Nishant Jindal

Rk:- (1) If $AB = B \cdot A$ then A & B are commute.

(2) If A & B are not commute then $AB \neq BA$

(3) If $AB = -BA$ then A & B are anti commute

(4) $I^n = I$

Q If A & B are 2 Matrices such that they are commute.

then show that $\forall n \in N$ $AB^n = B^n A$

$\to AB = BA$

$LHS = A \cdot B^n = (AB)B^{n-1}$

$= BAB^{n-1} = B(A \cdot B) \cdot B^{n-2}$

$= BB \cdot A \cdot B^{n-2} = B^2(AB)B^{n-3}$

$= B^2 \cdot BA \cdot B^{n-3} = B^3(AB)B^{n-4}$

$\vdots$

$= B^n \cdot A = RHS$

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Q Find all Possible matrices of order 2 which commute with matrix $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

let B is a matrix of order 2 which is also commute to A

$\Rightarrow AB = BA$ let $B = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} a+c & b+d \\ c & d \end{bmatrix} = \begin{bmatrix} a & a+b \\ c & c+d \end{bmatrix}$$

$a+c=a$	$\cancel{b}+d=a+\cancel{b}$	$c=c$ ✓	$\cancel{d}=c+\cancel{d}$
$a=a$ ✓	$a=d$	$0=0$ ✓	$c=0$

$\therefore B = \begin{bmatrix} \alpha & \beta \\ 0 & \alpha \end{bmatrix}$

∞ Matrices Possible

Ex:- $\begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix}, \begin{bmatrix} 3 & 1 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 4 & 1 \\ 0 & 4 \end{bmatrix}$

$\begin{bmatrix} -1 & 1 \\ 0 & -1 \end{bmatrix}$

Nishant Jindal

$\hat{Q}3$ $A=\begin{bmatrix}2 & 1\\4 & 1\end{bmatrix}$ $B=\begin{bmatrix}3 & 4\\2 & 3\end{bmatrix}$, $C=\begin{bmatrix}3 & -4\\-2 & 3\end{bmatrix}$ then

$$tr(A)+tr\left(\frac{A(BC)}{2}\right)+tr\left(\frac{A\cdot(BC)^2}{4}\right)+tr\left(\frac{A\cdot(BC)^3}{8}\right)+\dots = ?$$

Interesting

$B\cdot C=\begin{bmatrix}3 & 4\\2 & 3\end{bmatrix}\begin{bmatrix}3 & -4\\-2 & 3\end{bmatrix}$

$=\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}=I$

Demand

$tr(A)+tr\left(\frac{A\cdot I}{2}\right)+tr\left(\frac{A\cdot I^2}{4}\right)+\dots$

$= tr(A)+\frac{1}{2}tr(A)+\frac{1}{4}tr(A)+\frac{1}{8}tr\,A \dots$

$= tr\,A\left(1+\frac{1}{2}+\frac{1}{2^2}+\frac{1}{2^3}+\dots\infty\right)$

$= \frac{1}{1-\frac{1}{2}}\cdot tr(A) = 2\,tr\,A = 2\times 3 = 6$

$\frac{a}{1-r}$

RK

(5) $KAB=(KA)B=K(AB)$

(6) $ABC = A(BC) = (AB)C \neq (AC)B$

(7) $A\cdot(B+C) = A\cdot B + A\cdot C$

(8) $(A+B)(C+d) = A\cdot(C+d)+B(C+d)$

(9) $(A+B)^2 = (A+B)\cdot(A+B)$

$= A\cdot(A+B)+B\cdot(A+B)$

$= A^2+AB+BA+B^2$

(10) If $A\cdot B = A\cdot C \not\Rightarrow B=C$

(11) If $AB=AC \Rightarrow A\cdot B - AC = 0$

$\Rightarrow A\cdot(B-C)=0$ ✓

(12) If $A^2=A \Rightarrow A^2-A=0 \Rightarrow A\cdot(A-I)=0$ *

Nishant Jindal

(17) When A & B are Commute

$AB = BA$

(A) $(A+B)^2 = A^2 + AB + BA + B^2$

$= A^2 + AB + AB + B^2$

$(A+B)^2 = A^2 + 2AB + B^2$

(B) $(A+B)\cdot(A-B)$

Normally

$A(A-B) + B(A-B)$

$A^2 - AB + BA - B^2$

Commute Hota to....

$A^2 - \cancel{AB} + \cancel{AB} - B^2$

$(A+B)(A-B) = A^2 - B^2$

(13) Pre $AB - C$ Post $\cdot A = 0$

$\Rightarrow A(B-C) = 0$ ✗

Com. नहीं तो सकते।

(14) If $A\cdot B = 0 \not\Rightarrow A = 0$ OR $B = 0$

(15) If $A\cdot B = A\cdot C$

$M = N \times A$ (Post)

$MA = NA$

$\Rightarrow AB - AC = 0$

$\Rightarrow A\cdot(B-C) = 0$

$\not\Rightarrow A = 0$ or $B - C = 0$

$B = C$

(16) If $\underline{A\cdot B = 0}$ & A, B non null matrix

$|A\cdot B| = |0|$

$|A|\cdot|B| = 0 \Rightarrow |A| = 0$ or $|B| = 0$ or Both

* When A & B are Com. Normal Algebric formula works

R_K

(5) $KAB = (KA)B = K(AB)$

(6) $ABC = A(BC) = (AB)C \neq (AC)B$

(7) $A\cdot(B+C) = A\cdot B + A\cdot C$

(8) $(A+B)(C+d) = A\cdot(C+d) + B(C+d)$

(9) $(A+B)^2 = (A+B)\cdot(A+B)$

$= A\cdot(A+B) + B\cdot(A+B)$

$= A^2 + AB + BA + B^2$

(10) If $A\cdot B = A\cdot C \not\Rightarrow B = C$

(11) If $AB = AC \Rightarrow A\cdot B - AC = 0$

$\Rightarrow A\cdot(B-C) = 0$ ✓

(12) If $A^2 = A \Rightarrow A^2 - A = 0 \Rightarrow A\cdot(A - I) = 0$ *

Nishant Jindal

(18)* If A & B are commute then B.T. formula is works.

$$(A+B)^n = {}^nC_0 A^n B^0 + {}^nC_1 A^{n-1} B + {}^nC_2 A^{n-2} B^2 + \dots + {}^nC_n A^0 B^n.$$

Q If $A = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}$ & $B = \begin{bmatrix} a & 1 \\ b & -1 \end{bmatrix}$ Such that $(A+B)^2 = A^2+B^2$

then ordered Pair $(a,b) = ?$

Ans (1,4)

$\cancel{A^2} + AB + BA + \cancel{B^2} = \cancel{A^2} + \cancel{B^2}$ (given)

$AB = -BA \Rightarrow A \cdot B + BA = 0$

$$\begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix} \cdot \begin{bmatrix} a & 1 \\ b & -1 \end{bmatrix} + \begin{bmatrix} a & 1 \\ b & -1 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix} = 0$$

$$\begin{bmatrix} a-b & 2 \\ 2a-b & 3 \end{bmatrix} + \begin{bmatrix} a+2 & -a-1 \\ b-2 & -b+1 \end{bmatrix} = 0$$

$$\begin{bmatrix} 2a-b+2 & -a+1 \\ 2a-2 & -b+4 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$2a-b+2=0$

$-a+1=0 \Rightarrow a=1$

$-b+4=0 \Rightarrow b=4$

$2a-2=0$

$2-2=0$

Q 2 Matrices P & Q $P^3 = Q^3$ & $P^2Q = Q^2P$, $\boxed{P \neq Q}$ find $|P^2+Q^2| = ?$

5 Mains

$P-Q \neq 0$

$|P-Q| \neq 0$

$P^3 = Q^3$

$P^2Q = Q^2P$

$P^3 - P^2Q = Q^3 - Q^2P$

$P^2(P-Q) = Q^2(Q-P)$

$P^2(P-Q) - Q^2(Q-P) = 0$

$P^2(P-Q) + Q^2(P-Q) = 0$

$(P^2+Q^2) \cdot (P-Q) = 0$

$|(P^2+Q^2) \cdot (P-Q)| = 0$

$|P^2+Q^2| \, |P-Q| = 0 \Rightarrow |P^2+Q^2| = 0$ OR $|P-Q| = 0$

$\therefore |P^2+Q^2| = 0$

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Q A & B are 2 Matrix such that $AB = B$ & $BA = A$ then $A^2 + B^2 = ?$

$2AB \quad A+B \quad 2BA \quad AB.$

Demam $A^2 + B^2$

$= A \cdot A + B \cdot B$

$= A\,BA + B \cdot AB$

$= (A\,B)A + (B\,A)B$

$= BA + AB$

$= A + B$

① $A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$

$A^2 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} = A$

$A^2 = A \times A$

$A^3 = A^2 = A$

$A^4 = A^3 = A^2 = A$

$A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \; I + A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$

$(I+A)^2 = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} \times \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2^2 & 0 \\ 0 & 1 \end{bmatrix}$

$(I+A)^3 = \begin{bmatrix} 2^3 & 0 \\ 0 & 1 \end{bmatrix}$

Fundaz

$^nC_0 + {}^nC_1 + {}^nC_2 + \dots + {}^nC_n = 2^n$

$^{10}C_0 + {}^{10}C_1 + {}^{10}C_2 + \dots + {}^{10}C_{10} = 2^{10}$

Q₁ Mains + Adv

① B.T लगाना ② $A \cdot I = I \cdot A$

If $(I+A)^{10} = x \cdot A + y \cdot I$ & $A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ then $x + y = ?$

Com.

$(I+A)^{10} = {}^{10}C_0 I^{10} A^0 + {}^{10}C_1 I^9 A + {}^{10}C_2 I^8 A^2 + {}^{10}C_3 I^7 A^3 + \dots + {}^{10}C_{10} A^{10}$

$= I + {}^{10}C_1 A + {}^{10}C_2 A^2 + {}^{10}C_3 A^3 + \dots + {}^{10}C_{10} A^{10}$

$= I + \{ {}^{10}C_1 A + {}^{10}C_2 A + {}^{10}C_3 A + \dots + {}^{10}C_{10} A \}$

$= I + A \{ {}^{10}C_1 + {}^{10}C_2 + {}^{10}C_3 + \dots + {}^{10}C_{10} \}$

$= I + A \{ 2^{10} - {}^{10}C_0 \}$

$= I + A \cdot (2^{10} - 1) = x \cdot A + y\, I$

$y = 1, \; x = 2^{10} - 1$

$x + y = 2^{10} - 1 + 1 = 2^{10}$ Ans

Nishant Jindal

Q If $A \cdot B = 0$ & $B \cdot C = I$ then $(A+B)^2 \cdot (A+C)^2 = ?$

$A \cdot B = 0$

$ABC = ?$

$= (AB)C$

$= 0 \cdot C = 0$

$A \cdot (BC) = 0$

$A \cdot I = 0$

$A = 0$

Demand $\equiv (A+B)^2 \cdot (A+C)^2$

$= (0+B)^2 (0+C)^2$

$= B^2 \cdot C^2 = B(B \cdot C)C$

$= B \cdot I \cdot C = BC = I$

Q find Sq. Root of form $\begin{bmatrix} 0 & a \\ b & 0 \end{bmatrix}$ for matrix $\begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix}$, $a, b \in I$

$\begin{bmatrix} 0 & a \\ b & 0 \end{bmatrix} \begin{bmatrix} 0 & a \\ b & 0 \end{bmatrix} = \begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix}$

demand $= \sqrt{\begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix}}$

$= \begin{bmatrix} 0 & a \\ b & 0 \end{bmatrix}$

$\begin{bmatrix} ab & 0 \\ 0 & ab \end{bmatrix} = \begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix}$

$ab = -2$; $a, b \in I$

$a = 1$, $b = -2$ or $a = -1$ $b = 2$

$\begin{bmatrix} 0 & 1 \\ -2 & 0 \end{bmatrix}$ or $\begin{bmatrix} 0 & -1 \\ 2 & 0 \end{bmatrix}$

$a = 2$ $b = -1$ OR $a = -2$ $b = 1$

$\begin{bmatrix} 0 & 2 \\ -1 & 0 \end{bmatrix}$ $\begin{bmatrix} 0 & -2 \\ 1 & 0 \end{bmatrix}$

Rk.

1) For every Sq^r Matrix A there exist a Determinant

denoted by $|A|$ or $\underline{\underline{\det A}}$

$|A| = 0$ — Singular Matrix.

$|A| \neq 0$ — NonSingular.

(2) Properties of determinant

(A) $|KA| = K^n |A|$ — $\underline{n = \text{order of } A}$

(B) $|A \cdot B| = |A| \cdot |B|$; $|A \cdot B \cdot C| = |A| \cdot |B| \, |C|$

(C) $|A^2| = |A|^2$ as $\underline{|A^2|} = |A \cdot A| = |A|\,|A| = \underline{|A|^2}$

$|A^3| = |A|^3$

$|A|^n = |A^n|$

Q If $A = \begin{bmatrix} \alpha & 2 \\ 2 & \alpha \end{bmatrix}$; $|A^3| = 125$ find α & $\left|-\frac{A}{2}\right| = ?$

(1) $|A| = \begin{vmatrix} \alpha & 2 \\ 2 & \alpha \end{vmatrix} = \alpha^2 - 2^2 = \alpha^2 - 4$

(2) $|A^3| = 125$

$|A|^3 = (5)^3 \Rightarrow |A| = 5$

$\alpha^2 - 4 = 5$

$\alpha^2 = 9 \Rightarrow \alpha = 3, -3$

(3) demand $= \left|-\frac{A}{2}\right| = \left(-\frac{1}{2}\right)^2 |A|$ — $n = 2$

$= \frac{1}{4} \times 5 = \frac{5}{4}$

Nishant Jindal

(3) Charactristic Eqn

1) $|A-xI|=0$ is Ch. Eqn of sq matrix A.

Ex: $A=\begin{bmatrix}2 & -1\\ 1 & 2\end{bmatrix}$ find Ch. Eq

$I=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

$x\cdot I=\begin{bmatrix}x & 0\\ 0 & x\end{bmatrix}$

$A-xI=\begin{bmatrix}2 & -1\\ 1 & 2\end{bmatrix}-\begin{bmatrix}x & 0\\ 0 & x\end{bmatrix}$

$A-xI=\begin{bmatrix}2-x & -1\\ 1 & 2-x\end{bmatrix}$

Ch. Eqn: $|A-xI|=\begin{vmatrix}2-x & -1\\ 1 & 2-x\end{vmatrix}=0 \Rightarrow$

$(2-x)^2-(-1)=0$

$x^2-4x+4+1=0$

$x^2-4x+5=0$ (Ch. Eqn for Matrix A)

2) Ch. Eqn is always Satisfied by its Matrix.

Q $A=\begin{bmatrix}3 & 0\\ 2 & 1\end{bmatrix}$ find Ch. Eqn.

$\begin{vmatrix}3 & 0\\ 2 & 1\end{vmatrix}=3\times1-2\times0=3.$

$|A-xI|=\begin{vmatrix}3-x & 0\\ 2 & 1-x\end{vmatrix}=0 \Rightarrow (3-x)(1-x)-0=0$

$\Rightarrow x^2-4x+3=0$ is Ch. Eqn of A

$A^2-4A+3I=0 \quad \times A^{-1}$ (Pre)

2) $f(A)=A^2-4A+3I$ is Poly of Matrix A

Trick

Ch. Eqn $\to$ $x^2-(\text{SOR})\,x+(\text{POR})=0$

SOR = Tr A, POR $\to$ det A

$x^2-4x+3=0$

Fayda $A^{-1}A^2-4A^{-1}A+3A^{-1}I=0$

$A-4I+3A^{-1}=0$

$A^{-1}=\left(\dfrac{4I-A}{3}\right)$

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Symmetric & Skew symm. Matrix.

(1) A sq^r Matrix A is symm Matrix.

if $A^T = A$ (2) if $a_{ij} = a_{ji}$

Ex $A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \Rightarrow A^T = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} = A$

$\therefore A = \text{Symm.}$

$a_{11} = a_{11}$

$a_{12} = a_{21}$

$a_{21} = a_{12}$

$a_{22} = a_{22}$

Ex $A = \begin{bmatrix} a & h & g \\ h & b & f \\ g & f & c \end{bmatrix}$

$\Rightarrow A^T = \begin{bmatrix} a & h & g \\ h & b & f \\ g & f & c \end{bmatrix} = A \Rightarrow$ [symm]

(2) If $A^T = -A$ then A is skew symm.

(2) When $a_{ij} = -a_{ji}$

(3) diag elements = 0

$i = j$

(Skew) $\therefore a_{ii} = -a_{ii}$

$2a_{ii} = 0$

$\boxed{a_{ii} = 0}$

Ex $A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

$A^T = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} = -\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} = -A$

Ex: $A = \begin{bmatrix} 0 & -h & g \\ h & 0 & -f \\ -g & f & 0 \end{bmatrix}$ then $A^T = ?$

$A^T = \begin{bmatrix} 0 & h & -g \\ -h & 0 & f \\ g & -f & 0 \end{bmatrix} = -\begin{bmatrix} 0 & -h & g \\ h & 0 & -f \\ -g & f & 0 \end{bmatrix}$

$= -A$ (Skew)

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Q $\underline{A+A^T}$ is skew or Symm?

let $B = A + A^T$

$(A+B)^T = A^T + B^T$

$B^T = (A+A^T)^T$

$= A^T + (A^T)^T$

$= A^T + A$

$B^T = B$

Symm.

Q $\underline{A-A^T}$ is skew / Sym

Let $B = A - A^T$

$B^T = (A - A^T)^T$

$= A^T - (A^T)^T$

$= A^T - A$

$= -(A - A^T)$

$B^T = -B$ ($\underline{\text{Symm}}$)