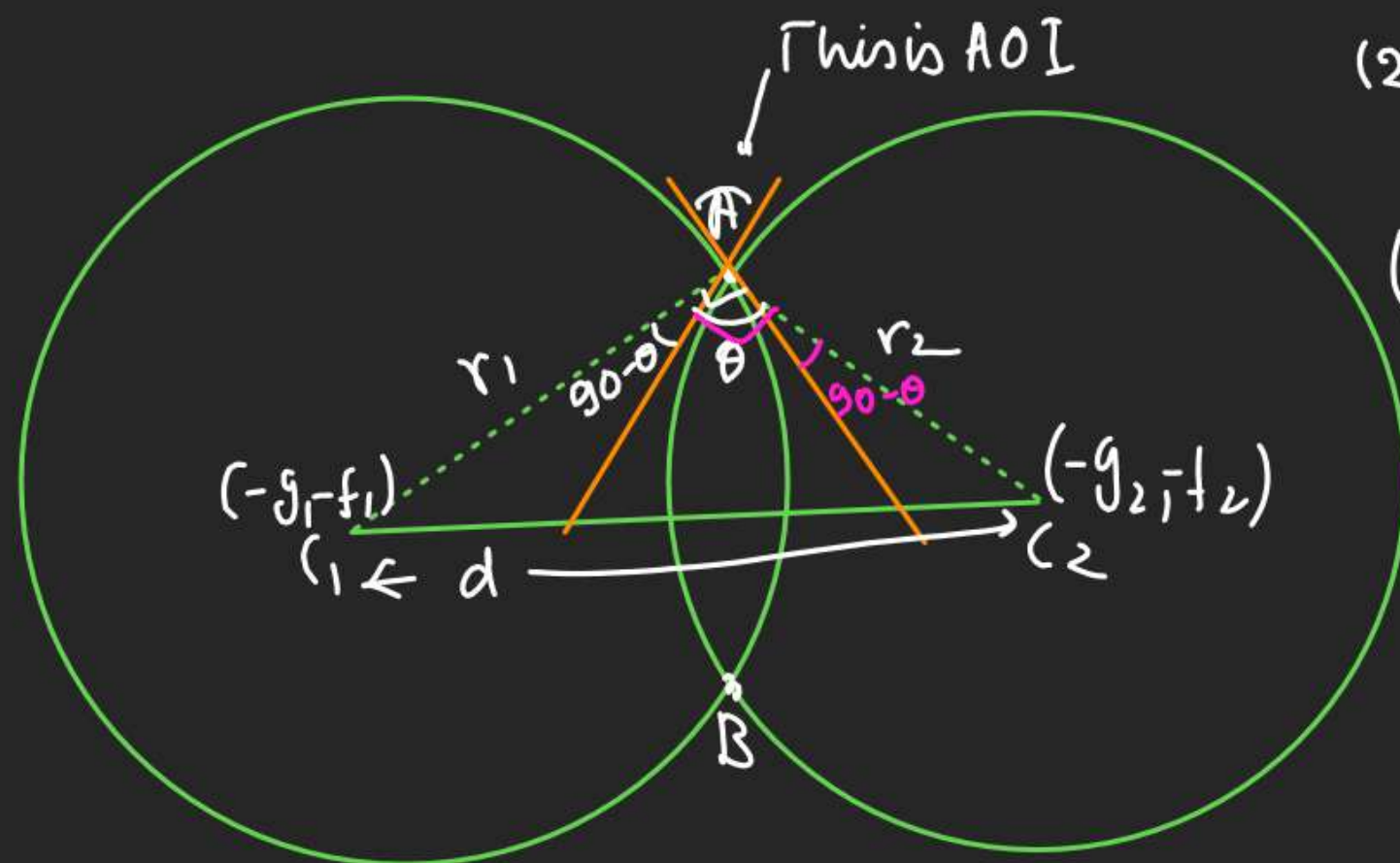


# Angle of Intersection of 2 Circles.



(1) AOI Bet<sup>n</sup> 2 Curves is AO Int. bet<sup>n</sup> tangents at POI

$$(2) \angle C_1 A C_2 = 90 - \theta + \theta + 90 - \theta = 180 - \theta$$

(3)  $\triangle C_1 A C_2 \rightarrow r_1, r_2, d$  Available.

$$\cos(180 - \theta) = \frac{r_1^2 + r_2^2 - d^2}{2r_1 r_2}$$

This is  
direct  
formula

$$\cos \theta = \frac{d^2 - r_1^2 - r_2^2}{2r_1 r_2}$$

for AOI Bet<sup>n</sup> 2 Circles.

(4) Special case  $\rightarrow$  When  $\theta = \frac{\pi}{2}$  then  $S_1, S_2$  are known as Orthogonal Circles.

$$\theta = \frac{\pi}{2} \Rightarrow \cos \theta = 0 \Rightarrow 0 = \frac{d^2 - r_1^2 - r_2^2}{2r_1 r_2} \Rightarrow r_1^2 + r_2^2 = d^2$$

central form!

$$(\sqrt{g_1^2 + f_1^2 - c_1})^2 + (\sqrt{g_2^2 + f_2^2 - c_2})^2 = (\sqrt{(g_1 - g_2)^2 + (f_1 - f_2)^2})^2$$

$$+ c_1 + c_2 = + 2g_1 g_2 + 2f_1 f_2$$

Cond<sup>n</sup> of orthogonality Standard form

Q Circles  $x^2+y^2+x+y=0$

&  $x^2+y^2+x-y=0$  Intersect

d = at an angle of ?

$$\sqrt{0^2+1^2} \begin{cases} C_1 = (-\frac{1}{2}, -\frac{1}{2}) & r_1 = \sqrt{\frac{1}{4} + \frac{1}{4} - 0} = \frac{1}{\sqrt{2}} \\ C_2 = (-\frac{1}{2}, +\frac{1}{2}) & r_2 = \sqrt{\frac{1}{4} + \frac{1}{4} - 0} = \frac{1}{\sqrt{2}} \end{cases}$$

$$\cos \theta = \frac{d^2 - r_1^2 - r_2^2}{2r_1 r_2} = \frac{1^2 - \frac{1}{2} - \frac{1}{2}}{2 \times \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}}} = 0$$

$\theta = \frac{\pi}{2} \Rightarrow S_1 \& S_2$  are Orthogonal.

RK ① Line & Circle are orthogonal then Line is normal of Circle.

(2) If angle bet<sup>n</sup> tangent is  $\theta$  then angle bet<sup>n</sup> their Normal is  $\theta$  also

Q A Circle Passes thro Origin

& has its centre at  $y=x$  If

it cuts  $x^2+y^2-4x-6y+10=0$  orthogonally then Eq<sup>n</sup> of Circle?

① Let Circle  $\rightarrow x^2+y^2+2gx+2fy+c=0$   
(centre  $(-g, -f)$  lies on  $y=x$   
 $-g = -f \Rightarrow g = f$

② Centre  $= (-g, -g)$  &  $(-0)$

$S_1 \therefore$  Circle  $\rightarrow x^2+y^2+2gx+2gy=0$

$S_2 \rightarrow x^2+y^2-4x-6y+10=0$

(centre  $(2, 3)$ ,  $r = \sqrt{4+9-10} = \sqrt{3}$ )

(3)  $S_1 + S_2 \Rightarrow 2g, g_2 + 2f, f_2 = c_1 + c_2$

$$2g(-2) + 2g(-3) = 0 + 10$$

$$-5g = 5 \Rightarrow g = -1$$

$$\Rightarrow S_1: x^2+y^2-2x-2y=0$$

Q If Line  $2x+y=b$  is orthogonal to  $x^2+y^2-2x-2y=0$  then  $b=?$

1) As Line is orthogonal to Circle  $\Rightarrow$  Line is Normal  $\Rightarrow$  It will pass from centre.

2)  $x^2+y^2-2x-2y=0$   
(centre  $(1, 1)$ )

(3)  $2 \times 1 + 1 = b$

$$\boxed{b=3}$$

Q 2 given circles

$$x^2 + y^2 + ax + by + c = 0$$

$$\& x^2 + y^2 + 2gx + 2fy + c = 0$$

will intersect orthogonally  
then find cond<sup>n</sup>.

$$(1) (g, f) = \left(\frac{a}{2}, \frac{b}{2}\right)$$

$$(2) (g, f) = \left(\frac{d}{2}, \frac{e}{2}\right)$$

Cond<sup>n</sup> of orthogonality

$$2g_1g_2 + 2f_1f_2 = c_1 + c_2$$

$$2 \cdot \frac{a}{2} \times \frac{d}{2} + 2 \cdot \frac{b}{2} \cdot \frac{e}{2} = c + f$$

$$ad + be = 2c + 2f \text{ or } \text{Required Cond<sup>n</sup>}$$

Q If circles of same radius

a & centres at (2, 3) & (5, 6)

Cut orthogonally then a = ?

$$S_1: (x-2)^2 + (y-3)^2 = a^2$$

$$S_2: (x-5)^2 + (y-6)^2 = a^2$$

$$r_1^2 + r_2^2 = d^2 \leftarrow \text{Cond<sup>n</sup> of orthogonality}$$

$$a^2 + a^2 = \left(\sqrt{(5-2)^2 + (6-3)^2}\right)^2$$

$$2a^2 = 18$$

$$a = \pm 3$$

Q A circle S passes thru P(-1, 1) & is

orthogonal to circle  $S_1: (x-1)^2 + y^2 = 16$  &  $S_2: x^2 + y^2 = 1$   
then A) Rad = 8 B) Rad = 7 C) Centre (-1, 1)  
(D) Centre (-8, 1)

Let circle S is  $x^2 + y^2 + 2gx + 2fy + c = 0$  P.T.  
 $0 + 1 + 0 + 2f + c = 0 \Rightarrow 2f + c = -1$  (0, 1)

$$(2) S \perp S_1 \Rightarrow S_1: x^2 + y^2 - 2x - 15 = 0$$

$$2(g)(-1) + 2(f)(0) = (-1) + (-15)$$

$$-2g = c - 15$$

$$(3) S \perp S_2: 2(g)(0) + 2(f)(0) = (-1) + (c)$$

$$c = 1$$

$$\therefore -2g = -15 \Rightarrow g = 7$$

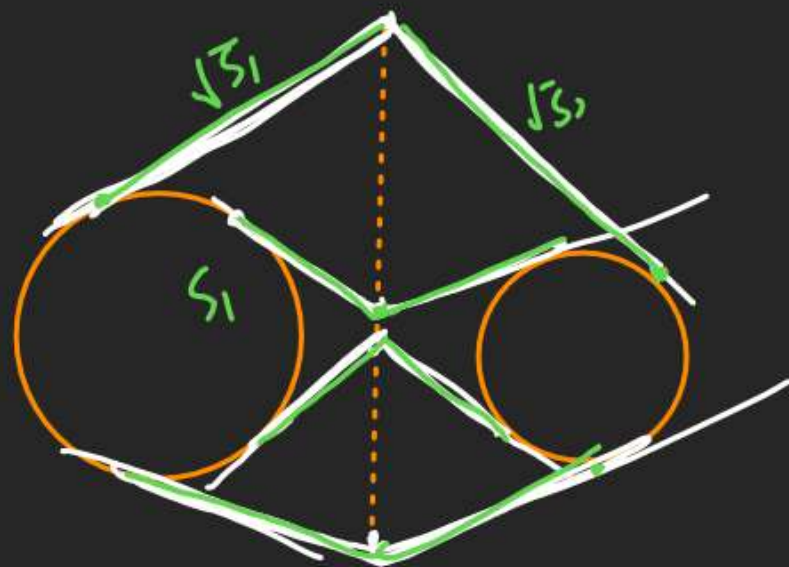
$$\& 2f + 1 = -1 \Rightarrow f = -1 \quad \left. \begin{array}{l} g = 7 \\ f = -1 \end{array} \right\} \text{Centre } (g, f) = (7, -1)$$

$$\text{Radius} = \sqrt{7^2 + (-1)^2} = 7$$

# Radical Axis & Radical Centre.

Def 1 Radical Axis of 2 circles is the Locus of a pt. whose Power w.r.t Both circles is  $\text{Eq}^l$ .

Def 2 R.A. is Locus of Pt. from which Length of tangent to Both circles are  $\text{Eq}^l$ .



(2) So for R.A.

$$\sqrt{S_1} = \sqrt{S_2}$$

$$\Rightarrow S_1 = S_2$$

$$\Rightarrow S_1 - S_2 = 0 \text{ in R.A. for } S_1 \text{ \& } S_2$$

Q If 2 circles are

$$S_1: x^2 + y^2 + 2x - 3y - 11 = 0$$

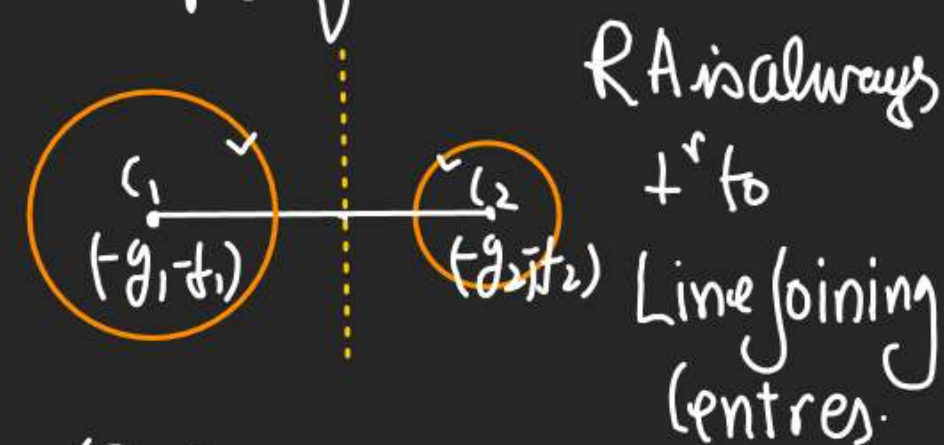
$$\& S_2: x^2 + y^2 = 4 \text{ find R.A.}$$

$$\& \text{R.A. in } S_1 - S_2 = 0$$

$$2x - 3y - 7 = 0$$

$$2x - 3y = 7 \text{ in Rad. Axis}$$

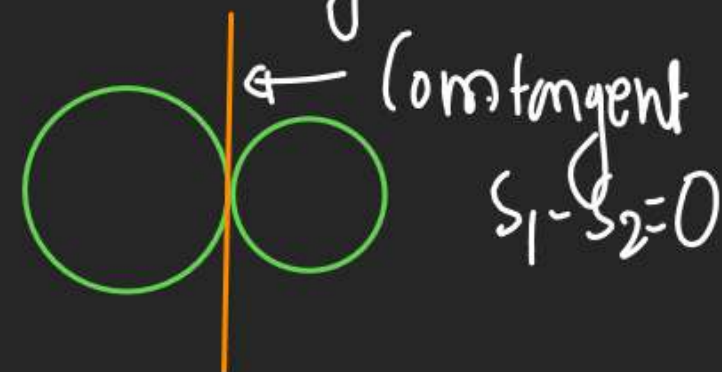
(3)\* Slope of R.A.



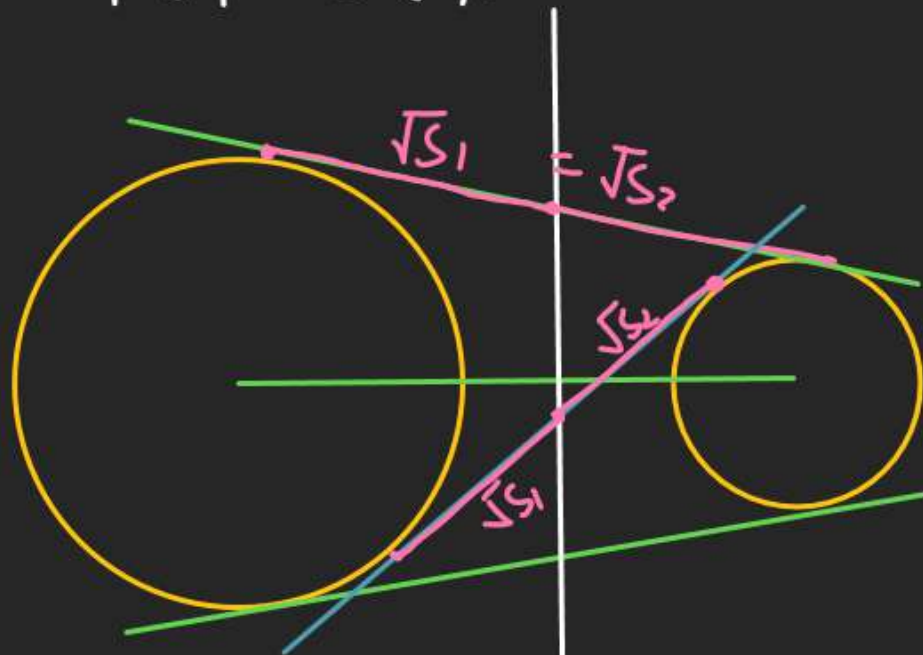
$$(S.L)_{C_1, C_2} = \frac{-f_2 + f_1}{-g_2 + g_1}$$

$$\therefore (S.L)_{R.A.} = - \left( \frac{g_1 - g_2}{f_1 - f_2} \right)$$

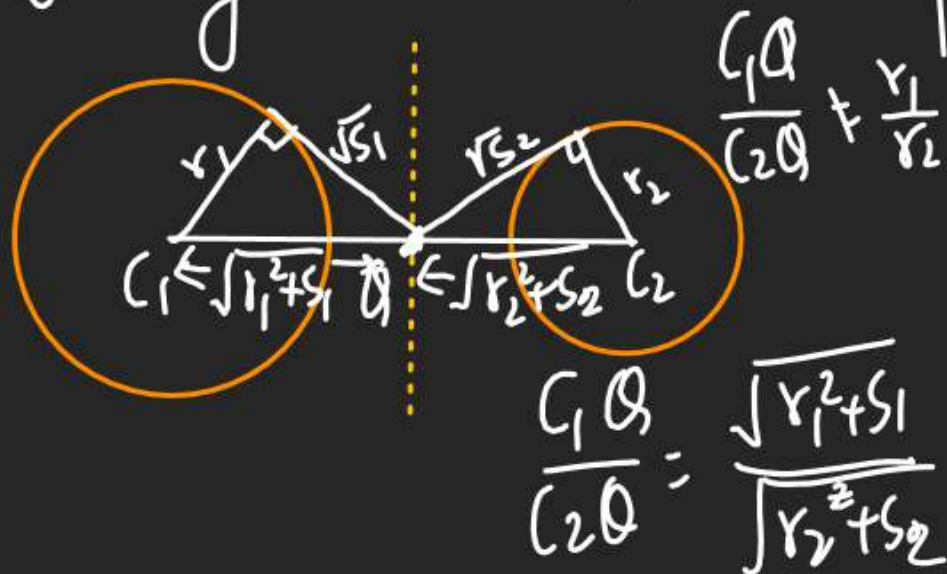
(4)\* When 2 circles touches. R.A. is com. tangent.



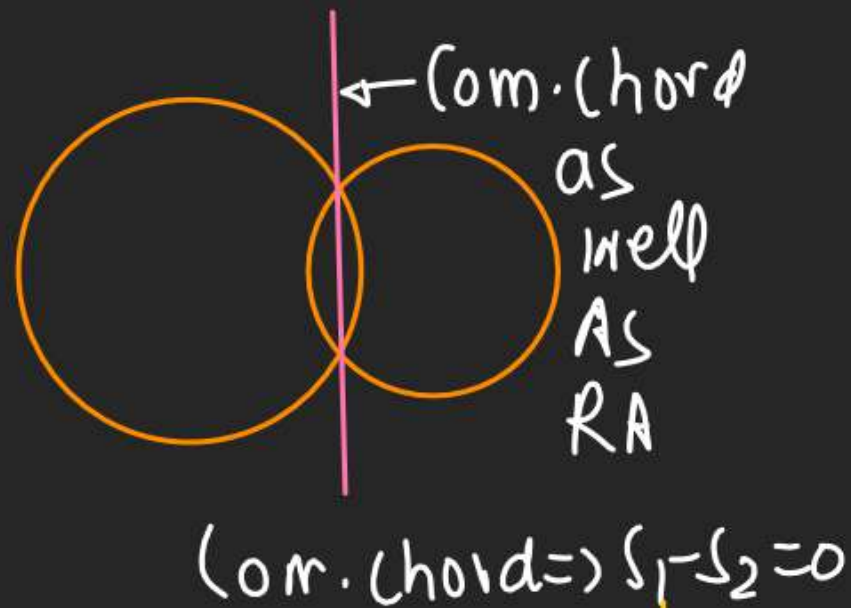
(1) RA always Biseects  
TCT & DCT.



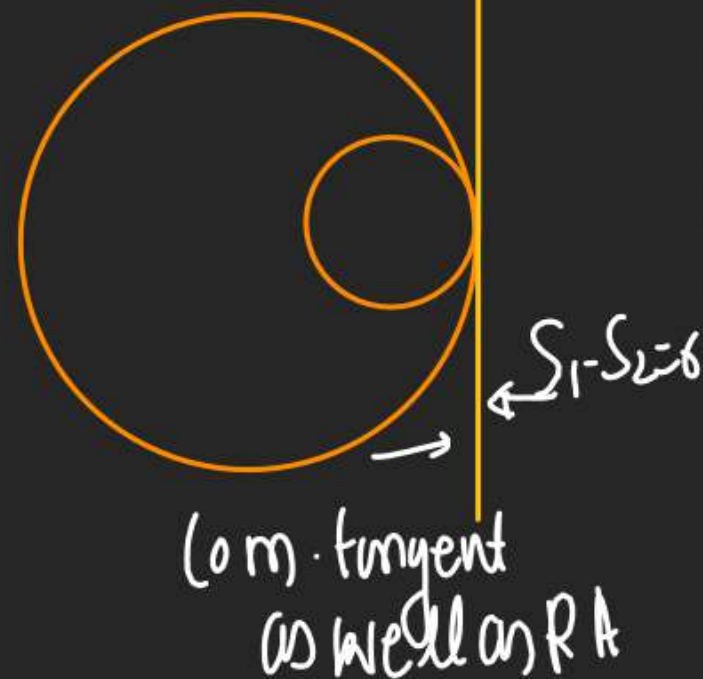
(8) R.A. does not divide line  
Joining centres in  $r_1 : r_2$



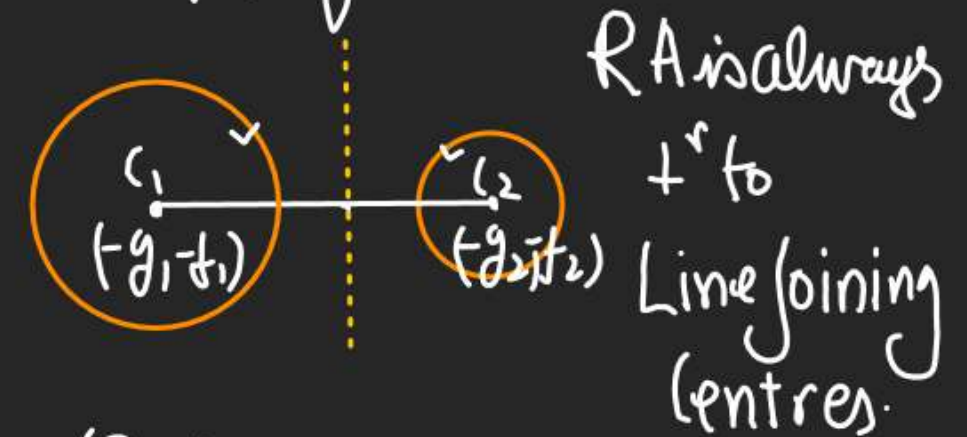
(5) When 2 Circles Intersect  
then Com. Chord is R.A.



(6)



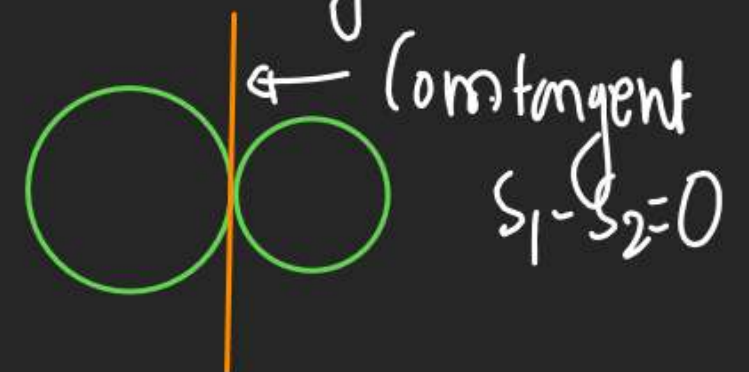
(3)\* Slope of RA.



$$(S_1)_{C_1, C_2} = \frac{-f_2 + t_1}{-g_2 + g_1}$$

$$\therefore (S_1)_{RA} = - \left( \frac{g_1 - g_2}{f_1 - f_2} \right)$$

(4)\* When 2 circles touches.  
RA is Com. tangent.



Q Tangents are drawn to circle.

$S_1: x^2 + y^2 = 12$  at Pts where it

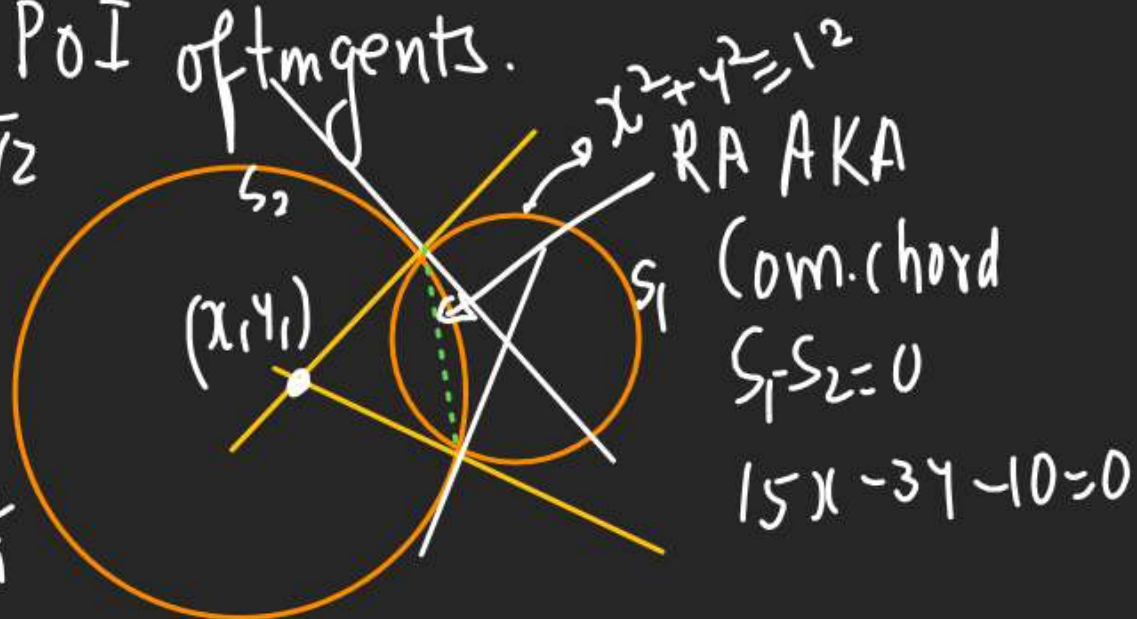
meet circle  $S_2: x^2 + y^2 - 15x + 3y - 2 = 0$

Ind POI of agents.

$$(0,0) \quad r_1 \approx \sqrt{12}$$
$$\begin{pmatrix} 15 & -3 \\ 2 & 1 \end{pmatrix}$$

$$Y = \sqrt{\frac{225}{4} + \frac{9}{4} + 2}$$

$$= \sqrt{\frac{242}{4}}$$



(2) This com. chord is  $OC$ .

$$f_0 r(x, y) - \bar{f} = 0$$

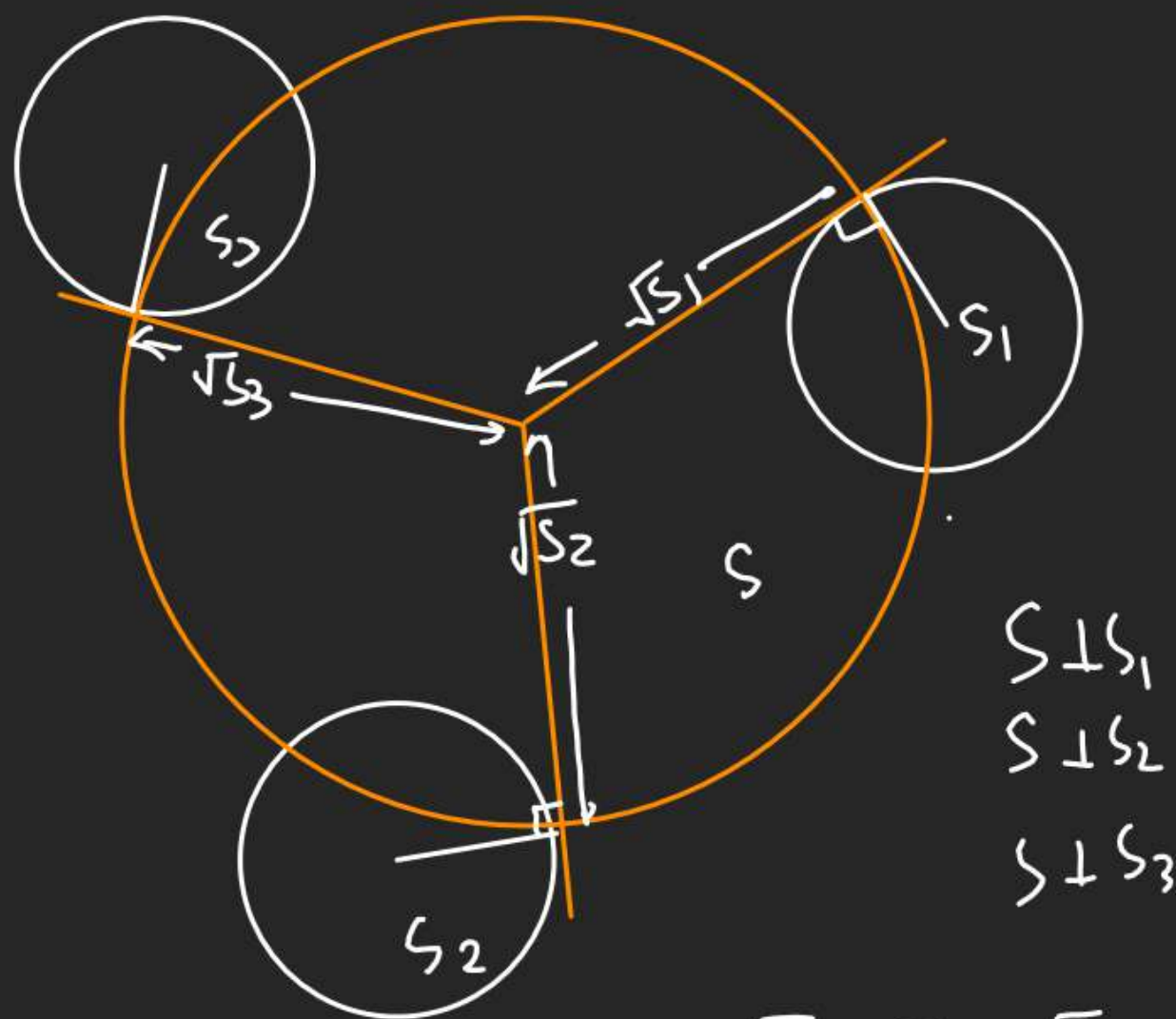
$$2x_1 + 4y_1 - 12 = 0$$

$$15x - 34 - 10 = 0$$

$$\frac{x_1}{15} = \frac{y_1}{-3} = \frac{+126}{+105}$$

$$x_1 = 18, y_1 = -\frac{18}{5}$$

$$(18, -\frac{18}{5})$$



$$\begin{aligned} S &\perp S_1 \\ S &\perp S_2 \\ S &\perp S_3 \end{aligned}$$

$$\sqrt{S_1} = \sqrt{S_2} = \sqrt{S_3}$$

$$\begin{aligned} S_1 - S_2 &= 0 \rightarrow \textcircled{1} \\ S_2 - S_3 &= 0 \rightarrow \textcircled{2} \end{aligned} \left. \vphantom{\begin{aligned} S_1 - S_2 &= 0 \rightarrow \textcircled{1} \\ S_2 - S_3 &= 0 \rightarrow \textcircled{2} \end{aligned}} \right\} \text{R.C.}$$