Nishant Jindal

248.

$$\lim_{n\to\infty} \frac{\cancel{n^3}^{\,n} \left(1 - \frac{100}{n} + \frac{1}{n^3}\right)}{\cancel{n^2}\left(100 + \frac{15}{n}\right)} = \infty$$

$$\lim_{n\to\infty} \frac{-n}{2(n+2)} = -\frac{1}{2}$$

260.

$$\frac{\frac{1}{1-\frac{1}{2}}}{\frac{1}{1-\frac{1}{3}}}$$

62.

$$\lim_{n\to\infty}\left(\frac{n(n+1)}{2(n+2)} - \frac{n}{2}\right) = \frac{n\left((\cancel{n}+1) - (\cancel{n}+2)\right)}{2(n+2)}$$

Nishant Jindal

265. $\lim_{n\to\infty} \frac{1}{1\cdot 3}+\frac{1}{3\cdot 5}+\frac{1}{5\cdot 7}+\cdots\frac{1}{(2n-1)(2n+1)}$

$$\sum_{r=1}^{n}\frac{1}{(2r-1)(2r+1)}$$

$$\frac{1}{2}\sum_{r=1}^{n}\frac{(2r+1)-(2r-1)}{(2r-1)(2r+1)}=\frac{1}{2}\sum_{r=1}^{n}\left(\frac{1}{2r-1}-\frac{1}{2r+1}\right)$$

$r=1 \swarrow$ $r=1$ $\quad$ $\swarrow r=n$

$$\frac{1}{2}\left(1-\frac{1}{2n+1}\right)$$

$$\boxed{\frac{1}{2}}$$

Nishant Jindal

270. $\lim_{x\to 1} \frac{x}{\underbrace{1-x}}$

LHL $= \frac{x}{\underbrace{1-x}_{0^+}} \to \infty$.

RHL $= \frac{x}{1-x} \to -\infty$.

$1-x \to 0^-$

279. $\frac{3(x^2-\cancel{4})+(x-4)^2}{3(x-1)(x-4)(x-2)} = \frac{4x^2-8x+4}{3(x-1)(x-2)(x-4)} = \frac{4(x-1)}{3(x-2)(x-4)} = 0$.

278. $\lim_{x\to 2}\left(\frac{1}{x(x-2)^2} - \frac{1}{(x-2)(x-1)}\right)$

$= \lim_{x\to 2} \frac{x-1-x(x-2)}{x(x-1)(x-2)^2} = \frac{3x-1-x^2}{x(x-1)(x-2)^2} \to \infty$

Nishant Jindal

86.

$$\lim_{x\to\infty} \frac{x^3(2x+1) - x^2(2x^2-1)}{(2x^2-1)(2x+1)}$$

$$= \lim_{x\to\infty} \frac{x^3+x^2}{(2x^2-1)(2x+1)} = \frac{1+\frac{1}{x}}{\left(2-\frac{1}{x^2}\right)\left(2+\frac{1}{x}\right)} = \frac{1}{4}$$

97

$$\lim_{x\to 1} \frac{\sqrt{x}\left(x^{3/2}-1\right)}{\sqrt{x}-1} = \lim_{x\to 1} \sqrt{x}\left(x+\sqrt{x}+1\right) = 3.$$

Nishant Jindal

91.

$$\frac{\sqrt[5]{x^7+3}+\sqrt[4]{2x^3-1}}{\sqrt[6]{x^8+x^7+1}-x}$$

$$=\lim_{x\to\infty}\frac{x^{\frac{7}{5}}\left(\sqrt[5]{1+\frac{3}{x^7}}+\frac{1}{x^{13/20}}\sqrt[4]{2-\frac{1}{x^3}}\right)}{x^{4/3}\left(\sqrt[6]{1+\frac{1}{x}+\frac{1}{x^8}}-\frac{1}{x^{1/3}}\right)}\to\infty.$$

Nishant Jindal

302. $\lim_{x\to 1} \dfrac{x^{\frac{1}{n}}-1}{x^{\frac{1}{m}}-1} = \lim_{x\to 1} \dfrac{\left(\dfrac{x^{\frac{1}{n}}-1^{\frac{1}{n}}}{x-1}\right)}{\dfrac{x^{\frac{1}{m}}-1^{\frac{1}{m}}}{x-1}} = \dfrac{\frac{1}{n}(1)^{\frac{1}{n}-1}}{\frac{1}{m}(1)^{\frac{1}{m}-1}} = \dfrac{m}{n}.$

$$\boxed{\lim_{x\to a} \frac{f(x)-f(a)}{x-a} = f'(a)}$$

$x = t^{mn}$

$$\lim_{t\to 1} \frac{t^m-1}{t^n-1} = \frac{(t-1)(1+t+t^2+\cdots+t^{m-1})}{(t-1)(1+t+t^2+\cdots+t^{n-1})}$$

Nishant Jindal

304. $\lim_{x\to 1} \frac{\sqrt[3]{7+x^3} - \sqrt{3+x^2}}{x-1}$

$$(7+x^3)^2 - (3+x^2)^3 = \left(14x^3 + 22 - 27x^2 - 9x^4\right) = (x-1)(-9x^3+5x^2-22x-22)$$

$3(3^2)x^2$

$$\lim_{x\to 1} \frac{(7+x^3)^2-(3+x^2)^3}{\left((7+x^3)^{5/3} + (7+x^3)^{4/3}(3+x^2)^{1/2} + \cdots\right)(x-1)}$$

$$\frac{-9+5-22-22}{2^5\times 6} = \frac{-48}{32\times 6} = -\frac{3}{2\times 6} = -\frac{1}{4}$$

$$\frac{(7+x^3)^{1/3}-2}{x-1} + \frac{2-\sqrt{3+x^2}}{x-1}$$

$$\frac{\cancel{x^3-1}\;\; x^2+x+1}{\cancel{(x-1)}\left((7+x^3)^{2/3}+2^2+2(7+x^3)^{1/3}\right)} + \frac{\cancel{1-x^2}\;\; -(1+x)}{\left(2+\sqrt{3+x^2}\right)\cancel{(x-1)}}$$

$$\frac{3}{12} - \frac{1}{2} = \frac{1}{4} - \frac{1}{2} = -\frac{1}{4}.$$

$$f(x) = (7+x^3)^{1/3} - (3+x^2)^{1/2}$$

$$\lim_{x\to 1} \frac{(7+x^3)^{1/3} - (3+x^2)^{1/2}}{x-1} = \lim_{h\to 0} \frac{(8+h^3+3h^2+3h)^{1/3} - (4+h^2+2h)^{1/2}}{h}$$

$x = 1+h$

$$2\left[\frac{\left(1+\frac{h^3+3h^2+3h}{8}\right)^{1/3} - \left(1+\frac{h^2+2h}{4}\right)^{1/2}}{h}\right]$$

$$2\left[\frac{\left(\cancel{1}+\frac{1}{3}\left(\frac{h^3+3h^2+3h}{8}\right)+\cdots\right) - \left(\cancel{1}+\frac{1}{2}\left(\frac{h^2+2h}{4}\right)+\cdots\right)}{h}\right] = 2\left(\frac{1}{8}-\frac{1}{4}\right) = \frac{1}{4}-\frac{1}{2} = -\frac{1}{4}$$

$$\lim_{x\to 1} \frac{f(x)-\cancel{f(1)}^{=0}}{x-1} = f'(1)$$

$$= \frac{1}{3}(7+x^3)^{-2/3}\,3x^2 - \frac{1}{2}(3+x^2)^{-1/2}(2x)\Big|_{x=1}$$

Nishant Jindal

$$= -\frac{c}{b}$$

305'

$$ax^2 + bx + c = 0$$

$$\frac{-4\cancel{a}c}{\left(\sqrt{b^2-4ac}+b\right)2\cancel{a}}$$

$a \to 0$

$$bx + c = 0$$

$$\boxed{x = -\frac{c}{b}}$$

$$\frac{-b+\sqrt{b^2-4ac}}{2a}$$

$\to \infty$

$$\frac{-b-\sqrt{b^2-4ac}}{2a}$$ < 0

$2a \to 0$

$\boxed{b > 0}$ ✓

Nishant Jindal

Sandwich / Squeeze Play Theorem

Let $f(x), g(x), h(x)$ satisfy inequality on interval $(a-h, a) \cup (a, a+h)$

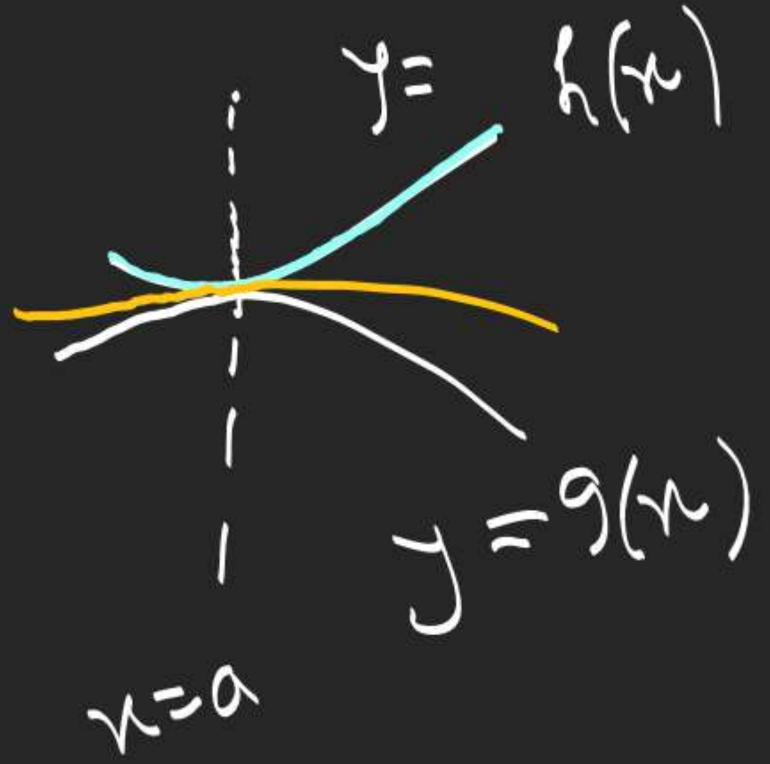
$$g(x) < f(x) < h(x)$$ and

$$\lim_{x \to a} g(x) = l, \quad \lim_{x \to a} h(x) = l.$$

$$\Rightarrow \lim_{x \to a} f(x) = l.$$

Nishant Jindal

$x \in (a-h, a) \cup (a, a+h)$



$g(x) \le f(x) \le h(x)$

$g(x) < f(x) < h(x)$

Nishant Jindal

1.

$$\lim_{x\to 0}\left(\underset{\downarrow\\0}{x^2}\ \underset{[-1,1]}{\underline{\cos\frac{1}{x}}}\right)=0$$

$$\lim_{x\to 0}\cos\frac{1}{x} \text{ not exist.}$$

$$-1\le \cos\frac{1}{x}\le 1$$

$$-x^2\le x^2\cos\frac{1}{x}\le x^2$$

$$\lim_{x\to 0}(-x^2)=0$$

$$\lim_{x\to 0}(x^2)=0$$

$$\Rightarrow \lim_{x\to 0}\left(x^2\cos\frac{1}{x}\right)=0$$

$[\cdot]$ = G.I.F

2.

$$\lim_{x\to 0}\left(\frac{x}{5}\left[\frac{2}{3x}\right]\right)$$

$\frac{1}{x}$ $\underline{100}$ $\underline{10}$

$$\lim_{x\to 0}\frac{x}{5}\left(\frac{2}{3x}-\left\{\frac{2}{3x}\right\}\right)=\lim_{x\to 0}\left(\frac{2}{15}-\frac{x}{5}\left\{\frac{2}{3x}\right\}\right)$$

$$=\frac{2}{15}-0=2/15$$

Nishant Jindal

$$\frac{x}{5}\left[\frac{2}{3x}\right]$$

$$\frac{2}{3x}-1 < \left[\frac{2}{3x}\right] \leq \frac{2}{3x}$$

$$\frac{x}{5}\left(\frac{2}{3x}-1\right) < \frac{x}{5}\left[\frac{2}{3x}\right] \leq \frac{2}{3x}\frac{x}{5}$$

$$\lim_{x \to 0}\left(\frac{2}{15}-\frac{x}{5}\right) = \frac{2}{15}$$

$$\lim_{x \to 0} \frac{2}{15} \to \frac{2}{15}$$

Nishant Jindal

3. $\lim_{n\to\infty}\left(\frac{n}{n^2+1}+\frac{n}{n^2+2}+\frac{n}{n^2+3}+\cdots+\frac{n}{n^2+n}\right)$

$\lim_{x\to a}\left(f_1 g_1 + f_2 g_2 + f_3 g_3 + \cdots f_n g_n\right)$

$\cancel{l_1 f_1 + l_2 f_2 + \cdots}$

$\frac{1}{n\left(1+\frac{1}{n^2}\right)} + \frac{1}{n\left(1+\frac{2}{n^2}\right)} + \frac{1}{n\left(1+\frac{3}{n^2}\right)}$

$\cancel{\frac{1}{n}+\frac{1}{n}+\frac{1}{n}+\cdots}$

$\lim(f+g) = \lim f + \lim g$

$\frac{n}{n} = 1$

Nishant Jindal

$$\frac{n}{n^2+n}+\frac{n}{n^2+n}+\dots+\frac{n}{n^2+n} < \frac{n}{n^2+1}+\frac{n}{n^2+2}+\frac{n}{n^2+3}+\dots\frac{n}{n^2+n} < \frac{n}{n^2+1}+\frac{n}{n^2+1}+\frac{n}{n^2+1}+\dots+\frac{n}{n^2+1}$$

$$\Rightarrow \boxed{\lim_{n\to\infty}\sum_{r=1}^{n}\frac{n}{n^2+r}=1}$$

$$\frac{n}{n^2+2} < \frac{n}{n^2+1}$$

$$\frac{n^2}{n^2+n} < \sum_{r=1}^{n}\frac{n}{n^2+r} < \frac{n^2}{n^2+1}$$

$$\frac{n}{n^2+1} > \frac{n}{n^2+n}$$

$$\lim_{n\to\infty}\frac{n^2}{n^2+n}=\lim_{n\to\infty}\frac{1}{1+\frac{1}{n}}=1$$

$$\lim_{n\to\infty}\frac{n^2}{n^2+1}=\lim_{n\to\infty}\frac{1}{1+\frac{1}{n^2}}=1$$

$$\boxed{\text{Ex-4 (ITF)}}$$