Nishant Jindal

Q If $|a|=2,\ |b|=3$ & $\vec{a}\cdot\vec{b}=0$

find $|a\times(a\times(a\times(a\times b)))| = ?$

$$|a\times(a\times(\cancel{(a\cdot b)}\vec{a}-(a\cdot a)\vec{b})))|$$

$$|a\times(a\times(-4b))|$$

$$|\vec{a}\times(-4(\vec{a}\times\vec{b}))|$$

$$|-4\times\vec{a}\times(\vec{a}\times\vec{b}))|.$$

$$|-4(\cancel{(a\cdot b)}\vec{a}-(a\cdot a)\vec{b})|$$

$$|-4\times(-4\vec{b})|$$

$$=16|\vec{b}| = 16\times 3$$

$$= 48.$$

Scalar Prod of 4 Vector.

$$(\vec{a}\times\vec{b})\cdot(\vec{c}\times\vec{d})$$

$$\vec{a}\cdot(\vec{b}\times(\vec{c}\times\vec{d}))$$

$$\vec{a}\cdot((b\cdot d)\vec{c}-(b\cdot c)\vec{d})$$

$$(\vec{b}\cdot\vec{d})(\vec{a}\cdot\vec{c})-(\vec{b}\cdot\vec{c})(\vec{a}\cdot\vec{d})$$

$$\begin{vmatrix} \vec{a}\cdot\vec{c} & \vec{a}\cdot\vec{d} \\ b\cdot\vec{c} & \vec{b}\cdot\vec{d} \end{vmatrix}$$

$$(\vec{m}\times\vec{n})\cdot(\vec{b}\times\vec{r})$$

$$= \begin{vmatrix} \vec{m}\cdot\vec{b} & m\cdot\vec{r} \\ n\cdot\vec{b} & n\cdot\vec{r} \end{vmatrix}$$

Q $(b\times c)\cdot(a\times d)+(c\times a)\cdot(b\times d) + (a\times b)\cdot(c\times d) = ?$

$$\begin{vmatrix} b\cdot a & b\cdot d \\ c\cdot a & c\cdot d \end{vmatrix} + \begin{vmatrix} c\cdot b & c\cdot d \\ a\cdot b & a\cdot d \end{vmatrix} + \begin{vmatrix} a\cdot c & a\cdot d \\ b\cdot c & b\cdot d \end{vmatrix}$$

$$= abcd - abcd + abcd - abcd + abcd - abcd$$

$$= 0$$

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Reciprocal system of vectors

① for $\vec{a}, \vec{b}, \vec{c}$

$\vec{a'}, \vec{b'}, \vec{c'}$ is available.

Such that $\vec{a} \cdot \vec{a'} = 1$

& $\vec{b} \cdot \vec{b'} = 1$, $\vec{c} \cdot \vec{c'} = 1$

(2) $\vec{a} \cdot \vec{a'} = 1$

then $\vec{a'} = \dfrac{\vec{b} \times \vec{c}}{[a\,b\,\vec{c}]}$

$$\vec{a} \cdot \vec{a'} = \frac{a \cdot (\vec{b} \times \vec{c})}{[a\,b\,c]} = \frac{[a\,b\,c]}{[a\,b\,c]} = 1$$

$$\text{Sly } \vec{b'} = \frac{\vec{c} \times \vec{a}}{[a\,b\,c]}, \quad \vec{c'} = \frac{\vec{a} \times \vec{b}}{[a\,b\,c]}$$

Q $[a\,b\,c] \cdot [a'\ b'\ c'] = ?$

$$[a\,b\,c] \cdot \left[\frac{b \times c}{[a\,b\,c]} \quad \frac{c \times a}{[a\,b\,c]} \quad \frac{a \times b}{[a\,b\,c]}\right]$$

$$\frac{\cancel{[a\,b\,c]}}{\cancel{[a\,b\,c]}^3}[b \times c \quad c \times a \quad a \times b]$$

$$\frac{1}{\cancel{[a\,b\,c]}^2}[\cancel{b \times} a] \times \cancel{[} \cancel{\times} a\,b] = 1$$

* Sh. that $\hat{i}, \hat{j}, \hat{k}$ are Reciprocal vectors of self.

$\hat{i} \cdot \hat{i'} = 1$

$$\hat{i'} = \frac{\hat{j} \times \hat{k}}{[\hat{i}\,\hat{j}\,\hat{k}]} = \frac{\hat{i}}{1} = \hat{i}$$

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Vector Eqn. (Mains)

If dot Product & cross product both are given in a Qs. We solve both given Product by using dot & cross Product again by appropriate vectors & solve Qs.

Q. Let $\vec{a} = \hat{i} - \hat{j}$, $\vec{c} = \hat{i} - \hat{j} - \hat{k}$ then $\boxed{\text{vector } \vec{b}}$ Satisfying

$|a| = \sqrt{2}$

$a \times b + c = 0$ & $\vec{a} \cdot \vec{b} = 3$ is?

$\vec{a} \times \vec{c}$

$$\begin{vmatrix} i & j & k \\ 1 & -1 & 0 \\ 1 & -1 & -1 \end{vmatrix} = \langle 1, +1, 0 \rangle$$

$a \times b + c = 0 \} \times \vec{a}$ (Pre)

$a \times (a \times b) + \vec{a} \times \vec{c} = 0$

$(a \cdot b)\vec{a} - (a \cdot a)\vec{b} + \vec{a} \times \vec{c} = 0$

$3\vec{a} - 2\vec{b} + \vec{a} \times \vec{c} = 0$

$$\vec{b} = \frac{3\vec{a} + \vec{a} \times \vec{c}}{2}$$

$$\vec{b} = \frac{(3\hat{i} - 3\hat{j}) + \hat{i} + \hat{j}}{2}$$

$\vec{b} = 2\hat{i} - \hat{j}$

Q. Vector $\vec{x}$ Satisfying

$\vec{A} \cdot \vec{x} = c$ & $\vec{A} \times \vec{x} = \vec{B}$ is

$\vec{A} \times \vec{x} = \vec{B} \} \times \vec{A}$ (Pre)

$\vec{A} \times (\vec{A} \times \vec{x}) = \vec{A} \times B$

$(\vec{A} \cdot \vec{x})\vec{A} - (\vec{A} \cdot \vec{A})\vec{x} = \vec{A} \times \vec{B}$

$c\vec{A} - |A|^2 \vec{x} = \vec{A} \times \vec{B}$

$$= \boxed{\vec{x} = \frac{c\vec{A} - \vec{A} \times \vec{B}}{|A|^2}}$$

Q. If $p\vec{x} + \vec{x} \times \vec{a} = \vec{b}$ then P.T.

$x \times a = \vec{b} - p\vec{x}$

$$\vec{x} = \frac{p^2\vec{b} + (\vec{b} \cdot \vec{a})\vec{a} - p(\vec{b} \times \vec{a})}{p(p^2 + \vec{a}^2)}$$

(1) $p\vec{x} + \vec{x} \times \vec{a} = \vec{b} \} \cdot \vec{a}$

$p(\vec{a} \cdot \vec{x}) + 0 = \vec{a} \cdot \vec{b}$

(2) $p\vec{x} + \vec{x} \times \vec{a} = \vec{b} \} \times \vec{a}$ (Pre)

$p(\vec{a} \times \vec{x}) + \vec{a} \times (\vec{x} \times \vec{a}) = \vec{a} \times \vec{b}$

$p(p\vec{x} - \vec{b}) + |a|^2 x - (\vec{a} \cdot \vec{x})\vec{a} = \vec{a} \times \vec{b}$

$p^2\vec{x} - p\vec{b} + a^2\vec{x} - \left(\frac{\vec{a} \cdot \vec{b}}{p}\right)\vec{a} = \vec{a} \times \vec{b}$

$(p^2 + a^2)\vec{x} = p\vec{b} + \left(\frac{\vec{a} \cdot \vec{b}}{p}\right)\vec{a} + \vec{a} \times \vec{b}$

$$\vec{x} = \frac{p^2\vec{b} + (\vec{a} \cdot \vec{b})\vec{a} - p(\vec{b} \times \vec{a})}{p(p^2 + a^2)}$$

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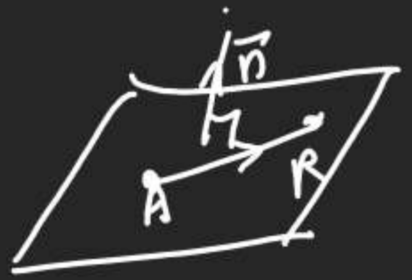
3D (3Qs.) Adv
(1-2) Mains.

1) Plane :- Let A be a pt. on Surface such that every pt. P on the Surface AP is $\perp^r$ to some fixed line & This fixed Line is called $\vec{n}$ (normal vector)

(2) Vector form = (Scalar Dot Prod form)



$\vec{AR} \cdot \vec{n} = 0$

$\boxed{(\vec{r} - \vec{a}) \cdot \vec{n} = 0} \rightarrow$ (A)

$(\vec{r} - \text{Fix pt}) \cdot \text{normal} = 0$

$\boxed{\vec{r} \cdot \vec{n} = d} \rightarrow$ (B)

(3) Cartesian form.

A) $ax + by + cz = d$

Where a, b, c = DR of normal vector

B) If Plane in P.T. fix pt. (x_1, y_1, z_1) & normal to Plane's DR = $\langle a, b, c \rangle$

then EOP $\rightarrow a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$

(4) A) Eq^n of YZ Plane $\rightarrow x = 0$

Eq^n of Plane $\parallel$ to YZ Plane $\Rightarrow x = \pm d$

B) Eq^n of XZ Plane $\Rightarrow y = 0$

(C) ——— XY Plane $\Rightarrow z = 0$

(d)* Eq^n of Plane $\parallel^{r}$ to X Axis then $a = 0$

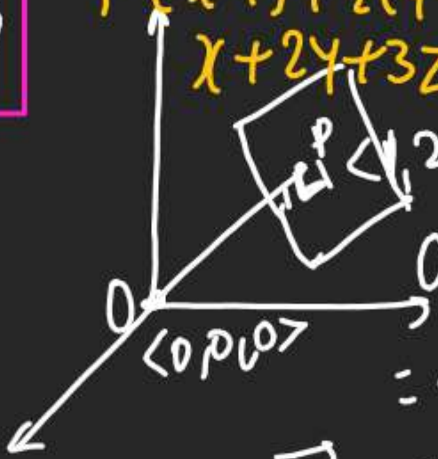
$\Rightarrow by + cz + d = 0$

Q Find EOP P.T. $P\langle 1, 2, 3 \rangle$ & $\perp^r$ to OP?

$a(x-1) + b(y-2) + c(z-3)$

$1 \cdot (x-1) + 2(y-2) + 3(z-3) = 0$

$x + 2y + 3z = 1 + 4 + 9$



OP's DR

$= \langle 1-0, 2-0, 3-0 \rangle$

$\vec{n} = \langle 1, 2, 3 \rangle$

as $\vec{n}$ is $\parallel^{rt}$ to OP

$x + 2y + 3z = 14.$

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(C) Coff of x, y, z in Cart. form.

are D.R. of normal

(D) If 2 Planes P_1 & P_2 are given.

$P_1: a_1x+b_1y+c_1z+d_1=0$

$P_2: a_2x+b_2y+c_2z+d_2=0$

(i) If $P_1 \parallel P_2 \Rightarrow \frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}\neq\frac{d_1}{d_2}$

(ii) P_1, P_2 Coincident

$\Rightarrow \frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}=\frac{d_1}{d_2}$

(iii) $P_1 \perp P_2$

$a_1a_2+b_1b_2+c_1c_2=0$

(iv) angle betn P_1 & P_2

$\cos\theta=\frac{n_1\cdot n_2}{|n_1||n_2|}$

if $P_1 \perp P_2 \Rightarrow \theta=90^\circ$

$\cos 90^\circ=0$

$\frac{n_1 n_2}{|n_1||n_2|}=0$

$n_1\cdot n_2=0$

$\boxed{a_1a_2+b_1b_2+c_1c_2=0}$

Q EOP PT $\langle 1,0,-2\rangle$ & $\perp$ to Plane.

$P_1: 2x+y-z=2$, $P_2: x-y-z=3$.

$\vec{n}=\vec{n}_{P_1}\times\vec{n}_{P_2}=\begin{vmatrix} i & j & k \\ 2 & 1 & -1 \\ 1 & -1 & -1 \end{vmatrix}$

$\vec{n}=\langle \underset{a}{-2}, \underset{b}{+1}, \underset{c}{-3}\rangle$

$-2(x-1)+1\cdot(y-0)-3(z+2)=0$

$-2x+y-3z-4=0 \Rightarrow 2x-y+3z+4=0$

* If a Line is equally Inclined to

x, y, z Axis $\rightarrow \alpha=\beta=\gamma$ (D. Angle)

$\cos\alpha=\cos\beta=\cos\gamma=$ (D.C.)

$\cos^2\alpha+\cos^2\beta+\cos^2\gamma=1 \Rightarrow 3\cos^2\alpha=1$

$\cos\alpha=\pm\frac{1}{\sqrt{3}}=\cos\beta=\cos\gamma$

$l, m, n=\langle\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\rangle \rightarrow$ D.R. $=\langle 1,1,1\rangle$

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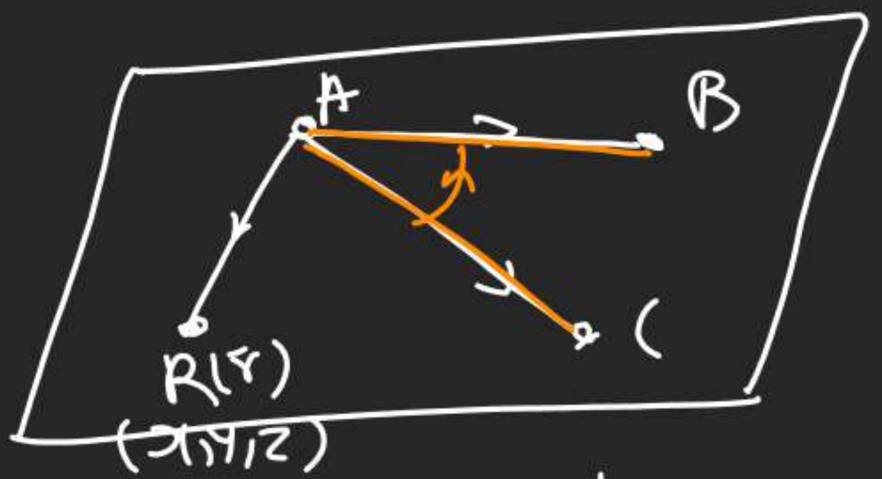
(5) EOP P.T. 3 given Pts.



(1) $\vec{AR}$ $\vec{AB}$ $\vec{AC}$ are Coplanar.

(2) $[\vec{AR}\ \vec{AB}\ \vec{AC}]=0$

$\vec{AR}\cdot(\vec{AB}\times\vec{AC})=0$

$(\vec{r}-\vec{a})$

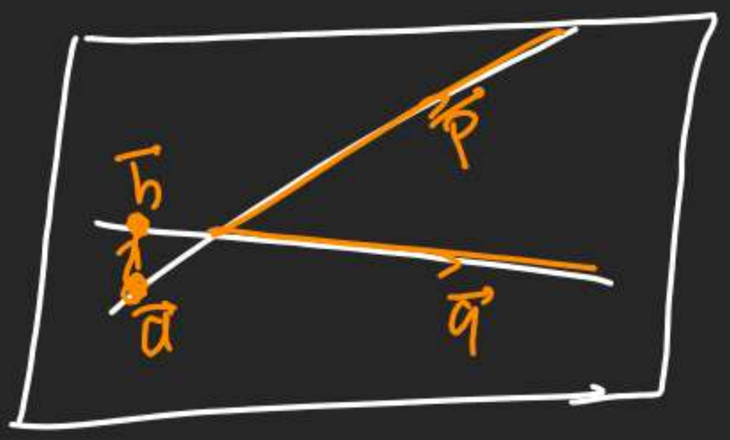
$$= \begin{vmatrix} x-a_1 & y-b_1 & z-c_1 \\ a_2-a_1 & b_2-b_1 & c_2-c_2 \\ a_3-a_1 & b_3-b_1 & c_3-c_1 \end{vmatrix} = 0$$

$A=\langle a_1, b_1, c_1\rangle$

$B=\langle a_2\ b_2\ c_2\rangle$

$C=\langle a_3\ b_3\ c_3\rangle$

(6) EOP having 2 Intersecting Lines



$L_1: \vec{r}=\vec{a}+\lambda\vec{p}$

$L_2: \vec{r}=\vec{b}+\mu\vec{q}$

$[\vec{b}-\vec{a}\ \ \vec{q}\ \ \vec{p}]=0$

$\to x_1, y_1, z_1$

$L_1: \frac{x-x_1}{a_1}=\frac{y-y_1}{b_1}=\frac{z-z_1}{c_1}$

$x_2 y_2 z_2$

$L_2: \frac{x-x_2}{a_2}=\frac{y-y_2}{b_2}=\frac{z-z_2}{c_2}$

$$= \begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$$

(7) $P: \vec{r}=\vec{a}+t\vec{p}+s\vec{q}$

is Parametric form of Plane.

for normal: $\vec{p}\times\vec{q}$

for fix pt $\Rightarrow \vec{a}$

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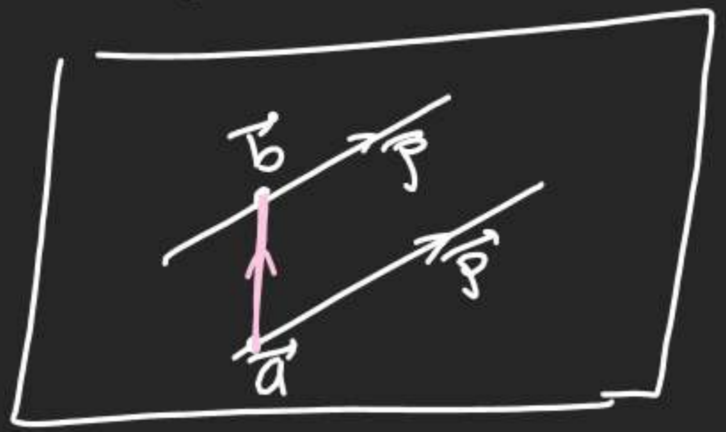
(8) EOP When 2 $\parallel^{rl}$ Lines are given



$L_1: \vec{r} = \vec{a} + \lambda \vec{p}$

$L_2: \vec{r} = \vec{b} + \mu \vec{p}$

normal $= \vec{p} \times (\vec{b} - \vec{a})$

$(\vec{r} - \vec{a}) \cdot \vec{p} \times (\vec{b} - \vec{a}) = 0$

$[\vec{AR} \;\; \vec{p} \;\; \vec{AB}] = 0$

$$\begin{vmatrix} x - a_1 & y - b_1 & z - c_1 \\ b_1 & p_2 & p_3 \\ a_2 - a_1 & b_2 - b_1 & c_2 - c_1 \end{vmatrix} = 0$$