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Calculas based questions

KINEMATICS

Q.7 The relation between time t and distance x is $t = ax^2 + \beta x$ where α and β are constants. The retardation is

(A) $2\alpha v^3$

(B) $2\beta v^3$

(C) $2\alpha\beta v^3$

(D) $2b^2v^3$

α.

Differentiating w.r.t x.

$\frac{1}{v} = \left(\frac{dt}{dx}\right) = (\alpha . 2x + \beta(1))$

$\frac{1}{v} = (2\alpha x + \beta)$

Differentiating both Side w.r.t (t).

$\frac{d}{dt}\left(\frac{1}{v}\right) = 2\alpha\left(\frac{dx}{dt}\right)$

$\frac{d}{dv}\left(\frac{1}{v}\right) \times \left(\frac{dv}{dt}\right) = 2\alpha v.$

$\frac{d}{dv}\left(\frac{1}{v}\right) = \frac{d}{dv}(v^{-1})$

$= (-1)\, v^{-1-1}$

$= \left(-\frac{1}{v^2}\right)$

$-\frac{1}{v^2}(a) = 2\alpha v.$

$\boxed{a = -2\alpha v^3}$

Retardation

Magnitude $= 2\alpha v^3$

Calculas based questions

KINEMATICS

Q.10 For motion of an object along the x-axis, the velocity v depends on the displacement x as $v = (3x^2 - 2x)$, then what is the acceleration at $x = 2\ \text{m}$.

(A) $48\ \text{m s}^{-2}$

(B) $80\ \text{m s}^{-2}$

(C) $18\ \text{m s}^{-2}$

(D) $10\ \text{m s}^{-2}$

$v \rightarrow f(x)$.

$$\frac{dv}{dx} = 3\frac{d}{dx}(x^2) - 2\frac{d}{dx}(x).$$

$$= (6x - 2)$$

$$a = v\left(\frac{dv}{dx}\right)$$

$$a = (3x^2 - 2x)(6x - 2)$$

$$a_{x=2m} = [3(2)^2 - 2(2)][6 \times 2 - 2]$$

$$= 8 \times 10 = 80\ \text{m s}^{-2}$$

Q.11 A point moves in a straight line so that its displacement x metre at time t second is given by $x^2 = 1 + t^2$. Its acceleration in ms^{-2} at time t second is

(A) $\frac{1}{x^3}$

(B) $\frac{-t}{x^3}$

(C) $\frac{1}{x} - \frac{t^2}{x^3}$

(D) $\frac{1}{x} - \frac{1}{x^2}$

Differentiating both Side w.r.t time.

$$\frac{d}{dt}(x^2) = \frac{d}{dt}(1) + \frac{d}{dt}(t^2)$$

$$\frac{d}{dx}(x^2) \times \left(\frac{dx}{dt}\right) = 0 + 2t$$

$$\cancel{2}x \cdot (v) = \cancel{2}t$$

$$xv = t$$

$$v = \left(\frac{t}{x}\right)$$

$$\frac{dv}{dt} = \frac{d}{dt}\left(t/x\right) = \frac{x\frac{d}{dt}(t) - t\left(\frac{dx}{dt}\right)}{x^2}$$

$$a = \frac{x - t \times v}{x^2}$$

$$a = \frac{1}{x} - \frac{t}{x^2} \times \frac{t}{x} = \frac{1}{x} - \frac{t^2}{x^3}$$

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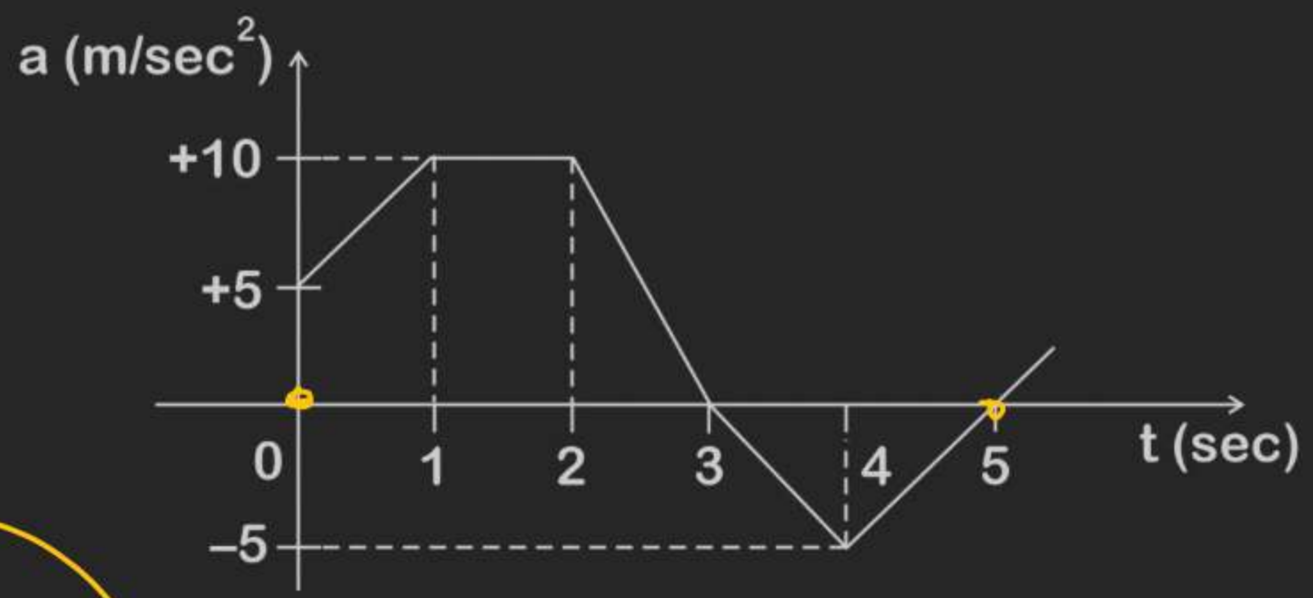
KINEMATICS

H-W.

Q. Acceleration-time graph is given in the figure. Find the change in velocity and average acceleration for the time interval $0 \rightarrow 5\text{sec.}$



$$a = \frac{dv}{dt}$$

$$\int_{t_1}^{t_2} a\,dt = \int_{v_i}^{v_f} dv$$

Area under the curve $= \Delta V$.

KINEMATICS

Q. The velocity of a particle at $t = 0$ is $\vec{u} = 4\hat{i} + 3\hat{j}\ \text{m/sec}$ and a constant acceleration is $\vec{a} = 6\hat{i} + 4\hat{j}\ \text{m/sec}^2$. Find the velocity and displacement of the particle at $t = 2\text{sec}$.

$$\vec{S} = \vec{u}t + \frac{1}{2}\vec{a}t^2$$

$$\vec{S} = (4\hat{i} + 3\hat{j})2 + \frac{1}{2}(6\hat{i} + 4\hat{j})4$$

$$\vec{S} = 8\hat{i} + 6\hat{j} + 12\hat{i} + 8\hat{j}$$

$$\boxed{\vec{S} = 20\hat{i} + 14\hat{j}}$$

$$\vec{v} = \vec{u} + \vec{a}t$$

$$\vec{v} = (4\hat{i} + 3\hat{j}) + (6\hat{i} + 4\hat{j})2$$

$$\vec{v} = (4\hat{i} + 3\hat{j}) + 12\hat{i} + 8\hat{j}$$

$$\boxed{\vec{v} = 16\hat{i} + 11\hat{j}}$$

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Kinematics Equation in Vector form :-

$$\boxed{\begin{aligned} \vec{v} &= \vec{u} + \vec{a}t \\ \vec{S} &= \vec{u}t + \frac{1}{2}\vec{a}t^2 \\ \vec{v}\cdot\vec{v} &= \vec{u}\cdot\vec{u} + 2\vec{a}\cdot\vec{S} \end{aligned}}$$

$$\vec{A}\cdot\vec{A} = A\cdot A\cos 0$$
$$= A^2$$

KINEMATICS

Q. The acceleration of a moving body at any time ' t ' is given by $\vec{a} = (4t)\hat{i} + (3t^2)\hat{j}\ m/sec^2$. If $\vec{u} = 0$ then find the velocity of the particle at 4sec.

Sol^n

$a_x \qquad a_y$

$\vec{u} = 0$

$u_x\hat{i} + u_y\hat{j} = 0\hat{i} + 0\hat{j}$

$a_x = 4t$

$\frac{dV_x}{dt} = 4t$

$\int_0^{V_x} dV_x = 4\int_0^t t\,dt$

$\Rightarrow \boxed{V_x = \frac{4t^2}{2} = 2t^2}$

$a_y = 3t^2$

$\frac{dV_y}{dt} = 3t^2$

$\int_0^{V_y} dV_y = 3\int_0^t t^2\,dt$

$\boxed{V_y = \frac{3t^3}{3} = t^3}$

$\vec{V} = V_x\hat{i} + V_y\hat{j}$

$\vec{V} = (2t^2)\hat{i} + t^3\hat{j}$

$\vec{V}_{t=4sec} = [2(4)^2]\hat{i} + (4)^3\hat{j}$

$= (32\hat{i} + 64\hat{j})$

$|\vec{V}|_{t=4sec} = \sqrt{(32)^2 + (32\times2)^2}$

$= 32\sqrt{5}\ m/s$

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Locus

$V_x = 2t^2$ | $V_y = t^3$

$\Downarrow$

$\frac{dx}{dt} = 2t^2$ | $\frac{dy}{dt} = t^3$

$\int_0^x dx = 2\int_0^t t^2 dt$ | $\int_0^y dy = \int_0^t t^3 dt$

$x = \left(\frac{2t^3}{3}\right)$ | $y = \left(\frac{t^4}{4}\right)$

$t^4 = (4y)$

$t = (4y)^{1/4}$

$$\boxed{x = \frac{2}{3}(4y)^{3/4}} \Rightarrow \text{locus}$$

2nd Method

$\vec{u} = 0$

$\vec{a} = (4t)\hat{i} + (3t^2)\hat{j}$

$\frac{d\vec{V}}{dt} = (4t)\hat{i} + (3t^2)\hat{j}$

$\int_0^V d\vec{V} = 4\int_0^4 (t\,dt)\hat{i} + 3\int_0^4 (t^2 dt)\hat{j}$

$\vec{V} = ??$

Irodov.

H.W

KINEMATICS

Q. A point moves in the $(x-y)$ plane according to the law $x = a\sin\omega t, y = a(1-\cos\omega t)$, where ' a ' and ω are positive constant. Find :

(a) the distance' s ' traversed by the point during the time ' t '.

(b) the angle between the point's velocity and acceleration vectors.

KINEMATICS

Note :- For trajector, Locus or path we Calculate $\boxed{y \to f(x)}$

Q. A particle moves in xy plane with a velocity given by $\vec{v} = (8t - 2)\hat{i} + 2\hat{j}$. If it passes through the point $(14, 4)$ at $t = 2sec$, then give equation of the path.

Solⁿ :-

$V_x = (8t - 2)$

$\frac{dx}{dt} = (8t-2)$

$\int dx = \int (8t-2)dt$

$x = [8\int t\,dt - 2\int dt] + c$

$x = \left(8\left(\frac{t^2}{2}\right) - 2t\right) + c$

$x = 4t^2 - 2t + c$

At $t = 2sec$, $x = 14$

$14 = 16 - 4 + c$

$14 = 12 + c$

$\boxed{c = 2}$

$V_y = 2$

$\frac{dy}{dt} = 2$

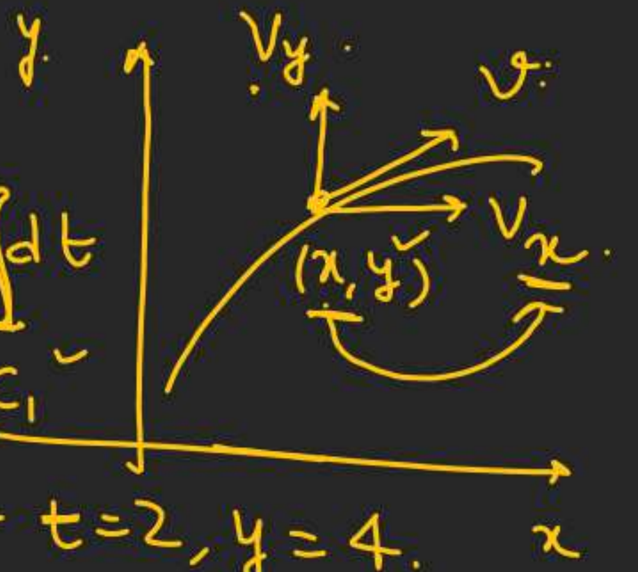
$\int dy = 2\int dt$

$y = 2t + c_1$

At $t = 2$, $y = 4$.

$4 = 4 + c_1$

$\boxed{c_1 = 0}$



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$$x = 4t^2 - 2t + 2 \quad \Big| \quad y = 2t$$

$$t = \left(\frac{y}{2}\right)$$

$$x = \cancel{4}\left(\frac{y^2}{\cancel{4}}\right) - 2\left(\frac{y}{2}\right) + 2$$

$$\boxed{x = y^2 - y + 2}$$

$\hookrightarrow$ Parabolic function

KINEMATICS

Q. A particle having a velocity $v = v_0$ at $t = 0$ is decelerated at the rate $|a| = \alpha\sqrt{v}$, where α is a positive constant.

(a) The particle comes to rest at $t = \frac{2\sqrt{v_0}}{\alpha}$

(b) The particle will come to rest at infinity.

(c) The distance travelled by the particle is $\frac{2v_0^{3/2}}{\alpha}$.

(d) The distance travelled by the particle is $\frac{2}{3}\frac{v_0^{3/2}}{\alpha}$.

Mark the correct option:

$$a = -\alpha\sqrt{v}$$

$$\Downarrow$$

$$\frac{dv}{dt} = -\alpha\sqrt{v}$$

$$\int_{v_0}^{v} \frac{dv}{\sqrt{v}} = -\alpha\int_0^t dt$$

$$\int_{v_0}^{v} v^{-1/2}\,dv = -\alpha t$$

$$\frac{\left[v^{-\frac{1}{2}+1}\right]_{v_0}^{v}}{\left(-\frac{1}{2}+1\right)} = -\alpha t$$

$\int a\,dt$

$a \to f(v)$ $\quad$ $v \to f(t)$

$$\left[\sqrt{v}\right]_{v_0}^{v} = -\frac{\alpha t}{2}$$

$$\boxed{\sqrt{v} - \sqrt{v_0} = \frac{-\alpha t}{2}}$$

$$\Downarrow$$

$$0 + \sqrt{v_0} = +\frac{\alpha t}{2}$$

$$t = \frac{2\sqrt{v_0}}{\alpha}$$

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$$a = -\alpha\sqrt{v}$$

$$\Downarrow$$

$$v\frac{dv}{ds} = -\alpha\sqrt{v}$$

$$\int_{v_0}^{0}\sqrt{v}\,dv = -\alpha\int_{0}^{s}ds$$

$$\int_{v_0}^{0} v^{1/2}\,dv = (-\alpha s)$$

$$\frac{\left[v^{3/2}\right]_{v_0}^{0}}{3/2} = -\alpha s$$

$$\frac{2}{3}\left[0 - v_0^{3/2}\right] = -\alpha s$$

$$\cancel{-}\frac{2}{3}v_0^{3/2} = \cancel{-}\alpha s$$

$$\boxed{s = \frac{2}{3\alpha}\left(v_0^{3/2}\right)}$$

KINEMATICS

H.W:

Q. (i) A particle is moving in three dimension. Its position vector is given by

$$\vec{r} = 6\hat{i} + (3 + 4t)\hat{j} - \left(3 + 2t - t^2\right)\hat{k}$$

Distance are in meters, and the time, t, in seconds.

(a) What is the velocity vector at $t = +3$?

(b) What is the speed (in m/sec) at $t = +3$?

(c) What is the acceleration vector and what is its magnitude (in m/sec^2) at $t = +3$?

(ii) Now the particle is moving only along the z-axis, and its position is given by, $\left(t^2 - 2t - 3\right)\hat{k}$ at what time does the particle stand still?

H.W :

Q. A motor boat of mass ' m ' moves along a lake with velocity v_0. At the moment $t = 0$ the engine of the boat is shut down. Assuming the resistance of water to be proportional to the velocity of the boat $F = -kv$, find-

(a) how long the motorboat moved with the shut down engine.

(b) the velocity of the motor boat as a function of the distance covered with the shut-down engine, as well as the total distance covered till it stops completely.

(c) the mean velocity of the motor boat over the time interval (beginning with the moment $t = 0$), during which its velocity decreases to $(1/\eta)$ times.