

$$\lim_{x \rightarrow 1^-} \frac{h(x)+1}{3x+3} = \frac{h(1)+1}{6}$$

$$\lim_{x \rightarrow 1^+} \frac{f(x)}{2} = \frac{f(1)}{2}$$

$$\frac{\sin^2\left(2\pi(2^{x-1}-1)\right)}{4\pi^2(2^{x-1}-1)^2} \cdot \frac{-\cos(2\pi(2^{x-1}-1))}{\cos(\)} = 2$$

$\rho_n(\)$

$$\underset{x \leq a}{\lim} f(x) = f(\underline{a}) f(\underline{1}) \Rightarrow \boxed{f(1) = 1}$$

$$\underset{x \rightarrow a}{\lim} f(x) = \underset{x \rightarrow a}{\lim} f(x \cdot 1) = f(1) \underset{x \rightarrow a}{\lim} f(x)$$

$$\begin{aligned} f(a) &= f(1) \\ \text{if } f(0) &= f^2(0) \\ &= 0, 1 \end{aligned} \quad \rightarrow \quad \underset{x \rightarrow a}{\lim} f\left(\frac{a-x}{a}\right) = f(a) \underset{x \rightarrow a}{\lim} f\left(\frac{-x}{a}\right)$$

$$\begin{aligned} f(-1) &= -1 \\ \frac{f(-1)}{f(-1)} &\neq f(1) = 1 \\ \boxed{f(-1) = \pm 1} & \quad \text{LHL} = \text{RHL} \\ & \quad = 0 \\ & \quad f(0) = \end{aligned}$$

$$f(a) \neq f(1)$$

$$\underset{x \rightarrow 0}{\lim} f(h) = RHL$$

$$\underset{h \rightarrow 0}{\lim} f(-h) = LHL = \underline{f(-1)} \underset{h \rightarrow 0}{\lim} f(h)$$

$$\lim_{x \rightarrow \frac{\pi}{2}^-} \lim_{n \rightarrow \infty} \frac{\ln(\tan x)}{1 + (\tan x)^n} = \lim_{x \rightarrow \frac{\pi}{2}^-} \ln \tan x = 0$$

$$\lim_{x \rightarrow \frac{\pi}{2}^+} \lim_{n \rightarrow \infty} \frac{\ln(\tan x)}{1 + (\tan x)^n}$$

$$\lim_{x \rightarrow \frac{\pi}{2}^+} \lim_{n \rightarrow \infty} \frac{\sin\left(\frac{x}{2^n} - \frac{\pi}{2}\right)}{\cos\frac{x}{2^n} \cos\frac{x}{2^{n-1}}} = 0 = 0$$

$$g(x) = \frac{(x+1)(x^2 - 2x - 1)}{(x+1) \dots}$$

$x \rightarrow \infty$

~~$(x+1)(x-2)$~~ \times

~~$(x+1)$~~

$$\lim_{x \rightarrow -1} = \infty$$

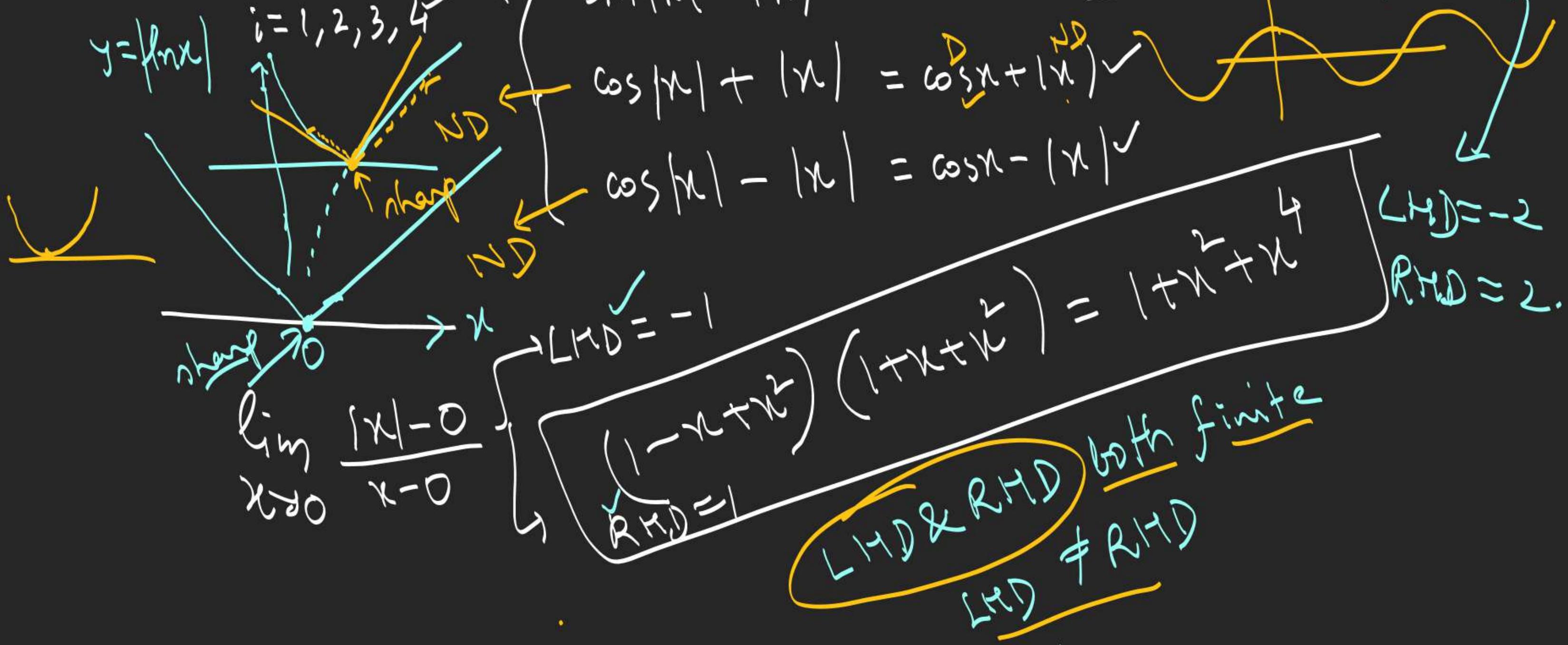
$$g(x) = k(x+1)$$

$$\lim_{x \rightarrow -1} \frac{(x+1)(\dots)}{k(x+1)} = \frac{1}{2}$$

$k = ?$

$$f(x) = \begin{cases} x & \sin|x| + |x| \\ D & \sin|x| - |x| \end{cases}$$

$\lim_{x \rightarrow 0} \frac{\sin|x| + |x| - 0}{x - 0} = \lim_{x \rightarrow 0} \left(\frac{\sin|x|}{|x|} + \frac{|x|}{x} \right)$



$\lim_{x \rightarrow 0} \frac{\sin(x) - x}{x} = 0$

sharp

$\lim_{x \rightarrow 0} \left(\frac{\sin(x) - x}{x} \right) \cdot \frac{x}{x} = 0$

$\lim_{x \rightarrow 0} \frac{\cos(x) + 1}{x} = \frac{1}{\sqrt{0}} = \infty$

$\lim_{x \rightarrow 0} \left(\frac{\cos(x) - 1}{x^2} \cdot \frac{x^2}{x^2} + \frac{1}{x^2} \cdot \frac{x^2}{x^2} \right) = \infty$

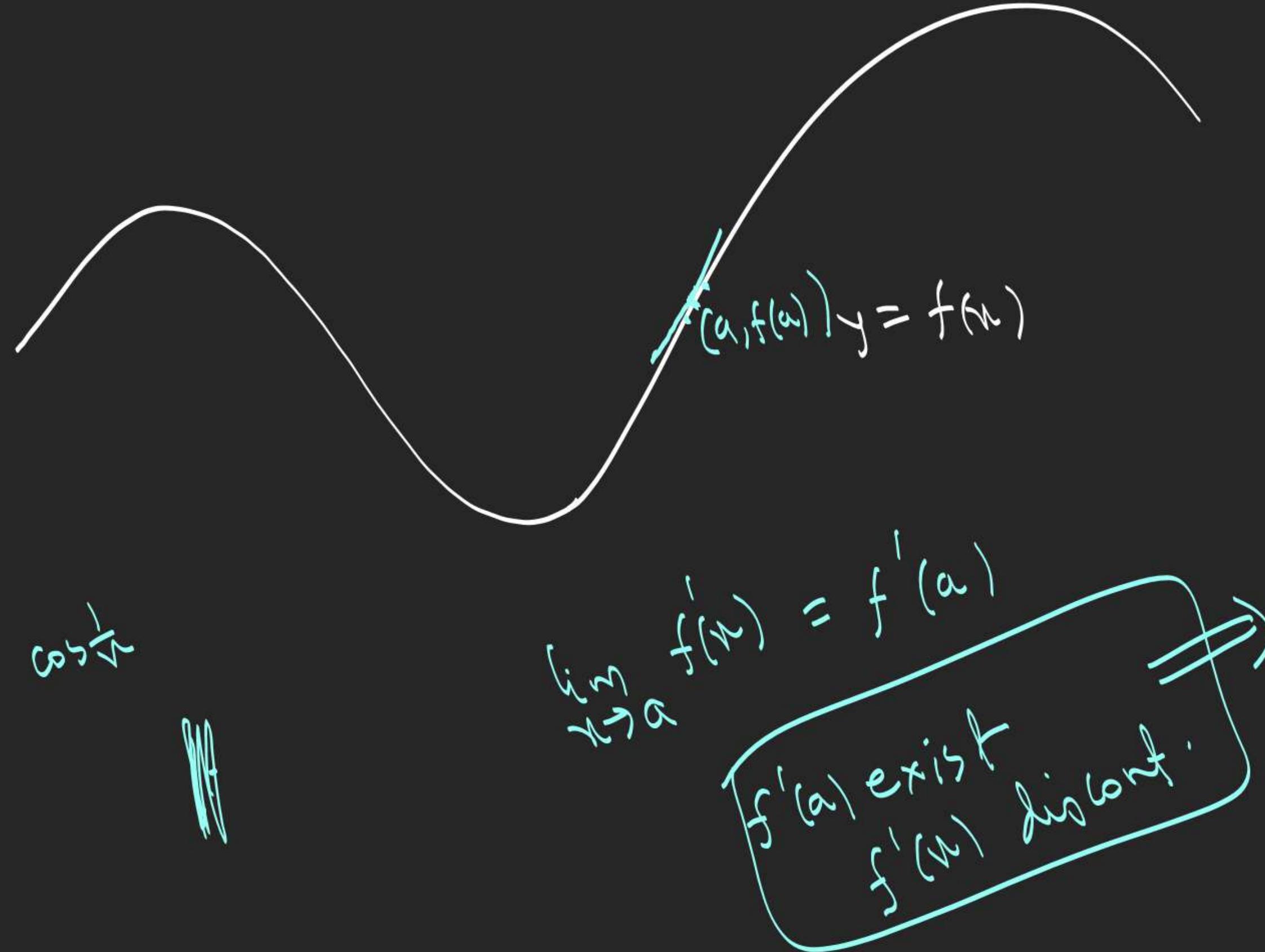
L'Hopital's Rule: $\lim_{x \rightarrow 0} \frac{\cos(x) - 1}{x^2} = -1$

R.H.S. ≈ 1

Note → If $f(x)$ is diff. at $x=a$, doesn't necessarily imply that $f'(x)$ is continuous at $x=a$.

$$f'(x)=g(x)$$

$f'(a)$ exist ✓
 $f'(x)$ may be ~~cont.~~ or discontinuous



$$f(x) = \begin{cases} x^2 \sin \frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$$

P.T. $f(x)$ is diff.
at $x=0$ & $f'(x)$

$$\lim_{x \rightarrow 0} f'(x) = \lim_{x \rightarrow 0} \left(\underbrace{2x \sin \frac{1}{x}}_0 - \underbrace{\cos \frac{1}{x}}_{[-1, 1]} \right) = \text{not exist. is discontinuous}$$

at $x=0$.

$$f'(0) = \lim_{x \rightarrow 0} \frac{x^2 \sin \frac{1}{x} - 0}{x - 0} = \lim_{x \rightarrow 0} x \sin \frac{1}{x} = 0$$

$$f'(x) = \begin{cases} 2x \sin \frac{1}{x} + x^2 \cos \frac{1}{x} \left(-\frac{1}{x^2}\right), & x \neq 0 \\ 0, & x = 0 \end{cases} \Rightarrow f'(x) = \begin{cases} 2x \sin \frac{1}{x} - \cos \frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$$

$$x \neq a \cdot \quad a \neq 0$$

$$f'(a) = \lim_{x \rightarrow a} \frac{x^2 \sin \frac{1}{x} - a^2 \sin \frac{1}{a}}{x - a} = \lim_{x \rightarrow a} \frac{x^2 \sin \frac{1}{x} - a^2 \sin \frac{1}{a} + a^2 \sin \frac{1}{x} - a^2 \sin \frac{1}{a}}{x - a}$$

$$\begin{aligned} & \lim_{x \rightarrow a} \left((x+a) \sin \frac{1}{x} + a^2 \right) \frac{2 \sin \left(\frac{1}{x} - \frac{1}{a} \right) \cos \left(\frac{1}{x} + \frac{1}{a} \right)}{2x} \frac{(x-a)}{2x} \\ &= 2a \sin \frac{1}{a} + a^2 \left(2 \cos \frac{1}{a} \right) \frac{1}{2a^2(-1)} \\ &= 2a \sin \frac{1}{a} - \cos \frac{1}{a}. \end{aligned}$$

