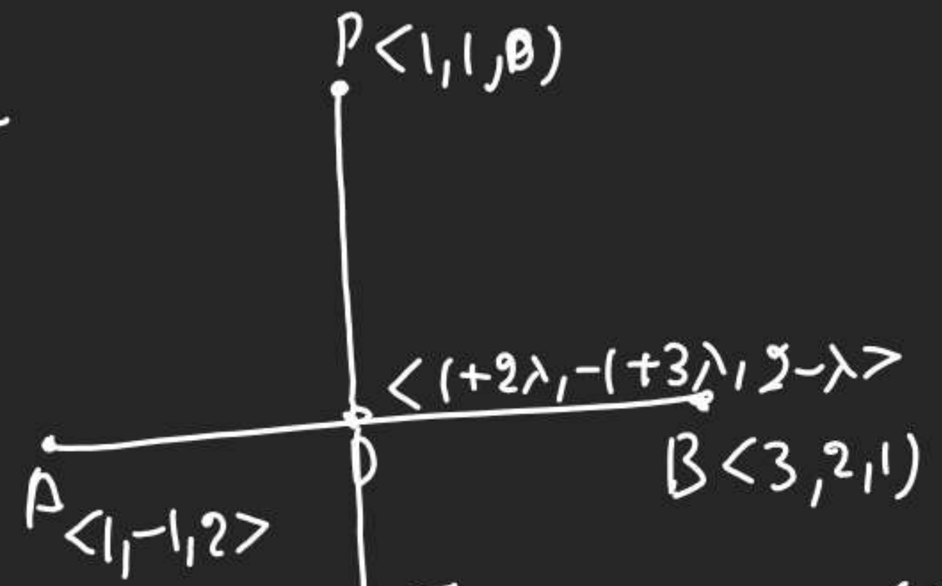
Nishant Jindal

Q① Foot of $\perp^n$ of Pt $\langle 1,1,0\rangle$ in Line Joining $\langle 1,-1,2\rangle$ & $\langle 3,2,1\rangle$

② find Image of $\langle 1,1,0\rangle$ in Above Line.

Parallelopipe



① Eqⁿ of AB $\Rightarrow \vec{r} = \langle 1,-1,2\rangle + \lambda\langle 2,3,-1\rangle$

$$\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-2}{-1}$$ } Cart. form of Line AB.

② Gen Pt $= D = \langle 1+2\lambda, -1+3\lambda, 2-\lambda\rangle$

(3) Dr of $PD = \langle 2\lambda, -2+3\lambda, 2-\lambda\rangle$

(4) $PD \perp AB \Rightarrow \langle 2\lambda, -2+3\lambda, 2-\lambda\rangle \cdot \langle 2,3,-1\rangle$

$4\lambda - 6 + 9\lambda - 2 + \lambda = 0$ $\quad 14\lambda = 8 \Rightarrow \lambda = \frac{4}{7}$

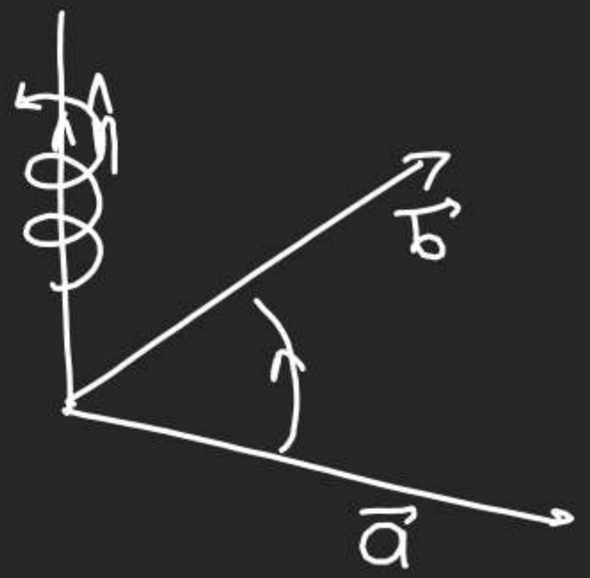
$\therefore D = \langle 1+\frac{8}{7}, -1+\frac{12}{7}, 2-\frac{4}{7}\rangle = \langle \frac{15}{7}, \frac{5}{7}, \frac{10}{7}\rangle$

$\frac{\alpha+1}{2} = \frac{15}{7}$ | $\frac{\beta+1}{2} = \frac{5}{7}$ | $\frac{\gamma+0}{2} = \frac{10}{7}$

$\alpha = \frac{30}{7} - 1$ | $\beta = \frac{10}{7} - 1$ | $\gamma = \frac{20}{7}$

$P' = \langle \frac{23}{7}, \frac{3}{7}, \frac{20}{7}\rangle$

Cross Product



1) If $\vec{a}$ & $\vec{b}$ are 2 vectors then their Cross Product will be $= \vec{a} \times \vec{b}$

(2) $\boxed{\vec{a} \times \vec{b} = |a||b| \sin\theta \cdot \hat{n}}$

$0^\circ \leq \theta < 180^\circ$

$1^{st}/2^{nd}$

$\sin\theta = +ve$

(3) $|\vec{a} \times \vec{b}| = \left| |a||b| \sin\theta\, \hat{n} \right|$

$= ||a||\,||b||\,|\sin\theta|\,|\hat{n}|$

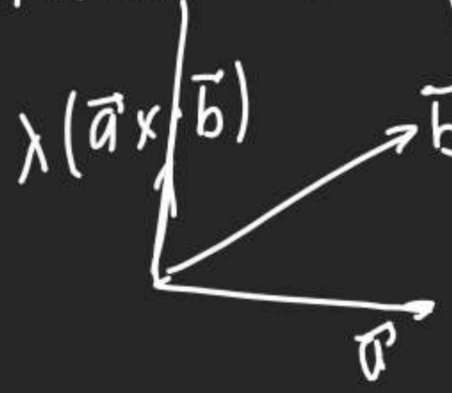
$= |a|\,|b| \sin\theta \cdot 1$

$|a||b| \sin\theta = |\vec{a} \times \vec{b}|$

(4) $\hat{n} = \dfrac{\vec{a} \times \vec{b}}{|a||b| \sin\theta}$

$\hat{n} = \dfrac{\vec{a} \times \vec{b}}{|\vec{a} \times \vec{b}|}$

$\hat{n}$ is Unit vector of $\vec{a} \times \vec{b}$



(5) $\sin\theta = \dfrac{|\vec{a} \times \vec{b}|}{|a||b|}$

& $\cos\theta = \dfrac{\vec{a} \cdot \vec{b}}{|a||b|}$

$S^2 + C^2 = 1$

$\dfrac{|a \times b|^2}{|a|^2 |b|^2} + \dfrac{(a \cdot b)^2}{|a|^2 |b|^2} = 1$ Lagrange's Identity

$$\boxed{\begin{aligned} |a \times b|^2 + (\vec{a} \cdot \vec{b})^2 &= |a|^2 |b|^2 \\ (\vec{a} \cdot \vec{b})^2 &= |a|^2 |b|^2 - |a \times b|^2 \end{aligned}}$$

It is establishing Relation betn Cross product

Iskull

$|a|^2 |b|^2 (1 - \sin^2\theta)$

Nishant Jindal

$$\boxed{(a\times b)^2 = \begin{vmatrix} \vec a\cdot\vec a & \vec a\cdot\vec b \\ \vec a\cdot\vec b & \vec b\cdot\vec b \end{vmatrix}}$$

L. Identity.

(6) 2 Lines.

$$\left.\begin{aligned} \vec r &= \boxed{\vec c} + \lambda\vec p \\ \vec r &= \boxed{\vec c} + \mu\vec q \end{aligned}\right\}$$ finding Line ⊥ to Both Line?

Both lines are Intersecting

at fix pt $\vec r = \vec c + t(\vec p\times\vec q)$

(7) $\vec a\times\vec b \neq \vec b\times\vec a$ (Generally)

But $\vec a\cdot\vec b = \vec b\cdot\vec a$.

Lekin $\vec a\times\vec b = -(\vec b\times\vec a)$

& $|a\times b| = |b\times a|$

(8) $\vec a\times\vec b = ?$

$\vec a = a_1\hat i + a_2\hat j + a_3\hat k$

$\vec b = b_1\hat i + b_2\hat j + b_3\hat k$

$$a\times b = \begin{vmatrix} i & j & k \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

Q $\vec a = \hat i - \hat j + \hat k$, $\vec b = \hat j + 3\hat k$

$a\times b = ?$

$$a\times b = \begin{vmatrix} i & j & k \\ 1 & -1 & 1 \\ 0 & 1 & 3 \end{vmatrix} = \langle -4, -3, 1\rangle = -4\hat i - 3\hat j + \hat k$$

(9) $\vec a\times\vec b$ is ⊥ to $\vec a$ & $\vec b$ both.

$(\vec a\times\vec b)\cdot\vec a = 0$

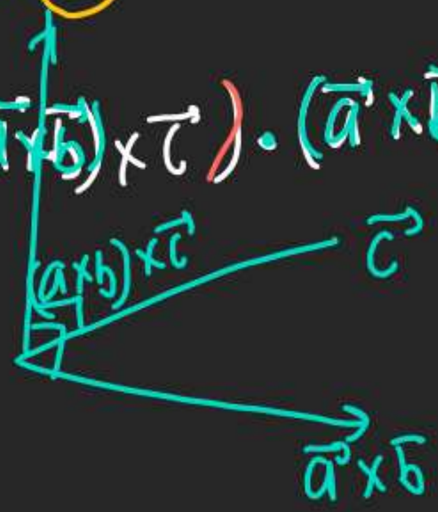
$(\vec a\times\vec b)\cdot\vec b = 0$

$\vec a\cdot(\vec a\times\vec b) = 0$

Q $((\vec a\times\vec b)\times\vec c)\cdot\vec c = ?$

$= 0$

Q $((\vec a\times\vec b)\times\vec c)\cdot(\vec a\times\vec b) = 0$



Nishant Jindal

Q If $|a| = 1, |b| = 1, |c| = 2$

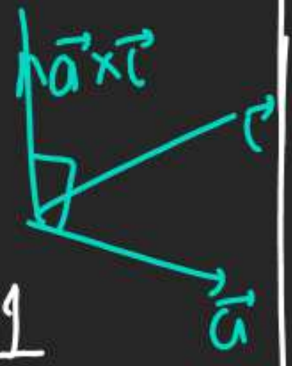
find angle betⁿ $\vec{a}$ & $\vec{c}$

if $\vec{a}\times(\vec{a}\times\vec{c}) - \vec{b} = 0$

$\vec{a}\times(\vec{a}\times\vec{c}) = \vec{b}$

$|a\times(a\times c)| = |b|$

$|a|\,|a\times c|\cdot\sin 90° = 1$



$|a\times c| = 1$

$|a||c|\sin\theta = 1$

$1\times 2\sin\theta = 1$

$\sin\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{6}$

$$\boxed{\vec{a}\,\hat{}\,\vec{c} = \frac{\pi}{6}}$$

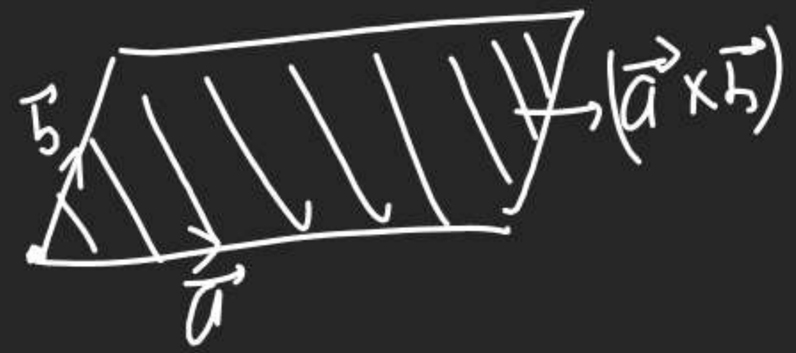
$\vec{a}\cdot(\vec{m}\times\vec{a}) = 0$

as $\vec{a}\perp\vec{m}\times\vec{a}$

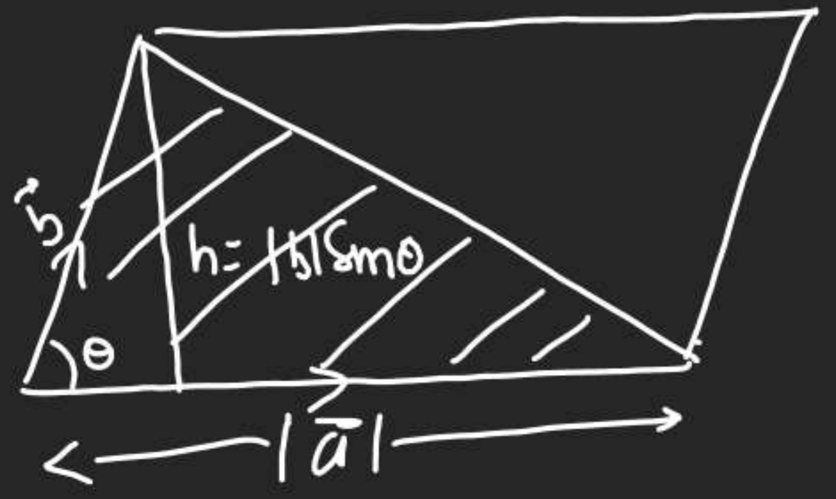
(10) <u>Geometrical Interpretation of $\vec{a}\times\vec{b}$</u>

1) $(\vec{a}\times\vec{b})$ is vector area of ||gm having adjacent side $\vec{a}$ & $\vec{b}$

2)



(3)



$\triangle$ area $= \frac{1}{2}\times|a||b|\sin\theta$

$= \frac{1}{2}|\vec{a}\times\vec{b}|$

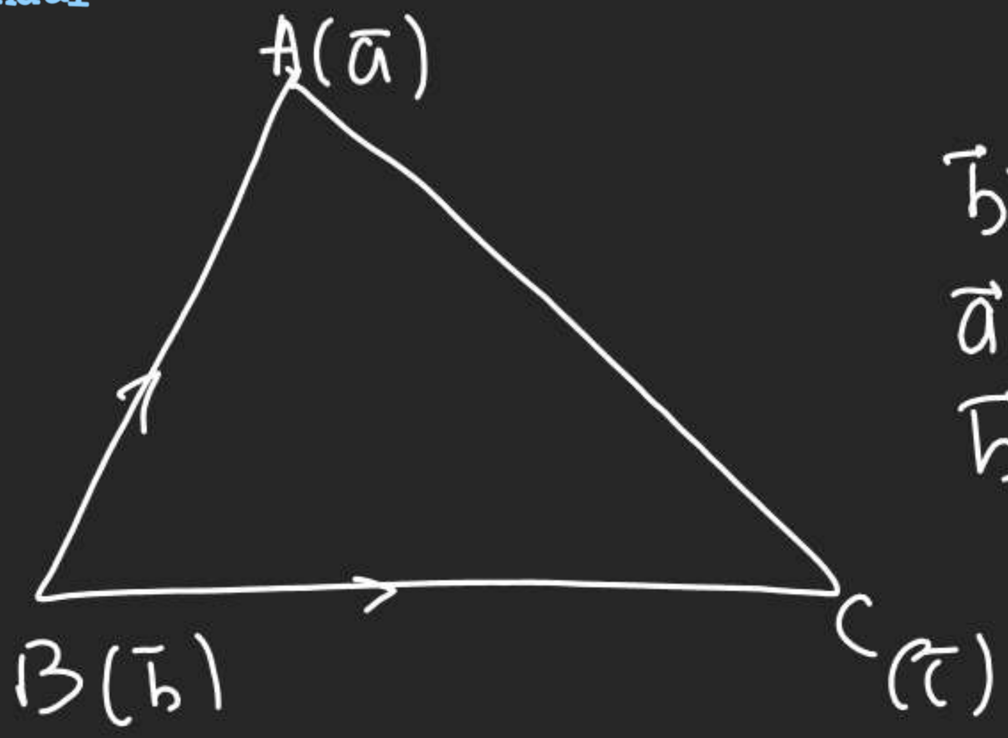
$\therefore$ ||gm's area $= 2\times\frac{1}{2}|\vec{a}\times\vec{b}|$

$= |\vec{a}\times\vec{b}|$

vector area $= \vec{a}\times\vec{b}$

Nishant Jindal

(4)



$\vec{b} \times \vec{b} = 0$

$\vec{a} \times \vec{b} = |a||b| \sin\theta \cdot \hat{n}$

$\vec{b} \times \vec{b} = |b||b| \underline{\sin 0} \cdot \hat{n} = 0$

Δ's Area = ?

$\Delta\text{'s area} = \frac{1}{2} \left| \vec{BC} \times \vec{BA} \right|$

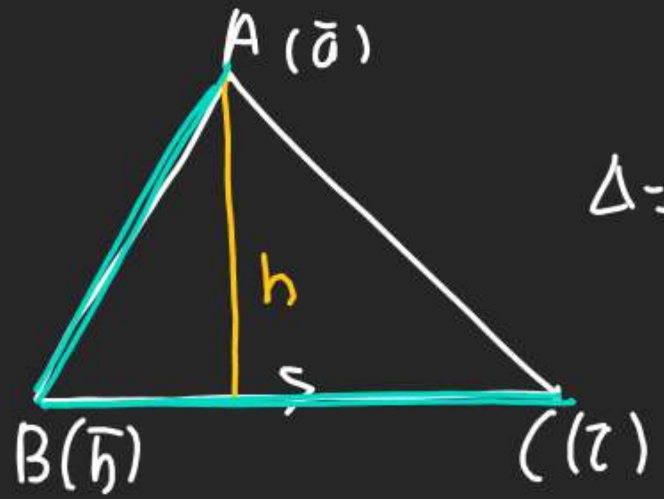
$= \frac{1}{2} \left| (\vec{c} - \vec{b}) \times (\vec{a} - \vec{b}) \right|$

$= \frac{1}{2} \left| c \times a - c \times b - b \times a + \cancel{\vec{b} \times \vec{b}} \right|$

$= \frac{1}{2} \left| a \times b + b \times c + c \times a \right|$

Q A) find ht of Δ having vertices

A(ā), B(b̄), C(c̄)



$\Delta = \frac{1}{2} \left| a \times b + b \times c + c \times a \right|$

$\cancel{\frac{1}{2}}\, \text{Base} \times \text{ht} = \cancel{\frac{1}{2}} \left| a \times b + b \times c + c \times a \right|$

$|\vec{c} - \vec{b}| \times \text{ht} = |a \times b + b \times c + c \times a|$

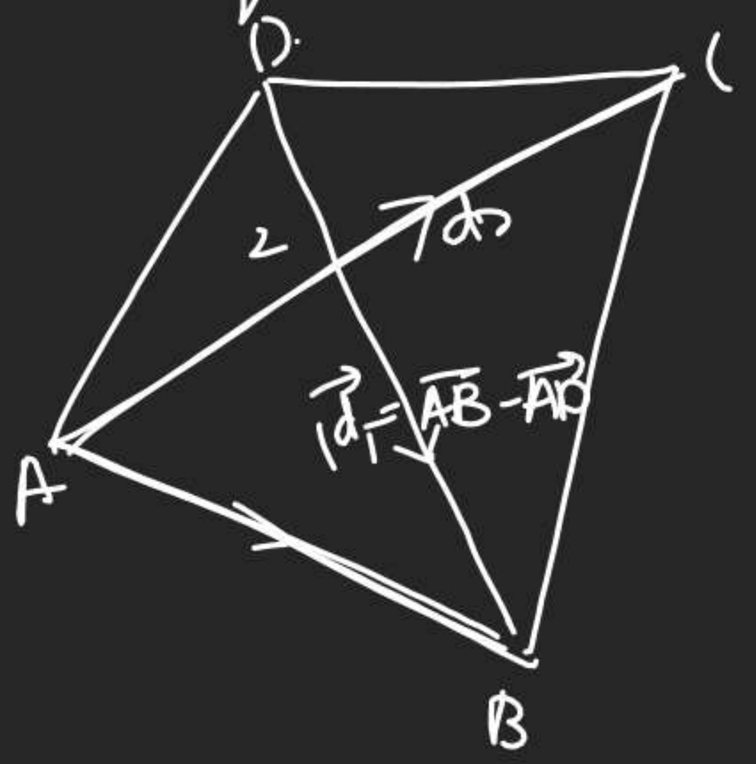
$\text{ht} = \dfrac{|a \times b + b \times c + c \times a|}{|c - b|}$

(B) find Unit vector $\perp^r$ to the plane containing 3 pts ā, b̄, c̄.

$\hat{n} = \dfrac{\vec{BC} \times \vec{BA}}{|\vec{BC} \times \vec{BA}|}$

Nishant Jindal

Q Area of Quad ABCD



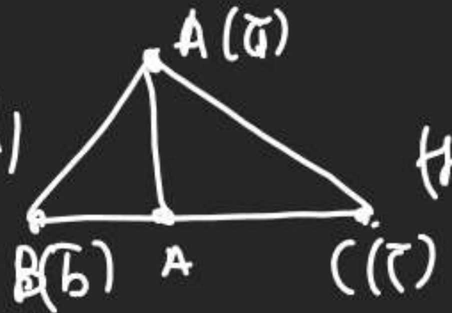
$$\Delta = \frac{1}{2}\left|(\vec{AB}\times\vec{AC}) + (\vec{AC}\times\vec{AD})\right|$$

$$= \frac{1}{2}\left|\vec{AB}\times\vec{AC} - \vec{AD}\times\vec{AC}\right|$$

$$= \frac{1}{2}\left|(\vec{AB}-\vec{AD})\times\vec{AC}\right|$$

$$= \frac{1}{2}\left|\vec{d_1}\times\vec{d_2}\right|$$

Rk. A1



then $\Delta = \frac{1}{2}|a\times b + b\times c + c\times a|$

If $A(\vec{a}), B(\vec{b}), C(\vec{c})$ are collinear

then $|a\times b + b\times c + c\times a| = 0$

$\Rightarrow a\times b + b\times c + c\times a = 0$

(B) $\vec{a}\times\vec{i} = 0$

$\hat{i}\times\hat{i} = \hat{j}\times\hat{j} = \hat{k}\times\hat{k} = 0$

$\hat{i}\times\hat{j} = \hat{k}$

$\hat{j}\times\hat{k} = \hat{i}$

$\hat{i}\times\hat{k} = -\hat{j}$

$2(a_1^2+a_2^2+a_3^2)$

$= 2|a|^2$

Q $(a\times\hat{i})\cdot\hat{i} + (a\times\hat{j})\cdot\hat{j} + (a\times\hat{k})\cdot\hat{k} = ?$

$0 + 0 + 0$

$= 0$

Q $(a\times\hat{i})^2 + (a\times\hat{j})^2 + (a\times\hat{k})^2$

$|\quad|^2 + |\quad|^2 + |\quad|$

$\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

$\vec{a}\times\hat{i} = (a_1\hat{i} + a_2\hat{j} + a_3\hat{k})\times\hat{i}$

$= 0 - a_2\hat{k} + a_3\hat{j}$

$a\times\hat{j} = (a_1\hat{i} + a_2\hat{j} + a_3\hat{k})\times\hat{j}$

$= a_1\hat{k} + 0 - a_3\hat{i}$

$a\times\hat{k} = (a_1\hat{i} + a_2\hat{j} + a_3\hat{k})\times\hat{k}$

$= -a_1\hat{j} + a_2\hat{i} + 0$

$|a\times\hat{i}|^2 + |a\times\hat{j}|^2 + |a\times\hat{k}|^2 = a_2^2 + a_3^2 + a_1^2 + a_3^2 + a_1^2 + a_2^2$

Nishant Jindal

M_2 $(a\times i)^2+(a\times j)^2+(a\times k)^2$

$(a\times b)^2=|a|^2|b|^2-(a\cdot b)^2$

$(a\times i)^2=|a|^2|\hat{i}|^2-(a\cdot\hat{i})^2=|a|^2-a_1^2$

$(a\times j)^2=|a|^2|j|^2-(\vec{a}\cdot\hat{j})^2=|a|^2-a_2^2$

$(a\times k)^2=|a|^2|k|^2-(a\cdot k)^2=|a|^2-a_3^2$

$|a|=\sqrt{a_1^2+a_2^2+a_3^2}$

$3|a|^2-\underbrace{(a_1^2+a_2^2+a_3^2)}$

$3|a|^2-|a|^2$

$=2|a|^2$

Q $(\vec{a}+\vec{b})\times(\vec{a}-\vec{b})=?$

$\cancel{a\times a}+-\vec{a}\times\vec{b}+b\times a+-\cancel{b\times b}$

$\vec{b}\times\vec{a}+b\times a$

$=2(b\times a)$

Rk $\Rightarrow$ 1) $\vec{a}\parallel\vec{b}$ then $\vec{a}\times\vec{b}=?$

$\vec{a}=\lambda\vec{b}$ So $\vec{a}\times\vec{b}$

$=\lambda(\vec{b}\times\vec{b})$

$=0$

$\therefore$ $\vec{a}\parallel\vec{b}$ then $\vec{a}\times\vec{b}=0$

But if $\vec{a}\times\vec{b}=0$

$\vec{a}=0$ $\quad$ $\vec{b}=0$ $\quad$ $\vec{a}\parallel\vec{b}$

(2) If Qs is giving $\underline{a\times b=c}$

$\underline{b\times c=a,\ c\times a=b}$

① $\vec{a},\vec{b},\vec{c}$ mutually $\perp^r$

① $a\times b=c$ $\quad$ ② $b\times c=a$

$|a\times b|=|c|$ $\quad$ $|b\times c|=|a|$

$\frac{|a\times b|}{|b\times c|}=\frac{|c|}{|a|}\Rightarrow\frac{|a|\cancel{|b|}\sin 90}{\cancel{|b|}|c|\sin 90}=\frac{|c|}{|a|}$

$\Rightarrow |a|^2=|c|^2=|b|^2$

$\Rightarrow |a|=|b|=|c|=K=1$

Best $\to$ $\vec{a},\vec{b},\vec{c}$ are $\underline{\hat{i},\hat{j},\hat{k}}$

Nishant Jindal

Q If $\bar a, \bar b, \vec c$ are 3 Non Zero vectors S.T $a \times b = c$, $b \times c = a$, $c \times a = b$ then $|a+b+c| = ?$

$= |\hat i + \hat j + \hat k| = \sqrt{1^2+1^2+1^2} = \sqrt{3}$

Q $\vec a, \vec b, \vec c$ are 3 Non Zero vectors. $\bar a \times \bar b = \bar c$, $\bar b \times \bar c = \bar a$, $c \times a = \bar b$ then $|\bar a| + 2|\vec b| - 3|\bar c| = ?$

$|\hat i| + 2|\hat j| - 3|\hat k|$

$= 1 + 2 \times 1 - 3 \times 1$

$= 0$

Q If $\vec a \times \vec b = \vec c \times \vec d$ & $\vec a \times \vec c = \vec b \times \vec d$ then $(\bar a - \bar d) \times (\bar b - \bar c) = ?$

$a \times b = c \times d$

$a \times c = b \times d$

$\vec a \times (\vec b - \vec c) = (\vec c - \vec b) \times \vec d$

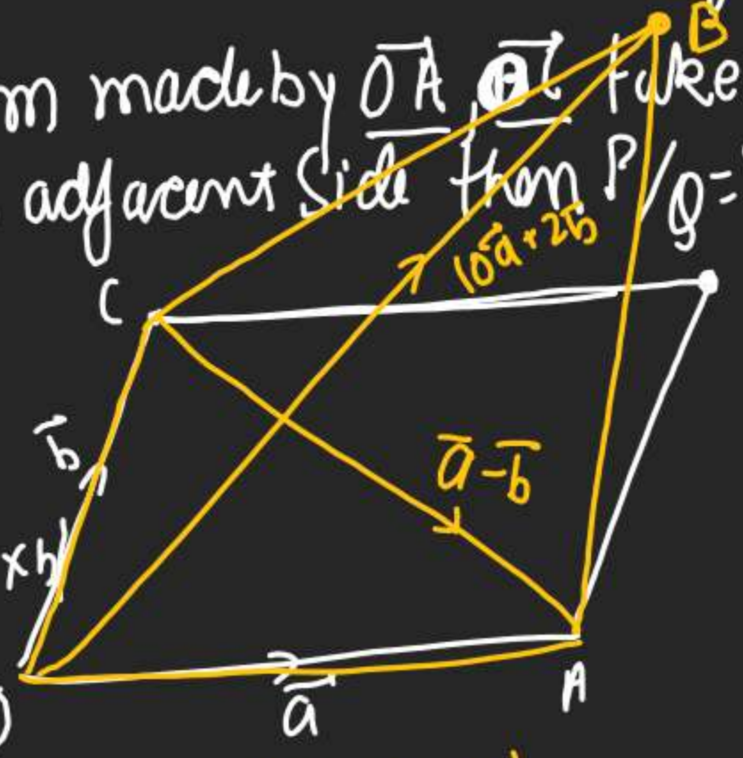
$\vec a \times (\vec b - \vec c) = d \times (\vec b - \vec c)$

$\vec a \times (\vec b - \vec c) - \vec d \times (\vec b - \vec c) = 0$

$(\bar a - \bar d) \times (\bar b - \bar c) = 0$

0 Ans

Q Let $OA = \bar a$, $OB = 10\bar a + 2\bar b$, $OC = \vec b$ where O, A, C 3 Non Collinear Pts. If P is Area of Quad OABC & Q is area of ||gm made by $\overline{OA}$, $\overline{OC}$ taken as adjacent Side then $P/Q = ?$



$Q = |a \times b|$

$P = \frac{1}{2}|((10\bar a + 2\bar b) \times (\bar a - \bar b)|$

$= \frac{1}{2}|0 - 10\bar a \times \bar b + 2\bar b \times a + 0|$

$= 6|\bar a \times \bar b|$

$\frac{P}{Q} = \frac{6|\cancel{a \times b}|}{|\cancel{a \times b}|} = 6$

Nishant Jindal

1) $\bar{a} = \hat{i} + 2\hat{j}$, $\bar{b} = 3\hat{i} - 5\hat{k}$

$\bar{r} \times \bar{a} = \bar{a} \times \vec{b}$ &

$\bar{r} \times \vec{b} = \bar{b} \times \vec{a}$ find Unit vector in dir. of $\vec{r}$?

$\bar{r} \times \vec{a} = \vec{a} \times \vec{b}$

$\vec{r} \times \bar{b} = -(\bar{a} \times \bar{b})$

$\vec{r} \times \vec{b} = -(\vec{r} \times \bar{a})$

$\vec{r} \times \vec{b} + (\bar{r} \times \bar{a}) = 0$

$\vec{r} \times (\bar{a} + \bar{b}) = 0$

$\vec{r} \parallel (\vec{a} + \vec{b})$

$\vec{r} = \lambda(\bar{a} + \bar{b}) = \lambda(4\hat{i} + 2\hat{j} - 5\hat{k})$

$$\hat{r} = \frac{\cancel{\lambda}(4\hat{i} + 2\hat{j} - 5\hat{k})}{\cancel{\lambda}\sqrt{16+4+25}} = \frac{1}{\sqrt{45}}(4\hat{i} + 2\hat{j} - 5\hat{k})$$