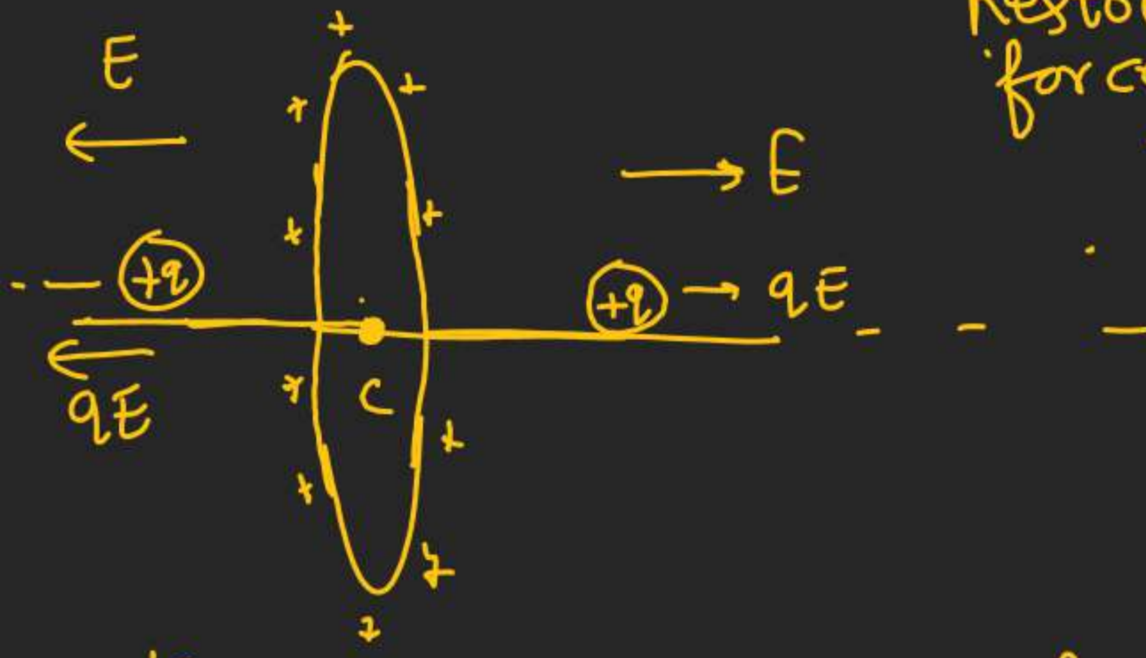
ELECTRIC FIELD

$$E = \frac{KQx}{(x^2+R^2)^{3/2}} \qquad x = 0 \qquad E = 0$$

❖ **Discuss the Motion of $+q$ and $-q$ when it is displaced slightly from center of ring.**



$+q \rightarrow$ center is a point of unstable equilibrium

Restoring force

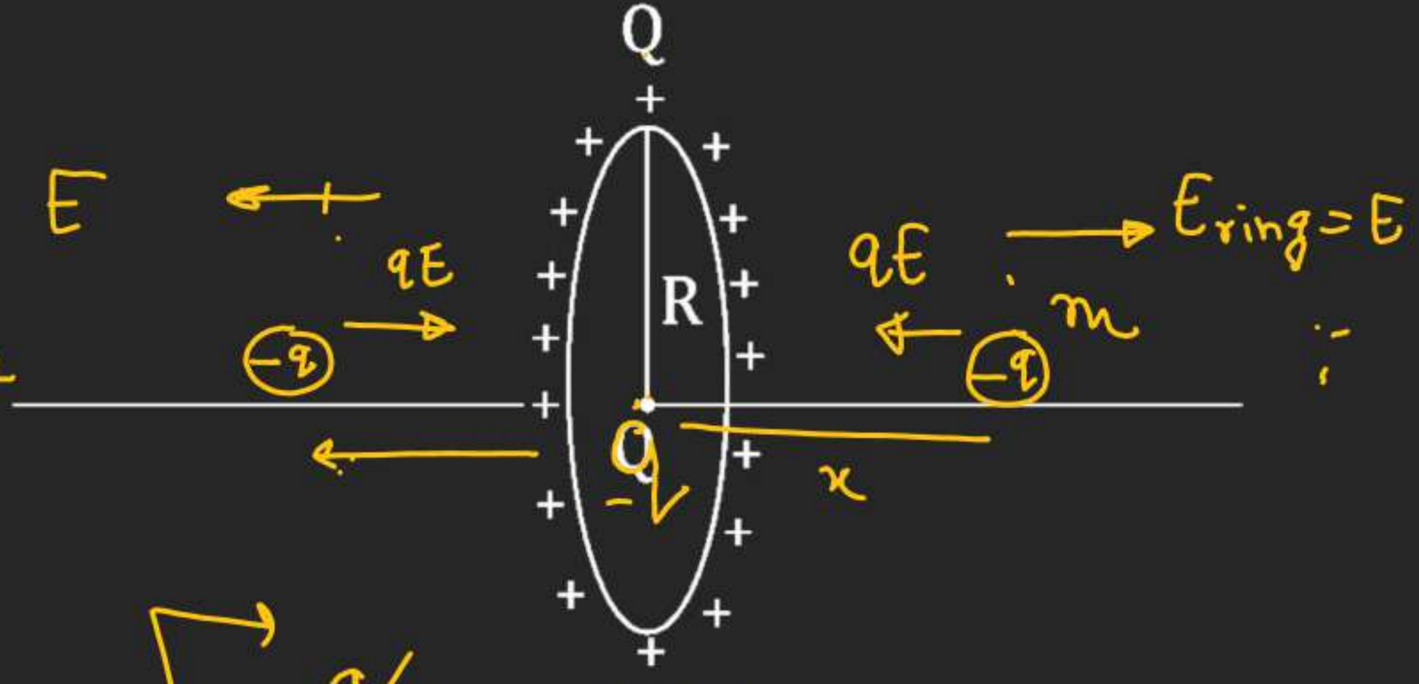
$$F_r = -qE_{ring}$$

$$F_r = -q\frac{KQx}{(x^2+R^2)^{3/2}}$$

If $\boxed{x << R}$

$$F_r = -\frac{KQqx}{R^3\left(1+\underbrace{\frac{x^2}{R^2}}_{\to 0}\right)^{3/2}}$$

$$F_r = -\frac{KQq}{R^3}x$$



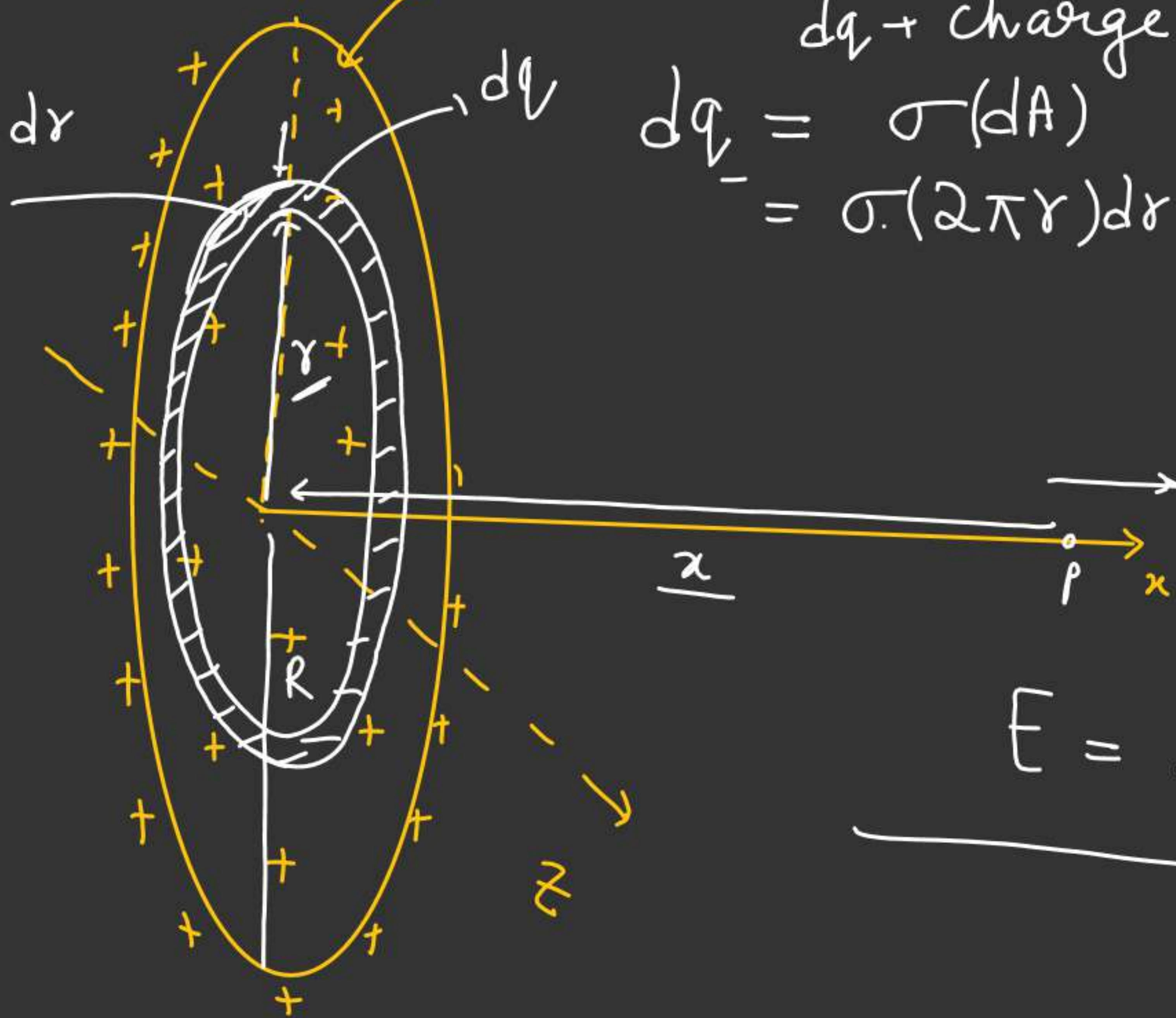
$$a = -\frac{KQq}{mR^3}x$$

$$a = -\omega^2 x$$

$$\omega = \sqrt{\frac{KQq}{mR^3}}, \quad T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{mR^3}{KQq}}$$

(x) Electric field due to Uniformly Charged disc at its axis

$\sigma =$ constant



$dE \rightarrow$ Electric field due to ring having radius 'r'

$dq \rightarrow$ charge on the ring.

$$dq = \sigma(dA)$$
$$= \sigma(2\pi r)dr$$

$$dE = \frac{K\,dq\,x}{(x^2+r^2)^{3/2}}$$

$$dE = \frac{1\,\sigma(\cancel{2\pi} r)dr\,x}{\cancel{4\pi}\varepsilon_0\,(x^2+r^2)^{3/2}}$$

$$\int_0^E dE = \int_0^R \frac{\sigma}{2\varepsilon_0}\frac{x\,r\,dr}{(x^2+r^2)^{3/2}}$$

$$E = \frac{\sigma x}{2\varepsilon_0}\int_0^R \frac{r\,dr}{(x^2+r^2)^{3/2}}$$

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$$E = \frac{\sigma x}{2\varepsilon_0}\int_0^R \frac{r\,dr}{(x^2+r^2)^{3/2}}$$

put $x^2+r^2 = t$

Differentiating both side w.r.t r.

$$\frac{d}{dr}(x^2+r^2) = \frac{dt}{dr}$$

$$2r = \frac{dt}{dr}$$

$$\boxed{r\,dr = \frac{dt}{2}}$$

$$E = \frac{\sigma x}{4\varepsilon_0}\int_0^R \frac{dt}{(t)^{3/2}}$$

$$E = \frac{\sigma x}{4\varepsilon_0}\int_0^R t^{-3/2}\cdot dt$$

$$\int x^n dx = \frac{x^{n+1}}{n+1}$$

$$E = \frac{\sigma x}{4\varepsilon_0}\left[\frac{t^{-\frac{3}{2}+1}}{\left(-\frac{3}{2}+1\right)}\right]_0^R$$

$$E = \frac{2\times\sigma x}{4\varepsilon_0}\left[\frac{-1}{\sqrt{t}}\right]_0^R$$

$$E = -\frac{\sigma x}{2\varepsilon_0}\left[\frac{1}{\sqrt{t}}\right]_0^R$$

$$E = \frac{-\sigma x}{2\varepsilon_0}\left[\frac{1}{\sqrt{x^2+r^2}}\right]_0^R$$

$$E = \frac{-\sigma x}{2\varepsilon_0}\left[\frac{1}{\sqrt{x^2+R^2}} - \frac{1}{x}\right]$$

$$\boxed{E = \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{x}{\sqrt{x^2+R^2}}\right]}$$

$x = 0$

$$\boxed{E_{\text{center of disc}} = \frac{\sigma}{2\varepsilon_0}}$$

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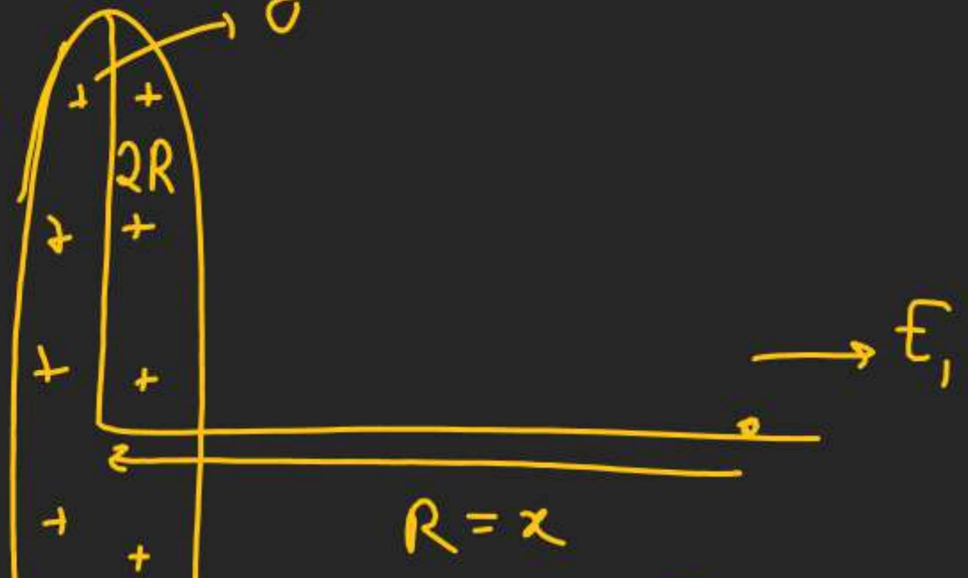
ELECTRIC FIELD

❖ **Electric field of an annular disc**

Superposition principle

Original body

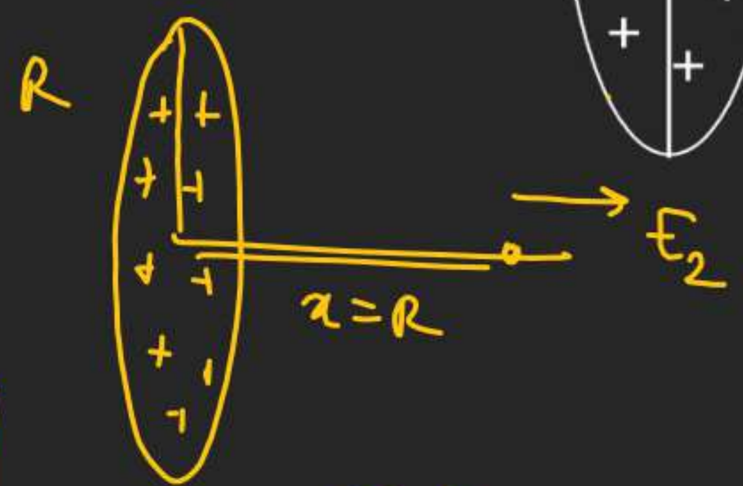


$$E_1 = \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{R}{\sqrt{R^2+4R^2}}\right]$$

$$E_1 = \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{R}{\sqrt{5}R}\right]$$

$$E_1 = \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{1}{\sqrt{5}}\right]$$

Cut body

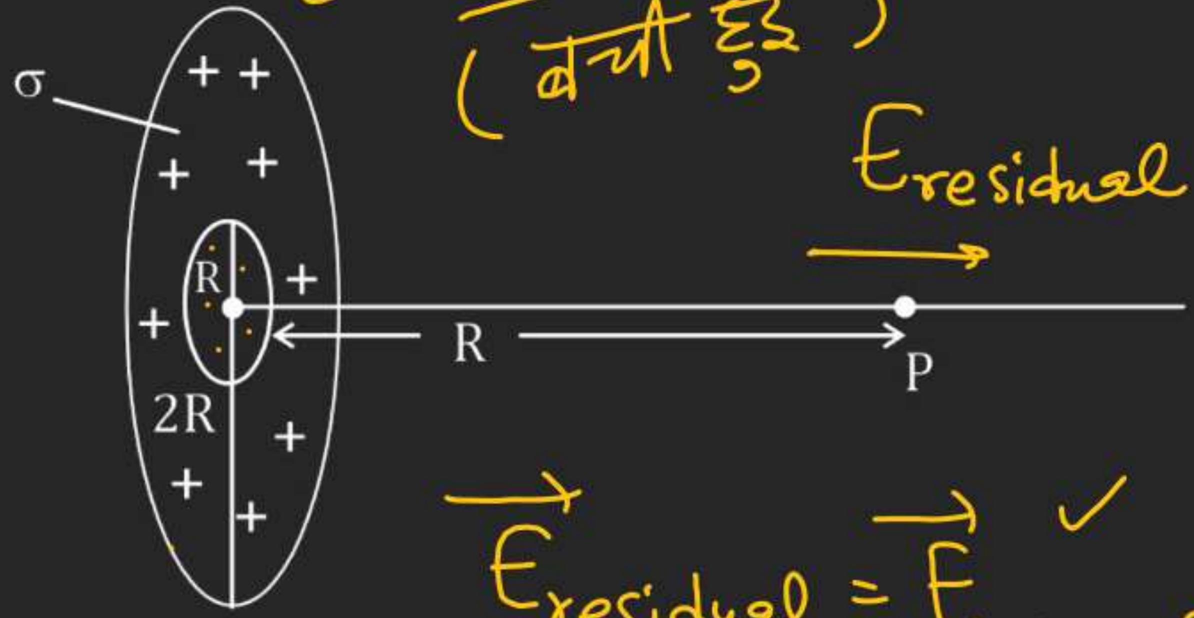


$$E_2 = \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{R}{\sqrt{R^2+R^2}}\right]$$

$$E_2 = \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{1}{\sqrt{2}}\right]$$

Residual body

(बची हुई)



$$\vec{E}_{residual} = \vec{E}_{original} - \vec{E}_{cut}$$

$$= (E_1 - E_2)\,\hat{i}$$

$$= \frac{\sigma}{2\varepsilon_0}\left[\left(1-\frac{1}{\sqrt{5}}\right) - \left(1-\frac{1}{\sqrt{2}}\right)\right]\hat{i}$$

$$= \frac{\sigma}{2\varepsilon_0}\left[\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{5}}\right]\hat{i}$$

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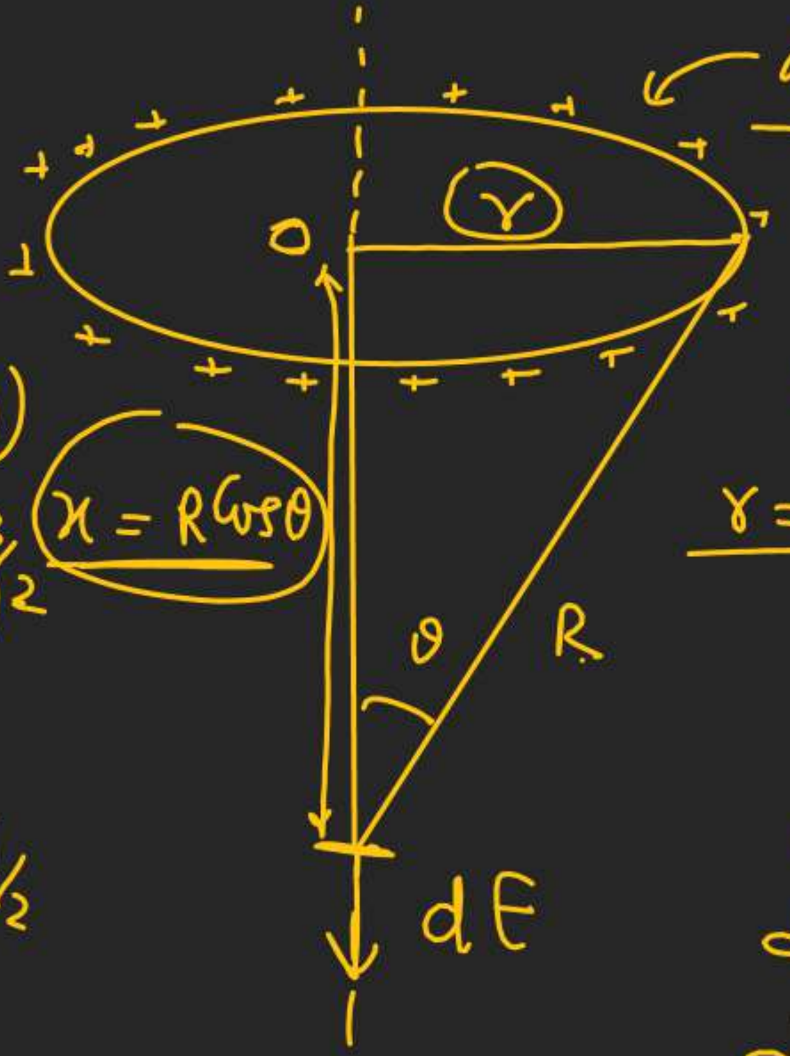
ELECTRIC FIELD

❖ **Electric field of a Uniformly charge Conducting hemispherical shell at its Center:-**

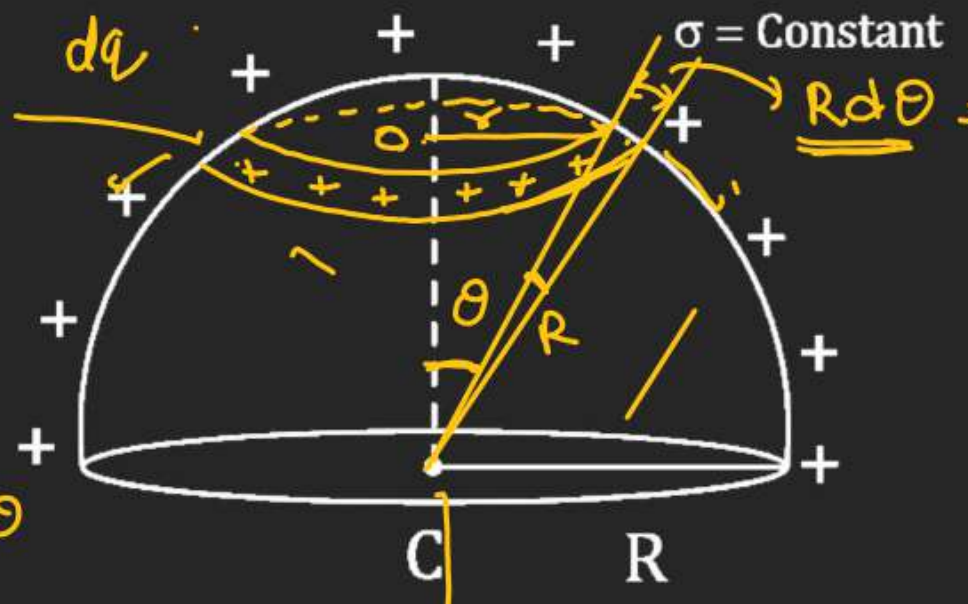
$$dE = \frac{K\,dq\,x}{(x^2+r^2)^{3/2}}$$

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{(\sigma\cdot 2\pi R^2\sin\theta\cdot d\theta)(R\cos\theta)}{(R^2\cos^2\theta+R^2\sin^2\theta)^{3/2}}$$

$$x = R\cos\theta$$



$$r = R\sin\theta$$



$$R\,d\theta = dr$$

$$dE = \frac{\sigma}{2\varepsilon_0}\frac{\cancel{R^3}\sin\theta\cdot\cos\theta\cdot d\theta}{\cancel{R^3}(\sin^2\theta+\cos^2\theta)^{3/2}}$$

$$dE = \frac{\sigma}{4\varepsilon_0}\,2\sin\theta\cdot\cos\theta\cdot d\theta$$

$$\int_0^{E} dE = \frac{\sigma}{4\varepsilon_0}\int_0^{\pi/2}\sin 2\theta\cdot d\theta.$$

$dq = \sigma\,dA$

$dE \rightarrow$ due to ring

$$dq = \sigma(2\pi r)\,dr$$

$$dq = \sigma(2\pi)(R\sin\theta)\,R\,d\theta$$

$$dq = \sigma\,2\pi R^2\sin\theta\cdot d\theta$$

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$$E = \frac{\sigma}{4\varepsilon_0}\int_0^{\pi/2} \sin 2\theta \cdot d\theta$$

$$E = \frac{\sigma}{4\varepsilon_0}\left[\frac{-\cos 2\theta}{2}\right]_0^{\pi/2}$$

$$E = \frac{\sigma}{8\varepsilon_0}\left[-\cos 2\left(\frac{\pi}{2}\right) - \left(-\cos 0\right)\right]$$

$$E = \frac{\sigma}{8\varepsilon_0}\left[1+1\right]$$

$$\boxed{E = \frac{\sigma}{4\varepsilon_0}}$$

$k =$ constant

$$\int \sin kx \, dx = \left[\frac{-\cos kx}{k}\right]$$

$$\int \cos kx \cdot dx = \left[\frac{\sin kx}{k}\right]$$

R

$$\sigma = \left(\frac{Q}{2\pi R^2}\right)$$

$$E = \frac{Q}{2\pi R^2} \times \frac{1}{4\varepsilon_0}$$

$$\boxed{E = \frac{Q}{8\pi\varepsilon_0 R^2}}$$

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ELECTRIC FIELD

H.W $\rho = \rho_0 r$ → radial distance

❖ **Electric field of a uniformly charge non Conducting hemisphere at its center**

$dV = $ (Area of differential element) thickness

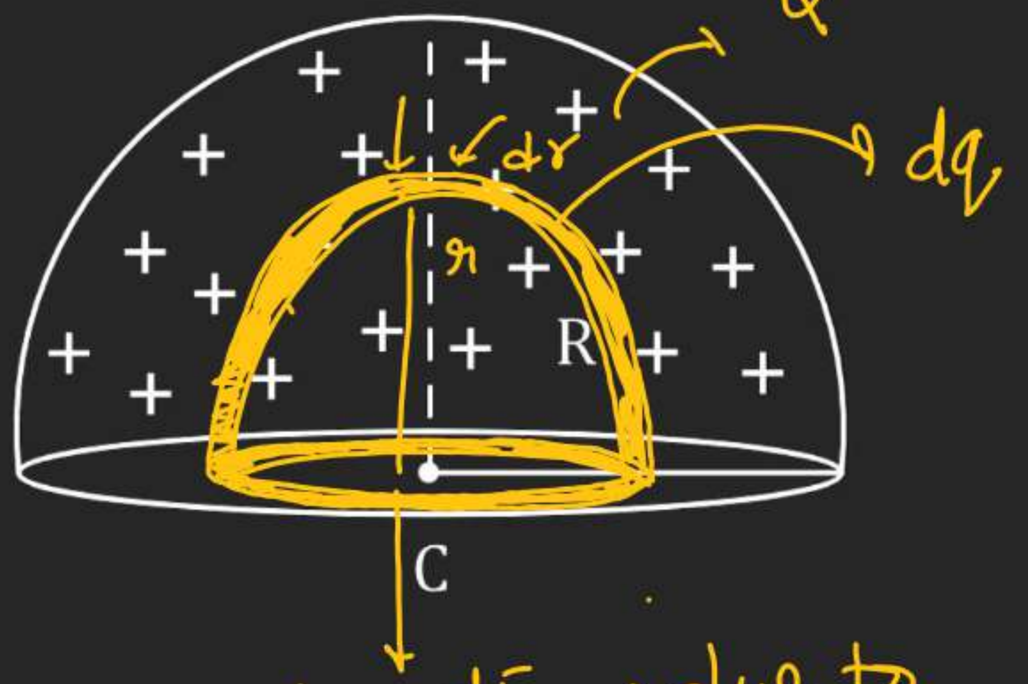
$dq = \rho \, dV$

Differential Volume of hemispherical shell

$dV = (2\pi r^2)\,dr$

$\rho = \dfrac{Q}{\frac{2}{3}\pi R^3}$

$\rho = \left(\dfrac{3Q}{2\pi R^3}\right)$



$dq = (\rho \, 2\pi r^2 \, dr)$

$dE = \dfrac{dq}{8\pi\varepsilon_0 r^2} = \dfrac{\rho \, 2\pi r^2 \, dr}{8\pi\varepsilon_0 r^2}$

dE → due to hemispherical shell of radius 'r' & thickness dr.

$$\int_0^E dE = \frac{\rho}{4\varepsilon_0}\int_0^R dr \Rightarrow \boxed{E = \frac{\rho R}{4\varepsilon_0}} \Rightarrow E = \frac{3Q}{8\pi\varepsilon_0 R^3}$$

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ELECTROSTATICS

Due to ring on its axis :-

$$E = \frac{KQx}{(x^2+R^2)^{3/2}}$$

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{D\frac{d}{dx}(N) - N\frac{d}{dx}(D)}{D^2}$$

For E to be maximum or minimum

$$\frac{dE}{dx} = 0$$

$$\frac{d}{dx}\left[\frac{KQx}{(x^2+R^2)^{3/2}}\right] \Rightarrow KQ\frac{d}{dx}\left[\frac{x}{(x^2+R^2)^{3/2}}\right] = 0$$

$$\frac{(x^2+R^2)^{3/2}\cdot\frac{d}{dx}(x) - x\frac{d}{dx}(\underbrace{x^2+R^2}_{t})^{3/2}}{(x^2+R^2)^3} = 0$$

$$(x^2+R^2)^{3/2} - \frac{3x}{\cancel{2}}(x^2+R^2)^{1/2} \times \cancel{2}x = 0$$

$$(x^2+R^2)^{3/2} = 3(x^2+R^2)^{1/2}x^2$$

$$(x^2+R^2) = 3x^2$$

$$R^2 = 2x^2$$

$$\boxed{x = \pm\frac{R}{\sqrt{2}}}$$

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ELECTROSTATICS

$$E = \frac{KQx}{(x^2+R^2)^{3/2}}$$

odd function

$$\boxed{f(-x) = -f(x)}$$

It is Symmetrical about origin

At $x = 0$, $E = 0$

At $x \to \infty$

$$E = \frac{KQ\cancel{x}}{x^{\cancel{3}}\left[1+\frac{R^2}{x^2}\right]^{3/2}}$$

$$E = \frac{KQ}{x^2\left[1+\frac{R^2}{x^2}\right]^{3/2}}$$

$KQ \to \frac{1}{\infty}$, $\frac{R^2}{x^2} \to \frac{1}{\infty}$

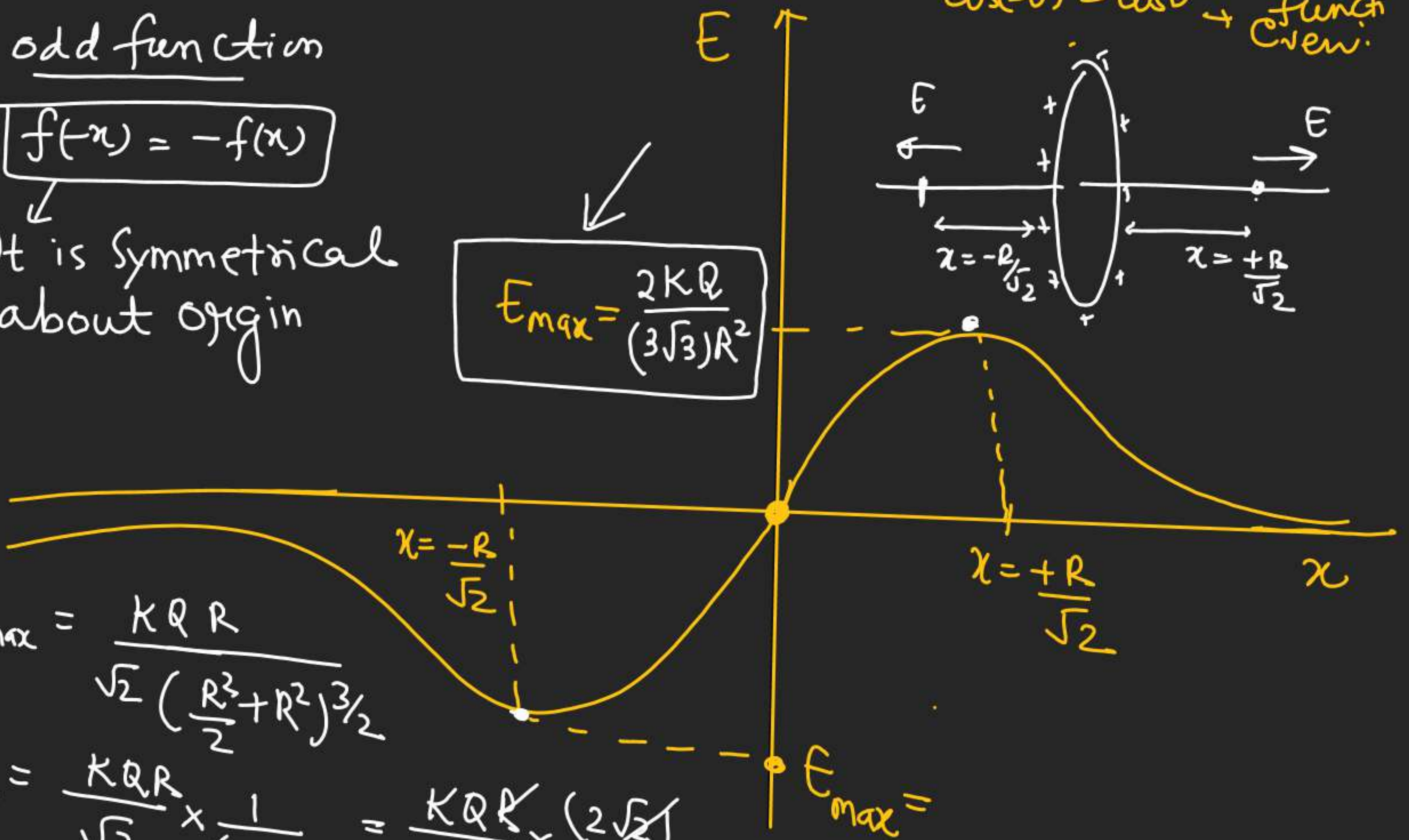
$x \to \infty$

$E \to 0$

$\sin(-\theta) = -\sin\theta \to$ odd function

$\cos(-\theta) = \cos\theta \to$ even.

$$\boxed{E_{max} = \frac{2KQ}{(3\sqrt{3})R^2}}$$



$$E_{max} = \frac{KQR}{\sqrt{2}\left(\frac{R^2}{2}+R^2\right)^{3/2}}$$

$$E_{max} = \frac{KQR}{\sqrt{2}} \times \frac{1}{\left(\frac{3R^2}{2}\right)^{3/2}} = \frac{KQ\cancel{R}}{\cancel{\sqrt{2}}} \times \frac{(2\cancel{\sqrt{2}})}{R^{\cancel{3}}(3\sqrt{3})} = \frac{2KQ}{(3\sqrt{3})R^2}$$