Nishant Jindal

Pr-3

Q $\int_0^4 [\sqrt{x}]\,dx$

$x \to 0-4$

$\sqrt{x} \to 0-2$

$\sqrt{x} \to 0-1-2$

$x \to 0-1-4$

$$= \int_0^1 0\,dx + \int_1^4 1\,dx$$

$$= 0 + 1\cdot(x)_1^4 = 3$$

Q $\int_0^1 [4x]\,dx$

M2

$4x = t$, $dx = \frac{dt}{4}$

x	t
0	0
1	4

$$= \frac{1}{4}\int_0^4 [t]\,dt$$

$$= \frac{1}{4}\left\{ \int_0^1 0\,dt + \int_1^2 1\,dt + \int_2^3 2\,dt + \int_3^4 3\,dt \right\}$$

$$= \frac{1}{4}\left[0 + 1\cdot(t)_1^2 + 2(t)_2^3 + 3(t)_3^4\right]$$

$$= \frac{1}{4}\left[0 + 1\cdot(2-1) + 2(3-2) + 3(4-3)\right]$$

$$= \frac{1}{4}(0+1+2+3) = \frac{6}{4} = \frac{3}{2}$$

Q $\int_0^1 \sin([x]+[2x])\,dx$

$x \to 0-1$

$2x \to 0-2$

$= \quad 2x \to 0-1-2$

limit $x \to 0-\frac{1}{2}-1$

$$\Rightarrow \int_0^{1/2} \sin(0+0)\,dx + \int_{1/2}^1 \sin(0+1)\,dx$$

$$0 + \sin 1\cdot(x)_{1/2}^1$$

$$= \frac{\sin 1}{2} \text{ Ans}$$

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① $\int_0^1 \{3x\}-7 \cdot dx$

$\Rightarrow \int_0^1 3x-[3x]-7 \cdot dx \qquad 3x=t$

$\Rightarrow \frac{3x^2}{2}\Big|_0^1 - 7x\Big|_0^1 - \int_0^1 [3x] \cdot dx$

$\Rightarrow \left(\frac{3}{2}-0\right)-7(1-0)-\frac{1}{3}\int_0^3 [t]\,dt$

$= \frac{3}{2}-7-\frac{1}{\cancel{3}}\left\{\frac{(\cancel{3})(3-1)}{2}\right\}$

$= \frac{3}{2}-8 = -\frac{13}{2}$

Q $\int_0^1 (\{2x\}-1)(\{3x\}-7) \cdot dx$

$\Rightarrow \int_0^1 (2x-[2x]-1)(3x-[3x]-7)\,dx$

$x \to 0-1$

$2x \to 0-2 \qquad 3x \to 0-3$

$2x \to 0-1-2 \qquad 3x \to 0-1-2-3$

$x \to 0-\frac{1}{2}-1 \qquad x \to 0-\frac{1}{3}-\frac{2}{3}-1$

$x \to \frac{2}{3}-1$

$2x \to \frac{4}{3}-2$

$3x \to 2-3$

$\Rightarrow \int_0^{1/3} (2x-0-1)(3x-0-7)\,dx + \int_{1/3}^{1/2} (2x-0-1)(3x-1-7)\,dx + \int_{1/2}^{2/3} (2x-1-1)(3x-1-7)\,dx + \int_{2/3}^{1} (2x-1-1)(3x-2-7)\,dx$

Multiply & Solve

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Qs of [f(x)] Based on Graph.

Q $\int_0^\infty \left[\frac{2}{e^x}\right] dx$

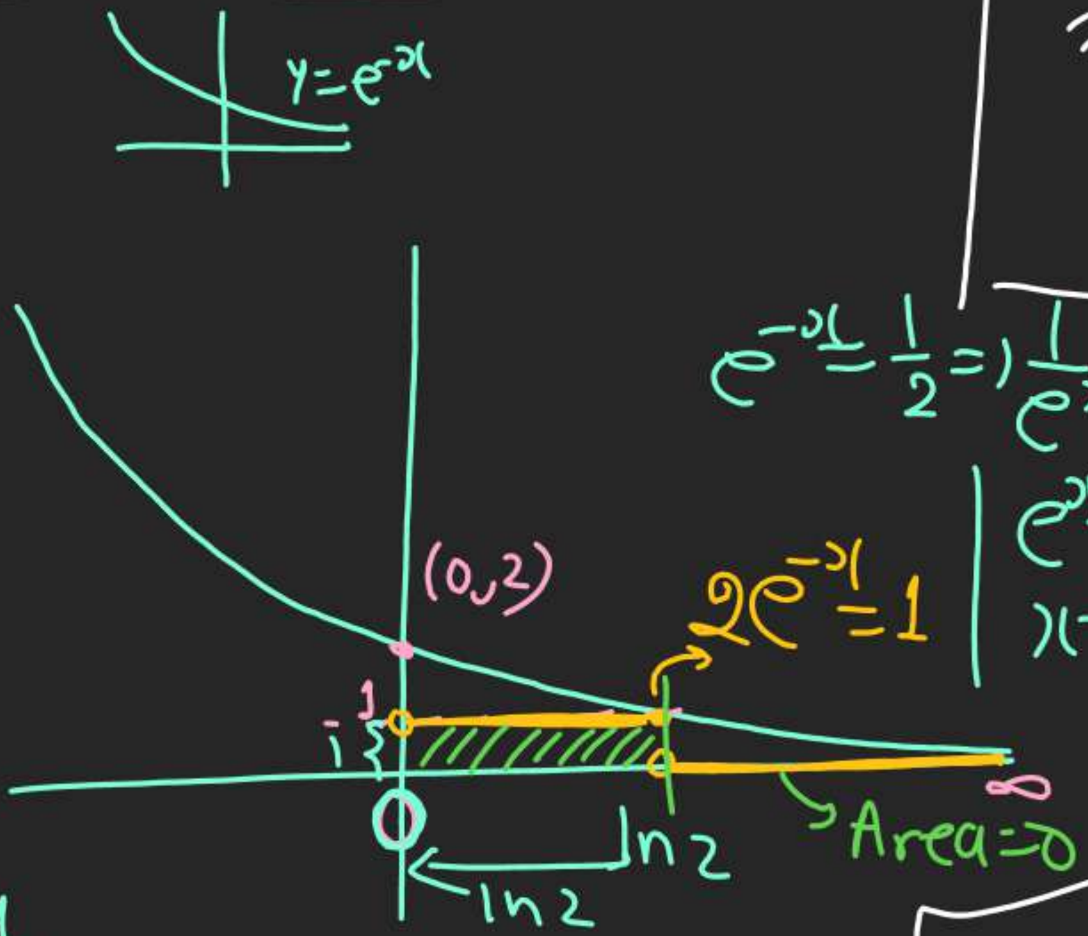
$\Rightarrow \int_0^\infty [2 \cdot e^{-x}]\, dx$

$x \to 0 - \infty$

$= \text{Area}$

$= \ln 2 \times 1$

$= \ln 2$



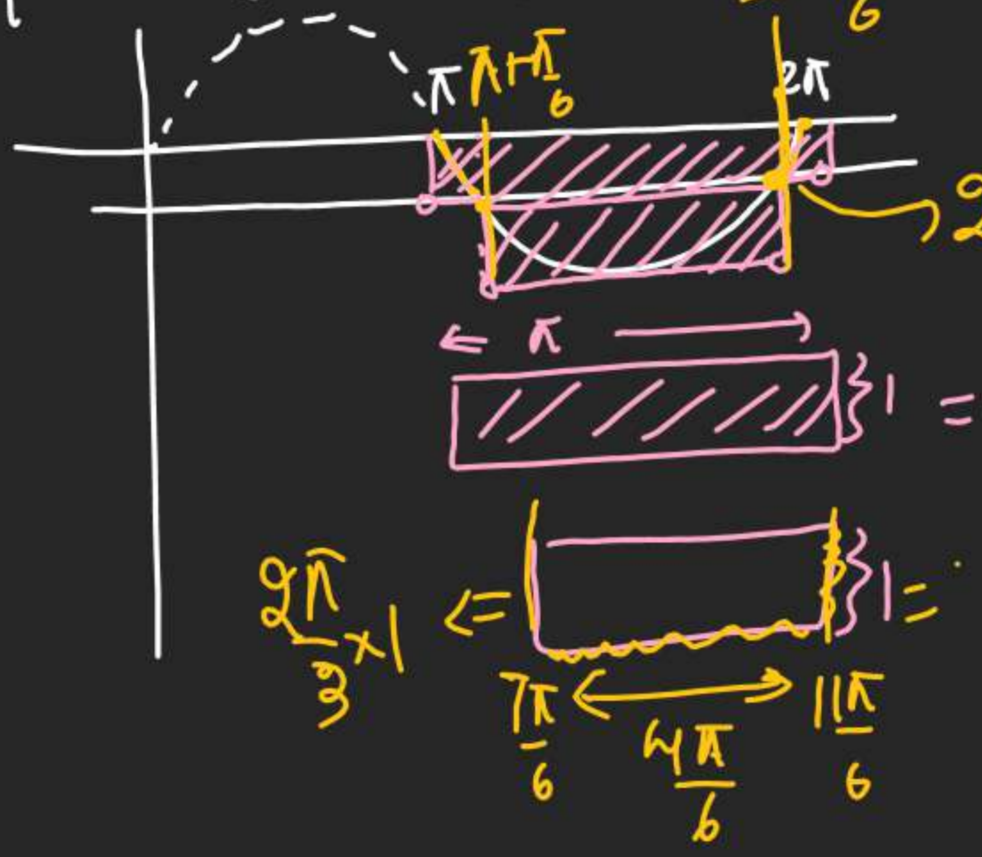
$e^{-x} = \frac{1}{2} \Rightarrow \frac{1}{e^x} = \frac{1}{2}$

$2e^{-x} = 1$

$e^x = 2$

$x = \ln 2$

Q $\int_\pi^{2\pi} [2\sin x] \cdot dx$



$2\sin x = -1$

$\sin x = -\frac{1}{2}$

$x = \pi + \frac{\pi}{6}$

$x = 2\pi - \frac{\pi}{6}$

Total Area $= -\left(\pi + \frac{2\pi}{3}\right) = -\left(\frac{5\pi}{3}\right)$

Below X Axis

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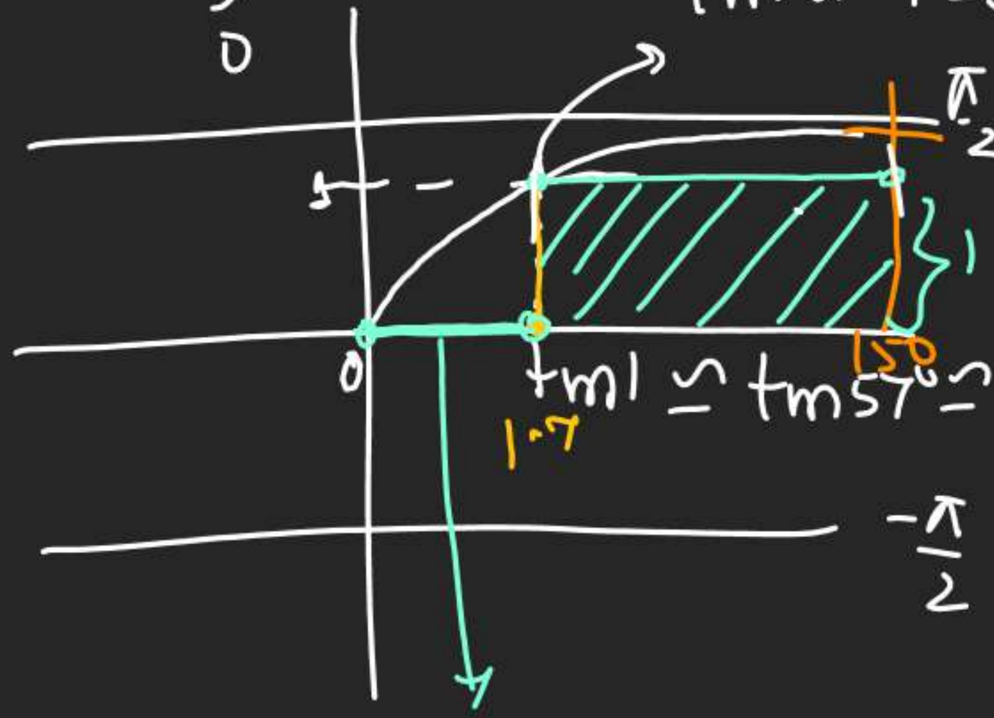
Q $\int_0^{150} [\tan^{-1}x]\, dx$

$\tan^{-1}x = 1 \Rightarrow x = \tan 1$



$\tan 1 \simeq \tan 57° \simeq \tan 60° = 1.7$

Area $= 0 + (150 - \tan 1) \times 1$

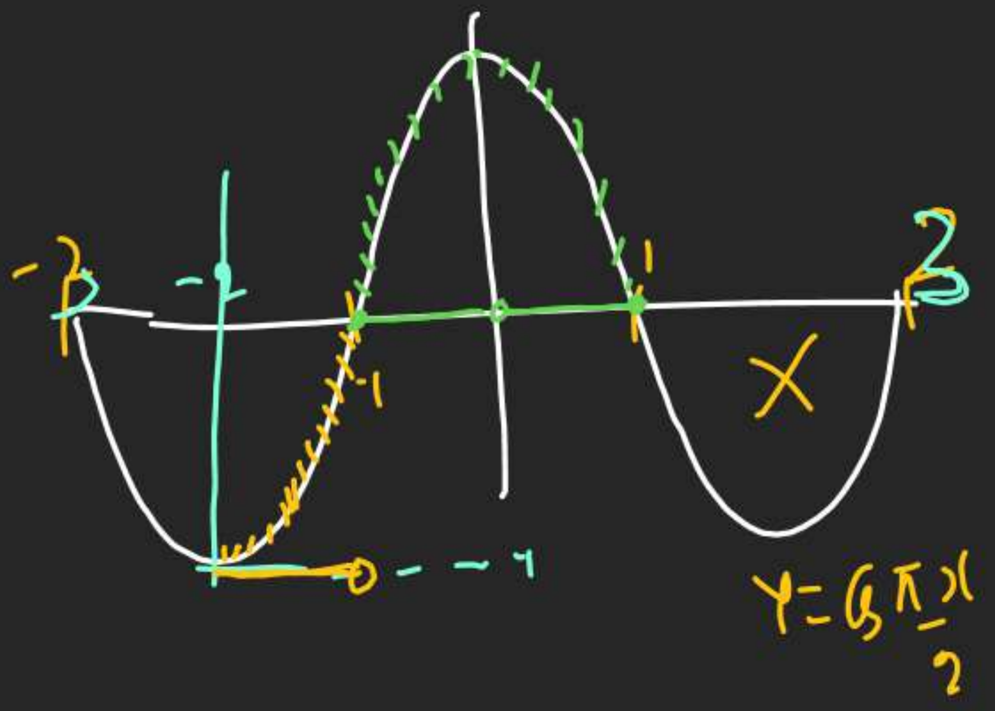
$= 150 - \tan 1$ Ans

Q $\int_{-2}^{1} \left[x\left[1+\cos\frac{\pi x}{2}\right]+1\right] dx$

$\cos x$ is 1-1 onto $\rightarrow \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$\cos\frac{\pi x}{2} \longrightarrow \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, $\frac{-\pi x}{2}$, $\frac{\pi}{2}$

$\Leftarrow (-1, 1)$



$\int_{-2}^{-1}\left[x\left[1+(-1)\right]+1\right]dx + \int_{-1}^{1}\left[x\left[1+0\right]+1\right]dx$

$\int_{-2}^{-1} dx + \int_{-1}^{1} [x+1]\, dx = (x)_{-2}^{-1} + \int_{-1}^{1} [x]\, dx + (x)_{-1}^{1}$

$= 1 + 2 + \int_{-1}^{1} [x]\, dx = 1 + 2 - 1 = 2$

$\int_{-1}^{0} -1\, dx + \int_0^1 0 \cdot dx$

$-(x)_{-1}^{0}$

$-(0+1)$

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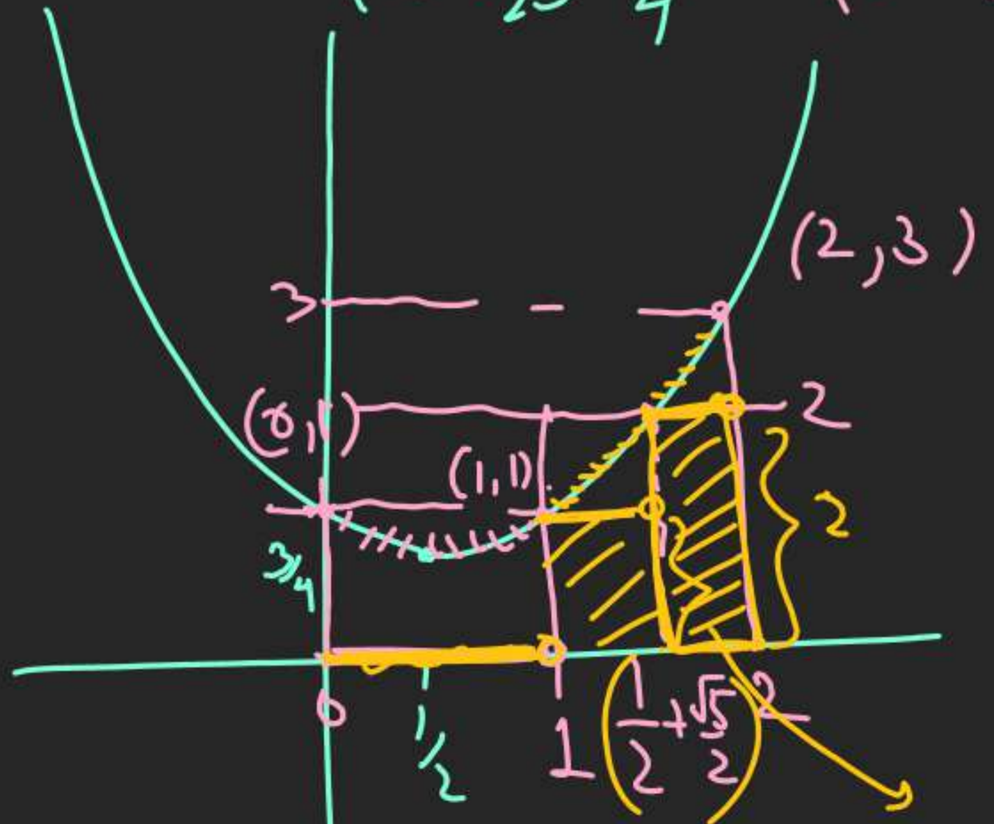
Q $\int_0^2 [x^2-x+1]\,dx$

$(x-1/2)^2+\frac{3}{4}$

$(x-1/2)^2+\frac{3}{4}=2$

$(x-1/2)^2=\frac{5}{4}=\left(\frac{\sqrt5}{2}\right)^2$

$x=\frac{1}{2}+\frac{\sqrt5}{2}$



Area $= 0 + 1\times\left(\frac{1}{2}+\frac{\sqrt5}{2}-1\right) + 2\times\left(2-\left(\frac{1}{2}+\frac{\sqrt5}{2}\right)\right)$

Pr4 (King's Property)

$$\text{①}\quad \int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$$

$$\text{Aox. ②}\quad \int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$$

King Kab lagau?

[ ] { }, Sym, defined fxn Na ho to Bche hue Sabhi fxn Pe rsbse Phle King hi try Karte huin

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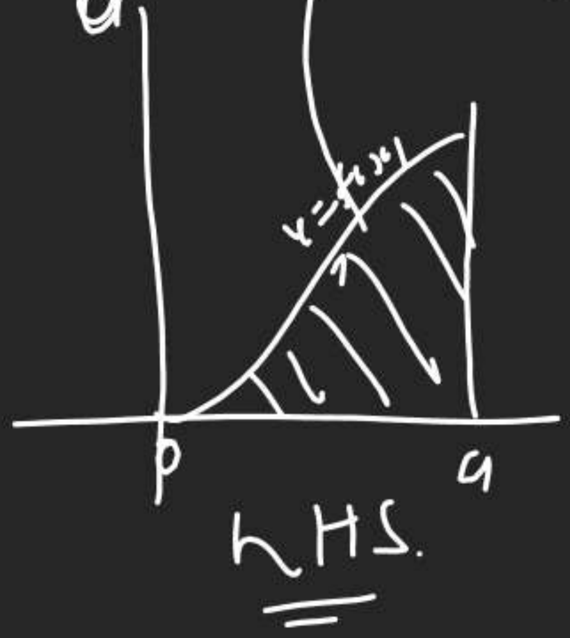
Q $\int_0^a f(x)\cdot dx = \int_0^a f(a-x)\cdot dx$



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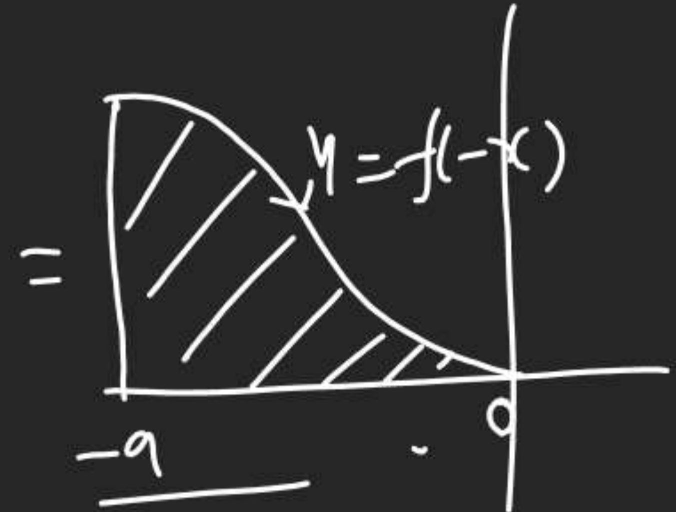

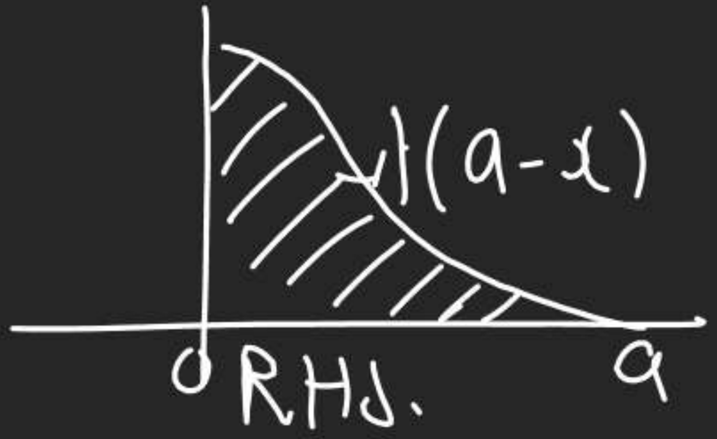

Q $I = \int_0^{\pi/2} \log\left(\frac{4+3\sin x}{4+3\cos x}\right)\cdot dx \to A$

Prop $\left(x \to \frac{\pi}{2} - x\right)$

$I = \int_0^{\pi/2} \log\left(\frac{4+3\sin\left(\frac{\pi}{2}-x\right)}{4+3\cos\left(\frac{\pi}{2}-x\right)}\right)\cdot dx$

$I = \int_0^{\pi/2} \log\left(\frac{4+3\cos x}{4+3\sin x}\right)\cdot dx \to B$

King & Add

$A+B$

$2I = \int_0^{\pi/2} \log\left\{\left(\frac{\cancel{4+3\sin x}}{\cancel{4+3\cos x}}\right)^{1}\times\left(\frac{\cancel{4+3\cos x}}{\cancel{4+3\sin x}}\right)\right\} dx$

$= 0 \Rightarrow I = 0$

Q $I = \int_0^{\pi} \frac{dx}{1+3^{\cos x}}$

Nishant Jindal

Q $I = \int_0^{\pi} \frac{1 \cdot dx}{1+3^{6x}} \to A$

Pr 4 $\downarrow (x \to \pi - x)$ King

$$= \int_0^{\pi} \frac{dx}{1+3^{6(\pi-x)}}$$

$$= \int_0^{\pi} \frac{dx}{1+3^{-6x}}$$

A+B

$$I = \int_0^{\pi} \frac{3^{6x} \cdot dx}{1+3^{6x}} \to B$$

$$2I = \int_0^{\pi} \frac{\cancel{1+3^{6x}}}{\cancel{1+3^{6x}}} dx = (x)_0^{\pi} = \pi$$

$$I = \frac{\pi}{2}$$

Q $I = \int_0^{\pi/2} \frac{\sin 8x \cdot \log \cot x}{\cot 2x} \cdot dx \to A$

Pr 4 $\downarrow (x \to \frac{\pi}{2} - x)$

$$I = \int_0^{\pi/2} \frac{\sin 8(\frac{\pi}{2}-x) \cdot \log(\cot(\frac{\pi}{2}-x))}{\cot 2(\frac{\pi}{2}-x)}$$

$$= \int_0^{\pi/2} \frac{\sin(4\pi - 8x) \cdot \log \tan x}{\cot(\pi - 2x)} dx$$

$$I = \int_0^{\pi/2} \frac{+\sin 8x \log \tan x}{+\cot 2x} dx \to B$$

King + Add

$$2I = \int_0^{\pi/2} \frac{\sin 8x \cdot \log \cot x + \sin 8x \cdot \log \tan x}{\cot 2x} \cdot dx = \int_0^{\pi/2} \frac{\sin 8x (\log(\overset{1}{\cancel{\cot x \times \tan x}}))}{\cot 2x} dx$$

$2I = 0 \Rightarrow I = 0$

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Q $I=\int_0^{\pi} e^{\sin^2 x}\cdot \tan^2 x\cdot \cos^3(2n+1)x\cdot dx$ $n\in I$

↓ Put $n=1$

$I=\int_0^{\pi} e^{\sin^2 x}\tan^2 x\cdot \cos^3(3x)\,dx \to (A)$

P4 ↓ $(x\to \pi-x)$

$=\int_0^{\pi} e^{(\sin x)^2}(-\tan x)^2(-\cos 3x)^3\cdot dx$

(A+B) $I=-\int_0^{\pi} e^{\sin^2 x}\tan^2 x\,\cos^3 3x\cdot dx \to (B)$

$2I=0 \Rightarrow I=0$

Q $I=\int_0^{\pi/2}\frac{\cos x-\sin x}{1+\sin x\cdot\cos x}\cdot dx$

Q $I=\int_0^{\pi/2}\frac{\sin^3 x-\cos^3 x}{\sin^3 x+\cos^3 x}\cdot dx$

Q $I=\int_0^{\pi/2}\frac{a\sin x+b\cos x}{\sin x+\cos x}\cdot dx$

Q $I=\int_0^{\pi/2}\frac{\sin^n x}{\sin^n x+\cos^n x}\cdot dx$

1) Base has $\sin$ & $\cos x$

2) Limit is $(0,\frac{\pi}{2})$

3) King → $\sin x\to\cos x$, $\cos x\to\sin x$

(1) $I=\int_0^{\pi/2}\frac{\cos x-\sin x}{1+\sin x\cdot\cos x}\cdot dx \to (A)$

P4 $(x\to \frac{\pi}{2}-x)$

$=\int_0^{\pi/2}\frac{\sin x-\cos x}{1+\cos x\sin x}\cdot dx \to (B)$

(A)+(B)

$\Rightarrow 2I=\int_0^{\pi/2}\frac{\cancel{\sin x}-\cancel{\cos x}+\cancel{\cos x}-\cancel{\sin x}}{1+\sin x\cdot\cos x}dx$

$=0$

$\boxed{I=0}$

Q $I = \int_0^{\pi/2} \frac{\sin^n x \cdot dx}{\sin^n x + \cos^n x} \quad (n \in R)$

$\downarrow$ pr 4 $\left(x \to \frac{\pi}{2} - x\right)$

$$I = \int_0^{\pi/2} \frac{\cos^n x \cdot dx}{\cos^n x + \sin^n x}$$

$$2I = \int_0^{\pi/2} \frac{\cancel{\sin^n x + \cos^n x} \cdot dx}{\cancel{\sin^n x + \cos^n x}}$$

$$2I = (x)_0^{\pi/2} = \frac{\pi}{2}$$

$$I = \frac{\pi}{4}$$

Set 1 (Result)

1) $\int_0^{\pi/2} \frac{\sin^n x \cdot dx}{\sin^n x + \cos^n x} = \frac{\pi}{4}$

2) $\int_0^{\pi/2} \frac{\cos^n x \cdot dx}{\sin^n x + \cos^n x} = \frac{\pi}{4}$

3) $\int_0^{\pi/2} \frac{dx}{1 + \tan^n x} = \frac{\pi}{4}$

4) $\int_0^{\pi/2} \frac{dx}{1 + \cot^n x} = \frac{\pi}{4}$

5) $\int_0^{\pi/2} \frac{\sec^n x \cdot dx}{\sec^n x + \operatorname{cosec}^n x} = \frac{\pi}{4}$

Q $\int_0^{\pi/2} \frac{dx}{1 + \sqrt{\cot x}}$

$= \frac{\pi}{4}$

Q $\int_0^{\pi/2} \frac{dx}{1 + \tan^{1/3} x}$

$= \frac{\pi}{4}$

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Q $I = \int_0^{\pi/2} \log \tan\theta \cdot d\theta \rightarrow (A)$

Prt $\downarrow (\theta \to \frac{\pi}{2} - \theta)$

$I = \int_0^{\pi/2} \log (\cot\theta \cdot d\theta \rightarrow (B)$

King & Add

$2I = \int_0^{\pi/2} \log(\cancel{\tan\theta} \times \cancel{\cot\theta}) \cdot d\theta$

$2I = 0$

$I = 0$

Q $I = \int_0^{\pi/4} \log(1+\tan\theta)\, d\theta$

Prt $\downarrow \theta \to \frac{\pi}{4} - \theta$

$= \int_0^{\pi/4} \log\left(1+\tan\left(\frac{\pi}{4}-\theta\right)\right) \cdot d\theta$

$= \int_0^{\pi/4} \log\left(1 + \frac{1-\tan\theta}{1+\tan\theta}\right) \cdot d\theta$

$= \int_0^{\pi/4} \log\left(\frac{2}{1+\tan\theta}\right) dx$

$I = \int_0^{\pi/4} \log 2\, d\theta - \int_0^{\pi/4} \log(1+\tan\theta) \cdot d\theta$

$I = \log 2 (\theta)_0^{\pi/4} - I \Rightarrow 2I = \frac{\pi}{4} \log 2$

$\boxed{I = \frac{\pi}{8} \ln 2}$

Set 2

1) $\int_0^{\pi/2} \ln \tan\theta \cdot d\theta = \int_0^{\pi/2} \ln \cot\theta \cdot d\theta = 0$

2) $\int_0^{\pi/4} \ln(1+\tan\theta)\, d\theta = \frac{\pi}{8} \ln 2$

Q $\int_0^1 \frac{\log(1+x)}{1+x^2} \cdot dx$

$\boxed{x = \tan\theta}$

$dx = \sec^2\theta \cdot d\theta$

x	θ
0	0
1	$\frac{\pi}{4}$

$I = \int_0^{\pi/4} \frac{\log(1+\tan\theta) \cdot \cancel{\sec^2\theta}\, d\theta}{\cancel{(1+\tan^2\theta)}}$

$= \frac{\pi}{8} \ln 2$

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Q $I = \int_0^{\infty} \frac{x \cdot dx}{(1+x)(1+x^2)}$

$x = \tan\theta$

$dx = \sec^2\theta \cdot d\theta$

x	θ
0	0
∞	$\frac{\pi}{2}$

$I = \int_0^{\pi/2} \frac{\tan\theta \cdot \cancel{\sec^2\theta} \cdot d\theta}{(1+\tan\theta)\cancel{(1+\tan^2\theta)}}$

$= \int_0^{\pi/2} \frac{\sin\theta \cdot d\theta}{\sin\theta + \cos\theta}$ Set 1

$I = \frac{\pi}{4}$

Q HW. $\int_0^{a} \frac{dx}{x + \sqrt{a^2 - x^2}}$

Objective in D. I

1, 2, 3 — 12

15 - 37

} 10 Qs good