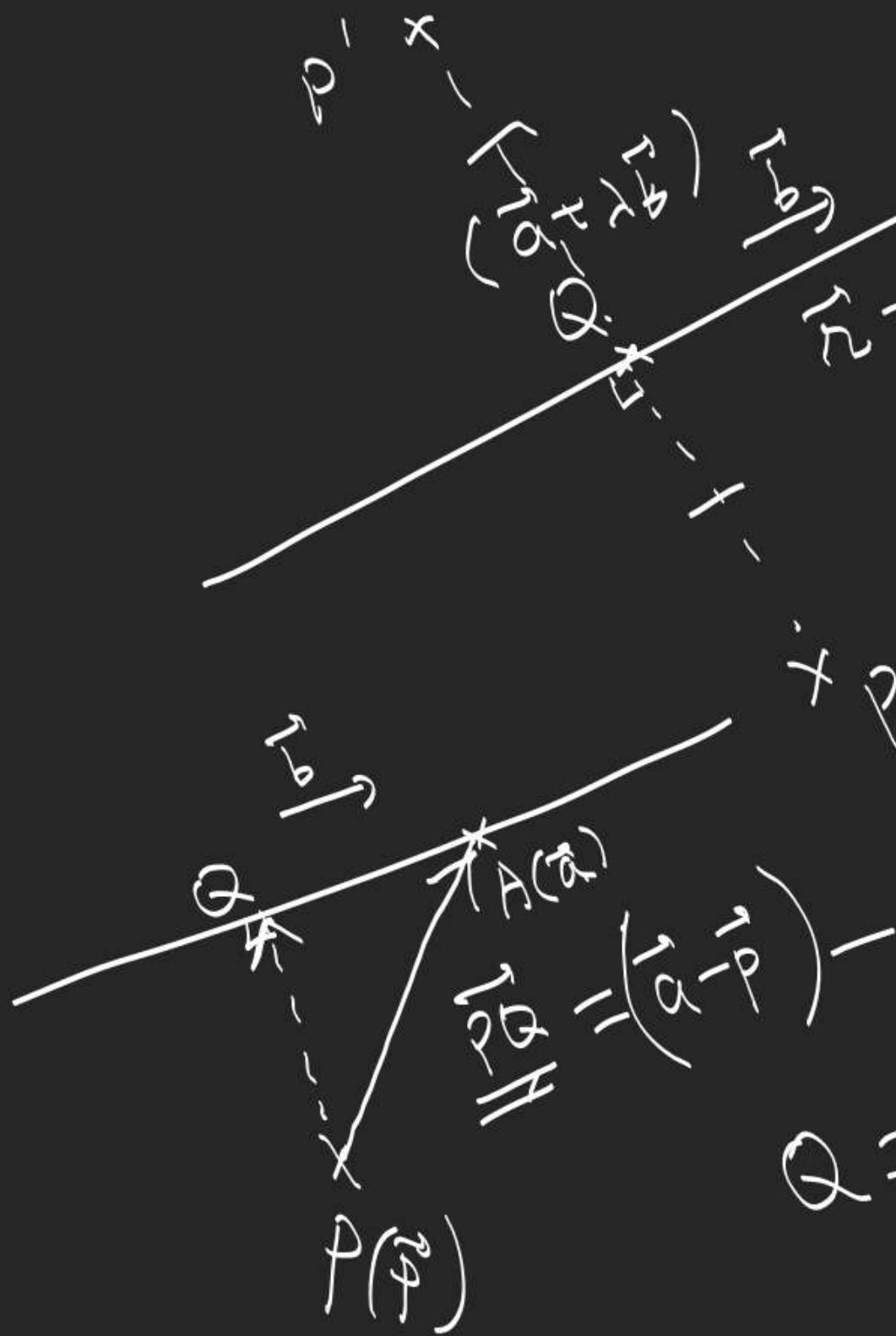


$$f'\left(\frac{x+y}{2}\right) = \frac{f'(x)+f'(y)}{2}$$

$$f(y+\frac{1}{2}) - f(y) = f'\left(y + \frac{1}{2}\right)$$



$$\vec{PQ} \cdot \vec{b} = 0$$

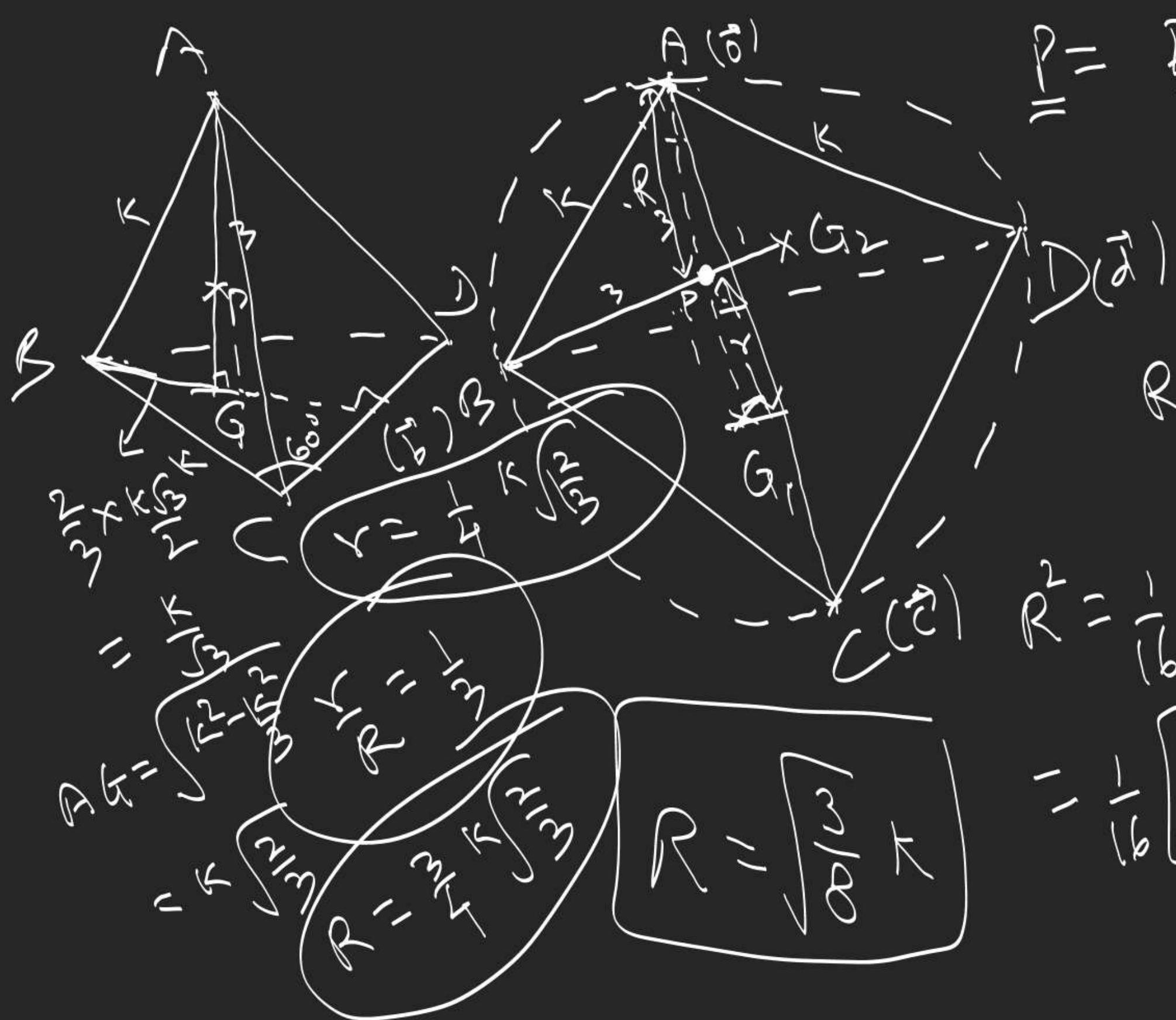
$$(\vec{a} + \lambda \vec{b} - \vec{p}) \cdot \vec{b} = 0$$

17

$$\lambda = \frac{(\vec{p} - \vec{a}) \cdot \vec{b}}{|\vec{b}|^2}$$

$$Q = \vec{a} + \left(\frac{(\vec{p} - \vec{a}) \cdot \vec{b}}{\|\vec{b}\|^2} \right) \vec{b}$$

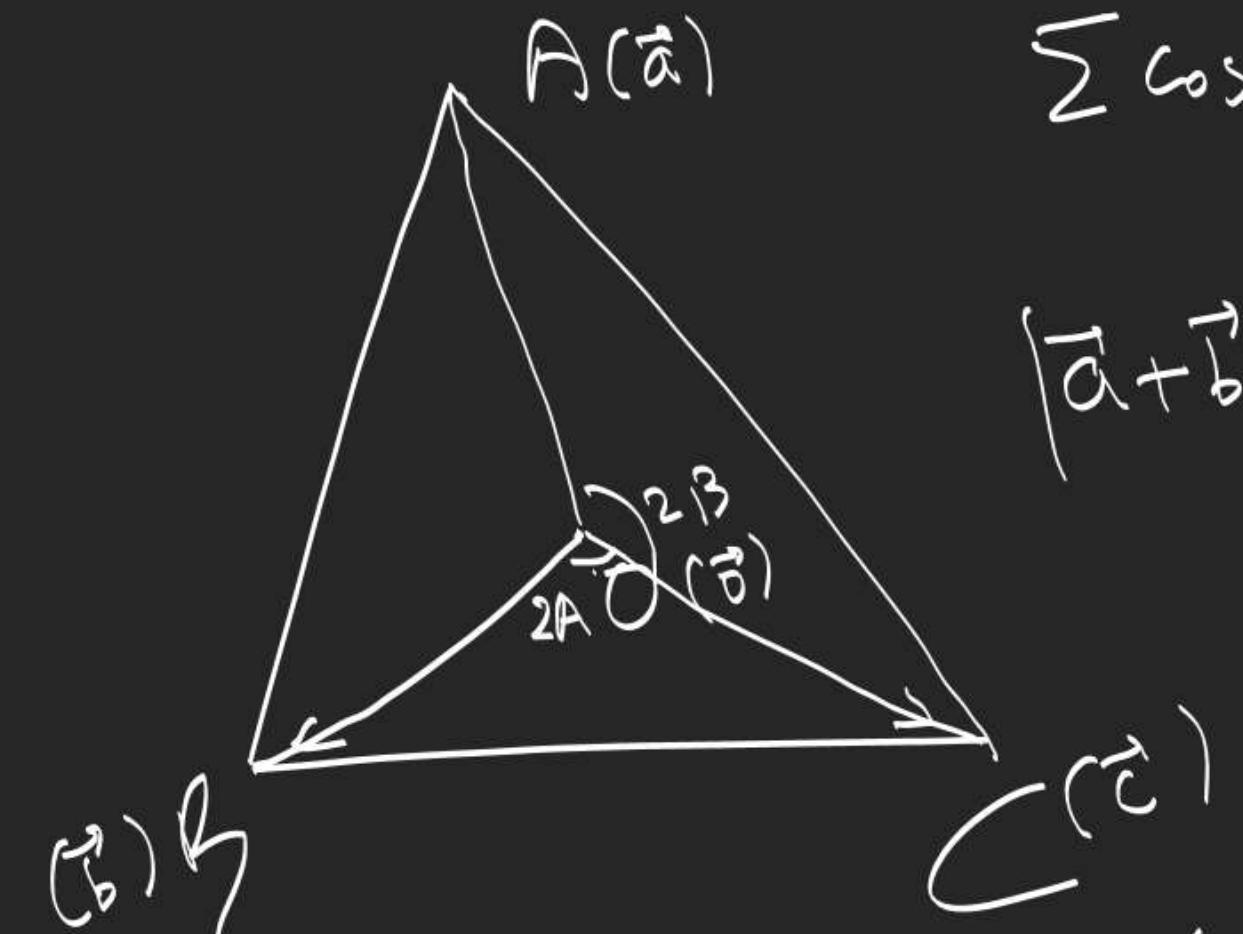
$$P' = \frac{Q(Q) - I(P)}{2} = 2 \left(\vec{a} + \frac{(\vec{p} - \vec{a}) \cdot \vec{b}}{|\vec{b}|^2} \right) -$$



$$\bar{P} = \frac{\bar{b} + \bar{c} + \bar{d}}{4}$$

$$R = \left(\begin{array}{c} A \\ B \end{array} \right) = \left[\begin{array}{c} B + C + D \\ \hline \end{array} \right]$$

$$\begin{aligned}
 R^2 &= \frac{1}{6} \left[\sum |z_i|^2 + 2 \sum \vec{z}_i \cdot \vec{z} \right] \\
 &= \frac{1}{6} \left[3k^2 + 2 \left(kx \left(x \frac{1}{2} \times 3 \right) \right) \right] \\
 &= \frac{6}{5} k^2
 \end{aligned}$$



$$\begin{aligned}
 |\vec{OG}|^2 &= \frac{1}{9} (\vec{a} + \vec{b} + \vec{c})^2 = \frac{1}{9} (3R^2 + 2R \underbrace{\sum_{\text{cos } 2A}}_{(-2\sin^2 A)}) \\
 &= \frac{1}{9} (9R^2 - (\vec{a} + \vec{b} + \vec{c})^2)
 \end{aligned}$$

$$\sum \cos 2A$$

$$|\vec{a} + \vec{b} + \vec{c}|^2 = \sum |\vec{a}|^2 + 2 \sum \vec{a} \cdot \vec{b} \geq 0$$

$$3R^2 + 2R \underbrace{\sum \cos 2A}_{> 0} \geq 0$$

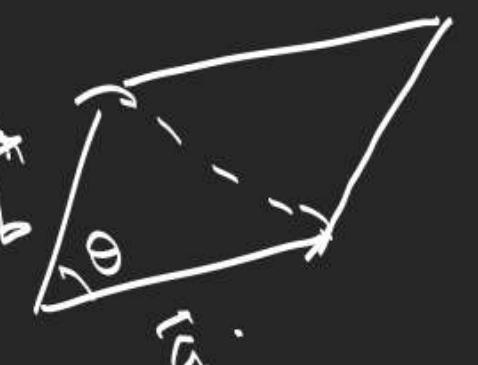
Vector (Cross) Product

$$\vec{a} \times \vec{b} = |\vec{a}| |\vec{b}| \sin\theta \hat{n}$$

$$\theta = \vec{a} \wedge \vec{b}$$

\hat{n} = unit vector along $\vec{a} \times \vec{b}$

$(\vec{a} \times \vec{b})$ = Area of ||gm with
 \vec{a}, \vec{b} as adjacent sides



$$\vec{a} \times \vec{b} = -\vec{b} \times \vec{a}$$

$$\vec{a} \times (\vec{b} + \vec{c}) = \vec{a} \times \vec{b} + \vec{a} \times \vec{c}$$

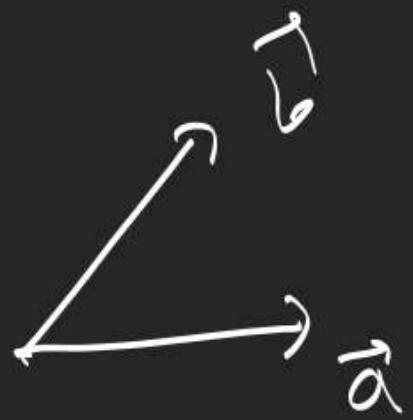
$$\hat{i} \times \hat{i} = \vec{0} = \hat{j} \times \hat{j} = \hat{k} \times \hat{k}$$

$$\hat{i} \times \hat{j} = \hat{k}$$

$$\hat{j} \times \hat{i} = -\hat{k}$$

$$\vec{a} = a_1 \hat{i} + a_2 \hat{j} + a_3 \hat{k}$$

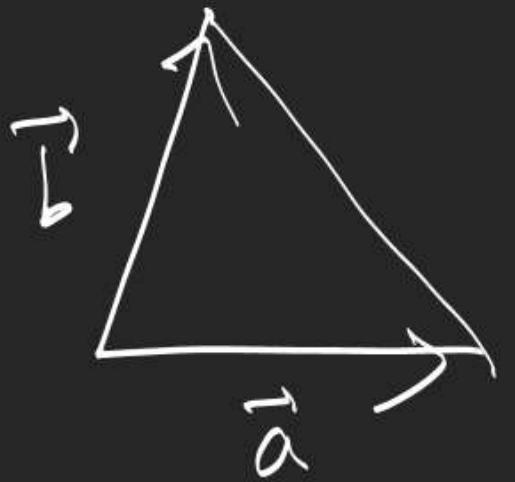
$$\vec{b} = b_1 \hat{i} + b_2 \hat{j} + b_3 \hat{k}$$



$$\vec{a} \times \vec{b} = \vec{0} \\ \Rightarrow \vec{a} = \vec{0} \text{ or } \vec{b} = \vec{0}$$

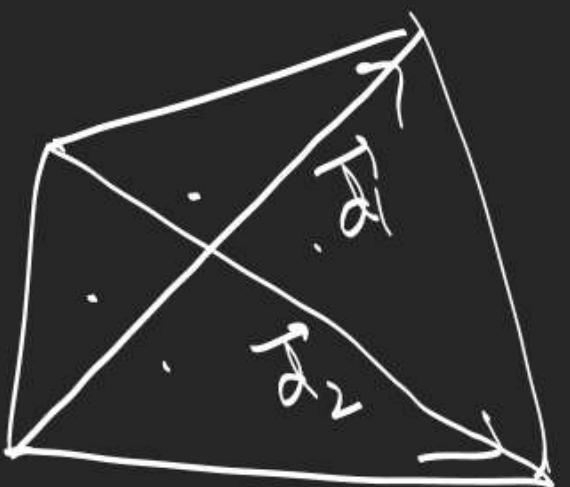
$$\text{or} \\ \vec{a} = \lambda \vec{b}$$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$



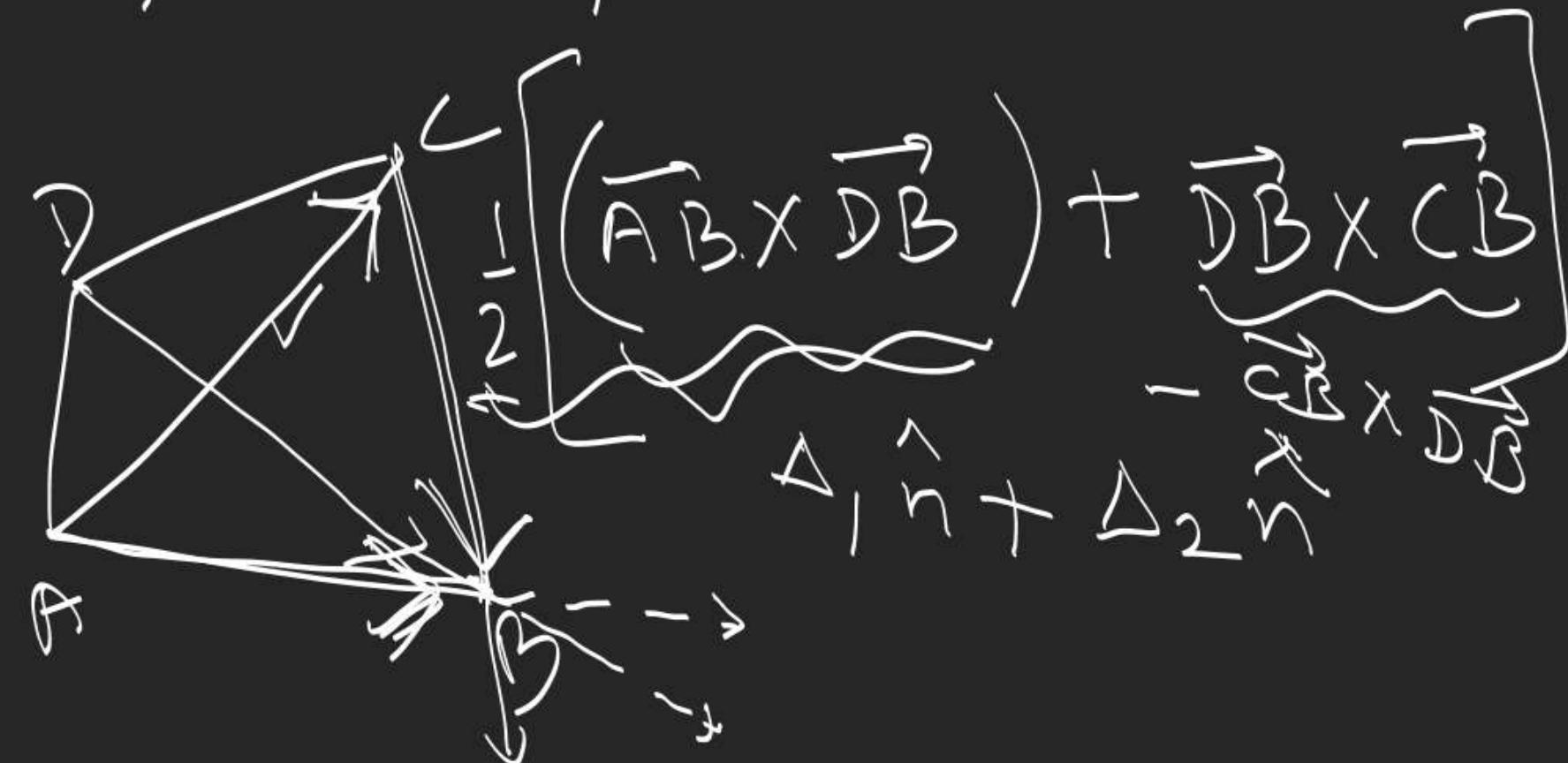
$$\text{Area of } \triangle = \frac{1}{2} |\vec{a} \times \vec{b}|$$

$\vec{a} \times \vec{b}$ is \perp to plane containing \vec{a}, \vec{b}



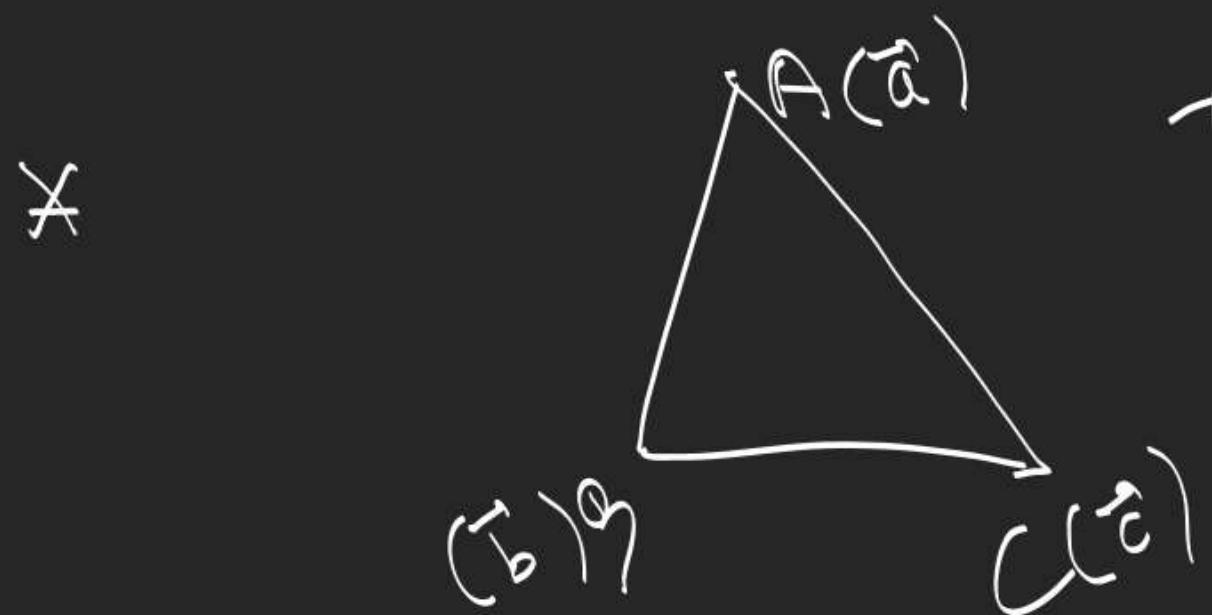
$$\frac{1}{2} |\vec{d}_1 \times \vec{d}_2| = \text{Area of quadrilateral}$$

$$\begin{aligned} \sum (\vec{AB} - \vec{CB}) \times \vec{DB} &= \\ &= \frac{1}{2} (\vec{AC} \times \vec{DB}) \end{aligned}$$



* Unit vector \perp an to plane containing
two non collinear vectors \vec{a}, \vec{b}

$$= \pm \frac{\vec{a} \times \vec{b}}{|\vec{a} \times \vec{b}|}$$



$$\boxed{\text{Area of } \triangle ABC = \frac{1}{2} |\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}|}$$

$$\frac{1}{2} |(\vec{b} - \vec{a}) \times (\vec{c} - \vec{b})| = \frac{1}{2} |\vec{b} \times \vec{c} + \vec{c} \times \vec{a} + \vec{a} \times \vec{b}|$$

Condition of collinearity
of 3 points with P.V. $\vec{a}, \vec{b}, \vec{c}$
 $\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a} = \vec{0}$

- Lagrangian Identity

$$(\vec{a} \times \vec{b})^2 = |\vec{a}|^2 |\vec{b}|^2 - (\vec{a} \cdot \vec{b})^2$$

1. Find the eqn. of line thru the point with p.v.

$2\hat{i} + 3\hat{j}$ and Lsn to vectors $\vec{A} = \hat{i} + 2\hat{j} + 3\hat{k}$ and

$$\vec{B} = 3\hat{i} + 4\hat{j} + 5\hat{k}$$

$$\begin{aligned}\hat{r} &= 2\hat{i} + 3\hat{j} + \lambda(\hat{i} - 2\hat{j} + \hat{k}) \\ \vec{R} &= \hat{i} - 3\hat{j} + 7\hat{k} - 5(\hat{i} + \hat{j} + \hat{k}) \\ &= -4\hat{i} - 8\hat{j} + 2\hat{k}\end{aligned}$$

2. Find the unknown vector \vec{R} satisfying

$$\vec{R} \times \vec{B} = \vec{C} \times \vec{B}$$

$$\vec{R} \cdot \vec{A} = 0$$

$$\vec{B} = \hat{i} + \hat{j} + \hat{k}, \quad \vec{C} = 4\hat{i} - 3\hat{j} + 7\hat{k}$$

$$(\vec{R} - \vec{C}) \times \vec{B} = \vec{0}$$

$$\vec{R} - \vec{C} = \lambda \vec{B}$$

$$\vec{A} = 2\hat{i} + \hat{k}$$

$$\begin{aligned}(\vec{C} + \lambda \vec{B}) \cdot \vec{A} &= 0 \\ \vec{C} \cdot \vec{A} + \lambda \vec{B} \cdot \vec{A} &= 0 \\ \Rightarrow 15 + \lambda(3) &= 0 \\ \lambda &= -5\end{aligned}$$

3. In $\triangle ABC$, M is midpoint of AB and D be foot of internal bisector of $\angle C$ on AB.

$$(i) \text{ P.T. } \frac{\text{area of } \triangle CDM}{\text{area of } \triangle ABC} = \frac{|a-b|}{2(a+b)}$$

Vector
 $\underline{Ex-1} (1-10)$

$$(ii) \text{ P.T. } \cos \angle DCM = \frac{(a+b) \cos \frac{C}{2}}{\sqrt{a^2 + b^2 + 2ab \cos C}}$$

\therefore



$$\frac{BD}{DC} = \frac{CE}{EA} = \frac{AF}{FB} = n$$

find the ratio $\frac{\triangle DEF}{\triangle ABC}$