EXERCISE - 3 (JEE-MAIN)

1. If $D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+x & 1 \\ 1 & 1 & 1+y \end{vmatrix}$ for $x \neq 0, y \neq 0$ then D is- **[AIEEE - 2007]**

(A) Divisible by both x and y (B) Divisible by x but not y

(C) Divisible by y but not x (D) Divisible by neither x nor y

2. Let a, b, c be any real numbers. Suppose that there are real numbers x, y, z not all zero such that $x = cy + bz, y = az + cx$ and $z = bx + ay$, then $a^2 + b^2 + c^2 + 2abc$ is equal to **[AIEEE - 2008]**

(A) 2 (B) -1 (C) 0 (D) 1

3. Let a, b, c be such that $b(a + c) \neq 0$. **[AIEEE - 2009]**

If $\begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix} + \begin{vmatrix} a+1 & b+1 & c-1 \\ a-1 & b-1 & c+1 \\ (-1)^{n+2}a & (-1)^{n+1}b & (-1)^n c \end{vmatrix} = 0$, then the value of n is :-

(A) Any odd integer (B) Any integer (C) Zero (D) Any even integer

4. Consider the system of linear equations : $x_1 + 2x_2 + x_3 = 3, 2x_1 + 3x_2 + x_3 = 3, 3x_1 + 5x_2 + 2x_3 = 1$. The system has **[AIEEE - 2010]**

(A) Infinite number of solutions (B) Exactly 3 solutions

(C) A unique solution (D) No solution

5. The number of values of k for which the linear equations $4x + ky + 2z = 0, kx + 4y + z = 0, 2x + 2y + z = 0$ possess a non-zero solution is : **[AIEEE - 2011]**

(A) 1 (B) zero (C) 3 (D) 2

6. If the trivial solution is the only solution of the system of equations $x - ky + z = 0, kx + 3y - kz = 0, 3x + y - z = 0$ Then the set of all values of k is: **[AIEEE - 2011]**

(A) $\{2, -3\}$ (B) $R - \{2, -3\}$ (C) $R - \{2\}$ (D) $R - \{-3\}$

7. The number of values of k, for which the system of equations : **[JEE(Main)-2013]**

$(k + 1)x + 8y = 4k, kx + (k + 3)y = 3k - 1$ has no solution, is -

(A) infinite (B) 1 (C) 2 (D) 3

8. If $\alpha, \beta \neq 0$, and $f(n) = \alpha^n + \beta^n$ and

$\begin{vmatrix} 3 & 1+f(1) & 1+f(2) \\ 1+f(1) & 1+f(2) & 1+f(3) \\ 1+f(2) & 1+f(3) & 1+f(4) \end{vmatrix} = K(1-\alpha)^2(1-\beta)^2(\alpha-\beta)^2$, then K is equal to :

(A) $\alpha\beta$ (B) $\frac{1}{\alpha\beta}$ (C) 1 (D) -1 **[JEE(Main)-2014]**

9. The set of all values of λ for which the system of linear equations : **[JEE(Main)-2015]**

$2x_1 - 2x_2 + x_3 = \lambda x_1, 2x_1 - 3x_2 + 2x_3 = \lambda x_2, -x_1 + 2x_2 = \lambda x_3$ has a non-trivial solution

(A) contains two elements (B) contains more than two elements

(C) is an empty set (D) is a singleton

10. The system of linear equations

$x + \lambda y - z = 0, \lambda x - y - z = 0, x + y - \lambda z = 0$ has a non-trivial solution for : **[JEE(Main)-2016]**

(A) exactly three values of λ. (B) infinitely many values of λ.

(C) exactly one value of λ (D) exactly two values of λ.

11. If S is the set of distinct value of ' b ' for which the following system of linear equations :

$x + y + z = 1 \quad \Rightarrow \quad x + ay + z = 1 \quad \Rightarrow \quad ax + by + z = 0$

has no solution, then S is : **[JEE(Main) -2017]**

(A) a singleton (B) an empty set

(C) an infinite set (D) a finite set containing two or more elements

12. If the system of linear equations

$x + ky + 3z = 0 \quad \Rightarrow \quad 3x + ky - 2z = 0 \Rightarrow \quad 2x + 4y - 3z = 0$

has a non-zero solution (x, y, z), then $\frac{xz}{y^2}$ is equal to **[JEE(Main) -2018]**

(A) 30 (B) -10 (C) 10 (D) -30

13. If $\begin{vmatrix} x-4 & 2x & 2x \\ 2x & x-4 & 2x \\ 2x & 2x & x-4 \end{vmatrix} = (A + Bx)(x - A)^2$, then the ordered pair (A, B) is equal to:

(A)$(4,5)$ (B) $(-4,-5)$ (C) $(-4,3)$ (D) $(-4,5)$

[JEE(Main) -2018]

14. The system of linear equations. **[JEE(Main) -2019]**

$x + y + z = 2 \quad \Rightarrow 2x + 3y + 2z = 5 \quad \Rightarrow 2x + 3y + (a^2 - 1)z = a + 1$

(A) has infinitely many solutions for $a = 4$ (B) is inconsistent when $|a| = \sqrt{3}$

(C) is inconsistent when $a = 4$ (D) has a unique solution for $|a| = \sqrt{3}$

15. If the system of linear equations **[JEE(Main) -2019]**

$x - 4y + 7z = g \quad \Rightarrow 3y - 5z = h \quad \Rightarrow -2x + 5y - 9z = k$ is consistent, then :

(A) $g + 2h + k = 0$ (B) $2g + h + k = 0$ (C) $g + h + k = 0$ (D) $g + h + 2k = 0$

16. Let $d \in R$, and $A = \begin{bmatrix} -2 & 4+d & (\sin\theta)-2 \\ 1 & (\sin\theta)+2 & d \\ 5 & (2\sin\theta)-d & (-\sin\theta)+2+2d \end{bmatrix}$, $\theta \in [0,2\pi]$. If the minimum value of det (A) is 8 , then a value of d is: **[JEE(Main) -2019]**

(A) -7 (B) -5 (C) $2(\sqrt{2}+1)$ (D) $2(\sqrt{2}+2)$

17. If the system of equations

$x+y+z=5 \quad \Rightarrow x+2y+3z=9 \quad \Rightarrow x+3y+\alpha z=\beta$

has infinitely many solutions, then $\beta-\alpha$ equals : **[JEE(Main) -2019]**

(A) 18 (B) 21 (C) 5 (D) 8

18. The number of values of $\theta \in (0,\pi)$ for which the system of linear equations $x+3y+7y=0$; $-x+4y+7z=0$; $(\sin 3\theta)x+(\cos 2\theta)y+2z=0$ has a non-trivial solution, is – **[JEE(Main) -2019]**

(A) four (B) one (C) three (D) two

19. If the system of linear equations **[JEE(Main) -2019]**

$2x+2y+3z=a \quad \Rightarrow 3x-y+5z=b \quad \Rightarrow x-3y+2z=c$

where a, b, c are non-zero real numbers, has more than one solution, then :

(A) $a+b+c=0$ (B) $b+c-a=0$ (C) $b-c+a=0$ (D) $b-c-a=0$

20. If $\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)(x+a+b+c)^2, x \neq 0$ and $a+b+c \neq 0$,

then x is equal to : **[JEE(Main) -2019]**

(A) abc (B) $2(a+b+c)$ (C) $-2(a+b+c)$ (D) $-(a+b+c)$

21. Let A and B be two invertible matrices of order 3×3. If $\det(ABA^T) = 8$and $\det(AB^{-1}) = 8$, then $\det(BA^{-1}B^T)$ is equal to : **[JEE(Main) -2019]**

(A) $\frac{1}{4}$ (B) $\frac{1}{16}$ (C) 1 (D) 16

22. An ordered pair (α,β) for which the system of linear equations. **[JEE(Main) -2019]**

$(1+\alpha)x+\beta y+z=2 \quad \Rightarrow \alpha x+(1+\beta)y+z=3 \quad \Rightarrow \alpha x+\beta y+2z=2$

has a unique solutions is :

(A) $(-3,1)$ (B) $(-4,2)$ (C) $(1,-3)$ (D) $(2,4)$

23. The set of all values of λ for which the system of linear equations **[JEE(Main) -2019]**

$x-2y-2z=\lambda x \quad \Rightarrow x+2y+z=\lambda y \quad \Rightarrow -x-y=\lambda z$

has a non-trivial solution:

(A) contains exactly two elements (B) is a singleton

(C) contains more than two elements (D) is an empty set

24. The greatest value of $c \in R$ for which the system of linear equations

$x-cy-cz=0 \quad \Rightarrow cx-y+cz=0 \quad \Rightarrow cx+cy-z=0$

has a non-trivial solution, is **[JEE(Main) -2019]**

(A) -1 (B) ½ (C) 2 (D) 0

25. Let the numbers 2, b, c be in an A.P. and $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & b & c \\ 4 & b^2 & c^2 \end{bmatrix}$. If det (A) $\in [2,16]$, then c lies in the interval : **[JEE(Main) -2019]**

(A) $\left(2+2^{3/4}, 4\right)$ (B) $\left[3, 2+2^{3/4}\right]$ (C) $[4,6]$ (D) $[2,3)$

26. Let α and β be the roots of the equation $x^2 + x + 1 = 0$. Then for $y \neq 0$ in **R**,

$\begin{vmatrix} y+1 & \alpha & \beta \\ \alpha & y+\beta & 1 \\ \beta & 1 & y+\alpha \end{vmatrix}$ is equal to : **[JEE(Main) -2019]**

(A) $y^3 - 1$ (B) y^3 (C) $y(y^2 - 1)$ (D) $y(y^2 - 3)$

27. If the system of equations $2x + 3y - z = 0, x + ky - 2z = 0$ and $2x - y + z = 0$ has a non-trivial solution (x, y, z) then $\frac{x}{y} + \frac{y}{z} + \frac{z}{x} + k$ is equal to **[JEE(Main) -2019]**

(A) $\frac{3}{4}$ (B) $-\frac{1}{4}$ (C) $\frac{1}{2}$ (D) -4

28. If $\Delta_1 = \begin{vmatrix} x & \sin\theta & \cos\theta \\ -\sin\theta & -x & 1 \\ \cos\theta & 1 & x \end{vmatrix}$ and $\Delta_2 = \begin{vmatrix} x & \sin 2\theta & \cos 2\theta \\ -\sin 2\theta & -x & 1 \\ \cos 2\theta & 1 & x \end{vmatrix}$, $x \neq 0$, then for all $\theta \in \left(0, \frac{\pi}{2}\right)$: **[JEE(Main) -2019]**

(A) $\Delta_1 + \Delta_2 = -2(x^3 + x - 1)$ (B) $\Delta_1 - \Delta_2 = -2x^3$

(C) $\Delta_1 - \Delta_2 = x(\cos 2\theta - \cos 4\theta)$ (D) $\Delta_1 + \Delta_2 = -2x^3$

29. If the system of linear equations **[JEE(Main) -2019]**

$x + y + z = 5$ $\Rightarrow x + 2y + 2z = 6$ $\Rightarrow x + 3y + \lambda z = \mu, (\lambda, \mu \in R)$

has infinitely many solutions, then the value of $\lambda + \mu$ is :

(A) 7 (B) 10 (C) 12 (D) 9

30. The sum of the real roots of the equation $\begin{vmatrix} x & -6 & -1 \\ 2 & -3x & x-3 \\ -3 & 2x & x+2 \end{vmatrix} = 0$, is equal to :

(A) 0 (B) 6 (C) 1 (D) -4 **[JEE(Main) -2019]**

31. Let λ be a real number for which the system of linear equations **[JEE(Main) -2019]**

$x + y + z = 6$ $\Rightarrow 4x + \lambda y - \lambda z = \lambda - 2$ $\Rightarrow 3x + 2y - 4z = -5$

has infinitely many solutions. Then λ is a root of the quadratic equation :

(A) $\lambda^2 + \lambda - 6 = 0$ (B) $\lambda^2 - 3\lambda - 4 = 0$ (C) $\lambda^2 + 3\lambda - 4 = 0$ (D) $\lambda^2 - \lambda - 6 = 0$

32. If [x] denotes the greatest integer $\leq$ x, then the system of linear equations

$[\sin\ \theta]x + [-\cos\theta]y = 0$ $\Rightarrow [\cot\ \theta]x + y = 0$ **[JEE(Main) -2019]**

(A) have infinitely many solutions if $\theta \in \left(\frac{\pi}{2}, \frac{2\pi}{3}\right)$ and has a unique solution if $\theta \in \left(\pi, \frac{7\pi}{6}\right)$

(B) has a unique solution if $\theta \in \left(\frac{\pi}{2}, \frac{2\pi}{3}\right) \cup \left(\pi, \frac{7\pi}{6}\right)$.

(C) have infinitely many solutions if $\theta \in \left(\frac{\pi}{2}, \frac{2\pi}{3}\right) \cup \left(\pi, \frac{7\pi}{6}\right)$.

(D) has a unique solution if $\theta \in \left(\frac{\pi}{2}, \frac{2\pi}{3}\right)$ and have infinitely many solutions if $\theta \in \left(\pi, \frac{7\pi}{6}\right)$.

33. A value of $\theta \in (0, \pi/3)$, for which $\begin{vmatrix} 1+\cos^2\theta & \sin^2\theta & 4\cos 6\theta \\ \cos^2\theta & 1+\sin^2\theta & 4\cos 6\theta \\ \cos^2\theta & \sin^2\theta & 1+4\cos 6\theta \end{vmatrix} = 0$, is :

(A) $\frac{7\pi}{36}$ (B) $\frac{\pi}{9}$ (C) $\frac{\pi}{18}$ (D) $\frac{7\pi}{24}$ **[JEE(Main) -2019]**

34. If α is a roots of equation $x^2 + x + 1 = 0$ and $A = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix}$ then A^{31} equal to:

(JEE Main 2020)

(A) A (B) A^2 (C) A^3 (D) A^4

35. The maximum value of

$f(x) = \begin{vmatrix} \sin^2 x & 1+\cos^2 x & \cos 2x \\ 1+\sin^2 x & \cos^2 x & \cos 2x \\ \sin^2 x & \cos^2 x & \sin 2x \end{vmatrix}, x \in R$ is: **(JEE Main 2021)**

(A) $\sqrt{7}$ (B) $\frac{3}{4}$ (C) $\sqrt{5}$ (D) 5

36. The system of equations

$-kx + 3y - 14z - 25$

$-15x + 4y - kz = 3$

$-4x + y + 3z = 4$

is consistent for all k in the set **(JEE Main 2022)**

(A) R (B) $R - \{-11, 13\}$ (C) $R - \{13\}$ (D) $R - \{-11, 11\}$

37. If the system of equations

$x + 2y + 3z = 3$

$4x + 3y - 4z = 4$

$8x + 4y - \lambda z = 9 + \mu$

has infinitely many solutions, then the ordered pair (λ, μ) is equal to **(JEE Main 2023)**

(A) $\left(\frac{72}{5}, \frac{21}{5}\right)$ (B) $\left(\frac{-72}{5}, \frac{-21}{5}\right)$ (C) $\left(\frac{72}{5}, \frac{-21}{5}\right)$ (D) $\left(\frac{-72}{5}, \frac{21}{5}\right)$

EXERCISE - 4 (JEE-ADVANCED)

1. (a) Consider three point $P = (-\sin(\beta-\alpha), -\cos\beta), Q = (\cos(\beta-\alpha), \sin\beta)$ and $R = (\cos(\beta-\alpha+\theta), \sin(\beta-\theta))$, where $0 < \alpha, \beta, \theta < \pi/4$

(A) P lies on the line segment RQ
(B) Q lies on the line segment PR
(C) R lies on the line segment QP
(D) P, Q, R are non collinear

(b) Consider the system of equations $x - 2y + 3z = -1; -x + y - 2z = k; x - 3y + 4z = 1$.

Statement-I : The system of equations has no solution for $k \neq 3$. and

Statement-II : The determinant $\begin{vmatrix} 1 & 3 & -1 \\ -1 & -2 & k \\ 1 & 4 & 1 \end{vmatrix} \neq 0$, for $k \neq 3$. **[JEE 2008, 3+3]**

(A) Statement-I is true, Statement-II is true ; Statement-II is correct explanation for Statement-I.

(B) Statement-I is true, Statement-II is true ; Statement-II is NOT a correct explanation for statement-I.

(C) Statement-I is true, Statement-II is false.

(D) Statement-I is false, Statement-II is true.

2. The number of all possible values of θ, where $0 < \theta < \pi$, for which the system of equations

$(y + z)\cos 3\theta = (xyz)\sin 3\theta$

$x\sin 3\theta = \frac{2\cos 3\theta}{y} + \frac{2\sin 3\theta}{z}$ $\Rightarrow$ $(xyz)\sin 3\theta = (y + 2z)\cos 3\theta + y\sin 3\theta$

have a solution (x_0, y_0, z_0) with $y_0 z_0 \neq 0$, is **[JEE 2010,3]**

3. Which of the following values of α satisfy the equation

$\begin{vmatrix} (1+\alpha)^2 & (1+2\alpha)^2 & (1+3\alpha)^2 \\ (2+\alpha)^2 & (2+2\alpha)^2 & (2+3\alpha)^2 \\ (3+\alpha)^2 & (3+2\alpha)^2 & (3+3\alpha)^2 \end{vmatrix} = -648\alpha$? **[JEE(Advanced)-2015, 4M, -2M]**

(A) -4 (B) 9 (C) -9 (D) 4

4. The total number of distinct $x \in R$ for which $\begin{vmatrix} x & x^2 & 1+x^3 \\ 2x & 4x^2 & 1+8x^3 \\ 3x & 9x^2 & 1+27x^3 \end{vmatrix} = 10$ is

[JEE(Advanced)-2016, 3(0)]

5. Let $a, \lambda, m \in R$. Consider the system of linear equations

$ax + 2y = \lambda$ $\Rightarrow$ $3x - 2y = \mu$

Which of the following statement(s) is(are) correct ? **[JEE(Advanced)-2016, 4(-2)]**

(A) If $a = -3$, then the system has infinitely many solutions for all values of λ and μ

(B) If $a \neq -3$, then the system has a unique solution for all values of λ and μ

(C) If $\lambda + \mu = 0$, then the system has infinitely many solutions for $a = -3$

(D) If $\lambda + \mu \neq 0$, then the system has no solution for $a = -3$

6. Let α, β and γ be real numbers. Consider the following system of linear equations

$x + 2y + z = 7$

$x + \alpha z = 11$

$2x - 3y + \beta z = \gamma$ **[JEE(Advanced)-2023]**

Match each entry in List-I to the correct entries in List-II.

List-I	List-II
(P) If $\beta = \frac{1}{2}(7\alpha - 3)$ and $\gamma = 28$, then the system has	(1) a unique solution
(Q) If $\beta = \frac{1}{2}(7\alpha - 3)$ and $\gamma \neq 28$, then the system has	(2) no solution
(R) If $\beta \neq \frac{1}{2}(7\alpha - 3)$ where $\alpha = 1$ and $\gamma \neq 28$, then the system has	(3) infinitely many solutions
(S) If $\beta \neq \frac{1}{2}(7\alpha - 3)$ where $\alpha = 1$ and $\gamma = 28$, then the system has	(4) $x = 11, y = -2$ and $z = 0$ as a solution
	(5) $x = -15, y = 4$ and $z = 0$ as a solution

The correct option is:

(A) (P) → (3), (Q) → (2), (R) → (1), (S) → (4)

(B)(P) → (3), (Q) → (2), (R) → (5), (S) → (4)

(C)(P) → (2), (Q) → (1), (R) → (4), (S) → (5)

(D) (P) → (2), (Q) → (1), (R) → (1), (S) → (3)

7. Let $|M|$ denote the determinant of a square matrix M. Let $g: \left[0, \frac{\pi}{2}\right] \to \mathbb{R}$ be the function defined by **[JEE(Advanced)-2023]**

$$g(\theta) = \sqrt{f(\theta) - 1} + \sqrt{f\left(\frac{\pi}{2} - \theta\right) - 1}$$

where $f(\theta) = \frac{1}{2}\begin{vmatrix} 1 & \sin\theta & 1 \\ -\sin\theta & 1 & \sin\theta \\ -1 & -\sin\theta & 1 \end{vmatrix} + \begin{vmatrix} \sin\pi & \cos\left(\theta + \frac{\pi}{4}\right) & \tan\left(\theta - \frac{\pi}{4}\right) \\ \sin\left(\theta - \frac{\pi}{4}\right) & -\cos\frac{\pi}{2} & \log_e\left(\frac{4}{\pi}\right) \\ \cot\left(\theta + \frac{\pi}{4}\right) & \log_e\left(\frac{\pi}{4}\right) & \tan\pi \end{vmatrix}$

Let $p(x)$ be a quadratic polynomial whose roots are the maximum and minimum values of the function $g(\theta)$, and $p(2) = 2 - \sqrt{2}$. Then, which of the following is/are TRUE ?

(A) $p\left(\frac{3+\sqrt{2}}{4}\right) < 0$ (B) $p\left(\frac{1+3\sqrt{2}}{4}\right) > 0$ (C) $p\left(\frac{5\sqrt{2}-1}{4}\right) > 0$ (D) $p\left(\frac{5-\sqrt{2}}{4}\right) < 0$

EXERCISE - 3 (JEE-MAIN)

1.	A	**2.**	D	**3.**	A	**4.**	D	**5.**	D	**6.**	B	**7.**	B
8.	C	**9.**	A	**10.**	A	**11.**	A	**12.**	C	**13.**	D	**14.**	B
15.	B	**16.**	B	**17.**	D	**18.**	D	**19.**	D	**20.**	C	**21.**	B
22.	D	**23.**	B	**24.**	B	**25.**	C	**26.**	B	**27.**	C	**28.**	D
29.	B	**30.**	A	**31.**	D	**32.**	A	**33.**	B	**34.**	C	**35.**	C
36.	D	**37.**	C										

EXERCISE - 4 (JEE-ADVANCED)

1.	(a) D; (b) A	**2.**	3	**3.**	B,C	**4.**	2	**5.**	B,C,D	**6.**	A
7.	AC										