

Assume to be simple pendulum

$$l_{\text{eff}} = 3R$$

$$T = 2\pi \sqrt{\frac{3R}{g}}$$

About point
of suspension

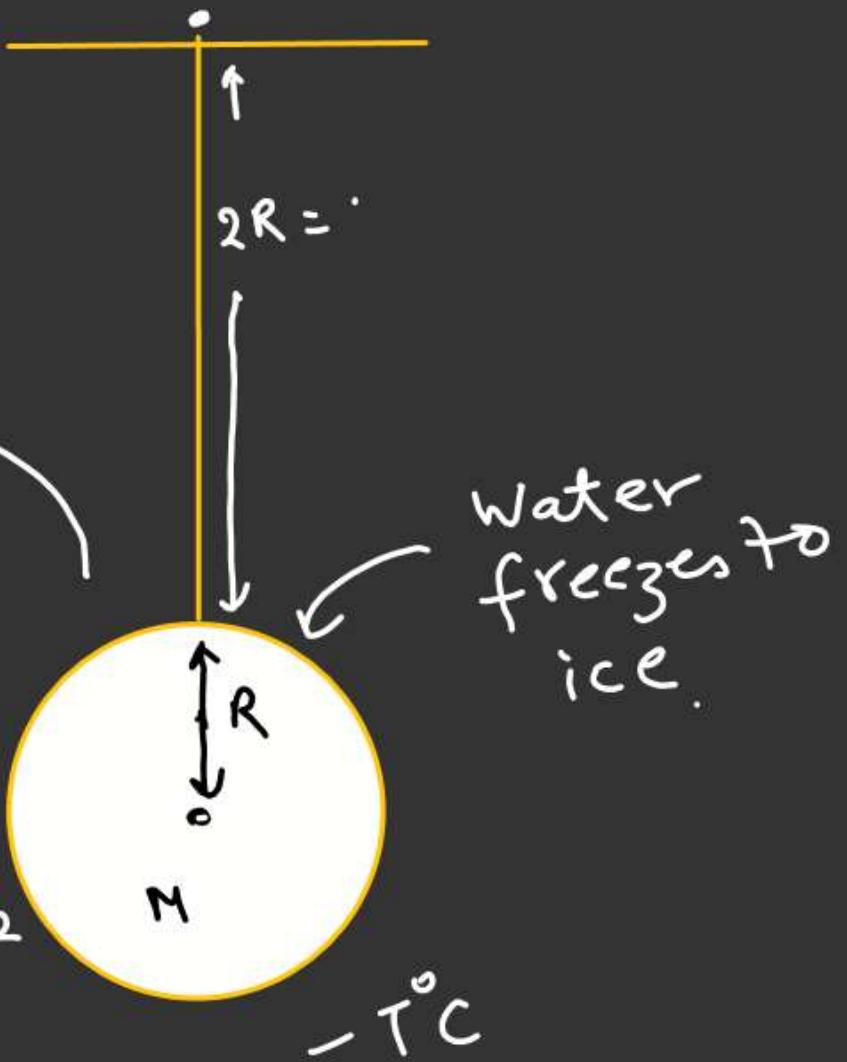
$$T = 2\pi \sqrt{\frac{47MR^2}{15MgR}}$$

Physical
pendulum

$$T = 2\pi \sqrt{\frac{I}{Mgd}}$$

$$d = 3R$$

$$I = \frac{2}{5}MR^2 + M(3R)^2$$



$$I = \frac{2}{5}MR^2 + 9MR^2$$

$$I = \left(\frac{47MR^2}{5}\right)$$

$$= 2\pi \sqrt{\frac{47R}{15g}}$$

Trolley can move horizontally on the parallel track

Find time period of String-bob System if .

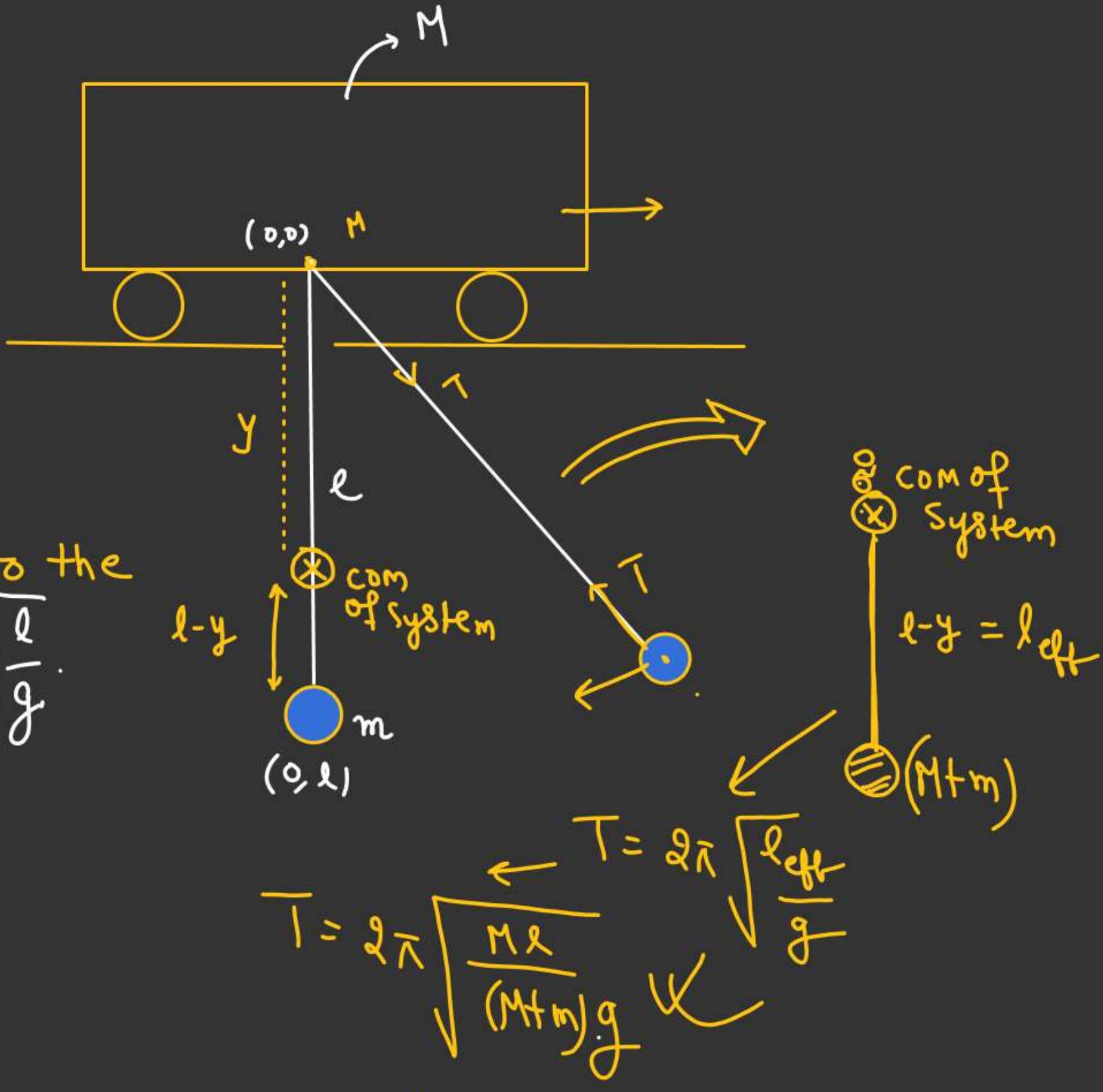
1) Bob oscillate along the plane of trolley.

2) Bob oscillate perpendicular to the plane of trolley. $\rightarrow T = 2\pi \sqrt{\frac{l}{g}}$

$$COM \text{ of system } (\Delta X) = 0$$

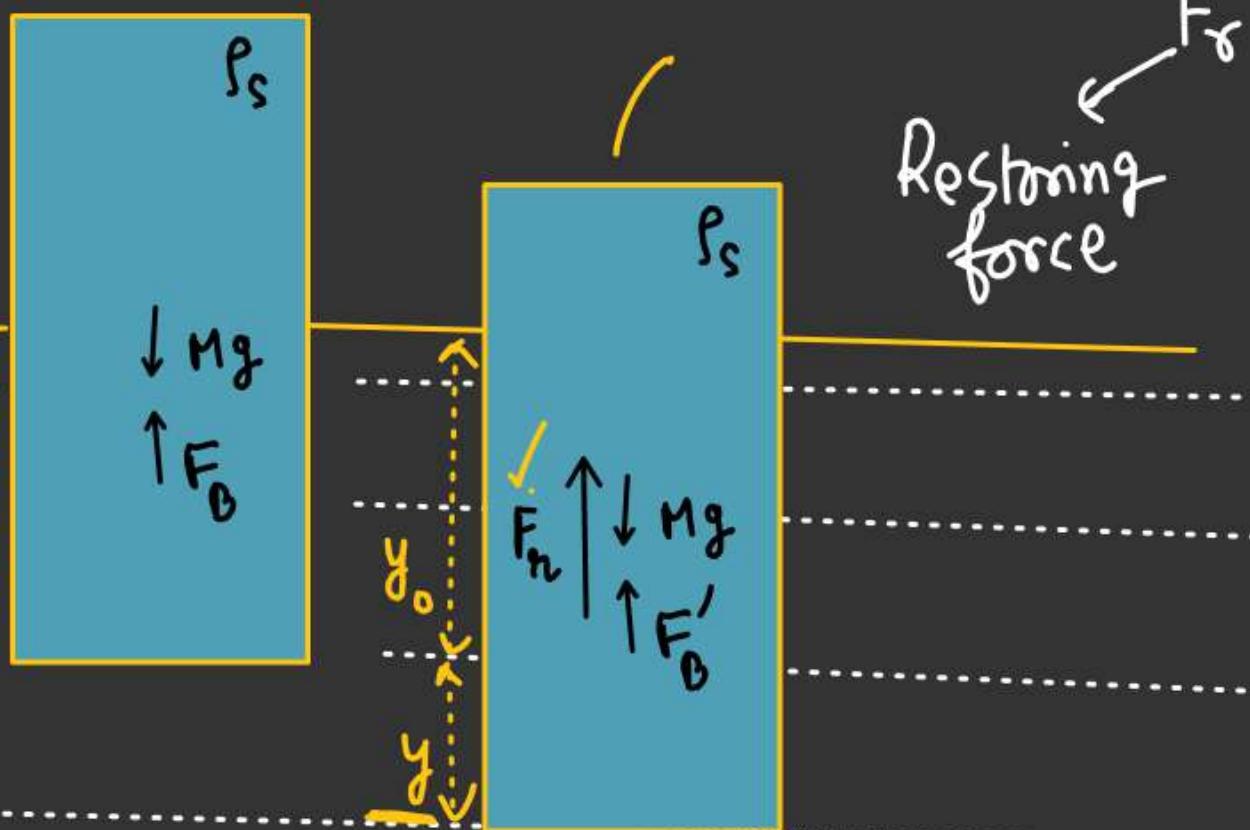
$$y = \left(\frac{ml}{M+m} \right)$$

$$\frac{l-y}{l_{eff}} = \frac{l - \frac{ml}{M+m}}{M+m} = \left(\frac{Ml}{M+m} \right)$$



S.H.M. in Fluid.

Cylinder, M, h.

A = Cross-sectional
Area of
Cylinder.Mean
Position

At Equilibrium.

$$F_B = Mg \Rightarrow \rho_L A y_0 g = (Ah) \rho_s g \quad \textcircled{1}$$

After cylinder pushed extra y distance

$$F_r = -(F'_B - Mg)$$

$$F_r = -[A(y+y_0)\rho_L g - Ah\rho_s g]$$

$$F_r = -[(A\rho_L g)y + (A y_0 \rho_L g - Ah\rho_s g)] \quad \textcircled{2}$$

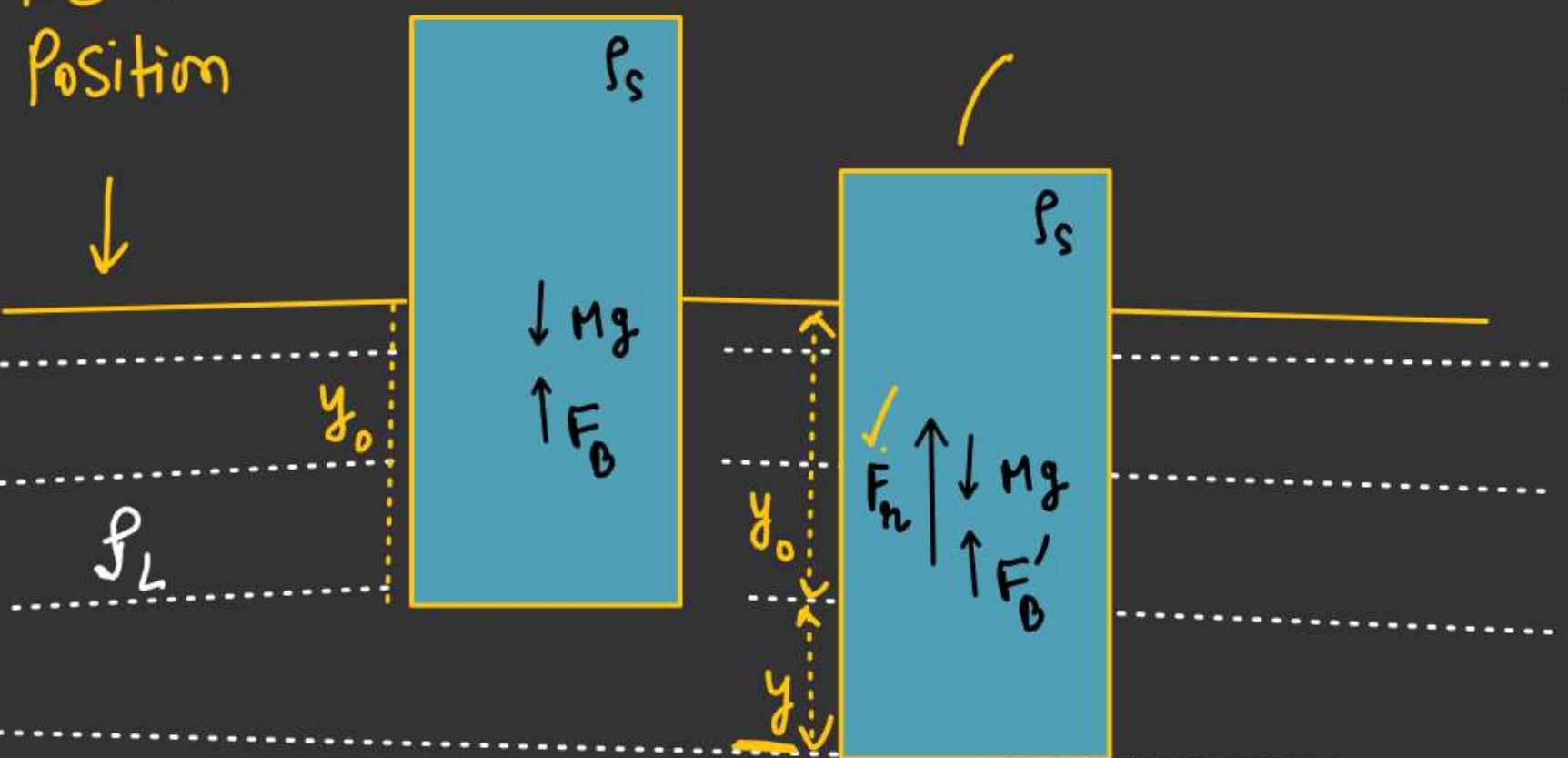
$$\underline{\underline{F_r}} = -(A\rho_L g)y$$

Extra buoyant
force responsible for
restoring.

S.H.M. in Fluid.

Cylinder, M, h.
A = Cross-sectional
Area of
Cylinder.

Mean
Position



$$F_y = -(S_L A g) y$$

$$a = -\frac{\rho_L A g y}{M}$$

$$a = -\frac{\rho_L A g y}{\rho_S A h}$$

$$a = -\frac{\rho_L g}{\rho_S h} y \rightarrow ④$$

From ③ & ④

$$a = -\left(\frac{g}{y_0}\right) y$$

$$a = -\omega^2 y$$

$$\omega = \sqrt{\frac{g}{y_0}}$$

A + Equilibrium

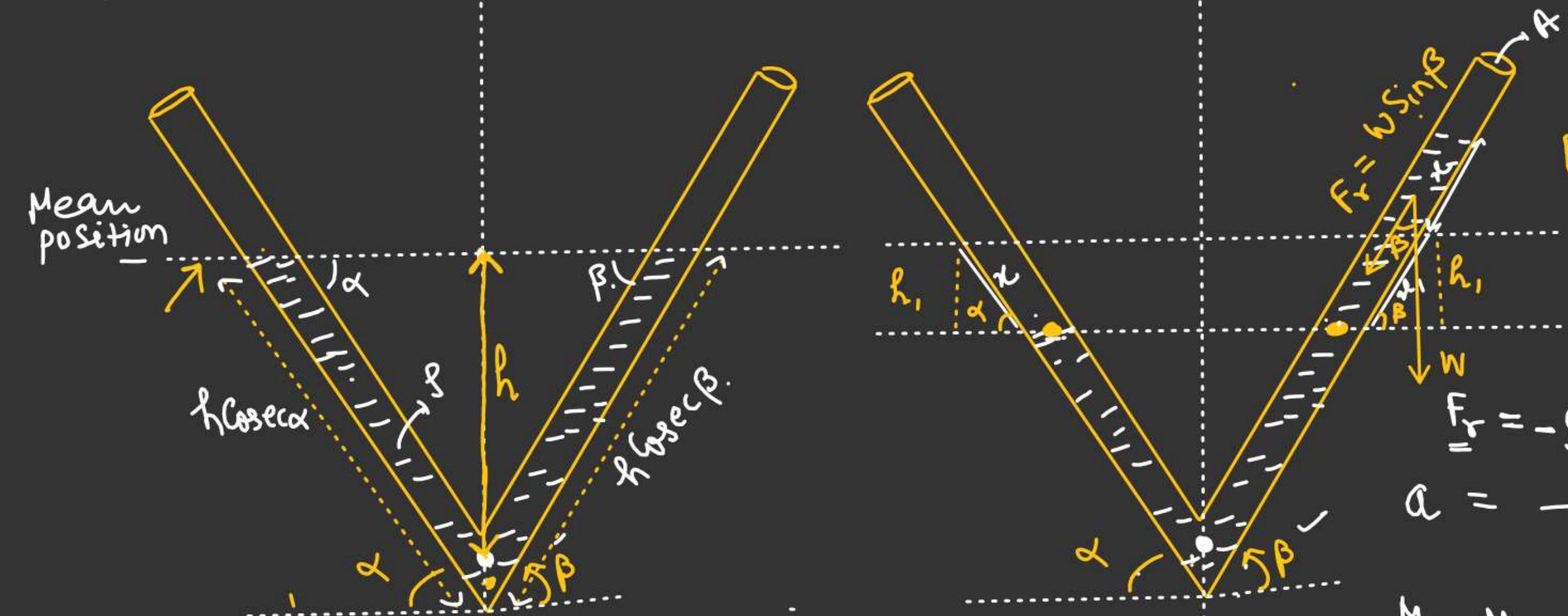
$$(y_0 A) \rho_L g = A h \rho_S g$$

$$\frac{\rho_L}{\rho_S h} = \frac{1}{y_0} \quad ③$$

~~22~~

$$T = 2\pi \sqrt{\frac{y_0}{g}}$$

Only depends on
length of submerged
part at equilibrium



α & β
inclination of
both arm of tube
from horizontal.

$$M = \rho A (h \operatorname{cosec} \alpha + \operatorname{cosec} \beta)$$

$$M = \rho A h (\operatorname{cosec} \alpha + \operatorname{cosec} \beta)$$

F_y = Component of weight of $(x+x_1)$ length of the liquid along the tube.

$$F_y = -\rho A (x+x_1) g \sin \beta$$

$$a = -\frac{\rho A (x+x_1) g \sin \beta}{M} \quad \text{--- (1)}$$

M = Mass of liquid.

$$h_1 = x \sin \alpha = x_1 \sin \beta$$

$$x_1 = \left(\frac{x \sin \alpha}{\sin \beta} \right) \rightarrow \text{put in (1)}$$

$$\underline{F_r} = -\rho A(x+x_1)g \sin\beta$$

$$a = -\frac{\rho A(x+x_1)g \sin\beta}{M} \quad \text{(1)}$$

M : Mass of liquid.

$$h_1 = x \sin\alpha = x_1 \sin\beta$$

$$x_1 = \left(\frac{x \sin\alpha}{\sin\beta} \right) \rightarrow \text{put in (1)}$$

$$a = -\frac{\cancel{\rho A g} \sin\beta \left(x + \frac{x \sin\alpha}{\sin\beta} \right)}{\cancel{\rho A h} (\cosec\alpha + \cosec\beta)}$$

$$M = \rho A h (\cosec\alpha + \cosec\beta)$$

$$a = -\frac{g \sin\beta \left(\frac{\sin\alpha + \sin\beta}{\sin\beta} \right) x}{\frac{1}{\sin\alpha} + \frac{1}{\sin\beta}}$$

$$a = -\left(\frac{g \sin\alpha \sin\beta}{h} \right) x$$

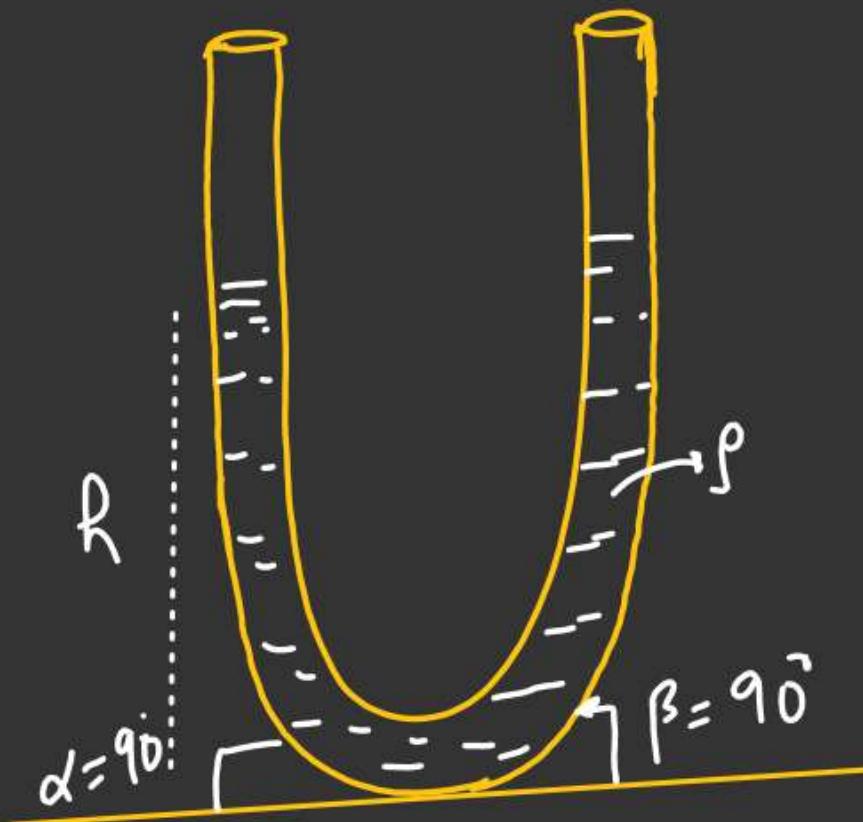
$$a = -\omega^2 x$$

$$\omega = \sqrt{\frac{g \sin\alpha \sin\beta}{h}}$$

Ans

$$T = 2\pi \sqrt{\frac{h}{g \sin\alpha \sin\beta}}$$

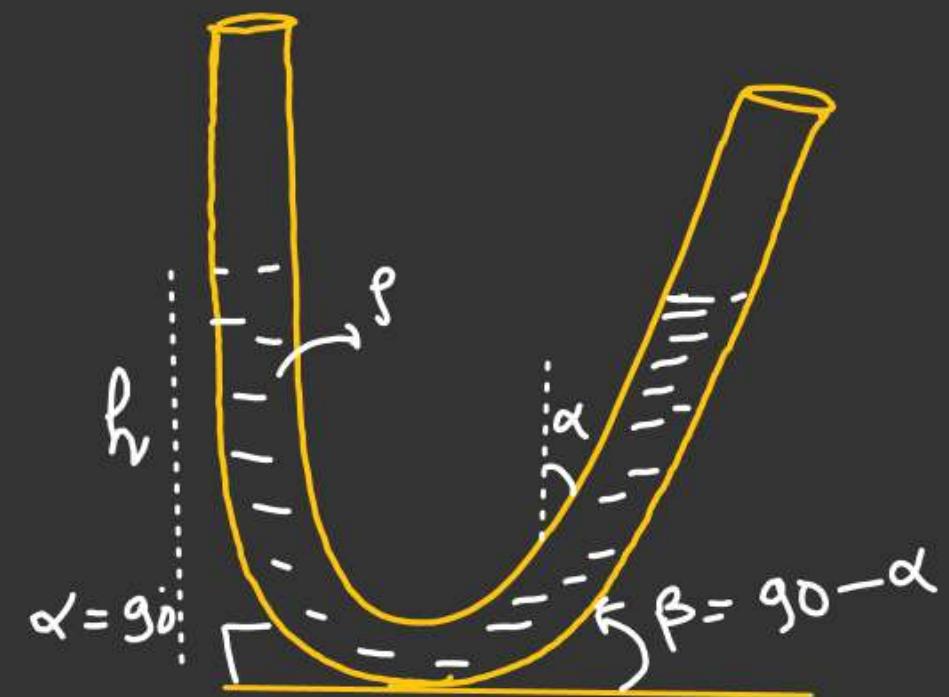
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$$T = ??$$

$$T = 2\pi \sqrt{\frac{h}{g \sin 90^\circ \cdot \sin 90^\circ}}$$

$$T = 2\pi \sqrt{\frac{h}{g}}$$



$$T = 2\pi \sqrt{\frac{h}{g \sin 90^\circ \sin(90 - \alpha)}}$$

$$T = 2\pi \sqrt{\frac{h}{g \cos \alpha}}$$

Sphere of radius r have pure rolling motion.

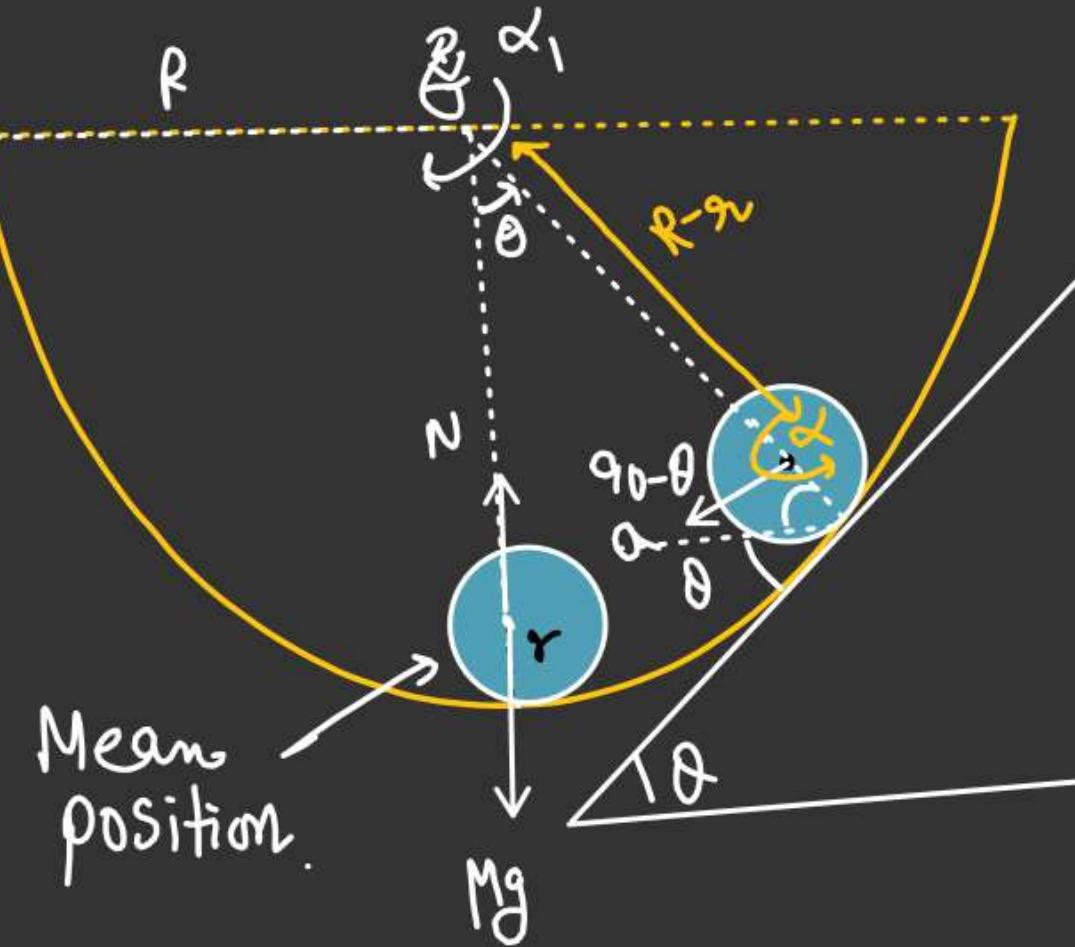
Prove that Sphere perform S.H.M & find its time period.

$$\ddot{\alpha} = \left(\frac{g \sin \theta}{1 + \frac{I}{MR^2}} \right) \quad I = \frac{2}{5} MR^2 \\ \Downarrow \quad \sin \theta \approx \theta.$$

$\alpha = r\dot{\theta} \Rightarrow$ pure rolling - ①

$$\ddot{\alpha}_1 \leftarrow R-r \quad \alpha = (R-r)\dot{\alpha}_1 \quad \text{②} \\ \gamma\dot{\alpha} = (R-r)\dot{\alpha}_1 \\ \alpha = \frac{(R-r)}{\gamma} \dot{\alpha}_1 \\ \alpha_t = \alpha$$

$\alpha_1, \alpha, \theta \rightarrow$ for S.H.M



$$\ddot{a} = \left(\frac{g \sin \theta}{1 + \frac{I}{MR^2}} \right) \quad \textcircled{1}$$

\Downarrow

$$I = \frac{2}{5} MR^2$$

$$\sin \theta \approx \theta.$$

From $\textcircled{1}$.

$$\gamma \alpha = \left(\frac{g}{1 + \frac{2}{5} \frac{1}{MR^2}} \right) \theta$$

 $a = \gamma \alpha \Rightarrow$ pure rolling - $\therefore \textcircled{2}$

$$\alpha_1 \leftarrow \begin{array}{l} R-\gamma \\ \searrow \end{array} \quad a = (R-\gamma) \alpha_1 - \textcircled{3}$$

$$\begin{aligned} \gamma \alpha &= (R-\gamma) \alpha_1 \\ \alpha &= \left(\frac{R-\gamma}{\gamma} \right) \alpha_1 \end{aligned} \quad] \text{ from } \textcircled{2} \text{ & } \textcircled{3}$$

$$\cancel{\frac{x(R-\gamma)}{x}} \alpha_1 = \left(\frac{5g}{7} \right) \theta$$

$$\alpha_1 = \frac{5g}{7(R-\gamma)} \theta$$

$$\alpha_1 = \omega^2 \alpha$$

Angular \Leftarrow frequency

$$\omega = \sqrt{\frac{5g}{7(R-\gamma)}} \Rightarrow T = 2\pi \sqrt{\frac{7(R-\gamma)}{5g}}$$

M-2
H.W

Try by Energy Method.

