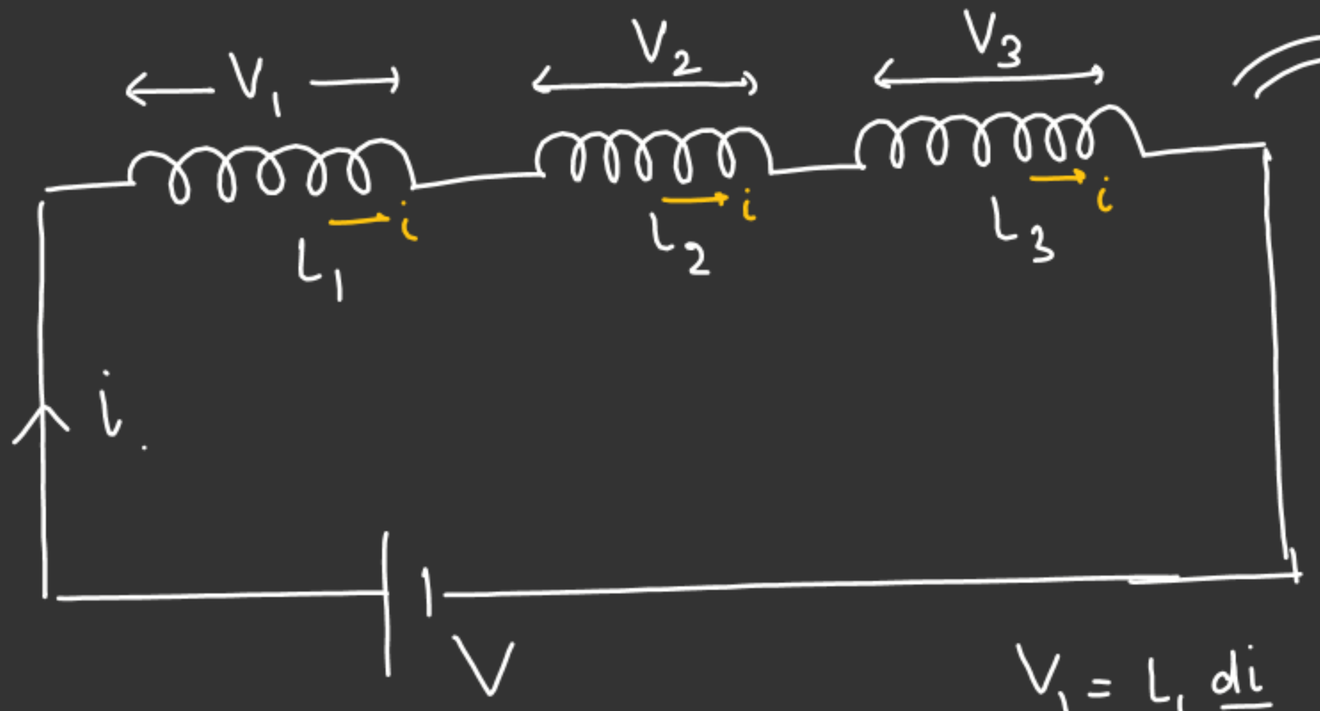
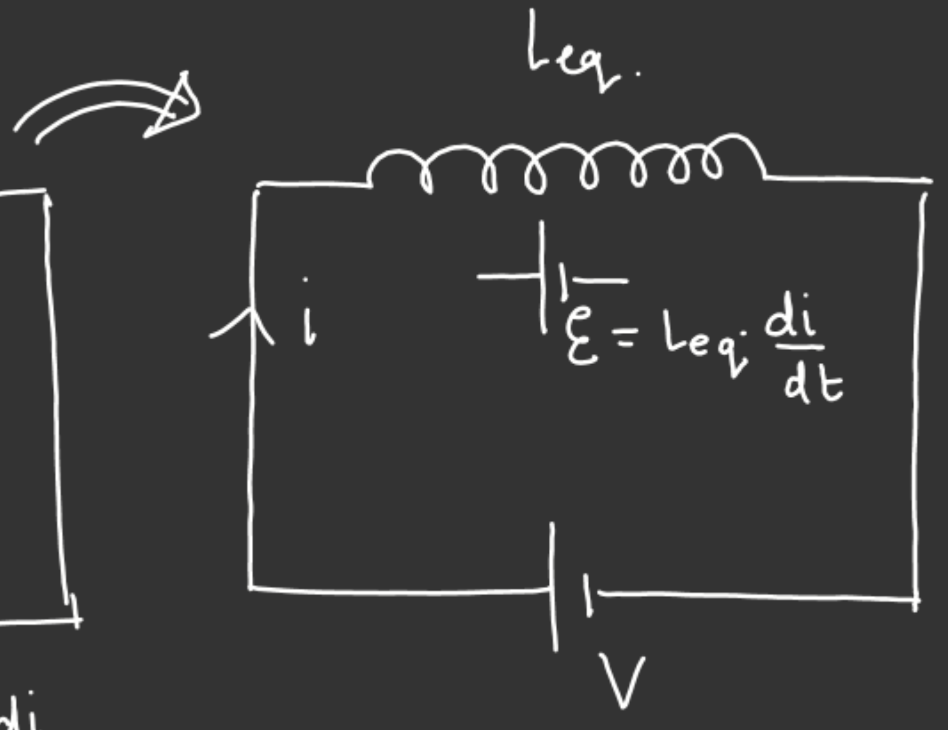
Nishant Jindal

Series Combination of Inductor





$$V_1 = L_1 \frac{di}{dt}$$

$$V_2 = L_2\, di/dt$$

$$V_3 = L_3\, di/dt$$

$$V = L_{eq} \frac{di}{dt}$$

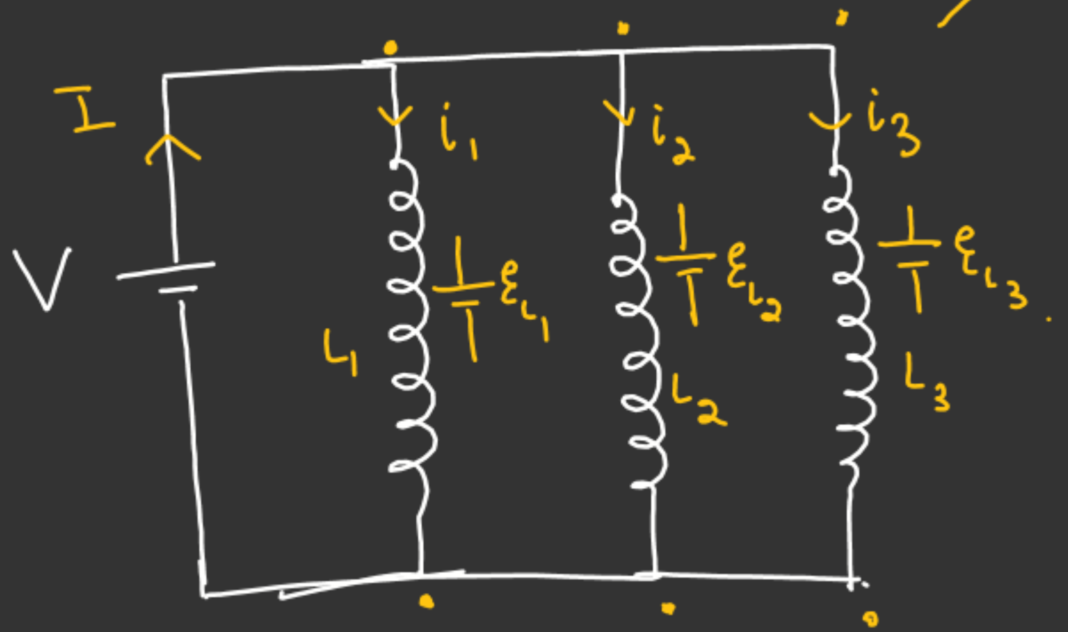
$$V = V_1 + V_2 + V_3 .$$

$$L_{eq}\left(\frac{di}{dt}\right) = L_1 \frac{di}{dt} + L_2 \frac{di}{dt} + L_3 \frac{di}{dt}$$

$$L_{eq} = (L_1 + L_2 + L_3)$$ ✓

Nishant Jindal

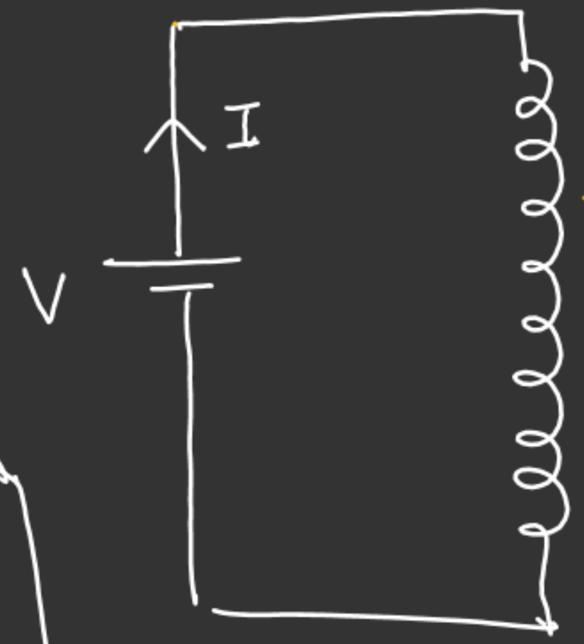
** Parallel Combination



$$\varepsilon_{L_1} = L_1 \frac{di_1}{dt}$$

$$\varepsilon_{L_2} = L_2 \frac{di_2}{dt}$$

$$\varepsilon_{L_3} = L_3 \frac{di_3}{dt}$$



$$L_{eq} \cdot \frac{di}{dt} = V$$

$$\Downarrow$$

$$\frac{di}{dt} = \frac{V}{L_{eq}}$$

$$V = \varepsilon_{L_1} = \varepsilon_{L_2} = \varepsilon_{L_3}$$

$$\frac{di_1}{dt} = \frac{\varepsilon_{L_1}}{L_1} = \frac{V}{L_1}$$

$$\frac{di_2}{dt} = \frac{\varepsilon_{L_2}}{L_2} = \frac{V}{L_2}$$

$$\frac{di_3}{dt} = \frac{\varepsilon_{L_3}}{L_3} = \frac{V}{L_3}.$$

$$I = I_1 + I_2 + I_3.$$

Differentiating both side w.r.t time

$$\frac{dI}{dt} = \frac{dI_1}{dt} + \frac{dI_2}{dt} + \frac{dI_3}{dt}$$

$$\frac{V}{L_{eq}} = \frac{V}{L_1} + \frac{V}{L_2} + \frac{V}{L_3}$$

$$\boxed{\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}}$$

Nishant Jindal

General L-R Ckt

Growth of Current

$$I = I_0 \left(1 - e^{-t/\tau}\right)$$

$I_0 \rightarrow$ [Maximum Current at ∞]

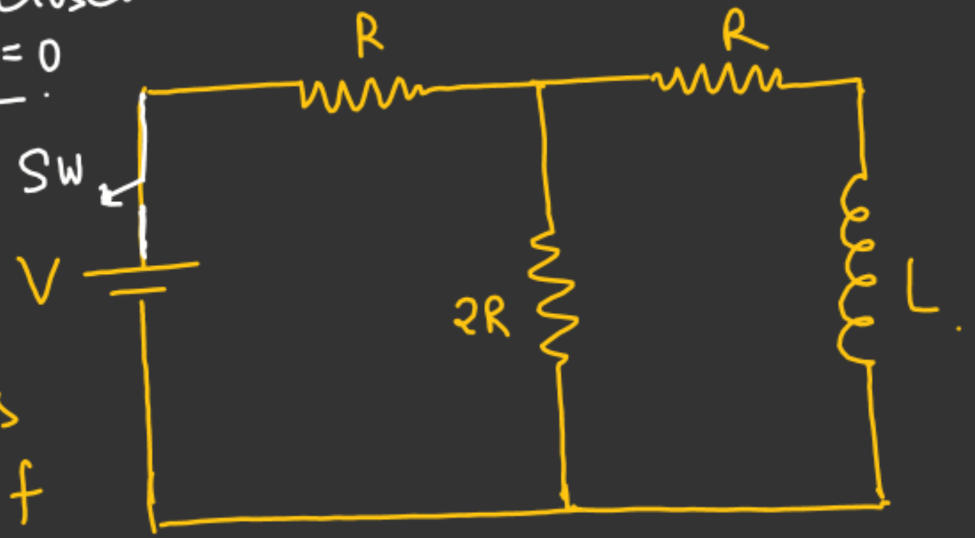
Steady State Current

[At this inductor behave as zero resistance wire]

$$\tau = \left(\frac{L}{R_{eq}}\right)$$

$R_{eq} \rightarrow$ Across inductor

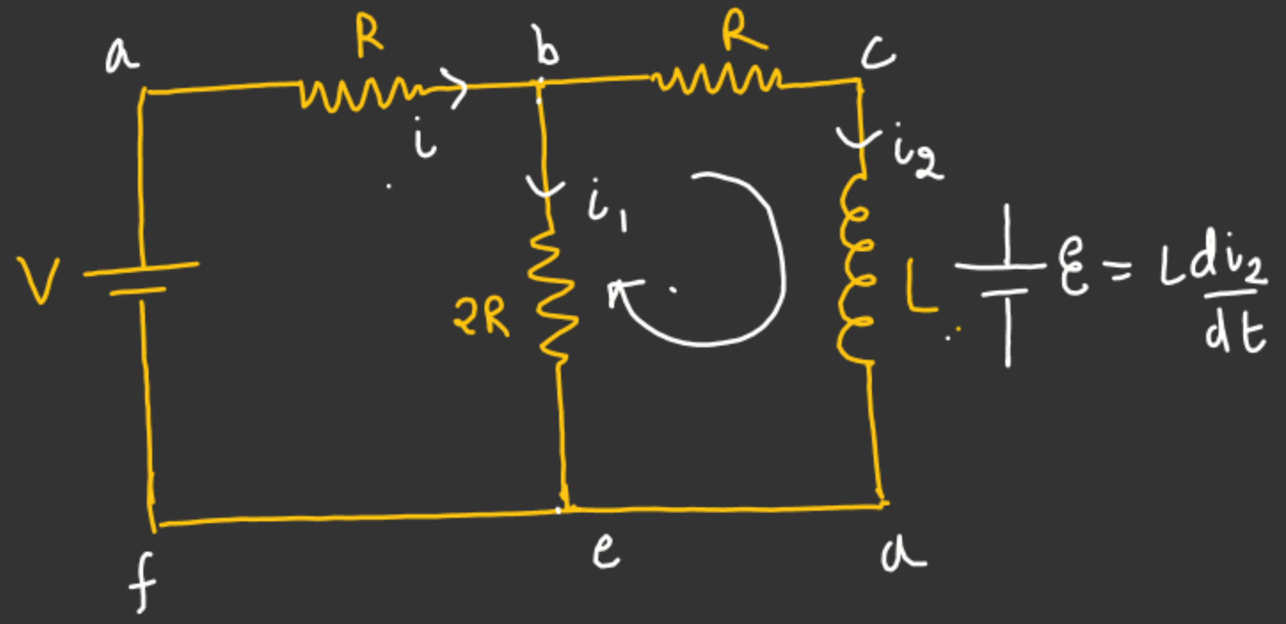
Sw closed at $t=0$



a) Find Current in the inductor as a function of time.

b) Total Current in the Ckt as a function of time.

Sol^n :-



$$i = i_1 + i_2 \quad \text{— (1)}$$

KVL in loop abcdefa

$$V - iR - i_2R - L\frac{di_2}{dt} = 0$$

KVL in loop bcdeb

$$-i_2R - L\frac{di_2}{dt} + i_1 2R = 0$$

$$i_1(2R) = i_2R + L\frac{di_2}{dt}$$

$$i_1 = \frac{i_2}{2} + \frac{L}{2R}\left(\frac{di_2}{dt}\right) \quad \text{— (2)}$$

From (1) $i - i_2 = i_1$ put in (2)

$$i - i_2 = \frac{i_2}{2} + \frac{L}{2R}\left(\frac{di_2}{dt}\right)$$

$$i = \frac{3i_2}{2} + \frac{L}{2R}\left(\frac{di_2}{dt}\right)$$

$$V - \left(\frac{3i_2}{2} + \frac{L}{2R}\frac{di_2}{dt}\right)R - i_2R - L\frac{di_2}{dt} = 0$$

$$V - \frac{3}{2}i_2R - i_2R - \left(\frac{L}{2} + L\right)\frac{di_2}{dt} = 0$$

$$V - \frac{5}{2}i_2R - \frac{3L}{2}\frac{di_2}{dt} = 0$$

Nishant Jindal

Sol^n:-

$i = i_1 + i_2$ — ①

KVL in loop abcdefa

$V - iR - i_2R - L\dfrac{di_2}{dt} = 0$

KVL in loop bcaeb

$-i_2R - L\dfrac{di_2}{dt} + i_1 2R = 0$

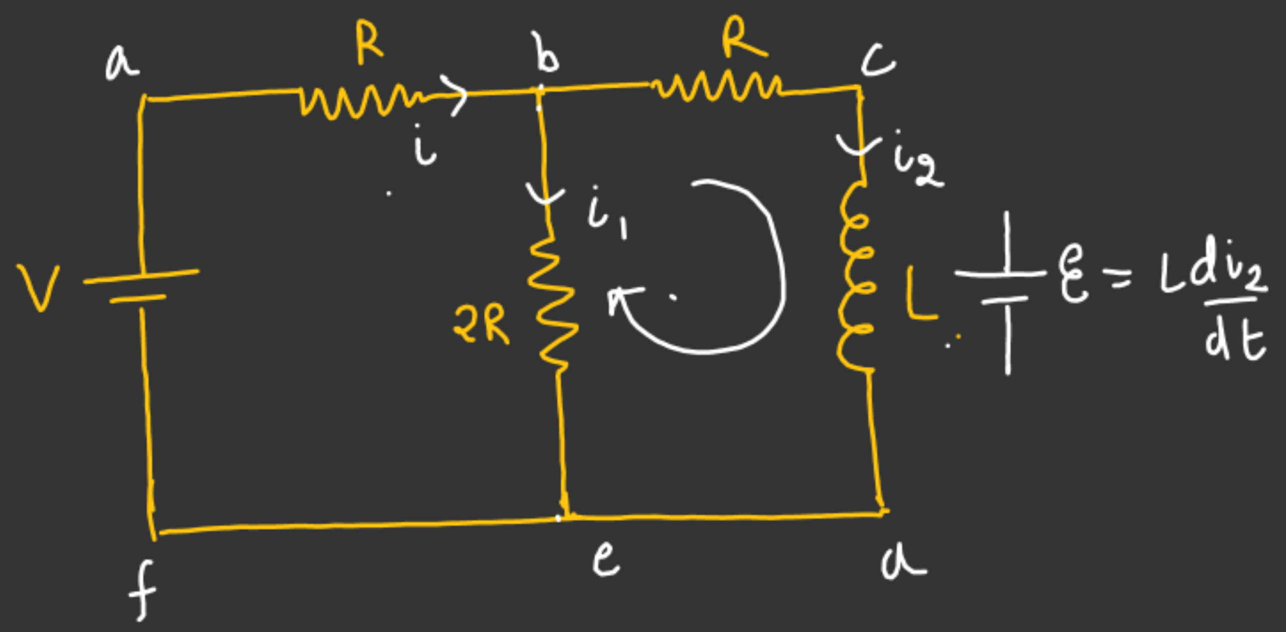
$i_1(2R) = i_2R + L\dfrac{di_2}{dt}$

$i_1 = \dfrac{i_2}{2} + \dfrac{L}{2R}\left(\dfrac{di_2}{dt}\right)$ — ②

From ① $i - i_2 = i_1$ put in ②

$i - i_2 = \dfrac{i_2}{2} + \dfrac{L}{2R}\left(\dfrac{di_2}{dt}\right)$

$i = \dfrac{3i_2}{2} + \dfrac{L}{2R}\left(\dfrac{di_2}{dt}\right)$



$V - \left(\dfrac{3i_2}{2} + \dfrac{L}{2R}\dfrac{di_2}{dt}\right)R - i_2R - L\dfrac{di_2}{dt} = 0$

$V - \dfrac{3}{2}i_2R - i_2R - \left(\dfrac{L}{2} + L\right)\dfrac{di_2}{dt} = 0$

$V - \dfrac{5}{2}i_2R - \dfrac{3L}{2}\dfrac{di_2}{dt} = 0$

$(2V - 5Ri_2) = 3L\dfrac{di_2}{dt}$

Nishant Jindal

$$(2V - 5Ri_2) = 3L\frac{di_2}{dt}$$

$$\int_0^{i_2} \frac{di_2}{(2V - 5Ri_2)} = \frac{1}{3L}\int_0^t dt$$

$$\frac{\ln(2V - 5Ri_2)\Big|_0^{i_2}}{-5R} = \frac{1}{3L}t$$

$$\ln\left(\frac{2V - 5Ri_2}{2V}\right) = -\frac{5R}{3L}t$$

$$2V - 5Ri_2 = 2V e^{-\frac{5R}{3L}t}$$

$$2V\left(1 - e^{\frac{-5R}{3L}t}\right) = 5R\,i_2$$

$$i_2 = \frac{2V}{5R}\left(1 - e^{\frac{-5R}{3L}t}\right)$$

$$i = i_0\left(1 - e^{-t/\tau}\right)$$

$(i_0)_{max}$ in inductor (At the time of Steady state)

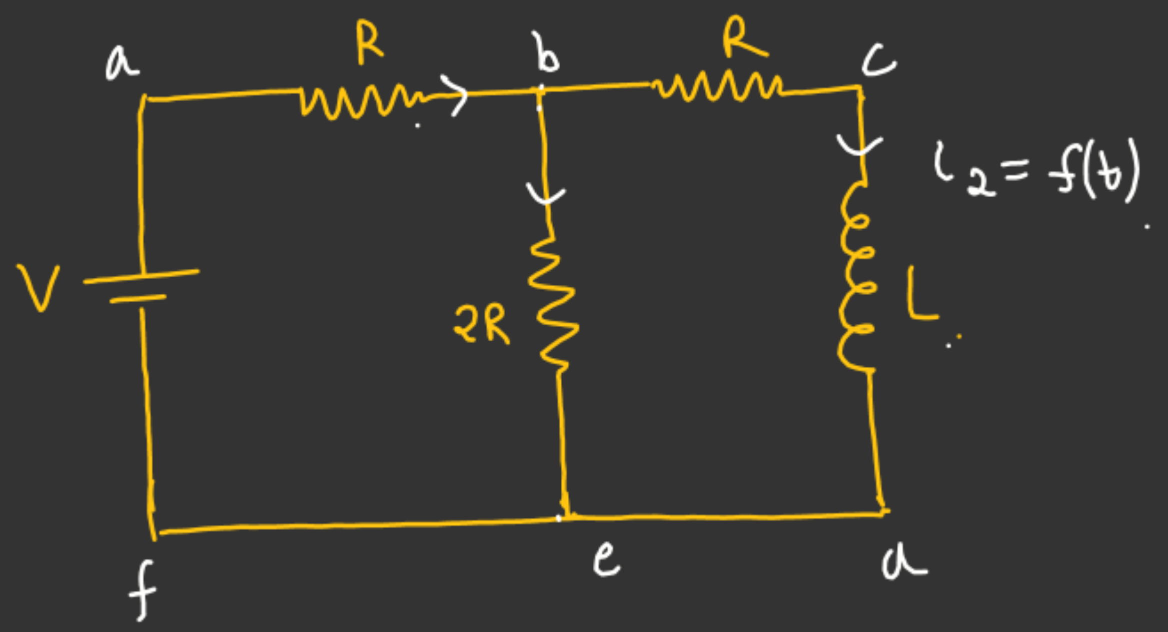
$$= \frac{2V}{5R}$$

$$\tau = \left(\frac{3L}{5R}\right) \checkmark$$

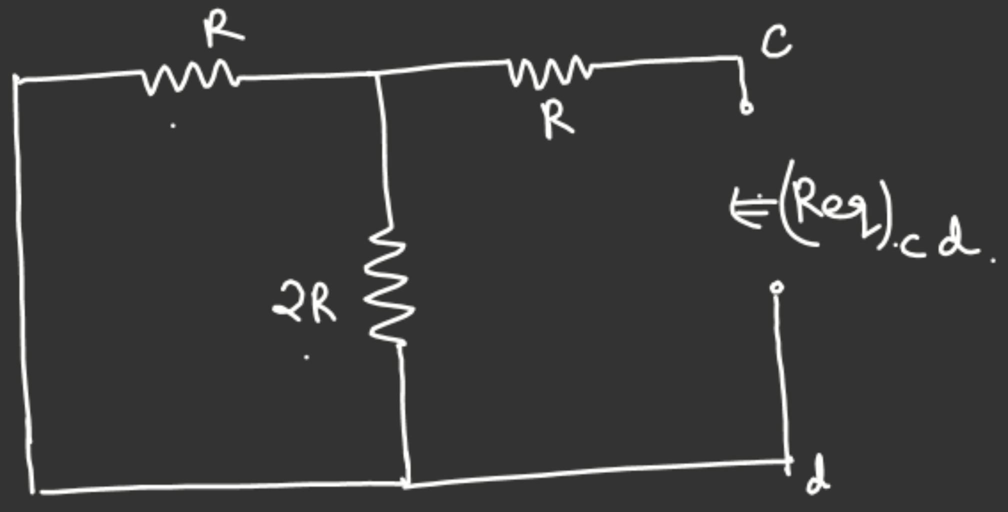
Time Constant

Nishant Jindal

M-2: Find τ.



(i) Req across inductor.
for this short the battery.



$$(R_{eq})_{c-d} = \frac{R \cdot 2R}{R+2R} + R$$

$$= \frac{2R}{3} + R$$

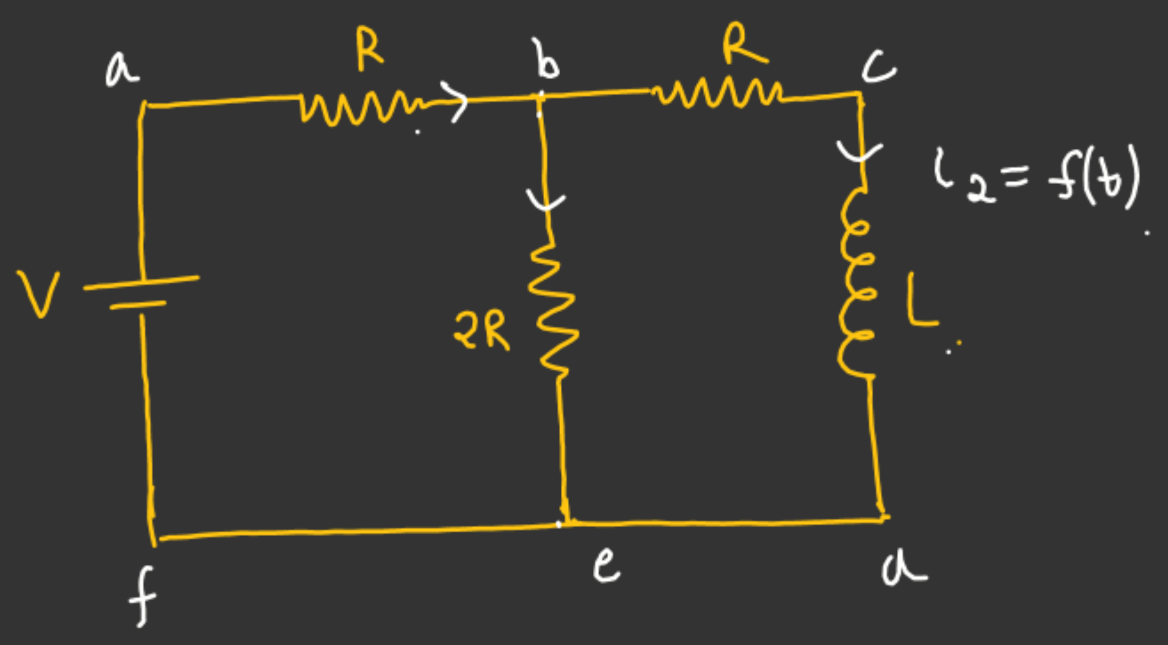
$$= \left(\frac{5R}{3}\right).$$

$$\tau_{ckt} = \frac{L}{(R_{eq})_{\text{across } L}}$$

$$\tau_{ckt} = \frac{L}{\frac{5R}{3}} = \left(\frac{3L}{5R}\right) \text{ Ans.}$$

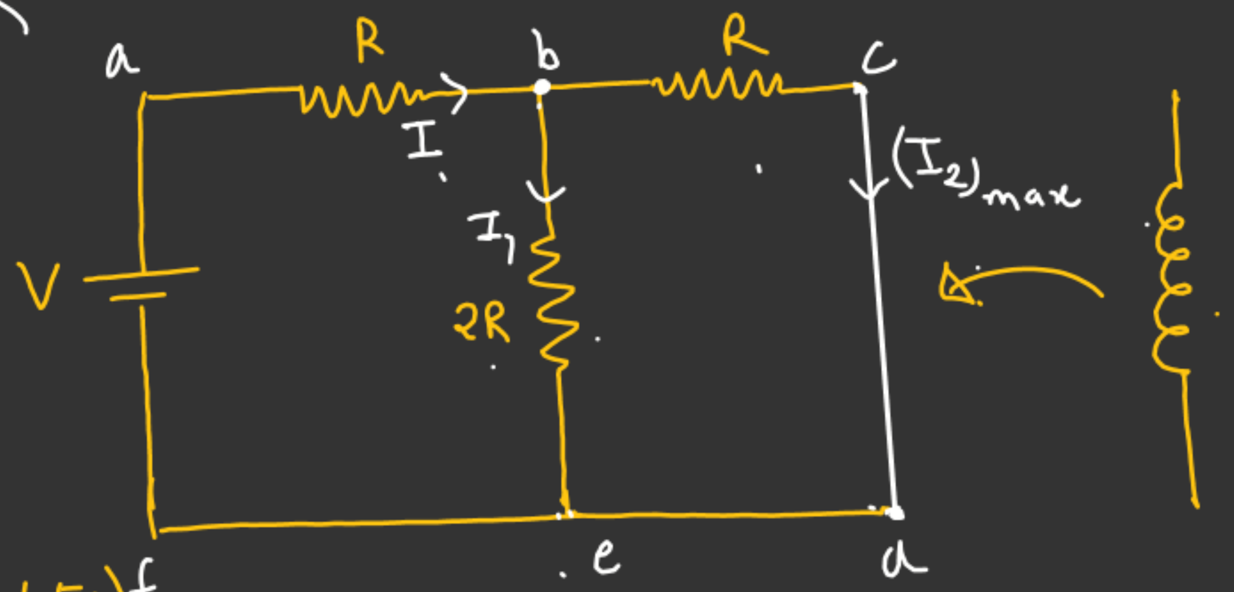
Nishant Jindal

M-2. $i_2 = f(t)$ ✓



$(i_2)_{max}$ at the time of Steady State.

At that time inductor behave as a zero resistance wire.



$$R_{eq} = \frac{2R \cdot R}{2R+R} + R$$

$$R_{eq} = \frac{2R}{3} + R = \left(\frac{5R}{3}\right)$$

$$I = \frac{V}{(5R/3)} = \left(\frac{3V}{5R}\right)$$

K.V.L in loop abcdefa

$$V - IR - (I_2)_{max} R = 0$$

$$(I_2)_{max} R = V - IR$$

$$= \left(V - \frac{3V}{5}\right) = \left(\frac{2V}{5}\right)$$

$$(I_2)_{max} = \left(\frac{2V}{5R}\right)$$

$$I_2 = \frac{2V}{5R}\left(1 - e^{-\frac{t5R}{3L}}\right)$$

Ans ✓

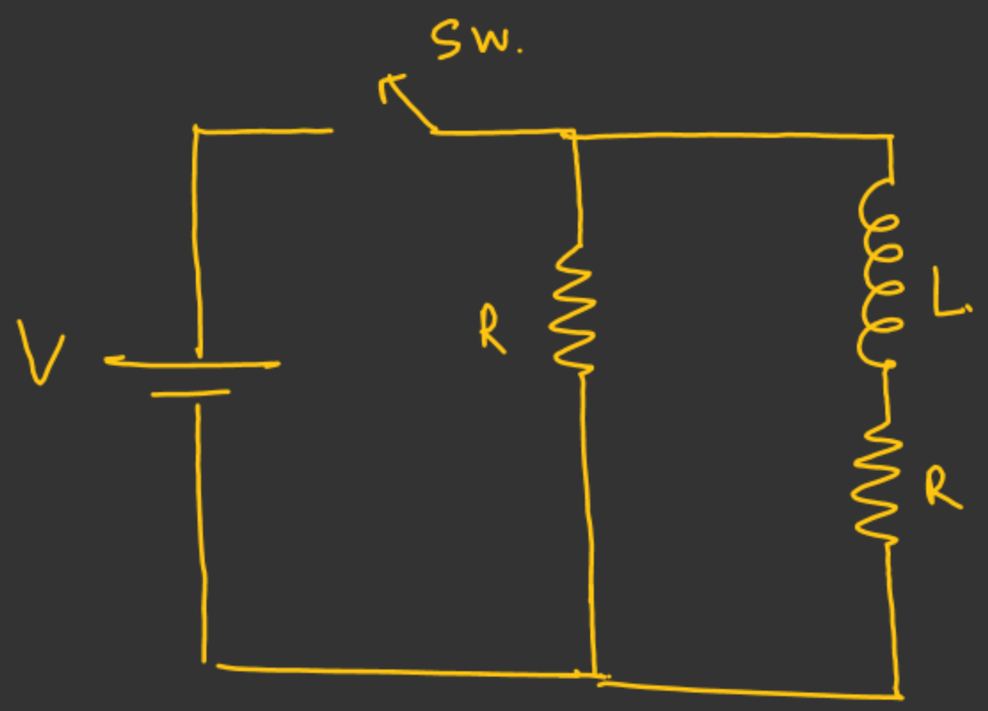
Nishant Jindal

QQ:

$V = 12$ Volt.

$R = 2\Omega$.

$L = 400 mH$.



Switch is closed at $t = 0$.

a) potential drop across L as a function of time.

Imp

b) After Steady achived, SW again opened.

i) find Current in resistor just after SW is reopened.

ii) find Current in resistor as a function of time after SW reopened.

Nishant Jindal

$V = 12$ Volt.

$R = 2\,\Omega$.

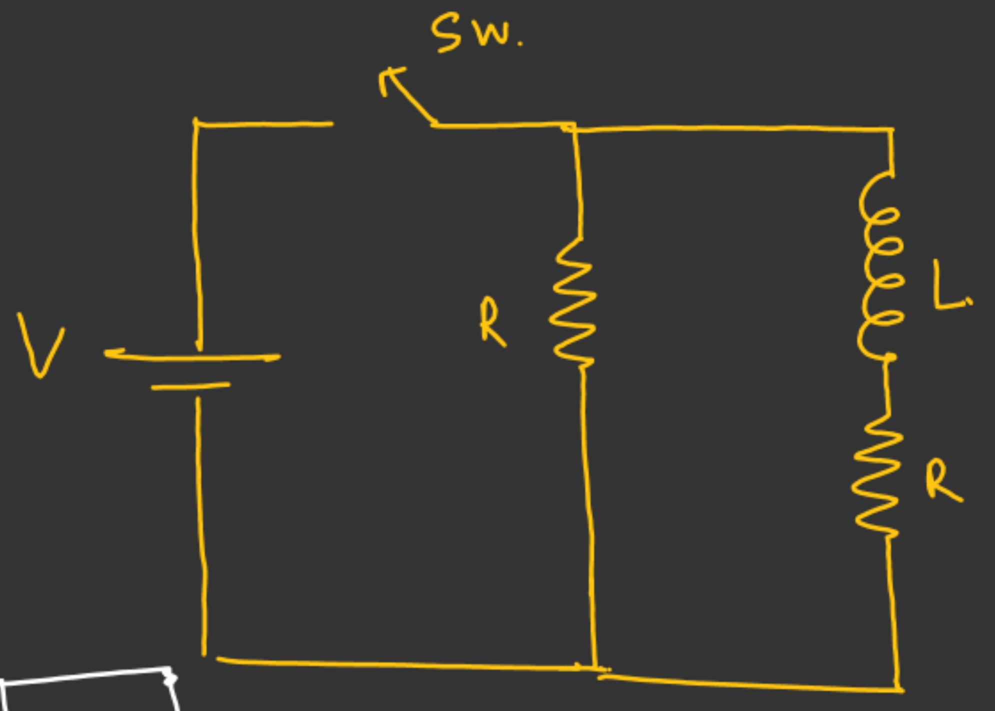
$L = 400\,mH$.

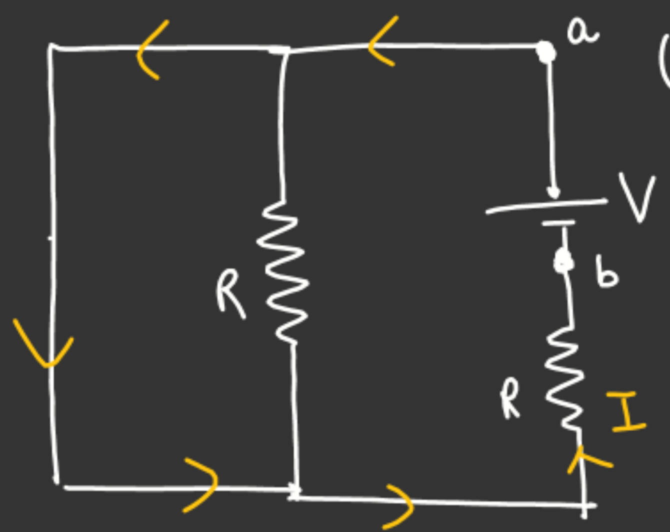
a) $\mathcal{E}_L = f(t)$.

$\tau = ??$

i_{max} in the inductor.

Inductor behave as zero resistance wire.



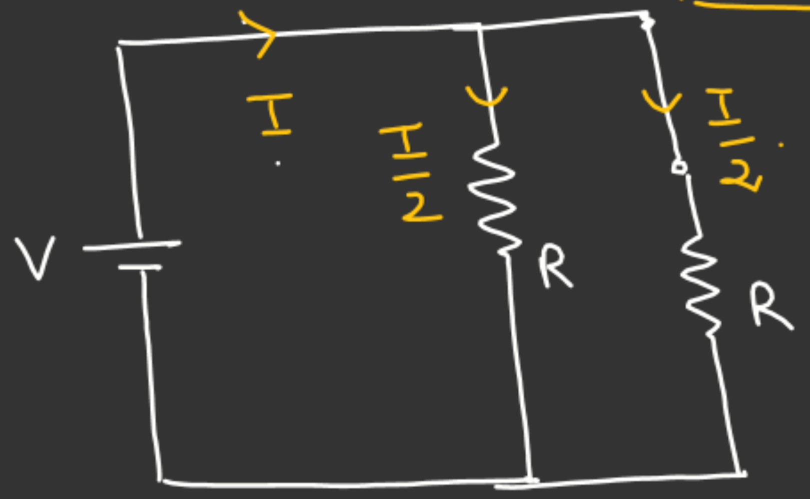


$(R_{eq})_{a-b} = ?$

$(R_{eq})_{a-b} = R$

$\tau = \dfrac{400 \times 10^{-3}}{2}$

$\tau = \dfrac{2}{10} = \dfrac{1}{5} = 0.2$ sec.



$I = \dfrac{V}{R/2} = \dfrac{2V}{R} = \dfrac{2 \times 12}{2} = 12$ Amp.

i_{max} in the inductor $= \underline{6 \text{ Amp.}}$ ✓.

[Current in the inductor as a function of time ← $i_L = 6(1 - e^{-5t})$

Nishant Jindal

$$i_L = 6\left(1 - e^{-5t}\right)$$

$$\mathcal{E}_L = -L\frac{d(i_L)}{dt}$$

$$\mathcal{E}_L = -L\left[-6\frac{d}{dt}\left(e^{-5t}\right)\right]$$

$$\mathcal{E}_L = +6L\left(e^{-5t}\right)(-5)$$

$$\mathcal{E}_L = -30L\left(e^{-5t}\right)$$

$$\mathcal{E}_L = -30 \times 400 \times 10^{-3}\, e^{-5t}$$

$$\mathcal{E}_L = -12e^{-5t}$$

$$|\mathcal{E}_L| = \left(12e^{-5t}\right) \checkmark$$

Nishant Jindal

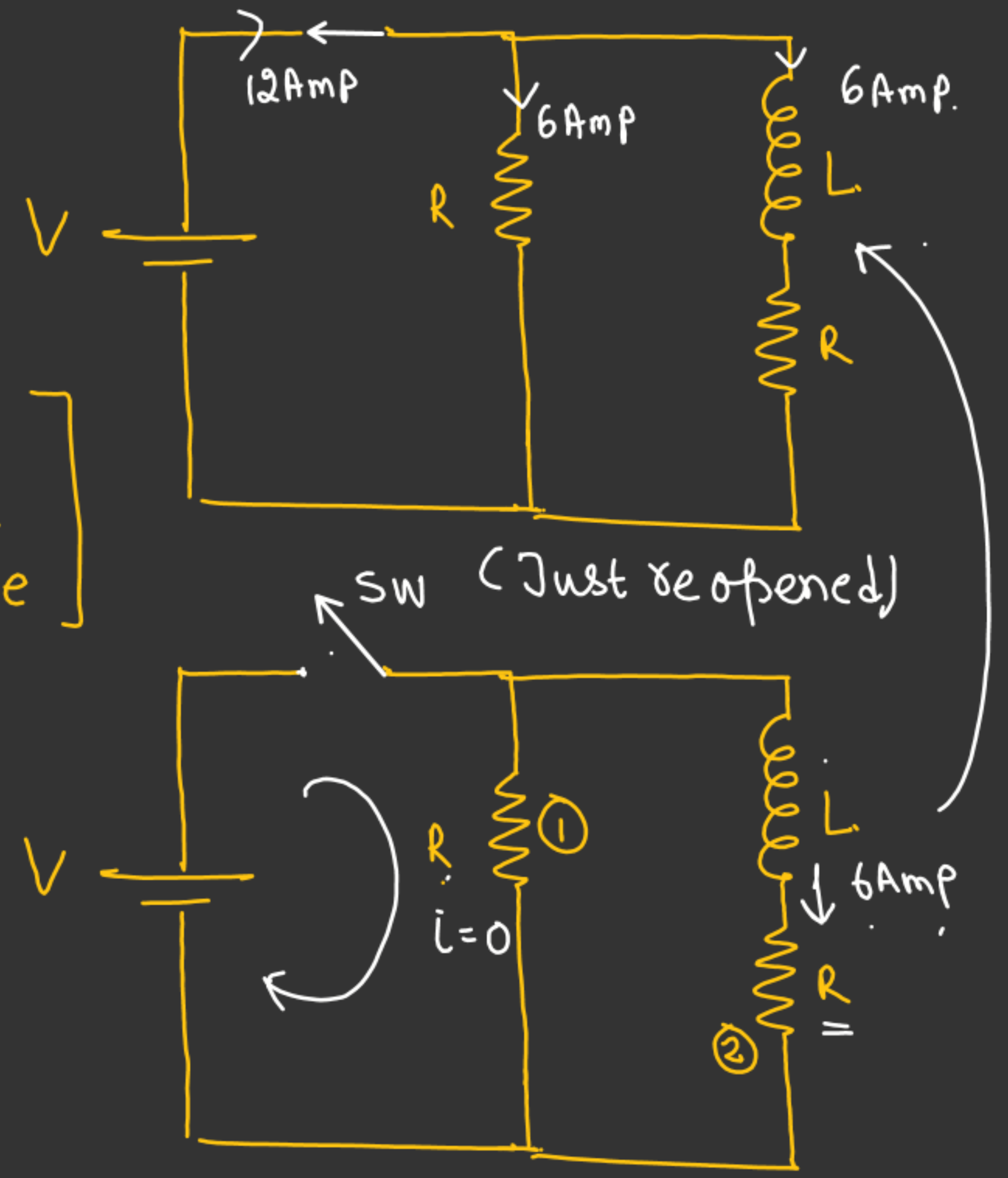
Imp

b) After steady achived, SW again opened.

i) find current in resistor just after SW is reopened. ✓

ii) find current in resistor as a function of time after SW reopened.

At the time of steady state.



Note:- [Inductor has a property of inertia so, just after SW open. the state of inductor is same as just before the SW opened.]

$R_1 = 0$, $R_2 = 6\text{Amp}.$

Nishant Jindal

(b) (ii) For decay of Current

$$i = i_0 e^{-t/\tau} \checkmark$$

i_0 = Current just after SW is reopend.

$i_0 = 6\,\text{Amp}.$

$$\tau = \frac{L}{2R} = \frac{400\times10^{-3}}{2\times2} = \left(\frac{1}{10}\right)$$

$$\boxed{i = 6e^{-10t}}$$