Nishant Jindal

Q $\sqrt{1+x+y}\cdot\frac{dy}{dx} = x+y-1$ Solve?

$x+y+1=t^2$

$1+\frac{dy}{dx} = 2t\frac{dt}{dx}$

$\frac{dy}{dx} = 2t\frac{dt}{dx} - 1$

$t\times\left(2t\frac{dt}{dx}-1\right) = t^2-2$

$2t\frac{dt}{dx} - 1 = \frac{t^2-2}{t}$

$2t\frac{dt}{dx} = \frac{t^2-2}{t}+1$

$= \frac{t^2+t-2}{t}$

$\int\frac{2t^2\,dt}{t^2+t-2} = \int dx$

Q $2\int\frac{\cancel{t^2+t-2}}{\cancel{t^2+t-2}} - \frac{(t-2)}{t^2+t-2}$

$2t - 2\int\frac{(t-2)dt}{\underbrace{t^2+t-2}} = x+C$

$\int\frac{\text{Linear}}{\text{Quad}}$

Solve urself.

Method 3 When $\frac{dy}{dx} = \frac{a_1x+b_1y+c_1}{a_2x+b_2y+c_2}$ is given.

When $\frac{a_1}{a_2} = \frac{b_1}{b_2}$ ($\parallel^{rel}$ lines)

Q $\frac{dy}{dx} = \frac{x-y+3}{2x-2y+5}$ Solve?

$x-y=t$

$1-\frac{dy}{dx} = \frac{dt}{dx}$

$\frac{dy}{dx} = 1-\frac{dt}{dx}$

$1-\frac{dt}{dx} = \frac{t+3}{2t+5}$

$1-\frac{t+3}{2t+5} = \frac{dt}{dx}$

$\frac{t+2}{2t+5} = \frac{dt}{dx} \Rightarrow \int\frac{2t+5}{t+2}dt = \int dx$

$\int\frac{2(t+2)+1}{(t+2)}dt \Rightarrow 2t+\ln|t+2| = x+C$

$2(x-y)+\ln|x-y+2| = x+C$

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Method 4 Polar Substitution

$x^2+y^2=r^2$

assume

$x=r\cos\theta, y=r\sin\theta$

$x^2+y^2=r^2$

Partial diff ①

$2x\,dx+2y\,dy=2r\,dr$

Qs में → $\boxed{x\,dx+y\,dy=r\,dr}$ ये मिलेगा।

② $x=r\cos\theta, y=r\sin\theta$; $\cos\theta=\frac{x}{r}$

$\frac{y}{x}=\tan\theta$

$\Rightarrow \frac{x\cdot dy-y\cdot dx}{x^2}=\sec^2\theta\cdot d\theta$

$\frac{x\,dy-y\,dx}{\cancel{x^2}}=\frac{r^2}{\cancel{x^2}}d\theta$

$\boxed{x\,dy-y\,dx=r^2d\theta}$

$x^2-y^2=r^2$

$x=r\sec\theta, y=r\tan\theta$

① $x^2-y^2=r^2$

$2x\,dx-2y\,dy=2r\,dr$

$\boxed{x\,dx-y\,dy=r\,dr}$ → Qs में मिलेगा

② $\frac{y}{x}=\sin\theta$

$\frac{x\cdot dy-y\cdot dx}{x^2}=\cos\theta\,d\theta$

$x\,dy-y\,dx=x^2\cdot\cos\theta\cdot d\theta$

$\boxed{x\,dy-y\,dx=r^2\sec\theta\,d\theta}$

2 Combo Can be Used.

$\boxed{\begin{aligned}x\,dx+y\,dy&=r\,dr\\ x\,dy-y\,dx&=r^2d\theta\end{aligned}}$

$x=r\cos\theta, y=r\sin\theta$

$\boxed{\begin{aligned}x\,dx-y\,dy&=r\,dr\\ x\,dy-y\,dx&=r^2\sec\theta\,d\theta\end{aligned}}$

$x=r\sec\theta, y=r\tan\theta$

Q $\frac{\boxed{x\,dx-y\,dy}}{x\,dy-y\,dx}=\sqrt{\frac{1+(x^2-y^2)}{x^2-y^2}}$

Isko deakh के 2^{nd} Combo

$\frac{\cancel{r}\,dr}{r^{\cancel{2}}\sec\theta\,d\theta}=\sqrt{\frac{1+r^2}{\cancel{r^2}}}$

$\int\frac{dr}{\sqrt{1+r^2}}=\int\sec\theta\,d\theta$ → Replace r & θ

$\ln|r+\sqrt{1+r^2}|=\ln\left|\tan\left(\frac{\pi}{4}+\frac{\theta}{2}\right)\right|+\ln c$

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Q $\dfrac{\boxed{x\,dx+y\,dy}}{\sqrt{x^2+y^2}} = \dfrac{y\,dx-x\,dy}{x^2}$

Combo 1 Isko dekh K combo1 decide kiya

$\dfrac{r\,dr}{\sqrt{r^2}} = \dfrac{-r^2 d\theta}{r^2\cos^2\theta}$

$\int dr = -\int \sec^2\theta\, d\theta$

$\Rightarrow r = -\tan\theta + c$

$\Rightarrow \sqrt{x^2+y^2} = -\dfrac{y}{x} + c$

Method 4 Exact differential

Partial diff^n Based

Solve

(1) $d(x+y) = dx+dy$

(2) $d(x-y) = dx-dy$

(3) $d(x^2) = 2x\cdot dx$

(4) $d(y^2) = 2y\cdot dy$

(5) $d\left(\dfrac{y}{x}\right) = \dfrac{x\cdot dy - y\cdot dx}{x^2}$

(6) $d(x\cdot y) = x\cdot dy + y\cdot dx$

(7) $d(x^2\cdot y) = x^2\cdot dy + y\cdot 2x\cdot dx$

(8) $d(x^2+y^2) = 2x\,dx + 2y\,dy$

(9) $d(\sin\theta) = \cos\theta\cdot d\theta$

(10) $d(\sqrt{x^2+y^2}) = \dfrac{1\times(2x\,dx+2y\,dy)}{2\sqrt{x^2+y^2}}$

2 Combo Can be Used.

$\boxed{\begin{gathered} x\,dx+y\,dy = r\,dr \\ x\,dy - y\,dx = r^2 d\theta \end{gathered}}$

$x = r\cos\theta,\ y = r\sin\theta$

$\boxed{\begin{gathered} x\,dx-y\,dy = r\,dr \\ x\,dy - y\,dx = r^2\sec\theta\, d\theta \end{gathered}}$

$x = r\sec\theta,\ y = r\tan\theta$

Q $\dfrac{\boxed{x\,dx-y\,dy}}{x\,dy-y\,dx} = \sqrt{\dfrac{1+(x^2-y^2)}{x^2-y^2}}$

Isko dekh ke 2nd Combo

$\dfrac{r\,dr}{r^2\sec\theta\, d\theta} = \sqrt{\dfrac{1+r^2}{r^2}}$

$\int \dfrac{dr}{\sqrt{1+r^2}} = \int \sec\theta\, d\theta$ → Replace r & θ

$\ln|r+\sqrt{1+r^2}| = \ln\left|\tan\left(\dfrac{\pi}{4}+\dfrac{\theta}{2}\right)\right| + \ln c$

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Q Solve.

$(1+x\sqrt{x^2+y^2})dx+(-1+\sqrt{x^2+y^2})y\,dy=0$

$(dx-y\,dy)+\sqrt{x^2+y^2}(x\,dx+y\,dy)=0$

" $+\frac{\sqrt{x^2+y^2}}{2}(2x\,dx+2y\,dy)=0$

$\int dx-\int y\,dy+\int\frac{\sqrt{x^2+y^2}}{2}\cdot d(x^2+y^2)=0$

$x-\frac{y^2}{2}+\frac{1}{2}\times\frac{2}{3}(x^2+y^2)^{3/2}=C$

$\int\frac{\sqrt{x}}{2}\cdot dx$

$\frac{1}{2}\cdot\frac{2}{3}\cdot x^{3/2}$

(1) $d(x+y)=dx+dy$

(2) $d(x-y)=dx-dy$

(3) $d(x^2)=2x\cdot dx$

(4) $d(y^2)=2y\cdot dy$

(5) $d\left(\frac{y}{x}\right)=\frac{x\cdot dy-y\cdot dx}{x^2}$

(6) $d(x\cdot y)=x\cdot dy+y\cdot dx$

(7) $d(x^2\cdot y)=x^2\cdot dy+y\cdot 2x\cdot dx$

(8) $d(x^2+y^2)=2x\,dx+2y\,dy$

(9) $d(\sin\theta)=\cos\theta\cdot d\theta$

(10) $d(\sqrt{x^2+y^2})=\frac{1\times(\cancel{2}x\,dx+\cancel{2}y\,dy)}{\cancel{2}\sqrt{x^2+y^2}}$

Methods

Exact differential

Partial diff^n Based

Q $y\,dx+(x+x^2y)dy=0$ Solve.

$\Rightarrow (y\,dx+x\,dy)+x^2y\,dy=0$ $\boxed{\text{Self}}$

$\Rightarrow \frac{d(x\cdot y)}{x^2y^2}+\frac{\cancel{x^2}y\,dy}{\cancel{x^2}y^2}=0$

$\Rightarrow \int\frac{d(x\cdot y)}{(xy)^2}+\int\frac{dy}{y}=\int 0$

$-\frac{1}{(x\cdot y)}+\ln y=C$ Ans

Q

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Q Sol. of

$$\frac{x}{x^2+y^2}\,dy = \left(\frac{y}{x^2+y^2}-1\right)dx$$

$$\frac{x\,dy - y\,dx}{x^2+y^2} = -dx$$

$$\frac{\boxed{x\,dy - y\,dx}}{\boxed{x^2}\left(1+\left(\frac{y}{x}\right)^2\right)} = -dx$$

$$\int \frac{d\left(\frac{y}{x}\right)}{1+\left(\frac{y}{x}\right)^2} = -\int dx$$

$$\int \frac{dt}{1+t^2} \qquad \tan^{-1}\left(\frac{y}{x}\right) = -x + C$$

(1) $d(x+y) = dx + dy$

(2) $d(x-y) = dx - dy$

(3) $d(x^2) = 2x\cdot dx$

(4) $d(y^2) = 2y\cdot dy$

(5) $d\left(\frac{y}{x}\right) = \frac{x\cdot dy - y\cdot dx}{x^2}$

(6) $d(x\cdot y) = x\cdot dy + y\cdot dx$

(7) $d(x^2\cdot y) = x^2\cdot dy + y\cdot 2x\cdot dx$

(8) $d(x^2+y^2) = 2x\,dx + 2y\,dy$

(9) $d(\sin\theta) = \cos\theta\cdot d\theta$

(10) $d\left(\sqrt{x^2+y^2}\right) = \frac{1\times(\cancel{2}x\,dx + \cancel{2}y\,dy)}{\cancel{2}\sqrt{x^2+y^2}}$

$$\int \frac{\sqrt{x}}{2}\cdot dx$$

$$\frac{1}{2}\cdot\frac{2}{3}\cdot x^{3/2}$$

Q Main $(x - x^3)\,dy = (y + yx^2 - 3x^4)\,dx$; $x > 2$

then $y(3) = 3$ find $y(4) = ?$

$$(x\,dy - y\,dx) - (x^3\,dy + y\,x^2\,dx) + 3x^4\,dx = 0$$

$$\frac{x\,dy - y\,dx}{x^2} - \left(\frac{\cancel{x^3}\,dy + y\,\cancel{x^2}\,dx}{\cancel{x^2}}\right) + \frac{3\cancel{x^4}^{\,x^2}}{\cancel{x^2}}\,dx = 0$$

$$d\left(\frac{y}{x}\right) - (x\,dy + y\,dx) + 3x^2\,dx = 0$$

$$\int d\left(\frac{y}{x}\right) - \int d(x\cdot y) + \int 3x^2\,dx = 0$$

$$\frac{y}{x} - (x\cdot y) + \cancel{3}\cdot\frac{x^3}{\cancel{3}} = C$$

(2) $y(3) = 3$: $\frac{\cancel{3}}{\cancel{3}} - (3\times 3) + 3^3 = C \Rightarrow C = 19$

$$\frac{y}{x} - (xy) + x^3 = 19$$

(3) $y(4) = ?$

$$\frac{y}{4} - 4y + 64 = 19 \quad \text{find } y = ?$$

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Q Sol. of

$$\frac{x}{x^2+y^2}\,dy = \left(\frac{y}{x^2+y^2}-1\right)dx$$

$$\frac{x\,dy - y\,dx}{x^2+y^2} = -dx$$

$$\frac{\boxed{x\,dy - y\,dx}}{\boxed{x^2}\left(1+\left(\frac{y}{x}\right)^2\right)} = -dx$$

$$\int \frac{d\left(\frac{y}{x}\right)}{1+\left(\frac{y}{x}\right)^2} = -\int dx$$

$$\int \frac{dt}{1+t^2} \qquad \tan^{-1}\left(\frac{y}{x}\right) = -x + c$$

(1) $d(x+y) = dx + dy$

(2) $d(x-y) = dx - dy$

(3) $d(x^2) = 2x\cdot dx$

(4) $d(y^2) = 2y\cdot dy$

(5) $d\left(\frac{y}{x}\right) = \frac{x\cdot dy - y\cdot dx}{x^2}$

(6) $d(x\cdot y) = x\cdot dy + y\cdot dx$

(7) $d(x^2\cdot y) = x^2\cdot dy + y\cdot 2x\cdot dx$

$\int \frac{\sqrt{x}}{2}\cdot dx$

$\frac{1}{2}\cdot\frac{2}{3}\cdot x^{3/2}$

Q Mains $\frac{dy}{dx} = 1 + x\cdot e^{y-x}$; $-\sqrt{2} < x < \sqrt{2}$, $y(0)=0$

then Min. value of $y(x)$, $x \in (-\sqrt{2}, \sqrt{2}) = ?$

$dy = dx + x\cdot e^{y-x}\,dx$

$dy - dx = x\cdot e^{y-x}\,dx \rightarrow -e^{-t}$

$\Rightarrow \frac{(dy-dx)}{e^{y-x}} = x\cdot dx \Rightarrow \int \frac{d(y-x)}{e^{y-x}} = \int x\,dx$

$\Rightarrow -e^{x-y} = \frac{x^2}{2} + c \Rightarrow -1 = 0 + c$

$\Rightarrow -e^{x-y} = \frac{x^2}{2} - 1 \Rightarrow e^{x-y} = 1 - \frac{x^2}{2}$

$\Rightarrow x - y = \ln\left(1 - \frac{x^2}{2}\right) \Rightarrow y = x - \ln\left(1 - \frac{x^2}{2}\right)$

$$\frac{dy}{dx} = 1 + \frac{+\frac{2x}{2}}{\left(1 - \frac{x^2}{2}\right)} = 1 + \frac{2x}{2-x^2}$$

$$= \frac{2 - x^2 + 2x}{2 - x^2}$$

$$\frac{dy}{dx} = \frac{x^2 - 2x + 1 - 3}{x^2 - 2} = \frac{(x-1)^2 - (\sqrt{3})^2}{x^2 - 2}$$

$$= \frac{(x-(1+\sqrt{3}))(x-(1-\sqrt{3}))}{(x-\sqrt{2})(x+\sqrt{2})}$$

Max, Min, Max, Min

$-\sqrt{2}$, $1-\sqrt{3}$, $\sqrt{2}$, $1+\sqrt{3}$

$\frac{4-2\sqrt{3}}{2} = 2-\sqrt{3}$

Min at $x = 1-\sqrt{3}$.

Min value: $y = (1-\sqrt{3}) - \ln(\sqrt{3}-1)$