DPP-01

AREA

Q. Find the areas of the triangles the coordinates of whose angular points are respectively:

1. $(1,3), (-7,6)$ and $(5,-1)$.

2. $(0,4), (3,6)$ and $(-8,-2)$.

3. $(5,2), (-9,-3)$ and $(-3,-5)$.

4. $(a, b+c), (a, b-c)$ and $(-a, c)$.

5. $(a, c+a), (a, c)$ and $(-a, c-a)$.

6. $(a\cos\phi_1, b\sin\phi_1), (a\cos\phi_2, b\sin\phi_2)$ and $(a\cos\phi_3, b\sin\phi_3)$.

7. $(am_1^2, 2am_1), (am_2^2, 2am_2)$ and $(am_3^2, 2am_3)$.

8. $\{am_1m_2, a(m_1+m_2)\}, \{am_2m_3, a(m_2+m_3)\}$ and $\{am_3m_1, a(m_3+m_1)\}$

9. $\left\{am_1, \frac{a}{m_1}\right\}, \left\{am_2, \frac{a}{m_2}\right\}$ and $\left\{am_3, \frac{a}{m_3}\right\}$.

Q. Prove (by shewing that the area of the triangle formed by them is zero) that the following sets of three points are in a straight line:

10. $(1,4), (3,-2)$, and $(-3,16)$.

11. $\left(-\frac{1}{2}, 3\right), (-5,6)$ and $(-8,8)$.

12. $(a, b+c), (b, c+a)$, and $(c, a+b)$.

Q. Find the areas of the quadrilaterals the coordinates of whose angular points, taken in order, are:

13. $(1,1), (3,4), (5,-2)$ and $(4,-7)$.

14. $(-1,6), (-3,-9), (5,-8)$, and $(3,9)$.

15. If O be the origin, and if the coordinates of any two points P_1 and P_2 be respectively (x_1, y_1) and (x_2, y_2), prove that: $OP_1 \cdot OP_2 \cdot \cos P_1OP_2 = x_1x_2 + y_1y_2$.

16. Find the area of the pentagon whose vertices are $A(1,1), B(7,21), C(12,2), D(7,-3)$, and $E(0,-3)$.

17. Four points $A(6,3), B(-3,5), C(4,-2)$, and $D(x, 2x)$ are given in such a way that $\frac{(\text{Area of } \triangle DBC)}{(\text{Area of } \triangle ABC)} = 1/2$. Find x.

18. Given three points $P(2,3), Q(4,-2)$ and $R(\alpha, 0)$.
(i) Find the value of α if $PR + RQ$ is minimum
(ii) Find the value of α if $|PR - RQ|$ is maximum.

TRANSFORMATION OF AXES

19. At what point should the origin be shifted if the coordinates of a point $(4,5)$ become $(-3,9)$?

20. If the origin is shifted to the point $(1,-2)$ without the rotation of the axes, what do the following equations become?
(i) $2x^2 + y^2 - 4x + 4y = 0$
(ii) $y^2 - 4x + 4y + 8 = 0$

21. Shift the origin to a suitable point so that the equation $y^2 + 4y + 8x - 2 = 0$ will not contain a term in y and the constant term.

22. Point $P(-2,3)$ goes through following transformations in succession:
(i) reflection in line $y = x$
(ii) translation of 4 units to the right
(iii) translation of 5 units up
(iv) reflection in y-axis
Find the coordinates of final position of the point.

ROTATION OF AXES

23. The axes are rotated through an angle of $\pi/3$ in the anticlockwise direction with respect to $(0,0)$. Find the coordinates of point $(4,2)$ (w.r.t. old coordinate system) in the new coordinates system.

24. The equation of a curve referred to a given system of axes is $3x^2 + 2xy + 3y^2 = 10$. Find its equation if the axes are rotated about the origin through an angle of $45°$.

25. Without rotating the original coordinate axes, to which point should origin be transferred, so that the equation $x^2 + y^2 - 4x + 6y - 7 = 0$ is changed to an equation which contains no term of first degree?

CENTRES OF TRIANGLE

26. The vertices of a triangle are $A(-1,-7)$, $B(5,1)$ and $C(1,4)$. If the internal angle bisector of $\angle B$ meets the side AC in D, then find the length of AD.

27. The vertices of a triangle are $A(x_1, x_1\tan\theta_1)$, $B(x_2, x_2\tan\theta_2)$, and $C(x_3, x_3\tan\theta_3)$. If the circumcenter of $\triangle ABC$ coincides with the origin and $H(a,b)$ is the orthocenter, show that

$$\frac{a}{b} = \frac{\cos\theta_1 + \cos\theta_2 + \cos\theta_3}{\sin\theta_1 + \sin\theta_2 + \sin\theta_3}$$

28. If (x_i, y_i), $i = 1,2,3$, are the vertices of an equilateral triangle such that

$(x_1 + 2)^2 + (y_1 - 3)^2 = (x_2 + 2)^2 + (y_2 - 3)^2 = (x_3 + 2)^2 + (y_3 - 3)^2$, then

find the value of $\frac{x_1+x_2+x_3}{y_1+y_2+y_3}$.

ANSWER KEY

1. 10

2. 1

3. 29

4. 2ac

5. a^2

6. $2ab\sin\frac{\phi_2-\phi_3}{2}\sin\frac{\phi_3-\phi_1}{2}\sin\frac{\phi_1-\phi_2}{2}$

7. $a^2(m_2-m_3)(m_3-m_1)(m_1-m_2)$

8. $\frac{1}{2}a^2(m_2-m_3)(m_3-m_1)(m_1-m_2)$

9. $\frac{1}{2}a^2(m_2-m_3)(m_3-m_1)(m_1-m_2)\div m_2m_2m_3$

13. $20\frac{1}{2}$

14. 96

15.

16. 146 sq. units

17. $\frac{11}{6}$

18. (i) $\alpha = 16/5$ (ii) $\alpha = 8$

19. $(7,-4)$.

20. (i) $2X^2+Y^2=6$ (ii) $Y^2=4X$

21. $(3/4,-2)$.

22. $(-7,3)$

23. $(2+\sqrt{3}\,,-2\sqrt{3}+1)$

24. $2X^2+Y^2=5$

25. $(2,-3)$

26. $\frac{10\sqrt{2}}{3}$

27. $\frac{\cos\theta_1+\cos\theta_2+\cos\theta_3}{\sin\theta_1+\sin\theta_2+\sin\theta_3}$

28. $-2/3$