Nishant Jindal

$$\frac{a}{r} + a + ar = 70$$

$$4\frac{a}{r} + 4ar = 2(5a)$$

$$4(70-a) = 10a$$

$$ar(r^2-1)$$

$$(ar-ar^2)^2 + (ar^2-a)^2 + (ar^3-ar)^2$$

$$= a^2(1-r)^2\left[r^2 + (r^2+2r+1) + r^2(r^2+2r+1)\right]$$

$$a^2(1-r)^2\left(r^4 + \boxed{2r^3} + 3r^2 + 2r + 1\right)$$

$$a^2(1-r)^2(r^2+r+1)^2$$

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$$r^m < \frac{1-r^{2m+1}}{(1-r)(2m+1)}$$

$$\frac{1+r+r^2+r^3+\cdots+r^{2m}}{2m+1} > \left(1\cdot r\cdot r^2\cdots r^{2m}\right)^{\frac{1}{2m+1}}$$

$$= r^m.$$

$$0<r<1$$

Nishant Jindal

Harmonic Progression

$T_1, T_2, T_3, T_4, \ldots, T_n \longrightarrow$ H.P.

$\Rightarrow \frac{1}{T_1}, \frac{1}{T_2}, \frac{1}{T_3}, \frac{1}{T_4}, \ldots, \frac{1}{T_n}$ are in A.P.

$$\frac{1}{a}, \frac{1}{a+d}, \frac{1}{a+2d}, \frac{1}{a+3d}, \ldots \frac{1}{a+(n-1)d} \longrightarrow \text{H.P.}$$

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Harmonic Mean of 2 numbers a, b.

$H = $ HM of a, b.

$$\boxed{a, H, b \text{ are in H.P.}}$$

$\frac{1}{a}, \frac{1}{H}, \frac{1}{b} \rightarrow$ A.P.

$$\frac{2}{H} = \frac{1}{a} + \frac{1}{b}$$

$$\boxed{H = \frac{2ab}{a+b}}$$

HM of 'n' no.s $a_1, a_2, \ldots, a_n$

$H = $ HM of $a_1, a_2, \ldots, a_n$

$$\boxed{H = \frac{n}{\left(\frac{1}{a_1} + \frac{1}{a_2} + \frac{1}{a_3} + \cdots + \frac{1}{a_n}\right)}}$$

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Insert 'n' HMs between two no.s a, b

$$\boxed{a, H_1, H_2, H_3, \ldots, H_n, b \text{ form HP}}$$

$$\Rightarrow \frac{1}{a}, \frac{1}{H_1}, \frac{1}{H_2}, \frac{1}{H_3}, \ldots, \frac{1}{H_n}, \frac{1}{b} \Rightarrow \underline{\underline{A \cdot P}}$$

$$\frac{1}{H_r} = \frac{1}{a} + r\left(\frac{\frac{1}{b} - \frac{1}{a}}{n+1}\right)$$

Nishant Jindal

$$AM \geq GM \geq HM$$

$$\frac{x_1+x_2+\cdots+x_n}{n} \geq (x_1x_2x_3\cdots x_n)^{\frac{1}{n}} \geq \frac{n}{\frac{1}{x_1}+\frac{1}{x_2}+\cdots\frac{1}{x_n}}$$

where $x_1, x_2, \ldots, x_n > 0$

Equality holds if $x_1 = x_2 = x_3 = \cdots = x_n$.

$$\frac{n}{\sum \frac{1}{x_i}} \leq (x_1x_2\cdots x_n)^{\frac{1}{n}} \Leftarrow \frac{\frac{1}{x_1}+\frac{1}{x_2}+\frac{1}{x_3}+\cdots+\frac{1}{x_n}}{n} \geq \left(\frac{1}{x_1}\frac{1}{x_2}\cdots\frac{1}{x_n}\right)^{\frac{1}{n}}$$

Nishant Jindal

1. If the 3^{rd}, 6^{th} and last terms of an H.P. are $\frac{1}{3}, \frac{1}{5}, \frac{3}{203}$ respectively. Find the number of terms.

$\frac{1}{a}, \frac{1}{a+d}, \frac{1}{a+2d}, \ldots$

$a+2d=3$

$a+5d=5$

$d=\frac{2}{3}, \; a=\frac{5}{3}$

$a+(n-1)d=\frac{203}{3}=\frac{5}{3}+(n-1)\frac{2}{3}$

$\Rightarrow \boxed{n=100}$

2. If m^{th} term of an H.P. is n and n^{th} term is 'm', then P.T. its $(m+n)^{th}$ term is $\frac{mn}{m+n}$.

$a+(m-1)d=\frac{1}{n}$

$a+(n-1)d=\frac{1}{m}$

$\Rightarrow (m-n)d=\frac{1}{n}-\frac{1}{m} \Rightarrow \boxed{d=\frac{1}{mn}}$

$\boxed{a=\frac{1}{mn}}$

$T_{m+n}=\frac{1}{a+(m+n-1)d}=\frac{mn}{1+m+n-1}$

$=\frac{mn}{m+n}$

Nishant Jindal

3. If $a_1, a_2, a_3, \cdots, a_n$ are in H.P., then P.T.

$$a_1 a_2 + a_2 a_3 + a_3 a_4 + \cdots + a_{n-1} a_n = (n-1) a_1 a_n.$$

$\frac{1}{a_1}, \frac{1}{a_2}, \frac{1}{a_3}, \cdots, \frac{1}{a_n} \rightarrow$ A.P.

$$\frac{1}{a_n} = \frac{1}{a_1} + (n-1)d$$

$$\Rightarrow \frac{1}{a_n} - \frac{1}{a_1} = (n-1)d$$

$$\frac{a_1 - a_n}{d} = (n-1)a_1 a_n$$

$$\boxed{d = \frac{1}{a_2} - \frac{1}{a_1}} = \frac{1}{a_3} - \frac{1}{a_2} = \frac{1}{a_4} - \frac{1}{a_3} = \cdots = \frac{1}{a_n} - \frac{1}{a_{n-1}}$$

$$a_1 a_2 = \frac{a_1 - a_2}{d}, \quad a_2 a_3 = \frac{a_2 - a_3}{d}, \quad \cdots, \quad a_n a_{n-1} = \frac{a_{n-1} - a_n}{d}$$

$$a_1 a_2 + \cdots + a_{n-1} a_n = \frac{a_1 - a_2}{d} + \frac{a_2 - a_3}{d} + \frac{a_3 - a_4}{d} + \cdots + \frac{a_{n-1} - a_n}{d} = \frac{a_1 - a_n}{d}$$

$$= (n-1) a_1 a_n.$$

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4. If 'a' is the AM of b, c ; 'b' is the G.M of a, c , then P.T. c is the HM of a & b .

$$a = \frac{b+c}{2}$$

$$c = \frac{2ab}{a+b}$$

$$b^2 = ac$$

$$2ab = b^2 + bc$$

$$2ab = ac + bc$$

$$\Rightarrow c = \frac{2ab}{a+b}$$

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5. If a, b, c are in A.P., p, q, r are in H.P and ap, bq, cr are in G.P., then P.T.

$$\frac{p}{r}+\frac{r}{p}=\frac{a}{c}+\frac{c}{a}$$

$$b^2q^2 = apcr$$

$$\left(\frac{a+c}{\cancel{2}}\right)^2\left(\frac{\cancel{2}pr}{p+r}\right)^2 = apcr$$

$$\frac{(a+c)^2}{ac} = \frac{(p+r)^2}{pr} \Rightarrow \frac{a}{c}+\frac{c}{a}=\frac{p}{r}+\frac{r}{p}$$

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6. If a, b, c are 3 distinct positive real numbers in H.P., then P.T. $a^n + c^n > 2b^n$, $n \in N$.

$$\frac{a^n + c^n}{2} > \sqrt{a^n c^n} = \left(\sqrt{ac}\right)^n > b^n$$

$$\sqrt{ac} > b$$

$$\left(\sqrt{ac}\right)^n > b^n$$

Nishant Jindal

Method of Difference → (diff. of consecutive terms are in A.P. or G.P.)

$$S = T_1 + T_2 + T_3 + T_4 + T_5 + \cdots + T_n \quad (1)$$

$$S = \quad T_1 + T_2 + T_3 + T_4 + \cdots + T_{n-1} + T_n \quad (2)$$

(1) − (2)

$$0 = T_1 + (T_2 - T_1) + (T_3 - T_2) + (T_4 - T_3) + \cdots + (T_n - T_{n-1}) - T_n$$

$$T_n = T_1 + \underbrace{\left[(T_2 - T_1) + (T_3 - T_2) + \cdots + (T_n - T_{n-1})\right]}$$

$$S = \sum_{r=1}^{n} T_r$$

Nishant Jindal

1. $S = 3 + 8 + 15 + 24 + 35 + \dots + T_n$ ①

$S = \quad 3 + 8 + 15 + 24 + \dots + T_{n-1} + T_n$ ②

① − ② $\quad 0 = \underbrace{(3 + 5 + 7 + 9 + 11 + \dots +)}_{n\text{ terms}} - T_n$

$\sum_{r=1}^{n} \left((r+1)^2 - 1\right)$

$\Rightarrow T_n = 3 + 5 + 7 + 9 + \dots + n\text{ term.}$

$= \frac{n}{2}\left(6 + (n-1)2\right) = n(3 + n - 1)$

$= n(n+2)$

$S = \sum_{r=1}^{n} r(r+2) = \frac{n(n+1)(2n+1)}{6} + 2\frac{n(n+1)}{2}$

$= \frac{n(n+1)}{6}\left[2n + 1 + 3\right]$

$\frac{n(n+1)(n+2)}{3} =$

2.

$$S = 5 + 7 + 13 + 31 + 85 + \cdots + T_n$$

$$S = \quad 5 + 7 + 13 + 31 + \cdots + T_{n-1} + T_n$$

$$T_n = 5 + (\underbrace{2 + 6 + 18 + 54 + \cdots}_{n-1 \text{ terms}}) = 5 + \frac{\cancel{2}(3^{n-1} - 1)}{\cancel{(3-1)}}$$

$$T_n = 3^{n-1} + 4.$$

$$S = \sum_{r=1}^{n} (3^{r-1} + 4) = \frac{1(3^n - 1)}{3-1} + 4n$$

$$= \frac{3^n - 1}{2} + 4n.$$

3. $1+\left(1+\frac{1}{3}\right)+\left(1+\frac{1}{3}+\frac{1}{3^2}\right)+\left(1+\underbrace{\frac{1}{3}+\frac{1}{3^2}+\frac{1}{3^3}}\right)+\dots+$ n terms.

$$= \sum_{r=1}^{n}\left(1+\frac{1}{3}+\frac{1}{3^2}+\frac{1}{3^3}+\dots\frac{1}{3^{r-1}}\right) = \sum_{r=1}^{n}\frac{1\left(1-\frac{1}{3^r}\right)}{1-\frac{1}{3}}$$

$$= \frac{3}{2}\sum_{r=1}^{n}\left(1-\frac{1}{3^r}\right)$$

$$= \frac{3}{2}\left(n-\frac{\frac{1}{3}\left(1-\frac{1}{3^n}\right)}{1-\frac{1}{3}}\right)$$

$$= \frac{3}{2}\left(n-\frac{1}{2}\left(1-\frac{1}{3^n}\right)\right)$$

Ex-6(a) → 6, 8, 9, 14, 15, 16, 17, 18, 20, 21