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# Matrix & Determinant

## Matrix:

**Definition** – ① It is Rectangular Arrangement of real & Complex NO.

② It has mn NO., having m Rows & n Columns

(3) It is Rep by Capital letters A, B, C & elements Inside Rep. by Small

(4) $A = [a_{ij}]_{m \times n}$ denotes matrix A

(5) here $a_{ij}$ = element of $i^{th}$ Row & $j^{th}$ column

$a_{13}$ = element of $1^{st}$ Row & $3^{rd}$ column

$a_{22}$ = ________ $2^{nd}$ Row & $2^{nd}$ Column

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(6) $$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ a_{31} & a_{32} & a_{33} & \cdots & a_{3n} \\ \vdots & \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mn} \end{bmatrix}_{m\times n}$$

Order $= m \times n$

No. of elements $= mn$

(7) Matrix is Rep. by $[\ \ ]$ or $(\ \ )$

(8) When Matrix is Sqr shape = order = $n \times n$ = n rows & n Columns

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ a_{31} & a_{32} & a_{33} & \cdots & a_{3n} \\ \vdots & & & & \\ a_{n1} & a_{n2} & a_{n3} & \cdots & a_{nn} \end{bmatrix}$$

$i<j$

$i=j$

$\boxed{i>j}$

$a_{31}$

$i=3$, $j=1$

Principle Diagonal exists only in Sqr Matrix

Principal Diagonal

$a_{11} \rightarrow i=j=1$

$a_{22} = i=2=j$

$a_{33} = i=3=j$

$\vdots$

all elements $i=j$

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Q. $A = \begin{pmatrix} 3 & -4 & 10 \\ -2 & 7 & 0 \\ 5 & -6 & 9 \end{pmatrix}$

$a_{13} = 10$

$a_{31} = 5$

$a_{21} = -2$

$a_{13} = 10$

$a_{22} = 7$

Order = $3 \times 3$

No. of elements = 5

Sqr / Rectangular

Principle diagonal elements = 3, 7, 9

(9) Order of Matrix –

$\boxed{\text{Order} = \text{No. of Rows} \times \text{No. of columns}}$

Q. If Matrix Consisting of total elements as 12 then how many different order of matrices are possible?

different orders :

$1 \times 12$

$12 \times 1$

$6 \times 2$

$2 \times 6$

$3 \times 4$

$4 \times 3$

6 diff. orders possible

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Adv.
Q. Using 6 digit No. how many different Matrices can be formed?

let Nos. are a, b, c, d, e, f

different orders

$1 \times 6$ 720

$6 \times 1$ 720

$2 \times 3$ 720

$3 \times 2$ 720

4 Matrices 2880 ways

2×3

6 choices

$$A = \begin{bmatrix} \bigcirc^{6} & \bigcirc^{5} & \bigcirc^{4} \\ \bigcirc^{3} & \bigcirc^{2} & \bigcirc^{1} \end{bmatrix}_{2 \times 3}$$

$6 \times 5 \times 4 \times 3 \times 2 \times 1$

$= \lfloor 6 = 720$ ways

to fill $2 \times 3$ Matrix.

Q. Using letter a, a, a, b, b, c how many distinc Matrices are possible?

Nos. are a, a, a, b, b, c → 6 No

diff. orders

$1 \times 6 \rightarrow 60$ ways

$6 \times 1 \rightarrow 60$ ways

$2 \times 3 \rightarrow 60$ ways

$3 \times 2 \rightarrow 60$ ways

240 ways

$$A = \begin{bmatrix} \bigcirc & \bigcirc & \bigcirc \\ \bigcirc & \bigcirc & \bigcirc \end{bmatrix}_{2 \times 3}$$

6 places of a, a, a, b, b, c

$$\frac{\lfloor 6}{\lfloor 3 \, \lfloor 2} = \frac{720}{6 \times 2} = 60$$

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Q. Find No. of $2 \times 2$ Matrix whose

$a_{ij} = 1$ or $-1$

& $a_{11} \cdot a_{21} + a_{12} \cdot a_{22} = 0$

① Order $= 2$ ② Elements $\rightarrow 1$ or $-1$

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}_{2 \times 2}$$

$$\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} -1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} -1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} -1 & 1 \\ -1 & -1 \end{bmatrix} \begin{bmatrix} -1 & -1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ -1 & -1 \end{bmatrix}$$

$-1 \times -1 + -1 \times 1$

$1 + -1 = 0$

8 matrices possible

Q. How many matrices of $2 \times 2$ Order in possible using 1 or $-1$

2 calories

$$\begin{bmatrix} \boxed{\bigcirc} & \boxed{\bigcirc} \\ \bigcirc & \bigcirc \end{bmatrix}$$

$2 \times 2 \times 2 \times \quad = 16$

PYQ Record $\rightarrow$ All time Best

1) M & D $\longrightarrow$ ③
2) Vector & 3D $\longrightarrow$ ③
3) Def. Int. $\longrightarrow$ ③

AUC + DE

Matrix

Adv.

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Q. Construct matrix of 3x2 order if $a_{ij} = \left|\frac{i-2j}{2}\right|$

$$A = \begin{array}{c} a_{11} \\ \\ a_{21} \\ \\ a_{31} \end{array} \begin{bmatrix} \left|\frac{1-2\times1}{2}\right| & \left|\frac{1-2\times2}{2}\right| \\ \left|\frac{2-2\times1}{2}\right| & \left|\frac{2-2\times2}{2}\right| \\ \left|\frac{3-2\times1}{2}\right| & \left|\frac{3-2\times2}{2}\right| \end{bmatrix}_{3\times2} \begin{array}{c} a_{12} \\ \\ a_{22} \\ \\ a_{32} \end{array}$$

$$A = \begin{bmatrix} \frac{1}{2} & \frac{3}{2} \\ 0 & +1 \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

Q. Construct a 2x3 matrix whose elements are $a_{ij} = \frac{i+2j}{3}$

$$A = \begin{array}{ccc} a_{11} & a_{12} & a_{13} \\ \begin{bmatrix} \left(\frac{1+2\times1}{3}\right) & \left(\frac{1+2\times2}{3}\right) & \left(\frac{1+2\times3}{3}\right) \\ \left(\frac{2+2\times1}{3}\right) & \left(\frac{2+2\times2}{3}\right) & \left(\frac{2+2\times3}{3}\right) \end{bmatrix}_{2\times3} \\ a_{21} & a_{22} & a_{23} \end{array}$$

$$A = \begin{bmatrix} 1 & 5/3 & 7/3 \\ 4/3 & 2 & 8/3 \end{bmatrix}$$

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Q. 3x3 Matrix

$$a_{ij} = \begin{cases} \frac{i-j}{2}, & \boxed{i>j} \to \text{lower } \Delta \\ i^2+j^2, & i=j \\ \frac{i+j}{2}, & i<j \end{cases}$$

$$A = \begin{bmatrix} (1^2+1^2)_{a_{11}} & \left(\frac{1+2}{2}\right)_{a_{12}} & \left(\frac{1+3}{2}\right)_{a_{13}} \\ \left(\frac{2-1}{2}\right)_{a_{21}} & (2^2+2^2)_{a_{22}} & \left(\frac{2+3}{2}\right)_{a_{23}} \\ \left(\frac{3-1}{2}\right)_{a_{31}} & \left(\frac{3-2}{2}\right)_{a_{32}} & (3^2+3^2)_{a_{33}} \end{bmatrix}_{3\times 3}$$

$$A = \begin{pmatrix} 2 & 3/2 & 2 \\ 1/2 & 8 & 5/2 \\ 1 & 1/2 & 18 \end{pmatrix}$$

Types of Matrix

1) <u>Row Matrix</u> $\to$ $1\times n$ order

Row = 1

$$A = \begin{bmatrix} 3 & 2 & -1 & 4 \end{bmatrix}_{1\times 4}$$

(2) Column Matrix $\Rightarrow$ 1 Column

$m\times 1$ order type

$$A = \begin{bmatrix} -3 \\ 2 \\ -1 \\ 4 \end{bmatrix}_{4\times 1}$$

Row & Column matrices are also known Vectors

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3) Null Matrix = Zero Matrix

$a_{ij} = 0$

$$A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \text{ or } A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

But $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \neq \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

4) Square Matrix → (1) when Row = Column

$m = n$

$$A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}_{3 \times 3}$$

it has Pr. diag.

3rd order

(5) Trace of Matrix:

A) It is Rep by $Tr(A)$

B) It is Sum diag. elements

$$A = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix}$$

C) $Tr(A) = a + e + i$

D) $Tr(A) = \sum_{i=1}^{n} a_{ii} = a_{11} + a_{22} + \text{----} + a_{nn}$

(E) $A = \begin{bmatrix} 2 & 6 & 1 \\ 15 & 9 & 0 \\ -7 & 3 & -8 \end{bmatrix}$, $Tr(A) = ?$

$Tr(A) = 2 + 9 - 8 = 3$

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## (F) Properties of $Tr(A)$

(1) $Tr(KA) = K\,Tr(A)$

(2) $Tr(A \pm B) = Tr(A) \pm Tr(B)$

(3) $Tr(A) = Tr(A^T)$

(4) $Tr(A \cdot B) = Tr(B \cdot A)$ if A & B are of same order

$A^T$ = Transpose of A → Row & Col. interchanged

$A = \begin{bmatrix} 1 & 3 \\ -2 & 7 \end{bmatrix}$ then $A^T = ?$

$A^T = \begin{bmatrix} 1 & -2 \\ 3 & 7 \end{bmatrix}$

Q. $A = \begin{bmatrix} x-2 & e^x & -\sin x \\ \cos x^2 & x^2-x+3 & \ln|x| \\ 0 & \tan^{-1} x & x-7 \end{bmatrix}$

if $Tr(A) = 0$, find $x$?

$Tr(A) = x - 2 + x^2 - x + 3 + x - 7 = 0$

$x^2 + x - 6 = 0$

$(x+3)(x-2) = 0$

$x = -3, 2$