

$$\emptyset \quad S = \frac{1}{\cancel{1} \cdot \cancel{3} \cdot 5} + \frac{1}{\cancel{3} \cdot 5 \cdot 7} + \frac{1}{\cancel{5} \cdot 7 \cdot 9} + \dots n \text{ terms.}$$

Odd $\cancel{1 \cdot 3 \cdot 5 \cdot 7 \cdot 9 \dots}$

$$S = \sum_{r=1}^{\infty} \frac{1}{(2r-1)(2r+1)(2r+3)}$$

diff = 4 Barish.

$$= \frac{1}{4} \sum_{r=1}^{\infty} \frac{1}{(2r-1)(2r+1)} - \frac{1}{(2r+1)(2r+3)}$$

$$= \frac{1}{4} \left\{ \frac{1}{1 \cdot 3} - \frac{1}{3 \cdot 5} \right. \\ \left. + \frac{1}{3 \cdot 5} - \frac{1}{5 \cdot 7} \right. \\ \left. \vdots \right. \\ \left. \frac{1}{(2n-1)(2n+1)} - \frac{1}{(2n+1)(2n+3)} \right\} \\ \Rightarrow \frac{1}{4} \left\{ \frac{1}{1 \cdot 3} - \frac{1}{(2n+1)(2n+3)} \right\}$$

$$= \sum_{r=1}^n \frac{1}{(r)(r+1)(r+2)}$$

dubt - 2

$$= \frac{1}{2} \left(\sum \frac{1}{(r)(r+1)} - \frac{1}{(r+1)(r+2)} \right) = \sum \frac{r^2 + 4r + 4}{(r)(r+1)(r+2)(r+3)}$$

$$\begin{aligned}
 &= \frac{1}{2} \left\{ \frac{1}{1 \cdot 2} - \frac{1}{2 \cdot 3} \right. \\
 &\quad \left. + \frac{1}{2 \cdot 3} - \frac{1}{3 \cdot 4} \right. \\
 &\quad \left. + \frac{1}{3 \cdot 4} - \frac{1}{4 \cdot 5} \right. \\
 &\quad \left. + \dots + \frac{1}{(n)(n+1)} - \frac{1}{(n+1)(n+2)} \right\} \\
 &= \frac{1}{2} \left(\frac{1}{1 \cdot 2} - \frac{1}{(n+1)(n+2)} \right)
 \end{aligned}$$

$\Rightarrow \sum \frac{(r^2 + 3r)}{(r)(r+1)(r+2)(r+3)} + \frac{r^2}{(r)(r+1)(r+2)(r+3)} + \frac{4}{(r)(r+1)}$
 $\Rightarrow \sum \frac{1}{(r+1)(r+2)} + \sum \frac{1}{(r+1)(r+2)(r+3)} + 4 \sum \frac{1}{(r)(r+1)(r+2)(r+3)}$
 $\Rightarrow \frac{1}{1}(\sum \frac{1}{r+1} - \frac{1}{n+1}) + \frac{1}{2} \left(\sum \frac{1}{(r+1)(r+2)} - \frac{1}{(r+2)(r+3)} \right) + 4 \sum \frac{1}{(r)(r+1)(r+2)(r+3)}$

$$\text{Q) } S = \frac{4}{1 \cdot 2 \cdot 3} + \frac{5}{2 \cdot 3 \cdot 4} + \frac{6}{3 \cdot 4 \cdot 5} + \dots n \text{ terms.}$$

$$S_n = \sum T_r = \sum \frac{(r+3)}{(r)(r+1)(r+2)}$$

$$= \sum \frac{(r+2)}{(r)(r+1)\cancel{(r+2)}} + \underbrace{\frac{1}{(r)(r+1)(r+2)}}_3$$

$$\Rightarrow \sum \frac{1}{(r)(r+1)} + \frac{1}{2} \left(\sum \frac{1}{(r)(r+1)} - \frac{1}{(r+1)(r+2)} \right)$$

$$= \frac{1}{1} \left(\sum \frac{1}{r} - \frac{1}{r+1} \right) + \frac{1}{2} \left(\sum \frac{1}{1 \cdot 2} - \frac{1}{(n+1)(n+2)} \right)$$

$$= \frac{1}{1} \left(1 - \frac{1}{n+1} \right) + \frac{1}{2} \left(\frac{1}{1 \cdot 2} - \frac{1}{(n+1)(n+2)} \right)$$

$$\text{Q} \quad \frac{1}{2 \cdot 3 \cdot 4} + \frac{1}{3 \cdot 4 \cdot 5} + \frac{1}{4 \cdot 5 \cdot 6} + \dots + \frac{1}{(100 \cdot 101 \cdot 102)} = \frac{K}{101} \quad \text{find } K.$$

$$\sum T_r = \sum_{r=1}^{gg} \frac{1}{(r+1)(r+2)(r+3)}$$

$$= \frac{1}{2} \sum_{r=1}^{gg} \frac{1}{(r+1)(r+2)} - \frac{1}{(r+2)(r+3)}$$

$$= \frac{1}{2} \left(\frac{1}{2 \cdot 3} - \frac{1}{(n+2)(n+3)} \right)$$

$$= \frac{1}{2} \left(\frac{1}{2 \cdot 3} - \frac{1}{(101)(102)} \right) = \frac{K}{101}$$

$$K=?$$

Q Let n^{th} term of any Seqⁿ is given by

$$T_n = \frac{1^3 + 2^3 - 3^3 + 4^3 + \dots + (2n)^3}{(n)(4n+3)} \quad \text{then } \sum_{r=1}^{120} T_r = ?$$

$$T_n = \frac{2(2^3 + 4^3 + 6^3 - \dots - (2n)^3) - (1^3 + 2^3 + 3^3 + \dots + (2n)^3)}{(n)(4n+3)}$$

$$= \frac{2 \times 8(1^3 + 2^3 + 3^3 + \dots + (n)^3) - (1^3 + 2^3 + \dots + (2n)^3)}{(n)(4n+3)}$$

$$= \frac{164 \frac{(n^3)(n+1)^2}{4} - \frac{(2n)^2(2n+1)^2}{4}}{(n)(4n+3)} = \frac{n^2((2n+2)^2 - (2n+1)^2)}{(n)(4n+3)}$$

$$= \frac{n(4n+3)}{(4n+3)}$$

$$\sum n = \frac{(n)(n+1)}{2}$$

$$= \frac{15 \times 16}{2}$$

$$= 120$$

Himmat

$$Q \quad \frac{2^3 - 1^3}{1 \times 7} + \frac{4^3 - 3^3 + 2^3 - 1^3}{2 \times 11} + \frac{6^3 - 5^3 + 4^3 - 3^3 + 2^3 - 1^3}{3 \times 15} + \dots + \frac{30^3 - 29^3 + 28^3 - 27^3 + \dots + 2^3 - 1^3}{15 \times 63} = ?$$

$$\frac{8-1}{7} + \frac{64-27+8-1}{2 \times 11} + \frac{216-125+64-27+8-1}{3 \times 15} - - \times \swarrow \searrow$$

$$1 + 2 + 3 + \dots + 15 = \frac{15 \times 16}{2} = 120 \leftarrow$$

$$Q = \frac{1}{3^2 - 1} + \frac{1}{5^2 - 1} + \frac{1}{7^2 - 1} + \dots + \frac{1}{(201)^2 - 1} = ?$$

$$T_n = \sum \frac{1}{(2r+1)^2 - 1^2} = \sum \frac{1}{(2r-1+1)(2r-1-1)}$$

$$\sum \frac{1}{(2r)} \frac{1}{(2r-2)} = \frac{1}{4} \sum \frac{1}{(r)(r-1)}$$

$$\frac{1}{4} \times \frac{1}{r} \sum \frac{1}{r-1} - \frac{1}{r} \text{ Open & Sol.}$$

Value of $\sum_{n=1}^r \frac{3}{(4n-1)(4n+3)}$

$$\frac{3}{4} \left(\frac{1}{3} - \cancel{\frac{1}{7}} \right) = \frac{1}{3} \left(\frac{1}{3} - \frac{1}{4n+3} \right)$$

1) There are Some Series in which are directly not AP.

G.P or H.P

2) $S = t_1 + t_2 + t_3 + t_4 + t_5 + t_6 + \dots + t_n$

$t_1, t_2, t_3, t_4, t_5 \rightarrow 1^{\text{st}} \text{ Order diff}$

$s_1, s_2, s_3, s_4 \rightarrow 2^{\text{nd}} \text{ Order diff}$

$p_1, p_2, p_3 \rightarrow 3^{\text{rd}} \text{ Order diff}$

(3) If 3^{rd} Order difference is same then n^{th} term

will be $T_n = t_1 + K_1(n-1) + S_1 \frac{(n-1)(n-2)}{1 \cdot 2} + P_1 \frac{(n-1)(n-2)(n-3)}{1 \cdot 2 \cdot 3}$

(4) If P^{th} Order difference in H.P. In with C.R. = R then

$$T_n = A \cdot r^{n-1} + f(n)$$

where $f(n)$ is Poly of degree (P_1)

Ex $\boxed{P=3} \Rightarrow 3^{\text{rd}} \text{ Order difference in H.P.}$

$$T_n = A \cdot r^{n-1} + \boxed{bn^2 + (n+d)}$$

Q Find Sum of n terms of

$$3+7+13+21+\dots$$

$$S = \underbrace{3+7+13+21}_{4} + \underbrace{31+43}_{6} + \dots$$

$$\begin{array}{cccccc} 4 & 6 & 8 & 10 & 12 \\ 2 & 2 & 2 & 2 & 2 \end{array} \rightarrow 2^{\text{nd}} \text{ Order diff}$$

$$\begin{aligned} \text{constant} \\ S_n &= \sum T_n = \sum 3 + 4(n-1) + 2 \frac{(n-1)(n-2)}{1 \cdot 2} \\ &= \sum 3 + 4n - 4 + n^2 - 3n + 2 \\ &= \sum n^2 + n + 1 \quad \text{Solve} \\ &\Rightarrow \underline{\frac{(n)(n+1)(2n+1)}{6}} + \underline{\frac{(n)(n+1)}{2}} + n \end{aligned}$$

Q) Find sum of n terms of Series.

$$1 + 4 + 10 + 20 + \dots$$

$$1 + 4 + 10 + 20 + \dots$$

$$\begin{array}{r} 3 \\ \downarrow \\ 6 \\ \hline 12 \end{array} \rightarrow \begin{array}{l} (1-1) \rightarrow 0 \text{ order Poly} \\ 1^{\text{st}} \text{ order diffn} \\ \text{G.P.} \end{array}$$

$$\therefore S_n = \sum T_n = \sum 3 \cdot 2^{n-1} - 4 = 3 \sum 2^{n-1} - \sum 4$$

$$T_n = a \cdot (2)^{n-1} + b$$

$$T_1 = a \cdot 2^0 + b \Rightarrow a + b = 1$$

$$T_2 = a \cdot 2^{2-1} + b \quad \left| \begin{array}{l} 2a + b = 4 \\ -a = -3 \end{array} \right.$$

$$4 = 2a + b$$

$$\begin{array}{l} a = 3 \\ b = 2 \end{array}$$

$$T_n = 3 \cdot 2^{n-1} - 2$$

$$3(2^0 + 2^1 + 2^2 + \dots + 2^{n-1}) - 2 \sum 1$$

$$3 \cdot \frac{(1 \cdot (2)^n - 1)}{(2-1)} - 2 \times n$$

$$3 \cdot 2^n - 3 - 2n$$

Q) $S = 2 + 12 + 36 + 80 + 150 + 252 + \dots$ n terms.

2	12	36	80	150	252
10	24	44	70	102	
14	20	26	32		
6	6	6			

$$\overbrace{\text{Const (3rd Order)}}^{2^{\text{nd}} \text{ order Poly}} \quad \overbrace{\frac{(n-1)(n-2)}{1 \cdot 2} + \frac{8(n-1)(n-2)(n-3)}{1 \cdot 2 \cdot 3}}$$

$$T_n = n^3 - n^2$$

$$S_n = \sum n^3 + \sum n^2$$

$$\textcircled{1} S = \textcircled{9} + \textcircled{16} + 2^9 + 5^4 + 10^3 + \dots$$

7 13 25 49
6 12 24

$$\rightarrow \text{GP} \rightarrow 2^{\frac{n}{3+1}} \text{ deg poly.}$$

$$T_n = a(2)^{n-1} + b^n + c$$

$$T_1 = a \cdot 2^0 + b + c = 9 \Rightarrow a + b + c = 9.$$

$$T_2 = a \cdot 2^1 + 2b + c = 16 \Rightarrow 2a + 2b + c = 16.$$

$$T_3 = a \cdot 2^2 + 3b + c = 29 \Rightarrow 4a + 3b + c = 29$$

$$\text{Find } a, b, c = (6, 1, 2)$$

\Leftrightarrow
Find S_n

Relation betn AM, GM & HM.

$$\textcircled{1} G^2 = AH$$

$$\textcircled{2} \boxed{AM \geq GM \geq HM} \rightarrow \text{for +ve No. Only}$$

$$\frac{a_1 + a_2 + \dots + a_n}{n} \geq \left(a_1 a_2 \dots a_n \right)^{\frac{1}{n}} \geq \left(\frac{1}{a_1} + \frac{1}{a_2} + \dots + \frac{1}{a_n} \right)^{-1}$$

Q If $x \geq 0$ then P.T. $\frac{1+x}{2} \geq \sqrt{x}$

If $1, x$ are 2 elements then

$$AM \geq HM$$

$$\frac{1+x}{2} \geq \left(\frac{1+x}{2}\right)^{\frac{1}{2}}$$

$$1+x \geq 2\sqrt{x} \quad \text{(J.I.P.)}$$

Q If $x > 0$ then P.T. $\sqrt{x} + \sqrt{\frac{1}{x}} \geq 2$

elements $\rightarrow x, \frac{1}{x}$

$$AM \geq HM$$

$$\frac{x + \frac{1}{x}}{2} \geq \left(x \cdot \frac{1}{x}\right)^{\frac{1}{2}}$$

$$x + \frac{1}{x} \geq 2$$

$$x + \frac{1}{x} \geq 2$$

Q If $x, y \in R^+$ then P.T.

$$x^2 + y^2 \geq 2xy$$

elements x^2, y^2

$$AM \geq HM$$

$$\frac{x^2 + y^2}{2} \geq \left(x^2 \cdot y^2\right)^{\frac{1}{2}}$$

$$x^2 + y^2 \geq 2xy \quad \cancel{(\text{J.I.P.})}$$

Q If $x, y \in R^+$ then P.T.

$$2(x^2 + y^2) \geq (x+y)^2$$

We already know.

$$x^2 + y^2 \geq 2xy \quad (\text{adding } x^2 + y^2)$$

$$x^2 + y^2 + (x^2 + y^2) \geq 2xy + x^2 + y^2$$

$$2(x^2 + y^2) \geq (x+y)^2 \quad (\text{J.I.P.})$$

Q If $x, y \in R^+$ then P.T. $\frac{1}{x} + \frac{1}{y} \geq \frac{4}{x+y}$

elements $\rightarrow \frac{1}{x}, \frac{1}{y}$

AM \geq HM.

$$\frac{\frac{1}{x} + \frac{1}{y}}{2} \geq \sqrt{\frac{1}{x} + \frac{1}{y}}$$

$$\frac{\frac{1}{x} + \frac{1}{y}}{2} \geq \frac{2}{x+y} \quad (2)$$

$$\frac{1}{x} + \frac{1}{y} \geq \frac{4}{x+y} \quad (\text{H.P.})$$

Q If $a, b, x \in R^+$ then $ax + \frac{b}{x} \geq \dots$?

elements $\rightarrow ax, \frac{b}{x}$

AM \geq HM.

$$2\bar{ab}$$

$$\frac{ax + \frac{b}{x}}{2} \geq \left(ax \cdot \frac{b}{x} \right)^{\frac{1}{2}}$$

$$ax + \frac{b}{x} \geq 2\sqrt{ab}$$

Q If $x, y, z \in R^+$ then P.T.

$$x^2 + y^2 + z^2 \geq xy + yz + zx$$

from 3rd ab + pr. up we come to know

$$x^2 + y^2 \geq 2xy$$

$$y^2 + z^2 \geq 2yz$$

$$z^2 + x^2 \geq 2zx$$

$$\underline{x^2 + y^2 + z^2 \geq xy + yz + zx}$$

S.L.P.

$\forall x, y, z \in \mathbb{R}^+$ P.T.

$$(x+y)(y+z)(z+x) \geq 8xyz$$

$$x+y \geq 2\sqrt{xy} \quad (\text{AM} \geq \text{GM})$$

$$y+z \geq 2\sqrt{yz}$$

$$z+x \geq 2\sqrt{zx}$$

$$\text{Multiply } (x+y)(y+z)(z+x) \geq 2 \times 2 \times 2 \sqrt{xy} \times \sqrt{yz} \times \sqrt{zx}$$

$$\geq 8(xyz)$$

H.P.