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4. (b) $f(3x) = x + f(3x-3)$

$$f(300) = 100 + f(297)$$

$297 \to 3(99)$

(a) $f(f(x)) = \dfrac{-f(x)}{1+f(x)}$

$f(x_1) = 3$

$$f(f(x_1)) = \frac{-f(x_1)}{1+f(x_1)}$$

$$\boxed{f(3) = \frac{-3}{1+3}}$$

$$f(300) = 100 + 99 + f(\underbrace{294}_{3\times 98})$$

$$= 100 + 99 + 98 + f(\underbrace{291}_{3\times 97})$$

$$\vdots$$

$$= 100 + 99 + 98 + \ldots + 3 + 2 + f(3\times 1)$$

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$$f\left(\frac{1-x}{1+x}\right) = x$$

$$\frac{1-x}{1+x} = \frac{t}{1} \Rightarrow x = \frac{1-t}{1+t}$$

$$\frac{D-N}{D+N}$$

$$f(t) = \frac{1-t}{1+t}$$

$$\boxed{f(x) = \frac{1-x}{1+x}}$$

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7. $f(x) = \sqrt{\underbrace{ax^2+bx}_{\geq 0}}$

ax^2+bx

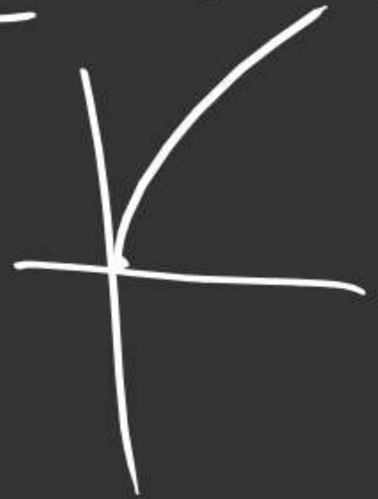
$b > 0$

$\boxed{a = -4} \Leftarrow \frac{b^2}{a^2} = \frac{-b^2}{4a} \Leftarrow -\frac{b}{a} = \sqrt{\frac{-b^2}{4a}}$

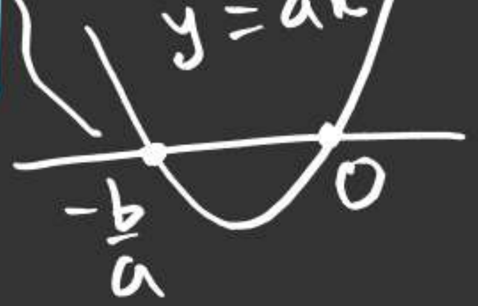
If $\boxed{a = 0}$ ✓ $\boxed{a = \{-4, 0\}}$

$f(x) = \sqrt{bx}$

$D_f = [0, \infty) = R_f$



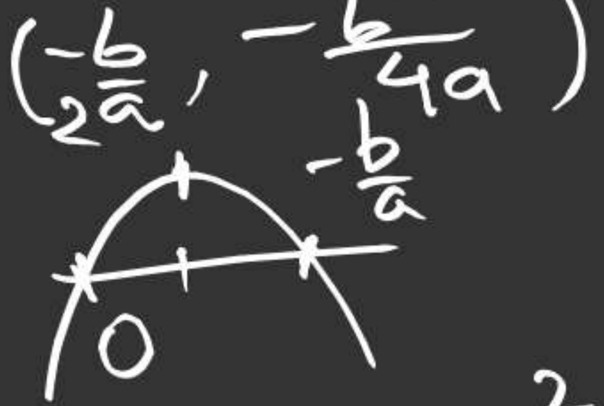
$a > 0$ ✗



$D_f = \left(-\infty, -\frac{b}{a}\right] \cup [0, \infty)$

$R_f = [0, \infty)$

$a < 0$



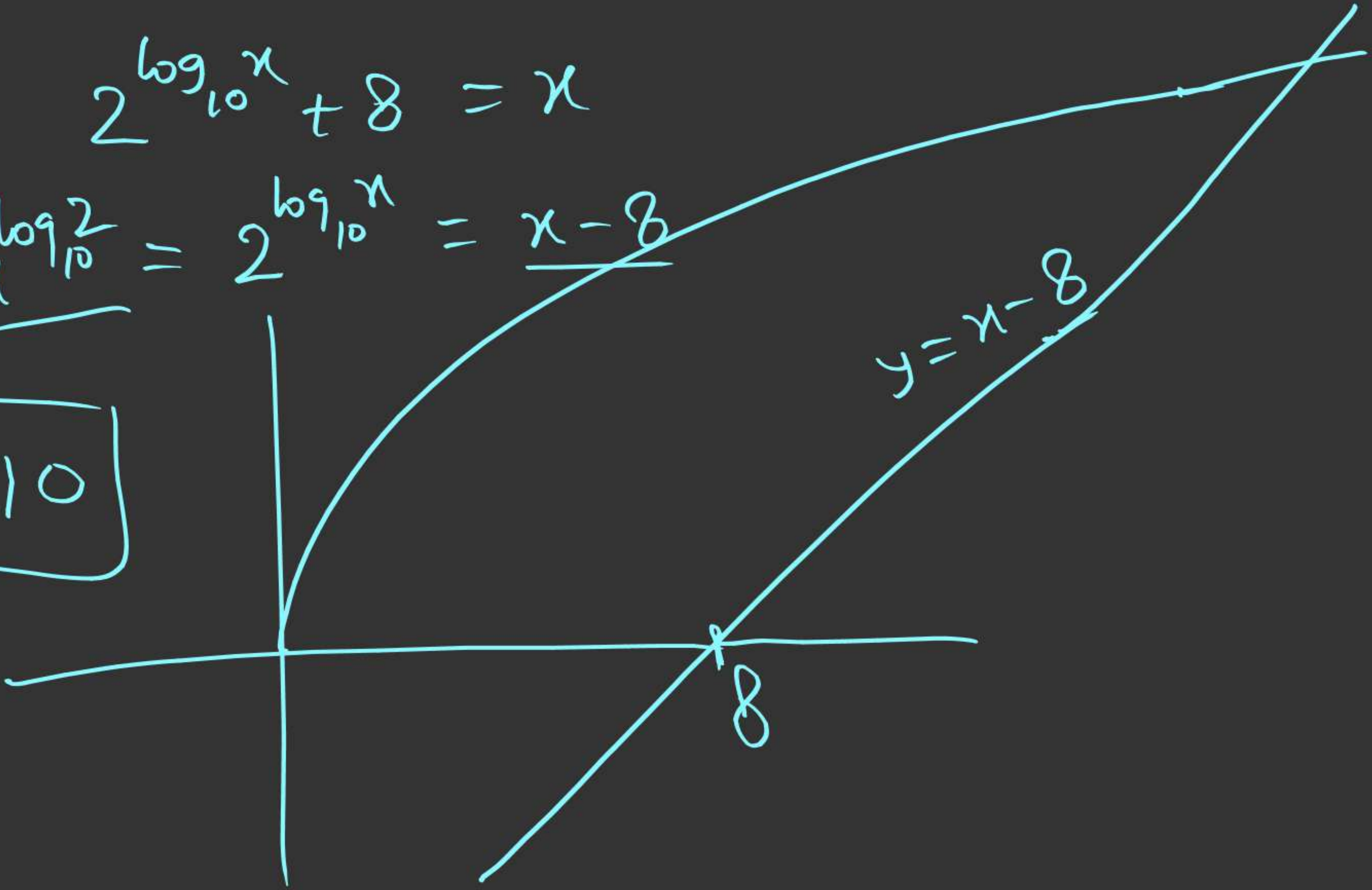
$D_f = \left[0, -\frac{b}{a}\right]$

$R_f = \left[0, \sqrt{\frac{-b^2}{4a}}\right]$

q. $2^{\log_{10} x} + 8 = x$

$$x^{\log_{10} 2} = 2^{\log_{10} x} = x - 8$$

$$\boxed{x = 10}$$



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$\boxed{P(1)=1}$ $P(x) = (x-1)Q_1(x) + 1$

$P(4)=10$ $P(x) = (x-4)Q_2(x) + 10$

$$P(x) = (x-1)(x-4)Q(x) + ax+b$$

$$\boxed{\begin{aligned} 1 &= a+b \\ 10 &= 4a+b \end{aligned}}$$

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Periodic Function

period:

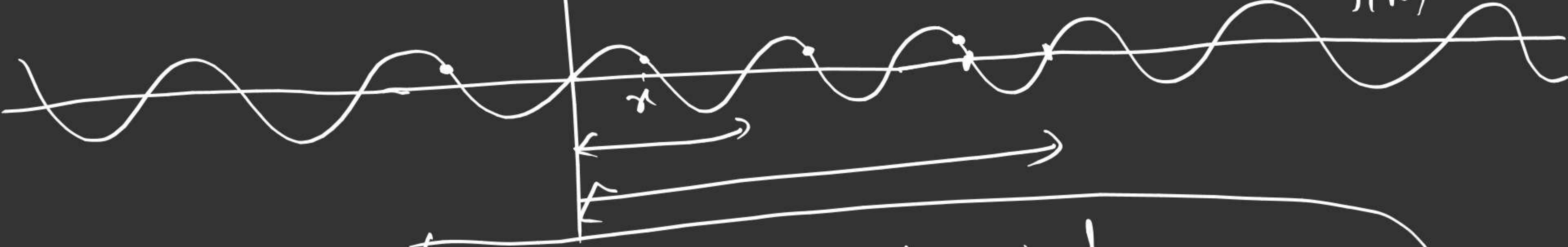
$f(x+T) = f(x) \quad \forall x \in D_f$, where

T is the least positive constant. Then function is periodic with fundamental period 'T'.

$\sin(x+2\pi) = \sin x = \sin(x+4\pi)$
$= \sin(x+6\pi) = \ldots$

fundamental period $= 2\pi$

$$f(x+2T) = f(\underbrace{x+T}_{t} + T) = f(x+T)$$
$$= f(x)$$



$T \rightarrow$ fundamental period

$$f(x+nT) = f(x) \quad \forall x \in D_f \,, \quad n \in I.$$

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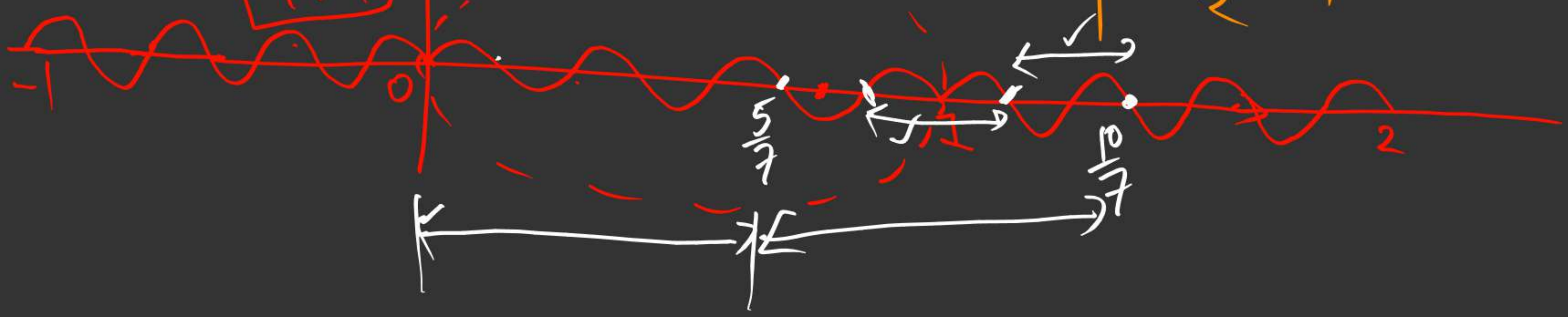
$f(x) = \sin(2\pi\{x\}) \rightarrow T = 1$ $\qquad \{\cdot\} = FPF$

$T = \frac{1}{2} \leftarrow f(x) = \sin(4\pi\{x\})$

$T = \frac{1}{3} \leftarrow f(x) = \sin(6\pi\{x\})$

$f(x) = \sin(7\pi\{x\})$

$\boxed{T = 1}$



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Note → ① $f(x) = \text{const}$, $x \in R$

② $f(x) = g(x) + h(x)$ $\quad \boxed{T = \text{LCM of } (T_1, T_2)}$

$g(x) \downarrow T_1$, $h(x) \downarrow T_2$

③ $y = f(x) \to T$

$y = f(ax+b) \to \dfrac{T}{|a|}$

$g(x) \to T_1, 2T_1, 3T_1, 4T_1, 5T_1, \ldots$

$h(x) \to T_2, 2T_2, 3T_2, 4T_2, 5T_2, 6T_2, 7T_2, \ldots$

Let $2T_1 = 5T_2$

$f(x+2T_1) = f(x) = g(x+2T_1) + h(x+5T_2)$

$= g(x) + h(x)$

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T_1	T_2	LCM of (T_1, T_2)
rational	rational	exist.
rational	irrational	not exist.
irrational	irrational	exist if their cause of being irrational is same.

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$$\text{LCM}\left(\frac{3}{7}, \frac{8}{5}\right) = \boxed{\frac{3}{7}n_1 = \frac{8}{5}n_2}$$

$(n_1)_{least}, (n_2)_{least}$

$$\text{LCM} = \frac{3}{7} \times 56 = 24$$

$$= \frac{8}{5} \times 15 = 24$$

$$\text{LCM} = 12 \times 3 \quad \overline{|\,12, 18}$$

$$\text{LCM} = 12n_1 = 18n_2$$

$$2n_1 = 3n_2$$

$$(n_1)_{min} = 3$$

$$15n_1 = 56n_2$$

$$(n_1)_{least} = 56$$

$$(n_2)_{least} = 15$$

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$$5n_1 \neq \sqrt[3]{2}\, n_2$$

$$\mathrm{LCM}\left(5\sqrt{3},\ 3\sqrt{3}\right) = 15\sqrt{3}$$

$$\frac{n_1}{n_2} \cancel{=} \frac{\sqrt{2}}{\sqrt{3}} \qquad n_1\sqrt{3} \neq n_2\sqrt{2}$$

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$$f(x) = g(x) + h(x)$$

$$\downarrow \qquad \downarrow T_1 \qquad \downarrow T_2$$

T

$$\boxed{T = \text{LCM of } (T_1, T_2) \text{ may fail in interconvertible functions}}$$

$g(x+T) = g(x)$

$h(x+T) = h(x)$

$g(x+T') = h(x)$

$h(x+T') = g(x)$

$T' < T$

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$$\sin^4\left(\frac{\pi}{2}+x\right)+\cos^4\left(\frac{\pi}{2}+x\right)=\cos^4 x+\sin^4 x.$$

$$\underline{1:}\quad f(x)=\underset{\pi}{\underset{\downarrow}{\sin^4 x}}+\underset{\pi}{\underset{\downarrow}{\cos^4 x}}$$

$$T=\frac{\pi}{2}$$

$n\in N$

$$\sin^{2n}x,\ \cos^{2n}x,\ \tan^{2n}x,\ \cot^{2n}x,\ \underset{\pi}{\underset{\downarrow}{\sec^{2n}x}},\ \mathrm{cosec}^{2n}x$$

$$\underbrace{\sin^{2n+1}x,\ \cos^{2n+1}x,\ \sec^{2n+1}x,\ \mathrm{cosec}^{2n+1}x}_{2\pi},\ \underbrace{\tan^{2n+1}x,\ \cot^{2n+1}x}_{\pi}$$

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$y = f(x) \rightarrow T$ ✓

$y = f(ax+b) \rightarrow \boxed{T'}$

$= g(x)$

$g(x+T') = g(x)$

$f(a(x+T')+b) = f(ax+b)$

$f(ax+b+aT') = f(ax+b)$

$f(t+aT') = f(t)$

$f(ax+b) \longrightarrow \dfrac{T}{|a|}$

$\boxed{aT' = T}$

$T' = \dfrac{T}{a}$

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$$f(x) = \sin^4 x + \cos^4 x$$

$$= 1 - \frac{1}{2}\sin^4 2x$$

$$\sin 2x \rightarrow \boxed{\pi}$$

$$\sin^4 2x \rightarrow \boxed{\frac{\pi}{2}}$$

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2.

$f(x) = |\sin x| + |\cos x|$

π π

$T = \frac{\pi}{2}$

3. $f(x$

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3: $f(x) = \sin\left(\frac{2}{3}x\right) + \cos\left(\frac{3}{5}x\right)$

$\frac{2\pi}{2/3} = 3\pi$

$\frac{2\pi}{3/5} = \frac{10\pi}{3}$

Ex-I (remaining)

~~RT~~ Ex-II (1,2,3,4)

$T = 3\pi n_1 = \frac{10\pi}{3} n_2 \Rightarrow 9n_1 = 10n_2$

$(n_1)_{min} = 10$

$LCM\left(\frac{a}{b}, \frac{c}{d}\right) = \frac{LCM\,(a,c)}{HCF\,(b,d)}$

$\boxed{T = 3\pi(10) = 30\pi}$