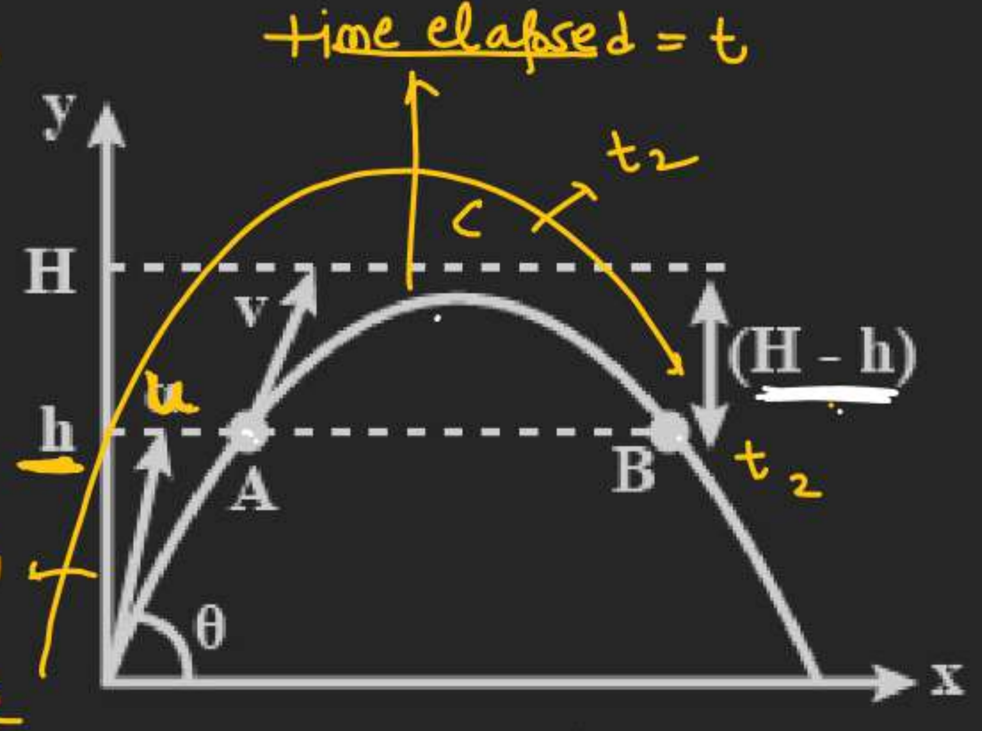
Nishant Jindal

Projectile motion

Q. Figure shows two positions A and B at the same height h above the ground. If the maximum height of the projectile is H, then determine the time t elapsed between the positions A and B in terms of H. & h ✓



Sol^n

$$h = (u\sin\theta)t - \frac{1}{2}gt^2 \qquad H = \frac{u^2\sin^2\theta}{2g}$$

$$t^2 - \left(\frac{2u\sin\theta}{g}\right)t + \frac{2h}{g} = 0 \qquad u\sin\theta = \sqrt{2gH}$$

$$t_1 + t_2 = \frac{2u\sin\theta}{g} = \frac{2}{g}\sqrt{2gH} = \sqrt{\frac{8H}{g}}$$

$$t_1 t_2 = \left(\frac{2h}{g}\right)$$

$$(t_2 - t_1) = \sqrt{(t_1+t_2)^2 - 4t_1t_2}$$

$$(t_2 - t_1) = \sqrt{\frac{8H}{g} - \frac{8h}{g}} = \boxed{\sqrt{\frac{8}{g}(H-h)}} \ **$$

Nishant Jindal

Projectile motion

Q. From a point on the ground at a distance a from the foot of a pole, a ball is thrown at an angle of 45°, which just touches the top of the pole and strikes the ground at a distance of b, on the other side of it. Find the height of the pole.

Sol^n :→

$R = (a+b)$

At $45°$ → $R \to R_{max}$.

$$R_{max} = \frac{u^2 \sin(2\times 45°)}{g}$$

$$(a+b) = R_{max} = \left(\frac{u^2}{g}\right) \quad —①$$

$$\boxed{u^2 = g(a+b)}$$

$$y = \left[x\tan\theta - \frac{g}{2u^2\cos^2\theta}\cdot x^2\right]$$

$$h = (a\tan 45°) - \frac{g}{2\, g(a+b)\cos^2 45}\times a^2$$

2nd method

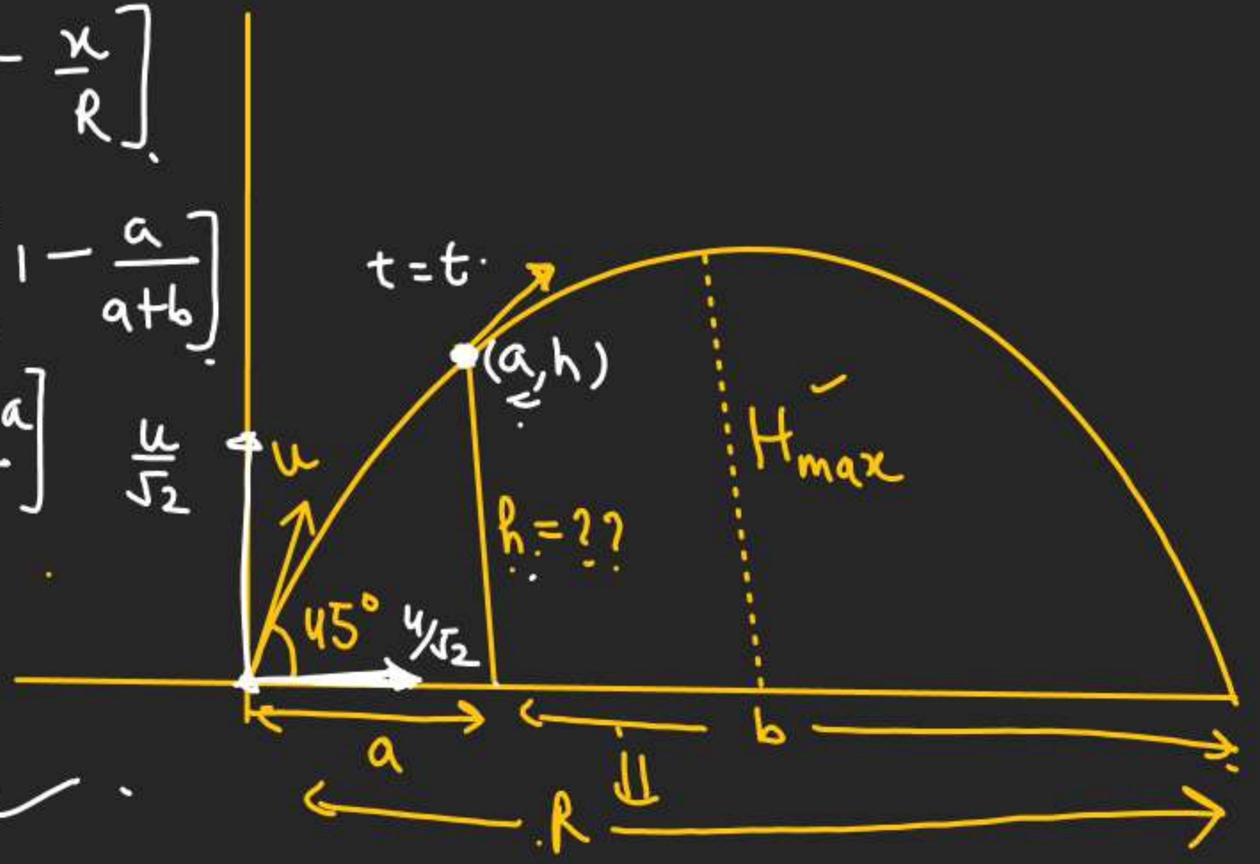
$$y = x\tan\theta\left[1 - \frac{x}{R}\right]$$

$$h = a\tan 45°\left[1 - \frac{a}{a+b}\right]$$

$$h = a\left[\frac{(a+b)-a}{a+b}\right]$$

$$h = a\left[\frac{b}{a+b}\right]$$

$$h = \left(\frac{ab}{a+b}\right)$$



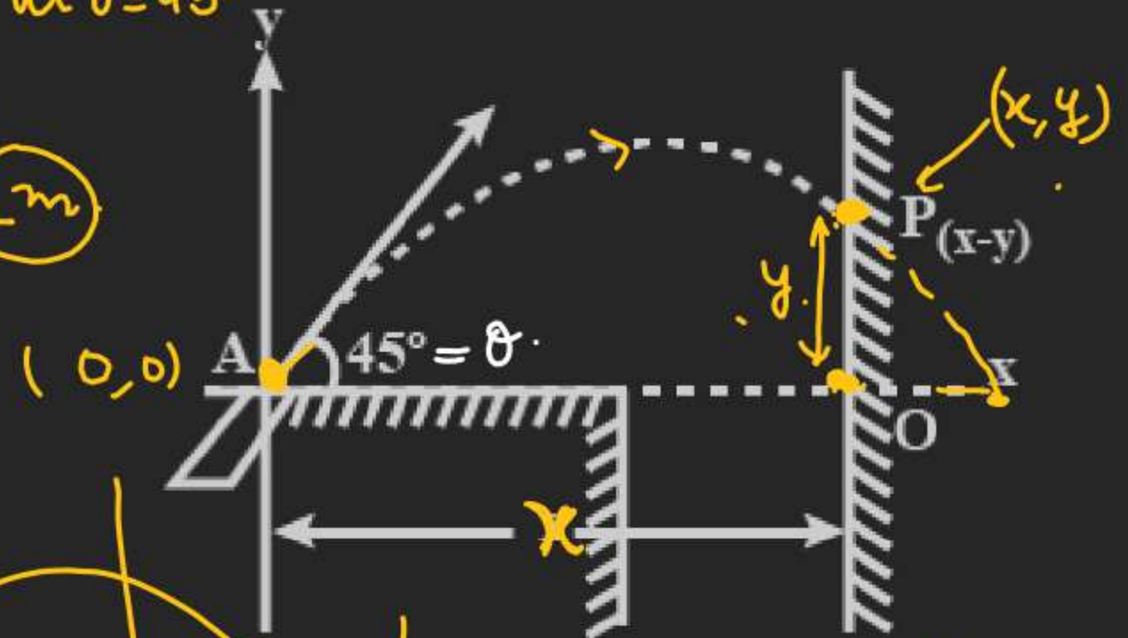
Projectile motion

Q. A jet of water is projected at an angle $\theta = 45^\circ$ with horizontal from point A which is situated at a distance $x = OA =$ (a) $1/2$ m, (b) 2 m from a vertical wall. If the speed of projection is $v_0 = \sqrt{10}\ \text{m s}^{-1}$, find point P of striking of the water jet with the vertical wall.

$y = x\tan\theta\left[1 - \frac{x}{R}\right]$

$R = \frac{u^2}{g}$ at $\theta = 45^\circ$

$R = \left(\frac{10}{10}\right) = 1\text{m}$



(a) $x = \frac{1}{2} \rightarrow (\theta = 45^\circ)$

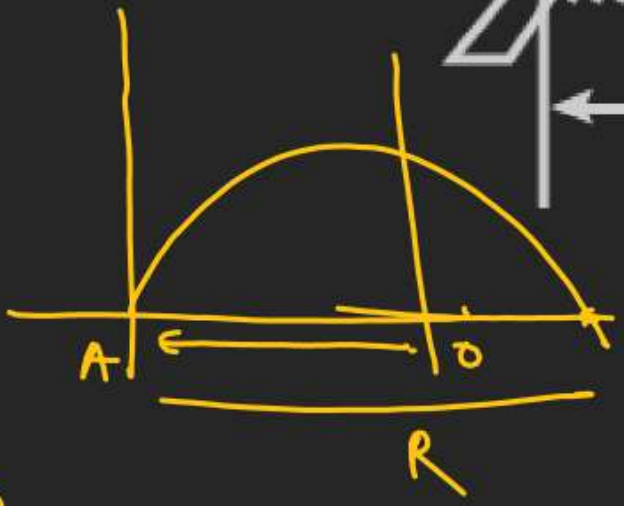
$y = \frac{1}{2}(\tan 45)\left[1 - \frac{1}{2\times 1}\right]$

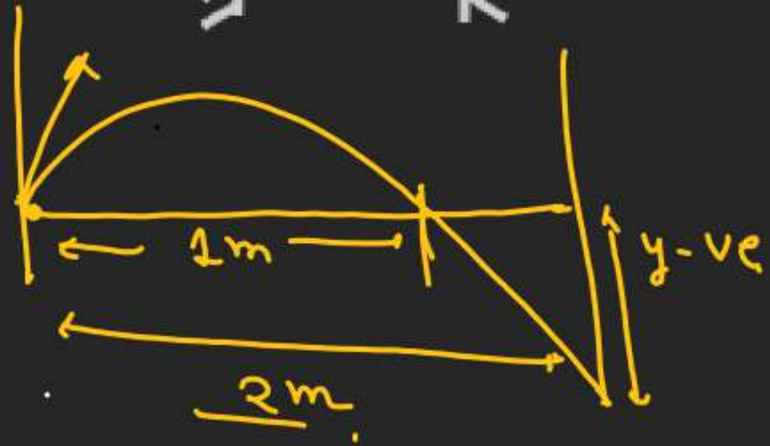
$y = \frac{1}{2}\times\frac{1}{2} = \frac{1}{4}\text{ m}$

(b) $x = 2\text{m}$

$y = 2\times 1\left[1 - \frac{2}{1}\right] = (-)2\text{m})$

$-ve = ??$



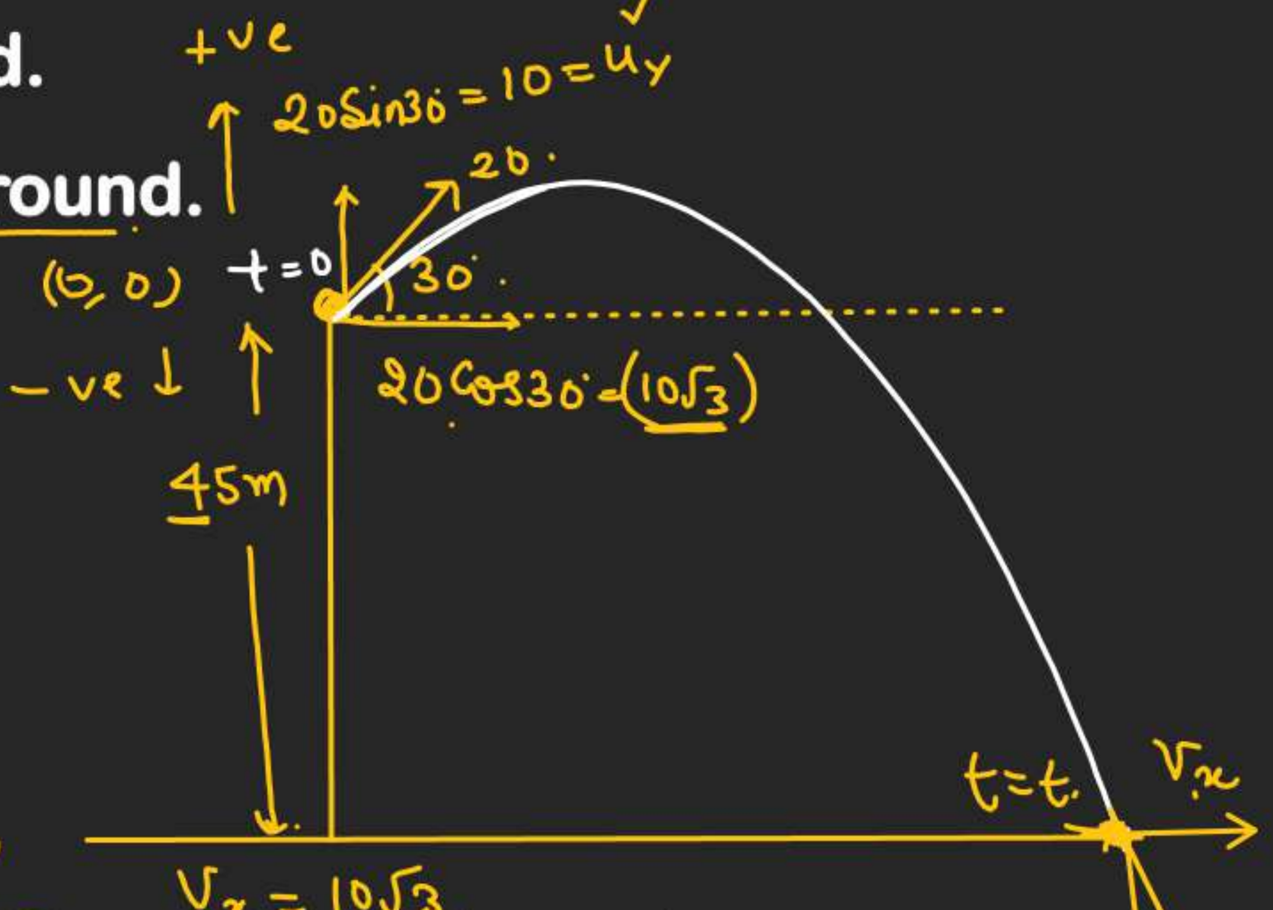


Projectile motion

Q. A ball is thrown from the top of a building $45 \mathrm{~m}$ high with a speed $20 \mathrm{~m} \mathrm{~s}^{-1}$ above the horizontal at an angle of 30°. Find

a. The time taken by the ball to reach the ground.

b. The speed of ball just before it touches the ground.



Sol^n:-

$$-45 = 10t - \frac{1}{2}\times 10 \times t^2 \qquad y = u_y t - \frac{1}{2} g t^2$$

$$-45 = 10t - 5t^2$$

$$t^2 - 2t - 9 = 0$$

$$t = \frac{2 \pm \sqrt{4+36}}{2(1)} = \frac{2 \pm \sqrt{40}}{2} = \frac{\cancel{2}(1 \pm \sqrt{10})}{\cancel{2}}$$

$$t \to (1+\sqrt{10}) \checkmark \qquad t \to (1-\sqrt{10}) \times$$

$$V_x = 10\sqrt{3}$$

$$V_y = (u_y - gt) = 10 - 10 \times (1+\sqrt{10})$$

$$V_y = -(10\sqrt{10})$$

$$V = \sqrt{v_x^2 + v_y^2} = \sqrt{(10\sqrt{3})^2 + (10\sqrt{10})^2} = 10\sqrt{3+10} = 10\sqrt{13} \text{ Ans}$$

Nishant Jindal

Projectile motion

H.W.

Q. A body is thrown at an angle θ_0 with the horizontal such that it attains a speed equal to $\sqrt{\frac{2}{3}}$ times the speed of projection when the body is at half of its maximum height. Find the angle θ_0.

Projectile motion

$\theta = 30°$ ← $\theta \to$ Missing ??

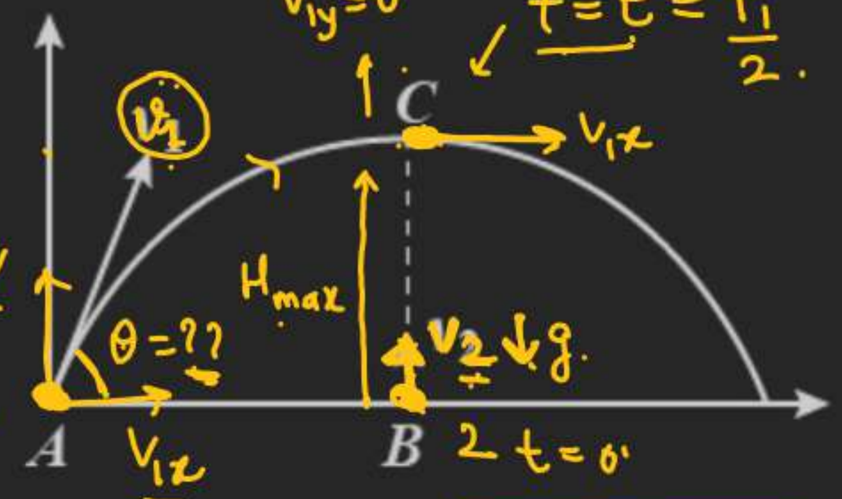
Q. A body is projected with velocity v_1 from the point A as shown in Fig. At the same time, another body is projected vertically upwards from B with velocity v_2. The point B lies vertically below the highest point of first particle. For both the bodies to collide, v_2/v_1 should be = ??

a. 2

b. $\sqrt{\frac{3}{2}}$

c. 0.5 ✓

d. 1



$t = t = \frac{T_1}{2}$

Take

Collision time $T_1 = \left(\frac{2V_{1y}}{g}\right)$

$t = \frac{T_1}{2}$

$t = \left(\frac{V_{1y}}{g}\right)$ ✓

$H_{max} = \left(\frac{u_{1y}^2}{2g}\right)$

For particle-2

$H_{max} = v_2 t - \frac{1}{2} g t^2$

$\frac{V_{1y}^2}{2g} = V_2\left(\frac{V_{1y}}{g}\right) - \frac{1}{2} g \left(\frac{V_{1y}}{g}\right)^2$

$\frac{V_{1y}^2}{2\cancel{g}} + \frac{V_{1y}^2}{2\cancel{g}} = \frac{V_2 \cdot V_{1y}}{\cancel{g}} \Rightarrow \boxed{V_{1y} = u_2}$ ✓

$\tan 30° = \frac{V_{1y}}{V_{1x}}$

$V_{1x} = \sqrt{3} V_{1y} = \sqrt{3} u_2$

$u_1 = \sqrt{(u_{1y}^2 + u_{1x}^2)}$

$u_1^2 = u_2^2 + 3u_2^2$

$u_1^2 = 4u_2^2 \Rightarrow (u_1/u_2) = \sqrt{4} = 2/1$

$\frac{v_2}{v_1} = \frac{1}{2}$

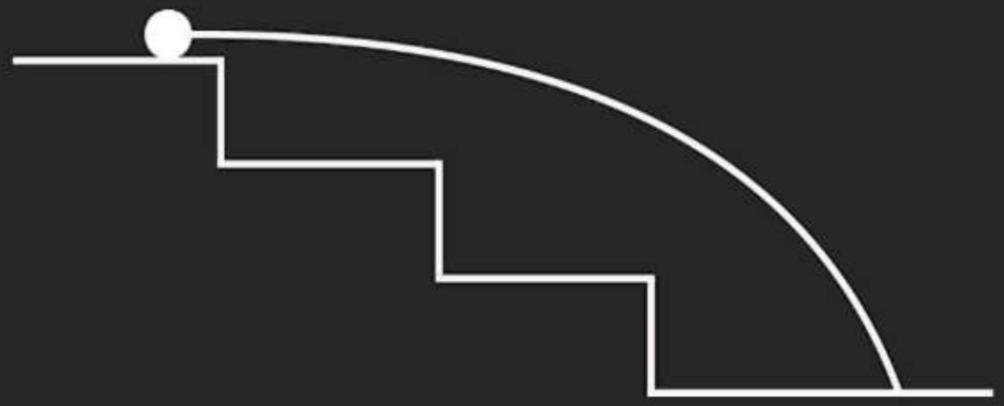
V_{1y} — V — 30° — V_{1x}

Nishant Jindal

Projectile motion

H.W.

Q. A staircase contains three steps each $10\ \text{cm}$ high and $20\ \text{cm}$ wide. What should be the minimum horizontal velocity of the ball rolling off the uppermost plane so as to hit directly the lowest plane? (in ms^{-1})



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Projectile motion

H.W

Q. A particle is thrown over a triangle from one end of a horizontal base and grazing the vertex falls on the other end of the base. If α and β be the base angles and θ be the angle of projection, prove that $\tan\theta = \tan\alpha + \tan\beta$.

Nishant Jindal

Projectile motion

H.W

Q. A student and his friend while experimenting for projectile motion with a stop-watch, taken some approximate readings. As one throws a stone in air at some angle, other observes that after 2.0 s it is moving at an angle 30° to the horizontal and after 1.0 s, it is travelling horizontally. Determine the magnitude and the direction of initial velocity of the stone.

Nishant Jindal

Projectile motion

H.W

Q. A particle moves in the plane xy with constant acceleration a directed along the negative y-axis. The equation of motion of the particle has the form $y = k_1 x - k_2 x^2$, where k_1 and k_2 are positive constants. Find the velocity of the particle at the origin of coordinates.

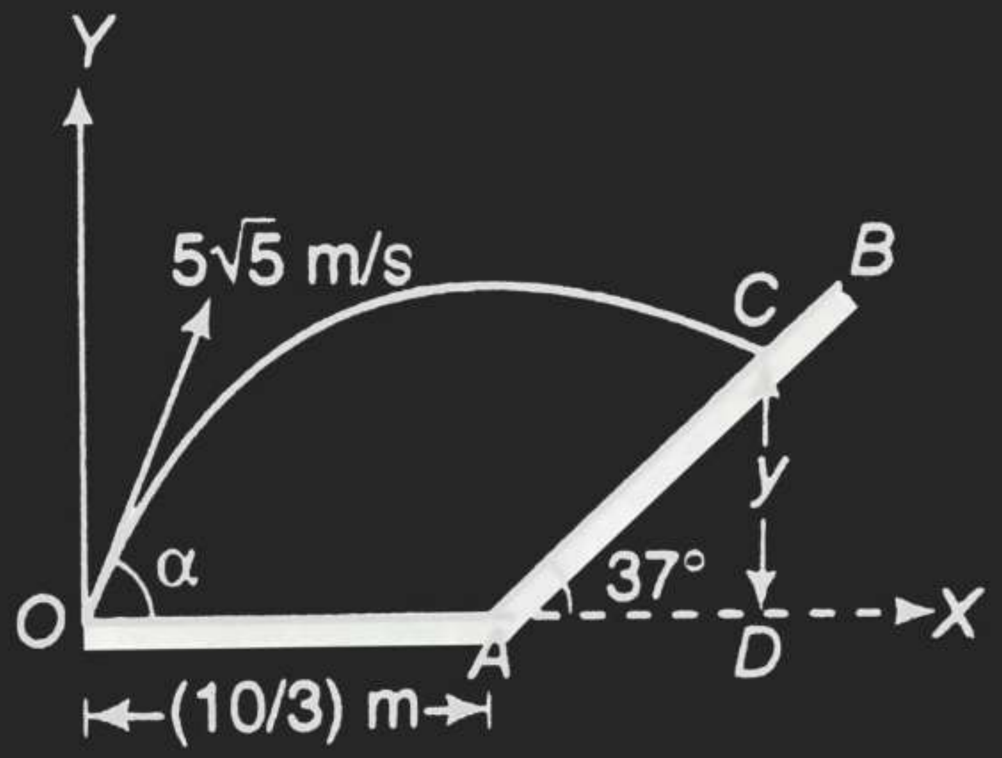
Projectile motion

H-W

Q. A particle is projected from point O on the ground with velocity $u = 5\sqrt{5}\ m/s$ at angle $\alpha = \tan^{-1}(0.5)$. It strikes at a point C on a fixed smooth plane AB having inclination of $37°$ with horizontal as shown in Fig. If the particle does not rebound, calculate

(a) coordinates of point C in reference to coordinate system as shown in the figure.

(b) maximum height from the ground to which the particle rises. $(g = 10\ m/s^2)$.



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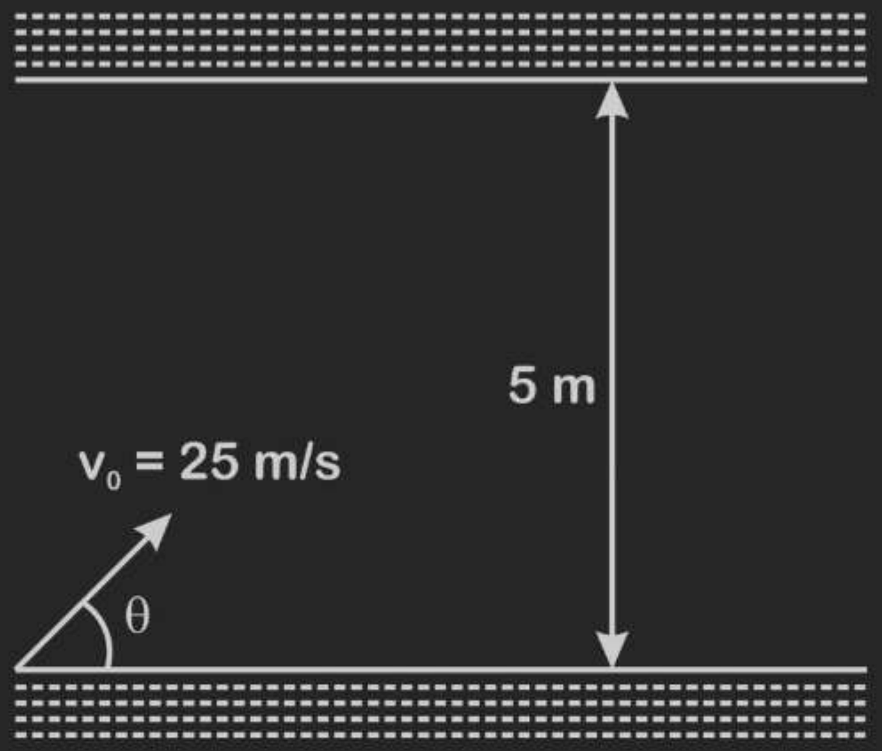
Projectile motion

H.W.

Q. A projectile is launched with a speed $v_B = 25\ \mathrm{m/s}$ from the floor of a $5\ \mathrm{m}$ high tunnel as shown in figure. Determine the maximum horizontal range R of the projectile and the corresponding launch angle θ.



Projectile motion

H.W.

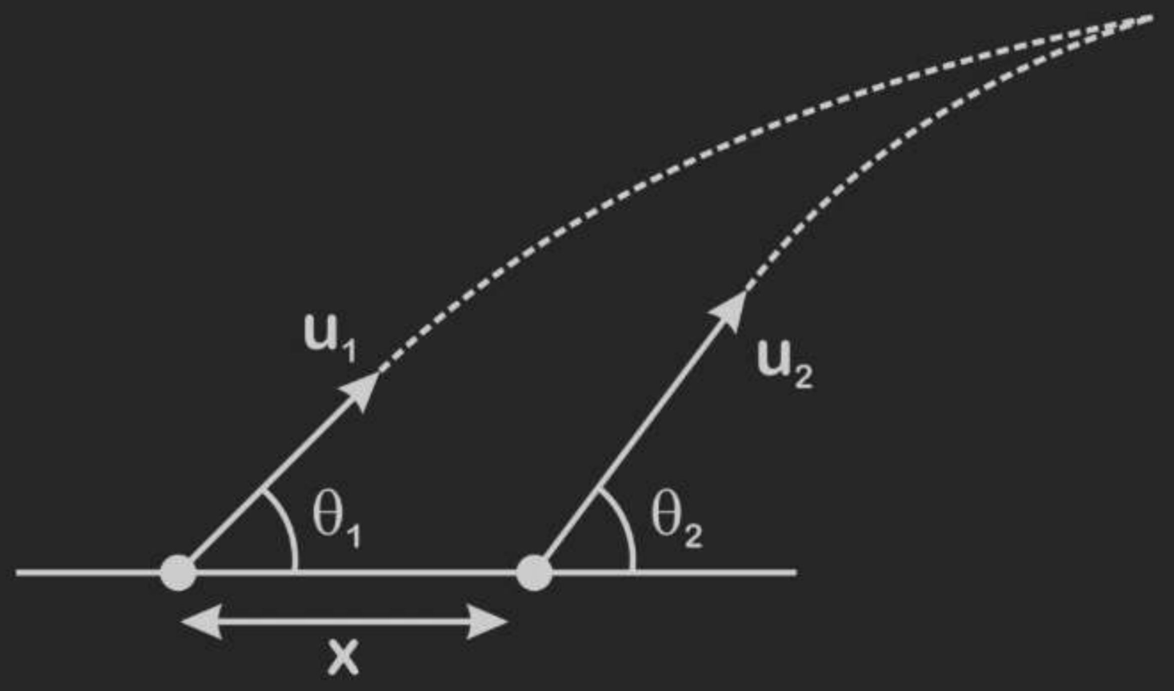
Q. Two particles are projected simultaneously from the level ground as shown in figure. They may collide after a time :

(a) $\frac{x\sin\theta_2}{u_1}$

(b) $\frac{x\cos\theta_2}{u_2}$

(c) $\frac{x\sin\theta_2}{u_1\sin(\theta_2-\theta_1)}$

(d) $\frac{x\sin\theta_1}{u_2\sin(\theta_2-\theta_1)}$



Nishant Jindal

HW

Q. A particle is projected from the ground. If the equation of the trajectory is $y = x - \frac{x^2}{2}$, then the time of flight is:

(a) $\frac{2}{\sqrt{g}}$

(b) $\frac{3}{\sqrt{g}}$

(c) $\frac{9}{\sqrt{g}}$

(d) $\sqrt{\frac{2}{g}}$

Nishant Jindal

H.W.

Q. A projectile moves from the ground such that its horizontal displacement is $x = Kt$ and vertical displacement is $y = Kt(1 - \alpha t)$, where K and α are constants and t is time. Find out total time of flight (T) and maximum height attained (Y_{max}) its

(a) $T = \alpha, Y_{max} = \frac{K}{2\alpha}$

(b) $T = \frac{1}{\alpha}, Y_{max} = \frac{2K}{\alpha}$

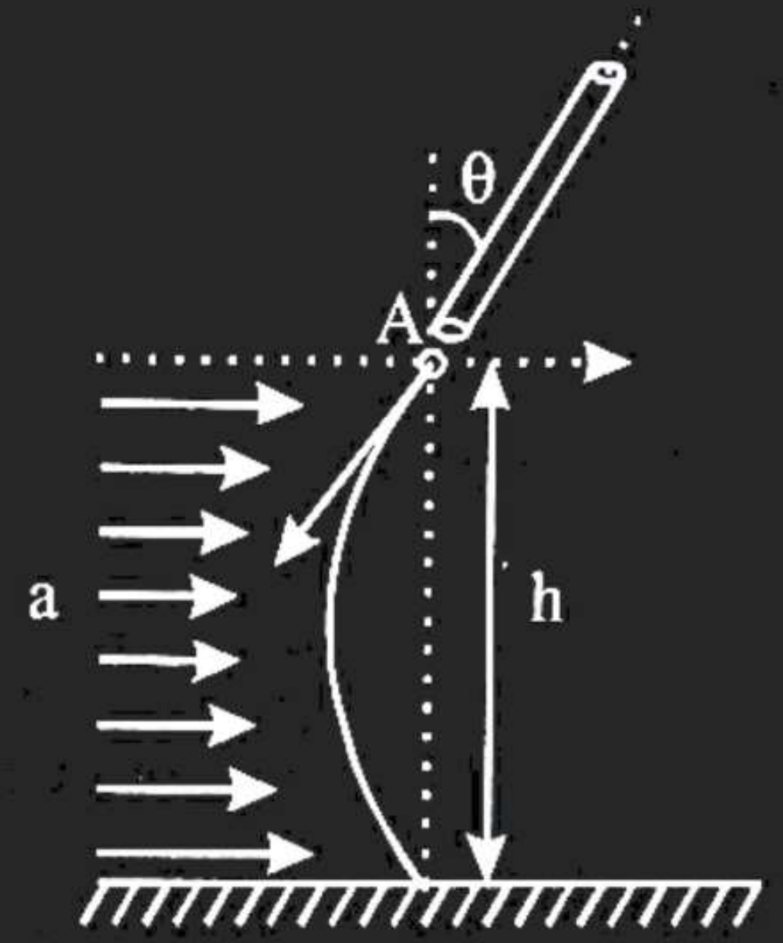
(c) $T = \frac{1}{\alpha}, Y_{max} = \frac{K}{6\alpha}$

(d) $T = \frac{1}{\alpha}, Y_{max} = \frac{K}{4\alpha}$

Projectile motion

H.W

Q. A particle is ejected from the tube at A with a velocity v at an angle θ with the vertical y-axis. A strong horizontal wind gives the particle a constant horizontal acceleration a in the x-direction. If the particle strikes the ground at a point directly under its released position and the downward y-acceleration is taken as g then



(a) $h = \dfrac{2v^2 \sin\theta \cos\theta}{a}$

(b) $h = \dfrac{2v^2 \sin\theta \cos\theta}{g}$

(c) $h = \dfrac{2v^2}{g} \sin\theta \left(\cos\theta + \dfrac{a}{g} \sin\theta\right)$

(d) $h = \dfrac{2v^2}{a} \sin\theta \left(\cos\theta + \dfrac{g}{a} \sin\theta\right)$

Nishant Jindal

Projectile motion

HW :

Q. Trajectories are shown in figure are for three kicked footballs, ignoring the effect of the air on the footballs. If T_1, T_2 and T_3 are their respective time of flights then:

(a) $T_1 > T_3$

(b) $T_1 < T_3$

(c) $T_2 = \frac{T_1+T_3}{2}$

(d) $T_1 = T_2 = T_3$