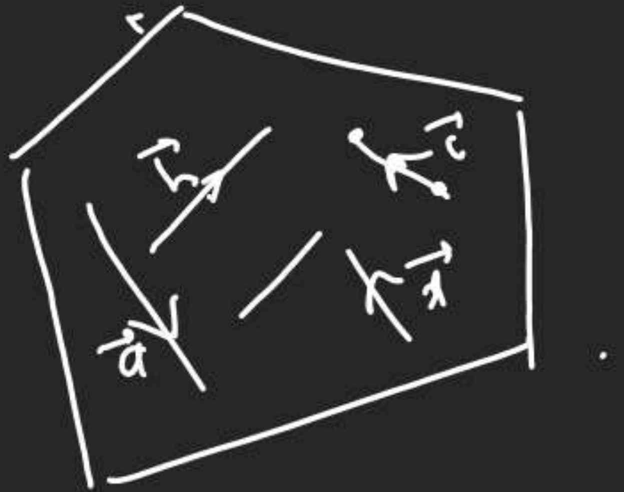
Nishant Jindal

Coplanar Vectors.

① Vectors are coplanar if their line segment are $\parallel^{re}$ to same plane.



(2) any 2 vectors in space are always coplanar.

(3) 3 vectors may or may not be coplanar.

(4) $\vec{a}, \vec{b}, \vec{c}, \vec{d}$ lying in same plane then for unit vector $(\hat{n})$ $\perp^r$ to plane will have Relation
$\hat{n}\cdot\vec{a} = n\cdot\vec{b} = \hat{n}\cdot\vec{c} = \hat{n}\cdot\vec{d} = 0$
as $\hat{n}$ is $\perp^r$ to all of them.

(5) Colinear Vectors are always coplanar.

(6) coplanar Vector are also known as Linearly Dependent (बाद में पढ़ो) vectors

(7) Collinear Vector = Coplanar = L.D.

(8) If $\vec{a}, \vec{b}, \vec{c}$ are coplanar vectors then any vector can be Represented as Linear combination of other two.

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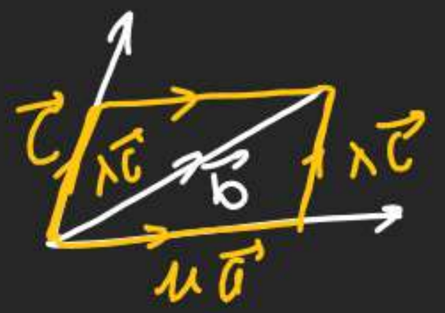
$\vec{c} = \frac{2\vec{a}+3\vec{b}}{5}$

$2\vec{a}+3\vec{b}-5\vec{c}=0$



$\vec{b} = \lambda\vec{c}+\mu\vec{a}$

$\vec{b}$ is Rep. as L.C. of $\vec{a}$ & $\vec{c}$.

(9) $\vec{a} = a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$

$\vec{b} = b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$

$\vec{c} = c_1\hat{i}+c_2\hat{j}+c_3\hat{k}$

$b_1\hat{i}+b_2\hat{j}+b_3\hat{k} = \lambda c_1\hat{i}+\lambda c_2\hat{j}+\lambda c_3\hat{k}+\mu a_1\hat{i}+\mu a_2\hat{j}+\mu a_3\hat{k}$

$b_1 = \lambda c_1+\mu a_1 \mid b_2 = \lambda c_2+\mu a_2 \mid b_3 = \lambda c_3+\mu a_3$

By Solving this we can get λ, μ

Confirm value of λ & μ in 3^{rd} eq^n

if value of λ & μ is satisfying 3^{rd} ① neither $\vec{a}, \vec{b}, \vec{c}$ are Coplanar Vector

(10) A, B, C, D having p.v. $\vec{a}, \vec{b}, \vec{c}, \vec{d}$ will be coplanar.

$\vec{AB} = \vec{b}-\vec{a}$

$\vec{BC} = \vec{c}-\vec{b}$

$\vec{CD} = \vec{d}-\vec{c}$

$\vec{AB}, \vec{BC}, \vec{CD}$ are in same plane

$b-a = l(\vec{c}-\vec{b})+m(\vec{d}-\vec{c})$

$l(\vec{c}-\vec{b})+m(\vec{d}-\vec{c})-(\vec{b}-\vec{a})=0$

$1\cdot\vec{a}+\vec{b}(-l-1)+\vec{c}(l-m)+m\vec{d}=0$

$x\vec{a}+y\vec{b}+z\vec{c}+t\vec{d}=0$

If $\vec{a}, \vec{b}, \vec{c}, \vec{d}$ lie in same plane then $x\vec{a}+y\vec{b}+z\vec{c}+t\vec{d}=0$ & $x+y+z+t=0$

(11)** $\vec{a}, \vec{b}, \vec{c}$ are coplanar then $\left|\quad\right| = 0$

$\cancel{1}+\cancel{-l}-\cancel{1}+\cancel{l}-\cancel{m}+\cancel{m}=0$

$\Rightarrow 0=0$

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$\quad\quad A \quad\quad\quad\quad\quad B \quad\quad\quad\quad C$

Q $-2a+3b+5c,\ a+2b+3c,\ 7a-c$

are collinear or not?

$\vec{AB} = \vec{B} - \vec{A}$

$= 3\vec{a} - \vec{b} - 2\vec{c}$

$\vec{BC} = \vec{C} - \vec{B}$

$= 6\vec{a} - 2\vec{b} - 4\vec{c}$

$2\vec{AB} = \vec{BC}$

$\lambda \vec{AB} = \vec{BC} \rightarrow \vec{AB}$ colline to $\vec{BC}$

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Q If $\vec{a}, \vec{b}, \vec{c}$ are noncoplanar vector.

$3\vec{a}-7\vec{b}-4\vec{c}$, $3\vec{a}-2\vec{b}+\vec{c}$

$\vec{a}+\vec{b}+2\vec{c}$ are coplanar?

$$\Delta = \begin{vmatrix} 3 & -7 & -4 \\ 3 & -2 & 1 \\ 1 & 1 & 2 \end{vmatrix}\begin{matrix} 3 & -7 \\ 3 & -2 \\ 1 & 1 \end{matrix}$$

$$= (-12-7-12)-(8+3-42)$$

$$= -42+42$$

$$= 0$$

Q a, b, c non coplanar then P.T.

4 Pts $\overset{A}{6\vec{a}+2\vec{b}-\vec{c}}$, $\overset{B}{2\vec{a}-\vec{b}+3\vec{c}}$

$\overset{C}{-\vec{a}+2\vec{b}-4\vec{c}}$, $\overset{D}{-12\vec{a}-\vec{b}-3\vec{c}}$ are coplanar.?

$\vec{AB} = -4a-3b+4\vec{c}$

$\vec{BC} = -3a+3b-7c$

$\vec{CD} = 11a-3\vec{b}+\vec{c}$

$$\begin{array}{r} 21 \\ 21 \\ \hline 231 \end{array}$$

$$\Delta = \begin{vmatrix} -4 & -3 & 4 \\ -3 & 3 & -7 \\ -11 & -3 & 1 \end{vmatrix}\begin{matrix} -4 & -3 \\ -3 & 3 \\ -11 & -3 \end{matrix}$$

$$(-12-231+36)-(-132-84+9)$$

$$-207+207 = 0$$

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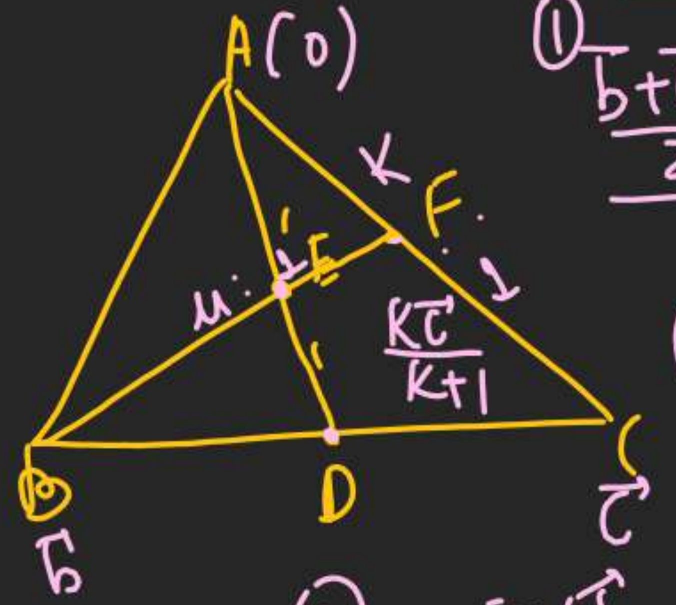
Q Median of $\triangle ABC$ from A gets Bisected at E & BE meets AC at F find AF : AC.



① $\dfrac{\frac{\vec{b}+\vec{c}}{2}+\frac{\vec{0}}{2}}{2}=\dfrac{\vec{b}+\vec{c}}{4}$

② $\dfrac{\vec{AF}}{\vec{FC}}=\dfrac{K}{1}$

③ F $\frac{K\vec{c}}{K+1}$

$\mu : 1$

E

B $(\vec{b})$

$\dfrac{\frac{\mu K\vec{c}}{K+1}+\vec{b}}{\mu+1}=\dfrac{\vec{b}+\vec{c}}{4}$

⑤ $\dfrac{\vec{b}}{\mu+1}=\dfrac{\vec{b}}{4}$

$\mu+1=4$

Ⓒ $\dfrac{\mu K}{(K+1)(\mu+1)}=\dfrac{1}{4}$

$\dfrac{3K}{(K+1)\times\cancel{4}}=\dfrac{1}{\cancel{4}}$

$3K=K+1$

$2K=1$

$K=1/2$

$\dfrac{\vec{AF}}{\vec{FC}}=\dfrac{1}{2}$

A

1

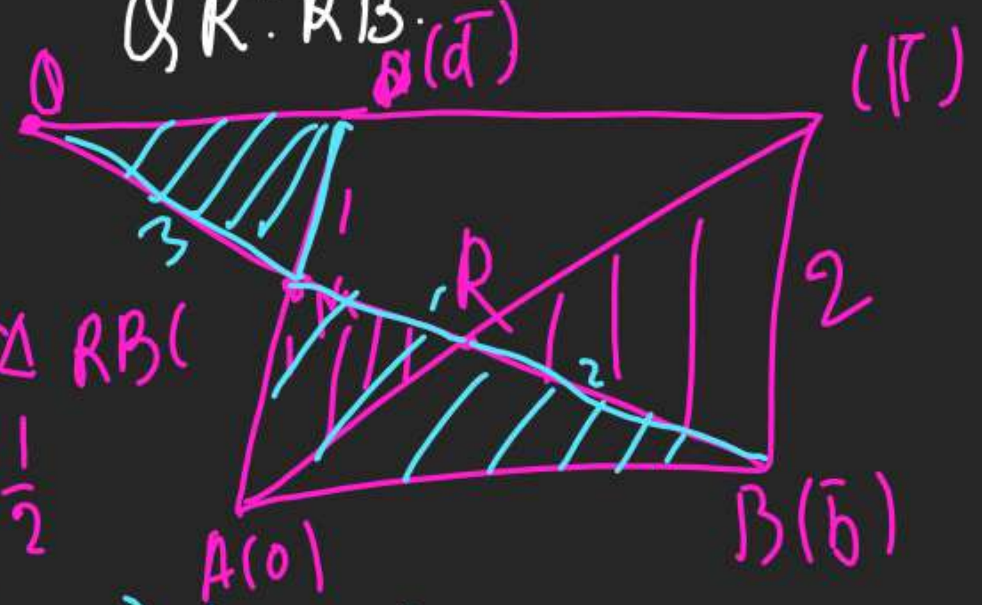
F

2

C

$\dfrac{AF}{AC}=\dfrac{1}{3}$

Q Consider a llgm ABCD. M is midPt of AD. BM when extended to meets CD at Q. AC Intersects BQ at R. Find QR : RB.



① $\triangle MRA \sim \triangle RBC$

$\dfrac{MA}{RB}=\dfrac{1}{2}$

② $\dfrac{QM}{MB}=\dfrac{3}{3}=\dfrac{1}{1}$ $\triangle QMD \sim \triangle MBA$

③ $\dfrac{QR}{RB}=\dfrac{QM+MR}{RB}=\dfrac{3+1}{2}=\dfrac{2}{1}$

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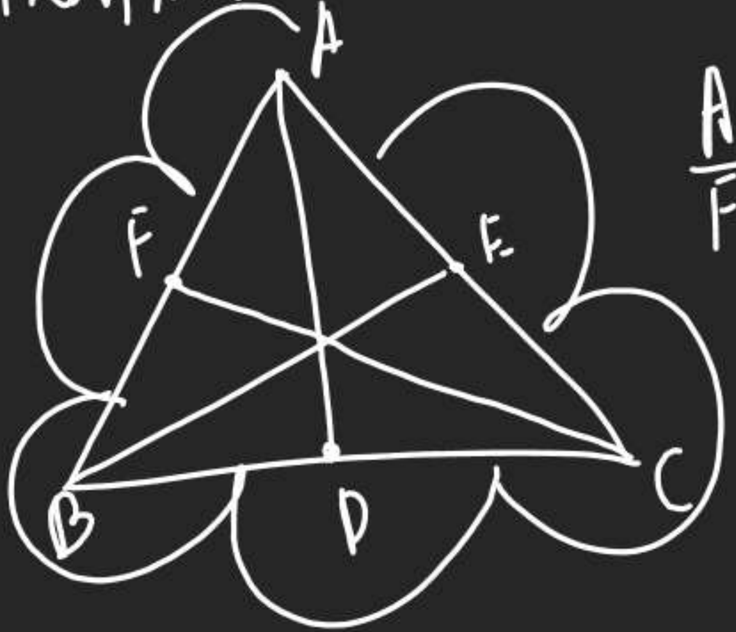
CEVA's Theorem

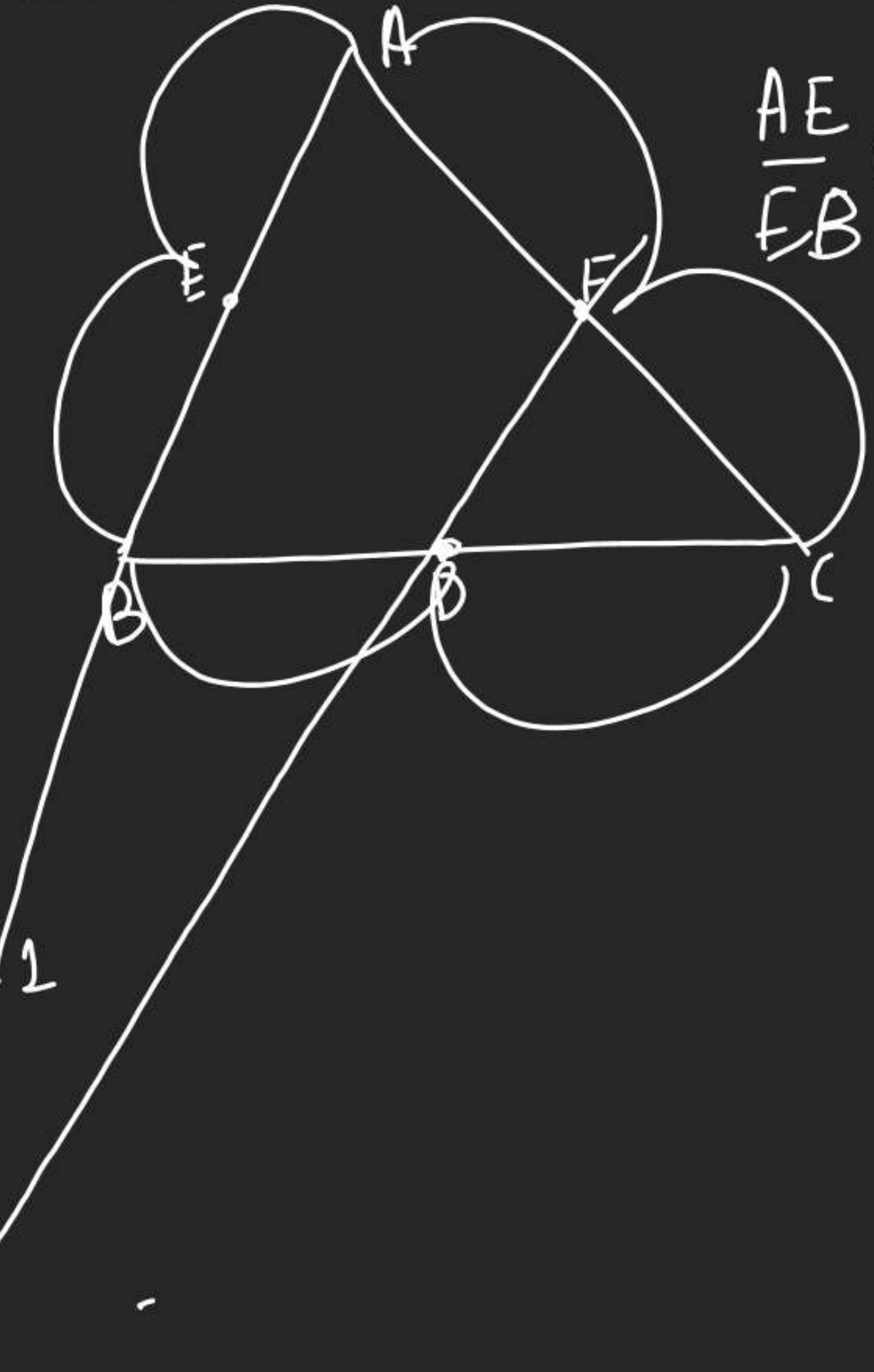
CEVIAN LINES.

1) Given in $\triangle ABC$ let lines AG, BG, CG are drawn from Vertices to a com. pt. G to meet opposite sides at F, D & E then acc. to CEVA's Thm



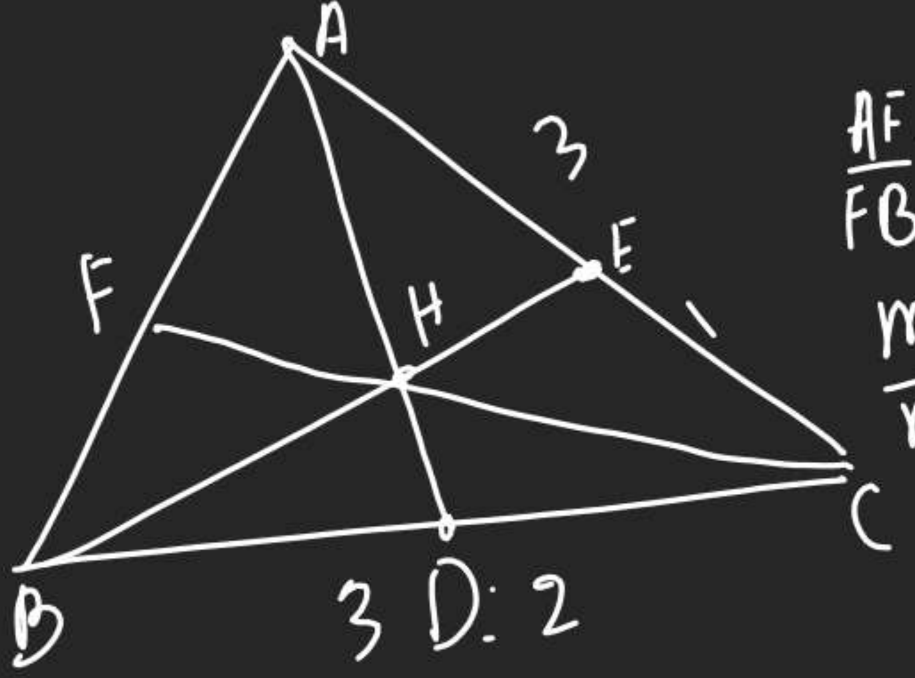
$$\frac{AF}{FB} \times \frac{BD}{DC} \times \frac{CE}{EA} = 1$$

MENALAUS Thm



$$\frac{AE}{EB} \times \frac{BD}{DC} \times \frac{CF}{FA} = -1$$

Q In a △ABC D divides BC in Ratio 3:2, E divides AC in 3:1. Line AD & BE meets at H. CH Meets AB at F. Find Ratio in which F divides AB.



$$\frac{AF}{FB} \times \frac{BD}{DC} \times \frac{CE}{EA} = 1$$

$$\frac{m}{n} \times \frac{\cancel{3}}{2} \times \frac{1}{\cancel{3}} = 1$$

$$\frac{m}{n} = \frac{2}{1}$$

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Linear Combination of Vectors

1) Rem $2\vec{a}+3\vec{b}-5\vec{c}=0$
is L.C. of $\vec{a}, \vec{b}, \vec{c}$
Where $2, 3, -5$ are Scalars.

(2) If $\vec{a}_1, \vec{a}_2, \vec{a}_3 - - - \vec{a}_n$ are n vectors.
& $\lambda_1, \lambda_2, \lambda_3 - - - \lambda_n$ are n Scalars.
then $\lambda_1 a_1 + \lambda_2 a_2 + \lambda_3 a_3 - - + \lambda_n a_n$ is
L.C. of vectors.

(3) $\vec{c} = x\vec{a} + y\vec{b} + z\vec{d}$
here $\vec{c}$ is in L.C. of $\vec{a}$, $\vec{b}$ & $\vec{d}$

Imp. If Linear Combination $= 0$

- Linearly Independent vector
- Linearly Dep. Vecto.

Linearly Dependent Vectors

A) If $\lambda_1\vec{a}_1 + \lambda_2\vec{a}_2 + \lambda_3\vec{a}_3 + - - + \lambda_n\vec{a}_n = 0$
& $\lambda_1 + \lambda_2 + \lambda_3 + \cdot\cdot - + \lambda_n = 0$
then $\vec{a}_1, \vec{a}_2, \vec{a}_3 - - - \vec{a}_n$ are L.D. vectors.

(B) L.D. = Collinear // Coplanar.

Linearly Indep. Vectors.

If $\lambda_1\vec{a}_1 + \lambda_2\vec{a}_2 + - - + \lambda_n\vec{a}_n = 0$ & $\lambda_1 = \lambda_2 = \lambda_3 = \sim = \lambda_n = 0$
then $\vec{a}_1, \vec{a}_2 - - a_n$ are L.I vectors

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Q $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = 4\hat{i} + 3\hat{j} + 4\hat{k}$

$\vec{c} = \hat{i} + \alpha\hat{j} + \beta\hat{k}$ are L. D. Vectors.

& $|\vec{c}| = \sqrt{3}$ find α, β.

Coplanar.

DPP-1
प्रश्न

1 $\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 4 & 3 & 4 \\ 1 & \alpha & \beta \end{vmatrix} = 0$

$(3\beta + 4 + \cancel{4\alpha}) - (3 + \cancel{4\alpha} + 4\beta) = 0$

$-\beta + 1 = 0 \Rightarrow \boxed{\beta = 1}$

2 $|c| = \sqrt{3}$

$\sqrt{1 + \alpha^2 + \beta^2} = \sqrt{3}$

$\alpha^2 + 1^2 + 1 = 3$

$\alpha^2 = 1 \Rightarrow \alpha = \pm 1$

$(1, 1)$ या $(-1, 1)$