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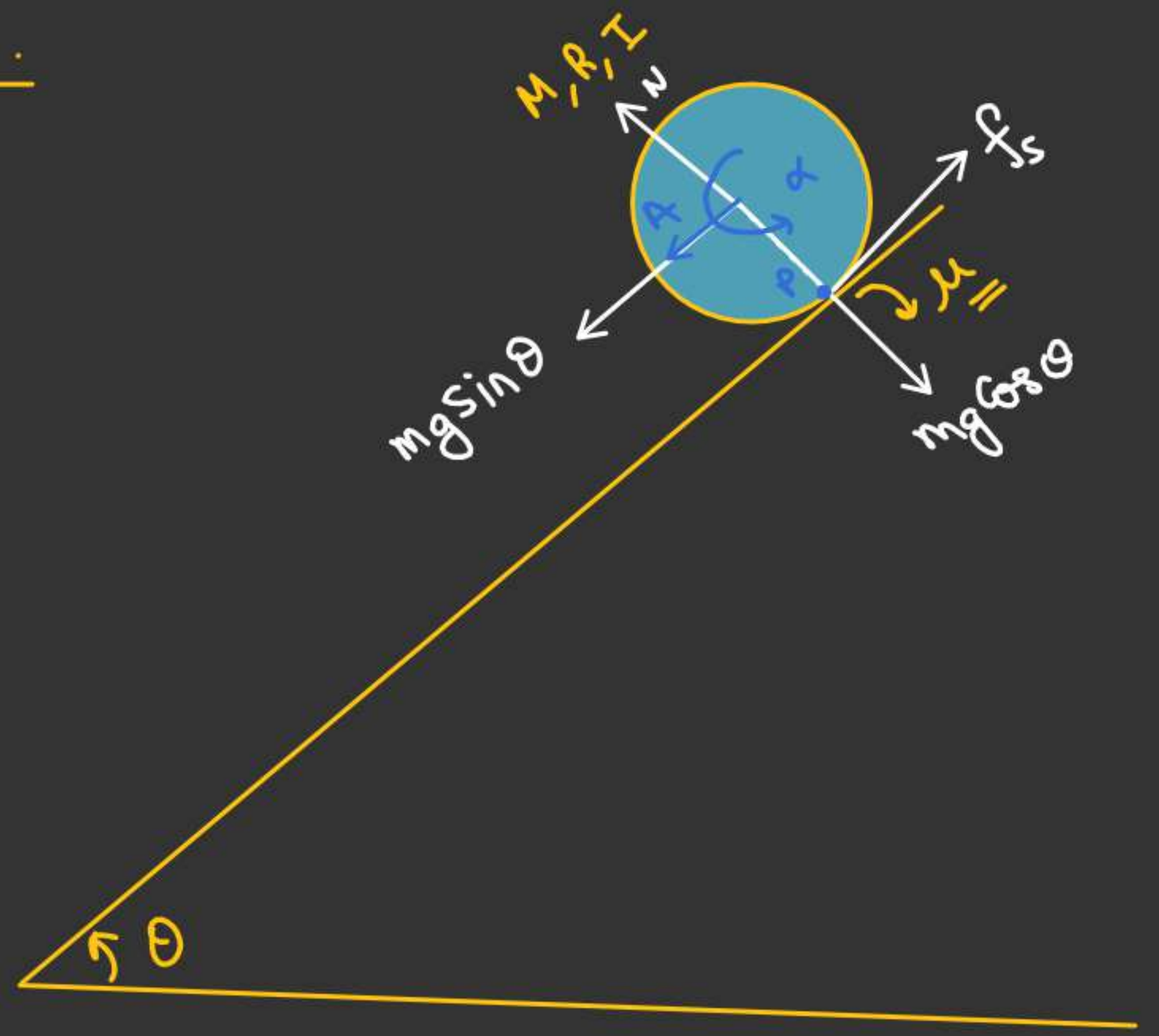
Rolling on an inclined plane.

Body starts pure rolling at the time when it is released.



At the time of pure rolling

$$A = R\alpha \quad -(1)$$

Equation for translational motion

$$mg\sin\theta - f_s = mA \quad -(2)$$

Equation for Rotational motion

$$f_s \cdot R = I\alpha \quad -(3)$$

Put $f_s = \frac{I\alpha}{R}$ in (2)

$$mg\sin\theta = mA + \frac{I\alpha}{R}$$

$\alpha = A/R$

$$mg\sin\theta = mA + \frac{IA}{R^2}$$

$$\cancel{m}g\sin\theta = \cancel{m}A\left[1 + \frac{I}{mR^2}\right]$$

$$\boxed{A = \frac{g\sin\theta}{1 + \frac{I}{mR^2}}}$$

$(N = mg\cos\theta)$ ✓

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$$f_s = mg\sin\theta - mA$$

$$f_s = mg\sin\theta - \frac{mg\sin\theta}{1+\frac{I}{mR^2}}$$

$$f_s = mg\sin\theta\left[\frac{\cancel{1}+\frac{I}{mR^2}-\cancel{1}}{1+I/mR^2}\right]$$

$$f_s = mg\sin\theta\left[\frac{I/mR^2}{1+I/mR^2}\right]$$

$$\boxed{f_s = \frac{mg\sin\theta}{1+\frac{mR^2}{I}}}$$

μ_{min} for pure rolling.

$$f_s \leq (f_s)_{max}$$

$$\frac{mg\sin\theta}{1+\frac{mR^2}{I}} \leq \mu\, mg\cos\theta.$$

$$\mu \geq \frac{\tan\theta}{1+\frac{mR^2}{I}}$$

$$\boxed{\mu_{min} = \frac{\tan\theta}{1+\frac{mR^2}{I}}}$$

<u>Note</u> :→ Rolling on an inclined plane f_s always acts.

$$-(W_{f_s})_{translational} = (W_{f_s})_{Rotational}.$$

So. $(W_{f_s})_{net} = 0$.

And we can conserved energy in case of rolling on an inclened plane.

⇒ [If f_k acts ie slipping then we cannot Conserve energy

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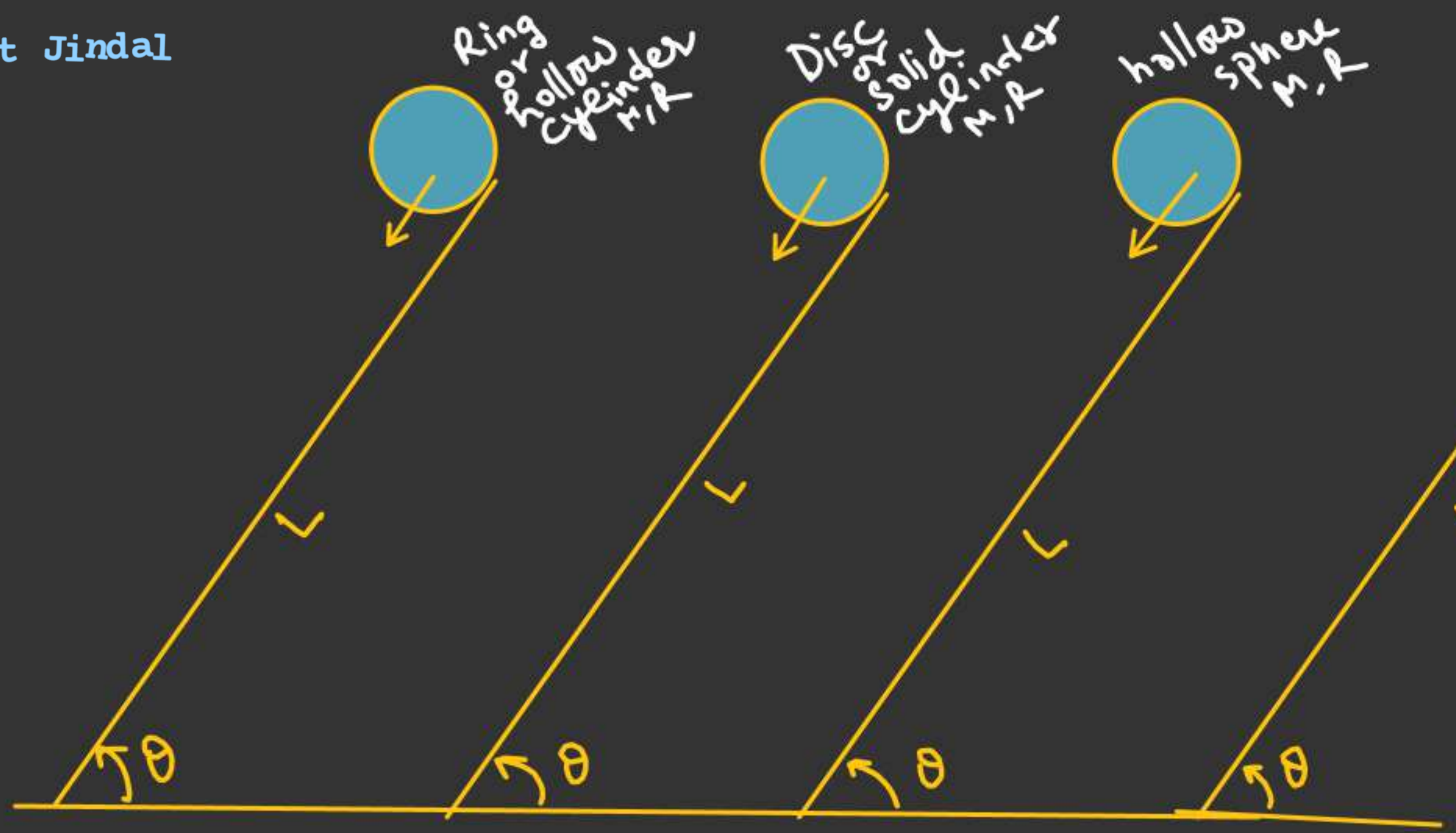

All the bodies released at $t=0$ from same vertical height h.

① Arrange in increasing order of time taken to reach ground.

$$A = \frac{g\sin\theta}{1+\frac{I}{MR^2}}$$

$$L = \frac{1}{2}At^2$$

$$t = \sqrt{\frac{2L}{A}}$$

$$\left(t \propto \frac{1}{A}\right)$$

$$A_{\substack{\text{Ring}\\ \text{or hollow cylinder}}} = \frac{g\sin\theta}{2} \simeq 0.5\, g\sin\theta$$

$$A_{\substack{\text{disc}\\ \text{or Solid cylinder}}} = \frac{2}{3}g\sin\theta \simeq 0.66\, g\sin\theta$$

$$A_{\substack{\text{hollow}\\ \text{Sphere}}} = \frac{3}{5}g\sin\theta \simeq 0.6\, g\sin\theta$$

$$A_{\substack{\text{solid}\\ \text{Sphere}}} = \frac{5}{7}g\sin\theta \simeq 0.7\, g\sin\theta$$

$$A_{\text{solid sphere}} > A_{\substack{\text{disc or}\\ \text{solid cylinder}}} > A_{\substack{\text{hollow}\\ \text{sphere}}} > A_{\text{Ring}}$$

$$t_{\text{solid sphere}} < t_{\substack{\text{disc}\\ \text{or solid cylinder}}} < t_{\substack{\text{hollow}\\ \text{sphere}}} < t_{\text{ring}}$$

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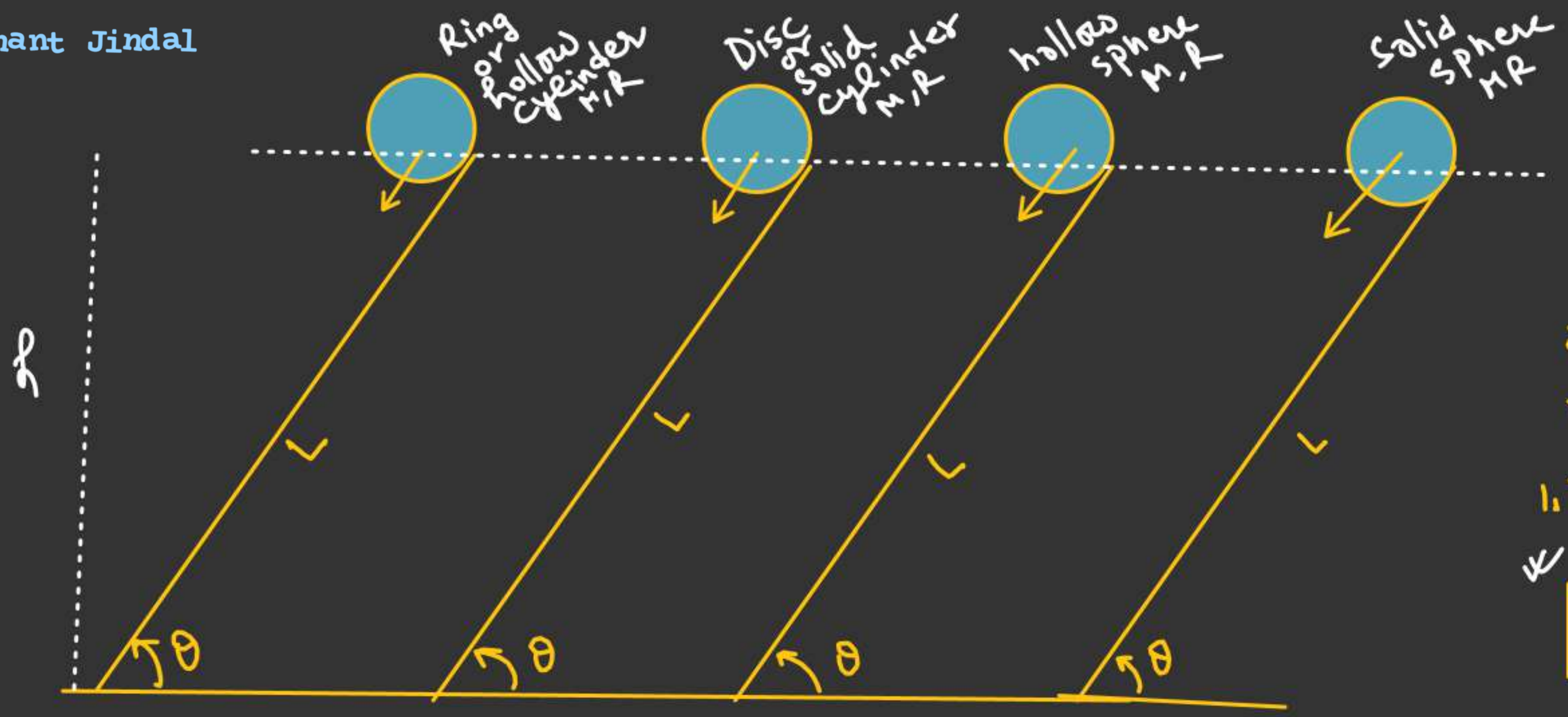

Arrange in increasing order of their
1) Translational Energy
2) Rotational Energy
3) Total Energy
When they reaches ground.

$$\boxed{U_T = \text{Same} = mgh.}$$

$$V^2 = \cancel{u^2}^{\,0} + 2AL$$

$$V = \sqrt{2AL} \qquad V \propto \sqrt{A}$$

$$A_{\text{solid sphere}} > A_{\text{disc or Solid Cylinder}} > A_{\text{hollow sphere}} > A_{\text{Ring}}$$

$$(K.E_T)_{\text{solid sphere}} > (K.E_T)_{\text{disc or Solid Cylinder}} > (K.E_T)_{\text{hollow sphere}} > (K.E_T)_{\text{Ring}}$$

$$(K.E_T) + (K.E)_{\text{Rotational}} = \underset{\text{constant}}{(mgh)}$$

$$(K.E)_{\text{Rotational}} = mgh - (K.E)_T$$

$$(K.E_{\text{Rotational}}) < (K.E_{\text{Rotational}}) < (K.E_{\text{Rotational}}) < (K.E_{\text{Rotational}})$$

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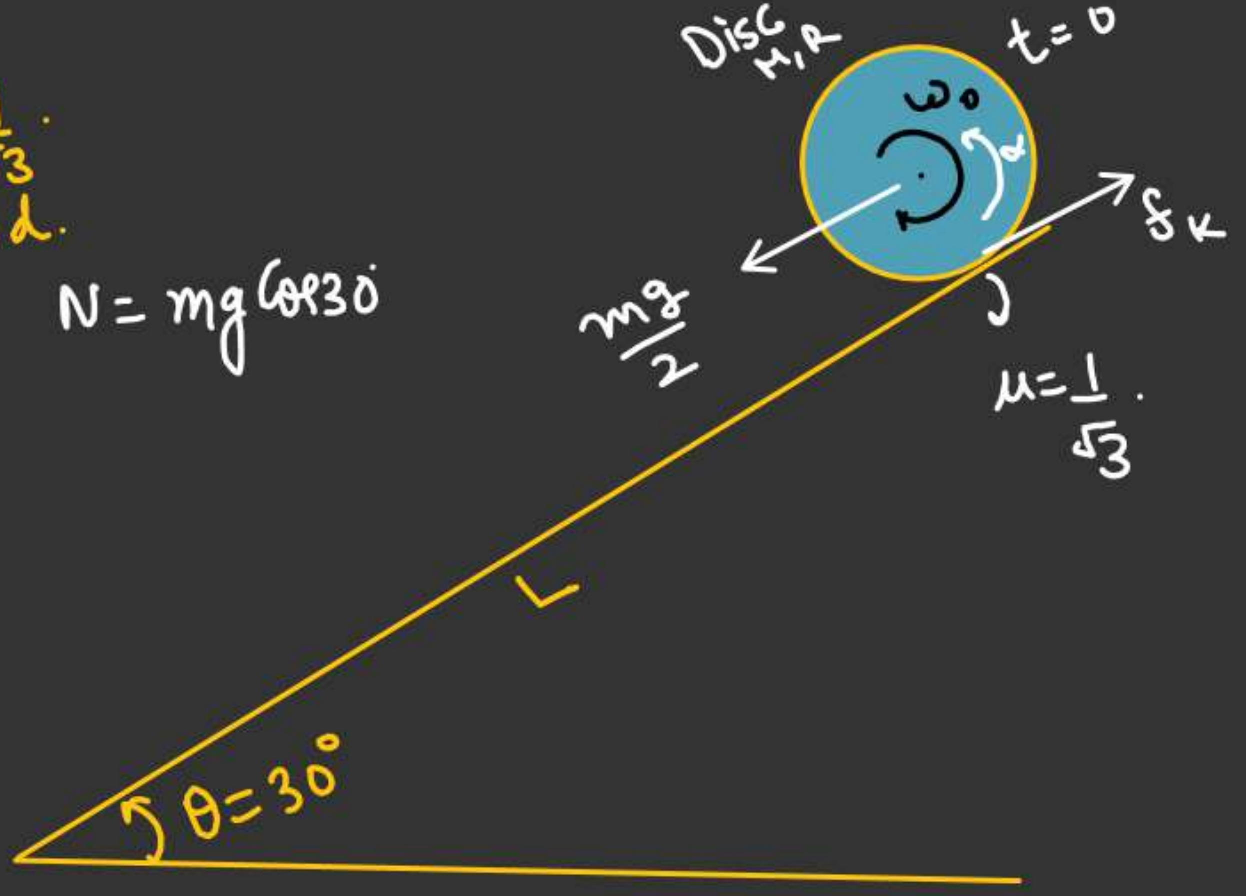
Disc given a spin with angular velocity ω_0 kept on a inclined plane whose $\mu = \frac{1}{\sqrt{3}}$.
Find time taken by disc to reach the ground.



$N = mg\cos 30^\circ$

$$f_K = \mu \cdot mg\cos 30^\circ$$
$$= \frac{1}{\sqrt{3}} \times mg \times \frac{\sqrt{3}}{2}$$
$$\left(f_K = \frac{mg}{2}\right) \Rightarrow \text{No translational only Rotation}$$

$$\alpha = \frac{f_K \cdot R}{I} = \frac{\mu mg\cos 30^\circ \times R}{\frac{MR^2}{2}}$$
$$\alpha = \frac{\frac{mg}{2} \times R}{\frac{MR^2}{2}}$$
$$\alpha = \left(g/R\right)$$

let, t_1 be the time when disc stop Spinning.

$$0 = \omega_0 - \alpha t_1$$
$$t_1 = \left(\frac{\omega_0}{\alpha}\right) = \left(\frac{\omega_0 R}{g}\right)$$

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- After disc stop Spinning.

$$(\mu_{min})_{disc} = \left(\frac{\tan\theta}{1+\frac{mR^2}{I}}\right)$$

For disc $I = \frac{MR^2}{2}$

$$(\mu_{min})_{disc} = \frac{1}{\sqrt{3}} \times \frac{1}{3}$$

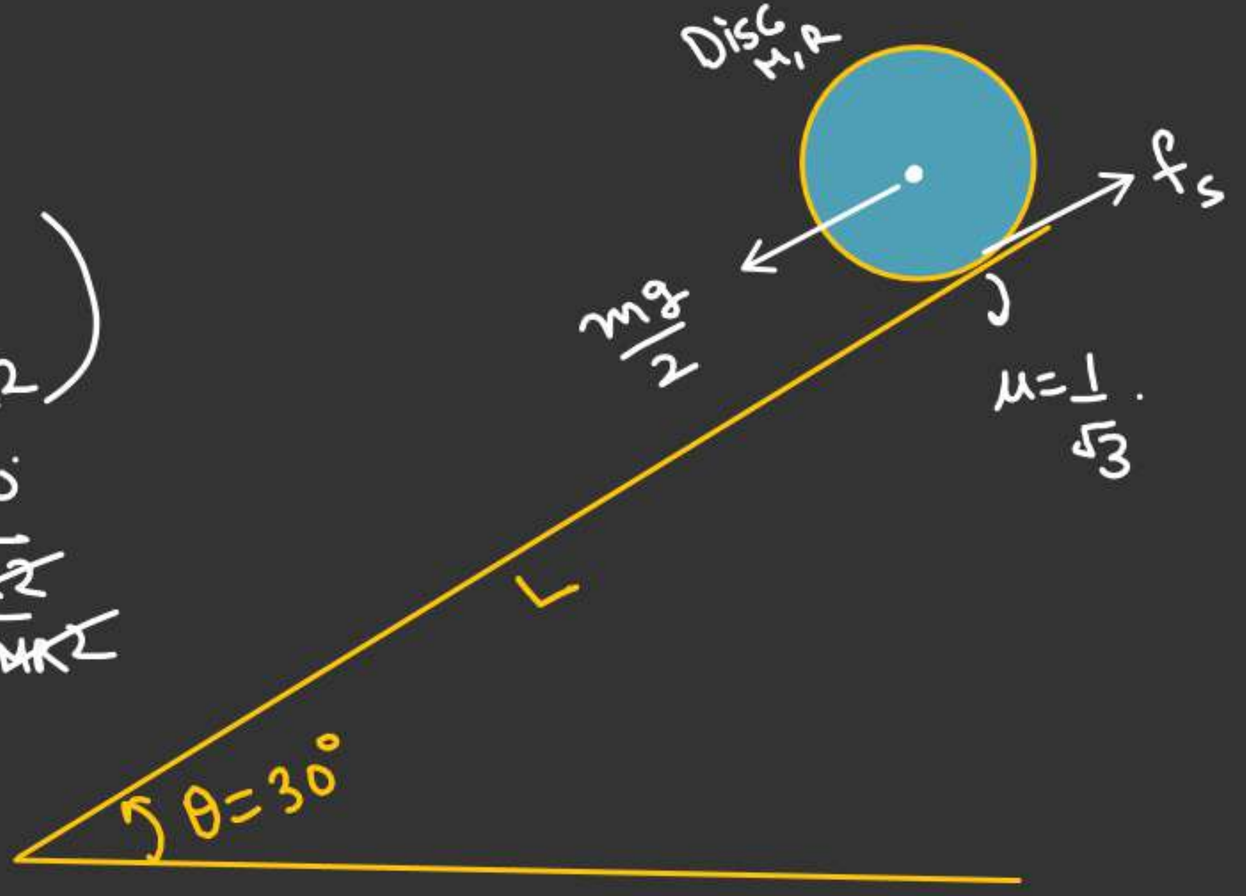
$$= \left(\frac{1}{3\sqrt{3}}\right)$$

$$\mu_{given} > (\mu_{min})_{disc}$$

$$A = \left(\frac{g\sin\theta}{1+\frac{I}{mR^2}}\right)$$

$$A_{disc} = \frac{g\sin 30^\circ}{1+\frac{\cancel{MR^2}}{2\cdot\cancel{MR^2}}}$$

$$A_{disc} = \left(g/3\right)$$



let Time taken by disc to reach the ground with rolling motion. be t_2

$$L = \frac{1}{2}At_2^2 \Rightarrow t_2 = \sqrt{\frac{2L}{A}}$$

$$\Rightarrow t_2 = \sqrt{\frac{6L}{g}}$$

$$T = t_1 + t_2$$

$$= \left[\frac{\omega_0 R}{g} + \sqrt{\frac{6L}{g}}\right]$$

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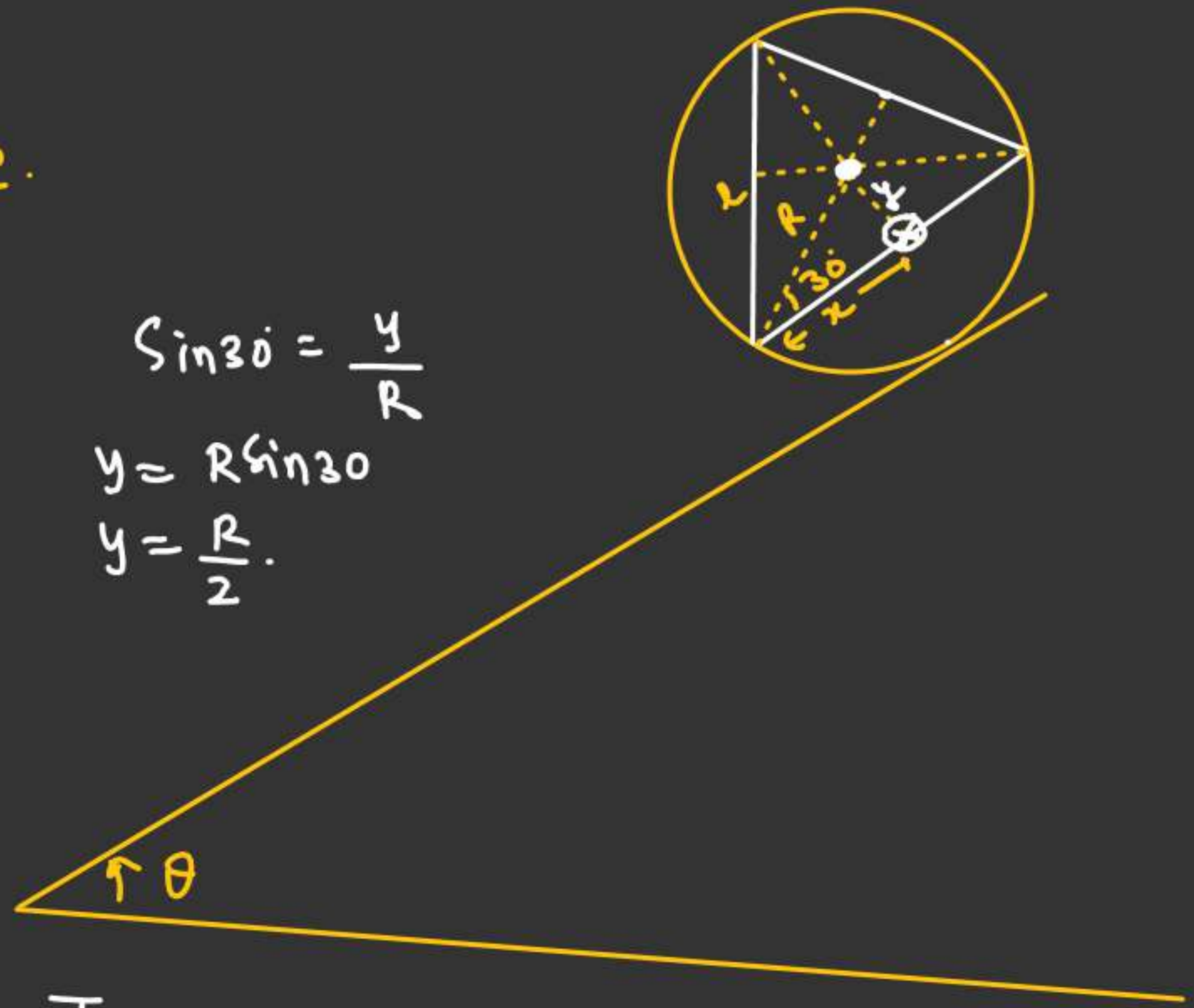
Three rods of mass m welded to form an equilateral triangle. Ring is light and have radius R. Find μ_{min} So that ring starts pure rolling.



$\sin 30^\circ = \frac{y}{R}$

$y = R\sin 30$

$y = \frac{R}{2}$.

$\cos 30^\circ = \frac{x}{R}$

$x = R\cos 30^\circ = \frac{\sqrt{3}R}{2}$.

$l = 2x$.

$l = 2 \times \frac{\sqrt{3}R}{2}$

$(l = \sqrt{3}R)$

$$\mu_{min} = \frac{\tan\theta}{\left(1 + \frac{MR^2}{I}\right)} \qquad M = 3m$$

$$\mu_{min} = \frac{\tan\theta}{1 + \frac{3mR^2}{\frac{3mR^2}{2}}} = \left(\frac{\tan\theta}{3}\right)$$

$$I_{system} = \left[\frac{ml^2}{12} + my^2\right] \times 3$$

$$= \left[\frac{m}{12}(\sqrt{3}R)^2 + m\left(\frac{R}{2}\right)^2\right] \times 3$$

$$= \left(\frac{3mR^2}{12} + \frac{mR^2}{4}\right) \times 3$$

$$= \left(\frac{3mR^2}{2}\right) \checkmark$$

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Rolling $\rightarrow$ $\underline{AB}$ path.

Find velocity of Cylinder. When it reaches to ground.

$\underline{AB = BC}$ ✓.