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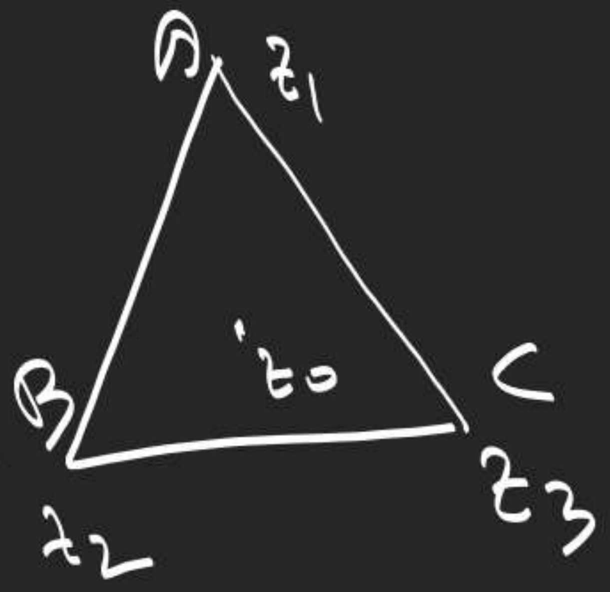
1.

$$(z_2 - z_1)e^{i\pi/3} = z_3 - z_1 \quad -①$$



$$② - \quad z_2 - z_1 = (z_2 - z_3)e^{i\pi/3}$$

$$① \times ② \quad (z_2 - z_1)^2 = (z_3 - z_1)(z_2 - z_3)$$

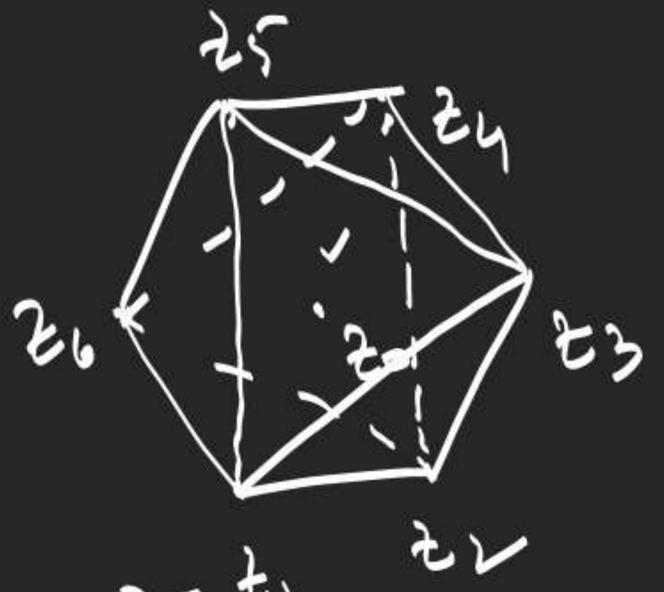
$$\boxed{\checkmark \sum z_1^2 = \sum z_1 z_2}$$

$$2\sum z_1^2 = 2\sum z_1 z_2$$

$$3\sum z_1^2 = \sum z_1^2 + 2\sum z_1 z_2 = (z_1 + z_2 + z_3)^2 = (3z_0)^2$$

$$3z_0^2 = \sum z_1^2$$

2.



$$z_1^2 + z_3^2 + z_5^2 = 3z_0^2$$

$$z_2^2 + z_4^2 + z_6^2 = 3z_0^2$$

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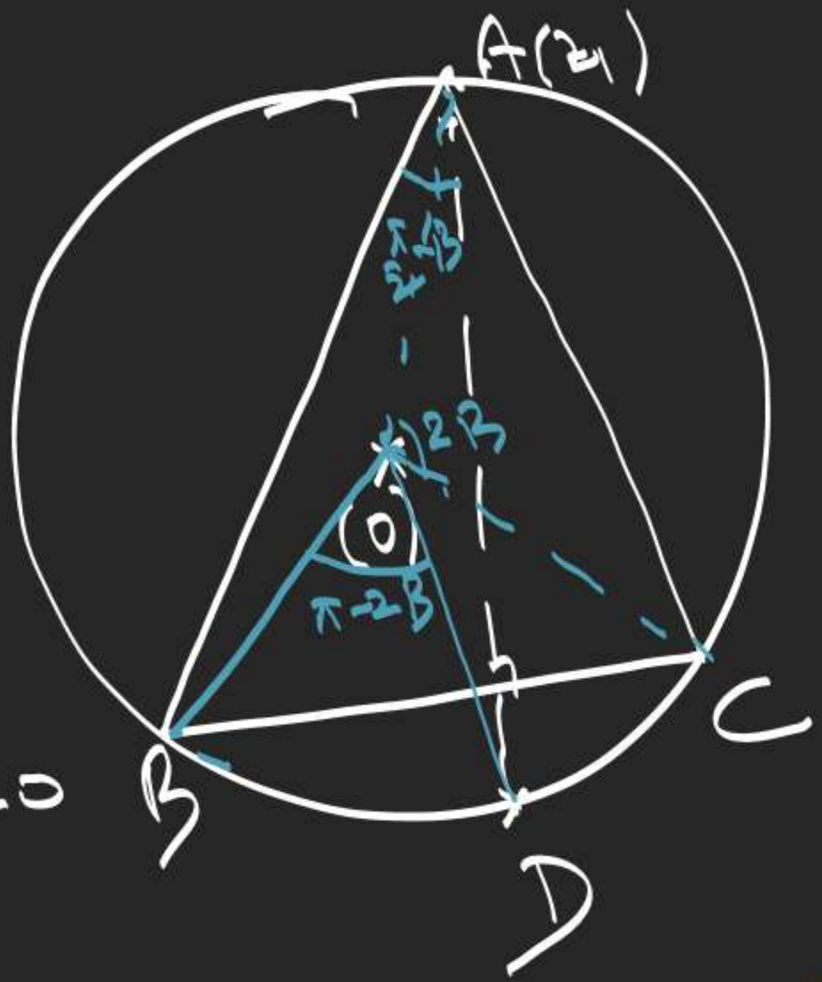
3.

$$\frac{z_D - z_1}{z_2 - z_3} + \frac{\bar{z}_D - \bar{z}_1}{\bar{z}_2 - \bar{z}_3} = 0$$

$$\left(\quad \right) + \frac{\frac{1}{z_D} - \frac{1}{z_1}}{\frac{1}{z_2} - \frac{1}{z_3}} = 0$$

$$\frac{z_D - z_1}{z_2 - z_3}\left(1 + \frac{z_2 z_3}{z_D z_1}\right) = 0$$



$$z_D = z_2 e^{i(\pi - 2B)}$$

$$z_D = -z_2 e^{-i2B}$$

$$z_1 = z_3 e^{i2B}$$

$$z_D z_1 = -z_2 z_3$$

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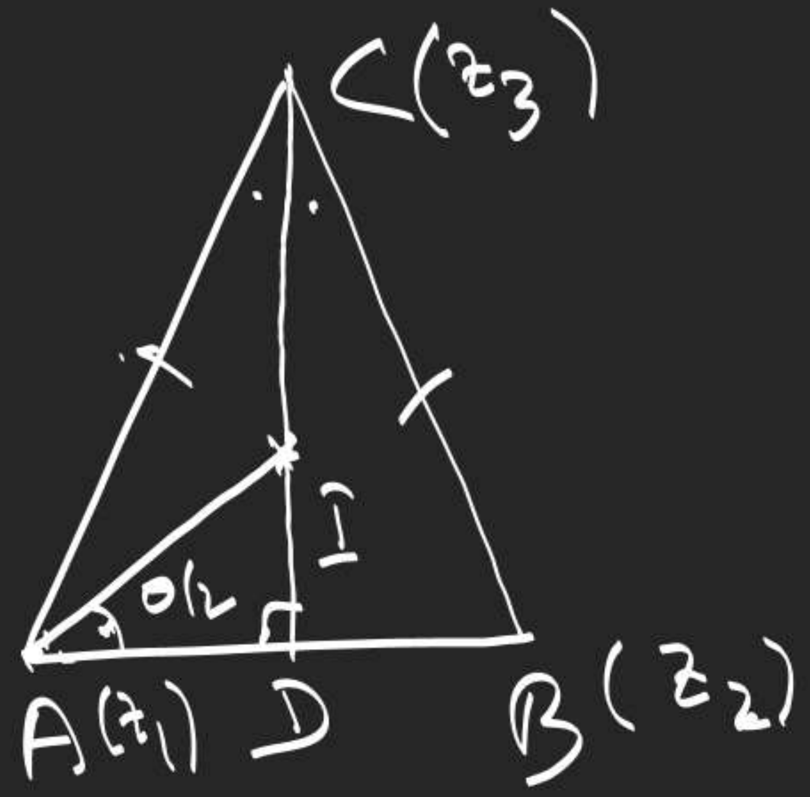
4.



$$\frac{(z_2-z_1)}{AB}e^{i\frac{\theta}{2}} = \frac{z_4-z_1}{AI}$$

$$\frac{z_3-z_1}{AC} = \frac{z_4-z_1}{AI}e^{i\frac{\theta}{2}}$$

$$\frac{(z_2-z_1)(z_3-z_1)}{(AB)(AC)} = \frac{(z_4-z_1)^2}{AI^2} \Rightarrow (z_2-z_1)(z_3-z_1) = \frac{(AB)(AC)}{(AI)^2}(z_4-z_1)^2$$

$$\left(2\frac{AD}{AI}\right)\frac{AC}{AD}\frac{AD}{AI}$$

$$= \frac{2\left(\cos\frac{\theta}{2}\right)^2}{\cos\theta} = \frac{1+\cos\theta}{\cos\theta}$$

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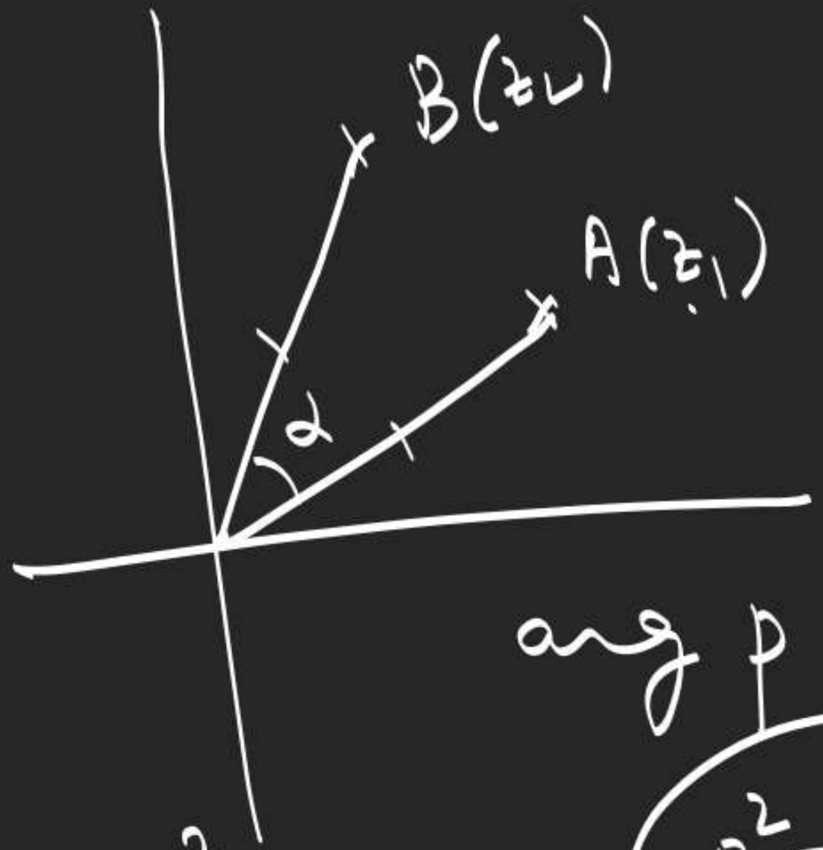
5.



$$\boxed{z_1 e^{i\alpha} = z_2}$$

$\arg p \; ? \; \arg q$

$p^2 = 4q\cos^2\frac{\alpha}{2}$ $\Rightarrow$ $\arg p^2 = \arg q$
$= 2\arg p$

$\frac{p^2}{q} = 4\cos^2\frac{\alpha}{2}$

$\downarrow$

$$LHS = \frac{(z_1+z_2)^2}{z_1 z_2} = \frac{z_1}{z_2} + \frac{z_2}{z_1} + 2 = \underline{e^{-i\alpha} + e^{i\alpha}} + 2 = 2\cos\alpha + 2$$

$$= 4\cos^2\frac{\alpha}{2}$$

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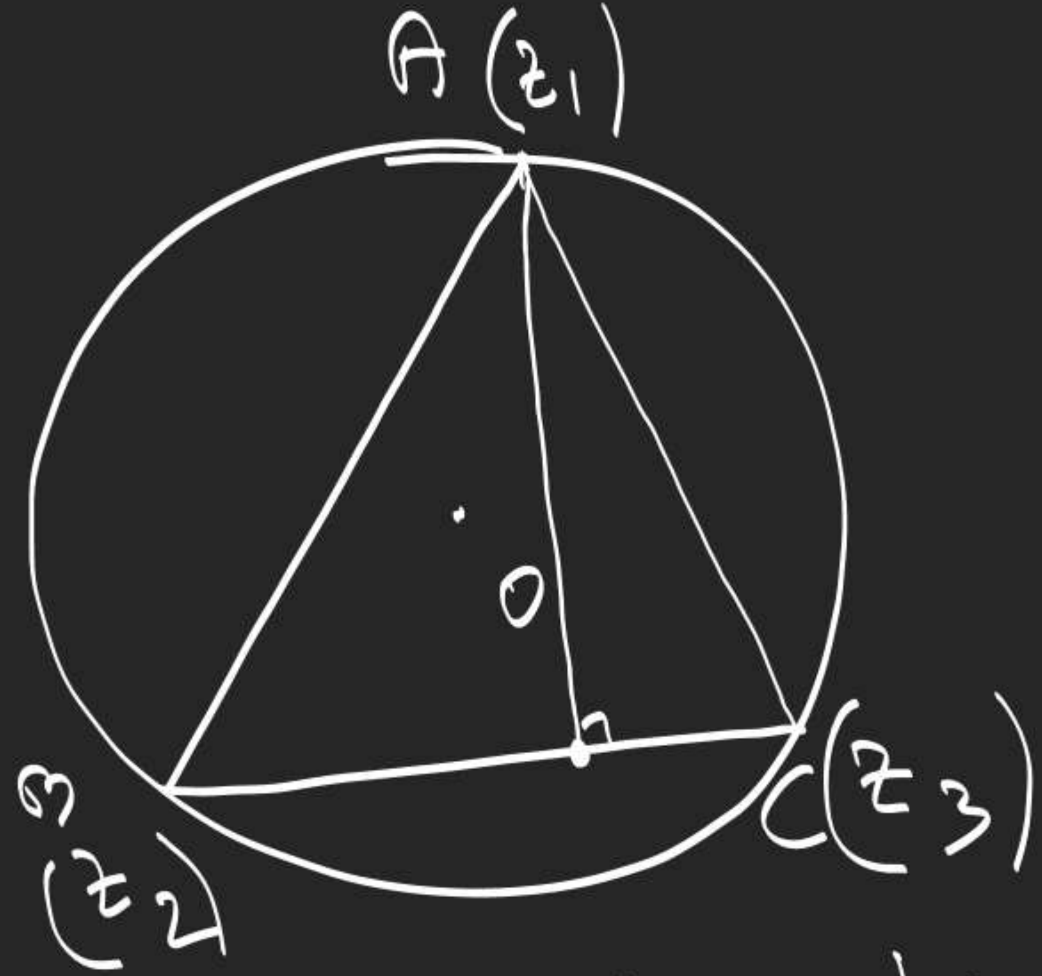
6.

$$\min_{a\in R}\left|\left(a z_2+(1-a)z_3\right)-z_1\right|$$



$$\boxed{h=\frac{bc}{2R}}$$

$$\underline{\underline{\Delta}}=\frac{1}{2}ah=\frac{abc}{4R}$$

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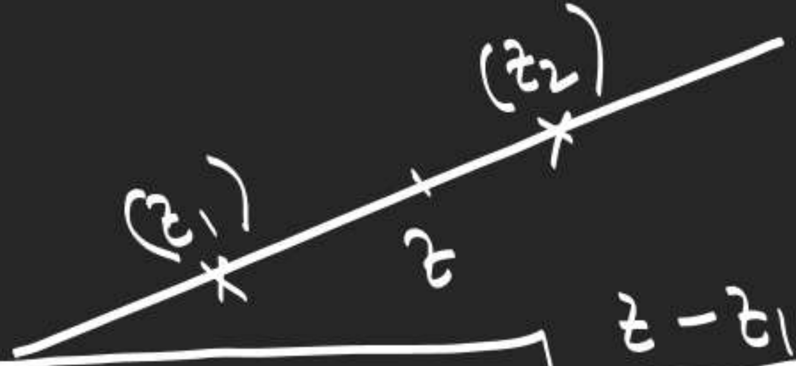
Straight Line



$$\frac{z-z_1}{z_2-z_1} \in R$$

$$\frac{z-z_1}{z_2-z_1} = \lambda \quad , \lambda \in R$$

$$z = z_1 + \lambda(z_2 - z_1) \quad , \lambda \in R$$

$$\Rightarrow \frac{z-z_1}{z_2-z_1} = \frac{\bar{z}-\bar{z}_1}{\bar{z}_2-\bar{z}_1} \Rightarrow z(\bar{z}_2-\bar{z}_1) - \bar{z}(z_2-z_1) - z_1\bar{z}_2 + \bar{z}_1 z_2 = 0$$

$$-i(z_2-z_1)\bar{z} + i(\bar{z}_2-\bar{z}_1)z + i(\bar{z}_1 z_2 - z_1\bar{z}_2) = 0$$

$$\begin{vmatrix} z & \bar{z} & 1 \\ z_1 & \bar{z}_1 & 1 \\ z_2 & \bar{z}_2 & 1 \end{vmatrix} = 0$$

$$\boxed{\alpha\bar{z} + \bar{\alpha}z + \beta = 0 \quad \beta \in R}$$

Coeff of $\bar{z}$ is $\perp$ar to line

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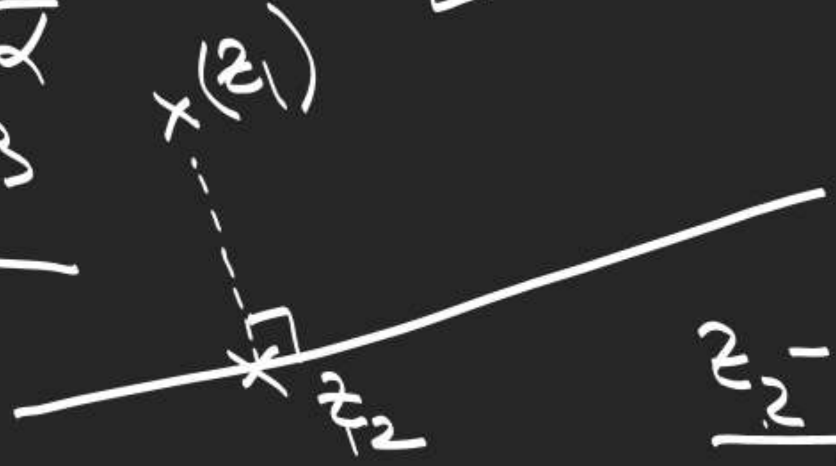
1.

$(z_1)\ P\ *$

$L:\ \alpha \bar{z} + \bar{\alpha} z + \beta = 0$, $\beta \in R$

find ① foot of ⊥ of P on L

② image of P from 'L'



$z_1' = 2z_2 - z_1$

$= \frac{\bar{\alpha} z_1 - \alpha \bar{z}_1 - \beta}{\bar{\alpha}} - z_1$

$z_1' = \frac{-\alpha \bar{z}_1 - \beta}{\bar{\alpha}}$

$\bar{\alpha} z_1' + \alpha \bar{z}_1 + \beta = 0$
or
$\alpha \bar{z}_1' + \bar{\alpha} z_1 + \beta = 0$

$\frac{z_2 - z_1}{\alpha} = \frac{\bar{z}_2 - \bar{z}_1}{\bar{\alpha}} \Rightarrow \bar{\alpha} z_2 - \bar{\alpha} z_1 - \alpha \bar{z}_2 + \alpha \bar{z}_1 = 0$ —①

$\alpha \bar{z}_2 + \bar{\alpha} z_2 + \beta = 0$ —②

① + ② $\quad 2\bar{\alpha} z_2 - \bar{\alpha} z_1 + \alpha \bar{z}_1 + \beta = 0$

$z_1' \quad z_2 = \frac{\bar{\alpha} z_1 - \alpha \bar{z}_1 - \beta}{2\bar{\alpha}}$

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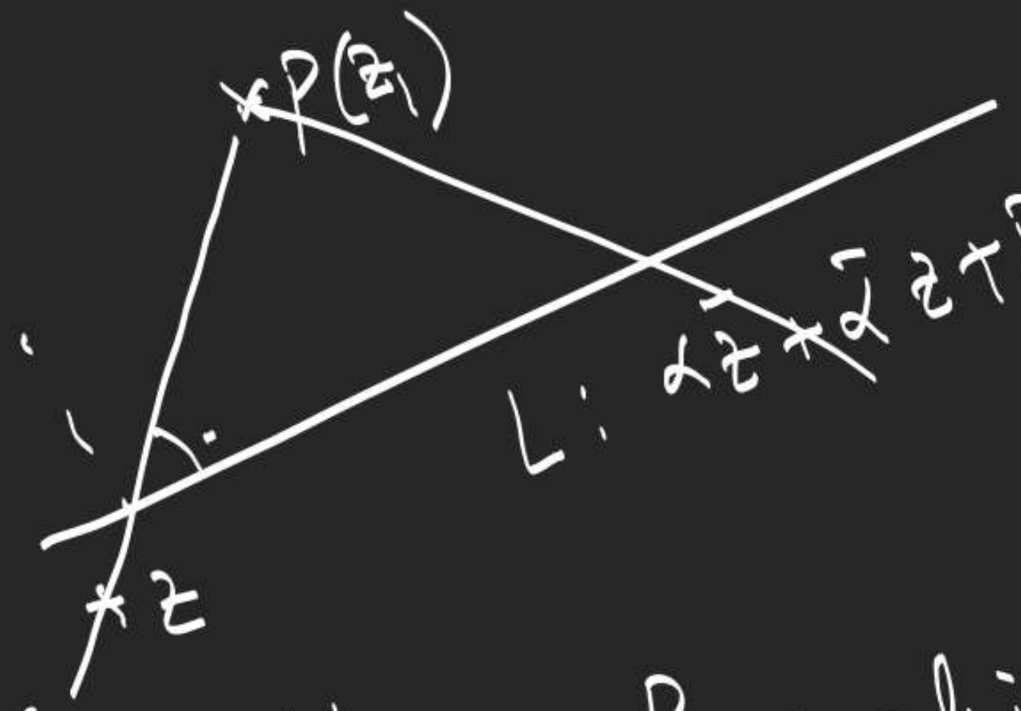
Q.



$L: \alpha\bar{z} + \bar{\alpha}z + \beta = 0$, $\beta \in R$

Find eqn. of line thru P making angle $\frac{\pi}{4}$ with 'L'.

$$\frac{z - z_1}{|z - z_1|} = \frac{\alpha}{|\alpha|} e^{\pm i\frac{\pi}{4}}$$

$$\frac{z - z_1}{\bar{z} - \bar{z}_1} = \pm i \frac{\alpha}{\bar{\alpha}}$$

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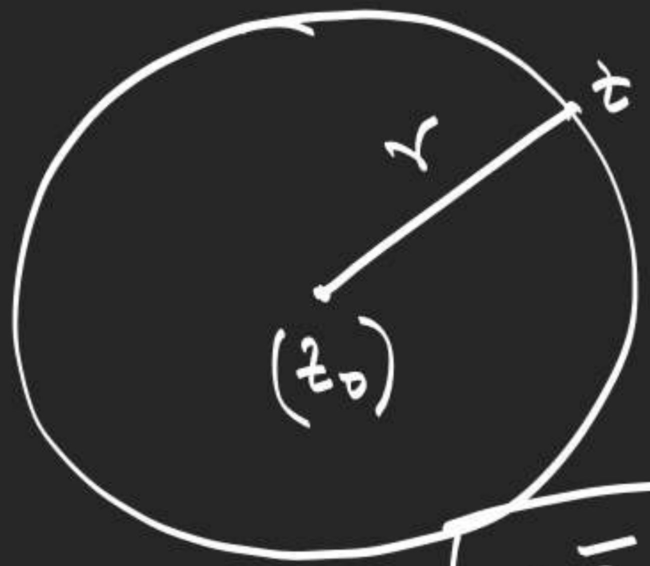
Circle



$$|z - z_0| = r$$

$$(z - z_0)(\bar{z} - \bar{z}_0) = r^2$$

$$z\bar{z} - \bar{z}_0 z - z_0 \bar{z} + z_0 \bar{z}_0 - r^2 = 0$$

$$\boxed{\begin{array}{l} z\bar{z} + \alpha\bar{z} + \bar{\alpha} z + \beta = 0, \quad \beta \in R \\ \text{Centre} = -\text{coeff of } \bar{z} = -\alpha \\ \text{radius} = \sqrt{\alpha\bar{\alpha} - \beta} \end{array}}$$

$$z_0 \bar{z}_0 - r^2 = \beta$$

$$(-\alpha)(-\bar{\alpha}) - r^2 = \beta$$

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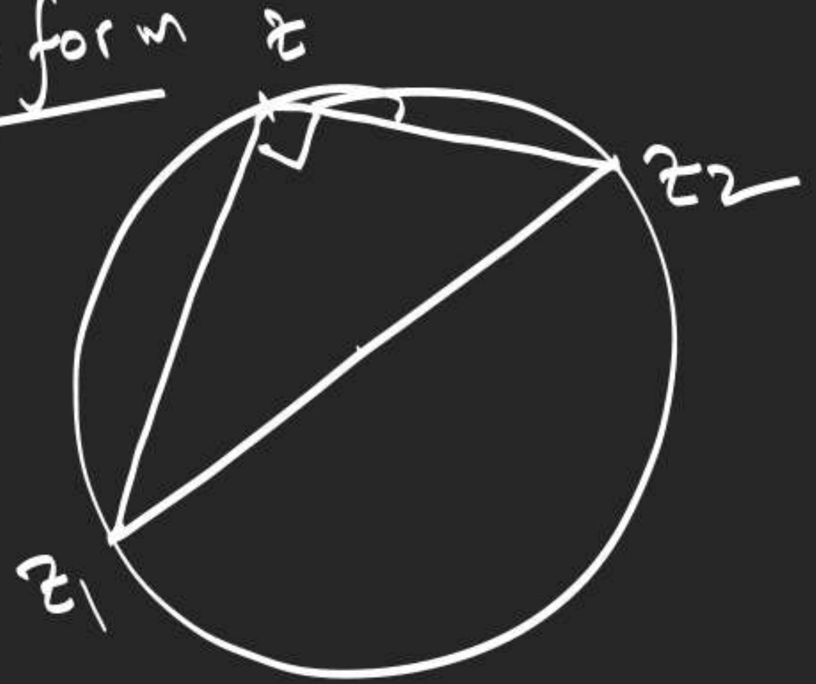
Diametric form



$$\frac{z-z_1}{z-z_2} \text{ purely imag.}$$

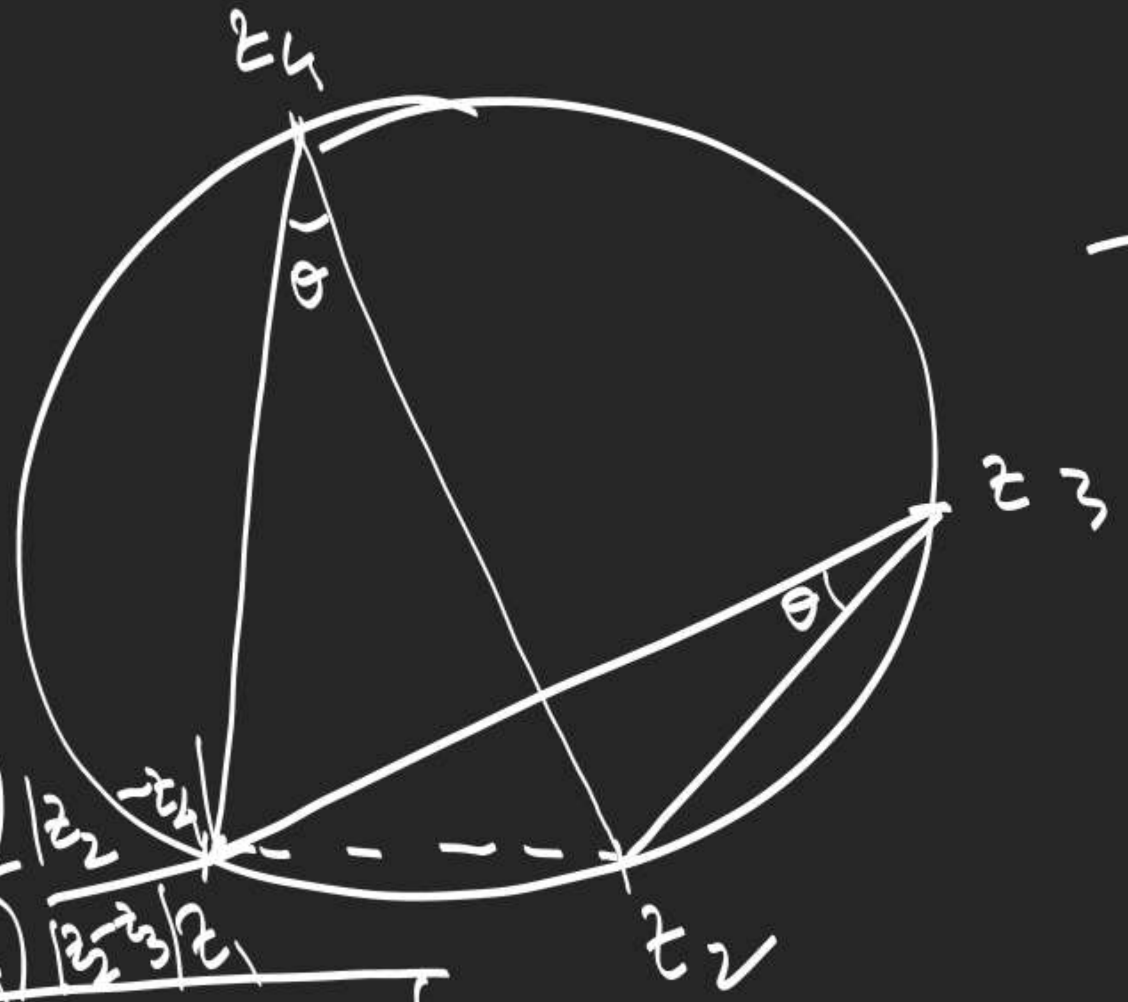
$$\frac{z-z_1}{z-z_2} + \frac{\bar{z}-\bar{z}_1}{\bar{z}-\bar{z}_2} = 0$$

Nishant Jindal

Condition for points z_1, z_2, z_3, z_4 to be concyclic

$$\frac{(z_1 - z_3)}{|z_1 - z_3|} e^{i\theta} = \frac{z_2 - z_3}{|z_2 - z_3|} \quad -①$$

$$\frac{(z_1 - z_4)}{|z_1 - z_4|} e^{i\theta} = \frac{(z_2 - z_4)}{|z_2 - z_4|} \quad -②$$



$$\frac{①}{②} \quad \frac{(z_1 - z_3)\,|z_1 - z_4|}{(z_1 - z_4)\,|z_1 - z_3|} = \frac{(z_2 - z_3)\,|z_2 - z_4|}{(z_2 - z_4)\,|z_2 - z_3|}$$

$$\boxed{\frac{(z_1 - z_3)(z_2 - z_4)}{(z_1 - z_4)(z_2 - z_3)} \in R}$$

Nishant Jindal

1. Find angle b/n 2 intersecting circles

$z\bar{z} + \alpha_1 \bar{z} + \bar{\alpha}_1 z + \beta_1 = 0 \qquad i = 1, 2 \qquad , \beta_1, \beta_2 \in R.$

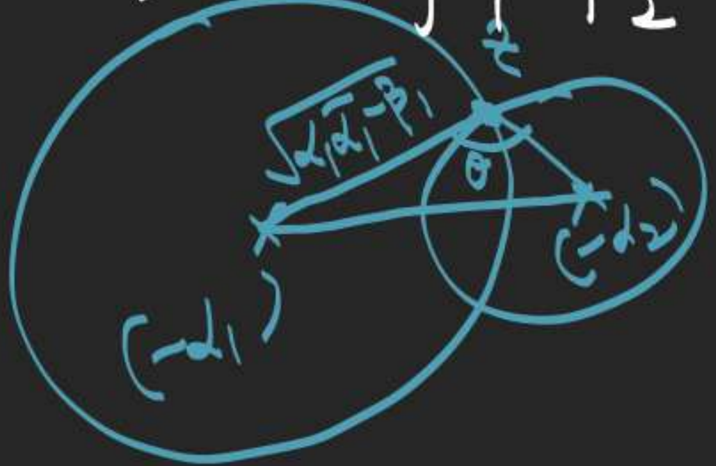
$z\bar{z} + \alpha_2 \bar{z} + \bar{\alpha}_2 z + \beta_2 = 0$

Also find the eqn of their common chord.

$$(\alpha_1 - \alpha_2)\bar{z} + (\bar{\alpha}_1 - \bar{\alpha}_2)z + \beta_1 - \beta_2 = 0.$$

Common chord

$$\cos\theta = \left| \frac{\alpha_1\bar{\alpha}_1 - \beta_1 + |\alpha_2|^2 - \beta_2 - |\alpha_1 - \alpha_2|^2}{2\sqrt{|\alpha_1|^2 - \beta_1}\sqrt{|\alpha_2|^2 - \beta_2}} \right|$$



$$\frac{\alpha_1\bar{\alpha}_2 + \bar{\alpha}_1\alpha_2 - \beta_1 - \beta_2}{2\sqrt{}\ \sqrt{}}$$

Nishant Jindal

1. Complex Numbers

2. Parabola → Ex-2, Ex-3

3. Ellipse → Ex-2, Ex-3

4. Hyperbola → Ex-2, Ex-3