendicular to each other, then the eccentricity of the ellipse is **[JEE Advanced - 2020]**

(A) $\frac{1}{\sqrt{2}}$ (B) $\frac{1}{2}$ (C) $\frac{1}{3}$ (D) $\frac{2}{5}$

29. Let E be the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$. For any three distinct points P, Q and Q' on E, let M(P, Q) be the mid-point of the line segment joining P and Q, and M(P, Q') be the mid-point of the line segment joining P and Q'. Then the maximum possible value of the distance between M(P, Q) and M(P, Q'), as P, Q and Q' vary on E, is ___. **[JEE Advanced - 2021]**

ANSWER KEY

EXERCISE–I

1. (a) $20x^2 + 45y^2 - 40x - 180y - 700 = 0$; (b) $3x^2 + 5y^2 = 32$

8. $x + y - 5 = 0,\ x + y + 5 = 0$

9. $\theta = \frac{\pi}{3}$ or $\frac{5\pi}{3}$; $4x \pm \sqrt{33}\,y - 32 = 0$

10. 24 sq.units

11. $\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$

14. $55\sqrt{2}$ sq. units

16. $\frac{18a}{17}$

20. 85

EXERCISE–II

1. $(a^2 - b^2)^2 x^2y^2 = a^2 (a^2 + b^2)^2 y^2 + 4 b^6x^2$

4. 186

5. $bx + a\sqrt{3}\,y = 2ab$

8. 80

9. (b) 8/3, (c) 4

12. $\sqrt{r^2 - b^2}$

13. $12x + 5y = 48$; $12x - 5y = 48$

15. 19

EXERCISE–III

1. **(a)** A ; **(b)** B, D ; **(c)** $25y^2 + 4x^2 = 4x^2y^2$

2. $(x - 1)^2 + y^2 = 11/3$

4. Locus is an ellipse with foci as the centres of the circles C_1 a nd C_2.

5. $a^2p^2 + b^2q^2 = r^2\sec^2\frac{\pi}{8} = (4 - 2\sqrt{2})r^2$

7. **(a)** C ; **(b)** A

8. C

9. (a) A, (b) AB = $\frac{14}{\sqrt{3}}$

10.	D	**11.**	C	**12.**	D	**13.**	C	**14.**	A				
15.	C	**16.**	B	**17.**	9	**18.**	D	**19.**	A	**20.**	A	**21.**	A, B
22.	4	**23.**	A	**24.**	C	**25.**	D	**26.**	A, C	**27.**	B,D	**28.**	A
29.	4.00												