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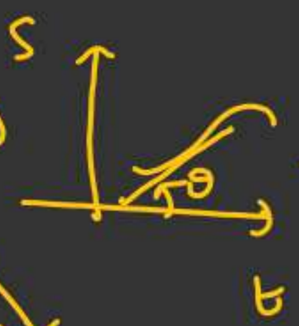
$\longrightarrow$

$ds/dt = v \Rightarrow$



$\tan\theta = \frac{ds}{dt}$

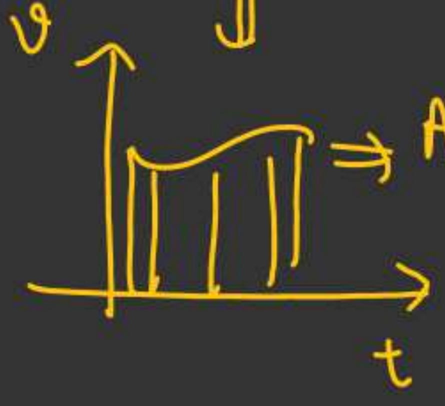
$\Downarrow$

Slope of s-vs-t graph gives velocity

$$S \to f(t) \xrightarrow{ds/dt = v} v \to f(t)$$

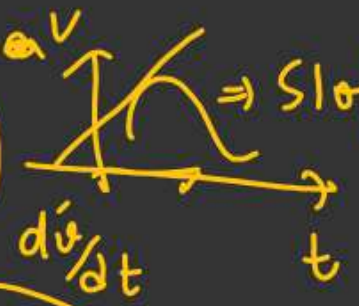
$$v \to f(t) \xrightarrow{\int v\,dt} S \to f(t)$$

$\Downarrow$



$\Rightarrow$ Area under v vs t graph = displacement. Δs

$\left(a = \frac{dv}{dt}\right)$



$\Rightarrow$ Slope $\to$ velocity

$$v \to f(t) \xrightarrow{dv/dt} a \to f(t)$$

$$a \to f(t) \xrightarrow{\int a\,dt} v \to f(t)$$

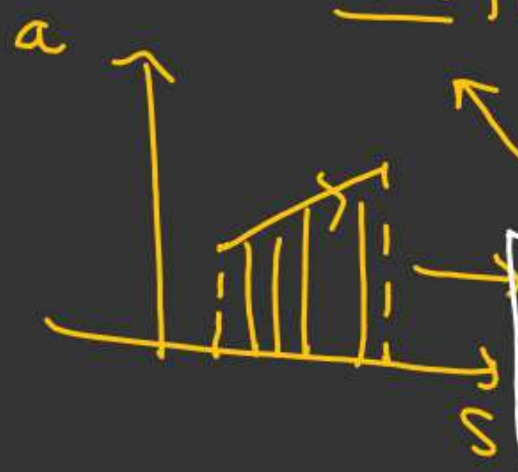
$\Rightarrow$ Area $= \Delta v$

$$\int a\,ds = \int_u^v v\,dv$$

$$v \to f(s) \xrightarrow{a = v\frac{dv}{ds}} a \to f(s)$$

$$a \to f(s) \xrightarrow{\int a\,ds} v \to f(s)$$

$$A = \frac{v^2 - u^2}{2}$$



$$\boxed{\frac{v^2 - u^2}{2} = \text{Area}}$$ **

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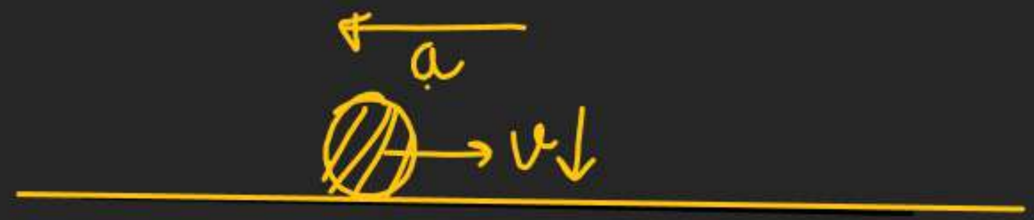
KINEMATICS

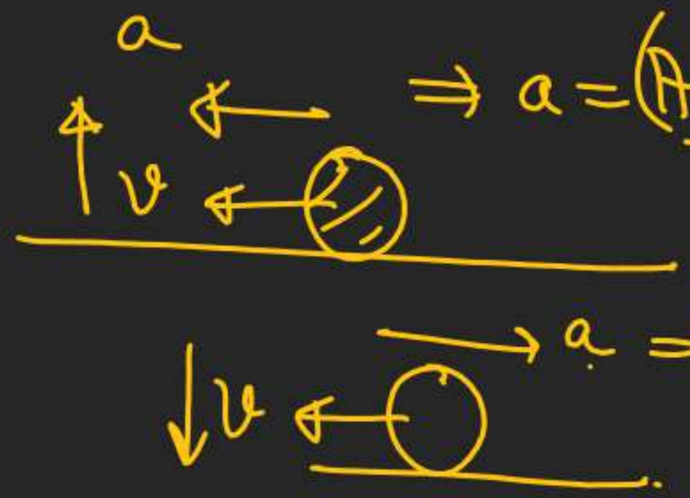
Calculas based questions

Q.1 A particle starts moving rectilinearly (one dimension) at time $t = 0$ such that its velocity v changes with time t according to the equation $\boxed{v = t^2 - t,}$ where t is in seconds and v in $\mathrm{m\,s^{-1}}$. Find the time interval for which the particle retards.

Retardation :-



a → Retardation



⇒ a = (Accleration)

a = (Retardation)

$v = (t^2 - t)$

$a = \dfrac{dv}{dt} = (2t - 1)$

At $t = 0$

$\left[\begin{array}{l} a = -1\,m/s^2 \\ v = 0\,m/s \end{array}\right.$

$\Delta t \rightarrow \left[\begin{array}{l} 0.5\,sec \\ \text{to } 1\,Sec \end{array}\right]$

↳ Retard

At $t = 1\,sec$

$a = 1\,m/s^2$

$a = 0.$

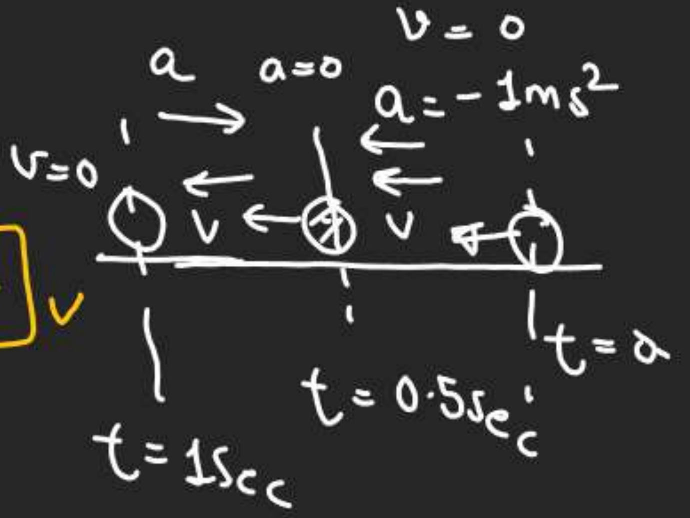
$(2t - 1) = 0$

$\boxed{t = \frac{1}{2}\,Sec}$ ✓

Retardation Continue untill and unless $v = 0.$

$v = 0.$

$t(t-1) = 0.$, $t = 0$, $t = 1\,sec$



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$v = (t^2 - t)$ ⇒ (Parabola) opening upward. ⇒ Slope of v-t graph gives accleration.

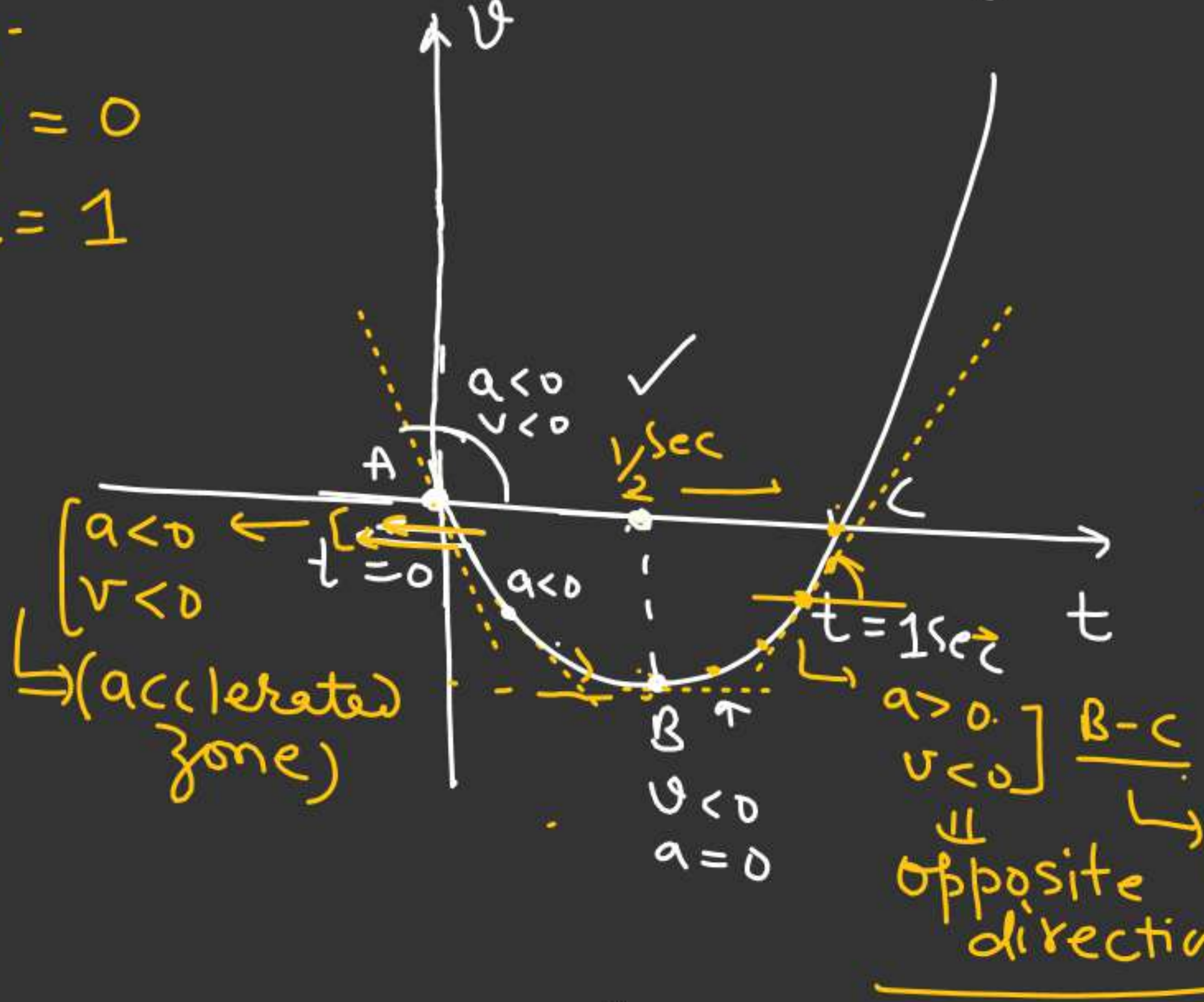
roots

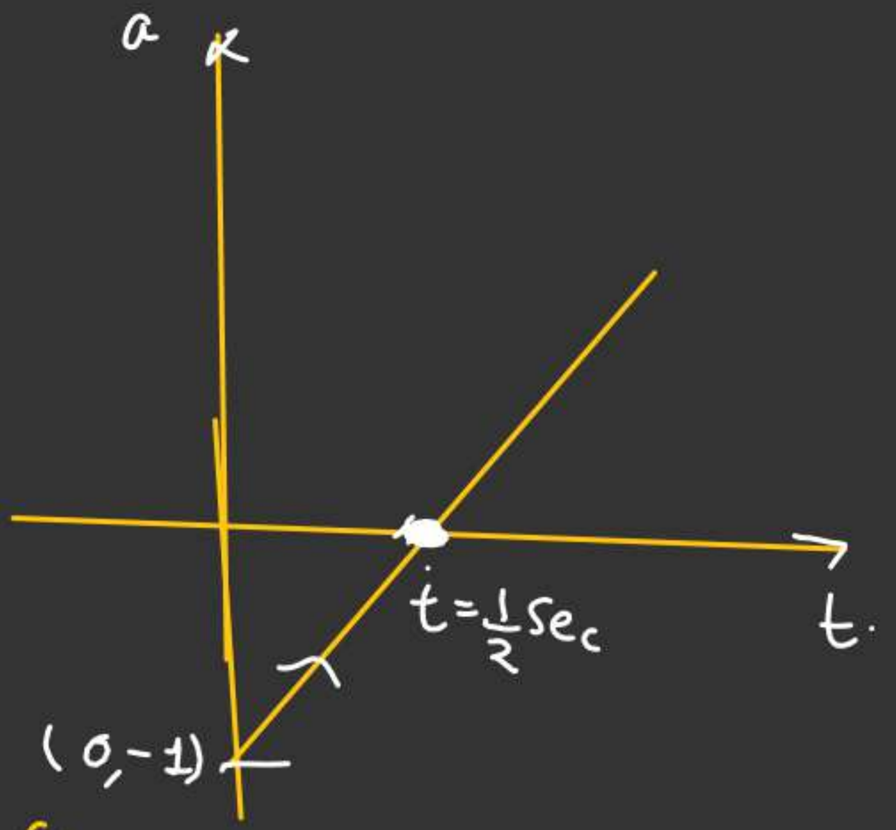
$v = 0$

$t(t-1) = 0$

$t = 0, \; t = 1$

$a = \frac{dv}{dt} = (2t - 1)$





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Calculas based questions

KINEMATICS

0.

Q.2 The position of a particle moving along x-axis is related to time t as follow:

$\boxed{x = 2t^2 - t^3}$, where x is in meters and t is in seconds.

Displacement

(A) What is the maximum positive displacement of the particle along the x axis and at what instant does it attain it?

$\boxed{x_{t=0} = 0}$ Initial position

(B) Describe the motion of the particle.

(C) What is the distance covered in the first three seconds?

(D) What is its displacement in the $\boxed{\text{first four seconds}}$?

$x_{t=4\,sec} = 2\times(4)^2 - (4)^3 = 32 - 64 = -32\,m$

e. What is the particle's average speed and average velocity in the first 3 seconds?

$V_{avg\ speed} = \left(\dfrac{\text{Total distance}}{\text{time}}\right) = \left(\dfrac{\frac{64}{27}+9}{3}\right)$, $V_{avg\ velocity} = \dfrac{-9}{3} = -3\,m/s$

f. What is the particles instantaneous acceleration at the instant of its maximum positive x displacement?

g. What is the average acceleration between the interval $\boxed{t = 2\text{ s to } t = 4\text{ s}}$?

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$$\boxed{x = 2t^2 - t^3}$$

$$v = \frac{dx}{dt} = (4t - 3t^2)$$

$$\boxed{a = (4 - 6t)}$$

$$v = 0$$

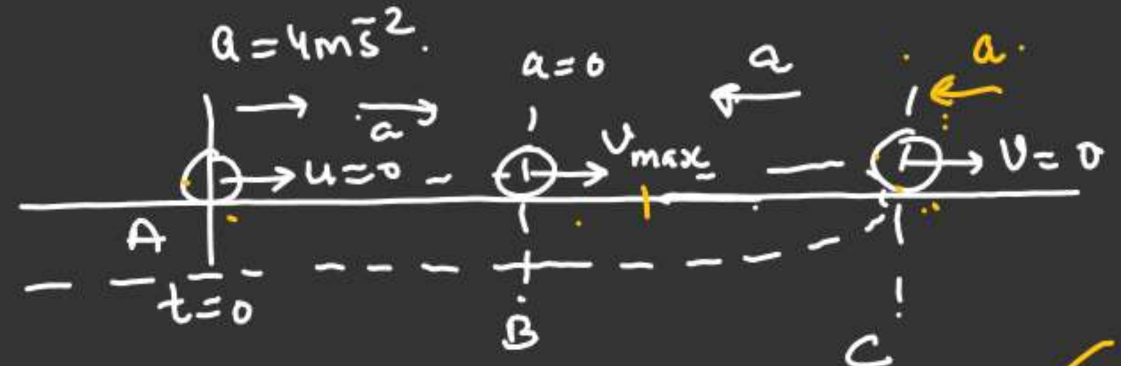
$$t(4 - 3t) = 0$$

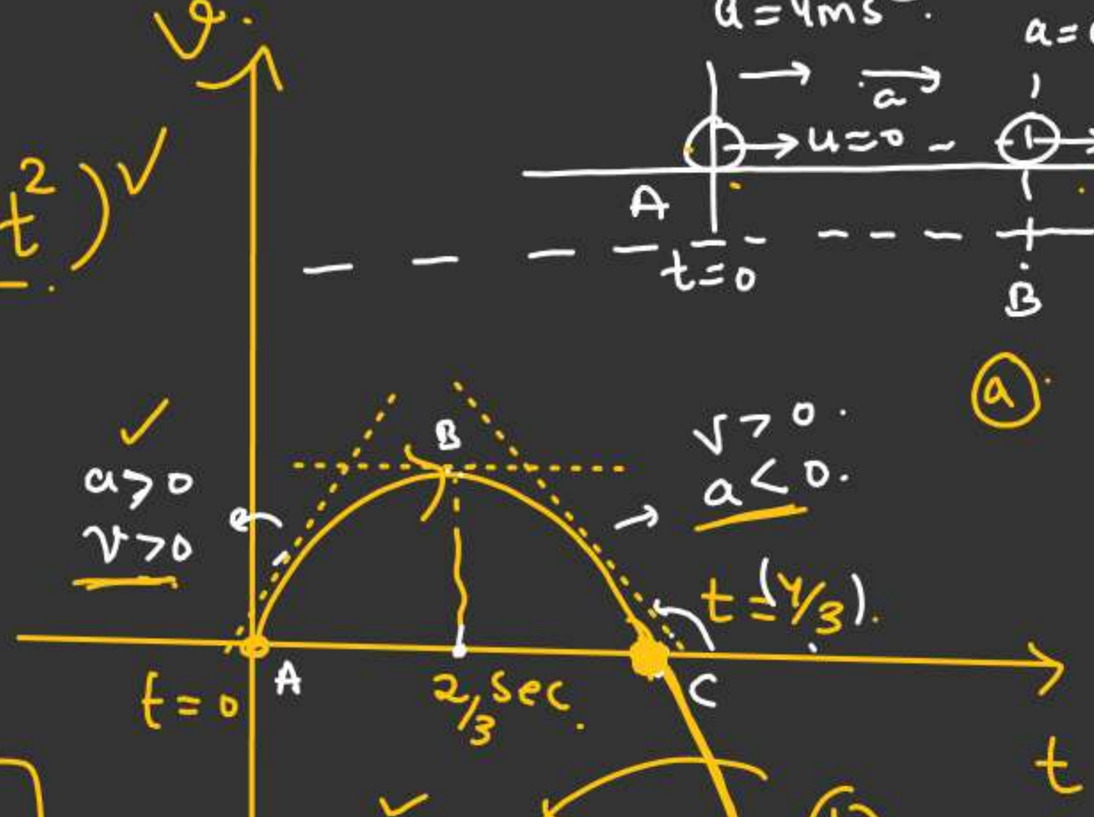
$$\boxed{t = 0,\ t = \frac{4}{3}\text{ sec}}$$

$$a_{t=\frac{4}{3}\text{sec}} = \left(4 - 6 \times \frac{4}{3}\right)$$

$$= (4 - 8)$$

$$= -4\text{ m s}^{-2}$$





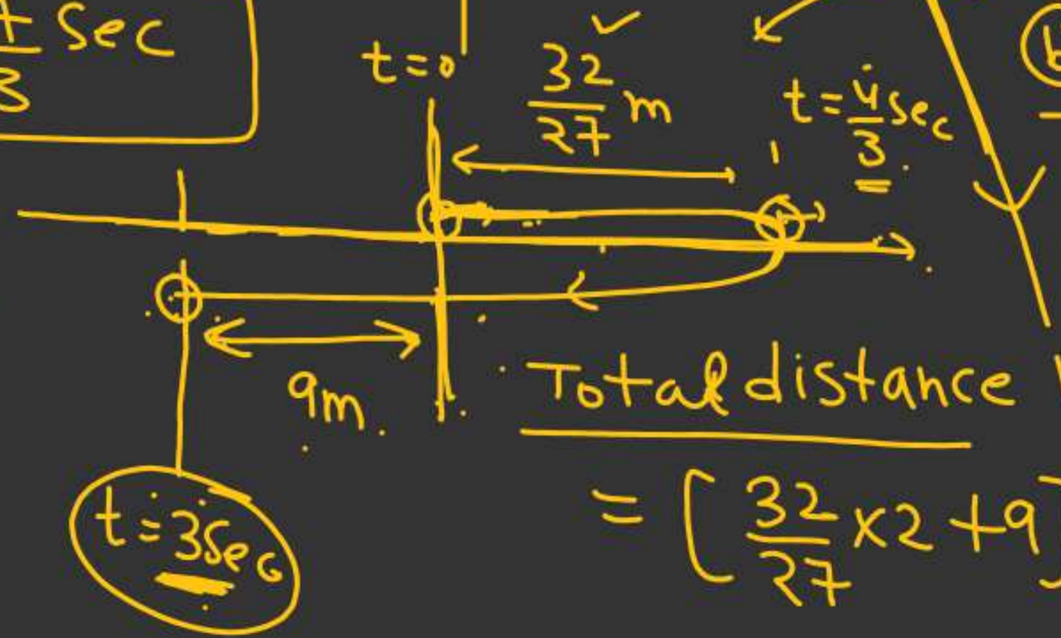
(a) At $t = \frac{4}{3}$ sec

$x_{max} = ??$

$$x_{(t = \frac{4}{3}\text{sec})} = 2 \times \left(\frac{4}{3}\right)^2 - \left(\frac{4}{3}\right)^3$$

$$= \left(\frac{4}{3}\right)^2 \left[2 - \frac{4}{3}\right]$$

$$= \frac{16}{9}\left[\frac{2}{3}\right] = \frac{32}{27}\text{ m}$$



(b) — $t_1 = \frac{4}{3}$ sec

$x_1 = \frac{32}{27}$ m

$t_2 = 3$ sec

$$x_2 = 2(3)^2 - (3)^3$$

$$= 18 - 27 = -9\text{ m}$$

Total distance

$$= \left[\frac{32}{27} \times 2 + 9\right] = \left(\frac{64}{27} + 9\right)$$

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Avg accleration :-

$a = (4 - 6t)$

$a \to f(t)$.

$v = (4t - 3t^2)$

$v_{t=2sec} = (4\times2) - 3(2)^2$

$= 8 - 12$

$= -4\,m/s$

$\vec{a}_{avg} = \dfrac{\vec{\Delta v}}{\Delta t}$

$v_{t=4sec} = (4\times4) - 3(4)^2$

$= (16) - (16\times3)$

$= 16(1-3)$

$= -32\,m/s$.

$\vec{a}_{avg} = \dfrac{\vec{v}_f - \vec{v}_i}{\Delta t}$

$= \dfrac{-32\hat{i} - (-4\hat{i})}{2}$

$= \dfrac{(-32+4)}{2}\hat{i} = \dfrac{-28}{2}\hat{i}$.

$= - 14\,m/s^2\,\hat{i}$.

Calculas based questions

KINEMATICS

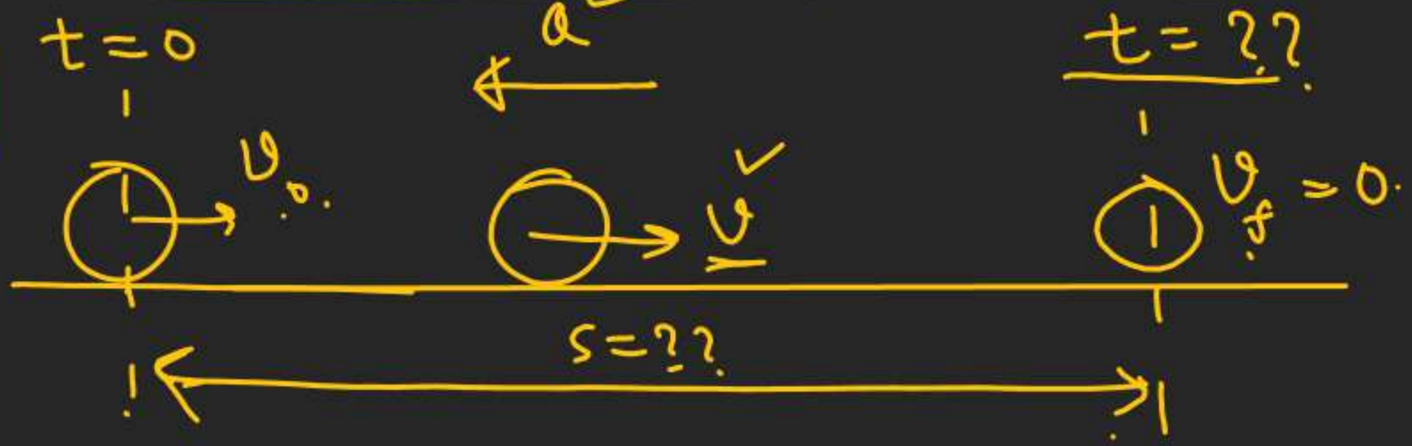
Q.3 A particle retards from a velocity v_0 while moving in a straight line. If the magnitude of deceleration is directly proportional to the square root of the speed of the particle, find its average velocity for the total time of its motion.

Sol^n

$a \propto \sqrt{v}$

$a = -K\sqrt{v}$

$\boxed{v \rightarrow f(t)}$



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1st Method

$$a = -k\sqrt{v}$$

$$\frac{dv}{dt} = -k\sqrt{v}$$

$$\int_{v_0}^{v} \frac{dv}{\sqrt{v}} = -k\int_0^t dt$$

$$\int_{v_0}^{v} v^{-1/2}\, dv = -k\int_0^t dt$$

$$\frac{\left[v^{1/2}\right]_{v_0}^{v}}{1/2} = -kt$$

$$2\left[\sqrt{v} - \sqrt{v_0}\right] = -kt$$

$$\sqrt{v} - \sqrt{v_0} = \left(\frac{-kt}{2}\right)$$

$$\sqrt{v} = \left(\sqrt{v_0} - \frac{k}{2}t\right)$$

$$v = \left(\sqrt{v_0} - \frac{k}{2}t\right)^2$$

$v = 0$

$t = ??$

$$v_{avg} = \left(\frac{\int_0^t v\, dt}{\int_0^t dt}\right)$$