5.

$$\underset{x^2-4x-5}{\overset{\geq 0 \quad \overset{\geq 0}{\boxed{x^4}}+\overset{\geq 0}{\boxed{x^2}}+1}{}} < 0 \implies \frac{1}{(x-5)(x+1)} < 0$$

$$\implies \boxed{x \in (-1, 5)}$$

2. $(x^2-9)(\overset{>0}{\cancel{x^2+7}}) \leq 0$

$(x-3)(x+3) \leq 0$

$$\boxed{x \in [-3, 3]}$$

4.

$$2(x-2) < -(x^2+1)$$

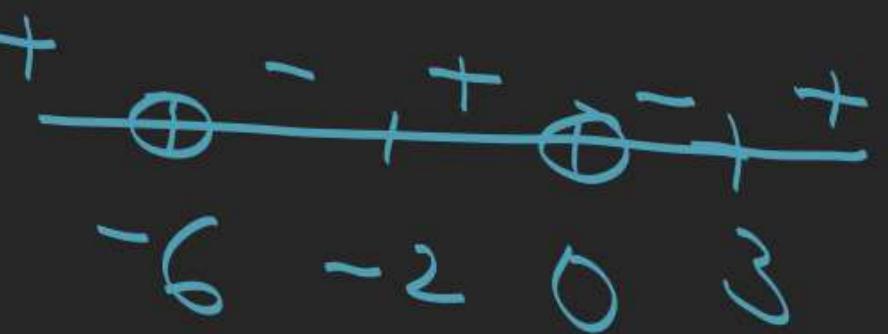$$x^2 + 2x - 3 < 0$$

$$(x+3)(x-1) < 0$$

$$x \in (-3, 1)$$

6.

$$\frac{\overbrace{1+3x^2}^{>0}}{2x^2-21x+40}{}<0$$

$$-5x-16x$$

$$\frac{1}{(2x-5)(x-8)}<0$$

$$\boxed{x \in \left(\frac{5}{2},\ 8\right)}$$

8. $\dfrac{(x-3)(x+2)}{x(x+6)} \geq 0$

$$\overset{+}{\underset{-6}{\oplus}}\ \overset{-}{\underset{-2}{}}\ \overset{+}{\underset{0}{\oplus}}\ \overset{-}{\underset{3}{}}\ \overset{+}{}$$

$$x \in (-\infty, -6) \cup [-2, 0)$$
$$\cup [3, \infty)$$

16.

$$\frac{(x-2)(x-4)(x-7)}{(x+2)(x+4)(x+7)} - 1 > 0$$

$$\frac{(x^3 - 13x^2 + 50x - 56) - (x^3 + 13x^2 + 50x + 56)}{(x+2)(x+4)(x+7)} > 0$$
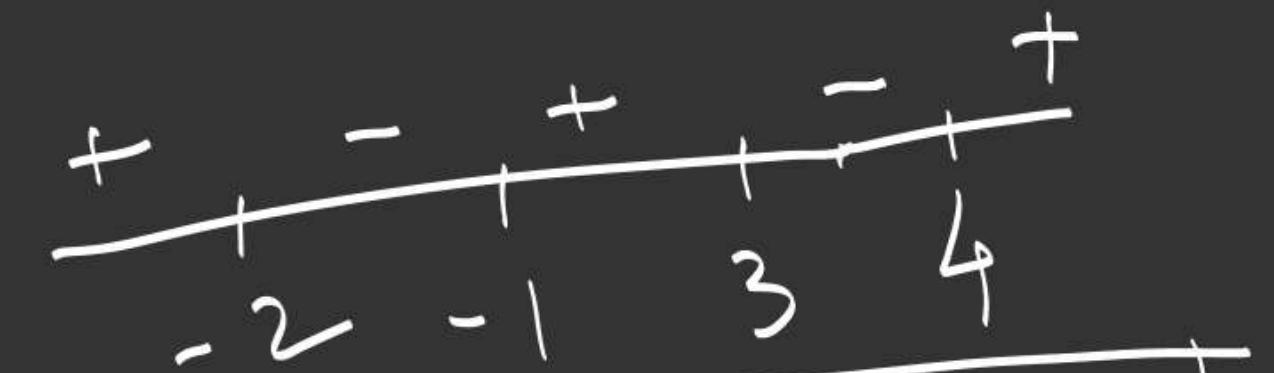
$$\frac{}{(x+2)(x+4)(x+7)} < 0$$

$$\frac{-26x^2 - 112}{(x+2)(x+4)(x+7)} > 0$$

$$\frac{1}{(x+2)(x+4)(x+7)} < 0$$

$$\overset{-}{\underset{-7}{\big|}} \overset{+}{} \overset{-}{\underset{-4}{\big|}} \overset{+}{\underset{-2}{\big|}}$$

$$x \in (-\infty, -7) \cup (-4, -2)$$

15.

$$\frac{20 + 10(x-3) + (x-3)(x-4)}{(x-3)(x-4)} > 0$$

$$\frac{x^2 + 3x + 2}{(x-3)(x-4)} > 0$$

$$\frac{(x+1)(x+2)}{(x-3)(x-4)} > 0$$

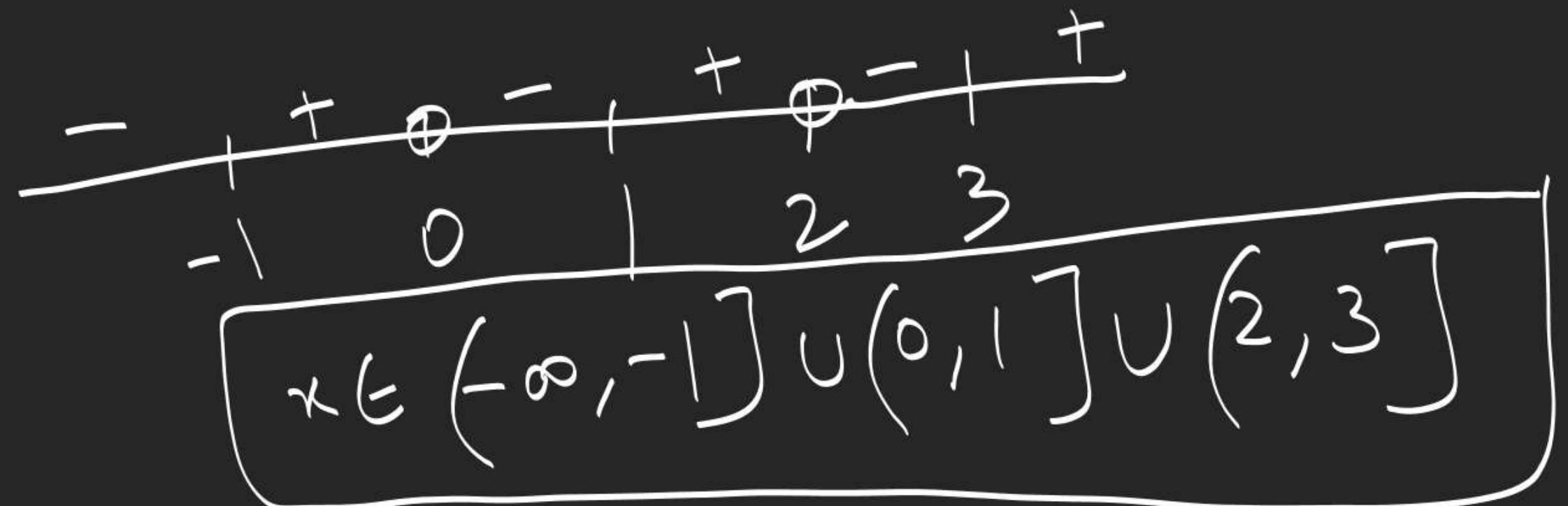$$x \in (-\infty, -2) \cup (-1, 3) \cup (4, \infty)$$

$\underline{20'}$

$$\frac{(2x-2)(x-3)(x+1)}{x(x-2)} \leq 0$$

$\boxed{x^2+3x+4}$

$\left(x+\frac{3}{2}\right)^2+\frac{7}{4}$

$$x \in (-\infty, -1] \cup (0, 1] \cup (2, 3]$$

$$\sqrt{x^2} = |x| = \begin{cases} x & x \geqslant 0 \\ -x & x < 0 \end{cases}$$

$$|a+b| = |a| + |b| \quad \text{if} \quad a \geqslant 0$$

$$|a+b| = |a| - |b| \qquad 6 = |-2 \times 3| = |-2||3|$$

$$\text{if } ab \leq 0$$

Properties

$$|ab| = |a||b| \qquad a+b$$

$$\left|\frac{a}{b}\right| = \frac{|a|}{|b|}$$

$$\Big||a| - |b|\Big| \leq |a+b| \leq |a| + |b| \qquad a+7$$

$$|x-3| = \begin{cases} x-3 & \text{if } x-3 \geq 0 \\ -(x-3) & \text{if } x-3 < 0 \end{cases}$$

$$|N| = \begin{cases} N & N \geq 0 \\ -N & N < 0 \end{cases}$$

$$|x-3| = \begin{cases} x-3 & \text{if } x \geq 3 \\ 3-x & \text{if } x < 3 \end{cases}$$

$$|3 - 2x| = |(-1)(2x - 3)| = |-1||2x - 3|$$

$$= |2x - 3| \qquad \boxed{|-6| = |6|}$$

$$= \begin{cases} 2x - 3 & x \geq \dfrac{3}{2} \\ \\ 3 - 2x & x < \dfrac{3}{2} \end{cases}$$

$$|0| = 0$$

$$|0| = -0$$

$$|x| = \begin{cases} x & x \geq 0 \\ \\ -x & x \leq 0 \end{cases}$$

1.

$|x| = 8$, $x = ?$

$x = 8, -8$

$|x|^2 = 8^2$

$x^2 = 64$

$(x-8)(x+8) = 0$

$x = 8, -8$

2. $\quad |5 - 2x| = 7$

$2x - 5 = 7$ or -7

$\boxed{x = 6 \text{ or } -1}$

3.
$$|x-3| + 2|x-2| = 6$$

$$|x-3| = \begin{cases} x-3 & x \geqslant 3 \\ 3-x & x < 3 \end{cases}$$

$$|x-2| = \begin{cases} x-2 & x \geqslant 2 \\ 2-x & x < 2 \end{cases}$$

If $x < 2$ ✓

and

$$3-x + 2(2-x) = 6$$ ✓

$$\boxed{x = \frac{1}{3}} \Rightarrow Ans$$

If $2 \leq x \leq 3$ OR

$$(3-x) + 2(x-2) = 6$$

rejected ← $x = 7$ ✗

OR

If $x \geqslant 3$

$$x-3 + 2(x-2) = 6 \Rightarrow \boxed{x = \frac{13}{3}}$$

$$\boxed{x = \frac{1}{3}, \frac{13}{3}} \downarrow Ans$$

4. $\quad 2|x| - |x+3| + |x-4| = 14$

<u>Case I</u>
$x \leq -3$

$2(-x) - (-(x+3)) + (-(x-4)) = 14$

$-2x + x + 3 + 4 - x = 14$

$$\boxed{x = -\frac{7}{2}}$$

<u>Case II</u> $\quad -3 \leq x \leq 0$

$-2x - (x+3) + (4-x) = 14$

$x = \frac{13}{-4}$ (reject)

(number line)
$-3 \quad 0 \quad 4$

$$\boxed{x = -\frac{7}{2}, \frac{21}{2}}$$

<u>Case III</u> $\quad 0 \leq x \leq 4$

$2x - (x+3) + (4-x) = 14$

$0 = 13$

<u>no solution</u>

<u>Case IV</u> $\quad x \geq 4$

$2x - (x+3) + (x-4) = 14$

$$\boxed{x = \frac{21}{2}}$$

$$5' \quad \frac{1}{|x|-3} < \frac{1}{2} \qquad \boxed{(-\infty, -5) \cup (-3, 3) \cup (5, \infty)}$$

Case I $\qquad \boxed{x \leq 0}$ $\qquad\qquad$ OR $\quad \underset{\Downarrow}{Ans}$ $\qquad\qquad$ Case II $\quad \boxed{x \geq 0}$

& $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ &

$$\frac{1}{-x-3} < \frac{1}{2} \qquad\qquad\qquad \frac{x-5}{2(x-3)} > 0 \Leftarrow \frac{5-x}{2(x-3)} < 0 \qquad \frac{1}{x-3} - \frac{1}{2} < 0$$

$$\overset{+}{\underset{-5}{\quad}} \overset{-}{\underset{-3}{\quad}} \overset{+}{\quad}$$

$$\frac{1}{2} + \frac{1}{x+3} > 0$$

$$\frac{x+5}{2(x+3)} > 0 \Rightarrow x \in \boxed{(-\infty, -5) \cup (-3, \infty)}$$

$$\overset{+}{\underset{3}{\quad}} \overset{-}{\underset{5}{\quad}} \overset{+}{\quad}$$

$$\Rightarrow x \in \boxed{(-\infty, -5) \cup (-3, 0]} \qquad\qquad x \in \boxed{(-\infty, 3) \cup (5, \infty)}$$

$$x \in \boxed{[0, 3) \cup (5, \infty)}$$

1. $|x-7)-3|x-2|+4|x+8|+|x)=21$

2. $\dfrac{|x+2|-x}{x}<2$

3. $\dfrac{x^2-7|x|+10}{x^2-6x+9}<0$

4. $\dfrac{x^2+6x-7}{|x+4|}<0$

5. $\dfrac{x^2-5x+6}{|x|+7}<0$

7. $\dfrac{|x-3|}{x^2-5x+6}\geq2$

6. $\dfrac{|x-1|}{x+2}<1$