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KINEMATICS

Q. A car starts moving along a line, first with acceleration $a = 5\ \mathrm{ms^{-2}}$ starting from rest then uniformly and finally decelerating at the same rate ' a ', comes to rest. The total time of motion is $\tau = 25\ \mathrm{s}$, the average velocity during the time is equal to $\langle v \rangle = 72\ \mathrm{km/hr}$. How long does the particle move uniformly?

$$\frac{\text{Total displacement}}{\text{Total time}} = \langle v \rangle$$

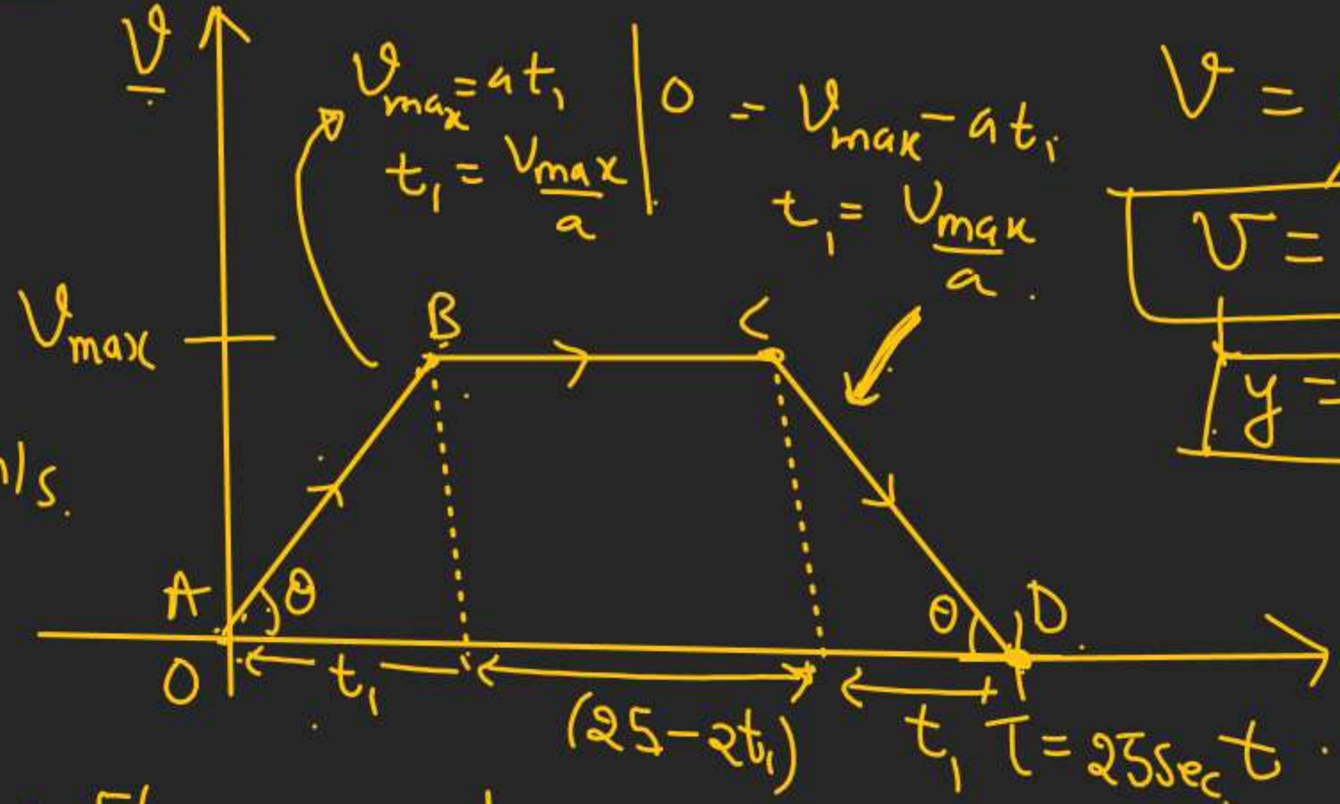
$$\frac{1}{2}\left[(25-2t_1)+25\right] \times V_{max} = \left(72 \times \frac{5}{18}\right) \mathrm{m/s}$$

$$(50-2t_1) \times V_{max} = (20 \times 50)$$

Slope of OA $= a$

$\tan\theta = a$

$$\frac{V_{max}}{t_1} = 5 \quad (2) \Rightarrow V_{max} = 5t_1$$



$v = \cancel{u} + at$

$\boxed{v = at}$

$\boxed{y = mx}$

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$$(50-2t_1)\times 5t_1 = 1000$$

$$(50-2t_1)\times t_1 = 200$$

$$50t_1 - 2t_1^2 = 200$$

$$t_1^2 - 25t_1 + 100 = 0$$

$$t_1^2 - 20t_1 - 5t_1 + 100 = 0$$

$$t_1(t_1-20) - 5(t_1-20) = 0$$

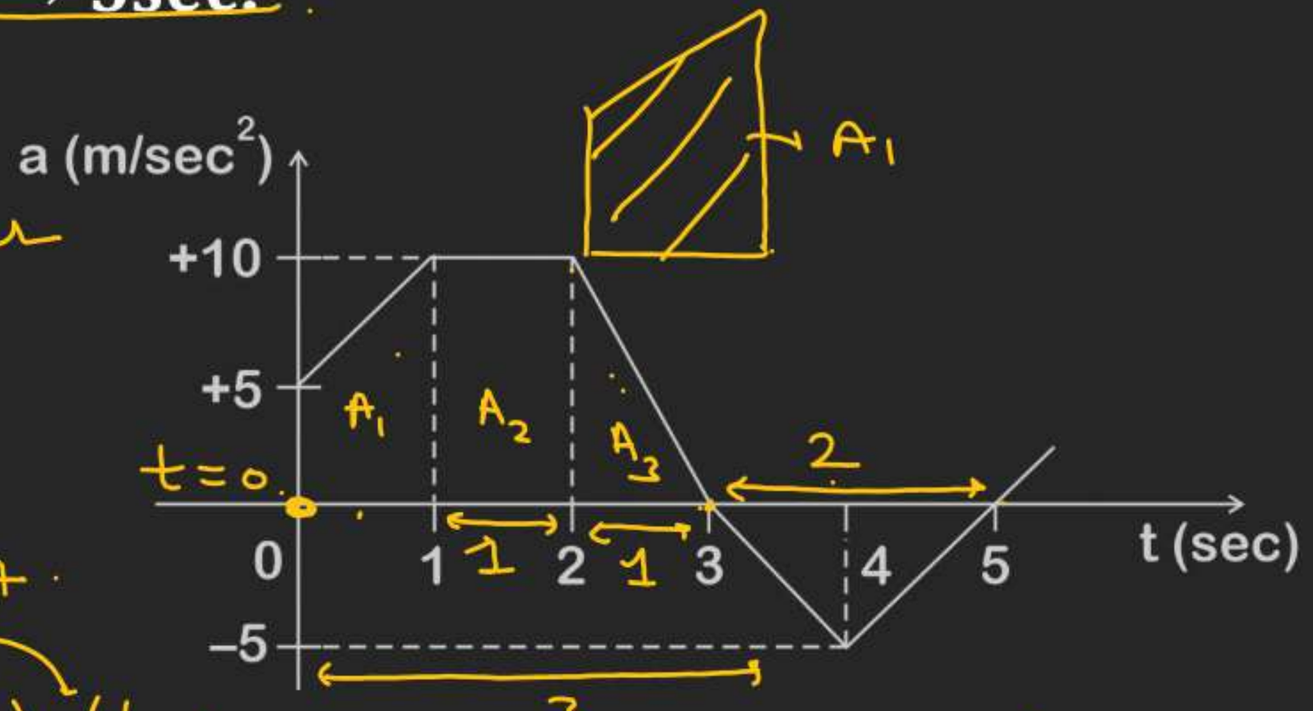
$$\boxed{t_1 = 5\,\text{sec},\quad t_1 = 20\,\text{sec}}$$ ✓

$$t_1 = 5\,\text{sec}$$

$\Rightarrow$ Time for uniform velocity motion $= 25 - 2t_1 = \underline{15\,\text{sec}}$ ✓

KINEMATICS

Q. Acceleration-time graph is given in the figure. Find the change in velocity and average acceleration for the time interval $0 \rightarrow 5\text{sec}$.



$$a = \frac{dv}{dt} \Rightarrow \int_u^v dv = \int_0^t a\,dt$$ → Area under a – vs t Curve

$$\Downarrow$$

$$(v - u) =$$

$$\Downarrow$$

$$\Delta v = (A_1 + A_2 + A_3) - A_4$$

$$= \frac{1}{2}[10+5]\times 1 + (10\times 1) + \left(\frac{1}{2}\times 1\times 10\right) - \frac{1}{2}\times 2\times 5$$

$$= \frac{15}{2} + 10 + \cancel{5} - \cancel{5}$$

$$= 10 + 7.5 = 17.5 \text{ m/s Ans}$$

Avg acceleration

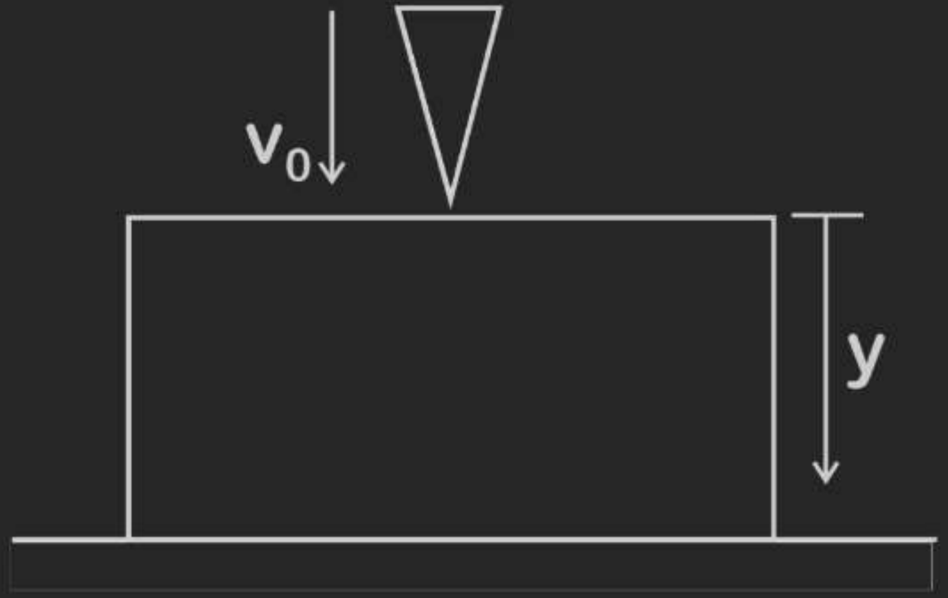
$$= \left(\frac{\Delta v}{\Delta t}\right)$$

$$= \frac{17.5}{5} =$$

KINEMATICS

H-W

Q. The cone falling with a speed v_0 strikes and penetrates the block of packing material (figure). The acceleration of the cone after impact is $a = g - cy^2$ where ' c ' is a positive constant and ' y ' is the penetration distance. If the maximum penetration depth is observed to be y_m, determine the constant ' c '.

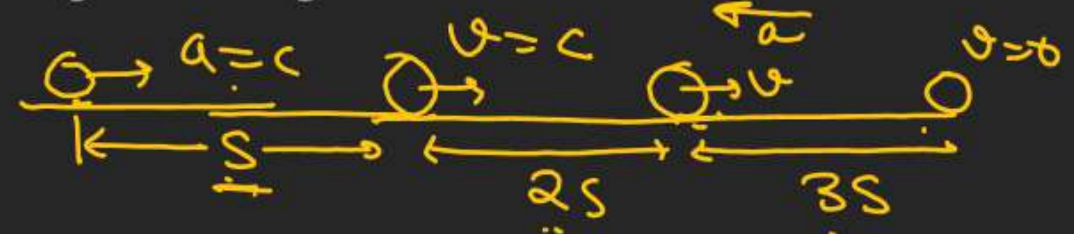


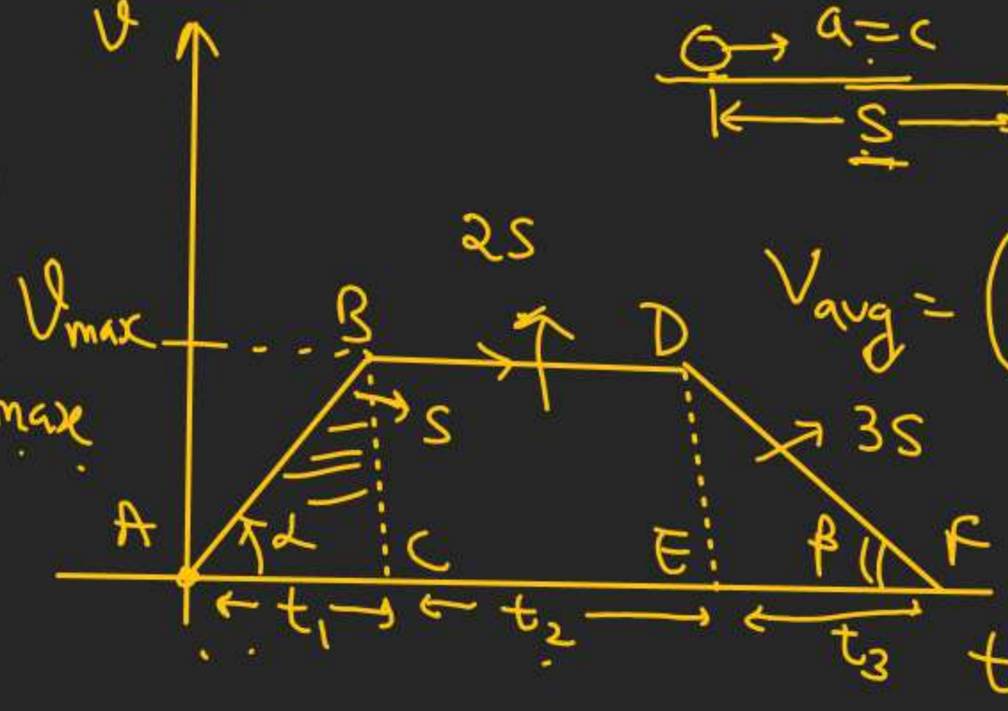
H.W:

Q. A bullet is fired horizontally on a fixed wooden block of length ' l ' as shown in the figure. It penetrates the block and emerges from its back face with velocity $[(v_0/\eta)(\eta > 1)]$. Resistance offered by the block against penetration is proprtional to the square of instantaneous velocity of the bullet. Find the time of penetration.

KINEMATICS

Q. A particle starts from rest and traverses a distance 's' with a uniform acceleration and then moves uniformly with the acquired velocity over a further distance $2s$. Finally it comes to rest after moving through a further distance 3 s under uniform retardation. Assuming the entire path is a straight line, find the ratio of the average speed over the journey to the maximum speed on the way.





$$6S = (\text{Area of trapezium})$$

$$6S = \frac{1}{2}\left[t_2 + (t_1+t_2+t_3)\right] \times V_{max}$$

$$S = \text{Area of } \Delta ABC$$

$$S = \frac{1}{2} \times t_1 \times V_{max}$$

$$t_1 = \left(\frac{2S}{V_{max}}\right)$$

$$V_{avg} = \left(\frac{6S}{t_1+t_2+t_3}\right)$$

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[$2S$ = Area of rectangle

$$2S = v_{max} \times t_2$$

$$t_2 = \left(\frac{2S}{v_{max}}\right)$$

Area of $\triangle DEF = 3S$.

$$\frac{1}{2} \times t_3 \times v_{max} = 3S$$

$$t_3 = \left(\frac{6S}{v_{max}}\right)$$

$$\text{Avg Speed} = \frac{6S}{t_1 + t_2 + t_3}$$

$$v_{avg} = \left(\frac{6S}{\frac{2S}{v_{max}} + \frac{2S}{v_{max}} + \frac{6S}{v_{max}}}\right)$$

$$v_{avg} = \left(\frac{6S}{10S}\right) v_{max}$$

$$\frac{v_{avg}}{v_{max}} = \frac{6}{10} = \frac{3}{5}$$

3:5 Ans ✓

KINEMATICS

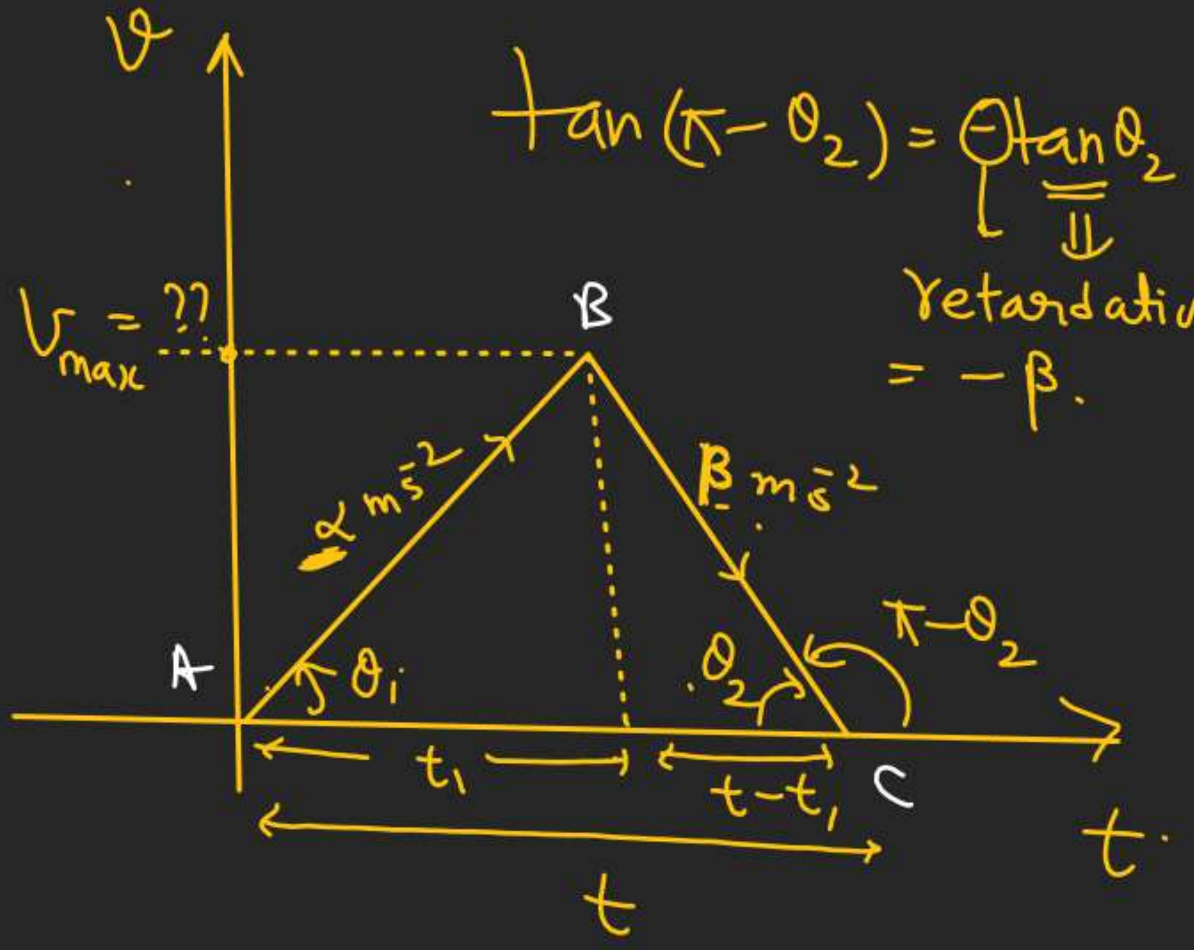
$v = \alpha t$ $\quad u = 0$

Q. A car accelerates from rest at a constant rate α for sometime after which it decelerates at a constant rate β to come to rest. If the total time lapse is ' t ' second, evaluate (i) the maximum velocity reached and (ii) the total distance travelled.

$$\tan\theta_1 = \alpha$$
$$\Downarrow$$
$$\frac{V_{max}}{t_1} = \alpha \Rightarrow t_1 = \left(\frac{V_{max}}{\alpha}\right) \quad \text{—(1)}$$

$$\tan\theta_2 = \beta$$
$$\frac{V_{max}}{t - t_1} = \beta$$
$$\frac{V_{max}}{\beta} = t - t_1 \quad \text{—(2)}$$

$$\frac{V_{max}}{\beta} + t_1 = t$$
$$\frac{V_{max}}{\beta} + \frac{V_{max}}{\alpha} = t$$
$$V_{max} = \left[\frac{\alpha\beta t}{\alpha+\beta}\right] \checkmark$$



$$\tan(\pi - \theta_2) = -\tan\theta_2$$
$$\Downarrow$$
retardation $= -\beta$

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Total distance:- Area of ΔABC

$$= \frac{1}{2} \times t \times \underline{v_{max}}$$

$$= \frac{1}{2} \times t \times \left(\frac{\alpha\beta t}{\alpha+\beta}\right)$$

$$= \frac{\alpha\beta t^2}{2(\alpha+\beta)} \checkmark$$

KINEMATICS

$F = ma \Rightarrow a = (F/m)$

Q. An object is thrown upward with an initial velocity v_0. The air drag on the object is assumed to be proportional to the velocity as shown in the figure. The intercept on time axis is; (λ is constant)

(A) $\ln\left(2 + \frac{\lambda v_0}{g}\right)$

(B) $\frac{1}{\lambda}\ln\left(1 + \frac{\lambda v_0}{g}\right)$ ✓

(C) none of these.

(D) can't be ascertained.

$F_{\text{air resistance}} \propto v$

$F_{\text{air resistance}} = -Kv$

$a = -\frac{(F_{\text{air resistance}} + mg)}{m}$

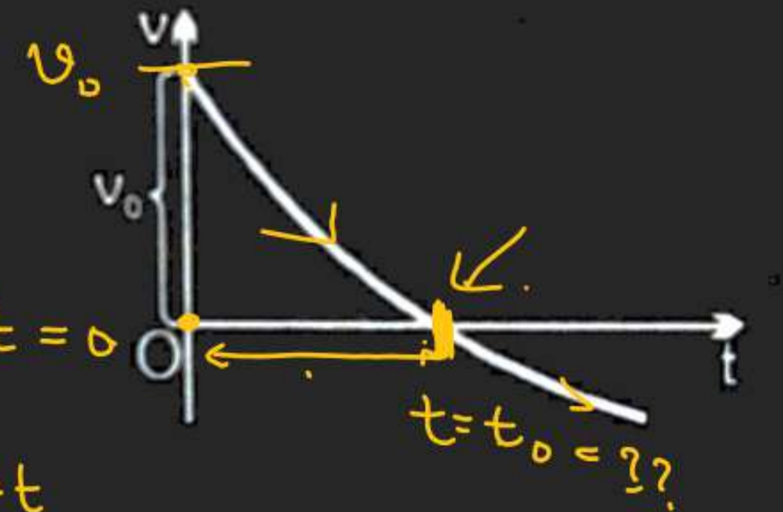
$a = -\frac{(Kv + mg)}{m}$

$$\boxed{a = -\left[\frac{K}{m}v + g\right]}$$

$t = t_0$ $\quad v = 0$

$t = t$ $\quad mg$ $\quad F_{\text{air resistance}}$

v_0 $\quad t = 0$



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$$a = -\left[\frac{K}{m}v + g\right]$$

$$\Downarrow$$

$$\frac{dv}{dt} = -\left[\frac{K}{m}v + g\right]$$

$$\int_{v_0}^{0} \frac{dv}{\left(\frac{K}{m}v + g\right)} = -\int_{0}^{t_0} dt$$

$\frac{K}{m} \Downarrow b$, $v \Downarrow x$, $g \Downarrow a$

$v = 0$, $t = t_0$

v_0, $t = 0$

$$\frac{\ln\left[\frac{K}{m}v + g\right]_{v_0}^{0}}{\left(\frac{K}{m}\right)} = -t_0 \Rightarrow \ln\left[\frac{g}{\frac{K}{m}v_0 + g}\right] = -\frac{K}{m}t_0$$

$$\Rightarrow \ln(g) - \ln\left(\frac{K}{m}v_0 + g\right) = -\frac{K}{m}t_0$$

$$\int \frac{dx}{x} = \ln x$$

$$\int \frac{dx}{a + bx} = \frac{1}{b}\ln(a + bx)$$

$$-\ln(a/b) = \ln(b/a)$$

$$t_0 = -\frac{m}{K}\ln\left(\frac{g}{\frac{K}{m}v_0 + g}\right)$$

$$t_0 = \frac{m}{K}\ln\left(\frac{\frac{K}{m}v_0 + g}{g}\right)$$

$$\boxed{t_0 = \frac{1}{\lambda}\ln\left(\frac{\lambda v_0 + g}{g}\right)}$$

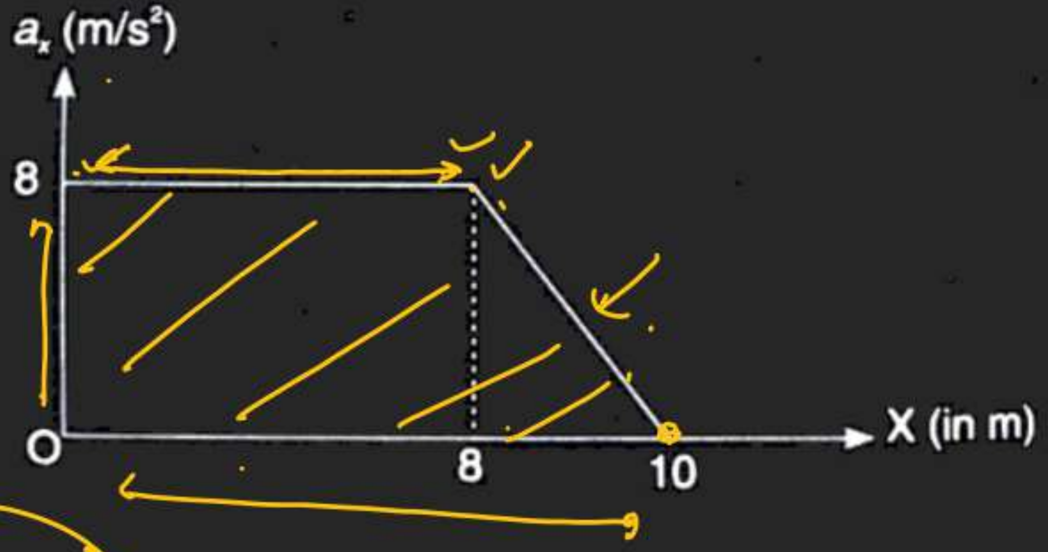
$\frac{K}{m} = \lambda$ $\Downarrow$ Constant

$$t_0 = \frac{1}{\lambda}\ln\left(\frac{\lambda v_0}{g} + 1\right) \checkmark$$

KINEMATICS

$\to u=0$

Q. A particle starts from rest (at $x = 0$) when an acceleration is applied to it. The acceleration of the particle changes with its co-ordinate as shown in the fig. Find the speed of the particle at $x = 10\text{m}$



$$a = v\frac{dv}{ds}$$

$$\int_0^s a\,ds = \int_u^v v\,dv$$

Area under a vs s graph $= \dfrac{v^2-u^2}{2}$

Area of trapezium $= \dfrac{v^2}{2}$

$$\frac{1}{\cancel{2}}[10+8]\times 8 = \frac{v^2}{\cancel{2}}$$

$$18\times 8 = v^2$$

$$V = \sqrt{8\times 18} = \sqrt{9\times 2\times 4\times 2} = \sqrt{144}$$

$$V = 12\,\text{m/s}$$

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KINEMATICS

Q.1 A point traversed half a circle of radius $R = 160\ \text{cm}$ during time interval $\tau = 10.0\ \text{s}$. [Irodov]

Calculate the following quantities averaged over that time:

(a) the mean velocity $\langle v \rangle$;

(b) the modulus of the mean velocity vector $|\langle \mathbf{v} \rangle|$;

(c) the modulus of the mean vector of the total acceleration $|\langle \mathbf{w} \rangle|$ if the point moved with constant tangent acceleration.

KINEMATICS

Q.2 A radius vector of a particle varies with time t as $r = = at(1 - \alpha t)$, where a is a constant vector and α is a positive factor Find: [Irodov]

(a) the velocity v and the acceleration w of the particle as functions of time;

(b) the time interval Δt taken by the particle to return to the initial points, and the distance s covered during that time.

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KINEMATICS

Q.3 At the moment $t = 0$ a particle leaves the origin and moves in the positive direction of the x axis. Its velocity varies with time as $v = v_0(1 - t/\tau)$, where v_0 is the initial velocity vector whose modulus equals $v_0 = 10.0\ \mathrm{cm/s}$; $\tau = 5.0\ \mathrm{s}$. Find:

[Irodov]

(a) the x coordinate of the particle at the moments of time 6.0 , 10 , and $20\ s$;

(b) the moments of time when the particle is at the distance $10.0\ \mathrm{cm}$ from the origin;

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KINEMATICS

Q.4 The velocity of a particle moving in the positive direction of the x axis varies as $v = \alpha\sqrt{x}$, where α is a positive constant. Assuming that at the moment $t = 0$ the particle was located at the point $x = 0$, find: [Irodov]

(a) the time dependence of the velocity and the acceleration of the particle;

(b) the mean velocity of the particle averaged over the time that the particle takes to cover the first s metres of the path.

KINEMATICS

Q.5 A point moves rectilinearly with deceleration whose modulus depends on the velocity v of the particle as $w = \alpha\sqrt{v}$, where a is a positive constant. At the initial moment the velocity of the point is equal to v_0. What distance will it traverse before it stops? What time will it take to cover that distance?

[Irodov]

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KINEMATICS

Q.6 A radius vector of a point A relative to the origin varies with time t as $r = ati - bt^2j$, where a and b are positive constants, and i and j are the unit vectors of the x and y axes. Find: [Irodov]

(a) the equation of the point's trajectory $y(x)$; plot this function;

(b) the time dependence of the velocity v and acceleration w vectors, as well as of the moduli of these quantities;

(c) the time dependence of the angle α between the vectors w and v;

(d) the mean velocity vector averaged over the first t seconds of motion, and the modulus of this vector.

KINEMATICS

Q.7 A point moves in the plane xy according to the law $x = at$, $y = at(1 - \alpha t)$, where a and α are positive constants, and t is time. Find: **[Irodov]**

(a) the equation of the point's trajectory $y(x)$; plot this function;

(b) the velocity v and the acceleration w of the point as functions of time;

(c) the moment t_0 at which the velocity vector forms an angle $\pi/4$ with the acceleration vector.

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Q.8 A point moves in the plane xy according to the law $x =$

$= a\sin\omega t, y = a(1 - \cos\omega t)$, where a and ω are positive constants. Find: [Irodov]

(a) the distance s traversed by the point during the time τ;

(b) the angle between the point's velocity and acceleration vectors.

KINEMATICS

Q.9 A particle moves in the plane xy with constant acceleration w directed along the negative y axis. The equation of motion of the particle has the form $y = ax - bx^2$, where a and b are positive constants. Find the velocity of the particle at the origin of coordinates. [Irodov]