Projectile Motion

Q. An inclined plane makes an angle $\theta_0 = 30°$ with the horizontal. A particle is projected from this plane with a speed of 5 m s^{-1} at an angle of elevation $\beta = 30°$ with the horizontal as shown in Fig.

a. Find the range of the particle on the plane when it strikes the plane.

b. Find the range of the particle for $\beta = 120°$.

$\theta = 60°$

$$T = \frac{2 u_y}{g_{eff}}$$

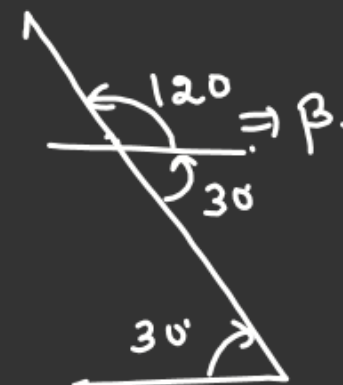$$T = \frac{2 \times 5 \sin 60°}{g \sin 60°} = 1 \sec$$

Along the Inclined plane:-
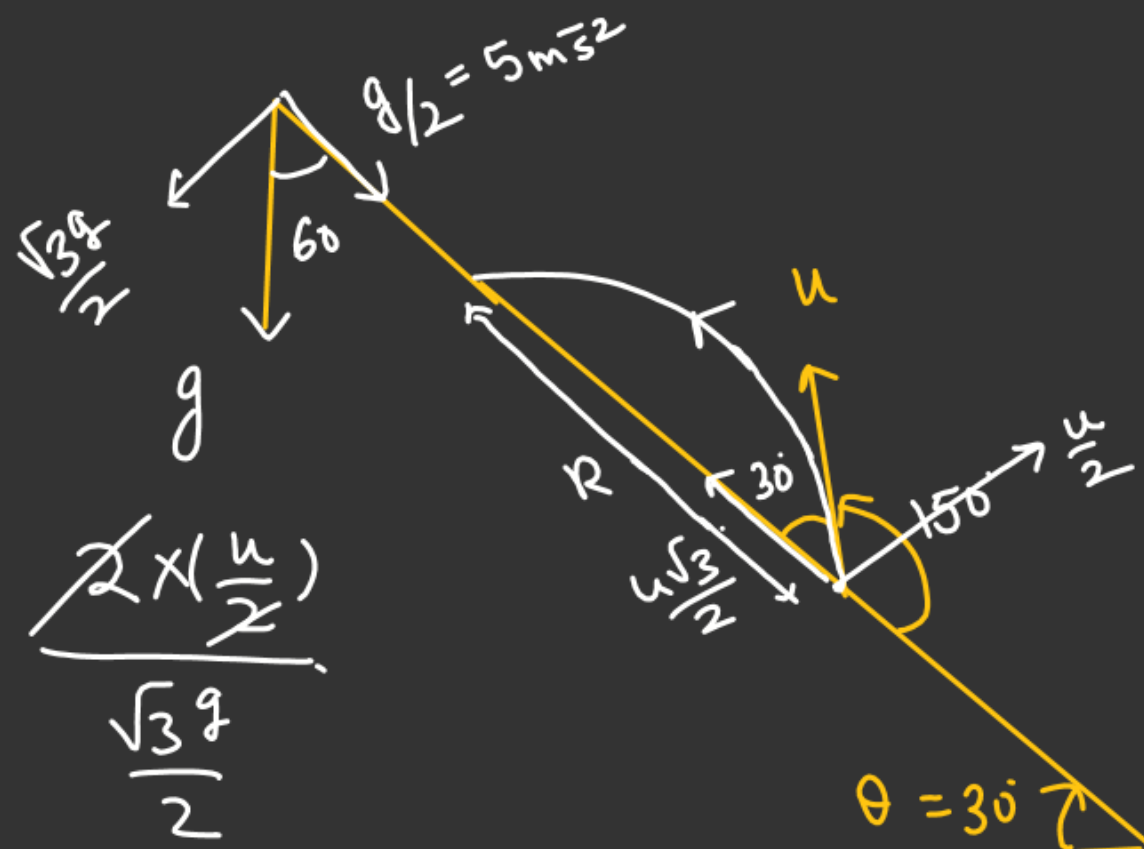
$$R = (5 \cos 60°) T + \frac{1}{2} (g \cos 60°) T^2$$

$$R = (5 \times \frac{1}{2} \times 1) + \frac{1}{2} \times 10 \times \frac{1}{2} \times (1)^2$$

$$R = \frac{5}{2} + \frac{5}{2} = 5m$$

$5 \sin 60°$

$g \sin 60°$

5m/s

$60°$ $60°$

g

$g \cos 60°$

$5 \cos 60°$

$30°$

u 5 ms^{-1}

$\beta = 30°$

θ

$30°$

$30° = \theta_0$

(b)

$g/2 = 5 ms^2$

$\frac{\sqrt{3}g}{2}$

60

g

u

R

30

$\frac{u}{2}$

150

$\frac{u\sqrt{3}}{2}$

$\theta = 30$

120 $\Rightarrow \beta$.

30

30
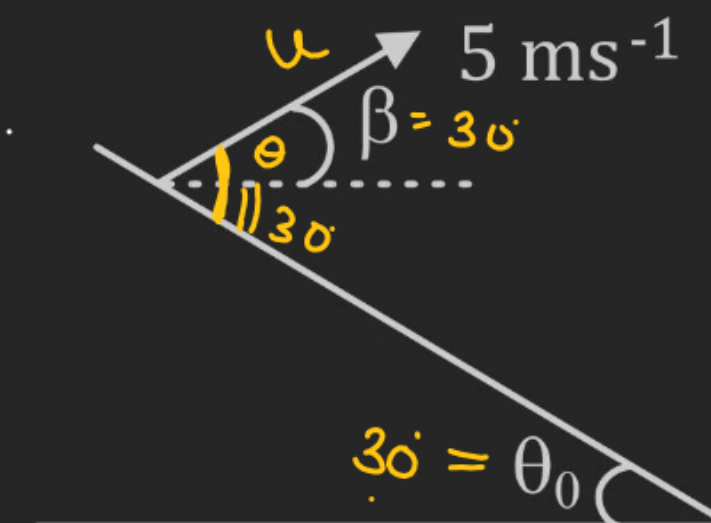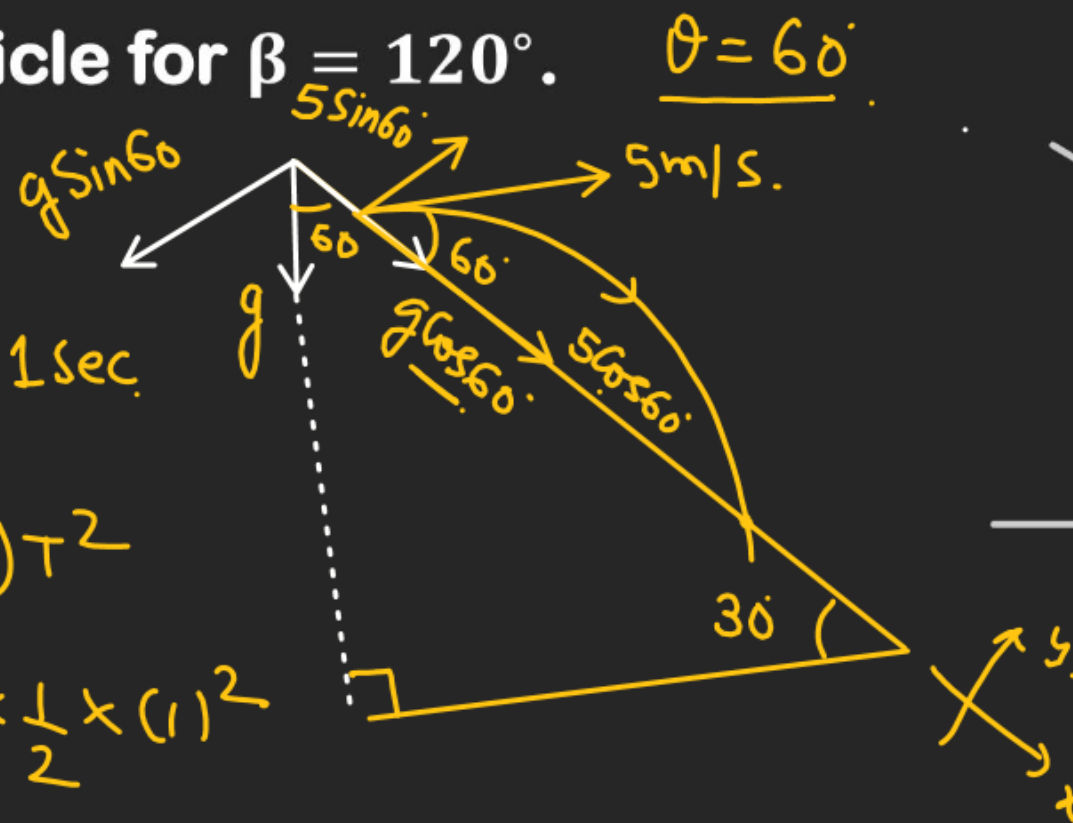
$$T = \frac{2 \times \left(\frac{u}{2}\right)}{\frac{\sqrt{3}g}{2}}$$

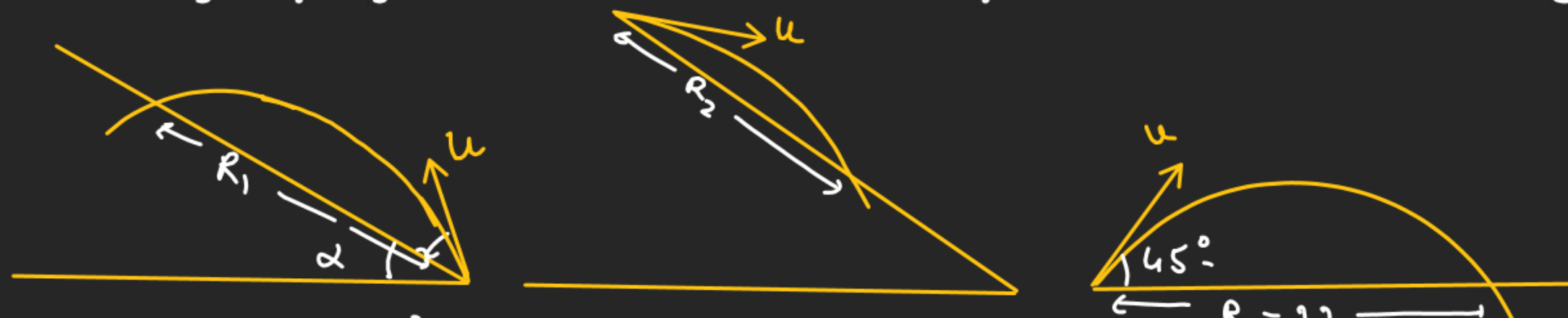$$T = \frac{2u}{\sqrt{3}g} = \left(\frac{1}{\sqrt{3}} \sec\right) \checkmark$$

$$R = \left(\frac{\sqrt{3}u}{2}\right)T - \frac{1}{2} \times 5 \times T^2$$

$$R = \left(\frac{\sqrt{3}}{2} \times 5 \times \frac{1}{\sqrt{3}}\right) - \frac{1}{2} \times 5 \times \left(\frac{1}{3}\right)$$

$$= \left(\frac{5}{2} - \frac{5}{6}\right) = \frac{15 - 5}{6} = \frac{10}{6} = \boxed{\frac{5}{3} m}$$

Projectile Motion

Q. A body has <u>maximum range R_1 when projected up the plane</u>. The same body when projected down the inclined plane, it has maximum range R_2. Find the maximum horizontal range. Assume equal speed of projection in each case and the body is projected onto the inclined plane in the line of the greatest slope.
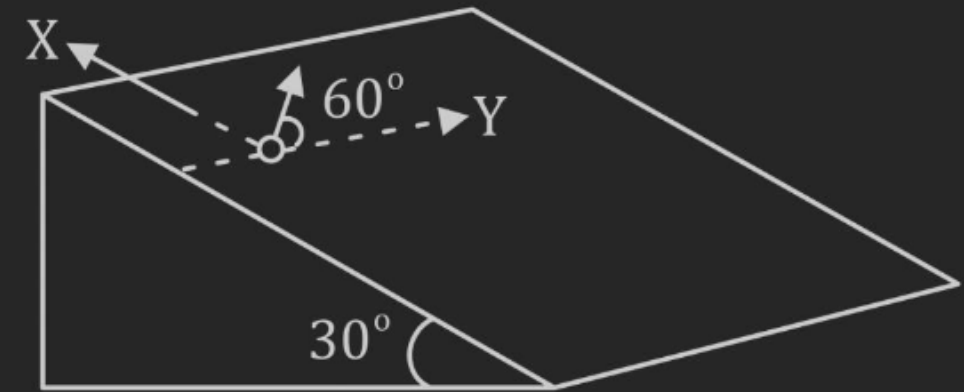

$$R_1 = \frac{u^2}{g(1+\sin\alpha)},$$

$$1 + \sin\alpha = \left(\frac{u^2}{R_1 g}\right) = \frac{R}{R_1} \quad -\text{①}$$

$$R_2 = \frac{u^2}{g(1-\sin\alpha)}$$

$$1 - \sin\alpha = \left(\frac{u^2}{g R_2}\right) = \frac{R}{R_2} \quad -\text{②}$$

$$45° \qquad R = ?? \atop \text{max}, \quad R_{max} = \left(\frac{u^2}{g}\right)$$

$$\frac{\text{①} + \text{②}}{2} = \qquad R\left(\frac{1}{R_1} + \frac{1}{R_2}\right) \Rightarrow R = \left(\frac{2 R_1 R_2}{R_1 + R_2}\right)$$

Projectile Motion

Q. A small sphere is projected with a velocity of 3 ms^{-1} in a direction 60° from the horizontal y-axis, on the smooth inclined plane (Fig.) The motion of sphere takes place in the x – y plane. Calculate the magnitude v of its velocity after 2 s.
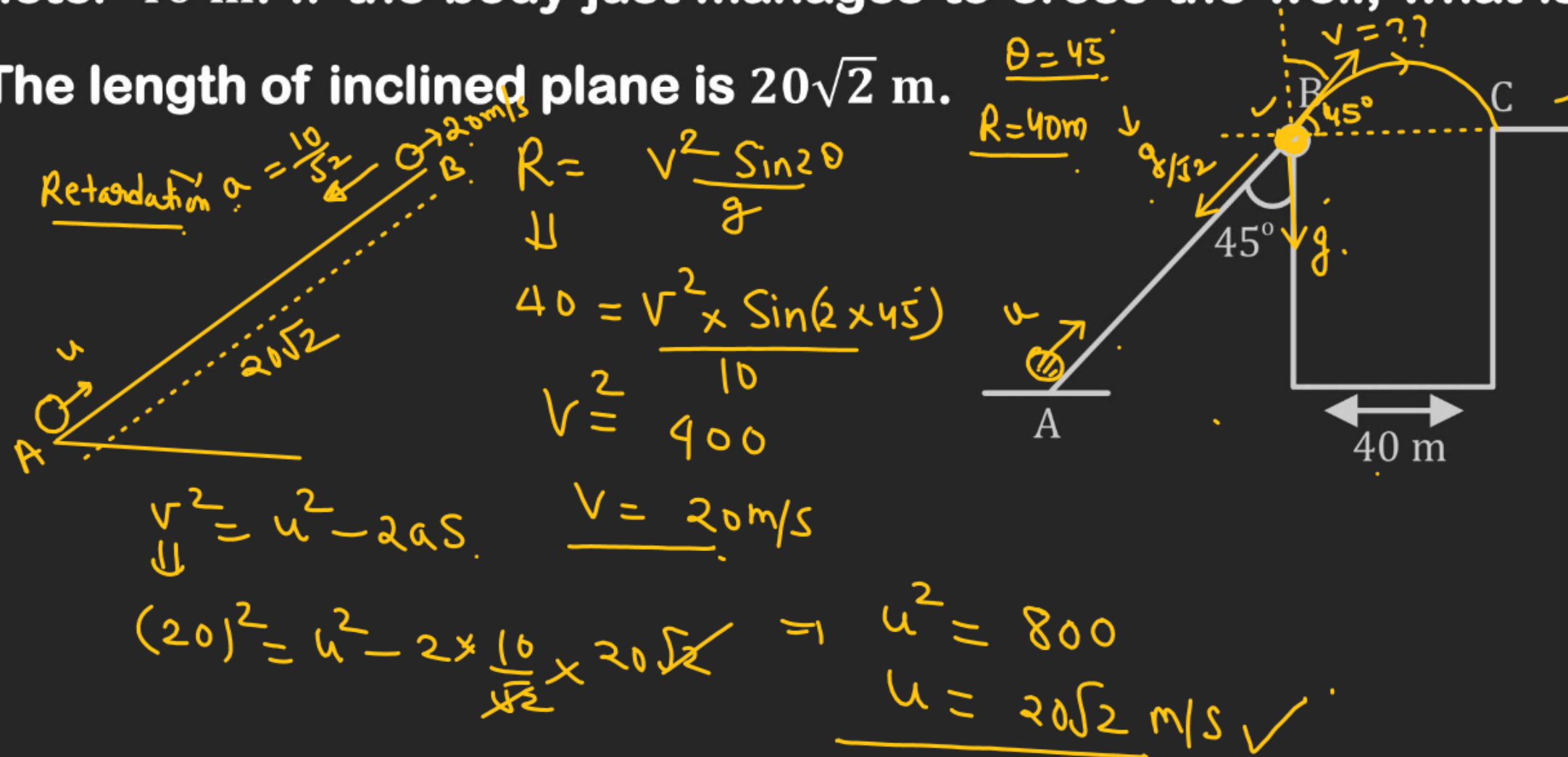
Projectile Motion

Q. A body is projected up along a smooth inclined plane with velocity u from the point A as shown in Fig. The angle of inclination is $45°$ and the top is connected to a well of diameter 40 m. If the body just manages to cross the well, what is the value of u ? The length of inclined plane is $20\sqrt{2}$ m.

(A) 40 ms^{-1}

(B) $40\sqrt{2}$ ms^{-1}

(C) 20 m s^{-1}

(D) $20\sqrt{2}$ ms^{-1}

Retardation $a = \dfrac{10}{\sqrt{2}}$

$\theta = 45°$

$R = 40m$

$v = ???$

$R = \dfrac{v^2 \sin 2\theta}{g}$

$40 = \dfrac{v^2 \times \sin(2 \times 45)}{10}$

$v^2 = 400$

$v = 20 m/s$

$v^2 = u^2 - 2as$

$(20)^2 = u^2 - 2 \times \dfrac{10}{\sqrt{2}} \times 20\sqrt{2}$

$\Rightarrow u^2 = 800$

$u = 20\sqrt{2} \ m/s \ \checkmark$

$20\sqrt{2}$

$45° \quad g$

A

40 m

$45°$

$\dfrac{g}{\sqrt{2}}$

Projectile Motion

$$T = \left(\frac{2u_y}{g_{eff}}\right)$$

Q. In Fig., the time taken by the projectile to reach from A to B is t. Then the distance AB is equal to

$g\cos 60 = 5$

$\left[\begin{array}{l}\theta = 60 - 30 \\ \theta = 30°\end{array}\right]$

(A) $\dfrac{ut}{\sqrt{3}}$ ✓

(B) $\dfrac{\sqrt{3}ut}{2}$

(C) $\sqrt{3}ut$

(D) $2ut$

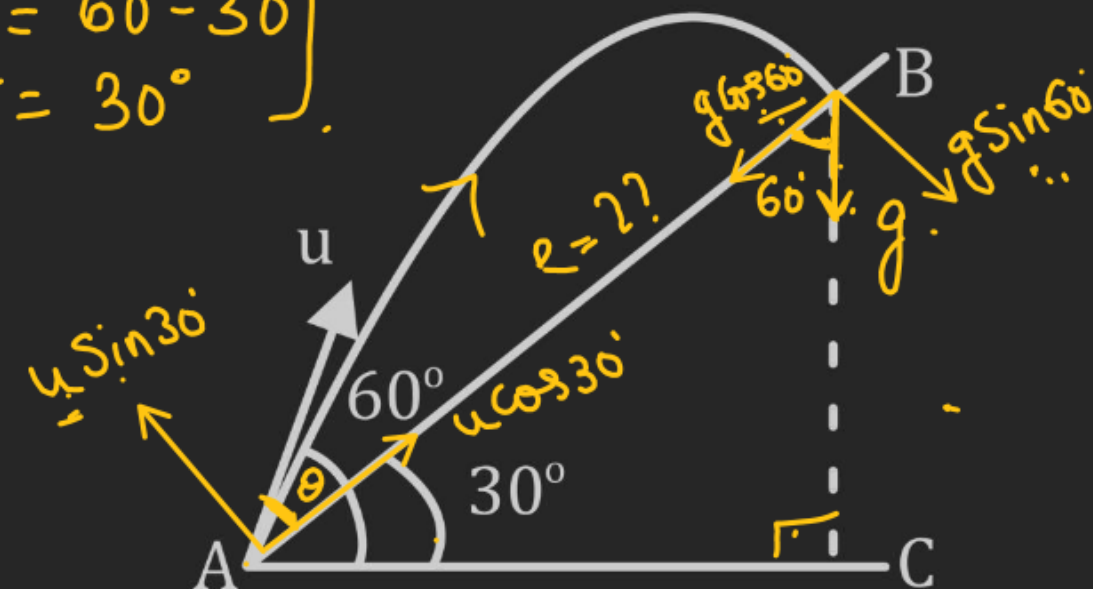$R = \left(\dfrac{\sqrt{3}u}{2}\right)t - \dfrac{1}{2} \times 5 \times t^2$

$R = \left(\dfrac{\sqrt{3}u}{2}\right)t - \dfrac{5t^2}{2}$

$R = \dfrac{\sqrt{3}}{2}(ut) - \dfrac{5t}{2}(t)$

$R = \dfrac{\sqrt{3}}{2}(ut) - \dfrac{5t}{2}\left(\dfrac{u}{5\sqrt{3}}\right)$

$R = \dfrac{\sqrt{3}}{2}(ut) - \left(\dfrac{ut}{2\sqrt{3}}\right) = \dfrac{ut}{2}\left(\sqrt{3} - \dfrac{1}{\sqrt{3}}\right) = \dfrac{ut}{2\sqrt{3}}(3-1) = \dfrac{ut}{\sqrt{3}}$

$t = \dfrac{2 \times \dfrac{u}{2}}{\left(\dfrac{\sqrt{3} \times 10}{2}\right)} = \dfrac{u}{10 \times \dfrac{\sqrt{3}}{2}} = \left(\dfrac{u}{5\sqrt{3}}\right)$

$g\cos 60$

$60°$

$u\cos 30$

$R = ?$

$30°$

$u\sin 30$

$u\sin 30$

$g\sin 60$

g

$\sqrt{g^2}\sin 60$

A, B, C, u, $60°$, θ

H-W   I·I·T Adv.

Projectile Motion

Q. Two inclined planes OA and OB having inclinations $30°$ and $60°$ with the horizontal respectively intersect each other at O, as shown in figure. A particle is projected from point P with velocity $u = 10\sqrt{3}\,\dfrac{m}{s}$ along a direction perpendicular to plane OA. If the particle strikes plane OB perpendicular at Q. Calculate

(A) time of flight

(B) velocity with which the particle strikes the plane OB.

(C) height h of point P from point O.

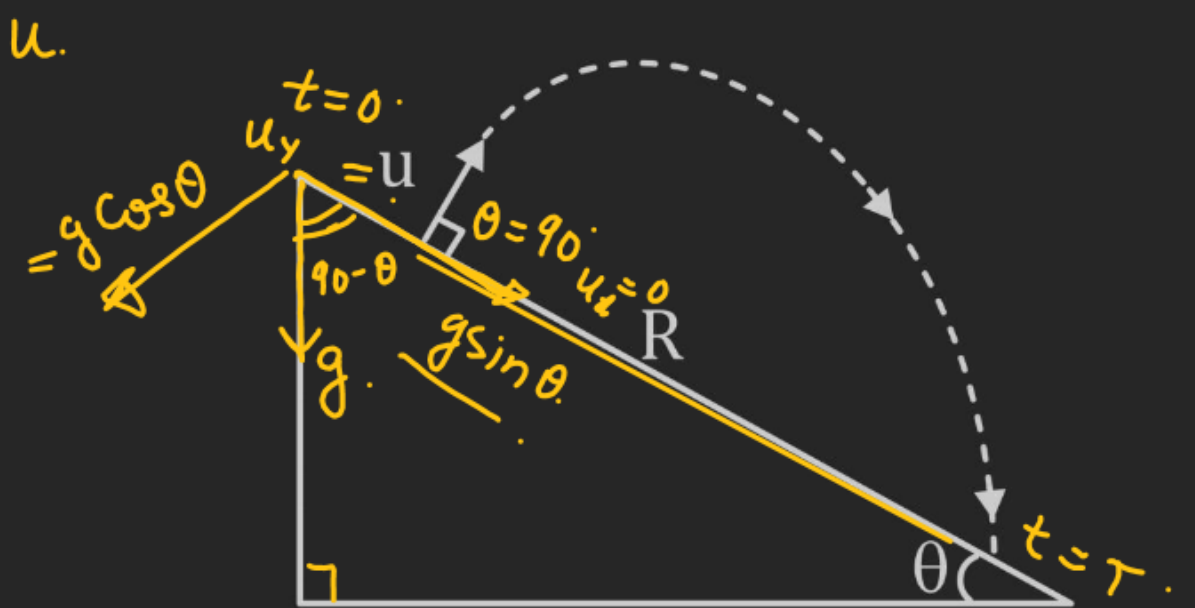(D) distance PQ. (Take $g = 10\ m/s^2$)
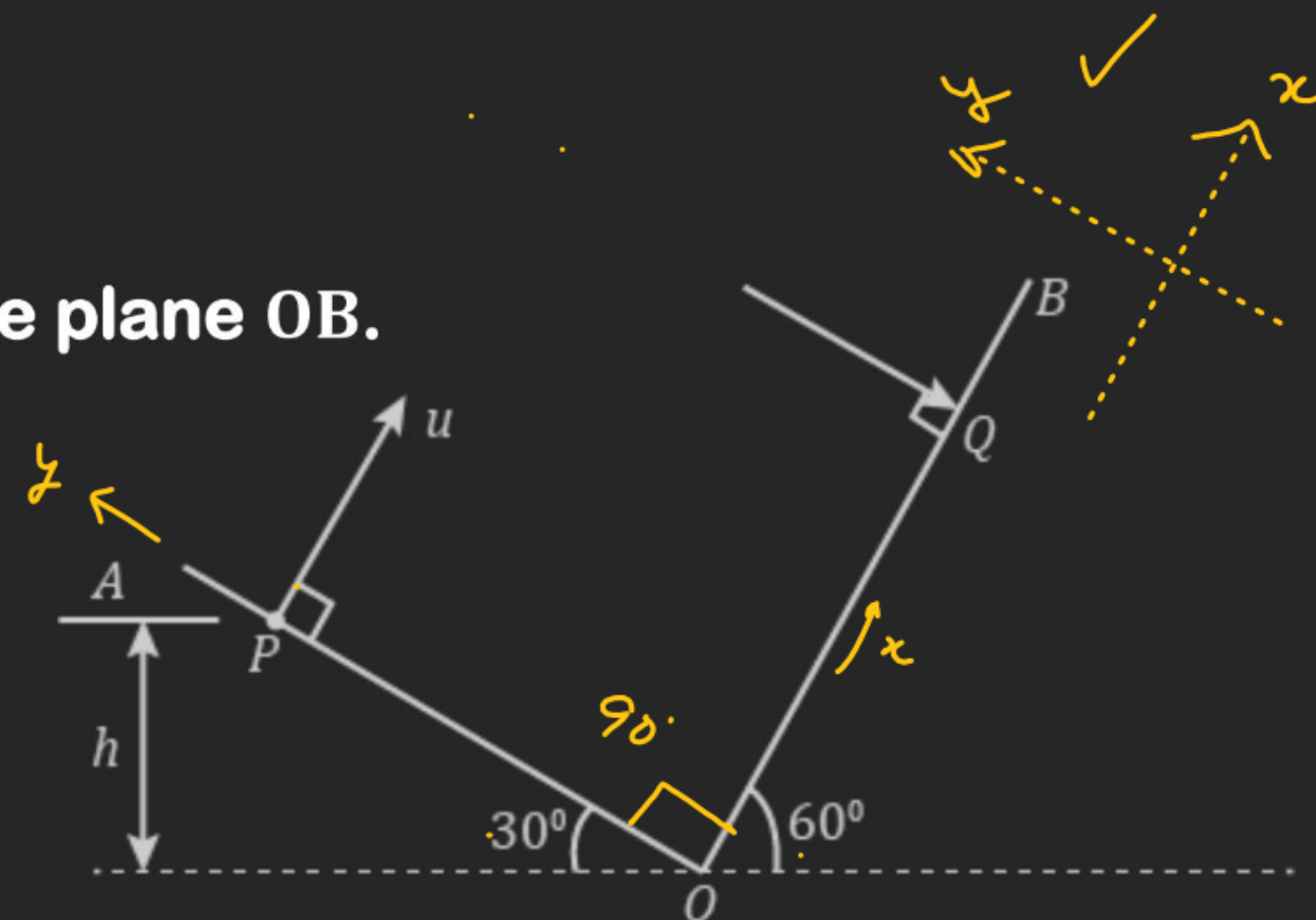
Projectile Motion

Q. A projectile is fired with a velocity u at right angles to the slope, which is inclined at an angle θ with the horizontal. Derive an expression for the distance R to the point of impact.

$$u_y = u$$

$$T = \left[\frac{2u}{g\cos\theta} \right]$$

$$R = \frac{1}{2} \times (g\sin\theta) \times T^2$$

$$R = \frac{1}{2} \times g\sin\theta \times \frac{4u^2}{g^2\cos^2\theta}$$

$$R = \left(\frac{2u^2\sin\theta}{g\cos^2\theta} \right) \Rightarrow R = \frac{2u^2}{g}(\tan\theta)(\sec\theta)$$

H.W.

Q. Cannon A is located on a plain a distance L from a wall of height H. On top of this wall is an identical cannon (cannon B). Ignore air resistance throughout this problem. Also ignore the size of the cannons relative to L and H. The two groups of gunners aim the cannons directly at each other. They fire at each other simultaneously, with equal muzzle speed v_0. What is the value of v_0 for which the two cannon balls collide just as they hit the ground?
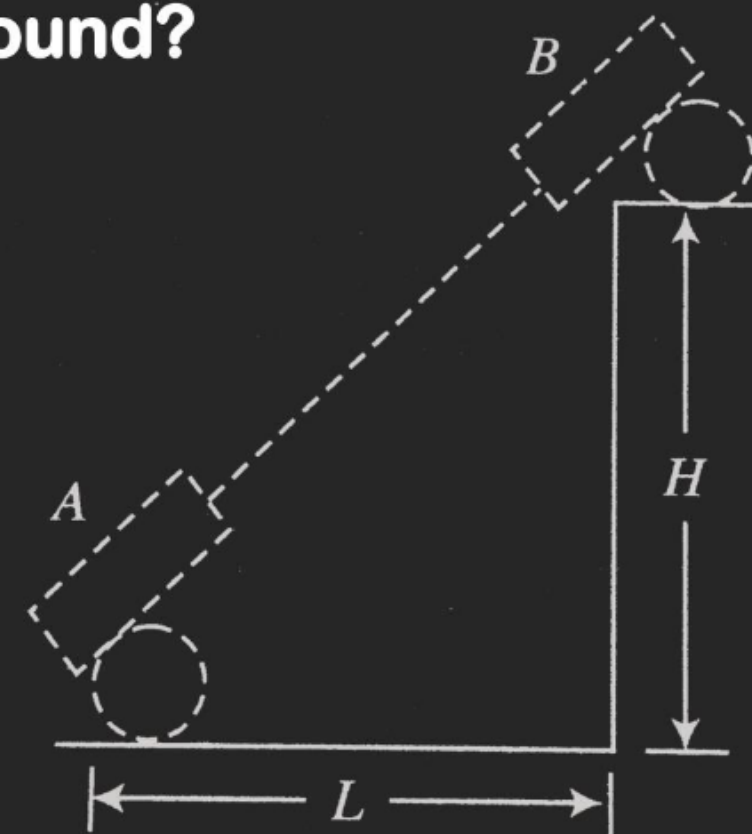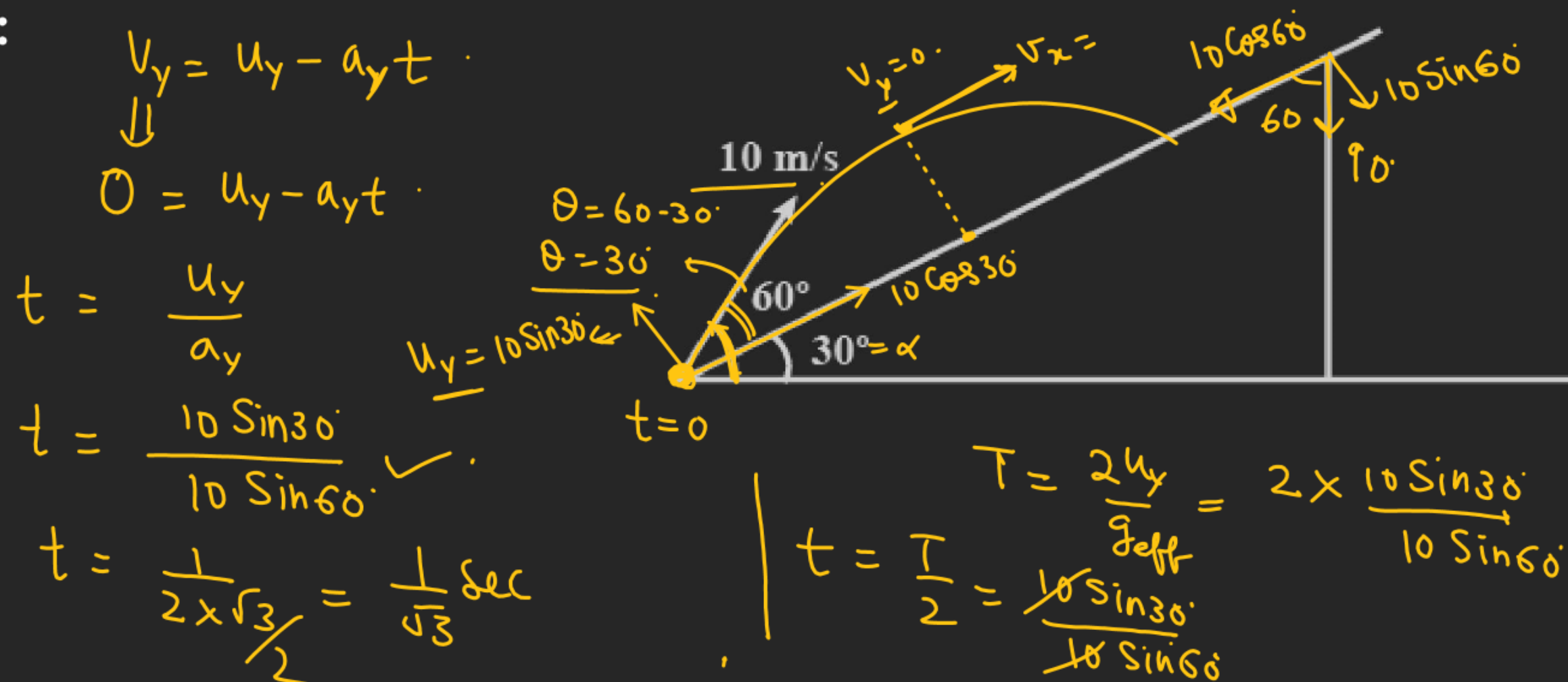
Projectile Motion

Q. A particle is thrown at time $t = 0$ with a velocity of 10 m/s at an angle of $60°$ with the horizontal from a point on an incline plane, making an angle of $30°$ with the horizontal. The time when the velocity of the projectile becomes parallel to the incline is:

(A) $\dfrac{2}{\sqrt{3}}$ sec

(B) $\dfrac{1}{\sqrt{3}}$ sec

(C) $\sqrt{3}$ sec

(D) $\dfrac{1}{2\sqrt{3}}$ sec

$V_y = U_y - a_y t$

$0 = U_y - a_y t$

$t = \dfrac{U_y}{a_y}$

$t = \dfrac{10 \sin 30}{10 \sin 60}$

$t = \dfrac{1}{2 \times \sqrt{3}/2} = \dfrac{1}{\sqrt{3}} \sec$

$\theta = 60 - 30$

$\theta = 30$

$U_y = 10 \sin 30$

10 m/s

$60°$

$10 \cos 30$

$30° = \alpha$

$t = 0$

$V_y = 0$

$V_x = 10 \cos 60$

60

$10 \sin 60$

90

$T = \dfrac{2U_y}{g_{eff}} = \dfrac{2 \times 10 \sin 30}{10 \sin 60}$

$t = \dfrac{T}{2} = \dfrac{10 \sin 30}{10 \sin 60}$

Projectile Motion

Q. Find range of projectile which is projected perpendicular to the incline plane with velocity 20 m/s as shown in figure:

(A) 75 m ✓

(B) 40 m

(C) 45 m

(D) 50 m

$$T = \frac{2u_y}{g_{eff}}$$

$$T = \frac{2 \times 20^2}{10 \times \frac{4}{5}} = 5\,sec$$

$$R = \left(\frac{1}{2}a_x T^2\right) = \frac{1}{2} \times 10 \times \frac{3}{5} \times (5)^2$$

$$R = 75\,m$$