DPP 05

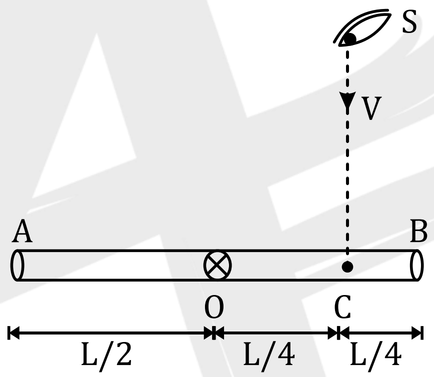
Angular momentum & Energy conservation

1. A disc of moment of inertia 4 kgm^2 is spinning freely at 3$rads^{-1}$. A second disc of moment of inertia 2 kgm^2 slides down the spindle and they rotate together.

(a) The angular velocity of the combination is____rads^{-2}.

(b) The change in kinetic energy (magnitude only) of the system is _____ J.

2. A homogeneous rod AB of length L = 1.8 m and mass M is pivoted at the centre O in such a way that it can rotate freely in the vertical plane (shown in Figure). The rod is initially in the horizontal position. An insect S of the same mass M falls vertically with speed V on the point C, midway between the points O and B. Immediately after falling, the insect moves towards the end B such that the rod rotates with a constant angular velocity $\frac{(k+5)V}{kL}$. Then the value of k is _____.



3. A uniform circular disc of mass m and radius a is rotating with constant angular velocity ω in a horizontal plane about a vertical axis through its centre A. A particle P of mass 2m is placed gently on the disc at a point distant $\frac{a}{2}$ from A. If the particle does not slip on the disc, the new angular velocity of the rotating system is $\left(\omega_2 = \frac{\omega}{\beta}\right)$. Then β is_____.

4. A wheel of moment of inertia I and radius R is rotating about its axis at an angular speed ω_0. It picks up a stationary particle of mass m at its edge. Find the new angular speed of the wheel.

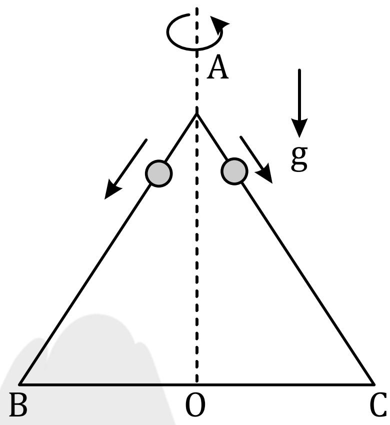
(A) $\left(\omega_2 = \frac{I\omega_0}{I+mR^2}\right)$

(B) $\left(\omega_2 = \frac{I\omega_0}{I+2mR^2}\right)$

(C) $\left(\omega_2 = \frac{I\omega_0}{I+3mR^2}\right)$

(D) $\left(\omega_2 = \frac{I\omega_0}{I+5mR^2}\right)$

5. An equilateral triangle ABC formed from a uniform wire has two small identical beads initially located at A. The triangle is set rotating about the vertical axis AO. Then the beads are released from rest simultaneously and allowed to slide down, one along AB and the other along AC as shown. Neglecting frictional effects, the quantities that are conserved as beads slide down are



(A) angular velocity and total energy (kinetic and potential).

(B) total angular momentum and total energy.

(C) angular velocity and moment of inertia about the axis of rotation.

(D) total angular momentum and moment of inertia about the axis of rotation.

6. In a simple pendulum, the breaking strength of the string is double the weight of the bob. The bob is released from rest when the string is horizontal. The string breaks when it makes an angle θ with the vertical

(A) $\theta = \cos^{-1}\left(\frac{1}{3}\right)$

(B) $\theta = \cos^{-1}\left(\frac{2}{3}\right)$

(C) $\theta = 60°$

(D) $\theta = 0$

7. A solid sphere and a hollow sphere of equal mass and radius are placed over a rough horizontal surface after rotating them about their respective centre of mass with same angular velocity ω_0. Once the pure rolling starts let v_1 and v_2 be the linear speeds of their centres of mass. Then

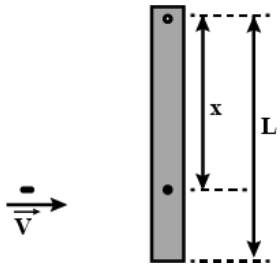
(A) $v_1 = v_2$

(B) $v_1 > v_2$

(C) $v_1 < v_2$

(D) data is insufficient

8. A rod of mass m and length L, pivoted at one of its ends, is hanging vertically. A bullet of the same mass moving at speed v strikes the rod horizontally at a distance x from its pivoted end and gets embedded in it. The combined system now rotates with angular speed ω about the pivot. The maximum angular speed ω_M is achieved for $x = x_M$. Then



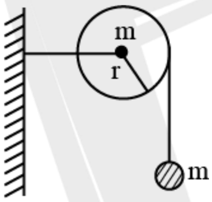
(A) $\omega = \frac{3vx}{L^2+3x^2}$

(B) $\omega = \frac{12vx}{L^2+12x^2}$

(C) $x_M = \frac{L}{\sqrt{3}}$

(D) $\omega_M = \frac{v}{2L}\sqrt{3}$

9. As shown in the figure, a bob of mass m is tied by a massless string whose other end portion is wound on a fly wheel (disc) of radius r and mass m. When released from rest the bob starts falling vertically. When it has covered a distance of h, the angular speed of the wheel will be



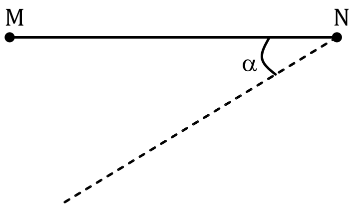
(A) $r\sqrt{\frac{3}{2gh}}$

(B) $r\sqrt{\frac{3}{4gh}}$

(C) $\frac{1}{r}\sqrt{\frac{4gh}{3}}$

(D) $\frac{1}{r}\sqrt{\frac{2gh}{3}}$

10. A thin rod MN, free to rotate in the vertical plane about the fixed end N, is held horizontal. When the end M is released the speed of this end, when the rod makes an angle a with the horizontal, will be proportional to (see figure)



(A) cos α (B) sin α (C) $\sqrt{\cos\alpha}$ (D) $\sqrt{\sin\alpha}$

11. A thin uniform rod of length l and mass m is swinging freely about a horizontal axis passing through its end. Its maximum angular speed is ω. Its centre of mass rises to a maximum height of

(A) $\frac{1}{3}\frac{l^2\omega^2}{g}$ (B) $\frac{1}{6}\frac{l\omega}{g}$ (C) $\frac{1}{2}\frac{l^2\omega^2}{g}$ (D) $\frac{1}{6}\frac{l^2\omega^2}{g}$

ANSWER KEY

1. (a) 2 (b) 6

2. 7

3. 2

4. (A)

5. (B)

6. (B)

7. (C)

8. (A, C, D)

9. (C)

10. (D)

11. (D)