Nishant Jindal

Rolle's Thm Qs MindMap

3 Types Qs

Eqⁿ + Eqⁿ [a, b]	Eqⁿ + Condⁿ [a, b]	Only one Eqⁿ [a, b]
① Check which one is fxn & which one is derivate. (2) Apply Root Thry	① $\int$ Eqⁿ = f(x) Assume (2) Apply Root Thry Using Condⁿ	① $\int$ Eqⁿ = f(x) Assume ② Apply Root Thry

Q If $\boxed{a+b+c=0}$ then S.T.

Condⁿ + Eqⁿ

$3ax^2+2bx+c=0$ has atleast one root in $[0,1]$ ← ये f'(x) होगा

let $f(x) = \int 3ax^2+2bx+c \cdot dx$

$= \cancel{3}a\frac{x^3}{\cancel{3}} + \cancel{2}b\cdot\frac{x^2}{\cancel{2}} + cx$

$f(x) = ax^3+bx^2+cx$

$[0,1]$ $f(0) = 0$

$f(1) = a+b+c = 0$

Acc. to Root Thry $f(x) = ax^3+bx^2+cx$ & $f'(x) = 3ax^2+2bx+c$ & betⁿ 2 Roots of $f(x)$ $\exists$ atleast one Root of $f'(x)$ will lie.

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Q If $\underline{2a+3b+6c=0}$ $a,b,c \in R$

Cond^n + Eq^n

Then S.T. Eq^n $\underline{ax^2+bx+c=0} \rightarrow \frac{d}{dx} f(x)$

has atleast one Root in bet^n $\underline{0 \& 1}$.

① let $f(x) = \int ax^2+bx+c\,dx$

$$= \frac{ax^3}{3} + \frac{bx^2}{2} + cx$$

② $f(0) = 0$

$$f(1) = \frac{a}{3} + \frac{b}{2} + c = \frac{2a+3b+6c}{6} = 0$$

$\Rightarrow$ Bet^n Root of $f(x) = [0,1]$

$\exists$ atleast one Root of $f'(x)$

$= ax^2+bx+c$ will lie.

MONOTONOCITY

Q. If $\frac{C_0}{1}+\frac{C_1}{2}+\frac{C_2}{3}=0$, where $C_0 C_1 C_2$ are all real, then the quadratic equation

Condn + Eqn

$C_2x^2+C_1x+C_0=0$ has

(A) at least one root in $(0,1)$

(B) one root in $(1,2)$ and the other in $(3,4)$

(C) one root in $(-1,1)$ and the other in $(-5,-2)$

(D) both roots imaginary

$$f(x)=\int C_2x^2+C_1x+C_0\,dx$$

option try $(0,1)$

$$f(x)=C_2\frac{x^3}{3}+C_1\frac{x^2}{2}+C_0x$$

$$\left.\begin{aligned} f(0)&=0\\ f(1)&=\frac{C_2}{3}+\frac{C_1}{2}+C_0=0 \end{aligned}\right\}$$

Betn $(0,1)$ $\exists$ atleast one Root of $f'(x)=C_2x^2+C_1x+C_0$ will lie.

MONOTONOCITY

Q. If $\frac{a_0}{n+1} + \frac{a_1}{n} + \frac{a_2}{n-1} + \cdots + \frac{a_{n-1}}{2} + a_n = 0$, then the equation $a_0x^n + a_1x^{n-1} + \cdots + a_{n-1}x + a_n = 0$ - has, in the interval $(0, 1)$,

(Cond + Eqn)

(A) exactly one root

~~(B)~~ at least one root

(C) at most one root

(D) No root.

① $f(x) = \int a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \cdots + a_{n-1}x + a_n$

$f(x) = a_0\frac{x^{n+1}}{n+1} + \frac{a_1x^n}{n} + \frac{a_2x^{n-1}}{n-1} + \cdots + \frac{a_{n-1}x^2}{2} + a_nx$

$f(0) = 0$, $f(1) = \frac{a_0}{n+1} + \frac{a_1}{n} + \frac{a_2}{n-1} \cdots + a_n = 0$

Betn $(0,1)$ $\exists$ atleast 1 Root of $f'(x) = a_0x^n + a_1x^{n-1} \cdots a_n =$ will be.

MONOTONOCITY

Eqn + Eqn

Q. If the polynomial equation $\overset{f(x)}{a_nx^n + a_{n-1}x^{n-1} + \cdots + a_2x^2 + a_1x + a_0} = 0$; n positive integer, has two different real roots α and β, then between α and β, the equation $a_nx^{n-1} + (n-1)a_{n-1}x^{n-2} + \cdots + a_1 = 0$ has

(A) exactly one root

(B) at most one root

(C) at least one root

(D) No root

$f(x) = a_nx^{n-1} + a_{n-1}x^{n-1} + \cdots + a_2x^2 + a_1x + a_0 < \begin{matrix}\alpha \\ \beta\end{matrix}$

$f'(x) = a_nx^{n-1} + (n-1)a_{n-1}x^{n-2} + \cdots + a_1$

Acc to Root Try $\exists$ atleast one Root of $f'(x)$ will lie.

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Q let $f''(x)$ exists $\forall x \in R$

& $f(x_1) = f(x_2) = f(x_3)$ where

$x_1 < x_2 < x_3$ Show that

$f''(c) = 0$ for $c \in (x_1, x_3)$

① $f''(x)$ exists

A) fxn $f(x)$ Cont^s & diff^{ble}

B) $f'(x)$ also Cont^s & diff^{ble}

$f(x)$ in (x_1, x_2)

① ✓ ② ✓

3) $f(x_1) = f(x_2)$

RMVT → $f'(c_1) = 0$

$c_1 \in (x_1, x_2)$

$f(x)$ in (x_2, x_3)

① ✓ ② ✓

(3) $f(x_2) = f(x_3)$

RMVT → $f'(c_2) = 0$

$c_2 \in (x_2, x_3)$

$\boxed{f'(x)}$ in (x_1, x_3)

① ✓ ② ✓

③ $f(c_1) = f(c_2) = 0$

RMVT $f''(c) = 0$ $c \in (c_1, c_2)$

$f''(c) = 0$ in $c \in (x_1, x_3)$

[J.1.P.]

Q For a twice diff^{ble} fxn

$f: R \to R$ with

$f(0) = f(1) = f'(0) = 0$

A) $f''(x) = 0$ at some pt. $x \in (0,1)$

B) $f''(0) = 0$

(C) $f''(x) \neq 0$ at every pt $x \in (0,1)$

(d) $f''(x) = 0$ $x \in (0,1)$

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① $f(x), x \in (0,1)$

① Cont ✓ ② diff

③ $f(0) = f(1)$

$c_1 \in (0,1) \to f'(c_1) = 0$

$f'(x)$

② $f'(x)$

① Cont^s ✓ ② diff ✓

③ $f'(0) = f'(c_1) = 0$

$c \in (0, c_1) \Rightarrow f''(c) = 0$

$\Rightarrow f''(x) = 0$ at some pt in $(0,1)$

Q For a twice diff^{ble} fxn $f: R \to R$ with $f(0) = f(1) = f'(0) = 0$

3) A) $f''(x) = 0$ at some pt $x \in (0,1)$

B) $f''(0) = 0$

(C) $f''(x) \neq 0$ at everypt $x \in (0,1)$

(d) $f''(x) = 0$ $x \in (0,1)$

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Lagrange's Mean Value Thm.

If $f(x)$ in Interval $[a,b]$ satisfy following 2 cond^n

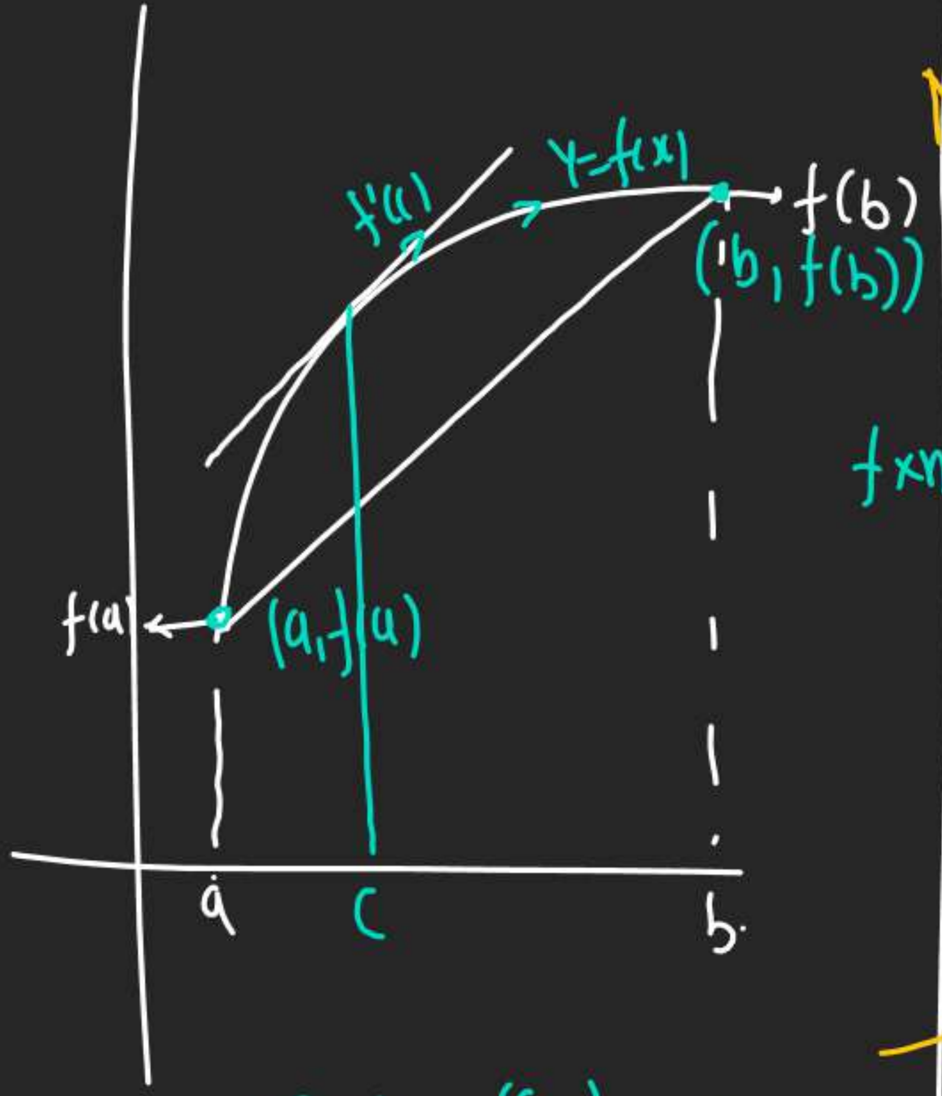
① $f(x)$ is conts in $[a,b]$

② $f(x)$ is diff^{ble} in (a,b)

then acc to LMVT $\exists$ atleast one pt. $x=c$ where

$$f'(c)=\frac{f(b)-f(a)}{b-a}$$

When $f(a)=f(b)$ then $f'(c)=0$ [R MVT]



$(Sl)_T=(Sl)_{chord}$

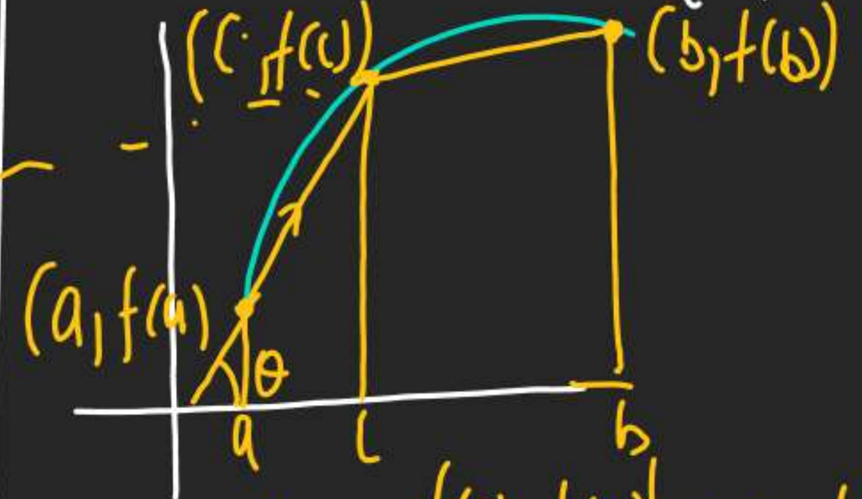
$f'(c)=\frac{f(b)-f(a)}{b-a}$

Q let f be any fxn on $[a,b]$ & twice diff^{ble} on (a,b). If for all $x\in(a,b)$ $f'(x)>0$, $f''(x)<0$ then for any $c\in(a,b)$, $\frac{f(c)-f(a)}{f(b)-f(c)}$ is gr. then.

Mains

fxn ↑   con. down

~~(A)~~ $\frac{c-a}{b-c}$ (B) $\frac{b-c}{c-a}$ (C) $\frac{b+c}{c-a}$ (D) 1



$\tan\theta=\frac{f(c)-f(a)}{c-a}$ | $\tan\phi=\frac{f(b)-f(c)}{b-c}$

$\theta>\phi$

$\tan\theta>\tan\phi$

$\frac{f(c)-f(a)}{c-a}>\frac{f(b)-f(c)}{b-c}$ → $\frac{f(c)-f(a)}{f(b)-f(c)}>\frac{c-a}{b-c}$

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Q Verify LMVT for

$f(x) = -x^2+4x+5,\ x \in [-1,1]$

① $f(x)$ = Poly ⇒ Conts & diff.

2) LMVT Satisfied then

Acc. to LMVT ∃ atleast one pt c such that

$$f'(c) = \frac{f(1)-f(-1)}{1-(-1)}$$

$$-2c+4 = \frac{\cancel{8-0}\ 4}{\cancel{2}}$$

$c=0 \in [-1,1]$

So LMVT Verified.

Q Find c Using LMVT

$f(x) = x^3-4x^2+8x+11$

$x \in [0,1]$.

① ✓ ② ✓ LMVT Satisfied

$$f'(c) = \frac{f(1)-f(0)}{1-0}$$

$3c^2-8c+8 = 16-11$

$3c^2-8c+3=0$

$$c = \frac{8 \pm \sqrt{64-36}}{6}$$

$$= \frac{8 \pm 2\sqrt{7}}{6}$$

$$= \frac{4+\sqrt{7}}{3},\ \frac{4-\sqrt{7}}{3}$$

✗ Not in (0,1) ✓

Q $f(x) = \begin{cases} \dfrac{\sin x}{x} & x \neq 0 \\ 1 & x = 0 \end{cases}$ ($\lim_{x\to 0} \frac{\sin x}{x} = 1$)

LMVT applicable or not.

① Conts

② $$f'(x) = \lim_{x\to 0} \frac{\frac{\sin x}{x} - 1}{1-0}$$

$$= \lim_{x\to 0} \frac{\sin x - x}{x} \quad \frac{0}{0}$$

$$= \lim_{x\to 0} \frac{\cos x - 1}{1} = 0$$

diff^ble

LMVT applicable ✓

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Q Let $f(x)$ be a twice diff^{ble} fxn on $(1,6)$

$f(2)=8$, $f'(2)=5$, $\boxed{f'(x)\geq 1}$, $\boxed{f''(x)\geq 4}$

$\forall x \in (1,6)$ then.

A) $f(5)+f'(5)\geq 28$ ✓

B) $f(5)+f'(5)\leq 26$

C) $f'(5)+f''(5)\leq 20$

D) $f(5)\leq 10$

$f'(x)\geq 1$ $\quad (2,5)\in(1,6)$

$$\frac{f(5)-f(2)}{5-2}\geq 1$$

$f(5)-8\geq 3$

$f(5)\geq 11$

$f(5)+f'(5)\geq 28$

$f''(x)\geq 4$

$$\frac{f'(5)-f'(2)}{5-2}\geq 4$$

$f'(5)-5\geq 12$

$f'(5)\geq 17$

Cauchy's MVT.

If $f(x)$ & $h(x)$ are 2 fxn in $[a,b]$ such that

(1) Both are cont^{s} in $[a,b]$

(2) Both are diff in (a,b)

then acc to CMVT. $\exists$ a pt c

Such that $\dfrac{f'(c)}{h'(c)}=\dfrac{f(b)-f(a)}{h(b)-h(a)}$

MONOTONOCITY

Q. $f(x)$ and $g(x)$ are differentiable functions for $0 \le x \le 2$ such that $f(0) = 5, g(0) = 0, f(2) = 8, g(2) = 1$. Show that there exists a number c satisfying $0 < c < 2$ and $f'(c) = 3\,g'(c)$.

$$\frac{f'(c)}{g'(c)} = \frac{f(2)-f(0)}{g(2)-g(0)} = \frac{8-5}{1-1} = 3$$

$$f'(c) = 3g'(c)$$

MONOTONOCITY

Q. If the function $f(x) = x^3 - 6ax^2 + 5x$ satisfies the conditions of Lagrange's mean theorem for the interval $[1, 2]$ and the tangent to the curve $y = f(x)$ at $x = 7/4$ is parallel to the chord joining the points of intersection of the curve with the ordinates $x = 1$ and $x = 2$. Then the value of a is

(A) $35/16$

(B) $35/48$

(C) $7/16$

(D) $5/16$

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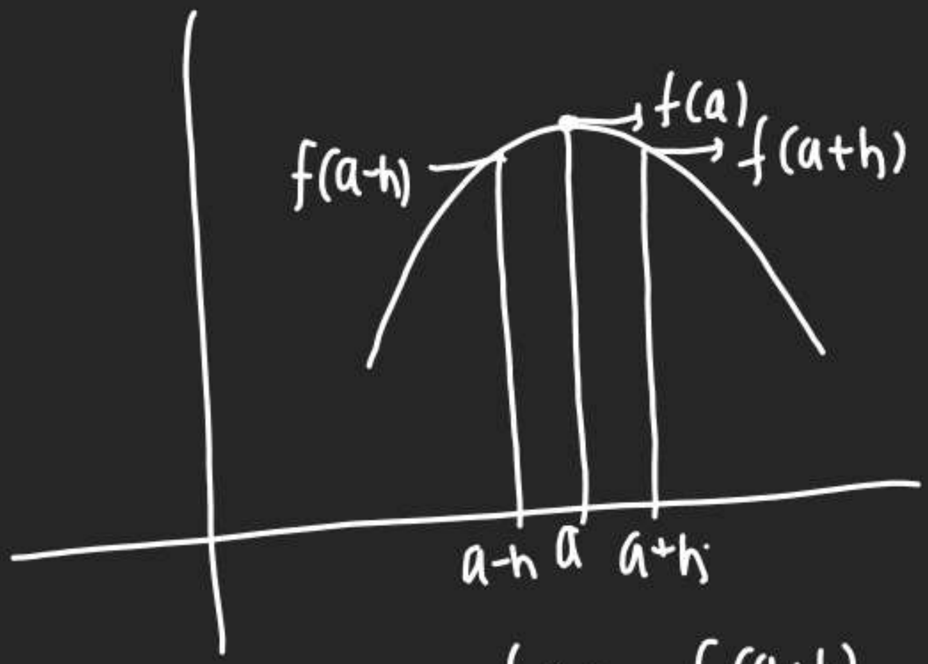
Maxima & Minima (In ABD this most Qs generating chapter)

1) Maxima & Minima Pts are also known as Pt of Extrema

① Local Maxima = Relative Max.

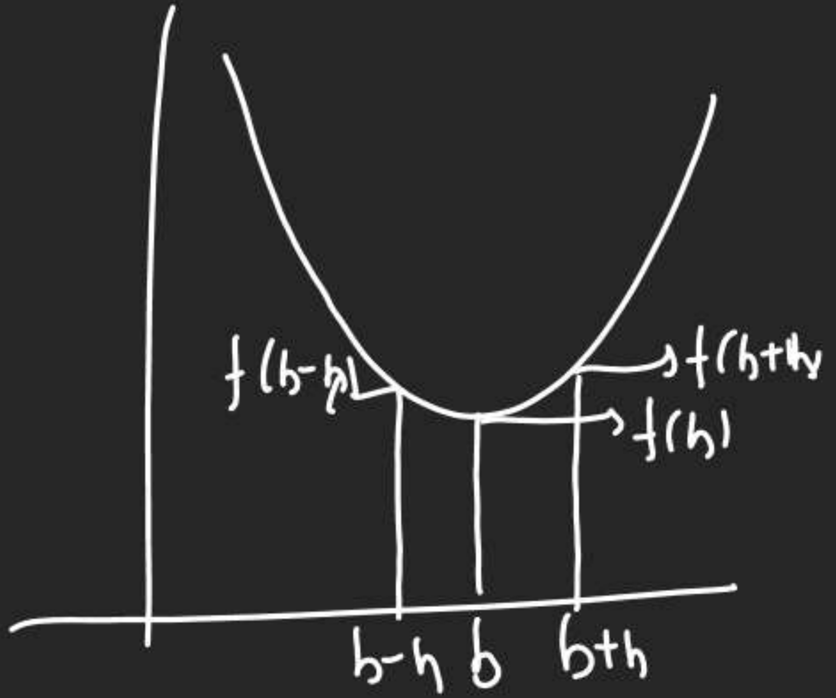


$f(a) > f(a+h)$

$f(a) > f(a-h)$

$f(x)$ having max^{a} at $x = a$

② Local Minima

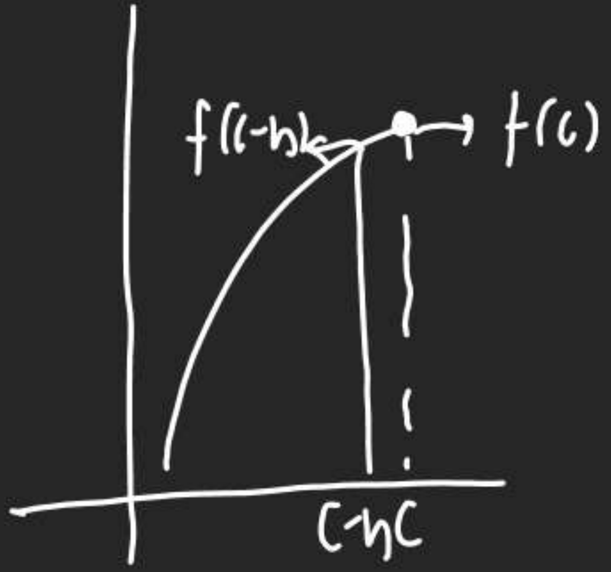


$f(b) < f(b-h)$

$f(b) < f(b+h)$

$\therefore x = b$ is L. Min

2) One Sided Extrema



$f(c) > f(c-h)$

$\therefore x = c$ is Pt of Max^{a}

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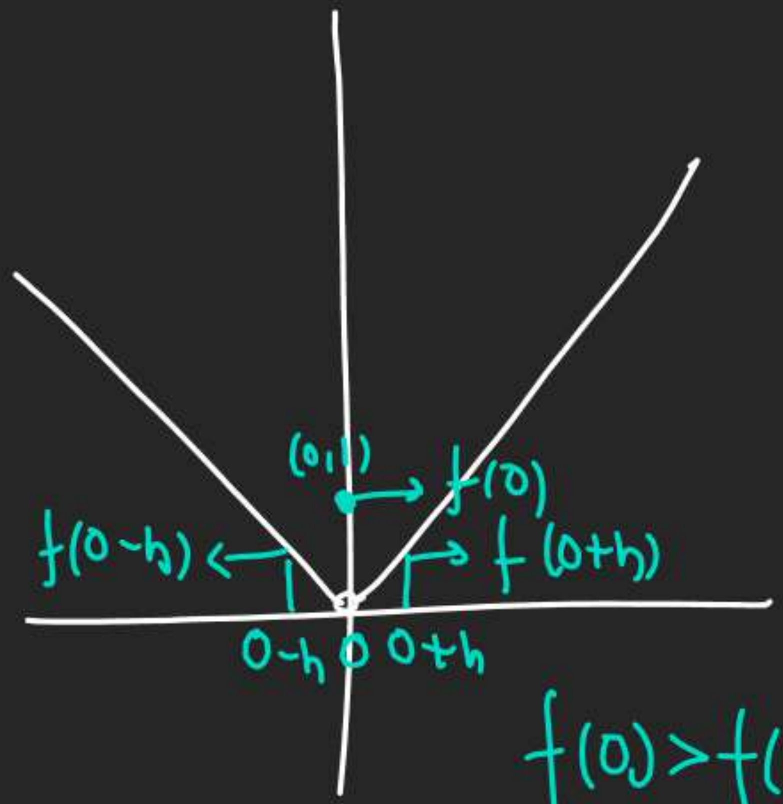
Q $f(x) = \begin{cases} |x| & x \neq 0 \\ 1 & x = 0 \end{cases}$

Check for extrema at $x=0$.



$f(0) > f(0+h)$

$f(0) > f(0-h)$

$\therefore$ $x=0$ is Pt of Max^m.

Q $f(x) = \begin{cases} \{-x\} & -1 \leq x < 0 \\ 1-x^2 & 0 \leq x \leq 1 \\ [x] & 1 < x \leq 2 \end{cases}$

Find Pts of LMax, LMin.

Complete monotonicity

- HW