

$$\int \frac{dx}{x^2 + a^2}$$

$$\frac{1}{a} \tan^{-1} \frac{x}{a}$$

$$\begin{aligned}\int \sec x dx &= \ln |\sec x - \tan x| \\ &= \ln |\tan \frac{x}{2}| + C\end{aligned}$$

$$\int \frac{dx}{\sqrt{x^2 + a^2}} = \ln(x + \sqrt{x^2 + a^2})$$

$$\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1} \frac{x}{a} + C$$

$$\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \ln \left| \frac{a+x}{a-x} \right| + C$$

$$\begin{aligned}&\int \sqrt{a^2 - x^2} dx = \left(\frac{x}{2} \right) \int a^2 - x^2 + \left(\frac{a^2}{2} \right) \boxed{\sin^{-1} \frac{x}{a} + C} \\&\int \frac{dx}{x^2 + a^2} = \boxed{\frac{1}{a}} \boxed{\tan^{-1} \frac{x}{a}} + C \\&\int \frac{dx}{\sqrt{x^2 + a^2}} = \boxed{\ln} \left(x + \sqrt{x^2 + a^2} \right)\end{aligned}$$

$$\int \frac{dx}{3+x^2}$$

$$\int \frac{dx}{x^2 + (\sqrt{3})^2} \rightarrow \int \frac{dx}{x^2 + a^2}$$

$$= \frac{1}{\sqrt{3}} \tan^{-1} \frac{x}{\sqrt{3}} + C$$

$$\int \frac{dx}{3-x^2} \rightarrow \int \frac{dx}{(\sqrt{3})^2 - x^2} \leftarrow \int \frac{dx}{a^2 - x^2}$$

$$= \frac{1}{2\sqrt{3}} \ln \left| \frac{\sqrt{3}+x}{\sqrt{3}-x} \right| + C$$

$$\int \frac{dx}{\sqrt{4-x^2}} \rightarrow \int \sqrt{a^2-x^2} dx$$

$$= \frac{x}{2} \sqrt{2^2 - x^2} + \frac{2^2}{2} \sin^{-1} \frac{x}{2} + C$$

$$\int \frac{dx}{\sqrt{19-x^2}} \rightarrow \int \sqrt{a^2-x^2} dx$$

$$= \frac{x}{2} \sqrt{19-x^2} + \frac{19}{2} \sin^{-1} \frac{x}{\sqrt{19}} + C$$

$$\int \sqrt{x^2 - 19} dx = \int \sqrt{x^2 - (\sqrt{19})^2} dx$$

$$= \frac{x}{2} \sqrt{x^2 - 19} - \frac{19}{2} \ln \left(x + \sqrt{x^2 - 19} \right) + C$$

$$\begin{aligned}
 14) \int \frac{1 + \cos^2 x}{1 + \cos 2x} dx \\
 &= \int \frac{1 + \cos^2 x}{2 \cos^2 x} dx \\
 &\Rightarrow \frac{1}{2} \int \frac{dx}{\cos^2 x} + \int \frac{\cos^2 x}{2 \cos^2 x} dx \\
 &\Rightarrow \frac{1}{2} \int \sec^2 x dx + \int \frac{1}{2} dx \\
 &\Rightarrow \frac{1}{2} \tan x + \frac{x}{2} + C
 \end{aligned}$$

$$\begin{aligned}
 &5 \int a^x \cdot e^x dx \quad \leftarrow \int (ae)^x dx \rightarrow \int A^x dx = \frac{A^x}{\ln A} \\
 &= \frac{(ae)^x}{\ln ae} + C \quad \text{Trick} \\
 15) \int 8m x d(8mx) \quad &8mx = y \\
 &\int y dy = \frac{y^2}{2} + C \\
 &= \frac{8m^2 x^2}{2} + C
 \end{aligned}$$

$$\begin{aligned}
 20) \int \tan^3 x \cdot d(\tan x) \\
 &\int y^3 dy = \frac{y^4}{4} + C
 \end{aligned}$$

$$= \frac{m^4}{4} + C$$

$$\begin{aligned}
 Q8 \quad 3 \cdot 4 \int x^{-17} dx \\
 &\quad -17+1
 \end{aligned}$$

$$(3 \cdot 4) \frac{x}{-17+1} + C$$

$$(3 \cdot 4) \frac{x}{-17+1} + C$$

$$\text{Q7} \int \frac{dh}{\sqrt{2gh}} \xrightarrow{\text{h var}} g = \text{const.}$$

$$\frac{1}{\sqrt{2g}} \int \frac{dh}{\sqrt{h}}$$

$$\frac{2\sqrt{h}}{\sqrt{2g}} + C$$

$$\text{Q9} \int \frac{\sqrt{x}}{x^3} - \frac{x^{3/2} e^x}{x^5} + \frac{x^2}{x^3} dx$$

$$\int x^{-5/2} dx - \int e^x dx + \int \frac{1}{x} dx$$

GIR Sir Pro Army 12th.

$\text{Q23} \int (8-3x)^{\frac{6}{5}+1} dx$ $\int \frac{8mx}{(8-3x)^{\frac{1}{5}}} dx = \frac{(8-3x)^{\frac{6}{5}+1}}{\frac{11}{5} \times (-3)} + C_1$ $\int \frac{8mx}{(8-3x)^{\frac{1}{5}}} \times \frac{1}{(8-3x)} dx$ $\int \sec x \tan x dx$ $\int x^{-3/2} - 2 \cdot x^{-1/2} + x^{1/2} dx$	$\text{Q26} \int \frac{8mx}{(8-3x)^{\frac{1}{5}}} dx = \frac{(8-3x)^{\frac{6}{5}+1}}{\frac{11}{5} \times (-3)} + C_1$ $\int \frac{1}{x^5} - 2 \cdot \frac{1}{x^3} + \frac{1}{x^2} dx$
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37x 12) $\int \frac{dx}{\sqrt{8-3x^2}} = \frac{1}{\sqrt{3}} \int \frac{dx}{\sqrt{\frac{8}{3}-x^2}}$

$$= \frac{1}{\sqrt{3}} * \sin^{-1} \frac{x}{\sqrt{\frac{8}{3}}} + C$$

$$\textcircled{Q} \quad \int \sin x \, dx$$

$$\textcircled{M1} \quad \frac{d(\sin x)}{dx} = \cos x$$

$$d(\sin x) = \cos x \, dx$$

$$\int \sin x \cdot (\cos x \cdot dx)$$

$$\frac{1}{2} \int 2 \sin x \cdot (\cos x \cdot dx)$$

$$\frac{1}{2} \int \sin 2x \, dx$$

$$-\frac{1}{2} \frac{\cos(2x)}{2} + C \quad \text{Ans 1}$$

$$\Rightarrow \frac{2 \sin^2 x - 1}{4} + C$$

$$\Rightarrow \frac{\sin 2x}{2} + C'$$

$$\stackrel{M2}{=} \sin x - y$$

$$\int y \cdot dy$$

$$= \frac{y^2}{2} + C$$

$$= \frac{\sin^2 x}{2} + C$$

Ans 2

\int Trigo fxn can give 2 or more different answers.

$$\int \frac{dx}{x^2 + a^2}$$

$$\frac{1}{a} \tan^{-1} \frac{x}{a}$$

$$\int \frac{dx}{\sqrt{x^2 + a^2}}$$

$$\ln(x + \sqrt{x^2 + a^2})$$

$$\int \frac{dx}{x^2 - a^2}$$

$$\frac{1}{2a} \ln \left| \frac{x-a}{x+a} \right|$$

$$\int \frac{dx}{\sqrt{a^2 - x^2}}$$

$$\ln(x + \sqrt{a^2 - x^2})$$

$$\int \frac{dx}{a^2 - x^2}$$

$$\frac{1}{2a} \ln \left| \frac{a+x}{a-x} \right|$$

$$\int \frac{dx}{\sqrt{a^2 - x^2}}$$

$$\sin^{-1} x$$

$$Q \int \frac{dx}{4x^2+5} \rightarrow \int \frac{dx}{(2x)^2 + (\sqrt{5})^2} \rightarrow \int \frac{dx}{x^2 + a^2}$$

$$= \frac{1}{2\sqrt{5}} \tan^{-1} \frac{2x}{\sqrt{5}} + C$$

$$Q \int \frac{dx}{7x^2+3} = \int \frac{dx}{(\sqrt{7}x)^2 + (\sqrt{3})^2} \rightarrow \int \frac{dx}{x^2 + a^2}$$

$$\sqrt{7}\sqrt{3} \tan^{-1} \frac{\sqrt{7}x}{\sqrt{3}} + C$$

$$Q \int \frac{dx}{\sqrt{4x^2+3}} = \int \frac{dx}{\sqrt{(2x)^2 + (\sqrt{3})^2}} \rightarrow \int \frac{dx}{\sqrt{x^2 + a^2}}$$

$$= \frac{1}{2} \ln \left\{ 2x + \sqrt{4x^2 + 3} \right\} + C$$

$$\int \sqrt{1-x^2} dx \quad \left| \begin{array}{l} x = \frac{1}{2} \sqrt{a^2 - x^2} \\ x = \frac{a}{2} \sin \theta \end{array} \right. + \frac{a^2}{2} \sin^{-1} \frac{x}{a} + C$$

$$Q \int \frac{dx}{\sqrt{4-3x^2}} \rightarrow \int \frac{dx}{\sqrt{(\sqrt{2})^2 - (\sqrt{3}x)^2}} \rightarrow \int \frac{dx}{\sqrt{a^2 - x^2}}$$

$$= \frac{1}{\sqrt{3}} \sin^{-1} \frac{\sqrt{3}x}{2} + C$$

$$\frac{x}{2} \sqrt{1-x^2} + \frac{1}{2} \sin^{-1} \frac{x}{1} + C$$

$$\oint \frac{dx}{\sqrt{2x^2 - 1}}$$

$$\int \frac{dx}{\sqrt{(\sqrt{2}x)^2 - 1^2}} \rightarrow \int \frac{dx}{\sqrt{x^2 - a^2}}$$

$$\frac{1}{\sqrt{2}} \ln \left\{ \sqrt{2}x + \sqrt{2(x^2 - 1)} \right\} + C$$

$$\oint \sqrt{x^2 - 4x + 6} dx$$

$$\int \frac{\sqrt{x^2 - 4x + 4 + 2} dx}{\sqrt{(x-2)^2 + (\sqrt{2})^2}} \rightarrow \int \sqrt{x^2 + a^2} dx$$

$$\frac{1}{2} \left(\frac{x-2}{2} \sqrt{x^2 - 4x + 6} + \left(\frac{\sqrt{2}}{2}\right)^2 \ln \left\{ (x-2) + \sqrt{(x-2)^2 + (\sqrt{2})^2} \right\} \right) + C$$

$$\oint \sqrt{3x^2 + 4} dx \rightarrow \int \sqrt{x^2 + a^2}$$

$$\int \sqrt{(\sqrt{3}x)^2 + 2^2} dx$$

$$\frac{1}{\sqrt{3}} \left(\frac{\sqrt{3}x}{2} \sqrt{3x^2 + 4} + \frac{2^2}{2} \ln \left\{ \sqrt{3}x + \sqrt{3x^2 + 4} \right\} \right) + C$$

$$\begin{aligned}
 & Q \int x \cdot (3x+5)^7 dx \\
 & \left. \begin{aligned}
 & \frac{1}{3} \int 3x \cdot (3x+5)^7 dx \\
 & = \frac{1}{3} \int ((3x+5) - 5) \cdot (3x+5)^7 dx \\
 & = \frac{1}{3} \int (3x+5)^8 - 5(3x+5)^7 dx \\
 & = \frac{1}{8} \int (3x+5)^8 dx - \frac{5}{3} \int (3x+5)^7 dx \\
 & = \frac{1}{8} \frac{(3x+5)^9}{9 \times 3} - \frac{5}{3} \frac{(3x+5)^8}{8 \times 3} + C
 \end{aligned} \right| \stackrel{\text{Q}}{=} \begin{aligned}
 & \int x \cdot (2x+3)^5 dx \\
 & \frac{1}{2} \int (2x+3) - 3 \cdot (2x+3)^5 dx \\
 & = \frac{1}{2} \int (2x+3)^6 - \frac{3}{2} \int (2x+3)^5 dx \\
 & = \frac{1}{2} \frac{(2x+3)^7}{7 \times 2} - \frac{3}{2} \times \frac{(2x+3)^6}{6 \times 2} + C
 \end{aligned}
 \end{aligned}$$

$$Q \int \frac{dx}{x^2 + \cancel{x^2} + 2}$$

$$\int \frac{dx}{(x+1)^2 - 1^2 + 2}$$

$$\int \frac{dx}{Q(+1)^2 + 1^2} \rightarrow \int \frac{dx}{x^2 + R^2}$$

$$\frac{1}{4} \tan^{-1} \frac{x+1}{1} + C$$

$\boxed{13} \xrightarrow{\frac{13}{2}} x^2$ $\left(\frac{13}{2}\right)^2 - (x - \frac{13}{2})^2$	$\boxed{9} \xrightarrow{\frac{9}{2}} x^2$ $\left(\frac{9}{2}\right)^2 - (x - \frac{9}{2})^2$
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$1) \quad x^2 + \cancel{13} x + 14$ $= (x + \frac{13}{2})^2 - (\frac{13}{2})^2 + 14$	$2) \quad x^2 - \cancel{9} x + 3$ $(x - \frac{9}{2})^2 - (\frac{9}{2})^2 + 3$
$3) \quad 2x^2 + 7x + 5$ $2 \left\{ x^2 + \cancel{\left(\frac{7}{2}\right)} x + \frac{5}{2} \right\}$	$2 \left\{ (x + \frac{7}{4})^2 - \left(\frac{7}{4}\right)^2 + \frac{5}{2} \right\}$

$$Q \int \frac{dx}{4x^2 - 4x - 3}$$

$$\frac{1}{4} \int \frac{dx}{x^2 - \left(x - \frac{3}{4}\right)} \quad -\frac{1}{4} - \frac{3}{4} = -1$$

$$\frac{1}{4} \int \frac{dx}{(x - \frac{1}{2})^2 - (\frac{1}{2})^2 - \frac{3}{4}}$$

$$\frac{1}{4} \int \frac{dx}{(x - \frac{1}{2})^2 - (1)^2} \int \frac{dx}{x^2 - a^2}$$

$$\frac{1}{4} + \frac{1}{2a} \ln \left| \frac{(x - \frac{1}{2}) - 1}{(x - \frac{1}{2}) + 1} \right| + C$$

Q

$$\int \frac{dx}{\sqrt{x^2 - a^2}}$$

$$\int \frac{dx}{\sqrt{(\frac{1}{2})^2 - (x - \frac{1}{2})^2}} \quad \int \frac{dx}{\sqrt{a^2 - x^2}}$$

$$= \frac{1}{1} \sin^{-1} \left(\frac{x - \frac{1}{2}}{\frac{1}{2}} \right) + C$$

in Qs of Q Eqn always try to make Per. Sq &
use basic
formu

$$Q \int \frac{dx}{\sqrt{2a^2(x - a)^2}}$$

$$\int \frac{dx}{\sqrt{a^2 - (x - a)^2}}$$

$$\sin^{-1} \frac{x - a}{a} + C$$

MJA AA Jatu Agrrrrr-----

$$\oint \int \frac{1+2x^2}{x^2(1+x^2)} dx$$

$$= \int \frac{(1+x)^2}{x((1+x)^2)} dx$$

$$\int \frac{1+x^2}{x^2(1+x^2)} + \frac{x^2}{x^2(1+x^2)}$$

$$\Rightarrow \int \frac{1+x^2}{x(1+x^2)} + \frac{2x}{x((1+x)^2)} dx$$

$$\int \frac{dx}{x^2} + \int \frac{dx}{1+x^2}$$

$$\int \frac{dx}{x} + 2 \int \frac{dx}{1+x^2}$$

$$-\frac{1}{x} + \frac{1}{1} \tan^{-1} x + C$$

$$\ln |x| + 2 \tan^{-1} x + C$$

$$\tan^{-1} x - \frac{1}{x} + C$$

Nr's Deg ≥ Dr's Deg

If Nr's Deg ≥ Dr's Deg then divide.

$$Q \int \frac{x^4}{x^2+1} dx \quad \text{①}$$

$$\int \frac{x^4 - 1 + 1}{x^2+1}$$

$$\int \frac{\cancel{x^4-1}}{\cancel{x^2+1}} + \frac{1}{x^2+1} dx$$

$$\frac{x^3}{3} - x + \frac{1}{1} + \ln \frac{1}{1+x} + C$$

$$Q \int \frac{x^4}{x-1} dx$$

$$\int \frac{x^4 - 1 + 1}{(x-1)} +$$

$$\int \frac{x^4 - 1}{(x-1)} + \int \frac{1}{x-1} dx$$

$$\int \frac{(x^2+1)(x^2-1)}{(x-1)} + \ln|x-1|$$

$$\int x^3 + x^2 + x + 1 dx + \ln|x-1|$$

$$\Rightarrow \frac{x^4}{4} + \frac{x^3}{3} + \frac{x^2}{2} + x + \ln|x-1| + C$$

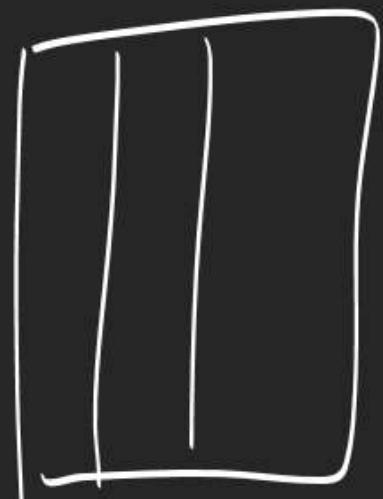
$$(x^2-1) = (x-1)(x+1)$$

$$(x^3-1) = (x-1)(x^2+x+1)$$

$$(x^4-1) = (x-1)(x^3+x^2+x+1)$$

$$(x^7-1) = (x-1)(x^6+x^5+x^4+x^3+x^2+x+1)$$

$$\int \frac{(\sqrt{x}+1)(x^2-\sqrt{x})}{x(\sqrt{x}+x+\sqrt{x})} dx$$



Mukund: $\int \frac{(\sqrt{x}+1)(\sqrt{x})(x^{3/2}-1)}{(\sqrt{x})(x+\sqrt{x}+1)} dx$

Nr Dey: $\int \frac{(\sqrt{x}+1)((\sqrt{x})^3-1)}{(\sqrt{x})^2+\sqrt{x}+1} dx$
 $\int \frac{(x+1)(x-1)(x^2+1)}{(x^2+1)(x^3+1)} dx$
 $\int \frac{(x^2-1)(x^2+1)}{(x^2+1)(x^3+1)} dx$

$$\int (x-1) dx = \frac{x^2}{2} - x + C$$

$$\begin{aligned} x^6+1 &= (x^2)^3+1^3 \\ &= (x^2+1)(x^4-x^2+1) \end{aligned}$$

$$\int \frac{x^4+1}{x^6+1} dx = \tan^{-1} x + \frac{1}{3} \operatorname{tm}^1(x^3) + C$$

$$\int \frac{x^4+1}{(x^2+1)(x^4-x^2+1)} dx$$

$$\int \frac{x^4-x^2+1}{(x^2+1)(x^4-x^2+1)} + \frac{x^2}{(x^2+1)(x^4-x^2+1)} dx$$

$$\int \frac{dx}{x^2+1} + \frac{1}{3} \int \frac{3x^2 dx}{x^6+1}$$

$$\tan^{-1} x + \frac{1}{3} \int \frac{d(x^3)}{(x^3)^2 + 1^2} = \tan^{-1} x + \frac{1}{3} \int \frac{dy}{y^2 + 1^2} = \tan^{-1} x + \frac{1}{3} \tan^{-1} y$$