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Matrix

(7) Cofactor

1) Cofactor is Rep by C_{ij}

& Minor of a_{ij} is Rep by M_{ij}

2) $\boxed{C_{ij} = (-1)^{i+j} \cdot M_{ij}}$

3) Sign Notation

$$\begin{vmatrix} + & - & + \\ - & + & - \\ + & - & + \end{vmatrix}$$

$$\Delta = \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}$$

Cofactor of $b = -\begin{vmatrix} d & f \\ g & i \end{vmatrix}$

Cofactor of $h = -\begin{vmatrix} a & c \\ d & f \end{vmatrix}$

C_{13} = Cofactor of 1st Row, 3 Col.'s Element

= Cofactor of a_{13}

$= (-1)^{1+3} M_{13}$

$C_{13} = M_{13}$

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Q. $\Delta = \begin{vmatrix} -1 & 2 & 2 \\ 0 & -5 & 3 \\ 6 & 7 & -9 \end{vmatrix}$

$C_{13} = (-1)^{1+3} M_{13} = M_{13} = \begin{vmatrix} 0 & -5 \\ 6 & 7 \end{vmatrix} = 0+30 = 30$

$C_{21} = (-1)^{2+1} M_{21} = -M_{21} = -\begin{vmatrix} 2 & 2 \\ 7 & -9 \end{vmatrix} = -18-14 = -32$

$C_{32} = (-1)^{3+2} M_{32} = -M_{32} = -\begin{vmatrix} -1 & 2 \\ 0 & 3 \end{vmatrix} = +3$

Q_2 If M_{31} in determinant $\begin{vmatrix} 0 & 1 & \sec\alpha \\ \tan\alpha & -\sec\alpha & \tan\alpha \\ \cancel{1} & \cancel{0} & \cancel{1} \end{vmatrix}$ is 1 then $\alpha = ?$

$\alpha \in [0, \pi]$

$\begin{vmatrix} 1 & \sec\alpha \\ -\sec\alpha & \tan\alpha \end{vmatrix} = 1 \Rightarrow \tan\alpha + \sec^2\alpha = 1 \Rightarrow \tan\alpha + \tan^2\alpha = 0$

$\tan\alpha(1+\tan\alpha) = 0$

$\tan\alpha = 0$ OR $\tan\alpha = -1$

$\alpha = 0, \pi,$ $\quad \alpha = \frac{3\pi}{4}$

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Adjoint of Matrix.

let $A = [a_{ij}]$ be a sqr Matrix.

then. (1) The matrix obtained by Replacing each element of A by Corresponding Cofactor is called Cofactor Matrix known as $C = [C_{ij}]$

$C_{ij} = $ Cof. of a_{ij}

2) Transpose of Cofactor Matrix is called Adjoint of Matrix A & denoted by adj A

$\text{adj } A = [d_{ij}]_n$; $\boxed{d_{ij} = C_{ji}}$

Q $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ find Adj A = ?

$\begin{bmatrix} + & - \\ - & + \end{bmatrix}$

SC SC

$C = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix} \Rightarrow \text{Adj } A = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}^T$

$\text{Adj } A = \begin{bmatrix} C_{11} & C_{21} \\ C_{12} & C_{22} \end{bmatrix}$

Q $A = \begin{bmatrix} 1 & 2 \\ 5 & -6 \end{bmatrix}$ find Adj A = ?

$\text{Adj } A = \begin{bmatrix} -6 & -2 \\ -5 & 1 \end{bmatrix}$

$C = \begin{bmatrix} +(-6) & -5 \\ -2 & +1 \end{bmatrix} = \begin{bmatrix} -6 & -5 \\ -2 & 1 \end{bmatrix}$

$\text{Adj } A = \begin{bmatrix} -6 & -5 \\ -2 & 1 \end{bmatrix}^T = \begin{bmatrix} -6 & -2 \\ -5 & 1 \end{bmatrix}$

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Q $A = \begin{bmatrix} 0 & 1 \\ -2 & 3 \end{bmatrix}$ find Adj A = ?

$\text{Adj } A = \begin{bmatrix} 3 & -1 \\ 2 & 0 \end{bmatrix}$

Q $A = \begin{bmatrix} 0 & 1 & -1 \\ 2 & 1 & 3 \\ 0 & 5 & 4 \end{bmatrix}$ find Adj A

$0\text{'} \, CF = + \begin{vmatrix} 1 & 3 \\ 5 & 4 \end{vmatrix} = -11$

$1\text{'s} \, CF = - \begin{vmatrix} 2 & 3 \\ 0 & 4 \end{vmatrix} = -8$

$-1 \quad = +10$

$2\text{'s} \, CF = -9$

$1\text{'s} \, CF = 0$

$3\text{'s} \, CF = -0$

$0 \quad CF = +4$

$5 \quad " = -2$

$4 \quad " = +-2$

$C = \begin{bmatrix} -11 & -8 & 10 \\ -9 & 0 & 0 \\ 4 & -2 & -2 \end{bmatrix}$

$\text{Adj } A = \begin{bmatrix} -11 & -9 & 4 \\ -8 & 0 & -2 \\ 10 & 0 & -2 \end{bmatrix}$

Trick for 3^{rd} Order.

1) 1^{st} 2 Col. Dubara

2) 1^{st} 2 Row Dubara

3) 1^{st} R 1^{st} Col Delete

~~0~~	~~1~~	~~-1~~	~~0~~	~~1~~
~~2~~	1	3	2	1
~~0~~	5	4	0	5
~~0~~	1	-1	0	1
~~2~~	1	3	2	1

$\text{Adj } A = \begin{bmatrix} -11 & -9 & 4 \\ -8 & 0 & -2 \\ 10 & 0 & -2 \end{bmatrix}$

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Q $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & -3 \\ 1 & 2 & 3 \end{bmatrix}$ find Adj A?

$$\begin{array}{c|cccc} 2 & 1 & -3 & 2 & 1 \\ 1 & 2 & 3 & 1 & 2 \\ 1 & 1 & 1 & 1 & 1 \\ 2 & 1 & -3 & 2 & 1 \end{array}$$

$$\text{Adj } A = \begin{bmatrix} 9 & -1 & -4 \\ -5 & 2 & 5 \\ 3 & -1 & -1 \end{bmatrix}$$

Inverse of Matrix.

① If A, B are Sqr Matrix of order n

$|A| \neq 0$

2) $A \cdot B = I_n = B \cdot A$ then B is Multiplication Inverse of A i.e. $B = A^{-1}$

** 3) $A \cdot (\text{adj} A) = |A| \cdot I_n \rightarrow$ Later $\times A^{-1}$

$(A^{-1} A)(\text{adj } A) = A^{-1} |A| I_n.$

$\text{adj } A = A^{-1} |A|$

$\dfrac{\text{adj } A}{|A|} = A^{-1}$

$$\boxed{A^{-1} = \frac{\text{adj } A}{|A|}}$$

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Q Inverse of Matrix $A = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$

$$A^{-1} = \frac{\text{adj } A}{|A|}$$

(1) $|A| = \begin{vmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{vmatrix} = 1\begin{vmatrix} 4 & 3 \\ 3 & 4 \end{vmatrix} - 3\begin{vmatrix} 1 & 3 \\ 1 & 4 \end{vmatrix} + 3\begin{vmatrix} 1 & 4 \\ 1 & 3 \end{vmatrix}$

$= 1(7) - 3(1) + 3(-1)$

$= 7 - 3 - 3 = 1$

(2) adj $A = \begin{array}{|ccccc} \cancel{1} & \cancel{3} & \cancel{3} & \cancel{1} & \cancel{3} \\ 1 & 4 & 3 & 1 & 4 \\ 1 & 3 & 4 & 1 & 3 \\ 1 & 3 & 3 & 1 & 3 \\ 1 & 4 & 3 & 1 & 4 \end{array} = \begin{bmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$

$$A^{-1} = \frac{1}{1}\begin{bmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$$

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Q If $A = \begin{pmatrix} 1 & 2 \\ 3 & -5 \end{pmatrix}$ & $B = \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$ & X be a Matrix.

Such that $A = BX$ then $X = ?$

$A = BX \quad \times B^{-1}$ (Pre)

$B^{-1}A = (B^{-1}B) \cdot X$

$B^{-1}A = I \cdot X$

$X = B^{-1}A$

(1) $|B| = 2$

(2) $\text{Adj}\,B = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$

(3) $B^{-1} = \dfrac{\text{Adj}\,B}{|B|} = \dfrac{1}{2}\begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$

$B^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1/2 \end{bmatrix}$

$X = B^{-1}A$

$= \begin{bmatrix} 1 & 0 \\ 0 & 1/2 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & -5 \end{bmatrix}$

$X = \begin{bmatrix} 1 & 2 \\ 3/2 & -5/2 \end{bmatrix}$

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Most Imp. Qs for Practice

Q1) $A = \begin{bmatrix} -1 & 2 & 5 \\ 2 & -3 & 1 \\ -1 & 1 & 1 \end{bmatrix}$ find A^{-1}

2) Solve Eqn

$$-x + 2y + 5z = 2$$
$$2x - 3y + z = 15$$
$$-x + y + z = -3$$

$$\begin{bmatrix} -1 & 2 & 5 \\ 2 & -3 & 1 \\ -1 & 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 15 \\ -3 \end{bmatrix}$$

$A . X = B$ $\quad \times A^{-1}$ (Pre)

$(A^{-1}A)\, x = A^{-1}.B$

$x = A^{-1}B$

$A^{-1}B$ Miss करने का Sochna bhi mat!!!

① $|A| = \begin{vmatrix} -1 & 2 & 5 \\ 2 & -3 & 1 \\ -1 & 1 & 1 \end{vmatrix} = -1(-4) - 2(3) + 5(-1)$

$= 4 - 6 - 5 = -7$

2) Adj A

$$\begin{array}{c|cccc} \cancel{-1} & \cancel{2} & \cancel{5} & \cancel{-1} & \cancel{2} \\ \hline 2 & -3 & 1 & 2 & -3 \\ -1 & 1 & 1 & -1 & 1 \\ -1 & 2 & 5 & -1 & 2 \\ 2 & -3 & 1 & 2 & -3 \end{array} = \begin{bmatrix} -4 & 3 & 17 \\ -3 & 4 & 11 \\ -1 & -1 & -1 \end{bmatrix}$$

3) $A^{-1}.B = \frac{1}{-7}\begin{bmatrix} -4 & 3 & 17 \\ -3 & 4 & 11 \\ -1 & -1 & -1 \end{bmatrix}\begin{bmatrix} 2 \\ 15 \\ -3 \end{bmatrix} = \frac{1}{-7}\begin{bmatrix} -8+45-51 \\ -6+60-33 \\ -2-15+3 \end{bmatrix}$

$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{-1}{7}\begin{bmatrix} -14 \\ 21 \\ -14 \end{bmatrix} = \begin{bmatrix} 2 \\ -3 \\ 2 \end{bmatrix} \Rightarrow x = 2,\ y = -3,\ z = 2$

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Q Mains 2019

$$\begin{bmatrix}1 & 1\\0 & 1\end{bmatrix}\cdot\begin{bmatrix}1 & 2\\0 & 1\end{bmatrix}\cdot\begin{bmatrix}1 & 3\\0 & 1\end{bmatrix}\cdots\begin{bmatrix}1 & n-1\\0 & 1\end{bmatrix}=\begin{bmatrix}1 & 78\\0 & 1\end{bmatrix}$$

find Inverse of $\begin{bmatrix}1 & n\\0 & 1\end{bmatrix}=?$

$$\begin{bmatrix}1 & 1+2\\0 & 1\end{bmatrix}\begin{bmatrix}1 & 3\\0 & 1\end{bmatrix}\begin{bmatrix}1 & 4\\0 & 1\end{bmatrix}$$

$$\begin{bmatrix}1 & 3+1+2\\0 & 1\end{bmatrix}\begin{bmatrix}1 & 4\\0 & 1\end{bmatrix}$$

$$\begin{bmatrix}1 & 1+2+3+4\\0 & 1\end{bmatrix} \quad - - - -$$

$$\begin{bmatrix}1 & n-1\\0 & 1\end{bmatrix}=\begin{bmatrix}1 & 1+2+3+\cdots+(n-1)\\0 & 1\end{bmatrix}=\begin{bmatrix}1 & \frac{(n-1)(\cancel{n-1}+1)}{2}\\0 & 1\end{bmatrix}=\begin{bmatrix}1 & \frac{(n)(n-1)}{2}\\0 & 1\end{bmatrix}$$

(1) $\frac{n^2-n}{2}=78 \Rightarrow n^2-n=156.$

$n^2-13n+12n-156=0$

$n=13,\ \cancel{-12}$

(13) Inverse of $\begin{bmatrix}1 & 13\\0 & 1\end{bmatrix}=\frac{1}{1}\begin{bmatrix}1 & -13\\0 & 1\end{bmatrix}=\begin{bmatrix}1 & -13\\0 & 1\end{bmatrix}$

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Q Matrix $A = \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}$ find a, b Such that

$A^2 + aI + bI = 0$ Also find A^{-1}?

(M1) Use $|A - xI| = 0$ Char. Eqn.

(M2) Use Trick

$a = Tr\ A = 4$

$b = |A| = \begin{vmatrix} 3 & 2 \\ 1 & 1 \end{vmatrix} = 1$

(2) find A^{-1} =, $A^2 - 4A + I = 0 \times A^{-1}$

$A^{-1} \cdot A^2 - 4(A^{-1}A) + A^{-1} \cdot I = 0$

$A - 4I + A^{-1} = 0 \Rightarrow A^{-1} = 4I - A = \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix} - \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ -1 & 3 \end{bmatrix}$

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Value of Determinant

$A^2 - \mathrm{Tr}\,A \cdot A + |A| = 0$

$$\Delta = \begin{vmatrix} \log_3 8 & \log_3 512 \\ \log_2 \sqrt{3} & \log_4 9 \end{vmatrix} = ?$$

$$= \left(\log_3 8 \times \log_4 9\right) - \log_2 \sqrt{3} \times \log_3 512$$

$$= \frac{\log 8}{\log 3} \times \frac{\log 9}{2\log 2} - \frac{\log \sqrt{3}}{\log 2} \times \frac{\log 512}{\log 3}$$

$$= \frac{3\cancel{\log 2}}{\cancel{\log 3}} \times \frac{\cancel{2}\cancel{\log 3}}{\cancel{2}\cancel{\log 2}} - \frac{\frac{1}{2}\cancel{\log 3}}{\cancel{\log 2}} \times \frac{9\cancel{\log 2}}{\cancel{\log 3}}$$

$$= \frac{\cancel{3}6}{\cancel{1}2} - \frac{9}{2} = \frac{-3}{2}$$

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Q $\Delta = \begin{vmatrix} 1 & -3 & 5 \\ 2 & -1 & 0 \\ -7 & 6 & 8 \end{vmatrix} = ?$

SaurasMethod

$$\begin{matrix} 1 & -3 & 5 & 1 & -3 \\ 2 & -1 & 0 & 2 & -1 \\ -7 & 6 & 8 & -7 & 6 \end{matrix}$$

$(-8+0+60)-(35+0+-48)$

$52+13=65$

BagulaMethod 

$$\begin{vmatrix} 1 & -3 & 5 \\ 2 & -1 & 0 \\ -7 & 6 & 8 \end{vmatrix}$$

$[-8+0+60]-[35+0+-48]$

$52+(13)=65$