



DPP - 03

Solution

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1. If $A + B = 225^\circ$, then find the value of $\frac{\cot A}{1+\cot A} \times \frac{\cot B}{1+\cot B}$.

Ans. $\frac{1}{2}$

$$\text{Sol. } \frac{\cot A}{1+\cot A} \times \frac{\cot B}{1+\cot B} = \frac{1}{(1+\tan A)(1+\tan B)} = \frac{1}{\tan A + \tan B + 1 + \tan A \tan B}$$

$$\text{Now, } \tan(A + B) = \tan 225^\circ \Rightarrow \tan A + \tan B = 1 - \tan A \tan B$$

Hence, from Eq. (i)

$$\text{R.H.S. } = \frac{1}{1 + \tan A \tan B + 1 + \tan A \tan B} = \frac{1}{2}$$

2. If $\tan A - \tan B = x$ and $\cot B - \cot A = y$, then find the value of $\cot(A - B)$.

Ans. $\frac{1}{x} + \frac{1}{y}$

$$\begin{aligned} \text{Sol. } \cot(A - B) &= \frac{1}{\tan(A - B)} \\ &= \frac{1 + \tan A \tan B}{\tan A - \tan B} \\ &= \frac{1}{\tan A - \tan B} + \frac{\tan A \tan B}{\tan A - \tan B} \\ &= \frac{1}{\tan A - \tan B} + \frac{1}{\cot B - \cot A} = \frac{1}{x} + \frac{1}{y} \end{aligned}$$

3. Prove that $\frac{\tan^2 2\theta - \tan^2 \theta}{1 - \tan^2 2\theta \tan^2 \theta} = \tan 3\theta \tan \theta$

$$\begin{aligned} \text{Sol. } \frac{\tan^2 2\theta - \tan^2 \theta}{1 - \tan^2 2\theta \tan^2 \theta} &= \left(\frac{\tan 2\theta - \tan \theta}{1 + \tan 2\theta \tan \theta} \right) \left(\frac{\tan 2\theta + \tan \theta}{1 - \tan 2\theta \tan \theta} \right) \\ &= \tan(2\theta - \theta) \tan(2\theta + \theta) = \tan 3\theta \tan \theta \end{aligned}$$

4. If $A + B = 45^\circ$, show that $(1 + \tan A)(1 + \tan B) = 2$.

Sol. Since $A + B = 45^\circ$, $\tan(A + B) = \tan 45^\circ = 1$

$$\Rightarrow \frac{\tan A + \tan B}{1 - \tan A \tan B} = 1 \quad \Rightarrow \tan A + \tan B = 1 - \tan A \tan B$$

$$\Rightarrow \tan A + \tan B + \tan A \tan B = 1$$

$$\Rightarrow \tan A(1 + \tan B) + \tan B = 1$$

$$\Rightarrow \tan A(1 + \tan B) + 1 + \tan B = 1 + 1$$

$$\Rightarrow (1 + \tan B)(\tan A + 1) = 2$$

$$\Rightarrow (1 + \tan A)(1 + \tan B) = 2$$



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5. If $\tan A = 1/2, \tan B = 1/3$, then prove that $\cos 2A = \sin 2B$.

Sol. $\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B} = \frac{\frac{1}{2} + \frac{1}{3}}{1 - \frac{1}{2} \times \frac{1}{3}} = 1$

$\Rightarrow A + B = 45^\circ$, therefore $2A = 90^\circ - 2B \Rightarrow \cos 2A = \sin 2B$

6. Find the maximum value of $1 + \sin\left(\frac{\pi}{4} + \theta\right) + 2 \sin\left(\frac{\pi}{4} - \theta\right)$ for all real values of θ .

Ans. $1 + \sqrt{5}$.

Sol. $1 + \sin\left(\frac{\pi}{4} + \theta\right) + 2 \sin\left(\frac{\pi}{4} - \theta\right) = 1 + \sin\left(\frac{\pi}{4} + \theta\right) + 2 \cos\left(\frac{\pi}{2} - \left(\frac{\pi}{4} - \theta\right)\right)$
 $= 1 + \sin\left(\frac{\pi}{4} + \theta\right) + 2 \cos\left(\frac{\pi}{4} + \theta\right)$

Hence, the maximum value is $1 + \sqrt{5}$.

7. Find the value of $\cos\frac{\pi}{12}\left(\sin\frac{5\pi}{12} + \cos\frac{\pi}{4}\right) + \sin\frac{\pi}{12}\left(\cos\frac{5\pi}{12} - \sin\frac{\pi}{4}\right)$

Ans. $\frac{3}{2}$

- Sol.** $\cos 15^\circ(\sin 75^\circ + \cos 45^\circ) + \sin 15^\circ(\cos 75^\circ - \sin 45^\circ) = \sin(75^\circ + 15^\circ) + \cos(45^\circ + 15^\circ)$

$$= 1 + \frac{1}{2} = \frac{3}{2}$$

8. If $\cos(\alpha + \beta) + \sin(\alpha - \beta) = 0$ and $\tan \beta \neq 1$, then find the value of $\tan \alpha$.

Ans. -1

- Sol.** $\cos \alpha \cos \beta - \sin \alpha \sin \beta + \sin \alpha \cos \beta - \cos \alpha \sin \beta = 0$

$$\Rightarrow \cos \alpha (\cos \beta - \sin \beta) + \sin \alpha (\cos \beta - \sin \beta) = 0$$

$$\Rightarrow (\cos \beta - \sin \beta)(\cos \alpha + \sin \alpha) = 0$$

If $\cos \beta - \sin \beta = 0 \Rightarrow \tan \beta = 1$, which is not possible

$$\therefore \sin \alpha + \cos \alpha = 0$$

$$\therefore \tan \alpha = -1$$

9. If $\sin A + \cos 2A = 1/2$ and $\cos A + \sin 2A = 1/3$, then find the value of $\sin 3A$.

Ans. -59/72

- Sol.** Squaring and adding,

$$1 + 1 + 2 \sin A \cos 2A + 2 \cos A \sin 2A = (1/4) + (1/9) = (13/36) \Rightarrow \sin 3A = -59/72$$

10. If $\sin x + \sin y + \sin z = 0 = \cos x + \cos y + \cos z$, then find the value of expression

$$\cos(\theta - x) + \cos(\theta - y) + \cos(\theta - z).$$

Ans. 0

- Sol.** $\cos \theta (\sum \cos x) + \sin \theta (\sum \sin x) = 0$