## EXERCISE - I

## SECTION - A & B

## NUMBER SYSTEM / BASIC OPERATIONS

**1.** Find the least value of $n(n \in N)$, for which $\left(\frac{1+i}{1-i}\right)^n$ is real

(A) 1 (B) 2 (C) 3 (D) 4

**2.** The number of solutions of the system of equations $\text{Re}\,(z^2) = 0, |z| = 2$ is

(A) 4 (B) 3 (C) 2 (D) 1

**3.** If $(a + ib)^5 = \alpha + i\beta$ then $(b + ia)^5$ is equal to

(A) $\beta + i\alpha$ (B) $\alpha - i\beta$ (C) $\beta - i\alpha$ (D) $-\alpha - i\beta$

**4.** Let $z(\neq 2)$ be a complex number such that $\log_{1/2} |z - 2| > \log_{1/2} |z|$, then

(A) $\text{Re}\,(z) > 1$ (B) $\text{Im}\,(z) > 1$ (C) $\text{Re}\,(z) = 1$ (D) $\text{Im}\,(z) = 1$

## SECTION - C

## ALGEBRA OF COMPLEX NUMBER

**5.** In one root of the quadratic equation $(1 + i)x^2 - (7 + 3i)x + (6 + 8i) = 0$ is $4 - 3i$, then the other root must be

(A) $1 + I$ (B) $4 + 3i$ (C) $1 - I$ (D) $4i + 3$

## SECTION - D:

## CONJUGATE

**6.** If $z(\neq -1)$ is a complex number such that $\frac{z-1}{z+1}$ is purely imaginary, then $|z|$ is equal to

(A) 1 (B) 2 (C) 3 (D) 5

**7.** If $(1 + i)z = (1 - i)\bar{z}$ then z is

(A) $t(1 - i), t \in R$ (B) $t(1 + i), t \in R$ (C) $\frac{t}{1+i}, t \in R^+$ (D) None of these

## SECTION -E:

## MODULUS

**8.** If $|z - 2| \geq |z - 4|$ then correct statement is-

(A) $R(z) \geq 3$ (C) $R(z) \geq 2$ (B) $R(z) \leq 3$ (D) $R(z) \leq 2$

**9.** If $|z_1-1|<1, |z_2-2|<2, |z_3-3|<3$ then $|z_1+z_2+z_3|$

(A) is less than 6 (B) is more than 3

(C) is less than 12 (D) lies between 6 and 12

**10.** If $iz^3+z^2-z+i=0$, then $|z|$ equals

(A) 4 (B) 3 (C) 2 (D) 1

## SECTION -F:
## ARGUMENT

**11.** Let z and w are two non zero complex number such that $|z|=|w|$, and $\text{Arg}(z)+\text{Arg}(w)=\pi$ then-

(A) $z=w$ (B) $z=\bar{w}$ (C) $\bar{z}=\bar{w}$ (D) $z=-\bar{w}$

**12.** Let z, w be complex numbers such that $\bar{z}+i\bar{w}=0$ and $\arg\ zw=\pi$. Then $\arg\ z$ equals-

(A) $\pi/4$ (B) $\pi/2$ (C) $3\pi/4$ (D) $5\pi/4$

**13.** If $z_1=-3+5i; z_2=-5-3i$ and z is a complex number lying on the line segment joining $z_1 \& z_2$, then arg (z) can be

(A) $-\frac{3\pi}{4}$ (B) $-\frac{\pi}{4}$ (C) $\frac{\pi}{6}$ (D) $\frac{5\pi}{6}$

**14.** If z and ω are two non- zero comlex numbers such that $|z\omega|=1$, and $\text{Arg}(z)-\text{Arg}(\omega)=\frac{\pi}{2}$, then $\bar{z}\omega$ is equal to-

(A) $-I$ (B) 1 (C) -1 (D) i

## SECTION - G, H, I & J
## CARTESIAN FORM / POLAR FORM / EULER'S FORM / DEMOVIRE'S THEOREM & APPLICATION

**15.** If $z_\gamma=\cos\left(\frac{\pi}{2^r}\right)+i\sin\left(\frac{\pi}{2^r}\right)$, $r=1,2,\ldots\ldots$ then $z_1\ z_2 z_3\ \ldots.$ is equal to

(A) -1 (B) i (C) $-I$ (D) 1

**16.** The expression $\left[\frac{1+i\tan\alpha}{1-i\tan\alpha}\right]^n-\frac{1+i\tan n\alpha}{1-i\tan n\alpha}$ when simplified reduces to

(A) zero (B) $2\sin\ n\alpha$ (C) $2\cos\ n\alpha$ (D) None of these

## SECTION -K

## ROOTS OF UNITY

**17.** If $\alpha$ is non real and $\alpha = \sqrt[5]{1}$ then the value of $2^{1-a^{1-2}+a^{2}+a^{-2}-a^{-4}|}$ is equal to

(A) 4 (B) 2 (C) 1 (D) None of these

**18.** Number of roots of the equation $z^{10} - z^{5} - 992 = 0$ with real part negative is

(A) 3 (B) 4 (C) 5 (D) 6

## SECTION -L

## CUBE ROOTS OF UNITY

**19.** If the cube roots of unity are $1, \omega, \omega^2$ then the roots of the equation $(x-1)^3 + 8 = 0$, are

(A) $-1, -1 + 2\omega, -1 - 2\omega^2$ (B) $-1, -1, -1$

(C) $-1, 1 - 2\omega, 1 - 2\omega^2$ (D) $-1, 1 + 2\omega, 1 + 2\omega^2$

**20.** If $\alpha$ and $\beta$ are the imaginary cube roots of unity then find the value of $\alpha^{100} + \beta^{100} + \frac{1}{\alpha^{100}.\beta^{100}}$

(A) 2 (B) 4 (C) 0 (D) 3

**21.** If $p = a + b\omega + c\omega^2$; $q = b + c\omega + a\omega^2$ and $r = c + a\omega + b\omega^2$ where $a, b, c \neq 0$ and $\omega$ is the complex cube root of unity then

(A) $p + q + r = a + b + c$ (B) $p^2 + q^2 + r^2 = a^2 + b^2 + c^2$

(C) $p^2 + q^2 + r^2 = 2(pq + qr + rp)$ (D) None of these

**22.** If $i = \sqrt{-1}$, then $4 + 5\left(-\frac{1}{2} + \frac{i\sqrt{3}}{2}\right)^{334} + 3\left(-\frac{1}{2} + \frac{i\sqrt{3}}{2}\right)^{365}$ is equal to

(A) $1 - i\sqrt{3}$ (B) $-1 + i\sqrt{3}$ (C) $i\sqrt{3}$ (D) $-i\sqrt{3}$

**23.** Let $\omega = -\frac{1}{2} + i\frac{\sqrt{3}}{2}$. Then the value of the determinant $\begin{vmatrix} 1 & 1 & 1 \\ 1 & -1-\omega^2 & \omega^2 \\ 1 & \omega^2 & \omega^4 \end{vmatrix}$ is

(A) $3\omega$ (B) $3\omega(\omega - 1)$ (C) $3\omega^2$ (D) $3\omega(1 - \omega)$

**24.** $\omega$ is an imaginary cube root of unity. If $(1 + \omega^2)^m = (1 + \omega^4)^m$, then the least positive integral value of m is

(A) 6 (B) 5 (C) 4 (D) 3

**25.** If a, b, c are integers not all equal and $\omega$ is cube root of unity ($\omega \neq 1$), then the minimum value of $|a + b\omega + c\omega^2|$ is

(A) 0 (B) 1 (C) $\sqrt{3}/2$ (D) ½

**26.** If $x^2 + x + 1 = 0$ then the numerical value of the expression

$\left(x + \frac{1}{x}\right)^2 + \left(x^2 + \frac{1}{x^2}\right)^2 + \left(x^3 + \frac{1}{x^3}\right)^2 + \left(x^4 + \frac{1}{x^4}\right)^2 \left(x^{27} + \frac{1}{x^{27}}\right)^2$ is

(A) 54 (B) 36 (C) 27 (D) 18

**27.** If $1, \alpha, \alpha^2, \ldots\ldots, \alpha^{n-1}$ are nth roots of unity. The value of $(3-\alpha)(3-\alpha^2)(3-\alpha^3)\ldots(3-\alpha^{n-1})$ is

(A) n (B) 0 (C) $\frac{3^n-1}{2}$ (D) $\frac{3^n+1}{2}$

**28.** If $w \neq 1$ is nth root of unity, then value of $\sum_{k=1}^{n-1} \left|z_1 + w^k z_2\right|^2$ is

(A) $n(z_1|^2 + |z_2|^2)$ (B) $|z_1|^2 + |z_2|^2$ (C) $(|z_1| + |z_2|)^2$ (D) $n(|z_1| + |z_2|)^2$

**29.** If $w(\neq 1)$ is a cube root of unity, then $\begin{vmatrix} 1 & 1+i+\omega^2 & \omega^2 \\ 1-i & -1 & \omega^2 - 1 \\ -i & -i+\omega-1 & -1 \end{vmatrix}$ equals

(A) 0 (B) 1 (C) i (D) $\omega$

**30.** The product of cube roots of -1 is equal to

(A) -2 (B) 0 (C) -1 (D) 4

## SECTION - M, N & O

## GEOMETRY / DISTANCE FORMULA / SECTION FORMULA

**31.** The curve represented by $\text{Re}(z)^2 = 4$ is

(A) a parabola (B) an ellipse (C) a circle (D) a rectangular hyperbola

**32.** If $|z_1| = |z_2| = |z_3| = 1$ and $z_1, z_2, z_3$ are represented by the vertices of an equilateral triangle then

(A) $z_1 + z_2 + z_3 = 0$ (B) $z_1 z_2 z_3 = 1$

(C) $z_1 z_2 + z_2 z_3 + z_3 z_1 = 1$ (D) None of these

**33.** The curve represented by $|z| = \text{Re}(z) + 2$ is

(A) a straight line (C) an ellipse (B) a circle (D) Parabola

**34.** Let A, B, C represent the complex numbers $z_1, z_2, z_3$ respectively on the complex plane, If the circumcentre of the triangle ABC lies at the origin, then the orthocentre is represented by the complex number

(A) $z_1 + z_2 - z_3$
(B) $z_2 + z_3 - z_1$
(C) $z_3 + z_1 - z_2$
(D) $z_1 + z_2 + z_3$

**35.** If $|z^2 - 1| = |z^2| + 1$, then z lies on

(A) Parabola
(B) the imaginary axis
(C) a circle
(D) an ellipse

**36.** If the complex numbers iz, z and z + iz represent the three vertices of a triangle then the area of the triangle is

(A) $\frac{1}{2}|z - 1|$
(B) $|z|^2$
(C) $\frac{1}{2}|z|^2$
(D) $|z - 1|^2$

**37.** Complex number $z_1, z_2$ and $z_3$ in AP

(A) lie on ellipse
(B) lie on a parabola
(C) lie on line
(D) lie on circle

**38.** For all complex numbers $z_1, z_2$ satisfying $|z_1| = 12$ and $|z_2 - 3 - 4i| = 5$, the minimum value of $|z_1 - z_2|$ is

(A) 0
(B) 2
(C) 7
(D) 13

**39.** The complex numbers $z_1, z_2$ and $z_3$ satisfying $\frac{z_1 - z_1}{z_2 - z_3} = \frac{1 - i\sqrt{3}}{2}$ are the vertices of a triangle which is

(A) of area zero
(B) right angled isosceles
(C) equilateral
(D) obtuse angled isosceles

**SECTION-S:**

**MIXED PROBLEMS**

**40.** The equation of the radical axis of the two circles represented by the equations, $|z - 2| = 3$ and $| z - 2 - 3i] = 4$ on the complex plane is

(A) $3iz - 3i\bar{z} - 2 = 0$
(B) $3iz - 3i\bar{z} + 2 = 0$
(C) $iz - i\bar{z} + 1 = 0$
(D) $2iz - 2i\bar{z} + 3 = 0$

**41.** If $z_1, z_2, z_3, \dots, z_n$ lie on the circle $|z| = 2$, then the value of

$E = |z_1 + z_2 + \cdots \dots + z_n| - 4\left|\frac{1}{z_1} + \frac{1}{z_2} + \cdots . + \frac{1}{z_2}\right|$ is.

(A) 0 (B) n (C) $-$n (D) $\frac{n}{2}$

## EXERCISE – II (LEVEL-I)

### MODULUS

**1.** If $z_1, z_2, z_3$ are complex number such that $|z_1| = |z_2| = |z_3| = \left|\frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3}\right| = 1$, then $|z_1 + z_2 + z_3|$ is.

(A) equal to 1 (B) less than 1 (C) greater than 3 (D) equal to 3

**2.** If $|z_1| = 2, |z_2| = 3, |z_3| = 4$ and $|2z_1 + 3z_2 + 4z_3| = 4$ then absolute value of $8z_2z_3 + 27z_3z_1 + 64z_1z_2$ equals

(A) 24 (B) 48 (C) 72 (D) 96

**3.** If $|z| = \max\{|z-1|, |z+1|\}$ then

(A) $|z + \bar{z}| = 1/2$ (B) $z + \bar{z} = 1$ (C) $|z + \bar{z}| = 1$ (D) None of these

### ARGUMENT

**4.** If $z$ is a complex number such that $|z| = 4$ and $\arg(z) = \frac{5\pi}{6}$, then z is equal to

(A) $-2\sqrt{3} + 2i$ (B) $2\sqrt{3} + i$ (C) $2\sqrt{3} - 2i$ (D) $-\sqrt{3} + i$

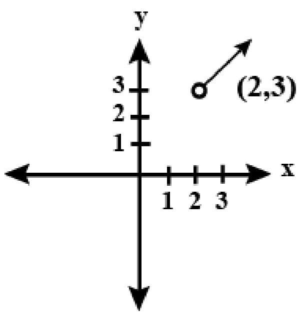
**5.** The argument of the complex number $\sin\frac{6\pi}{5} + i\left(1 + \cos\frac{6\pi}{5}\right)$ is.

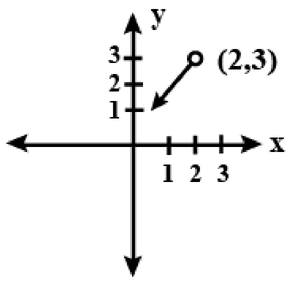
(A) $\frac{6\pi}{5}$ (B) $\frac{5\pi}{6}$ (C) $\frac{9\pi}{10}$ (D) $\frac{2\pi}{5}$

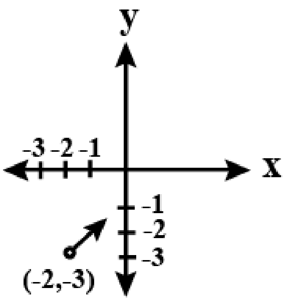
**6.** If $z_1$ and $z_2$ are two non-zero complex numbers such that $|z_1 + z_2| = |z_1| + |z_2|$, then $\arg z_1$ - $\arg z_2$ is equal to –

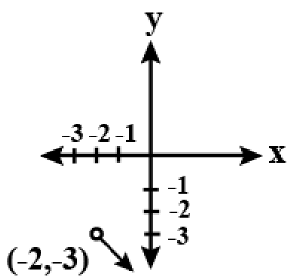
(A) $\frac{\pi}{2}$ (B) $-\pi$ (C) 0 (D) $\frac{-\pi}{2}$

**7.** If $\text{Arg}(z - 2 - 3i) = \frac{\pi}{4}$, then the locus of $z$ is

(A) 

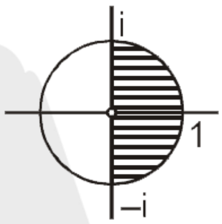

(B) 

(C) 


(D) 


**GEOMETRY**

**8.** The locus of z which lies in shaded region is best represented by




(A) $|z| \leq 1, \frac{-\pi}{2} \leq \arg\, z \leq \frac{\pi}{2}$

(B) $|z| = 1, \frac{-\pi}{2} \leq \arg\, z \leq \frac{\pi}{2}$

(C) $|z| \geq 0, 0 \leq \arg\, z \leq \frac{\pi}{2}$

(D) $|z| \leq 1, \frac{\pi}{2} \leq \arg\, z \leq \pi$

**9.** If $z_1 \& z_2$ are two complex number & if $\arg \frac{z_1+z_2}{z_1-z_2} = \frac{\pi}{2}$ but $|z_1 + z_2| \neq |z_1 - z_2|$ then the figure formed by the points represented by $0, z_1, z_2 \& z_1 + z_2$ is

(A) a parallelogram but not a rectangle or a rhombus

(B) a rectangle but not a square

(C) a rhombus but not a square

(D) a square

**ROTATION**

**10.** If magnitude of a complex number $4 - 3i$ is tripled and is rotated by an angle $\pi$ anticlockwise then resulting complex number would be

(A) $-12 + 9i$ (B) $12 + 9\text{i}$ (C) $7 - 6\text{i}$ (D) $7 + 6i$

**STRAIGHT LINE & CIRCLE**

**11.** The points $z_1, z_2, z_3, z_4$ in the complex plane are the vertices of a parallelogram taken in order if and only if

(A) $z_1 + z_4 = z_2 + z_3$

(B) $z_1 + z_3 = z_2 + z_4$

(C) $z_1 + z_2 = z_3 + z_4$

(D) None of these

**12.** The equation $|z-1|^2+|z+1|^2=2$ represents

(A) a circle of radius ' 1 '
(B) a straight line
(C) the ordered pair (0,0)
(D) None of these

**13.** The points $z_1=3+\sqrt{3}i$ and $z_2=2\sqrt{3}+6i$ are given on a complex plane. The complex number lying on the bisector of the angle formed by the vectors $z_1$ and $z_2$ is

(A) $z=\frac{(3+2\sqrt{3})}{2}+\frac{\sqrt{3}+2}{2}I$
(B) $z=5+5i$
(C) $z=3+i$
(D) $5+3i$

**14.** The points of intersection of the two curves $|z-3|=2$ and $|z|=2$ in an argand plane are

(A) $\frac{1}{2}(7\pm i\sqrt{3})$
(B) $\frac{1}{2}(3\pm i\sqrt{7})$
(C) $\frac{3}{2}\pm i\sqrt{\frac{7}{2}}$
(D) $\frac{7}{2}\pm i\sqrt{\frac{3}{2}}$

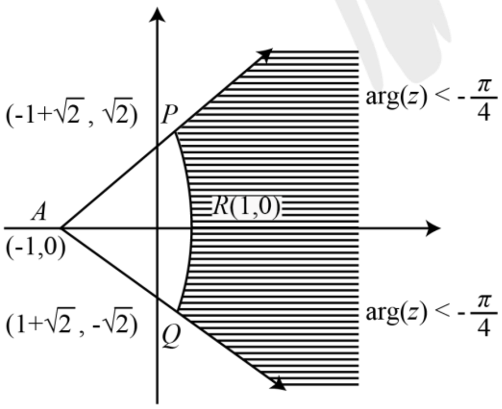
**15.** The region of Argand diagram defined by $|z-1|+|z+1|\leq 4$ is

(A) interior of an ellipse
(B) exterior of a circle
(C) interior and boundary of an ellipse
(D) interior and boundary of a circle.

**16.** If $w=\frac{z}{z-\frac{1}{3}i}$ and $|w|=1$, then $z$ lies on -

(A) an ellipse
(B) a circle
(C) a straight line
(D) a parabola

**17.** The locus of z which lies in shaded region (excluding the boundaries) is best represented by




(A) $z:|z+1|>2$ and $|\arg(z+1)|<\pi/4$.
(B) $z:|z-1|>2$ and $|\arg(z-1)|<\pi/4$
(C) $z:|z+1|<2$ and $|\arg(z+1)|<\pi/2$

(D) z: $|z-1|<2$ and $|\arg(z-1)|<\pi/2$

**18.** If $z=x+iy$ then the equation of a straight line $Ax+By+C=0$ where $A, B, C \in R$, can be written on the complex plane in the form $\bar{a}z+a\bar{z}+2C=0$ where ' $a$ ' is equal to

(A) $\frac{(A+iB)}{2}$ (B) $\frac{A-iB}{2}$ (C) $A+Ib$ (D) None of these

**MIXED PROBLEMS**

**19.** Points $z_1 \& z_2$ are adjacent vertices of a regular octagon. The vertex $z_3$ adjacent to $z_2 (z_3 \neq z_1)$ is represented by

(A) $z_2+\frac{1}{\sqrt{2}}(1 \pm i)(z_1+z_2)$ (B) $z_2+\frac{1}{\sqrt{2}}(1+i)(z_1-z_2)$

(C) $z_2+\frac{1}{\sqrt{2}}(1 \pm i)(z_2-z_1)$ (D) None of these

**20.** If $|z-2-3i|+|z+2-6i|=4$ where $i=\sqrt{-1}$ then locus of P(z) is

(A) an ellipse

(B) $\phi$

(C) segment joining the point $2+3i; -2+6i$

(D) None of these

**21.** Let $z_1$ and $z_2$ be to non real complex cube roots of unity and $|z-z_1|^2+|z-z_2|^2=\lambda$ be the equation of a circle with $z_1, z_2$ as ends of a diameter then the value of $\lambda$ is

(A) 4 (B) 3 (C) 2 (D) $\sqrt{2}$

**22.** The number of solutions of the equation in $z$, if $z\bar{z}-(3+i)z-(3-i)\bar{z}-6=0$ is

(A) 0 (B) 1 (C) 2 (D) infinite

## EXERCISE - II (LEVEL-II)

### BASIC OPERATIONS

1. The value of $i^n + i^{-n}$, for $i = \sqrt{-1}$ and $n \in I$ is

(A) $\frac{2^n}{(1-i)^{2n}} + \frac{(1+i)^{2n}}{2^n}$ (B) $\frac{(1+i)^{2n}}{2^n} + \frac{(1-i)^{2n}}{2^n}$

(C) $\frac{(1+i)^{2n}}{2^n} + \frac{2^n}{(1-i)^{2n}}$ (D) $\frac{2^n}{(1+i)^{2n}} + \frac{2^n}{(1-i)^{2n}}$

### MODULUS

2. If $z$ is a complex number then the equation $z^2 + z|z| + |z^2| = 0$ is satisfied by ( $\omega$ and $\omega^2$ are imaginary cube roots of unity)

(A) $z = k\omega$ where $k \in R$ (B) $\mathrm{z} = \mathrm{k}\omega^2$ where k is positive real

(C) $\mathrm{z} = \mathrm{k}\omega$ where k is positive real (D) $\mathrm{z} = \mathrm{k}\omega^2$ where $\mathrm{k} \in \mathrm{R}$

### ARGUMENT

3. If z satisfies the inequality $|\mathrm{z} - 1 - 2\mathrm{i}| \leq 1$, then

(A) $\min(\arg(\mathrm{z})) = \tan^{-1}\left(\frac{3}{4}\right)$ (B) $\max(\arg(\mathrm{z})) = \frac{\pi}{2}$

(C) $\min(|z|) = \sqrt{5} - 1$ (D) $\max(|z|) = \sqrt{5} + 1$

### POLAR FORM

4. If $2\cos\ \theta = x + \frac{1}{x}$ and $2\cos\ \varphi = y + \frac{1}{y}$, then

(A) $x^n + \frac{1}{x^n} = 2\cos\ (n\theta)$ (B) $\frac{x}{y} + \frac{y}{x} = 2\cos\ (\theta - \varphi)$

(C) $xy + \frac{1}{xy} = 2\cos\ (\theta + \varphi)$ (D) None of these

### CUBE ROOTS OF UNITY

5. If $g(x)$ and $h(x)$ are two real polynomials such that the polynomial $g(x^3) + xh(x^3)$ is divisible by $x^2 + x + 1$, then

(A) $g(1) = h(1) = 0$ (B) $g(1) = h(1) \neq 0$

(C) $g(1) = -h(1)$ (D) $g(1) + h(1) = 0$

### GEOMETRY

6. The equation $|z - i| + |z + i| = k, k > 0$, can represent

(A) an ellipse if $k > 2$ (B) line segment if $k = 2$

(C) an ellipse if $\mathrm{k} = 4$ (D) line segment if $k = 1$

7. The equation $\| z + i| - |z - i \| = k$ represents

(A) a hyperbola if $0 < \mathrm{k} < 2$ (B) a pair of ray if $k > 2$

(C) a straight line if $\mathrm{k} = 0$ (D) a pair of ray if $\mathrm{k} = 2$

**8.** ABCD is a square, vertices being taken in the anticlockwise sense. If A represents the complex number z and the intersection of the diagonals is the origin then

(A) B represents the complex number iz
(B) D represents the complex number $i\bar{z}$
(C) B represents the complex number $i\bar{z}$
(D) D represents the complex number – iz

**STRAIGHT LINE**

**9.** POQ is a straight line through the origin O, P and Q represent the complex number a + ib and c + id respectively and OP = OQ. Then

(A) $|a+ib| = |c+id|$
(B) $a+c=b+d$
(C) arg (a + i b) = arg (c + id)
(D) None of these

**MIXED PROBLEMS**

**10.** If $|z_1+z_2|^2 = |z_1|^2 + |z_2|^2$ then

(A) $\frac{z_1}{z_2}$ is purely real
(B) $\frac{z_1}{z_2}$ is purely imaginary
(C) $z_1\bar{z}_2 + z_2\bar{z}_1 = 0$
(D) amp $\frac{z_1}{z_2}$ may be equal to $\frac{\pi}{2}$

### EXERCISE - III SUBJECTIVE JEE ADVANCED

**1.** Show that the product, $\left[1+\left(\frac{1+i}{2}\right)\right]\left[1+\left(\frac{1+i}{2}\right)^2\right]\ldots\ldots\left[1+\left(\frac{1+i}{2}\right)^{2^n}\right]$

$=\left(1-\frac{1}{2^{2^n}}\right)(1+i)$ where $n \geq 2$.

**2.** Without expanding the determinant at any stage, find $K \in R$ such that

$\begin{vmatrix} 4i & 8+i & 4+3i \\ -8+i & 16i & i \\ -4+Ki & i & 8i \end{vmatrix}$ has purely imaginary value.

**3.** Given that $x, y \in R$ solve

(a) $(x+2y)+i(2x-3y)=5-4i$

(b) $(x+iy)+(7-5i)=9+4i$

(c) $x^2-y^2-i(2x+y)=2i$

(d) $(2+3i)x^2-(3-2i)y=2x-3y+5i$

(e) $4x^2+3xy+(2xy-3x^2)i=4y^2-(x^2/2)+(3xy-2y^2)i$

**4.** Prove that, with regard to the quadratic equation $z^2+(p+ip')z+q+iq'=0$; where $p, p', q, q'$ are all real.

(a) If the equation has one real root then $q'^2-pp'q'+pq'^2=0$

(b) If the equation has two equal roots then $p^2-p'^2=4q \,\&\, pp'=2q'$.

State whether these equal roots are real or complex.

**5.** (a) Let Z is complex satisfying the equation, $z^2-(3+i)z+m+2i=0$, where $m \in R$. Suppose the equation has a real root, then find the value of $m$

(b) a, b, c are real numbers in the polynomial, $P(Z)=2Z^4+aZ^3+bZ^2+cZ+3$. If two roots of the equation $P(Z)=0$ are 2 and $i$, then find the value of ' $a$ '.

**6.** Find the real values of the parameter ' $a$ ' for which at least one complex number $z=x+iy$ satisfies both the equality $|z-ai|=a+4$ and the inequality $|z-2|<1$.

**7.** If $|z_1|=|z_2|=\cdots..=|z_n|=1$ then show that

(i) $\bar{z}_1=\frac{1}{z_1}$

(ii) $|z_1+z_2+\cdots..+z_n|=\left|\frac{1}{z_1}+\frac{1}{z_2}+\cdots\ldots+\frac{1}{z_n}\right|$.

And hence interpret that the centroid of polygon with $2n$ vertices $z_1, z_2 \ldots..z_n, \frac{1}{z_1}, \frac{1}{z_2}, \ldots.\frac{1}{z_n}$ (need not be in order) lies on real axis.

**8.** If $\alpha \& \beta$ are any two complex numbers, prove that

(i) $|\alpha+\beta|^2+|\alpha-\beta|^2=2(|\alpha|^2+|\beta|^2)$

(ii) $\left|\alpha-\sqrt{\alpha^2-\beta^2}\right|+\left|\alpha+\sqrt{\alpha^2-\beta^2}\right|=|\alpha+\beta|+|\alpha-\beta|$.

**9.** Prove that identity, $|1-z_1\bar{z}_2|^2-|z_1-z_2|^2=(1-|z_1|^2)(1-|z_2|^2)$

**10.** Among the complex numbers $z$ satisfying the condition $|z+3-\sqrt{3}i|=\sqrt{3}$, find the number having the least positive argument.

**11.** Given, $z=\cos\frac{2\pi}{2n+1}+i\sin\frac{2\pi}{2n+1}$, ' $n$ ' a positive integer, find the equation whose roots are, $\alpha=z+z^3+\cdots.+z^{2n-1}$ & $\beta=z^2+z^4+\cdots\ldots+z^{2n}$.

**12.** (i) If $\alpha=\mathrm{e}^{\frac{2\mathrm{si}}{7}}$ and $\mathrm{f(x)}=\mathrm{A}_0+\sum_{\mathrm{k}=1}^{20}\mathrm{A}_\mathrm{k}\mathrm{x}^\mathrm{k}$, then find the value of, $f(x)+f(\alpha x)+\cdots\ldots+f(\alpha^6x)$ independent of $\alpha$.

(ii) Let $\alpha+i\beta;\alpha,\beta\in\mathrm{R}$, be a root of the equation $\mathrm{x}^3+\mathrm{qx}+\mathrm{r}=0,;\mathrm{q},\mathrm{r}\in\mathrm{R}$. Find a real cubic equation, independent of $\alpha\&\beta$, whose one root is $2\alpha$.

**13.** If A, B and C are the angles of a triangle

$D=\begin{vmatrix} e^{-2iA} & e^{iC} & e^{iB} \\ e^{iC} & e^{-2iB} & e^{iA} \\ e^{iB} & e^{iA} & e^{-2iC} \end{vmatrix}$ where $i=\sqrt{-1}$ then find the value of $D$.

**14.** If n is a positive integer, prove the following

(i) $(1+\cos\ \theta+i\sin\ \theta)^n+(1+\cos\ \theta-i\sin\ \theta)^n=2^{n+1}\cos^n\frac{\theta}{2}\cos\frac{n\theta}{2}$

(ii) $(1+i)^n+(1-i)^n=2^{\frac{n}{2}+1}\cdot\cos\frac{n\pi}{4}$

**15.** Find all those roots of the equation $z^{12}-56z^6-512=0$ whose imaginary part is positive.

**16.** Let a complex number $\alpha,\alpha\neq1$, be a root of the equation $z^{p+q}-z^p-z^q+1=0$, where $p,q$ are distinct primes. Show that either $1+\alpha+\alpha^2+.+\alpha^{p-1}=0$ or $1+\alpha+\alpha^2+\cdots\ldots.+\alpha^{q-1}=0$, but not both together.

**17.** Solve $(z-1)^4-16=0$. Find sum of roots. Locate roots, sum of roots and centroid of polygon formed by roots in complex plane.

**18.** If $\omega$ is the fifth root of unity then find value of $\log_2\left|1+\omega+\omega^2+\omega^3-\frac{1}{\omega}\right|$

**19.** If $\omega$ is an imaginary cube root of unity then prove that

(a) $(1+\omega-\omega^2)^3-(1-\omega+\omega^2)^3=0$

(b) $(1-\omega+\omega^2)^5+(1+\omega-\omega^2)^5=32$

(c) If $\omega$ is the cube root of unity, Find the value of $(1+5\omega^2+\omega^4)(1+5\omega^4+\omega^2)(5\omega^3+\omega+\omega^2)$.

**20.** If $\omega$ is a cube root of unity, prove that

(i) $(1+\omega-\omega^2)^3-(1-\omega+\omega^2)^3=0$

(ii) $\frac{\mathrm{a}+\mathrm{b}\omega+\mathrm{c}\omega^2}{\mathrm{c}+\mathrm{a}\omega+\mathrm{b}\omega^2}=\omega^2$

(iii) $(1-\omega)(1-\omega^2)(1-\omega^4)(1-\omega^8)=9$

**21.** If $x = a + b; y = a\omega + b\omega^2; z = a\omega^2 + b\omega$, show that

(i) $xyz = a^3 + b^3$

(ii) $x^2 + y^2 + z^2 = 6ab$

(iii) $x^3 + y^3 + z^3 = 3(a^3 + b^3)$

**22.** (a) $(1 + w)^7 = A + Bw$ where w is the imaginary cube root of of a unity and $A, B \in R$, find the ordered pair (A, B).

(b) The value of the expression;

1. $(2 - w)(2 - w^2) + 2 \cdot (3 - w)(3 - w^2) + +(n - 1) \cdot (n - w)(n - w^2)$, where $w$ is an imaginary cube root of unity is

**23.** If $w$ is an imaginary cube root of unity then prove that

(a) $(1 - w + w^2)(1 - w^2 + w^4)(1 - w^4 + w^8)$ ... to $2n$ factors $= 2^{2n}$.

(b) If $w$ is a complex cube root of unity, find the value of $(1 + w)(1 + w^2)(1 + w^4)(1 + w^8)$ to n factors.

**24.** Interpret the following locii in $z \in C$.

(a) $1 < |z - 2i| < 3$

(b) $\text{Im}\ (z) \geq 1$

(c) $\text{Arg}\ (z - a) = \pi/3$ where $a = 3 + 4i$

**25.** If $|z - 2 + i| \leq 2$, then find the greatest and least value of $|z|$.

**26.** If $|z + 3| \leq 3$ then find minimum and maximum values of

(i) $|z|$

(ii) $|z - 1|$

(iii) $|z + 1|$

**27.** If O is origin and affixes of P, Q, R are respectively z, iz, z + iz. Locate the points on complex plane. If $\triangle$ PQR = 200 then find

(i) $|z|$

(ii) sides of quadrilateral OPRQ

**28.** Plot the region represented by $\text{Re}\ (z) \leq 2, \text{Im}\ (z) \leq 2$ and $\frac{\pi}{8} \leq \arg\ (z) \leq \frac{3\pi}{8}$.

**29.** Let 1: $\text{Arg}\left(\frac{z-8i}{z+6}\right) = \pm\frac{\pi}{2}$

11: $\text{Re}\left(\frac{z - 8i}{z + 6}\right) = 0$

Show that locus of $z$ in 1 or II lies on $x^2+y^2+6x-8y=0$ Hence show that locus of $z$ can also be represented by $\frac{z-8i}{z+6}+\frac{\bar{z}-8i}{\bar{z}+6}=0$. Further if locus of $z$ is expressed as $|z+3-4i|=R$, then find $R$.

**30.** If a & b are real numbers between 0&1 such that the points $z_1=a+i, z_2=1+\mathrm{bi}\ \& z_3=0$ from an equilateral triangle, then find the values of ' $a$ ' and ' $b$ '.

**31.** (a) Find all non-zero complex numbers $Z$ satisfying $\overline{\mathrm{Z}}=\mathrm{i}\mathrm{Z}^2$.

(b) If the complex numbers $z_1, z_2, \ldots \ldots . . z_n$ lie on te unit circle $|z|=1$ then show that $|z_1+z_2+\cdots . .+z_n|=|z_1^{-1}+z_2^{-1}+\cdots \ldots+z_n^{-1}|$.

**32.** Find the Cartesian equations of the locus of ' $z$ ' in the complex plane satisfying $|z-4|+|z+4|=16$.

**33.** Prove that the complex numbers $z_1$ and $z_2$ and the origin form an isosceles triangle with vertical angle $2\pi/3$ if $z_1^2+z_2^2+z_1 z_2^*=0$

**34.** If the complex number P(w) lies on the standard unit circle in an Argand's plane and $z=(aw+b)(w-\mathrm{c})\ ^{-1}$ then, find the locus of $z$ and interpret it. Given a, b, c are real.

**35.** (a) Let $z=x+iy$ be a complex number, where $x$ and $y$ are real numbers. Let $A$ and $B$ be the sets. defied by $\mathrm{A}=\{\mathrm{z}||\mathrm{z}\ |\leq 2\}$ and $\mathrm{B}=|\mathrm{z}|(1-i)z+(1+i)\bar{z}\geq 4$ ). Find the area of region $A \cap B$.

(b) For all real numbers $x$, let the mapping $f(x)=\frac{1}{x-i}$ (where $i=\sqrt{-1}$ ). If there exist real number $a, b, c$ and $d$ for which $f(a), f(b), f(c)$ and $f(d)$ form a square on the complex plane. Find the area of the square.

**36.** Given that, $|z-1|=1$, where ' $z$ ' is a point on the argand plane. Show that $\frac{z-2}{z}=i\tan(\arg\ z)$.

**37.** Dividing $f(z)$ by $z-i$, we get the remainder $i$ and dividing it by $z+i$, we get the remainder $1+i$. Find the remainder upon the division of f(z) by $\mathrm{z}^2+1$.

**38.** If the biquadratic $x^4 + ax^3 + bx^2 + cx + d = 0 (a, b, c, d \in R)$ has 4 non real roots, two with sum $3 + 4i$ and the other two with product $13 + i$. Find the value of ' $b$ '.

**39.** If $z_1, z_2$ are the roots of the equation $az^2 + bz + c = 0$, with $a, b, c > 0$; $2b^2 > 4ac > b^2$; $z_1 \in$ third quadrant ; $z_2 \in$ second quadrant in the argand's plane then, show that

$$\arg\left(\frac{z_1}{z_2}\right) = 2\cos^{-1}\left(\frac{b^2}{4ac}\right)^{1/2}$$

**40.** Find the set of points on the argand plane for which the real part of the complex number $(1 + i)z^2$ is positive where $z = x + iy, x, y \in R$ and $i = \sqrt{-1}$.

**41.** If $Z_r$, r = 1,2,3, ... .2 m, mεN are roots of the equation $\mathrm{Z}^{2\,\mathrm{m}} + \mathrm{Z}^{2\,\mathrm{m}-1} + \mathrm{Z}^{2\,\mathrm{m}-2} + \cdots .. + \mathrm{Z} + 1 = 0$ then prove that $\sum_{r=1}^{2m} \frac{1}{Z_s - 1} = -m$

**42.** Show that all the roots of the equation $\left(\frac{1+ix}{1-ix}\right)^n = \frac{1+ia}{1-ia}$ a $\in R$ are real and distinct.

**COMPREHENSION TYPE**

The equation $z^n - 1 = 0$ has $n$ roots which are called the nth roots of unity. The $n$, nth rotos of unity are $1, \alpha, \alpha^2, ,\alpha^{n-1}$ which are in GP, where $\alpha = \cos\left(\frac{2\pi}{n}\right) + i\sin\left(\frac{2\pi}{n}\right)$; $i = \sqrt{-1}$ then we have following results:

(i) $\sum_{r=0}^{n} \alpha^2 = 0$ or $\sum_{r=0}^{n-1} \cos\left(\frac{2\pi r}{n}\right) = 0$ and

$$\sum_{n=0}^{\infty-1} \sin\left(\frac{2\pi}{n}\right) = 0$$

(ii) $z^n - 1 = \sum_{r=0}^{n-1} \alpha^r = 0$

(iii) $\prod_{r=0}^{n-1} \alpha^{\mathrm{r}} = (-1)^{\mathrm{n}-1}$

(iv) $\sum_{r=0}^{n-1} \alpha^{kr} = \begin{cases} n, \text{ if } k \text{ is multiple of } n \\ 0, \text{ if } k \text{ is not multiple of } n \end{cases}$

**43.** The value of $\sum_{r=1}^{n-1} \frac{1}{(2-\alpha^r)}$ is equal to

(A) $(n-2)2^n$ (B) $\frac{(\mathrm{n}-2)2^{\mathrm{n}-1}+1}{2^{\mathrm{n}}-1}$ (C) $\frac{(\mathrm{n}-2)2^{\mathrm{n}-1}}{2^{\mathrm{n}}-1}$ (D) $\frac{(\mathrm{n}-1)2^{\mathrm{n}-1}}{2^{\mathrm{n}}-1}$

**44.** The algebraic sum of perpendicular distances from the points $1, \alpha, \alpha^2, \alpha^3, \ldots \alpha^{n-1}$ to the line $\bar{a}z + a\bar{z} + \mathrm{b} = 0$, (where a is complex number and b is real) is equal to

(A) $\frac{\mathbf{n}}{2|\mathrm{a}|}$ (B) $\frac{\mathrm{n}|\mathrm{b}|}{2\mathrm{a}}$ (C) $\frac{\mathrm{nb}}{|\mathrm{a}|}$ (D) $\frac{nb}{2|a|}$

**MATCH THE COLUMN**

**45.** **Column-I**

**(A)** If $|a| < 1; \lambda_1 \geq 0$ for $i = 1,2,3, \ldots\ldots n$ and $\lambda_1 + \lambda_2 + \cdots + \lambda_n = 1$ and $\omega$ is a complex cube root of unity, then $|\lambda_1 a_1 \omega + \lambda_2 a_2 \omega^2 + \cdots + \lambda_n a_n \omega^2|$ cannot exceed

**(B)** If Re $(z) < 0$, then the value of $(1 + z + z^2 + \cdots + z^n)$ can not exceed

**(C)** If $\omega(\neq 1)$ is a cube root of unity, then $\frac{1}{\sqrt{3}}|1 + 2\omega + 3\omega^2 + \cdots + 3n\omega^{3n-1}|$ $(\mathrm{n} \in \mathrm{N})$ cannot exceed

**(P)** $|z|^2 + \frac{1}{|z|}$

**(Q)** 2

**(R)** $n$

**(S)** 1

**(T)** $|a_1| + |a_2| + \cdots |a_n|$

**EXERCISE-IV (Level-I)**

**1.** If $z^2 + z + 1 = 0$, where $z$ is a complex number, then the value of $\left(z+\frac{1}{z}\right)^2 + \left(z^2+\frac{1}{z^2}\right)^2 + \left(z^3+\frac{1}{z^3}\right)^2 + + \left(z^6+\frac{1}{z^6}\right)^2$ is- **[AIEEE 2006]**

(A) 54 (B) 6 (C) 12 (D) 18

**2.** If $|z+4| \leq 3$, then the maximum and minimum value of $|z+1|$ are - **[AIEEE - 2007]**

(A) 4,1 (B) 4,0 (C) 6,1 (D) 6,0

**3.** The conjugate of a complex number is $\frac{1}{i-1}$. Then that complex number is- **[AIEEE - 2008]**

(A) $\frac{1}{i+1}$ (B) $\frac{-1}{i+1}$ (C) $\frac{1}{i-1}$ (D) $\frac{-1}{i-1}$

**4.** If $\left|Z-\frac{4}{Z}\right| = 2$, then the maximum value of $|Z|$ is equal to : **[AIEEE 2009]**

(A) $\sqrt{5}+1$ (B) 2 (C) $2+\sqrt{2}$ (D) $\sqrt{3}+1$

**5.** The number of complex numbers $Z$ such that $|z-1| = |z+1| = |z-i|$ equals -**[AIEEE 2010]**

(A) 0 (B) 1 (C) 2 (D) 3

**6.** If $\alpha$ and $\beta$ are the roots of the equation $x^2 - x + 1 = 0$, then $\alpha^{200} + \beta^{200m} =$ **[AIEEE - 2010]**

(A) -2 (B) -1 (C) 1 (D) 2

**7.** Let $\alpha, \beta$ be real and $z$ be a complex number. If $z^2 + \alpha z + \beta = 0$ has two distinct roots on the line Re $z = 1$, then it is necessary that : **[AIEEE 2011]**

(A) $\beta \in (0,1)$ (B) $\beta \in (-1,0)$ (C) $|\beta| = 1$ (D) $\beta \in (1,\infty)$

**8.** If $z \neq 1$ and $\frac{z^2}{z-1}$ is real, then the point represented by the complex number $z$ lies:

**[AIEEE 2012]**

(A) either on the real axis or on a circle not passing through the origin.

(B) On the imaginary axis.

(C) either on the real axis or on a circle passing through the origin.

(D) on a circle with centre at the origin.

**9.** If $z$ is a complex number of unit modulus and argument $\theta$, then arg $\left(\frac{1+z}{1+\bar{z}}\right)$ equals

**[JEE-MAIN 2013]**

(A) $\theta$ (B) $\pi-\theta$ (C) $-\theta$ (D) $\frac{\pi}{2}-\theta$

**10.** If $z$ is a complex number such that $|z| \geq 2$, then the minimum value of $\left|z+\frac{1}{2}\right|$ :

**[JEE-MAIN 2014]**

(A) is equal to $\frac{5}{2}$

(B) lies in the interval $(1,2)$

(C) is strictly greater than $\frac{5}{2}$

(D) is strictly greater than $\frac{3}{2}$ but less than $\frac{5}{2}$

**11.** A complex number $z$ is the said to be unimodular if $|z|=1$. Suppose $z_1$ and $z_2$ are complex number such that $\frac{z_1-2z_2}{2-z_1\bar{z}_2}$ is unimodular and $z_2$ is not unimodular. **[JEE-MAIN 2015]**

Then the point $z_1$ lies on a :

(A) circle of radius 2 .

(B) circle of radius $\sqrt{2}$.

(C) straight line parallel to $x$-axis

(D) straight line parallel to $y$-axis

**12.** A value of $\theta$ for which $\frac{2+3i\sin\theta}{1-2i\sin\theta}$ is purely imaginary, is : **[JEE - MAIN 2016]**

(A) $\frac{\pi}{6}$ (B) $\sin^{-1}\left(\frac{\sqrt{3}}{4}\right)$ (C) $\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$ (D) $\frac{\pi}{3}$

**13.** Let $\omega$ be a complex number such that $2w+1=z$ where $z=\sqrt{-3}$. If $\begin{vmatrix} 1 & 1 & 1 \\ 1 & -\omega^2-1 & \omega^2 \\ 1 & \omega^2 & \omega^7 \end{vmatrix}=3k$,

then $k$ is equla to : **[JEE - MAIN 2017]**

(A) $-z$ (B) $z$ (C) -1 (D) 1

**EXERCISE-IV (Level-II)**

**1.** If $w = \alpha + i\beta$ where $\beta \neq 0$ and $z \neq 1$, satisfies the condition that $\frac{w-\bar{w}z}{1-z}$ is purely real, then the set of the values of $z$ is **[JEE 2006, 3]**

(A) $\{z: |z| = 1\}$ (B) $\{z: z = \bar{z}\}$ (C) $\{z: z \neq 1\}$ (D) $\{z: |z| = 1, z \neq 1\}$

**2.** (a) A man walks a distance of 3 units from the origin towards the north-east (N45°E) direction. Form there, he walks a distance of 4 units towards the north-west (N45°W) direction to reach a point $P$. Then the position of P in the Argand plane is **[JEE 2007, 3+3]**

(A) $3e^{i\pi/4} + 4i$ (B) $(3 - 4i)e^{i/4}$ (C) $(4 + 3i)e^{in/4}$ (D) $(3 + 4i)e^{i/4}$

(b) If $|z| = 1$ and $z \neq \pm 1$, then all the values of $\frac{z}{1-z^2}$ lie on

(A) a line not passing through the origin (B) $|z| = \sqrt{2}$

(C) the $x$-axis (D) the $y$-axis

**3.** (a) A particle $P$ starts from the point $z_0 = 1 + 2i$, where $i = \sqrt{-1}$. It moves first horizontally away from origin by 5 units and then vertically away from origin by 3 units to reach a point $z_1$. From $z_1$ the particle moves $\sqrt{2}$ units in the direction of the vector $\hat{\imath} + \hat{\jmath}$ and then it moves through an angle $\frac{\pi}{2}$ in anticlockwise direction on a circle with centre at origin, to reach a point $z_2$. The point $z_2$ is given by **[JEE 2008, 3+4+4+4]**

(A) $6 + 7i$ (B) $-7 + 6i$ (C) $7 + 6i$ (D) $-6 + 7i$

(b) Comprehension (3 questions together)

Let A, B, C be three sets of complex numbers as defined below

$A = \{z: \text{lm } z \geq 1\}$

$B = \{z: |z - 2 - i| = 3\}$

$C = \{z: \text{Re}((1 - i)z) = \sqrt{2}\}$.

(i) The number of elements in the set $A \cap B \cap C$ is

(A) 0 (B) 1 (C) 2 (D) $\infty$

(ii) Let z be any point in $A \cap B \cap C$. Then $|z + 1 - i|^2 + |z - 5 - i|^2$ lies between

(A) 25&29 (B) 30&34 (C) 35&39 (D) 40&44

(iii) Let z be any point in $A \cap B \cap C$ and let w be any point satisfying $|w - 2 - i| < 3$. Then, $|z| - |w| + 3$ lies between

(A) $-6 \& 3$ (B) $-3 \& 6$ (C) $-6 \& 6$ (D) $-3 \& 9$

**4.** Let $z = x + iy$ be a complex number where $x$ and $y$ are integers. Then the area of the rectangle whose vertices are the roots of the equation $\overline{ZZ}^3 + z\bar{z}^3 = 350$ is **[JEE 2009]**

(A) 48 (B) 32 (C) 40 (D) 80

**5.** Let $z = \cos\theta + i\sin\theta$. Then the value of $\sum_{m=1}^{15} \text{Im}\,(z^{2m-1})$ at $\theta = 2°$ is

(A) $1/\sin 2°$ (B) $1/3\sin 2°$ (C) $1/2\sin 2°$ (D) $1/4\sin 2°$

**6.** Let p and q be real numbers such that $p \neq 0, p^3 \neq q$ and $p^3 = -q$. If $\alpha$ and $\beta$ are nonzero complex numbers satisfying $\alpha + \beta = -p$ and $\alpha^3 + \beta^3 = q$, then a quadratic equation having $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$ as its roots is **[JEE 2010]**

(A) $(p^3 + q)x^2 - (p^3 + 2q)x + (p^3 + q) = 0$

(B) $(p^3 + q)x^2 - (p^3 - 2q)x + (p^3 + q) = 0$

(C) $(p^3 - q)x^2 - (5p^3 - 2q)x + (p^3 - q) = 0$

(D) $(p^3 - q)x^2 + (5p^3 + 2q)x + (p^3 - q) = 0$

**7.** Let $\omega$ be a complex cube root of unity with $\omega \neq 1$. A fair die is thrown three times. If $r_1, r_2$ and $r_3$ are the numbers obtained on the die, then the probability that $\omega^{\tau_1} + \omega^{t_2} + \omega^{t_1} = 0$ is **[JEE 2010]**

(A) $\frac{1}{18}$ (B) $\frac{1}{9}$ (C) $\frac{2}{9}$ (D) $\frac{1}{36}$

**8.** Let $z_1$ and $z_2$ be two distinct complex numbers and let $z = (1 - t)z_1 + tz_2$ for some real number $t$ with $0 < t < 1$. If $Arg(w)$ denotes the principal argument of a nonzero complex number $w$, then **[JEE 2010]**

(A) $|z - z_1| + |z - z_2| = |z_1 - z_2|$ (B) $\text{Arg}\,(z - z_1) = \text{Arg}\,(z - z_2)$

(C) $\begin{vmatrix} z - z_1 & \bar{z} - \bar{z}_1 \\ z_2 - z_1 & \bar{z}_2 - \bar{z}_1 \end{vmatrix} = 0$ (D) $\text{Arg}\,(z - z_1) = \text{Arg}\,(z_2 - z_1)$

**9.** Let $\omega$ be the complex number $\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}$. Then the number of distinct complex numbers $z$ satisfying $\begin{vmatrix} z+1 & \omega & \omega^2 \\ \omega & z+\omega^2 & 1 \\ \omega^2 & 1 & z+\omega \end{vmatrix} = 0$ is equal to **[JEE 2010]**

**10.** Match the statement in Column I with those in Column II.

[Note: Here z takes values in the complex plane and Im $z$ and Re $z$ denote, respectively, the imaginary part and the real part of z]. **[JEE 2010]**

| **Column-I** | **Column-II** |
|---|---|
| (A) The set of points z satisfying $\lvert z - i\lvert z\rvert\rvert = \lvert z + i\lvert z\rvert\rvert$ is contained in or equal to | (P) an ellipse with eccentricity $\frac{4}{5}$ |
| (B) The set of points z satisfying $\lvert z+4\rvert + \lvert z-4\rvert = 10z$ is contained in or equal to | (Q) the set of points satisfying $\text{Im } z = 0$ |
| (C) If $\lvert w\rvert = 2$, then the set of points $z = w - \frac{1}{w}$ is contained in or equal to | (R) the set of points z satisfying $\lvert \text{Im } z\rvert \le 1$ |
| | (S) the set of points $z$ satisfying $\lvert \text{Re } z\rvert \le 2$ |
| (D) If $\lvert w\rvert = 1$, then the set of points $z = w + \frac{1}{w}$ is contained in or equal to | (T) the set of points z satisfying $\lvert z\rvert < 3$ |

**Paragraph for Question Nos. 11 to 13**

Let a, b and c be three real numbers satisfying

$$[a \quad b \quad c]\begin{bmatrix} 1 & 9 & 7 \\ 8 & 2 & 7 \\ 7 & 3 & 7 \end{bmatrix} = [0 \quad 0 \quad 0] \text{……..(E)}$$ **[JEE 2011]**

**11.** If the point P(a, b, c), with reference to (E), lies on the plane $2x + y + z = 1$, then the value of $7a + b + c$ is

(A) 0 (B) 12 (C) 7 (D) 6

**12.** Let $\omega$ be a solution of $x^3 - 1 = 0$ with Im $(\omega) > 0$, If a $= 2$ with $b$ and $c$ satisfying (E), then the value of $\frac{3}{\omega^4} + \frac{1}{\omega^3} + \frac{3}{\omega^7}$ is equal to

(A) -2 (B) 2 (C) 3 (D) -3

**13.** Let $b = 6$, with a and $c$ satisfying (E). If $\alpha$ and $\beta$ are the roots of the quadratic equation $ax^2 + bx + c = 0$, then $\sum_{n=0}^{\infty}\left(\frac{1}{\alpha} + \frac{1}{\beta}\right)^n$ is

(A) 6 (B) 7 (C) 6/7 (D) $\infty$

**14.** If $z$ is any complex number satisfying $|z - 3 - 2i| \leq 2$, then the minimum value of $|2z - 6 + 5i|$ is **[JEE 2011]**

**15.** Let $\omega \neq 1$ be cube root of unity and $S$ be the set of all non-singular matrices of the form $\begin{bmatrix} 1 & a & b \\ \omega & 1 & c \\ \omega^2 & \omega & 1 \end{bmatrix}$ where each of $a, b$ and $c$ is either $\omega$ or $\omega^2$. Then the number of distinct matrices in the set S is **[JEE 2011]**

(A) 2 (B) 6 (C) 4 (D) 8

**16.** Let $\omega = e^{i/3}$, and a, b, c, x, y, z be non-zero complex numbers such that a + b + c = x

$a + b\omega + c\omega^2 = y$

$a + b\omega^2 + c\omega = z$

Then the value of $\frac{|x|^2+|y|^2+|z|^2}{|a|^2+|b|^2+|c|^2}$ is **[JEE 2011]**

**17.** Match the statements given in Column I with the values given in Column II **[JEE 2011]**

| **Column – I** | **Column - II** |
|---|---|
| (A) If $\vec{a} = \hat{j} + \sqrt{3}\hat{k}, \vec{b} = \hat{j} + \sqrt{3}\hat{k}$ and $\vec{c} = 2\sqrt{3}\hat{k}$ form a triangle, then the internal angle of the triangle between $\vec{a}$ and $\vec{b}$ is | (P) $\frac{\pi}{6}$ |
| (B) If $\int_a^b (f(x) - 3x)dx = a^2 - b^2$, then the value of $f\left(\frac{\pi}{6}\right)$ is | (Q) $\frac{2\pi}{3}$ |
| (C) The value of $\frac{\pi^2}{\ln 3}\int_{\pi/6}^{5/6}(\sec(\pi x)dx$ is | (R) $\frac{\pi}{3}$ |

(D) The mximum value of $\left|\text{Arg}\left(\frac{1}{1-z}\right)\right|$ for $|z| = 1, z \neq 1$ is given by

(S) $\pi$

(T) $\frac{\pi}{2}$

**18.** Match the statements given in Column I with the intervals/union of intervals given in Column II **[JEE 2011]**

**Column – I**

(A) The $\left\{\text{Re}\left(\frac{2iz}{1-x^2}\right) : zix = \text{comples}\right.$ number, $| \ z - t, z + \cdots + 1\}$ is

(B) The domain of the function $f(x) = \sin^{-1}\left(\frac{8(3)^{x-2}}{1-3^{2(x-1)}}\right)$ is

(C) If $f(\theta) = \begin{vmatrix} 1 & \tan\ \theta & 1 \\ -\tan\ \theta & 1 & \tan\ \theta \\ -1 & -\tan\ \theta & 1 \end{vmatrix}$, then the set $\left\{f(\theta): 0 \leq \theta < \frac{\pi}{2}\right\}$ is

(D) If $f(x) = x^{3/2}(3x - 10), x \geq 0,$ then $f(x)$ is increasing in

**Column - II**

(P) $(-\infty, -1) \cup (1, \infty)$

(Q) $(-\infty, 0) \cup (0, \infty)$

$(R)[2, \infty)$

(S) $(-\infty, -1] \cup [1, \infty)$

(T) $(-\infty, 0] \cup [2, \infty)$

**19.** Let $z$ be a complex number such that the imaginary part of $z$ is nonzero and $a = z^2 + z + 1$ is real. Then a cannot take the value **[JEE 2012]**

(A) -1 (B) $\frac{1}{3}$ (C) $\frac{1}{2}$ (D) $\frac{3}{4}$

**20.** Let complex numbers $\alpha$ and $\frac{1}{\bar{\alpha}}$ lie on circles $(x - x_0)^2 + (y - y_0)^2 = r^2$ and $(x - x_0)^2 + (y - y_0)^2 = 4r^2$, respectively. If $z_0 = x_0 + y_0$ satisfies the equation $2|z_0|^2 = r^2 + 2$, then $|\alpha| =$ **[JEE 2013]**

(A) $\frac{1}{\sqrt{2}}$ (B) $\frac{1}{2}$ (C) $\frac{1}{\sqrt{7}}$ (D) $\frac{1}{3}$

**21.** Let $w = \frac{\sqrt{3}+i}{2}$ and $P = \{w^n : n = 1,2,3, \dots \dots\}$. Further $\mathrm{H}_1 = \left\{\mathrm{z} \in \mathrm{C}: \mathrm{Rez} > \frac{1}{2}\right\}$ and $\mathrm{H}_2 = \left\{z \in C: Rez < \frac{-1}{2}\right\}$, where $C$ is the set of all complex numbers. If $\mathrm{z}_1 \in \mathrm{P} \cap \mathrm{H}_1, \mathrm{z}_2 \in \mathrm{P} \cap \mathrm{H}_2$ and O represents the origin, then $\angle \mathrm{z}_1 \mathrm{O} \mathrm{z}_2 =$ **[JEE 2013]**

(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{6}$ (C) $\frac{2\pi}{3}$ (D) $\frac{5\pi}{6}$

**22.** Let $\omega$ be a complex cube root of unity with $\omega \neq 1$ and $P = [P_{ij}]$ be a $n \times n$ matrix with $P_{ij} = \omega^{i+j}$. Then $P^2 \neq 0$, when $n =$ **[JEE 2013]**

(A) 57 (B) 55 (C) 58 (D) 56

**Paragraph for Question Nos. 23 to 24**

Let $S = S_1 \cap S_2 \cap S_3$, where$S_1 = \{z \in C : |z| < 4\}, = \left\{z \in C : \text{Im}\left[\frac{z-1+\sqrt{3}i}{1-\sqrt{3}i}\right] > 0\right\}$ and $S_3 = \{z \in C : \text{Re } z > 0\}$. **[JEE 2013]**

**23.** Area of S =

(A) $\frac{10\pi}{3}$ (B) $\frac{20\pi}{3}$ (C) $\frac{16\pi}{3}$ (D) $\frac{32\pi}{3}$

**24.** $\min_z |t - 3i - z| =$

(A) $\frac{2-\sqrt{3}}{2}$ (B) $\frac{2+\sqrt{3}}{2}$ (C) $\frac{3-\sqrt{3}}{2}$ (D) $\frac{3+\sqrt{3}}{2}$

**25.** Let $z_1 = \cos\left(\frac{2k\pi}{10}\right) + i\sin\left(\frac{2k\pi}{10}\right)$; $k = 1,2,\ldots,9$ **[JEE 2014]**

| **List I** | **List II** |
|---|---|
| (P) For each $z_2$ there exists a $z$ such that $z_1 \cdot z_2 = 1$ | (1) true |
| (Q) There exists a $k \in \{1,2,\ldots,9\}$ such that $z_1 \cdot z = z_k$ has no solution z in the set of complex numbers. | (2) False |
| (R) $\frac{|1-z_1||1-z_2|\ldots|1-z_9|}{10}$ equal | (3) 1 |
| (S) $1 - \sum_{k=1}^{9} \cos\left(\frac{2k\pi}{10}\right)$ equal | (4) 2 |

| | **P** | **Q** | **R** | **S** |
|---|---|---|---|---|
| (A) | 1 | 2 | 4 | 3 |
| (B) | 2 | 1 | 3 | 4 |
| (C) | 1 | 2 | 3 | 4 |
| (D) | 2 | 1 | 4 | 3 |

**26.** **Column I** **Column II** **[JEE 2015]**

(A) In $R^2$, if the magnitude of the projection vector of the vector $\alpha\hat{\mathbf{i}}+\beta\hat{\mathbf{j}}$ on $\sqrt{3}\hat{\imath}+\hat{\jmath}$ is $\sqrt{3}$ and if $\alpha=2+\sqrt{3}\beta$, then possible valué(s) of $|\alpha|$ is (are)

(B) Let a and b be real numbers such that the function $f(x)=\begin{cases}-3ax^2-2, & x<1\\ bx+a^2, & x\geq 1\end{cases}$ is differentiable for all $x\in R$. Then possible value(s) of a is (are)

(C) Let $\omega\neq 1$ be a complex cube root of unity. If $(3-3\omega+2\omega^2)^{4+3}+(2+3\omega-3\omega^2)^{4+3}$ $+(-3+2\omega+3\sigma^2)^{40-3}=0$, then possible value(s) of $n$ is (are)

(D) Let the harmonic mean of two positive real numbers $a$ and $b$ be 4 . If $q$ is positive real number such that a, $5,q,b$ is an arithmetic number such that a, 5, q, b is an arithmetic progression, then the value(s) of $|q-a|$ is (are),

(P) 1

(Q) 2

(R) 3

(S) 4

(T) 5

**27.** For any integer $k$, let $\alpha_2=\left(\frac{k\pi}{7}\right)+i\sin\left(\frac{k\pi}{7}\right)$, where $\mathrm{i}=\sqrt{-1}$. The value of the expression $\frac{\sum_{k=1}^{12}|\alpha_{k+1}-\alpha_k|}{\sum_{k=1}^{3}|\alpha_{kk-1}-\alpha_{4k-2}|}$ is **[JEE 2015]**

**28.** Let $\mathrm{a},\mathrm{b}\in\mathrm{R}$ and $\mathrm{a}^2+\mathrm{b}^2\neq 0$. Suppose $\mathrm{S}=\left\{z\in C:\frac{1}{a+ibt},t\in R,t\neq 0\right\}$; where $i=\sqrt{-1}$. If $z=x+iy$ and $z\in S$, then $(x,y)$ lies on **[JEE 2016]**

(A) the circle with radius $\frac{1}{2a}$ and centre $\left(\frac{1}{2a},0\right)$ for $a>0,b\neq 0$

(B) the eirele with radius $-\frac{1}{2a}$ and centre $\left(-\frac{1}{2a},0\right)$ for $a<0,b\neq 0$

(C) the $x$-axis for $a\neq 0,b=0$

(D) the $y$-axis for $a=0,b\neq 0$

**29.** Let $z = \frac{-1+\sqrt{3}i}{2}$, where $i = \sqrt{-1}$, and $r, s \in [1, 2,3]$. Let $P = \begin{bmatrix} (-z)^t & z^{2t} \\ z^{2*} & z' \end{bmatrix}$ and $I$ be the identity matrix of order 2 . Then the total number of ordered pairs $(r, s)$ for which $P^2 = -1$ is

**[JEE 2016]**

**30.** Let $a, b, x$ and $y$ be real numbers such that $a - b = 1$ and $y \neq 0$. If the complex number $z = x +$ iy satisfies Im $\left(\frac{az+b}{z+1}\right) - y$, then which of the following is(are) possible value(s) fo x ?

**[JEE 2017]**

(A) $1 - \sqrt{1 + y^2}$

(B) $-1 - \sqrt{1 - y^2}$

(C) $1 + \sqrt{1 + y^2}$

(D) $-1 + \sqrt{1 - y^2}$

**31.** Let $s, t, r$ be non - zero complex numbers and $L$ be the set of solutions $z = x + iy (x, y \in R, i = \sqrt{-1})$ of the equation $sz + t\bar{z} + r = 0$, where $\bar{z} = x - iy$. Then, which of the following statement (s) is (are) TRUE? **[JEE ADVANCE 2018]**

(A) If L has exactly one element, then $|s| \neq |t|$

(B) If $|s| = |t|$, then $L$ has infinitely many elements

(C) The number of elements in L $(z: |z - 1 + 1| + 5\}$ is at most 2

(D) If L has more than one element, then L has infinitely many elements

**32.** For a non-zero complex number $z$, let arg $(z)$ denote the principal argument with $-\pi <$ arg $(z) \leq \pi$. Then, which of the following statement(s) is (are) False? **[JEE ADVANCE 2018]**

(A) arg $(-1 - i) = \frac{\pi}{4}$, where $i = \sqrt{-1}$

(B) The function $f: R \to (-\pi, \pi]$, defined by $f(t) = \arg(-1 + it)$ for all $t \in R$, is continuous at all points of $R$, where $i = \sqrt{-1}$

(C) For any two non-zero complex numbers $z_1$ and $z_2$, arg $\left(\frac{z_1}{z_2}\right) - \arg(z_1) + \arg(z_2)$ is an integer multiple of $2\pi$

(D) For any three given distinct complex numbers $z_1, z_2$ and $z_3$, the locus of the point $z$ satisfying the condition

arg $\left(\frac{(z-z_1)(z_2-z_3)}{(z-z_3)(z_2-z_1)}\right) = \pi$ lies on a straight line

**33.** Let S be the set of all complex numbers z satisfying $|z^2 + z + 1| = 1$. Then which of the following statements is/are TRUE? **[JEE ADVANCE 2020]**

(A) $\left|z + \frac{1}{2}\right| \leq \frac{1}{2}$ for all $z \in S$

(B) $|z| \leq 2$ for all $z \in S$

(C) $\left|z + \frac{1}{2}\right| \geq \frac{1}{2}$ for all $z \in S$

(D) The set S has exactly four elements

**34.** For a complex number $z$, let Re $(z)$ denote the real part of $z$. Let $S$ be the set of all complex numbers $z$ satisfying $z^4 - |z|^4 = 4iz^2$, where $i = \sqrt{-1}$. Then the minimum possible value of $|z_1 - z_2|^2$, where $z_1, z_2 \in S$ with Re $(z_1) > 0$ and Re $(z_2) < 0$, is **[JEE ADVANCE 2020]**

**35.** Let $\theta_1, \theta_2, \ldots, \theta_{10}$ be positive valued angles (in radian) such that $\theta_1 + \theta_2 + \cdots + \theta_{10} = 2\pi$. Define the complex numbers $z_1 = e^{in}, z_k = z_{k-1}e^{ik}$ for $k = 2,3, \ldots, 10$, where $i = \sqrt{-1}$. Consider the statements $P$ and $Q$ given below: **[JEE ADVANCE 2021]**

$P: |z_2 - z_1| + |z_3 - z_2| + \cdots . + |z_{10} - z_9| + |z_1 - z_{10}| \leq 2\pi$

$Q: |z_2^2 - z_1^2| + |z_3^2 - z_2^2| + \cdots . + |z_{10}^2 - z_9^2| + |z_1^2 - z_{10}^2| \leq 4\pi$

Then

(A) $P$ is TRUE and $Q$ is FALSE

(B) $Q$ is TRUE and P IS FALSE

(C) Both $P$ and $Q$ are TRUE

(D) Both $P$ and $Q$ are FALSE

**36.** For any complex number $w = c + id$, let arg $(w) \in (-\pi, \pi]$, where $i = \sqrt{-1}$, Let $\alpha$ and $\beta$ be real numbers such that for all complex numbers $z = x + iy$ satisfying arg $\left(\frac{z+\alpha}{z+\beta}\right) = \frac{\pi}{4}$, the ordered pair $(x, y)$ lies on the circle $x^2 + y^2 + 5x - 3y + 4 = 0$ Then which of the following statements is (are) TRUE ? **[JEE ADVANCE 2021]**

(A) $\alpha = -1$

(B) $\alpha\beta = 4$

(C) $\alpha\beta = -4$

(D) $\beta = 4$

**37.** Let $z$ be a complex number with non-zero imaginary part. If $\frac{2+3z+4z^2}{2-3z+4z^2}$ is a real number, then the value of $|z|^2$ is **[JEE ADV. 2022]**

**38.** Let $\bar{z}$ denote the complex conjugate of a complex number $z$ and let $i = \sqrt{-1}$. In the set of complex numbers, the number of distinct roots of the equation $\bar{z} - z^2 = i(\bar{z} + z^2)$ is .

**[JEEADV.2022]**

**39.** Let $\bar{z}$ denote the complex conjugate of a complex number $z$. If $z$ is a non-zero complex number for which both real and imaginary parts of $(\bar{z})^2 + \frac{1}{z^2}$ are integers, then which of the following is/are possible value(s) of $|z|$? **[JEE ADV. 2022]**

(A) $\left(\frac{43+3\sqrt{205}}{2}\right)^{\frac{1}{4}}$ (B) $\left(\frac{7+\sqrt{33}}{4}\right)^{\frac{1}{4}}$ (C) $\left(\frac{9+\sqrt{65}}{4}\right)^{\frac{1}{4}}$ (D) $\left(\frac{7+\sqrt{13}}{6}\right)^{\frac{1}{4}}$

## EXERCISE - I

### JEE Main

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **1.** | D | **2.** | A | **3.** | A | **4.** | A | **5.** | A | **6.** | A |
| **7.** | A | **8.** | A | **9.** | C | **10.** | D | **11.** | D | **12.** | C |
| **13.** | D | **14.** | A | **15.** | A | **16.** | A | **17.** | A | **18.** | C |
| **19.** | C | **20.** | C | **21.** | C | **22.** | C | **23.** | B | **24.** | D |
| **25.** | B | **26.** | A | **27.** | C | **28.** | A | **29.** | A | **30.** | C |
| **31.** | D | **32.** | A | **33.** | D | **34.** | D | **35.** | B | **36.** | C |
| **37.** | C | **38.** | B | **39.** | C | **40.** | B | **41.** | A | | |

## EXERCISE - II

### JEE Advance

### Single correct Option - type Questions

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **1.** | A | **2.** | D | **3.** | D | **4.** | A | **5.** | C | **6.** | C |
| **7.** | A | **8.** | A | **9.** | C | **10.** | A | **11.** | B | **12.** | C |
| **13.** | B | **14.** | B | **15.** | C | **16.** | C | **17.** | A | **18.** | C |
| **19.** | C | **20.** | B | **21.** | B | **22.** | D | | | | |

### Multiple correct Option - type Questions

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **1.** | B, D | **2.** | B,C | **3.** | A, B, C, D | **4.** | A, B, C | **5.** | A, C, D |
| **6.** | A, B, C | **7.** | A, C, D | **8.** | A,D | **9.** | A, B | **10.** | B, C, D |

## EXERCISE - III

### Subjective - type Questions

**2.** 3

**3.** **(a)** $x = 1, y = 2$;**(b)** $(2,9)$;**(c)** $(-2,2)$ or $\left(-\frac{2}{3}, -\frac{2}{3}\right)$;**(d)** $(1,1)$ $\left(0, \frac{5}{2}\right)$;

**(e)** $x = K, y = \frac{3K}{2}, K \in R$

**5.** **(a)** 2 **(b)** $-\frac{11}{2}$

**6.** $\left(-\frac{21}{10}, -\frac{5}{6}\right]$

**10.** $-\frac{3}{2} + \frac{3\sqrt{3}}{2} i$

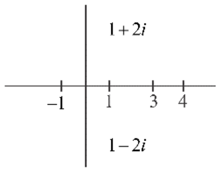
**11.** $z^2 + z + \frac{\sin^2 n\theta}{\sin^2 \theta} = 0$, where $\theta = \frac{2\pi}{2n+1}$

**12.** **(i)** $7A_0 + 7A_7x^7 + 7A_{14}x^{14}$ **(ii)** $x^3 + qx - r = 0$

**13.** -4

**15.** $\pm 1 + i\sqrt{3}, \frac{(\pm\sqrt{3}+i)}{\sqrt{2}}, \sqrt{2}i$

**17.** $z = -1, 3, 1 - 2i, 1 + 2i$




Sum $= 4$, Centroid $= 1$

**18.** 1

**19.** **(c)** 64

**22.** **(a)** $(1,1)$ **(b)** $\left[\frac{n(n+1)}{2}\right]^2 - n$

**23.** **(b)** 1 ; if n is even, $-w^2$; if n is odd

**24.** **(a)** The region between the concentric circles with centre at $(0,2)$& radii 1 and 3 units

**(b)** The part of the complex plane on or above the line $y = 1$

**(c)** a ray emanating from the point $(3 + 4i)$ directed away from the origin & having equation, $\sqrt{3}x - y - 3\sqrt{3} = 0, x > 3$

**25.** $\sqrt{5} + 2 \& \sqrt{5} - 2$

**26.** **(i)** 0,6

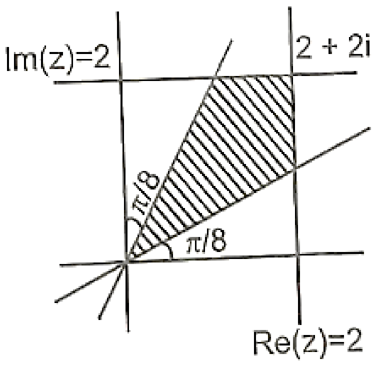
**(ii)** 1,7

**(iii)** 0,5

**27.** **(i)** $|z| = 20$

**(ii)** $OP = OQ = PR = QR = 20$

**28.**




**29.** 5

**30.** $a = b = 2 - \sqrt{3}$

**31.** **(a)** $\frac{\sqrt{3}}{2} - \frac{i}{2}, -\frac{\sqrt{3}}{2} - \frac{i}{2}, i;$

**32.** $\frac{x^2}{64} + \frac{y^2}{48} = 1$

**34.** $(1 - c^2)|z|^2 - 2(a + bc)(\text{Re } z) + a^2 - b^2 = 0$

35. **(a)** $\pi - 2$;

**(b)** ½

37. $\frac{iz}{2} + \frac{1}{2} + i$

38. 51

40. required set is constituted by the angles without their boundaries, whose sides are the straight lines $y = (\sqrt{2} - 1)x$ and $y + (\sqrt{2} + 1)x = 0$ containing the x-axis.

**Comprehension - based Questions**

43. B **44.** D

**Matrix Match - type Questions**

45. (A) $-$Q, S, T; (B) $-$P; (C)-R

**EXERCISE - IV**

**Previous Year's Question**

**JEE Main**

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **1.** | C | **2.** | D | **3.** | B | **4.** | A | **5.** | B | **6.** | C |
| **7.** | D | **8.** | C | **9.** | A | **10.** | B | **11.** | A | **12.** | C |
| **13.** | 1 | | | | | | | | | | |

**JEE Advanced**

**1.** D **2.** (a) D ; (b) D **3.** (a) D;(b) (i) B;(ii) C; (iii) D

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **4.** | A | **5.** | D | **6.** | B | **7.** | C | **8.** | A,C,D | **9.** | 1 |

**10.** (A) $-$Q, R; (B) $-$P; (C) $-$P, S, T;(D) $-$P, Q, R, S

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **11.** | D | **12.** | A | **13.** | B | **14.** | 5 | **15.** | A | **16.** | 3 |

**17.** (A) $-$Q; (B) $-$P; (C) $-$S; (D) $-$T

**18.** (A) $-$S; (B) $-$ T; (C) $-$ R; (D) $-$R

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **19.** | D | **20.** | C | **21.** | C,D | **22.** | B,C,D | **23.** | B | **24.** | C |

**25.** C **26.** (A) $\rightarrow$ P, Q;(B) $\rightarrow$ P, Q;(C) $\rightarrow$ P, Q, S, T;(D) $\rightarrow$ Q, T

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **27.** | 4 | **28.** | A, C,D | **29.** | 1 | **30.** | B,D | **31.** | A,C,D | **32.** | A, B, D |
| **33.** | BC | **34.** | 8 | **35.** | C | **36.** | BD | **37.** | 0.5 | **38.** | 4 |
| **39.** | A | | | | | | | | | | |