Q Gr. Int. less than or Equal to.

Adv 2022

$$\int_1^2 \log_2(x^3+1)\,dx + \int_1^{\log_2 9} (2^x-1)^{1/3}\,dx$$

$$= [2\log_2 9 - 1]$$

$$= [2\times 3\cdot 2 - 1]$$

$$= [6\cdot 4 - 1]$$

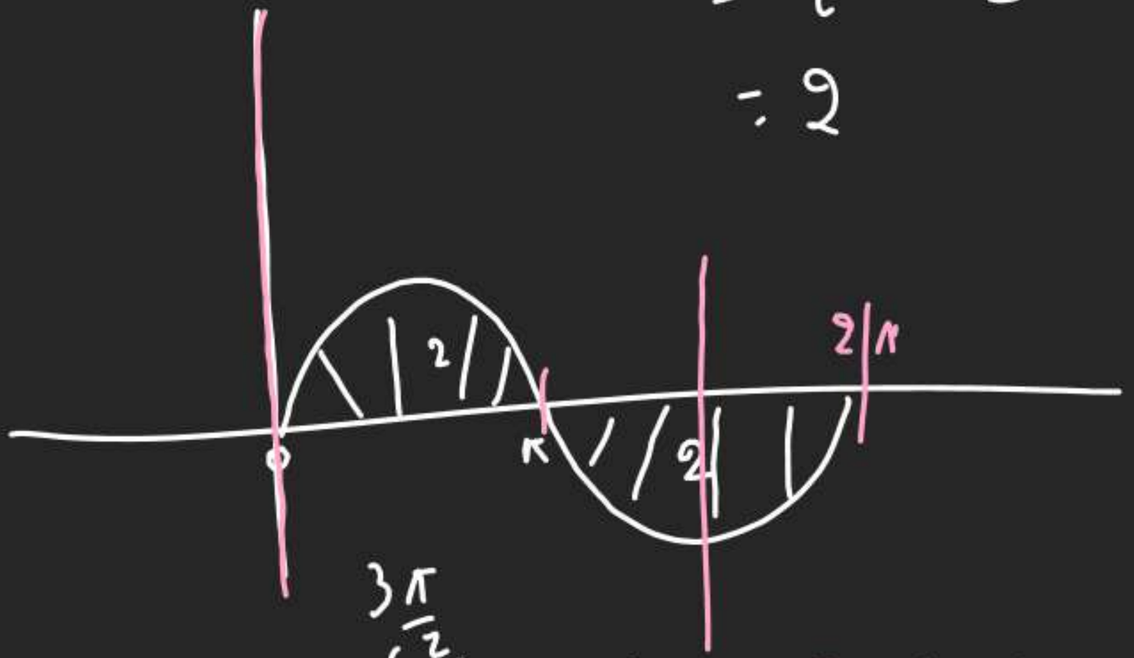
$$= [5\cdot 4] = 5$$

10) $$\int_0^{\pi} \sin x\cdot dx = -[\cos x]_0^{\pi}$$

$$= -[\cos\pi - \cos 0]$$

$$= -[-1-1]$$

$$= 2$$



$$\int_0^{\frac{3\pi}{2}} \sin x\cdot dx = 2 - 1 = 1$$

Q $$\int_0^{2023\pi} \sin x\,dx = \int_0^{2022\pi} \sin x\,dx + \int_{2022\pi}^{2023\pi}$$

$$0 + 2 = 2$$

Nishant Jindal

Q $\displaystyle\int_{-1}^{1} \frac{dx}{x^2+2x+2}$

$\displaystyle\int_{-1}^{1} \frac{dx}{(x^2+2x+1)+1}$

$\displaystyle\int_{-1}^{1} \frac{dx}{(x+1)^2+1}$ $\qquad \displaystyle\int \frac{dx}{1+x^2} = \tan^{-1} x$

$= \tan^{-1}(x+1)\Big|_{-1}^{1}$

$= \tan^{-1} 2 - \tan^{-1}(0)$

$= \tan^{-1} 2$

Q $\displaystyle\int_{0}^{\pi/4} \frac{dx}{1+\cos 2x}$

$= \displaystyle\int_{0}^{\pi/4} \frac{dx}{2\cos^2 x}$

$= \frac{1}{2}\displaystyle\int \sec^2 x \cdot dx$

$= \frac{1}{2}\left(\tan x\right)_{0}^{\pi/4}$

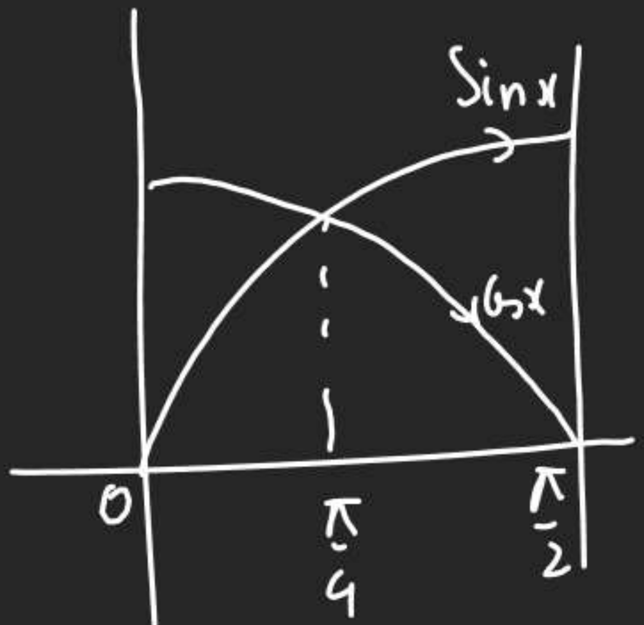
$= \frac{1}{2}\left(\tan\frac{\pi}{4} - \tan 0\right)$

$= \frac{1}{2}$

Q $\displaystyle\int_{\pi/4}^{\pi/2} \sqrt{1-\sin 2x} \cdot dx$

$\displaystyle\int_{\pi/4}^{\pi/2} \sqrt{(\sin x - \cos x)^2} \cdot dx$

$\displaystyle\int_{\pi/4}^{\pi/2} |\underset{\oplus}{\sin x - \cos x}| \cdot dx$



$= \displaystyle\int_{\pi/4}^{\pi/2} \sin x - \cos x \cdot dx$

$\sin x > \cos x$

$= -\left[\cos x + \sin x\right]_{\pi/4}^{\pi/2}$

$\sin x - \cos x > 0$

$= -\left[\left(\cos\frac{\pi}{2} + \sin\frac{\pi}{2}\right) - \left(\cos\frac{\pi}{4} + \sin\frac{\pi}{4}\right)\right]$

$= -\left[1 - \sqrt{2}\right] = \sqrt{2} - 1$

Nishant Jindal

Q $\displaystyle\int_{-1}^{1/2} \frac{e^x(2-x^2)}{(1-x)\sqrt{1-x^2}}\,dx$

$\Rightarrow \displaystyle\int_{-1}^{1/2} e^x\left\{\frac{1}{(1-x)\sqrt{1-x^2}} + \frac{\cancel{1-x^2}\ \sqrt{1-x^2}}{(1-x)\cancel{\sqrt{1-x^2}}}\right\}dx$

$\Rightarrow \displaystyle\int_{-1}^{1/2} e^x\left\{\underbrace{\frac{1}{(1-x)\sqrt{1-x^2}}}_{f'} + \underbrace{\sqrt{\frac{1+x}{1-x}}}_{f}\right\}dx$

$= e^x\sqrt{\frac{1+x}{1-x}}\Bigg|_{-1}^{+1/2}$

$= \left\{e^{1/2}\sqrt{\frac{3/\cancel{2}}{1/\cancel{2}}} - e^{-1}\sqrt{\frac{0}{2}}\right\} = \sqrt{3e}$

$y = \sqrt{\frac{1+x}{1-x}}$

$\frac{dy}{dx} = \frac{1}{2\sqrt{\frac{1+x}{1-x}}} \times \frac{(1-x)\cdot 1 + (1+x)(+1)}{(1-x)^2}$

$= \sqrt{\frac{1-x}{1+x}} \times \frac{1}{\cancel{(1-x)^2}\,(1-x)\sqrt{1-x}} = \frac{1}{(1-x)\sqrt{1-x^2}}$

Nishant Jindal

Q $\int_{1/2}^{2}\left(1+x-\frac{1}{x}\right)e^{x+\frac{1}{x}}\cdot dx$

$\int_{1/2}^{2}\left(\underbrace{e^{1/x}+\underbrace{x\cdot e^{1/x}}_{f}-\frac{1}{x}e^{1/x}}_{f'}\right)\cdot e^{x}\cdot dx$

$= e^{x}\left(x\cdot e^{1/x}\right)$

$= x\cdot e^{x+\frac{1}{x}}\Big|_{1/2}^{2}$

$= 2e^{5/2}-\frac{1}{2}e^{5/2}$

$= \frac{3}{2}e^{5/2}$

Q $\int_{a}^{b} \underset{v}{x}\cdot \underset{u'}{f'(x)}+\underset{uv'}{f(x)}\cdot dx$

$= x\cdot f(x)\Big|_{a}^{b}$

$= bf(b)-af(a)$

Q $\int_{1}^{2} x^{x}(1+x+x\ln x)\cdot dx$

$= \int_{1}^{2} x^{x}+x^{x}\cdot x(1+\ln x)\cdot dx$

$= \int_{1}^{2} \underset{f}{x^{x}}+x\cdot \underbrace{x^{x}(1+\ln x)}_{f'}\cdot dx$ 98%

$+x\times f'$

$= x\cdot x^{x}\Big|_{1}^{2} = 2\cdot 2^{2}-1\cdot 1^{1}$

$= 7$

Q $\int_{1}^{e} e^{x}(x+1)\ln x\cdot dx = a+b\cdot e^{e}$

find $a+b$?

$= \int_{1}^{e} e^{x}(x\ln x+\ln x)\cdot dx$

$= \int_{1}^{e} e^{x}\{\underbrace{x\ln x-1}_{f}+\underbrace{\ln x+1}_{f'}\}\,dx$

B

$e^{x}(x\ln x-1)\Big|_{1}^{e}$

$e^{e}(e\ln e-1)-e(0-1)$

$e^{e}(e-1)+e$

$a=e$, $b=e+1=1$

$a+b=1$

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Q $\displaystyle\int_0^1 \frac{2e^{2x}+1+(3+x)e^{x}}{(e^{2x}+2e^{x}+xe^{x}+x+1)^2}\,dx$

$$\boxed{e^{2x}+2e^{x}+x\cdot e^{x}+x+1=t}$$

$$(2\cdot e^{2x}+2e^{x}+x e^{x}+e^{x}+1)dx=dt$$

$$\int_0^1 \frac{(2e^{2x}+3e^{x}+xe^{x}+1)dx}{(e^{2x}+2e^{x}+xe^{x}+x+1)^2}$$

$$=\int_4^{e^2+3e+2}\frac{dt}{t^2}$$

x	t
0	$e^0+2e^0+0\cdot e^0+0+1=1+2+1=4$
1	$e^2+2e+e+1+1=e^2+3e+2$

$$=-\frac{1}{t}\Big|_4^{e^2+3e+2}$$

$$=-\frac{1}{e^2+3e+2}+\frac{1}{4}$$

Nishant Jindal

Q $\displaystyle\int_0^{2a} \frac{dx}{\sqrt{2a x - x^2}}$

a

$\boxed{3}x - x^2 = \left(\frac{3}{2}\right)^2 - \left(x - \frac{3}{2}\right)^2$

$\to \frac{3}{2}$

$\boxed{5}x - x^2 = \left(\frac{5}{2}\right)^2 - \left(x - \frac{5}{2}\right)^2$

$\frac{5}{2}$

Q

$$\int_0^{2a} \frac{dx}{\sqrt{a^2 - (x-a)^2}} \to \int \frac{dx}{\sqrt{A^2 - x^2}} = \sin^{-1}\frac{x}{a}$$

$$= \sin^{-1}\left(\frac{x-a}{a}\right)\Big|_0^{2a}$$

$$= \sin^{-1}\left(\frac{a}{a}\right) - \sin^{-1}\left(\frac{-a}{a}\right)$$

$$= \sin^{-1}(1) - \sin^{-1}(-1)$$

$$= \sin^{-1}(1) + \sin^{-1}(1)$$

$$\frac{\pi}{2} + \frac{\pi}{2} = \pi$$