DPP – 02

SOLUTION

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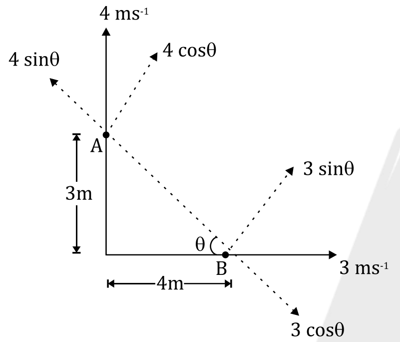
1. (a) $\tan\ \theta = \frac{3}{4}$

v_{sep} = relative velocity along line AB

$\Rightarrow v_{sep} = 3\cos\ \theta + 4\sin\ \theta$

$\Rightarrow v_{sep} = 3\left(\frac{4}{5}\right) + 4\left(\frac{3}{5}\right) = \frac{24}{5} = 4.8\ \text{ms}^{-1}$

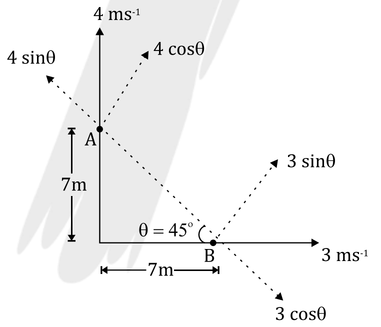


(b) $\theta = 45°$

v_{sep} = relative velocity along line AB

$\Rightarrow v_{sep} = 3\cos\theta + 4\sin\theta$

$\Rightarrow v_{sep} = 3\left(\frac{1}{\sqrt{2}}\right) + 4\left(\frac{1}{\sqrt{2}}\right) = \frac{7}{\sqrt{2}}\ \text{ms}^{-1}$



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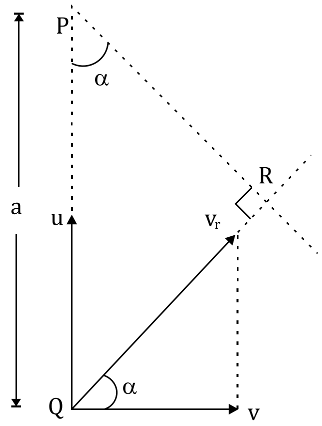
2. Assuming P to be at rest, particle Q is moving with velocity v_r in the direction shown in figune Components of 0, along and perpendicular to PQ are u and v respectively. In the figure

$\sin\ \alpha = \frac{u}{v_t}, \cos\ \alpha = \frac{v}{v_r}$



(i) The closest distance between the particles is PR So,

$$s_{min} = PR = PQ\cos\ \alpha = (a)\left(\frac{v}{v_r}\right)$$

$$\Rightarrow S_{min} = \frac{av}{v_r}$$

(ii) Time after which they arrive at their nearest distance is

$$t = \frac{QR}{v_r} = \frac{(PQ)\sin\ \alpha}{v_r} = \frac{(a)\left(\frac{u}{v_r}\right)}{v_r} = \frac{aut}{v_r^2}$$

3. (a) The velocity of swimmer with respect to river $v_{sr} = 5kmh^{-1}$, this is greater than the river flow velocity, therefore, he can cross the river directly (along the shortest path). The angle of swim must be

$$\theta = \frac{\pi}{2} + \sin^{-1}\left(\frac{v_r}{v_{sr}}\right) = 90^\circ + \sin^{-1}\left(\frac{v_r}{v_{sr}}\right)$$

$$\theta = 90^\circ + \sin^{-1}\left(\frac{3}{5}\right) = 90^\circ + 37^\circ$$

$\theta = 127^\circ$ w.r.t. the river flow or 37° w.r.t. perpendicular in backward direction

(b) Resultant velocity will be

$$v_s = \sqrt{v_{sr}^2 - v_r^2} = \sqrt{5^2 - 3^2} = 4kmh^{-1}$$

Along the direction perpendicular to the river flow.

(c) Time taken to cross the

$$t = \frac{d}{\sqrt{v_{sr}^2 - v_r^2}} = \frac{1\ km}{4kmh^{-1}} = \frac{1}{4}\ h = 15\ min$$

4. Velocity of Aeroplan while flying from P to Q is

$v_d = v + u$

$\Rightarrow t_{PQ} = \frac{1}{v+u}$

Velocity of Aeroplan while flying from Q to R is

$v_0 = \sqrt{v^2 - u^2}$

$$\Rightarrow t_{QR} = \frac{1}{\sqrt{v^2 - u^2}}$$

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Velocity of Aeroplan while flying from R to S is

$v_a = v - \|$

$\Rightarrow t_{Rs} = \frac{1}{v-u}$

Velocity of Aeroplan while flying from S to P is

$v_a = \sqrt{v^2 - u^2}$

$\Rightarrow t_{SP} = \frac{1}{\sqrt{v^2-u^2}}$

Total time

$t = t_{PQ} + t_{QR} + t_{RS} + t_{SP}$

$$\Rightarrow 1 = \frac{1}{v+u} + \frac{1}{\sqrt{v^2-u^2}} + \frac{1}{v-u} + \frac{1}{\sqrt{v^2-u^2}}$$

$\Rightarrow t = \frac{2a}{v^2-u^2}\left(v + \sqrt{v^2-u^2}\right)$

5. : Given, $\theta = 45^\circ$, velocity of girl, $|\vec{v}_G| = OA = 15\sqrt{2}$ km h^{-1} Here, $|\vec{v}_{RG}| = OB =$ velocity of rain with respect to the girl and $|\vec{v}_R| = OC =$ velocity of rain with respect to ground

Now, for speed of rain with respect to the moving girl, From

$\triangle$ OCB, $\frac{OB}{CB} = \cot\ 45^\circ \therefore OB = CB\cot\ 45^\circ = v_G \times \cot\ 45^\circ$ or $v_{RG} = 15\sqrt{2} \times 1 = 15\sqrt{2}$ km h^{-1} or $v_{RG} = \frac{30}{\sqrt{2}}$ km h^{-1} Therefore, this velocity is same as that of the velocity of the girl.

6. Focal length, $f = 10$ cm; Distance of car B from car A, $u = 1.9$ m $\Rightarrow u = -190$ cm

The mirror is convex in nature, as it is rear view mirror. $V_{B/A} = 40\ m\ s^{-1} = V_{0/m} =$ velocity of object with respect to mirror.

Velocity of image with respect to mirror is $V_{1/m} = -m^2 V_{0/m}$

The magnification of mirror is

$$m = \frac{f}{f-u} = \frac{10}{10+190} = \frac{10}{200} = \frac{1}{20}$$

$V_{1/m} = -\left(\frac{1}{20}\right)^2 \times 40 = 0.1$ m s

7. Let train A moving along positive x-axis. Then velocity of train A w.r.t. ground,

$\vec{v}_{Ag} = 36\ km/h\hat{i} = 10\ m/s(\hat{i})$

Velocity of train B w.r.t. ground, $\vec{v}_{Bg} = 72\ km/h(-\hat{i}) = -20\ m/s(\hat{i})$

Velocity of the person w.r.t train A,

$\vec{v}_{pA} = 1.8\ km/h(-\hat{i}) = -0.5\ m/s(\hat{i})$

velocity of the person w.r.t train B,

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$\vec{v}_{pB} = \vec{v}_{pg} + \vec{v}_{pB} = \vec{v}_{pA} + \vec{v}_{Ag} - \vec{v}_{Bg}$

$\vec{v}_{pB} = -0.5(\hat{\imath}) + 10(\hat{\imath}) - (-20)\hat{\imath} = (29.5\hat{\imath})$

$\therefore$ Velocity of the person observed from train B $= 29.5\ \text{m s}^{-1}$.

8. In the first case,

$\tan 60^\circ = \frac{v_{\text{rain, car}}}{v_{\text{car, road}}} = \frac{v_r}{v}$

In the second case, $\tan 45^\circ = \frac{v_T}{(1+\beta)v}$

Solving the eqn's (i) and (ii), we have, $\frac{\sqrt{3}}{1} = 1 + \beta$ or $\beta = 0.73$

9. The total distance to be travelled by the train is $60 + 120 = 180$ m.

When the trains are moving in the same direction, relative velocity is

$v_1 - v_2 = 80 - 30 = 50\ \text{kmhr}^{-1}$.

So time taken to cross each other, $t_1 = \frac{180}{50 \times \frac{10^3}{3600}} = \frac{18 \times 18}{25}$ s

When the trains are moving in opposite direction relative velocity,

$|v_1 - (-v_2)| = 80 + 30 = 110\ \text{kmhr}^{-1}$

So time taken to cross each other

$$t_2 = \frac{180}{110 \times \frac{1000}{3600}} = \frac{18 \times 36}{110}\ \text{s; Ratio } \frac{t_1}{t_2} = \frac{\frac{18 \times 18}{15}}{\frac{18 \times 36}{110}} = \frac{11}{5}$$

10. Let the velocity of swimmer which is equal to velocity of river be v.

For both velocity vectors of same magnitude, the resultant would pass exactly midway. V_M should be the bisector of v_1 and v_2.

$\therefore\ \theta = 30^\circ$

11. Velocity of swimmer in still water,

$v_m = 12$ km/h

Velocity of water flowing, $v_r = 6$ km/h

From diagram,

$v_m \sin\theta = v_r$

$v_m \sin\theta = 6$

$12 \sin\theta = 6$ or $\sin\theta = \frac{1}{2}$

$\theta = \frac{\pi}{6} = 30^\circ$

$\therefore$ Direction with respect to flow of water

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$= 90^\circ + 30^\circ = 120^\circ$

12. Velocity along river, $v_R = 10\sin\ 30^\circ$

$= 10 \times \frac{1}{2} = 5\ \text{m/s}$

13. $x = 10 + 8t - 3t^2, y = 5 - 8t^3$

$v_x = 8 - 6t, v_y = -24t^2; \vec{v}_{x(t=1\ s)} = 2\hat{\imath}, \vec{v}_{y(t=1\ s)} = -24\hat{\jmath}$

So, the required speed $v_{net} = \sqrt{v_x^2 + v_y^2} = \sqrt{4 + 576} = \sqrt{580}$

$\therefore\ v = 580\ \text{m s}^{-1}$