

$$\exists \cdot f'(\frac{\pi}{2}^+) = \lim_{x \rightarrow \frac{\pi}{2}^+} \frac{x + (x - \frac{\pi}{2})^2 - x}{(x - \frac{\pi}{2})} = 0.$$

$$f'(\frac{\pi}{2}^-) = \lim_{x \rightarrow \frac{\pi}{2}^-} \frac{1 + |\sin x| - 2}{x - \frac{\pi}{2}} = \lim_{x \rightarrow \frac{\pi}{2}^-} \frac{\sin x - \sin \frac{\pi}{2}}{x - \frac{\pi}{2}}$$

$\lim_{h \rightarrow 0} \frac{f(3+h^2) - f(3-h^2)}{(3+h^2) - (3-h^2)}$

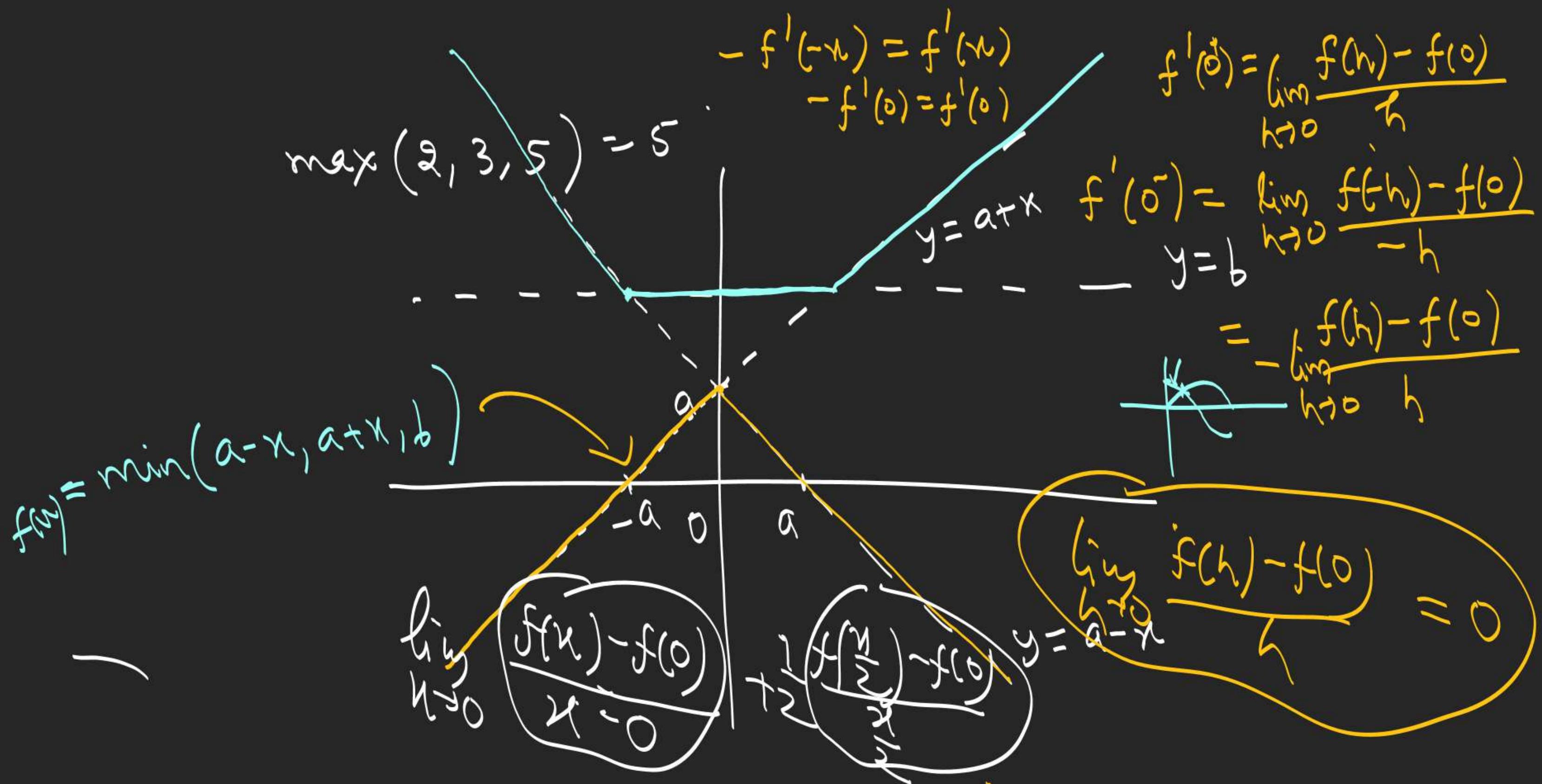
$= \lim_{h \rightarrow 0} \frac{f(3+h^2) - f(3)}{h^2} + \frac{f(3-h^2) - f(3)}{-h^2}$

$= \cos \frac{\pi}{2} = 0$

$x \in (-\infty, -\sqrt{2}] \cup [\sqrt{2}, \infty)$

$x \in (-\sqrt{2}, \sqrt{2})$

$f(x) = \begin{cases} x^2 & x \in (-\infty, -\sqrt{2}] \cup [\sqrt{2}, \infty) \\ 2 & x \in (-\sqrt{2}, \sqrt{2}) \end{cases}$



1.

$$h > 0 \checkmark$$

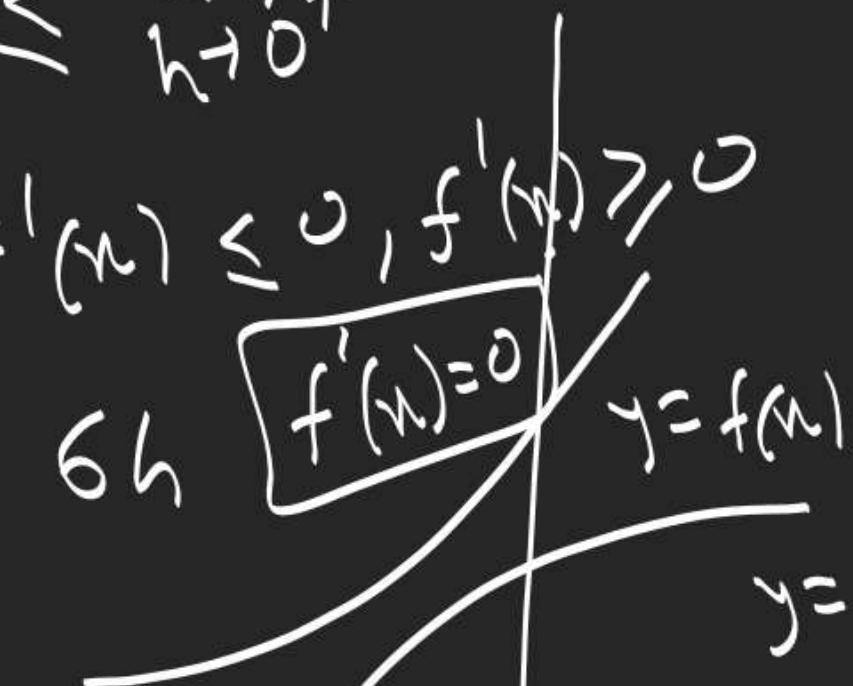
$$\frac{f(x+h) - f(x)}{h} \leq 6h$$

$$\lim_{h \rightarrow 0^+} \frac{f(x+h) - f(x)}{h} \leq \lim_{h \rightarrow 0^+} 6h \Rightarrow f'(x^+) \leq 0$$

$$h < 0$$

$$\frac{f(x+h) - f(x)}{h} \geq 6h$$

$$f'(x) \leq 0, f'(x) \geq 0$$



$$f(x) > g(x)$$

$$\lim_{x \rightarrow a} f(x) > \lim_{x \rightarrow a} g(x)$$

$$\lim_{h \rightarrow 0^-} \frac{f(x+h) - f(x)}{h} \geq \lim_{h \rightarrow 0^-} (6h)$$

$$f'(x^-) \geq 0$$

$$\lim_{x \rightarrow 0} \frac{(x^2 + 2|x|) - 0}{x - 0} = \lim_{x \rightarrow 0} \left(x + \frac{2|x|}{x} \right) \begin{cases} LHD = -2 \\ RHD = 2. \end{cases}$$

$$f(x) = \begin{cases} 1+x & x > 0 \\ \left(0, \frac{\pi}{2}\right] \\ \frac{1}{3} \sin^{-1} x & x \in [0, 1] \\ \frac{1}{3} \sin^{-1} x & x \in [-1, 0) \end{cases}$$

$f'(0) = \lim_{x \rightarrow 0} \frac{(\sin^{-1} x) \cos \frac{1}{x} - 0}{x}$

$\underset{x \rightarrow 0}{\text{RHS}} = \lim_{x \rightarrow 0} \frac{\sqrt{1 - (\frac{\sin^{-1} x}{x})^2} \cos \frac{1}{x}}{x} = 0.$

1. Let $F(x) = f(x)g(x)h(x)$. If for some $x = x_0$,

$$F'(x_0) = 21, F(x_0), f'(x_0) = 4f(x_0), g'(x_0) = -7g(x_0)$$

and $h'(x_0) = k h(x_0)$, find k .

$$\frac{F'}{F} = \sum \frac{f'}{f} \Rightarrow 21 = 4 - 7 + k \Rightarrow \boxed{k=24}$$

2. If $f(x) = (1+x)(3+x^2)^{1/2}(9+x^3)^{1/3}$, find $f'(-1)$

$$f'(-1) = (3+(-1))^2(9+(-1)^3)^{1/3} = 4.$$

3. Let $f(0)=1, g(0)=2, h(0)=3$ and $(fg)'(0)=6$

$(gh)'(0)=4$ and $(hf)'(0)=5$, find $(fgh)'(0) = \boxed{16}$

$$(fg' + f'g)(0) = 6 \Rightarrow \underline{h(0)} (\underline{f} \underline{g'} + f'g)(0) = 6 \times 3$$

$$(gh' + g'h)(0) = 4 \Rightarrow \underline{f(0)} (gh' + \underline{g'} h)(0) = 4 \times 1$$

$$(hf' + h'f)(0) = 5 \Rightarrow g(0) (hf' + \underline{h'} f)(0) = 5 \times 2$$

$$\begin{aligned} \textcircled{(fgh)'} &= \frac{(fg)(gh)(hf)}{2(fgh)^2}(0) = \left(\frac{6}{1 \times 2} + \frac{5}{2 \times 1} + \frac{5}{1 \times 3} \right) (fgh)'(0) = 3^2 \\ &\quad (fgh)'(0) = \frac{1 \times 2 \times 3}{2} \left(\frac{6}{2} + \frac{5}{1} + \frac{5}{3} \right) \end{aligned}$$

Ex: 2] $y = \frac{\sec x + \tan x - 0}{\tan x - \sec x + 1}$, find $\frac{dy}{dx}$ at $x = \frac{\pi}{4}$.

$$y = \frac{(\sec x + \tan x) - (\sec^2 x - \tan^2 x)}{\tan x - \sec x + 1} = \sec x + \tan x$$

$$\frac{dy}{dx} = \sec x \tan x + \sec^2 x.$$

Ex: 1] $y = \frac{x^3 + x^2 + x}{1 + x^2}$, find $\frac{dy}{dx}$.

$$y = \frac{(x+1)(x+1) - 1}{1+x^2} = x+1 - \frac{1}{x^2+1}$$

$$\frac{dy}{dx} = 1 - \frac{(x^2+1)(0) - 1(2x)}{(x^2+1)^2} = 1 + \frac{2x}{(x^2+1)^2}$$

$$\frac{d}{dx} \left(e^{x^2 \cos x} \right) = \frac{d}{dt} (e^t) \frac{dt}{dx} = e^{x^2 \cos x} \frac{d}{dx} (x^2 \cos x)$$

$$= e^{x^2 \cos x} (2x \cos x - x^2 \sin x)$$

$$y = f(x)$$

$$t = g(x)$$

$$\frac{dy}{dx} = \frac{dy}{dt} \frac{dt}{dx}$$

$$\frac{d}{dx} \left(\ln^3(\tan^2(x^4)) \right) = \frac{d}{dt} t^3 \underbrace{\frac{d}{dx} \left(\ln(\tan t) \right)}_{\tan^2(x^4)}$$

$$= 3 \ln^2(\tan x^4) \frac{d}{dt} \ln t \frac{d}{dx}$$

$$t = \ln(\tan x)$$

$$\tan^2(x^4)$$

$$= 3 \ln^2(\tan x^4) \frac{1}{\tan^2(x^4)} \frac{d}{dt} (t^2) \frac{d}{dx} \tan(x^4)$$

$$\frac{d}{dx} \left(\sec^2(f^3(x)) \right)$$

$$= 3 \ln^2(\tan x^4) \frac{1}{\tan^2(x^4)} \frac{2 \tan(x^4)}{f'(x^4)} \frac{d}{dx} (f^3(x))$$

$$\frac{3 \ln^2(\tan x^4) \sec^2(x^4) f'(x^4)}{\tan^2(x^4)}$$

$$D\left(e^{\sqrt{\sin(\ln(x^2+7)^5)}}\right)$$

$$= e^{\sqrt{\sin(\ln(x^2+7)^5)}} \times \frac{1}{2\sqrt{\sin(\ln(x^2+7)^5)}} \times \cos(\ln(x^2+7)^5) \times \frac{5x^2}{(x^2+7)}$$

Logarithmic Differentiation

$$y = f(x) = e^{g(x) \ln f(x)}$$

$$\frac{dy}{dx} = \frac{f'_1}{f_1} + \frac{f'_2}{f_2} x - \dots$$

$$y = (f_1 f_2 f_3 \dots f_n)(x)$$

$$\ln y = \ln f_1 + \ln f_2 + \dots$$

$$\ln y = g(x) \ln f(x)$$

$$\frac{1}{y} \frac{dy}{dx} = g'(x) \ln f(x) + \frac{g(x)f'(x)}{f(x)}$$

$$\text{Given } f(x) = \prod_{n=1}^{100} (x-n)^{n(101-n)}$$

$\sum_{r=1}^{\infty}$ (Differentiability)

$$\frac{f'(x)}{f(x)} = \sum_{r=1}^{100} \frac{r(101-r)}{x-r} \Rightarrow \frac{f'(101)}{f(101)-1} = \sum_{r=1}^{100} \frac{r(101-r)}{101-r}$$

$$\sum_{r=1}^{100} \frac{g'_r}{g_r} = \frac{r(101-r)(x-r)}{(x-r)^{r(101-r)}} = 5050$$

$$\ln f(x) =$$

$$\sum_{n=1}^{100} n(101-n) \ln(x-n)$$

$$\frac{f'(x)}{f(x)} = \sum_{n=1}^{100} \frac{n(101-n)}{x-n}$$

Q. If $y = x^{\tan x} + (\sin x)^{\cos x}$, find $\frac{dy}{dx}$.

$$y = e^{\tan x \ln x} + e^{\cos x \ln(\sin x)}$$

$$y' = x^{\tan x} \left(\sec^2 x \ln x + \frac{\tan x}{x} \right) + (\sin x)^{\cos x} \left(-\sin x \ln(\sin x) + \cos x \frac{\cos x}{\sin x} \right)$$

$$y_1 = x^{\tan x}$$

$$y_2 = (\sin x)^{\cos x}$$

$$\frac{dy}{dx} = \frac{dy_1}{dx} + \frac{dy_2}{dx}$$

$$\ln y_1 = \tan x \ln x$$

$$\frac{1}{y_1} y_1' = \sec^2 x \ln x + \frac{\tan x}{x} \Rightarrow \frac{dy_1}{dx} = y_1 \left(\sec^2 x \ln x + \frac{\tan x}{x} \right)$$