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THERMODYNAMICS

Polytropic process

$$\boxed{PV^x = C} \qquad x \neq 1$$

Work done in polytropic

$$\int dw = \int_{V_i}^{V_f} P \cdot dV$$

$$W = \int_{V_i}^{V_f} \frac{C}{V^x} dV = C \int_{V_i}^{V_f} \frac{dV}{V^x}$$

$$W = C \frac{\left[V^{-x+1}\right]_{V_i}^{V_f}}{(-x+1)}$$

$$W = \frac{C}{1-x}\left[V_f^{-x+1} - V_i^{-x+1}\right]$$

$$W = \frac{C}{1-x}\left[\frac{V_f}{V_f^x} - \frac{V_i}{V_i^x}\right]$$

$$P_i V_i^x = P_f V_f^x = C$$

$$P_i = \frac{C}{V_i^x}, \quad P_f = \frac{C}{V_f^x}$$

$$W = \frac{P_f V_f - P_i V_i}{1-x} \Rightarrow W = \frac{P_i V_i - P_f V_f}{x-1}$$

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$$\boxed{W = \frac{P_i V_i - P_f V_f}{\gamma - 1}}$$

$$P_i V_i = nRT_i$$
$$P_f V_f = nRT_f$$

(gas in Ideal)

$$\boxed{W = \frac{nR(T_i - T_f)}{(\gamma - 1)}}$$

$$\Rightarrow W = \frac{-nR}{(\gamma-1)}(T_f - T_i)$$

$$\boxed{dW = \frac{-nR}{\gamma-1}\, dT}$$

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Molar heat Capacity of any polytropic process $PV^x = C$

$$C = \frac{1}{n}\left(\frac{dQ}{dT}\right)$$

For any process

$$\boxed{\frac{1}{n}\left(\frac{dU}{dT}\right) = C_V}$$

$$\boxed{dU = nC_V dT}$$

From 1^{st} Law of thermodynamics

$$dQ = dU + dW$$

$$\frac{dQ}{dT} = \frac{dU}{dT} + \frac{dW}{dT}$$

$$\frac{1}{n}\left(\frac{dQ}{dT}\right) = \frac{1}{n}\left(\frac{dU}{dT}\right) + \frac{1}{n}\left(\frac{dW}{dT}\right)$$

$$C = C_V -$$

$$dW = -\frac{nR}{(x-1)}dT$$

$$\frac{1}{n}\frac{dW}{dT} = \frac{-R}{(x-1)} = \frac{R}{1-x}$$

$$\boxed{C = C_V + \frac{R}{1-x}}$$

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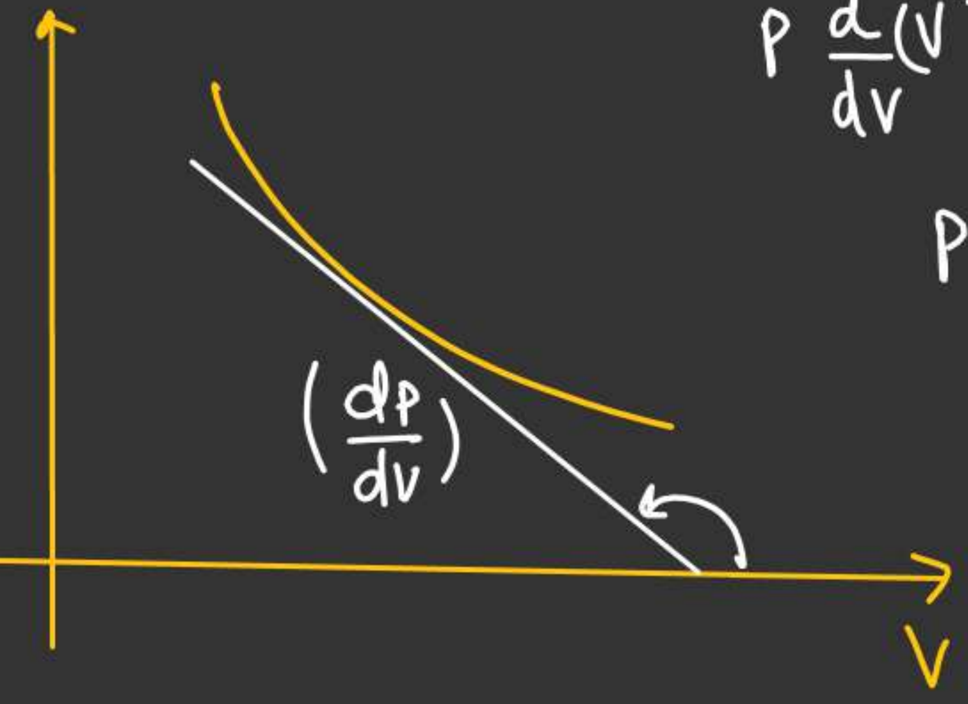
graph of $PV^x = C$.



$$P\frac{d}{dv}(V^x) + V^x\cdot\frac{dP}{dv} = 0$$

$$P x V^{x-1} + V^x\frac{dP}{dv} = 0$$

$$xP\cdot V^{x-1} = -V^x\cdot\frac{dP}{dV}$$

$$\boxed{\frac{-xP}{V} = \frac{dP}{dV}}$$

↳ Slope in P–V Curve of $PV^x = C$

** Bulk Modulus of a gas undergoes according to process $PV^x = C$.

$$B = \frac{-dP}{\left(\frac{dV}{V}\right)}$$

$$\boxed{B = xP}$$

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1 mole of a mono atomic gas undergoes according to process $TV^2 = C$. ΔT be the change in temp.

$\Delta U = nC_v \Delta T$

$= \frac{3}{2} R \Delta T$

Find

1. $\Delta Q = ?$
2. $\Delta W = ?$
3. $C = ?$
4. $B = ?$

Since gas is ideal

$PV = RT \quad (n=1)$

$T = \left(\frac{PV}{R}\right)$

$PV^3 = (C \cdot R)$

$PV^3 = C'$

Compair with $PV^x =$ Const $\Rightarrow x = 3$

$C = C_v + \frac{R}{1-x}$

$(C_v)_{mono} = \frac{3}{2} R$

$C = \frac{3}{2} R + \frac{R}{1-3}$

$C = \frac{3}{2} R - \frac{R}{2}$

$C = R$ A.

$\Delta Q = nC\Delta T$

$= (1) R\Delta T$

$\boxed{\Delta Q = R\Delta T}$

$\Delta W = \frac{nR\Delta T}{1-x}$

$= \frac{R\Delta T}{1-3}$

$= \left(-\frac{R\Delta T}{2}\right)$

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Q:

$$U = KV^{\alpha}$$

U = Internal energy.
K & α are constant.

A Ideal gas whose adiabatic coff^n is γ undergoes according to process in which $U = KV^{\alpha}$.

Find

a) $\Delta W_{gas} = ??$ ✓

b) $\Delta Q = ??$ ✓

c) $C = ??$

Sol^n.

$$nC_vT = KV^{\alpha}.$$

By Ideal gas Equation

$$PV = nRT$$

$$\frac{PV}{R} = nT$$

$$C_v\left(\frac{PV}{R}\right) = KV^{\alpha}$$

$$PV^{1-\alpha} = \left(\frac{K \cdot R}{C_v}\right) = \frac{KR}{R}(\gamma-1) = K(\gamma-1)$$

$x = (1-\alpha)$
$(PV^{x} = C)$

$$W = -\frac{nR(\Delta T)}{(x-1)} = \frac{-nR\Delta T}{1-\alpha-1}$$

$$\longrightarrow W = \left(\frac{nR\Delta T}{\alpha}\right) J$$

$$\frac{C_p}{C_v} = \gamma$$

$$C_p - C_v = R$$

$$C_p = \frac{\gamma R}{\gamma-1}, \quad C_v = \frac{R}{\gamma-1}$$

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$$\Delta Q = \Delta U + \Delta W$$

$$\Delta U = nC_v \Delta T$$

$$= \left(\frac{nR\Delta T}{\gamma - 1}\right)$$

$$\Delta Q = \left(\frac{nR\Delta T}{\gamma-1} + \frac{nR\Delta T}{\alpha}\right)$$

$$\Delta Q = nR\Delta T\left(\frac{1}{\alpha} + \frac{1}{\gamma - 1}\right)$$

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$$dQ = dU + dW$$

$$\underbrace{\frac{1}{n}\frac{dQ}{dT}}_{\Downarrow\ C} = \underbrace{\frac{1}{n}\frac{dU}{dT}}_{\Downarrow\ C_V} + \left(\frac{dW}{dT}\right)\frac{1}{n} \rightarrow \boxed{C = C_V + \frac{RT}{V}\left(\frac{dV}{dT}\right)}$$

$$dW = PdV$$

$$PV = nRT$$

$$P = \left(\frac{nRT}{V}\right)$$

$$dW = nRT\left(\frac{dV}{V}\right)$$

Use when Relation b/w T & V given & we have to find C = ??

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\# 1 Mole of an ideal gas whose molar heat capacity at Constant pressure is given by C_p undergoes a process where T & V related as.

$T = T_0 + \alpha V$ where T_0 & α are Constant.

a) Find $C = ??$

b) Amount of heat transfered when volume of gas changes from V_1 to V_2.

$$\left(C = C_p + \frac{RT_0}{\alpha V} \right)$$

↓ function of Volume

Sol^n

$$T = T_0 + \alpha V$$

$$\frac{dT}{dV} = \alpha$$

$$\frac{dV}{dT} = \frac{1}{\alpha} \quad \text{— (II)}$$

$$C = C_v + \frac{RT}{V}\left(\frac{dV}{dT}\right) \quad \text{— (I)}$$

From (I) & (II)

$$C = C_v + \frac{RT}{V}\frac{1}{\alpha}$$

$$C = C_v + \frac{RT}{\alpha V}$$

$$C = C_v + \frac{R}{\alpha V}(T_0 + \alpha V)$$

$$C = (C_v + R) + \frac{RT_0}{\alpha V}$$

$$\left(C = C_p + \frac{RT_0}{\alpha V} \right)$$

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$$\left(C = C_v + \frac{RT_0}{\alpha V}\right) \qquad (T = T_0 + \alpha V)$$

By 1^{st} Law of thermodynamics

$$\Delta Q = \Delta U + \Delta W.$$

$$\int_0^{\Delta W} dw = \int_{V_1}^{V_2} \underline{P}\, dV$$

By Ideal gas equation

$$PV = RT \quad (n=1)$$

$$P = \frac{RT}{V} = \frac{R}{V}(T_0 + \alpha V)$$

$$P = \left(\frac{RT_0}{V} + \alpha R\right)$$

$$\Delta W = \int_{V_1}^{V_2} \left(\frac{RT_0}{V} + \alpha R\right) dV$$

$$\Delta W = RT_0 \ln\left(\frac{V_2}{V_1}\right) + \alpha R (V_2 - V_1) \checkmark$$

$$\Delta U = nC_v \Delta T \quad (n=1)$$

$$C_p = \frac{\gamma R}{\gamma - 1}$$

$$= C_v \Delta T$$

$$= \frac{R}{\gamma - 1}(T_f - T_i)$$

$$= \frac{R}{\gamma - 1}\left(T_0 + \alpha V_2 - (T_0 + \alpha V_1)\right)$$

$$= \frac{\alpha R}{\gamma - 1}(V_2 - V_1) \checkmark$$

$$= \alpha C_v (V_2 - V_1)$$

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$$\Delta Q = \Delta U + \Delta W$$

$$= \underline{\alpha C_v (V_2 - V_1)} + \frac{RT_0}{} \ln\left(\frac{V_2}{V_1}\right) + \underline{\alpha R (V_2 - V_1)}$$

$$= (C_v + R)\,\alpha (V_2 - V_1) + RT_0 \ln\left(\frac{V_2}{V_1}\right)$$

$$= \underline{\alpha C_p (V_2 - V_1) + RT_0 \ln\left(\frac{V_2}{V_1}\right)}$$

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#

$$C = C_V + \alpha T$$

Find Equation of the process in terms T & V.

Compair

$$C = C_V + \frac{RT}{V}\left(\frac{dV}{dT}\right)$$

$$\frac{R}{V}\frac{dV}{dT} = \alpha$$

$$\int \frac{dV}{V} = \frac{\alpha}{R}\int dT$$

$$\ln V + \ln C_0 = \frac{\alpha}{R}T$$

$$C_0 V = e^{\frac{\alpha}{R}T}$$

$$V = \frac{1}{C_0} e^{\frac{\alpha}{R}T}$$

$$V e^{-\frac{\alpha}{R}T} = \frac{1}{C_0}$$

$$\boxed{V e^{-\frac{\alpha}{R}T} = \text{Constant}}$$

✓