Nishant Jindal

141. $2\log_{10}^2 x = 1 + 2\log_{10} x$

$3t - (3 + t^2) + 1 = 0 \quad t \geq 0$

$2t^2 - 2t - 1 = 0$

$t = \frac{2 \pm \sqrt{4+8}}{4} = \frac{1 \pm \sqrt{3}}{2} = \log_{10} x$

$x = 10^{\frac{1+\sqrt{3}}{2}}, \ 10^{\frac{1-\sqrt{3}}{2}}$

147. $\boxed{x = 5}$

$2\log_x (x-2)$

$x^{2\log_x(x-2)} = 9$

$= \left(x^{\log_x (x-2)}\right)^2$

$(x-2)^2 = 9$

$\Rightarrow x - 2 = \pm 3$

$x = -1, 5$

reject

144. $\frac{1}{2}(\log_5 x - 1)\log_5 x = 1$

$t^2 - t - 2 = 0 = (t-2)(t+1)$

$\log_5 x = 2, -1$

$\boxed{x = 25, \frac{1}{5}}$

Nishant Jindal

148.

$$\left(\frac{\log x}{2}\right)^{\log^2 x + \log x - 2} = \frac{\log x}{2}$$

or 2

$\frac{\log x}{2} = 1$ ✓

$\log^2 x + (\quad 2)\log x - 2 = 1$

check

$$\boxed{a^b = a^c}$$

$\boxed{b = c}$ or $\boxed{a = 1}$ or

$t^2 + 2t - 3 = 0$ ✓

$a = -1$ or $a = 0$

$\boxed{\log_{10} x = -3, 1}$

-2

reject ← $\frac{\log x}{2} = 0$

1 0 not defined.

$x = 10^{-3}, 10, 100$

Nishant Jindal

154. $\left(a^{\log_b x}\right)^2 - 5x^{\log_b a} + 6 = 0$

$t^2 - 5t + 6 = 0$

$x^{\log_b a} = 2, 3$

$x = 2^{\frac{1}{\log_b a}}, 3^{\frac{1}{\log_b a}}$

$= 2^{\log_a b}, 3^{\log_a b}$

$\log_x \sqrt{5} = \frac{1}{2}, 1$

$\log_{\sqrt{5}} x = 2, 1$

$\boxed{x = 5, \sqrt{5}}$

157.

$$\log_4\left(\frac{x^2-1}{(x-1)^2}\right) = \log_4 |x-4|$$

$$\frac{x+1}{x-1} = |x-4|$$

$\boxed{x>4}$ ✓

$(x+1) = x^2-5x+4$

$x^2-6x+3=0$

$x = 3 \pm \sqrt{6}$

$\boxed{x = 3+\sqrt{6}}$ ✓

$x<4$

$x+1 = (4-x)(x-1)$

$x+1 = -x^2+5x-4$

$x^2-4x+5=0$

$\frac{6 \pm \sqrt{24}}{2}$

$\downarrow$

ϕ

$(x-2)^2+1$

Nishant Jindal

159

$$2\log_4(4-x) = \log_2 2^4 - \log_2(-2-x)$$

$$\log_2(4-x) = \log_2\left(\frac{16}{-2-x}\right)$$

$$4-x = \frac{16}{-2-x}$$

$$(2+x)(x-4) = 16$$

$$x^2-2x-24=0$$

$x = \cancel{6}, -4$

$$\boxed{x=-4}$$

Nishant Jindal

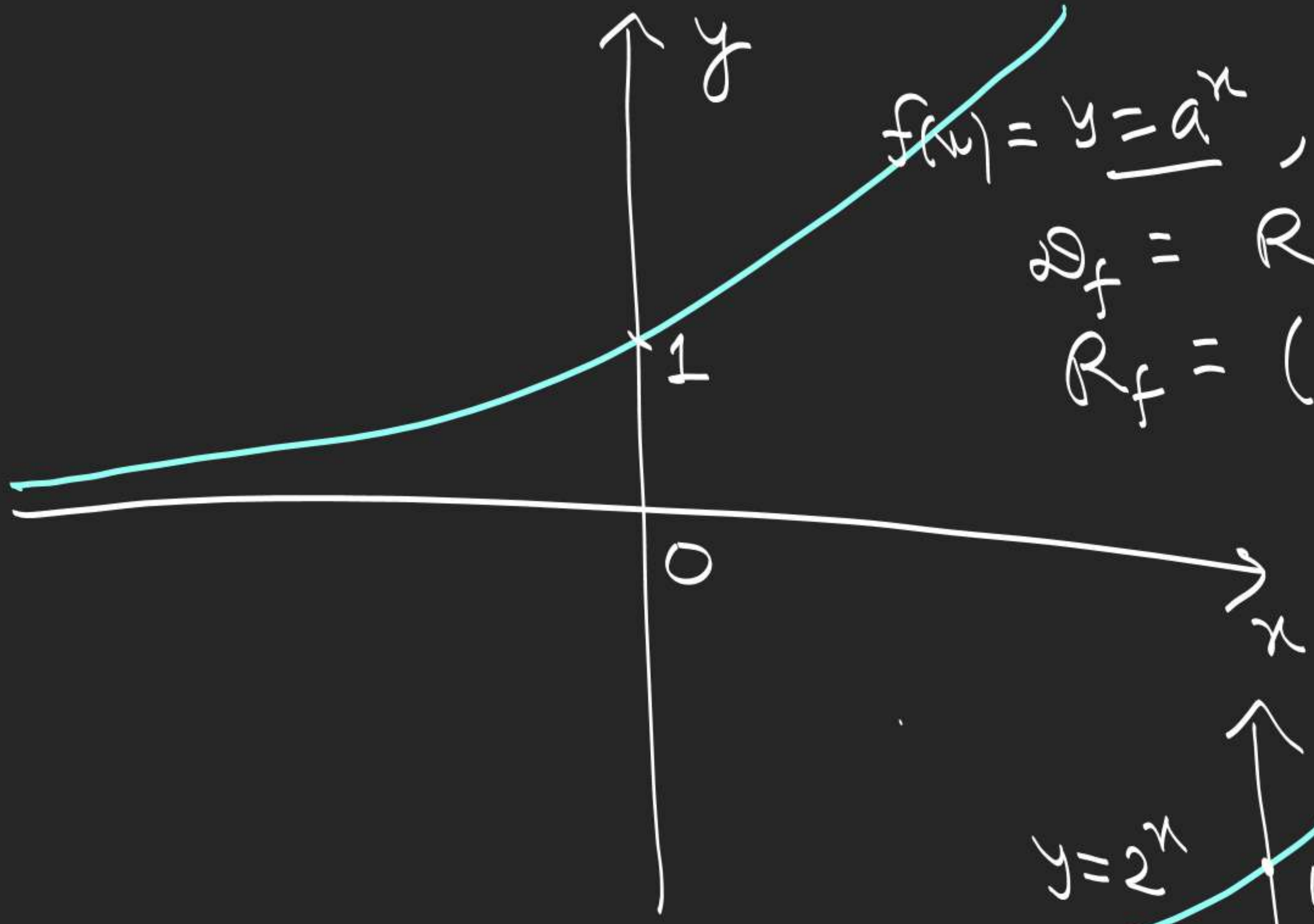
$$x_1 > x_2$$

$$\Rightarrow \log_a x_1 > \log_a x_2 \quad , \quad a > 1$$

$$\log_a x_1 < \log_a x_2 \quad , \quad 0 < a < 1$$

$$a^{x_1} > a^{x_2} \quad , \quad a > 1$$

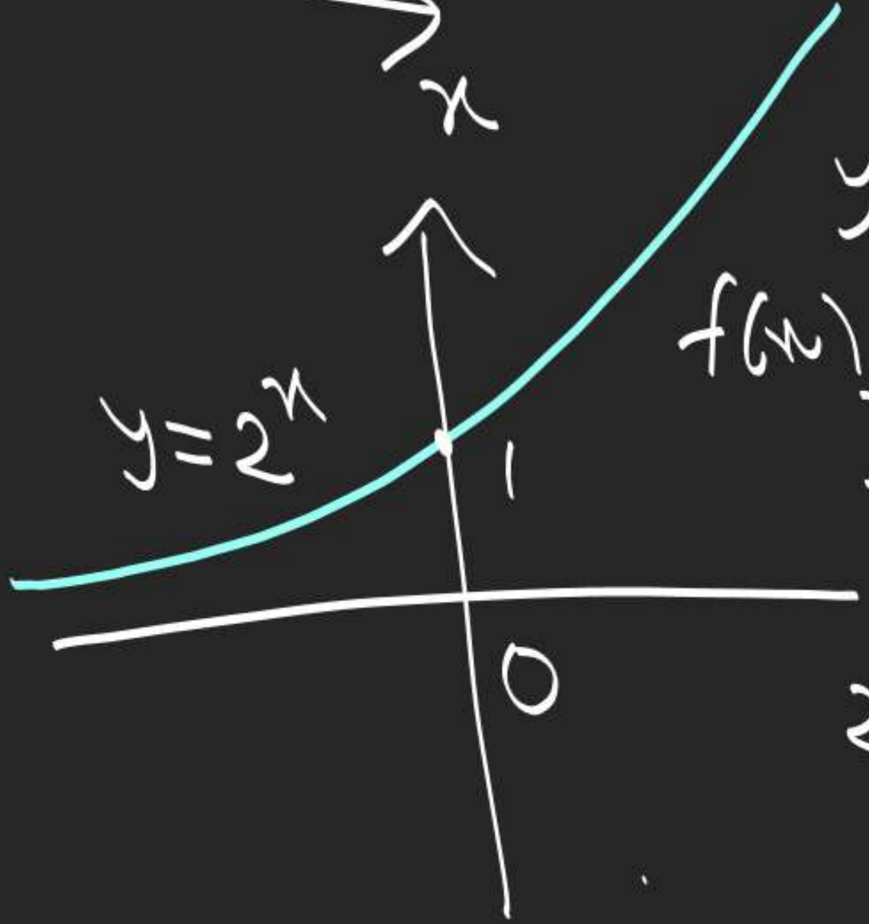
$$a^{x_1} < a^{x_2} \quad , \quad 0 < a < 1$$



$f(x) = y = a^x$, $a > 1$ ✓

$D_f = \mathbb{R}$

$R_f = (0, \infty)$



$y = 2^x = f(x)$

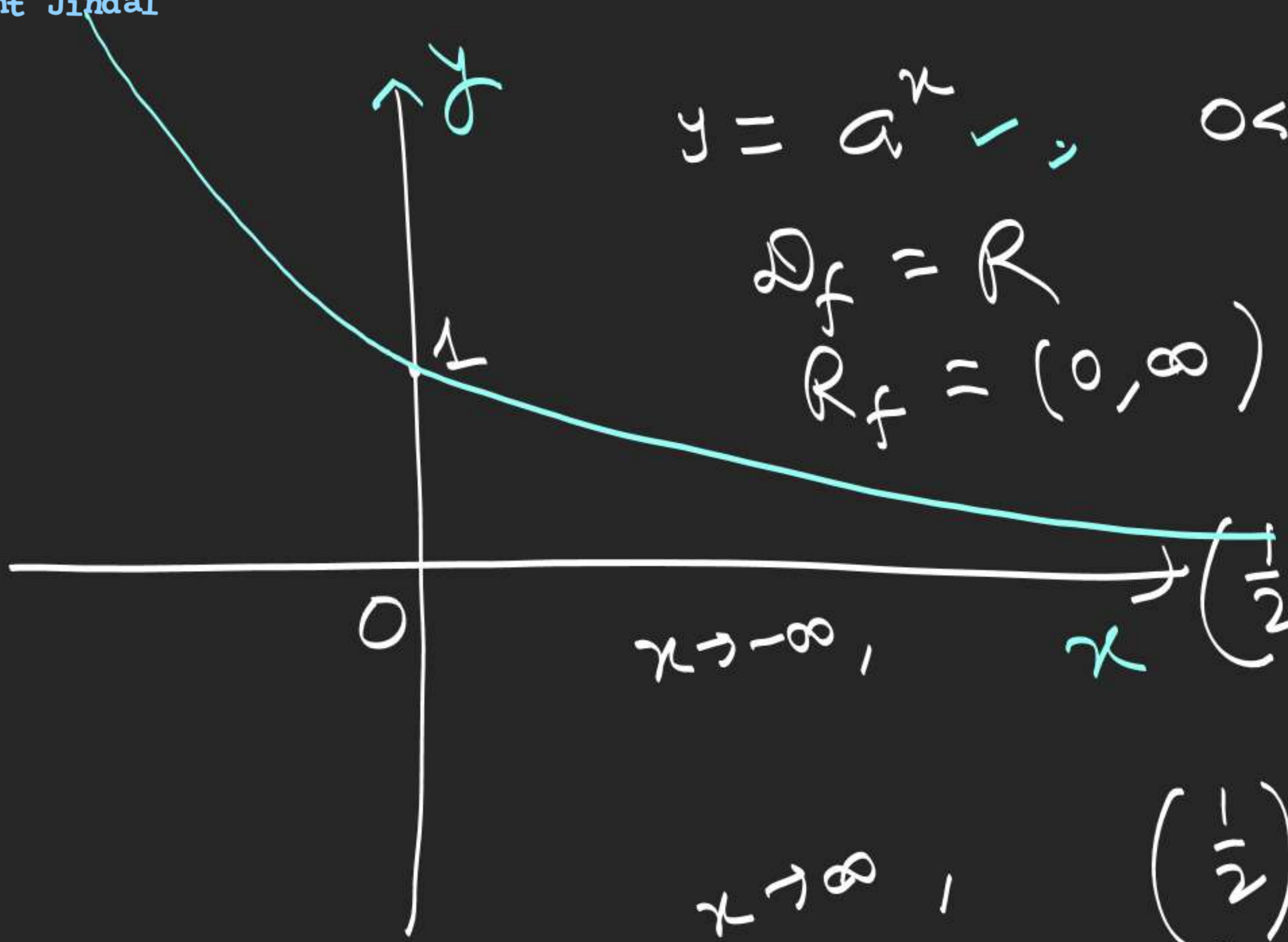
$f(x)$ increases

$x \to \infty,\ 2^x \to \infty$

$x \to -\infty,\ 2^x \to 0$

$2^{-1} = \frac{1}{2},\ 2^{-2} = \frac{1}{4}$

Nishant Jindal



$y = a^x$, $0 < a < 1$ ✓

$D_f = R$

$R_f = (0, \infty)$

$x \to -\infty$, $\left(\frac{1}{2}\right)^x \to \infty$

$\left(\frac{1}{2}\right)^{-\infty} \to 2^{\infty} \to \infty$

$x \to \infty$, $\left(\frac{1}{2}\right)^x \to 0$

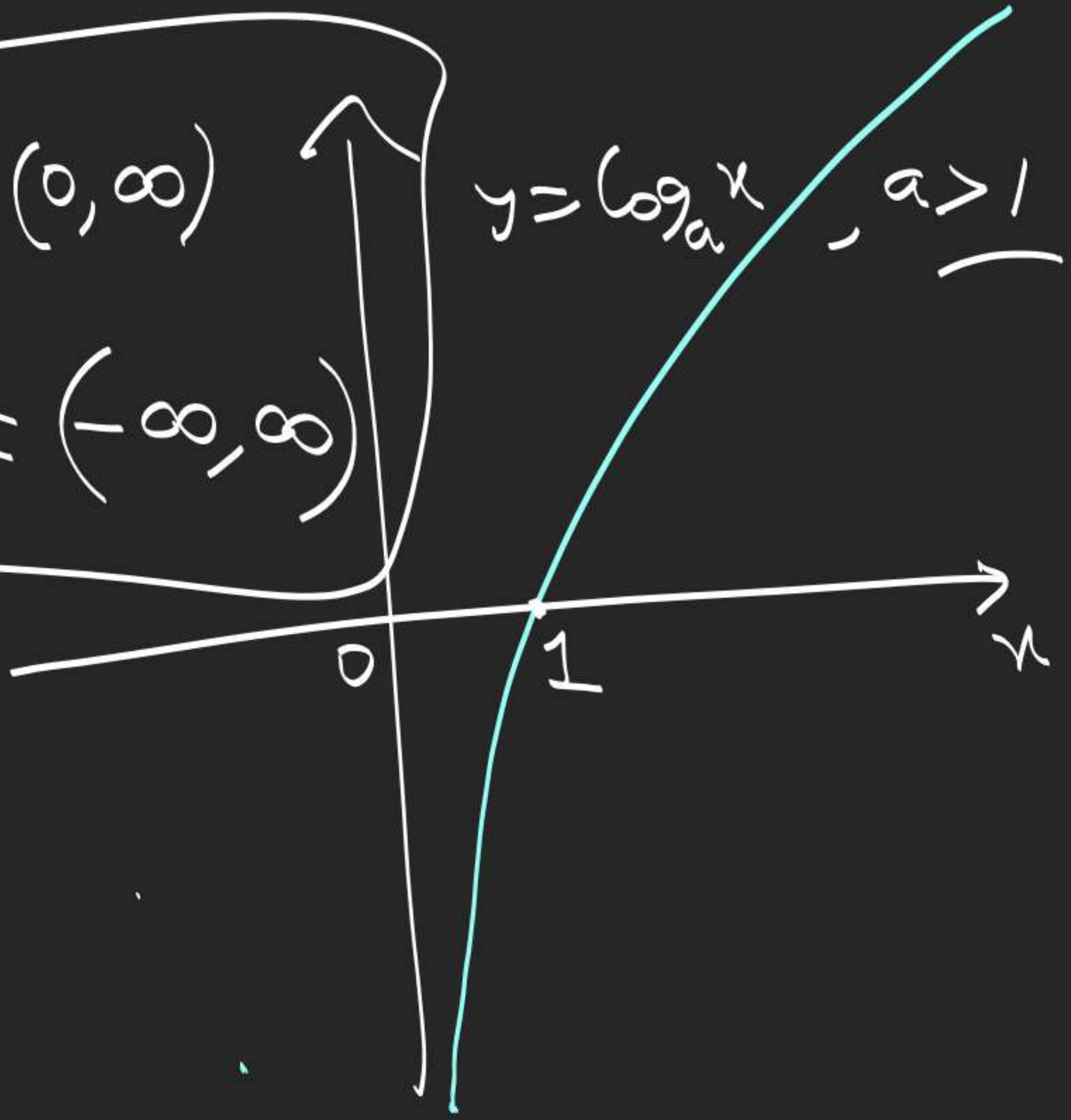
$3^x = 22\pi \Rightarrow x = \log_3(22\pi)$

$x = 3.\ldots$

Nishant Jindal

$f(x) = \log_a x \quad , \; a > 1$

$$\boxed{\begin{gathered} \mathcal{D}_f = (0, \infty) \\ R_f = (-\infty, \infty) \end{gathered}}$$



$x \to 0^+ , \; \log_a x \to -\infty$

$x \to \infty , \; \log_a x \to \infty .$

$x = 0.0000$

$\log_a x = y$

$a^y = x = 0.000\ldots$

$2^y = 0.00\ldots$

$2^y = \infty , \; y \to \infty$

Nishant Jindal

$f(x) = \log_a x \qquad 0<a<1$

$x \to 0.000\ldots, \ \log_a x \to \infty$

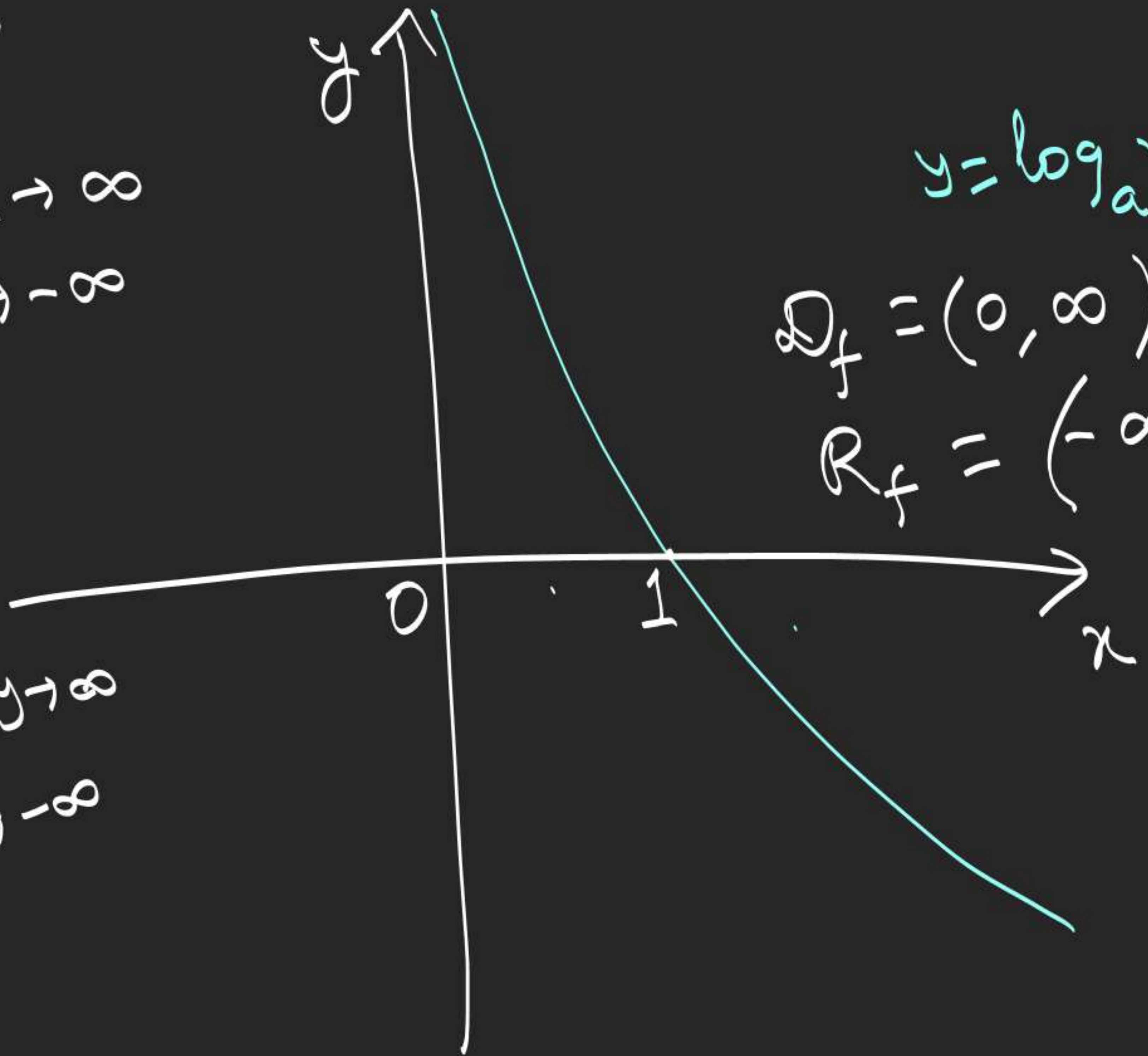
$x \to \infty, \ \log_a x \to -\infty$

$\log_a x = y$

$x = a^y$

$a^y \to 0.00\ldots, \ y \to \infty$

$a^y \to \infty, \ y \to -\infty$



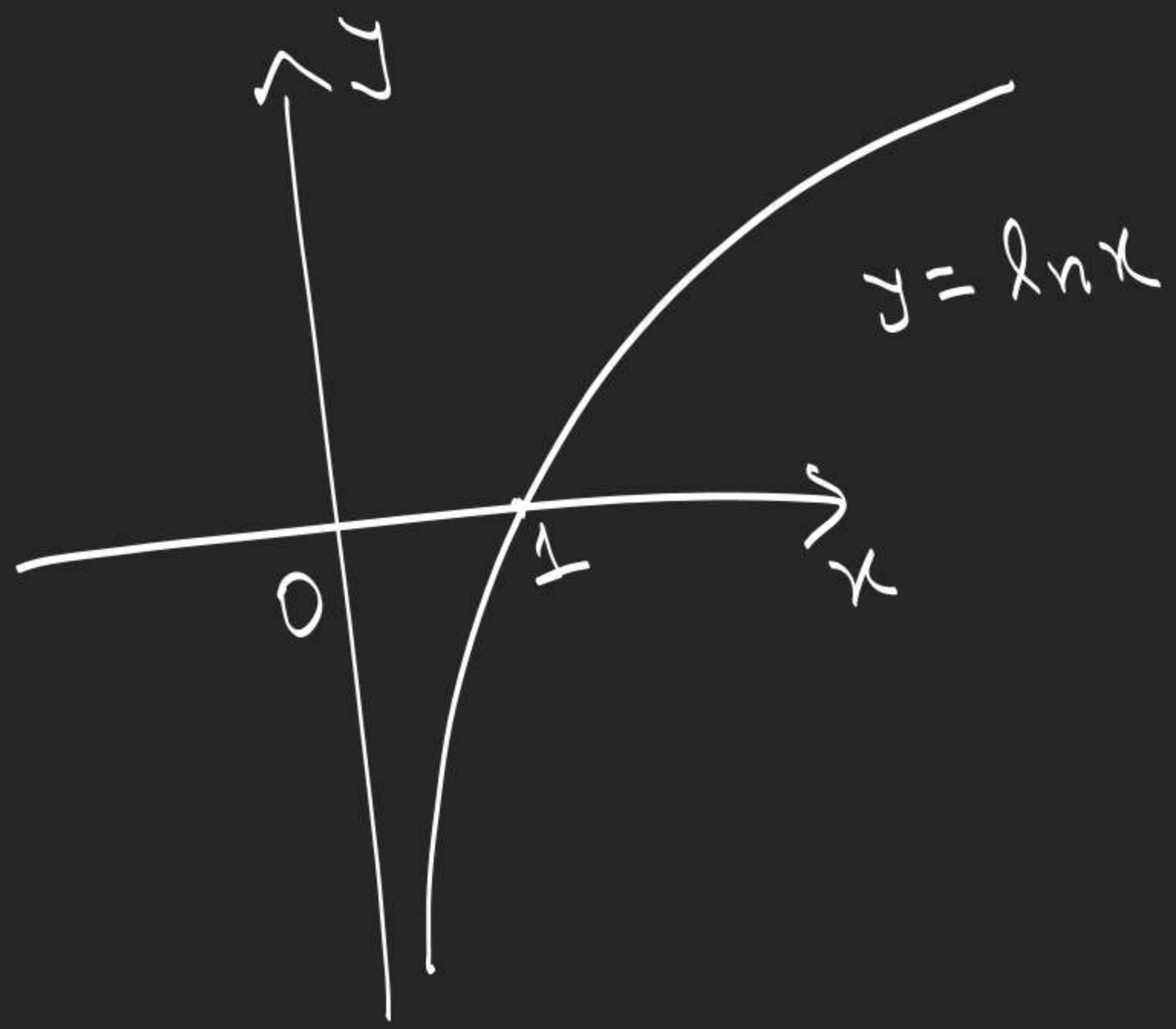
$y = \log_a x, \ 0<a<1$

$D_f = (0, \infty)$

$R_f = (-\infty, \infty)$

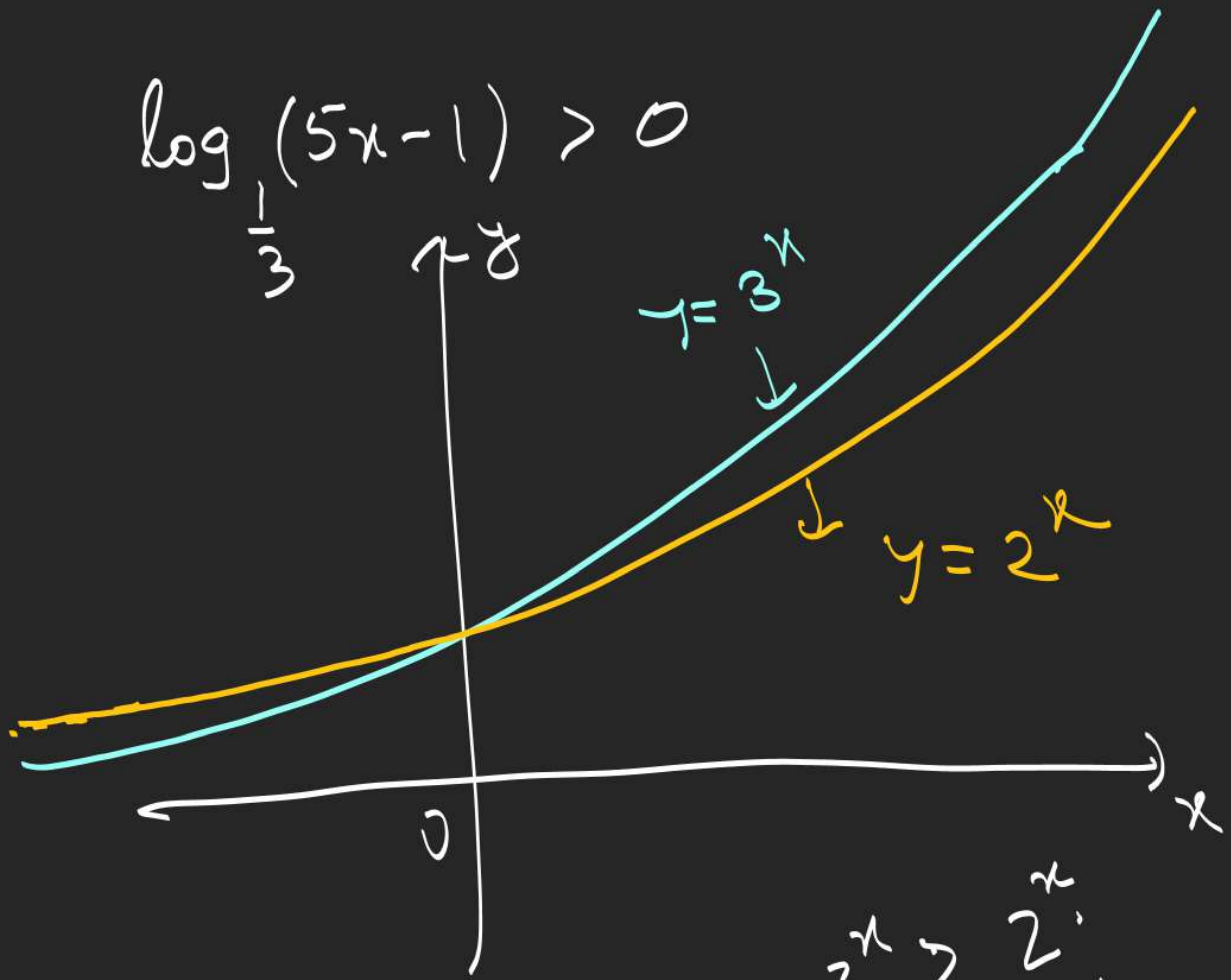
Nishant Jindal

$$y = \log x = \ln x = \log_e x$$



Nishant Jindal

1. $\log_{\frac{1}{3}}(5x-1) > 0$



$3^x > 2^x$

$\left(\frac{3}{2}\right)^x > 1$ $\quad x > 0$

$3 < 1$ $\quad x < 0$

Nishant Jindal

1.

$$\log_{\frac{1}{3}}(5x-1) > 0$$

$$\Rightarrow \log_{\frac{1}{3}}(5x-1) > \log_{\frac{1}{3}} 1$$

$$0 < 5x-1 < 1$$

$$5x-1 > 0 \Rightarrow \boxed{x > \frac{1}{5}}$$

$$5x-1 < 1 \Rightarrow \boxed{x < \frac{2}{5}}$$

$$\boxed{x \in \left(\frac{1}{5}, \frac{2}{5}\right)}$$

Q.

$$\log_7\left(\frac{2x-6}{2x-1}\right) > 0 = \log_7 1$$

161-175

264, 265, 266, 267, 268

$$\frac{2x-6}{2x-1} > 1 \quad \& \quad \frac{2x-6}{2x-1} > 0$$

$$\boxed{x \in \left(-\infty, \frac{1}{2}\right)}$$

Ans.

$$\boxed{\frac{2x-6}{2x-1} > 1}$$

$$\frac{2x-6}{2x-1} - 1 > 0$$

$$\frac{-5}{2x-1} > 0 \Rightarrow \frac{5}{2x-1} < 0$$

$$\boxed{x < \frac{1}{2}}$$