Nishant Jindal

Trigonometry

(1) $\sin 18^\circ = \frac{\sqrt{5}-1}{4}$

(2) $\cos 36^\circ = \frac{\sqrt{5}+1}{4}$

(3) $\sin 72^\circ = \cos 18^\circ = \frac{\sqrt{10+2\sqrt{5}}}{4}$

(4) $\sin 36^\circ = \cos 54^\circ = \frac{\sqrt{10-2\sqrt{5}}}{4}$

(5) $\cos 15^\circ = \frac{\sqrt{3}+1}{2\sqrt{2}}$

(6) $\sin 15^\circ = \frac{\sqrt{3}-1}{2\sqrt{2}}$

(7) $\tan 15^\circ = \frac{\sqrt{3}-1}{\sqrt{3}+1} = 2-\sqrt{3}$

(8) $\cot 15^\circ = \tan 75^\circ = 2+\sqrt{3}$

(9) $\tan\left(\frac{\pi}{8}\right) = \tan\left(22\frac{1}{2}^\circ\right) = \sqrt{2}-1$

(10) $\tan\left(67\frac{1}{2}^\circ\right) = \sqrt{2}+1$

(11) $\cos\theta \cdot \cos 2\theta \cdot \cos 4\theta \cdot \cos 8\theta = \frac{\sin(2^n A)}{2^n \sin(A)}$

(12) If $A+B = \frac{\pi}{4} \Rightarrow (1+\tan A)(1+\tan B) = 2$

(13) $\tan 3A \cdot \tan 2A \cdot \tan A = \tan 3A - \tan 2A - \tan A$

(14) $\sin(A)+\sin(A+d)+\sin(A+2d) \ldots \sin(A+(n-1)d)$
$= \frac{\sin\left(n \cdot \frac{D}{2}\right)}{\sin\left(\frac{d}{2}\right)} \times \sin\left(\frac{1^{st}+Last}{2}\right)$

(15) $\cos(A) \ldots \cos(A+(n-1)d) = \frac{\sin\left(\frac{n(D)}{2}\right)}{\sin\left(\frac{D}{2}\right)} \times \cos\left(\frac{1^{st}+Last}{2}\right)$

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Trigonometry

(16) $\tan\theta \cdot \tan(60-\theta) \cdot \tan(60+\theta) = \frac{\tan 3\theta}{4}$

(17) $\cos\theta \cdot \cos(60-\theta)\cos(60+\theta) = \frac{\cos 3\theta}{4}$

(18) $\tan\theta \cdot \tan(60-\theta)\tan(60+\theta) = \tan 3\theta$

(19) $\tan\theta + \tan(60+\theta) + \tan(120+\theta) = 3\tan 3\theta$

(20) $\cot\theta + \cot(60+\theta) + \cot(120^\circ+\theta) = 3\cot 3\theta$

(19) $\tan\theta + \tan(60^\circ+\theta) + \tan(120^\circ+\theta)$

$$\tan\theta + \frac{\tan 60 + \tan\theta}{1-\tan 60^\circ \tan\theta} + \frac{\tan 120^\circ + \tan\theta}{1-\tan 120^\circ \cdot \tan\theta}$$

$$\tan\theta + \left(\frac{\sqrt{3}+\tan\theta}{1-\sqrt{3}\tan\theta}\right) + \left(\frac{-\sqrt{3}+\tan\theta}{1+\sqrt{3}\tan\theta}\right)$$

$$\frac{\tan\theta(1-3\tan^2\theta) + (\sqrt{3}+\tan\theta)(1+\sqrt{3}\tan\theta) + (1-\sqrt{3}\tan\theta)(-\sqrt{3}+\tan\theta)}{(1-\sqrt{3}\tan\theta)(1+\sqrt{3}\tan\theta)}$$

$$\Rightarrow \frac{\tan\theta - 3\tan^3\theta + \cancel{\sqrt{3}} + 3\tan\theta + \tan\theta + \cancel{\sqrt{3}\tan^2\theta} - \cancel{\sqrt{3}} + \tan\theta + 3\tan\theta - \cancel{\sqrt{3}\tan^2\theta}}{1-3\tan^2\theta}$$

$$= \frac{9\tan\theta - 3\tan^3\theta}{1-3\tan^2\theta} = 3\left(\frac{3\tan\theta - \tan^3\theta}{1-3\tan^2\theta}\right)$$

$$= 3\tan 3\theta$$

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Trigonometry

Q $\cot 16^\circ \cdot \cot 44^\circ + \cot 44^\circ \cdot \cot 76^\circ - \cot 76^\circ \cdot \cot 16^\circ$

$\cot 16^\circ \cdot \cot 44^\circ \cdot \cot 76^\circ \{ \tan 76^\circ + \tan 16^\circ - \tan 44^\circ \}$

$\cot 16^\circ \cdot \cot(60^\circ - 16^\circ) \cot(60+16^\circ) \{ \tan 16^\circ + \tan(60+16^\circ) + \tan(120^\circ + 16^\circ) \}$

$\cancel{\cot(3 \times 16^\circ)} \{ 3 \cancel{\tan(3 \times 16^\circ)} \} = 3$

$\tan(120^\circ + 16^\circ) = \tan(136^\circ) = \tan(\pi - 44^\circ)$

$= -\tan 44$

Nahi Aaya.

$\cot 16^\circ \cdot \cot 44^\circ \cdot \cancel{\cot 76^\circ} \times \cancel{\tan 76^\circ}$

Nishant Jindal

Trigonometry

Trigo Identities

यार

(1) If $(A+B+C) = n\pi$ then

$$\tan(A+B+C) = \tan n\pi$$

$$\frac{\tan A+\tan B+\tan C-\tan A\tan B\tan C}{1-(\tan A\tan B+\tan B\tan C+\tan C\tan A)} = 0$$

$$\boxed{\tan A+\tan B+\tan C = \tan A\cdot\tan B\cdot\tan C}$$

$$\boxed{\sum \tan A = \prod \tan A}$$

(2) If $(A+B+C) = (2n+1)\frac{\pi}{2}$

$\tan n\pi = 0$

& $\tan \text{odd}\frac{\pi}{2} \to \infty$

$$\tan(A+B+C) = \tan(2n+1)\frac{\pi}{2}$$

$$\frac{\tan A+\tan B+\tan C-\tan A\tan B\tan C}{1-(\tan A\cdot\tan B+\tan B\tan C+\tan C\tan A)} \to \frac{1}{0}$$

$$1-(\tan A\cdot\tan B+\tan B\cdot\tan C+\tan C\cdot\tan A) = 0$$

$$\Rightarrow \boxed{\tan A\cdot\tan B+\tan B\tan C+\tan C\tan A = 1}$$

$$\boxed{\sum \tan A\cdot\tan B = 1}$$

Nishant Jindal

Trigonometry

Q If $\boxed{A+B+C=\pi}$ then P.T.

(1) $\sum \tan A = \prod \tan A$ //

(2) $\sum \cot A \cot B = 1$

(3) $\sum \tan A/2 \cdot \tan B/2 = 1$ //

(4) $\sum \cot A/2 = \prod \cot \frac{A}{2}$

$\boxed{A+B+C=\pi} \Rightarrow \frac{A}{2}+\frac{B}{2}+\frac{C}{2} = \frac{\pi}{2}$

$\Rightarrow \tan\frac{A}{2}\cdot\tan\frac{B}{2}+\tan\frac{B}{2}\cdot\tan\frac{C}{2}+\tan\frac{C}{2}\tan\frac{A}{2}=1$

3 $\boxed{\sum \tan\frac{A}{2}\tan\frac{B}{2}=1}$

$A+B+C=\pi$ hai then

$\tan A+\tan B+\tan C = \tan A\cdot\tan B\cdot\tan C$.

(1) $\boxed{\sum \tan A = \prod \tan A}$

(2) $\dfrac{(\tan A)+(\tan B)+(\tan C)}{\tan A\cdot\tan B\cdot\tan C} = \dfrac{\cancel{\tan A\cdot\tan B\cdot\tan C}}{\cancel{\tan A\cdot\tan B\cdot\tan C}}$

$\cot B\cdot\cot C+\cot A\cdot\cot C+\cot A\cdot\cot B = 1$

2) $\boxed{\sum \cot A\cdot\cot B=1}$

$\frac{A}{2}+\frac{B}{2}+\frac{C}{2}=\frac{\pi}{2}$

$\left(\tan\frac{A}{2}\cdot\tan\frac{B}{2}\right)+\left(\tan\frac{B}{2}\cdot\tan\frac{C}{2}\right)+\left(\tan\frac{C}{2}\cdot\tan\frac{A}{2}\right)=1$

$\div\left(\tan\frac{A}{2}\cdot\tan\frac{B}{2}\cdot\tan\frac{C}{2}\right)$

$\frac{1}{\tan\frac{C}{2}}+\frac{1}{\tan\frac{A}{2}}+\frac{1}{\tan\frac{B}{2}}=$

$\cot\frac{A}{2}\cdot\cot\frac{B}{2}\cdot\cot\frac{C}{2}$

$\cot\frac{A}{2}+\cot\frac{B}{2}+\cot\frac{C}{2}=\cot\frac{A}{2}\cdot\cot\frac{B}{2}\cdot\cot\frac{C}{2}$

$\sum\cot\frac{A}{2}=\prod\cot\frac{A}{2}$

Nishant Jindal

Trigonometry

$$\frac{A}{2}+\frac{B}{2}+\frac{C}{2}=\frac{\pi}{2}$$

$$\left(\tan\frac{A}{2}\cdot\tan\frac{B}{2}\right)+\left(\tan\frac{B}{2}\cdot\tan\frac{C}{2}\right)+\left(\tan\frac{C}{2}\cdot\tan\frac{A}{2}\right)=1$$

(LHS, RHS दोनों को $\tan\frac{A}{2}\cdot\tan\frac{B}{2}\cdot\tan\frac{C}{2}$ से divid

$$\frac{\cancel{\tan\frac{A}{2}\cdot\tan\frac{B}{2}}}{\cancel{\tan\frac{A}{2}\cdot\tan\frac{B}{2}}\cdot\tan\frac{C}{2}}+\frac{\cancel{\tan\frac{B}{2}\cdot\tan\frac{C}{2}}}{\tan\frac{A}{2}\cdot\cancel{\tan\frac{B}{2}\cdot\tan\frac{C}{2}}}+\frac{\cancel{\tan\frac{A}{2}}\cdot\cancel{\tan\frac{C}{2}}}{\cancel{\tan\frac{A}{2}}\cdot\tan\frac{B}{2}\cdot\cancel{\tan\frac{C}{2}}}=\frac{1}{\left(\tan\frac{A}{2}\right)\tan\frac{B}{2}\cdot\tan\frac{C}{2}}$$

$$\cot\frac{C}{2}+\cot\frac{A}{2}+\cot\frac{B}{2}=\cot\frac{A}{2}\cdot\cot\frac{B}{2}\cdot\cot\frac{C}{2}$$

$$\sum\cot\frac{A}{2}=\prod\cot\frac{A}{2}$$

Nishant Jindal

Trigonometry

Q $A+B+C=\pi$

$$\boxed{\begin{array}{l}(1)\ \cos 2A+\cos 2B+\cos 2C=-1-4\cos A\cdot\cos B\cos C\\ (2)\ \sin 2A+\sin 2B+\sin 2C=4\sin A\cdot\sin B\cdot\sin C.\\ (3)\ \tan A+\tan B+\tan C=\tan A\cdot\tan B\cdot\tan C.\end{array}}$$

$A+B+C=\pi$

(1) $A+B=\pi-C$ (2) $C=\pi-(A+B)$

(3) Sum / diff → Product (4) Common (Mahol)

Practice

Long (1) $\cos 2A+\cos 2B+\cos 2C$

Sum Prod $2\cos(A+B)\cdot\cos(A-B)+2\cos^2 C-1$

$2\cos(\pi-C)\cos(A-B)+2\cos^2 C-1$

Com $-2\cos C\cdot\cos(A-B)+2\cos^2 C-1$

$-1-2\cos C\{\cos(A-B)-\cos C\}$

$-1-2\cos C\{\cos(A-B)-\cos(\pi-(A+B))\}$

$-1-2\cos C\{\cos(A-B)+\cos(A+B)\}=-1-2\cos C\times 2\cos A\cdot\cos B=-1-4\cos A\cos B\cos C$

Nishant Jindal

Trigonometry

Long Pract

Q If $A+B+C=\pi$

then P.T. $\sin 2A+\sin 2B+\sin 2C=4\sin A\cdot\sin B\cdot\sin C$

— (1) Sum / diff → Prod

(2) $A+B=\pi-C$ / $C=\pi-(A+B)$

(3) Com (Method)

LHS

$2\sin(A+B)\cos(A-B)+2\sin C\cos C$

$2\sin(\pi-C)\cos(A-B)+2\sin C\cos C$

$2\sin C\cdot\cos(A-B)+2\sin C\cos C$

$2\sin C\{\cos(A-B)+\cos C\}$

$2\sin C\{\cos(A-B)+\cos(\pi-(A+B))\}$

$2\sin C\{\cos(A-B)-\cos(A+B)\}$

$2\sin C\cdot 2\sin A\cdot\sin B=4\sin A\cdot\sin B\cdot\sin C=RHS$

$\cos(A-B)=\cos A\cdot\cos B+\sin A\cdot\sin B$

$-\cos(A+B)=\cos A\cdot\cos B-\sin A\cdot\sin B$

$\cos(A-B)-\cos(A+B)=2\sin A\cdot\sin B$

$\cos\left(\frac{A-B}{2}\right)-\cos\left(\frac{A+B}{2}\right)=2\sin\frac{A}{2}\cdot\sin\frac{B}{2}$

Nishant Jindal

Trigonometry

Q If $A+B+C=\pi$ then P.T.

$$\cos A+\cos B+\cos C = 1+4\sin\frac{A}{2}\cdot\sin\frac{B}{2}\cdot\sin\frac{C}{2}$$

① Sum/diff → Prod

② Com (Mahol)

③ $\frac{A+B}{2} = \frac{\pi - C}{2} = \frac{\pi}{2}-\frac{C}{2}$

④ $C = \pi-(A+B)$

$\frac{C}{2} = \frac{\pi}{2}-\left(\frac{A+B}{2}\right)$

$$2\cos\left(\frac{A+B}{2}\right)\cdot\cos\left(\frac{A-B}{2}\right)+\cos C$$

$$2\cos\left(\frac{\pi}{2}-\frac{C}{2}\right)\cdot\cos\left(\frac{A-B}{2}\right)+1-2\sin^2\frac{C}{2}$$

$$2\sin\frac{C}{2}\cos\left(\frac{A-B}{2}\right)-2\sin^2\frac{C}{2}+1$$

$$1+2\sin\frac{C}{2}\left\{\cos\left(\frac{A-B}{2}\right)-\sin\left(\frac{C}{2}\right)\right\}$$

$$1+2\sin\frac{C}{2}\left\{\cos\left(\frac{A-B}{2}\right)-\sin\left(\frac{\pi}{2}-\left(\frac{A+B}{2}\right)\right)\right\}$$

$$1+2\sin\frac{C}{2}\left\{\cos\left(\frac{A-B}{2}\right)-\cos\left(\frac{A+B}{2}\right)\right\} = 1+2\sin\frac{C}{2}\times 2\sin\frac{A}{2}\cdot\sin\frac{B}{2} = 1+4\sin\frac{A}{2}\cdot\sin\frac{B}{2}\cdot\sin\frac{C}{2}$$

Nishant Jindal

Trigonometry

Ex 1 Q 20

Q $A+B+C=\pi$ then P.T.

Normal Level.

क्या इस level तक आना है।

$$\sin\frac{A}{2}+\sin\frac{B}{2}+\sin\frac{C}{2}=1+4\sin\left(\frac{\pi-A}{4}\right)\cdot\sin\left(\frac{\pi-B}{4}\right)\cdot\sin\left(\frac{\pi-C}{4}\right)$$

$A+B+C=\pi$

$B+C=\pi-A$

$\frac{B+C}{4}=\frac{\pi-A}{4}$

$$RHS \Rightarrow 1+4\sin\left(\frac{B+C}{4}\right)\cdot\sin\left(\frac{A+C}{4}\right)\cdot\sin\left(\frac{A+B}{4}\right)$$

$$1+2\left\{2\sin\left(\frac{B+C}{4}\right)\cdot\sin\left(\frac{A+C}{4}\right)\right\}\cdot\sin\left(\frac{A+B}{4}\right)$$

$$1+2\left\{\cos\left(\frac{B-A}{4}\right)-\cos\left(\frac{A+B+2C}{4}\right)\right\}\cdot\sin\left(\frac{A+B}{4}\right)$$

$$1+2\cos\left(\frac{B-A}{4}\right)\cdot\sin\left(\frac{A+B}{4}\right)-2\cos\left(\frac{A+B+2C}{4}\right)\cdot\sin\left(\frac{A+B}{4}\right)$$

$$\cancel{1}+\left\{\sin\left(\frac{B}{2}\right)+\sin\left(+\frac{A}{2}\right)\right\}+\left\{\cancel{\sin\left(\frac{A+B+C}{2}\right)}+\sin\left(\frac{C}{2}\right)\right\}$$

$$\sin\frac{A}{2}+\sin\frac{B}{2}+\sin\frac{C}{2}=LHS$$