DPP – 06

Solution

1. By Ampere's Circuital Law, the current inside the Amperian Loop passing through the point P is

$$I' = I\left(\frac{(3R/2)^2 - R^2}{(2R)^2 - R^2}\right) = \frac{5}{12}I$$

Since $\oint \vec{B}\cdot d\vec{l} = \mu_0 I'$

$$\Rightarrow\ B(2\pi r) = \mu_0 I',\ \text{where } r = \frac{3R}{2}$$

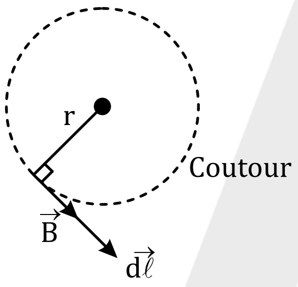
$$\Rightarrow\ 2\pi B\left(\frac{3R}{2}\right) = \mu_0\left(\frac{5}{12}I\right)$$

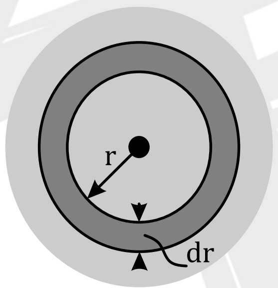
$$\Rightarrow\ B = \frac{5\mu_0 I}{36\pi R}$$

$\therefore K = 5$

2. As per Ampere's Circuital Law

$$\int \vec{B}\cdot d\vec{l} = \mu_0(I_{total})$$





Stream of Electrons

Since, $I_{total} = \int \vec{j}\cdot d\vec{A} = \int jdA$, where $dA = 2\pi r dr$

$$\Rightarrow\ B(2\pi r) = \mu_0 \int jdA$$

$$\Rightarrow\ B(2\pi r) = \mu_0 \int j(2\pi r dr)$$

But $B = br^{\alpha}$

$$\Rightarrow\ 2\pi b r^{\alpha+1} = \mu_0 \int j(2\pi r dr)$$

Differentiating both sides, we get

$$\Rightarrow\ 2\pi b(\alpha+1)r^{\alpha}dr = \mu_0(j2\pi r dr)$$

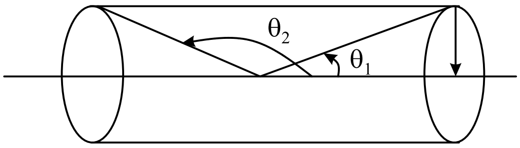
$$\Rightarrow\ j = \frac{b(\alpha+1)}{\mu_0}r^{\alpha-1}$$

$\therefore \beta = 1$

3. $B = \frac{1}{2}\mu_0 n i[\cos\ \theta_1 - \cos\ \theta_2] \Rightarrow\ n\ = \frac{1000}{0.4} = 2500$ per meter

$i = 5 \times 10^{-3}$ A.

(i)



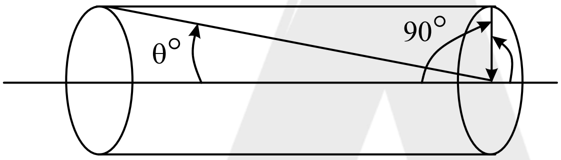
$$\cos\ \theta_1 = \frac{0.2}{\sqrt{(0.3)^2 + (0.2)^2}} = \frac{0.2}{\sqrt{0.13}}$$

$$\cos\theta_2 = \frac{-0.2}{\sqrt{0.13}} \qquad \ldots \text{(i)}$$

$$\Rightarrow\ B = \frac{1}{2} \times (4 \times \pi \times 10^{-7}) \times 2500 \times 5 \times 10^{-3}\, \frac{2 \times 0.2}{\sqrt{0.13}}$$

$$= \frac{\pi \times 10^{-5}}{\sqrt{13}}\ \text{T} \qquad \ldots \text{(ii)}$$

(ii)



At the end ...(iii)

$$\cos\ \theta_1 = \frac{0.4}{\sqrt{(0.3)^2 + (0.4)^2}} = 0.8$$

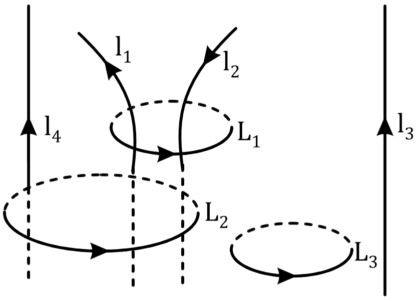
$$\cos\theta_2 = \cos 90^\circ = 0 \qquad \ldots \text{(iv)}$$

$$B = \frac{1}{2} \times (4 \times \pi \times 10^{-7}) \times 2500 \times 5 \times 10^{-3} \times 0.8 \Rightarrow B = 2\pi \times 10^{-6}\ \text{Wb/m}^2$$

4. For L_1 $\oint \vec{B} \cdot \vec{d\ell} = \underline{\mu_0(I_1 - I_2)}$

Here I_1 is taken positive because magnetic lines of force produced by I_1 is anti clockwise as seen from top. I_2 produces lines of $\vec{B}$ in clockwise sense as seen from top. The sense of $\vec{d\ell}$ is anticlockwise as seen from top.

For L_2 $\oint \vec{B} \cdot \vec{d\ell} = \mu_0(I_1 - I_2 + I_4)$



For L_3 $\oint \vec{B} \cdot \vec{d\ell} = 0$

5. According to Ampere's law, we have

$\oint \vec{B} \cdot d\vec{l} = \mu_0 I$

For loop A, $\oint \vec{B} \cdot d\vec{l} = \mu_0(0) = 0$

For loop B, $\oint \vec{B} \cdot d\vec{l} = \mu_0(I + I - I) = \mu_0 I$

For loop C, $\oint \vec{B} \cdot d\vec{l} = -\mu_0 I$

For loop D, $\oint \vec{B} \cdot d\vec{l} = -\mu_0 I$

$\Rightarrow B > A > C = D$

6. Applying Ampere's Law to the Rectangular Loop 12341 shown, we get

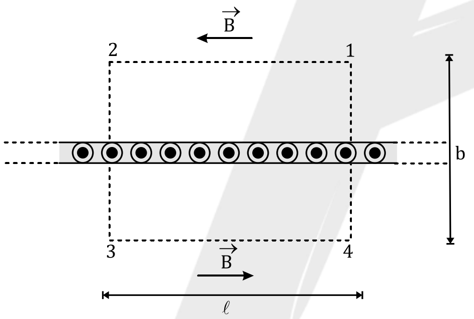
$$\oint \vec{B} \cdot d\vec{\ell} = \int_1^2 \vec{B} \cdot d\vec{\ell} + \int_2^3 \vec{B} \cdot d\vec{\ell} + \int_3^4 \vec{B} \cdot d\vec{\ell} + \int_4^1 \vec{B} \cdot d\vec{\ell}$$

For 14 and 32 (or 41 and 23),

$\vec{B} \perp d\vec{\ell}$ and hence $\int \vec{B} \cdot d\vec{\ell} = 0$

For 1 to 2 and 3 to 4 $\vec{B} \parallel d\vec{\ell}$

$\Rightarrow \oint \vec{B} \cdot \overrightarrow{d\ell} = 2B\ell$



According to Ampere's Circuital Law,

$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I_{enc}$

$\Rightarrow 2B\ell = \mu_0(\lambda\ell)$

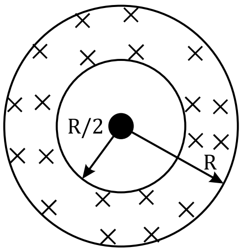
$\Rightarrow B = \dfrac{\mu_0\lambda}{2}$

7. From Ampere's Circuital Law, we get

$B_1(2\pi x) = \mu_0 I \Rightarrow B_1 = \dfrac{\mu_0 I}{2\pi x}$

Again, using Ampere's Circuital Law, we get

$B_2(2\pi)(2x) = \mu_0(I + I) \Rightarrow B_2 = \dfrac{\mu_0 I}{2\pi x} = B_1$

8. 

Let r be the distance of a point from centre, then For $r \le \frac{R}{2}$, using Ampere's circuital law,

$\int \vec{B} \cdot dl$

$\Rightarrow Bl = \mu_0(I_{in})$

$\Rightarrow B(2\pi r) = \mu_0(I_{in})$

$\Rightarrow B = \frac{\mu_0}{2\pi}\frac{I_{in}}{r}$ ... (i)

Since, $I_{in} = 0$

$\Rightarrow B = 0$

If J be the current per unit area, then for $\frac{R}{2} \le r \le R$, we have

$$I_{in} = \left[\pi r^2 - \pi\left(\frac{R}{2}\right)^2\right] J$$

Substituting in equation (1), we have

$$B = \frac{\mu_0}{2\pi}\frac{\left(\pi r^2 - \pi\frac{R^2}{4}\right)J}{r} \Rightarrow B = \frac{\mu_0 J}{2r}\left(r^2 - \frac{R^2}{4}\right)$$

At $r = \frac{R}{2}, B = 0$

At $r = R, B = \frac{3\mu_0 JR}{8}$

For $r \ge R$, we have

$I_{in} = I_{Total} = I(\text{ say })$

Therefore, substituting in equation (1), we have

$$B = \frac{\mu_0 I}{2\pi r} \Rightarrow B \propto \frac{1}{r}$$