Definite Integration. (Extreme Imp. Chapter) $\equiv$ AOD

$20\% + 80\%$
old   New.

$\Leftarrow$ 2Qs $\longrightarrow$
$\Leftarrow$ 3QS $\longrightarrow$

① $\int f(x) \cdot dx = g(x) + C$

then $\int\limits_{a}^{b} f(x) \cdot dx = \left[g(x) + C \right]_{a}^{b}$

$= (g(b) + C) - (g(a) + C)$

$$\boxed{\int\limits_{a}^{b} f(x) \cdot dx = g(b) - g(a)}$$ No. "C"

$\Rightarrow$ So it in definite Int

(2) $\int\limits_{a}^{b} f(x) \cdot dx$ Can be defined only

when $f(x)$ is defined in (a,b)

Ex: $\int\limits_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \tan x \cdot dx = $ Undefined

as. $x = \frac{\pi}{2} \in \left(\frac{\pi}{4}, \frac{3\pi}{4} \right)$
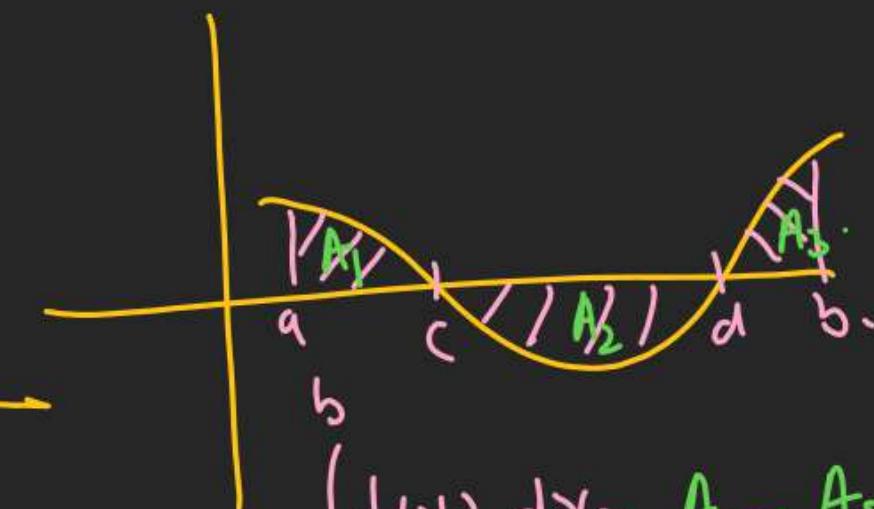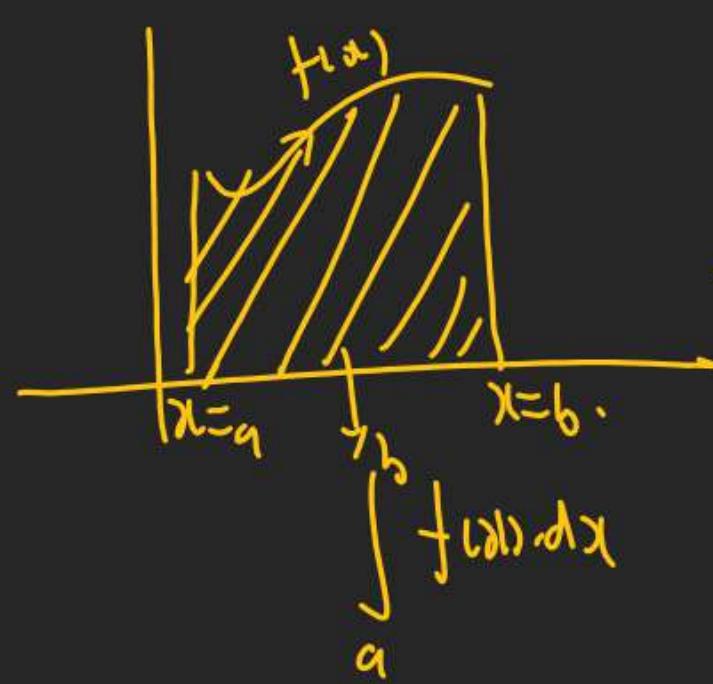
(3) If $f(x)$ is not defined at $x = a$ & $x = b$.
then also $\int\limits_{a}^{b} f(x)$ can be evaluated

4) Geometrical Meaning

$$\int_a^b f(x)dx$$ geometrically Implies.

algebric Sum of areas. enclosed
bet^n Continuous fxn $f(x)$, x(Axis
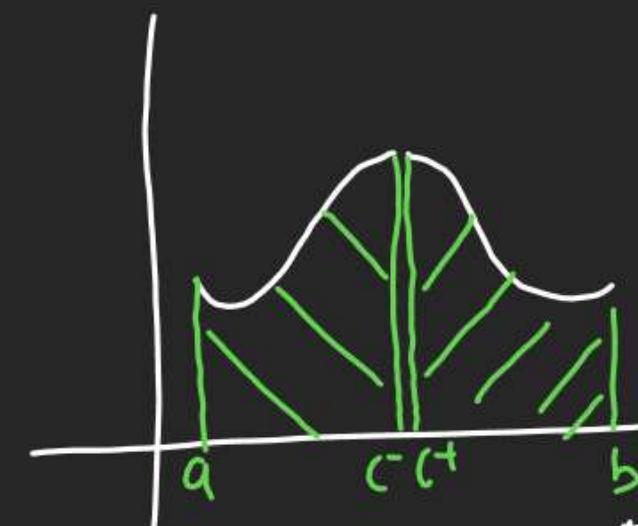
& $x=a$, $x=b$.

$x=a$ $x=b$.

$$\int_a^b f(x)dx$$

$$\int_a^b f(x)\cdot dx = A_1 - A_2 + A_3$$

(5) If fxn is discontinuous at $x=c$ then.

$$\int_a^b f(x)\cdot dx = \int_a^{c^-} f(x)dx + \int_{c^+}^b f(x)dx$$

a $c^- c^+$ b
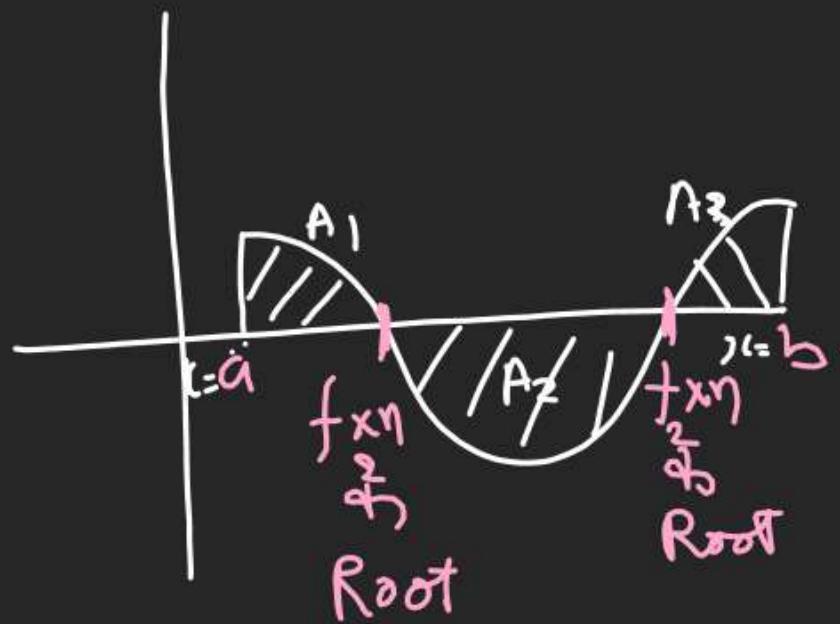
Q $$\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \tan x \cdot dx = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}^-} \tan x \cdot dx + \int_{\frac{\pi}{2}^+}^{\frac{3\pi}{4}} \tan x \cdot dx$$

(6) If $\int_a^b f(x)\cdot dx = 0$ then fxn has at least one

APKI $A_1 - A_2 + A_3 = 0$ ___ Root in (a,b)

NZRme

A_1 A_3

$x = a$ $x = b$

fxn $\frac{2}{3}$ fxn $\frac{2}{3}$

A_2

Root   Root

(7) If $f(x) > 0$ in (a,b) then $\int_a^b f(x)\, dx > 0$

If $f(x) < 0$ in (a,b) then $\int_a^b f(x)\cdot dx < 0 \rightarrow$

if fxn is above x-axis then area will be +ve

(8) $\int_a^b f(x)\cdot d(g(x)) = \int_{g^{-1}(a)}^{g^{-1}(b)} f(x)\cdot g'(x)\cdot dx$

$x = g^{-1}(a) \leftarrow g(x) = a$

$x = g^{-1}(b) \quad g(x) = b$

Q $\int_{1/e}^e x^2 \cdot d(\ln x)$

$\Rightarrow \int_{1/e}^e x^2 \cdot \frac{1}{x}\cdot dx$

$\left[\frac{x^2}{2}\right]_{1/e}^e$

$= \left(\frac{e^2}{2} - \frac{e^{-2}}{2}\right)$

$\dfrac{d(g(x))}{dx} = g'(x)$

$d(g(x)) = g'(x)\cdot dx$

$\dfrac{d(\ln x)}{dx} = \frac{1}{x}$

$d(\ln x) = \frac{1}{x}\cdot dx$

$\ln x = -1 \quad | \quad \ln x = 1$

$\log_e x = -1 \quad | \quad \log_e x = 1$

$x = e^{-1} \quad | \quad x = e^1$

Q. $\int_{-1}^{1} \frac{d}{dx}\left(\cot^{-1}\frac{1}{x}\right)\cdot dx$

Ans

$x \in (-1,1)$

$\frac{1}{x}$ is D.C. at $x=0$

$\int_{-1}^{0^-} \frac{d}{dx}\left(\cot^{-1}\left(\frac{1}{x}\right)\right) dx + \int_{0^+}^{1} \frac{d}{dx}\left(\cot^{-1}\frac{1}{x}\right)dx$

$=\ \left(\cot^{-1}\frac{1}{x}\right)\Big|_{-1}^{0-h} + \left(\cot^{-1}\frac{1}{x}\right)\Big|_{0^+}^{1}$

$\Rightarrow \left\{\cot^{-1}\left(\frac{1}{-h}\right) - \cot^{-1}(-1)\right\} + \left\{\cot^{-1}(1) - \cot^{-1}\left(\frac{1}{h}\right)\right\}$

$\cot^{-1}(-\infty) - \left(\pi - \cot^{-1}(1)\right)\right\} + \left\{\frac{\pi}{4} - \cot^{-1}(+\infty)\right\}$

$\pi - \pi + \frac{\pi}{4} + \frac{\pi}{4} - 0 = \frac{\pi}{2}$

M2 $\int \frac{d}{dx}\left(\cot^{-1}\left(\frac{1}{x}\right)\right)\cdot dx$

$= \left(\cot^{-1}\left(\frac{1}{x}\right)\right)\Big|_{-1}^{1}$

$= \cot^{-1}(1) - \cot^{-1}(-1)$

$= \frac{\pi}{4} - \left(\pi - \cot^{-1}(1)\right)$

$= -\frac{\pi}{2}$

(9) If $g(x)$ is Inverse fxn of $f(x)$ $\{g(f(x))=x\}$ & $f(a)=c$

$f(b)=d$

then $\int_{a}^{b} f(x)\cdot dx + \int_{c}^{d} g(x)\,dx =$ Upr·Upr − Niche Niche

$= bd - ac$

$LHS = \int_{a}^{b} f(x)\cdot dx + \int_{c}^{d} g(f(x))\cdot d(f(x))$

$f(x)=c$
$f(x)=d$

$= \int_{a}^{b} f(x)\cdot dx + \int_{a}^{b} x\cdot f'(x)\,dx$

Multiply → IBP

$= \int_{a}^{b} f(x)\cdot dx + \left[(x\cdot f(x))_{a}^{b} - \int_{a}^{b} 1\cdot f(x)\,dx \right]$

$b\cdot f(b) - a\cdot f(a) = b\cdot d - a\cdot c$

① $\widehat{GT}$ fxn $\int$ me ho ② 2nd, 1st Ka Inverse ho.

then Use Above formula.

Q $\int_{0}^{1} e^{x}\cdot dx + \int_{1}^{e} \ln x\cdot dx = ?$

$\ln x$ is Inverse fxn of e^{x}

$= 1\times e - 0\times 1$

$= e$

Adr (3yr)

Q $\int_{0}^{?} e^{\sqrt{e^{x}}}\,dx + \int_{e}^{?} 2\ln(\ln x)\cdot dx = ?$

$= 1\times e^{\sqrt{e}} - 0\times e = e^{\sqrt{e}}$

$y = e^{\sqrt{e^{x}}} \Rightarrow \log_{e} y = \log_{e} e^{\sqrt{e^{x}}}$

$\ln y = \sqrt{e^{x}}$

$(\ln y)^{2} = e^{x}$

$\ln(\ln y)^{2} = \log_{e} e^{x}$

$x = 2\ln(\ln y)$

$f^{-1}(x) = 2\ln(\ln x)$

Q $\displaystyle\int_0^1 e^{x^2}(x-\alpha)\cdot dx = 0$ then.

A) $\alpha > 0$ ~~B) $0 < \alpha < 1$~~ () $\alpha < 1$ D) $\alpha > 1$

(M1) ~~$\displaystyle\int x\cdot e^{x^2}dx - \alpha\int e^{x^2}dx$~~

Definite Int का Answer $= 0 \Rightarrow$ +ve Area $=$ -ve Area.
NKh1 Graph.

$x = e$

$e^{x^2}(x-\alpha) = 0$

$e^{x^2}\cdot(x-\alpha)$ का
ek Root

$x - \alpha = 0$

$x = \alpha$

$\therefore 0 < \alpha < 1$