Nishant Jindal

2 Types of Qs.

(1) Eqⁿ of family of Lines

Passing thru pt of Intersection of Lines $L_1=0$ & $L_2=0$ is $L_1+\lambda L_2=0$

Q3 $x(1+3\lambda)+y(1+4\lambda)+2+6\lambda=0$

P.T. $(1,3) \Rightarrow 1+3\lambda+3(1+4\lambda)+2+6\lambda=0$

$21\lambda=-6 \Rightarrow \lambda=-\frac{2}{7}$

$x\left(1-\frac{6}{7}\right)+y\left(1-\frac{8}{7}\right)+2-\frac{12}{7}=0$

$7x-2y+2=0$

(2) Converse of Type 1

Given → One Line $L=0$ To Find → Fixed Pt

If any Line $L=0$ can be represented in the form of $L_1+\lambda L_2=0$ then $L=0$ will always Passes thru a Fix Pt. & that Fixed Pt is Pt. of Intersection of $L_1=0$ & $L_2=0$.

Type 1

Q1. Find EOL P.T. POI of Lines.

$L_1: x+y+2=0$, $L_2: 3x+4y+6=0$ & Satisfies

3 अलग-2 Qs है

(1) Line $\parallel^{r}$ $L_3: 3x-2y+7=0$

(2) Whose y intercept $=1$

(3) Line in P.T. $(1,3)$

(1) Any Line P.T. POI of L_1 & L_2 is

$(x+y+2)+\lambda(3x+4y+6)=0 \rightarrow (A)$

$x(1+3\lambda)+y(1+4\lambda)+(2+6\lambda)=0$

$\rightarrow Sl = \frac{-(1+3\lambda)}{(1+4\lambda)}$

Q1 Line $\parallel$ to $3x-2y+7=0 \rightarrow Sl = \frac{+3}{+2}$

$\frac{-1-3\lambda}{1+4\lambda}=\frac{3}{2} \Rightarrow -2-6\lambda=3+12\lambda$

$\Rightarrow 18\lambda=-5 \Rightarrow \lambda=\frac{-5}{18}$

EOL $x\left(1-\frac{15}{18}\right)+y\left(1-\frac{20}{18}\right)+\left(2-\frac{30}{18}\right)=0$

$3x-2y+6=0$

Q2 y int $\Rightarrow$ Put $x=0$

$y=\frac{-2-6\lambda}{1+4\lambda}=$ y int $=1$

$-2-6\lambda=1+4\lambda$

$\Rightarrow 10\lambda=-3 \Rightarrow \lambda=\frac{-3}{10}$

Put this in (A)

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Q2 If a, b, c are in AP then P.T. the Lines $ax+by+c=0$ P.T. a $\boxed{\text{Fixed Pt.}}$ find that Fix Pt.

Anyhow Using A.P & Line make $L_1 + \lambda L_2 = 0$

M1

① a, b, c AP

$2b = a + c$ — Mix

② Line $ax+by+c=0$

$ax + \left(\frac{a+c}{2}\right)y + c = 0$

$2ax + ay + cy + 2c = 0$

$a(2x+y) + c(y+2) = 0$

$(2x+y) + \frac{c}{a}(y+2) = 0$

$L_1 + \lambda L_2 = 0$

$L_1: 2x+y=0 \Rightarrow x = 1$

$L_2: y+2=0 \Rightarrow y = -2$

$(1, -2)$ in Fixed Pt.

M2 Match constant Point

Line $ax+by+c=0$

Shrt → $a - 2b + c = 0$

match coff of a & b

$x = 1 \quad y = -2$

$(1, -2)$ A

Line + given Condⁿ make combination of 2 Line Inherent

Q3 If $a^2 + 9b^2 = 6ab + 4c^2$

P.T. Line $ax+by+c=0$ P.T. One or 2 fixed Pts.

Condⁿ $a^2 + 9b^2 - 6ab = 4c^2$

$(a-3b)^2 - (2c)^2 = 0$

$(a-3b-2c)(a-3b+2c) = 0$

2 Condⁿ exists

Cond 1

$a - 3b - 2c = 0$ } Not Med

$ax + by + c = 0$

$-\frac{a}{2} + \frac{3}{2}b + c = 0$

$ax + by + c = 0$

$x = -\frac{1}{2},\ b = +\frac{3}{2}$

$\left(-\frac{1}{2}, +\frac{3}{2}\right)$

Cond 2

$\frac{a}{2} - \frac{3}{2}b + c = 0$

$ax + by + c = 0$

$x = \frac{1}{2},\ y = -\frac{3}{2}$

$\left(\frac{1}{2}, -\frac{3}{2}\right)$

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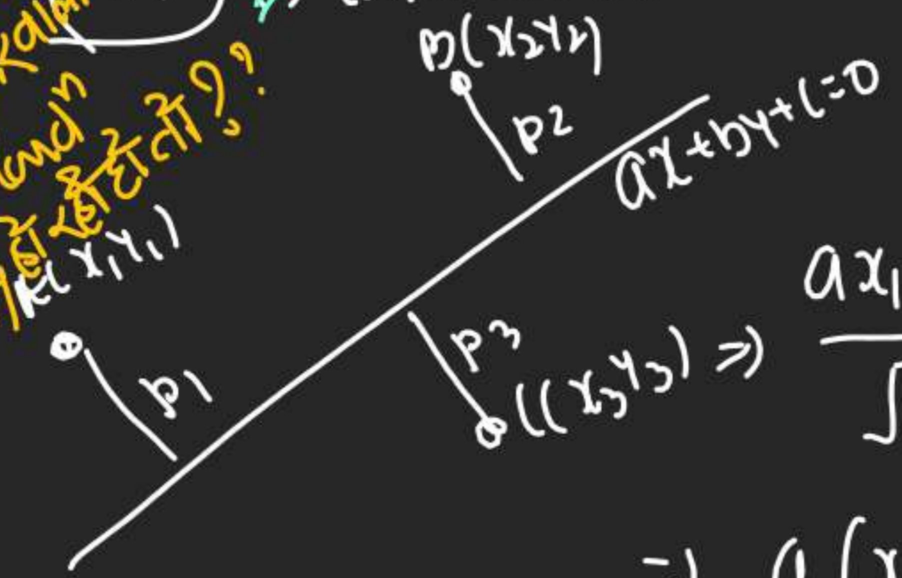
Q If algebric Sum of ⊥r's from 3 Non Collinear Pts A (x_1, y_1) B $(x_2 y_2)$ C $(x_3 y_3)$ on a Variable line Vanishes.

P.T. (A) Centroid B) Orthocentre (C) Incentre (D) Circumcentre.

किसी Fix Pt से Pass करेगी अगर ऊपर वाली ⊥r की Cond^n Satisfy हो जाती हो?



Cond^n

$$p_1 + p_2 + p_3 = 0$$

$$\Rightarrow \frac{ax_1+by_1+c}{\sqrt{a^2+b^2}} + \frac{ax_2+by_2+c}{\sqrt{a^2+b^2}} + \frac{ax_3+by_3+c}{\sqrt{a^2+b^2}} = 0$$

$$\Rightarrow a(x_1+x_2+x_3) + b(y_1+y_2+y_3) + 3c = 0$$

$$a\left(\frac{x_1+x_2+x_3}{3}\right) + b\left(\frac{y_1+y_2+y_3}{3}\right) + c = 0$$

$$ax \quad + \quad by \quad + c = 0$$

$$x = \frac{x_1+x_2+x_3}{3}, \quad y = \frac{y_1+y_2+y_3}{3}$$

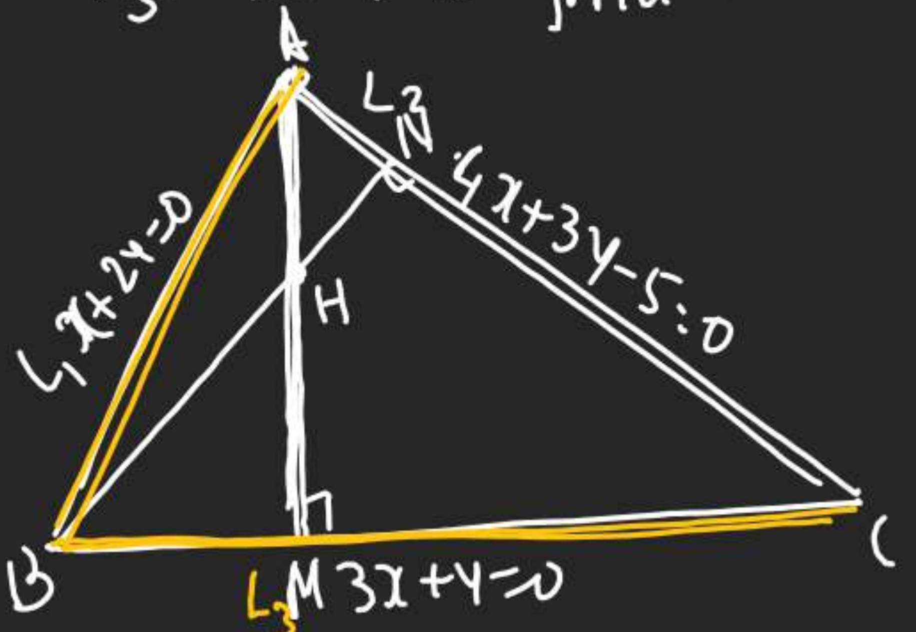
$$\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right) - \text{Centroid}$$

Q5 If Eq^n Sides of △ are

2nd Method

$L_1: x+2y=0$ $L_2: 4x+3y-5=0$

$L_3: 3x+y=0$ find orthocentre of △.



① Altitude AM → $L_1 + \lambda L_2 = 0$

$m_{AM} \times m_{BC} = -1 \Rightarrow \lambda$ आएगा

AM आएगा

② Altitude BN → $L_3 + \mu L_2 = 0$

$m_{BN} \times m_{AC} = -1 \Rightarrow \mu$ आएगा

BN आएगा

Solve AM & BN.

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Pair of Straight Line

2 Case

 PoI = origin

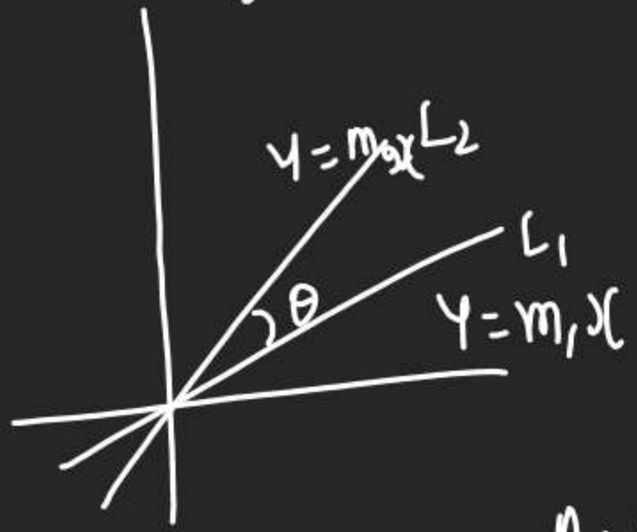
 PoI other than origin.

① Eqⁿ of Pair of Lines P.T. origin {Homogeneous form}

1) We are producting 2 Eqⁿ of Lines Simultaneously.

2) It in Product of 2 Lines.

3) In hom. form 2 Lines must Pass thru Origin.



$$(m_1x - y)(m_2x - y) = 0$$

$$m_1m_2x^2 - m_1xy - m_2xy + y^2 = 0$$

$$m_1m_2x^2 - (m_1+m_2)xy + y^2 = 0$$

$$Ax^2 + 2Hxy + By^2 = B(y - m_1x)(y - m_2x)$$

$$B\left(y^2 + \frac{2H}{B}xy + \frac{A}{B}x^2\right) = B\left(y^2 - (m_1+m_2)xy + m_1m_2x^2\right)$$

$$\boxed{-\frac{2H}{B} = m_1 + m_2 \;\Big|\; m_1m_2 = \frac{A}{B}}$$

(4)
$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1m_2}\right|$$

$$= \frac{|m_1 - m_2|}{|1 + m_1m_2|} = \frac{\sqrt{(m_1+m_2)^2 - 4m_1m_2}}{|1 + m_1m_2|}$$

$$= \frac{\sqrt{\frac{4H^2}{B^2} - \frac{4A}{B}}}{\left|1 + \frac{A}{B}\right|}$$

$$\boxed{\tan\theta = \frac{2\sqrt{H^2 - AB}}{|B + A|}}$$

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(6) A) If $A+B=0$ $\qquad \tan\theta = \dfrac{2\sqrt{H^2-AB}}{|A+B|}$

$\tan\theta \to \infty$

$L_1 \perp L_2$

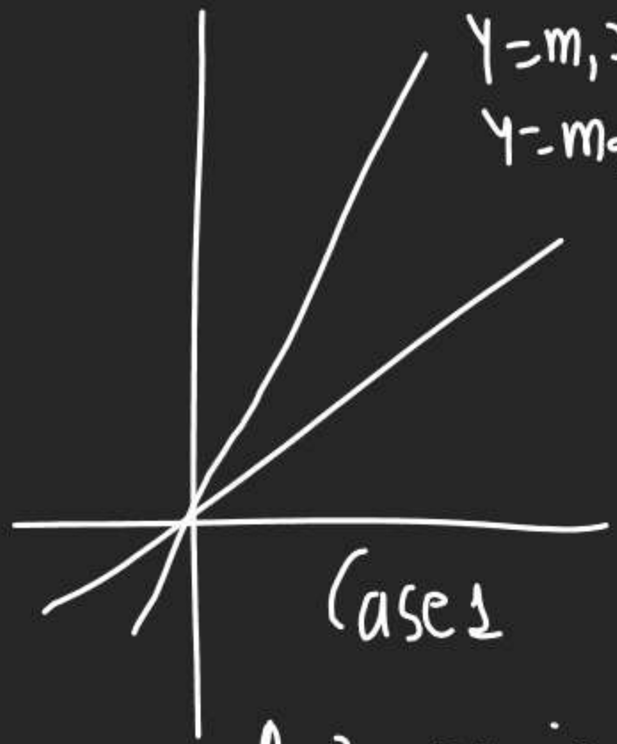
B) If $L_1 \parallel L_2 \Rightarrow \theta = 0$

$\tan\theta = 0$

$\dfrac{2\sqrt{H^2-AB}}{|A+B|} = 0 \Rightarrow \boxed{H^2 = AB}$

Q Lines are Intersecting at

Case ① at origin $\qquad$ Case ② other than Origin.



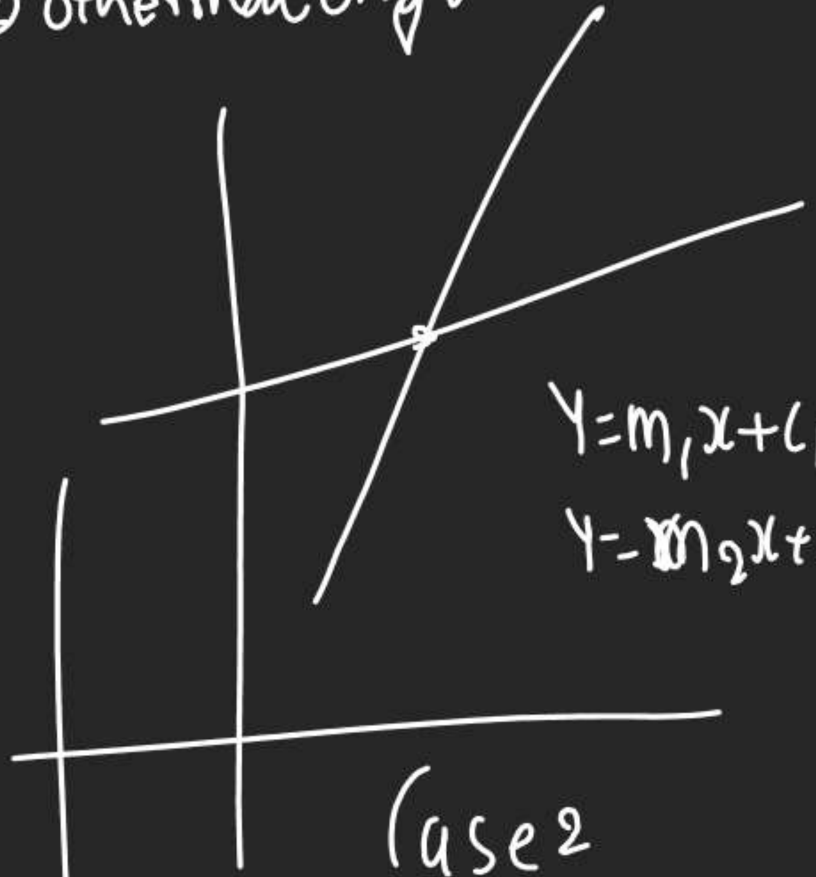
$Y = m_1 x$

$Y = m_2 x$

Case 1

$Ax^2 + 2Hxy + By^2 = 0$

Hom. form



$Y = m_1 x + c_1$

$Y = m_2 x + c_2$

Case 2

Prod

$\underbrace{Ax^2 + 2Hxy + By^2 + 2gx + 2fy + c = 0}_{\text{Non Hom. form}}$

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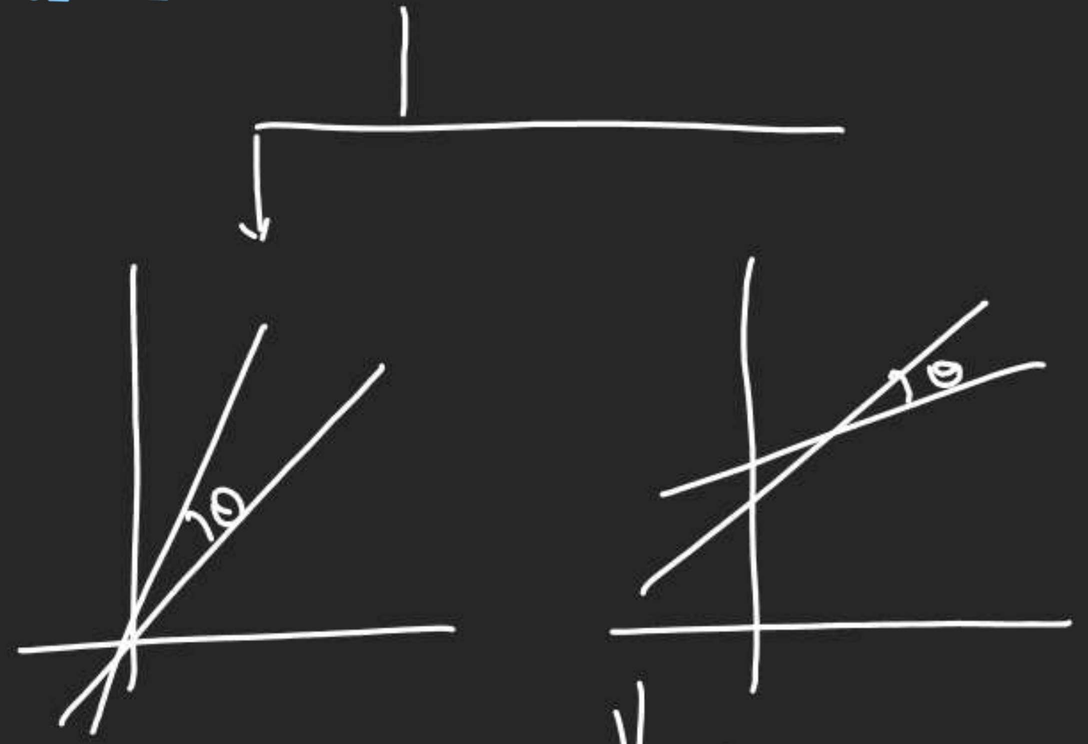


1) $Ax^2+2Hxy+By^2=0$

2) $\tan\theta = \dfrac{2\sqrt{H^2-AB}}{|A+B|}$

3) $A+B=0$ $L_1 \perp L_2$

4) $L_1 \parallel L_2 \Rightarrow AB=H^2$

1) $Ax^2+2Hxy+By^2+2gx+2fy+c=0$

2) $\tan\theta = \dfrac{2\sqrt{H^2-AB}}{|A+B|}$

3) $A+B=0 \Rightarrow L_1 \perp L_2$

4) $L_1 \parallel L_2$ $H^2=AB$

Q Find all Lines Rep. by

$$f(x,y)=(2x^2+xy-y^2)-x+5y-6=0$$

$$Ax^2+2Hxy+By^2+2gx+2fy+c=0$$

$A=2$
$B=-1$
$C=-6$
$g=-1/2$
$f=5/2$
$H=1/2$

1) Attack & factorise Hom. part

$2x^2+xy-y^2$

$(2x-y)(x+y)$

(2) add c_1 & c_2 in Factors & compare.

$$(2x-y+c_1)(x+y+c_2)=2x^2+xy-y^2-x+5y-6$$

x: $-1=c_1+2c_2$

y: $5=c_1-c_2$

$-1=c_1+2c_2$

$6=-3c_2$

$c_2=-2$

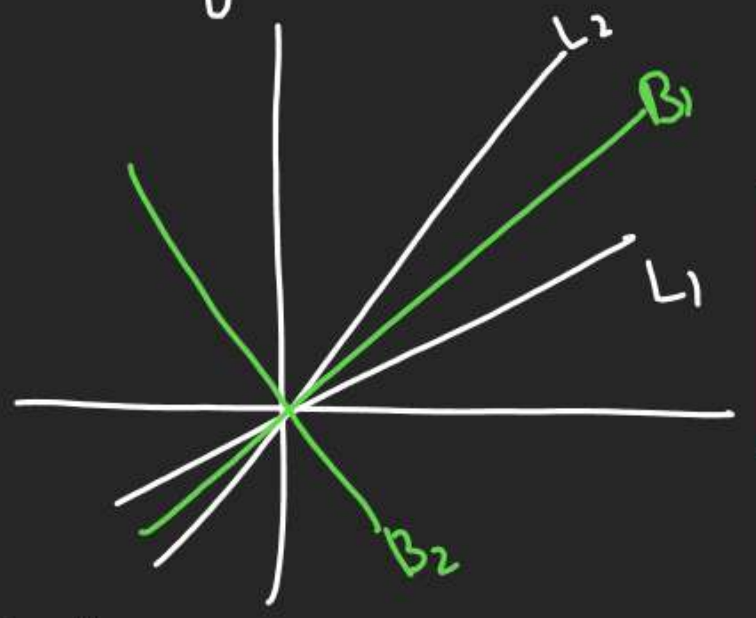
$c_1-4=-1 \Rightarrow c_1=3$

Const: $-6=c_1c_2$

(3) L_1: $\boxed{2x-y+3=0}$

L_2: $\boxed{x+y-2=0}$

* Eqⁿ of Pair of Bisector of Lines given by $ax^2+2hxy+by^2=0$



Joint Eqⁿ of B_1B_2

$$\boxed{\frac{x^2-y^2}{a-b}=\frac{xy}{h}}$$

Q If Sum of Slopes of Lines given by $x^2-2cxy-7y^2=0$ is 4 times of their Product find "c".

$x^2-2cxy-7y^2=0$

$Ax^2+2Hxy+By^2=0$

$A=1$

$B=-7$

$H=-c$

$m_1+m_2=4m_1m_2$

$-\frac{2H}{\cancel{B}}=4\frac{A}{\cancel{B}}$

$-2(-c)=4\times 1$

$c=2$

Q Find angle betⁿ Lines

$2x^2+5xy+3y^2+6x+7y+4=0$

$\tan\theta=\frac{2\sqrt{H^2-AB}}{A+B}$

$A=2$

$B=3$

$H=5/2$

$=\frac{2\sqrt{\frac{25}{4}-6}}{5}=\frac{2\times\frac{1}{2}}{5}=\frac{1}{5}$

$\theta=\tan^{-1}\frac{1}{5}$

Q Find value of a for which $ax^2+5xy+2y^2=0$ has 2 ⊥ʳ Lines.

Lines are ⊥ʳ $\Rightarrow A+B=0$

$a+2=0 \Rightarrow a=-2$

Q

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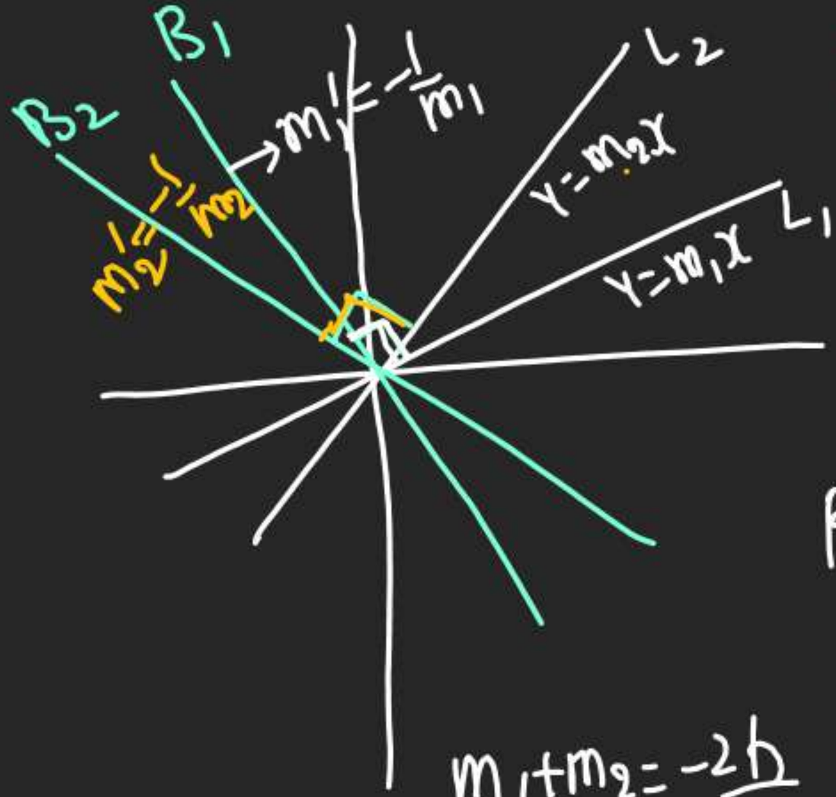
Q If both Lines P.T. origin & $\perp^r$ to Lines inherent in $ax^2+2hxy+by^2=0$ then find Eq^n of Lines.

Chapter → Homogenisation → Sheet diss

St. line → 20Qs



Joint Eq^n of B_1B_2

$L_1L_2 \rightarrow ax^2+2hxy+by^2=0$

$B_1 \rightarrow y = \frac{-1}{m_1}x \;\Big|\; B_2 \rightarrow y = \frac{-1}{m_2}x$

Prod $\left(y+\frac{1}{m_1}x\right)\left(y+\frac{1}{m_2}x\right)=0$

$y^2+\frac{1}{m_1m_2}x^2+\left(\frac{1}{m_1}+\frac{1}{m_2}\right)xy=0$

$y^2+\frac{1}{m_1m_2}x^2+\left(\frac{m_1+m_2}{m_1m_2}\right)xy=0$

$m_1+m_2 = -\frac{2h}{b}$

$m_1m_2 = \frac{a}{b}$

$m_1m_2y^2+x^2+(m_1+m_2)xy=0$

$\frac{a}{b}y^2+x^2+-\frac{2h}{b}xy=0 \Rightarrow bx^2+ay^2-2hxy=0$