RELATION FUNCTION

Live class ⑫ → Notes → 4BSIR → Sheet1.

10) Fractional Part fxn = FPF

A) $f(x) = \{x\}$

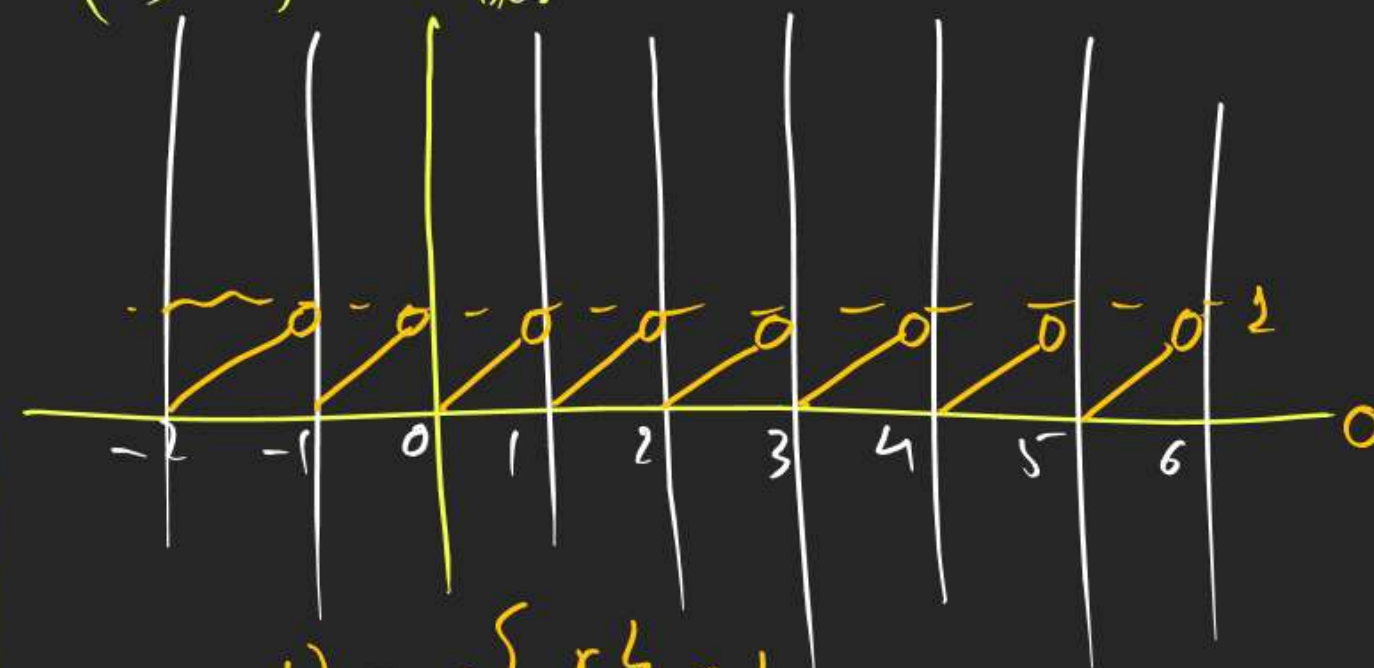B) $\{2.49\} = .49$

$\{-2.49\} = 1 - .49 = .51$

$\{3\} = 0$

$\{-105\} = 0$

(c) $f(x) = \{x\}$

1) $0 \leq \{x\} < 1$

2) $x \in R$, ③ $R_f \in [0,1)$

Prop.

1) $\boxed{0 \le \{x\} < 1}$

2) $\{x\} \ge 0$

3) $\{I\} = 0$

4) $\{x\} = 0 \Rightarrow \boxed{x = I}$

5) $\{x\} > 0$ given $\Rightarrow$ $\{x\} \ne 0$

$x \in R - \{I\}$

Q Dom of $y = \log_e \{x\}$.

$e > 0, e \ne 1, \{x\} > 0$

$\Rightarrow \{x\} \ne 0$

$\Rightarrow x \in R - \{I\}$.

Q $\underset{<1}{\{x\}} + \underset{<1}{\{\sin x\}} = 2$ find x?

$< 2 \ne 2$

No sol.

(6)

$$3 \cdot 8 = 3 + \cdot 8$$

$$3 \cdot 8 = [3 \cdot 8] + \{3 \cdot 8\}$$

$$4 \cdot 2 = [4 \cdot 2] + \{4 \cdot 2\}$$

$$-5 \cdot 8 = [-5 \cdot 8] + \{-5 \cdot 8\}$$

Imp

$$x = [x] + \{x\}$$

$$x - [x] = \{x\}$$

$$x - \{x\} = [x]$$

$$x \geqslant [x] > x - 1$$

$$x - [x] \geqslant 0 \qquad x - [x] < 1$$

$$\{x\} \geqslant 0 \qquad \{x\} < 1$$

$$0 \leq \{x\} < 1$$

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Q find Dom of $y = \sqrt{x - \lceil x \rceil}$?

$x - \lceil x \rceil \geqslant 0$

$\{x\} \geqslant 0 \rightarrow$ True

for $\boxed{x \in R}$

Q Df of $y = \dfrac{1}{\sqrt{x - \lceil x \rceil}}$

$x - \lceil x \rceil > 0$

$\{x\} > 0$

$\Rightarrow \{x\} \neq 0$

$\Rightarrow x \in \underline{R - I}$ ⓐ

RELATION FUNCTION

Q $f(x) = \sin^{-1}\{x\}$ find D_f?

$-1 \leq \{x\} \leq 1$
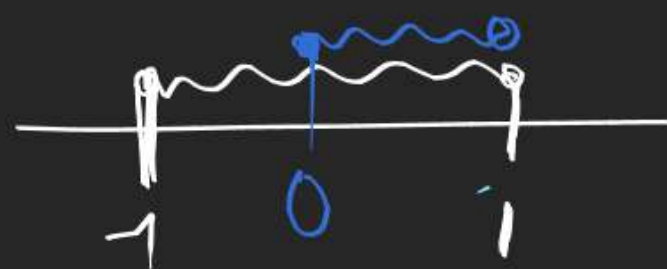
Classic

Blue Band is Perfectly
Coming in White Band.

$\Rightarrow x \in R$

Prob7 $\quad \{x+n\} = \{x\}$

$\{x-3\} = \{x\}$

$\{x+7\} = \{x\}$

Q $\{x+\lceil x \rceil\} = ?$
$\qquad\quad\; x$
$= \{x\}$

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Q If $Y = [x] + \dfrac{\sum\limits_{r=1}^{200} \{x+r\}}{200}$ find $\int Y \cdot dx = ?$

Nots
Jarur

$$Y = [x] + \frac{\{x+1\} + \{x+2\} + \{x+3\} + \cdots + \{x+200\}}{200}$$

$\xleftarrow{\ 200\ } \xrightarrow{\quad 200 \quad}$

$$= [x] + \frac{\{x\} + \{x\} + \{x\} + \cdots + \{x\}}{200}$$

$$Y = [x] + \frac{200\{x\}}{200} = [x] + \{x\}.$$

$$Y = x =)$$

$$\int Y \cdot dx = \int x \cdot dx = \frac{x^2}{2} + C$$

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① Range of $y = x - [x-2]$

$$= x - [x] + 2$$

$$y = \{x\} + 2$$

$$y \in [0,1) + 2$$

$$R_f = y \in [2,3)$$

Q Range of $y = x + 3 - [x-5]$

$$y = x + 3 - [x] + 5$$

$$= x - [x] + 8$$

$$y = \{x\} + 8$$

$$y \in [0,1) + 8$$

$$y \in [8,9)_{A_r}$$

RELATION FUNCTION

Q If $g(x) = 1 + \boxed{x - [x]}$ & $f(x) = \begin{cases} -1 & x < 0 \\ 0 & x = 0 \\ 1 & x > 0 \end{cases}$ find $f(g(x)) = ?$

$= 1 + \{x\}$

$\downarrow$

$= 1 + [0, 1)$
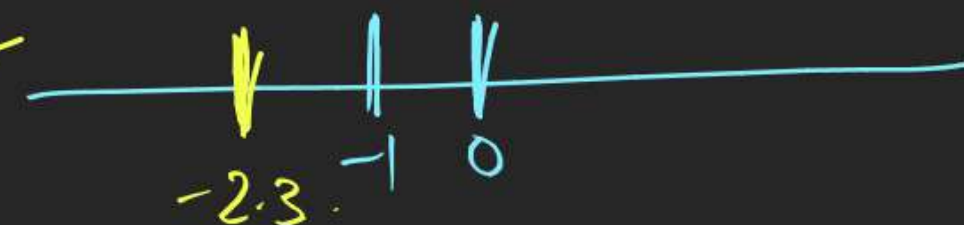
$g(x) \in [1, 2)$

$\oplus$ Pos

$\begin{array}{ccc} 0 & 1 & 2 \\ & Pos & \end{array}$

$f(g(x)) = \begin{cases} +1 & \cancel{g(x) < 0} \\ 0 & \cancel{g(x) = 0} \\ \perp & g(x) > 0 \checkmark \end{cases}$

$f(g(x)) = 1$

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$$Q \quad f(x) = \begin{cases} 1 + |x| & x < -1 \\ [x] & x \geq -1 \end{cases} \quad find \ f(f(-2.3)) = ?$$

$$f(-2.3) = \begin{cases} 1 + |-2.3| & -2.3 < -1 \ \checkmark \\ [-2.3] & -2.3 \geq -1 \end{cases}$$

$$-2.3 \quad -1 \quad 0$$

$$f(-2.3) = 1 + 2.3 = \underline{3.3}$$

$$Demand \ f(f(-2.3)) = f(3.3) = \begin{cases} 1 + |3.3| & 3.3 < -1 \\ [3.3] & 3.3 \geq -1 \ \checkmark \end{cases}$$
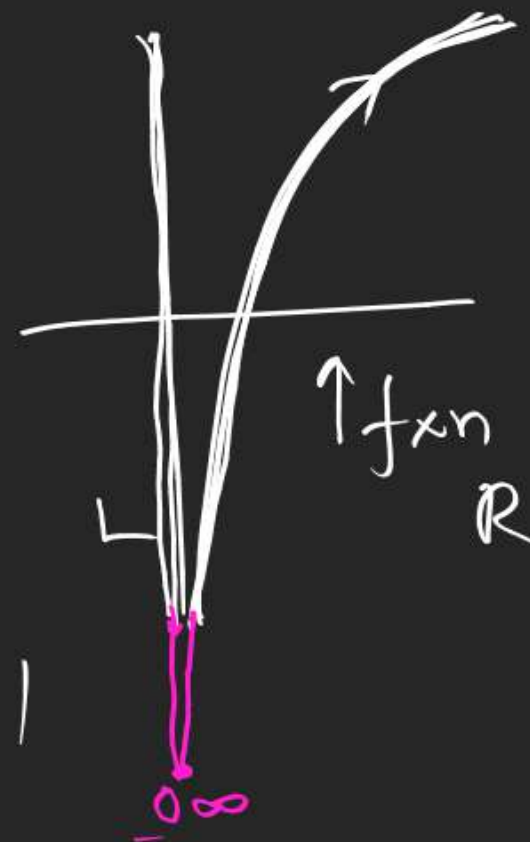
$$f(f(-2.3)) = [3.3] = 3$$

Range

J J H S J J H

Q Range of $y = \log_e \{x\}$

$$0 \leq \{x\} < 1$$

$$\log 0 \leq \log_e \{x\} < \log 1$$

$\log at 0 \quad -\infty \leq y < 0$

$$y \in (-\infty, 0)$$

$\uparrow fxn$

$-\infty$

$L \qquad R$

RELATION FUNCTION

Q Range of $y = \sin \{x\}$.

$$\downarrow\uparrow H \downarrow\uparrow\downarrow H$$

$$0 \leq \{x\} < 1$$

$$\sin 0 \leq \sin \{x\} < \sin 1$$

$$0 \leq y < \sin 1$$

$$\therefore \; [0, \sin 1)$$

$$[0, 1) = [\overset{\circ}{0}, 57°)$$
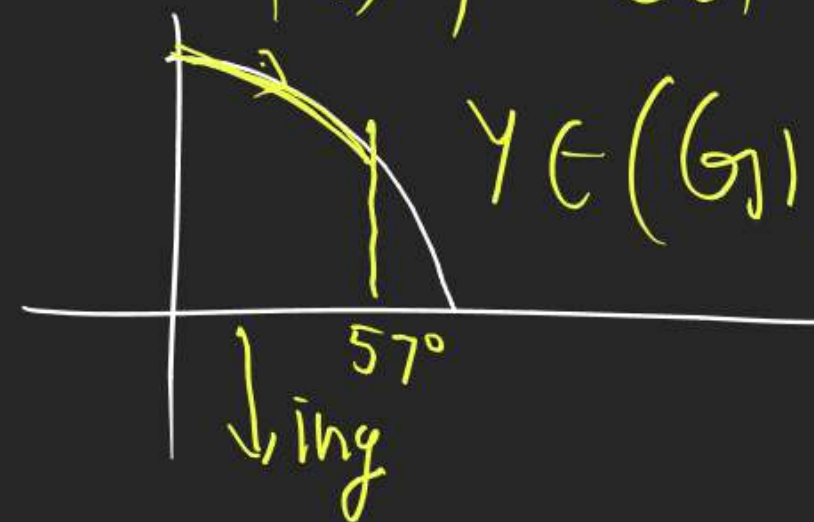

$$\underset{\uparrow ing}{57°}$$

Q $f(x) = \cos \{x\}$ R_f?

$$0 \leq \{x\} < 1$$

$$\cos 0 \geq \cos \{x\} > \cos 1$$

$$1 \geq y > \cos 1$$

$$y \in (\cos 1, 1]$$

$$\underset{\downarrow ing}{57°}$$

RELATION FUNCTION

Graph of $Y = J\{x\}$

$Y = \sin\{x\}$ graph

$[0,1)$ me Cut kro

$\pi \simeq 3.14$

$\dfrac{\pi}{2} \simeq 1.57$

Graph of $Y = G\{x\}$

$Y \in (G_1, 1]$

$\sin 1$

1.57

$\dfrac{\pi}{2}$

$\sin 1$

$-1 \quad 0 \quad 1 \quad 2 \quad 3$

$R_f \in [0, \sin 1)$

$\dfrac{\pi}{2}$

1.57

$-2 \quad -1 \quad 0 \quad 1 \quad 2 \quad 3$
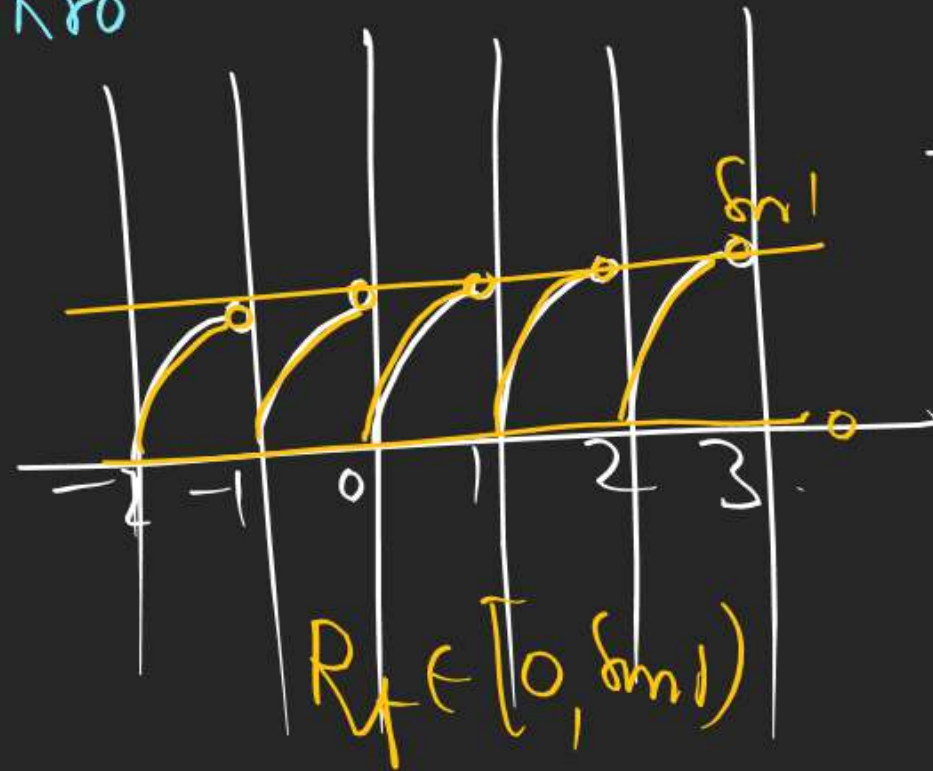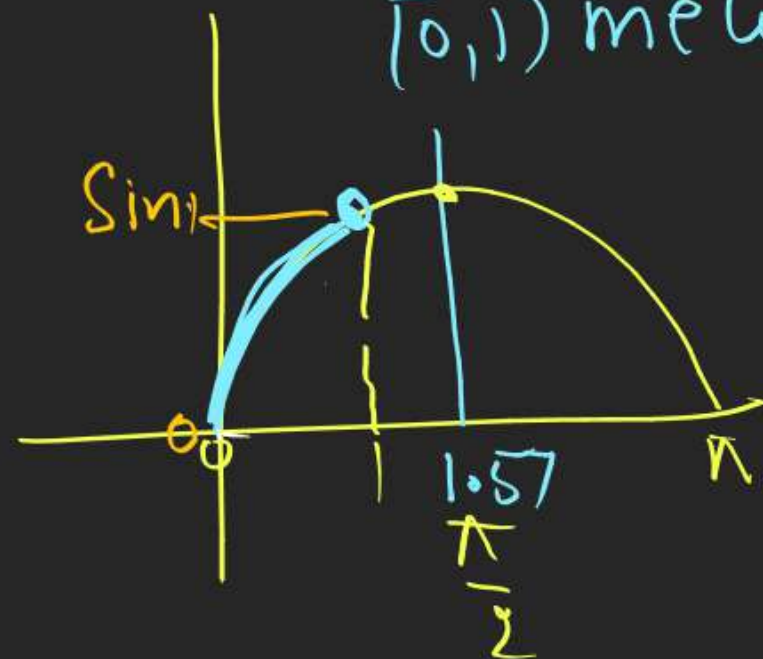
G_1
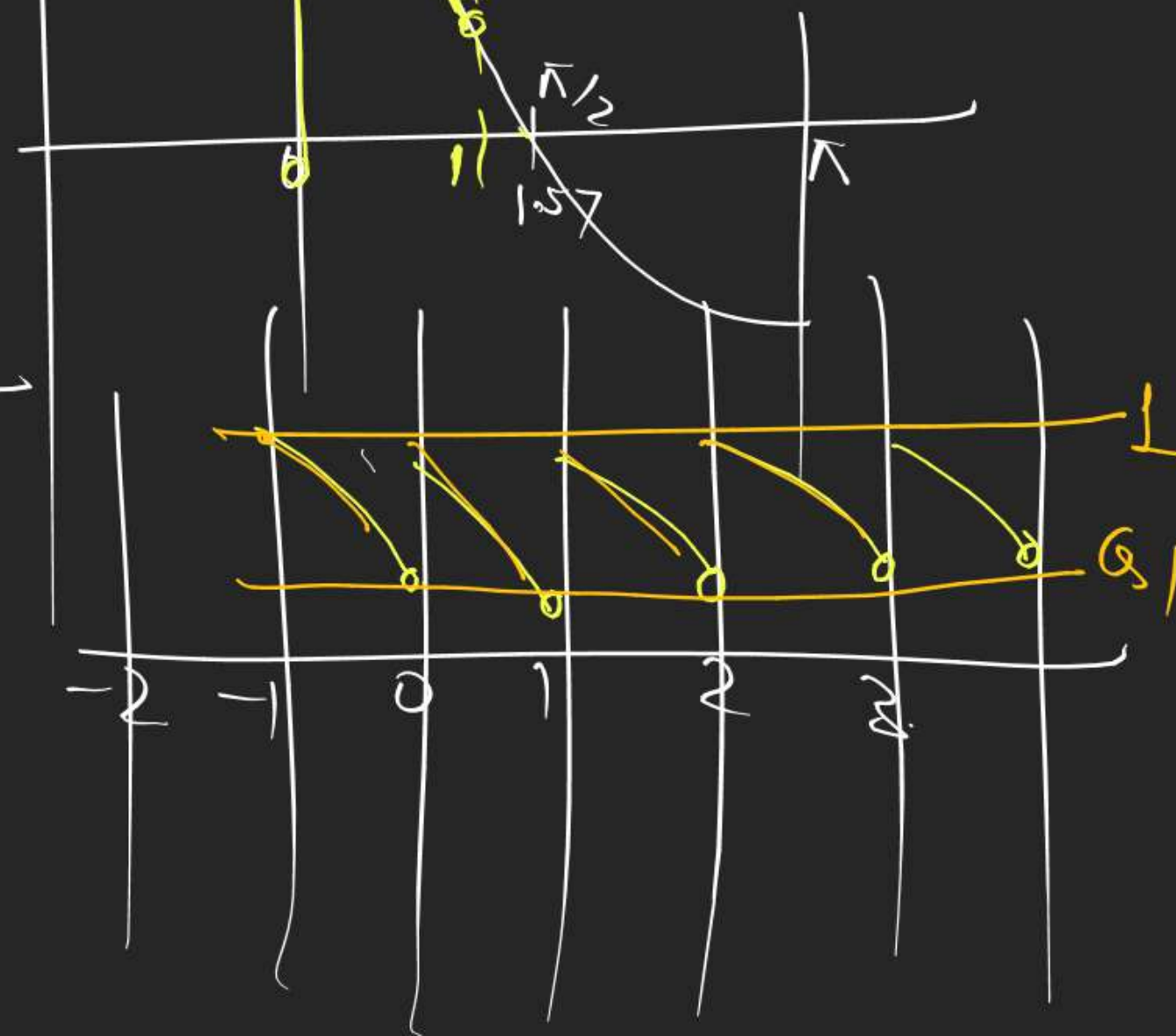
RELATION FUNCTION

Q Graph of $y = \log\{x\}$
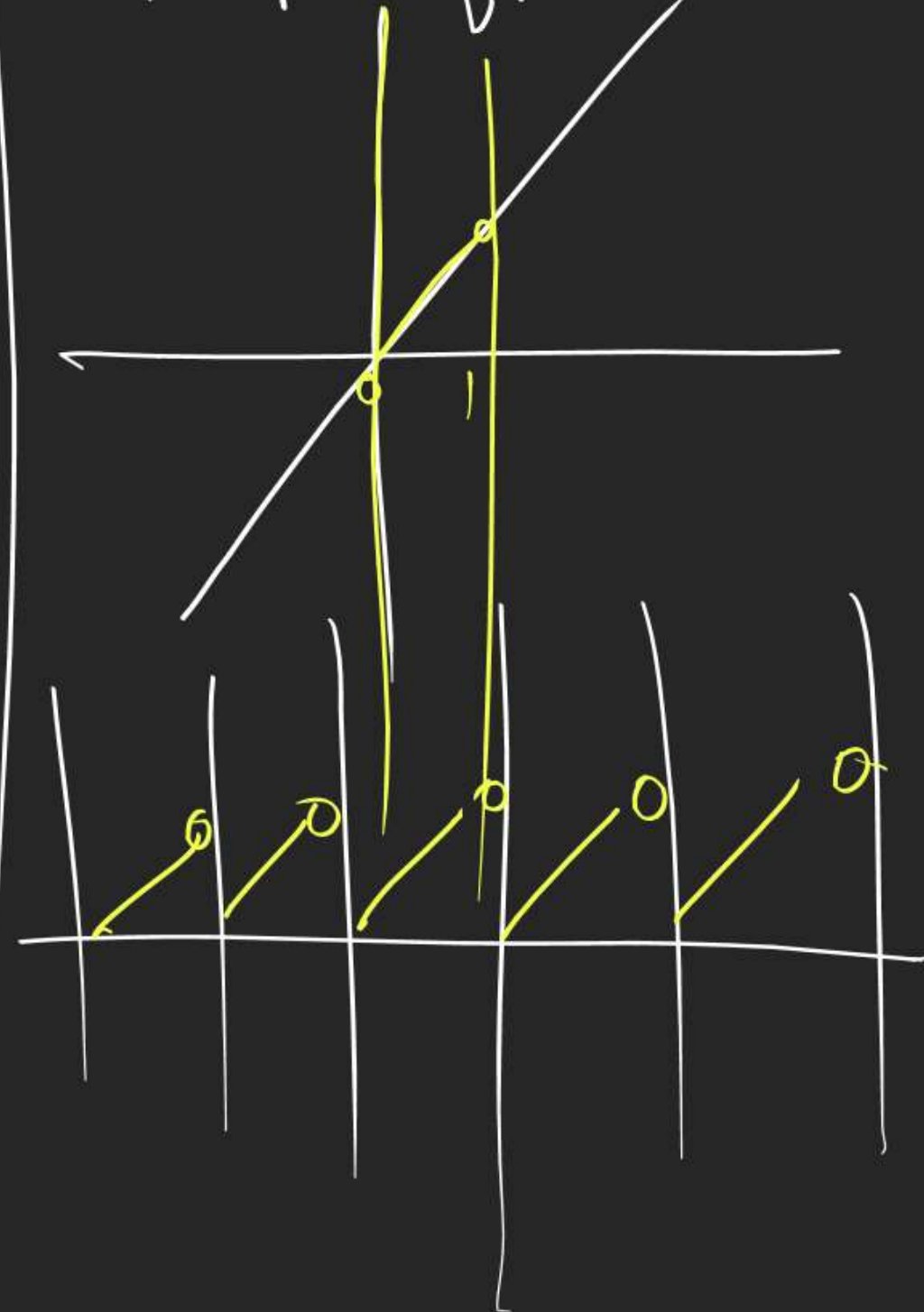

Graph of $y = \{x\}$

$$y \in (-\infty, 0)$$

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$Q \quad y = e^{\{x\}}$

$y \in [1, e)$

$D_f \to x \in R$

$y = e^{\{x\}}$

$y = e' = e^1$

$y = e^0 = 1$
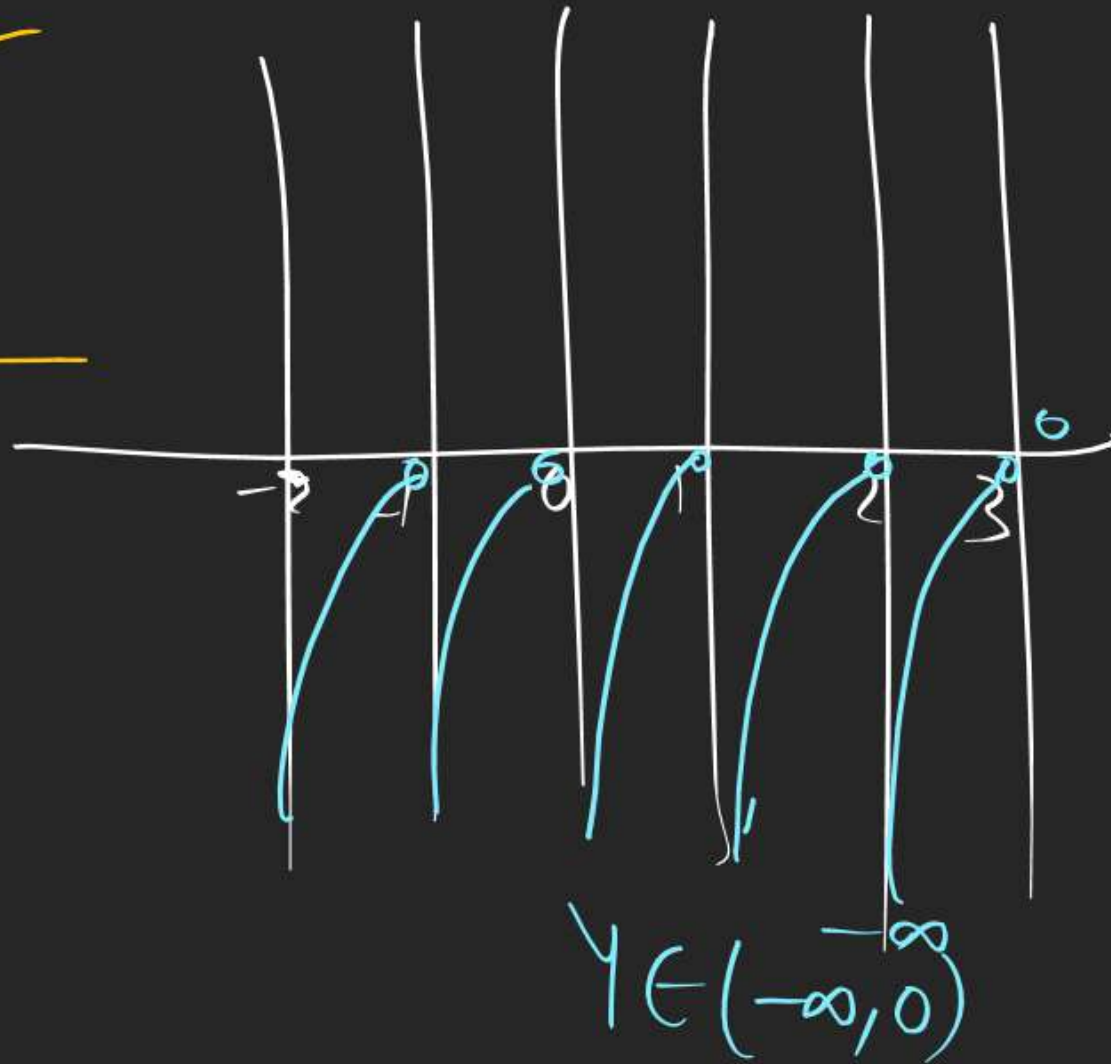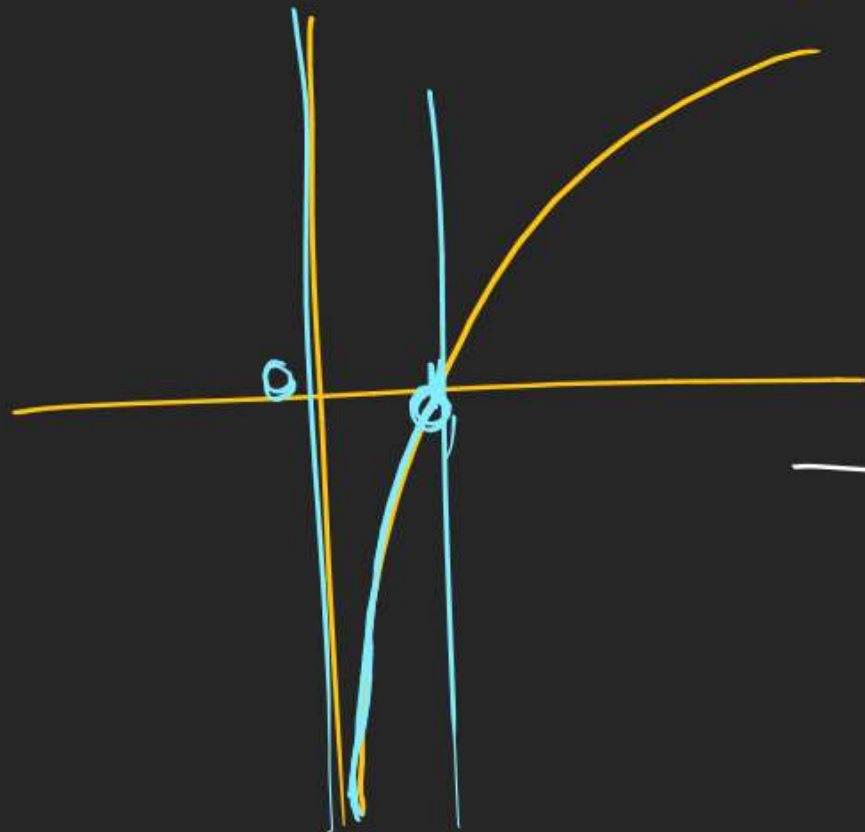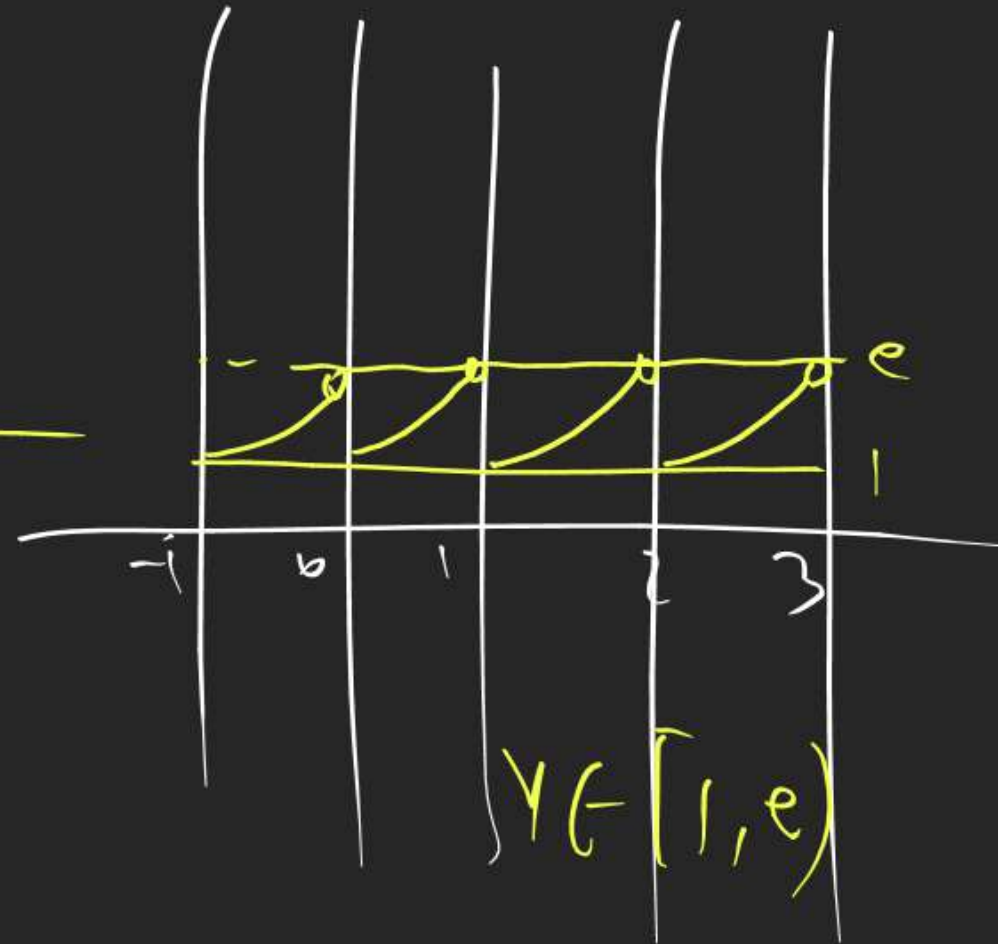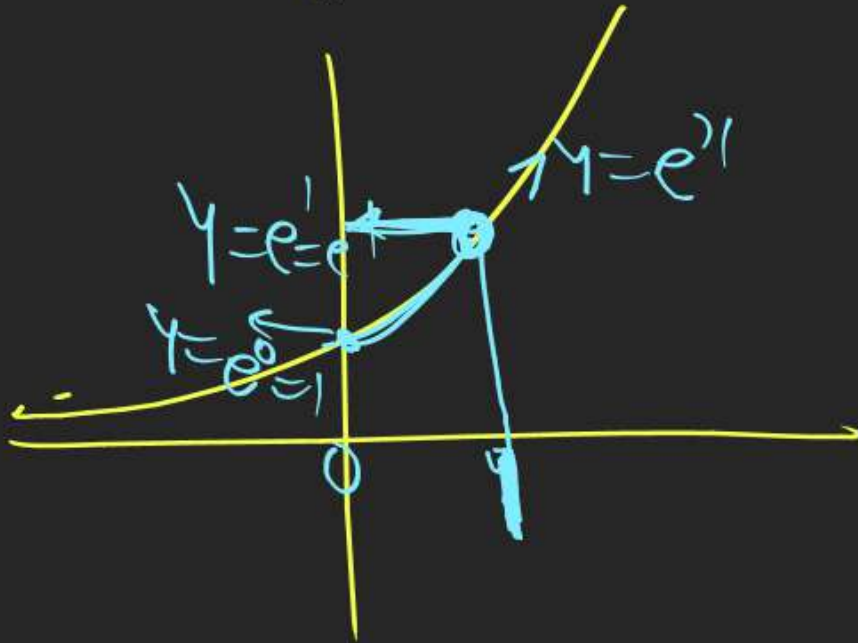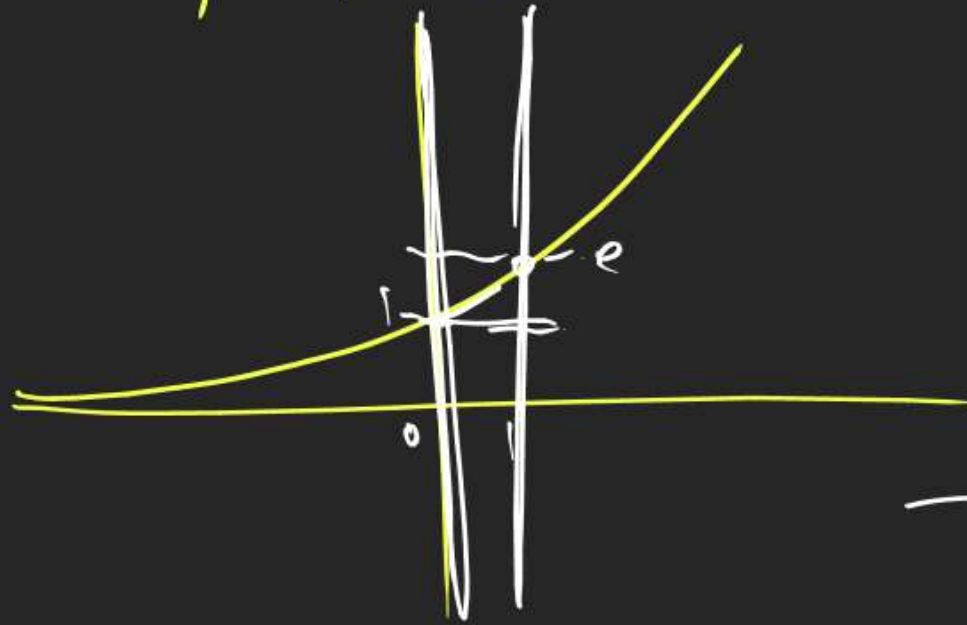
$$y = \frac{\{x\}}{1+\{x\}}$$

$$= \frac{(\{x\}+1)-1}{(\{x\}+1)}$$

Q find Domain & Range of $y = \frac{x-[x]}{1-[x]+x}$

$$y = \frac{\{x\}}{1+\{x\}} = \{x\} \times \frac{1}{\{x\}+1}$$

$$y = 1 - \frac{1}{1+\{x\}}$$

$0 \leq \{x\} < 1$

$1 \leq 1+\{x\} < 2$

$1 \geq \frac{1}{1+\{x\}} > \frac{1}{2}$

$$y = 1 - \frac{1}{1+\{x\}}$$

$$y \in \left[0, \frac{1}{2}\right)$$

$$0 \leq \boxed{1 - \frac{1}{1+\{x\}}} < \frac{1}{2} \leq -1 \leq \frac{-1}{1+\{x\}} < -\frac{1}{2}$$

R

$x \in R$

$\{x\}+1 \neq 0$

$\boxed{\{x\} \neq -1}$

$x \in R$

$$y = 1 - \frac{1}{1+\{x\}}$$

$$J J N S J J 1 7$$

$$y = \frac{\{x\}}{1+\{x\}} = \frac{(\{x\}+1)-1}{(\{x\}+1)} = 1 - \frac{1}{1+\{x\}}$$

$$\sqrt{\sqrt{1+\sqrt{\#}}}$$

$$0 \leq \{x\} < 1$$

$$1 \leq 1+\{x\} < 2$$

$$1 \geq \frac{1}{1+\{x\}} > \frac{1}{2}$$

$$-1 \leq \frac{-1}{1+\{x\}} < \frac{-1}{2}$$

$$0 \leq \boxed{1 - \frac{1}{1+\{x\}}} < \frac{1}{2}$$

$$0 \leq y < \frac{1}{2}$$

$$R_f \quad y \in \left[0, \frac{1}{2}\right)$$

RELATION FUNCTION

$x, [x]$ & $\{x\}$ Based Qs

Q If $4\{x\} = x + [x]$ find $x = ?$

① Convert x into $\{x\} + [x]$

$$4\{x\} = \{x\} + [x] + [x]$$

$$3\{x\} = 2[x]$$

② find value of $\{x\}$ & put it in $[0, 1)$

$$\boxed{\{x\} = \frac{2[x]}{3}} \Rightarrow 0 \leq \frac{2[x]}{3} < 1$$

$$0 \leq [x] < \frac{3}{2}$$

3) Read it & find $[x]$ then find $\{x\}$

then add them.

$$0 \leq [x] < \frac{3}{2}$$

$[x] = 0$	$[x] = 1$
$\{x\} = \dfrac{2 \times 0}{3}$	$\{x\} = \dfrac{2 \times 1}{3}$
$= 0$	$= \dfrac{2}{3}$
$x = 0$	$x = 1 + \dfrac{2}{3} = \dfrac{5}{3}$

$$x \in \left\{ 0, \frac{5}{3} \right\}$$

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Q If $2x + 3\{x\} = 4[x] - 2$ find x?

① $2\{x\} + 2[x] + 3\{x\} = 4[x] - 2$

$5\{x\} = 2[x] - 2$

$$\boxed{\{x\} = \frac{2([x] - 1)}{5}}$$

② $0 \leq \frac{2([x] - 1)}{5} < 1$

$0 \leq [x] - 1 < \frac{5}{2}$

$1 \leq [x] < \frac{7}{2}$

$1 \leq [x] < \frac{7}{2}$

$[x] = 1$

$\{x\} = 2\frac{(1-1)}{5}$

$= 0$

$x = 1 + 0$

$= 1$

$[x] = ②$

$\{x\} = 2\frac{(2-1)}{5}$

$= \boxed{\frac{2}{5}}$

$x = 2 + \frac{2}{5}$

$= \frac{12}{5}$

$[x] = 3$

$\{x\} = 2\frac{(3-1)}{5}$

$= \frac{4}{5}$

$x = 3 + \frac{4}{5}$

$= \frac{19}{5}$

$x \in \left\{1, \frac{12}{5}, \frac{19}{5}\right\}$

Q If $[x] \{x\} = 1$ find Sol. Set?

Adv

$$\{x\} = \frac{1}{[x]}$$

$$x - [x] = \frac{1}{[x]}$$

$$x = [x] + \frac{1}{[x]} > 2$$

Sol set = $(2, \infty)$

$[x] \cdot \boxed{\{x\}} = 1$

$\underset{(0,1)}{\downarrow}$ $\oplus$ (+ve)

+ve
होना पड़ेगा

$\{x\} = \frac{1}{[x]} = \frac{1}{\oplus}$

$[x] = \frac{1}{[x]}$

$[x]^2 = 1$

$[x] = 1$

Q No of sol. of eqn $e^{2x} + e^x - 2 = \left[\{x^2 + 10x + 11\}\right] = ?$

$$\left[\left[(0,1)\right]\right] = O$$

$$e^{2x} + e^x - 2 = 0$$

$$(e^x)^2 + e^x - 2 = 0$$

$$(e^x + 2)(e^x - 1) = 0$$

$$e^x = -2 \quad \& \quad e^x = 1 = e^0$$

$$\otimes \qquad\qquad x = 0$$

$$\Big\}\quad x = \{0\}$$

11) $\underline{\text{Signum fxn.}}$

A) Rep by $y = Sgn(x)$

B) $Sgn(x) = \begin{cases} \dfrac{|x|}{x} \\ \\ 0 \end{cases}$

$x \neq 0 \begin{array}{l} \nearrow x > 0 \\ \searrow x < 0 \end{array}$

$x = 0$

$Sgn(x) = \begin{cases} \dfrac{\cancel{x}}{\cancel{x}} = 1 \\ \\ \dfrac{-\cancel{x}}{\cancel{x}} = -1 \\ \\ 0 \end{cases}$

$x > 0$

$x < 0$

$x = 0$

$y = Sgn x = \begin{cases} 1 & x > 0 \\ -1 & x < 0 \\ 0 & x = 0 \end{cases}$

$Sgn(\underset{\oplus}{3}) = 1$

$Sgn(\underset{-ve}{-1.8}) = -1$

$Sgn(0) = 0$

Q $y = Sgn \{x\}$ Range.

$= Sgn [0,1)$

$0 \quad 1$

$= Sgn \, 0, \; Sgn \, \overset{\oplus}{(\underline{0},1)}$

$R_f = Y = \{0, 1\}$

$Y = Sgn \, x = \begin{cases} 1 \\ 0 \\ -1 \end{cases}$

$\begin{aligned} x &> 0 \\ x &= 0 \\ x &< 0 \end{aligned}$

$y = 1$

$y = -1$

$x < 0 \qquad\qquad x > 0$
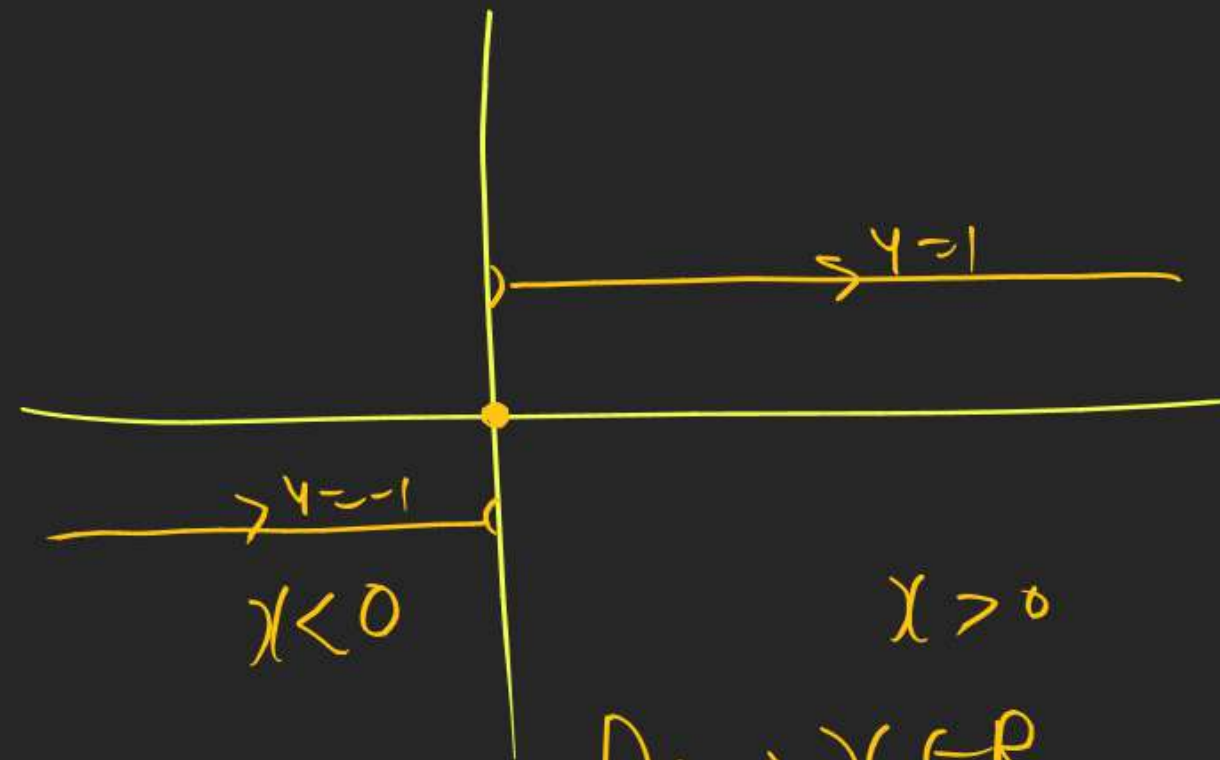
$D_f \rightarrow x \in R$

$R_f \rightarrow \{-1, 0, 1\}$

Q $Y = Sgn(e^x)$ find R_f?

$e^x \in (0, \infty)$

$Y = Sgn(0, \infty)$

$Y = 1 \therefore R_f \quad Y \in \{1\}$

$Y = Sgn \; x^2$

$x^2 \in [0, \infty)$

$Y = Sgn[0, \infty)$

$= Sgn \, 0, \, Sgn(0, \infty)$

$= 0, 1$

$\therefore R_f \in \{0, 1\}$