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$$(a+md)(a+rd) = (a+nd)^2$$

$$a(m+r-2n) = (n^2-mr)\,d$$

$$\frac{a}{d} = \frac{n^2-mr}{m+r-2n} = \frac{n^2-mr}{\frac{2mr}{n}-2n}$$

$a,\ A_1,\ A_2,\ b$

$\frac{1}{a},\ \boxed{\frac{1}{A_1},\ \frac{1}{A_2}},\ \frac{1}{b}$

$$n = \frac{2mr}{m+r}$$

$a,\ G_1,\ G_2,\ b$

$$T_n = c(2n-1)+b$$

$$\frac{1}{H_1}+\frac{1}{H_2} = \frac{1}{a}+\frac{1}{b}$$

$$\frac{H_1+H_2}{H_1H_2} = \frac{a+b}{ab} = \frac{A_1+A_2}{G_1G_2}$$

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$a, A_1, A_2, \dots A_n, b$

$A_1 = p$

$p = a + \frac{b-a}{n+1} = \frac{na+b}{n+1}$

$a, H_1, H_2, \dots, H_n, b$

$H_1 = q$

$q = \frac{(n+1)ab}{nb+a}$

$\frac{1}{a} + \frac{\frac{1}{b} - \frac{1}{a}}{n+1} = \frac{1}{q} = \frac{1}{a} + \frac{a-b}{(n+1)ab} = \frac{(n+1)b + a - b}{(n+1)ab}$

$q\left(a + n\left((n+1)p - na\right)\right) = (n+1)a\left((n+1)p - na\right)$

$n(n+1)a^2 + (\quad\quad)a + (\quad\quad) = 0$

$\boxed{D \geq 0}$ ✓

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Series

$T_1 + T_2 + T_3 + T_4 + T_5 + \cdots + T_n$.

$= (T_1' - T_1'') + (T_2' - T_2'') + \cdots + (T_n' - T_n'')$

$S_\infty = \frac{1}{3}\left(\frac{1}{1\cdot2\cdot3}\right) = \frac{1}{18}$, $n \to \infty$

$\frac{1}{3}\left(\frac{1}{1\cdot2\cdot3} - \frac{1}{(n+1)(n+2)(n+3)}\right)$

1. $\frac{1}{1\cdot2\cdot3\cdot4} + \frac{1}{2\cdot3\cdot4\cdot5} + \frac{1}{3\cdot4\cdot5\cdot6} + \frac{1}{4\cdot5\cdot6\cdot7} + \cdots$ upto n terms.

$= \sum_{r=1}^{n} \frac{1}{r(r+1)(r+2)(r+3)} = \frac{1}{3}\sum_{r=1}^{n} \frac{(r+3)-r}{r(r+1)(r+2)(r+3)}$

$= \frac{1}{3}\sum_{r=1}^{n}\left(\frac{1}{r(r+1)(r+2)} - \frac{1}{(r+1)(r+2)(r+3)}\right) = \frac{1}{3}\left(\frac{1}{1\cdot2\cdot3} - \frac{1}{(n+1)(n+2)(n+3)}\right)$

$r=1$, $r=n$

$\frac{1}{3}\left[\left(\frac{1}{1\cdot2\cdot3} - \cancel{\frac{1}{2\cdot3\cdot4}}\right) + \left(\cancel{\frac{1}{2\cdot3\cdot4}} - \cancel{\frac{1}{3\cdot4\cdot5}}\right) + \left(\cancel{\frac{1}{3\cdot4\cdot5}} - \cancel{\frac{1}{4\cdot5\cdot6}}\right) + \cdots \left(\cancel{\frac{1}{n(n+1)(n+2)}} - \frac{1}{(n+1)(n+2)(n+3)}\right)\right]$

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2: $\frac{1}{1\cdot3\cdot5}+\frac{1}{3\cdot5\cdot7}+\frac{1}{5\cdot7\cdot9}+\frac{1}{7\cdot9\cdot11}+\cdots$ upto n terms.

$$=\sum_{r=1}^{n}\frac{1}{(2r-1)(2r+1)(2r+3)}=\frac{1}{4}\sum_{r=1}^{n}\frac{(2r+3)-(2r-1)}{(2r-1)(2r+1)(2r+3)}$$

$$=\frac{1}{4}\sum_{r=1}^{n}\left(\frac{1}{(2r-1)(2r+1)}-\frac{1}{(2r+1)(2r+3)}\right)$$

$$\downarrow r=1 \qquad \downarrow r=n$$

$$S_\infty=\frac{1}{12}$$

$$=\frac{1}{4}\left(\frac{1}{1\cdot3}-\frac{1}{(2n+1)(2n+3)}\right)$$

$$\frac{1}{4}\left[\left(\frac{1}{1\cdot3}-\cancel{\frac{1}{3\cdot5}}\right)+\left(\cancel{\frac{1}{3\cdot5}}-\cancel{\frac{1}{5\cdot7}}\right)+\left(\cancel{\frac{1}{5\cdot7}}-\cancel{\frac{1}{7\cdot9}}\right)+\cdots\left(\cancel{\frac{1}{(2n-1)(2n+1)}}-\frac{1}{(2n+1)(2n+3)}\right)\right]$$

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3. $\frac{3}{1\cdot2\cdot4}+\frac{4}{2\cdot3\cdot5}+\frac{5}{3\cdot4\cdot6}+\frac{6}{4\cdot5\cdot7}+\cdots$ upto n terms.

$$=\sum_{r=1}^{n}\frac{(r+2)}{r(r+1)(r+3)}=\sum_{r=1}^{n}\frac{(r+2)^2}{r(r+1)(r+2)(r+3)}=\sum_{r=1}^{n}\frac{r^2+4r+4}{r(r+1)(r+2)(r+3)}$$

$$r^2+4r+4 = r^2+3r+\underline{r+4} \qquad r+3+1$$

$$=\sum_{r=1}^{n}\frac{1}{(r+1)(r+2)}+\sum_{r=1}^{n}\frac{1}{r(r+1)(r+2)}+\sum_{r=1}^{n}\frac{1}{r(r+1)(r+2)(r+3)}$$

$$(r+2)-(r+1) \qquad \frac{(r+2)-r}{2} \qquad \frac{(r+3)-r}{3}$$

$$=\sum_{r=1}^{n}\left(\frac{1}{r+1}-\frac{1}{r+2}\right)+\frac{1}{2}\sum_{r=1}^{n}\left(\frac{1}{r(r+1)}-\frac{1}{(r+1)(r+2)}\right)+\frac{1}{3}\sum_{r=1}^{n}\left(\frac{1}{r(r+1)(r+2)}-\frac{1}{(r+1)(r+2)(r+3)}\right)$$

$$r=1 \quad r=n \quad r=1 \quad r=n \quad r=1 \quad r=n$$

$$=\left(\frac{1}{2}-\frac{1}{n+2}\right)+\frac{1}{2}\left(\frac{1}{1\cdot2}-\frac{1}{(n+1)(n+2)}\right)+\frac{1}{3}\left(\frac{1}{1\cdot2\cdot3}-\frac{1}{(n+1)(n+2)(n+3)}\right)$$

$$S_\infty=\frac{1}{2}+\frac{1}{4}+\frac{1}{18}$$

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4. $1\cdot2\cdot3\cdot4\cdot5 + 2\cdot3\cdot4\cdot5\cdot6 + 3\cdot4\cdot5\cdot6\cdot7 + 4\cdot5\cdot6\cdot7\cdot8 + \cdots$ upto n terms.

$$= \sum_{r=1}^{n} r(r+1)(r+2)(r+3)(r+4)$$

$$= \frac{1}{6}\sum_{r=1}^{n} r(r+1)(r+2)(r+3)(r+4)\Big((r+5)-(r-1)\Big)$$

$$= \frac{1}{6}\sum_{r=1}^{n}\Big(\underbrace{r(r+1)(r+2)(r+3)(r+4)(r+5)}_{r=n} - \underbrace{(r-1)r(r+1)(r+2)(r+3)(r+4)}_{r=1}\Big)$$

$$= \frac{1}{6}\Big(n(n+1)(n+2)(n+3)(n+4)(n+5) - 0\Big)$$

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5. $\frac{1}{1\cdot 3}+\frac{2}{1\cdot 3\cdot 5}+\frac{3}{1\cdot 3\cdot 5\cdot 7}+\frac{4}{1\cdot 3\cdot 5\cdot 7\cdot 9}+\ldots$ upto n terms.

$$=\sum_{r=1}^{n}\frac{r}{1\cdot 3\cdot 5\cdot 7\cdot 9\cdots(2r-1)(1+2r)} \qquad r=\frac{(2r+1)-1}{2}$$

$$=\frac{1}{2}\sum_{r=1}^{n}\left(\frac{1}{1\cdot 3\cdot 5\cdot 7\cdots(2r-1)}-\frac{1}{1\cdot 3\cdot 5\cdot 7\cdots(2r-1)(2r+1)}\right)$$

$r=1$; $r=n$

$$=\frac{1}{2}\left(\frac{1}{1}-\frac{1}{1\cdot 3\cdot 5\cdot 7\cdots(2n-1)(2n+1)}\right)$$

$$\frac{1}{2}\left[\left(\frac{1}{1}-\cancel{\frac{1}{3}}\right)+\left(\cancel{\frac{1}{1\cdot 3}}-\cancel{\frac{1}{1\cdot 3\cdot 5}}\right)+\left(\cancel{\frac{1}{1\cdot 3\cdot 5}}-\cancel{\frac{1}{1\cdot 3\cdot 5\cdot 7}}\right)+\ldots\left(\cancel{\frac{1}{1\cdot 3\cdot 5\cdots(2n-1)}}-\frac{1}{1\cdot 3\cdot 5\cdot 7\cdots(2n+1)}\right)\right]$$

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6. $\frac{1}{2\cdot4}+\frac{1\cdot3}{2\cdot4\cdot6}+\frac{1\cdot3\cdot5}{2\cdot4\cdot6\cdot8}+\frac{1\cdot3\cdot5\cdot7}{2\cdot4\cdot6\cdot8\cdot10}+\cdots$ upto n terms.

$$=\sum_{r=1}^{n}\frac{1\cdot3\cdot5\cdot7\cdots(2r-1)}{2\cdot4\cdot6\cdot8\cdots(2r)(2r+2)}=\sum_{r=1}^{n}\frac{1\cdot3\cdot5\cdot7\cdots(2r-1)\left((2r+2)-(2r+1)\right)}{2\cdot4\cdot6\cdot8\cdots2r(2r+2)}$$

$$=\sum_{r=1}^{n}\left(\frac{1\cdot3\cdot5\cdots(2r-1)}{2\cdot4\cdot6\cdots2r}-\frac{1\cdot3\cdot5\cdot7\cdots(2r-1)(2r+1)}{2\cdot4\cdot6\cdot8\cdots2r(2r+2)}\right)$$

$r=1$ $\quad$ $r=n$

$$=\frac{1}{2}-\frac{1\cdot3\cdot5\cdot7\cdots(2n-1)(2n+1)}{2\cdot4\cdot6\cdot8\cdots2n(2n+2)}$$

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7. $\frac{5}{1\cdot 2}\frac{1}{3}+\frac{7}{2\cdot 3}\frac{1}{3^2}+\frac{9}{3\cdot 4}\frac{1}{3^3}+\frac{11}{4\cdot 5}\frac{1}{3^4}+\cdots+$ upto n terms.

$$=\sum_{r=1}^{n}\frac{(2r+3)}{r(r+1)}\frac{1}{3^r}=\sum_{r=1}^{n}\frac{3(r+1)-r}{r(r+1)}\frac{1}{3^r}$$

$$=\sum_{r=1}^{n}\left(\frac{1}{r}\frac{1}{3^{r-1}}-\frac{1}{r+1}\frac{1}{3^r}\right)=\frac{1}{1}-\frac{1}{(n+1)}\frac{1}{3^n}$$

$\downarrow r=1$ $\quad\downarrow r=n$

$5+(r-1)2$

Ex-29(a) → $\boxed{3-20}$

Ex-29(b) → 9, 11, 12, 19, 20,

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$$\frac{1}{2\cdot 4}+\frac{1\cdot 3}{2\cdot 4\cdot 6}+\frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6\cdot 8}+\cdots\cdots\frac{1\cdot 3\cdot 5\cdots(2n-1)}{2\cdot 4\cdot 6\cdots 2n(2n+2)}$$

$$T_n=\frac{2n-1}{2n+2}T_{n-1}$$

$$(2n+2)T_n=(2n-1)T_{n-1}$$

$$(2n+1)T_n-(2n-1)T_{n-1}=-T_n$$

$n=2$ $\quad \cancel{5T_2}-3T_1=-T_2$ $\quad$ Add

$n=3$ $\quad \cancel{7T_3}-\cancel{5T_2}=-T_3$

$n=4$ $\quad \cancel{9T_4}-\cancel{7T_3}=-T_4$

$\vdots$

$n=n$ $\quad (2n+1)T_n-\cancel{(2n-1)T_{n-1}}=-T_n$

$$(2n+1)T_n-3T_1=-(T_2+T_3+T_4+\cdots+T_n)$$

$$(2n+1)T_n-4T_1=-(T_1+T_2+\cdots+T_n)$$

$$S_n=4T_1-(2n+1)T_n$$

$$=4\left(\frac{1}{2\cdot 4}\right)-\frac{(2n+1)\,1\cdot 3\cdot 5\cdots(2n-1)}{2\cdot 4\cdot 6\cdots 2n(2n+2)}$$