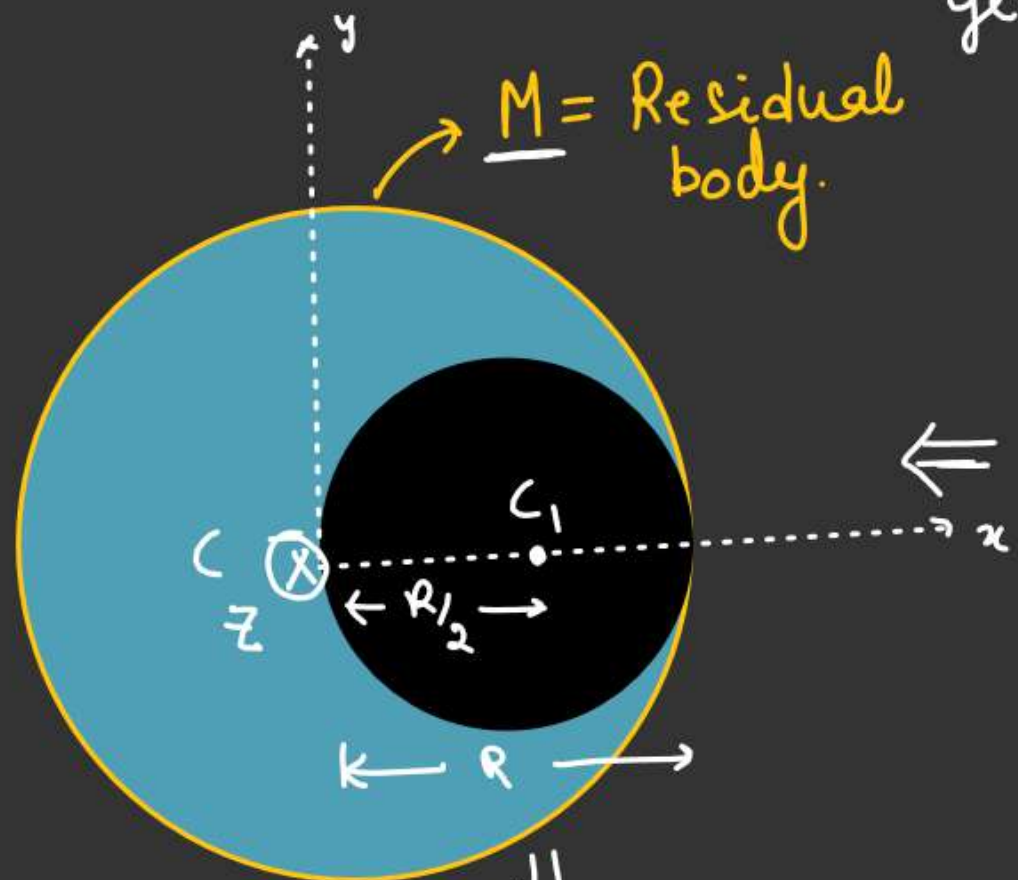


M-I of Residual body

$$I_{\text{Residual}} = I_{\text{Original body about given axis of Rotation}} - I_{\text{Cut body about given axis of Rotation}}$$

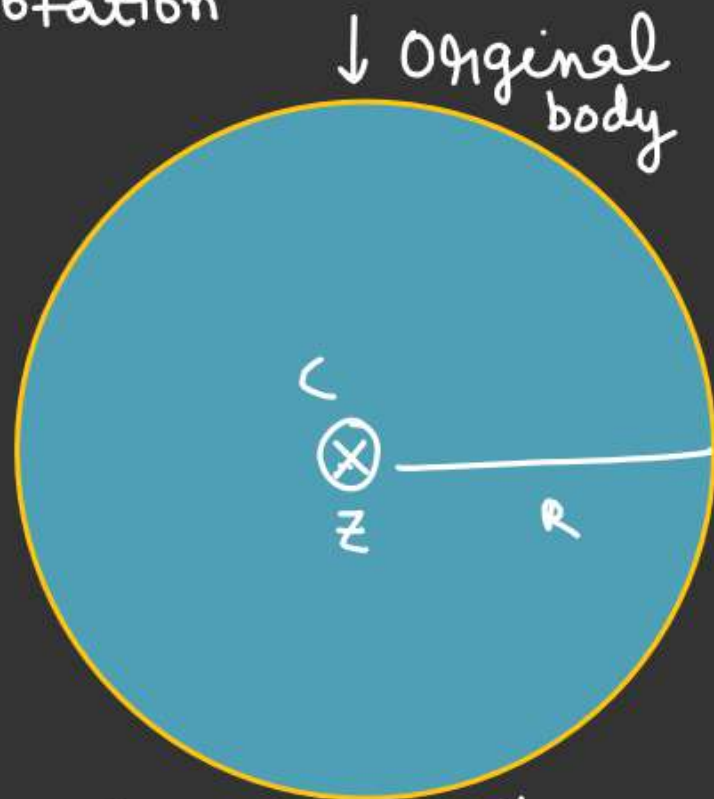


$M = \text{Residual body.}$

Residual body

$$\sigma = \frac{M}{\pi R^2 - \pi (R/2)^2}$$

$$\sigma = \frac{4M}{3\pi R^2}$$

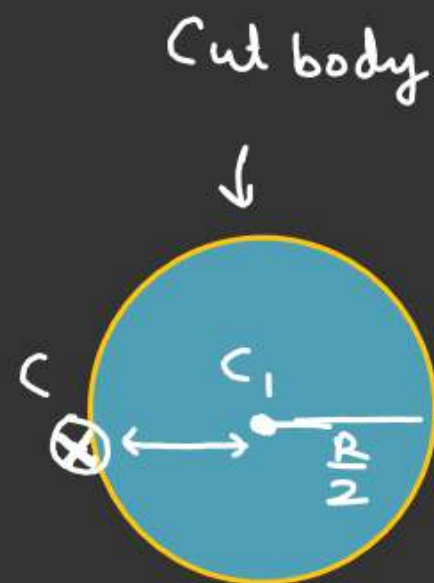


Original body

$$M_1 = \sigma \times \pi R^2$$

$$M_1 = \frac{4M}{3}$$

$$I_1 = \frac{M_1 R^2}{2} = \left(\frac{2MR^2}{3} \right)$$



Cut body

$$I_{C_1} = \frac{\frac{M}{3} (R/2)^2}{2}$$

$$M_2 = \sigma \left(\frac{\pi R^2}{4} \right)$$

$$I_2 = \left(\frac{M}{3} \left(\frac{R}{2} \right)^2 \cdot \frac{1}{2} \right) = \frac{4M}{3\pi R^2} \times \frac{\pi R^2}{4}$$

Cut about

$$= \frac{MR^2}{24} + \frac{MR^2}{12}$$

$$= \frac{MR^2}{8}$$

$$\begin{aligned} I_{\text{residual}} &= I_1 - I_2 \\ &= \frac{2}{3}MR^2 - \frac{MR^2}{8} \\ &= \frac{16MR^2 - 3MR^2}{24} \\ &= \left(\frac{13MR^2}{24} \right) \end{aligned}$$

From a hollow sphere a vertical cylinder of radius $R/2$ is cut.
Find M.I of the remaining body about vertical axis passing through center of sphere.

$$\begin{aligned} dm &= \text{mass of ring} \\ &= \sigma (2\pi R \sin\phi) R d\phi \\ dm &= \sigma 2\pi R^2 \sin\phi d\phi \end{aligned}$$

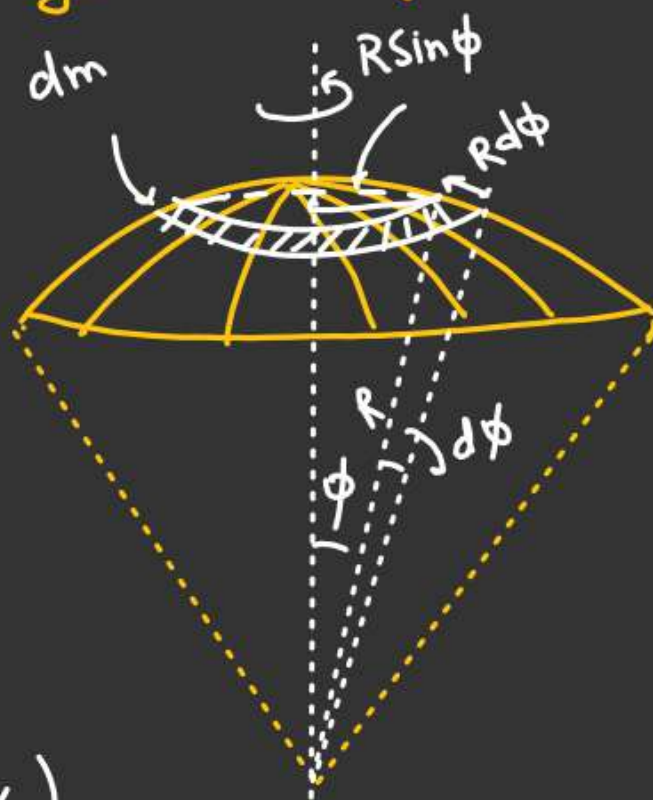
$dI \rightarrow$ M.I of dm

$$dI = dm (R \sin\phi)^2$$

$$I_{\text{cut}} \quad dI = \sigma (2\pi R^2 \sin\phi d\phi) (R^2 \sin^2\phi)$$

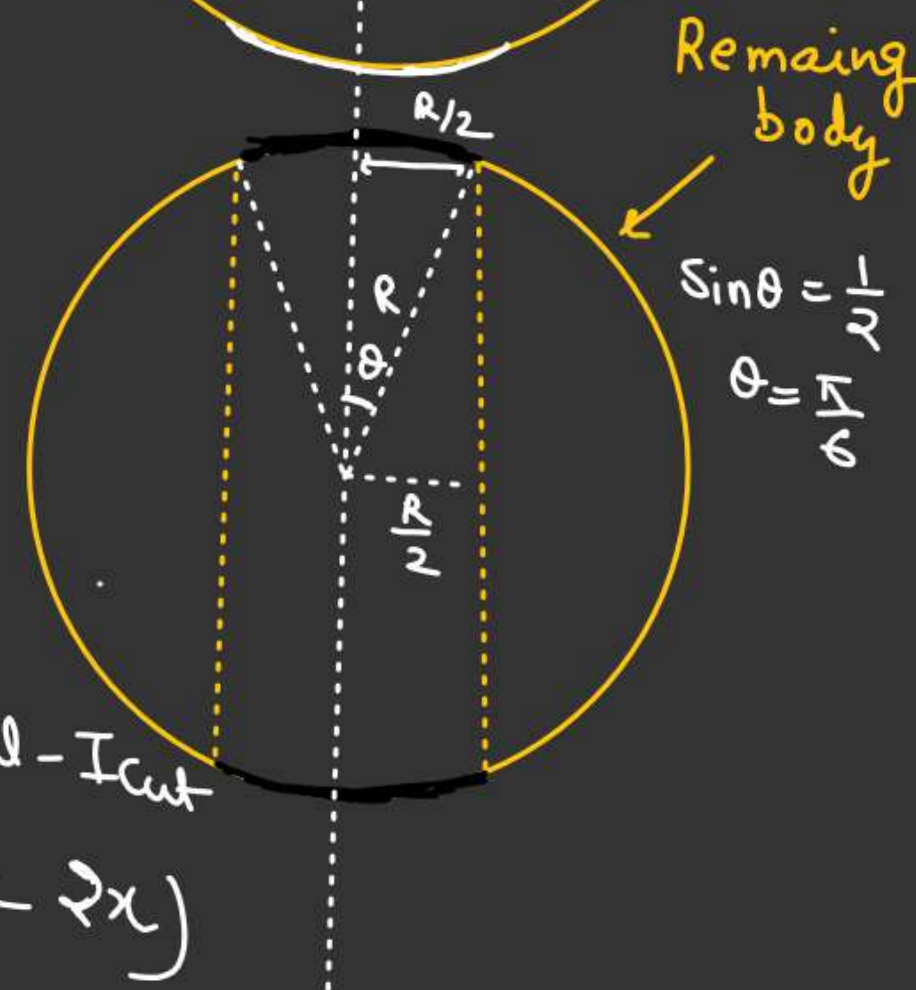
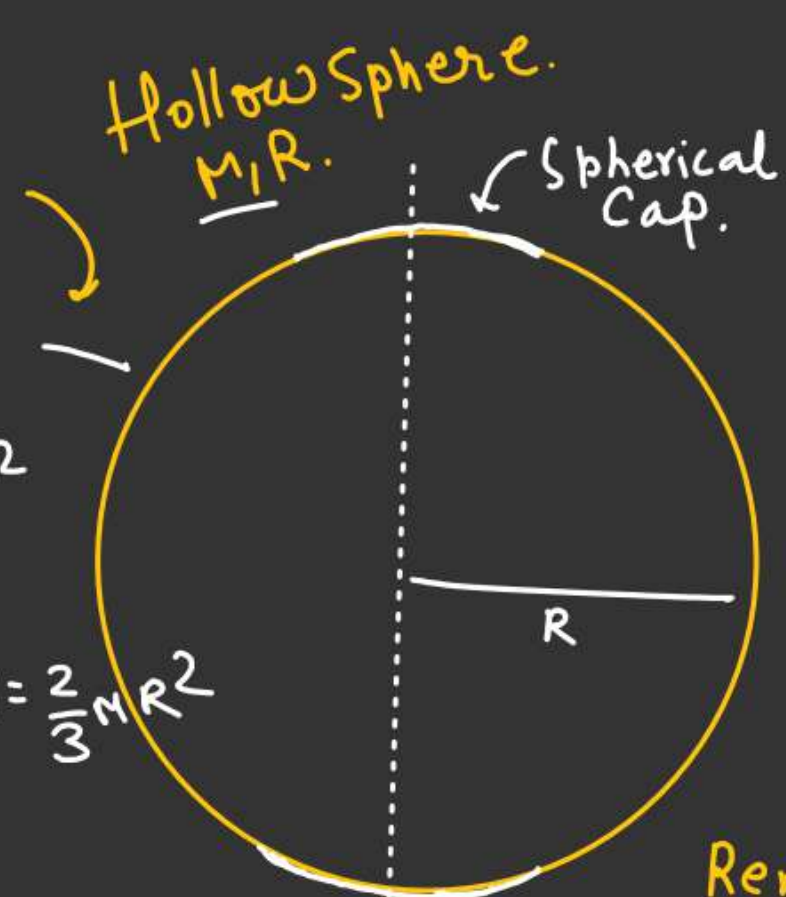
$$\int_0^{\pi/2} dI = \sigma 2\pi R^4 \int_0^{\pi/2} \sin^3\phi d\phi \quad \sin 3\phi = 3\sin\phi - 4\sin^3\phi$$

$$= \frac{M}{4\pi R^2} \times 2\pi R^4 \int_0^{\pi/2} \left(\frac{3\sin\phi - \sin 3\phi}{4} \right) d\phi = x \ll$$



$$\sigma = \frac{M}{4\pi R^2}$$

$$I_{\text{original}} = \frac{2}{3} M R^2$$



$$\begin{aligned} I_{\text{residual}} &= I_{\text{original}} - I_{\text{cut}} \\ &= \left(\frac{2}{3} M R^2 - 2x \right) \end{aligned}$$

TORQUE

(Moment of force) $[\theta = \text{Angle b/w } \vec{r} \text{ \& } \vec{F}]$

Torque about a point

$$\vec{\tau}_{F/C} = (\vec{r} \times \vec{F}) \rightarrow |\vec{\tau}_{F/C}| = r F \sin \theta$$

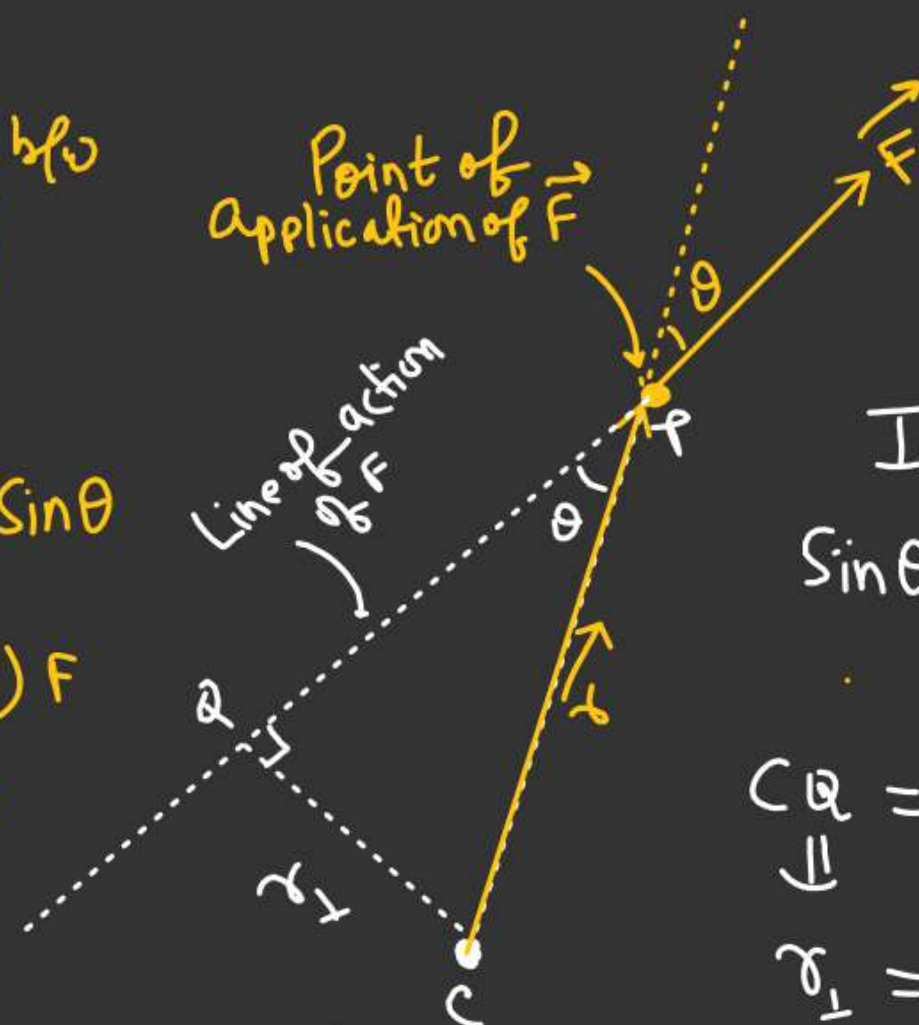
\vec{r} = Position vector joining the point from where we have to calculate the torque to the point of application of force F

$$= (r \sin \theta) F$$

$$= r_{\perp} F$$

$$|\vec{\tau}_{F/C}| = r_{\perp} F$$

$\Rightarrow \vec{\tau}$ is perpendicular to the plane containing \vec{r} \& \vec{F}



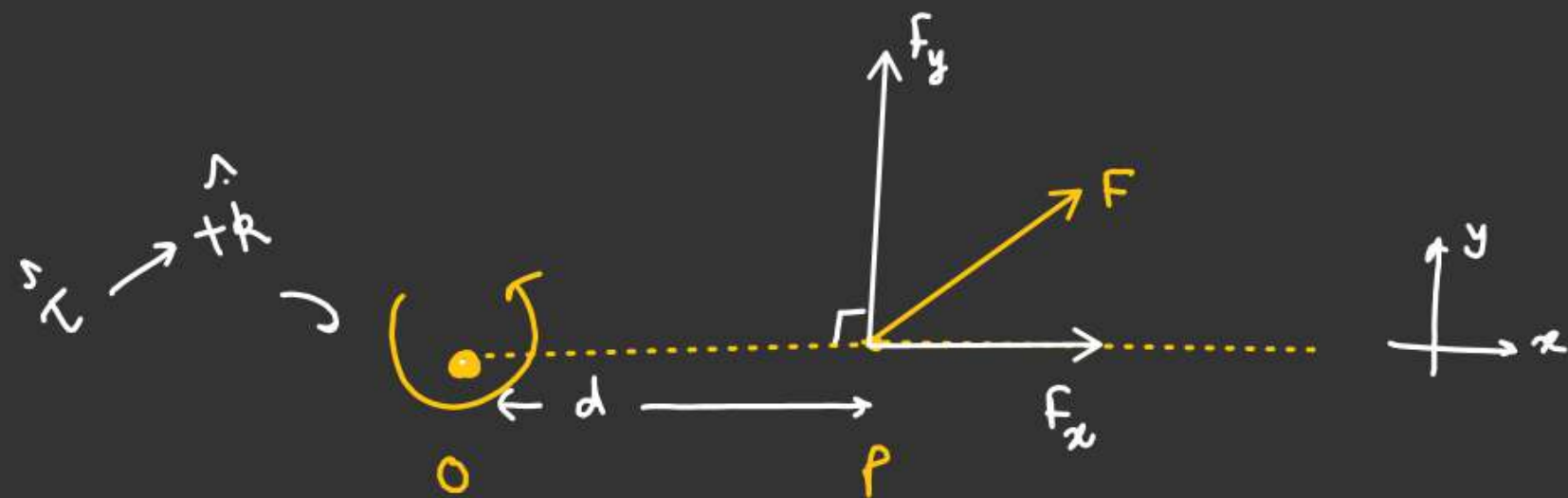
In ΔPCQ

$$\sin \theta = \frac{CQ}{PC}$$

$$CQ = PC \sin \theta$$

$$r_{\perp} = |\vec{r}| \sin \theta$$

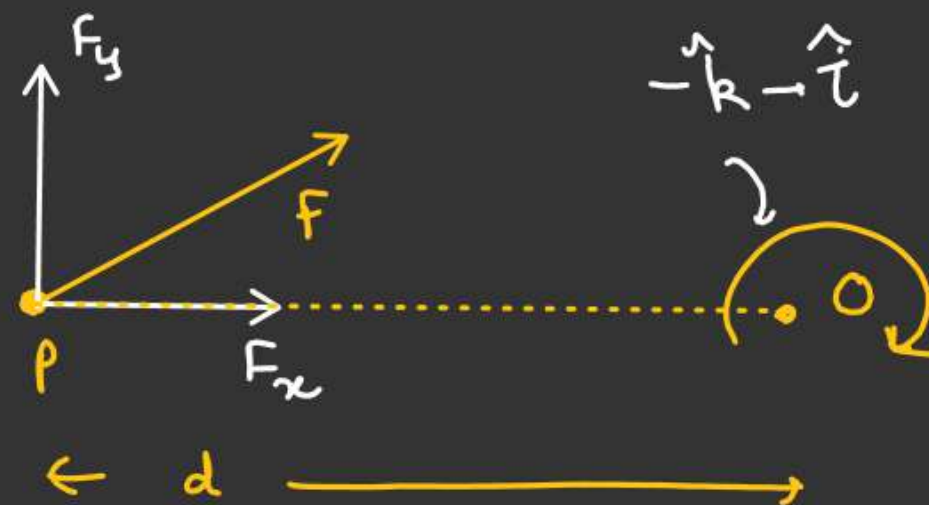
$r_{\perp} \rightarrow$ Perpendicular distance from the point where we have to calculate the torque to the line of action of F



$$|\vec{\tau}_{F/O}| = |\vec{\tau}_{F_y/O}| = (F_y \cdot d)$$

$$\vec{\tau}_{F_x/O} = 0$$

line of action of F_x
passing through O
so $\gamma_{\perp} = 0$



$$|\vec{\tau}_{F/O}| = |\vec{\tau}_{F_y/O}| = F_y \cdot d$$

$$\vec{\tau} = (F_y d)(-\hat{k})$$

Find

$$\vec{\tau}_{F/O} = ?$$

$$\vec{\tau}_{F/A} = ?$$

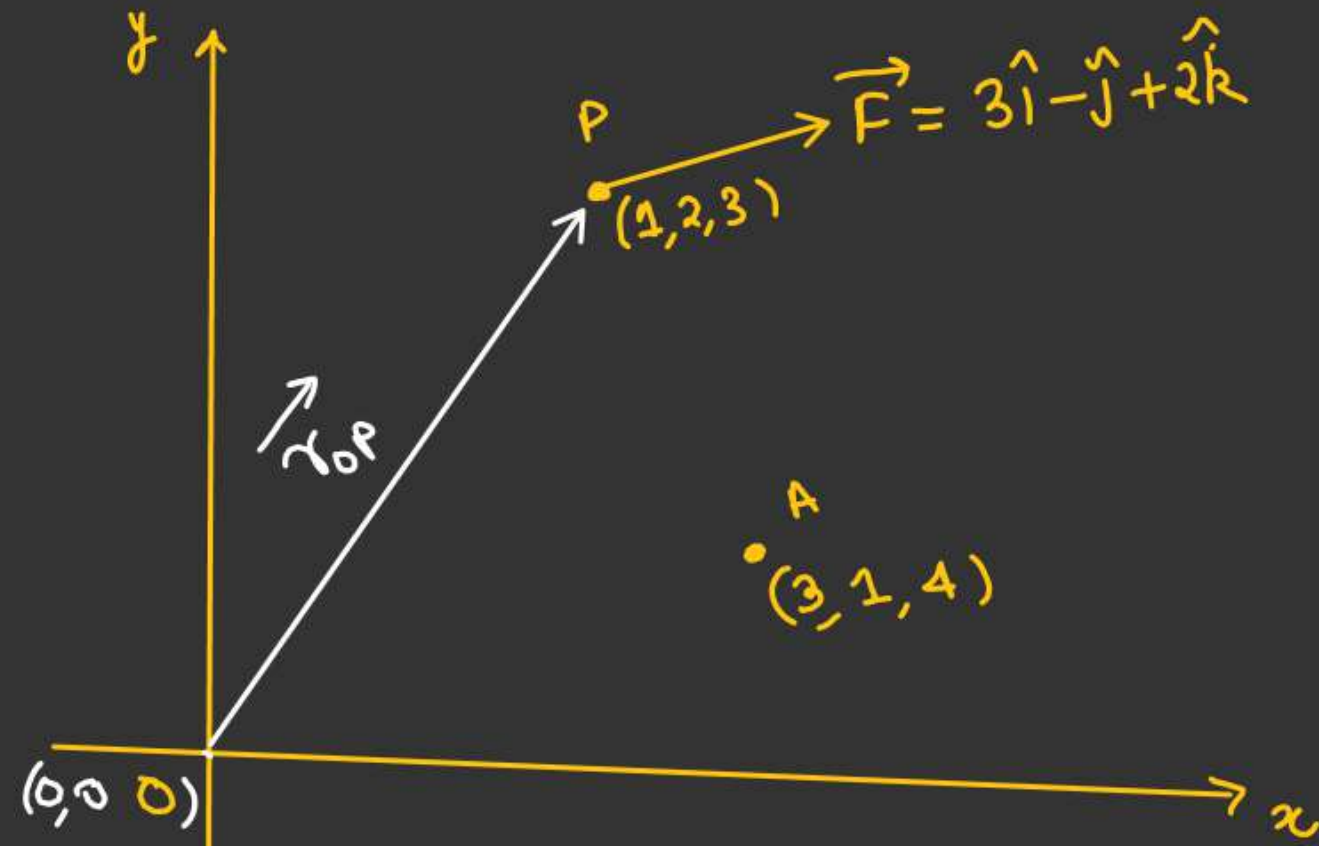
$$\begin{aligned}\vec{\tau}_{F/O} &= \vec{r}_{OP} \times \vec{F} \\ &= (\hat{i} + 2\hat{j} + 3\hat{k}) \times (3\hat{i} - \hat{j} + 2\hat{k})\end{aligned}$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 3 & -1 & 2 \end{vmatrix} = \hat{i}(4 - (-3)) - \hat{j}(2 - 9) + \hat{k}(-1 - 6)$$

$$= \underline{7\hat{i} + 7\hat{j} - 7\hat{k}}$$

$$|\vec{\tau}_{F/O}| = \underline{7\sqrt{3} \text{ N-m}}$$

$$\vec{r}_{OP} = \hat{i} + 2\hat{j} + 3\hat{k}$$



Find $\vec{\tau}_{F/A} = ?$

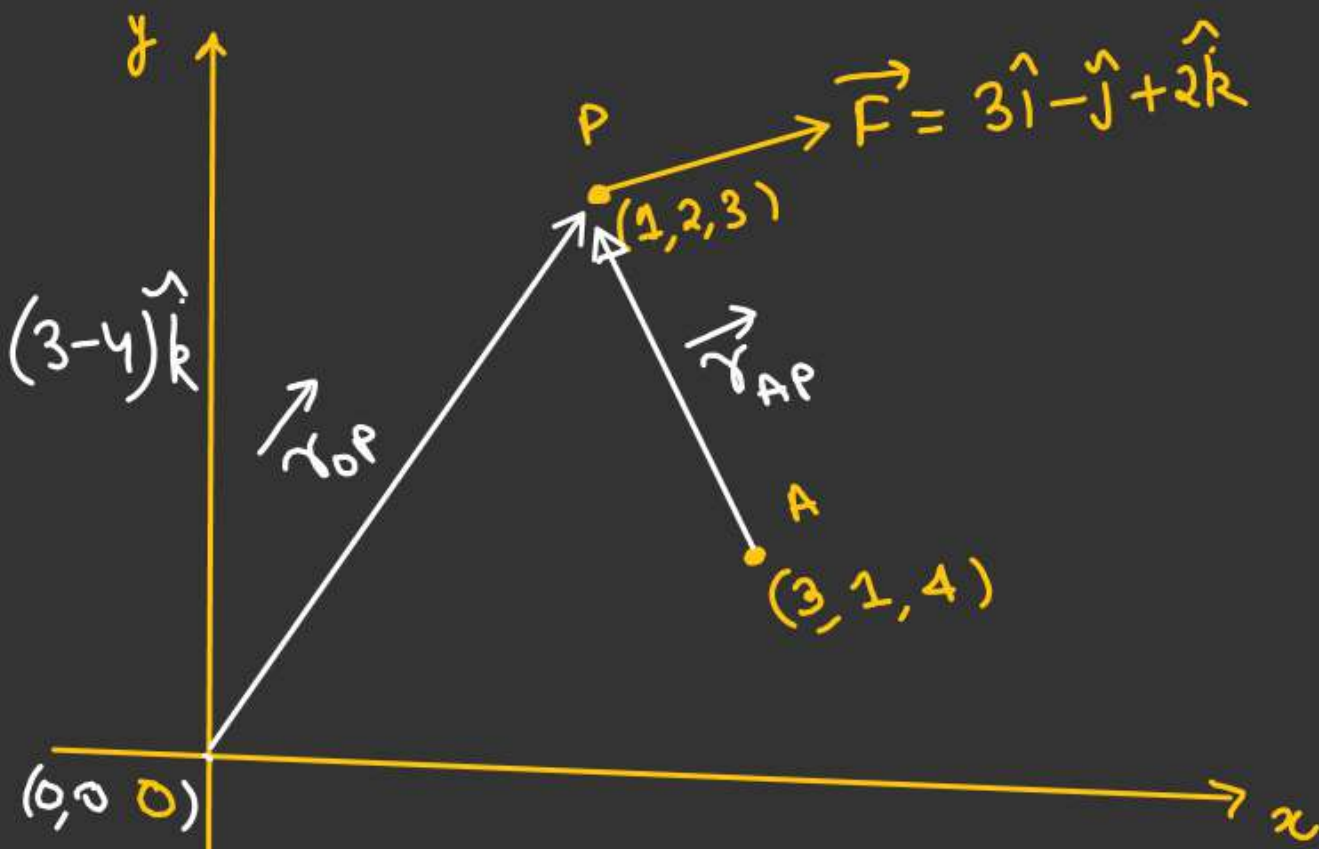
$$\vec{\tau}_{F/A} = (\vec{r}_{AP} \times \vec{F})$$

$$\Rightarrow \vec{r}_{AP} = (1-3)\hat{i} + (2-1)\hat{j} + (3-4)\hat{k}$$

$$\vec{\tau}_{F/A} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2 & 1 & -1 \\ 3 & -1 & 2 \end{vmatrix} = (-2\hat{i} + \hat{j} - \hat{k})$$

$$= \hat{i}(2-1) - \hat{j}(-4+3) + \hat{k}(2-3)$$

$$= \underline{\hat{i} + \hat{j} - \hat{k}}$$



Find net torque acting on the body about O

$$\vec{\tau}_{F_1/O} = F_1 r_{\perp} = (2 \times 1)(-\hat{k})$$

$$\vec{\tau}_{F_2/O} = 0$$

$$\vec{\tau}_{F_3/O} = (2 \times 2)(-\hat{k}) = -4\hat{k}$$

$$\vec{\tau}_{\text{net}/O} = \underline{-6\hat{k} \text{ N-m}}$$

