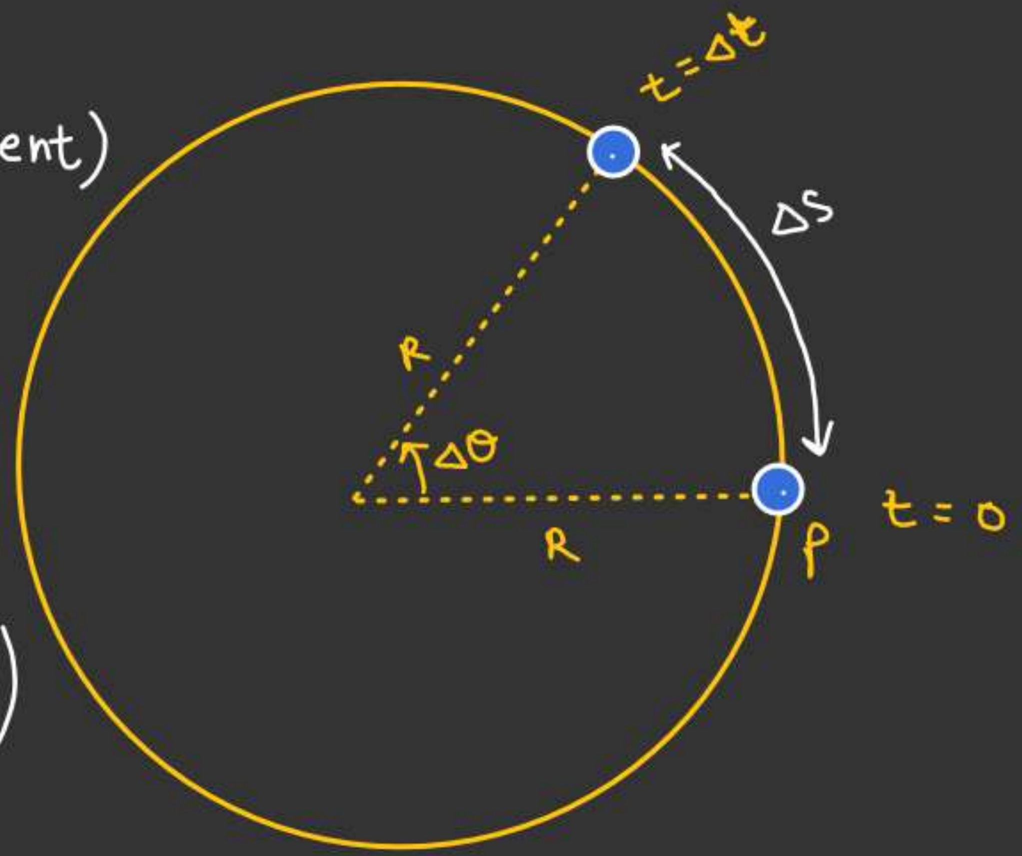
Nishant Jindal

CIRCULAR MOTION

Kinematics of Circular Motion :→

θ = (Angular displacement)



$$\Delta S = R\Delta\theta$$

$$\left(\frac{\Delta S}{\Delta t}\right) = R\left(\frac{\Delta\theta}{\Delta t}\right)$$

$$\Downarrow$$

(Speed) → $\boxed{v = R\omega}$

$$\Downarrow$$

$$\left(\frac{dv}{dt}\right) = R\left(\frac{d\omega}{dt}\right)$$

$$\Downarrow$$

$$\boxed{a_t = R\alpha}$$

$$\omega_{avg} = \left(\frac{\Delta\theta}{\Delta t}\right)$$

$$\omega_{inst} = \lim_{\Delta t \to 0}\left(\frac{\Delta\theta}{\Delta t}\right) = \left(\frac{d\theta}{dt}\right)$$

$$\boxed{\omega_{inst} = \frac{d\theta}{dt}}$$

$$\Downarrow$$

Rate of change of angular displacement w.r.t time.

Nishant Jindal

$$\boxed{a_t = R\alpha}$$

a_t = Tangential accleration.

$$\boxed{a_t = \frac{d}{dt}|\vec{v}|}$$

$$\boxed{\alpha = \frac{d\omega}{dt}}$$

$\Downarrow$

Angular accleration $\Big\} \Rightarrow$ Rate of Change of angular velocity is Called angular accleration.

$$\alpha = \frac{d\omega}{dt} = \frac{d\omega}{d\theta} \times \left(\frac{d\theta}{dt}\right)$$

$$\boxed{\alpha = \omega \frac{d\omega}{d\theta}}$$

$$\alpha = \frac{d\omega}{dt} = \frac{d}{dt}\left(\frac{d\theta}{dt}\right)$$

$$\boxed{\alpha = \frac{d^2\theta}{dt^2}}$$

Nishant Jindal

(*) Uniform Circular Motion

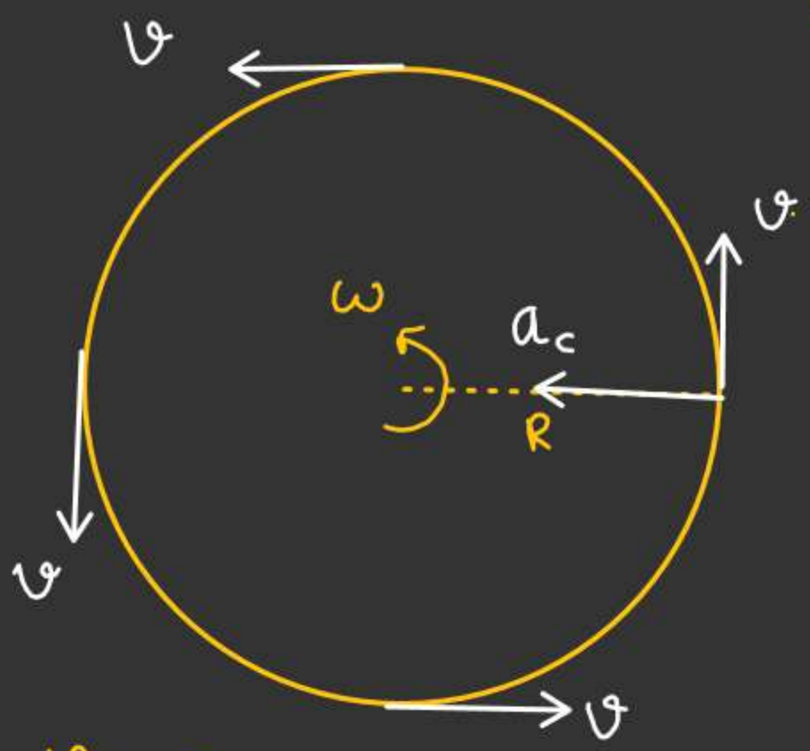
↳ Speed = constant; (Direction Changing)

⇓

Due to change in direction velocity changing.

⇓

Due to change in direction of velocity there is an accleration always towards the center of the circle called Centripetal accleration or radial accleration



$v = R\omega$

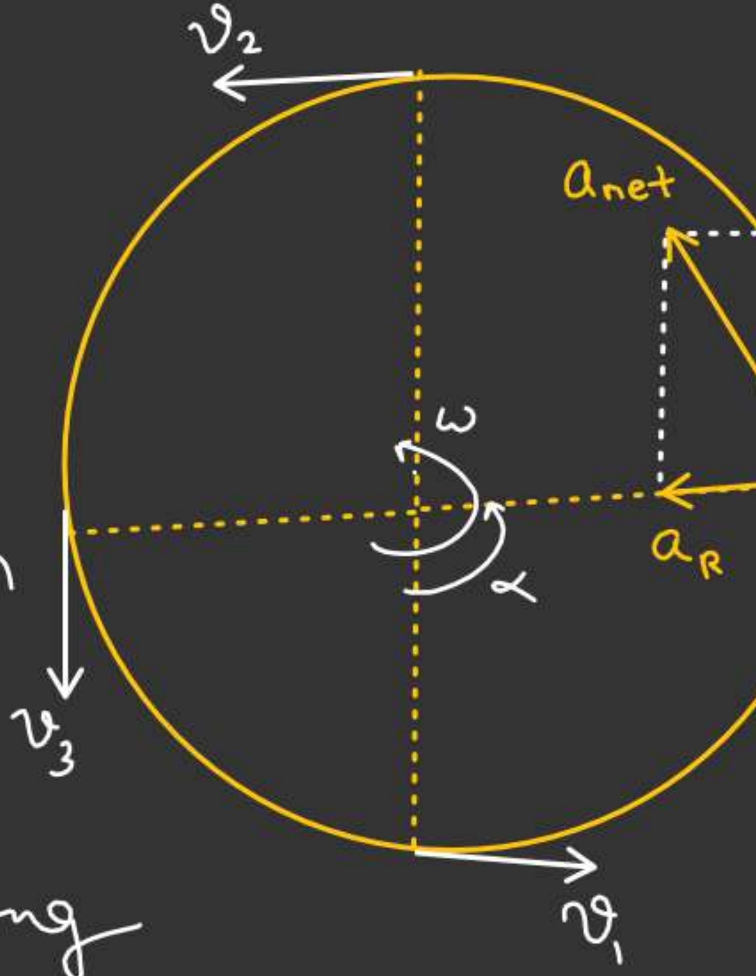
$v = c$
$\omega = c$

$$\boxed{a_c = \frac{v^2}{R} = \omega^2 R}$$

Nishant Jindal

** Non-Uniform Circular Motion :→

↳ Speed and direction of velocity both Changing



⇒ Due to change in direction of Velocity the accleration is towards the Center and is Called Centripetal accleration

$$\boxed{a_c = \frac{v^2}{R} = \omega^2 R}$$

$$a_{net} = \sqrt{a_t^2 + a_R^2}$$

$$a_{net} = \sqrt{(\alpha R)^2 + (\omega^2 R)^2}$$

⇒ Due to change in the Speed we have an accleration along tangential direction Called tangential accleration

$$\boxed{a_t = \frac{d}{dt}|v| = \frac{d}{dt}(R\omega) = R\frac{d\omega}{dt} = R\alpha}$$

Nishant Jindal

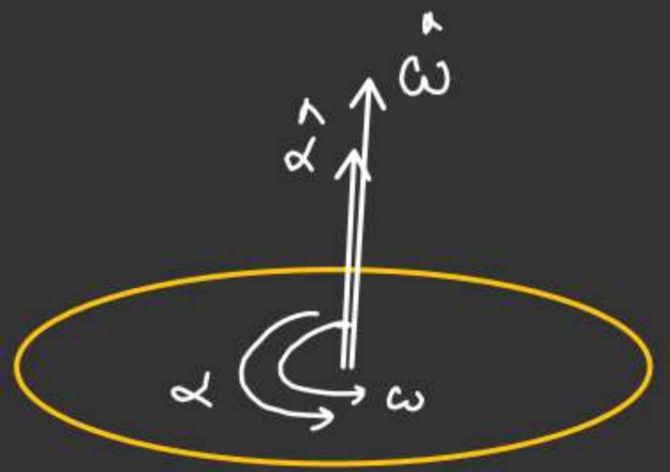
Kinematics Equations for Circular Motion:—

$v = u + at$

$S = ut + \frac{1}{2}at^2$

$v^2 = u^2 + 2aS$

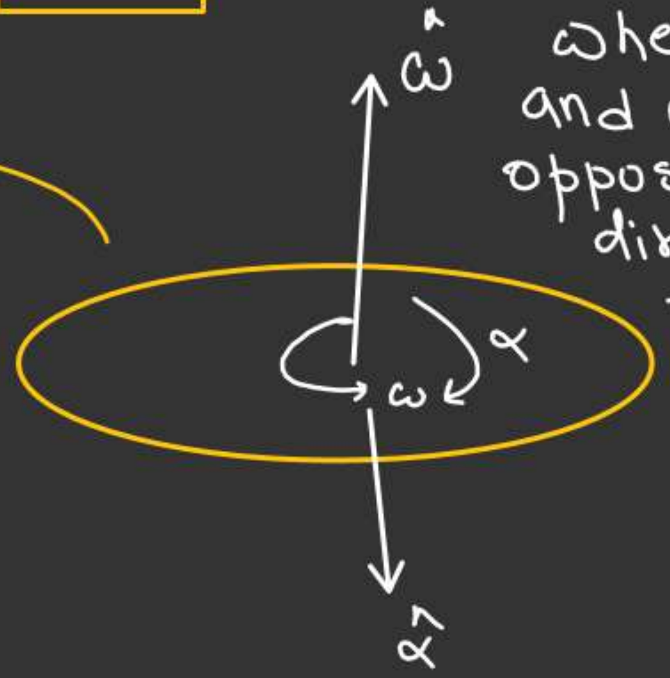
$$\boxed{\begin{aligned} \omega &= \omega_0 + \alpha t \\ \theta &= \omega_0 t + \tfrac{1}{2}\alpha t^2 \\ \omega^2 &= \omega_0^2 + 2\alpha\theta \end{aligned}}$$



when α & ω in Same direction then $(+\alpha)$

$\boxed{\alpha = C}$

$$\boxed{\begin{aligned} \omega &= \omega_0 - \alpha t \\ \theta &= \omega_0 t - \tfrac{1}{2}\alpha t^2 \\ \omega^2 &= \omega_0^2 - 2\alpha\theta \end{aligned}}$$



when α and ω in opposite direction then $\alpha \rightarrow (-ve)$

Nishant Jindal

$$\omega = f(t)$$

$$\alpha = \frac{d\omega}{dt}$$

$$\int_{\omega_0}^{\omega} d\omega = \int_0^t \underline{\alpha}\, dt$$

$$\omega - \omega_0 = \alpha \int_0^t dt$$

$$\omega - \omega_0 = \alpha t$$

$$\boxed{\omega = \omega_0 + \alpha t}$$

At $t = t$, $\omega = \omega$, θ. | At $t = 0$, $\omega = \omega_0$, $\theta = 0$

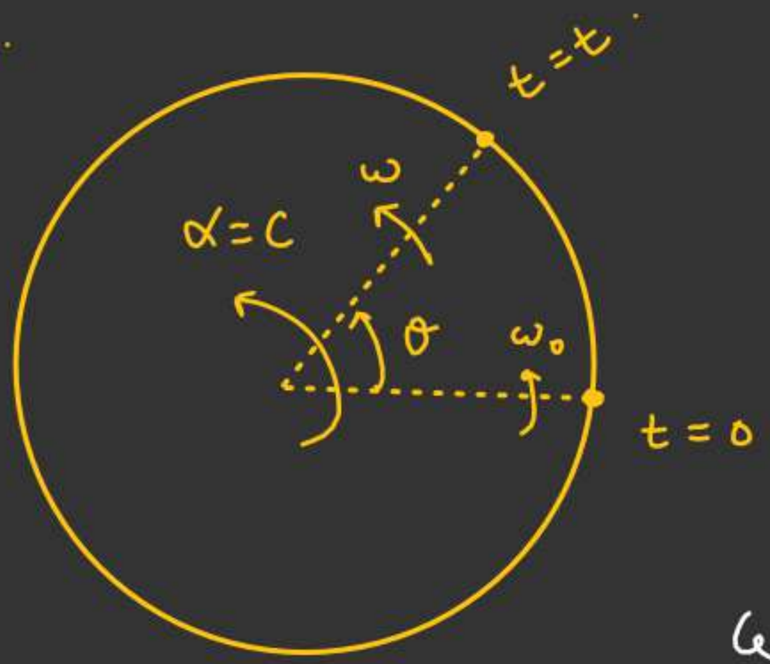
$$\theta = f(t)$$

$$\omega = \frac{d\theta}{dt}$$

$$\int d\theta = \int \omega\, dt$$

$$\int_0^\theta d\theta = \int_0^t (\omega_0 + \alpha t)\, dt$$

$$\boxed{\theta = \omega_0 t + \frac{1}{2}\alpha t^2}$$



$$\omega = \left(\frac{d\theta}{dt}\right)$$ radian, second

$\omega \rightarrow$ radian/second

$\alpha = \frac{d\omega}{dt} \rightarrow \frac{\text{radian/second}}{\text{second}}$

$(\alpha = \text{radian/sec}^2)$

Nishant Jindal

$$\omega^2 = \omega_0^2 + 2\alpha\theta$$

$$\alpha = \frac{d\omega}{dt} = \omega\frac{d\omega}{d\theta}$$

$$\int_0^\theta \alpha\, d\theta = \int_{\omega_0}^{\omega} \omega\, d\omega$$

$$\alpha\int_0^\theta d\theta = \frac{\omega^2 - \omega_0^2}{2}$$

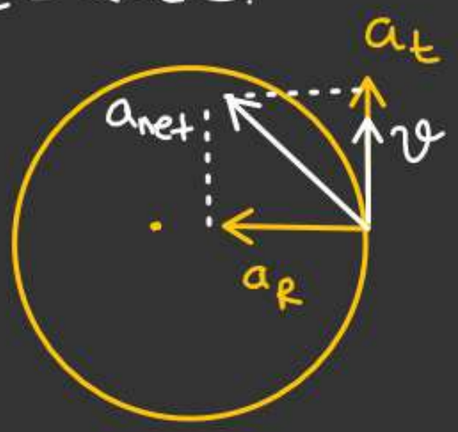
$$2\alpha\theta = \omega^2 - \omega_0^2$$

$$\boxed{\omega^2 = \omega_0^2 + 2\alpha\theta}$$

$\theta = t^3 + t^2 + 2t + 1$

(i) Find α & ω at $t = 2\,sec$

(ii) If $R = 2m$ then find.
a_t, a_R and a_{net} at $t = 2\,sec$.



$$\omega = \frac{d\theta}{dt} = (3t^2 + 2t + 2)$$

$$\omega_{t=2sec} = 12 + 4 + 2 = 18\ rad/sec$$

$$\alpha = \frac{d\omega}{dt} = (6t + 2)$$

$$\alpha_{t=2sec} = 14\ rad/sec^2$$

At $t = 2\,sec$

$$a_t = R\alpha$$

$$a_t = (2 \times 14)$$

$$= 28\ m/s^2$$

$$a_R = \omega^2 R$$

$$= (18)^2 \times 2$$

$$a_{net} = \sqrt{a_R^2 + a_t^2}$$

Nishant Jindal

$$\alpha = -K\omega^2 \qquad K = \text{Constant.}$$

α = angular accleration.

Find $\begin{bmatrix} \omega = f(t) \\ \theta = f(t) \end{bmatrix}$ At $t = 0$, $\omega = \omega_0$.

$$\omega \frac{d\omega}{d\theta} = -K\omega^2$$

$$\frac{\omega\, d\omega}{\omega^2} = -K\, d\theta$$

$$\int_{\omega_0}^{\omega} \frac{d\omega}{\omega} = -K\int_0^{\theta} d\theta$$

$$\ln\left(\frac{\omega}{\omega_0}\right) = -K\theta$$

$$\frac{\omega}{\omega_0} = e^{-K\theta}$$

$$\omega = \omega_0 e^{-K\theta}$$

$$\frac{d\theta}{dt} = \omega_0 e^{-K\theta}$$

$$\int_0^{\theta} \frac{d\theta}{e^{-K\theta}} = \omega_0 \int_0^{t} dt$$

$$\int_0^{\theta} e^{K\theta} \cdot d\theta = \omega_0 t$$

$$\left[\frac{e^{K\theta}}{K}\right]_0^{\theta} = \omega_0 t$$

$$e^{K\theta} - 1 = K\omega_0 t$$

$$e^{K\theta} = (1 + K\omega_0 t)$$

$$K\theta = \ln(1 + K\omega_0 t)$$

$$\boxed{\theta = \frac{1}{K}\ln(1 + K\omega_0 t)}$$

Nishant Jindal

$\omega = (a + bt)$ (a & b are constant)

At $t = 0,\ \theta = 0$.

$$\begin{cases} \theta = f(t) \\ \alpha = f(t) \end{cases}$$

$$\omega = \frac{d\theta}{dt}$$

$$(a + bt) = \frac{d\theta}{dt}$$

$$\int_0^\theta d\theta = \int_0^t (a + bt)\,dt$$

$$\theta = \left[at + \frac{bt^2}{2} \right]$$

Ans.

$$\alpha = \frac{d\omega}{dt}.$$

$$\alpha = \frac{d}{dt}(a + bt)$$

$$\alpha = \frac{d}{dt}(a) + b\frac{d}{dt}(t)$$

$[\alpha = b]$ Ans

$\Downarrow$

Constant.

Nishant Jindal

Q: A particle performing a Curvilinear motion have velocity vector as

$$\vec{v} = (t^2\hat{i} + t\hat{j})$$

Find a_t, a_R & a_{net} at $t = 1\,sec$.

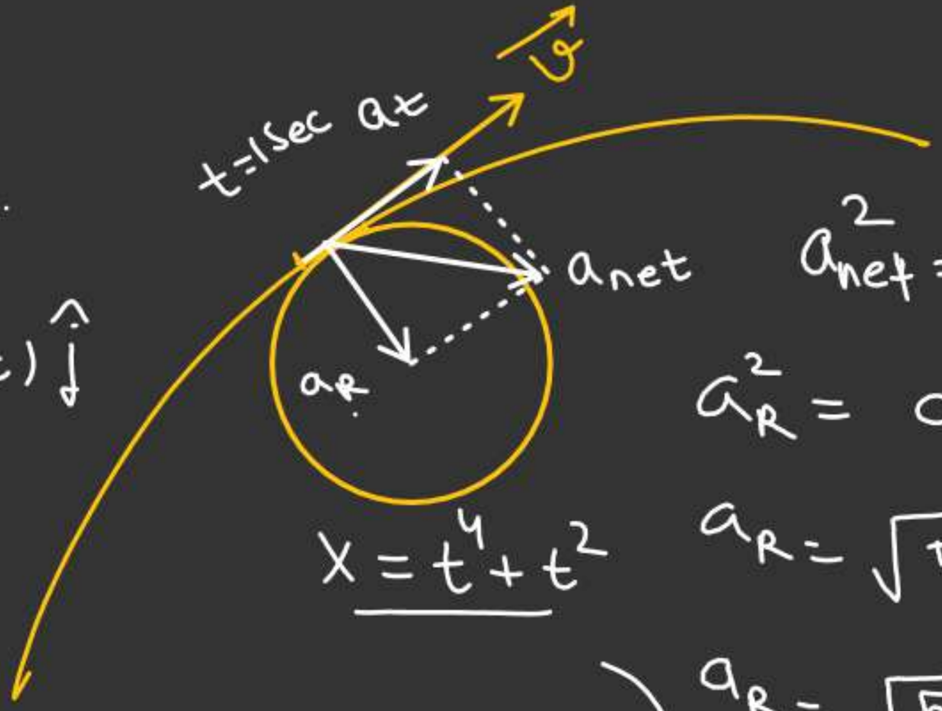
Sol^n

$$\vec{a}_{net} = \frac{d\vec{v}}{dt} = \frac{d}{dt}(t^2)\,\hat{i} + \frac{d}{dt}(t)\,\hat{j}$$

$$\vec{a}_{net} = (2t)\,\hat{i} + \hat{j}$$

$$|\vec{a}_{net}| = \sqrt{4t^2 + 1}$$

$$|\vec{a}_{net}|_{t=1sec} = \sqrt{4(1)+1} = \sqrt{5}\ m/s^2$$



$$a_{net}^2 = a_R^2 + a_t^2$$

$$a_R^2 = a_{net}^2 - a_t^2$$

$$a_R = \sqrt{5 - \left(\frac{3}{\sqrt{2}}\right)^2}$$

$$a_R = \sqrt{5 - \frac{9}{2}}$$

$$a_R = \left(\frac{1}{\sqrt{2}}\right) m s^{-2}$$

$$x = t^4 + t^2$$

$$a_t = \frac{d|\vec{v}|}{dt} = \frac{d}{dt}\left[\sqrt{\underbrace{t^4 + t^2}_{x}}\right]$$

$$a_t = \frac{d}{dx}(\sqrt{x}) \times \frac{dx}{dt} = \frac{1}{2\sqrt{x}} \times \frac{d}{dt}(t^4 + t^2)$$

$$a_t = \frac{1}{2\sqrt{t^4+t^2}}(4t^3 + 2t) = \left(\frac{2t^3 + t}{\sqrt{t^4 + t^2}}\right)$$

$$(a_t)_{t=1sec} = \left(\frac{3}{\sqrt{2}}\right) m/s^2 \longleftarrow$$