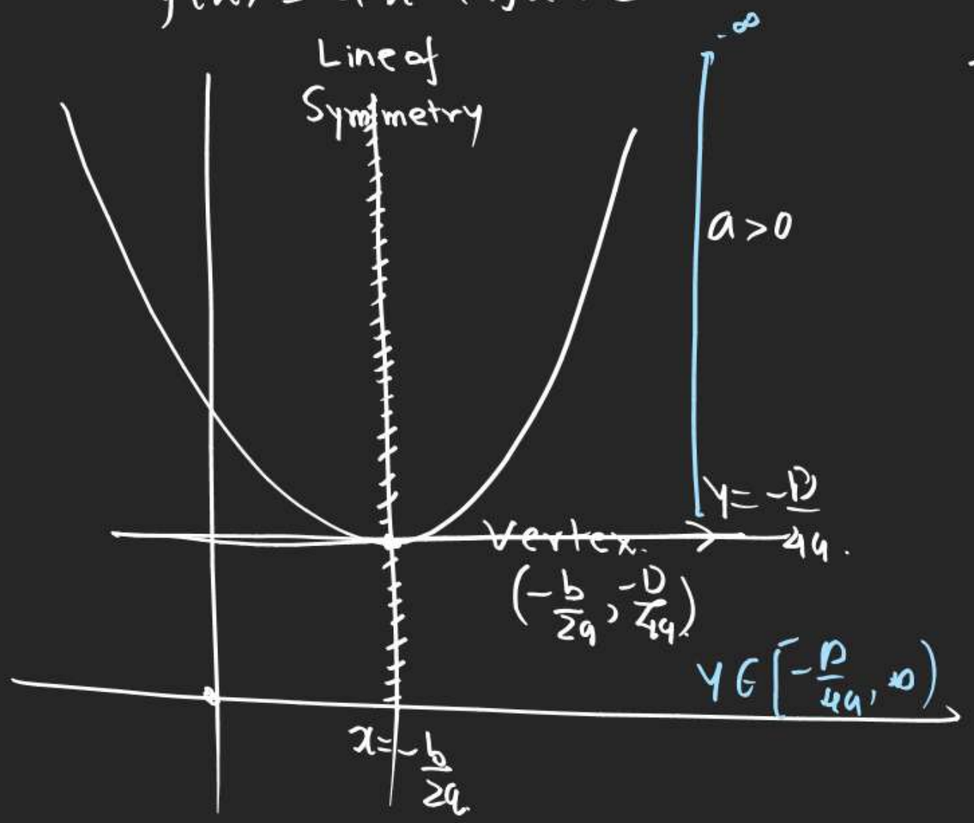
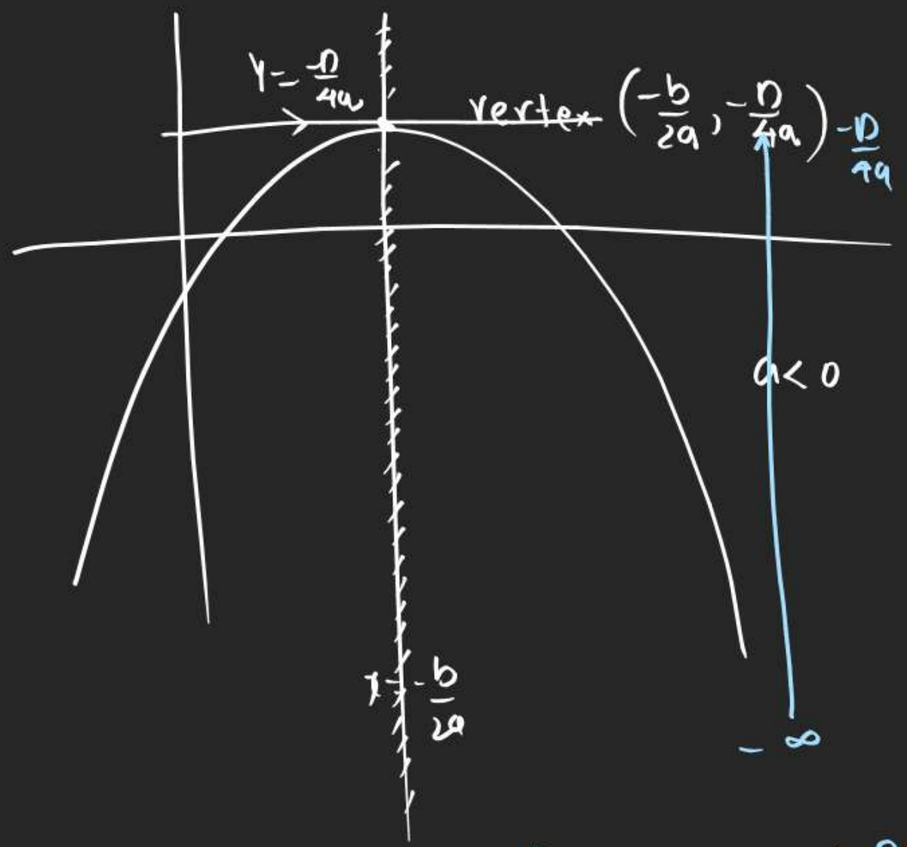
Nishant Jindal

QUADRATIC EQUATION

Range of Q fxn.

$f(x) = ax^2 + bx + c$





Range $\in \left(-\infty, \frac{-D}{4a}\right]$

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QUADRATIC EQUATION

$y = x^2 - 3x + 11$'s Rf

$a = 1, b = -3, c = 11$

$a > 0$ (+)

$y \in \left[\frac{4ac - b^2}{4a}, \infty\right)$

$y \in \left[\frac{4 \times 1 \times 11 - (-3)^2}{4 \times 1}, \infty\right)$

$y \in \left[\frac{35}{4}, \infty\right)$

$y = -2x^2 + x + 6$'s Rf

$a = -2, b = 1, c = 6$

$a < 0$

$y \in \left(-\infty, -\frac{D}{4a}\right]$

$y \in \left(-\infty, \frac{4ac - b^2}{4a}\right]$

$y \in \left(-\infty, \frac{4 \times -2 \times 6 - (1)^2}{4 \times -2}\right]$

$y \in \left(-\infty, \frac{-49}{-8}\right]$

$y \in \left(-\infty, \frac{49}{8}\right]$

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QUADRATIC EQUATION

Q Find Range of $y = 2x^2 - 3x + 1$

$a = 2, b = -3, c = 1$

> 0

$y \in \left[-\frac{D}{4a}, \infty\right)$

$y \in \left[\frac{4 \times 2 \times 1 - (-3)^2}{4 \times 2}, \infty\right)$

$y \in \left[-\frac{1}{8}, \infty\right)$

Q let a & b are Roots of $x^2 + ax + b = 0$ then Least value of $x^2 + ax + b = 0$?

$x^2 + ax + b = 0 \overset{a}{\underset{b}{\rightleftarrows}}$

$a + b = -\frac{a}{1}$ | $a \cdot b = \frac{b}{1}$

$1 + b = -1$ | $a \cdot b = b$

$b = -2$ | $a = 1$

Actual Eqn

$x^2 + 1 \cdot x - 2 = 0$

$x^2 + x - 2 = 0$

$a = 1, b = 1, c = -2$

$y \in \left[\frac{4 \times 1 \times -2 - (1)^2}{4 \times 1}, \infty\right)$

$y \in \left[-\frac{9}{4}, \infty\right)$

Least value $= -\frac{9}{4}$

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QUADRATIC EQUATION

Range Under Restricted Domain

Q Range of $y = x^2 + x + 1$ in $x \in [0, 2]$

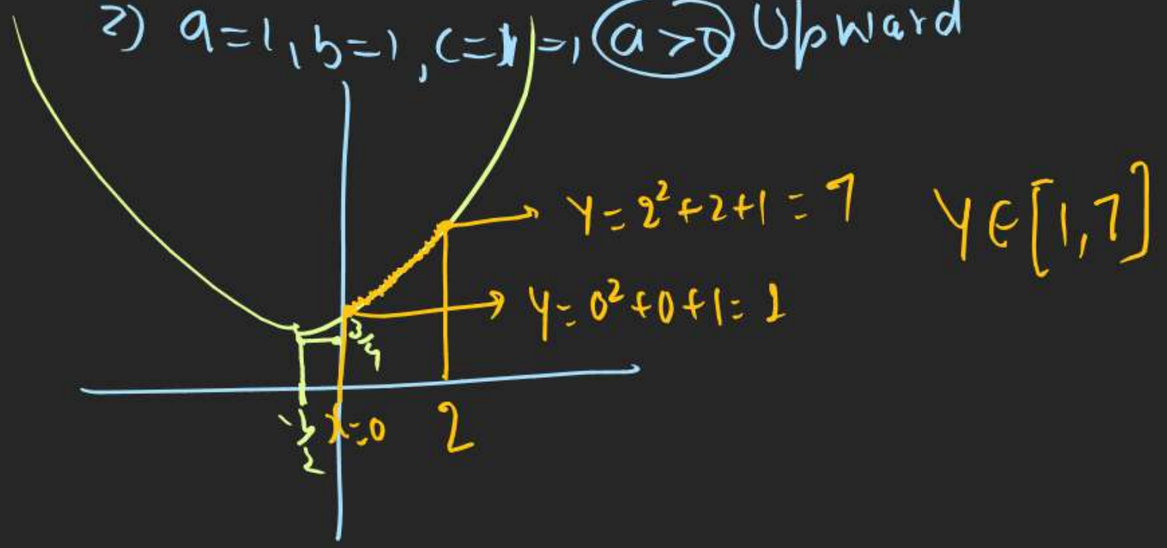
When x is Restricted Try to make Graph

1) $\frac{dy}{dx} = 2x + 1 = 0$

$x = -\frac{1}{2}$

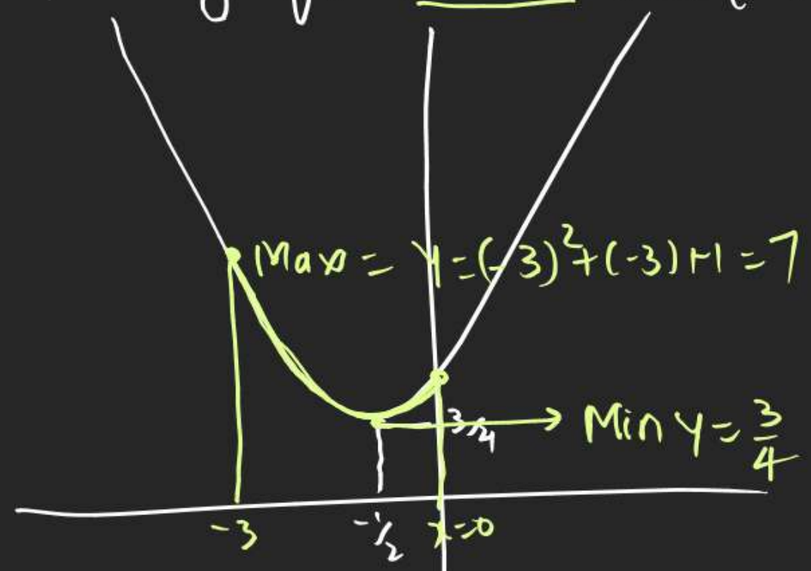
$y = \left(-\frac{1}{2}\right)^2 + -\frac{1}{2} + 1 = \frac{3}{4}$ } Vertex $= \left(-\frac{1}{2}, \frac{3}{4}\right)$

2) $a = 1, b = 1, c = 1 = 1$ $(a > 0)$ Upward



$y \in [1, 7]$

Q Range of $y = x^2 + x + 1$ in $x \in [-3, 0]$



$y \in \left[\frac{3}{4}, 7\right]$

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QUADRATIC EQUATION

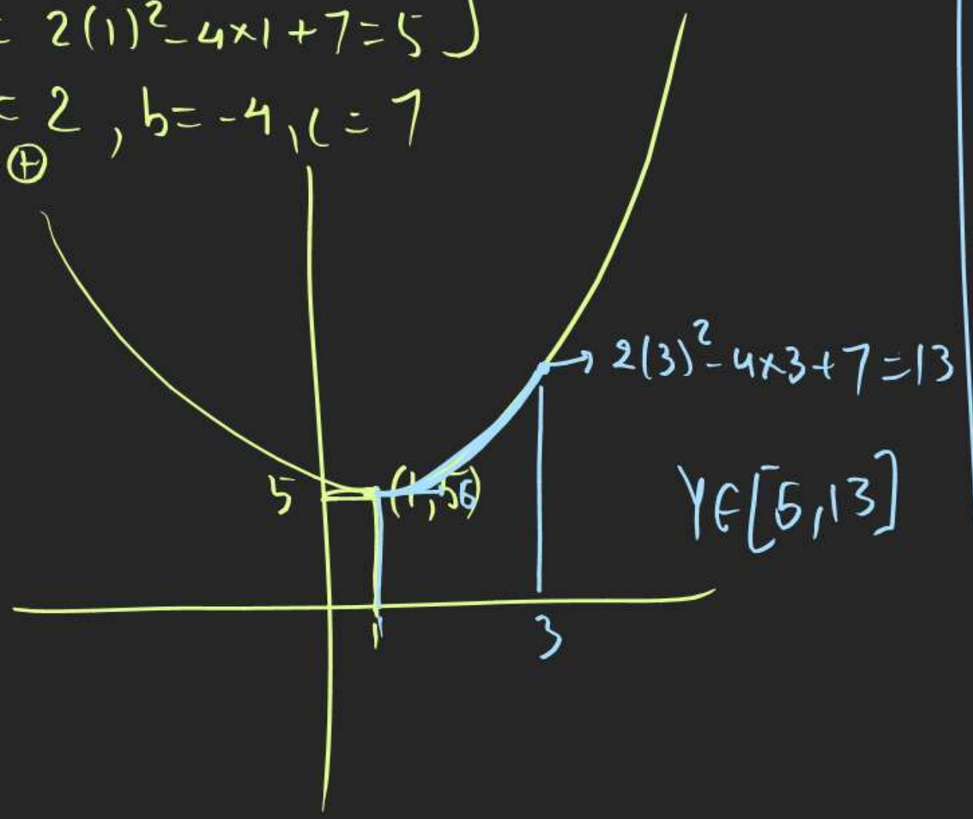
Q $y = 2x^2 - 4x + 7$ Range in $x \in [1, 3]$

1) $\frac{dy}{dx} = 4x - 4 = 0 \Rightarrow x = 1$

$y = 2(1)^2 - 4\times1 + 7 = 5$

Vertex $= (1, 5)$

2) $a = 2$ (+), $b = -4$, $c = 7$



$y \in [5, 13]$

Q $y = \sin^2 x - 2\sin x + 2$ find Range?

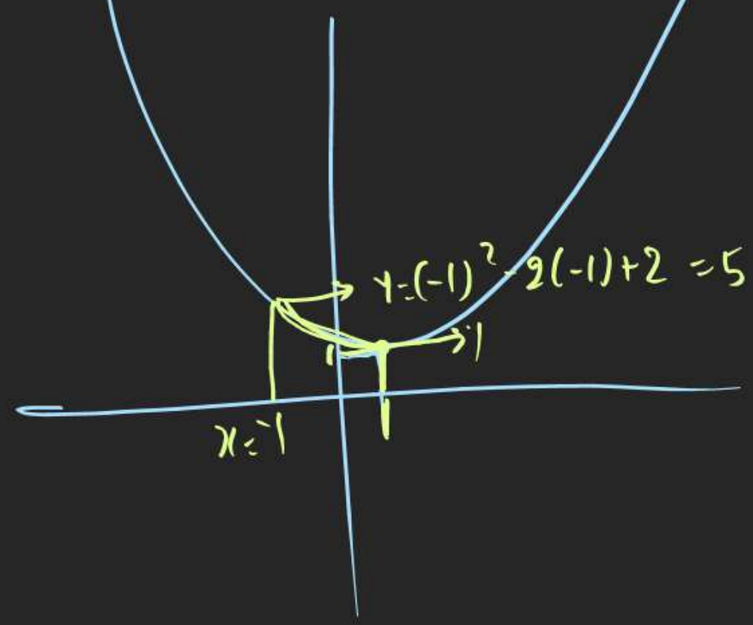
let $\sin x = t$

$y = t^2 - 2t + 2$; $t = \sin x \in [-1, 1]$

$\frac{dy}{dt} = 2t - 2 = 0 \Rightarrow t = 1$

$y = 1^2 - 2\times1 + 2 = 1$

Vertex $(1, 1)$



$y \in [1, 5]$

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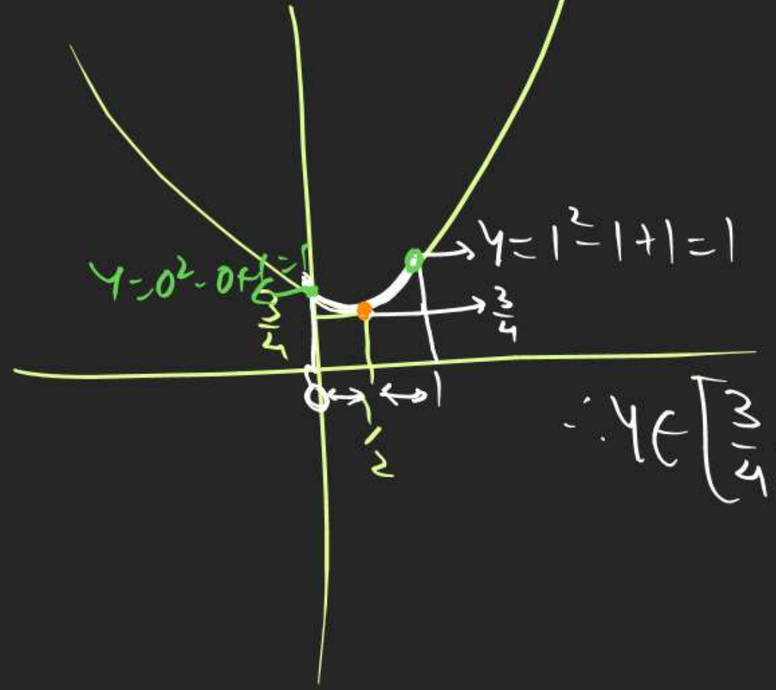
Q $y = \{x\}^2 - \{x\} + 1$ find Range?

let $y = t^2 - t + 1$; $t = \{x\} \in [0, 1)$ fotore

$\frac{dy}{dt} = 2t - 1 = 0$

$t = 1/2$

$y = (\frac{1}{2})^2 - \frac{1}{2} + 1 = \frac{3}{4}$ } Ver $= (\frac{1}{2}, \frac{3}{4})$



$\therefore y \in [\frac{3}{4}, 1]$

Range of Rational Expression.

① Linear/Linear ② Quad/Quad ③ Linear/Quad ④ Quad/Linear.

① Range of $y = \frac{ax+b}{cx+d}$ type.

Q Range of $y = \frac{4x-1}{7x-2}$

① Cross multiply & find x in terms of y

$7xy - 2y = 4x - 1$

$7xy - 4x = 2y - 1 \Rightarrow x(7y-4) = 2y - 1$

$x = \frac{2y-1}{(7y-4)}$

② find y where $Dr = 0$ & Restrict it

$7y - 4 \neq 0 \Rightarrow y \neq \frac{4}{7} \Rightarrow y \in (-\infty, \infty) - \{\frac{4}{7}\}$

Range

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QUADRATIC EQUATION

Q Range of $y = \frac{3x+4}{2x+7}$

$2xy + 7y = 3x + 4$

$2xy - 3x = -7y + 4$

$x(2y - 3) = -7y + 4$

$x = \frac{-7y+4}{(2y-3)}$

$2y - 3 \neq 0 \Rightarrow y \neq \frac{3}{2}$

$y \in R - \left\{\frac{3}{2}\right\}$

Short Method

$y = \frac{3x+4}{2x+7}$

Range $y \in R - \left\{\frac{3}{2}\right\}$

Q $y = \frac{1x+3}{1x-2}$ Range?

$y \in R - \left\{\frac{1}{1}\right\}$

$y \in R - \{1\}$

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(B) When $\frac{Q}{Q} \to \frac{L}{L}$

Q Find Range of $y = \frac{x^2-5x+4}{x^2+2x-3}$

$$y = \frac{(x-4)(x-1)}{(x+3)(x-1)} \quad x \neq 1$$

$x \neq 1$

$$y = \frac{1-4}{1+3} \neq \frac{-3}{4}$$

$\hookrightarrow y \in R - \{1, -\frac{3}{4}\}$

Q $y = \frac{2x^2-5x+3}{4x^2+x-5}$, Range? 3 Lec More

$$y = \frac{(2x-3)(x-1)}{(4x+5)(x-1)} \quad x \neq 1$$

$y \in R - \{\frac{2}{4}, \frac{-1}{9}\} \Rightarrow y \in R - \{\frac{1}{2}, \frac{-1}{9}\}$

$x \neq 1$

$$y = \frac{2\times1-3}{4\times1+5} \neq \frac{-1}{9}$$

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(1) Range of $y = \frac{Q}{Q}$

Q $y = \frac{x^2 - x + 1}{x^2 + x + 1}$ Range?

1) First Cross Multiply & make Quad in x.

$x^2y + xy + y = x^2 - x + 1$

$x^2y - x^2 + xy + x + y - 1 = 0$

$\Rightarrow x^2(y-1) + x(y+1) + (y-1) = 0$ → Q in x

$y - 1 \neq 0$

$y \neq 1$

Check $y \neq 1$

$1 = \frac{x^2 - x + 1}{x^2 + x + 1}$

$x^2 + x + 1 = x^2 - x + 1$

$2x = 0$

$x = 0$

$D \geq 0$

$(y+1)^2 - 4(y-1)(y-1) \geq 0$

$(y+1)^2 - (2(y-1))^2 \geq 0$

$(y+1+2(y-1))(y+1-2(y-1)) \geq 0$

$(3y-1)(-y+3) \geq 0 \Rightarrow (3y-1)(y-3) \leq 0$

$\frac{1}{3} \leq y \leq 3 \Rightarrow y \in \left[\frac{1}{3}, 3\right]$

Q $y = \frac{x^2 + x + 2}{x^2 + x + 1}$ find Rf

$x^2y + xy + y = x^2 + x + 2$

$x^2(y-1) + x(y-1) + (y-2) = 0$

$y - 1 \neq 0$

$y \neq 1$

Check

$1 = \frac{x^2 + x + 2}{x^2 + x + 1}$

$x^2 + x + 1 = x^2 + x + 2$

$1 = 2$

$y \neq 1$

$D \geq 0$

$(y-1)^2 - 4(y-1)(y-2) \geq 0$

$y^2 - 2y + 1 - 4(y^2 - 3y + 2) \geq 0$

$-3y^2 + 10y - 7 \geq 0$

$3y^2 - 10y + 7 \leq 0$

$3y^2 - 3y - 7y + 7 \leq 0$

$(3y-7)(y-1) \leq 0$

$1 \leq y \leq \frac{7}{3}$

$y \in \left(1, \frac{7}{3}\right]$

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Q Find Range of $y = \frac{3x^2+9x+17}{3x^2+9x+7}$

Method 2

$$y = \frac{(3x^2+9x+7)+10}{(3x^2+9x+7)}$$

$$y = 1 + \frac{10}{3x^2+9x+7}$$

$$\frac{1}{4} \le 3x^2+4x+7 < \infty$$

$$4 \ge \frac{1}{3x^2+4x+7} > \frac{1}{\infty}$$

$$40 \ge \frac{1\times 10}{3x^2+4x+7} > 0$$

$$41 \ge \frac{10}{3x^2+4x+7} + 1 > 1 \Rightarrow 1 < y \le 41 \quad y \in (1, 41]$$

$y = 3x^2+9x+7$ find Range

$a = 3, b = 9, c = 7$

$a > 0$

$$y \in \left[\frac{4ac-b^2}{4a}, \infty\right)$$

$$y \in \left[\frac{4\times 3\times 7-(9)^2}{4\times 3}, \infty\right)$$

$$y \in \left[\frac{1}{4}, \infty\right)$$

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QUADRATIC EQUATION

Q $y=\dfrac{6x^2-5x-3}{x^2-2x+6} \le 4$ then find least & gr. value of $y=4x^2$

$$\frac{6x^2-5x-3}{(x^2-2x+6)} \le 4$$

(x^2-2x+6) ⊕

factor नहीं होगा

Check $(-2)^2-4\times1\times6 = -20 = -ve$

$a=1\oplus$

$$6x^2-5x-3 \le 4x^2-8x+24$$

$$2x^2+3x-27 \le 0$$

$$2x^2+9x-6x-27 \le 0$$

$$x((2x+9)-3(2x+y) \le 0$$

$$(x-3)(2x+9) \le 0$$

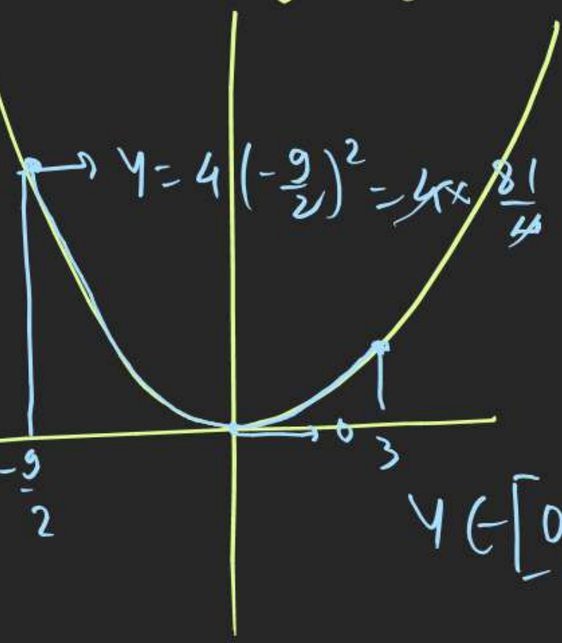
$$-\frac{9}{2} \le x \le 3$$

$y=\frac{Q}{Q}$ & $y=\frac{L}{Q}$ have same Method

25, 26 Ex 2 Ex 3 Q6 Q15, Ex 6

Range of $y=4x^2$; $x\in\left[-\frac{9}{2},3\right]$

$[-4.5,3]$



$y\in[0,81]$