

**Formula**

(i)  $\sin(A + B) = \sin A \cos B + \sin B \cos A$

$$\sin(A - B) = \sin A \cos B - \sin B \cos A$$

$$\cos(A + B) = \cos A \cos B - \sin A \sin B$$

$$\cos(A - B) = \cos A \cos B + \sin A \sin B$$

$$\sin(A + B) \sin(A - B) = \sin^2 A - \sin^2 B$$

$$\cos(A + B) \cos(A - B) = \cos^2 A - \sin^2 B$$

(ii)  $\sin \frac{\pi}{12} = \sin 15^\circ = \frac{\sqrt{3}-1}{2\sqrt{2}} = \cos 75^\circ = \cos \frac{5\pi}{12}$

$$\cos \frac{\pi}{12} = \cos 15^\circ = \frac{\sqrt{3}+1}{2\sqrt{2}} = \sin 75^\circ = \sin \frac{5\pi}{12}$$

$$\tan \frac{\pi}{12} = \tan 15^\circ = 2 - \sqrt{3} = \cot 75^\circ = \cot \frac{5\pi}{12}$$

$$\tan 75^\circ = 2 + \sqrt{3} = \cot 15^\circ$$

(iii)  $2 \sin A \cos B = \sin(A + B) + \sin(A - B)$

$$2 \cos A \cos B = \cos(A + B) + \cos(A - B)$$

$$2 \sin A \sin B = \cos(A - B) - \cos(A + B)$$

$$\sin C + \sin D = 2 \sin \frac{C + D}{2} \cos \frac{C - D}{2}$$

$$\sin C - \sin D = 2 \sin \frac{C - D}{2} \cos \frac{C + D}{2}$$

$$\cos C + \cos D = 2 \cos \frac{C + D}{2} \cos \frac{C - D}{2}$$

$$\cos C - \cos D = 2 \sin \frac{D - C}{2} \sin \frac{C + D}{2}$$

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

$$\cot(A + B) = \frac{\cot A \cot B - 1}{\cot A + \cot B}$$

$$\cot(A - B) = \frac{\cot A \cot B + 1}{\cot B - \cot A}$$

$$\tan(A + B) - \tan A - \tan B = \tan(A + B) \tan A \tan B$$



(iv)  $\tan A + \tan B = \frac{\sin(A+B)}{\cos A \cos B}$

$$\tan A - \tan B = \frac{\sin(A-B)}{\cos A \cos B}$$

$$\cot A + \cot B = \frac{\sin(A+B)}{\sin A \sin B}$$

$$\cot A - \cot B = \frac{\sin(B-A)}{\sin A \sin B}$$

(v)  $\tan(\theta_1 + \theta_2 + \theta_3 + \dots + \theta_n) = \frac{s_1 - s_3 + s_5 - s_7 + \dots}{1 - s_2 + s_4 - s_6 + \dots}$

$$s_1 = \sum \tan \theta_1$$

$$s_2 = \sum \tan \theta_1 \tan \theta_2$$

$$s_3 = \sum \tan \theta_1 \tan \theta_2 \tan \theta_3$$

(vi)  $\sin 2\theta = 2 \sin \theta \cos \theta = \frac{2 \tan \theta}{1 + \tan^2 \theta}$

(vii)  $\cos 2\theta = \cos^2 \theta - \sin^2 \theta$

$$= 2 \cos^2 \theta - 1 = 1 - 2 \sin^2 \theta = \frac{1 - \tan^2 \theta}{1 + \tan^2 \theta}$$

(viii)  $1 + \cos 2\theta = 2 \cos^2 \theta$

$$1 - \cos 2\theta = 2 \sin^2 \theta$$

$$1 + \sin 2\theta = (\cos \theta + \sin \theta)^2$$

$$1 - \sin 2\theta = (\cos \theta - \sin \theta)^2$$

(ix)  $\tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta}$

(x)  $\sin 3\theta = 3 \sin \theta - 4 \sin^3 \theta$

$$\cos 3\theta = 4 \cos^3 \theta - 3 \cos \theta$$

$$\tan 3\theta = \frac{3 \tan \theta - \tan^3 \theta}{1 - 3 \tan^2 \theta}$$

(xi)  $\sin^4 \theta + \cos^4 \theta = 1 - \frac{1}{2} \sin^2 2\theta$

$$\sin^6 \theta + \cos^6 \theta = 1 - \frac{3}{4} \sin^2 2\theta$$

$$\cos \theta \cos 2\theta \cos 2^2 \theta \cos 2^3 \theta \dots \cos 2^{n-1} \theta$$

$$= \frac{\sin 2^n \theta}{2^n \sin \theta}$$



$$(xii) \sin \theta \sin \left( \frac{\pi}{3} - \theta \right) \sin \left( \frac{\pi}{3} + \theta \right) = \frac{1}{4} \sin 3\theta$$

$$\cos \theta \cos \left( \frac{\pi}{3} - \theta \right) \cos \left( \frac{\pi}{3} + \theta \right) = \frac{1}{4} \cos 3\theta$$

$$\tan \theta \tan \left( \frac{\pi}{3} - \theta \right) \tan \left( \frac{\pi}{3} + \theta \right) = \tan 3\theta$$

### Conditional Identities

if  $\tan A + \tan B + \tan C = \tan A \tan B \tan C$

$\Rightarrow A + B + C = n\pi, n \in I.$

(i) If  $A + B + C = \pi$ , then

$$\tan A + \tan B + \tan C = \tan A \tan B \tan C$$

$$\cot A \cot B + \cot B \cot C + \cot C \cot A = 1$$

$$\tan \frac{A}{2} \tan \frac{B}{2} + \tan \frac{B}{2} \tan \frac{C}{2} + \tan \frac{C}{2} \tan \frac{A}{2} = 1$$

$$\cot \frac{A}{2} + \cot \frac{B}{2} + \cot \frac{C}{2} = \cot \frac{A}{2} \cot \frac{B}{2} \cot \frac{C}{2}$$

$$(ii) \sin 18^\circ = \sin \frac{\pi}{10} = \frac{\sqrt{5}-1}{4} = \cos 72^\circ = \cos \frac{2\pi}{5}$$

$$\cos 36^\circ = \cos \frac{\pi}{5} = \frac{\sqrt{5}+1}{4} = \sin 54^\circ = \sin \frac{3\pi}{10}$$

$$\tan 22.5^\circ = \tan \frac{\pi}{8} = \sqrt{2} - 1 = \cot \frac{3\pi}{8} = \cot(67.5^\circ)$$

$$\tan 67.5^\circ = \tan \frac{3\pi}{8} = \sqrt{2} + 1 = \cot \frac{\pi}{8} = \cot 22.5^\circ$$

$$(iii) \sin \alpha + \sin(\alpha + \beta) + \sin(\alpha + 2\beta) + \sin(\alpha + 3\beta) + \dots + \sin(\alpha + (n-1)\beta)$$

$$= \frac{\sin \left( \frac{n\beta}{2} \right)}{\sin \left( \frac{\beta}{2} \right)} \sin \left( \frac{2\alpha + (n-1)\beta}{2} \right)$$

$$\cos \alpha + \cos(\alpha + \beta) + \cos(\alpha + 2\beta) + \cos(\alpha + 3\beta) + \dots + \cos(\alpha + (n-1)\beta)$$

$$= \frac{\sin \left( \frac{n\beta}{2} \right)}{\sin \left( \frac{\beta}{2} \right)} \cos \left( \frac{2\alpha + (n-1)\beta}{2} \right)$$