KEY CONCEPTS

DEFINITIONS :

1. **PERMUTATION :** Each of the arrangements in a definite order which can be made by taking some or all of a number of things is called a **PERMUTATION**.

2. **COMBINATION :** Each of the groups or selections which can be made by taking some or all of a number of things without reference to the order of the things in each group is called a **COMBINATION**.

FUNDAMENTAL PRINCIPLE OF COUNTING :

If an event can occur in 'm' different ways, following which another event can occur in '*n*' different ways, then the total number of different ways of simultaneous occurrence of both events in a definite order is $m \times n$. This can be extended to any number of events.

RESULTS :

(i) A Useful Notation : $n! = n(n-1)(n-2)\ldots\ldots 3.\ 2.\ 1$; $n! = n.(n-1)!$
$0! = 1! = 1$; $(2n)! = 2^n.\ n!\ [1.\ 3.\ 5.\ 7\ldots(2n-1)]$
Note that factorials of negative integers are not defined.

(ii) If ${}^nP_r$ denotes the number of permutations of n different things, taking r at a time, then

${}^nP_r = n(n-1)(n-2)\ldots\ldots(n-r+1) = \dfrac{n!}{(n-r)!}$ Note that , ${}^nP_n = n!$.

(iii) If ${}^nC_r$ denotes the number of combinations of n different things taken r at a time, then

${}^nC_r = \dfrac{n!}{r!(n-r)!} = \dfrac{{}^nP_r}{r!}$ where $r \le n$; $n \in N$ and $r \in W$.

(iv) The number of ways in which (m + n) different things can be divided into two groups containing m & n things respectively is : $\dfrac{(m+n)!}{m!n!}$ If m = n, the groups are equal & in this case the number of subdivision is $\dfrac{(2n)!}{n!\ n!\ 2!}$; for in any one way it is possible to interchange the two groups without obtaining a new distribution. However, if 2n things are to be divided equally between two persons then the number of ways $= \dfrac{(2n)!}{n!\ n!}$.

(v) Number of ways in which (m + n + p) different things can be divided into three groups containing m , n & p things respectively is $\dfrac{(m+n+p)!}{m!\ n!\ p!}$, $m \ne n \ne p$.

If $m = n = p$ then the number of groups $= \dfrac{(3n)!}{n!\ n!\ n!\ 3!}$.

However, if 3n things are to be divided equally among three people then the number of ways $= \dfrac{(3n)!}{(n!)^3}$.

(vi) The number of permutations of *n* things taken all at a time when *p* of them are similar & of one type, q of them are similar & of another type, *r* of them are similar & of a third type & the remaining $n-(p+q+r)$ are all different is : $\dfrac{n!}{p!\,q!\,r!}$.

(vii) The number of circular permutations of n different things taken all at a time is ; $(n-1)!$. If clockwise & anti–clockwise circular permutations are considered to be same, then it is $\dfrac{(n-1)!}{2}$.

Note : Number of circular permutations of n things when p alike and the rest different taken all at a time distinguishing clockwise and anticlockwise arrangement is $\dfrac{(n-1)!}{p!}$.

(viii) Given n different objects, the number of ways of selecting atleast one of them is , ${}^nC_1 + {}^nC_2 + {}^nC_3 + ... + {}^nC_n = 2^n - 1$. This can also be stated as the total number of combinations of n distinct things.

(ix) Total number of ways in which it is possible to make a selection by taking some or all out of p + q + r +...... things , where p are alike of one kind, q alike of a second kind , r alike of third kind & so on is given by : $(p + 1)(q + 1)(r + 1)........ -1$.

(x) Number of ways in which it is possible to make a selection of m + n + p = N things , where p are alike of one kind , m alike of second kind & n alike of third kind taken r at a time is given by coefficient of x^r in the expansion of

$$(1 + x + x^2 + ... + x^p)(1 + x + x^2 + ... + x^m)(1 + x + x^2 + ... + x^n).$$

Note : Remember that coefficient of x^r in $(1 - x)^{-n} = {}^{n+r-1}C_r$ $(n \in N)$. For example the number of ways in which a selection of four letters can be made from the letters of the word **PROPORTION** is given by coefficient of x^4 in $(1 + x + x^2 + x^3)(1 + x + x^2)(1 + x + x^2)(1 + x)(1 + x)(1 + x)$.

(xi) Number of ways in which n distinct things can be distributed to p persons if there is no restriction to the number of things received by men = p^n.

(xii) Number of ways in which n identical things may be distributed among p persons if each person may receive none , one or more things is ; ${}^{n+p-1}C_n$.

(xiii) **a.** ${}^nC_r = {}^nC_{n-r}$; ${}^nC_0 = {}^nC_n = 1$; **b.** ${}^nC_x = {}^nC_y \Rightarrow x = y$ or $x + y = n$

c. ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$

(xiv) ${}^nC_r$ is maximum if : (a) $r = \frac{n}{2}$ if n is even. (b) $r = \frac{n-1}{2}$ or $\frac{n+1}{2}$ if n is odd.

(xv) Let $N = p^a \cdot q^b \cdot r^c .....$ where p , q , r...... are distinct primes & a , b , c..... are natural numbers then:

(a) The total numbers of divisors of N including 1 & N is = $(a + 1)(b + 1)(c + 1).....$

(b) The sum of these divisors is

$$= (p^0 + p^1 + p^2 + ... + p^a)(q^0 + q^1 + q^2 + ... + q^b)(r^0 + r^1 + r^2 + ... + r^c)....$$

(c) Number of ways in which N can be resolved as a product of two

factors is = $\begin{cases} \frac{1}{2}(a+1)(b+1)(c+1).... & \text{if N is not a perfect square} \\ \frac{1}{2}[(a+1)(b+1)(c+1)....+1] & \text{if N is a perfect square} \end{cases}$

(d) Number of ways in which a composite number N can be resolved into two factors which are relatively prime (or coprime) to each other is equal to 2^{n-1} where n is the number of different prime factors in N.

(xvi) Grid Problems and tree diagrams.

DEARRANGEMENT :

Number of ways in which n letters can be placed in n directed letters so that no letter goes into its own

envelope is = $n!\left[\frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} + ... + (-1)^n \frac{1}{n!}\right]$.

(xvii) Some times students find it difficult to decide whether a problem is on permutation or combination or both. Based on certain words / phrases occuring in the problem we can fairly decide its nature as per the following table :

PROBLEMS OF COMBINATIONS	PROBLEMS OF PERMUTATIONS
■ Selections , choose	■ Arrangements
■ Distributed group is formed	■ Standing in a line seated in a row
■ Committee	■ Problems on digits
■ Geometrical problems	■ Problems on letters from a word

EXERCISE–I

1. The straight lines l_1 , l_2 & l_3 are parallel & lie in the same plane. A total of m points are taken on the line l_1 , n points on l_2 & k points on l_3. How many maximum number of triangles are there whose vertices are at these points ?

2. How many five digits numbers divisible by 3 can be formed using the digits 0, 1, 2, 3, 4, 7 and 8 if each digit is to be used atmost once.

3. There are 2 women participating in a chess tournament. Every participant played 2 games with the other participants. The number of games that the men played between themselves exceeded by 66 as compared to the number of games that the men played with the women. Find the number of participants & the total numbers of games played in the tournament.

4. All the 7 digit numbers containing each of the digits 1, 2, 3, 4, 5, 6, 7 exactly once, and not divisible by 5 are arranged in the increasing order. Find the $(2004)^{th}$ number in this list.

5. 5 boys & 4 girls sit in a straight line. Find the number of ways in which they can be seated if 2 girls are together & the other 2 are also together but separate from the first 2.

6. A crew of an eight oar boat has to be chosen out of 11 men five of whom can row on stroke side only, four on the bow side only, and the remaining two on either side. How many different selections can be made?

7. An examination paper consists of 12 questions divided into parts A & B.
Part-A contains 7 questions & Part–B contains 5 questions. A candidate is required to attempt 8 questions selecting atleast 3 from each part. In how many maximum ways can the candidate select the questions ?

8. In how many ways can a team of 6 horses be selected out of a stud of 16 , so that there shall always be 3 out of $A\ B\ C\ A'\ B'\ C'$, but never $A\ A'$, $B\ B'$ or $C\ C'$ together.

9. During a draw of lottery, tickets bearing numbers 1, 2, 3,......, 40, 6 tickets are drawn out & then arranged in the descending order of their numbers. In how many ways, it is possible to have 4^{th} ticket bearing number 25.

10. Find the number of distinct natural numbers upto a maximum of 4 digits and divisible by 5, which can be formed with the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 each digit not occuring more than once in each number.

11. The Indian cricket team with eleven players, the team manager, the physiotherapist and two umpires are to travel from the hotel where they are staying to the stadium where the test match is to be played. Four of them residing in the same town own cars, each a four seater which they will drive themselves. The bus which was to pick them up failed to arrive in time after leaving the opposite team at the stadium. In how many ways can they be seated in the cars ? In how many ways can they travel by these cars so as to reach in time, if the seating arrangement in each car is immaterial and all the cars reach the stadium by the same route.

12. There are n straight lines in a plane, no 2 of which parallel , & no 3 pass through the same point. Their point of intersection are joined. Show that the number of maximum fresh lines thus introduced is
$$\frac{n(n-1)(n-2)(n-3)}{8}.$$

13. In how many ways can you divide a pack of 52 cards equally among 4 players. In how many ways the cards can be divided in 4 sets, 3 of them having 17 cards each & the 4^{th} with 1 card.

14. A firm of Chartered Accountants in Bombay has to send 10 clerks to 5 different companies, two clerks in each. Two of the companies are in Bombay and the others are outside. Two of the clerks prefer to work in Bombay while three others prefer to work outside. In how many ways can the assignment be made if the preferences are to be satisfied.

15. A train going from Cambridge to London stops at nine intermediate stations. 6 persons enter the train during the journey with 6 different tickets of the same class. How many different sets of ticket may they have had?

16. How many arrangements each consisting of 2 vowels & 2 consonants can be made out of the letters of the word **'DEVASTATION'**?

17. Find the number of ways in which the letters of the word 'KUTKUT' can be arranged so that no two alike letters are together.

18. Find the number of words each consisting of 3 consonants & 3 vowels that can be formed from the letters of the word **"Circumference".** In how many of these c's will be together.

19. There are 5 white , 4 yellow , 3 green , 2 blue & 1 red ball. The balls are all identical except for colour. These are to be arranged in a line in 5 places. Find the number of distinct arrangements.

20. How many 4 digit numbers are there which contains not more than 2 different digits?

21. In how many ways 8 persons can be seated on a round table

(a) If two of them (say A and B) must not sit in adjacent seats.

(b) If 4 of the persons are men and 4 ladies and if no two men are to be in adjacent seats.

(c) If 8 persons constitute 4 married couples and if no husband and wife, as well as no two men, are to be in adjacent seats?

22. A flight of stairs has 10 steps. A person can go up the steps one at a time, two at a time, or any combination of 1's and 2's. Find the total number of ways in which the person can go up the stairs.

23. Each of 3 committees has 1 vacancy which is to be filled from a group of 6 people. Find the number of ways the 3 vacancies can be filled if ;

(i) Each person can serve on atmost 1 committee.

(ii) There is no restriction on the number of committees on which a person can serve.

(iii) Each person can serve on atmost 2 committees.

24. How many ten digit whole numbers satisfy the following property they have 2 and 5 as digits, and there are no consecutive 2's in the number (i.e. any two 2's are separated by at least one 5).

25. In how many other ways can the letters of the word **MULTIPLE** be arranged;

(i) without changing the order of the vowels

(ii) keeping the position of each vowel fixed &

(iii) without changing the relative order/position of vowels & consonants.

26. 12 persons are to be seated at a square table, three on each side. 2 persons wish to sit on the north side and two wish to sit on the east side. One other person insists on occupying the middle seat (which may be on any side). Find the number of ways they can be seated.

27. How many integers between 1000 and 9999 have exactly one pair of equal digit such as 4049 or 9902 but not 4449 or 4040?

28. Determine the number of paths from the origin to the point (9, 9) in the cartesian plane which never pass through (5, 5) in paths consisting only of steps going 1 unit North and 1 unit East.

29. (i) Prove that : ${}^{n}P_{r} = {}^{n-1}P_{r} + r.\,{}^{n-1}P_{r-1}$

(ii) If ${}^{20}C_{r+2} = {}^{20}C_{2r-3}$ find ${}^{12}C_{r}$

(iii) Find the ratio ${}^{20}C_{p}$ to ${}^{25}C_{r}$ when each of them has the greatest value possible.

(iv) Prove that ${}^{n-1}C_{3} + {}^{n-1}C_{4} > {}^{n}C_{3}$ if $n > 7$.

(v) Find r if ${}^{15}C_{3r} = {}^{15}C_{r+3}$

30. There are 20 books on Algebra & Calculus in our library. Prove that the greatest number of selections each of which consists of 5 books on each topic is possible only when there are 10 books on each topic in the library.

EXERCISE–II

1. Find the number of ways in which 3 distinct numbers can be selected from the set $\{3^1, 3^2, 3^3, ... 3^{100}, 3^{101}\}$ so that they form a G.P.

2. There are counters available in 7 different colours. Counters are all alike except for the colour and they are atleast ten of each colour. Find the number of ways in which an arrangement of 10 counters can be made. How many of these will have counters of each colour.

3. For each positive integer k, let S_k denote the increasing arithmetic sequence of integers whose first term is 1 and whose common difference is k. For example, S_3 is the sequence 1, 4, 7, 10...... Find the number of values of k for which S_k contain the term 361.

4. Find the number of 7 lettered words each consisting of 3 vowels and 4 consonants which can be formed using the letters of the word "DIFFERENTIATION".

5. A shop sells 6 different flavours of ice-cream. In how many ways can a customer choose 4 ice-cream cones if
(i) they are all of different flavours
(ii) they are not necessarily of different flavours
(iii) they contain only 3 different flavours
(iv) they contain only 2 or 3 different flavours?

6. There are ***n*** triangles of positive area that have one vertex A(0, 0) and the other two vertices whose coordinates are drawn independently with replacement from the set {0, 1, 2, 3, 4} e.g. (1, 2), (0, 1), (2, 2) etc. Find the value of ***n***.

7. There are 2n guests at a dinner party. Supposing that the master and mistress of the house have fixed seats opposite one another, and that there are two specified guests who must not be placed next to one another. Show that the number of ways in which the company can be placed is $(2n - 2)!.(4n^2 - 6n + 4)$.

8. (a) How many divisors are there of the number x = 21600. Find also the sum of these divisors.
(b) In how many ways the number 7056 can be resolved as a product of 2 factors.
(c) Find the number of ways in which the number 300300 can be split into 2 factors which are relatively prime.

9. How many 15 letter arrangements of 5 A's, 5 B's and 5 C's have no A's in the first 5 letters, no B's in the next 5 letters, and no C's in the last 5 letters.

10. How many different ways can 15 Candy bars be distributed between Ram, Shyam, Ghanshyam & Balram, if Ram cannot have more than 5 candy bars & Shyam must have at least two. Assume all Candy bars to be alike.

11. Find the number of ways in which the number 30 can be partitioned into three unequal parts, each part being a natural number. What this number would be if equal parts are also included.

12. In an election for the managing committee of a reputed club , the number of candidates contesting elections exceeds the number of members to be elected by r (r > 0). If a voter can vote in 967 different ways to elect the managing committee by voting atleast 1 of them & can vote in 55 different ways to elect (r – 1) candidates by voting in the same manner. Find the number of candidates contesting the elections & the number of candidates losing the elections.

13. Find the number of three digits numbers from 100 to 999 inclusive which have any one digit that is the average of the other two.

14. A man has 3 friends. In how many ways he can invite one friend everyday for dinner on 6 successive nights so that no friend is invited more than 3 times.

15. Find the number of distinct throws which can be thrown with 'n' six faced normal dice which are indistinguishable among themselves.

16. There are 15 rowing clubs; two of the clubs have each 3 boats on the river; five others have each 2 and the remaining eight have each 1; find the number of ways in which a list can be formed of the order of the 24 boats, observing that the second boat of a club cannot be above the first and the third above the second. How many ways are there in which a boat of the club having single boat on the river is at the third place in the list formed above?

17. Consider a 7 digit telephone number 336 - 7624 which has the property that the first three digit prefix, 336 equals the product of the last four digits. How many seven digit phone numbers beginning with 336 have this property, e.g. (336–7624)

18. An 8 oared boat is to be manned by a crew chosen from 14 men of which 4 can only steer but can not row & the rest can row but cannot steer. Of those who can row, 2 can row on the bow side. In how many ways can the crew be arranged.

19. How many 6 digits odd numbers greater than 60,0000 can be formed from the digits 5, 6, 7, 8, 9, 0 if

(i) repetitions are not allowed **(ii)** repetitions are allowed.

20. Find the sum of all numbers greater than 10000 formed by using the digits 0 , 1 , 2 , 4 , 5 no digit being repeated in any number.

21. The members of a chess club took part in a round robin competition in which each plays every one else once. All members scored the same number of points, except four juniors whose total score were 17.5. How many members were there in the club? Assume that for each win a player scores 1 point , for draw 1/2 point and zero for losing.

22. There are 3 cars of different make available to transport 3 girls and 5 boys on a field trip. Each car can hold up to 3 children. Find

(a) the number of ways in which they can be accomodated.

(b) the numbers of ways in which they can be accomodated if 2 or 3 girls are assigned to one of the cars.

In both the cases internal arrangement of children inside the car is considered to be immaterial.

23. Find the number of three elements sets of positive integers {a, b, c} such that $a \times b \times c = 2310$.

24. Find the number of integer betwen 1 and 10000 with at least one 8 and at least one 9 as digits.

25. In Indo–Pak one day International cricket match at Sharjah , India needs 14 runs to win just before the start of the final over. Find the number of ways in which India just manages to win the match (i.e. scores exactly 14 runs) , assuming that all the runs are made off the bat & the batsman can not score more than 4 runs off any ball.

26. A man goes in for an examination in which there are 4 papers with a maximum of m marks for each paper; show that the number of ways of getting 2m marks on the whole is $\frac{1}{3}(m + 1)(2m^2 + 4m + 3)$.

EXERCISE–III

1. How many different nine digit numbers can be formed from the number 223355888 by rearranging its digits so that the odd digits occupy even positions ? [JEE 2000]

(A) 16 (B) 36 (C) 60 (D) 180

2. Let T_n denote the number of triangles which can be formed using the vertices of a regular polygon of 'n' sides. If $T_{n+1} - T_n = 21$, then 'n' equals: [JEE2001]

(A) 5 (B) 7 (C) 6 (D) 4

3. The number of arrangements of the letters of the word BANANA in which the two N's do not appear adjacently is [JEE 2002]

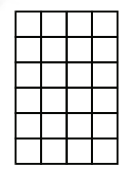
(A) 40 (B) 60 (C) 80 (D) 100

4. Number of points with integral co-ordinates that lie inside a triangle whose co-ordinates are (0, 0), (0, 21) and (21,0) [JEE 2003]

(A) 210 (B) 190 (C) 220 (D) None

5. Using permutation or otherwise, prove that $\frac{(n^2)!}{(n!)^n}$ is an integer, where n is a positive integer. [JEE 2004]

6. A rectangle with sides $2m - 1$ and $2n - 1$ is divided into squares of unit length by drawing parallel lines as shown in the diagram, then the number of rectangles possible with odd side lengths is [JEE 2005]



(A) $(m + n + 1)^2$ (B) 4^{m+n-1}

(C) m^2n^2 (D) $mn(m + 1)(n + 1)$

7. If *r*, *s*, *t* are prime numbers and p, q are the positive integers such that their LCM of p, q is is $r^2t^4s^2$, then the numbers of ordered pair of (p, q) is [JEE 2006, 3]

(A) 252 (B) 254 (C) 225 (D) 224

8. The letters of the word **COCHIN** are permuted and all the permutations are arranged in an alphabetical order as in an English dictionary. The number of words that appear before the word **COCHIN** is

(A) 360 (B) 192 (C) 96 (D) 48 [JEE 2007, 3]

9. Consider all possible permutations of the letters of the word ENDEANOEL

Match the statements / Expression in **Column-I** with the statements / Expressions in **Column-II**.

Column-I	**Column-II**
(A) The number of permutations containing the word ENDEA is	(P) 5!
(B) The number of permutations in which the letter E occurs in the first and the last position is	(Q) 2 × 5!
(C) The number of permutations in which none of the letters D, L, N occurs in the last five positions is	(R) 7 × 5!
(D) The number of permutations in which the letters A, E, O occurs only in odd positions is	(S) 21 × 5!

[JEE 2008, 6]

10. The number of seven digit integers, with sum of the digits equal to 10 and formed by using the digits 1, 2 and 3 only, is [JEE 2009]

(A) 55 (B) 66 (C) 77 (D) 88

11. Let S = {1, 2, 3, 4}. The total number of unordered pairs of disjoint subsets of S is equal to [JEE - 2010]

(A) 25 (B) 34 (C) 42 (D) 41

12. The total number of ways in which 5 balls of different colours can be distributed among 3 persons so that each person gets at least one ball is [JEE - 2012]
(A) 75 (B) 150 (C) 210 (D) 243

13. Let T_n be the number of all possible triangles formed by joining vertices of an n-sided regular polygon. If $T_{n+1} - T_n = 10$, then the value of n is **[IIT JEE Main 2013]**
(A) 8 (B) 7 (C) 5 (D) 10

14. Let $n_1 < n_2 < n_3 < n_4 < n_5$ be positive integers such that $n_1 + n_2 + n_3 + n_4 + n_5 = 20$. Then the number of such distinct arrangements $(n_1, n_2, n_3, n_4, n_5)$ is **[IIT JEE Adv. 2014]**

15. Let $n \geq 2$ be an integer. Take n distinct points on a circle and join each pair of points by a line segment. Colour the line segment joining every pair of adjacent points by blue and the rest by red. If the number of red and blue line segments are equal, then the value of n is **[IIT JEE Adv. 2014]**

16. Six cards and six envelopes are numbered 1, 2, 3, 4, 5, 6, and cards are to be placed in envelopes so that each envelope contains exactly one card and no card is placed in the envelope bearing the same number and moreover the card numbered 1 is always placed in envelope numbered 2. Then the number of ways it can be done is : **[IIT JEE Adv. 2014]**
(A) 264 (B) 265 (C) 53 (D) 67

17. The number of integers greater than 6,000 that can be formed, using the digits 3, 5, 6, 7 and 8, without repetition, is: **[IIT JEE Main 2015]**
(A) 72 (B) 216 (C) 192 (D) 120

18. Let A and B be two sets containing four and two elements respectively. Then the number of subsets of the set A × B, each having at least three elements is : **[IIT JEE Main 2015]**
(A) 510 (B) 219 (C) 256 (D) 275

19. Let n be the number of ways in which 5 boys and 5 girls can stand in a queue in such a way that all the girls stand consecutively in the queue. Let m be the number of ways in which 5 boys and 5 girls can stand in a queue in such a way that exactly four girls stand consecutively in the queue. Then the value of $\frac{m}{n}$ is
[IIT JEE Advanced 2015]

20. If all the words (with or without meaning) having five letters, formed using the letters of the word SMALL and arranged as in a dictionary ; then the position of the word SMALL is : **[IIT JEE Main 2016]**
(A) 46^{th} (B) 59^{th} (C) 52^{nd} (D) 58^{th}

21. A debate club consists of 6 girls and 4 boys. A team of 4 members is to be selected from this club including the selection of a captain (from among these 4 members) for the team. If the team has to include at most one boy, them the number of ways of selecting the team is : **[IIT JEE Advanced 2016]**
(A) 380 (B) 320 (C) 260 (D) 95

22. A man X has 7 friends, 4 of them are ladies and 3 are men. His wife Y also has 7 friends, 3 of them are ladies and 4 are men. Assume X and Y have no common friends. Then the total number of ways in which X and Y together can throw a party inviting 3 ladies and 3 men, so that 3 friends of each of X and Y are in this party, is : **[IIT JEE Main 2017]**
(A) 469 (B) 484 (C) 485 (D) 468

23. Let S = {1, 2, 3, ..., 9}. For k = 1, 2, ... 5, let N_k be the number of subsets of S, each containing five elements out of which exactly k are odd. Then $N_1 + N_2 + N_3 + N_4 + N_5 =$ **[JEE Advanced 2017]**

(A) 125 (B) 252 (C) 210 (D) 126

24. Words of length 10 are formed using the letters A, B, C, D, E, F, G, H, I, J. Let x be the number of such words where no letter is repeated; and let y be the number of such words where exactly one letter is repeated twice and no other letter is repeated. Then $\frac{y}{9x} =$ **[JEE Advanced 2017]**

25. From 6 different novels and 3 different dictionaries, 4 novels and 1 dictionary are to be selected and arranged in a row on a shelf so that the dictionary is always in the middle. The number of such arrangmenets is :

(A) at least 750 but less than 1000 (B) at least 1000 **[JEE Main 2018]**

(C) less than 500 (D) at least 500 but less than 750

26. In a high school, a committee has to be formed from a group of 6 boys $M_1, M_2, M_3, M_4, M_5, M_6$ and 5 girls G_1, G_2, G_3, G_4, G_5.

(i) Let α_1 be the total number of ways in which the committee can be formed such that the committee has 5 members, having exactly 3 boys and 2 girls.

(ii) Let α_2 be the total number of ways in which the committee can be formed such that the committee has at least 2 members, and having an equal number of boys and girls.

(iii) Let α_3 be the total number of ways in which the committee can be formed such that the committee has 5 members, at least 2 of them being girls.

(iv) Let α_4 be the total number of ways in which the committee can be formed such that the committee has 4 members, having at least 2 girls and such that both M_1 and G_1 are **NOT** in the committee together. **[JEE Advanced 2018]**

	List - I		List-II
(P)	The value of α_1 is	(1)	136
(Q)	The value of α_2 is	(2)	189
(R)	The value of α_3 is	(3)	192
(S)	The value of α_4 is	(4)	200
		(5)	381
		(6)	461

The correct option is :

(A) $P \to 4;\ Q \to 6;\ R \to 2;\ S \to 1$ (B) $P \to 1;\ Q \to 4;\ R \to 2;\ S \to 3$

(C) $P \to 4;\ Q \to 6;\ R \to 5;\ S \to 2$ (D) $P \to 4;\ Q \to 2;\ R \to 3;\ S \to 1$

27. The number of 5 digit numbers which are divisible by 4, with digits from the set {1, 2, 3, 4, 5} and the repetition of digits is allowed, is _____________ . **[JEE Advanced 2018]**

28. Five persons A, B, C, D and E are seated in a circular arrangement. If each of them is given a hat of one of the three colours red, blue and green, then the number of ways of distributing the hats such that the persons seated in adjacent seats get different coloured hats is___. **[JEE Advanced 2019]**

29. An engineer is required to visit a factory for exactly four days during the first 15 days of every month and it is mandatory that **no** two visits take place on consecutive days. Then the number of all possible ways in which such visits to the factory can be made by the engineer during 1-15 June 2021 is _____ **[JEE Advanced 2020]**

30. In a hotel, four rooms are available. Six persons are to be accommodated in these rooms in such a way that each of these rooms contains at least one person and at most two persons. Then the number of all possible ways in which this can be done is _____. **[JEE Advanced 2020]**

ANSWER KEY

EXERCISE–I

1. ${}^{m+n+k}C_3 - ({}^{m}C_3 + {}^{n}C_3 + {}^{k}C_3)$ **2.** 744 **3.** 13, 156 **4.** 4316527 **5.** 43200 **6.** 145

7. 420 **8.** 960 **9.** ${}^{24}C_2 \cdot {}^{15}C_3$ **10.** 1106 **11.** $12!\ ;\ \dfrac{11!\,.\,4!}{(3!)^4\,2!}$

13. $\dfrac{52!}{(13!)^4}\ ;\ \dfrac{52!}{3!(17!)^3}$ **14.** 5400 **15.** ${}^{45}C_6$ **16.** 1638 **17.** 30 **18.** 22100 , 52

19. 2111 **20.** 576 **21.** (a) $5 \cdot (6!)$, (b) $3! \cdot 4!$, (c) 12 **22.** 89

23. 120, 216, 210 **24.** 143 **25.** **(i)** 3359; **(ii)** 59; **(iii)** 359 **26.** 2 ! 3 ! 8 !

27. 3888 **28.** 30980 **29.** **(ii)** 792 ; **(iii)** 143/4025 ; **(v)** r = 3

EXERCISE–II

1. 2500 **2.** 7^{10}; (49/6) 10! **3.** 24 **4.** 532770 **5.** (i) 15, (ii) 126, (iii) 60, (iv) 105

6. 256 **8.** **(a)** 72; 78120; **(b)** 23; **(c)** 32 **9.** 2252 **10.** 440 **11.** 61, 75

12. 10, 3 **13.** 121 **14.** 510 **15.** ${}^{n+5}C_5$ **16.** $\dfrac{24!}{(3!)^2(2!)^5}\ ;\ {}^{8}C_1 \cdot \dfrac{23!}{(3!)^2(2!)^5}$

17. 84 **18.** $4\,.\,(4!)^2\,.\,{}^{8}C_4\,.\,{}^{6}C_2$ **19.** 240 , 15552 **20.** 3119976 **21.** 27

22. (a) 1680; (b) 1140 **23.** 40 **24.** 974 **25.** 1506

EXERCISE–III

1. C **2.** B **3.** A **4.** B **6.** C **7.** C **8.** C

9. (A) P; (B) S; (C) Q; (D) Q **10.** C **11.** D **12.** B **13.** C

14. 7 **15.** 5 **16.** C **17.** C **18.** B **19.** 5

20. D **21.** A **22.** C **23.** D **24.** 5 **25.** B

26. C **27.** 625 **28.** 30.00 **29.** 495.00 **30.** 1080.00