Nishant Jindal

$\cos^{-1}x+\cos^{-1}y=\cos^{-1}\left(xy-\sqrt{1-x^2}\sqrt{1-y^2}\right)$

Q If $\cos^{-1}\left(\frac{x}{a}\right)+\cos^{-1}\left(\frac{y}{b}\right)=\alpha$.

then S.T. $\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{2xy\cos\alpha}{ab}=\sin^2\alpha$

$$\cos^{-1}\left(\frac{xy}{ab}-\sqrt{1-\frac{x^2}{a^2}}\sqrt{1-\frac{y^2}{b^2}}\right)=\alpha$$

$$\frac{xy}{ab}-\sqrt{1-\frac{x^2}{a^2}}\sqrt{1-\frac{y^2}{b^2}}=\cos\alpha.$$

$$\left(\frac{xy}{ab}-\cos\alpha\right)^2=\left(\sqrt{\left(1-\frac{x^2}{a^2}\right)}\sqrt{\left(1-\frac{y^2}{b^2}\right)}\right)^2$$

$$\cancel{\frac{x^2y^2}{a^2b^2}}+\cos^2\alpha-\frac{2xy\cos\alpha}{ab}=1-\frac{y^2}{b^2}-\frac{x^2}{a^2}+\cancel{\frac{x^2y^2}{a^2b^2}}$$

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{2xy\cos\alpha}{ab}=1-\cos^2\alpha.$$

$$=\sin^2\alpha$$

$\cos^{-1}(y)+\cos^{-1}(bxy)=\frac{\pi}{2}-\sin^{-1}(ax)$

Q $\sin^{-1}(ax)+\cos^{-1}(y)+\cos^{-1}(bxy)=\frac{\pi}{2}$ then $(x,y)$

Adv

| Col 1 | | Col 2 |
|---|---|---|
| A) $a=1 \quad b=0$ | → | P) $x^2+y^2=1$ |
| B) $a=1 \quad b=1$ | → | Q) $(x^2-1)(y^2-1)=0$ |
| C) $a=1 \quad b=2$ | | R) $y=x$ |
| d) $a=2 \quad b=2$ | | S) $(4x^2-1)(y^2-1)=0$ |

$$\cos^{-1}\left(bxy^2-\sqrt{1-y^2}\sqrt{1-b^2x^2y^2}\right)=\cos^{-1}(ax)$$

$$\left(bxy^2-ax\right)^2=\left(\sqrt{1-y^2}\sqrt{1-b^2x^2y^2}\right)^2$$

$$\cancel{b^2x^2y^4}+a^2x^2-2abx^2y^2=1-y^2-b^2xy^2+\cancel{b^2x^2y^4}$$

$a=1,\ b=0 \quad x^2=1-y^2 \Rightarrow x^2+y^2=1$

$a=1,\ b=1 \quad x^2-2x^2y^2=1-y^2-xy^2$

Nishant Jindal

Q If $\underbrace{\cos^{-1}x}_{A} + \underbrace{\cos^{-1}y}_{B} + \underbrace{\cos^{-1}z}_{C} = \pi$ then. $\Rightarrow x = \cos A$

S.T $\rightarrow$ $x^2+y^2+z^2+2xyz=1$

given $\cos^{-1}x = A \Rightarrow x = \cos A$

$y = \cos B$

$\cos^{-1}x + \cos^{-1}y + \cos^{-1}z = \pi z = \cos C$

$\cos^{-1}x + \cos^{-1}y = \underline{\pi - \cos^{-1}z}$

$\cos^{-1}\left(xy - \sqrt{1-x^2}\sqrt{1-y^2}\right) = \cos^{-1}(-z)$

$xy - \sqrt{1-x^2}\sqrt{1-y^2} = -z$

sqr $(xy+z)^2 = \left(\cancel{\sqrt{1-x^2}}\,\cancel{\sqrt{1-y^2}}\right)^{\cancel{2}}$

$x^2y^2+z^2+2xyz = (1-x^2)(1-y^2)$

$\cancel{x^2y^2}+z^2+2xyz = 1-x^2-y^2+\cancel{x^2y^2}$

$x^2+y^2+z^2+2xyz=1$

Trigo Me $\rightarrow$ If $A+B+C=\pi$

$\cos 2A + \cos 2B + \cos 2C = -1-4\cos A\cos B\cos C$

$\sin 2A + \sin 2B + \sin 2C = 4\sin A\sin B\sin C$

$\tan A + \tan B + \tan C = \tan A\tan B\tan C$

$(2\cos^2A-1)+(2\cos^2B-1)+(2\cos^2C-1) = -1-4\cos A\cos B\cos C$

$2x^2-1+2y^2-1+2z^2\cancel{-1} = \cancel{-1}-4xyz$

$2x^2+2y^2+2z^2+4xyz=2$

$x^2+y^2+z^2+2xyz=1$

Nishant Jindal

Q If $0 < x, y, z < 1$

& $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \pi$

then $x + y + z = ?$

$\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \pi$

$A + B + C = \pi$

$\tan A + \tan B + \tan C = \tan A \cdot \tan B \tan C$

$\cos A = \sqrt{1 - \sin^2 A}$

$\cos A = \sqrt{1 - x^2}$

$x + y + z = \boxed{xyz}$

A

Q If $\sin^{-1}x + \sin^{-1}y + \sin^{-1}z = \pi$

then $x\sqrt{1-x^2} + y\sqrt{1-y^2} + z\sqrt{1-z^2} = ?$

$\sin^{-1}x + \sin^{-1}y + \sin^{-1}z = \pi$

$A + B + C = \pi$

$\sin 2A + \sin 2B + \sin 2C = 4 \sin A \sin B \sin C$

$\cancel{2} \sin A \cos A + \cancel{2} \sin B \cos B + \cancel{2} \sin C \cos C$

$= \cancel{4}^{2} \sin A \sin B \sin C$

$x\sqrt{1-x^2} + y\sqrt{1-y^2} + z\sqrt{1-z^2}$

$= \boxed{2xyz}$

A.

Nishant Jindal

*** **Prop → 7 →** $ITF(TF) = T^{-1}(T(x))$

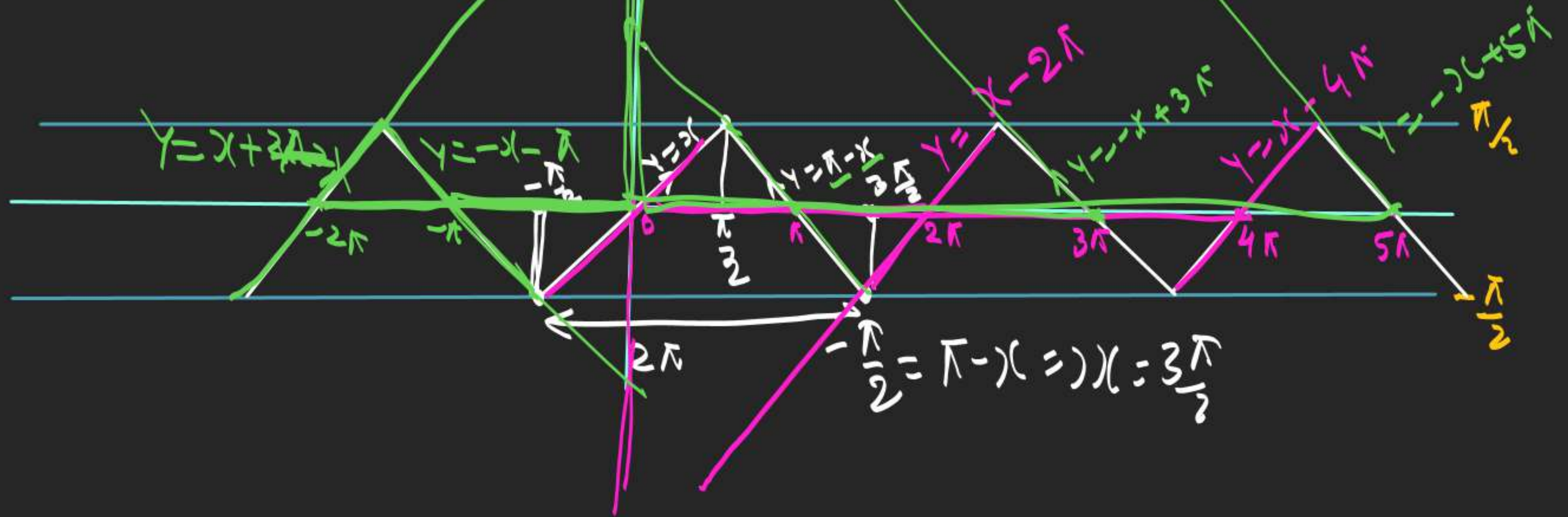
$\sin(\sin^{-1}x) = \boxed{x}$ $P(N) = N$ | $\sin^{-1}(\sin x)$ $N(P) = Per$

① $y = \sin^{-1}(\boxed{\sin x}) \rightarrow T = 2\pi$

① Dom → $-1 \le \sin x \le 1 \rightarrow x \in R$

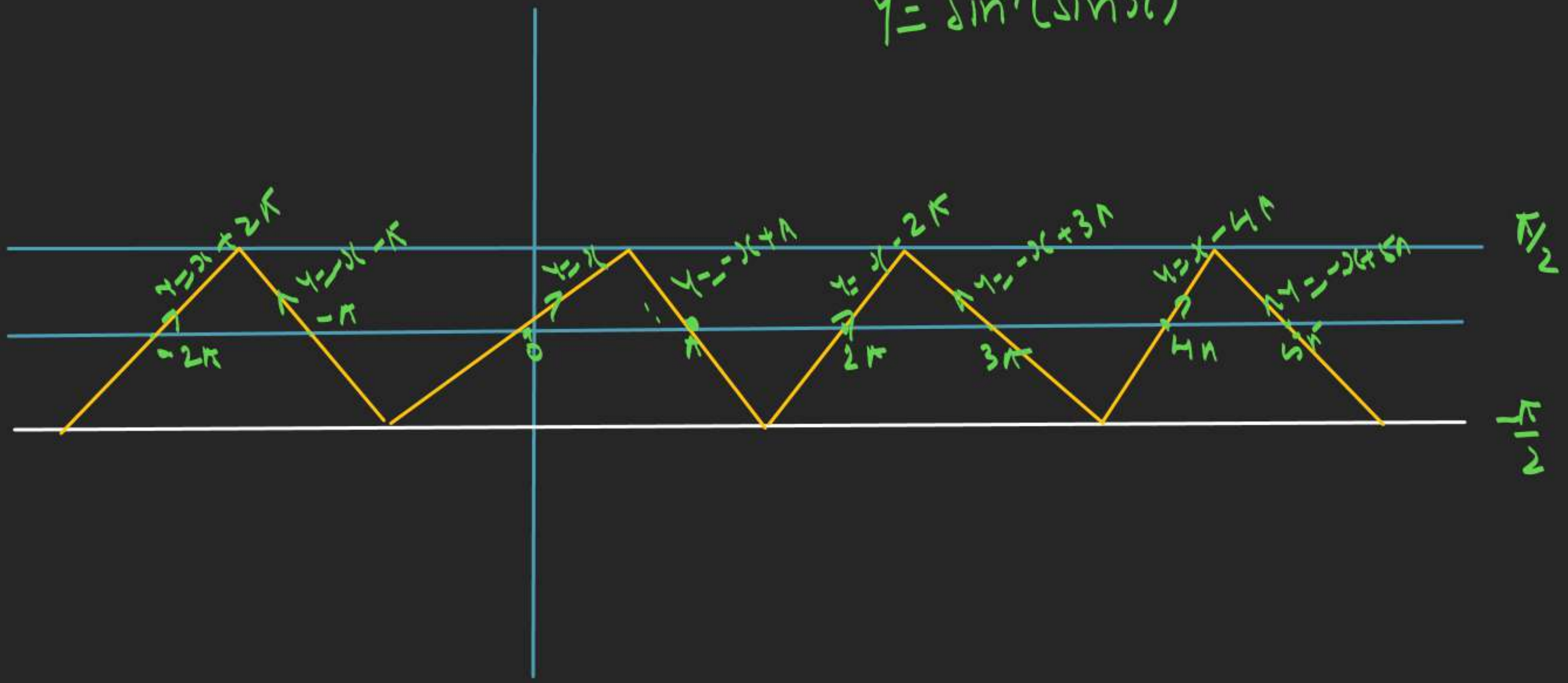
② Range $= [-\frac{\pi}{2}, \frac{\pi}{2}]$

(3) $\sin y = \sin y \Rightarrow y = n\pi + (-1)^n x$ —
- $n = 0$: $y = x$
- $n = -1$: $y = \pi - x = -x + \pi$




$-\frac{\pi}{2} = \pi - x \Rightarrow x = \frac{3\pi}{2}$

$Y = \sin^{-1}(\sin x)$



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(2) $y = \cos^{-1}(\cos x)$

1) Dom. $-1 \le \cos x \le 1 \Rightarrow x \in R$

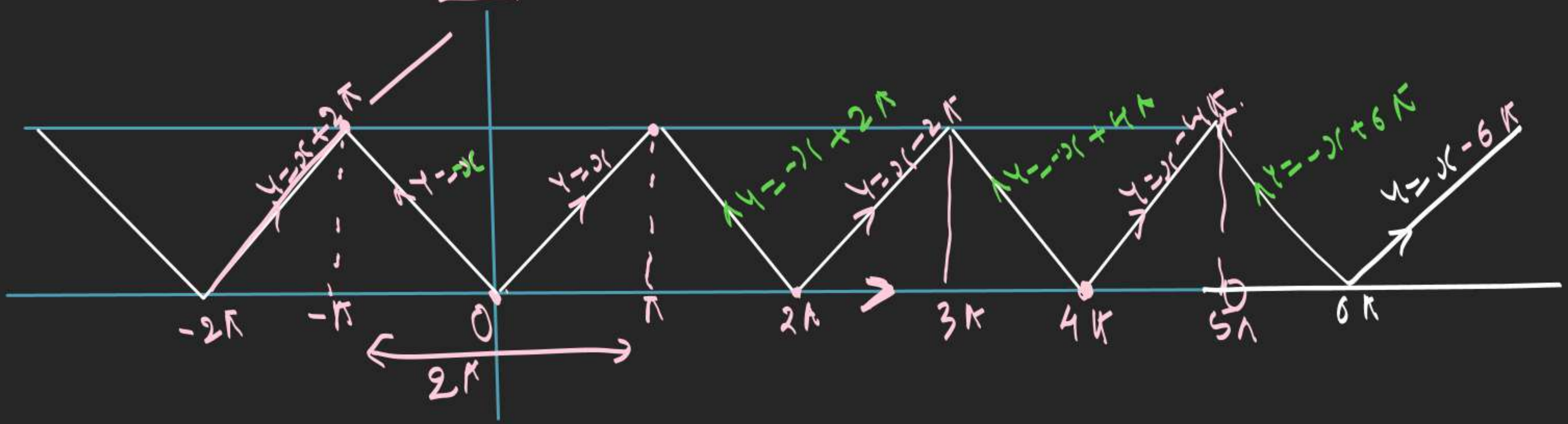
2) Range $y \in [0, \pi]$

3) $\cos y = \cos x \Rightarrow y = 2n\pi \pm x$ — $n=0$: $y = x$; $n=0$: $y = -x$

4) Period $= T = 2\pi$

$y = \cancel{2n\pi} \pm x$

$y = \pm x$



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3) $y = \tan^{-1}(\tan x)$

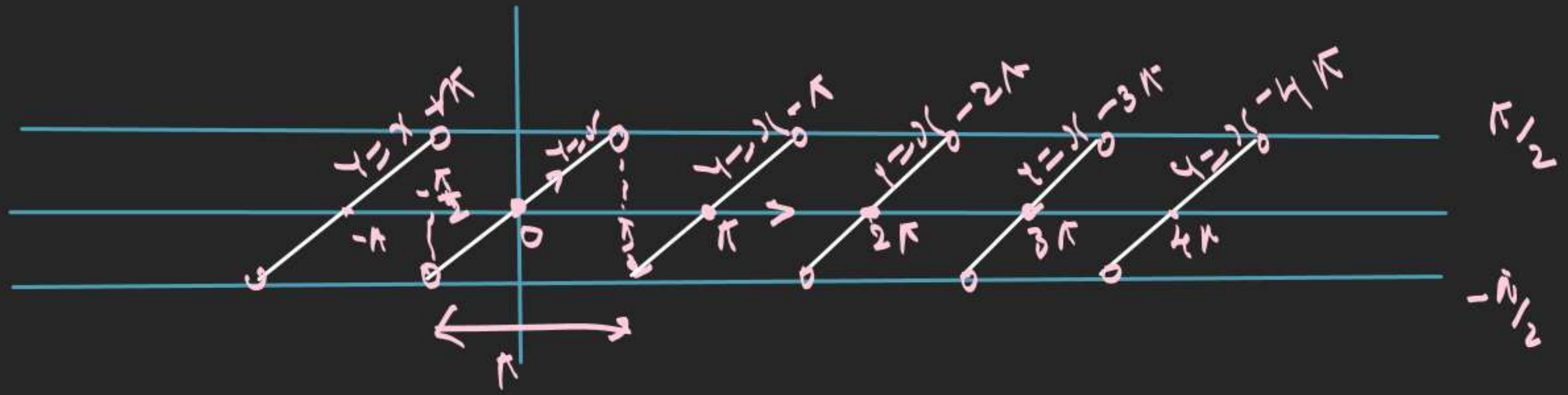
1) Dom $\quad \tan x$ ⇒ Dom ⇒ $\dfrac{\sin x}{\cos x}$ ⇒ $\cos x \neq 0 \Rightarrow x \neq (2n+1)\dfrac{\pi}{2}$

$x \in R - (2n+1)\dfrac{\pi}{2}$

2) $y \in \left(-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right)$

3) $\tan y = \tan x \Rightarrow y = n\pi + x \xrightarrow{n=0} \boxed{y = x}$

4) Period = $\pi$



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(4) $y = \underset{R}{\underline{\cot^{-1}}}(\underline{\cot x})$

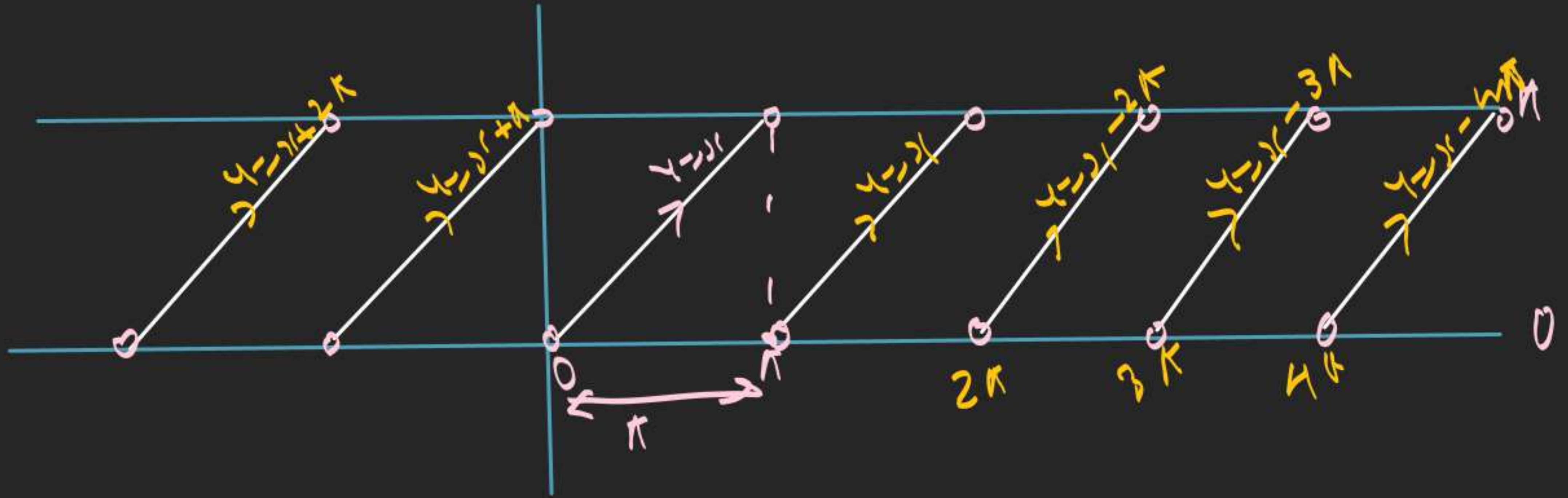
1) Dom → $\cot x$ of Dom $= \frac{\cos x}{\boxed{\sin x}} \rightarrow \sin x \neq 0 \quad x \neq n\pi$

$x \in R - (n\pi)$

2) Range $\in (0, \pi)$

3) $\cot y = \cot x \Rightarrow \tan y = \tan x \Rightarrow y = n\pi + x \xrightarrow{n=0} \boxed{y = x}$

4) <u>Period</u> $= \pi$



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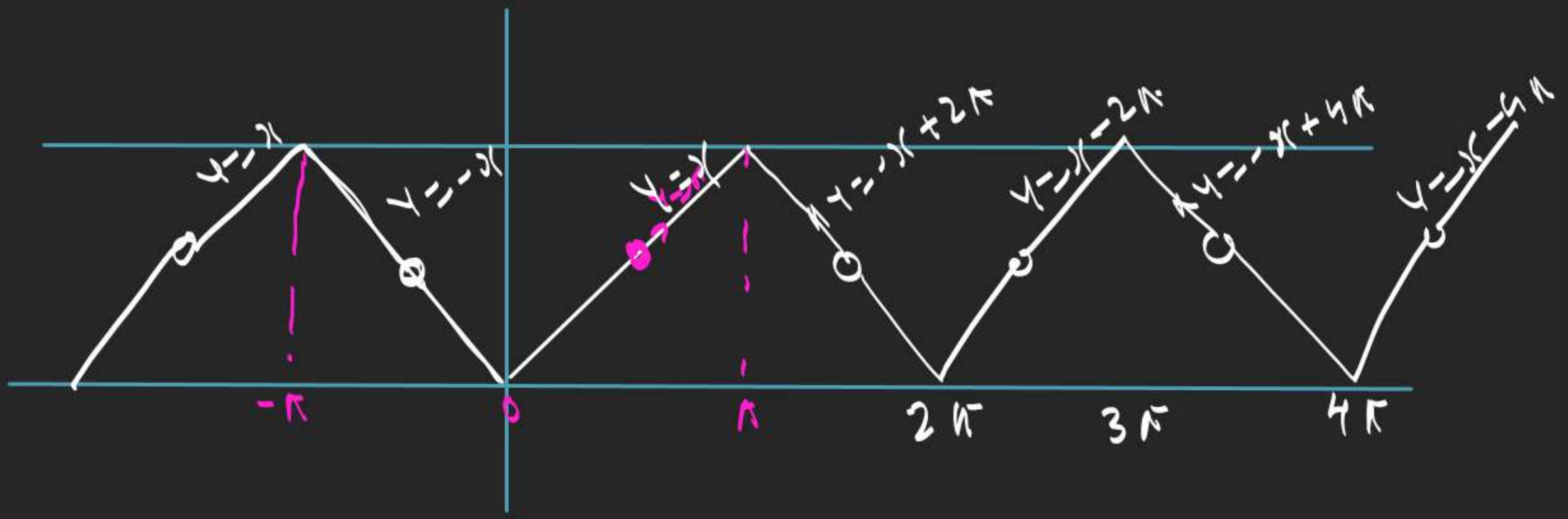
5) $y = \sec^{-1}(\sec x)$

1) Dm. $\to |\sec x| \geq 1 \Rightarrow x \in R$ $\;\; \to \frac{1}{\cos x} \to \cos x \neq 0 \Rightarrow x \neq (2n+1)\frac{\pi}{2}$

2) Range $\to [0, \pi] - \{\frac{\pi}{2}\}$

3) Period $= 2\pi$

4) $\sec y = \sec x \Rightarrow \cos y = \cos x \Rightarrow y = 2n\pi \pm x \to y = \pm x$ ($n=0$)



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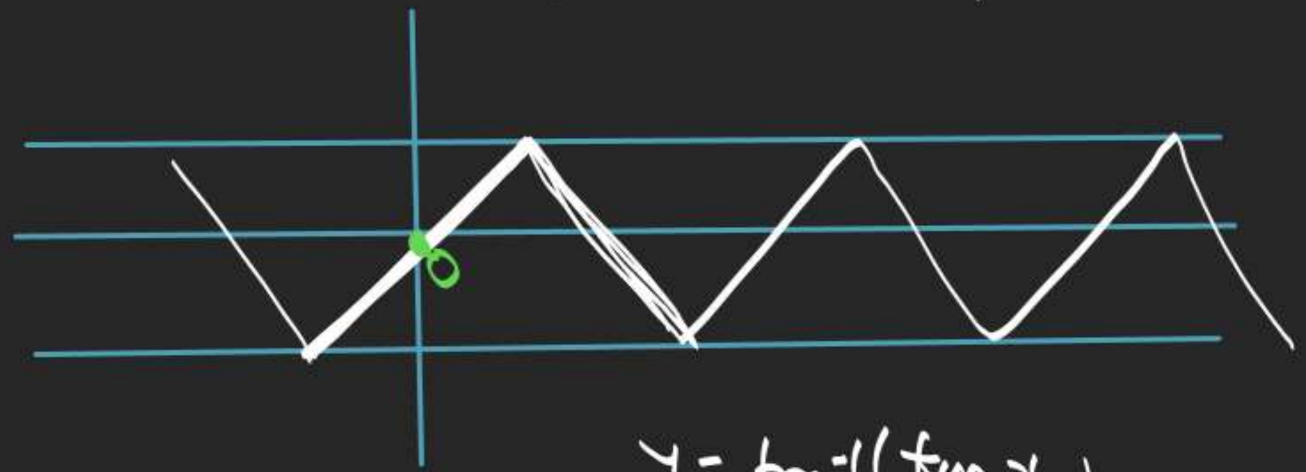
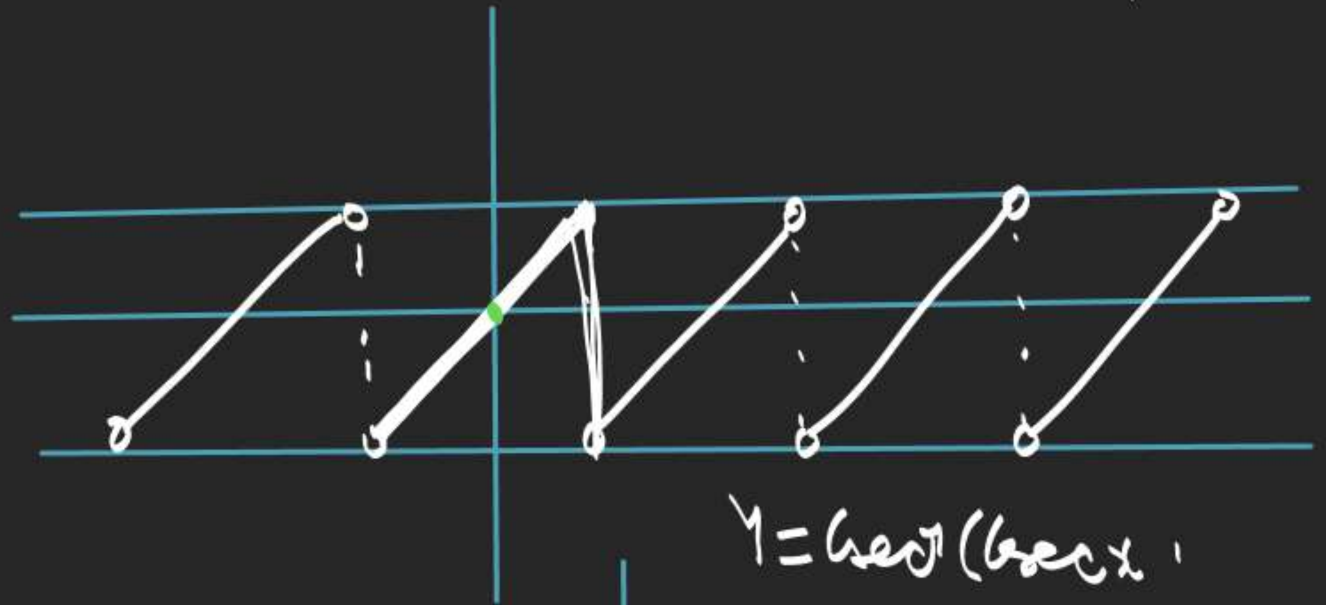
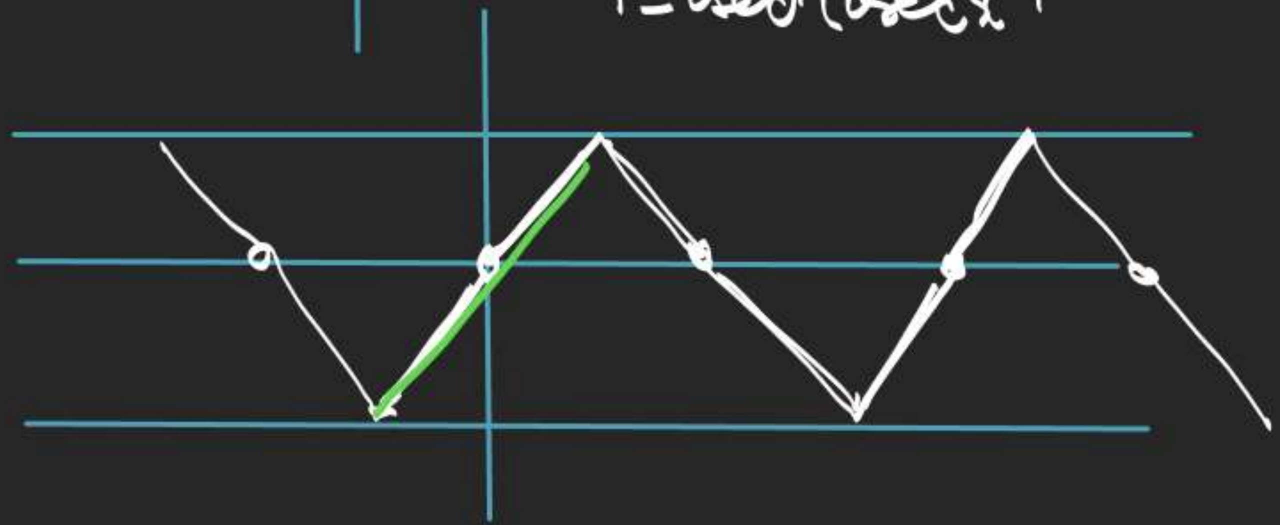
① $y = \sin^{-1}(\sin x)$



$y = \tan^{-1}(\tan x)$

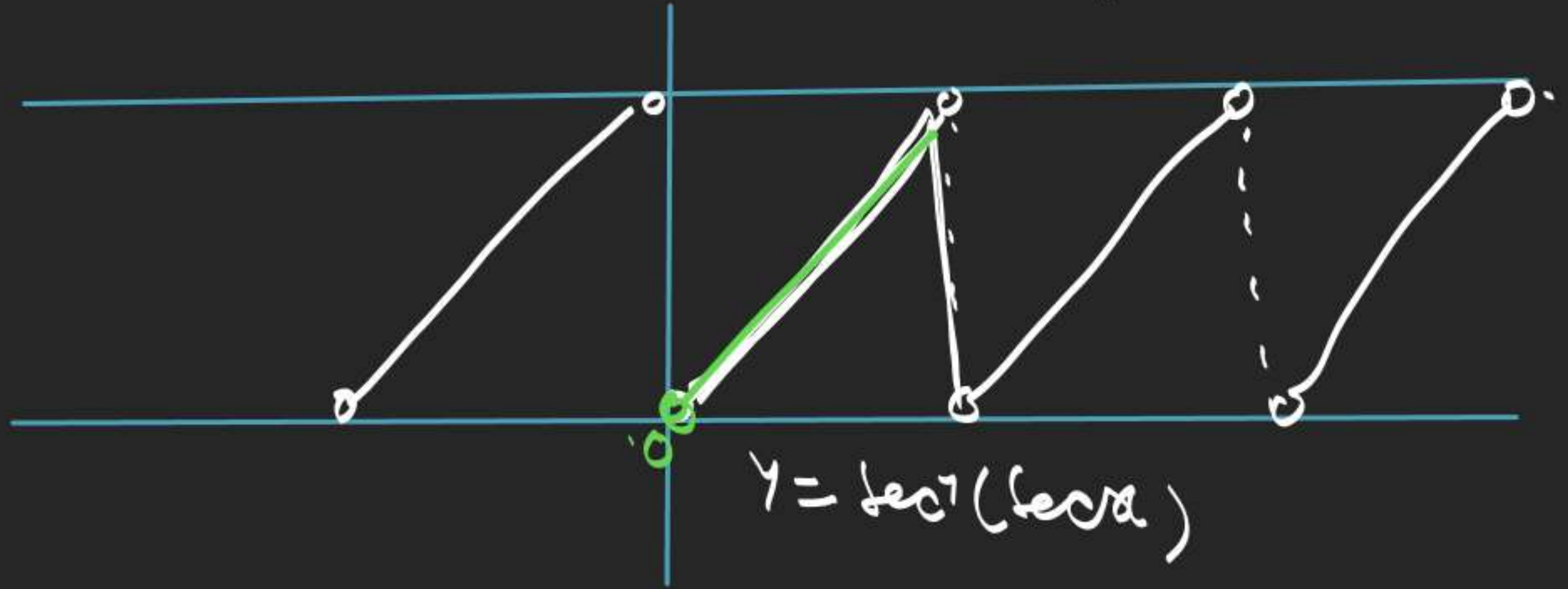


$y = \csc^{-1}(\csc x)$

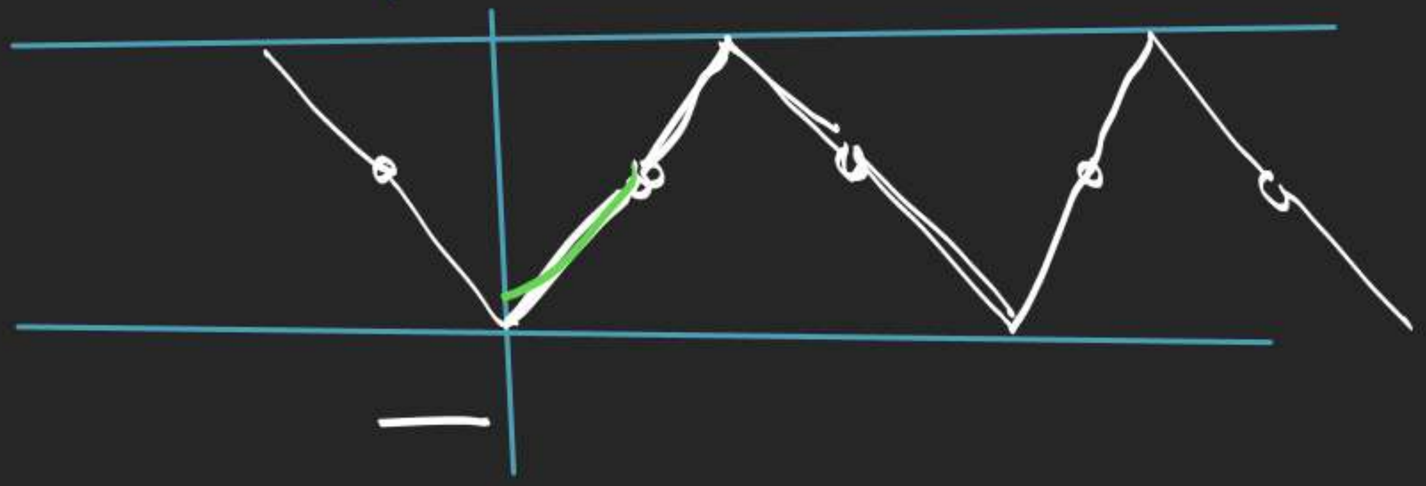


2) $y = \cos^{-1}(\cos x)$

$y = \cot^{-1}(\cot x)$

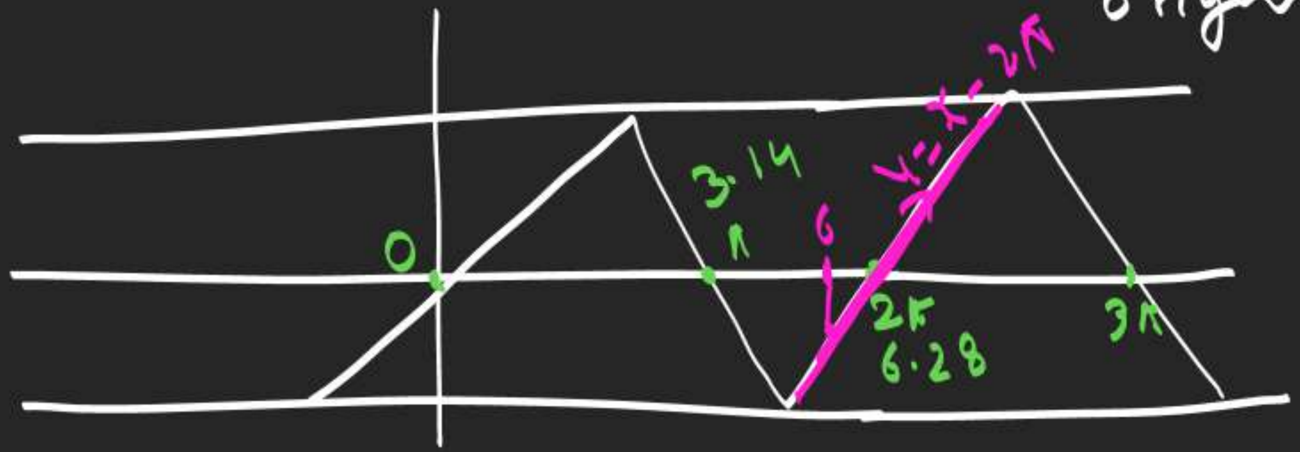


$y = \sec^{-1}(\sec x)$

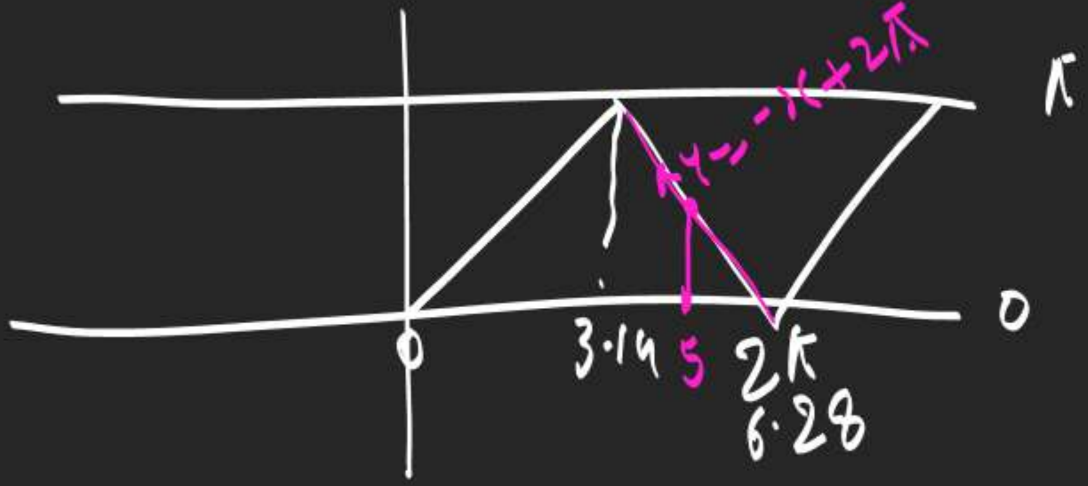


Q $y = \sin^{-1}(\sin 6) = ?$ $\quad 6 - 2\pi$ ✓

$\sin^{-1}(\sin x) \rightarrow$ Tambu origin

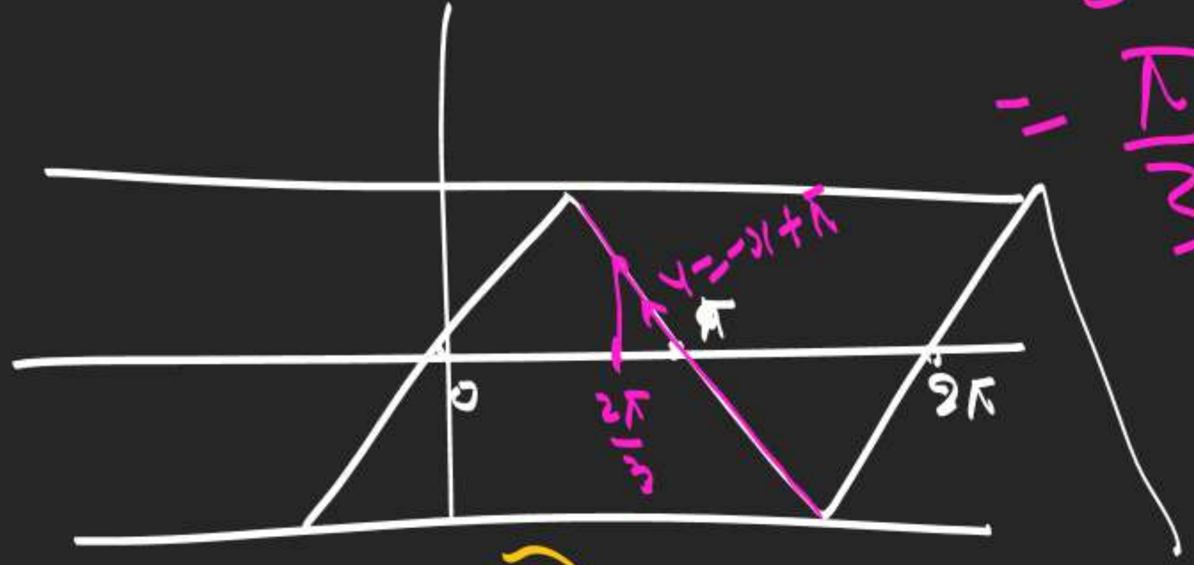



Q $y = \cos^{-1}(\cos 5) = 2\pi - 5$



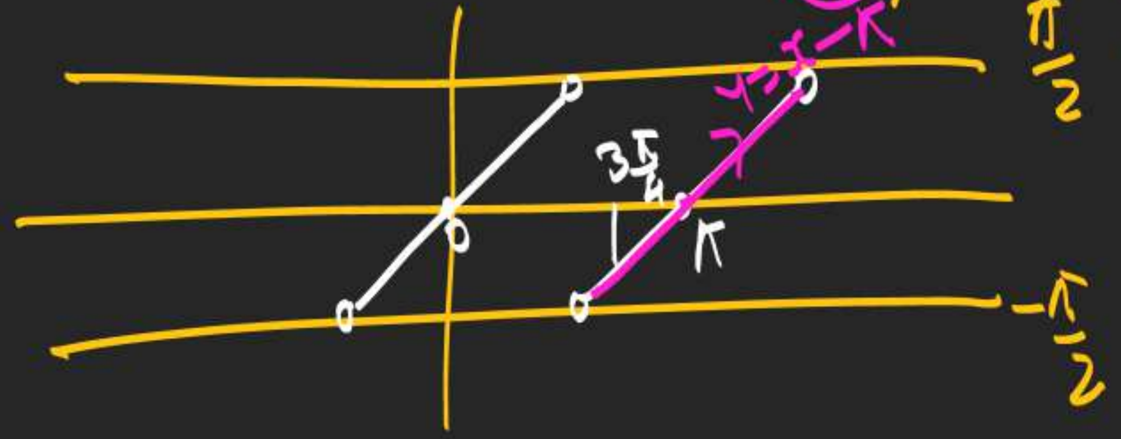

$y = \sin^{-1}\left(\sin \frac{2\pi}{3}\right) = -\frac{2\pi}{3} + \pi$

$= \frac{\pi}{3}$




$y = \tan^{-1}\left(\tan \frac{3\pi}{4}\right) = \frac{3\pi}{4} - \pi$

$= -\frac{\pi}{4}$



Nishant Jindal

1) $\sin^{-1}(\sin 3)$, $\sin^{-1}(\sin 10)$

$\sin^{-1}(\sin 7)$

2) $\cos^{-1}(\cos 2)$ $\cos^{-1}(\cos 3)$

$\cos^{-1}(\cos 10)$

3) $\tan^{-1}(\tan \frac{3\pi}{4})$ $\tan^{-1}(\tan \frac{\pi}{4})$

$\tan^{-1}(\tan 9)$

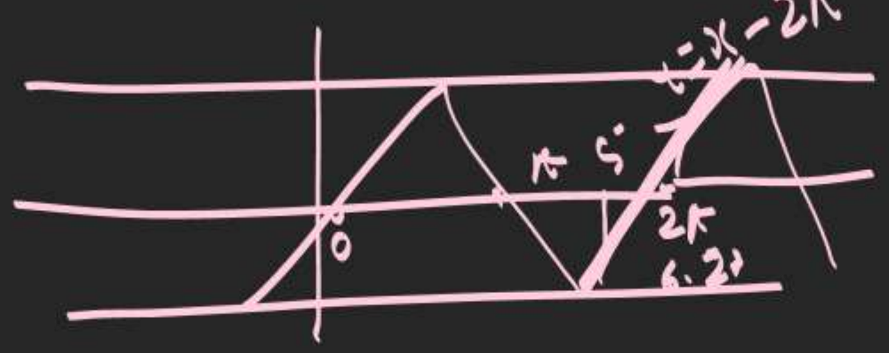
Q $\cos^{-1}(\sin(-5))$

$\frac{\pi}{2} - \sin^{-1}(\sin(-5))$

$\frac{\pi}{2} - \sin^{-1}(-\sin 5)$

$\frac{\pi}{2} + \sin^{-1}(\sin 5) = \frac{\pi}{2} + 5 - 2\pi$

$\boxed{5 - \frac{3\pi}{2}}$



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Q $f:[0,4\pi] \to [0,\pi]$ be defined by $f(x) = \cos^{-1}(\cos x)$

Adv

The No. of Pts of $x \in [0,4\pi]$ Satisfying Eqn $g(x) = \frac{10-x}{10}$ is

$x=10$

$g(x) = 1 - \frac{x}{10} \to$

$x=10$

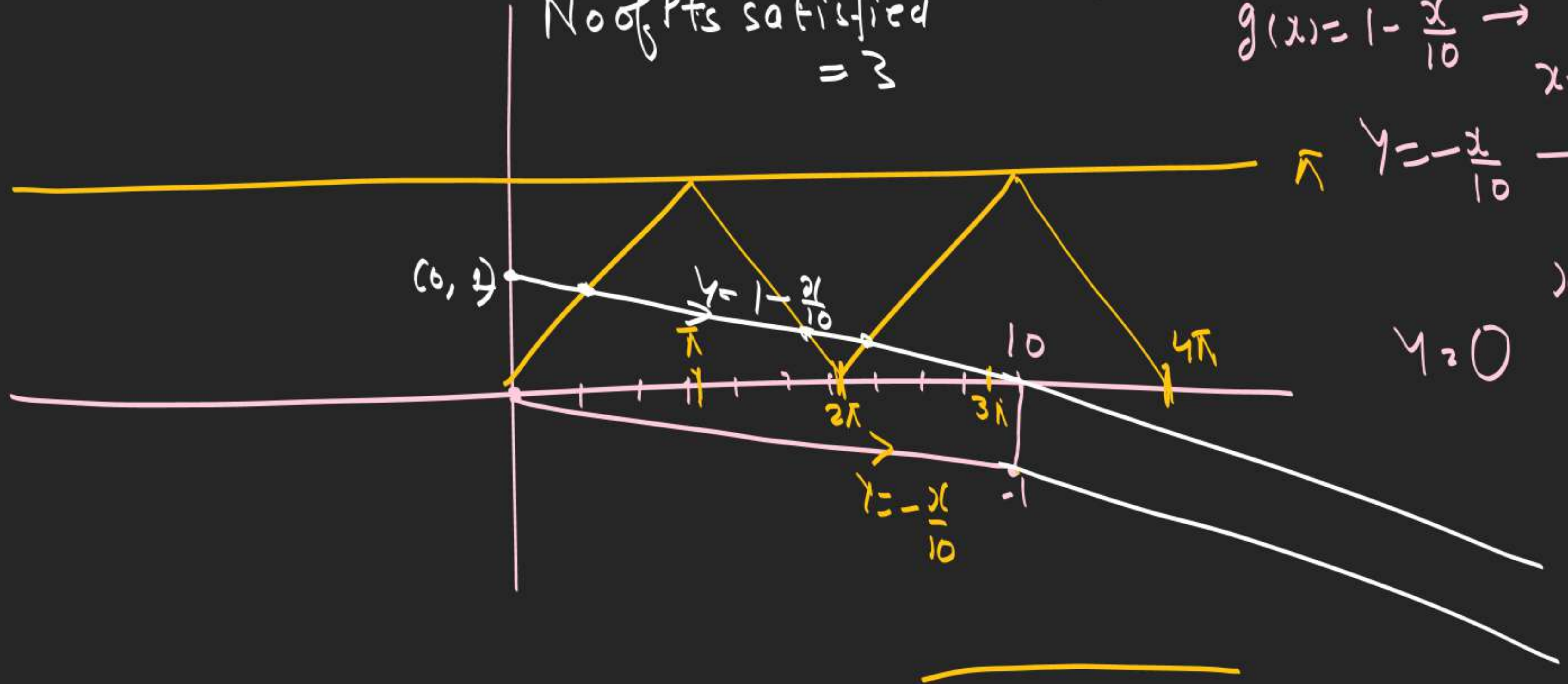
$y = -\frac{x}{10} \to y = -\frac{10}{10}$

$= -1$

$x=0$

$y=0$

No of Pts satisfied $= 3$



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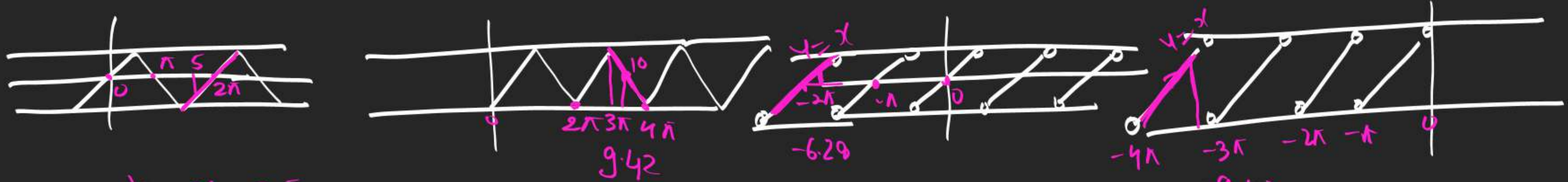
Q $\sin^{-1}(\sin 5) + \cos^{-1}(\cos \boxed{10}) + \tan^{-1}(\tan(-6)) + \cot^{-1}(\cot(-10))$




$y = x - 2\pi$ $\quad$ $y = -x + 4\pi$ $\quad$ $y = x + 2\pi$ $\quad$ $y = +x + 4\pi$

$5 - 2\pi \quad -10 + 4\pi \quad -6 + 2\pi + (-10) + 4\pi$

$8\pi - 21$