Nishant Jindal

1. (a) $\sin(-\theta) = -\sin\theta,$

   (b) $\cos(-\theta) = \cos\theta,$

   (c) $\tan(-\theta) = -\tan\theta,$

   (d) $\cot(-\theta) = -\cot\theta,$

   (e) $\operatorname{cosec}(-\theta) = -\operatorname{cosec}\theta,$

   (f) $\sec(-\theta) = \sec\theta,$

2. (a) $\sin(90^\circ - \theta) = \cos\theta,$

   (b) $\cos(90^\circ - \theta) = \sin\theta,$

   (c) $\tan(90^\circ - \theta) = \cot\theta,$

   (d) $\cot(90^\circ - \theta) = \tan\theta,$

   (e) $\sec(90^\circ - \theta) = \operatorname{cosec}\theta,$

   (f) $\operatorname{cosec}(90^\circ - \theta) = \sec\theta,$

3. (a) $\sin(90^\circ + \theta) = \cos\theta$,

(b) $\cos(90^\circ + \theta) = -\sin\theta$,

(c) $\tan(90^\circ + \theta) = -\cot\theta$,

(d) $\cot(90^\circ + \theta) = -\tan\theta$,

(e) $\sec(90^\circ + \theta) = -\operatorname{cosec}\theta$,

(f) $\operatorname{cosec}(90^\circ - \theta) = \sec\theta$,

4. (a) $\sin(180^\circ - \theta) = \sin\theta$,

(b) $\cos(180^\circ - \theta) = -\cos\theta$,

(c) $\tan(180^\circ - \theta) = -\tan\theta$,

(d) $\cot(180^\circ - \theta) = -\cot\theta$,

(e) $\sec(180^\circ - \theta) = -\sec\theta$,

(f) $\operatorname{cosec}(180^\circ - \theta) = \operatorname{cosec}\theta$,

5. (a) $\sin(180^\circ + \theta) = -\sin\theta$,

(b) $\cos(180^\circ + \theta) = -\cos\theta$,

(c) $\tan(180^\circ + \theta) = \tan\theta$,

(d) $\cot(180^\circ + \theta) = \cot\theta$,

(e) $\sec(180^\circ + \theta) = -\sec\theta$,

(f) $\operatorname{cosec}(180^\circ + \theta) = -\operatorname{cosec}\theta$

6. (a) $\sin(270^\circ - \theta) = -\cos\theta$,

(b) $\cos(270^\circ - \theta) = -\sin\theta$,

(c) $\tan(270^\circ - \theta) = \cot\theta$,

(d) $\cot(270^\circ - \theta) = \tan\theta$,

(e) $\sec(270^\circ - \theta) = -\operatorname{cosec}\theta$,

(f) $\operatorname{cosec}(270^\circ - \theta) = -\sec\theta$,

7. (a) $\sin(360^\circ - \theta) = -\sin\theta$,

(b) $\cos(360^\circ - \theta) = \cos\theta$,

(c) $\tan(360^\circ - \theta) = -\tan\theta$,

(d) $\cot(360^\circ - \theta) = -\cot\theta$,

(e) $\sec(360^\circ - \theta) = \sec\theta$,

(f) $\operatorname{cosec}(360^\circ - \theta) = -\operatorname{cosec}\theta$,

8. (a) $\sin(360^\circ + \theta) = \sin\theta$,

(b) $\cos(360^\circ + \theta) = \cos\theta$,

(c) $\tan(360^\circ + \theta) = \tan\theta$,

(d) $\cot(360^\circ + \theta) = \cot\theta$,

(e) $\sec(360^\circ + \theta) = \sec\theta$,

(f) $\operatorname{cosec}(360^\circ + \theta) = \operatorname{cosec}\theta$

9. (a) $\sin 18^\circ = \frac{\sqrt{5}-1}{4}$

(b) $\cos 36^\circ = \frac{\sqrt{5}+1}{4}$

(c) $\sin\frac{\pi}{12} = \sin 15^\circ = \frac{\sqrt{3}-1}{2\sqrt{2}}$

(d) $\cos\frac{\pi}{12} = \cos 15^\circ = \frac{\sqrt{3}+1}{2\sqrt{2}}$

(e) $\tan\frac{\pi}{12} = \tan 15^\circ = 2-\sqrt{3}$

(f) $\sin\frac{5\pi}{12} = \sin 75^\circ = \frac{\sqrt{3}+1}{2\sqrt{2}}$

(g) $\cos\frac{5\pi}{12} = \cos 75^\circ = \frac{\sqrt{3}-1}{2\sqrt{2}}$

(h) $\tan\frac{5\pi}{12} = \tan 75^\circ = 2+\sqrt{3}$

(i) $\tan\frac{\pi}{8} = \tan 22\frac{1}{2}^\circ = \sqrt{2}-1$

(j) $\tan\frac{3\pi}{8} = \tan 67\frac{1}{2}^\circ = \sqrt{2}+1$

(k) $\cot\frac{3\pi}{8} = \cot 67\frac{1}{2}^\circ = \sqrt{2}-1$

10. $\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

11. $\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

12. $\sin^2 A - \sin^2 B = \cos^2 B - \cos^2 A = \sin(A+B) \cdot \sin(A-B)$

13. $\cos^2 A - \sin^2 B = \cos^2 B - \sin^2 A = \cos(A+B) \cdot \cos(A-B)$

14. $\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$

15. $\cot(A \pm B) = \frac{\cot A \cot B \mp 1}{\cot B \pm \cot A}$

16. $\sin 2A = 2\sin A \cos A = \frac{2\tan A}{1+\tan^2 A}$

17. $\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A = \frac{1-\tan^2 A}{1+\tan^2 A}$

18. $\tan 2A = \frac{2\tan A}{1-\tan^2 A}$

19. $1 - \cos 2A = 2\sin^2 A$

20. $1 + \cos 2A = 2\cos^2 A$

21. $\frac{1-\cos 2A}{1+\cos 2A} = \tan^2 A$

22. $\sin 3A = 3\sin A - 4\sin^3 A$

23. $\cos 3A = 4\cos^3 A - 3\cos A$

24. $\tan 3A = \frac{3\tan A - \tan^3 A}{1-3\tan^2 A}$

25. $\sin C + \sin D = 2\sin\frac{C+D}{2}\cos\frac{C-D}{2}$

26. $\sin C - \sin D = 2\cos\frac{C+D}{2}\sin\frac{C-D}{2}$

27. $\cos C + \cos D = 2\cos\frac{C+D}{2}\cos\frac{C-D}{2}$

28. $\cos C - \cos D = -2\sin\frac{C+D}{2}\sin\frac{C-D}{2}$

29. $2\sin A\cos B = \sin(A+B) + \sin(A-B)$

30. $2\cos A\sin B = \sin(A+B) - \sin(A-B)$

31. $2\cos A\cos B = \cos(A+B) + \cos(A-B)$

32. $2\sin A\sin B = \cos(A-B) - \cos(A+B)$

33. $\tan A - \tan B = \frac{\sin(A-B)}{\cos A\cos B}$

34. $\cot A - \cot B = \frac{\sin(B-A)}{\sin A\sin B}$

35. $\tan A + \tan B = \frac{\sin(A+B)}{\cos A\cos B}$

36. $\cot A + \cot B = \frac{\sin(B+A)}{\sin A\sin B}$

37. $\tan(A+B) - \tan A - \tan B = \tan(A+B)\tan A\tan B$

Nishant Jindal

38.(a) $\tan(A+B+C) = \frac{\tan A+\tan B+\tan C-\tan A\tan B\tan C}{1-\tan A\tan B-\tan B\tan C-\tan C\tan A} = \frac{s_1-s_3}{1-s_2}$

(b) $\tan(A_1+A_2+A_3+\cdots \ldots\ldots A_n) = \frac{s_1-s_3+s_5-s_7+\cdots\ldots}{1-s_2+s_4-s_6+\cdots\ldots}$

where $s_1 = \sum_{i=1}^{n} \tan A_i$; $s_2 = \sum \tan A_1 \tan A_2$;

39. $\sin\alpha + \sin(\alpha+\beta) + \sin(\alpha+2\beta) + \cdots \ldots + \sin(\alpha+\overline{n-1}\beta) = \frac{\sin\frac{n\beta}{2}}{\sin\frac{\beta}{2}} \sin\left(\alpha + \frac{n-1}{2}\beta\right)$

40. $\cos\alpha + \cos(\alpha+\beta) + \cos(\alpha+2\beta) + \cdots \ldots + \cos(\alpha+\overline{n-1}\beta) = \frac{\sin\frac{n\beta}{2}}{\sin\frac{\beta}{2}} \cos\left(\alpha + \frac{n-1}{2}\beta\right)$

Nishant Jindal

41. If $A + B + C = \pi$, then

(a) $\Sigma \tan A = \Pi \tan A$

(b) $\Sigma \tan \frac{A}{2} \tan \frac{B}{2} = 1$

(c) $\Sigma \cot A \cot B = 1$

(d) $\Sigma \cot \frac{A}{2} = \Pi \cot \frac{A}{2}$

(e) $\Sigma \sin 2A = 4\Pi \sin A$

(f) $\Sigma \cos A = 1 + 4\Pi \sin \frac{A}{2}$

1. Prove that $\frac{\cos 8A\cos 5A-\cos 12A\cos 9A}{\sin 8A\cos 5A+\cos 12A\sin 9A} = \tan 4A$

2. If $\sin A = \frac{3}{5}, \cos B = \frac{-12}{13}$, where $\frac{\pi}{2} < A < \pi$ and $\frac{\pi}{2} < B < \pi$, then $\sin(A+B)$ equals

(A) $\frac{56}{65}$ (B) $\frac{-56}{65}$ (C) $\frac{33}{65}$ (D) $\frac{-33}{65}$

3. The value of expression $\frac{\cos 68^\circ}{\sin 56^\circ \cdot \sin 34^\circ \cdot \tan 22^\circ}$ is equal to

(A) 1 (B) 2 (C) 3 (D) 4

4. Prove that $\cos^4\frac{\pi}{8} + \cos^4\frac{3\pi}{8} + \cos^4\frac{5\pi}{8} + \cos^4\frac{7\pi}{8} = \frac{3}{2}$

5. If $\tan x = \frac{3}{4}, \pi < x < \frac{3\pi}{2}$, then $\tan\frac{x}{2}$ is equal to

(A) -3 (B) $\frac{1}{3}$ (C) $\frac{-1}{2}$ (D) $\frac{-1}{3}$

6. Prove that $(4\cos^2 9^* - 3)(4\cos^2 27^\circ - 3) = \tan 9^*$,

7. If $\alpha + \beta = \gamma$. prove that $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1 + 2\cos\alpha\cos\beta\cos\gamma$.

8. If $\alpha + \beta = \gamma$. prove that $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1 + 2\cos\alpha\cos\beta\cos\gamma$.

9. Calculate without using trigonometric tables:

(a) $4\cos 20^\circ - \sqrt{3}\cot 20^\circ$

(b) $\frac{2\cos 40^\circ - \cos 20^\circ}{\sin 20^\circ}$

(c) $\cos^6\frac{\pi}{16} + \cos^6\frac{3\pi}{16} + \cos^6\frac{5\pi}{16} + \cos^6\frac{7\pi}{16}$

(d) $\tan 10^\circ - \tan 50^\circ + \tan 70^\circ$

10. Given that $(1 + \tan 1^\circ)(1 + \tan 2^\circ) \ldots (1 + \tan 45^\circ) = 2^n$, find n.

11. If A, B, C denote the angles of a triangle ABC then prove that the triangle is right angled if and only If $\sin 4A + \sin 4B + \sin 4C = 0$

12. (a) If $y = 10\cos^2 x - 6\sin x\cos x + 2\sin^2 x$, then find the greatest & least value of y.

(b) If $y = 1 + 2\sin x + 3\cos^2 x$, find the maximum & minimum values of $y \forall x \in R$.

(c) If $a \le 3\cos\left(\theta + \frac{\pi}{3}\right) + 5\cos\theta + 3 \le b$, find a and b.

13. If the expression $\cos^2\frac{\pi}{11} + \cos^2\frac{2\pi}{11} + \cos^2\frac{3\pi}{11} + \cos^2\frac{4\pi}{11} + \cos^2\frac{5\pi}{11}$ has the value equal to $\frac{p}{q}$ in its Lowes form : then find $(p+q)$.

14. Prove that: $\cos^2\alpha + \cos^2(\alpha+\beta) - 2\cos\alpha \cdot \cos\beta\cos(\alpha+\beta) = \sin^2\beta$

15. Prove that:

(a) $\tan 20^\circ, \tan 40^\circ, \tan 60^\circ, \tan 80^\circ = 3$

(b) $\tan 9^\circ - \tan 27^\circ - \tan 63^\circ + \tan 81^\circ = 4.$

(c) $\sin^4\frac{\pi}{16} + \sin^4\frac{3\pi}{16} + \sin^4\frac{5\pi}{16} + \sin^4\frac{7\pi}{16} = \frac{3}{2}$

16. Find the positive integers p, q, r, s satisfying $\tan\frac{\pi}{24} = (\sqrt{p} - \sqrt{q})(\sqrt{r} - s)$.

17. If the value of the expression $\sin 25^\circ \cdot \sin 35^\circ \cdot \sin 85^\circ$ can be expressed as $\frac{\sqrt{a}+\sqrt{b}}{c}$ where a, b, c N and are in their lowest form, find the value of $(a + b + c)$.

Nishant Jindal

Calculus $\rightarrow$ motion, growth

$\downarrow$

rate of change

velocity, acceleration, ...

Nishant Jindal

Cartesian Product of set A with B

$A \times B$ is set of all possible ordered pairs of elements from set A & B.

$$A \times B = \{(a,b) \mid a \in A, b \in B\}$$

$$B \times A = \{(b,a) \mid a \in A, b \in B\}.$$

Nishant Jindal

$$A = \{1, 2\} \quad , \quad B = \{a, b, c\}$$

$$A \times B = \{(1,a), (1,b), (1,c), (2,a), (2,b), (2,c)\}$$

$$n(A \times B) = 2 \times 3 = 6$$

$$n(A) = m_1 \quad , \quad n(B) = m_2$$

$$n(A \times B) = m_1 m_2$$

Nishant Jindal

<u>Function</u> $\longrightarrow$ is relation between various objects

Volume of cube is the function of its side.

$$V(x) = x^3$$

Nishant Jindal

Function

$f : A \to B$ is a rule that assign to every point $x \in A$, a unique element in B denoted by $f(x)$

$$(x, f(x)) \in f$$

Nishant Jindal

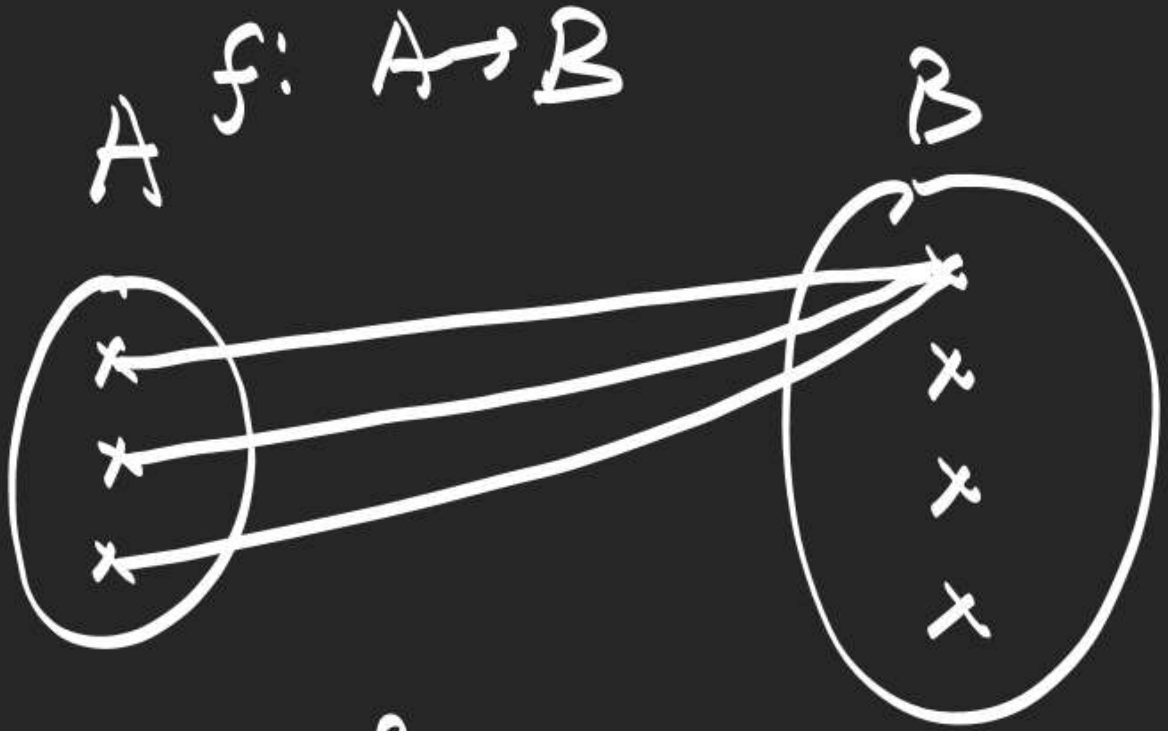
$f : A \to B$ is a function if

1. $f \subseteq A \times B$

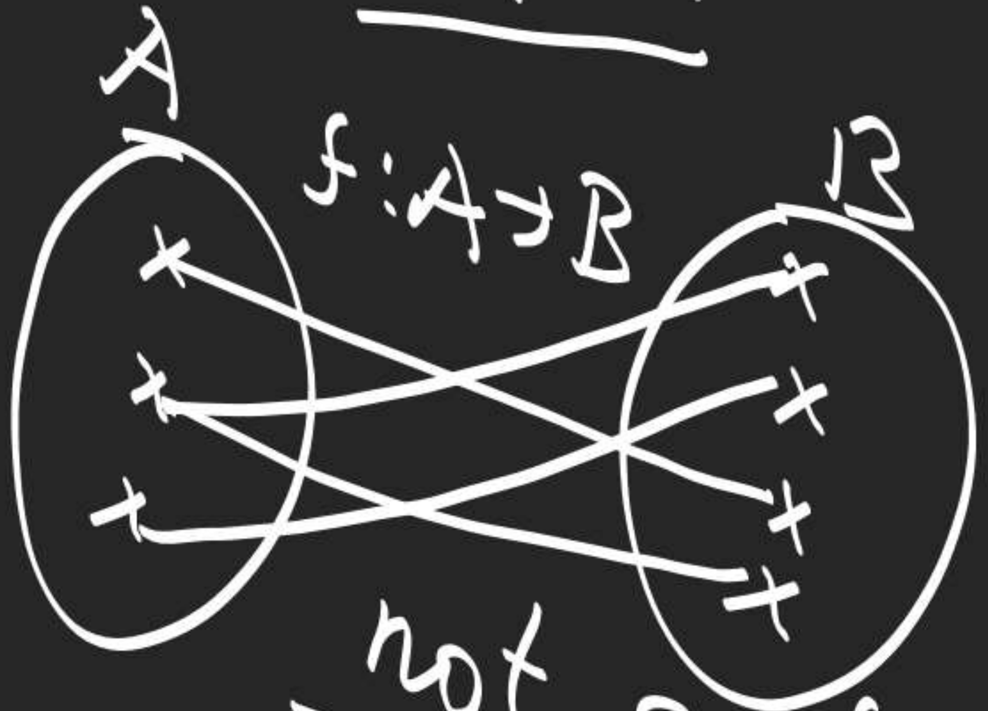
2. $\forall a \in A$, there exists $b \in B$, such that $(a, b) \in f$

3. if $(a, b) \in f$, $(a, c) \in f$

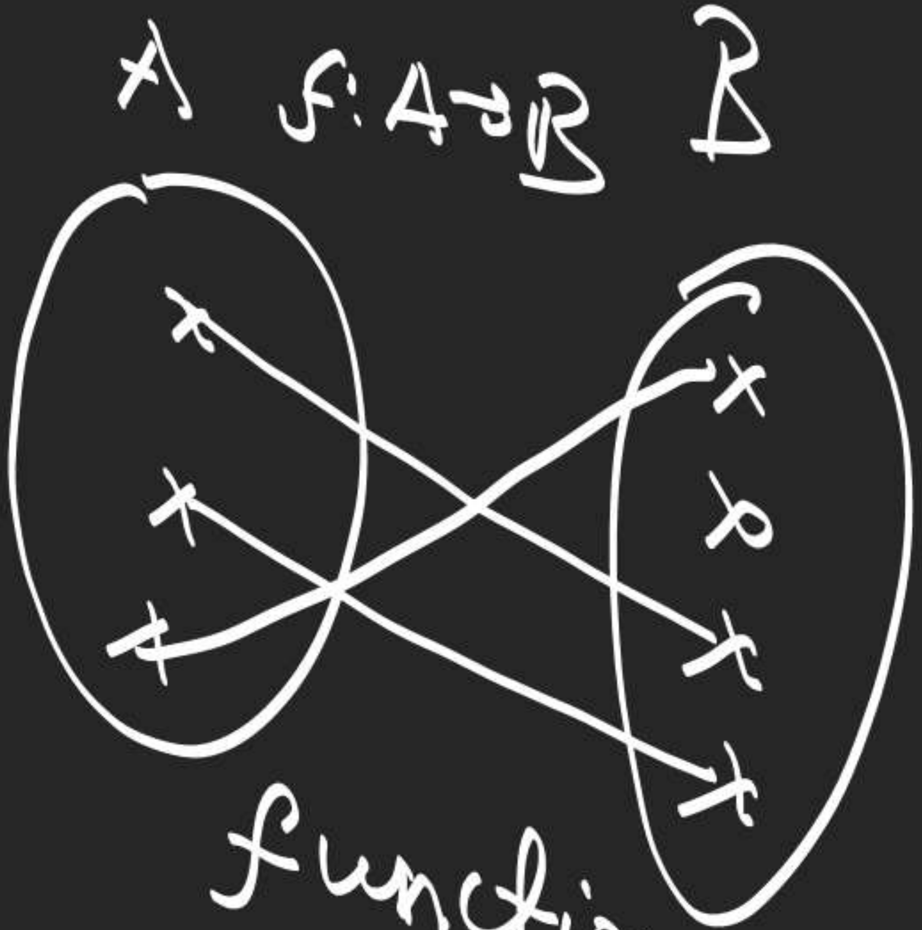
$\Rightarrow b = c$



function



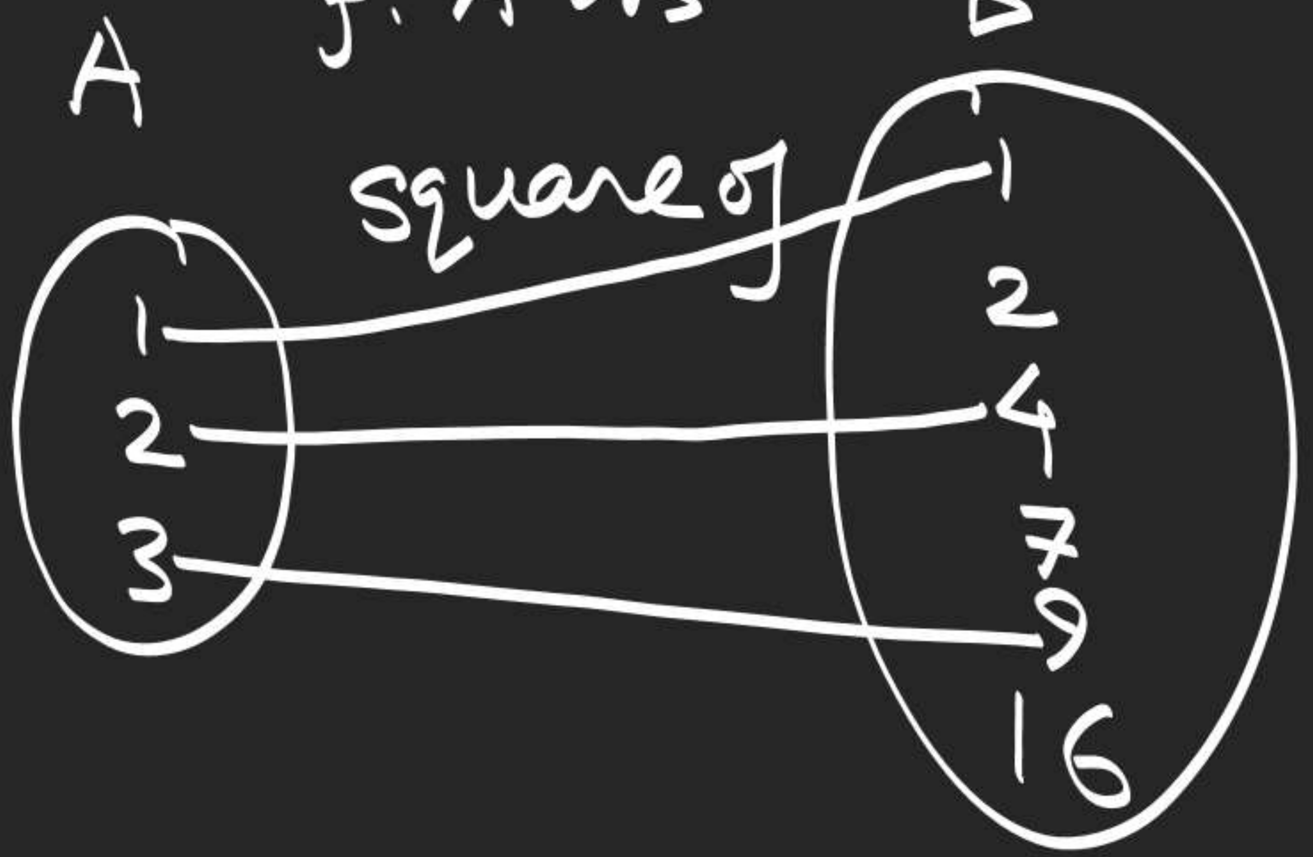
not a function



function

Nishant Jindal

$f : A \to B$



$\boxed{y = f(x)}$ $x \in A,\ y \in B$

$f = \{(1,1), (2,4), (3,9)\}$

square of 1 = $1^2 = 1$

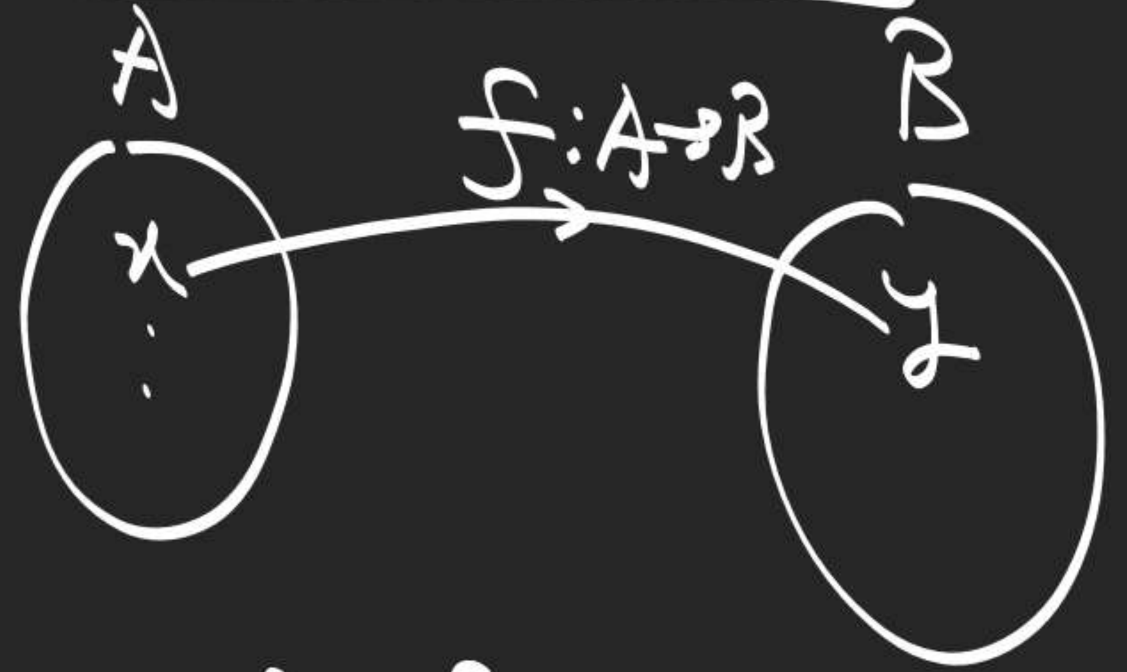
square of 2 = $2^2 = 4$

— " — 3 = $3^2 = 9$

$y = f(x) = x^2$

$(x, y) \in f$

$$\boxed{y = f(x)}$$



$$f = \{(x, y)$$

Nishant Jindal

$y = f(x)$

<u>Domain of a function 'f'</u>

Set of all real values of x for which $f(x)$ is defined . is called Domain of 'f'.

Nishant Jindal

1. $f(x) = \dfrac{1}{x-2}$

$$D_f = R - \{2\}$$

2. $f(x) = \sqrt{(\sin x) - 1}$

$$\boxed{D_f = \left\{2n\pi + \frac{\pi}{2}\right\} \quad n \in I}$$

$\sin x = 1$

$\sin x - 1 \geq 0$

$\sin x \geq 1$

Nishant Jindal

③ $f(x) = \sin(x^2) + \dfrac{1}{\sqrt{x^2-3x+2}}$

$x \in R$

$(x-1)(x-2) > 0$

$x \in (-\infty, 1) \cup (2, \infty)$

$$\boxed{D_f = (-\infty, 1) \cup (2, \infty)}$$

Nishant Jindal

4. $f(x) = \log_{(x-3)} (16 - x^2)$

$x - 3 > 0$ & $x - 3 \neq 1$, & $16 - x^2 > 0$

$\boxed{x > 3}$ $\boxed{x \neq 4}$, $\boxed{x \in (-4, 4)}$

$\boxed{D_f = (3, 4)}$

$$b = f(a)$$

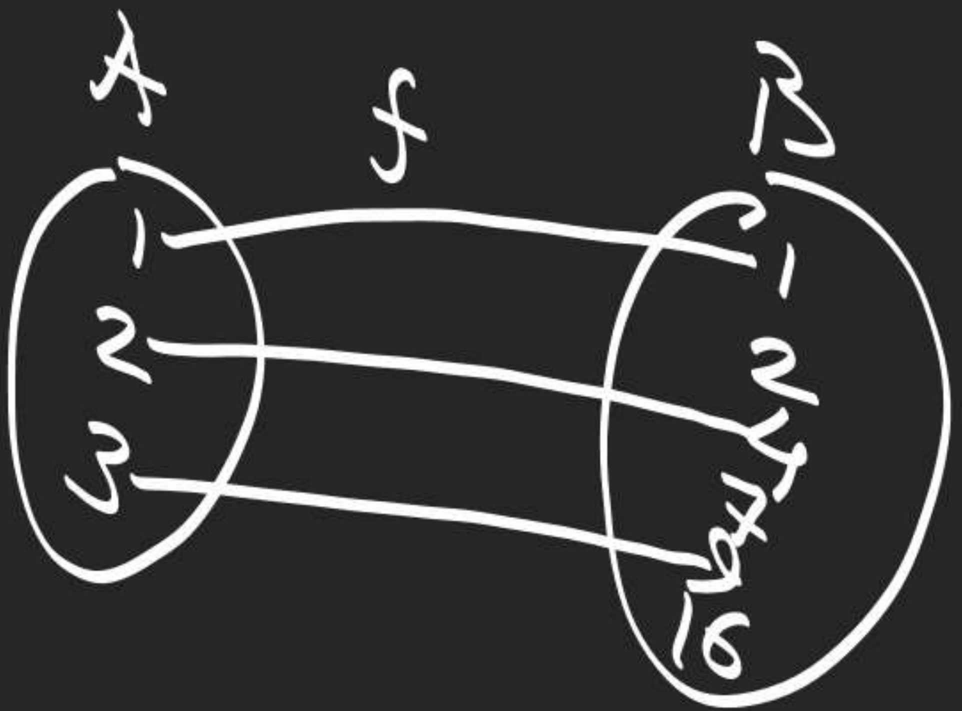
b is called the image of 'a' under the rule f

a is called the pre image of b under the rule f.

Range of function 'f'

Set of all images is called range.

$f(x)=x^2$



$f = \{(1,1),(2,4),(3,9)\}$

$\boxed{R_f = \{1,4,9\}}$

Nishant Jindal

$$f(x) = \frac{1}{x}$$

$$D_f = R - \{0\}$$

$$R_f = R - \{0\} = (-\infty, 0) \cup (0, \infty)$$

$$\frac{1}{\sqrt[3]{2}} = y$$

$$y = f(x) \rightarrow \text{domain}$$

$$x = \sqrt[3]{2},\ \frac{1}{\sqrt[3]{2}}\ \text{range}$$

Nishant Jindal

$$f(x) = \sqrt{(x-2)}$$

$$D_f = [2, \infty)$$

$$R_f = [0, \infty)$$

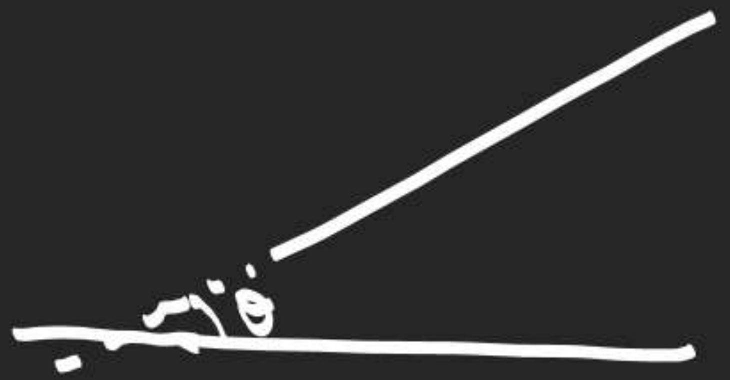
$$R^+ \cup \{0\}$$

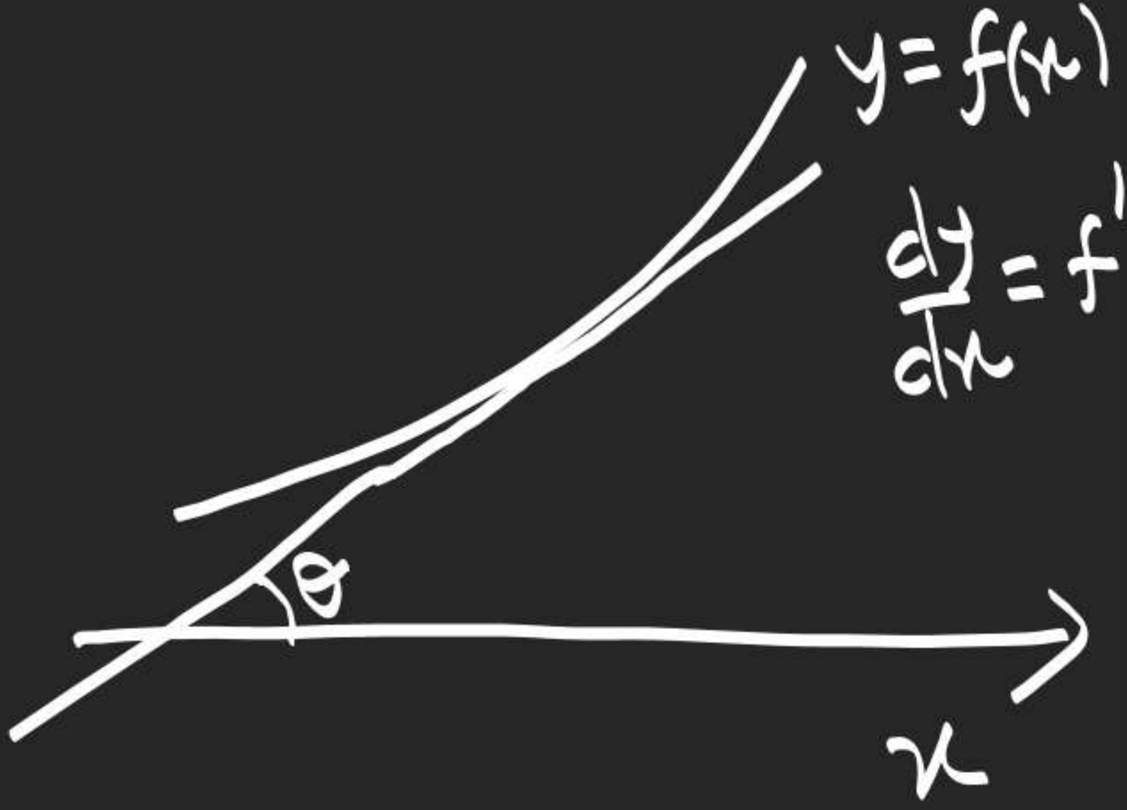
Nishant Jindal

Graph

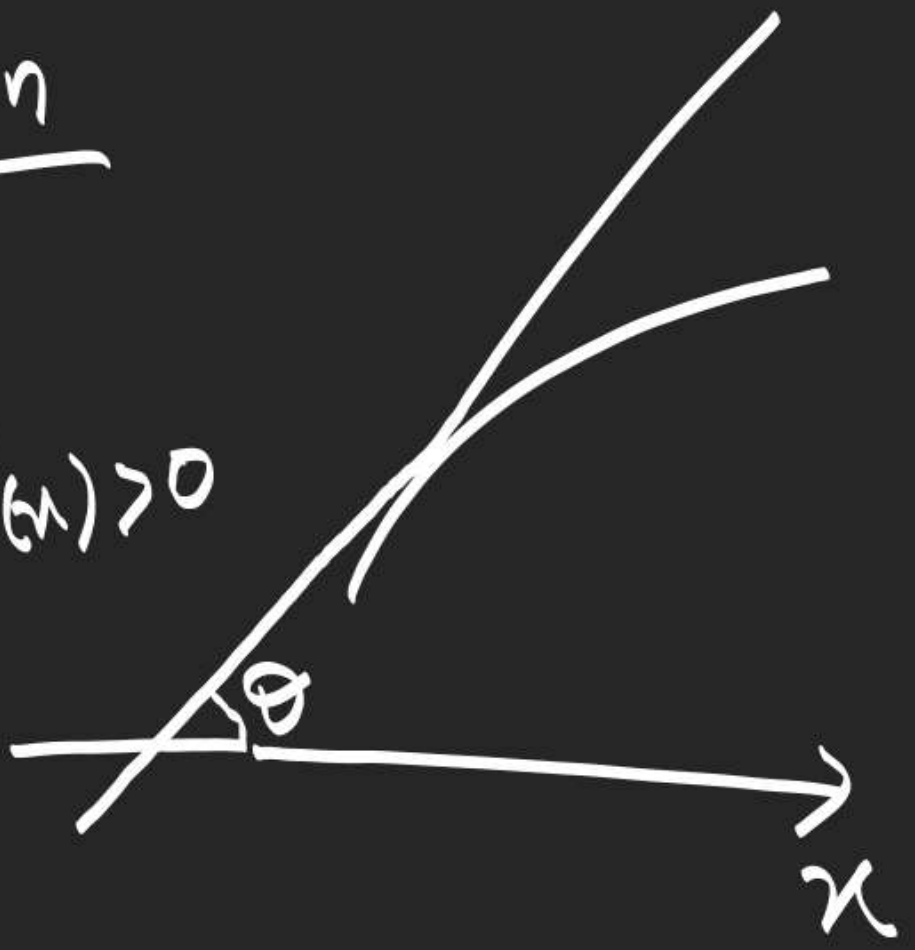
1. Domain of function
2. Intervals of increase/decrease
3. Concavity → to be continued
4. Sketch the graph

Increase of function





$$\frac{dy}{dx} = f'(x) > 0$$



If $f'(x) > 0 \Rightarrow f$ is increasing f is increasing

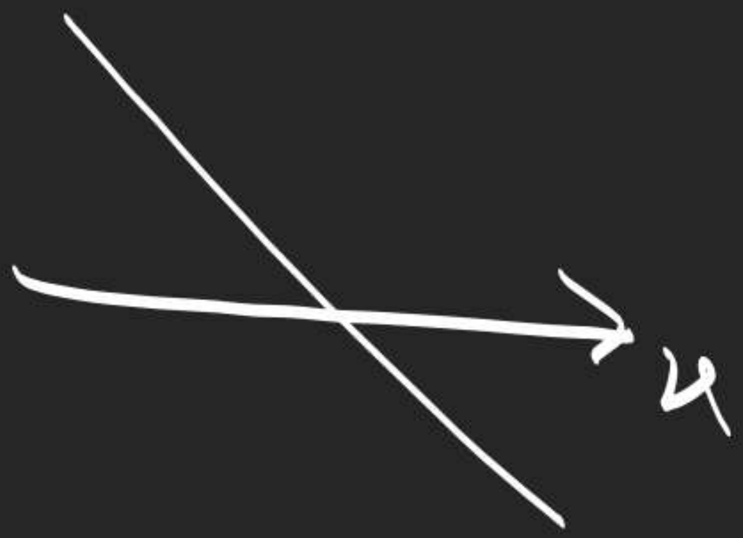
Concavity

f

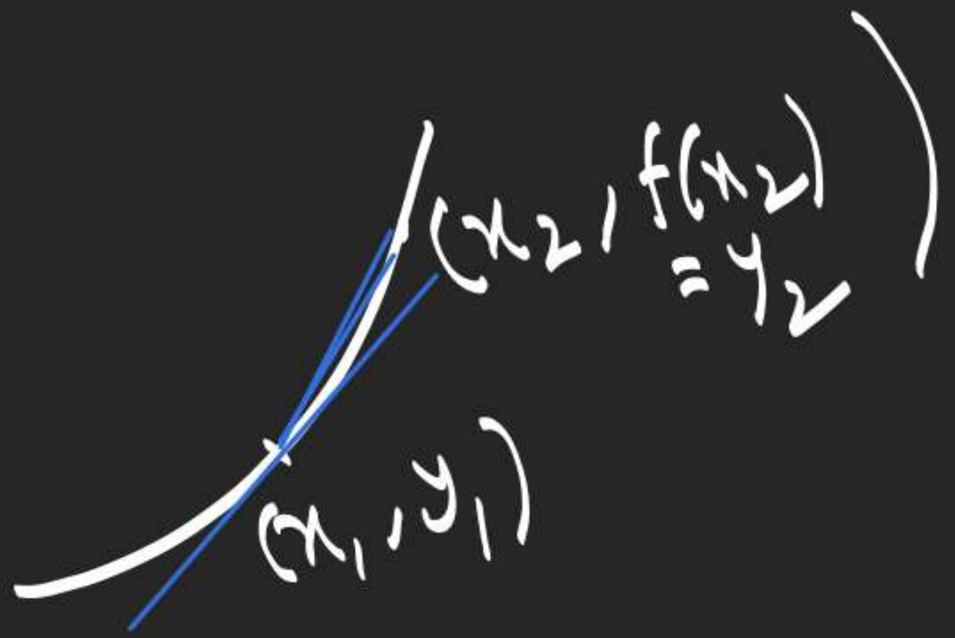
Nishant Jindal

Decreasing

$$f'(x) < 0 \Rightarrow f \text{ is decreasing}$$



Nishant Jindal



$$\frac{dy}{dx} = \lim_{x_2 \to x_1} \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$