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Algebra of matrices

Addition

$$A_{m\times n} + B_{m\times n} = \{a_{ij} + b_{ij}\}$$

If $A + B = O$

A, B mutually additive inverse to each other.

Scalar Multiplication

$$2A = \{2a_{ij}\}$$

$$KA = \{Ka_{ij}\}$$

$$A = \begin{bmatrix} a_{11} \\ a_{21} \end{bmatrix}$$

$$KA = \begin{bmatrix} Ka_{11} \\ Ka_{21} \end{bmatrix}$$

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$$|kA_{n\times n}| = k^n|A|$$

$$\begin{bmatrix} K & 0 & 0 \\ 0 & K & 0 \\ 0 & 0 & K \end{bmatrix} = K\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = KI.$$

Equality of two matrices

$A = B$ ✓

$$a_{ij} = b_{ij} \quad \forall i, j$$

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1: Let $A = \begin{bmatrix} 1 & 2 & 0 \\ -2 & 1 & 3 \\ 0 & -1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -2 & 2 \\ 2 & 3 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

find $2A + 3B - 5I$

$$= \begin{bmatrix} 0 & -2 & 6 \\ 2 & 6 & 9 \\ 0 & 1 & -1 \end{bmatrix}$$

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$\underline{2}$. Find the matrices X & Y if

$$\underset{\downarrow A}{2X - Y} = \begin{pmatrix} 6 & -6 & 0 \\ -4 & 2 & 1 \end{pmatrix} \text{ and } \underset{\downarrow B}{X + 2Y} = \begin{pmatrix} 3 & 2 & 5 \\ -2 & 1 & -7 \end{pmatrix}$$

$$X = \frac{1}{5}(2A + B) = \begin{bmatrix} 3 & -2 & 1 \\ -2 & 1 & -1 \end{bmatrix}$$

$$Y = \frac{1}{5}(2B - A) = \begin{pmatrix} 0 & 2 & 2 \\ 0 & 0 & -3 \end{pmatrix}$$

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3. A matrix has 12 elements. Find number of possible orders it can have.

$12 = 2^2 3$

1×12 12×1
2×6 6×2
3×4 4×3

$\boxed{3 \times 2}$

4. Solve the equation $\begin{bmatrix} x & 2y & 3z \end{bmatrix} - 2\begin{bmatrix} y & z & -x \end{bmatrix} + 3\begin{bmatrix} -z & x & y \end{bmatrix}$

$$\begin{bmatrix} x-2y-3z & 2y-2z+3x & 3z+2x+3y \end{bmatrix} = \begin{bmatrix} -12 & 1 & 17 \end{bmatrix}$$

$$x-2y-3z = -12$$
$$2y-2z+3x = 1$$
$$3z+2x+3y = 17$$

$$\boxed{(x,y,z) = (1,2,3)}$$

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Multiplication of Matrices

$$(AB)_{ij}$$

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ \vdots & & & & \\ a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mn} \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1p} \\ b_{21} & b_{22} & \cdots & b_{2p} \\ b_{31} & & & \\ \vdots & & & \\ b_{n1} & & & \end{bmatrix} = \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} + a_{13}b_{31} + \cdots a_{1n}b_{n1} \\ \\ \\ \end{bmatrix}$$

$A_{m\times n}\, B_{n\times p}$ no. of columns of A = no. of rows of B.

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$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ \vdots & \vdots & & \vdots \\ a_{i1} & a_{i2} & a_{i3} \cdots & a_{in} \\ \vdots & & & \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1j} & \cdots & b_{1p} \\ b_{21} & b_{22} & \cdots & b_{2j} & \cdots & b_{2p} \\ b_{31} & b_{32} & \cdots & b_{3j} & \cdots & b_{3p} \\ \vdots & & & \vdots & & \\ & & & b_{nj} & \cdots & b_{np} \\ b_{n1} & b_{n2} & \cdots & & & \end{bmatrix} = \begin{bmatrix} \cdot & \cdot & \cdot & \cdot \\ & & & \\ & a_{i1}b_{1j} + a_{i2}b_{2j} + \cdots + a_{in}b_{nj} & & \\ & & & \end{bmatrix}$$

$A_{m\times n}$, $B_{n\times p}$

$\boxed{A_{m\times n}}$

$$(AB)_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \cdots + a_{in}b_{nj}$$

$$\boxed{(AB)_{ij} = \sum_{r=1}^{n} a_{ir}b_{rj}}$$

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Properties

- $AB = BA$ not necessarily true

$AB \quad BA$

$A_{2\times 4} B_{4\times 3}$

$B_{4\times 3}, A_{2\times 4}$

AB defined

BA not defined.

$A_{2\times 4} B_{4\times 2} = (AB)_{2\times 2}$

$B_{4\times 2} A_{2\times 4} = (BA)_{4\times 4}$

$A_{2\times 2} B_{2\times 2} = (AB)_{2\times 2}$

$B_{2\times 2} A_{2\times 2} = (BA)_{2\times 2}$

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1) $AB = BA$, then A & B commute.

$AB = -BA$, then A & B anticommute.

- Distributive is true

$$A_{m\times n}(B+C)_{n\times p} = AB + AC \qquad A_{m\times n},\ B_{n\times p},\ C_{n\times p}$$

$$\left(A(B+C)\right)_{ij} = \sum_{r=1}^{n} A_{ir}(B+C)_{rj} = \sum_{r=1}^{n} A_{ir}(B_{rj}+C_{rj}) = \sum_{r=1}^{n} A_{ir}B_{rj} + \sum_{r=1}^{n} A_{ir}C_{rj}$$

$$(AB+AC)_{ij} = (AB)_{ij} + (AC)_{ij} =$$

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- <u>Associative is true</u>

$$A(BC) = (AB)C$$

$$A_{m\times n}(BC)_{n\times p}$$

$$A_{m\times n},\ B_{n\times q},\ C_{q\times p}$$

$$\big(A(BC)\big)_{ij} = \sum_{r=1}^{n} A_{ir}(BC)_{rj} = \sum_{r=1}^{n}\left(A_{ir}\left(\sum_{s=1}^{q} B_{rs}C_{sj}\right)\right)$$

$$= \sum_{r=1}^{n}\sum_{s=1}^{q} A_{ir}B_{rs}C_{sj} = \sum_{s=1}^{q}\sum_{r=1}^{n} A_{ir}B_{rs}C_{sj}, \quad \big((AB)C\big)_{ij}$$

$$2\sum_{i=1}^{n} i = \sum_{i=1}^{n} 2i$$

$$= \sum_{s=1}^{q}\left(C_{sj}\sum_{r=1}^{n} A_{ir}B_{rs}\right) = \sum_{s=1}^{q} C_{sj}(AB)_{is}$$

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Positive Integral powers of square matrix

$$A^2 = AA$$

$$A^3 = (AA)A = A^2A = AA^2$$

$$A^m A^n = A^{m+n} \rightarrow \underbrace{AA \cdots A}_{m+n \text{ times}} = \underbrace{(AA \cdots A)}_{m \text{ times}}\underbrace{(AA \cdots A)}_{n \text{ times}} = A^m A^n$$

$$(A^m)^n = A^{mn}$$

$$(A^m)^n = \underbrace{A^m A^m \cdots A^m}_{n \text{ times}} = \underbrace{AA \cdots A}_{mn \text{ times}} = \underbrace{(A \cdots A)}_{m \text{ times}}\underbrace{(A \cdots A)}_{m \text{ times}}\underbrace{(A \cdots A)}_{m \text{ times}} \cdots \underbrace{(A \cdots A)}_{m \text{ times}}$$

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Idempotent matrix

If $A^2 = A$, then A is idempotent matrix

Involutary matrix

$A^2 = I \Rightarrow A$ is involutary matrix

Nilpotent matrix

$A^p = O$, & $A^{p-1} \neq O$, then A is said to be nilpotent having index 'p'.

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Periodic matrix

$A^{p+1} = A$, then A is periodic with period 'p'.

$$A^{1+2p} = \boxed{A^{1+p}} A^{p} = A A^{p} = A^{p+1} = A.$$

$\vdots$

$n \in N$, $A^{1+np} = A$.