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Binomial Theorem for natural number index

$$\boxed{\begin{aligned}(a+b)^n &= {}^nC_0 a^n + {}^nC_1 a^{n-1} b + {}^nC_2 a^{n-2} b^2 + \dots + {}^nC_r a^{n-r} b^r \\ &\quad + \dots + {}^nC_{n-1} a b^{n-1} + {}^nC_n b^n \\ &= T_1 + T_2 + T_3 + \dots + T_{n+1}\end{aligned}}$$

$$\boxed{(a+b)^n = \sum_{r=0}^{n} {}^nC_r a^{n-r} b^r}$$

$(a+b)(a+b)(a+b)(a+b)$

$T_{r+1} = {}^nC_r a^{n-r} b^r$

General term

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$(2-3x)^8$

$(2-3x)^8 = \sum_{r=0}^{8} {}^8C_r\, 2^{8-r} (-3x)^r$

Put $x=1$ $\quad (2-3)^8 = \sum_{r=0}^{8} {}^8C_r\, 2^{8-r} (-3)^r$

- 6th Term of $(2-3x)^8 = {}^8C_5\, 2^{8-5} (-3x)^5$

- Coeff. of 6th term $= {}^8C_5\, 2^3 (-3)^5$

- Binomial coeff. of 6th term $= {}^8C_5$

Find sum of all coefficients of $(2-3x)^8 = {}^8C_0 2^8 + {}^8C_1 2^7(-3) + {}^8C_2 2^6(-3)^2 + \dots {}^8C_8(-3)^8$

$= (2-3)^8$

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$$(2+3x-7z)^{15} = \sum a_r\, 2^{r_1}(3x)^{r_2}(-7z)^{r_3}$$

Find sum of all coeff.

put $x=1,\ z=1$

$$= (2+3-7)^{15} = -2^{15}$$

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Middle term of $(a+b)^n$, $n \in N$.

$$T_1 + T_2 + T_3 + \dots + \dots + T_{n+1}$$

$$= \begin{cases} T_{\frac{n+1}{2}},\ T_{\frac{n+3}{2}} & n \text{ is odd} \\ T_{\frac{n}{2}+1} & n \text{ is } \underline{\text{even}} \end{cases}$$

$\frac{n-1}{2}+1$

$\left({}^nC_r\right)_{max} =$

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1. Find x if 3^{rd} term in the expansion $\left(x + x^{\log_{10} x}\right)^5$ is equal to 10^6.

$$T_3 = {}^5C_2\, x^3 \left(x^{\log_{10} x}\right)^2 \Rightarrow 10\, x^{3+2\log_{10} x} = 10^6$$

$2+1$

$$x^{3+2\log_{10} x} = 10^5$$

$$\left(3+2\log_{10} x\right)\log_{10} x = 5$$

$$2t^2 + 3t - 5 = 0 = (2t+5)(t-1)$$

$-2t+5t$

$$\log_{10} x = -\frac{5}{2}, 1$$

$$x = 10^1, 10^{-5/2}$$

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2. Find middle term in expansion of

$\left(2x^2 - \frac{1}{3x}\right)^{11}$.

$\boxed{12}$

$T_6 = {}^{11}C_5 (2x^2)^6 \left(-\frac{1}{3x}\right)^5$

$T_7 = {}^{11}C_6 (2x^2)^5 \left(-\frac{1}{3x}\right)^6$

3. Find the term containing x^3 in $\left(2x^2 - \frac{1}{3x}\right)^6$.

$T_{r+1} = {}^6C_r (2x^2)^{6-r} \left(-\frac{1}{3x}\right)^r = {}^6C_r\, 2^{6-r} \left(-\frac{1}{3}\right)^r x^{12-3r}$

$12 - 3r = 3$

$\Rightarrow \boxed{r = 3}$

$T_4 = {}^6C_3\, 2^3 \left(-\frac{1}{3}\right)^3 x^3$

HW → P&C → Ex - I (1-20)