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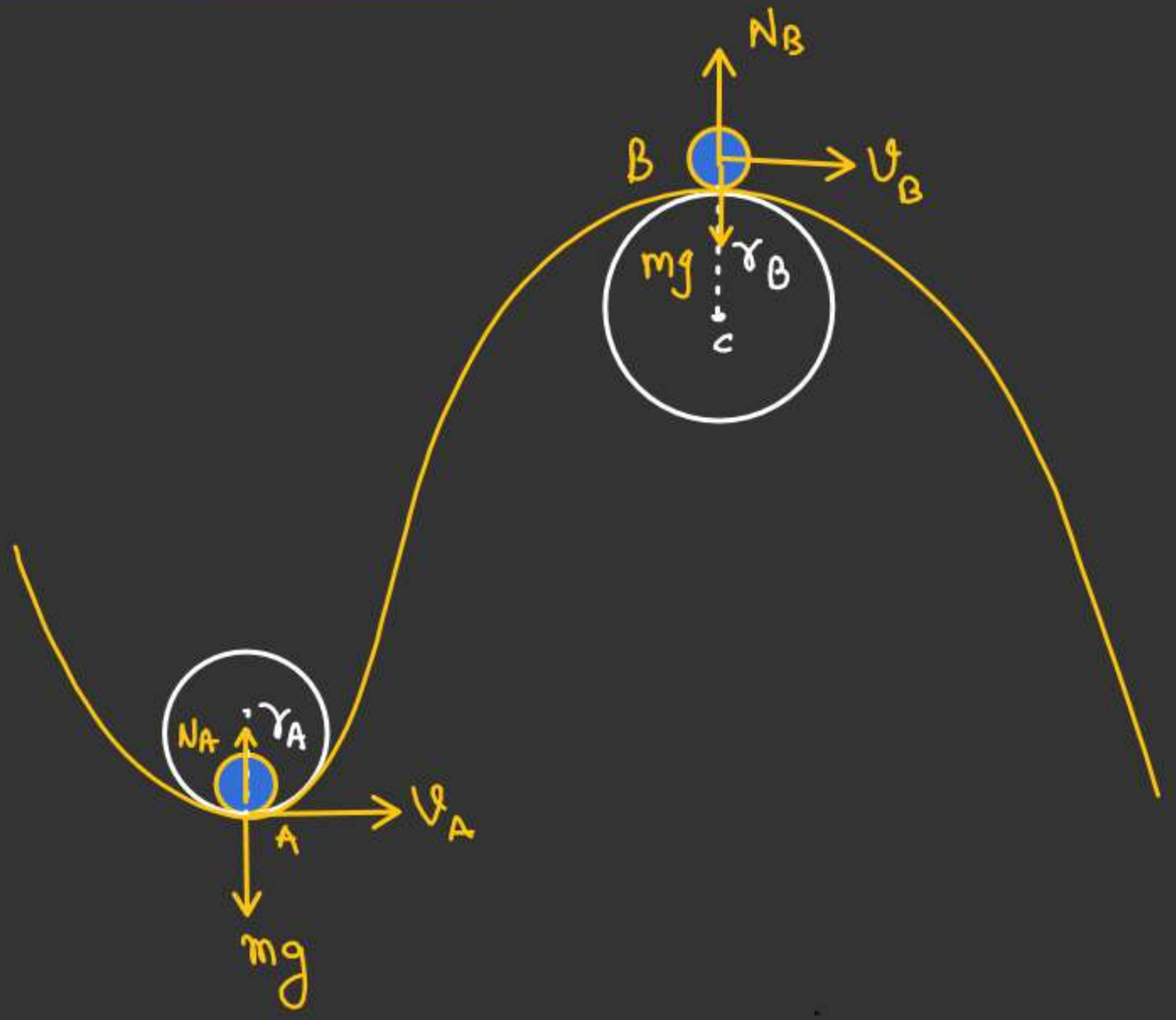
**

Concept of Radius of Curvature

For A $\Rightarrow$ $N_A - mg = \dfrac{mv_A^2}{R_A}$

For B $\Rightarrow$ $mg - N_B = \dfrac{mv_B^2}{R_B}$



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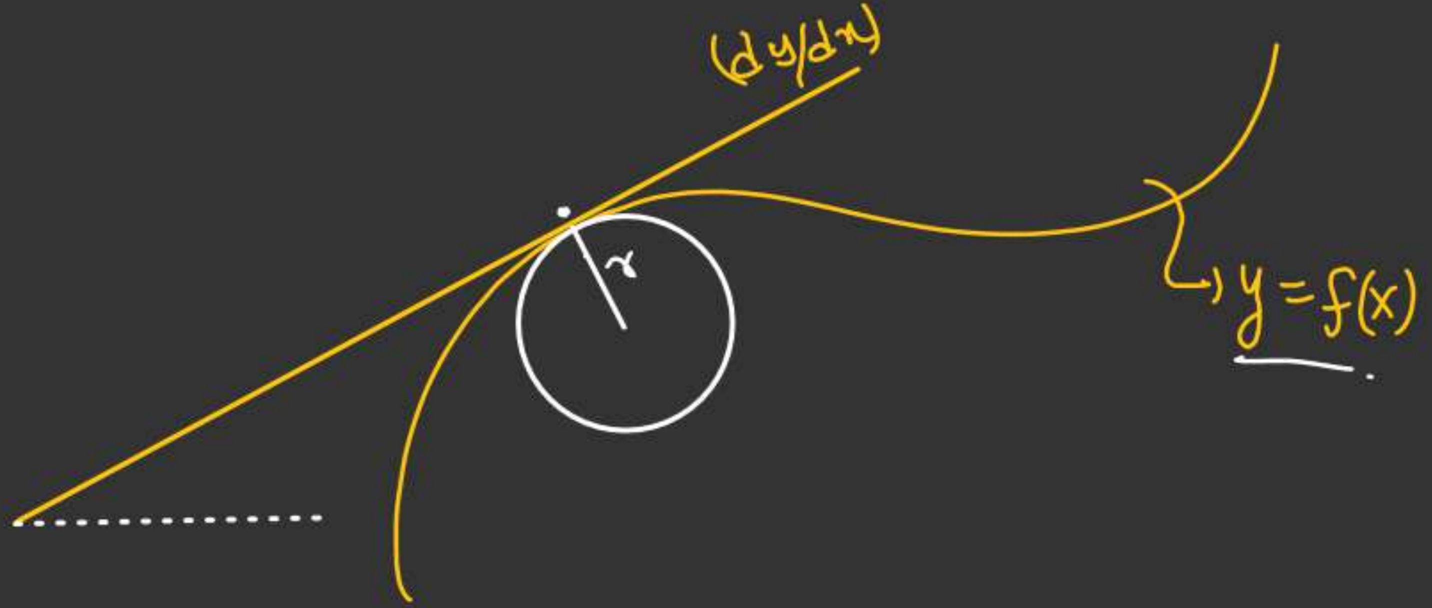
General Equation for radius of Curvature:-

$$\boxed{r = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left(\frac{d^2y}{dx^2}\right)}}$$



Irodov questions

CIRCULAR MOTION

Constant Speed

Q.1 A particle moves along the plane trajectory $y(x)$ with velocity v whose modulus is constant. Find the acceleration of the particle at the point $x = 0$ and the curvature radius of the trajectory at that point if the trajectory has the form

(a) of a parabola $y = ax^2$;

(b) of an ellipse $(x/a)^2 + (y/b)^2 = 1$; a and b are constants here.

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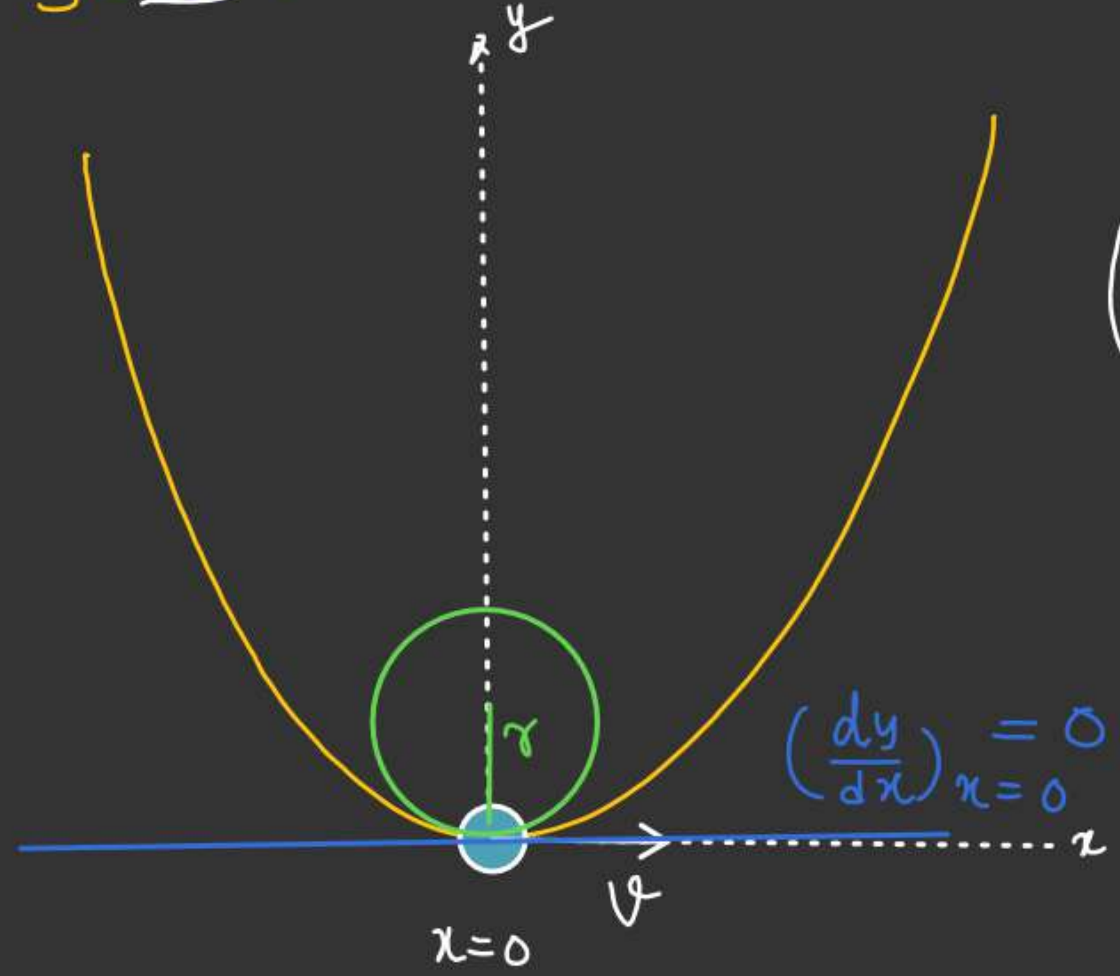
a) $y = ax^2$.



$$\frac{dy}{dx} = (2ax)$$

$$\left(\frac{dy}{dx}\right)_{x=0} = 0.$$

$$\left(\frac{d^2y}{dx^2}\right) = (2a)$$

$$r = \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\left(\frac{d^2y}{dx^2}\right)}$$

$$r_{x=0} = \frac{(1)^{3/2}}{2a}$$

$$\boxed{r_{x=0} = \frac{1}{2a}}$$

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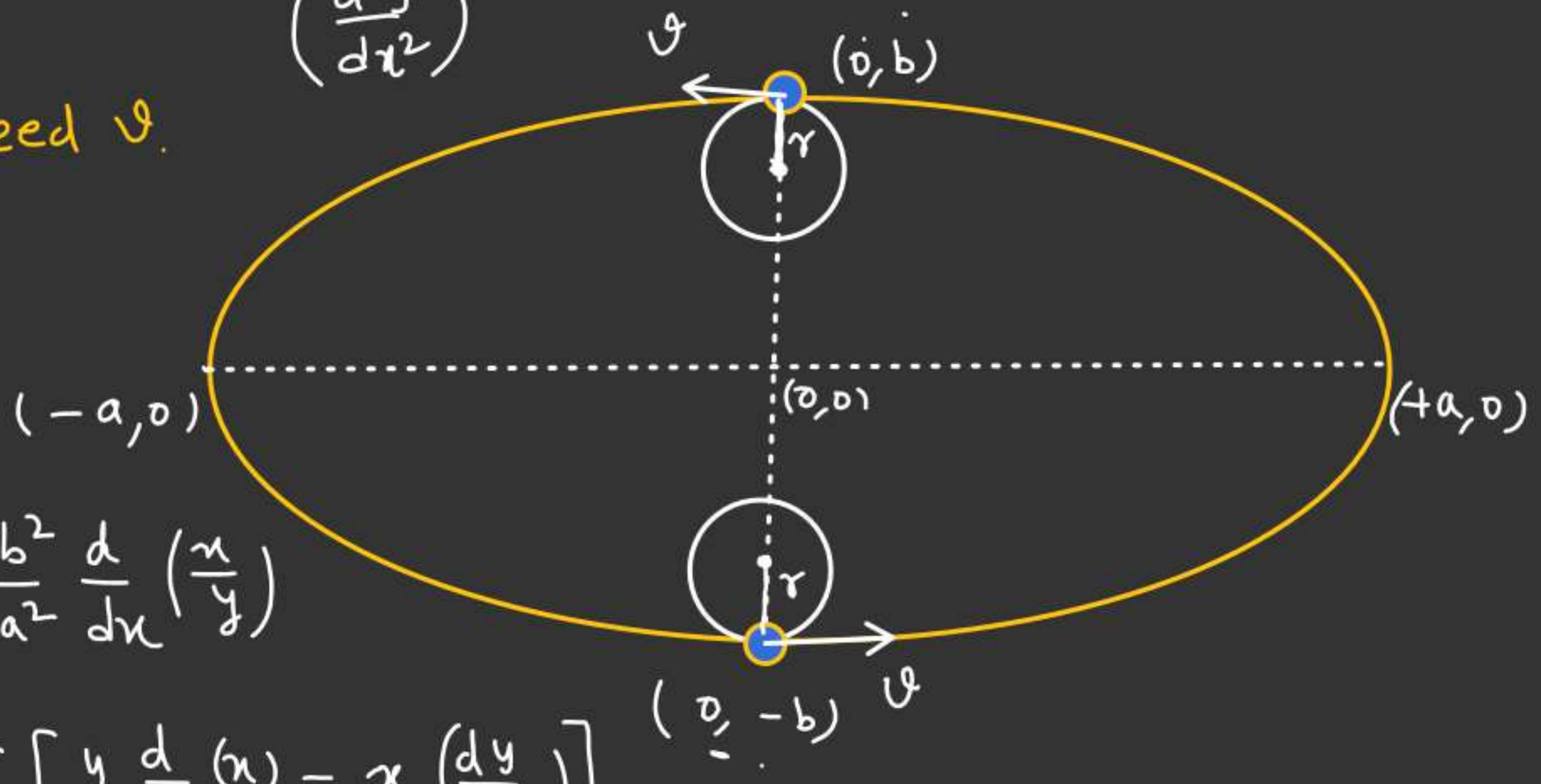
(b) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ (Ellipse)

$$r = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\left(\frac{d^2y}{dx^2}\right)}$$

radius of Curvature at $x = 0$.
particle moving with Constant Speed v.



Sol^n

$$\frac{y^2}{b^2} = 1 - \frac{x^2}{a^2}$$

$$y^2 = b^2 - \frac{b^2}{a^2}x^2$$

$$\cancel{2}y\frac{dy}{dx} = -\frac{b^2}{a^2} \times \cancel{2}x$$

$$\frac{dy}{dx} = -\left(\frac{b^2}{a^2}\right)\left(\frac{x}{y}\right)$$

$$\left(\frac{dy}{dx}\right)_{x=0} = 0$$

$$\frac{d^2y}{dx^2} = -\frac{b^2}{a^2}\frac{d}{dx}\left(\frac{x}{y}\right)$$

$$\frac{d^2y}{dx^2} = \frac{-b^2}{a^2}\left[\frac{y\frac{d}{dx}(x) - x\left(\frac{dy}{dx}\right)}{y^2}\right]$$

$$\frac{d^2y}{dx^2} = \frac{-b^2}{a^2}\frac{\left[y - x\left(\frac{dy}{dx}\right)\right]}{y^2}$$

$$\left(\frac{d^2y}{dx^2}\right)_{0,b} = \left(-\frac{b}{a^2}\right) ✓$$

$$r_{(0,b)} = \ominus\left(\frac{a^2}{b}\right)$$

$$r_{(0,-b)} = \left(+a^2/b\right)$$

CIRCULAR MOTION

Q.2 A point moves in the plane so that its tangential acceleration $w_\tau = a$, and its normal acceleration $w_n = bt^4$, where a and b are positive constants, and t is time. At the moment $t = 0$ the point was at rest. Find how the curvature radius R of the point's trajectory and the total acceleration w depend on the distance covered s.

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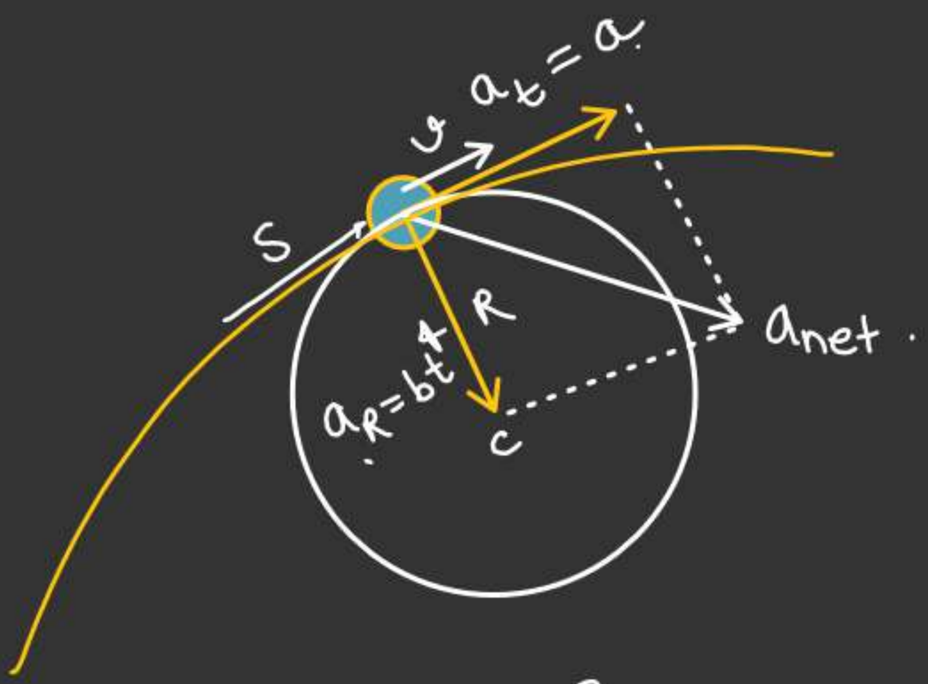
$$\left[\begin{array}{l} a_{net} = f(s) \\ R = f(s). \end{array}\right.$$

At $t = 0$; $v = 0$



$$v^2 = \cancel{u^2}^{\,0} + 2a_t S$$

$$v = \sqrt{2as}$$

$$v = \cancel{u}^{\,0} + a_t t$$

$$\sqrt{2as} = at$$

$$t = \sqrt{\frac{2s}{a}}$$

$$a_R = bt^4.$$

$$\frac{v^2}{R} = bt^4$$

$$R = \frac{v^2}{bt^4}$$

$$R = \frac{2as}{b\left(\frac{2s}{a}\right)^2}$$

$$R = \frac{2a\cancel{s}}{b} \times \frac{a^2}{4s^{\cancel{2}}}$$

$$R = \left(\frac{a^3}{2bs}\right)$$

$$a_R = b\left(\frac{2s}{a}\right)^{4/2}$$

$$a_R = \left(\frac{4bs^2}{a^2}\right)$$

$$a_{net}^2 = a_t^2 + a_r^2$$

$$a_{net}^2 = a^2 + \left(\frac{4bs^2}{a^2}\right)^2$$

$$a_{net} = \sqrt{a^2 + \frac{16b^2s^4}{a^4}}$$

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Irodov questions

CIRCULAR MOTION

Imp.

Q.3 A balloon starts rising from the surface of the Earth. The ~~ascension~~ ascent rate is constant and equal to v_0. Due to the wind the balloon gathers the horizontal velocity component $v_x = ay$, where a is a constant and y is the height of ascent. Find how the following quantities depend on the height of ascent:

(a) the horizontal drift of the balloon $x(y)$;

(b) the total, tangential, and normal accelerations of the balloon.

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$a_y = 0$, $\quad V_0 = \text{Constant} = \dfrac{dy}{dt}$

$\vec{a}_{net} = a_x\hat{i} + a_y\hat{j}$

$a_{net} = a_x$.

$v_x = ay$

Differentiating both side w.r.t time

$$\frac{dv_x}{dt} = a\left(\frac{dy}{dt}\right)$$

$$a_x = av_0.$$

$y = v_0 t$ — ①

$v_x = ay$

$v_x = av_0 t$

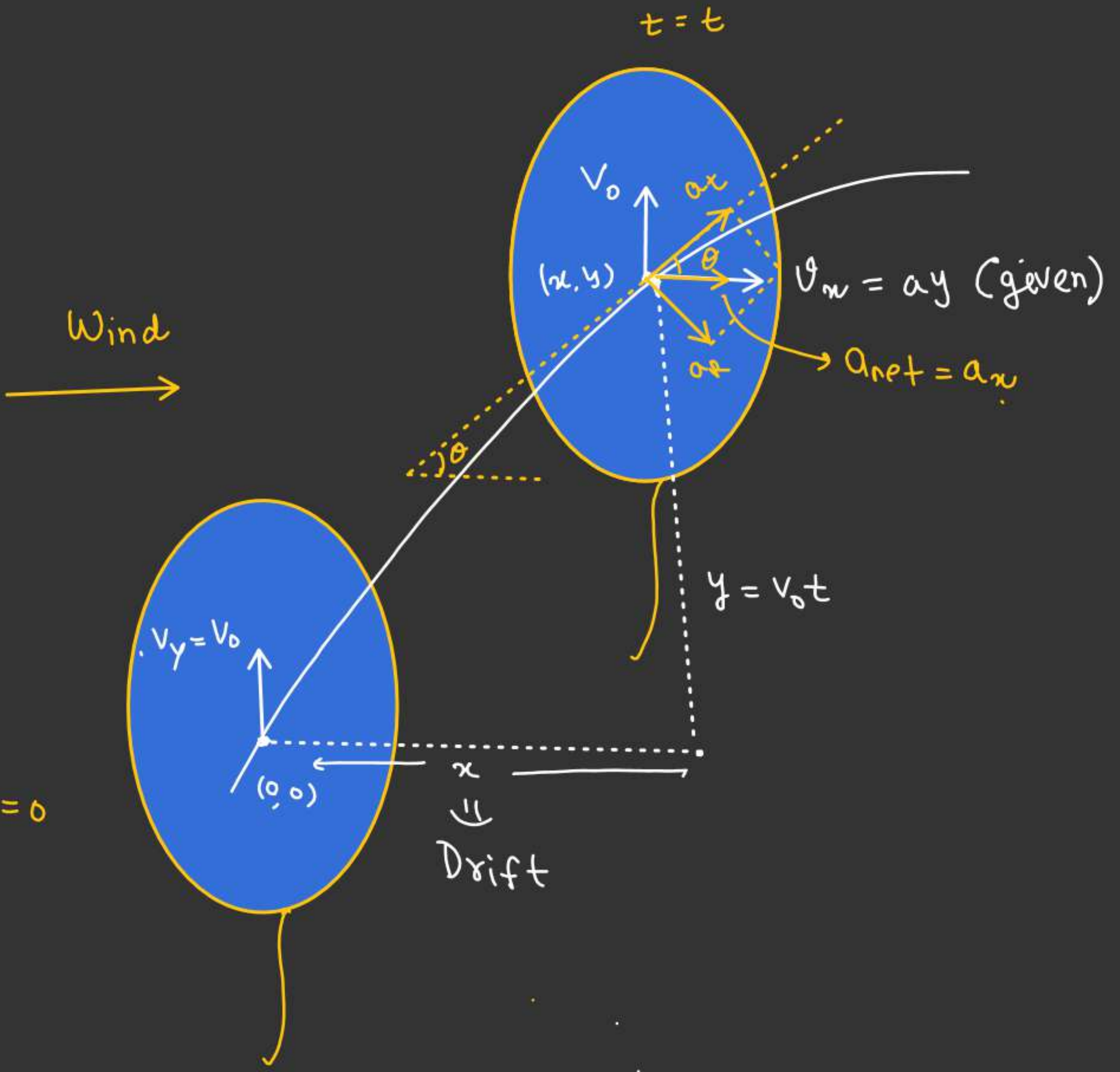
$$\int_0^x dx = av_0 \int_0^t t\,dt$$

$x = \dfrac{av_0}{2}t^2$ — ② $t=0$

From ① & ②

$$x = \frac{av_0}{2}\left(\frac{y}{v_0}\right)^2$$

$$\boxed{x = \frac{a}{2v_0}(y^2)}$$



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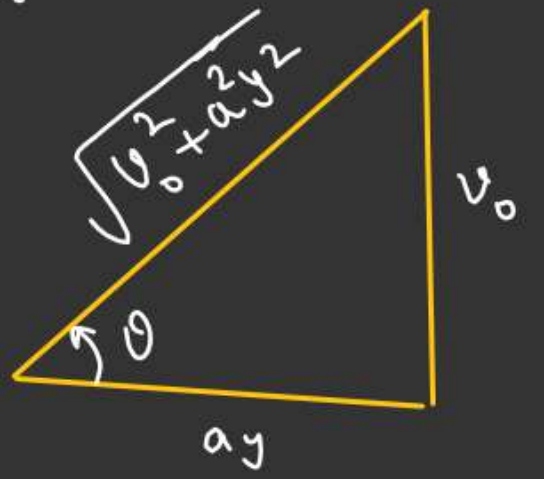
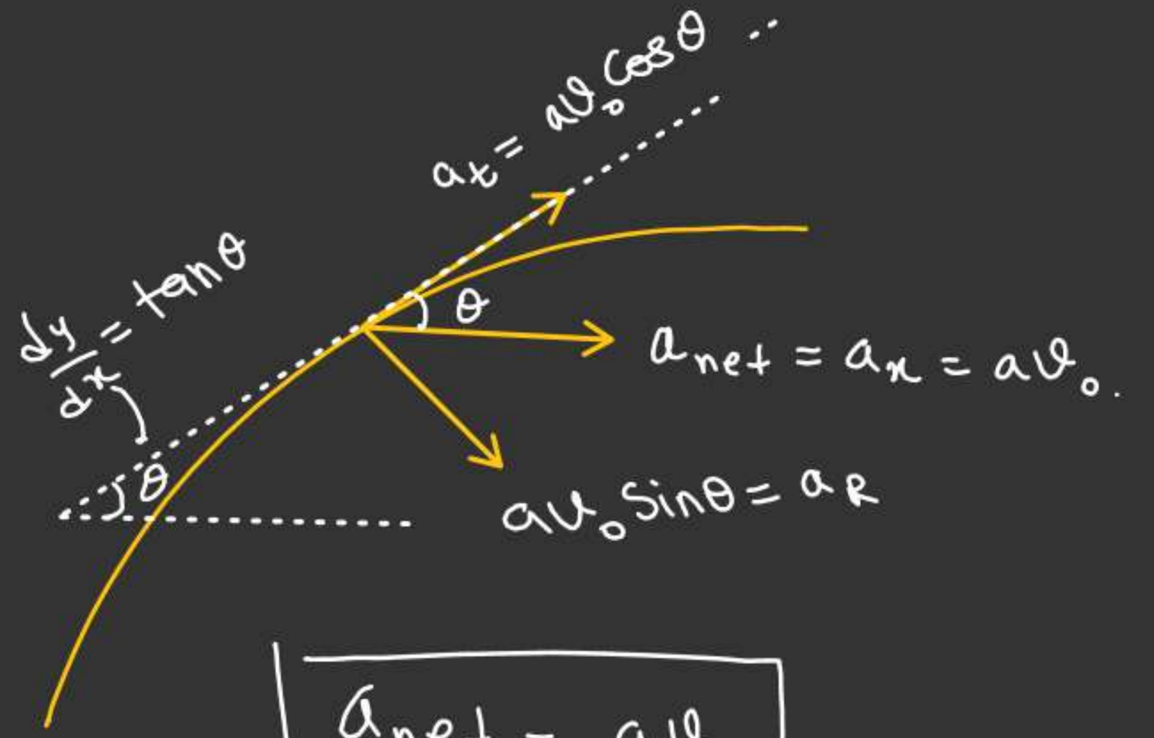
$$x = \frac{a}{2v_0} y^2$$

$$\frac{d}{dx}(x) = \frac{a}{2v_0}(2y)\left(\frac{dy}{dx}\right)$$

$$\tan\theta = \frac{dy}{dx} = \left(\frac{v_0}{ay}\right)$$





$$a_t = av_0\cos\theta$$

$$a_t = av_0\left(\frac{ay}{\sqrt{v_0^2+a^2y^2}}\right)$$

$$a_t = \frac{a^2 . v_0 y}{\sqrt{v_0^2+a^2y^2}}$$

$$a_R = av_0\sin\theta$$

$$a_R = av_0\left(\frac{v_0}{\sqrt{v_0^2+a^2y^2}}\right) = \left(\frac{av_0^2}{\sqrt{v_0^2+a^2y^2}}\right)$$

$$\boxed{a_{net} = av_0}$$

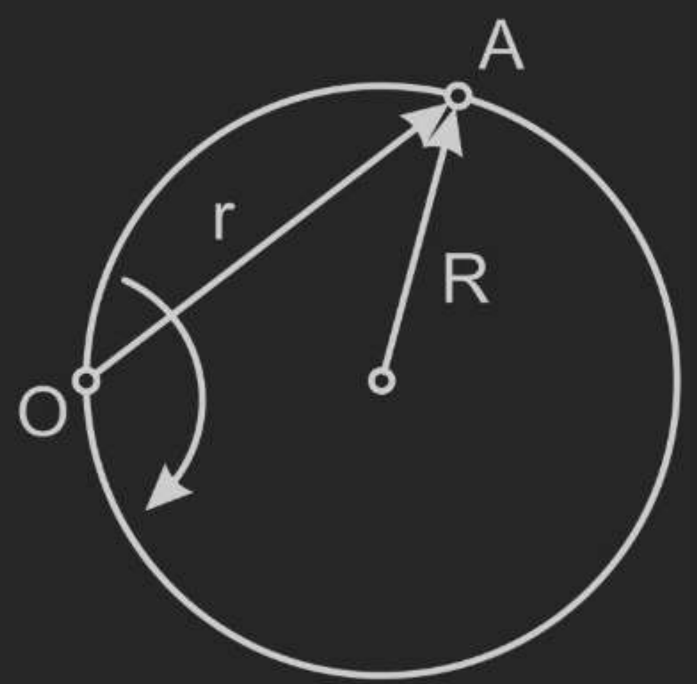
$\Downarrow$

Constant $\Rightarrow$ independent of y.

CIRCULAR MOTION

Q.4 A particle A moves along a circle of radius $R = 50\ \mathrm{cm}$ so that its radius vector r relative to the point O (Fig.) rotates with the constant angular velocity $\omega = 0.40\mathrm{rad/s}$. Find the modulus of the velocity of the particle, and the modulus and direction of its total acceleration.

H.W



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Irodov questions

CIRCULAR MOTION

Q.5 A solid body rotates with deceleration about a stationary axis with an angular deceleration $\beta \propto \sqrt{\omega}$, where ω is its angular velocity. Find the mean angular velocity of the body averaged over the whole time of rotation if at the initial moment of time its angular velocity was equal to ω_0.

H.W

CIRCULAR MOTION

H-W

Q.6 A solid body starts rotating about a stationary axis with an angular acceleration $\beta =$ at, where $a = 2.0 \cdot 10^{-2} \mathrm{rad/s^3}$. How soon after the beginning of rotation will the total acceleration vector of an arbitrary point of the body form an angle $\alpha = 60^\circ$ with its velocity vector?