DPP - 3

Integration

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1. (a) $\int x^5 dx = \frac{x^{5+1}}{5+1} + c = \frac{x^6}{6} + c$

(b) $\int \frac{1}{\sqrt{x}} dx = \int x^{-1/2} dx$

$= \frac{x^{-1/2+1}}{-\frac{1}{2}+1} = \frac{x^{1/2}}{1/2}$

$= 2\sqrt{x}.$

(c) $\int \frac{1}{r^2} dr = \int r^{-2} dr$

$= \frac{r^{-2+1}}{-2+1}$

$= \frac{-1}{r}$

2. $Y = x^2 + x + 1$

$\int Y dx = \int (x^2 + x + 1) dx$

$= \frac{x^3}{3} + \frac{x^2}{2} + x$

3. $Y = 5\sin x$

$\int Y dx = \int 5\sin x dx$

$= 5[-\cos x]$

$= -5\cos x.$

4. $Y = e^x + \frac{1}{x} + 8$

$\int Y dx = \left(e^x + \frac{1}{x} + 8\right) dx$

$= e^x + \ln x + 8x$

5. (a) $\int \cos(3x + 4) dx$

$= \frac{\sin(3x + 4)}{3}$

(b) $\int \frac{1}{(4t-1)} dx$

$= \frac{\ln(4t - 1)}{4}$

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6. $\int (x^e + e^x + e^e)dx$

$= \frac{x^{e+1}}{e+1} + e^x + e^e \cdot x$

7. (a) $Y = \int \sin^2 x dx$

$\cos 2\theta = \cos^2\theta - \sin^2\theta$

$\cos 2\theta = 1 - 2\sin^2\theta$

$2\sin^2\theta = 1 - \cos 2\theta$

$\sin^2\theta = \frac{1}{2}(1 - \cos 2\theta)$

$Y = \int \frac{1}{2}(1 - \cos 2x)dx$

$= \frac{1}{2}\left[x - \frac{\sin 2x}{2}\right]$

(b) $\cos^2 x dx$

$Y = \frac{1}{2}\int (\cos 2x + 1)dx$

$= \frac{1}{2}\left(x + \frac{\sin 2x}{2}\right)$

$\frac{\cos 2\theta + 1}{2} = \cos^2\theta$

8. (a) $\int_0^{\pi} \cos 2x dx$

$= \left[\frac{\sin 2x}{2}\right]_0^{\pi}$

$= \frac{1}{2}[\sin 2\pi - \sin 0]$

$= 0$

(b) $\int_2^4 4x dx$

$= 4\left[\frac{x^2}{2}\right]_2^4$

$= 2[16 - 4] = 24$

(c) $S_{\infty}^{0} e^{-t}dt = [-e^{-t}]_{\infty}^{0}$

$= [-e^0 + e^{-\infty}]$

$= [-1 + 0] = -1$

(d) $\int_{\infty}^{R} \frac{GMm}{r^2} dr = GMm\left[-\frac{1}{r}\right]_{\infty}^{R} = GMm\left[-\frac{1}{R} + \frac{1}{\infty}\right] = \frac{-GMm}{R}$

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9. (a) $\int_0^6 YdY$ = area under the Y − x curve from 0 to 6

$$\text{Area} = 20 + \frac{1}{2} \times 2 \times 5 = 25$$

(b) $\int_0^{10} YdY$ = area under the Y − x curve.

$$\Rightarrow 20 + 5 + \frac{1}{2} \times 4 \times (-3)$$

$$= 20 + 5 - 6 = 19$$

10. $= \int x^3 dx = \left[\frac{x^4}{4}\right]_{-1}^{2}$

$$= \frac{1}{4}[16 - 1]$$

$$= \frac{15}{4}$$

11. $\int x^n dx$

$$= \int x^{-1} dx.$$

$$= \int \frac{1}{x} dx$$

$$= \log_e x$$

12. $\int (x^5 + x^7 + x^9) dx$

$$= \frac{x^6}{6} + \frac{x^8}{8} + \frac{x^{10}}{10}.$$

13. $\int_a^b \frac{dx}{2} = [2\ln x]_a^b$

$$= [\ln x^2]_a^b$$

$$= \ln \frac{b^2}{a^2}$$

14. $f(x) = x^2$

$$\int_2^3 x^2 dx = \left[\frac{x^3}{3}\right]_2^3$$

$$= \left[\frac{27}{3} - \frac{8}{3}\right]$$

$$= \frac{19}{3} = 6.33$$