

KEY CONCEPTS

1. The symbol $\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$ is called the determinant of order two. Its value is given by :

$$D = a_1 b_2 - a_2 b_1$$

2. The symbol $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ is called the determinant of order three.

Its value can be found as : $D = a_1 \begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} - a_2 \begin{vmatrix} b_1 & c_1 \\ b_3 & c_3 \end{vmatrix} + a_3 \begin{vmatrix} b_1 & c_1 \\ b_2 & c_2 \end{vmatrix}$ OR

$$D = a_1 \begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} - b_1 \begin{vmatrix} a_2 & c_2 \\ a_3 & c_3 \end{vmatrix} + c_1 \begin{vmatrix} a_2 & b_2 \\ a_3 & b_3 \end{vmatrix} \dots \dots \dots \text{and so on .}$$

In this manner we can expand a determinant in 6 ways using elements of ; R_1, R_2, R_3 or C_1, C_2, C_3 .

3. Following examples of short hand writing large expressions are:

- (i) The lines :

$$a_1 x + b_1 y + c_1 = 0 \dots \dots (1)$$

$$a_2 x + b_2 y + c_2 = 0 \dots \dots (2)$$

$$a_3 x + b_3 y + c_3 = 0 \dots \dots (3)$$

are concurrent if, $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0.$

Condition for the consistency of three simultaneous linear equations in 2 variables.

- (ii) $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represents a pair of straight lines if :

$$abc + 2fgh - af^2 - bg^2 - ch^2 = 0 = \begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix}$$

- (iii) Area of a triangle whose vertices are $(x_r, y_r); r = 1, 2, 3$ is :

$$D = \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} \text{ If } D = 0 \text{ then the three points are collinear.}$$

- (iv) Equation of a straight line passing through (x_1, y_1) & (x_2, y_2) is $\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0$

4. MINORS :

The minor of a given element of a determinant is the determinant of the elements which remain after deleting the row & the column in which the given element stands. For example, the minor of a_1 in (Key Concept 2)

is $\begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix}$ & the minor of b_2 is $\begin{vmatrix} a_1 & c_1 \\ a_3 & c_3 \end{vmatrix}.$

Hence a determinant of order two will have " 4 minors" & a determinant of order three will have "9 minors".

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5. COFACTOR :

If M_{ij} represents the minor of some typical element then the cofactor is defined as :

$C_{ij} = (-1)^{i+j} \cdot M_{ij}$; Where i & j denotes the row & column in which the particular element lies.

Note that the value of a determinant of order three in terms of 'Minor' & 'Cofactor' can be written as :

$$D = a_{11}M_{11} - a_{12}M_{12} + a_{13}M_{13} \text{ or } D = a_{11}C_{11} + a_{12}C_{12} + a_{13}C_{13} \text{ \& so on}$$

6. PROPERTIES OF DETERMINANTS :

P-1: The value of a determinant remains unaltered, if the rows & columns are inter changed.

$$\text{e.g. if } D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = D'$$

D & D' are transpose of each other . If $D' = -D$ then it is **Skew Symmetric** determinant but $D' = D \Rightarrow 2D = 0 \Rightarrow D = 0 \Rightarrow$ Skew symmetric determinant of third order has the value zero.

P-2: If any two rows (or columns) of a determinant be interchanged , the value of determinant is changed in sign only. e.g.

$$\text{Let } D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \text{ \& } D' = \begin{vmatrix} a_2 & b_2 & c_2 \\ a_1 & b_1 & c_1 \\ a_3 & b_3 & c_3 \end{vmatrix} \text{ Then } D' = -D.$$

P-3: If a determinant has any two rows (or columns) identical , then its value is zero.

$$\text{e.g. Let } D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_1 & b_1 & c_1 \\ a_3 & b_3 & c_3 \end{vmatrix} \text{ then it can be verified that } D = 0.$$

P-4: If all the elements of any row (or column) be multiplied by the same number, then the determinant is multiplied by that number.

$$\text{e.g. If } D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \text{ and } D' = \begin{vmatrix} Ka_1 & Kb_1 & Kc_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \text{ Then } D' = KD$$

P-5: If each element of any row (or column) can be expressed as a sum of two terms then the determinant can be expressed as the sum of two determinants. e.g.

$$\begin{vmatrix} a_1 + x & b_1 + y & c_1 + z \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} + \begin{vmatrix} x & y & z \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

P- 6: The value of a determinant is not altered by adding to the elements of any row (or column) the same multiples of the corresponding elements of any other row

$$\text{(or column). e.g. Let } D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \text{ and}$$

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$$D' = \begin{vmatrix} a_1 + ma_2 & b_1 + mb_2 & c_1 + mc_2 \\ a_2 & b_2 & c_2 \\ a_3 + na_2 & b_3 + nb_2 & c_3 + nc_2 \end{vmatrix}. \text{ Then } D' = D$$

Note : that while applying this property **ATLEAST ONE ROW (OR COLUMN)** must remain unchanged .

P- 7: If by putting $x = a$ the value of a determinant vanishes then $(x - a)$ is a factor of the determinant.

7. MULTIPLICATION OF TWO DETERMINANTS :

$$(i) \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} \times \begin{vmatrix} l_1 & m_1 \\ l_2 & m_2 \end{vmatrix} = \begin{vmatrix} a_1l_1 + b_1l_2 & a_1m_1 + b_1m_2 \\ a_2l_1 + b_2l_2 & a_2m_1 + b_2m_2 \end{vmatrix}$$

Similarly two determinants of order three are multiplied.

$$(ii) \text{ If } D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \neq 0 \text{ then, } D^2 = \begin{vmatrix} A_1 & B_1 & C_1 \\ A_2 & B_2 & C_2 \\ A_3 & B_3 & C_3 \end{vmatrix} \text{ where } A_i, B_i, C_i \text{ are cofactors}$$

Proof: Consider $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \times \begin{vmatrix} A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \\ C_1 & C_2 & C_3 \end{vmatrix} = \begin{vmatrix} D & 0 & 0 \\ 0 & D & 0 \\ 0 & 0 & D \end{vmatrix}$

Note: $a_1A_2 + b_1B_2 + c_1C_2 = 0$ etc.

$$\text{Therefore, } D \times \begin{vmatrix} A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \\ C_1 & C_2 & C_3 \end{vmatrix} = D^3 \Rightarrow \begin{vmatrix} A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \\ C_1 & C_2 & C_3 \end{vmatrix} = D^2 \text{ or } \begin{vmatrix} A_1 & B_1 & C_1 \\ A_2 & B_2 & C_2 \\ A_3 & B_3 & C_3 \end{vmatrix} = D^2$$

8. SYSTEM OF LINEAR EQUATION (IN TWO VARIABLES) :

(i) Consistent Equations : Definite & unique solution . [intersecting lines]

(ii) Inconsistent Equation : No solution . [Parallel line]

(iii) Dependent equation : Infinite solutions . [Identical lines]

Let $a_1x + b_1y + c_1 = 0$ & $a_2x + b_2y + c_2 = 0$ then :

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2} \Rightarrow \text{Given equations are inconsistent}$$

$$\& \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \Rightarrow \text{Given equations are dependent}$$

9. CRAMER'S RULE : [Simultaneous Equations Involving Three Unknowns]

Let , $a_1x + b_1y + c_1z = d_1 \dots (I)$; $a_2x + b_2y + c_2z = d_2 \dots (II)$; $a_3x + b_3y + c_3z = d_3 \dots (III)$

Then, $x = \frac{D_1}{D}, Y = \frac{D_2}{D}, Z = \frac{D_3}{D}$.

$$\text{Where } D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}; D_1 = \begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix}; D_2 = \begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix} \& D_3 = \begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix}$$

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NOTE :

- (a) If $D \neq 0$ and atleast one of $D_1, D_2, D_3 \neq 0$, then the given system of equations are consistent and have unique non trivial solution .
- (b) If $D \neq 0$ & $D_1 = D_2 = D_3 = 0$, then the given system of equations are consistent and have trivial solution only .
- (c) If $D = D_1 = D_2 = D_3 = 0$, then the given system of equations are consistent and have infinite solutions.

In case $\left. \begin{array}{l} a_1x + b_1y + c_1z = d_1 \\ a_2x + b_2y + c_2z = d_2 \\ a_3x + b_3y + c_3z = d_3 \end{array} \right\}$ represents these parallel planes then also

$D = D_1 = D_2 = D_3 = 0$ but the system is inconsistent.

- (d) If $D = 0$ but atleast one of D_1, D_2, D_3 is not zero then the equations are inconsistent and have no solution.

If x, y, z are not all zero, the condition for $a_1x + b_1y + c_1z = 0$; $a_2x + b_2y + c_2z = 0$ &

$a_3x + b_3y + c_3z = 0$ to be consistent in x, y, z is that $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$.

Remember that if a given system of linear equations have **Only Zero** Solution for all its variables then the given equations are said to have **TRIVIAL SOLUTION**.

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PROFICIENCYTEST-01

1. $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} =$

2. $\begin{vmatrix} 1 & 5 & \pi \\ \log_e e & 5 & \sqrt{5} \\ \log_{10} 10 & 5 & e \end{vmatrix} =$

3. $\begin{vmatrix} 19 & 17 & 15 \\ 9 & 8 & 7 \\ 1 & 1 & 1 \end{vmatrix} =$

4. The value of the determinant $\begin{vmatrix} 4 & -6 & 1 \\ -1 & -1 & 1 \\ -4 & 11 & -1 \end{vmatrix}$ is :

5. The value of the determinant $\begin{vmatrix} 31 & 37 & 92 \\ 31 & 58 & 71 \\ 31 & 105 & 24 \end{vmatrix}$ is :

6. $\begin{vmatrix} a-b & b-c & c-a \\ x-y & y-z & z-x \\ p-q & q-r & r-p \end{vmatrix} =$

7. $\begin{vmatrix} 1 & a & a^2 - bc \\ 1 & b & b^2 - ac \\ 1 & c & c^2 - ab \end{vmatrix} =$

8. $\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+x & 1 \\ 1 & 1 & 1+y \end{vmatrix} =$

9. The roots of the equation $\begin{vmatrix} 1 & 4 & 20 \\ 1 & -2 & 5 \\ 1 & 2x & 5x^2 \end{vmatrix} = 0$ are

10. If $a \neq b \neq c$, the value of x (independent of a, b, c) which satisfies the equation

$\begin{vmatrix} 0 & x-a & x-b \\ x+a & 0 & x-c \\ x+b & x+c & 0 \end{vmatrix} = 0$, is:

11. If $a + b + c = 0$, then the solution of the equation $\begin{vmatrix} a-x & c & b \\ c & b-x & a \\ b & a & c-x \end{vmatrix} = 0$ is :

12. If $\begin{vmatrix} x+1 & 3 & 5 \\ 2 & x+2 & 5 \\ 2 & 3 & x+4 \end{vmatrix} = 0$, then $x =$

13. $\begin{vmatrix} 1 & a & b \\ -a & 1 & c \\ -b & -c & 1 \end{vmatrix} =$

14. $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix} =$

15. $\begin{vmatrix} 0 & a & -b \\ -a & 0 & c \\ b & -c & 0 \end{vmatrix} =$

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PROFICIENCY TEST-02

1. $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} =$

2. $\begin{vmatrix} a+b & a+2b & a+3b \\ a+2b & a+3b & a+4b \\ a+4b & a+5b & a+6b \end{vmatrix} =$

3. $\begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix} =$

4. The roots of the equation $\begin{vmatrix} 1+x & 1 & 1 \\ 1 & 1+x & 1 \\ 1 & 1 & 1+x \end{vmatrix} = 0$ are

5. One of the roots of the given equation $\begin{vmatrix} x+a & b & c \\ b & x+c & a \\ c & a & x+b \end{vmatrix} = 0$ is :

(A) $a + b + c$

(B) $-(a + b + c)$

(C) $a^2 + b^2 + c^2$

(D) $-(a^2 + b^2 + c^2)$

6. $\begin{vmatrix} x+1 & x+2 & x+4 \\ x+3 & x+5 & x+8 \\ x+7 & x+10 & x+14 \end{vmatrix} =$

7. $\begin{vmatrix} 1/a & a^2 & bc \\ 1/b & b^2 & ca \\ 1/c & c^2 & ab \end{vmatrix} =$

8. $\begin{vmatrix} b^2+c^2 & a^2 & a^2 \\ b^2 & c^2+a^2 & b^2 \\ c^2 & c^2 & a^2+b^2 \end{vmatrix} =$

9. $\begin{vmatrix} 1+x & 1 & 1 \\ 1 & 1+y & 1 \\ 1 & 1 & 1+z \end{vmatrix} =$

10. If $\begin{vmatrix} y+z & x & y \\ z+x & z & x \\ x+y & y & z \end{vmatrix} = k(x+y+z)(x-z)^2$, then $k =$

11. If -9 is a root of the equation $\begin{vmatrix} x & 3 & 7 \\ 2 & x & 2 \\ 7 & 6 & x \end{vmatrix} = 0$ then the other two roots are:

12. If a, b, c are unequal what is the condition that the value of the following determinant is zero

$$\Delta = \begin{vmatrix} a & a^2 & a^3+1 \\ b & b^2 & b^3+1 \\ c & c^2 & c^3+1 \end{vmatrix}$$

13. The value of the determinant $\begin{vmatrix} 1 & a & b+c \\ 1 & b & c+a \\ 1 & c & a+b \end{vmatrix}$ is :

14. If a, b and c are non zero numbers, then $\Delta = \begin{vmatrix} b^2c^2 & bc & b+c \\ c^2a^2 & ca & c+a \\ a^2b^2 & ab & a+b \end{vmatrix}$ is equal to :

15. If $\begin{vmatrix} 1 & k & 3 \\ 3 & k & -2 \\ 2 & 3 & -1 \end{vmatrix} = 0$, then the value of k is :

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PROFICIENCY TEST-03

1. If $\Delta = \begin{vmatrix} a & b & c \\ x & y & z \\ p & q & r \end{vmatrix}$, then $\begin{vmatrix} ka & kb & kc \\ kx & ky & kz \\ kp & kq & kr \end{vmatrix} =$
2. If $\begin{vmatrix} x^2 + x & x + 1 & x - 2 \\ 2x^2 + 3x - 1 & 3x & 3x - 3 \\ x^2 + 2x + 3 & 2x - 1 & 2x - 1 \end{vmatrix} = Ax - 12$, then the value of A is :
3. The roots of the equation $\begin{vmatrix} 3 - x & -6 & 3 \\ -6 & 3 - x & 3 \\ 3 & 3 & -6 - x \end{vmatrix} = 0$ are :
4. $\begin{vmatrix} \sin^2 x & \cos^2 x & 1 \\ \cos^2 x & \sin^2 x & 1 \\ -10 & 12 & 2 \end{vmatrix} =$
5. If $D_p = \begin{vmatrix} p & 15 & 8 \\ p^2 & 35 & 9 \\ p^3 & 25 & 10 \end{vmatrix}$, then $D_1 + D_2 + D_3 + D_4 + D_5 =$
6. If $\begin{vmatrix} 4 & 1 \\ 2 & 1 \end{vmatrix}^2 = \begin{vmatrix} 3 & 2 \\ 1 & x \end{vmatrix} - \begin{vmatrix} x & 3 \\ -2 & 1 \end{vmatrix}$, then $x =$
7. If a, b, c are in A.P., then the value of $\begin{vmatrix} x+2 & x+3 & x+a \\ x+4 & x+5 & x+b \\ x+6 & x+7 & x+c \end{vmatrix}$ is :
8. If $\Delta = \begin{vmatrix} x & y & z \\ p & q & r \\ a & b & c \end{vmatrix}$, then $\begin{vmatrix} x & 2y & z \\ 2p & 4q & 2r \\ a & 2b & c \end{vmatrix}$ equals
9. If $\begin{vmatrix} a & b & c \\ m & n & p \\ x & y & z \end{vmatrix} = k$, then $\begin{vmatrix} 6a & 2b & 2c \\ 3m & n & p \\ 3x & y & z \end{vmatrix} =$
10. If $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 5$; then the value of $\begin{vmatrix} b_2c_3 - b_3c_2 & c_2a_3 - c_3a_2 & a_2b_3 - a_3b_2 \\ b_3c_1 - b_1c_3 & c_3a_1 - c_1a_3 & a_3b_1 - a_1b_3 \\ b_1c_2 - b_2c_1 & c_1a_2 - c_2a_1 & a_1b_2 - a_2b_1 \end{vmatrix}$ is:
11. If $\begin{vmatrix} (b+c)^2 & a^2 & a^2 \\ b^2 & (c+a)^2 & b^2 \\ c^2 & c^2 & (a+b)^2 \end{vmatrix} = kabc(a+b+c)^3$, then the value of k is :
12. If A, B, C be the angles of a triangle, then $\begin{vmatrix} -1 & \cos C & \cos B \\ \cos C & -1 & \cos A \\ \cos B & \cos A & -1 \end{vmatrix} =$
13. $\begin{vmatrix} (a^x + a^{-x})^2 & (a^x - a^{-x})^2 & 1 \\ (b^x + b^{-x})^2 & (b^x - b^{-x})^2 & 1 \\ (c^x + c^{-x})^2 & (c^x - c^{-x})^2 & 1 \end{vmatrix} =$
14. The value of the determinant $\begin{vmatrix} 1 & \cos(\alpha - \beta) & \cos \alpha \\ \cos(\alpha - \beta) & 1 & \cos \beta \\ \cos \alpha & \cos \beta & 1 \end{vmatrix}$ is :
15. If $\begin{vmatrix} y+z & x-z & x-y \\ y-z & z+x & y-x \\ z-y & z-x & x+y \end{vmatrix} = kxyz$, then the value of k is :

EXERCISE-I

1. (a) Prove that the value of the determinant $\begin{vmatrix} -7 & 5+3i & \frac{2}{3}-4i \\ 5-3i & 8 & 4+5i \\ \frac{2}{3}+4i & 4-5i & 9 \end{vmatrix}$ is real.

(b) Prove that the value of the determinant $\begin{vmatrix} 1 & a & a^2-bc \\ 1 & b & b^2-ca \\ 1 & c & c^2-ab \end{vmatrix} = 0$

(c) On which one of the parameter out of a, p, d or x, the value of the determinant

$\begin{vmatrix} 1 & a & a^2 \\ \cos(p-d)x & \cos px & \cos(p+d)x \\ \sin(p-d)x & \sin px & \sin(p+d)x \end{vmatrix}$ does not depend

2. Without expanding as far as possible, prove that

(a) $\begin{vmatrix} a^2+2a & 2a+1 & 1 \\ 2a+1 & a+2 & 1 \\ 3 & 3 & 1 \end{vmatrix} = (a-1)^3$, (b) $\begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ x^3 & y^3 & z^3 \end{vmatrix} = [(x-y)(y-z)(z-x)(x+y+z)]$

3. If $\begin{vmatrix} x^3+1 & x^2 & x \\ y^3+1 & y^2 & y \\ z^3+1 & z^2 & z \end{vmatrix} = 0$ and x, y, z are all different then, prove that $xyz = -1$.

4. Using properties of determinants or otherwise evaluate $\begin{vmatrix} 18 & 40 & 89 \\ 40 & 89 & 198 \\ 89 & 198 & 440 \end{vmatrix}$.

5. Prove that $\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^3$.

6. If $D = \begin{vmatrix} a & b & c \\ c & a & b \\ b & c & a \end{vmatrix}$ and $D' = \begin{vmatrix} b+c & c+a & a+b \\ a+b & b+c & c+a \\ c+a & a+b & b+c \end{vmatrix}$ then prove that $D' = 2D$.

7. Prove that $\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} = (1+a^2+b^2)^3$.

8. Prove that $\begin{vmatrix} a & b-c & c+b \\ a+c & b & c-a \\ a-b & b+a & c \end{vmatrix} = (a+b+c)(a^2+b^2+c^2)$.

9. Show that the value of the determinant $\begin{vmatrix} \tan(A+P) & \tan(B+P) & \tan(C+P) \\ \tan(A+Q) & \tan(B+Q) & \tan(C+Q) \\ \tan(A+R) & \tan(B+R) & \tan(C+R) \end{vmatrix}$ vanishes for

all values of A, B, C, P, Q & R where $A+B+C+P+Q+R=0$

10. Factorise the determinant $\begin{vmatrix} bc & bc'+b'c & b'c' \\ ca & ca'+c'a & c'a' \\ ab & ab'+a'b & a'b' \end{vmatrix}$.

11. Prove that $\begin{vmatrix} (\beta+\gamma-\alpha-\delta)^4 & (\beta+\gamma-\alpha-\delta)^2 & 1 \\ (\gamma+\alpha-\beta-\delta)^4 & (\gamma+\alpha-\beta-\delta)^2 & 1 \\ (\alpha+\beta-\gamma-\delta)^4 & (\alpha+\beta-\gamma-\delta)^2 & 1 \end{vmatrix} = -64(\alpha-\beta)(\alpha-\gamma)(\alpha-\delta)(\beta-\gamma)(\beta-\delta)(\gamma-\delta)$

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12. For a fixed positive integer n , if $D = \begin{vmatrix} n! & (n+1)! & (n+2)! \\ (n+1)! & (n+2)! & (n+3)! \\ (n+2)! & (n+3)! & (n+4)! \end{vmatrix}$ then show that $\left[\frac{D}{(n!)^3} - 4\right]$ is divisible by n .

13. Solve for x $\begin{vmatrix} x+2 & 2x+3 & 3x+4 \\ 2x+3 & 3x+4 & 4x+5 \\ 3x+5 & 5x+8 & 10x+17 \end{vmatrix} = 0$

14. If $p + q + r = 0$, prove that $\begin{vmatrix} pa & qb & rc \\ qc & ra & pb \\ rb & pc & qa \end{vmatrix} = pqr \begin{vmatrix} a & b & c \\ c & a & b \\ b & c & a \end{vmatrix}$.

15. If a, b, c are all different & $\begin{vmatrix} a & a^3 & a^4 - 1 \\ b & b^3 & b^4 - 1 \\ c & c^3 & c^4 - 1 \end{vmatrix} = 0$, then prove that, $abc(ab + bc + ca) = a + b + c$.

16. Show that, $\begin{vmatrix} a^2 + \lambda & ab & ac \\ ab & b^2 + \lambda & bc \\ ac & bc & c^2 + \lambda \end{vmatrix}$ is divisible by λ^2 and find the other factor.

17. (a) Without expanding prove that $\begin{vmatrix} bc & a & a^2 \\ ca & b & b^2 \\ ab & c & c^2 \end{vmatrix} = \begin{vmatrix} 1 & a^2 & a^3 \\ 1 & b^2 & b^3 \\ 1 & c^2 & c^3 \end{vmatrix}$.

(b) $\begin{vmatrix} a^2 & b^2 & c^2 \\ (a+1)^2 & (b+1)^2 & (c+1)^2 \\ (a-1)^2 & (b-1)^2 & (c-1)^2 \end{vmatrix} = 4 \begin{vmatrix} a^2 & b^2 & c^2 \\ a & b & c \\ 1 & 1 & 1 \end{vmatrix}$.

18. Solve for x : $\begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ x-4 & 2x-9 & 3x-16 \\ x-8 & 2x-27 & 3x-64 \end{vmatrix} = 0$

19. If $D_r = \begin{vmatrix} 2^{r-1} & 2(3^{r-1}) & 4(5^{r-1}) \\ x & y & z \\ 2^n - 1 & 3^n - 1 & 5^n - 1 \end{vmatrix}$ then prove that $\sum_{r=1}^n D_r = 0$

20. In a $\triangle ABC$, determine condition under which $\begin{vmatrix} \cot \frac{A}{2} & \cot \frac{B}{2} & \cot \frac{C}{2} \\ \tan \frac{B}{2} + \tan \frac{C}{2} & \tan \frac{C}{2} + \tan \frac{A}{2} & \tan \frac{A}{2} + \tan \frac{B}{2} \\ 1 & 1 & 1 \end{vmatrix} = 0$

21. Prove that: $\begin{vmatrix} (a-p)^2 & (a-q)^2 & (a-r)^2 \\ (b-p)^2 & (b-q)^2 & (b-r)^2 \\ (c-p)^2 & (c-q)^2 & (c-r)^2 \end{vmatrix} = \begin{vmatrix} (1+ap)^2 & (1+aq)^2 & (1+ar)^2 \\ (1+bp)^2 & (1+bq)^2 & (1+br)^2 \\ (1+cp)^2 & (1+cq)^2 & (1+cr)^2 \end{vmatrix}$

22. Prove that $\begin{vmatrix} (a_1 - b_1)^2 & (a_1 - b_2)^2 & (a_1 - b_3)^2 \\ (a_2 - b_1)^2 & (a_2 - b_2)^2 & (a_2 - b_3)^2 \\ (a_3 - b_1)^2 & (a_3 - b_2)^2 & (a_3 - b_3)^2 \end{vmatrix} = 2(a_1 - a_2)(a_2 - a_3)(a_3 - a_1)(b_1 - b_2)(b_2 - b_3)(b_3 - b_1)$

23. If $ax_1^2 + by_1^2 + cz_1^2 = ax_2^2 + by_2^2 + cz_2^2 = ax_3^2 + by_3^2 + cz_3^2 = d$

and $ax_2x_3 + by_2y_3 + cz_2z_3 = ax_3x_1 + by_3y_1 + cz_3z_1 = ax_1x_2 + by_1y_2 + cz_1z_2 = f$,

then prove that $\begin{vmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{vmatrix} = (d-f) \left[\frac{d+2f}{abc}\right]^{1/2} \quad (a, b, c \neq 0)$

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24. If $S_r = \alpha^r + \beta^r + \gamma^r$ then show that $\begin{vmatrix} S_0 & S_1 & S_2 \\ S_1 & S_2 & S_3 \\ S_2 & S_3 & S_4 \end{vmatrix} = (\alpha - \beta)^2(\beta - \gamma)^2(\gamma - \alpha)^2$.

25. If $u = ax^2 + 2bxy + cy^2$, $u' = a'x^2 + 2b'xy + c'y^2$. Prove that

$$\begin{vmatrix} y^2 & -xy & x^2 \\ a & b & c \\ a' & b' & c' \end{vmatrix} = \begin{vmatrix} ax + by & bx + cy \\ a'x + b'y & b'x + c'y \end{vmatrix} = -\frac{1}{y} \begin{vmatrix} u & u' \\ ax + by & a'x + b'y \end{vmatrix}.$$



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EXERCISE-II

1. Solve using Cramer's rule : $\frac{4}{x+5} + \frac{3}{y+7} = -1$ & $\frac{6}{x+5} - \frac{6}{y+7} = -5$.
2. Solve the following using Cramer's rule and state whether consistent or not.
 - (a) $x + y + z - 6 = 0$
 $2x + y - z - 1 = 0$
 $x + y - 2z + 3 = 0$
 - (b) $7x - 7y + 5z = 3$
 $3x + y + 5z = 7$
 $2x + 3y + 5z = 5$
3. Solve the system of equations ; $\left. \begin{array}{l} z + ay + a^2x + a^3 = 0 \\ z + by + b^2x + b^3 = 0 \\ z + cy + c^2x + c^3 = 0 \end{array} \right\}$
4. For what value of K do the following system of equations possess a non trivial (i.e. not all zero) solution over the set of rationals Q ?
 $x + Ky + 3z = 0$, $3x + Ky - 2z = 0$, $2x + 3y - 4z = 0$
 For that value of K , find all the solutions of the system.
5. Given $x = cy + bz$; $y = az + cx$; $z = bx + ay$ where x, y, z are not all zero , prove that
 $a^2 + b^2 + c^2 + 2abc = 1$
6. Given $a = \frac{x}{y-z}$; $b = \frac{y}{z-x}$; $c = \frac{z}{x-y}$ where x, y, z are not all zero, prove that : $1 + ab + bc + ca = 0$.
7. If $\sin p \neq \cos q$ and x, y, z satisfy the equations
 $x \csc p - y \sin p + z = \cos q + 1$
 $x \sin p + y \csc p + z = 1 - \sin q$
 $x \cos(p + q) - y \sin(p + q) + z = 2$
 then find the value of $x^2 + y^2 + z^2$.
8. Investigate for what values of λ, μ the simultaneous equations $x + y + z = 6$;
 $x + 2y + 3z = 10$ & $x + 2y + \lambda z = \mu$ have ;
 (a) A unique solution.
 (b) An infinite number of solutions.
 (c) No solution.
9. For what values of p , the equations : $x + y + z = 1$; $x + 2y + 4z = p$ & $x + 4y + 10z = p^2$ have a solution ? Solve them completely in each case.
10. Solve the equations : $Kx + 2y - 2z = 1$, $4x + 2Ky - z = 2$, $6x + 6y + Kz = 3$ considering specially the case when $K = 2$.

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11. Let a, b, c, d are distinct numbers to be chosen from the set $\{1, 2, 3, 4, 5\}$. If the least possible positive solution for x to the system of equations $\begin{cases} ax + by = 1 \\ cx + dy = 2 \end{cases}$ can be expressed in the form $\frac{p}{q}$ where p and q are relatively prime, then find the value of $(p + q)$.
12. If $bc + qr = ca + rp = ab + pq = -1$ show that $\begin{vmatrix} ap & a & p \\ bq & b & q \\ cr & c & r \end{vmatrix} = 0$.
13. If the following system of equations $(a - t)x + by + cz = 0$, $bx + (c - t)y + az = 0$ and $cx + ay + (b - t)z = 0$ has non-trivial solutions for different values of t , then show that we can express product of these values of t in the form of determinant.
14. Show that the system of equations $3x - y + 4z = 3$, $x + 2y - 3z = -2$ and $6x + 5y + \lambda z = -3$ has atleast one solution for any real number λ . Find the set of solutions of $\lambda = -5$.

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EXERCISE-III

1. If α, β, γ are the roots of $x^3 + px^2 + q = 0$, where $q \neq 0$, then $\Delta = \begin{bmatrix} 1/\alpha & 1/\beta & 1/\gamma \\ 1/\beta & 1/\gamma & 1/\alpha \\ 1/\gamma & 1/\alpha & 1/\beta \end{bmatrix}$ equals
 (A) p/q (B) q/p (C) pq (D) 0
2. If $\Delta = \begin{vmatrix} \sqrt{6} & 2i & 3 + \sqrt{6} \\ \sqrt{12} & \sqrt{3} + \sqrt{8}i & 3\sqrt{2} + \sqrt{6}i \\ \sqrt{18} & \sqrt{2} + \sqrt{12}i & \sqrt{27} + 2i \end{vmatrix}$ then Δ is ($i^2 = -1$)
 (A) a negative integer (B) a natural number
 (C) an irrational number (D) an imaginary number
3. If x, y, z are different from zero and $\Delta = \begin{vmatrix} a & b-y & c-z \\ a-x & b & c-z \\ a-x & b-y & c \end{vmatrix} = 0$, then the value of the expression $\frac{a}{x} + \frac{b}{y} + \frac{c}{z}$ is
 (A) 0 (B) -1 (C) 1 (D) 2
4. If $p + q + r = a + b + c = 0$, then the determinant $\Delta = \begin{vmatrix} pa & qb & rc \\ qc & ra & pb \\ rb & pc & qa \end{vmatrix}$ equals
 (A) 0 (B) 1 (C) $pa + qb + rc$ (D) $abcpqr$
5. If $p \neq a, q \neq b, r \neq c$ and the system of equations
 $px + ay + az = 0$
 $bx + qy + bz = 0$
 $cx + cy + rz = 0$
 has a non-trivial solution, then the value of $\frac{p}{p-a} + \frac{q}{q-b} + \frac{r}{r-c}$ is
 (A) -1 (B) 0 (C) 1 (D) 2
6. If the system of linear equations $x + y + z = 6$, $x + 2y + 3z = 14$ and $2x + 5y + \lambda z = \mu$, ($\lambda, \mu \in \mathbb{R}$) has more than one solution, then
 (A) $\lambda \neq 8, \mu \in \mathbb{R}$ (B) $\lambda = 8, \mu \neq 36$
 (C) $\lambda = 8, \mu = 36$ (D) $\lambda = a, \mu \in \mathbb{R}$
7. If a, b, c are positive and not all equal, then the value of the determinant $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$ is always
 (A) > 0 (B) 0 (C) < 0 (D) none of these
8. Let $px^4 + qx^3 + rx^2 + sx + t =$
 $\begin{vmatrix} x^2 + 3x & x - 1 & x + 3 \\ x + 1 & -2x & x - 4 \\ x - 3 & x + 4 & 3x \end{vmatrix}$ be an identity, where p, q, r, s, t are constants. Then $t =$
 (A) 0 (B) 1 (C) 2 (D) -1

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9. If $\begin{vmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{vmatrix} = \lambda \begin{vmatrix} a^2 & b^2 & c^2 \\ a & b & c \\ 1 & 1 & 1 \end{vmatrix}$, then $\lambda =$
- (A) 1 (B) -1 (C) 2 (D) -2
10. The number of real values of λ for which the system of equations $\lambda x + y + z = 0$, $x - \lambda y - z = 0$, $x + y - \lambda z = 0$ will have nontrivial solution is
- (A) 0 (B) 1 (C) 2 (D) 3
11. If x, y, z be non-unity positive numbers, then $\begin{vmatrix} 1 & \log_x y & \log_x z \\ \log_y x & 1 & \log_y z \\ \log_z x & \log_z y & 1 \end{vmatrix} =$
- (A) 0 (B) 1 (C) -1 (D) value depends on x, y, z
12. $\begin{vmatrix} xp + y & x & y \\ yp + z & y & z \\ 0 & xp + y & yp + z \end{vmatrix} = 0$ if
- (A) x, y, z are in A.P (B) x, y, z are in G.P
(C) x, y, z are in H.P (D) xy, yz, zx are in A.P
13. The number of distinct real roots of the equation $\begin{vmatrix} \sin x & \cos x & \cos x \\ \cos x & \sin x & \cos x \\ \cos x & \cos x & \sin x \end{vmatrix} = 0$ in $\left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$ is
- (A) 0 (B) 1 (C) 2 (D) 3
14. The value of λ such that the system $x - 2y + z = -4$, $2x - y + 2z = 2$, $x + y + \lambda z = 4$ has infinitely many solutions is
- (A) 0 (B) 1 (C) -1 (D) none of these
15. If the equations : $x + ay - z = 0$, $2x - y + az = 0$, $ax + y + 2z = 0$; have nontrivial solutions , then a can't be
- (A) 2 (B) -2 (C) $1 + \sqrt{3}$ (D) $1 - \sqrt{3}$

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EXERCISE-IV

1. If $a > 0$ and discriminant of $ax^2 + 2bx + c$ is $-ve$, then [AIEEE 2002]

$$\begin{vmatrix} a & b & ax+b \\ b & c & bx+c \\ ax+b & bx+c & 0 \end{vmatrix}$$
 is equal to
 (A) $+ve$ (B) $(ac - b^2)(ax^2 + 2bx + c)$
 (C) $-ve$ (D) 0
2. If the system of linear equations [AIEEE 2003]
 $x + 2ay + az = 0$; $x + 3by + bz = 0$; $x + 4cy + cz = 0$; has a non-zero solution, then a, b, c .
 (A) Satisfy $a + 2b + 3c = 0$ (B) Are in A.P.
 (C) Are in G.P. (D) Are in H.P.
3. If $a_1, a_2, a_3, \dots, a_n, \dots$ are in G.P., then the value of the determinant [AIEEE 2004]

$$\begin{vmatrix} \log a_n & \log a_{n+1} & \log a_{n+2} \\ \log a_{n+3} & \log a_{n+4} & \log a_{n+5} \\ \log a_{n+6} & \log a_{n+7} & \log a_{n+8} \end{vmatrix}$$

 (A) -2 (B) 1 (C) 2 (D) 0
4. The system of equations [AIEEE 2005]
 $\alpha x + y + z = \alpha - 1$
 $x + \alpha y + z = \alpha - 1$
 $x + y + \alpha z = \alpha - 1$
 has infinite solutions, if α is
 (A) -2 (B) Either -2 or 1 (C) not -2 (D) 1
5. If $a^2 + b^2 + c^2 = -2$ and $f(x) = \begin{vmatrix} 1 + a^2x & (1 + b^2)x & (1 + c^2)x \\ (1 + a^2)x & 1 + b^2x & (1 + c^2)x \\ (1 + a^2)x & (1 + b^2)x & 1 + c^2x \end{vmatrix}$, then $f(x)$ is a polynomial of [AIEEE 2005]
 degree
 (A) 1 (B) 0 (C) 3 (D) 2
6. If $a_1, a_2, a_3, \dots, a_n$ are in G.P., then the determinant [AIEEE 2005]

$$\Delta = \begin{vmatrix} \log a_n & \log a_{n+1} & \log a_{n+2} \\ \log a_{n+3} & \log a_{n+4} & \log a_{n+5} \\ \log a_{n+6} & \log a_{n+7} & \log a_{n+8} \end{vmatrix}$$
 is equal to
 (A) 1 (B) 0 (C) 4 (D) 2
7. If $D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+x & 1 \\ 1 & 1 & 1+y \end{vmatrix}$ for $x \neq 0, y \neq 0$, then D is [AIEEE 2007]
 (A) Divisible by x but not y (B) Divisible by y but not x
 (C) Divisible by neither x nor y (D) Divisible by both x and y

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8. Let $A = \begin{vmatrix} 5 & 5\alpha & \alpha \\ 0 & \alpha & 5\alpha \\ 0 & 0 & 5 \end{vmatrix}$. If $|A^2| = 25$, then $|\alpha|$ equals. [AIEEE 2007]
 (A) $1/5$ (B) 5 (C) 5^2 (D) 1
9. Let a, b, c be any real numbers. Suppose that there are real numbers x, y, z not all zero such that $x = cy + bz$, $y = az + cx$, and $z = bx + ay$. Then $a^2 + b^2 + c^2 + 2abc$ is equal to [AIEEE 2008]
 (A) 2 (B) -1 (C) 0 (D) 1
10. Let a, b, c be such that $b(a + c) \neq 0$ if [AIEEE 2009]
 $\begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix} + \begin{vmatrix} a+1 & b+1 & c-1 \\ a-1 & b-1 & c+1 \\ (-1)^{n+2}a & (-1)^{n+1}b & (-1)^nc \end{vmatrix} = 0$, then the value of n is :
 (A) Any even integer (B) Any odd integer
 (C) Any integer (D) Zero
11. Consider the system of linear equations : [AIEEE 2010]
 $x_1 + 2x_2 + x_3 = 3$
 $2x_1 + 3x_2 + x_3 = 3$
 $3x_1 + 5x_2 + 2x_3 = 1$
 Then system has
 (A) Exactly 3 solutions (B) A unique solution
 (C) No solution (D) Infinite number of solutions
12. The number of values of k for which the linear equation $4x + ky + 2z = 0$, $kx + 4y + z = 0$ and $2x + 2y + z = 0$ possess a non-zero solution is [AIEEE 2011]
 (A) 2 (B) 1 (C) Zero (D) 3
13. The number of values of k , for which the system of equations : [JEE Main 2013]
 $(k + 1)x + 8y = 4k$
 $kx + (k + 3)y = 3k - 1$
 has no solution, is :
 (A) 3 (B) infinite (C) 1 (D) 2
14. If $\alpha, \beta \neq 0$, and $f(n) = \alpha^n + \beta^n$ and [JEE Main 2014]
 $\begin{vmatrix} 3 & 1+f(1) & 1+f(2) \\ 1+f(1) & 1+f(2) & 1+f(3) \\ 1+f(2) & 1+f(3) & 1+f(4) \end{vmatrix} = K(1 - \alpha)^2(1 - \beta)^2(\alpha - \beta)^2$, then K is equal to :
 (A) -1 (B) $\alpha\beta$ (C) $\frac{1}{\alpha\beta}$ (D) 1

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15. The set of all values of λ for which the system of linear equations : [JEE Main 2015]
 $2x_1 - 2x_2 + x_3 = \lambda x_1$
 $2x_1 - 3x_2 + 2x_3 = \lambda x_2$
 $-x_1 + 2x_2 = \lambda x_3$
 has a non-trivial solution,
 (A) contains more than two elements (B) is an empty set
 (C) is a singleton (D) contains two elements
16. The system of linear equations [JEE Main 2016]
 $x + \lambda y - z = 0$
 $\lambda x - y - z = 0$
 $x + y - \lambda z = 0$
 has a non-trivial solution for :
 (A) infinitely many values of λ . (B) exactly one value of λ
 (C) exactly two values of λ (D) exactly three values of λ
17. If S is the set of distinct values of 'b' for which the following system of linear equations [JEE Main 2017]
 $x + y + z = 1$
 $x + ay + z = 1$
 $ax + by + z = 0$
 has no solution, then S is :
 (A) a finite set containing two or more elements (B) a singleton
 (C) an empty set (D) an infinite set
18. If the system of linear equations [JEE Main 2018]
 $x + ky + 3z = 0$
 $3x + ky - 2z = 0$
 $2x + 4y - 3z = 0$
 has a non-zero solution (x, y, z) , then $\frac{xz}{y^2}$ is equal to
 (A) 30 (B) -10 (C) 10 (D) -30
19. If $\begin{vmatrix} x-4 & 2x & 2x \\ 2x & x-4 & 2x \\ 2x & 2x & x-4 \end{vmatrix} = (A+Bx)(x-A)^2$, then the ordered pair (A, B) is equal to [JEE Main 2018]
 (A) (4,5) (B) (-4, -5) (C) (-4,3) (D) (-4,5)

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20. Let S be the set of all real values of k for which the system of linear equations

$$x + y + z = 2$$

$$2x + y - z = 3$$

$$3x + 2y + kz = 4$$

has a unique solution. Then S is :

[JEE Main 2018]

- (A) an empty set (B) equal to $\{0\}$ (C) equal to \mathbb{R} (D) equal to $\mathbb{R} - \{0\}$

21. If the system of linear equations

$$x + ay + z = 3$$

$$x + 2y + 2z = 6$$

$$x + 5y + 3z = b$$

has no solution, then :

[JEE Main 2018]

- (A) $a = -1, b = 9$ (B) $a = -1, b \neq 9$
(C) $a \neq -1, b = 9$ (D) $a = 1, b \neq 9$

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EXERCISE-V

1. (a) If $f(x) = \begin{vmatrix} 1 & x & x+1 \\ 2x & x(x-1) & (x+1)x \\ 3x(x-1) & x(x-1)(x-2) & (x+1)x(x-1) \end{vmatrix}$ then $f(100)$ is equal to:

- (A) 0 (B) 1 (C) 100 (D) -100

(b) Let a, b, c, d be real numbers in G.P. If u, v, w satisfy the system of equations,

$$u + 2v + 3w = 6$$

$$4u + 5v + 6w = 12$$

$$6u + 9v = 4$$

then show that the roots of the equation,

$$\left(\frac{1}{u} + \frac{1}{v} + \frac{1}{w}\right)x^2 + [(b-c)^2 + (c-a)^2 + (d-b)^2]x + u + v + w = 0$$

$$\text{and } 20x^2 + 10(a-d)^2x - 9 = 0$$

are reciprocals of each other.

[JEE '99, 2 + 10 out of 200]

2. If the system of equations $x - Ky - z = 0$, $Kx - y - z = 0$ and $x + y - z = 0$ has a non zero solution, then the possible values of K are

[JEE 2000 (Screening)]

- (A) -1, 2 (B) 1, 2 (C) 0, 1 (D) -1, 1

3. Prove that for all values of θ , $\begin{vmatrix} \sin \theta & \cos \theta & \sin 2\theta \\ \sin \left(\theta + \frac{2\pi}{3}\right) & \cos \left(\theta + \frac{2\pi}{3}\right) & \sin \left(2\theta + \frac{4\pi}{3}\right) \\ \sin \left(\theta - \frac{2\pi}{3}\right) & \cos \left(\theta - \frac{2\pi}{3}\right) & \sin \left(2\theta - \frac{4\pi}{3}\right) \end{vmatrix} = 0$

[JEE 2000 (Mains)]

4. Find the real values of r for which the following system of linear equations has a non-trivial solution. Also find the non-trivial solutions :

[REE 2000 (Mains)]

$$2rx - 2y + 3z = 0$$

$$x + ry + 2z = 0$$

$$2x + rz = 0$$

5. Solve for x the equation $\begin{vmatrix} a^2 & a & 1 \\ \sin(n+1)x & \sin nx & \sin(n-1)x \\ \cos(n+1)x & \cos nx & \cos(n-1)x \end{vmatrix} = 0$ [REE 2001 (Mains)]

6. Test the consistency and solve them when consistent, the following system of equations for all values of λ

[REE 2001 (Mains)]

$$x + y + z = 1$$

$$x + 3y - 2z = \lambda$$

$$3x + (\lambda + 2)y - 3z = 2\lambda + 1$$

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7. Let a, b, c be real numbers with $a^2 + b^2 + c^2 = 1$. Show that the equation [JEE 2001 (Mains)]

$$\begin{vmatrix} ax - by - c & bx + ay & cx + a \\ bx + ay & -ax + by - c & cy + b \\ cx + a & cy + b & -ax - by + c \end{vmatrix} = 0 \text{ represents a straight line.}$$

8. The number of values of k for which the system of equations [JEE 2002 (Screening), 3]

$$(k + 1)x + 8y = 4k$$

$$kx + (k + 3)y = 3k - 1$$

has infinitely many solutions is

- (A) 0 (B) 1 (C) 2 (D) infinite

9. The value of λ for which the system of equations

$$2x - y - z = 12, x - 2y + z = -4, x + y + \lambda z = 4 \text{ has no solution is [JEE 2004 (Scr.)]}$$

- (A) 3 (B) -3 (C) 2 (D) -2

10. (a) Consider three points $P = (-\sin(\beta - \alpha), -\cos\beta)$, $Q = (\cos(\beta - \alpha), \sin\beta)$ and

$$R = (\cos(\beta - \alpha + \theta), \sin(\beta - \theta)), \text{ where } 0 < \alpha, \beta, \theta < \pi/4$$

- (A) P lies on the line segment RQ (B) Q lies on the line segment PR
(C) R lies on the line segment QP (D) P, Q, R are non collinear

(b) Consider the system of equations

$$x - 2y + 3z = -1$$

$$-x + y - 2z = k$$

$$x - 3y + 4z = 1$$

STATEMENT-1 : The system of equations has no solution for $k \neq 3$.

STATEMENT-2 : The determinant $\begin{vmatrix} 1 & 3 & -1 \\ -1 & -2 & k \\ 1 & 4 & 1 \end{vmatrix} \neq 0$, for $k \neq 3$.

(A) Statement-1 is True, Statement-2 is True ; statement-2 is a correct explanation for statement-1

(B) Statement-1 is True, Statement-2 is True ; statement-2 is NOT a correct explanation for statement-1

(C) Statement-1 is True, Statement-2 is False

(D) Statement-1 is False, Statement-2 is True

[JEE 2008, 3 + 3]

11. Which of the following values of α satisfy the equation

[IIT Advance - 2015]

$$\begin{vmatrix} (1 + \alpha)^2 & (1 + 2\alpha)^2 & (1 + 3\alpha)^2 \\ (2 + \alpha)^2 & (2 + 2\alpha)^2 & (2 + 3\alpha)^2 \\ (3 + \alpha)^2 & (3 + 2\alpha)^2 & (3 + 3\alpha)^2 \end{vmatrix} = -648\alpha ?$$

- (A) -4 (B) 9 (C) -9 (D) 4

(MATHEMATICS)

DETERMINANT

12. The total number of distinct $x \in \mathbb{R}$ for which $\begin{vmatrix} x & x^2 & 1+x^3 \\ 2x & 4x^2 & 1+8x^3 \\ 3x & 9x^2 & 1+27x^3 \end{vmatrix} = 10$ is :

[IIT Advance - 2016]

13. Let $\alpha, \lambda, \mu \in \mathbb{R}$. Consider the system of linear equations

[IIT Advance - 2016]

$$\alpha x + 2y = \lambda$$

$$3x - 2y = \mu$$

Which of the following statement(s) is(are) correct?

- (A) If $\alpha = -3$, then the system has infinitely many solutions for all values of λ and μ .
 (B) If $\alpha \neq -3$, then the system has a unique solution for all values of λ and μ .
 (C) If $\lambda + \mu = 0$, then the system has infinitely many solutions for $\alpha = -3$
 (D) If $\lambda + \mu \neq 0$, then the system has no solution for $\alpha = -3$

14. If $f(x) = \begin{vmatrix} \cos(2x) & \cos(2x) & \sin(2x) \\ -\cos x & \cos x & -\sin x \\ \sin x & \sin x & \cos x \end{vmatrix}$, then

[IIT Advance - 2017]

- (A) $f'(x) = 0$ at exactly three points in $(-\pi, \pi)$
 (B) $f(x)$ attains its maximum at $x = 0$
 (C) $f(x)$ attains its minimum at $x = 0$
 (D) $f'(x) = 0$ at more than three points in $(-\pi, \pi)$

(MATHEMATICS)

DETERMINANT

ANSWER KEY

PROFICIENCY TEST-01

1. $(a-b)(b-c)(c-a)$ 2. 0 3. 0 4. -25 5. 0
6. 0 7. 0 8. xy 9. $x = 2, -1$
10. $x = 0$ 11. $x = 0, \pm \sqrt{\frac{3}{2}(a^2 + b^2 + c^2)}$
12. $1, -9$ 13. $1 + a^2 + b^2 + c^2$
14. $(a-b)(b-c)(c-a)(a+b+c)$ 15. 0

PROFICIENCY TEST-02

1. $-(a^3 + b^3 + c^3 - 3abc)$ 2. 0 3. $4abc$ 4. $0, -3$
5. B 6. -2 7. 0 8. $4a^2b^2c^2$
9. $xyz \left(1 + \frac{1}{x} + \frac{1}{y} + \frac{1}{z}\right)$ 10. 1 11. 2, 7 12. $abc = -1$
13. 0 14. 0 15. $\frac{33}{8}$

PROFICIENCY TEST-03

1. $k^3\Delta$ 2. 24 3. $\{-9, 0, 9\}$ 4. 0 5. -28000 6. 6
7. 0 8. 4Δ 9. $6k$ 10. 25 11. 2 12. 0
13. 0 14. 0 15. 8

EXERCISE-I

1. (c) p 4. -1 10. $(ab' - a'b)(bc' - b'c)(ca' - c'a)$
13. $x = -1$ or $x = -2$ 16. $\lambda^2(a^2 + b^2 + c^2 + \lambda)$ 18. $x = 4$
20. Triangle ABC is isosceles.

EXERCISE-II

1. $x = -7, y = -4$ 2. (a) $x = 1, y = 2, z = 3$; consistent (b) inconsistent
3. $x = -(a + b + c), y = ab + bc + ca, z = -abc$
4. $K = \frac{33}{2}, x : y : z = -\frac{15}{2} : 1 : -3$ 7. 2
8. (a) $\lambda \neq 3$ (b) $\lambda = 3, \mu = 10$ (c) $\lambda = 3, \mu \neq 10$
9. $x = 1 + 2K, y = -3K, z = K$, when $p = 1$; $x = 2K, y = 1 - 3K, z = K$ when $p = 2$; where $K \in \mathbb{R}$
10. If $K \neq 2, \frac{x}{2(K+6)} = \frac{y}{2K+3} = \frac{z}{6(K-2)} = \frac{1}{2(K^2+2K+15)}$, If $K = 2$, then $x = \lambda, y = \frac{1-2\lambda}{2}$ and $z = 0$ where $\lambda \in \mathbb{R}$
11. 19 13. $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$

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14. If $\lambda \neq -5$ then $x = \frac{4}{7}$; $y = -\frac{9}{7}$ and $z = 0$;

If $\lambda = 5$ then $x = \frac{4-5K}{7}$; $y = \frac{13K-9}{7}$ and $z = K$ where $K \in \mathbb{R}$

EXERCISE-III

1. D 2. A 3. D 4. A 5. D 6. C 7. C
8. A 9. B 10. B 11. A 12. B 13. B 14. D
15. A

EXERCISE-IV

1. C 2. D 3. D 4. D 5. D 6. B 7. D
8. A 9. D 10. B 11. C 12. A 13. C 14. D
15. D 16. D 17. B 18. C 19. D 20. D 21. B

EXERCISE-V

1. (a) A 2. D 4. $r = 2$; $x = k$; $y = k/2$; $z = -k$ where $k \in \mathbb{R} - \{0\}$
5. $x = n\pi, n \in \mathbb{I}$
6. If $\lambda = 5$, system is consistent with infinite solution given by $z = K$,
 $y = \frac{1}{2}(3K + 4)$ and $x = -\frac{1}{2}(5K + 2)$ where $K \in \mathbb{R}$
If $\lambda \neq 5$, system is consistent with unique solution given by $z = \frac{1}{3}(1 - \lambda)$; $x = \frac{1}{3}(\lambda + 2)$ and $y = 0$.
8. B 9. D 10. (a) D; (b) A 11. BC 12. 2 13. BCD
14. BD