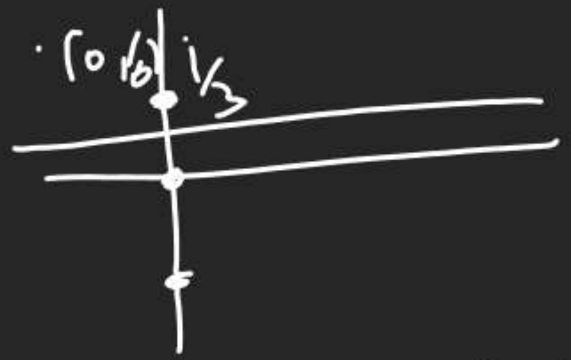
Nishant Jindal

Q1 If $|w| = \dfrac{z}{z-\frac{i}{3}}$ & $|w|=1$

then z lies on ....?

1) Par. 2) St. L 3) Circle 4) Ellipse.

(M1) $\left|\dfrac{z}{z-\frac{i}{3}}\right| = 1 \Rightarrow |z-0| = |z-\frac{i}{3}|$



Q2 If $|z|=1$, $z \neq \pm 1$ value of $\dfrac{z}{1-z^2}$ lies on.

(M1) $z = x+iy$, $\boxed{x^2+y^2 = 1}$ Solve

(M2) $z = i$ $\quad \dfrac{z}{1-z^2} = \dfrac{i}{2}$ lies on. ~ Imy. Axis

Q3 If z_1, z_2 C.N. Such that

$\dfrac{z_1 - 2z_2}{2 - z_1\bar{z}_2}$ is Unimodular.

find $|z_1|$ if $|z_2| \neq 1$

(1) $\left|\dfrac{z_1-2z_2}{2-z_1\bar{z}_2}\right| = 1$

$\Rightarrow |z_1 - 2z_2|^2 = |2 - z_1\bar{z}_2|^2$

Sq $\Rightarrow (z_1 - 2z_2)(\bar{z}_1 - 2\bar{z}_2) = (2 - z_1\bar{z}_2)(2 - \bar{z}_1 z_2)$

$\Rightarrow |z_1|^2 - \cancel{2\bar{z}_1 z_2} - \cancel{2z_1\bar{z}_2} + 4|z_2|^2 = 4 - \cancel{2\bar{z}_1 z_2} - \cancel{2z_1\bar{z}_2} + |z_1|^2|z_2|^2$

$\Rightarrow |z_1|^2(1 - |z_2|^2) + 4(|z_2|^2 - 1) = 0$

$(|z_1|^2 - 4)(|z_2|^2 - 1) = 0$

$|z_1| = 2$

Q4 If $|z|=1$ & $\boxed{w=\dfrac{z-1}{z+1}}$ $(z\neq -1)$

then $\text{Re}(w) = ?$

A) 0, B) $\dfrac{1}{|z+1|^2}$, C) $\dfrac{1}{|z+1|}\cdot\dfrac{1}{(z+1)^2}$ D) $\dfrac{\sqrt{2}}{|z+1|^2}$

$$z=\frac{1+w}{1-w} \Rightarrow |z|=1$$

$$|1+w|^2=|1-w|^2$$

$$\Rightarrow 1+|w|^2+2\text{Re}(1\cdot w) = 1+|w|^2-2\text{Re}(1\cdot w)$$

$$4\,\text{Re}(w)=0$$

$$\text{Re}(w)=0$$

Q5 If $|z_1|=|z_2|=1$ & $z_1z_2\neq -1$

then P.T. $z=\dfrac{z_1+z_2}{1+z_1z_2}$ is Purely Real

$z=\bar{z}$ Prove

$$\bar{z}=\frac{\bar{z}_1+\bar{z}_2}{1+\bar{z}_1\bar{z}_2}$$

$$=\frac{\frac{1}{z_1}+\frac{1}{z_2}}{1+\frac{1}{z_1z_2}}$$

$$\bar{z}=\frac{z_1+z_2}{1+z_1z_2}=z$$

(H.P.)

$|z_1|=1$

$$\frac{1}{z_1}=\frac{\bar{z}_1}{|z_1|^2}$$

$$\frac{1}{z_1}=\bar{z}_1 \;*$$

Q6 $z_1, z_2, z_3 \ldots z_n$ are C.N. Such that $z_1\, z_2 \ldots z_n$ are lying on circle of centre $(0,0)$ & Rad $=1$ If $w=\left(\sum_{k=1}^{n} z_k\right)\left(\sum_{k=1}^{n}\frac{1}{z_k}\right)$

then P.T. (1) w is Real

(2) $0\leq w\leq n^2$

(1) $w=(z_1+z_2+\cdots+z_n)\left(\dfrac{1}{z_1}+\dfrac{1}{z_2}+\cdots+\dfrac{1}{z_n}\right)$

(2)

$|z_1|=|z_2|=|z_3|\ldots=|z_n|=1$

(3) $w=(z_1+z_2\ldots z_n)(\bar{z}_1+\bar{z}_2\ldots\bar{z}_n)$

$$=(z_1+z_2\ldots z_n)\cdot\overline{(z_1+z_2\ldots z_n)}$$

$$w=|z_1+z_2+\cdots+z_n|^2=\text{Real Positive C.N.}$$

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Q 7 If $\overset{A}{z}, \overset{B}{iz}, \overset{C}{i^2 z}$ are Vertices of Δ then Nature of Δ?

$AB = |z_1 - z_2| = |z - iz| = |z|\,|1-i| = \sqrt{2}\,|z|$

$BC = |z_2 - z_3| = |iz + z| = |z|\,|1+i| = \sqrt{2}\,|z|$

$AC = |z_1 - z_3| = |z + z| = 2|z|$

$AB^2 + BC^2 = AC^2$

Rt. Isosceles.

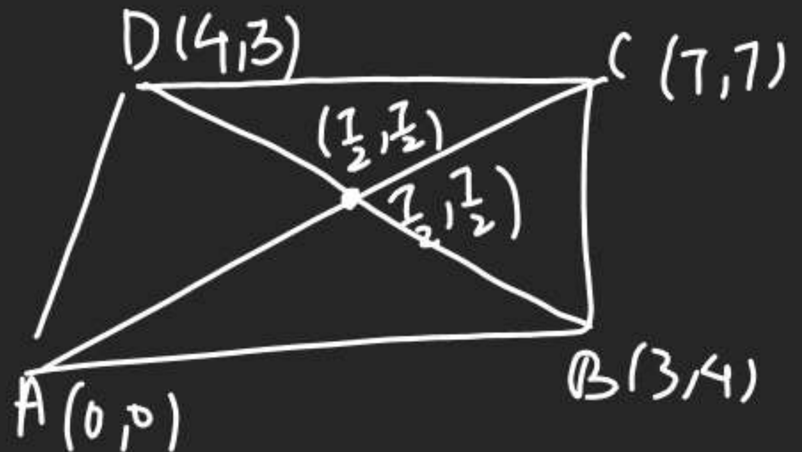
Q 8 Distance betⁿ $z_1 = 2+3i,\ z_2 = -1+4i$?

$|z_1 - z_2| = |3 - i| = \sqrt{10}$

Q 9 $0,\ 3+4i,\ 7+7i,\ 4+3i$ are Vertices of Quad. ... Rect, Sqⁿ, Rhom, ||gm
x x

$A(0,0)\quad B(3,4)\quad C(7,7)$

$D(4,3)$



1) Mid Pt. Coincide.

2) $AC \perp BD$ Check

$m_{BD} = \frac{4-3}{3-4} = -1$

$m_{AC} = \frac{7-0}{7-0} = 1$

$m_1 \times m_2 = -1$

(3) $\left.\begin{array}{l} AC = \sqrt{49+49} \\ BD = \sqrt{2} \end{array}\right\}$ Rhombus

Q 10 If $\left|\frac{6z-i}{2+3iz}\right| \le 1$ then P.T. $|z| \le \frac{1}{3}$?

$|6z - i| \le |2+3iz|$

Sq² $|6z-i|^2 \le |2+3iz|^2$

$(6z-i)(6\bar{z}+i) \le (2+3iz)(2-3i\bar{z})$

$36|z|^2 + 6iz - 6i\bar{z} + 1 \le 4 - 6i\bar{z} + 6iz + 9|z|^2$

$27|z|^2 \le 3$

$|z|^2 \le \frac{1}{9}$

$|z| \le \frac{1}{3}$

H.P.

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Q If $|z|\le 1$, $|w|\le 1$
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then P.T.

$$(z-w)^2 \le \underbrace{(|z|-|w|)^2}_{(r_1-r_2)^2} + \underbrace{(\operatorname{Arg} z - \operatorname{Arg} w)^2}_{(\theta_1-\theta_2)^2}$$

① $z = r_1 e^{i\theta_1}$, $w = r_2 e^{i\theta_2}$

$\sin x \le x$
$\sin^2 x \le x^2$

$$(z-w)^2 = r_1^2 + r_2^2 - 2r_1r_2\cos(\theta_1-\theta_2)$$

$$= (r_1^2+r_2^2-2r_1r_2) + 2r_1r_2(1-\cos(\theta_1-\theta_2))$$

$$= (r_1-r_2)^2 + 4r_1r_2\sin^2\left(\frac{\theta_1-\theta}{2}\right)$$

$$(z-w)^2 \le (r_1-r_2)^2 + 4\left(\frac{\theta_1-\theta_2}{2}\right)^2$$

$$\le \underbrace{(|z|-|w|)^2 + (\operatorname{Arg} z - \operatorname{Arg} w)^2}_{\text{H.P.}}$$

$$(r_1-r_2)^2 + 4\underbrace{|z|}_{\le 1}\underbrace{|w|}_{\le 1}\underbrace{\sin^2\left(\frac{\theta_1-\theta_2}{2}\right)}_{\le\left(\frac{\theta_1-\theta}{2}\right)^2}$$

$$\le 4\left(\frac{\theta_1-\theta_2}{2}\right)^2 + (r_1-r_2)^2$$

Q u, v, w C.N.S.T.
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$|u|\le 1$, $\boxed{|v|=1}$, $w = \dfrac{v\cdot(u-z)}{(\bar{u}z-1)}$

If $|w|\le 1$ then P.T. $|z|\le 1$

① $|v\cdot(u-z)| \le |\bar{u}z-1|$

$\Rightarrow |v(u-z)|^2 \le |\bar{u}z-1|^2$

$\Rightarrow (v(u-z))(\bar{v}(\bar{u}-\bar{z})) \le (\bar{u}z-1)(u\bar{z}-1)$

$\Rightarrow \underbrace{v\bar{v}}_{|v|}(u\bar{u} - \cancel{u\bar{z}} - \cancel{z\bar{u}} + z\bar{z}) \le u\bar{u}z\bar{z} - \cancel{\bar{u}z} - \cancel{u\bar{z}} + 1$

$\Rightarrow |u|^2 + |z|^2 - |u|^2|z|^2 - 1 \le 0$

$(|u|^2-1) - |z|^2(|u|^2-1) \le 0$

$(|u|^2-1)(1-|z|^2) \le 0$

$|u|\le 1$
$|u|^2\le 1$
$-$ve Aayega

+ve hogya.

$1-|z|^2 \ge 0$
$|z|^2\le 1$
$|z|\le 1$ H.P.

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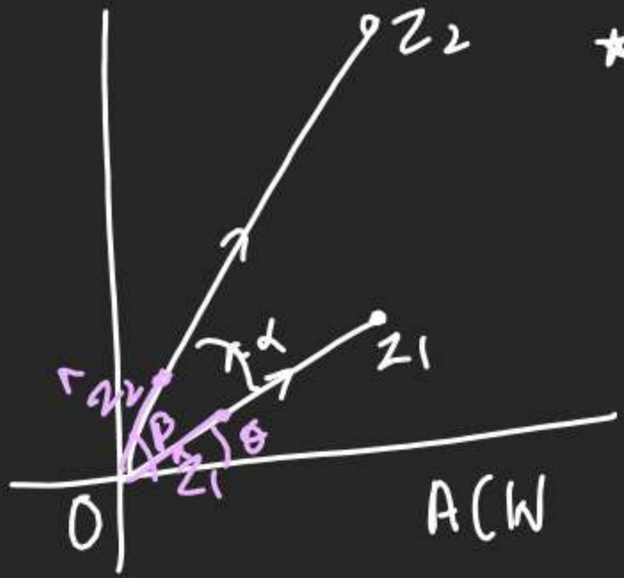
RoTaTion TheoRem.

1) here we think in vectors Basically.

2) 2 vectors are Same.
=) Mag. Same & direction Same.

3)



* Rt. now mag z_1 & z_2 are different
* But if we make them Unit vectors mag. will become Same.

$z_1 = |z_1| \cdot e^{i\theta}$

$\hat{z}_1 = e^{i\theta}$

ACW: $\alpha = +ve$

C.W.: $\alpha = -ve$

* z_1 's Unit vector $= \dfrac{z_1}{|z_1|}$

$z_2 = |z_2| \cdot e^{i\beta}$

$\hat{z}_2 = \dfrac{z_2}{|z_2|} = e^{i\beta}$

$\beta = \alpha + \theta \Rightarrow e^{i\beta} = e^{i(\alpha+\theta)}$

$e^{i\beta} = e^{i\alpha} \cdot e^{i\theta}$

$\dfrac{z_2}{|z_2|} = \dfrac{z_1}{|z_1|} \cdot e^{i\alpha}$

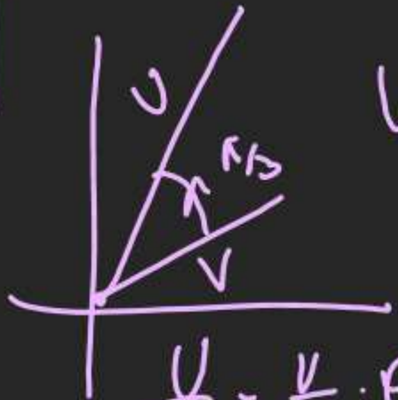
$\hat{z}_2 = \hat{z}_1 \cdot e^{i\alpha}$

Ex:- 14

$\hat{U} = \hat{W} \cdot e^{i\frac{\pi}{4}}$

$\dfrac{U}{|U|} = \dfrac{W}{|W|} \cdot e^{i\frac{\pi}{4}}$

Ex: 15



U in term of V?

V को Rotate $\frac{\pi}{3}$ A

$\dfrac{U}{|U|} = \dfrac{V}{|V|} \cdot e^{i\frac{\pi}{3}}$

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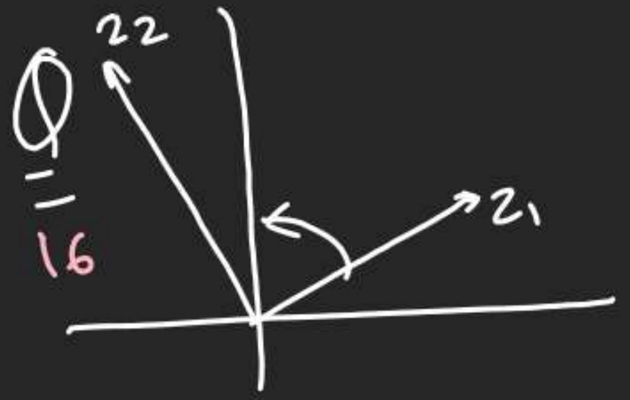
Q 16



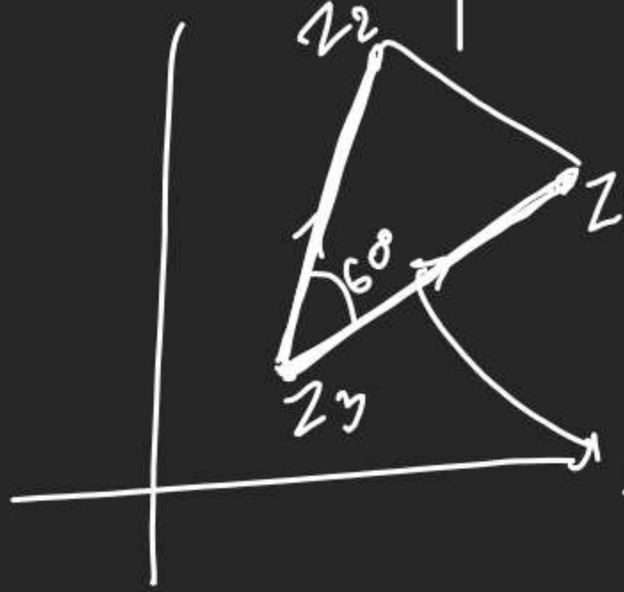
$z_2 \perp z_1$

find z_2 in terms of z_1

$\hat{z}_2 = \hat{z}_1 \cdot e^{i\frac{\pi}{2}}$

$\frac{z_2}{|z_2|} = \frac{z_1}{|z_1|} \cdot \left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)$

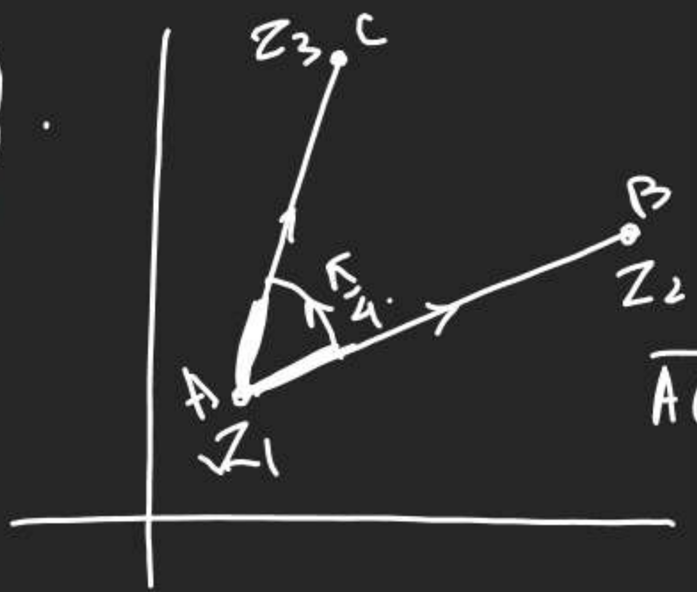
$\hat{z}_2 = \hat{z}_1 \cdot i$



$\frac{z_2 - z_3}{|z_2 - z_3|} = \frac{(z_1 - z_3)}{|z_1 - z_3|} e^{i(60°)}$

Δ is eq^l Δ

Q. 17



$\vec{AC} = C - A$
$= z_3 - z_1$
$\vec{AB} = B - A$
$=$

Connect z_1, z_2, z_3 ?

$\hat{AC} = \hat{AB} \cdot e^{i\frac{\pi}{4}}$

$\frac{z_3 - z_1}{|z_3 - z_1|} = \frac{z_2 - z_1}{|z_2 - z_1|} \cdot e^{i\frac{\pi}{4}}$

Q 18 z_1, z_2, z_3 are C.N.

Satisfying $\frac{z_1 - z_3}{z_2 - z_3} = \frac{1 - i\sqrt{3}}{2}$

are vertices of . . . . Δ.

① $\frac{z_1 - z_3}{z_2 - z_3} = \frac{1 - i\sqrt{3}}{2}$

$\Rightarrow \frac{z_2 - z_3}{z_1 - z_3} = \frac{2}{1 - i\sqrt{3}} \times \frac{1 + i\sqrt{3}}{1 + i\sqrt{3}} = \frac{1 + i\sqrt{3}}{2}$

$= \frac{1}{2} + i\frac{\sqrt{3}}{2}$

$\frac{z_2 - z_3}{z_1 - z_3} = \cos 60° + i\sin 60° = e^{i\frac{\pi}{3}}$

① $\left|\frac{z_2 - z_3}{z_1 - z_3}\right| = 1 \Rightarrow |z_2 - z_3| = |z_1 - z_3|$

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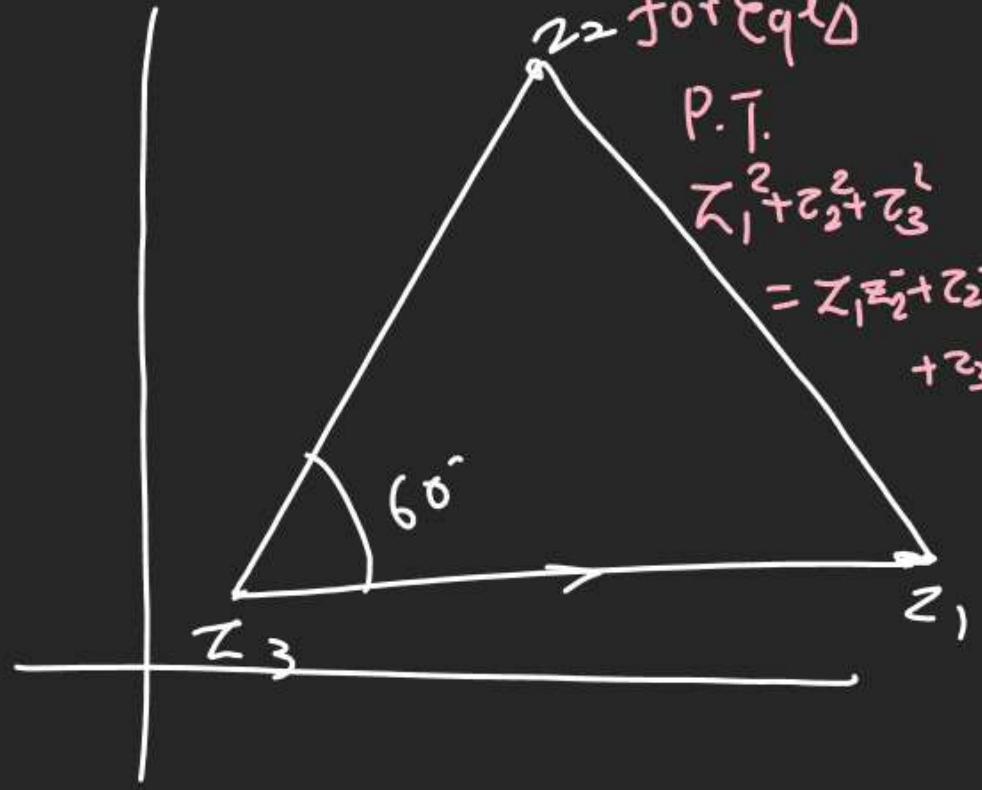
Extension of Prev Qs. 19 for Eq. Δ

P.T. $z_1^2+z_2^2+z_3^2 = z_1z_2+z_2z_3+z_3z_1$



$$\frac{z_2-z_3}{z_1-z_3} = e^{i\frac{\pi}{3}} = \frac{1}{2} + \frac{i\sqrt{3}}{2}$$

$$\frac{z_2-z_3}{z_1-z_3} - \frac{1}{2} = \frac{i\sqrt{3}}{2}$$

$$\frac{2z_2-2z_3-z_1+z_3}{\cancel{2}(z_1-z_3)} = \frac{i\sqrt{3}}{\cancel{2}}$$

$$(2z_2-z_1-z_3)^2 = (i\sqrt{3}(z_1-z_3))^2$$

$$z_1^2+z_2^2+z_3^2 = z_1z_2+z_2z_3+z_3z_1$$

$$\Rightarrow \boxed{\sum z_1^2 = \sum z_1z_2}$$

Eq Δ की निशानी

Extensions

$$2z_1^2+2z_2^2+2z_3^2 = 2z_1z_2+2z_2z_3+2z_3z_1$$

Q 20 $(z_1-z_2)^2+(z_2-z_3)^2+(z_3-z_1)^2=0$

या

$$\frac{1}{z_1-z_2}+\frac{1}{z_2-z_3}+\frac{1}{z_3-z_1}=0$$