Nishant Jindal

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ITF

A) $\sin^{-1}\left(\frac{1}{2}\right) = \theta$ whose sine value is $\frac{1}{2}$

$\theta = \frac{\pi}{6}$.

B) $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = \theta$ whose cosine value $\frac{\sqrt{3}}{2}$

$= \frac{\pi}{6}$

(C) $\sin^{-1}\left(\frac{1}{\sqrt{3}}\right) = \frac{\pi}{6}$.

(D) $\cot^{-1}\left(\frac{1}{\sqrt{3}}\right) = \frac{\pi}{3}$

(E) $\sin^{-1}(1) = \frac{\pi}{2}$

(F) $\cos^{-1}(1) = 0$

(G) $\sin^{-1}\left(\frac{\pi}{2}\right)$ = Not Possible

as $\sin\theta \leq 1 \neq 1.57$

(H) $\sin^{-1}(\pi) \neq 0$

$\sin^{-1}(3.14)$ - Not Possible

(I) $\tan^{-1}(1) = \frac{\pi}{4}$

(J) $\sec^{-1}\left(\frac{2}{\sqrt{3}}\right) = \frac{\pi}{6}$

(K) $\cos^{-1}(0) = \frac{\pi}{2}$

(L) $\sin^{-1}(0) = 0$

(M) $\sec^{-1}(0)$ Not Possible

$\sec\theta \leq -1$, $\sec\theta \geq 1$

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Raazt

1) $\sin^{-1}x, \cos^{-1}, \tan^{-1}x$ -- Rep. angle θ

2) Value of $\sin^{-1}x, \cos^{-1}x, \tan^{-1}x$ - -

is always numerically Least

3) $\sin^{-1}(x) = \text{arc}\sin x$

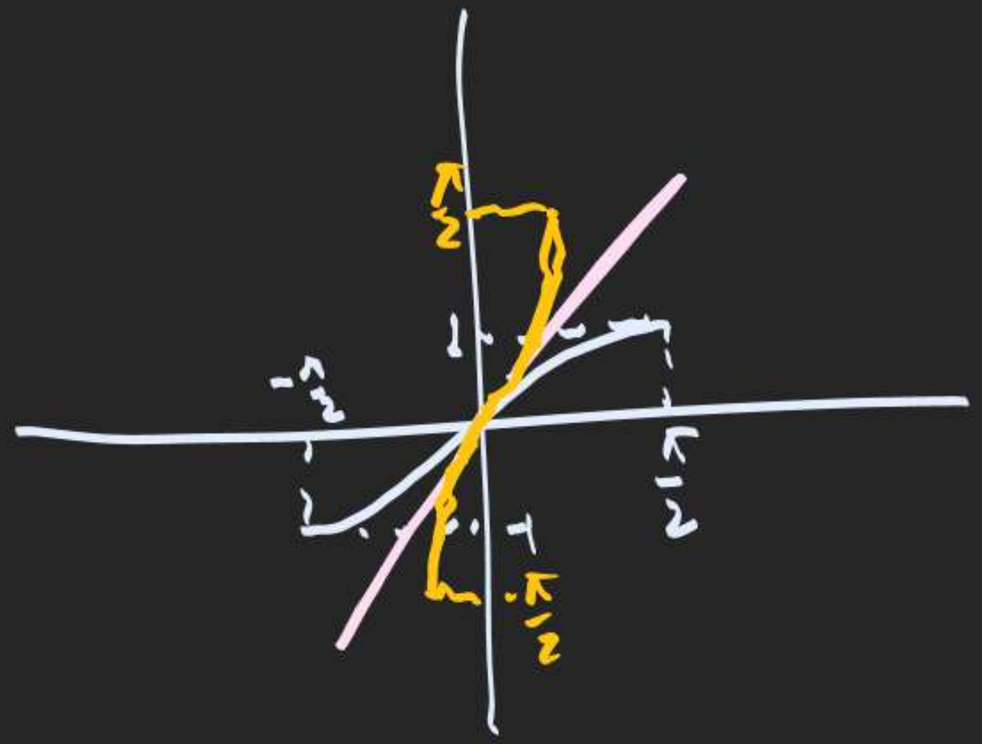
$\cos^{-1}x = \text{arc}\cos x$

(4) If 2 angle's whose modulus is same

then we take always +ve.

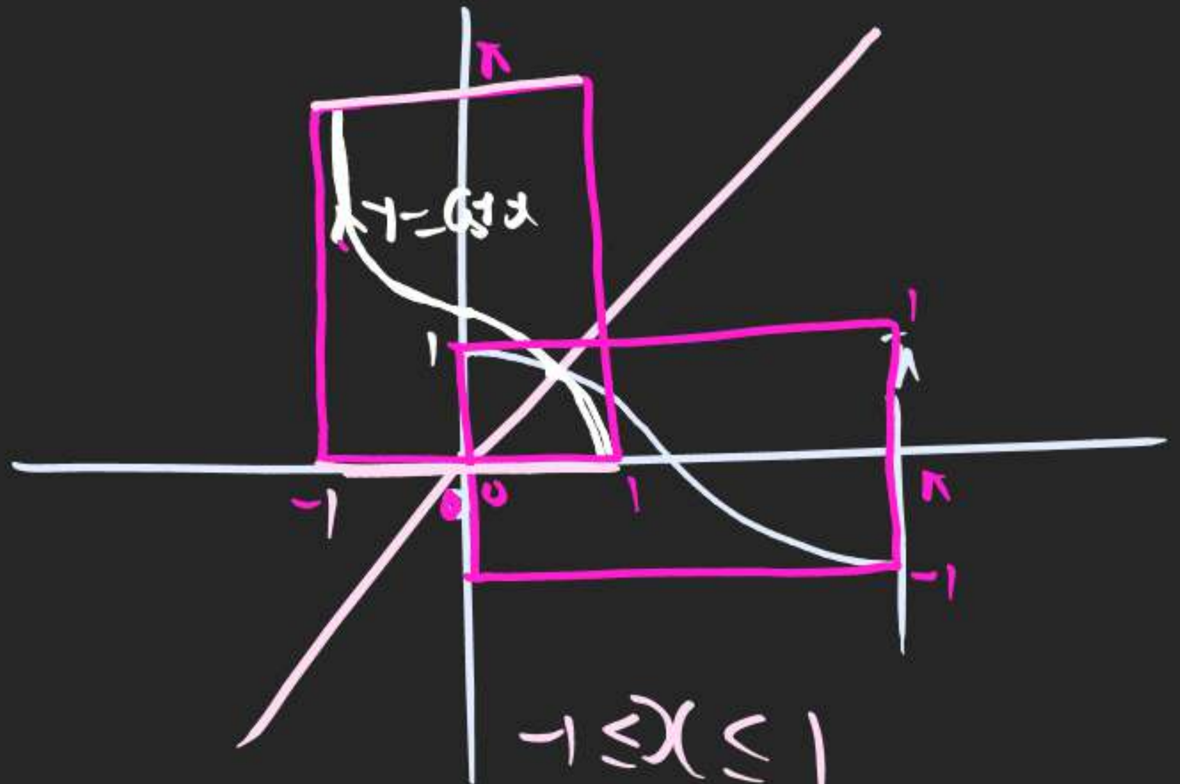
A) $f : \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \to [-1, 1]$ $f(x) = \sin x$

$f^{-1} [-1, 1] \to \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ $f(x) = \sin^{-1}x$



2) $f: \boxed{[0, \pi]} \to [-1, 1] \quad f(x) = \cos x.$

$f^{-1}: [-1, 1] \to [0, \pi] \quad f(x) = \cos^{-1} x$



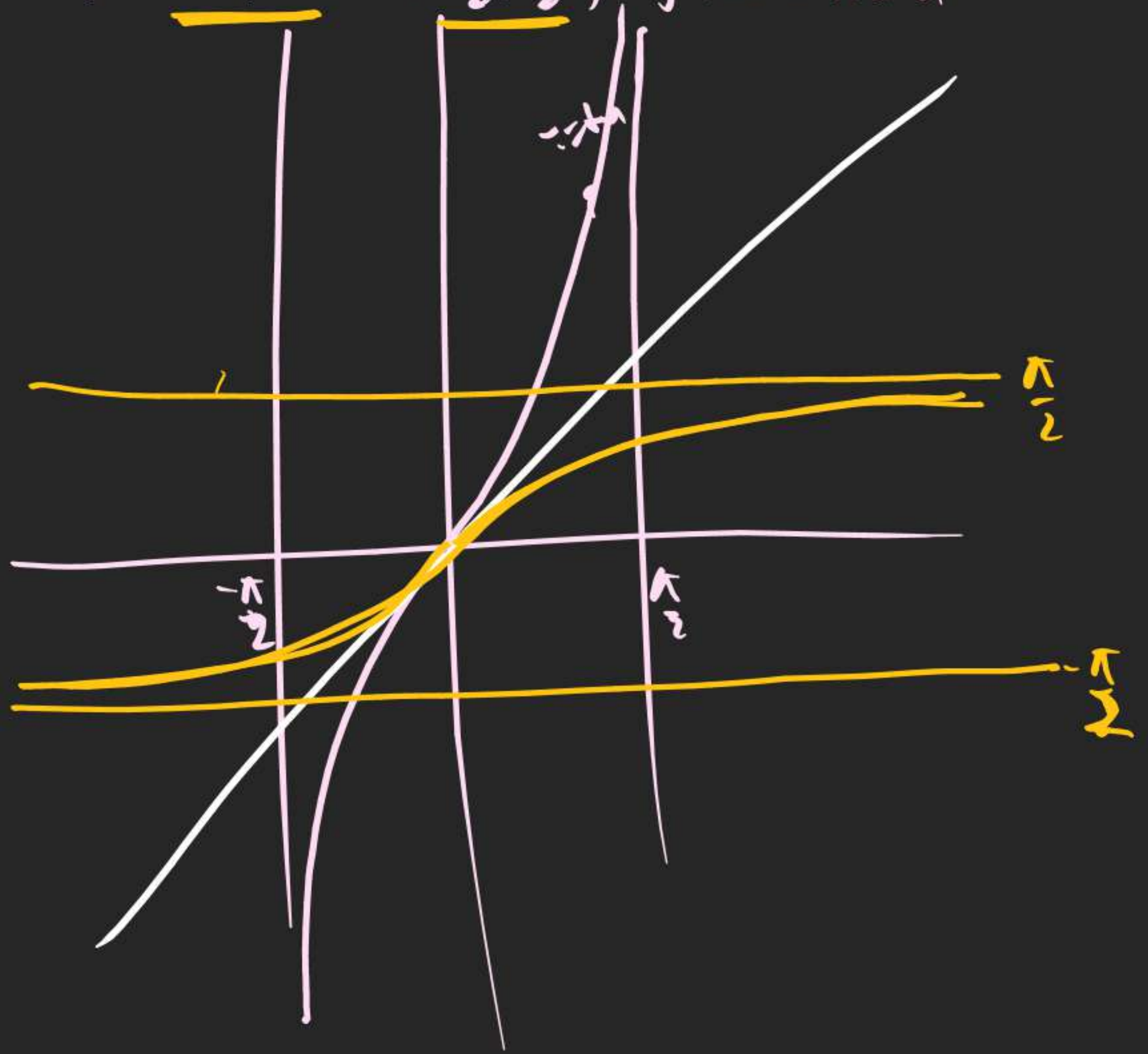
$-1 \le x \le 1$

$0 \le y \le \pi$

$0 \le \cos^{-1} x \le \pi$

(3) $f: \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \to R \quad f(x) = \tan x$

$f: (-\infty, \infty) \to \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \quad f(x) = \tan^{-1} x$



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Domain & Range of ITF

$y = \sin^{-1}x$	$-1 \le x \le 1$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
$y = \cos^{-1}x$	$-1 \le x \le 1$	$[0, \pi]$
$y = \tan^{-1}x$	R	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
$y = \cot^{-1}x$	R	$(0, \pi)$
$y = \operatorname{cosec}^{-1}x$	$x \le -1 \cup x \ge 1$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$
$y = \sec^{-1}x$	$x \le -1 \cup x \ge 1$	$[0, \pi] - \left\{\frac{\pi}{2}\right\}$

Domain

1) $y = \sin^{-1}(f(x)) / \cos^{-1}(f(x))$

$-1 \le f(x) \le 1$ Solve

2) $y = \underbrace{\tan^{-1}}_{R} f(x) / \underbrace{\cot^{-1}}_{R}(f(x))$ Ka dom.

$= f(x)$ Ka domain

3) $y = \sec^{-1} f(x) / \operatorname{cosec}^{-1} f(x)$

$|f(x)| \ge 1$

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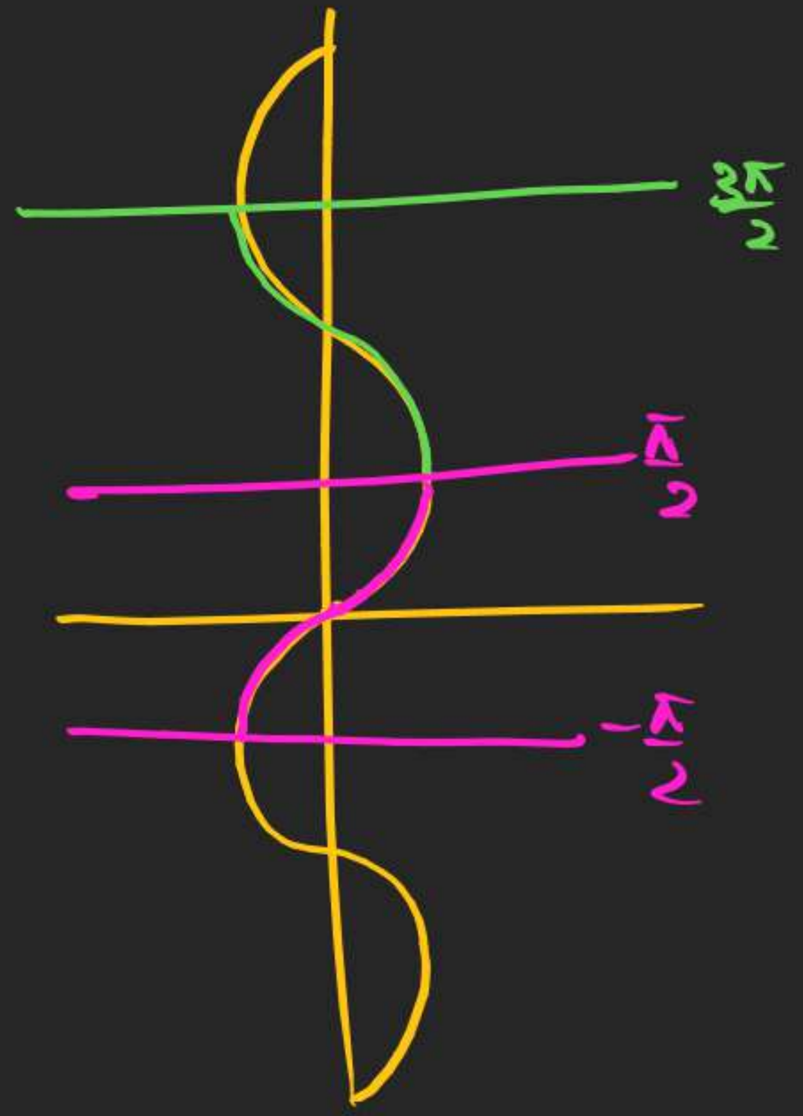
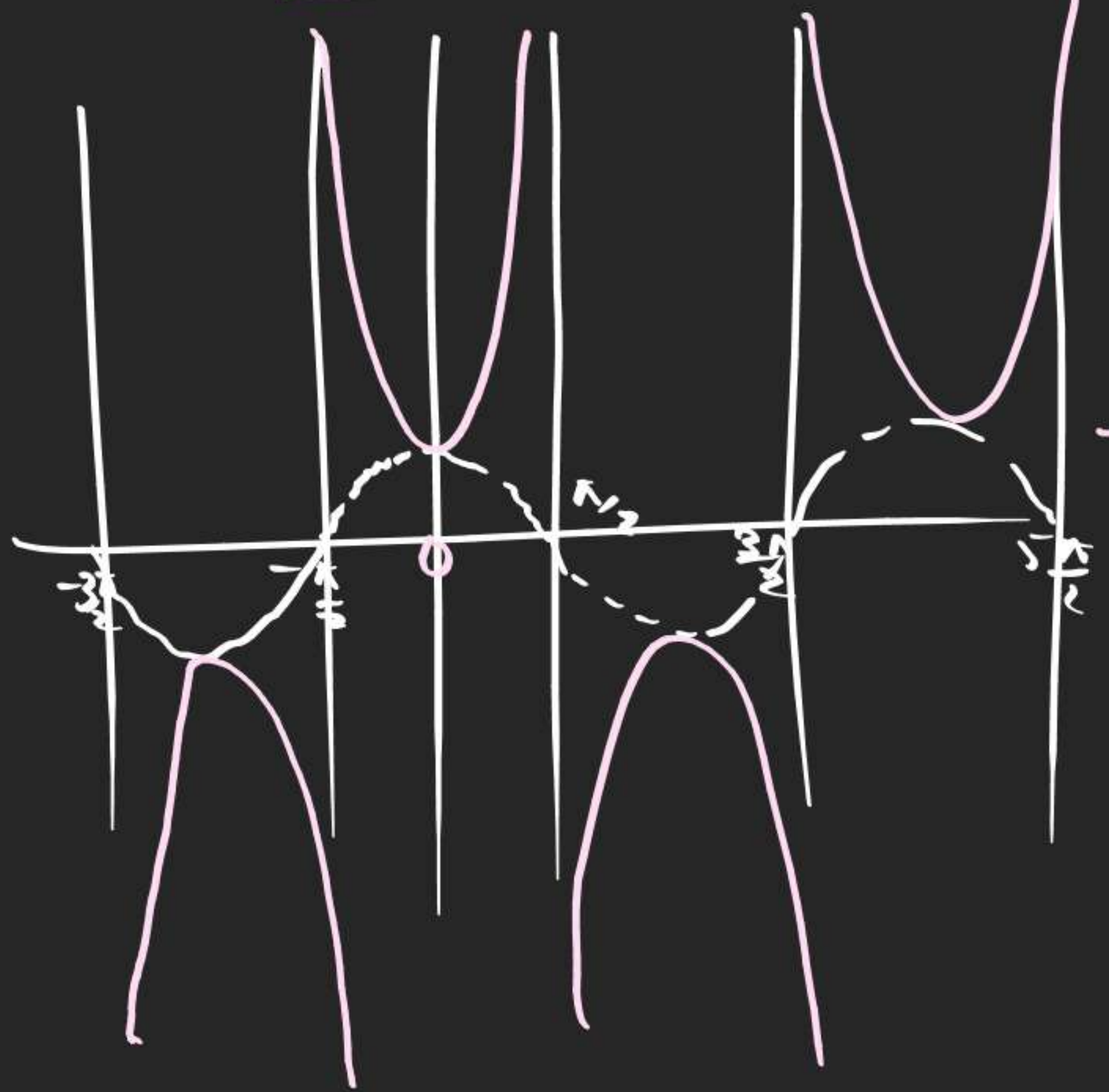
another way to draw

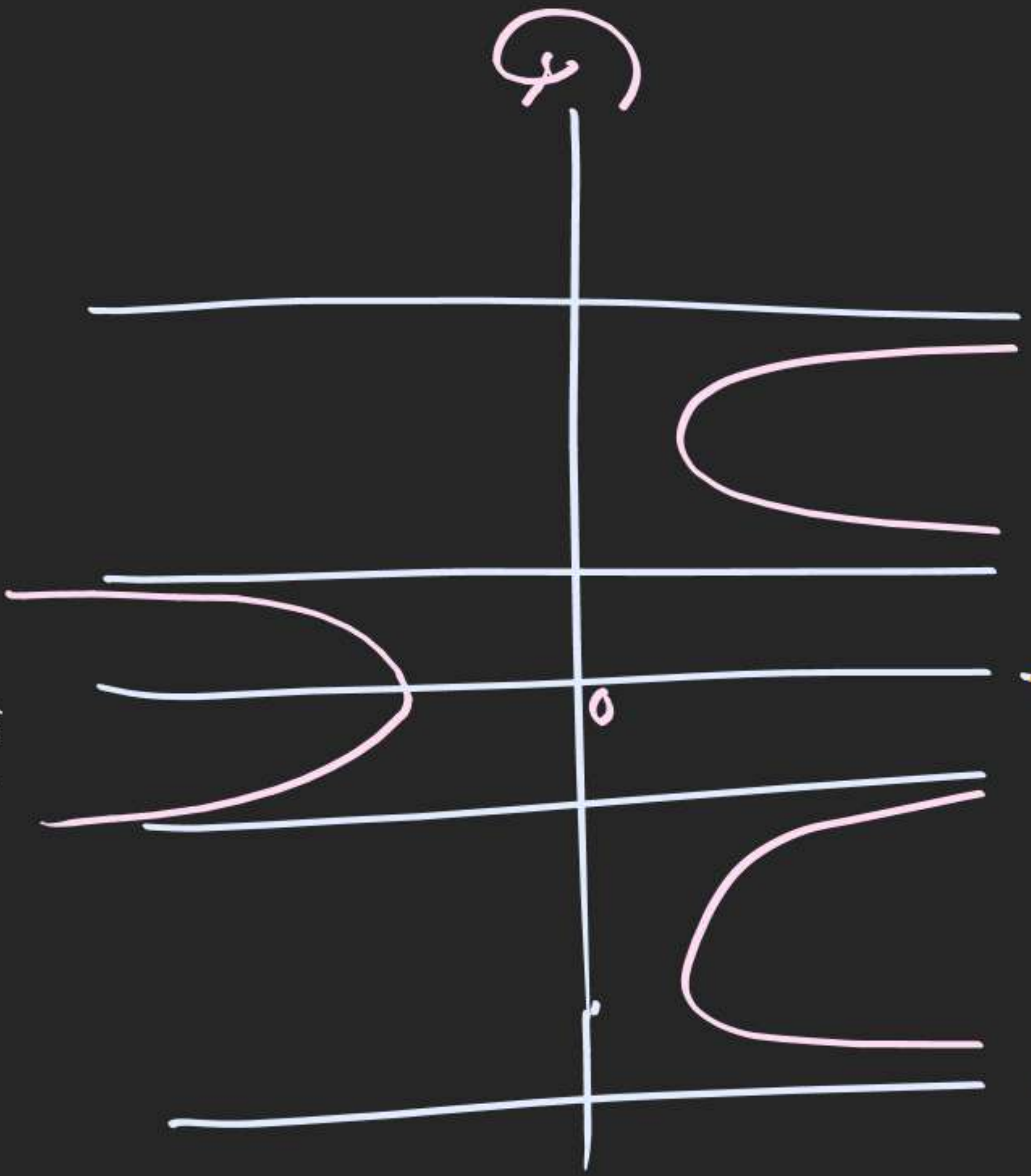
Graph of Inverse fxn

$x = \sin y$

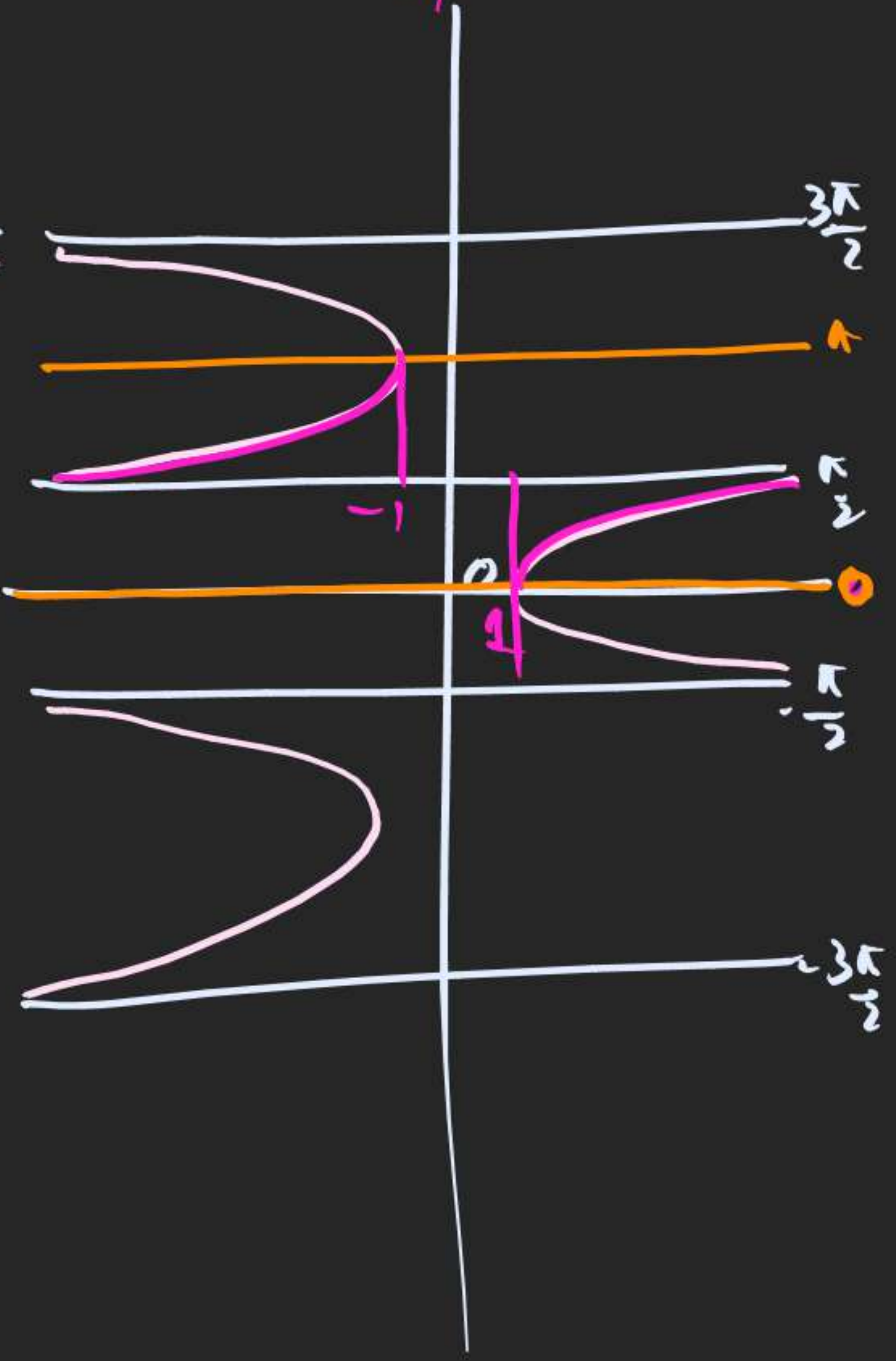


$y = \sec^{-1} x$





$x = \sec y$



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Domain & Range Based

(1) $y = \sin^{-1}(2x)$ find D_f

$-1 \le 2x \le 1$

$-\frac{1}{2} \le x \le \frac{1}{2}$ A

(3) $y = \tan^{-1}(2x + \frac{1}{3})$ D_f?

R

Dom of $y = 2x + \frac{1}{3}$ (R)

Linear Poly

$x = -\frac{1}{6}$

(3) $y = \sec^{-1}(2x + \frac{1}{3})$'s D_f

$2x + \frac{1}{3} \le -1 \quad \cup \quad 2x + \frac{1}{3} \ge 1$

$2x \le -\frac{4}{3} \quad \cup \quad 2x \ge \frac{2}{3}$

$x \le -\frac{2}{3} \quad \cup \quad x \ge \frac{1}{3}$

$x \in (-\infty, -\frac{2}{3}] \cup [\frac{1}{3}, \infty)$

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IIT

Q $y=\sqrt{\sin^{-1}(2x)+\frac{\pi}{6}}$. D_f

$\sin^{-1}(2x)+\frac{\pi}{6}\geq 0$ | $-1\leq 2x\leq 1$ | R

$\frac{\pi}{2}\geq \sin^{-1}(2x)\geq -\frac{\pi}{6}$ | $\boxed{-\frac{1}{2}\leq x\leq \frac{1}{2}}$

$\sin\left(\frac{\pi}{2}\right)\geq 2x\geq \sin\left(-\frac{\pi}{6}\right)$

$1\geq 2x\geq -\frac{1}{2}$

$\boxed{\frac{1}{2}\geq x\geq -\frac{1}{4}}$

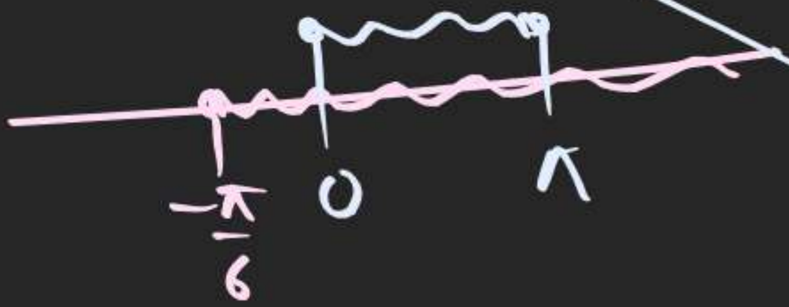
$-\frac{\pi}{2}\leq \sin^{-1}(f(x))\leq \frac{\pi}{2}$

$x\in\left[-\frac{1}{4},\frac{1}{2}\right]$

Q $y=\sqrt{\cos^{-1}(2x)+\frac{\pi}{6}}$ D_f? $D_f\to\left[-\frac{1}{2},\frac{1}{2}\right]$

$\cos^{-1}(2x)+\frac{\pi}{6}\geq 0$ | $-1\leq 2x\leq 1$

$\cos^{-1}(2x)\geq -\frac{\pi}{6}$ | $\boxed{-\frac{1}{2}\leq x\leq \frac{1}{2}}$



$0\leq \cos^{-1}(2x)\leq \pi$

$0\leq \cos^{-1}2x\leq \pi$

$\cos 0\geq 2x\geq \cos\pi$

$1\geq 2x\geq -1$

$\frac{1}{2}\geq x\geq -\frac{1}{2}$

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Q D_f of $f(x) = \sin^{-1}\frac{1}{|x^2-1|} + \frac{1}{\sqrt{\sin^2 x + \sin x + 1}}$

$-1 \le \frac{1}{|x^2-1|} \le 1$ (+) (+)

$-ve \le +ve$

Ignore

$0 < \frac{1}{|x^2-1|} \le 1$

$\infty > \boxed{|x^2-1| \geq 1}$

$|x^2-1| \geq 1$

$x^2 \cancel{-1} \le \cancel{-1}$ $\cup$ $x^2-1 \geq 1$

$\boxed{x^2 \le 0}$

$x^2=0$

$\boxed{x=0}$

$x^2-2 \geq 0$

$(x-\sqrt{2})(x+\sqrt{2}) \geq 0$

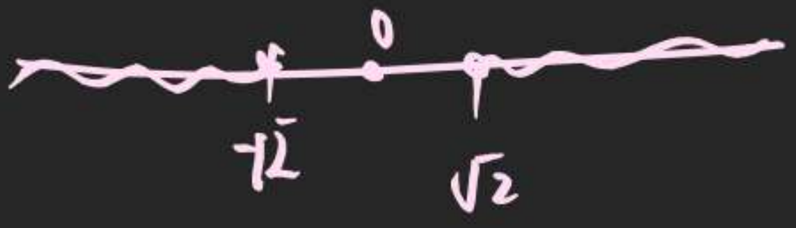
$x \le -\sqrt{2} \cup x \geq \sqrt{2}$

$\sin^2 x + \sin x + 1 > 0$

$D = (1)^2 - 4 \times 1 \times 1$

$= -3 < 0$

always $x \in \mathbb{R}$



$x \in (-\infty, -\sqrt{2}] \cup [\sqrt{2}, \infty) \cup \{0\}$

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Q D_f of $y = \sin^{-1}(\log_4 x^2)$

$$-1 \le \cancel{\log_4} x^2 \le 1$$

$$4^{-1} \le x^2 \le 4^1$$

$$\frac{1}{4} \le x^2 \le 4$$

$$\frac{1}{2} \le \sqrt{x^2} \le 2$$

$$\frac{1}{2} \le |x| \le 2$$

$$x \in \left[-2, -\frac{1}{2}\right] \cup \left[\frac{1}{2}, 2\right]$$

Q $y = \sin^{-1}\left(\frac{x-3}{2}\right) + \log_{10}(4-x)$ D_f?

$$-1 \le \frac{x-3}{2} \le 1$$

$$-2 \le x-3 \le 2$$

$$1 \le x \le 5$$

$$4 - x > 0$$

$$\underline{x < 4}$$

$$x \in [1, 4)$$

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Q D_f of $y = \cos^{-1}\left(\frac{x^2+1}{2x}\right)$

$-1 \le \frac{x^2+1}{2x} \le 1$ Ulta

$\left|\frac{x^2+1}{2x}\right| \le 1$

$\frac{|x^2+1|}{|2x|} \le 1$ $\quad x \ne 0$

$\frac{x^2+1}{2|x|} \le 1$

$|x|^2+1 \le 2|x|$

$|x|^2 - 2|x| + 1 \le 0$

$(|x|-1)^2 \le 0$

$(|x|-1)^2 = 0$

$|x|-1 = 0$

$|x| = 1$

$\boxed{x = 1, -1}$

$x \in \{1, -1\}$

Q Range of $y = \cos^{-1}\left(\frac{x^2+1}{2x}\right)$

as Dom $\ne R$

$\therefore$ Range depends on dom

$x = 1 \to y = \cos^{-1}\left(\frac{1^2+1}{2\times1}\right) = \cos^{-1}(1)$

$= 0$

$x = -1 \to y = \cos^{-1}\left(\frac{(-1)^2+1}{2\times-1}\right)$

$= \cos^{-1}(-1)$

$= \pi$

$y \in \{0, \pi\}$

Q $f(x) = \operatorname{cosec}^{-1}(\cos x)$ Exist for $x \in$?

$\cos x \le -1 \quad \cup \quad \cos x \geq 1$

$\cos x = -1 \qquad \cos x = 1$

$x = \pi \qquad x = 0$

$x \in \{0, \pi\}$

Q $f(x) = \sin^{-1} x + \cos^{-1} x + \tan^{-1} x \quad D_f$:

$-1 \le x \le 1 \quad | \quad -1 \le x \le 1 \quad | \quad x \in R.$

$x \in [-1, 1]$

Q $f(x) = \sin^{-1}x + \tan^{-1}x + \sec^{-1}x$ D_f

$-1 \le x \le 1$ | R | $x \le -1 \cup x \ge 1$

$x \in \{-1, 1\}$

Q Range of $f(x) = \sin^{-1}x + \tan^{-1}x + \sec^{-1}x$

Answe

$x = 1 \rightarrow y = \sin^{-1}(1) + \tan^{-1}(1) + \sec^{-1}(1)$

$= \frac{\pi}{2} + \frac{\pi}{4} + 0 = \frac{3\pi}{4}$

$x = -1 \Rightarrow y = \sin^{-1}(-1) + \tan^{-1}(-1) + \sec^{-1}(-1)$

$= -\frac{\pi}{2} + -\frac{\pi}{4} + \pi = \frac{\pi}{4}$

$\therefore y \in \left\{\frac{\pi}{4}, \frac{3\pi}{4}\right\}$

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Q D_f of $y = e^{\sin^{-1}\frac{x}{2}} + \tan^{-1}\left[\frac{x}{2}-1\right] + \ln\sqrt{x-[x]}$

R   R   R   Poly R

$-1 \le \frac{x}{2} \le 1$

$\boxed{-2 \le x \le 2}$

R

$x-[x] > 0$

$\{x\} > 0$

$\Rightarrow \{x\} \ne 0$

$x \notin I$

$x \in R - I$

$x \in (-2,2) - \{-1,0,1\}$

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Q R_f of $f(x) = 2\sin^{-1}(3x-5) + \frac{\pi}{2}$

$$-\frac{\pi}{2} \le \sin^{-1}(3x-5) \le \frac{\pi}{2}$$

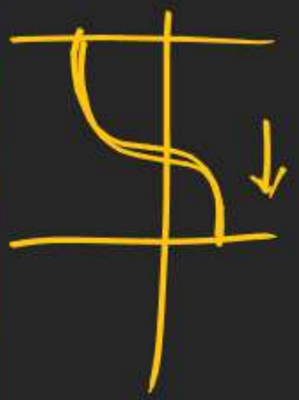
$$-\pi \le 2\sin^{-1}(3x-5) \le \pi$$

$$-\frac{\pi}{2} \le \underbrace{2\sin^{-1}(3x-5) + \frac{\pi}{2}} \le \frac{3\pi}{2}$$

$$y \in \left[-\frac{\pi}{2}, \frac{3\pi}{2}\right]$$

Q Find R_f of

$$y = 3\cot^{-1}(-x^2) - \frac{\pi}{2}$$



JJHSDH

$$0 \le x^2 < \infty$$

$$0 \ge -x^2 > -\infty$$

$$\cot^{-1}(0) \le \cot^{-1}(-x^2) < \cot^{-1}(-1)$$

$$\frac{\pi}{2} \le \cot^{-1}(-x^2) < \pi$$

$$\frac{3\pi}{2} \le 3\cot^{-1}(-x^2) < 3\pi$$

$$\underline{\pi \le 3\cot^{-1}(-x^2) - \frac{\pi}{2} < \frac{5\pi}{2}}$$

$$y \in \left[\pi, \frac{5\pi}{2}\right)$$

Q R_f of $y = \cos^{-1}(2x - x^2)$

$$2x - x^2 = -(x^2 - 2x + 1) + 1$$

$$= 1 - (x-1)^2$$

$$\infty > (x-1)^2 \geq 0$$

$$-\infty < -(x-1)^2 \leq 0$$

$$-\infty < 1 - (x-1)^2 \leq 1$$

$$\cos^{-1}(-1) > \cos^{-1}(2x - x^2) \geq \cos^{-1} 1$$

$$\pi > y \geq 0 \therefore y \in [0, \pi)$$

Q R_f of $y = \cos^{-1}\left(\frac{x^4 + x^2 + 1}{x^2 + x + 1}\right)$

$$\frac{\cancel{x^4 + x^2 + 1}}{\cancel{x^2 + x + 1}} = x^2 - x + 1$$

$$= \left(x - \tfrac{1}{2}\right)^2 - \left(\tfrac{1}{2}\right)^2 + 1$$

$$= \left(x - \tfrac{1}{2}\right)^2 + \frac{3}{4} \geq \frac{3}{4}$$

$$\frac{3}{4} \leq \frac{x^4 + x^2 + 1}{x^2 + x + 1} < \infty$$

$$\cos^{-1}\left(\tfrac{3}{4}\right) \geq \cos^{-1}\left(\frac{x^4 + x^2 + 1}{x^2 + x + 1}\right) > \cos^{-1}(1)$$

$$y \in \left[0, \cos^{-1}\left(\tfrac{3}{4}\right)\right]$$

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RELATION FUNCTION

Q. The range of the function $y = \frac{8}{9-x^2}$ is

Dme

(A) $(-\infty, \infty) - \{\pm 3\}$

(B) $\left[\frac{8}{9}, \infty\right)$

(C) $\left(0, \frac{8}{9}\right)$

(D) $(-\infty, 0) \cup \left[\frac{8}{9}, \infty\right)$

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RELATION FUNCTION

Q. For the function $f(x)=\frac{e^x+1}{e^x-1}$, if $n(d)$ denotes the number of integers which are not in its domain and $\boxed{n(r)}$ denotes the number of integers which are not in its range, then $n(d)+n(r)$ is equal to

(A) 2

(B) 3

(C) 4

(D) Infinite

$$y=\frac{e^x+1}{e^x-1}$$

$$e^x\cdot y-y=e^x+1$$

$$e^x(y-1)=1+y$$

$$\boxed{e^x}=\left(\frac{y+1}{y-1}\right)>0$$

$\{-1,0,1\}$ $n(r)=3$ $+$ $n(d)=1$

4

$$y=\frac{e^x+1}{e^x-1}$$

$$e^x-1\neq 0$$

$$e^x\neq 1$$

$$e^x\neq e^0$$

$$\boxed{x\neq 0}$$

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RELATION FUNCTION

Q. Number of integral values of x in the domain of function

$$f(x) = \sqrt{\ln|\ln|x||} + \sqrt{7|x| - |x|^2 - 10} \text{ is equal to}$$

(A) 4

(B) 5

(C) 6

(D) 7

$\cancel{\ln}|\ln|x|| \geq 0$

$|\ln|x|| \geq 1$

$\cancel{\ln}|x| \leq -1 \cup \cancel{\ln}|x| \geq 1$

$|x| \leq e^{-1} \cup |x| \geq e$

$|x| \leq \frac{1}{e}$

$7|x| - |x|^2 - 10 \geq 0$

$|x|^2 - 7|x| + 10 \leq 0$

$(|x|-2)(|x|-5) \leq 0$

$2 \leq |x| \leq 5$

$x \in [-5, -e] \cup [e, 5]$

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RELATION FUNCTION

Q. If a polynomial function ' f ' satisfies the relation

$$\log_2(f(x)) = \log_2\left(2 + \frac{2}{3} + \frac{2}{9} + \cdots .\infty\right) \cdot \log_3\left(1 + \frac{f(x)}{f\left(\frac{1}{x}\right)}\right) \text{ and } f(10) = \boxed{1001}$$

then the value of $f(20)$ is

(A) 2002

(B) 7999

(C) 8001

(D) 16001

$f(x) = 1 + x^3$

$n = 3$

$f(x) = 1 + x^n$

$f(10) = 1 + 10^n = 1001$

$\log_2 2\left(1 + \frac{1}{3} + \frac{1}{3^2} \cdots\right) \times \log_3\left(\frac{f(x) + f(1/x)}{f(1/x)}\right)$

$\left(\frac{1}{1 - 1/3}\right)$

$\log_2 2\left(\frac{3}{2}\right)$

$f(x) \cdot f\left(\frac{1}{x}\right) = f(x) + f(1/x)$

$\log_2 f(x) = \frac{\log 3}{\log 2} \times \frac{\log\left(\frac{f(x)+f(1/x)}{f(1/x)}\right)}{\log 3}$

$\Rightarrow \log_2 f(x) = \log_2\left(\frac{f(x)+f(1/x)}{f(1/x)}\right)$