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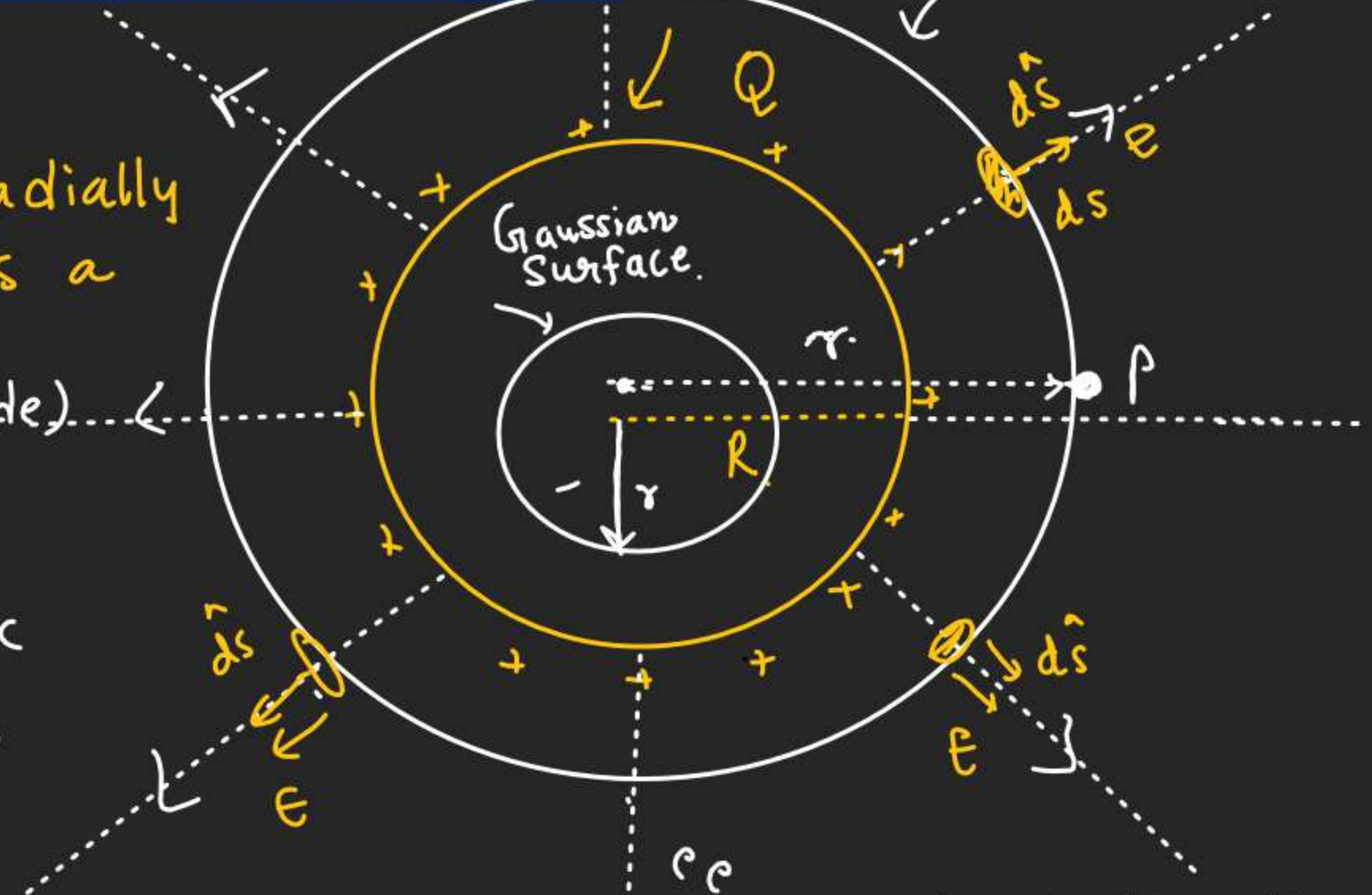
APPLICATION OF GAUSS'S LAW

Electric field due to conducting solid sphere.

⇒ In Sphere Electric field lines always radially out ward. So, Gaussian Surface is a Sphere.



$r<R$ (Inside)

$q_{enc} = 0$

$$\oint \vec{E} \cdot ds = \frac{q_{enc}}{\varepsilon_0} = 0$$

$$\boxed{E_{inside} = 0}$$

$r>R$ (out side)

$\vec{E} \parallel \hat{ds}$

$$\oint \vec{E} \cdot \vec{ds} = \frac{q_{enc}}{\varepsilon_0}$$

$$E \oint ds = \frac{Q}{\varepsilon_0}$$

$$E \cdot 4\pi r^2 = \frac{Q}{\varepsilon_0} \Rightarrow \boxed{E_{outside} = \frac{Q}{4\pi\varepsilon_0 r^2}}$$

"For outside it will behave as a point charge"

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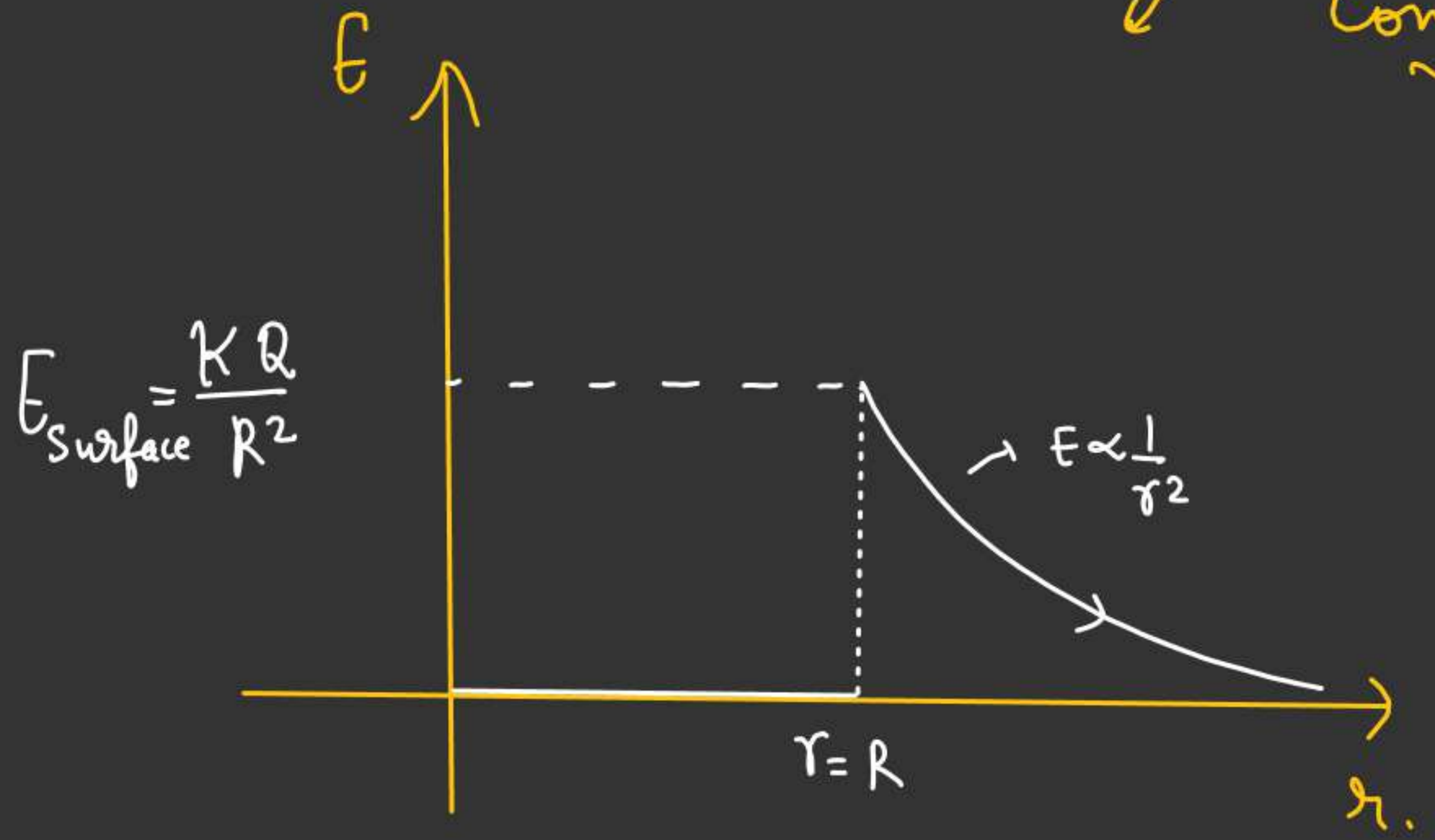
Conducting Sphere



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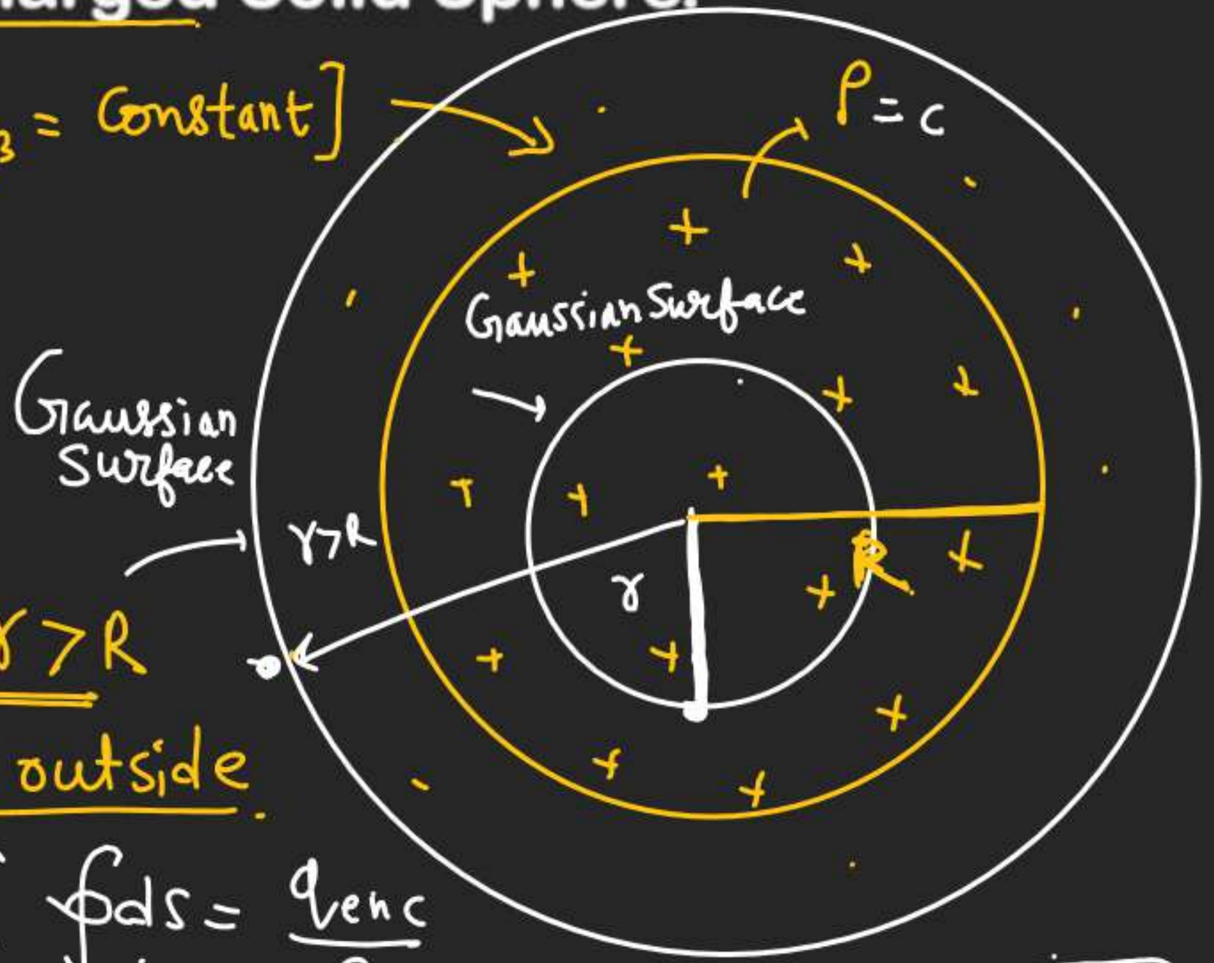
APPLICATION OF GAUSS'S LAW

Electric field due to non-conducting uniformly Charged Solid Sphere.

(Charge distributed in the Volume)

$$\left[\rho = \frac{Q}{\frac{4}{3}\pi R^3} = \text{Constant}\right]$$



Inside :-

$r < R$ ✓ ⇒ (Spherical Gaussian Surface)

$\vec{E} \parallel d\vec{s}$

$$\oint \vec{E}\cdot d\vec{s} = \oint E\cdot ds = \frac{q_{enc}}{\varepsilon_0}$$

$$E \oint ds = \frac{\rho \frac{4}{3}\pi r^3}{\varepsilon_0}$$

$$E \cdot \cancel{4\pi r^2} = \frac{\cancel{4}\rho \cancel{\pi r^3}}{3\varepsilon_0}$$

XX

$$\boxed{E_{inside} = \frac{\rho r}{3\varepsilon_0}}$$

$$\boxed{E_{inside} \propto r}$$

$r > R$

↳ outside

$$E \oint ds = \frac{q_{enc}}{\varepsilon_0}$$

$$E \times 4\pi r^2 = \frac{\rho \frac{4}{3}\pi R^3}{\varepsilon_0}$$

$$\Rightarrow \boxed{E_{outside} = \frac{\rho R^3}{3\varepsilon_0 r^2}}$$

outside

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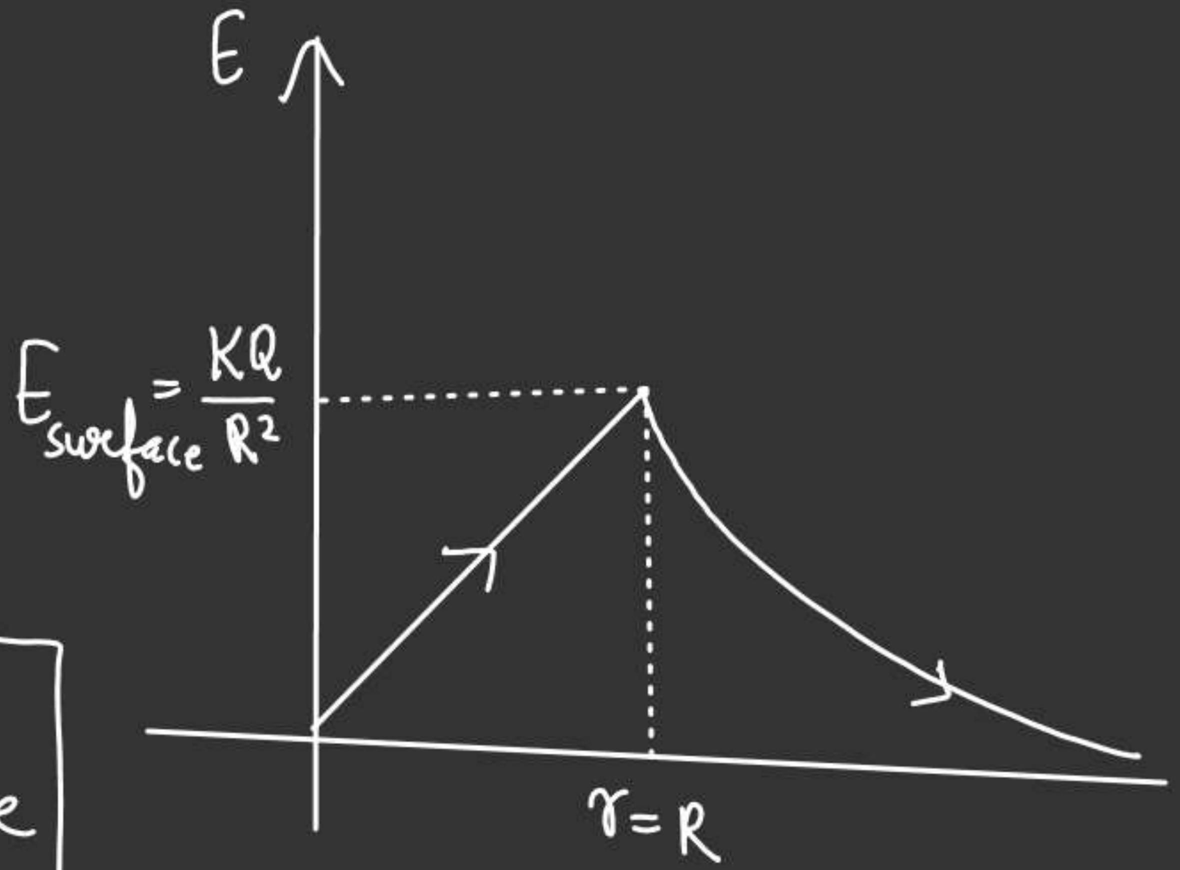
$$E_{outside} = \left(\frac{\rho R^3}{3\varepsilon_0 r^2}\right) = \frac{\cancel{3}Q}{4\pi \cancel{R^3}} \times \frac{\cancel{R^3}}{\cancel{3}\varepsilon_0 r^2}$$

$$Q = \rho \cdot \frac{4}{3}\pi R^3$$

$$\rho = \left(\frac{3Q}{4\pi R^3}\right)$$

$$\boxed{E_{outside} = \frac{Q}{4\pi\varepsilon_0 r^2}}$$

↳ A Uniformly Charged non-Conducting solid-Sphere behave as point charge.



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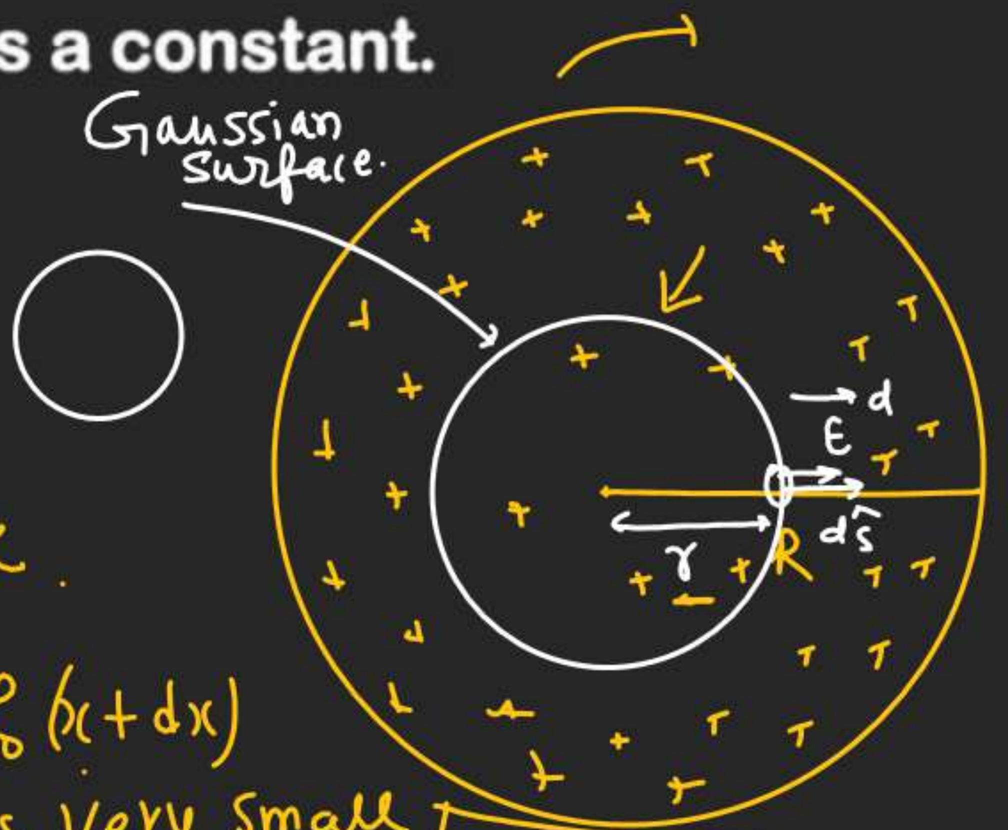
APPLICATION OF GAUSS'S LAW

$[\rho \neq C]$

Electric field due to non-conducting n on-uniformly Charge solid Sphere whose volume charge density is $\boxed{\rho = \rho_0 r.}$ where ρ_0 is a constant.

$q_{enc} = ??$

$dq \rightarrow$ Change on the Spherical Shell having radius x, and thickness 'dx'



$$\rho_x = \rho_0 x$$

$$\rho_{x+dx} = \rho_0 (x+dx)$$

Since 'dx' is very small

$$\rho_x = \rho_{x+dx} = c$$

$dq = \rho_x \cdot dV$ → Differential Volume of shell

$$dq = (\rho_0 x)\, 4\pi x^2 dx$$

$$\int_0^{q_{enc}} dq = \rho_0 4\pi \int_0^r x^3 dx = \rho_0 4\pi \left(\frac{r^4}{4}\right) = \rho_0 \pi r^4$$

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By Gauss's law

$$\oint \vec{E}\cdot d\vec{s} = \frac{q_{enc}}{\varepsilon_0}$$

$\vec{E} \parallel d\vec{s}$

(Surface integral of Gaussian surface)

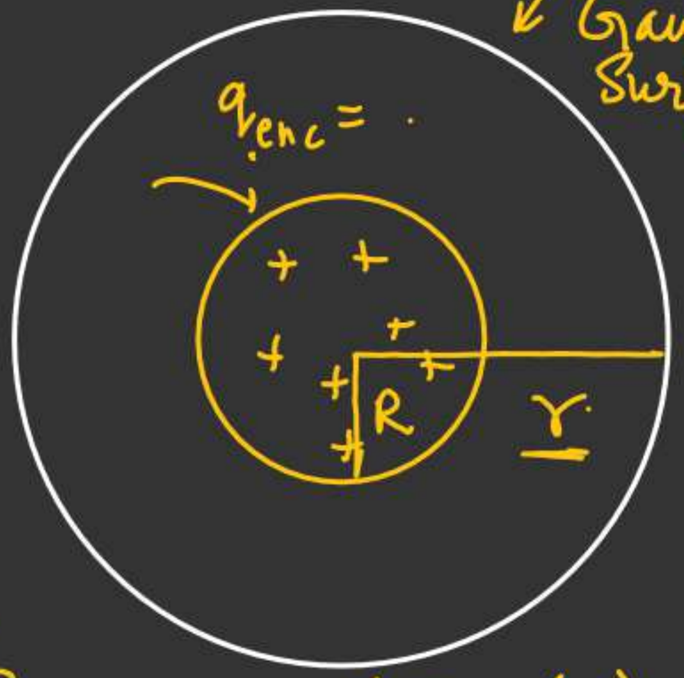
$$E \oint ds = \frac{q_{enc}}{\varepsilon_0}$$

$$E\cdot 4\cancel{\pi}\cancel{r^2} = \frac{\rho_0 \cancel{\pi} r^{\cancel{4}}}{\varepsilon_0}$$

$$\boxed{E = \frac{\rho_0 r^2}{4\varepsilon_0}}$$

$r > R$

Gaussian Surface.



$$E\times 4\pi r^2 = \left(\frac{\rho_0 \pi R^4}{\varepsilon_0}\right)$$

$$E_{outside} = \left(\frac{\rho_0 R^4}{4\varepsilon_0 r^2}\right)$$

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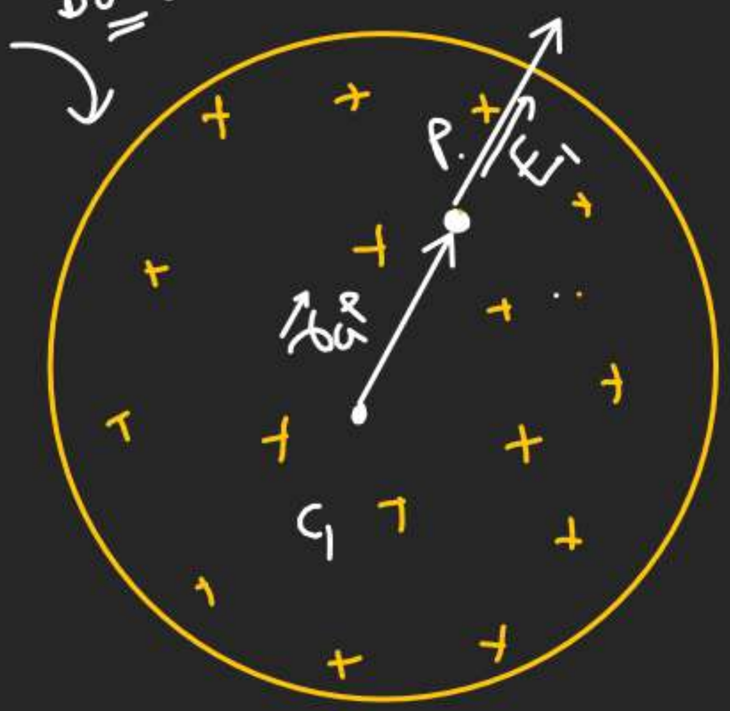
APPLICATION OF GAUSS'S LAW

Electric field inside the cavity of a uniformly Charge non- Conducting Solid Sphere.

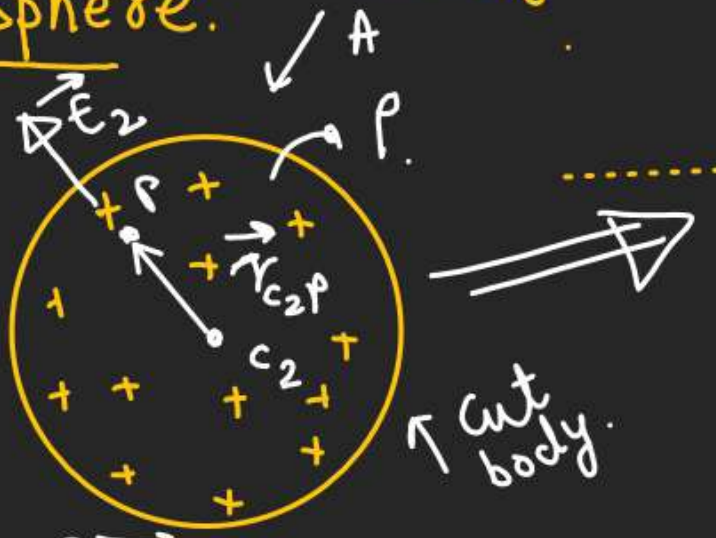
$\rho = C$



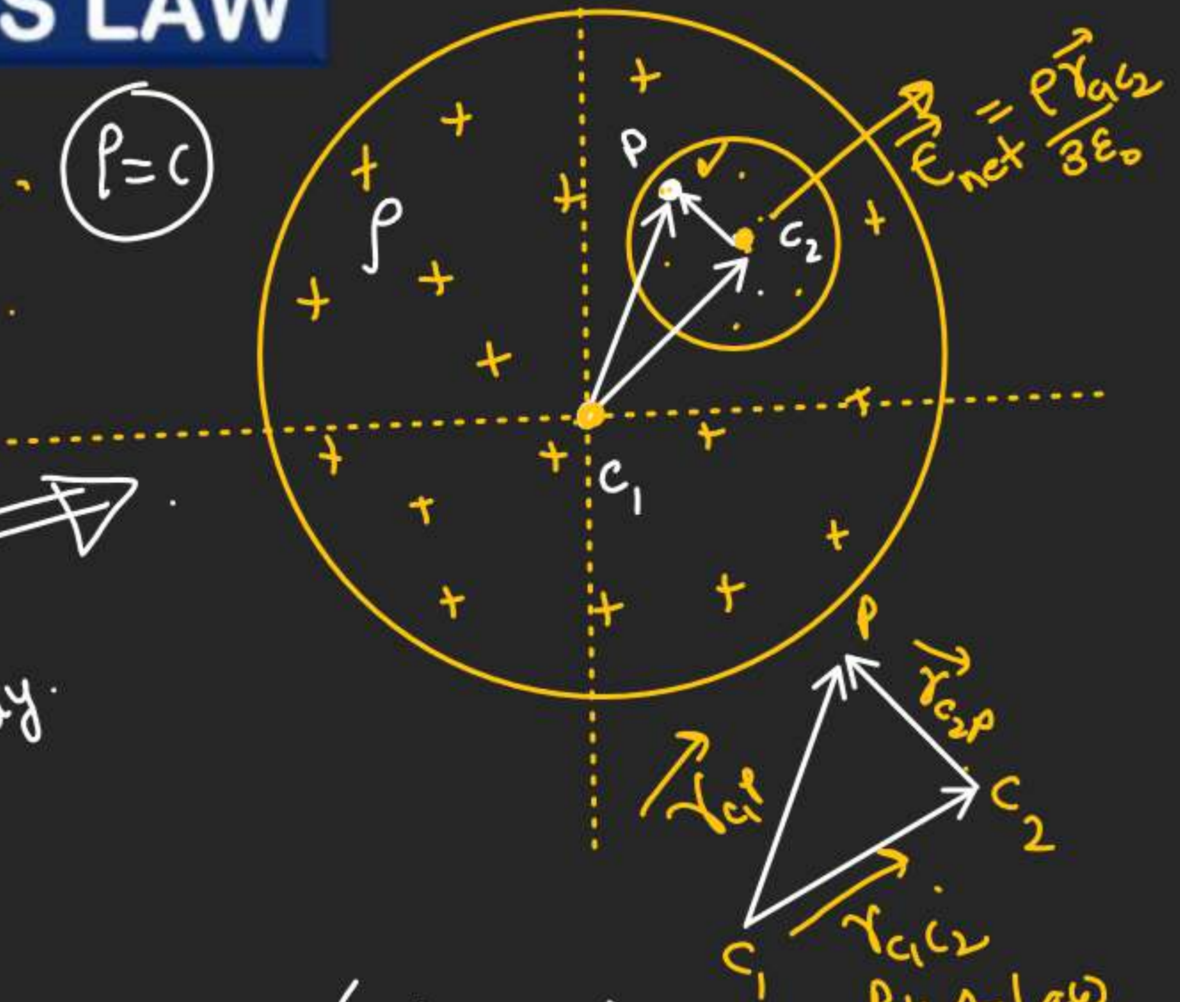
Uniformly Charge non-Conducting Solid- Sphere.

Note: Inside the Cavity field is uniform

$$\vec{E_1} = \frac{\rho\,\vec{r}_{c_1p}}{3\varepsilon_0}$$



$$\vec{E_2} = \frac{\rho\,\vec{r}_{c_2p}}{3\varepsilon_0}$$



$$\vec{E}_{residual} = \vec{E_1} - \vec{E_2} = \frac{\rho}{3\varepsilon_0}\left(\vec{r}_{c_1p} - \vec{r}_{c_2p}\right) = \frac{\rho}{3\varepsilon_0}\vec{r}_{c_1c_2}$$

$\vec{E}_{residual}$ or remaning body; $\vec{E_1}$ (Orginal body); $\vec{E_2}$ (Cut body)

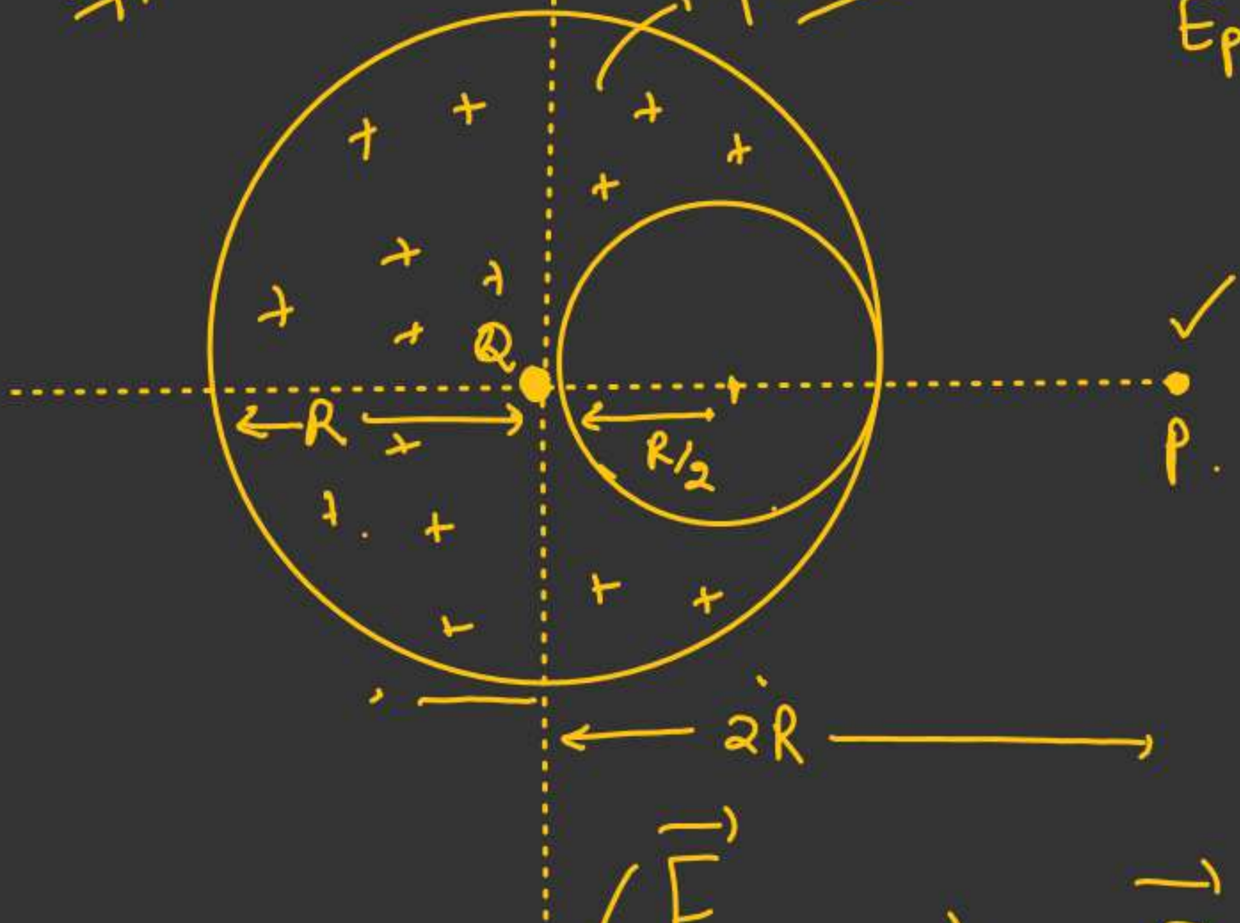
By Δ-Law of vector addition.

$$\vec{r}_{c_1c_2} + \vec{r}_{c_2p} = \vec{r}_{c_1p}$$

$$\vec{r}_{c_1c_2} = \vec{r}_{c_1p} - \vec{r}_{c_2p}$$

#.

ρ = Constant.



Find
$E_P = ??$, $E_Q = ??$

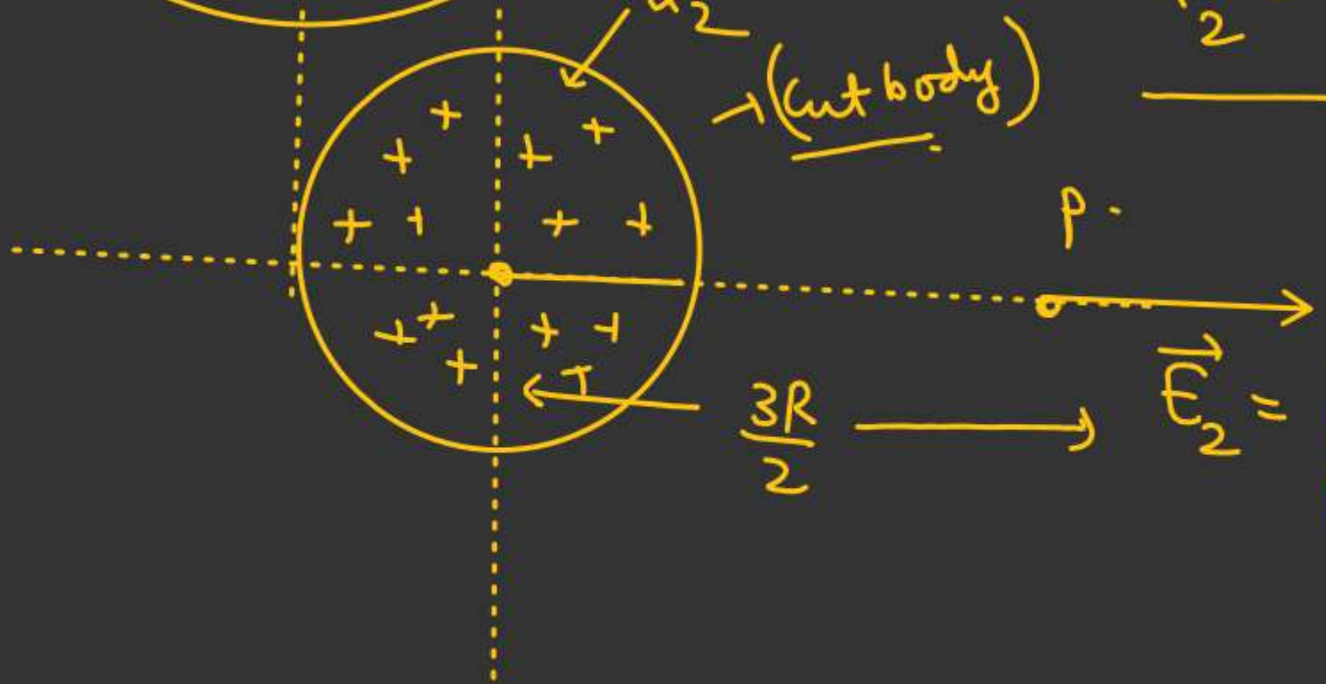
$$\left(\vec{E}_{\substack{\text{remaning}\\ \text{body}}}\right)_P = \vec{E}_1 - \vec{E}_2$$

$Q_1 = \rho \cdot \frac{4}{3}\pi R^3$

(orginal body)

$2R - \frac{R}{2} = 3R/2$

$$\vec{E}_1 = \frac{KQ_1}{(2R)^2}(+\hat{i})$$



(cut body)

$Q_2 = \rho \cdot \frac{4}{3}\pi \left(\frac{R}{2}\right)^3$

$$\vec{E}_2 = \frac{KQ_2}{\left(\frac{3R}{2}\right)^2}(\hat{i})$$

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APPLICATION OF GAUSS'S LAW

H-W

An infinitely long solid cylinder of radius R has a uniform volume charge density ρ. It has a spherical cavity of radius $R/2$ with its centre on the axis of the cylinder, as shown in the figure. Find the magnitude of the electric field at the point P, which is at a distance $2R$ from the axis of the cylinder

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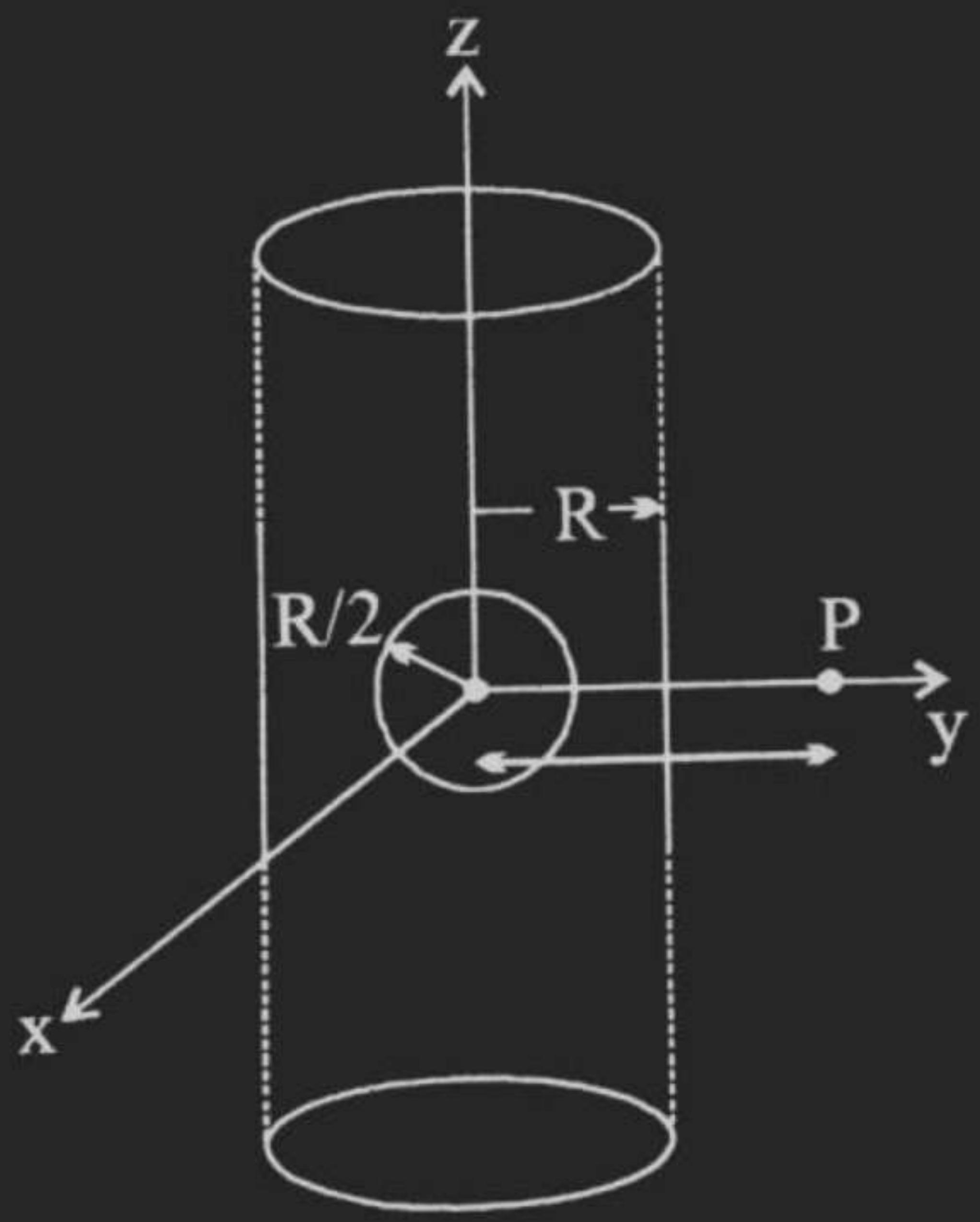
APPLICATION OF GAUSS'S LAW



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APPLICATION OF GAUSS'S LAW

H.W :

A ball of radius R carries a positive charge whose volume density depends on a separation r from the ball's centre as $\rho = \rho_0(1 - r/R)$, where ρ_0 is a constant. Assuming the permittivities of the ball and the environment is equal to unity, find :

(a) The magnitude of the electric field strength as a function of the distance r both inside and outside the ball,

(b) The maximum intensity E_{max} and the corresponding distance r_m