

$$\underline{24.} \quad 1 + (\cos^2 A - \sin^2 B) + \cos^2 C + \underline{2 \cos A \cos B \cos C}$$

$$= 1 + \cos(A-B) \cos(A+B) + \underline{\cos^2 C} + \frac{(\cos(A-B) + \cos(A+B))}{\underline{\cos C}}$$

$$= 1 + \left(\cos C + \cos(A+B) \right) \left(\cos C + \cos(A-B) \right)$$

$$= 1 + \left(2 \cos \left(\frac{A+B+C}{2} \right) \cos \left(\frac{A+B-C}{2} \right) \right) \left(2 \cos \left(\frac{C+A-B}{2} \right) \cos \left(\frac{C-A+B}{2} \right) \right)$$

$$= 1 + 4 \cos S \cos(S-C) \cos(S-B) \cos(S-A)$$

$$\begin{aligned}
 \underline{25.} \quad (1) \quad & 2 \cos \frac{\alpha+\beta}{2} \cos \frac{\alpha-\beta}{2} + 2 \cos \frac{\gamma+\delta}{2} \cos \frac{\gamma-\delta}{2} + 4 \cos \frac{2\pi - (\alpha+\beta)}{2} = \pi - \frac{\alpha+\beta}{2} \\
 & = 2 \cos \frac{\alpha+\beta}{2} \left(\cos \frac{\alpha-\beta}{2} - \cos \frac{\gamma-\delta}{2} \right) + 4 \cos \frac{\alpha+\beta}{2} \cos \frac{\alpha+\gamma}{2} \cos \frac{\delta+\alpha}{2} \\
 & \quad \frac{\pi}{2} - \frac{\alpha+\delta}{2} \leftarrow \sin \frac{2\pi - 2(\alpha+\delta)}{4} \sin \frac{\alpha-\beta+\gamma-\delta}{4} + 4 \cos \frac{\alpha+\beta}{2} \cos \frac{\alpha+\gamma}{2} \cos \frac{\alpha+\delta}{2} \\
 & = 4 \cos \frac{\alpha+\beta}{2} \sin \frac{2\pi - 2(\alpha+\delta)}{4} \sin \frac{\alpha-\beta+\gamma-\delta}{4} + 4 \cos \frac{\alpha+\beta}{2} \cos \frac{\alpha+\gamma}{2} \cos \frac{\alpha+\delta}{2} \\
 & \quad \cos \frac{\gamma+\delta}{2} \leftarrow \sin \frac{(\alpha+\gamma) - (2\pi - \alpha - \gamma)}{2} = \sin \left(\frac{\alpha+\gamma}{2} - \frac{\pi}{2} \right) \\
 & \quad = -\cos \left(\frac{\alpha+\gamma}{2} \right)
 \end{aligned}$$

34.

$$x = \tan A, y = \tan B, z = \tan C$$

$$\sum \tan A = \prod \tan A$$

$$\Rightarrow A + B + C = n\pi \quad n \in \mathbb{I}$$

$$3A + 3B + 3C = \boxed{3n\pi} = k\pi, \quad k \in \mathbb{I}$$

$$\sum \tan 3A = \prod \tan 3A$$

35.

$$\boxed{\sum \tan 2A = \prod \tan 2A}$$

$$\begin{aligned}
 & \underline{1} \quad \sin \theta - \sin 2\theta + \sin 3\theta - \sin 4\theta + \sin 5\theta - \sin 6\theta + \dots \\
 & \quad \quad \quad \theta + (\theta - \pi) \quad \quad \quad = \sin 3\theta \quad \quad \quad \text{upto } n \text{ terms.} \\
 & \quad \quad \quad - \sin 2\theta \quad \quad \quad - \sin(\pi - 3\theta) \\
 & \quad \quad \quad \theta + 2(\theta - \pi) \\
 & \sin \theta + \sin(\theta + (\theta + \pi)) + \sin(\theta + 2(\theta + \pi)) + \sin(\theta + 3(\theta + \pi)) \\
 & \quad \quad \quad + \sin(\theta + 4(\theta + \pi)) + \dots + \sin(\theta + (n-1)(\theta + \pi)) \\
 & = \frac{\sin\left(\frac{n(\theta + \pi)}{2}\right)}{\sin\left(\frac{\theta + \pi}{2}\right)} \sin\left(\frac{2\theta + (n-1)(\theta + \pi)}{2}\right)
 \end{aligned}$$

$$\begin{aligned}
 & 2. \quad \tan \frac{x}{2} \sec x + \tan \frac{x}{2^2} \sec \frac{x}{2} + \tan \frac{x}{2^3} \sec \frac{x}{2^2} + \tan \frac{x}{2^4} \sec \frac{x}{2^3} + \dots \\
 &= \frac{\sin \frac{x}{2} = \frac{x - \frac{x}{2}}{2}}{\cos \frac{x}{2} \cos x} + \frac{\sin \frac{x}{2^2} = \frac{\frac{x}{2} - \frac{x}{2^2}}{2}}{\cos \frac{x}{2^2} \cos \frac{x}{2}} + \frac{\sin \frac{x}{2^3} = \frac{\frac{x}{2^2} - \frac{x}{2^3}}{2}}{\cos \frac{x}{2^3} \cos \frac{x}{2^2}} + \dots + \frac{\sin \frac{x}{2^n} = \frac{\frac{x}{2^{n-1}} - \frac{x}{2^n}}{2}}{\cos \frac{x}{2^n} \cos \frac{x}{2^{n-1}}}
 \end{aligned}$$

$$\frac{\sin \frac{x}{2}}{\cos \frac{x}{2} \cos x} = \frac{\sin \left(x - \frac{x}{2} \right)}{\cos \frac{x}{2} \cos x} = \tan x - \tan \frac{x}{2}$$

$$\begin{aligned}
 &= \left(\tan x - \cancel{\tan \frac{x}{2}} \right) + \left(\cancel{\tan \frac{x}{2}} - \cancel{\tan \frac{x}{2^2}} \right) + \left(\cancel{\tan \frac{x}{2^2}} - \cancel{\tan \frac{x}{2^3}} \right) \\
 &+ \left(\cancel{\tan \frac{x}{2^3}} - \cancel{\tan \frac{x}{2^4}} \right) + \dots + \left(\cancel{\tan \frac{x}{2^{n-2}}} - \cancel{\tan \frac{x}{2^{n-1}}} \right) + \left(\cancel{\tan \frac{x}{2^{n-1}}} - \tan \frac{x}{2^n} \right) \\
 &= \tan x - \tan \frac{x}{2^n}
 \end{aligned}$$

$$\begin{aligned}
 \underline{3.} \quad & \operatorname{cosec} x + \operatorname{cosec}(2x) + \operatorname{cosec}(2^2 x) + \operatorname{cosec}(2^3 x) + \dots + \operatorname{cosec}(2^{n-1} x) \\
 &= \frac{\sin \frac{x}{2}}{\sin x \sin \frac{x}{2}} + \frac{\sin x}{\sin 2x \sin x} + \frac{\sin 2x}{\sin(2^2 x) \sin(2x)} + \dots + \frac{\sin 2^{n-2} x}{\sin(2^{n-1} x) \sin(2^{n-2} x)} \\
 &= \frac{\sin(x - \frac{x}{2})}{\sin x \sin \frac{x}{2}} + \frac{\sin(2x - x)}{\sin 2x \sin x} + \frac{\sin(2^2 x - 2x)}{\sin 2^2 x \sin(2x)} + \dots + \frac{\sin(2^{n-1} x - 2^{n-2} x)}{\sin 2^{n-1} x \sin 2^{n-2} x} \\
 &= \left(\cot \frac{x}{2} - \cancel{\cot x} \right) + \left(\cancel{\cot x} - \cot 2x \right) + \left(\cancel{\cot 2x} - \cot 2^2 x \right) + \dots + \left(\cancel{\cot 2^{n-2} x} - \cot 2^{n-1} x \right) \\
 &= \cot \frac{x}{2} - \cot 2^{n-1} x.
 \end{aligned}$$

4. $\sin x \sec 3x + \sin 3x \sec 9x + \sin 9x \sec 27x + \sin 27x \sec 81x + \dots$

$$\frac{\sin 2x}{2 \cos 3x \cos x} + \frac{\sin 6x}{2 \cos 9x \cos 3x} + \frac{\sin 18x}{2 \cos 27x \cos 9x} + \dots + \frac{\sin(3^{n-1}x) \sec(3^n x)}{2 \cos 3^n x \cos 3^{n-1} x}$$

$\sin 2 \cdot (3^{n-1}x)$

$$= \frac{1}{2} \left(\frac{\sin(3x-x)}{\cos 3x \cos x} + \frac{\sin(9x-3x)}{\cos 9x \cos 3x} + \frac{\sin(27x-9x)}{\cos 27x \cos 9x} + \dots + \frac{\sin(3^n x - 3^{n-1}x)}{\cos 3^n x \cos 3^{n-1} x} \right)$$

$$= \frac{1}{2} \left[\cancel{(\tan 3x - \tan x)} + \cancel{(\tan 9x - \tan 3x)} + \cancel{(\tan 27x - \tan 9x)} + \dots + \cancel{(\tan 3^n x - \tan 3^{n-1}x)} \right]$$

$$= \frac{1}{2} (\tan 3^n x - \tan x)$$

$$\boxed{a \sin x + b \cos x} = \sqrt{a^2 + b^2} \left(\frac{a}{\sqrt{a^2 + b^2}} \sin x + \frac{b}{\sqrt{a^2 + b^2}} \cos x \right)$$

$$\frac{a}{\sqrt{a^2 + b^2}} = \cos \theta$$

$$\frac{b}{\sqrt{a^2 + b^2}} = \sin \theta$$

$$\alpha \sin x + \beta \cos x$$

$$\alpha^2 + \beta^2 = 1$$

$$\leq 1 \leq 1$$

$$a \sin x + b \cos x = \sqrt{a^2 + b^2} \left(\sin x \cos \theta + \sin \theta \cos x \right)$$

$$= \sqrt{a^2 + b^2} \sin(x + \theta) \in [-\sqrt{a^2 + b^2}, \sqrt{a^2 + b^2}]$$

$$\sin x - \cos x = \sqrt{1^2 + 1^2} \left(\underbrace{\frac{1}{\sqrt{2}}}_{\cos \frac{\pi}{4}} \sin x - \underbrace{\frac{1}{\sqrt{2}}}_{\sin \frac{\pi}{4}} \cos x \right)$$

$$= \sqrt{2} \sin\left(x - \frac{\pi}{4}\right)$$

$$\sin x - \cos x \in [-\sqrt{2}, \sqrt{2}]$$

$$f(x) = \sqrt{3} \cos x + \sin x = 2 \left(\underbrace{\frac{\sqrt{3}}{2}}_{\cos \frac{\pi}{6}} \cos x + \underbrace{\frac{1}{2}}_{\sin \frac{\pi}{6}} \sin x \right) = 2 \sin \left(x + \frac{\pi}{3} \right)$$

$$R_f = [-2, 2]$$

$$2 \left(\underbrace{\frac{\sqrt{3}}{2}}_{\cos \frac{\pi}{6}} \cos x + \underbrace{\frac{1}{2}}_{\sin \frac{\pi}{6}} \sin x \right)$$

$$= 2 \cos \left(x - \frac{\pi}{6} \right)$$

$\cos \frac{\pi}{6}$
HW

PT-1, PT-2

$\Sigma x - I (2, 3, 4, 5, 6, 7, 8, 9, 10, 11)$

leaving Q. 1