

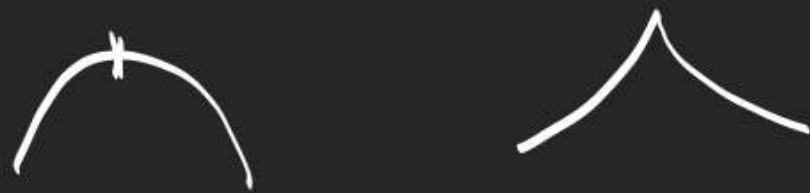
# Relative / Local Maximum of function of point $x=a$

$$\text{If } f(x) \leq f(a) \quad \forall x \in (a-\delta, a+\delta), \delta > 0$$

$\delta$  very small

$\Rightarrow f$  has local maximum at  $x=a$

Continuous (non constant)



Sign scheme  
of  $f'(x)$

$\begin{array}{c} \text{Critical point} \\ \downarrow \\ \begin{array}{c} + \quad - \\ \hline a-h \quad x=a \quad a+h \end{array} \end{array}$

→ First Derivative  
test

$f$  is strictly  $\uparrow$   $(a-\delta, a)$   
 $\downarrow$   $(a, a+\delta)$

## Second Derivative Test.

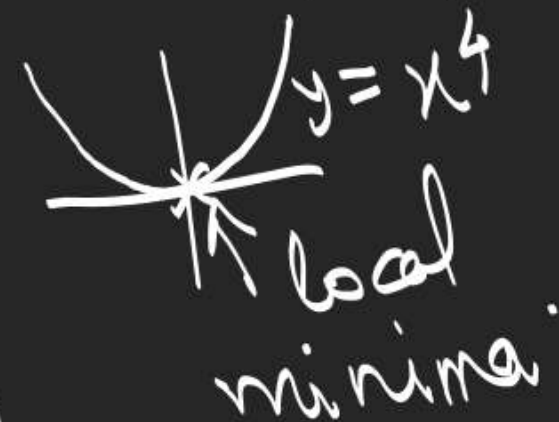
$$\text{If } f'(a) = 0 \text{ \& } f''(a) < 0$$

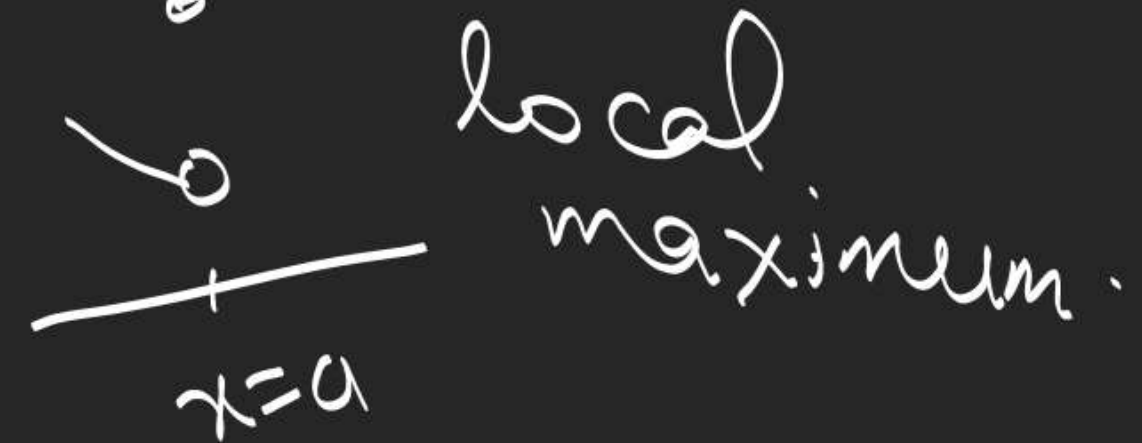
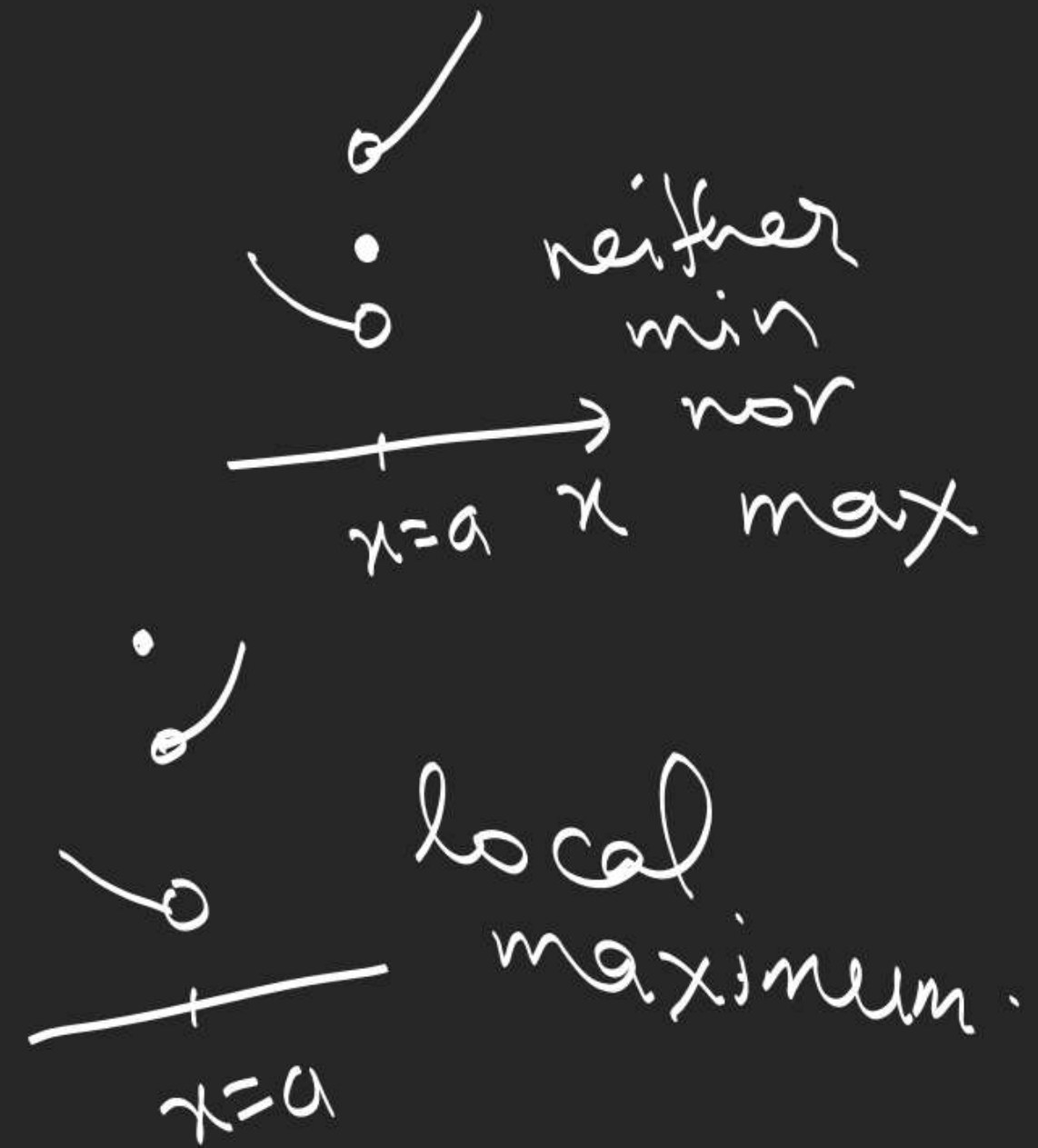
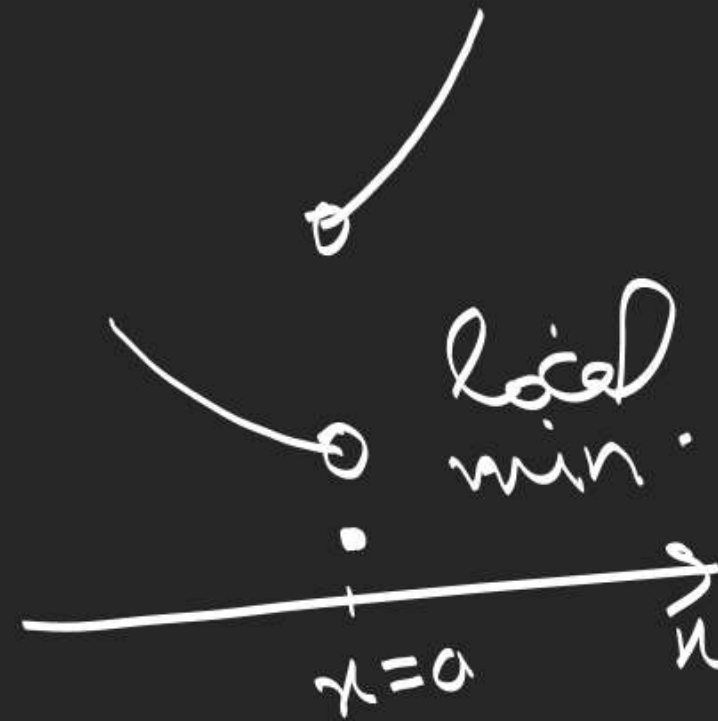
$\Rightarrow x=a$  is local maximum.

Method fails if  $f''(a) = 0$  or Doesn't exist.



$$f(x) = \begin{cases} x^4 & \rightarrow \text{minimum} \\ -x^4 & \rightarrow \text{maximum} \\ x^3 & \rightarrow \text{neither max. nor min} \end{cases} \text{ at } x=0$$





$$f(x) = \begin{cases} x^{4/3} & \rightarrow \underline{\text{min}} \\ -x^{4/3} & \rightarrow \text{max} \\ x^{7/5} & \rightarrow \text{neither min nor max} \end{cases} \text{ at } x=0$$



Monotonicity

$\{x \in I \mid (1 \leq x \leq 5)\}$   
 $-\{1, 3\}$