8.

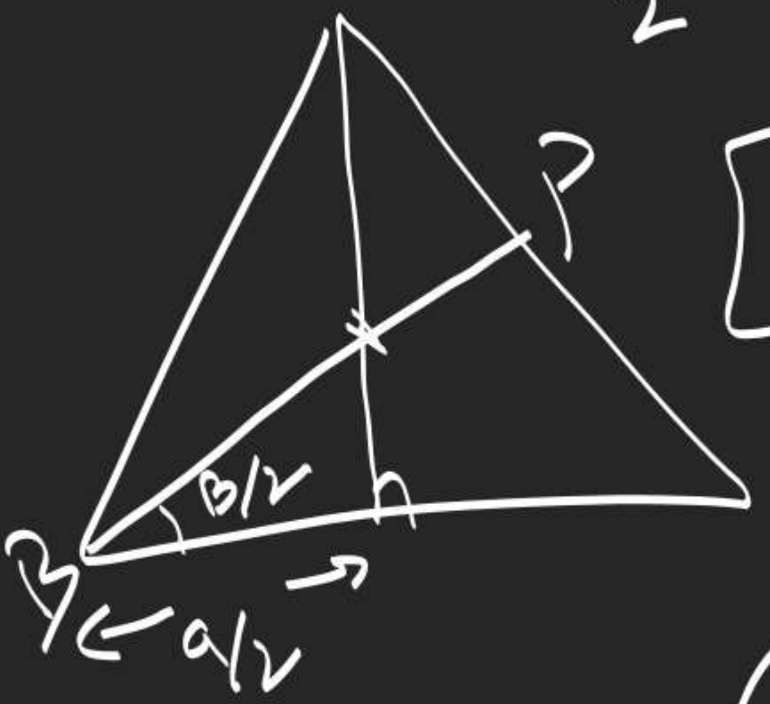
$x \to 0^-$ $\quad \ln 2$

$x \to \frac{\pi}{2}^+$ $\quad \ln 2$

$$\frac{2ac\cos\frac{B}{2}}{a+c}$$

$$\boxed{b=c}$$

$$G \circ G(x) = G(G(-x))$$
$$= G(-G(x))$$
$$= -G(G(x))$$



$B \to 0$

$BP =$

$$\frac{[\sin x]-2+\{\cos x\}}{\{x\}-1}$$

$\downarrow 0$

$x \to 0^-$

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$$f(0) = \lim_{n\to\infty} x\left(\frac{3}{2} + \left[\cos x\right]\left(\sqrt{n^2+1} - \sqrt{n^2-3n+1}\right)\right)$$

$$= 0 \quad [\cos x]\, n\left(\sqrt{1+\frac{1}{n^2}} - \sqrt{1-\frac{3}{n}+\frac{1}{n^2}}\right)$$

$x \in \left[0, \frac{\pi}{2}\right)$

$f(x) = \frac{3}{2}x$

$x \in \left(0, \frac{\pi}{2}\right)$ $\boxed{f(x) = \frac{3}{2}x}$

$\cos x \in [0, 1)$

$$\lim_{n\to\infty}\left(0 \quad n\right) = \lim_{n\to\infty} 0 = 0$$

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$$f(x^5) = \underbrace{(x^4+x^3+x^2+x+1)}_{f(x)} Q(x) + \boxed{\alpha x^3+\beta x^2+\gamma x+\delta}.$$

$$i=1,2,3,4 \quad f(x_i^5) = 0 + \alpha x_i^3 + \beta x_i^2 + \gamma x_i + \delta = f(1) = 5$$

$$x^4+x^3+x^2+x+1 = 0 \quad x_1,\ x_2,\ x_3,\ x_4$$

$$\boxed{x^5-1=0}$$

$$x^4+\dots 1$$

$$\alpha x^3+\beta x^2+\gamma x+\delta-5=0 \quad x_1\ x_2\ x_3\ x_4$$

$$\alpha=\beta=\gamma=\delta-5=0.$$

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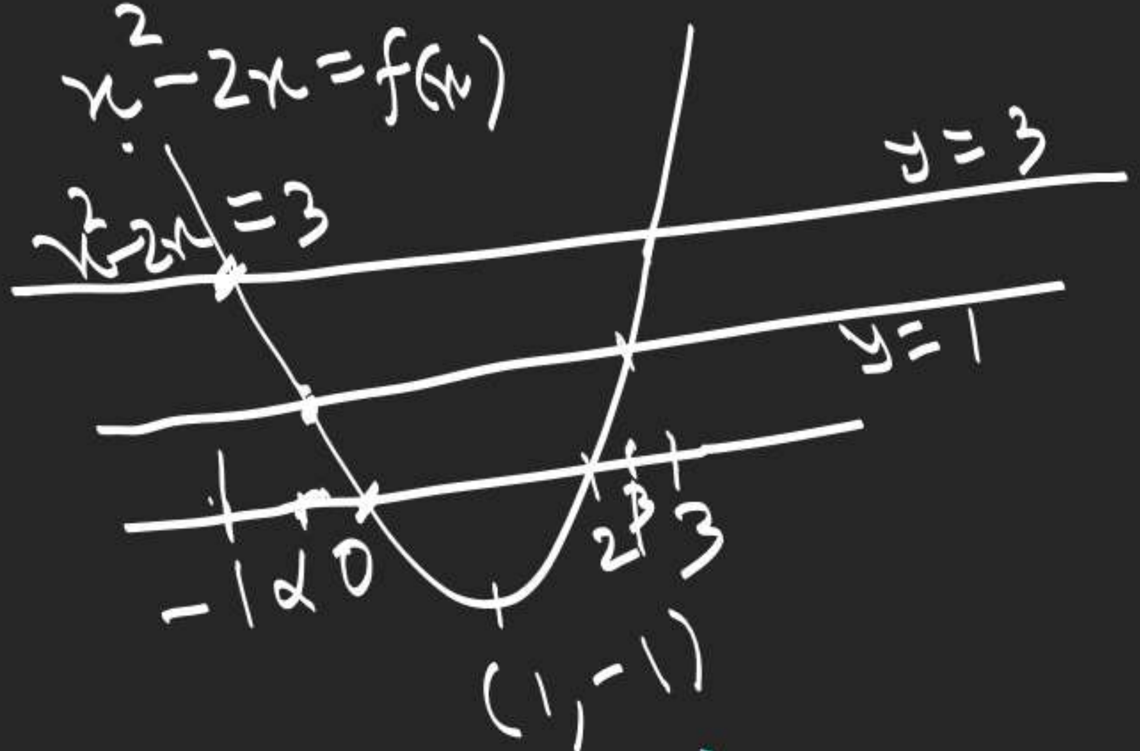
$$f(f(f(f(c)))) = 3$$



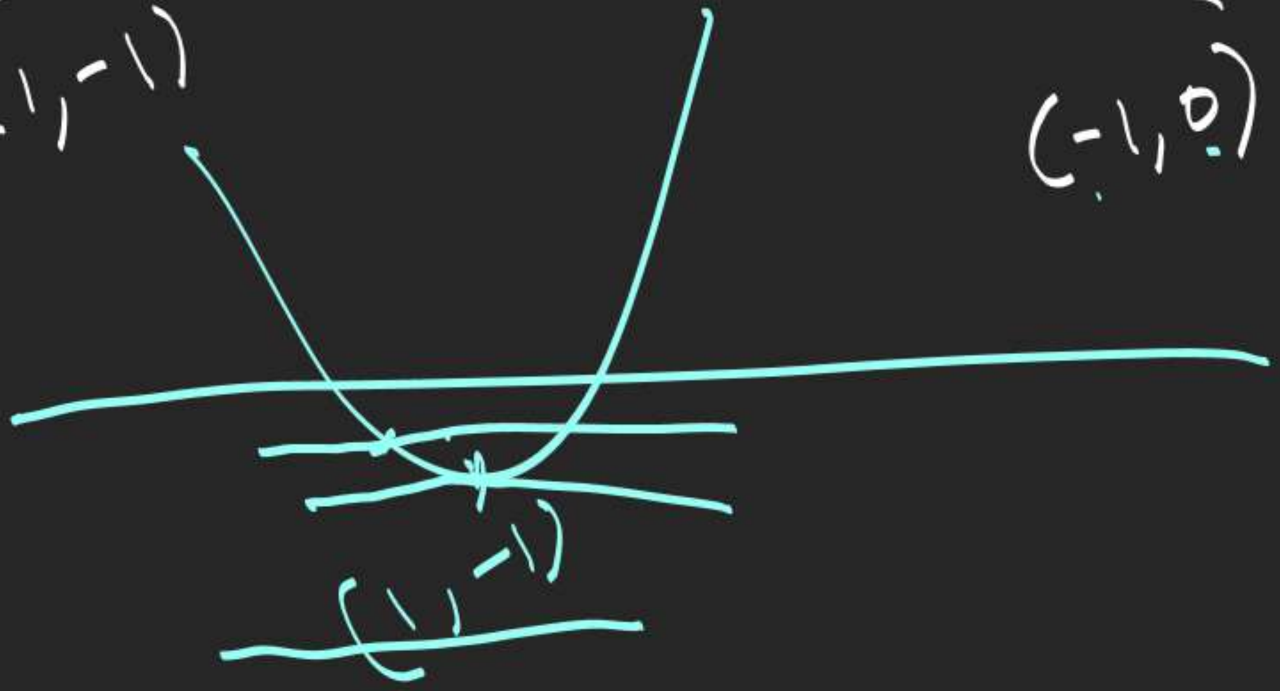
$$f(f(f(c))) = -1 \text{ or } f(f(f(c))) = 3$$

$$\Rightarrow f(f(c)) = 1 \text{ or } f(f(c)) = -1, 3$$

2 2 2 1 2

$$f(c) = \alpha, \beta, \quad f(c) = 1, -1, 3$$

$(-1, 0)$ $(2, 3)$



$$f(x) = k$$

9

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$$\lim_{x\to 0^+} \frac{\sin^{-1}(1-\{x\})\cos^{-1}(1-\{x\})}{\sqrt{\{x\}}\,(1-\{x\})}\sqrt{2} = \frac{\pi}{2\sqrt{2}}\sqrt{2} = \frac{\pi}{2}$$

$$\lim_{x\to 0^-} \frac{\sin^{-1}(1-\{x\})\cos^{-1}(1-\{x\})}{\sqrt{\{x\}}\,(1-\{x\})}$$

$$= \frac{\pi}{2\sqrt{2}}$$

$\theta\to 0^+$

$$\frac{\theta}{\sqrt{1-\cos\theta}} = \frac{1}{\sqrt{\frac{1-\cos\theta}{\theta^2}}} = \sqrt{2}$$

$\sqrt{2}$

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$$\lim_{t\to 0} e^{\left(\frac{\cos\left(2\pi\left(1-(1+t)^{-a}\right)\right)-1}{(\quad)^2}\right)}$$

$$4\pi^2\left(\frac{(1+t)^{-a}-1^{-a}}{(1+t)-1}\right)^2$$

$$\left(\frac{x}{1+x}\right)^a = \left(1+\frac{1}{x}\right)^{-a}$$

$$e^{-\frac{1}{2}4\pi^2(-a)^2}$$

Nishant Jindal

9.

$$x_n = \frac{x_{n-1} + x_{n-2}}{2}$$

$$x_2 = \frac{x_1 + x_0}{2}$$

$$x_3 = \frac{x_2 + x_1}{2}$$

$$x_4 = \frac{x_3 + x_2}{2}$$

$$x_5 = \frac{x_4 + x_3}{2}$$

$\vdots$

$$\frac{x_{n-2}}{} = \frac{x_{n-3} + x_{n-4}}{2}$$

$$x_{n-1} = \frac{x_{n-2} + x_{n-3}}{2}$$

$$x_n = \frac{x_{n-1} + x_{n-2}}{2}$$

$$\boxed{x_n + \frac{x_{n-1}}{2} = x_1 + \frac{x_0}{2}}$$

$$\lim_{n\to\infty}\left(x_n + \frac{x_{n-1}}{2}\right) = x_1 + \frac{x_0}{2}$$

$$= \frac{3}{2}\lim_{n\to\infty} x_n$$

Nishant Jindal

$$x_n = \frac{x_{n-1} + x_{n-2}}{2} \qquad x_2 = \frac{x_1 + x_0}{2}$$

$$\boxed{a^2 = 3}$$

$$x_2 - x_1 = -\left(\frac{x_1 - x_0}{2}\right) \qquad x_n - x_1 = (x_1 - x_0)\underbrace{\left(-\frac{1}{2} + \frac{1}{2^2} - \frac{1}{2^3} + \dots\right)}_{n-1 \text{ terms}}$$

$$e^{-a^2}$$

$$x_3 - x_2 = -\frac{1}{2}(x_2 - x_1) = \frac{1}{2^2}(x_1 - x_0)$$

$$a + b = 0$$

$$e^{\frac{(ax - ax^2)^a}{x-1}}$$

$$x_4 - x_3 = -\frac{1}{2}(x_3 - x_2) = -\frac{1}{2^3}(x_1 - x_0)$$

$$\vdots$$

$$e^{-a^2 x}$$

$$x_n - x_{n-1} = \qquad = \frac{(-1)^{n-1}}{2^{n-1}}(x_1 - x_0)$$

Nishant Jindal

$$= \lim_{n\to\infty} \frac{1x + 2x + 3x + \cdots (nx) -}{n^2} \quad \frac{\{1x\} + \{2x\} + \cdots \{nx\}}{n^2} \downarrow 0$$

$$\frac{n(n+1)}{2n^2} x$$

$$= \frac{x}{2}$$

$$rx - 1 < [rx] \le rx$$

$$\frac{n(n+1)x}{2} - n = \sum_{r=1}^{n} (rx - 1) < \sum [rx] \le \left(\sum_{r=1}^{n} r\right) x = \frac{n(n+1)}{2} x.$$

Nishant Jindal

$$\lim_{n\to\infty} \sum_{n=1}^{n} 3^{n-1}\left(3\sin\frac{x}{3^n} - \sin\frac{x}{3^{n-1}}\right)\frac{1}{4}$$

$$\lim_{n\to\infty} \frac{1}{4} \sum_{n=1}^{n} \left(3^n \sin\frac{x}{3^n} - 3^{n-1}\sin\frac{x}{3^{n-1}}\right)$$

$n = n$ $\qquad$ $n = 1$

$$\frac{1}{4} \lim_{n\to\infty} \left(x\,\frac{3^n \sin\frac{x}{3^n}}{x} - \sin x \right) = \frac{x - \sin x}{4}$$

Nishant Jindal

13. $$\frac{\sqrt{\frac{\cos 2x+(1+3x)^{1/3}}{2}}-1}{x}+\frac{1-\left(\cos^3 x-\ln(1+x)\right)^{1/3}}{x}$$

$$\frac{\left(\cos 2x-1\right)+(1+3x)^{\frac{1}{3}}-1}{2\left(\sqrt{}+1\right)x}+\frac{(1-\cos x)(1+\cos x+\cos^2 x)+\ln(1+x)}{x\left(1+(\cos^3 x-\ln(1+x))^{2/3}+()^{1/3}\right)}$$

$$\boxed{\frac{1}{4}+\frac{1}{3}}$$

$$3.\ \frac{(1+3x)^{1/3}-1^{1/3}}{3x}=1$$

Nishant Jindal

$$f(x) = \lim_{x\to 0} \frac{\sqrt{\frac{\cos 2x + (1+3x)^{1/3}}{2}} - \sqrt[3]{\cos^3 x - \ln(1+x)} - f(0)}{x - 0}$$

$\boxed{14 \text{ to } 17}$ ✓

Ex - II

$$= f'(0)$$

$$= \frac{1}{2}\left(\frac{\cos 2x + (1+3x)^{1/3}}{2}\right)^{-\frac{1}{2}} \frac{1}{2}\left(-2\sin 2x + \frac{1}{3}(1+3x)^{-2/3} 3\right)$$

$$- \frac{1}{3}\left(\cos^3 x - \ln(1+x)\right)^{-2/3}\left(3\cos^2 x(-\sin x) - \frac{1}{1+x}\right)$$

$$= \frac{1}{4} + \frac{1}{3}.$$