

(MATHEMATICS)

INDEFINITE INTEGRATION

DPP-04 (INTEGRATION BY SUBSTITUTION)

1. Evaluate the following:

(i) $\int \frac{\sec^2 x}{3 + \tan x} dx$ (ii) $\int \frac{e^x - e^{-x}}{e^x + e^{-x}} dx$ (iii) $\int \frac{1 - \tan x}{1 + \tan x} dx$ (iv) $\int \frac{1}{1 + e^{-x}} dx$

2. Evaluate the following:

(i) $\int \left(x + \frac{1}{x}\right)^{\frac{1}{2}} \left(\frac{x^2 - 1}{x^2}\right) dx$ (ii) $\int \frac{\sqrt{2 + \log x}}{x} dx$ (iii) $\int \frac{(\sin^{-1} x)^3}{\sqrt{1 - x^2}} dx$

3. Evaluate $\int \frac{1}{1 - \tan x} dx$ 4. Evaluate $\int \frac{\log\left(\tan \frac{x}{2}\right)}{\sin x} dx$

5. Evaluate $\int \sec^p x \tan x dx$ 6. Evaluate $\int \frac{\log_e (x + \sqrt{x^2 + 1})}{\sqrt{x^2 + 1}} dx$.

7. Evaluate $\int \frac{2x - \sqrt{\sin^{-1} x}}{\sqrt{1 - x^2}} dx$. 8. Evaluate $\int (x^6 + x^4 + x^2) \sqrt{2x^4 + 3x^2 + 6} dx$.

INTEGRATION OF FUNCTION $f(g(x)) \cdot g'(x)$

9. Evaluate $\int \cos^3 x \sqrt{\sin x} dx$, 10. Evaluate $\int 2^{2^{2^x}} 2^{2^x} 2^x dx$

11. Evaluate $\int \frac{e^{\sqrt{x}} \cos(e^{\sqrt{x}})}{\sqrt{x}} dx$ 12. Find $\int \frac{e^x(1+x)}{\cos^2(xe^x)} dx$ 13. $\int 5^{x + \tan^{-1} x} \cdot \left(\frac{x^2 + 2}{x^2 + 1}\right) dx$

14. $\int \frac{a^{\sqrt{x}}}{\sqrt{x}} dx$ is equal to

(A) $\frac{a^{\sqrt{x}}}{\sqrt{x}} + c$ (B) $\frac{2a^{\sqrt{x}}}{\ln a} + c$ (C) $2a^{\sqrt{x}} \cdot \ln a + c$ (D) $\frac{a^{\sqrt{x}}}{\ln a} + c$

15. $\int 5^{5^{5^x}} \cdot 5^{5^x} \cdot 5^x dx$ is equal to

(A) $\frac{5^{5^x}}{(\log 5)^3} + c$ (B) $5^{5^{5^x}} (\ln 5)^3 + c$ (C) $\frac{5^{5^{5^x}}}{(\log 5)^3} + c$ (D) $\frac{5^{5^{5^x}}}{\ln 5} + c$

16. $\int \frac{\sqrt{\tan x}}{\sin x \cos x} dx$ is equal to

(A) $2\sqrt{\tan x} + c$ (B) $2\sqrt{\cot x} + c$ (C) $\frac{\sqrt{\tan x}}{2} + c$ (D) $\frac{\sqrt{\sec x}}{2} + c$

17. If $\int \frac{2^x}{\sqrt{1 - 4^x}} dx = K \sin^{-1}(2^x) + C$, then K is equal to

(A) $\ln 2$ (B) $\frac{1}{2} \ln 2$ (C) $\frac{1}{2}$ (D) $\frac{1}{\ln 2}$

18. $\int \frac{e^x - 1}{e^x + 1} dx$ is equal to

(A) $\ln(e^x + \sqrt{e^{2x} - 1}) - \sec^{-1}(e^x) + C$ (B) $\ln(e^x + \sqrt{e^{2x} - 1}) + \sec^{-1}(e^x) + C$
(C) $\ln(e^x - \sqrt{e^{2x} - 1}) - \sec^{-1}(e^x) + C$ (D) A and B both

19. $\int \sqrt{\sec x - 1} dx$ is equal to

(A) $2 \ln \left(\cos \frac{x}{2} + \sqrt{\cos^2 \frac{x}{2} - \frac{1}{2}} \right) + C$ (B) $2 \ln \left(\cos \frac{x}{2} + \sqrt{\cos^2 \frac{x}{2} - \frac{1}{2}} \right) + C$
(C) $-2 \ln \left(\cos \frac{x}{2} + \sqrt{\cos^2 \frac{x}{2} - \frac{1}{2}} \right) + C$ (D) $\ln \left(\cos \frac{x}{2} + \sqrt{\cos^2 \frac{x}{2} - \frac{1}{2}} \right) + C$

(MATHEMATICS)

INDEFINITE INTEGRATION

20. $\int \frac{1}{\cos^6 x + \sin^6 x} dx$ is equal to
 (A) $\tan^{-1} (\tan x + \cot x) + c$ (B) $-\tan^{-1} (\tan x + \cot x) + c$
 (C) $\tan^{-1} (\tan x - \cot x) + c$ (D) $-\tan^{-1} (\tan x - \cot x) + c$
21. $\int \frac{dx}{\cos^3 x \sqrt{\sin 2x}}$ is equal to
 (A) $\frac{\sqrt{2}}{5} (\tan x)^{\frac{5}{2}} + 2\sqrt{\tan x} + c$ (B) $\frac{\sqrt{2}}{5} (\tan^2 x + 5)\sqrt{\tan x} + c$
 (C) $\frac{\sqrt{2}}{5} (\tan^2 x + 5)\sqrt{2\tan x} + c$ (D) $\sqrt{2}(\tan^2 x + 5)\sqrt{2\tan x} + c$
22. If $\int \frac{dx}{\sqrt{\sin^3 x \cos^5 x}} = a\sqrt{\cot x} + b\sqrt{\tan^3 x} + c$ where c is an arbitrary constant of integration then the values of 'a' and 'b' are respectively
 (A) -2 & $\frac{2}{3}$ (B) 2 & $-\frac{2}{3}$ (C) 2 & $\frac{2}{3}$ (D) $2, 2$
23. $\int \frac{1}{\sqrt{\sin^3 x \cos x}} dx$ is equal to
 (A) $\frac{-2}{\sqrt{\tan x}} + c$ (B) $2\sqrt{\tan x} + c$ (C) $\frac{2}{\sqrt{\tan x}} + c$ (D) $-2\sqrt{\tan x} - c$
24. $\int \frac{\ln |x|}{x\sqrt{1+\ln |x|}} dx$ is equal to
 (A) $\frac{2}{3}\sqrt{1+\ln |x|}(\ln |x| - 2) + c$ (B) $\frac{2}{3}\sqrt{1+\ln |x|}(\ln |x| + 2) + c$
 (C) $\frac{1}{3}\sqrt{1+\ln |x|}(\ln |x| - 2) + c$ (D) $\frac{1}{3}\sqrt{1+\ln |x|}(3\ln |x| + 2) + c$
25. $\int \frac{\cos 2x}{(\sin x + \cos x)^2} dx$ is equal to
 (A) $\frac{-1}{\sin x + \cos x} + c$ (B) $\ln (\sin x + \cos x) + c$
 (C) $\ln (\sin x - \cos x) + c$ (D) $\ln (\sin x + \cos x)^2 + c$
26. $\int \sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}} dx$ is equal to
 (A) $\sqrt{x}\sqrt{1-x} - 2\sqrt{1-x} + \cos^{-1}(\sqrt{x}) + c$ (B) $\sqrt{x}\sqrt{1-x} + 2\sqrt{1-x} + \cos^{-1}(\sqrt{x}) + c$
 (C) $\sqrt{x}\sqrt{1-x} - 2\sqrt{1-x} + \cos^{-1}(\sqrt{x}) + c$ (D) $\sqrt{x}\sqrt{1-x} + 2\sqrt{1-x} - \cos^{-1}(\sqrt{x}) + c$
27. If $\int \frac{1}{x\sqrt{1-x^3}} dx = a \ln \left| \frac{\sqrt{1-x^3}-1}{\sqrt{1-x^3}+1} \right| + b$, then a is equal to
 (A) $1/3$ (B) $2/3$ (C) $-1/3$ (D) $-2/3$
28. $\int \frac{xdx}{\sqrt{1+x^2+\sqrt{(1+x^2)^3}}}$ is equal to
 (A) $\frac{1}{2} \ln (1 + \sqrt{1+x^2}) + c$ (B) $2\sqrt{1+\sqrt{1+x^2}} + c$
 (C) $2(1 + \sqrt{1+x^2}) + c$ (D) None of these

(MATHEMATICS)

INDEFINITE INTEGRATION

29. $\int \tan^3 2x \sec 2x \, dx$ is equal to

(A) $\frac{1}{3} \sec^3 2x - \frac{1}{2} \sec 2x + c$

(B) $-\frac{1}{6} \sec^3 2x - \frac{1}{2} \sec 2x + c$

(C) $\frac{1}{6} \sec^3 2x - \frac{1}{2} \sec 2x + c$

(D) $\frac{1}{3} \sec^3 2x + \frac{1}{2} \sec 2x + c$

30. If $\int x^{13/2} \cdot (1 + x^{5/2})^{1/2} dx = A(1 + x^{5/2})^{7/2} + B(1 + x^{5/2})^{5/2} + C(1 + x^{5/2})^{3/2}$, then

(A) $A = -\frac{4}{35}, B = -\frac{8}{25}, C = \frac{4}{15}$

(B) $A = \frac{4}{35}, B = -\frac{8}{25}, C = -\frac{4}{15}$

(C) $A = \frac{4}{35}, B = -\frac{8}{25}, C = \frac{4}{15}$

(D) $A = -\frac{4}{35}, B = -\frac{8}{25}, C = -\frac{4}{15}$

31. $\int \sqrt{\frac{1 - \cos x}{\cos \alpha - \cos x}} dx$ where $0 < \alpha < x < \pi$, is equal to

(A) $2 \ln \left(\cos \frac{\alpha}{2} - \cos \frac{x}{2} \right) + c$

(B) $\sqrt{2} \ln \left(\cos \frac{\alpha}{2} - \cos \frac{x}{2} \right) + c$

(C) $2\sqrt{2} \ln \left(\cos \frac{\alpha}{2} - \cos \frac{x}{2} \right) + c$

(D) $-2 \sin^{-1} \left(\frac{\cos \frac{x}{2}}{\cos \frac{\alpha}{2}} \right) + c$

32. $\int \frac{1}{[(x-1)^3(x+2)^5]^{1/4}} dx$ is equal to

(A) $\frac{4}{3} \left(\frac{x-1}{x+2} \right)^{1/4} + c$

(B) $\frac{4}{3} \left(\frac{x+1}{x-2} \right)^{1/4} + c$

(C) $\frac{1}{3} \left(\frac{x-1}{x+2} \right)^{1/4} + c$

(D) $\frac{1}{3} \left(\frac{x+1}{x-2} \right)^{1/4} + c$

(MATHEMATICS)

INDEFINITE INTEGRATION

ANSWER KEY

1. (i) $\log |3 + \tan x| + C$ (ii) $\log_e (e^x + e^{-x}) + C$ (iii) $\log_e |\cos x + \sin x| + C$
(iv) $\log_e (1 + e^x) + C$
2. (i) $\frac{2}{5} \left(x + \frac{1}{x} \right)^{\frac{5}{2}} + C$ (ii) $\frac{2(2 + \log x)^{\frac{3}{2}}}{3} + c$ (iii) $\frac{(\sin^{-1} x)^4}{4} + c$
3. $-\frac{1}{2} \log_e |\cos x - \sin x| + \frac{1}{2} x + c$ 4. $\frac{[\log(\tan \frac{x}{2})]^2}{2} + C$ 5. $\frac{\sec^p x}{p} + c$
6. $\frac{(\log_e (x + \sqrt{x^2 + 1}))^2}{2} + c$ 7. $2(1 - x^2)^{\frac{1}{2}} - \frac{2}{3} (\sin^{-1} x)^{\frac{3}{2}} + c$
8. $\frac{1}{18} (2x^6 + 3x^4 + 6x^2)^{\frac{3}{2}} + c$ 9. $\frac{2}{3} \sin^{\frac{3}{2}} x - \frac{2}{7} \sin^{\frac{7}{2}} x + c$ 10. $\frac{1}{(\log 2)^3} 2^{2^{2^x}} + C$
11. $2 \sin e^{\sqrt{x}} + c$ 12. $\tan z + c = \tan(xe^r) + c$ 13. $\frac{5^x + \tan^{-1} x}{\log_e 5} + c$
14. (B) 15. (C) 16. (A) 17. (D) 18. (A) 19. (C) 20. (C)
21. (B) 22. (A) 23. (A) 24. (A) 25. (B) 26. (A) 27. (A)
28. (B) 29. (C) 30. (C) 31. (D) 32. (A)