Nishant Jindal

Reduced Syllabus मे

Abhi Bat nahi Karenge.

De-Moivere's Thm

1) $(\cos\theta + i\sin\theta)^n = \begin{cases} \cos n\theta + i\sin n\theta & \text{if } n \in I \\ (\cos n\theta + i\sin n\theta) \text{ \& more Answers} & \text{if } n \in Q - \{I\} \end{cases}$

(2) $(x+iy)^n = (|z|e^{i\theta})^n = |z|^n \cdot e^{in\theta}$

Ex:-> $(\cos\pi + i\sin\pi)^3 = (e^{i\pi})^3 = e^{i3\pi} = \cos 3\pi + i\sin 3\pi$
$= -1 + 0$
$= -1$

Ex:- $(\cos\pi + i\sin\pi)^{\sqrt{2}} \neq \cos\sqrt{2}\pi + i\sin\sqrt{2}\pi$

$\sqrt{2} \notin I$

$\sqrt{2} \notin Q - \{I\}$

DMT will not

DMT will be applied only for Rational Deg.

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Q $(1+i)^{100} = \left(\sqrt{2}\, e^{i\frac{\pi}{4}}\right)^{100} = (\sqrt{2})^{100} \cdot e^{i25\pi}$

$(1,1)$

$\operatorname{Arg} z = \tan^{-1}\left|\frac{1}{1}\right| = \frac{\pi}{4}$

$= 2^{50} \cdot (\cos 25\pi + i\sin 25\pi)$

$= 2^{50}(-1+0)$

$= -2^{50}$

Basic Qs

① $(\cos\theta + i\sin\theta)^n = ?$

$\cos n\theta + i\sin n\theta$

(2) $(\cos\theta - i\sin\theta)^n$

$\Rightarrow (\cos(-\theta) + i\sin(-\theta))^n = \cos(-n\theta) + i\sin(-n\theta)$

$= \cos n\theta - i\sin n\theta$

(3) $(\sin\theta + i\cos\theta)^n = ?$

$\left(\cos\left(\frac{\pi}{2}-\theta\right) + i\sin\left(\frac{\pi}{2}-\theta\right)\right)^n$

DMT

$\cos\left(\frac{n\pi}{2} - n\theta\right) + i\sin\left(\frac{n\pi}{2} - n\theta\right)$

$e^{i\left(\frac{n\pi}{2} - n\theta\right)}$

(4) $(\sin\theta + i\cos\theta)^6$

$\left(\cos\left(\frac{\pi}{2}-\theta\right) + i\sin\left(\frac{\pi}{2}-\theta\right)\right)^6$

DMT

$(\cos(3\pi - 6\theta) + i\sin(3\pi - 6\theta))$

$(-\cos 6\theta + i\sin 6\theta)$

(5) P.T.

$\prod_{K=1}^{6} \operatorname{cis}\theta_K = \operatorname{cis}\sum_{K=1}^{6}\theta_K$

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P.T.

Q $\prod_{K=1}^{6} \text{Cis}(\theta_K) = \text{Cis} \sum_{K=1}^{6} \theta_K$

$(\cos\theta_1 + i\sin\theta_1) \cdot (\cos\theta_2 + i\sin\theta_2) \cdot (\cos\theta_3 + i\sin\theta_3) \cdots (\cos\theta_6 + i\sin\theta_6)$

$e^{i\theta_1 + i\theta_2 + i\theta_3 \cdots + i\theta_6} = e^{i(\theta_1 + \theta_2 + \cdots \theta_6)} = e^{i\left(\sum_{K=1}^{6} \theta_K\right)}$

$= \text{Cis} \sum_{K=1}^{6} \theta_K$ RHS

(6) $(1 + \cos\theta + i\sin\theta)^{10} = ?$

$= \left(2\cos^2\frac{\theta}{2} + i\,2\sin\frac{\theta}{2} \cdot \cos\frac{\theta}{2}\right)^{10}$

$= 2^{10}\cos^{10}\frac{\theta}{2}\left(\cos\frac{\theta}{2} + i\sin\frac{\theta}{2}\right)^{10} = 2^{10}\cos^{10}\frac{\theta}{2}(\cos 5\theta + i\sin 5\theta)$ ← DMT

(7) $\dfrac{1 + \cos\theta + i\sin\theta}{1 + \cos\theta - i\sin\theta} = ?$

$= \dfrac{2\cos^2\frac{\theta}{2} + i\,2\sin\frac{\theta}{2} \cdot \cos\frac{\theta}{2}}{2\cos^2\frac{\theta}{2} - i\,2 \cdot \sin\frac{\theta}{2} \cdot \cos\frac{\theta}{2}}$

$= \dfrac{\cos\frac{\theta}{2} + i\sin\frac{\theta}{2}}{\cos\frac{\theta}{2} - i\sin\frac{\theta}{2}} = \dfrac{e^{i\theta/2}}{e^{-i\theta/2}} = e^{i\theta}$

$= \cos\theta + i\sin\theta$

Q $\left(\dfrac{1 + \cos\theta + i\sin\theta}{1 + \cos\theta - i\sin\theta}\right)^n = ?$

$\cos n\theta + i\sin n\theta$

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n^{th} Root of C.N.

$$Z = (x+iy)^{1/n}$$

$$Z = \left(|z| e^{i\theta}\right)^{1/n}$$

$$= |z|^{1/n} \cdot e^{i\left(\frac{\theta + 2K\pi}{n}\right)}$$

$$\frac{\theta}{n} + \frac{2\cancel{n}\pi}{\cancel{n}}$$

Rad $\quad K = 0, 1, 2, 3, 4 \ldots (n-1)$

(1) all n Roots lie on a Circle of Rad $= |z|^{1/n}$

$K=1 \rightarrow Z = |z|^{1/n} \cdot e^{i\left(\frac{\theta + 2\pi}{n}\right)}$

$= |z|^{1/n} \cdot e^{i\left(\frac{\theta}{n} + \frac{2\pi}{n}\right)}$

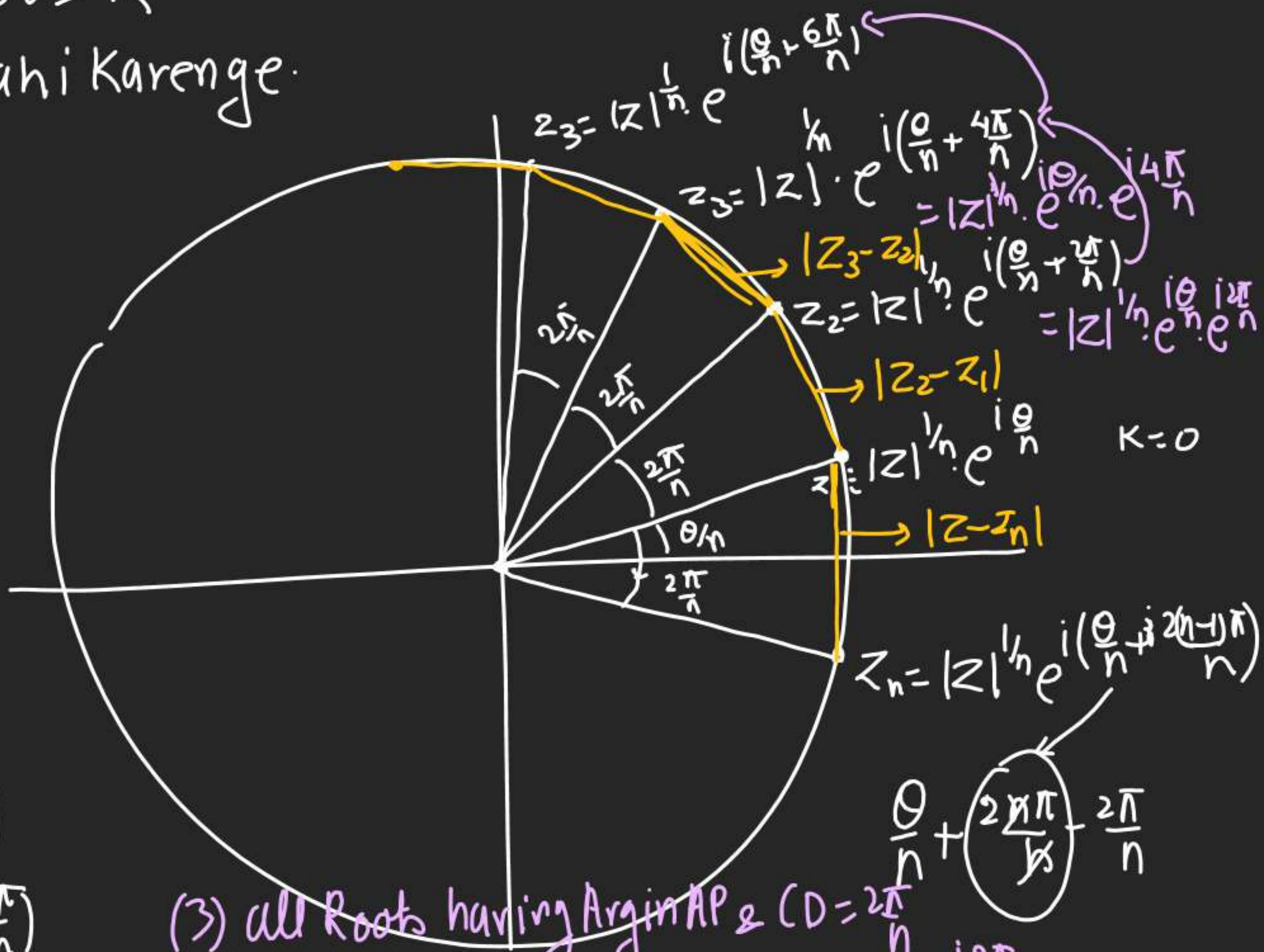
(2) But all of them are vertice of n side Polygon

whose Side $= |z_3 - z_2| = |z_2 - z_1|$

$= |z - z_n|$

$K=2 \quad Z = |z|^{1/n} \cdot e^{i\left(\frac{\theta}{n} + \frac{4\pi}{n}\right)}$

$K = n-1 \quad Z = |z|^{1/n} \cdot e^{i\left(\frac{\theta}{n} + \frac{2(n-1)\pi}{n}\right)}$



$$\frac{\theta}{n} + \frac{2\cancel{n}\pi}{\cancel{n}} - \frac{2\pi}{n}$$

(3) all Roots having Arg in AP & $CD = \frac{2\pi}{n}$

But Roots are in G.P $\rightarrow$ C.R $= e^{\frac{i2\pi}{n}}$

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Q For any Int. K

$\alpha_K = \cos\frac{K\pi}{7} + i\sin\frac{K\pi}{7} \to \alpha_K = 1\cdot e^{i\left(\frac{0+2K\pi}{14}\right)}$

Value of Exp. $\dfrac{\sum_{K=1}^{12}|\alpha_{K+1}-\alpha_K|}{\sum_{K=1}^{3}|\alpha_{4K-1}-\alpha_{4K-2}|} = \dfrac{12K}{3K} = 4$ A.

Circle पर स्थित whose Rad = 1 $\quad |z_2^2 - z_1^2|$

14 Sides Polygon

$= |z_2-z_1||z_2+z_1| \le |z_2-z_1|(|z_1|+|z_2|)$

$\le 2|z_2-z_1|$

$K_1 \to Nr = |\alpha_2-\alpha_1| = K$

$Dr = |\alpha_3-\alpha_2| = K$

Q Let $\theta_1, \theta_2, \theta_3 \dots \theta_{10}$ be +ve Angles such that $\theta_1+\theta_2+\theta_3 \dots + \theta_{10} = 2\pi$

Defining C.N. $z_1 = 1e^{i\theta_1}$, $z_K = z_{K-1}e^{i\theta_K}$ for $K = 1, 2, 3 \dots 10$ Conside Statements P & Q

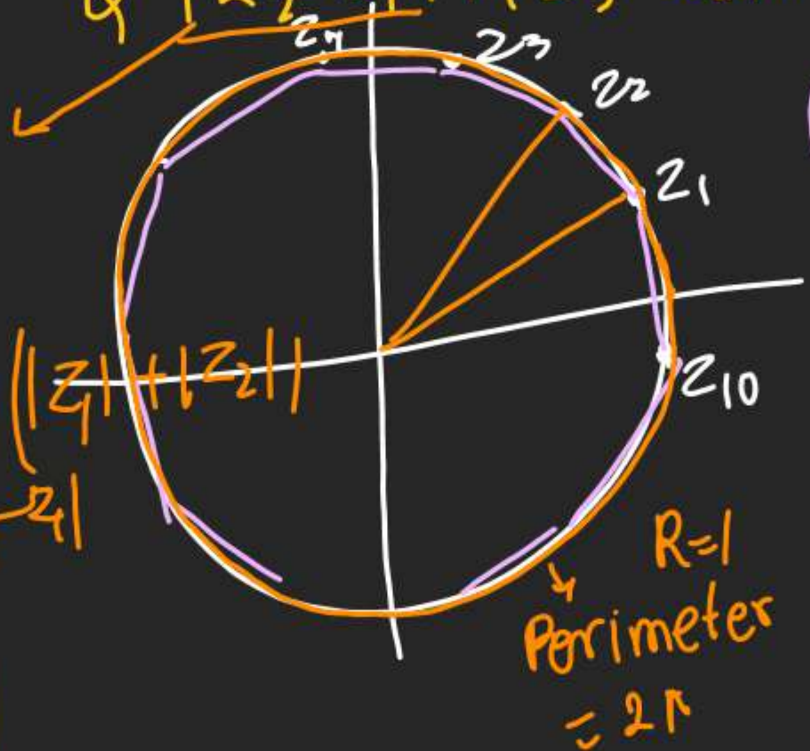
P: $|z_2-z_1|+|z_3-z_2|+ \dots + |z_1-z_{10}| \le 2\pi$

Q: $|z_2^2-z_1^2|+|z_3^2-z_2^2|+ \dots + |z_1^2-z_{10}^2| \le 4\pi \le 4\pi$

$\le 2|z_2-z_1|+2|z_3-z_2|+\dots+2|z_1-z_{10}|$

$|z_2-z_1|+|z_3-z_2|+\dots+|z_1-z_{10}|$

$\le 2\pi$



P is Correct

Q is also Correct

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Q How to find n^{th} Root of C.N.

Ex:→ $Z = (1+i)^{1/3}$ → 3 Roots ऽiji

1) Write z in Exp. form → $|z| \cdot e^{i\theta}$

$$Z = \left(\sqrt{2} \cdot e^{i\frac{\pi}{4}}\right)^{1/3}$$

2) Now add $2K\pi$ in Arg & Use DMT

$$Z = (\sqrt{2})^{1/3} \cdot \left(e^{i\left(\frac{\pi}{4}+2K\pi\right)}\right)^{1/3}$$

$$= (\sqrt{2})^{1/3} \cdot e^{i\left(\frac{\frac{\pi}{4}+2K\pi}{3}\right)}$$

$K = 0, 1, 2$

3) Roots $z_1 = \underbrace{2^{1/6} e^{i\frac{\pi}{12}}}_{\text{LPA}}, \; z_2 = 2^{1/6} e^{i\frac{9\pi}{12}}, \; z_3 = 2^{1/6} e^{i\frac{17\pi}{12}}$

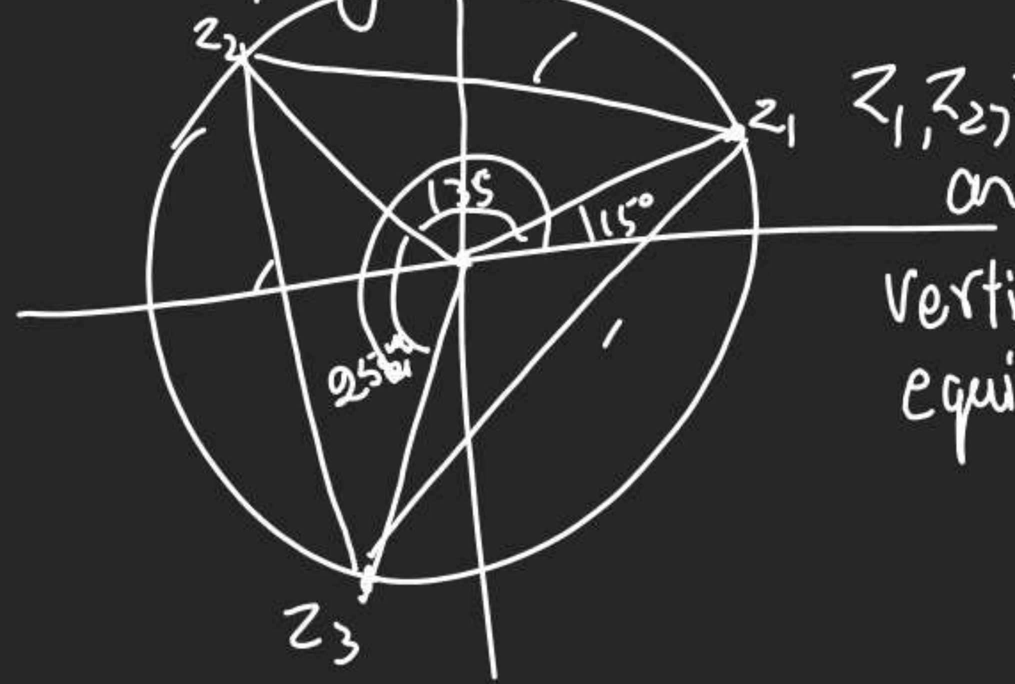
LPA
Least +ve Arg.

4) Real Part -ve a/c Root?

$$\underset{\substack{\downarrow \\ 1^{st} \\ (+,+)}}{2^{1/6} \cdot e^{i\overset{15^\circ}{\frac{\pi}{12}}}}, \quad \underset{\substack{2^{nd} \\ (-,+)}}{2^{1/6} \cdot e^{i\overset{135^\circ}{\frac{9\pi}{12}}}}, \quad \underset{\substack{3^{rd} \\ (-,-)}}{2^{1/6} \cdot e^{i\overset{255}{\frac{17\pi}{12}}}}$$

2 Roots $= z_2, z_3$

(5) Diagram → lying on Circle Rad $= 2^{1/6}$



z_1, z_2, z_3 are vertices of equilateral Δ.

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Cube Root of Unity

Q. $(1)^{1/3} = ?$

$Z = (1)^{1/3}$

$= (1\cdot e^{i0})^{1/3}$

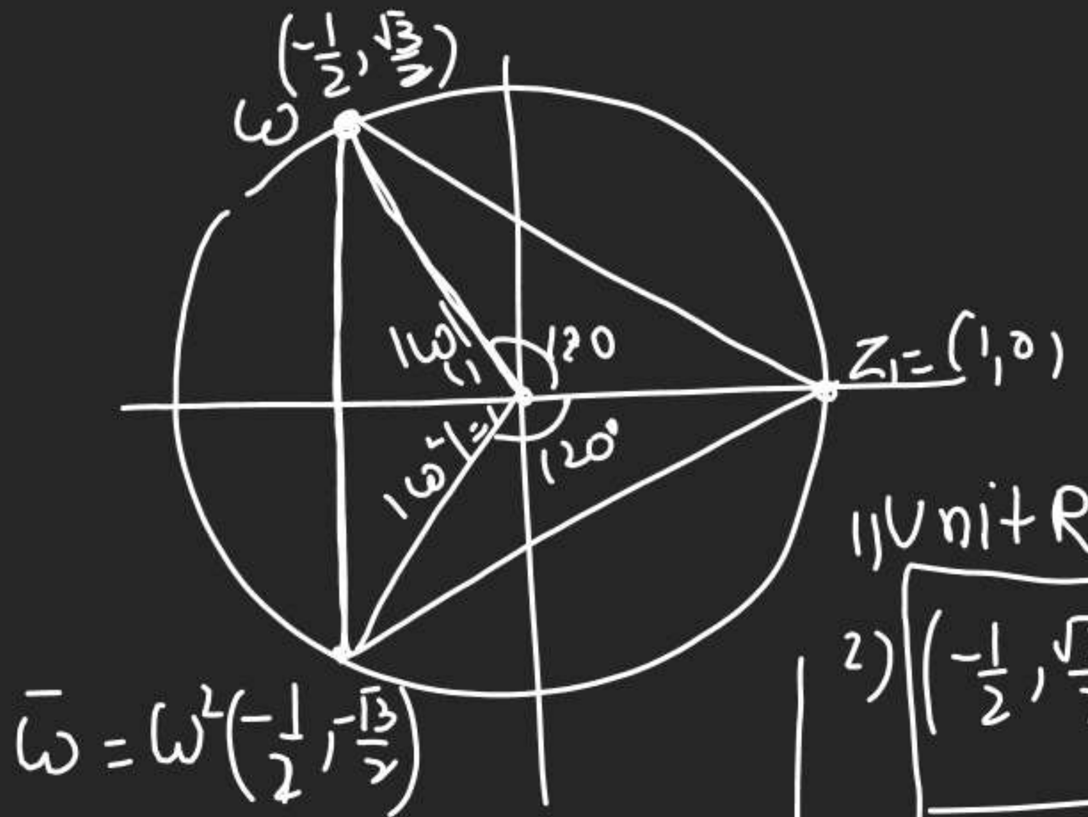
$= 1^{1/3}\cdot e^{i\left(\frac{0+2K\pi}{3}\right)}$

$K = 0, 1, 2$

$K=0$: $Z_1 = 1\cdot e^{i0} = 1+0i$
$(1,0)$

$K=1$: $Z_2 = 1\cdot e^{i\frac{2\pi}{3}}$ (120°)
$= (\cos 120^\circ + i\sin 120^\circ)$
$= \left(-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)$

$K=2$: $Z_3 = 1\cdot e^{i\frac{4\pi}{3}}$ (240°)
$= (\cos 240^\circ + i\sin 240^\circ)$
$\left(-\frac{1}{2} - i\frac{\sqrt{3}}{2}\right)$



6) $Z = (1)^{1/3}$
$\Rightarrow Z^3 = 1$
$\Rightarrow Z^3 - 1 = 0$
$\Rightarrow (Z-1)(Z^2+Z+1) = 0$
$Z = 1$ | ω, ω^2

$Z^2 + Z + 1 = (Z-\omega)(Z-\omega^2)$

$x^2 + x + 1 = (x-\omega)(x-\omega^2)$

1) Unit Rad Circle

2)

$\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right) = \omega$	$-\omega = \left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$
$\left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right) = \omega^2$	$-\omega^2 = \left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

3) ω & ω^2 are image of each other.

4) $|\omega| = |\omega^2| = 1$

(5) $\frac{1}{\omega} = \frac{\bar{\omega}}{|\omega|^2} \Rightarrow \frac{1}{\omega} = \bar{\omega} = \omega^2$

$\boxed{\omega^3 = 1} \Rightarrow \omega^{3K} = 1$
$K \in I$

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(7) $1+\omega+\omega^2=0$

$1+\omega=-\omega^2$ $\quad$ $1+\omega^2=-\omega$

Ex $\omega^7=?$

$(\omega^3)^2\cdot\omega$

$=\omega$

(8) $\bar{\omega}=\omega^2$, $\bar{\omega}^2=\omega$ | $\frac{1}{\omega}=\omega^2$

(9) $\omega^{3n}=1$, $\omega^{3n+1}=\omega$, $\omega^{3n+2}=\omega^2$

(10) $|\omega|=|\omega^2|=1$

(11) $|\omega-1|=|\omega^2-1|=|\omega-\omega^2|$

(12) $a^3+b^3=(a+b)\underline{(a^2-ab+b^2)}$

$=(a+b)(a+b\omega)(a+b\omega^2)$

$(a^2+ab\omega^2+ab\omega+b^2\omega^3)$

$(a^2+b^2+ab(\omega+\omega^2))$

(a^2+b^2-ab)

(13) $a^3-b^3=(a-b)(a^2+ab+b^2)$

$a^3-b^3=(a-b)(a-b\omega)(a-b\omega^2)$

(C) $a^3+b^3+c^3-3abc$

$=(a+b+c)(a^2+b^2+c^2-ab-bc-ca)$

$=\cancel{(a+b+c)}(a+b+c)(a+b\omega+c\omega^2)(a+b\omega^2+c\omega)$

(12) $1+\omega^k+\omega^{2k}=\begin{cases}0 & k\neq 3n\\ 3 & k=3n\end{cases}$

$1+\omega^2+\omega^4$

$1+\omega^2+\omega$

$=0$

$1+\omega^3+\omega^6$

$1+1+1$

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Q $z+\frac{1}{z}=1$ find $z^{2014}+z^{-2013}$

$\downarrow$

$z^2-z+1=0$

$(-z)^2+(-z)+1=0 \begin{cases} \omega \\ \omega^2 \end{cases}$

$\Rightarrow z=-\omega,-\omega^2$

Demand $=(-\omega)^{2014}+(-\omega)^{-2013}$

$=\omega^{2014}-\frac{1}{\omega^{2013}}$

$=\left((\omega^3)^{671}\right)\cdot\omega-\frac{1}{1}$

$=\omega-1$