Nishant Jindal

Limits of form 1^∞

1^∞

$$\lim_{x\to a}\left(f(x)\right)^{g(x)} = e^{\lim_{x\to a}(f(x)-1)g(x)}$$

, where $\lim_{x\to a} f(x) = 1$ &

$\lim_{x\to a} g(x) = \infty$ or $-\infty$

$$= \lim_{x\to a}\left(\left(1+(f(x)-1)\right)^{\frac{1}{f(x)-1}}\right)^{(f(x)-1)g(x)}$$

$$= e^{\lim_{x\to a}(f(x)-1)g(x)}$$

Nishant Jindal

1. $$\lim_{n\to\infty}\left(5^{\frac{1}{n}}+3^{\frac{1}{n}}-1\right)^{n}$$

$$= e^{\lim_{n\to\infty}\left(5^{\frac{1}{n}}+3^{\frac{1}{n}}-2\right)n}$$

$$= e^{\lim_{n\to\infty}\frac{\left(5^{\frac{1}{n}}-1\right)}{\frac{1}{n}}+\frac{\left(3^{\frac{1}{n}}-1\right)}{\frac{1}{n}}}$$

$$= e^{\ln 5+\ln 3}$$

$$= \boxed{15}$$

2. $$\lim_{x\to 0}\left(\frac{5}{2+\sqrt{9+x}}\right)^{\operatorname{cosec} x}$$

$$e^{\lim_{x\to 0}\frac{3-\sqrt{9+x}}{\left(2+\sqrt{9+x}\right)\sin x}} = e^{\lim_{x\to 0}\frac{-x}{\sin x\left(2+\sqrt{9+x}\right)\left(3+\sqrt{9+x}\right)}}$$

$$= e^{-\frac{1}{30}}$$

Test

P&C

Binomial

St. line

Circle

Function

ITF

Limits

Nishant Jindal

3. $\lim_{x\to 0}\left(\cos mx\right)^{\frac{n}{x^2}} = e^{\lim_{x\to 0}\left(\cos(mx)-1\right)\frac{n}{x^2}} = e^{\lim_{x\to 0}\frac{\cos(mx)-1}{(mx)^2}\, m^2 n} = e^{-\frac{m^2 n}{2}}$

4. $\lim_{n\to\infty}\left(\frac{a-1+\sqrt[n]{b}}{a}\right)^n$ $a>0, b>0, n\in N$

$e^{\lim_{n\to\infty}\left(\frac{b^{\frac{1}{n}}-1}{a}\right)n} = e^{\frac{1}{a}\lim_{n\to\infty}\frac{b^{\frac{1}{n}}-1}{\frac{1}{n}}} = e^{\frac{1}{a}\ln b} = b^{\frac{1}{a}}$

$e^{\ln\left(b^{\frac{1}{a}}\right)}$

Nishant Jindal

5: $\lim_{x\to\infty}\left(\frac{1^{\frac{1}{x}}+2^{\frac{1}{x}}+3^{\frac{1}{x}}+\cdots+n^{\frac{1}{x}}}{n}\right)^{nx}$, $n\in N$.

$$e^{\lim_{x\to\infty}\left(\frac{(1^{\frac{1}{x}}-1)+(2^{\frac{1}{x}}-1)+(3^{\frac{1}{x}}-1)+\cdots+(n^{\frac{1}{x}}-1)}{\cancel{n}\ 1/x}\right)\cancel{n}}$$

$$e^{(\ln 1+\ln 2+\ln 3+\cdots+\ln n)}=n!$$

$a^n b^n=(ab)^n$

$$\therefore \left(\left(1^{\frac{1}{x}}\,2^{\frac{1}{x}}\,3^{\frac{1}{x}}\cdots n^{\frac{1}{x}}\right)^{\frac{1}{n}}\right)^{nx}=n!$$

6. $$\lim_{x\to 0}\left(\sin^2\left(\frac{\pi}{2-ax}\right)\right)^{\sec^2\left(\frac{\pi}{2-bx}\right)}$$

$$e^{\lim\limits_{x\to 0} -\left(\frac{\cos\left(\frac{\pi}{2-ax}\right)}{\cos\left(\frac{\pi}{2-bx}\right)}\right)^2} = e^{\lim\limits_{x\to 0} -\left(\frac{\sin\left(\frac{\pi}{2}-\frac{\pi}{2-ax}\right)}{\sin\left(\frac{\pi}{2}-\frac{\pi}{2-bx}\right)}\right)^2}$$

$$= e^{-\lim\limits_{x\to 0}\left(\frac{\frac{\sin\left(\frac{-\pi ax}{2(2-ax)}\right)}{\frac{-\pi ax}{2(2-ax)}}\cdot\left(\frac{-\pi ax}{2(2-ax)}\right)}{\frac{\sin\left(\frac{-\pi bx}{2(2-bx)}\right)}{\left(\frac{-\pi bx}{2(2-bx)}\right)}\cdot\left(\frac{-\pi bx}{2(2-bx)}\right)}\right)^2}$$

$$= e^{-\frac{a^2}{b^2}}$$

Nishant Jindal

7. $\lim_{x\to\infty}\left(x^2 \sin\left(\ln\sqrt{\cos\frac{\pi}{x}}\right)\right)$

$$\lim_{x\to\infty} \cancel{x^2}\left(\frac{\sin\left(\ln\sqrt{\cos\frac{\pi}{x}}\right)}{\ln\sqrt{\cos\frac{\pi}{x}}}\right)\left(\frac{\frac{1}{2}\ln\left(1+\left(\cos\frac{\pi}{x}-1\right)\right)}{\left(\cos\frac{\pi}{x}-1\right)}\right)\left(\frac{\left(\cos\frac{\pi}{x}-1\right)}{\left(\frac{\pi}{x}\right)^2}\right)\left(\frac{\pi}{\cancel{x}}\right)^2$$

$\to 1 \quad \to 1 \quad \to -\frac{1}{2}$

$$= -\frac{\pi^2}{4}$$

Nishant Jindal

8.

$$\lim_{n\to\infty}\left(\left(\frac{n}{n+1}\right)^{\alpha}+\sin\frac{1}{n}\right)^{n}$$

$$\left(1+\frac{1}{n}\right)^{-\alpha}-1$$

$$\frac{1}{n}=t$$

$$= e^{\lim\limits_{n\to\infty}\left(\frac{\sin\frac{1}{n}}{\frac{1}{n}}+\frac{\left(\frac{n}{n+1}\right)^{\alpha}-1}{\frac{1}{n}}\right)}$$

$$= e^{\lim\limits_{t\to 0}\left(\frac{\sin t}{t}+\frac{(1+t)^{-\alpha}-1}{t}\right)} = e^{1-\alpha}$$

$$=-\alpha$$

$$-\alpha(1)^{-\alpha-1}$$

$$\lim_{x\to y}\frac{x^{-\alpha}-1^{-\alpha}}{x-1}$$

$$=\lim_{t\to 0}\frac{(1+t)^{-\alpha}-1^{-\alpha}}{(1+t)-1}$$

$$\frac{\cancel{1}-\alpha t+\frac{(-\alpha)(-\alpha-1)}{2!}t^2+\dots\ \cancel{-1}}{t}$$

Nishant Jindal

Q: $\lim_{x \to 1} \left(\frac{1+x}{2+x} \right)^{\frac{1-\sqrt{x}}{1-x}} = \lim_{x \to 1} \left(\frac{1+x}{2+x} \right)^{\frac{1}{1+\sqrt{x}}}$

$\frac{1}{2} \leftarrow x \to 0$

$1 \leftarrow x \to \infty$

$= \sqrt{\frac{2}{3}}$

Nishant Jindal

1. $\lim_{x\to 0}\left(\frac{1-\cos x+\underline{2\sin x}-\sin^3 x-x^2+3x^4}{\tan^3 x-6\sin^2 x+x-5x^3}\right)$

$$\frac{x\frac{(1-\cos x)}{x^2}+\frac{2\sin x}{x}-\frac{\sin^3 x}{x^3}x^2-x+3x^3}{x^2\frac{\tan^3 x}{x^3}-6\frac{\sin^2 x}{x^2}x+1-5x^2}=2.$$

2. $\lim_{n\to\infty}\left(\frac{5^{n+1}+3^n-2^{2n}}{5^n+2^n+3^{n+3}}\right)=\lim_{n\to\infty}\frac{\cancel{5^n}\left(5+\left(\frac{3}{5}\right)^n-\left(\frac{4}{5}\right)^n\right)}{\cancel{5^n}\left(1+\left(\frac{2}{5}\right)^n+27\left(\frac{3}{5}\right)^n\right)}$

$=\boxed{5}$

Nishant Jindal

3. $\lim_{x \to 0} \frac{\cot^{-1}\left(\frac{1}{x}\right)}{x}$

4. $\lim_{x \to \frac{\pi}{8}^+} \left(\tan\left(\frac{\pi}{8}+x\right)\right)^{\tan 2x}$

H.W.

$\boxed{351 - 376}$

$\boxed{379 - 392}$

PT-1

PT-2