DPP-02

SOLUTION

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1. If in the expansion of $(x+a)^n$, then sum of the odd terms is A and that of even terms is B, then the value of $(x^2-a^2)^n$ is

(A) A^2+B^2 (B) A^2-B^2 (C) $4AB$ (D) $2(A-B)$

Ans. **(B)**

Sol. As given, $(x+a)^n = A+B \Rightarrow (x-a)^n = A-B$

$\therefore (x^2-a^2)^n = (x+a)^n(x-a)^n = (A+B)\cdot(A-B) = A^2-B^2$

2. In the expansion of $(x+1/x)^{2n}(n \in N)$, the middle term is

(A) $\frac{1\cdot3\cdot5.....(2n-1)}{\lfloor n}2^n$ (B) $\frac{\lfloor 2n}{\lfloor n}$ (C) $\frac{\lfloor 2n}{\lfloor n}2^n$ (D) $\frac{\lfloor n}{\lfloor 2n}$

Ans. **(A)**

Sol. Middle term is $\left(\frac{2n}{2}+1\right)^{th}$ term.

required term $= T_{n+1} = {}^{2n}C_n \cdot \frac{1\cdot2\cdot3\cdot4\cdot...\cdot n(n+1)\cdot...\cdot(2n)}{\lfloor n \lfloor n}$

$= \frac{[1\cdot3,5\cdot...\cdot(2n-1)](2,4,6\cdot...\cdot2n)}{\lfloor n\lfloor n}$

$= \frac{1\cdot3\cdot5\cdot...\cdot(2n-1)}{\lfloor n}2^n$

3. The value of $(\sqrt{2}+1)^6+(\sqrt{2}-1)^6$ is equal to

(A) 198 (B) -198 (C) 99 (D) -99

Ans. **(A)**

Sol. Exp. $= 2$ [the sum of odd terms in the expansion of $(\sqrt{2}+1)^6$] $= 2[T_1+T_3+T_5+T_7]$

$= 2\left[{}^6C_0(\sqrt{2})^6 + {}^6C_2(\sqrt{2})^4 + {}^6C_4(\sqrt{2})^2 + {}^6C_6\right]$

$= 2[8+60+30+1] = 198$

4. If in the expansion of $(x^4-1/x^3)^{15}, x^{-17}$ occurs in the nth term, then

(A) $r=10$ (B) $r=11$ (C) $r=12$ (D) $r=13$

Ans. **(C)**

Sol. If x^{-17} occurs in T_{p+1}, then using formula

$p = \frac{15(D)-(-17)}{4+3} = 11$

$r = p+1 = 11+1 = 12.$

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5. The value of the middle term in the expansion of $\left(\frac{x\sqrt{y}}{3}-\frac{3}{y\sqrt{x}}\right)^{12}$

(A) $C(12,7)x^3y^{-3}$ (B) $C(12,6)x^3y^{-3}$ (C) $C(12,7)x^{-3}y^3$ (D) $C(12,6)x^{-3}y^3$

Ans. (B)

Sol. Middle term $= T_{6+1} = {}^{12}C_6\left(\frac{x\sqrt{y}}{3}\right)^6\left(-\frac{3}{y\sqrt{x}}\right)^6 = C(12,6)x^3y^{-3}$

6. If the sum of all the coefficients in the expansion of $\left(x^{3/2}+x^{-1/3}\right)^n$ is 128 , then the coefficient of x^5 will be

(A) 7 (B) 21 (C) 35 (D) 45

Ans. (C)

Sol. $2^n = 128 \Rightarrow n = 7$. If x^5 occurs in T_{r+1}, then

$r = \dfrac{7(3/2)-5}{3/2+1/3} = 3.$

$\Rightarrow$ reqd. coef $= {}^7C_3 = 35.$

7. The value of $(\sqrt{3}+1)^{2n}-(\sqrt{3}-1)^{2n}, n \in \mathbf{N}$ is

(A) an irrational number (B) a rational number

(C) an even integer (D) an odd integer

Ans. (A)

Sol. In the expansion of the expression every term contains odd powers of $\sqrt{3}$, hence its value is an irrational number.

8. If $n \in N$, then $(\sqrt{5}+1)^{2n+1}-(\sqrt{5}-1)^{2n+1}$ is

(A) an even integer (B) an odd integer

(C) irrational (D) rational

Ans. (A)

Sol. Exp. $= 2[T_2+T_4+\cdots+T_{2n+2}]$

$= 2\left[{}^{2n+1}C_1(\sqrt{5})^{2n}+{}^{2n+1}C_3(\sqrt{5})^{2n-2}+\cdots+{}^{2n+1}C_{2n+1}(\sqrt{5})^0\right]$

$= 2[$ an integer $]$

$=$ an even integer.

9. The middle term in the expansion of $(1-3x+3x^2-x^3)^6$ is

(A) ${}^{18}C_{10}x^{10}$ (B) ${}^{18}C_9(-x)^9$ (C) ${}^{18}C_0x^9$ (D) $-{}^{18}C_{10}x^{10}$

Ans. (B)

Sol. Exp. $= [(1-x)^3]^6 = (1-x)^{18}$.

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10. The number of rational terms in the expansion of $\left(7^{1/3}+11^{1/9}\right)^{6561}$ is equal to

(A) 730 (B) 721 (C) 728 (D) none of these

Ans. **(A)**

Sol. $T_{r+1}={}^{6561}C_r\cdot\left(7^{1/3}\right)^{6561-r}\cdot\left(11^{1/9}\right)^r={}^{6561}C_r\cdot 7^{2187}\cdot 7^{-r/3}\cdot 11^{r/9}, 0\le r\le 6561.$

Now T_{r+1} will be rational when $r/3$ and $r/9$ both are integers. This is possible only when r is a multiple of 9 . But multiples of 9 from 0 to 6561 are $0,9,18,27,\ldots,6561$.

These are in AP, so their number n is given by

$6561=0+(n-1)9\Rightarrow n=730.$

11. If $a=(\sqrt{3}+1)^7$, then [a] is equal to (where [ ] is the greatest integer function)

(A) 1138 (B) 1137 (C) 1136 (D) 968

Ans. **(C)**

Sol. $(\sqrt{3}+1)^7-(\sqrt{3}-1)^7=2[T_2+T_4+T_6+T_8]$

$=2\left[{}^7C_1(\sqrt{3})^6+{}^7C_3(\sqrt{3})^4+{}^7C_5(\sqrt{3})^2+{}^7C_7(\sqrt{3})^0\right]$

$=2[189+315+63+1]=1136$

$\Rightarrow(\sqrt{3}+1)^7=1136+(\sqrt{3}-1)^7$

$\Rightarrow[a]=1136.[\because 0<\sqrt{3}-1<1]$

12. The remainder when 2^{2000} is divided by 17 is

(A) 8 (B) 11 (C) 2 (D) 1

Ans. **(D)**

Sol. $2^{2000}=(2^4)^{500}=(17-1)^{500}$

$=17^{500}-{}^{909}C_1.17^{499}+{}^{500}C_2.17^{496}-\cdots-{}^{500}C_{499}(17)+(-1)^{500}$

$=($ a multiple of $17)+1$

$\therefore$ remainder $=1.$

13. In the expansion of $(4-3x)^7$ when $x=2/3$, the numerically greatest term is

(A) T_4 (B) T_3 (C) T_5^4 (D) none of these

Ans. **(B)**

Sol. Here numerically $x=4,\ a=3(2/3)=2, n=7.$

$\frac{(n+1)a}{x+a}=\frac{8\cdot 2}{4+2}=\frac{8}{3}=2\frac{2}{3}.$

$\Rightarrow$ the numerically greatest term $=T_{2+1}=T_3.$

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14. If a_1, a_2, a_3, a_4 are coefficients of T_2, T_3, T_4 and T_5 respectively in $(1+x)^n$; then $\frac{a_1}{a_1+a_2} + \frac{a_3}{a_3+a_4}$ equals

(A) $\frac{a_2}{a_2+a_3}$ (B) $\frac{2a_2}{a_2+a_3}$ (C) $\frac{-a_2}{a_2+a_3}$ (D) $\frac{a_2}{2(a_2+a_3)}$

Ans. **(B)**

Sol. If y occurs in T_{r+1}, then $r = \frac{5(B)-1}{2+1} = 3$

$\therefore$ coef. of $y = {}^5C_3(c)^3 = 10c^3$.

15. In the expansion of $\left(\frac{1}{2}x^{1/3} + x^{-1/5}\right)^8$, the term independent of x is

(A) T_5 (B) T_6 (C) T_7 (D) T_8

Ans. **(B)**

Sol. As above $r = \frac{8(1/3)-0}{1/3+1/5} = 5$.

So required term $= T_{5+1} = T_6$.

16. The value of $(\sqrt{5}+1)^5 - (\sqrt{5}-1)^5$ is

(A) 352 (B) 252 (C) 452 (D) 532

Ans. **(A)**

Sol. Exp $= 2[T_2 + T_4 + T_6]$

$= 2\left[{}^5C_1(\sqrt{5})^4 + {}^5C_3(\sqrt{5})^2 + {}^5C_5(\sqrt{5})^0\right] = 2[125+50+1] = 352$

17. If three consecutive coefficients in the expansion of $(1+x)^n$ are 28,56 and 70 , then the value of n is

(A) 4 (B) 6 (C) 8 (D) 10

Ans. **(C)**

Sol. ${}^nC_{r-1} = 28,\ {}^nC_r = 56,\ {}^nC_{r+1} = 70$

$\frac{{}^nC_r}{{}^nC_{r-1}} = \frac{n-r+1}{r} = 2 \Rightarrow \frac{{}^nC_{r+1}}{{}^nC_r} = \frac{n-r}{r+1} = \frac{5}{4}$

They given $n+1 = 3r$ and $4n+9r = 5$.

18. The number of terms in the expansion of $[(x+3y)^2(3x-y)^2]^3$ is

(A) 14 (B) 28 (C) 32 (D) 56

Ans. **(B)**

Sol. Exp. $= [(x+3y)(3x-y)]^6 = (3x^2+8xy-3y)^6$.

Now since number of terms in the expansion of

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$(a+b+c)^n = \frac{(n+1)(n+2)}{2}$.

$\therefore$ In the given expansion number of terms $= \frac{7\times 8}{2} = 28$.

19. The middle term in the expansion of $(1+x)^{2n} (n \in \mathbf{N})$ is

(A) $\frac{1\cdot 3\cdot 5.....\cdot(2n+1)}{\lfloor n} 2^n x^n$

(B) $\frac{1\cdot 3\cdot 5.....\cdot(2n-1)}{\lfloor n} 2^n x^n$

(C) $\frac{1\cdot 3\cdot 5.....\cdot(2n-1)}{\lfloor n} 2^{n-1} x^n$

(D) $\frac{1\cdot 3\cdot 5.....\cdot(2n+1)}{\lfloor n} 2^{n+1} x^n$

Ans. **(B)**

Sol. Middle term $= T_{n+1} = {}^{2n}C_n x^n$

$$= \frac{1\cdot 2\cdot 3\cdot 4\cdot n(n+1)\cdot \ldots\cdot(2n-1)\cdot 2n}{\lfloor n \lfloor n} x^n$$

$$= \frac{1\cdot 3\cdot 5\cdot \ldots\cdot (2n-1)[2\cdot 4\cdot 6\cdot \ldots\cdot 2n]}{\lfloor n \lfloor n} x^n$$

$$= \frac{1\cdot 3\cdot 5\cdot \ldots\cdot (2n-1)2^n \lfloor n}{\lfloor n \lfloor n} x^n = \frac{1\cdot 3\cdot 5\cdot \ldots\cdot (2n-1)}{\lfloor n} 2^n x^n.$$

20. In the expansion of $\left(\frac{2\sqrt{x}}{5} - \frac{1}{2x\sqrt{x}}\right)^{11}$, the term independent of x is

(A) no term (B) 5th term (C) 6th term (D) 11th term

Ans. **(A)**

Sol. If T_{n-1} is the required term, then $r = \frac{11(1/2)}{1/2+3/2} = \frac{11}{4} \notin N$.

21. In the expansion of $\left(x^2 - \frac{1}{3x}\right)^9$, the term without x is equal to

(A) 28/81 (B) $-28/243$ (C) 28/243 (D) none of these

Ans. **(C)**

Sol. If T_{r+1} is the term without x, then $r = \frac{9(2)}{2+1} = 6$.

$\therefore$ required term $= T_7 = {}^9C_6 (x^2)^3 \left(-\frac{1}{3x}\right)^6 = \frac{9\cdot 8\cdot 7}{1\cdot 2\cdot 3}\cdot\frac{1}{3^6} = \frac{28}{243}$.

22. The middle term in the expansion of $\left(\frac{2x^2}{3} + \frac{3}{2x^2}\right)^{10}$ is

(A) 251 (B) 252 (C) 250 (D) none of these

Ans. **(B)**

Sol. Here $n = 10$ is even; so that middle term $= 10/2 + 1 = 6$ th term.

$\therefore$ Middle term $= T_6 = {}^{10}C_3 \left(\frac{2x^2}{3}\right)^5 \left(\frac{2}{2x^2}\right)^5 = \frac{10\cdot 9\cdot 8\cdot 7\cdot 6}{1\cdot 2\cdot 3\cdot 4\cdot 5} = 252$.

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23. In how many terms in the expansion of $\left(x^{1/5}+y^{1/10}\right)^{55}$ do not have fractional power of the variable

(A) 6 (B) 7 (C) 8 (D) 10

Ans. **(A)**

Sol. $T_{r+1}={}^{15}C_r\left(x^{1/5}\right)^{55-r}\left(y^{1/10}\right)^r={}^{55}C_r x^{(55-n)/5}y^{r\backslash 10}$.

Now powers of x and y are not fractional when $r=0,10,20,30,40,50$. Hence six terms do not have fractional power.

ANSWER KEY

1.	(B)	**2.**	(A)	**3.**	(A)	**4.**	(C)	**5.**	(B)	**6.**	(C)	**7.**	(A)
8.	(A)	**9.**	(B)	**10.**	(A)	**11.**	(C)	**12.**	(D)	**13.**	(B)	**14.**	(B)
15.	(B)	**16.**	(A)	**17.**	(C)	**18.**	(B)	**19.**	(B)	**20.**	(A)	**21.**	(C)
22.	(B)	**23.**	(A)										