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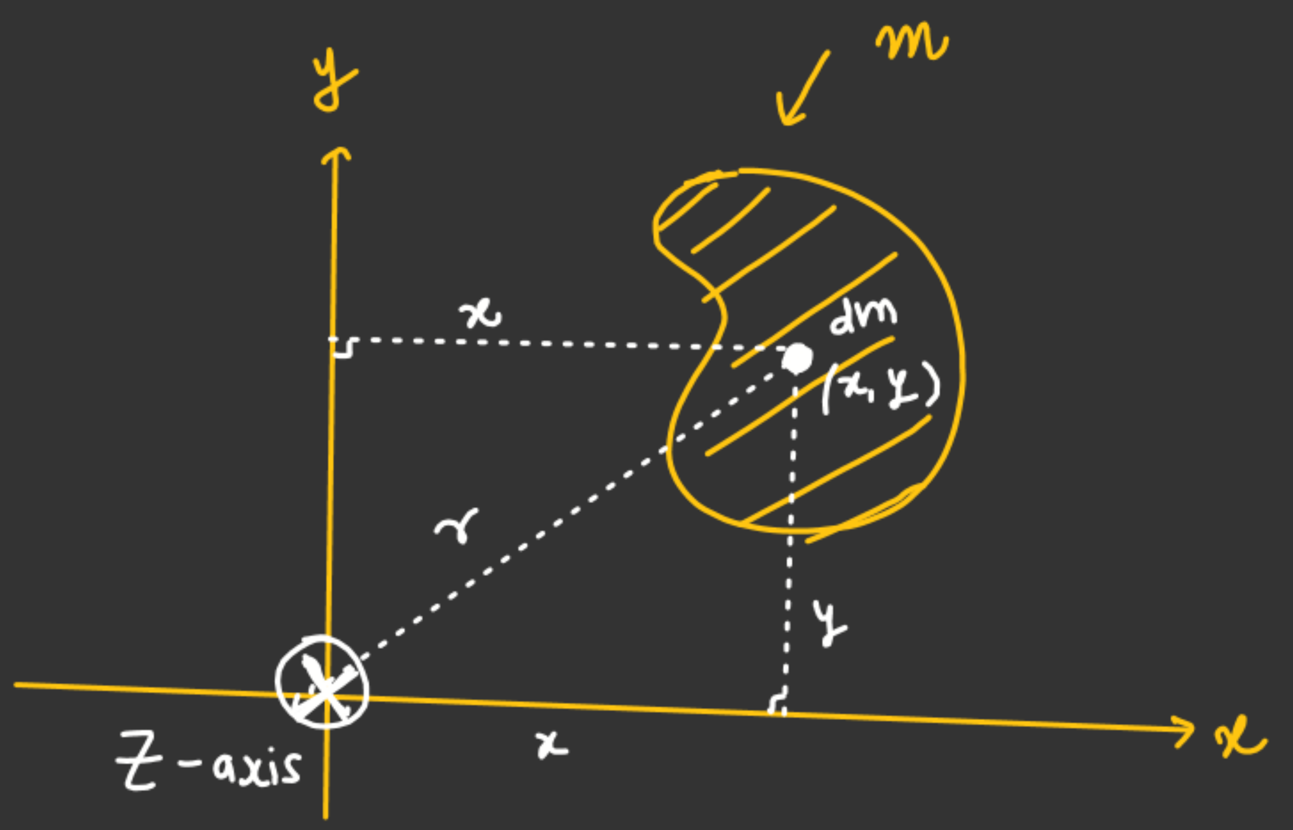
Theorems on M.I

Perpendicular axis theorem

↳ (Valid for 2-Dimensional body)



dI = M.I of dm mass about z-axis.

$$dI = dm\, r^2$$

$$dI = dm\,(x^2+y^2) \Rightarrow \boxed{I_z = I_x + I_y} \quad \left[r = \sqrt{x^2+y^2}\right]$$

↳ For planar body.

$$\int dI = \int dm\, x^2 + \int dm\, y^2 \Rightarrow I_z + I_y = I_x$$

$$(I_{body})_z = (I_{body})_x + (I_{body})_y \Rightarrow I_z + I_x = I_y$$

(*) All three mutual ⊥ axis must pass through common point.

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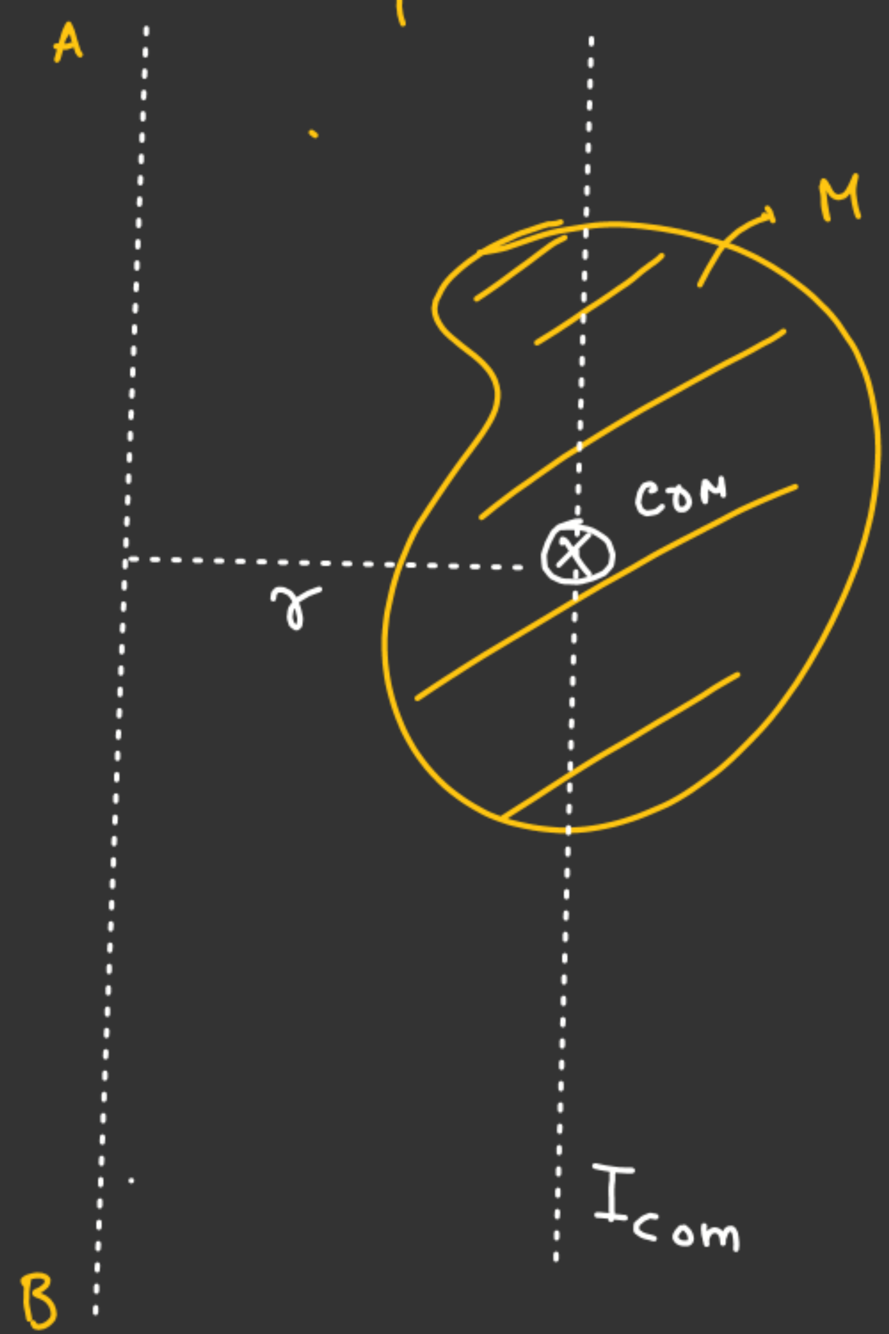
Parallel axis theorem

(Valid for 1-D, 2-D, 3-D)

$$\boxed{I_{AB} = I_{com} + Mr^2}$$



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Application

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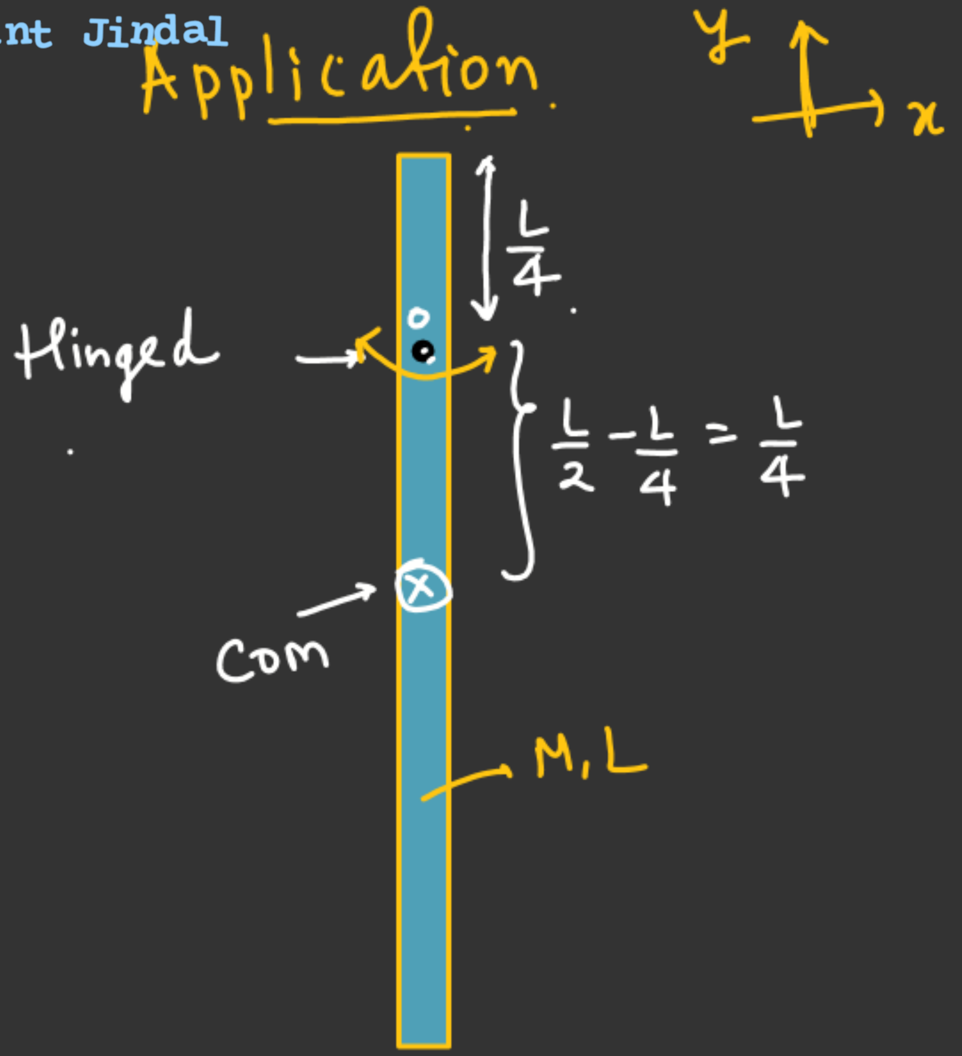
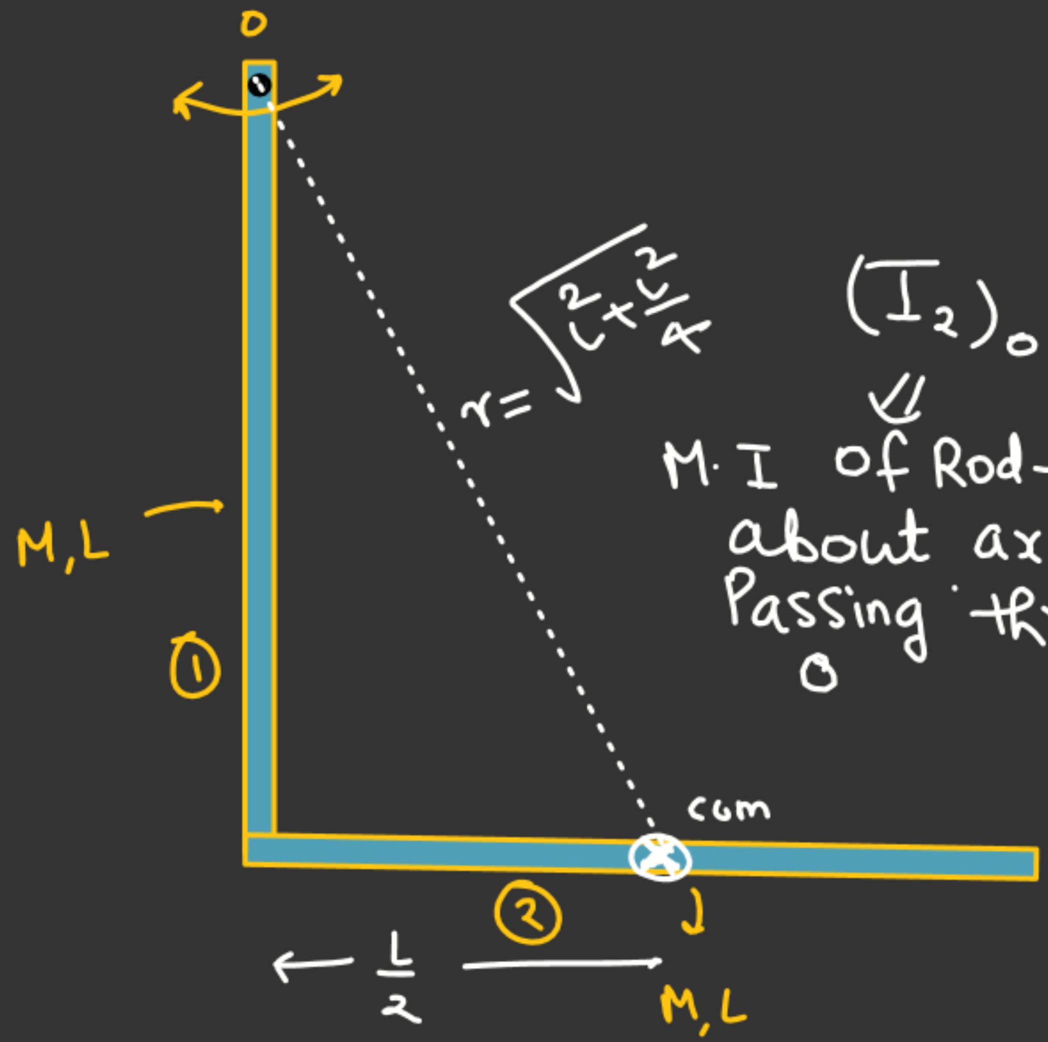


$$I_o = I_{com} + M\left(\frac{L}{4}\right)^2$$

$$= \left(\frac{ML^2}{12} + \frac{ML^2}{16}\right)$$

$$= \left(\frac{7ML^2}{48}\right)$$



$$(I_1)_o = \frac{ML^2}{3}$$

$$(I_2)_o = (I_{com}) + M\left(\sqrt{\frac{5L^2}{4}}\right)^2$$

M.I of Rod-2 about axis Passing through O

$$= \frac{ML^2}{12} + \frac{5ML^2}{4}$$

$$= \frac{ML^2 + 15ML^2}{12}$$

$$= \frac{16ML^2}{12}$$

$$= \frac{4ML^2}{3} \text{ Ans}$$

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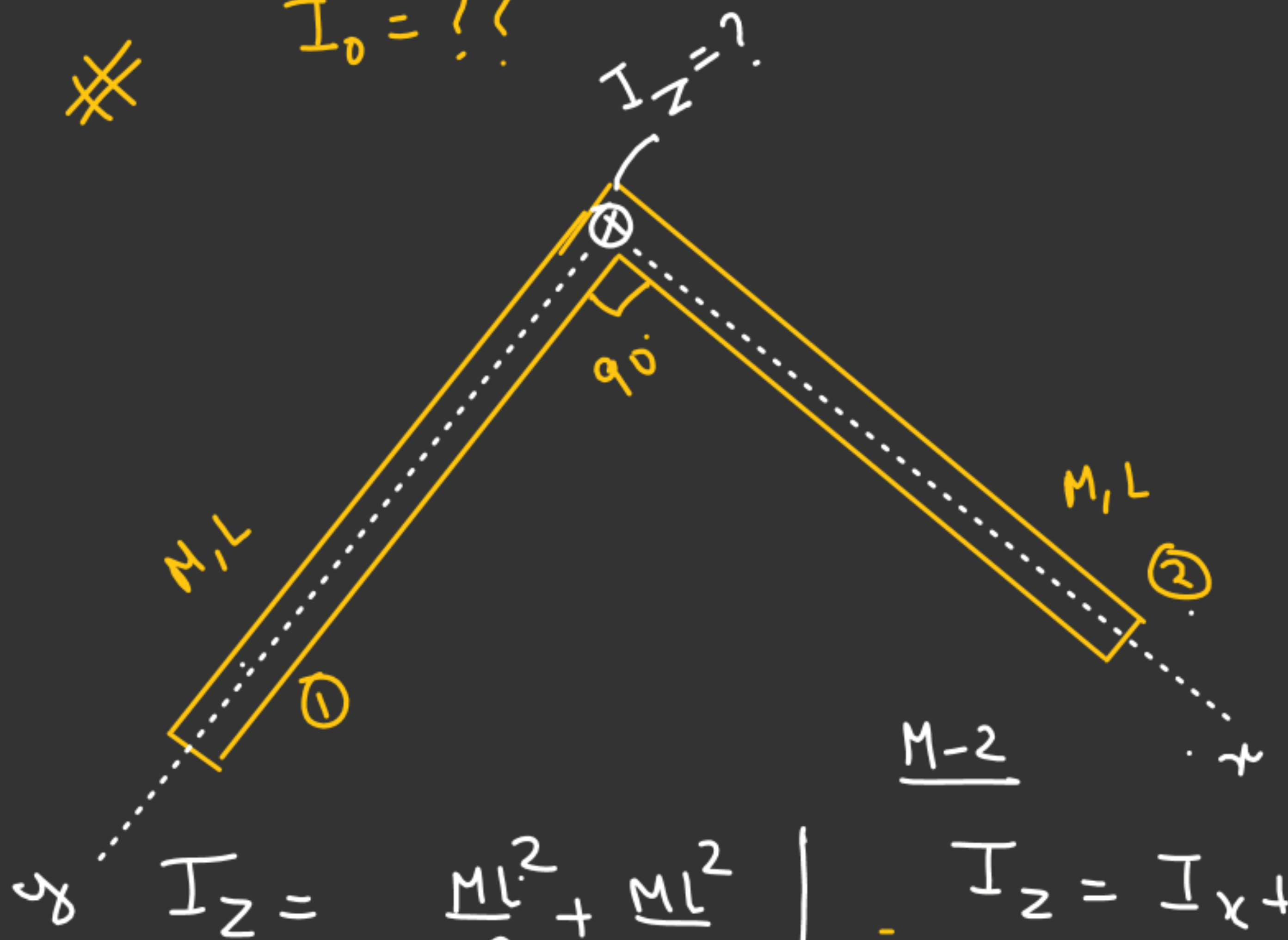
$I_0 = ??$



$$I_z = \frac{ML^2}{3} + \frac{ML^2}{3}$$

$$= \left(\frac{2ML^2}{3}\right)$$

<u>M-2</u>

$$I_z = I_x + I_y$$

$$(I_x)_{Rod-2} = 0$$

$$(I_x)_{Rod-1} = \frac{ML^2}{3}$$

$$(I_y)_{Rod-1} = 0$$

$$(I_y)_{Rod-2} = \frac{ML^2}{3}$$

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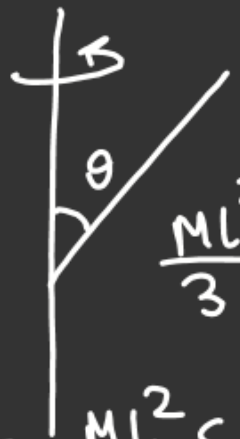
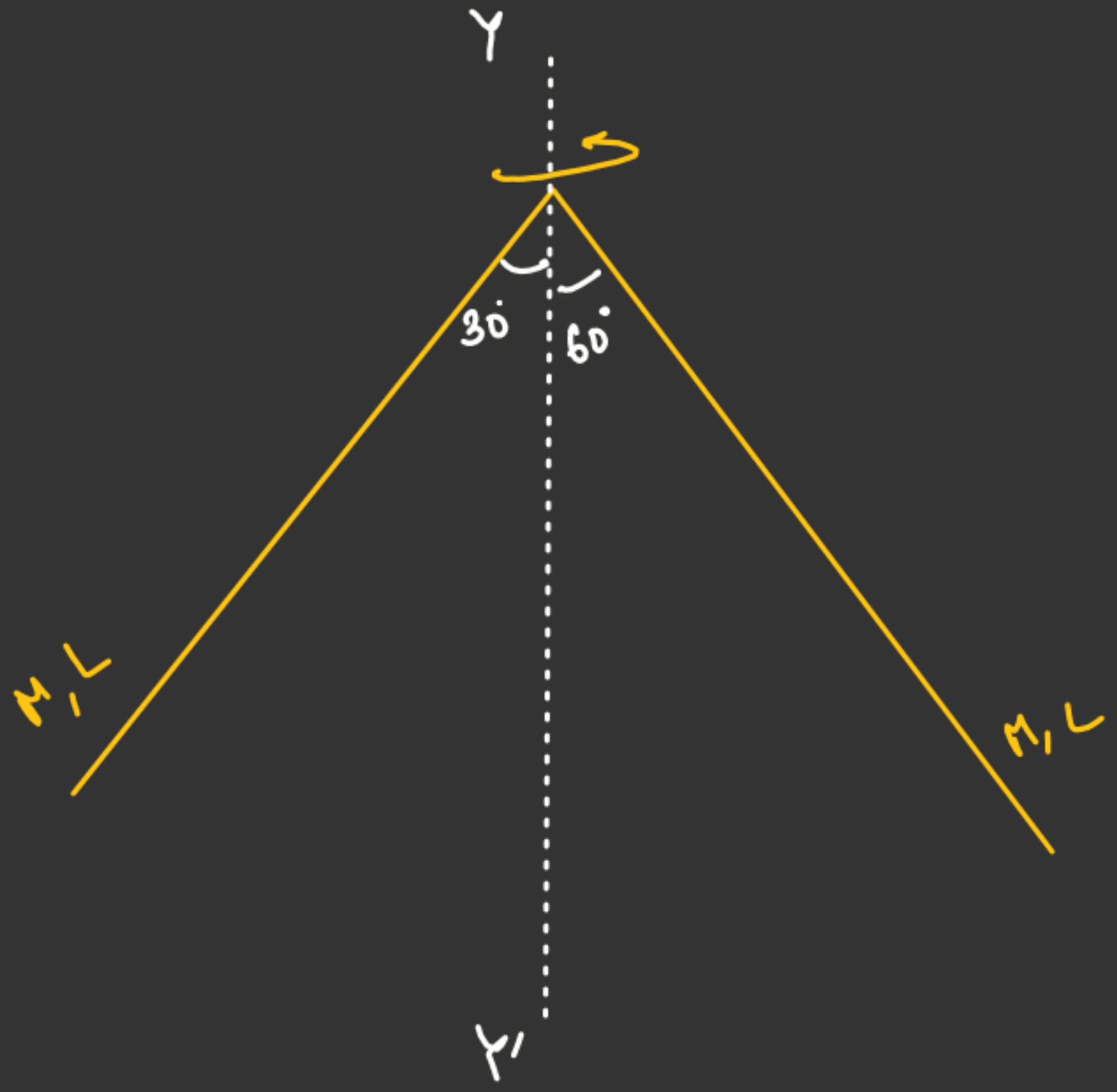
$I_{YY'} = ??$



$\frac{ML^2}{3}\sin^2\theta$



$$I_{YY'} = \frac{ML^2}{3}\sin^2 30^\circ + \frac{ML^2}{3}\sin^2 60^\circ$$

$$= \left(\frac{ML^2}{3} \times \frac{1}{4}\right) + \frac{ML^2}{3} \times \left(\frac{\sqrt{3}}{2}\right)^2$$

$$= \frac{ML^2}{12} + \frac{3ML^2}{12}$$

$$= \left(\frac{ML^2}{3}\right)$$

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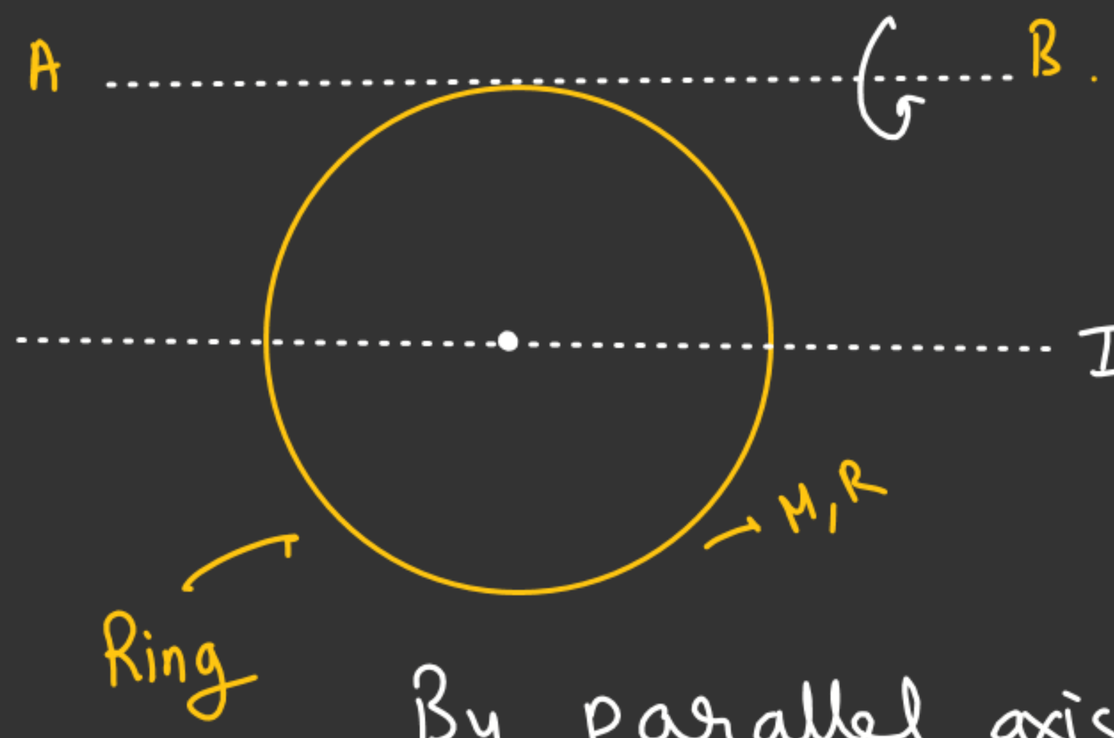
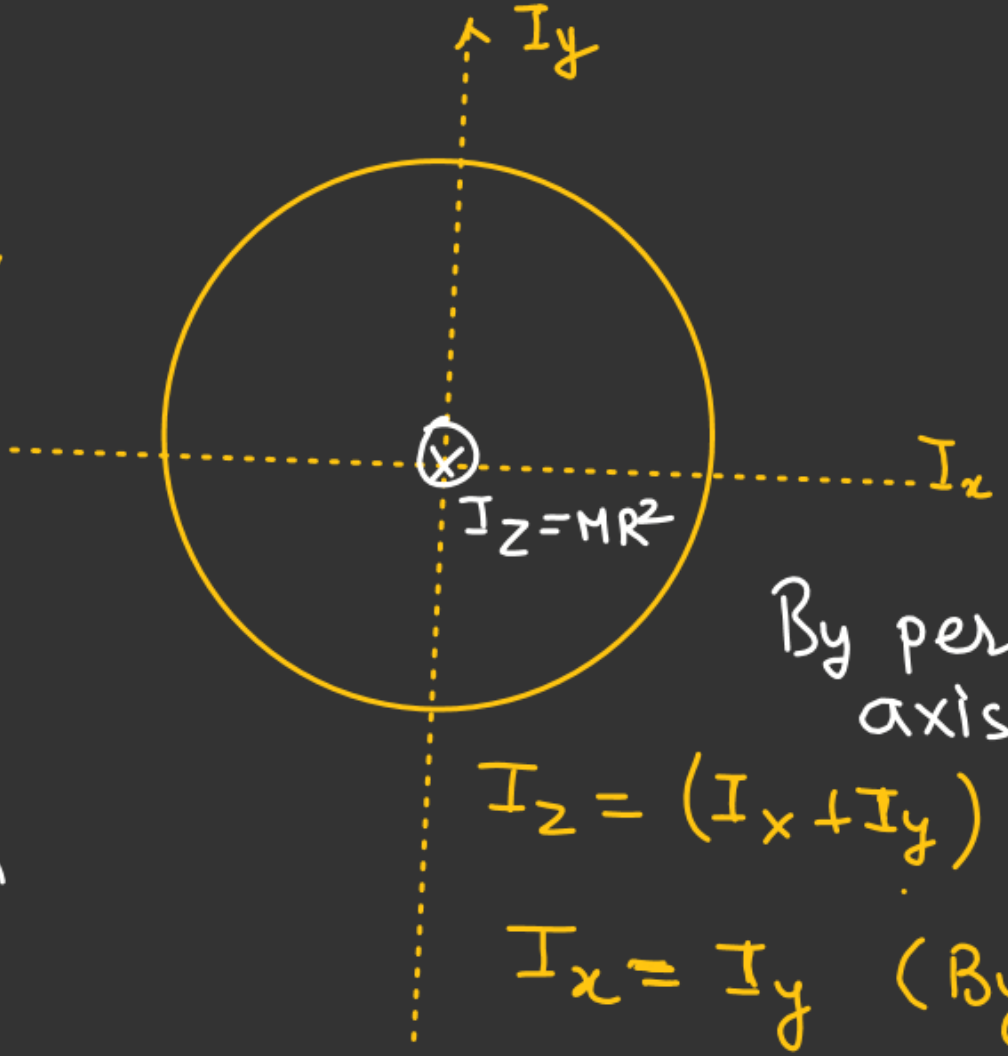
$I_{AB} = ??$



$I_{com} = \frac{MR^2}{2}$

By parallel axis theorem

$$I_{AB} = I_{com} + MR^2$$
$$= \frac{MR^2}{2} + MR^2$$
$$= \frac{3}{2} MR^2$$



By perpendicular axis theorem

$$I_z = (I_x + I_y)$$
$$I_x = I_y \quad \text{(By Symmetry)}$$
$$I_z = I_x + I_x$$
$$2I_x = I_z$$
$$I_x = \frac{I_z}{2} = \frac{MR^2}{2}.$$

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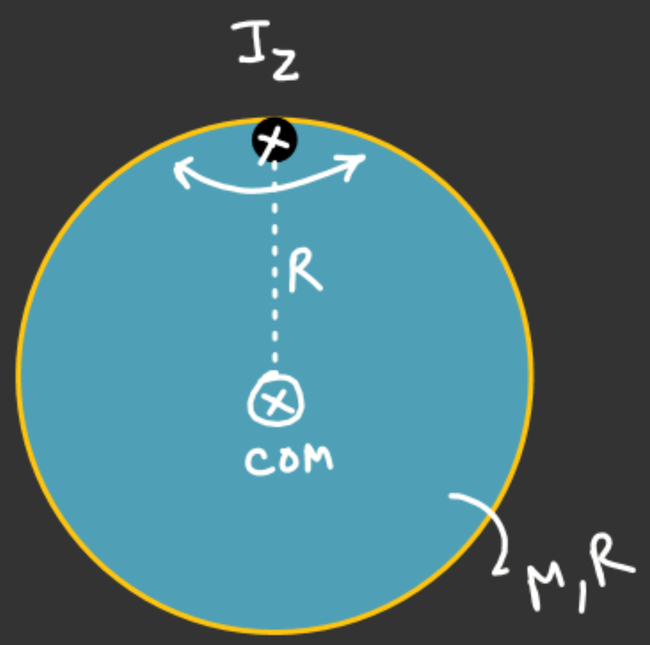
DISC



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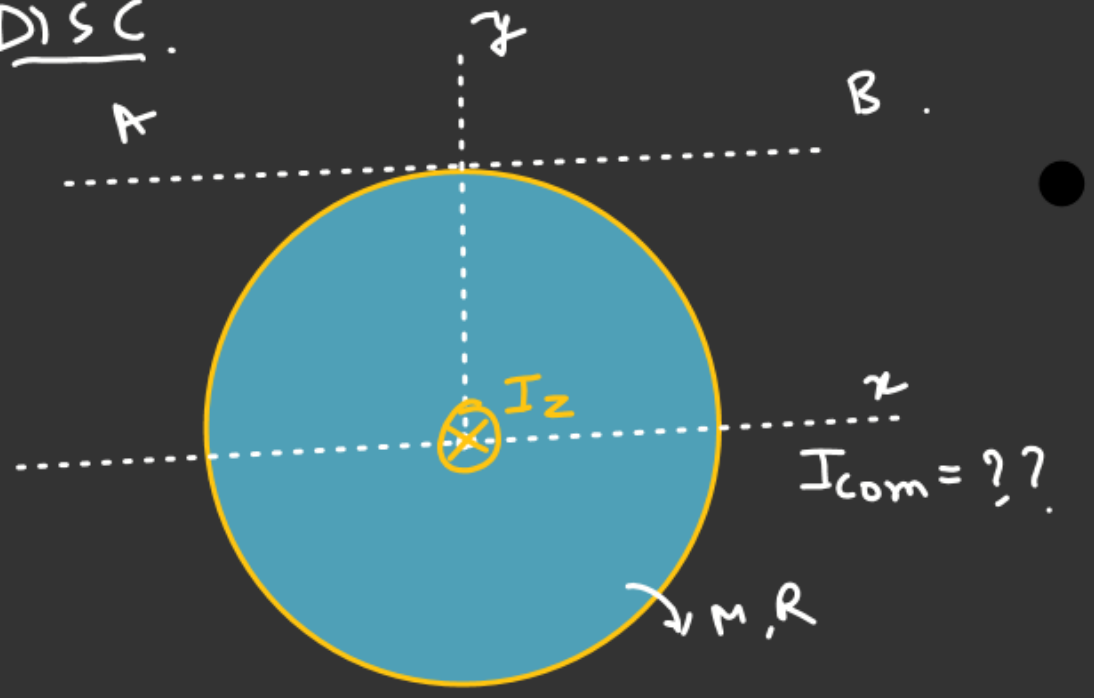
Parallel axis theorem

$$I_z = I_{com} + MR^2$$

$$= \frac{MR^2}{2} + MR^2$$

$$= \frac{3}{2}MR^2$$

DISC.



$$I_y + I_x = I_z$$

$$I_x = I_y$$

$$2I_x = I_z$$

$$I_x = \frac{I_z}{2} = \frac{MR^2}{2} \times \frac{1}{2} = \left(\frac{MR^2}{4}\right)$$

$$I_{AB} = I_x + MR^2$$

$$= \frac{MR^2}{4} + MR^2$$

$$= \frac{5}{4}MR^2$$

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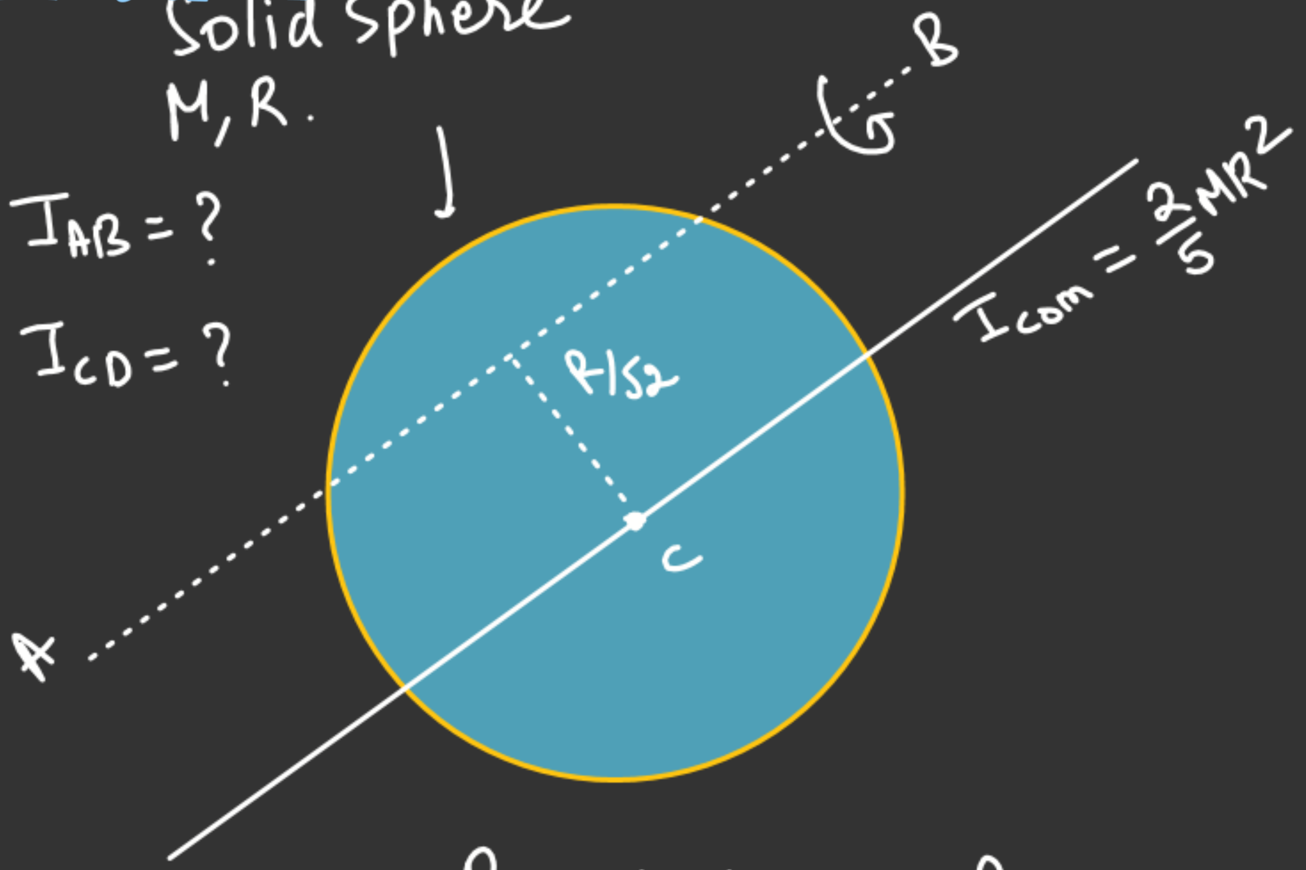
Solid Sphere
M, R.

$I_{AB} = ?$

$I_{CD} = ?$

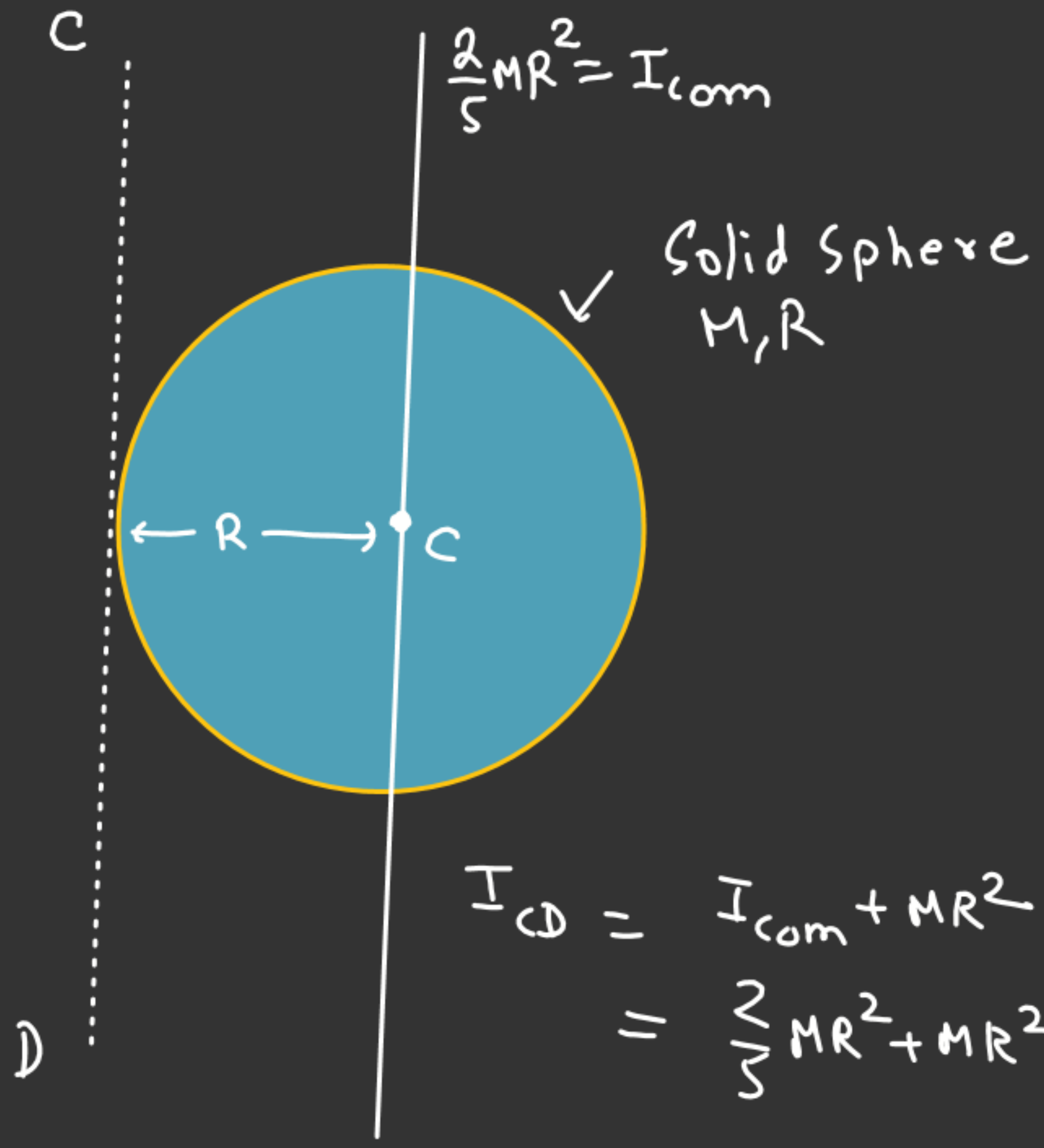


Parallel axis theorem

$$I_{AB} = \frac{2}{5}MR^2 + M\left(R/\sqrt{2}\right)^2$$

$$= \frac{2}{5}MR^2 + \frac{MR^2}{2}$$

$$= \frac{4MR^2 + 5MR^2}{10} = \left(\frac{9MR^2}{10}\right)$$



$$I_{CD} = I_{com} + MR^2$$

$$= \frac{2}{5}MR^2 + MR^2$$

$$= \frac{7}{5}MR^2$$

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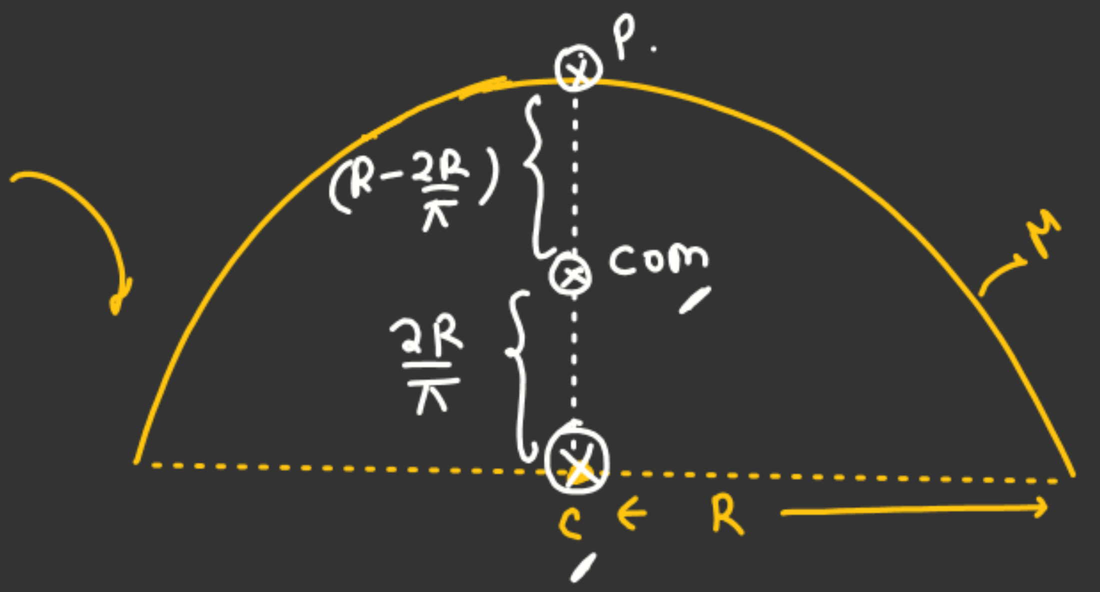
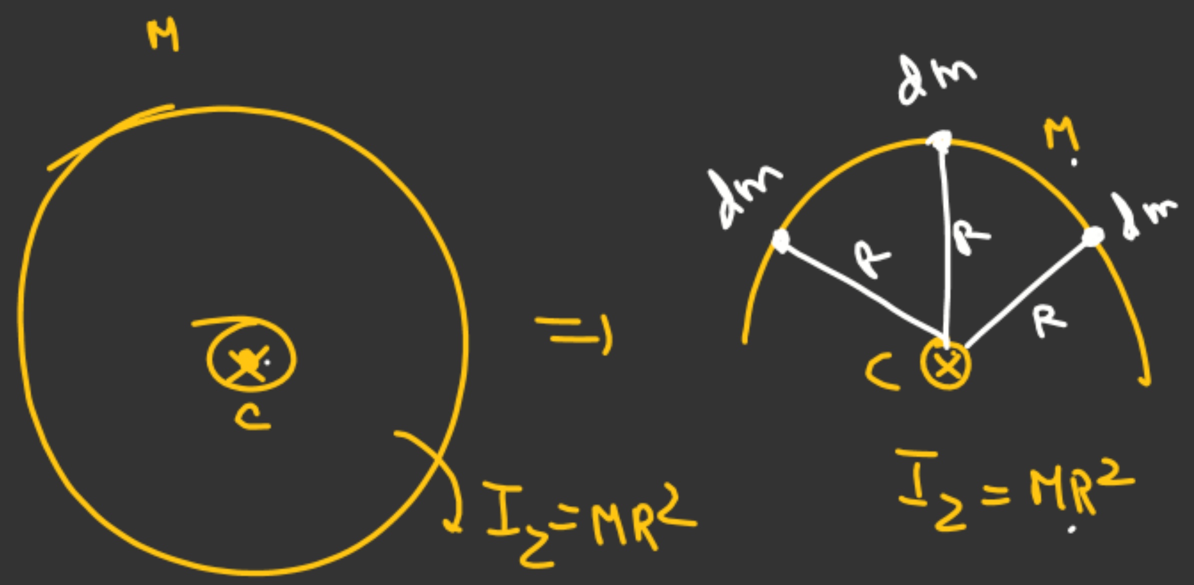
⋕ Semi Circular wire. hinged about P.





$$I_c = MR^2$$

$$I_c = I_{com} + M\left(\frac{2R}{\pi}\right)^2$$

$$I_{com} = MR^2 - M\left(\frac{2R}{\pi}\right)^2$$

$$I_p = I_{com} + m\left(R - \frac{2R}{\pi}\right)^2$$

$$I_p = MR^2 - M\left(\frac{2R}{\pi}\right)^2 + M\left(R^2 + \left(\frac{2R}{\pi}\right)^2 - \frac{4R^2}{\pi}\right)$$

$$I_p = \left(2MR^2 - \frac{4MR^2}{\pi}\right)$$

$$I_p = 2MR^2\left(1 - \frac{2}{\pi}\right) \text{ Ans}$$

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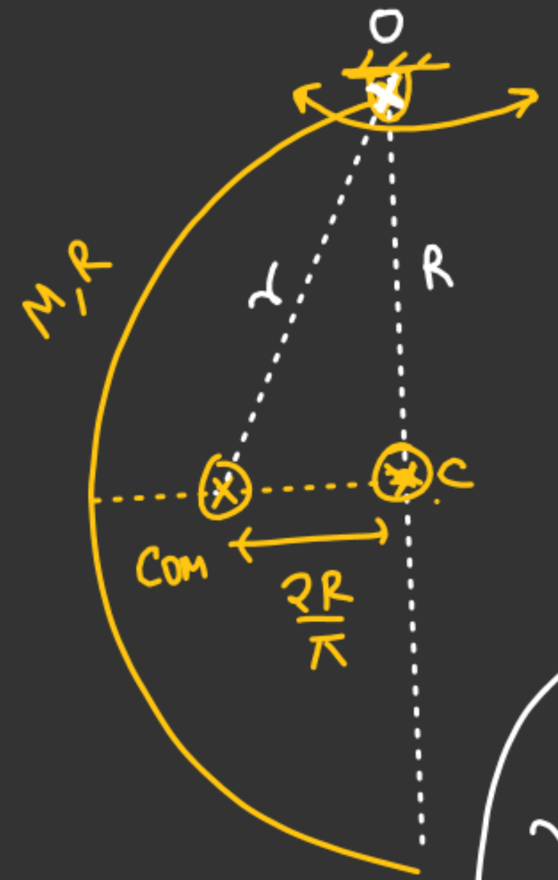

$I_c = MR^2$.

$I_{COM} + M\left(\frac{2R}{\pi}\right)^2 = MR^2$

$I_{COM} = MR^2 - M\left(\frac{2R}{\pi}\right)^2$

$I_o = I_{com} + mr^2$

$I_o = MR^2 - M\left(\frac{2R}{\pi}\right)^2 + m\left[R^2 + \left(\frac{2R}{\pi}\right)^2\right]$

$r = \sqrt{R^2 + \left(\frac{2R}{\pi}\right)^2}$

$I_o = MR^2 - \cancel{M\left(\frac{2R}{\pi}\right)^2} + mR^2 + \cancel{m\left(\frac{2R}{\pi}\right)^2}$

$I_o = 2MR^2$

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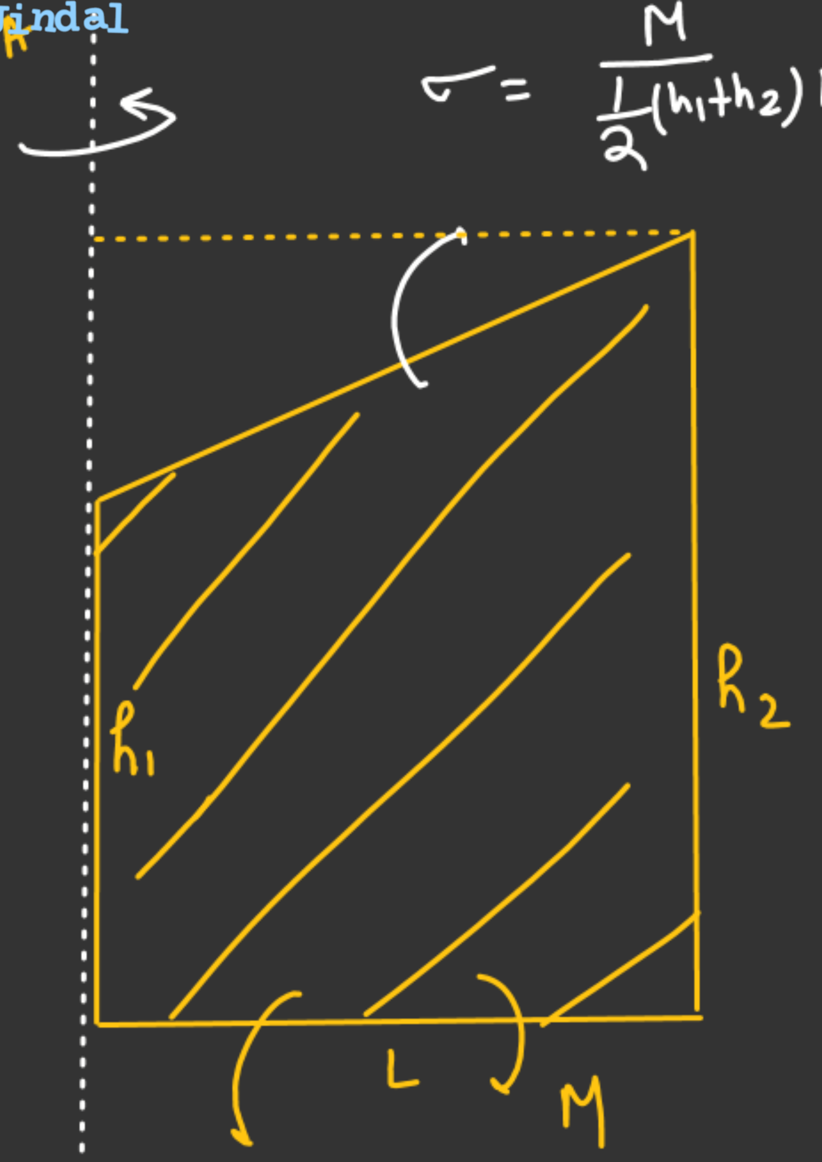
$$\sigma = \frac{M}{\frac{1}{2}(h_1+h_2)L} = \frac{2M}{(h_1+h_2)L}$$



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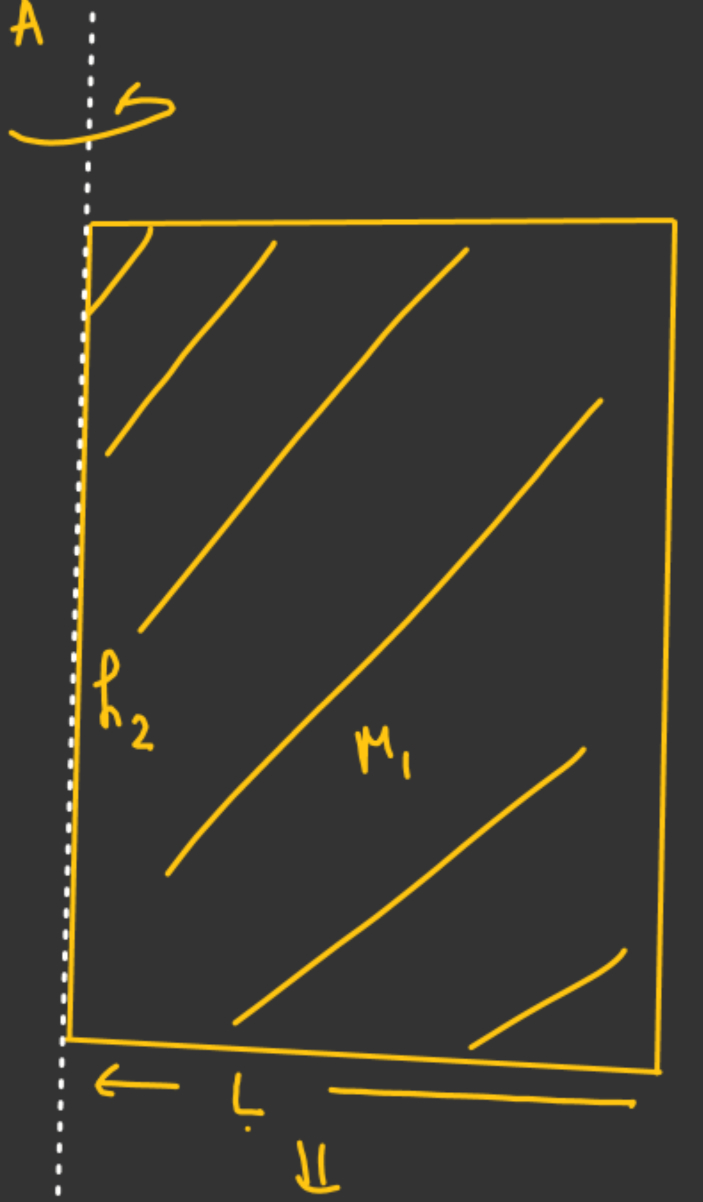

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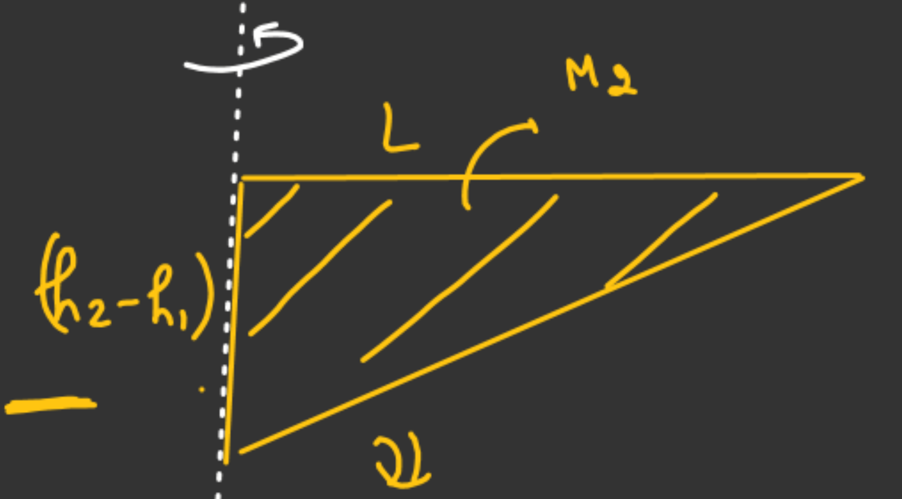

Lamina

$$(I_{AB}) = (I_1 - I_2)$$

$$I_1 = \frac{M_1 L^2}{3} = \frac{2}{3}\left(\frac{ML^2 h_2}{h_1+h_2}\right)$$

$$M_1 = \sigma(L h_2)$$

$$= \frac{2M}{(h_1+h_2)L} \times L h_2 = \left(\frac{2Mh_2}{h_1+h_2}\right)$$

$$I_2 = \frac{M_2 L^2}{6}$$

$$M_2 = \sigma \cdot \frac{1}{2} \times (h_2-h_1) L$$

$$= \frac{\cancel{2}M}{(h_1+h_2)\cancel{L}} \times \frac{1}{\cancel{2}}(h_2-h_1)\cancel{L}$$

$$= \frac{M(h_2-h_1)}{(h_1+h_2)}$$

$$I_2 = \left[\frac{ML^2(h_2-h_1)}{h_1+h_2}\right]$$

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Q-9. to Q-16