DPP-4

1. Solve the following inequalities

(i) $\text{arc cot}^2 x - 5 \text{ arc cot } x + 6 > 0$

(ii) $\arcsin x > \arccos x$

(iii) $\tan^2(\arcsin x) > 1$

Ans. (i) $x \in (-\infty, \cot 3) \cup (\cot 2, \infty)$

(ii) $1 \geq x > \frac{1}{\sqrt{2}}$

(iii) $x \in \left(-1, \frac{-1}{\sqrt{2}}\right) \cup \left(\frac{1}{\sqrt{2}}, 1\right)$

2. Solve the following inequalities

$4\arctan \tan^2 x - 8\arctan x + 3 < 0$&

$4 \text{ arc cot } x - \text{arc cot}^2 x - 3 \geq 0$

Ans. $\tan x \in \left(\frac{1}{2}, \frac{3}{2}\right)$

3. find the sum of series

(i) $\sin^{-1}\frac{1}{\sqrt{2}} + \sin^{-1}\frac{\sqrt{2}-1}{\sqrt{6}} + \cdots + \sin^{-1}\frac{\sqrt{n}-\sqrt{n-1}}{\sqrt{n(n+1)}} + \cdots \infty$

(ii) $\text{Tan}^{-1}\frac{1}{3} + \text{Tan}^{-1}\frac{2}{9} + \cdots + \text{Tan}^{-1}\frac{2^{n-1}}{1+2^{2n-1}} + \cdots \infty$

(iii) $\cot^{-1} 7 + \cot^{-1} 13 + \cot^{-1} 21 + \cot^{-1} 31 + \cdots$ ... to nterms.

(iv) $\text{Tan}^{-1}\frac{1}{x^2+x+1} + \text{Tan}^{-1}\frac{1}{x^2+3x+3} + \text{Tan}^{-1}\frac{1}{x^2+5x+7} + \text{Tan}^{-1}\frac{1}{x^2+7x+13} + \cdots$ to nterms.

(v) $\text{Tan}^{-1}\frac{1}{2} + \text{Tan}^{-1}\frac{1}{8} + \text{Tan}^{-1}\frac{1}{18} + \text{Tan}^{-1}\frac{1}{32} + \cdots \infty$

Ans. (i) $\frac{\pi}{2}$.

(ii) $\text{Tan}^{-1} 2^n - \frac{\pi}{4}$

(iii) $\frac{\pi}{4}$

(iv) $\text{Tan}^{-1}(x+n) - \text{Tan}^{-1} x$

(v) $\frac{\pi}{4}$

4. find the value of Tan $\left\{\frac{1}{2}\sin^{-1}\left(\frac{2x}{1+x^2}\right)+\frac{1}{2}\cos^{-1}\left(\frac{1-y^2}{1+y^2}\right)\right\}$, if $x > y > 1$.

5. If $\alpha = 2\tan^{-1}\left(\frac{1+x}{1-x}\right)$ & $\beta = \sin^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ for $0 < x < 1$, then Prove that $\alpha + \beta = \pi$. What the value of $\alpha + \beta$ will be if $x > 1$?

6. If $x \in \left[-1, \frac{-1}{2}\right]$ then express the functions $f(x) = \sin^{-1}(3x - 4x^3) + \cos^{-1}(4x^3 - 3x)$ in the form of $a\cos^{-1} x + b\pi$, where a and b are rational numbers.

7. Prove that the equations $(\sin^{-1} x)^3 + (\cos^{-1} x)^3 = \alpha\pi^3$ has no roots for $\alpha < \frac{1}{32}$ and $\alpha > \frac{7}{8}$

8. If $\cos^{-1}(2x^2 - 1) = 2\pi - 2\cos^{-1} x$, then

(A) $x \in [-1,0]$ (B) $x \in [0,1]$

(c) $x \in \left[0, \frac{1}{\sqrt{2}}\right]$ (D) $x \in \left[\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right]$

Ans. **(A)**

9. The value of $\cot\left(\sum_{n=1}^{19}\cot^{-1}\left(1+\sum_{p=1}^{n}2p\right)\right)$ is:

(a) $\frac{21}{19}$ (b) $\frac{19}{21}$ (c) $\frac{22}{23}$ (d) $\frac{23}{22}$

Ans. **(A)**

10. The value of $\tan^{-1}\left[\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right]$, $|x| < \frac{1}{2}, x \neq 0$, is equal to

(a) $\frac{\pi}{4}+\frac{1}{2}\cos^{-1} x^2$ (b) $\frac{\pi}{4}+\cos^{-1} x^2$

(c) $\frac{\pi}{4}-\frac{1}{2}\cos^{-1} x^2$ (d) $\frac{\pi}{4}-\cos^{-1} x^2$

Ans. **(A)**