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1. Examine whether the following functions are even or odd or none.

(i) $f(x) = \log\left(x + \sqrt{1 + x^2}\right)$

Ans. odd

Sol. $f(x) = \log\left(x + \sqrt{1 + x^2}\right)$

$f(-x) = \log\left(-x + \sqrt{1 + x^2}\right)$

$f(-x) = \log\left[\left(\sqrt{1 + x^2} - x\right)\left(\frac{\sqrt{1 + x^2} + x}{\sqrt{1 + x^2} + x}\right)\right]$

$f(-x) = \log\left[\frac{1 + x^2 - x^2}{\sqrt{1 + x^2} + x}\right]$

$f(-x) = \log\left[\frac{1}{\sqrt{1 + x^2} + x}\right]$

$f(-x) = -\log\left(\sqrt{1 + x^2} + x\right)$

$\Rightarrow f(-x) = -f(x)$

Hence, f is an odd function.

(ii) $f(x) = \frac{x(a^x+1)}{a^x-1}$

Ans. **even**

Sol. $f(x) = \frac{x(a^x+1)}{a^x-1}$

$f(-x) = \frac{-x(a^{-x} + 1)}{a^{-x} - 1}$

$\Rightarrow f(-x) = \frac{-x(a^x + 1)}{1 - a^x}$

$\Rightarrow f(-x) = \frac{x(a^x + 1)}{a^x - 1}$

$\Rightarrow f(-x) = f(x)$

Hence, f is an even function.

(iii) $f(x) = \frac{x}{e^x-1} + \frac{x}{2} + 1$

Ans. **even**

Sol. Given $f(x) = \frac{x}{e^x-1} + \frac{x}{2} + 1$

$\Rightarrow f(x) = \frac{x+xe^x}{2(e^x-1)} + 1$

first of all find out the value off $(-x)$

$f(-x) = \frac{-x}{e^{-x} - 1} + \frac{-x}{2} + 1$

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$\Rightarrow f(-x)=\frac{-x}{\frac{1}{e^x}-1}+\frac{-x}{2}+1 \Rightarrow f(-x)=\frac{-xe^x}{1-e^x}-\frac{x}{2}+1$

$\Rightarrow f(-x)=\left[\frac{(x+xe^x)}{2(e^x-1)}\right]+1 \Rightarrow f(-x)=f(x)$

since, $f(-x)=f(x)$ which imply that the given function is even.

(iv) $f(x)=\frac{(1+2^x)^7}{2^x}$

Ans. neither even nor odd

Sol. $f(x)=\frac{(1+2^x)^7}{2^x}$

$$f(-x)=\frac{(1+2^{-x})^7}{2^{-x}}=\frac{(2^x+1)^7/(2^x)^7}{y/(2^x)}\neq f(x)$$

(v) $f(x)=\frac{\sec x+x^2-9}{x\sin x}$

Ans. **even**

Sol. $f(-x)=\frac{\sec(-x)+x^2-9}{-x\cdot\sin(-x)}$

$$=\frac{\sec x+x^2-9}{x\cdot\sin x}$$
$=f(x)$

Hence, f is an even function.

(vi) $f(x)=\sqrt{1+x+x^2}-\sqrt{1-x+x^2}$

Ans. **odd**

Sol. $f(-x)=\sqrt{1-x+x^2}-\sqrt{1+x+x^2}$

$=-\left(\sqrt{1+x+x^2}-\sqrt{1-x+x^2}\right)$
$=-f(x)$

Hence, f is an odd function.

(vii) $f(x)=\begin{cases} x|x| & , \quad x\leq -1 \\ [1+x]-[x-1] & , \quad -1<x<1 \\ -x|x| & , \quad x\geq 1 \end{cases}$

Ans. **even**

Sol. $f(x)=\begin{cases} -x^2, & x\leq -1 \\ 1+[x]-(y)+1, & -1\leq x<1 \\ -x^2, & x\geq 1 \end{cases}$

$f(-x)=f(x)$

Hence, f is even function

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(viii) $f(x)=\frac{2x(\sin x+\tan x)}{2\left[\frac{x+2\pi}{\pi}\right]-3}$

where [∗] denotes greatest integer function.

Ans. **odd**

Sol. $f(-x)=\frac{-2x(-\sin x-\tan x)}{2\left[\frac{-x+2\pi}{\pi}\right]-3}$

$$=\frac{2x(\sin x+\tan x)}{2\left[-\frac{x}{\pi}+2\right]-3}$$

$$=\frac{2x(\sin x+\tan x)}{4+2\left[\frac{-x}{\pi}\right]-3}$$

$$=\frac{2x(\sin x+\tan x)}{1+2\left[-\frac{x}{\pi}\right]}=\frac{2x(\sin x+\tan x)}{1-2-2\left[\frac{x}{\pi}\right]}$$

$$=\frac{2x(\sin x+\tan x)}{-\left(1+2\left[\frac{x}{\pi}\right]\right)}$$

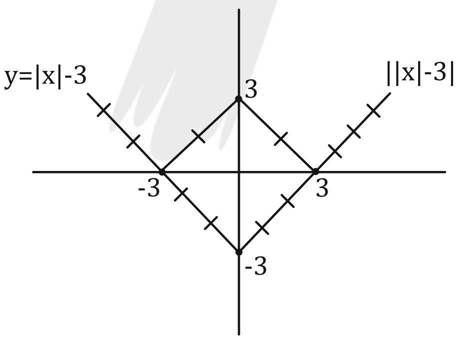
$$=-f(x)$$

Hence, f is odd function.

2. Make the graph of the following functions

(i) $f(x)=||x|-3|$

Sol. $f(x)=||x|-3|$



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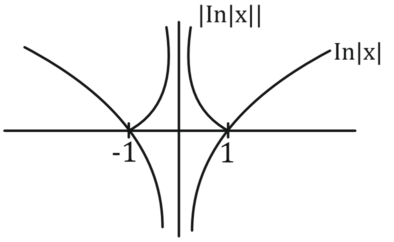
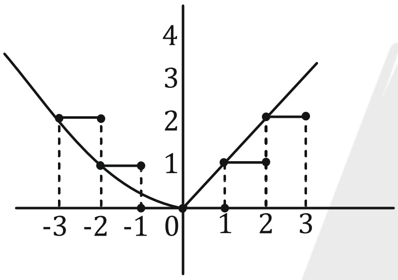
(ii) $f(x) = |\ln|x||$

Sol. $F(x) = |\ln|x||$

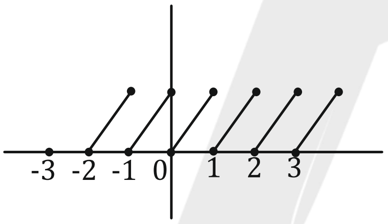


(iii) $f(x) = [|x|]$

Sol.



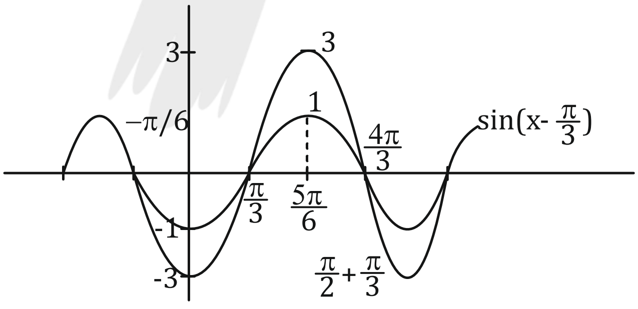
(iv) $f(x) = |\{x\}|$



Sol.

(v) $f(x) = 3\sin\left(x - \frac{\pi}{3}\right)$

Sol.



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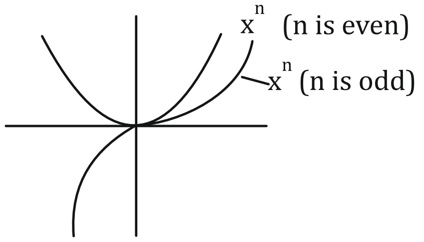
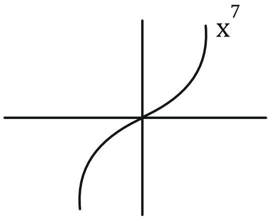
(vi) $f(x) = \frac{x^8}{x}$

Sol. $f(x) = \frac{x^8}{x} = x^7 (x \neq 0)$

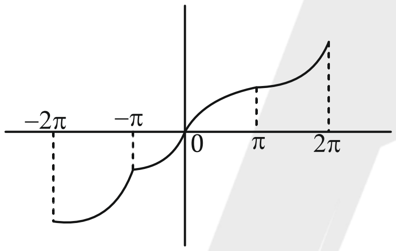




(vii) $f(x) = x + \sin x$

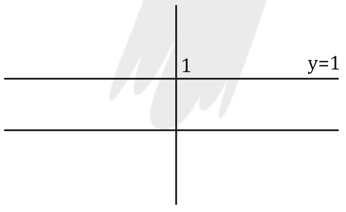
Sol. $f'(x) = 1 + \cos x$ $\quad$ $0 - \pi$

$0 \le 1 + \cos x \le 2$ $\quad$ $2 \to 0$



(viii) $f(x) = (\sin x)^0$

Sol. $f(x) = (\sin x)^0 = 1$



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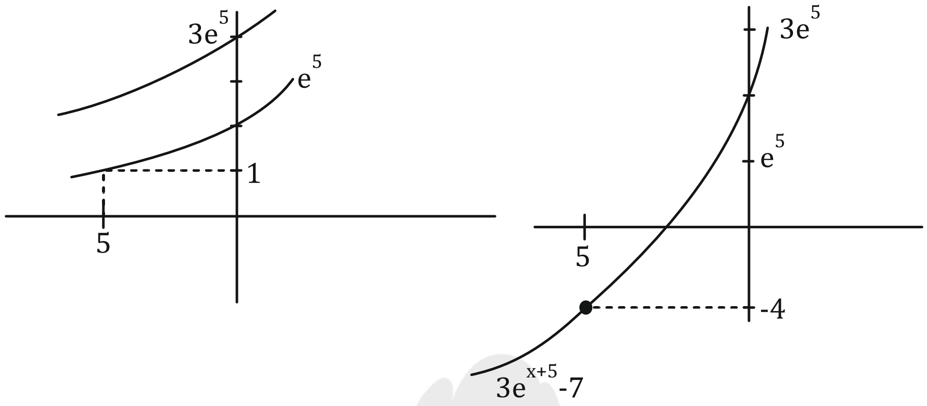
(ix) $f(x) = 3e^{x+5} - 7$

Sol.



$x = -5$

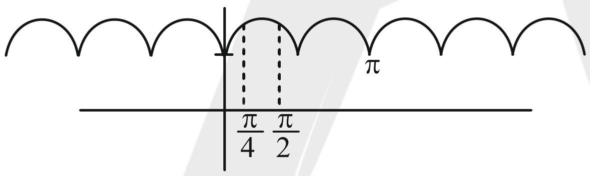
$f(-5) = 3 - 7 = -4$

(x) $f(x) = |\sin x| + |\cos x|$

Sol. $f(x) = |\sin x| + |\cos x|$, $x \in$ I quadrant.

$= \sin x + \cos x$

$= \sqrt{2}\sin\left(x + \frac{\pi}{4}\right)$



3. If $f(x) = \frac{4^x}{4^x+2}$, then show that $f(x) + f(1-x) = 1$

Sol. L.H.S $= f(x) + f(1-x)$

$$= \frac{4^x}{4^x+2} + \frac{4^{1-x}}{4^{1-x}+2} = \frac{4^x}{4^x+2} + \frac{4/4^x}{\frac{4}{4^x}+2}$$

$$= \frac{4^x}{4^x+2} + \frac{4}{4+2\cdot 4^x} = \frac{4^x}{4^x+2} + \frac{2}{2+4^x}$$
$$= \frac{4^x+2}{4^x+2} = 1 = \text{R.H.S}$$

4. Find the period of the following functions (where [∗] denotes greatest integer function)

(i) $f(x) = 2 + 3\cos(x-2)$

Ans. **2π**

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Sol. Note: $\underline{f+g} \longrightarrow \text{L.C.M.}(P_1, P_2)$

$f(x) = 2 + 3\cos(x-2)$

$\downarrow \qquad \downarrow$

$1 \qquad 2\pi$

L.C.M. $\to 2\pi$

(ii) $f(x) = \sin 3x + \cos^2 x + |\tan x|$

Ans. **2π**

Sol. $\downarrow \qquad \downarrow \qquad \downarrow$

$\frac{2\pi}{3} \qquad \frac{\pi}{1} \qquad \frac{\pi}{T}$

$= \frac{2\pi}{1}\ \frac{(\text{L.C.M of nemurator})}{(\text{H.C.F. of denominator})}$

$= 2\pi$

(iii) $f(x) = \sin\frac{\pi x}{4} + \sin\frac{\pi x}{3}$

Ans. **24**

Sol. $\downarrow \qquad \downarrow$

$\frac{2\pi}{\frac{\pi}{4}}, \qquad \frac{2\pi}{\frac{\pi}{3}}$

$8 \qquad 6$

Period=24

(iv) $f(x) = \cos\frac{3}{5}x - \sin\frac{2}{7}x.$

Ans. **70π**

Sol. $\frac{2\pi}{\frac{3}{5}} \qquad \frac{2\pi}{\frac{2}{7}}$

$\frac{10\pi}{3} \qquad \frac{14\pi}{2}$

$= \frac{70\pi}{1} = 70\pi$

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(v) $f(x) = [\sin 3x] + |\cos 6x|$

Ans. $\frac{2\pi}{3}$

Sol. $\frac{2\pi}{3}$ $\frac{\pi}{6}$

Period $= \frac{2\pi}{3}$

(vi) $f(x) = \frac{1}{1-\cos x}$

Ans. 2π

Sol. $= \frac{1}{2\sin^2 x/2}$

$= \frac{1}{2}\text{cosec}^2 x/2$

$\frac{\pi}{\frac{1}{2}} = 2\pi$

(vii) $f(x) = \frac{\sin 12x}{1+\cos^2 6x}$

Ans. $\frac{\pi}{6}$

Sol. $\sin 12x \to \frac{2\pi}{12}$

$\cos^2 6x \to \frac{\pi}{6}$

$\frac{\pi}{6}$

(viii) $f(x) = \sec^2 x + \text{cosec}^3 x$

Ans. 2π

Sol. L.C.M π 2π

2π

5. Find the period of the following functions.

(i) $f(x) = 1 - \frac{\sin^2 x}{1+\cot x} - \frac{\cos^2 x}{1+\tan x}$

Ans. π

Sol $f(x) = 1 - \frac{\sin^2 x}{1+\cot x} - \frac{\cos^2 x}{1+\tan x} = \frac{\sin 2x}{2}$

period is $\frac{2\pi}{2} = \pi$

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(ii) $f(x) = \log(2 + \cos 3x)$

Ans. $\frac{2\pi}{3}$

Sol. $\frac{2\pi}{3}$

(iii) $f(x) = \tan\frac{\pi}{2}[x]$,

where [∗] denotes greatest integer function

Ans. **2**

Sol. $f(x + T) = f(x)$

$$\tan\frac{\pi}{2}[x + T] = \tan\frac{\pi}{2}[x]$$

$$\frac{\pi}{2}[x + T] = n\pi + [x]$$

$$T = 2$$

(iv) $f(x) = e^{\ln \sin x} + \tan^3 x - \operatorname{cosec}(3x - 5)$

Ans. **2π**

Sol. $\sin x + \tan^3 x - \operatorname{cosec}(3x - 5)$

$\frac{2\pi}{1}$ $\frac{\pi}{1}$ $\frac{2\pi}{3}$

$$\text{L.C.M} = \frac{2\pi}{1} = 2\pi$$

(v) $f(x) = \frac{1}{2}\left(\frac{|\sin x|}{\cos x} + \frac{\sin x}{|\cos x|}\right)$

Ans. 2π

Sol. $\frac{|\sin x|}{\cos x} - \pi, 2\pi$

$$\frac{\sin x}{|\cos x|} - \pi, 2\pi$$

$$\text{L.C. M} = 2\pi$$

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Answer Key

1.	**(i)** odd	**(ii)** even	**(iii)** even	**(iv)** neither even nor odd
	(v) even	**(vi)** odd	**(vii)** even	**(viii)** odd
4.	**(i)** 2π	**(ii)** 2π	**(iii)** 24	**(iv)** 70π
	(v) $\frac{2\pi}{3}$	**(vi)** 2π	**(vii)** $\pi/12$	**(viii)** 2π
5.	**(i)** π	**(ii)** $\frac{2\pi}{3}$	**(iii)** 2	**(iv)** 2π
	(v) 2π			