

Express each of the complex number given in the Exercises 1 to 10 in the form $a + ib$.

1. $(5i) \left(-\frac{3}{5}i\right)$
2. $i^9 + i^{19}$
3. i^{-39}
4. $3(7 + i7) + i(7 + i7)$
5. $(1 - i) - (-1 + i6)$
6. $\left(\frac{1}{5} + i\frac{2}{5}\right) - \left(4 + i\frac{5}{2}\right)$
7. $\left[\left(\frac{1}{3} + i\frac{7}{3}\right) + \left(4 + i\frac{1}{3}\right)\right] - \left(-\frac{4}{3} + i\right)$
8. $(1 - i)^4$
9. $\left(\frac{1}{3} + 3i\right)^3$
10. $\left(-2 - \frac{1}{3}i\right)^3$
11. Evaluate $\sum_{n=1}^{13} (i^n + i^{n+1})$, where $n \in \mathbf{N}$.
12. Evaluate: $\left[i^{18} + \left(\frac{1}{i}\right)^{25}\right]^3$.

ALGEBRA OF COMPLEX NUMBER

Find the multiplicative inverse of each of the complex numbers given in the Exercises 13 to 15.

13. $4 - 3i$
14. $\sqrt{5} + 3i$
15. $-i$
16. Let $x, y \in \mathbf{R}$, then $x + iy$ is a non real complex number if:
 - (A) $x = 0$
 - (B) $y = 0$
 - (C) $x \neq 0$
 - (D) $y \neq 0$
17. If $a + ib = c + id$, then
 - (A) $a^2 + c^2 = 0$
 - (B) $b^2 + c^2 = 0$
 - (C) $b^2 + d^2 = 0$
 - (D) $a^2 + b^2 = c^2 + d^2$
18. Express the following expression in the form of $a + ib$:

$$\frac{(3 + i\sqrt{5})(3 - i\sqrt{5})}{(\sqrt{3} + \sqrt{2}i) - (\sqrt{3} - i\sqrt{2})}$$
19. Reduce $\left(\frac{1}{1-4i} - \frac{2}{1+i}\right)\left(\frac{3-4i}{5+i}\right)$ to the standard form.
20. If $\left(\frac{1+i}{1-i}\right)^m = 1$, then find the least positive integral value of m ,
21. For a positive integer n , find the value of $(1 - i)^n \left(1 - \frac{1}{i}\right)^n$
22. If $x + iy = \frac{a+ib}{a-ib}$, prove that $x^2 + y^2 = 1$
22. For any two complex numbers z_1 and z_2 , prove that $\operatorname{Re}(z_1 z_2) = \operatorname{Re} z_1 \operatorname{Re} z_2 - \operatorname{Im} z_1 \operatorname{Im} z_2$
24. Let $z_1 = 2 - i, z_2 = -2 + i$. Find
 - (i) $\operatorname{Re} \left(\frac{z_1 z_2}{\bar{z}_1}\right)$, (ii) $\operatorname{Im} \left(\frac{1}{z_1 \bar{z}_1}\right)$.
25. Find the real numbers x and y if $(x - iy)(3 + 5i)$ is the conjugate of $-6 - 24i$.

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26. If $(x + iy)^3 = u + iv$, then show that $\frac{u}{x} + \frac{v}{y} = 4(x^2 - y^2)$.
27. If $\left(\frac{1+i}{1-i}\right)^3 - \left(\frac{1-i}{1+i}\right)^3 = x + iy$, then find (x, y) .
28. If $\frac{(1+i)^2}{2-i} = x + iy$, then find the value of $x + y$.
29. If $\left(\frac{1-i}{1+i}\right)^{100} = a + ib$, then find (a, b) .
30. If $a = \cos \theta + i \sin \theta$, find the value of $\frac{1+a}{1-a}$.
31. If $\frac{(a^2+1)^2}{2a-i} = x + iy$, what is the value of $x^2 + y^2$?
32. The real value of θ for which the expression $\frac{1+i \cos \theta}{1-2i \cos \theta}$ is a real number is:
 (A) $n\pi + \frac{\pi}{4}$ (B) $n\pi + (-1)^n \frac{\pi}{4}$
 (C) $2n\pi \pm \frac{\pi}{2}$ (D) none of these.
33. If $(a + ib)^5 = \alpha + i\beta$ then $(b + ia)^5$ is equal to
 (A) $\beta + i\alpha$ (B) $\alpha - i\beta$ (C) $\beta - i\alpha$ (D) $-\alpha - i\beta$
34. If $z (\neq -1)$ is a complex number such that $\frac{z-1}{z+1}$ is purely imaginary, then $|z|$ is equal to
 (A) 1 (B) 2 (C) 3 (D) 5
35. A value of θ for which $\frac{2+3i \sin \theta}{1-2i \sin \theta}$ is purely imaginary, is : [JEE - MAIN 2016]
 (A) $\frac{\pi}{6}$ (B) $\sin^{-1} \left(\frac{\sqrt{3}}{4}\right)$ (C) $\sin^{-1} \left(\frac{1}{\sqrt{3}}\right)$ (D) $\frac{\pi}{3}$

CONJUGATE

36. Find the conjugate of $\frac{(3-2i)(2+3i)}{(1+2i)(2-i)}$.
37. What is the conjugate of $\frac{2-i}{(1-2i)^2}$?
38. The conjugate of a complex number is $\frac{1}{i-1}$. Then that complex number is- [AIEEE - 2008]
 (A) $\frac{1}{i+1}$ (B) $\frac{-1}{i+1}$ (C) $\frac{1}{i-1}$ (D) $\frac{-1}{i-1}$
39. $\sin x + i \cos 2x$ and $\cos x - i \sin 2x$ are conjugate to each other for:
 (A) $x = n\pi$ (B) $x = \left(n + \frac{1}{2}\right) \frac{\pi}{2}$
 (C) $x = 0$ (D) No value of x
40. If $z = x + iy$ lies in the third quadrant, then $\frac{\bar{z}}{z}$ also lies in the third quadrant if
 (A) $x > y > 0$ (B) $x < y < 0$
 (C) $y < x < 0$ (D) $y > x > 0$

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41. If $x - iy = \sqrt{\frac{a-ib}{c-id}}$ prove that $(x^2 + y^2)^2 = \frac{a^2 + b^2}{c^2 + d^2}$.
42. If $(1 + i)z = (1 - i)\bar{z}$, then show that $z = -i\bar{z}$.
43. The value of $(z + 3)(\bar{z} + 3)$ is equivalent to
 (A) $|z + 3|^2$ (B) $|z - 3|$
 (C) $z^2 + 3$ (D) None of these
44. A real value of x satisfies the equation $\left(\frac{3-4ix}{3+4ix}\right) = \alpha - i\beta$ ($\alpha, \beta \in \mathbf{R}$) if $\alpha^2 + \beta^2 =$
 (A) 1 (B) -1 (C) 2 (D) -2

MODULUS

45. Find the modulus of $\frac{1+i}{1-i} - \frac{1-i}{1+i}$.
46. Find the number of non-zero integral solutions of the equation $|1 - i|^x = 2^x$.
47. If $(a + ib)(c + id)(e + if)(g + ih) = A + iB$, then show that
 $(a^2 + b^2)(c^2 + d^2)(e^2 + f^2)(g^2 + h^2) = A^2 + B^2$
48. If $|z_1| = |z_2|$, is it necessary that $z_1 = z_2$?
49. Find $\left|(1 + i)\frac{(2+i)}{(3+i)}\right|$
50. If z is a complex number, then
 (A) $|z^2| > |z|^2$ (B) $|z^2| = |z|^2$ (C) $|z^2| < |z|^2$ (D) $|z^2| \geq |z|^2$
51. If $z_1 = 2 - i, z_2 = 1 + i$, find $\left|\frac{z_1 + z_2 + 1}{z_1 - z_2 + 1}\right|$.
52. If $a + ib = \frac{(x+i)^2}{2x^2+1}$, prove that $a^2 + b^2 = \frac{(x^2+1)^2}{(2x^2+1)^2}$.
53. If α and β are different complex numbers with $|\beta| = 1$, then find $\left|\frac{\beta-\alpha}{1-\bar{\alpha}\beta}\right|$
54. If $\frac{z-1}{z+1}$ is a purely imaginary number ($z \neq -1$), then find the value of $|z|$.
55. If $|z_1| = 1$ ($z_1 \neq -1$) and $z_2 = \frac{z_1-1}{z_1+1}$, then show that the real part of z_2 is zero.
56. If $|z_1| = |z_2| = \dots = |z_n| = 1$, then show that $|z_1 + z_2 + z_3 + \dots + z_n| = \left|\frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3} + \dots + \frac{1}{z_n}\right|$.
57. Where does z lie, if $\left|\frac{z-5i}{z+5i}\right| = 1$.
58. The complex number z which satisfies the condition $\left|\frac{i+z}{i-z}\right| = 1$ lies on
 (A) circle $x^2 + y^2 = 1$ (B) the x-axis
 (C) the y-axis (D) the line $x + y = 1$.

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59. If $f(z) = \frac{7-z}{1-z^2}$, where $z = 1 + 2i$, then $|f(z)|$ is
 (A) $\frac{|z|}{2}$ (B) $|z|$ (C) $2|z|$ (D) none of these.
60. If $|z - 2| \geq |z - 4|$ then correct statement is-
 (A) $R(z) \geq 3$ (B) $R(z) \leq 3$ (C) $R(z) \geq 2$ (D) $R(z) \leq 2$
61. If $|z_1 - 1| < 1$, $|z_2 - 2| < 2$, $|z_3 - 3| < 3$ then $|z_1 + z_2 + z_3|$
 (A) is less than 6 (B) is more than 3
 (C) is less than 12 (D) lies between 6 and 12
62. If $iz^3 + z^2 - z + i = 0$, then $|z|$ equals
 (A) 4 (B) 3 (C) 2 (D) 1
63. If $|z_1| = 2$, $|z_2| = 3$, $|z_3| = 4$ and $|2z_1 + 3z_2 + 4z_3| = 4$ then absolute value of $8z_2z_3 + 27z_3z_1 + 64z_1z_2$ equals
 (A) 24 (B) 48 (C) 72 (D) 96
64. If z is a complex number such that $|z| \geq 2$, then the minimum value of $\left|z + \frac{1}{z}\right|$:
 [JEE-MAIN 2014]
 (A) is equal to $\frac{5}{2}$ (B) lies in the interval $(1, 2)$
 (C) is strictly greater than $\frac{5}{2}$ (D) is strictly greater than $\frac{3}{2}$ but less than $\frac{5}{2}$

ARGUMENT

65. Find principal argument of $(1 + i\sqrt{3})^2$.
66. The value of $\arg(x)$ when $x < 0$ is:
 (A) 0 (B) $\frac{\pi}{2}$ (C) π (D) none of these
67. The argument of the complex number $\sin \frac{6\pi}{5} + i \left(1 + \cos \frac{6\pi}{5}\right)$ is.
 (A) $\frac{6\pi}{5}$ (B) $\frac{5\pi}{6}$ (C) $\frac{9\pi}{10}$ (D) $\frac{2\pi}{5}$
68. z_1 and z_2 are two complex numbers such that $|z_1| = |z_2|$ and $\arg(z_1) + \arg(z_2) = \pi$, then show that $z_1 = -\bar{z}_2$.
69. If z_1, z_2 and z_3, z_4 are two pairs of conjugate complex numbers, then find $\arg\left(\frac{z_1}{z_4}\right) + \arg\left(\frac{z_2}{z_3}\right)$.
70. If for complex numbers z_1 and z_2 , $\arg(z_1) - \arg(z_2) = 0$, then show that $|z_1 - z_2| = |z_1| - |z_2|$
71. Fill in the blanks of the following
 (i) For any two complex numbers z_1, z_2 and any real numbers a, b ,
 $|az_1 - bz_2|^2 + |bz_1 + az_2|^2 = \dots$

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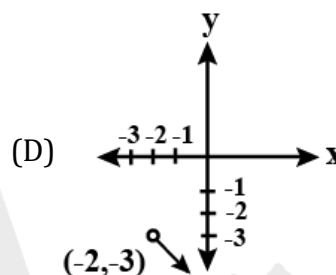
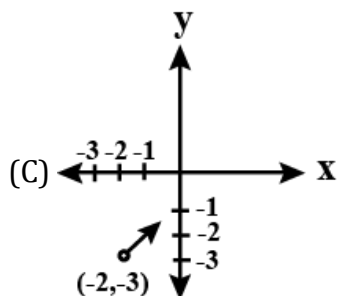
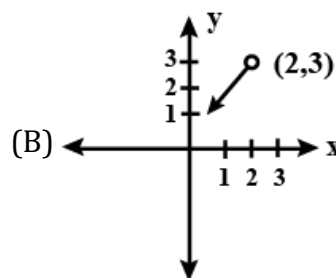
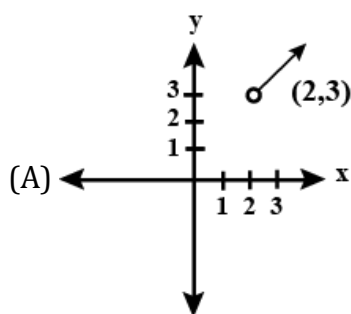
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- (ii) The value of $\sqrt{-25} \times \sqrt{-9}$ is
- (iii) The number $\frac{(1-i)^3}{1-i^3}$ is equal to
- (iv) The sum of the series $i + i^2 + i^3 + \dots$ upto 1000 terms is
- (v) Multiplicative inverse of $1 + i$ is
- (vi) If z_1 and z_2 are complex numbers such that $z_1 + z_2$ is a real number, then $z_2 = \dots$
- (vii) $\arg(z) + \arg(\bar{z})$ ($\bar{z} \neq 0$) is
- (viii) If $|z + 4| \leq 3$, then the greatest and least values of $|z + 1|$ are and
72. Find z if $|z| = 4$ and $\arg(z) = \frac{5\pi}{6}$.
73. Which of the following is correct for any two complex numbers z_1 and z_2 ?
- (A) $|z_1 z_2| = |z_1| |z_2|$ (B) $\arg(z_1 z_2) = \arg(z_1) \cdot \arg(z_2)$
- (C) $|z_1 + z_2| = |z_1| + |z_2|$ (D) $|z_1 + z_2| \geq |z_1| - |z_2|$
74. $|z_1 + z_2| = |z_1| + |z_2|$ is possible if
- (A) $z_2 = \bar{z}_1$ (B) $z_2 = \frac{1}{z_1}$
- (C) $\arg(z_1) = \arg(z_2)$ (D) $|z_1| = |z_2|$
75. Let z and w are two non zero complex number such that $|z| = |w|$, and $\arg(z) + \arg(w) = \pi$ then-
- (A) $z = w$ (B) $z = \bar{w}$ (C) $\bar{z} = \bar{w}$ (D) $z = -\bar{w}$
76. Let z, w be complex numbers such that $\bar{z} + i\bar{w} = 0$ and $\arg zw = \pi$. Then $\arg z$ equals-
- (A) $\pi/4$ (B) $\pi/2$ (C) $3\pi/4$ (D) $5\pi/4$
77. If $z_1 = -3 + 5i$; $z_2 = -5 - 3i$ and z is a complex number lying on the line segment joining z_1 & z_2 , then $\arg(z)$ can be
- (A) $-\frac{3\pi}{4}$ (B) $-\frac{\pi}{4}$ (C) $\frac{\pi}{6}$ (D) $\frac{5\pi}{6}$
78. If z and ω are two non- zero complex numbers such that $|z\omega| = 1$, and $\arg(z) - \arg(\omega) = \frac{\pi}{2}$, then $\bar{z}\omega$ is equal to-
- (A) -1 (B) 1 (C) -1 (D) i
79. If z_1 and z_2 are two non-zero complex numbers such that $|z_1 + z_2| = |z_1| + |z_2|$, then $\arg z_1 - \arg z_2$ is equal to -
- (A) $\frac{\pi}{2}$ (B) $-\pi$ (C) 0 (D) $-\frac{\pi}{2}$

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80. If $\text{Arg}(z - 2 - 3i) = \frac{\pi}{4}$, then the locus of z is



81. If z is a complex number of unit modulus and argument θ , then $\arg\left(\frac{1+z}{1+\bar{z}}\right)$ equals

[JEE-MAIN 2013]

- (A) θ (B) $\pi - \theta$ (C) $-\theta$ (D) $\frac{\pi}{2} - \theta$

COMPLEX NUMBER EQUATION

Short Answer Type

82. Solve the equation $|z| = z + 1 + 2i$.
83. The number of solutions of the system of equations $\text{Re}(z^2) = 0$, $|z| = 2$ is
(A) 4 (B) 3 (C) 2 (D) 1
84. Let $z (\neq 2)$ be a complex number such that $\log_{1/2} |z - 2| > \log_{1/2} |z|$, then
(A) $\text{Re}(z) > 1$ (B) $\text{Im}(z) > 1$ (C) $\text{Re}(z) = 1$ (D) $\text{Im}(z) = 1$

SQUARE ROOT OF A COMPLEX NUMBER

85. In one root of the quadratic equation $(1 + i)x^2 - (7 + 3i)x + (6 + 8i) = 0$ is $4 - 3i$, then the other root must be
(A) $1 + i$ (B) $4 + 3i$ (C) $1 - i$ (D) $4i + 3$

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ANSWER KEY

- | | | |
|---|---|-------------------------------------|
| 1. $3 + i0$ | 2. $0 + i0$ | 3. $0 + i1$ |
| 4. $14 + 28i$ | 5. $2 - 7i$ | 6. $-\frac{19}{5} - \frac{21i}{10}$ |
| 7. $\frac{17}{3} + i\frac{5}{3}$ | 8. $-4 + i0$ | 9. $-\frac{242}{27} - 26i$ |
| 10. $\frac{-22}{3} - i\frac{107}{27}$ | 11. $-1 + i$ | 12. $2 - 2i$ |
| 13. $\frac{4}{25} + i\frac{3}{25}$ | 14. $\frac{\sqrt{5}}{14} - i\frac{3}{14}$ | 15. $0 + i1$ |
| 16. (D) | 17. (D) | 18. $0 - i\frac{7\sqrt{2}}{2}$ |
| 19. $\frac{307+599i}{442}$ | 20. (4) | 21. (2^n) |
| 22. (1) | 24. (i) $\frac{-2}{5}$, (ii) 0 | 25. $x = 3, y = -3$ |
| 27. $(0, -2)$ | 28. $\frac{2}{5}$ | 29. $(1, 0)$ |
| 30. $i \cot \frac{\theta}{2}$ | 31. $\frac{(a^2+1)^4}{4a^2+1}$ | 32. (C) |
| 33. (A) | 34. (A) | 35. (C) |
| 37. $\frac{-2}{25} - i\frac{11}{25}$ | 38. (B) | 39. (D) |
| 40. (B) | 42. $\frac{3}{2} - 2i$ | 43. (A) |
| 44. (A) | 45. (2) | 46. (0) |
| 48. (No) | 49. (1) | 50. (B) |
| 51. $(\sqrt{2})$ | 53. (1) | 54. (1) |
| 57. (Real axis) | 58. (B) | 59. (A) |
| 60. (A) | 61. (C) | 62. (D) |
| 63. (D) | 64. (B) | 65. $(\frac{2\pi}{3})$ |
| 66. (C) | 67. (C) | 69. (0) |
| 71. (i) $(a^2 + b^2)(z_1 ^2 + z_2 ^2)$ | (ii) -15 | |
| (iii) -2 (iv) 0 (v) $\frac{1}{2} - \frac{i}{2}$ | (vi) \bar{z}_1 (vii) 0 | |
| (viii) 6 and 0 | (ix) a circle | |
| 72. $-2\sqrt{3} + 2i$ | 73. (A) | 74. (C) |
| 75. (D) | 76. (C) | 77. (D) |
| 78. (A) | 79. (C) | 80. (A) |
| 81. (A) | 82. $\frac{3}{2} - 2i$ | 83. (A) |
| 84. (A) | 85. (A) | |