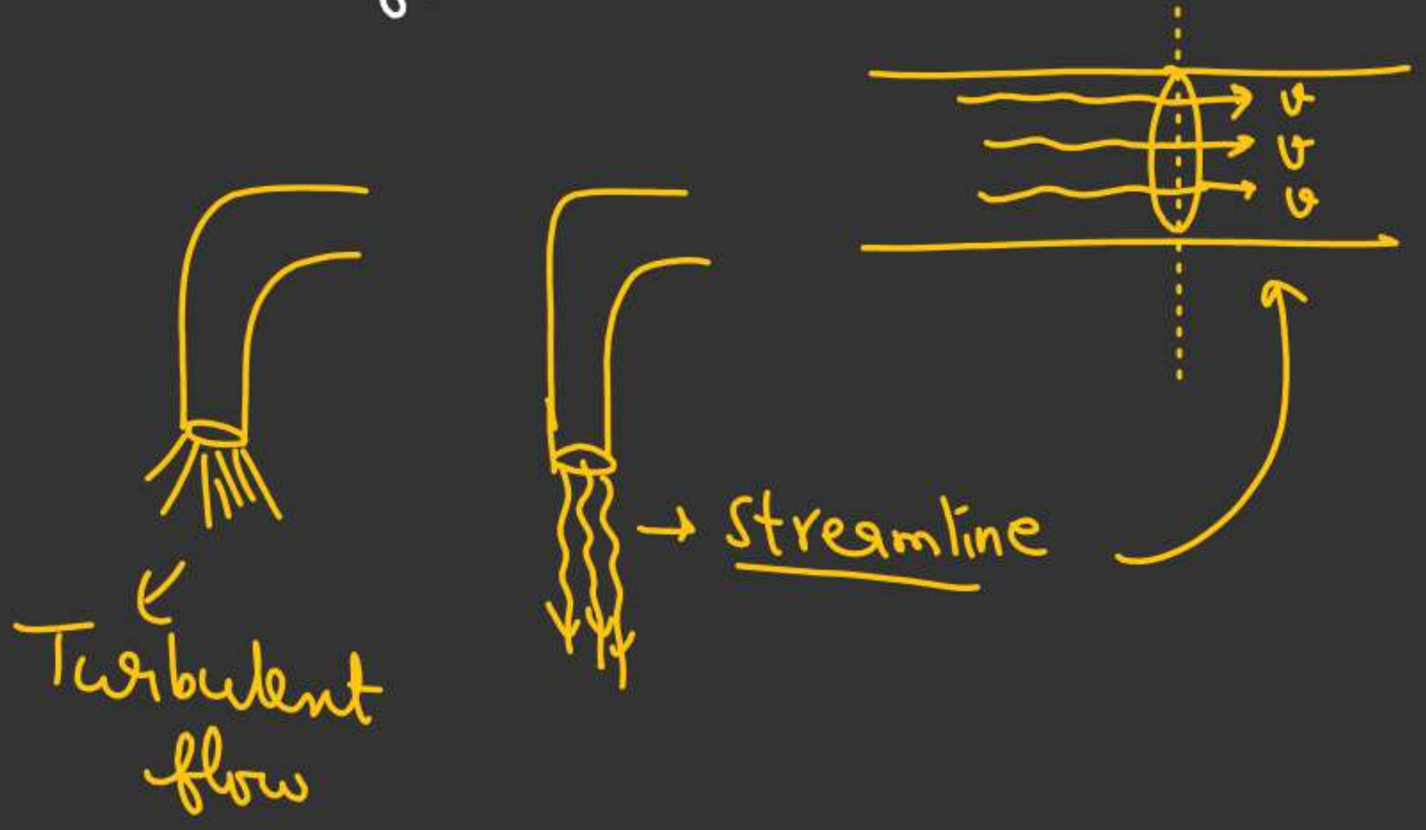
Nishant Jindal

FLUID DYNAMICS

Assumptions:-

For Ideal liquid

- In Compressible → (Density of the liquid through-out its volume remain constant)
- Non-Viscious. → (Any two Consecutive layers doesn't apply any tangential force)
- Stream line flow. →



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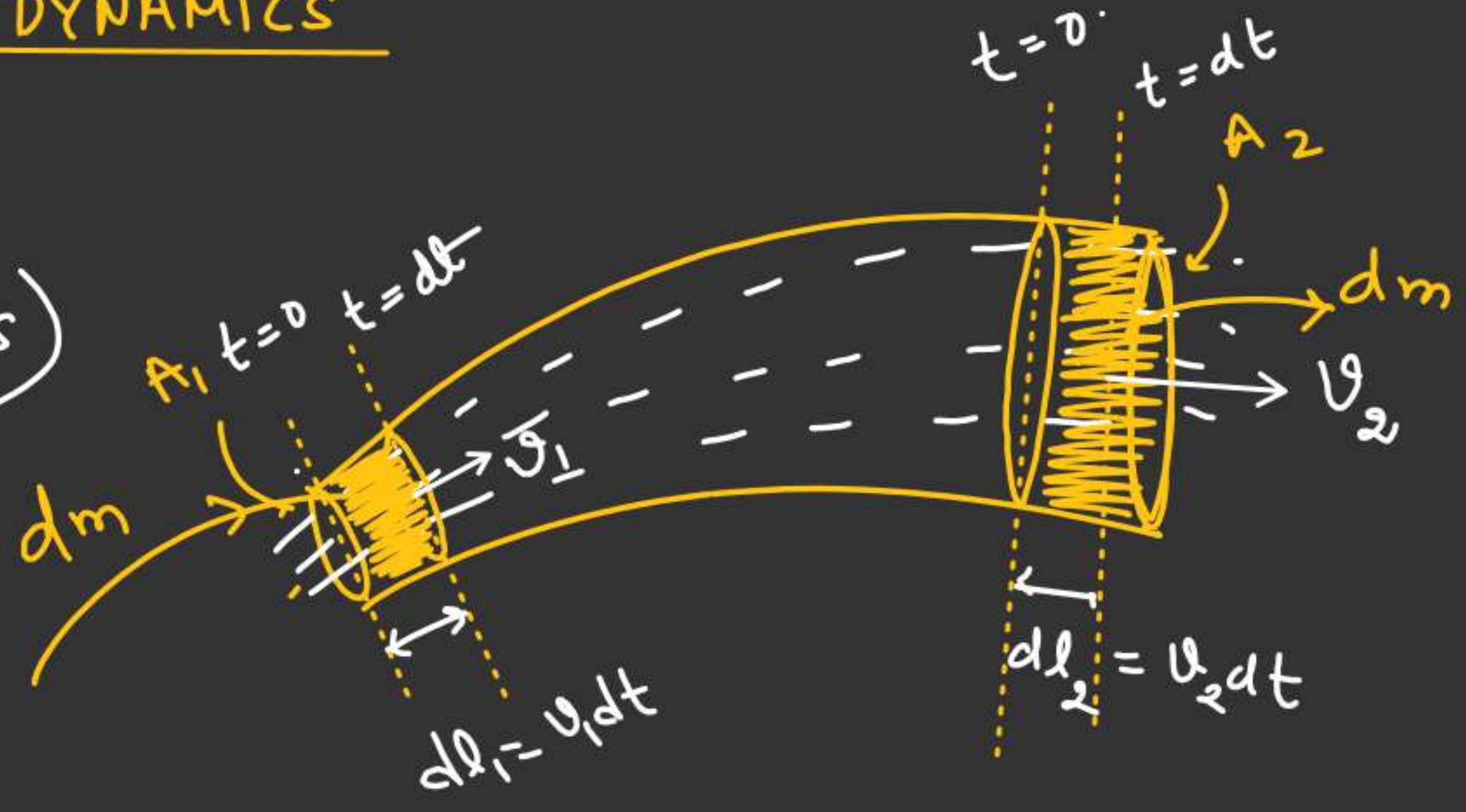
FLUID DYNAMICS

** LAW OF CONTINUITY

(Based on Conservation of mass)



$$dm = \rho A_1 \underline{dl_1} = \rho A_2 \underline{dl_2}$$

$$\Rightarrow \rho A_1 v_1 dt = \rho A_2 v_2 dt$$

$$\Rightarrow \boxed{A_1 v_1 = A_2 v_2}$$

$m^2 \times \frac{m}{s}$

$\Downarrow$

m^3/s

$$\boxed{A_1 v_1 = A_2 v_2 = \frac{dV}{dt}}$$

$\frac{dV}{dt}$ = Volume flow rate

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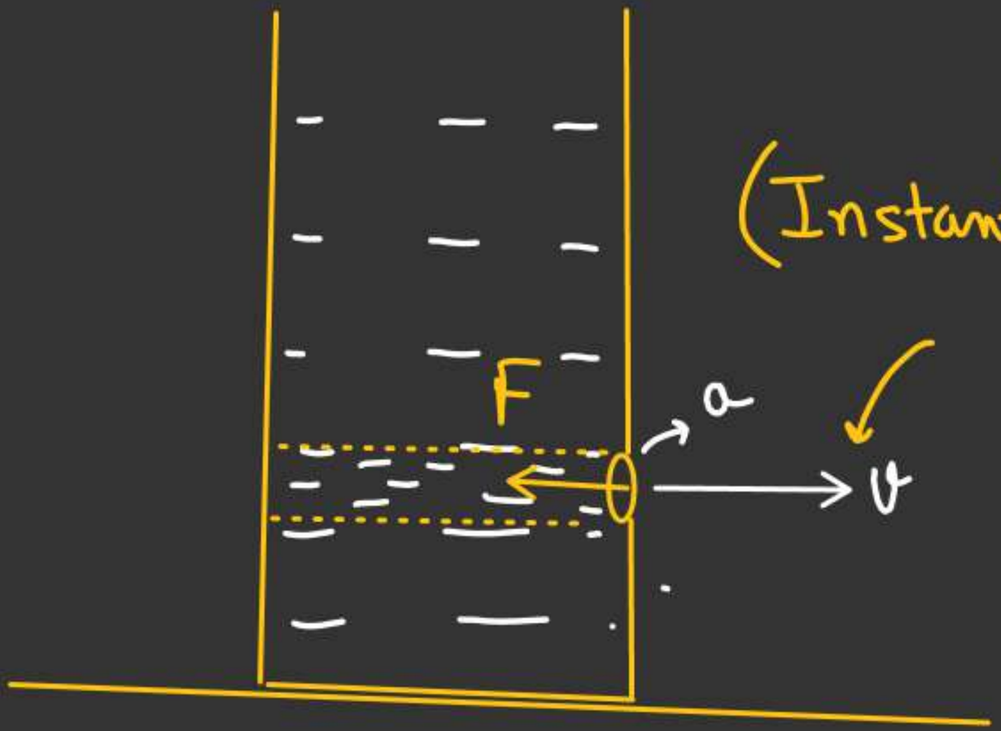
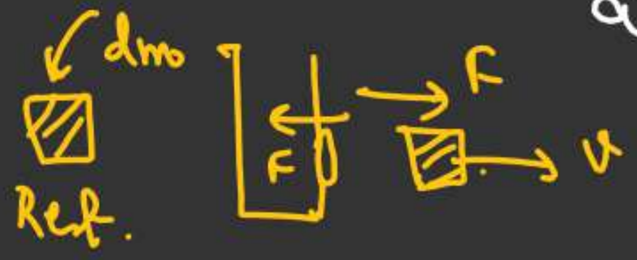
FLUID DYNAMICS

Hydrostatic thrust when liquid exit from a Small hole



a = Crosectional area of hole



By Continuity

$$\frac{dV}{dt} = C$$

$$V = \left(\frac{m}{\rho}\right)$$

$$V = \frac{1}{\rho}\left(\frac{dm}{dt}\right)$$

$$\rho a v = \frac{dm}{dt}$$

$$\left(\rho = \frac{m}{V}\right)$$

$$F = \frac{dp}{dt}$$

$$F = \frac{d(mv)}{dt}$$

$$F = v\left(\frac{dm}{dt}\right)$$

$$\boxed{F = \rho a v^2}$$

$v \rightarrow$ Relative to container

ρ = density of liquid

a = Crossectional area of hole

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FLUID DYNAMICS

Find net hydrostatic torque activing on the vessel.

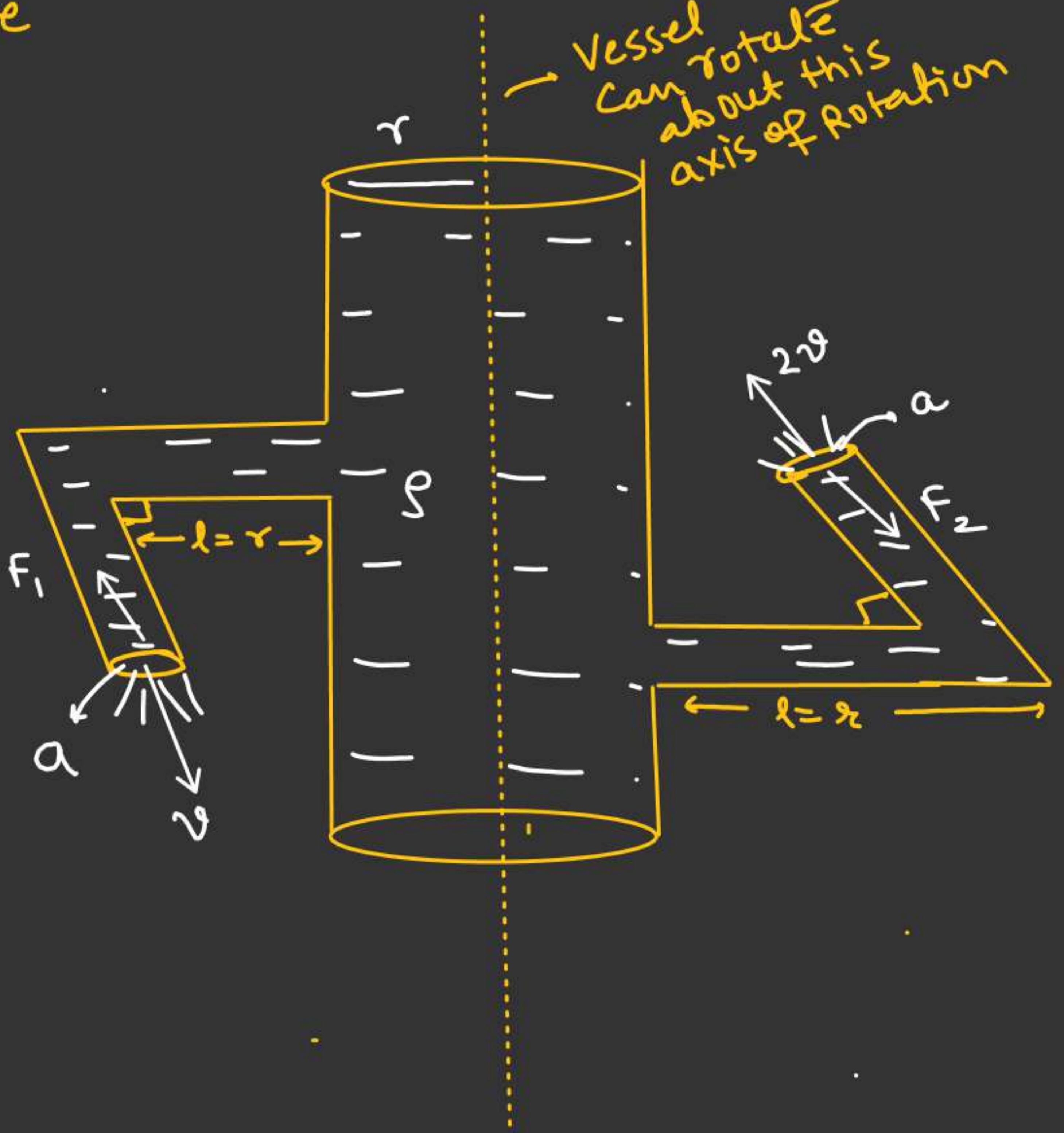
$$\tau = F_1(2r) + F_2(2r)$$

$$= (F_1 + F_2)\,2r$$

$$= [\rho a v^2 + \rho a (2v)^2]\,2r$$

$$= (5\rho a v^2)\cdot(2r)$$

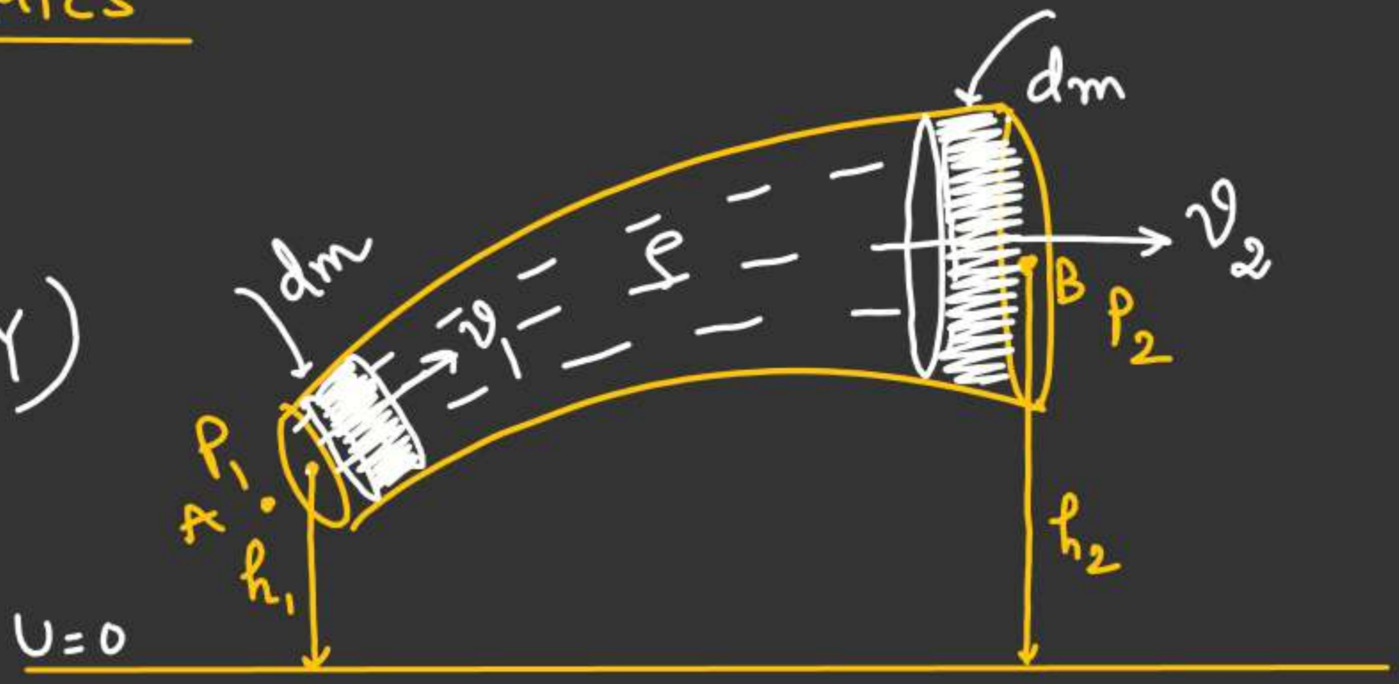
$$= \underline{10\rho a v^2 r}$$



BERNOULLI'S EQUATION

↳ (Based on Conservation of Energy)

$$\boxed{P + \frac{1}{2}\rho v^2 + \rho g h = C}$$



$$\boxed{P_1 + \rho g h_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2}$$

$\frac{1}{2}\rho v^2 =$ K·E per unit Volume

$\rho = \frac{m}{V}$

$d(K \cdot E) = \frac{1}{2}(dm)v^2$

$d(K \cdot E) = \frac{1}{2}(\rho dV)v^2$

$av = \frac{dV}{dt} = \frac{1}{\rho}\frac{dm}{dt}$

$\rho dV = dm$

K·E per Unit volume ← $\frac{d(K \cdot E)}{dV} = \frac{1}{2}\rho v^2$

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FLUID DYNAMICS

Application of Bernoulli's

Velocity of Efflux

Bernoulli's b/w point 1 and 2

$$\cancel{P_{atm}} + \rho g h + \frac{1}{2}\rho v_1^2 = \cancel{P_{atm}} + \frac{1}{2}\rho v_2^2$$

$$\rho g h = \frac{1}{2}\rho\left(v_2^2 - v_1^2\right)$$

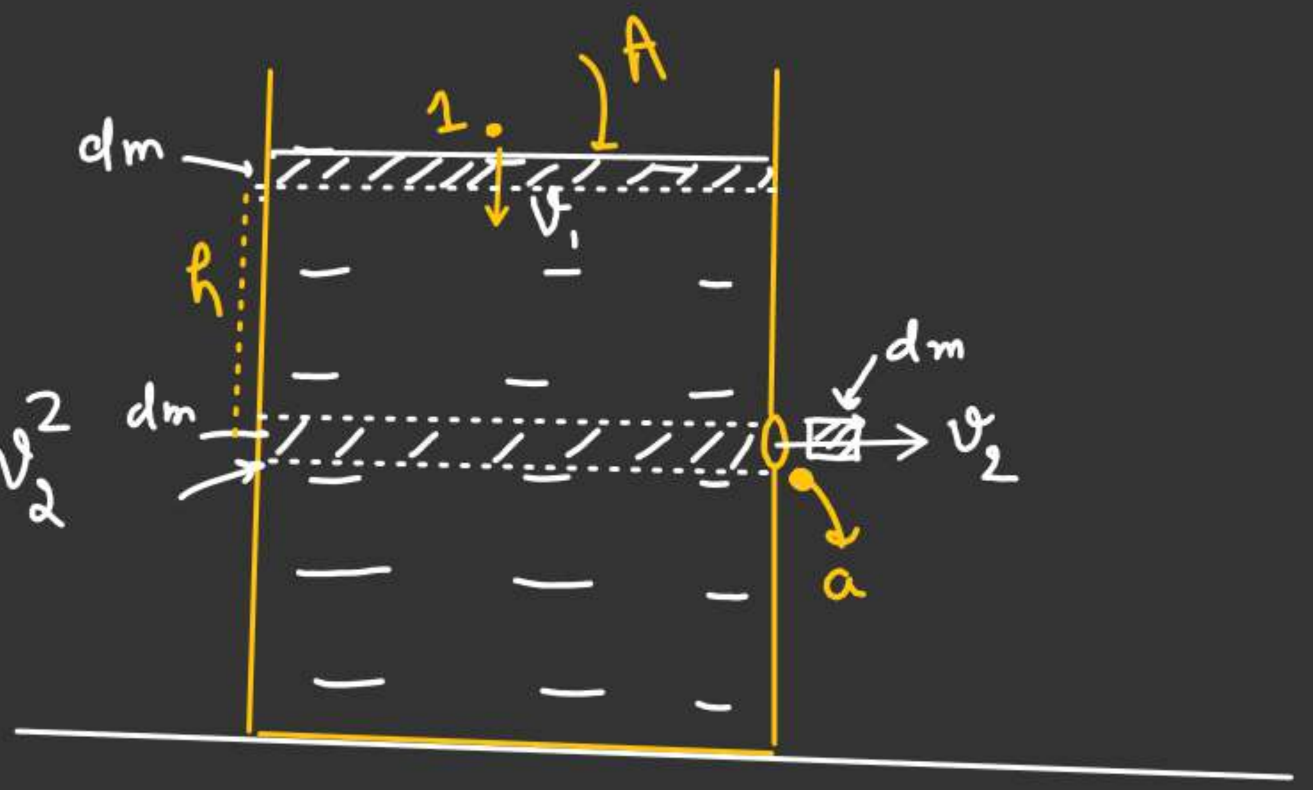
Law of Continuity

$$A v_1 = a v_2$$

$$v_1 = \left(\frac{a v_2}{A}\right)$$

$$\cancel{\rho} g h = \frac{1}{2}\cancel{\rho} v_2^2\left(1 - \frac{a^2}{A^2}\right)$$

$$\sqrt{\frac{2gh}{\left(1 - \frac{a^2}{A^2}\right)}} = v_2$$



A = Crossectional area of container

a = Crossectional area of hole

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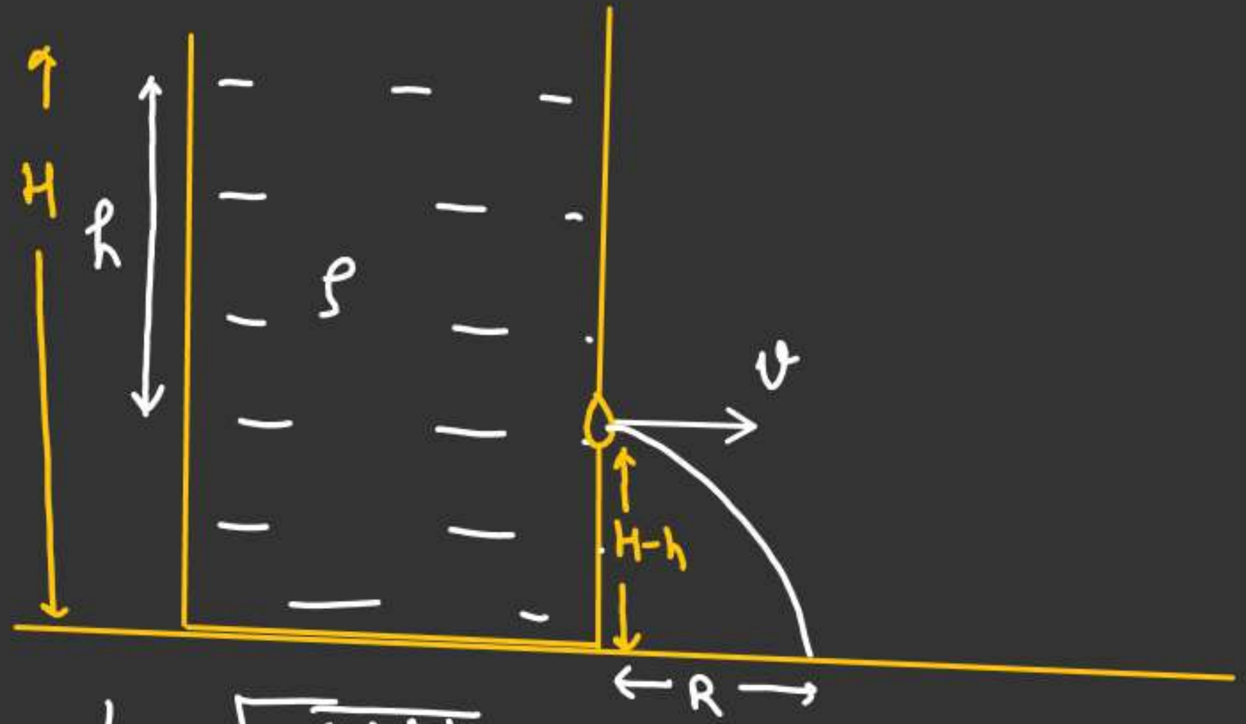
FLUID DYNAMICS

Application of Bernoulli's

Velocity of Efflux

$$\sqrt{\frac{2gh}{\left(1-\frac{a^2}{A^2}\right)}} = v_2$$

h = depth of the orifice from the liquid surface



if $\underline{A >> a}$.

$$\boxed{v_2 = \sqrt{2gh}}$$ ✓

↓

velocity of Efflux

$R = v \times t$

$t = \sqrt{\frac{2(H-h)}{g}}$

$R = \left(\sqrt{2gh}\right)\sqrt{\frac{2(H-h)}{g}}$

$$\boxed{R = 2\sqrt{h(H-h)}}$$

$R \rightarrow f(h)$

For R to be max

$\frac{dR}{dh} = 0$

$\Rightarrow \boxed{h = \frac{H}{2}}$

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FLUID DYNAMICS

$$R_1 = 2\sqrt{h_1 (H-h_1)}$$

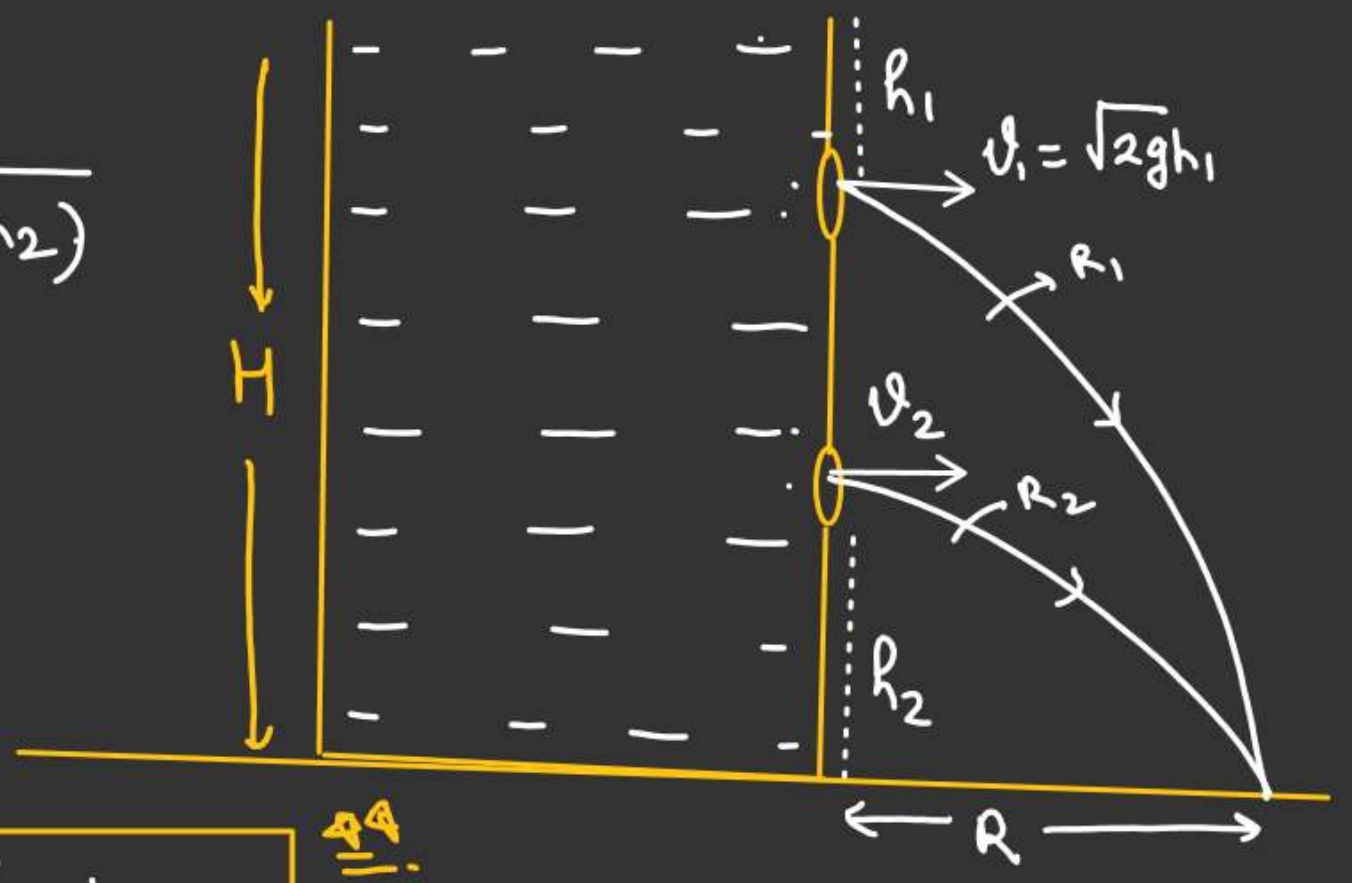
$$R_1 = R_2 = R$$

$$v_2 = \sqrt{2g(H-h_2)}$$

$$R_2 = 2\sqrt{(H-h_2)\,(H-(H-h_2))}$$

$$= 2\sqrt{(H-h_2)h_2}$$

$$R_1 = R_2$$



$$\boxed{h_1 = h_2}$$

$$h_1 (H-h_1) = (H-h_2)h_2$$

$$h_1 H - h_2 H = h_1^2 - h_2^2$$

$$(h_1-h_2)H = (h_1-h_2)(h_1+h_2)$$

$$(h_1-h_2)\underbrace{\left[H-(h_1+h_2)\right]}_{\neq 0} = 0$$

FLUID DYNAMICS

Bernoulli's in two or more than two immisible liquid

$\underline{1 \rightarrow 4}$

$$\cancel{P_{atm}} + \underbrace{\rho_1 g h_1 + \rho_2 g h_2 + \rho_3 g h_3} = \cancel{P_{atm}} + \frac{1}{2}\rho_3 v^2$$

$$v = \sqrt{\frac{2g(\rho_1 h_1 + \rho_2 h_2 + \rho_3 h_3)}{\rho_3}}$$