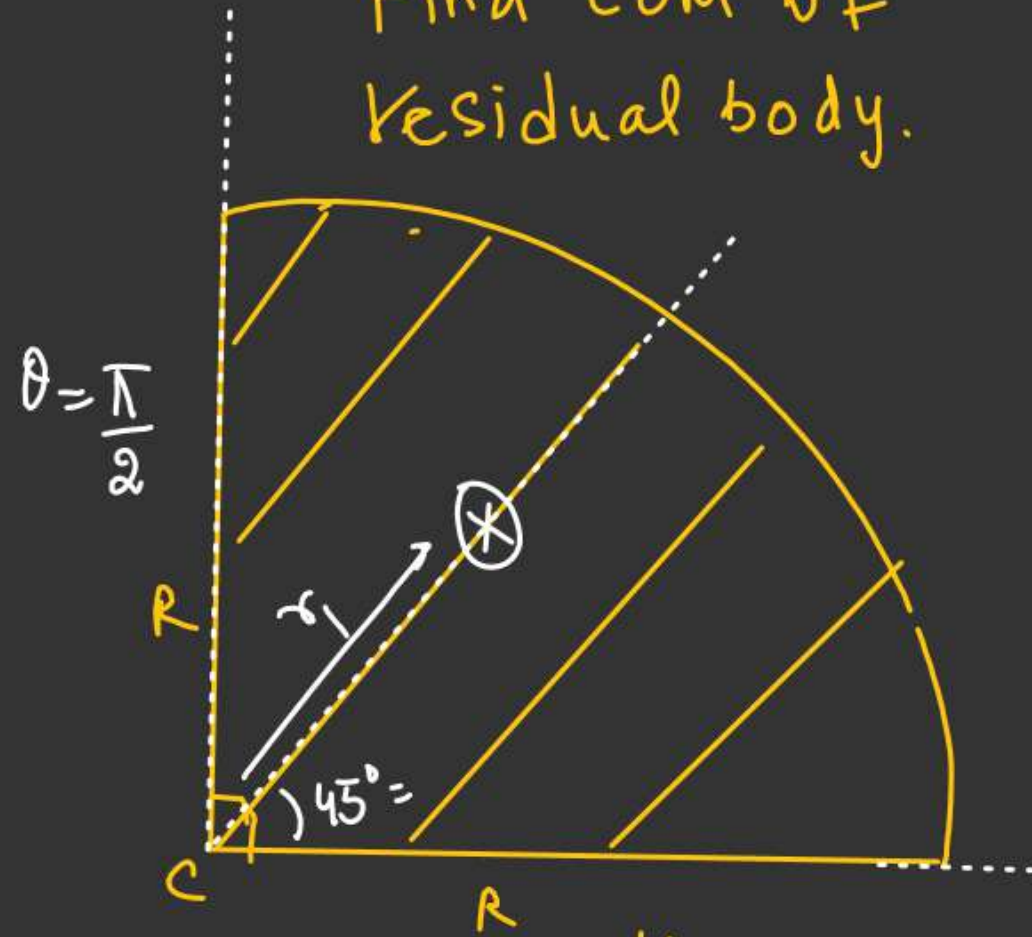
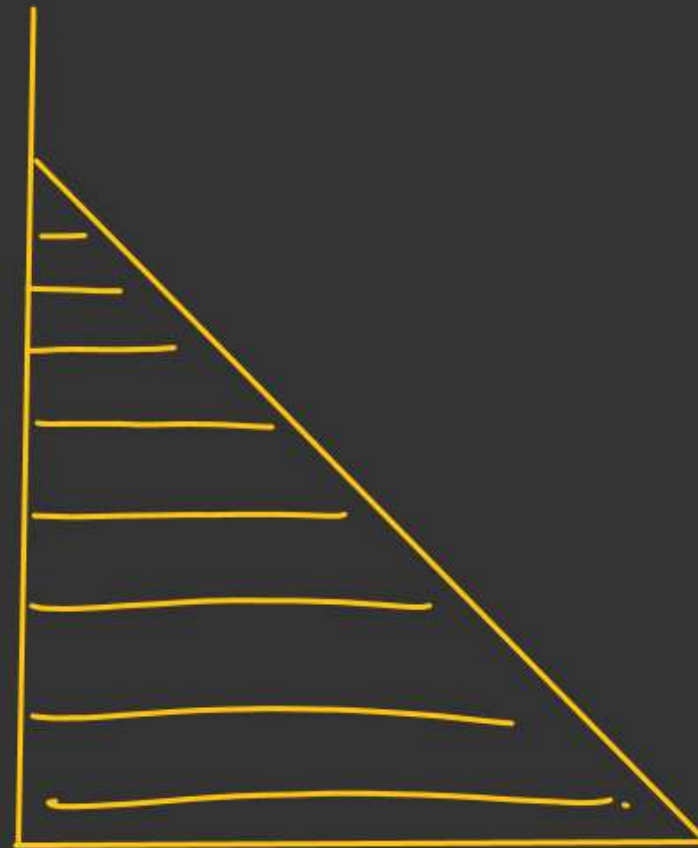


# H.W.: A triangle is cut from a sector of radius  $R$ .

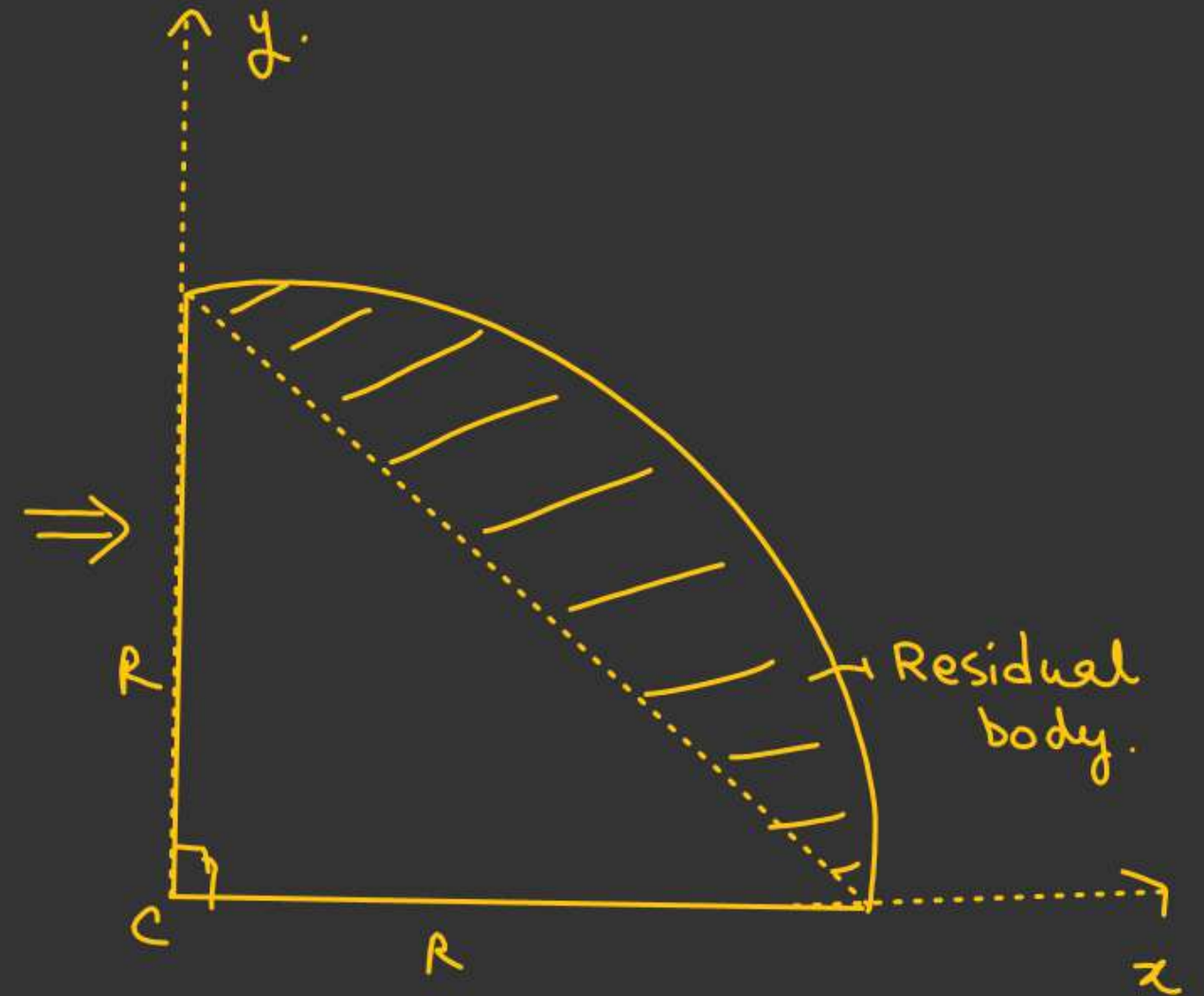
Find COM of residual body.

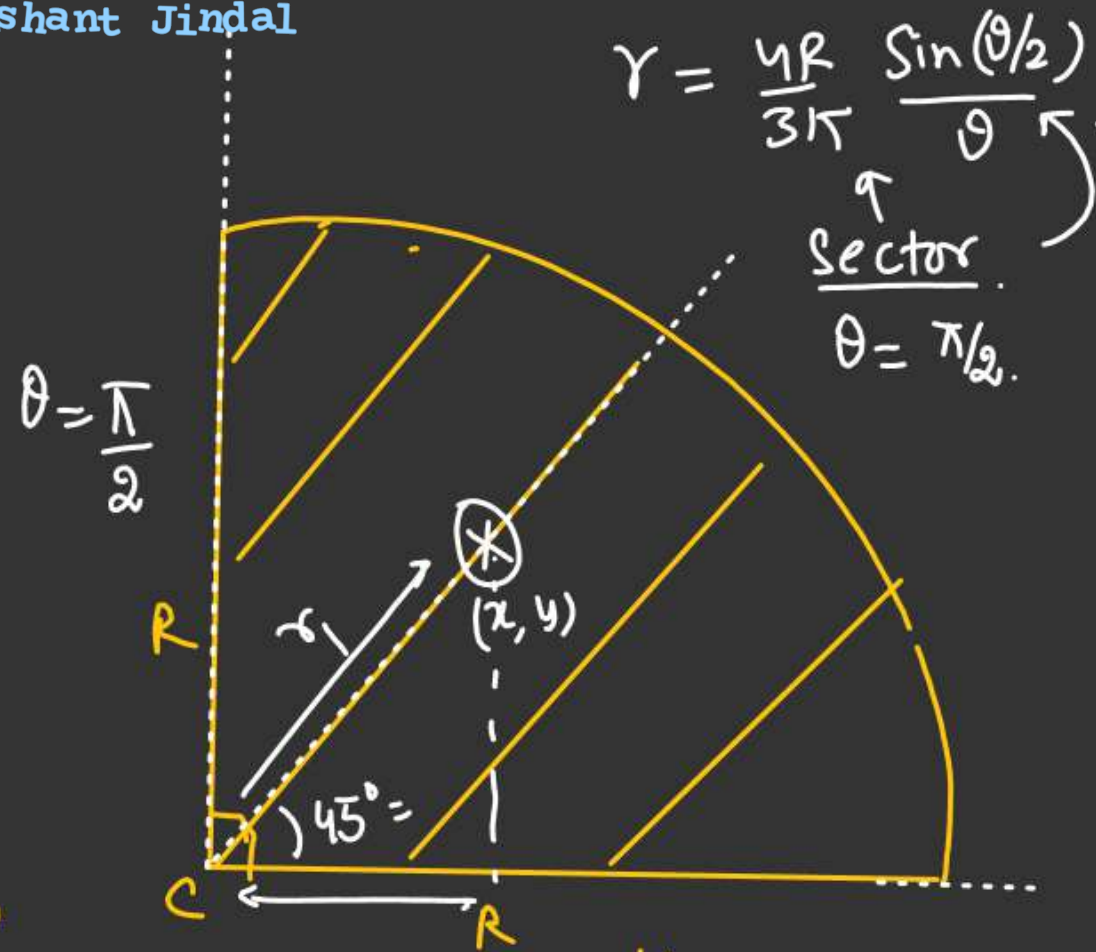


Original body



Cut body



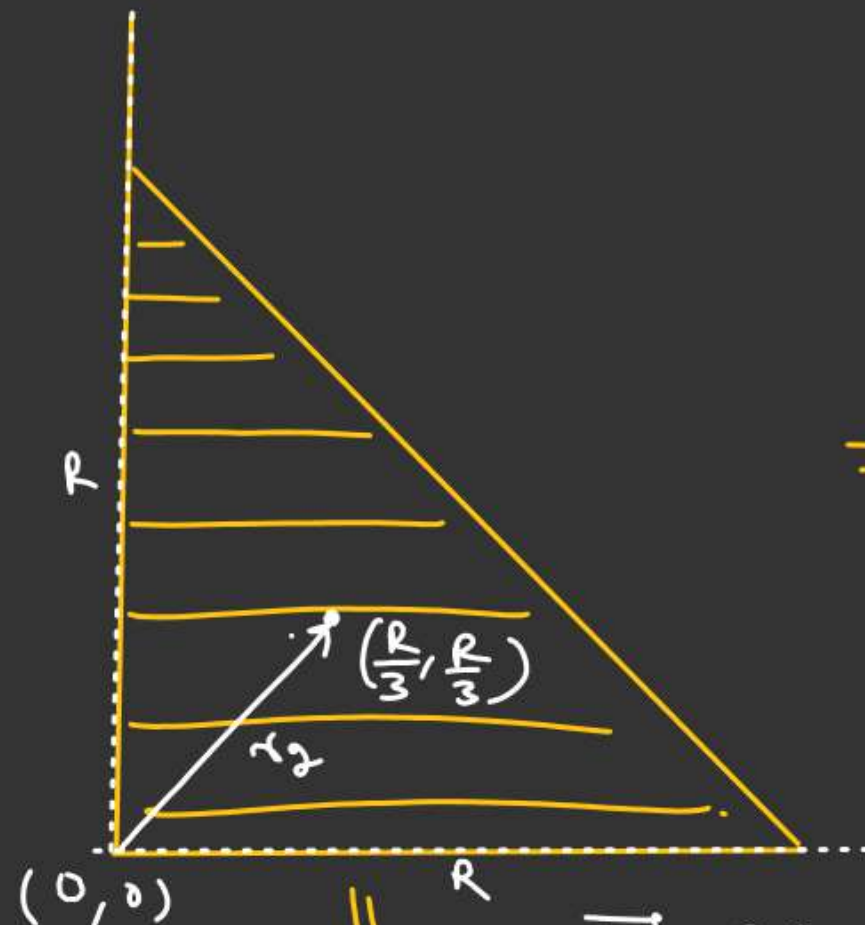


Original body

$$r_1 = \frac{4R}{3} \frac{\sin(\pi/4)}{\pi/2} \quad \vec{r}_1 = \frac{8R}{3\pi\sqrt{2}} (\cos 45^\circ \hat{i} + \sin 45^\circ \hat{j})$$

$$r_1 = \left( \frac{8R}{3\pi} \times \frac{1}{\sqrt{2}} \right) \quad \vec{r}_1 = \frac{4R}{3\pi} (\hat{i} + \hat{j})$$

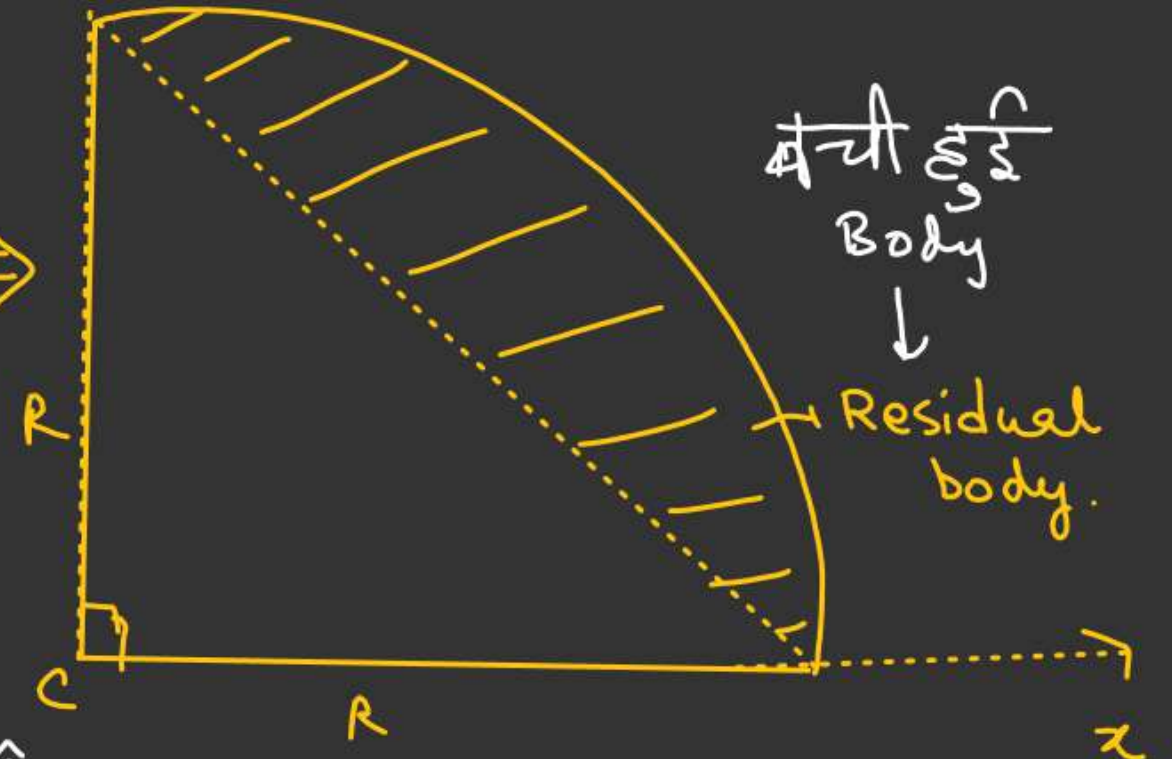
$$A_1 = \frac{R^2}{2} \pi/2 = \left( \frac{R^2}{4} \pi \right)$$



Cut body

$$\vec{r}_2 = \frac{R}{3} \hat{i} + \frac{R}{3} \hat{j}$$

$$A_2 = \frac{1}{2} \times R \times R = \frac{R^2}{2}$$



$$\vec{r}_{\text{residual}} = \frac{(A_1 \vec{r}_1 - A_2 \vec{r}_2)}{A_1 - A_2}$$

$$= \frac{\left[ \frac{\pi R^2}{4} \times \frac{4R}{3\pi} (\hat{i} + \hat{j}) \right] - \left[ \frac{R^2}{2} \times \frac{R}{3} (\hat{i} + \hat{j}) \right]}{\left( \frac{\pi R^2}{4} - \frac{R^2}{2} \right)}$$

?

H.W#  $M =$  Mass of Residual body ✓A Solid Sphere of radius  $R/2$  is cut from a Solid Sphere of radius  $R$ .

Find COM of Residual body.

$$\vec{X}_{\text{com}} = \left( \frac{V_1 \vec{R}_1 - V_2 \vec{R}_2}{V_1 - V_2} \right)$$

$$\vec{R}_1 = 0$$

$$\vec{R}_2 = +\frac{R}{2} \hat{i}$$

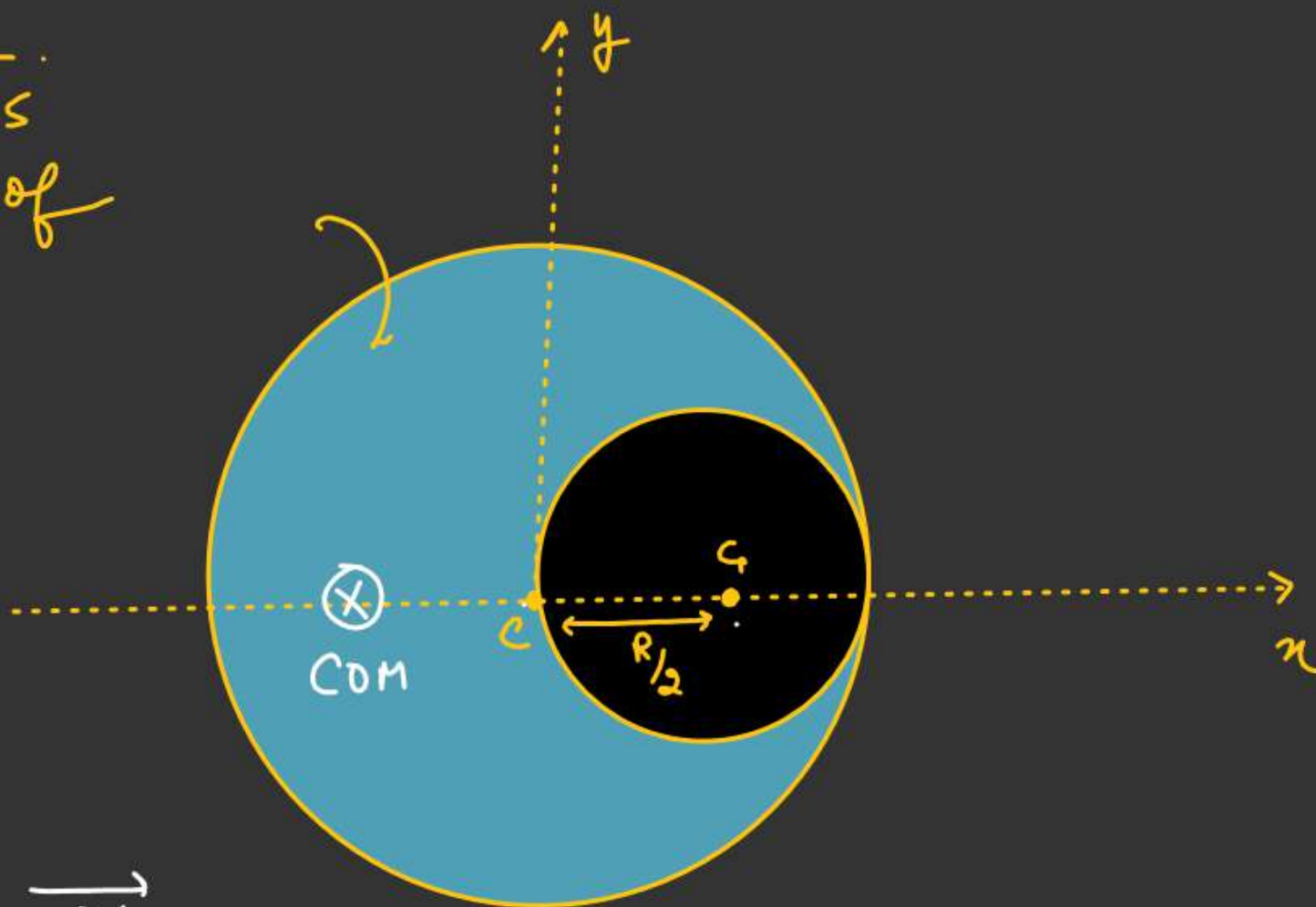
$$V_1 = \left( \frac{4}{3} \pi R^3 \right)$$

$$V_2 = \frac{4}{3} \pi \left( \frac{R}{2} \right)^3$$

$$V_2 = \left( \frac{4}{3} \pi R^3 \right) \frac{1}{8}$$

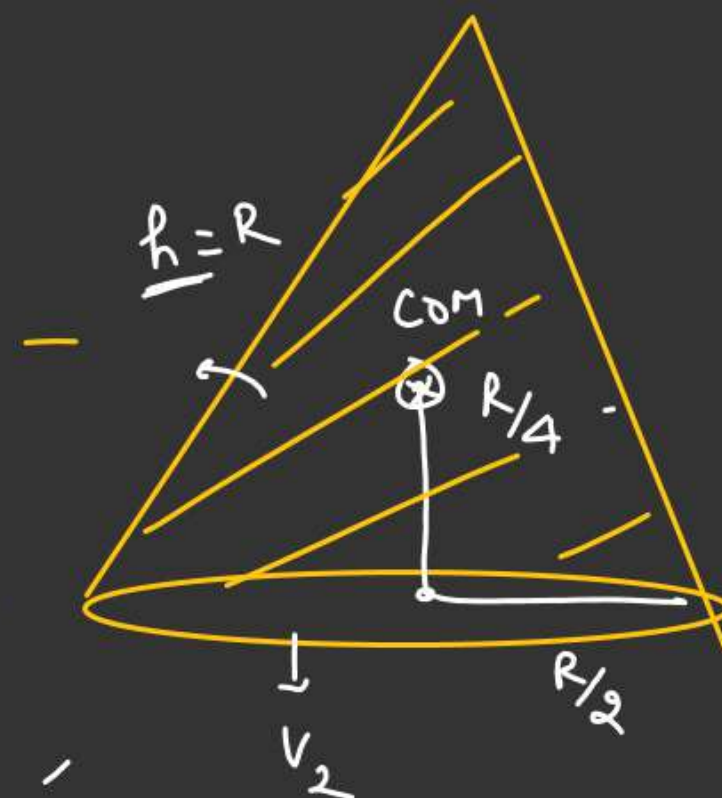
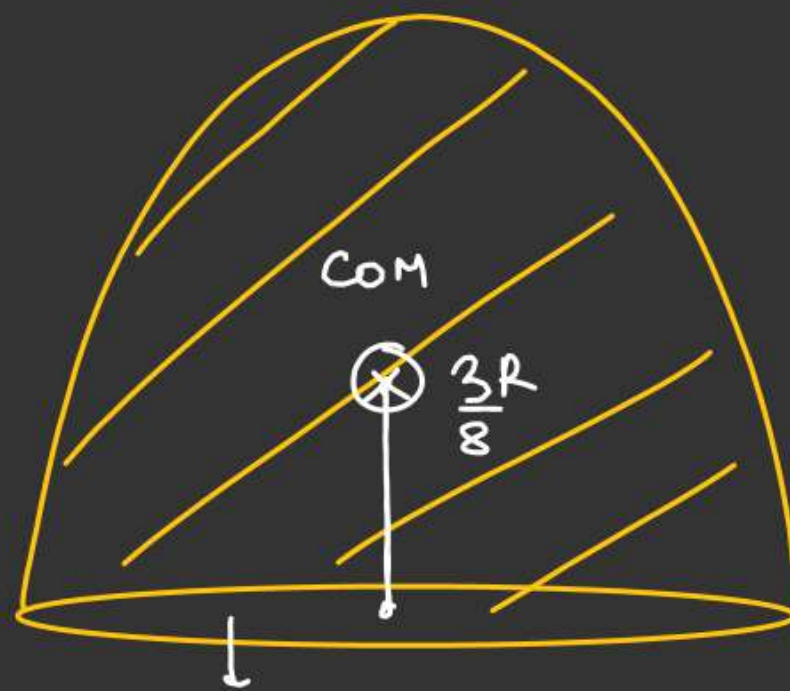
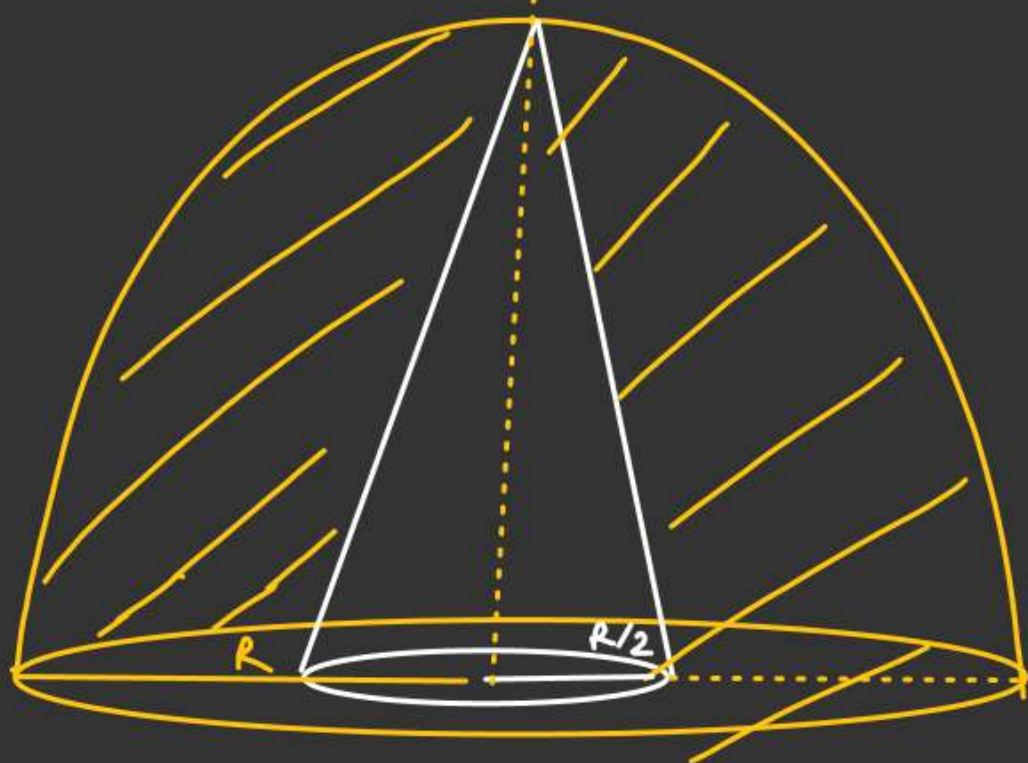
$$\vec{X}_{\text{com}} = \frac{-\left( \frac{4}{3} \pi R^3 \right) \frac{1}{8} \times \frac{R}{2} \hat{i}}{\frac{4}{3} \pi R^3 \left( 1 - \frac{1}{8} \right)} = -\frac{R/16}{7/8} \hat{i}$$

$$\vec{X}_{\text{com}} = -\frac{R}{14} \hat{i}$$



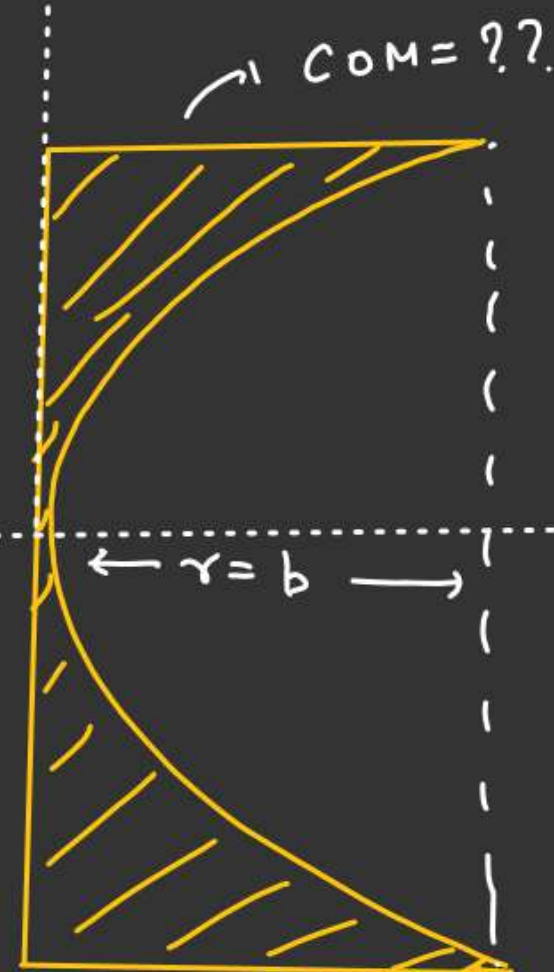
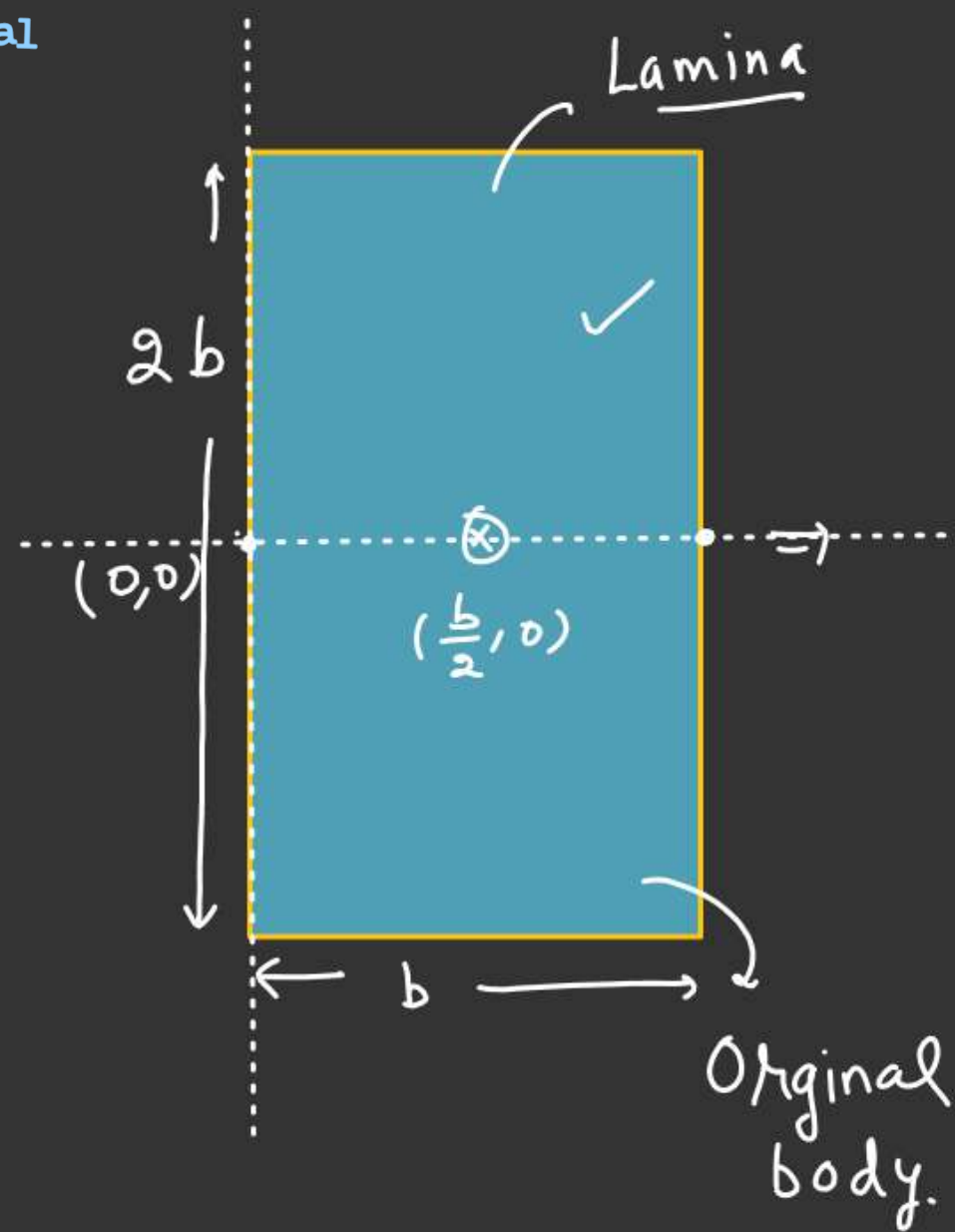
AS

Find COM of remaining body if a cone of radius  $R/2$  is cut from a solid hemisphere.



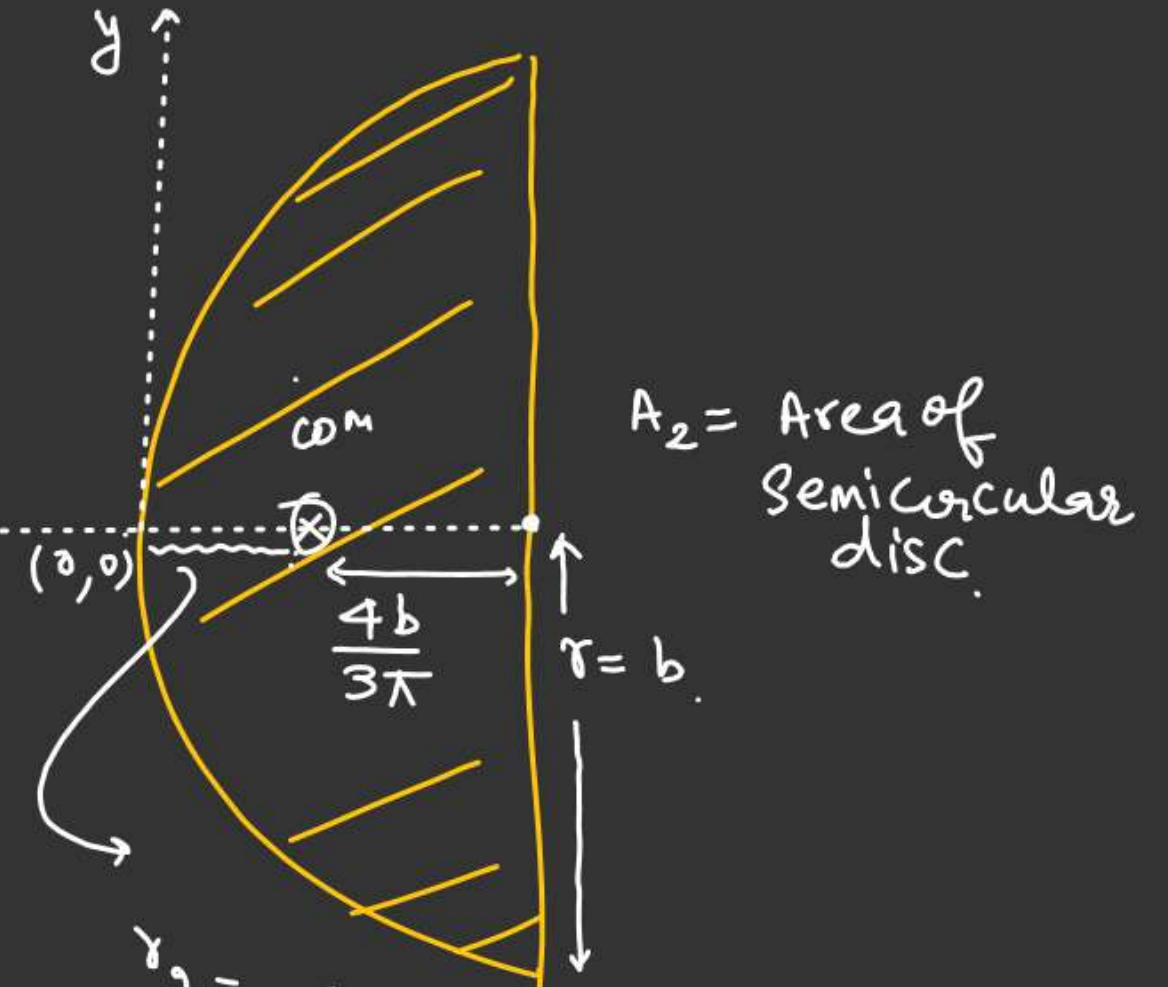
$$y = \frac{\left(\frac{2}{3}\pi R^3\right)\left(\frac{3R}{8}\right) - \left[\frac{1}{3}\pi \left(\frac{R}{2}\right)^2 \times R\right] \times \left(\frac{R}{4}\right)}{\left[\frac{2}{3}\pi R^3 - \frac{\pi}{3}\left(\frac{R}{2}\right)^2 \times R\right]} =$$

$$\frac{33R}{56} \rightarrow \text{Check ??}$$



$$X_{com} =$$

$$\frac{\begin{matrix} A_1 & r_1 & A_2 & r_2 \\ \downarrow & \downarrow & \downarrow & \downarrow \\ (2b)(b)(\frac{b}{2}) - (\frac{\pi b^2}{2})(b - \frac{4b}{3\pi}) \end{matrix}}{(2b^2 - \frac{\pi b^2}{2})}$$



Q 4

## Motion of COM : $\rightarrow$

$$\vec{r}_{com} = \left( \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2 + \dots + m_n \vec{r}_n}{m_1 + m_2 + \dots + m_n} \right)$$

Differentiating both side w.r.t time.

$$\frac{d}{dt}(\vec{r}_{com}) = \frac{m_1 \frac{d}{dt}(\vec{r}_1) + m_2 \frac{d}{dt}(\vec{r}_2) + \dots + m_n \frac{d}{dt}(\vec{r}_n)}{(m_1 + m_2 + \dots + m_n)}$$

$\Downarrow$

$$\vec{v}_{com} = \frac{m_1 \vec{v}_1 + m_2 \vec{v}_2 + \dots + m_n \vec{v}_n}{(m_1 + m_2 + \dots + m_n)}$$

$\Downarrow$

$$\frac{d}{dt}(\vec{v}_{com}) = \frac{m_1 \frac{d}{dt} \vec{v}_1 + m_2 \frac{d}{dt} \vec{v}_2 + \dots + m_n \frac{d}{dt} \vec{v}_n}{m_1 + m_2 + \dots + m_n}$$

$$\vec{a}_{com} = \left( \frac{m_1 \vec{a}_1 + m_2 \vec{a}_2 + \dots + m_n \vec{a}_n}{m_1 + m_2 + \dots + m_n} \right)$$

$$\vec{A}_{com} = \frac{m_1 \vec{a}_1 + m_2 \vec{a}_2 + \dots + m_n \vec{a}_n}{(m_1 + m_2 + \dots + m_n)}$$

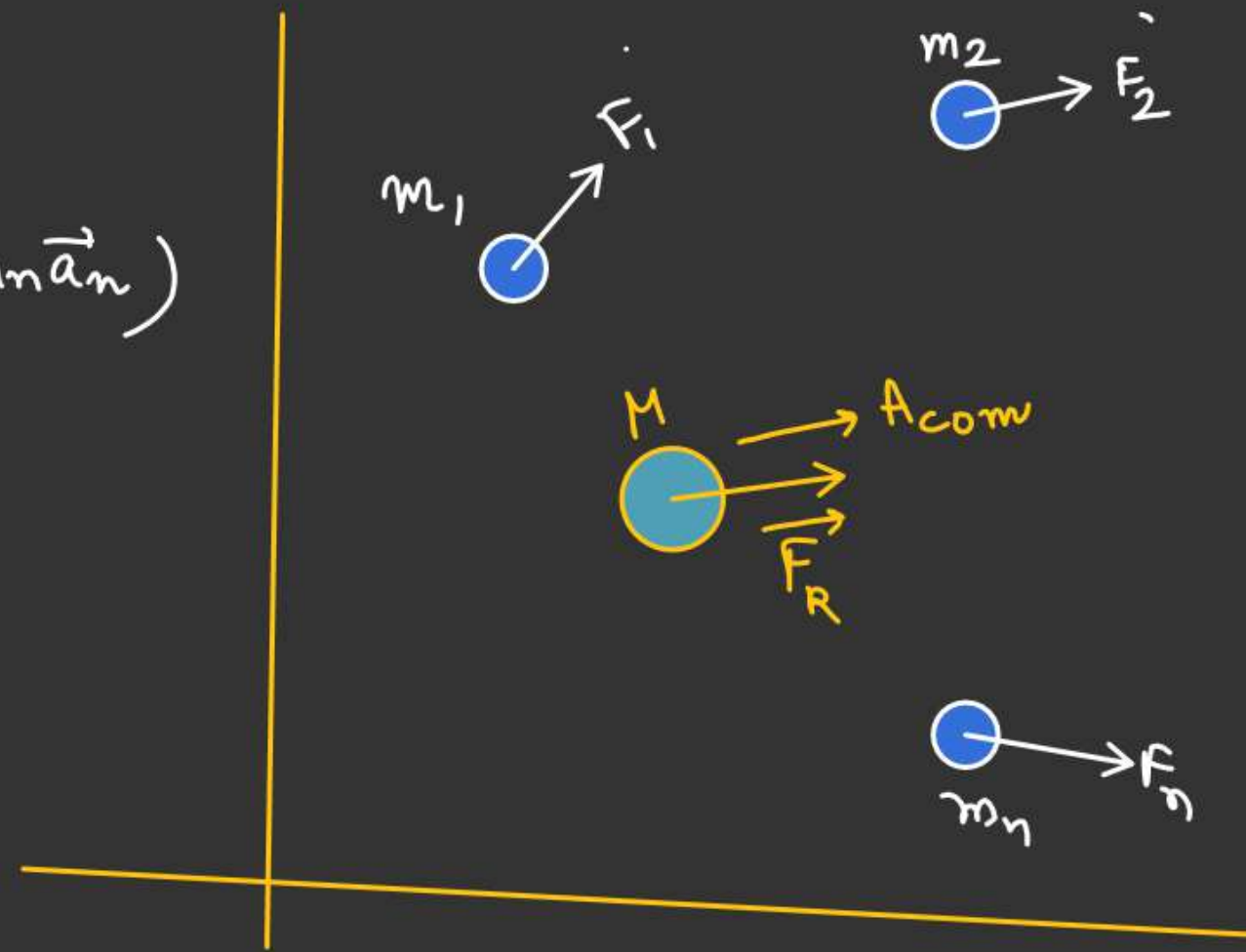
$$(m_1 + m_2 + \dots + m_n) \vec{A}_{com} = (m_1 \vec{a}_1 + m_2 \vec{a}_2 + \dots + m_n \vec{a}_n)$$

$\Downarrow$ 
 $\Downarrow$

$$M \vec{A}_{com} = (\vec{F}_1 + \vec{F}_2 + \dots + \vec{F}_n)$$

$\Downarrow$

$$M \vec{A}_{com} = \vec{F}_R$$



$$\vec{v}_{\text{com}} = \frac{m_1 \vec{v}_1 + m_2 \vec{v}_2 + \dots + m_n \vec{v}_n}{(m_1 + m_2 + \dots + m_n)}$$

$$(m_1 + m_2 + \dots + m_n) \vec{v}_{\text{com}} = m_1 \vec{v}_1 + m_2 \vec{v}_2 + \dots + m_n \vec{v}_n$$

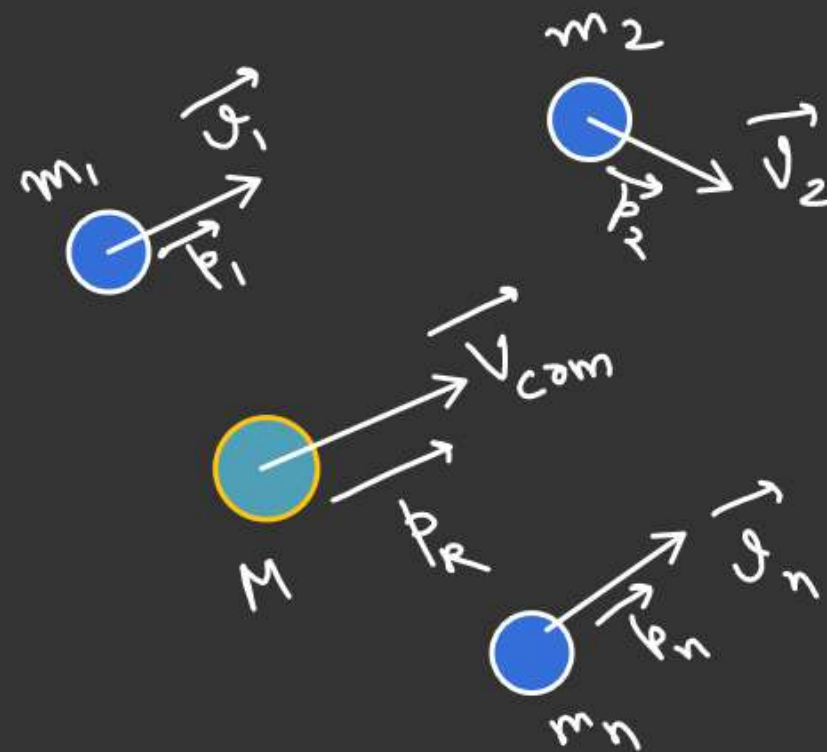
⇓

$$\underline{\underline{M \vec{v}_{\text{com}}}}$$

$$= (\vec{p}_1 + \vec{p}_2 + \dots + \vec{p}_n)$$

$$\Downarrow$$

$$\vec{p}_R \text{ or } \vec{p}_{\text{net}}$$



AA:

If  $\vec{F}_R = 0$  ✓  $F_R = \text{Resultant force.}$

or  
Net Force

$$M \vec{A}_{\text{com}} = 0$$

$$\vec{A}_{\text{com}} = 0 \quad \checkmark$$

$$\frac{d\vec{V}_{\text{com}}}{dt} = 0$$

$$(\vec{V}_{\text{com}})_i = (\vec{V}_{\text{com}})_f$$

$$M(\vec{V}_{\text{com}})_i = M(\vec{V}_{\text{com}})_f$$

⇓

$$(\vec{p}_i)_{\text{system}} = (\vec{p}_f)_{\text{system}}$$

⇒ Momentum of the system conserved.

$$\vec{F}_R = 0, \quad \vec{A}_{\text{com}} = 0$$

$$(\vec{V}_{\text{com}})_i = (\vec{V}_{\text{com}})_f$$

If  $(\vec{V}_{\text{com}})_i = 0 \Rightarrow (\vec{V}_{\text{com}})_f = 0$

⇓

$$\frac{d(\vec{R}_{\text{com}})}{dt} = 0$$

$$(\vec{R}_{\text{com}})_i = (\vec{R}_{\text{com}})_f$$

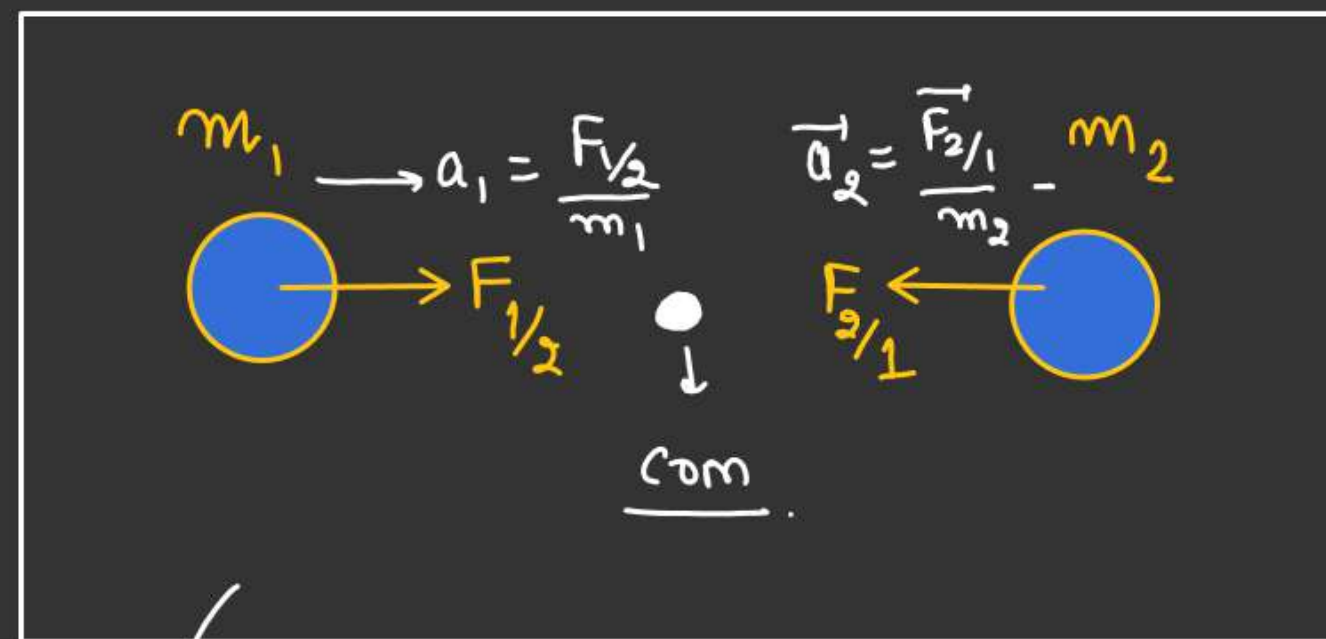
$$\Rightarrow \Delta \vec{R}_{\text{com}} = 0$$

Q.2:

$F$  = Gravitational force  
of interaction  
b/w two particle.

$$\vec{F}_{1/2} = -\vec{F}_{2/1} \checkmark$$

$$|\vec{F}_{1/2}| = |\vec{F}_{2/1}| = \frac{Gm_1m_2}{r^2}$$



System boundary.

$$\vec{A}_{com} = \frac{m_1 \vec{a}_1 + m_2 \vec{a}_2}{m_1 + m_2}$$

$$\vec{A}_{com} =$$

$$\frac{m_1 \left( \frac{\vec{F}_{1/2}}{m_1} \right) + m_2 \left( \frac{\vec{F}_{2/1}}{m_2} \right)}{m_1 + m_2} = \frac{\vec{F}_{1/2} + \vec{F}_{2/1}}{m_1 + m_2} = 0$$

As  $(\vec{F}_{1/2} = -\vec{F}_{2/1})$

$$F_{net} = 0$$

$$A_{com} = 0$$

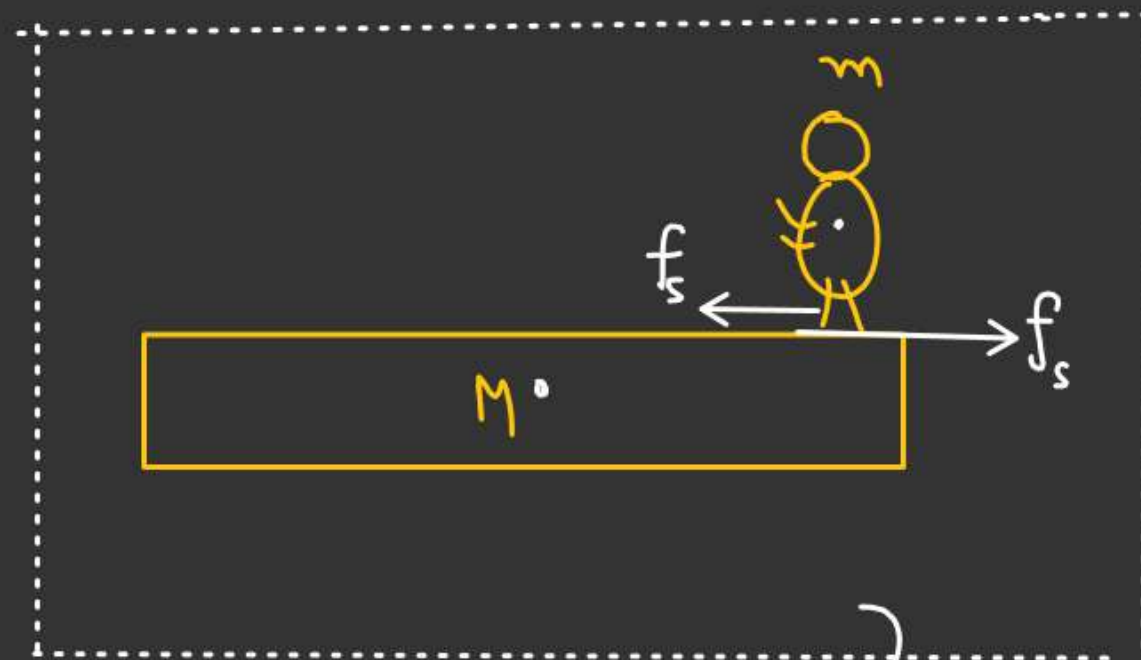
$$\vec{V}_{com} = \text{constant}$$

$$\vec{P}_{com} = \text{constant}$$

MAN - PLANK SYSTEM

Man starts walking on the plank.

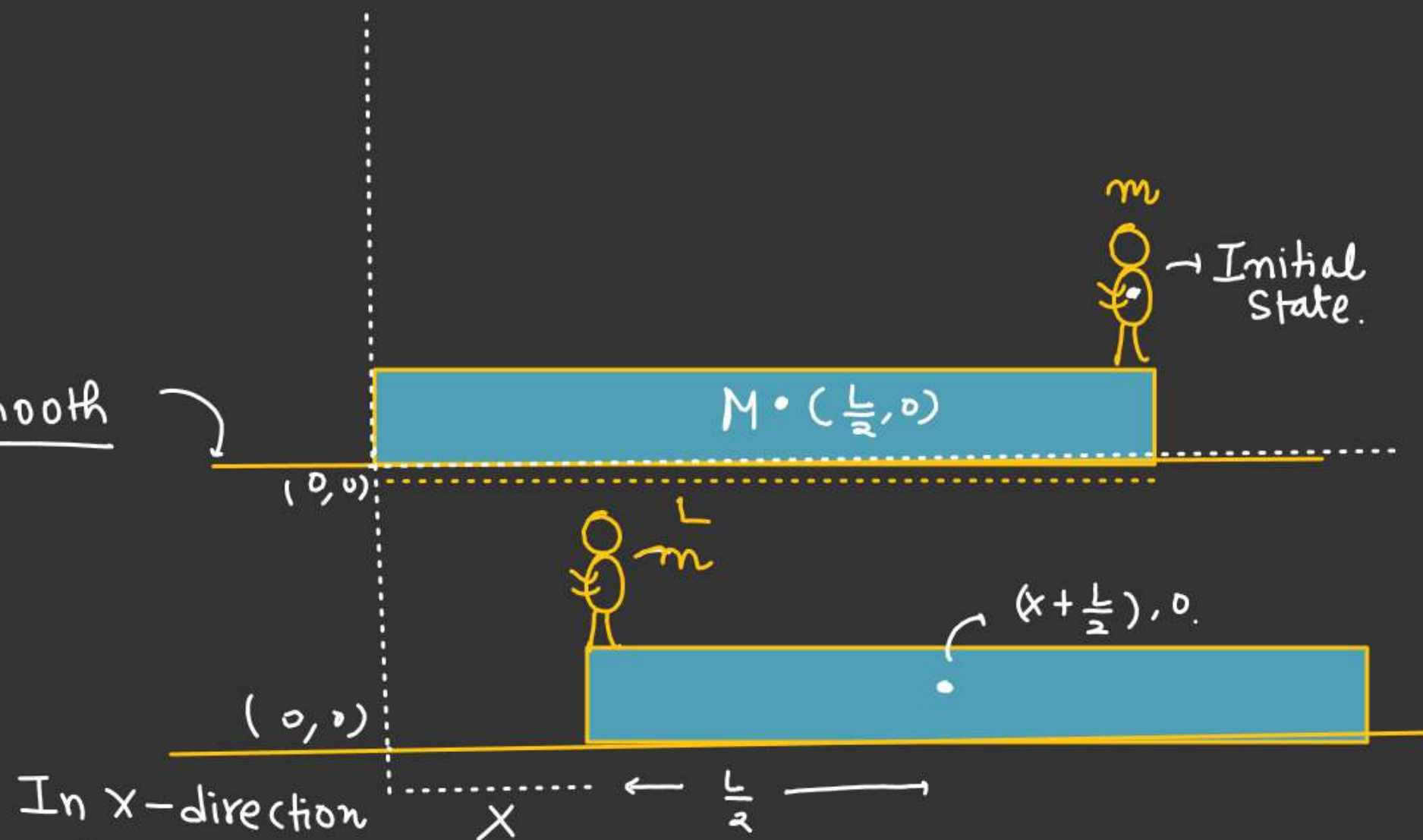
Find displacement of plank when reaches the other end of the plank.



System boundary

→ (Man + Plank)

Smooth



In x-direction

$$F_{net} = 0$$

$$A_{com} = 0$$

$$V_{com} = 0$$

$$\Rightarrow (R_{com})_i = (R_{com})_f$$

$$\Rightarrow (X_{com})_i = (X_{com})_f$$

In x-direction

$$\left. \begin{aligned} F_{\text{net}} &= 0 \\ A_{\text{com}} &= 0 \\ V_{\text{com}} &= 0 \end{aligned} \right\} \Rightarrow$$

$$(R_{\text{com}})_i = (R_{\text{com}})_f$$

$$(X_{\text{com}})_i = (X_{\text{com}})_f$$

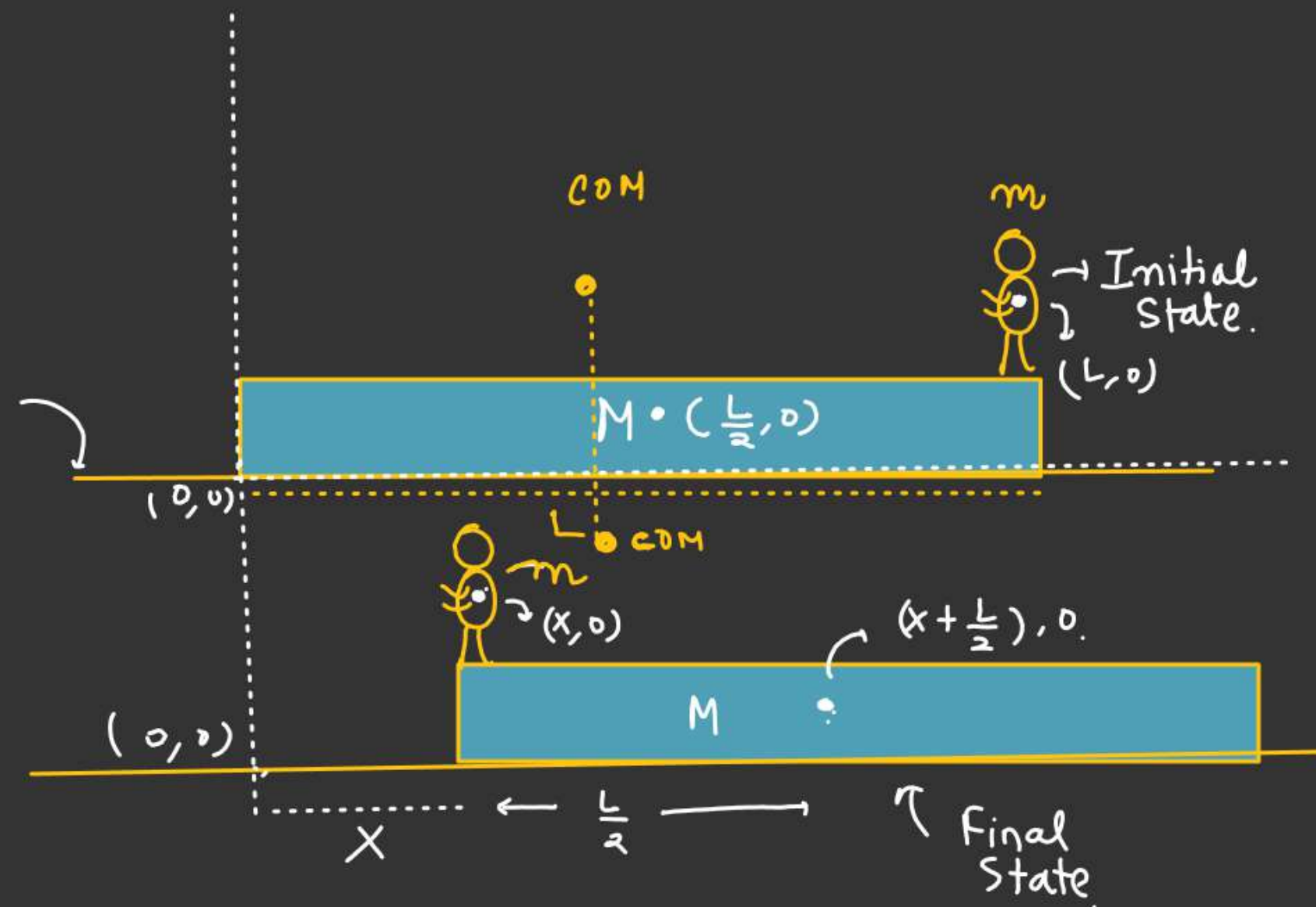
$$(X_{\text{com}})_i = \frac{M\left(\frac{L}{2}\right) + (mL)}{M+m}$$

$$(X_{\text{com}})_f = \frac{mx + M\left(\frac{L}{2} + x\right)}{M+m}$$

$$(X_{\text{com}})_i = (X_{\text{com}})_f$$

$$\cancel{M\left(\frac{L}{2}\right)} + mL = mx + \cancel{M\frac{L}{2}} + Mx$$

$$\frac{mL}{M+m} = x \quad \checkmark$$



H.W

D.P.P  $\rightarrow$  (1)

↳ Module questions. Mention in D.P.P-1.

H.C.V Page-No (159  $\rightarrow$  160)

Q.No  $\rightarrow$  1 to 11 ✓.