$$\mathrm{cosec}^2 x = 3 \sec^2 y \Rightarrow \sqrt{3} |\sec y| = |\mathrm{cosec}(x)|$$
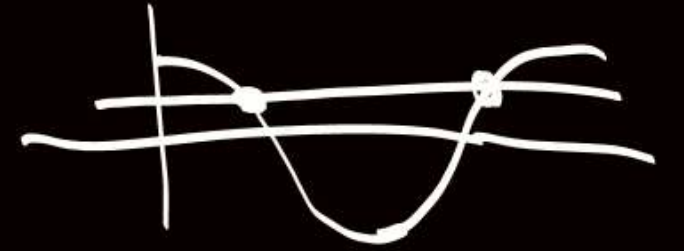
$$2 \mathrm{cosec} x + \sqrt{3} |\sec y| = 6$$

$$\alpha - 2\pi \in (-3\pi, -\pi)$$

$$\alpha \in (-\pi, \pi) \qquad \beta = \alpha + 2\pi \quad \times$$

$$\alpha + 2\pi \in (\pi, 3\pi)$$

$$2 \mathrm{cosec} x + |\mathrm{cosec} x| = 6$$

$$\boxed{\mathrm{cosec} x = 2}$$

$$-\pi \leq \alpha \leq \pi$$

$$-\pi \leq -\beta \leq \pi$$

$$\boxed{\beta = \alpha}$$

$$\frac{\alpha - \beta}{\downarrow}$$

$$[-2\pi, 2\pi]$$

$$\boxed{2\alpha \in [-2\pi, 2\pi]}$$

$$\alpha \rightarrow \boxed{4}$$

$$\alpha - \beta = -2\pi, 0, 2\pi$$

$$\cos(\alpha + \beta) = \frac{1}{e}$$

$$\cos 2\alpha = \frac{1}{e}$$

1. A line thru $A(-5,-4)$ meets the lines

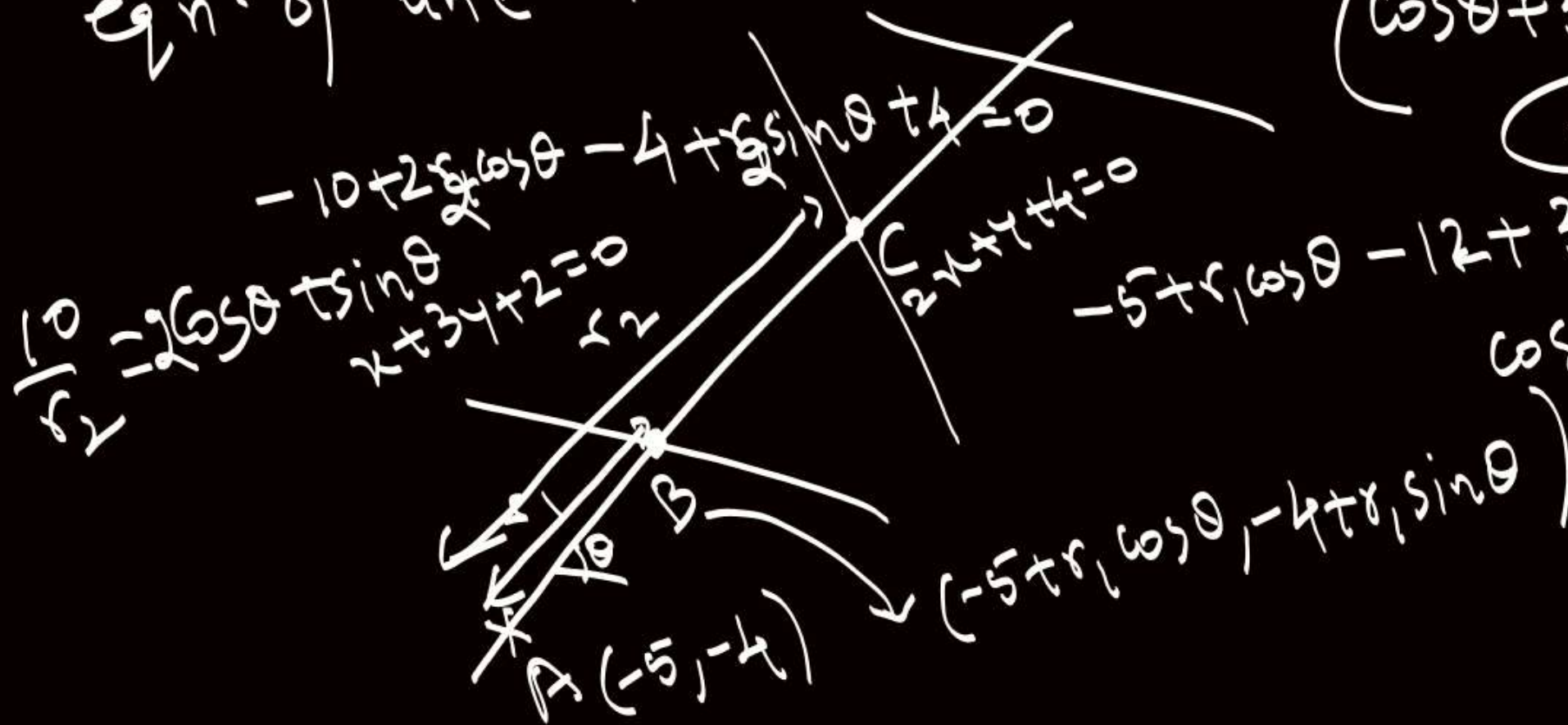$x+3y+2=0$, $2x+y+4=0$ and $x-y-5=0$ at B, C & D

respectively. If $\left(\dfrac{15}{AB}\right)^2 + \left(\dfrac{10}{AC}\right)^2 = \left(\dfrac{6}{AD}\right)^2$, then find the

eqn. of line.

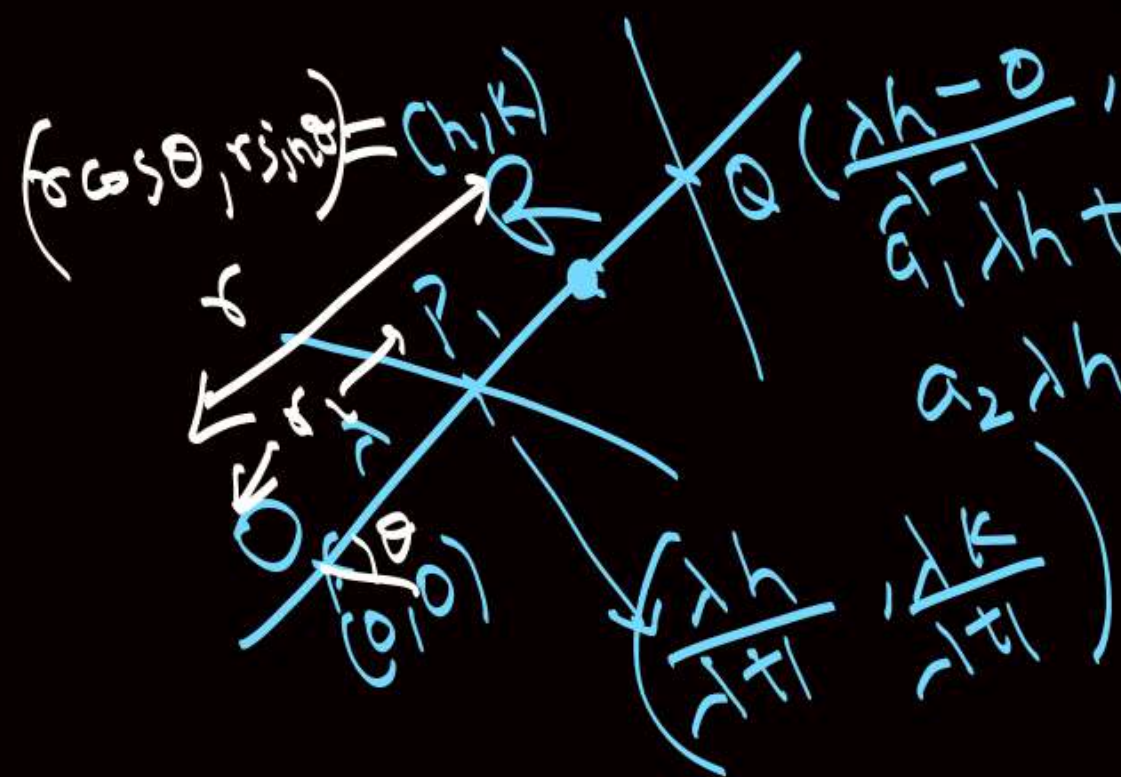$(\cos\theta+3\sin\theta)^2 + (2\cos\theta+\sin\theta)^2 = (\cos\theta-\sin\theta)^2$

$-10+2r_2\cos\theta-4+r_2\sin\theta+4=0$

$\boxed{\tan\theta = \dfrac{-7}{3}}$

$-5+r_1\cos\theta-12+3r_1\sin\theta+2=0$

$\dfrac{10}{r_2}=2\cos\theta+\sin\theta$

$x+3y+2=0$

r_2

C

$2x+y+4=0$

$\cos\theta+3\sin\theta = \dfrac{15}{r_1}$

B

$\sqrt{r_1}$ θ

$A(-5,-4)$

$(-5+r_1\cos\theta, -4+r_1\sin\theta)$

$\boxed{y+4 = \dfrac{-2}{3}(x+5)}$

2. A variable line thru origin 'O' meets two fixed lines $a_1 x + b_1 y + c_1 = 0$ and $a_2 x + b_2 y + c_2 = 0$ at P & Q. On it is taken a point R. If $\dfrac{2}{OR} = \dfrac{1}{OP} + \dfrac{1}{OQ}$, then P.T. locus of R is a straight line.

$$\frac{2}{r} = \frac{1}{r_1} + \frac{1}{r_2}$$

$$\frac{1}{r_1} + \frac{1}{r_2} = -\frac{a_1}{c_1}\cos\theta - \frac{b_1}{c_1}\sin\theta - \frac{a_2}{c_2}\cos\theta - \frac{b_2}{c_2}\sin\theta$$

$$a_1 r_1 \cos\theta + b_1 r_1 \sin\theta + c_1 = 0$$

$$-\frac{a_1}{c_1}\cos\theta - \frac{b_1}{c_1}\sin\theta$$

$$= \frac{2}{r}$$

$(r\cos\theta, r\sin\theta) = (h,k)$

R

$Q\left(\dfrac{\lambda h - 0}{\lambda - 1}, \dfrac{\lambda k}{\lambda - 1}\right)$

$(r\cos\theta, r\sin\theta)$

$(0,0)$

$\left(\dfrac{\lambda h}{\lambda + 1}, \dfrac{\lambda k}{\lambda + 1}\right)$

$a_1 \lambda h + b_1 \lambda k + c_1 (\lambda + 1) = 0$

$a_2 \lambda h + b_2 \lambda k + c_2 (\lambda - 1) = 0$

$\lambda (a_1 h + b_1 k + c_1) = -c_1$
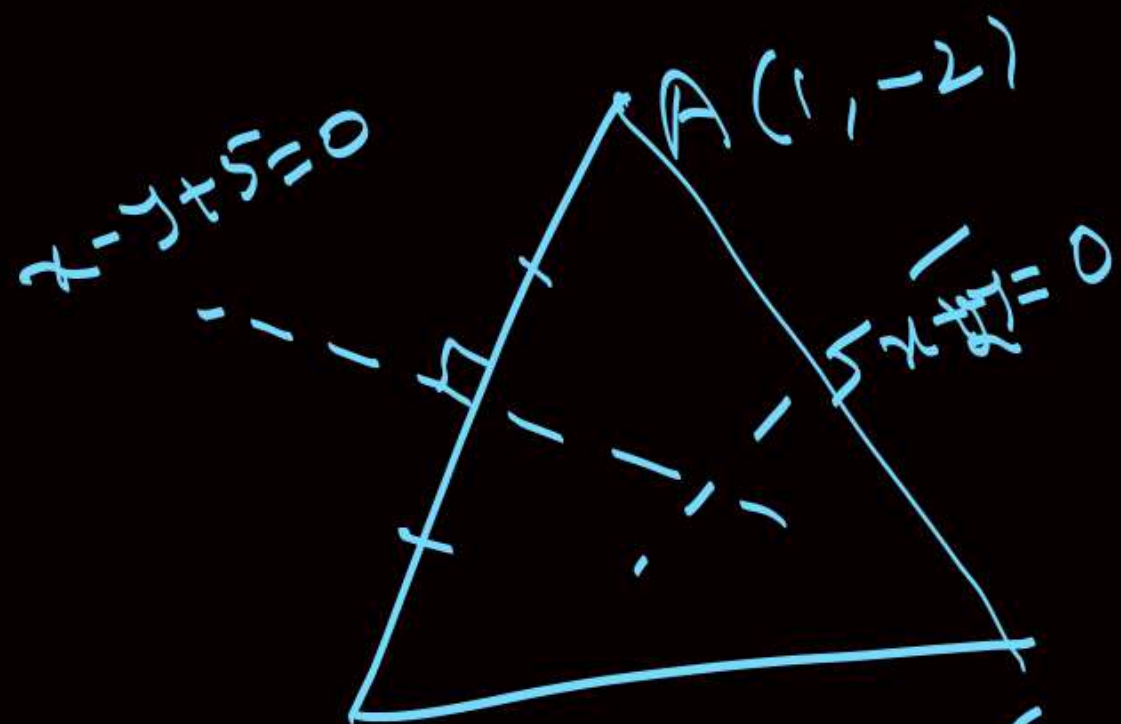
$\lambda (a_2 h + b_2 k + c_2) = c_2$

$$\boxed{\left(-\frac{a_1}{c_1} - \frac{a_2}{c_2}\right)h + \left(-\frac{b_1}{c_1} - \frac{b_2}{c_2}\right)k = \frac{2}{r}}$$

$$\frac{(a_1 h + b_1 k + c_1)}{(a_2 h + b_2 k + c_2)} = -\frac{c_1}{c_2}$$

3. Equations of perpendicular bisectors of the sides AB and AC of a triangle ABC are $x - y + 5 = 0$ and $x + 2y = 0$. If $A = (1, -2)$, find the eqn. of BC.

$$\frac{x-1}{1} = \frac{y+2}{-1} = -2\left(\frac{1+2+5}{1^2+(-1)^2}\right) = -8$$

$A(1,-2)$

$x - y + 5 = 0$

$x + 2y = 0$

B

$B = (-7, 6)$

$C = \left(\frac{11}{5}, \frac{2}{5}\right)$

$$\frac{x-1}{1} = \frac{y+2}{2} = -2\left(\frac{1-4}{5}\right) = \frac{6}{5}$$

$2x - 5 \rightarrow 8, 10, 25, 27,$

$2x - 6 \rightarrow 7, 8, 10, 14, 17, 19,$

$2x - 7 \rightarrow 7$

$SOT \rightarrow PT - 2$

4. If lines $px + by + c = 0$ are concurrent.
$$ax + qy + c = 0$$
$$ax + by + r = 0 \quad \text{find} \quad \frac{p}{p-a} + \frac{q}{q-b} + \frac{r}{r-c}.$$

$$\begin{vmatrix} p & b & c \\ a & q & c \\ a & b & r \end{vmatrix} = 0 = \begin{vmatrix} p & b & c \\ a-p & q-b & 0 \\ a-p & 0 & r-c \end{vmatrix} = p(q-b)(r-c) + b(p-a)(r-c)$$
$$+ c(p-a)(q-b)$$

$$\frac{p}{p-a} + \frac{bq+q}{q-b} + \frac{c-r+r}{r-c} = 0$$

$$\frac{p}{p-a} + \frac{q}{q-b} + \frac{r}{r-c} = 2$$