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$$\frac{x+\ln\left(\sqrt{x^2+1}-x\right)}{x^3} = \frac{1+\dfrac{\frac{x}{\sqrt{x^2+1}}-1}{\sqrt{x^2+1}-x}}{3x^2} = \frac{1-\frac{1}{\sqrt{x^2+1}}}{3x^2} = \frac{1}{3\sqrt{x^2+1}\left(\sqrt{x^2+1}+1\right)} = \boxed{\frac{1}{6}}$$

$$x \to 0^+$$

$$\frac{x-\ln\left(\sqrt{x^2+1}+x\right)}{x^3}$$

$$e^{(x^x-1)\ln x} = e^{\frac{e^{x\ln x}-1}{x\ln x}\cdot x\ln^2 x} = e^0 = 1$$

$$x\ln x$$

$$\frac{\ln^2 x}{1/x} = \frac{2\ln x}{-\frac{1}{x}} = -2x\ln x \to 0$$

$$\frac{\ln x}{1/x} \qquad \frac{1/x}{-\frac{1}{x^2}} \to 0$$

$$\frac{\ln\left(1+\frac{e^x}{\sqrt{x^2+1}+x}-1\right)}{(\quad)}$$

$$\frac{e^x-x-\sqrt{x^2+1}}{\left(\sqrt{x^2+1}+x\right)x^3}$$

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$$\lim_{x\to 0^+} \frac{6x(\ln\sin x - \ln x) + x^2}{\left(\frac{x-\sin x}{x^3}\right)\left(\frac{1-\cos x}{x^2}\right) x^4}$$

$$12\left(\frac{6\left(\frac{1}{\tan x} - \frac{1}{x}\right) + 2x}{4x^3}\right) = 3\lim_{x\to 0} \frac{3(x-\tan x) + x^2\tan x}{x^5 \frac{\tan x}{x}}$$

$$\frac{\left(\frac{x-\sin x}{x^3}\right)\left(\frac{x^{5999} + x^{5998}\sin x + \cdots + \sin^{5999} x}{x^{5999}}\right)}{\left(\frac{\sin x}{x}\right)^{6000}}$$

$$\frac{3\left(x - x - \frac{x^3}{3} - \frac{2}{15}x^5 - \cdots\right) + x^2\left(x + \frac{x^3}{3} + \frac{2}{15}x^5 \cdots\right)}{x^5}$$

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$$\frac{1-\cos 3x\cos 9x\cos 27x\cdots\cos 3^n x}{1-\cos\frac{x}{3}\cos\frac{x}{9}\cos\frac{x}{27}\cdots\cos\frac{x}{3^n}}$$

$$\frac{\frac{3^2\sin 3x\cos 9x\cdots}{3x}+\cos 3x\frac{(9^2\sin 9x)}{9x}\cdots+\cos 3x\cos 3^2x\cdots\cos 3^{n-1}x\frac{(3^n\sin 3^n x)}{3^n x}\times 3^n}{\frac{\frac{1}{3^2}\sin\frac{x}{3}\cos\frac{x}{9}\cdots}{\frac{x}{3}}+\frac{\cos\frac{x}{3}\left(\frac{1}{9^2}\sin\frac{x}{9}\right)\cos\frac{x}{27}}{\frac{x}{9}}\cdots+\frac{\cos\frac{x}{3}\cos\frac{x}{9}\cdots\cos\frac{x}{3^{n-1}}\left(\frac{1}{3^n}\sin\frac{x}{3^n}\right)}{\frac{x}{3^n}}}$$

$$\frac{3^2+3^4+3^6+\cdots 3^{2n}}{\frac{1}{3^2}+\frac{1}{3^4}+\frac{1}{3^6}+\cdots+\frac{1}{3^{2n}}}$$

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System of Equations

$a_1x+b_1y+c_1z=d_1$
$a_2x+b_2y+c_2z=d_2$
$a_3x+b_3y+c_3z=d_3$

$|A|\neq 0$
unique soln.

$\boxed{X = A^{-1}B}$

$|A|=0$, & $(\text{adj}A)B=O$
infinite solution

Exception
$x+y+z=1$
$x+y+z=2$
$x+y+z=3$

$|A|=0$ & $(\text{adj}A)B\neq O$
no solution

$(k, 2k, 3k) = (x, y, z)$

$$\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$$

$(2k, 5, k)$

$AX = B$

$\Rightarrow \underbrace{(\text{adj}A)A}X = (\text{adj}A)B$

$|A|X = (\text{adj}A)B$

$x=f(k)$, $y=g(k)$

Infinite soln
$z=k$

$a_1x+b_1y=d_1-c_1k$
$a_2x+b_2y=d_2-c_2k$

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Consistent → system of eqn. has atleast one solution.

Inconsistent → if system of eqn. has no solution.

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1.

$$x+y+z=6$$
$$x-y+z=2$$
$$2x+y-z=1$$

$$A=\begin{bmatrix}1 & 1 & 1\\ 1 & -1 & 1\\ 2 & 1 & -1\end{bmatrix} \qquad \text{adj}\,A=\begin{bmatrix}0 & 2 & 2\\ 3 & -3 & 0\\ 3 & 1 & -2\end{bmatrix}$$

$$|A| = 3+3=6$$

$$X = A^{-1}B = \frac{1}{6}\begin{bmatrix}0 & 2 & 2\\ 3 & -3 & 0\\ 3 & 1 & -2\end{bmatrix}\begin{bmatrix}6\\ 2\\ 1\end{bmatrix} = \frac{1}{6}\begin{bmatrix}6\\ 12\\ 18\end{bmatrix} = \begin{bmatrix}1\\ 2\\ 3\end{bmatrix}$$

$$\boxed{(x,y,z)=(1,2,3)}$$

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Q. Let $A^n = \begin{bmatrix} 2 & 1 \\ 1 & 0 \end{bmatrix}^n = [a_{ij}]$ is a 2×2 matrix

find $\lim\limits_{n\to\infty}\left(\dfrac{a_{12}}{a_{22}}\right)$

$$\frac{\frac{1}{2\sqrt2}}{\frac{1}{2}-\frac{1}{2\sqrt2}} = \frac{1}{\sqrt2-1} = \boxed{\sqrt2+1}$$

$$B^2 = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} = 2I$$

$B^3 = 2B$

$B^4 = 2B^2 = 2^2 I$

$B^5 = 2^2 B$

$B^6 = 2^3 I$

$$(I+B)^n = {}^nC_0 I + {}^nC_1 B + {}^nC_2 B^2 + {}^nC_3 B^3 + \dots + {}^nC_n B^n$$

$$= I\left({}^nC_0 + {}^nC_2\, 2 + {}^nC_4\, 2^2 + {}^nC_6\, 2^3 + \dots\right)$$

$$+ B\left({}^nC_1 + {}^nC_3\, 2 + {}^nC_5\, 2^2 + {}^nC_7\, 2^3 + \dots\right)$$

$$A^n = I\,\frac{(1+\sqrt2)^n + (1-\sqrt2)^n}{2} + B\,\frac{(1+\sqrt2)^n - (1-\sqrt2)^n}{2\sqrt2}$$

$$\lim_{n\to\infty} \frac{\dfrac{(1+\sqrt2)^n + (1-\sqrt2)^n}{2\sqrt2}}{\dfrac{(1+\sqrt2)^n + (1-\sqrt2)^n}{2} - \dfrac{(1+\sqrt2)^n - (1-\sqrt2)^n}{2\sqrt2}}$$