

# Complex Number

$$z = x + iy \quad \text{where } x, y \in \mathbb{R}$$

iota       $i = \sqrt{-1}$

$$\operatorname{Re}(z) = x$$

$$\operatorname{Im}(z) = y$$

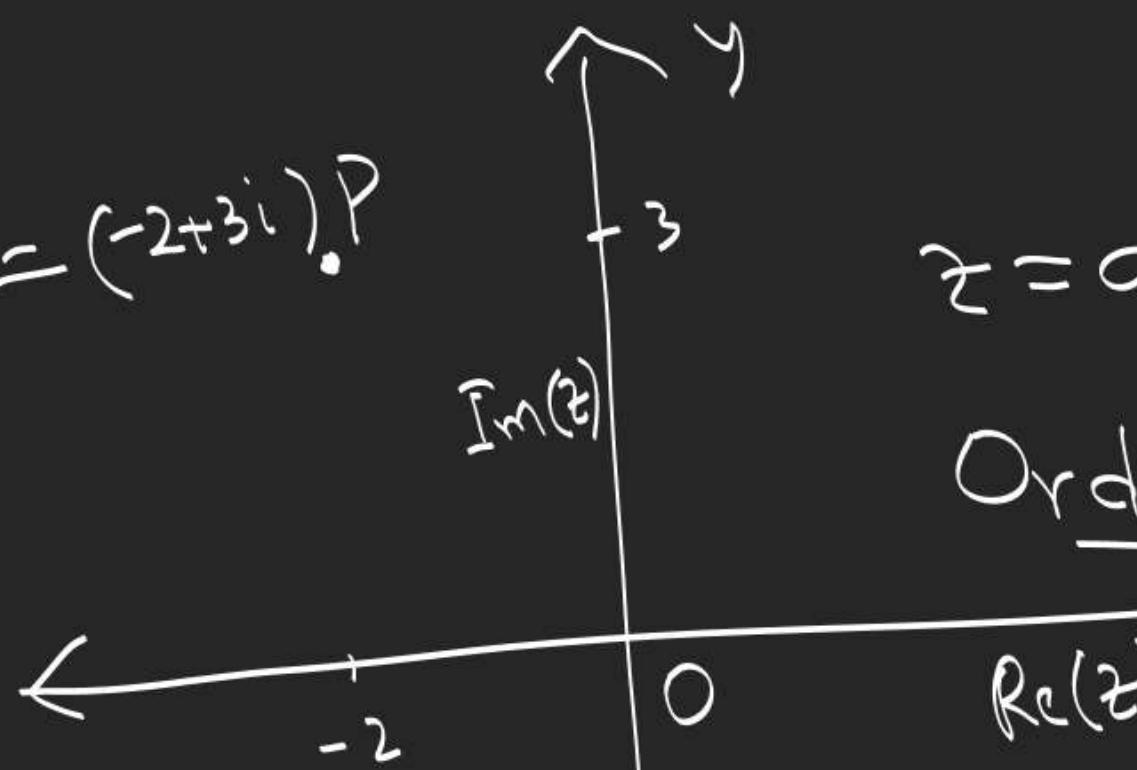
$z=0$  is purely real or purely imaginary  
 $2+3i$   
 $-2i$

- If  $y=0 \Rightarrow z$  is purely real.
- If  $x=0, \Rightarrow z$  is purely imaginary
- If  $y \neq 0 \Rightarrow z$  is imaginary number

# Representation of Complex Number

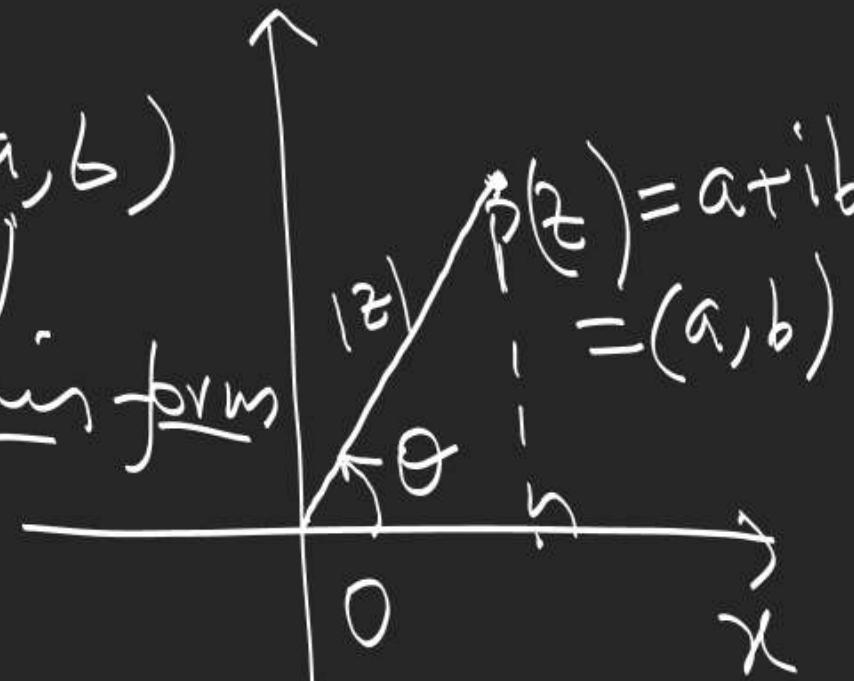
(Argand Plane / Complex plane)

$$(-2, 3) = (-2 + 3i)$$



$$z = a + bi = (a, b)$$

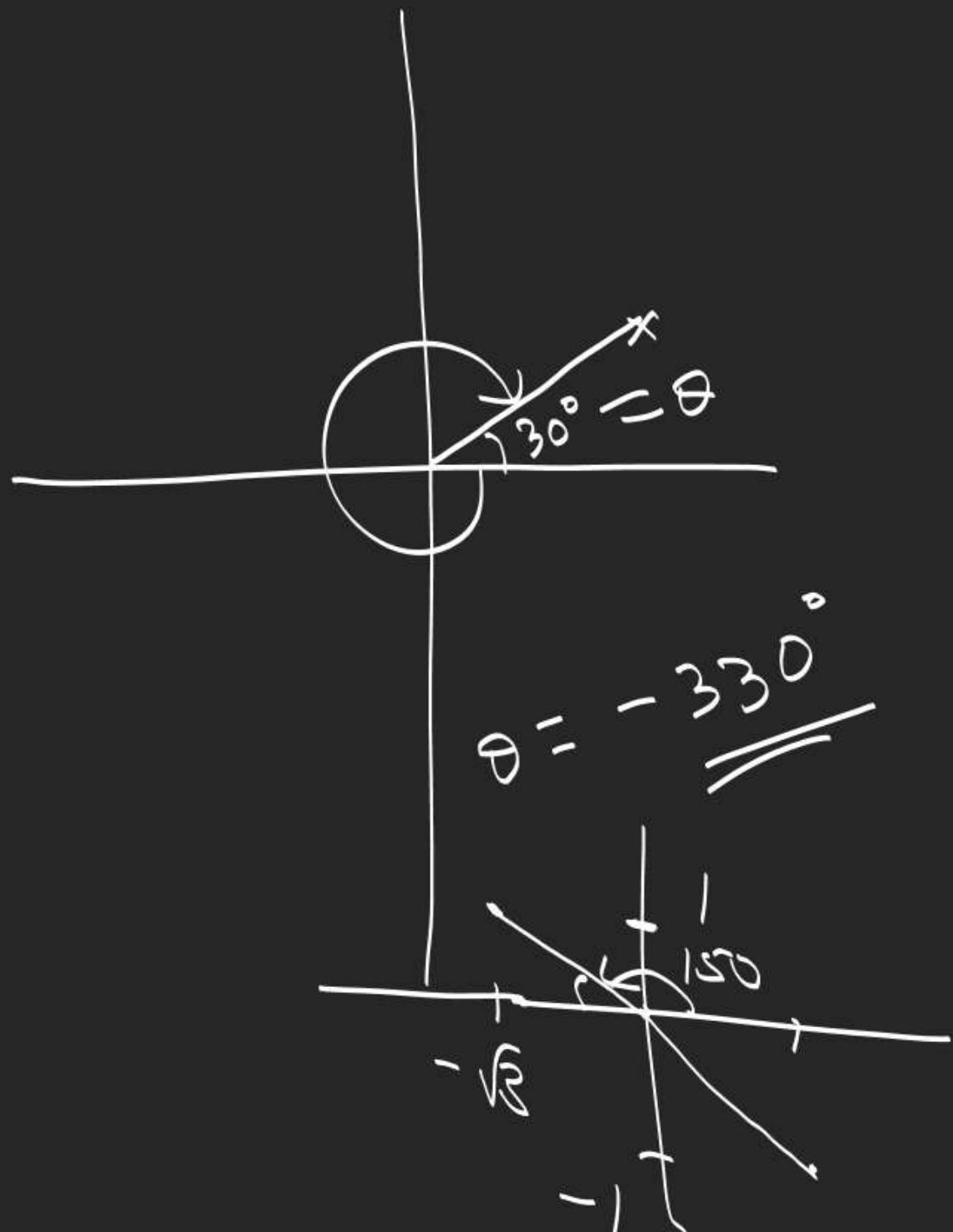
Ordered pair form



$$\theta = \text{argument of } z = \arg(z)$$

$$\tan \theta = \frac{b}{a}$$

$$|z| \text{ is non negative real number} \leftarrow |z| = \sqrt{a^2 + b^2} \quad |z| = \begin{cases} \text{Modulus} \\ \text{Magnitude} \end{cases}$$



Principle argument of  
Complex number

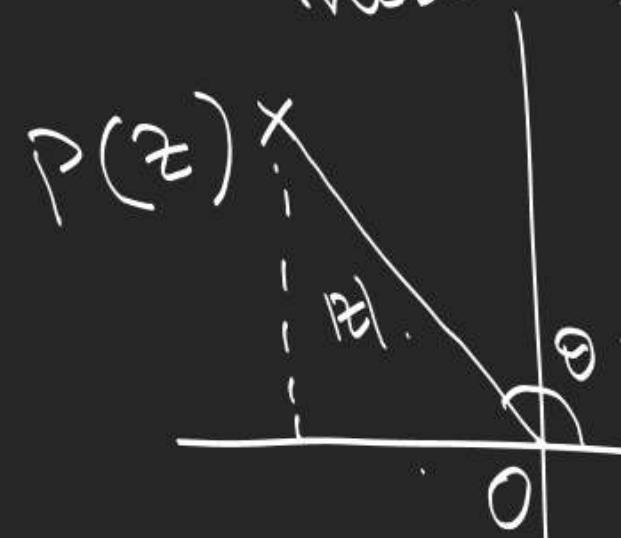
$$\theta \in (-\pi, \pi]$$

principle arg. of

- ①  $z = 1 - i$        $\theta = -\frac{\pi}{4}$
- ②  $z = -\sqrt{3} + i$        $\theta = \frac{4\pi}{3}$

Note  $\rightarrow$  ①

②  $z = 0$  has modulus '0' and argument not defined.



$$z = \vec{OP}$$

$$\begin{aligned} z &= |z|(\cos\theta + i\sin\theta) = |z| e^{i\theta} \\ &= |z| \text{cis } \theta \end{aligned}$$

$$z = |z|\cos\theta + i|z|\sin\theta$$

$$z = |z|(\cos\theta + i\sin\theta)$$

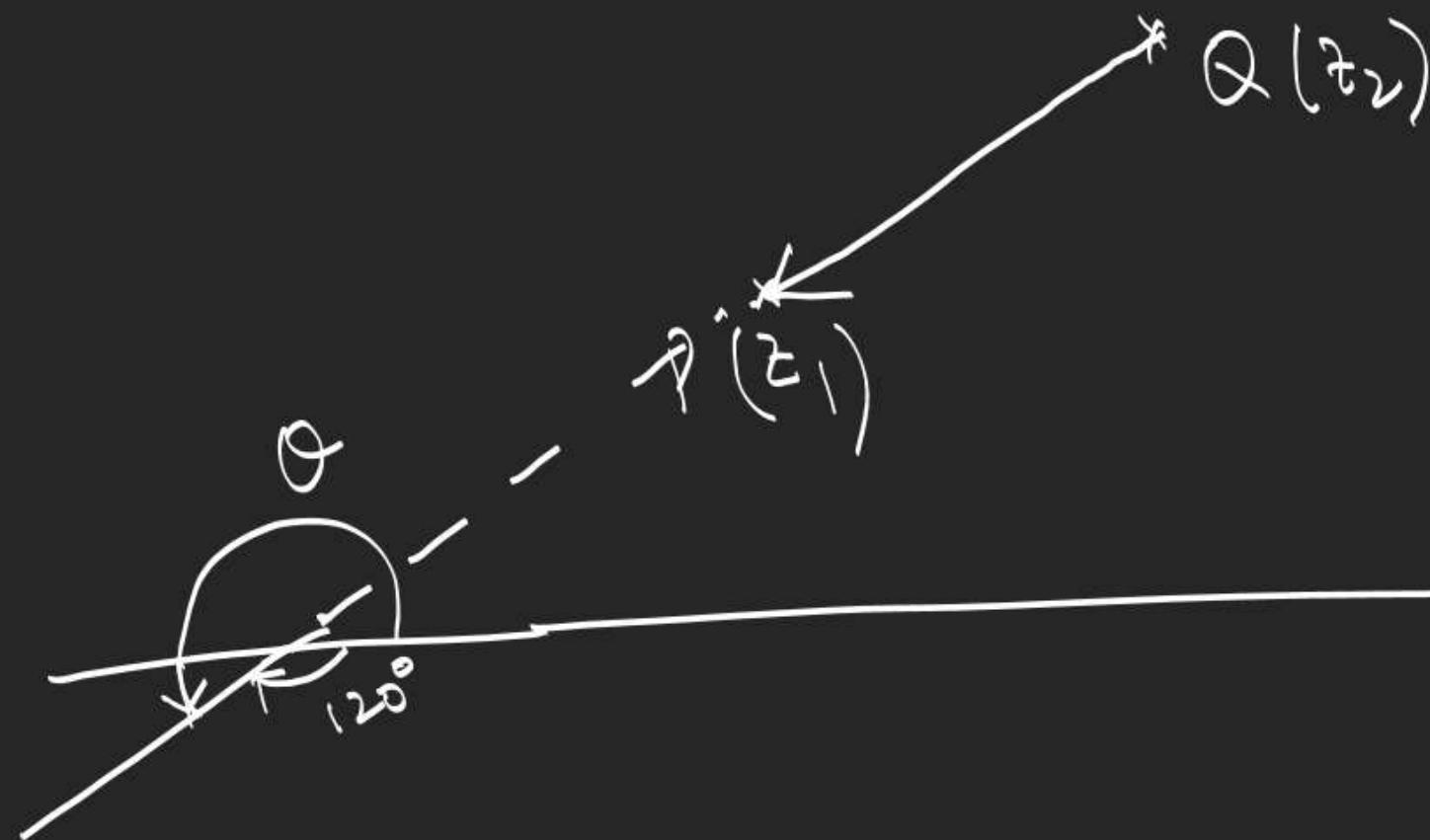
$\downarrow$  Circular form



Trigonometric form

$$(|z|, \theta) = \left(2, \frac{\pi}{3}\right)$$

$$\begin{aligned} z &= 2\cos\frac{\pi}{3} + i 2\sin\frac{\pi}{3} \\ &= 1 + i\sqrt{3} \end{aligned}$$



$$\arg(z_1 - z_2) = ?$$

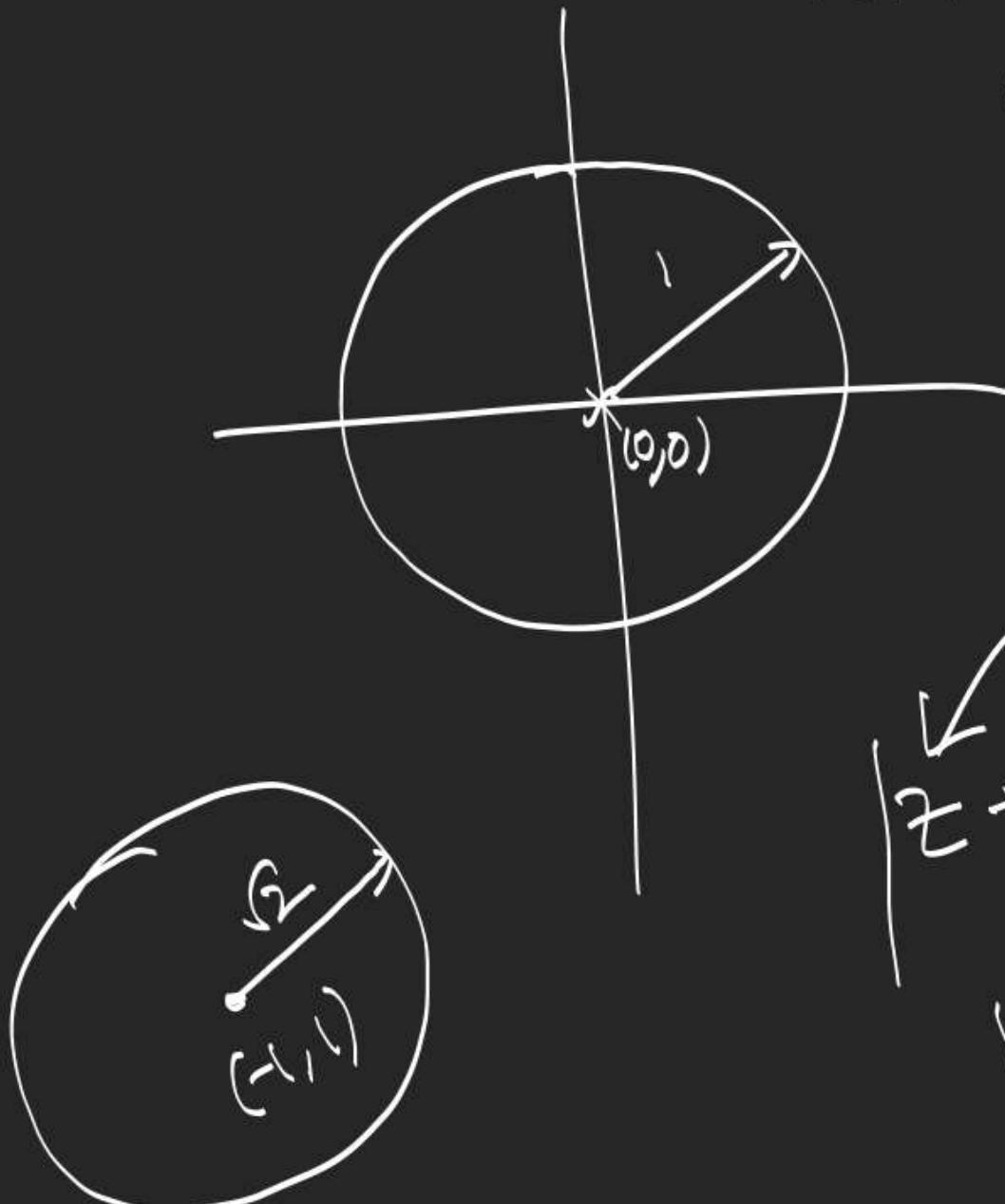
$$|z_1 - z_2| = ?$$

$$z_1 - z_2 = \overrightarrow{QP}$$

$$|\overrightarrow{PQ}| = |z_1 - z_2|$$

Represent 'z' in Argand plane

Satisfying  $|z| = 1$

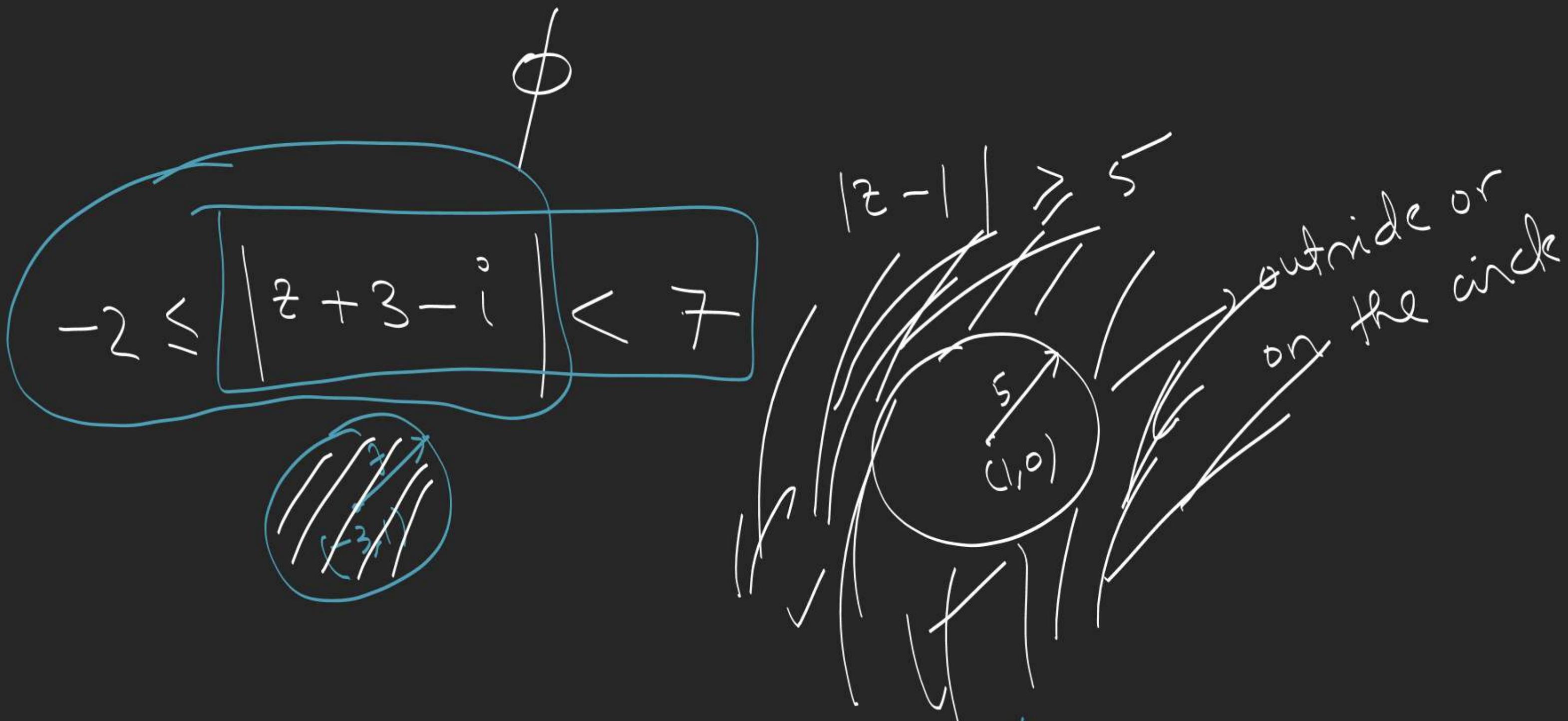


Lies on circle with  
centre  $(0,0)$  and radius 1.

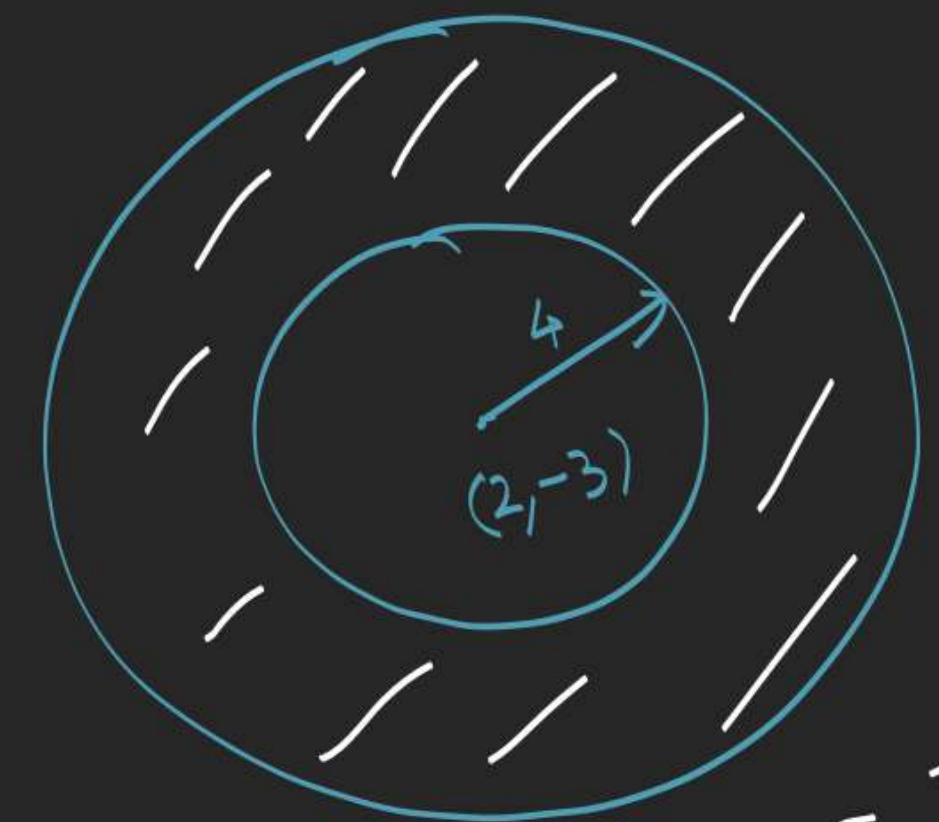
$$\begin{cases} z + 1 - i \\ z - (-1 + i) \end{cases} = \sqrt{2}$$

$$\begin{aligned} |(x+1)+i(y-1)| &= \sqrt{2} \\ [(x+1)^2 + (y-1)^2]^{1/2} &= 2 \end{aligned}$$

$$|z+2-i| = -5$$



$$4 < |z - 2 + 3i| \leq 7$$



Find area of region  
formed by  $|z|$  satisfying

$$\text{Area} = \pi (7^2 - 4^2)$$

$\pi 7^2 (\text{remaining})$   
leave  $1/4 \pi 8^2$   $\pi 8^2 (1 - 1)$