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$$\frac{\cos x - \sin x}{\cos x\,(1-\sqrt{2}\sin x)} = \frac{(\cos^2 x - \sin^2 x)(1+\sqrt{2}\sin x)}{\cos x\,(\cos x + \sin x)(1-2\sin^2 x)}$$

$$2 \qquad \frac{\sin\left(\frac{\pi}{6\cdot 2^{n+1}}\right)}{\frac{\pi}{6\cdot 2^{n+1}}}$$

$$= \frac{\pi}{3}$$

$$\frac{\sqrt{1-\sin 2x}}{\frac{\pi}{6}\sqrt{1+\sqrt{\sin 2x}}\;4\left(\frac{\pi}{4}-x\right)} = \frac{\left|\sqrt{2}\sin\left(-x+\frac{\pi}{4}\right)\right|}{4\sqrt{1+\sqrt{\sin 2x}}\left(\frac{\pi}{4}-x\right)}$$

$$\sqrt{x^2} = |x|$$

$$\lim_{x\to 0}\left(e^{\frac{\ln(1+x)-x}{x^2}}\right)$$

$$e^{-\frac{1}{2}}$$

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$$\lim_{x\to 0} \frac{2\left[\left(\cancel{x}+\cancel{\frac{x^3}{3}}+\frac{2}{15}x^5+\cdots\right)-\left(\cancel{\frac{x}{1!}}-\cancel{\frac{x^3}{3!}}+\frac{x^5}{5!}-\cdots\right)\right]\cancel{-x^3}}{x^5}$$

$$2\left(\frac{2}{15}+\frac{1}{120}\right)$$

$$1-2\sin^2\frac{x}{4}$$

$$\frac{2\sin x(1-\cos x)-x^3\cos x}{x^5\cos x} = \frac{8\sin^3\frac{x}{2}\cos\frac{x}{2}-x^3\left(1-2\sin^2\frac{x}{2}\right)}{x^5\cos x}$$

$$=\left(\frac{8\left(\sin^3\frac{x}{2}-\left(\frac{x}{2}\right)^3\right)}{x^5\cos x}\right)+\frac{2\sin^2\frac{x}{2}}{x^2\cos x}-\left(\frac{16\sin^3\frac{x}{2}\sin^2\frac{x}{2}}{x^5\cos x}\right)$$

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$$= \boxed{\ln a}$$

12. $$\frac{(1+x)(1+x^2)(1+x^4)\cdots(1+x^{2^{n-1}})(1-x)}{(1-x)} = \frac{1-x^{2^n}}{1-x}$$

$$n^2 \, a^{\frac{1}{n+1}} \frac{\left(a^{\frac{1}{n}-\frac{1}{n+1}} - 1\right)}{n(n+1)\left(\frac{1}{n(n+1)}\right)}$$

$$\lim_{x\to 2} \frac{a^x - 1}{x} = \frac{a^2-1}{2}$$

$$\lim_{n\to\infty} \frac{\left(1 - \left(\frac{1}{5^{2^{n+1}}}\right)\right)}{1-\frac{1}{5}}$$

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15.

$$\frac{2(\cos ax - 1) - \left(\underbrace{\cos(b+c)x}_{-1} + \underbrace{\cos(b-c)x}_{-1}\right)}{bcx^2\,\dfrac{\sin bx}{bx}\,\dfrac{\sin cx}{cx}}$$

$$\frac{2\left(-\dfrac{a^2}{2}\right) + \left(\dfrac{(b+c)^2}{2} + \dfrac{(b-c)^2}{2}\right)}{bc}$$

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$$\lim_{x\to 0}\left\{\frac{x^2}{\sin x\tan x}\right\} = 0$$

$$\frac{x^2\left(1-\tan^4\frac{x}{2}\right)}{4\tan^2\frac{x}{2}}$$

$$\frac{1}{1^+} \to 1^-$$

$$\frac{1}{\cos x\sec^2 x}$$

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4. (ii) $\lim_{x\to\infty}\left(\sqrt{x^2-x+1}-ax-b\right)=0$

$a>0$

$$\lim_{x\to\infty}\frac{x^2-x+1-(ax+b)^2}{\sqrt{x^2-x+1}+ax+b}$$

$$\lim_{x\to\infty}\frac{x^2(1-a^2)-(2ab+1)x+1-b^2}{\sqrt{x^2-x+1}+ax+b}$$

$\boxed{1-a^2=0}$

$\boxed{a=1}$

$$\frac{-1-2b}{2}=\lim_{x\to\infty}\frac{\cancel{x}\left(-(2ab+1)+\frac{1-b^2}{x}\right)}{\cancel{x}\left(\sqrt{1-\frac{1}{x}+\frac{1}{x^2}}+a+\frac{b}{x}\right)}$$

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$\boxed{\text{G-10 (Ex-II)}}$

If $\lim_{x\to 0}\left(\frac{f(x)}{x^2}\right) = -2$, find

$f(x) - [f(x)] = 0-(-1)$

① $\lim_{x\to 0} \frac{f(x)}{x^2}\, x^2 = 0$

② $\lim_{x\to 0} [f(x)] = -1$

③ $\lim_{x\to 0} \{f(x)\} = 1$

not exist

④ $\lim_{x\to 0}\left(\frac{f(x)}{x}\right) = 0$

not exist

⑤ $\lim_{x\to 0}\left[\frac{f(x)}{x}\right]$

⑥ $\lim_{x\to 0}\left\{\frac{f(x)}{x}\right\}$

$[\cdot] = G.I.F$, $\{\cdot\} = FPF$

LHL = 0

RHL = −1

$\frac{f(x)}{x} = \frac{f(x)}{x^2}\cdot x$

$\frac{f(x)}{x^2} \to -2$, $x \to 0^-$

$f(x) = \frac{f(x)}{x^2}\, x^2 \to 0^-$

$\frac{f(x)}{x^2} \to -2$, $x^2 \to 0^+$

LHL = 0

RHL = 1