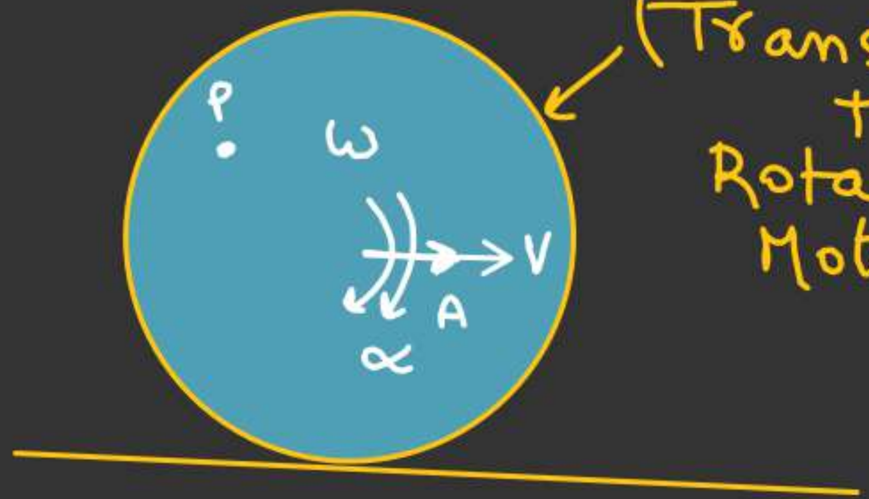
Nishant Jindal

Rolling Motion

- Rolling Motion a Combination of (translational + Rotational) Motion for body like ring, disc, Sphere or cylinder.

- $$\vec{X}_{P/E} = \vec{X}_{P/com} + \vec{X}_{com/E}$$
  $$\vec{v}_{P/E} = \vec{v}_{P/com} + \vec{v}_{com/E}$$
  $$\vec{A}_{P/E} = \vec{A}_{P/com} + \vec{A}_{com/E}$$



(Translational + Rotational Motion)

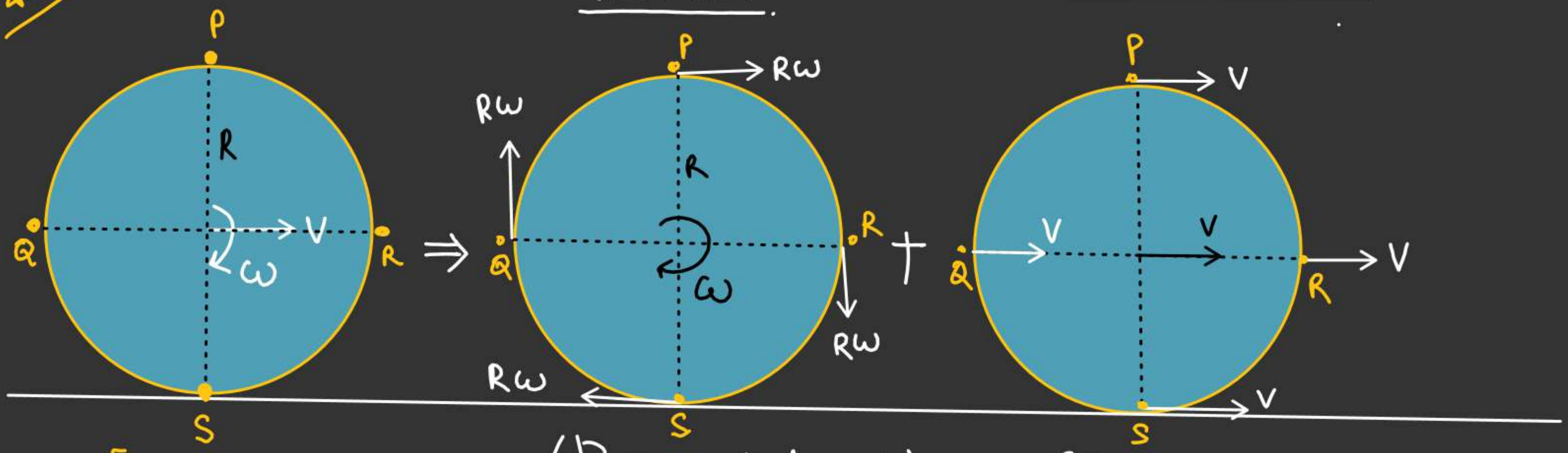
Note:-
- w.r.t COM pure rotational Motion
- COM w.r.t earth has pure translational Motion

Rolling Motion

44. General

W.r.t COM

COM w.r.t Earth



(Pure rotational)

(Pure translational Motion)

$$\vec{v}_P = \vec{v}_{P/com} + \vec{v}_{com/E} = R\omega\hat{i} + V\hat{i} = (V+R\omega)\hat{i}$$

$$\vec{v}_Q = R\omega\hat{j} + V\hat{i} \Rightarrow |\vec{v}_Q| = \sqrt{V^2+R^2\omega^2}$$

$$\vec{v}_R = -R\omega\hat{j} + V\hat{i} \Rightarrow \sqrt{R^2\omega^2+v^2} = |v_R|$$

$$v_S = V\hat{i} - R\omega\hat{i} = (v-R\omega)\hat{i}$$

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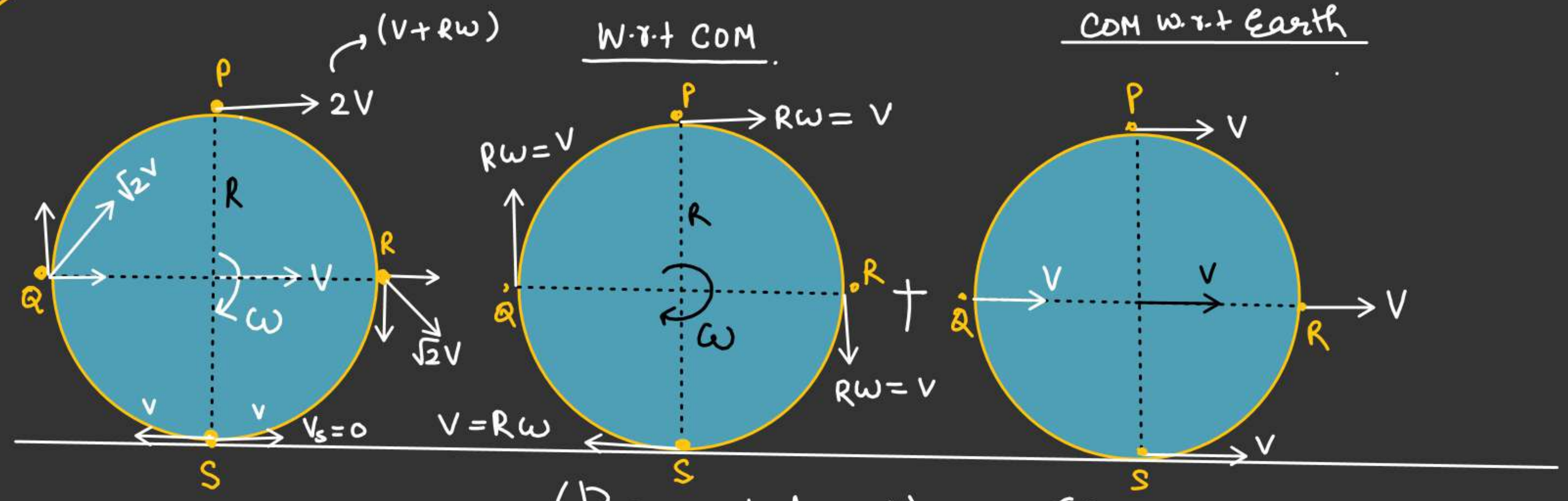
4. Pure Rolling :- No relative slipping of point of contact



(Pure rotational) + (Pure translational Motion)

$$\left(\frac{dx}{dt}\right) = R\left(\frac{d\theta}{dt}\right)$$

$$\boxed{X = R\theta}$$

$$V_{s/E} = 0 \Rightarrow \text{(Pure Rolling)}$$

$$V - R\omega = 0$$

$$\boxed{V = R\omega}$$

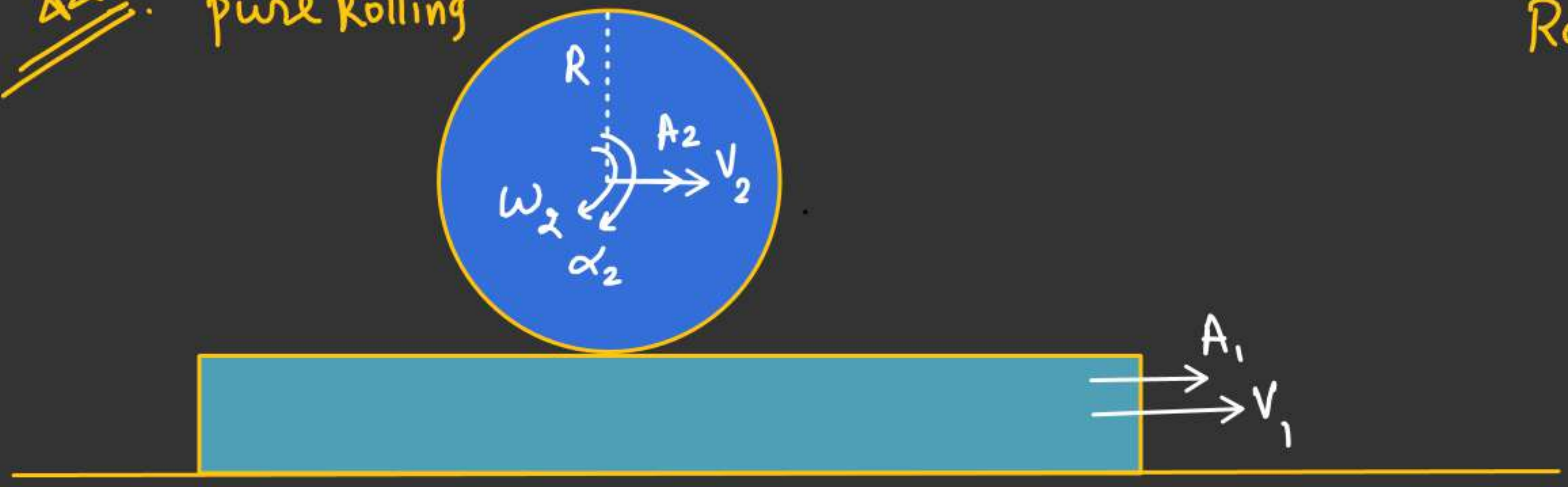
$$\frac{dV}{dt} = R\frac{d\omega}{dt}$$

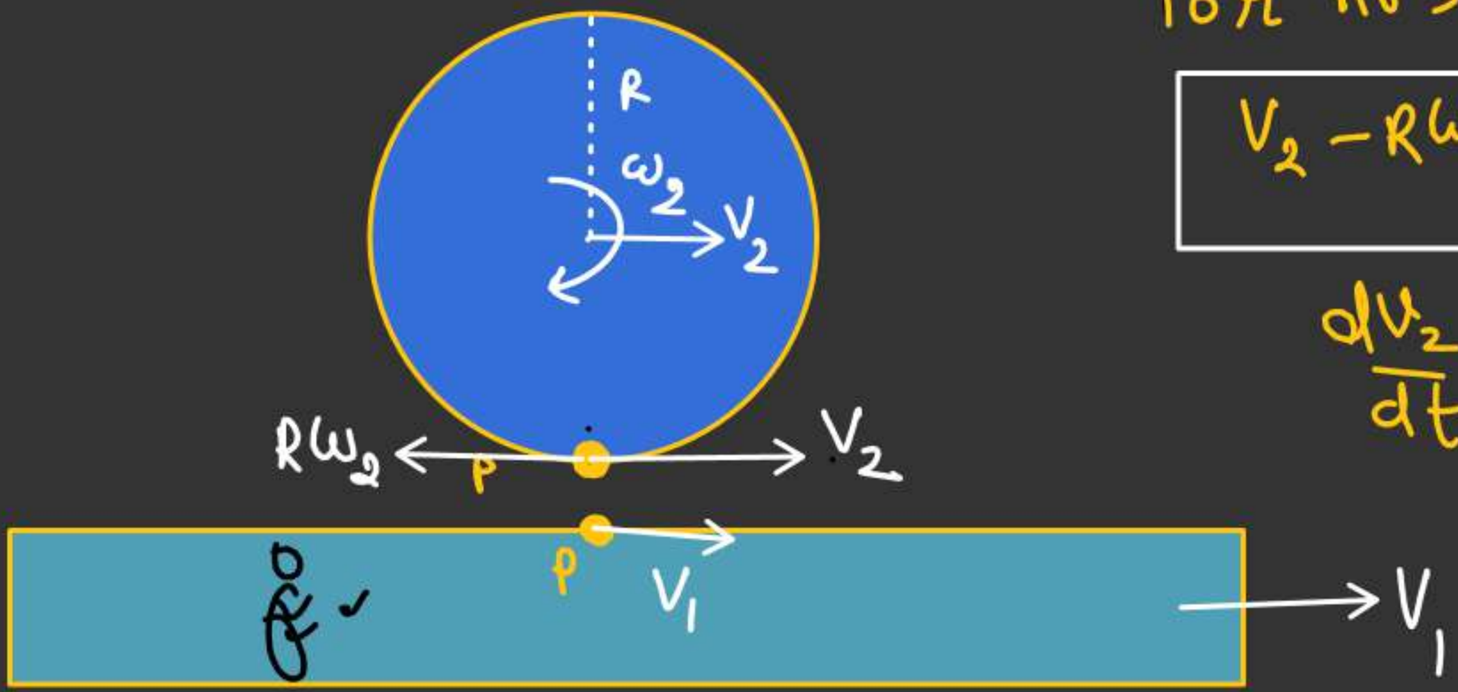
$$\boxed{A = R\alpha}$$

Rolling Motion

Condition of pure Rolling

Rolling → pure Rolling.





For no slipping of disc

$$\boxed{V_2 - R\omega_2 = v_1}$$

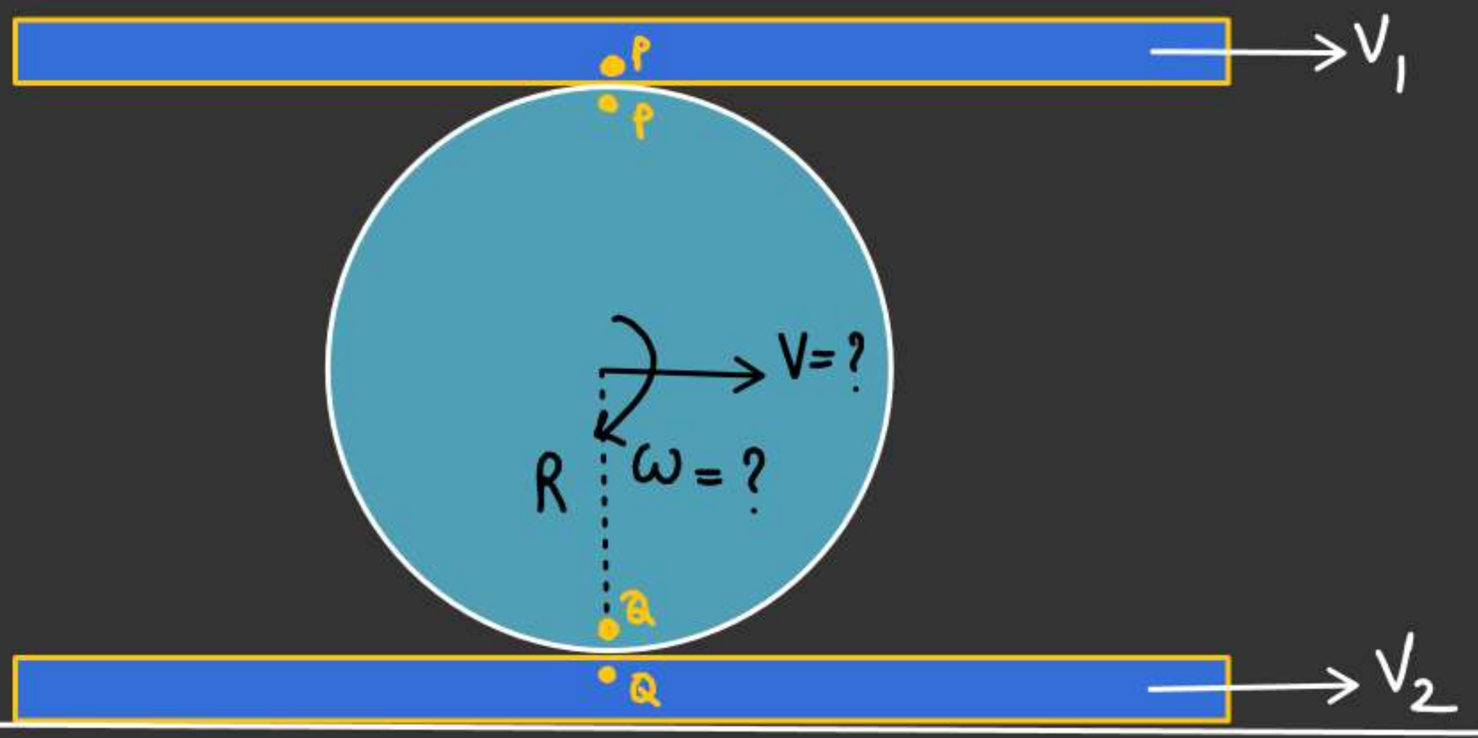
$$\frac{dV_2}{dt} - R\frac{d\omega_2}{dt} = \frac{dv_1}{dt}$$

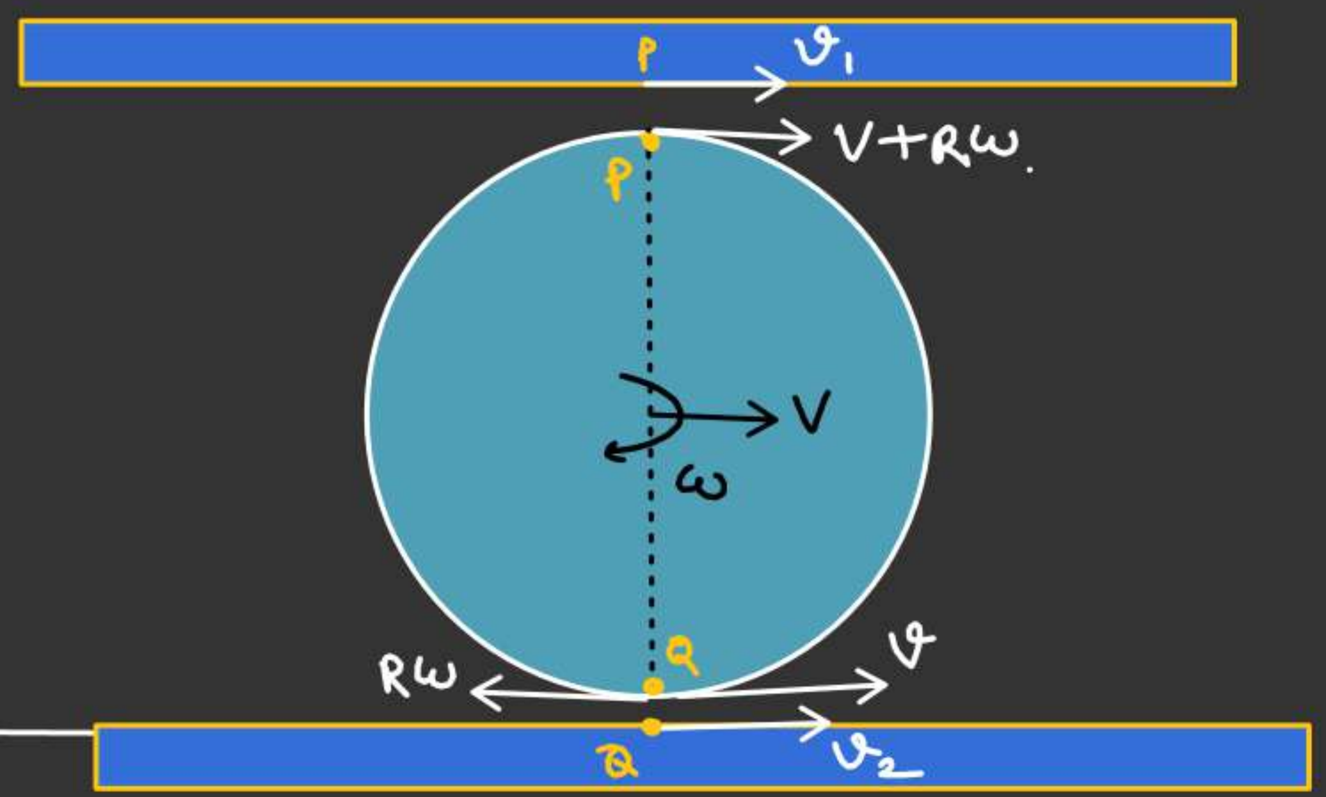
$$\boxed{A_2 - R\alpha_2 = A_1}$$

Rolling Motion

No slipping b/w planks & cylinder.

$v = ?\ ,\ \omega = ?$





No Slipping for point P.

$$V + R\omega = v_1 \quad -(1)$$

No Slipping for point Q

$$v - R\omega = v_2 \quad -(2)$$

(1) + (2)

$$2V = v_1 + v_2$$

$$\boxed{V = \left(\frac{v_1 + v_2}{2}\right)}$$

$$R\omega = V_1 - v = v_1 - \left(\frac{v_1 + v_2}{2}\right)$$

$$R\omega = \frac{v_1 - v_2}{2}$$

$$\boxed{\omega = \frac{v_1 - v_2}{2R}}$$

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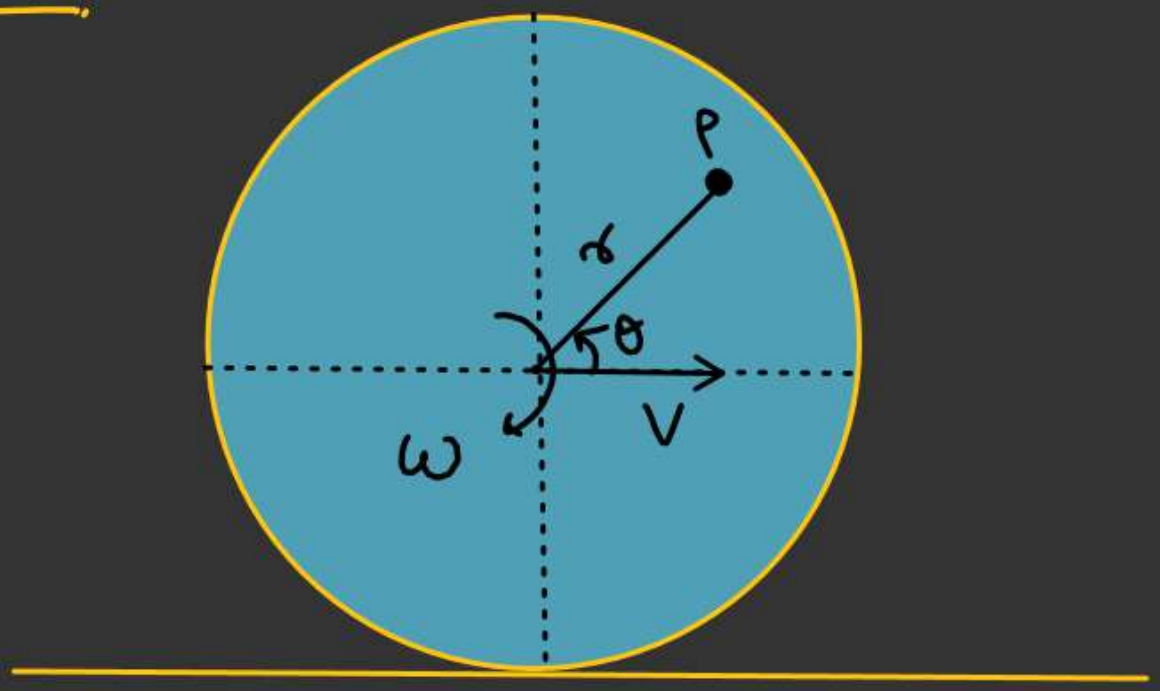
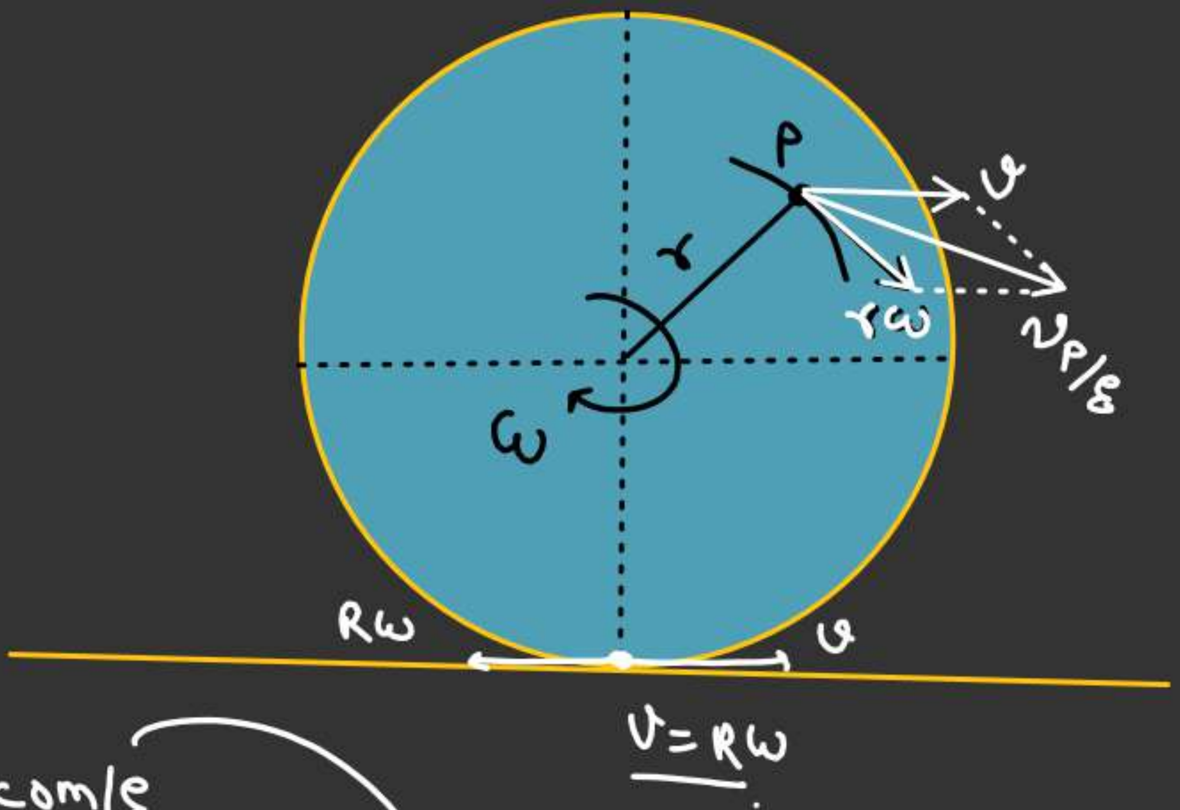
Rolling Motion

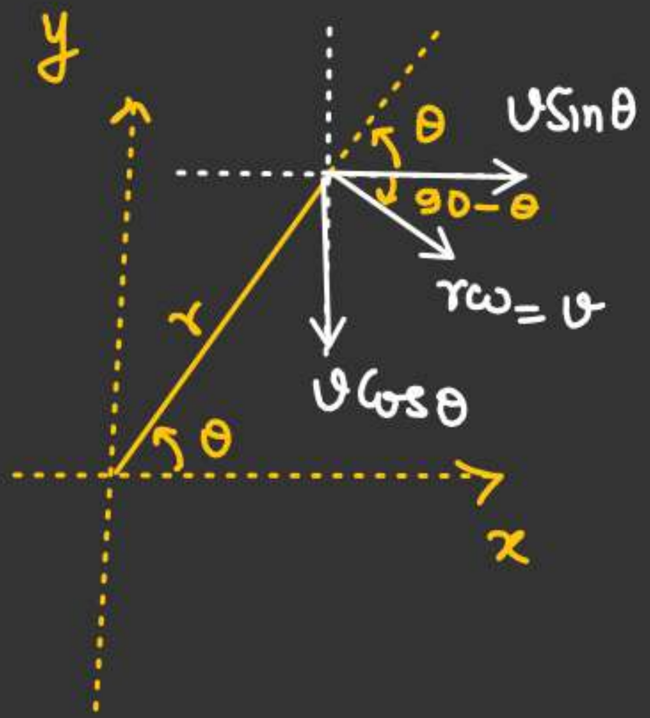
General velocity vector of any point on the body performing pure Rolling

$V = R\omega$ (Pure Rolling)

Case 1:-







$$\vec{v}_{P/E} = \vec{v}_{P/com} + \vec{v}_{com/E}$$

$$= \left(v\sin\theta\,\hat{i} - v\cos\theta\,\hat{j}\right) + v\hat{i}$$

$$\vec{v}_{P/E} = (v\sin\theta + v)\hat{i} - (v\cos\theta)\hat{j}$$

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Rolling Motion

\#4 :

$$\vec{v}_{P/E} = (v\sin\theta + v)\hat{i} - (v\cos\theta)\hat{j}$$

$$|\vec{v}_{P/E}| = \sqrt{v^2(1+\sin\theta)^2 + v^2\cos^2\theta}$$

$$= v\sqrt{1+\sin^2\theta + 2\sin\theta + \cos^2\theta}$$

$$= v\sqrt{2+2\sin\theta}$$

$$= \sqrt{2}\,v\left(\sqrt{1+\sin\theta}\right) \checkmark$$

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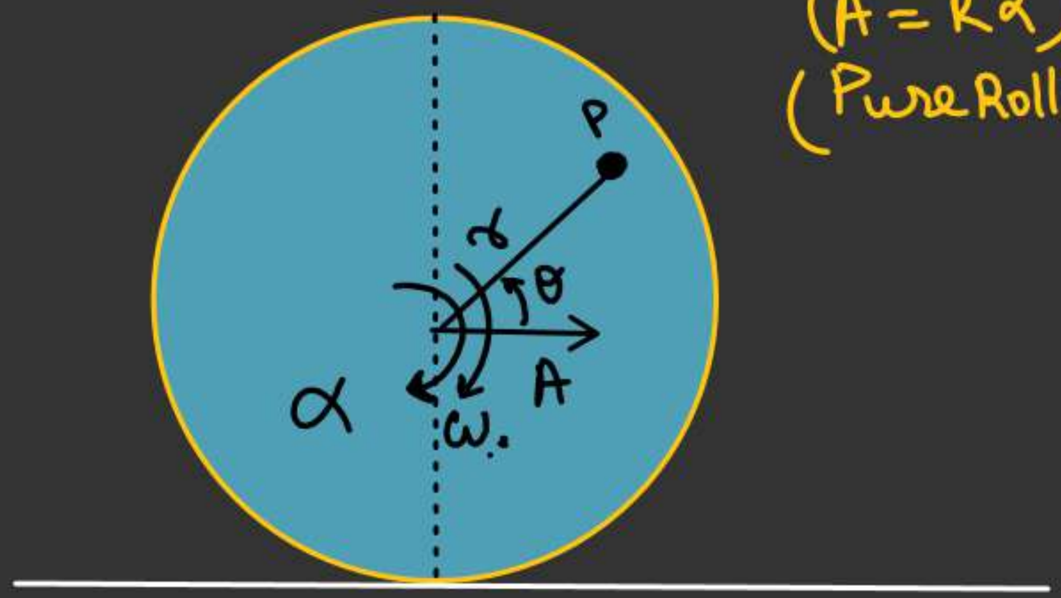
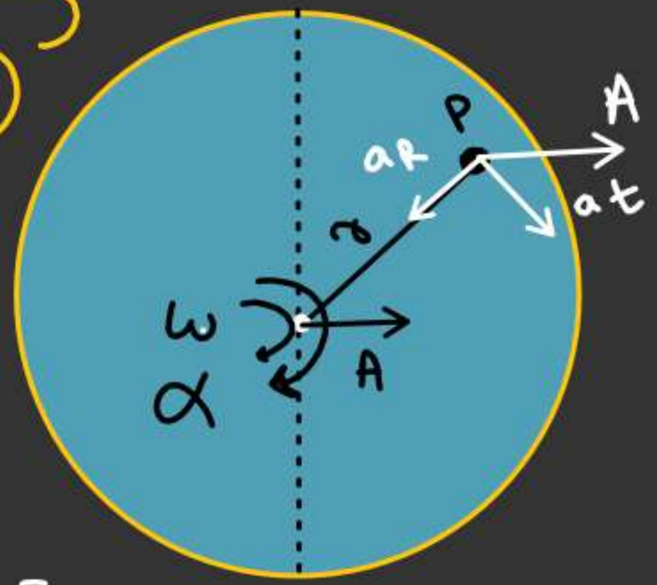
Rolling Motion

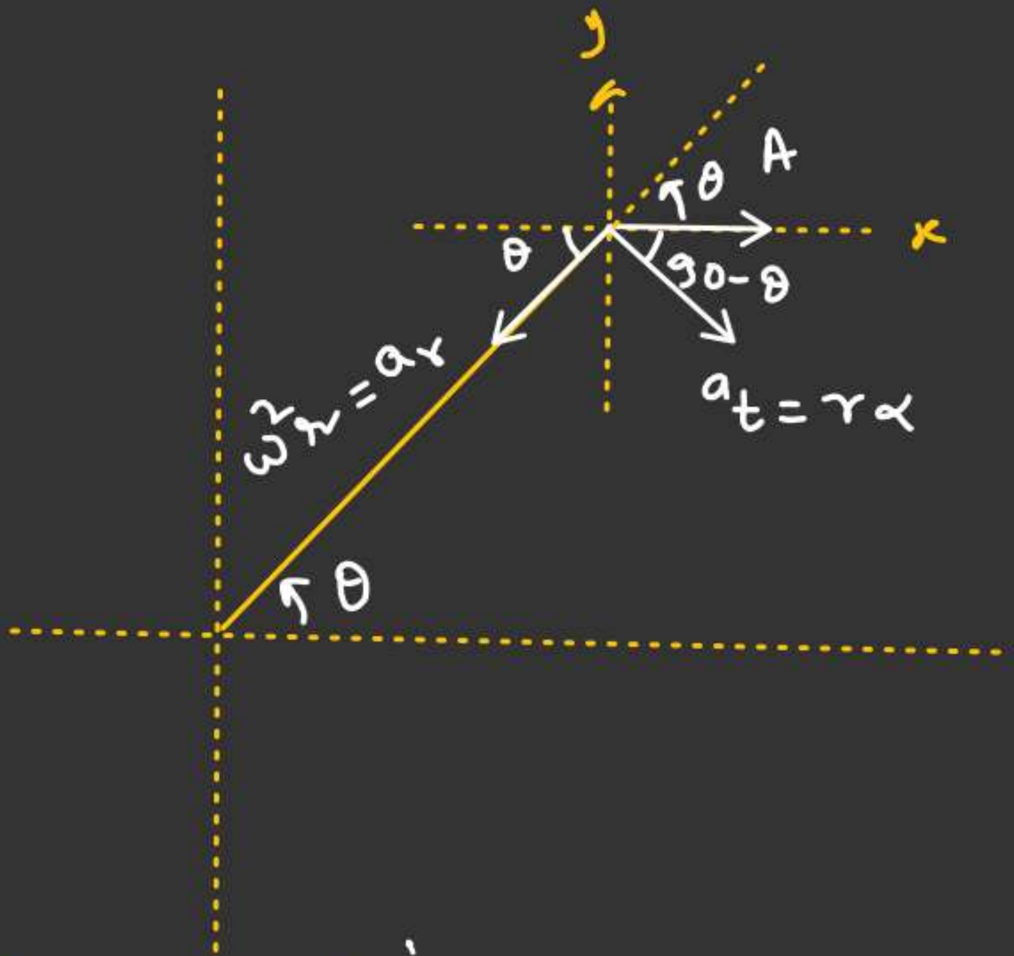
Case - 2

A & α Constant

$(A = R\alpha)$ (Pure Rolling)







$$a_R = \omega^2 r$$
$$a_t = r\alpha$$

$$\vec{a}_{P/E} = (\vec{a}_{P/E})_x + (\vec{a}_{P/E})_y$$

$$= (A + \underbrace{r\alpha}_{A} \sin\theta - \omega^2 r\cos\theta)\hat{i}$$

$$- (r\alpha\cos\theta + \underbrace{a_r}_{\omega^2 r}\sin\theta)\hat{j}$$

$$(\vec{a}_{P/E})_x = (A + a_t\sin\theta - a_R\cos\theta)\hat{i}$$

$$= (A + r\alpha\sin\theta - \omega^2 r\cos\theta)\hat{i}$$

$$(\vec{a}_{P/E})_y = -(r\alpha\cos\theta + \omega^2 r\sin\theta)\hat{j}$$

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Rolling Motion

H.W :

H.C.V

Rotational Mechanics

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Q. No – (21) ⟶ (32) → (Torque)

Q. No → (46) ⟶ (64) → (A.M)
+
(A.M.C)