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3: $|x^2+x| < 2$

$(x^2+x)^2 < 4 \Rightarrow (x^2+x-2)(\underbrace{x^2+x+2}_{>0}) < 0$

$(x+2)(x-1)(x^2+x+2) < 0$

$\boxed{x \in (-2,1)}$

$|x^2-1| + |x^2-2| = 1$

$x^2 < 1$

$1-x^2+2-x^2=1$

$x^2=1$

ϕ

$\boxed{1 \le x^2 \le 2}$

$x^2-1+2-x^2=1$

$\boxed{x \in [-\sqrt{2},-1] \cup [1,\sqrt{2}]}$

$x^2 > 2$

$x^2-1+x^2-2=1$

$x^2=2$

ϕ

$(x-6)(x-2)$

$x^2 = \boxed{t}$

1

t

2

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8. $\left(\dfrac{3|x|-2}{|x|-1}\right)^2 - 4 \geq 0$ $\quad \big|\underbrace{\ \ }_{\geq 0}\big| + \sqrt{\underbrace{\qquad}_{\geq 0}}$ $\quad \sqrt{64} = 8$

$\boxed{x=1}$

$$\frac{\big((3|x|-2)-(2|x|-2)\big)\big(3|x|-2+2|x|-2\big)}{(|x|-1)^2} \geq 0$$

$$\frac{|x|\,(5|x|-4)}{(|x|-1)^2} \geq 0$$

$\frac{4}{5} \leq |x| < 1$

$\boxed{\frac{16}{25} \leq x^2 < 1}$

$|x| \in \{0\} \cup \left[\frac{4}{5}, 1\right) \cup (1, \infty)$

$+ \quad 0 \quad - \quad \frac{4}{5} \quad + \quad 1 \quad +$

$x \in (-\infty, -1) \cup \left(-1, -\frac{4}{5}\right] \cup \left[\frac{4}{5}, 1\right) \cup (1, \infty) \cup \{0\}$.

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$$2ax^3+bx^2+cx+d=0 \quad (1)$$

$$2ax^2+3bx+4c=0 \quad (2)$$

$$x\times(2)-(1)$$

$$2bx^2+3cx-d=0$$

$$\frac{x^2}{-3bd-12c^2}=\frac{x}{\;}=\frac{1}{\;}$$

$$ac$$

$$\boxed{x=\frac{1}{a},}\quad ba$$

$$\frac{1}{a},\ ba$$

$$x^2-a(b+c)x+a^2bc=0$$

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α, β $\qquad$ $\alpha, \frac{1}{\beta}$ $\qquad$ $x(y-1) = -y$

$y = \frac{x}{1+x}$

$\alpha+\beta = p$ $\qquad$ $\alpha+\frac{1}{\beta} = a$

$x = -\frac{y}{y-1}$

$\alpha\beta = q$ $\qquad$ $\frac{\alpha}{\beta} = b$

$\frac{-y^3}{(y-1)^3} + \frac{py}{y-1} + q = 0$

$\alpha^2 = bq$ ✓

$\beta - \frac{1}{\beta} = p-a$ ✓

$q-b = \alpha\left(\beta-\frac{1}{\beta}\right)$ ✓

$(q-b)^2 = \alpha^2\left(\beta-\frac{1}{\beta}\right)^2 = bq(p-a)^2$

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Sequence or Progression

Set of numbers satisfying a definite pattern

$\{1, -1, 1, -1, 1, -1, 1, -1, \ldots\}$

$\{1, 4, 9, 16, 25, 36, \ldots\}$

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Arithmetic Progression (AP)

Sequence of numbers whose consecutive terms have the same difference.

$\boxed{AP} \rightarrow \{a, a+d, a+2d, a+3d, \ldots\ldots\}$

d = common difference

first term $\rightarrow T_1 = a$

common difference $= d$

n^{th} term, $\boxed{T_n = a+(n-1)d}$

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Sum of first 'n' terms of A.P.

(1) $S = a + (a+d) + (a+2d) + (a+3d) + \dots + (a+(n-2)d) + (a+(n-1)d)$

(2) $S = (a+(n-1)d) + (a+(n-2)d) + (a+(n-3)d) + \dots + (a+d) + a$

(1) + (2)

$\Rightarrow 2S = (2a+(n-1)d) + (2a+(n-1)d) + (2a+(n-1)d) + \dots (2a+(n-1)d)$

$= n(2a+(n-1)d)$

$\boxed{S = \frac{n}{2}(2a+(n-1)d)} = \boxed{\frac{n}{2}(\text{First} + \text{last term})}$

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Note → 1. Sum of terms equidistant from beginning and end is the same.

$T_1, T_2, T_3, \dots T_n$

$T_1 + T_n = T_2 + T_{n-1} = T_3 + T_{n-2} = \dots \dots$

$(T_1 + d) + (T_n - d)$

$T_1 + 2d \qquad T_n - 2d$

$T_1 = a$

$a + 2d + a + (n-3)d$

$= 2a + (n-1)d$

2. $T_n = S_n - S_{n-1}$

T_n → nth term

$\{T_1, T_2, T_3, \dots, T_n\}$

S_n → sum of first n terms

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3. $T_1, T_2, T_3, \ldots, T_n \rightarrow A.P.$

$kT_1, kT_2, kT_3, \ldots, kT_n \rightarrow A.P.$

$k+T_1, k+T_2, k+T_3, \ldots, k+T_n \rightarrow A.P.$

$T_1' - T_1, T_2' - T_2, T_3' - T_3, \ldots \rightarrow A.P.$

4. $T_1, T_2, T_3, T_4, \ldots, T_n \rightarrow A.P.$

$T_1', T_2', T_3', T_4', \ldots, T_n' \rightarrow A.P.$

$T_1' + T_1, T_2' + T_2, T_3' + T_3, \ldots, T_n' + T_n \rightarrow A.P.$

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5. 3 terms in A.P. $\longrightarrow$ $a-d,\ a,\ a+d$

4 terms in A.P $\longrightarrow$ $a-3d,\ a-d,\ a+d,\ a+3d$

5 terms in A.P. $\longrightarrow$ $a-2d,\ a-d,\ a,\ a+d,\ a+2d$

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Q. If p^{th}, q^{th} and r^{th} term of an A.P. are respectively a, b and c, then P.T.

$$a(q-r)+b(r-p)+c(p-q)=0$$

$T_1 = A$

common diff $= D$

$$\big(A+(p-1)D\big)(q-r)+\big(A+(q-1)D\big)(r-p)+\big(A+(r-1)D\big)(p-q)$$

$$= A(q-r+r-p+p-q)+D\big(p(q-r)+q(r-p)+r(p-q)\big)$$

$$-D(q-r+r-p+p$$

$$0 =$$

1. In an A.P. $T_p = q$, $T_q = p$, find the r^{th} term

$a+(p-1)d = q$ ①

$a+(q-1)d = p$ ②

① − ② $\Rightarrow (p-q)d = q-p \Rightarrow \boxed{d=-1}$

$a = q+p-1$

$T_r = a+(r-1)d$

$= q+p-1-(r-1)$

$= q+p-r$

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3: The sum of first 3 terms of an A.P. is 27 and the sum of their squares is 293. Find the sum of first 'n' terms.

$3a = 27 \Rightarrow \boxed{a = 9}$

$a-d, a, a+d$

$(9-d)^2 + 9^2 + (9+d)^2 = 293$

$3(9)^2 + 2d^2 = 293$

$d^2 = 25$

$d = 5, -5$

$d = 5, T_1 = 4$

$S_n = \frac{n}{2}(2(4) + (n-1)5)$

$\boxed{S_n = \frac{n}{2}(3 + 5n)}$

$d = -5, T_1 = 14$

$S_n = \frac{n}{2}(2(14) + (n-1)(-5))$

$\boxed{S_n = \frac{n}{2}(33 - 5n)}$

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4. If $S_1, S_2, S_3, \ldots, S_p$ are the sums of first 'n' terms of 'p' arithmetic series whose first terms are 1, 2, 3, 4, ... and whose common difference are 1, 3, 5, 7, ... Then

P.T. $S_1 + S_2 + S_3 + \ldots + S_p = \frac{np}{2}(np+1)$ $\quad 1+(p-1)2$

$\frac{n}{2}(p+np^2) = \boxed{\frac{np}{2}(1+np)}$

$\frac{n}{2}\left(2(1)+(n-1)1\right) + \frac{n}{2}\left(2(2)+(n-1)(3)\right) + \frac{n}{2}\left(2(3)+(n-1)5\right) + \ldots$

$+ \frac{n}{2}\left(2(p)+(n-1)(2p-1)\right)$

$= \frac{n}{2}\left[2(1+2+3+\ldots p) + (n-1)\left(1+3+5+7+\ldots+(2p-1)\right)\right]$

$= \left[2 \times \frac{p}{2}(1+p) + (n-1)\frac{p}{2}(1+2p-1)\right]\frac{n}{2} = \frac{n}{2}\left(p(p+1)+(n-1)p^2\right)$

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$f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$ $a_0, a_1, \cdots, a_n \in I$.

$\underline{f(0)}$ & $f(1)$ are odd.

$K \in I$.

$f(2K) = \underbrace{a_n(2K)^n + a_{n-1}(2K)^{n-1} + \cdots + a_1(2K)}_{\text{Even}} + \underbrace{\cancel{a_0}}_{\text{odd}} \neq 0$. $\xrightarrow{} f(0)$

$f(2K+1) = a_n(2K+1)^n + a_{n-1}(2K+1)^{n-1} + \cdots + a_1(2K+1) + a_0$

$\boxed{\lambda_i \in I}$

$= a_n(2\lambda_n + 1) + a_{n-1}(2\lambda_{n-1}+1) + \cdots + a_1(2\lambda_1 + 1) + a_0$

$= \text{Even} + (a_n + a_{n-1} + a_{n-2} + \cdots + a_1 + a_0) \neq 0$.

$\downarrow f(1) \to \text{odd}$