

10. $\cos(A+B) + \sin(A-B)$

$$= \sin\left(\frac{\pi}{2} + A+B\right) + \sin(A-B)$$

$$= 2 \sin\left(\frac{\frac{\pi}{2} + 2A}{2}\right) \cos\left(\frac{\frac{\pi}{2} + 2B}{2}\right)$$

$$= \frac{2(2\sin 2\theta \cos 2\theta) \cos 4\theta}{\sin 2\theta}$$

13. $\frac{\tan 5\theta + \tan 3\theta}{\tan 5\theta - \tan 3\theta}$

$$= \frac{\frac{\sin 5\theta}{\cos 5\theta} + \frac{\sin 3\theta}{\cos 3\theta}}{\frac{\sin 5\theta}{\cos 5\theta} - \frac{\sin 3\theta}{\cos 3\theta}} = \frac{\left(\frac{\sin 8\theta}{\cancel{\cos 5\theta} \cancel{\cos 3\theta}}\right)}{\left(\frac{\sin 2\theta}{\cancel{\cos 5\theta} \cancel{\cos 3\theta}}\right)}$$

$$= \frac{\sin 8\theta}{\sin 2\theta} = \frac{\sin(4\theta + 4\theta)}{\sin 2\theta}$$

$$= \frac{2 \sin 4\theta \cos 4\theta}{\sin 2\theta} = \frac{2 \sin(2\theta + 2\theta) \cos 4\theta}{\sin 2\theta}$$

$$\underline{11.} \quad \frac{-2 \sin A \sin 2A}{2 \sin A \cos 2A} + \frac{2 \sin A \sin 3A}{2 \sin A \cos 3A}$$

$$= \frac{\sin 3A}{\cos 3A} - \frac{\sin 2A}{\cos 2A}$$

$$= \frac{\sin 3A \cos 2A - \sin 2A \cos 3A}{\cos 2A \cos 3A} = \frac{\sin A}{\cos 2A \cos 3A}$$

$$\underline{18.} \quad \frac{(\sin(A-C) + \sin(A+C)) + 2\sin A}{(\sin(B-C) + \sin(B+C)) + 2\sin B}$$

$$= \frac{2\sin A \cos C + 2\sin A}{2\sin B \cos C + 2\sin B} = \frac{\cancel{2}\sin A (\cancel{\cos C} + 1)}{\cancel{2}\sin B (\cancel{\cos C} + 1)}$$

$$= \frac{\sin A}{\sin B}$$

$$\tan A + \tan B = \frac{\sin A}{\cos A} + \frac{\sin B}{\cos B} = \frac{\sin A \cos B + \sin B \cos A}{\cos A \cos B}$$

$$\tan A + \tan B = \frac{\sin(A+B)}{\cos A \cos B}$$

$$\tan A - \tan B = \frac{\sin(A-B)}{\cos A \cos B}$$

$$\cot A + \cot B = \frac{\sin(A+B)}{\sin A \sin B}$$

$$\cot A - \cot B = \frac{\sin(B-A)}{\sin A \sin B}$$

$$19. \frac{(\sin A - \sin 5A) + (\sin 9A - \sin 13A)}{(\cos A - \cos 5A) - (\cos 9A - \cos 13A)}$$

$$\boxed{1569}$$

$$\boxed{1155} = \frac{(-2\cancel{\sin 2A} \cos 3A) + (-2\cancel{\sin 2A} \cos 11A)}{}$$

$$\min(1155, 1569) = \frac{(2\cancel{\sin 2A} \sin 3A) - (2\cancel{\sin 2A} \sin 11A)}{\cos 3A + \cos 11A}$$

$$= \frac{2\cancel{\cos 7A} \cos 4A}{2\sin 4A \cancel{\cos 7A}}$$

$$\boxed{\text{rank} < \min(1155, 1569)} \quad \frac{\sin 11A - \sin 3A}{\cos 3A + \cos 11A} = \cot 4A$$

$$3 = \max(2, 3) \quad \min(2, 3) = 2$$

$$\begin{aligned}
 & \underline{24.} \quad \frac{(\cos(A+B+C) + \cos(-A+B+C)) + (\cos(A-B+C) + \cos(A+B-C))}{(\sin(A+B+C) + \sin(-A+B+C)) - (\sin(A-B+C) - \sin(A+B-C))} \\
 &= \frac{2\cancel{\cos(B+C)}\cancel{\cos A} + 2\cancel{\cos A}\cos(B-C)}{2\cancel{\sin(B+C)}\cancel{\cos A} - 2\sin(C-B)\cancel{\cos A}} \\
 &= \frac{\cos(B+C) + \cos(B-C)}{\sin(B+C) + \sin(B-C)} = \frac{2\cos B \cos C}{2\sin B \cos C} = \cot B.
 \end{aligned}$$

$$\underline{30.} \quad \cos\left(\theta + \left(n - \frac{3}{2}\right)\phi\right) - \cos\left(\theta + \left(n + \frac{3}{2}\right)\phi\right)$$

$$= 2 \sin \frac{3\phi}{2} \sin(\theta + n\phi) \quad \underline{28.}$$

$$28 \quad \underline{27.} \quad (\sin 50^\circ - \sin 70^\circ) + \sin 10^\circ$$

$$= -2 \sin 10^\circ \cancel{\cos 60^\circ} + \sin 10^\circ$$

\swarrow
 $\frac{1}{2}$

$$= -\sin 10^\circ + \sin 10^\circ = 0$$

$$\begin{aligned} \underline{28.} \quad & \frac{\sin 10^\circ + \sin 20^\circ + \sin 40^\circ + \sin 50^\circ}{(\sin 10^\circ + \sin 50^\circ) + (\sin 20^\circ + \sin 40^\circ)} \\ &= 2 \sin 30^\circ \cos 20^\circ + 2 \sin 30^\circ \cos 10^\circ \\ &= \cos 20^\circ + \cos 10^\circ \\ &= \sin 70^\circ + \sin 80^\circ \end{aligned}$$

$$\begin{aligned}\tan(A+B) &= \frac{\sin(A+B)}{\cos(A+B)} = \frac{\sin A \cos B + \cos A \sin B}{\cos A \cos B - \sin A \sin B} \\&= \left(\frac{\sin A \cos B + \cos A \sin B}{\cos A \cos B} \right) \div \left(\frac{\cos A \cos B - \sin A \sin B}{\cos A \cos B} \right) \\&= \frac{\tan A + \tan B}{1 - \tan A \tan B}\end{aligned}$$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\cot(A+B) = \frac{(1 - \tan A \tan B) / \tan A \tan B}{(\tan A + \tan B) / \tan A \tan B}$$

$$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

$$\cot(A+B) = \frac{\cot A \cot B - 1}{\cot A + \cot B}$$

$$\cot(A-B) = \frac{\cot A \cot B + 1}{\cot B - \cot A}$$

$$\tan(A+B) - \tan A - \tan B = \tan(A+B)\tan A \tan B$$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\tan(A+B) - \tan(A+B)\tan A \tan B = \tan A + \tan B$$

$$\tan(A+B) - \tan A - \tan B = \tan(A+B)\tan A \tan B$$

$$\underline{1.} \quad \cot 16^\circ \cot 44^\circ + \cot 44^\circ \cot 76^\circ - \cot 76^\circ \cot 16^\circ = ?$$

$$= (\cot 16^\circ \cot 44^\circ - 1) + (\cot 44^\circ \cot 76^\circ - 1) - (\cot 76^\circ \cot 16^\circ + 1) + 3$$

$$\cot(A+B) = \frac{\cot A \cot B - 1}{\cot A + \cot B}$$

$$\cot(A-B) = \frac{\cot A \cot B + 1}{\cot B - \cot A}$$

$$= \cot(16^\circ + 44^\circ)(\cot 16^\circ + \cot 44^\circ) + \cot 120^\circ(\cot 44^\circ + \cot 76^\circ) - \cot(76^\circ - 16^\circ)(\cot 16^\circ - \cot 76^\circ) + 3$$

$$= \frac{1}{\sqrt{3}}(\cot 16^\circ + \cot 44^\circ) - \frac{1}{\sqrt{3}}(\cot 44^\circ + \cot 76^\circ) - \frac{1}{\sqrt{3}}(\cot 16^\circ - \cot 76^\circ) + 3$$

3

$$= \frac{1}{\sqrt{3}}[\cot 16^\circ + \cot 44^\circ - \cot 44^\circ - \cot 76^\circ - \cot 16^\circ + \cot 76^\circ] + 3$$

2. P.T. $\tan 54^\circ = \frac{\cos 9^\circ + \sin 9^\circ}{\cos 9^\circ - \sin 9^\circ}$

$$\tan(45^\circ + 9^\circ) = \frac{\tan 45^\circ + \tan 9^\circ}{1 - \tan 45^\circ \tan 9^\circ} = \frac{1 + \tan 9^\circ}{1 - \tan 9^\circ}$$

$$= \frac{1 + \frac{\sin 9^\circ}{\cos 9^\circ}}{1 - \frac{\sin 9^\circ}{\cos 9^\circ}}$$
$$= \frac{\cos 9^\circ + \sin 9^\circ}{\cos 9^\circ - \sin 9^\circ}$$

H.W
Ex-15 (leave Q.15)
Ex-16 (Q.5, 6, 7)