Nishant Jindal

# Trigonometry

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4) $$\frac{\sin 7A - \sin A}{\sin 8A - \sin 2A} = \frac{\cancel{2}\cos(4A)\,\cancel{\sin(3A)}}{\cancel{2}\cos(5A)\,\cancel{\sin(3A)}}$$

$$= \cos 4A \sec 5A$$

(5) $$\frac{\cos 2B + \cos 2A}{\cos 2B - \cos 2A} = \frac{\cancel{2}\cos(B+A)\cos(B-A)}{-\cancel{2}\sin(\underset{D}{B-A})\sin(\underset{S}{B+A})} = \cot(B-A)\cdot\cot(B+A)$$

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# Trigonometry

S L Loney

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7) $\dfrac{\sin A + \sin 2A}{\cos A - \cos 2A} = \dfrac{\cancel{2}\sin\left(\frac{3A}{2}\right)\cos\left(+\frac{A}{2}\right)}{+\cancel{2}\sin\left(\frac{3A}{2}\right)\sin\left(+\frac{A}{2}\right)} = \cot\frac{A}{2}$

Q10 $\cos(A+B) + \sin(A-B) = 2\sin(45^\circ + A)\cdot\cos(45^\circ + B)$ Copy

$\sin\left(\frac{\pi}{2} - (A+B)\right) + \sin(A-B) = 2\sin\left(\quad\right)\cdot\sin\left(\quad\right)$

11 $\dfrac{\cos 3A - \cos A}{\sin 3A - \sin A} + \dfrac{\cos 2A - \cos 4A}{\sin 4A - \sin 2A} = \dfrac{-\cancel{2}\sin(2A)\,\cancel{\sin(A)}}{\cancel{2}\cos(2A)\,\cancel{\sin(A)}} + \dfrac{+\cancel{2}\sin(3A)\,\cancel{\sin(A)}}{\cancel{2}\cos(3A)\,\cancel{\sin(A)}}$

$\dfrac{\sin 3A}{\cos 3A} - \dfrac{\sin 2A}{\cos 2A} = \dfrac{\sin 3A\cos 2A - \sin 2A\cos 3A}{\cos 3A\cdot\cos 2A} = \dfrac{\sin(3A-2A)}{\cos 2A\cos 3A} = \dfrac{\sin A}{\cos 3A\cos 2A}$

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# Trigonometry

(13) $\dfrac{\tan 5\theta + \tan 3\theta}{\tan 5\theta - \tan 3\theta} = 4\cos 2\theta \cdot \cos 4\theta$

Thodi der K Liye

★★ $\sin 2\theta = 2\sin\theta \cdot \cos\theta$

$\sin 4\theta = 2\sin 2\theta \cos 2\theta$

$\sin 8\theta = 2\sin 4\theta \cdot \cos 4\theta$

$\sin 16\theta = 2\sin 8\theta \cos 8\theta$

$$\dfrac{\dfrac{\sin 5\theta}{\cos 5\theta} + \dfrac{\sin 3\theta}{\cos 3\theta}}{\dfrac{\sin 5\theta}{\cos 5\theta} - \dfrac{\sin 3\theta}{\cos 3\theta}} = \dfrac{\dfrac{\sin 5\theta \cdot \cos 3\theta + \sin 3\theta \cdot \cos 5\theta}{\cancel{\cos 3\theta \cos 5\theta}}}{\dfrac{\sin 5\theta \cdot \cos 3\theta - \sin 3\theta \cos 5\theta}{\cancel{\cos 3\theta \cos 5\theta}}} = \dfrac{\sin(5\theta + 3\theta)}{\sin(5\theta - 3\theta)}$$

$$= \dfrac{\sin 8\theta}{\sin 2\theta} = \dfrac{2(\sin 4\theta)\cdot \cos(4\theta)}{\sin 2\theta}$$

$$= \dfrac{2(2\cancel{\sin 2\theta}\cos 2\theta)\cdot \cos 4\theta}{\cancel{\sin 2\theta}}$$

$= 4\cos 2\theta \cdot \cos 4\theta$ RHS

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# Trigonometry

Q 15 Copy

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$$\frac{\sin(\theta+\phi) - 2\sin\theta + \sin(\theta-\phi)}{\cos(\theta+\phi) - 2\cos\theta + \cos(\theta-\phi)} = \tan\theta$$

$$\frac{\{\sin(\theta+\phi) + \sin(\theta-\phi)\} - 2\sin\theta}{\{\cos(\theta+\phi) + \cos(\theta-\phi)\} - 2\cos\theta}$$

$$\frac{2\sin\left(\frac{\theta+\cancel{\phi}+\theta-\cancel{\phi}}{2}\right)\cos\left(\frac{\cancel{\theta}+\phi-\cancel{\theta}+\phi}{2}\right) - 2\sin\theta}{2\cos\left(\frac{\theta+\cancel{\phi}+\theta-\cancel{\phi}}{2}\right)\cos\left(\frac{\cancel{\theta}+\phi-\cancel{\theta}+\phi}{2}\right) - 2\cos\theta}$$

$$\frac{2\sin\theta\cdot\cos(\phi) - 2\sin\theta}{2\cos\theta\cdot\cos(\phi) - 2\cos\theta} = \frac{\cancel{2}\sin\theta(\cancel{\cos\phi-1})}{\cancel{2}\cos\theta(\cancel{\cos\phi-1})} = \tan\theta$$

17 Copy

19 CCMM

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# Trigonometry

24) $$\frac{\cos(A+B+C)+\cos(-A+B+C)+\cos(A-B+C)+\cos(A+B-C)}{\sin(A+B+C)+\sin(-A+B+C)-\sin(A-B+C)+\sin(A+B-C)} = \cot B$$

$$\frac{2\cos\left(\frac{A+B+\cancel{C}+A+B-\cancel{C}}{2}\right)\cos\left(\frac{\cancel{A}+\cancel{B}+C-\cancel{A}-\cancel{B}+C}{2}\right)+2\cos\left(\frac{-\cancel{A}+\cancel{B}+C+\cancel{A}-\cancel{B}+C}{2}\right)\cdot\cos\left(\frac{-A+B+\cancel{C}-A+B-\cancel{C}}{2}\right)}{2\sin\left(\frac{A+B+\cancel{C}+A+B-\cancel{C}}{2}\right)\cos\left(\frac{\cancel{A}+\cancel{B}+C-\cancel{A}-\cancel{B}+C}{2}\right)+2\cos\left(\frac{-\cancel{A}+\cancel{B}+C+\cancel{A}-\cancel{B}+C}{2}\right)\sin\left(\frac{-A+B+\cancel{C}-A+B-\cancel{C}}{2}\right)}$$

$$\frac{\cos(A+B)\cos C+\cos(B-A)\cdot\cos C}{\sin(A+B)\cos C+\cos C\sin(B-A)} = \frac{\cos(A+B)+\cos(B-A)}{\sin(A+B)+\sin(B-A)}$$

$$\cot B$$

$$\frac{2\cos\left(\frac{\cancel{A}+B+B-\cancel{A}}{2}\right)\cos\left(\frac{A+\cancel{B}-\cancel{B}+A}{2}\right)}{2\sin\left(\frac{\cancel{A}+B+B-\cancel{A}}{2}\right)\cos\left(\frac{A+\cancel{B}-\cancel{B}+A}{2}\right)} \quad \frac{\cancel{2}\cos B\,\cancel{\cos A}}{\cancel{2}\sin B\,\cancel{\cos A}}$$

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# Trigonometry

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Q27 $\underline{\sin 50^\circ - \sin 70^\circ} + \sin 10^\circ = 0$

$2\cos(60^\circ)\sin(-10^\circ) + \sin 10^\circ$

$\cancel{2} \times \frac{1}{\cancel{2}} \times (-\sin 10^\circ) + \sin 10^\circ$

$-\sin 10^\circ + \sin 10^\circ = 0$

Q28 $\sin 10^\circ + \sin 20^\circ + \sin 40^\circ + \sin 50^\circ = \sin 70^\circ + \sin 80^\circ$

LHS $2\sin(30^\circ)\cos(-20^\circ) + 2\sin(30^\circ)\cos(-10^\circ)$

$\cancel{2} \times \frac{1}{\cancel{2}} \cos 20^\circ + \cancel{2} \times \frac{1}{\cancel{2}} \times \cos(10^\circ) = \underline{\cos 20^\circ + \cos 10^\circ}$ Comp.

$= \underline{\sin 70^\circ + \sin 80^\circ}$

RHS

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# S.L. Loney EX. XIV

31 $\sin(\theta + (n-\tfrac{1}{2})\phi) + \sin(\theta + (n+\tfrac{1}{2})\phi) = $

$$2\sin\left(\frac{\theta + n\phi - \cancel{\tfrac{1}{2}\phi} + \theta + n\phi + \cancel{\tfrac{1}{2}\phi}}{2}\right)\cos\left(\frac{\overbrace{\cancel{\theta} + \cancel{n\phi} - \tfrac{1}{2}\phi - \cancel{\theta} - \cancel{n\phi} - \tfrac{1}{2}\phi}^{-\frac{\phi}{2}}}{2}\right)$$

$$2\sin(\theta + n\phi)\cdot\cos\left(+\frac{\phi}{2}\right)$$

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Sum/Diff → Prod & Prod given → Sum/diff

$$\sin C + \sin D = 2\sin\left(\frac{C+D}{2}\right)\cdot\cos\left(\frac{C-D}{2}\right)$$

$$\sin C - \sin D = 2\cos\left(\frac{C+D}{2}\right)\cdot\sin\left(\frac{C-D}{2}\right)$$

$$\cos C + \cos D = 2\cos\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)$$

$$\cos C - \cos D = -2\sin\left(\frac{C+D}{2}\right)\cdot\sin\left(\frac{C-D}{2}\right)$$

$$2\sin(A)\cdot\cos(B) = \sin(A+B) + \sin(A-B)$$

$$2\cos(A)\cdot\sin(B) = \sin(A+B) - \sin(A-B)$$

$$2\cos(A)\cdot\cos(B) = \cos(A+B) + \cos(A-B)$$

$$2\sin(A)\cdot\sin(B) = \cos(A-B) - \cos(A+B)$$

Prod → Sum/Diff

Q $2\cos\theta\cdot\sin\phi = ?$

$$\sin(\theta+\phi) - \sin(\theta-\phi)$$

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Q $\dfrac{\sin A \cdot \sin 2A + \sin 3A \cdot \sin 6A + \sin 4A \cdot \sin 13A}{\sin A \cdot \cos 2A + \sin 3A \cos 6A + \sin 4A \cos 13A} = ?$

$\dfrac{2\sin A \sin 2A + 2\sin 3A \cdot \sin 6A + 2\sin 4A \sin 13A}{2\sin A \cos 2A + 2\sin 3A \cos 6A + 2\sin 4A \cos 13A}$

$\dfrac{\{\cos(-A) - \cancel{\cos(3A)}\} + \{\cancel{\cos(-3A)} - \cancel{\cos(9A)}\} + \{\cos(\cancel{-9A}) - \cos(17A)\}}{\{\sin(3A) + \sin(-A)\} + \{\sin(9A) + \sin(-3A)\} + \{\sin(17A) + \sin(-9A)\}}$

$\dfrac{\cos A - \cos 17A}{\cancel{\sin 3A} - \sin A + \cancel{\sin 9A} - \cancel{\sin 3A} + \sin 17A - \cancel{\sin 9A}} = \dfrac{\cos A - \cos 17A}{\sin 17A - \sin A}$

$+\dfrac{\cancel{2}\sin\left(\cancel{\frac{+16A}{2}}\right)\sin\left(\frac{18A}{2}\right)}{\cancel{2}\cos\left(\frac{18A}{2}\right)\sin\left(\cancel{\frac{16A}{2}}\right)}$

$= \tan 9A$

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$\cos\cancel{\frac{10\pi}{13}} + \cos\cancel{\frac{8\pi}{13}} \quad - \cos\cancel{\frac{10\pi}{13}} - \cos\cancel{\frac{8\pi}{13}} = \underline{0\ 0}$

Q $2\cos\frac{\pi}{13}\cdot\cos\frac{9\pi}{13}$

$= \cos\left(\frac{\pi}{13}+\frac{9\pi}{13}\right) + \cos\left(\frac{\pi}{3}-\frac{9\pi}{13}\right)^{\cos\left(+\frac{8\pi}{13}\right)}$

$= \cos\left(\frac{10\pi}{13}\right) + \cos\left(\frac{8\pi}{13}\right)$ $\qquad 3 < \frac{13}{2} \Rightarrow 3 < 6.5$

Q $2\cos\frac{\pi}{13}\cdot\cos\frac{9\pi}{13} + \cos\frac{3\pi}{13} + \cos\frac{5\pi}{13} = ?$

$\cos\frac{10\pi}{13} + \cos\left(\frac{8\pi}{13}\right) + \cos\frac{3\pi}{13} + \cos\frac{5\pi}{13}$

$\quad " \qquad " \quad + \cos\left(\frac{13\pi-10\pi}{13}\right) + \cos\left(\frac{13\pi-8\pi}{13}\right)$

$\cos\frac{10\pi}{13} + \cos\left(\frac{8\pi}{13}\right) + \cos\left(\boxed{\pi}-\frac{10\pi}{13}\right) + \cos\left(\pi-\frac{8\pi}{13}\right)$

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# Trigonometry

Q P.T.

$\sin\alpha + \sin\left(\alpha+\frac{2\pi}{3}\right) + \sin\left(\alpha+\frac{4\pi}{3}\right) = 0$

$2\sin\left(\alpha+\frac{2\pi}{3}\right)\cdot\cos\left(+\frac{2\pi}{3}\right) + \sin\left(\alpha+\frac{2\pi}{3}\right)$

$\cancel{2}\sin\left(\alpha+\frac{2\pi}{3}\right)\cdot \times -\frac{1}{\cancel{2}} + \sin\left(\alpha+\frac{2\pi}{3}\right)$

$-\cancel{\sin\left(\alpha+\frac{2\pi}{3}\right)} + \cancel{\sin\left(\alpha+\frac{2\pi}{3}\right)} = 0$

$\cos 120^\circ = \cos(90 + 30^\circ)$

$= -\sin 30^\circ$

$= -1/2$

Q $\cos\alpha + \cos\left(\alpha+\frac{2\pi}{3}\right) + \cos\left(\alpha+\frac{4\pi}{3}\right) = 0$ (heck

$\cancel{2}\cos\left(\alpha+\frac{2\pi}{3}\right)\cos\left(+\frac{2\pi}{3}\right) + \cos\left(\alpha+\frac{2\pi}{3}\right)$

$-\cancel{\cos\left(\alpha+\frac{2\pi}{3}\right)} + \cancel{\cos\left(\alpha+\frac{2\pi}{3}\right)} = 0$

# Trigonometry

Q If $a \sin\theta = b\sin\left(\theta + \frac{2\pi}{2}\right) = c\sin\left(\theta+\frac{4\pi}{3}\right)$

then $ab + bc + cA = ?$

$a\sin\theta = b\sin\left(\theta+\frac{2\pi}{3}\right) = c\sin\left(\theta+\frac{4\pi}{3}\right) = K$ let

$\sin\theta = \frac{K}{a}$ | $\sin\left(\theta+\frac{2\pi}{3}\right) = \frac{K}{b}$ | $\sin\left(\theta+\frac{4\pi}{3}\right) = \frac{K}{c}$

from Previous QS. $\rightarrow \sin\theta + \sin\left(\theta+\frac{2\pi}{3}\right) + \sin\left(\theta+\frac{4\pi}{3}\right) = 0$

$$\frac{K}{a} + \frac{K}{b} + \frac{K}{c} = 0$$

$$\Rightarrow \frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 0 \Rightarrow \frac{ab+bc+ca}{abc} = 0$$

$$\boxed{ab+bc+ca = 0}$$

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Q $x\cos\theta = y\cos\left(\theta+\frac{2\pi}{3}\right) = z\cos\left(\theta+\frac{4\pi}{3}\right)$

then $xy+yz+zx = ?$

$x\cos\theta = y\cos\left(\theta+\frac{2\pi}{3}\right) = z\cos\left(\theta+\frac{4\pi}{3}\right) = K$ let

$\cos\theta = \frac{K}{x}$ | $\cos\left(\theta+\frac{2\pi}{3}\right) = \frac{K}{y}$ | $\cos\left(\theta+\frac{4\pi}{3}\right) = \frac{K}{z}$.

$\cos\theta + \cos\left(\theta+\frac{2\pi}{3}\right) + \cos\left(\theta+\frac{4\pi}{3}\right) = 0$

$\frac{K}{x}+\frac{K}{y}+\frac{K}{z} = 0 \Rightarrow \frac{1}{x}+\frac{1}{y}+\frac{1}{z} = 0$

$\Rightarrow \frac{xy+yz+zx}{xyz} = 0 \Rightarrow xy+yz+zx = 0$

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# Trigonometry

$2\theta+\cancel{\phi}-\cancel{\phi}$

Q If $\boxed{y\sin\phi = x\sin(2\theta+\phi)}$ then S.T. $(x+y)\cdot\underline{\cot(\theta+\phi)} = (y-x)\cot\theta$

$$\frac{y}{x} = \frac{\sin(2\theta+\phi)}{\sin\phi}$$

Ratio / Ratio → C & D

$$\frac{y+x}{y-x} = \frac{\sin(2\theta+\phi)+\sin\phi}{\sin(2\theta+\phi)-\sin\phi} = \frac{\cancel{2}\sin(\theta+\phi)\cdot\cos(\theta)}{\cancel{2}\cos(\theta+\phi)\sin(\theta)}$$

$$\frac{y+x}{y-x} = \frac{\cot\theta}{\cot(\theta+\phi)}$$

$$(y+x)\cot(\theta+\phi) = (y-x)\cot\theta$$

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# Trigonometry

Q If $\tan(A+B) = 3\tan A$ then $\sin(2A+B) = ?$

$$\frac{\tan(A+B)}{\tan A} = \frac{3}{1}$$

$$\frac{\sin(A+B)\cos A}{\cos(A+B)\cdot\sin A} = \frac{3}{1} \quad \text{C \& D}$$

$$\frac{\sin(A+B)\cos A + \cos(A+B)\sin A}{\sin(A+B)\cos A - \cos(A+B)\sin A} = \frac{3+1}{3-1}$$

$$\frac{\sin\{(A+B)+A\}}{\sin\{(\cancel{A}+B)-\cancel{A}\}} = \frac{\cancel{4}}{\cancel{2}}2 \Rightarrow \sin(2A+B) = \boxed{2\sin B}$$

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# Trigonometry

Q If $\tan(\alpha+\theta) = n\tan(\alpha-\theta)$ then $\frac{\sin 2\alpha}{\sin 2\theta} = ?$

HW
S L Loney Ex 15

$$\frac{\tan(\alpha+\theta)}{\tan(\alpha-\theta)} = \frac{n}{1}$$

$$\frac{\sin(\alpha+\theta)\,\cos(\alpha-\theta)}{\cos(\alpha+\theta)\cdot\sin(\alpha-\theta)} = \frac{n}{1} \quad C\&D$$

$$\frac{\sin(\alpha+\theta)\cdot\cos(\alpha-\theta)+\cos(\alpha+\theta)\cdot\sin(\alpha-\theta)}{\sin(\alpha+\theta)\cdot\cos(\alpha-\theta)-\cos(\alpha+\theta)\cdot\sin(\alpha-\theta)} = \frac{n+1}{n-1}$$

$$\frac{\sin\{(\alpha+\cancel{\theta})+(\alpha-\cancel{\theta})\}}{\sin\{(\cancel{\alpha}+\theta)-(\cancel{\alpha}-\theta)\}} = \frac{n+1}{n-1} \Rightarrow \frac{\sin 2\alpha}{\sin 2\theta} = \frac{n+1}{n-1}$$