DPP – 6

Solution

1. Tension will be maximum at lowest position.

$T_{max} - mg = m\omega^2 R$

$\Rightarrow 30 - 0.5 \times 10 = 0.5\omega^2 \times 2$

$\omega^2 = \dfrac{25}{0.5 \times 2} \Rightarrow \omega = 5\text{rad/sec.}$

2. Tension at lowest point = 3mg

So, velocity at lowest point

$T - mg = \dfrac{mv^2}{\ell} \Rightarrow 3mg - mg$

$= \dfrac{mv^2}{\ell} \Rightarrow v = \sqrt{2g\ell}$

Velocity at maximum displacement is zero.

$\dfrac{1}{2}mv^2 = mg\ell(1 - \cos\theta)$

$\Rightarrow \cos\theta = 0 \Rightarrow \theta = 90°$

3. (i) minimum velocity at lowest point to complete the circular motion is $v = \sqrt{5gr}$

energy conservation

$\dfrac{1}{2}mv^2 = mgh$

$\dfrac{1}{2}m\left(\sqrt{5gr}\right)^2 = mgh$

$\Rightarrow h = \dfrac{5}{2}r$

(ii) h is double then velocity at h position is

$mg2h - mg\,2r = \frac{1}{2}mv^2$ (Energy conservation)

$v = \sqrt{6gr}$

Normal at highest point.

$F_R = N + mg = \dfrac{m\left(\sqrt{6gr}\right)^2}{r}$

$F_R = 6mg$

4. Using energy conservation:

$\dfrac{1}{2}mv_B^2 = mgh$

$$v_B = \sqrt{\frac{2mgh}{m}}$$

$$v_B = \sqrt{2hg} \qquad \text{... (i)}$$

Also, to complete vertical circle

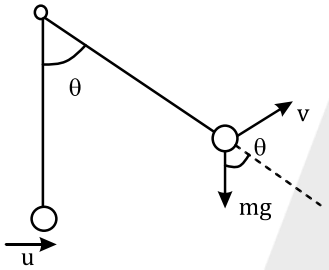
$$v_B = \sqrt{5gR} \qquad \text{... (ii)}$$

$$\therefore R = \frac{2}{5}h = 2cm$$

5. For circular motion in vertical plane normal reaction is minimum at highest point and it is zero, minimum speed of motorbike is

$$mg = \frac{mv^2}{R} \Rightarrow v = \sqrt{gR}$$

6.



$$T - mg\cos\theta = \frac{mv^2}{r} \qquad \text{... (i)}$$

energy conservation.

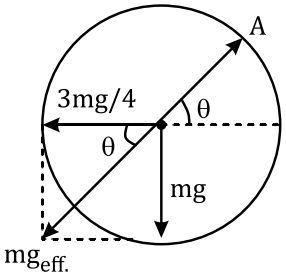
$$\frac{1}{2}mu^2 = \frac{1}{2}mv^2 + mgr(1-\cos\theta)$$

From (i) and (ii)

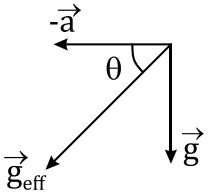
$$T = \frac{mu^2}{r} + 3mg\cos\theta - 2mg$$

For $\theta = 30° \ \& \ 60° \Rightarrow T_1 > T_2$

7.



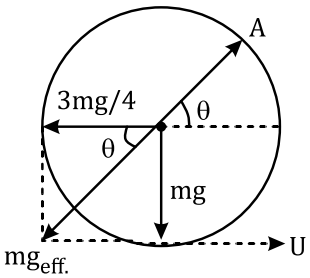
$$\vec{g}_{eff} = \vec{g} - \vec{a}$$



Tension would be minimum when its (tension) is along $\vec{g}_{eff}$

$$\tan\theta = \frac{mg}{\frac{3}{4}mg} = \frac{4}{3} \qquad \therefore \theta = 53°$$

8.

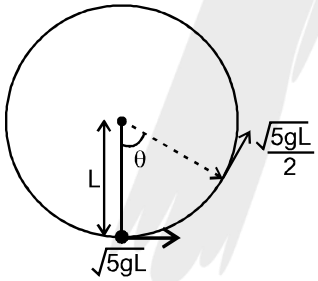


$$V_{min} = \sqrt{\ell g_{eff}} = \sqrt{\ell\frac{5}{4}g} = \sqrt{\frac{5\ell g}{2}}$$

9. $T_{max} = 6\ mg_{eff}$ $\left(g_{eff} = \frac{5}{4}g\right)$

$= \frac{15}{2}mg$

10.



By energy conservation,

$$\frac{1}{2}mu^2 = \frac{1}{2}mv^2 + mg\ell(1-\cos\theta)$$

$$V^2 = U^2 - 2g(L - L\cos\theta)$$

$$\frac{5gL}{4} = 5gL - 2gL(1-\cos\theta)$$

$$5 = 20 - 8 + 8\cos\theta$$

$$\cos\theta = -\frac{7}{8}$$

$$\frac{3\pi}{4} < \theta < \pi$$