Nishant Jindal

Trigonometry

$a\sin\theta + b\cos\theta$ type fxn's Range

$$-\sqrt{a^2+b^2} \le a\sin\theta + b\cos\theta \le \sqrt{a^2+b^2}$$

(1) $y = 3\sin x - 4\cos x$'s R_f

$$\left[-\sqrt{3^2+(-4)^2}, \sqrt{3^2+(-4)^2}\right]$$

$$y \in [-5, 5]$$

(2) $y = \boxed{5\sin\theta - 12\cos\theta} + 7$'s R_f

$a = 5, b = -12$

$$y \in \left[-\sqrt{5^2+(-12)^2}, \sqrt{5^2+(-12)^2}\right] + 7$$

$$y \in [-13, 13] + 7 \Rightarrow y \in [-6, 20]$$

$a = 1, b = -1$

(3) $y = \sin x - \cos x$'s R_f?

$$\left[-\sqrt{1^2+(-1)^2}, \sqrt{1^2+(-1)^2}\right]$$

$$y \in [-\sqrt{2}, \sqrt{2}]$$

(4) $y = \log_{\sqrt{2}}(\sin x - \cos x + 2\sqrt{2})$'s R_f

fxn(fxn)

Aaye then check Behaviour of fxn outside.

$$\boxed{\sin x - \cos x} + 2\sqrt{2}$$

$$\left[-\sqrt{1^2+(-1)^2}, \sqrt{1^2+(-1)^2}\right] + 2\sqrt{2}$$

$$[-\sqrt{2}, \sqrt{2}] + 2\sqrt{2}$$

$$y \in [\sqrt{2}, 3\sqrt{2}]$$

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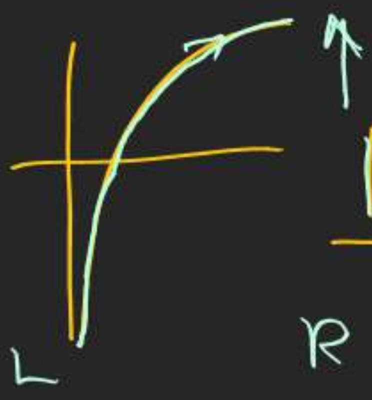
$$Y = \log_{\sqrt{2}}(\sin x - \cos x + 2\sqrt{2})$$

$$\sin x - \cos x + 2\sqrt{2} \in [\sqrt{2}, 3\sqrt{2}]$$



$$\log_{\sqrt{2}}(\sin x - \cos x + 2\sqrt{2}) \in [\log_{\sqrt{2}}\sqrt{2}, \log_{\sqrt{2}}3\sqrt{2}]$$

$$Y \in [1, \log_{\sqrt{2}}3\sqrt{2}]$$

Q $Y = \log_2\left[\frac{3\sin x - 4\cos x + 15}{10}\right]$, DR+

$$3\sin x - 4\cos x \in [-\sqrt{3^2+(-4)^2}, \sqrt{3^2+(-4)^2}]$$

$$3\sin x - 4\cos x \in [-5, 5]$$

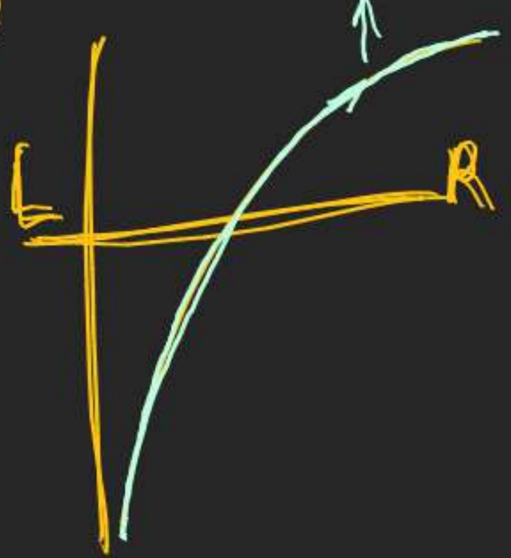
$$3\sin x - 4\cos x + 15 \in [-5, 5] + 15$$

$$3\sin x - 4\cos x + 15 \in [10, 20]$$

$$\frac{3\sin x - 4\cos x + 15}{10} \in \left[\frac{10}{10}, \frac{20}{10}\right]$$

$$\frac{3\sin x - 4\cos x + 15}{10} \in [1, 2]$$

$$\boxed{\log_2}\left[\frac{3\sin x - 4\cos x + 15}{10}\right] \in [\log_2 1, \log_2 2]$$



$$Y \in [0, 1]$$

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Trigonometry

Q $y = \sin\left(x+\frac{\pi}{3}\right) + 3\cos\left(x-\frac{\pi}{3}\right)$ R_f

Yha $a\sin\theta + b\cos\theta$
Nahi lagega Kyunki
Argument Same Nhi hai

$y = \sin x \cos\frac{\pi}{3} + \cos x \sin\frac{\pi}{3}$

$+3\left(\cos x \cos\frac{\pi}{3} + \sin x \sin\frac{\pi}{3}\right)$

$= \left(\frac{1}{2}\sin x + \frac{\sqrt{3}}{2}\cos x\right) + \frac{3}{2}\cos x + \frac{3\sqrt{3}}{2}\sin x$

$= \left(\frac{1}{2} + \frac{3\sqrt{3}}{2}\right)\sin x + \left(\frac{\sqrt{3}}{2} + \frac{3}{2}\right)\cos x$

$y = \underbrace{\left(\frac{3\sqrt{3}+1}{2}\right)}_{a}\sin x + \underbrace{\left(\frac{3+\sqrt{3}}{2}\right)}_{b}\cos x$

$\left[-\sqrt{\left(\frac{3\sqrt{3}+1}{2}\right)^2 + \left(\frac{3+\sqrt{3}}{2}\right)^2}, \sqrt{}\right]$

$y \in \left[-\sqrt{\frac{27+1+9+3+6\sqrt{3}+6\sqrt{3}}{4}}, \sqrt{}\right]$

$y \in \left[-\sqrt{\frac{40+12\sqrt{3}}{4}}, \sqrt{\frac{40+12\sqrt{3}}{4}}\right]$

$y \in \left[-\sqrt{10+3\sqrt{3}}, \sqrt{10+3\sqrt{3}}\right]$

Trigonometry

Making Perfect Sqr.

1) $x^2 - \boxed{4}x + \boxed{9}$

$\boxed{4} \rightarrow 2$

$(x-2)^2 - 2^2 + 9$

1) x^2's Cofficient should be 1

2) Check Coff^t of x & make half of it

2) $3x^2 + 7x + 5 \rightarrow \frac{7}{6}$

$3\left(x^2 + \boxed{\frac{7}{3}}x + \boxed{\frac{5}{3}}\right)$

$3\left(\left(x + \frac{7}{6}\right)^2 - \left(\frac{7}{6}\right)^2 + \frac{5}{3}\right)$

(3) $4x^2 - x + 2 \rightarrow \frac{1}{8}$

$4\left(x^2 - \boxed{\frac{1}{4}}x + \boxed{\frac{2}{4}}\right)$

$4\left(\left(x - \frac{1}{8}\right)^2 - \left(\frac{1}{8}\right)^2 + \frac{2}{4}\right)$

4) $7x^2 + 12x - 13 \rightarrow \frac{6}{7}$

$7\left(x^2 + \boxed{\frac{12}{7}}x - \frac{13}{7}\right)$

$7\left(\left(x + \frac{6}{7}\right)^2 - \left(\frac{6}{7}\right)^2 - \frac{13}{7}\right)$

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Trigonometry

Q $y = \cos^2 x - \boxed{4}\cos x + 13$'s R_f)

$\boxed{4} \rightarrow 2$

$y = (\cos x - 2)^2 - 2^2 + 13$

$y = (\cos x - 2)^2 + 9$

$\cos x = 0$	$\cos x = 1$	$\cos x = -1$
$(0-2)^2 + 9$	$(1-2)^2 + 9$	$(-1-2)^2 + 9$
13	10	18
	Min	Max

$y \in [10, 18]$

Q $(\cos x - 2)^2$ can give value 0 or Not?

$\cos x - 2 = 0$

$\boxed{\cos x = 2}$

$\underrightarrow{-1 \le 2 \le 1}$

X

Trigonometry

Q $y = \cos^2 x - \boxed{2}\cos x + \boxed{13}$'s Range.

$\boxed{2} \to 1$

$$= (\cos x - 1)^2 - 1^2 + 13$$

$$y = (\underline{\cos x - 1})^2 + 12$$

	$\cos x = 0$	$\cos x = 1$	$\cos x = -1$
Min $= 0$	$(0-1)^2 + 12$	$(1-1)^2 + 12$	$(-1-1)^2 + 12$
$0 + 12$	13	12	16

$$\left.\begin{matrix} \text{Min} = 12 \\ \text{Max} = 16 \end{matrix}\right\} y \in [12, 16]$$

Q $(\cos x - 1)^2$ Can give Zero or Not?

$$\cos x - 1 = 0$$

$\cos x = 1$ Deta hai

$$-1 \le 1 \le 1$$

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Trigonometry

Q $y = \tan^2 x - 2\tan x + 13$ is R_+)

$$= (\tan x - 1)^2 - 1^2 + 13.$$

$$= \underline{(\tan x - 1)}^2 + 12$$

$0 + 12$ $(\tan x - 1)^2 + 12$

Min. ∞

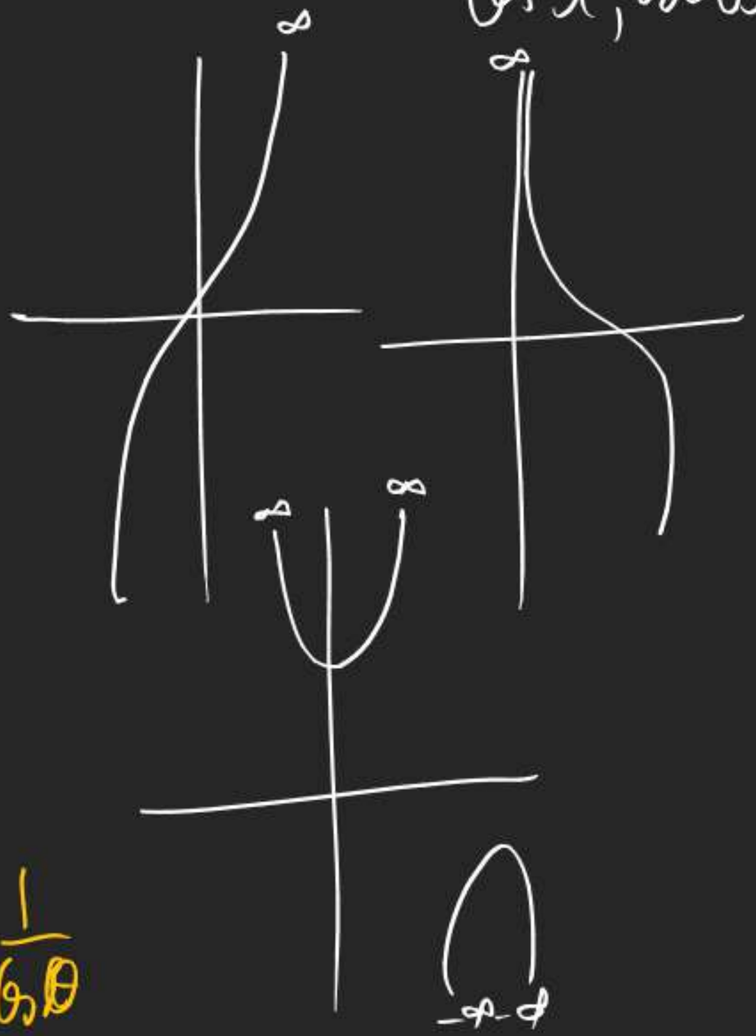
$\therefore [12, \infty)$

Q $(\tan x - 1)^2$ can give zero or Not?

$\tan x - 1 = 0$

$\underline{\tan x = 1}$

$\left.\begin{matrix}\tan x, \sec x \\ \cot x, \operatorname{cosec} x\end{matrix}\right\}$ Max^m ∞



a of Reciprocal $= \frac{1}{a}$

$\underline{\tan\theta}$ ———— $= \underline{\cot\theta}$

$\sin\theta$ ———— $= \operatorname{cosec}\theta$

$\cos\theta$ —— $\sec\theta = \frac{1}{\cos\theta}$

Trigonometry

Q $y = \sin^2 x - 20\cos x + 1$'s R_f ?

$$y = 1 - \cos^2 x - 20\cos x + 1$$

$$= -\cos^2 x - 20\cos x + 2$$

$$= -\left[\cos^2 x + \boxed{20}\cos x - 2\right] \quad 10$$

$$= -\left[(\cos x + 10)^2 - 10^2 - 2\right]$$

$$= -\left[(\cos x + 10)^2 - 102\right]$$

$$y = 102 - (\cos x + 10)^2$$

Q $(\cos x + 10)^2$ (can give 0 or not)

$\cos x = -10$ Not Possible

$$y = 102 - (\cos x + 10)^2$$

$102 - 81$

$\cos x = 0$ $\quad$ $\cos x = 1$ $\quad$ $\cos x = -1$

$y = 102 - (0+10)^2$ $\quad$ $102 - (1+10)^2$ $\quad$ $102 - (-1+10)^2$

$= 2$ $\quad$ -19 $\quad$ 21

$102 - 121$

$$\boxed{\begin{array}{l} \text{Min} = -19 \\ \text{Max} = 21 \end{array}}$$

$R_f \in [-19, 21]$

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Next year

Kam aayega

Q $y = 3 + (\cos x - 2)^2$ is R_f?

$\cos x = 0$ | $\cos x = 1$ | $\cos x = -1$

$3 + (0-2)^2$ | $3 + (1-2)^2$ | $3 + (-1-2)^2$

7 | $\boxed{4}$ | 12

Min $= 4$

Max $= 12$

$[4, 12]$

$\cos 2\theta \to \cos^2\theta - \sin^2\theta$

$\to 1 - 2\sin^2\theta$

$\to 2\cos^2\theta - 1$

($(\cos x - 2)^2$ can give 0 or not?)

Not: $\cos x = 2$ Not Poss.

$\frac{17}{8} - \cancel{2} \times \frac{49}{\cancel{16}8}$

$= -\frac{32}{8} = -4$

Q $y = \boxed{\cos 2x} + 3\sin x$ is R_f?

$y = 1 - 2\sin^2 x + 3\sin x$

$= -(2\sin^2 x - 3\sin x - 1)$

$= -2\left[\sin^2 x - \left(\frac{3}{2}\right)\sin x - \frac{1}{2}\right]$ $\to \frac{3}{4}$

$= -2\left[\left(\sin x - \frac{3}{4}\right)^2 - \left(\frac{3}{4}\right)^2 - \frac{1}{2}\right]$

$-\frac{9}{16} - \frac{8}{16}$

$= -\frac{17}{16}$

$= -2\left[\left(\sin x - \frac{3}{4}\right)^2 - \frac{17}{16}\right]$

$y = \frac{17}{8} - 2\left(\sin x - \frac{3}{4}\right)^2$

$y = \frac{17}{8} - 2\left(\sin x - \frac{3}{4}\right)^2$

$\frac{17}{8} - 2 \times 0 = \frac{17}{8}$ | $\frac{17}{8} - 2\left(0 - \frac{3}{4}\right)^2$, $\frac{17}{8} - \frac{9}{8} = 1$ | $\frac{17}{8} - 2\left(1 - \frac{3}{4}\right)^2$, $\frac{17}{8} - \frac{1}{8} = 2$ | $\frac{17}{8} - 2\left(-1 - \frac{3}{4}\right)^2 = -4$

$R_f \in \left[-4, \frac{17}{8}\right]$

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Trigonometry

When fxn are Reciprocal

Concept :- AM ≥ GM if No. are +ve

AM of $a, b = \frac{a+b}{2}$

GM of $a, b = \sqrt{ab}$

$\frac{a+b}{2} \geq \sqrt{ab}$

Q $y = a^2\tan^2\theta + b^2\cot^2\theta$ find R_f ?

Max → ∞

$y = a^2\tan^2\theta + \frac{b^2}{\tan^2\theta}$ ← Reciprocal

AM ≥ GM

$\frac{a^2\tan^2\theta + \frac{b^2}{\tan^2\theta}}{2} \geq \sqrt{a^2\cancel{\tan^2\theta} \times \frac{b^2}{\cancel{\tan^2\theta}}}$

$a^2\tan^2\theta + b^2\cot^2\theta \geq 2ab$

$(a^2\tan^2\theta + b^2\cot^2\theta) \in [2ab, \infty)$

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Trigonometry

Q $y = 4\operatorname{cosec}^2 x + 9\sec^2 x$ R_f

$= 4(1+\cot^2 x) + 9(1+\tan^2 x)$ $\{a^2\tan^2\theta + b^2\cot^2\theta\}$

$= \boxed{13} + \{4\cot^2 x + 9\tan^2 x\}$

$4\cot^2 x + 9\tan^2 x \in [2\times2\times3, \infty)$

$4\cot^2 x + 9\tan^2 x \in [12, \infty)$

$4\cot^2 x + 9\tan^2 x + 13 \in [25, \infty)$