ALTERNATING CURRENT

H-W

Q.4 In a series $L - R$ circuit ($L = 35mH$ and $R = 11\Omega$), a variable emf source $(V = V_0 \sin \omega t)$ of $V_{rms} = 220\,V$ and frequency 50 Hz is applied. Find the current amplitude in the circuit and phase of current with respect to voltage. Draw current-time graph on given graph ($\pi = 22/7$). **(2004)**

Sol^n

$I_0 = ??$ $\dfrac{\mathcal{E}_0}{\sqrt{2}} = (V_{rms}) \Rightarrow \mathcal{E}_0 = 220\sqrt{2}$

$I_0 = \dfrac{\mathcal{E}_0}{|Z|}$

$\omega L = X_L$

$|Z| = \sqrt{(11)^2 + (11)^2}$

$= (11)\sqrt{2}$

$I_0 = \dfrac{220\sqrt{2}}{11\sqrt{2}} = 20\,Amp.$

$\tan\phi = \dfrac{X_L}{R}$

$\tan\phi = 1$

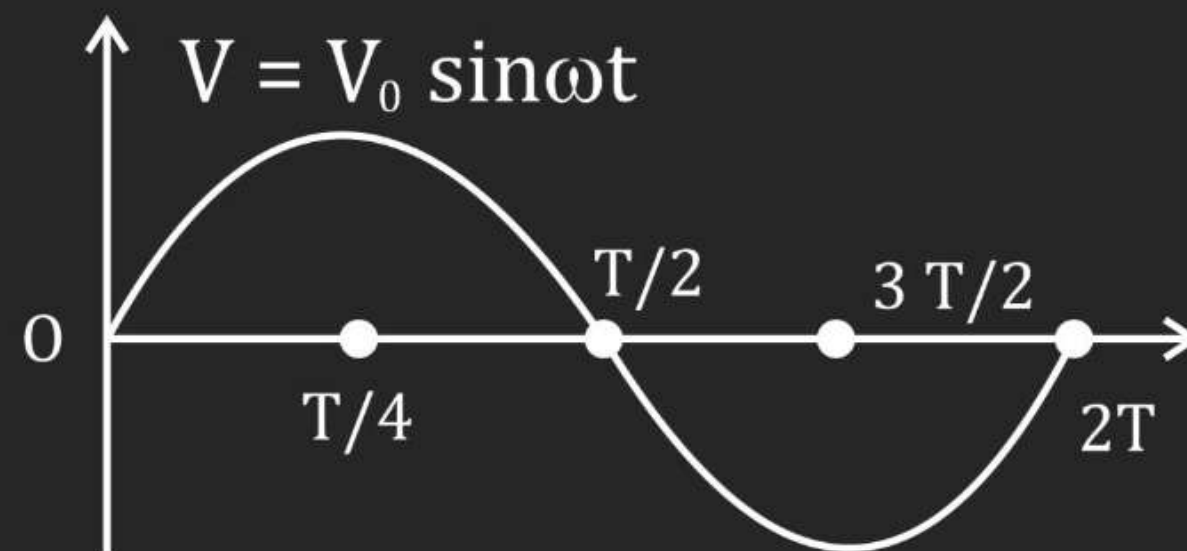$\left(\phi = \dfrac{\pi}{4}\right)$

$X_L = \omega L$
$= 2\pi f L$
$= 2 \times \pi \times 50 \times 35 \times 10^{-3}$
$X_L = 11\,\Omega$

$V = V_0 \sin\omega t$

0, T/4, T/2, 3 T/2, 2T

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Q.5 The instantaneous voltages at three terminals marked X, Y and Z are given by

$$V_X = V_0 \sin \omega t,$$

$$V_Y = V_0 \sin \left(\omega t + \frac{2\pi}{3} \right) \text{ and } V_Z = V_0 \sin \left(\omega t + \frac{4\pi}{3} \right)$$

An ideal voltmeter is configured to read rms value of the potential difference between its terminals. It is connected between points X and Y and then between Y and Z. The reading(s) of the voltmeter will be
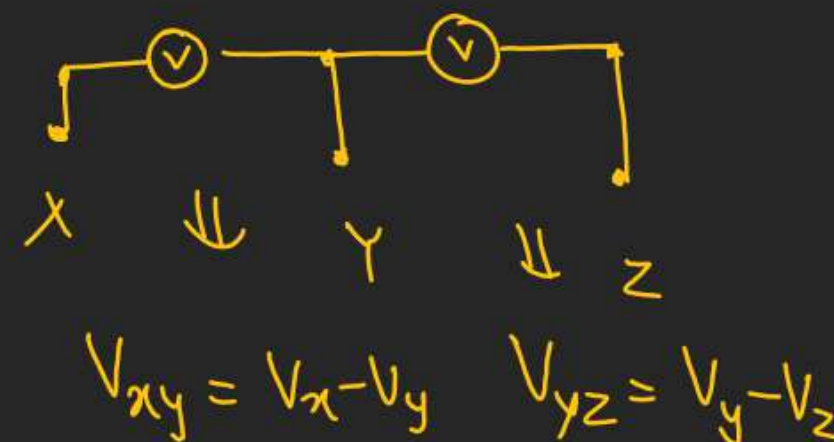
(A) independent of the choice of the two terminals

(B) $\left(V_{XY}^{rms} \right) = V_0$

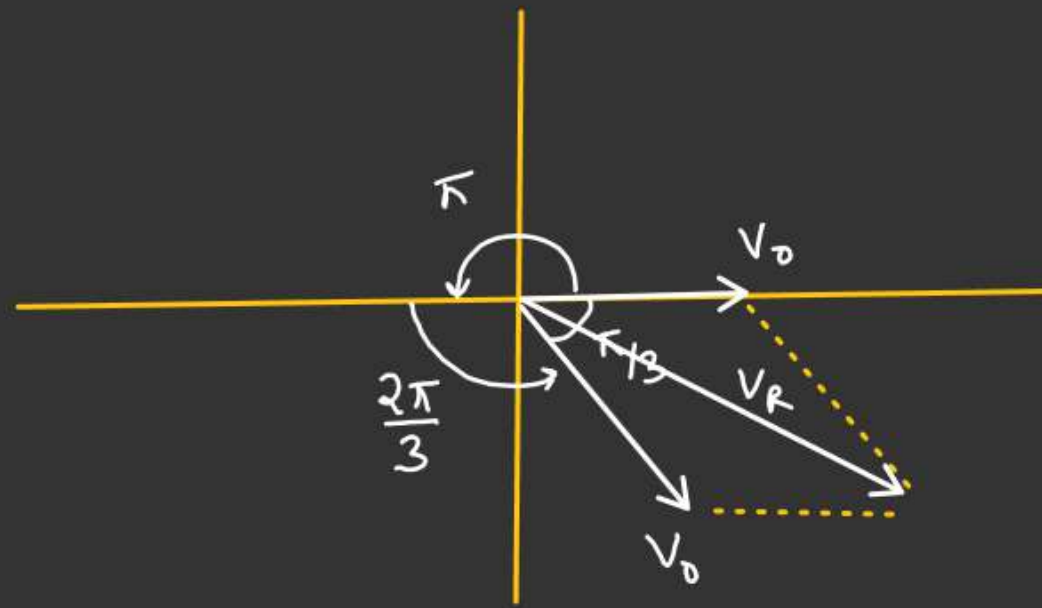(A, D)

(C) $V_{YZ}^{rms} = V_0 \sqrt{\frac{1}{2}}$

(D) $V_{XY}^{rms} = V_0 \sqrt{\frac{3}{2}}$

$X \quad Y \quad Z$

$V_{xy} = V_x - V_y \qquad V_{yz} = V_y - V_z$

$$V_x = V_0 \sin \omega t$$

$$V_y = V_0 \sin\left(\omega t + \frac{2\pi}{3}\right)$$

$$V_2 = V_0 \sin\left(\omega t + \frac{4\pi}{3}\right)$$

$$\underline{V_x - V_y} = V_x + (-V_y)$$

$$= (V_0 \sin \omega t) + \left[-V_0 \sin\left(\omega t + \frac{2\pi}{3}\right)\right]$$

$$= V_0 \sin \omega t + V_0 \sin\left[\omega t + \frac{2\pi}{3} + \pi\right]$$

$$= V_0 \sin \omega t + V_0 \sin\left(\omega t + \frac{5\pi}{3}\right)$$
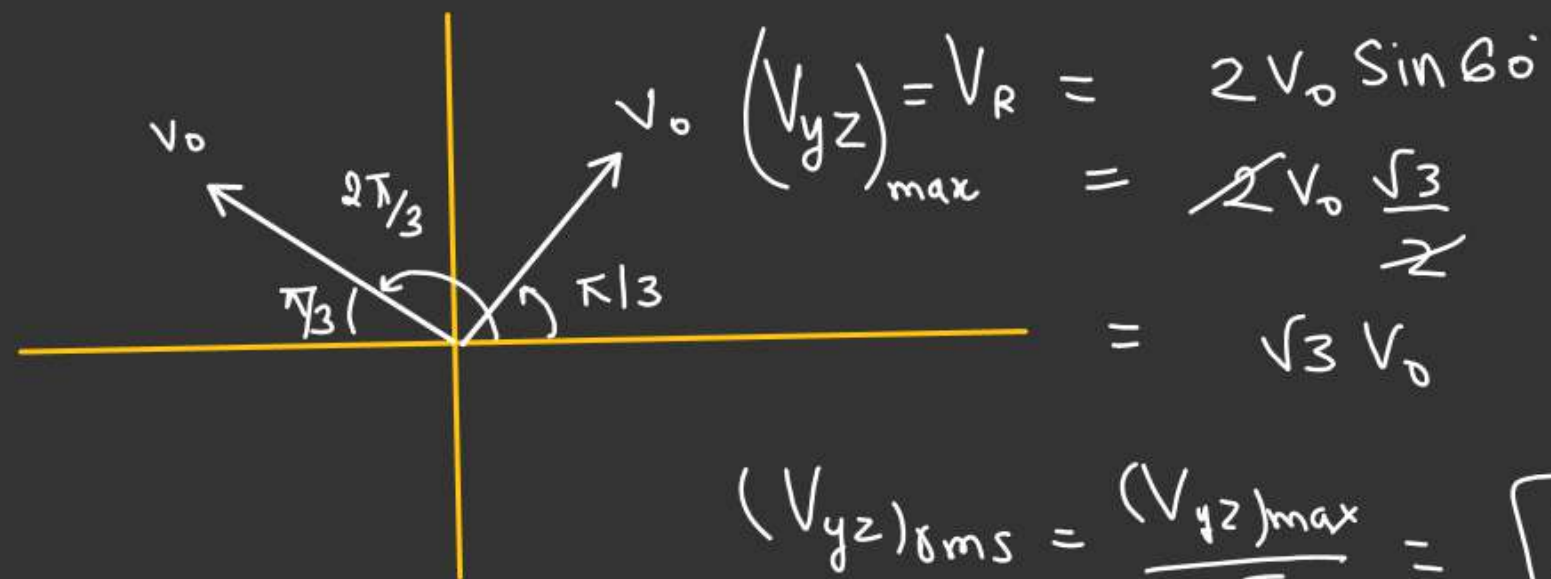
$$V_R = \sqrt{V_0^2 + V_0^2 + 2V_0^2 \cos(\pi/3)}$$

$$V_R = \sqrt{3}\, V_0$$

$$V_{xy} = \sqrt{3}\, V_0$$

$$(V_{xy})_{rms} = \left(\frac{V_{xy}}{\sqrt{2}}\right) = \sqrt{\frac{3}{2}}\, V_0$$

$$V_y - V_z = V_y + (-V_z)$$

$$= V_0 \sin\left(\omega t + \frac{2\pi}{3}\right) + V_0 \sin\left(\omega t + \frac{4\pi}{3} + \pi\right)$$

$$= V_0 \sin\left(\omega t + \frac{2\pi}{3}\right) + V_0 \sin\left(\omega t + \frac{7\pi}{3}\right)$$

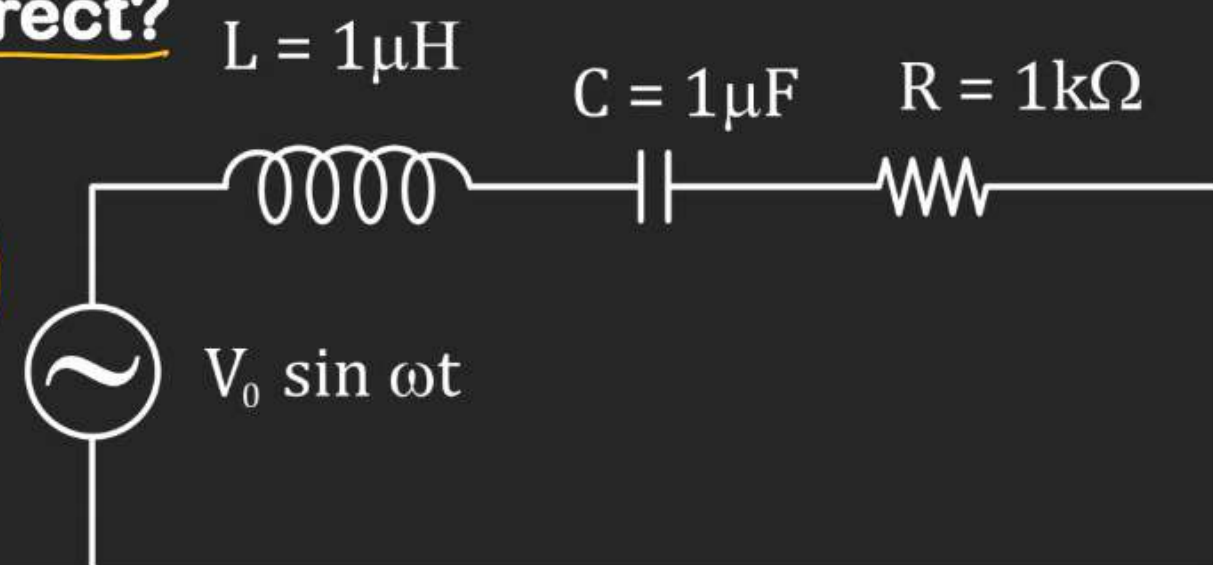$$\left(V_{yz}\right)_{max} = V_R = 2V_0 \sin 60°$$

$$= 2 V_0 \frac{\sqrt{3}}{2}$$

$$= \sqrt{3} V_0$$

$$\left(V_{yz}\right)_{rms} = \frac{\left(V_{yz}\right)_{max}}{\sqrt{2}} = \sqrt{\frac{3}{2}} V_0 .$$

Nishant Jindal

Q.6 In the circuit shown, $L = 1\mu H, C = 1\mu F$ and $R = 1k\Omega$. They are connected in series with an a.c. source $V = V_0 \sin \omega t$ as shown. Which of the following options is/are correct? **(2017)**

$$L = 1\mu H \quad C = 1\mu F \quad R = 1k\Omega$$

$$\omega_o = \frac{1}{\sqrt{LC}}$$

$$\omega_o = \frac{1}{\sqrt{1 \times 10^{-6} \times 1 \times 10^{-6}}} = (10^6)$$

$V_0 \sin \omega t$

$$X_L = \omega L$$

$$\omega \uparrow \quad X_L \uparrow \quad \begin{bmatrix} \omega \to \infty \\ X_C \to 0 \\ X_L \to \infty \end{bmatrix}$$

$$X_C = \frac{1}{\omega C}$$

$$\omega \to 0, \quad X_C \to \infty$$

(A) At $\omega \sim 0$ the current flowing through the circuit becomes nearly zero

(B) The frequency at which the current will be in phase with the voltage is independent of R

(C) The current will be in phase with the voltage if $\omega = 10^4 \text{rads}^{-1}$.

(D) At $\omega \gg 10^6 \text{rads}^{-1}$, the circuit behaves like a capacitor. (Inductive)

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Q.7 At time $t = 0$, terminal A in the circuit shown in the figure is connected to B by a key and an alternating current $I(t) = I_0 \cos(\omega t)$, with $I_0 = 1\,A$ and $\omega = 500\,rads^{-1}$ starts

$7\pi/6$

$-\sqrt{2}$

flowing in it with the initial direction shown in the figure. At $t = \dfrac{7\pi}{6\omega}$, the key is switched from B to D. Now onwards only A and D are connected. A total charge Q flows from the battery to charge the capacitor fully. If $C = 20\mu F, R = 10\Omega$ and the battery is ideal with emf of $50\,V$, identify the correct statement(s). **(2014)**

(A) Magnitude of the maximum charge on the capacitor

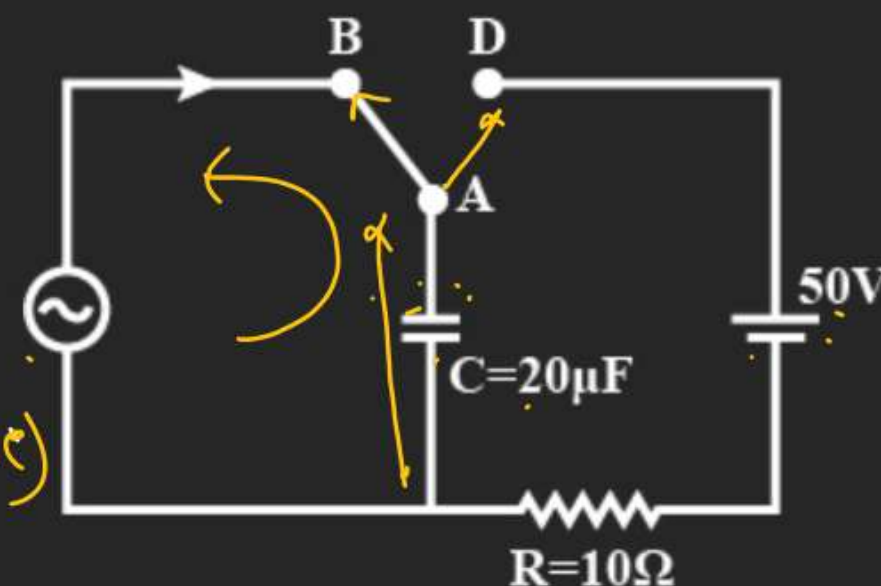before $t = \dfrac{7\pi}{6\omega}$ is $1 \times 10^{-3}\,C$. ✗

(B) The current in the left part of the circuit

just before $t = \dfrac{7\pi}{6\omega}$ is clockwise. ✗ (anti clockwise)

(C) Immediately after A is connected to D,

the current in R is $10\,A$. ✓

(D) $Q = 2 \times 10^{-3}\,C$.

B   D

A

50V

C=20μF

R=10Ω

$$I = I_0 \cos \omega t \qquad \left[0 < t < \frac{7\pi}{6\omega} \right]$$

For Charge to be maximum

At $t = \frac{\pi}{2\omega}$ Charge will be maximum.

$$\frac{dq}{dt} = 0$$

$$I = 0$$

$$I_0 \cos \omega t = 0$$

$$\int_0^q dq = I_0 \int_0^t \cos \omega t \cdot dt$$

$$\cos \omega t = 0$$

$$\omega t = (2n+1)\frac{\pi}{2}$$

$$q = \frac{I_0}{\omega} \sin \omega t$$

$$\underline{\underline{t = (2n+1)\frac{\pi}{2\omega}}}$$

$$q_{max} = \frac{I_0}{\omega} = \frac{1}{500} = 0.2 \times 10^{-2}$$

$$q_{max} = \left(2 \times 10^{-3} \, C \right) \checkmark$$

$$n = 0, 1, 2, 3 \ldots$$

$$I = I_0 \cos \omega t$$

$$\frac{dq}{dt} = \frac{I_0 \cos \omega t}{7\pi/6\omega}$$
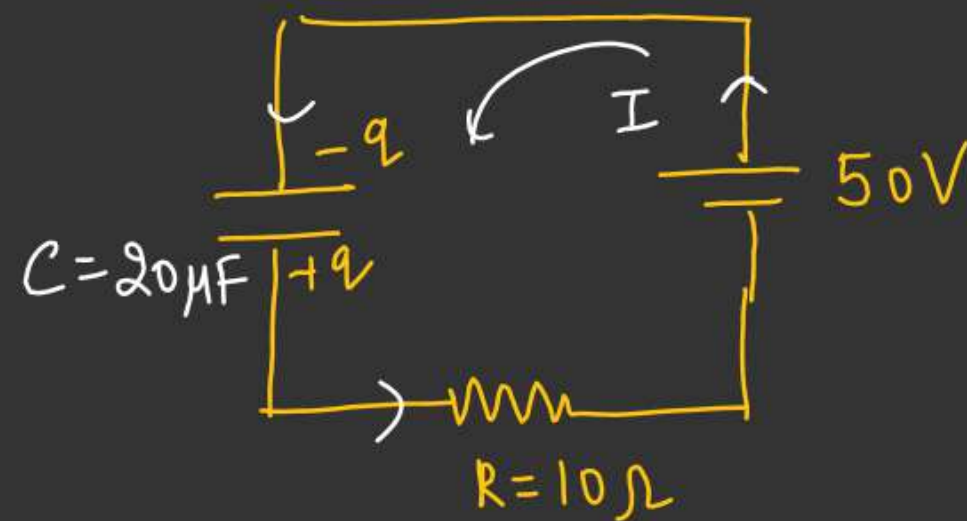
$$\int_0^{q} dq = I_0 \int_0^{} \cos \omega t \cdot dt$$

$$q = \frac{I_0}{\omega} \left[\sin \omega t \right]_0^{\frac{7\pi}{6\omega}}$$

$$q = \frac{I_0}{\omega} \sin\left(\frac{7\pi}{6}\right)$$

$$q = \frac{1}{500} \times \left(-\frac{1}{2}\right)$$

$$q = \underline{-(10)^{-3} C}$$

Total charge

$$= 10^{-3} - (-10^{-3})$$

$$= \underline{2 \times 10^{-3} C}$$

$$C = 20 \mu F \quad | +q \quad | -q \quad \text{I} \quad \uparrow \quad 50V$$

$$R = 10 \Omega$$

$$50 + q/_c - iR = 0$$

$$50 + \frac{10^{-3}}{20 \times 10^{-6}} = i \times 10$$

$$5 + \frac{10^3}{200}$$

$$5 + \frac{10}{2} = 10 = \overset{\circ}{i}$$

Charge flow at steady state.

$$= CV = 20 \times 10^{-6} \times 50$$
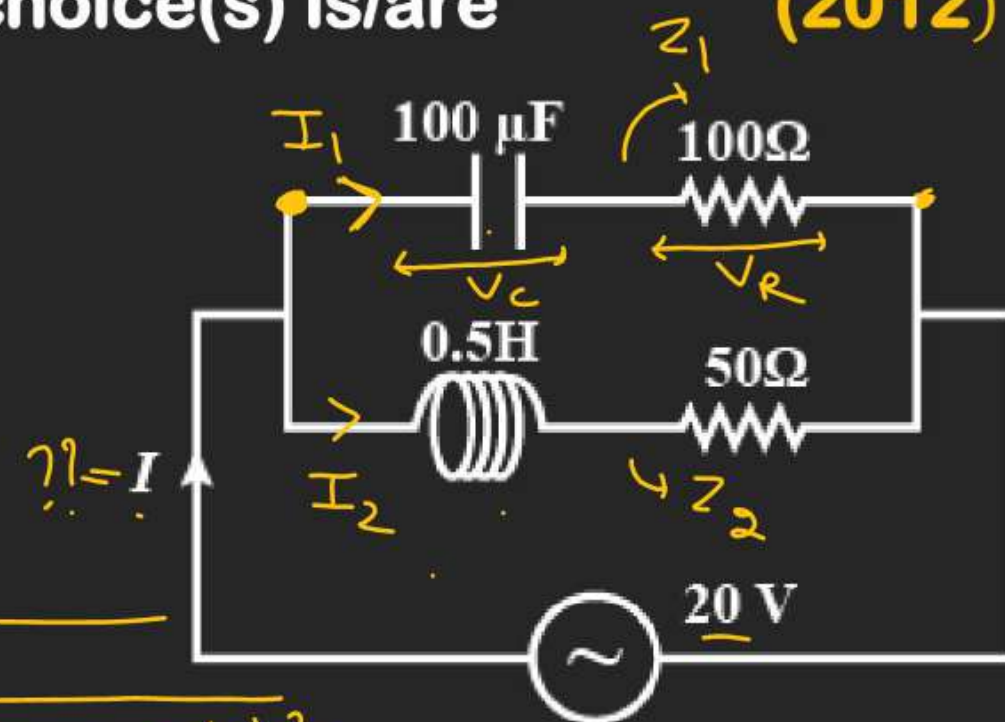
$$= \underline{10^{-3}}$$

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Q.8 In the given circuit, the AC source has $\omega = 100\,\text{rad/s}$. Considering the inductor and capacitor to be ideal, the correct choice(s) is/are **(2012)**

(A) the current through the circuit, I is 0.3 A.

(B) the current through the circuit, I is $0.3\sqrt{2}$ A.

(C) the voltage across 100Ω resistor $= 10\sqrt{2}$ V.

(D) the voltage across 50Ω resistor $= 10$ V.

$$\left(V_R\right)_{100\Omega} = \frac{1}{5\sqrt{2}} \times 100$$

$$= \frac{20}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$$

$$= 10\sqrt{2}$$

$$Z_1 = \sqrt{R^2 + \frac{1}{\omega^2 c^2}} = \sqrt{(100)^2 + \frac{1}{(100)^2 \times (100 \times 10^{-6})^2}}$$

$$I_1 = \frac{20}{Z_1} = \frac{20}{100\sqrt{2}} = 100\sqrt{2}$$

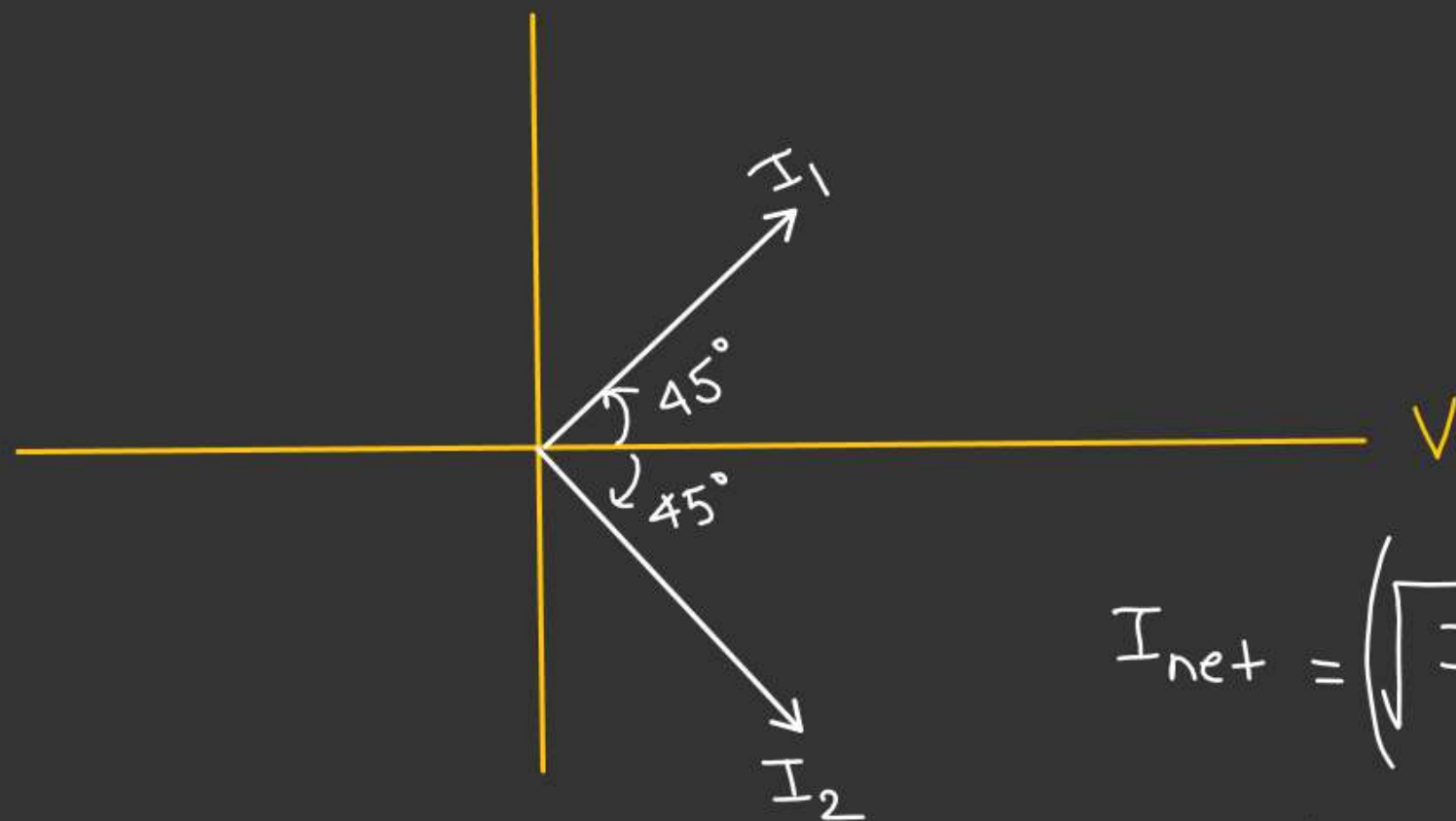$$I_1 = \frac{1}{5\sqrt{2}}$$

$$\left(V_R\right)_{50\Omega} = I_2 \times 50$$

$$= \frac{2}{5\sqrt{2}} \times 50 = 10\sqrt{2} ✓$$

$$I_2 = \frac{20}{50\sqrt{2}} = \left(\frac{2}{5\sqrt{2}}\right)$$

$$V_c^2 + V_R^2 = (20)^2$$

$$Z_2 = \sqrt{(50)^2 + (100)^2 (5 \times 10^{-1})^2}$$

$$Z_2 = 5\sqrt{10^2 + 10^2}$$

$$= 50\sqrt{2}$$

$45°$

$45°$

I_1

I_2

V

$$I_{net} = \left(\sqrt{I_1^2 + I_2^2} \right)$$

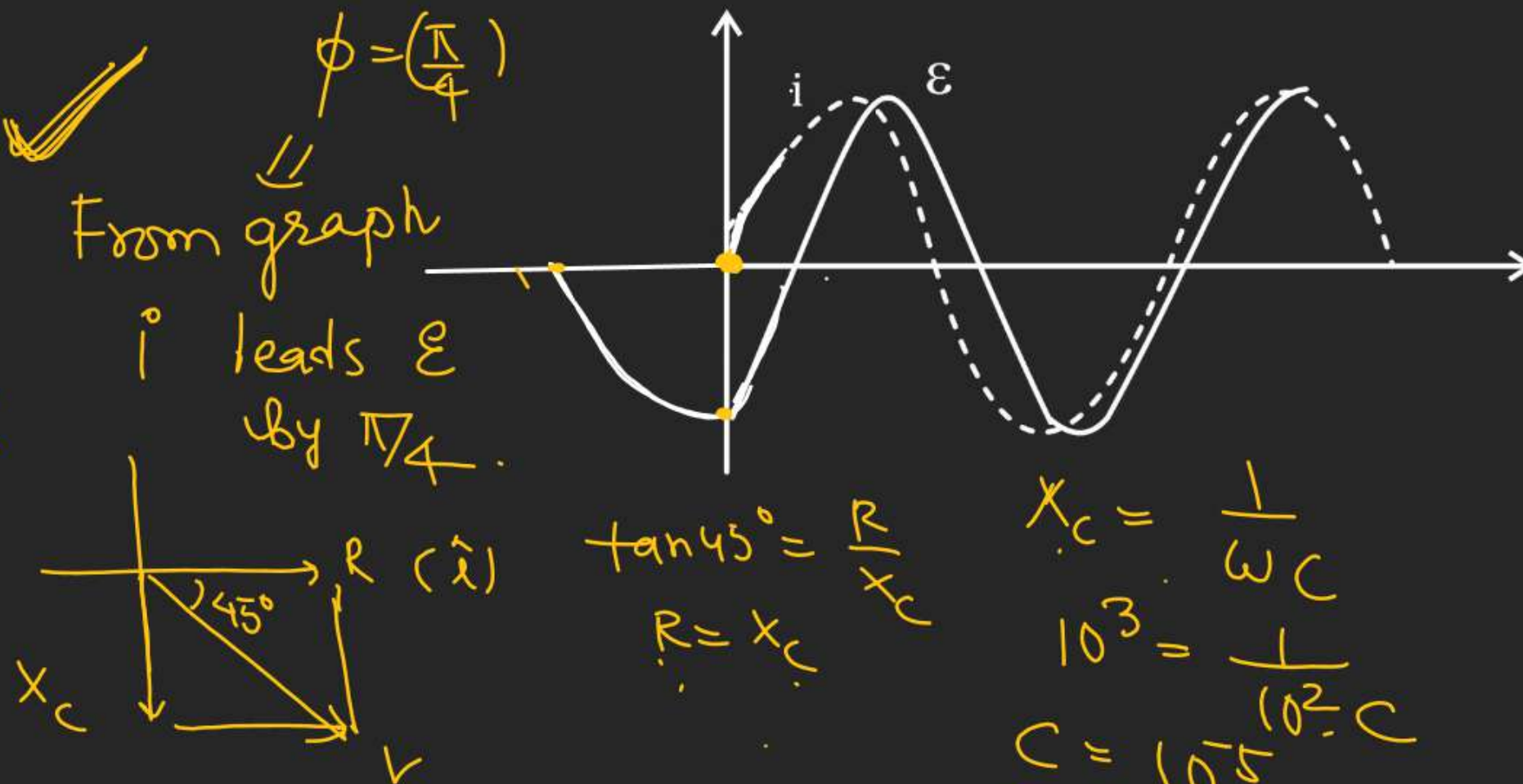$$= \underline{\hspace{3cm}}.$$

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$\omega = 100$

Q.9 When an AC source of emf $\varepsilon = E_0\sin(100t)$ is connected across a circuit, the phase difference between the emf e and the current i in the circuit is observed to be $\pi/4$, as shown in the diagram. If the circuit consists possibly only of $R - C$ or $R - L$ or $L - C$ in series, find the relationship between the two elements.

(A) $R = 1k\Omega, C = 10\mu F$

(B) $R = 1k\Omega, C = 1\mu F$

(C) $R = 1k\Omega, L = 10H$

(D) $R = 1k\Omega, L = 1H$.

$\phi = \left(\dfrac{\pi}{4}\right)$

From graph

i leads ε by $\pi/4$.

$\tan 45° = \dfrac{R}{X_C}$

$R = X_C$

$X_C = \dfrac{1}{\omega C}$

$10^3 = \dfrac{1}{10^2 \cdot C}$

$C = 10^{-5}$

$C = 10\mu F$

45° R (i) X_C V

H.W

Q.1 Determine the rms value of a semi-circular current wave which has a maximum value of a.

(A) $(1/\sqrt{2})a$

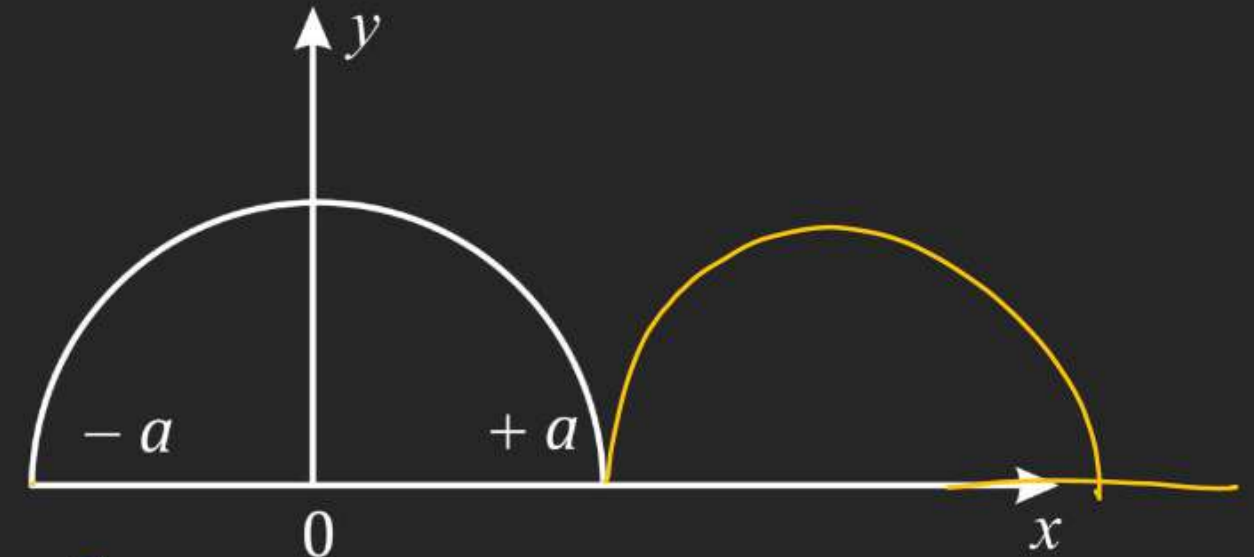(B) $\sqrt{(3/2)}a$

(C) $\sqrt{(2/3)}a$

(D) $(1/\sqrt{3})a$

$$i_{rms} = \sqrt{\overline{i^2}}$$

$$y^2 + x^2 = a^2$$

$$y^2 = (a^2 - x^2)$$

$$y = \left(\sqrt{a^2 - x^2}\right)$$

$$y^2 = (a^2 - x^2)$$

$$\frac{\displaystyle\int_{-a}^{a} y^2 \, dx}{\displaystyle\int_{-a}^{a} dx} = \left(\overline{y^2}\right)$$

$$y_{rms} = \sqrt{\overline{y^2}}$$
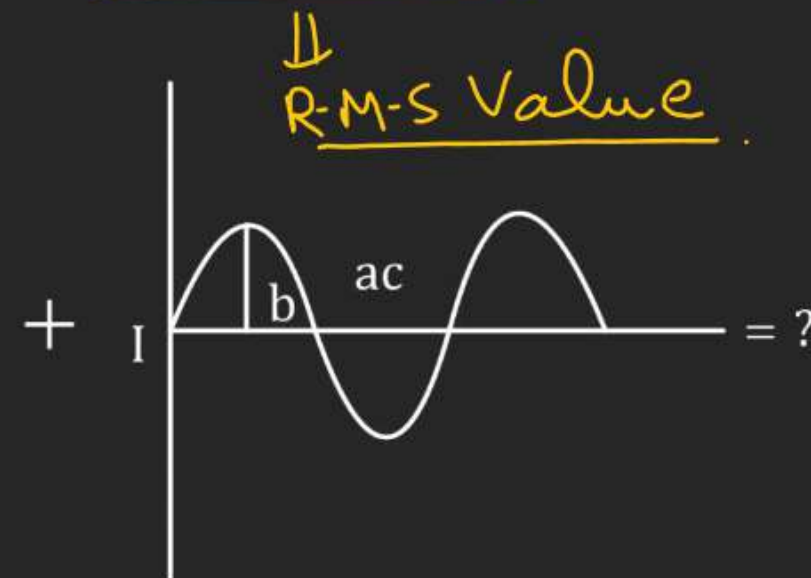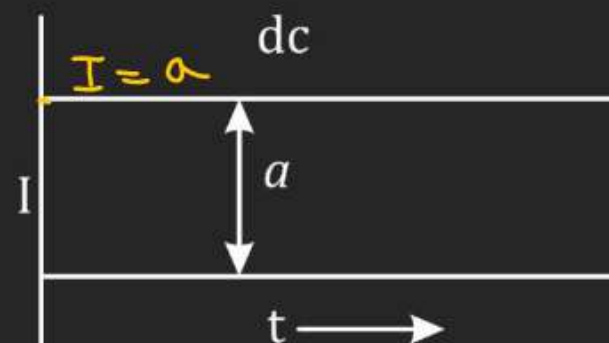
ALTERNATING CURRENT

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Q.2 If a direct current of value a ampere is superimposed on an alternative current $I = \underline{b\sin \omega t}$ flowing through a wire, what is the <u>effective value</u> of the resulting current in the circuit?

(A) $\left[a^2 - \frac{1}{2}b^2\right]^{1/2}$

(B) $\left[a^2 + b^2\right]^{1/2}$

(C) $\left[\frac{a^2}{2} + b^2\right]^{1/2}$

(D) $\left[a^2 + \frac{1}{2}b^2\right]^{1/2}$

$I = a$ dc

a

t →

$\sqcap$
R-M-S Value

b ac

+ $= ?$

$I_R = \textcircled{a} + \textcircled{b} \sin\omega t$

$I_R^2 = (a + b\sin\omega t)^2 = \left(a^2 + b^2\sin^2\omega t + 2ab\sin\omega t\right)$

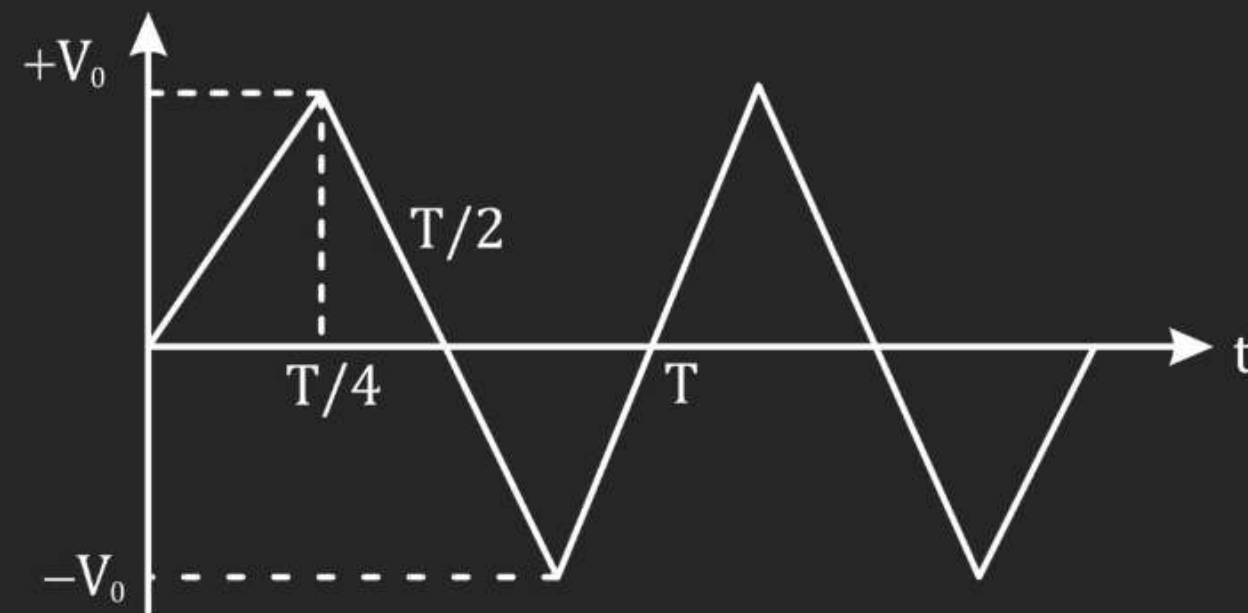$$\overline{I_R^2} = \int_0^{\frac{2\pi}{\omega}} (a^2 + b^2\sin^2\omega t + 2ab\sin\omega t)\, dt = a^2 + \frac{b^2}{2}$$

$I_{rms} = \sqrt{a^2 + \frac{b^2}{2}}$

$\frac{2\pi}{\omega}\int_0^{} dt$

H.W

Q.3 The voltage time $(V - t)$ graph for triangular wave having peak value V_0 is as shown in

H-w — . Fig in Q-3 .

ALTERNATING CURRENT

Q.4 The rms value of V in time interval from $t = 0$ to $T/4$ is

(A) $\dfrac{V_0}{\sqrt{3}}$

(B) $\dfrac{V_0}{2}$

(C) $\dfrac{V_0}{\sqrt{2}}$

(D) None of these

H.W Fig in Q.No-3.

Q.5 In the above question, the average value of voltage (V) in one time period will be

(A) $\dfrac{V_0}{\sqrt{3}}$

(B) $\dfrac{V_0}{2}$

(C) $\dfrac{V_0}{\sqrt{2}}$

(D) 0

H-W

ALTERNATING CURRENT

For problems 6 – 8

A series LCR circuit containing a resistance of 120Ω has angular resonance frequency $4 \times 10^5 \text{rads}^{-1}$. At resonance the voltages across resistance and inductance are 60 V and 40 V. respectively.

6. The value of inductance L is

(A) 0.1mH (B) 0.2mH (C) 0.35mH (D) 0.4mH

7. The value of capacitance C is

(A) $\dfrac{1}{32}\mu\text{F}$ (B) $\dfrac{1}{16}\mu\text{F}$ (C) $32\mu\text{F}$ (D) $16\mu\text{F}$

8. At what frequency, the current in the circuit lags the voltage by $45°$?
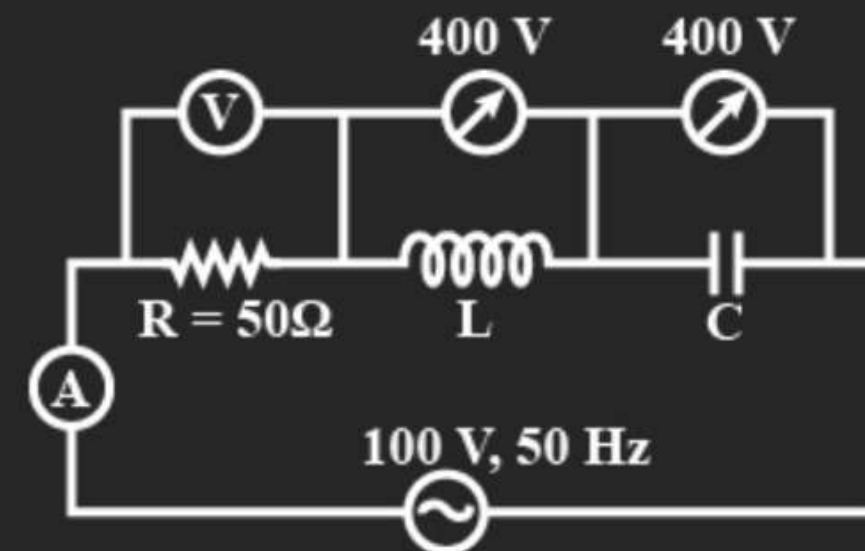
(A) $4 \times 10^5 \text{rads}^{-1}$ (B) $3 \times 10^5 \text{rad}$

(C) $8 \times 10^5 \text{rads}^{-1}$ (D) $2 \times 10^5 \text{rads}^{-1}$

ALTERNATING CURRENT

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Q.9 In the series LCR circuit (Fig), the voltmeter and ammeter readings are:

(A) $V = 100$ V, $I = 2$ A
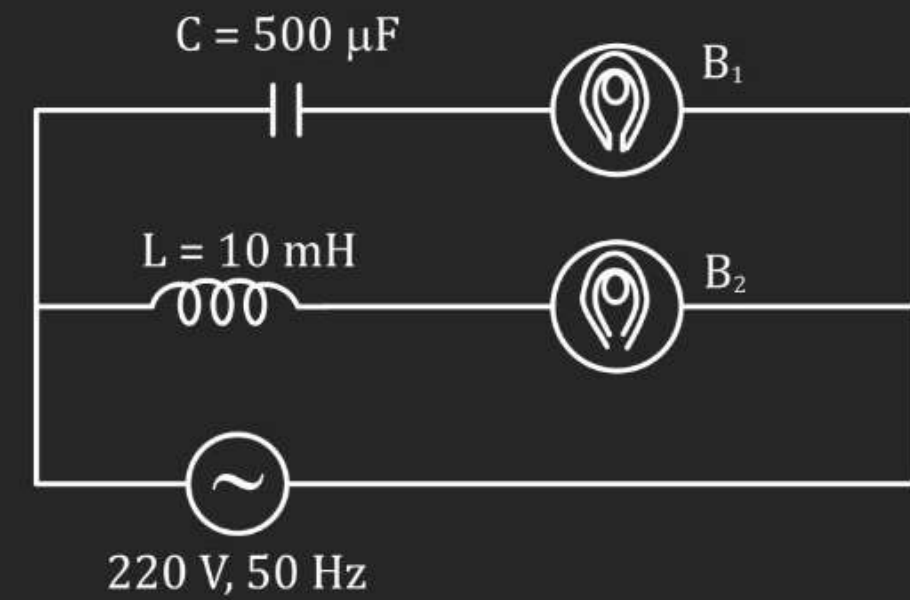
(B) $V = 100$ V, $I = 5$ A

(C) $V = 1000$ V, $I = 2$ A

(D) $V = 300$ V, $I = 1$ A

Q.10 In the circuit shown in Fig, if both the bulbs B_1 and B_2 are identical,

(A) their brightness will be the same

(B) B_2 will be brighter than B_1

(C) B_1 will be brighter than B_2

(D) only B_2 will glow because the capacitor has

infinite impedance

C = 500 µF

B_1

L = 10 mH

B_2

220 V, 50 Hz

Q.10 The circuit given in Fig. has a resistance less choke coil L and a resistance R. The voltages across R and L are also given in the figure. The virtual value of the applied voltage is

(A) 100 V

(B) 200 V

(C) 300 V

(D) 400 V

AC source