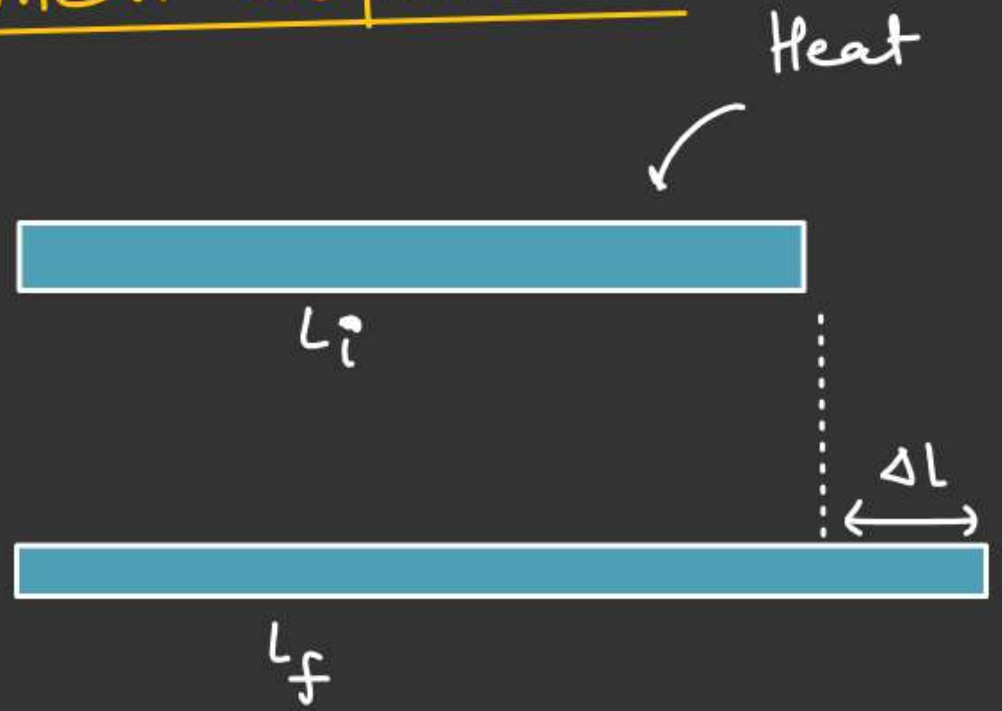
Thermal Expansion

Linear Expansion



$$\boxed{L_f = L_i(1+\alpha\Delta T)}$$

α = coffⁿ of linear expansion

$$L_f = L_i + L_i\alpha\Delta T$$

$$L_f - L_i = L_i\alpha\Delta T$$

$$\boxed{\Delta L = L_i\alpha\Delta T}$$

$$\left(\frac{\Delta L}{L_i}\right) = \alpha\Delta T$$

Fractional Change in length

$\left(\frac{\Delta L}{L_i}\times 100\right) \rightarrow$ Percentage change in length.

$$\boxed{\frac{dl}{l} = \alpha\, dT}$$

$$\Delta T = (T_f - T_i)$$

Thermal Expansion

$$\left(\frac{\Delta L}{L_i}\right) = \alpha \Delta T$$

Fractional Change in length

$$\left(\frac{\Delta L}{L_i} \times 100\right) \rightarrow$$ Percentage change in length.

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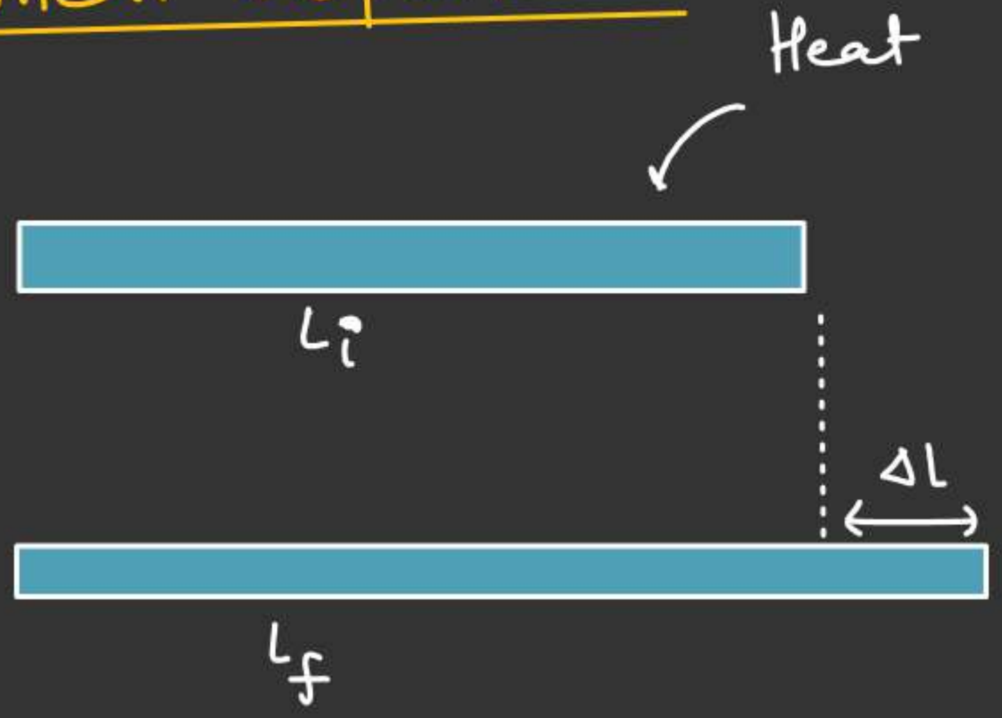
Linear Expansion



$$\boxed{L_f = L_i(1 + \alpha \Delta T)}$$

α = coff^n of linear expansion

$$L_f = L_i + L_i \alpha \Delta T$$

$$L_f - L_i = L_i \alpha \Delta T$$

$$\boxed{\Delta L = L_i \alpha \Delta T}$$

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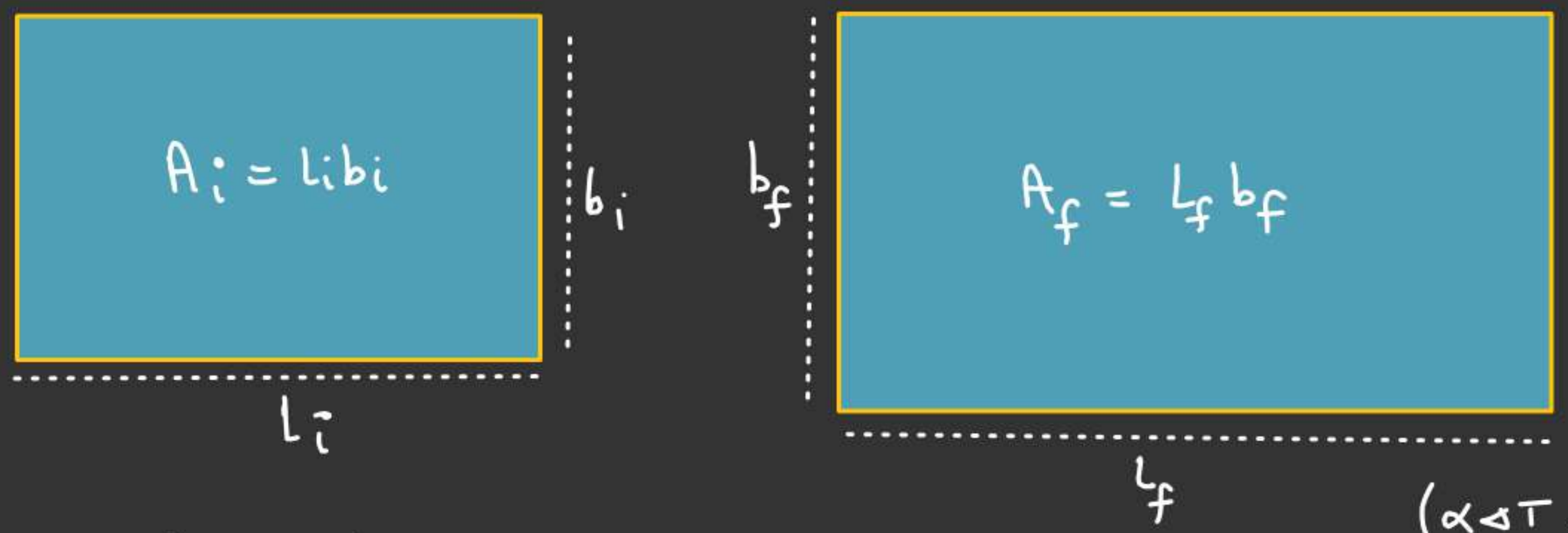
Areal Expansion (2-Dimensional)

Heat

Isotropic → (Expansion in all direction same)



$(\alpha \Delta T << 1)$

$$l_f = l_i(1+\alpha\Delta T)$$

$$b_f = b_i(1+\alpha\Delta T)$$

$$l_f b_f = l_i b_i (1+\alpha\Delta T)^2$$

$$A_f = A_i(1+\alpha\Delta T)^2$$

$$A_f = A_i(1+\underbrace{2\alpha}_{\beta}\Delta T)$$

β = coffn of Areal expansion

$$\boxed{A_f = A_i(1+\beta\Delta T)}$$

$$\boxed{\beta = 2\alpha}$$

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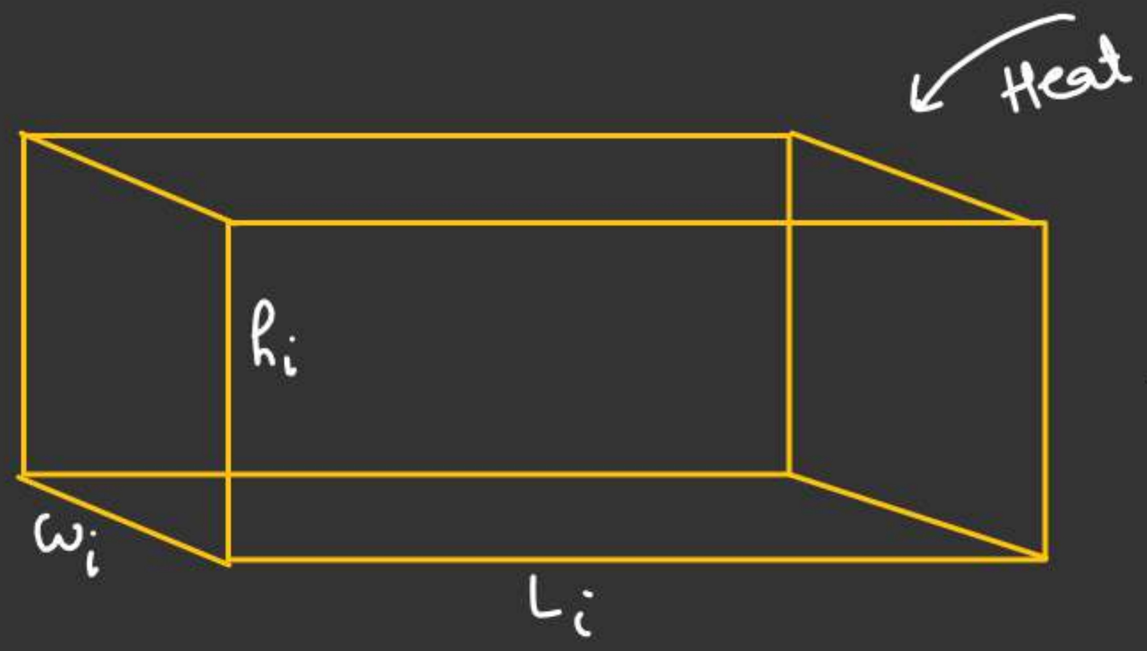
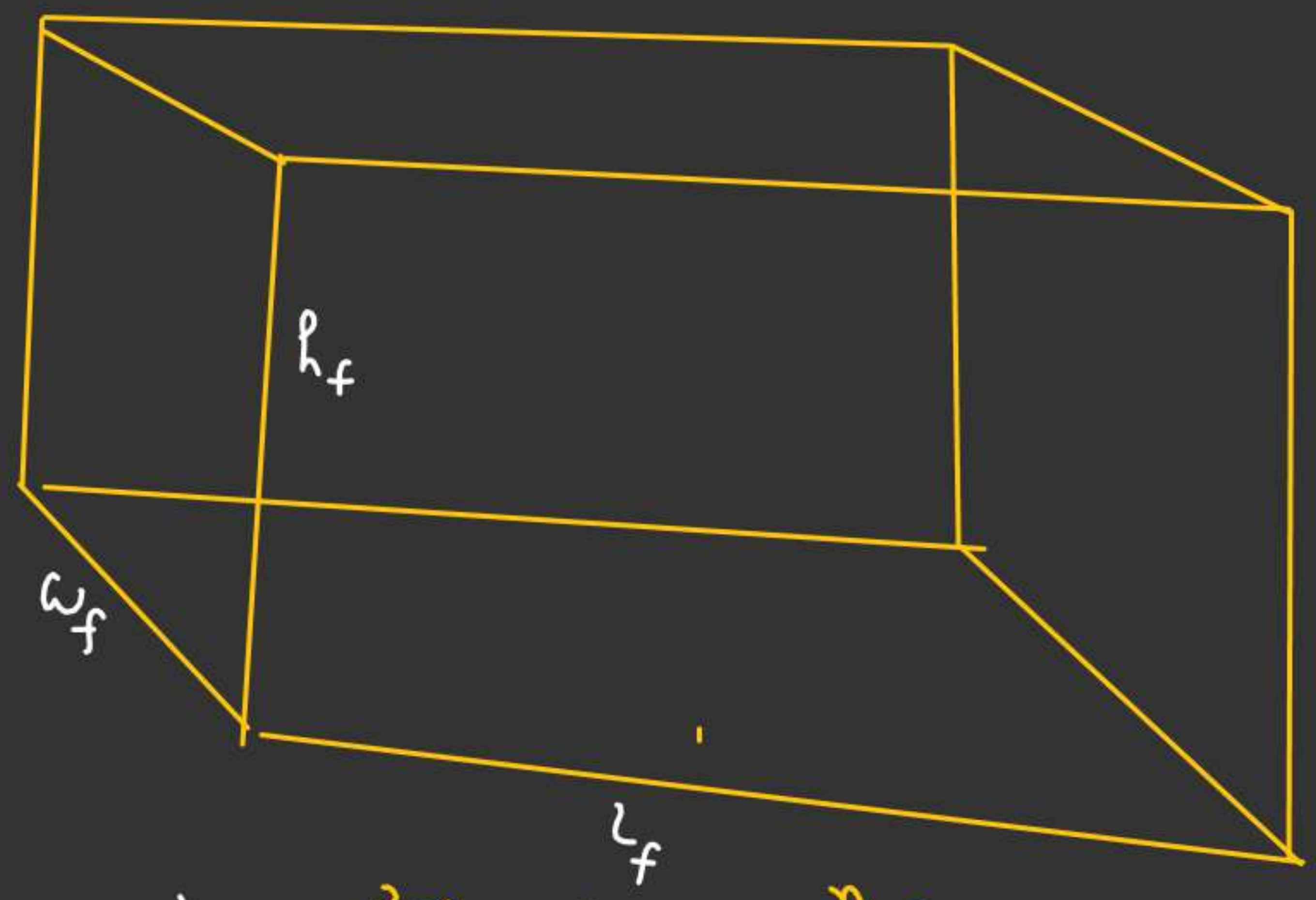
Volume Expansion (3-D expansion)





$$L_f = L_i(1+\alpha\Delta T)$$

$$h_f = h_i(1+\alpha\Delta T)$$

$$w_f = w_i(1+\alpha\Delta T)$$

$$(1 >> \alpha\Delta T)$$

$$L_f h_f w_f = L_i w_i h_i (1+\alpha\Delta T)^3$$

$$\boxed{V_f = V_i(1+3\alpha\Delta T)}$$

$3\alpha = \gamma \rightarrow$ coff^n of Volume expansion

$$\boxed{V_f = V_i(1+\gamma\Delta T)}$$

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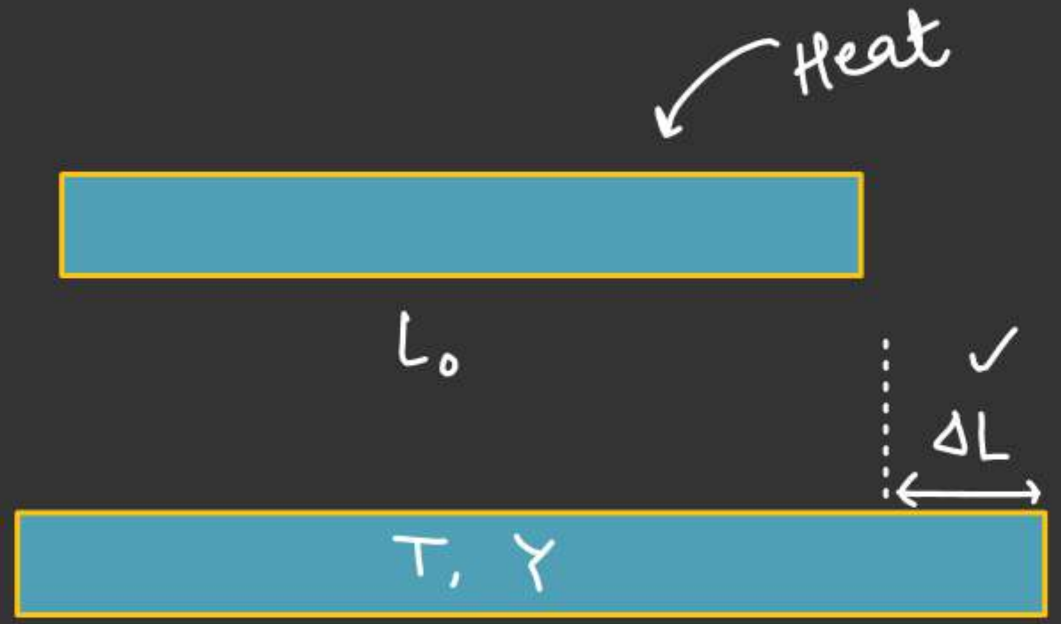
THERMAL STRESS

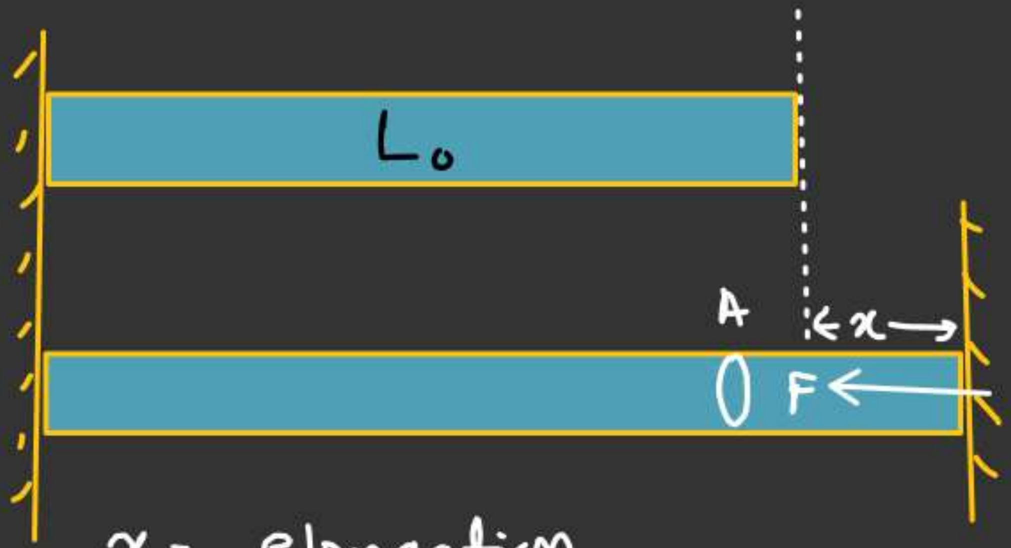
$\boxed{\dfrac{\text{Stress}}{\text{Strain}} = Y}$ Stress = F/A, Strain = $\dfrac{\Delta L}{L}$

↳ Thermal Strain = $\dfrac{\text{(Unachived length)}}{\text{(Initial length)}}$

↳ Thermal Stress = Y (thermal strain)



{ Here, rod is free to elongate so no thermal stress & thermal strain



x = elongation

ΔL = total elongation when rod is free to elongation

$(\Delta L - x)$ = unachived elongation

Thermal Strain = $\left(\dfrac{\Delta L - x}{L_0}\right)$

Thermal Stress = $Y \cdot \left(\dfrac{\Delta L - x}{L_0}\right)$

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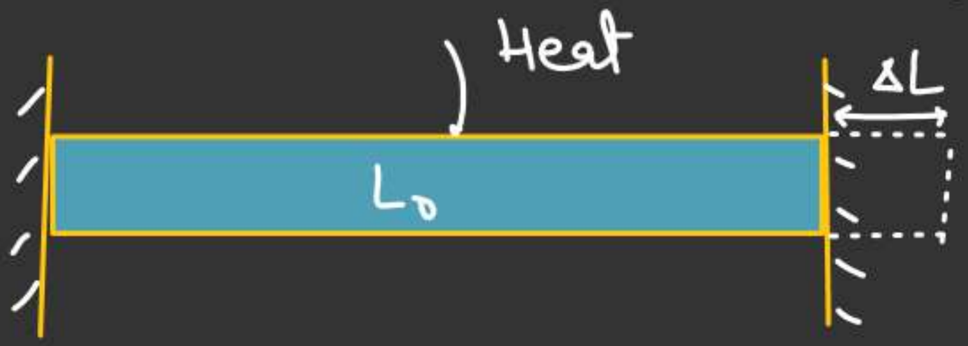
$$\text{Thermal Stress} = \left(\frac{\Delta L - x}{L_0}\right)$$

if $x = \Delta L$. $\Rightarrow$ Thermal stress $= 0$.

if $x = 0$, unachived elongation $= \Delta L$



$$\text{Thermal Strain} = \left(\frac{\Delta L}{L}\right) \checkmark$$

$$\text{Thermal Stress} = \left(Y\frac{\Delta L}{L}\right) \checkmark$$

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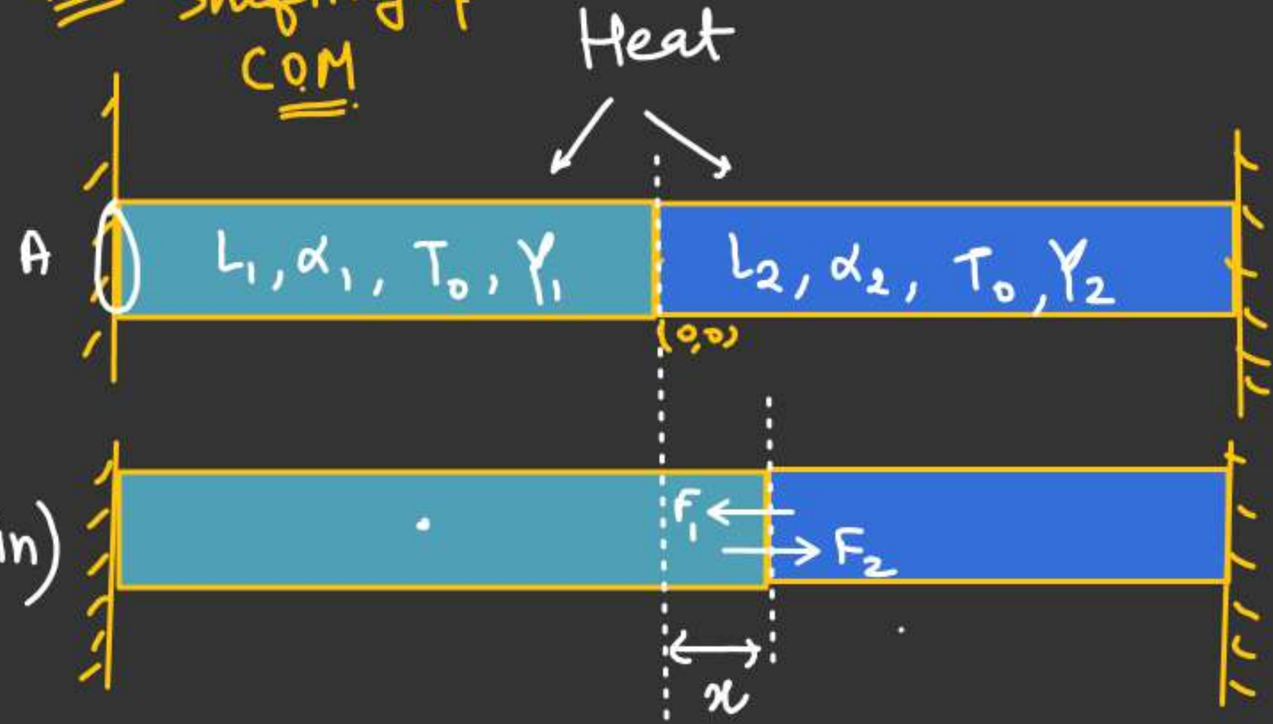
Both the rod fixed at their end with rigid support after heating find the Shift of junction

H.W. Shifting of COM



Sol^n:- Junction Shifting Stop when

$$\left(\frac{F_1}{A}\right) = \left(\frac{F_2}{A}\right)$$

Stress = Y(strain)

ΔL_1 = Total elongation in the rod-1
ΔL_2 = Total elongation in the rod-2 } $\Rightarrow$ when both free to expansion

$$Y_1 (\text{Strain})_1 = Y_2 \cdot (\text{Strain})_2$$

$$Y_1 \left(\frac{\Delta L_1 - x}{L_1}\right) = Y_2 \frac{(\Delta L_2 + x)}{L_2}$$

$$\left(Y_1\left(\frac{\Delta L_1}{L_1}\right) - \frac{Y_2 \Delta L_2}{L_2} \right) = \left(\frac{Y_2}{L_2} + \frac{Y_1}{L_1}\right) x$$

$$\frac{(Y_1\alpha_1 - Y_2\alpha_2)\Delta T}{\left(\frac{Y_2}{L_2} + \frac{Y_1}{L_1}\right)} = x$$

$$(L_1') = L_1(1 + \alpha_1 \Delta T)$$

$$(L_1' - L_1) = L_1 \alpha \Delta T$$

$$\frac{\Delta L_1}{L_1} = (\alpha_1 \Delta T)$$

$$\frac{\Delta L_2}{L_2} = \alpha_2 \Delta T$$

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$$\frac{(Y_1\alpha_1 - Y_2\alpha_2)\Delta T}{\left(\frac{Y_2}{l_2}+\frac{Y_1}{l_1}\right)} = x$$

if $Y_1\alpha_1 > Y_2\alpha_2 \Rightarrow$ Junction shifting right side

if $Y_1\alpha_1 = Y_2\alpha_2 \Rightarrow x = 0$ No junction shifting.

if $Y_1\alpha_1 < Y_2\alpha_2 \Rightarrow$ Junction shifting in left direction.

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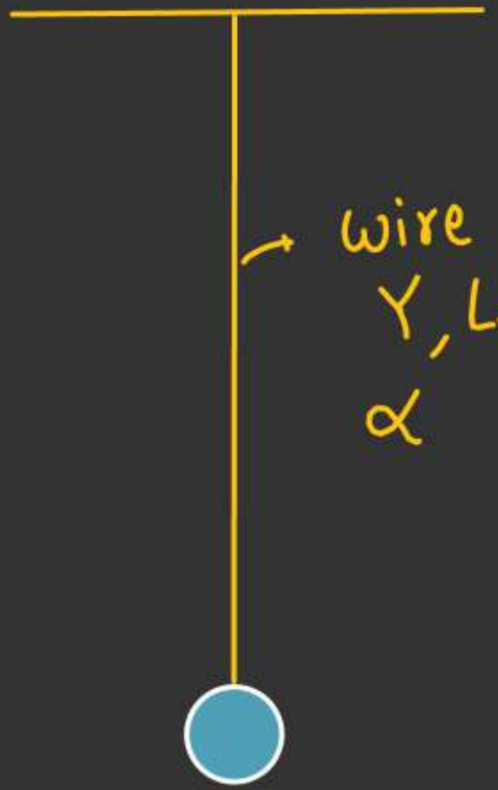
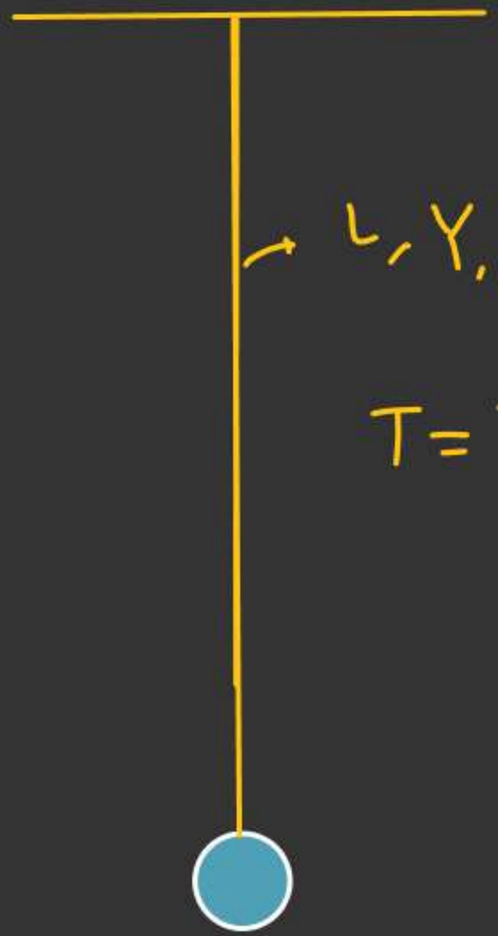
$\Delta\theta$:

$$T = 2\pi\sqrt{L/g}$$



T_0 = Time period at θ_0°C



T = Time period at θ°C

$$L = L_0(1+\alpha\Delta\theta)$$

$$T = 2\pi\sqrt{\frac{L}{g}}$$

$$T = 2\pi\sqrt{\frac{L_0(1+\alpha\Delta\theta)}{g}}$$

$$T = \underbrace{2\pi\sqrt{\frac{L_0}{g}}}\,(1+\alpha\Delta\theta)^{1/2}$$

$$T = T_0(1+\alpha\Delta\theta)^{1/2}$$

$$T = T_0\left(1+\frac{\alpha\Delta\theta}{2}\right)$$

$1 >> \alpha\Delta\theta$

$$\frac{\Delta T}{T_0} = \frac{T-T_0}{T_0} = \frac{\alpha\Delta\theta}{2}$$

$$\left(\frac{\Delta T}{T_0}\right) = \left(\frac{\alpha\Delta\theta}{2}\right)$$

$\Downarrow$ Fraction change in time period

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$$\frac{\Delta t}{t} = \left(\frac{\Delta T}{T}\right) = \left(\frac{\alpha \Delta\theta}{2}\right)$$

$$\boxed{\Delta t = \frac{\alpha \Delta\theta}{2} \times \underbrace{t}_{\text{(Total time)}}}$$