DPP 01

Solution

1. $E_y = 540\sin\ \pi \times 10^4\ (x - ct)\ \ v/m$

$E_0 = 540\ V/m$

So, $B_0 = \frac{E_0}{c} = \frac{540}{3\times10^8} = 180 \times 10^{-8}$

$B_0 = 1.8 \times 10^{-6} = 18 \times 10^{-7}\ T$

2. $\because E = BC$

$C \rightarrow$ speed of light

$E = 2 \times 10^8 \times 3 \times 10^8 = 6$

$E = 6\ V/m$

3. Given,

$B_y = 5 \times 10^{-6}\sin\ 1000\pi(5x - 4 \times 10^8 t)T$

Comparing with general equation,

$B_y = B_0\sin\ (kx - \omega t)$

$B_0 = 5 \times 10^{-6}\ T, k = 5, \omega = 4 \times 10^8 rad/s$

Since, $E_0 = vB_0$ where v is speed of wave.

As speed of wave, $v = \frac{\omega}{k}$

$\therefore v = \frac{4 \times 10^8}{5} = 8 \times 10^7\ m\ s^{-1}$

$\therefore E_0 = 8 \times 10^7 \times 5 \times 10^{-6} = 4 \times 10^2\ V\ m^{-1}$

4. Given, electric field,

$E_y = 900\sin\omega\left(t - \frac{x}{c}\right)$ ... (i)

We know that, $E = E_0 \sin\omega\left(t - \frac{x}{c}\right)$ ... (ii)

On comparing equation (i) and (ii) we have, $E_0 = 900\ V/m$

Maximum magnetic field $B_0 = \frac{E_0}{c}$

$\therefore B_0 = \frac{900}{3 \times 10^8} = 3 \times 10^{-6}\ T$

Electric force, $F_E = qE_0$, and magnetic force,

$F_B = qvB_0$

$\therefore \frac{F_E}{F_B} = \frac{qE_0}{qvB_0} = \frac{E_0}{vB_0}$

$$= \frac{900}{3 \times 10^7 \times 3 \times 10^{-6}} = \frac{10}{1}$$

6. $\mu_2 = \frac{c_{air}}{c_2}; \frac{\mu_2}{\mu_{air}}$

$$= \frac{c}{c_2}; \frac{\sqrt{\mu_{r_2}\varepsilon_{r_2}}}{1} = \frac{c}{c_2}$$

$$\frac{\sqrt{1 \times 9}}{1} = \frac{c}{c_2}; c_2 = \frac{c}{3} = \frac{3 \times 10^8}{3} = 1 \times 10^8 \text{ m/s}$$

7. Given : Area $= 36 \text{ cm}^2$

Average force, $F = 7.2 \times 10^{-9}$ N

Time period, $t = 20 \text{ min} = 20 \times 60 = 1200\text{sec}$

Energy flux, $I = \frac{Fc}{A}; I = 0.06 \text{ W/cm}^2$

8. $\mu_r = 1.61, \ \varepsilon_r = 6.44$

$B = 4.5 \times 10^{-2}$

$$\because \frac{C}{V} = \sqrt{\mu_r \varepsilon_r} = \sqrt{1.61 \times 6.44}$$

$$V = \frac{C}{\sqrt{1.61 \times 6.44}} = \frac{3 \times 10^8}{\sqrt{1.61 \times 6.44}}$$

$V = 9.32 \times 10^7$ m/s.

$E = VB = 9.32 \times 10^7 \times 4.5 \times 10^{-2}$

$E = 4.2 \times 10^6$

9. Here : power of the bulb, $P = 200$ W

Efficiency, $\eta = 3.5\%$, Distance, $r = 4$ m

Let the magnetic field is B_0.

Power by bulb, $P' = 3.5\%$ of P

$$= \frac{3.5}{100} \times 200 = 7 \text{ W}$$

$$\text{Intensity, } I = \frac{P'}{4\pi r^2} = \frac{7}{4\pi(4)^2} = 0.0348 \text{ W/m}^2$$

$$\text{Now, } I = \frac{B_0^2 c}{2\mu_0}; B_0 = \sqrt{\frac{I \times 2\mu_0}{c}} = \sqrt{\frac{0.0348 \times 2 \times 4\pi \times 10^{-7}}{3 \times 10^8}} \Rightarrow B_0 = 1.71 \times 10^{-8} \text{ T}$$

10. Here, $E = 56.5 \sin(\omega)\left(t - \frac{x}{c}\right)$

Intensity, $I = \frac{1}{2}\varepsilon_0 E_0^2 c$

$I = \frac{1}{2} \times 8.85 \times 10^{-12} \times (56.5)^2 \times 3 \times 10^8$

$I = \frac{1}{2} \times 8.85 \times 3 \times 10^{-4} \times 56.5 \times 56.5$

$I = 4.24\ W/m^2$

11. Let the relative permittivity is ε_r

$v = \frac{1}{\sqrt{\mu_r \varepsilon_r}\sqrt{\mu_0 \varepsilon_0}}$; $v = \frac{c}{\sqrt{\mu_r \varepsilon_r}}$

$\mu_r \varepsilon_r = \left(\frac{c}{v}\right)^2 = \left(\frac{3 \times 10^8}{2 \times 10^8}\right)^2$

$= \frac{9}{4}; \varepsilon_r = \frac{9}{4} = 2.25$

12. $\vec{E} = 301.6 \sin(kz - \omega t)(-\hat{a}_x) + 452.4 \sin(kz - \omega t)\hat{a}_y$

$\vec{B} = \frac{301.6}{C} \sin(kz - \omega t)(-\hat{a}_y) + \frac{452.4}{C} \sin(kz - \omega t)(-\hat{a}_x)$

$\vec{H} = \frac{\vec{B}}{\mu_0} = \frac{301.6}{\mu C} \sin(kz - \omega t)(-\hat{a}_y) + \frac{452.4}{\mu C} \sin(kz - \omega t)(-\hat{a}_x)$

$\vec{H} = -0.8 \sin(kz - \omega t)\hat{a}_y - 1.2 \sin(kz - \omega t)\hat{a}_x$

For direction

$\vec{E} \times \vec{B}$ is direction of $\vec{C}$

For first part $\hat{E} = -\hat{\imath}, \hat{B} = ?$

$\hat{E} \times \hat{B} = \hat{k} \Rightarrow \hat{B} = -\hat{\jmath}$

Similarly for second

$\hat{E} = \hat{\jmath}, \hat{B} = ?$

$\hat{E} \times \hat{B} = \hat{k} \Rightarrow \hat{B} = -\hat{\imath}$

13. In a material medium, speed of EM wave is, $v = \frac{1}{\sqrt{\mu\varepsilon}}$ where μ and ε are permeability and permittivity of the medium.

$\therefore\ v = \frac{1}{\sqrt{\mu_0 \mu_r \varepsilon_0 \varepsilon_r}} = \frac{c}{\sqrt{\mu_r \varepsilon_r}}.$

So, statement II is false.

14. Given, $E_0 = 2.25\ V/m$,

$B_0 = 1.5 \times 10^{-8}\ T$

Amplitude of electric field of the electromagnetic wave is given by $E_0 = cB_0$,

$$\text{or } c = \frac{E_0}{B_0} = \frac{2.25}{1.5 \times 10^{-8}} = 1.5 \times 10^8 \text{ m/s}$$

Now, distance travelled by the signal to reach the radar after reflection

$= 2 \times 3 \text{ km} = 6 \text{ km}$

$$\therefore \text{ Time taken, } t = \frac{6 \times 10^3}{1.5 \times 10^8} = 4 \times 10^{-5} \text{ s}$$

15. $B_0 = \frac{E_0}{c} = \frac{60}{3\times10^8} = 2 \times 10^{-7}$ T

$\hat{E} \times \hat{B}$ must be direction of propagation.

So, $\hat{B} \to z -$ axis

$$k = \frac{2\pi}{\lambda} = \frac{\pi}{4} \times 10^3 \text{ m}^{-1}$$

$$E_y = 60\sin\left[\frac{\pi}{4} \times 10^3(x - 3 \times 10^8 t)\right]\hat{j}\text{Vm}^{-1}$$

$$B_z = 2 \times 10^{-7}\sin\left[\frac{\pi}{4} \times 10^3(x - 3 \times 10^8 t)\right]\widehat{kT}$$

16. Electric field,

$E = 20\cos(2 \times 10^{10}t - 200x)$V/m

On comparing with, $E = E_0\cos(\omega t - kx)$

we get, $k = 200$; $\omega = 2 \times 10^{10}$;

$$v = \frac{\omega}{k} = \frac{2 \times 10^{10}}{200} = 10^8 \text{ m s}$$

as, $v = \frac{1}{\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu_0\mu_r\varepsilon_0\varepsilon_r}}$;

Given, $\mu_r = 1$

$$\text{So, } v = \frac{1}{\sqrt{\mu_0\varepsilon_0\varepsilon_r}}; \varepsilon_r = \frac{1}{v^2\mu_0\varepsilon_0}$$

$$= \frac{1}{10^{16} \times 4\pi \times 10^{-7} \times 8.85 \times 10^{-12}} = \frac{1000}{4\pi \times 8.85} = 9; \text{ So, } \varepsilon_r = 9$$

17. $\frac{E_0}{C} = B_0$

$F_{max} = eB_0V$

$$= 1.6 \times 10^{-19} \times \frac{800}{3 \times 10^8} \times 3 \times 10^7 = 12.8 \times 10^{-18} \text{ N}$$

18. $V = \frac{\omega}{K} = \frac{10\times10^{10}}{500} = 2 \times 10^8$

$V = \frac{2C}{3}$

19. $V(t) = 20\sin\ \omega t, v = 50\ Hz,$

$d = 2\ mm, A = 1\ m^2$

$C = \frac{\varepsilon_0 A}{d} = \frac{8.85 \times 10^{-12} \times 1}{2 \times 10^{-3}}\ F$

$X_C = \frac{1}{\omega C} = \frac{1}{2\pi v C}$

$X_C = \frac{1 \times 2 \times 10^{-3}}{2 \times 3.14 \times 50 \times 8.85 \times 10^{-12}}\Omega$

$i_0 = \frac{V_0}{X_C} = \frac{20 \times 2 \times 3.14 \times 50 \times 8.85 \times 10^{-12}}{2 \times 10^{-3}}$

$i_0 = 27.29\mu A$

20. The direction of propagation of wave is

$\hat{n} = \vec{E} \times \vec{B} = E_0\hat{i} \times B_0\hat{k} = -E_0B_0\hat{j}$

So, EM wave propagates along $-\hat{j}$.