HOMEWORK-04(Solution)
(A.G.P.)

1. If $3+\frac{1}{4}(3+d)+\frac{1}{4^2}(3+2d)+\cdots.+\text{upto }\infty=8$ then the value of d is

(A) 9 (B) 5 (C) 1 (D) 3

Ans. **(A)**

Sol. Let $S=3+\frac{1}{4}(3+d)+\frac{1}{4^2}(3+2d)+\cdots\rightarrow(1)$

Multiplying (1) by $\frac{1}{4}$

We get

$$\frac{1}{4}S=\frac{3}{4}+\frac{1}{4^2}(3+d)+\frac{1}{4^3}(3+3d)+\cdots\rightarrow(2)$$

(1) − (2) gives

$$\left(1-\frac{1}{4}\right)S=3+\frac{d}{4}\left(1+\frac{1}{4}+\frac{1}{4^2}+\cdots\right)$$

$$\Rightarrow\frac{3}{4}S=3+\frac{d}{4}\left(\frac{1}{1-\frac{1}{4}}\right)\Rightarrow\frac{3}{4}S=3+\frac{d}{4}\left(\frac{4}{3}\right)\quad\Rightarrow\frac{3}{4}S=3+\frac{d}{3}\Rightarrow\frac{3}{4}S=\frac{9+d}{3}\Rightarrow S=\left(\frac{9+d}{3}\right)\frac{4}{3}$$

$$\therefore 8=\left(\frac{9+d}{3}\right)\frac{4}{3}\Rightarrow d=9$$

2. If $x>0$, and $\log_2 x+\log_2(\sqrt{x})+\log_2(\sqrt[4]{x})+\log_2(\sqrt[8]{x})+\log_2(\sqrt[16]{x})+\cdots=4$, then x equals

(A) 2 (B) 3 (C) 4 (D) 5

Ans. **(C)**

Sol. We can write the given equation as

$$\log_2\left(x^{1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\cdots}\right)=4$$

$$\Rightarrow\log_2(x^2)=4\Rightarrow x^2=2^4\Rightarrow x=4$$

3. The positive integer $\mathbf{n}$ for which $2\times2^2+3\times2^3+4\times2^4+\cdots..+n\times2^n=2^{n+10}$ is

(A) 510 (B) 511 (C) 512 (D) 513

Ans. **(D)**

Sol. $2^{n+10}=2\cdot2^2+3\cdot2^3+4\cdot2^4+\cdots+n\cdot2^n\rightarrow(1)$

Multiply (1) by 2

$2^{n+11}=2\cdot2^3+3\cdot2^4+4\cdot2^5+\cdots+n\cdot2^{n+1}\rightarrow(2)$

Subtracting (2) from (1) by shifting one place, we get

$-2^{n+10}=2\cdot2^2+2^3+2^4+\cdots+2^n-n\cdot2^{n+1}\rightarrow(3)$

$n\cdot2^{n+1}-2^{n+10}=2+2^1+2^2+\cdots+2^n$

$$\Rightarrow n\cdot2^n-2^9\cdot2^n=1+\frac{2^n-1}{2-1}$$

$\Rightarrow n-1=2^9$

$n=513$

4. If $b+c,c+a,a+b$ are inH.P., then a^2,b^2,c^2 will be in-

(A) A.P. (B) G.P. (C) H.P. (D) None of these

Ans. **(A)**

Sol. **Given** $\frac{1}{b+c}\frac{1}{c+a'}\frac{1}{a+b}$ **are in A.P.**

Multiplying by $(a+b)(b+c)(c+a)$, **we get**

$\Rightarrow(a+b)(c+a),(a+b)(b+c),(b+c)(c+a)$ **are in A.P.**

$\Rightarrow a^2+(ab+bc+ca),b^2+(ab+bc+ca),c^2+(ab+bc+ca)$ **are in A.P.**

$\Rightarrow a^2,b^2,c^2$ **are in A.P.**

5. If first and second terms of a HP are a and b, then its n^{th} term will be-

(A) $\frac{ab}{a+(n-1)ab}$ (B) $\frac{ab}{b+(n-1)(a+b)}$

(C) $\frac{ab}{b+(n-1)(a-b)}$ (D) $\frac{ab}{(a+(n+1)ab)}$

Ans. (C)

6. If a, b, c are in A.P., then $\frac{bc}{ca+ab}, \frac{ca}{bc+ab}, \frac{ab}{bc+ca}$ are in-

(A) A.P. (B) G.P. (C) H.P. (D) None of these

Ans. (C)

Sol. **a, b, c are in A.P**

$\frac{a}{abc}, \frac{b}{abc}, \frac{c}{abc}$ **are in A.P**

$\frac{1}{bc}, \frac{1}{ca}, \frac{1}{ab}$ **are in A.P**

$\frac{ab+bc+ca}{bc}, \frac{ab+bc+ca}{ca}, \frac{ab+bc+ca}{ab}$ **are also in A.P**

$\frac{ab+bc+ca}{bc} - 1, \frac{ab+bc+ca}{ca} - 1, \frac{ab+bc+ca}{ab} - 1$ **are also in A.P**

$\frac{ab+ca}{bc}, \frac{ab+bc}{ca}, \frac{bc+ca}{ab}$ **are also in A.P**

$\frac{bc}{ab+ca}, \frac{ca}{ab+bc}, \frac{ab}{bc+ca}$ **are in H.P**

Hence, this is the answer.

7. $\{a_n\}$ and $\{b_n\}$ are two sequences given by

$a_n = (x)^{1/2^n} + (y)^{1/2^n}$ and $b_n = (x)^{1/2^n} - (y)^{1/2^n}$

for all $n \in N$. The value of $a_1 a_2 a_3 \ldots \ldots .. a_n$ is equal to

(A) $x - y$ (B) $\frac{x+y}{b_n}$ (C) $\frac{x-y}{b_n}$ (D) $\frac{xy}{b_n}$

Ans. (C)

Sol. $a_n = \frac{\left(x^{1/2^n}+y^{1/2^n}\right)\left(x^{1/2^n}-y^{1/2^n}\right)}{\left(x^{1/2^n}-y^{1/2^n}\right)}$

$$a_n = \frac{\left(x^{1/2^{n-1}} - y^{1/2^{n-1}}\right)}{\left(x^{1/2^n} - y^{1/2^n}\right)} = \frac{b_{n-1}}{b_n}$$

$$a_1 a_2 a_3 \ldots \ldots a_n = \frac{b_0}{b_1} \times \frac{b_1}{b_2} \times \frac{b_2}{b_3} \times \ldots \ldots \times \frac{b_{n-1}}{b_n}$$

$$a_1 a_2 a_3 \ldots \ldots . a_n = \frac{b_0}{b_n} = \frac{(x-y)}{b_n}$$

8. Show that in any arithmetic progression

$a_1, a_2, a_3 \ldots a_1{}^2 - a_2{}^2 + a_3{}^2 - a_4{}^2 + \cdots . + a_{2K-1}^2$

$a_{2K} = [K/(2K-1)]\left(a_1{}^2 - a_{2K}^2\right)$.

Sol. $a_1^2 - a_2^2 = (a_1 - a_2)(a_1 + a_2) = -d(a_1 + a_2)$

Similarly for each of k brackets formed out of 2k given terms.

$$\therefore S = -dS_{2k} = -d \cdot \frac{2k}{2}[a_1 + a_{2k}]$$

$= -dk\left[\frac{a_1^2-a_{2k}^2}{a_1-a_{2k}}\right] = -dk\frac{(a_1^2-a_{2k}^2)}{a_1-\{a_1+(2k-1)d\}}$

$= \frac{k}{2k-1}(a_1^2 - a_{2k}^2)$

9. For any three positive real numbers a, b and c,
$9(25a^2 + b^2) + 25(c^2 - 3ac) = 15\,b(3a + c)$. Then:
(A) b,c and a are in G.P.
(B) b,c and a are in A.P.
(C) a, b and c are in A.P.
(D) a,b and c are in G.P.

Ans. **(B)**

Sol. $9(25a^2 + b^2) + 25(c^2 - 3ac) = 15b(3a + c)$

$225a^2 + 9b^2 + 25c^2 - 75ac = 45ab + 15bc$

$225a^2 + 9b^2 + 25c^2 - 75ac - 45ab - 15bc = 0$

$(15a)^2 + (2b)^2 + (5c)^2 - (15a)(5c) - (15a)(3b) - (3b)(5c) = 0$

$(15a)^2 + (2b)^2 + (5c)^2 = (15a)(5c) + (15a)(3b) + (3b)(5c)$

$15a = 3b = 5c = k$

$b = 5a$ and $c = 3a$

Hence, $a = \frac{k}{15}, b = \frac{k}{3}$ and $c = \frac{k}{5}$

$\therefore b - a = \frac{4k}{15}$

and $c - a = \frac{2k}{15}$

and $b - c = \frac{2k}{15}$

$\therefore c - a = b - c$

$\Rightarrow 2c = a + b$

Hence, b, c and a are in A.P.

10. If $(10)^9 + 2(11)^1(10)^8 + 3(11)^2(10)^7 + \cdots \ldots\ldots + 10(11)^9 = k(10)^9$, then $\mathbf{k}$ is equal to
(A) $\frac{121}{10}$
(B) $\frac{441}{100}$
(C) 100
(D) 110

Ans. **(C)**

Sol. Let the sum be denoted by S then

$$S = 10^9 + 2 \times 11 \times 10^8 + 3 \times 11^2 \times 10^7 + \cdots .. + 10 \times 11^9$$

$$\text{or, } \frac{11}{10} S = 11 \times 10^8 + 2 \times 11^2 \times 10^7 + \cdots \ldots \ldots + 9 \times 11^9 + 11 \times 11^9$$

on subtracting we get

$$\frac{S}{10} = 10^9 + 11 \times 10^8 + 11^2 \times 10^7 + \cdots \ldots + 11^8 \times 10 + 11^9 - 11^{10}$$

we can see that except the last term all the terms are in G.P with $a = 10^9$ and $r = \frac{11}{10}$

$$\text{Hence } \Rightarrow \frac{S}{10} = \frac{10^9\left(\underline{11^{10^{10}}} - 1\right)}{\frac{11}{10} - 1} - 11^{10} = 10^{10}\left(\frac{11^{10}}{10} - 1\right) - 11^{10}$$

$$\Rightarrow S = 10 \times 10^{10} = 100(10)^9$$

Therefore, $k = 100$

(A.M.G.M.H.M.)

11. If x, y, z are AM, GM and HM of two positive distinct numbers respectively, then correct statement is -

(A) $x < y < z$ (B) $y < x < z$
(C) $z < y < x$ (D) $z < x < y$

Ans. **(C)**

Sol. $x, y, z \rightarrow AM, GM$ **&** HM **of 2 numbers**

let two numbers be arb

$$\therefore x = \frac{a+b}{2} \; y = \sqrt{ab} \; \& z = \frac{2ab}{a+b}$$

$$x - y = \frac{a+b}{2} - \sqrt{ab}$$

$$= \frac{a+b-2\sqrt{ab}}{2}$$

$$= \frac{(\sqrt{a})^2+(\sqrt{b})^2-2\sqrt{ab}}{2} = \frac{(\sqrt{a}-\sqrt{b})^2}{2}$$

$$\Rightarrow x - y \geqslant 0 \; \Rightarrow \; x \geqslant y$$

$$y - z = \sqrt{ab} - \frac{2ab}{a+b}$$

$$= \sqrt{ab}\left(1 - \frac{2\sqrt{ab}}{a+b}\right)$$

$$= \frac{\sqrt{ab}(a+b-2\sqrt{ab})}{a+b}$$

$$= \frac{\sqrt{ab}(\sqrt{a}-\sqrt{b})^2}{a+b} \geqslant 0$$

$$\Rightarrow \; y - z \geqslant 0 \Rightarrow y \geqslant z$$

from (1) & (2) $x \geqslant y \geqslant z$

12. The A.M. of two positive numbers exceeds the GM by 5, and the GM exceeds the H.M. by 4 . Then the numbers are-

(A) 10,40 (B) 10,20 (C) 20,40 (D) 10,50

Ans. **(A)**

Sol. Given

A.M $= \mathbf{5 + GM - (1)}$

G.M $= \mathbf{4 +}$ H.M - (2)

We know

$\mathbf{G \cdot M^2 = AM \cdot HM}$

$\mathbf{GM^2 = (5 + GM)(GM - 4)}$

$\mathbf{GM^2 = 5GM - 20 + GM^2 - 4GM}$

$\mathbf{\therefore GM = 20}$

$\mathbf{AM = 25}$(from $(\mathbf{1})$)

$\sqrt{ab} = 20, \dfrac{a+b}{2} = 25 \Rightarrow a + b = 50$

$\Rightarrow ab = 400 \ldots (3)$

$\Rightarrow (a-b)^2 = (a+b)^2 - 4ab$

$= 50^2 - 4 \times 400$

$= 2500 - 1600$

$= 900$

$\therefore a - b = \sqrt{900} = 30 \ldots (4)$

from $(3)\&(4)$

$a + b = 50$

$a - b = 30$

$a = 80$

$a = 40 \& b = 10$

$\therefore$ ***The*** numbers are $(40, 10)$.

$\therefore$ The numbers are $(40, 10)$.

13. If A,G & 4 are A.M, G.M & H.M of two numbers respectively and $2A + G^2 = 27$, then the numbers are-

(A) 8,2 (C) 6,3 (B) 8,6 (D) 6,4

Ans. **(C)**

Sol. $HM = \frac{2ab}{a+b} = 4$

$\frac{2G^2}{2A} = 4$

$\Rightarrow G^2 = 4A$

$AM = \frac{a+b}{2} = A$

$\Rightarrow 2A = a + b$

$GM = \sqrt{ab} = G$

$\Rightarrow G^2 = ab$

Given,

$2A + G^2 = 27$

$2A + 4A = 27$

$6A = 27$

$6\left(\dfrac{a+b}{2}\right) = 27$

$\Rightarrow a + b = 9.$

$\dfrac{2ab}{a+b} = 4 \Rightarrow \dfrac{2ab}{9} = 4$

$ab = 18$

$b = \dfrac{18}{a} \ldots \ldots \ldots \ldots (2)$

substituting (2) in (1), we get,

$a + \dfrac{18}{a} = 9$

$\Rightarrow a^2 - 9a + 18 = 0$

$(a-3)(a-6) = 0$

$a = 3, 6$

when $a = 3, b = \frac{18}{3} = 6$

when $a = 6, b = \frac{18}{6} = 3$

$\therefore 3, 6$

14. If **A**, **G** & **H** are respectively the A.M., G.M. & H.M. of three positive numbers $\boldsymbol{a}, \boldsymbol{b}$, & c then the equation whose roots are **a**, **b** & **c** is given by

(A) $x^3 - 3Ax^2 + 3G^3x - G^3 = 0$ (B) $x^3 - 3Ax^2 + 3(G^3/H)x - G^3 = 0$

(C) $x^3 + 3Ax^2 + 3(G^3/H)x - G^3 = 0$ (D) $x^3 - 3Ax^2 - 3(G^3/H)x + G^3 = 0$

Ans. **(B)**

Sol. $A = a + b + c/3$

$G = \sqrt[3]{abc}$

$$H = \frac{3}{1/a + 1/b + 1/c}$$

$a + b + c = 3A, ab + bc + ca = 3G^3/H, abc = G^3$

so the equation is $x^3 - 3Ax^2 + 3\left(\frac{G^3}{H}\right)x - G^3 = 0$

15. If $x^2 + 9y^2 + 25z^2 = xyz\left(\frac{15}{x} + \frac{5}{y} + \frac{3}{z}\right)$, then x, y and z are in

(A) AGP (B) GP (C) **AP** (D) **HP**

Ans. **(D)**

Sol. $x^2 + 9y^2 + 25z^2 = xyz\left(\frac{15}{x} + \frac{5}{y} + \frac{3}{z}\right)$

$\Rightarrow x^2 + 9y^2 + 25z^2 = 3xy + 15yz + 5xz$

$\Rightarrow (x^2 - 6xy + 9y^2) + (9y^2 - 30yz + 25z^2) + (25z^2 - 10xz + x^2) = 0$

$\Rightarrow (x - 3y)^2 + (3y - 5z)^2 + (5z - x)^2 = 0$

Therefore, $x - 3y = 0; 3y - 5z = 0; 5z - x = 0$

$\Rightarrow x = 3y = 5z = k$

$\Rightarrow x = k; y = \frac{k}{3}; z = \frac{k}{5}$

now $\frac{2}{y} = \frac{6}{k}$ **and** $\frac{1}{x} + \frac{1}{z} = \frac{6}{k}$

$$\frac{1}{x} + \frac{1}{z} = \frac{2}{y}$$

Therefore, x, y, z are in H.P

16. If G_1 and G_2 are two geometric means and A is the arithmetic means inserted between two positive numbers then the value of $\frac{G_1^2}{G_2} + \frac{G_2^2}{G_1}$ is

(A) **A/2** (B) **A** (C) **2 A** (D) **3 A**

Ans. **(C)**

Sol. **Now if two geometric means are inserted between a^3 and b^3 that means b^3 is the fourth term of that GP.**

So $a^3r^3 = b^3$

$r = b/a$

$G1 = a^3r = a^2 b$

$G2 = a^3r^2 = a\, b^2$

$(G1)^2/G2 + (G2)^2/G1 = [a^4\, b^2/ab^2] + [a^2\, b^4/a^2\, b] = a^3 + b^3 = 2\, A$

17. If sum of A.M. and H.M. between two positive numbers is 25 and their GM is 12, then the sum of numbers is-

(A) 9 (B) 18 (C) 32 (D) 18 or 32

Ans. (C)

Sol. Let a and b are the numbers.

$\text{A.M} = \frac{a+b}{2}$

$\text{H.M} = \frac{2ab}{a+b}$

$\text{G.M} = \sqrt{ab}$

But given, $AM + HM = 25$ and $GM = 12$

$\Rightarrow \frac{a+b}{2} + \frac{2ab}{a+b} = 25$ and $ab = 144$

Let $a + b = t$

$\Rightarrow \frac{t}{2} + \frac{288}{t} = 25$

$\Rightarrow t^2 - 50t + 576 = 0$

Solving above quadratic gives

$t = -18$ or $t = 32$

As a, b are positive numbers.

$\therefore t = a + b = 32$

Hence, option C.

18. The A.M. of two numbers is 34 and GM is 16 , the numbers are-

(A) 2 and 64 (B) 64 and 3 (C) 64 and 4 (D) 64 and 8

Ans. (C)

Sol. Let a, b be the numbers.

Then according to the problem,

$\frac{a+b}{2} = 34$

[Arithmetic mean]

or, $a + b = 68$

And $\sqrt{ab} = 16$

[Geometric mean]

or, $ab = 256$
or, $a(68 - a) = 256$
Using (1)
or, $a^2 - 68a + 256 = 0$
or, $(a - 64)(a - 4) = 0$
or, $a = 64, 4$ then $b = 4, 64$.
So the required numbers are 64 and 4 .

19. The ratio between the GM's of the roots of the equations $ax^2 + bx + c = 0$ and $\ell x^2 + mx + n = 0$ is-

(A) $\sqrt{\frac{b\ell}{an}}$ (B) $\sqrt{\frac{c\ell}{an}}$ (C) $\sqrt{\frac{an}{c\ell}}$ (D) $\sqrt{\frac{cn}{a\ell}}$

Ans. (B)

Sol. If α and β are the roots of the equation $ax^2 + bx + c = 0$

Then, $\alpha\beta = \frac{c}{a}$

$$\text{G.M.} = \sqrt{\alpha\beta} = \pm\sqrt{\frac{c}{a}}$$

If h and k are the roots of $1x^2 + mx + n = 0$ then,

$$hk = \frac{n}{l}$$

$$\text{G.M.} = \sqrt{hk} = \pm\sqrt{\frac{n}{I}}$$

$$\text{Ratio} = \pm\frac{\sqrt{\frac{c}{a}}}{\sqrt{\frac{n}{1}}} = \pm\sqrt{\frac{cl}{an}}$$

20. Using the relation A.M. 2 G.M. Prove that

(i) $\tan\theta + \cot\theta \geq 2$; if $0 < \theta < \frac{\pi}{2}$

(ii) $(x^2y + y^2z + z^2x)(xy^2 + yz^2 + zx^2) > 9x^2y^2z^2$.
Where x, y, z are different real no.

(iii) $(a + b)\cdot(b + c)\cdot(c + a) \geq 8abc$; if a, b, c are positive real numbers.

Sol. (i) $\frac{(x^2y+y^2z+z^2x)(xy^2+yz^2+zx^2)}{9} =$

$$\frac{x^3y^3 + 3x^2y^2z^2 + x^4yz + y^4xz + y^3z^3 + z^4yx + z^3x^3}{9} = \text{A.M}$$

$$\sqrt[9]{x^3y^3 \times (x^2y^2z^2)^3 \times x^4yz \times y^4xz \times z^4yx \times y^3z^3 \times z^3x^3} = x^2y^2z^2 = G\cdot M$$

As A.M $\geq$ G.M,

$$\frac{(x^2y + y^2z + z^2x)(xy^2 + yz^2 + zx^2)}{9} \geq x^2y^2z^2$$

$\Rightarrow (x^2y+y^2z+z^2x)(xy^2+yz^2+zx^2) \geq 9x^2y^2z^2$

(ii) $\dfrac{(a+b)(b+c)(c+a)}{8} = \dfrac{(ab+ac+b^2+bc)(c+a)}{8} =$

$\dfrac{2abc+ac^2+b^2c+bc^2+a^2b+a^2c+b^2a}{8} = A\cdot M$

$\sqrt[8]{(abc)^2 \times ac^2 \times a^2c \times b^2c \times bc^2 \times a^2b \times ab^2} = abc = G.M$

As A.M $\geq G\cdot M,$

$\dfrac{(a+b)(b+c)(c+a)}{8} \geq abc$

$\Rightarrow (a+b)(b+c)(c+a) \geq 8abc > abc$

21. If **a**, **b**, **c** are sides of triangle then prove that (i) $b^2c^2+c^2a^2+a^2b^2 \geq abc(a+b+c)$
(ii) $(a+b+c)^3 > 27(a+b-c)(c+a-b)(b+c-a)$

Sol. **Let, a, b and c are positive real numbers and sides of the triangle. Then from the rule of forming triangle we have,**

$(a+b>c), (c+a>b), (b+c>a)$

$\Rightarrow (a+b-c>0), (c+a-b>0), (b+c-a>0).$

Now we take the positive real numbers $(a+b-c), (c+a-b), (b+c-a)$ and apply

A.M. $\geq$ G.M.

$\Rightarrow \frac{(a+b-c)+(c+a-b)+(b+c-a)}{3} \geq \sqrt[3]{(a+b-c)(c+a-b)(b+c-a)}$

$\Rightarrow \frac{(a+b+c)}{3} \geq \sqrt[3]{(a+b-c)(c+a-b)(b+c-a)}$

$\Rightarrow \left\{\frac{(a+b+c)}{3}\right\}^3 \geq (a+b-c)(c+a-b)(b+c-a)$

$\Rightarrow (a+b+c)^3 \geq 27(a+b-c)(c+a-b)(b+c-a)$

22. The arithmetic mean of two numbers is 6 and their geometric mean **G** and harmonic mean **H** satisfy the relation $G^2+3H=48$. Find the two numbers.

Ans. $a=4, b=8$

Sol. **Let two numbers be a, b.**
Arithmetic mean is 6 .

$\therefore \frac{a+b}{2} = 6 \Rightarrow a+b=12.$

$G=(ab)^{1/2}$

$H=\frac{2ab}{a+b}$

Given: $G^2+3H=48 \Rightarrow \left((ab)^{1/2}\right)^2+\frac{3\times 2ab}{a+b}=48$

$$ab+\frac{6ab}{a+b}=48$$

Put value of $a+b$ by (1),

$$ab+\frac{6ab}{12}=48 \Rightarrow ab=32$$

So, $a+b=12$ and $ab=32$ means $a=4$ and $b=8$.
Hence two numbers are 4 and 8 .

23. Let A_1, G_1, H_1 denote the arithmetic, geometric and harmonic means, respectively, of two distinct positive numbers. For $\mathbf{n} \geq \mathbf{2}$, Let $\mathbf{A}_{\mathbf{n}-\mathbf{1}}$ and $\mathbf{H}_{\mathbf{n}-\mathbf{1}}$ have arithmetic, **231**% metric and harmonic means as A_n, G_n, H_n respectively
(a) Which one of the following statements is correct ?
(A) $G_1>G_2>G_3>\cdots$ .....
(B) $G_1<G_2<G_3<\cdots$ ....
(C) $G_1=G_2=G_3=$
(D) $G_1<G_3<G_5<$ and $G_2>G_4>G_6>\cdots$ ...
(b) Which one of the following statement is correct ?
(A) $A_1>A_2>A_3>\cdots$ ....
(B) $A_1<A_2<A_3<$
(C) $A_1>A_3>A_5>\cdots$... and $A_2<A_4<A_6<\cdots$ ....
(D) $A_1<A_3<A_5<\cdots$... and $A_2>A_4>A_6>\cdots$ ......
(c) Which one of the following statement is correct?
(A) $H_1>H_2>H_3>$
(B) $H_1<H_2<H_3<$
(C) $H_1>H_3>H_5>\cdots$... and $H_2<H_4<H_6<$
(D) $H_1<H_3<H_5<\cdots$...and $H_2>H_4>H_6>\cdots$ ....

Ans. **(A) C,(B)A(C)B**

Sol. **we know $A_1>G_1>H_1$**

It is given that $\dot{A}_2'$ is the A.M of A_1&H_1 and $A_1>H_1$
$\therefore\ A_1>A_2>H_1$
$\dot{A}_3'$ is the $A\cdot M$ of $\dot{A}_2'$ and H_2 and $A_2>H_2$
$\therefore\ A_2>A_3>H_2$
Hence, $A_1>A_2>A_3>\cdots$

24. The minimum value of the sum of real numbers $a^{-5}, a^{-4}, 3a^{-3}, 1, a^8$ and a^{10} with $a > 0$ is

Ans. 8

Sol. Since, $AM \geq GM$

$$\Rightarrow \frac{\frac{1}{a^5}+\frac{1}{a^4}+\frac{1}{a^3}+\frac{1}{a^3}+\frac{1}{a^3}+1+a^8+a^{10}}{8} \geq \left(\frac{1}{a^5}\times\frac{1}{a^4}\times\frac{1}{a^3}\times\frac{1}{a^3}\times\frac{1}{a^3}\times 1\times a^8\times a^{10}\right)^{\frac{1}{8}}$$

$$\Rightarrow \frac{\frac{1}{a^5}+\frac{1}{a^4}+\frac{1}{a^3}+\frac{1}{a^3}+\frac{1}{a^3}+1+a^8+a^{10}}{8} \geq (1)^{\frac{1}{8}}$$

$$\Rightarrow \frac{1}{a^5}+\frac{1}{a^4}+\frac{1}{a^3}+\frac{1}{a^3}+\frac{1}{a^3}+1+a^8+a^{10} \geq 8(1)^{\frac{1}{8}}$$

$$\Rightarrow \frac{1}{a^5}+\frac{1}{a^4}+\frac{1}{a^3}+\frac{1}{a^3}+\frac{1}{a^3}+1+a^8+a^{10} \geq 8$$

(Miscellaneous & V_nMethod)

25. The sum of n term of the series $1(1!)+2(2!)+3(3!)+\cdots$

(A) $(n+1)!-1$ (B) $(n-1)!-1$

(C) $(n-1)!+1$ (D) $(n+1)!+1$

Ans. (A)

Sol. Let $S_n = 1.1!+2.2!+3.3!+4.4!+\cdots+n\cdot n!$

$= (2-1)1!+(3-1)2!+(4-1)3!+\cdots+((n+1)-1)n!$

$= (2.1!-1!)+(3.2!-2!)+(4.3!-3!)+\cdots+((n+1)n!-n!)$

$= (2!-1!)+(3!-2!)+(4!-3!)+\cdots+((n+1)!-n!)$

$= (n+1)!-1! = (n+1)!-1$

26. If p is positive, then the sum to infinity of the series, $\frac{1}{1+p}-\frac{1-p}{(1+p)^2}+\frac{(1-p)^2}{(1+p)^3}-\cdots\ldots$ is

(A) 1/2 (B) ¾ (C) 1 (D) ¼

Ans. (A)

Sol. $a = \frac{1}{1+P}$

$r = -\frac{1-P}{1+P}$

Sum to infinity $= \frac{a}{1-r} = \frac{\frac{1}{1+P}}{1+\frac{1-P}{1+P}} = \frac{\frac{1}{1+P}}{\frac{1+P+1-P}{1+P}} = \frac{1}{2}$

27. The sum of infinite series $1-\frac{3}{2}+\frac{5}{4}-\frac{7}{8}+\cdots$ is-

(A) 2/9 (B) 2/3 (C) $-2/9$ (D) 9/2

Ans. (A)

Sol. **Given:- Series** $1-\frac{3}{2}+\frac{5}{4}-\frac{7}{8}--$

To find:- Sum of infinity series

$$S = 1+\left(-\frac{3}{2}\right)+\frac{5}{4}+\left(-\frac{7}{8}\right)\ldots\infty$$

Multiplying bs of (1) with $\left(-\frac{1}{2}\right)$

$$\left(-\frac{1}{2}\right)s = \left(\frac{-1}{2}\right) + \frac{3}{4} + \left(\frac{-5}{8}\right) + \frac{7}{16} \cdots \infty - 2$$

Subtracting (2) from (1)

$$S - (-\frac{1}{2}s\Big) = 1 + \left(-\frac{3}{2}\right) - \left(-\frac{1}{2}\right) + \left(\frac{5}{4} - \frac{3}{4}\right) + \left(-\frac{7}{8}\right) - \left(-\frac{5}{8}\right) + \cdots \infty$$

$$\Rightarrow s + \frac{1}{2}s = 1 - \frac{3}{2} + \frac{1}{2} + \frac{2}{4} - \frac{7}{8} + \frac{5 + \cdots \infty}{8}$$

$$\Rightarrow \frac{3}{2}s = 1 + (-1) + \frac{1}{2} - \frac{1}{4} + \cdots \infty$$

$$\Rightarrow \frac{3}{2}s = 1 + \left[(-1) + \frac{1}{2} + \left(-\frac{1}{4}\right) + \cdots \infty\right]$$

$$\Rightarrow \frac{3}{2}s = 1 + s\infty$$

$$\Rightarrow \frac{3}{2}s = 1 - \frac{2}{3}$$

$$\Rightarrow \frac{3}{2}s = \frac{3-2}{3}$$

$$\Rightarrow \frac{3}{2}S = \frac{1}{3}$$

$$\Rightarrow S = \frac{1}{3} \times \frac{2}{3}$$

$$\Rightarrow S = \frac{2}{9}$$

$\therefore$ Sum of infinity series $1 - \frac{3}{2} + \frac{5}{4} - \frac{7}{8}$ is $\frac{2}{9}$ $\therefore$ Option A is correct

28. Find the sum of n terms of the series the r^{th} term of which is $(2r+1)2^r$.

(A) $n \cdot 2^{n+1} - 2^n + 2$ (B) $n \cdot 2^{n+2} - 2^{n+1} + 2$

(C) $n \cdot 2^{n+2} + 2^{n+1} - 2$ (D) None of these

Ans. **(B)**

Sol. $(2r+1)2^r = r2^{r+1} + 2^r$

$$\text{Let } S_1 = \sum_{r=1}^{n} 2^r = \frac{2(2^n - 1)}{2 - 1} = 2^{n+1} - 2$$

$$S_2 = \sum_{r=1}^{n} r2^{r+1} = 1 \cdot 2^2 + 2 \cdot 2^3 + \cdots \ldots . n \cdot 2^{n+1} \ldots . . \text{(i)}$$

Multiply both sides by 2 , we get

$$2\,S_2 = \sum_{r=1}^{n} r2^{r+1} = 0 + 1 \cdot 2^3 + 2 \cdot 2^4 + \cdots \ldots (n-1) \cdot 2^{n+1} + n \cdot 2^{n+2} \ldots \ldots$$

Subtracting both equations (i) and (ii), we get

$$-S_2 = \sum_{r=1}^{n} r2^{r+1} = 2^2 + 2^3 + \cdots \ldots \ldots .2^{n+1} - n \cdot 2^{n+2} - S_2 = \frac{4[2^n - 1]}{2-1} - n2^{n+2}$$

$$S_2 = -(4[2^n - 1]) + n2^{n+2}$$

Total sum $= S_1 + S_2 = 2^{n+1} - 2 - 4[2^n - 1] + n2^{n+2} = n \cdot 2^{n+2} - 2^{n+1} + 2$

29. If $H_n = 1 + \frac{1}{2} + \frac{1}{3} + \cdots \ldots \ldots . + \frac{1}{n}$, then value of $1 + \frac{3}{2} + \frac{5}{3} + \cdots \ldots \ldots \ldots . . + \frac{2n-1}{n}$ is

(A) $2n - H_n$
(B) $2n + H_n$
(C) $H_n - 2n$
(D) $H_n + n$

Ans. (A)

Sol. $H_n = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots \ldots + \frac{1}{n}$

$$S_n = 1 + \frac{3}{2} + \frac{5}{2} + \cdots \ldots + \frac{2n-1}{n}$$

$$T_n = \frac{2n-1}{n}$$

$$T_n = \left(2 - \frac{1}{n}\right)$$

$$\sum T_n = S_n = \sum\left(2 - \frac{1}{n}\right)$$

$$= 2n - \sum\frac{1}{n}$$

$$S_n = 2n - H_n$$

30. The sum of all possible products of first **n** natural numbers taken two at a time is

(A) $\frac{1}{24}n(n+1)(n-1)(3n+2)$
(B) $\frac{n(n+1)(2n+1)}{6}$
(C) $\frac{n(n+1)(2n-1)(n+3)}{24}$
(D) $\frac{n(n^2+1)(3n+2)}{24}$

Ans. (A)

Sol. $(b_1 + b_2 + b_3 + \cdots + b_n)^2 = b_1^2 + b_2^2 + \cdots + b_n^2 + 2\sum_{i=j} b_ib_j$

taking $b_1 = 1,\ b_2 = 2, \ldots, b_n = n$

$\therefore (1 + 2 + 3 + \cdots + n)^2 = 1^2 + 2^2 + 3^2 + \cdots + n^2 + 2\sum$ **(Product of number taken two at a time)**

$$\Rightarrow 2\sum b_ib_j = (1 + 2 + 3 + \cdots + n)^2 - \sum_{n=1}^{n} n^2 = \frac{n^2(n+1)^2}{4} - \frac{n(n+1)(2n+1)}{6} =$$

$$\frac{n(n^2-1)(3n+2)}{12}$$

$\Rightarrow \sum b_i b_j = \frac{n(n^2-1)(3n+2)}{24}$

31. Find the sum of the n terms of the series whose nth term is
(i) $n(n+2)$
(ii) $3^n - 2^n$

Ans. (i) $\frac{1}{6}n(n+1)(2n+7)$, (ii) $1/2(3^{n+1}+1) - 2^{n+1}$

Sol. $T_n = 3^n - 2^n$

Now, $\Sigma T_n = \Sigma 3^n - \Sigma 2^n$

We know sum of n terms of a G.P. is

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

where, a = first term of G P.

r = common ratio of the G.P.

Now, $\Sigma 3^n = \frac{3(3^n-1)}{3-1}$ $[a = 3, r = 3]$

$$= \frac{3^{n+1} - 3}{2}$$

Similarly, $\Sigma 2^n = \frac{2(2^n-1)}{2-1}$ $[a = 2, r = 2]$

$$= 2^{n+1} - 2$$

Using (2) and (3) in equation (1),

$$\Sigma T_n = \frac{3^{n+1} - 3}{2} - (2^{n+1} - 2)$$

$$\Sigma T_n = \frac{3^{n+1}}{2} - 2^{n+1} + \frac{1}{2}$$

32. Find the sum of the series
$\frac{5}{13} + \frac{55}{(13)^2} + \frac{555}{(13)^3} + \frac{5555}{(13)^4} \ldots \ldots ..$ up to ∞

Ans. $\frac{65}{36}$

Sol. $S = \frac{5}{13} + \frac{55}{13^2} + \frac{555}{13^3} + \frac{5555}{13^4} + \cdots \ldots (1)$

Now multiply by $\frac{1}{13}$, we get

$$\frac{1}{13}\,s = \frac{5}{13^2} + \frac{55}{13^3} + \frac{555}{13^4} + \cdots ..$$

Subtracting (2) from (1)

$$s - \frac{1}{13}\,s\left[\frac{5}{13} + \frac{55}{13^2} + \frac{555}{13^3} + \cdots ..\right] - \left[\frac{5}{13^2} + \frac{55}{13^3} + \cdots ..\right]$$

$$\frac{12\,s}{13} = \frac{5}{13} + \left(\frac{55}{13^2} + \frac{5}{13^2}\right) + \left(\frac{555}{13^3} + \frac{55}{13^3}\right) + \cdots ..$$

$$\frac{12\,s}{13} = \frac{5}{13} + \frac{50}{13^2} + \frac{500}{13^3} + \cdots .$$

$$= \frac{\frac{5}{13}}{1 - \frac{10}{13}}$$

$$= \frac{\frac{5}{13}}{1 - \frac{10}{13}}$$

$$\frac{12\,s}{13} = \frac{5}{3}$$

$$\Rightarrow s = \frac{65}{36}$$

33. Sum of the series to n terms and to infinity :

$$1^2 - \frac{2^2}{5} + \frac{3^2}{5^2} - \frac{4^2}{5^3} + \frac{5^2}{5^4} - \frac{6^2}{5^5} + \cdots \ldots \infty.$$

Ans. $\frac{25}{54}$

34. Find the sum of the **n** terms and to infinity of the sequence

$$\frac{1}{1 + 1^2 + 1^4} + \frac{2}{1 + 2^2 + 2^4} + \frac{3}{1 + 3^2 + 3^4} + \cdots \ldots ..$$

Ans. $\frac{n(n+1)}{2(n^2+n+1)}; S_\infty = \frac{1}{2}$

Sol. **Let T_n be the n^{th} term of the series**

$$\frac{1}{1 + 1^2 + 1^4} + \frac{2}{1 + 2^2 + 2^4} + \frac{3}{1 + 3^2 + 3^4} + \cdots$$

$$T_n = \frac{n}{1+n^2+n^4} = \frac{n}{(1+n^2)^2 - n^2}$$

$$= \frac{n}{(n^2+n+1)(n^2-n+1)} = \frac{1}{2}\left[\frac{1}{n^2-n+1} - \frac{1}{n^2+n+1}\right]$$

$= \frac{1}{2}\left[\frac{1}{1+(n-1)n} - \frac{1}{1+n(n+1)}\right]$

Now, $\sum_{r=1}^{n} T_r = \frac{1}{2}\left[\frac{1}{1} - \frac{1}{1+1.2}\right] + \frac{1}{2}\left[\frac{1}{1+1.2} - \frac{1}{1+2.3}\right]$

$$+\frac{1}{2}\left[\frac{1}{1+2.3} - \frac{1}{1+3.4}\right] + \cdots + \frac{1}{2}\left[\frac{1}{1+(n-1)n} - \frac{1}{1+n(n+1)}\right]$$

$$= \frac{1}{2}\left[1 - \frac{1}{1+n(n+1)}\right] = \frac{n(n+1)}{2(n^2+n+1)}$$

35. Find the sum of the first n terms of the sequence:

$$1 + 2\left(1+\frac{1}{n}\right) + 3\left(1+\frac{1}{n}\right)^2 + 4\left(1+\frac{1}{n}\right)^3 + \cdots ....$$

Ans. n^2

Sol. $S_m = 1 + 2\left(1+\frac{1}{n}\right) + \cdots \ldots\ldots + \infty$

$$\Rightarrow S_m\left(1+\frac{1}{n}\right) = \left(1+\frac{1}{n}\right) + 2\left(1+\frac{1}{n}\right)^2 + \cdots .. + \infty$$

$$S_m - S_m\left(1+\frac{1}{n}\right) = -\frac{S_m}{n}$$

$$-\frac{S_m}{n} = 1 + \left(1+\frac{1}{n}\right) + \left(1+\frac{1}{n}\right)^2 + \cdots \ldots\ldots + \infty$$

$$-\frac{S_m}{n} = \frac{1}{1-\left(1+\frac{1}{n}\right)} = -n$$

$$S_m = n^2$$

36. Find the n^{th} term and the sum of n terms of the sequence

(i) $1 + 5 + 13 + 29 + 61 + \cdots.$

(ii) $6 + 13 + 22 + 33 + \cdots \ldots$

Ans. **(i)** $2^{n+1} - 3; 2^{n+2} - 4 - 3n$ **(ii)** $n^2 + 4n + 1; (1/6)n(n+1)(2n+13) + n$

Sol. **(i)** $S = 1 + 5 + 13 + 29 + 61 + T_n$

$$S = 1 + 5 + 13 + 29 + \cdots \ldots . T_n$$

$$0 = 1 + 4 + 8 + 16 + 32 - T_n$$

$4 + 8 + 16 + 32 \ldots \ldots .$ **sum of GP with** $r = 2$ **and** $a = 4$

$$T_n = 1 + 4(2^{n-1} - 1) = 2^{n+1} - 3$$

$$S_n = \sum(2^{n+1} - 3) = (2^2 + 2^3 + 2^{n+1}) - 3n$$

$$= 2^2\left(\frac{2^n - 1}{2 - 1}\right) - 3n = 2^{n+2} - 4 - 3n$$

(ii) $S = 6 + 13 + 22 + 33 + .. T_n$

$S = 6 + 13 + 22 + \cdots T_n$

$T_n = 6 + 7 + 9 + 11 \ldots \ldots$

$= 1 + 5 + 7 + 9 + 11 \ldots \ldots$

$5 + 7 + 9 + 11 \ldots \ldots$ **sum of AP**

$$T_n = 1 + \frac{n}{2}(10 + (n-1)2) = n^2 + 4n + 1$$

$$S_n = \sum T_n = \frac{1}{6} n(n+1)(2n+13) + n$$

37. If the sum

$\sqrt{1 + \frac{1}{1^2} + \frac{1}{2^2}} + \sqrt{1 + \frac{1}{2^2} + \frac{1}{3^2}} + \sqrt{1 + \frac{1}{3^2} + \frac{1}{4^2}} + \cdots \ldots \ldots + \sqrt{1 + \frac{1}{(1999)^2} + \frac{1}{(2000)^2}}$ equal to $n - 1/n$ where $n \in N$. Find n.

Ans. $n = 2000$

Sol. Note that $1 + \frac{1}{n^2} + \frac{1}{(n+1)^2} = \frac{n^4+2n^3+3n^3+2n+1}{n^2(n+1)^2} = \frac{(n^2+n+1)^2}{n^2(n+1)^2} = \left(\frac{n^2+n+1}{n(n+1)}\right)^2 = \left(1 + \frac{1}{n} - \frac{1}{n+1}\right)^2$.

Thus, $\sum_{n=1}^{2007} \sqrt{1 + \frac{1}{n^2} + \frac{1}{(n+1)^2}} = \sum_{n=1}^{2007} \left[1 + \frac{1}{n} - \frac{1}{n+1}\right] = \sum_{n=1}^{2007} 1 + \sum_{n=1}^{2007} \left[\frac{1}{n} - \frac{1}{n+1}\right] = 2007 + 1 - \frac{1}{2008} = 2008 - \frac{1}{2008}$.

38. Two distinct, real infinite geometric series each have a sum of 1 and have the same second term. The third term of one of the series is $1/8$. If the second term of both the series can be written in the form $\frac{\sqrt{m}-n}{p}$, where m, n and p are positive integers and m is not divisible by the square of any prime, find the value of $100m + 10n + p$.

201%

Ans. 518

Sol. **let first series is**

$S1 = a + ar1 + ar1^2 + ar1^3 + ar1^4$

and second series is

$s1 = b + br2 + br2^2 + br2^3 + br2^4$

given $ar1 = br2$

$S1 = 1 = a/1 - r1$

or $a = 1 - r1$

third term $ar1^2 = 1/8$

put value of a

$(1 - r1)r1^2 = 1/8$

or $r1^3 + r1^2 + 1/8 = 0$
by inspection one value of $r1 = 1/2$
devide above equation be $r1 - 1/2 = 0$
remaining 2 roots are the roots of equation $4r1^2 - 2r1 - 1 = 0$

$$r1 = \frac{1 \pm \sqrt{5}}{4}$$

second term $= ar1 = (1 - r1)r1$

$= r1 - r1^2$

$= r1 - (r1/2 + 1/4)$

$= r1/2 - 1/4$

put value of $r1 = (1 + \sqrt{5})/4$

second term $= (\sqrt{5} - 1)/8$ other value of r will not give this form

so $m = 5, n = 1, p = 8$

$100m + 10n + p = 518$

39. One of the roots of the equation

$52000x^6 + 100x^5 + 10x^3 + x - 2 = 0$ is of the form $\frac{m+\sqrt{n}}{r}$, where m is non zero integer and n and r are relatively prime natural numbers. Find the value of $\mathbf{m + n + r}$.

Ans. 200

40. Statement 1: The sum of the series
$1 + (1 + 2 + 4) + (4 + 6 + 9) + (9 + 12 + 16) + \cdots \ldots$
$\ldots . +(361 + 380 + 400)$ is 8000.
Statement 2: $\sum_{k=1}^{n}(k^3 - (k-1)^3) = n^3$, for any natural number $\mathbf{n}$.
[AIEEE- 2012]
(A) Statement 1 is true, Statement 2 is true, Statement 2 is not a correct explanation for statement 1 .
(B) Statement 1 is true, Statement 2 is false.
(C) Statement 1 is false, Statement 2 is true.
(D) Statement 1 is true, Statement 2 is true,
Statement 2 is a correct explanation for statement 1.

Ans. (D)

Sol. $T_k = (k-1)^2 + k(k-1) + k^2 = \frac{((k-1)^3 - k^3)}{(k-1) - k} = k^3 - (k-1)^3$

$S_k = \sum_{k=1}^{n} k^3 - (k^3 - 1 - 3k(k-1)) = \sum_{k=1}^{n} 3k^2 - 3k + 1$

$$= \frac{3n(n+1)(2n+1)}{6} - \frac{3k(k+1)}{2} + n = n^3$$

$S_{20} = 20^3 = 8000$

41. The sum of first 20 terms of the sequence $0.7, 0.77, 0.777$, ….. is :

(A) $\frac{7}{81}(179+10^{-20})$ (B) $\frac{7}{9}(99+10^{-20})$

(C) $\frac{7}{81}(179-10^{-20})$ (D) $\frac{7}{9}(99-10^{-20})$

Ans. (A)

Sol. $0.7+0.77+\cdots 20 \text{ terins}$

$0.7(0.1+0.11+\cdots 20 \text{ texin})$

$\frac{0.9}{9.}\frac{0.9+0.99t \ldots 20\text{ter } 1)}{9}(0)$

$$\frac{7}{90}\left(01-\frac{1}{10}+1-\frac{1}{100}+\cdots 20 \text{ terios}\right)$$

$$\frac{7}{90}\left(20-\left(\frac{1}{10}+\frac{1}{(10)^2}+\cdots 20 \text{ terese}\right)\right)$$

$$\frac{7}{90}\left(20-\left((1/10)\left(\frac{1-(1/10)^{20})}{(1-1/10)}\right)\right.$$

$$\frac{7}{90}\left(20-\frac{1}{10}\frac{10}{9}(1-1/10^{20})\right.$$

$$\frac{7}{90}\left(20-\frac{1}{9}(1-10^{-20})\right)$$

$$\frac{7}{9}\left(20-\frac{1}{9}+\frac{1}{9}10^{-20}\right)=\frac{7}{90}\left(\frac{179}{9}+\frac{1}{9}10^{-20}\right)$$

$$=7(179+10^{-20})$$

$$81$$

42. The sum of first 9 terms of the series $\frac{1^3}{1}+\frac{1^3+2^3}{1+3}+\frac{1^3+2^3+3^3}{1+3+5}+\cdots$.. is :

(A) 142 (B) 192 (C) 71 (D) 96

Ans. (D)

Sol. $\frac{1^3}{1}+\frac{1^3+2^3}{1+3}+\frac{1^3+2^3+3^3}{1+3+5}+\cdots\ldots.9 \text{ terms}$

$$\text{General term } = \text{tn} = \frac{\left(\frac{n(n+1)}{2}\right)^2}{n^2}$$

$$=\frac{(n+1)^2}{4}$$

$$\text{the } \sum_{n=1}^{9}\frac{(n^2+2n+1)}{4}$$

$$=\frac{1}{4}\sum_{n=1}^{9} n(n+1)(2n)$$

$$=\frac{1}{4}\left[\sum_{n=1}^{9}+2\sum_{n=1}^{9}n+\sum 1\right]$$

$$=\frac{1}{4}\left[\frac{n(n+1)(2n+1)}{6}+\frac{2n(n+1)}{2}+n\right]^{9}$$

$$\Rightarrow \mathbf{n}=9$$

$$\therefore \frac{1}{4}\left[\frac{9\times 10\times 19}{6}+9\times 10+9\right]$$

$$=\frac{1}{4}[285+90+9]$$

$$=\frac{1}{x}\times 384=96$$

43. If the sum of the first ten terms of the series $\left(1\frac{3}{5}\right)^2+\left(2\frac{2}{5}\right)^2+\left(3\frac{1}{5}\right)^2+4^2+\left(4\frac{4}{5}\right)^2+\cdots\ldots$ is $\frac{16}{5}$ **m**, then **m** is equal to :

(A) 101 (B) 100 (C) 99 (D) 102

Ans. **(A)**

Sol. **This can be written as**

$$\left(\frac{8}{5}\right)^2+\left(\frac{12}{5}\right)^2+\left(\frac{16}{5}\right)^2+\left(\frac{20}{5}\right)^2+\cdots\ldots$$

We can see that the numbers are in AP

$$a_{10}=10^{\text{th}}\ \text{term}\ =\frac{8}{5}+\frac{4}{5}\times(10-1)=\frac{44}{5}$$

Hence, Sum of first ten terms is

$$\left(\frac{8}{5}\right)^2+\left(\frac{12}{5}\right)^2+\left(\frac{16}{5}\right)^2+\left(\frac{20}{5}\right)^2+\cdots\ldots\left(\frac{44}{5}\right)^2$$

Common term is $a_k=\frac{8}{5}+(k-1)\frac{4}{5}=\frac{4}{5}(k+1)$

$$\sum_{k=1}^{n}\left[\frac{4}{5}(k+1)\right]^2=\frac{16}{25}\left[\sum_{k=1}^{n}k^2+\sum_{k=1}^{n}2k+\sum_{k=1}^{n}1\right]=\frac{16}{25}\left[\frac{(n)(n+1)(2n+1)}{6}+2\times\frac{(n)(n+1)}{2}+n\right]$$

and so, $\sum_{k=1}^{10}\left[\frac{4}{5}(k+1)\right]^2=\frac{16}{25}\left[\frac{(10)(10+1)(2\times 10+1)}{6}+2\times\frac{(10)(10+1)}{2}+10\right]=$

$$\frac{16\times 505}{25}=\frac{16}{5}\times 101$$

Therefore, m $=101$

44. Let A be the sum of the first 20 terms and B be the sum of the first 40 terms of the series $1^2 + 2 \cdot 2^2 + 3^2 + 2 \cdot 4^2 + 5^2 + 2 \cdot 6^2 + \cdots$.. If $B - 2A = 100\lambda$, then λ is equal to :

(A) 496 (B) 232 (C) 248 (D) 464

Ans. **(C)**

Sol. $B = 1^2 + 2.2^2 + 3^2 + 2.4^2 + \cdots. + 2.40^2$

$A = 1^2 + 2.2^2 + 3^2 + 2.4^2 + \cdots. + 2.20^2$

$B = 1^2 + 3^3 + 5^2 + \cdots + 39^2 + 2[2^2 + 4^2 + \cdots + 40^2]$

$A = 1^2 + 3^2 + 5^2 + \cdots + 19^2 + 2[2^2 + 4^2 + \cdots + 20^2]$

Sum of square of first n odd natural number

$$= \frac{n(2n+1)(2n-1)}{3}$$

sum of square of first n even natural number

$$= \frac{2n(n+1)(2n+1)}{3}$$

$$B = \left[\frac{n(2n+1)(2n-1)}{3}\right]_{n=20} + 2\left[\frac{2n(n+1)(2n+1)}{3}\right]_{n=20}$$

$$A = \left[\frac{n(2n+1)(2n-1)}{3}\right]_{n=10} + 2\left[\frac{2n(n+1)2n+1)}{3}\right]_{n=10}$$

$$B = \frac{20(41)(39)}{3} + 2\left[\frac{(400(21)(41)}{3}\right]$$

$$B = \frac{100860}{3}$$

$$A = \frac{10(21)(19)}{3} + 2\left[\frac{20(11)(21)}{3}\right]$$

$$A = \frac{13230}{3}$$

$$B - 2A = \left[\frac{100860 - 26460}{3}\right] = \frac{74400}{3}$$

$$B - 2A = 24800 = 100\lambda$$

$$\lambda = 248$$

45. Let V_r denote the sum of first r terms of an arithmetic progression (A.P.) whose first term is r and the common difference is $(2r-1)$.

Let $T_r = V_{r+1} - V_r - 2$ and $Q_r = T_{r+1} - T_r$ for $r = 1, 2, ..$

(a) The sum $V_1 + V_2 + \cdots .... + V_n$ is

(A) $\frac{1}{12}n(n+1)(3n^2 - n + 1)$ (B) $\frac{1}{12}n(n+1)(3n^2 + n + 2)$

(C) $\frac{1}{2}n(2n^2 - n + 1)$ (D) $\frac{1}{3}(2n^3 - 2n + 3)$

(b) T_f is always

(A) an odd number (B) an even number

(C) a prime number (D) a composite number

Ans. (A) B,(B)B

Sol. $V_r = \frac{r}{2}(r + (r-1)(2r-1))$

$$= \frac{1}{2}(2r^3 - r^2 + r)$$

$$V_1 + V_2 + V_3 + \cdots \ldots = \sum V_r$$

$$\sum V_r = \frac{1}{2}(2\sum r^3 - \sum r^2 + \sum r)$$

$$= \frac{1}{2}\left(2 \times \frac{r^2(r+1)^2}{4} - \frac{r(r+a)(2r+1)}{6} + \frac{r(r+1)}{2}\right)$$

$$= \frac{1}{12} r(r+1)(3r^2 + r + 2)$$

for $r = n$ (i.e · sum upto n terms)

$$= \frac{1}{12} n(n+1)(3n^2 + n + 2)$$

46. Let $S_k, K = 1, 2, \ldots, 100$ denote the sum of the infinite geometric series whose first term is $\frac{k-1}{k!}$ and the common ratio is $1/k$. Then the value of $\frac{100^2}{100!} + \sum_{k=1}^{100} |(k^2 - 3k + 1)S_k|$ is

Ans. 3

Sol. $S_k = \frac{\frac{k-1}{k!}}{1-\frac{1}{k}} = \frac{1}{(k-1)!}$, for $k > 1$

$$\sum_{k=2}^{100} \left|(k^2 - 3k + 1)\frac{1}{(k-1)!}\right|$$

$$= \sum_{k=2}^{100} \left|\frac{(k-1)^2 - k}{(k-1)!}\right|$$

$$= \sum_{k=2}^{100} \left|\frac{k-1}{(k-2)!} - \frac{k}{(k-1)!}\right|$$

$$= \left|\frac{1}{0!} - \frac{2}{1!}\right| + \left|\frac{2}{1!} - \frac{3}{2!}\right| + \left|\frac{3}{2!} - \frac{4}{3!}\right| + \cdots$$

$$= \frac{2}{1!} - \frac{1}{0!} + \frac{2}{1!} - \frac{3}{2!} + \frac{3}{2!} - \frac{4}{3!} + \cdots + \frac{99}{98!} - \frac{100}{99!}$$

$$= 1 + \frac{2}{1!} - \frac{100}{99!}$$

$$= 3 - \frac{100}{99!}$$

Hence the value of $\frac{100^2}{100!} + \sum_{k=1}^{100} |(k^2 - 3k + 1)S_k|$

$= \frac{100}{99!} + 3 - \frac{100}{99!}$

$= 3$

47. Let $a_1, a_2, a_3, \ldots$ be a sequence of positive integers in arithmetic progression with common difference 2. Also, let $b_1, b_2, b_3, \ldots$ be a sequence of positive integers in geometric progression with common ratio 2. If $a_1 = b_1 = c$, then the number of all possible values of $\mathbf{c}$, for which the equality $2(a_1 + a_2 + \cdots + a_n) = b_1 + b_2 + \cdots + b_n$ holds for some positive integer $\mathbf{n}$, is

Ans. 1

Sol. $2 \cdot \frac{n}{2}[2C + (n-1)2] = C \cdot \frac{2^{n-1}}{2-1}$

$C(2n - 2^n + 1) = -2(n-1)n$

$C = \frac{2n-2n^2}{2n-2^n+1} \geq 1$

$\frac{2n^2-2n}{2^n-2n-1} \geq 1$

$$\frac{n-1}{\frac{2^n-1}{2n} - 1} \geq 1$$

On checking for $n = 1, 2, 3, \ldots$. $C = 12$ for $n = 3$ so $C = 12$ and only one value of c.

48. Find the sum

$$2017 + \frac{1}{4}\left(2016 + \frac{1}{4}\left(2015 + \cdots + \frac{1}{4}\left(2 + \frac{1}{4}(1)\right) \ldots\right)\right)$$

Sol. We have

$$S = 2017 + \frac{1}{4} \times 2016 + \frac{1}{4^2} \times 2015 + \cdots + \frac{1}{4^{2016}} \times 1$$

Clearly, this is A.G.P.

$$\therefore \frac{1}{4}S = \frac{1}{4} \times 2017 + \frac{1}{4^2} \times 2016 + \cdots + \frac{1}{4^{2016}} \times 2 + \frac{1}{4^{2017}} \times 1$$

Subtracting (2) from (1), we get

$$\frac{3}{4}S = 2017 - \frac{1}{4} - \frac{1}{4^2} - \cdots - \frac{1}{4^{2016}} - \frac{1}{4^{2017}}$$

$$\frac{3}{4}S = 2017 - \left(\frac{\frac{1}{4}\left(1 - \frac{1}{4^{2017}}\right)}{\left(1 - \frac{1}{4}\right)}\right)$$

$$S = \frac{4}{3}(2017) - \frac{4}{9}\left(1 - \frac{1}{4^{2017}}\right)$$

49. Find the sum $1 + 2\left(1 + \frac{1}{50}\right) + 3\left(1 + \frac{1}{50}\right)^2 + \cdots$ **50** terms

Sol. Let $n = 50$

Let S be the sum of n terms of the given series and $x = 1 + \frac{1}{n}$. Then,

$S = 1 + 2x + 3x^2 + 4x^3 + \cdots + nx^{n-1}$

$\Rightarrow xS = x + 2x^2 + 3x^3 + \cdots + (n-1)x^{n-1} + nx^n$

$\therefore S - xS = 1 + [x + x^2 + \cdots + x^{n-1}] - nx^n$

$\Rightarrow S(1-x) = \frac{1-x^n}{1-x} - nx^n$

$\Rightarrow S\left(-\frac{1}{n}\right) = -n\left[1 - \left(1 + \frac{1}{n}\right)^n\right] - n\left(1 + \frac{1}{n}\right)^n$

$\Rightarrow \frac{1}{n}S = n - n\left(1 + \frac{1}{n}\right)^n + n\left(1 + \frac{1}{n}\right)^n$

$\Rightarrow \frac{1}{n}S = n$

$\Rightarrow\ S = n^2 = (50)^2 = 2500$

50. Find the sum $1 + \frac{1}{1+2} + \frac{1}{1+2+3} + \cdots . + \frac{1}{1+2+3+\cdots+n}$

Sol. $T_r = \frac{1}{1+2+3+\cdots+r}$

$= \frac{2}{r(r+1)}$

$= 2\frac{(r+1)-r}{r(r+1)}$

$= 2\left(\frac{1}{r} - \frac{1}{r+1}\right)$

$= 2(V(r) - V(r+1))$, where $V(r) = \frac{1}{r}$

$\therefore \sum_{r=1}^{n} T_r = 2\sum_{r=1}^{n} (V(r) - V(r+1))$

$= 2(V(1) - V(n+1))$

$= 2\left(1 - \frac{1}{n+1}\right)$

$= \frac{2n}{n+1}$

51. Find the sum to n terms of the series $\frac{1}{1\times 3} + \frac{1}{3\times 5} + \frac{1}{5\times 7} + \cdots$

Sol. $T_r = \frac{1}{(2r-1)(2r+1)}$

$= \frac{1}{2}\left(\frac{1}{2r-1} - \frac{1}{2r+1}\right)$

$= \frac{1}{2}(V(r-1) - V(r))$, where $V(r) = \frac{1}{2r+1}$

$\therefore \sum_{r=1}^{n} T_r = \sum_{r=1}^{n} \frac{1}{2}(V(r-1) - V(r))$

$= \frac{1}{2}(V(0) - V(n))$

$= \frac{1}{2}\left(1 - \frac{1}{2n+1}\right)$

$= \frac{n}{2n+1}$

52. Find the sum to n terms of the series

$$\frac{1}{1+1^2+1^4} + \frac{2}{1+2^2+2^4} + \frac{3}{1+3^2+3^4} + \cdots$$

Sol. Here $T_r = \frac{r}{1+r^2+r^4}$

$= \frac{1}{2} \times \frac{(r^2+r+1)-(r^2-r+1)}{(r^2+r+1)(r^2-r+1)}$

$= \frac{1}{2}\left[\frac{1}{r^2-r+1} - \frac{1}{r^2+r+1}\right]$

$= \frac{1}{2}[V(r-1) - V(r)]$

$\therefore \sum_{r=1}^{n} T_r = \frac{1}{2}\sum_{r=1}^{n} (V(r-1) - V(r))$

$= \frac{1}{2}(V(0) - V(n))$

$= \frac{1}{2}\left[1 - \frac{1}{n^2+n+1}\right]$

$= \frac{n^2+n}{2(n^2+n+1)}$

53. Find the sum $\sum_{r=1}^{n} \frac{1}{r(r+1)(r+2)(r+3)}$.

Also, find $\sum_{r=1}^{\infty} \frac{1}{r(r+1)(r+2)(r+3)}$.

Sol. $T_r = \frac{1}{r(r+1)(r+2)(r+3)}$

$= \frac{r+3-r}{3(r(r+1)(r+2)(r+3))}$

$= \frac{1}{3}\left(\frac{1}{r(r+1)(r+2)} - \frac{1}{(r+1)(r+2)(r+3)}\right)$

$= \frac{1}{3}(V(r) - V(r+1))$, where $V(r) = \frac{1}{r(r+1)(r+2)}$

$\therefore$ Required sum $= \sum_{r=1}^{n} \frac{1}{3}(V(r) - V(r+1))$

$= \frac{1}{3}(V(1) - V(n+1))$

$= \frac{1}{3}\left(\frac{1}{6} - \frac{1}{(n+1)(n+2)(n+3)}\right)$

$\sum_{r=1}^{\infty} \frac{1}{r(r+1)(r+2)(r+3)}$

$= \lim_{n\to\infty} \frac{1}{3}\left(\frac{1}{6} - \frac{1}{(n+1)(n+2)(n+3)}\right) = \frac{1}{3} \times \frac{1}{6} = \frac{1}{18}$

54. Find the sum of the series $\sum_{r=1}^{99}\left(\frac{1}{r\sqrt{r+1}+(r+1)\sqrt{r}}\right)$.

Sol. $T_r = \frac{1}{\sqrt{r}\sqrt{r+1}[\sqrt{r}+\sqrt{r+1}]} = \frac{\sqrt{r+1}-\sqrt{r}}{\sqrt{r}\sqrt{r+1}}$

$= \frac{1}{\sqrt{r}} - \frac{1}{\sqrt{r+1}}$

$= V(r) - V(r+1)$, where $V(r) = \frac{1}{\sqrt{r}}$

$\therefore$ Required sum, $\sum_{r=1}^{99} (V(r) - V(r+1)) = V(1) - V(100)$

$= 1 - \frac{1}{\sqrt{100}}$

$= 1 - \frac{1}{10} = \frac{9}{10}$

55. Find the sum of the series $\frac{1}{3^2+1} + \frac{1}{4^2+2} + \frac{1}{5^2+3} + \frac{1}{6^2+4} + \cdots \infty$.

Sol. $T_r = \frac{1}{r^2+(r-2)} = \frac{1}{(r+2)(r-1)}$, where $n = 3, 4, 5, \ldots$

$= \frac{1}{3}\left[\frac{1}{r-1} - \frac{1}{r+2}\right]$

$= \frac{1}{3}[V(r) - V(r+3)]$, where $V(r) = \frac{1}{r-1}$

$\therefore$ Sum of $\mathbf{n}$ terms of the series,

$$\sum_{r=3}^{n} T_r = \sum_{r=3}^{n} \frac{1}{3}[V(r) - V(r+3)]$$

$= \frac{1}{3}[V(3) + V(4) + V(5) - V(n+1) - V(n+2) - V(n+3)]$

$= \frac{1}{3}\left[\frac{1}{2} + \frac{1}{3} + \frac{1}{4} - \frac{1}{n} - \frac{1}{n+1} - \frac{1}{n+2}\right]$

$\therefore \sum_{r=3}^{\infty} T_r = \frac{1}{3}\left[\frac{1}{2} + \frac{1}{3} + \frac{1}{4}\right] = \frac{13}{36}$

56. Find the sum of the series $\frac{2}{1\times2} + \frac{5}{2\times3} \times 2 + \frac{10}{3\times4} \times 2^2 + \frac{17}{4\times5} \times 2^3 + \cdots$ upto n terms.

Sol. $T_r = \frac{r^2+1}{r(r+1)} 2^{r-1}$

$= \frac{2r^2-(r^2-1)}{r(r+1)} 2^{r-1}$

$= \frac{175\%}{r+1} - \frac{r-1}{r}\Big) 2^{r-1} = \frac{r\cdot 2^r}{r+1} - \frac{(r-1)\cdot 2^{r-1}}{r}$

$= V(r) - V(r-1)$, where $V(r) = \frac{r\cdot 2^r}{r+1}$

$\therefore$ Required sum, $\sum_{r=1}^{n} T_r = \sum_{r=1}^{n} (V(r) - V(r-1))$

$= V(n) - V(0)$

$= \frac{n}{n+1} 2^n - 0 = \frac{n}{n+1} 2^n$

57. If $\sum_{r=1}^{n} T_r = \frac{n}{8}(n+1)(n+2)(n+3)$, then find $\sum_{r=1}^{n} \frac{1}{T_r}$.

Sol. $T_n = \sum_{r=1}^{n} T_r - \sum_{r=1}^{n-1} T_r$

$= \frac{n(n+1)(n+2)(n+3)}{8} - \frac{(n-1)n(n+1)(n+2)}{8}$

$= \frac{n(n+1)(n+2)}{2}$

$\therefore \frac{1}{T_r} = \frac{2}{r(r+1)(r+2)} = \frac{r+2-r}{r(r+1)(r+2)}$

$\therefore \sum_{r=1}^{n} \frac{1}{T_r} = \sum_{r=1}^{n} (V(r) - V(r+1))$

$= V(1) - V(n+1)$

$= \frac{1}{2} - \frac{1}{(n+1)(n+1)(r+2)} = \frac{1}{2(n+1)(n+2)}$

58. Let $S = \frac{\sqrt{1}}{1+\sqrt{1}+\sqrt{2}} + \frac{\sqrt{2}}{1+\sqrt{2}+\sqrt{3}} + \frac{\sqrt{3}}{1+\sqrt{3}+\sqrt{4}} + \cdots + \frac{\sqrt{n}}{1+\sqrt{n}+\sqrt{n+1}} = 10$

Then find the value of n.

Sol. $T_r = \frac{\sqrt{r}}{1+\sqrt{r}+\sqrt{r+1}} = \frac{\sqrt{r}\{1+\sqrt{r}-\sqrt{r+1}\}}{1+r+2\sqrt{r}-(r+1)}$

$= \frac{1}{2}\{1 + \sqrt{r} - \sqrt{r+1}\}$

$\therefore S_n = \frac{1}{2}(n + 1 - \sqrt{n+1}) = 10$

Let $\sqrt{n+1} = x$

$\therefore x^2 - x = 20$

$\Rightarrow x^2 - x - 20 = 0 \Rightarrow x = \sqrt{n+1} = 5$

$\therefore n = 24$

59. The sum $\sum_{r=2}^{\infty} \frac{1}{r^2-1}$ is equal to

(A) 1 (C) $4/3$ (B) $3/4$ (D) $3/2$

Ans. **(B)**

Sol. **Solution: Given data:** $\sum_{r=2}^{\infty} \frac{1}{r^2-1}$

The General term $(T) = \frac{1}{r^2-1}$

$= \frac{1}{(\sigma+1)(r-1)} \cdots \left[\begin{matrix} a^2 - b^2 = (a+b) \\ (4-b) \end{matrix}\right]$

$= \frac{(r+1)-(r-1)}{(r+1)(r-1)} = \frac{1}{2}\left[\frac{1}{r-1} - \frac{01}{r+1}\right]$

$\therefore \sum_{r=2}^{\infty} T = \frac{1}{2}\left[\frac{1}{1} - \frac{1}{3} + \frac{1}{2} - \frac{1}{4} + \frac{1}{3} - \frac{1}{5} - \frac{1}{6} + \cdots \infty\right]$

$= \frac{1}{2}\left[1 + \frac{1}{2}\right] = \frac{1}{2}\left[\frac{3}{2}\right]$

$\therefore \sum_{r2} \frac{1}{r^2 - 1} = \frac{3}{4}$

60. Sum of the series $\mathbf{S = 1^2 - 2^2 + 3^2 - 4^2 + \cdots .. - 2002^2 + 2003^2}$ is

(A) 2007006 (B) 1005004

(C) 2000506 (D) 200700

Ans. (A)

Sol. $\mathbf{S = 1^2 - 2^2 + 3^2 - 4^2 + \cdots \ldots - 2002^2 + 2003^2}$

$\Rightarrow \mathbf{S = \sum_{k=1}^{2003} (-1)^{k+1} k^2}$

$\Rightarrow \mathbf{S = \sum_{k=1}^{1002} (2k-1)^2 - \sum_{k=1}^{1001} (2k)^2}$

$\Rightarrow \mathbf{S = [(2 \times 1002) - 1]^2 + \sum_{k=1}^{1001} [(2k)^2 - (2k)^2 - 4k + 1]}$

$\Rightarrow \mathbf{S = (2003)^2 - \sum_{k=1}^{1001} (4k-1) = (2003)^2 - 4 \times \frac{1001(1001+1)}{2} + 1001 = 2007006}$

61. If $\mathbf{1^2 + 2^2 + \cdots \ldots + n^2 = 1015}$, then value of $\boldsymbol{n}$ is

(A) 15 (B) 14 (C) 13 (D) 12

Ans. (B)

Sol. Given $\mathbf{1^2 + 2^2 + \cdots + n^2 = 1015}$

we know $\mathbf{1^2 + 2^2 + \cdots + n^2 = \frac{n(n-1)(2n-1)}{6}}$.

$\mathbf{n \frac{n-1)(2n-1)}{6} = 1015}$

$\mathbf{2n^3 - 3n^2 + n - 6090 = 0}$

$\mathbf{(n-15)(2n^2 + 27n + 400) = 0}$

Hence $\boldsymbol{n} = \mathbf{15}$& other roots are imaginary

$\therefore \ [\boldsymbol{n} = \mathbf{15}]$

62. If $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \cdots$ upto $\infty = \frac{\pi^2}{6}$, then $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots .. =$

(A) $\boldsymbol{\pi^2/12}$ (B) $\boldsymbol{\pi^2/24}$ (C) $\boldsymbol{\pi^2/8}$ (D) $\boldsymbol{\pi^2/4}$

Ans. (C)

Sol. We have $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots ..$ upto ∞

$= \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \frac{1}{5^2} + \frac{1}{6^2} \ldots ..$ upto ∞

$-\frac{1}{2^2}\left[1 + \frac{1}{2^2} + \frac{1}{3^2} + \cdots ..\right] = \frac{\pi^2}{6} - \frac{1}{4}\left(\frac{\pi^2}{6}\right) = \frac{\pi^2}{8}$