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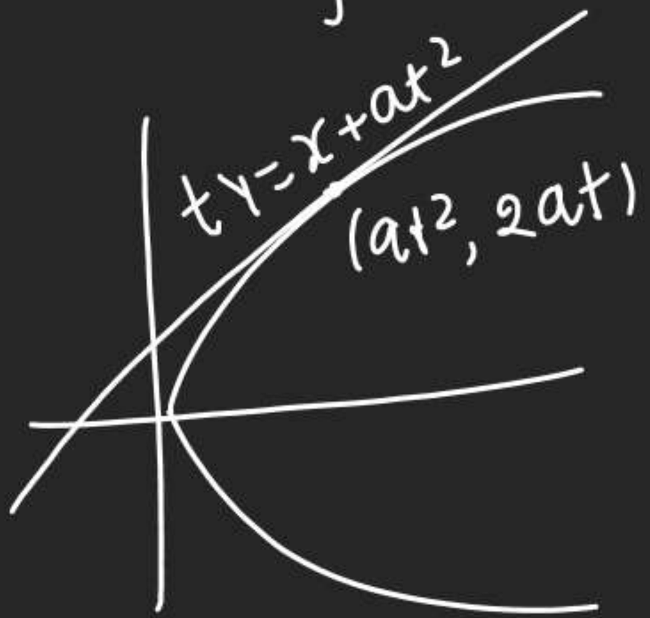
Eq^n of tangent. $Y^2 = 4ax$

3 forms

Par. form



Cart. form

(x, y_1)

$T = 0$

$$\boxed{YY_1 = 2a(x + x_1)}$$

Slope form

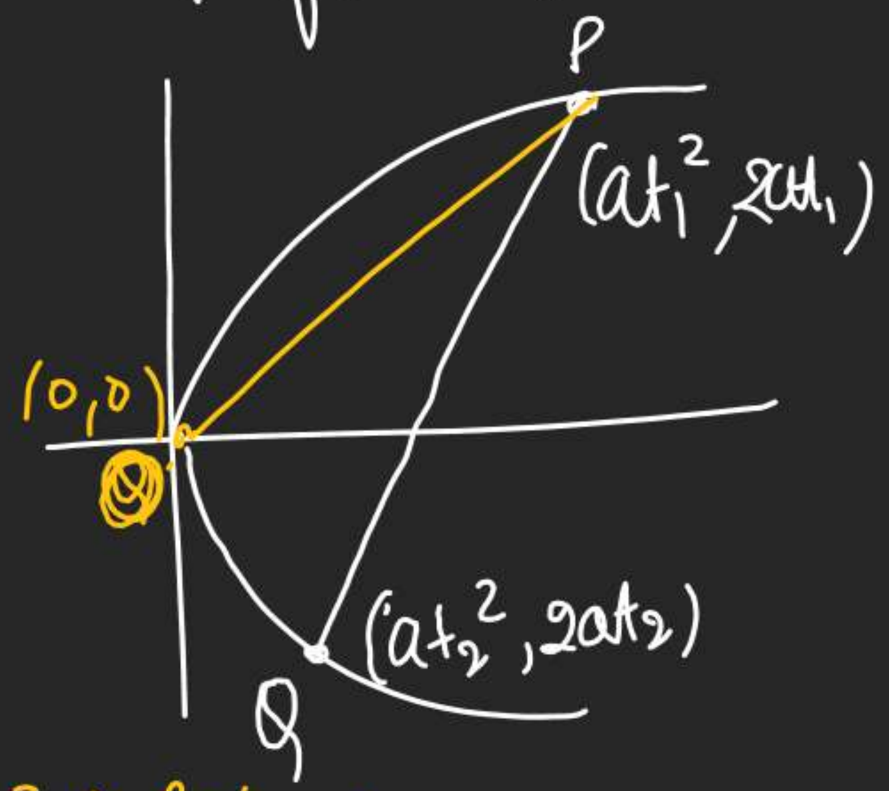
Cond^n of tangency

$Y = mx + c$ touches $Y^2 = 4ax$

if $c = \frac{a}{m}$

$$Y = mx + \frac{a}{m}$$

Eq^n of chord



Eq^n of chord

1) $2x - Y(t_1 + t_2) + 2at_1t_2 = 0$

2) $m_{PQ} = \frac{2}{t_1 + t_2}$

3) $m_{OP} = \frac{2}{t_1}$

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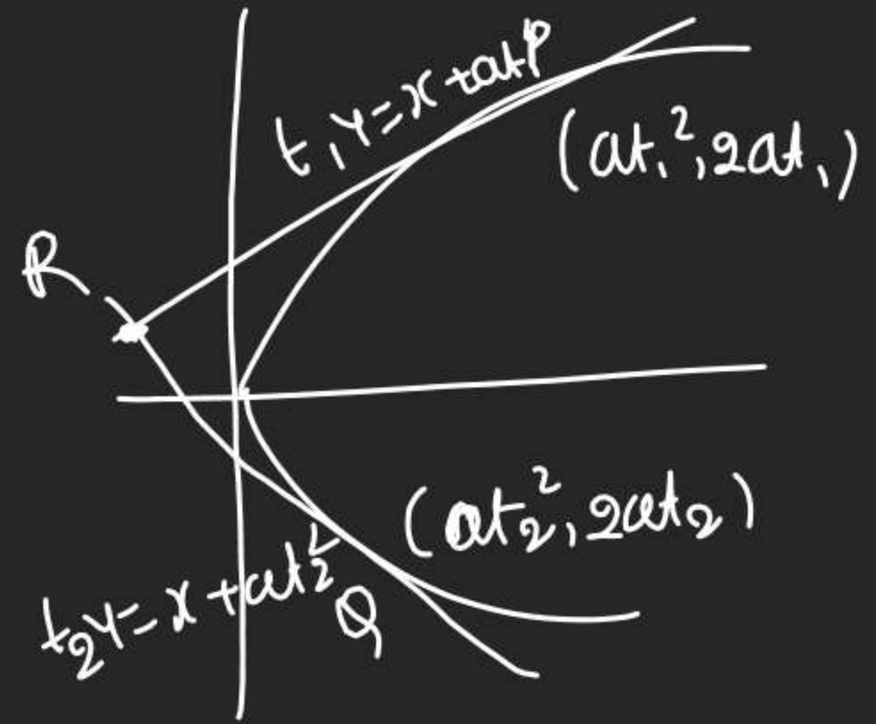
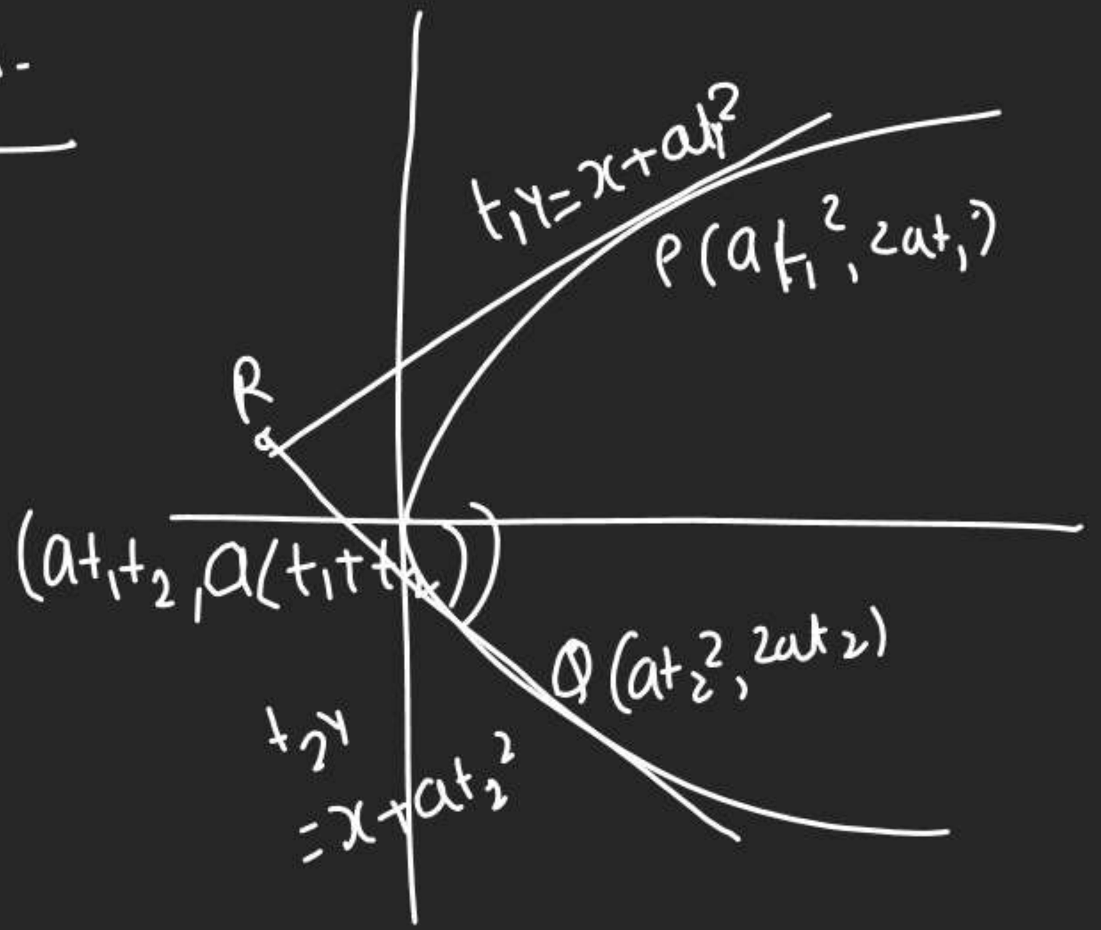
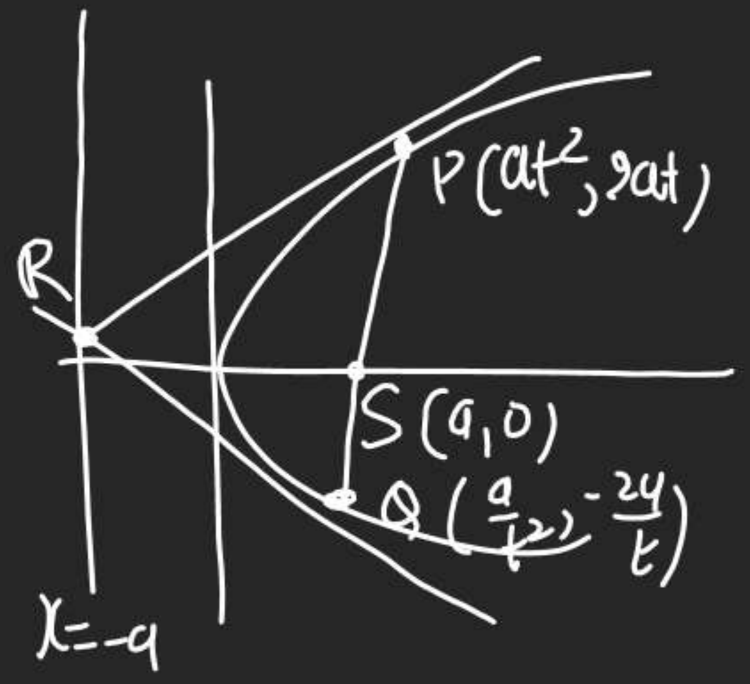
* Pt. of Intersection of 2 tangents.







$$R = (at_1t_2, a(t_1+t_2))$$

$$R = \left(-a, a\left(t - \frac{1}{t}\right)\right)$$

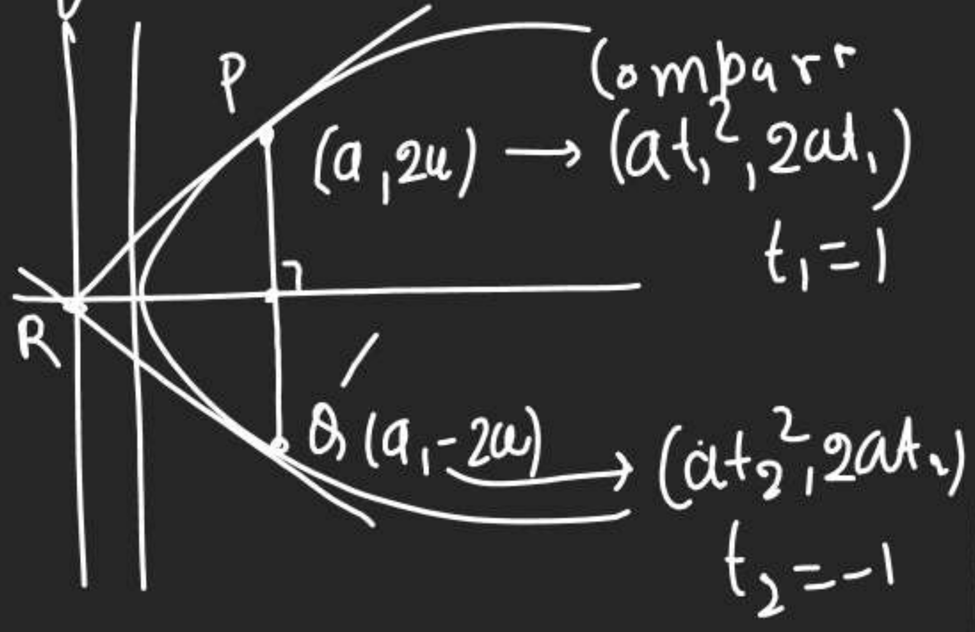
$$x - t_1 y + at_1^2 = 0$$
$$x - t_2 y + at_2^2 = 0$$
$$- \quad + \quad -$$
$$y(t_2 - t_1) = a(t_2^2 - t_1^2)$$
$$y = a(t_1 + t_2)$$
$$x - at_1(\cancel{t_1} + t_2) + \cancel{at_1^2} = 0 \Rightarrow x = at_1t_2$$

* If tangents are drawn at the end Pt. of Focal Chord.

$$\downarrow$$
$$t_1 t_2 = -1$$

Tangents at the end Pt of Focal Chord always Intersect at Directrix.

Q Tangents at the end Pt of L.R intersect at?



$R = (at_1t_2, a(t_1+t_2))$

$= (a\times 1\times -1, a(1+-1))$

$= (-a, 0)$

$(1+t^2)(1+\frac{1}{t^2}) + (-\frac{1}{t}-t)(t+\frac{1}{t}) = 0$

$(1+t^2)(1+\frac{1}{t^2}) - (t+\frac{1}{t})^2 = 0$

Q Relation of PoI of tangents wrt P & Q?

$P(at_1^2, 2at_1)$

$Q(at_2^2, 2at_2)$

$R = (at_1t_2, a(t_1+t_2))$

X Coord $= at_1t_2 = \sqrt{at_1^2 \times at_2^2}$

= GM of abscissa of P & Q

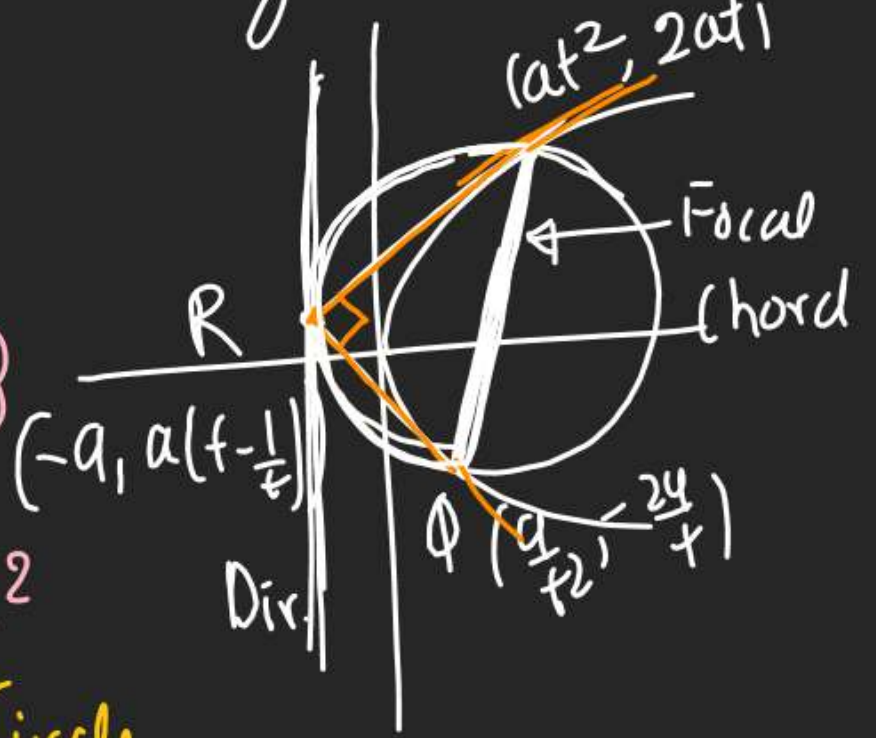
Y Coord $= a(t_1+t_2) = \frac{2at_1+2at_2}{2}$

= AM of ordinates.

Position of Circles

① Circle made by taking Focal Chord as diameter always touches directrix

GJJB



Circle

$(x-at^2)(x-\frac{a}{t^2}) + (y-2at)(y+\frac{2a}{t}) = 0$

Satisfied by R

$(-a-at^2)(-a-\frac{a}{t^2}) + (a(t-\frac{1}{t})-2at)(a(t-\frac{1}{t})+\frac{2a}{t})$

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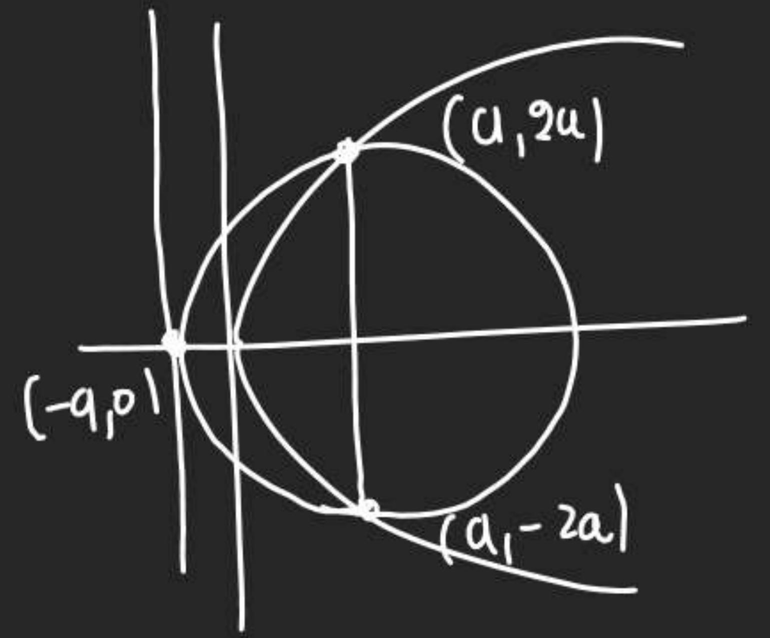
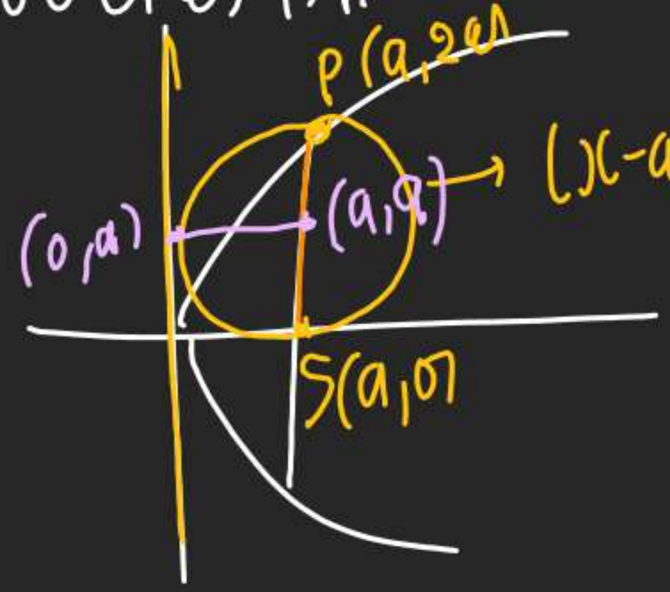
(2) Circle drawn at L.R. taking diameter touches Foot of dir.



(3) Circle taken Semi LR as diameter touches T.V.

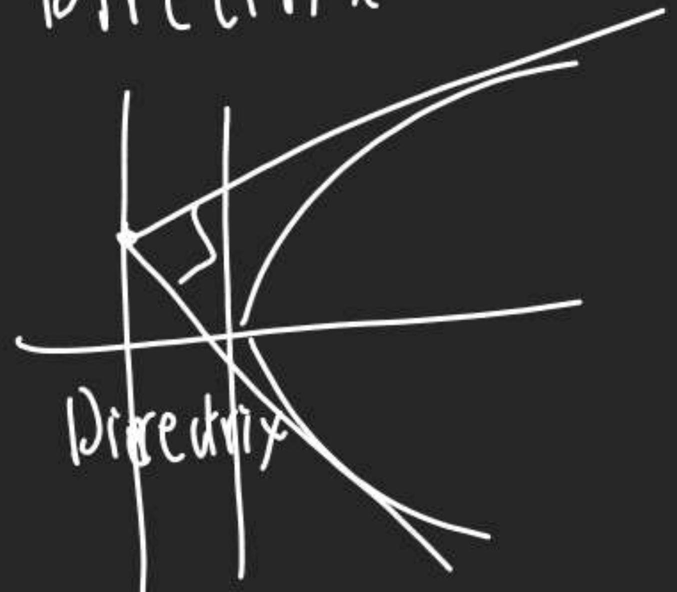


$(x-a)(x-a)+(y-2a)(y-0)=0$

(0,a) Satisfies.

(4) If tangents are Intersecting each other at 90° then Locus of Point of Intersection is known as Director Circle.

But in Parabola it is a STL. So we call it Directrix.



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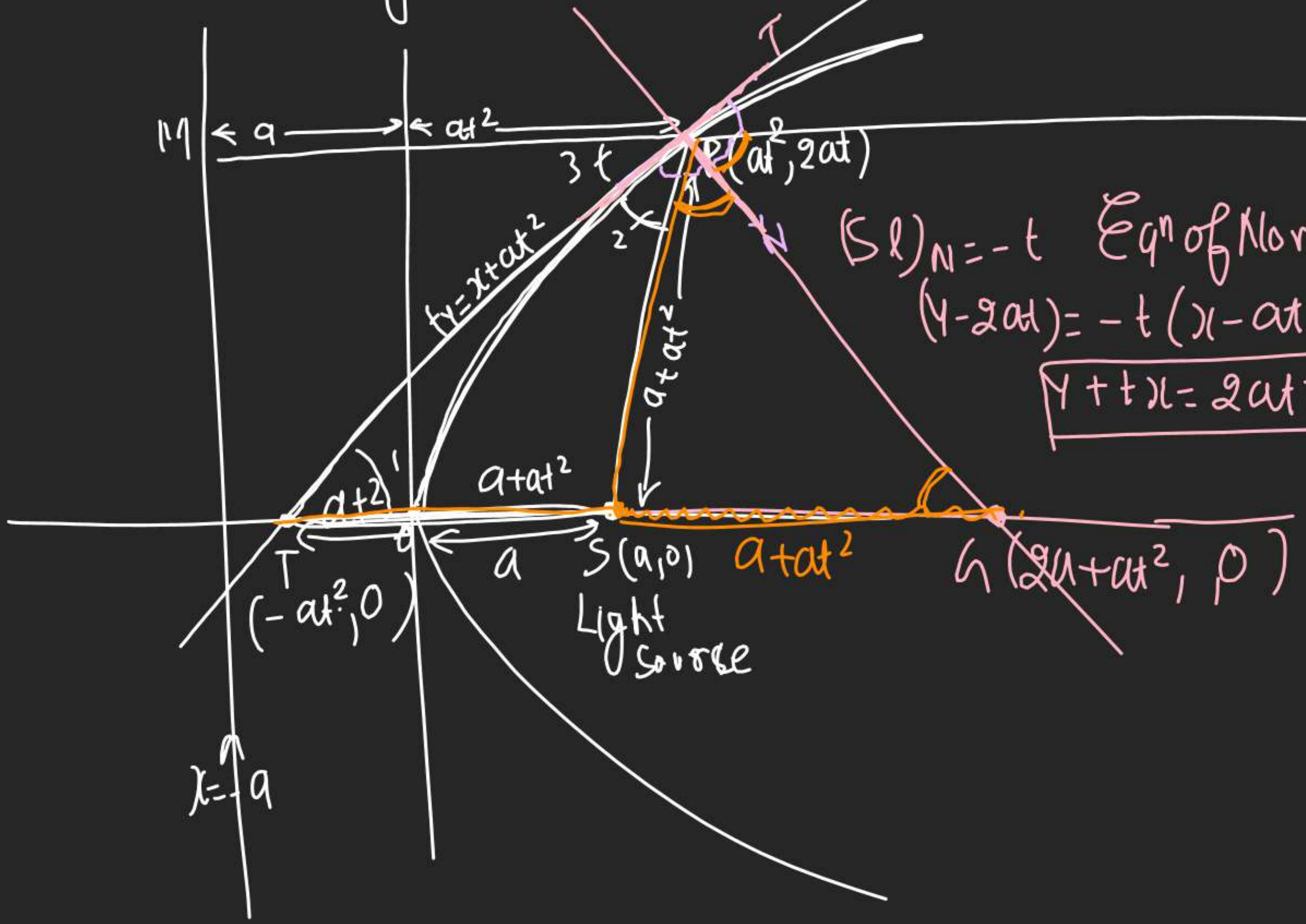
* Super Diagram



$(Sl)_N = -t$ Eq^n of Normal

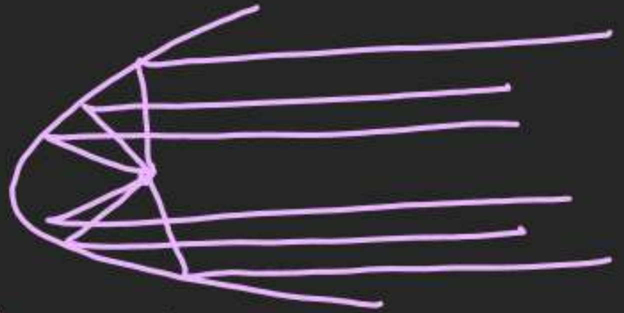
$(y-2at) = -t(x-at^2)$

$$\boxed{y + tx = 2at + at^3}$$

1) $SP = a + at^2$
$= PM$

2) $\angle 1 = \angle 2$ & $\angle 1 = \angle 3$
$\Rightarrow \angle 2 = \angle 3$

PT is angle Bisector Line



(3) $y^2 = 4ax$

$$2y\frac{dy}{dx} = 4a$$

$$\left.\frac{dy}{dx}\right|_{(at^2,\,2at)} = \frac{2a}{y} = \frac{2a}{2at} = \frac{1}{t}$$

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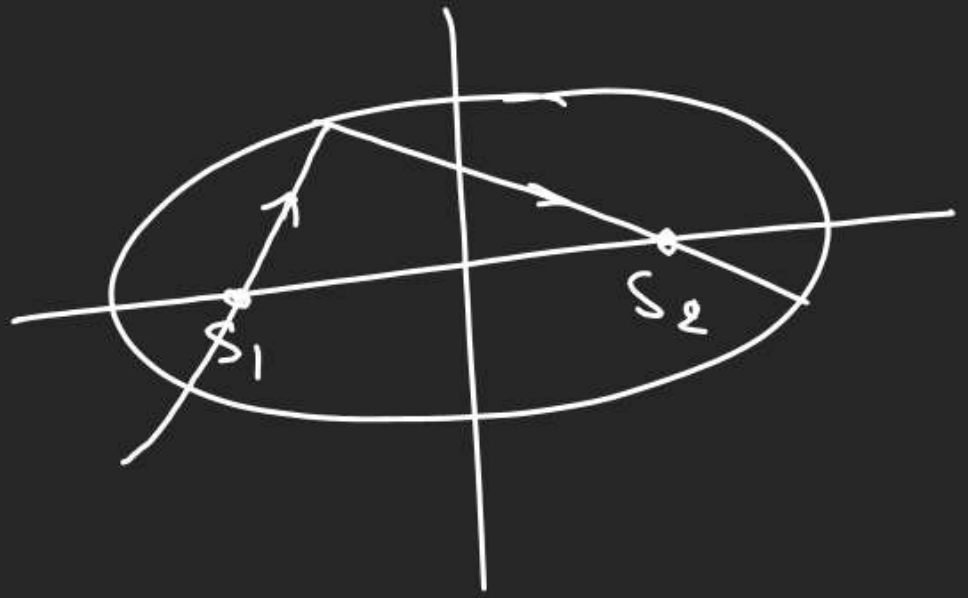
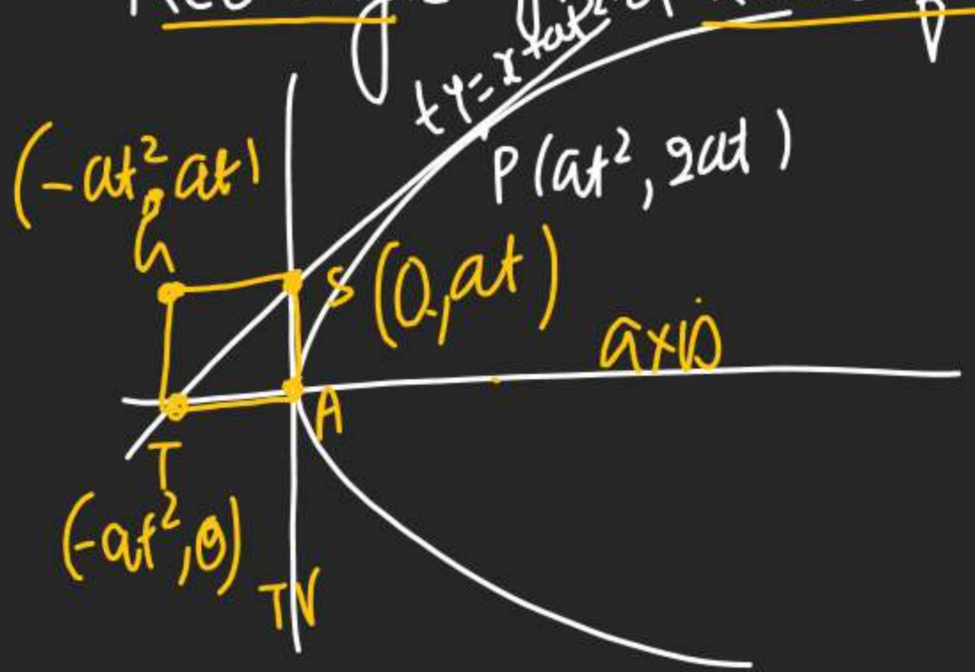
Optical Prop. of Parabola.

A) Every Line $\parallel^{rl}$ to Axis after Reflection Passes thru S.

Optical Prop. of Ellipse.



Q If tangent at any Pt. to Parabola $y^2=4ax$ meets axis at Pt. T & meets TV at "S" where A is vertex of Parabola then after completing TASG as a Rectangle find Locus of "G"?



Let G is (h, k)

$h = -at^2$ | $k = at$

$t = \frac{k}{a}$

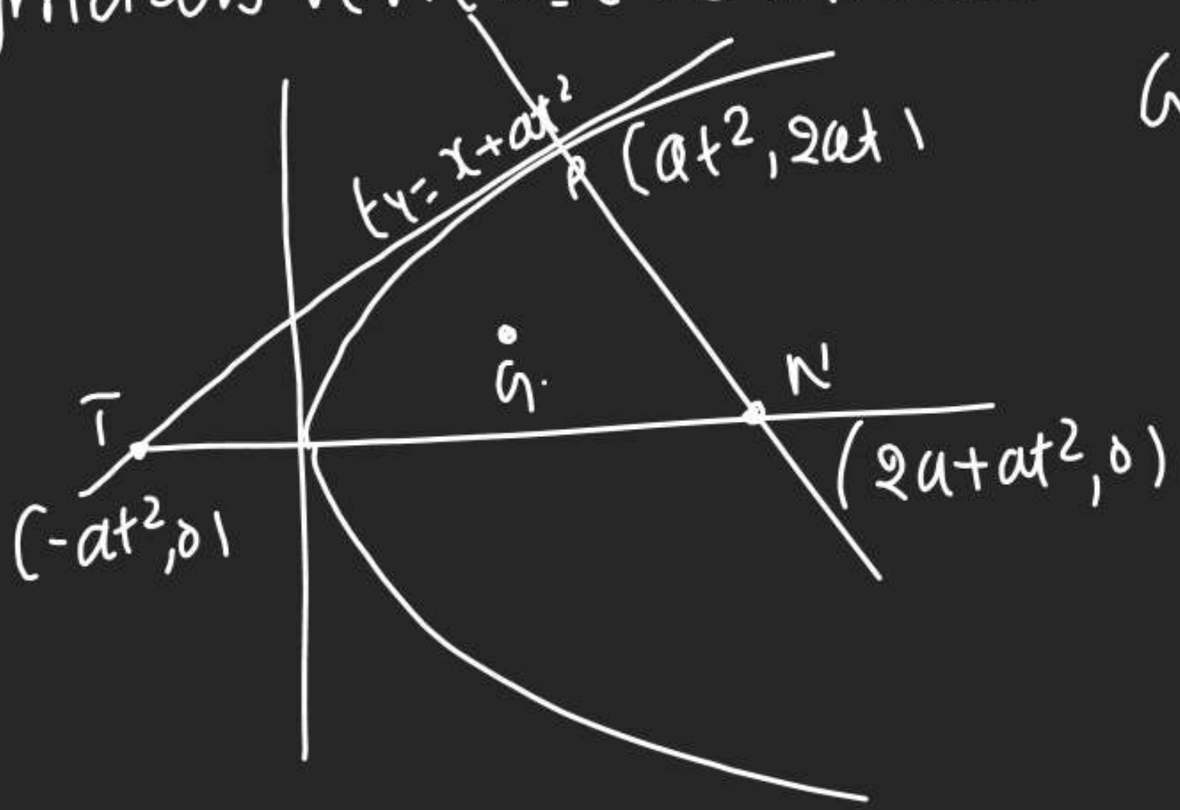
$h = -a \times \frac{k^2}{a^2}$

$k^2 = -ah$

$\boxed{y^2 = -ax}$

Locus is Parabola

Q Tangent & Normal to Parabola. $y^2=4ax$ at Pt. P meets X Axis at T & N find Locus of Centroid of ΔPTN. If answer is a Parabola. find its Vertex . . . & Directrix



$G = (h, k)$

$$G = \left(\frac{\cancel{-at^2} + \cancel{at^2} + 2a + at^2}{3}, \frac{0+0+2at}{3}\right)$$

$$h = \frac{2a+at^2}{3} \quad \Big| \quad k = \frac{2at}{3}$$

$$t = \frac{3k}{2a}$$

$$h = \frac{2a + a\left(\frac{9k^2}{4a^2}\right)}{3}$$

$$3h = 2a + \frac{9k^2}{4a}$$

$$\Rightarrow \frac{4a}{9}(3h - 2a) = k^2$$

$$(y-0)^2 = 4a\left(\frac{x}{3} - \frac{2a}{9}\right) \rightarrow \text{Parabola.}$$

Vertex $\rightarrow X = 0$

$Y = 0$

$$(y-0)^2 = \frac{4a}{3}\left(x - \frac{2a}{3}\right)$$

Vertex: $X = 0,\ Y = 0$

$$x - \frac{2a}{3} = 0 \quad \Big| \quad y - 0 = 0$$

$$x = \frac{2a}{3} \quad \Big| \quad y = 0$$

$$\left(\frac{2a}{3}, 0\right)$$

Directrix $X = -A$

$$\left(x - \frac{2a}{3}\right) = -\frac{a}{3}$$

$$x = \frac{a}{3}$$

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Q If Line $lx+my+n=0$ touches $y^2=4ax$ then $ln = \dots$?

If Line touches Parabola then Combine Eqⁿ के D = 0

$$\left(-\frac{lx-n}{m}\right)^2 = 4ax$$

$$l^2x^2+n^2+2lnx = 4am^2x$$

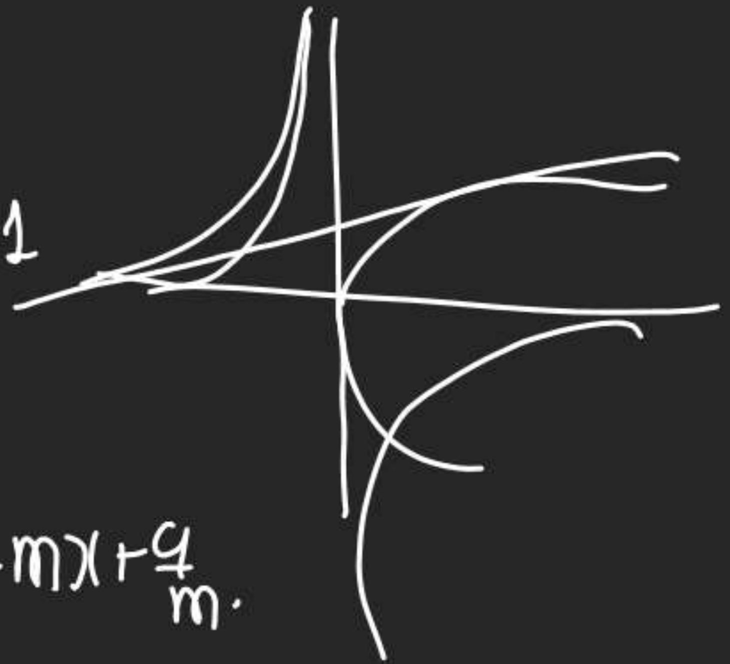
$$l^2x^2+(2ln-4am^2)x+n^2=0$$

D=0 $\quad (2ln-4am^2)^2 = 4l^2n^2$

$$\cancel{4l^2n^2}+16a^2m^4-16alnm^2=\cancel{4l^2n^2}$$

$$am^2(am^2-ln)=0 \Rightarrow \boxed{ln=am^2}$$

Q Eqⁿ of Com tangent to Parabola $y^2=8x$ & $xy=-1$



$\downarrow$

$a=2$

tangent to $y^2=4ax$ is $y=mx+\frac{a}{m}$.

$\therefore$ tangent will be $\rightarrow y=mx+\frac{2}{m}$

$$-\frac{1}{x}=mx+\frac{2}{m}$$

$$\Rightarrow -m = m^2x^2+2x$$

$$m^2x^2+m+2x=0 \rightarrow x \text{ में Quad}$$

D=0 $\quad (2)^2 = 4m^2m \Rightarrow \boxed{m=1}$

$\therefore$ Com. tangent $\rightarrow \boxed{y=x+2}$