Nishant Jindal

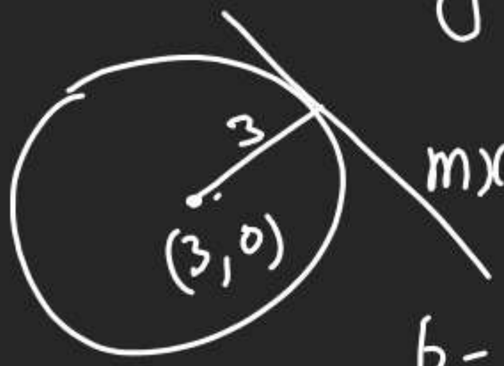
Q Eqⁿ of Com. tangent touching

Circle $(x-3)^2+y^2=9$ & $y^2=4x$

above X Axis $\qquad \downarrow$ $a=1$

$EOT \Rightarrow y = mx + \frac{1}{m}$

This is also tangent to Circle.



$mx - y + \frac{1}{m} = 0$

$p = r$

$$\frac{\left|3m - 0 + \frac{1}{m}\right|}{\sqrt{m^2 + (-1)^2}} = 3$$

$$\left(3m + \frac{1}{m}\right)^2 = \left(3\sqrt{1+m^2}\right)^2$$

$$\cancel{9m^2} + \frac{1}{m^2} + 6 = 9 + \cancel{9m^2}$$

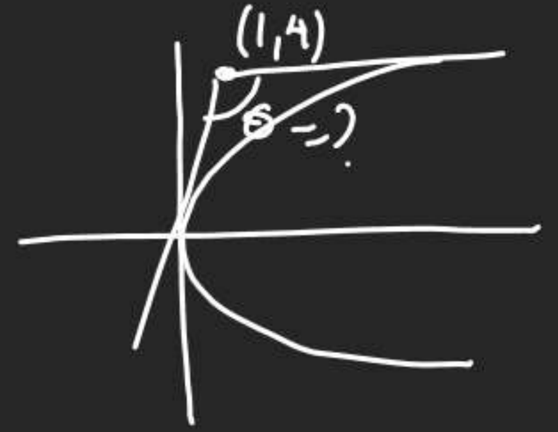
$$\frac{1}{m^2} = 3$$

$$m^2 = \frac{1}{3} \Rightarrow m = \frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}.$$

$$\therefore EOT \Rightarrow y = \frac{x}{\sqrt{3}} + \sqrt{3}$$

Q. Find angle betⁿ tangents to Parabola $y^2 = 4x \rightarrow a=1$ from Pt. $(1,4)$.

$$y^2 - 4x = 4^2 - 4\times 1 > 0$$



$\therefore EOT \Rightarrow y = mx + \frac{1}{m}$

$(1,4)$ Satisfy करेगा

$$4 = m + \frac{1}{m}$$

$$m^2 + 1 = 4m$$

$$m^2 - 4m + 1 = 0$$

$$m = \frac{4 \pm \sqrt{16-4}}{2} = 2 \pm \sqrt{3}$$

$$\tan\theta = \left|\frac{\cancel{2}+\sqrt{3} - (\cancel{2} - \sqrt{3})}{1 + (2+\sqrt{3})(2-\sqrt{3})}\right|$$

$$= \frac{\cancel{2}\sqrt{3}}{\cancel{2}} \Rightarrow \theta = 60^\circ / 120^\circ$$

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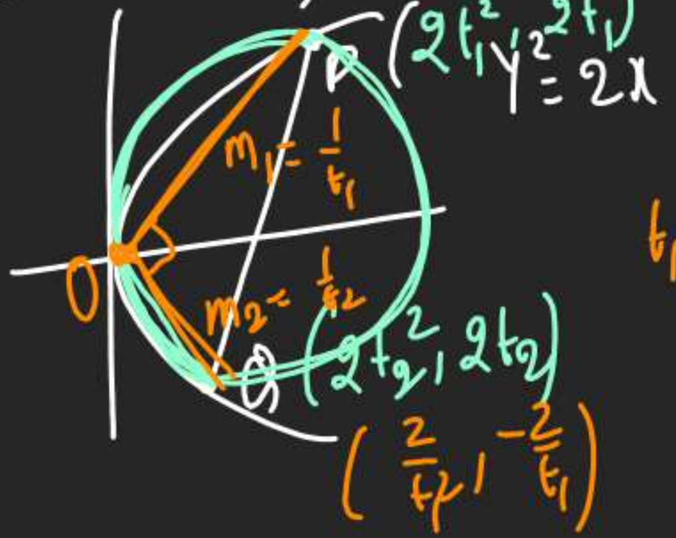
Q Let P & Q be distinct Pts on Parabola $y^2 = 2x$ Such that a Circle with PQ as diameter P.T. vertex O of the Parabola. If P lies in 1^{st} Quads the area of Δ ΔOPQ is $3\sqrt{2}$ find Coord. of P



$t_1 t_2 = -1$

$$\frac{1}{2}\begin{vmatrix} 0 & 0 \\ \frac{2}{t_1^2} & -\frac{2}{t_1} \\ 2t_1^2 & 2t_1 \\ 0 & 0 \end{vmatrix} = 3\sqrt{2}$$

$$\frac{4}{t_1} + 4t_1 = 6\sqrt{2}$$

$$\frac{2}{t_1} + 2t_1 = 3\sqrt{2}$$

$$2t_1^2 - 3\sqrt{2}t_1 + 2 = 0$$

$$t_1 = \frac{3\sqrt{2} \pm \sqrt{18-16}}{4}$$

$$t_1 = \frac{3\sqrt{2}+\sqrt{2}}{4} = \frac{1}{\sqrt{2}}$$

$$t_1 = \sqrt{2}$$

(I) $(4, 2\sqrt{2})$ //

$P = (2t_1^2, 2t_1)$

(II) $= (1, \sqrt{2})$ //

$$\frac{4}{t_1} + 4t_1 = -6\sqrt{2}$$

$$\frac{2}{t_1} + 2t_1 = -3\sqrt{2}$$

$$2t_1^2 + 3\sqrt{2}t_1 + 2 = 0$$

$$t_1 = \frac{-3\sqrt{2} \pm \sqrt{18-16}}{2\times 2}$$

$$= \frac{-3\sqrt{2}+\sqrt{2}}{2\times 2} \quad \Big| \quad \frac{-3\sqrt{2}-\sqrt{2}}{4}$$

$\ominus$

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Pt. of tangency.

Type of Parabola	Pt. of tangency $(at^2, 2at)$	EOT.
$y^2 = 4ax$	$\left(\frac{a}{m^2}, \frac{2a}{m}\right)$	$y = mx + \frac{a}{m}$.
$y^2 = -4ax$	$\left(\frac{-a}{m^2}, \frac{-2a}{m}\right)$	$y = mx - \frac{a}{m}$.
$x^2 = 4ay$	$(2am, am^2)$	$y = mx - am^2$
$x^2 = -4ay$	$(-2am, -am^2)$	$y = mx + am^2$

Cart. form

$tx + y = 2at + at^3$

Slope for

$t \to -m$

$y - mx = -2am - am^3$

$$\boxed{y = mx - 2am - am^3}$$

Normal है Very Usefull

$m = -t$

tangent है this is Useless

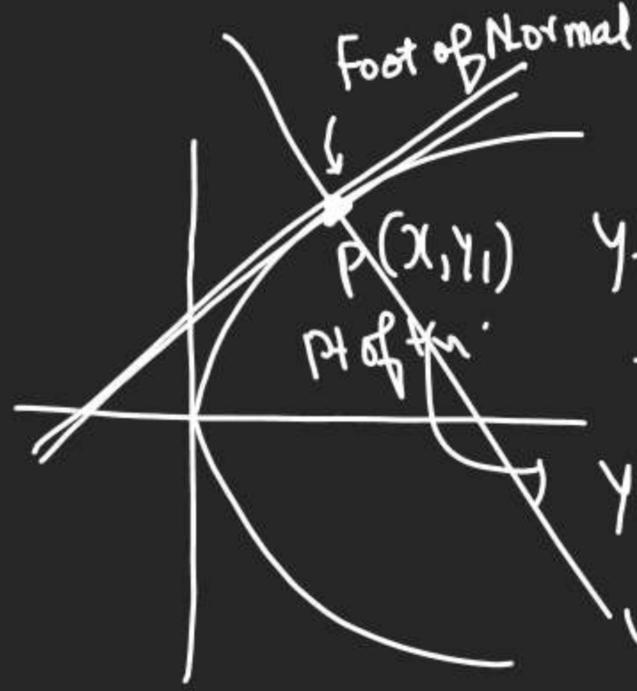
Par. form

$ty = x + at^2$

$y = mx + \frac{a}{m}$

$y = \frac{x}{t} + at$

$m = \frac{1}{t}$



$y^2 = 4ax$

$y = mx + \frac{a}{m}$ ← Slope form tangent

$yy_1 = 2a(x + x_1)$ ← tangent Cart. for.

$yy_1 = 2ax + 2ax_1$

$\frac{y_1}{1} = \frac{2a}{m} = \frac{2ax_1}{a/m}$

$y_1 = \frac{2a}{m}$

$\frac{2a}{m} = 2x_1 m$

$x_1 = \frac{a}{m^2}$

$\therefore P(x_1, y_1) = \left(\frac{a}{m^2}, \frac{2a}{m}\right)$

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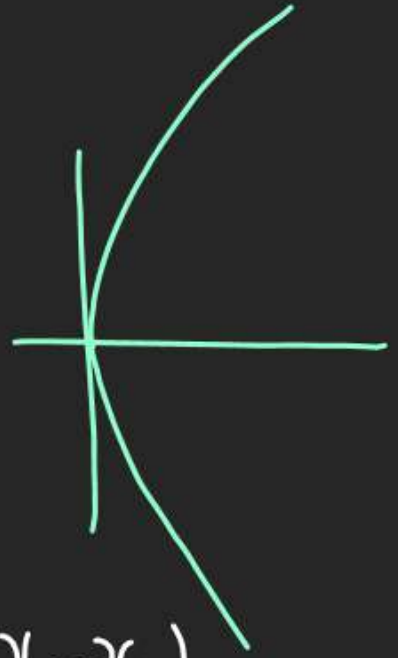
Normal



① Cart. form (x_1, y_1)

$$(y - y_1) = -\frac{y_1}{2a}(x - x_1)$$

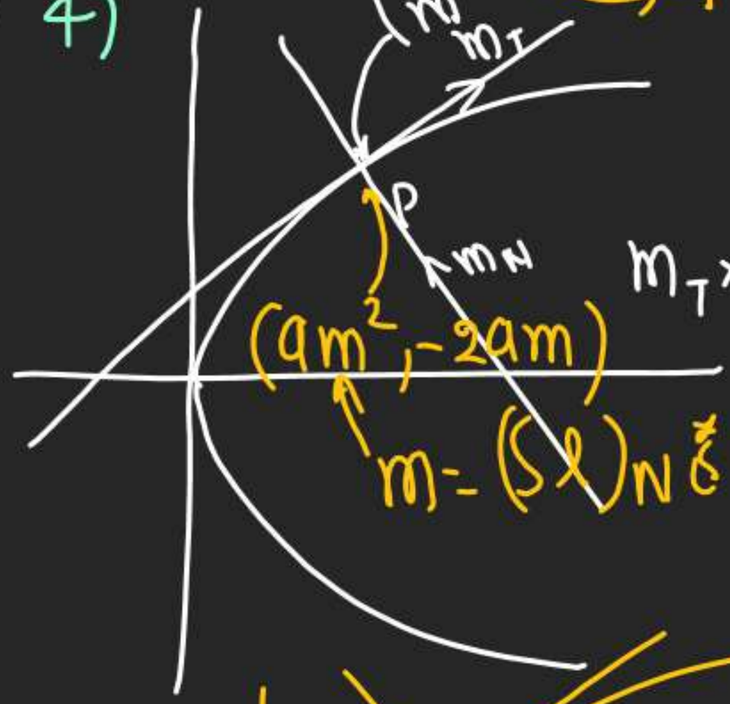
(2) Slope form.

$$y = mx - 2am - am^3.$$

(3) Par. form $(at^2, 2at)$

$$tx + y = 2at + at^3$$

4)



$\left(\frac{a}{m^2}, \frac{2a}{m}\right) \rightarrow$ से $m = (Sl)_T$

m_T

P

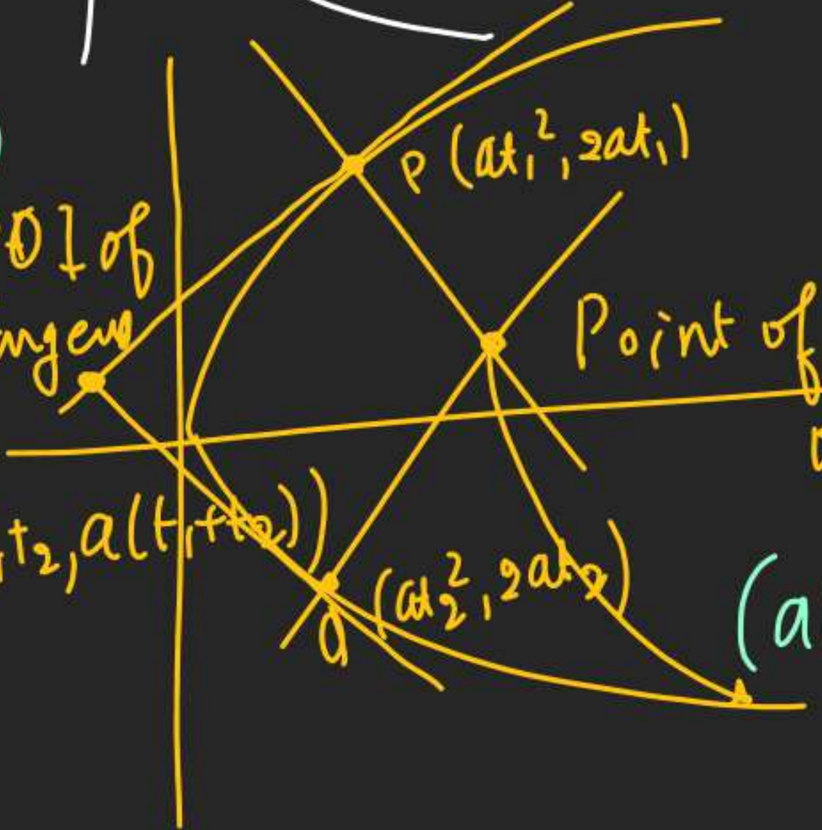
m_N

$m_T \times m_N = -1$

$(am^2, -2am)$

$m = (Sl)_N$ है।

(5)



$P\,(at_1^2, 2at_1)$

PoI of Tangents

$(at_1t_2, a(t_1+t_2))$

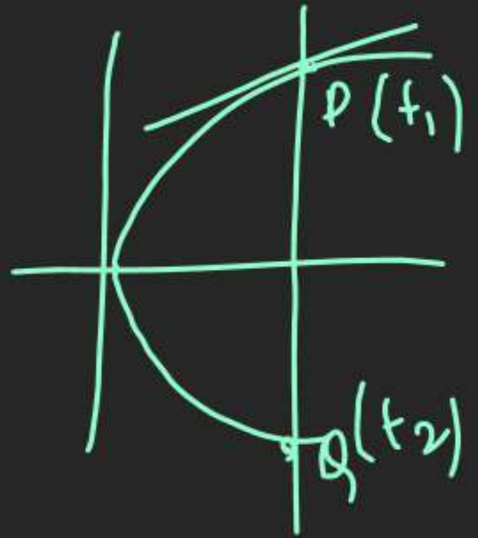
$Q\,(at_2^2, 2at_2)$

Point of Intersection of Norm

$$\left(a(t_1^2+t_2^2+t_1t_2+2), -at_1t_2(t_1+t_2)\right)$$

(6) Q s की gehraai se dekho

Normal की Jagah Normal chord to है।



$P(t_1)$

$Q(t_2)$

$$t_2 = -t_1 + \frac{2}{t_1}$$

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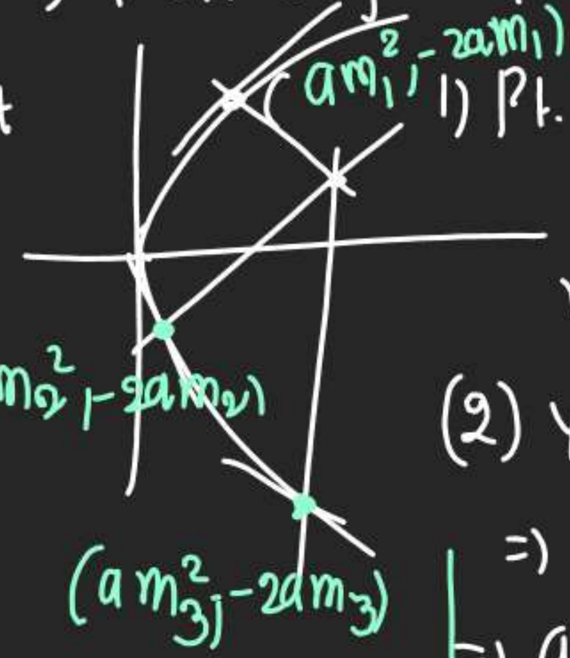
Q If 2 of 3 feet of Normals drawn from a pt. on $y^2=4ax$ are $(1,2)$, $(1,-2)$ then 3^{rd} foot = ?

$m \to -t$



1) Pt. Inside Parabola Can draw max^m 3 Normals

(2) $y = mx - 2am - am^3$

$\Rightarrow am^3 + 2am - mx + y = 0$ — m_1, m_2, m_3

$\Rightarrow am^3 + 0\cdot m^2 + (2a - x)m + y = 0$

(1) SOR $= m_1+m_2+m_3 = -\frac{b}{a} = 0$

(2) SOPORT2AT $= \sum m_1 m_2 = \frac{c}{a} = \frac{2a-x}{a}$

(3) POR $= m_1 m_2 m_3 = -\frac{d}{a} = -\frac{y}{a}$

Sum of y Coord

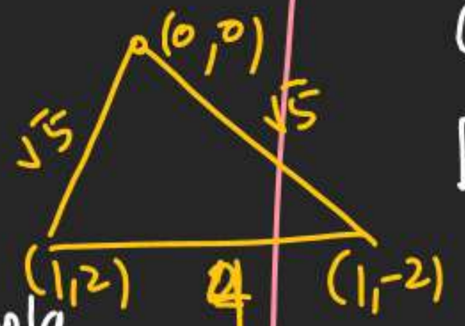
$= -2am_1 - 2am_2 - 2am_3$

$= -2a(m_1+m_2+m_3)$

$2 + -2 + \alpha = 0 \Rightarrow \alpha = 0 \Rightarrow m_3 = 0$

$= (0,0)$

$(a\cdot 0^2, -2a\times 0)$



Q (3,0) is the pt. from which 3 Normals are drawn to $y^2=4x$ which meet Parabola in Pt. P, Q, R then

(1) Area $\triangle PQR$ $= 2$ (2) Radius of Circumcircle of $\triangle PQR$

(3) Centroid of $\triangle PQR$

(4) Circumcentre

(1) $a=1 \Rightarrow$ EON $\Rightarrow y = mx - 2m - m^3$ P.T. $(3,0)$

$\Rightarrow 0 = 3m - 2m - m^3$

$\Rightarrow m^3 - m = 0 \Rightarrow m = 0, 1, -1$

(2) $(am_1^2, -2am_1)$ $(am_2^2, -2am_2)$ $(am_3^2, -2am_3)$

$(0,0)$ $(1,-2)$ $(1,2)$

(1) $\Delta = \frac{1}{2}\begin{vmatrix} 0 & 0 \\ 1 & -2 \\ 1 & 2 \\ 0 & 0 \end{vmatrix} = 2$

(2) $R = \frac{abc}{4\Delta} = \frac{\sqrt{5}\sqrt{5}\cancel{4}}{\cancel{4}\times 2} = \frac{5}{2}$

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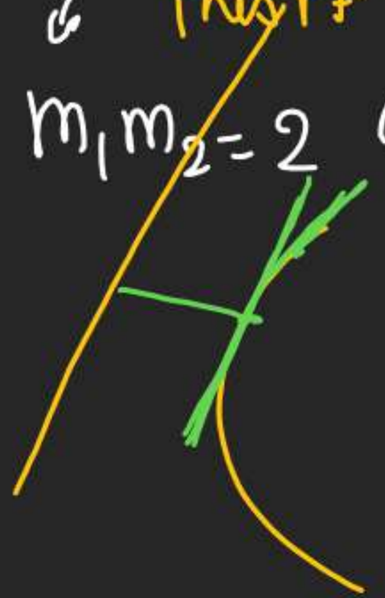
Q If 2 Normals are drawn from any pt. to $y^2 = 4ax$ make angle α & β with axis such that $\boxed{\tan\alpha \cdot \tan\beta = 2}$ find Locus of this Pt.

$m_1 m_2 = 2$ $\quad am^3 + 0\cdot m^2 + (2a - x)m + y = 0$



$\boxed{m_1 m_2} m_3 = -\frac{y}{a}$

$2 m_3 = -\frac{y}{a}$

$m_3 = -\frac{y}{2a}$

$a\left(-\frac{y}{2a}\right)^3 + (2a - x)\left(-\frac{y}{2a}\right) + y = 0$

$-\frac{y^3}{8a^2} - \cancel{y} + \frac{xy}{2a} + \cancel{y} = 0$

$+\frac{y^2}{\cancel{4}8a^{\cancel{2}}} = +\frac{x}{\cancel{2a}}$

$y^2 = 4ax$

Q Let P be the Pt. on Parabola $y^2 = 4x$ which is at Shortest distance from the centre S of the circle $x^2 + y^2 - 4x - 16y + 64 = 0$. Let Q be the Pt. on circle dividing the line segment SP Internally then

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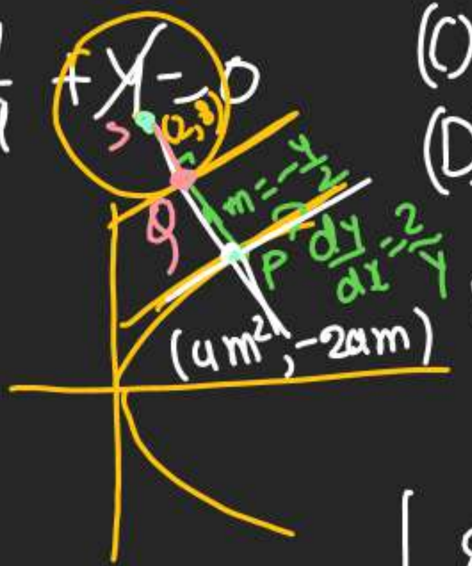
(1) $SP = 2\sqrt{5}$ (2) $\frac{SQ}{QP} = \frac{\sqrt{5}+1}{2}$

(C) x Int. of Normal = 6

(D) $(SQ)_T$ to Circle at $Q = \frac{1}{2}$



$(2, 8)$ $\quad r = \sqrt{4 + 64 - 64} = 2$

$y = mx - 2m - m^3$

$(2, 8)$ Pass

$8 = 2\cancel{m} - 2\cancel{m} - m^3$

$m = -2$

$P = (am^2, -2am)$

$= (4, 4)$

$SP = \sqrt{(4-2)^2 + (4-8)^2}$

$= \sqrt{4 + 16} = 2\sqrt{5}$

(2) $\frac{SQ}{QP} = \frac{2}{2\sqrt{5} - 2}$

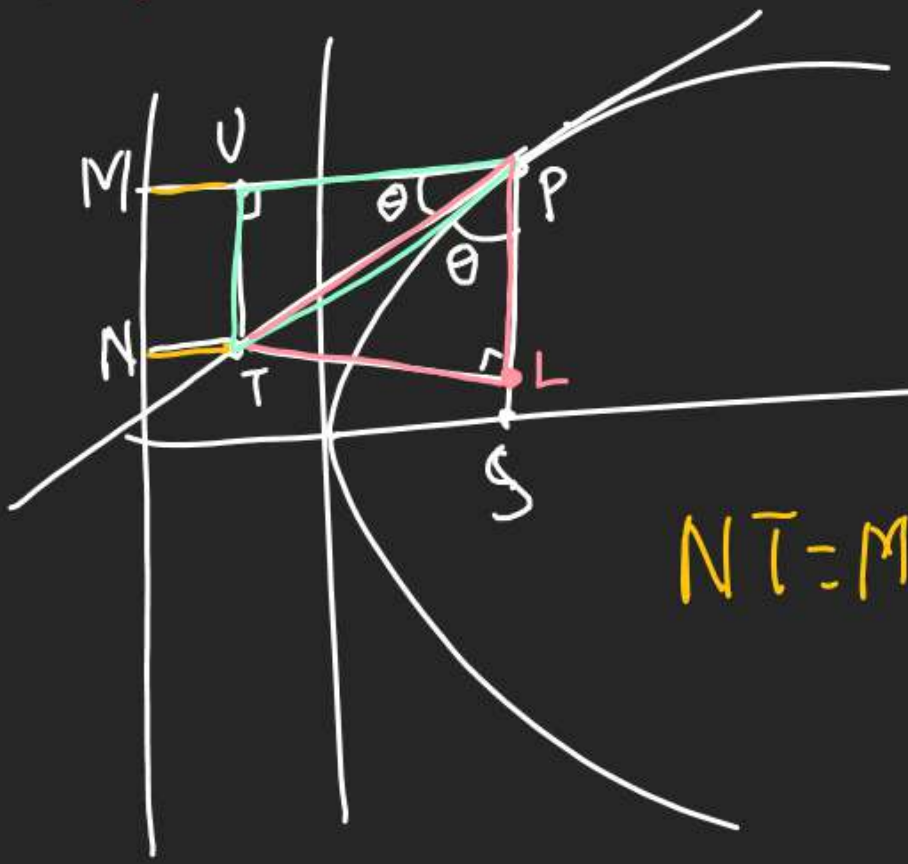
$= \frac{1}{\sqrt{5} - 1} = \frac{\sqrt{5}+1}{4}$

Nishant Jindal

Q T is a Pt. on tangent to Par. $y^2 = 4ax$ at Pt. P. If TL & TN are Normals on Focal chord (PS) & Directrix Resp.

A) $SL = 2(TN)$ (B) $3(SL) = 2(TN)$

(C) $SL = TN$ (D) $2(SL) = 3(TN)$



$NT = MU$

$SP = PM$

$\Delta PTU \cong \Delta PTS$

$\cancel{PL} + LS = \cancel{PU} + MU$

$SL = TN$