

$$t_1 t_2 = -1$$

$$\textcircled{1} - y t_1 = x + a + a t_1^2$$

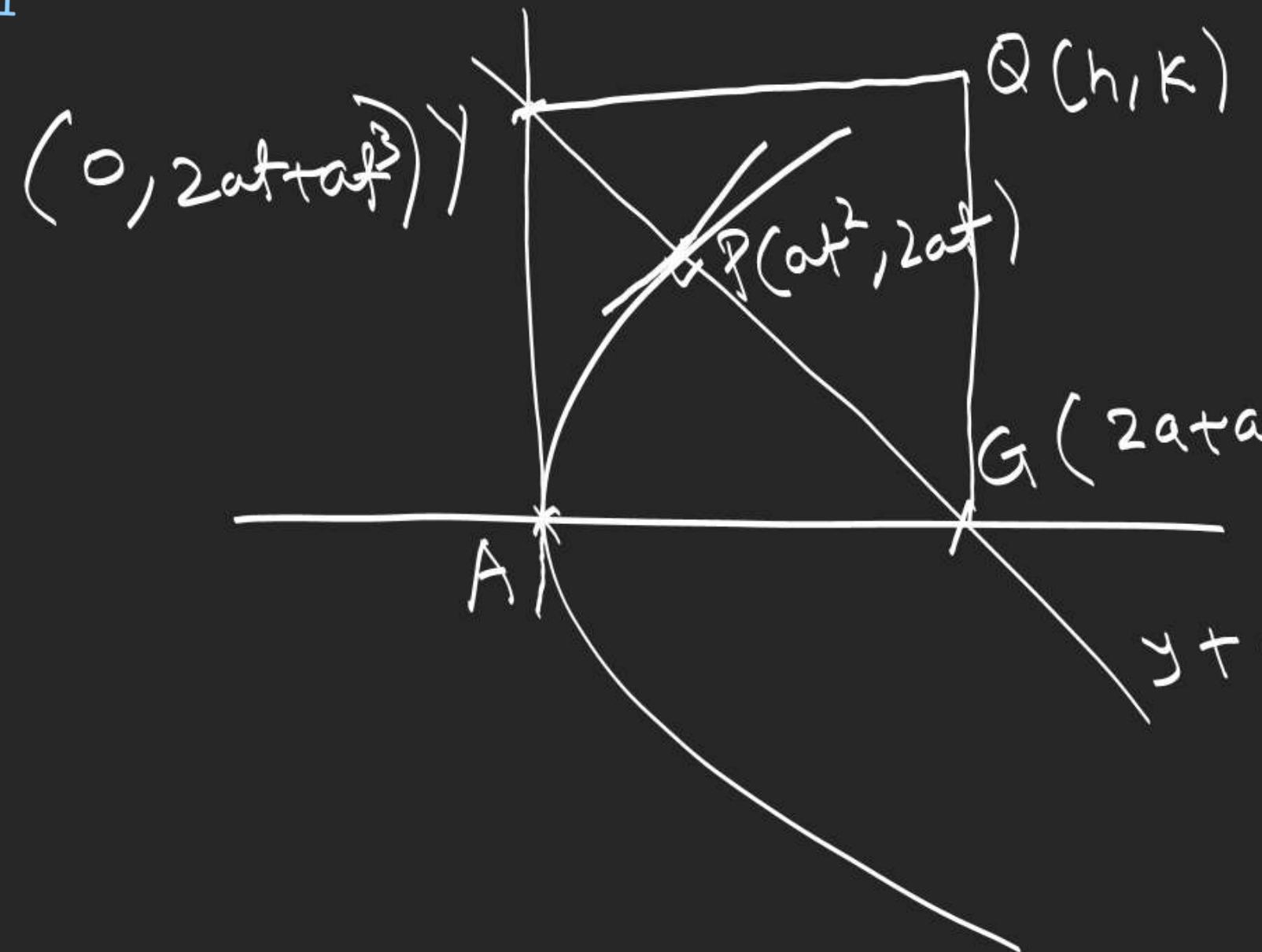
$$\textcircled{2} - y t_2 = x + a' + a' t_2^2$$

$$\textcircled{1} \times t_2 - \textcircled{2} \times t_1$$

$$0 = x(t_2 - t_1) + (a t_2 - a' t_1) + (-a t_1 + a' t_2)$$

$$y^2 = \underbrace{q(a-a')}_{\substack{(t_2-t_1)(x+a+a')}} = 0$$

$$y^2 = \underbrace{q'(a)}_{\substack{(-a)}} \quad \boxed{x = -a - a'}$$



$$h = 2a + at^2$$

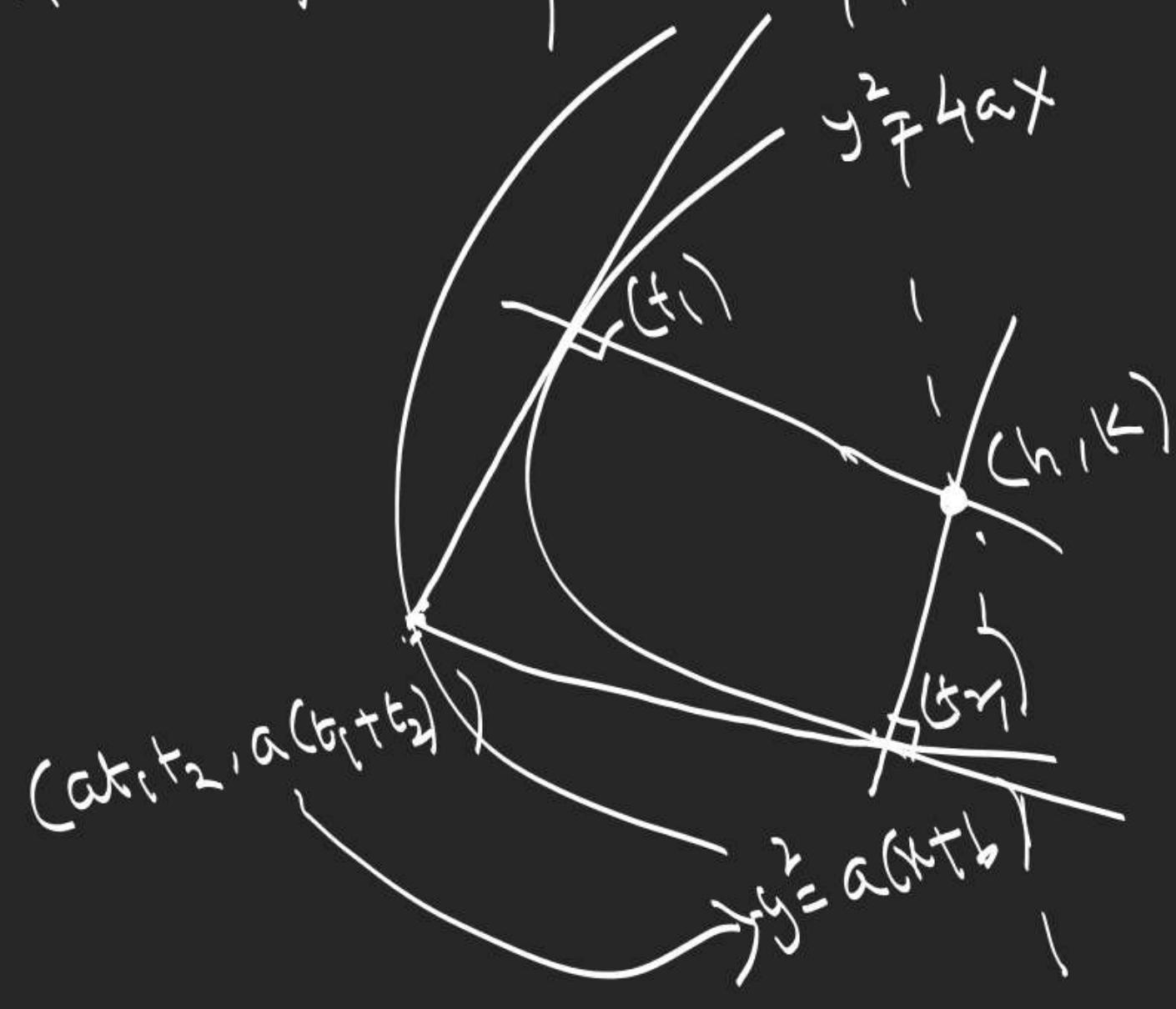
$$k = 2at + at^3$$

$$\frac{k}{h} = t$$

$$y + tx = 2at + at^3$$

$$h = 2a + a \left( \frac{k^2}{h^2} \right)$$

1. If tangents are drawn to  $y^2 = 4ax$  from any point  $P$  on parabola  $y^2 = a(x+b)$ , then show that normals drawn at their point of contact meet on a line.



$$\begin{aligned} h &= a(2 + t_1^2 + t_2^2 + t_1 t_2) \\ k &= -at_1 t_2(t_1 + t_2) \\ \boxed{R &= 2a+b} \end{aligned}$$

$$\begin{aligned} a(t_1^2 + t_2^2 + 2t_1 t_2) &= a(at_1 t_2 + b) \\ a(t_1^2 + t_2^2 + t_1 t_2) &= b \end{aligned}$$

Q. P.T. two parabolas  $y^2 = 4ax$  and  $y^2 = 4c(x-b)$

can't have a common normal other than axis

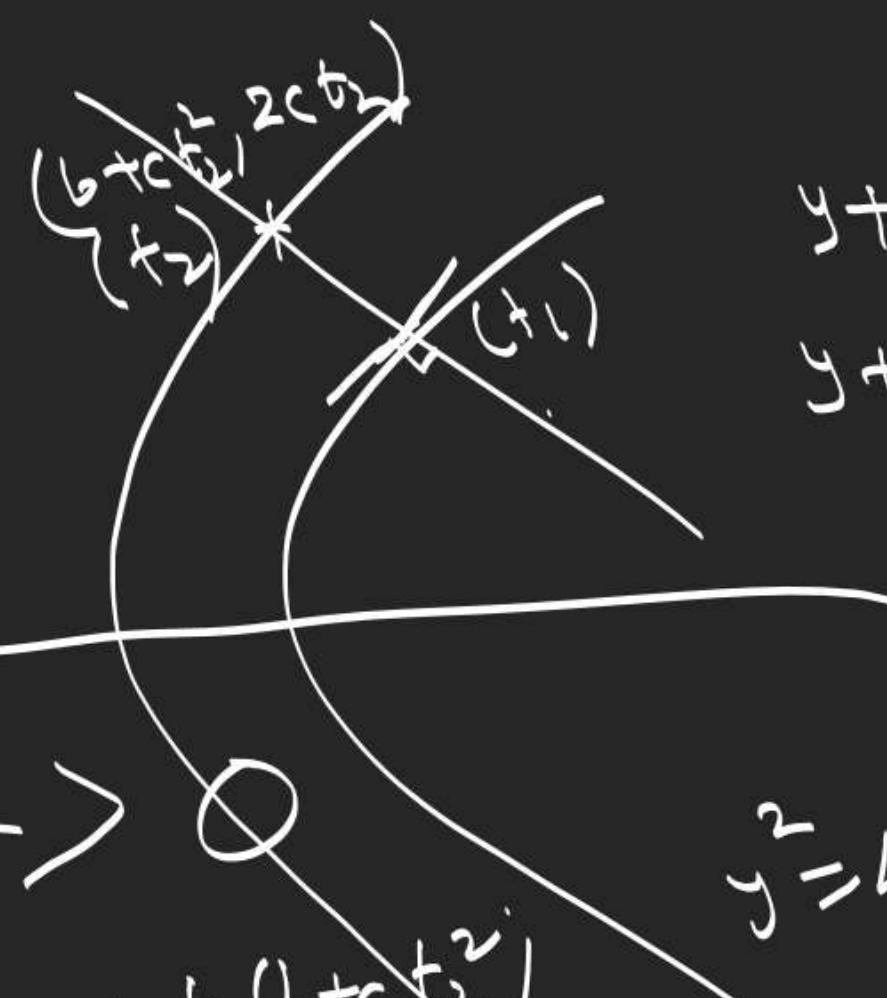
unless

$$\frac{b}{a-c} > 2$$

$$t_1^2 = \frac{b}{a-c} - 2 > 0$$

$$y + t_2 x = 2ct_2 + t_2(b+ct_2^2)$$

$$y + t_1 x = 2at_1 + at_1^3$$



$$m = \frac{2c}{2c-a} = \frac{1}{t_2} = \frac{2(c-a)t_1 + at_1^3}{(2c+b)t_2 + ct_2^3}$$

$$y + t_1 x = 2at_1 + at_1^3$$

$$y + t_2(x-b) = 2ct_2 + ct_2^3$$

$$1 = \frac{t_1}{t_2} = \frac{2at_1 + at_1^3}{(2c+b)t_2 + ct_2^3}$$

$$t_2 = t_1$$

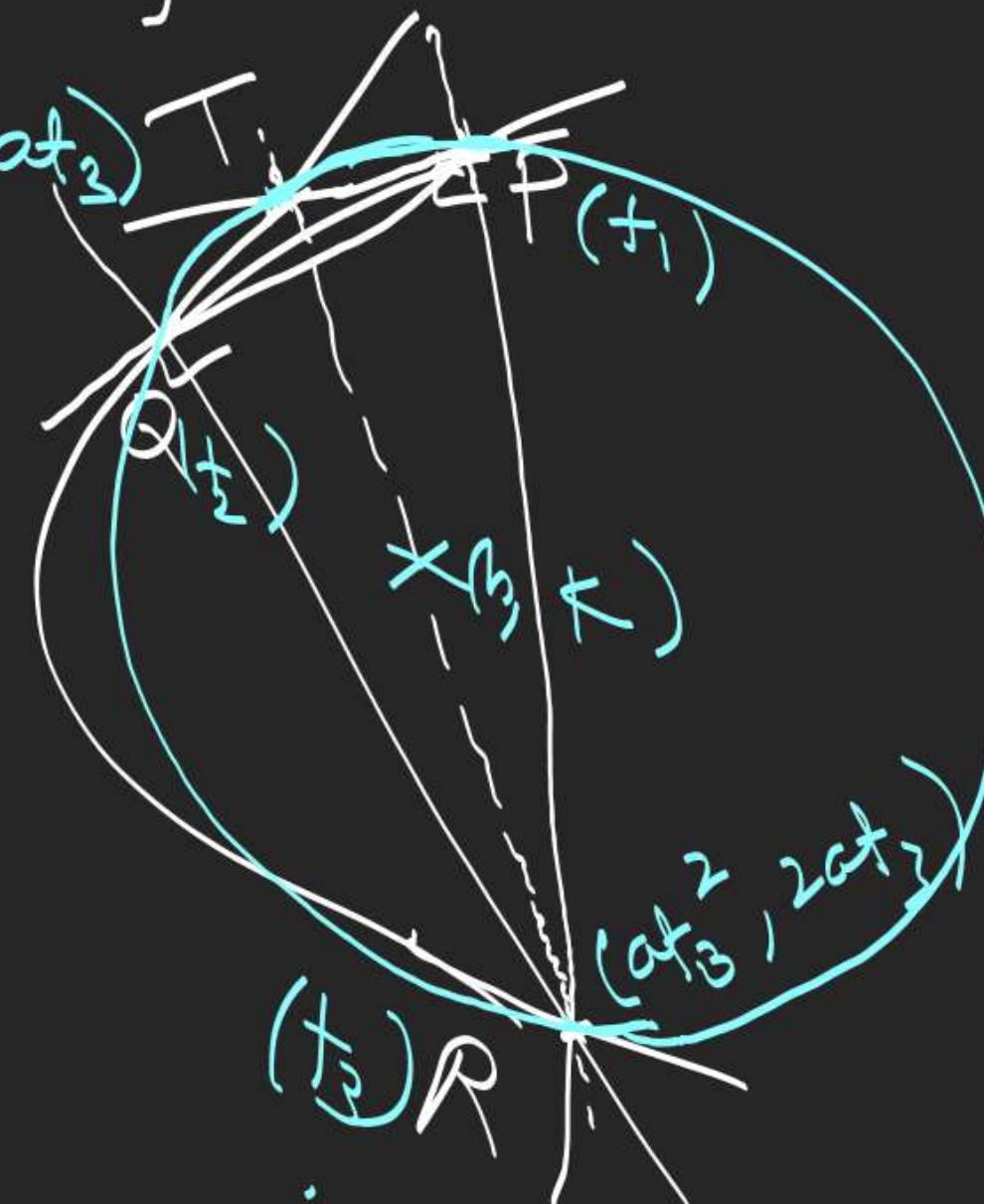
3. TP and TQ are tangents to parabola  $y^2 = 4ax$  and normals at P & Q meet at a point R on  $y^2 = 4ax$ .  
 P.T. centre of circle circumscribing  $\triangle TPQ$  lies

on parabola  $2y^2 = a(n-a)$

$$2h = 2a + a\left(\frac{2K}{a}\right)^2$$

$$\begin{aligned} 2h &= 2a + at_3^2 \\ 2K &= at_3 \end{aligned}$$

$$\begin{aligned} t_1 + t_2 + t_3 &= 0 \\ t_1 t_2 &= 2. \end{aligned}$$



4. Find the locus of point which is such that two of the normals through it to parabola  $y^2 = 4ax$  are at right angles.

$$a\left(-\frac{k}{a}\right)^3 + (2a-h)\left(-\frac{k}{a}\right) - k = 0$$

$$\frac{k-2at}{h-at^2} = -t$$

$$at^3 + (2a-h)t - k = 0$$

$$t_1 t_2 t_3 = -t_3 = \frac{k}{a}$$

$$t_1 t_2 = -1$$

$$(h, k) = \left( a(t_1^2 + t_2^2 + t_1 t_2 + 2), -at_1 t_2(t_1 + t_2) \right)$$

$$k = (t_1 + t_2)a$$

$$h = a\left(2 + \frac{(t_1 + t_2)^2}{t_1 t_2} - t_1 t_2\right)$$

$$h = a\left(3 + \left(\frac{k}{a}\right)^2\right)$$

Chord of Contact

$\sigma(\alpha, \beta)$  w.r.t.  $y^2 = 4ax$

P  
 $(\alpha, \beta)$

$$S = y^2 - 4ax$$

$$T = y\beta - 2a(x + \alpha)$$

$$T=0$$

AB

$y^2 = 4ax$   
A  
 $(x_1, y_1)$

Tangent at A  
 $yy_1 = 2a(x + x_1)$   
Point  $(\alpha, \beta)$

$$\beta y_1 = 2a(x + x_1)$$

$$\beta y_2 = 2a(x + x_2)$$

$$\beta y = 2a(x + x) \quad \begin{matrix} (\alpha, \beta) \\ (x_1, y_1) \\ (x_2, y_2) \end{matrix}$$

Chord whose midpoint is given

$$y - \beta = \frac{2a}{\beta} (x - \alpha)$$

$$\beta y - 2ax = \beta^2 - 2a\alpha$$

$$\beta y - 2a(x + \alpha) = \beta^2 - 4ax$$

$$\boxed{\Gamma = S}$$



$$\beta = \frac{2a(t_1 + t_2)}{2}$$

$$t_1 t_2 = \frac{\beta}{a}$$

$$m = \frac{2}{t_1 + t_2}$$

$$m = \frac{2a}{\beta}$$

# Pair of Tangents

