

## Relative velocity

Find the closest distance  
of approach b/w A & B

$\vec{V}_{A/B} = V_1 \hat{j}$   
 $\vec{V}_{B/A} = -V_2 \hat{i}$   
 $\vec{V}_{A/B} = \vec{V}_A - \vec{V}_B$   
 $= V_1 \hat{j} - (-V_2) \hat{i}$   
 $\vec{V}_{A/B} = V_1 \hat{j} + V_2 \hat{i}$

$$|\vec{V}_{A/B}| = \sqrt{v_1^2 + v_2^2}$$


$$\tan\theta = \frac{V_2}{V_1}$$

2

B CRes

$d_c$  = (closest distance,

B<sub>C</sub>=

$$\sqrt{v_1^2 + v_2^2}$$

In  $\triangle ABC$

$$\sin \theta = \frac{BC}{AB}$$

$$BC = AB \sin S$$

$$BC = d \sin \theta$$

$$d_c = \frac{d_{V_2}}{\sqrt{V_1^2 + V_2^2}}$$

2 Time for Closest distance

$$t = \frac{AC}{|V_A/V_B|} = \frac{d \cos \theta}{\sqrt{V_x^2 + V_y^2}}$$

$$t = \frac{d_{V_1} |V_{A/B}|}{\sqrt{V_1^2 + V_2^2}} = \left( \frac{d_{V_1}}{\sqrt{V_1^2 + V_2^2}} \right)$$

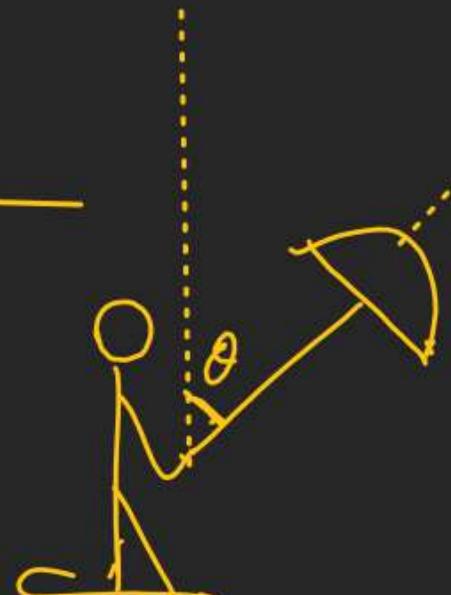
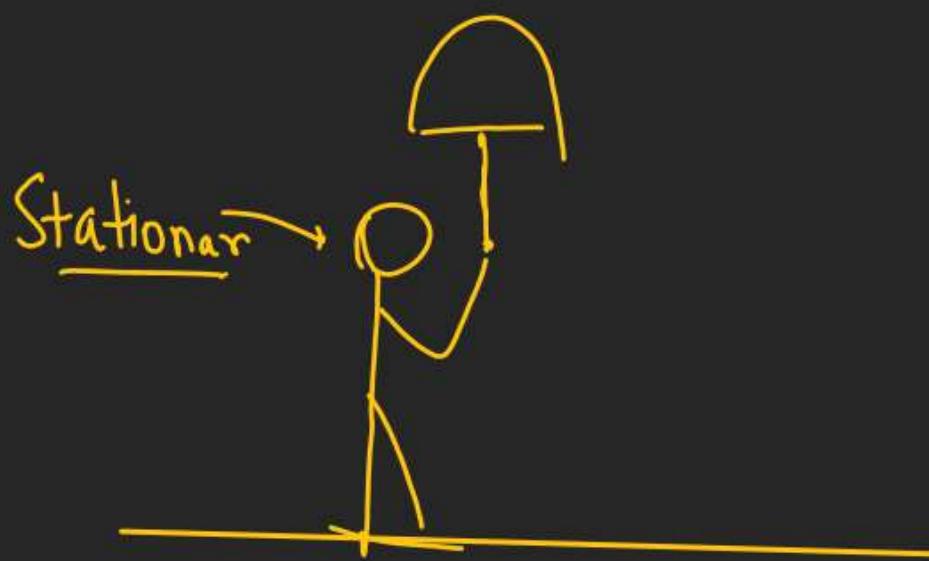
# Relative velocity

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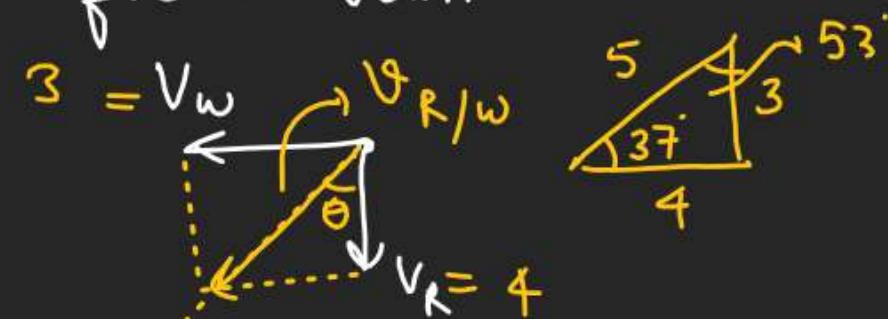
## Rain - man problem



$$\downarrow V_R = 4 \text{ m/s}$$



Wind starts blowing in West direction with velocity 3 m/s. Find the angle from vertical man rotates its hand to protect from rain

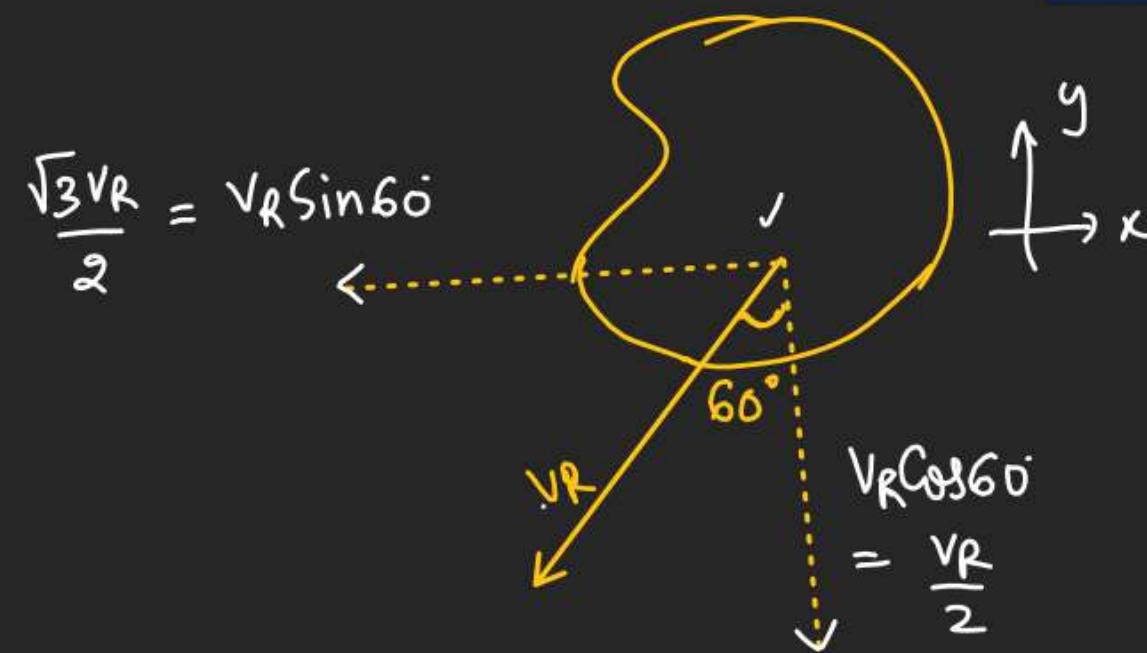


$$\tan \theta = \frac{V_w}{V_R}$$

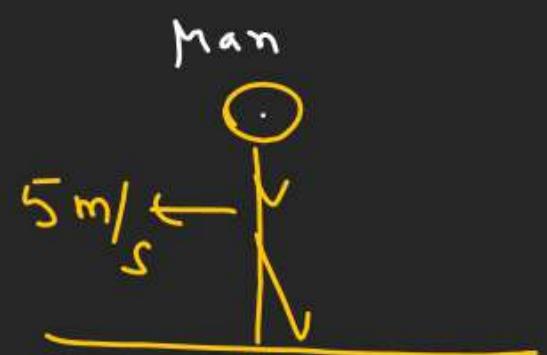
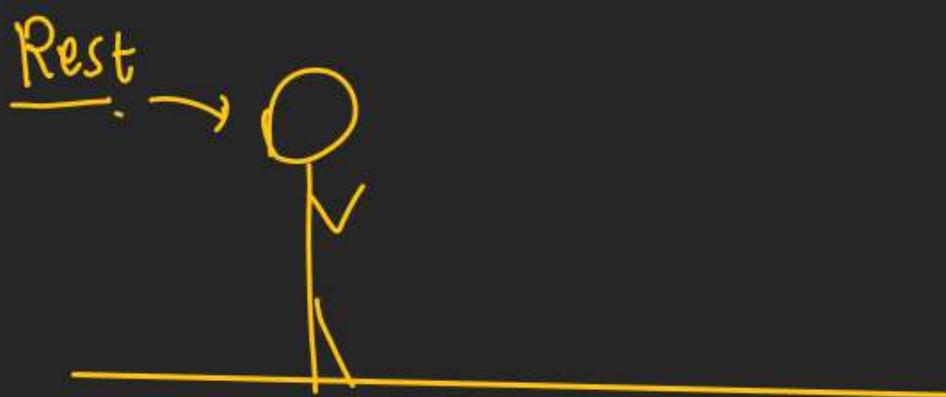
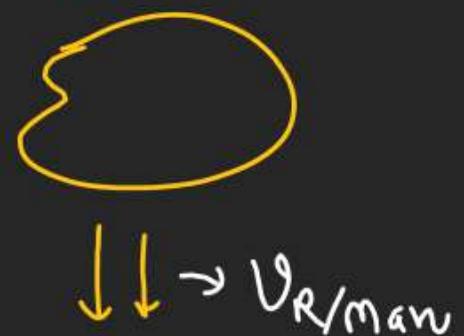
$$\tan \theta = \frac{3}{4}$$

37°

# Relative velocity



when man starts running with velocity 5m/s. Rain drops seem's to fall vertically downward. Find the speed of the rain.



According to question

$$\begin{aligned}
 \underline{\overrightarrow{V}_{R/\text{Man}}} &= \underline{\overrightarrow{V}_{R/\text{E}}} - \underline{\overrightarrow{V}_{\text{Man}/\text{E}}} \\
 &= -\frac{\sqrt{3}V_R}{2}\hat{i} - \frac{V_R}{2}\hat{j} + 5\hat{i} \\
 \underline{\overrightarrow{V}_{R/\text{M}}} &= \underbrace{\left(5 - \frac{\sqrt{3}V_R}{2}\right)\hat{i}} - \underbrace{\frac{V_R}{2}\hat{j}}.
 \end{aligned}$$

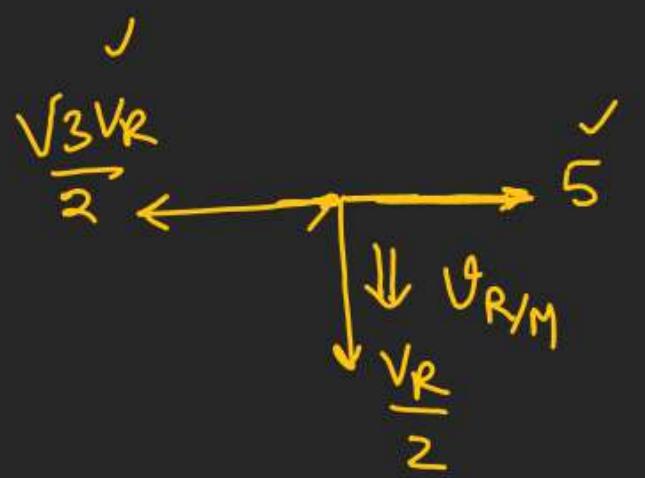
$$\begin{aligned}
 (\underline{V_{R/M}})_x &= 0 \\
 5 - \frac{\sqrt{3}V_R}{2} &= 0 \Rightarrow V_R = \frac{10}{\sqrt{3}} \text{ m/s} \checkmark
 \end{aligned}$$

# Relative velocity

(brick).

$$S = \frac{\sqrt{3} V_R}{2}$$

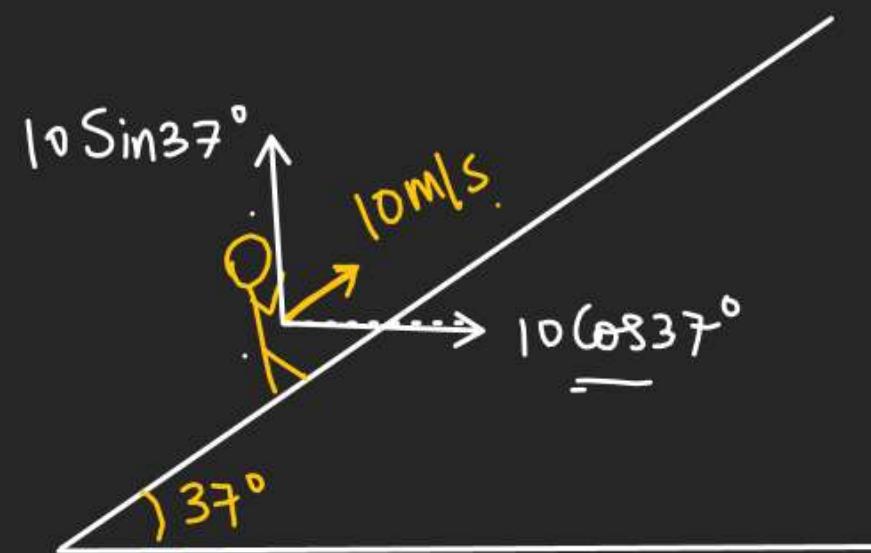
$$\sqrt{R} = \left( \frac{10}{\sqrt{3}} \text{ m/s} \right)$$



# Relative velocity

# W.r.t man rains drops falling vertically downward with speed 5 m/s.

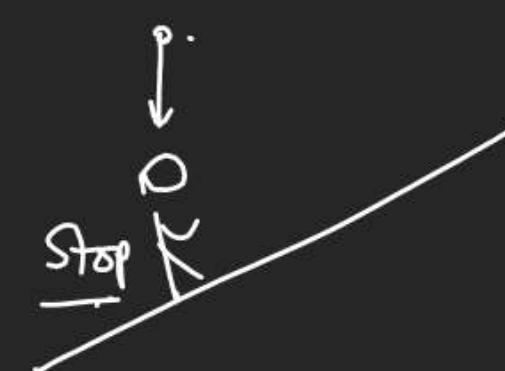
find. Speed of rain drop.



$$\begin{aligned}\vec{V}_{M/E} &= 10 \cos 37^\circ \hat{i} + 10 \sin 37^\circ \hat{j} \\ \underline{\vec{V}_{R/E} = a\hat{i} - b\hat{j}} \quad &= 10 \times \frac{4}{5} \hat{i} + 10 \times \frac{3}{5} \hat{j} \\ &= 8\hat{i} + 6\hat{j}\end{aligned}$$

$$\begin{aligned}\vec{V}_{R/M} &= \vec{V}_{R/E} - \vec{V}_{M/E} \\ &= a\hat{i} - b\hat{j} - 8\hat{i} + 6\hat{j} \\ &= (a-8)\hat{i} + (b-6)\hat{j} = (a-8)\hat{i} - (b-6)\hat{j}\end{aligned}$$

According to question  $(V_{R/M})_x = 0$   $\vec{(V_{R/M})} = - (b-6)\hat{j}$  || (Speed)



$$\vec{V}_{R/E} = 8\hat{i} - 11\hat{j} \quad a = 8$$

$$b-6 = 5 \\ b = 11$$

$$|\vec{V}_{R/E}| = \sqrt{64+121} = \sqrt{185} \text{ m/s}$$

# Relative velocity

**Q.7** A girl standing on road holds her umbrella at  $45^\circ$  with the vertical to keep the rain away. If she starts running without umbrella with a speed of  $15\sqrt{2}\text{ kmh}^{-1}$ , the rain drops hit her head vertically. The speed of rain drops with respect to the moving girl is:

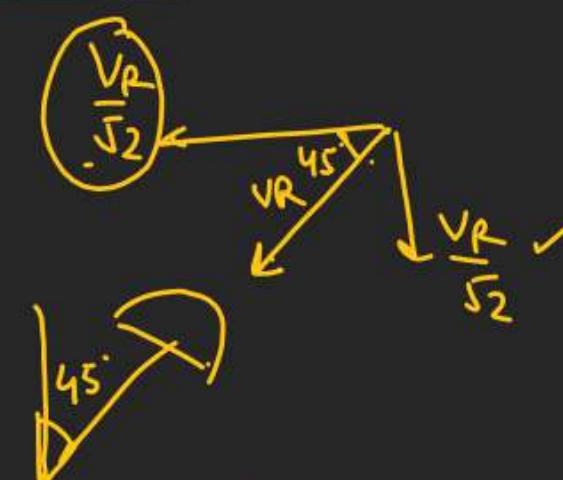
**[June 27, 2022 (I)]**

(A)  $30 \text{ kmh}^{-1}$

(B)  $\frac{25}{\sqrt{2}} \text{ kmh}^{-1}$

(C)  $\frac{30}{\sqrt{2}} \text{ kmh}^{-1}$

(D)  $25 \text{ kmh}^{-1}$

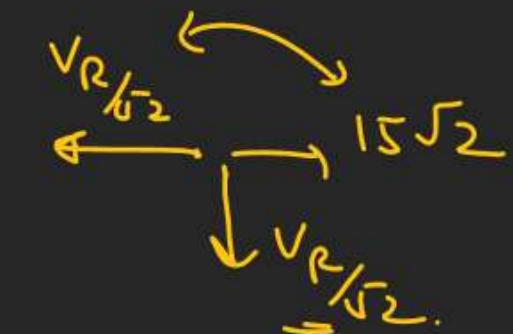


$$\begin{aligned} V_{R/\text{girl}} &= \frac{V_R}{\sqrt{2}} \\ &= \frac{30}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \end{aligned}$$

$$\downarrow V_{R/\sqrt{2}} = V_{R/\text{girl}}$$

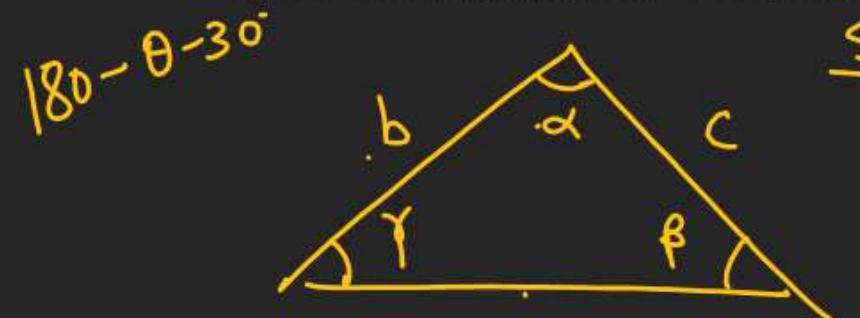
$$15\sqrt{2} = \frac{V_R}{\sqrt{2}}$$

$$\underline{V_R = 30 \text{ km/h}}.$$



# Relative velocity

**Q.8** A swimmer wants to cross a river from point A to point B. Line AB makes an angle of  $30^\circ$  with the flow of river. Magnitude of velocity of the swimmer is same as that of the river. The angle  $\theta$  with the line AB should be  $\underline{\underline{\underline{\theta}}}$  °, so that the swimmer reaches point B.



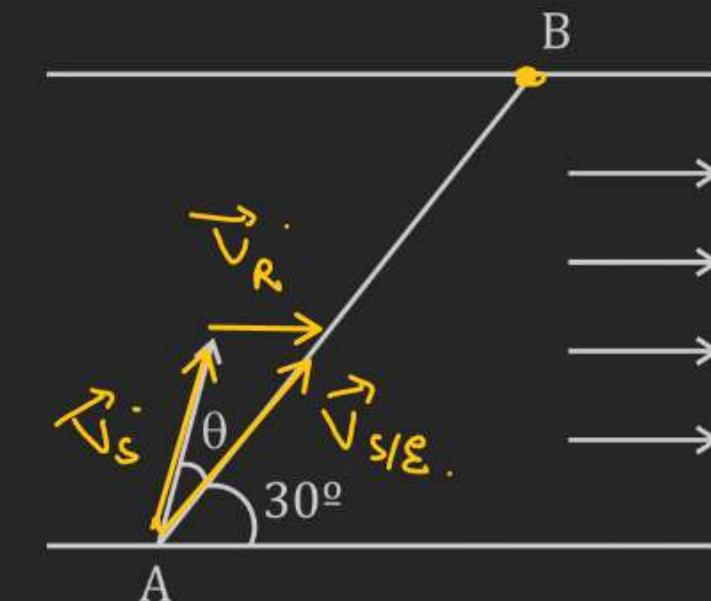
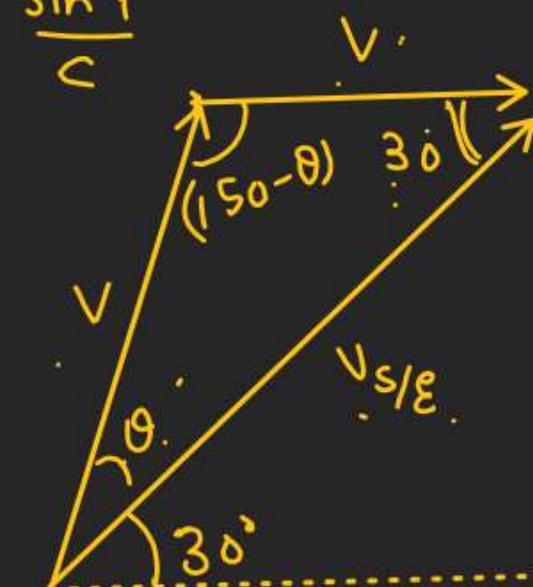
$$\frac{\sin \alpha}{a} = \frac{\sin \beta}{b} = \frac{\sin \gamma}{c}$$

$$\frac{\sin 30}{v} = \frac{\sin (150 - \theta)}{v_{s/\epsilon}} = \frac{\sin \theta}{v}$$

$$\sin \theta = \sin 30$$

$$\theta = 30$$

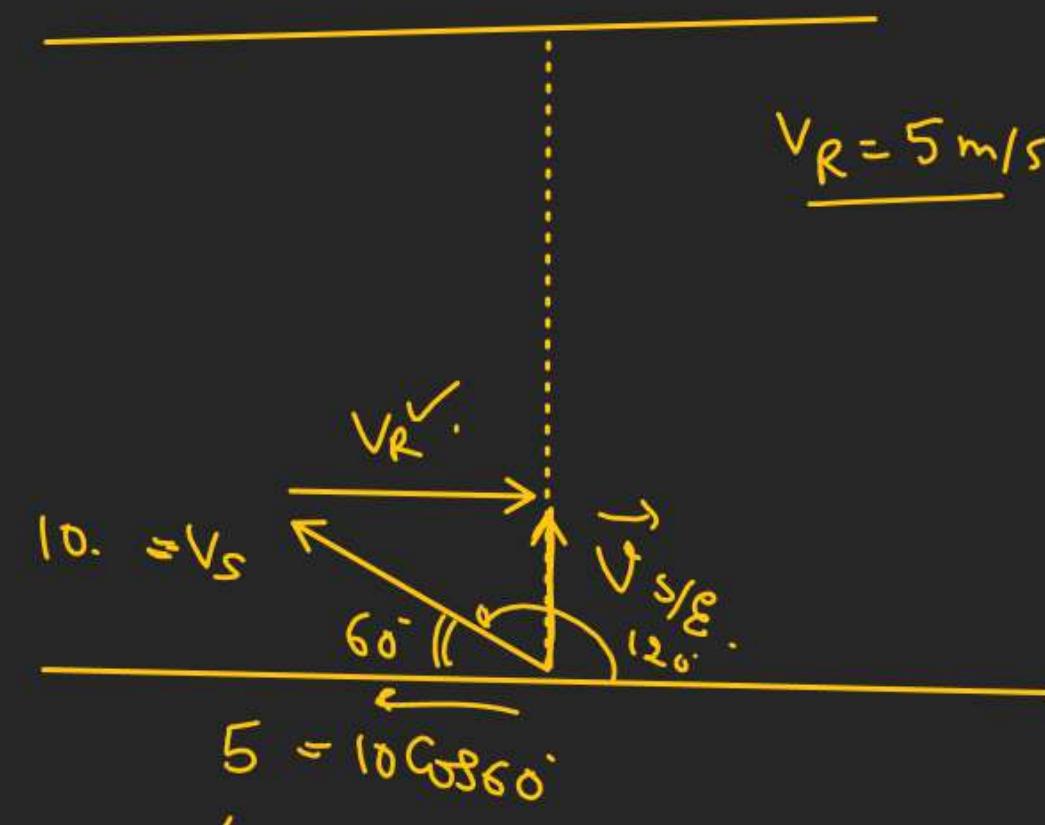
$v_R = v_s = v$  [NA, July 27, 2021 (II)]



## Relative velocity

Q.9 A person is swimming with a speed of 10 m/s at an angle of  $120^\circ$  with the flow and reaches to a point directly opposite on the other side of the river.  
The speed of the flow is 'x' m / s. The value of 'x' to the nearest integer is

[March 18, 2021 (I)]



# Relative velocity

**Q.10** When a car is at rest, its driver sees raindrops falling vertically. When driving the car with speed  $v$ , he sees that raindrops are coming at an angle  $60^\circ$  from the horizontal. On further increasing the speed of the car to  $(1 + \beta)v$ , this angle changes to  $45^\circ$ . The value of  $\beta$  is close to: [Sep. 06, 2020 (II)]

- (A) 0.73
- (B) 0.41
- (C) 0.37
- (D) 0.50

Diagram illustrating the relative velocity of raindrops as seen by a driver in a moving car.

Initial state (at rest): Raindrop falls vertically downwards with velocity  $V_R$ .

State 1 (car moves with speed  $v$ ): Raindrop has a horizontal component  $V_R/C$  and a vertical component  $V_R$ . The angle between the raindrop's velocity vector and the vertical is  $60^\circ$ .

State 2 (car moves with speed  $(1 + \beta)v$ ): Raindrop has a horizontal component  $V_R/C$  and a vertical component  $V_R$ . The angle between the raindrop's velocity vector and the vertical is  $45^\circ$ .

From the car's perspective:

$$\tan 60^\circ = \frac{V_R}{V}$$

$$V_R = \sqrt{3}V \quad \text{(Equation 1)}$$

$$\tan 45^\circ = \frac{V_R}{V_1}$$

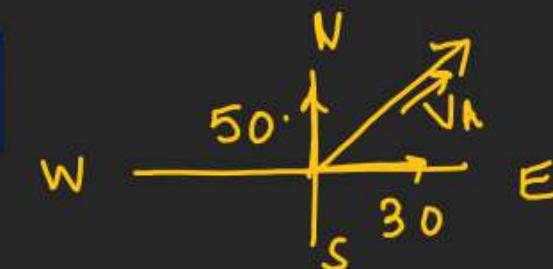
$$V_R = V_1 \quad \text{(Equation 2)}$$

$$F_{\text{from } 1 + 2} \quad V_1 = \sqrt{3}V$$

$$1 + \beta = \sqrt{3}$$

$$\beta = \sqrt{3} - 1 = 0.73$$

# Relative velocity



**Q.11** Ship A is sailing towards north-east with velocity  $\bar{v} = 30\hat{i} + 50\hat{j}$  km/hr where  $\hat{i}$  points east and  $\hat{j}$  north. Ship B is at a distance of 80 km east and 150 km north of Ship A and is sailing towards west at 10 km/hr. A will be at minimum distance from B in:

- (A) 4.2 hrs.
- (B) 2.6 hrs.
- (C) 3.2 hrs.
- (D) 2.2 hrs.

[8 April 2019 I]

