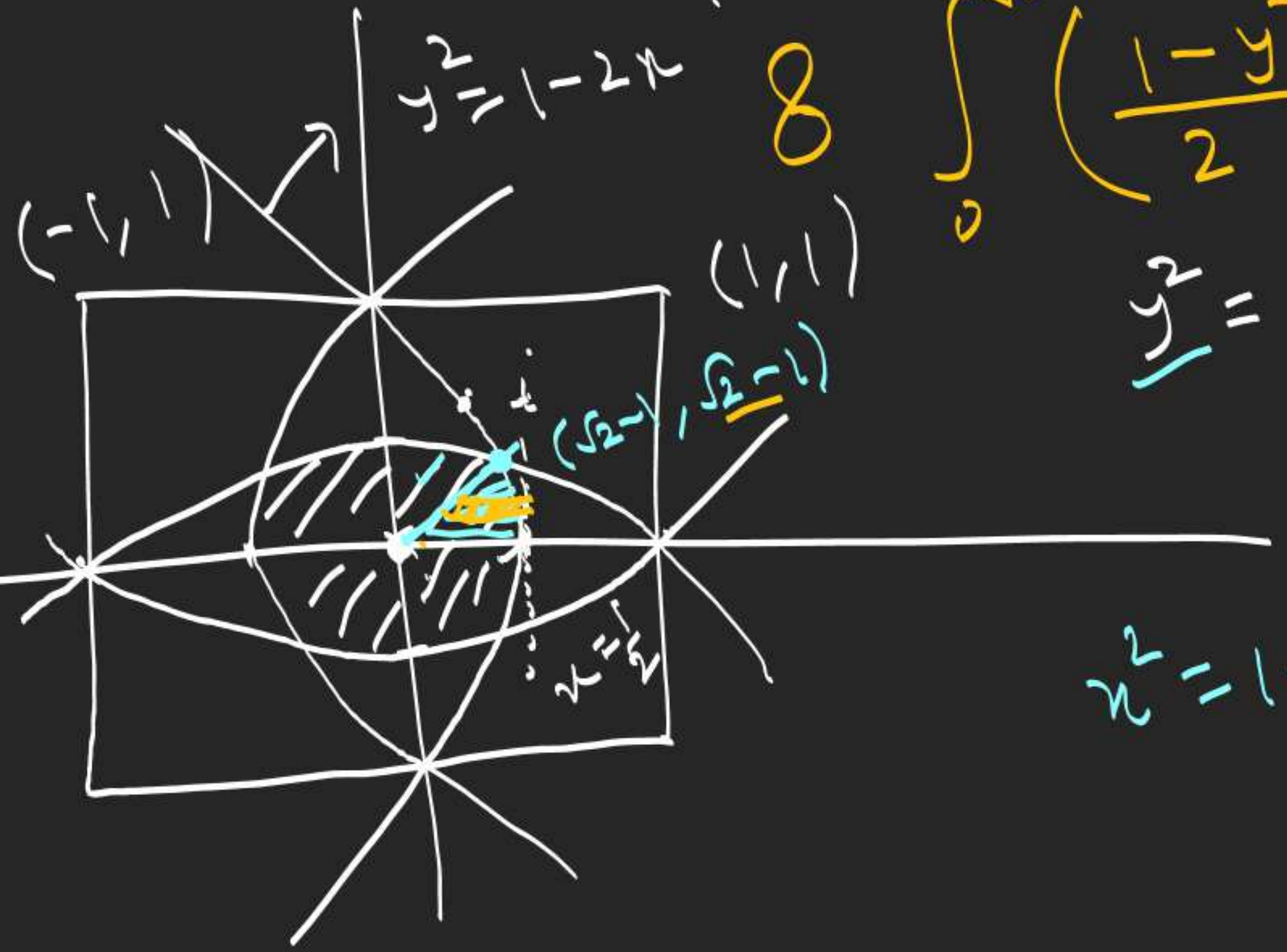
Nishant Jindal

1. Consider a square with vertices at $(1,1)$, $(-1,1)$, $(-1,-1)$, and $(1,-1)$. Let S be the region consisting of all points inside square which are nearer to origin than to any edge. Find the area of region S.

$$x^2+y^2 < |x-1|^2$$

$$\boxed{y^2 < 1-2x}$$



$$8\int_0^{\sqrt{2}-1}\left(\frac{1-y^2}{2}-y\right)dy = 8\left[\frac{\sqrt{2}-1}{2}-\frac{(\sqrt{2}-1)^2}{2}-\frac{(\sqrt{2}-1)^3}{6}\right]$$

$$y^2 = 2\left(\frac{1}{2}-x\right) = 1-2x.$$

$$x^2 = 1-2x \Rightarrow (x+1)^2 = 2$$

$$x = \sqrt{2}-1$$

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Q. 8 players of equal skill enter for a knockout tournament. They are drawn in pairs for next round. Find the probability that two given players play each other in the course of the tournament.

$\boxed{P_1 P_2}$

$$\frac{1}{7} + \left(\frac{6}{7} \times \frac{1}{2} \times \frac{1}{2}\right)\frac{1}{3} + \left(\frac{6}{7} \times \frac{1}{2} \times \frac{1}{2}\right)\left(\frac{2}{3} \times \frac{1}{2} \times \frac{1}{2}\right) \times 1 = \frac{1}{4}$$

P_1

$$\frac{1}{7} + \frac{{}^6C_2}{{}^8C_4} \times \frac{1}{3} + \frac{1}{{}^8C_2} \times 1$$

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3. Find the ratio of the areas in which the curve

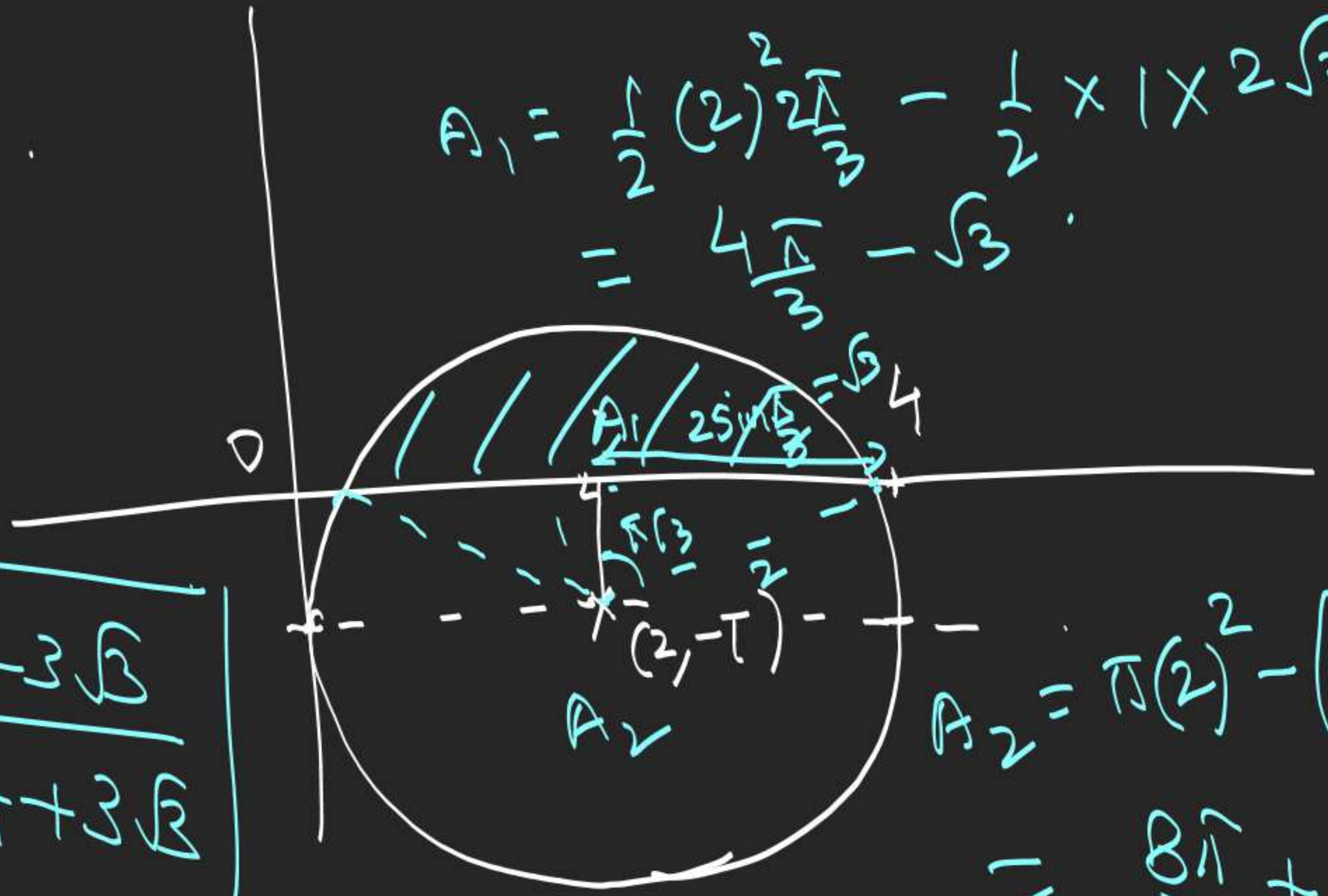
$y = \left[\frac{x^3}{100} + \frac{x}{35}\right]$, $[\cdot] = \text{G.I.F}$, divides the circle

$x^2 + y^2 - 4x + 2y + 1 = 0$.

$\boxed{y = 0}$

$A_1 = \frac{1}{2}(2)^2 \frac{2\pi}{3} - \frac{1}{2} \times 1 \times 2\sqrt{3}$

$= \frac{4\pi}{3} - \sqrt{3}$.



$A_2 = \pi(2)^2 - \left(\frac{4\pi}{3} - \sqrt{3}\right)$

$= \frac{8\pi}{3} + \sqrt{3}$

$$\boxed{A_1 : A_2 = \frac{4\pi - 3\sqrt{3}}{8\pi + 3\sqrt{3}}}$$

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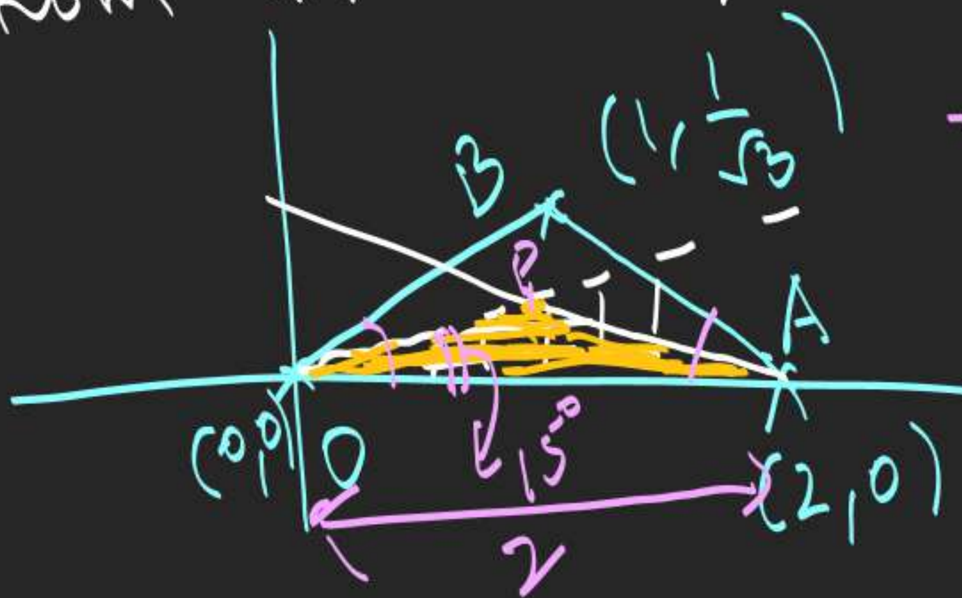
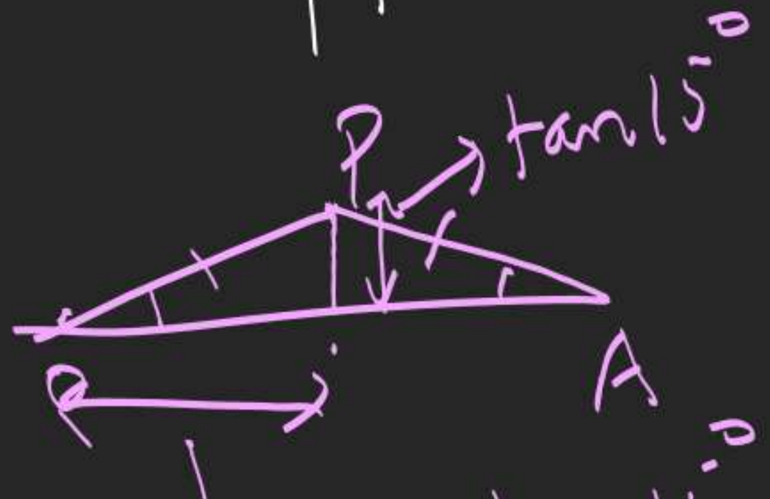
4. Let $O(0,0)$, $A(2,0)$, $B\left(1,\frac{1}{\sqrt{3}}\right)$ be the vertices of ΔOAB. Let R be the region consisting of all points P inside ΔOAB which satisfy $d(P,OA) \le \min\{d(P,OB), d(P,AB)\}$. Sketch region R and find its area. ($d(P,L)$ denote $\perp$ar distance of point P from line L).

$$(P,OA) \le (P,OB)$$

$$(P,OA) \le (P,AB)$$





$$\Delta = \frac{1}{2} \times 2 \times \tan 15^\circ$$
$$= 2-\sqrt{3}$$

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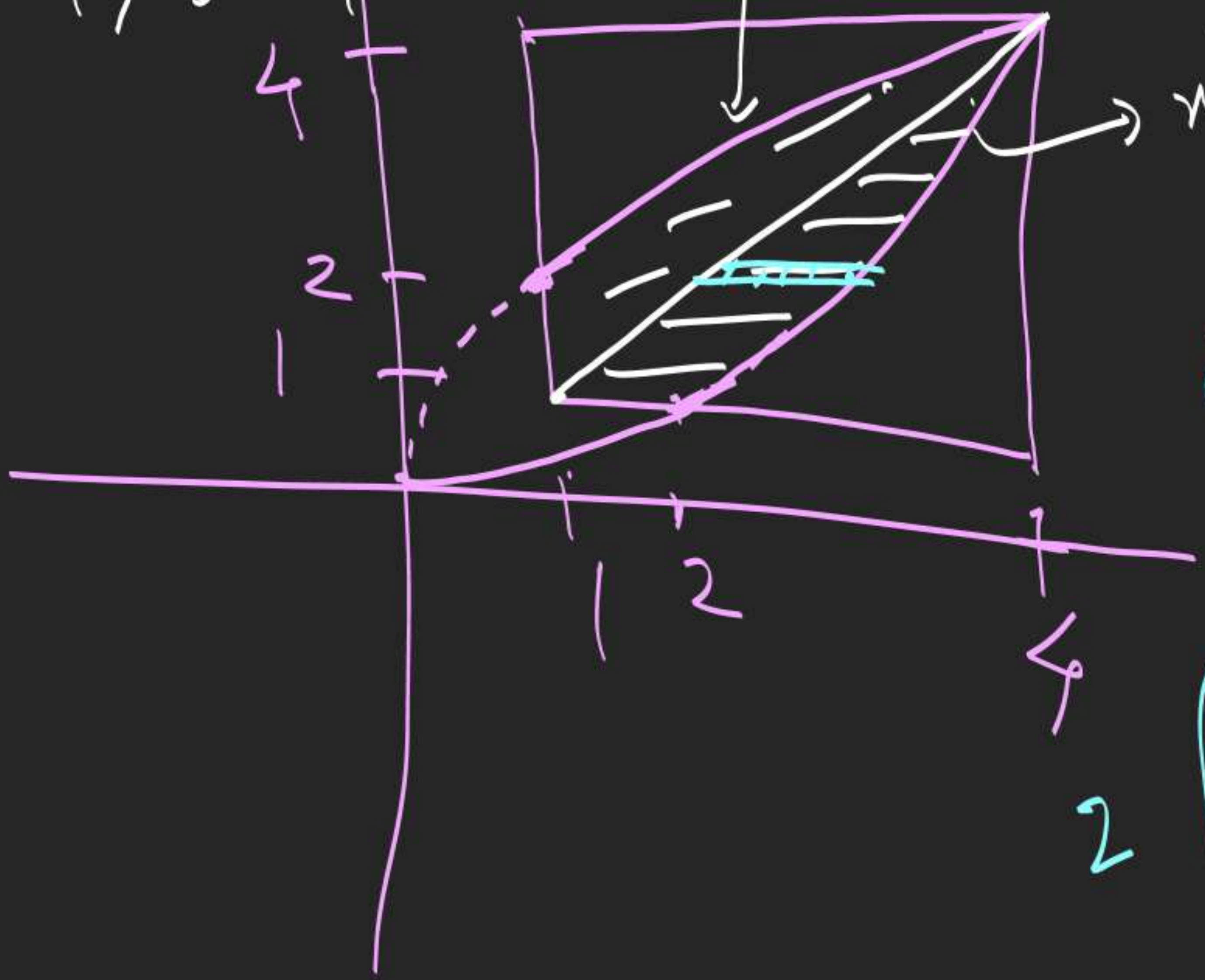
5. Consider the curves $C_1 : y^2 = 4[\sqrt{y}]x$ and $C_2 : x^2 = 4[\sqrt{x}]y$, $[\cdot] = $ G.I.F. Find area enclosed b/n two curves C_1, C_2 within the square formed by lines $x=1$, $y=1$, $x=4$, $y=4$.



$$2\int_1^4 \left(2\sqrt{y} - y\right) dy$$

DPP-3, Ex-3 (remaining)

$$2\left[\frac{4}{3}(8-1) - \frac{1}{2}(16-1)\right]$$

$$= 2\left(\frac{28}{3} - \frac{15}{2}\right) = \frac{11}{3}.$$