Nishant Jindal

WAVE

- **Mechanical Wave** (Needs medium for propagation)
  - **Transverse wave** (Medium particles oscillates perpendicular to wave propagation)
  - **Longitudinal wave** (Medium particle oscillate along the direction of wave propagation)
- **E.M wave** (Doesn't required Medium for propagation)

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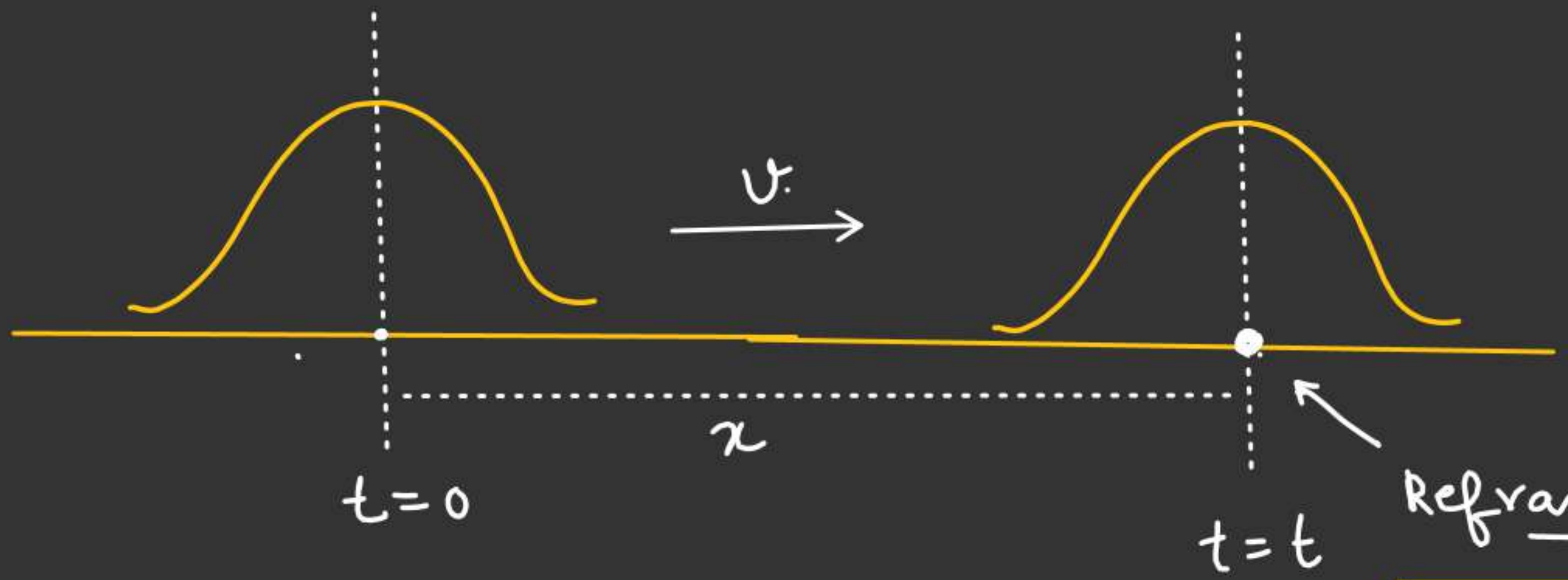
General Equation of travelling wave

$y = f(x)$

$y = f(x-a)$ $\qquad$ $y = f(x+a)$



Refrance point

Equation of travelling wave in +ve x-direction

$$\boxed{y = f\left(t - \frac{x}{v}\right)}$$

$$\boxed{y = g(vt - x)}$$

$y = f(t)$

$$\boxed{\begin{aligned} y &= f\left(t + \frac{x}{v}\right) \\ y &= g(vt + x) \end{aligned}}$$

Equation of travelling wave in −ve x-direction

$y = f\left(\frac{vt - x}{v}\right)$

$y = g(vt - x)$

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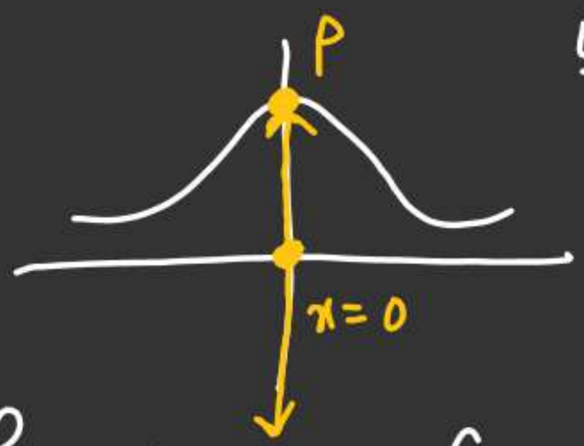
$$y = f\left(t - \frac{x}{v}\right) \rightarrow$$
$$y = g(vt - x) \rightarrow$$

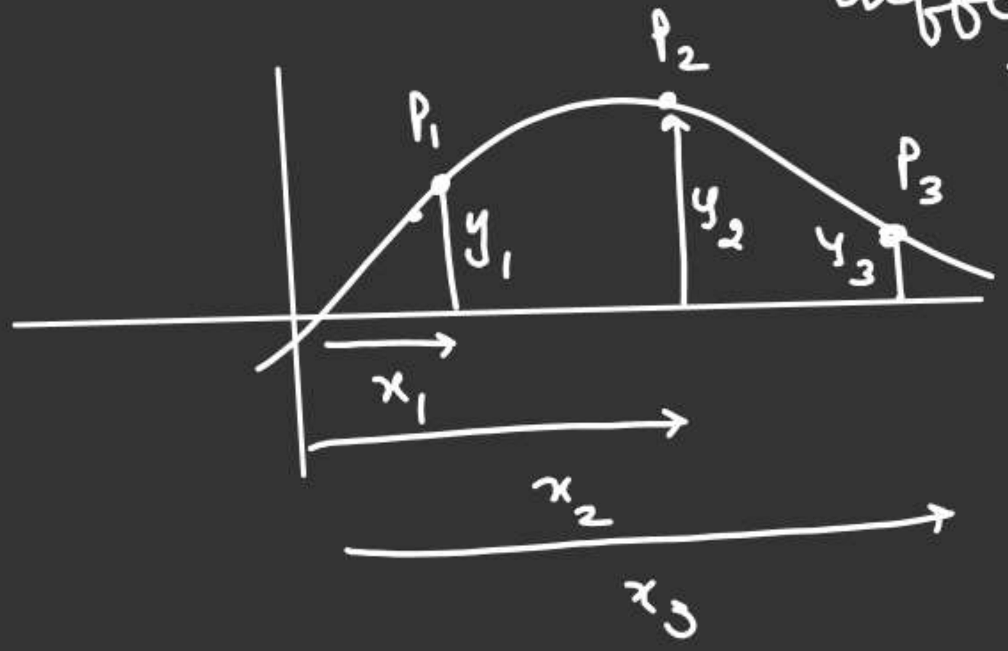
$\Rightarrow$ if x is fixed let $x = 0$, $\underline{y = f(t)}$



$\hookrightarrow$ gives information of particular particle

if $\underline{t \text{ is fixed}}$, we can locate the position of different particles

i.e it gives shape of wave pulse.



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(Which on represent travelling wave Equation)

$$y = Ae^{\left(t - \frac{x}{v}\right)^2} \checkmark$$

$$y = e^{\left(t^2 - x^2/v^2\right)} \times$$

$$y = A\sin\left(t - x/v\right) \checkmark$$

$$y = A\sin^2\left(t - x/v\right) \checkmark$$

$$y = A\sin\left(x^2 - v^2t^2\right) \times$$

WAVE

General Equation of travelling wave

$$y = f\left(t - \frac{x}{v}\right)$$

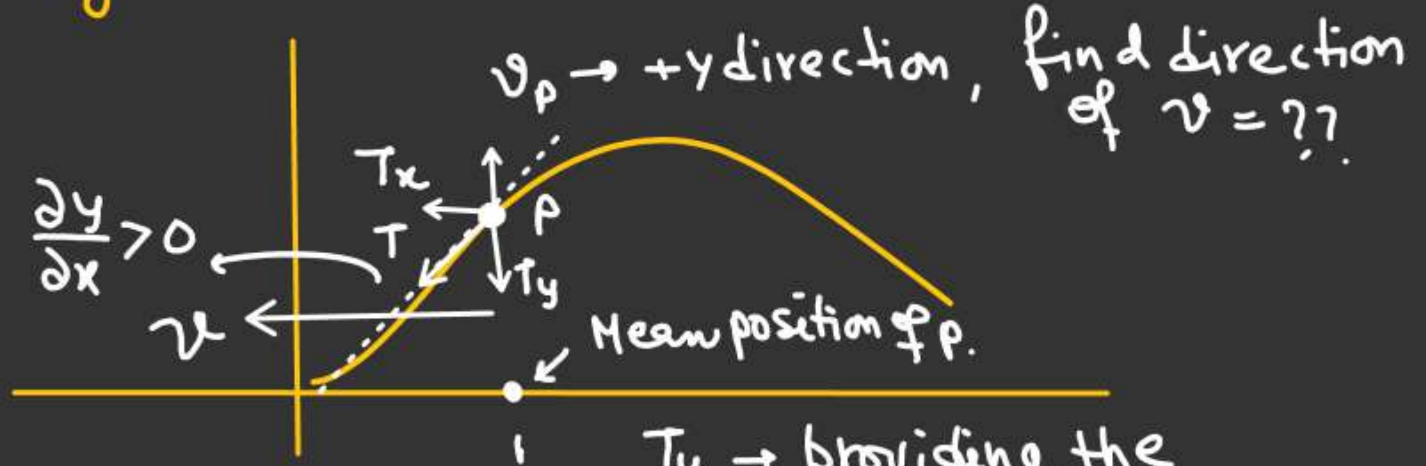
$$\frac{\partial y}{\partial t} = f'\left(t - \frac{x}{v}\right)\frac{\partial (t)}{\partial t}$$

$$\frac{\partial y}{\partial t} = f'\left(t - \frac{x}{v}\right) \quad \text{--- (1)}$$

$$\frac{\partial y}{\partial x} = f'\left(t - \frac{x}{v}\right)\left(-\frac{1}{v}\right)$$

$$\frac{\partial y}{\partial x} = -\frac{1}{v} f'\left(t - \frac{x}{v}\right) \quad \text{--- (2)}$$

$$\boxed{\underbrace{\frac{\partial y}{\partial x}}_{\text{Slope of tangent on the wave pulse}} = -\underbrace{\frac{1}{v}}_{\text{(wave propagation) velocity}}\underbrace{\left(\frac{\partial y}{\partial t}\right)}_{\text{(velocity of particle)}}}$$



$T_y \to$ providing the restoring force

$\frac{\partial y}{\partial t} = v_p > 0$, $\frac{\partial y}{\partial x} > 0$,

from Equation

$$\underbrace{\left(\frac{\partial y}{\partial x}\right)}_{+ve} = -\frac{1}{v}\underbrace{\left(\frac{\partial y}{\partial t}\right)}_{+ve} \Rightarrow v < 0$$

WAVE

General Equation of travelling wave

$$y = f\left(t - \frac{x}{v}\right)$$

$$\frac{\partial y}{\partial t} = f'\left(t - \frac{x}{v}\right)\frac{\partial (t)}{\partial t}$$

$$\frac{\partial y}{\partial t} = f'\left(t - \frac{x}{v}\right)$$

Again differentiating w.r.t time

$$\frac{\partial^2 y}{\partial t^2} = f''\left(t - \frac{x}{v}\right) \quad -(1)$$

$$\frac{\partial y}{\partial x} = f'\left(t - \frac{x}{v}\right)\left(-\frac{1}{v}\right)$$

$$\frac{\partial y}{\partial x} = -\frac{1}{v} f'\left(t - \frac{x}{v}\right)$$

Again differentiating w.r.t x

$$\frac{\partial^2 y}{\partial x^2} = -\frac{1}{v} f''\left(t - \frac{x}{v}\right)\left(-\frac{1}{v}\right)$$

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2} f''\left(t - \frac{x}{v}\right) \quad -(2)$$

From (1) & (2)

$$\boxed{\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\left(\frac{\partial^2 y}{\partial t^2}\right)}$$

$\downarrow$

General differential Equation of a travelling wave

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Sine wave travelling in a String (Transverse wave)

(Displacement of particle)

$$\boxed{y = A\sin(\omega t - Kx)} \quad -(1)$$

or

$$\boxed{y = A\sin(Kx - \omega t)}$$

→ Travelling in +ve x-direction.

Travelling in +ve x-direction

$K = \frac{\omega}{v}$, $\left(K = \frac{2\pi}{\lambda}\right)$

(Wave No)

$$y = -A\sin(\omega t - Kx)$$

$$y = A\sin(\omega t - Kx + \pi) \quad -(2)$$

$\Delta\phi = \pi$ b/w Eq^n (1) & (2)

$$y = A\sin\left(\frac{2\pi}{T}t - \frac{2\pi}{\lambda}x\right)$$

$$\boxed{y = A\sin 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right)}$$

$$y = A\sin\omega\left(t - \frac{k}{\omega}x\right)$$

$$\boxed{y = A\sin\omega\left(t - \frac{x}{v}\right)}$$

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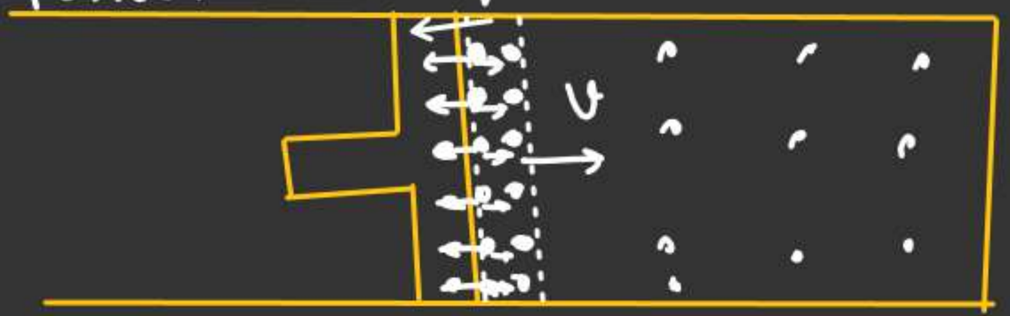
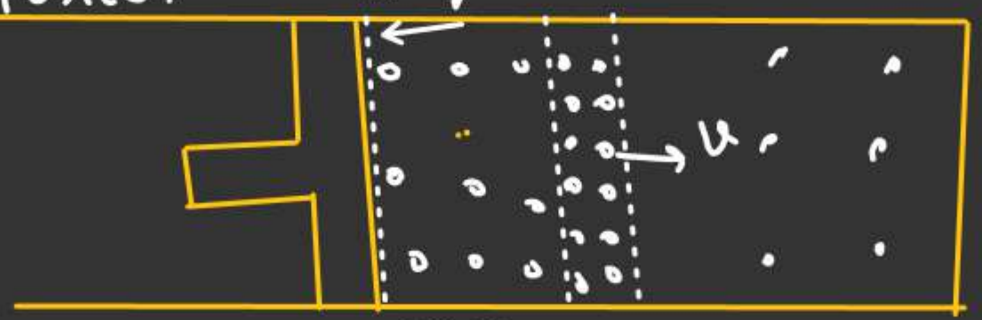
Equation of Longitudinal wave

→ In Longitudinal wave disturbance propagate in the medium in the form of Compression and rarefraction zone.

Restoring force → PA / $P_0 + P$ → P → excess pressure.



Restoring force → PA /



Rarefraction zone → Displacement of particle maximum from their mean position or pressure minimum

$$\boxed{S = S_0 \sin(\omega t - Kx)}$$

S → Displacement of Medium particle

S_0 → Maximum displacement of Medium particle.

$$P = P_0 \sin\left(\omega t - kx + \frac{\pi}{2}\right)$$

$$\boxed{P = P_0 \cos(\omega t - Kx)}$$

Phase defference of $\frac{\pi}{2}$

P → Excess Pressure

P_0 → Excess pressure Amplitude

$$\boxed{P_0 = BKS_0}$$

B → Bulk Modulus

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★★ Wave propagation velocity in transverse wave

$$\boxed{v = \sqrt{\frac{T}{\mu}}}$$

T = Tension in the String

μ = linear mass density of the string

$\left(\mu = \frac{m}{l}\right)$

$T \rightarrow$ Tension represent elastic property

$\mu \rightarrow$ Represent inertial property

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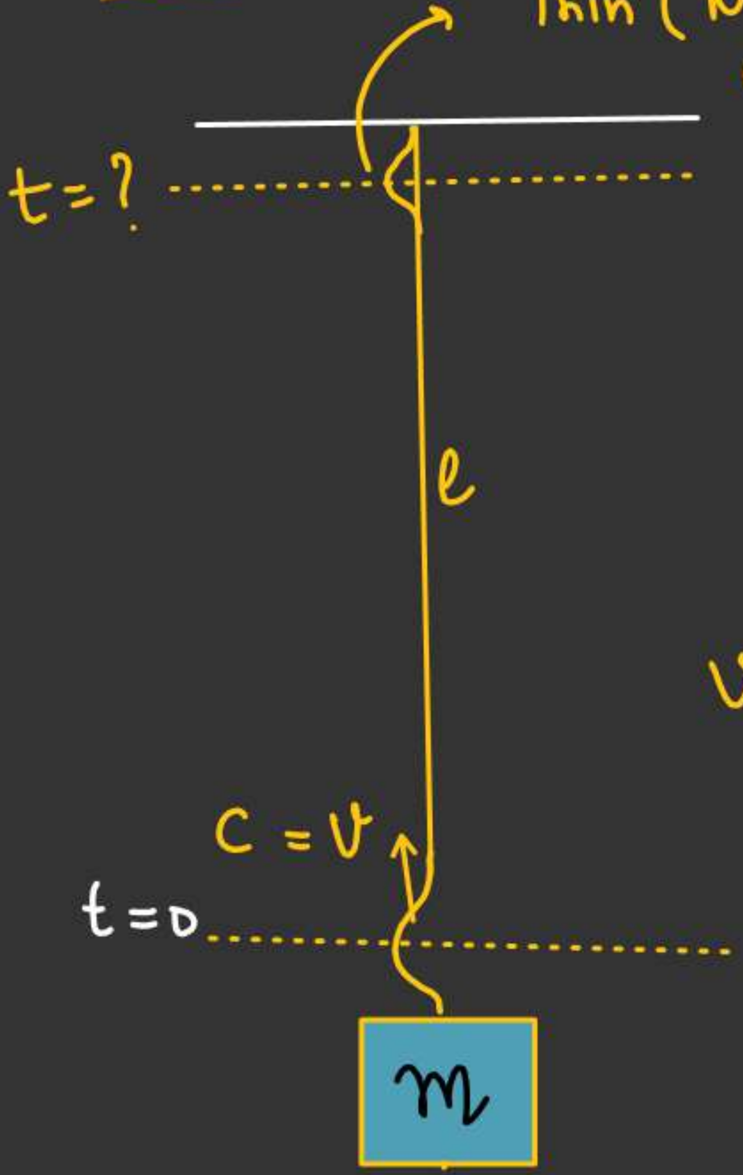
Case-1
Thin (No tension due to self weight)



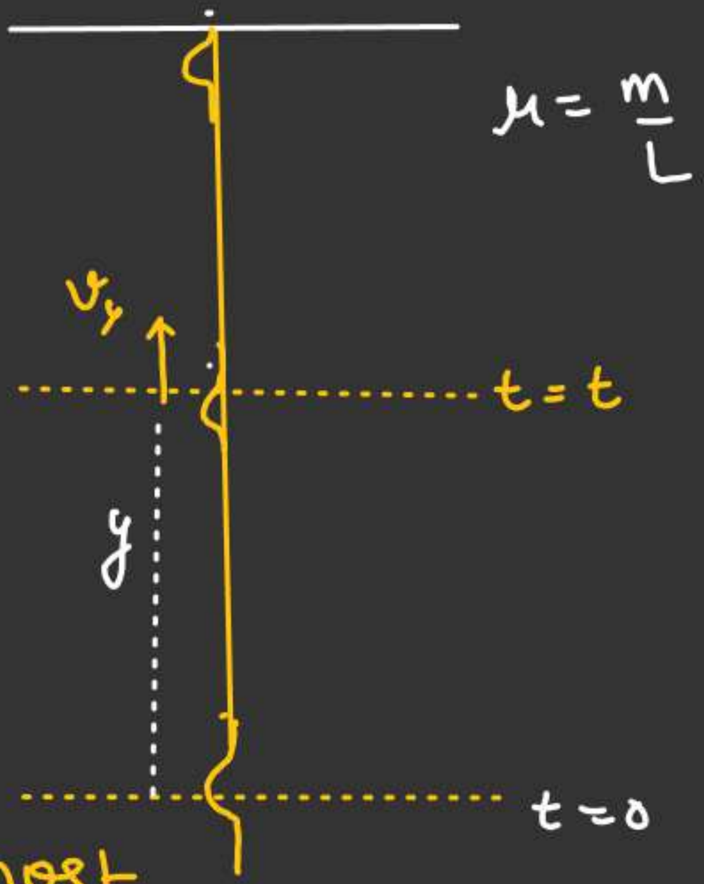
$$l = vt$$

$$t = \frac{l}{v}$$

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{Mg}{\mu}}$$

$$t = l\sqrt{\frac{\mu}{Mg}}$$

Case-2
Thick Rope (uniform)



$$v_y = \sqrt{\frac{T_y}{\mu}}$$

$$T_y = \mu y g$$

$$v_y = \sqrt{\frac{\mu y g}{\mu}}$$

$$v_y = \sqrt{yg}$$

Time to reach the wave pulse at top most point

$$\frac{dy}{dt} = \sqrt{yg}$$

$$\int_0^l \frac{dy}{\sqrt{y}} = \sqrt{g}\int_0^T dt$$

$$2\left[\sqrt{y}\right]_0^l = \sqrt{g}\,T$$

$$2\sqrt{\frac{l}{g}} = T$$