Nishant Jindal

Continuity.

(Online)

← Self Bound →

→ Preventive

Regular.

↓

Limit

200Qs

online

(2) Cont^y at Integral Pt.

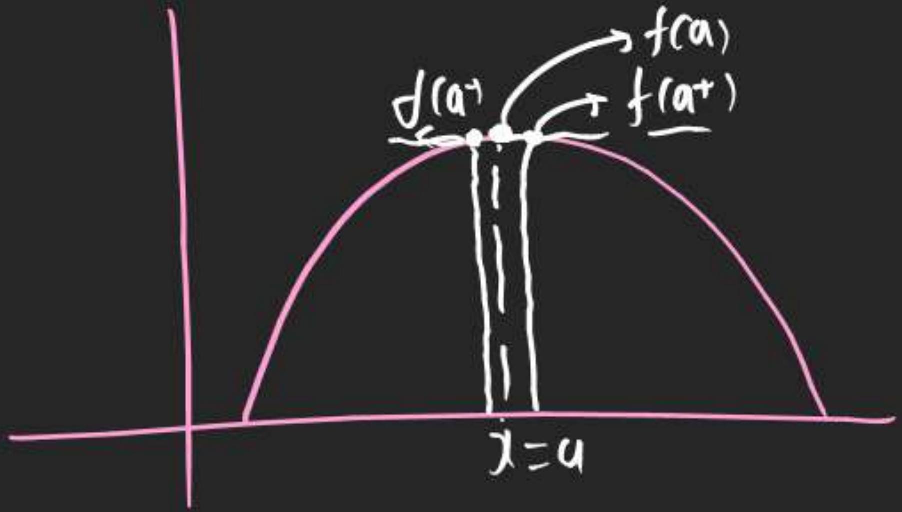
↓

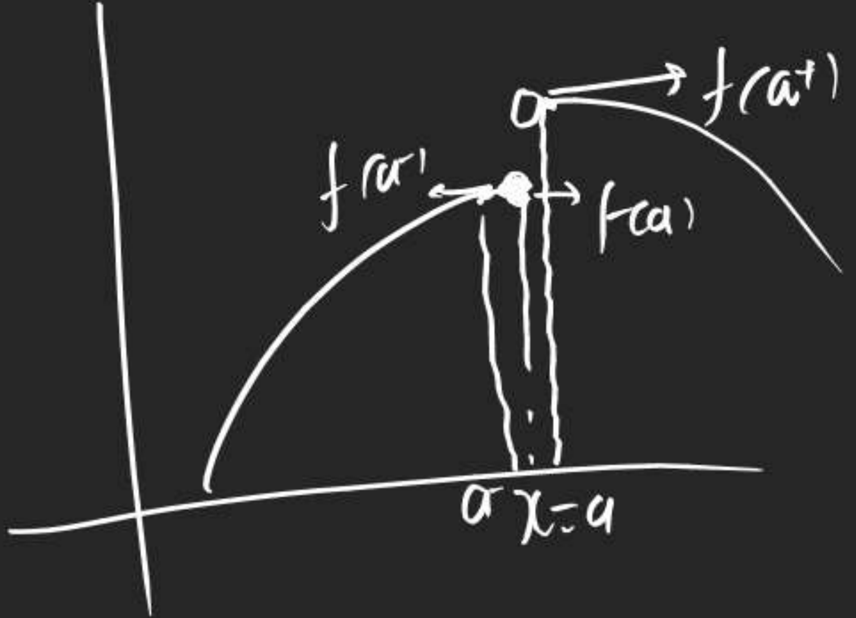
Interior

A fxn $y = f(x)$ is said to be Cont^s at $x = a$ if

$$\boxed{\underset{x=a}{\text{LHL}} = \underset{x=a}{\text{RHL}} = f(a)}$$

$$\boxed{\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x) = f(a)}$$

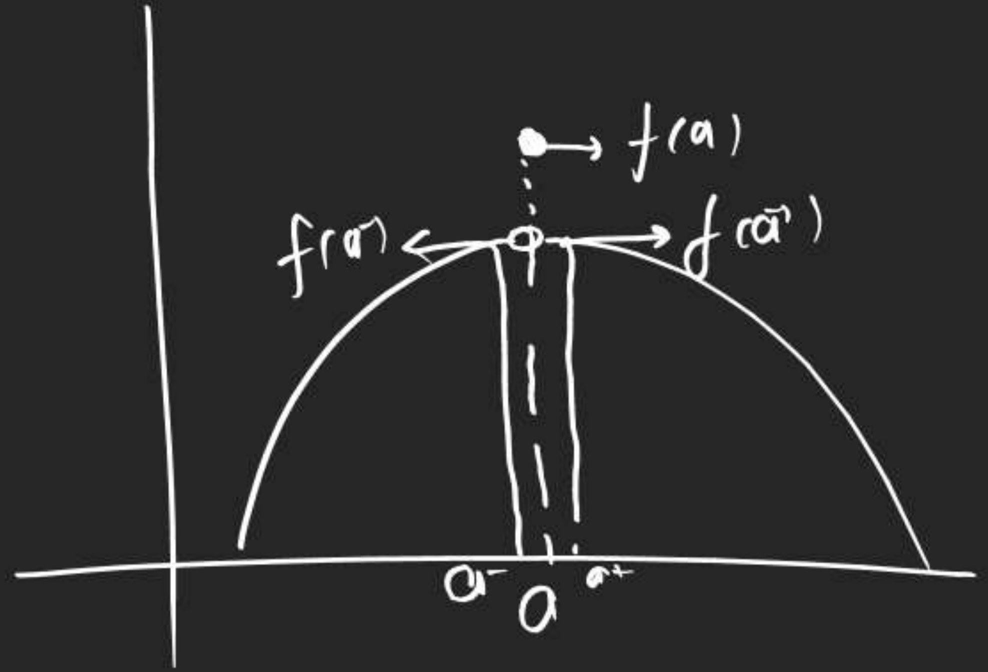




$f(a^-) = f(a) \neq f(a^+)$

fxn is not Cont^s

at $x = a$



fxn is D.C.

$f(a^-) \neq f(a) \neq f(a^+)$

But $f(a^+) = f(a^-)$

RHL = RHL

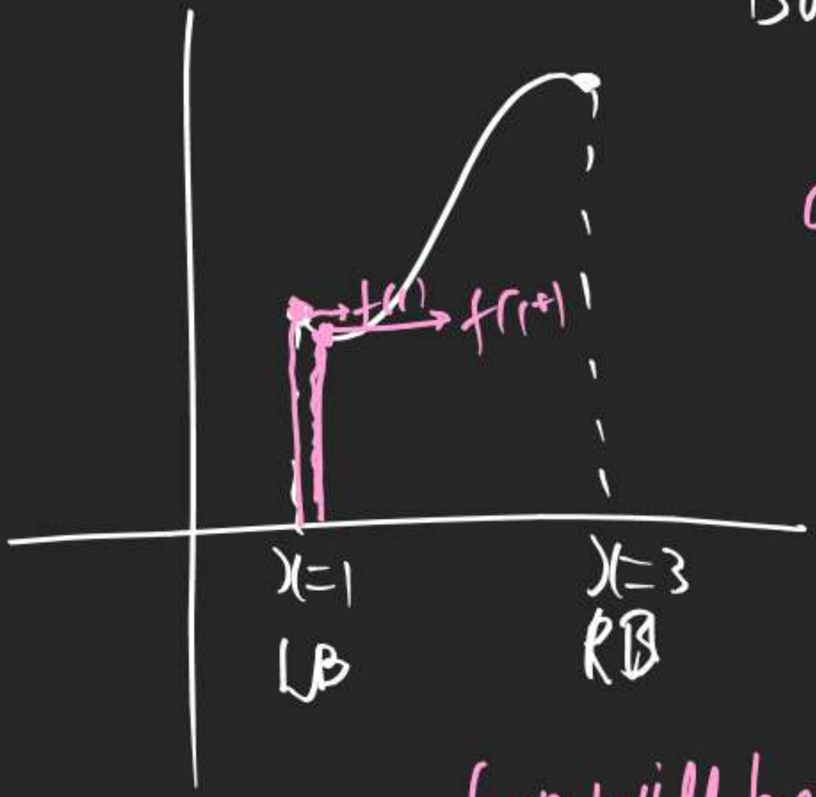
(3) Cont^y at Boundary Pt.

$x \in [1, 3]$ has 2 Boundaries

L.B RB

$x = 1$ & $x = 3$

is left Boundary Right Boundary



at $x=1$ only $f(1^+)$ & $f(1)$ is available.

So we will check these 2 only.

fxn will be cont^s at $x=1$ when

$f(1) = f(1^+)$

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a fxn is cont at $x=3$ (RB)

if $f(3) = f(3^-)$

(4) Cont^y in (a, b)

A fxn is Said to be cont in $x \in (a, b)$ if it is cont at every pt. betn $x = a$ & $x = b$.

Ex:→ Is $y = [x]$ cont in $\boxed{(4, 6)}$?

$y = [x]$ is D.C. at every Integer

⇒ it is discont at $x = 5$ ⇒ It is said to be D.C. at $x \in (4, 6)$

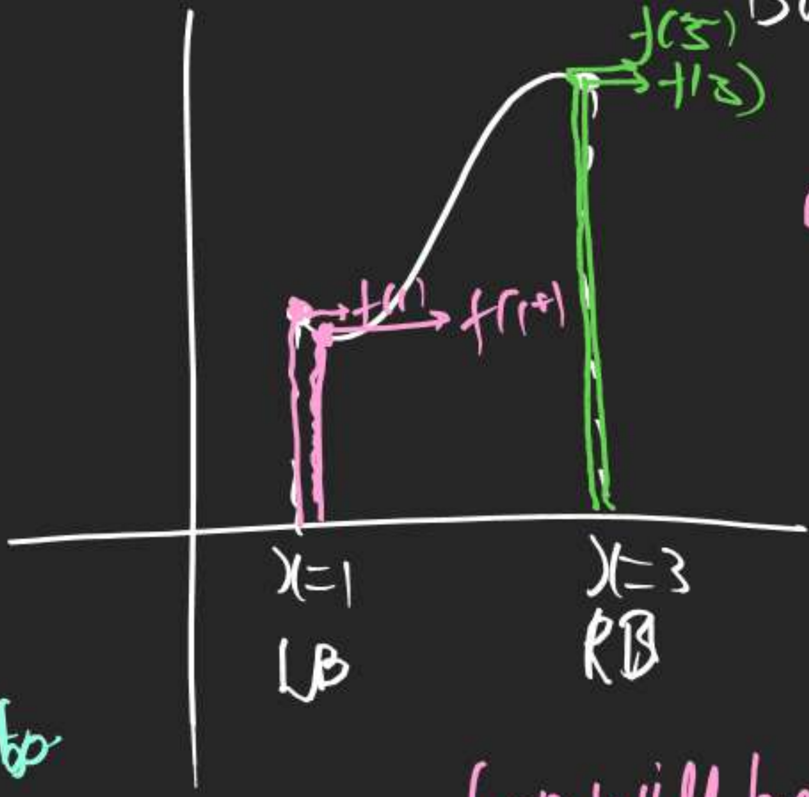
(3) Cont^y at Boundary Pt.

$x \in [1, 3]$ has 2 Boundaries $x = 1$ & $x = 3$.

↓ L.B ↓ RB

$x=1$ is left Boundary

$x=3$ Right Boundary



at $x = 1$ only $f(1^+)$ & $f(1)$ is available.

So we will check these 2 only.

fxn will be cont at $x = 1$ when $f(1) = f(1^+)$

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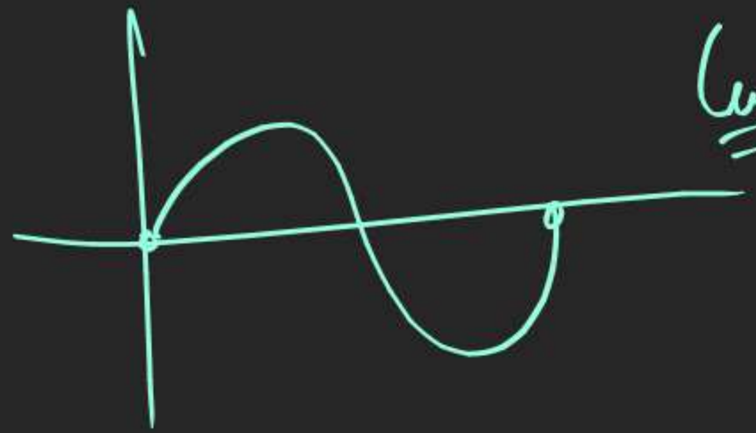
Q $y=[x]$ is D.C. at $(4,5)$?

as there is no Integer in $(4,5)$

$\Rightarrow$ It is completely cts.

Q $y=\sin x$ is cts in $(0,2\pi)$?



Cts

Q $y=\cot x$ is cts in $(0,2\pi)$

$y=\frac{\cos x}{\sin x} \rightarrow \boxed{\sin x=0}$ at $x=n\pi$

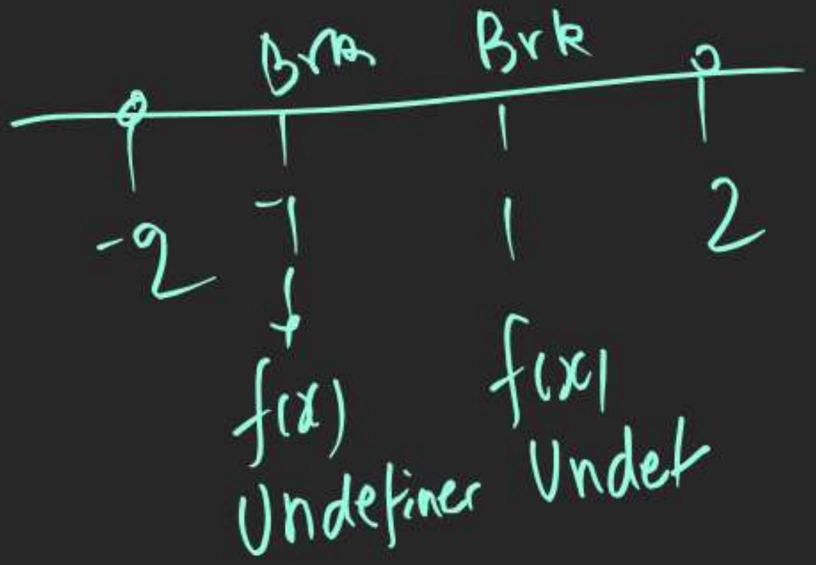
$x=\pi \in (0,2\pi)$

$x=\pi$ $f(x)=\cot x$ has a Break (D.C.)

$\Rightarrow x\in(0,2\pi)$ me D.C

Q $f(x)=\frac{x^3+1}{x^2-1}$ is cts / D.C. in $x\in(-2,2)$

$x^2-1=0 \Rightarrow x=1,-1$



D.C. in $(-2,2)$

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(5) Cont^y in $[a,b]$

$f(x)$ is said to be Cont^s in $[a,b]$

if ① it is Cont^s in (a,b)

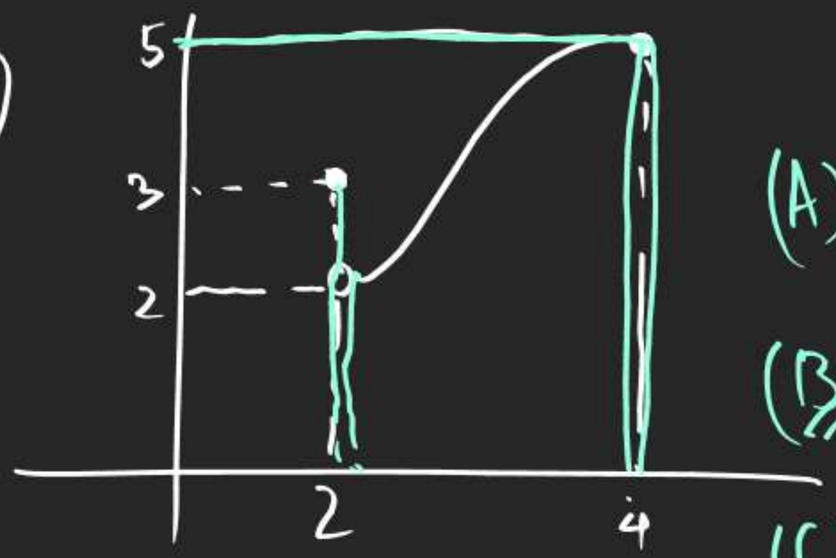
② Cont^s at $\boxed{x=a}$ $\rightarrow f(a)=f(a^+)$

LB

③ Cont^s at $\boxed{x=b}$ $\rightarrow f(b)=f(b^-)$

RB

Q



(A) fxn is Cont^s in $(2,4)$

(B) fxn is Cont^s in $(2,4]$

(C) fxn is Cont^s in $[2,4)$

(D) fxn is Cont^s in $[2,4]$

① $f(2)=3$

$f(2^+)=2$

D.C. at $x=2$

② fxn is Cont^s in $(2,4)$

③ $f(4)=5$

$f(4^-)=5$

Cont^s at $x=4$

fxn is Cont^s in $(2,4]$

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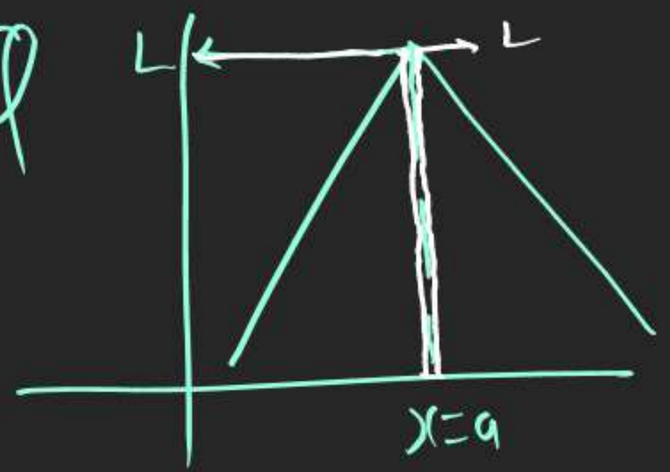
Q



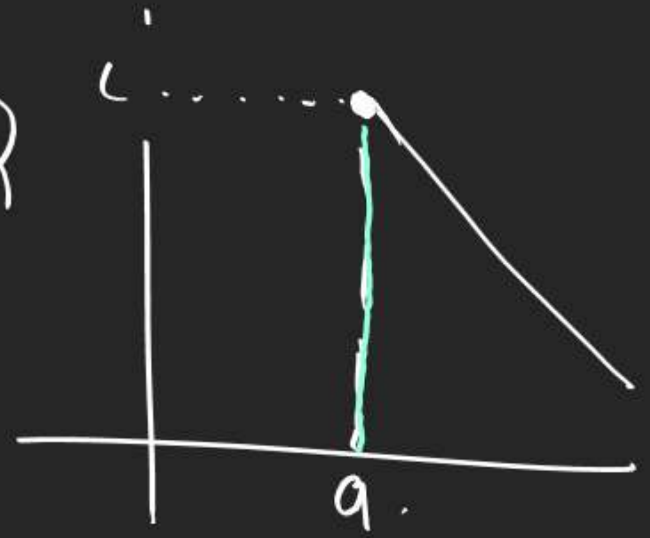
$f(a) = L$

$f(a^+) = L$

$f(a^-) = L$

} fxn cont's at $x=a$

Q



$f(a) = L$

$f(a^+) = L$

$f(a^-) = \times$

only $f(a)$ & $f(a^+)$ will be checked

Cont's $f(a) = f(a^+)$

Cont'y has mainly 3 Kinds of Qs.

(1) Type ≥ ≤ = type

$$f(x) = \begin{cases} x^2+9 & x>1 \\ 1/x & x=1 \\ 3-x & x<1 \end{cases}$$

(2) ≠ a, = a type

$$f(x) = \begin{cases} \dfrac{\sin^2(ax)}{x^2+3} & x \neq 0 \\ -2 & x = 0 \end{cases}$$

(3) Subjective type

$$f(x) = \frac{\sqrt{1+px^2} - \sqrt{1-px^2}}{x}, \quad (x \neq 0)$$

the $f(0)$

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Q $f(x) = \begin{cases} \frac{x}{[x]} & \boxed{x \neq 0} \\ 0 & x = 0 \end{cases}$ Check:

Type2

1) Take limit at $x \to a$ $(x \neq a)$

2) Match Ur Answer with $x = a$

Cont^y at $x = 0$

L.V. $\Rightarrow \lim_{x \to 0} \frac{x}{[x]}$ Aakarshan Mahsus

TWIST

LHL

$\lim_{x \to 0^-} \frac{x}{[x]}$

$x = 0 - h$.

$\lim_{h \to 0} \frac{-h}{[0-h]}$

$\lim_{h \to 0} \frac{+h}{+1} = 0 = L.V.$

$f(0) = 0$ Conts

RHL

$\lim_{x \to 0^+} \frac{x}{[x]}$

$\underline{x = 0 + h}$

$\lim_{h \to 0} \frac{h}{[h]}$

Dom

$[x] \neq 0$

$x \notin [0, 1)$

$0 + h \in [0, 1)$ & we can not take it

Type2

Q $f(x) = \begin{cases} \lim_{x \to 3} \frac{x^2 - 9}{x - 3} = 6 & x \neq 3 \\ 6 & x = 3 \end{cases}$

Check Cont^y at $x = 3$?

$\lim_{x \to 3} \frac{\cancel{x^2 - 9}^{\,x+3}}{\cancel{x - 3}} = 3 + 3 = 6$

LV = 6

& $f(3) = 6$

L.V. = f(3)

Cont^s at $x = 3$.

Q $f(x) = \begin{cases} \lim_{x \to 0} \frac{\sin x}{x} = 1 & x \neq 0 \\ 1 & x = 0 \end{cases}$

(Check Cont^y at $x = 0$)

L.V. $\Rightarrow \lim_{x \to 0} \frac{\sin x}{x} = 1$

$f(0) = 1$

$\Rightarrow$ L.V. $= f(0) = 1$ fxn in Cont^s

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Q $f(x) = \begin{cases} \dfrac{\sin x}{[x]} & x \neq 0 \\ 0 & x = 0 \end{cases}$

Cont[y] nahi Puchh Rha

$\leftarrow \lim_{x \to} f(a) = ?$

$\lim_{x \to 0} \dfrac{\sin x}{[x]} \rightarrow$ Aakarshan Mahsus

$\therefore \lim_{x \to 0} f(x) = 0$

LHL $\boxed{x = 0 - h}$

$\lim_{h \to 0} \dfrac{\sin(-h)}{[-h]}$

$\lim_{h \to 0} \dfrac{+\sin h}{+1} = 0$

RHL $\boxed{0 + h}$

$\dfrac{\sin h}{[0+h]}$

X

$\lim_{h \to 0} \dfrac{\sin h}{0} \to \infty$

Exact

$[x] \neq 0$

$x \in [0, 1)$

$h \in [0, 1) \to$ Cannot be taken

Q Type 2

$f(x) = \begin{cases} \lim_{x \to 2} (x-2) \cdot \sin \dfrac{1}{(x-2)} & \boxed{x \neq 2} \\ 2 & x = 2 \end{cases}$

Is fxn Cont[s] at $x = 2$?

L.V $\Rightarrow \lim_{x \to 2} (x-2) \sin \dfrac{1}{x-2} = 0 \times \sin \dfrac{1}{0}$

$= 0 \times \sin \infty$

$= 0 \times$ [Any Value bet[n] -1 to $+1$]

$\left.\begin{array}{l} \text{L.V.} = 0 \\ f(2) = 2 \end{array}\right\}$ Not match D.C.

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Q $f(x)=\begin{cases}\lim_{x\to 2}(x-2)\cdot\sin\frac{1}{(x-2)} = 0 & x\neq 2\\ K & x=2\end{cases}$

If fxn Cnt^s at $x=2$ then $K=?$

If fxn Cnt^s at $x=2$

then $f(2)=$ L.V.

$\Rightarrow \boxed{K=0}$

Q Type 2

$f(x)=\begin{cases}\lim_{x\to 2}(x-2)\cdot\sin\frac{1}{(x-2)} & \boxed{x\neq 2}\\ 2 & x=2\end{cases}$

Is fxn Cnt^s at $x=2$?

L.V $\Rightarrow \lim_{x\to 2}(x-2)\sin\frac{1}{x-2} = 0\times\sin\frac{1}{0}$

$= 0\times\sin\infty$

$= 0\times$ [Any Value bet^n -1 to $+1$]

$\left.\begin{array}{l}\text{L.V.} = 0\\ f(2)=2\end{array}\right\}$ Not match

D.C.

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as fxn Cnt^s at $x=a \Rightarrow LHL = RHL = f(a)$

Q $f(x) = \begin{cases} x^2 & x>1 \\ 1 & x=1 \\ \frac{1}{x} & x<1 \end{cases}$

Type 1

1) L.V. of fxn in front of $x>a$ is RHL
2) L.V. of fxn in front of $x<a$ is LHL
(3) $f(a)$
(4) Compare

Check Cnt^y at $x=1$?

LHL

$\lim_{x\to 1^-} \frac{1}{x}$

$\lim_{h\to 0} \frac{1}{1-h}$

$\frac{1}{1-0} = 1$

RHL

$\lim_{x\to 1^+} x^2$

$\lim_{h\to 0} (1+h)^2$

$(1+0)^2$

$=1$

$f(1)=1$

$f(1) = LHL = RHL$

Cnt^s at $x=1$

Q Type 2 $f(x) = \begin{cases} \frac{8^x - 2^x - 4^x + 1}{x^2} & x>0 \\ e^x \sin x + \pi x + \lambda \ln 4 & x \le 0 \end{cases}$

find λ if fxn in Cnt^s at $x=0$

LHL

$\lim_{x\to 0^-} e^x \sin x + \pi x + \lambda \ln 4$

$e^0 \sin 0 + \pi \cdot 0 + \lambda \ln 4$

$1 \times 0 + 0 + \lambda \ln 4$

$\lambda \ln 4$

RHL

$\lim_{x\to 0^+} \frac{(2\cdot 4)^x - 2^x - 4^x + 1}{x^2}$

$\lim_{x\to 0} \frac{2^x \cdot 4^x - 2^x - 4^x + 1}{x^2}$

$\frac{2^x(4^x - 1) - 1(4^x - 1)}{x^2}$

$\lim_{x\to 0^+} \left(\frac{4^x - 1}{x}\right)\left(\frac{2^x - 1}{x}\right) = \ln 4 \cdot \ln 2$

$\Rightarrow \lambda \ln 4 = \ln 4 \times \ln 2$

$\Rightarrow \boxed{\lambda = \ln 2}$

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Q $f(x) = \begin{cases} \boxed{x} + a\sqrt{2}\,\boxed{\sin x} & 0 \le x < \frac{\pi}{4} \\ 2x\cot x + b & \frac{\pi}{4} \le x < \frac{\pi}{2} \\ a\cos 2x - b\sin x & \frac{\pi}{2} \le x \le \pi \end{cases}$

TP1, LHL, TP.2, RHL

if fxn is cont^s in $x \in [0, \pi]$ then find a, b.

(1) as fxn is given cont^s

& We also feel that fxn are not very confusing

So we will check at turning pt. only

(2) here Turning pt. are $x = \frac{\pi}{4}$ & $\frac{\pi}{2}$

cont^s at $x = \frac{\pi}{4}$

$f\left(\frac{\pi}{4}\right) = 2\left(\frac{\pi}{4}\right)\cdot\cot\frac{\pi}{4} + b = \frac{\pi}{2} + b$

$f\left(\frac{\pi}{4}^+\right) = \lim_{h\to 0} 2\left(\frac{\pi}{4}+h\right)\cdot\cot\left(\frac{\pi}{4}+h\right) + b = \frac{\pi}{2}\times 1 + b = \frac{\pi}{2} + b$

LHL $f\left(\frac{\pi}{4}^-\right) = \lim_{h\to 0}\left(\frac{\pi}{4}-h\right) + a\sqrt{2}\sin\left(\frac{\pi}{4}-h\right)$

$= \frac{\pi}{4} + a\cancel{\sqrt{2}} \times \frac{1}{\cancel{\sqrt{2}}} = \frac{\pi}{4} + a$

$\frac{\pi}{2} + b = \frac{\pi}{4} + a \Rightarrow \boxed{a - b = \frac{\pi}{4}}$

$\boxed{x = \frac{\pi}{2}}$

$f\left(\frac{\pi}{2}^-\right) = 2\left(\frac{\pi}{2}-h\right)\cdot\cot\left(\frac{\pi}{2}-h\right) + b = \pi \times 0 + b = b$

RHL $f\left(\frac{\pi}{2}^+\right) = a\cos\left(\cancel{2}\times\frac{\pi}{\cancel{2}}\right) - b\sin\left(\frac{\pi}{2}^+\right) = -a - b$

$f\left(\frac{\pi}{2}\right) = a\cos\left(2\times\frac{\pi}{2}\right) - b\sin\frac{\pi}{2} = -a - b$

$-a - b = -a - b = b$

$-a = 2b$
$a - b = \frac{\pi}{4}$

$-3b = \frac{\pi}{4}$

$b = -\frac{\pi}{12}$

$a = -2 \times -\frac{\pi}{12} = \frac{\pi}{6}$

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Q $f(x) = \begin{cases} \boxed{x} + a\sqrt{2}\,\boxed{\sin x} & 0 \le x < \frac{\pi}{4} \\ 2x\cot x + b & \frac{\pi}{4} \le x \le \frac{\pi}{2} \\ a\cos 2x - b\sin x & \frac{\pi}{2} \le x \le \pi \end{cases}$

TP.1 LHL TP.2 RHL

if fxn is cnt^s in $x \in [0, \pi]$ then find a, b.

$f(x) = \begin{cases} x + a\sqrt{2}\sin x & 0 \le x < \frac{\pi}{4} \\ 2x\cot x + b & \frac{\pi}{4} \le x < \frac{\pi}{2} \\ a\cos 2x - b\sin x & \frac{\pi}{2} \le x \le \pi \end{cases}$

$x = \frac{\pi}{4}$

$LHL = \frac{\pi}{4} + a\sqrt{2} \times \frac{1}{\sqrt{2}}$

$a + \frac{\pi}{4}$

$RHL = 2 \times \frac{\pi}{4}\cot\frac{\pi}{4} + b$

$= \frac{\pi}{2} + b$

$a + \frac{\pi}{4} = \frac{\pi}{2} + b$

$\boxed{a - b = \frac{\pi}{4}}$

$-3b = \frac{\pi}{4}$

$\boxed{b = -\frac{\pi}{12}}$

$x = \frac{\pi}{2}$

$LHL = 2 \times \frac{\pi}{2}\cancel{\cot\frac{\pi}{2}} + b$

$= b$

$RHL = a\cos\pi - b\sin\frac{\pi}{2}$

$= -a - b$

$-a - b = b$

$\boxed{a = -2b}$

$\boxed{a = \frac{\pi}{6}}$

Q $f: R \to R$

$$f(x) = \begin{cases} \sin x - e^x & \boxed{x \le 0} \\ a + [-x] & \boxed{0 < x} < 1 \\ 2x - b & x \ge 1 \end{cases}$$

$x = 0 + h$

$0 + 3 = 3$

Cont^s in $x \in R$ then $a + b = ?$

$a + [-.99]$

$x = 0$

LHL $= \sin 0 - e^0 = -1$

RHL $= a + [-h] = a - 1$

$a - \cancel{1} = -\cancel{1}$

$a = 0$

$x = 1$

LHL $= a + [-(1-h)] = a - 1$

RHL $= 2 \times 1 - b$

$a - 1 = 2 - b$

$0 - 1 = 2 - b$

$\boxed{b = 3}$