## HOMEWORK–1

**1.** If $y = \frac{a + bx^{\frac{3}{2}}}{x^{\frac{5}{4}}}$ & $\frac{dy}{dx}$ vanishes when $x = 5$ then $\frac{a}{b} =$

(A) $\sqrt{3}$ (B) 2 (C) $\sqrt{5}$ (D) 3

**2.** If $\frac{d}{dx}\left(\frac{1 + x^2 + x^4}{1 + x + x^2}\right) = ax + b$ then the value of a and b are respectively

(A) 2 and 1 (B) –2 and 1 (C) 2 and –1 (D) 3 and 1

**3.** If $y = x - x^2$, then the derivative of $y^2$ w.r.t. $x^2$ is

(A) $2x^2 + 3x - 1$ (B) $2x^2 - 3x + 1$ (C) $2x^2 + 3x + 1$ (D) $2x^2 + 5x + 1$

**4.** The differential coefficient of $a^{\sin^{-1}x}$ w.r.t. $\sin^{-1}x$ is –

(A) $a^{\sin^{-1}x} \log_e a$ (B) $a^{\sin^{-1}x}$ (C) $\frac{a^{\sin^{-1}x}}{\sqrt{(1-x^2)}}$ (D) $a^{\sin^{-1}x}\sqrt{(1 - x^2)}$

**5.** The value of derivative of $\tan^{-1}\left(\frac{2x\sqrt{1 - x^2}}{1 - 2x^2}\right)$ w.r.t $\sec^{-1}\left(\frac{1}{2x^2-1}\right)$ at $x = 1/2$ equals-

(A) 1 (B) -1 (C) 0 (D) 2

**6.** If $y = \cos^{-1}(\cos x)$ then $\frac{dy}{dt}$ at $x = \frac{5\pi}{4}$ is equal to

(A) 1 (B) -1 (C) $\frac{1}{\sqrt{2}}$ (D) $-\frac{1}{\sqrt{2}}$

**7.** If $8f(x) + 6f\left(\frac{1}{x}\right) = x + 5$ and $y = x^2 f(x)$, then $\frac{dy}{dx}$ at $x = -1$ is equal to

(A) 0 (B) $\frac{1}{14}$ (C) $-\frac{1}{14}$ (D) 14

**8.** If $f(x) = x^n$, then the value of $f(1) - \frac{f'(1)}{1!} + \frac{f''(1)}{2!} - \frac{f'''(1)}{3!} + \cdots .. + \frac{(-1)^n f^n(1)}{n!}$ is

(A) $2^n$ (B) $2^{n-1}$ (C) 0 (D) 1

**9.** If f is differentiable in $(0, 6)$ & $f'(4) = 5$ then $\text{Lim}_{x\to 2} \frac{f(4) - f(x^2)}{2-x} =$

(A) 5 (B) $\frac{5}{4}$ (C) 10 (D) 20

**10.** If $u = ax + b$ then $\frac{d^n}{dx^n}(f(ax + b))$ is equal to

(A) $\frac{d^n}{du^n}(f(u))$ (B) $a\frac{d^n}{du^n}(f(u))$ (C) $a^n \frac{d^n}{du^n}(f(u))$ (D) $a^{-n}\frac{d^n}{du^n}(f(u))$

**11.** Let f(x) be a polynomial in x. Then the second derivative of $f(e^x)$, is

(A) $f''(e^x) \cdot e^x + f'(e^x)$
(B) $f''(e^x) \cdot e^{2x} + f'(e^x) \cdot e^{2x}$
(C) $f'(e^x)e^{2x}$
(D) $f''(e^x) \cdot e^{2x} + f'(e^x) \cdot e^x$

**12.** If $y = f(x)$ is an odd differentiable function defined on $(-\infty, \infty)$ such that $f'(3) = -2$, then $f'(-3)$ equals

(A) 4 (B) 2 (C) -2 (D) 0

**13.** If g is inverse of f and $f'(x) = \frac{1}{1+x^n}$, then $g'(x)$ equals –

(A) $1 + x^n$ (B) $1 + (f(x))^n$ (C) $1 + (g(x))^n$ (D) $1 - x^n$

**14.** Derivative of $\log_e(\log_e |\sin x|)$ with respect to x at $x = \frac{\pi}{6}$ is

(A) $-\frac{\sqrt{3}}{\log_e 2}$ (B) $\frac{\sqrt{3}}{\log_e 2}$ (C) $-\frac{\sqrt{3}}{2\log 2}$ (D) does not exist

**15.** If $f(x) = f'(x) + f''(x) + f'''(x) + f''''(x) \dots \dots \dots \infty$ also $f(0) = 1$ and $f(x)$ is a differentiable function indefinitely then $f(x)$ has the value

(A) $e^x$ (C) $e^{x/2}$ (B) $e^{2x}$ (D) $e^{4x}$

**16.** If $f(x) = |(x-4)(x-5)|$, then $f'(x)$ is

(A) $-2x + 9$, for all $x \in R$ (B) $2x - 9$ if $x > 5$

(C) $-2x + 9$ if $4 < x < 5$ (D) not defined for $x = 4, 5$

**17.** If f is twice differentiable such that $f''(x) = -f(x)$ and $f'(x) = g(x)$. If $h(x)$ is twice differentiable function such that $h'(x) = [f(x)]^2 + [g(x)]^2$. If $h(0) = 2$, $h(1) = 4$, then the equation $y = h(x)$ represents

(A) a curve of degree 2 (B) a curve passing through the origin

(C) a straight line with slope 2 (D) a straight line with y intercept equal to 2.

**18.** Two functions f & g have first & second derivatives at $x = 0$ satisfy the relations,

$f(0) = \frac{2}{g(0)}$, $f'(0) = 2g'(0) = 4g(0)$, $g''(0) = 5f'(0) = g(0) = 3$ then

(A) if $h(x) = \frac{f(x)}{g(x)}$ then $h'(0) = \frac{32}{9}$ (B) if $k(x) = f(x) \cdot g(x) \sin x$ then $k'(0) = 2$

(C) $\text{Lim}_{x\to 0} \frac{g'(x)}{f'(x)} = \frac{1}{2}$ (D) $f'(x) = g'(x)$

**19.** Differentiate the following functions with respect to x.

(i) $x^{2/3} + 7e - \frac{5}{x} + 7\tan x$ (ii) $\ln \tan\left(\frac{\pi}{4} + \frac{x}{2}\right)$

(iii) $\frac{\sin x - x\cos x}{x\sin x + \cos x}$ (iv) $\tan\left(\tan^{-1}\sqrt{\frac{1-\cos x}{1+\cos x}}\right)$

**20.** Differentiate $x^2 . \ln x . e^x$ with respect to x.

**21.** If $\cos\frac{x}{2} \cdot \cos\frac{x}{2^2} \cdot \cos\frac{x}{2^3} \dots \infty = \frac{\sin x}{x}$ then find the value of $\frac{1}{2^2}\sec^2\frac{x}{2} + \frac{1}{2^4}\sec^2\frac{x}{2^2} + \frac{1}{2^6}\sec^2 + \dots \infty$.

**22.** Let f, g and h are differentiable functions. If $f(0) = 1$; $g(0) = 2$; $h(0) = 3$ and the derivatives of their pair wise products at $x = 0$ are $(fg)'(0) = 6$; $(gh)'(0) = 4$ and $(hf)'(0) = 5$ then compute the value of $(fgh)'(0)$.

**23.** If $f : R \to R$ is a function such that $f(x) = x^3 + x^2 f'(1) + x f''(2) + f'''(3)$ for all $x \in R$, then prove that $f(2) = f(1) - f(0)$.

**Paragraph for Question Nos. 24 to 26**

$f(x)$ is a polynomial function $f: R \to R$ such that $f(2x) = f'(x)f''(x)$.

**24.** The value of $f(3)$ is

(A) 4 (B) 12 (C) 15 (D) 11

**25.** $f(x)$ is :

(A) one-one and onto (B) one-one and into

(C) many-one and onto (D) many-one and into

**26.** Equation $f(x) = x$ has:

(A) three real and positive roots (B) three real and negative roots

(C) one real root (D) three real such that sum of roots is zero

**27.** Let $f: (-1,1) \to R$ be a differentiable function with $f(0) = -1$ and $f'(0) = 1$.

Let $g(x) = [f(2f(x) + 2)]^2$, then $g'(0) =$ **[AIEEE 2010]**

(A) 4 (C) 0 (B) -4 (D) -2

**28.** If $f(x) = \frac{2x-4}{x^2-1}$ and $f'(x) = \frac{p}{(x^2-1)^2}$, then p equals-

(A) $x^2 - 8x - 2$ (B) $-2x^2 + 8x + 2$

(C) $4x + 2$ (D) $-2x^2 + 8x - 2$

**29.** If $y = \sqrt{\frac{1-\cos x}{1+\cos x}}, x \in (0, \pi)$ then $\frac{dy}{dx}$ equals-

(A) $\frac{1}{2}\sec^2 x/2$ (B) $\frac{1}{2}\operatorname{cosec}^2 x/2$

(C) $\sec^2 x/2$ (D) $\operatorname{cosec}^2 x/2$

**30.** $\frac{d}{d\theta}\left\{\tan^{-1}\left(\frac{1-\cos\theta}{\sin\theta}\right)\right\}$ equals-

(A) 1/2 (B) 1 (C) $\sec\theta$ (D) $\operatorname{cosec}\theta$

**31.** $d/dx(\sec x^\circ)$ equals -

(A) $\sec x \tan x$ (B) $\sec x^\circ \tan x^\circ$

(C) $\left(\frac{\pi}{180}\right) \sec x^\circ \tan x^\circ$ (D) $\left(\frac{\pi}{180}\right) \sec x \tan x$

**32.** If $a\cos^2(x + y) = b$, then $dy/dx$ equals-

(A) 2 (B) -2 (C) 1 (D) -1

**33.** If $y = \log\sqrt{\frac{1-\sin x}{1+\sin x}}$, then $dy/dx$ equals-

(A) $\sec x$ (B) $-\sec x$

(C) $\operatorname{cosec} x$ (D) $\sec x \tan x$

**34.** If $y = \log_{10}(\sin\ x)$, then dy/dx equals-

(A) $\sin x \log_{10} e$ (B) $\cos x \log_{10} e$

(C) $\cot x \log_{10} e$ (D) $\cot x$

**35.** The derivative of $x|x|$ is-

(A) $2x$ (B) $-2x$ (C) $2|x|$ (D) Does not exist

## ANSWER KEY

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|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **1.** | **(C)** | **2.** | **(C)** | **3.** | **(B)** | **4.** | **(A)** | **5.** | **(B)** | **6.** | **(B)** | **7.** | **(C)** |
| **8.** | **(C)** | **9.** | **(D)** | **10.** | **(C)** | **11.** | **(D)** | **12.** | **(C)** | **13.** | **(C)** | **14.** | **(D)** |
| **15.** | **(B)** | **16.** | **(BCD)** | **17.** | **(CD)** | **18.** | **(ABC)** | | | | | | |

**19.** **(i)** $\frac{2}{3}x^{-\frac{1}{3}} + \frac{5}{x^2} + 7\sec^2 x$ **(ii)** $\sec x$

**(iii)** $\frac{x^2}{(x\sin x + \cos x)^2}$ **(iv)** $\frac{1}{2}\sec^2\left(\frac{x}{2}\right)$

**20.** $x^2 \ln x \cdot e^x + xe^x + 2xe^x \ln x$ **21.** $\operatorname{cosec}^2 x - \frac{1}{x^2}$

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|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **22.** | **16** | **24.** | **(B)** | **25.** | **(A)** | **26.** | **(D)** | **27.** | **(B)** | **28.** | **(D)** | **29.** | **(A)** |
| **30.** | **(A)** | **31.** | **(C)** | **32.** | **(D)** | **33.** | **(B)** | **34.** | **(C)** | **35.** | **(C)** | | |