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PERMUTATION & COMBINATION

Q. A student solves the equation $^{n}C_2 = 10$ using the following steps, but finds the solution yields decimal answer and therefore he must not be correct which step did he make the mistake?

$$^{n}C_r = \frac{\lfloor n}{\lfloor r \lfloor n-r} \Rightarrow {}^{n}C_2 = \frac{\lfloor n}{\lfloor 2 \lfloor n-2}$$

(A) Step-1: $\frac{n!}{(n-2)!} = 10$ $\times$

(B) Step-2: $n! = 10(n-2)!$

(C) Step-3: $n(n-1)(n-2)! = 10(n-2)!$

(D) Step-4: $n(n-1) = 10 \Rightarrow n^2 - n - 10 = 0$

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Q. For some natural N, the number of positive integral ' x ' satisfying the equation,

$1! + 2! + 3! + \cdots \ldots + (x!) = (N)^2$ is

(A) none

(B) one

(C) two ✓

(D) infinite

$\lfloor 1 = 1$

$\lfloor 2 = 2$

$\lfloor 3 = 6$

$\lfloor 4 = 24$

$\lfloor 5 = 120$

$\lfloor 6 = 720$ ✓

$\lfloor 7 = 5040$

$x = 1 \quad \lfloor 1 = 1 = (N)^2$ ✓

$x = 2 \quad \lfloor 1 + \lfloor 2 = 3 = (N)^2$ ✗

$x = 3 \quad \lfloor 1 + \lfloor 2 + \lfloor 3 = 9 = (N)^2$ ✓

$x = 4 \quad \lfloor 1 + \lfloor 2 + \lfloor 3 + \lfloor 4 = \boxed{33}\ (N)^2$ ✗

$x = 5 \quad \underline{\lfloor 1 + \lfloor 2 + \lfloor 3 + \lfloor 4} + \lfloor 5$

$33 + 120 = 15\boxed{3} \quad N^2$ ✗

$+ \lfloor 6 = \sim\sim \boxed{3}$

$+ \lfloor 7 = \sim\sim \boxed{3}$

$\lfloor 4$ के बाद unit digit = 3

आएगा

Not Perfect sq

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Q. All possible three digits even numbers which can be formed with the condition that if 5 is one of the digit, then 7 is the next digit is :

(A) 5

(B) 325

(C) 345

(D) 365

Even

Starting with 5 → | 5 | 7 | 0/2/4/6/8 |

$1 \times 1 \times 5 = 5$

When 5 is in Middle | | 5 | 7 | $= 0$

×

Not an Even No.

5 last | | | 5 | $= 0$

Not an even no.

When they are not having any 5

| 5× 0× | 5× | 0/2/4/6/8 |

$8 \times 9 \times 5$

$= 360$

$= 365$ Ans

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Q. How many of the 900 three digit numbers have at least one even digit?

(A) 775

(B) 875

(C) 450

(D) 750

Total - No Even digit

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Q. Out of seven consonants and four vowels, the number of words of six letters, formed by taking four consonants and two vowels is (Assume that each ordered group of letter is a word):

(A) 210

(B) 462

(C) 151200

(D) 332640

7C, 4V

$${}^7C_4 \times {}^4C_2 \times \lfloor 6$$

7 में से 4 Conso Select then 4 में से 2 vowel Select then Arrange them at 6 Place

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PERMUTATION & COMBINATION

Q. Find the number of natural numbers less than 1000 and divisible by 5 can be formed with the ten digits, each digit not occurring more than once in each number.

Single digit double digit Triple digit

1

$\boxed{5} + \boxed{\ \ \mid 0} + \boxed{0\times\ 5\times \mid 5} + \boxed{0\times\ 5\times \mid 0\ \ 5\times \mid 5} + \boxed{0\times \mid \ \ \mid 0}$

$9 \times 1 \quad + \quad 8 \times 1 \quad + \quad 8 \times 8 \times 1 \quad + \quad 9 \times 8 \times 1$

$+9 + 8 \times 1$

$1 + 9 + 8 + 64 + 72 = 154$

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Q. The set of values of r simultaneously satisfying the system of equations

$P(5, r) = 2 \cdot P(6, r-1)$ and $5 \cdot P(4, r) = 6 \cdot P(5, r-1)$, is

(A) an empty set

(B) a singleton set //

(C) a set consisting of two elements

(D) a set consisting of three elements.

$${}^nP_r = \frac{\lfloor n}{\lfloor n-r}$$

$$P(5,r) = {}^5P_r = \frac{\lfloor 5}{\lfloor 5-r}$$

$${}^5P_r = 2\,{}^6P_{r-1}$$

$$\frac{\lfloor 5}{\lfloor 5-r} = 2 \times \frac{\lfloor 6}{\lfloor 6-r+1}$$

$$\frac{\cancel{\lfloor 5}}{\lfloor 5-r} = \frac{2\times 6\times \cancel{\lfloor 5}}{\lfloor 7-r}$$

$$\lfloor 7-r = 12 \lfloor 5-r$$

$$5 \times {}^4P_r = 6 \times {}^5P_{r-1}$$

$$\frac{5\times\lfloor 4}{\lfloor 4-r} = \frac{6\times\lfloor 5}{\lfloor 5-r+1}$$

$$\frac{5\,\cancel{\lfloor 4}}{\lfloor 4-r} = \frac{6\times 5\,\cancel{\lfloor 4}}{\lfloor 6-r}$$

$$5\,\lfloor 6-r = 30\,\lfloor 4-r$$

$$\frac{\lfloor 7-r}{5\,\lfloor 6-r} = \frac{12\,\lfloor 5-r}{30\,\lfloor 4-r}$$

$$\frac{(7-r)\,\cancel{\lfloor 6-r}}{\cancel{5}\,\cancel{\lfloor 6-r}} = \frac{\cancel{12}^{\,2}\times(5-r)\,\cancel{\lfloor 4-r}}{\cancel{30}_{\,6}\,\cancel{\lfloor 4-r}}$$

$$7-r = 10-2r$$

$$r = 3$$

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Q. Let P_n denotes the number of permutations of n distinct things taken all at a time and $x_n = {}^{n+5}C_4 - \left(\frac{143}{96}\right)\left(\frac{P_{n+5}}{P_{n+3}}\right)$ (where $n \in N$). The possible value of n for which x_n is negative, can be

(A) 1

(B) 2

(C) 3

(D) 4

$P_n = \lfloor n$

$^nP_r = \frac{\lfloor n}{\lfloor n-r}$

$^nC_r = \frac{\lfloor n}{\lfloor r \lfloor n-r}$

$$x_n = \frac{\lfloor n+5}{\lfloor 4 \lfloor n+1} - \frac{143}{96} \times \frac{\lfloor n+5}{\lfloor n+3}$$

$$= \frac{(n+5)(n+4)(n+3)(n+2)\cancel{\lfloor n+1}}{24\cancel{\lfloor n+1}} - \frac{143}{96} \times \frac{(n+5)(n+4)\cancel{\lfloor n+3}}{\cancel{\lfloor n+3}}$$

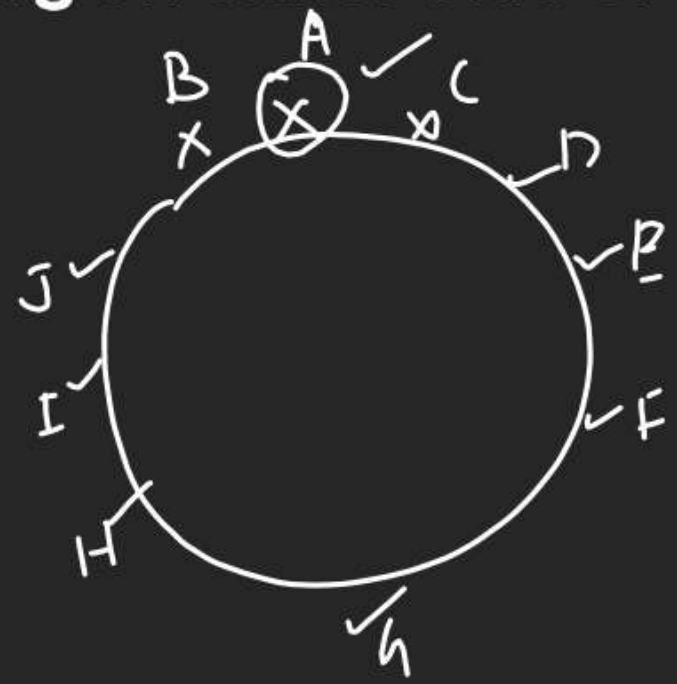
$$= \frac{(n+4)(n+5)}{24}\left\{(n+3)(n+2) - \frac{143}{4}\right\}$$

$n=1 \rightarrow \left\{(4\times3) - \frac{143}{4}\right\}$

$n=2 \rightarrow \left\{5\times4 - \frac{143}{4}\right\}$

$n=3 \rightarrow \left\{6\times5 - \frac{143}{4}\right\}$

Q. 10 people are sitting around a circular table, each one shaking a hand with everyone else except from the people sitting on either side of him. Find the number of handshakers.



$$^7C_2 = \frac{7\times 6}{2\times 1} = 21$$

A - J
A - I
A - H
A - G
A - F
A - E
A - D

B

A → 7 handshakes
B → 7
C → 7
⋮
J - 7

$$\frac{70 \text{ handshakes}}{2} = 35$$

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Factorial Notation

$|\underline{0} = 1$

$|\underline{1} = 1$

$|\underline{2} = 2$

$\vdots$

$2n!$

$|\underline{6} = \underbrace{6\times5\times4\times3\times2\times1}_{\leftarrow 6 \text{ बार} \rightarrow}$

Q Find Product of 50 odd Natural No.

$|\underline{2n} = \underbrace{(2n)(2n-1)(2n-2)(2n-3)(2n-4)\cdots 4\cdot3\cdot2\cdot1}_{\leftarrow 2n \text{ बार} \rightarrow}$

$= \underbrace{\{(2n)(2n-2)(2n-4)\cdots 6\cdot4\cdot2\}}_{\leftarrow n \text{ बार} \rightarrow}\underbrace{\{(2n-1)(2n-3)\cdots5\cdot3\cdot1\}}_{\leftarrow n \text{ बार} \rightarrow}$

$|\underline{2n} = 2^n\{(n)(n-1)(n-2)\cdots3\cdot2\cdot1\}\{1\cdot3\cdot5\cdot7\cdots(2n-1)\}$

$= 2^n\,|\underline{n}\ \underbrace{\{1\cdot3\cdot5\cdot7\cdots(2n-1)\}}_{\leftarrow n \text{ odd No.} \rightarrow}$

$$\boxed{1\cdot3\cdot5\cdot7\cdots(2n-1) = \frac{|\underline{2n}}{2^n\,|\underline{n}}}$$

$\underbrace{1\cdot3\cdot5\cdot7\cdots99}_{\leftarrow 50 \text{ odd Nat.} \rightarrow} = \dfrac{|\underline{100}}{2^{50}\,|\underline{50}}$

Exponant of Prime No. in $|\underline{n}$

1) $2, 3, 2^2, 5, 2\times3, 7, 2^3, 3^2, 2\times5$

$11, 2\times2\times3, 13, 2\times7, 3\times5, 2\times2\times2\times2$

$17, 2\times3\times3, 19, \boxed{2\times2\times5} - - - -$

$20 = 2^2\times5^1$

Every +ve Int. is a Prime No or Product of Prime No.

(2) We can break every $|\underline{n}$ into Prime No. that Process is known as Prime factorisation

$|\underline{n} = 2^x\cdot3^y\cdot5^z\cdot7^w\cdot11^t \cdots$

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Q If $\lfloor 100 = 2^m \cdot I$ (I = odd Int.) then m?

$\lfloor 2n = 2^n \lfloor n \{1 \cdot 3 \cdot 5 - - - (2n-1)\}$

$\lfloor 2\times 50 = 2^{50} \cdot \lfloor 50 \{1 \cdot 3 \cdot 5 \cdot 7 - - - 99\}$ (2x25)

$= 2^{50} \cdot 2^{25} \cdot \lfloor 25 \underbrace{\{1 \cdot 3 \cdot 5 - - 99\}\{1 \cdot 3 \cdot 5 - - 49\}}_{} \rightarrow I$

$= 2^{75} \times 25 \lfloor 24 \times \{ \underbrace{\quad}_{\text{odd No}} \}$ (2x12)

$= 2^{75} \times 2^{12} \lfloor 12 \{ \quad \}$ (2x6)

$= 2^{87} \times 2^{6} \lfloor 6 \{ \quad \}$ (2x3)

$= 2^{93} \times 2^{3} \lfloor 3 \{ \quad \}$

$= 2^{96} \times 3 \times 2 \{ \quad \}$

$= 2^{97} \times \underbrace{\{ \quad I \quad \}}_{\text{odd}}$

$m = 97$

Meaning of Qs.

$\lfloor 100 = 2^{97} \times \overline{\underline{\text{odd} - - I}}$

Whenever we will dissolve $\lfloor 100$ into Primefactors it must be having 97 times 2.

$\left[\frac{100}{128}\right] = [\text{kuch km}] = 0$

directformula for Exponent of Prime in $\lfloor n$

$E_p \lfloor n = \left[\frac{n}{p}\right] + \left[\frac{n}{p^2}\right] + \left[\frac{n}{p^3}\right] \sim$

Q $E_2 \lfloor 100 = \left[\frac{100}{2}\right] + \left[\frac{100}{2^2}\right] + \left[\frac{100}{2^3}\right] + \left[\frac{100}{2^4}\right] + \left[\frac{100}{2^5}\right] + \left[\frac{100}{2^6}\right]$ (12.5)

$= 50 + 25 + 12 + 6 + 3 + 1 + 0 + 0 + \sim$

$= 97$

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Q) Exponent of 3 in $\lfloor 100$?

$$E_3 \lfloor 100 = \left[\frac{100}{3}\right] + \left[\frac{100}{9}\right] + \left[\frac{100}{27}\right] + \left[\frac{100}{81}\right]$$

$$= [33.33] + [11.11] + [3.--] + [1.---]$$

$$= 33 + 11 + 3 + 1 = 48.$$

Q) Exponent of 5 in $\lfloor 100$?

$$= \left[\frac{100}{5}\right] + \left[\frac{100}{25}\right] + \text{—}$$

$$= 20 + 4 = 24$$

final Meaning & Uses-

1) $\lfloor 100 = 2^{97} \times 3^{48} \times 5^{24} \times 7^{16} \sim -$

2) Find No of Cyphers in $\lfloor 100$
or
No of zeroes in $\lfloor 100$

Base on Exponent formula.

Zero means degree of 10.
of 2×5

$$\lfloor 100 = 2^{24} \times 5^{24} \times 2^{73} \times 3^{48} - - -$$

$$= (10)^{24} \times -$$

24 Zeroes.

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Q Find Exponent of 18 in $\lfloor 100$

$18 \downarrow (2 \times 3^2)$

$\lfloor 100 = 2^{97} \times (3^2)^{24} \times 5^{24} \times 7^{16} - - \cdot$

$= 2^{24} \times (3^2)^{24} \times 2^{73}$

$= (2 \times 3^2)^{24} \times$ ——

$= (18)^{24} \times -$ ——

Exponent of 18 is 24

Q Exponent of 18 in $^{100}C_{50}$?