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$$\int \frac{e^x\,dx}{\sqrt{e^x-1}}$$

$$\int \frac{(x+2)\,dx}{\sqrt{x^2+2x}}$$

$$-\int \frac{e^x\,dx}{e^x\sqrt{e^x-1}}$$

$$t^2 = e^x - 1$$

$$\int \frac{x^2\,x^3\,dx}{\sqrt{x^4+4}}$$

$$\int \frac{x^3\left(\frac{1}{x^3}-1\right)^{1/2}dx}{x^4}$$

$$t^2$$

$$= \int \frac{t\,t\,dt}{t^2+1}$$

$$\frac{1}{2}\int 2x\sqrt{(x^2)^2+4}\;dx$$

$$\left(x^{3/2}+\frac{1}{x^{1/2}}\right)\tan^{-1}x$$

$$\int \frac{dx}{\sqrt{x}}\int\left(\frac{3}{2}x^{\frac{1}{2}}-\frac{1}{2x^{3/2}}\right)\tan^{-1}x\,dx$$

II I

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Trigonometric forms

T-I $\int \frac{dx}{a\sin^2x+b}$, $\int \frac{dx}{a\cos^2x+b}$, $\int \frac{dx}{a\sin^2x+b\sin x\cos x+c\cos^2x}$

$\int \frac{\quad}{a\sin^2x+b\cos^2x+c}$

$\int \frac{\sec^2x\,dx}{a\tan^2x+b\tan x+c}$

$\boxed{\text{Put } \tan x = t}$

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T-II

$$\int \frac{dx}{a\sin x+b}, \quad \int \frac{dx}{a\cos x+b}, \quad \int \frac{dx}{a\sin x+b\cos x+c}.$$

Put $\tan\frac{x}{2} = t$.

$$\frac{1}{2}\sec^2\frac{x}{2}\,dx = dt$$

$$dx = \frac{2\,dt}{1+t^2}$$

$$\int \frac{\frac{2\,dt}{1+t^2}}{a\left(\frac{2t}{1+t^2}\right)+b\frac{(1-t^2)}{(1+t^2)}+c}.$$

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T-III

$$\int \frac{(p\sin x + q\cos x + r)\,dx}{(a\sin x + b\cos x + c)}$$

$p\sin x + q\cos x + r \rightarrow N(x)$

$a\sin x + b\cos x + c \rightarrow D(x)$

$$N(x) = k_1 D(x) + k_2 D'(x) + k_3 .$$

$$= k_1 x + k_2 \ln|D(x)| + k_3 \int \frac{dx}{D(x)}$$

$\tan\frac{x}{2} = t$

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1. $$\int \frac{dx}{(3\sin x - 4\cos x)^2} = \frac{1}{3}\int \frac{3\sec^2 x\,dx}{(3\tan x - 4)^2}$$

$$= -\frac{1}{3(3\tan x - 4)} + C.$$

$\boxed{\tan\alpha = \frac{3}{4}}$ $\quad 5\cos(x+\alpha)$

$$= \frac{1}{25}\int \sec^2(x+\alpha)\,dx$$

$$= \frac{1}{25}\tan(x+\alpha) + C.$$

2. $$\int \frac{dx}{\cos x(5+3\cos x)} = \int\left(\frac{1}{5}\sec x - \frac{3}{5}\frac{1}{5+3\cos x}\right)dx.$$

$\frac{5+3\cos x - 3\cos x}{5}$

$\tan\frac{x}{2} = t.$

$$= \frac{1}{5}\ln|\sec x + \tan x| - \frac{3}{5}\int \frac{\frac{2dt}{1+t^2}}{5+\frac{3(1-t^2)}{1+t^2}}$$

$$\int \frac{dt}{4+t^2}$$

$$= \frac{1}{5}\ln|\sec x + \tan x| - \frac{3}{10}\tan^{-1}\left(\frac{\tan\frac{x}{2}}{2}\right) + C.$$

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3. $\displaystyle\int \frac{(6+3\sin x+14\cos x)\,dx}{(3+4\sin x+5\cos x)}$

$6+3\sin x+14\cos x = k_1(3+4\sin x+5\cos x) + k_2(4\cos x-5\sin x)+k_3$

$3 = 4k_1 - 5k_2$

$14 = 5k_1 + 4k_2$

$k_1 = 2,\ k_2 = 1$

$6 = 3k_1 + k_3 \Rightarrow k_3 = 0$

$= 2x + \ln|3+4\sin x+5\cos x| + C$.

4. $\displaystyle\int \frac{\sin 2x\,dx}{(\sin^4 x+\cos^4 x)} = \int \frac{2\tan x\sec^2 x\,dx}{(1+\tan^4 x)} = \tan^{-1}(\tan^2 x) + C$.

$\displaystyle\int \frac{\sin 2x\,dx}{1-\frac{1}{2}(1-\cos^2 2x)} = -\int \frac{-2\sin 2x\,dx}{1+\cos^2 2x} = -\tan^{-1}(\cos 2x) + C$.

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5.

$$\int \frac{\sin x\, dx}{(e^x - \sin x - \cos x)}$$

$$1-(\sin x+\cos x)e^{-x} = t$$

$$e^{-x}\left(\sin x+\cos x-(\cos x-\sin x)\right)dx = dt$$

$$-2\sin x e^{-x}dx = dt$$

$$= -\frac{1}{2}\int \frac{-2e^{-x}\sin x\, dx}{\left(1-(\sin x+\cos x)e^{-x}\right)}$$

$$= -\frac{1}{2}\ln\left|1-e^{-x}(\sin x+\cos x)\right| + C.$$

$$\frac{1}{2}\int \frac{-(e^x-\sin x-\cos x)+(e^x-\cos x+\sin x)}{(e^x-\sin x-\cos x)}$$

Nishant Jindal

Integrals of form

$$\int \frac{(x^2+1)\,dx}{(x^4+kx^2+1)} \quad ; \quad \int \frac{(x^2-1)\,dx}{(x^4+kx^2+1)} \quad ; \quad \int \frac{x^2\,dx}{(x^4+kx^2+1)} \quad ; \quad \int \frac{dx}{(x^4+kx^2+1)}$$

$$\int \frac{\left(1+\frac{1}{x^2}\right)dx}{x^2+\frac{1}{x^2}+k}$$

$$\int \frac{\left(1+\frac{1}{x^2}\right)dx}{\left(x-\frac{1}{x}\right)^2+k+2}$$

$$\int \frac{\left(1-\frac{1}{x^2}\right)dx}{x^2+\frac{1}{x^2}+k}$$

$$= \int \frac{\left(1-\frac{1}{x^2}\right)dx}{\left(x+\frac{1}{x}\right)^2+k-2}$$

$$\frac{1}{2}\left[\int \frac{(x^2+1)\,dx}{x^4+kx^2+1} \pm \int \frac{(x^2-1)\,dx}{x^4+kx^2+1}\right]$$

Nishant Jindal

Integrals of form

$$\int f(\sin 2x)(\cos x+\sin x)\,dx \quad ; \quad \int f(\sin 2x)(\cos x-\sin x)\,dx \; ;$$

$$\sin 2x = 1-(\sin x-\cos x)^2 \qquad\qquad \sin 2x = (\sin x+\cos x)^2-1$$

$$\int f(\sin 2x)\cos x\,dx \quad ; \quad \int f(\sin 2x)\sin x\,dx$$

$$\frac{1}{2}\left[\int f(\sin 2x)(\cos x+\sin x)\,dx \pm \int f(\sin 2x)(\cos x-\sin x)\,dx\right]$$

Nishant Jindal

1. $$\int \frac{(x+1)^2\,dx}{(x^4+x^2+1)}$$

$$\int \frac{(x^2+1)\,dx}{(x^4+x^2+1)} + \int \frac{2x\,dx}{x^4+x^2+1}$$

$$= \int \frac{\left(1+\frac{1}{x^2}\right)dx}{\left(x-\frac{1}{x}\right)^2+3} + \int \frac{2x\,dx}{\left(x^2+\frac{1}{2}\right)^2+\frac{3}{4}}$$

$$\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{x-\frac{1}{x}}{\sqrt{3}}\right) + \frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{x^2+\frac{1}{2}}{\frac{\sqrt{3}}{2}}\right) + C$$

2. $$\int \frac{x^{17}\,dx}{(1+x^{24})} =$$

$$= \frac{1}{6}\int \frac{6x^{12}\,x^5\,dx}{1+x^{24}}$$

$$\boxed{x^6 = t}$$

$$= \frac{1}{6}\int \frac{t^2\,dt}{1+t^4}$$

$$= \frac{1}{12}\left[\int \frac{(t^2+1)\,dt}{1+t^4} + \int \frac{(t^2-1)\,dt}{1+t^4}\right]$$

$$\frac{1}{12}\int \frac{\left(1+\frac{1}{t^2}\right)dt}{\left(t-\frac{1}{t}\right)^2+2} + \int \frac{\left(1-\frac{1}{t^2}\right)dt}{\left(t+\frac{1}{t}\right)^2-2}$$

$$\frac{1}{12}\left\{\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{x^6-x^{-6}}{\sqrt{2}}\right) + \frac{1}{2\sqrt{2}}\ln\left|\frac{x^6+x^{-6}-\sqrt{2}}{x^6+x^{-6}+\sqrt{2}}\right|\right]$$

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3. $$\int \frac{(x^2+3)\,dx}{(x^4+8x^2+9)} = \int \frac{\left(1+\frac{3}{x^2}\right)dx}{\left(x^2+\frac{9}{x^2}+8\right)} = \int \frac{\left(1+\frac{3}{x^2}\right)dx}{\left(x-\frac{3}{x}\right)^2+14}$$

$$\int \frac{3(t^2+1)\sqrt{3}\,dt}{9t^4+9+8\times 3t^2} \qquad x = \sqrt{3}\,t.$$

$$= \frac{1}{\sqrt{14}} \tan^{-1}\left(\frac{x-\frac{3}{x}}{\sqrt{14}}\right) + C.$$

4. $$\int \left(\sqrt{\cot x} + \sqrt{\tan x}\right) dx$$

$$= \int \sqrt{2}\,\frac{(\cos x+\sin x)\,dx}{\sqrt{\sin 2x}} = \sqrt{2}\int \frac{(\cos x+\sin x)\,dx}{\sqrt{1-(\sin x-\cos x)^2}}$$

$$= \sqrt{2}\,\sin^{-1}(\sin x-\cos x) + C.$$

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$$\int \sqrt{\tan x}\, dx$$

$\tan x = t^2$

$\sec^2 x\, dx = 2t\, dt$

$dx = \dfrac{2t\, dt}{1+t^4}$

$$= \int \frac{2t^2\, dt}{1+t^4} = \int \frac{(t^2+1)\,dt}{1+t^4} + \frac{(t^2-1)\,dt}{1+t^4}$$

$$\frac{1}{2}\int \left(\sqrt{\tan x} + \sqrt{\cot x}\right) dx + \frac{1}{2}\int \left(\sqrt{\tan x} - \sqrt{\cot x}\right) dx$$

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5. $\int \frac{dx}{\operatorname{cosec}x+\cos x}$

6. $\int \frac{\cos x\,dx}{\sqrt{8-\sin 2x}}$

$\int \frac{\sin x\,dx}{1+\sin x\cos x}$

$= \int \frac{2\sin x\,dx}{2+\sin 2x}$

$= \int \frac{(\cos x+\sin x)\,dx}{3-(\sin x-\cos x)^2} - \int \frac{(\cos x-\sin x)\,dx}{1+(\sin x+\cos x)^2}$

2021, 2031, 2034, 2035, 2054, 2055, 2061, 2063,

2068-2075, 2090-2131