

$$\tan \theta = \frac{3}{2}$$

$$\tan \alpha = \frac{2}{3}$$

$$\tan(\theta - \alpha) = m$$

(0,0)

1714

(2,4)



$$x^2$$

$$\int \sqrt[5]{x^3 + 2} dx$$



$$f(x+1) = 2f(x)$$

$$m \neq -1$$

$$f(a) \sum_{k=1}^n f(k)$$

$$m = -1$$

$$x \in (x_0 - \delta, x_0 + \delta)$$

$$2(2^n - 1)f(a)$$

$$\infty$$

$$[f(1), f(-1)]$$



Q31

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} a^2 + bc & (a+d)b \\ c(a+d) & cb + d^2 \end{bmatrix}$$

$$a^2 + bc = 1 \quad \checkmark$$

$$P^2(P-Q) = Q^2(Q-P) \quad (a+d)b = 0$$

$$= -Q^2(P-Q) \quad c(a+d) = 0$$

$$\underline{a+d=0} \quad \text{or}$$

$$a+d \neq 0$$

$$b = 0$$

$$c = 0$$

$$a^2 = 1$$

$$d^2 = 1$$

$$(a, d) = (1, 1)$$

$$(-1, -1)$$

$$(P^2 + Q^2)(P-Q) = 0 \quad \text{LHS}$$

$$a(-d) + bc = 1$$

$$cb + d^2 = 1 \quad \checkmark$$

6x6

$$\begin{pmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{pmatrix} \begin{pmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{pmatrix} = \begin{pmatrix} a_1^2 + b_1c_1 & a_1a_2 + b_1c_2 & a_1a_3 + b_1c_3 \\ a_2a_1 + b_2c_1 & a_2^2 + b_2c_2 & a_2a_3 + b_2c_3 \\ a_3a_1 + b_3c_1 & a_3a_2 + b_3c_2 & a_3^2 + b_3c_3 \end{pmatrix}$$

$$\frac{1.}{\int (\tan^3 x - x \tan^2 x) dx = \int \tan^2 x (\tan x - x) dx = \int t dt$$

$$= \frac{(\tan x - x)^2}{2} + C \quad dt = (\sec^2 x - 1) dx = \tan^2 x dx$$

$$\frac{2.}{\int \frac{\sec^4 x dx}{\sqrt{\tan x}} = \int \frac{(1 + \tan^2 x) \sec^2 x dx}{\sqrt{\tan x}} = \int \left(\frac{1}{\sqrt{\tan x}} + \tan^{\frac{3}{2}} x \right) \sec^2 x dx$$

DE, Seq,
Comp Angle,
Trig. Eqn.
SOT
Determinant

$$\frac{3.}{\int \frac{\ln^2 \left(\frac{x}{x+1} \right) dx}{x(x+1)} = \int t^2 dt$$

$$\int \frac{(1+t^4)}{t} \frac{dt}{2t}$$

$$\left(\frac{1}{x} - \frac{1}{x+1} \right) dx = dt$$

$$\tan^2 x = t$$

$$2 \tan x \sec^2 x dx = dt$$

$$\sec^2 x dx = \frac{1}{2t} dt$$

$$4. \quad \frac{1}{(a-b)} \int \frac{(a-b) \sin 2x \, dx}{(a \sin^2 x + b \cos^2 x)^2} = - \frac{1}{(a-b)(a \sin^2 x + b \cos^2 x)} + C.$$

$$\int \frac{dt}{t^2}$$

$$5. \quad \frac{1}{2} \int \frac{2 \left(x + e^x (\sin x + \cos x) + \sin x \cos x \right) dx}{(x^2 + 2e^x \sin x - \cos^2 x)^2} = - \frac{1}{2(x^2 + 2e^x \sin x - \cos^2 x)} + C$$

$$\underline{6.} \quad \int \left(\frac{\tan x + \sec x - 1}{\tan x - \sec x + 1} \right) dx \stackrel{\sec^2 x - \tan^2 x}{=} \int (\tan x + \sec x) dx = \ln |\sec x| + \ln |\sec x + \tan x| + C.$$

$$\underline{7.} \quad \int \frac{dx}{(\sqrt{3} \sin x + \cos x)} = \int \frac{dx}{2 \sin(x + \frac{\pi}{6})} = \frac{1}{2} \ln \left| \tan \left(\frac{x}{2} + \frac{\pi}{12} \right) \right| + C.$$

$$\underline{8.} \quad \int \frac{dx}{\sin x \cos^2 x} = \int \frac{\sin^2 x + \cos^2 x}{\sin x \cos^2 x} dx = \int (\tan x \sec x + \sec x) dx$$

$$= \int \frac{\sin x}{(1 - \cos^2 x) \cos^2 x} dx = \int \left(\frac{1}{1 - \cos^2 x} + \frac{1}{\cos^2 x} \right) \sin x dx = \sec x + \ln |\csc x - \cot x| + C.$$

$$9. \int \frac{dx}{\sec x + \csc x} = \frac{1}{2} \int \frac{2 \sin x \cos x dx}{(\sin x + \cos x)} = \frac{1}{2} \int \frac{((\sin x + \cos x)^2 - 1) dx}{(\sin x + \cos x)}$$

$$\boxed{\Sigma x - \text{V (1-15)}} = \frac{1}{2} \int \left(\sin x + \cos x - \frac{1}{\sqrt{2}} \csc\left(x + \frac{\pi}{4}\right) \right) dx$$

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$$= \frac{1}{2} \left(-\cos x + \sin x - \frac{1}{\sqrt{2}} \ln \left| \tan\left(\frac{x}{2} + \frac{\pi}{8}\right) \right| \right) + C.$$

$$\int \frac{1 \cdot dx}{(e^x + 1)} = - \int \frac{-e^{-x} dx}{1 + e^{-x}} = -\ln|1 + e^{-x}| + C.$$

$$\int \frac{(1 + e^x) - e^x}{(e^x + 1)} dx = \int \left(1 - \frac{e^x}{1 + e^x} \right) dx = x - \ln|1 + e^x| + C$$

$$11. \int \frac{dx}{(1 + 3e^x + 2e^{2x})} \xrightarrow{2(e^x + 1) - (2e^x + 1)} \int \left(\frac{2}{2e^x + 1} - \frac{1}{e^x + 1} \right) dx$$

$$= \int \frac{dx}{(2e^x + 1)(e^x + 1)} = -2 \ln|2 + e^{-x}| + \ln|1 + e^{-x}| + C = \int \left(\frac{2e^{-x}}{2 + e^{-x}} + \frac{-e^{-x}}{1 + e^{-x}} \right) dx$$

