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2 (")

$$\log_5 \underbrace{\log_3}\ > 0$$

$$(x-2)(2x+1)(x+4)>0$$

$$\log_3 \log_2 (\quad) > 1$$

$$\log_2 (\quad) > 3$$

$$2x^3+5x^2-14x-8>0$$

$$(x-2)(2x^2+9x+4)>0$$

$$+x+8x$$

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FUNCTIONS

(iii) $\sqrt{\underbrace{x^2-5x-24}_{\geq 0}} \geq x+2$

$\boxed{D_f = (-\infty, -3]}$

$\boxed{(-\infty,-3] \cup [8,\infty)}$

if $x+2 < 0$

$x < -2$

$\boxed{x \in (-\infty,-3]}$

OR

if $x+2 \geq 0$

$\cancel{x^2} - 5x - 24 > \cancel{x^2} + 4x + 4$

$x < -\frac{28}{9}$

$\boxed{x \in \phi}$

$4 > 3$

$4 > -5$

$4 > -4$

$4 >$

FUNCTIONS

(iv) $\dfrac{1-5^x}{7^{-x}-7} \geq 0$

$$\boxed{D_f = (-\infty,-1) \cup [0,\infty)}$$

$\Downarrow$

Ans.

$\boxed{x \leq 0} \Leftarrow 5^x \leq 1 \Leftarrow 1-5^x \geq 0$ & $7^{-x}-7>0$

$-x>1 \Rightarrow \boxed{x<-1}$

$\boxed{x \in (-\infty,-1)}$

OR

$1-5^x \leq 0$ & $7^{-x}-7<0$

$x \geq 0$ $\quad$ $-x<1$

$x \in [0,\infty)$ $\quad$ $x>-1$

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(vi) $100x > 0 \quad , \neq 1$

$x \in \left(0, \frac{1}{100}\right) \cup \left(\frac{1}{100}, \infty\right)$

& $\frac{2\log_{10}x + 1}{-x} > 0$

$2\log_{10}x + 1 < 0$

$\log_{10}x < -\frac{1}{2} \Rightarrow \boxed{x < \frac{1}{\sqrt{10}}}$

(ix) $|x|(|x| - 1) \geq 0$

$|x| \in \{0\} \cup [1, \infty)$

$D_f \in \left(\left(0, \frac{1}{100}\right) \cup \left(\frac{1}{100}, \frac{1}{\sqrt{10}}\right)\right)$

$D_f = (-3, -1] \cup \{0\} \cup [1, 3)$

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(x)

$$\boxed{D_f = \{4\} \cup [5, \infty)}$$

$$(x-5)(x+2) \geq 0$$

$$(-\infty, -2] \cup [5, \infty)$$

$$x - 3 \geq 0$$

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FUNCTIONS

(11) $\log_x(\cos 2\pi x) \geq 0$

$$\boxed{D_f = \left(0, \frac{1}{4}\right) \cup \left(\frac{3}{4}, 1\right) \cup \{n\} \quad n \in N - \{1\}}$$

If $\underline{0 < x < 1}$ OR $x > 1$

$0 < \cos(2\pi x) \leq 1$

$2\pi x \in (0, 2\pi)$

$2\pi x \in \left(0, \frac{\pi}{2}\right) \cup \left(\frac{3\pi}{2}, 2\pi\right)$

$x \in \left(0, \frac{1}{4}\right) \cup \left(\frac{3}{4}, 1\right)$

$\cos 2\pi x \geq 1$

$=$

$2\pi x = 2n\pi$

$x = n \quad , n \in N - \{1\}$

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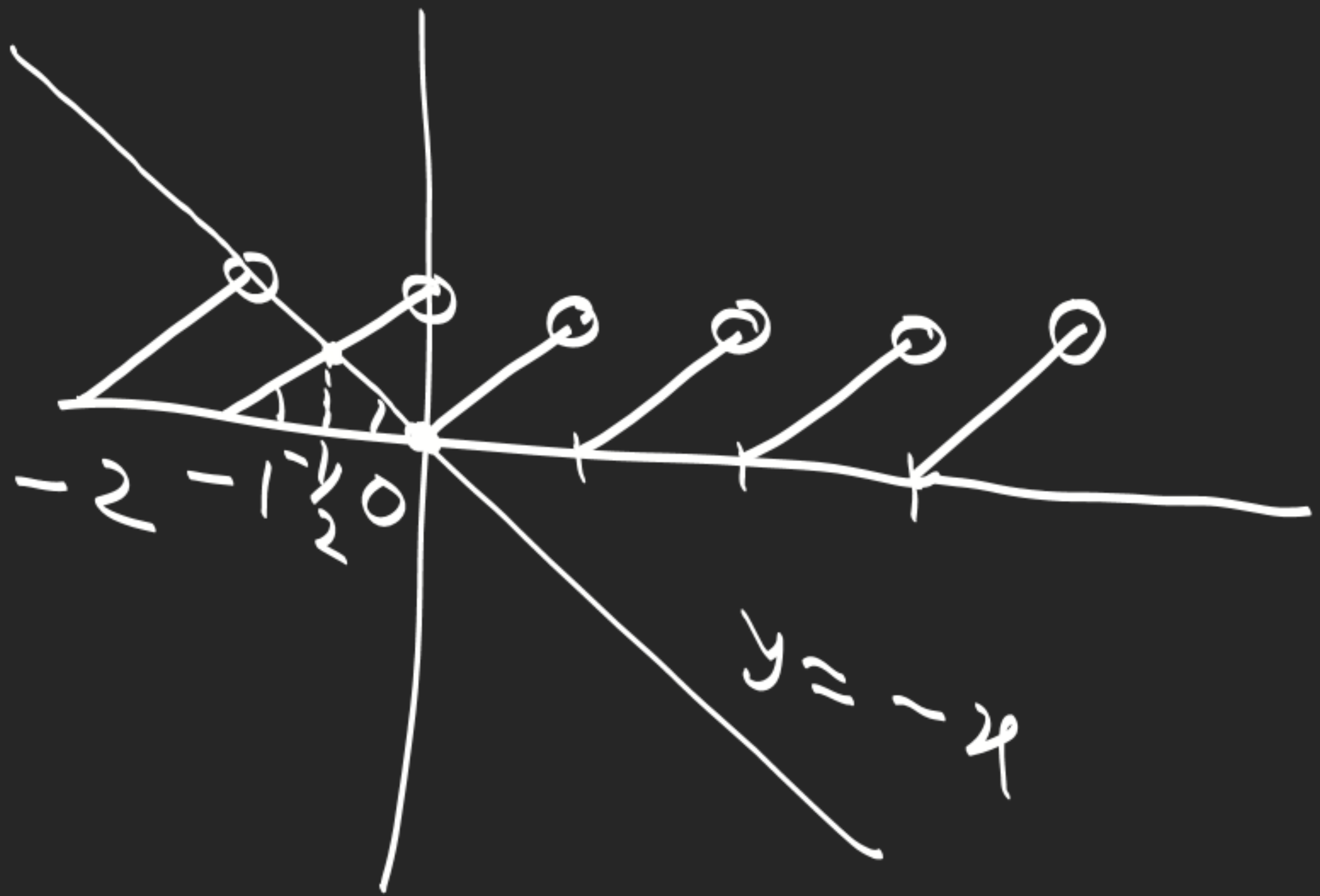
(XV)

$$2x - [x] \neq 0$$

$$x + \{x\} = 0$$

$$\{x\} = -x$$



$$\boxed{D_f = R - \{-\tfrac{1}{2}, 0\}}$$

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Classification of function

- Onto (Surjective)
  - One to one (Injective)
  - Many to one (not injective)
- Into (not surjective)
  - One to one
  - Many to one

Bijective.

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Onto (Surjective) Function

$f : A \to B$ is surjective if every element in set B is mapped/connected to atleast one element in A.

$$\boxed{R_f = CoD_f}$$

Into

Atleast one element in B is not connected to any element in A.

FUNCTIONS

1-1 (Injective) function

$f: A \to B$ is 1-1 if every element in B is connected to exactly one element in A or not connected at all.

M-1 (not injective)

At least one element in B is connected to more than one element in A.

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Bijective / Invertible / Non Singular Function

$f : A \to B$ is bijective if it is surjective & injective both.

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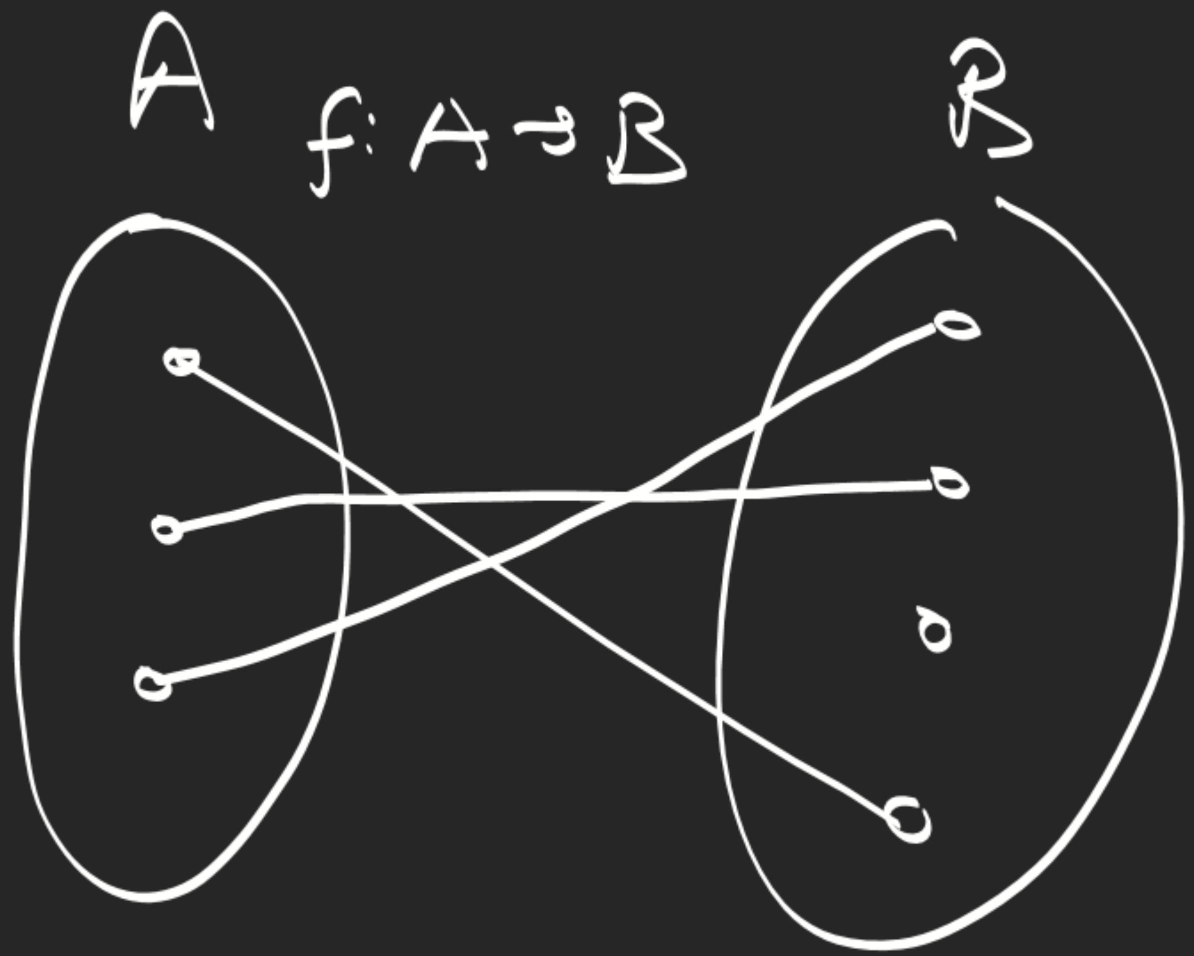
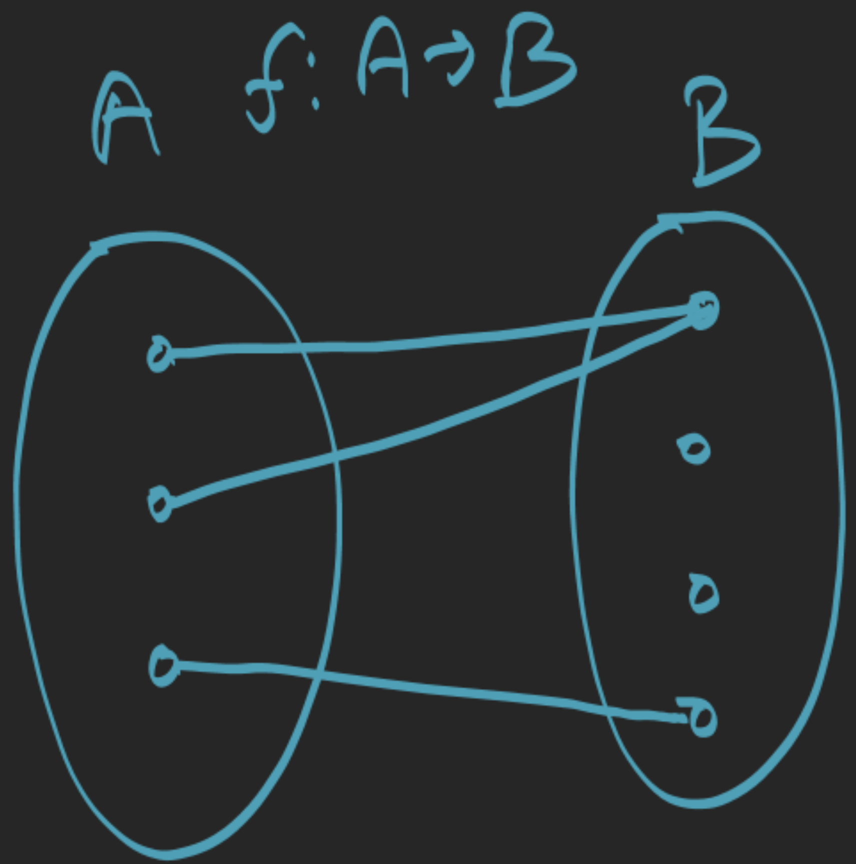


Into & 1-1



Into & M-1

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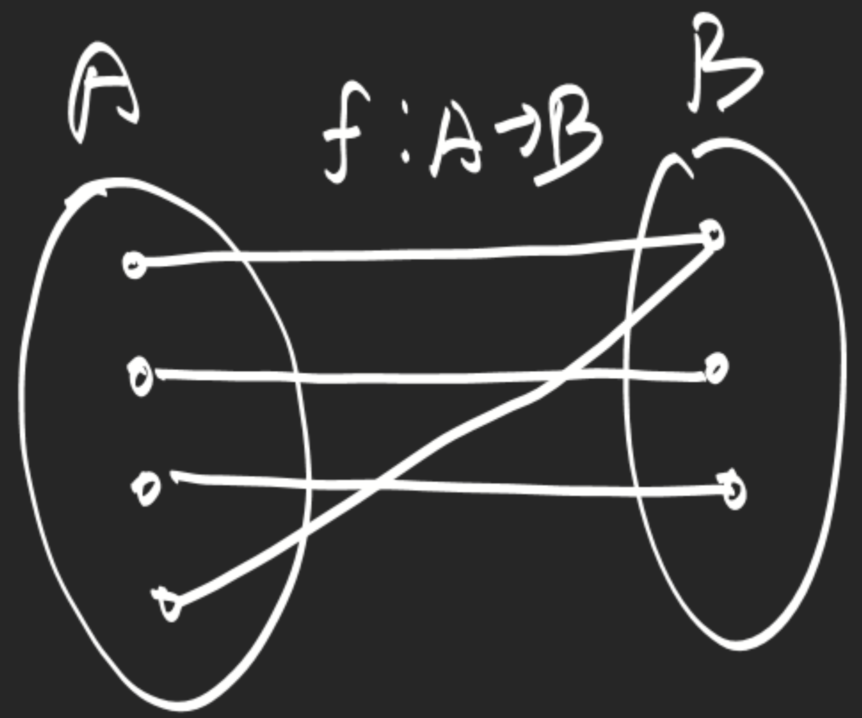
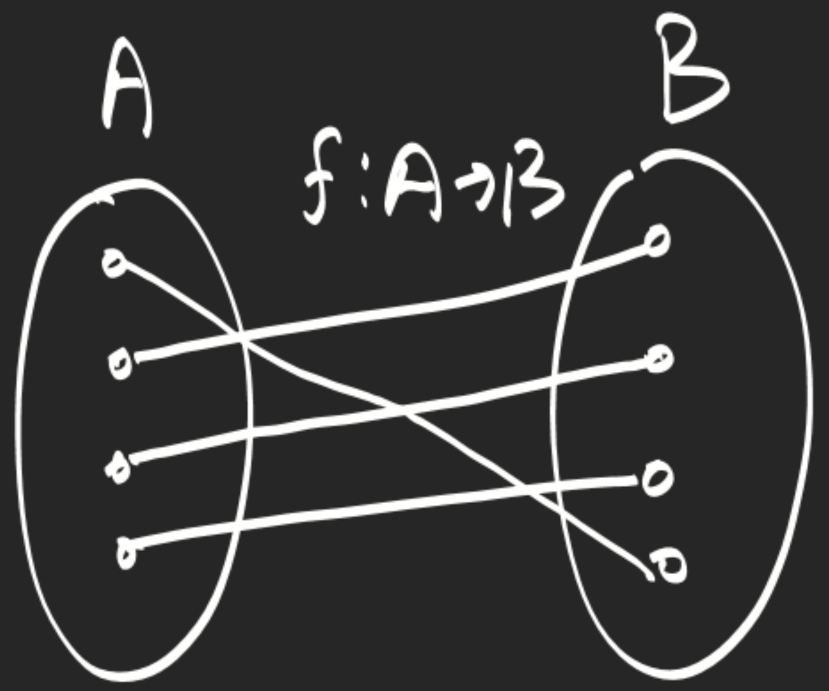
FUNCTIONS



Onto, M-1



Onto, 1-1

Bijective

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1-1

Let $f(x_1) = f(x_2)$

$\Rightarrow x_1 = x_2$

then f is 1-1

$f(x) = x^3$

$x_1^3 = x_2^3$

$(x_1 - x_2)(x_1^2 + x_1 x_2 + x_2^2) = 0$

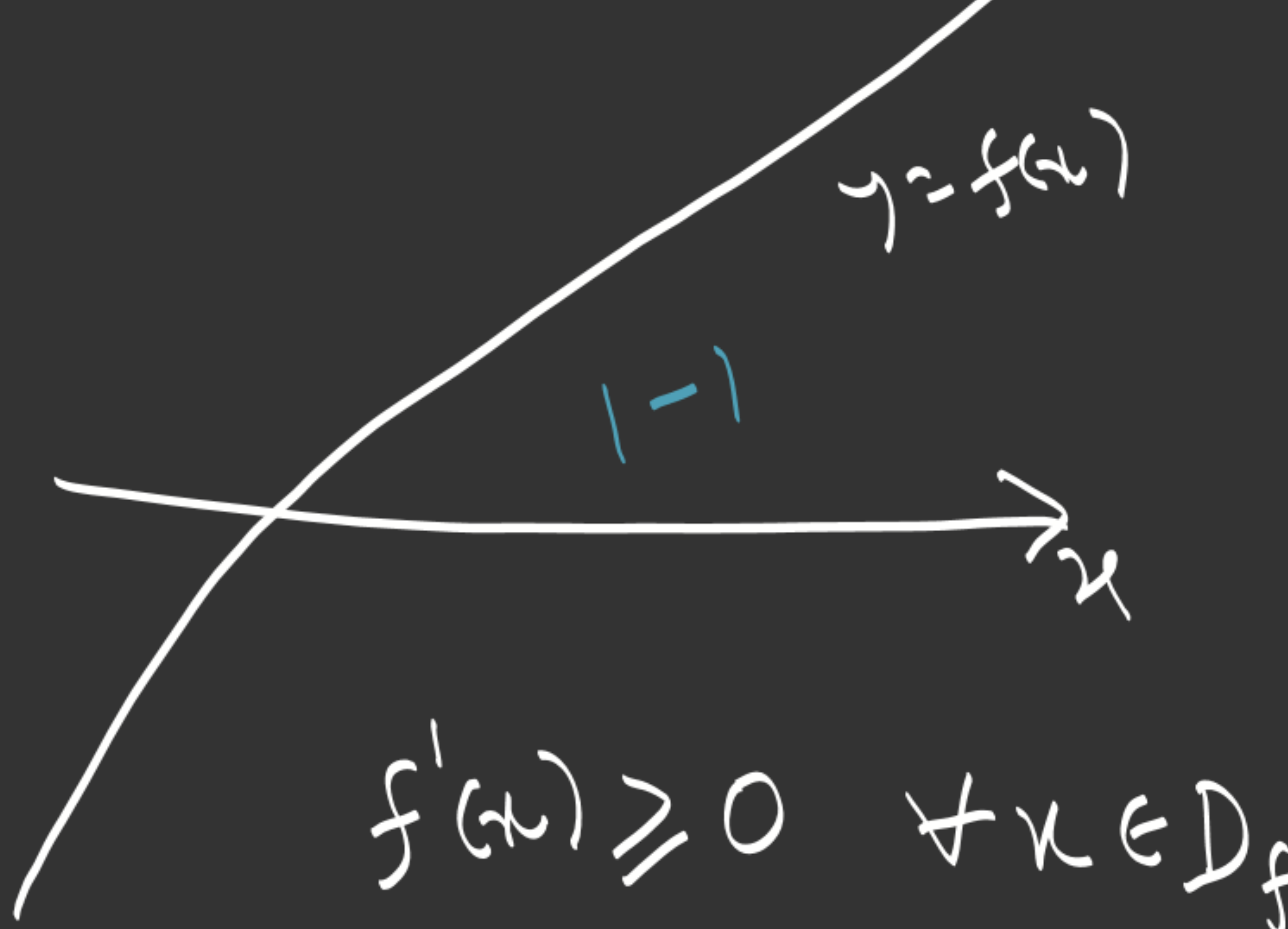
$\Rightarrow \boxed{x_1 = x_2}$

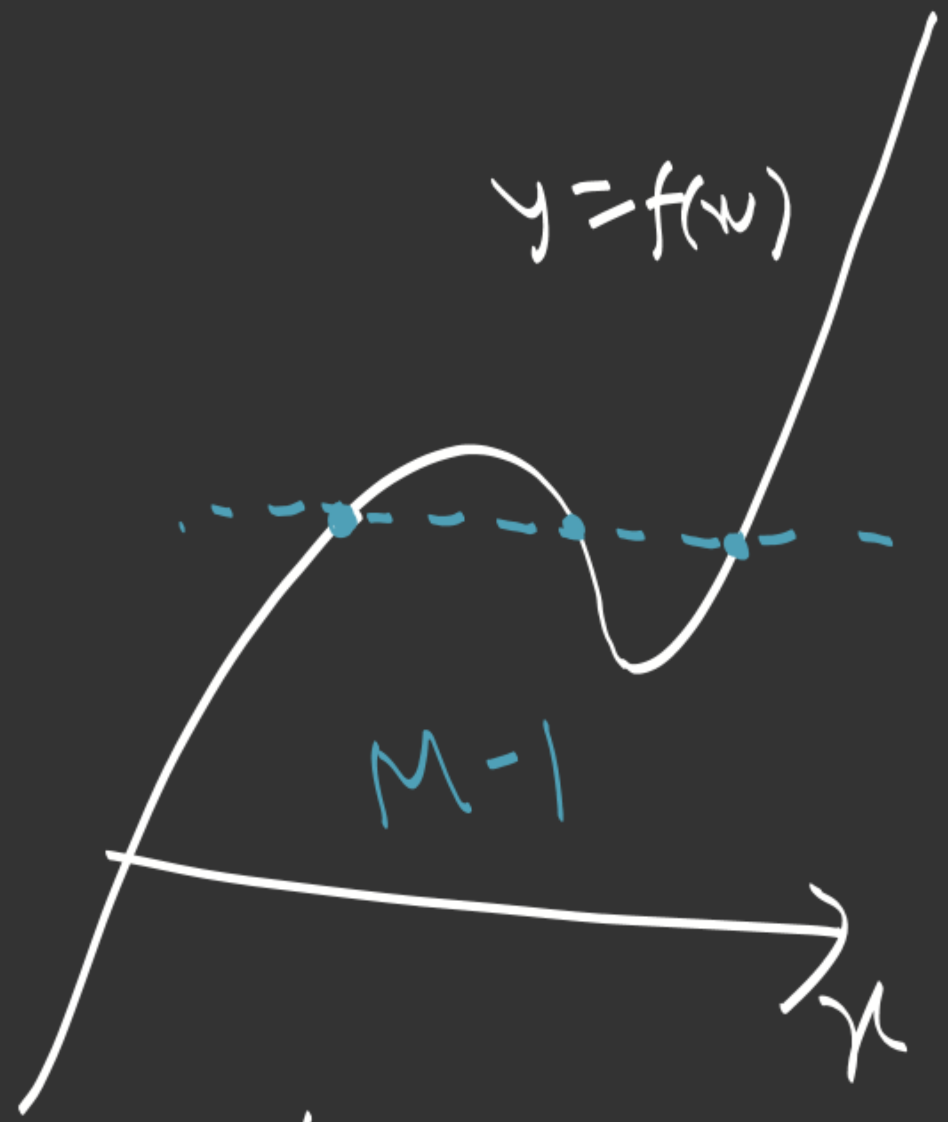
$x_2 \neq 0$

$\left(\frac{x_1}{x_2}\right)^2 + \frac{x_1}{x_2} + 1 = 0$

1-1

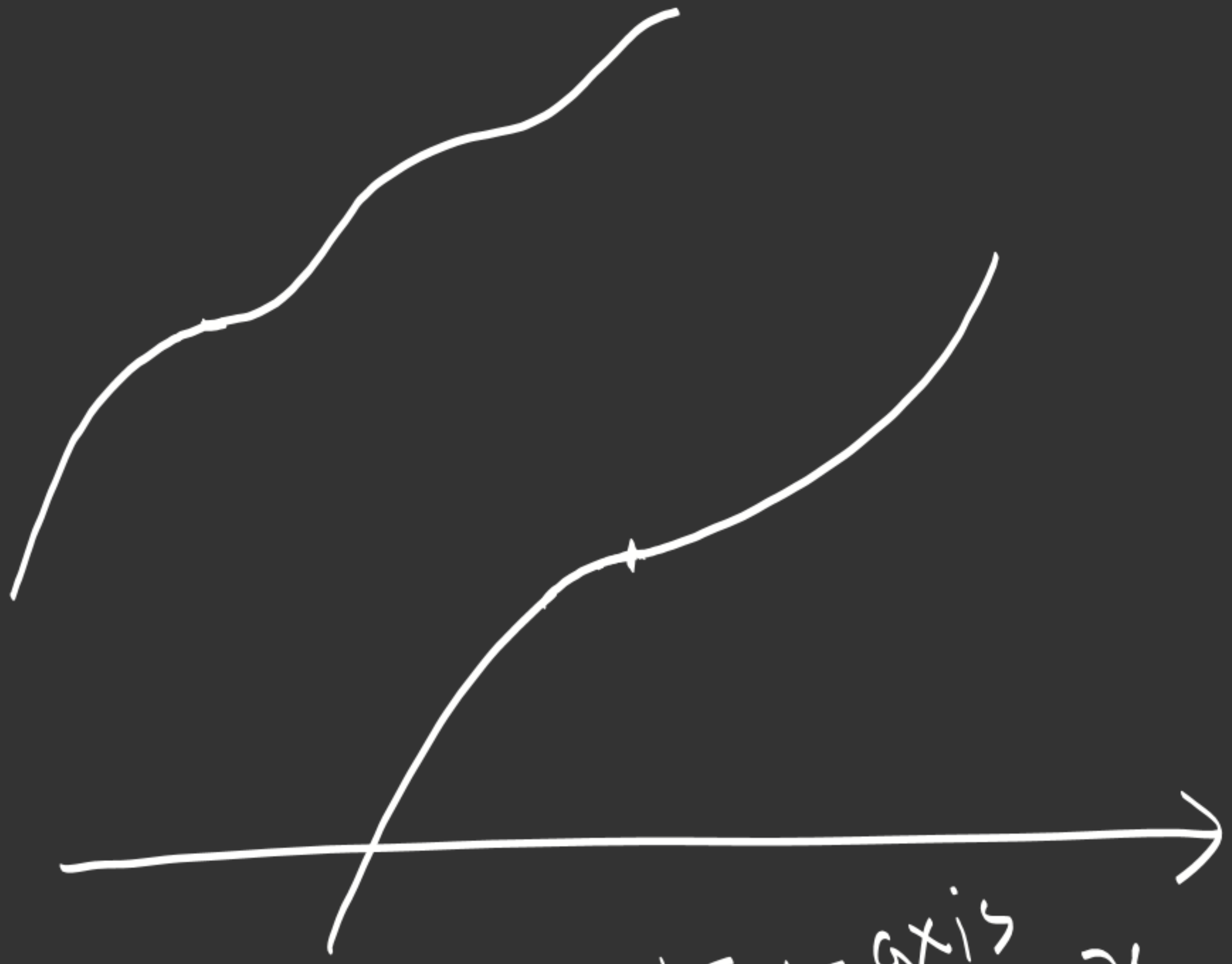
If $f(x)$ is continuous & $f'(x)$ exists



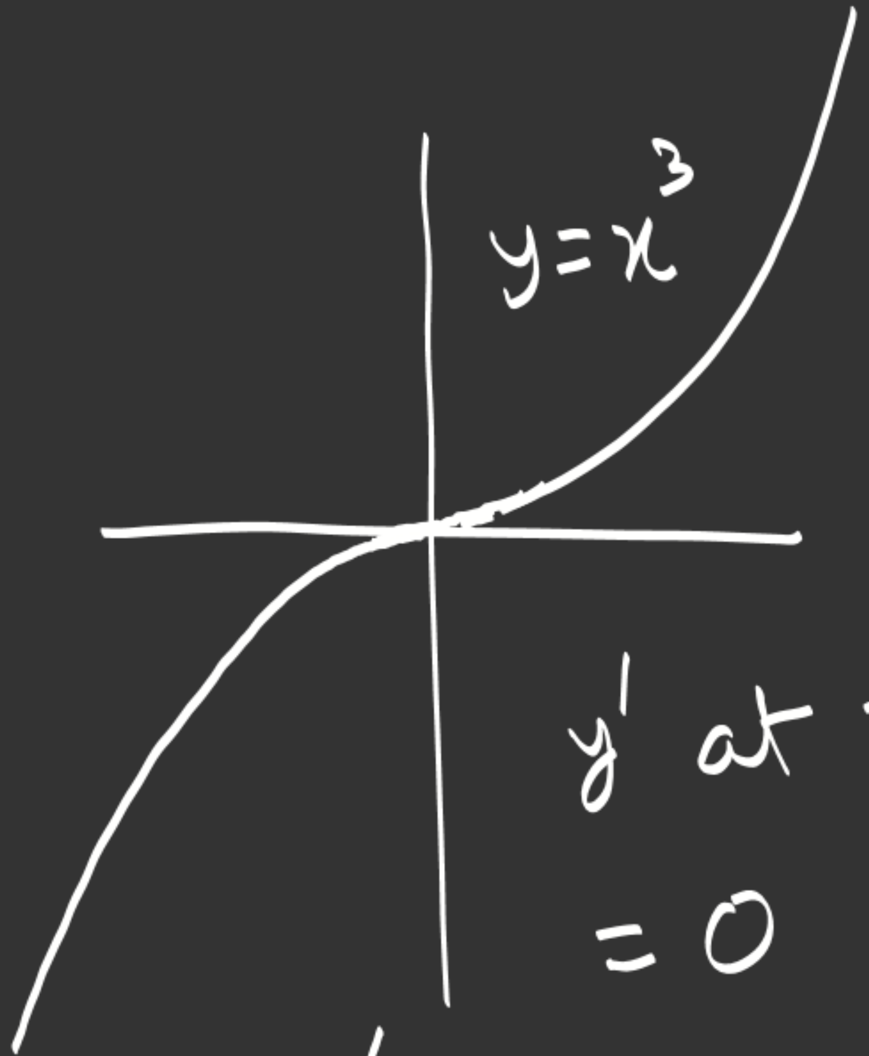


$f'(x) \geq 0 \ \forall x \in D_f$ & $f'(x)=0$ occurs at instants only.

f is 1-1 $\Leftarrow$ or $f'(x) \leq 0 \ \forall x \in D_f$ & $f'(x)=0$ holds at instants



Any line $\parallel$ to x-axis will intersect at one point only or not at all



y' at $x = 0$
$= 0$

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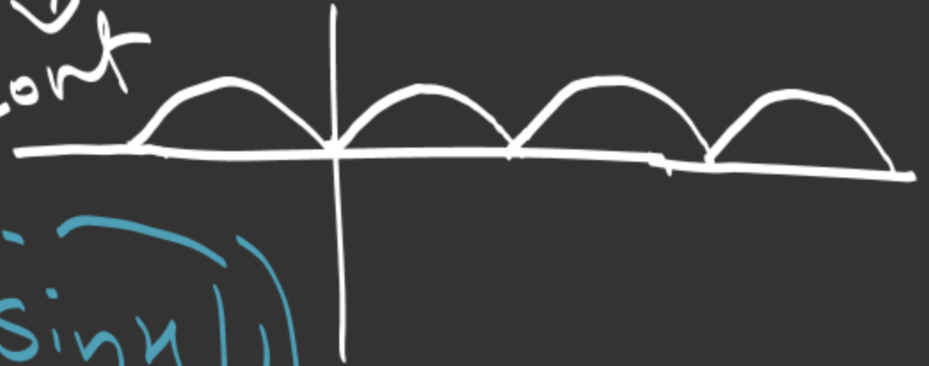
$\rightarrow f$ is onto & $1-1 \Rightarrow$ bijective

1. $f: R \to R$, $f(x) = 2x - |\sin x|$

$2x \to$ cont

$|\sin x| \to$ cont



$f(x)$ is cont.

$x \to -\infty$, $y \to -\infty$

$y = x\left(2 - \frac{|\sin x|}{x}\right)$

$\frac{|\sin x|}{x} \to 0$

$f(x) = \begin{cases} 2x - \sin x \\ 2x + \sin x \end{cases}$

$f'(x) = 2 - \cos x$ or $2 + \cos x$

$\boxed{f'(x) > 0}$

2. $f: R \to R$, $f(x) = x^3 - 2x^2 + 5x + 3$

$\boxed{f'(x) > 0}$

$x \to \infty$, $y \to \infty$

onto

$\boxed{R_f = R}$

$$h(x) = \frac{f(x)}{g(x)}$$

cont.

$f(x) \to$ cont.

$g(x) \to$ cont.

$g(x) \neq 0$

cont.

$$h(x) = f(x) - g(x)$$

$h(x) \to$ cont.

$f(x) \to$ cont.

$g(x) \to$ cont.

$$h(x) = f(x)\,g(x)$$

$h(x) \to$ cont.

$f(x) \to$ cont.

$g(x) \to$ cont.

Continuous

If we can draw the graph of $f(x)$ without raising pen, then it is said to be continuous

$\downarrow$ cont.

$$f(x) + g(x) = h(x)$$

$h(x) \to$ cont.

$f(x) \to$ cont.

$g(x) \to$ cont.

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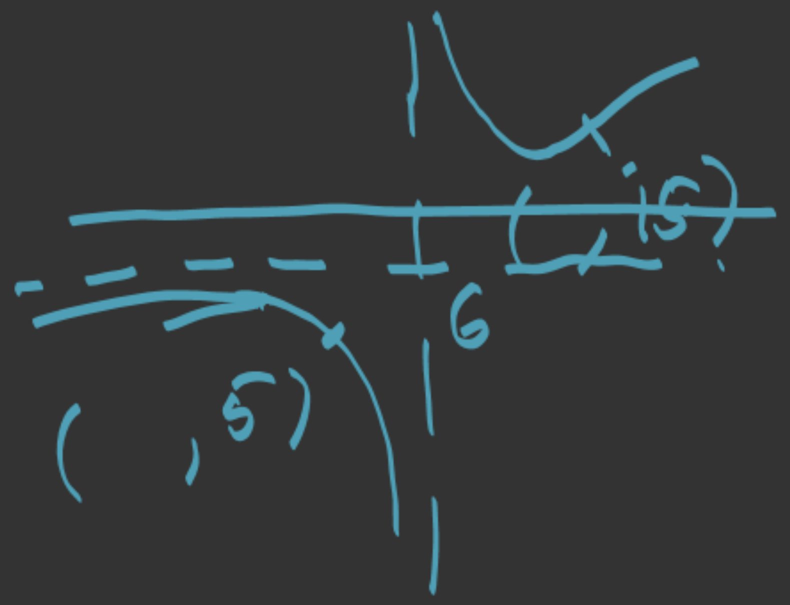
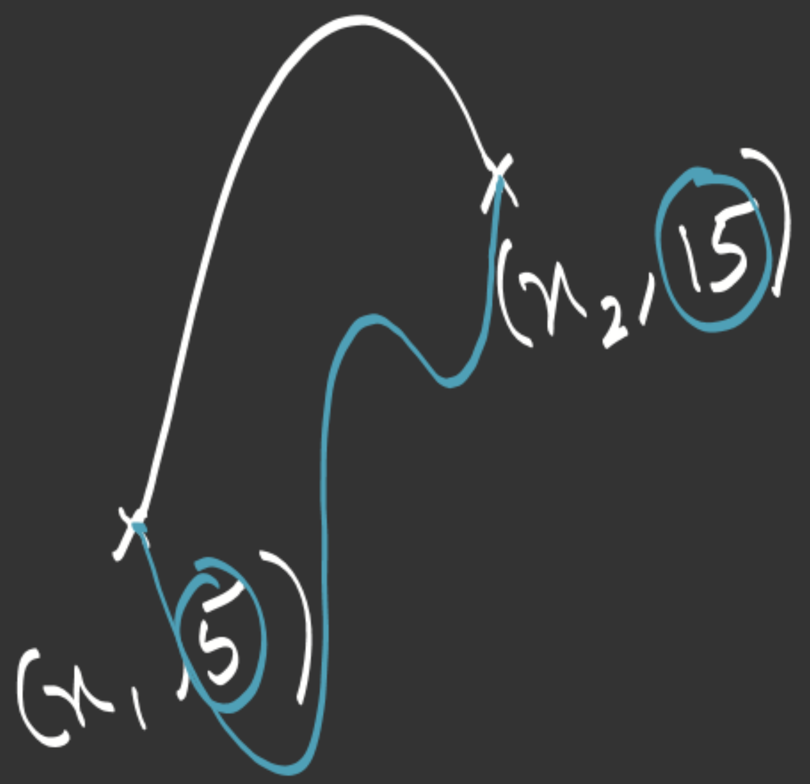


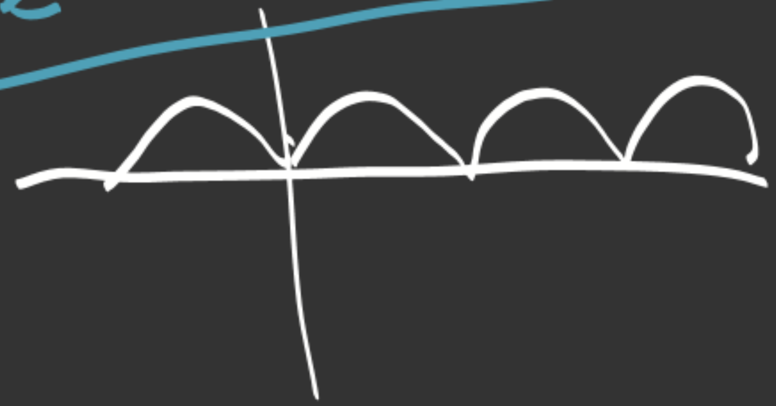
Cont.



$(x_1, 5)$, $(x_2, 15)$, $x_1 < x_2$

$\exists\, c \in (x_1, x_2)$, $f(c) = 7$

Intermediate Value Theorem



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$\boxed{f \text{ is } 1\text{-}1 \text{ \& onto}}$

Q: $f: R \to R$, $f(x) = x^3 - 2x^2 + 5x + 3$

f is cont.

$$f(x) = x^3\left(1 - \frac{2}{x} + \frac{5}{x^2} + \frac{3}{x^3}\right)$$

$x \to -\infty, y \to -\infty$

$x \to \infty, y \to \infty$

$R_f = (-\infty, \infty)$

$f'(x) = 3x^2 - 4x + 5 > 0$

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3. $f : R \to R$, $f(x) = x^3 + x^2 + 3x + \sin x$

4. $f : R \to R$, $f(x) = \dfrac{2x^2 - x + 5}{7x^2 + 2x + 10}$

6. $f : B \to A$

5. A → 4 element
B → 6 elements

(6) Bijective
(4) Onto
(8) Into

Find no. of $f : A \to B$

1. functions $f : A \to B$
2. — " — which is 1-1
3. M-1

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PT - 1 (remaining)

PT - 2 → Q 1