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$x^2+5x+2=0 \rightarrow \alpha, \beta.$

New Eqn → New Roots $\alpha+1, \beta+1$

$y=\alpha+1 \Rightarrow \alpha=y-1$

$\alpha^2+5\alpha+2=0$

$(y-1)^2+5(y-1)+2=0$

$y^2-2y+1+5y-5+2=0$

$y^2+3y-2=0 \Rightarrow \boxed{x^2+3x-2=0}$

(2) New Roots → $5\alpha-3, 5\beta-3$

$y=5\alpha-3 \Rightarrow \alpha=\frac{y+3}{5}$

$\left(\frac{y+3}{5}\right)^2+\cancel{5}\left(\frac{y+3}{\cancel{5}}\right)+2=0$

$(y+3)^2+25y+125=0$

$y^2+31y+134=0$

$x^2+31x+134=0$

$x^2+5x+2=0 \rightarrow \alpha, \beta.$

Eqn whose Roots

1) $\alpha+1, \beta+1$

2) $5\alpha-3, 5\beta-3.$

3) $\alpha+\frac{1}{\beta}, \beta+\frac{1}{\alpha}.$

$y=\alpha+\frac{1}{\beta}$

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3) New Root $\Rightarrow \alpha+\frac{1}{\beta}, \beta+\frac{1}{\alpha}$.

By Method 1

$$x^2-(SOR)x+POR=0$$

$$x^2-\left(\alpha+\beta+\frac{1}{\alpha}+\frac{1}{\beta}\right)x+\left(\alpha+\frac{1}{\beta}\right)\left(\beta+\frac{1}{\alpha}\right)=0$$

$$x^2-\left((\alpha+\beta)+\left(\frac{\alpha+\beta}{\alpha\beta}\right)\right)x+\alpha\beta+\frac{1}{\alpha\beta}+2=0$$

$$x^2-\left(-5+\frac{-5}{2}\right)x+2+\frac{1}{2}+2=0$$

$\alpha+\beta=-5$

$\alpha\cdot\beta=2$ $\quad x^2+5x+2=0 \rightarrow \alpha, \beta$

Eqn whose Roots

1) $\alpha+1, \beta+1$

2) $5\alpha-3, 5\beta-3$.

3) $\alpha+\frac{1}{\beta}, \beta+\frac{1}{\alpha}$.

$y=\alpha+\frac{1}{\beta}$

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QUADRATIC EQUATION

Q Let $a, b \in R$ be Such that eqⁿ

2022 Mains

$ax^2 - 2bx + 15 = 0$ has Repeated Roots α.

Qs If α & β are Roots of $x^2 - 2bx + 21 = 0$ then $\alpha^2 + \beta^2 = ?$

$$\boxed{\text{Repeated Roots} = \text{Equal Root} = \text{Identical Root} = \text{Coincident Roots}}$$

$ax^2 - 2bx + 15 = 0 \to \alpha, \alpha$

$2\alpha = \frac{2b}{a}$

$a = \frac{b}{\alpha}$

$\alpha^2 = \frac{15}{a}$

$\alpha^{\cancel{2}} = \frac{15 \times \cancel{\alpha}}{b}$

$\alpha = \frac{15}{b}$

$x^2 - 2bx + 21 = 0 \to \alpha, \beta$

$\alpha^2 - 2b\alpha + 21 = 0$

$\left(\frac{15}{b}\right)^2 - 2\cancel{b} \times \frac{15}{\cancel{b}} + 21 = 0$

$\frac{225}{b^2} - 9 = 0 \Rightarrow \frac{\cancel{225}^{25}}{b^2} = \cancel{9}$

$b = 5$

Demand $= \alpha^2 + \beta^2$

$= (\alpha + \beta)^2 - 2\alpha\beta$

$= (2b)^2 - 2 \times 21$

$= (10)^2 - 42 = 58$

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QUADRATIC EQUATION

Eqn Vs Identity.

1) If any Poly (deg n) has $\boxed{\text{No of Roots} > \text{its degree}}$

than it is an Identity.

Ex: → $(x+2)^2 = x^2 + 4x + 4$

$x = -1 \rightarrow (-1+2)^2 = (-1)^2 + 4 \times -1 + 4$

$1 = 1 - \cancel{4} + \cancel{4}$ ✓

$x = 3 \rightarrow (3+2)^2 = 3^2 + 4 \times 3 + 4$

$25 = 9 + 12 + 4$ ✓

$x = 0 \rightarrow (0+2)^2 = 0^2 + 4 \times 0 + 4$

$4 = 4$ ✓

$x^2 + 4x + 4 = x^2 + 4x + 4$

$x^2 - x^2 + 4x - 4x + 4 - 4 = 0$

$x^2(1-1) + 4x(1-1) + 4(1-1) = 0$

$Ax^2 + Bx + C = 0$

$\boxed{\text{Q Eqn } Ax^2 + Bx + C = 0 \text{ becomes an Identity when } A = B = C = 0}$

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QUADRATIC EQUATION

Q Value of P for which $(p^2-4)x^2+(p^2-3p+2)x+(p^2-5p+6)=0$ is an Identity.

It is an Identity when.

$p^2-4=0$

$p=\boxed{2},-2$

$p^2-3p+2=0$

$(p-2)(p-1)=0$

$p=1,\boxed{2}$

$p^2-5p+6=0$

$(p-2)(p-3)=0$

$p=\boxed{2},3.$

$\boxed{p=2}$

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QUADRATIC EQUATION

Q. P.T.

$$\frac{(x-a)(x-b)}{(c-a)(c-b)}+\frac{(x-b)(x-c)}{(a-b)(a-c)}+\frac{(x-c)(x-a)}{(b-c)(b-a)}=1$$

Is it a Quad? Quad?? Quad??

as it is clear that No of values of x Satisfying > deg of Poly

So it is an Identity

① By Observation It is a Quad ⇒ ∴ It must has 2 Roots

(2) put $x=a$ $\frac{(a-a)\cancel{(a-b)}}{\cancel{(c-a)}(c-b)}+\frac{\cancel{(a-b)}\cancel{(a-c)}}{\cancel{(a-b)}\cancel{(a-c)}}+\frac{(a-c)\cancel{(a-a)}}{\cancel{(b-c)}(b-c)}=1 \Rightarrow 1=1$

put $x=b$ $\frac{(b-a)\cancel{(b-b)}}{\cancel{(c-a)}(c-b)}+\frac{(b-b)\cancel{(b-c)}}{\cancel{(a-b)}(a-c)}+\frac{\cancel{(b-c)}\cancel{(b-a)}}{\cancel{(b-c)}\cancel{(b-a)}}=1 \Rightarrow 1=1$

$x=c \rightarrow$ $\frac{(c-a)(c-b)}{(c-a)(c-b)}+\frac{(c-b)\cancel{(c-c)}}{\cancel{(a-b)}(a-c)}+\frac{(c-c)\cancel{(c-a)}}{\cancel{(b-c)}(b-a)}=1 \Rightarrow 1=1$

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QUADRATIC EQUATION

Q Find the condition for which $ax^2+bx+c=0$ has ① one Root sq^n of another.

$ax^2+bx+c=0 \to \alpha,\ \alpha^2$

let Roots are α & α^2

① $\alpha+\alpha^2=-\frac{b}{a}$

② $\alpha\cdot\alpha^2=\frac{c}{a}$

$\alpha^3=\frac{c}{a}$

$(\alpha+\alpha^2)^3=-\frac{b^3}{a^3} \Rightarrow \alpha^3+(\alpha^2)^3+3\alpha\cdot\alpha^2(\alpha+\alpha^2)=-\frac{b^3}{a^3}$

$\alpha^3+(\alpha^3)^2+3\alpha^3(\alpha+\alpha^2)=-\frac{b^3}{a^3}$

$\frac{c}{a}+\frac{c^2}{a^2}+3\frac{c}{a}\left(-\frac{b}{a}\right)=-\frac{b^3}{a^3}$

$a^2c+ac^2-3abc=-b^3$

$$\boxed{a^2c+ac^2+b^3=3abc}$$

is the Required Cond^n for one Root sq^r of other.

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QUADRATIC EQUATION

$(a+b)^3 = a^3+b^3+3ab(a+b)$

$ax^2+bx+c=0 \to \alpha, \alpha^2$

$\alpha+\alpha^2 = -\frac{b}{a}$ | $\alpha\cdot\alpha^2 = \frac{c}{a}$

$\alpha^3 = \frac{c}{a}$

$(\alpha+\alpha^2)^3 = -\frac{b^3}{a^3}$

$\alpha^3+(\alpha^2)^3+3\cdot\underline{\alpha\cdot\alpha^2}(\alpha+\alpha^2) = -\frac{b^3}{a^3}$

LCM a^3

$\frac{c}{a}+\frac{c^2}{a^2}+3\frac{c}{a}\left(-\frac{b}{a}\right) = -\frac{b^3}{a^3}$

$a^2c+ac^2-3abc = -b^3$

$\underline{a^2c+ac^2+b^3 = 3abc}$

(2) When one Root is K times of other.

$ax^2+bx+c=0 \to \alpha, K\alpha$

① $\alpha+K\alpha = -\frac{b}{a}$ ② $\alpha\cdot K\alpha = \frac{c}{a}$

$\alpha(1+K) = -\frac{b}{a}$ | $\boxed{\alpha^2}K = \frac{c}{a}$

$\alpha = \frac{-b}{a(1+K)}$

$\Rightarrow \frac{b^2}{a^2(1+K)^2}\times K = \frac{c}{a}$

$$\boxed{\Rightarrow \frac{b^2}{ac} = \frac{(1+K)^2}{K}}$$

Required Condⁿ When one in K times of other.

QUADRATIC EQUATION

Q Find Cond^n for $ax^2+bx+c=0$

① When one Root is sq^n of other.

Cond^n $\rightarrow a^2c+ac^2+b^3=3abc$

② When one Root is K times of other.

$$\frac{b^2}{ac}=\frac{(1+K)^2}{K}$$

Q If one Root of $x^2-x-K=0$ is sq^n of other then K?

$a=1,\ b=-1,\ c=-K$

$a^2c+ac^2+b^3=3abc$

$1^2\cdot(-K)+1\cdot(-K)^2+(-1)^3=3\times1\times-1\times-K$

$-K+K^2-1=3K$

$\Rightarrow K^2-4K-1=0$

$$\Rightarrow K=\frac{4\pm\sqrt{16+4}}{2}$$

$=2\pm\sqrt{5}$

$K=2+\sqrt{5}$ or $K=2-\sqrt{5}$

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QUADRATIC EQUATION

Q If one Root of $x^2+px+q=0$ is sq of other then find $p^3+q^2+q(1-3p)=$?

Ans $=0$

$$x^2+px+q=0$$

$a=1,\ b=p,\ c=q$

$$a^2c+ac^2+b^3=3abc$$

$$1^2\cdot q+1\cdot q^2+p^3=3\times1\times p\times q$$

$$p^3+q^2+q-3pq=0$$

$$p^3+q^2+q(1-3p)=0$$

Q If one Root of Q Eqn $x^2-30x+K=0$ is sq of other then $K=$?

Mains

$$x^2-30x+K=0 \quad \begin{matrix}\to\alpha\\ \to\alpha^2\end{matrix}$$

$\alpha+\alpha^2=30$

Yhan se α aa Jayega!!

$\alpha\cdot\alpha^2=K$

$\alpha^3=K$

$\alpha^2+\alpha-30=0$

$(\alpha+6)(\alpha-5)=0$

$\alpha=5,-6$

$K=(5)^3,(-6)^3$

$=125,\ 216$

QUADRATIC EQUATION

Q If α, β are Roots of Eqn $\rightarrow 5x^2+mx+12=0$

good

Which are in Ratio $\boxed{2:3}$ then $m=?$

$$5x^2+mx+12=0 \rightarrow \alpha,\ \boxed{\frac{2}{3}}\alpha$$

K

$a=5,\ b=m,\ c=12$

$$\Rightarrow \frac{(m)^2}{5\times 12} = \frac{\left(\frac{2}{3}+1\right)^2}{\frac{2}{3}}$$

$$\Rightarrow m^2 = \cancel{60}^{10} \times \frac{25}{\cancel{9}_{3}} \times \frac{3}{\cancel{2}}$$

$$m^2 = 250$$

$$m = \pm\sqrt{250}$$

$\beta = \frac{2}{3}\alpha$.

$$\frac{\beta}{\alpha} = \frac{2/3\,\cancel{\alpha}}{\cancel{\alpha}} = \frac{2}{3}$$

When one root is K times of other

$$\alpha,\ \boxed{K}\alpha \rightarrow \frac{b^2}{ac} = \frac{(K+1)^2}{K}$$

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QUADRATIC EQUATION

100 → 30 Qs बाकी, 70 done

Q Find value of a for which one Root of Eqⁿ → $(a^2-5a+3)x^2+(3a-1)x+2=0$ is twice as Large as other.

$$\frac{b^2}{ac}=\frac{(K+1)^2}{K}$$

$$\underbrace{(a^2-5a+3)}_{a}x^2+\underbrace{(3a-1)}_{b}x+\underbrace{2}_{c}=0 \to \alpha, 2\alpha \quad K=2$$

$$\frac{(3a-1)^2}{(a^2-5a+3)2}=\frac{(2+1)^2}{2}$$

$$\cancel{9a^2}-6a+1=\cancel{9a^2}-45a+27$$

$$39a=26$$

$$a=\frac{2}{3}$$

Q Find all Integral values of a for which Q Eqⁿ $(x-a)(x-10)+1=0$ has Integral Roots?

Note
1) Roots = Int. चाहिए
⇓
Value of x is Integer.

(2) an x is Integer

$x-10=$ Integer
I - I होगा

(3) $a=$ Integer demanded

$x-a=$ Integer होगा
I - I

$(x-a)(x-10)=-1$

Integer × Int = −1

$x-a=1$ & $x-10=-1$

$\boxed{x=9}$

$9-a=1$

$\boxed{a=8}$

$x-a=-1$ & $x-10=1$

$\boxed{x=11}$

$11-a=-1$

$\boxed{a=12}$