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SUBJECTIVE (JEE ADVANCED)

1. If $P=\int_0^{\infty}\frac{x^2}{1+x^4}dx;\ Q=\int_0^{\infty}\frac{xdx}{1+x^4}$ and $R=\int_0^{\infty}\frac{dx}{1+x^4}$ then prove that

(a) $Q=\frac{\pi}{4}$

(b) $P=R$

(c) $P-\sqrt{2}Q+R=\frac{\pi}{2\sqrt{2}}$

2. $\int_0^{1}\frac{x^2\cdot\ln x}{\sqrt{1-x^2}}dx$

3. $\int_0^{2\pi}\frac{dx}{2+\sin 2x}$

4. $\int_0^{2\pi}e^x\cos\left(\frac{\pi}{4}+\frac{x}{2}\right)dx$

5. Evaluate : $\int_0^{1}e^{\ln\tan^{-1}x}\cdot\sin^{-1}(\cos x)dx$.

6. Find the range of the function, $f(x)=\int_{-1}^{1}\frac{\sin xdt}{1-2t\cos x+t^2}$.

7. Evaluate $I_n=\int_1^{e}(\ell n^n x)dx$ hence find I_3.

8. $\int_0^{\sqrt{3}}\sin^{-1}\frac{2x}{1+x^2}dx$

9. For $a\geq 2$, if the value of the definite integral $\int_0^{\infty}\frac{dx}{a^2+\left(x-\frac{1}{x}\right)^2}$ equals $\frac{\pi}{5050}$. Find the value of a.

10. $\int_0^{1}(\{2x\}-1)(\{3x\}-1)dx$, where $\{*\}$ denotes fractional part of x.

11. Find the value of the definite integral $\int_0^{\pi}|\sqrt{2}\sin x+2\cos x|dx$

12. Evaluate the integral $\int_3^{5}\left(\sqrt{x+2\sqrt{2x-4}}+\sqrt{x-2\sqrt{2x-4}}\right)dx$

13. Evaluate $\int_{-\pi/3}^{\pi/3}\frac{\pi+4x^3}{2-\cos\left(|x|+\frac{\pi}{3}\right)}dx$

14. Evaluate the definite integral, $\int_{-1}^{1}\frac{(2x^{332}+x^{998}+4x^{1668}\cdot\sin x^{691})}{1+x^{666}}dx$.

15. $\int_{-2}^{2}\frac{x^2-x}{\sqrt{x^2+4}}dx$

16. $\int_0^{\pi/4}\frac{xdx}{\cos x(\cos x+\sin x)}$

17. $\int_0^{\pi/2}\frac{a\sin x+b\cos x}{\sin\left(\frac{\pi}{4}+x\right)}dx$

18. $\int_0^{\pi}\frac{(ax+b)\sec x\tan x}{4+\tan^2 x}dx\ (a,b>0)$

19. $\int_0^{\pi}\frac{(2x+3)\sin x}{(1+\cos^2 x)}dx$

20. $\int_0^{2a}x\sin^{-1}\left[\frac{1}{2}\sqrt{\frac{2a-x}{a}}\right]dx$

21. If f, g, h be continuous function on $[0, a]$ such that $f(a-x) = f(x), g(a-x) = -g(x)$ and $3\,h(x) - 4\,h(a-x) = 5$, then prove that, $\int_0^a f(x)g(x)h(x) = 0$

22. If f is an even function then prove that $\int_0^{\pi/2} f(\cos 2x)\cos xdx = \sqrt{2}\int_0^{\pi/4} f(\sin 2x)\cos xdx$

23. Evaluate $\int_0^1 \frac{1}{(5+2x-2x^2)(1+e^{(2-4x)})}dx$

24. Evaluate $\int_0^{\pi} \frac{xdx}{1+\cos \alpha \sin x}$

25. $\int_0^{\pi/4} \frac{x^2(\sin 2x-\cos 2x)}{(1+\sin 2x)\cos^2 x}dx$

26. $\int_0^{\pi} e^{\cos^2 x}\cos^3 (2n+1)xdx, n \in I$

27. Evaluate : $\int_0^{\pi} e^{|\cos x|}\left(2\sin\left(\frac{1}{2}\cos x\right) + 3\cos\left(\frac{1}{2}\cos x\right)\right)\sin xdx$

28. If f(x) is an odd function defined on $\left[-\frac{T}{2}, \frac{T}{2}\right]$ and has period T, then prove that $\phi(x) = \int_0^x f(t)dt$ is also periodic with period T.

29. Evaluate $\int_{-1}^{2}\{2x\}dx$ (where $\{*\}$ denotes fractional part function)

30. If $y = x^1\int^x \ln tdt$, find $\frac{dy}{dx}$ at $x = e$.

31. $\lim_{n\to\infty} n^2\int_{-1/n}^{1/n}(2006\sin x + 2007\cos x)|x|dx.$

32. Prove that following inequalities

(i) $\frac{\sqrt{3}}{8} < \int_{\pi/4}^{\pi/3}\frac{\sin x}{x}dx < \frac{\sqrt{2}}{6}$

(ii) $4 \le \int_1^3 \sqrt{(3+x^3)}dx \le 2\sqrt{30}$

33. Prove the inequalities

(a) $\frac{\pi}{6} < \int_0^1 \frac{dx}{\sqrt{4-x^2-x^3}} < \frac{\pi\sqrt{2}}{8}$

(b) $2e^{-1/4} < \int_0^2 e^{x^2-x}dx < 2e^2$

(c) $a < \int_0^{2\pi}\frac{dx}{10+3\cos x} < b$ then find a& b

(d) $\frac{1}{2} \le \int_0^2 \frac{dx}{2+x^2} \le \frac{5}{6}$

34. Evaluate

(i) $\lim_{n\to\infty}\sum_{r=1}^{n-1}\frac{1}{\sqrt{n^2-r^2}}$

(ii) $\lim_{n\to\infty}\frac{3}{n}\left[1+\sqrt{\frac{n}{n+3}}+\sqrt{\frac{n}{n+6}}+\sqrt{\frac{n}{n+9}}+\cdots+\sqrt{\frac{n}{n+3(n-1)}}\right]$

35. Evaluate

(a) $\text{limit}_{n\to\infty}\frac{1}{n}\left[\frac{1}{n+1}+\frac{2}{n+2}+\cdots\ldots+\frac{3n}{4n}\right]$

(b) $\text{limit}_{n\to\infty}\left[\frac{n!}{n^n}\right]^{1/n}$

(c) For positive integers n, let

$A_n = \frac{1}{n}\{(n+1)+(n+2)+\cdots\ldots(n+n)\}$,

$B_n = \{(n+1)(n+2)\ldots\ldots\ldots\ldots(n+n)\}^{1/n}$.

If $\frac{A_n}{B_n} = \frac{ae}{b}$ where $a, b \in N$ and relatively prime find the value of $(a+b)$.

36. Supose $g(x)$ is the inverse of $f(x)$ and $f(x)$ has a domain $x \in [a, b]$. Given $f(a) = \alpha$ and $f(b) = \beta$, then find the value of $\int_a^b f(x)dx + \int_\alpha^\beta g(y)dy$ in terms of a, b, α and β.

37. If $f(x) = 5^{g(x)}$ and $g(x) = \int_2^{x^2} \frac{t}{\ell n(1+t^2)} dt$ then find the value of $f'(\sqrt{2})$

38. Solve the equation for y as a function of x, satisfying $x.\int_0^x y(t)dt = (x+1)\int_0^x t \cdot y(t)dt$, where $x > 0$, given $y(1) = 1$.

39. Evaluate, $I = \int_0 2\sin(pt)\sin(qt)dt$, if :

(i) p&q are different roots of the equation, $\tan x = x$.

(ii) p&q are equal and either is root of the equation $\tan x = x$.

40. $\int_0^{\pi/2} \sin 2x \cdot \arctan(\sin x)dx$

41. $\int_1^2 \frac{(x^2-1)dx}{x^3 \cdot \sqrt{2x^4-2x^2+1}} = \frac{u}{v}$ where u and v are in their lowest form. Find the value of $\frac{(1000)u}{v}$.

42. A function r is defined in $[-1,1]$ as $f'(x) = 2x\sin\frac{1}{x} - \cos\frac{1}{x}; x \neq 0; f(0) = 0; f(1/\pi) = 0$. Discuss the continuity and derivability of f at $x = 0$.

43. Let $f(x) = \begin{cases} -1 & \text{if} \quad -2 \leq x \leq 0 \\ |x-1| & \text{if} \quad 0 < x \leq 2 \end{cases}$ and

$g(x) = \int_{-2}^{x} f(t)dt.$

Test the continuity and differentiability of $g(x)$ in $(-2,2)$.

44. Let f and g be function that are differentiable for all real numbers x and that have the following properties

(i) $f'(x) = f(x) - g(x)$

(ii) $g'(x) = g(x) - f(x)$

(iii) $f(0) = 5$

(iv) $g(0) = 1$

(a) Prove that $f(x) + g(x) = 6$ for all x.

(b) Find $f(x)$ and $g(x)$.

45. Evaluate, $\int_0^1 |x-t| \cdot \cos \pi t dt$ where ' x ' is any real number

46. If $f(x) = \frac{\sin x}{x} \forall x \in (0, \pi]$, prove that, $\frac{\pi}{2}\int_0^{\pi/2} f(x)f\left(\frac{\pi}{2}-x\right)dx = \int_0^\pi f(x)dx$

47. If $n > 1$, evaluate $\int_0^\infty \frac{dx}{\left(x+\sqrt{1+x^2}\right)^n}$

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48. $\int_0^1 \frac{\sin^{-1}\sqrt{x}}{x^2-x+1}dx$

49. Let $f(x) = \begin{cases} 1-x & \text{if } 0 \le x \le 1 \\ 0 & \text{if } 1 < x \le 2 \\ (2-x)^2 & \text{if } 2 < x \le 3 \end{cases}$. Define the function

$F(x) = \int_0^x f(t)dt$ and show that F is continuous in [0,3] and differentiable in (0,3).

50. Let $u = \int_0^{\pi/4} \left(\frac{\cos x}{\sin x+\cos x}\right)^2 dx$ and $v = \int_0^{\pi/4} \left(\frac{\sin x+\cos x}{\cos x}\right)^2 dx$. Find the value of $\frac{v}{u}$.

51. $\int_{-2}^{0} (x^3 + 3x^2 + 3x + 3 + (x+1)\cos(x+1))dx$ is equal to

COMPREHENSION 52 TO 54

If function f(x) is continuous in the interval (a, b) and having same definition between a and b, then we can find $\int_a^b f(x)dx$ if f(x) is discontiuous and not same definition between a and b, then we must break the interval such that f(x) becomes continuous and having same definition in the breaking intervals.

Now, if f(x) is discontinuous at $x = c(a < c < b)$, then $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$ and also if f(x) is discontinuous at $x = a$ in (0,2a), then we can write

$$\int_0^{2a} f(x)dx = \int_0^a \{f(a-x) + f(a+x)\}dx$$

On the basis of above information, answere the following questions :

52. $\int_0^{10} \left[\frac{x^2+2}{x^2+1}\right] dx$ (where [.] denotes greatest integer function) is equal to

(A) 0 (B) 2 (C) 5 (D) None of these

53. $\int_0^1 \sin([x] + [2x])dx$ (where [.] denotes the greatest integer function) is equalt to

(A) $\sin 1$ (B) $\sin\left(\frac{3}{2}\right)$ (C) $\frac{\sin 1}{2}$ (D) $\frac{\sin 2}{3}$

54. $\int_{-1}^{1} [|x|] d\left(\frac{1}{1+e^{-1/x}}\right)$ (where [.] denotes the greates integer functions) is equal to

(A) -3 (B) -2 (C) -1 (D) None of these

MATRIX MATCH TYPE

55.

Column-I	**Column-II**
(A) The value of $\int_\alpha^{\pi/2-\alpha} \frac{d\theta}{1+\cot^n\theta}$ Where, $0 < \alpha < \frac{\pi}{2}, n > 0$ is	(P) $\frac{\pi}{2}$
	(Q) $\frac{\pi}{4} - \alpha$
(B) The value of	(R) $2\pi^2 - 2\pi\alpha$

$\int_{-\pi}^{\pi} \frac{\sin^2 x}{1+\alpha^x} dx, \alpha > 0$ is | (S) dependent of α

(C) The value of | (T) independent of n

$\int_{\alpha}^{2\pi-\alpha} \frac{x\sin^{2n} x}{\sin^{2n} x + \cos^{2n} x} dx$

PREVIOUS YEAR (JEE MAIN)

56. The value of the integral, $\int_3^6 \frac{\sqrt{x}}{\sqrt{9-x}+\sqrt{x}} dx$ is - **[AIEEE 2006]**

(A) $\frac{3}{2}$ (B) 2 (C) 1 (D) $\frac{1}{2}$

57. $\int_{-3\pi/2}^{-\pi/2} [(x+\pi)^3 + \cos^2 (x+3\pi)]dx$ is equal to - **[AIEEE 2006]**

(A) $(\pi^4/32) + (\pi/2)$ (B) $\pi/2$ (C) $(\pi/4) - 1$ (D) $\pi^4/32$

58. $\int_0^{\pi} xf(\sin x)dx$ is equal to- **[AIEEE 2006]**

(A) $\pi\int_0^{\pi} f(\sin x)dx$ (B) $\frac{\pi}{2}\int_0^{\pi/2} f(\sin x)dx$

(C) $\pi\int_0^{\pi/2} f(\cos x)dx$ (D) $\pi\int_0^{\pi} f(\cos x)dx$

59. The value of $\int_1^a [x]f'(x)dx, a > 1$, where $[x]$ denotes the greatest integer not exceeding x is **[AIEEE 2006]**

(A) $[a]f(a) - \{f(1) + f(2) + \cdots + f([a])\}$ (B) $[a]f([a]) - \{f(1) + f(2) + \cdots + f(a)\}$

(C) $af([a]) - \{f(1) + f(2) + \cdots + f(a)\}$ (D) $a\,f(a) - \{f(1) + f(2) + \cdots + f([a])\}$

60. Let $F(x) = f(x) + f\left(\frac{1}{x}\right)$, where $f(x) = \int_1^x \frac{\log t}{1+t} dt$. Then $F(e)$ equals **[AIEEE 2007]**

(A) $\frac{1}{2}$ (B) 0 (C) 1 (D) 2

61. The solution for x of the equation $\int_{\sqrt{2}}^{x} \frac{dt}{t\sqrt{t^2-1}} = \frac{\pi}{12}$ is **[AIEEE 2007]**

(A) 2 (B) π (C) $\sqrt{3}/2$ (D) $2\sqrt{2}$

62. Let $I = \int_0^1 \frac{\sin x}{\sqrt{x}} dx$ and $J = \int_0^1 \frac{\cos x}{\sqrt{x}} dx$. Then which one of the following is true? **[AIEEE 2008]**

(A) $I < \frac{2}{3}$ and $J < 2$ (B) $I < \frac{2}{3}$ and $J > 2$ (C) $I > \frac{2}{3}$ and $J < 2$ (D) $I > \frac{2}{3}$ and $J > 2$

63. $\int_0^{\pi} [\cot x]dx$ where [.] denotes the greatest integer function, is equal to **[AIEEE 2009]**

(A) $\frac{\pi}{2}$ (B) 1 (C) -1 (D) $-\frac{\pi}{2}$

64. Let $p(x)$ be a function defined on R such that $p'(x) = p'(1-x)$, for all $x \in [0,1]$, $p(0) = 1$ and $p(1) = 41$. Then $\int_0^1 p(x)dx$ equals - **[AIEEE 2010]**

(A) $\sqrt{41}$ (B) 21 (C) 41 (D) 42

65. The value of $\int_{0}^{1} \frac{8\log(1+x)}{1+x^{2}} dx$ is **[AIEEE 2011]**

(A) $\pi\log 2$ (B) $\frac{\pi}{8}\log 2$ (C) $\frac{\pi}{2}\log 2$ (D) $\log 2$

66. If $g(x) = \int_{0}^{x} \cos 4t dt$, then $g(x+\pi)$ equals : **[AIEEE 2012]**

(A) $g(x) - g(\pi)$ (B) $g(x) \cdot g(\pi)$ (C) $\frac{g(x)}{g(\pi)}$ (D) $g(x) + g(\pi)$

67. Statement - I : The value of the interval $\int_{\pi/6}^{\pi/3} \frac{dx}{1+\sqrt{\tan x}}$ is equal to $\frac{\pi}{6}$. **[JEE-MAIN 2013]**

Statement - II : $\int_{a}^{b} f(x)dx = \int_{a}^{b} f(a+b-x)dx$.

(A) If both Statement - I and Statement - II are true, and Statement - II is the correct explanation of Statement- I.

(B) If both Statement-I and Statement - II are true but Statement - II is not the correct explanation of Statement-I.

(C) If Statement-I is true but Statement - II is false.

(D) If Statement-I is false but Statement-II is true.

68. The intercepts on x-axis made by tangents to the curve, $y = \int_{0}^{x} |t| dt, x \in R$, which are parallel to the line $y = 2x$, are equal to : **[JEE-MAIN 2013]**

(A) \pm 3 (B) \pm 4 (C) \pm 1 (D) \pm 2

69. The integral $\int_{0}^{\pi} \sqrt{1 + 4\sin^{2}\frac{x}{2} - 4\sin\frac{x}{2}} dx$ equals: **[JEE-MAIN 2014]**

(A) $\pi - 4$ (B) $\frac{2\pi}{3} - 4 - 4\sqrt{3}$ (C) $4\sqrt{3} - 4$ (D) $4\sqrt{3} - 4 - \frac{\pi}{3}$

70. The integral $\int_{2}^{4} \frac{\log x^{2}}{\log x^{2} + \log(36-12x+x^{2})} dx$ is equal to : **[JEE-MAIN 2015]**

(A) 1 (B) 6 (C) 2 (D) 4

71. The value of $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\sin^{2} x}{1+2^{x}} dx$ is **[JEE-MAIN 2018]**

(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{8}$ (C) $\frac{\pi}{2}$ (D) 4π

PREVIOUS YEAR (JEE ADVANCED)

72. Let $y = f(x)$ be a twice differentiable, non-negaitve function defined on $[a, b]$. The area $\int_a^b f(x)dx, b > a$ bounded by $y = f(x)$, the x-axis and the ordinates at $x = a$ and $x = b$ can be approximated as

$$\int_a^b f(x)dx \cong \frac{(b-a)}{2}\{f(a) + f(b)\}.$$

Since $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx, c\hat{I}(a, b)$, a

better approximation to $\int_a^b f(x)dx$ can be written

as $\int_a^b f(x)dx = \frac{(c-a)}{2}\{f(a) + f(c)\} + \frac{(b-c)}{2}\{f(c) + f(b)\}$

If $c = \frac{a+b}{2}$, then this gives : **[JEE 2006]**

$\int_a^b f(x)dx = \frac{b-a}{4}\{f(a) + 2f(c) + f(b)\}$,...............(1)

(a) given above is

(A) $\frac{\pi}{8\sqrt{2}}(1+\sqrt{2})$ (B) $\frac{\pi}{4\sqrt{2}}(1+\sqrt{2})$

(C) $\frac{\pi}{8}(1+\sqrt{2})$ (D) $\frac{\pi}{4}(1+\sqrt{2})$

(b) If $\text{Lim}_{t\to a}\left\{\frac{\int_0^t f(x)dx - \frac{(t-a)}{2}(f(t)+f(a))}{(t-a)^3}\right\} = 0$, for each fixed a, then f(x) is a polynomial of degree utmost

(A) 4 (B) 3 (C) 2 (D) 1

(c) If $f''(x) < 0, x \in (a, b)$, then at the point $C(c, f(c))$ on $y = f(x)$ for which F(c) is a maximum, $f'(c)$ is given by

(A) $f'(c) = \frac{f(b)-f(a)}{b-a}$ (B) $f'(c) = \frac{f(b)-f(a)}{a-b}$

(C) $f'(c) = \frac{2(f(b)-f(a))}{b-a}$ (D) $f'(c) = 0$

73. Find the value of $\frac{5050\int_0^1 (1-x^{50})^{100}dx}{\int_0^1 (1-x^{50})^{101}dx}$ **[JEE 2006]**

74. **(a)** $\lim_{x\to\frac{\pi}{4}} \frac{\int_2^{x^2} f(t)dt}{x^2-\frac{\pi^2}{16}}$ equals **[JEE 2007]**

(A) $\frac{8}{\pi}f(2)$ (B) $\frac{2}{\pi}f(2)$ (C) $\frac{2}{\pi}f\left(\frac{1}{2}\right)$ (D) $4f(2)$

(b) Match the integrals in Column I with the values in Column II.

Column I	Column II
(A) $\int_{-1}^1 \frac{dx}{1+x^2}$	(P) $\frac{1}{2}\log\left(\frac{2}{3}\right)$
(B) $\int_0^1 \frac{dx}{\sqrt{1-x^2}}$	(Q) $2\log\left(\frac{2}{3}\right)$

(C) $\int_2^3 \frac{dx}{1-x^2}$ (R) $\frac{\pi}{3}$

(D) $\int_1^2 \frac{dx}{x\sqrt{x^2-1}}$ (S) $\frac{\pi}{2}$

75. Let $S_n = \sum_{k=0}^{n} \frac{n}{n^2+kn+k^2}$ and $T_n = \sum_{k=1}^{n-1} \frac{n}{n^2+kn+k^2}$, for $n = 1,2,3, \ldots \ldots \ldots$ Then, **[JEE 2008]**

(A) $S_n < \frac{\pi}{3\sqrt{3}}$ (B) $S_n > \frac{\pi}{3\sqrt{3}}$ (C) $T_n < \frac{\pi}{3\sqrt{3}}$ (D) $T_n > \frac{\pi}{3\sqrt{3}}$

76. (a) Let f be a non-negative function defined on the interval [0,1].

If $\int_0^x \sqrt{1-(f'(t))^2}dt = \int_0^x f(t)dt, 0 \le x \le 1$, and $f(0) = 0$, then **[JEE 2009]**

(A) $f\left(\frac{1}{2}\right) < \frac{1}{2}$ and $f\left(\frac{1}{3}\right) > \frac{1}{3}$ (B) $f\left(\frac{1}{2}\right) > \frac{1}{2}$ and $f\left(\frac{1}{3}\right) > \frac{1}{3}$

(C) $f\left(\frac{1}{2}\right) < \frac{1}{2}$ and $f\left(\frac{1}{3}\right) < \frac{1}{3}$ (D) $f\left(\frac{1}{2}\right) > \frac{1}{2}$ and $f\left(\frac{1}{3}\right) < \frac{1}{3}$

(b) If $I_n = \int_{-\pi}^{\pi} \frac{\sin nx}{(1+\pi^x)\sin x}dx, n = 0,1,2, \ldots \ldots$ then

(A) $I_n = I_{n+2}$ (B) $\sum_{m=1}^{10} I_{2m+1} = 10\pi$

(C) $\sum_{m=1}^{10} I_{2m} = 0$ (D) $I_n = I_{n+1}$

(c) Let $f: R \rightarrow R$ be a continuous function which satisfies $f(x) = \int_0^x f(t)dt$. Then the value of $f(\ln 5)$ is

77. **(a)** The value of $\lim_{x\to 0} \frac{1}{x^3}\int_0^x \frac{t\ell n(1+t)}{t^4+4}dt$ is **[JEE 2010]**

(A) 0 (B) $\frac{1}{12}$ (C) $\frac{1}{24}$ (D) $\frac{1}{64}$

(b) The value(s) of $\int_0^1 \frac{x^4(1-x)^4}{1+x^2}dx$ is (are)

(A) $\frac{22}{7} - \pi$ (B) $\frac{2}{105}$ (C) 0 (D) $\frac{71}{15} - \frac{3\pi}{2}$

(c) Let f be a real-valued function defined on the interval $(-1,1)$ such that

$e^{-x}f(x) = 2 + \int_0^x \sqrt{t^4+1}dt$, for all xÎ$(-1,1)$, and let f^{-1} be the inverse function of f.

Then $(f^{-1})'(2)$ is equal to

(A) 1 (B) 1/3 (C) ½ (D) 1/e

(d) For any real number x, let [x] denote the largest integer less than or equal to x. Let f be a real valued function defined on the interval $[-10,10]$ by

$$f(x) = \begin{cases} x - [x] & \text{if } [x] \text{ is odd} \\ 1 + [x] - x & \text{if } [x] \text{ is even} \end{cases}$$

Then the value of $\frac{\pi^2}{10}\int_{-10}^{10} f(x)\cos \pi x dx$ is

78. The value of $\int_{\sqrt{\ln 2}}^{\sqrt{\ln 3}} \frac{x\sin x^2}{\sin x^2+\sin(\ln 6-x^2)}dx$ is **[JEE 2011]**

(A) $\frac{1}{4}\ln\frac{3}{2}$ (B) $\frac{1}{2}\ln\frac{3}{2}$ (C) $\ln\frac{3}{2}$ (D) $\frac{1}{6}\ln\frac{3}{2}$

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79. Let $f:[1,\infty)\to[2,\infty)$ be a differentiable function such that $f(1)=2$.
If $6\int_1^x f(t)dt = 3xf(x) - x^3$ for all $x \geq 1$, then the value of $f(2)$ is **[JEE 2011]**

80. The value of the integral $\int_{-\pi/2}^{\pi/2}\left(x^2 + \ln\frac{\pi+x}{\pi-x}\right)\cos\ xdx$ is

(A) 0 (B) $\frac{\pi^2}{2} - 4$ (C) $\frac{\pi^2}{2} + 4$ (D) $\frac{\pi^2}{2}$

81. Let $f:\left[\frac{1}{2},1\right]\to R$ (the set of all real numbers) be a positive, non-constant and differentiable function such that $f'(x) < 2f(x)$ and $f\left(\frac{1}{2}\right) = 1$. Then the value of $\int_{1/2}^1 f(x)dx$ lies in the interval

(A) $(2e-1,2e)$ (B) $(e-1,2e-1)$ (C) $\left(\frac{e-1}{2}, e-1\right)$ (D) $\left(0,\frac{e-1}{2}\right)$ **[JEE 2013]**

82. For $a \in R$ (the set of all real numbers), $a^1 - 1$,
$\lim_{n\to\infty}\frac{(1^a+2^a+\cdots+n^a)}{(n+1)^{a-1}[(na+1)+(na+2)+\cdots+(na+n)]} = \frac{1}{60}$ Then $a =$ **[JEE 2013]**

(A) 5 (B) 7 (C) $\frac{-15}{2}$ (D) $\frac{-17}{2}$

83. Let $f:(0,\infty)\to R$ be given by $f(x)\int_{\frac{1}{x}}^{x} e^{-\left(t+\frac{1}{t}\right)}\frac{dt}{t}$. Then **[JEE 2014]**

(A) $f(x)$ is monotonically increasing on $[1,\infty)$

(B) $f(x)$ is monotonically decreasing on $(0,1)$

(C) $f(x) + f\left(\frac{1}{x}\right) = 0$, for all $x \in (0,\infty)$

(D) $f(2^x)$ is an odd function of x on R

84. The value of $\int_0^1 4x^3\left\{\frac{d^2}{dx^2}(1-x^2)^5\right\}dx$ is **[JEE 2014]**

85. Let $f:[0,2]\to R$ be a function which is continuous on $[0,2]$ and is differentiable on $(0,2)$ with $f(0)=1$. Let $F(x) = \int_0^{x^2} f(\sqrt{t})dt$ for $x \in [0,2]$. If $F'(x) = f'(x)$ for all $x \in (0,2)$, then $F(2)$ equals

(A) $e^2 - 1$ (B) $e^4 - 1$ (C) $e - 1$ (D) e^4

86. The following integral $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}(2\text{cos}\quad\text{ec}\ x)^{17}dx$ is equal to **[JEE 2014]**

(A) $\int_0^{\log(1+\sqrt{2})} 2(e^u + e^{-u})^{16}du$ (B) $\int_0^{\log(1+\sqrt{2})}(e^u + e^{-u})^{17}du$

(C) $\int_0^{\log(1+\sqrt{2})}(e^u - e^{-u})^{17}du$ (D) $\int_0^{\log(1+\sqrt{2})} 2(e^u - e^{-u})^{16}du$

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87. **List-I** **List-II**

(P) The number of polynomials f(x) with non-negative integer coefficients of degree ≤ 2, satisfying $f(0) = 0$ and $\int_0^1 f(x)dx = 1$, is — 1. 8

(Q) The number of points in the interval $[-\sqrt{13}, \sqrt{13}]$ at which $f(x) = \sin(x^2) + \cos(x^2)$ attains its maximum value is — 2. 2

(R) $\int_{-2}^{2} \frac{3x^2}{(1+e^x)} dx$ equals — 3. 4

(S) $\frac{\left(\int_{-\frac{1}{2}}^{\frac{1}{2}} \cos 2x \log\left(\frac{1+x}{1-x}\right)dx\right)}{\left(\int_0^{\frac{1}{2}} \cos 2x \log\left(\frac{1+x}{1-x}\right)dx\right)}$ equal — 4. 0

	P	Q	R	S
(A)	3	2	4	1
(B)	2	3	4	1
(C)	3	2	1	4
(D)	2	3	14	

[JEE 2014]

88. Let $f: R \to R$ be a function defined by $f(x) = \begin{cases} [x], & x \le 2 \\ 0, & x > 2 \end{cases}$, where [x] is the greatest integer less than or equal to x,

If $I = \int_{-1}^{2} \frac{xf(x^2)}{2+f(x+1)} dx$, then the value of $(4I - 1)$ is. **[JEE 2015]**

89. If $a = \int_0^1 \left(e^{9x+3\tan^{-1} x}\right)\left(\frac{12+9x^2}{1+x^2}\right) dx$ where $\tan^{-1} x$ takes only principal values, then the value of $\left(\log_e |1 + \alpha| - \frac{3\pi}{4}\right)$ is **[JEE 2015]**

90. Let $f: R \to R$ be a continuous odd fucntion, which vanishes exactly at one point and $f(1) = \frac{1}{2}$. Suppose that $F(x) = \int_{-1}^{x} f(t)dt$ for all $x \in [-1,2]$ and $G(x) = \int_{-1}^{x} t|f(f(t))|dt$ for all $x \in [-1,2]$. If $\lim_{x\to 1} \frac{F(x)}{G(x)} = \frac{1}{14}$, then the value of $f\left(\frac{1}{2}\right)$ **[JEE 2015]**

91. Let $f(x) = 7\tan^8 x + 7\tan^6 x - 3\tan^4 x - 3\tan^2 x$ for all $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$. Then the correct expression(s) is(are). **[JEE 2015]**

(A) $\int_0^{\pi/4} xf(x)dx = \frac{1}{12}$ (B) $\int_0^{\pi/4} f(x)dx = 0$

(C) $\int_0^{\pi/4} xf(x)dx = \frac{1}{6}$ (D) $\int_0^{\pi/4} f(x)dx = 1$

92. Let $f'(x) = \frac{192x^3}{2+\sin^4 \pi x}$ for all $x \in R$ with $f\left(\frac{1}{2}\right) = 0$.

If $m \le \int_{1/2}^{1} f(x)dx \le M$, then the possible values of m and M are

(A) $m = 13, M = 24$ (B) $m = \frac{1}{4}, M = \frac{1}{2}$ (C) $m = -11, M = 0$ (D) $m = 1, M = 12$

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Paragraph 93 to 94

Let $F: R \to R$ be a thrice differentiable function.

Suppose that $F(1) = 0$, $F(3) = -4$ and $F'(x) < 0$ for all $x \in (1/2, 3)$.

Let $f(x) = xF(x)$ for all $x \in R$.

93. The correct statement(s) is(are)

(A) $f'(1) < 0$ (B) $f(2) < 0$

(C) $f'(x) \neq 0$ for any xÎ(1,3) (D) $f'(x) = 0$ for some xÎ(1,3)

94. If $\int_1^3 x^2 F'(x)dx = -12$ and $\int_1^3 x^3 F''(x)dx = 40$, then the correct expression(s) is(are)

(A) $9f'(3) + f'(1) - 32 = 0$ (B) $\int_1^3 f(x)dx = 12$ **[JEE 2016]**

(C) $9f'(3) - f'(1) + 32 = 0$ (D) $\int_1^3 f(x)dx = -12$

95. The total number of distinct $x \in [0,1]$ for which $\int_0^x \frac{t^2}{1+t^4} dt = 2x - 1$ is **[JEE 2016]**

96. Let $f(x) = \lim_{n\to\infty} \left(\frac{n^n (x+n)\left(x+\frac{n}{2}\right)\ldots\left(x+\frac{n}{n}\right)}{n!(x^2+n^2)\left(x^2+\frac{n^2}{4}\right)\ldots\left(x^2+\frac{n^2}{n^2}\right)} \right)^{\frac{x}{n}}$, for all $x > 0$. Then **[JEE 2016]**

(A) $f\left(\frac{1}{2}\right) \geq f(1)$ (B) $f\left(\frac{1}{3}\right) \leq f\left(\frac{2}{3}\right)$

(C) $f'(2) \leq 0$ (D) $\frac{f'(3)}{f(3)} \geq \frac{f'(2)}{f(2)}$

97. Let $f: R \to R$ be a differentiable function such that $f(0) = 0$, $f\left(\frac{\pi}{2}\right) = 3$ and $f'(0) = 1$. if $g(x) = \int_x^{\frac{\pi}{2}} [f'(t)\operatorname{cosec} t - \cot t \operatorname{cosec} t f(t)]dt$ for $x \in \left(0, \frac{\pi}{2}\right]$, then $\lim_{x\to 0} g(x) =$ **[JEE 2017]**

98. For each positive integer n, let $y_n = \frac{1}{n}((n+1)(n+2)\ldots(n+n))^{\frac{1}{n}}$.

For $x \in R$, let $[x]$ be the greatest integer less than or equal to x. If $\lim_{n\to\infty} y_n = L$, then the value of $[L]$ is **[JEE Adv. 2018]**

99. The value of the integral $\int_0^{\frac{1}{2}} \frac{1+\sqrt{3}}{((x+1)^2(1-x)^6)^{\frac{1}{4}}} dx$ is **[JEE Adv. 2018]**

100. If $I = \frac{2}{\pi} \int_{-\pi/4}^{\pi/4} \frac{dx}{(1+e^{\sin x})(2-\cos 2x)}$ then $27I^2$ equals **[JEE Adv. 2019]**

101. For

For $a \in R |a| > 1$, let

$$\lim_{n\to\infty} \left(\frac{1+\sqrt[3]{2}+\cdots+\sqrt[3]{n}}{n^{7/3}\left(\frac{1}{(an+1)^2}+\frac{1}{(an+2)^2}+\cdots+\frac{1}{(an+n)^2}\right)} \right)$$

Then the possible value(s) of a is/are [JEE Adv. 2019]

(A) 7 (B) -6 (C) 8 (D) -9

102. The value of the integral $\int_0^{\pi/2} \frac{3\sqrt{\cos\theta}}{(\sqrt{\cos\theta}+\sqrt{\sin\theta})^5} d\theta$ equals _ **[JEE Adv. 2019]**

103. Let $f:\left[-\frac{\pi}{2},\frac{\pi}{2}\right] \to \mathbb{R}$ be a continuous function such that $f(0)=1$ and $\int_0^{\frac{\pi}{3}} f(t)dt = 0$ Then which of the following statements is (are) TRUE ? **[JEE Adv. 2021]**

(A) The equation $f(x) - 3\cos 3x = 0$ has at least one solution in $\left(0,\frac{\pi}{3}\right)$

(B) The equation $f(x) - 3\sin 3x = \frac{6}{\pi}$ has at least one solution in $\left(0,\frac{\pi}{3}\right)$

(C) $\lim_{x\to 0} \frac{x\int_0^x f(t)dt}{1-e^{x^2}} = -1$

(D) $\lim_{x\to 0} \frac{\sin x\int_0^x f(t)dt}{x^2} = -1$

Paragraph for Question Nos. 104 and 105

Let $f_1:(0,\infty)\to\mathbb{R}$ and $f_2:(0,\infty)\to\mathbb{R}$ be defined by $f_1(x)=\int_0^x \prod_{j=1}^{21}(t-j)^j dt, x>0$ and $f_2(x) = 98(x-1)^{50} - 600(x-1)^{49} + 2450, x>0$, where, for any positive integer n and real numbers $a_1, a_2, a_n, \prod_{i=1}^{n} a_i$ denotes the product of a_1, a_2, a_n. Let m_1 and n_1 respectively, denote the number of points of local minima and the number of points of local maxima of function $f_L i = 1,2$, in the interval $(0,\infty)$. **[JEE Adv. 2021]**

104. The value of $2m_1 + 3n_1 + m_1 n_1$ is____.

105. The value of $6m_2 + 4n_2 + 8m_2 n_2$ is_____.

Paragraph for Question Nos. 106 and 107

Let $g_1:\left[\frac{\pi}{8},\frac{3\pi}{8}\right] \to \mathbb{R}, i = 1,2$

$f:\left[\frac{\pi}{8},\frac{3\pi}{8}\right] \to \mathbb{R}$ be functions such that $g_1(x) = 1, g_2 = |4x-\pi|$ and $f(x) = \sin^2 x$, for all $x \in \left[\frac{\pi}{8},\frac{3\pi}{8}\right]$.

Define $S_i = \int_{\frac{\pi}{8}}^{\frac{3\pi}{8}} f(x)\cdot g_i(x)dx, i = 1,2$ **[JEE Adv. 2021]**

106. The value of $\frac{16\, S_1}{\pi}$ is

107. The value of $\frac{48\, S_2}{\pi^2}$ is

Paragraph for Question Nos. 108 and 109 **[JEE Adv. 2021]**

$\Psi_1:[0,\infty)\to\mathbb{R}, \Psi_2:[0,\infty)\to\mathbb{R}$,

such that $f(0) = g(0) = 0$,

$\Psi_1(x) = e^{-x} + x, x \geq 0$,

$\Psi_2(x) = x^2 - 2x - 2e^{-x} + 2, x \geq 0$,

$f(x) = \int_{-x}^{x} (|t| - t^2)e^{-t^2}, g(x) = \int_0^{x^2} \sqrt{t}e^{-t}, x > 0$

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108. Which of the following statements is TRUE ?

(A) $f(\sqrt{\ln 3}) + g(\sqrt{\ln 3}) = \frac{1}{3}$

(B)For every $x > 1$, there exists an $\alpha \in (1, x)$ such that $\Psi_1(x) = 1 + \alpha x$

(C) For every $x > 0$, there exists a $\beta \in (1, x)$ such that $\Psi_2(x) = 2x(\Psi_1(\beta) - 1)$

(D) f is an increasing function on the interval $\left[0, \frac{3}{2}\right]$

109. Which of the following statements is TRUE ?

(A) $\Psi_1(x) \leq 1$, fo all $x > 0$

(B) $\Psi_2(x) \leq 0$, for all $x > 0$

(C) $f(x) \geq 1 - e^{-x^2} - \frac{2}{3}x^3 + \frac{2}{5}x^5$, for all $x \in \left(0, \frac{1}{2}\right)$

(D) $g(x) \leq \frac{2}{3}x^3 - \frac{2}{5}x^5 + \frac{1}{7}x^7$, for all $x \in \left(0, \frac{1}{2}\right)$

110. For any real number x, let [x] denote the largest integer less than or equal to x.

If $I = \int_0^{10} \left[\sqrt{\frac{10x}{x+1}}\right] dx$, then the value of 9I is **[JEE Adv. 2021]**

111. Consider the equation $\int_1^e \frac{(\log_e x)^{\frac{1}{2}}}{x\left(a - (\log_e x)^{\frac{3}{2}}\right)} dx = 1$

$a \in (-\infty, 0) \cup (1, \infty)$

Which of the following statements is/are TRUE?

(A) No a satisfies the above equation

(B) An integer a satisfies the above equation

(C) An irrational number a satisfies the above equation

(D) More than one a satisfies the above equation **[JEE Adv, 2022]**

112. The greatest integer less than or equal to $\int_1^2 \log_2 (x^3 + 1)dx + \int_1^{\log_2 9} (2^x - 1)^{\frac{1}{3}} dx$ is

[JEE Adv. 2022]

113. For positive integer n, define

$f(n) = n + \frac{16+5n-3n^2}{4n+3n^2} + \frac{32+n-3n^2}{8n+3n^2} + \frac{48-3n-3n^2}{12n+3n^2} + \cdots + \frac{25n-7n^2}{7n^2}$. Then, the

value of $\lim_{n\to\infty} f(n)$ is equal to 7 **[JEE Adv. 2022]**

(A) $3 + \frac{4}{3}\log_e 7$

(B) $4 - \frac{3}{4}\log_e \left(\frac{7}{3}\right)$

(C) $4 - \frac{4}{3}\log_e \left(\frac{7}{3}\right)$

(D) $3 + \frac{3}{4}\log_e 7$

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ANSWER KEY

2. $\frac{\pi}{8}(1-\ell N4)$ **3.** $\frac{2\pi}{\sqrt{3}}$ **4.** $-\frac{3\sqrt{2}}{5}(e^{2\pi}+1)$

5. $\frac{\pi^2}{8}-\frac{\pi}{4}\times(1+\ln\ 2)+\frac{1}{2}$ **6.** $\left\{-\frac{\pi}{2},\frac{\pi}{2}\right\}$ **7.** $6-2e$

8. $\frac{\pi\sqrt{3}}{3}$ **9.** 2525 **10.** $\frac{19}{72}$

11. $2\sqrt{6}$ **12.** $2\sqrt{2}+\frac{4}{3}(3\sqrt{3}-2\sqrt{2})$ **13.** $\frac{4\pi}{\sqrt{3}}\tan^{-1}\left(\frac{1}{2}\right)$

14 $\frac{\pi+4}{666}$ **15.** $4\sqrt{2}-4(\ln(1+\sqrt{2}))$ **16.** $\frac{\pi}{8}\ln 2$ 17. $\frac{\pi(a+b)}{2\sqrt{2}}$

18. $-\frac{(a\pi+2b)\pi}{3\sqrt{3}}$ **19.** $\frac{\pi(\pi+3)}{2}$ **20.** $\frac{\pi a^2}{4}$

23. $\frac{1}{2\sqrt{11}}\ln\frac{\sqrt{11}+1}{\sqrt{11}-1}$ **24.** $I=\begin{cases}\frac{\pi\alpha}{\sin\alpha} & \text{if } \alpha\in(0,\pi)\\ \frac{\pi}{\sin\alpha}(\alpha-2\pi) & \text{if } \alpha\in(\pi,2\pi)\end{cases}$

25. $\frac{\pi^2}{16}-\frac{\pi}{4}\ln\ 2$ **26.** 0 **27.** $\frac{24}{5}\left[e\cos\frac{1}{2}+e\sin\frac{1}{2}-1\right]$

29. **(i)** $\frac{3}{2}$ **30.** $1+e$ **31.** 2007

34. **(i)** $\frac{\pi}{2}$ **(ii)** 2 **35.** **(a)** $3-\ln\ 4$; **(b)** $\frac{1}{e}$ (c) 11 **36.** $b\beta-a\alpha$

37. $4\sqrt{2}$ **38.** $y=x^{-3}e^{\left(1-\frac{1}{x}\right)}$ **39.** **(i)** 0 **(ii)** $\frac{p^2}{1+p^2}$

40. $\frac{\pi}{2}-1$ **41.** 125 **42.** cont. & diff. at $x=0$

43. $g(x)$ is cont. in $(-2,2)$; $g(x)$ is diff.

at $x=1$ & not diff. at $x=0$. Note that ; $g(x)=\begin{cases}-(x+2) & \text{for } -2\le x\le 0\\ -2+x-\frac{x^2}{2} & \text{for } 0<x<1\\ \frac{x^2}{2}-x-1 & \text{for } 1\le x\le 2\end{cases}$

44. (b) $f(x)=2e^{2x}+3$ & $g(x)=3-2e^{2x}$

45. $-\frac{2}{\pi^2}\cos\ \pi x$ for $0<x<1$; $\frac{2}{\pi^2}$ for $x\ge 1$ & $-\frac{2}{\pi^2}$ for $x\le 0$

47. $\frac{n}{n^2-1}$ 48. $\frac{\pi^2}{6\sqrt{3}}$ **50.** 4 **51.** 4 **52.** D **53.** C **54.** D

55. A → Q,S,T ; B → P,T ; C → S **56.** A **57.** B **58.** C **59.** A

60. A **61.** A **62.** A **63.** D **64.** B **65.** A **66.** A,D

67. D **68.** C **69.** D **70.** A **71.** A **72.** **(a)** C, **(b)** D, **(c)** A

73. 5051 **74.** **(a)** A, **(b)** (A)–S; (B)–S ; (C)–P; (D)–R **75.** A,D

76. **(a)** C, **(b)** A,B,C **(c)** 0 **77.** **(a)** B, **(b)** A, **(c)** B, **(d)** 4 **78.** A **79.** 6

80. B **81.** D **82.** B,D **83.** A,C,D **84.** 2 **85.** B **86.** A

87. D **88.** 0 **89.** 9 **90.** 7 **91.** A, B **92.** D **93.** A, B, C

94. C,D **95.** 1 **96.** B,C **97.** 2 **98.** 1 **99.** 2 **100.** 4

101. C,D **102.** 0.5 **103.** ABC **104.** 57 **105.** 6 **106.** 2 **107.** 1.5

108. C **109.** D **110.** 182 **111.** CD **112.** 5 **113.** B