Nishant Jindal

Q $\int e^x \frac{(x+1)}{(x+2)^2} \cdot dx$

$\int e^x \left(\frac{(x+2)-1}{(x+2)^2}\right) dx$

$\int e^x \left(\underset{f}{\frac{1}{x+2}} - \underset{f'}{\frac{1}{(x+2)^2}}\right) dx$

$= \frac{e^x}{x+2} + C$

Q $\int \sin(\log_e x) + \cos(\log_e x)\, dx$

$\log_e x = t$

$x = e^t$

$dx = e^t \cdot dt$

$\int (\underset{f}{\sin t} + \underset{f'}{\cos t}) e^t \cdot dt$

$\Rightarrow e^t \cdot \sin t + C$

$\Rightarrow x \cdot \sin(\log x) + C.$

RK When some fxn comes at the place of x. then take that fxn $= t$

Q $\int e^{\tan^{-1} x} \left[\frac{1+x+x^2}{1+x^2}\right] dx$

$\tan^{-1} x = t$

$x = \tan t$

$dx = \sec^2 t \cdot dt$

$\Rightarrow \int e^t \left(\frac{1+\tan t+\tan^2 t}{\cancel{1+\tan^2 t}}\right) \cancel{\sec^2 t}\, dt$

$\int e^t (\underset{f}{\tan t} + \underset{f'}{\sec^2 t}) dt = e^t \cdot (\tan t) + C = e^{\tan^{-1} x} \cdot x + C$

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Q $\int e^{-x}(1-\tan x)\sec x \cdot dx$

$\int e^{-x}(\sec x - \sec x \tan x)\cdot dx$

$-x = t \Rightarrow x = -t$

$dx = -dt$

$\Rightarrow -\int e^{t}(\sec(-t) - \sec(-t)\cdot \tan(-t))dt$

$-\int e^{t}(\underset{f}{\sec t} + \underset{f'}{\sec t \tan t})dt$

$= -e^{t}.\sec t + C$

$= -e^{-x}\sec(-x) + C$

$= -e^{-x}\sec x + C$

When $\int e^{x}\left(f(x) + \frac{\text{Something}}{\text{Else}}\right)$ is given.

Q $\int e^{x}(x^2 + x)\,dx$

$\int e^{x}(\underbrace{x^2 - x + 1}_{f} \underbrace{+2x - 1}_{f'})dx$

Diff D B B

$\Rightarrow e^{x}(x^2 - x + 1) + C$

Q $\int e^{x}\left(\log x + \frac{1}{x^2}\right).dx$

$\int e^{x}\left(\underbrace{\log x - \frac{1}{x}}_{f} \underbrace{+\frac{1}{x} + \frac{1}{x^2}}_{f'}\right)dx = e^{x}\left(\log x - \frac{1}{x}\right) + C$

D D B

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Q $\int e^x(x^3-3x^2-5x)\cdot dx$

$$\int e^x(\underbrace{x^3-6x^2+7x-7}_{f}+3x^2-12x+7)\cdot dx$$

$$e^x(x^3-6x^2+7x-7)+C$$

$$y=\sqrt{\frac{1+x^n}{1-x^n}}$$

$$y'=\frac{1}{2\sqrt{\frac{1+x^n}{1-x^n}}}\times\frac{(1-x^n)\cdot n\cdot x^{n-1}-(1+x^n)(-n\cdot x^{n-1})}{(1-x^n)^2}$$

$$\cdot=\frac{\cancel{\sqrt{1-x^n}}}{2\sqrt{1+x^n}}\times\frac{n\cdot x^{n-1}-\cancel{n\cdot x^{2n-1}}+n\cdot x^{n-1}+\cancel{n\cdot x^{2n-1}}}{(1-x^n)\cancel{(1+x^n)}\sqrt{1-x^n}}$$

Q $\int e^x\left(\frac{1+nx^{n-1}-x^{2n}}{(1-x^n)\sqrt{1-x^{2n}}}\right)\cdot dx$

$$\Rightarrow\int e^x\left(\frac{1-x^{2n}}{(1-x^n)\sqrt{1-x^{2n}}}+\frac{n\,x^{n-1}}{(1-x^n)\sqrt{1-x^{2n}}}\right)\cdot dx$$

$$\Rightarrow\int e^x\left(\frac{1+x^n}{\sqrt{(1-x^n)(1+x^n)}}+\frac{n\cdot x^{n-1}}{(1-x^n)\sqrt{1-x^{2n}}}\right)\cdot dx$$

$$\Rightarrow\int e^x\left(\underbrace{\sqrt{\frac{1+x^n}{1-x^n}}}_{f}+\underbrace{\frac{n\cdot x^{n-1}}{(1-x^n)\sqrt{1-x^{2n}}}}_{f'}\right)\cdot dx$$

$$e^x\cdot\sqrt{\frac{1+x^n}{1-x^n}}+C$$

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Integration By Parts.

1) When 2 or more fxns are multiplied. We use IBP.

2) $\int U \cdot V \, dx = U \cdot \int v \cdot dx - \int \left(\frac{du}{dx} \cdot \int v \cdot dx \right) dx$

3) for deciding U, V we use I L A T E

(4) If I L A T E fails then we take "V" as a fxn whose Integration is easier.

Q $I = \int \overset{U}{\underset{A}{x}} \cdot \overset{V}{\underset{E}{e^x}} \cdot dx$

$= x \cdot \int e^x \cdot dx - \int \left(\frac{d(x)}{dx} \cdot \int e^x \cdot dx \right) dx$

$I = x \cdot e^x - \int 1 \cdot e^x \cdot dx$

$= x e^x - e^x + C$

Q $I = \int \overset{U}{\underset{A}{x}} \cdot \overset{V}{\underset{T}{\sin x}} \cdot dx$

$= x \int \sin x \cdot dx - \int \left(\frac{d(x)}{dx} \cdot \int \sin x \cdot dx \right) dx$

$= -x \cos x + \int 1 \cdot (+\cos x) \cdot dx$

$= -x \cos x + \int \cos x \cdot dx$

$= -x \cos x + \sin x + C$

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Q $\int f(x)\cdot g''(x) - g(x)\cdot f''(x)\cdot dx$

$$\int \underset{U}{f(x)}\cdot \underset{V}{\underbrace{g''(x)}_{\text{Int. करेंगे}}}\cdot dx - \int \underset{U}{g(x)}\cdot \underset{V}{\underbrace{f''(x)}_{\text{Int करेंगे}}}\, dx$$

$$\Rightarrow f(x)\cdot\int \underline{g''(x)\cdot dx} - \int\left(f'(x)\cdot\int g''(x)\cdot dx\right)dx$$

$$-\left\{g(x)\int f''(x)\,dx - \int\left(g'(x)\cdot\int f''(x)\,dx\right)dx\right\}$$

$$\Rightarrow f(x)\cdot g'(x) - \cancel{\int f'(x)\cdot g'(x)\cdot dx}$$

$$-\left\{g(x)\, f'(x) - \cancel{\int g'(x)\, f'(x)\cdot dx}\right\}$$

$$\Rightarrow f\cdot g' - g\cdot f' + C$$

Q $I = \int \underset{A}{\overset{V}{x}}\cdot \underset{L}{\overset{U}{\ln x}}\cdot dx$ ILATE

$$= \ln x\int x\cdot dx - \int\left(\frac{1}{x}\cdot\int x\cdot dx\right)dx$$

$$= \frac{x^2}{2}\cdot\ln x - \int\frac{1}{x}\cdot\frac{x^2}{2}\cdot dx$$

$$= \frac{x^2}{2}\ln x - \frac{1}{2}\cdot\frac{x^2}{2} + C$$

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Q $I = \int \overset{V\cdot}{\underset{A\cdot}{x^2}} \cdot \overset{U}{\underset{L}{\ln x}} \cdot dx$

$= \ln x \cdot \int x^2 \cdot dx - \int \left(\frac{1}{x} \cdot \int x^2 \cdot dx\right) dx$

$= \frac{x^3}{3} \cdot \ln x - \int \frac{1}{\cancel{x}} \cdot \frac{x^{\cancel{3}\,2}}{3} \cdot dx$

$= \frac{x^3}{3} \cdot \ln x - \frac{1}{3} \cdot \frac{x^3}{3} + C$

Q $I = \int \overset{A\cdot}{\underset{U}{x^2}} \cdot \overset{E}{\underset{V}{e^{2x}}} \cdot dx$

$= x^2 \cdot \int e^{2x} \cdot dx - \int \left(2x \cdot \int e^{2x} \cdot dx\right) dx$

$= \frac{x^2 \cdot e^{2x}}{2} - \cancel{2} \int x \cdot \frac{e^{2x}}{\cancel{2}} \cdot dx$

$I = \frac{x^2 \cdot e^{2x}}{2} - \int \overset{U}{\underset{A}{x}} \cdot \overset{V}{\underset{E}{e^{2x}}} \cdot dx$ IBP Prod.

$= \frac{x^2 \cdot e^{2x}}{2} - \left\{ x \cdot \int e^{2x} \cdot dx - \int \left(1 \cdot \int e^{2x} \cdot dx\right) dx \right\}$

$= \frac{x^2 \cdot e^{2x}}{2} - \left\{ x \cdot \frac{e^{2x}}{2} - \int \frac{e^{2x}}{2} \cdot dx \right\}$

$= \frac{x^2 \cdot e^{2x}}{2} - \frac{x \cdot e^{2x}}{2} + \frac{1}{2} \frac{e^{2x}}{2} + C$

Q.

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Q $I=\int \cos(\ln x)\,dx$

$\ln x = t$

$x = e^t$

$dx = e^t\,dt$

$I = \int \underset{E}{\overset{V}{e^t}} \cdot \underset{T}{\overset{U}{\cos t}} \cdot dt$

$= \cos t \int e^t\,dt + \int \left((+\sin t) \cdot \int e^t\,dt \right) dx$

$= e^t \cos t + \int \overset{V}{e^t} \cdot \overset{U}{\sin t}\,dt$

$+ \left\{ \sin t \cdot \int e^t\,dt - \int \left(\cos t \cdot \int e^t\,dt \right) dt \right\}$

$I = e^t \cos t + \left\{ e^t \sin t - \int e^t \cos t\,dt \right\}$

$I = e^t \cos t + e^t \sin t - I \Rightarrow 2I = e^t(\sin t + \cos t)$

$I = \frac{e^t}{2}(\sin t + \cos t) = \frac{x}{2}\left(\sin(\ln x) + \cos(\ln x)\right) + C$

Short method for IBP

for

$$\boxed{\begin{array}{c} \int \text{Alg} \times \text{Exp} \\ \text{OR} \\ \int \text{Alg} \times \text{Trigo} \end{array}} \Rightarrow \text{Successive Integration}$$

$I = \int \overset{U}{x} \cdot \overset{V}{e^{2x}}\,dx$

$= x \cdot \left(\frac{e^{2x}}{2} \right) - 1 \cdot \left(\frac{e^{2x}}{4} \right) + C$

I

I

D

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Q $I=\int \underset{U}{x^3}\cdot \underset{V}{e^{3x}}\cdot dx$

$= x^3\cdot\left(\frac{e^{3x}}{3}\right)-3x^2\cdot\left(\frac{e^{3x}}{9}\right)+6x\left(\frac{e^{3x}}{27}\right)-6\left(\frac{e^{3x}}{81}\right)+C$

Q $I=\int \underset{U}{x^2}\cdot \underset{V}{\sin 2x}\cdot dx$

$= x^2\cdot\left(-\frac{\cos 2x}{2}\right)-2x\left(-\frac{\sin 2x}{4}\right)+2\left(+\frac{\cos 2x}{8}\right)+C$

Q $I=\int \underset{U}{x^5}\cdot \underset{V}{e^x}\cdot dx$

$= e^x\left(x^5-5x^4+20x^3-60x^2+120x-120\right)+C$

Q $I=\int (x^3-x)\cdot e^x\cdot dx$

$e^x\left((x^3-x)-(3x^2-1)+(6x)-6\right)+C$

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Q. $I=\int \frac{x\cdot\tan^{-1}x}{(1+x^2)^{3/2}}\,dx$

yad.

$\tan^{-1}x=t$

$x=\tan t$

$dx=\sec^2 t\cdot dt$

$=\int \frac{\tan t\cdot t\sec^2 t\,dt}{(1+\tan^2 t)^{3/2}}$

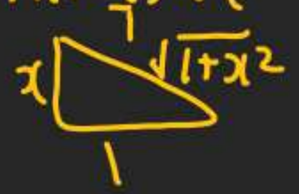
$=\int \frac{t\cdot\tan t\cdot\cancel{\sec^2 t}\cdot dt}{(\cancel{\sec^2 t})^{\cancel{3/2}}\sec t}$

$=\int t\cdot\frac{\sin t}{\cancel{\cos t}}\times\frac{\cancel{\cos t}}{1}$

$\Rightarrow \int \overset{u}{t}\cdot\overset{v}{\sin t}\,dt$

$t\cdot(-\cos t)-1\cdot(-\sin t)+C$

$-\tan^{-1}x\cdot\cos(\tan^{-1}x)+\sin(\tan^{-1}x)+C$



$-\tan^{-1}x\cdot\cancel{\cos}\left(\cancel{\cos^{-1}}\frac{1}{\sqrt{1+x^2}}\right)+\cancel{\sin}\left(\cancel{\sin^{-1}}\frac{x}{\sqrt{1+x^2}}\right)+C$

$-\frac{\tan^{-1}x}{\sqrt{1+x^2}}+\frac{x}{\sqrt{1+x^2}}+C$

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Board.

Q $I = \int \cos\sqrt{x}\cdot dx$

$\sqrt{x} = t$

$x = t^2$

$dx = 2t\,dt$

$= 2\int \cos t \cdot t\,dt$

I

$= 2\left[t\cdot(\sin t) - 1\cdot(-\cos t)\right] + C$

$= 2\left[\sqrt{x}\sin\sqrt{x} + \cos\sqrt{x}\right] + C$

Q $\int \frac{\sqrt{x^2+1}\cdot\left[\ln(x^2+1) - 2\ln x\right]}{x^4}\,dx$

$\int \frac{\sqrt{x^2+1}}{x^4}\left(\log\left(\frac{x^2+1}{x^2}\right)\right)\cdot dx$

$\Rightarrow \int \sqrt{\frac{x^2+1}{x^2}}\ \log\left(1+\frac{1}{x^2}\right)\cdot\frac{1}{x^3}\cdot dx$

$\Rightarrow \int \sqrt{1+\frac{1}{x^2}}\cdot\log\left(1+\frac{1}{x^2}\right)\cdot\frac{1}{x^3}\cdot dx$

$1+\frac{1}{x^2} = t$

$-\frac{2}{x^3}\cdot dx = dt$

$\frac{dx}{x^3} = \frac{-dt}{2}$

$-\frac{1}{2}\int \sqrt{t}\cdot\ln t\cdot dt$ IBP

$I = -\frac{1}{2}\left[\ln t\cdot\int\sqrt{t}\cdot dt - \int\left(\frac{1}{t}\cdot\int\sqrt{t}\cdot dt\right)dt\right]$

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★ Rk

When Integration has log fxn or Inverse fxn
We take "1" as 2^{nd} fxn.

$$\int \log fxn \times 1 \cdot dx \quad OR \quad \int \text{Inverse fxn} \times 1\, dx$$

Q $I = \int \sin^{-1}x \cdot dx$

$$= \int \underset{U}{\sin^{-1}x} \cdot \underset{V}{1} \cdot dx$$

$$= \sin^{-1}x \cdot \int 1 \cdot dx - \int \left(\frac{1}{\sqrt{1-x^2}} \cdot \int 1 \cdot dx \right) dx$$

$$= x \cdot \sin^{-1}x - \int \frac{x}{\sqrt{1-x^2}} \cdot dx$$

$$= x \sin^{-1}x + \left(+\sqrt{1-x^2}\right) + C$$

Q $I = \int \ln x \cdot dx$

$$= \int \ln x \cdot 1 \cdot dx$$

$$= \ln x \cdot \int 1 \cdot dx - \int \left(\frac{1}{x} \cdot \int 1 \cdot dx \right) dx$$

$$= x \cdot \ln x - \int \frac{1}{x} \cdot x \cdot dx$$

$$\boxed{\int \ln x \cdot dx = x \ln x - x + C}$$

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Q. $\int \ln(\sqrt{1+x} - \sqrt{1-x}) \cdot dx$

$\Rightarrow \int \ln(\sqrt{1+x} - \sqrt{1-x}) \cdot 1 \cdot dx$

Q $\int \ln(1+x^2) \cdot dx$

$= \int \ln(1+x^2) \cdot 1 \cdot dx$

Q $\int \ln(x+\sqrt{x^2+a^2}) \cdot dx$

$\int \underbrace{\ln(x+\sqrt{x^2+a^2})}_{u} \cdot \overset{v}{1} \cdot dx$

$= \ln(x+\sqrt{x^2+a^2}) \int 1 \cdot dx - \int \left(\frac{1}{\sqrt{x^2+a^2}} \cdot \int 1 \cdot dx\right) dx$

$= x \cdot \ln(x+\sqrt{x^2+a^2}) - \int \frac{x}{\sqrt{x^2+a^2}} dx$

$x \ln(x+\sqrt{x^2+a^2} - \sqrt{x^2+a^2} + c$

AW Sheet

45, 46, 47, 48, 49

50, 53

11 Q's, 21, 22, 23, 24

26, 29, 30, 31

38, 40, 41, 42, 43, 44