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** String - pully constrain :→

Constrain Motion :-

⇒ If one body is about to move then other body is bound to move.

⇒ By the help of Constrain motion we can find the relation b/w displacement, velocity & accleration b/w the bodies which are in Constrain with each other.

let, x_A and x_B be the displacement of two bodies which are in Constrain Motion & $x_A = K x_B$ (K = Constain)

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$$\boxed{x_A = K x_B}$$ [At instant]

[K = Constant]

Differentiating both side w.r.t time.

$$\left(\frac{dx_A}{dt}\right) = K\left(\frac{dx_B}{dt}\right)$$

$$\boxed{v_A = \underline{K} v_B}$$

$\Downarrow$

if K is constant

$$\frac{dv_A}{dt} = K\frac{dv_B}{dt}$$

$$\boxed{a_A = K a_B}$$

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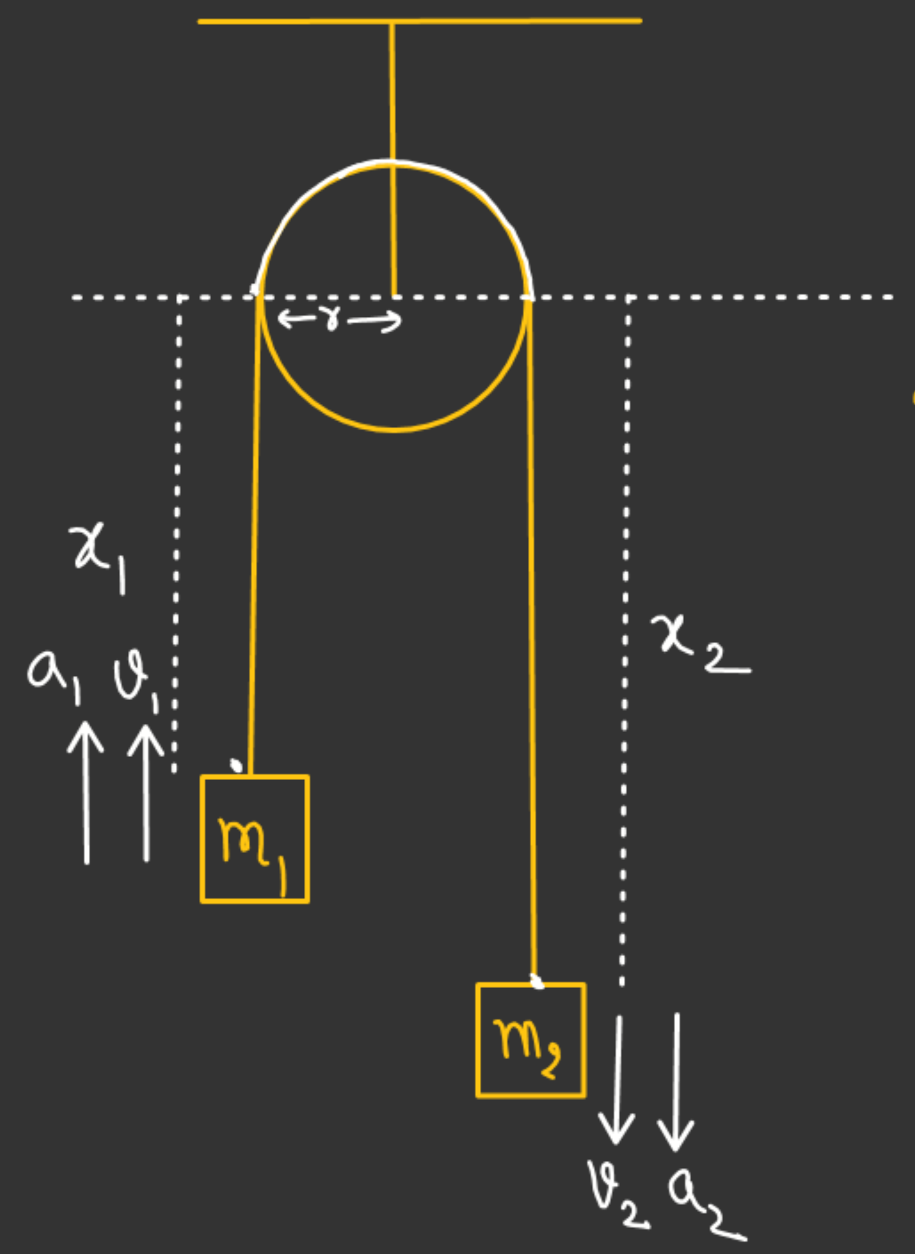
Pully Constrain:-

M-1 (Basic Method) $\Rightarrow$ String is inextensible.
i.e length of the string will not change.

$$\boxed{L = \text{Constant}}$$



$$L = x_1 + \pi r + x_2$$

Differentiating above relation w.r.t time.

$$\frac{d(L)}{dt} = \left(\frac{dx_1}{dt}\right) + \frac{d}{dt}(\pi r) + \frac{d}{dt}(x_2)$$

$$0 = -v_1 + 0 + v_2$$

$$\boxed{v_2 = v_1} \longrightarrow \frac{dv_2}{dt} = \frac{dv_1}{dt} \Rightarrow \boxed{a_2 = a_1}$$

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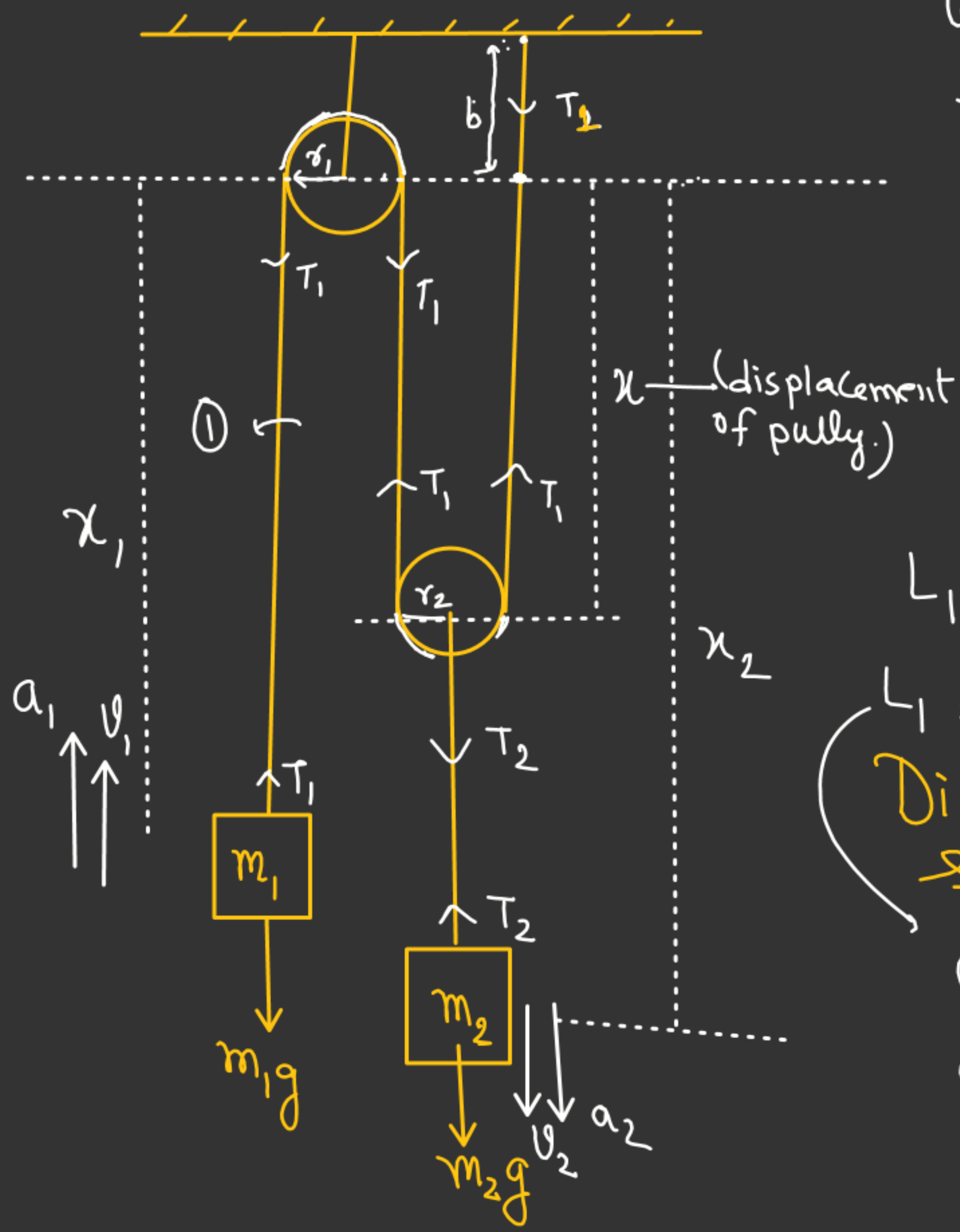

let, L_2 and L_1 be the total length of string ② & ①

$$L_1 = x_1 + \pi r_1 + 2x + \pi r_2 + b$$

$$L_2 = x_2 - x$$

$$x = (x_2 - L_2)$$

$$L_1 = x_1 + \pi r_1 + 2(x_2 - L_2) + \pi r_2 + b.$$

$$L_1 = x_1 + 2x_2 + \underbrace{\pi r_1 - 2L_2 + \pi r_2 + b}_{\Rightarrow \text{ constant.}}$$

Differentiating both side w.r.t time.

$$0 = \left(\frac{dx_1}{dt}\right) + 2\left(\frac{dx_2}{dt}\right) + 0$$

$$0 = -v_1 + 2v_2$$

$$v_1 = 2v_2$$

$$\boxed{a_1 = 2a_2} \rightarrow ①$$

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given

$m_1 = m$ ✓

$m_2 = 3m$ ✓

Constrain relation

$\boxed{a_1 = 2a_2}$ — (1)

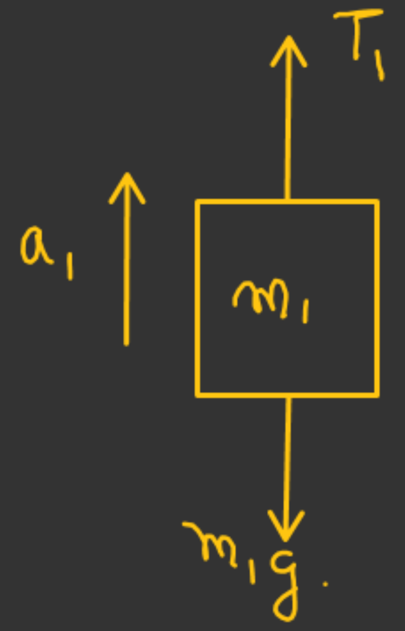
If $a_2 = a$ ✓

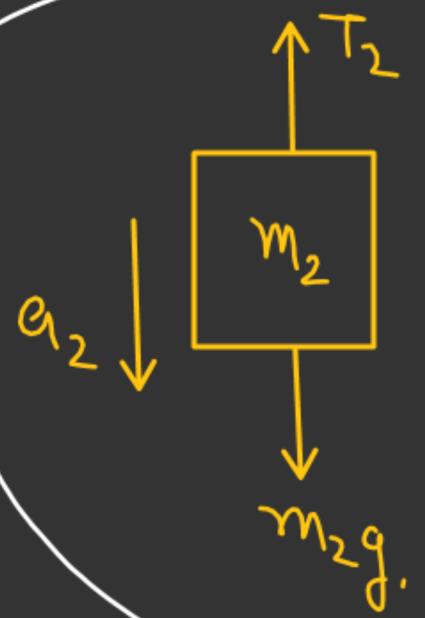
$a_1 = 2a$

For m_1

$T_1 - m_1 g = m_1 a_1$ — (2)

$m_2 g - T_2 = m_2 a_2$ — (3)





$\Rightarrow$ Massless pully

$2T_1 = T_2$ ✓

If $T_1 = T$, $T_2 = 2T$

$[T - mg = m\,2a]$ — (4)

$3mg - 2T = 3m\,a$ — (5)

2(4) + (5)

$3mg - 2mg = 7ma$

$mg = 7ma$

$a = g/7 \ ms^{-2}$

Accleration of (3m)

Accleration of $m = 2g/7 \ ms^{-2}$ ✓

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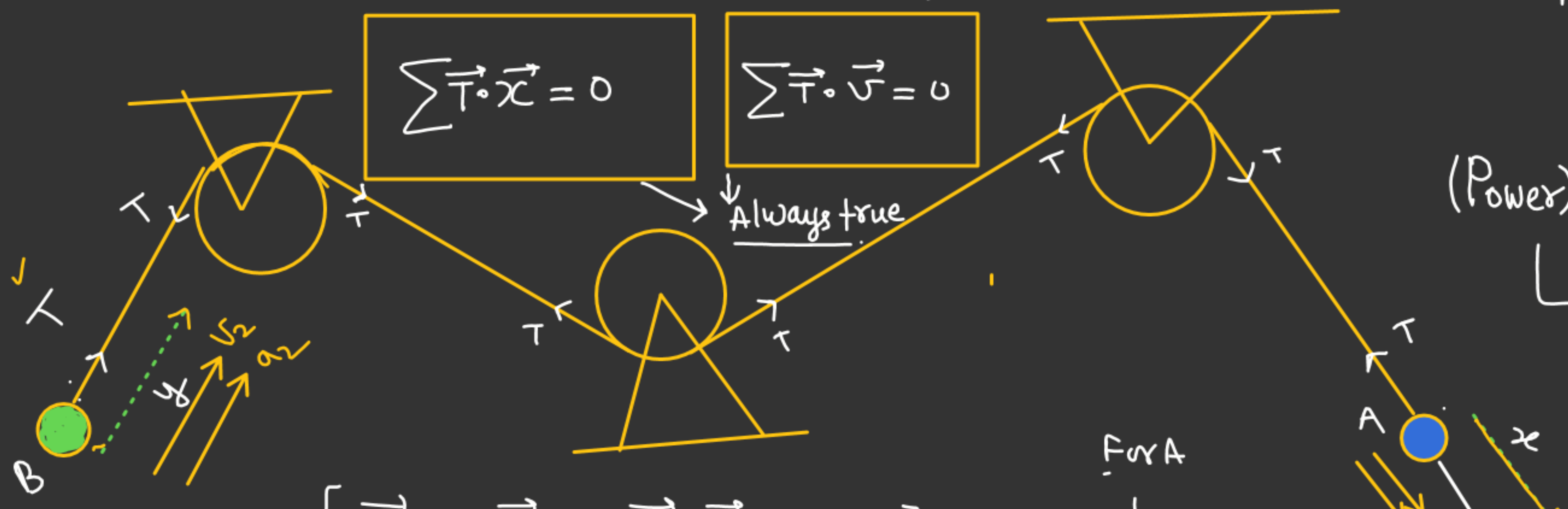
Q: M-2.

Virtual work-done Method (Tension trick)

Concept :- Since, for whole system tension is an internal force so.

$$\boxed{\sum \vec{T} \cdot \vec{x} = 0} \qquad \boxed{\sum \vec{T} \cdot \vec{v} = 0}$$

Always true



$$\left[\begin{array}{l} \vec{T} \uparrow\uparrow \vec{x} \Rightarrow \vec{T} \cdot \vec{x} = Tx \\ \vec{T} \uparrow\downarrow \vec{x} \Rightarrow \vec{T} \cdot \vec{x} = -Tx \end{array}\right]$$

For A

$$-Tx + Ty = 0$$

$$\boxed{x = y}$$

$$-Tv_1 + Tv_2 = 0$$

$$\boxed{v_1 = v_2}$$

Dot-product

$$\vec{F} \cdot \vec{S} = W$$

F ∥ S

$$W = \vec{F} \cdot \vec{S}$$

$$(\text{Power}) = \frac{dW}{dt} = \vec{F} \cdot \frac{d\vec{s}}{dt}$$

$$\hookrightarrow P = \vec{F} \cdot \vec{v}$$

$$\boxed{\sum \vec{T} \cdot \vec{a} = 0}$$

If θ changing then not applicable.

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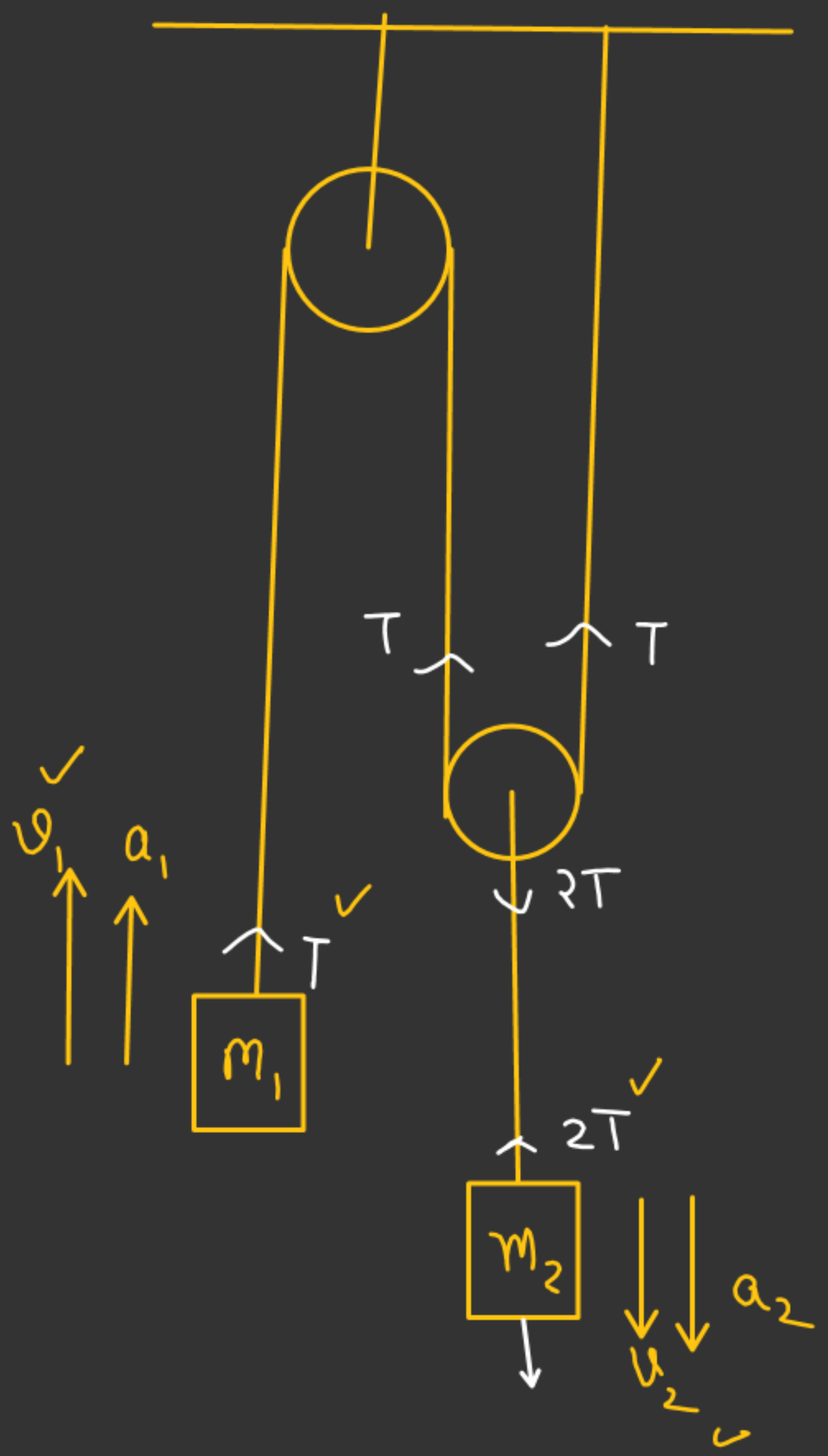

$$\sum \vec{T} \cdot \vec{v} = 0$$

$$\Downarrow$$

$$T v_1 \cos 0^\circ + 2T v_2 \cos \pi = 0$$

$$v_1 - 2v_2 = 0$$

$$\boxed{v_1 = 2v_2}$$

$$\boxed{a_1 = 2a_2}$$

Not always

$$\Downarrow$$

$$\sum \vec{T} \cdot \vec{a} = 0$$

$$T a_1 \cos 0^\circ + 2T \cos \pi \, a_2 = 0$$

$$a_1 - 2a_2 = 0$$

$$\boxed{a_1 = 2a_2}$$

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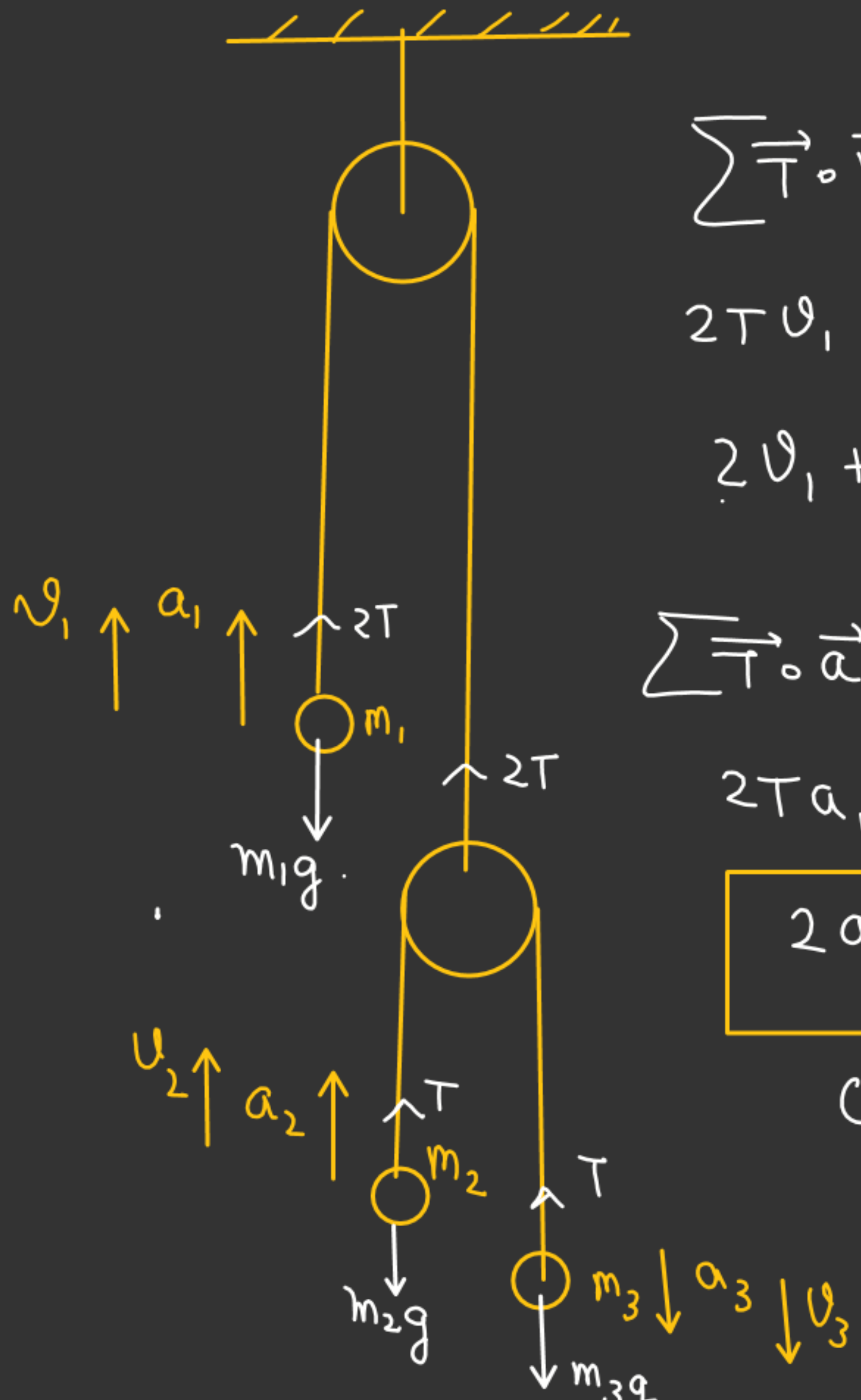

$$\sum \vec{T}\cdot\vec{v} = 0.$$

$$2Tv_1 + Tv_2 - Tv_3 = 0$$

$$2v_1 + v_2 = v_3$$

$$\sum \vec{T}\cdot\vec{a} = 0.$$

$$2Ta_1 + Ta_2 - Ta_3 = 0.$$

$$\boxed{2a_1 + a_2 = a_3 \quad -(1)}$$

$\Downarrow$

Constrain Relation.

For m_1.

$$2T - m_1 g = m_1 a_1 \quad -(2)$$

For m_2

$$T - m_2 g = m_2 a_2 \quad -(3)$$

For m_3

$$m_3 g - T = m_3 a_3 \quad -(4)$$

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Newton's law of Motion

H.C.V : (Page-No-79)

1 to 14

40, 41, 42

Test Syllabus (23^{rd} July. Jee Mains.)

→ 50% Kinematics

→ 50% (Law of Motion)

(Before pully or constrain lecture)

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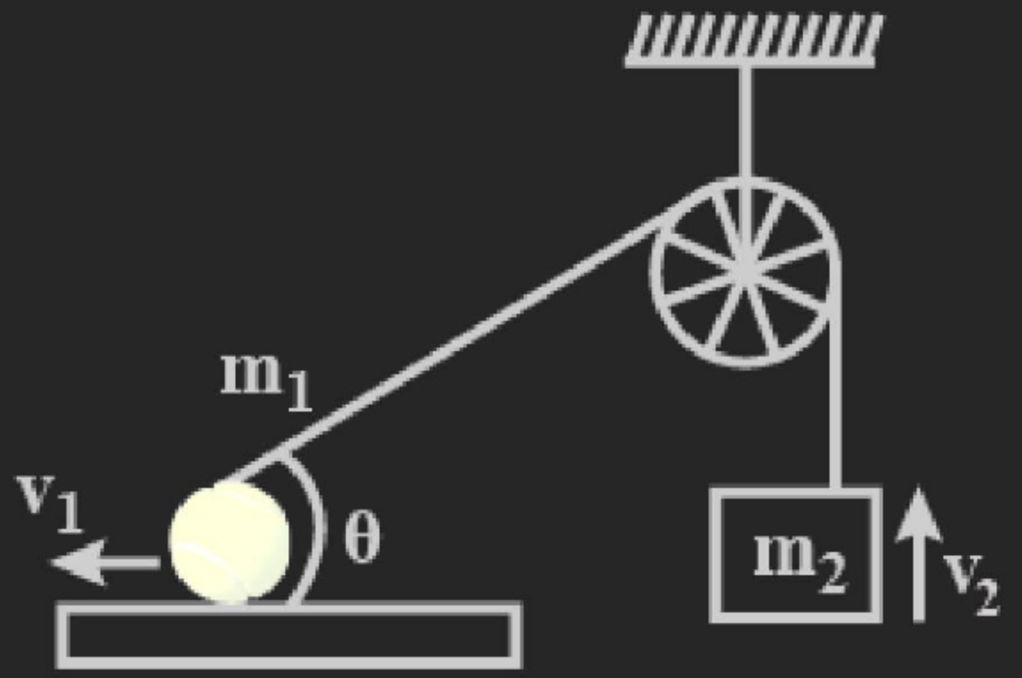
String - Pully constrain

Law of Motion

Q.1 In Fig., a ball of mass m_1 and a block of mass m_2 are joined together with an inextensible string. The ball can slide on a smooth horizontal surface. If v_1 and v_2 are the respective speeds of the ball and the block, then determine the constraint relation between the two.

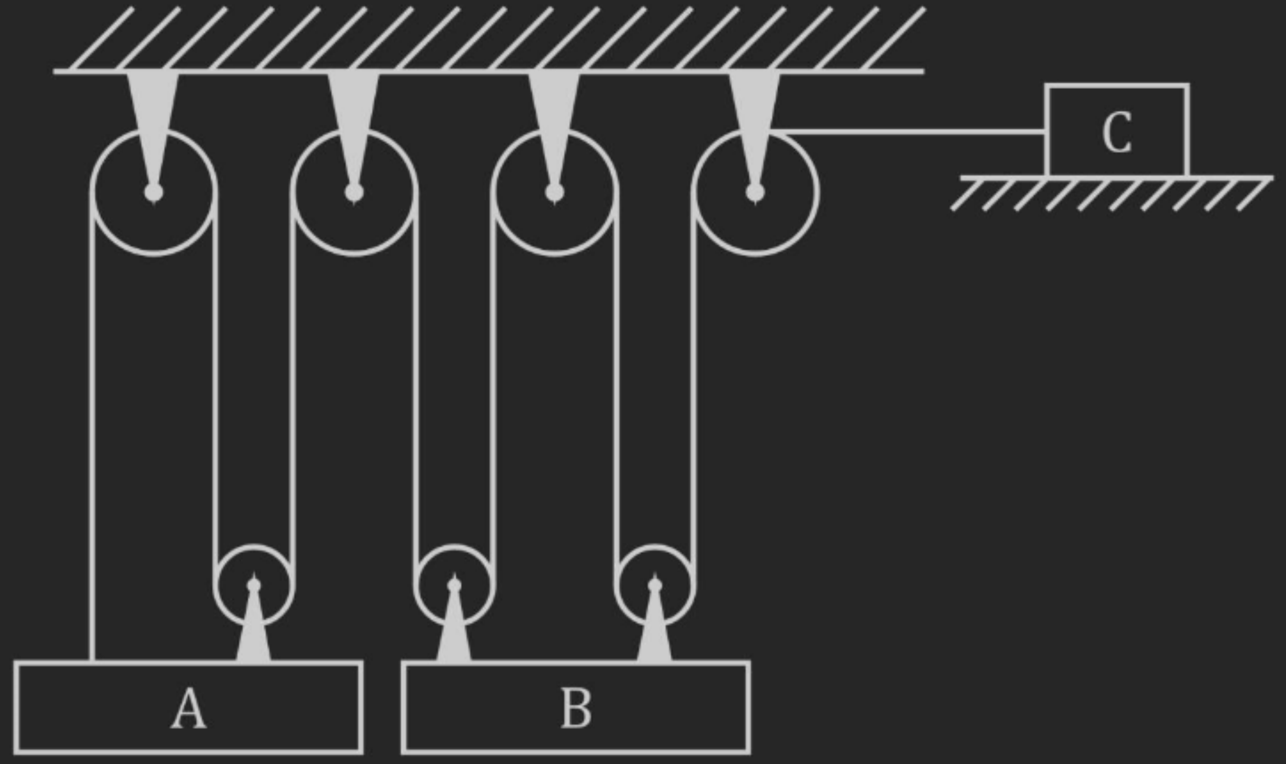


String - Pully constrain

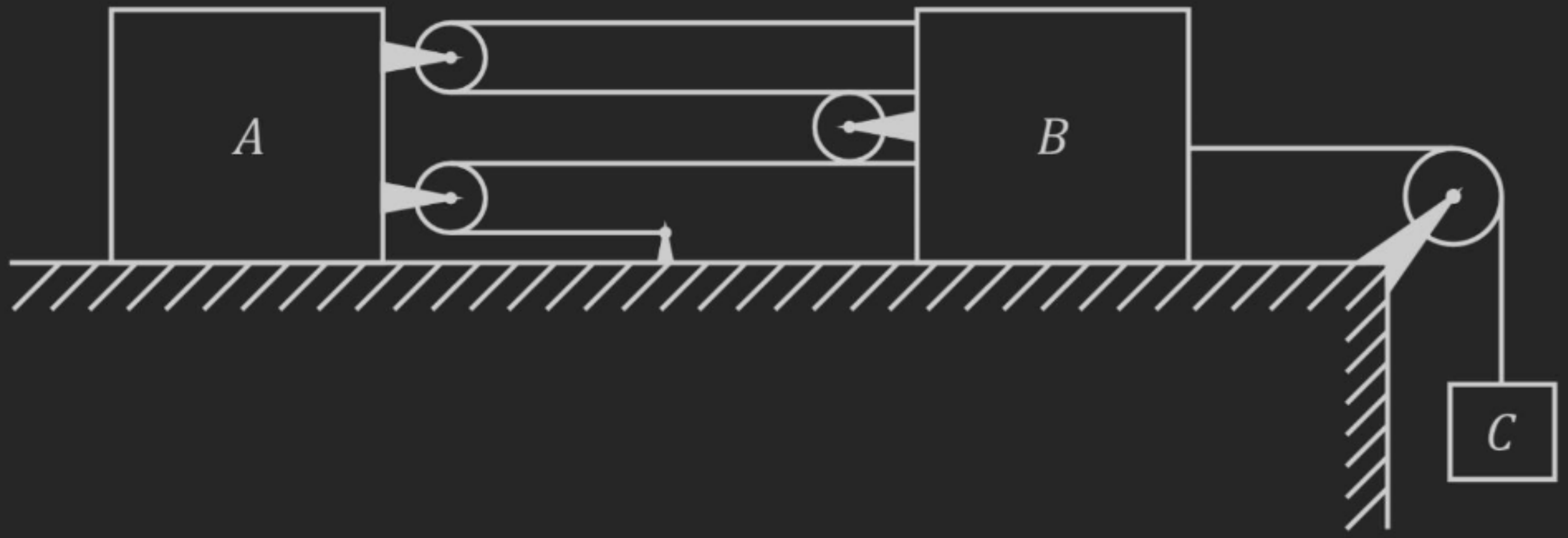
Law of Motion

Q.2 Block B shown in figure., moves downward with a constant velocity of $20\ \mathrm{cm/s}$. At $t = 0$, block A is moving upward with a constant acceleration, and its velocity is $3\ \mathrm{cm/s}$. If at $t = 3\ \mathrm{s}$ blocks C has moved $27\ \mathrm{cm}$ to the right, determine the velocity of block C at $t = 0$ and the acceleration of A and C.

$\left[a_A = 20\ \mathrm{cm/s^2} \uparrow;\ a_C = 60\ \mathrm{cm/s^2} \rightarrow;\ v_C = 71\ \mathrm{cm/sec} \leftarrow\right]$



Q.3 Block C shown in figure is going down at acceleration $2\ \mathrm{m/s^2}$. Find the acceleration of blocks A and B.



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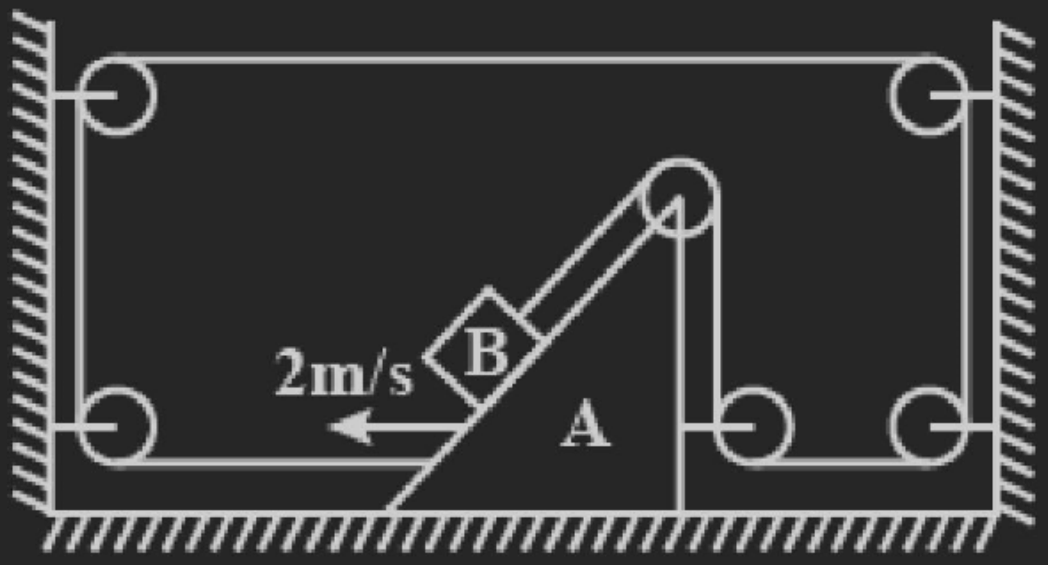
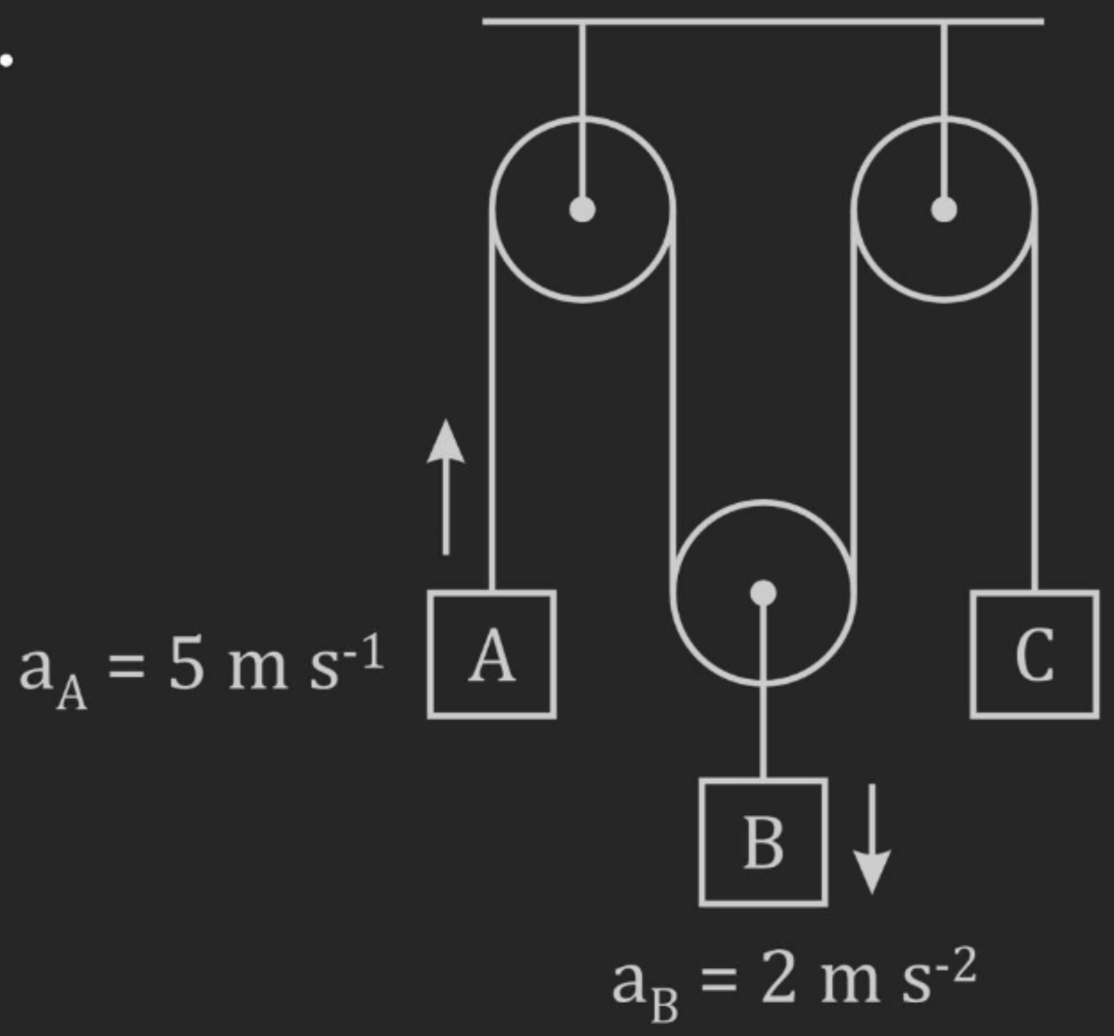
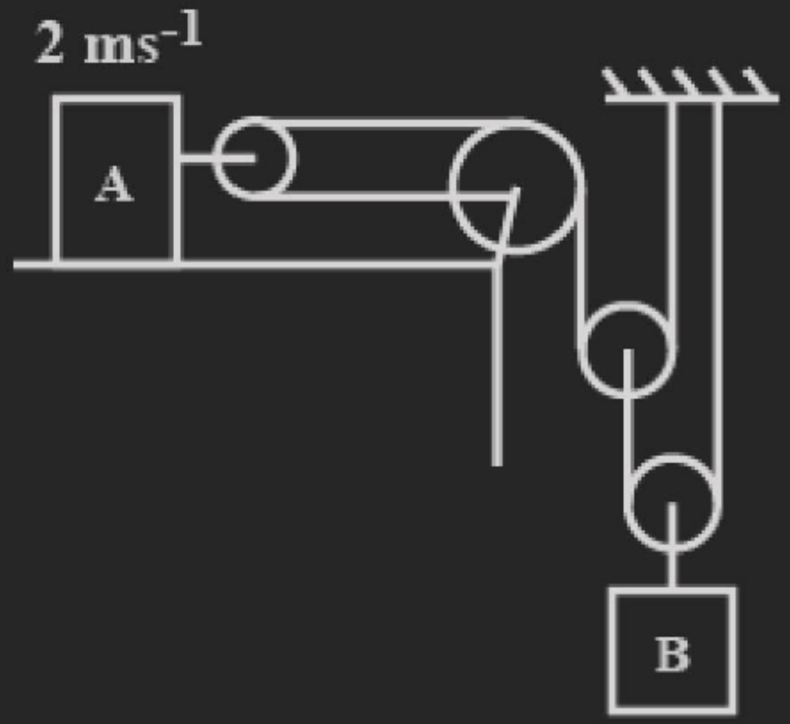
Q.4 System is shown in Fig. and wedge is moving toward left with speed 2 m s^{-1}. Find the velocity of the block B.



Q.5 For the system as shown in Fig., find the acceleration of C. The accelerations of A and B with respect to ground are marked.



Q.6 In Fig., find the acceleration of B, if the acceleration of A is 2 m s^{-2}.



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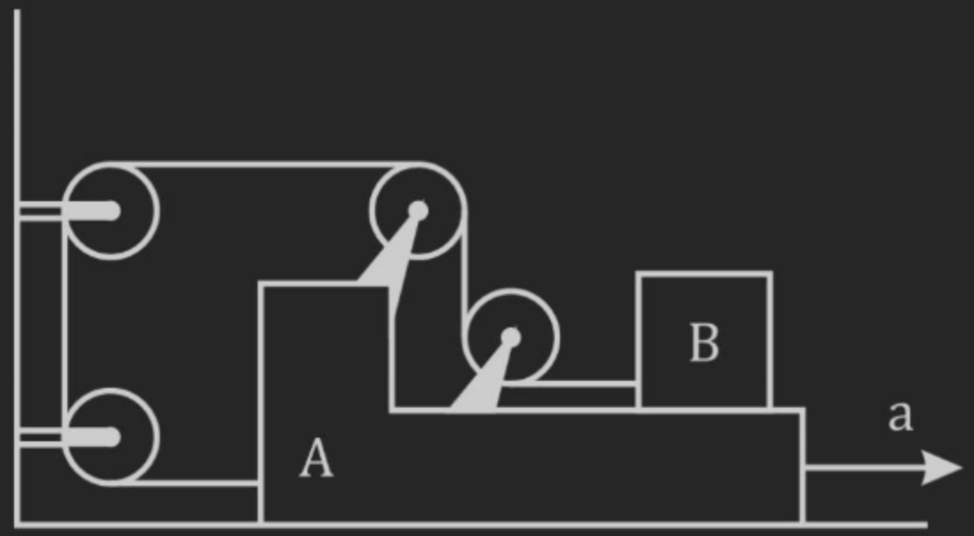
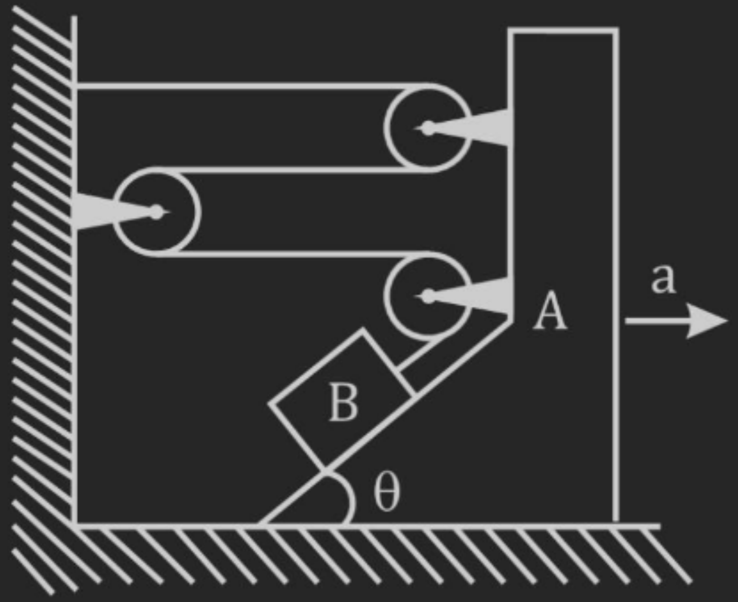
Q.7 Find the acceleration of block B as shown in Fig. (a) and (b) relative to block A and relative to ground if block A is moving toward right with acceleration a.

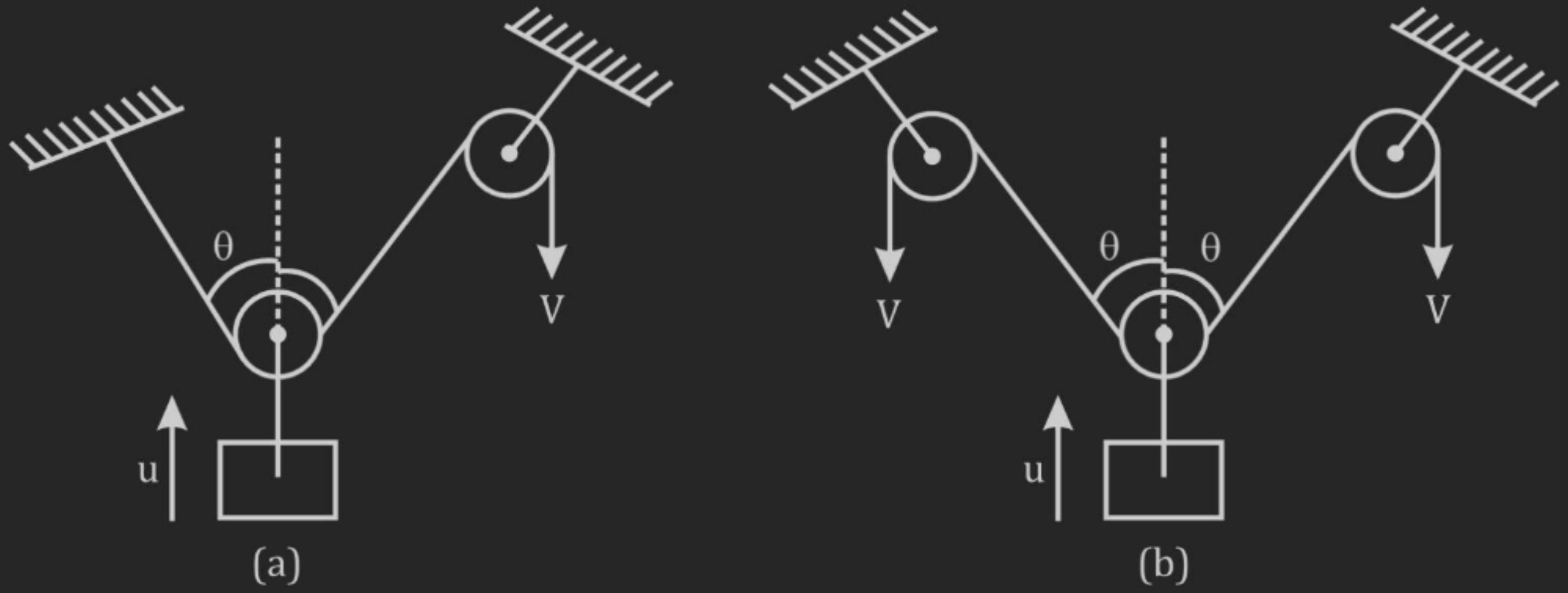


(a)

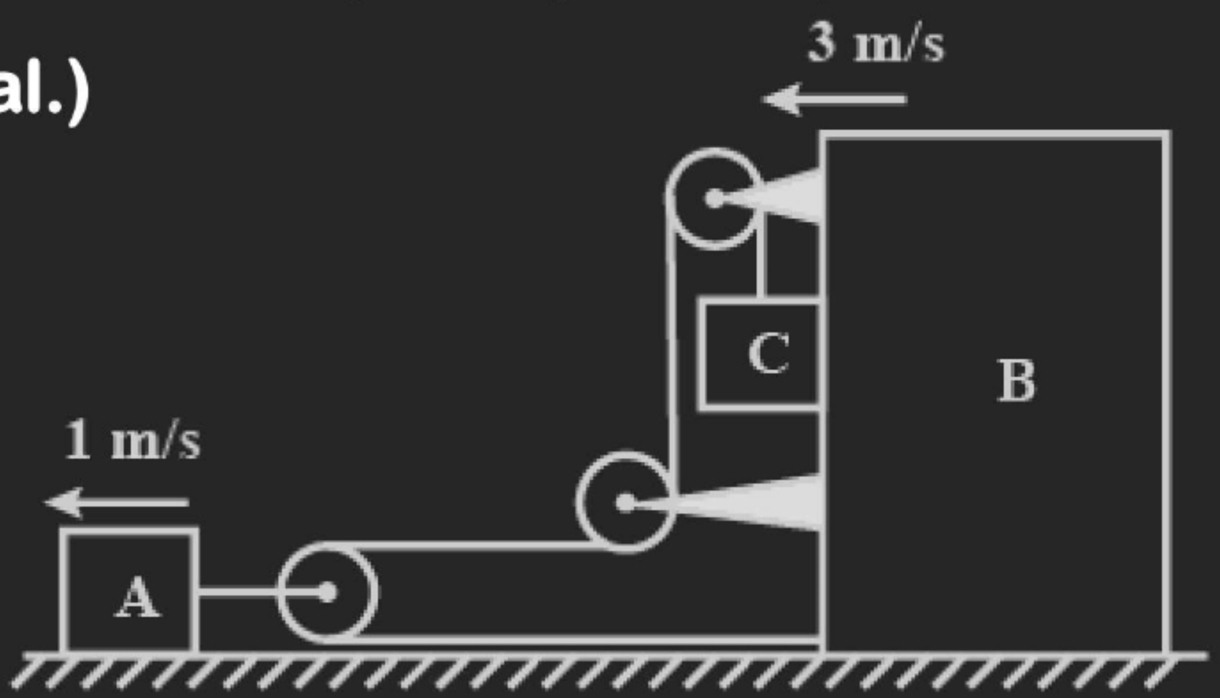


(b)

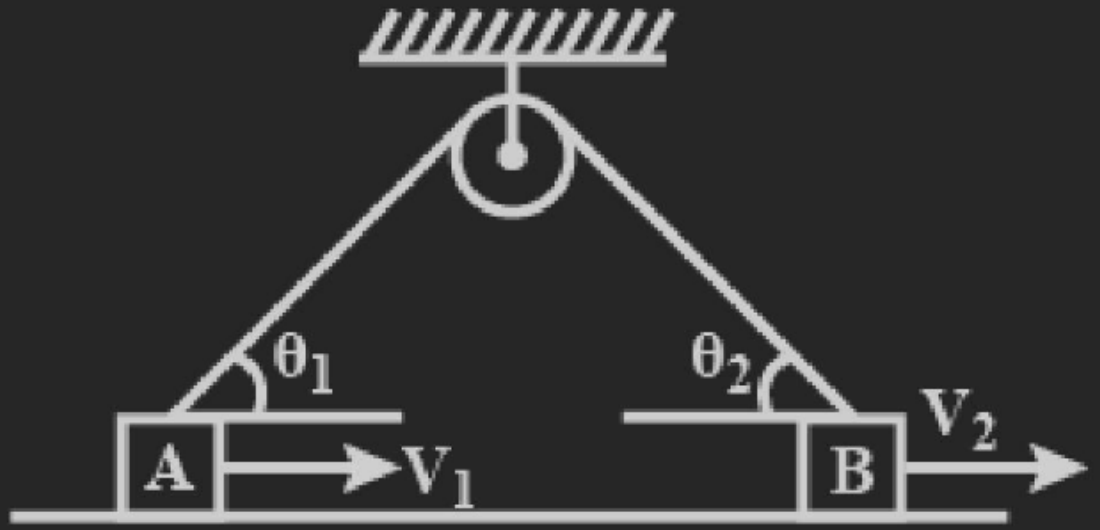
Q.8 If the string is inextensible, determine the velocity u of each block in terms of v and θ.



Q.9 The velocities of A and B are shown in Fig. Find the speed (in m s^{-1}) of block C. (Assume that the pulleys and string are ideal.)



Q.10 In Fig., blocks A and B move with velocities v_1 and v_2 along horizontal direction. Find the ratio of v_1 / v_2.



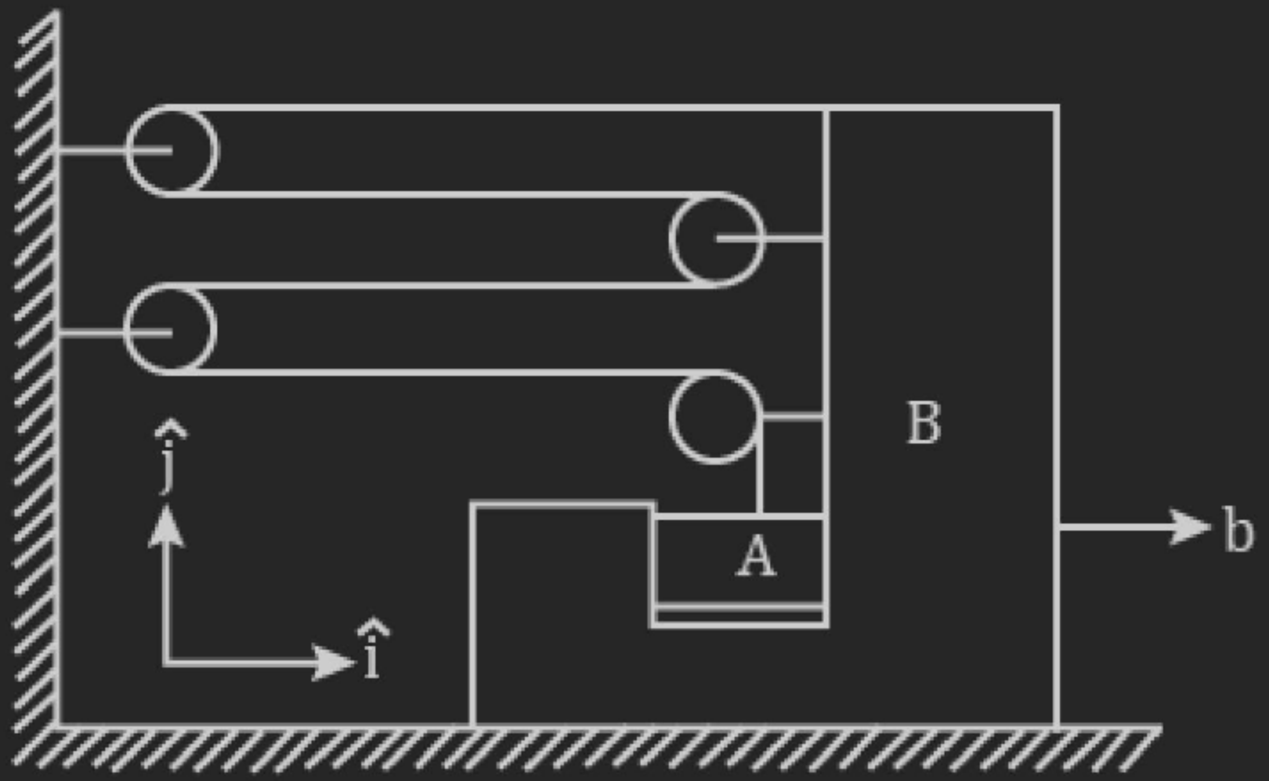
(A) $\frac{\sin\theta_1}{\sin\theta_2}$

(B) $\frac{\sin\theta_2}{\sin\theta_1}$

(C) $\frac{\cos\theta_2}{\cos\theta_1}$

(D) $\frac{\cos\theta_1}{\cos\theta_2}$

Q.11 If block B moves towards right with acceleration b, find the net acceleration of block A.



(A) $b\hat{i} + 4b\hat{j}$

(B) $b\hat{i} + b\hat{j}$

(C) $b\hat{i} + 2b\hat{j}$

(D) None of these

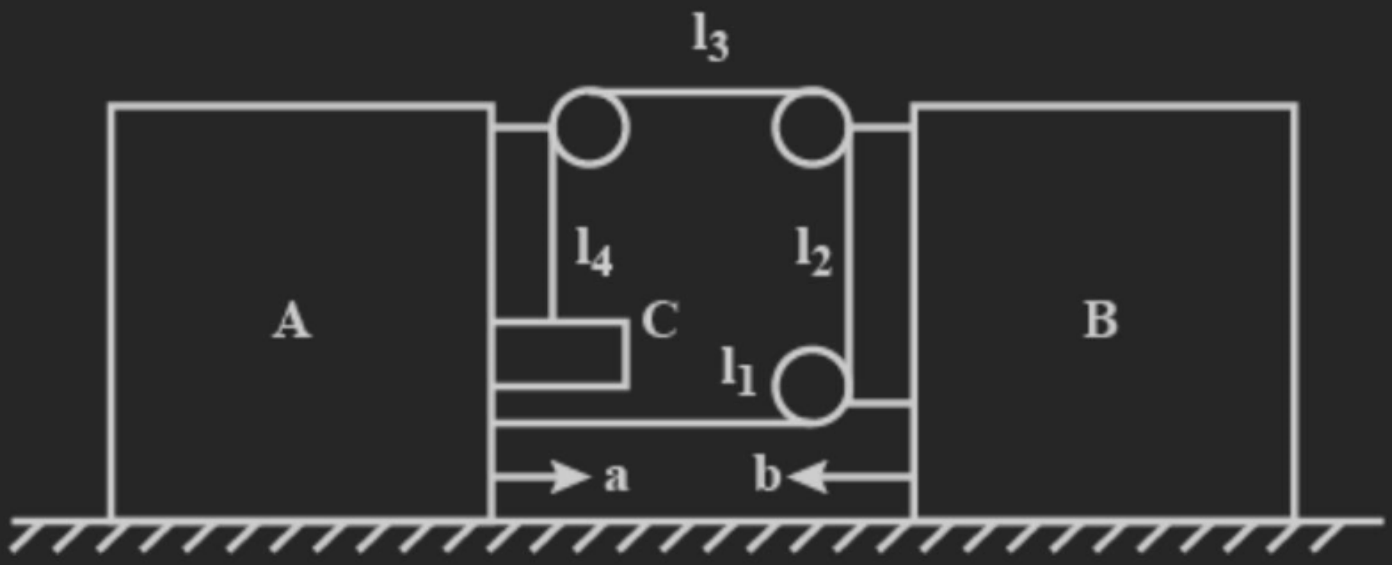
Q.12 If the blocks A and B are moving towards each other with accelerations a and b as shown in Fig., find the net acceleration of block C.

(A) $a\hat{i} - 2(a+b)\hat{j}$

(B) $-(a+b)\hat{j}$

(C) $a\hat{i} - (a+b)\hat{j}$

(D) None of these



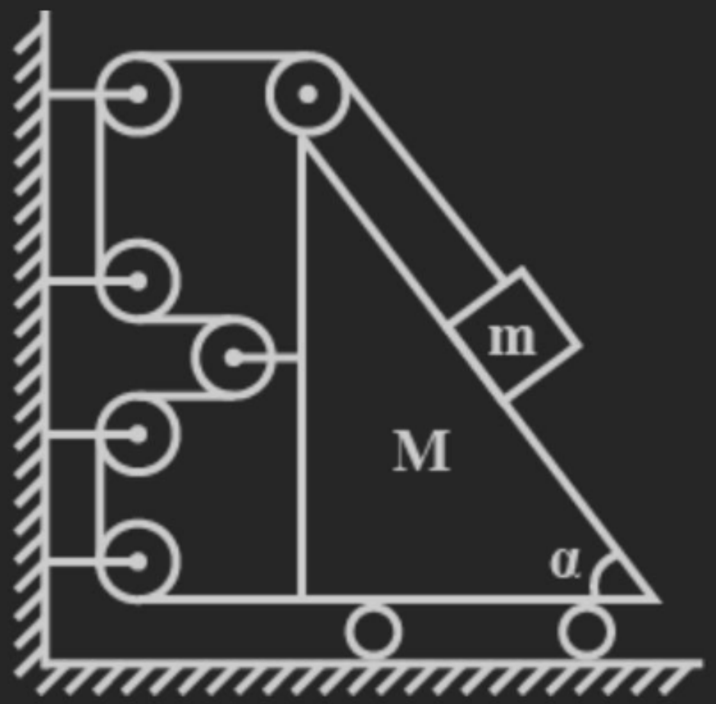
Q.13 If the acceleration of wedge in the shown arrangement is a m s^{-2} towards left, then at this instant, acceleration of the block (magnitude only) would be

(A) $4a\ ms^{-2}$

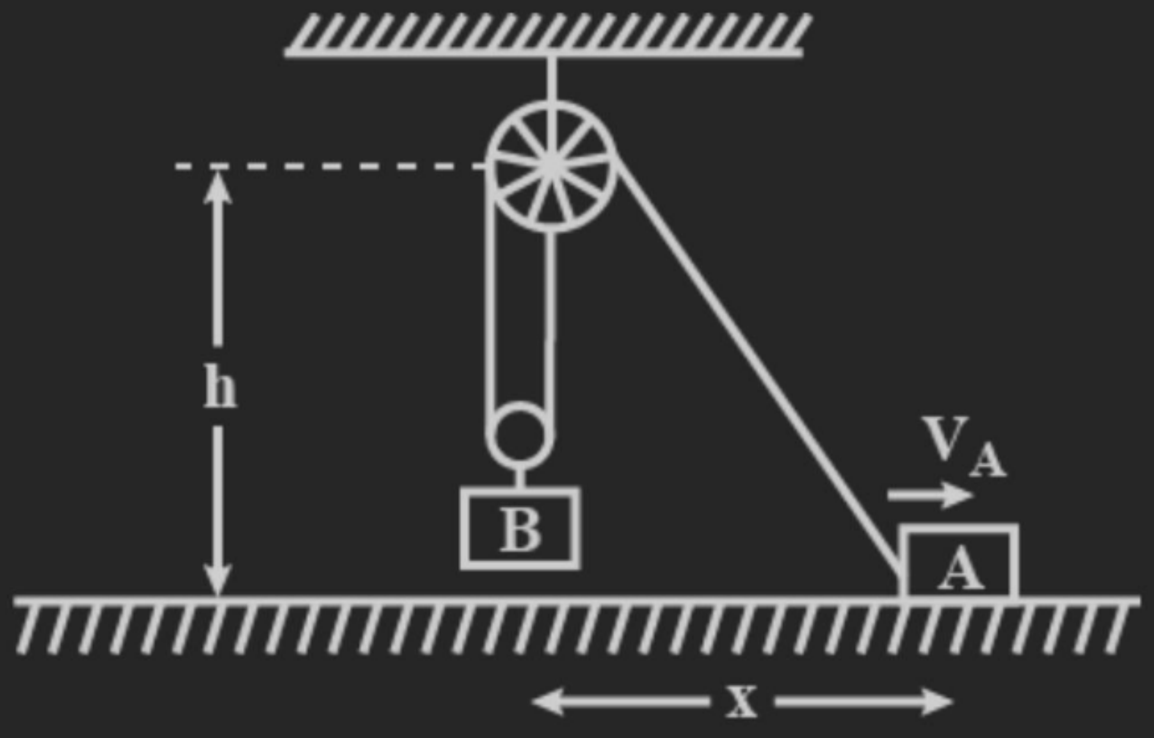
(B) $a\sqrt{17-8\cos\alpha}\ m\ s^{-2}$

(C) $(\sqrt{17})a\ m\ s^{-2}$

(D) $\sqrt{17}\cos\frac{\alpha}{2}\times a\ m\ s^{-2}$



Q.14 If block A is moving horizontally with velocity v_A, then find the velocity of block B at the instant as shown in Fig.



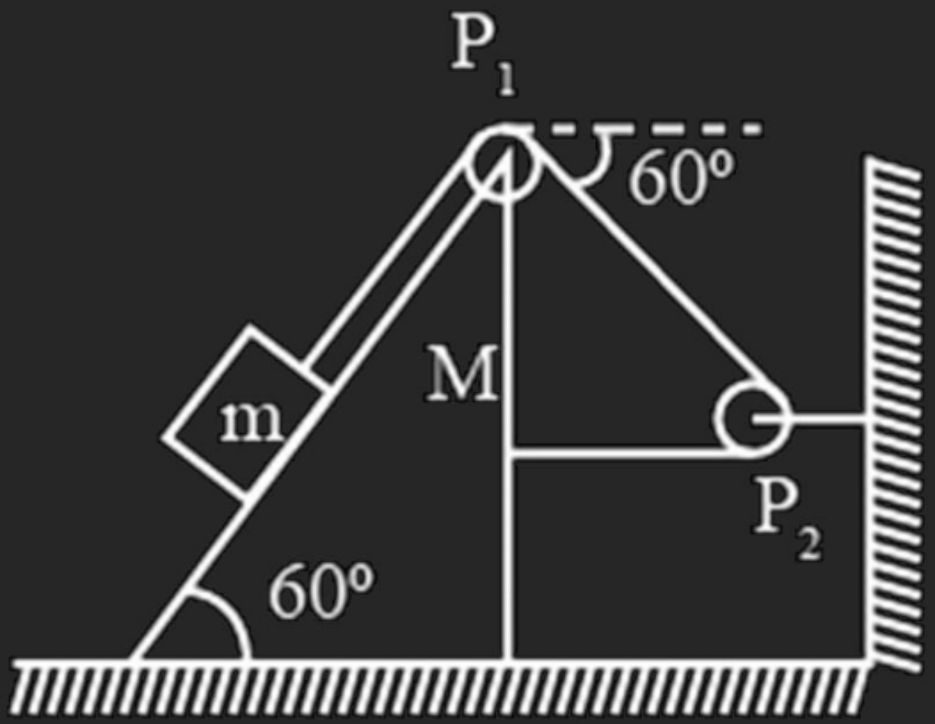
(A) $\frac{hv_A}{2\sqrt{x^2+h^2}}$

(B) $\frac{xv_A}{\sqrt{x^2+h^2}}$

(C) $\frac{xv_A}{2\sqrt{x^2+h^2}}$

(D) $\frac{hv_A}{\sqrt{x^2+h^2}}$

Q.15 In the arrangement shown in Fig., the block of mass m = 2 kg lies on the wedge of mass M = 8 kg. The initial acceleration of the wedge, if the surfaces are smooth, is



(A) $\frac{\sqrt{3}g}{23}\,ms^{-2}$

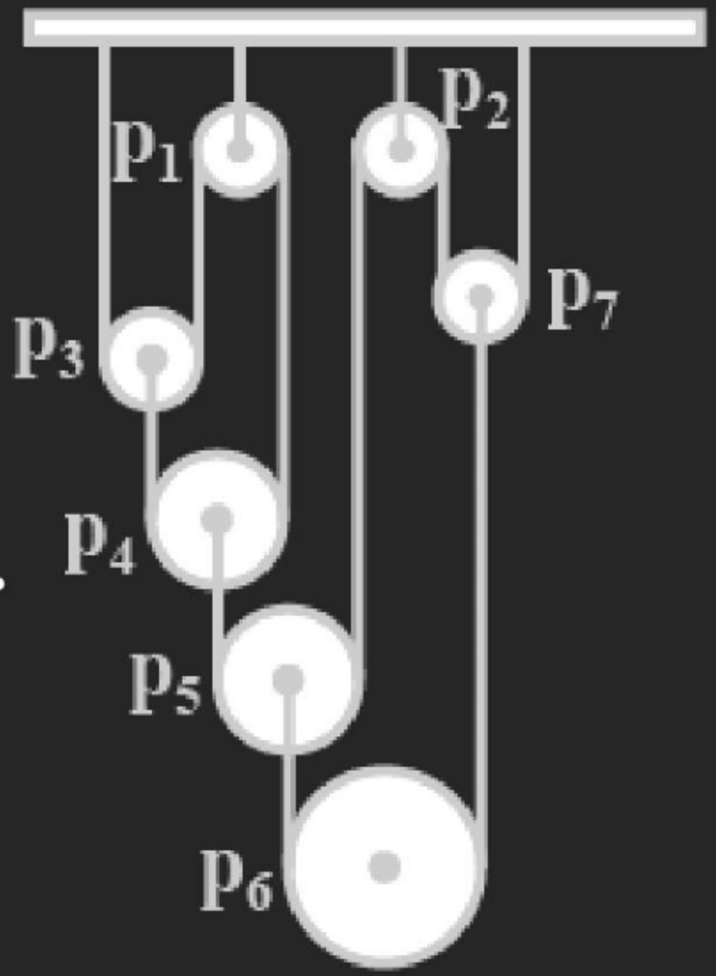
(B) $\frac{3\sqrt{3}g}{23}\,ms^{-2}$

(C) $\frac{3g}{23}\ m\ s^{-2}$

(D) $\frac{g}{23}\ m\ s^{-2}$

Law of Motion

Q.16 Seven pulleys are connected with the help of three light strings as shown in Fig. Consider P_3, P_4, P_5 as light pulleys and pulleys P_6 and P_7 have masses m each. For this arrangement, mark the correct statement(s).



(A) Tension in the string connecting P_1, P_3, and P_4 is zero.

(B) Tension in the string connecting P_1, P_3 and P_4 is $mg/3$.

(C) Tensions in all the three strings are same and equal to zero.

(D) Acceleration of P_6 is g downwards and that of P_7 is g upwards.