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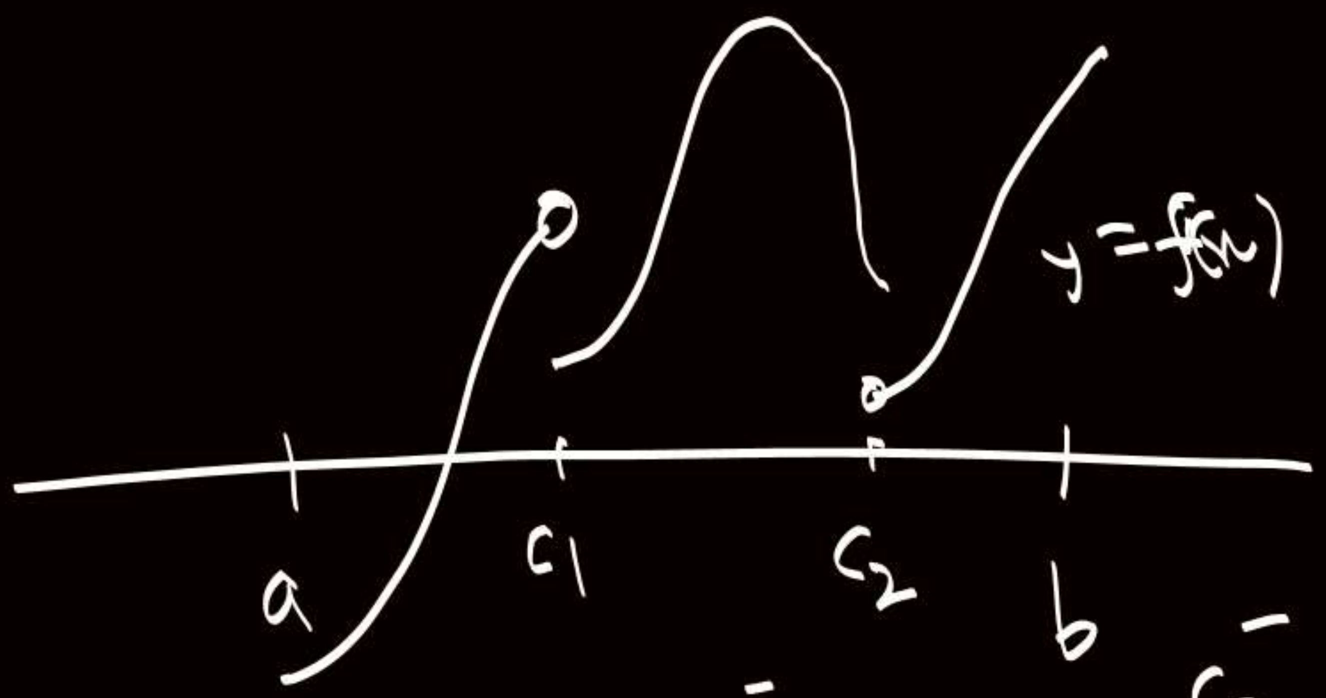
Let $f(x)$ has finite no. of discontinuities



$$\int_a^b f(x)\,dx = \int_a^{c_1^-} f(x)\,dx + \int_{c_1^+}^{c_2^-} f(x)\,dx + \int_{c_2^+}^{b} f(x)\,dx.$$

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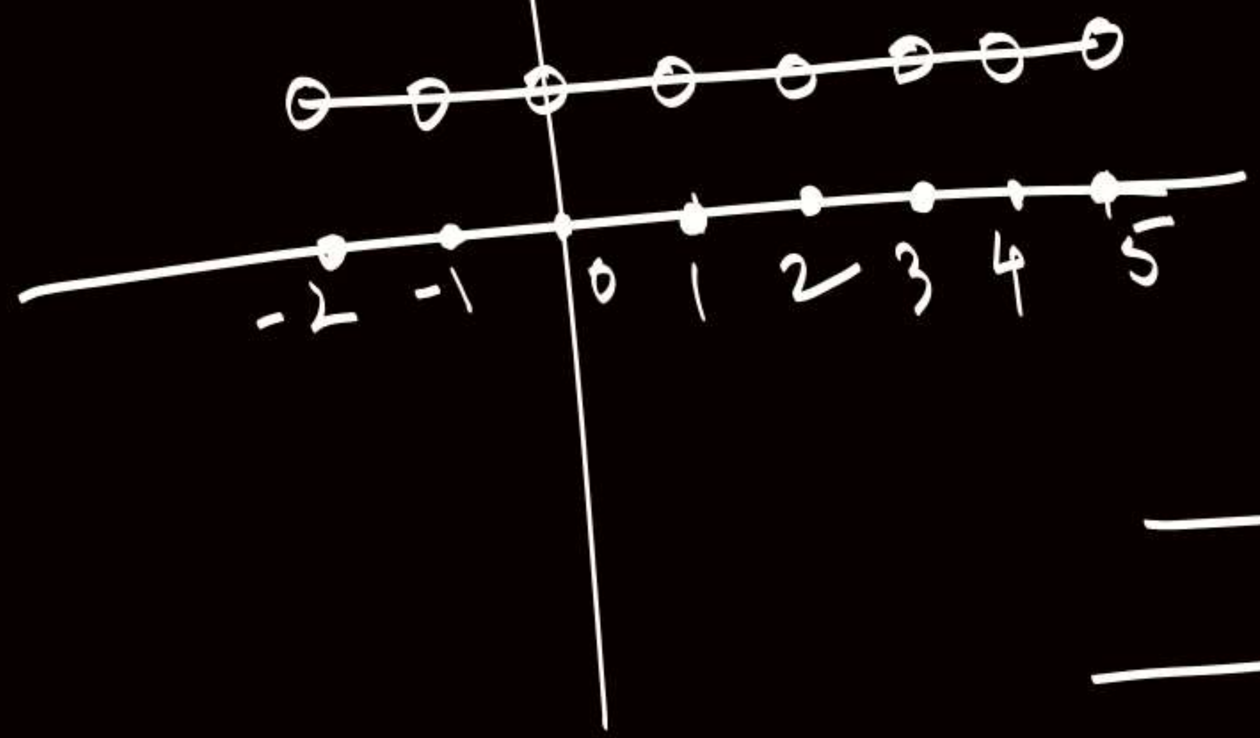
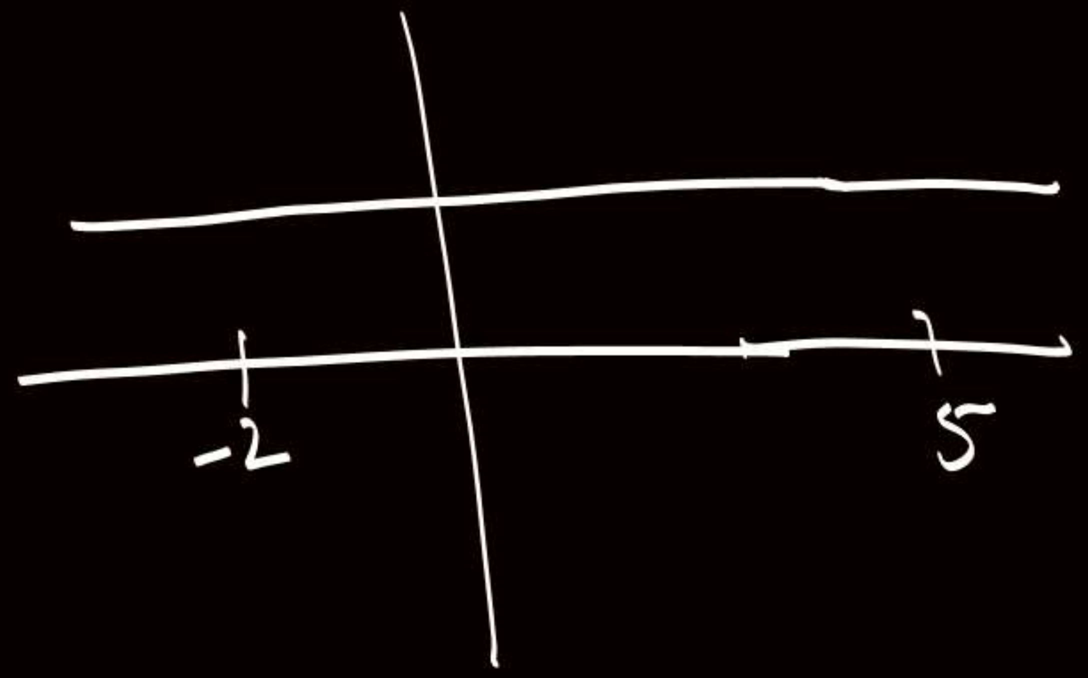
$\{\cdot\} = FPF$

$$\int_{-2}^{5} \operatorname{sgn}\{x\}\,dx = \int_{-2}^{5} 1\,dx = 7$$





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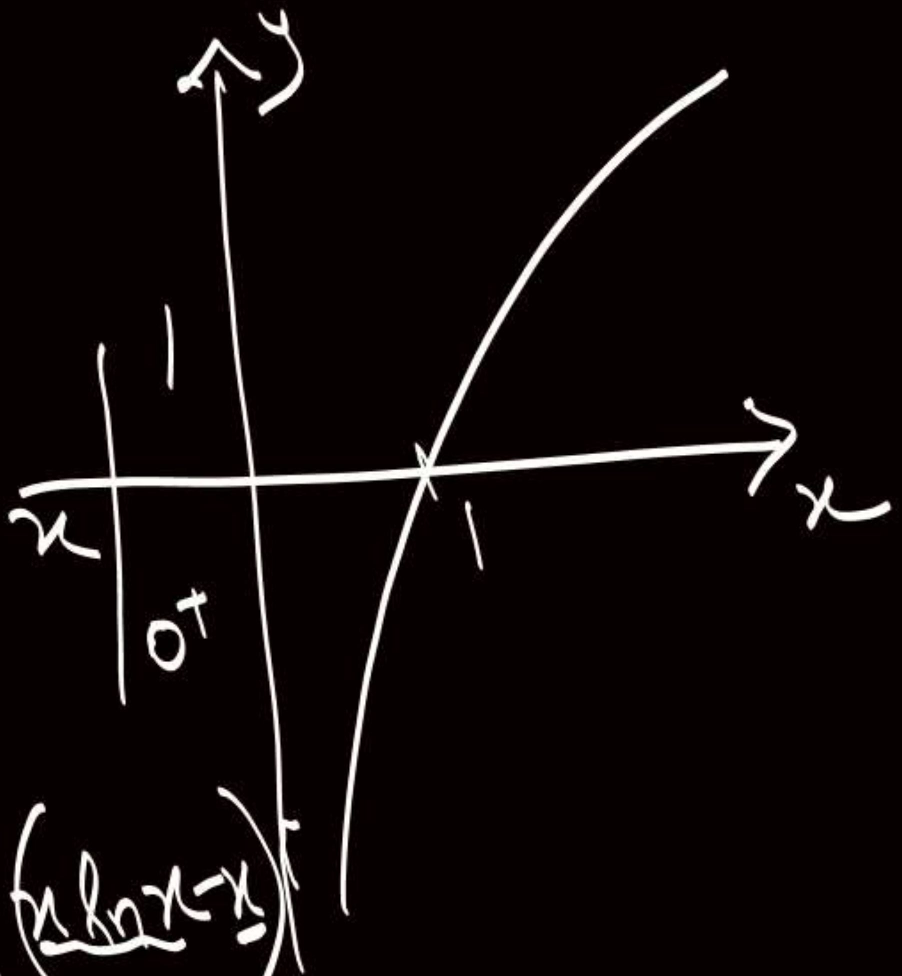
DI^n Limiting Value.

$$\int_0^1 \ln x \, dx = x\ln x - x \Big|_{0^+}^{1}$$



$$= (1\ln 1 - 1) - \lim_{x\to 0^+}(x\ln x - x)$$

$$= -1 - \lim_{x\to 0^+}\left(\frac{\ln x}{\frac{1}{x}} - x\right) \qquad \frac{\frac{1}{x}}{-\frac{1}{x^2}}$$

$$= -1$$

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$$\int_0^{\pi/2} \sin x\,dx = 1 = \int_0^{\pi/2} \cos x\,dx$$

$$\int_0^{\frac{\pi}{2}} \sin^2 x\,dx = \frac{\pi}{4} = \int_0^{\frac{\pi}{2}} \cos^2 x\,dx$$

$$\int_0^{\pi/2} \sin^3 x\,dx = \frac{2}{3} = \int_0^{\frac{\pi}{2}} \cos^3 x\,dx$$

$$\int_0^{\frac{\pi}{2}} \sin^4 x\,dx = \frac{3\pi}{16} = \int_0^{\frac{\pi}{2}} \cos^4 x\,dx$$

$$\int_0^{\pi/2} \left(1 - \cos^2 x\right) \sin x\,dx$$

$$= 1 + \left.\frac{\cos^3 x}{3}\right|_0^{\pi/2}$$

$$= 1 + \frac{0-1}{3}$$

$$= \frac{2}{3}$$

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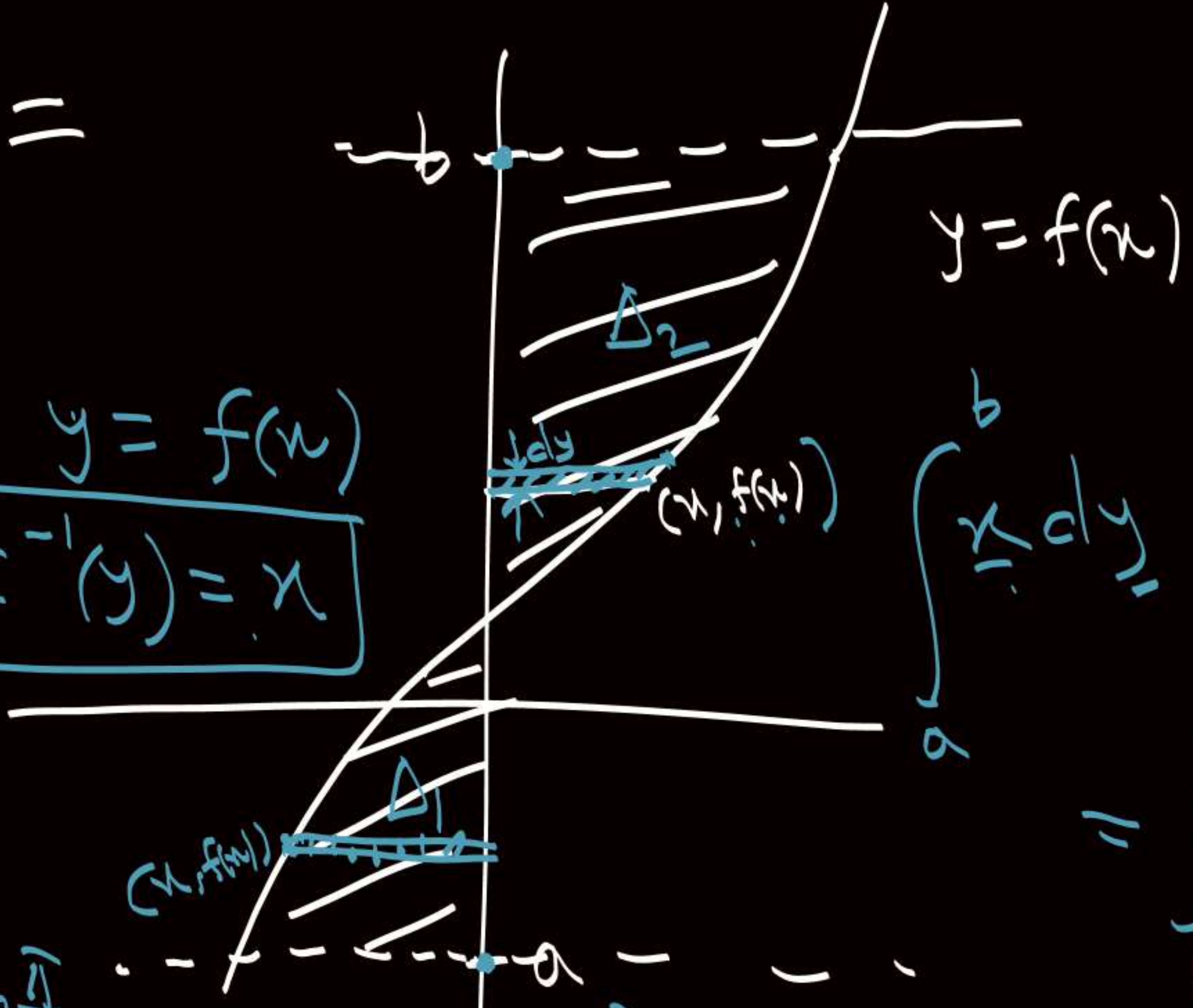
$$\int_a^b f^{-1}(x)\,dx =$$



y = f(x)

$$\boxed{f^{-1}(y) = x}$$

$$\int_a^b x\,dy = -\Delta_1 + \Delta_2$$

$$= \int_a^b f^{-1}(y)\,dy$$

$$\int_0^{\frac{\pi}{2}} \sin t\,dt = \int_0^{\pi/2} \sin u\,du = \int_a^b f^{-1}(x)\,dx$$

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1. $2\int_3^8 \frac{\sin\sqrt{x+1}}{2\sqrt{x+1}}\,dx$

2.

$= -2\cos\sqrt{x+1}\Big|_3^8 = -2\left(\cos 3 - \cos 2\right)$

$\sqrt{x+1} = t$ $\qquad \frac{1}{2\sqrt{x+1}}\,dx = dt$

$2\int_2^3 \sin t\,dt = 2\left(\cos 2 - \cos 3\right)$

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2.
$$\int_{\alpha}^{\beta} \frac{dx}{\sqrt{(x-\alpha)(\beta-x)}} \quad , \beta > \alpha .$$

$$\alpha\cos^2\theta + \beta\sin^2\theta = x$$

$$x-\alpha = (\beta-\alpha)\sin^2\theta$$

$$\beta-x = (\beta-\alpha)\cos^2\theta$$

$$dx = 2(\beta-\alpha)\sin\theta\cos\theta\, d\theta .$$

$$\int_0^{\pi/2} \frac{2(\beta-\alpha)\sin\theta\cos\theta\, d\theta}{\sqrt{(\beta-\alpha)^2\sin^2\theta\cos^2\theta}}$$

$$\int_0^{\pi/2} \frac{2(\beta-\alpha)\sin\theta\cos\theta\, d\theta}{(\beta-\alpha)\sin\theta\cos\theta} = \pi .$$

$$\int_{-\pi}^{\frac{3\pi}{2}} \frac{2(\beta-\alpha)\sin\theta\cos\theta\, d\theta}{(\beta-\alpha)\,|\sin\theta|\,|\cos\theta|}$$

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3.
$$\int_1^e \underbrace{(x+1)e^x}_{\text{II}} \underbrace{\ln x}_{\text{I}}\, dx = xe^x \ln x \Big|_1^e - \int_1^e \cancel{x}e^x \frac{1}{\cancel{x}}dx$$

$$= e^e e - 0 - e^x \Big|_1^e$$

$$= e e^e - \left(e^e - e^1\right)$$

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4.
$$\int_2^4 \frac{\sqrt{x^2-4}}{x^4}\,dx$$

$$\frac{1}{8}\int_2^4 \frac{8}{x^3}\sqrt{1-\frac{4}{x^2}}\,dx$$

$$\frac{1}{8}\times\frac{2}{3}\left(1-\frac{4}{x^2}\right)^{3/2}\Bigg|_2^4$$

$$= \frac{1}{12}\left(\frac{3}{4}\right)^{3/2} = \frac{3\sqrt{3}}{12\times 8} = \frac{\sqrt{3}}{32}$$

5:
$$\int_{-1}^{1} x^2\, d(\ln x)$$

$$d(\ln x) = \frac{1}{x}dx$$

$$\ln x = -1$$

$$\boxed{x = e^{-1}}$$

$$= \int_{e^{-1}}^{e^1} x^2\,\frac{1}{x}\,dx = \frac{x^2}{2}\Bigg|_{e^{-1}}^{e^1}$$

$$= \frac{e^2 - e^{-2}}{2}$$

$$\ln x = t \Rightarrow x = e^t$$

$$= \int_{-1}^{1} e^{2t}\,dt = \frac{e^{2t}}{2}\Bigg|_{-1}^{1}$$

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$$\int_{-1}^{1}\left(\frac{d}{dx}\left(\cot^{-1}\frac{1}{x}\right)\right)dx = \cot^{-1}\frac{1}{x}\Big|_{-1}^{0^-} + \cot^{-1}\frac{1}{x}\Big|_{0^+}^{1}$$

$$\int_{-1}^{0^-} + \int_{0^+}^{1}$$

$$= \left(\pi - \frac{3\pi}{4}\right) + \left(\frac{\pi}{4} - 0\right)$$

$$\int f'(x)\,dx = f(x)$$

$$= \frac{\pi}{2}$$

$$\boxed{\text{Ex-V (Monotonocity)}}$$