

# FUNCTIONS

3:  $f: \mathbb{R} \rightarrow \mathbb{R}, f(x) = x^3 + x^2 + 3x + \sin x \rightarrow$  continuous

$$f(x) = x^3 \left( 1 + \frac{1}{x} + \frac{3}{x^2} + \frac{\sin x}{x^3} \right)$$

$x \rightarrow -\infty, y \rightarrow -\infty$

$$R_f = \mathbb{R}$$

$x \rightarrow \infty, y \rightarrow \infty$

$$f'(x) = 3x^2 + 2x + 3 + \cos x = \underbrace{3x^2 + 2x + 2}_{>0} + \underbrace{(1 + \cos x)}_{\geq 0} > 0$$

Bijective

Q.  $f: \mathbb{R} \rightarrow \mathbb{R}$ ,  $f(x) = \frac{2x^2 - x + 5}{7x^2 + 2x + 10}$

Int o & M-1

$\lim_{x \rightarrow \infty} f(x) = \frac{\infty}{\infty}$  (Indeterminate form)

Continuous  $\Rightarrow$   $\lim_{x \rightarrow c} f(x) = f(c)$

$y = \frac{1}{2}$

$\lim_{x \rightarrow \infty} f(x) = \frac{2 - \frac{1}{x} + \frac{5}{x^2}}{7 + \frac{2}{x} + \frac{10}{x^2}}$

$\lim_{x \rightarrow -\infty} f(x) = \frac{2 - \frac{1}{x} + \frac{5}{x^2}}{7 + \frac{2}{x} + \frac{10}{x^2}}$

$\lim_{x \rightarrow 0} f(x) = \frac{5}{10} = \frac{1}{2}$

$$\frac{1}{2} = \frac{2x^2 - x + 5}{7x^2 + 2x + 10} \Rightarrow 3x^2 + 4x = 0$$

$x = 0, -\frac{4}{3}$

M-1

$\therefore \dots - \frac{4}{3} - \frac{1}{2} \dots \frac{1}{2} = \frac{4}{7}$

Q.

$$f: A \rightarrow B$$



6 elements

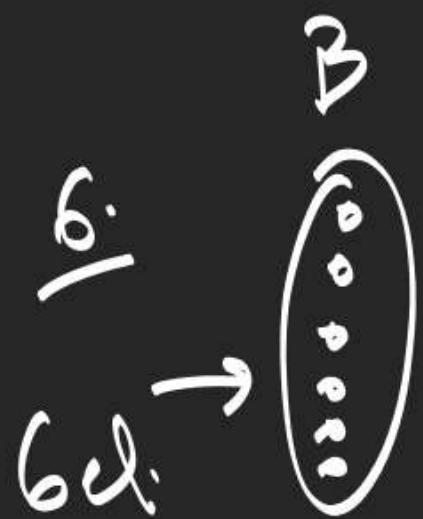
①  $6^4 \rightarrow$

② ( $1-1$ ) fns

$$\textcircled{3} \quad M-1 = 6^4 - (1-1) = 6^4 - {}^6P_4$$

- ④ Onto fns = 0
- ⑤ Into fns =  $6^4$
- ⑥ Bijective = 0

# FUNCTIONS



Distribute 6 dist. looks over 4 loops  
each loop get at least one

④ Onto

$$1113 \rightarrow \frac{6!}{(1!)^3 3! 3!} \times 4!$$

$$1122 \rightarrow \frac{6!}{(1!)^2 (2!)^2 2! 2!} \times 4!$$

①  $4^6$

②  $1-1 \text{ fn} = 0$

③  $M-1 = 4^6$

$E_i = B_i \text{ get none}$   
 $i=1, 2, 3, 4$

$$\text{Onto} = 4^6 - n(E_1 \cup E_2 \cup E_3 \cup E_4)$$

$$= 4^6 - \left( {}^4 C_1 3^6 - {}^4 C_2 2^6 + {}^4 C_3 1^6 \right)$$

Info

3: (iv)  leave

$$y = f(x) = \frac{x}{1+x}$$

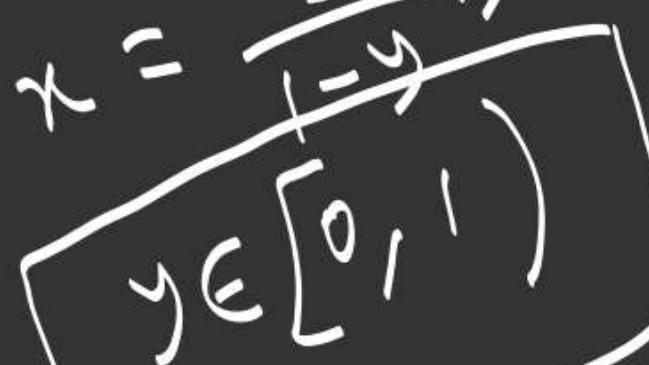
$$R_f = (-1, 1)$$

I)   
 $x \geq 0$

$$y = \frac{x}{1+x}$$

$$y + xy = x$$

$$x = \frac{y}{1-y} \geq 0$$



$y \in [0, 1]$

OR II)  $x < 0$

$$y = \frac{x}{1-x}$$

$$y - xy = x$$

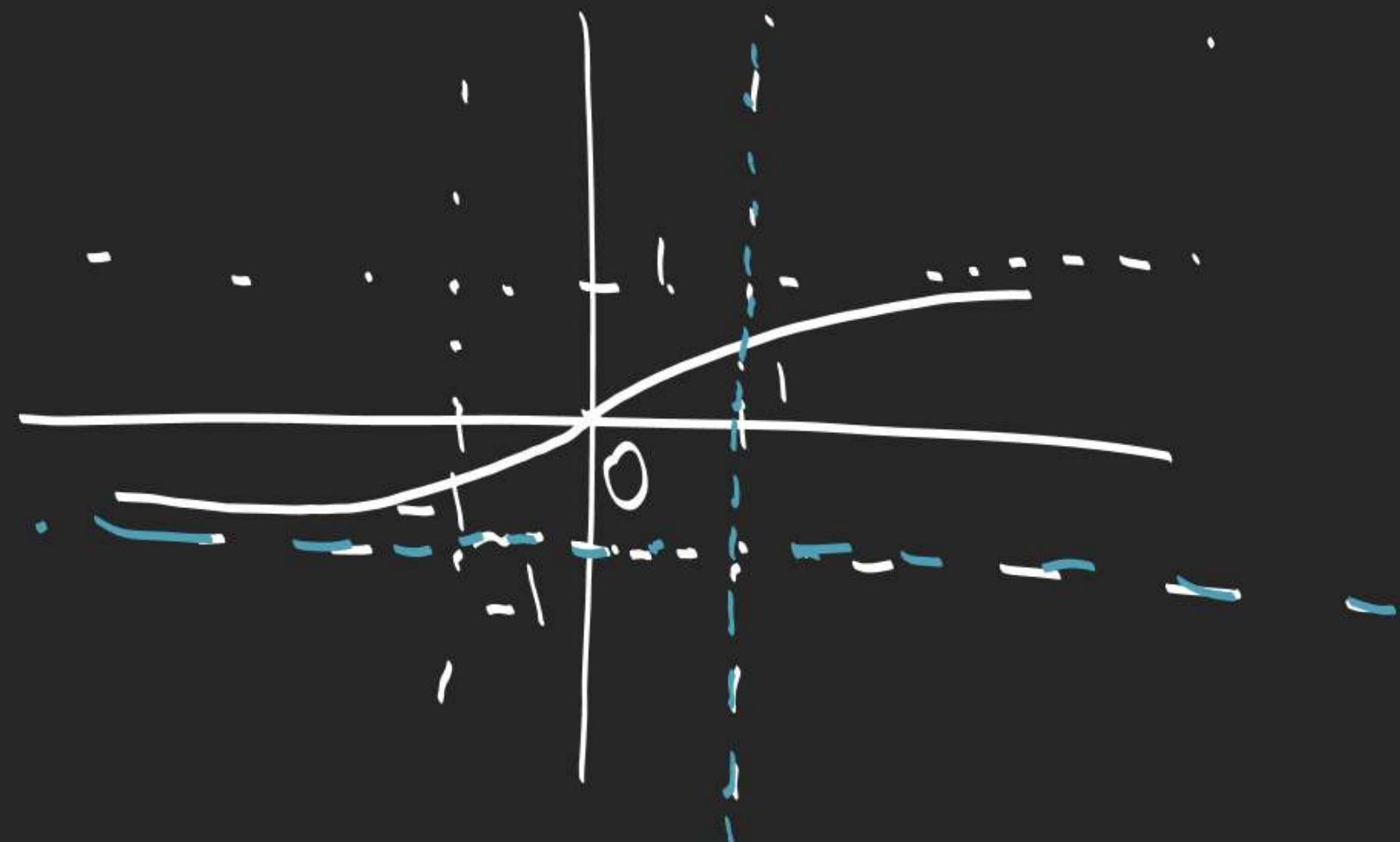
$$x = \frac{y}{1+y} < 0$$

$y \in (-1, 0)$

Ans.

## FUNCTIONS

$$y = \frac{x}{1+|x|} = \begin{cases} \frac{x}{1+x} = 1 - \frac{1}{1+x} & x \geq 0 \\ \frac{x}{1-x} = -1 - \frac{1}{x-1} & x < 0 \end{cases} \Rightarrow \begin{cases} y = -\frac{1}{x} & x \geq 0 \\ y = -\frac{1}{x} & x < 0 \end{cases}$$



5.

$$f(x) = 2|\sin x| - 3|\cos x|$$

$$y_{\max} = 2 - 0$$

$$y_{\min} = 0 - 3 = -3$$

$$R_f = [-3, 2]$$

6. (ii)

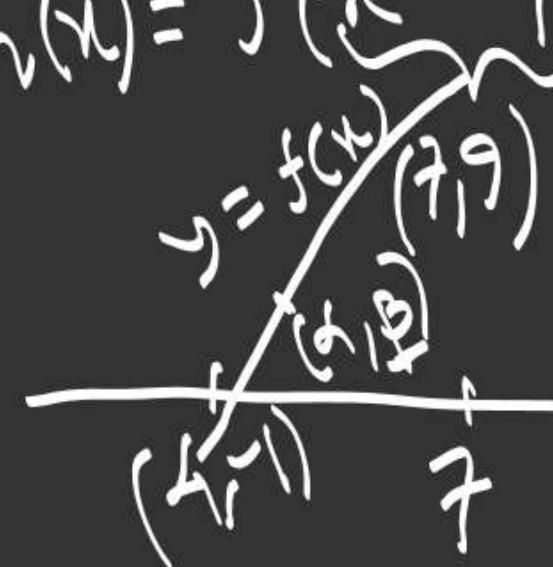
$$D_x = [4, 7]$$

$$R_f = [-1, 9]$$

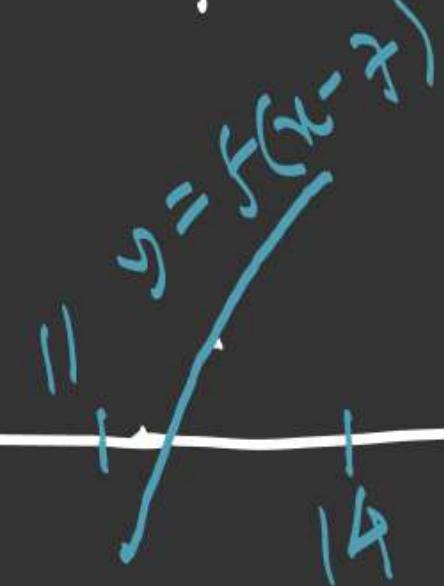
$$x \leq x - 7 \leq 7$$

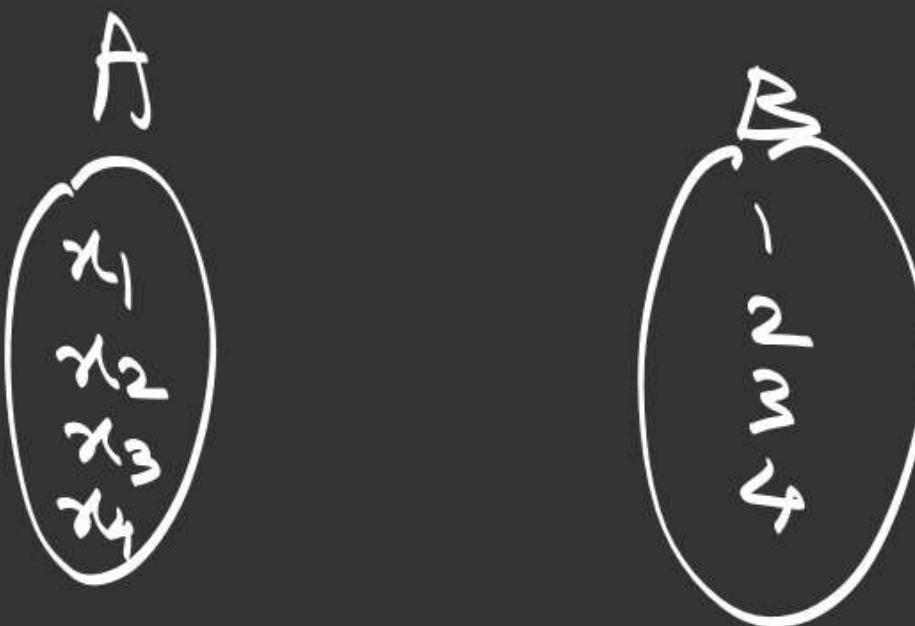
$$h(x) = f(x - 7) \quad x \in [11, 14]$$

$$R_f = [-1, 9]$$



$$h(x) = f(x - 7)$$





Find ① No. of bijective function defined from  
 $A \rightarrow B$   
Ans  $\rightarrow 24 = 4!$

②  $\sim 11$  such that  $f(x_i) \neq i$   
~~such that  $f(x_i) \neq i$~~   
 $\Rightarrow$  ~~many to one~~  $x_i, i=1, 2, 3, 4$ .

**FUNCTIONS**Composite Function

$$f: A \rightarrow B , g: C \rightarrow D$$

$$(f \circ g)(x) = f(g(x))$$

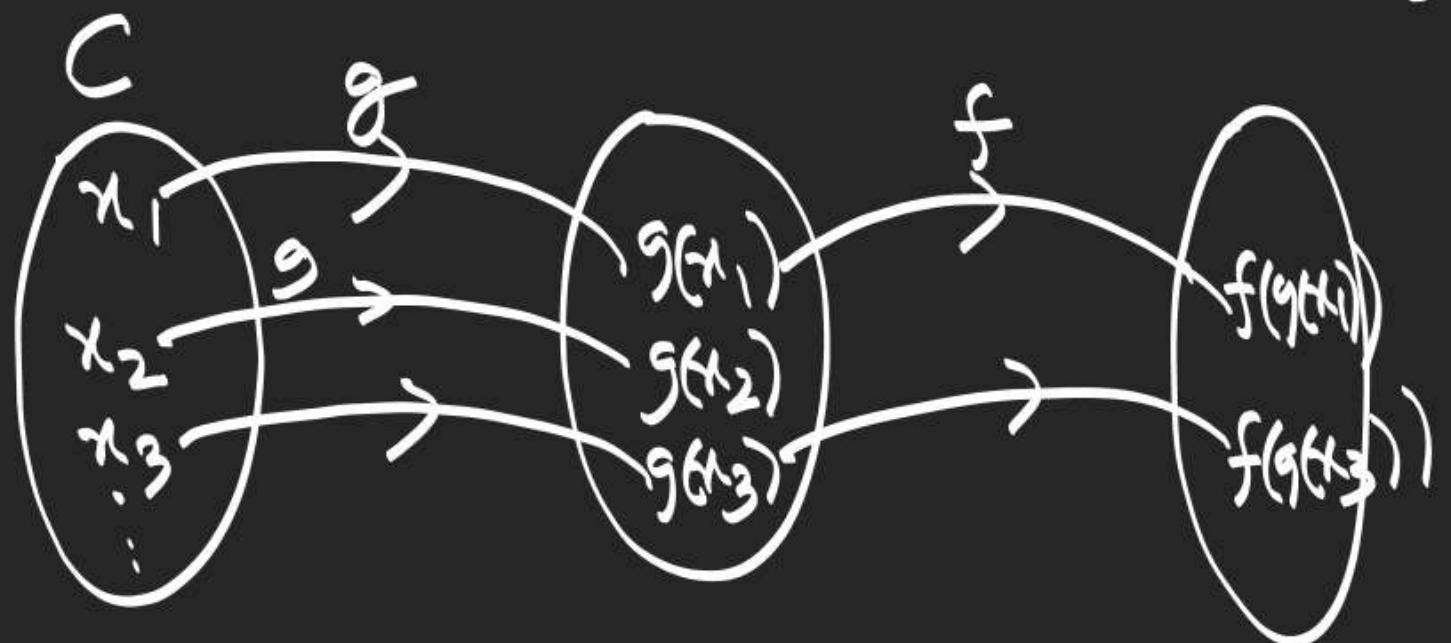
$$(f \circ f)(x) = f(f(x))$$

## FUNCTIONS

$$f: A \rightarrow B, g: C \rightarrow D$$

$$f \circ g(x) = f(g(x))$$

$$D_{f \circ g} = \{x \mid x \in D_g \text{ & } g(x) \in D_f\}$$



$$\therefore f(x) = \begin{cases} 1-x & \text{if } 0 \leq x \leq 1 \\ x+2 & \text{if } 1 < x < 2 \\ 4-x & \text{if } 2 \leq x \leq 4 \end{cases}$$

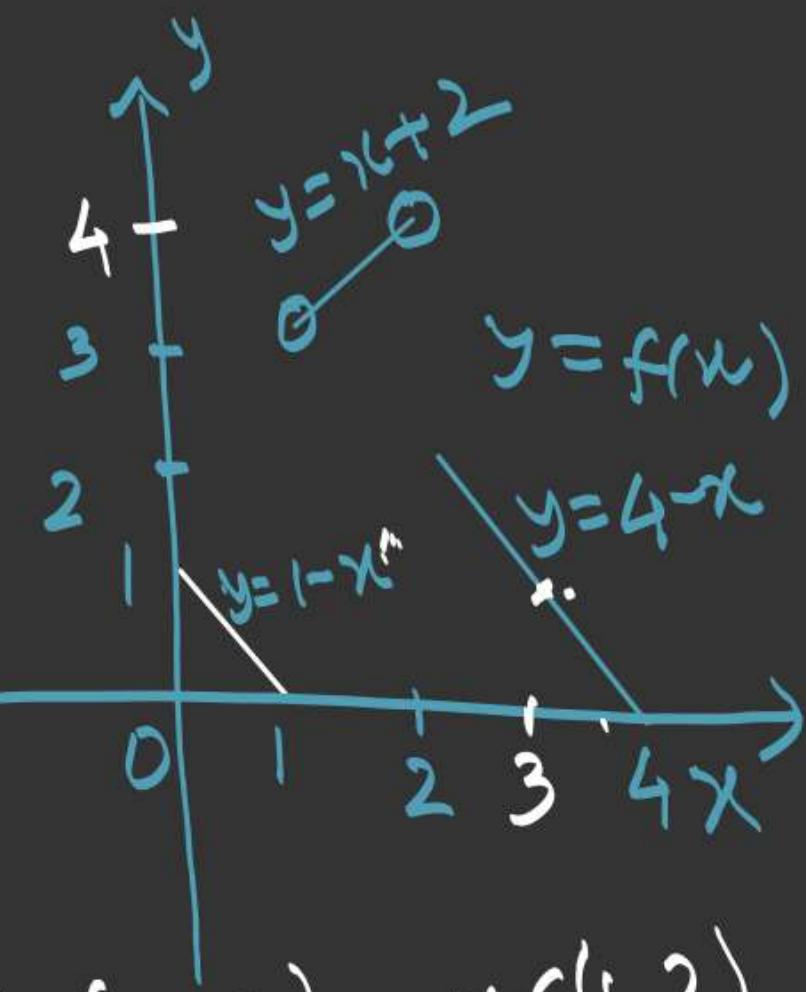
Find  $f \circ f(x)$

$$f(f(x)) = \begin{cases} 1-f(x) & 0 \leq f(x) \leq 1 \\ f(x)+2 & 1 < f(x) < 2 \end{cases}$$

$$4-f(x) \quad 2 \leq f(x) \leq 4$$

$$f(f(x)) = \begin{cases} 4-(x+2) & ; x \in (1, 2) \\ 1-(1-x) & ; x \in [0, 1] \\ 1-(4-x) & ; x \in [3, 4] \\ 4-(x+2) & ; x \in (2, 3) \\ 4-2 & ; x=2 \end{cases}$$

$\Rightarrow f \circ f = [0, 4]$



$$\begin{cases} 4-(x+2) & x \in (1, 2) \\ 4-2 & ; x=2 \end{cases}$$

$$f(x) = \begin{cases} x^2 - 1 & -2 \leq x \leq 1 \\ x^2 + 3 & 1 < x \leq 3 \end{cases}$$

$$f(g(u)) = \begin{cases} g^2(u) - 1 & -2 \leq g(u) \leq 1 \\ g^2(u) + 3 & 1 < g(u) \leq 3 \end{cases}$$

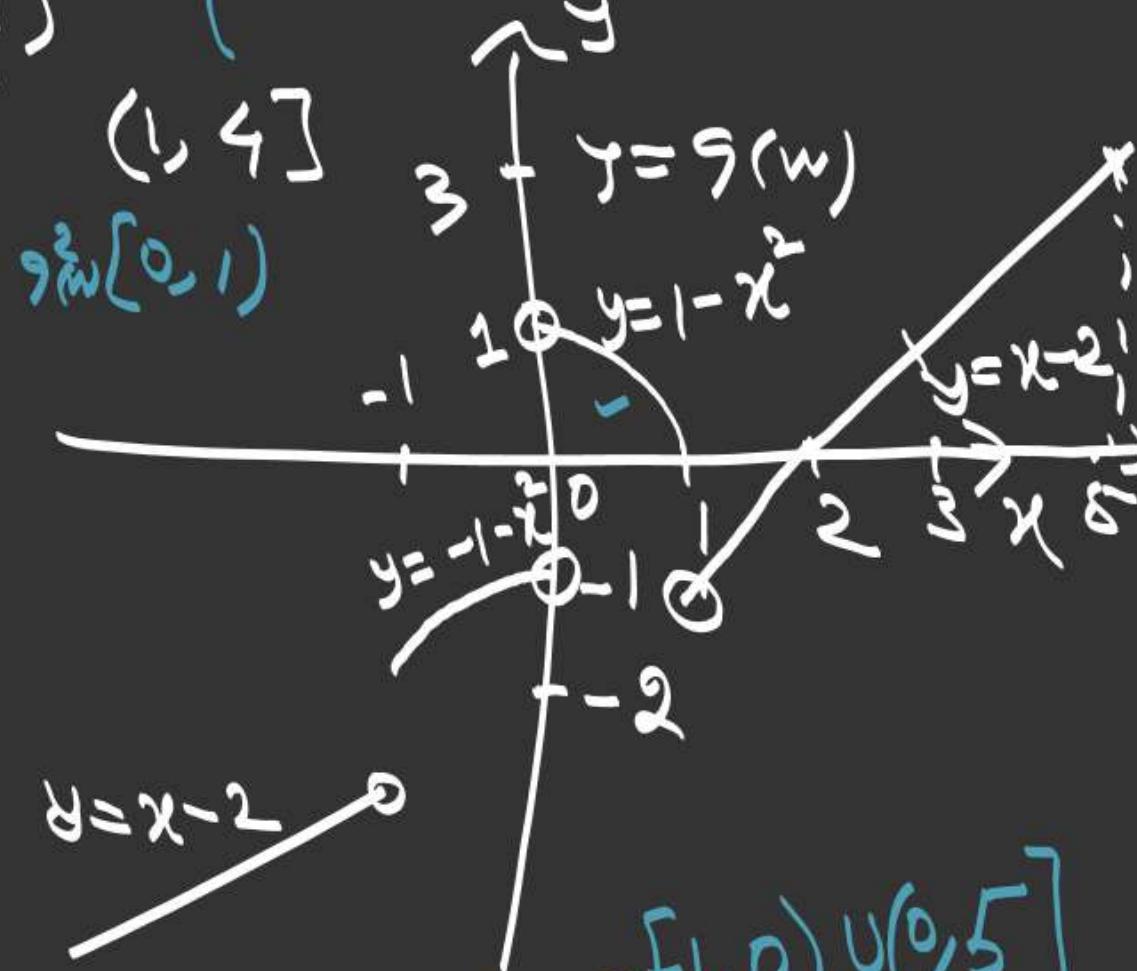
$$g(x) = \begin{cases} \frac{x}{|x|} - x^2 & 0 < |x| \leq 1 \\ x - 2 & |x| > 1 \end{cases}$$

~~Q3, 4~~ - 2

$$f(g(u)) = \begin{cases} (-1-x^2)^2 - 1 & u \in [-1, 0) \\ (-1-x^2)^2 - 1 & u \in (0, 3] \\ (x+2)^2 - 1 & u \in [1, 0) \\ (x+2)^2 + 3 & u \in (1, 3] \cup (3, 5] \end{cases}$$

leaving Q3, 4, 10

$$\begin{aligned} &[-1, 0) \cup (0, 1] \\ &0 < |x| \leq 1 \\ &|x| > 1 \end{aligned}$$



$$(1, 9]_{R_fog} = [1, 3] \cup (4, 12]$$

$$(1, 9]_{D_fog} = [1, 0) \cup (0, 5]$$

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