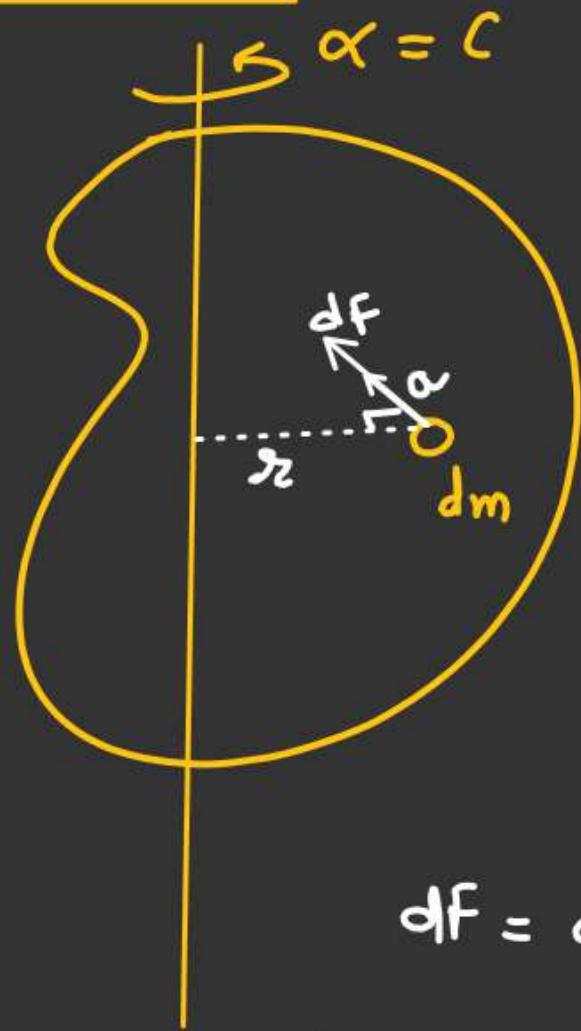




$$\vec{\tau} = I \vec{\alpha}$$



$$a = r\alpha$$

$$dF = dm a$$

$$dT = dF \cdot r$$

$$dT = dm a r$$

$$\int dT = \int dm r^2 \alpha$$

$$T_{\text{net}} = \left(\int dm r^2 \right) \alpha$$

\Downarrow
 (I_{body}
 about axis
 of rotation)

$$T_{\text{net}} = I \alpha$$

↳ Newton's 2nd Law

K.E of a hinged body

$$\tau = \rho \omega$$

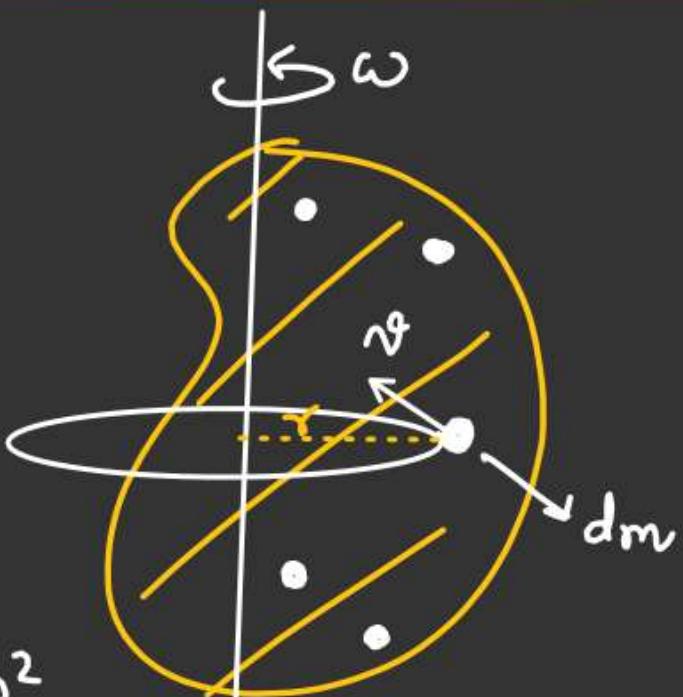
$$d(K.E) = \frac{1}{2} (dm) v^2$$

$$\int d(K.E) = \int \frac{1}{2} (dm \rho^2) \omega^2$$

$$K.E_{body} = \frac{1}{2} \omega^2 \left\{ \int dm \rho^2 \right\}$$

$K.E_{body} = \frac{1}{2} I_{axis \text{ of Rotation}} \cdot \omega^2$

(I_{body}) about axis of Rotation

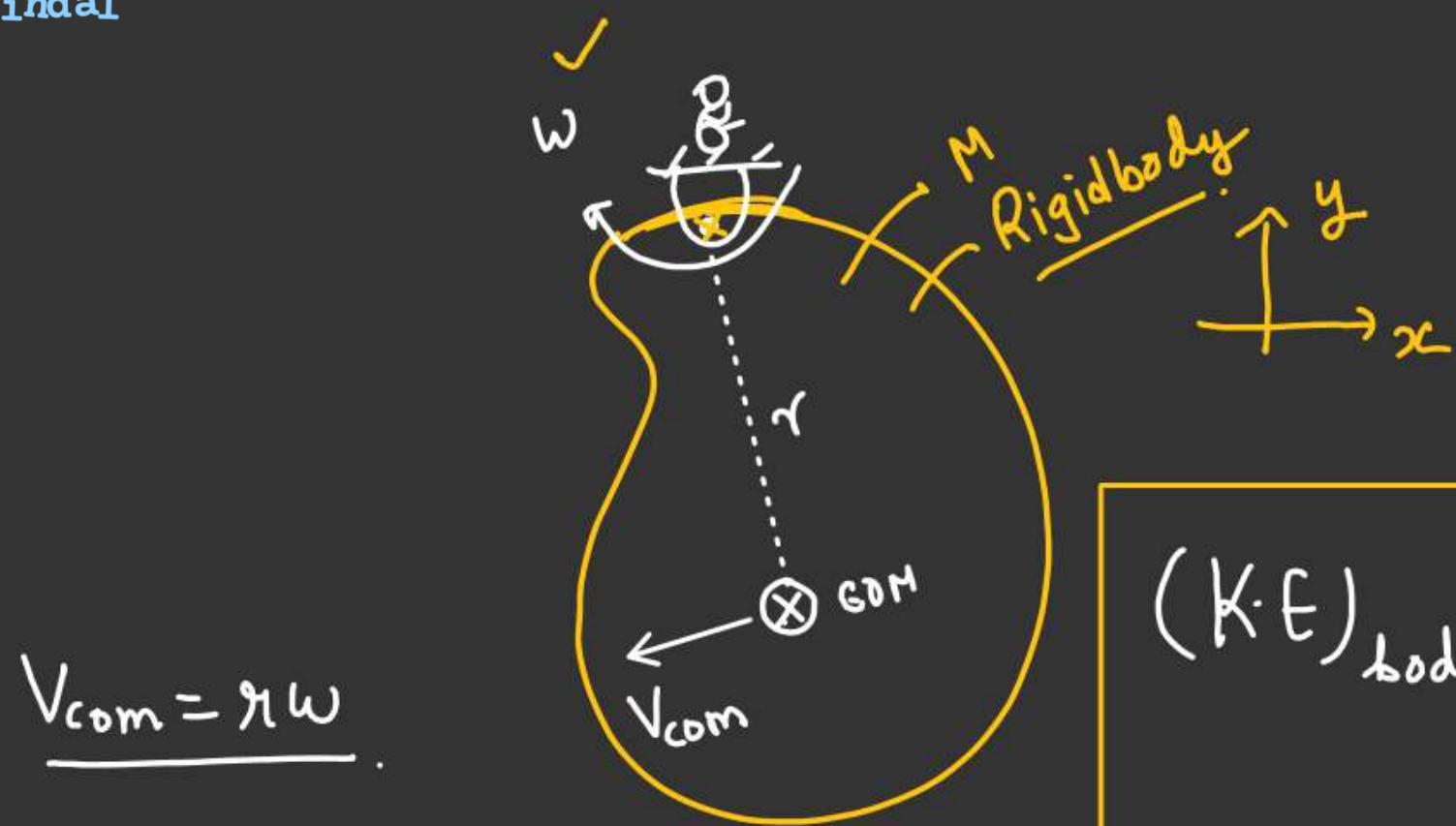


$$K.E_{Rod} = \frac{1}{2} I_{COM} \cdot \omega^2 = \frac{1}{2} \left(\frac{\pi L^2}{12} \right) \omega^2 = \frac{M L^2 \omega^2}{24}$$

$$\int \omega$$



$$K.E_{Rod} = \frac{1}{2} \left(\frac{\pi L^2}{3} \right) \cdot \omega^2 = \frac{M L^2}{6} \omega^2$$



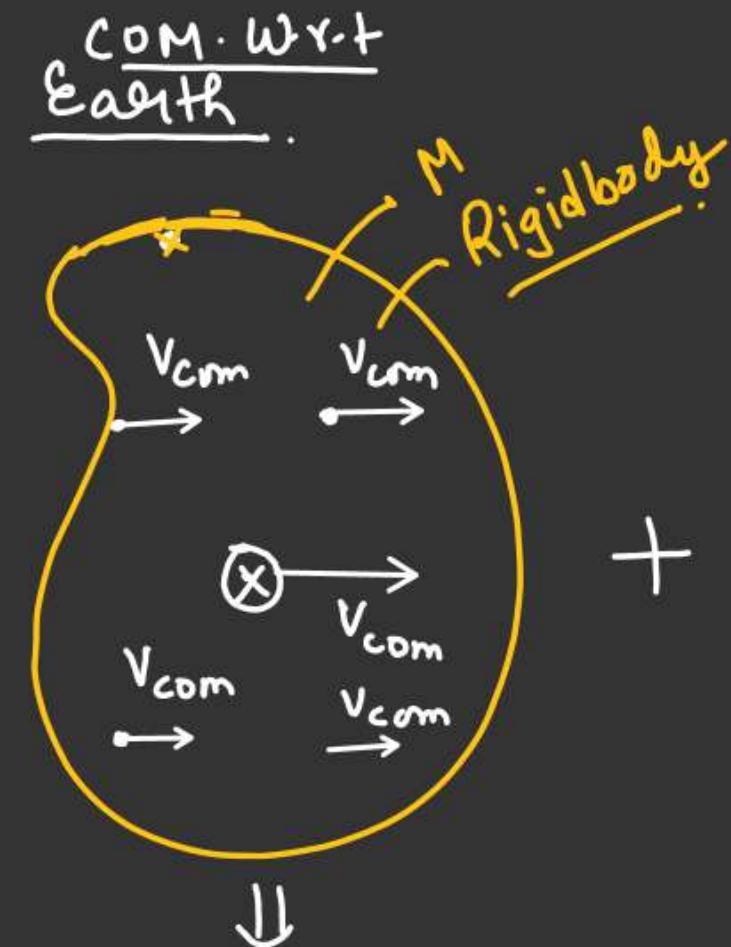
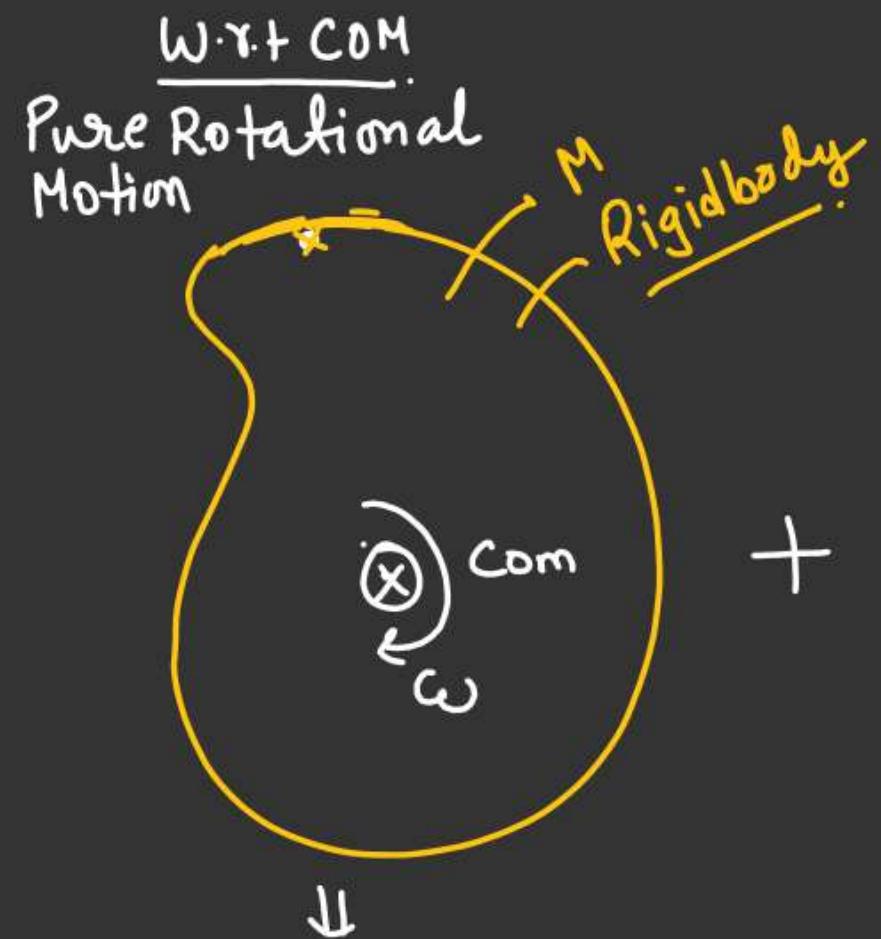
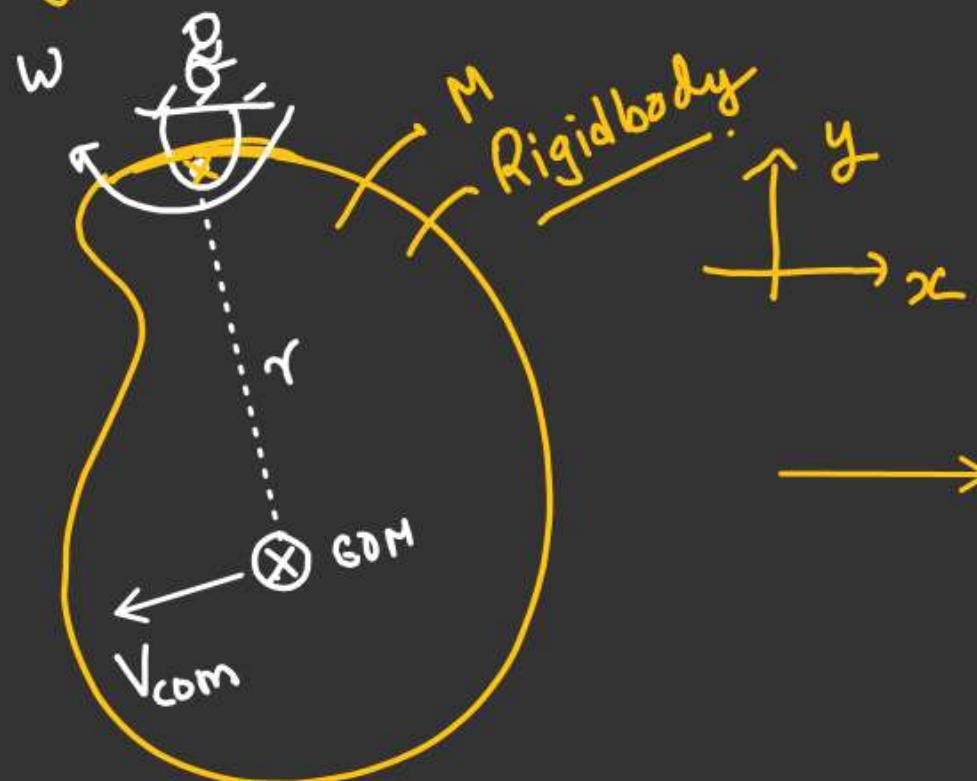
$$I_{\text{body}} \text{ about axis of rotation} = [I_{\text{com}} + m r^2]$$

$$(K.E)_{\text{body}} = \frac{1}{2} \left(I_{\text{body about axis of Rotation}} \right) \omega^2$$

$$K.E_{\text{body}} = \frac{1}{2} [I_{\text{com}} + m r^2] \omega^2$$

$$K.E_{\text{body}} = \frac{1}{2} I_{\text{com}} \omega^2 + \frac{1}{2} M V_{\text{com}}^2$$

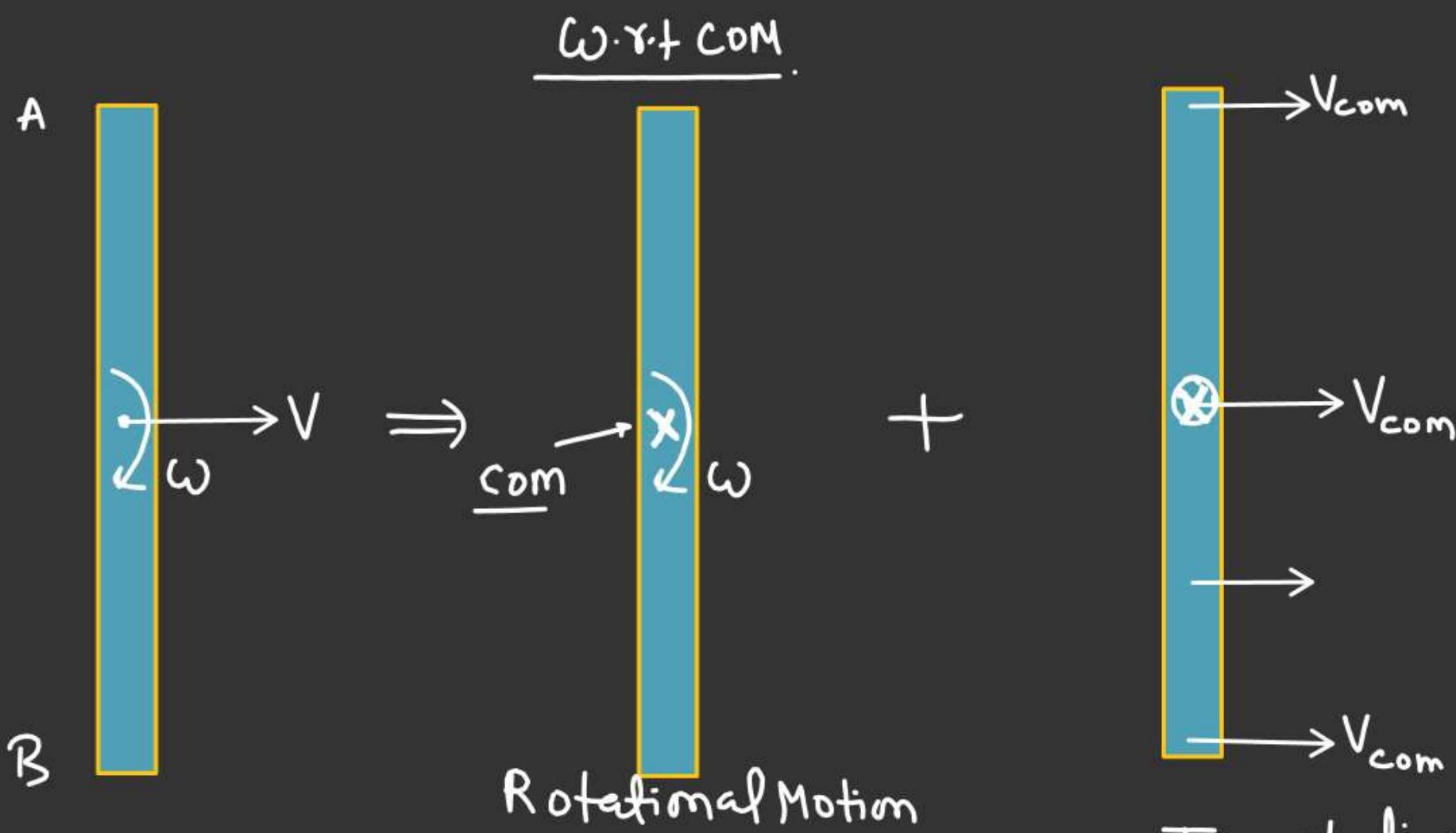
$$K.E_{\text{body}} = \frac{1}{2} (I_{\text{com}}) \omega^2 + \frac{1}{2} m r^2 \omega^2$$



$$K.E_{body} = \frac{1}{2} I_{com} \omega^2 + \frac{1}{2} M V_{com}^2$$

$$\leftarrow K.E_{Rotational} = \frac{1}{2} I_{com} \omega^2$$

$$K.E_{Translational} = \frac{1}{2} M V_{com}^2$$



Note:- When a body is free to rotate it always rotate about COM

$$\begin{aligned}
 & \text{K.E}_{\text{Rotational}} = \frac{1}{2} I_{COM} \cdot \omega^2 \\
 & = \frac{1}{2} \left(\frac{M L^2}{I_R} \right) \omega^2
 \end{aligned}$$

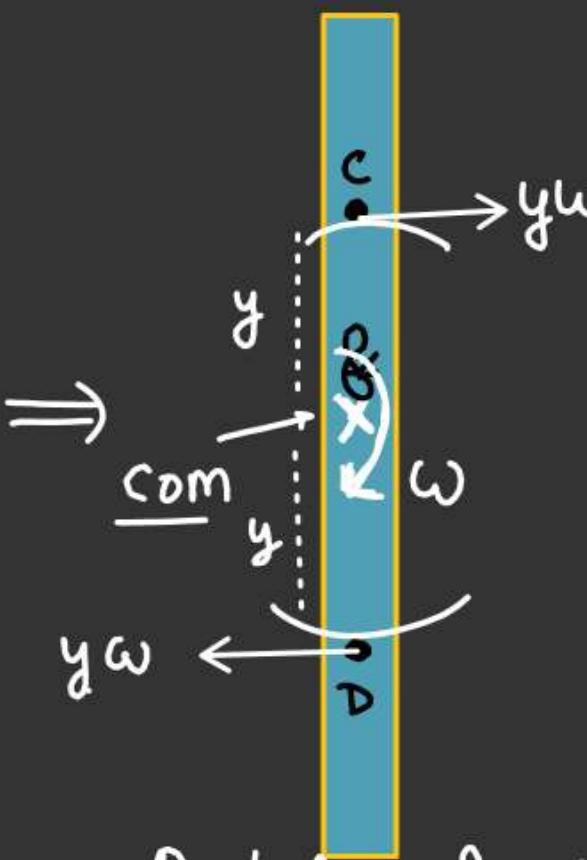
K-E_{Translational} = $\frac{1}{2} M V_{COM}^2$

$$|\vec{V}_c| = ??$$

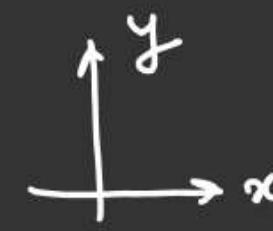
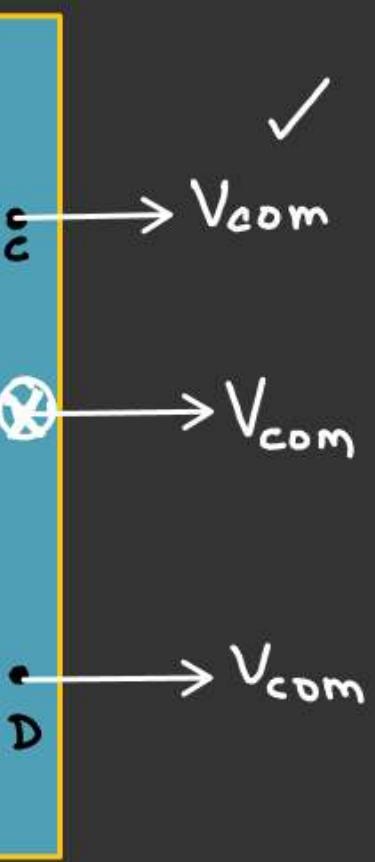
$$|\vec{a}_c| = ??$$



$\omega \cdot r + \text{COM}$



+



$$\vec{V}_c/\varepsilon = \vec{V}_{c/\text{COM}} + \vec{V}_{\text{COM}/\varepsilon}$$

$\circlearrowleft \rightarrow V_{\text{COM}}$
 $\otimes \rightarrow V_{c/\text{COM}}$

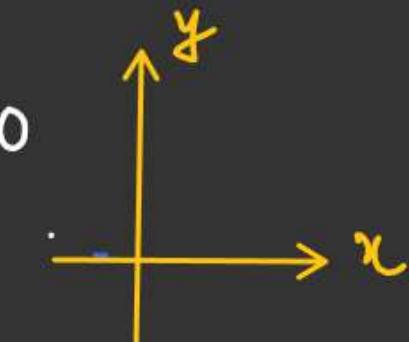
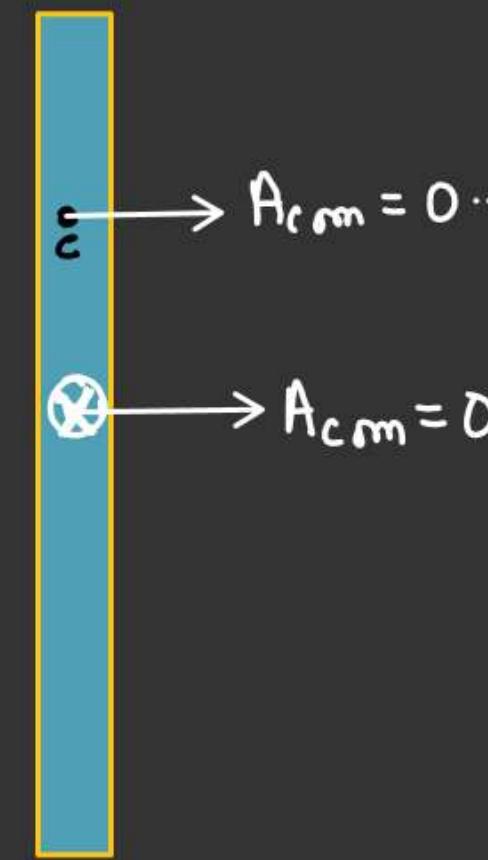
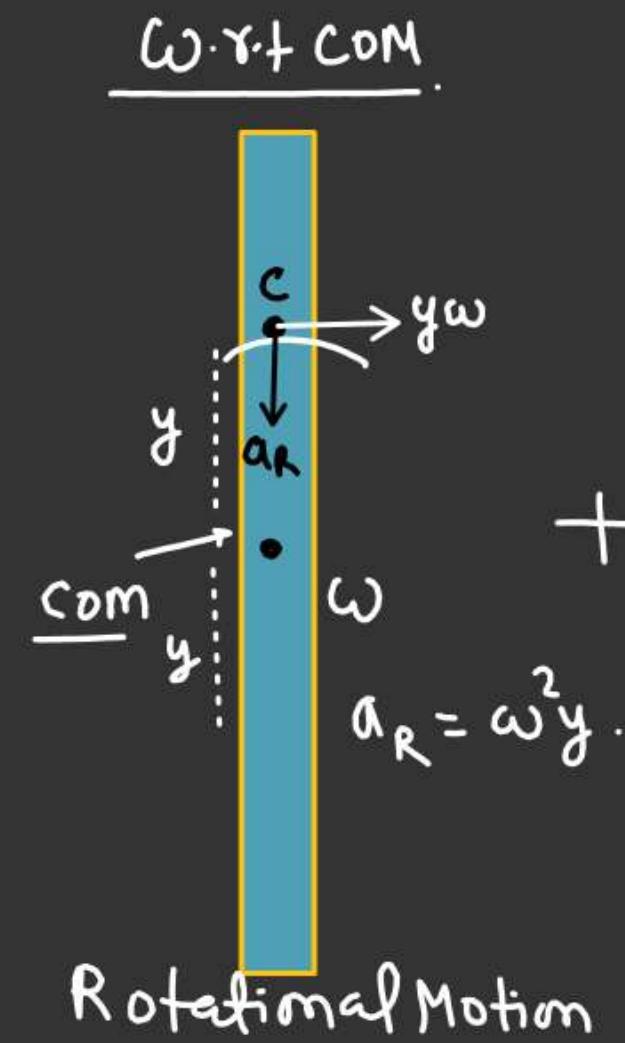
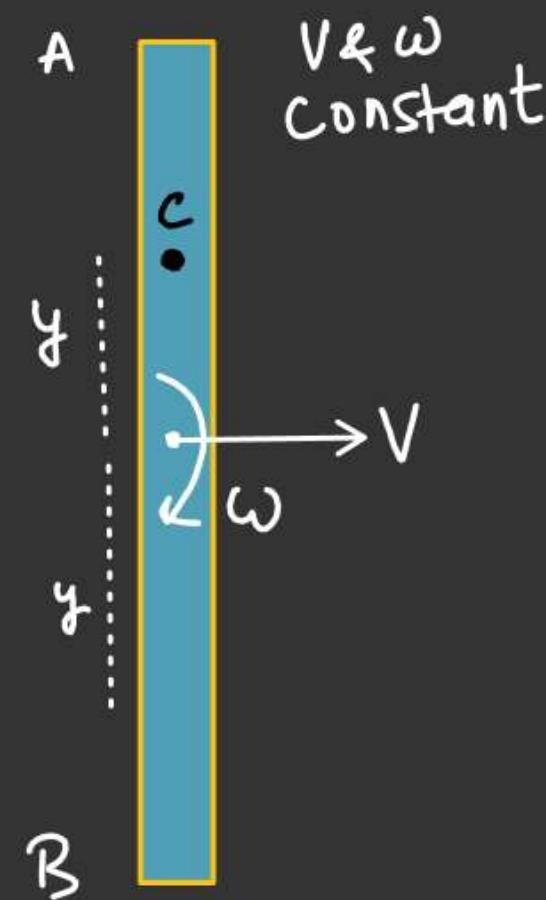
Translational
Motion.

$\frac{\odot}{\text{Earth}}$

$$\begin{aligned} \vec{V}_c/\varepsilon &= y\omega \hat{i} + V_{\text{COM}} \hat{i} \\ &= (V_{\text{COM}} + y\omega) \hat{i} \end{aligned}$$

$$\begin{aligned} \vec{V}_D/\varepsilon &= \vec{V}_{D/\text{COM}} + \vec{V}_{\text{COM}/\varepsilon} \\ &= -y\omega \hat{i} + V_{\text{COM}} \hat{i} \\ &= (V_{\text{COM}} - y\omega) \hat{i} \end{aligned}$$

$$|\vec{a}_c| = ?? \quad A$$

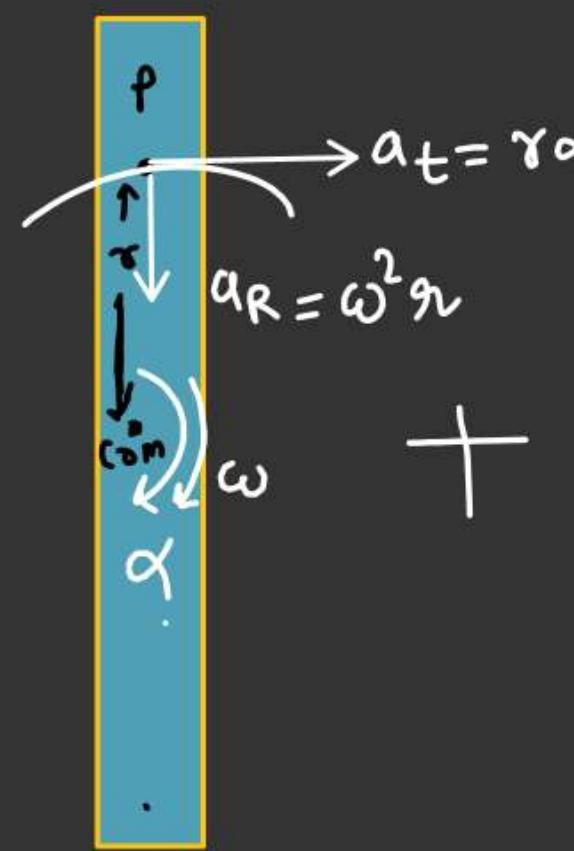


$$\vec{a}_c/\varepsilon = \vec{a}_{c/COM} + \vec{a}_{COM}/\varepsilon$$

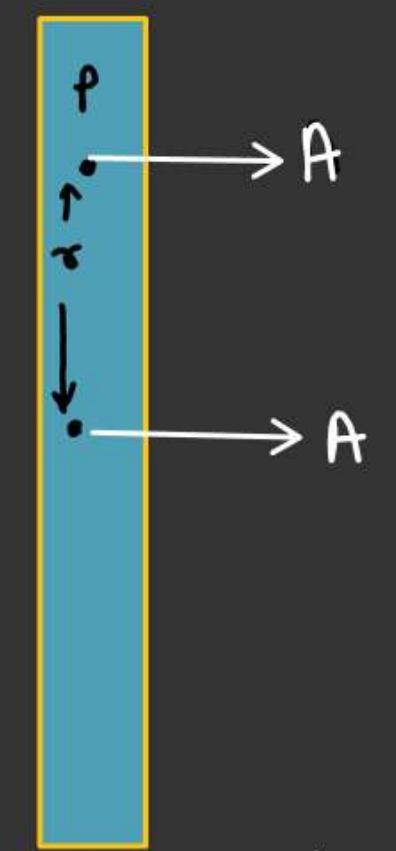
$$\vec{a}_c/\varepsilon = -\omega^2 y \hat{j} + \vec{0}$$

~~A & α~~ .

$A \& \alpha \rightarrow \text{Constant}$.
 $a_p = ? \quad a_q = ?$



Rotational Motion

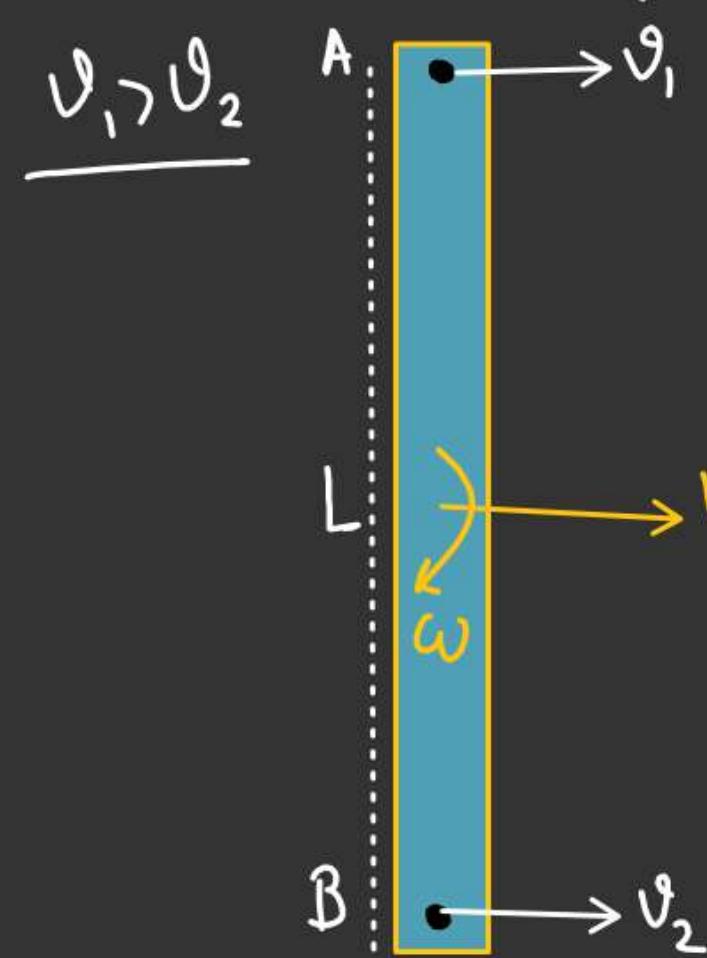


Translational Motion

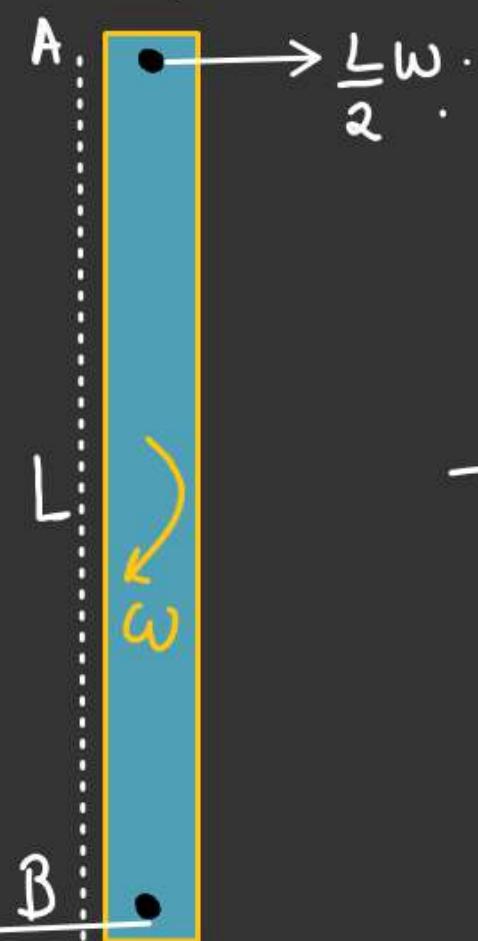
$$\vec{a}_{p/\text{com}} = \left(\begin{array}{l} (\text{com frame}) \\ \gamma\alpha\hat{i} - \omega^2 r\hat{j} \end{array} \right)$$

$$\vec{a}_{\text{com}/\epsilon} = A\hat{i}$$

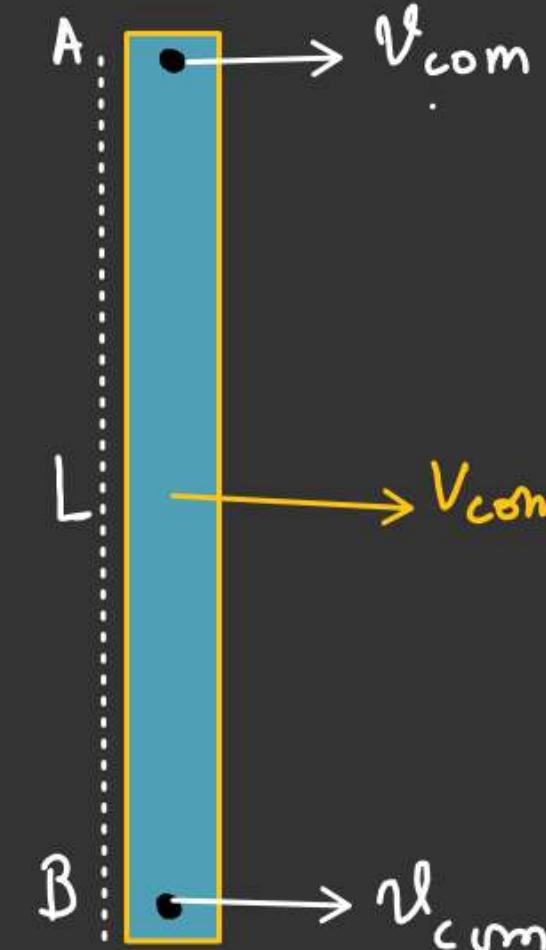
$$\begin{aligned} \vec{a}_{p/\epsilon} &= \vec{a}_{p/\text{com}} + \vec{a}_{\text{com}/\epsilon} \\ &= (\gamma\alpha\hat{i} - \omega^2 r\hat{j}) + A\hat{i} \\ &= \underline{(\underline{A + \gamma\alpha}\hat{i} - \omega^2 r\hat{j})}. \end{aligned}$$



$$v_{\text{com}} = ??, \omega = ??$$



+



$$v_1 = v_{\text{com}} + \frac{L}{2}\omega$$

$$v_2 = v_{\text{com}} - \frac{L}{2}\omega$$

$$v_1 + v_2 = 2v_{\text{com}}$$

$$v_{\text{com}} = \frac{v_1 + v_2}{2}$$

Rotational Motion
(COM frame)

$$\frac{L}{2}\omega = v_1 - v_{\text{com}} \\ = v_1 - \left(\frac{v_1 + v_2}{2} \right)$$

~~$$\frac{L}{2}\omega = \frac{v_1 - v_2}{2}$$~~

~~$$\omega = \frac{v_1 - v_2}{L}$$~~