PROBLEM SET-04

SOLUTION

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Q.1 Find the number of ways in which two squares can be selected from an 8 by 8 chess board of size 1×1 so that they are not in the same row or in the same column.

Ans. **(1568)**

Sol. 1^{st} square in 64 ways

2^{nd} square in 49 ways

$\therefore\ 64 \times 49$ (think!)

but we are counting the same selection twice hence required ways $= \frac{64\times49}{2} = 1568$

Q.2 Number of different natural numbers which are smaller than two hundred million & using only the digits 1 or 2 is:

(A) $(3)\cdot 2^8 - 2$ (B) $(3)\cdot 2^8 - 1$ (C) $2(2^9 - 1)$ (D) none

Ans. **(A)**

Sol. Two hundred million $= 2 \times 10^8$; $(2^1 + 2^2 + 2^3 + 2^4 + 2^5 + 2^6 + 2^7 + 2^8) + 2^8 = 766$

Q.3 5 Indian & 5 American couples meet at a party & shake hand. If no wife shakes hands with her own husband & no Indian wife shakes hands with a male, then the number of handshakes that takes place in the party is

(A) 95 (B) 110 (C) 135 (D) 150

Ans. **(C)**

Sol. ${}^{20}C_2 - (50 + 5) = 135$

Q.4 The number of n digit numbers which consists of the digits 1&2 only if each digit is to be used at least once, is equal to 510 then n is equal to:

(A) 7 (B) 8 (C) 9 (D) 10

Ans. **(C)**

Sol. $(2 \times 2 \times \ldots\ldots\ldots 2)$ n times-(when 1 or 2 is there at all the n places

$2^n - 2 = 510; 2^n = 512 \Rightarrow n = 9$

Q.5 Number of six digit numbers which have 3 digits even &3 digits odd, if each digit is to be used at most once is

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Ans. **(64800)**

Sol. Total = ${}^5C_3 \cdot {}^5C_3 \cdot 6!$. Since all digits 0,1,2, … … . .8,9 are equally likely at all places

$\Rightarrow$ required number $= \frac{{}^5C_3 \cdot {}^5C_3 \cdot 6!}{10} \cdot 9$ digits $<^{0|2|4|6|8}_{1|3|5|7|9}$

alternatively required number of ways $= \underbrace{{}^4C_3 \cdot {}^5C_3 \cdot 6!}_{\text{'0' excluded}} + \underbrace{{}^4C_2 \cdot {}^5C_3(6! - 5!)}_{\text{'0' included}}$

Q.6 Find the number of odd integers between 1000 and 8000 which have none of their digits repeated.

Ans. **(1736)**

Sol. Let the last place is 9

then we have $7 \cdot 8 \cdot 7 = 392$ (1^{st} place only 1 to 7 can come)

If the last place has 1|3|5 | 7 then

we have (6) (8) (7) (4) = 1344[0,9 or 8 can not come at unit's place]

Total $= 392 + 1344 = 1736$

Q.7 Number of four digit numbers with all digits different and containing the digit 7 is

(A) 2016 (B) 1828 (C) 1848 (D) 1884

Ans. **(C)**

Sol-1. 7 along with 3 other digit can be selected in ${}^9C_3$ ways and arranged in 4! ways.

Required number of ways $= {}^9C_3 \cdot 4! -$ (when ' 0 ' and 7 ' always included and ' 0 ' occupies the first place)

$= {}^9C_3 \cdot 4! - {}^8C_2 \cdot 3! = 84 \cdot 4 \cdot 3! - 28 \cdot 3! = 7 \cdot 3!\,[48 - 4] = 6 \cdot 7 \cdot 44 = 1848$ Ans.]

Sol-2. 0 excluded and 7 included ${}^8C_3$ e.g. 1237 $\rightarrow {}^8C_3 \cdot 4! = 56 \cdot 24$

' 0 ' included and 7 included ${}^8C_2$ e.g. 0712 $\rightarrow {}^8C_2 \cdot (4! - 3!) = 28 \cdot 18 = 28 \cdot 6 \cdot 11$ Ans.]

Sol-3. Total - when no 7 is included

$= 9 \cdot 9 \cdot 8 \cdot 7 - 8 \cdot 8 \cdot 7 \cdot 6$

$= 7 \cdot 8(81 - 48) = 1848$ Ans.

Sol-4. If 7 is at the $(1000)^{th}$ place then $9 \cdot 8 \cdot 7 = 504$

If 7 is not at the $(1000)^{th}$ place then ${}^3C_1 \cdot 8 \cdot 8 \cdot 7 = 1344$

$\therefore$ Total $= 1848$

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Q.8 A 5 digit number divisible by 3 is to be formed using the numerals 0,1,2,3,4&5 without repetition. The total number of ways this can be done is :

(A) 3125 (B) 600 (C) 240 (D) 216

Ans. **(D)**

Sol. reject 0 + reject 3 $\Rightarrow 5! + 4 \cdot 4! = 120 + 96 = 216$

Q.9 A committee of 5 is to be chosen from a group of 9 people. Number of ways in which it can be formed if two particular persons either serve together or not at all and two other particular persons refuse to serve with each other, is

(A) 41 (B) 36 (C) 47 (D) 76

Ans. **(A)**

Sol. 9 → AB; C / D separated; 5 other

AB included $^7C_3 - {^5C_1} = 30 \rightarrow$ ($^7C_3$ denotes any 3 from CD and 5 others – no. of ways when CD)

AB excluded $^7C_5 - {^5C_3} = 11$ is taken and one from remaining five)

|||ly when AB excluded

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Q.10 A question paper on mathematics consists of twelve questions divided into three parts A, B and C, each containing four questions. In how many ways can an examinee answer five questions, selecting atleast one from each part.

(A) 624 (B) 208 (C) 2304 (D) none

Ans. **(A)**

Sol. $3({^4C_2} \cdot {^4C_2} : {^4C_1}) + 3({^4C_1} \cdot {^4C_1} \cdot {^4C_3}) = 432 + 194 = 624$

Alternative: Total- [no of ways in which he does not select any question from any one section]

$^{12}C_5 - 3 \cdot {^8C_5}$; Note that $^4C_1 \cdot {^4C_1} \cdot {^4C_1} \cdot {^9C_2}$ is wrong think!

Q.11 Consider 8 vertices of a regular octagon and its centre. If T denotes the number of triangles and S denotes the number of straight lines that can be formed with these 9 points then T − S has the value equal to

(A) 52 (B) 56 (C) 48 (D) 44

Ans. **(A)**

Sol. $({^9C_3} - 4) - {^8C_2} = 52$