Nishant Jindal

DOT PRODUCT

Q.1 Find the dot product of two vectors $\vec{A} = 3\hat{i} + 2\hat{j} - 4\hat{k}$ and $\vec{B} = 2\hat{i} - 3\hat{j} - 6\hat{k}$

Sol^n.

$$\vec{A} \cdot \vec{B} = (3\hat{i} + 2\hat{j} - 4\hat{k}) \cdot (2\hat{i} - 3\hat{j} - 6\hat{k})$$

$$= 6 - 6 + 24$$

$$= \underline{24} \ \underline{\text{Ans}}.$$

DOT PRODUCT

Q.2 Find the value of m so that the vector $\underbrace{3\hat{i} - 2\hat{j} + \hat{k}}_{\vec{A}}$ may be perpendicular to the vector $\underbrace{2\hat{i} + 6\hat{j} + m\hat{k}}_{\vec{B}}$.

$\underline{\text{Sol}^n}$:- If two vectors are perpendicular then $\vec{A} \cdot \vec{B} = 0$.

$$(3\hat{i} - 2\hat{j} + \hat{k}) \cdot (2\hat{i} + 6\hat{j} + m\hat{k}) = 0$$

$$6 - 12 + m = 0$$

$$\boxed{m = 6}$$

DOT PRODUCT

Q.3 What is the angle between the following pair of vectors ?

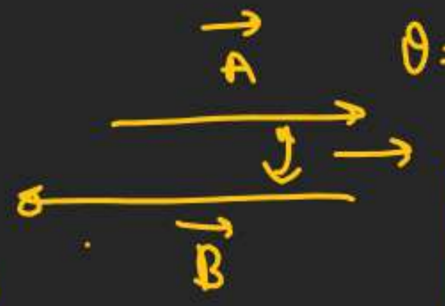
$\vec{A} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{B} = -2\hat{i} - 2\hat{j} - 2\hat{k}$ $\Rightarrow \vec{B} = -2(\hat{i}+\hat{j}+\hat{k})$

$\vec{B} = -2\vec{A}$

$$\cos\theta = \frac{\vec{A}\cdot\vec{B}}{|\vec{A}||\vec{B}|} = \frac{(\hat{i}+\hat{j}+\hat{k})\cdot(-2\hat{i}-2\hat{j}-2\hat{k})}{(\sqrt{3})\sqrt{3(2)^2}}$$

$$= \frac{-2-2-2}{6} = \frac{-6}{6} = -1$$

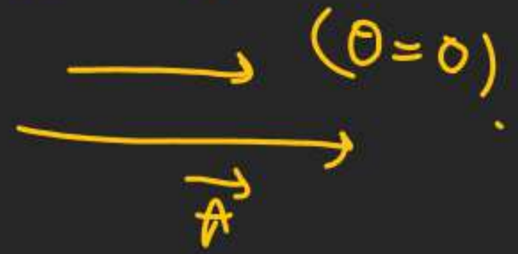
$\vec{B} = -2\vec{A}$.



$\cos\theta = -1$

$\boxed{\theta = 180^\circ}$

$\vec{B} = 2\vec{A}$

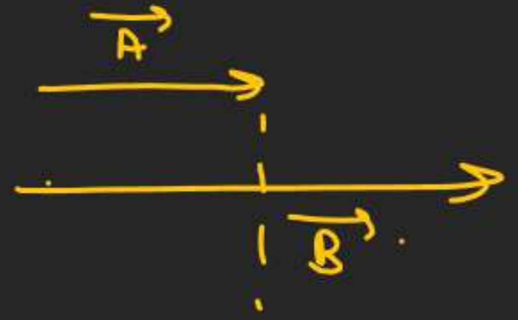


= Another Method

$\vec{A} = \lambda\vec{B}$

where λ is a Constant

$\vec{A} \parallel \vec{B}$.



DOT PRODUCT

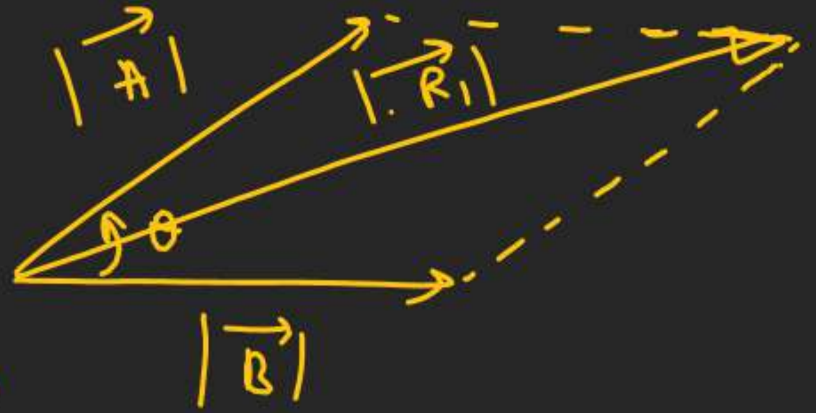
Q.4 If the sum of two unit vectors is a unit vector, then find the magnitude of their difference.

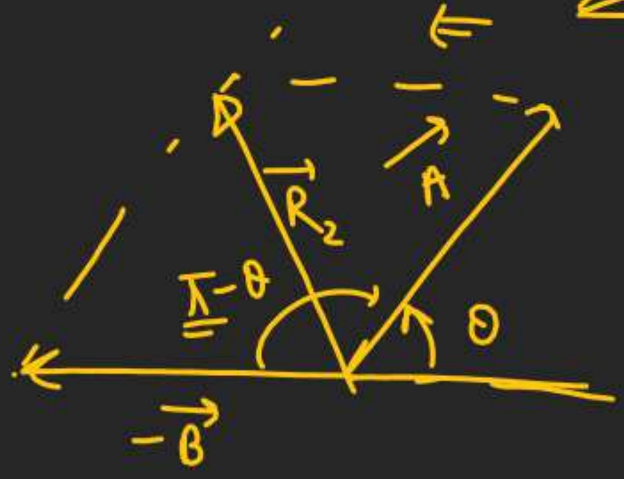
Let, $\hat{A}$ and $\hat{B}$ be two unit vectors.

$$|\hat{A}| = |\hat{B}| = 1.$$

$$\vec{R}_1 = \vec{A} + \vec{B}$$

$$\vec{R}_2 = (\vec{A} - \vec{B})$$





$$R_1 = \sqrt{A^2 + B^2 + 2AB\cos\theta}.$$

$$1 = \sqrt{1 + 1 + 2\cos\theta}.$$

$$1 = 2(1+\cos\theta)$$

$$\frac{1}{2} = 1 + \cos\theta.$$

$$\cos\theta = \left(-\frac{1}{2}\right) \Rightarrow \theta = 120°$$

$$|\vec{R}_2| = \sqrt{A^2 + B^2 + 2AB\cos 60°}$$

$$|\vec{R}_2| = \sqrt{1 + 1 + \cancel{2}\times\frac{1}{\cancel{2}}} = \sqrt{3} \text{ Ans}$$

DOT PRODUCT

$\theta \to$ Angle b/w $\vec{A}$ and $\vec{B}$.

Q.5 Prove that $(\vec{A} + 2\vec{B}) \cdot (2\vec{A} - 3\vec{B}) = 2A^2 + AB\cos\theta - 6B^2$

Sol^n:-

$$= 2(\vec{A}\cdot\vec{A}) - 3(\vec{A}\cdot\vec{B}) + 4(\vec{B}\cdot\vec{A}) - 6(\vec{B}\cdot\vec{B})$$

$$= 2A^2 - 3AB\cos\theta + 4BA\cos\theta - 6B^2$$

$$= 2A^2 + AB\cos\theta - 6B^2$$

$\vec{A}\cdot\vec{A} = A^2$
$\vec{A}\cdot\vec{B} = AB\cos\theta$.
$\vec{B}\cdot\vec{B} = B^2$

$\checkmark \; (\vec{A}\cdot\vec{B} = \vec{B}\cdot\vec{A}.)$

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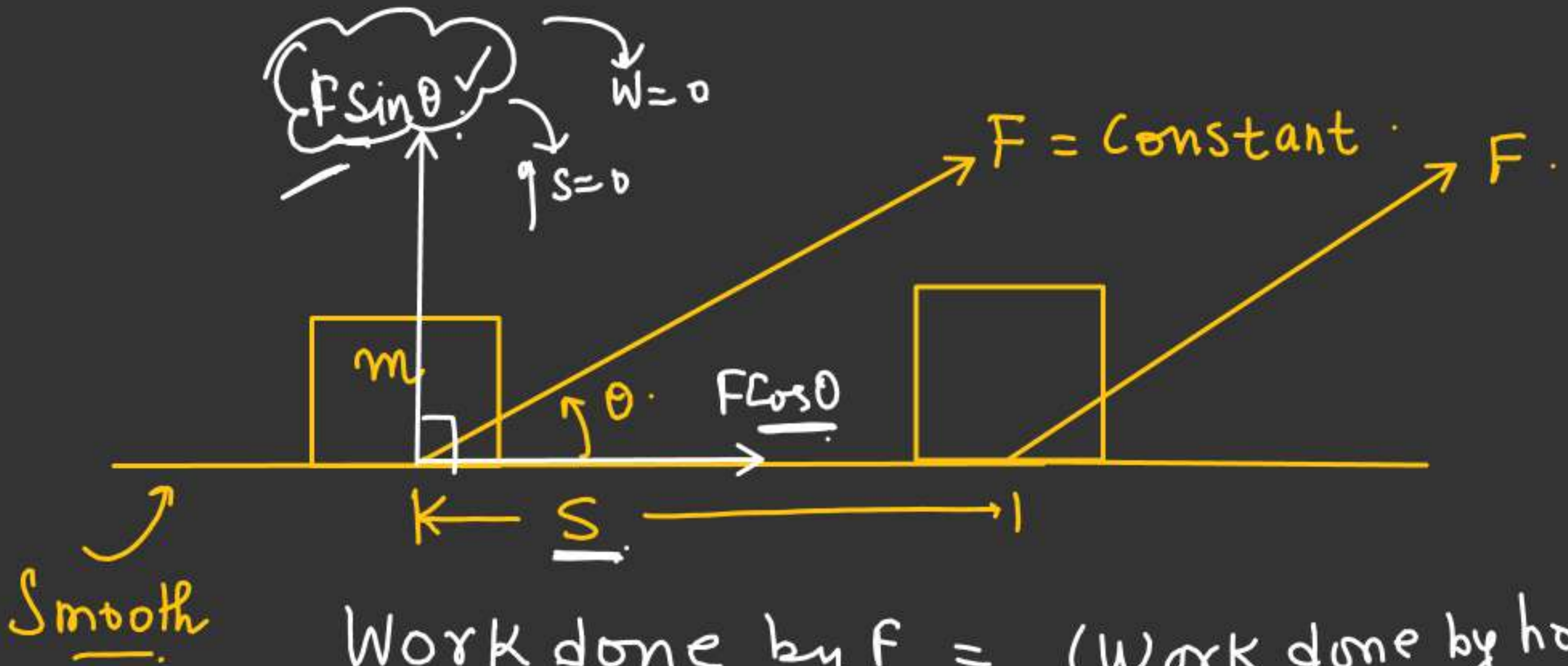
Application of Dot product :→



Work done by F = (Work done by horizontal Component)

$$= F\cos\theta \times S \qquad \vec{F}\cdot\vec{S}$$

$$\boxed{W = \vec{F}\cdot\vec{S}}$$

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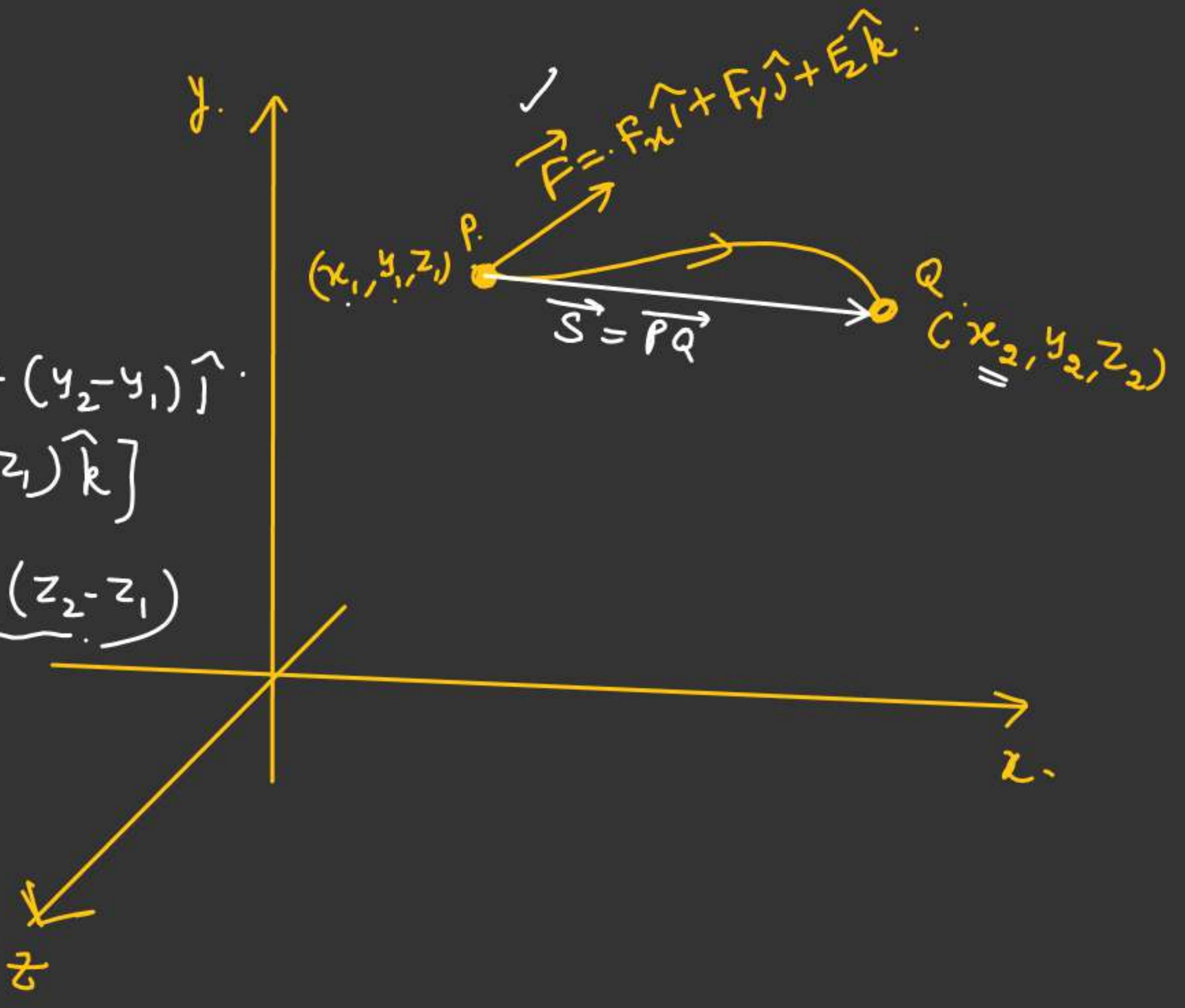
$$\vec{S} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$$

$$W = \vec{F} \cdot \vec{S}$$

$$W = (F_x\hat{i} + F_y\hat{j} + F_z\hat{k}) \cdot \left[(x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}\right]$$

$$W = F_x(x_2 - x_1) + F_y(y_2 - y_1) + F_z(z_2 - z_1)$$



DOT PRODUCT

Q.6 A body constrained to move along the z-axis of a co-ordinate system is subjected to a constant force $\vec{F}$ given by $\vec{F} = -\hat{i} + 2\hat{j} + 3\hat{k}$ newton where $\hat{i}, \hat{j}$, and $\hat{k}$ represent unit vectors along $x-, y$, and z-axes of the system, respectively. Calculate the $\boxed{\text{work done}}$ by this force in displacing the body through a distance of $\boxed{4\ \text{m}}$ along the z-axis.

Soln

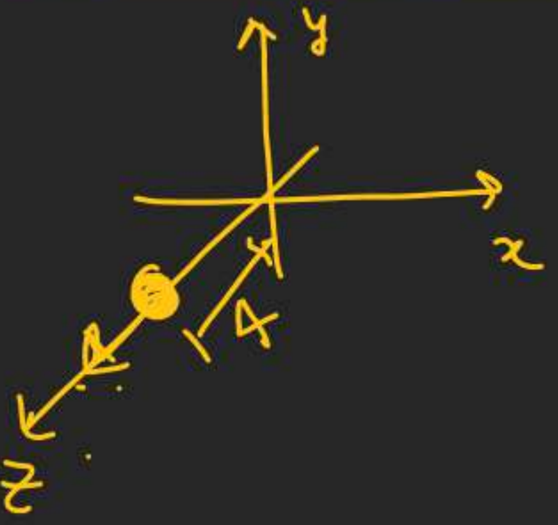
$\vec{S} = 4\hat{k}$

$\vec{F} = -\hat{i} + 2\hat{j} + 3\hat{k}$

$W = \vec{F} \cdot \vec{S} = (-\hat{i} + 2\hat{j} + 3\hat{k}) \cdot 4\hat{k}$

$= (-\hat{i} + 2\hat{j} + 3\hat{k}) \cdot (0\hat{i} + 0\hat{j} + 4\hat{k})$

$= 12\ J$



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DOT PRODUCT

H.W.

Q.7 By vector method, prove that if the diagonals of a parallelogram intersect perpendicularly, then the parallelogram is a rhombus.

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DOT PRODUCT

Q.8 $\hat{i}$ and $\hat{j}$ are unit vectors along x - and y-axes respectively. What is the magnitude and direction of the vectors $\hat{i}+\hat{j}$ and $\hat{i}-\hat{j}$? What are the components of a vector $\vec{A} = 2\hat{i} + 3\hat{j}$ along the direction $\hat{i}+\hat{j}$ and $\hat{i}-\hat{j}$?

$\vec{r}_1$ $\quad$ $\vec{r}_2$

Sol^n

$\vec{r}_1 = \hat{i}+\hat{j}$

$|\vec{r}_1| = \sqrt{(1)^2+(1)^2} = \sqrt{2}$

$\hat{r}_1 = \dfrac{\vec{r}_1}{|\vec{r}_1|} = \left(\dfrac{\hat{i}+\hat{j}}{\sqrt{2}}\right)$

$\vec{r}_2 = \hat{i}-\hat{j}$

$|\vec{r}_2| = \sqrt{(1)^2+(-1)^2} = \sqrt{2}$

$\hat{r}_2 = \left(\dfrac{\hat{i}-\hat{j}}{\sqrt{2}}\right)$

Component of $\vec{A}$ along $\vec{r}_1$

$$= \left(\frac{\vec{A}\cdot\vec{r}_1}{|\vec{r}_1|}\right) = \frac{(2\hat{i}+3\hat{j})\cdot(\hat{i}+\hat{j})}{\sqrt{2}}$$

$$= \frac{2+3}{\sqrt{2}} = 5/\sqrt{2}$$

Vector component of $\vec{A}$ along $\vec{r}_1$ $= \left(\dfrac{5}{\sqrt{2}}\right)\cdot(\hat{r}_1)$

$$= \frac{5}{\sqrt{2}}\left(\frac{\hat{i}+\hat{j}}{\sqrt{2}}\right) = \frac{5}{2}(\hat{i}+\hat{j})$$

DOT PRODUCT

→ Sides of a triangle

Q.9 If $\boxed{\vec{A} = \vec{B} + \vec{C}}$, and the magnitudes of $\vec{A}, \vec{B}, \vec{C}$ are 5,4 , and 3 units, then the angle between $\vec{A}$ and $\vec{C}$ is ✓

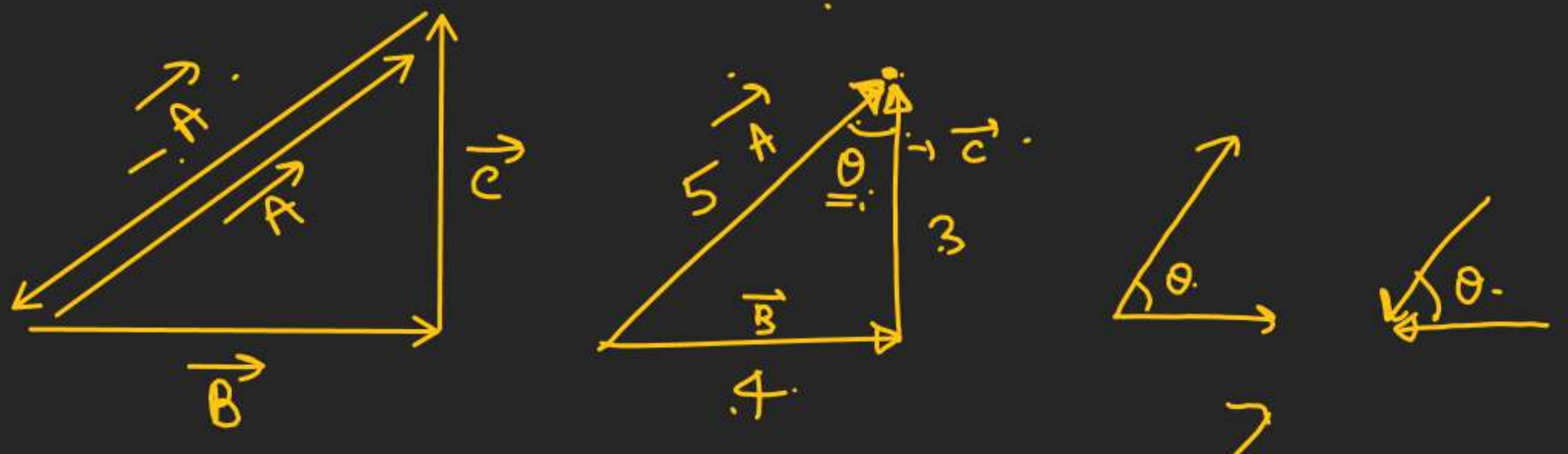
(A) $\cos^{-1}\left(\frac{3}{5}\right)$ ✓

(B) $\cos^{-1}\left(\frac{4}{5}\right)$

(C) $\sin^{-1}\left(\frac{3}{4}\right)$ ✗

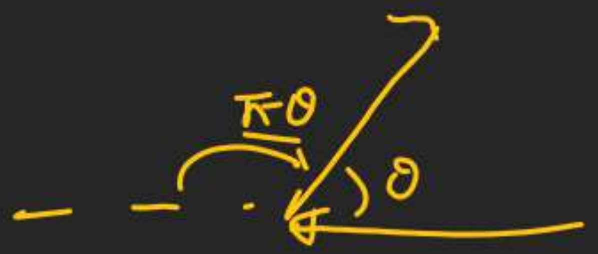
(D) $\frac{\pi}{2}$ ✗

$(\vec{B} + \vec{C}) + (-\vec{A}) = 0$ → right angle △ (Sides of △)



$\cos\theta = \frac{3}{5}$

$\theta = \cos^{-1}\left(\frac{3}{5}\right)$



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DOT PRODUCT

Q.10 Given: $\vec{A} = A\cos\theta\hat{i} + A\sin\theta\hat{j}$. A vector $\vec{B}$, which is perpendicular to $\vec{A}$, is given by

(A) $B\cos\theta\hat{i} - B\sin\theta\hat{j}$

(B) $B\sin\theta\hat{i} - B\cos\theta\hat{j}$

(C) $B\cos\theta\hat{i} + B\sin\theta\hat{j}$

(D) $B\sin\theta\hat{i} + B\cos\theta\hat{j}$

$AB\cos^2\theta - AB\sin^2\theta = AB(\cos^2\theta - \sin^2\theta) \neq 0$

$AB\sin\theta\cdot\cos\theta - AB\sin\theta\cdot\cos\theta = 0$

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DOT PRODUCT

Q.11 The projection of a vector $\vec{r} = 3\hat{i} + \hat{j} + 2\hat{k}$ on the $x - y$ plane has magnitude

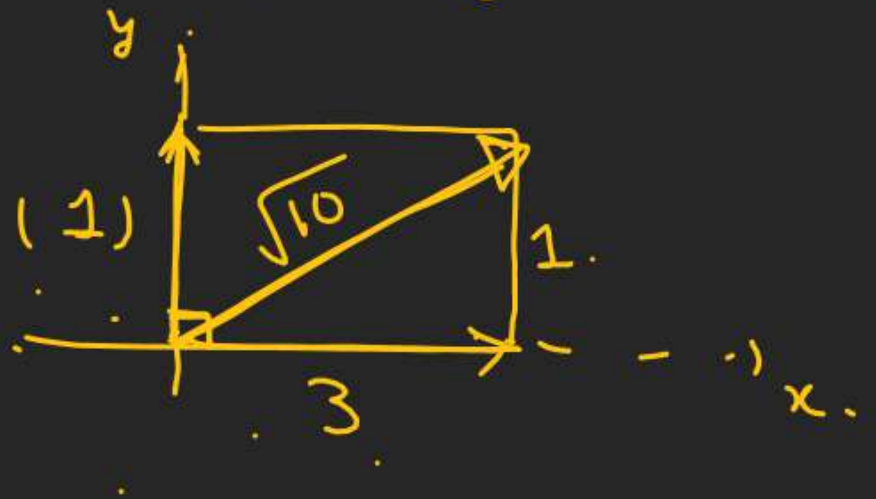
(A) 3 → projection along x-axis

(B) 4

(C) $\sqrt{14}$

(D) $\sqrt{10}$

(projection of $\vec{r}$ along x-axis) (projection of $\vec{r}$ along y-axis) (projection of $\vec{r}$ along z-axis)



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DOT PRODUCT

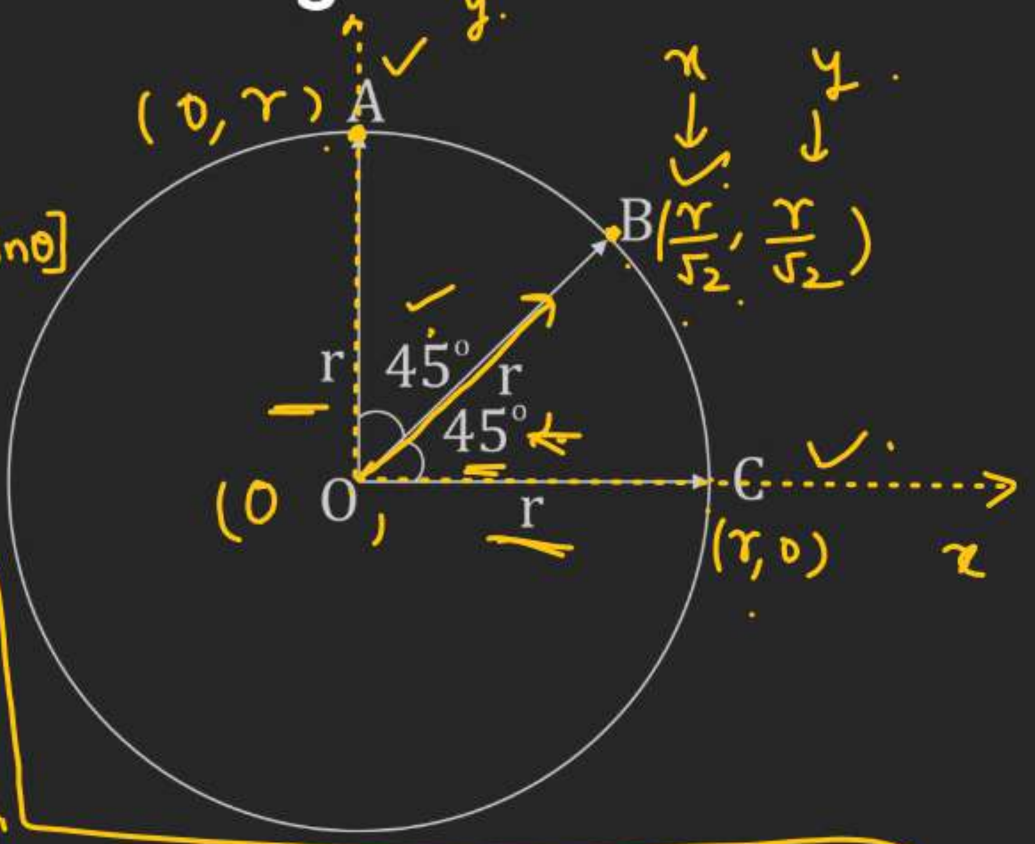
Q.12 The resultant of the three vectors $\overrightarrow{OA}$, $\overrightarrow{OB}$, and $\overrightarrow{OC}$ shown in Fig. is

(A) r

(B) 2r

(C) $r(1+\sqrt{2})$

(D) $r(\sqrt{2}-1)$



$$\vec{R} = \vec{OA} + \vec{OB} + \vec{OC}$$

$$\vec{R} = r\hat{j} + \left[\frac{r}{\sqrt2}\hat{i} + \frac{r}{\sqrt2}\hat{j}\right] + r\hat{i}$$

[r cosθ, r sinθ]

$$\vec{R} = \left(\frac{r}{\sqrt2} + r\right)\hat{i} + \left(r + \frac{r}{\sqrt2}\right)\hat{j}$$

$$|\vec{R}| = \sqrt{\left(\frac{r}{\sqrt2}+r\right)^2 + \left(\frac{r}{\sqrt2}+r\right)^2}$$

$$= \left(\frac{r}{\sqrt2}+r\right)(\sqrt2)$$

$$|\vec{R}| = r(\sqrt2+1)$$

Another Method

$\tan\theta = \frac{r}{r} = 1$

$\theta = 45°$

$\sqrt{r^2+r^2} = \sqrt2 r$

$\to (\sqrt2 r + r) = R$

$R = (\sqrt2+1)r$

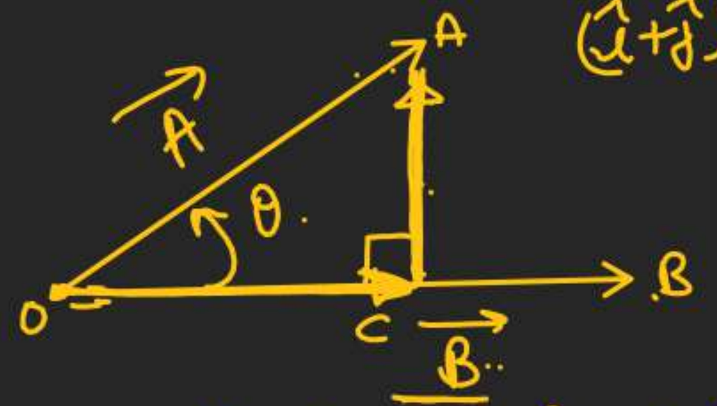
DOT PRODUCT

$(\hat{i}+\hat{j})\cdot(-\hat{i}+\hat{j}) = 0$

Q.13 Given two vectors $\vec{A} = 3\hat{i} + 4\hat{j}$ and $\vec{B} = \hat{i} + \hat{j}$. θ is the angle between $\vec{A}$ and $\vec{B}$.

$(\hat{i}+\hat{j})\cdot(\hat{i}-\hat{j}) = 0$

Which of the following statements is/are correct?



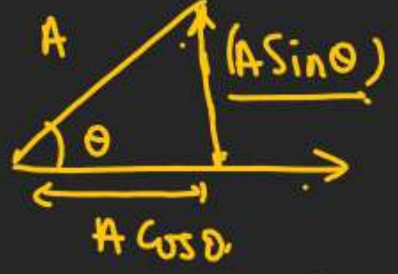
vector

(A) $|\vec{A}|\cos\theta\left(\frac{\hat{i}+\hat{j}}{\sqrt{2}}\right)$ is the component of $\vec{A}$ along $\vec{B}$

(B) $|\vec{A}|\sin\theta\left(\frac{\hat{i}-\hat{j}}{\sqrt{2}}\right)$ is the component of $\vec{A}$ perpendicular to $\vec{B}$.

Vector component of $\vec{A}$ along $\vec{B}$

(C) $|\vec{A}|\cos\theta\left(\frac{\hat{i}-\hat{j}}{\sqrt{2}}\right)$ is the component of $\vec{A}$ along $\vec{B}$.



$= \left\{\frac{\vec{A}\cdot\vec{B}}{|\vec{B}|}\right\}\cdot(\hat{B})$

(D) $|\vec{A}|\sin\theta\left(\frac{\hat{i}+\hat{j}}{\sqrt{2}}\right)$ is the component of $\vec{A}$ perpendicular to $\vec{B}$.

$= \frac{A\cancel{B}\cos\theta}{\cancel{B}} \times \hat{B}$

perpendicular component $= (A\sin\theta)(\hat{c}) \qquad = (A\cos\theta)\cdot\left(\frac{\hat{i}+\hat{j}}{\sqrt{2}}\right)$

$= A\sin\theta\left(\frac{\hat{i}-\hat{j}}{\sqrt{2}}\right)$

or

$A\sin\theta\left(\frac{-\hat{i}+\hat{j}}{\sqrt{2}}\right)$

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DOT PRODUCT

Q.14 A plane is inclined at an angle 30° with horizontal. The component of a vector $\vec{A} = -10\hat{k}$ perpendicular to this plane is: (here z-direction is vertically upwards)

(A) $5\sqrt{2}$

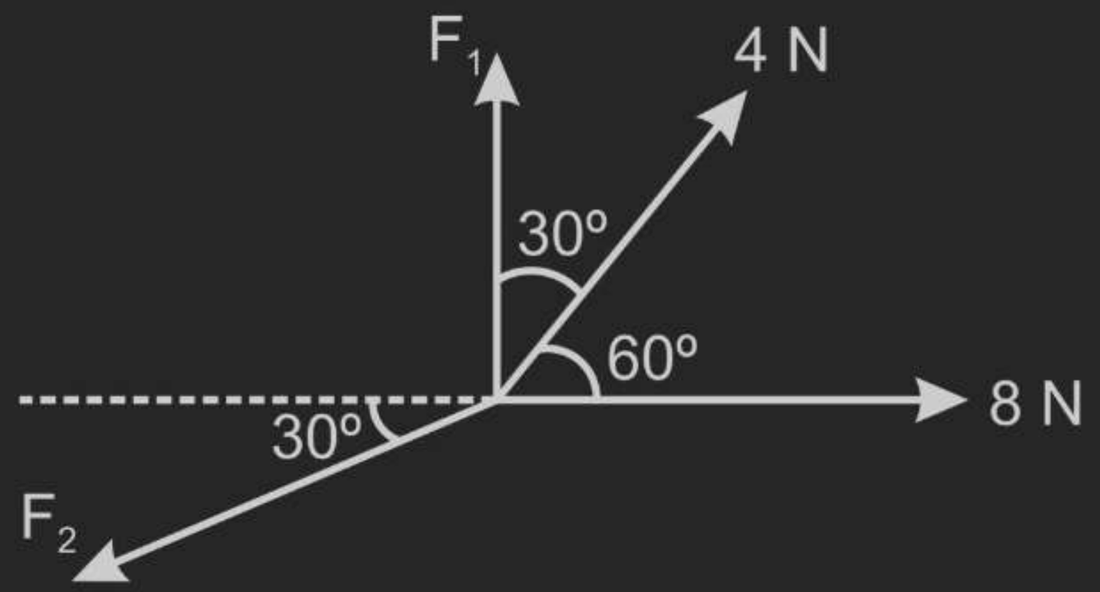
(B) $5\sqrt{3}$

(C) 5

(D) 2.5

DOT PRODUCT

Q.15 An object is in equilibrium under four concurrent forces in the directions shown in figure. Find the magnitude of $\overrightarrow{F_1}$ and $\vec{F}_2$.



DOT PRODUCT

Q.15 One end of a string $0.5\ \mathrm{m}$ long is fixed to a point A and the other end is fastened to a small object of weight $8\ \mathrm{N}$. The object is pulled aside by a horizontal force F, until it is $0.3\ \mathrm{m}$ from the vertical through A. Find the magnitudes of the tension T in the string and the force F.