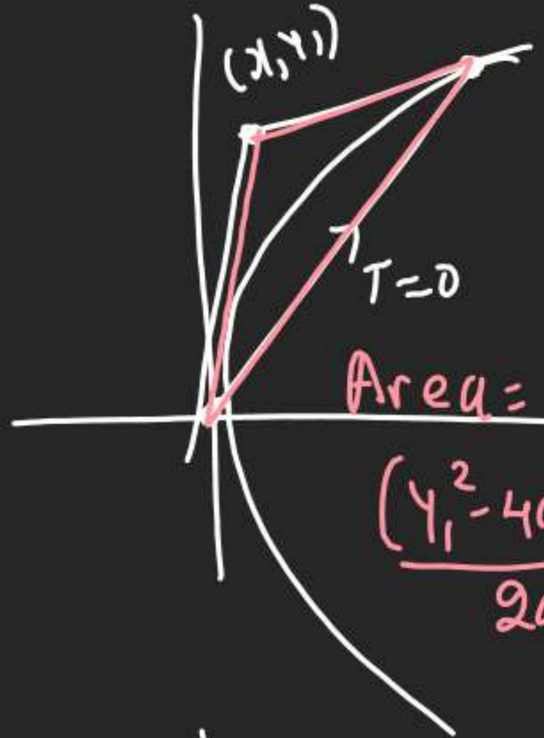
Nishant Jindal

COC = Chord of Contact

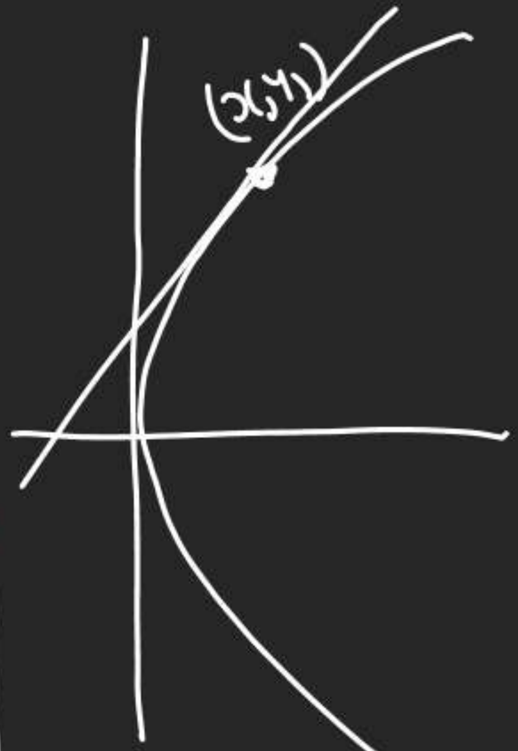
$\downarrow$

T=0  3 pts to Rem.

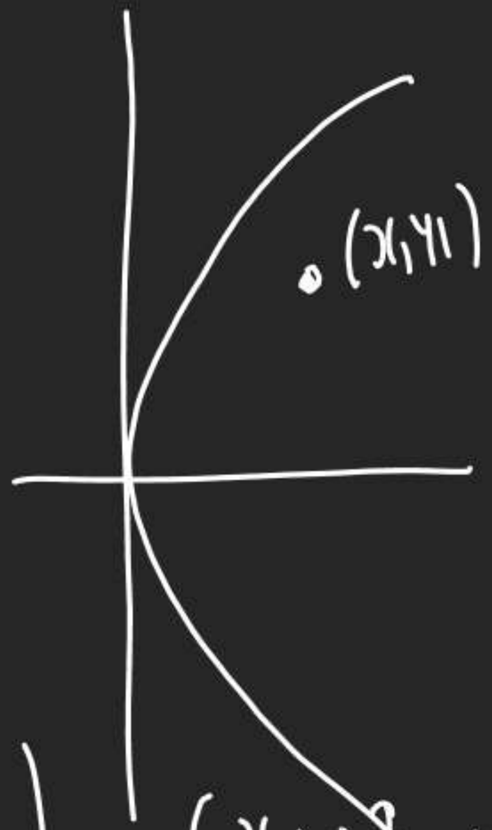



Area = $\dfrac{(y_1^2-4ax_1)^{3/2}}{2a}$

When Pt. outside Parabola then T=0 Rep. COC.




When Pt. $(x_1,y_1)$ comes on Parabola T=0 is EO




$(x_1,y_1)$ Inside Parabola then T=0 is Eqn of PoLaR.

T=0 mean change

$$\boxed{\begin{aligned} x^2 &\to xx_1 \\ y^2 &\to yy_1 \\ 2x &\to x+x_1 \\ 2y &\to y+y_1 \end{aligned}}$$

$$xy \to \frac{xy_1+yx_1}{2}$$

$$\frac{2}{4} = \frac{y_1}{1} = \frac{-2x_1}{9}$$

$$(x_1,y_1) \equiv \left(-\frac{9}{4}, \frac{1}{2}\right)$$

Q A Line $l: 4x+y=9$ Intersects $y^2=-4x$ at Pts A&B find PoI of tangents at A&B.


① For P AB Line is COC

$AB \to yy_1 = -2(x+x_1)$

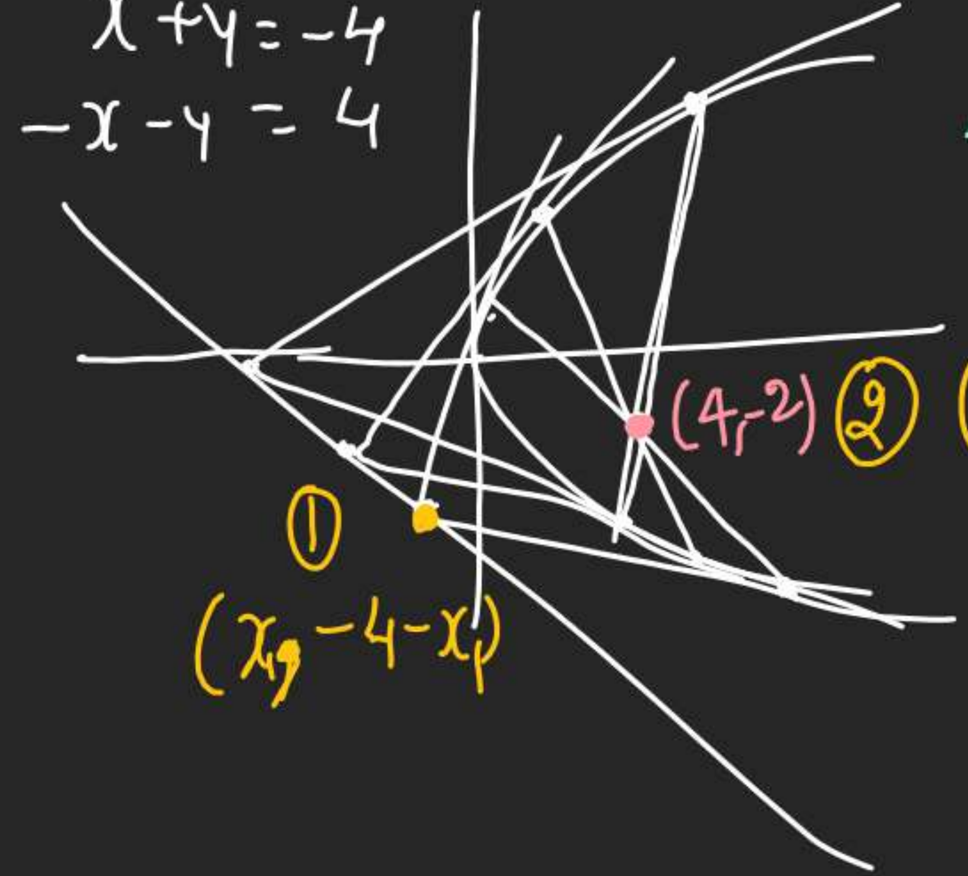
$2x + yy_1 = -2x_1$

$4x + y = 9$

Q Pair of tangents are drawn from every pt of $x+y+4=0$ to Par. $y^2=4x$ show that Each COC Passes thru a fixed pt.

$x+y=-4$

$-x-y=4$




Family of Lines $L_1+\lambda L_2=0$

② COC's Eqn

$Y(-4-x_1) = 2(x+x_1)$

$2x+4y+yx_1+2x_1=0$

$$\underbrace{2x+4y}_{L_1}+\underbrace{x_1}_{\lambda}\underbrace{(y+2)}_{L_2}=0$$

(3) $L_1:\ 2x+4y=0$

$L_2:\ y+2=0 \Rightarrow y=-2$

$x=+4$

So pt. is $(4,-2)$

Eqn of Chord having Mid Pt $(x_1,y_1)$

$y^2=4ax$

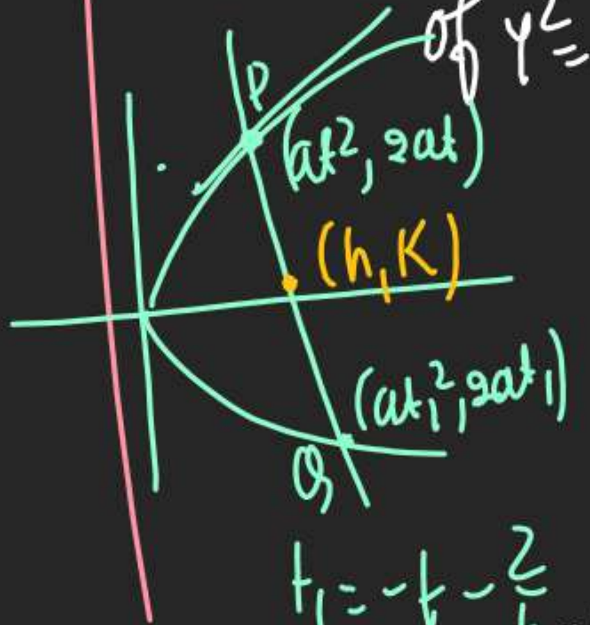
When Mid Pt is given then

Eqn of Chord $\Rightarrow T=S_1$

$$\boxed{yy_1-2a(x+x_1)=y_1^2-4ax_1}$$

$t_2=-t_1-\frac{2}{t_1}$

Q Find Locus of Mid Pt of normal chord of $y^2=4ax$.




$t_1=-t-\frac{2}{t}$

$h=\frac{at^2+at_1^2}{2}$

$h=\frac{at^2+a\left(t^2+\frac{4}{t^2}+4\right)}{2}$

$h=at^2+2a+\frac{2a}{t^2}$

$K=at+at_1$

$K=at+a\left(-t-\frac{2}{t}\right)$

$K=-\frac{2a}{t} \Rightarrow t=-\frac{2a}{K}$

$t=\ \ h=\frac{a\times 4a^2}{K^2}+2a+\frac{2a}{4a^2}\times K^2$

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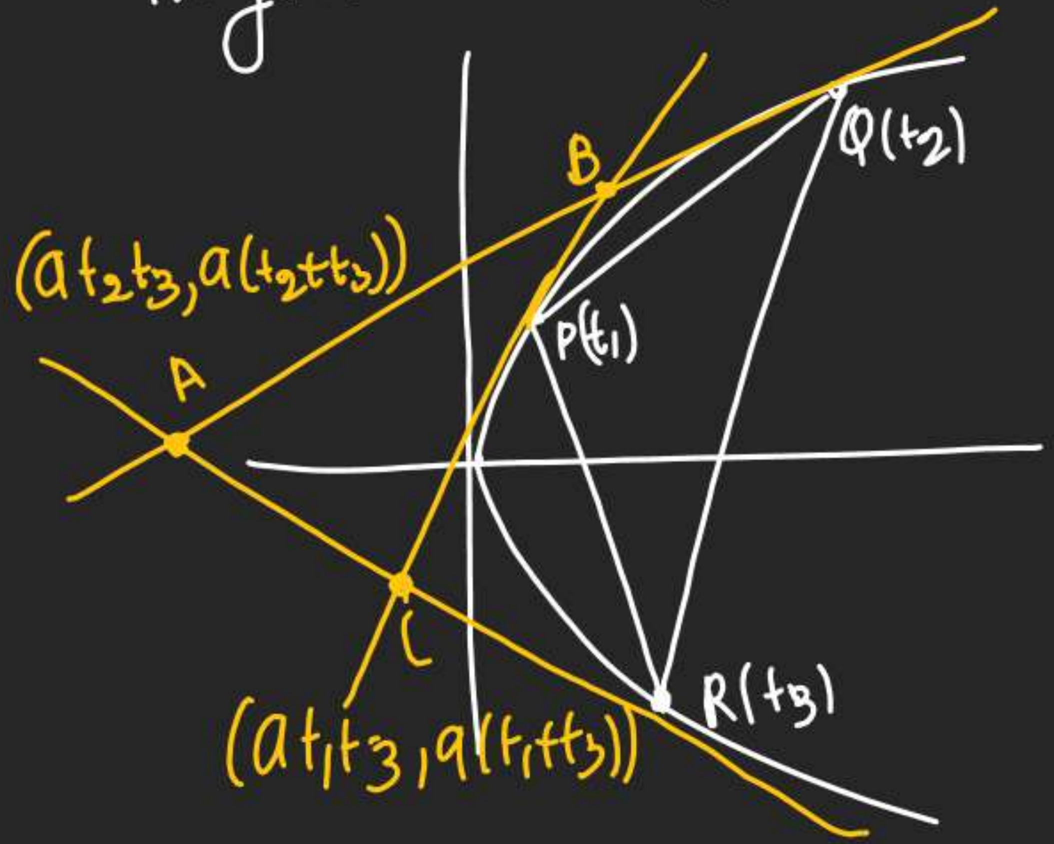
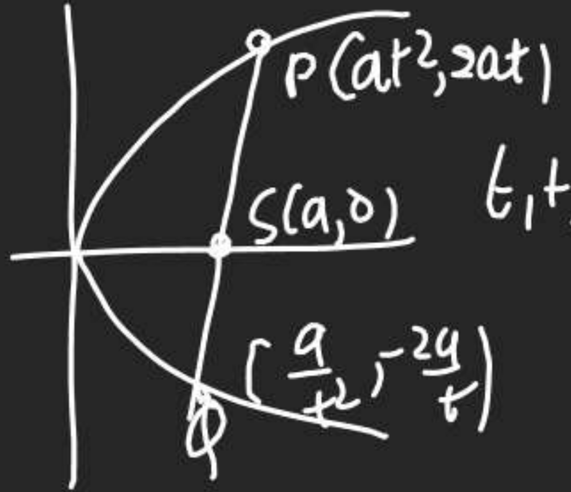
Rk.

Area of Δ made by 3 pts on Parabola is double the area of Δ made by tangents on these pts.




$$\frac{\frac{1}{2}\begin{vmatrix} at_1^2 & 2at_1 & 1 \\ at_2^2 & 2at_2 & 1 \\ at_3^2 & 2at_3 & 1 \end{vmatrix}}{\frac{1}{2}\begin{vmatrix} at_1t_2 & a(t_1+t_2) & 1 \\ at_2t_3 & a(t_2+t_3) & 1 \\ at_1t_3 & a(t_1+t_3) & 1 \end{vmatrix}} = 2$$

Q If $(t^2, 2t)$ is one end of Focal Chord of Par. $y^2=4x$ then Length of Focal chord = ?




$t_1t_2=-1$

$$\sqrt{\left(at^2-\frac{a}{t^2}\right)^2+\left(2at+\frac{2a}{t}\right)^2}$$

$$=\sqrt{\left(t^2-\frac{1}{t^2}\right)^2+\left(2t+\frac{2}{t}\right)^2}$$

$$=\sqrt{\left(t-\frac{1}{t}\right)^2\left(t+\frac{1}{t}\right)^2+4\left(t+\frac{1}{t}\right)^2}$$

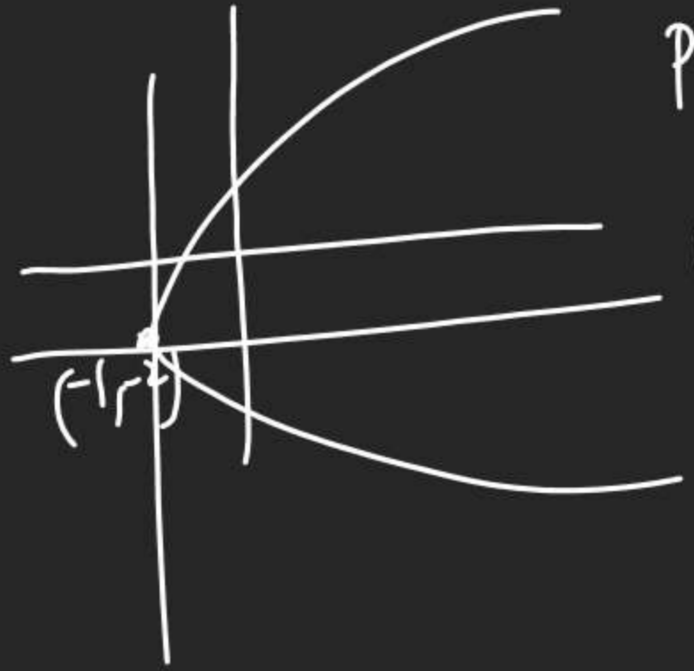
$$=\left(t+\frac{1}{t}\right)\sqrt{\left(t-\frac{1}{t}\right)^2+4}=\left(t+\frac{1}{t}\right)^2$$

Q Locus of PoI of ⊥r tangents to curve $y^2+4y-6x-2=0$

$y^2+4y+4 = 6x+2+4$

$(y+2)^2 = 6(x+1) \to 4A=6$

$Y^2 = 4AX$ $\qquad A=\frac{3}{2}$




PoI of tangent

$(at_1t_2,\ a(t_1+t_2))$

⊥r tangents का Locus = Directrix

$X=-a$

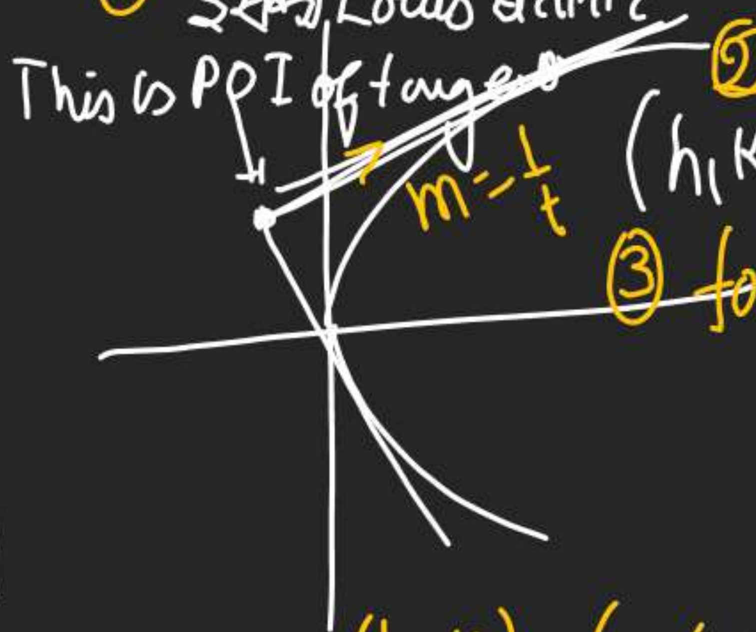
$x+1 = -\frac{3}{2}$

$x = -\frac{5}{2} \Rightarrow 2x+5=0$

Q Locus of a pt. S.T. 2 tangents drawn from it to Par. $y^2=4ax$ are such that slope of one is double of other is

A) $y^2=\frac{9}{2}ax$ B) $y^2=\frac{9}{4}ax$ (C) $y^2=9ax$ (D) $x^2=4ay$.

① उसका Locus बताना है




② $(h,k)=(at_1t_2,\ a(t_1+t_2))$

③ for a tangent Sl $= m = \frac{1}{t}$

$\frac{1}{t_1} = \frac{2}{t_2} \Rightarrow t_1 = \frac{t_2}{2}$

$\Rightarrow t_2 = 2t_1$

$(h,k) = (at_1(2t_1),\ a(t_1+2t_1))$

$(h,k) = (2at_1^2,\ 3at_1) \Rightarrow t_1 = \frac{k}{3a}$

$h = 2at_1^2 = 2a\times\frac{k^2}{9a^2} \Rightarrow y^2 = \frac{9a}{2}x$

Nishant Jindal

Q PQ is Normal chord of Par. $y^2 = 4ax$ at P

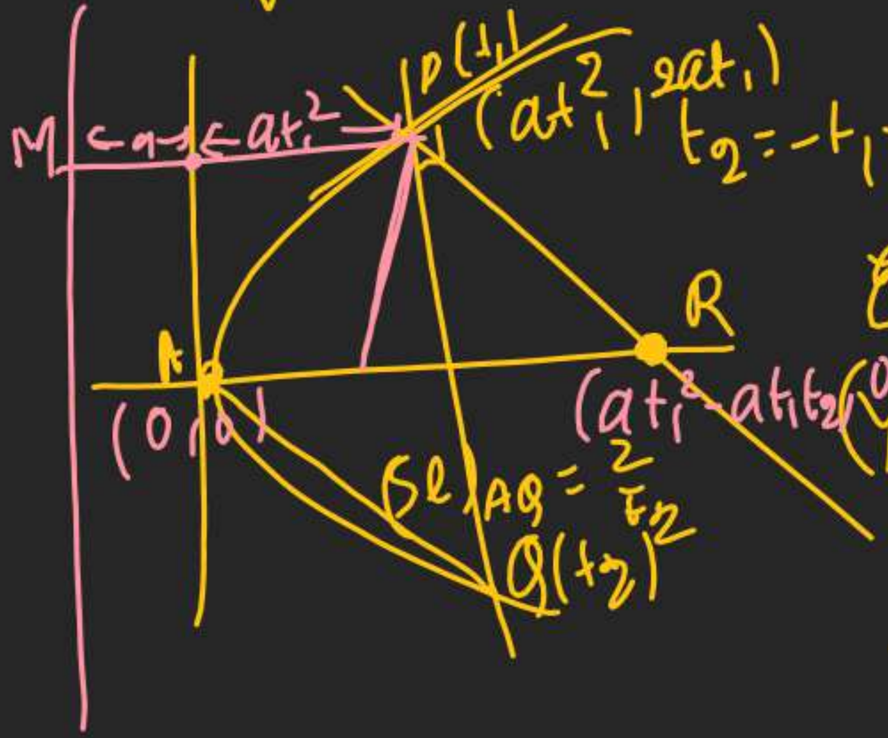
A being the vertex of Par. Thru P a line is drawn $\parallel^{re}$ to AQ meeting the x Axis in R

Then Length of AR =

$= a + at_1^2$ $\quad$ $2(a+at_1^2)$

A) LLR (B) Focal dis of P (C) 2x Focal dist of P

(D) dist of P from directrix.




Eq$^n$ PR

$$(y - 2at_1) = \frac{2}{t_2}(x - at_1^2)$$

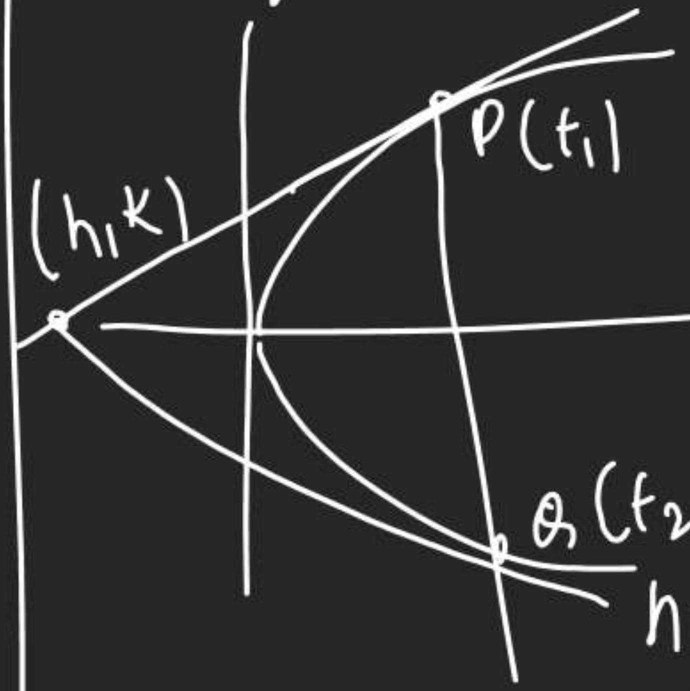
y = 0 Put

$$-2at_1 = \frac{2}{t_2}(x - at_1^2) \Rightarrow x - at_1^2 = -at_1t_2$$

$$x = at_1^2 - at_1t_2$$

$$AR = at_1^2 - at_1t_2$$
$$= at_1\left(t_1 + t_1 + \frac{2}{t_1}\right)$$
$$= 2at_1^2 + 2a$$
$$=$$

Q Locus of Intersection of tangents at the end of Normal chord of Parabola $y^2 = 4ax$ is




$$(h,k) \equiv (at_1t_2,\ a(t_1+t_2))$$

$$h = at_1t_2$$
$$= at_1\left(-t_1 - \frac{2}{t_1}\right)$$
$$h = -at_1^2 - 2a$$
$$h = -a \times \frac{4a^2}{k^2} - 2a$$

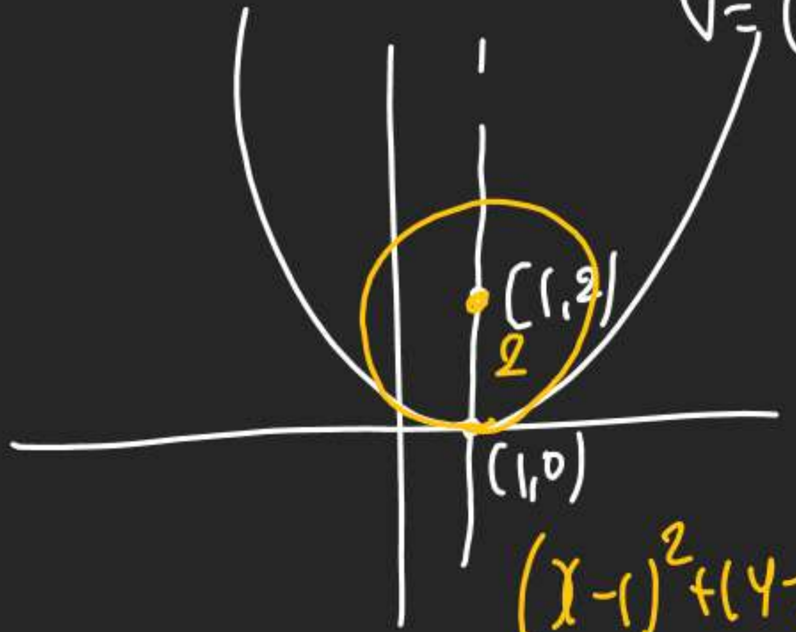
$$k = a\left(t_1 - t_1 - \frac{2}{t_1}\right)$$
$$k = \frac{-2a}{t_1}$$
$$t_1 = \frac{-2a}{k}$$

Q Eq$^n$ of circle drawn with Focus of Parabola $(x-1)^2-8y=0$ as its Centre and touching Par. at its vertex is .

$(x-1)^2=8y$

$X^2=4aY$ $\quad a=2$

$V=(1,0)$



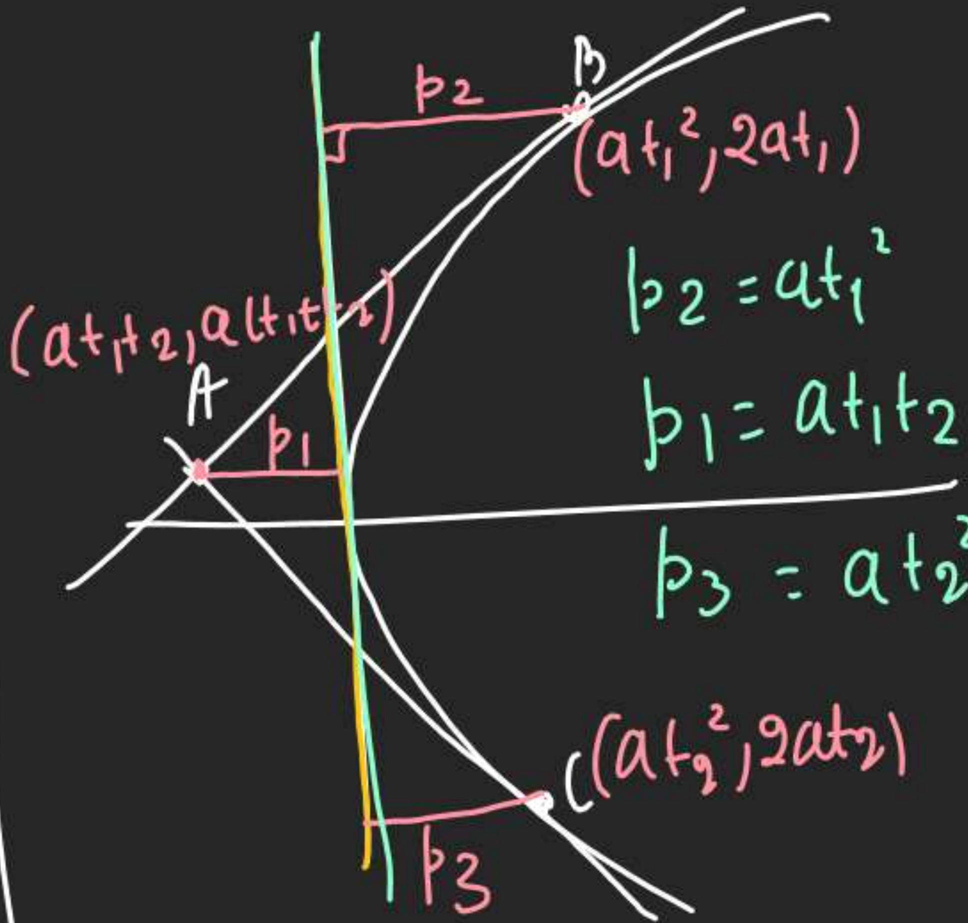

$(x-1)^2+(y-2)^2=2^2$

Q AB, AC are tangents to Par. $y^2=4ax$

$P_1, P_2, P_3$ are lengths of $\perp^r$ from A, B, C Resp. on $\boxed{\text{any tangent}}$ to the curve → 3$^{rd}$ tangent

Then $P_1\ P_2\ P_3$ are in AP GP HP - -

here 3$^{rd}$ tangent is TV




$p_2 = at_1^2$

$p_1 = at_1t_2$

$p_3 = at_2^2$

$p_1 = \sqrt{p_2p_3}$

$at_1t_2 = \sqrt{at_1^2 \cdot at_2^2}$

GP.