Nishant Jindal

Symm / Skew symm.

$A^T = A$ | $A^T = -A$

diag = 0.

$a_{ij} = -a_{ji}$

Q A is a Sqr Matrix then. $A \cdot A^T$ is Sym / Skew?

let $B = A \cdot A^T$

$B^T = (A \cdot A^T)^T$

$= (A^T)^T (A)^T$

$= A \cdot A^T$

$B^T = B$

$A \cdot A^T$ is (Symm)

Q A is a skew sym. then $B^T \cdot A \cdot B$ is ~ ?

$A^T = -A$

let $C = B^T \cdot A \cdot B$

$C^T = (B^T A B)^T$

$= B^T \cdot (A^T) (B^T)^T$

$= B^T \cdot (-A) B$

$= -B^T \cdot A B$

$C^T = -C$

$\therefore$ $B^T A B$ is Skew sym.

RK

① Det. value of all odd order Skew symm Matrix = 0 always.

(2) $A + A^T$ = Symm.

$A - A^T$ = Skew

(3) Every Sqr matrix can be expressed as a Sum of Symm & Skew symm.

$$A = \frac{(A + A^T)}{2} + \frac{(A - A^T)}{2}$$

A = Symm + Skew

(4) If A is symm matrix then A^n is also symm.

(5) If A is Skew Symm then $A^n = \begin{cases} \text{Symm} & n = \text{Even} \\ \text{Skew} & n = \text{odd} \end{cases}$

Proof $A =$ Symm. then $\boxed{A^n}$ - - - ?

$\downarrow$

$A^T = A$

let $B = A^n$

$B^T = (A^n)^T$

$= (A^T)^n$

$= (A)^n$

$B^T = B$

Symm

Proof $A =$ Skew then A^n - - -

$\boxed{A^T = -A}$

let $B = A^n$

$B^T = (A^n)^T$

$= (A^T)^n$

$$B^T = (-A)^n = \begin{cases} A^n = B & \text{Symm.} \quad n = \text{Even} \\ -A^n = -B & \text{Skew.} \quad n = \text{odd.} \end{cases}$$

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(b) If A is skew symm.

then (A) $A^{2n} =$ Symm. $n \in N$

(B) $\boxed{A^{2n+1}} =$ Skew $n \in N$

$K \cdot A =$ Skew.

Proof

$A^T = -A$

let $B = A^{2n+1}$

$B^T = (A^{2n+1})^T$

$= (A^T)^{2n+1}$

$= (-A)^{2n+1}$

$= -A^{2n+1}$

$B^T = -B$ skew

Q If A & B are 2 Symm Matrix.

(1) $A + B = \dots ?$

$A^T = A$ & $B^T = B$ (given)

let $C = A + B$

$C^T = (A+B)^T$

$C^T = A^T + B^T$

$= A + B$

$C^T = C$

Symm.

(2) $AB - BA = \dots ?$

let $C = AB - BA$

$C^T = (AB - BA)^T$

$= (AB)^T - (BA)^T$

$= B^T \cdot A^T - A^T \cdot B^T$

$= B \cdot A - A \cdot B$

$= -(AB - BA)$

$C^T = -C$

Skew

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Q If A & B are Skew then

(1) $AB - BA = ?$

$A^T = -A$, $B^T = -B$

let $C = AB - BA$

$C^T = (AB - BA)^T$

$= (AB)^T - (BA)^T$

$= B^T \cdot A^T - A^T B^T$

$= (-B)(-A) - (-A)(-B)$

$= BA - AB$

$= -(AB - BA)$

$= -C$ Skew

Q Check statement

Adv

A) If M is Symm then $N^T \cdot MN$ is Symm.

" Skew " $N^T \cdot MN$ is Skew?

$M^T = M$ Symm

$B = N^T \cdot MN$

$B^T = (N^T MN)^T$

$= N^T (M^T) (N^T)^T$

$= N^T MN$

$= B$

Symm.

$M^T = -M$ (Skew

$B = N^T \cdot MN$

$B^T = (N^T \cdot MN)^T$

$= N^T \cdot M^T \cdot (N^T)^T$

$= N^T (-M)(N)$

$= -N^T \cdot MN$

$B^T = -B$

(Skew)

(B) M & N = Symm.

then $MN - NM = ?$

Skew

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Q If X, Y are skew & Z is symm.

Adv then $Y^3 \cdot Z^4 - Z^4 \cdot Y^3 = ----$

$X^T = -X$, $Y^T = -Y$ & $Z^T = Z$

$B = Y^3 Z^4 - Z^4 \cdot Y^3$

$B^T = (Y^3 Z^4 - Z^4 \cdot Y^3)^T$

$= (Y^3 \cdot Z^4)^T - (Z^4 \cdot Y^3)^T$

$= (Z^4)^T \cdot (Y^3)^T - (Y^3)^T \cdot (Z^4)^T$

$= (Z^T)^4 (Y^T)^3 - (Y^T)^3 (Z^T)^4$

$= Z^4 (-Y)^3 - (-Y)^3 \cdot (Z)^4$

$= -Z^4 Y^3 + Y^3 \cdot Z^4$

$B^T = B$ (Symm)

(B) $X^{44} + Y^{44}$

Symm.

(C) $X^{23} + Y^{22}$

Skew

Q How many 3x3 Matrices entries with {0, 1, 2} are there? Adv for which sum of diagonal entries of $M^T \cdot M = 5$?

$$M^T \cdot M = \begin{bmatrix} a & d & g \\ b & e & h \\ c & f & i \end{bmatrix} \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = \begin{bmatrix} a^2+d^2+g^2 & - & - \\ - & b^2+e^2+h^2 & - \\ - & - & c^2+f^2+i^2 \end{bmatrix}$$

Sum $= a^2+b^2+c^2+d^2+e^2+f^2+g^2+h^2+i^2 = 5$ (Lanah)

Case ① $= 1^2+1^2+1^2+1^2+1^2+0^2+0^2+0^2+0^2 = 5 \rightarrow \frac{\cancel{9}}{\cancel{5}\,\cancel{4}} = \frac{9 \cdot 8 \cdot 7 \cdot 6}{4 \cdot 3 \cdot 2} = 126$

Case ② $= 2^2+1^2+0^2+0^2+0^2+0^2+0^2+0^2+0^2 = 5$ $\frac{\cancel{9}}{\cancel{7}} = 9 \cdot 8 = 72$

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Q. Adv 2016

$P = \begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 16 & 4 & 1 \end{bmatrix}$ & I be the Identity Mat. of order 3

If $Q = [q_{ij}]$ is a matrix such that $P^{50} - Q = I$

$P^{50} - I = Q$

then $\dfrac{q_{31} + q_{32}}{q_{21}}$

$P^2 = \begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 16 & 4 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 16 & 4 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 8 & 1 & 0 \\ 48 & 8 & 1 \end{bmatrix}$ $\rightarrow 16(1+2)$

$\dfrac{q_{31} + q_{32}}{q_{21}} = \dfrac{400 \times 51 + 200}{200} = 103$

$P^3 = \begin{bmatrix} 1 & 0 & 0 \\ 8 & 1 & 0 \\ 48 & 8 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 16 & 4 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 12 & 1 & 0 \\ 96 & 12 & 1 \end{bmatrix}$ $\rightarrow 16(1+2+3)$

$P^{50} - I = \begin{bmatrix} 1 & 0 & 0 \\ 200 & 1 & 0 \\ 400 \times 51 & 200 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 200 & 0 & 0 \\ 400 \times 51 & 200 & 0 \end{bmatrix} = Q$

$\overset{8}{\cancel{16}} \times \dfrac{50 \times 51}{\cancel{2}}$

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Q If A is of order 3 Matrix then

$|A - A^T| = ?$

A = 3 order (odd)

A^T = 3rd order (odd)

1) $A - A^T$ = odd order Matrix

2) $A - A^T$ = Skew symm

3) |Skew with odd| = 0

∴ Ans = 0

Q. If $A+B+C=\pi$ then $\begin{vmatrix} \sin(A+B+C) & \sin B & \cos C \\ -\sin B & 0 & \tan A \\ \cos(A+B) & -\tan A & 0 \end{vmatrix} = ?$

$\sin \pi = 0$

$A+B = \pi - C$

$\cos(A+B) = \cos(\pi - C)$

$= -\cos C$

$\begin{vmatrix} 0 & \sin B & \cos C \\ -\sin B & 0 & \tan A \\ -\cos C & -\tan A & 0 \end{vmatrix}$ → order = 3.

|Skew odd order| = 0

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Q If $C=\begin{pmatrix}1&4&6\\7&2&5\\9&8&3\end{pmatrix}\begin{pmatrix}0&2&3\\-2&0&4\\-3&-4&0\end{pmatrix}\begin{pmatrix}1&7&9\\4&2&8\\6&5&3\end{pmatrix}$

then trace of $C+C^3+C^5+----C^{99}=?$

$C = A \cdot B \cdot A^T$

B = Skew

$B^T=-B$

$(x^{23}+y^{23})$

$C^T=(ABA^T)^T$

$=(A^T)^T\cdot(B)^T(A)^T$

$=A(-B)A^T$

$=-ABA^T$

$C^T=-C \Rightarrow C$ is skew

$C^3, C^5, C^7, -- C^{99}$ = Skew

$(C+C^3+C^5+C^7+---C^{99})$ = diagonal element 0 आएंगे

$Tr(C+C^3+--\sim C^{99}) = 0$

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Special Types of Matrix.

(A) Idempotent

(1) $A^2 = A$

(2) $|A^2| = |A|$

$|A|^2 = |A|$

$|A|^2 - |A| = 0$

$|A|(|A|-1) = 0$

$|A| = 0$ या 1

(B) Involutory

1) $A^2 = I$

2) $|A|^2 = |I|$

$\Rightarrow |A|^2 = 1$

$\Rightarrow |A| = 1$ या -1

Non Singular Matrix.

$\Rightarrow$ Inverse PSBL

(C) Nilpotent

1) $A^K = 0$

2) $|A|^K = 0$

$|A| = 0$

Singular.

Non Invertible.

(D) Periodic

1) $\boxed{A^K = A} \rightarrow$ Period $= K-1$

$\star$ let $\boxed{A^4 = A}$

$A^4 = A \times A^3$

$A^7 = A^4 = A$

$A^4 = A \times A^6$

$A^{10} = A^7 = A$

$A = A^4 = A^7 = A^{10} = A^{13} = A^{16}$

here Period $= 3$

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(5) Orthogonal Matrix

If A is orthogonal then $\boxed{A^T \cdot A = I = A \cdot A^T}$

Popular Exp.

$$A = \begin{bmatrix} \sqrt{3}/2 & 1/2 \\ -1/2 & \sqrt{3}/2 \end{bmatrix}, \quad A = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Q If A & B orthogonal then P.T. (AB) is also orthogonal.?

$A A^T = I$ & $B \cdot B^T = I$ (given)

To Prove

$(AB) \cdot (AB)^T = I$

LHS $AB \cdot B^T \cdot A^T$

$A(B B^T) A^T$

$A \cdot I \cdot A^T = A A^T = I = RHS$

Q If X is orthogonal & $B = X A X^T$ then P.T.

$X \cdot X^T = I$

(1) $A = X^T \cdot B \cdot X$ (2) $B^{10} = X \cdot A^{10} \cdot X^T$

RHS

(1) $X^T \cdot B \cdot X$

$X^T \cdot (X A X^T) \cdot X$

$(X^T \cdot X) A (X^T \cdot X)$

$I A I = A = LHS$.

(2) $B^2 = X A (X^T \cdot X) A X^T$

$B^2 = X A \cdot I \cdot A \cdot X^T = X A^2 X^T$

$B^3 = B^2 \cdot B = X \cdot A^2 (X^T \cdot X) A \cdot X^T$

$B^3 = X A^2 I A \cdot X^T = X \cdot A^3 \cdot X^T$

$\therefore \boxed{B^{10} = X A^{10} \cdot X^T}$

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Determinant

→ Initially 5% Det then Back to Matrix to complete then come back for 95%.

(1) Determinant is value of Matrix.

(2) Determinat of Matrix A is possible only when A is sq^r Matrix.

(3) $|2| \rightarrow$ Det of 2
1^{st} Orde.

(4) $\Delta = \begin{vmatrix} a & b \\ c & d \end{vmatrix} \Rightarrow 2^{nd}$ Order
2×2

$\Delta = ad - bc$

(5) $\Delta = \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}$ 3^{rd} Order.

(6) Minor of det

M_{11} = Minor of a_{11} = by deleting Row & Column attached to a_{11} & find det value

Minor of $a = \begin{vmatrix} e & f \\ h & i \end{vmatrix}$, Minor of $f = \begin{vmatrix} a & b \\ g & h \end{vmatrix}$
$= (ah - gb)$

Minor of g
$= \begin{vmatrix} b & c \\ e & f \end{vmatrix}$
$= (bf - ce)$

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(7) Cofactor

1) Cofactor is Rep by C_{ij}

& Minor of a_{ij} is Rep by M_{ij}

2) $\boxed{C_{ij} = (-1)^{i+j} \cdot M_{ij}}$

3) Sign Notation

$$\begin{vmatrix} + & - & + \\ - & + & - \\ + & - & + \end{vmatrix}$$

$$\Delta = \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}$$

$$\text{Cofactor of } b = -\begin{vmatrix} d & f \\ g & i \end{vmatrix}$$

$$\text{Cofactor of } h = -\begin{vmatrix} a & c \\ d & f \end{vmatrix}$$