HOMEWORK-04

(A.G.P.)

1. If $3 + \frac{1}{4}(3 + d) + \frac{1}{4^2}(3 + 2d) + \cdots. + \text{upto } \infty = 8$ then the value of d is

(A) 9 (B) 5 (C) 1 (D) 3

2. If $x > 0$, and $\log_2 x + \log_2 (\sqrt{x}) + \log_2 (\sqrt[4]{x}) + \log_2 (\sqrt[8]{x}) + \log_2 (\sqrt[16]{x}) + \cdots = 4$, then x equals

(A) 2 (B) 3 (C) 4 (D) 5

3. The positive integer n for which $2 \times 2^2 + 3 \times 2^3 + 4 \times 2^4 + \cdots.. + n \times 2^n = 2^{n+10}$ is

(A) 510 (B) 511 (C) 512 (D) 513

4. If $b + c, c + a, a + b$ are inH.P., then a^2, b^2, c^2 will be in-

(A) A.P. (B) G.P. (C) H.P. (D) None of these

5. If first and second terms of a HP are a and b, then its n^{th} term will be-

(A) $\frac{ab}{a+(n-1)ab}$ (B) $\frac{ab}{b+(n-1)(a+b)}$

(C) $\frac{ab}{b+(n-1)(a-b)}$ (D) $\frac{ab}{(a+(n+1)ab)}$

6. If a, b, c are in A.P., then $\frac{bc}{ca+ab}, \frac{ca}{bc+ab}, \frac{ab}{bc+ca}$ are in-

(A) A.P. (B) G.P. (C) H.P. (D) None of these

7. $\{a_n\}$ and $\{b_n\}$ are two sequences given by

$a_n = (x)^{1/2^n} + (y)^{1/2^n}$ and $b_n = (x)^{1/2^n} - (y)^{1/2^n}$

for all $n \in N$. The value of $a_1 a_2 a_3 \ldots \ldots .. a_n$ is equal to

(A) $x - y$ (B) $\frac{x+y}{b_n}$ (C) $\frac{x-y}{b_n}$ (D) $\frac{xy}{b_n}$

8. Show that in any arithmetic progression

$a_1, a_2, a_3 \ldots a_1{}^2 - a_2{}^2 + a_3{}^2 - a_4{}^2 + \cdots. + a_{2K-1}^2 - a_{2K} = [K/(2K - 1)](a_1{}^2 - a_{2K}^2)$.

9. For any three positive real numbers a, b and c,

$9(25a^2 + b^2) + 25(c^2 - 3ac) = 15\,b(3a + c)$. Then:

(A) b,c and a are in G.P. (B) b,c and a are in A.P.

(C) a, b and c are in A.P. (D) a,b and c are in G.P.

10. If $(10)^9 + 2(11)^1(10)^8 + 3(11)^2(10)^7 + \cdots \ldots .. + 10(11)^9 = k(10)^9$, then k is equal to

(A) $\frac{121}{10}$ (B) $\frac{441}{100}$ (C) 100 (D) 110

(A.M.G.M.H.M.)

11. If x, y, z are AM, GM and HM of two positive distinct numbers respectively, then correct statement is -

(A) $x < y < z$ (B) $y < x < z$

(C) $z < y < x$ (D) $z < x < y$

12. The A.M. of two positive numbers exceeds the GM by 5, and the GM exceeds the H.M. by 4 . Then the numbers are-

(A) 10,40 (B) 10,20 (C) 20,40 (D) 10,50

13. If A,G & 4 are A.M, G.M & H.M of two numbers respectively and $2A + G^2 = 27$, then the numbers are-

(A) 8,2 (B) 8,6 (C) 6,3 (D) 6,4

14. If A, G & H are respectively the A.M., G.M. & H.M. of three positive numbers a, b, & c then the equation whose roots are a, b & c is given by

(A) $x^3 - 3Ax^2 + 3G^3x - G^3 = 0$ (B) $x^3 - 3Ax^2 + 3(G^3/H)x - G^3 = 0$

(C) $x^3 + 3Ax^2 + 3(G^3/H)x - G^3 = 0$ (D) $x^3 - 3Ax^2 - 3(G^3/H)x + G^3 = 0$

15. If $x^2 + 9y^2 + 25z^2 = xyz\left(\frac{15}{x} + \frac{5}{y} + \frac{3}{z}\right)$, then x, y and z are in

(A) AGP (B) GP (C) AP (D) HP

16. If G_1 and G_2 are two geometric means and A is the arithmetic means inserted between two positive numbers then the value of $\frac{G_1^2}{G_2} + \frac{G_2^2}{G_1}$ is

(A) A/2 (B) A (C) 2 A (D) 3 A

17. If sum of A.M. and H.M. between two positive numbers is 25 and their GM is 12, then the sum of numbers is-

(A) 9 (B) 18 (C) 32 (D) 18 or 32

18. The A.M. of two numbers is 34 and GM is 16 , the numbers are-

(A) 2 and 64 (B) 64 and 3 (C) 64 and 4 (D) 64 and 8

19. The ratio between the GM's of the roots of the equations $ax^2 + bx + c = 0$ and $\ell x^2 + mx + n = 0$ is-

(A) $\sqrt{\frac{b\ell}{an}}$ (B) $\sqrt{\frac{c\ell}{an}}$ (C) $\sqrt{\frac{an}{c\ell}}$ (D) $\sqrt{\frac{cn}{a\ell}}$

20. Using the relation A.M. 2 G.M. Prove that

(i) $\tan\theta + \cot\theta \geq 2$; if $0 < \theta < \frac{\pi}{2}$

(ii) $(x^2y + y^2z + z^2x)(xy^2 + yz^2 + zx^2) > 9x^2y^2z^2$.

Where x, y, z are different real no.

(iii) $(a + b) \cdot (b + c) \cdot (c + a) \geq 8abc$; if a, b, c are positive real numbers.

21. If a, b, c are sides of triangle then prove that (i) $b^2c^2 + c^2a^2 + a^2b^2 \geq abc(a + b + c)$

(ii) $(a + b + c)^3 > 27(a + b - c)(c + a - b)(b + c - a)$

22. The arithmetic mean of two numbers is 6 and their geometric mean G and harmonic mean H satisfy the relation $G^2 + 3H = 48$. Find the two numbers.

23. Let A_1, G_1, H_1 denote the arithmetic, geometric and harmonic means, respectively, of two distinct positive numbers. For $n \geq 2$, Let A_{n-1} and H_{n-1} have arithmetic, 231% metric and harmonic means as A_n, G_n, H_n respectively

(a) Which one of the following statements is correct ?

(A) $G_1 > G_2 > G_3 > \cdots$ ..... (B) $G_1 < G_2 < G_3 < \cdots$ ....

(C) $G_1 = G_2 = G_3 =$ (D) $G_1 < G_3 < G_5 <$ and $G_2 > G_4 > G_6 > \cdots$ ...

(b) Which one of the following statement is correct ?

(A) $A_1 > A_2 > A_3 > \cdots$ ....

(B) $A_1 < A_2 < A_3 <$

(C) $A_1 > A_3 > A_5 > \cdots$... and $A_2 < A_4 < A_6 < \cdots$ ....

(D) $A_1 < A_3 < A_5 < \cdots$... and $A_2 > A_4 > A_6 > \cdots$ ......

(c) Which one of the following statement is correct?

(A) $H_1 > H_2 > H_3 >$

(B) $H_1 < H_2 < H_3 <$

(C) $H_1 > H_3 > H_5 > \cdots$... and $H_2 < H_4 < H_6 <$

(D) $H_1 < H_3 < H_5 < \cdots$...and $H_2 > H_4 > H_6 > \cdots$ ....

24. The minimum value of the sum of real numbers $a^{-5}, a^{-4}, 3a^{-3}, 1, a^8$ and a^{10} with $a > 0$ is

(Miscellaneous & V_n Method)

25. The sum of n term of the series $1(1!) + 2(2!) + 3(3!) + \cdots$

(A) $(n + 1)! - 1$ (B) $(n - 1)! - 1$

(C) $(n - 1)! + 1$ (D) $(n + 1)! + 1$

26. If p is positive, then the sum to infinity of the series, $\frac{1}{1+p} - \frac{1-p}{(1+p)^2} + \frac{(1-p)^2}{(1+p)^3} - \cdots$ ... is

(A) 1/2 (B) ¾ (C) 1 (D) ¼

27. The sum of infinite series $1-\frac{3}{2}+\frac{5}{4}-\frac{7}{8}+\cdots$ is-
(A) 2/9 (B) 2/3 (C) −2/9 (D) 9/2

28. Find the sum of n terms of the series the r^{th} term of which is $(2r+1)2^r$.
(A) $n\cdot 2^{n+1}-2^n+2$ (B) $n\cdot 2^{n+2}-2^{n+1}+2$
(C) $n\cdot 2^{n+2}+2^{n+1}-2$ (D) None of these

29. If $H_n=1+\frac{1}{2}+\frac{1}{3}+\cdots\ldots\ldots+\frac{1}{n}$, then value of $1+\frac{3}{2}+\frac{5}{3}+\cdots\ldots\ldots\ldots+\frac{2n-1}{n}$ is
(A) $2n-H_n$ (B) $2n+H_n$
(C) H_n-2n (D) H_n+n

30. The sum of all possible products of first n natural numbers taken two at a time is
(A) $\frac{1}{24}n(n+1)(n-1)(3n+2)$ (B) $\frac{n(n+1)(2n+1)}{6}$
(C) $\frac{n(n+1)(2n-1)(n+3)}{24}$ (D) $\frac{n(n^2+1)(3n+2)}{24}$

31. Find the sum of the n terms of the series whose nth term is
(i) $n(n+2)$ (ii) 3^n-2^n

32. Find the sum of the series
$$\frac{5}{13}+\frac{55}{(13)^2}+\frac{555}{(13)^3}+\frac{5555}{(13)^4}\ldots\ldots\text{ up to }\infty$$

33. Sum of the series to n terms and to infinity :
$$1^2-\frac{2^2}{5}+\frac{3^2}{5^2}-\frac{4^2}{5^3}+\frac{5^2}{5^4}-\frac{6^2}{5^5}+\cdots\ldots\infty.$$

34. Find the sum of the n terms and to infinity of the sequence
$$\frac{1}{1+1^2+1^4}+\frac{2}{1+2^2+2^4}+\frac{3}{1+3^2+3^4}+\cdots\ldots$$

35. Find the sum of the first n terms of the sequence:
$$1+2\left(1+\frac{1}{n}\right)+3\left(1+\frac{1}{n}\right)^2+4\left(1+\frac{1}{n}\right)^3+\cdots\ldots$$

36. Find the n^{th} term and the sum of n terms of the sequence
(i) $1+5+13+29+61+\cdots$. (ii) $6+13+22+33+\cdots\ldots$

37. If the sum
$$\sqrt{1+\frac{1}{1^2}+\frac{1}{2^2}}+\sqrt{1+\frac{1}{2^2}+\frac{1}{3^2}}+\sqrt{1+\frac{1}{3^2}+\frac{1}{4^2}}+\cdots\ldots\ldots+\sqrt{1+\frac{1}{(1999)^2}+\frac{1}{(2000)^2}}$$ equal to
$n-1/n$ where $n\in N$. Find n.

38. Two distinct, real infinite geometric series each have a sum of 1 and have the same second term. The third term of one of the series is 1/8. If the second term of both the series can be written in the form $\frac{\sqrt{m}-n}{p}$, where m, n and p are positive integers and m is not divisible by the square of any prime, find the value of $100m+10n+p$. 201%

39. One of the roots of the equation
$52000x^6+100x^5+10x^3+x-2=0$ is of the form $\frac{m+\sqrt{n}}{r}$, where m is non zero integer and n and r are relatively prime natural numbers. Find the value of $m+n+r$.

40. Statement 1: The sum of the series
$1+(1+2+4)+(4+6+9)+(9+12+16)+\cdots\ldots$
$\ldots.+(361+380+400)$ is 8000.
Statement 2: $\sum_{k=1}^{n}(k^3-(k-1)^3)=n^3$, for any natural number n.
(A) Statement 1 is true, Statement 2 is true, Statement 2 is not a correct explanation for statement 1 .
(B) Statement 1 is true, Statement 2 is false.
(C) Statement 1 is false, Statement 2 is true.

(D) Statement 1 is true, Statement 2 is true,
Statement 2 is a correct explanation for statement 1.

41. The sum of first 20 terms of the sequence 0.7,0.77,0.777, ... is :
(A) $\frac{7}{81}(179+10^{-20})$ (B) $\frac{7}{9}(99+10^{-20})$
(C) $\frac{7}{81}(179-10^{-20})$ (D) $\frac{7}{9}(99-10^{-20})$

42. The sum of first 9 terms of the series $\frac{1^3}{1}+\frac{1^3+2^3}{1+3}+\frac{1^3+2^3+3^3}{1+3+5}+\cdots$.. is :
(A) 142 (B) 192 (C) 71 (D) 96

43. If the sum of the first ten terms of the series $\left(1\frac{3}{5}\right)^2+\left(2\frac{2}{5}\right)^2+\left(3\frac{1}{5}\right)^2+4^2+\left(4\frac{4}{5}\right)^2+\cdots\ldots$ is $\frac{16}{5}$ m, then m is equal to :
(A) 101 (B) 100 (C) 99 (D) 102

44. Let A be the sum of the first 20 terms and B be the sum of the first 40 terms of the series $1^2+2\cdot2^2+3^2+2\cdot4^2+5^2+2\cdot6^2+\cdots$.. If B -2 A $=100\lambda$, then λ is equal to :
(A) 496 (B) 232 (C) 248 (D) 464

45. Let V_r denote the sum of first r terms of an arithmetic progression (A.P.) whose first term is r and the common difference is $(2r-1)$.
Let $T_r=V_{r+1}-V_r-2$ and $Q_r=T_{r+1}-T_r$ for $r=1,2,..$
(a) The sum $V_1+V_2+\cdots\ldots.+V_n$ is
(A) $\frac{1}{12}n(n+1)(3n^2-n+1)$ (B) $\frac{1}{12}n(n+1)(3n^2+n+2)$
(C) $\frac{1}{2}n(2n^2-n+1)$ (D) $\frac{1}{3}(2n^3-2n+3)$
(b) T_f is always
(A) an odd number (B) an even number
(C) a prime number (D) a composite number

46. Let $S_k, K=1,2,\ldots.,100$ denote the sum of the infinite geometric series whose first term is $\frac{k-1}{k!}$ and the common ratio is $1/k$. Then the value of $\frac{100^2}{100!}+\sum_{k=1}^{100}|(k^2-3k+1)S_k|$ is

47. Let $a_1,a_2,a_3,\ldots$ be a sequence of positive integers in arithmetic progression with common difference 2. Also, let $b_1,b_2,b_3,\ldots$ be a sequence of positive integers in geometric progression with common ratio 2. If $a_1=b_1=c$, then the number of all possible values of c, for which the equality $2(a_1+a_2+\cdots+a_n)=b_1+b_2+\cdots+b_n$ holds for some positive integer n, is

48. Find the sum
$$2017+\frac{1}{4}\left(2016+\frac{1}{4}\left(2015+\cdots+\frac{1}{4}\left(2+\frac{1}{4}(1)\right)\ldots\right)\right)$$

49. Find the sum $1+2\left(1+\frac{1}{50}\right)+3\left(1+\frac{1}{50}\right)^2+\cdots$ 50 terms

50. Find the sum $1+\frac{1}{1+2}+\frac{1}{1+2+3}+\cdots.+\frac{1}{1+2+3+\cdots+n}$

51. Find the sum to n terms of the series $\frac{1}{1\times3}+\frac{1}{3\times5}+\frac{1}{5\times7}+\cdots$

52. Find the sum to n terms of the series
$$\frac{1}{1+1^2+1^4}+\frac{2}{1+2^2+2^4}+\frac{3}{1+3^2+3^4}+\cdots$$

53. Find the sum $\sum_{r=1}^{n}\frac{1}{r(r+1)(r+2)(r+3)}$.
Also, find $\sum_{r=1}^{\infty}\frac{1}{r(r+1)(r+2)(r+3)}$.

54. Find the sum of the series $\sum_{r=1}^{99}\left(\frac{1}{r\sqrt{r+1}+(r+1)\sqrt{r}}\right)$.

55. Find the sum of the series $\frac{1}{3^2+1}+\frac{1}{4^2+2}+\frac{1}{5^2+3}+\frac{1}{6^2+4}+\cdots\infty$.

56. Find the sum of the series $\frac{2}{1\times 2}+\frac{5}{2\times 3}\times 2+\frac{10}{3\times 4}\times 2^2+\frac{17}{4\times 5}\times 2^3+\cdots$ upto n terms.

57. If $\sum_{r=1}^{n} T_r=\frac{n}{8}(n+1)(n+2)(n+3)$, then find $\sum_{r=1}^{n}\frac{1}{T_r}$.

58. Let $S=\frac{\sqrt{1}}{1+\sqrt{1}+\sqrt{2}}+\frac{\sqrt{2}}{1+\sqrt{2}+\sqrt{3}}+\frac{\sqrt{3}}{1+\sqrt{3}+\sqrt{4}}+\cdots+\frac{\sqrt{n}}{1+\sqrt{n}+\sqrt{n+1}}=10$

Then find the value of n.

59. The sum $\sum_{r=2}^{\infty}\frac{1}{r^2-1}$ is equal to

(A) 1 (C) 4/3 (B) 3/4 (D) 3/2

60. Sum of the series $S=1^2-2^2+3^2-4^2+\cdots..-2002^2+2003^2$ is

(A) 2007006 (B) 1005004

(C) 2000506 (D) 200700

61. If $1^2+2^2+\cdots...+n^2=1015$, then value of n is

(A) 15 (B) 14 (C) 13 (D) 12

62. If $\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\cdots$ upto $\infty=\frac{\pi^2}{6}$, then $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\cdots..=$

(A) $\pi^2/12$ (B) $\pi^2/24$ (C) $\pi^2/8$ (D) $\pi^2/4$

ANSWER KEY
(A.G.P.)

1. (A) 2. (C) 3. (D) 4. (A) 5. (C) 6. (C) 7. (C)
9. (B) 10. (C)

(A.M.G.M.H.M.)

11. (C) 12. (A) 13. (C) 14. (B) 15. (D) 16. (C) 17. (C)
18. (C) 19. (B) 22. $a = 4, b = 8$ 23. (A) C,(B)A(C)B 24. 8

(MISCELLANEOUS & V_nMETHOD)

25. (A) 26. (A) 27. (A) 28. (B) 29. (A) 30. (A)
31. (i) $\frac{1}{6}n(n+1)(2n+7)$, (ii) $1/2(3^{n+1}+1)-2^{n+1}$ 32. $\frac{65}{36}$ 33. $\frac{25}{54}$
34. $\frac{n(n+1)}{2(n^2+n+1)}; s_\infty = \frac{1}{2}$ 35. n^2
36. (i) $2^{n+1}-3; 2^{n+2}-4-3n$ (ii) $n^2+4n+1; (1/6)n(n+1)(2n+13)+n$

37. $n = 2000$ 38. 518 39. 200 40. (D) 41. (A) 42. (D)
43. (A) 44. (C) 45. (A) B,(B)B 46. 3 47. 1
48. $\frac{4}{3}(2017)-\frac{4}{9}\left(1-\frac{1}{4^{2017}}\right)$
49. 2500 50. $\frac{2n}{n+1}$ 51. $\frac{n}{2n+1}$ 52. $\frac{n^2+n}{2(n^2+n+1)}$ 53. $\frac{1}{18}$ 54. $\frac{9}{10}$
55. $\frac{13}{36}$ 56. $\frac{n}{n+1}2^n$ 57. $\frac{n(n+3)}{2(n+1)(n+2)}$ 58. 24 59. (B) 60. (A) 61. (B)
62. (C)