Nishant Jindal

Q $\vec{x} = \hat{i}+\hat{j}$, $y = \hat{i}-\hat{j}$, $\vec{z} = \hat{i}+\hat{j}+\hat{k}$

find Proj of $\vec{x}\times\vec{y}$ on $\vec{z}$?

$$\text{Proj} = \frac{(x\times y)\cdot\vec{z}}{|z|} = \frac{[x\ y\ z]}{|z|}$$

$$[x\ y\ z] = \begin{vmatrix} 1 & 1 & 0 \\ 1 & -1 & 0 \\ 1 & 1 & 1 \end{vmatrix}$$

$[x\ y\ z]$

$= [y\ z\ x]$

$= [z\ x\ y]$

$|z| = \sqrt{3}$

DY

Q 2 $$\frac{\vec{a}\cdot(\vec{b}\times\vec{c})}{(\vec{c}\times\vec{a})\cdot\vec{b}} + \frac{\vec{b}\cdot(\vec{a}\times\vec{c})}{\vec{c}\cdot(\vec{a}\times\vec{b})} = ?$$

$$\frac{[a\,b\,c]}{[b\,c\,a]} + \frac{[b\,a\,c]}{[c\,a\,b]}$$

$$\frac{\cancel{[a\,b\,c]}}{\cancel{[a\,b\,c]}} + \frac{-\cancel{[a\,b\,c]}}{\cancel{[a\,b\,c]}} = 1 - 1$$

$= 0$

$[a\,b\,c] = [b\,c\,a] = [c\,a\,b]$

Q 3 $\alpha = \vec{a}\times\vec{b}$, $\beta = \vec{b}\times\vec{c}$, $\gamma = \vec{c}\times\vec{a}$

$\vec{a} = \hat{i}+2\hat{j}$, $\vec{b} = 2\hat{j}+3\hat{k}$, $\vec{c} = 3\hat{i}+\hat{k}$

find vol. of ∥ pipe having coturminous Edges α, β, γ ?

$V = [\alpha\ \beta\ \gamma] = [a\times b\quad b\times c\quad c\times a]$

$= [a\,b\,c]\times[b\,c\,a] = [a\,b\,c]^2$

$$= \begin{vmatrix} 1 & 2 & 0 \\ 0 & 2 & 3 \\ 3 & 0 & 1 \end{vmatrix}^2$$

$= 1(2) - 2(-9) + 0 = (20)^2$

$= 400$

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Q $\vec{p}=\dfrac{\vec{b}\times\vec{c}}{[abc]}$, $\vec{q}=\dfrac{\vec{c}\times\vec{a}}{[abc]}$, $\vec{r}=\dfrac{a\times b}{[abc]}$

find $(\vec{a}+\vec{b})\cdot\vec{p}+(\vec{b}+\vec{c})\cdot\vec{q}+(\vec{c}+\vec{a})\cdot\vec{r}=?$

$\vec{a}\cdot\vec{p}+\vec{b}\cdot\vec{p}+\vec{b}\,\vec{q}+\vec{c}\cdot\vec{q}+\vec{c}\,\vec{r}+\vec{a}\,\vec{r}$

$\vec{a}\cdot\dfrac{(b\times c)}{[abc]}+\cancel{\dfrac{b\cdot(b\times c)}{[abc]}}+\dfrac{b\cdot((\times a)}{[abc]}+\cancel{\dfrac{c\cdot((\times a)}{[abc]}}+ \text{- - -}$

$\dfrac{[abc]}{[abc]}+\dfrac{[abc]}{[abc]}+\dfrac{[abc]}{[abc]}=3$

Q $\vec{a}=a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$, $\vec{b}=b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$

$\vec{c}=c_1\hat{i}+c_2\hat{j}+c_3\hat{k}$

Value of $[n\vec{a}+\vec{b} \quad n\vec{b}+\vec{c} \quad n\vec{c}+\vec{a}]$

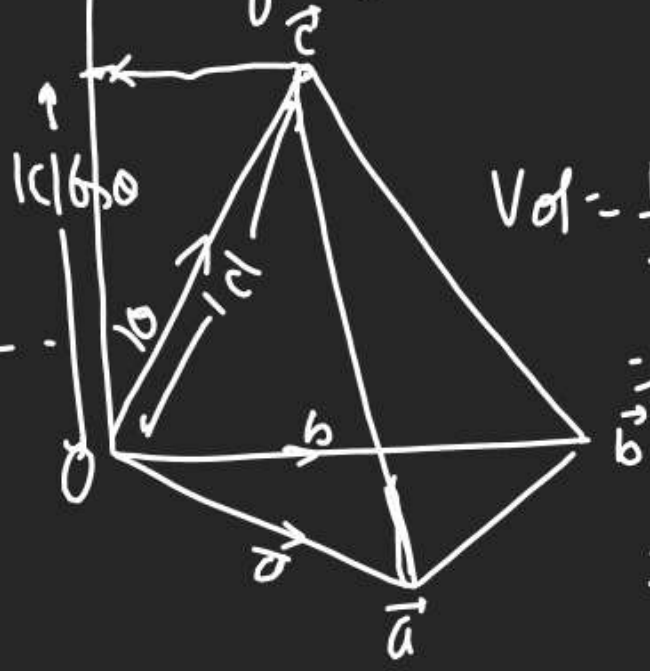
$[n\vec{a} \quad n\vec{b} \quad n\vec{c}]+[b\ c\ a]$

$n^3[abc]+[abc]$

$(n^3+1)[abc]$

Volume of ‖pipe $=[a\ b\ c]$

Vol. of tetrahedron



Vol $=\frac{1}{3}$ (base Area) × ht

$=\frac{1}{3}\times\frac{1}{2}(\vec{a}\times\vec{b})\cdot|c|\cos\theta$

$=\frac{1}{6}|a||b||c|\sin\phi\cos\theta$

$=\frac{1}{6}[a\ b\ c]$

(3) Volume of Prism

$V=\frac{1}{2}(\vec{a}\times\vec{b})\cdot\vec{m}$

Q If $L_1: \vec{r}=\vec{a}+\lambda\vec{b}$

$L_2: \vec{r}=\vec{c}+\mu\vec{d}$

are Skew lines find S.D $=$?

$$S.D=\left|\frac{(FP-FP)\cdot(DR\times DR)}{|DR\times DR|}\right|$$

$$=\frac{(\vec{a}-\vec{c})\cdot(\vec{b}\times\vec{d})}{|b\times d|}$$

$$=\frac{|[a\ b\ d]-[c\ b\ d]|}{|b\times d|}$$

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Q Find Cond^n if L_1 & L_2 are Intersecting.

$L_1: \vec{r} = \vec{a} + \lambda\vec{b}$

$L_2: \vec{r} = \vec{c} + \mu\vec{d}$

If L_1 & L_2 are Intersecting then S.D. = 0

$$\frac{|[a\,b\,d] - [c\,b\,d]|}{|b \times a|} = 0$$

$[a\,b\,d] = [c\,b\,d]$

Q P.T. Lines

$\vec{r} \times \vec{a} = b \times \vec{a}$ &

$\vec{r} \times \vec{b} = \vec{a} \times \vec{b}$ are Intersecting?

L_1 $(\vec{r} - \vec{b}) \times \vec{a} = 0$

$\vec{r} - \vec{b} \parallel \vec{a}$

$\vec{r} - \vec{b} = \lambda\vec{a}$

$L_1: \underline{\vec{r} = \vec{b} + \lambda\vec{a}}$

$L_2: \vec{r} \times \vec{b} = \vec{a} \times \vec{b}$

$(\vec{r} - \vec{a}) \times \vec{b} = 0$

$\vec{r} - \vec{a} = \mu\vec{b}$

L_2 $\underline{\vec{r} = \vec{a} + \mu\vec{b}}$

here $\Rightarrow [\vec{b}\,\vec{a}\,\vec{b}] = [\vec{a}\,\vec{a}\,\vec{b}]$

$0 = 0$

Lines Interset

Show that:

Q Any Vector $\vec{r}$ can be written as a Linear Combination of $\vec{r} = \frac{[r\,b\,c]}{[a\,b\,c]}\vec{a} + \frac{[r\,c\,a]}{[a\,b\,c]}\vec{b} + \frac{[r\,a\,b]}{[a\,b\,c]}\vec{c}$

(1) let $\vec{r} = x\vec{a} + y\vec{b} + z\vec{c}$ $\left.\right\}$ $\cdot(\vec{b} \times \vec{c})$, $\cdot(\vec{c} \times \vec{a})$, $\cdot(\vec{a} \times \vec{b})$

A) $[r\,b\,c] = x[a\,b\,c] + 0 + 0 \Rightarrow x = \frac{[r\,b\,c]}{[a\,b\,c]}$

B) $[r\,c\,a] = 0 + y[b\,c\,a] + 0 \Rightarrow y = \frac{[r\,c\,a]}{[a\,b\,c]}$

C) $[r\,a\,b] = 0 + 0 + z[c\,a\,b] \Rightarrow z = \frac{[r\,a\,b]}{[a\,b\,c]}$

$\therefore \vec{r} = \frac{[r\,b\,c]}{[a\,b\,c]}\vec{a} + \frac{[r\,c\,a]}{[a\,b\,c]}\vec{b} + \frac{[r\,a\,b]}{[a\,b\,c]}\vec{c}$

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Q Express $(\vec{b}\times\vec{c})$ in terms of $\vec{a},\vec{b},\vec{c}$?

$$(\vec{b}\times\vec{c}) = x\vec{a}+y\vec{b}+z\vec{c}\ \Big\}\ \begin{matrix}\cdot(b\times c)\\ \cdot(c\times a)\\ \cdot(a\times b)\end{matrix}$$

A) $|b\times c|^2 = x[a\,b\,c]+0+0 \longrightarrow x = \dfrac{|b\times c|^2}{[a\,b\,c]}$

B) $(b\times c)\cdot(c\times a) = 0+y[a\,b\,c]+0 \qquad y = \dfrac{(b\times c)\cdot(c\times a)}{[a\,b\,c]}$

(C) $(a\times b)\cdot(b\times c) = 0+0+z[c\,a\,b]$

$$z = \frac{(a\times b)\cdot(b\times c)}{[a\,b\,c]}$$

$$b\times c = \frac{|b\times c|^2}{[a\,b\,c]}\vec{a} + \frac{(b\times c)\cdot(c\times a)}{[a\,b\,c]}\cdot\vec{b} + \frac{(a\times b)\cdot(b\times c)}{[a\,b\,c]}\vec{c}$$

Q Express $\vec{a}$ in terms of $\vec{b}\times\vec{c},\ \vec{c}\times\vec{a},\ \vec{a}\times\vec{b}$.

$$\vec{a} = x(b\times c)+y(c\times a)+z(a\times b)\ \Big\}\ \begin{matrix}\cdot\vec{a}\\ \cdot\vec{b}\\ \cdot\vec{c}\end{matrix}$$

$|a|^2 = x[a\,b\,c]+0+0$

$\vec{a}\cdot\vec{b} = 0+y[b\,c\,a]+0$

$\vec{c}\cdot\vec{a} = 0+0+z[c\,a\,b]$

$$\vec{a} = \frac{|a|^2}{[a\,b\,c]}(\vec{b}\times c) + \frac{(\vec{a}\cdot b)}{[a\,b\,c]}(c\times a) + \frac{(c\cdot a)}{[a\,b\,c]}\cdot(a\times b)$$

Q $\vec{U} = \langle 2,-1,1\rangle$ $\vec{V} = \langle 1,1,1\rangle$ $\hat{W}$ find Max Value of $[U\ V\ W] = ?$

$\Rightarrow$ Volume $= \hat{W}\cdot(\vec{U}\times\vec{V})$

$$\vec{U}\times\vec{V} = \begin{vmatrix} i & j & k\\ 2 & -1 & 1\\ 1 & 1 & 1\end{vmatrix}$$

$= \langle -2,-1,3\rangle$

$Vol_{max} = |\hat{W}|\,|U\times V|\cdot\cos\theta$

$= 1\times\sqrt{4+1+9}\cdot\cos\theta_{Max}$

$= \sqrt{14}\times 1$

$= \sqrt{14}.$

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Q Let $\hat{U} = U_1\hat{i}+U_2\hat{j}+U_3\hat{k}$ be a Unit vector in R^3 & $\hat{W} = \frac{1}{\sqrt{6}}(\hat{i}+\hat{j}+2\hat{k})$. Given that there exist a vector $\vec{V}$ in R^3 Such that $|u\times v|=1$ & $\hat{W}\cdot(\hat{U}\times\hat{V})=1$ then WOTF is true.

A) There is exactly one choice for $\vec{V}$

B) ,, are ∞ many choices for $\vec{V}$ ✓✓

C) If $\hat{u}$ lies in XY Plane then $|U_1|=|U_2|$

D) If $\hat{U}$ lies in XZ plane then $2|U_1|=|U_3|$

(1) $\hat{W}\cdot(\vec{U}\times\vec{V})=1$

$|\hat{W}|\cdot|U\times V|\cdot\cos\theta=1$ — $\hat{W}$ & $(\vec{U}\times\vec{V})$

$1\times 1\times\cos\theta=1 \Rightarrow \theta=0$

$\hat{W} \parallel^{el} \vec{U}\times\vec{V}$

$\hat{W}$ $\perp^r$ to $\hat{U}$ & $\hat{W}$ is $\perp^r$ to $\vec{V}$

$\hat{W}\cdot\hat{U}=0$ $\quad\exists\ \infty$ many positions for $\vec{V}$

XY Plane में $\vec{U}$ है

$\Rightarrow \hat{U} = U_1\hat{i}+U_2\hat{j}$

$\hat{W} = \frac{1}{\sqrt{6}}(\hat{i}+\hat{j}+2\hat{k})$

$\hat{W}\cdot\hat{U} = \frac{1}{\sqrt{6}}U_1+\frac{U_2}{\sqrt{6}}+0$

$0 = U_1+U_2 \Rightarrow U_1=-U_2$

$\Rightarrow |U_1|=|U_2|$

(D) $\hat{U}$ lies in XZ Plane.

$\hat{U} = U_1\hat{i}+U_3\hat{k}$

$\hat{W}\cdot\hat{U} = \frac{U_1}{\sqrt{6}}+0+\frac{2U_3}{\sqrt{6}}$

$0 = U_1+2U_3 \Rightarrow U_1=-2U_3$

$\Rightarrow |U_1| = 2|U_3|$

VTP = Vector Triple Product

1) Cross Prod betn 3 Vector is Vector triple Prod.

> But it is Basically vector Prod of a vector & vector Product of remaining 2 vector

Ex:- $\vec{a}\times(\vec{b}\times\vec{c})$, $\vec{b}\times(\vec{c}\times\vec{a})$, $\vec{c}\times(\vec{a}\times\vec{b})$

(2) $\vec{a}\times(\vec{b}\times\vec{c})$ or $(\vec{b}\times\vec{c})\times\vec{a}$ both are V.T.P.

But $\vec{a}\times(\vec{b}\times\vec{c}) = -(\vec{b}\times\vec{c})\times\vec{a}$

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(3) $\vec{a}\times(\vec{b}\times\vec{c})$ is a vector in the Plane of $\vec{b}$ & $\vec{c}$ & $\perp^r$ to $\vec{a}$

Qs $\rightarrow$ find a Vector Coplanar to $\vec{a}$ & $\vec{b}$ & $\perp^r$ to $\vec{a}$

Such Vector $= \vec{a}\times(\vec{a}\times\vec{b})$

Qs is Like $\rightarrow$ Vector Coplanar to $\vec{b}$ & $\vec{c}$ & $\perp^r$ to $\vec{m}$

$$= \underset{1}{\vec{m}}\times(\underset{2}{\vec{b}}\times\underset{3}{\vec{c}})$$

$$= (\vec{m}\cdot\vec{c})\vec{b} - (\vec{m}\cdot\vec{b})\cdot\vec{c}$$

(4) $\underset{1}{\vec{a}}\times(\underset{2}{\vec{b}}\times\underset{3}{\vec{c}}) = 132 - 123$

$$= (\vec{a}\cdot\vec{c})\vec{b} - (\vec{a}\cdot\vec{b})\vec{c}$$

(5) $(\vec{b}\times\vec{c})\times\vec{a} = -\ \vec{a}\times(\vec{b}\times\vec{c})$

$$= -[(a\cdot c)\vec{b} - (a\cdot b)\vec{c})$$

$$\therefore (\vec{a}\cdot\vec{b})\vec{c} - (a\cdot c)\vec{b}$$

(6) $\hat{i}\times(\hat{j}\times\hat{k}) = (\hat{i}\cdot\hat{k})\hat{j} - (\hat{i}\cdot\hat{j})\hat{k}$

$$= 0 - 0 = 0$$

Q. $\vec{a} = \langle 1,0,-1\rangle$, $\vec{b} = \langle 1,1,-1\rangle$

find Unit vector of a vector Which is $\perp^r$ to $\vec{a}$ & in the Plane of $\vec{a}$ & $\vec{b}$?

Such vector $= \vec{a}\times(\vec{a}\times\vec{b})$

$|a|^2 = \sqrt{2}^2$

$$= (a\cdot b)\vec{a} - (\vec{a}\cdot\vec{a})\vec{b}$$

$$= (1+0+1)\vec{a} - 2\vec{b}$$

$$= 2\vec{a} - 2\vec{b}$$

$$= \langle 2,0,-2\rangle - \langle 2,2,-2\rangle$$

$$= \langle 0,-2,0\rangle = -2\hat{j}$$

Unit vector $= \dfrac{-2\hat{j}}{2} = -\hat{j}$

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Q $\vec{a}\times(\vec{b}\times\vec{c})=\frac{\vec{b}}{2}$ find angle betⁿ $\vec{a}\ \&\ \vec{c}$? If $\vec{a},\vec{b},\vec{c}$ are Unit vector

$(\vec{a}\cdot\vec{c})\vec{b}-(\vec{a}\cdot\vec{b})\vec{c}=\frac{\vec{b}}{2}+0\cdot\vec{c}$

$\vec{a}\cdot\vec{c}=\frac{1}{2}$

$|a||c|\cos\theta=\frac{1}{2}\Rightarrow\cos\theta=\frac{1}{2}$

$\theta=60^\circ=\frac{\pi}{3}$.

Q If $\vec{a}\times(\vec{b}\times\vec{c})=\frac{\vec{b}+\vec{c}}{2}$ find angle betⁿ $\vec{a},\vec{c}\ \&\ \vec{a},\vec{b}$

Q $\hat{i}\times(\vec{a}\times\hat{i})+\hat{j}\times(\vec{a}\times\hat{j})+\hat{k}\times(\vec{a}\times\hat{k})=?$

$(\hat{i}\cdot\hat{i})\vec{a}-(\vec{a}\cdot\hat{i})\hat{i}+(\hat{j}\cdot\hat{j})\vec{a}-(\vec{a}\cdot\hat{j})\hat{j}+(\hat{k}\cdot\hat{k})\vec{a}-(\vec{a}\cdot\hat{k})\hat{k}$

$\vec{a}-a_1\hat{i}+\vec{a}-a_2\hat{j}+\vec{a}-a_3\hat{k}$

$=3\vec{a}-(a_1\hat{i}+a_2\hat{j}+a_3\hat{k})$

$=3\vec{a}-\vec{a}=2\vec{a}$

Q $\vec{a}\times(\vec{b}\times\vec{c})=p\vec{a}+q\vec{b}+r\vec{c}$ find p,q,r?

$0\vec{a}+(\vec{a}\cdot\vec{c})\vec{b}-(\vec{a}\cdot\vec{b})\vec{c}=p\vec{a}+q\vec{b}+r\vec{c}$

$p=0,\ q=(\vec{a}\cdot\vec{c}),\ r=-(\vec{a}\cdot\vec{b})$

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Q $((\vec{a}\times\vec{b})\times(\vec{a}\times\vec{c}))\cdot\vec{d} = ?$

$\vec{U} = \vec{a}\times\vec{b}$

$(\vec{U}\times(a\times c))\cdot\vec{d}$

$((U\cdot\vec{c})\vec{a} - (U\cdot\vec{a})\vec{c})\cdot\vec{d}$

$(((a\times b)\cdot\vec{c})\vec{a} - ((a\times b)\cdot\vec{a})\vec{c})\cdot\vec{d}$

$([a\,b\,c]\,\vec{a} - [\cancel{a\,b\,a}]\vec{c})\cdot\vec{d}$

$[a\,b\,c]\cdot(\vec{a}\cdot\vec{d})$ Ans

Q $(a\times b)\times(\vec{r}\times\vec{c})$

$1 \quad 2 \quad 3 = 132 - 123$

$[a\,b\,c]\,\vec{r} - [a\,b\,r]\,\vec{c}$

Q $[(a\times b)\times(b\times c) \quad (b\times c)\times(c\times a) \quad (c\times a)\times(a\times b)]$

$[[a\,b\,c]\vec{b} - [\cancel{a\,b\,b}]c \quad [b\,c\,a]\vec{c} - [\cancel{b\,c\,c}]\vec{a} \quad [c\,a\,b]\vec{a} - [\cancel{c\,a\,a}]\vec{b}]$

$[a\,b\,c]^3[\vec{b} \quad \vec{c} \quad \vec{a}] = [a\,b\,c]^4$ Ans