

Basic Maths (Physics)

Formula

$\int x^n dx = \left(\frac{x^{n+1}}{n+1} \right) + C.$

$\int e^x dx = e^x + C.$

$\int \frac{dx}{x} = \ln x + C$

$\int \sin x dx = -\cos x + C$

$\int \cos x dx = \sin x + C$

$\Rightarrow \int e^{ax+b} dx = \frac{1}{a}(e^{ax+b}) + C.$

$\Rightarrow \int \frac{dx}{a+bx} = \frac{\ln(a+bx)}{b}$

$\Rightarrow \int \sin kx dx = -\frac{\cos kx}{k} + C$

$\Rightarrow \int \cos kx dx = \frac{\sin kx}{k} + C$

$\frac{d}{dx} \ln(x) = \frac{1}{x}$

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Definite Integration

Upper limit

$$\int_{x=a}^{x=b} f(x) dx = \left[g(x) \right]_a^b$$

lower limit

$$= g(b) - g(a)$$

$$\int f(x) dx = g(x)$$

$$= \left(\frac{2}{3} + 2 \right)$$

$\frac{8}{3}$ Sq. unit

Ex:-

$$y = (x^2 - 2x + 1)$$

$$\int y dx = \int (x^2 - 2x + 1) dx$$

$$= \int x^2 dx - 2 \int x dx + \int 1 dx$$

$$= \left[\frac{x^{2+1}}{2+1} \right]_{-1}^{+1} - 2 \left[\frac{x^{1+1}}{2} \right]_{-1}^{+1} + [x]_{-1}^{+1}$$

$$= \frac{1}{3} [x^3]_{-1}^{+1} - [x^2]_{-1}^{+1} + [x]_{-1}^{+1}$$

$$= \frac{1}{3} [(1)^3 - (-1)^3] - \{(1)^2 - (-1)^2\} + \{(1) - (-1)\}$$

$$= \frac{1}{3} [1+1] - \{0\} + \{2\}$$

$$\int x^n dx = \frac{x^{n+1}}{n+1}$$

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Integrate the functions.

$$\int x^n dx = \left(\frac{x^{n+1}}{n+1} \right).$$

$\int_0^1 x^{\frac{3}{2}} dx = \frac{\left[x^{\frac{3}{2}+1} \right]_0^1}{\frac{3}{2}+1} = \frac{2}{5} \left[x^{\frac{5}{2}} \right]_0^1 = \frac{2}{5} \left[(1)^{\frac{5}{2}} - (0)^{\frac{5}{2}} \right] = \frac{2}{5}$ Ans.

$\int_0^1 \left(\frac{1}{\sqrt{x}} + x^2 \right) dx = \int_0^1 (x^{-\frac{1}{2}} + x^2) dx = \int_0^1 x^{-\frac{1}{2}} dx + \int_0^1 x^2 dx = \left[\frac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} \right]_0^1 + \left[\frac{x^{2+1}}{2+1} \right]_0^1$

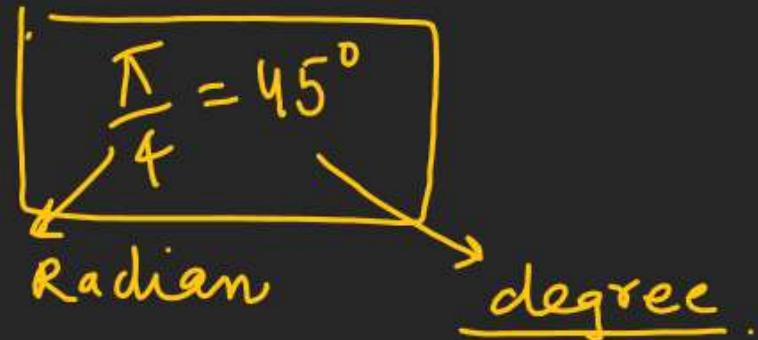
$\int_0^1 e^{3x+2} dx = \left[\frac{e^{3x+2}}{3} \right]_0^1 = \frac{1}{3} \left[e^{3(x_1)+2} - e^{3(x_0)+2} \right] = \left(\frac{e^5 - e^2}{3} \right) = \frac{7}{3}$ Ans.

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$$\int_0^{\frac{\pi}{4}} \sin 2x \, dx = ??$$

$$\begin{aligned}
 &= -\left[\frac{\cos 2x}{2} \right]_0^{\frac{\pi}{4}} \\
 &= -\frac{1}{2} \left[\cos 2\left(\frac{\pi}{4}\right) - \cos 0 \right] \\
 &= -\frac{1}{2} \left[0 - (+1) \right] = \boxed{\frac{1}{2}}
 \end{aligned}$$

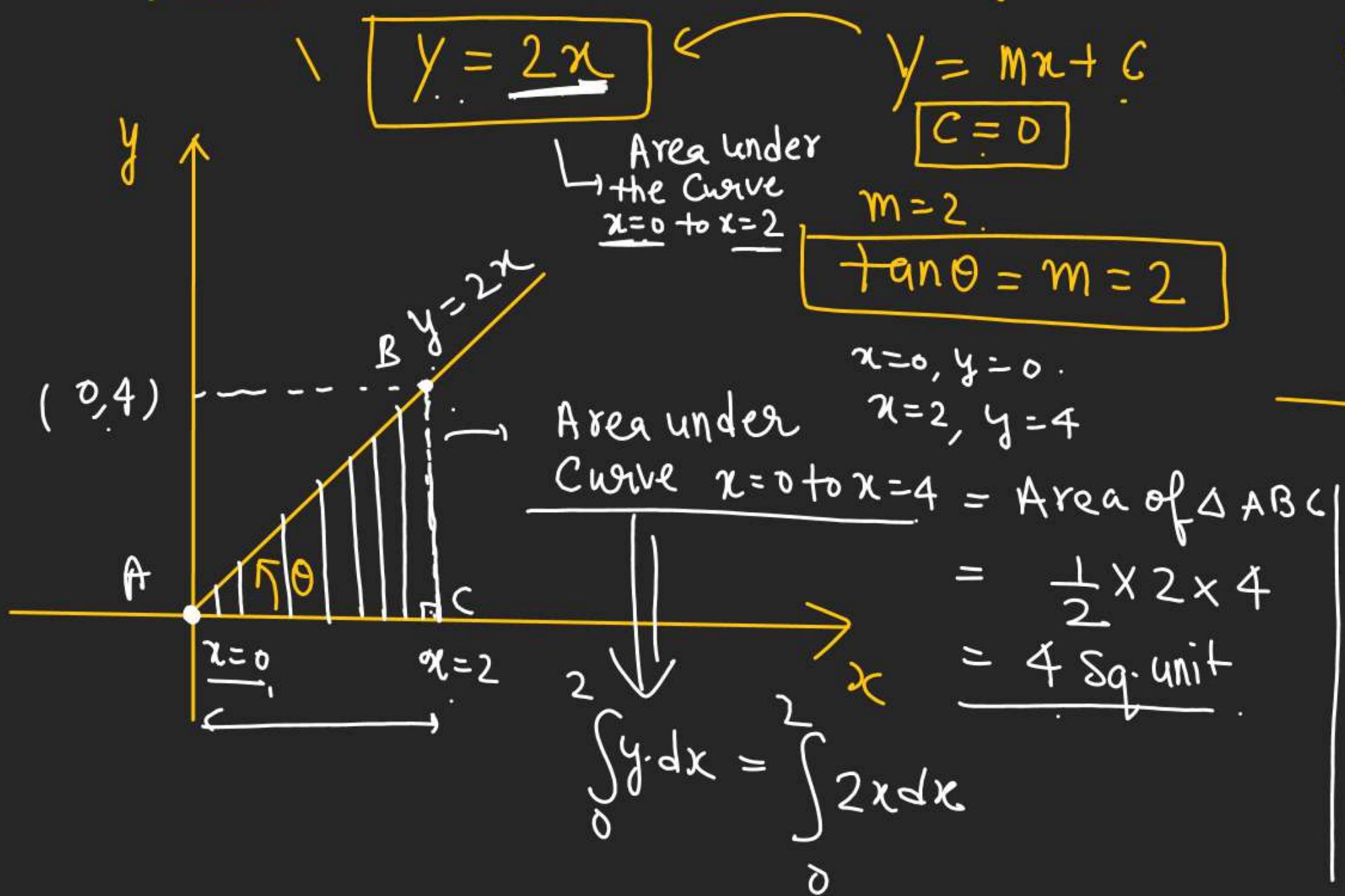


$$\begin{bmatrix}
 \cos \frac{\pi}{2} = 0. \\
 \downarrow \\
 \cos 90^\circ = 0 \\
 \cos 0 = +1
 \end{bmatrix}$$

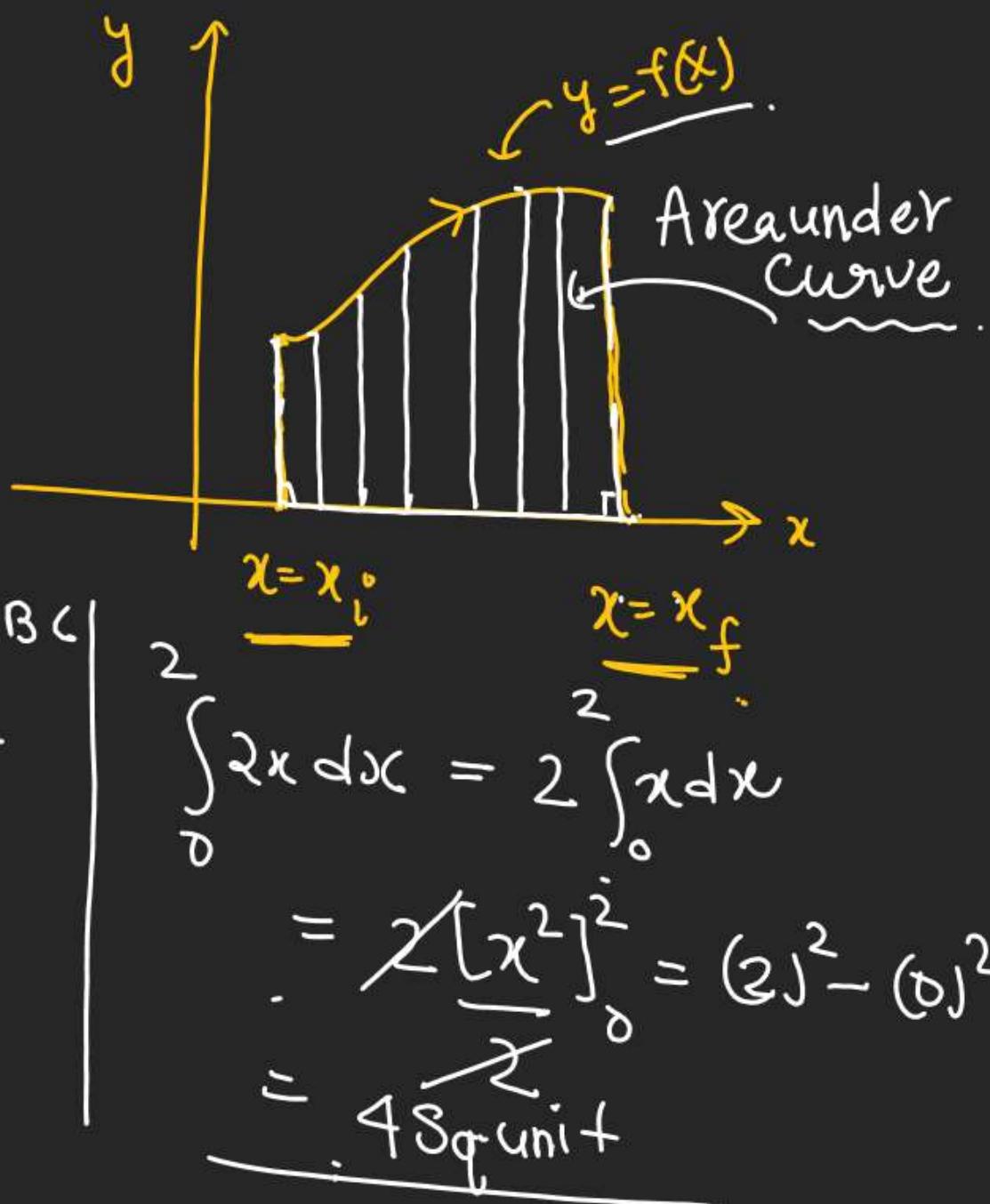
$$\int \sin Kx \, dx = -\frac{\cos Kx}{K}$$

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Geometrical meaning of definite integration.



Area under the Curve



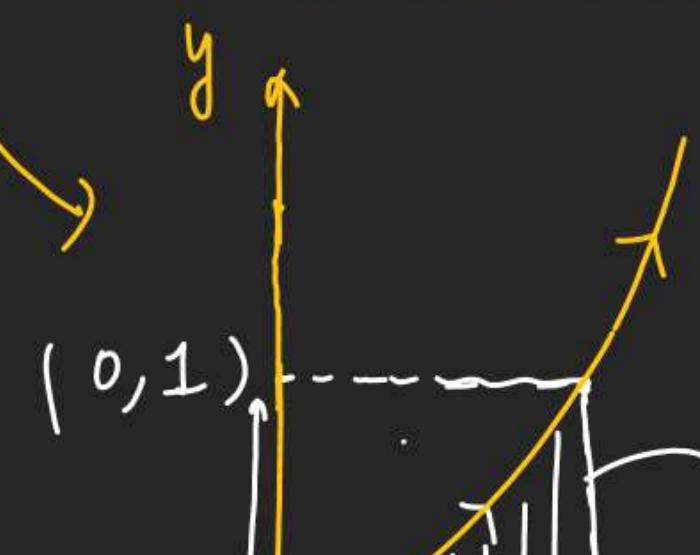
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$$x^2 = 4y$$

Find Area under the Curve from

$x=0$ to $x=2$



$$\boxed{y = \left(\frac{x^2}{4}\right)}$$

At $x=2, y=1$

$$\boxed{x^2 = 4ay}$$

Area under the Curve = $\int_0^2 \left(\frac{x^2}{4}\right) dx = \frac{1}{4} \int_0^2 x^2 dx$

$$= \frac{1}{4} \left[\frac{x^3}{3} \right]_0^2 = \frac{1}{12} [x^3]_0^2 = \frac{1}{12} [(2)^3 - (0)^3]$$

$$= \frac{8}{12} = \left(\frac{2}{3}\right) \text{ Sq. Unit}$$

$$\# \cdot \int_0^1 (e^{-x} + e^{+x}) dx \quad \begin{matrix} x=1 \\ e=1 \end{matrix}$$

$$= \int_0^1 e^{-x} dx + \int_0^1 e^x dx \quad \left(e^{-\frac{1}{e}} \right) A$$

$$= \frac{\left[e^{-x} \right]_0^1}{(-1)} + \frac{\left[e^x \right]_0^1}{1}$$

$$= -\left(e^1 - e^0 \right) + \left(e^1 - e^0 \right)$$

$$= -\left(\frac{1}{e} - 1 \right) + \left(e - 1 \right) = -\frac{1}{e} + 1 + e - 1$$

(★)

$$y = \underline{\underline{x^3 + 2x^2 + 1}}$$

$$\text{Find } \left(\frac{d^2y}{dx^2} \right)_{x=2} = ?? \quad \rightarrow \quad \frac{dy}{dx} = \underline{\underline{3x^2 + 4x}}$$

$$\frac{d^2y}{dx^2} = \underline{\underline{\frac{d}{dx} \left(\frac{dy}{dx} \right)}}$$

$$\begin{aligned} \frac{d^2y}{dx^2} &= \frac{d}{dx} (3x^2 + 4x) \\ &= 3 \frac{d}{dx}(x^2) + 4 \frac{d}{dx}(x) \end{aligned}$$

$$\left(\frac{d^2y}{dx^2} \right) = \underline{\underline{(6x+4)}} \quad \overbrace{\quad}^{4} \quad \overbrace{1}$$

$$\left(\frac{d^2y}{dx^2} \right)_{x=2} = (6 \times 2) + 4 = \underline{\underline{16}} \quad \textcircled{O}$$