

$$\underline{Q} \quad (\text{ii}) \quad \cos\left(\frac{\sin^{-1}x}{y}\right) = 0$$

$$\frac{\sin^{-1}x}{y} = (2n+1)\frac{\pi}{2} \quad n \in \mathbb{Z}$$

$y \neq 1$

$$\sin^{-1}x = (2n+1)\frac{\pi}{2} = -\frac{\pi}{2}, \frac{\pi}{2}$$

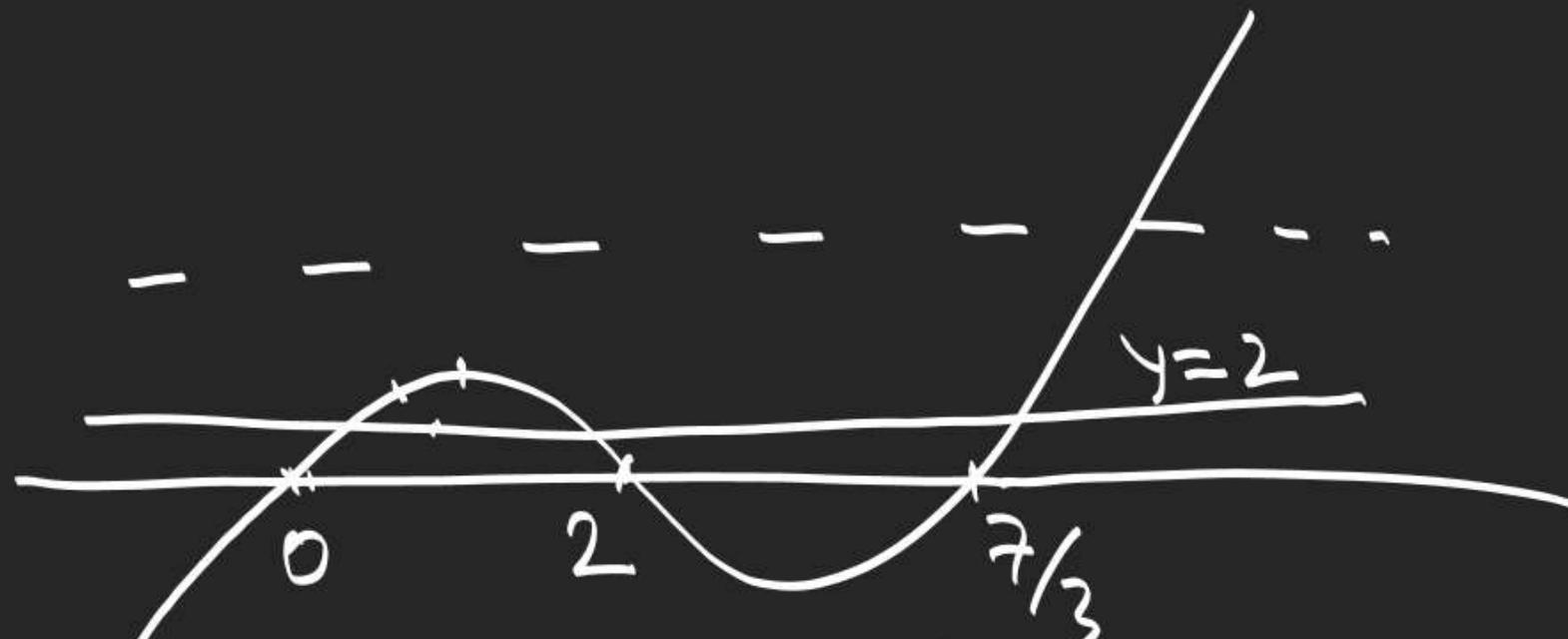
$$x = -1, 1$$

$$\begin{aligned} y &= -1 \\ -\sin^{-1}x &= (2n+1)\frac{\pi}{2} \\ &= -\frac{\pi}{2}, \frac{\pi}{2} \end{aligned}$$

$$\begin{aligned} y &= 2 \\ \frac{\sin^{-1}x}{2} &= (2n+1)\frac{\pi}{2} \quad x \neq 0 \end{aligned}$$

$$(x, y) = (-1, 1), (1, 1), (-1, -1), (1, -1)$$

11.



$$f(x) = r(x-2)(3x-7)$$

$$f(x) = 2$$

$$f(x) = 1(-1)(-4)$$

$$= 4$$

$$r, s, t > 0$$

$$r(x-2)(3x-7) - 2 = 0$$

$$y = 2 \quad \theta_1 + \theta_2 + \theta_3 \in (0, \frac{3\pi}{2})$$

$$\tan(\tan^{-1} r + \tan^{-1} s + \tan^{-1} t) = \frac{s_1 - s_3}{1 - s_2}$$

$$\begin{aligned} \tan^{-1} r + \tan^{-1} s + \tan^{-1} t &= \pi \\ \tan^{-1} r + \tan^{-1} s + \tan^{-1} t &= \frac{\pi}{2} \end{aligned}$$

$$\frac{1 - \sum rs}{1 - \sum rs} = -1$$

$$\underline{13.} \quad 2(\cos^{-1}x)^2 = a \cos^{-1}x + a^2$$

$$2t^2 - at - a^2 = 0 \quad \frac{\pi}{2} < 2 < \frac{2^2 + 4}{1+x^2} = 2 + \frac{2}{1+x^2} < 4 < 3\pi$$

$$-2at + at$$

$$(2t+a)(t-a) = 0$$

$$\cos^{-1}x = -\frac{a}{2}, \text{ or } \cos^{-1}x = a$$

$$-\frac{a}{2} \in (0, \pi] \quad \text{or} \quad a \in (0, \pi]$$

$$a \in [-2\pi, 0) \cup (0, \pi]$$

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots \infty , x \in \mathbb{R}$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots \infty , x \in (-1, 1]$$

$$\ln(1-x) = -x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} - \dots \infty , x \in [-1, 1)$$

$$\sin x = \frac{x}{1!} - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots \rightarrow \infty , x \in \mathbb{R}$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots \infty , x \in \mathbb{R}$$

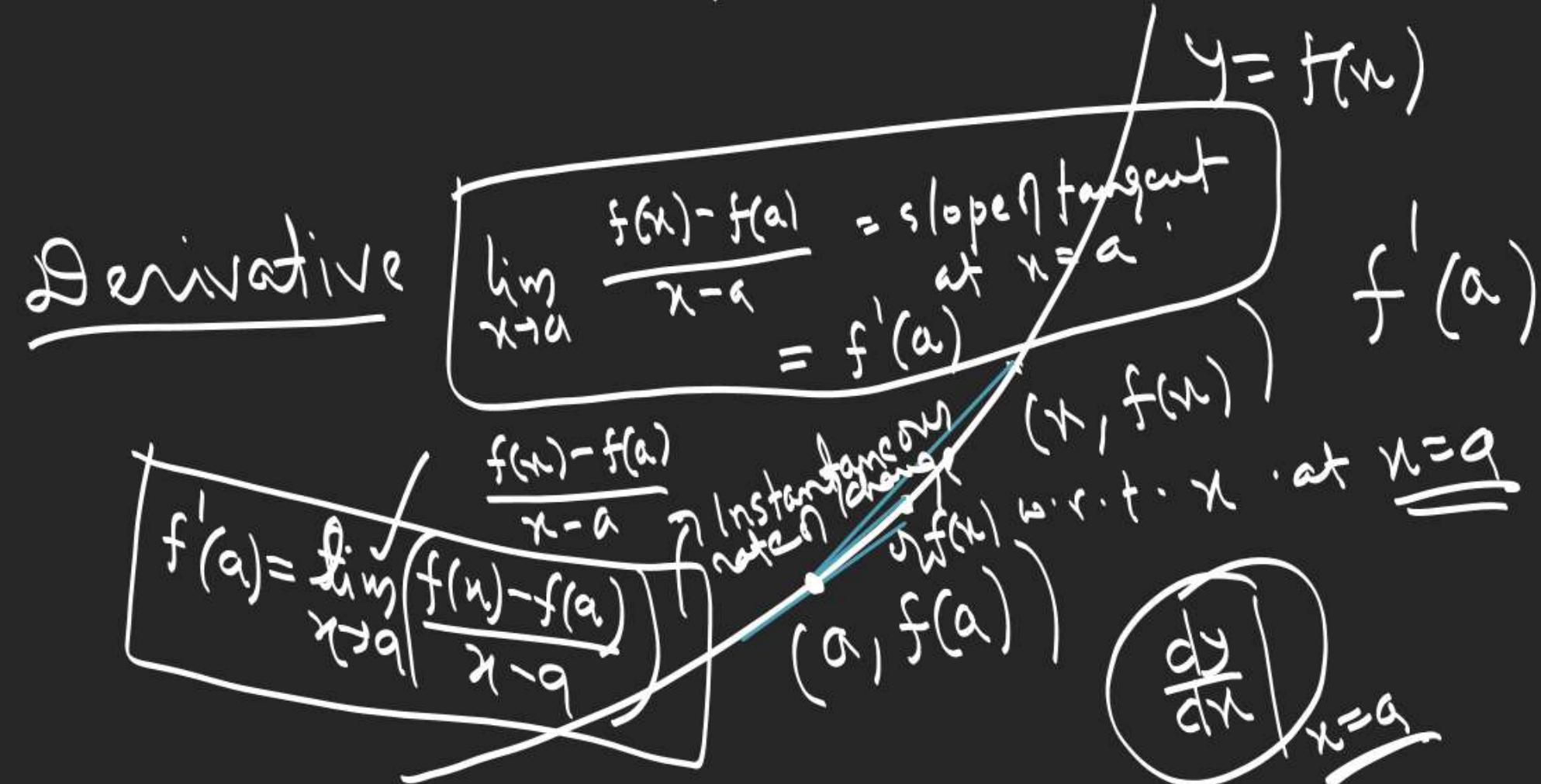
$$\tan x = x + \frac{x^3}{3} + \frac{2}{15}x^5 + \dots \infty$$

# Binomial theorem for any index

$n \in \mathbb{I}^-$

$\mathbb{Q} - \{\mathbb{I}\}$

$$(1+x)^n = 1 + nx + \frac{n(n-1)x^2}{2!} + \frac{n(n-1)(n-2)x^3}{3!} + \dots \infty$$



$$\begin{aligned}
 1. \quad \lim_{x \rightarrow \frac{\pi}{4}} \left( \frac{1 - \cot^3 x}{2 - \cot x - \cot^3 x} \right) &= \lim_{x \rightarrow \frac{\pi}{4}} \frac{(1 - \cot x)(1 + \cot^2 x + \cot x)}{(1 - \cancel{\cot x})(\cancel{\cot^2 x} + \cot x + 2)} \\
 &= \frac{1+1+1}{1+1+2} = \frac{3}{4} . \\
 &\quad \cot x = t
 \end{aligned}$$

$$\begin{aligned}
 2. \quad \lim_{x \rightarrow 2} \frac{2^x + 2^{3-x} - 6}{\sqrt{2^{-x}} - 2^{1-x}} &\quad \lim_{t \rightarrow 1} \frac{1 - t^3}{2 - t - t^3} \\
 \lim_{x \rightarrow 2} \frac{2^{2x} + 8 - 6 \cdot 2^x}{2^{x/2} - 2} &= \lim_{x \rightarrow 2} \frac{(2^x - 4)(2^x - 2)}{2^{x/2} - 2} = \lim_{x \rightarrow 2} \frac{(2^{x/2} + 2)(2^x - 2)}{2^{x/2} - 2} \\
 &= 4 \times 2 = 8
 \end{aligned}$$

$$\text{3: } \lim_{x \rightarrow 9} \left( \frac{3 - \sqrt{x}}{4 - \sqrt{2x-2}} \right)$$

$$\lim_{x \rightarrow 9} \frac{(9-x)(4 + \sqrt{2x-2})}{(3+\sqrt{x})(16-(2x-2))} = \lim_{x \rightarrow 9} \frac{4 + \sqrt{2x-2}}{2(3+\sqrt{x})} = \frac{8}{12} = \frac{2}{3}$$

$$\therefore \lim_{n \rightarrow \infty} \left( \frac{\sqrt[3]{n^3 - 2n^2 + 1} + \sqrt[3]{n^4 + 1}}{\sqrt[4]{n^6 + 6n^5 + 2} - \sqrt[5]{n^7 + 3n^3 + 1}} \right)$$

$$\begin{aligned} & \underset{n \rightarrow \infty}{\cancel{\lim}} \left( \frac{n^{3/2} \sqrt{1 - \frac{2}{n} + \frac{1}{n^3}} + n^{4/3} \sqrt[3]{1 + \frac{1}{n^4}}}{n^{3/2} \sqrt[4]{1 + \frac{6}{n} + \frac{9}{n^2}} - n^{7/5} \sqrt[5]{1 + \frac{3}{n^4} + \frac{1}{n^7}}} \right) = \underset{n \rightarrow \infty}{\cancel{\lim}} \left( \sqrt{1 - \frac{2}{n} + \frac{1}{n^3}} + \frac{1}{n^{1/6}} \sqrt[3]{1 + \frac{1}{n^4}} \right) \\ & \quad \cancel{n^{3/2}} \left( \sqrt[4]{1 + \frac{6}{n} + \frac{9}{n^2}} - \frac{1}{n^{1/10}} \sqrt[5]{1 + \frac{3}{n^4} + \frac{1}{n^7}} \right) \end{aligned}$$

$$= \frac{1}{1 - 0} = 1$$

5:

$$\lim_{x \rightarrow -\infty} \frac{\sqrt{x^2 + 1}}{(3x - 6)} = \lim_{x \rightarrow -\infty} \frac{(\sqrt{|x|}) \sqrt{1 + \frac{1}{x^2}}}{3x - 6}$$

$$= -\frac{1}{3}$$

$\sqrt{x^2} = |x|$

$$\text{Q: } \lim_{x \rightarrow \pm\infty} \left( \sqrt{x^2 - 2x - 1} - \sqrt{x^2 - 7x - 3} \right)$$

$$= \lim_{x \rightarrow \pm\infty} \frac{5x + 2}{\sqrt{x^2 - 2x - 1} + \sqrt{x^2 - 7x - 3}}$$

$$= \lim_{x \rightarrow \pm\infty} \frac{\cancel{x} \left( 5 + \frac{2}{\cancel{x}} \right)}{\cancel{x} \left( \sqrt{1 - \frac{2}{\cancel{x}} - \frac{1}{\cancel{x}^2}} + \sqrt{1 - \frac{7}{\cancel{x}} - \frac{3}{\cancel{x}^2}} \right)}$$

$$x \rightarrow -\infty, l = -\frac{5}{2}$$

$$x \rightarrow \infty, l = \frac{5}{2}$$

$$\begin{aligned}
 & \underline{\text{Ex}} \quad \lim_{x \rightarrow \infty} \left( \sqrt{4x^2 + x} - \sqrt{\frac{4x^3}{x+2}} \right) = \lim_{x \rightarrow \infty} \frac{\sqrt{4x^3 + 9x^2 + 2x} - \sqrt{4x^3}}{\sqrt{x+2}} \\
 &= \lim_{x \rightarrow \infty} \frac{(9x^2 + 2x)}{\sqrt{x+2} (\sqrt{4x^3 + 9x^2 + 2x} + \sqrt{4x^3})}
 \end{aligned}$$

$$\begin{aligned}
 & \underline{\text{Ex}} \quad \lim_{n \rightarrow \infty} \frac{1^2 + 2^2 + 3^2 + \dots + n^2}{n^3} = \lim_{n \rightarrow \infty} \frac{n(n+1)(2n+1)}{6n^3} = \frac{2}{6} = \frac{1}{3} \\
 & \qquad \qquad \qquad = \boxed{\frac{1}{3}}
 \end{aligned}$$

$$9. \lim_{x \rightarrow \infty} \left( \sqrt[3]{x^3 + 3x^2} - \sqrt{x^2 - 2x} \right)$$

$$\lim_{x \rightarrow \infty} x \left( \left(1 + \frac{3}{x}\right)^{\frac{1}{3}} - \left(1 - \frac{2}{x}\right)^{\frac{1}{2}} \right)$$

$$\lim_{x \rightarrow \infty} x \left[ \left( x + \frac{1}{3} \left( \frac{3}{x} \right) + \frac{\frac{1}{3} \left( \frac{1}{3} - 1 \right)}{2!} \left( \frac{3}{x} \right)^2 + \dots \infty \right) - \left( x + \frac{1}{2} \left( \frac{-2}{x} \right) + \frac{\frac{1}{2} \left( \frac{1}{2} - 1 \right)}{2!} \left( \frac{-2}{x} \right)^2 + \dots \infty \right) \right]$$

$$\lim_{x \rightarrow \infty} x \left( \frac{2}{x} + \frac{1}{x^2} (\dots) \right) = \lim_{x \rightarrow \infty} \left( 2 + \frac{1}{x} (\dots) \right) = 2$$

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$\sum x - 4 \rightarrow 1 - 10$