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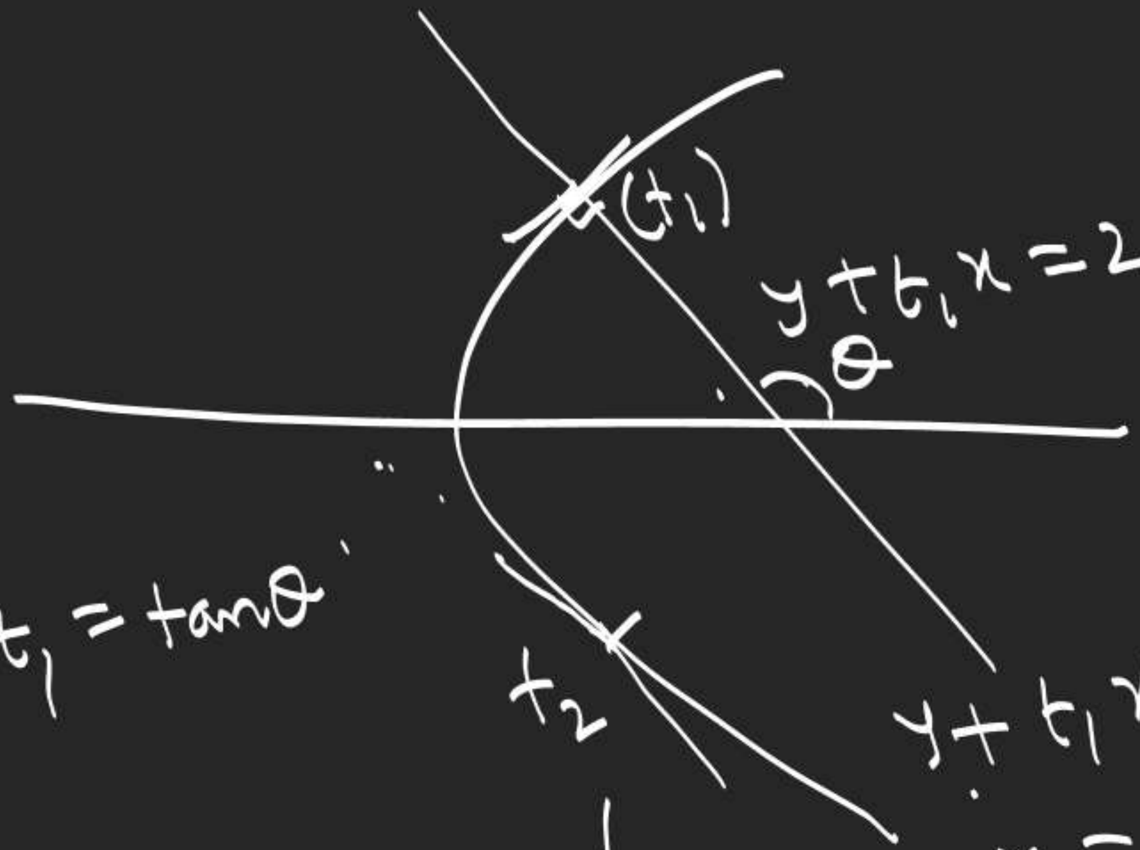


$$y + t_1 x = 2at_1 + at_1^3$$

$$y + t_1 x = -\frac{a}{t_1}$$

$$t_2 y - x = at_2^2$$

$$\frac{1}{t_2} = -t_1$$

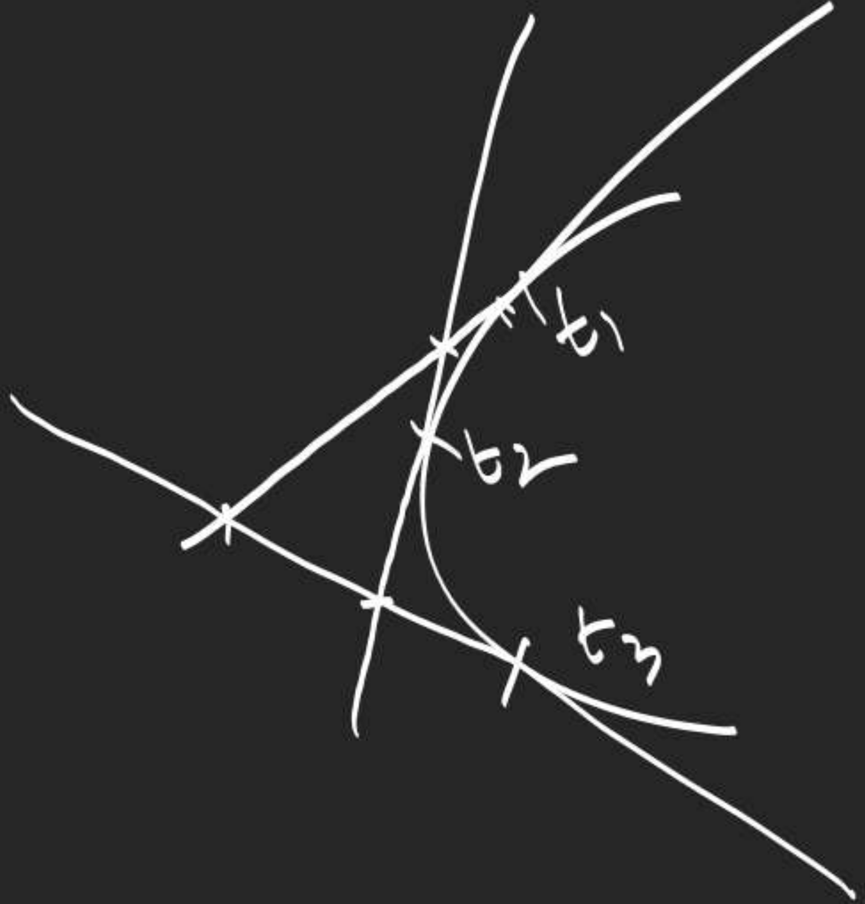
$$-\frac{y}{t_1} - x = \frac{a}{t_1^2}$$

$$-t_1 = \tan\theta$$

$$\frac{a\left(1+t_1^2\right)^{3/2}}{t_1}$$

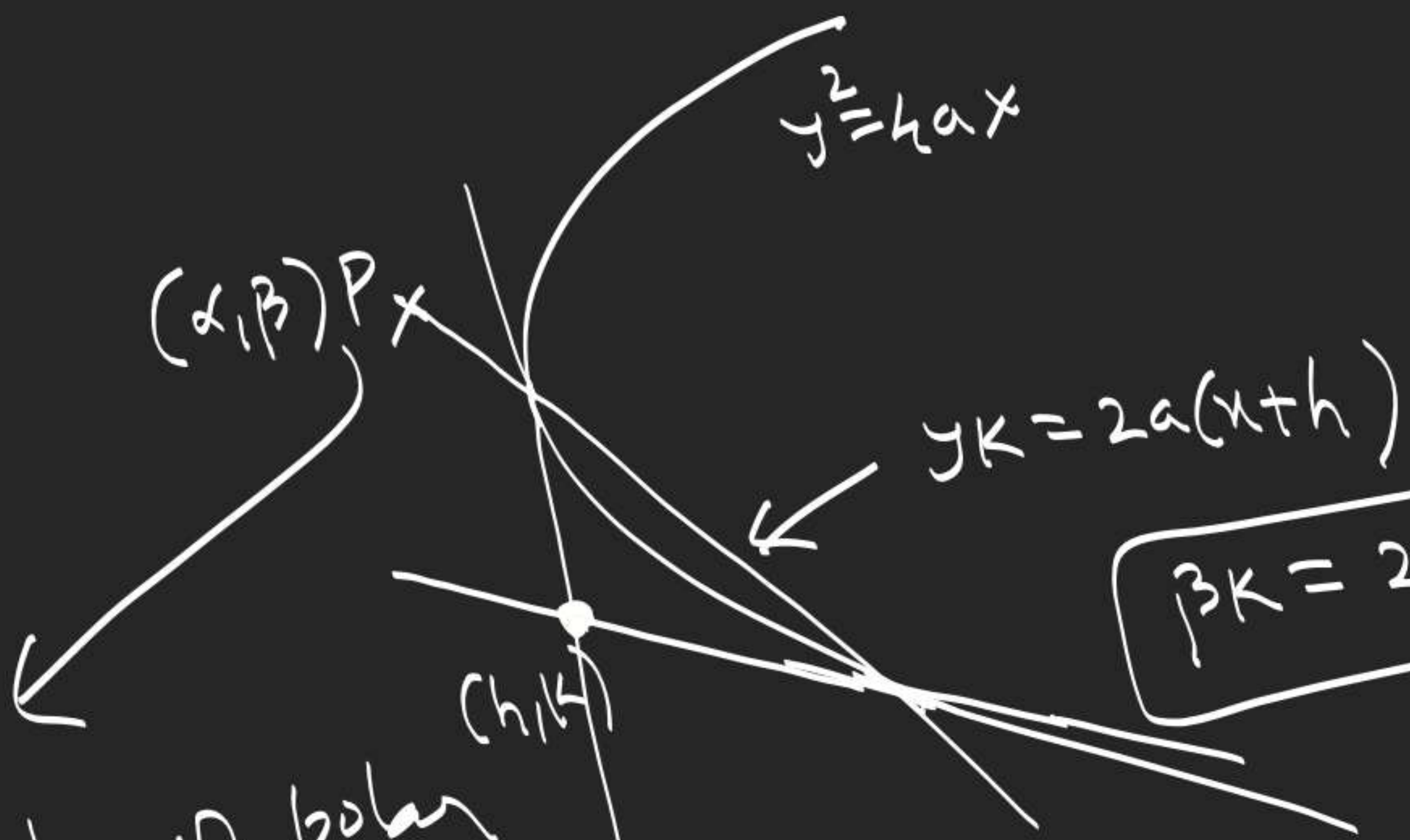
$$\left|\frac{at_1^4 + 2at_1^2 + a}{t_1\sqrt{1+t_1^2}}\right| = \frac{\left|2at_1 + at_1^3 + \frac{a}{t_1}\right|}{\sqrt{1+t_1^2}}$$

$$\left|\frac{a\sec^3\theta}{\tan\theta}\right|$$



$$\frac{a^2}{2}\left|\begin{matrix} t_1t_2 & t_1+t_2 & 1 \\ t_2t_3 & t_2+t_3 & 1 \\ t_3t_1 & t_3+t_1 & 1 \end{matrix}\right|$$

$$a^2\left|(t_1-t_2)(t_2-t_3)(t_3-t_1)\right|$$



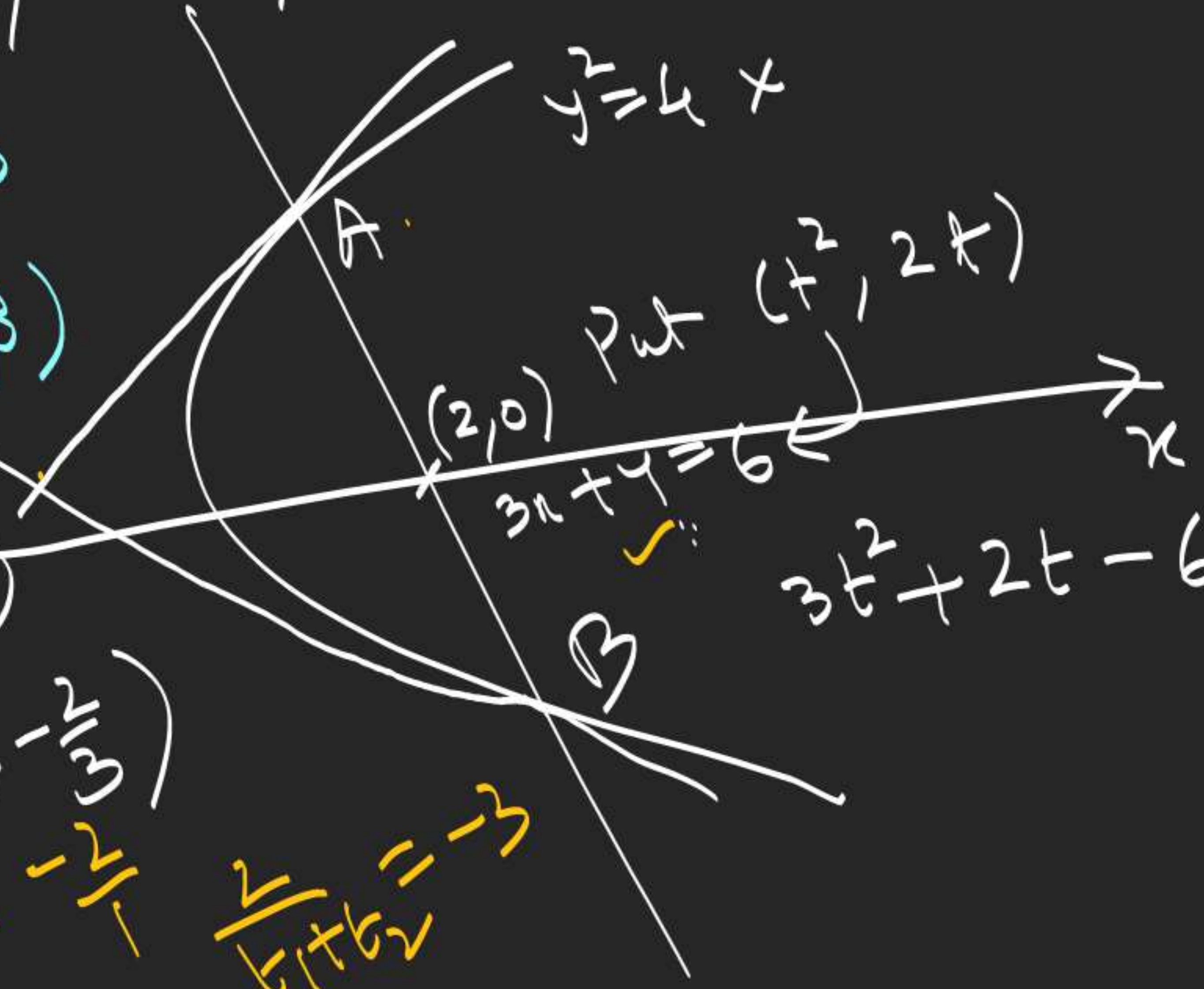
$$\boxed{\beta k = 2a(\alpha+h)}$$

Pole of polar w.r.t. given curve.

$\beta y = 2a(x+\alpha)$.

$\boxed{T=0} \rightarrow$ Polar

1. Line $3x+y=6$ intersects the parabola $y^2=4x$ at A & B. Find the coordinates of point of intersection of tangents drawn at A & B.



$$3t^2+2t-6=0 \quad \begin{cases} t_1 \\ t_2 \end{cases}$$

$$y\beta - 2(x+\alpha) = 0$$

$$y\beta - 2x = 2\alpha \quad (\alpha, \beta)$$

$$\frac{y}{1} = \frac{-2}{3} = \frac{2\alpha}{6}$$

$$(t_1t_2,\ t_1+t_2)$$

$$= \left(-2, -\frac{2}{3}\right)$$

$$t_1t_2 = \frac{-2}{1} \qquad \frac{2}{t_1+t_2} = -3$$

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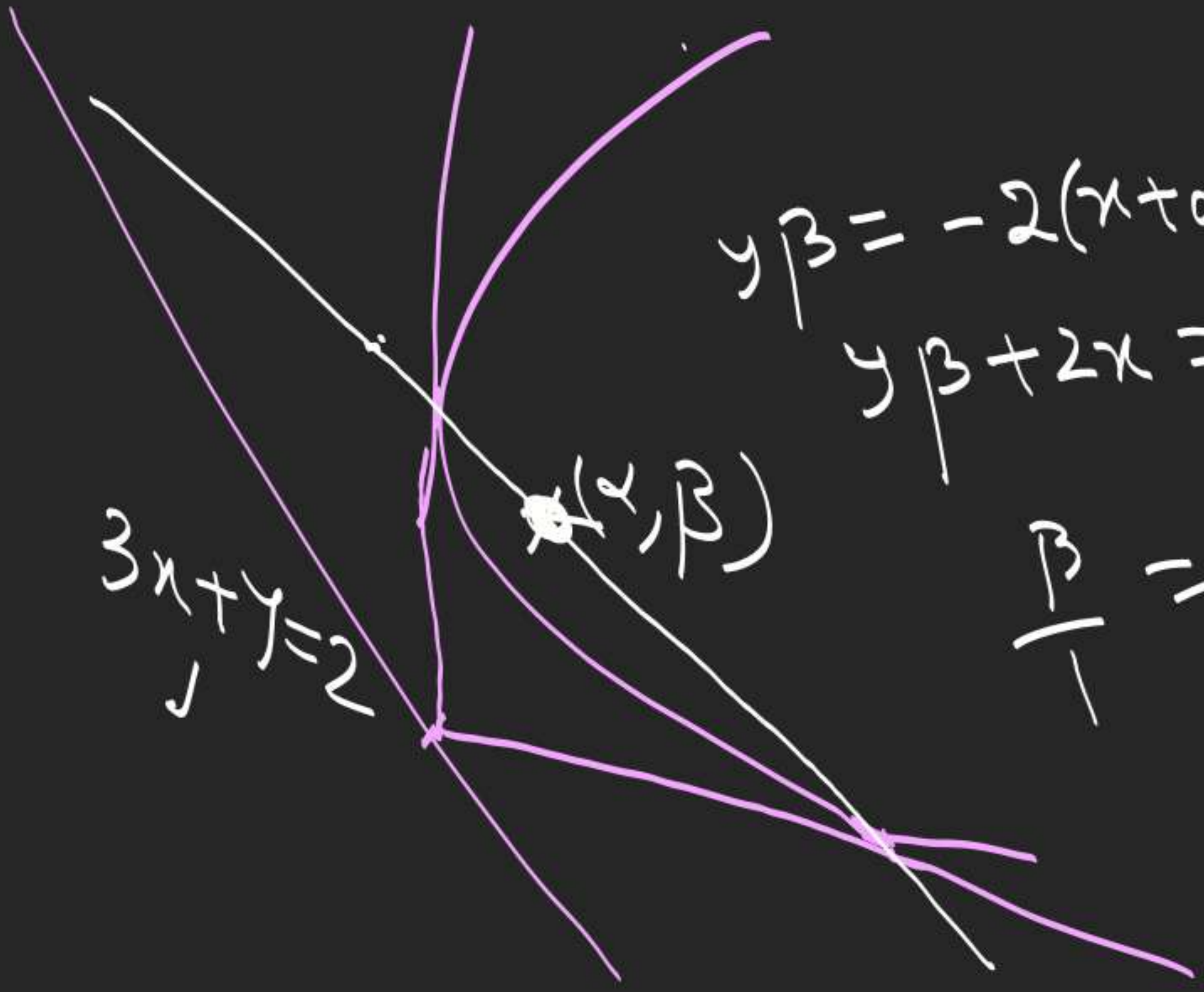
Q: Pair of tangents are drawn to parabola $y^2=-4x$ from every point on the line $3x+y=2$. P.T. their chord of contact passes thru a fixed point.



$y\beta=-2(x+\alpha)$

$y\beta+2x=-2\alpha$

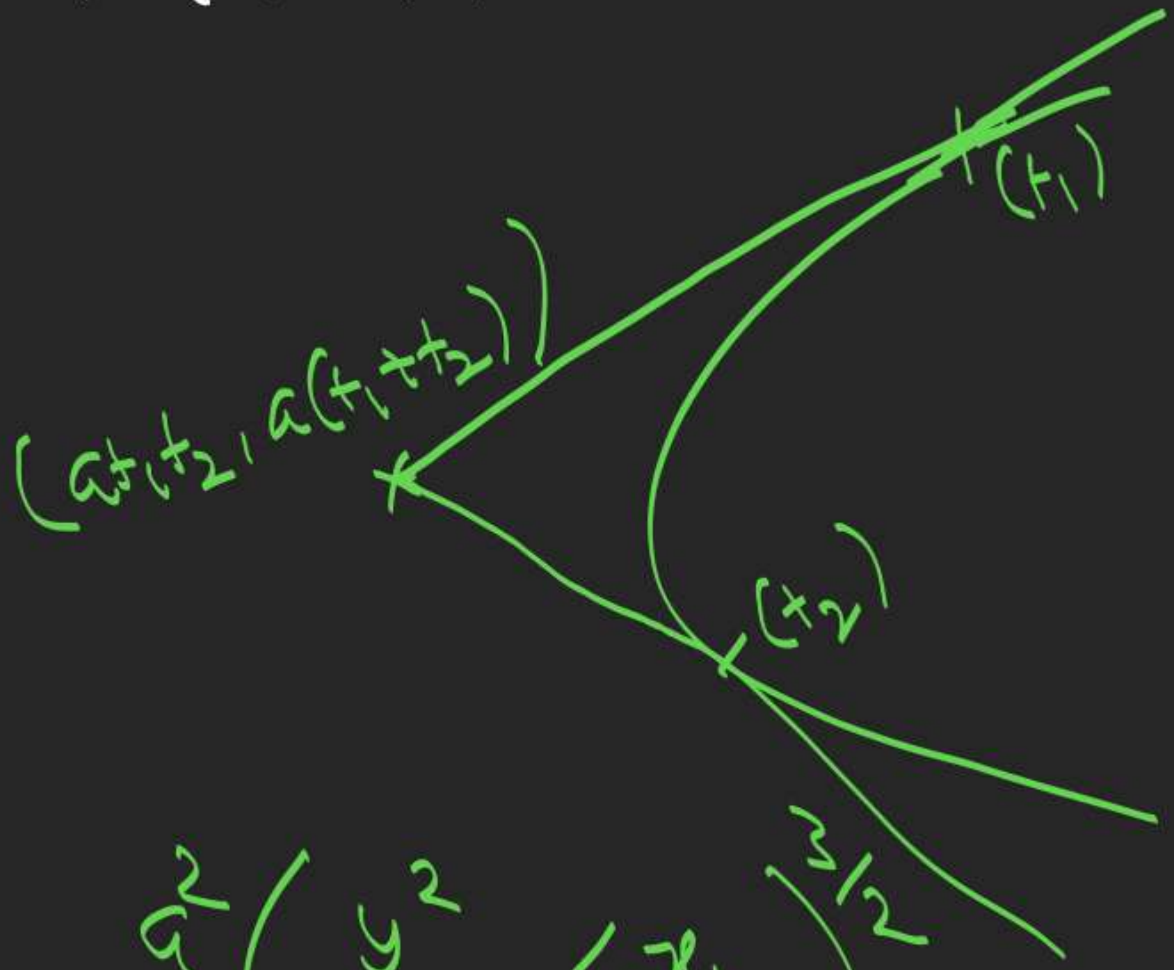
$\frac{\beta}{1}=\frac{2}{3}=\frac{-2\alpha}{2}$

$(\alpha,\beta)=\left(-\frac{2}{3},\frac{2}{3}\right)$

$y(2-3\lambda)=-2(x+\lambda)$

$2y+2x+\lambda(2-3y)=0$

3. P.T. area of ΔPAB formed by pair of tangents and their chord of contact drawn from point $P(x_1, y_1)$ to $y^2 = 4ax$ is $\left|\dfrac{(y_1^2 - 4ax_1)^{3/2}}{2a}\right|$



$$\frac{a^2}{2}\begin{vmatrix} t_1^2 & 2t_1 & 1 \\ t_1t_2 & t_1+t_2 & 1 \\ t_2^2 & 2t_2 & 1 \end{vmatrix}$$

$$= \frac{a^2}{2}(t_2-t_1)^2\begin{vmatrix} t_1^2 & 2t_1 & 1 \\ t_1 & 1 & 0 \\ t_2 & 1 & 0 \end{vmatrix}$$

$$= \frac{a^2}{2}\left|(t_2-t_1)^3\right|$$

$$\frac{a^2}{2}\left(\frac{y_1^2}{a^2} - 4\frac{x_1}{a}\right)^{3/2}$$

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4. Find the locus of middle point of chords of parabola $y^2=4ax$ which

(i) are normal to $y^2=4ax$.

(ii) subtend a constant 'α' at vertex.

(iii) are such that normals at their extremeties meet on parabola $y^2=4ax$.

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$2h = a(t_1^2 + t_2^2)$

$2k = 2a(t_1 + t_2)$

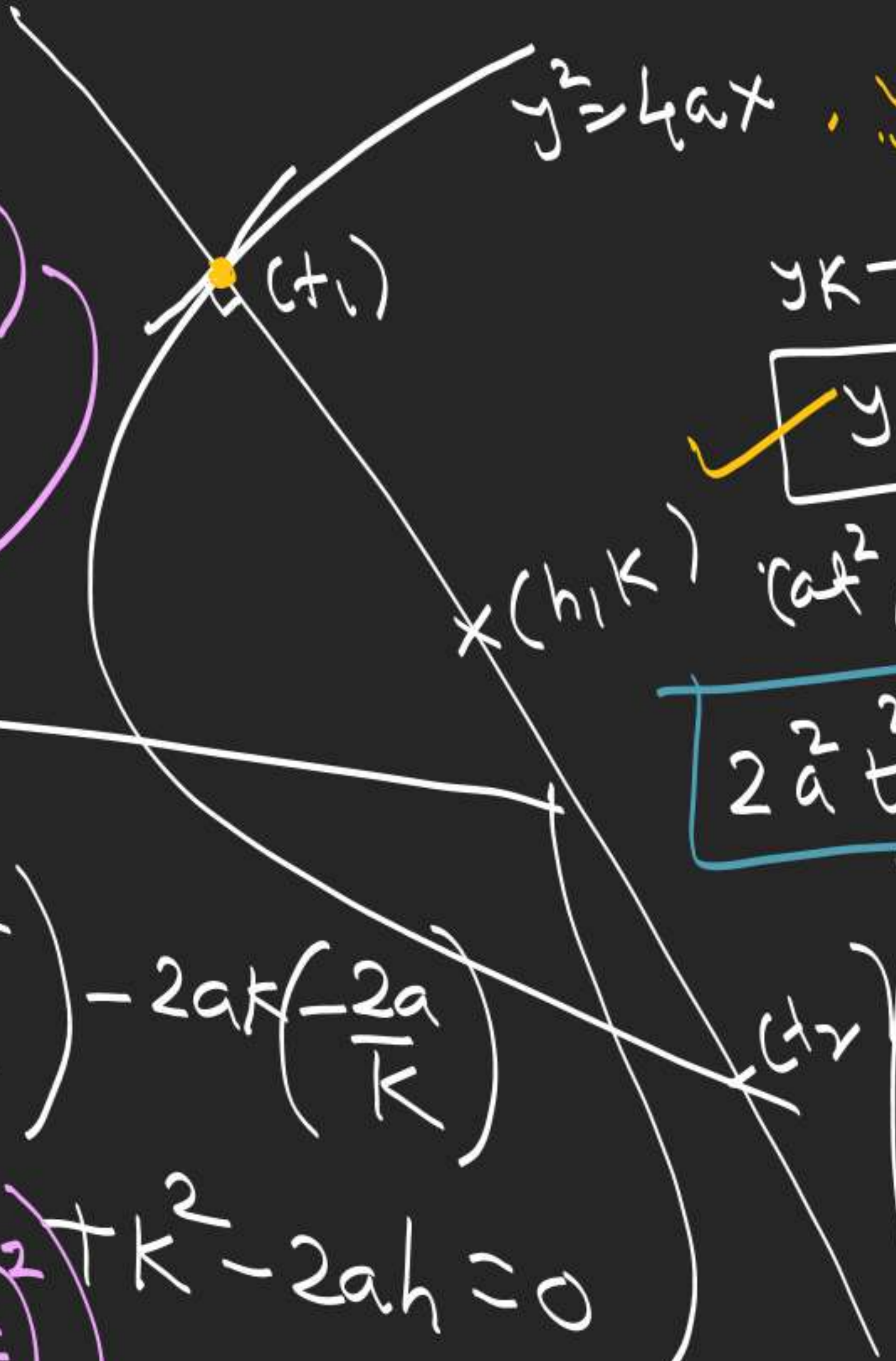
$t_2 = -t_1 - \frac{2}{t_1}$

$k = a\left(-\frac{2}{t_1}\right)$

$t_1 = -\frac{2a}{k}$

$t_2 = \frac{2a}{k} + \frac{k}{a}$

$2h = a\left(\frac{4a^2}{k^2} + \left(\frac{2a}{k} + \frac{k}{a}\right)^2\right)$



$y^2 = 4ax$, $y + t_1 x = 2at_1 + at_1^3$ — ①

$yk - 2a(x+h) = k^2 - 4ah$

$\boxed{yk - 2ax = k^2 - 2ah}$ — ②

$(at^2, 2at)$ — $\frac{1}{k} = \frac{t_1}{-2a} = \frac{2at_1 + at_1^3}{k^2 - 2ah}$

$\boxed{2a^2t^2 - 2akt + k^2 - 2ah = 0}$ t_1, t_2

$k^2 - 2ah = k\left(2a\left(\frac{-2a}{k}\right) + a\left(\frac{-8a^3}{k^3}\right)\right)$

$2a^2\left(\frac{4a^2}{k^2}\right) - 2ak\left(-\frac{2a}{k}\right) + k^2 - 2ah = 0$

$t_2 = -t_1 - \frac{2}{t_1}$

$\frac{k}{a} = -\frac{2}{t_1}$

$t_1 = -\frac{2a}{k}$

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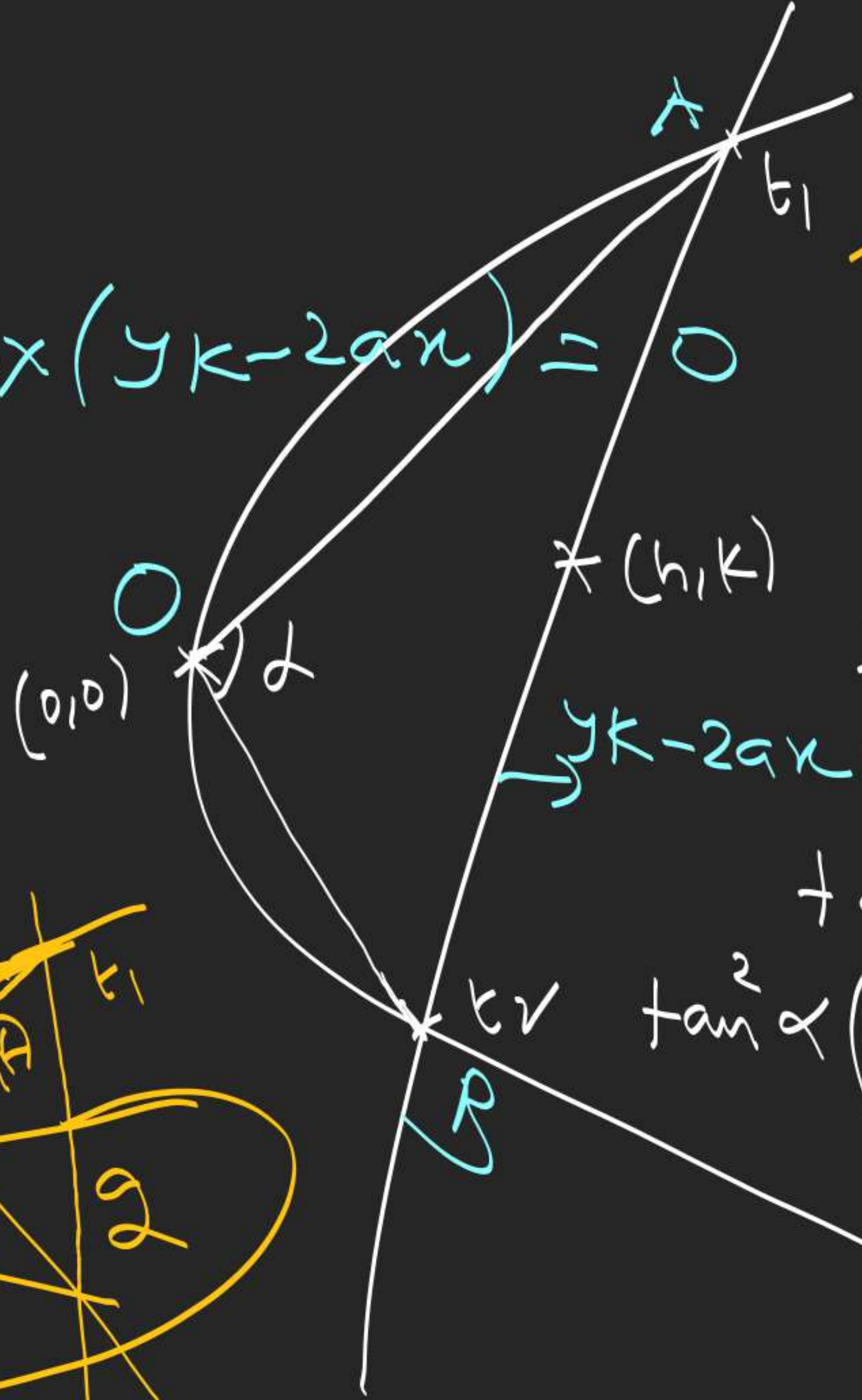
OA & OB

$$(k^2-2ah)y^2 - 4ax(yk-2ax) = 0$$

$\tan\alpha =$



$$\boxed{2a^2t^2 - 2akt + k^2 - 2ah = 0 \;<\; \begin{matrix} t_1 \\ t_2 \end{matrix}}$$

$$yk - 2ax = k^2 - 2ah$$

$$\tan\alpha = \frac{\frac{2}{t_1} - \frac{2}{t_2}}{1 + \frac{2}{t_1}\frac{2}{t_2}}$$

$$\tan^2\alpha\,(t_1t_2+4)^2 = 4\left((t_1+t_2)^2 - 4t_1t_2\right)$$

$$\tan^2\alpha\left(\frac{k^2-2ah}{2a^2}+4\right)^2 = 4\left(\frac{k^2}{a^2} - 4\left(\frac{k^2-2ah}{2a^2}\right)\right)$$

(iii)

$$t_1t_2 = 2$$

$$\frac{k^2-2ah}{2a^2} = 2$$

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Ex-28

10, 29, 14, 17, 23