Nishant Jindal

1. Using LMVT, P.T. $|\cos a - \cos b| \le |a-b|$ ✓

LMVT over $f(x) = \cos x$ in $[a,b]$

$\exists\ c \in (a,b)$, $\left|\dfrac{\cos b - \cos a}{b-a}\right| = |-\sin c| \le 1$

2. If $a<b$, show that a real number 'c' can be found in (a,b) s.t. $f(b)-f(a)=0$

$3c^2 = a^2+ab+b^2$

$f(x) = x^3 - (a^2+ab+b^2)x$ $\Rightarrow \exists\ c \in (a,b)$, $f'(c)=0$

LMVT over $f(x) = x^3$ in $[a,b]$

$\exists\ c \in (a,b)$, $3c^2 = \dfrac{b^3-a^3}{b-a} = b^2+a^2+ab$.

3: Use LMVT to P.T.

(i) $\tan x > x$ for $x \in \left(0, \frac{\pi}{2}\right)$

(ii) $e^x \geq 1 + x$ $\forall x \in R$.

$f(x) = e^x$

$x > 0$: $[0, x]$

$\frac{e^x - e^0}{x - 0} = \underbrace{e^c}_{>1}$ $\Rightarrow e^x - 1 > x$, $x > 0$

$x < 0$: LMVT in $[x, 0]$

$\frac{e^0 - e^x}{0 - x} = \underbrace{e^c}_{<1}$, $c \in (x, 0)$

$1 - e^x < -x$

$e^x > 1 + x$

$\forall x < 0$

LMVT over $f(x) = \tan x$ in $[0, x]$, $0 < x < \frac{\pi}{2}$

$\exists\, c \in (0, x)$, $\frac{\tan x - \tan 0}{x - 0} = \sec^2 c > 1$

$\boxed{\tan x > x}$

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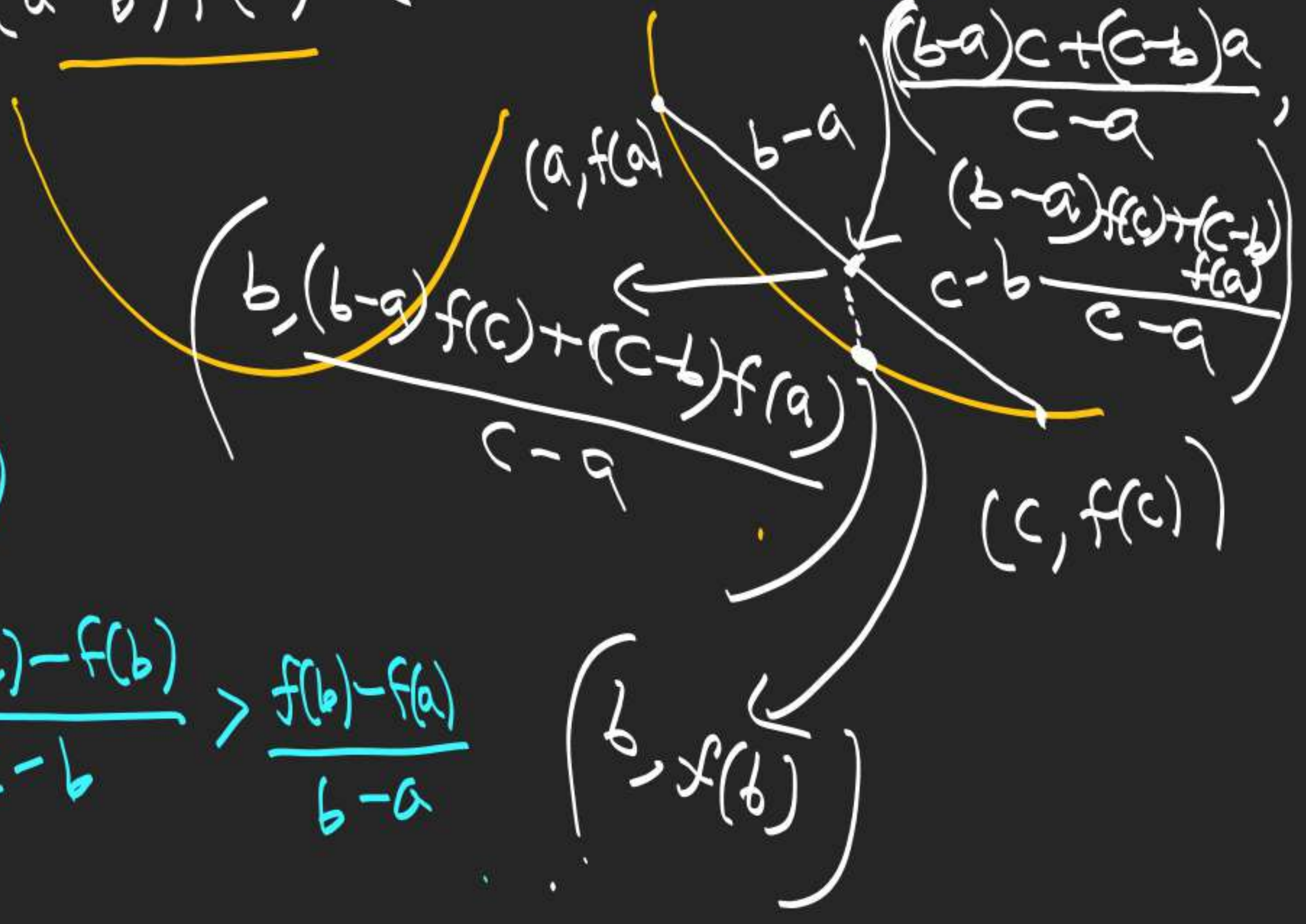
4. Let $a, b, c \in R$, $a < b < c$, $f(x)$ is continuous in $[a, c]$ and differentiable in (a, c). Also $f'(x)$ is strictly increasing in (a, c). P.T.

$$\frac{(b-a)f(c)+(c-b)f(a)}{c-a} > f(b)$$

$$(b-c)f(a)+(c-a)f(b)+(a-b)f(c) < 0$$



$\exists\, c_1 \in (a, b)$, $\quad f'(c_1) = \dfrac{f(b)-f(a)}{b-a}$

$\exists\, c_2 \in (b, c)$, $\quad f'(c_2) = \dfrac{f(c)-f(b)}{c-b}$

$$f'(c_2) > f'(c_1) \Rightarrow \frac{f(c)-f(b)}{c-b} > \frac{f(b)-f(a)}{b-a}$$

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2:00 - 3:30 pm

Thurs.

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5. Let $f:[0,4]\to R$ is a differentiable function. Then P.T.

$$f^2(4)-f^2(0) = 8f'(a)\underline{f(b)} \quad \text{for some } a,b\in\boxed{[0,4]}$$

IVT $\Rightarrow$ $\exists\, b\in[0,4]$, $f(b)=\dfrac{f(0)+f(4)}{2}$

$\underline{\text{LMVT}}$ $\exists\, a$, $a\in(0,4)$,

$$f'(a)=\frac{f(4)-f(0)}{4-0}$$

$\underline{\text{Diff}} \Rightarrow$ Ex-5 (remaining)

Ex-3