Nishant Jindal

$\sin x \to 2\pi$

$\sin^2 x \to \pi$

$\cos x \to 2\pi$

$\cos^3 x \to 2\pi$

$\tan^3 x \to \pi$

$\tan x \to \pi$

$|\sin^{32} x| \to \pi$

$|\sin^2 x| \to \pi$

$|\tan x| \to \pi$

$\sec x \to 2\pi$

$\sec^2 x \to \pi$

$\operatorname{cosec}^4 x \to \pi$

$\operatorname{cosec}^7 x \to 2\pi$

$\{x\} = 1$

$f(x) = T$

$k \times f(x) \pm k$

$k \cdot \frac{1}{f(x)}$ $|f(x)|$

$\sqrt{\frac{5\{x+3\}-7}{9}} \to 1$

$\sqrt{\frac{5(\sin^3(x+\frac{\pi}{2})-7)}{e^2+1}} \to 2\pi$

6

Q $f(x) = e^{\sqrt{\frac{2\sin(x+5)}{3}-13}}$ 2π

Q $f(x) = 3^{4\{x+16\}-\frac{\pi}{2}}$ 1

Q $f(x) = \frac{2}{e^{\sqrt{\frac{\pi(\sin(x+5))}{3}}}}$ $\to 2\pi$

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RELATION FUNCTION

Q $f(x) = e^{\sin^2 x + \sin^2\left(x+\frac{\pi}{3}\right) + \cos x \cdot \cos\left(x+\frac{\pi}{3}\right)} + 7$ $T = ?$

$= e^{\frac{5}{4}} + 7$

$f(x) =$ Constant $\rightarrow$ $T =$ Undefined

But Periodic

Q $f(x) = \sin^4 x + \cos^6 x$ T.

$\sin^E x + \cos^E x$

$T = \frac{\pi}{2}$

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RELATION FUNCTION

$\underline{\underline{T3}}$ When Constant is multiplied to x

If $f(x)$'s Period $= T$

$f(Kx)$ ——— $= \frac{T}{|K|}$

Q $\{x\} \to T = 1$

$\{3x\} \to T = \frac{1}{3}$

$\left\{\frac{x}{5}\right\} \to T = \frac{1}{1/5} = 5$

Q $\sin x \to 2\pi$

$\sin 2x \to \frac{2\pi}{2}$

$\sin nx \to \frac{2\pi}{n}$

Q $\tan^3 x \to \pi$

$\sqrt{\dfrac{\tan^3\left(\frac{x}{2}+5\right) - 7}{5}} \to T = \frac{\pi}{1/2} = 2\pi$

Q $\{x\} \to 1$

$\left\{-\frac{x}{4}\right\} = \frac{1}{\left|-\frac{1}{4}\right|} = \frac{1}{1/4}$

$= 4$

Q $y=\sqrt{\sin\left(\frac{x}{2}\right)-1}$ $\rightarrow T=\frac{2\pi}{1/2}=4\pi$

Q $y=\cos\frac{1}{3}\left(\boxed{x}-\frac{\pi}{9}\right)$ $T=\frac{2\pi}{1/3}=6\pi$

Q $y=e^{\{5x\}-1}$ $\rightarrow T=\frac{1}{5}$

Q $y=\sec^3\frac{x}{\lfloor n+3}$ $\rightarrow \frac{2\pi}{\frac{1}{\lfloor n+3}}=2\lfloor n+3\;\pi$

Q find T if

$f(x)=\underset{\text{नीचा}}{\text{Min}}\{\sin x,|x|\}$

(1) make graph of all fxn at same place

$\therefore \text{Min}\{\sin x,|x|\}=\sin x$

का $T=2\pi$

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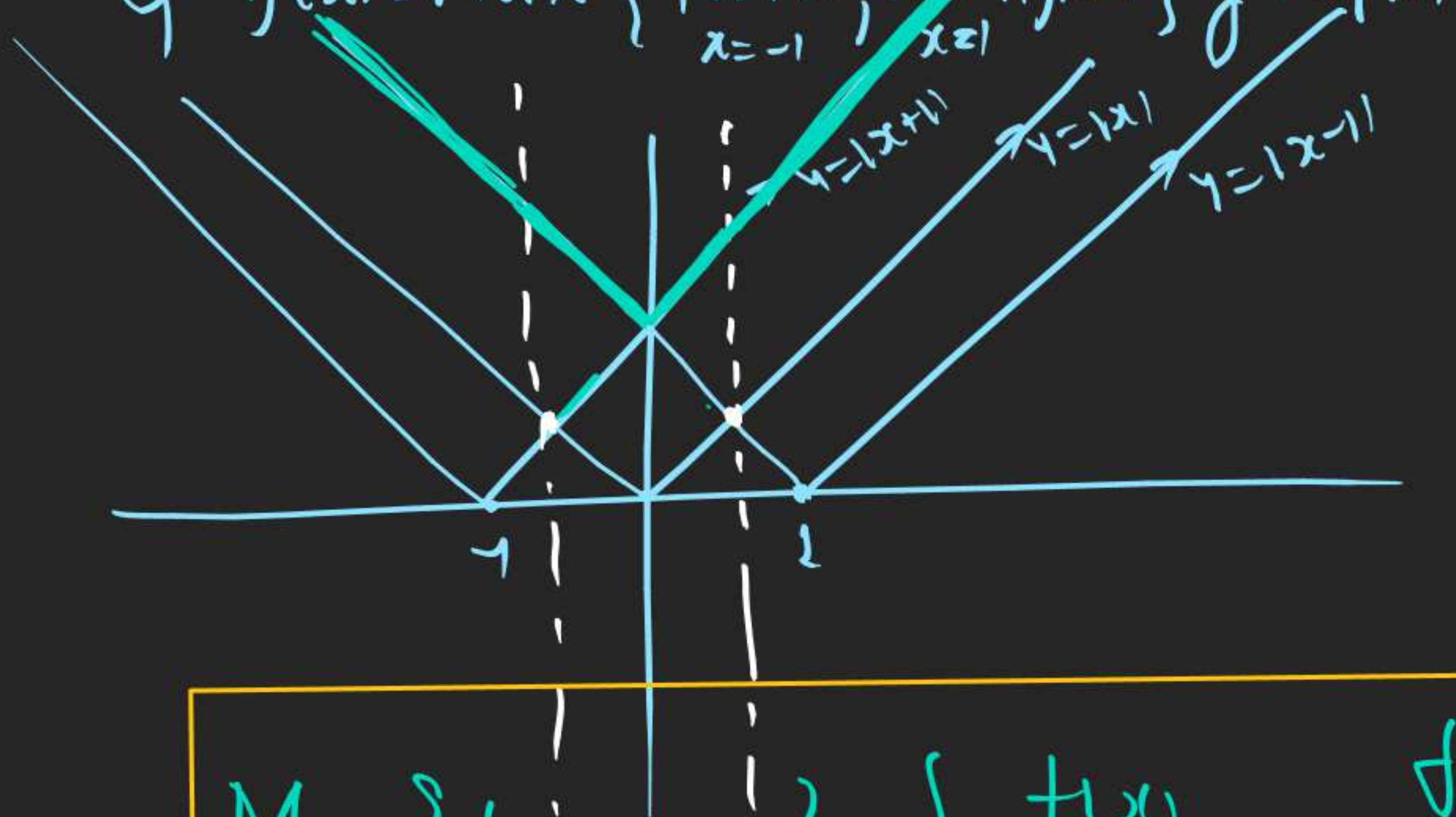
Q $f(x) = \text{Max}\{|x+1|, |x-1|, |x|\}$ graph

ऊँचा



$$\text{Max}\{f(x), g(x)\} = \begin{cases} f(x) & f(x) \geq g(x) \\ g(x) & g(x) > f(x) \end{cases}$$

Q If Period of $e^{3(x-[x])}$ is T_1 & $e^{3x-[3x]}$ is T_2 then $\frac{T_1}{2T_2} = ?$

1) $e^{3(x-[x])} = e^{3\boxed{\{x\}}} \rightarrow T_1 = 1$

2) $e^{3x-[3x]} = e^{\{3x\}} \rightarrow T_2 = \frac{1}{3}$

$$\frac{T_1}{2T_2} = \frac{1}{2 \times \frac{1}{3}} = \frac{3}{2}$$

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RELATION FUNCTION

Q $f(x) = \dfrac{\sin x + \sin 2x + \sin 4x + \sin 5x}{\cos x + \cos 2x + \cos 4x + \cos 5x}$

then $T = ?$

funda

$$\boxed{\begin{aligned} \sin C + \sin D &= 2\sin\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right) \\ \cos C + \cos D &= 2\cos\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right) \end{aligned}}$$

$$= \frac{2\sin(3x)\cos(\pm 2x) + 2\sin(3x)\cos(\pm x)}{2\cos(3x)\cos(\pm 2x) + 2\cos(3x)\cos(\pm x)}$$

$$= \frac{\cancel{2}\sin 3x\,\{\cancel{\cos 2x + \cos x}\}}{\cancel{2}\cos 3x\,\{\cancel{\cos 2x + \cos x}\}} = \tan 3x \rightarrow T = \frac{\pi}{3}$$

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T4 When 2 or more fxn are given.

$$h(x) = f(x) + g(x)$$

$$T = LCM(T_1, T_2)$$

Q $f(x) = \sin 2x + \cos 3x + \tan 4x$ find T

$$LCM\left\{\frac{\cancel{2}\pi}{\cancel{2}1}, \frac{2\pi}{3}, \frac{\pi}{4}\right\}$$

$$\frac{LCM(\pi, 2\pi, \pi)}{HCF(1, 3, 4)} = \frac{2\pi}{1} \therefore T = 2\pi$$

Q $f(x) = \sin^2\left(\frac{x}{2}\right) + \tan^2\left(\frac{x}{4}\right) + 10$ T?

$$\left\{\frac{\pi}{1/2}, \frac{\pi}{1/4}\right\}$$

$$LCM\{2\pi, 4\pi\} = 4\pi$$

Q $f(x) = \sin\frac{x}{\lfloor n} + \cos\frac{x}{\lfloor n+1}$ T = ?

$$\frac{2\pi}{\frac{1}{\lfloor n}}, \frac{2\pi}{\frac{1}{\lfloor n+1}}$$

$$LCM\{2\lfloor n\,\pi, 2\lfloor n+1\,\pi\}$$

$$T = 2\lfloor n+1\,\pi$$

Q $f(x) = \sin x + \{x\}$ $T = ?$

$\sin x \downarrow 2\pi$, $\{x\} \downarrow 1$

$LCM(2\pi, 1) \neq 2\pi$

$LCM(Q', Q) = $ PSBL Nhi

$\therefore$ fxn Non Periodic

$LCM(\underline{2, 4, 8})$
$= 8$
$(2, 2^2, \underline{2^3})$
$= 2^3$

Q $f(x) = \sin x + \boxed{x}$ $T = ?$

$\sin x \downarrow 2\pi$ (Periodic) + $\boxed{x} \downarrow$ NonPeriodic

NonPeriodic

T = Undefined

Q $f(x) = 2^{5\pi\{x\}} + \tan \pi[x]$ $T = ?$

$[x]$ Kya deta h?
Int = n deta

$= 2^{5\pi\{x\}} + \tan n\pi$

$f(x) = 2^{5\pi\{x\}} + 0$

$T = L$

Q $f(x) = \sin x + \tan\frac{x}{2} + \sin\frac{x}{2^2} + \tan\frac{x}{2^3} \ldots + \sin\frac{x}{2^{n-1}} + \tan\frac{x}{2^n}$

$2\pi, \frac{\pi}{1/2}, \frac{2\pi}{1/2^2}, \frac{\pi}{1/2^3} \ldots \frac{2\pi}{1/2^{n-1}}, \frac{\pi}{1/2^n}$

$T = LCM\{2\pi, 2\pi, 2^3\pi, 2^3\pi \ldots 2^n\pi, 2^n\pi\}$
$= 2^n\pi$

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$$[x] = x - \{x\}$$

$$\left\{-\frac{x}{4}\right\} \to \frac{1}{\left|-\frac{1}{4}\right|} = 4$$

Q $f(x) = \left[x+\frac{1}{2}\right] + \left[x-\frac{1}{2}\right] + 2[-x]$, $T = ?$

$$= \left(\cancel{x}+\cancel{\tfrac{1}{2}}\right) - \left\{x+\frac{1}{2}\right\} + \left(\cancel{x}-\cancel{\tfrac{1}{2}}\right) - \left\{x-\frac{1}{2}\right\} + 2\left(-\cancel{x} - \{-x\}\right)$$

$$f(x) = -\left\{x+\frac{1}{2}\right\} - \left\{x-\frac{1}{2}\right\} - 2\{-x\}$$

$$\left\{1,\ 1,\ \frac{1}{|-1|}\right\}$$

$$\mathrm{LCM}\{1, 1, 1\} = 1$$

Q $f(x) = \underbrace{[x] + \left[x + \frac{1}{3}\right] + \left[x + \frac{2}{3}\right]} - 3[x] + 10 \quad T = ?$

$f(x) = [3x] - 3[x] + 10$

$= \cancel{3x} - \{3x\} - 3(\cancel{x} - \{x\}) + 10$

$= -\{3x\} + \{x\} + 10$

$\mathrm{LCM}\left(\frac{1}{3}, \frac{1}{1}\right) = \frac{\mathrm{LCM}(1,1)}{\mathrm{HCF}(3,1)} = \frac{1}{1}$

$T = 1$

$2n\pi = 4\pi \Rightarrow n = 2$

$n = \pm 2$

$[x] + \left[x + \frac{1}{n}\right] + \left[x + \frac{2}{n}\right] + \cdots + \left[x + \frac{n-1}{n}\right]$

$= [nx]$

Q If $f(x) = \frac{\sin nx}{\sin \frac{x}{n}}$ is Periodic

& Period $= 4\pi$ find n?

$f(x) = \frac{\sin nx}{\sin \frac{x}{n}} \rightarrow \left(\frac{2\pi}{n}, \frac{2\pi}{1/n}\right)$

$= \mathrm{LCM}\left(\frac{2\pi}{n}, \frac{2n\pi}{1}\right) = \frac{\mathrm{LCM}(2\pi, 2n\pi)}{\mathrm{HCF}(n,1)}$

$T = 2n\pi$

T5 → Checking Method

When no method work

(1) Check $f\left(\frac{\pi}{2}+x\right)=f(x)$ if works then $T=\frac{\pi}{2}$ otherwise

(2) Check $f(\pi+x)=f(x)$ if works then $T=\pi$ otherwise

(3) $T=2\pi$

Q $f(x)=\underset{P(P)}{\cos(\cos x)}+\cos(\sin x)$ $T=?$

$f\left(\frac{\pi}{2}+x\right)=\cos\left(\cos\left(\frac{\pi}{2}+x\right)\right)+\cos\left(\sin\left(\frac{\pi}{2}+x\right)\right)$

$=\cos(+\sin x)+\cos(\cos x)=f(x)$

$T=\frac{\pi}{2}$

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RELATION FUNCTION

$|-a| = |a|$

Q $f(x) = \dfrac{|\sin x| + |\cos x|}{|\sin x - \cos x|}$ $T = ?$

$f\left(\frac{\pi}{2}+x\right) = \dfrac{|\cos x| + |-\sin x|}{|\cos x + \sin x|} \neq f(x)$

$f(\pi + x) = \dfrac{|\sin(\pi+x)| + |\cos(\pi+x)|}{|\sin(\pi+x) - \cos(\pi+x)|}$

$= \dfrac{|+\sin x| + |+\cos x|}{|-\sin x + \cos x|} = \dfrac{|\sin x| + |\cos x|}{|\sin x - \cos x|}$

$f(\pi+x) = f(x) \Rightarrow T = \pi$

T6

f(g(x)) Type

4 Kind

Check

Khatra

$N(P)$	$\boxed{P(P)}$	$P(N)$	$N(N)$
$= P$	P	$= N$	N

Q $f(x) = \sin\sqrt{x}$ $T = ?$

$P(N)$

$= N$

$T = \phi$

Q $f(x) = \cos(x^2)$ $T = ?$

$P(N) = N$ $T = \phi$

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Q $f(x) = \log(\cos 2x + 1)$ T.

$N(P) = $ Periodic

$T = \frac{2\pi}{2} = \pi$

$|\cos x|$

Q $f(x) = \frac{1}{1-\cos x}$

$= \frac{1}{2\sin^2\frac{x}{2}}$

$\frac{\pi}{1/2} = 2\pi$

Q $f(x) = [\sin 5x] + |\cos 3x|$

$N(P)$

Periodic

$LCM\left(\frac{2\pi}{5}, \frac{\pi}{3}\right) = \frac{LCM(2\pi, \pi)}{HCF(5,3)}$

$= \frac{2\pi}{1} = 2\pi$

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Composite & Inverse fxn.

1) If $f(x)$ & $g(x)$ are 2 fxn → $\left.\begin{array}{l} fog(x) \\ gof(x) \\ fof(x) \\ gog(x) \end{array}\right\}$ are Composite fxn.

2) $fog(x) = f(g(x))$

$gof(x) = g(f(x))$

$fof(x) = f(f(x))$

3) $fogof(x) = f(g(f(x)))$

$fofof(x) = f(f(f(x)))$

Q $f(x) = \dfrac{1-x}{1+x}$ then

A) $f(\sin\theta) = \dfrac{1-\sin\theta}{1+\sin\theta}$

B) $f(f(\sin\theta)) = \dfrac{1-f(\sin\theta)}{1+f(\sin\theta)}$

$$= \frac{1-\left(\frac{1-\sin\theta}{1+\sin\theta}\right)}{1+\left(\frac{1-\sin\theta}{1+\sin\theta}\right)} = \frac{2\sin\theta}{2} = \sin\theta$$

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RELATION FUNCTION

Q $f(\boxed{x}) = \frac{1}{1-x}$

① $fof(x) = ?$

$f(f(x)) = \frac{1}{1-f(x)} = \frac{1}{1-\frac{1}{1-x}} = \frac{1-x}{x-x-1} = \frac{x-1}{x}$

(2) $fofof(x)$

$= f(f(f(x))) = \frac{1}{1-f(f(x))} = \frac{1}{1-\frac{x-1}{x}}$

$= \frac{x}{x-x+1} = x$

Q $f(x) = x^3 - x$, $g(x) = \sin x$

A) $fog(x) = f(g(x)) = g^3(x) - g(x)$

$= \sin^3 x - \sin x$

(B) $gof(x) = g(f(x))$

$= \sin f(x)$

$= \sin(x^3 - x)$

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RELATION FUNCTION

Q $f(x) = \underline{(a - x^n)}^{1/n}$ then $fof(x) = ?$

$$fof(x) = f(f(x)) = \left(a - (f(x))^n\right)^{\frac{1}{n}}$$

$$= \left(a - \left((a - x^n)^{\cancel{1/n}}\right)^{\cancel{n}}\right)^{\frac{1}{n}}$$

$$= (\cancel{a} - \cancel{a} + x^n)^{\frac{1}{n}}$$

$$= (x^{\cancel{n}})^{\frac{1}{\cancel{n}}}$$

$$fof(x) = x$$

Q $g(x) = \left(4\cos^4 x - 2\boxed{\cos 2x} - \frac{1}{2}\boxed{\cos 4x} - x^7\right)^{\frac{1}{7}}$ then $g(g(100)) = ?$

$\cos 2\theta = 2\cos^2\theta - 1$

$$= \left(4\cos^4 x - 2(2\cos^2 x - 1) - \frac{1}{2}\left(2\cos^2(2x) - 1\right) - x^7\right)^{\frac{1}{7}}$$

$$= \left(4\cos^4 x - 4\cos^2 x + 2 - \cos^2(2x) + \frac{1}{2} - x^7\right)^{1/7}$$

$$= \left(4\cos^4 x - 4\cos^2 x + 2 - (2\cos^2 x - 1)^2 + \frac{1}{2} - x^7\right)^{\frac{1}{7}}$$

$$= \left(4\cancel{\cos^4 x} - 4\cancel{\cos^2 x} + 2 - 4\cancel{\cos^4 x} + 4\cancel{\cos^2 x} - 1 + \frac{1}{2} - x^7\right)^{1/7}$$

$g(x)$

$$(a - x^n)^{\frac{1}{n}}$$

$g(g(x)) =$

$$g(x) = \left(\frac{3}{2} - x^7\right)^{\frac{1}{7}}$$

$$g(g(x)) = x \Rightarrow g(g(100)) = 100$$

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Q If $f(x) = \dfrac{x}{\sqrt{1+x^2}}$

$f o f o f o f o f \cdots f(x) = ? = \dfrac{x}{\sqrt{1+nx^2}}$

$\xleftarrow{\quad n \text{ times} \quad}$

1) $f o f(x) = f(f(x)) = \dfrac{f(x)}{\sqrt{1+f^2(x)}} = \dfrac{\frac{x}{\sqrt{1+x^2}}}{\sqrt{1+\frac{x^2}{1+x^2}}}$

$\underline{\underline{4Qs}}$ / $\boxed{20\ Qs}$

$f o f(x) = \dfrac{x}{\sqrt{1+2x^2}}$

2) $f o f o f(x) = f(f(f(x))) = \dfrac{f(f(x))}{\sqrt{1+(f(f(x)))^2}} = \dfrac{\frac{x}{\sqrt{1+2x^2}}}{\sqrt{1+\frac{x^2}{1+2x^2}}} = \dfrac{x}{\sqrt{1+3x^2}}$