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Inequalities

Modulus function

Compound Angles

3. $\sqrt{\frac{(1-\sin\theta)^2}{\cos^2\theta}} = \left|\frac{1-\sin\theta}{\cos\theta}\right|$

$= \frac{(1-\sin\theta)}{|\cos\theta|}$

$\theta \in \left(0, \frac{\pi}{2}\right) \cup \left(\frac{3\pi}{2}, 2\pi\right)$ $\frac{1-\sin\theta}{\cos\theta}$

$\frac{\pi}{2}$, $\frac{1-\sin\theta}{-\cos\theta}$

$\pi - \frac{5\pi}{11}$

2. $\sum_{r=1}^{5} \cos^2\frac{r\pi}{11} = \frac{1}{2}\sum_{r=1}^{5}\left(1+\cos\frac{2r\pi}{11}\right)$

$\frac{1}{2}\left(5 \frac{2\sin\frac{5\pi}{11}\cos\frac{5\pi}{11}}{2\sin\frac{\pi}{11}}\right) = \frac{1}{2}\left(5 + \frac{\sin\frac{5\pi}{11}}{\sin\frac{\pi}{11}}\cos\frac{6\pi}{11}\right)$

$\frac{1}{2}\left(5 - \frac{\sin\frac{10\pi}{11}}{2\sin\frac{\pi}{11}}\right)$

$= \frac{1}{2}\left(5 - \frac{1}{2}\right)$

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$$a^3+b^3+c^3-3abc = (a+b+c)(a^2+b^2+c^2-ab-bc-ca)$$
$$= (a+b+c)\frac{1}{2}\left((a-b)^2+(b-c)^2+(c-a)^2\right)$$

$n\in N$
n is odd, $a^n+b^n = (a+b)(a^{n-1}-a^{n-2}b+a^{n-3}b^2-a^{n-4}b^3+\dots+b^{n-1})$

$n\in N$, $a^n-b^n = (a-b)(a^{n-1}+a^{n-2}b+a^{n-3}b^2+a^{n-4}b^3+\dots+ab^{n-2}+b^{n-1})$

If $a=b$, $a^n-b^n=0$

$$\begin{array}{r|l} & a^{n-1}+a^{n-2}b \\ \hline a-b & a^n-b^n \\ & a^n \quad -a^{n-1}b \\ \hline & -b^n+a^{n-1}b \\ & a^{n-1}b-a^{n-2}b^2 \\ \hline & -b^n+a^{n-2}b^2 \end{array}$$

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5:

$$\cos\alpha + \cos\beta + \cos\gamma = 0$$

$$\Rightarrow \cos^3\alpha + \cos^3\beta + \cos^3\gamma = 3\cos\alpha\cos\beta\cos\gamma$$

$$\sum \cos 3\alpha = 4\sum \cos^3\alpha - 3\sum \cos\alpha$$

$$= 12\cos\alpha\cos\beta\cos\gamma - 3(0)$$

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7:

$$2\sin^2\beta + 2\cos(\alpha+\beta)(\sin\alpha\sin\beta) + \cos(2\alpha+2\beta)$$

$$= 2\sin^2\beta + 2\cos(\alpha+\beta)\cos(\alpha-\beta) - \cancel{2\cos^2(\alpha+\beta)} + \cancel{2\cos^2(\alpha+\beta)} - 1$$

$$= 2\sin^2\beta + \underline{2(\cos^2\alpha} - \sin^2\beta) - \underline{1}$$

$$= \cos 2\alpha$$

(c) $1 - \frac{1}{2}\sin^2\frac{\pi}{8} + 1 - \frac{1}{2}\sin^2\frac{3\pi}{8}$

(b) $(\tan 9^\circ + \cot 9^\circ) - (\tan 27^\circ + \cot 27^\circ)$

$$\frac{2}{\sin 18^\circ} - \frac{2}{\cos 36^\circ} = \frac{1}{\sin 9^\circ \cos 9^\circ} - \frac{1}{\sin 27^\circ \cos 27^\circ}$$

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10. $X = \underbrace{\sin\left(\theta+\frac{7\pi}{12}\right)+\sin\left(\theta-\frac{\pi}{12}\right)}+\sin\left(\theta+\frac{3\pi}{12}\right)$

$= 2\sin\left(\theta+\frac{3\pi}{12}\right)\cos\frac{\pi}{3}+\sin\left(\theta+\frac{3\pi}{12}\right) = 2\sin\left(\theta+\frac{\pi}{4}\right)$

$\frac{1}{2}$

$Y = 2\cos\left(\theta+\frac{3\pi}{12}\right)\cos\frac{\pi}{3}+\cos\left(\theta+\frac{3\pi}{12}\right) = 2\cos\left(\theta+\frac{\pi}{4}\right)$

$\frac{X}{Y}-\frac{Y}{X} = \tan\left(\theta+\frac{\pi}{4}\right)-\cot\left(\theta+\frac{\pi}{4}\right) = \frac{\tan^2\left(\theta+\frac{\pi}{4}\right)-1}{\tan\left(\theta+\frac{\pi}{4}\right)}$

$\frac{-2}{-\cot 2\theta} = -\frac{2}{\tan\left(2\theta+\frac{\pi}{2}\right)} = -2\left(\frac{1-\tan^2\left(\theta+\frac{\pi}{4}\right)}{2\tan\left(\theta+\frac{\pi}{4}\right)}\right)$

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14.

$$\frac{\tan A}{\tan B \tan C} + \frac{\tan B}{\tan C \tan A} + \frac{\tan C}{\tan A \tan B}$$

$$= \frac{\tan^2 A + \tan^2 B + \tan^2 C}{\tan A \tan B \tan C}$$

$$= \frac{(\tan A + \tan B + \tan C)^2 - 2(\tan A \tan B + \tan B \tan C + \tan C \tan A)}{\tan A \tan B \tan C}$$

$$= \sum \tan A - 2 \sum \cot A$$

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16. (a) $$\frac{4\cos 20^\circ - \sqrt{3}\cos 20^\circ}{\sin 20^\circ} = \frac{4\cos 20^\circ \sin 20^\circ - \sqrt{3}\cos 20^\circ}{\sin 20^\circ}$$

$$= \frac{2\sin 40^\circ - \sqrt{3}\cos 20^\circ}{\sin 20^\circ}$$

$$= \frac{2\sin(60-20^\circ) - \sqrt{3}\cos 20^\circ}{\sin 20^\circ}$$

$$= \frac{2\left(\frac{\sqrt{3}}{2}\cos 20^\circ - \frac{1}{2}\sin 20^\circ\right) - \sqrt{3}\cos 20^\circ}{\sin 20^\circ}$$

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(d) $\tan 10^\circ - \tan(60^\circ - 10^\circ) + \tan(60^\circ + 10^\circ)$

$\tan 10^\circ = t$

$$= t - \frac{\sqrt{3} - t}{1 + \sqrt{3}t} + \frac{\sqrt{3} + t}{1 - \sqrt{3}t}$$

$$(1 + \tan 1^\circ)\left(1 + \frac{1 - \tan 1^\circ}{1 + \tan 1^\circ}\right) = 2.$$

$$= \frac{3(3t - t^3)}{(1 - 3t^2)}$$

17.
$$\left((1+\tan 1^\circ)(1+\tan(45-1))\right)\left((1+\tan 2^\circ)(1+\tan(45-2))\right)$$

$$\ldots \ldots \quad (1 + \tan 45^\circ)$$

$$= 2^{22} \times 2.$$

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$$(1+\tan 1^\circ)(1+\tan 2^\circ)(1+\tan 3^\circ)\cdots(1+\tan 44^\circ)(1+\tan 45^\circ)$$

$$(\sqrt{2})^{44}\,\frac{\sin 46^\circ}{\cos 1^\circ}\,\frac{\sin 47^\circ}{\cos 2^\circ}\cdots\frac{\sin 89^\circ}{\cos 44^\circ}\times 2$$

$$= \frac{(\sin 1^\circ+\cos 1^\circ)(\sin 2^\circ+\cos 2^\circ)(\sin 3^\circ+\cos 3^\circ)\cdots(\sin 44^\circ+\cos 44^\circ)\;2}{\cos 1^\circ \quad \cos 2^\circ \quad \cos 3^\circ \quad \cos 44^\circ}$$

$$= \frac{\sqrt{2}\,\cancel{\sin(46^\circ)}\,(\sqrt{2}\,\cancel{\sin 47^\circ})(\sqrt{2}\,\cancel{\sin 48^\circ})\cdots(\sqrt{2}\,\cancel{\sin 89^\circ})\;2}{\cancel{\sin 89^\circ}\;\cancel{\sin 88^\circ}\cdots\cancel{\sin 46^\circ}}$$

$$= (\sqrt{2})^{44}\times 2$$

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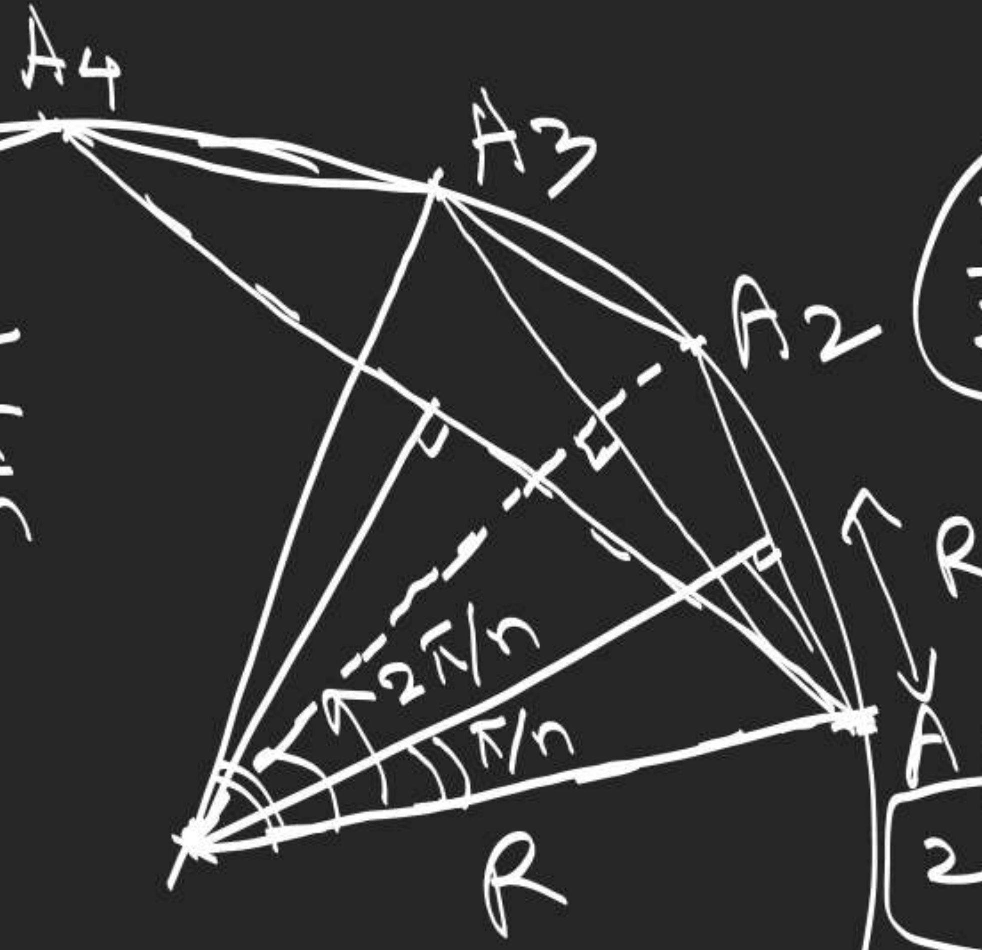
19.



$$\frac{1}{2R\sin\frac{\pi}{n}} = \frac{1}{2R\sin\frac{2\pi}{n}} + \frac{1}{2R\sin\frac{3\pi}{n}}$$

$$\frac{1}{\sin\frac{\pi}{n}} = \frac{1}{\sin\frac{2\pi}{n}} + \frac{1}{\sin\frac{3\pi}{n}}$$

$$\frac{1}{\sin\frac{\pi}{n}} - \frac{1}{\sin\frac{3\pi}{n}} = \frac{1}{\sin\frac{2\pi}{n}}$$

$$\frac{\sin\frac{3\pi}{n} - \sin\frac{\pi}{n}}{\sin\frac{\pi}{n}\sin\frac{3\pi}{n}} = \frac{1}{\sin\frac{2\pi}{n}} = \frac{2\cancel{\sin\frac{\pi}{n}}\cos\frac{2\pi}{n}}{\cancel{\sin\frac{\pi}{n}}\sin\frac{3\pi}{n}}$$

$$\Rightarrow \boxed{\sin\frac{3\pi}{n} = \sin\frac{4\pi}{n}}$$

$$\boxed{2\sin\frac{\pi}{2n}\cos\frac{7\pi}{2n} = 0}$$

$$\frac{\pi}{2n} = k\pi \quad \text{or} \quad \frac{7\pi}{2n} = (2k+1)\frac{\pi}{2}$$

$$\boxed{\frac{7}{n} = 2k+1}$$

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$x > 0$, $\quad x + \frac{1}{x} = \left((\sqrt{x})^2 + \left(\frac{1}{\sqrt{x}}\right)^2 - 2\sqrt{x}\frac{1}{\sqrt{x}} \right) + 2$

$$= \underbrace{\left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)^2}_{\geq 0} + 2 \geq 2$$

$$\boxed{x + \frac{1}{x} \in (-\infty, -2] \cup [2, \infty)}$$

$x + \frac{1}{x} \geq 2 \quad \forall x > 0$

$\downarrow$ for all

$\boxed{x + \frac{1}{x} = 2 \text{ if } x = 1}$

$x + \frac{1}{x} = -2$ if $x = -1$

$f(x) = x + \frac{1}{x}$, $x > 0$

$R_f = [2, \infty)$

If $x < 0$, $\quad x = -t$, $t > 0$

$x + \frac{1}{x} = -\left(t + \frac{1}{t}\right)$

$t > 0$, $\quad t + \frac{1}{t} \geq 2 \Rightarrow -\left(t + \frac{1}{t}\right) \leq -2$

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① Find the minimum value of

$$4x + \frac{9}{x} \quad \text{if} \quad x > 0$$

$$4x + \frac{9}{x} = \left(2\sqrt{x}\right)^2 + \left(\frac{3}{\sqrt{x}}\right)^2 - 2\left(2\sqrt{x}\right)\left(\frac{3}{\sqrt{x}}\right) + 12$$

$$= \left(2\sqrt{x} - \frac{3}{\sqrt{x}}\right)^2 + 12 \geq 12$$

$$4x + \frac{9}{x} = 12$$

if $2\sqrt{x} = \frac{3}{\sqrt{x}}$

$$\boxed{x = \frac{3}{2}}$$

② Find the maximum value of

$$x + \frac{16}{x}, \text{ if } x < 0$$

$x = -t$, $t > 0$

$$x + \frac{16}{x} = -\left(t + \frac{16}{t}\right) = -\underbrace{\left[\left(\sqrt{t} - \frac{4}{\sqrt{t}}\right)^2 + 8\right]}_{\geq 8} \leq -8.$$

$$x + \frac{16}{x} = -8. \text{ if } \sqrt{t} = \frac{4}{\sqrt{t}} \Rightarrow \boxed{t = 4}$$

$$\text{if } \boxed{x = -4}$$

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Find the minimum value of

1. $f(x) = 4\sin^2 x + \operatorname{cosec}^2 x$

$= (2\sin x - \operatorname{cosec} x)^2 + 4 \geq 4$

$f_{min} = 4$ if $2\sin x = \operatorname{cosec} x$

$\Rightarrow \sin^2 x = \frac{1}{2}$.

2. $f(x) = 8\sec^2 x + 18\operatorname{cosec}^2 x$

$= 8 + 18 + 8\tan^2 x + 18\cot^2 x$

$= 26 + (2\sqrt{2}\tan x - 3\sqrt{2}\cot x)^2 + 2(2\sqrt{2})(3\sqrt{2})$

$\geq 26 + 24 = 50$

$f(x) = 50$ if $2\sqrt{2}\tan x = 3\sqrt{2}\cot x$

$\tan^2 x = \frac{3}{2}$.

3. $f(x) = 18\sec^2 x + 8\cos^2 x$

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$$f(x) = 18\sec^2 x + 8\cos^2 x \quad > 0$$

$$= \left(3\sqrt{2}\sec x - 2\sqrt{2}\cos x\right)^2 + 24 \geq 24$$

$f(x) = 24$ if $3\sqrt{2}\sec x = 2\sqrt{2}\cos x$

$\rightarrow$ not possible. $\quad \cos^2 x = \frac{3}{2}$ $\times$

$$f(x) = 10\underbrace{\sec^2 x}_{\geq 1} + 8\left(\underbrace{\sec^2 x + \cos^2 x}_{\geq 2}\right) \geq 10(1) + 8(2) = 26$$

$f(x) = 26$ if $\sec^2 x = 1$

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1: (a)

$\boxed{\text{Ex 3 / Ex-4 (1-10)}}$

$$y = 10\cos^2 x - 6\sin x\cos x + 2\sin^2 x$$

$$= 2 + 8\cos^2 x - \underline{6\sin x\cos x}$$

$$= 2 + 4(1+\cos 2x) - 3\sin 2x$$

$$= 6 + \left(\underbrace{4\cos 2x - 3\sin 2x}_{-5 \le \quad \le 5}\right)$$

$$f(x) \in [6-5, 6+5]$$