

# Fundamentals of Mathematics

$$Q \quad |x-4| + |x-3| = 7$$

$$Q \quad |x-1| + |x-3| = 4$$

$$Q \quad |2x-1| + |x| = 3$$



$$x = \frac{4}{3}, -\frac{2}{3}$$

A)  $x > \frac{1}{2}$        $x = 1 \quad |2x-1| + |1|$   
 $2x-1 + x = 3 \quad 3x = 4 \Rightarrow x = \frac{4}{3} \quad \checkmark$

B)  $0 \leq x \leq \frac{1}{2}$        $x = \frac{1}{2} \quad |2x-1| + |\frac{1}{2}|$   
 $-(2x-1) + (\frac{1}{2}) = 3 \quad -2x = 2 \Rightarrow x = -2 \notin [0, \frac{1}{2}]$

-  $(2x-1) + (x) = 3 \quad -3x = 2 \quad x = -\frac{2}{3}$

-  $x = 2 \Rightarrow x = -2 \notin [0, \frac{1}{2}]$

C)  $x < 0 \quad |2x-1| + |-1|$   
 $-(2x-1) - x = 3 \quad -3x = 2 \quad x = -\frac{2}{3}$

-  $(2x-1) - x = 3 \quad -x = 2 \quad x = -2$

$x = -\frac{2}{3} \quad \checkmark$

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Modulus Eqn.

Approach 2

$$\text{Q} |x-1| = 4 \text{ from } |x|.$$

$$x-1 = 4, -4$$

$$x = 5, -3$$

$$\text{Q. } ||x-2|-3| = 4.$$

$$|x-2|-3 = 4, -4$$

$$|x-2| = 4+3, -4+3$$

$$|x-2| = 7, \text{ } \textcircled{X}$$

$$|x-2|=7 \Rightarrow x-2=7, -7 \quad \boxed{x=9, -5}$$

$$\text{APP2} \quad |x|^2 = x^2$$

$$\text{Q } x^2 + 7|x| + 10 = 0 \text{ find } x?$$

$$|x|^2 + 7|x| + 10 = 0$$

$$t^2 + 7t + 10 = 0$$

$$(t+2)(t+5) = 0$$

$$t = -2 \text{ or } t = -5$$

$$|x| = -2 \text{ or } |x| = -5$$

⊕ +ve      ⊕ -ve

$$x = \emptyset$$

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APP-3

$$|\int(x)| = \int(x) \Rightarrow \int(x) \geq 0$$

$$|f(\lambda)| = -f(\lambda) \Rightarrow f(\lambda) \leq 0$$

$$Q \quad |x+2| = -(x+2) \text{ find } x$$

$$x+2 \leq 0$$

$$\chi_{(-\infty, -2]}$$

$$|x| = \begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases}$$

$$Q | \chi + 4 | = (\chi + 4)$$

As it is

$$= 1 \times 47,0$$

)(-7,-4

$$\gamma \in [-\infty, \infty)$$

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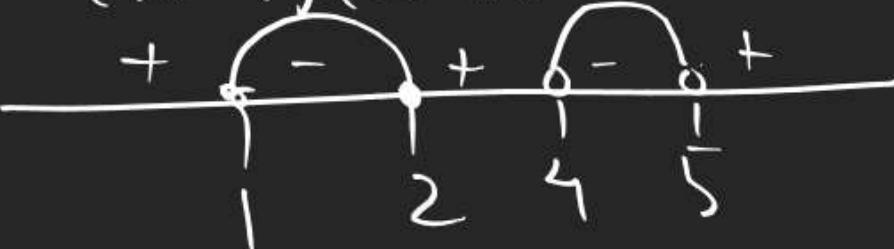
$\text{Q}$

$$\left| \frac{x^2 - 3x + 2}{x^2 - 9)(x+20) } \right| = - \left( \frac{x^2 - 3x + 2}{x^2 - 9)(x+20) } \right) \int dx$$

- Minus K Sath.

$$\frac{x^2 - 3x + 2}{x^2 - 9)(x+20)} \leq 0$$

$$\frac{(x-1)(x-2)}{(x-4)(x+5)}$$



$$x \in (-\infty, 1] \cup (4, 5)$$

APPY  
Prop

$|a+b| = |a| + |b|$  then  $a \cdot b \geq 0$

$\text{Q}$

$$|x-1| + |x+5| = |2x+4|$$

$$|a| + |b| = |(x-1) + (x+5)|$$

$|a+b|$

$$\Rightarrow a \cdot b \geq 0$$

$$|3| + |5| = |3+5|$$

$$|3| + |5| = |3+5|$$

$$|-3| + |-5| = |-3-5|$$

$$3 + 5 = |-8| = 8$$

$$|3| + |-5| = |3 + (-5)|$$

$$3 + 5 = |-2| = 2$$

$$(x-1)(x+5) \geq 0$$



$$x \in (-\infty, -5] \cup [1, \infty)$$

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## Angle Measurement

A) Sexagesimal System [Degree]

In this System we divide

a Rt. angle in 90 equal parts.

& each part is known as  $1^\circ$ .

(B) Centesimal System [Grade]

In this system we have divided a Rt. angle into 100 equal parts. Each part is known as "grade".

((C)) Circular System [Radian]

1 Radian is defined by an angle subtended at centre of a circle by an arc equal to its

Radius.

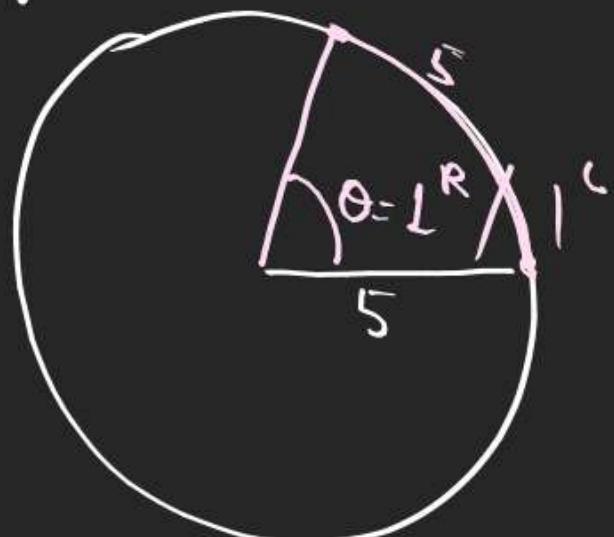
$$\text{Angle} = \frac{\text{Arc}}{\text{Radius}}$$

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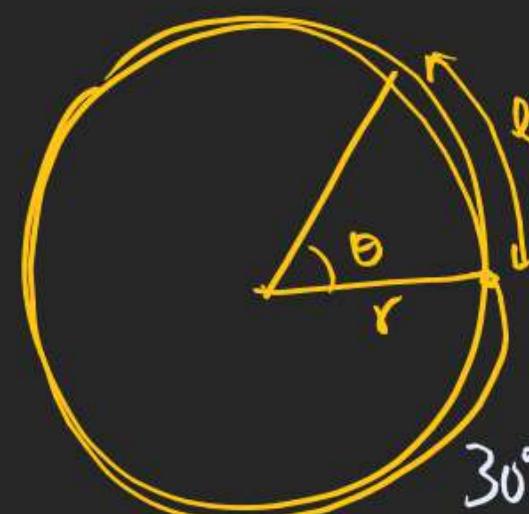
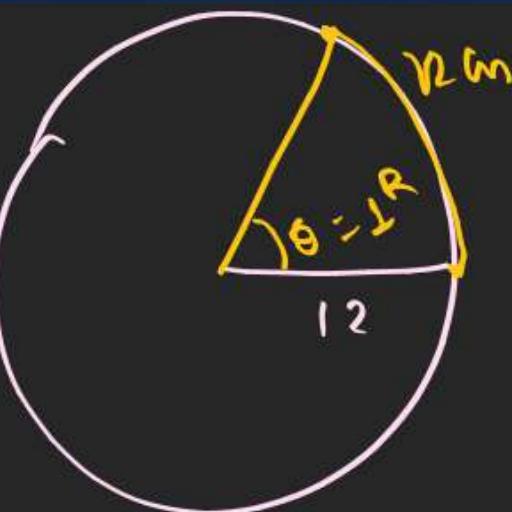
If  $l$  hem Arc = Radius

$$\Rightarrow \theta = \text{angle} = \frac{\text{Arc}}{\text{Radius}} = 1^{\text{R}}$$

$\Rightarrow$  If circle is of rad = 5  
then angle projected by  
Arc of 5 is 1 Rad



$$\begin{aligned} 1^{\text{C}} &\approx 57^{\circ} \\ \pi^{\text{C}} &= 180 \\ 2\pi^{\text{C}} &= 360^{\circ} \end{aligned}$$



$$\theta = \frac{l}{r} \Rightarrow$$

$$l = r\theta$$

$$1^{\text{C}} = \frac{\pi^{\text{C}}}{180}$$

$$30^{\circ} = \frac{\pi^{\text{C}}}{180} \times 30$$

If hem 2 complete rotation is done

$$l = 2\pi r$$

$$\theta = \frac{l}{r} = \frac{2\pi r}{r} = 2\pi^{\text{R}} = 360^{\circ}$$

$$\pi^{\text{C}} = 180^{\circ}$$

$$1^{\text{C}} \approx 57^{\circ}$$

$$1^{\text{C}} = \frac{180^{\circ}}{\pi} = \frac{180^{\circ}}{3.14} \approx 57^{\circ}$$

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Q) Convert from deg to Rad?

$$45^\circ \rightarrow 45 \times \frac{\pi}{180} = \frac{\pi}{4}$$

$$30^\circ \rightarrow 30 \times \frac{\pi}{180} = \frac{\pi}{6}$$

$$60^\circ \rightarrow 60 \times \frac{\pi}{180} = \frac{\pi}{3}$$

$$90^\circ \rightarrow 90 \times \frac{\pi}{180} = \frac{\pi}{2}$$

$$120^\circ \rightarrow 120 \times \frac{\pi}{180} = \frac{2\pi}{3}$$

$$150^\circ \rightarrow 150 \times \frac{\pi}{180} = \frac{5\pi}{6}$$

$$180^\circ \rightarrow \pi$$

$$135^\circ \rightarrow 135 \times \frac{\pi}{180} = \frac{3\pi}{4}$$

$$215^\circ \rightarrow 215 \times \frac{\pi}{180} = 4\frac{3\pi}{36}$$

$$300^\circ \rightarrow 300 \times \frac{\pi}{180} = 5\frac{\pi}{3}$$

$$330^\circ \rightarrow 330 \times \frac{\pi}{180} = 11\frac{\pi}{6}$$

(2)

Degree to Radian = Multiply by  $\frac{\pi}{180}$

Radian to Degree = Multiply by  $= \frac{180}{\pi}$

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## Some Important Deg & Rad.

Quadrant	Angle (Deg)	Angle (Rad)
1 <sup>st</sup>	0, 360°	
2 <sup>nd</sup>	90°	$\frac{\pi}{2}$
3 <sup>rd</sup>	180°	$\pi$
4 <sup>th</sup>	270°	$\frac{3\pi}{2}$
1 <sup>st</sup> Quad	30° → $\frac{\pi}{6}$	120° → $\frac{2\pi}{3}$
2 <sup>nd</sup> Quad	45° → $\frac{\pi}{4}$	135° → $\frac{3\pi}{4}$
3 <sup>rd</sup> Quad	60° → $\frac{\pi}{3}$	150° → $\frac{5\pi}{6}$
4 <sup>th</sup> Quad	75° → $\frac{5\pi}{12}$	180° → $\pi$
1 <sup>st</sup> Quad	90° → $\frac{\pi}{2}$	210° → $\frac{7\pi}{6}$
2 <sup>nd</sup> Quad	135° → $\frac{3\pi}{4}$	225° → $\frac{5\pi}{4}$
3 <sup>rd</sup> Quad	150° → $\frac{5\pi}{6}$	240° → $\frac{4\pi}{3}$
4 <sup>th</sup> Quad	180° → $\pi$	270° → $\frac{3\pi}{2}$
1 <sup>st</sup> Quad	300° → $\frac{5\pi}{3}$	
2 <sup>nd</sup> Quad	315° → $\frac{7\pi}{4}$	
3 <sup>rd</sup> Quad	330° → $\frac{11\pi}{6}$	
4 <sup>th</sup> Quad	360° → $2\pi$	

# Fundamentals of Mathematics

13 Lec

10<sup>th</sup> class

Imp Identities.

A)  $\sin^2\theta + \cos^2\theta = 1$

B)  $\sec^2\theta - \tan^2\theta = 1 \rightarrow ((\sec\theta - \tan\theta)(\sec\theta + \tan\theta) = 1)$

C)  $\sec^2\theta - \tan^2\theta = 1$

$$\frac{(\sec\theta - \tan\theta)}{(\sec\theta + \tan\theta)} = \frac{1}{(\sec\theta + \tan\theta)}$$

$$\sec\theta - \tan\theta = \frac{1}{\sec\theta + \tan\theta}$$

(D)  $\sec^2\theta - \tan^2\theta = (\sec\theta \cdot \cos\theta) \rightarrow P.T.Q.S$   
FTW

(E)  $\sec^2\theta - \tan^2\theta = \sec^2\theta \cdot \tan^2\theta$

$$\begin{aligned}
 & \sec^2\theta - \tan^2\theta \\
 &= \frac{\sin^2\theta}{\cos^2\theta} - \tan^2\theta \\
 &= \sin^2\theta \left( \frac{1}{\cos^2\theta} - 1 \right) \\
 &= \sin^2\theta \times \left( \frac{1 - \cos^2\theta}{\cos^2\theta} \right) \\
 &= \sin^2\theta \times \frac{\sin^2\theta}{\cos^2\theta} \\
 &= \underline{\tan^2\theta \times \sec^2\theta \text{ RHS}}
 \end{aligned}$$

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F)  
New

$$\sin^4 \theta + \cos^4 \theta = 1 - 2 \sin^2 \theta \cdot \cos^2 \theta$$

$$\text{LHS} \left\{ a^2 + b^2 + 2ab \right\} \\ \left\{ (\sin^2 \theta)^2 + (\cos^2 \theta)^2 + 2 \sin^2 \theta \cdot \cos^2 \theta \right\} - 2 \sin^2 \theta \cdot \cos^2 \theta$$

$$\left\{ \sin^2 \theta + \cos^2 \theta \right\}^2 - 2 \sin^2 \theta \cos^2 \theta$$

$$1 - 2 \sin^2 \theta \cos^2 \theta \text{ RHS}$$

(h) New

$$\sin^6 \theta + \cos^6 \theta = 1 - 3 \sin^2 \theta \cdot \cos^2 \theta$$

$$\text{LHS} \quad (\sin^2 \theta)^3 + (\cos^2 \theta)^3 \\ = (\sin^2 \theta + \cos^2 \theta)^3 - 3 \cdot \sin^2 \theta \cdot \cos^2 \theta (\sin^2 \theta + \cos^2 \theta) \\ = 1 - 3 \sin^2 \theta \cdot \cos^2 \theta$$

RHS

$$a^3 + b^3 + 3ab(a+b) = (a+b)^3$$

$$a^3 + b^3 = (a+b)^3 - 3ab(a+b)$$

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Q.  $\sec^4 x - 6\sec^2 x + 25 \tan^2 x + 26 \sec^2 x = \frac{15}{4}$

find  $\tan x = ?$

$$(\sec^4 x - 2 \sec^2 x + 1) - ((\sec^4 x - 2(\sec^2 x + 1)) = \frac{15}{4}$$

$$(\sec^2 x - 1)^2 - (\tan^2 x - 1)^2 = \frac{15}{4}$$

$$(\tan^2 x)^2 - (\tan^2 x)^2 = \frac{15}{4}$$

$$\tan^4 x - \frac{1}{\tan^4 x} = 4 - \frac{1}{4}$$

$$\begin{aligned}\sec^2 x - \tan^2 x &= 1 \\ \sec^2 x - 1 &= \tan^2 x\end{aligned}$$

$$(\sec^2 x - 1)^2 = 1$$

$$(\sec^2 x - 1) = \tan^2 x$$

$$(\tan^2 x - 1) = \frac{1}{\tan^2 x}$$

$$\tan^4 x = 4$$

$$\tan^2 x = \pm 2 \Rightarrow \tan^2 x = 2 \quad \text{or} \quad \tan^2 x = -2$$

$$\tan^2 x = 2$$

$$\tan x = \pm \sqrt{2}$$

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Q If  $\sin x + \delta m^2 x = 1$  then  $(\sin^3 x + 2 \sin^6 x + \sin^9 x) = ?$

$$\sin x = 1 - \delta m^2 x$$

$$\sin x = \sqrt{\sin^2 x}$$

$$\sin^2 x = (\sin x)^2$$

$$\sin^4 x = (\sin^2 x)^2$$

$$\sin^4 x + 2(\sin^2 x) \cdot (\sin^6 x) + \sin^8 x$$

$$\sin^4 x + 2(\sin^2 x) \cdot (\sin^2 x) + \sin^4 x$$

$$\sin^4 x + 2(\sin^2 x) \cdot (\sin^2 x) + \sin^4 x$$

$$\underbrace{(\sin^4 x + 2 \sin^2 x \cdot \sin^2 x + \sin^4 x)}$$

$$(\sin^2 x)^2 + 2 \sin^2 x \cdot \sin^2 x + (\sin^2 x)^2$$

$$\underbrace{(\sin^2 x + \sin x)^2} = 1^2 = 1$$

Kisi Ka Perfect Sqr. h?

# Fundamentals of Mathematics

Q If  $\sec x - \tan x = p$  then  $\sec x = ?$

$$\begin{aligned} \sec x - \tan x &= p \\ \sec x + \tan x &= \frac{1}{p} \end{aligned}$$

add

$$\frac{\sec x - \tan x + \sec x + \tan x}{2} = \frac{p + \frac{1}{p}}{2}$$

$$\sec x = \frac{1}{2} \left( p + \frac{1}{p} \right)$$

$$\sec x - \tan x = \frac{1}{\sec x + \tan x}$$

$$p = \frac{1}{\sec x + \tan x}$$

$$\sec x + \tan x = \frac{1}{p}$$

# Fundamentals of Mathematics

(Q) P.T.

$$1) \sqrt{\sec^2 \theta + \csc^2 \theta} = \tan \theta + \cot \theta$$

$$2) (\sec \theta - \csc \theta)(\sec \theta + \csc \theta)(\tan \theta + \cot \theta) = 1$$

$$3) \tan \theta + \sec \theta = 3 \quad \text{then } \tan \theta = ?$$

$$4) \text{If } 2 \tan \theta = 2 - \csc \theta \quad \text{then } \tan \theta = ?$$

$$5) 3 \sec^4 \theta + 8 = 10 \sec^2 \theta \quad \text{then } \tan \theta = ?$$

$$(6) \frac{\sin x + \csc x}{\csc^3 x} = \tan^3 x + \tan^2 x + \tan x + 1$$

$$\underline{\log K_i D^{PP} \rightarrow 3}$$