DPP - 7

SOLUTION

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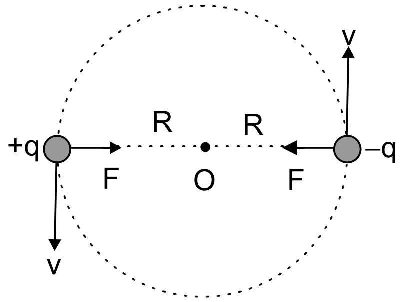
1. In this situation, the centripetal force is provided by the electrostatic force of attraction.

(a) By Newton's second law we get,



$$F = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{(2R)^2} = \frac{mv^2}{R}$$

$$\Rightarrow v = \sqrt{\frac{1}{4\pi\varepsilon_0}\frac{q^2}{(4R)m}}$$

(b) Kinectic energy of the system is $K = \frac{1}{2}mv^2 + \frac{1}{2}mv^2$

$$\Rightarrow K = \frac{q^2}{4\pi\varepsilon_0(4R)} = \frac{q^2}{16\pi\varepsilon_0 R}$$

(c) Potential energy of the system is $U = \frac{1}{4\pi\varepsilon_0}\frac{q(-q)}{2R}$

$$\Rightarrow U = \frac{-1}{4\pi\varepsilon_0}\frac{q^2}{(2R)} = -\frac{q^2}{8\pi\varepsilon_0 R}$$

(d) Total energy of the system is $E = K.E + P.E.$

$$\Rightarrow E = \frac{-q^2}{4\pi\varepsilon_0(4R)} = -\frac{q^2}{16\pi\varepsilon_0 R}$$

So, we observe that $T.E. = -(K.E) =.\frac{1}{2}(P.E)$

2. The work required to make an arrangement of charges is equal to potential energy of the system

$$\Rightarrow W = \frac{q^2}{4\pi\varepsilon_0 a} \times 3 = \frac{3q^2}{4\pi\varepsilon_0 a}$$

Work done by electric field equals the decrease in potential energy. So, we have

$$W_{\text{E Field}} = -\Delta U = U_i - U_f$$

Since $U_i = \frac{q^2}{4\pi\varepsilon_0(a)} \times 3$ and

$$U_f = \frac{q^2}{4\pi\varepsilon_0(2a)} \times 3$$

$$\Rightarrow W_{\text{E Field}} = U_f - U_f = \frac{3q^2}{4\pi\varepsilon_0 a} - \frac{3q^2}{8\pi\varepsilon_0 a} = \frac{3q^2}{8\pi\varepsilon_0 a}$$

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3. Interaction energy of any two point charges q_1 and q_2 at separation r is given by $\frac{q_1 q_2}{4\pi\varepsilon_0 r}$

Let interaction energy of the system in Figure (a), (b) and (c) be U_a, U_b and U_c respectively, then

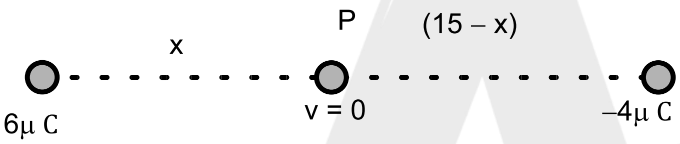
$$U_t = 4\frac{q^2}{4\pi\varepsilon_0 l} + 2\frac{q^2}{4\pi\varepsilon_0(\sqrt{2}l)} = \frac{q^2}{4\pi\varepsilon_0 l}(4+\sqrt{2})$$

$$U_b = 4\frac{-q^2}{4\pi\varepsilon_0 l} + 2\frac{q^2}{4\pi\varepsilon_0(\sqrt{2}l)} = \frac{q^2}{4\pi\varepsilon_0 l}(-4+\sqrt{2})$$

and $U_c = 2\frac{q^2}{4\pi\varepsilon_0 l} - \frac{2q^2}{4\pi\varepsilon_0 l} - \frac{2q^2}{4\pi\varepsilon_0(\sqrt{2}l)} = -\frac{\sqrt{2}q^2}{4\pi\varepsilon_0 l}$

4. CASE-1: Between the charges, let $V = 0$ at distance x from 6×10^{-6}C charge. So, we have

$$\frac{1}{4\pi\varepsilon_0}\left(\frac{6\times10^{-6}}{x} - \frac{4\times10^{-6}}{15-x}\right) = 0$$



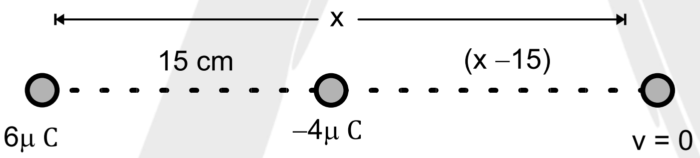
$\Rightarrow \frac{3}{x} = \frac{2}{15-x}$

$\Rightarrow x = 9$ cm

Outside the charges, let $V = 0$ at distance x from 6×10^{-6}C charge. So, we have

$$\frac{1}{4\pi\varepsilon_0}\left(\frac{6\times10^{-6}}{x} - \frac{4\times10^{-6}}{x-15}\right) = 0$$

$\Rightarrow x = 45$ cm

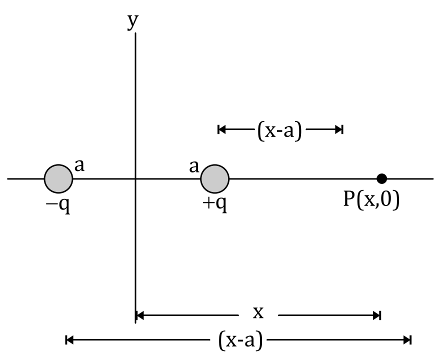


Note that the formula for potential used in the calculation required choosing potential to be zero at infinity.

5. The electric potential can be found by the superposition principle. At a point P(x, 0) on the x-axis, we have

$$V(x) = \frac{1}{4\pi\varepsilon_0}\frac{q}{|x-a|} + \frac{1}{4\pi\varepsilon_0}\frac{(-q)}{|x+a|}$$

$$\Rightarrow V(x) = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{|x-a|} - \frac{1}{|x+a|}\right]$$



The above expression is rewritten as

$$\frac{V(x)}{V_0} = \frac{1}{\left|\frac{x}{a}-1\right|} - \frac{1}{\left|\frac{x}{a}+1\right|}$$

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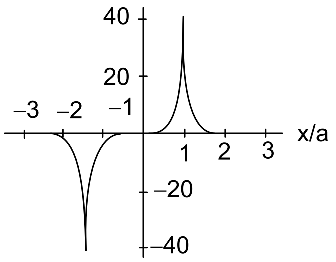
where $V_0 = \frac{q}{4\pi\varepsilon_0 a}$. The plot of the dimensionless electric potential as a function of $\frac{x}{a}$ is shown in figure.



As can be seen from the graph, V(x) diverges at $\frac{x}{a} = \pm 1$, where the charges are located.

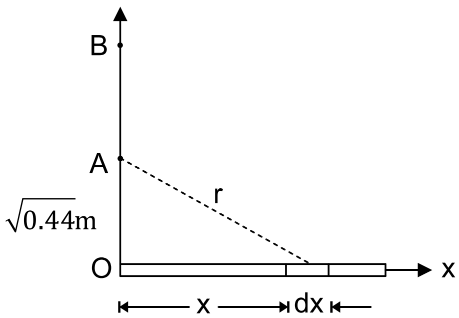
6. Let us calculate the potential due to the rod at A and then at B.

Potential at $A(0, \sqrt{0.44})$m

Potential at A due to rod is calculated by taking an infinitesimal element of length dx at a distance x from O(0,0) on the rod. Then

$$dV = \frac{dq}{4\pi\varepsilon_0 r} = \frac{\lambda dx}{4\pi\varepsilon_0\sqrt{OA^2 + x^2}}$$

$$\Rightarrow dV = \frac{kxdx}{4\pi\varepsilon_0\sqrt{0.44 + x^2}}$$



$$\Rightarrow \int_{V_0}^{V_4} dV = \frac{k}{4\pi\varepsilon_0}\left[\sqrt{0.44 + x^2}\Big|_0^{l=1\text{ m}}\right]$$

$$\Rightarrow V_A - V_0 = \frac{k}{4\pi\varepsilon_0}[\sqrt{1.44} - \sqrt{0.44}]$$

$OA = \sqrt{0.44}$ m = constant

OB = 1 = 1 m = constant

Potential at B(0,1)m

$$dV = \frac{dq}{4\pi\varepsilon_0 r'} = \frac{kxdx}{4\pi\varepsilon_0\sqrt{OB^2 + x^2}} = \frac{kxdx}{4\pi\varepsilon_0\sqrt{1 + x^2}}$$

$$\Rightarrow V_B - V_0 = \frac{k}{4\pi\varepsilon_0}\left[\sqrt{1 + x^2}\Big|_0^1\right]$$

$$\Rightarrow V_B - V_0 = \frac{k}{4\pi\varepsilon_0}(\sqrt{2} - 1)$$

Now, by definition, $W_{A\to B} = q(V_B - V_A)$

$$\Rightarrow W_{A\to B} = W = \frac{qk}{4\pi\varepsilon_0}[\sqrt{2} - 1 - \sqrt{1.44} + \sqrt{0.44}]$$

(Please observe, that V_0 will cancel in the process)

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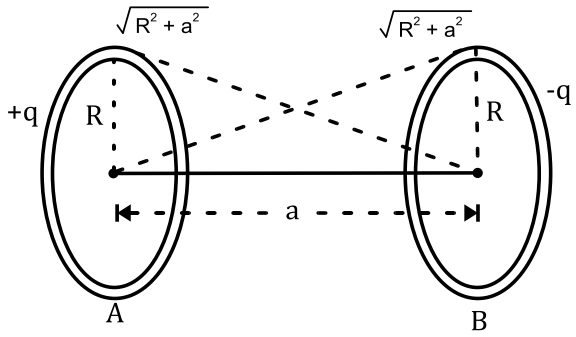
$$\Rightarrow W_{A\to B} = \frac{qk}{4\pi\varepsilon_0}[1.414 - 1 - 1.2 + 0.66]$$

$$\Rightarrow W_{A\to B} = 9\times10^9 \times 1000 \times 10^{-6} \times 10^{-9}[-0.126]$$

$$\Rightarrow W_{A\to B} = -1.1\times10^{-3}\ \text{J}$$

7. Net potential at centre of ring A is



$$V_A = \begin{pmatrix}\text{Potential at A}\\ \text{due to itself}\end{pmatrix} + \begin{pmatrix}\text{Potential at A}\\ \text{due to B}\end{pmatrix}$$

$$\Rightarrow V_A = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{R} + \frac{(-q)}{\sqrt{R^2+a^2}}\right)$$

$$\Rightarrow V_A = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{R} - \frac{1}{\sqrt{R^2+a^2}}\right]$$

Similarly net potential at centre of ring B is

$$V_B = \begin{pmatrix}\text{Potential at B}\\ \text{due to itself}\end{pmatrix} + \begin{pmatrix}\text{Poteniial at B}\\ \text{due to A}\end{pmatrix}$$

$$\Rightarrow V_B = \frac{q}{4\pi\varepsilon_0}\left[-\frac{1}{R} + \frac{1}{\sqrt{R^2+a^2}}\right]$$

Thus potential difference,

$$\Delta V = V_B - V_A$$

$$\Rightarrow \Delta V = \frac{q}{4\pi\varepsilon_0}\left[-\frac{1}{R} + \frac{1}{\sqrt{R^2+a^2}} - \frac{1}{R} + \frac{1}{\sqrt{R^2+a^2}}\right]$$

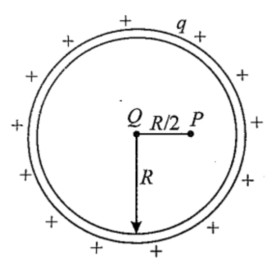
$$\Rightarrow \Delta V = \frac{2q}{4\pi\varepsilon_0}\left[\frac{1}{\sqrt{R^2+a^2}} - \frac{1}{R}\right]$$

$$\Rightarrow \Delta V = \frac{q}{2\pi\varepsilon_0}\left[\frac{1}{\sqrt{R^2+a^2}} - \frac{1}{R}\right]$$

Since, $W_{A\to B} = q_0(V_B - V_A)$

$$\Rightarrow W_{A\to B} = \frac{qq_0}{2\pi\varepsilon_0}\left[\frac{1}{\sqrt{R^2+a^2}} - \frac{1}{R}\right]$$

8. Electric potential at point P is given as



$$V_P = V_{QP} + V_{qP}$$

$$\Rightarrow V_P = \frac{KQ}{\left(\frac{R}{2}\right)} + \frac{Kq}{R}$$

$$\Rightarrow V_P = \frac{2Q}{4\pi\in_0 R} + \frac{q}{4\pi\in_0 R}$$

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9. At the point of closest approach, the KE of particle transforms into the PE of system. Thus by conservation c energy, we have

$$\Rightarrow \frac{1}{2}mv^2 + 0 = 0 + \frac{KqQ}{r} \Rightarrow r = \frac{2KqQ}{mv^2}$$

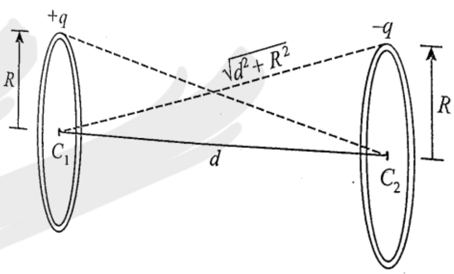
$$\Rightarrow r \propto \frac{1}{v^2}$$

If q was given a speed 2v then now closest approach distance will be

$$\frac{r_1}{r_2} = \left(\frac{v_2}{v_1}\right)^2 \Rightarrow r_2 = \left(\frac{v_1}{v_2}\right)^2 r_1 = \left(\frac{v_1}{2v}\right)^2 r = \frac{r}{4}$$

10. Electric potential at point C_2 is

$$V_{C_2} = \left(+\frac{Kq}{\sqrt{d^2+R^2}}\right) + \left(-\frac{Kq}{R}\right)$$



Electric potential at point C_1 is

$$V_{C_1} = V_{+q} + V_{-q}$$

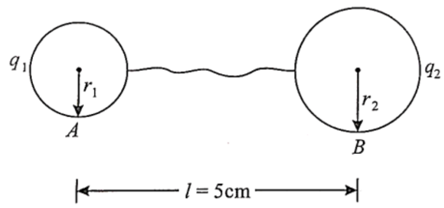
$$\Rightarrow V_{C_1} = \frac{Kq}{R} + \left(-\frac{Kq}{\sqrt{d^2+R^2}}\right)$$

Potential difference between centres of rings is

$$\Delta V = V_{C_1} - V_{C_2} = \frac{q}{2\pi\epsilon_0}\left(\frac{1}{R} - \frac{1}{\sqrt{d^2+R^2}}\right)$$

11. We can ignore induction effects on spheres due to charge on other sphere as separation $l > r_1, r_2$

If q_1 and q_2 are charges on A and B after connection, we have



$$V_A = V_B$$

$$\frac{Kq_1}{r_1} = \frac{Kq_2}{r_2}$$

Electric field magnitude on surfaces of A and B are

$$E_A = \frac{Kq_1}{r_1^2} \text{ and } E_B = \frac{Kq_2}{r_2} \Rightarrow \frac{E_A}{E_B} = \frac{r_2}{r_1} = \frac{2}{\cdot 1}$$

12. Due to symmetry $V_0 = 0$

At O net EF is in direction as shown in figure-1

After interchanging charges as given in que

Still $V_0 = 0$ (remain unchanged)

As shown in figure-2 direction of $\vec{E}$ is reversed.

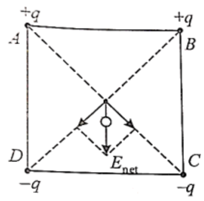


Figure-1

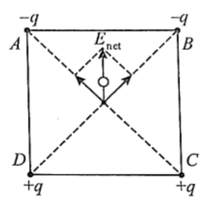


Figure-2

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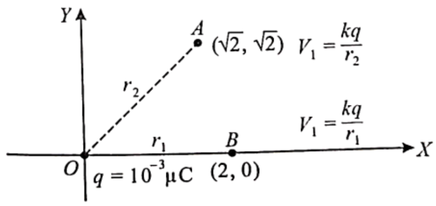
13. $r_1 = \sqrt{2^2 + 0^2} = 2$



$r_2 = \sqrt{(\sqrt{2})^2 + (\sqrt{2})^2} = 2$

As $r_1 = r_2$ so electric potentials at points A and B are equal

$\Rightarrow V_A = V_B$

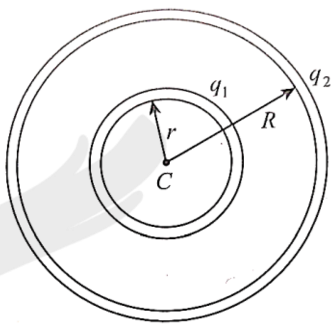
14. Work done in moving a charge q from point P to Q is given as

$$W = q(V_Q - V_P)$$

$$\Rightarrow \quad W = [100 \times (-1.6 \times 10^{-19})] \times [-4 - (10)]$$

$$\Rightarrow \quad W = 2.24 \times 10^{-16}\ J$$

15. As the surface charge densities of the two shells are along equal



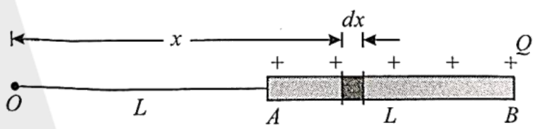
$$\Rightarrow \frac{q_1}{4\pi r^2} = \frac{q_2}{4\pi R^2} = \frac{Q}{4\pi(r^2 + R^2)}$$

Electric potential at common centre is given as

$$V_C = \frac{Kq_1}{r} + \frac{Kq_2}{R}$$

$$\Rightarrow V_C = \frac{1}{4\pi\epsilon_0}\left[\frac{Qr^2}{r(r^2 + R^2)} + \frac{QR^2}{R(r^2 + R^2)}\right] \Rightarrow V_C = \frac{(r + R)Q}{4\pi \in_0 (r^2 + R^2)}$$

16. Charge on element



$$dq = \frac{Q}{L}dx$$

Potential at O due to dq is given as

$$dV = \frac{Kdq}{x} = \frac{KQ}{Lx}dx$$

Potential at O due to whole rod AB is

$$V = \int dV = \int_{x=L}^{x=2L} \frac{KQ}{Lx}dx$$

$$\Rightarrow V = \frac{KQ}{L}[\ln\ x]_L^{2L} = \frac{KQ}{L}[\ln\ 2\,L \ln\ L]$$

$$\Rightarrow V = \frac{KQ}{L}\ln(2) = \frac{Q}{4\pi \in_0 L}\ln(2)$$

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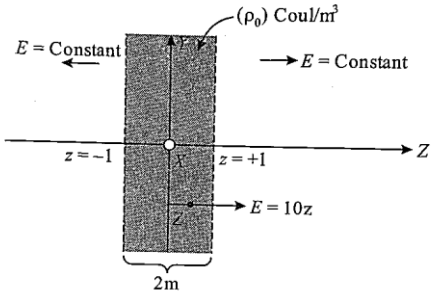
17. For $|Z| \leq \Big|m \;\; \vec{E} = -\frac{dV}{dZ}\hat{k} = 10\Big|Z\mid\hat{k}.\quad -1\text{ m} < Z < 1\text{ m}$ and for $|Z| \geq 1\text{ m}\ \vec{E} = -\frac{dV}{dZ}\hat{k} = \pm 10\hat{k}$

$Z > 1$ m and $Z < -1$ m constant EF



By analyzing the EF we can state that it is produced by a uniform charge distribution in a thick sheet of width 2 m as shown.

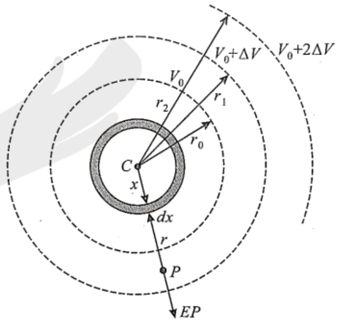
Due to a thick volume charged sheet, inside at a distance Z from central plane the EF is given as

$$E_{in} = \frac{\rho z}{\epsilon_0} = 10z$$

$\Rightarrow \rho_0 = 10\ \epsilon_0$ for $|z| \leq 1$ m $\Rightarrow \rho_0 = 0$ for $|z| \geq 1$ m

18. $r_2 - r_1 = r_1 - r_0 = r_N - r_{N-1}$

$\Delta V\ \Delta V\ \Delta V$ Potential differences



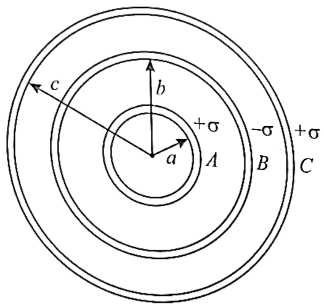
Only in uniform EF the separation betwe having same ΔV is constant.

To calculate EF at a distance r from centre of charge distribution then it is given by

$$E_P = \frac{kq}{r^2}$$

Where q → charge enclosed within sphere of rad $r = 0$ to $r = r$.

19. Charges on shells A, B and C are



$$q_A = +\sigma \times 4\pi a^2$$
$$q_B = -\sigma \times 4\pi b^2$$
$$q_C = -\sigma \times 4\pi e^2$$

'Potential of shell B due to all the shell charges is given as

$$V_B = V_{q_A} + V_{q_B} + V_{q_C}$$

$$\Rightarrow V_B = \frac{Kq_A}{b} + \frac{Kq_B}{b} + \frac{Kq_C}{C}$$

$$\Rightarrow V_B = \frac{1}{4\pi\epsilon_0}\left(\frac{\sigma 4\pi a^2}{b} - \frac{\sigma 4\pi b^2}{b} + \frac{\sigma 4\pi c^2}{c}\right)$$

$$\Rightarrow V_B = \frac{\sigma}{\epsilon_0}\left(\frac{a^2}{b} - b + c\right) = \frac{\sigma}{\epsilon_0}\left(\frac{a^2 - b^2}{b} + c\right)$$

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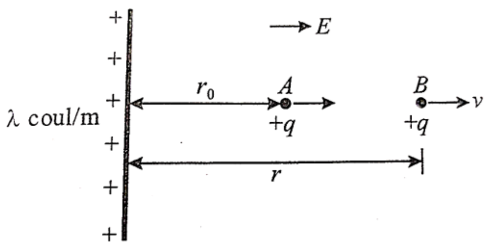
20. By work energy theorem, we use

$$\frac{1}{2}mv^2 = -q(V_A - V_B)$$



$$\Rightarrow V_A - V_B = \int_{r_0}^{r} E \cdot dr = \int_{r_0}^{r} \frac{\lambda}{2\pi\epsilon_0 r} dr = \frac{\lambda}{2\pi\epsilon_0}[\ln\ r]_{r_0}^{r}$$

$$\Rightarrow V_A - V_B = \frac{\lambda}{2\pi\epsilon_0}\ln\left(\frac{r}{r_0}\right)$$

From equation-(1), we use

$$\frac{1}{2}mv^2 = q\left[\frac{\lambda}{2\pi\ \epsilon_0}\ln\left(\frac{r}{r_0}\right)\right]$$

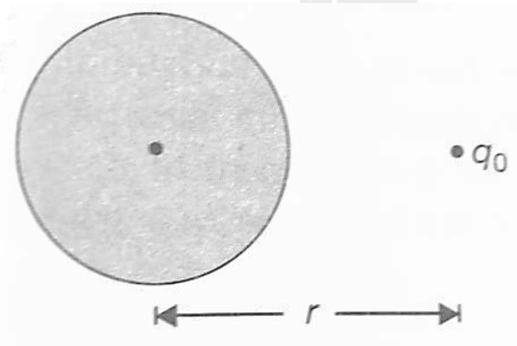
$$\Rightarrow v \propto \sqrt{\ln\left(\frac{r}{r_0}\right)}$$

21. Statement-1 is also practical experience based; so it is true.

Statement-2 is also true but is not the correct explanation of Statement-1. Correct explanation is "there is increase in normal reaction when the object is pushed and there is decrease in normal reaction when object is pulled".

Hence, the correct answer is (B).

22. Electric field due to q_0 is towards left and is $\frac{q_0}{4\pi\varepsilon_0 r^2}$ but electric field due to induced charge is towards right and will have same magnitude $\frac{q_0}{4\pi\varepsilon_0 r^2}$ so that electric field inside the sphere is zero.



Hence, the correct answer is (D).