Nishant Jindal

1) $\int \frac{1}{x} = \ln|x| + c$

2) $\int \frac{1}{x^2} = -\frac{1}{x}$

3) $\int \sqrt{x} = \frac{2}{3} x^{3/2}$

4) $\int \frac{1}{\sqrt{x}} = 2\sqrt{x}$

5) $\int a^x = \frac{a^x}{\ln a}$

6) $\int e^x = e^x$

$\int \sin x = -\cos x$

$\int \cos x = \sin x$

$\int \tan x = \ln|\sec x|$

$\int \cot x = \ln|\sin x|$

$\int \sec x = \ln|\sec x + \tan x|$

$\int \operatorname{cosec} x = \ln\left|\tan\left(\frac{\pi}{4} + \frac{x}{2}\right)\right|$

$= \ln|\operatorname{cosec} x - \cot x|$

$= \ln\left|\tan \frac{x}{2}\right|$

$\int \sec^2 x = \tan x$

$\int \operatorname{cosec}^2 x = -\cot x$

$\int \sec x \tan x = \sec x$

$\int \operatorname{cosec} x \cot x.$

$= -\operatorname{cosec} x + c$

Nishant Jindal

$$\int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a}$$

$$\int \frac{dx}{\sqrt{x^2+a^2}} = \ln\left|x+\sqrt{x^2+a^2}\right|$$

$$\int \sqrt{x^2+a^2}\,dx = \frac{x}{2}\sqrt{x^2+a^2} + \frac{a^2}{2}\ln\left|x+\sqrt{x^2+a^2}\right|$$

$$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right|$$

$$\int \frac{dx}{\sqrt{x^2-a^2}} = \ln\left|x+\sqrt{x^2-a^2}\right|$$

$$\int \sqrt{x^2-a^2}\,dx = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}\ln\left|x+\sqrt{x^2-a^2}\right|$$

$$\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\frac{x}{a}$$

$$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left|\frac{a+x}{a-x}\right| + c$$

$$\int \sqrt{a^2-x^2}\,dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a}.$$

HW

DPP 3-4 Urgent

Nishant Jindal

Q $\int \frac{\sin x}{\sin(x-a)}\,dx = \int \frac{\sin(x-a+a)}{\sin(x-a)}$

Q Q $\int \frac{dx}{2x^2-5}$

$\int \frac{dx}{(\sqrt{2}x)^2-(\sqrt{5})^2} \rightarrow \int \frac{dx}{x^2-a^2}$

$\frac{1}{2\sqrt{5}} \ln \left| \frac{\sqrt{2}x+\sqrt{5}}{\sqrt{2}x-\sqrt{5}} \right| + c$.

Q $\int \frac{dx}{x^2-2x+3}$.

$\int \frac{dx}{x^2-2x+1+2}$

$\int \frac{dx}{(x-1)^2+(\sqrt{2})^2} \rightarrow \int \frac{dx}{x^2+a^2}$

$\frac{1}{\sqrt{2}} \tan^{-1} \frac{x-1}{\sqrt{2}} + c$.

$\int \frac{dx}{x^2-x-2}$

$\int \frac{dx}{\underbrace{(x-2)(x+1)}_{3}}$

$\frac{1}{3} \ln \frac{x-2}{x+1} + c$.

Nishant Jindal

Substitution.

Q $\int \frac{dx}{x+x\ln x}$

$\int \frac{dx}{x(1+\ln x)}$

Remaining after com $=t$

$\int \frac{dt}{t}$

$\boxed{\begin{array}{l} 1+\ln x=t \\ \frac{1}{x}\,.dx=dt \end{array}}$

$= \ln|t|+C$

$= \ln|1+\ln x|+C$

Q $\int \frac{\cos 2x+x+1}{\sin 2x+x^2+2x}\,.dx$

$\boxed{\text{Normally we try } dr = t}$

$\sin 2x+x^2+2x=t$

$(2\cos 2x+2x+2)dx=dt$

$(\cos 2x+x+1)dx=\frac{dt}{2}$

$\frac{1}{2}\int \frac{dt}{t}$

$\frac{1}{2}\ln|t|+C$

$\frac{1}{2}\ln|\sin 2x+x^2+2x|+C$

Nishant Jindal

Q $\int (x+2)(x^2+4x+10)^5 dx$

$x^2+4x+10=t$

$(2x+4)dx=dt$

$(x+2)dx=\frac{dt}{2}$

$\frac{1}{2}\int t^5 \, dt$

$=\frac{t^6}{12}+C$

$=\frac{(x^2+4x+10)^6}{12}+C$

Q $\int \frac{10x^9+10^x\log_e 10}{x^{10}+10^x} dx$

$x^{10}+10^x=t$

$(10x^9+10^x\ln 10)dx=dt$

$\int \frac{dt}{t}$

$=\ln|t|+C \Rightarrow \ln|x^{10}+10^x|+C$

Q $\int \frac{x^{e-1}-e^{x-1}}{x^e-e^x} dx$

$x^e-e^x=t$

$(e.x^{e-1}-e^x)dx=dt$

$e\left(x^{e-1}-\frac{e^x}{e^1}\right)dx=dt$

$x^{e-1}-e^{x-1}dx=\frac{dt}{e}$

$\Rightarrow \frac{1}{e}\int \frac{dt}{t}$

$\frac{1}{e}\ln|x^e-e^x|+C$

Nishant Jindal

Q $\displaystyle\int \left\{\frac{f(x)\cdot g'(x) - g(x)\cdot f'(x)}{f(x)\cdot g(x)}\right\}\left[\ln g(x) - \ln f(x)\right] dx$

$\ln g(x) - \ln f(x) = t$

$\left(\dfrac{g'(x)}{g(x)} - \dfrac{f'(x)}{f(x)}\right) dx = dt$

$\dfrac{f(x)\cdot g'(x) - g(x)\cdot f'(x)}{f(x)\cdot g(x)}\, dx = dt$

$\displaystyle\int t\,dt = \frac{t^2}{2} + C = \frac{\ln^2 \frac{g(x)}{f(x)}}{2} + C$

try

$\boxed{\ln(f(x)) = t}$

Q $\displaystyle\int \frac{dx}{x+\sqrt{x}}$

$\displaystyle\int \frac{dx}{\sqrt{x}(\sqrt{x}+1)}$

Remaining

$\sqrt{x}+1 = t$

$\dfrac{dx}{2\sqrt{x}} = dt$

$\dfrac{dx}{\sqrt{x}} = 2\,dt$

$2\displaystyle\int \frac{dt}{t}$

$2\ln|\sqrt{x}+1| + C$

Nishant Jindal

Q $\int \frac{dx}{\sqrt{x+x\sqrt{x}}}$ OR $\int \frac{dx}{\sqrt{x+x^{3/2}}}$ OR $\int \frac{dx}{\sqrt{x+(\sqrt{x})^3}}$ ✓

$\int \frac{dx}{\sqrt{x}\sqrt{1+\sqrt{x}}}$

$1+\sqrt{x}=t$

$\frac{dx}{2\sqrt{x}}=dt$

$\frac{dx}{\sqrt{x}}=2dt$

$2\int \frac{dt}{\sqrt{t}}$

$2\times 2\sqrt{t}$

$\Rightarrow 4\sqrt{t}+C$

$\Rightarrow 4\sqrt{1+\sqrt{x}}+C$

Q Mains

$\int \frac{x\,dx}{\sqrt{(1+x^2)+(\sqrt{1+x^2})^3}}$

$\int \frac{x\,dx}{\sqrt{(1+x^2)+(1+x^2)\cdot\sqrt{1+x^2}}}$

$\int \frac{x\,dx}{\sqrt{1+x^2}\sqrt{1+\sqrt{1+x^2}}}$

$1+\sqrt{1+x^2}=t$

$\frac{2x\,dx}{2\sqrt{1+x^2}}=dt$

$\int \frac{dt}{\sqrt{t}}=2\sqrt{t}+C$

Nishant Jindal

Q $\int \frac{4x^2+x+1}{x^3-1}\,dx$

$x^3-1=t$

$3x^2\,dx=dt$

$\int \frac{3x^2\,dx}{x^3-1} + \int \frac{x^2+x+1}{x^3-1}\,dx$

$\int \frac{dt}{t} + \int \frac{\cancel{x^2+x+1}\,dx}{(x-1)(\cancel{x^2+x+1})}$

$\ln|x^3-1|+\ln|x-1|+C$

Q $\int \frac{1-x^7}{x(1+x^7)}\,dx$

$\int \frac{\cancel{1+x^7}}{x(\cancel{1+x^7})}\,dx - \int \frac{2x^{7}}{x(1+x^7)}\,dx$

$\int \frac{dx}{x} - 2\int \frac{x^6\,dx}{1+x^7}$

$1+x^7=t$

$7x^6\,dx=dt$

$x^6\,dx=\frac{dt}{7}$

$\ln|x| - \frac{2}{7}\int \frac{dt}{t}$

$\ln|x| - \frac{2}{7}\ln|1+x^7|+C$

Nishant Jindal

Q $\int \frac{2x+3}{\sqrt{x^2+1}}\,dx$

$\int \frac{2x\,dx}{\sqrt{x^2+1}} + \int \frac{3\,dx}{\sqrt{x^2+1^2}}$

$x^2+1=t$

$2x\,dx=dt$

$\int \frac{dt}{\sqrt{t}} + 3\int \frac{dx}{\sqrt{x^2+a^2}} \to \int \frac{dx}{\sqrt{x^2+a^2}}$

$2\sqrt{x^2+1} + 3\ln\left|x+\sqrt{x^2+1}\right|+C$

Q $\int \frac{\sec x\cdot\operatorname{cosec} x}{\ln\tan x}\,dx$

$\ln\tan x=t$

$\frac{1}{\tan x}\times\sec^2 x\cdot dx=dt$

$\frac{\cancel{\cos x}}{\sin x}\times\frac{1}{\cancel{\cos x}}\cdot\sec x\cdot dx=dt$

$\sec x\cdot\operatorname{cosec} x\cdot dx=dt$

$\int \frac{dt}{t}$

$\ln|\ln\tan x|+C$

Nishant Jindal

Q $\int 5^{5^{5^{x}}} 5^{5^{x}} \cdot 5^{x} \cdot dx$

$5^{5^{5^{x}}} = t$

$5^{5^{5^{x}}} \cdot \ln 5 \times (5^{5^{x}})' dx = dt$

$5^{5^{5^{x}}} \ln 5 \times 5^{5^{x}} \ln 5 \times (5^{x})' dx = dt$

$5^{5^{5^{x}}} \cdot 5^{5^{x}} (\ln 5)^2 5^{x} \cdot \ln 5 \cdot dx = dt$

$5^{5^{5^{x}}} \cdot 5^{5^{x}} \cdot 5^{x} (\ln 5)^3 dx = dt$

$5^{5^{5^{x}}} \cdot 5^{5^{x}} \cdot 5^{x} dx = \frac{dt}{(\ln 5)^3}$

$\frac{1}{(\ln 5)^3} \int dt$

$\frac{t}{(\ln 5)^3} + C$

$\frac{5^{5^{5^{x}}}}{(\ln 5)^3} + C$

Q $\int 2^{2^{2^{x}}} \cdot 2^{2^{x}} \cdot 2^{x} \cdot 2^{x} dx$

$\frac{\text{Bda}}{(\ln 2)^{\text{No. of term}}} + C$

$= \frac{2^{2^{2^{2^{x}}}}}{(\ln 2)^4} + C$

Q $\int \tan^3(2x) \cdot \sec 2x \cdot dx$

$\int \tan^2(2x) \cdot \sec(2x) \tan 2x \, dx$

$\int (\sec^2(2x) - 1) \sec 2x \cdot \tan 2x \, dx$

$\sec 2x = t$

$2 \sec 2x \cdot \tan 2x \, dx = dt$

$\frac{1}{2} \int (t^2 - 1) dt = \frac{1}{2} \left[\frac{t^3}{3} - t \right] + C$

Nishant Jindal

Q For $x^2 \neq n\pi + 1,\ n \in N$

then $\int x \sqrt{\dfrac{2\sin(x^2-1) - \sin 2(x^2-1)}{2\sin(x^2-1) + \sin 2(x^2-1)}}\, dx = ?$

BarBar = t

$x^2 - 1 = t$

$x\, dx = \dfrac{dt}{2}$

$$\frac{1}{2}\int \sqrt{\frac{2\sin t - \sin 2t}{2\sin t + \sin 2t}}\, dt$$

$2\sin t - 2\sin t \cos t$

$2\sin t(1-\cos t)$

$$\frac{1}{2}\int \sqrt{\frac{1-\cos t}{1+\cos t}}\, dt$$

$$\frac{1}{2}\int \sqrt{\frac{2\sin^2 t/2}{2\cos^2 t/2}}\, dt$$

$$\frac{1}{2}\int \tan\frac{t}{2}\, dt = \frac{1}{2}\,\frac{\ln\left|\sec\frac{t}{2}\right|}{\frac{1}{2}} + 1$$

Q $\displaystyle\int \frac{(2x+3)\,dx}{(x)(x+1)(x+2)(x+3)+1}$

$$\Rightarrow \int \frac{(2x+3)\,dx}{(x^2+3x)(x^2+3x+2)+1}$$

$x^2+3x = t$

$(2x+3)\,dx = dt$

$$\Rightarrow \int \frac{dt}{(t)(t+2)+1}$$

$$\Rightarrow \int \frac{dt}{t^2+2t+1} = \int \frac{dt}{(t+1)^2} \rightarrow \int \frac{1}{x^2} \text{ Jesa}$$

$$= -\frac{1}{(t+1)} + C$$

Nishant Jindal

Q $\int \frac{\sec^4 x}{\sqrt{\tan x}}\,dx$

$\tan x = t$

$\sec^2 x\,dx = dt$

$\Rightarrow \int \frac{\sec^2 x \cdot \sec^2 x \cdot dx}{\sqrt{\tan x}}$

$\Rightarrow \int \frac{(\tan^2 x+1)\cdot \sec^2 x\,dx}{\sqrt{\tan x}}$

$\Rightarrow \int \frac{(t^2+1)\,dt}{\sqrt{t}} = \int t^{3/2} + \frac{1}{\sqrt{t}}\,dt$

$= \frac{2t^{5/2}}{5} + 2\sqrt{t} + C$

Q* $\int \frac{dx}{\cos^3 x\sqrt{\sin 2x}}$

$\int \frac{dx}{\cos^3 x\sqrt{\frac{2\tan x}{1+\tan^2 x}}}$

$\frac{1}{\sqrt{2}}\int \frac{dx}{\cos^4 x\sqrt{\tan x}}$

$\frac{1}{\sqrt{2}}\int \frac{\sec^4 x\,dx}{\sqrt{\tan x}}$ (Pr. Q5)

Nishant Jindal

Yad.

Q $\int \frac{\sin x\, dx}{\sin x - \cos x}$

Yad

$\frac{1}{2}\int \frac{2\sin x\, dx}{(\sin x - \cos x)}$ → Hindi me Padho 2 Sin x

$\frac{1}{2}\left\{\int \frac{\sin x + \cos x\, dx}{\sin x - \cos x} + \int \frac{\sin x - \cos x}{\sin x - \cos x}\, dx\right\}$

$\frac{1}{2}\int \frac{dt}{t} + \int dx$

$\sin x - \cos x = t$

$(\cos x + \sin x)\, dx = dt$

$\frac{1}{2}\ln|\sin x - \cos x| + \frac{1}{2}x + C$

Q $\int \frac{\cos^2 x}{\sin^4 x} dx$

$\Rightarrow \int \frac{\cos^2 x}{\sin^2 x} \times \frac{1}{\sin^2 x} dx$

$\int \cot^2 x \cdot \operatorname{cosec}^2 x\, dx$

$\cot x = t$

$-\operatorname{cosec}^2 x\, dx = dt$

$-\int t^2 dt$

$-\frac{t^3}{3} + C$

$-\frac{\cot^3 x}{3} + C$

Q $\int \frac{\cos^4 x}{\sin^2 x} dx$ Smart

$\int \frac{(1 - \sin^2 x)^2}{\sin^2 x} dx$

$\int \frac{\sin^4 x - 2\sin^2 x + 1}{\sin^2 x} dx$

$\int \sin^2 x - 2 + \operatorname{cosec}^2 x\, dx$

$\left(\frac{x}{2} - \frac{\sin 2x}{4}\right) - 2x - \cot x + C$

10th

M2 $\int \cot^2 x \cdot \cos^2 x\, dx$

$\Rightarrow \int \cot^2 x - \cos^2 x\, dx$

$\Rightarrow \int \operatorname{cosec}^2 x - 1 - \int \frac{1}{2} + \frac{\cos 2x}{2} dx$

$\Rightarrow -\cot x - x - \frac{x}{2} - \frac{\sin 2x}{4} + C$

Nishant Jindal

Q $\int \left(\frac{x^2+\cos^2 x}{x^2+1}\right) \cdot \operatorname{cosec}^2 x \, dx$

$\int \left(\frac{(x^2+1)-\sin^2 x}{x^2+1}\right) \cdot \operatorname{cosec}^2 x \, dx$

$\int \cancel{\frac{x^2+1}{x^2+1}} \cdot \operatorname{cosec}^2 x - \frac{\cancel{\sin^2 x} \cdot \cancel{\operatorname{cosec}^2 x}}{x^2+1} \, dx$

$-\cot x - \tan^{-1} x + C$

$(\tan^{-1} x)' = \frac{1}{1+x^2}$

Bdi Bdi degree wala Qs

Q. $\int \frac{5x^4+4x^5}{(x^5+x+1)^2} \, dx$

Dr. Sbse bdi deg com Le Lo

$\int \frac{5x^4+4x^5 \cdot dx}{x^{10}(1+x^{-4}+x^{-5})^2}$

$\int \frac{5x^{-6}+4x^{-5} \cdot dx}{(x^{-5}+x^{-4}+1)^2}$

$x^{-5}+x^{-4}+1 = t$

$-5x^{-6}-4x^{-5} \cdot dx = dt$

$(5x^{-6}+4x^{-5})\,dx = -dt$

$-\int \frac{dt}{t^2}$

$= -\left(-\frac{1}{t}\right) + C$

Nishant Jindal

Q $\int \frac{3x^4+4x^3}{(x^4+x+1)^2}\,dx$

$\int \frac{3x^4+4x^3\,dx}{x^8(1+x^{-3}+x^{-4})^2}$

$\int \frac{3x^{-4}+4x^{-5}\,.dx}{(1+x^{-3}+x^{-4})^2}$

$1+x^{-3}+x^{-4}=t$

$-3x^{-4}-4x^{-5}\,dx=dt$

$-\int \frac{dt}{t^2}$

$=\frac{1}{t}+C$

Q Main 2022 $f(x)=\int \frac{5x^8+7x^6}{(x^2+1+2x^7)^2}\,dx$ & $f(0)=0$, $f(1)=\frac{1}{K}$ find K?

$\int \frac{5x^8+7x^6\,dx}{x^{14}(x^{-5}+x^{-7}+2)^2} = \int \frac{5x^{-6}+7x^{-8}\,dx}{(x^{-7}+x^{-5}+2)^2}$

$x^{-7}+x^{-5}+2=t$

$-7x^{-8}-5x^{-6}\,dx=dt$

$=-\int \frac{dt}{t^2}$

$f(x)=\frac{1}{x^{-7}+x^{-5}+2}+C = \frac{x^7}{1+x^2+2x^7}+C$

$f(0)=0+C=0 \Rightarrow C=0$

$\therefore f(x)=\frac{x^7}{1+x^2+2x^7}$

$f(1)=\frac{1}{1+1+2}=\frac{1}{4}=\frac{1}{K}$

$\boxed{K=4}$

Nishant Jindal

Q $\displaystyle\int \frac{dx}{\sqrt{\sin^3 x \cdot \sin(x+\alpha)}}$

$$\int \frac{dx}{\sqrt{\sin^3 x \cdot (\sin x \cos\alpha + \cos x \cdot \sin\alpha)}}$$

Sin Com.

$$\int \frac{dx}{\sin^2 x\sqrt{\cos\alpha + \cot x \cdot \sin\alpha}}$$

$$\int \frac{\operatorname{cosec}^2 x \, dx}{\sqrt{\cos\alpha + \cot x \cdot \sin\alpha}}$$

$\cos\alpha + \sin\alpha \cdot \cot x = t$

$0 + \sin\alpha(-\operatorname{cosec}^2 x)dx = dt$

$\operatorname{cosec}^2 x \, dx = \dfrac{dt}{-\sin\alpha}$

$$-\frac{1}{\sin\alpha}\int \frac{dt}{\sqrt{t}}$$

$$-\frac{1}{\sin\alpha} \times 2\sqrt{t} + C$$

Q $\displaystyle\int \frac{\sin^{3/2} x + \cos^{3/2} x}{\sqrt{\sin^3 x \cdot \cos^3 x \cdot \sin(x+\alpha)}} dx$

Split & Solve

Nishant Jindal

Q $\displaystyle\int \frac{\sec^2 x \tan x \, dx}{(\sec x+\tan x)^{100}}$

$\sec x+\tan x = t$

$\sec x - \tan x = \frac{1}{t}$

Add

$\sec x = \frac{1}{2}\left(t+\frac{1}{t}\right)$

Sub.

$\tan x = \frac{1}{2}\left(t-\frac{1}{t}\right)$

$\sec x+\tan x = t$

$\sec x \tan x + \sec^2 x \cdot dx = dt$

$\sec x(\sec x+\tan x)\, dx = dt$

$\sec x \, dx = \frac{dt}{t}$

$\displaystyle\int \frac{\frac{1}{2}\left(t+\frac{1}{t}\right)\times\frac{1}{2}\left(t-\frac{1}{t}\right)}{t\times t^{100}} dt$

$\displaystyle\frac{1}{4}\int \frac{t^2-\frac{1}{t^2}}{t^{101}} dt$

$\displaystyle\frac{1}{4}\int t^{-99}-t^{-103} dt$

$\displaystyle\frac{1}{4}\times\frac{t^{-98}}{-98}+\frac{t^{-102}}{102}+C$

HW

DPP-3,4

Urgent