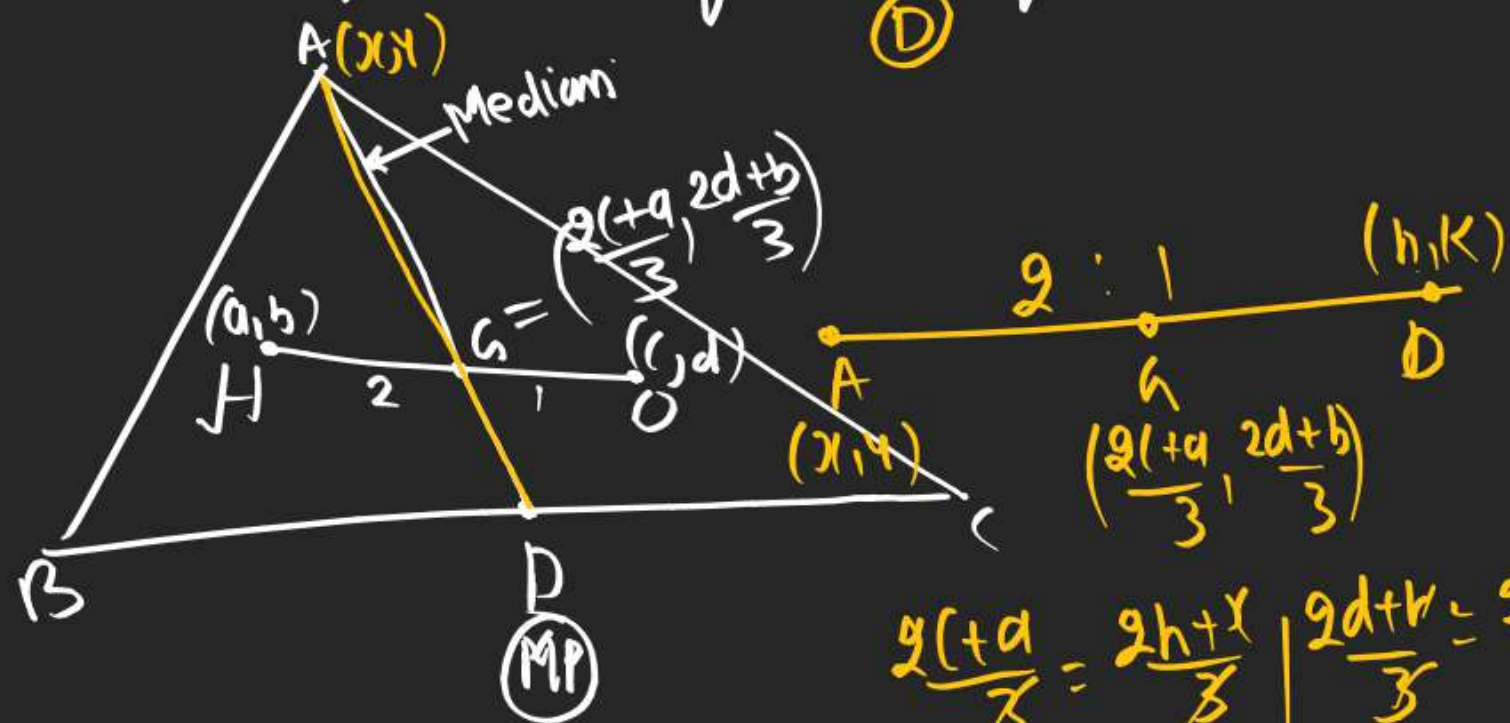


Q3 Orthocentre & Circumcentre of ΔABC are $(g, b), (c, d)$. If (oord. of vertex A are (x, y) . Find (oord. of Mid Pt. of BC) (D)



$$\frac{2g}{3} = \frac{2h+x}{3} \mid \frac{2d}{3} = \frac{2k+y}{3}$$

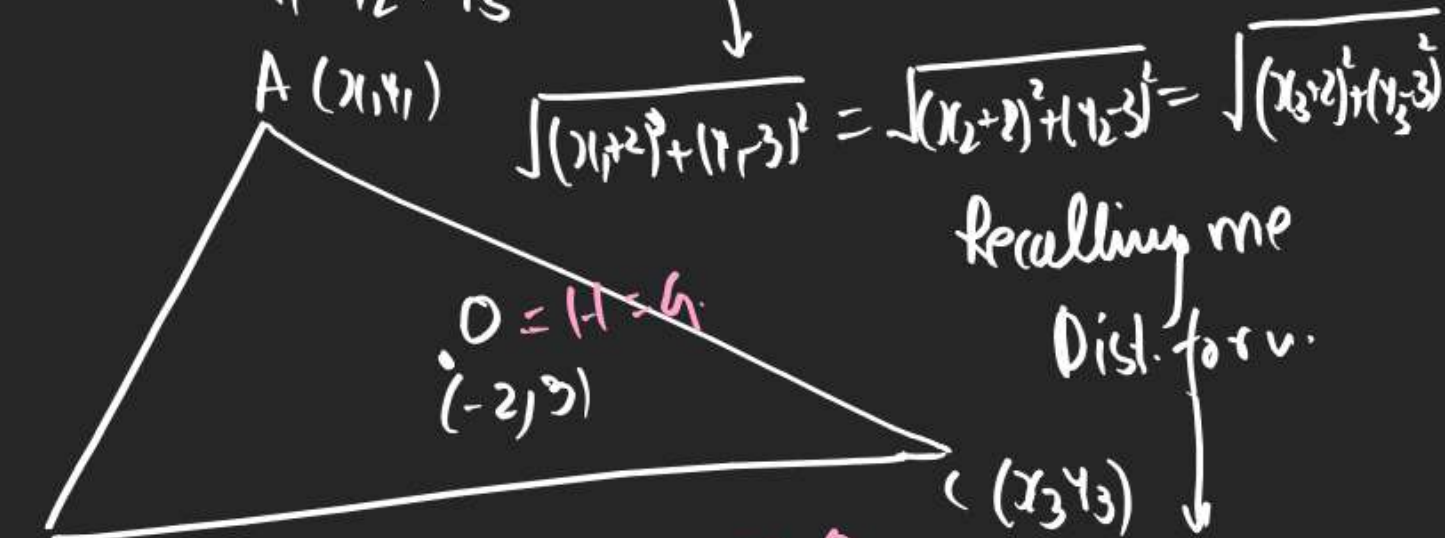
$$h = \frac{2g+x}{2} \text{ \& } k = \frac{2d+y}{2}$$

$$\therefore D = \left(\frac{2g+x}{2}, \frac{2d+y}{2} \right)$$

Q If $(x_i, y_i); i=1, 2, 3$ are vertices of Δ S.T.

$$(x_1+2)^2 + (y_1-3)^2 = (x_2+2)^2 + (y_2-3)^2 = (x_3+2)^2 + (y_3-3)^2$$

then $\frac{x_1+x_2+x_3}{y_1+y_2+y_3} = ?$



$$\sqrt{(x_1+2)^2 + (y_1-3)^2} = \sqrt{(x_2+2)^2 + (y_2-3)^2} = \sqrt{(x_3+2)^2 + (y_3-3)^2}$$

Recalling the Dist. formula

$$H = \left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3} \right) = (-2, 3)$$

$$\Rightarrow \frac{x_1+x_2+x_3}{y_1+y_2+y_3} = -\frac{2 \times 3}{3 \times 3} = -\frac{6}{9} = -\frac{2}{3}$$

Ratio = $-\frac{2}{3}$

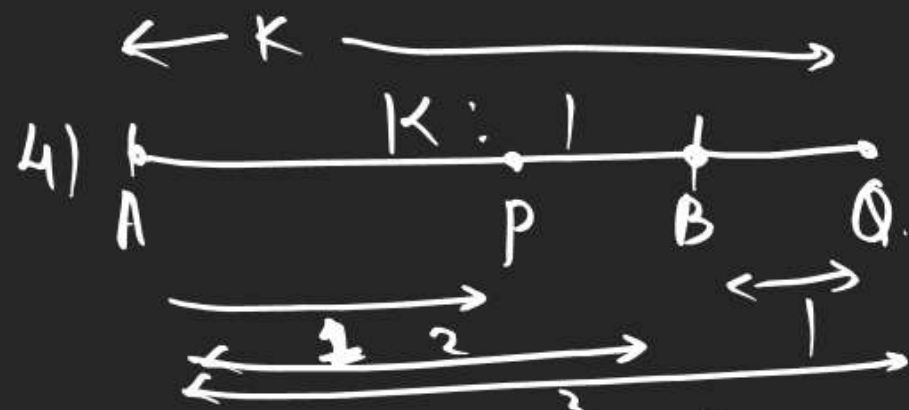
$(x_1, y_1), (x_2, y_2), (x_3, y_3)$ are all at $(-2, 3)$ & dist. dist. dist. $\Rightarrow (-2, 3)$ is circumcentre.

Harmonic Conjugate

① Here we are talking about 2 pts one which is dividing Internally & other dividing Externally.

(2) Both Pts are dividing in Same Ratio

(3) So if 2 pts P & Q divides line joining A & B in same Ratio Internally & Externally then P & Q will be called Harmonic conjugate to each other.



$$\frac{AP}{PB} = \frac{K}{1}$$

(Internal Div.)

$$\frac{AQ}{QB} = \frac{K}{1}$$

(External Div.)

$\therefore P$ & Q are H.C. to each other.

Proof $\frac{AP}{PB} = \frac{AQ}{QB} \Rightarrow \frac{AP}{AB - AP} = \frac{AQ}{AQ - AB}$

Reciprocal $\frac{AB - AP}{AP} = \frac{AQ - AB}{AQ} \Rightarrow \frac{AB}{AP} - 1 = 1 - \frac{AB}{AQ}$

$$AB \left(\frac{1}{AP} + \frac{1}{AQ} \right) = 2 \Rightarrow \frac{1}{AP} + \frac{1}{AQ} = \frac{2}{AB} \rightarrow H.P$$

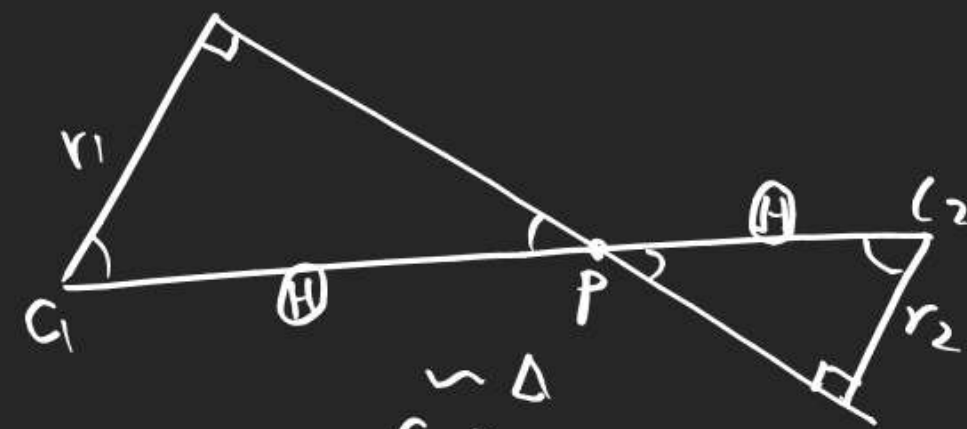
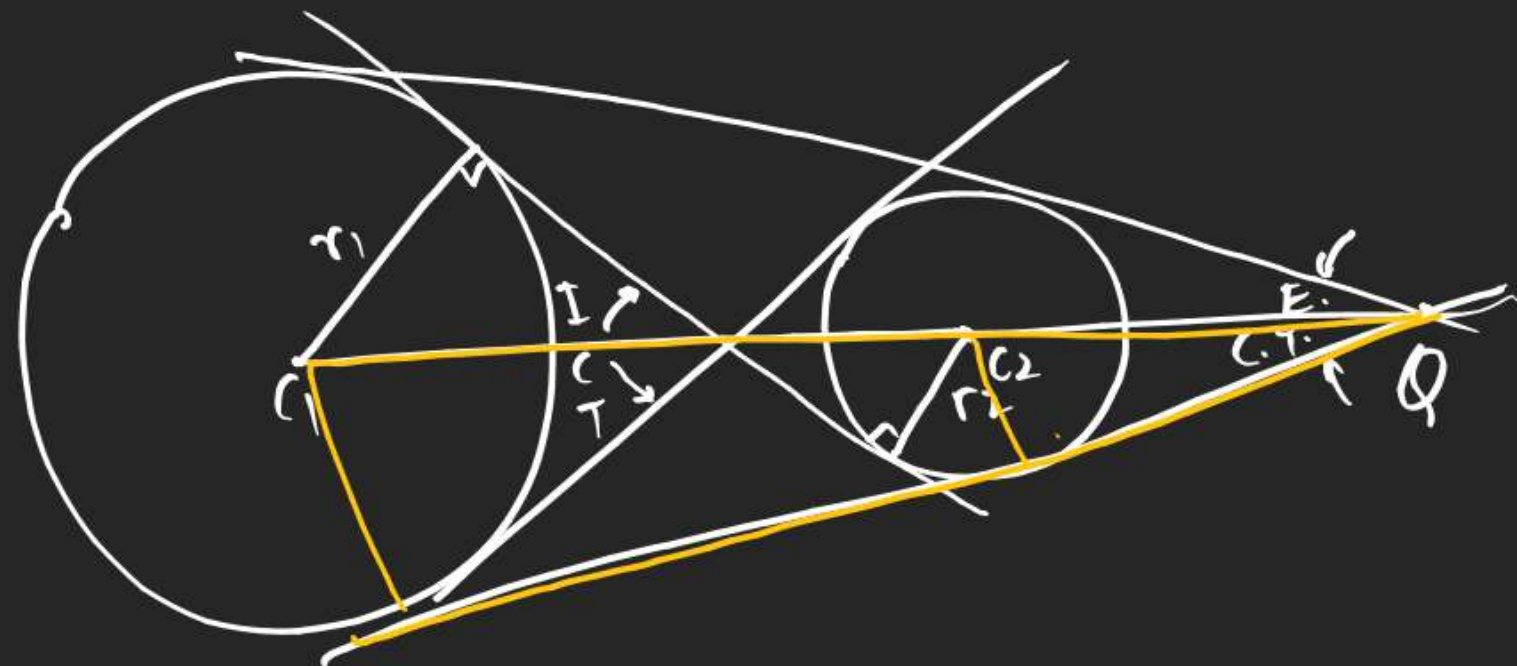
$$\frac{1}{Ch} + \frac{1}{Bch} = \frac{2}{Med.}$$

2 Imp Exp. of H.C.

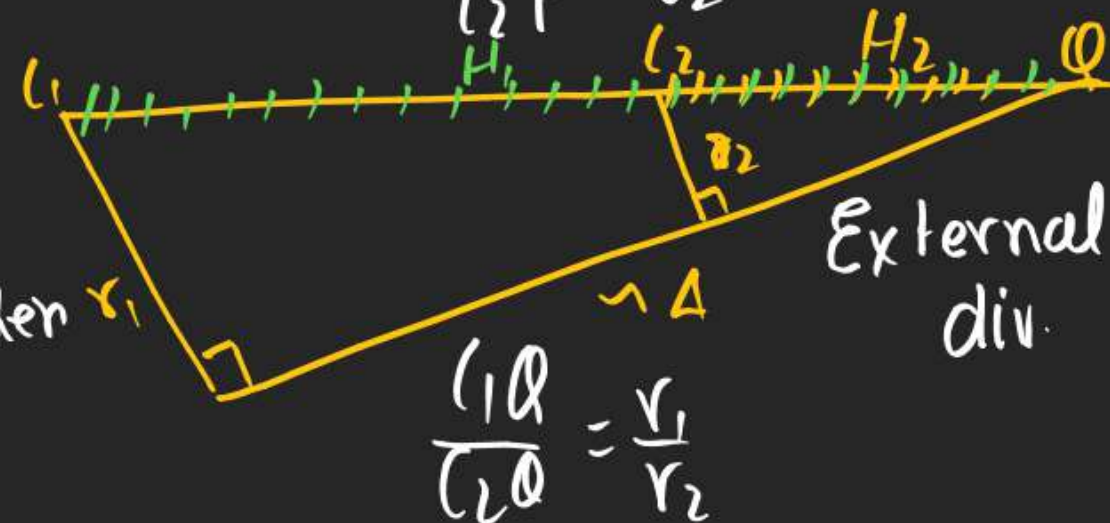
① Internal & External com. tangents of 2 circles.

② Internal & External Angle Bisector.

External & Int. com. tangents dividing the line joining Centre of 2 circles Externally & Internally in Ratio of their Radii.



$$\frac{C_1P}{C_2P} = \frac{r_1}{r_2} \quad \text{Int. div.}$$

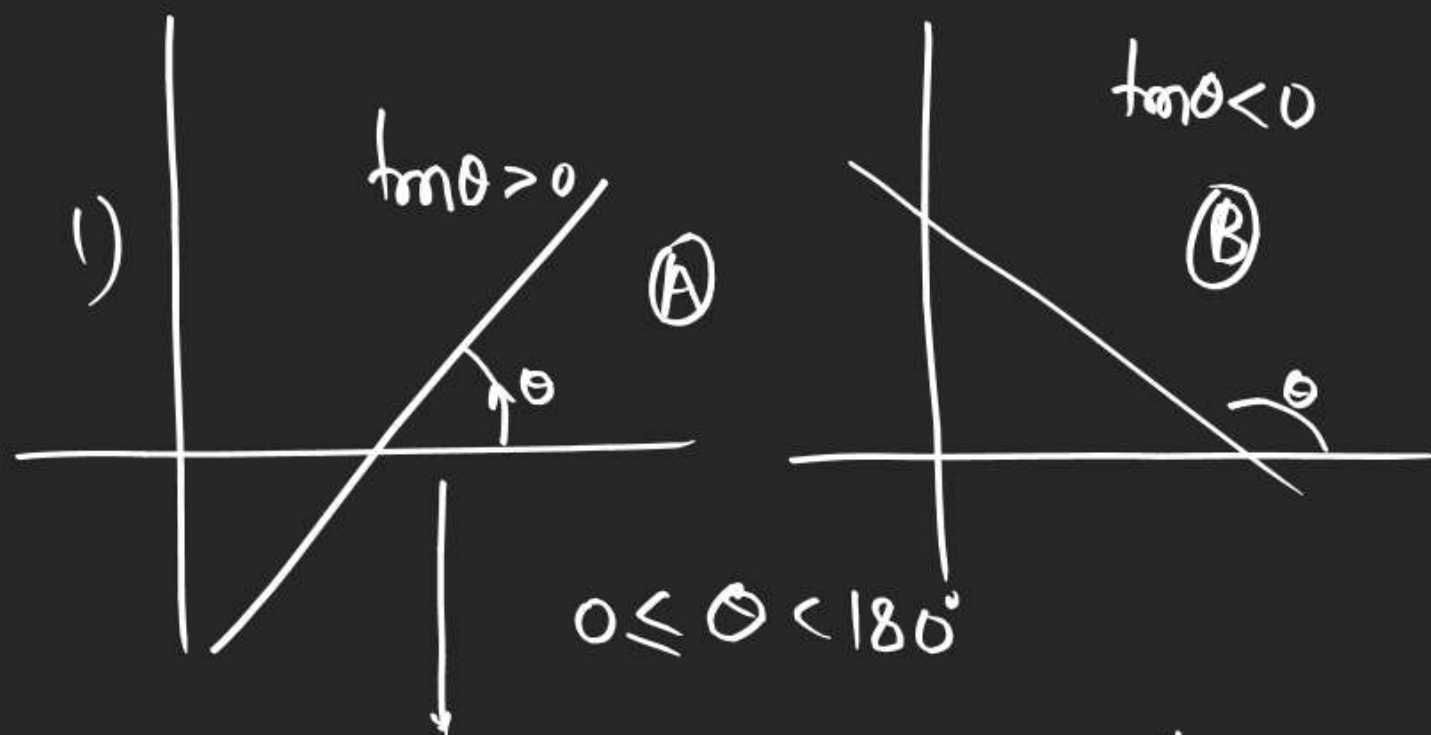


As Ratio for.
P & Q in $\frac{r_1}{r_2}$

\therefore P, Q are H.C. to each other

$$\frac{C_1Q}{C_2Q} = \frac{r_1}{r_2}$$

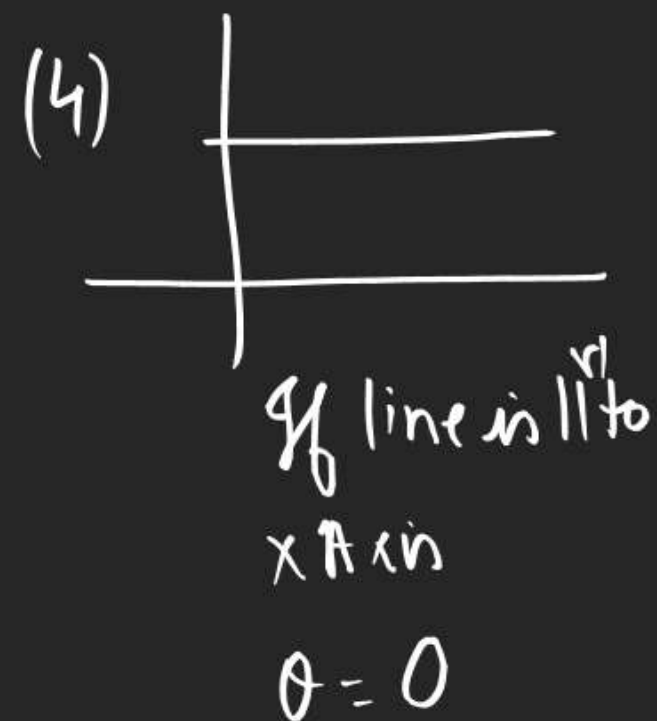
Slope of Line.



(2) $\tan \theta$ is Slope of Line & θ is Inclination.

(3) $\tan \theta > 0$ in (A) as θ is acute Angle $\theta \in (0, 90^\circ)$ $\therefore \tan \theta > 0$

& $\tan \theta < 0$ in (B) θ is obtuse Angle $\theta \in (90, 180)$ $\therefore \tan \theta < 0$



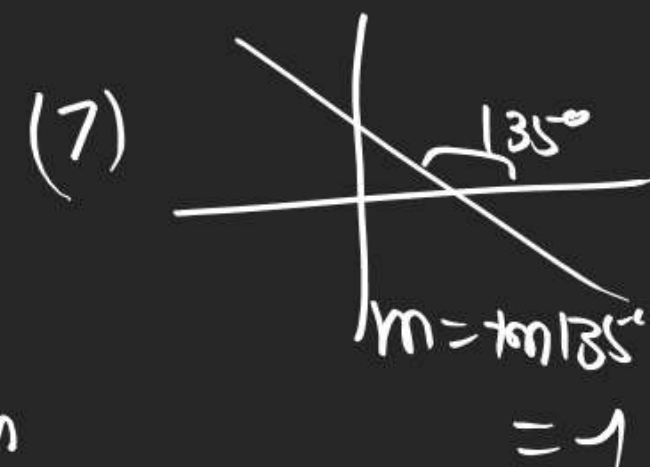
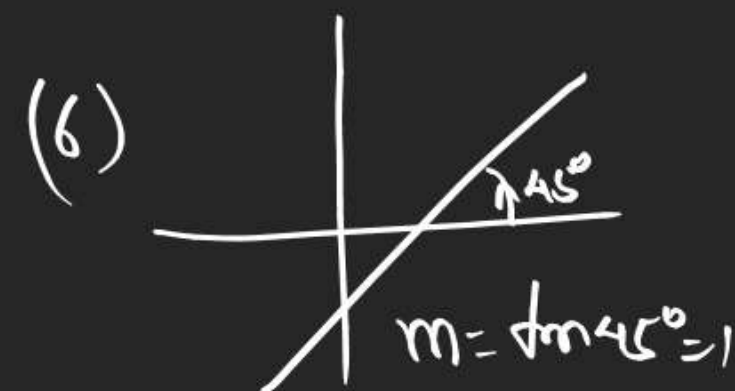
Slope: $m = \tan \theta = 0$

(5) If line is \perp to X Axis



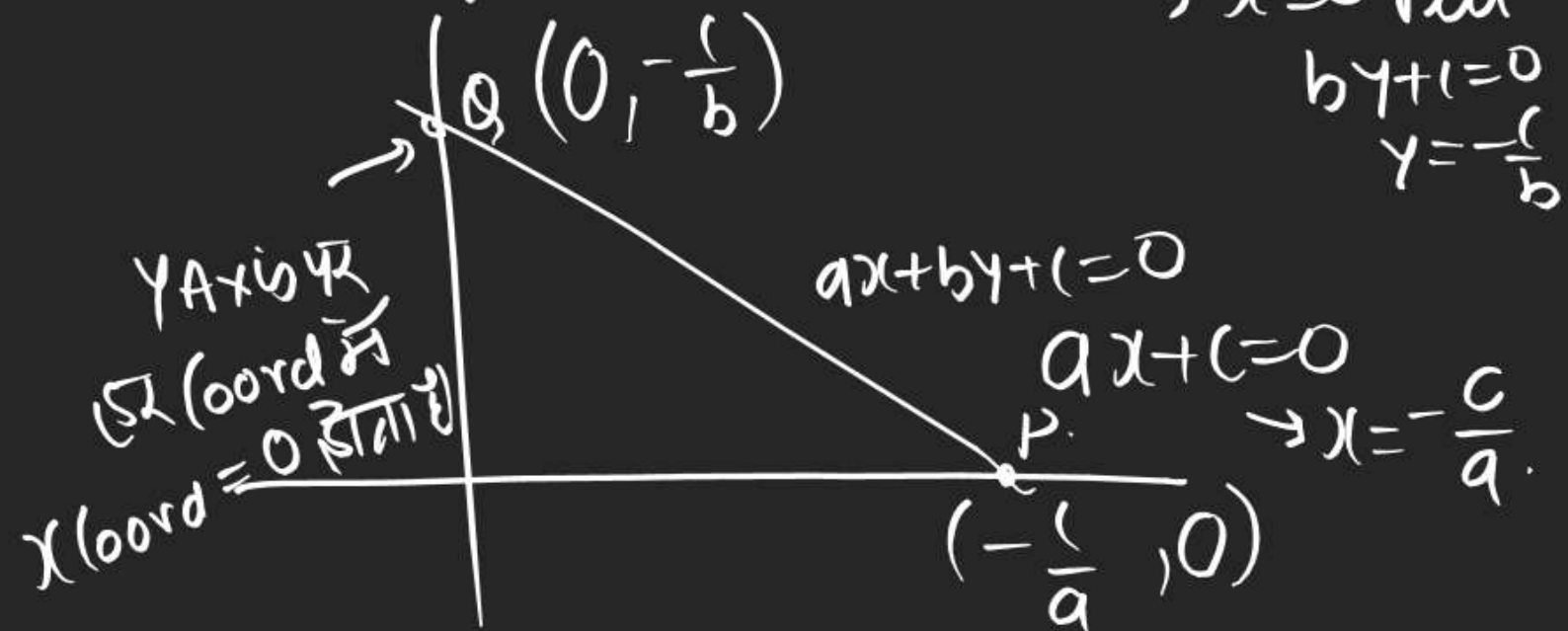
Slope: $m = \tan 90^\circ \rightarrow \infty$ Undefined.

\therefore for line \perp to X Axis Slope is called undefined



Slope $m = \frac{y_2 - y_1}{x_2 - x_1}$

(9) If Eqⁿ of line is $ax+by+c=0$ \rightarrow $x=0$ Put
 $by+c=0$
 $y=-\frac{c}{b}$



$$P = (-\frac{c}{a}, 0) \text{ \& } Q = (0, -\frac{c}{b})$$



$$m_{PQ} = \frac{-\frac{c}{b} - 0}{0 + (-\frac{c}{a})} = -\frac{c}{b} \times \frac{a}{c} = -\frac{a}{b}$$

(10) So Slope of line $ax+by+c=0$ is $-\frac{a}{b}$.

$$m_{\text{Line}} = -\frac{(\text{off of } x)}{(\text{off of } y)}$$

Q. $3x-2y+5=0$ find Slope of line

$$m = -\frac{3}{-2}$$

$$x \text{ off} = 3$$

$$y \text{ off} = -2$$

$$m = \frac{3}{2}$$

RK 1 If 2 lines are \parallel^r then their slope is same

$$m_1 = m_2$$

(2) If 2 line are \perp^r then $m_1 \times m_2 = -1$ (Psbl)

Q If line $3x - ay - 1 = 0$ is \perp^r to $(a+2)x - y + 3 = 0$ find a ?

$$m_1 = m_2 \text{ Hoga. } \left| \begin{array}{l} 3x - ay - 1 = 0 \rightarrow m_1 = -\frac{3}{-a} = \frac{3}{a} \\ (a+2)x - y + 3 = 0 \rightarrow m_2 = +\frac{(a+2)}{+1} \end{array} \right.$$

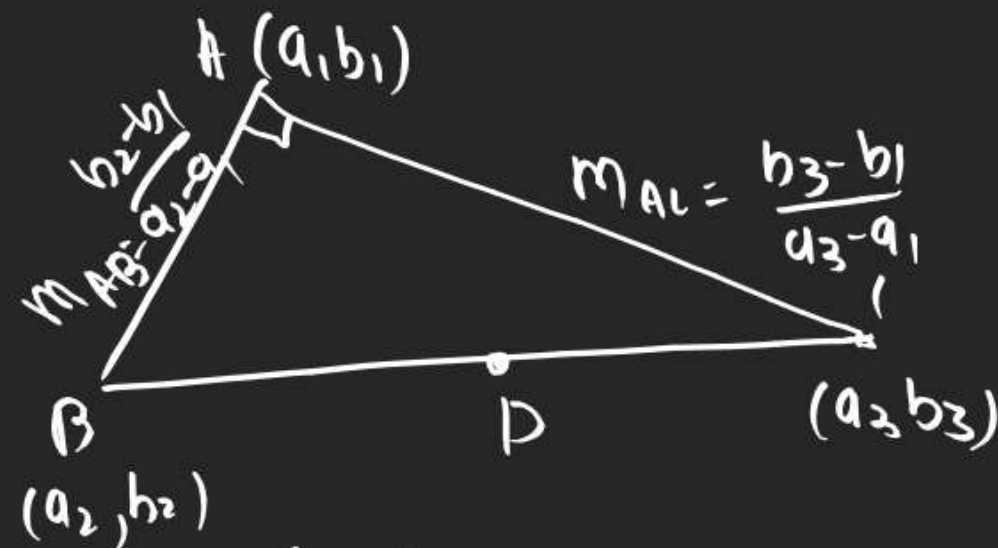
$$\frac{3}{a} = a+2$$

$$\Rightarrow a^2 + 2a - 3 = 0 \Rightarrow a^2 + 2a - 3 = 0$$

$$\Rightarrow (a+3)(a-1) = 0$$

$$\Rightarrow \underline{a = 1, -3}$$

Q If $(b_2 - b_1)(b_3 - b_1) + (a_2 - a_1)(a_3 - a_1) = 0$ then Show that circumcentre of Δ having vertices $(a_1, b_1), (a_2, b_2), (a_3, b_3)$ is $\left(\frac{a_2 + a_3}{2}, \frac{b_2 + b_3}{2}\right)$



$$\text{and} \rightarrow (b_2 - b_1)(b_3 - b_1) = -(a_2 - a_1)(a_3 - a_1)$$

$$\frac{(b_2 - b_1)}{(a_2 - a_1)} \cdot \frac{(b_3 - b_1)}{(a_3 - a_1)} = -1$$

$$m_{AB} m_{AC} = -1 \Rightarrow AB \perp AC$$

$\Rightarrow \Delta ABC$ is a right-angled Δ at A

Rt. angle Δ is a right-angled Δ with the orthocentre at the midpoint of the hypotenuse.

$$\therefore \text{Circumcentre} = D = \left(\frac{a_2 + a_3}{2}, \frac{b_2 + b_3}{2}\right)$$

LOCUS

A) Set of Pt. Which follow Q's (md^m)

B) Steps.

A) first assumption In whose locus is to be known.

let (h, k)

(B) Now follow Q's (md^m)

(C) after solving $(h \rightarrow x, k \rightarrow y)$

$\left. \begin{array}{l} \leftarrow k \\ 61 \rightarrow 66 \\ 67-69 \end{array} \right\} \underline{\underline{\text{done}}}$