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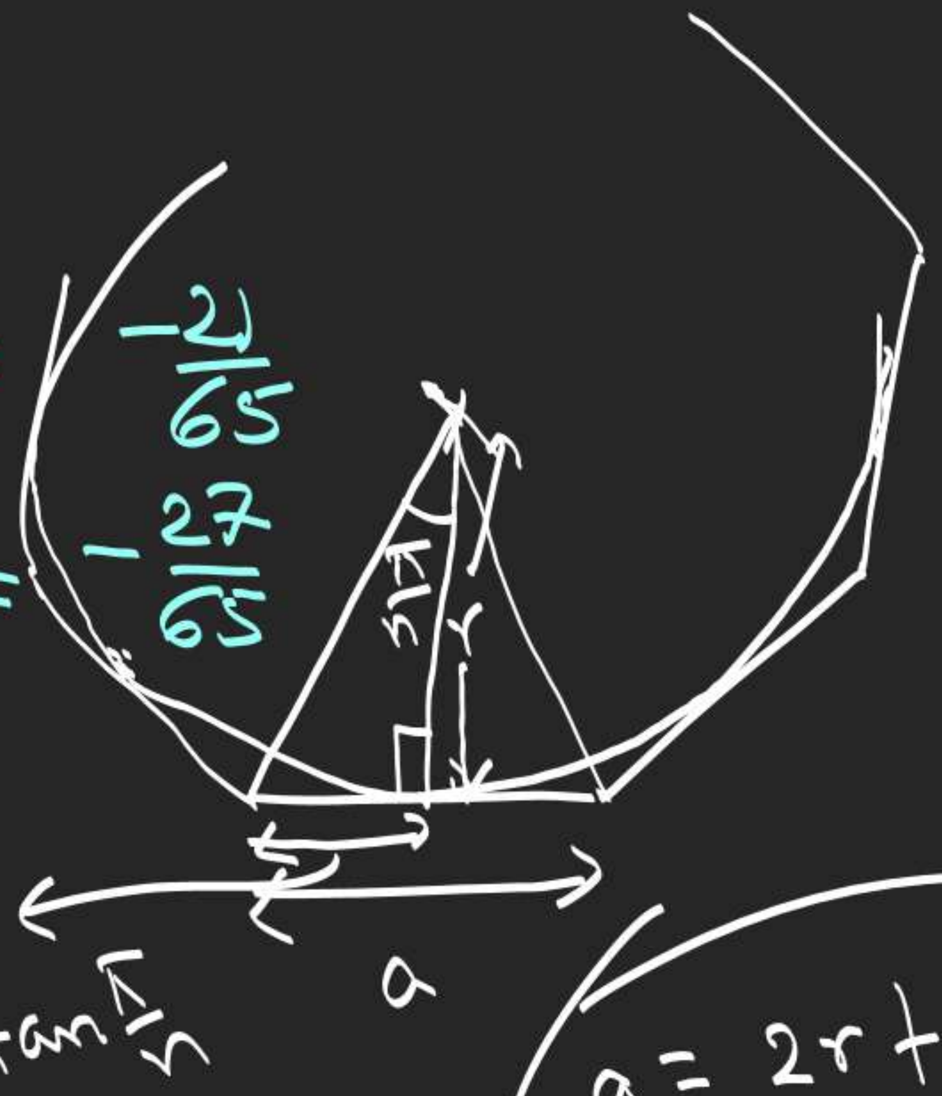
3. $2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2} = -\frac{21}{65}$

$2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2} = -\frac{27}{65}$

$4\cos^2\left(\frac{\alpha-\beta}{2}\right) =$



$-$

$$\boxed{\frac{r}{R} = \cos\frac{\pi}{n}}$$

$a = 2r\tan\frac{\pi}{n}$

$a = 2R\sin\frac{\pi}{n}$

$$r + R = \frac{a}{2}\cot\frac{\pi}{n} + \frac{a}{2}\operatorname{cosec}\frac{\pi}{n} = \frac{a}{2}\left(\frac{1+\cos\frac{\pi}{n}}{\sin\frac{\pi}{n}}\right)$$

$$= \frac{a}{2}\cot\frac{\pi}{2n}$$

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4. $u^2 = a^2+b^2+2\sqrt{(a^4+b^4)\underline{\sin^2\theta\cos^2\theta} + a^2b^2(\sin^4\theta+\cos^4\theta)}$

$$(a^4+b^4)\frac{\sin^2 2\theta}{4} + a^2b^2\left(1-\frac{1}{2}\sin^2 2\theta\right)$$

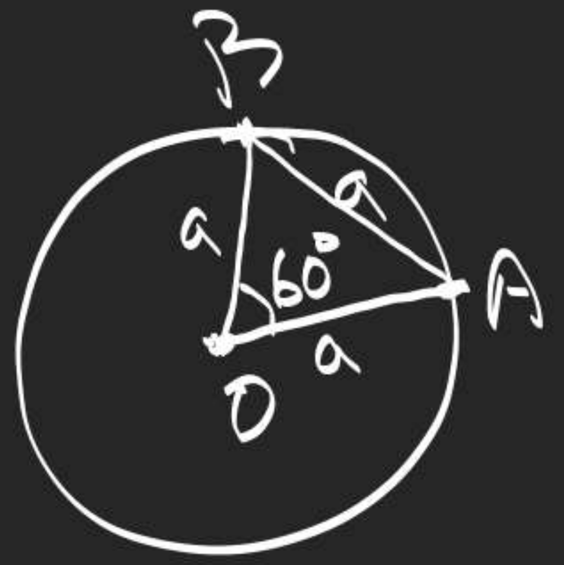
$0 \leq \quad \leq 1$

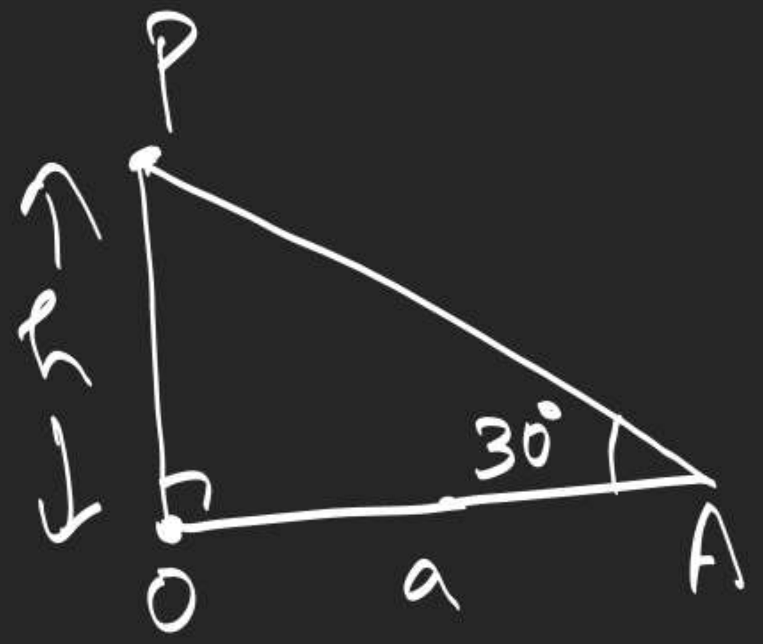
$$u^2 = (a^2+b^2) + 2\sqrt{\frac{\sin^2 2\theta}{4}(a^2-b^2)^2 + a^2b^2}$$

$$a^2+b^2+2\sqrt{a^2b^2} \leq u^2 \leq (a^2+b^2)+2\sqrt{\frac{(a^2-b^2)^2}{4}+a^2b^2}$$

$= (a+b)^2 \qquad\qquad = 2(a^2+b^2)$

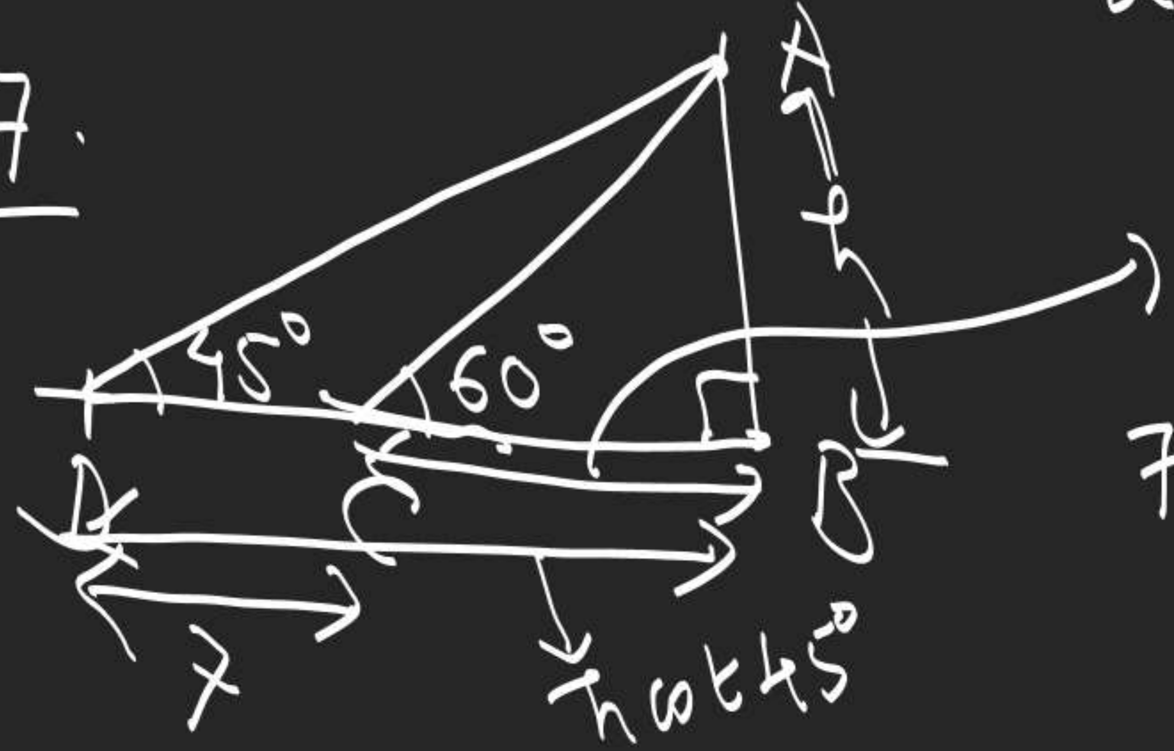
6.





$$\frac{h}{a} = \tan 30^\circ = \frac{1}{\sqrt{3}}$$

7.



$h\cot 60^\circ$

$$7 = h\left(1 - \frac{1}{\sqrt{3}}\right)$$

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8: $2(\cos\beta\cos\gamma + \cos\gamma\cos\alpha + \cos\alpha\cos\beta) + 2\sum \sin\beta\sin\gamma = -3$

$\left(\cos^2\alpha + \cos^2\beta + \cos^2\gamma + 2\sum\cos\alpha\cos\beta\right) + \left(\sum\sin^2\alpha + 2\sum\sin\alpha\sin\beta\right) = 0$

$-\frac{\pi}{4} \le \alpha - \beta \le \frac{\pi}{4}$

$0 \le \alpha - \beta \le \frac{\pi}{4}$

$\Rightarrow \sin(\alpha-\beta) = \frac{5}{13}$

$\underbrace{(\cos\alpha + \cos\beta + \cos\gamma)^2}_{\geq 0} + \underbrace{(\sin\alpha + \sin\beta + \sin\gamma)^2}_{\geq 0} = 0$

$0 \le \beta \le \frac{\pi}{4}$

$-\frac{\pi}{4} \le -\beta \le 0$

$0 \le \alpha \le \pi/4$

$0 \le \alpha + \beta \le \frac{\pi}{2}$

$$\boxed{\cos\alpha + \cos\beta + \cos\gamma = 0 = \sin\alpha + \sin\beta + \sin\gamma}$$

$0 \le \alpha \le \frac{\pi}{4}$

$0 \le \beta \le \frac{\pi}{4}$

$\tan((\alpha+\beta) + (\alpha-\beta))$

Nishant Jindal

$$\cos^4 x - \cos^2 x + 1$$

$$= \left(\cos^2 x - \frac{1}{2}\right)^2 + \frac{3}{4}$$

$$0 \le \cos^2 x \le 1$$

$$-\frac{1}{2} \le \cos^2 x - \frac{1}{2} \le \frac{1}{2}$$

$$0 \le \left(\cos^2 x - \frac{1}{2}\right)^2 \le \frac{1}{4}$$

$$\frac{3}{4} \le \left(\cos^2 x - \frac{1}{2}\right)^2 + \frac{3}{4} \le 1$$

12.

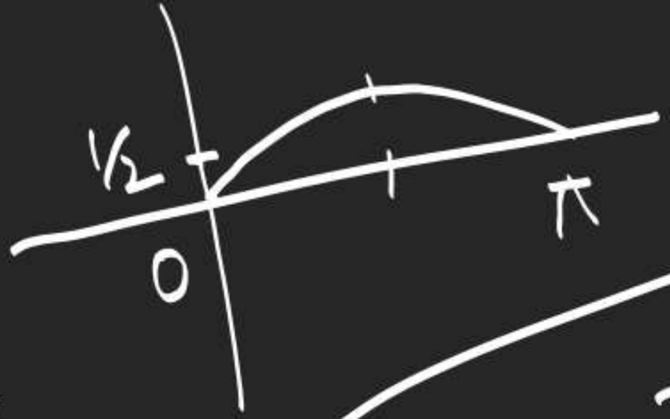
$$3\underbrace{\sin P}_{<\frac{1}{2}} + 4\cos Q = 6 \quad (1)$$

$$< \frac{3}{2} + 4$$

$$3\cos P + 4\sin Q = 1 \quad (2)$$

$(1)^2 + (2)^2$

$$9 + 16 + 24\sin(\underbrace{P+Q}_{\pi - R}) = 37$$



$$\sin R = \frac{1}{2}$$

$-4 \quad 4$

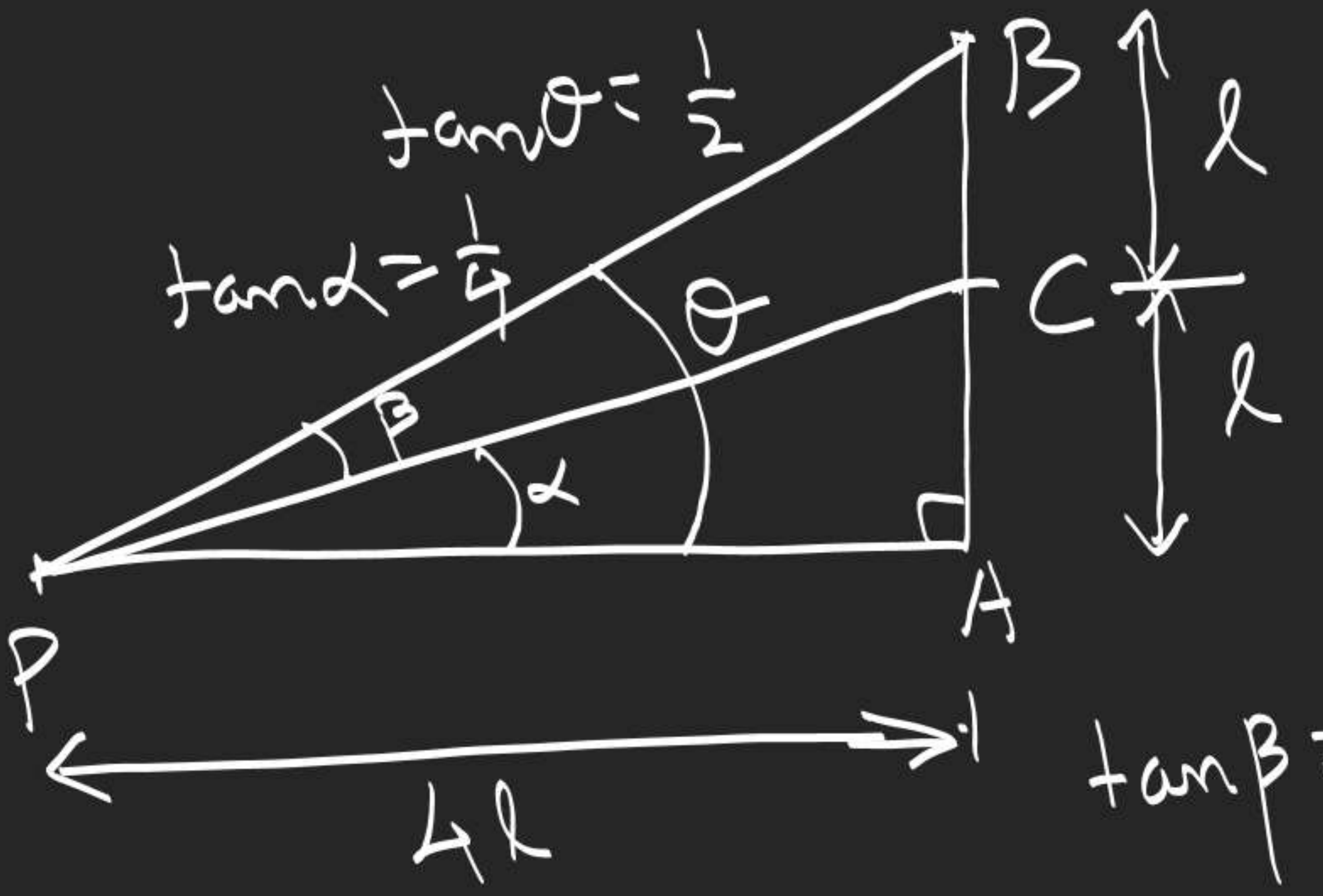
$$R = \frac{\pi}{6} \text{ or } \frac{5\pi}{6}$$

If $R = \frac{5\pi}{6}$, $\boxed{P, Q < \frac{\pi}{6}}$

rejected

Nishant Jindal

17.



$$\tan\beta = \tan(\theta - \alpha)$$

$$= \frac{\frac{1}{2} - \frac{1}{4}}{1 + \frac{1}{2}\frac{1}{4}}$$

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18.

$$5\tan^2 x - 5\cos^2 x = 2(2\cos^2 x - 1) + 9$$

$$5(\sec^2 x - 1) = 9\cos^2 x + 7$$

$$9\cos^2 x - 5\sec^2 x + 12 = 0$$

$$9\cos^4 x + 12\cos^2 x - 5 = 0$$

$$9\cos^4 x - 3\cos^2 x + 15\cos^2 x - 5 = 0$$

$$(3\cos^2 x + 5)(3\cos^2 x - 1) = 0$$

$$\boxed{\cos^2 x = \frac{1}{3}}$$

$$\cos 2x = \frac{2}{3} - 1 = -\frac{1}{3}$$

$$\cos 4x = 2\left(\frac{1}{9}\right) - 1$$

$$= -\frac{7}{9}.$$

Nishant Jindal

$$\boxed{a^2+b^2+c^2 \geq ab+bc+ca}$$

$$a^2+b^2+c^2-(ab+bc+ca)$$

$$= \frac{1}{2}\left((a-b)^2+(b-c)^2+(c-a)^2\right) \geq 0$$

$$\boxed{\begin{gathered} a^2+b^2+c^2 = ab+bc+ca \\ \text{if } a=b=c \end{gathered}}$$

Nishant Jindal

1. $\boxed{\text{In } \triangle ABC,}$ P.T

(i) $\cos A \cos B \cos C \leq \frac{1}{8}$

(ii) $\boxed{\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2} \leq \frac{1}{8}}$ ✓

(iii) $1 < \cos A + \cos B + \cos C \leq \frac{3}{2}$

Nishant Jindal

$$\frac{1}{2}\left(2\cos A\cos B\right)\cos C = \frac{1}{2}\Big(\cos(\underbrace{A+B}_{\pi-C}) + \cos(A-B)\Big)\cos C$$

$$= \frac{1}{2}\Big(-\cos C + \cos(A-B)\Big)\cos C$$

$$= -\frac{1}{2}\Big(-\cos(A-B)\cos C + \cos^2 C\Big)$$

$$= -\frac{1}{2}\left(\left(\cos C - \frac{1}{2}\cos(A-B)\right)^2 - \frac{1}{4}\cos^2(A-B)\right)$$

$$= \frac{1}{8}\cos^2(A-B) - \frac{1}{2}\left(\cos C - \frac{1}{2}\cos(A-B)\right)^2$$

$$\le \frac{1}{8}$$

$$\frac{1}{8} - \underbrace{\left[\frac{1}{8}\sin^2(A-B) + \frac{1}{2}\left(\cos C - \frac{1}{2}\cos(A-B)\right)^2\right]}_{\ge 0} \le \frac{1}{8}$$

$\cos A\cos B\cos C = \frac{1}{8}$

if $\sin(A-B) = 0$

& $\cos C - \frac{1}{2}\cos(A-B) = 0$

$A = B$ & $\cos C = \frac{1}{2}$

$\boxed{C = \frac{\pi}{3}}$

$A = B = C = \pi/3$

Nishant Jindal

$$\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2} = \frac{1}{2}\left(\cos\frac{A-B}{2}\sin\frac{C}{2} - \sin^2\frac{C}{2}\right)$$

$$= -\frac{1}{2}\left(\left(\sin\frac{C}{2} - \frac{1}{2}\cos\frac{A-B}{2}\right)^2 - \frac{1}{4}\cos^2\left(\frac{A-B}{2}\right)\right)$$

$$= \frac{1}{8}\cos^2\left(\frac{A-B}{2}\right) - \frac{1}{2}\left(\sin\frac{C}{2} - \frac{1}{2}\cos\frac{A-B}{2}\right)^2$$

$$= \frac{1}{8} - \left[\frac{1}{8}\sin^2\left(\frac{A-B}{2}\right) + \frac{1}{2}\left(\sin\frac{C}{2} - \frac{1}{2}\cos\frac{A-B}{2}\right)^2\right]$$

$$\leq \frac{1}{8}$$

$\prod \sin\frac{A}{2} = \frac{1}{8}$ if $\sin\frac{A-B}{2} = 0$ & $\sin\frac{C}{2} = \frac{1}{2}\cos\frac{A-B}{2}$

$$\boxed{A = B = C = \pi/3}$$

Nishant Jindal

$$\cos A+\cos B+\cos C = 2\cos\frac{A+B}{2}\cos\frac{A-B}{2}+\cos C$$

$$= 2\sin\frac{C}{2}\cos\frac{A-B}{2}+1-2\sin^2\frac{C}{2}$$

$$1+2\sin\frac{C}{2}\left(\cos\frac{A-B}{2}-\cos\frac{A+B}{2}\right)$$

$$= 1+4\underbrace{\sin\frac{A}{2}}_{>0}\underbrace{\sin\frac{B}{2}}_{>0}\underbrace{\sin\frac{C}{2}}_{>0} > 1$$

$$= -2\left(\sin^2\frac{C}{2}-\sin\frac{C}{2}\cos\frac{A-B}{2}\right)+1$$

$$= 1-2\left(\left(\sin\frac{C}{2}-\frac{1}{2}\cos\frac{A-B}{2}\right)^2-\frac{1}{4}\cos^2\frac{A-B}{2}\right)$$

$$= 1+\frac{1}{2}\cos^2\left(\frac{A-B}{2}\right)-2\left(\sin\frac{C}{2}-\frac{1}{2}\cos\frac{A-B}{2}\right)^2$$

$$\frac{3}{2} \geq = \frac{3}{2}-\underbrace{\left[\frac{1}{2}\sin^2\left(\frac{A-B}{2}\right)+2\left(\sin\frac{C}{2}-\frac{1}{2}\cos\frac{A-B}{2}\right)^2\right]}_{\geq 0}$$

$$\cos A+\cos B+\cos C=\frac{3}{2}$$

$$\sin\left(\frac{A-B}{2}\right)=0 \ \&$$

$$\sin\frac{C}{2}=\frac{1}{2}\cos\frac{A-B}{2}$$

$$\boxed{A=B=C=\frac{\pi}{3}}$$

$$\boxed{\text{Ex}-5}$$