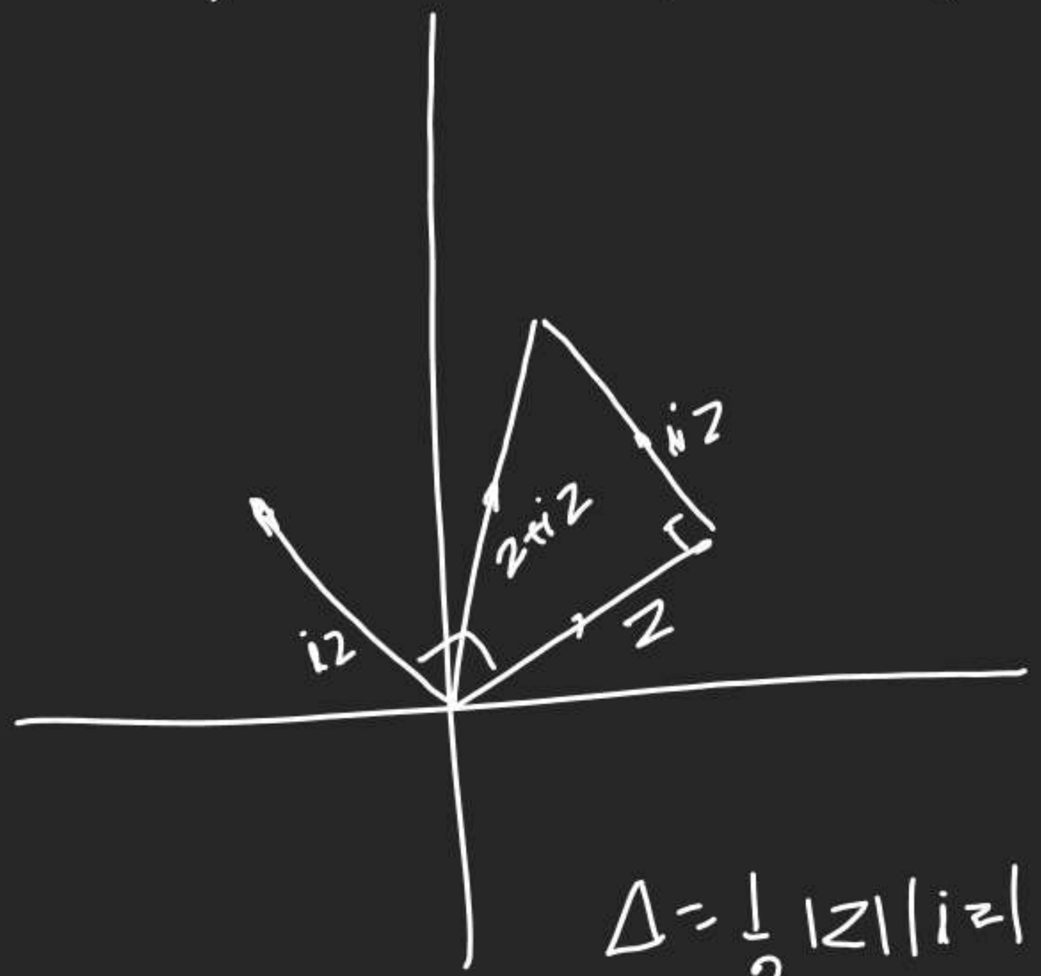
Nishant Jindal

Q Area of Δ formed by $z, iz, z+iz$.

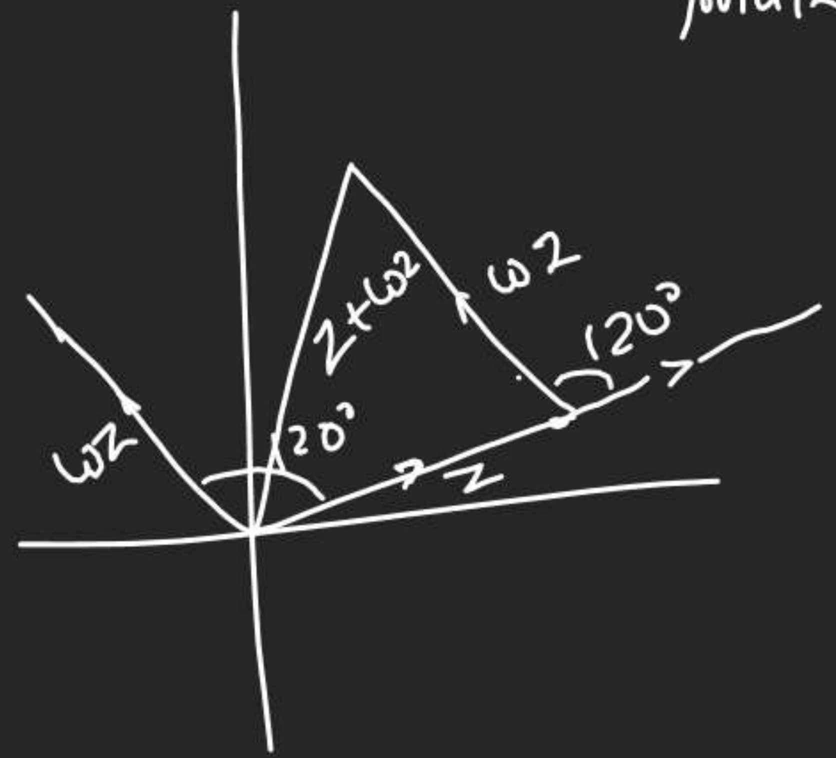


$$\Delta = \frac{1}{2}|z||iz|$$

$$= \frac{1}{2}|z||i||z|$$

$$= \frac{|z|^2}{2}$$

Q Area of Δ formed by $z, \omega z, z+\omega z$ is $16\sqrt{3}$. find $|z|$



$$\Delta = \frac{1}{2}|z||\omega z|\sin\frac{2\pi}{3}$$

$$= \frac{|z|^2|\omega|}{2} \cdot \frac{\sqrt{3}}{2} = 16\sqrt{3}$$

$$= |z|^2 = 64$$

$$|z| = 8$$

Q If a, b, c are distinct C.N. such that $\frac{a}{b} = \frac{b}{c} = \frac{c}{a} = \lambda$ find Sum of values of λ.

Sol

$$\frac{a}{b} \times \frac{b}{c} \times \frac{c}{a} = \lambda \cdot \lambda \cdot \lambda$$

$$\lambda^3 = 1 \begin{cases} 1 \\ \omega \\ \omega^2 \end{cases}$$

Roots of λ are ω & ω^2 only

1 can not be Root as a, b, c distinct

$\Rightarrow$ SOR $= \omega + \omega^2 = -1$

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Q If $z^2 = \bar{z} \cdot 2^{1-|z|}$ find $|z|$

C.N = C.N → Arg(C.N) = Arg(C.N), |C.N| = |C.N|

$|z|^2 = |\bar{z}| \cdot 2^{1-|z|}$

$|z|^2 = |z| \cdot 2^{1-|z|}$

Directly $|z| = 1$

By observation

Q Find all non-zero C.N. Satisfying $z^2 + z|z| + |z^2| = 0$

$x^2 + x + 1 = 0 < \begin{matrix}\omega \\ \omega^2\end{matrix}$

$z^2 + z|z| + |z|^2 = 0$

$\left(\frac{z}{|z|}\right)^2 + \left(\frac{z}{|z|}\right) + 1 = 0 < \begin{matrix}\omega \\ \omega^2\end{matrix}$

$\frac{z}{|z|} = \omega \mid \frac{z}{|z|} = \omega^2$

$z = |z|\omega$ or $z = |z|\omega^2$

Q If $(a+\omega)^{-1} + (b+\omega)^{-1} + (c+\omega)^{-1} + (d+\omega)^{-1} = 2\omega^{-1}$

& $(a+\omega^2)^{-1} + (b+\omega^2)^{-1} + (c+\omega^2)^{-1} + (d+\omega^2)^{-1} = 2\omega^{-2}$

$a, b, c, d \in R$, $\omega^3 = 1$

P.T (1) $\sum abc = 2$ (2) $\sum a = 2\prod a$

(3) $(a+1)^{-1} + (b+1)^{-1} + (c+1)^{-1} + (d+1)^{-1} = 2$

$2x^4 + (\sum a)x^3 - 2x - \sum a = 0 \rightarrow x = 1, \omega, \omega^2$

$(x^3 - 1)(2x + \sum a) = 0$

(3) also Proved

(1) $\frac{1}{a+x} + \frac{1}{b+x} + \frac{1}{c+x} + \frac{1}{d+x} = \frac{2}{x}$

$(x^3 + \quad) + (x^3 \quad) + (x^3 \quad) + (x^3 \quad) = \frac{2}{x}$

$(a+x)(b+x)(c+x)(d+x)$

$2x^4 + (\sum a)x^3 - (\sum abc)x - 2\prod a = 0$

$\omega \quad \omega^2 \quad \alpha \quad \beta$

$2\omega^4 + (\sum a)\omega^3 - (\sum abc)\omega - 2\prod a = 0 \rightarrow (1)$

$2\omega^8 + (\sum a)\omega^6 - (\sum abc)\omega^2 - 2\prod a = 0 \rightarrow (2)$

$\Rightarrow 2\omega + (\sum a) - (\sum abc)\omega - 2\prod a = 0 \rightarrow (1)$

$\Rightarrow 2\omega^2 + (\sum a) - (\sum abc)\omega^2 - 2\prod a = 0 \rightarrow (2)$

(1) − (2)

$2(\omega - \omega^2) + (\sum abc)(\omega^2 - \omega) = 0$

$\boxed{\sum abc = 2}$

$\cancel{2\omega} + \sum a - \cancel{2\omega} - 2\prod a = 0$

$\boxed{\sum a = 2\prod a}$

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$$(x-1)(x-2)(x-3)$$

$$= x^3 - x^2(1+2+3) + x(1\cdot2+2\cdot3+3\cdot1) - (1\cdot2\cdot3) = 0$$

$$= x^3 - 6x^2 + 11x - 6 = 0$$

$$(x+1)(x+2)(x+3) = x^3 + x^2(\Sigma 1) + x(\Sigma 1\cdot2) + \Pi 1$$

$$\frac{(x^3 + x^2(\Sigma a) + x(\Sigma ab) + \Pi a) + (x^3 \qquad) + (x^3 \qquad) + (x^3 \qquad)}{4x^3 + x^3(\Sigma a + \Sigma b + \Sigma c + \Sigma d) + (\qquad) x^2 + (\Pi \qquad) x} = \frac{2}{x}$$

$=$

Nishant Jindal

Q $z=(-1)^{1/3}$ $|z|=1$, $Arg(z)=\pi$

$$=\left(1\cdot e^{i\left(\frac{\pi+2k\pi}{3}\right)}\right)$$

$k=0,1,2$

$$z=e^{i\frac{\pi}{3}},\ e^{i\pi},\ e^{i\frac{5\pi}{3}}$$

$$=\left(\frac{1}{2},\frac{\sqrt{3}}{2}\right),(-1,0),\left(\frac{1}{2},-\frac{\sqrt{3}}{2}\right)$$

Q Find Roots

$$\bar{z}=i\cdot z^2$$

$$|\bar{z}|=|i\cdot z^2|$$

$$|z|=|z|^2 \Rightarrow |z|^2-|z|=0$$

$$|z|(|z|-1)=0$$

$|z|=0$ or $|z|=1$

$z=0$

$$|z|^2=1$$

$$z\cdot\bar{z}=1$$

$\bar{z}=\frac{1}{z}$ But $\bar{z}=iz^2$

$$\frac{1}{z}=iz^2$$

$$\Rightarrow z^3=\frac{1}{i}=-i$$

$z=(-i)^{1/3}$ DMT

$$=\left(1\cdot e^{-i(\pi/2)}\right)^{1/3}$$

$$=1^{1/3}\cdot e^{i\left(\frac{-\frac{\pi}{2}+2k\pi}{3}\right)}$$

$k=0,1,2.$

Find Roots

Q $z^5=\bar{z}$

$$|z|^5=|\bar{z}|=|z|$$

$$|z|^5-|z|=0$$

$$|z|(|z|^4-1)=0$$

$|z|=0$ & $z=0$

$|z|^4=1$

$\Rightarrow |z|=1$

$\Rightarrow |z|^2=1$

$z\bar{z}=1 \Rightarrow \bar{z}=\frac{1}{z}$. But $\bar{z}=z^5$

$$\Rightarrow \frac{1}{z}=z^5 \Rightarrow z^6=1$$

$z=(1)^{1/6}$ DMT.

Nishant Jindal

(1) $1+z+z^2+z^3+z^4+z^5=0$

(1) find Roots with -ve Real Part $\cos\theta$

(2) find $|z|$ (3) Prod of Root?

GP.

$$\frac{1\cdot(z^6-1)}{(z-1)}=0 \qquad (z=1)$$

$$\Rightarrow z^6=1 \qquad |z|=1$$

$$z=(1)^{1/6}=\left(e^{i\left(\frac{0+2K\pi}{6}\right)}\right) \quad K=0,1,2,3,4,5$$

e^{i0} (X), $e^{i\frac{\pi}{3}}$ (1) (+), $e^{i\frac{2\pi}{3}}$ 2nd R.P = -ve, $e^{i\frac{3\pi}{3}}$ (π) RP-ve, $e^{i\frac{4\pi}{3}}$ (3) RP-ve, $e^{i\frac{5\pi}{3}}$ 4th $\cos\theta$

n^{th} Root of Unity.

$$z^n=1 \qquad z=(1)^{1/n}=e^{i\left(\frac{2K\pi}{n}\right)} \qquad K=0,1,2,\dots,(n-1)$$

$$e^{i\frac{4\pi}{n}}=\left(e^{i\frac{2\pi}{n}}\right)^2$$

$$\boxed{z^n-1=0}$$

z^{n-1} ka coeff $=0$

$$e^{i0},\ e^{i\frac{2\pi}{n}},\ e^{i\frac{4\pi}{n}},\ e^{i\frac{6\pi}{n}}\ \dots\ e^{\frac{2i(n-1)\pi}{n}}$$

$$1,\ \alpha,\ \alpha^2,\ \alpha^3\ \dots\ \alpha^{n-1}$$

$$SOR=1+\alpha+\alpha^2+\alpha^3\dots+\alpha^{n-1}=0$$

$$(\cos 0+i\sin 0)+\left(\cos\frac{2\pi}{n}+i\sin\frac{2\pi}{n}\right)+\left(\cos\frac{4\pi}{n}+i\sin\frac{4\pi}{n}\right)\dots\left(\cos\frac{2(n-1)\pi}{n}+i\sin\frac{2(n-1)\pi}{n}\right)=0$$

$$\boxed{\sum_{K=0}^{(n-1)}\left(\cos\frac{2K\pi}{n}+i\sin\frac{2K\pi}{n}\right)=0}$$

$$\sum_{K=1}^{n-1}\cos\frac{2K\pi}{n}+i\sum_{K=1}^{n-1}\sin\frac{2K\pi}{n}=-1+i0$$