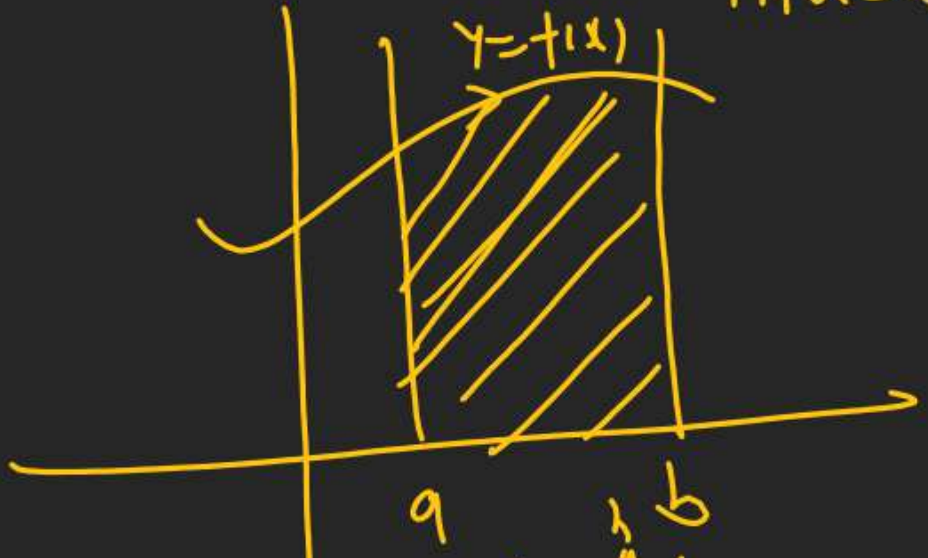
# Area under Curve. [2days]

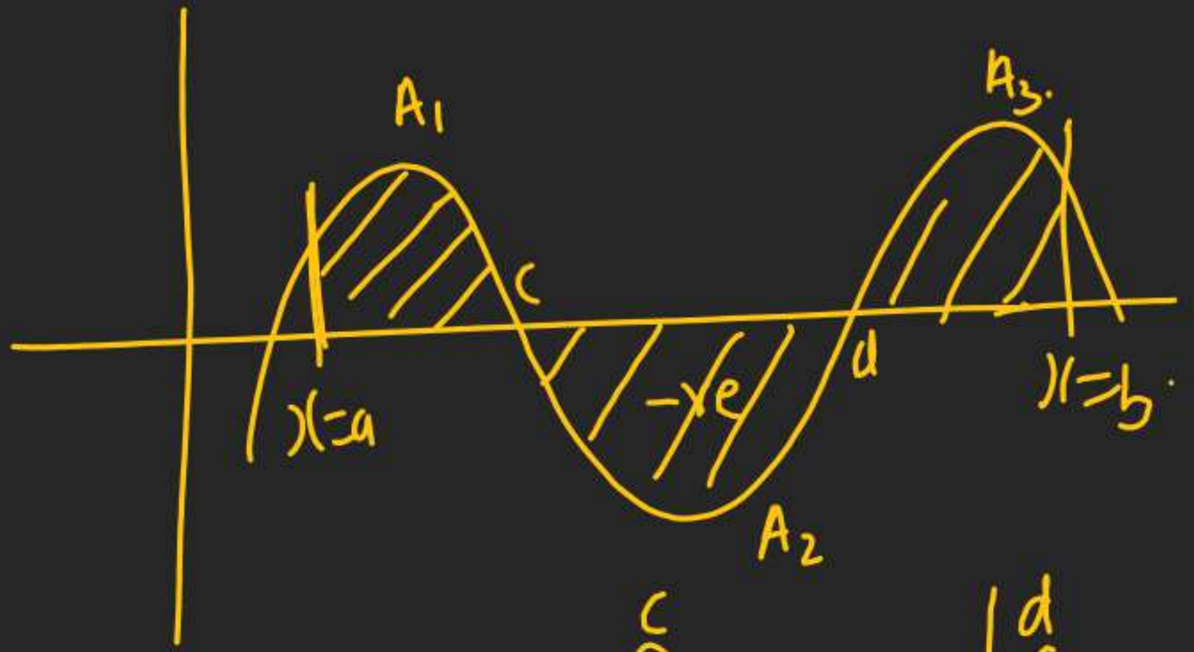
1Qs Sure.

① $\int_a^b f(x)\cdot dx$ = Algebric Sum of Area Under $y=f(x)$ in $x=a$, $x=b$ & x Axis




$A=\int_a^b f(x)dx$

Now in this Chapter. [Basic Graph]




$$\text{Area}=\int_a^c f(x)\,dx+\left|\int_c^d f(x)\cdot dx\right|+\int_d^b f(x)\cdot dx$$

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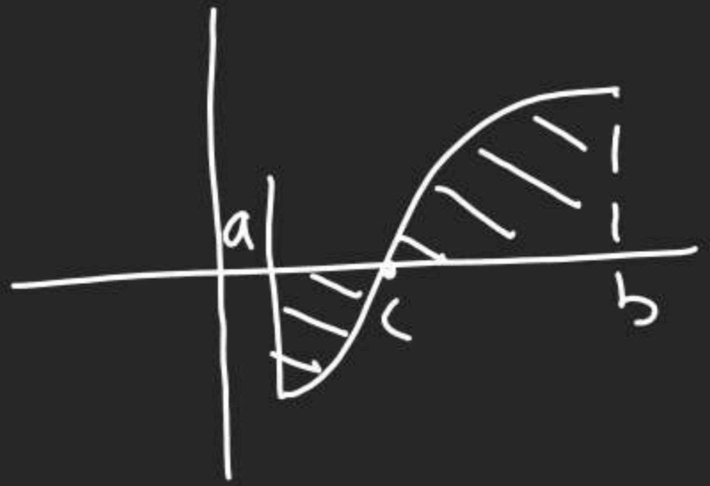
# Area Under Curve = AUC.

① Area Bounded betⁿ $y = f(x)$, $x = a$ & $x = b$, X Axis

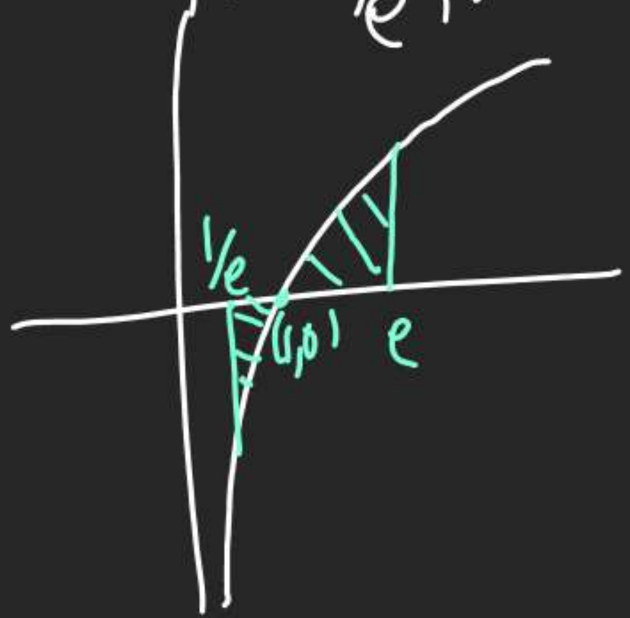



$$\text{Area} = \left|\int_a^c f(x)\,dx\right| + \int_c^b f(x)\,dx$$

$$\int y \cdot dx$$

ABB

Q $y = \ln x$, $x = 1/e$, $x = e$ & X Axis.




$$A = \left|\int_{1/e}^{1} \ln x \cdot dx\right| + \int_1^e \ln x\,dx$$

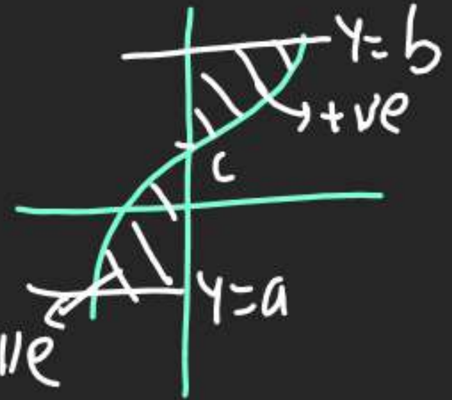
$$= \left|x\ln x - x\Big|_{1/e}^{1}\right| + x\ln x - x\Big|_1^e$$

$$\left|(-1) - \left(\frac{1}{e}\ln\frac{1}{e} - \frac{1}{e}\right)\right| + (e\cancel{\ln e} - \cancel{e}) - (-1)$$

$$\left|\frac{2}{e} - 1\right| + 1 = 1 - \frac{2}{e} + 1 = 2 - \frac{2}{e}.$$

(2) Area Bounded betⁿ $y = f(x)$

$y = a$, $y = b$ & Y Axis




$$\text{Area} = \left|\int_{y=a}^{y=c} x \cdot dy\right| + \int_c^b x \cdot dy$$

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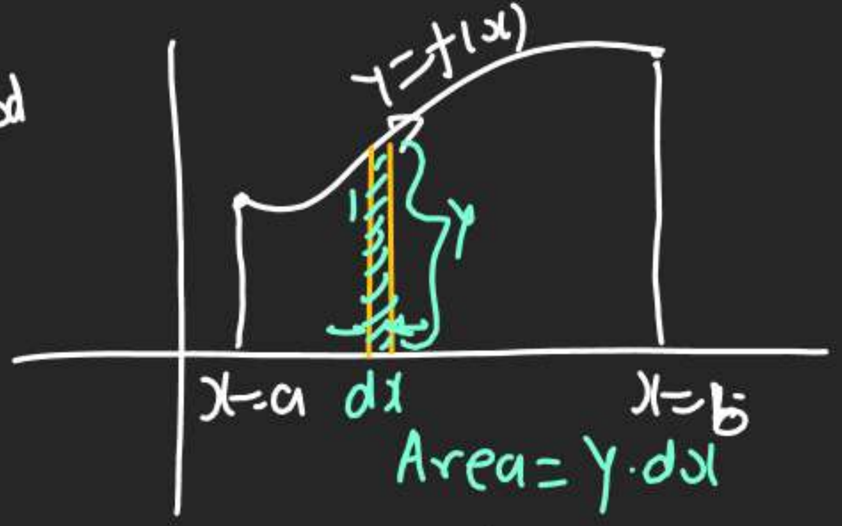
A) $y=f(x),\ x=a,\ x=b$, X Axis.

V.S Method



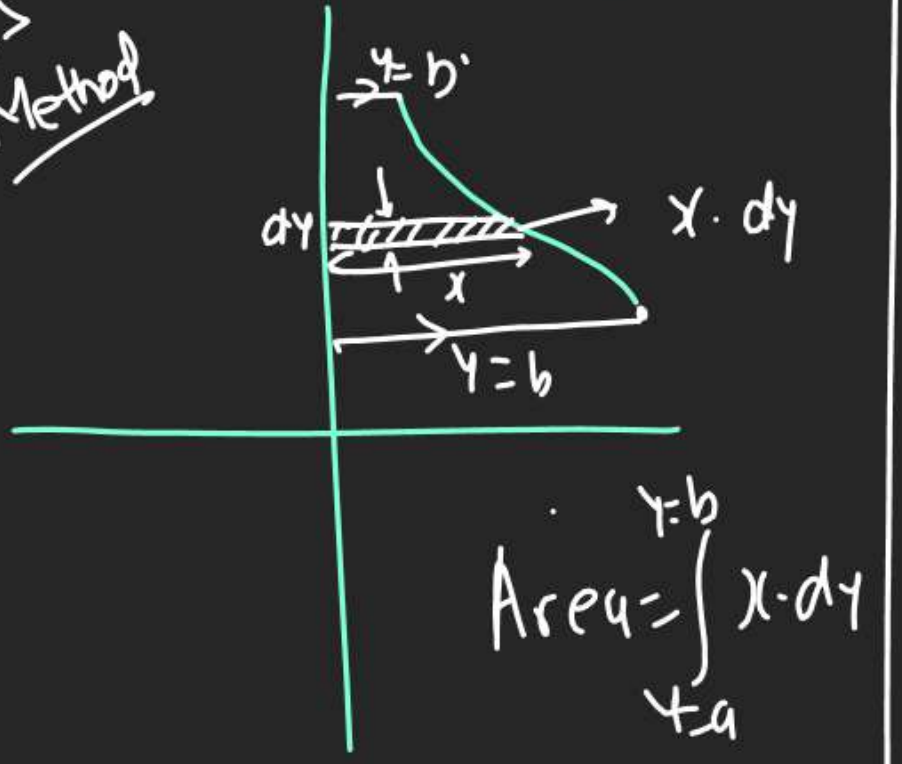

Area $= y \cdot dx$

$$\text{Area} = \int_a^b f(x)\,dx$$

Vertical strip

(B) $y=f(x),\ y=a,\ y=b$, y Axis

HS Method




$$\text{Area} = \int_{y=a}^{y=b} x \cdot dy$$

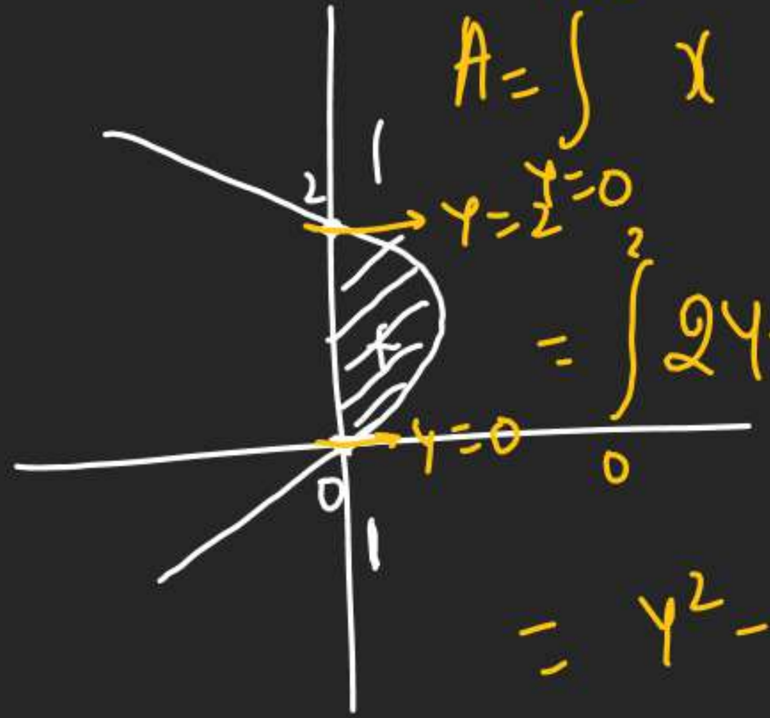
Horizontal strip.

$Q_2$ ABB.

$x = 2y - y^2$ & y Axis

$x = y(2-y)$

$= -(y)(y-2)$




$$A = \int_{y=0}^{y=2} x\ dy$$

$$= \int_0^2 2y - y^2 \cdot dy$$

$$= y^2 - \frac{y^3}{3}\Big|_0^2$$

$$= \left(4 - \frac{8}{3}\right) - 0$$

$$= \frac{4}{3}$$

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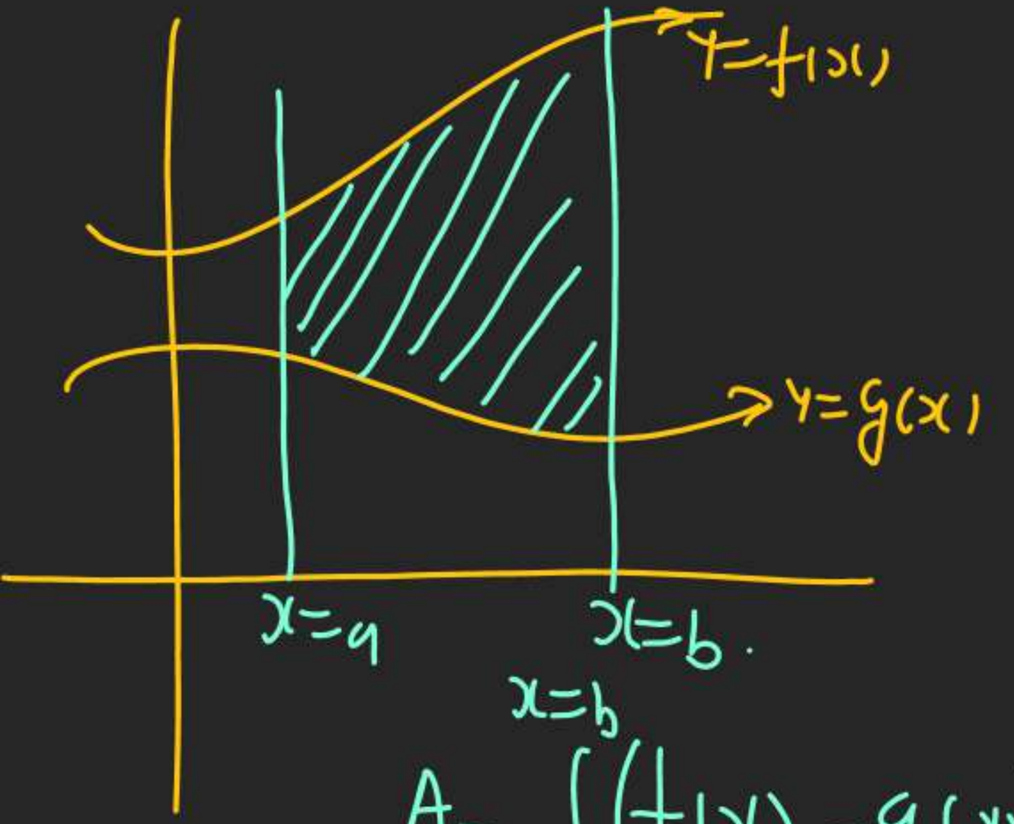
3) A.B.B 2 fxn.

$y=f(x),\ y=g(x),\ x=a,\ x=b$



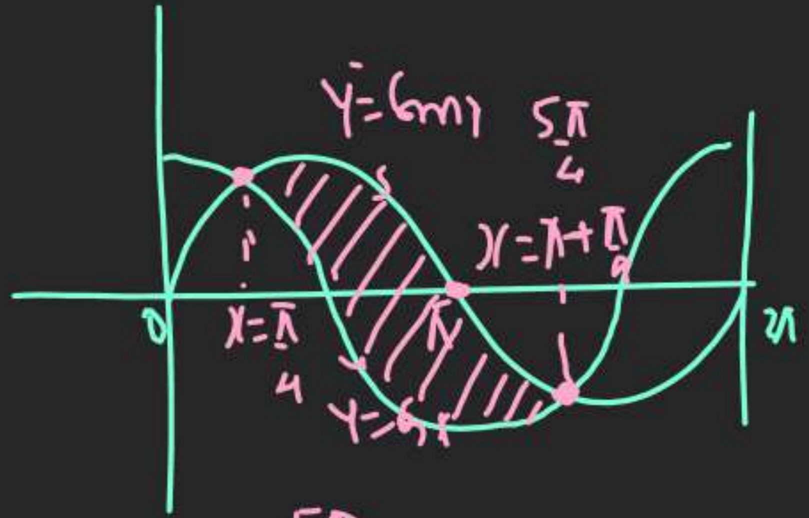

$$A=\int_{x=a}^{x=b}\left(f(x)-g(x)\right)dx$$

Upper - Lower

No mod Required

Q Find A.B.B.

$y=\sin x,\ y=\cos x$. betn 2 PoI




$$A=\int_{\pi/4}^{5\pi/4}(\sin x-\cos x)\,dx$$

$$-\cos x-\sin x\Big|_{\pi/4}^{5\pi/4}$$

$$=\left(-\cos\frac{5\pi}{4}-\sin\frac{5\pi}{4}\right)+\left(\cos\frac{\pi}{4}+\sin\frac{\pi}{4}\right) \to +\sqrt{2}+\sqrt{2}=2\sqrt{2}$$

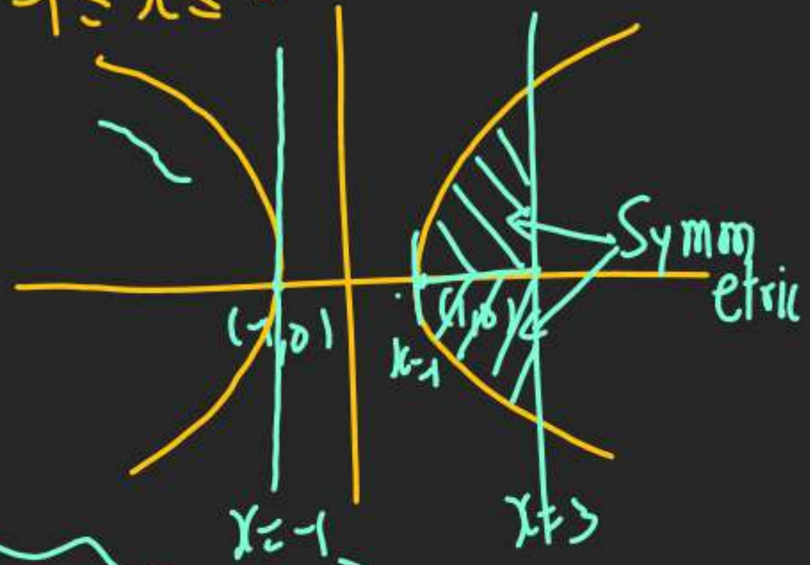
Q A.B.B.

$|x-1|\leq 2$ & $x^2-y^2=1$ → $x^2-1=y^2$, $y=\pm\sqrt{x^2-1}$

Ghoda

hyperbola


$-1\leq x\leq 3$




$$A=2\int_1^3\sqrt{x^2-1}\,dx$$

$$A=2\left[\frac{x}{2}\sqrt{x^2-1}-\frac{1}{2}\ln\left(x+\sqrt{x^2-1}\right)\right]_1^3$$

$$=2\left[\frac{3}{2}\times 2\sqrt{2}-\frac{1}{2}\ln(3+2\sqrt{2})\right]$$

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**Q** A.B.B. $y=f(x)$, $x=1$, $x=b$ is $(b-1)\sin(3b+4)$ find $f(x)$?

Main

$$A=\int_1^b f(x)\,dx=(b-1)\sin(3b+4)$$

diff$^n$ WRT "b"

NL

$$f(b)\cdot 1-0=3(b-1)\cos(3b+4)+\sin(3b+4)$$

$$f(x)=3(x-1)\cos(3x+4)+\sin(3x+4)$$

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**Q** If $f(x)$ is a Non-ve Cont$^s$ fxn S.T. A.B.B $y=f(x)$, X Axis $x=\frac{\pi}{4}$, $x=\beta>\frac{\pi}{4}$ is $\beta\sin\beta+\frac{\pi}{4}\cos\beta+\sqrt{2}\beta$ then $f\left(\frac{\pi}{2}\right)$?

Main

$$\int_{\pi/4}^{\beta} f(x)\,dx=\beta\sin\beta+\frac{\pi}{4}\cos\beta+\sqrt{2}\beta$$

Diff$^n$ WRT $\beta$

$$f(\beta)\cdot 1=\beta\cos\beta+\sin\beta-\frac{\pi}{4}\sin\beta+\sqrt{2}$$

$$f\left(\frac{\pi}{2}\right)=\cancel{\frac{\pi}{2}\cos\frac{\pi}{2}}+\sin\frac{\pi}{2}-\frac{\pi}{4}\sin\frac{\pi}{2}+\sqrt{2}$$

$$=1-\frac{\pi}{4}+\sqrt{2}$$

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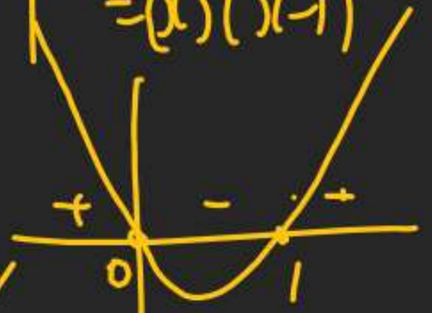
**Q.** Ratio In which ABB Parabolas $y=4x-x^2$, $y=x^2-x$ is divided by X Axis?

$y=x(4-x)=-(x)(x-4)$ | $y=x^2-x=(x)(x-1)$



POI $= x^2-x=4x-x^2$

$2x^2=5x$

$x=5/2$

① $A_1+A_2=\int_0^{5/2}(4x-x^2)-(x^2-x)=\int 5x \ldots = \frac{5x^2}{2}-\frac{2x^3}{3}\Big|_0^{5/2}=\left(\frac{125}{8}-\frac{125}{12}\right)=\frac{125}{24}$

② $A_1=\int_0^1 x^2-x\,dx=\frac{x^3}{3}-\frac{x^2}{2}\Big|_0^1=\frac{1}{3}-\frac{1}{2}=\left|-\frac{1}{6}\right|$ $\frac{1}{6}$

$A_1+A_2=\frac{125}{24}$, $A_1=\frac{1}{6}=\frac{4}{24}$

$A_2=\frac{121}{24}$ $\therefore A_1:A_2=4:121$

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Q ABB $x=-4y^2$ & $x-1=-5y^2$ is ?

$y^2=4ax$  , $y^2=-4ax$ 

$-\frac{(x-0)}{4} - (y$

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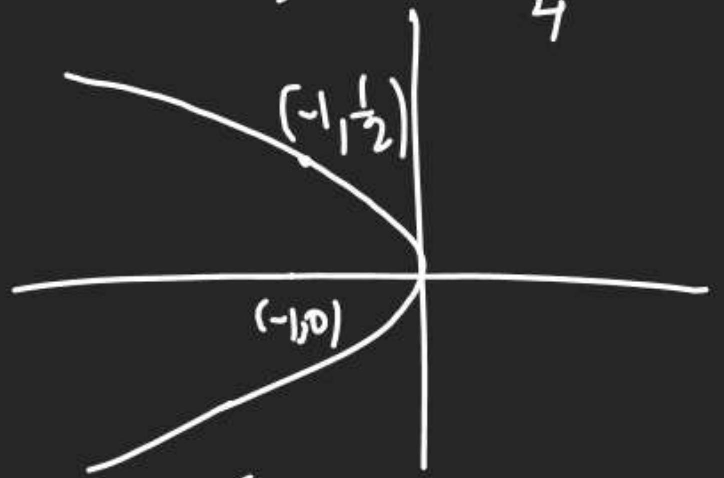
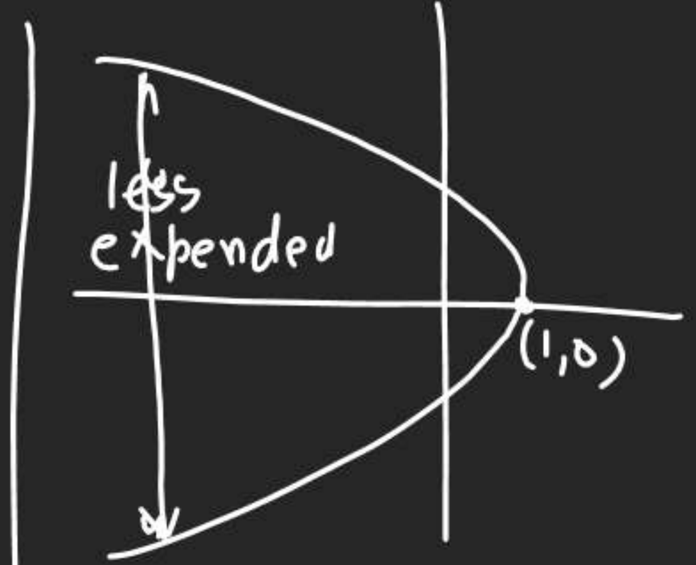
Q ABB $x=-4y^2$ & $x-1=-5y^2$

$y^2=-\frac{x}{4}$ $\qquad$ $y^2=-\frac{(x-1)}{5}$ → $(y-0)^2=-\frac{(x-1)}{5}$ → $(1,0)$

$y^2=-4ax$

$(y-0)^2=-\frac{(x-0)}{4}$

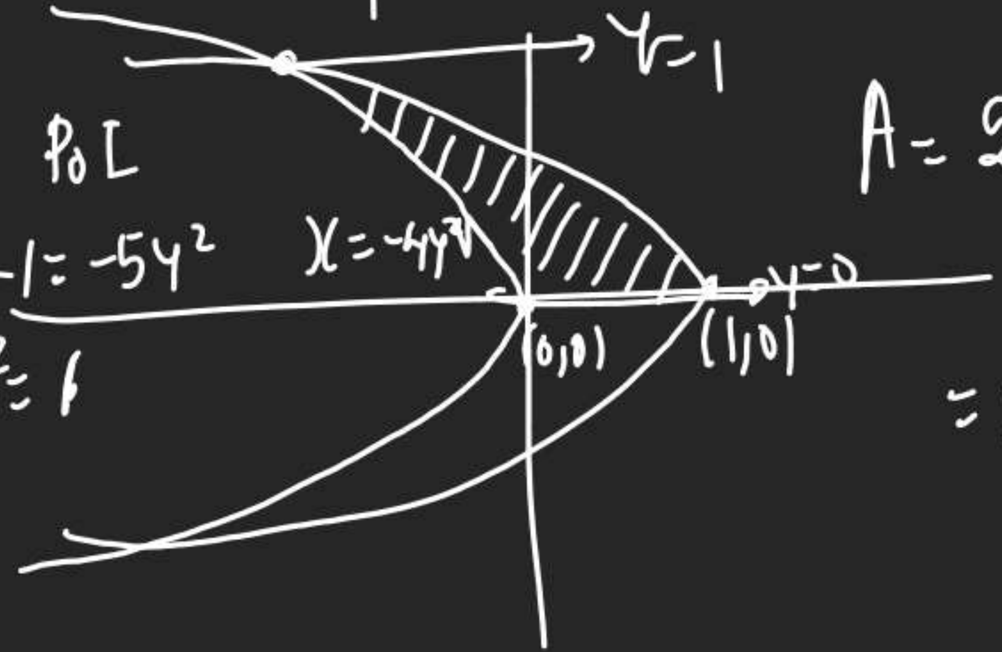



PoI

$-4y^2-1=-5y^2$

$y^2=1$




$A=2\int_0^1 (x_{Right}-x_{left})\,dy$

$=2\int_0^1 \left((1-5y^2)-(-4y^2)\right)dy$

$=2\int_0^1 (1-y^2)\,dy=2\left[y-\frac{y^3}{3}\right]_0^1=2\left[1-\frac{1}{3}\right]$

$=4/3$

Q Find Area of Region $[(x,y): x\geq 0,\ x+y\leq 3,\ x^2\leq 4y\ \&\ y\leq 1+\sqrt{x}]$

$y=mx+1$

Above x-Axis

line

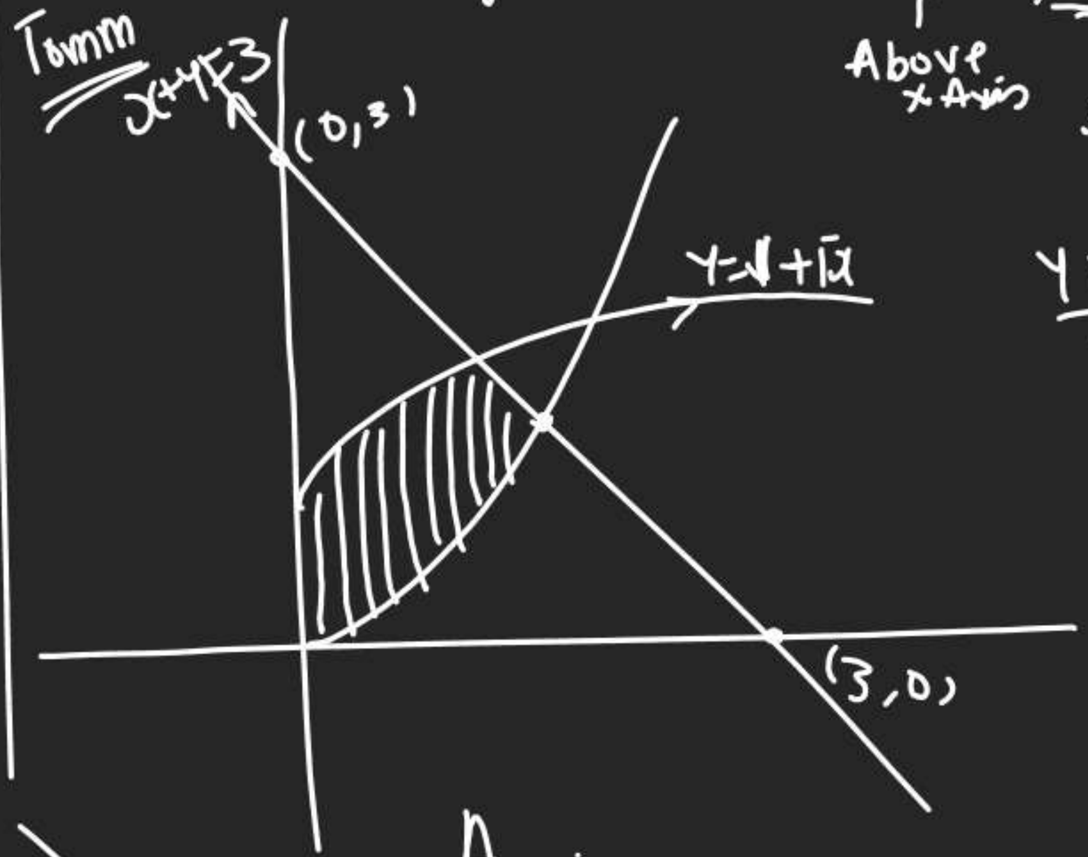
$x+y=3$

$y=3-x$

$x^2=4y$

$y\geq \frac{x^2}{4}$

graph?




A =