DPP-02

SOLUTIONS

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1. Find the value of $\sin\left(2\sin^{-1}\left(\frac{1}{4}\right)\right)$

Sol. Let $\sin^{-1}\left(\frac{1}{4}\right) = \theta$

$\Rightarrow \sin\theta = \frac{1}{4}$

Now, $\sin(2\theta) = 2\sin\theta.\cos\theta$

$= 2x\frac{1}{4}x\sqrt{1-\frac{1}{16}}$

$= \frac{\sqrt{15}}{8}$

2. Find the value of $\cos\left(2\cos^{-1}\left(\frac{1}{3}\right)\right)$

Sol. Let $\cos^{-1}\left(\frac{1}{3}\right) = \theta$

$\Rightarrow \cos\theta = \frac{1}{3}$

Now, $\cos(2\theta) = 2\cos^2\theta - 1$

$= \frac{2}{9} - 1 = -\frac{7}{9}$

3. Find the value of $\cos\left(2\tan^{-1}\left(\frac{1}{3}\right)\right)$

Sol. Let $\tan^{-1}\left(\frac{1}{3}\right) = \theta$

$\Rightarrow \tan\theta = \frac{1}{3}$

Now,

$\cos(2\theta) = \frac{1-\tan^2\theta}{1+\tan^2\theta} = \frac{1-\left(\frac{1}{3}\right)^2}{1+\left(\frac{1}{3}\right)^2} = \frac{9-1}{9+1} = \frac{8}{10} = \frac{4}{5}$

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4. Find the value of $\sin\left(\frac{1}{2}\cot^{-2}\left(\frac{3}{4}\right)\right)$

Sol. Let $\cot^{-1}\left(\frac{3}{4}\right) = \theta$ Now, $\sin\left(\frac{\theta}{2}\right)$

$$= \sqrt{\frac{1-\cos\theta}{2}} = \sqrt{\frac{1-\frac{\cos\theta}{\sin\theta\cdot\text{cosec}\,\theta}}{2}} = \sqrt{\frac{1-\frac{\cot\theta}{\text{cosec}\,\theta}}{2}} = \sqrt{\frac{1-\frac{\cot\theta}{\sqrt{1+\cot^2\theta}}}{2}}$$

$$= \frac{1}{\sqrt{2}}\sqrt{1-\frac{3/4}{\sqrt{1+9/16}}} = \frac{1}{\sqrt{2}}\sqrt{1-\frac{3}{5}} = \frac{2}{\sqrt{5}}$$

5. Find the value of $\tan^{-1}\left(\frac{3\pi}{4} - 2\tan^{-1}\left(\frac{3}{4}\right)\right)$

Sol. Let $\tan^{-1}\left(\frac{3}{4}\right) = \theta$

$\Rightarrow \tan\theta = \frac{3}{4}$

We have

$$\tan\left(\frac{3\pi}{4} - 2\theta\right) = \frac{\tan\left(\frac{3\pi}{4}\right) - \tan(2\theta)}{1+\tan\left(\frac{3\pi}{4}\right)\cdot\tan(2\theta)}$$

$$= \frac{-1-\tan(2\theta)}{1-\tan(2\theta)} = \frac{-1-\frac{2\tan\theta}{1-\tan^2\theta}}{1-\frac{2\tan\theta}{1-\tan^2\theta}} = \frac{\tan^2\theta-2\tan\theta-1}{1-\tan^2\theta-2\tan\theta} = \frac{\frac{9}{16}-\frac{6}{4}-1}{1-\frac{9}{16}-\frac{6}{4}} = \frac{41}{17}$$

6. Prove that $\sin\left(2\sin^{-1}\left(\frac{1}{2}\right)\right) = \frac{\sqrt{3}}{2}$

Sol. Let $\sin^{-1}\left(\frac{1}{2}\right) = \theta$

Then $\sin\theta = \frac{1}{2}$

Now, $\sin\left(2\sin^{-1}\left(\frac{1}{2}\right)\right) = \sin(2\theta) = 2\sin\theta\cos\theta = 2\times\frac{1}{2}\times\frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2}$

7. Prove the $\sin\left(3\sin^{-1}\left(\frac{1}{3}\right)\right) = \frac{23}{27}$

Sol. Let $\sin^{-1}\left(\frac{1}{3}\right) = \theta$

Then $\sin\theta = \frac{1}{3}$

Now, $\sin\left(3\sin^{-1}\left(\frac{1}{3}\right)\right)$

$$= \sin(3\theta) = 3\sin\theta - 4\sin^3\theta \quad = 3\cdot\frac{1}{3} - 4\cdot\left(\frac{1}{3}\right)^3 \quad = 1-\frac{4}{27} = \frac{23}{27}$$

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8. Prove that $\cos\left(\frac{1}{2}\cos^{-1}\left(\frac{1}{8}\right)\right)=\frac{3}{4}$

Sol. Let $\frac{1}{2}\cos^{-1}\left(\frac{1}{8}\right)=\theta$

Then $\cos^{-1}\left(\frac{1}{8}\right)=2\theta$

$\Rightarrow \cos(2\theta)=\frac{1}{8} \Rightarrow 2\cos^2\theta \mid 1=\frac{1}{8} \quad \Rightarrow 2\cos^2\theta=\frac{9}{8} \quad \Rightarrow \cos^2\theta=\frac{9}{16}$

$\Rightarrow \cos\theta=\frac{3}{4} \Rightarrow \cos\left(\frac{1}{2}\cos^{-1}\left(\frac{1}{8}\right)\right)=\frac{3}{4}$

9. Prove that $\cos\left(\frac{1}{2}\cos^{-1}\left(-\frac{1}{10}\right)\right)=\frac{3\sqrt{5}}{10}$

Sol. Let $\frac{1}{2}\cos^{-1}\left(-\frac{1}{10}\right)=\theta$

Then $\cos^{-1}\left(-\frac{1}{10}\right)=2\theta \quad \Rightarrow \quad \cos(2\theta)=\sin(\theta)\times\frac{\sqrt{7}}{\sqrt{2}}=\frac{\sqrt{63}}{12}$

$\Rightarrow \quad 2\cos^2-1=-\frac{1}{10} \Rightarrow 2\cos^2\theta-1=-\frac{1}{10}=\frac{9}{10}$

$\Rightarrow \quad \cos^2\theta=\frac{9}{20} \Rightarrow \cos\theta=\frac{3}{2\sqrt{5}} \Rightarrow \cos\theta=\frac{3}{2\sqrt{5}}=\frac{3\sqrt{5}}{2\sqrt{5}\times\sqrt{5}}=\frac{3\sqrt{5}}{10}$

$\Rightarrow \quad \cos\left(\frac{1}{2}\cos^{-1}\left(-\frac{1}{10}\right)\right)=\frac{3\sqrt{5}}{10}$

10. Prove that $\sin\left(\frac{1}{2}\cos^{-1}\left(\frac{1}{9}\right)\right)=\frac{2}{3}$

Sol. Let $\frac{1}{2}\cos^{-1}\left(\frac{1}{9}\right)=\theta$

$\Rightarrow \quad \cos^{-1}\left(\frac{1}{9}\right)=2\theta \Rightarrow \cos(2\theta)=\frac{1}{9} \Rightarrow 2\cos^2(\theta)=1+\frac{1}{9}=\frac{10}{9} \Rightarrow \cos^2(\theta)=\frac{5}{9}$

$\Rightarrow \quad \sin^2(\theta)=1-\frac{5}{9}=\frac{4}{9} \Rightarrow \sin(\theta)=\frac{2}{3} \Rightarrow \sin\left(\frac{1}{2}\cos^{-1}\left(\frac{1}{9}\right)\right)=\frac{2}{3}$

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11. Prove that $\sin\left(\frac{1}{4}\tan^{-1}\sqrt{63}\right)=\frac{1}{2\sqrt{2}}$

Sol. Let $\frac{1}{4}\tan^{-1}\left(\sqrt{63}\right)=\theta$

$\Rightarrow$ $\text{tna}^{-1}\left(\sqrt{63}\right)=4\theta$

$\Rightarrow$ $\tan(4\theta) = \sqrt{63}$

$\Rightarrow$ $\tan(4\theta) = \sqrt{63}$

$\Rightarrow$ $\frac{\sin(4\theta)}{\cos(4\theta)}=\sqrt{63}$ $\Rightarrow$ $\frac{\sin(4\theta)}{\sqrt{63}}=\frac{\cos(4\theta)}{1}=\frac{1}{8}$

Now, $\cos 4\theta=\frac{1}{8}$

$\Rightarrow$ $2\cos^2(2\theta)-1 = \frac{1}{8} \Rightarrow 2\text{son}^2(2\theta) = 1 + \frac{1}{8}=\frac{9}{8}$

$\Rightarrow$ $\cos^2(2\theta) = \frac{9}{16} \Rightarrow \cos(2\theta) = \frac{3}{4} \Rightarrow$ $2\cos^2(\theta)-1=\frac{3}{4} \Rightarrow$ $2\cos^2(\theta) = \frac{7}{4}$

$\Rightarrow$ $\cos^2(\theta) = \frac{7}{8} \Rightarrow \cos(\theta) = \frac{\sqrt{7}}{2\sqrt{2}} \Rightarrow$ Also, $\sin(4\theta) = \frac{\sqrt{63}}{8} \Rightarrow 2\sin(2\theta)\cos(2\theta) = \frac{\sqrt{63}}{8}$

$\Rightarrow$ $2\sin(2\theta) \times \frac{3}{4}=\frac{\sqrt{63}}{8}$, form (i)

$\Rightarrow$ $\sin(2\theta) = \frac{\sqrt{63}}{12} \Rightarrow$ $2\sin(\theta)\cos(\theta) = \frac{\sqrt{63}}{12}$, form (ii)

$\Rightarrow$ $2\sin(\theta) \times \frac{\sqrt{7}}{2\sqrt{2}}=\frac{\sqrt{63}}{12} \Rightarrow$ $\sin(\theta)\times\frac{\sqrt{7}}{\sqrt{2}}=\frac{\sqrt{63}}{12}$

$\Rightarrow$ $\sin(\theta) = \frac{3\sqrt{2}}{12} \Rightarrow$ $\sin(\theta) = \frac{\sqrt{2}}{4} \Rightarrow$ $\sin(\theta) = \frac{1}{2\sqrt{2}}$

$\Rightarrow$ $\sin\left(\frac{1}{4}\tan^{-1}\left(\sqrt{63}\right)\right)=\frac{1}{2\sqrt{2}}$

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12. Prove that $\cos\left(\frac{1}{4}\left(\tan^{-1}\left(\frac{24}{7}\right)\right)\right)=\frac{3}{\sqrt{10}}$

Sol. Let $\frac{1}{4}\tan^{-1}\left(\frac{24}{7}\right)=\theta$

$\Rightarrow \quad \tan^{-1}\left(\frac{24}{7}\right)=4\theta \Rightarrow \tan(4\theta)=\frac{24}{7}$

$\Rightarrow \quad \frac{\sin(4\theta)}{24}=\frac{\cos(4\theta)}{7}=\frac{1}{25}$

Now, $\cos(4\theta)=\frac{7}{25}$

$\Rightarrow \quad 2\cos^2(2\theta)-1=\frac{7}{25} \Rightarrow \quad 2\cos^2(2\theta)=1+\frac{7}{25}=\frac{32}{25}$

$\Rightarrow \quad \cos^2(2\theta)=\frac{32}{50} \Rightarrow \quad \cos(2\theta)=\sqrt{\frac{32}{50}}$

$\Rightarrow \quad 2\cos^2(\theta)-1=\sqrt{\frac{32}{50}}$

$\Rightarrow \quad 2\cos^2(\theta)=1+\frac{8}{10}=\frac{18}{10}$

$\Rightarrow \quad \cos^2(\theta)=\frac{9}{10} \Rightarrow \quad \cos(\theta)=\frac{3}{\sqrt{10}} \Rightarrow \quad \cos\left(\frac{1}{4}\left(\tan^{-1}\left(\frac{24}{7}\right)\right)\right)=\frac{3}{\sqrt{10}}$

13. Prove that $\tan\left(\frac{1}{2}\cos^{-1}\left(\frac{2}{3}\right)\right)=\frac{1}{\sqrt{5}}$

Sol. Let $\frac{1}{2}\cos^{-1}\left(\frac{2}{3}\right)=\theta$

$\Rightarrow \quad \cos^{-1}\left(\frac{2}{3}\right)2\theta \Rightarrow \quad \cos(2\theta)=\frac{2}{3}$

$\Rightarrow \quad \frac{1-\tan^2\theta}{1+\tan^2\theta}=\frac{2}{3} \Rightarrow \quad 2+2\tan^2\theta=3-3\tan^2\theta$

$\Rightarrow \quad 5\tan^2\theta=1 \Rightarrow \quad \tan^2\theta=\frac{1}{5}$

$\Rightarrow \quad \tan\theta=\frac{1}{\sqrt{5}} \Rightarrow \quad \tan\left(\frac{1}{2}\cos^{-1}\left(\frac{2}{3}\right)\right)=\frac{1}{\sqrt{5}}$

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14. Prove that $\tan\left(2\tan^{-1}\left(\frac{1}{5}\right)-\frac{\pi}{4}\right)=-\frac{7}{17}$

sol. We have $\tan\left(2\tan^{-1}\left(\frac{1}{5}\right)-\frac{\pi}{4}\right)=-\frac{7}{17}$

Let $2\tan^{-1}\left(\frac{1}{5}\right)=\theta$

$\Rightarrow \quad \tan^{-1}\left(\frac{\frac{2}{5}}{1-\frac{1}{25}}\right)=\theta \quad \Rightarrow \quad \tan\theta\left(\frac{\frac{2}{5}}{1-\frac{1}{25}}\right)=\frac{10}{24}=\frac{5}{12}$

Now, $\tan\left(\theta-\frac{\pi}{4}\right)=\frac{\tan\theta-1}{1+\tan\theta}=\frac{\frac{5}{12}-1}{\frac{5}{12}+1}=-\frac{7}{17}$

15. Prove that $\tan\left(\frac{3\pi}{4}-\frac{1}{4}\sin^{-1}\left(-\frac{4}{5}\right)\right)=\frac{1-\sqrt{5}}{2}$

Sol. Let $\frac{1}{4}\sin^{-1}\left(-\frac{4}{5}\right)=\theta$

$\Rightarrow \sin^{-1}\left(-\frac{4}{5}\right)=4\theta$

$\Rightarrow \sin(4\theta)=-\frac{4}{5}$

16. The domain of the function $f(x)=\sin^{-1}\left(\frac{x^2-3x+2}{x^2+2x+7}\right)$ is. **[Main, 2022]**

(A) $[1,\infty)$ (B) $(-1,2]$ (C) $[-1,\infty)$ (D) $(-\infty,2]$

Ans. **(C)**

Sol. Let $\frac{1}{4}\sin^{-1}\left(-\frac{4}{5}\right)=\theta$

$\Rightarrow \sin^{-1}\left(-\frac{4}{5}\right)=4\theta \Rightarrow \sin(4\theta)=-\frac{4}{5}$

$\Rightarrow \frac{2\tan(2\theta)}{1+\tan^2(2\theta)}=-\frac{4}{5}$

$\Rightarrow \frac{\tan(2\theta)}{1+\tan^2(2\theta)}=-\frac{2}{5}$

$\Rightarrow 2\tan^2(2\theta)+5\tan(2\theta)+2=0$

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$\Rightarrow \tan(2\theta) = -\frac{1}{2}, -2$

when $\tan(2\theta) = -\frac{1}{2}$

$\Rightarrow \frac{2\tan\ \theta}{1-\tan^2\ \theta} = -\frac{1}{2}$

$\Rightarrow \tan\ ^2\theta - 4\tan\ \theta - 1 = 0$

$\Rightarrow \tan\ \theta = 2 - \sqrt{5}$

Now, $\tan\left(\frac{3\pi}{4} - \frac{1}{4}\sin^{-1}\left(-\frac{4}{5}\right)\right)$

$= \tan\left(\frac{3\pi}{4} - \theta\right)$

$= \tan\left(\pi - \left(\frac{\pi}{4} + \theta\right)\right) = -\tan\left(\frac{\pi}{4} + \theta\right)$

$= -\left(\frac{1+\tan\ \theta}{1-\tan\ \theta}\right) = -\left(\frac{1+2-\sqrt{5}}{1-2+\sqrt{5}}\right) = \left(\frac{3-\sqrt{5}}{1-\sqrt{5}}\right)$

$= \left(\frac{(3-\sqrt{5})(1+\sqrt{5})}{-4}\right)$

$= -\frac{1}{4}(3 + 2\sqrt{5} - 5)$

$= -\frac{1}{4}(-2 + 2\sqrt{5})$

$= \left(\frac{1-\sqrt{5}}{2}\right)$

17. The domain of the function $\cos^{-1}\left(\frac{2\sin^{-1}\left(\frac{1}{4x^2-1}\right)}{\pi}\right)$ is: **[Main, 2022]**

(A) $R - \left\{-\frac{1}{2}, \frac{1}{2}\right\}$

(B) $(-\infty, -1] \cup [1, \infty) \cup \{0\}$

(C) $\left(-\infty, \frac{-1}{2}\right) \cup \left(\frac{1}{2}, \infty\right) \cup \{0\}$

(D) $\left(-\infty, \frac{-1}{\sqrt{2}}\right] \cup \left[\frac{1}{\sqrt{2}}, \infty\right) \cup \{0\}$

Ans. (D)

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Sol. Let $f(x) = \cos^{-1}\left(\frac{2\sin^{-1}\left(\frac{1}{4x^2-1}\right)}{\pi}\right)$

As we know the domain of $\cos^{-1} y$ is $[-1,1]$

$\Rightarrow -1 \le \left(\frac{2\sin^{-1}\left(\frac{1}{4x^2-1}\right)}{\pi}\right) \le 1$

$\Rightarrow -\frac{\pi}{2} \le \sin^{-1}\left(\frac{1}{4x^2-1}\right) \le \frac{\pi}{2}$

$\Rightarrow -1 \le \frac{1}{4x^2-1} \le 1$

So, $\frac{1}{4x^2-1} \ge -1$ and $\frac{1}{4x^2-1} \le 1$

$\Rightarrow \frac{1}{4x^2-1} + 1 \ge 0$ and $\frac{1}{4x^2-1} - 1 \le 0$

$\Rightarrow \frac{4x^2}{4x^2-1} \ge 0$ and $\frac{2-4x^2}{4x^2-1} \le 0$

$\Rightarrow \frac{x^2}{(2x-1)(2x+1)} \ge 0$

and $\frac{(1-\sqrt{2}x)(1+\sqrt{2}x)}{(2x-1)(2x+1)} \le 0$

$\Rightarrow x \in \left(-\infty, -\frac{1}{\sqrt{2}}\right) \cup \left(\frac{1}{\sqrt{2}}, \infty\right) \cup \{0\}$

and $x \in \left(-\infty, \frac{1}{\sqrt{2}}\right) \cup \left(-\frac{1}{2}, \frac{1}{2}\right) \cup \left(\frac{1}{\sqrt{2}}, \infty\right)$

$\Rightarrow x \in \left(-\infty, -\frac{1}{\sqrt{2}}\right) \cup \left(\frac{1}{\sqrt{2}}, \infty\right) \cup \{0\}$

18. Considering only the principal values of the inverse trigonometric functions, the domain of the function $f(x) = \cos^{-1}\left(\frac{x^2 - 4x + 2}{x^2 + 3}\right)$ is: **[Main, 2022]**

(A) $\left(-\infty, \frac{1}{4}\right]$ (B) $\left[-\frac{1}{4}, \infty\right)$ (C) $\left(-\frac{1}{3}, \infty\right)$ (D) $\left(-\infty, \frac{1}{3}\right]$

Ans. **(B)**

Sol. $\left|\frac{x^2+4x+2}{x^2+3}\right| \le 1$

$\Leftrightarrow (x^2 - 4x + 2)^2 \le (x^2 + 3)^2$

$\Leftrightarrow (x^2 - 4x + 2)^2 - (x^2 + 3)^2 \le 0$

$\Leftrightarrow (2x^2 - 4x + 5)(-4x - 1) \le 0 \Leftrightarrow -4x - 1 \le 0 \rightarrow x \ge -\frac{1}{4}$

19. Considering the principal values of the inverse trigonometric functions, the sum of all the solutions of the equation $\cos^{-1}(x) - 2\sin^{-1}(x) = \cos^{-1}(2x)$ is equal to:

(A) 0 (B) 1 (C) $\frac{1}{2}$ (D) $-\frac{1}{2}$ **[Main, 2022]**

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Ans. (A)

Sol. $\cos^{-1} x = 2\sin^{-1} x = \cos^{-1} 2x$

$\cos^{-1} x - 2\left(\frac{\pi}{2} - \cos^{-1} x\right) = \cos^{-1} 2x$

$\cos^{-1} x - \pi + 2\cos^{-1} x = \cos^{-1} 2x$

$3\cos^2 x = \pi + \cos^{-1} 2x$

$\cos(3\cos^{-1} x) = \cos(\pi + \cos^{-1} 2x)$

$4x^3 - 3x = -2x$

$4x^3 = x \Rightarrow x = 0, \pm\frac{1}{2}$

All satisfy the original equation

$\text{sum } = -\frac{1}{2} \text{ to } +\frac{1}{2} = 0$

20. The domain of the function $f(x) = \sin^{-1}[2x^2 - 3] + \log_2\left(\log_{\frac{1}{2}}(x^2 - 5x + 5)\right)$ where $[t]$ is the greatest integer function, is: **[Main, 2022]**

(A) $\left(-\sqrt{\frac{5}{2}}, \frac{5-\sqrt{5}}{2}\right)$ (B) $\left(\frac{5-\sqrt{5}}{2}, \frac{5+\sqrt{5}}{2}\right)$ (C) $\left(1, \frac{5-\sqrt{5}}{2}\right)$ (D) $\left[1, \frac{5+\sqrt{5}}{2}\right)$

Ans. (C)

Sol. $f(x) = \sin^{-1}[2x^2 - 3] + \log_2\left(\log_{\frac{1}{2}}(x^2 - 5x + 5)\right)$

$P_1: -1 \le [2x^2 - 3] < 1$

$\Rightarrow -1 \le 2x^2 - 3 < 2$

$\Rightarrow 2 < 2x^2 < 5$

$\Rightarrow 1 < x^2 < \frac{5}{2}$

$\Rightarrow P_1: x \in \left(-\sqrt{\frac{5}{2}}, -1\right) \cup \left(1, \sqrt{\frac{5}{2}}\right)$

$P_2: x^2 - 5x + 5 > 0$

$\Rightarrow \left(x - \left(\frac{5-\sqrt{5}}{2}\right)\right)\left(x - \left(\frac{5+\sqrt{5}}{2}\right)\right) > 0$

$P_3: \log_{\frac{1}{2}}(x^2 - 5x + 5) > 0$

$\Rightarrow x^2 - 5x - 5 < 1$

$\Rightarrow x^2 - 5x + 4 < 0$

$\Rightarrow P_3 : x \in (1,4)$

So, $P_1 \cap P_2 \cap P_3 = \left(1, \frac{5-\sqrt{5}}{2}\right)$

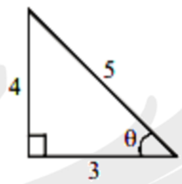
21. If the inverse trigonometric functions take principal values, then

$\cos^{-1}\left(\frac{3}{10}\cos\left(\tan^{-1}\left(\frac{4}{3}\right)\right) + \frac{2}{5}\sin\left(\tan^{-1}\left(\frac{4}{3}\right)\right)\right)$ is equal to: **[Main, 2022]**

(A) 0 (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{6}$

Ans. **(C)**

Sol. $\tan^{-1}\frac{4}{3} = \theta \Rightarrow \tan\theta = \frac{4}{3}$



$E = \cos^{-1}\left(\frac{3}{10}\cos\theta + \frac{2}{5}\sin\theta\right)$

$= \cos^{-1}\left(\frac{3}{10} \times \frac{3}{5} + \frac{2}{5} \cdot \frac{4}{5}\right)$

$= \cos^{-1}\left(\frac{9}{50} + \frac{8}{25}\right)$

$= \cos^{-1}\left(\frac{25}{50}\right)$

$= \cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$

22. The domain of the function $f(x) = \frac{\cos^{-1}\left(\frac{x^2-5x+6}{x^2-9}\right)}{\log_e(x^2-3x+2)}$ is **[Main, 2022]**

(A) $(-\infty, 1) \cup (2, \infty)$ (B) $(2, \infty)$

(C) $\left[-\frac{1}{2}, 1\right) \cup (2, \infty)$ (D) $\left[-\frac{1}{2}, 1\right) \cup (2, \infty) - \left\{\frac{3+\sqrt{5}}{2}, \frac{3-\sqrt{5}}{2}\right\}$

Ans. **(D)**

Sol. $-1 \le \frac{x^2-5x+6}{x^2-9} \le 1$

$\frac{x^2-5x+6}{x^2-9} - 1 \le 0$

$\frac{1}{x+3} \ge 0$

$x \in (-3, \infty) \ldots\ldots (1)$

$\frac{x^2-5x+6}{x^2-9} + 1 \ge 0$

$$\frac{2x+1}{x+3} \ge 0$$

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$x \in (-\infty, -3) \cup \left[-\frac{1}{2}, \infty\right)$ ... .(2)

after taking intersection

$x \in \left[-\frac{1}{2}, \infty\right)$

$x^2 - 3x + 2 > 0$

$x \in (-\infty, 1) \cup (2, \infty)$ $x^2 - 3x + 2 \neq 1$ $x \neq \frac{3 \pm \sqrt{5}}{2}$

after taking intersection of each solution

$[-1/2, 1) \cup (2, \infty) - \left\{\frac{3+\sqrt{5}}{2}, \frac{3-\sqrt{5}}{2}\right\}$

23. Let $x^* y = x^2 + y^3$ and $(x * 1) * 1 = x * (1 * 1)$.

Then a value of $2sin^{-1}\left(\frac{x^4+x^2-2}{x^4+x^2+2}\right)$ is

(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{2}$ (D) $\frac{\pi}{6}$ **[Main, 2022]**

Ans. **(B)**

Sol. $\because (x * 1) * 1 = x * (1 * 1)$

$(x^2 + 1) * 1 = x * (2)$

$(x^2 + 1)^2 + 1 = x^2 + 8$

$x^4 + x^2 - 6 = 0 \Rightarrow (x^2 + 3)(x^2 - 2) = 0$ $x^2 = 2$

$\Rightarrow 2sin^{-1}\left(\frac{x^4+x^2-2}{x^4+x^2+2}\right) = 2sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$

24. The domain of the function $f(x) = sin^{-1}\left(\frac{3x^2+x-1}{(x-1)^2}\right) + cos^{-1}\left(\frac{x-1}{x+1}\right)$ is **[Main, 2021]**

(A) $\left[0, \frac{1}{4}\right]$ (B) $[-2,0] \cup \left[\frac{1}{4}, \frac{1}{2}\right]$ (C) $\left[\frac{1}{4}, \frac{1}{2}\right] \cup \{0\}$ (D) $\left[0, \frac{1}{2}\right]$

Ans. **(C)**

Sol. $f(x) = sin^{-1}\left(\frac{3x^2+x-1}{(x-1)^2}\right) + cos^{-1}\left(\frac{x-1}{x+1}\right)$

$-1 \leq \frac{x-1}{x+1} \leq 1 \Rightarrow 0 \leq x < \infty$ ...(1)

$-1 \leq \frac{3x^2+x-1}{(x-1)^2} \leq 1 \Rightarrow x \in \left[\frac{-1}{4}, \frac{1}{2}\right] \cup \{0\}$ ...(2)

(1) & (2) $\Rightarrow$ Domain $= \left[\frac{1}{4}, \frac{1}{2}\right] \cup \{0\}$