

AA

COM of a Solid Cone

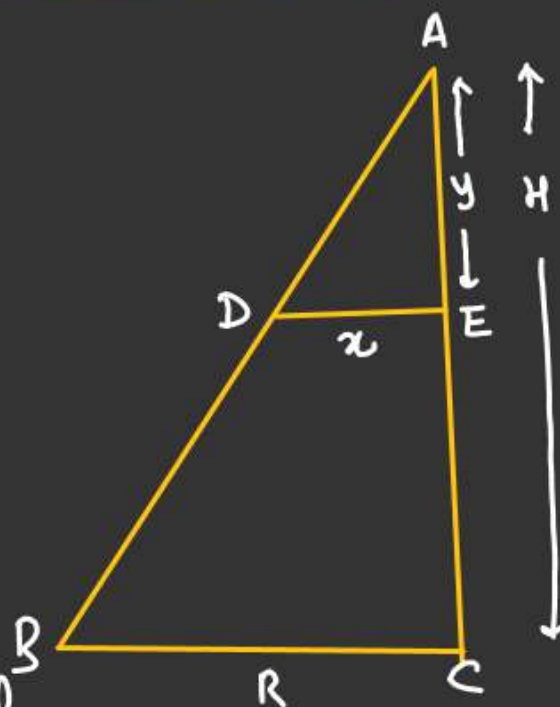
In $\triangle ADE$ & $\triangle ABC$

$$\frac{x}{R} = \frac{y}{H}$$

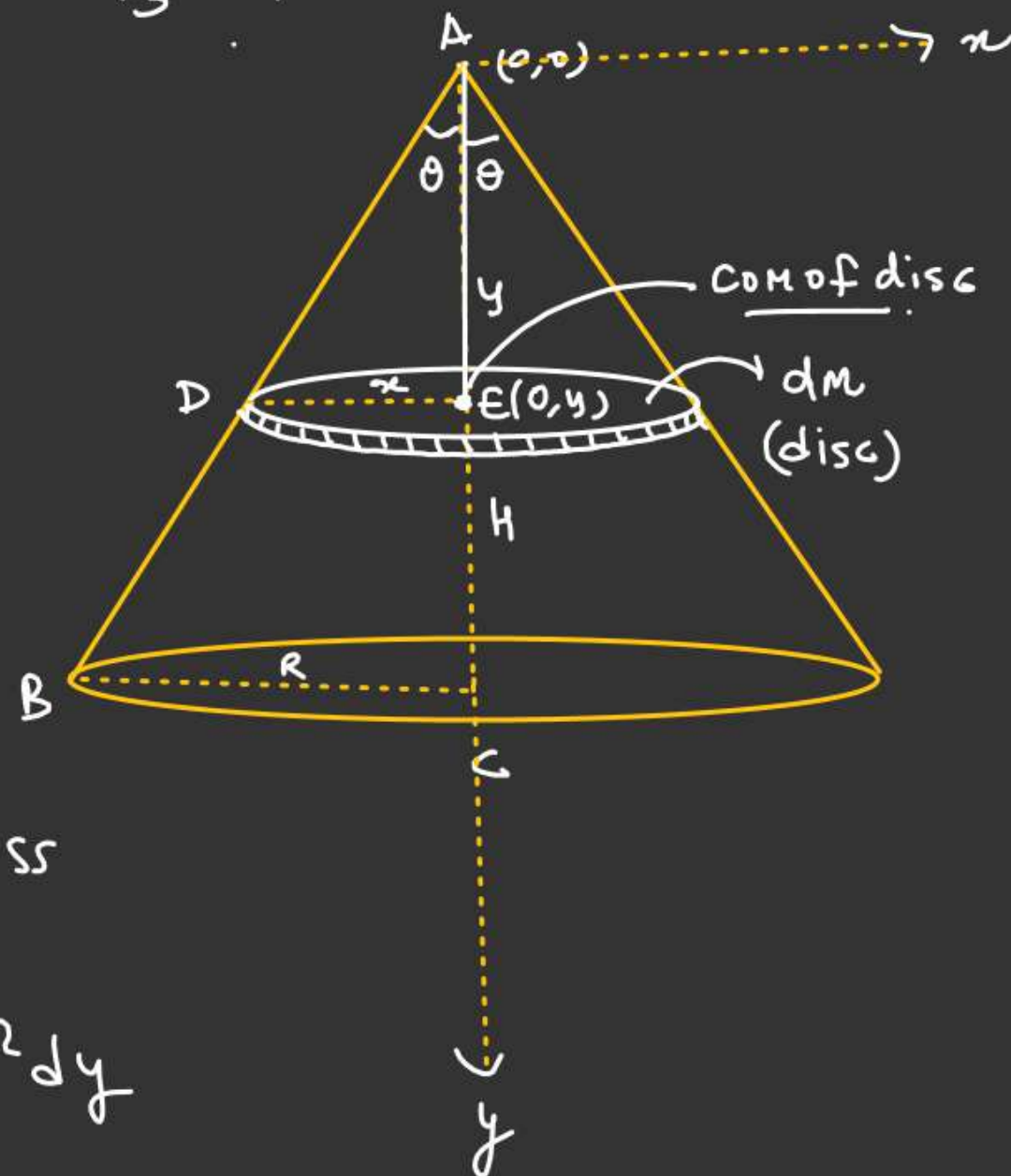
$$x = \left(\frac{R}{H} \cdot y\right)$$

$$dm = \left(\frac{3M}{\pi R^2 H}\right) \times \frac{dV}{\parallel}$$

differential
Volume of disc



$$\rho = \left(\frac{M}{\frac{1}{3}\pi R^2 H}\right)$$



$$dV = (\text{Area of differential element}) \times \text{thickness}$$

$$= (\pi x^2) dy$$

$$= \left(\frac{\pi R^2}{H^2} y^2 dy\right)$$

$$dm = \frac{3M}{\pi R^2 H} \times \frac{\pi R^2}{H^2} y^2 dy$$

$$dm = \left(\frac{3M}{H^3} y^2 dy\right) \checkmark$$

$$dm = \frac{3M}{H^3} y^2 dy$$

$$y_{\text{com}} = \frac{\int dm y}{\int dm} = \frac{\frac{3M}{H^3} \int_0^H y^3 dy}{M}$$

$$y_{\text{com}} = \frac{3}{H^3} \times \frac{H^4}{4}$$

$$y_{\text{com}} = \frac{3H}{4} \Rightarrow \text{From Apex}$$

$$y_{\text{com}} = H - \frac{3H}{4} = \frac{H}{4} \quad \text{From base}$$

COM of triangular lamina

Lamina = [Two dimensional having negligible thickness]

$$\tan \theta = \frac{x/2}{y} = \frac{b/2}{h}$$

$$x = \left(\frac{b}{h} y\right)$$

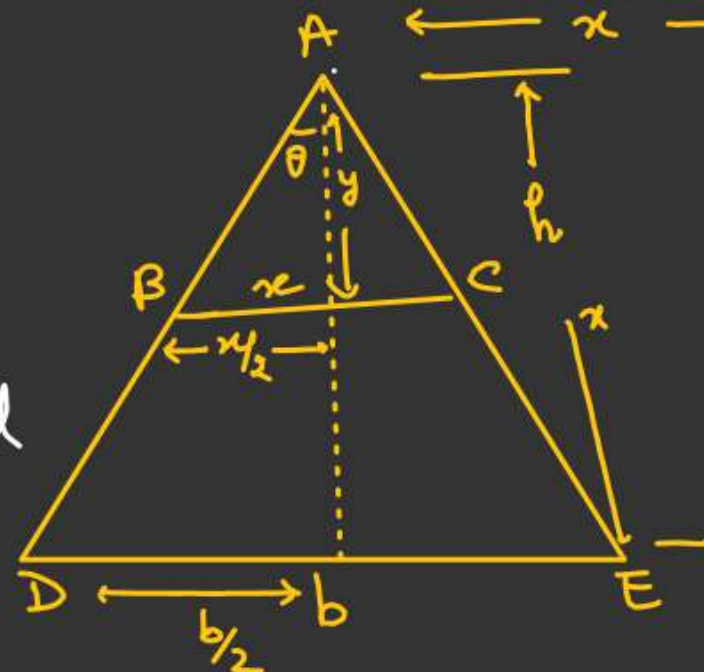
$$dm = \frac{M}{A} (dA) \quad \text{differential area of strip.}$$

$$= \frac{M}{\frac{1}{2}bh} x(x dy)$$

$$= \left(\frac{2M}{bh}\right) x \frac{b}{h} \times y$$

$$= \left(\frac{2M}{h^2} y\right)$$

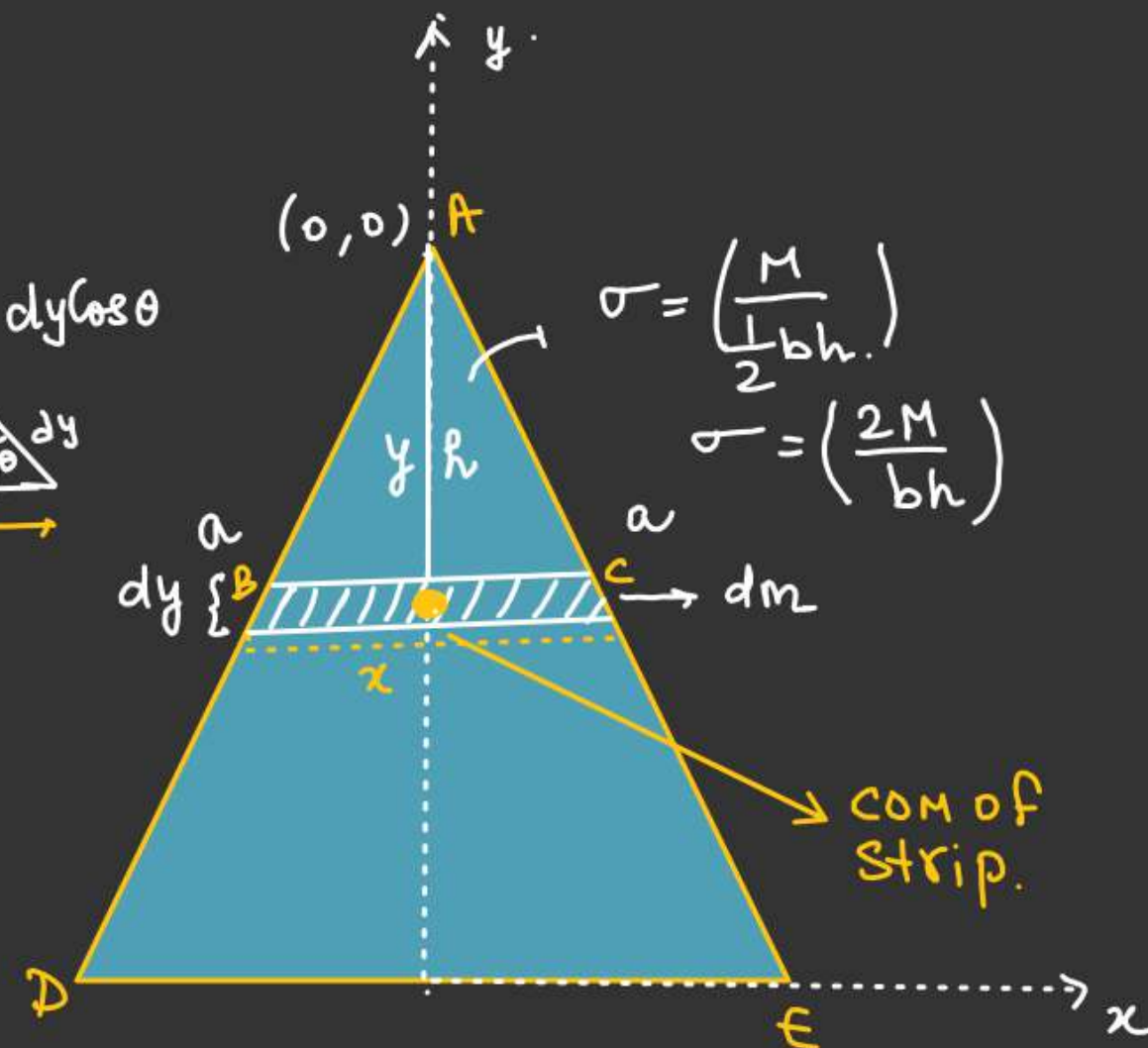
Assume to be rectangle.



$$y_{com} = \frac{\int_0^h dm \cdot y}{M} = \frac{2M}{h^2 \times M} \int_0^h y^2 dy = \left(\frac{2h}{3}\right)$$

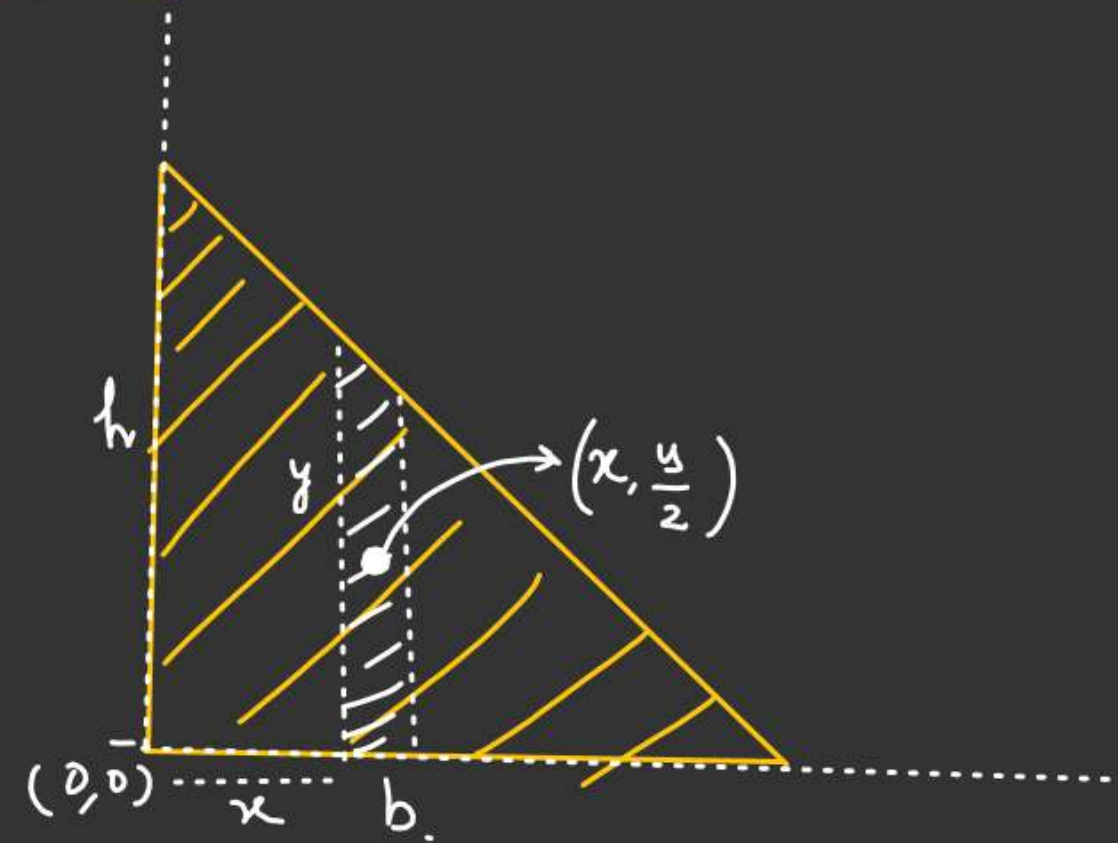
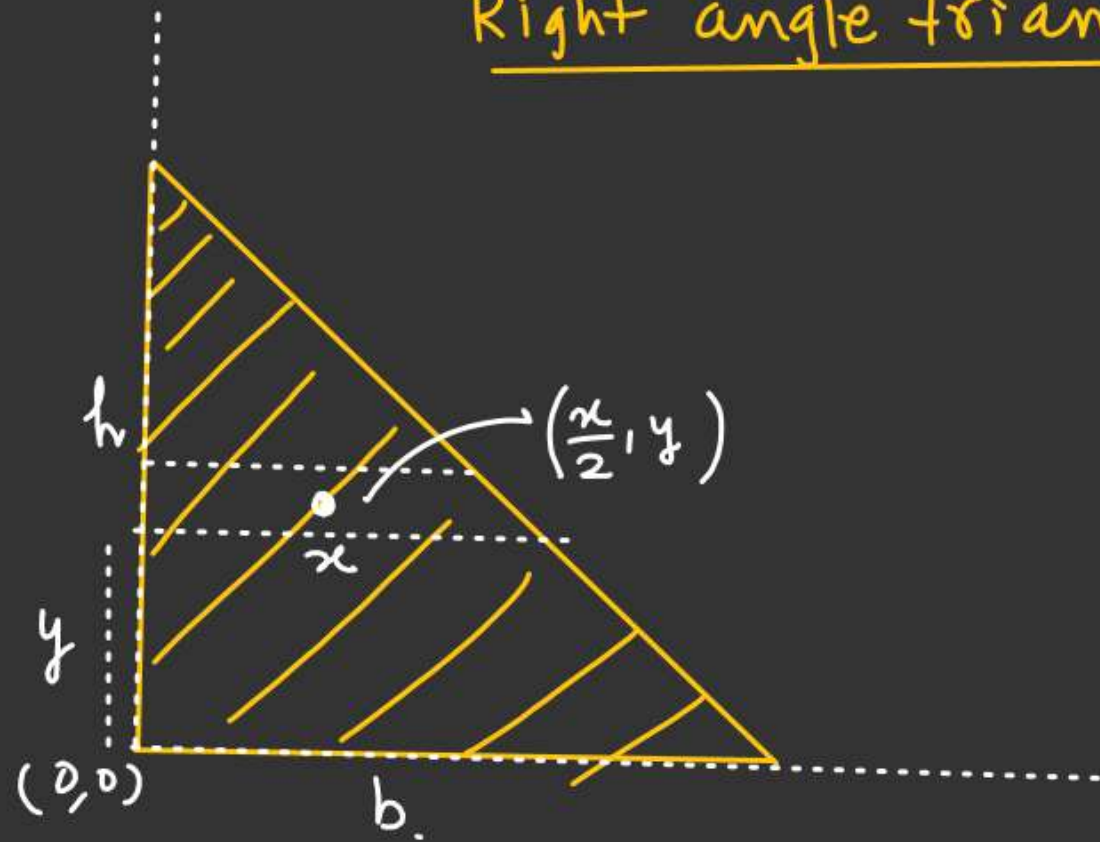
(From A)

From base = $h/3 \rightarrow$ (Same as hollow cone)



$$\sigma = \left(\frac{M}{\frac{1}{2}bh}\right)$$

$$\sigma = \left(\frac{2M}{bh}\right)$$

Right angle triangular lamina.

$$\text{COM} = \left[\frac{b}{3}, \frac{h}{3} \right] \checkmark$$

Find COM of the figure.

Both Cone and hemisphere have same density.

$\rho = \text{density}$

$M_1 = \text{Mass of Solid Cone.}$

$$M_1 = \int \frac{1}{3} \pi R^2 (2R) = \frac{2}{3} (\rho \pi R^3) \checkmark$$

$M_2 = \text{Mass of Solid hemisphere.}$

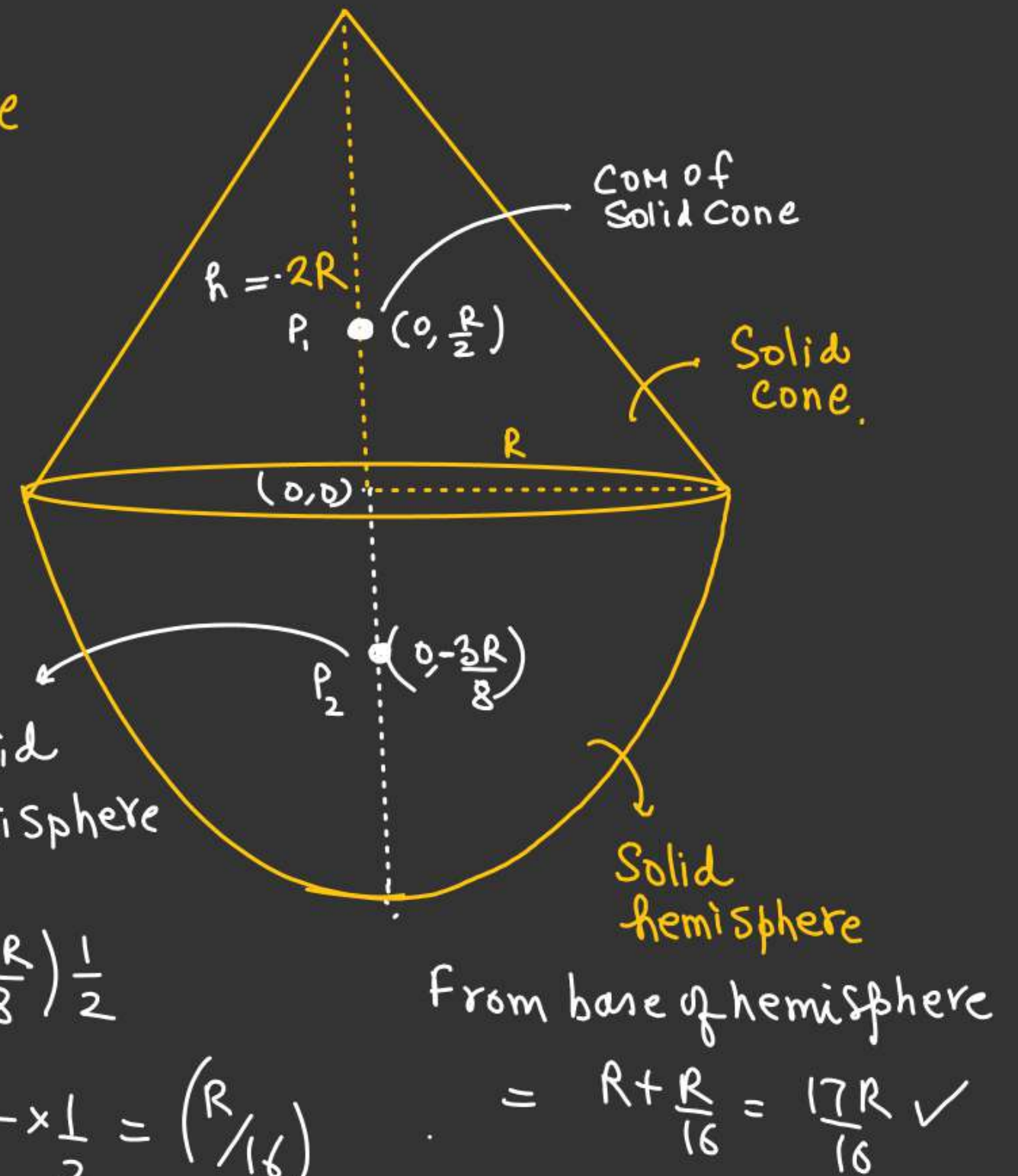
$$M_2 = \left(\rho \cdot \frac{2}{3} \pi R^3 \right) \checkmark$$

$$y_{\text{com}} = \frac{M_1 y_1 + M_2 y_2}{M_1 + M_2}$$

$$= \frac{\left(\frac{2}{3} \rho \pi R^3 \right) \frac{R}{2} + \frac{2}{3} \rho \pi R^3 \left(-\frac{3R}{8} \right)}{\left(\frac{2}{3} \rho \pi R^3 \times 2 \right)}$$

$$= \left(\frac{R}{2} - \frac{3R}{8} \right) \frac{1}{2}$$

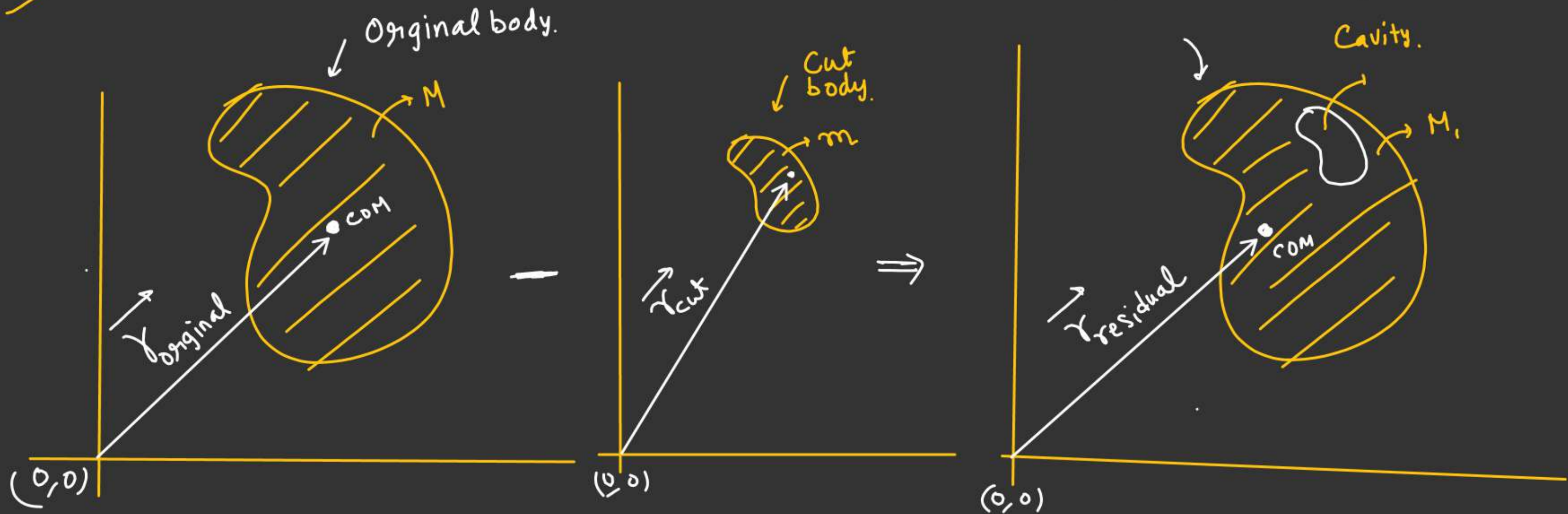
$$= \frac{4R - 3R}{8} \times \frac{1}{2} = \left(\frac{R}{16} \right)$$





com of Residual bodies.

Residual body \rightarrow (Remaining body.)



$$\vec{r}_{\text{residual}} = \left(\frac{M \vec{r}_{\text{original}} - m \vec{r}_{\text{cut}}}{(M - m)} \right) = \left(\frac{M \vec{r}_{\text{original}} + (-m) \vec{r}_{\text{cut}}}{M + (-m)} \right)$$

If body is lamina i.e 2-dimensional

$$M = \sigma \cdot A_1$$

$$m = \sigma \cdot A_2$$

$$\vec{r}_{\text{residual}} = \frac{M \vec{r}_{\text{original body}} - m \vec{r}_{\text{cut body}}}{M - m}$$

$$\vec{r}_{\text{residual}} = \left(\frac{A_1 \vec{r}_1 - A_2 \vec{r}_2}{A_1 - A_2} \right)$$

$A_1 =$ Area of original body

$A_2 =$ Area of cut body

$\vec{r}_1 =$ Position vector of COM of original body

$\vec{r}_2 =$ Position vector of COM of cut body.

If body is
3-dimensional

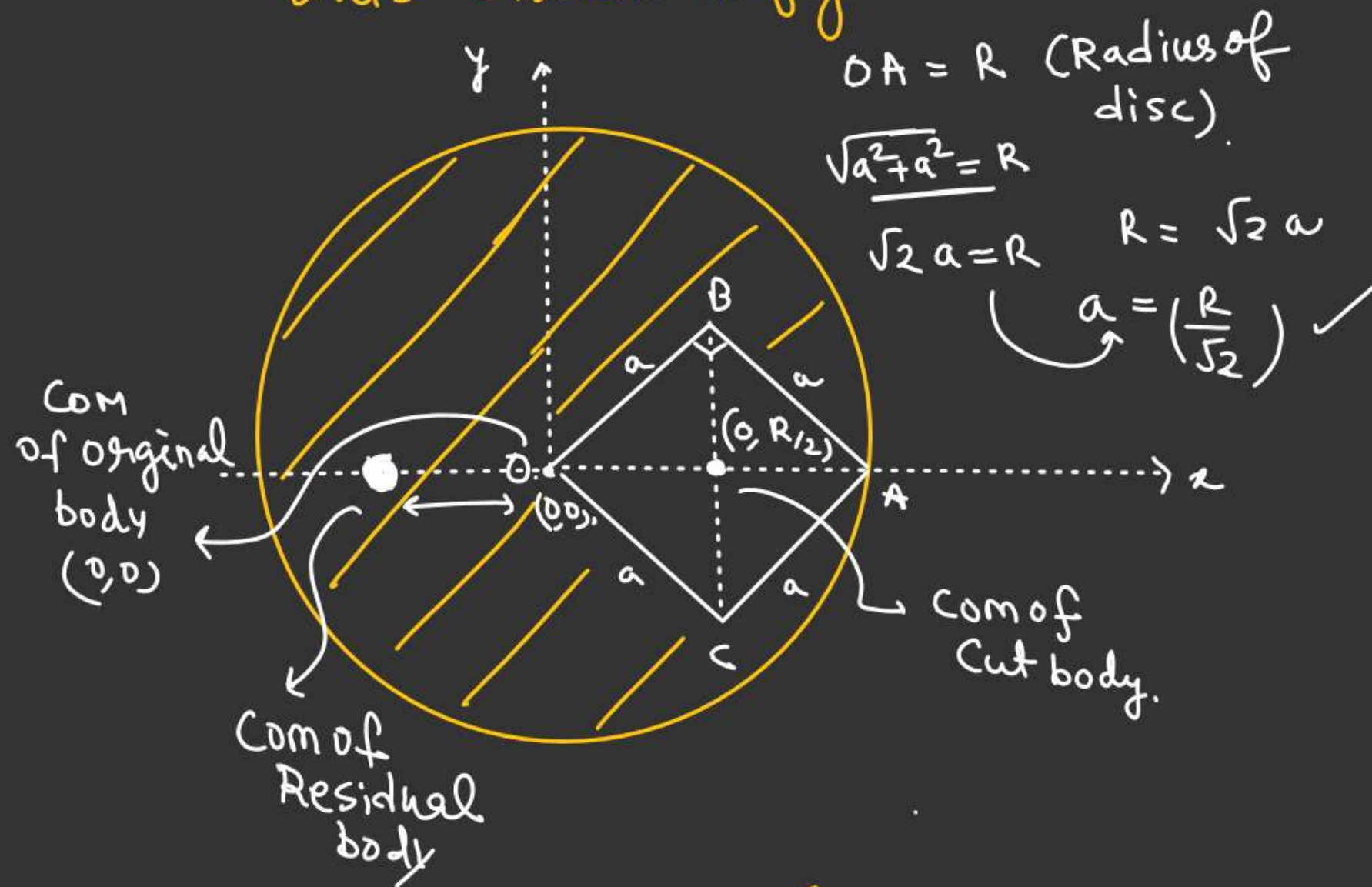
$$M = \rho V_1$$

$$m = \rho V_2$$

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$$\vec{r}_{\text{residual}} = \left(\frac{V_1 \vec{r}_1 - V_2 \vec{r}_2}{V_1 - V_2} \right)$$

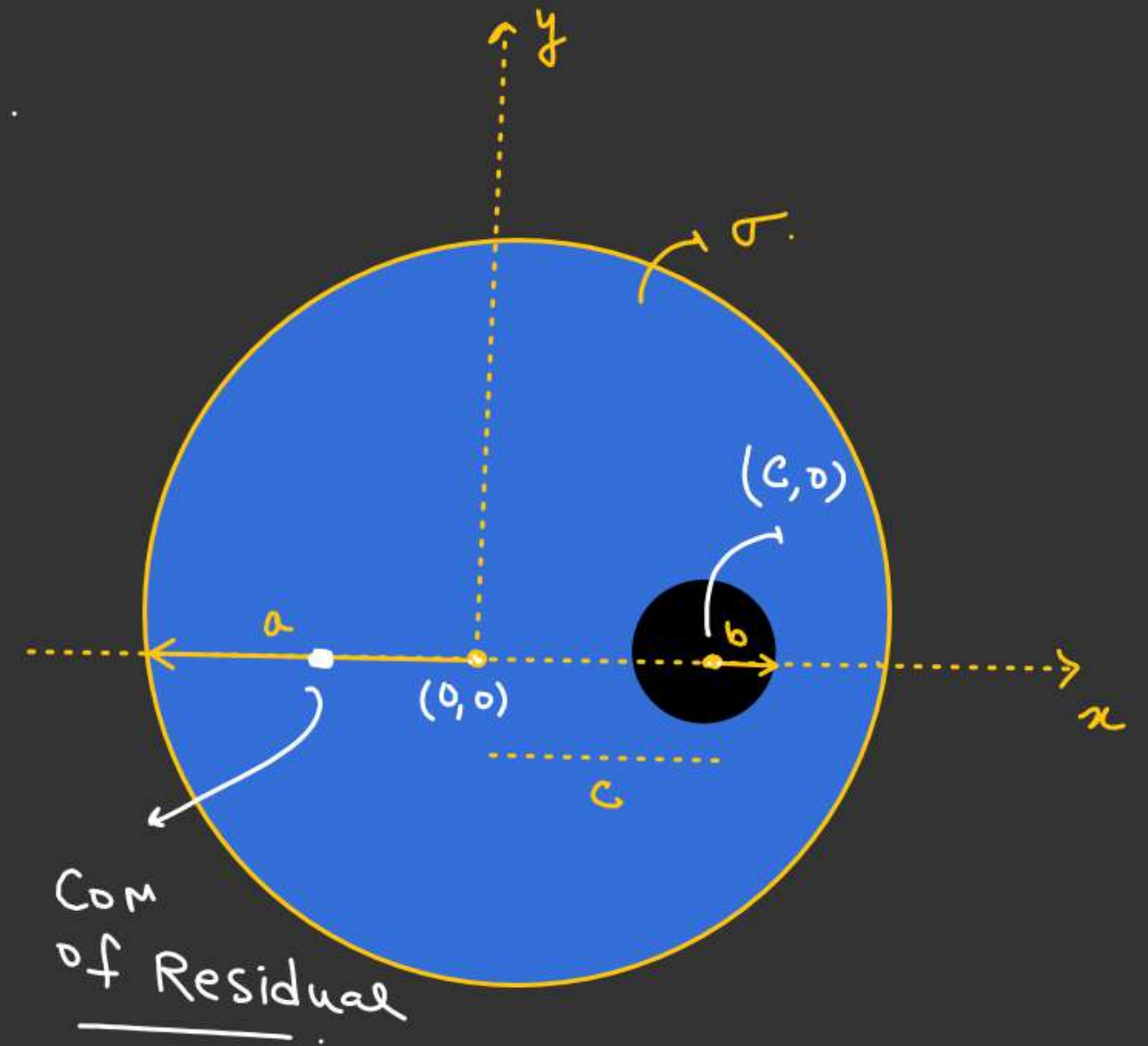
From a Circular lamina a Square is cut so that diagonal of Square be the radius of Circular lamina.
Find COM of residual body from the axis shown in fig.



$$\begin{aligned}
 \vec{r}_{\text{residual}} &= \frac{A_1 \vec{r}_1 - A_2 \vec{r}_2}{A_1 - A_2} \\
 &= \frac{(\pi R^2)(0\hat{i}) - \left(a^2 \cdot \frac{R}{2}\hat{i}\right)}{(\pi R^2 - a^2)} \\
 &= \frac{-\left(\frac{R}{\sqrt{2}}\right)^2 \times \frac{R}{2}\hat{i}}{\pi R^2 - \left(\frac{R}{\sqrt{2}}\right)^2} = \frac{\frac{R}{4}(-\hat{i})}{\left(\pi - \frac{1}{2}\right)} \\
 &= \frac{R}{2(2\pi - 1)}(-\hat{i})
 \end{aligned}$$

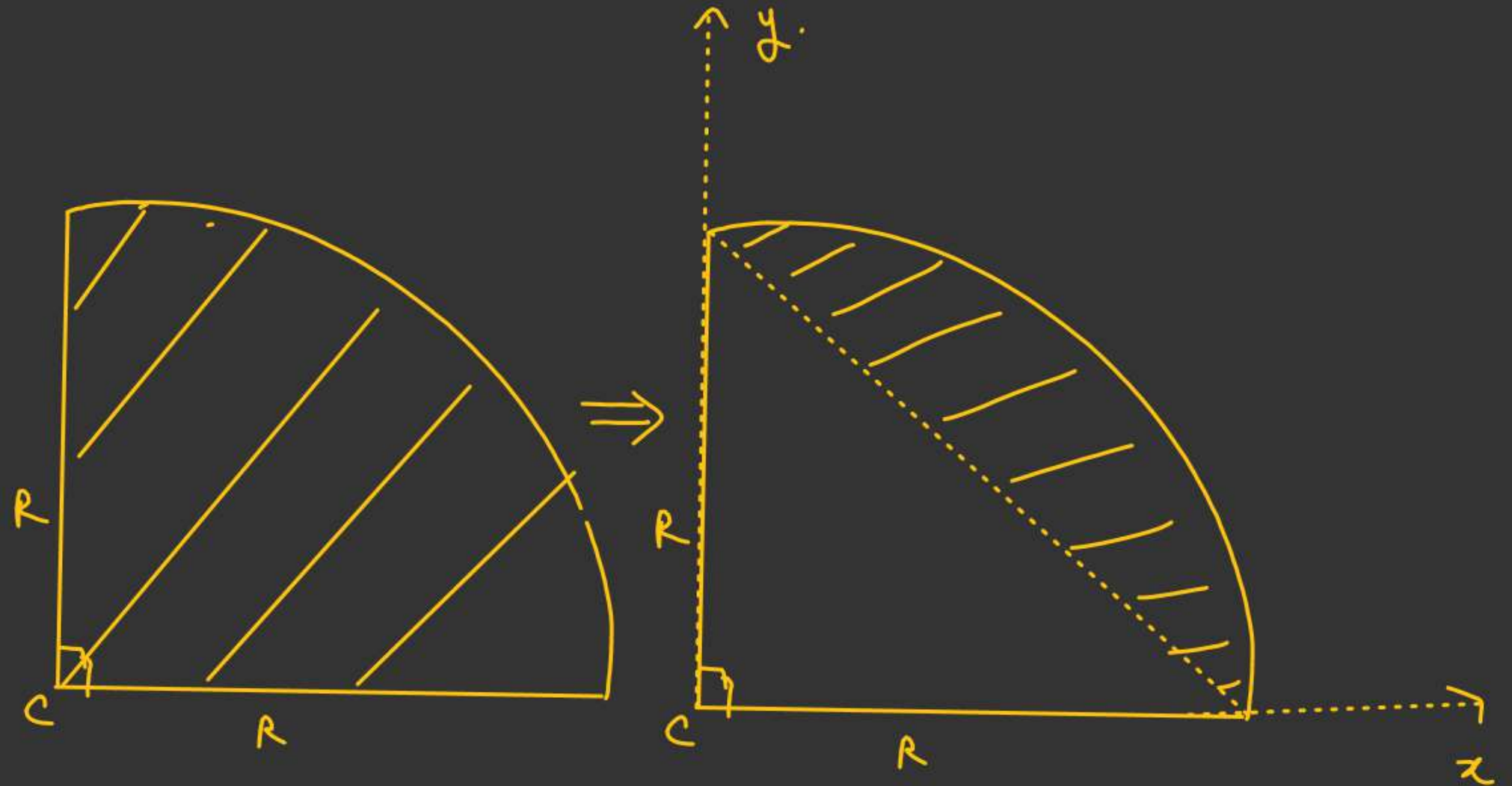
- # A small disc of radius b is cut from a larger disc of radius a .
find COM of residual body.

$$\begin{aligned}
 \vec{r}_{\text{residual}} &= \frac{A_1 \vec{r}_1 - A_2 \vec{r}_2}{A_1 - A_2} \\
 &= \frac{-A_2 \vec{r}_2}{A_1 - A_2} \\
 &= \left(\frac{\pi b^2}{\pi a^2 - \pi b^2} \right) (-c \hat{i}) \\
 &= \left(\frac{b^2 c}{a^2 - b^2} \right) (-\hat{i})
 \end{aligned}$$



H.W: A triangle is cut from a sector of radius R .

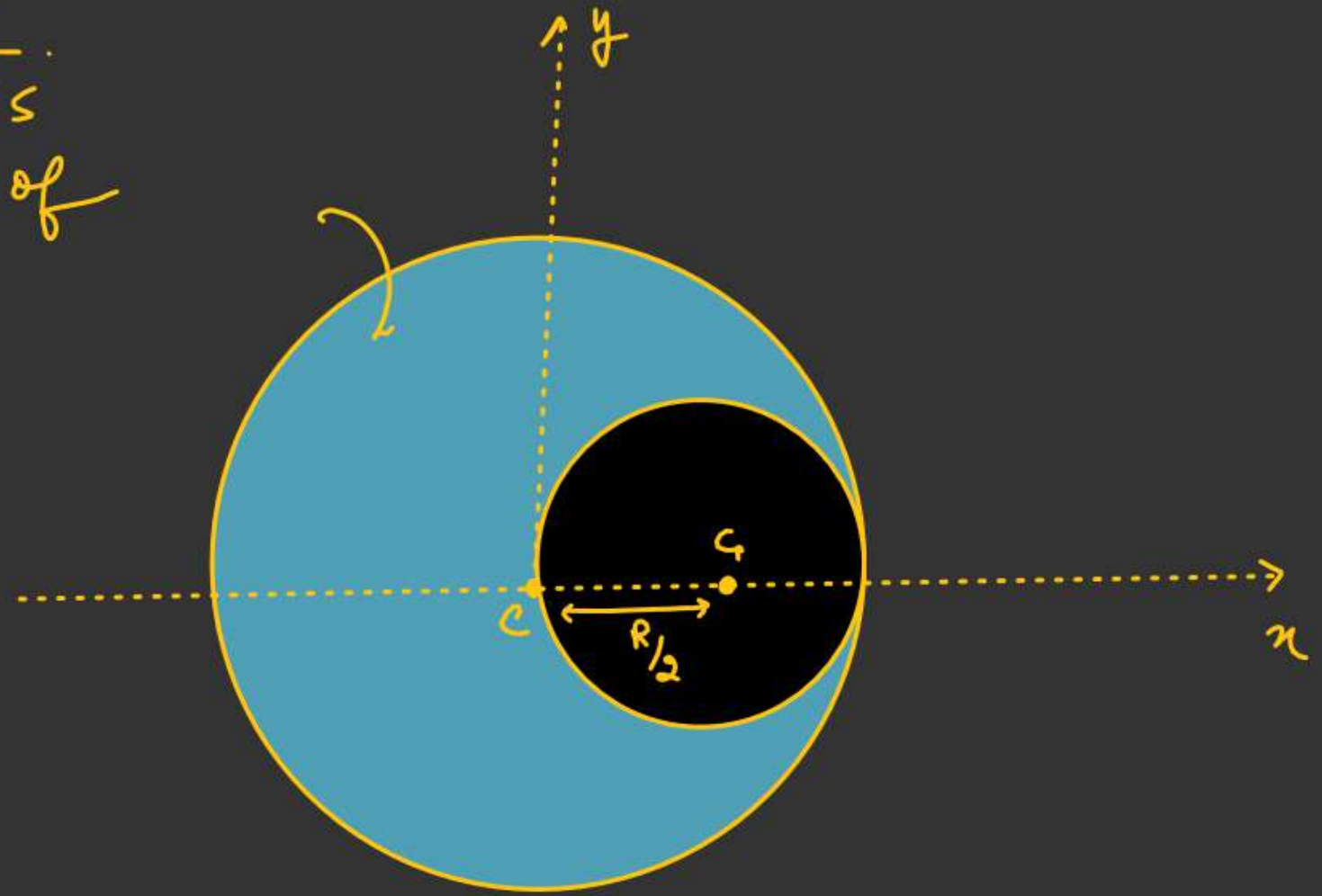
Find com of residual body.



H.W# $M =$ Mass of Residual body.

A Solid Sphere of radius $R/2$ is
Cut from a Solid Sphere of
radius R .

Find COM of Residual body.



$$dm =$$