Nishant Jindal

Trigonometry

Q. Value of $\sin^6\frac{\pi}{8}+\sin^6\frac{3\pi}{8}+\sin^6\frac{5\pi}{8}+\sin^6\frac{7\pi}{8}=?$

$$\sin^6\frac{\pi}{8}+\sin^6\frac{3\pi}{8}+\left(\sin\left(\frac{4\pi+\pi}{8}\right)\right)^6+\left(\sin\left(\frac{4\pi+3\pi}{8}\right)\right)^6$$

$$\sin^6\frac{\pi}{8}+\sin^6\frac{3\pi}{8}+\left(\sin\left(\frac{\pi}{2}+\frac{\pi}{8}\right)\right)^6+\left(\sin\left(\frac{\pi}{2}+\frac{3\pi}{8}\right)\right)^6$$

$$\sin^6\frac{\pi}{8}+\sin^6\frac{3\pi}{8}+\left(\cos^6\frac{\pi}{8}\right)+\cos^6\frac{3\pi}{8}$$

$$\left(1-3\sin^2\frac{\pi}{8}\cdot\cos^2\frac{\pi}{8}\right)+\left(1-3\sin^2\frac{3\pi}{8}\cdot\cos^2\frac{3\pi}{8}\right)$$

$$\left(1-\frac{3\times 4}{4}\sin^2\frac{\pi}{8}\cdot\cos^2\frac{\pi}{8}\right)+\left(1-\frac{3\times 4}{4}\sin^2\frac{3\pi}{8}\cos^2\frac{3\pi}{8}\right)$$

$$\left(1-\frac{3}{4}\times\left(\sin\frac{\pi}{4}\right)^2\right)+\left(1-\frac{3}{4}\left(\sin\frac{3\pi}{4}\right)^2\right)=\left(1-\frac{3}{4}\left(\frac{1}{\sqrt{2}}\right)^2\right)+\left(1-\frac{3}{4}\left(\frac{1}{\sqrt{2}}\right)^2\right)$$

$$=\left(1-\frac{3}{8}\right)+\left(1-\frac{3}{8}\right)=\frac{5}{8}+\frac{5}{8}=\frac{\cancel{25}}{\cancel{64}}$$

$5/4$

1) $\sin^6\theta+\cos^6\theta$

$=1-3\sin^2\theta\cos^2\theta$

2) $\boxed{4\sin^2\theta\cdot\cos^2\theta}$

$\overline{4}$

$=\frac{1}{4}(2\sin\theta\cos\theta)^2$

$\frac{(\sin 2\theta)^2}{4}$

Nishant Jindal

Trigonometry

Q find $\sin 18^\circ$ or $\cos 18^\circ$

$\theta = 18^\circ \rightarrow$ 1st Quadrant

1) $\theta = 18^\circ \rightarrow 2\theta = 36^\circ$, $3\theta = 54^\circ$

2) $2\theta + 3\theta = 90^\circ$

$2\theta = 90^\circ - 3\theta$

$\sin(2\theta) = \sin(90 - 3\theta)$

$2\sin\theta\cos\theta = \cos 3\theta$

$2\sin\theta\cos\theta = 4\cos^3\theta - 3\cos\theta$

$2\sin\theta = 4\cos^2\theta - 3$

$2\sin\theta = 4(1 - \sin^2\theta) - 3$

$2\sin\theta = 1 - 4\sin^2\theta$

$4\sin^2\theta + 2\sin\theta - 1 = 0$

$$\sin\theta = \frac{-2 \pm \sqrt{4 - 4\times4\times-1}}{2\times4}$$

$$\sin(18^\circ) = \frac{-2 \pm 2\sqrt{5}}{2\times4} \rightarrow \frac{-1+\sqrt{5}}{4} \checkmark, \quad \frac{-1-\sqrt{5}}{4} = -ve \times$$

$+$

$$\boxed{\sin 18^\circ = \frac{-1+\sqrt{5}}{4} = \cos 72^\circ}$$

Trigonometry

$\theta = 36^\circ$

① We know that $\sin 18^\circ = \dfrac{-1+\sqrt{5}}{4}$

② $\cos 2\theta = 1 - 2\sin^2\theta$

$$\cos 36^\circ = 1 - 2\sin^2 18^\circ$$

$$= 1 - 2\times\left(\frac{\sqrt{5}-1}{4}\right)^2$$

$$= 1 - \cancel{2}\times\frac{(5+1-2\sqrt{5})}{\cancel{16}\,8} = \frac{8-6+2\sqrt{5}}{8} = \frac{2+2\sqrt{5}}{8} = \frac{\sqrt{5}+1}{4}$$

$$\sin 36^\circ = \sqrt{1-\cos^2(36^\circ)}$$

$$= \sqrt{1-\left(\frac{\sqrt{5}+1}{4}\right)^2}$$

$$= \frac{\sqrt{10-2\sqrt{5}}}{4}$$

$$\boxed{\cos 36^\circ = \frac{\sqrt{5}+1}{4} = \sin 54^\circ}$$

$\sin 18^\circ$

$\sin 36^\circ$

$$\cos 18^\circ = \sqrt{1-\sin^2 18}$$

$$= \sqrt{1-\left(\frac{\sqrt{5}-1}{4}\right)^2}$$

$$= \sqrt{\frac{16-(5+1-2\sqrt{5})}{16}}$$

$$= \sqrt{\frac{10+2\sqrt{5}}{16}}$$

$$\cos 18^\circ = \frac{\sqrt{10+2\sqrt{5}}}{4} = \sin 72^\circ$$

Nishant Jindal

Trigonometry

$\cos 72^\circ$	$\sin 54^\circ$	$\cos 54^\circ$	$\sin 72^\circ$
$\sin 18^\circ$	$\cos 36^\circ$	$\sin 36^\circ$	$\cos 18^\circ$
$\frac{\sqrt{5}-1}{4}$	$\frac{\sqrt{5}+1}{4}$	$\frac{\sqrt{10-2\sqrt{5}}}{4}$	$\frac{\sqrt{10+2\sqrt{5}}}{4}$

$$\frac{16}{5-1} = \frac{16}{4} = 4$$

Q $\tan 9^\circ - \tan 27^\circ - \tan 63^\circ + \tan 81^\circ = ?$

$$\tan 9^\circ - \tan 27^\circ - \cot 27^\circ + \cot 9^\circ$$

$$\tan 9^\circ + \cot 9^\circ - (\tan 27^\circ + \cot 27^\circ)$$

$$\left(\frac{\sin 9^\circ}{\cos 9^\circ} + \frac{\cos 9^\circ}{\sin 9^\circ}\right) - \left(\frac{\sin 27^\circ}{\cos 27^\circ} + \frac{\cos 27^\circ}{\sin 27^\circ}\right)$$

$$\left(\frac{\sin^2 9^\circ + \cos^2 9^\circ}{\sin 9^\circ \cdot \cos 9^\circ}\right) - \left(\frac{\sin^2 27^\circ + \cos^2 27^\circ}{\sin 27^\circ \cdot \cos 27^\circ}\right)$$

$$\left(\frac{2}{2\sin 9^\circ \cos 9^\circ}\right) - \left(\frac{2}{2\sin 27^\circ \cos 27^\circ}\right)$$

$$\left(\frac{2}{\sin 18^\circ}\right) - \left(\frac{2}{\sin 54^\circ}\right) = \frac{2}{\sin 18^\circ} - \frac{2}{\cos 36^\circ}$$

$$= \frac{2\times 4}{\sqrt{5}-1} - \frac{2\times 4}{\sqrt{5}+1} = 8\left(\frac{\cancel{\sqrt{5}}+1-(\cancel{\sqrt{5}}-1)}{(\sqrt{5}-1)(\sqrt{5}+1)}\right)$$

Nishant Jindal

Trigonometry

$\cos 72° = $

$\sin 18°$	$\cos 36°$	$\cos 18°$	$\sin 36°$
$\frac{\sqrt{5}-1}{4}$	$\frac{\sqrt{5}+1}{4}$	$\frac{\sqrt{10+2\sqrt{5}}}{4}$	$\frac{\sqrt{10-2\sqrt{5}}}{4}$

Q $\sin^2 24° - \sin^2 6° = ?$

$\sin^2 A - \sin^2 B = \sin(A+B)\sin(A-B)$

$\sin(24+6)\cdot\sin(24-6)$

$\sin 30 \cdot \sin 18°$

$\frac{1}{2} \times \frac{\sqrt{5}-1}{4} = \frac{\sqrt{5}-1}{8}$

Q $\cos\frac{2\pi}{15}\cdot\cos\frac{4\pi}{15}\cdot\cos\frac{8\pi}{15}\cdot\cos\frac{14\pi}{15} = ?$

$2\pi =$

$\cos 24°\cdot\cos 48°\cdot\cos 96°\ \underline{\cos 168°}$

$\cos 24°\cdot\cos 48°\cdot\cos 96°\ \cos(\pi - 12°)$

$-\cos 24°\cdot\cos 48°\cdot\cos 96°\cdot\cos 12°$

$-\frac{1}{4}\left[(2\cos 24°\cdot\cos 96°)\cdot(2\cos 48°\cdot\cos 12°)\right]$

$-\frac{1}{4}\left[\{\cos(120°)+\cos(72°)\}\{\cos(60°)+\cos(36°)\}\right]$

$-\frac{1}{4}\left[\left\{-\frac{1}{2}+\frac{\sqrt{5}-1}{4}\right\}\left\{\frac{1}{2}+\frac{\sqrt{5}+1}{4}\right\}\right]$

$-\frac{1}{4}\left[\frac{(-4+2\sqrt{5}-2)}{8}\times\frac{(4+2\sqrt{5}+2)}{8}\right] = -\frac{1}{4}\left[\frac{(2\sqrt{5}-6)(2\sqrt{5}+6)}{64}\right]$

$= +\frac{1}{4}\left[\frac{+16}{64}\right] = \frac{1}{16}$

Nishant Jindal

Trigonometry

$\sin 18^\circ$	$\cos 36^\circ$	$\cos 18^\circ$	$\sin 36^\circ$
$\frac{\sqrt{5}-1}{4}$	$\frac{\sqrt{5}+1}{4}$	$\frac{\sqrt{10+2\sqrt{5}}}{4}$	$\frac{\sqrt{10-2\sqrt{5}}}{4}$

Q $\sin^2 48^\circ - \cos^2 12^\circ = ?$

$$= -\left(\cos^2 \overset{B}{12^\circ} - \sin^2 \overset{A}{48^\circ}\right)$$

$$= -\left(\cos(12^\circ+48^\circ) * \cos(48-12^\circ)\right\}$$

$$= -\cos 60^\circ \times \underline{\cos(36^\circ)}$$

$$= -\frac{1}{2} \times \frac{\sqrt{5}+1}{4} = \frac{-(\sqrt{5}+1)}{8}$$

$$\cos(A+B)\cdot\cos(A-B)$$

$$= \cos^2 A \cdot \cos^2 B - \sin^2 A \sin^2 B$$

$$= \cos^2 A\,(1-\sin^2 B) - (1-\cos^2 A)\sin^2 B$$

$$= \cos^2 A - \cancel{\cos^2 A \sin^2 B} - \sin^2 B + \cancel{\cos^2 A \sin^2 B}$$

$$= \cos^2 A - \sin^2 B$$

$$= (1 - s^2 A) - (1 - c^2 B)$$

$$\cos^2 B - \sin^2 A$$

Nishant Jindal

Trigonometry

$\frac{\pi}{5} = \frac{180}{5} = 36.$

Q $\cos 12^\circ + \cos 84^\circ + \cos 156^\circ + \cos 132^\circ = ?$

$(\cos 12^\circ + \cos 132^\circ) + (\cos 84^\circ + \cos 156^\circ)$

$2\cos(72)\cos(+60) + 2\cos(120^\circ)\cos\left(\frac{+\cancel{72}}{\cancel{2}}\right)$ 36.

$\cancel{2} \times \frac{1}{\cancel{2}} \cdot \sin 18^\circ + \cancel{2} \times -\frac{1}{\cancel{2}} \times \cos 36^\circ$

$\frac{\sqrt{5}-1}{4} - \left(\frac{\sqrt{5}+1}{4}\right)$

$= \frac{\cancel{\sqrt{5}} - 1 - \cancel{\sqrt{5}} - 1}{4} = -\frac{2}{4} = -\frac{1}{2}$

Q $\sin \frac{\pi}{5} \cdot \sin \frac{4\pi}{5} = ?$

$\sin 36^\circ \cdot \sin 144^\circ$

$\sin 36^\circ \cdot \sin(180^\circ - 36^\circ)$

$\sin 36^\circ \cdot \sin 36^\circ$

$(\sin 36)^2 = \left(\frac{\sqrt{10-2\sqrt{5}}}{4}\right)^2$

$= \frac{10-2\sqrt{5}}{16} = \frac{5-\sqrt{5}}{8}$

Nishant Jindal

Trigonometry

Q $\cos\frac{\pi}{5}\sin\frac{4\pi}{5} = ?$

$\cos 36^\circ \cdot \sin 144^\circ$

$\cos 36^\circ \cdot \sin(180 - 36^\circ)$

$\cos 36^\circ \cdot \sin 36^\circ$

$\frac{2\sin 36^\circ \cdot \cos 36^\circ}{2}$

$= \frac{\sin 72^\circ}{2} = \frac{\cos 18^\circ}{2} = \frac{\sqrt{10+2\sqrt5}}{4\times 2}$

Q $\cos^2\left(\frac{3\pi}{5}\right) + \cos^2\left(\frac{4\pi}{5}\right) = ?$

$1 - \sin^2\frac{3\pi}{5} + \cos^2\frac{4\pi}{5}$

$\Rightarrow 1 + \left(\cos^2\left(\overset{B}{\frac{4\pi}{5}}\right) - \sin^2\left(\overset{A}{\frac{3\pi}{5}}\right)\right)$

$\Rightarrow 1 + \cos\left(\frac{7\pi}{5}\right)\cdot\cos\left(-\frac{\pi}{5}\right)$

$\Rightarrow 1 + \cos\left(\frac{5\pi + 2\pi}{5}\right)\cdot\cos\left(\frac{\pi}{5}\right)$

$\Rightarrow 1 + \cos\left(\pi + \frac{2\pi}{5}\right)\cdot\cos\left(\frac{\pi}{5}\right)$

$\Rightarrow 1 + -\cos\frac{2\pi}{5}\cdot\cos\frac{\pi}{5} = 1 - \cos 36^\circ \cdot \cos 72^\circ$

$= 1 - \cos 36^\circ \cdot \sin 18^\circ$

$\cos^2 B - \sin^2 A$

$= \cos(A+B)\cdot\cos(A-B)$

$1 - \left(\frac{\sqrt5 - 1}{4}\right)\left(\frac{\sqrt5 + 1}{4}\right)$

$1 - \frac{4}{4\times 4} = \frac{3}{4}$

Nishant Jindal

Trigonometry

Continued Product of Cosine & Sine Series

Q $\cos 6^\circ \cdot \cos 12^\circ \cdot \cos 24^\circ \cdot \cos 48^\circ$

$$\frac{(2\sin 6^\circ \cos 6^\circ) \cdot \cos 12^\circ \cdot \cos 24^\circ \cdot \cos 48^\circ}{2\sin 6^\circ}$$

$$\frac{(2\sin 12^\circ \cdot \cos 12^\circ) \cdot \cos 24^\circ \cdot \cos 48^\circ}{2 \times 2\sin 6^\circ}$$

$$\frac{(2\sin 24^\circ \cdot \cos 24^\circ) \cdot \cos 48^\circ}{2 \times 4 \cdot \sin 6^\circ}$$

$$\frac{(2\sin 48^\circ \cdot \cos 48^\circ)}{2 \times 8 \sin 6^\circ} = \frac{\sin 96^\circ}{16 \cdot \sin 6^\circ} = \frac{\sin(90+6^\circ)}{16 \cdot \sin 6^\circ} = \frac{\cos 6^\circ}{16 \cdot \sin 6^\circ} = \frac{\cot 6^\circ}{16}$$

$\cos(\theta) \cdot \cos(2\theta) \cdot (\cos 2^2\theta)(\cos 2^3\theta) \ldots (\cos 2^{n-1}\theta)$

$$= \frac{\sin(2 \times L.A)}{2^{\text{no of term}} \cdot \sin(S.A)}$$

Q $\cos \boxed{6^\circ} \cos 12^\circ \cdot \cos 24 \cdot \cos \boxed{48^\circ}$

$$= \frac{\sin(2 \times 48^\circ)}{2^4 \cdot \sin(6)} = \frac{\sin 96^\circ}{2^4 \cdot \sin 6^\circ} = \frac{\cot 6^\circ}{2^4}$$

Nishant Jindal

Trigonometry

Q $\cos\frac{2\pi}{15}\cdot\cos\left(\frac{4\pi}{15}\right)\cdot\cos\left(\frac{8\pi}{15}\right)\cdot\cos\left(\frac{16\pi}{15}\right)=?$

$$\frac{\sin\left(2\times\frac{16\pi}{15}\right)}{2^4\sin\left(\frac{2\pi}{15}\right)}=\frac{\sin\left(\frac{32\pi}{15}\right)}{16\cdot\sin\left(\frac{2\pi}{15}\right)}$$

$$\frac{\sin\left(\frac{30\pi+2\pi}{15}\right)}{16\cdot\sin\left(\frac{2\pi}{15}\right)}=\frac{\sin\left(2\pi+\frac{2\pi}{15}\right)}{16\cdot\sin\left(\frac{2\pi}{15}\right)}$$

$$=\frac{\cancel{\sin\frac{2\pi}{15}}}{16\cdot\cancel{\sin\frac{2\pi}{15}}}=\frac{1}{16}$$

Sheet

Ex 1 (20)

Q1 If $\sin x+\sin^2 x=1$ then $\cos^2 x+\cos^4 x=?$