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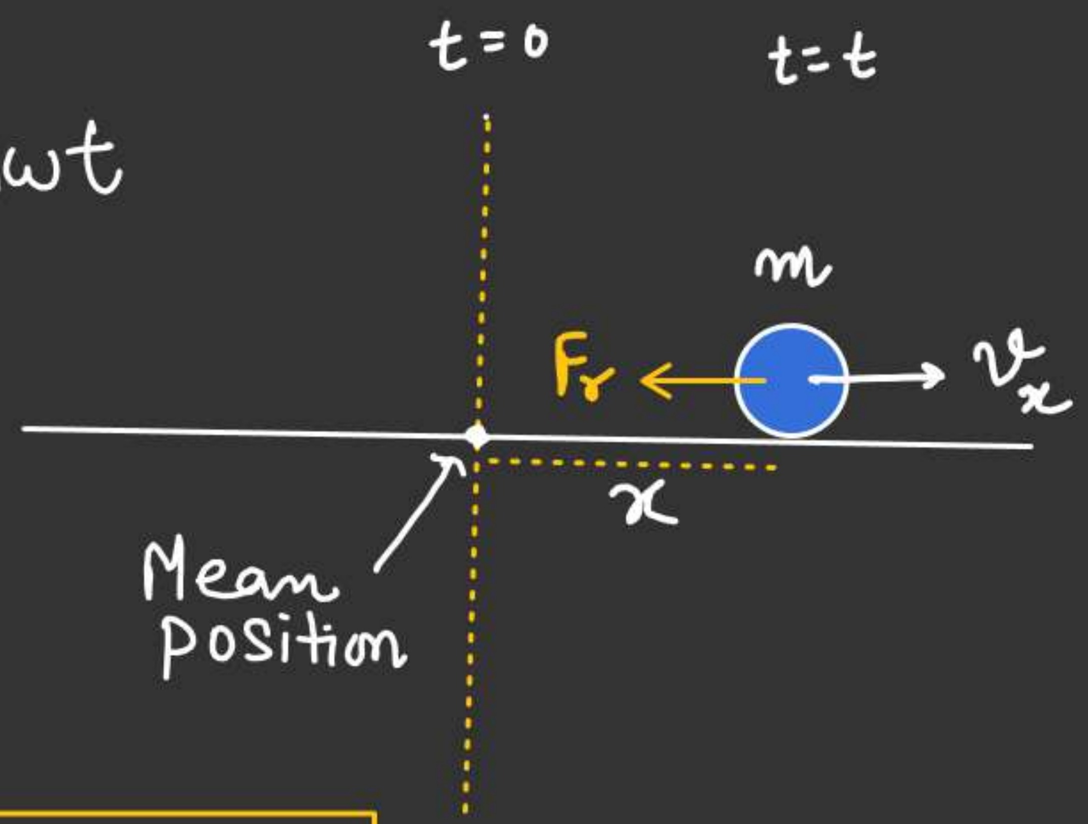
** Energy in Case of S·H·M

S·H·M

$$E_T = P.E + K.E$$

$$x = A\sin\omega t$$



P.E :- $F_r = -Kx$.

Let, dW_{system} be the work done for dx displacement

$$dW_{system} = -Kx \cdot dx$$

$$dU = -dW_{system} = Kx\,dx$$

$$\int_0^U dU = K\int_0^x x\,dx$$

$$\boxed{U = \frac{1}{2}Kx^2}$$

$$\omega^2 = \frac{K}{m} \Rightarrow K = m\omega^2$$

$$U = \frac{1}{2}KA^2\sin^2\omega t$$

$$\boxed{U = \frac{1}{2}m\omega^2A^2\sin^2\omega t}$$ **

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K.E in Case of S.H.M

$$x = A \sin\omega t$$

$$v = \frac{dx}{dt} = \underline{A\omega \cos\omega t}$$

$$K.E = \frac{1}{2} m v^2$$

$$\boxed{K.E = \frac{1}{2} m A^2 \omega^2 \cos^2\omega t}$$

Total Energy in Case of S.H.M

$$E_T = P.E + K.E$$

$$= \frac{1}{2} m\omega^2 A^2 \sin^2\omega t + \frac{1}{2} m\omega^2 A^2 \cos^2\omega t$$

$$= \frac{1}{2} m\omega^2 A^2 (\sin^2\omega t + \cos^2\omega t)$$

$$\boxed{E_T = \frac{1}{2} m\omega^2 A^2}$$

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S.H.M

$$P.E = \frac{1}{2}Kx^2$$

$y \qquad y = 4ax^2$

$$v = \omega\sqrt{A^2 - x^2}$$

$$K.E = \frac{1}{2}mv^2$$

$$= \frac{1}{2}m\omega^2(A^2 - x^2)$$

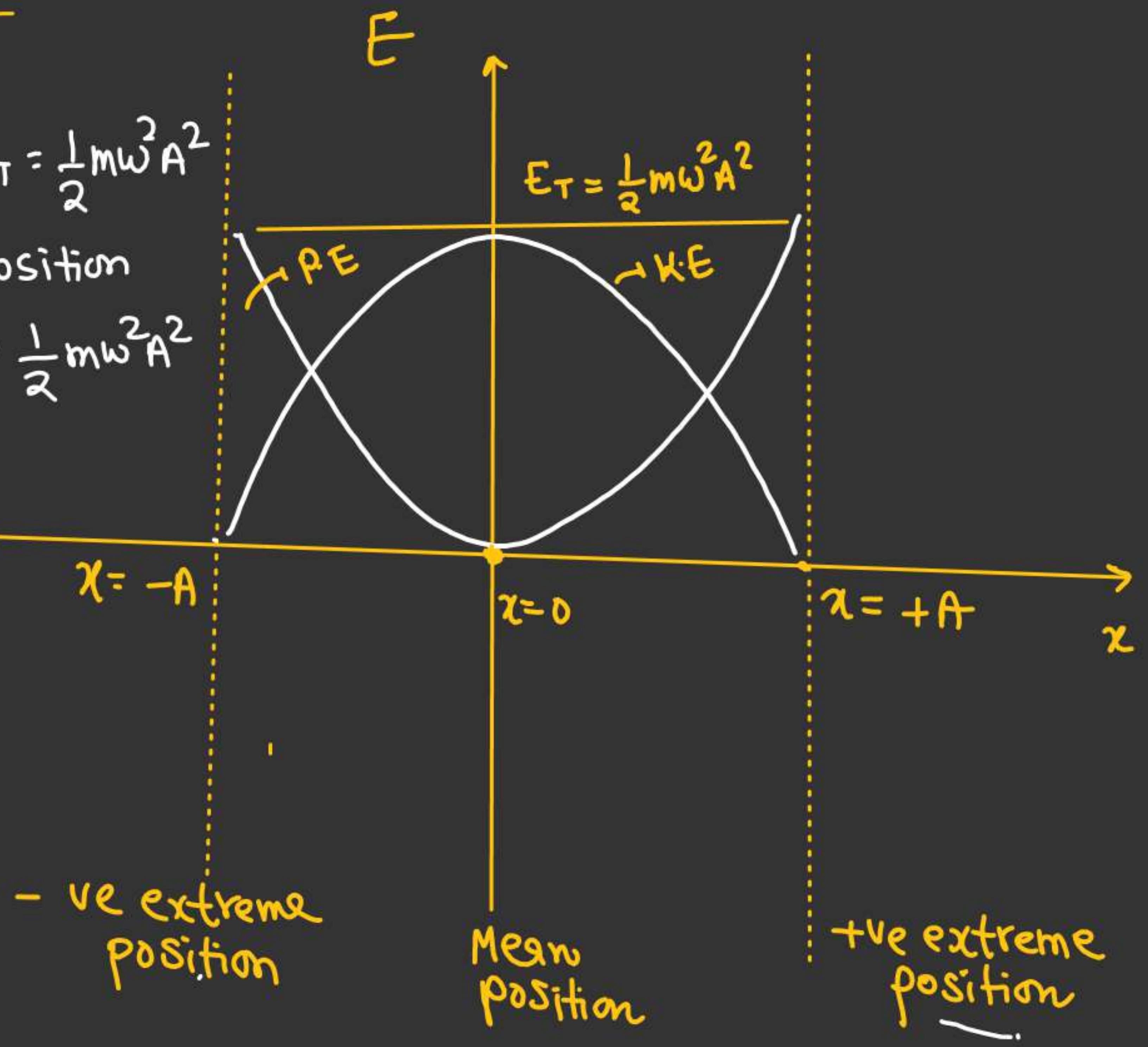
(Parabola opening downward)

At Mean

$$K.E_{max} = E_T = \frac{1}{2}m\omega^2A^2$$

At extreme position

$$P.E_{max} = E_T = \frac{1}{2}m\omega^2A^2$$



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S.H.M

Avg P.E & K.E in S.H.M (In one time period)

$$y = f(x)$$

$$y_{avg} = \frac{\int_{x_i}^{x_f} y \cdot dx}{\int_{x_i}^{x_f} dx}$$

$$P.E = \frac{1}{2} m\omega^2 A^2 \sin^2 \omega t.$$

$$T = \frac{2\pi}{\omega}.$$

$$P.E_{avg} = \frac{\frac{1}{2} m\omega^2 A^2 \int_0^T \sin^2 \omega t \cdot dt}{\int_0^T dt}$$

$$\cos 2\theta = 2\cos^2\theta - 1$$

$$= 1 - 2\sin^2\theta \Rightarrow \sin^2\theta = \frac{1 - \cos 2\theta}{2}$$

$$P.E_{avg} = \frac{\frac{1}{2} m\omega^2 A^2 \int_0^T \left(\frac{1 - \cos 2\omega t}{2}\right) dt}{\int_0^T dt}$$

$$= \frac{1}{2T} m\omega^2 A^2 \left[\frac{1}{2} \int_0^T dt - \frac{1}{2} \int_0^T \cos 2\omega t \cdot dt \right]$$

$$= \frac{m\omega^2 A^2}{2T} \left[\frac{1}{2} \times T - \left[\frac{\sin 2\omega t}{2\omega} \right]_0^{2\pi/\omega} \right]$$

$$\Downarrow$$

$$0.$$

$$\boxed{P.E_{avg} = \frac{m\omega^2 A^2}{4}}$$

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K·E avg in one time period

$$\boxed{K \cdot E_{avg} = \frac{1}{4} m \omega^2 A^2}$$

$\sin^2\theta$ or $\cos^2\theta$ → Period → π

$\sin\theta$ or $\cos\theta$ → Period $= 2\pi$

$$x = A \sin\omega t \rightarrow 2\pi$$

$$K \cdot E = \frac{1}{2} m \omega^2 A^2 \cos^2\omega t \rightarrow \pi$$

Note :- If f be the frequency of particle then frequency of energy i.e (oscillation of energy) be $2f$

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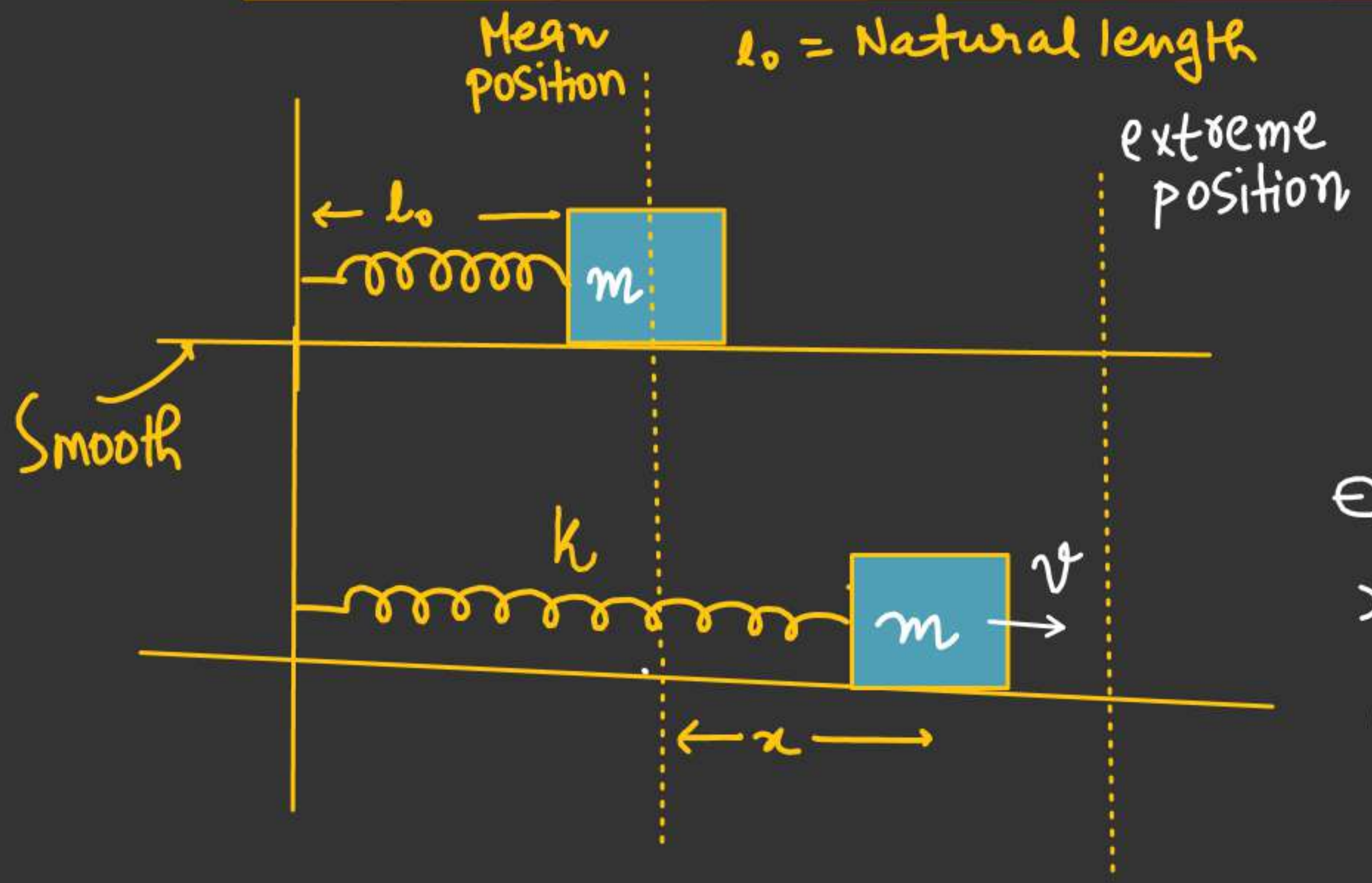
S.H.M

Time period of Spring-block System (ENERGY METHOD)



↳ Approach :- Write the total energy of particle at any intermediate position & differentiate it w.r.t time and apply $\frac{dE_T}{dt} = 0$

E_T when block is at a position x from mean position

$$E_T = \frac{1}{2}mv^2 + \frac{1}{2}Kx^2$$

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$$E_T = \frac{1}{2}mv^2 + \frac{1}{2}Kx^2$$

Differentiating both Side w.r.t time

$$\frac{dE_T}{dt} = \frac{1}{2}m\,\frac{d}{dt}(v^2) + \frac{1}{2}K\,\frac{d}{dt}(x^2)$$

$$= \frac{1}{2}m\left[\frac{d}{dv}(v^2) \times \frac{dv}{dt}\right] + \frac{1}{2}K\left[\frac{d}{dx}(x^2)\right] \times \left(\frac{dx}{dt}\right)$$

$$0 = \frac{1}{2}m\left[2v \times \frac{dv}{dt}\right] + \frac{1}{2}K(2x)\left(\frac{dx}{dt}\right)$$

$$0 = mv\left(\frac{dv}{dt}\right) + Kx\left(\frac{dx}{dt}\right)$$

$$m\cancel{v}\left(\frac{dv}{dt}\right) = -Kx\cancel{\left(\frac{dx}{dt}\right)}$$

$$\Downarrow$$

$$a$$

$$\boxed{a = -\frac{K}{m}x}$$

$$a = -\omega^2 x$$

$$\omega = \sqrt{\frac{K}{m}}$$

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{K}}$$

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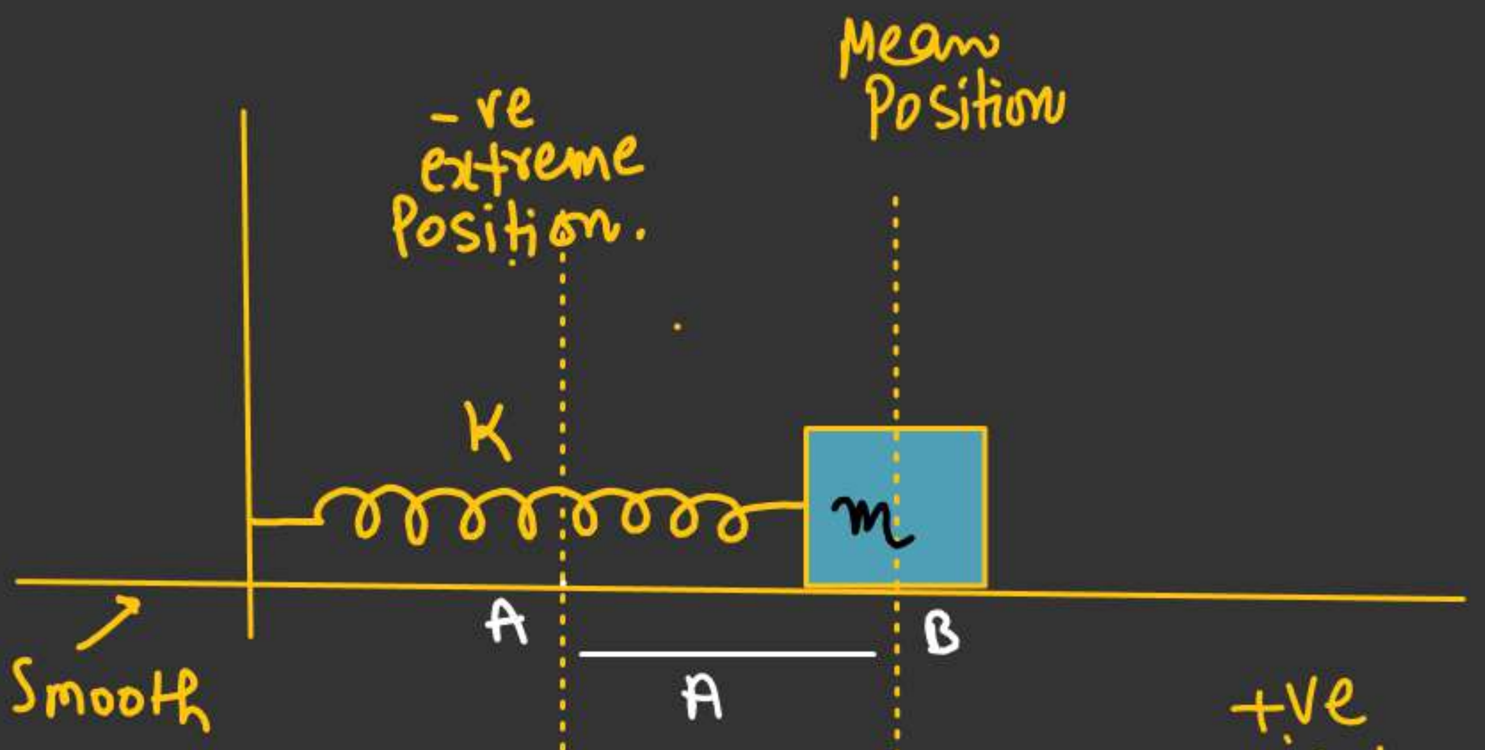
S.H.M

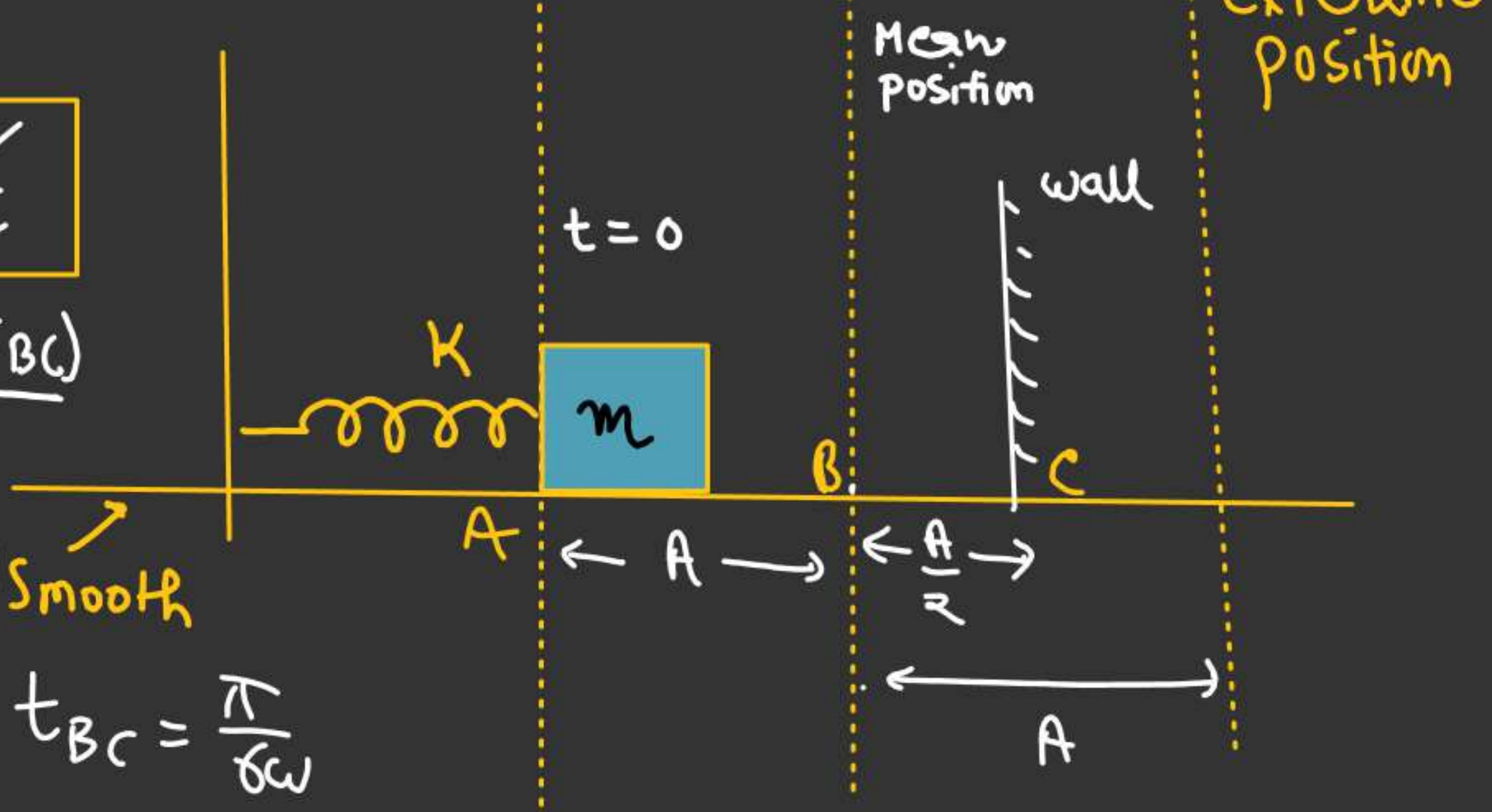
\# Collision b/w block & wall is perfectly elastic.

Block Compressed by A distance and released. Find the time period of block. [Wall is at a distance of A/2 from the mean position].

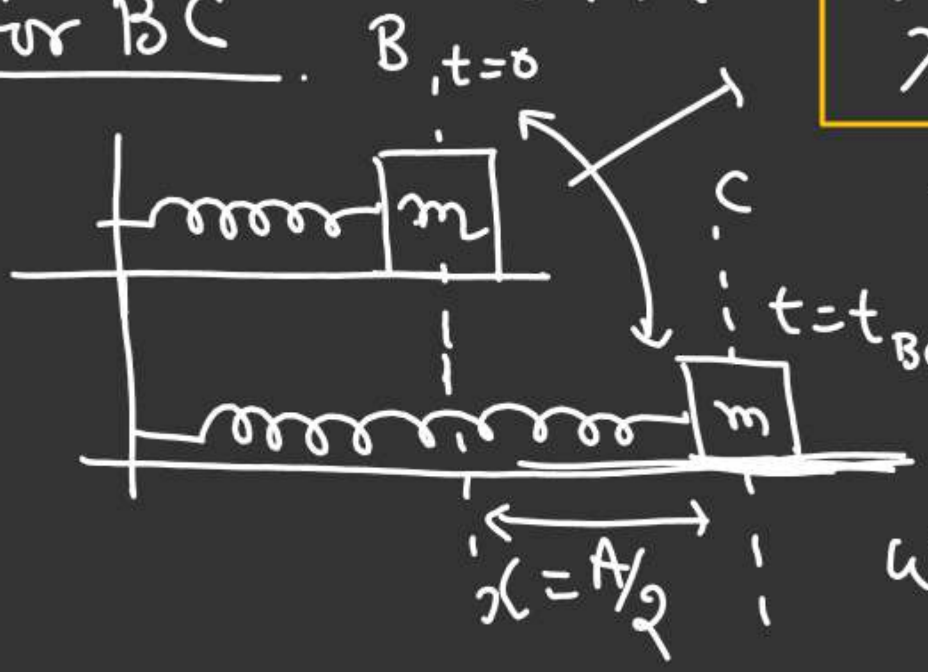


$$t_{AB} = T/4 = \frac{1}{4} \times 2\pi\sqrt{\frac{m}{K}}$$

$$t_{AB} = \frac{\pi}{2}\sqrt{\frac{m}{K}}$$



For BC



$$\boxed{x = A\sin\omega t}$$

$$\frac{\cancel{A}}{2} = \cancel{A}\sin(\omega t_{BC})$$

$$\omega t_{BC} = \sin^{-1}\left(\frac{1}{2}\right)$$

$$\omega t_{BC} = \frac{\pi}{6} \Rightarrow t_{BC} = \frac{\pi}{6\omega}$$

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$$T' = 2\left(t_{AB} + t_{BC}\right) \qquad \omega = \sqrt{\frac{K}{m}}$$

$$T' = 2\left(\frac{\pi}{2}\sqrt{\frac{m}{K}} + \frac{\pi}{6\omega}\right)$$

$$T' = 2\left[\frac{\pi}{2}\sqrt{\frac{m}{K}} + \frac{\pi}{6}\sqrt{\frac{m}{K}}\right]$$

$$T' = \pi\sqrt{\frac{m}{K}}\left[1 + \frac{1}{3}\right]$$

$$\left(T' = \frac{4\pi}{3}\sqrt{\frac{m}{K}}\right) \checkmark$$

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S.H.M

** Time period of two blocks & Spring System

Both the block released simultaneously at $t=0$ from a distance x_1 and x_2 from their mean position

$$x_0 = x_1 + x_2 \quad \text{(1)} \qquad x_0 = \text{(Total Elongation)}$$

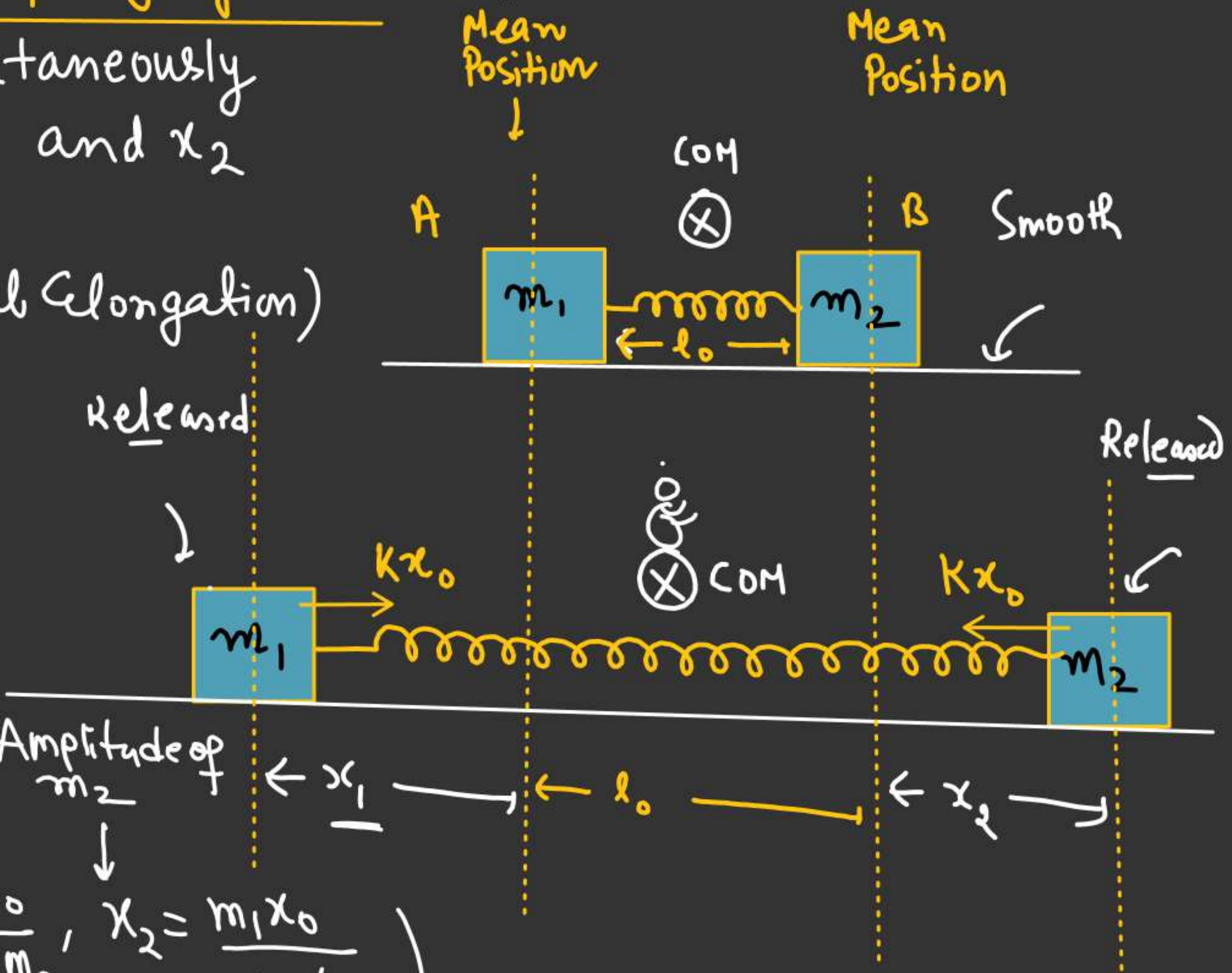
$$\Delta X_{com} = 0$$

$$\frac{m_2 x_2 - m_1 x_1}{m_1 + m_2} = 0$$

$$m_2 x_2 - m_1 x_1 = 0 \quad \text{(2)}$$

$$x_1 = \frac{m_2}{m_1} x_2 \rightarrow \text{Put in (1)}$$

Amplitude of m_1 $\quad \left(x_1 = \frac{m_2 x_0}{m_1 + m_2}, \; x_2 = \frac{m_1 x_0}{m_1 + m_2} \right)$ $\quad$ Amplitude of m_2



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S.H.M

$$F_r = -Kx_0$$

$$\frac{m_2 x_0}{m_1+m_2} = x_1$$

$$x_0 = \frac{x_1(m_1+m_2)}{m_2}$$

$$\frac{m_1 x_0}{m_1+m_2} = x_2$$

$$x_0 = \left(\frac{m_2+m_2}{m_1}\right)x_2 \checkmark$$

For m_1

$$F_r = -K\left(\frac{m_1+m_2}{m_2}\right)x_1$$

$$a_1 = \frac{F_r}{m_1} = -K\left(\frac{m_1+m_2}{m_1 m_2}\right)x_1$$

$$a = -\omega^2 x$$

$$\omega = \sqrt{\frac{K(m_1+m_2)}{m_1 m_2}} \Rightarrow \boxed{T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m_1 m_2}{K(m_1+m_2)}}}$$

For m_2

$$F_r = -Kx_0$$

$$F_r = -K\left(\frac{m_1+m_2}{m_1}\right)x_2$$

$$a_2 = \frac{F_r}{m_2} = -K\left(\frac{m_1+m_2}{m_1 m_2}\right)x_2$$

$$a = -\omega^2 x$$

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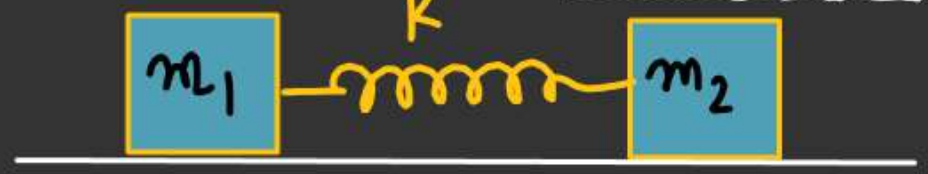
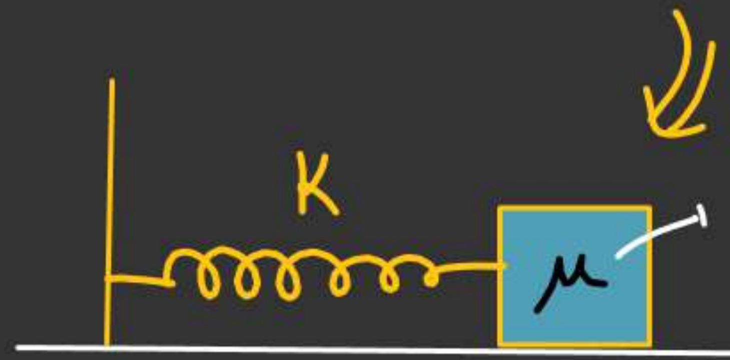
Two block System





$$\frac{m_1 m_2}{m_1+m_2}$$

$$T = 2\pi\sqrt{\frac{\mu}{K}}$$

$$\boxed{T = 2\pi\sqrt{\left(\frac{m_1 m_2}{m_1+m_2}\right)\frac{1}{K}}}$$

S·H·M