CIRCLE

SINGLE CORRECT ANSWER TYPE

1. S1: The locus of the centre of a circle which cuts a given circle orthogonally and also touches a given straight line is a parabola.

S2: Two circles $x^2+y^2+2ax+c=0$ and $x^2+y^2+2by+c=0$ touches iff $\frac{1}{a^2}+\frac{1}{b^2}=\frac{1}{c^2}$.

S3: The two circles which passes through $(0,a)$ and $(0,-a)$ and touch the straight line $y=mx+c$, will cut orthogonally if $c^2=a^2(2+m^2)$.

S4: The length of the common chord of the circles $(x-a)^2+y^2=a^2$ and $x^2+(y-b)^2=b^2$ is $\frac{ab}{\sqrt{a^2-b^2}}$.

(A) TFTF (B) TTFF

(C) TFTT (D) FFTT

2. P is a variable point on the line $L=0$. Tangents are drawn to the circle $x^2+y^2=4$ from P to touch it at Q and R. The parallelogram PQSR is completed.

If $L=2x+y-6=0$, then the locus of circumcentre of $\triangle PQR$ is

(A) $2x-y-4$ (B) $2x+y=3$

(C) $x-2y=4$ (D) $x+2y=3$

PARABOLA

SINGLE CORRECT ANSWER TYPE

3. A circle is described whose centre is the vertex and whose diameter is three-quarters of the latus rectum of the parabola $y^2=4ax$. If PQ is the common chord of the circle and the parabola and L_1L_2 is the latus rectum, then the area of the trapezium $PL_1\,L_2Q$ is

(A) $3\sqrt{2}a^2$ (B) $2\sqrt{2}a^2$

(C) $4a^2$ (D) $\left(\frac{2+\sqrt{2}}{2}\right)a^2$

MATCH THE COLUMN

4.

Column-I	Column-II
(A) Area of a triangle formed by the tangents drawn from a point $(-2,2)$ to the parabola $y^2=4(x+y)$ and their corresponding chord of contact is	(P) 8
(B) Length of the latus rectum of the conic $25\{(x-2)^2+(y-3)^2\}=(3x+4y-6)^2$ is	(Q) $4\sqrt{3}$

(C) If focal distance of a point on the parabola $y = x^2 - 4$ is $25/4$ and points are of the form $(\pm\sqrt{a}, b)$ then value of $a + b$ is

(D) Length of side of an equilateral triangle inscribed in a parabola $y^2 - 2x - 2y - 3 = 0$ whose one angular point is vertex of the parabola, is

(R) $\frac{12}{5}$

(T) $\frac{24}{5}$

ELLIPSE

MULTIPLE CORRECT ANSWER TYPE

5. If P is a point of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, whose focii are S and S′. Let $\angle PSS' = \alpha$ and $\angle PS'S = \beta$, then

(A) $PS + PS' = 2a$, if $a > b$

(B) $PS + PS' = 2b$, if $a < b$

(C) $\tan\frac{\alpha}{2}\tan\frac{\beta}{2} = \frac{1-e}{1+e}$

(D) $\tan\frac{\alpha}{2}\tan\frac{\beta}{2} = \frac{\sqrt{a^2-b^2}}{b^2}\left[a - \sqrt{a^2 - b^2}\right]$ when $a > b$

INTEGER TYPE

6. Origin O is the centre of two concentric circles whose radii are a & b respectively, $a < b$. A line OPQ is drawn to cut the inner circle in P& the outer circle in Q. PR is drawn parallel to the y-axis & QR is drawn parallel to the x-axis. The locus of R is an ellipse touching the two circles. If the focii of this ellipse lie on the inner circle, if eccentricity is $\sqrt{2}\lambda$, then find λ

HYPERBOLA

COMPREHENSION TYPE (7-8)

For the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ the normal at P meets the transverse axis AA′ in G and the conjugate axis BB′ in g and CF be perpendicular to the normal from the centre.

7. PF . PG = K CB^2 , then K =

(A) 2 (B) 1 (C) $\frac{1}{2}$ (D) 4

8. PF. Pg equals to

(A) CA^2 (B) CF^2 (C) CB^2 (D) CA . CB

FUNCTION

SINGLE CORRECT ANSWER TYPE

9. Let $f: \{x, y, z\} \rightarrow \{1,2,3\}$ be a one-one mapping such that only one of the following three statements is true and remaining two are false: $f(x) \neq 2, f(y) = 2, f(z) \neq 1$, then

(A) $f(x) > f(y) > f(z)$ (B) $f(x) < f(y) < f(z)$

(C) $f(y) < f(x) < f(z)$ (D) $f(y) < f(z) < f(x)$

INTEGER TYPE

10. If $f(x) + f(y) + f(xy) = 2 + f(x).f(y)$, for all real values of x & y and f(x) is a polynomial function with $f(4) = 17$, then find the value of $f(5)/14$, where $f(1) \neq 1$.

LIMIT OF FUNCTION

ASSERTION AND REASON TYPE

11. Statement 1: $\lim_{x\to\infty}\left(\frac{1^2}{x^3}+\frac{2^2}{x^3}+\frac{3^2}{x^3}+\cdots\ldots\ldots+\frac{x^2}{x^3}\right)=\frac{1}{3}$

Statement 2: $\lim_{x\to a}(f_1(x)+f_2(x)+\cdots\ldots.+f_n(x)) = \lim_{x\to a} f_1(x) + \lim_{x\to a} f_2(x) \ldots\ldots. + \lim_{x\to a} f_n(x)$ where $n \in \mathbb{N}$

(A) Statement 1 is true, statement 2 is true, statement 2 is a correct explanation for statement 1

(B) Statement 1 is true, statement 2 is true, statement 2 is NOT correct explanation for statement 1

(C) Statement 1 is true, statement 2 is false

(D) Statement 1 is false, statement 2 is true

INTEGER TYPE

12. Let $P = \frac{\left(1^4+\frac{1}{4}\right)\left(3^4+\frac{1}{4}\right)\left(5^4+\frac{1}{4}\right)\ldots\left((2n-1)^4+\frac{1}{4}\right)}{\left(2^4+\frac{1}{4}\right)\left(4^4+\frac{1}{4}\right)\left(6^4+\frac{1}{4}\right)\ldots\left((2n)^4+\frac{1}{4}\right)}$ and $\lim_{n\to\infty}(n^a P)$ exists, then find a

CONTINUITY & DERIVABILITY

SINGLE CORRECT ANSWER TYPE

13. Number of points where the function $f(x) = \max(|\tan x|, \cos|x|)$ is non differentiable in the interval $(-\pi, \pi)$ is

(A) 4 (B) 6 (C) 3 (D) 2

INTEGER TYPE

14. Let f(x) is differentiable function & $f(0) = 1$. Also if f(x) satisfies

$f(x + y + 1) = \left(\sqrt{f(x)} + \sqrt{f(y)}\right)^2 \ \forall\, x, y \in R$ and $f(x) = a(x + 1)^b$, then find $(a + b)$

METHOD OF DIFFERENTIANTION

MULTIPLE CORRECT ANSWER TYPE

15. Given $f(x)=-\frac{x^3}{3}+x^2\sin1.5a-x\sin a\cdot\sin2a-5\sin^{-1}(a^2-8a+17)$ then:

(A) $f'(x)=-x^2+2x\sin6-\sin4\sin8$

(B) $f'(\sin8)>0$

(C) $f'(x)$ is not defined at $x=\sin8$

(D) $f'(\sin8)<0$

APPLICATION OF DERIVATIVES

SINGLE CORRECT ANSWER TYPE

16. Tangent of acute angle between the curves $y=|x^2-1|$ and $y=|x^2-3|$ at their points of intersection is

(A) 0 (B) $\frac{4\sqrt{2}}{3}$ (C) $\frac{4\sqrt{2}}{7}$ (D) $2\sqrt{2}$

17. If $\frac{a_0}{n+1}+\frac{a_1}{n}+\frac{a_2}{n-1}+\cdots\ldots\ldots+\frac{a_{n-1}}{2}+a_n=0$, then the equation

$a_0x^n+a_1x^{n-1}+a_2x^{n-2}+\cdots\ldots.+a_{n-1}x+a_n=0$ has

(A) exactly one root in (0,1) (B) at least one root in (0,1)

(C) no root in (0,1) (D) at the most one root in (0,1)

INTEGER TYPE

18. A cubic f(x) vanishes at $x=-2$ and has relative minimum/maximum at $x=-1$ and $x=\frac{1}{3}$. If $\int_{-1}^{1}f(x)dx=\frac{14}{3}$, the cubic $f(x)=\lambda_1x^3+\lambda_2x^2-x+2$, then find $(\lambda_1+\lambda_2)$

INDEFINITE INTEGRATION

SINGLE CORRECT ANSWER TYPE

19. Evaluate $\int x^2\log(1-x^2)\cdot dx$ and hence find the value of:

$$\frac{1}{1.5}+\frac{1}{2.7}+\frac{1}{3.9}+\cdots\ldots\ldots.=$$

(A) $\frac{1}{4}\log2$ (B) $\frac{2}{7}-\frac{2}{3}\log2$ (C) $\frac{8}{9}-\frac{2}{3}\log2$ (D) $-\frac{2}{3}\log2$

INTEGER TYPE

20. $\int\left(\frac{x-1}{x+1}\right)\frac{dx}{\sqrt{x^3+x^2+x}}=\lambda\tan^{-1}\sqrt{\left(x+\frac{1}{x}+1\right)}+c$, then find λ

DEFINITE INTEGRATION

SINGLE CORRECT ANSWER TYPE

21. The absolute value of $\int_{10}^{18} \frac{\sin x}{1+x^8} dx$, is

(A) less than 10^{-7}

(B) more than 10^{-7}

(C) less than 10^{-6}

(D) more than 10^{-6}

MULTIPLE CORRECT ANSWER TYPE

22. If the value of the definite integral $\int_0^1 \frac{\sin^{-1}\sqrt{x}}{x^2-x+1} dx$ is $\frac{\pi^2}{\sqrt{n}}$ where $n \in N$, then the value of ' n ' is divisible by prime number

(A) 2 (B) 3 (C) 5 (D) 9

INTEGER TYPE

23. A function $f: R \to R$ satisfies the equation $f(x+y) = f(x) + f(y) \ \forall \ x, y \in R$ and is continuous throughout the domain. If $I_1 + I_2 + I_3 + I_4 + I_5 = 450$ where $I_n = n\int_0^n f(x)dx$ and $f(x) = \lambda x$, then find λ

AREA UNDER CURVE

SINGLE CORRECT ANSWER TYPE

24. If $f(x) = \sin x \ \forall \ x \in \left[0, \frac{\pi}{2}\right], f(x) + f(\pi - x) = 2 \ \forall \ x \in \left(\frac{\pi}{2}, \pi\right]$ and $f(x) = f(2\pi - x) \ \forall \ x \in (\pi, 2\pi]$, then the area enclosed by $y = f(x)$ and x - axis is

(A) π (B) 2π (C) 2 (D) 4

DIFFERENTIAL EQUATION

SINGLE CORRECT ANSWER TYPE

25. Solution of differential equation $x^2 = 1 + \left(\frac{x}{y}\right)^{-1} \frac{dy}{dx} + \frac{\left(\frac{x}{y}\right)^{-2}\left(\frac{dy}{dx}\right)^2}{2!} + \frac{\left(\frac{x}{y}\right)^{-3}\left(\frac{dy}{dx}\right)^3}{3!} + \cdots$. Is

(A) $y^2 = x^2(\ln x^2 - 1) + c$

(B) $y = x^2(\ln x - 1) + c$

(C) $y^2 = x(\ln x - 1) + c$

(D) $y = x^2 e^{x^2} + c$

INTEGER TYPE

26. Let the curve $y = f(x)$ passes through $(4, -2)$ satisfy the differential equation,

$y(x + y^3)dx = x(y^3 - x)dy$ & $y = g(x) = \int_{1/8}^{\sin^2 x} \sin^{-1}\sqrt{t}dt + \int_{1/8}^{\cos^2 x} \cos^{-1}\sqrt{t}dt, 0 \le x \le \frac{\pi}{2}$.

The area of the region bounded by curves, $y = f(x), y = g(x)$ and $x = 0$ is $\frac{\lambda}{8}\left(\frac{3\pi}{16}\right)^4$,

then find the value of λ.

PROBABILITY

SINGLE CORRECT ANSWER TYPE

27. S_1 : Two persons each make a single throw with a die. The probability they get equal values is P_1. Four persons each make a single throw and probability of exactly three being equal is P_2. Then P_1 greater than P_2.

S_2 : Each of A & B throw 2 dice, if A throws 9 , then B 's probability of throwing a higher number is $\frac{1}{6}$

S_3: If $P(A_1 \cup A_2) = 1 - P(A_1{}^C) \cdot P(A_2{}^C)$, then A_1 and A_2 are independent

S_4 : If the events A, B, C are independent, then A, B, $\bar{C}$ are independent

(A) T T T T
(B) TTFT
(C) TFTF
(D) F TTF

MULTIPLE CORRECT ANSWER TYPE

28. A bag initially contains one red & two blue balls. An experiment consisting of selecting a ball at random, noting its colour & replacing it together with an additional ball of the same colour. If three such trials are made, then:

(A) probability that atleast one blue ball is drawn is 0.9

(B) probability that exactly one blue ball is drawn is 0.2

(C) probability that all the drawn balls are red given that all the drawn balls are of same colour is 0.2

(D) probability that atleast one red ball is drawn is 0.6.

MATRICES & DETERMINANTS

SINGLE CORRECT ANSWER TYPE

29. If a determinant of order 3×3 is formed by using the numbers 1 or -1 then minimum value of determinant is

(A) -2 (B) -4 (C) 0 (D) -8

30. **Match the following**

Column – I	**Column - II**
(A) Let $\|A\| = \|a_i\|_{3\times3} \neq 0$. Each element a_{ij} is multiplied by k^{i-1}. Let $\|B\|$ the resulting determinant, where $k_1\|A\| + k_2\|B\| = 0$. Then $k_1 + k_2 =$	(p) 0
(B) The maximum value of a third order determinant each of its entries are \pm 1 equals	(q) 4
(C) $\begin{vmatrix} 1 & \cos\alpha & \cos\beta \\ \cos\alpha & 1 & \cos\gamma \\ \cos\beta & \cos\gamma & 1 \end{vmatrix} = \begin{vmatrix} 0 & \cos\alpha & \cos\beta \\ \cos\alpha & 0 & \cos\gamma \\ \cos\beta & \cos\gamma & 0 \end{vmatrix}$ if $\cos^2\alpha + \cos^2\beta + \cos^2\gamma =$	(r) 1
(D) $\begin{vmatrix} x^2+x & x+1 & x-2 \\ 2x^2+3x-1 & 3x & 3x-3 \\ x^2+2x+3 & 2x-1 & 2x-1 \end{vmatrix} = Ax + B$ where A and B are determinants of order3. Then A + 2 B =	(s) 2
	(t) $\begin{vmatrix} 1 & 2 \\ 2 & 4 \end{vmatrix}$

COMPLEX NUMBER

SINGLE CORRECT ANSWER TYPE

31. S1: If (z_1, z_2) and (z_3, z_4) are two pairs of non zero conjugate complex numbers then $\arg\left(\frac{z_1}{z_3}\right) + \arg\left(\frac{z_2}{z_4}\right) = \pi/2$

S2: If ω is an imaginary fifth root of unity, then $\log_2\left|1 + \omega + \omega^2 + \omega^3 - \frac{1}{\omega}\right| = 1$

S3: If z_1 and z_2 are two of the 8th roots of unity, such that $\arg\left(\frac{z_1}{z_2}\right)$ is least positive, then $\frac{z_1}{z_2} = \frac{1+i}{\sqrt{2}}$

S4: The product of all the fifth roots of -1 is equal to -1

(A) TTFT (B) TFFT (C) FFTF (D)FTIT

32. **Match the column :**

If z_1, z_2, z_3, z_4 are the roots of the equation $z^4 + z^3 + z^2 + z + 1 = 0$ then

Column-I	**Column - II**
(A) $\left\|\sum_{i=1}^{4} z_i^4\right\|$ is equal to	(p) 0
(B) $\sum_{i=1}^{4} z_i^5$ is equal to	(q) 4
(C) $\prod_{i=1}^{4}(z_i + 2)$ is equal to	(r) 1
(D) least value of $[\|z_1 + z_2\|]$ is	(s) 11
(Where [] represents greatest integer function)	(t) $\left\|4\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)\right\|$

VECTORS

MULTIPLE CORRECT ANSWER TYPE

33. If in $\triangle$ ABC, $\overrightarrow{AB} = \frac{\vec{u}}{|\vec{u}|} - \frac{\vec{v}}{|\vec{v}|}$ and $\overrightarrow{AC} = \frac{2\vec{u}}{|\vec{u}|}$, where $|\vec{u}| \neq |\vec{v}|$, then

(A) $1 + \cos 2A + \cos 2B + \cos 2C = 0$

(B) $\sin A = \cos C$

(C) projection of AC on BC equal to BC

(D) projection of AB on BC is equal to AB

MULTIPLE CORRECT ANSWER TYPE

34. A vector ($\vec{d}$) is equally inclined to three vectors $\vec{a} = \hat{\imath} - \hat{\jmath} + \hat{k}, \vec{b} = 2\hat{\imath} + \hat{\jmath}$ and $\vec{c} = 3\hat{\jmath} - 2\hat{k}$. Let $\vec{x}, \vec{y}, \vec{z}$ be three vector in the plane of $\vec{a}, \vec{b}; \vec{b}, \vec{c}; \vec{c}, \vec{a}$ respectively, then

(A) $\vec{x} \cdot \vec{d} = 14$

(B) $\vec{y} \cdot \vec{d} = 3$

(C) $\vec{z} \cdot \vec{d} = 0$

(D) $\vec{r} \cdot \vec{d} = 0$ where $\vec{r} = \lambda\vec{x} + \mu\vec{y} + \delta\vec{z}$

THREE-DIMENSIONAL GEOMETRY

SINGLE CORRECT ANSWER TYPE

35. Equation of the straight line in the plane $\vec{r} \cdot \vec{n} = d$ which is parallel to $\vec{r} = \vec{a} + \lambda\vec{b}$ and passes through the foot of perpendicular drawn from the point $P(\vec{a})$ to the plane $\vec{r} \cdot \vec{n} = d$ is

(where $\vec{n} \cdot \vec{b} = 0$)

(A) $\vec{r} = \vec{a} + \left(\frac{d - \vec{a}\cdot\vec{n}}{\vec{n}^2}\right)\vec{n} + \lambda\vec{b}$

(B) $\vec{r} = \vec{a} + \left(\frac{d - \vec{a}\cdot\vec{n}}{\vec{n}}\right)\vec{n} + \lambda\vec{b}$

(C) $\vec{r} = \vec{a} + \left(\frac{\vec{a}\cdot\vec{n} - d}{\vec{n}^2}\right)\vec{n} + \lambda\vec{b}$

(D) $\vec{r} = \vec{a} + \left(\frac{\vec{a}\cdot\vec{n} - d}{\vec{n}}\right)\vec{n} + \lambda\vec{b}$

36. If $P_1 : \vec{r} \cdot \vec{n}_1 - d_1 = 0, P_2 : \vec{r} \cdot \vec{n}_2 - d_2 = 0$ and $P_3 : \vec{r} \cdot \vec{n}_3 - d_3 = 0$ are three planes and $\vec{n}_1, \vec{n}_2$ and $\vec{n}_3$ are three non-coplanar vectors then, the three lines $P_1 = 0, P_2 = 0; P_2 = 0, P_3 = 0$ and $P_3 = 0, P_1 = 0$ are

(A) parallel lines

(B) coplanar lines

(C) coincident lines

(D) concurrent lines

INVERSE TRIGONOMETRIC FUNCTION

MATRIX - MATCH TYPE

37. [.] represents greatest integer function in parts (A), (B) and (C)

Column -1	**Column - II**
(A) If $f(x) = \sin^{-1}x$ and $\lim\limits_{x \to \frac{1}{2}^+} f(3x - 4x^3) = a - 3\lim\limits_{x \to \frac{1}{2}^+} f(x)$, then $[a] =$	(p) 2
(B) If $f(x) = \tan^{-1}g(x)$ where $g(x) = \frac{3x - x^3}{1 - 3x^2}$ and $\lim\limits_{h \to 0} \dfrac{f(a + 3h) - f(a)}{3h} = \dfrac{3}{1 + a^2}$, when $\dfrac{-1}{\sqrt{3}} < a < \dfrac{1}{\sqrt{3}}$, then find $\left[\lim\limits_{h \to 0} \frac{f\left(\frac{1}{2} + 6h\right) - f\left(\frac{1}{2}\right)}{6h}\right] =$	(q) 3
(C) If $\cos^{-1}(4x^3 - 3x) = a + b\cos^{-1}x$ for $-1 < x < \frac{-1}{2}$, then $[a + b + 2] =$	(r) 4
(D) If $f(x) = \cos^{-1}(4x^3 - 3x)$ and $\lim\limits_{x \to \frac{1}{2}^+} f'(x) = a$ and $\lim\limits_{x \to \frac{1}{2}^-} f'(x) = b$, then $a + b - 3 =$	(s) -2
	(t) -3

INTEGER TYPE

38. $\tan^{-1}\left[\frac{3\sin 2\alpha}{5 + 3\cos 2\alpha}\right] + \tan^{-1}\left[\frac{\tan\alpha}{4}\right] = \lambda\alpha$, then find the value of λ, where $-\frac{\pi}{2} < \alpha < \frac{\pi}{2}$.

ANSWER KEY

1.	(A)	2.	(B)	3.	(D)	4.	((A) $\to$ (r), (B) $\to$ (t), (C) $\to$ (p), (D) $\to$ (q))						
5.	(ABC)	6.	(1)	7.	(B)	8.	(A)	9.	(C)	10.	(9)	11.	(C)
12.	(2)	13.	(A)	14.	(3)	15.	(AD)	16.	(C)	17.	(B)	18.	(2)
19.	(C)	20.	(2)	21.	(A)	22.	(A, B)	23.	(4)	24.	(B)	25.	(A)
26.	(1)	27.	(A)	28.	(A,B,C,D)		29.	(B)					
30.	(A) – (p, t), (B) – (q), (C) – (r), (D) – (p, t)							31.	(D)				
32.	(A) $\to$ (r),(B) $\to$ (q, t),(C) $\to$ (s),(D) $\to$ (p)							33.	(A,B,C)				
34.	(C,D)	35.	(A)	36.	(D)	37.	(A) $\to$ (q) .(B) $\to$ (p),(C) $\to$ (s),(D) $\to$ (t)						
38.	1												