Nishant Jindal

Properties

* $a^{\log_a b} = b$, $a>0, a\neq 1, b>0$

* $\log_a m + \log_a n = \log_a (mn)$, $a>0, a\neq 1, m>0, n>0$

* $\log_a (m) - \log_a (n) = \log_a \left(\frac{m}{n}\right)$, $a>0, a\neq 1, m>0, n>0$

* $\log_a (m)^n = n \log_a (m)$, $a>0, a\neq 1, m>0$

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$$a^{\log_a b} = b$$

Let $\log_a b = x$

$\Rightarrow \quad a^x = b$

$$\boxed{a^{\log_a b} = b}$$

$$\log_a m + \log_a n = \log_a (mn)$$

$\log_a m = x \quad \& \quad \log_a n = y$

$\Rightarrow \quad a^x = m \qquad \Rightarrow \quad a^y = n$

$\frac{m}{n} = \frac{a^x}{a^y} = a^{x-y} \quad \Rightarrow \log_a\left(\frac{m}{n}\right) = x - y.$

$mn = a^x a^y = a^{x+y}$

$(mn) = (a)^{x+y}$

$\Rightarrow \quad x + y = \log_a (mn)$

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$$\log_a(m^n) = n\log_a m$$

Let $\log_a m = x$

$\Rightarrow m = a^x$

$\Rightarrow m^n = (a^x)^n$

$\Rightarrow m^n = a^{nx}$

$\Rightarrow \log_a(m^n) = nx = n\log_a m$

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Base Change Theorem

$$\log_a b = \frac{\log_c b}{\log_c a}$$

$a>0, a\neq 1, c>0, c\neq 1, b>0$

$$(\log_a b)(\log_b a) = 1$$

$$(\log_a b)(\log_b a) = \frac{\cancel{\log_c b}}{\cancel{\log_c a}} \; \frac{\cancel{\log_c a}}{\cancel{\log_c b}}$$

$$= 1$$

$$\log_a b = x$$

$$\Rightarrow a^x = b$$

$$\Rightarrow \log_c a^x = \log_c b \Rightarrow x \log_c a = \log_c b \Rightarrow x = \frac{\log_c b}{\log_c a}$$

$$4 = 4$$

$$\log_3 4 = \log_3 4$$

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* $\log_{(a^m)} b = \frac{1}{m}\log_a b$, $a>0, a\neq 1, b>0,$
  $m\neq 0$

* $a^{\log_b c} = c^{\log_b a}$

* $\log_{(a^m)}(b^n) = \frac{n}{m}\log_a b$

$n\log_{a^m} b = \frac{n}{m}\log_a b$

$\log_{a^m} b = \frac{1}{\log_b(a^m)}$

$= \frac{1}{m\log_b a}$

$= \frac{1}{m}\log_a b$

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$$a^{\log_b c} = c^{\log_b a}$$

$$a^{\log_b c} = a^{\frac{\log_a c}{\log_a b}} = \left(a^{\log_a c}\right)^{\frac{1}{\log_a b}}$$

$$\left(a^{\log_a c}\right) = c$$

$$= c^{\frac{1}{\log_a b}}$$

$$= c^{\log_b a}$$

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$$\left(\log_a(m_1) + \log_a(m_2)\right) + \log_a(m_3) + \log_a(m_4) = \log_a(m_1 m_2 m_3 m_4)$$

$$= \left(\log_a(m_1 m_2) + \log_a(m_3)\right) + \log_a(m_4)$$

$$= \log_a(m_1 m_2 m_3) + \log_a m_4$$

$$= \log_a(m_1 m_2 m_3 m_4)$$

$$\log_a x_1 + \log_a x_2 - \log_a y_1 - \log_a y_2 - \log_a y_3$$

$$\log_a\left(\frac{x_1 x_2}{y_1}\right) - \log_a y_2 - \log_a y_3$$

$$= \log_a \frac{x_1 x_2}{y_1 y_2} - \log_a y_3$$

$$= \log_a\left(\frac{x_1 x_2}{y_1 y_2 y_3}\right)$$

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1: Let $A = \log_{11}\left(11^{\log_{11} 1331}\right) = \log_{(11)^3} 1331^{11} = 3$

$B = \log_{385} 5 + \log_{385} 7 + \log_{385} 11 = \log_{385}(\overbrace{5\times 7\times 11}^{385}) = 1$

$C = \log_4\left(\log_2\left(\underbrace{\log_5 625}_{4}\right)\right) = \log_4\left(\log_2 4\right) = \log_4 2 = \frac{1}{2}$

$D = 10^{\log_{100} 16} = \left(10^{\frac{1}{2}\log_{10} 16}\right) = \left(\underbrace{10^{\log_{10} 16}}_{4}\right)^{\frac{1}{2}} = (16)^{\frac{1}{2}} = 4$

$100 = 10^2$

$D = 16^{\log_{100} 10}$

$= 16^{1/2} = 4.$

find $\frac{AB}{CD} = \frac{3\times 1}{\frac{1}{2}\times 4} = \boxed{\frac{3}{2}}$

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2. Simplify $\log_2(10) - \log_8(125) = \log_2 10 - \log_{2^3} 5^3$

$= \log_2 10 - \cancel{\tfrac{3}{3}} \log_2 5 = \log_2\left(\frac{10}{5}\right) = \log_2 2 = 1$

3. If $(\log_2 3)(\log_3 4)(\log_4 5) \cdots (\log_n (n+1)) = 10$, find 'n'.

$\log_2(n+1) = 10 \Rightarrow n+1 = 2^{10}$

$\boxed{n = 1023}$

$$\frac{\cancel{\log_c 3}}{\log_c 2} \cdot \frac{\cancel{\log_c 4}}{\cancel{\log_c 3}} \cdot \frac{\cancel{\log_c 5}}{\cancel{\log_c 4}} \cdot \frac{\cancel{\log_c 6}}{\cancel{\log_c 5}} \cdots \frac{\cancel{\log_c n}}{\cancel{\log_c (n-1)}} \cdot \frac{\log_c(n+1)}{\cancel{\log_c n}}$$

$$= \frac{\log_c(n+1)}{\log_c 2} = \log_2(n+1).$$

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Simplify 4: $\cancel{7^{\log_3 5}} + 3^{\log_5 7} - \cancel{5^{\log_3 7}} - 7^{\log_5 3}$

$= \boxed{0}$

5: $\log_{10} 2 + 16\log_{10}\left(\frac{16}{15}\right) + 12\log_{10}\frac{25}{24} + 7\log_{10}\left(\frac{81}{80}\right)$

$\log_{10} 2 + \log_{10}\left(\frac{16}{15}\right)^{16} + \log_{10}\left(\frac{25}{24}\right)^{12} + \log_{10}\left(\frac{81}{80}\right)^{7}$

$= \log_{10}\left(2\left(\frac{16}{15}\right)^{16}\left(\frac{25}{24}\right)^{12}\left(\frac{81}{80}\right)^{7}\right) = \log_{10}\left(2\,\frac{2^{64}}{3^{16}5^{16}}\,\frac{5^{24}}{2^{36}3^{12}}\,\frac{3^{28}}{2^{28}5^{7}}\right)$

$= \log_{10} 10 = 1$

6. $$\frac{1}{\log_3 2}+\frac{2}{\log_{9=3^2} 4=2^2}-\frac{3}{\log_{27=3^3} 8=2^3}=\frac{1}{\log_3 2}+\frac{2}{\log_3 2}-\frac{3}{\log_3 2}=0$$

7. $$\frac{\log_3(12)}{\log_{36}(3)}-\frac{\log_3(4)}{\log_{108}(3)}=\left(\log_3 12\right)\left(\log_3 36\right)-\left(\log_3 4\right)\log_3(108)$$

$$=\left(\log_3\left(3\cdot 2^2\right)\right)\left(\log_3 3^2 2^2\right)-\left(\log_3 2^2\right)\left(\log_3 3^3 2^2\right)$$

$$=\left(1+2\log_3 2\right)\left(2+2\log_3 2\right)-2\log_3 2\left(3+2\log_3 2\right)$$

$$=(1+2t)(2+2t)-2t(3+2t)=\boxed{2}$$

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2.

$$4\left(\cos\frac{2\pi}{7}\cos\frac{\pi}{7}\right)-1=\frac{\sin\frac{4\pi}{7}}{\sin\frac{\pi}{7}}-1$$

$$\pi-\frac{3\pi}{7}$$

$$=\frac{\sin\left(\frac{4\pi}{7}\right)-\sin\frac{\pi}{7}}{\sin\frac{\pi}{7}}=\frac{\sin\frac{3\pi}{7}-\sin\frac{\pi}{7}}{\sin\frac{\pi}{7}}$$

$$=\frac{2\sin\frac{\pi}{7}\cos\frac{2\pi}{7}}{\sin\frac{\pi}{7}}=2\cos\frac{2\pi}{7}$$

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3:

$$\left(\tan A+\cot A\right)+\left(\tan^2 A+\cot^2 A\right)+\left(\tan^3 A+\cot^3 A\right)=70.$$

$$\boxed{A+B=\frac{\pi}{2}}$$

$$\tan B=\tan\left(\frac{\pi}{2}-A\right)$$
$$=\cot A$$

$$\left(\underbrace{\tan A+\cot A}_{=t}\right)+\left(\tan A+\cot A\right)^2-2+\left(\tan A+\cot A\right)^3-3\left(\tan A+\cot A\right)=70$$

$$\tan A+\cot A=t.$$

$$t^3+t^2-2t-72=0.$$

$$(t-4)(t^2+5t+18)=0$$

Hint & Trial $\to 0,\pm1,\pm2,\pm3,\pm4,$
$\pm\frac{1}{2},\pm\frac{1}{3},$

$$\tan A+\cot A=4$$

$$\frac{1}{\sin A\cos A}=4 \Rightarrow \boxed{\sin 2A=\frac{1}{2}}$$

$\boxed{\text{Q4. Upto 15}} \to$ Ex II