Nishant Jindal

CONTINUITY

$e^{\infty} \to \infty$ $\quad e^{-\infty} \to 0$ $\quad e^{-1/h - 1/h} = e^{-2/h}$

Q Type 2 $f(x) = \begin{cases} \lim_{h\to 0} \dfrac{e^{1/x} - e^{-1/x}}{e^{1/x} + e^{-1/x}} & x \neq 0 \\ 0 & x = 0 \end{cases}$

Check Cont'y of fxn at $x=0$?

Chor fxn

Aakash Sharma

L.V.$\Rightarrow \lim_{x\to 0} \dfrac{e^{1/x} - e^{-1/x}}{e^{1/x} + e^{-1/x}}$

LHL $\boxed{x = 0-h}$

$\lim_{h\to 0} \dfrac{e^{-1/h} - e^{1/h}}{e^{-1/h} + e^{+1/h}}$

$\lim_{h\to 0} \dfrac{\cancel{e^{1/h}}(e^{-2/h} - 1)}{\cancel{e^{1/h}}(e^{-2/h} + 1)} = \dfrac{e^{-\infty} - 1}{e^{-\infty} + 1} = \dfrac{0-1}{0+1} = -1$

RHL $\boxed{x = 0+h}$

$e^{1/h}\left(1 - \dfrac{e^{-1/h}}{e^{1/h}}\right)$ / $e^{1/h}\left(1 + \dfrac{e^{-1/h}}{e^{1/h}}\right)$

$\lim_{h\to 0} \dfrac{e^{1/h} - e^{-1/h}}{e^{1/h} + e^{-1/h}} = \dfrac{\cancel{e^{1/h}}(1 - e^{-2/h})}{\cancel{e^{1/h}}(1 + e^{-2/h})}$

$\dfrac{1 - e^{-\infty}}{1 + e^{-\infty}} = \dfrac{1-0}{1+0} = 1$

L.V. = DNE

$\therefore$ fxn is D.C. at $x=0$

Q $f(x) = \begin{cases} \dfrac{e^{1/x} - e^{-1/x}}{e^{1/x} + e^{-1/x}} & x \neq 0 \\ K & x = 0 \end{cases}$

If fxn is Cont's at $x=0$ then K?

Sol If fxn is Cont's at $x=0$

then $K = \lim_{x\to 0} \dfrac{e^{1/x} - e^{-1/x}}{e^{1/x} + e^{-1/x}}$

LDNE

$\Rightarrow \therefore K = \phi$

Nishant Jindal

CONTINUITY

Q If $f(x) = \begin{cases} \lim\limits_{x \to 0} \dfrac{2^{1/x} - 1}{2^{1/x} + 1} & x \neq 0 \\ 1 & x = 0 \end{cases}$

Type 2

$2^{\infty} = \infty$

$2^{-\infty} = 0$

Check cont^y of fxn at $x = 0$.

Chor fxn Aakarshan

$$L.V. \Rightarrow \lim_{x \to 0} \frac{2^{1/x} - 1}{2^{1/x} + 1}$$

LHL $x = 0 - h$

$$\lim_{h \to 0} \frac{2^{-1/h} - 1}{2^{-1/h} + 1} = \frac{2^{-\infty} - 1}{2^{-\infty} + 1}$$

$$= \frac{0 - 1}{0 + 1} = -1$$

RHL $x = 0 + h$

$\rightarrow \infty$

$$\lim_{h \to 0} \frac{2^{1/h} - 1}{2^{1/h} + 1} = \lim_{h \to 0} \frac{\cancel{2^{1/h}}\left(1 - 2^{-1/h}\right)}{\cancel{2^{1/h}}\left(1 + 2^{-1/h}\right)}$$

$$= \frac{1 - 2^{-\infty}}{1 + 2^{-\infty}} = \frac{1 - 0}{1 + 0} = 1$$

LHL ≠ RHL

$\Rightarrow$ L. D. N. E.

$\Rightarrow$ Not Cont.

CONTINUITY

Repeated

Q Find $f(0)$ if Possible

Type 1

if $f(x) = \begin{cases} \dfrac{\ln \cos x}{\sqrt[4]{1+x^2} - 1} & x > 0 \text{ RHL} \\ \dfrac{e^{\sin x} - 1}{\ln(1+\tan 2x)} & x < 0 \text{ LHL} \end{cases}$

is cont^s at $x = 0$?

$f(0) = \text{LHL} = \text{RHL}$

$\frac{1}{2} \neq -2$

$\therefore f(0) = \text{D.N.E.}$

LHL $\lim\limits_{x \to 0^-} \cancel{\dfrac{e^{\sin x} - 1}{\sin x}} \times \dfrac{\sin x}{\tan 2x} \times \cancel{\dfrac{\tan 2x}{\ln(1+\tan 2x)}}$

$1 \times 1 \times \frac{x}{2x} = \frac{1}{2}$

RHL $\lim\limits_{x \to 0^+} \dfrac{\ln \cos x}{\underbrace{(1+x^2)^{1/4} - 1}_{BT}} = \lim\limits_{x \to 0^+} \dfrac{\ln(1-(1-\cos x))}{\cancel{1} + \frac{x^2}{4} \cancel{- 1}}$

$= \lim\limits_{x \to 0^+} \cancel{\dfrac{\ln(1-(1-\cos x))}{-(1-\cos x)}} \times \cancel{\dfrac{-(1-\cos x)}{x^2}} \times \dfrac{\cancel{x^2}}{\frac{\cancel{x^2}}{4}}$

$-2 \Leftarrow 1 \times -\frac{1}{2} \times 4$

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CONTINUITY

Q If $f(x)=\frac{3x^2+ax+a+3}{x^2+x-2}$ is cnt^s at $\boxed{x=-2}$

Type 3

then $f(-2)$ & $f(0)$ & $\boxed{a}=?$

In T-3 Qs.
if fxn is cnt^s
at $x=a$ (given)
then $f(a)=\lim_{x\to a} f(x)$

$$f(-2)=\lim_{x\to -2}\frac{3x^2+ax+a+3}{(x+2)(x-1)}\to\frac{0}{0}$$ Ban gya then Qs can be Saved otherwise ∞ is coming

$3x^2+ax+a+3=0$ at $x=-2$

$3(-2)^2-2a+a+3=0$

$12-a+3=0$

$\boxed{a=15}$

$$=\lim_{x\to -2}\frac{3x^2+15x+18}{x^2+x-2}\quad \frac{0}{0}\ \text{DL}$$

$$\lim_{x\to -2}\frac{6x+15}{2x+1}=\frac{-12+15}{-4+1}=\frac{3}{-3}$$

$f(-2)=-1$

③ $f(0)=?$

$$f(x)=\frac{3x^2+15x+18}{x^2+x-2}$$

$x=0$

$$f(0)=\frac{0+0+18}{0+0-2}=-9$$

CONTINUITY

Q A fxn is defined as $f(x) = \lim_{x\to 0} \frac{\cos(\sin x) - \cos x}{x^2}$; $\boxed{x \neq 0}$ & $f(0) = a$

Type 3

if fxn is cnt^s at $x = 0$ then $a = ?$

as fxn is given cnt^s at $x = 0$

$$\therefore f(0) = \lim_{x\to 0} \frac{\cos(\sin x) - \cos x}{x^2}$$

$$= \lim_{x\to 0} \frac{-2\sin\left(\frac{\sin x + x}{2}\right)\cdot \sin\left(\frac{\sin x - x}{2}\right)}{x^2}$$

$$= -2\lim_{x\to 0} \frac{\sin\left(\frac{\sin x + x}{2}\right)}{\left(\frac{\sin x + x}{2}\right)} \times \frac{\sin\left(\frac{\sin x - x}{2}\right)}{\left(\frac{\sin x - x}{2}\right)} \quad \frac{\frac{\sin x + x}{2} \times \frac{\sin x - x}{2}}{x^2}$$

$$= -2\times 1\times 1\times \lim_{x\to 0} \frac{\sin^2 x - x^2}{4x^2} \quad \frac{0}{0}$$

$$-2\lim_{x\to 0} \frac{x^2 - x^2}{4x^2}$$

$$-2\times 0$$

$$= 0$$

$$-2\lim_{x\to 0} \frac{2\sin x\cos x - 2x}{8x} \quad \frac{0}{0} \text{ DL}$$

$$-\frac{2}{8}\lim_{x\to 0} \frac{2\cos(2x) - 2}{1}$$

$$-\frac{2}{8}\times(2\times 1 - 2) = 0$$

$$\boxed{a = 0}$$

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CONTINUITY

$e^{\frac{a}{b}} = 1 = e^0 \Rightarrow a = 0$

Repeat

Q Find a & b ($=-1$) if $y = f(x)$ Cont^s at $x = \frac{\pi}{2}$ where

Type

$$f(x) = \begin{cases} \left(\frac{6}{5}\right)^{\frac{\tan 6x}{\tan 5x}} & 0 < x < \frac{\pi}{2} \text{ LHL} \\ b+2 & x = \frac{\pi}{2} \rightarrow f\left(\frac{\pi}{2}\right) \\ \left(1+|\cos x|\right)^{\frac{a|\tan x|}{b}} & \frac{\pi}{2} < x \le \pi \rightarrow \text{RHL} \end{cases}$$

$e^{\frac{a}{b} \times \frac{\sec^2 x}{\sec x \tan x}}$ $\quad \lim \frac{a}{b} \sin x = \lim_{x \to \frac{\pi}{2}} \frac{a}{b} \sin x$

$e^{\frac{a}{b \times 1}} = e^{\frac{a}{b}}$

is Cont^s at $x \in (0, \pi]$

as $f$xn is Cont^s in $(0, \pi]$

$\Rightarrow$ It will be Cont^s at $x = \frac{\pi}{2}$ also

$f\left(\frac{\pi}{2}^-\right) = f\left(\frac{\pi}{2}\right) = f\left(\frac{\pi}{2}^+\right)$

LHL $= f\left(\frac{\pi}{2}\right) =$ RHL

$1 = b+2 = e^{a/b}$

$$\text{LHL} \Rightarrow \lim_{x \to \frac{\pi}{2}^-} \left(\frac{6}{5}\right)^{\frac{\tan 6x}{\tan 5x}} = \lim_{h \to 0} \left(\frac{6}{5}\right)^{\frac{\tan 6\left(\frac{\pi}{2} - h\right)}{\tan 5\left(\frac{\pi}{2} - h\right)}}$$

$x = \frac{\pi}{2} - h$

$$\Rightarrow \lim_{h \to 0} \left(\frac{6}{5}\right)^{\frac{\tan(3\pi - 6h)}{\tan\left(\frac{5\pi}{2} - 5h\right)}} \Rightarrow \lim_{h \to 0} \left(\frac{6}{5}\right)^{\frac{-\tan 6h}{+\cot 5h}}$$

$$= \left(\frac{6}{5}\right)^{-\frac{\tan 0}{\cot 0}} = \left(\frac{6}{5}\right)^{\frac{0}{\infty}} = \left(\frac{6}{5}\right)^0 = 1$$

$\frac{5\pi}{2}$, 3π

$$\text{RHL} \lim_{\boxed{x \to \frac{\pi}{2}^+}} \left(1 + |\cos x|\right)^{\frac{a|\tan x|}{b}}$$

Quad $= 2^{nd}$

$\cos x = -ve$

$\tan x = -ve$

$$\lim_{x \to \frac{\pi}{2}^+} \left(1 - \cos x\right)^{\frac{-a \tan x}{b}} \rightarrow 1^\infty$$

$+\frac{a}{b} \tan x \left(\cancel{1} + \cos x - \cancel{1}\right)$

$e^{\lim_{x \to \frac{\pi}{2}} \frac{a}{b} \tan x \times \cos x} \rightarrow \infty \times 0$

$e^{\lim_{x \to \frac{\pi}{2}} \frac{a}{b} \times \frac{\tan x}{\sec x}} \quad \frac{\infty}{\infty}$

CONTINUITY

Q Find a, if $f(x) = \begin{cases} \dfrac{1-\cos 4x}{x^2} & x<0 \text{ LHL} \\ \dfrac{\sqrt{x}}{\sqrt{16+\sqrt{x}}-4} & x>0 \\ a & x=0 \end{cases}$ Cont^s at $x=0$

is Cont^s at $x=0$?

LHL $\lim\limits_{x\to 0^-} \dfrac{1-\cos 4x}{x^2} = \dfrac{4^2}{2} = 8$

$f(0) = f(0^+) = f(0^-)$

$a = 8 = 8$

$\Rightarrow \boxed{a=8}$

RHL $\lim\limits_{x\to 0^+} \dfrac{\sqrt{x}}{(16+\sqrt{x})^{1/2}-4} = \dfrac{\sqrt{x}}{4\left\{\left(1+\dfrac{\sqrt{x}}{16}\right)^{1/2}-1\right\}}$ BT

$= \lim\limits_{x\to 0^+} \dfrac{\sqrt{x}}{4\left\{\cancel{1}+\dfrac{\sqrt{x}}{32}-\cancel{1}\right\}} = \dfrac{32}{4} = 8$

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CONTINUITY

T2

Q Find A, B & K if $f(x)=\begin{cases} \dfrac{\sin 3x+A\sin 2x+B\sin x}{x^5} & x\neq 0 \\ K & x=0 \end{cases}$ is conts at $x=0$?

as fxn is conts $\Rightarrow K=\lim_{x\to 0}\dfrac{\sin 3x+A\sin 2x+B\sin x}{x^5}$

$$K=\lim_{x\to 0}\frac{\left(3x-\frac{(3x)^3}{\lfloor 3}+\frac{(3x)^5}{\lfloor 5}\right)+A\left(2x-\frac{(2x)^3}{\lfloor 3}+\frac{(2x)^5}{\lfloor 5}\right)+B\left(x-\frac{x^3}{\lfloor 3}+\frac{x^5}{\lfloor 5}\right)}{x^5}$$

$$=\lim_{x\to 0}\frac{\cancel{x}(3+2A+B)}{\cancel{x^5}x^4}-\frac{\frac{\cancel{x^3}}{6}(27+8A+B)}{\cancel{x^5}x^2}+\frac{\frac{\cancel{x^5}}{120}(243+32A+B)}{\cancel{x^5}}$$

$3+2A+B=0$

$27+8A+B=0$

$A=-4 \quad B=5$

$$K=\frac{243+32A+B}{120}=1$$

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CONTINUITY

Q If $f(x)$ is Cont^s at $x=0$

& $f\left(\frac{1}{(2023)^n}\right) = \cos((2020)^n) \cdot (2021)^{-n} + \frac{n^2+n+5}{n^2-n+1}$

find $f(0) = ?$

Demand Kyah!!

$f(0)$

fxn में 0 कैसे करेगा?

n को Kis value पर 0 करेगा

$$\lim_{n\to\infty} f\left(\frac{1}{(2023)^n}\right) = \lim_{n\to\infty} \frac{\cos(2020)^n}{(2021)^n} + \frac{n^2+n+5}{n^2-n+1}$$

$$f(0) = \frac{\cos\infty}{\infty} + \lim_{n\to\infty} \frac{n^2+n+5}{n^2-n+1} \quad \frac{\infty}{\infty}$$

$$= \frac{(-1 \text{ to } +1)}{\infty} + 1 = 0 + 1 = 1$$

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CONTINUITY

Finest

Q $f(x)=\begin{cases} 3-\left[\cot^{-1}\left(\dfrac{2x^3-3}{x^2}\right)\right]=0 & x>0 \quad \text{RHL} \\ 0 & x=0 \\ \{x^2\}\cos\left(e^{1/x}\right)= & x<0 \quad \text{LHL} \end{cases}$

$\{.4\}=.4$

$\{h\}=h=\{h^2\}=h^2$

Check Cont^y at $x=0$

Stttvaddyyy

LHL

$\lim_{x\to 0^-}\{x^2\}\cos\left(e^{\frac{1}{x}}\right)$

$x=0-h$

$\lim_{h\to 0}\{(-h)^2\}\cos\left(e^{\frac{1}{-h}}\right)$

$\lim_{h\to 0}\{h^2\}\cos\left(e^{-1/h}\right)$

$\lim_{h\to 0}h^2\cos\left(e^{-1/h}\right)=0\times\cos\left(e^{-\infty}\right)=0\times\cos(0)=0\times 1=0$

RHL $\lim_{x\to 0^+} 3-\left[\cot^{-1}\left(\dfrac{\overset{-ve}{\boxed{2x^3-3}}}{\underset{\to 0}{x^2}}\right)\right]$

$\lim_{x\to 0^+} 3-\left[\cot^{-1}(-\infty)\right]$

$\lim_{x\to 0^+} 3-[\pi]=3-[3.14]$

$=3-3$

$=0$

$x\to 0^+$

$x^3\to 0^+$

$2x^3\to 0$

$\underset{0}{\boxed{2x^3}}\underset{-3}{-3}\to -ve$

$\dfrac{2x^3-3}{0}=\dfrac{-ve}{0}\to-\infty$

$\text{RHL}=\text{LHL}=f(0)=0$

$\Rightarrow$ Conts

CONTINUITY

Q If $f(x)=\begin{cases} \lim\limits_{x\to 0}\dfrac{a^{2[x]+\{x\}}-1}{2[x]+\{x\}} & x\neq 0 \\ \log_e a & x=0 \end{cases}$ Check Cont^y at $x=0$?

T2

$$L.V. = \lim_{x\to 0}\frac{a^{2[x]+\{x\}}-1}{2[x]+\{x\}}$$

Aakarsham

$$LHL \Rightarrow \lim_{h\to 0}\frac{a^{2[-h]+\{-h\}}-1}{2[-h]+\{-h\}}$$

St. Limit Ahhhh!!

$$\lim_{h\to 0}\frac{a^{-2+1-h}-1}{-2+1-h} = \lim_{h\to 0}\frac{a^{-1-h}-1}{-1-h}$$

$$= \frac{a^{-1}-1}{-1} = 1-\frac{1}{a} \neq \log_e a$$

fxn is D.C.

$$RHL \lim_{h\to 0}\frac{a^{2[h]+\{h\}}-1}{2[h]+\{h\}}$$

$$\lim_{h\to 0}\frac{a^{0+h}-1}{h} = \log_e a$$

$LHL \neq f(0) = RHL$

It is D.C. But Rt. Cont^s

$\boxed{\text{Right hand Cont}^s}$

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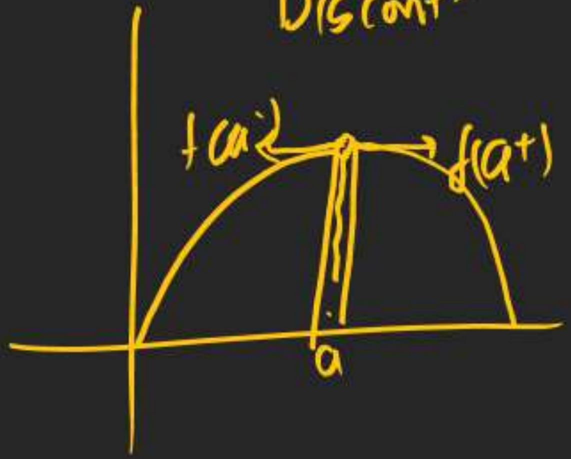
CONTINUITY

Type of Discontinuity

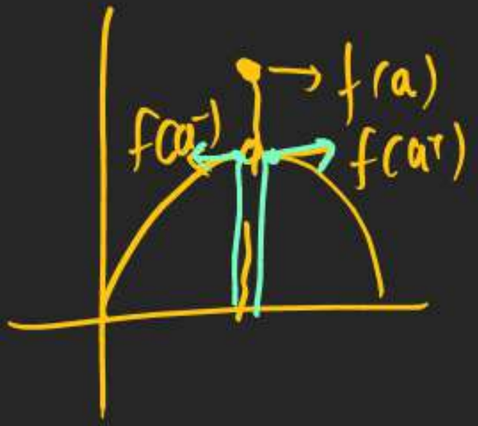
Removable
LHL = RHL

(1st Kind)

- Missing Pt. Disconts



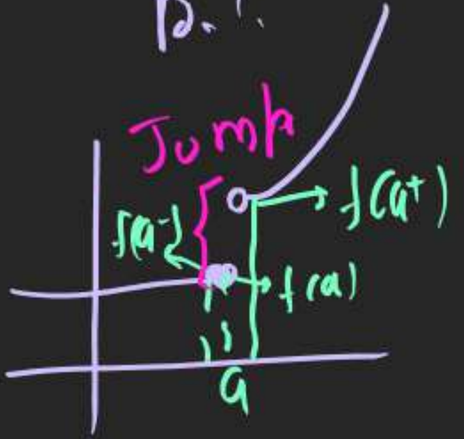
- Isolated Discont



Non Removable
LHL ≠ RHL

(2nd Kind)

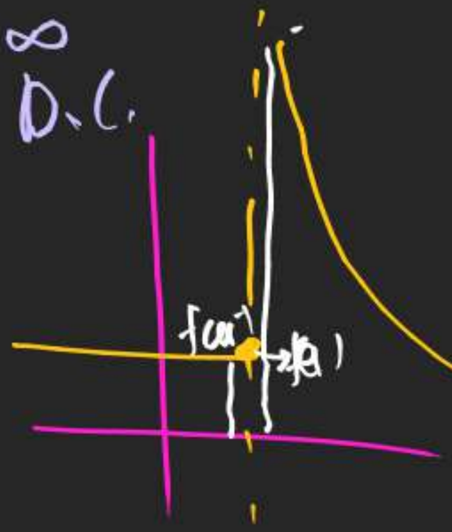
- finite D.C.



$f(a^-) \neq f(a^+)$

Jump = |LHL − RHL|

- ∞ D.C.



- Oscillating D.C