

Q find z if $4z^4 + 2z^3 + 2z^2 + z + 1 = 0$

$$(2z)^2 + 2z + 1 = 0 \quad \xrightarrow{\omega} \omega^2$$

$$z^2 + z + 1 = 0 \quad \xrightarrow{\omega} \omega^1$$

$$2z = \omega \quad \text{or} \quad 2z = \omega^2$$

$$z = \frac{\omega}{2} \quad \text{or} \quad z = \frac{\omega^2}{2}$$

find $|z|$

$$(z^4 + z^3 + z^2) + (z^2 + z + 1) = 0$$

$$z^2(z^2 + z + 1) + (z^2 + z + 1) = 0$$

$$(z^2 + z + 1)(z^2 + 1) = 0$$

$$z^2 + z + 1 = 0 \quad \text{or} \quad z^2 = -1$$

$$\downarrow$$

$$|\omega|, |\omega^2| \quad |z| = |\omega| = 1$$

$$|\omega^2| = 1 \quad |z| = |\omega| = 1$$

$$|z| = 1$$

$|z| = 1$

Q $x^2 + x + 1 = 0$ has Roots α & β find $\alpha^{19} + \beta^{19} = ?$

$$x^2 + x + 1 = 0 \quad \xrightarrow{\omega} \omega^1$$

$$\alpha = \omega, \beta = \omega^2$$

$$\alpha^{19} + \beta^{19} = \omega^{19} + (\omega^2)^{19}$$

$$= \omega^{19} + \omega^{38}$$

$$= (\omega^3)^6 \cdot \omega + (\omega^3)^{12} \cdot \omega^2$$

$$= \omega + \omega^2$$

$$= -1$$

Q If

$$\frac{(1+\omega - \omega^2)(1+\omega^2 - \omega^4)(1+\omega^4 - \omega^8) \dots 2n}{\prod} \text{ factors}$$

$$(-\omega^2 - \omega)(-\omega^2 - \omega^2)(-\omega - \omega)$$

$$(-2\omega^2)(-2\omega)(-2\omega^2)(-2\omega) \dots 2n \text{ factors}$$

$$(-2\omega)(-2\omega)(\dots n \text{ factors})$$

$$\times (-2\omega^2)(-2\omega^2) \dots n \text{ factors}$$

$$2^n \cdot \omega^n \times \omega^{2n} = 4^n (\omega^3)^n = 4^n$$

$$Q \frac{a+b\omega + \omega^2}{a\omega + b\omega^2 + 1} + \frac{a\omega + b\omega^2 + 1}{a+b\omega + \omega^2}$$

$$\frac{a+b\omega + \omega^2}{\omega(a+b\omega + \frac{1}{\omega})} + \frac{\omega(a+b\omega + \frac{1}{\omega})}{(a+b\omega + \omega^2)}$$

$$\frac{1}{\omega} + \omega = \omega \cdot \omega^2 = -1$$

$$-1 = 6\pi + i(8m)\pi$$

$$-1 = e^{i\pi}$$

$$(x-1)^3 + 3 = 0$$

$$\text{find Roots}$$

$$((-1)^{\frac{3}{2}} - 8)^{\frac{1}{3}}$$

$$(-1 - (-8))^{\frac{1}{3}}$$

$$= \left\{ \left| 8 \right|^{\frac{1}{3}} e^{i \frac{(\pi + 2k\pi)}{3}} \right\}_{K=0,1,2}$$

$$x-1 = 2 e^{i \frac{\pi}{3}} \cdot 2 \cdot e^{\frac{i\pi}{12}} \cdot e^{\frac{5\pi}{3}}$$

$$x-1 = 1 + 2 \cdot e^{\frac{i\pi}{3}} = \left| 1 + 2 \cdot e^{\frac{i\pi}{3}} \right| = 1 + 2 \cdot e^{\frac{i\pi}{3}} = -1$$

Q If $\alpha_1, \alpha_2, \dots, \alpha_{n-1}$ are n^{th} Roots of Unity

$$\textcircled{1} \quad n = (1-\alpha_1)(1-\alpha_2) \dots (1-\alpha_{n-1})$$

(other than 1) find

$$1) (1-\alpha_1)(1-\alpha_2) \dots (1-\alpha_{n-1})$$

$$2) (\omega-\alpha_1)(\omega-\alpha_2) \dots (\omega-\alpha_{n-1})$$

$$1, \alpha, \alpha^2, \alpha^3, \dots, \alpha^{n-1}$$

$$\chi = 1^n \rightarrow 1, \alpha_1, \alpha_2, \alpha_3, \dots, \alpha_{n-1}$$

$$\chi^n = 1 \Rightarrow \chi^n - 1 = 0$$

$$\chi^n - 1 = (\chi - 1)(\chi - \alpha_1)(\chi - \alpha_2) \dots (\chi - \alpha_{n-1})$$

$$\frac{\chi^n - 1}{\chi - 1} = (\chi - \alpha_1)(\chi - \alpha_2) \dots (\chi - \alpha_{n-1})$$

② Put $\chi = \omega$

$$1 + \omega + \omega^2 + \omega^3 + \dots + \omega^{n-1} = (\omega - \alpha_1)(\omega - \alpha_2) \dots (\omega - \alpha_{n-1})$$

In terms of HP

$$= \frac{\omega^n - 1}{\omega - 1} = \begin{cases} \frac{\omega - 1}{\omega - 1} = 0 & n = 3K \\ \frac{\omega^1 - 1}{\omega - 1} = 1 & n = 3K+1 \\ \frac{\omega^2 - 1}{\omega - 1} = \frac{\omega + 1}{-1} = -\omega^2 & n = 3K+2 \end{cases}$$

$$\omega^{3K+1} = \omega$$

$$\omega^{3K+2} = \omega^2$$

$$\therefore 1 + \omega + \omega^2 + \dots + \omega^{n-1} = (\chi - \alpha_1)(\chi - \alpha_2) \dots (\chi - \alpha_{n-1})$$

Put $\chi = 1$

$$\text{Q) } f(z) = \left(\frac{2\pi}{2n+1} + i \operatorname{Im} \frac{2\pi}{2n+1} \right)^{2n+1} \quad n = \text{+ve int}$$

find Eq whose roots are

$$\alpha = z + z^3 + \dots + z^{2n-1} \quad \beta = z^2 + z^4 + \dots + z^{2n}$$

← n terms up →

DMT
use
~~at k=0~~
already

$$1) \quad z = \left(\left(82\pi + i 8m\pi \right)^{\frac{1}{2n+1}} \right)^{2n+1} \text{ DMT}$$

$$\Rightarrow z^{2n+1} = 1 \Rightarrow z^{2n} = \frac{1}{z}$$

$$(2) \quad \alpha = \frac{z(z^2 - 1)}{(z^2 - 1)} = \frac{z(\frac{1}{z} - 1)}{(\frac{1}{z^2} - 1)} : \frac{z(z-1)}{z(z-1)}$$

$$\beta = \frac{z^2((z^2)^n - 1)}{(z^2 - 1)} : \frac{z^2(\frac{1}{z} - 1)}{(\frac{1}{z^2} - 1)} = -\frac{z}{z+1}$$

$$(3) \quad \alpha + \beta = -\frac{1}{z+1} - \frac{z}{z+1} = -\frac{(z+1)}{(z+1)} = -1$$

Answer → Eq in z with

$$82\theta_2 - \frac{1}{z}$$

200 Qs

100
150

+
+

Qs
Bonus

-
-

z+1

$$200 Qs = \frac{z}{(z+1)^2}$$

$$= \frac{z}{z^2 + 2z + 1}$$

$$= \frac{1}{z + \frac{1}{z} + 2}$$

$$z = 6\theta + i 8m\theta$$

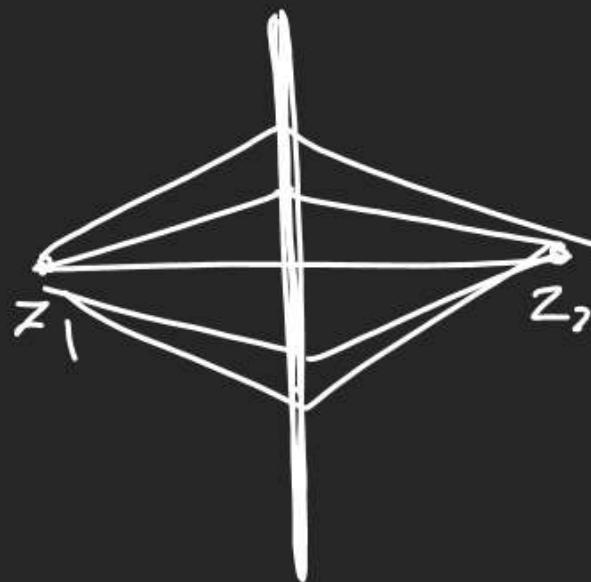
$$\frac{1}{z} = 6\theta - i 8m\theta$$

$$z + \frac{1}{z} = 12\theta$$

$$\alpha \cdot \beta = \frac{1}{2(1+6\theta)} = \frac{1}{482\theta_2}$$

$$z^2 + z + \frac{1}{482\theta_2} = 0$$

$$\textcircled{1} |z - z_1| = |z - z_2|$$



$$PA - PB < AB$$

$$\text{hyperbola } |z - z_1| - |z - z_2| < |z - z_2|$$

$$|z - z_1|^2 + |z - z_2|^2 = |z_1 - z_2|^2 \rightarrow \text{Circle} \rightarrow \text{Pythagorean}$$

$$\textcircled{2} |z + 5| + |z - 5| = 10 \text{ Rep?}$$

$P \quad A \quad PA + PB = AB$

$A \leftarrow 10 \rightarrow B$

$(-5, 0) \quad (5, 0)$

$\xrightarrow{\text{STL}}$

$$|z + 5| + |z - 5| = 8$$

$$PA + PB = 10 \quad \& \quad AB = 8$$

$$PA + PB > AB \rightarrow \text{Ellipse}$$

$$\textcircled{3} |3z - 2| + |3z + 2| = 4 \text{ Now,}$$

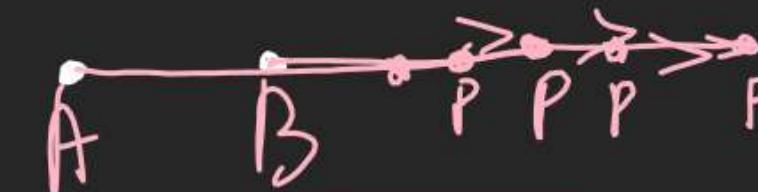
$$\textcircled{4} PA + PB = AB$$



St. L (AB)

$$\text{STL}_{\text{outside}} \Rightarrow |z - z_1| + |z - z_2| = |z_1 - z_2|$$

$$\textcircled{5} PA - PB = AB$$



$$\text{STL} \Rightarrow |z - z_2| - |z - z_1| = |z_1 - z_2|$$

$$\textcircled{6} PB - PA = AB$$



$$\textcircled{7} PA + PB > AB$$

No [Down]



$$\text{Ellipse } |z - z_1| + |z - z_2| > |z_1 - z_2|$$