Nishant Jindal

Q If $a, b, c, d \in R^+$ then P.T.

$$(a+b+c+d)\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+\frac{1}{d}\right) \geq 16.$$

a, b, c, d AM, HM

AM $\geq$ HM

$$\frac{a+b+c+d}{4} \geq \frac{4}{\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+\frac{1}{d}\right)}$$

$$(a+b+c+d)\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+\frac{1}{d}\right) \geq 16$$

Q $a, b, c, d \in R^+$

$$(a^2b+b^2c+c^2a)(ab^2+bc^2+ca^2) \geq 9a^2b^2c^2$$

(1) $$\frac{a^2b+b^2c+c^2a}{3} \geq (a^2 \cdot b \cdot b^2c \cdot c^2a)^{\frac{1}{3}}$$

$$a^2b+b^2c+c^2a \geq 3(a^3b^3c^3)^{\frac{1}{3}}$$

$$\geq 3abc \longrightarrow (1)$$

(2) $$\frac{ab^2+bc^2+ca^2}{3} \geq (ab^2 \cdot bc^2 \cdot ca^2)^{\frac{1}{3}}$$

$$ab^2+bc^2+ca^2 \geq 3((abc)^3)^{\frac{1}{3}} \longrightarrow (2)$$

1×2

$$(a^2b+b^2c+c^2a)(ab^2+bc^2+ca^2) \geq 9a^2b^2c^2$$ H.P.

Nishant Jindal

★ Rk **When to Use AM ≥ GM**

1) If Sum of No is given & Max of their Prod is asked we use AM ≥ GM.

2) If Prod is given & min. of Sum is asked. We use AM ≥ GM.

3) If Qs is of form $a\cdot f(x)+\frac{b}{f(x)}$ then also. We use AM ≥ GM.

Q If $a,b,c \in R^+$ Such that $\boxed{a+b+c=18}$ (Sum is given) then find max. Value of (1) $(abc)_{Max}$.

a,b,c AM ≥ GM

$$\frac{a+b+c}{3} \ge (a\cdot b\cdot c)^{\frac13} \Rightarrow \frac{\cancel{18}^{6}}{\cancel{3}} \ge (abc)^{\frac13} \Rightarrow 6^3 \ge (abc)$$

$abc \le 216$

↳ abc will always be less than to 216

⇒ Max.m of abc can be 216.

(2) $(a^2bc)_{Max}=?$ → Jitni deg utna Brk

$$\frac{\frac a2+\frac a2+b+c}{4} \ge \left(\frac a2\cdot\frac a2\cdot b\cdot c\right)^{\frac14}$$

$$\frac{\cancel{18}^{9}}{2\,\cancel{4}} \ge \left(\frac{a^2bc}{4}\right)^{\frac14}$$

$$\left(\frac{a^2bc}{4}\right) \le \left(\frac92\right)^4 \Rightarrow (a^2bc) \le 4\cdot\underbrace{\left(\frac92\right)^4}_{\text{Max.}}$$

(3) Max of a^2b^3c

$$\frac{\frac a2+\frac a2+\frac b3+\frac b3+\frac b3+c}{6} \ge \left(\frac a2\cdot\frac a2\cdot\frac b3\cdot\frac b3\cdot\frac b3\cdot c\right)^{\frac16}$$

$$\frac{a+b+c}{6} \ge \left(\frac{a^2}{4}\cdot\frac{b^3}{27}\cdot c\right)^{\frac16} \Rightarrow (3)^6 \ge \left(\frac{a^2b^3c}{4\cdot 27}\right)$$

$$a^2b^3c \le \underbrace{27\cdot4\cdot(3)^6}_{\text{Max of } a^2b^3c}$$

Nishant Jindal

Q If x, y, z are +ve Real No. & $x+y+z=7$ then gr. value of $x^2 y^3 z^2$?

Sum is give & Max of Prod is asked

$$\frac{\left(\frac{x}{2}+\frac{x}{2}\right)+\left(\frac{y}{3}+\frac{y}{3}+\frac{y}{3}\right)+\left(\frac{z}{2}+\frac{z}{2}\right)}{7} \geq \left(\frac{x}{2}\cdot\frac{x}{2}\cdot\frac{y}{3}\cdot\frac{y}{3}\cdot\frac{y}{3}\cdot\frac{z}{2}\cdot\frac{z}{2}\right)^{\frac{1}{7}}$$

$$\frac{7}{7} \geq \left(\frac{x^2y^3z^2}{4\times 27\times 4}\right)^{\frac{1}{7}}$$

$$\frac{x^2y^3z^2}{16\times 27} \leq (1)^7$$

$$x^2y^3z^2 \leq 1\times 16\times 27$$

Max of $x^2y^3z^2 = 16\times 27 = 432$

Q If $a_1 a_2 a_3 \dots a_{20}$ are AM betn 13 & 67 then maxm value of $\underline{a_1\cdot a_2\cdot a_3 \dots a_{20}}$ is?

Max value of Prod = ? AM ≥ GM

$13, \underbrace{a_1, a_2, a_3 \dots a_{20}}, 67$

Sum of AM = $a_1+a_2+a_3+\dots+a_{20}$

$= \frac{20}{2}(13+67)$

$= 20\times 40 = 800$

$$\frac{a_1+a_2+a_3+\dots+a_{20}}{20} \geq (a_1\cdot a_2\cdot a_3 \dots a_{20})^{\frac{1}{20}}$$

$$\frac{\cancel{800}^{40}}{\cancel{20}} \geq (a_1\cdot a_2\cdot a_3 \dots a_{20})^{\frac{1}{20}}$$

$a_1 a_2 \dots a_{20} \leq (40)^{20}$

Nishant Jindal

(1) If $a>0, b>0, c>0$, $abc=64$ then Min. value of (1) $a+b+c=?$

$$\frac{a+b+c}{3} \geq (a\cdot b\cdot c)^{\frac{1}{3}}$$

$$a+b+c \geq 3(64)^{\frac{1}{3}}$$

$$\geq 3(4^3)^{1/3}$$

$$a+b+c \geq 12$$

$$(a+b+c)_{Min}=12$$

(2) $(2a+3b+4c)_{Min}$

$$\frac{2a+3b+4c}{3} \geq (2a\times 3b\times 4c)^{\frac{1}{3}}$$

$$2a+3b+4c \geq 3(24abc)^{\frac{1}{3}}$$

$$\geq 3(24)^{\frac{1}{3}}\times 4$$

$$\therefore \boxed{Min = 12(24)^{\frac{1}{3}}}$$

Q. If $a_i>0 \ \forall\, i \in N$ such that $\prod_{i=1}^{n} a_i = 1$ then P.T.

$$(1+a_1)(1+a_2)(1+a_3)\cdots(1+a_n) \geq 2^n$$

$$\frac{1+a_1}{2} \geq \sqrt{1\cdot a_1} \qquad \frac{1+a_2}{2} \geq \sqrt{1\cdot a_2} \qquad \frac{1+a_3}{2} \geq \sqrt{1\cdot a_3}$$

$$1+a_1 \geq 2\sqrt{a_1} \qquad 1+a_2 \geq 2\sqrt{a_2} \qquad 1+a_3 \geq 2\sqrt{a_3}.$$

$$(1+a_1)(1+a_2)(1+a_3)\cdots(1+a_n) \geq \underbrace{2\cdot 2\cdot 2\cdots 2}_{n\text{ at }2}\cdot\sqrt{a_1 a_2 a_3\cdots a_n}$$

$$\geq 2^n\cdot\sqrt{1}$$

$$\geq 2^n$$

H.P.

Nishant Jindal

Q If $a_1, a_2 a_3 \ldots a_n$ are Real No. such that $a_1 \cdot a_2 \cdot a_3 \ldots a_n = C$ then min. value of

$a_1 + a_2 + a_3 + \ldots + a_{n-1} + 2a_n = ?$

← n elements →

$$\frac{(a_1 + a_2 + a_3 + \ldots + 2a_n)}{n} \geq (a_1 \cdot a_2 \cdot a_3 \ldots 2a_n)^{\frac{1}{n}}$$

$$(a \cdot b \cdot 2c)^2 = (2abc)^2$$

$$(a_1 + a_2 + a_3 + \ldots + 2a_n) \geq n(2 \cdot a_1 a_2 \cdot a_3 \ldots a_n)^{\frac{1}{n}}$$

$$\geq n(2c)^{\frac{1}{n}}$$

Ans.

Q Find Min. sum of No. $(a > 0)$

$$\frac{1}{a^5}, \frac{1}{a^4}, \frac{3}{a^3}, 1, a^8, a^{10}$$

Prod नहीं दिया

$$\frac{\frac{1}{a^5} + \frac{1}{a^4} + \frac{1}{a^3} + \frac{1}{a^3} + \frac{1}{a^3} + 1 + a^8 + a^{10}}{8} \geq \left(\frac{1}{a^5} \cdot \frac{1}{a^4} \cdot \frac{1}{a^3} \cdot \frac{1}{a^3} \cdot \frac{1}{a^3} \cdot 1 \cdot a^8 a^{10}\right)^{\frac{1}{8}}$$

$$\geq (1)^{1/8}$$

$$\frac{1}{a^5} + \frac{1}{a^4} + \frac{3}{a^3} + 1 + \frac{1}{a^8} + \frac{1}{a^{10}} \geq 8 \times 1$$

Min = 8

Nishant Jindal

Q If $a_i > 0$ $(i = 1, 2, 3, 4)$ Such that

$$\underline{501}\, a_1 + \underline{504}\, a_2 + \underline{505}\, a_3 + \underline{506}\, a_4 = 2016$$

& $256\, a_1 \cdot a_2 \cdot a_3 \cdot a_4 \geq \left(\sum_{r=1}^{4} a_r \right)^4$ then find.

Min value of $\sum_{r=1}^{4} a_r^2 = ?$

$$\left.\begin{array}{r} 501 \\ 504 \\ 505 \\ 506 \\ \hline 2016 \end{array}\right\}$$

$a_1 = a_2 = a_3 = a_4 = 1$

$$\sum_{r=1}^{4} a_r^2 = a_1^2 + a_2^2 + a_3^2 + a_4^2$$

$$= 1^2 + 1^2 + 1^2 + 1^2$$

$$= 4$$