

$$\underline{3.} \text{ (iv) } \cancel{\cos 24^\circ} + \cancel{\cos 55^\circ} + \cancel{\cos 125^\circ} + \cancel{\cos 204^\circ} + \cos 300^\circ$$

$\frac{180+24^\circ}{} \downarrow$
 $360-60$

$$\text{|| } \cancel{\sin 120^\circ} + \cancel{\cos 150^\circ} - \cancel{\tan 180^\circ} = 0$$

$\swarrow \quad \searrow$
 $\frac{\sqrt{3}}{2} \quad \frac{\sqrt{3}}{2}$

$$= \cos 60^\circ$$

$$\text{(vi) } \sin \left(\frac{1560 - 1440}{2} \right) + \cos \left(\frac{-3030 + 2880}{2} \right) + \tan \left(\frac{-1260 + 1080}{2} \right) = \frac{1}{2}$$

4. $2\sin^2 x + 2\sin x + \sin x + 1 = 0$

$x \in (0, \pi/2)$
 $\sin x = \frac{1}{3}$
 $(2\sin x + 1)(\sin x + 1) = 0$

$\sin x = -\frac{1}{2}, -1$



$\sin \beta = \frac{1}{4}$

$\beta \in (0, \frac{\pi}{2})$

$\frac{7\pi}{6}, \frac{11\pi}{6}, \frac{3\pi}{2}$

$180 + 30$
 $360 - 30$

$\pi + \alpha, 2\pi - \alpha$
 $\pi + \beta, 2\pi - \beta$

$\rightarrow 3\pi$
 $\sin x = -\frac{1}{2}$

$1+t^2 > 1$

$1-t^2 > 1$

6π $\left| \frac{1+t^2}{1-t^2} \right| > 1$

Find num of all values satisfying in $[0, 2\pi]$

$(3\sin x + 1)(4\sin x + 1) = 0$

Q10 \rightarrow leave

$$\frac{\sin A (\cos(B+C)) + \cos A \sin(B+C)}{\cos A \cos B \cos C}$$

$$= \frac{\sin A \cos B \cos C - \sin A \sin B \sin C + \cos A \sin B \cos C + \cos A \sin C \cos B}{\cos A \cos B \cos C}$$

$$\sin(A+B) = \frac{3}{5} \times \left(-\frac{12}{13}\right) + \frac{5}{13} \left(-\frac{4}{5}\right)$$

$$\underline{\text{Q.}} \quad \cot(A-B) = \frac{1 + \tan A \tan B}{\tan A - \tan B} = \frac{1 + \left(-\frac{4}{3}\right)}{x}$$

$$\frac{\frac{1}{\tan A} - \frac{1}{\tan B}}{-x} = y$$



$$\frac{|\sec \frac{\pi}{12}|}{\tan \frac{\pi}{12}} - \frac{|\csc \frac{\pi}{12}|}{\cot \frac{\pi}{12}}$$

$$= \frac{-2 \left(\cos \frac{\pi}{12} - \sin \frac{\pi}{12} \right)}{2 \sin \frac{\pi}{12} \cos \frac{\pi}{12}}$$

$$= \frac{2 \left(\frac{\sqrt{3}+1}{2\sqrt{2}} - \frac{\sqrt{3}-1}{2\sqrt{2}} \right)}{1}$$

$$= \frac{\cos 22^\circ}{\sin 34^\circ \cos 34^\circ} = \frac{2 \cos 22^\circ}{\sin 68^\circ} = 2$$

10. $\cos 68^\circ \cos 22^\circ$

$\sin 56^\circ$ (circled)
 $\cos 34^\circ$ (pointing to $\sin 56^\circ$)
 $\sin 34^\circ \sin 22^\circ$

$$\underline{12.} \quad \tan \frac{\pi}{2} = \frac{1 - \cos \pi}{\sin \pi} = \frac{1 - \left(-\frac{4}{5}\right)}{-\frac{3}{5}} = -3.$$

$\tan \pi =$



$$a \sin x + b \cos x$$

$$f(x) = \sin x + \sqrt{3} \cos x = 2 \left(\underbrace{\frac{1}{2}}_{\cos \frac{\pi}{3}} \sin x + \underbrace{\frac{\sqrt{3}}{2}}_{\sin \frac{\pi}{3}} \cos x \right) = 2 \sin \left(x + \frac{\pi}{3} \right)$$

find range

$$R_f = [-2, 2]$$

$$D_f = \mathbb{R}$$

1. Find the domain and range of

$$(i) f(x) = \frac{3\sin x - 4\cos x + 15}{10}$$

$$R_f = [1, 2]$$

$$D_f = \mathbb{R}$$

$$10 = 15 - 5 \leq 3\sin x - 4\cos x \leq 5$$

$$3\sin x - 4\cos x + 15 \leq 15 + 5 = 20$$

$$(ii) f(x) = \sin^2\left(\frac{15\pi}{8} - 4x\right) - \sin^2\left(\frac{17\pi}{8} - 4x\right)$$

$$= \sin\frac{\pi}{4} \sin(-8x) = -\sin 8x$$

$$\sin\left(-\frac{2\pi}{8}\right) \sin\left(\frac{32\pi}{8} - 8x\right) = -\sin 8x$$

$$D_f = \mathbb{R}$$

$$R_f = \left[-\frac{1}{2}, \frac{1}{2}\right]$$

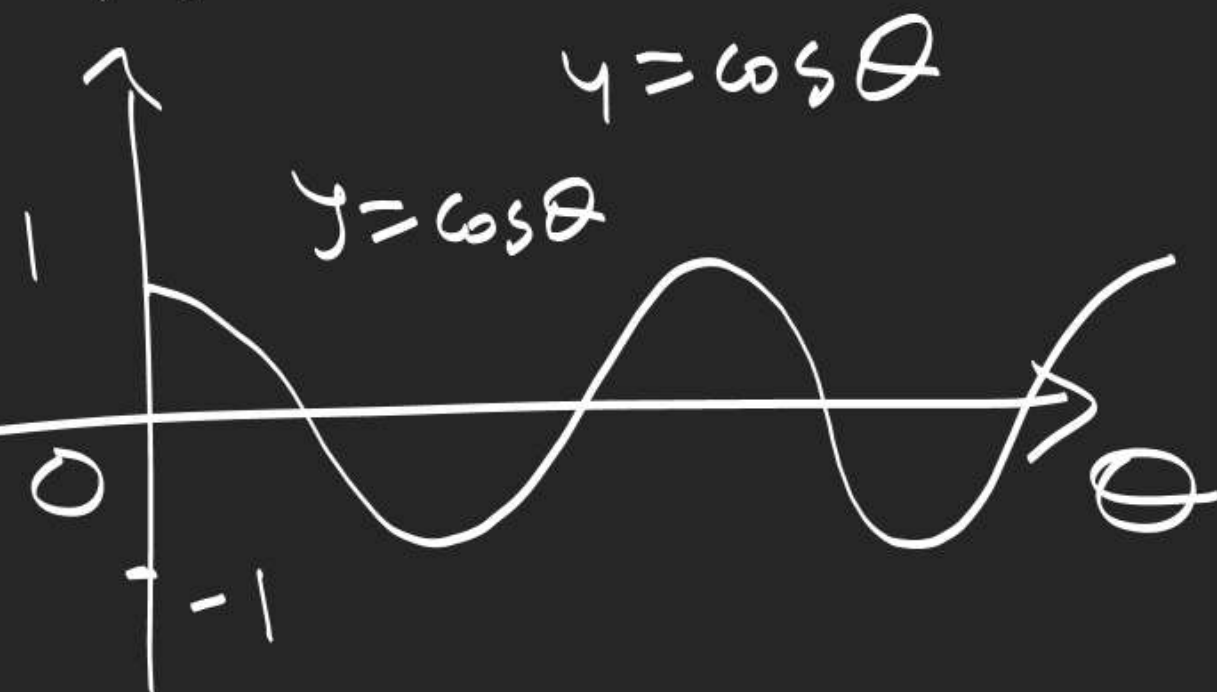
1. $f(x) = \cos(x^2)$, find Domain & Range

$$D_f = \mathbb{R}$$

$$R_f = [-1, 1]$$

$$x^2 \in [0, \infty)$$

$$\theta = x^2$$



$$f(x) = \tan(x^2)$$

$$\theta = x^2 \geq 0$$

$$y = \tan \theta$$

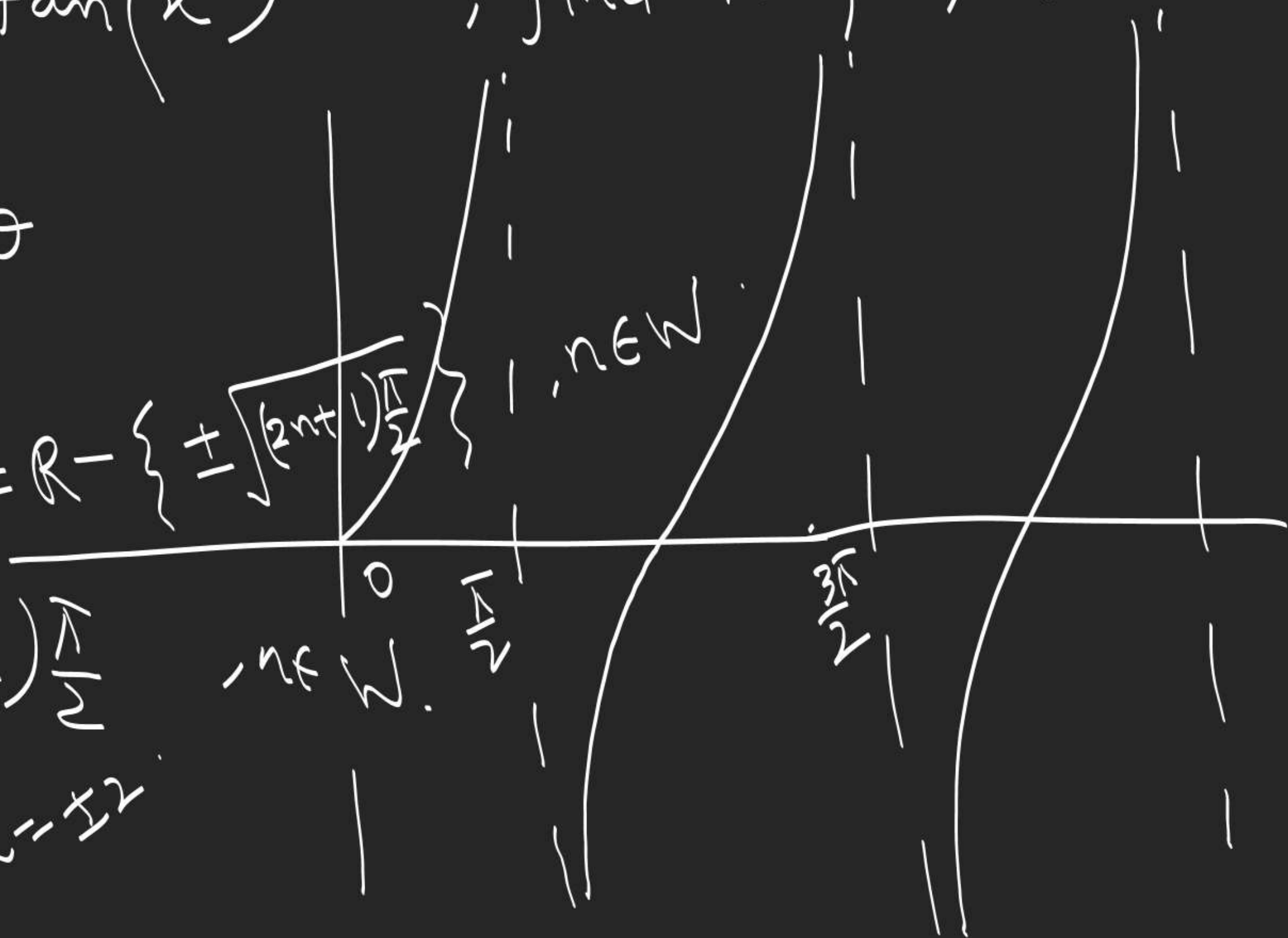
$$R_f = R$$

$$D_f = R - \left\{ \pm \sqrt{(2n+1)\frac{\pi}{2}} \right\} \quad n \in \mathbb{N}$$

$$x^2 = (2n+1)\frac{\pi}{2}$$

$$x = \pm \sqrt{(2n+1)\frac{\pi}{2}}$$

, find range & domain



2. Find domain & range of

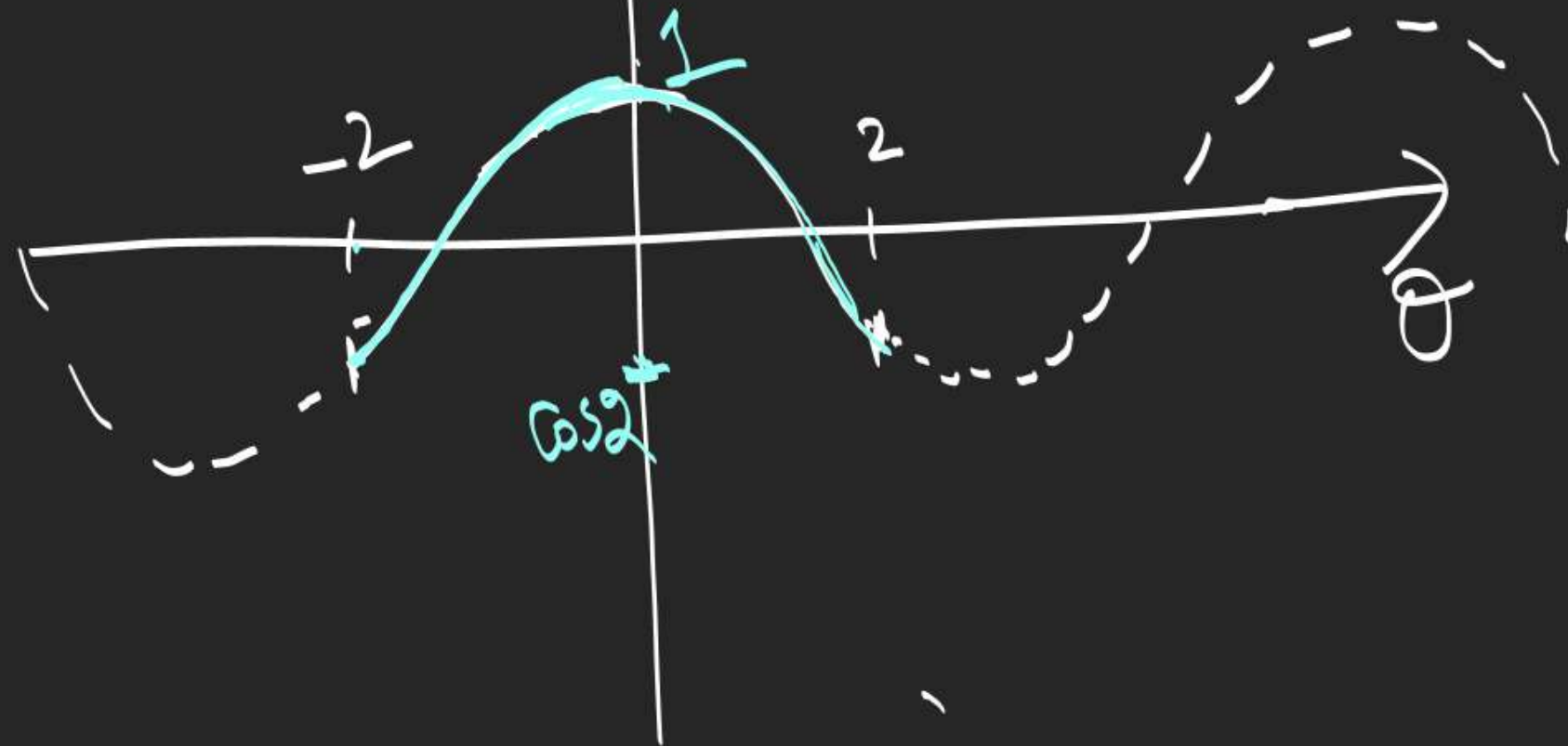
$$f(x) = \cos(2\sin x)$$

$$D_f = \mathbb{R}$$

$$f(\theta) = \cos \theta$$

$$\theta \in [-2, 2]$$

$$R_f = [\cos 2, 1] \checkmark$$



Find domain & range of

$$1. \quad f(x) = 3 \cos\left(x + \frac{\pi}{3}\right) + 5 \cos x + 3$$

$$\boxed{D_f = \mathbb{R}} \quad \boxed{[-4, 10]} = 3 \left(\frac{1}{2} \cos x - \frac{\sqrt{3}}{2} \sin x \right) + 5 \cos x + 3$$

$$R_f = [3 - 7, 3 + 7] = \left[\frac{13}{2} \cos x - \frac{3\sqrt{3}}{2} \sin x + 3 \right]$$

$$-7 = -\sqrt{\frac{169}{4} + \frac{27}{4}} \leq \frac{13}{2} \cos x - \frac{3\sqrt{3}}{2} \sin x \leq \sqrt{\frac{169}{4} + \frac{27}{4}} = 7$$

2. $f(x) = \sin\left(x + \frac{\pi}{3}\right) + 3\cos\left(x - \frac{\pi}{3}\right)$

$$= \frac{1}{2}\sin x + \frac{\sqrt{3}}{2}\cos x + 3\left(\frac{1}{2}\cos x + \frac{\sqrt{3}}{2}\sin x\right)$$

$$= \left(\frac{3+\sqrt{3}}{2}\right)\cos x + \left(\frac{3\sqrt{3}+1}{2}\right)\sin x$$

$$\frac{9+3+6\sqrt{3}+27+1}{4} \in \left[-\sqrt{\frac{(3+\sqrt{3})^2}{4} + \frac{(3\sqrt{3}+1)^2}{4}}, \sqrt{\frac{(3+\sqrt{3})^2}{4} + \frac{(3\sqrt{3}+1)^2}{4}} \right]$$

$$\frac{40+10\sqrt{3}}{4}$$

$$R_f = \left[-\sqrt{10+3\sqrt{3}}, \sqrt{10+3\sqrt{3}} \right]$$

3. $f(x) = \cos^2 x - 4 \cos x + 13$

$$\cos^2 x - 2(2) \cos x + 4 = (\cos x - 2)^2 + 9$$

$$R_f = [10, 18]$$

$$-1 \leq \cos x \leq 1$$

$$-3 \leq \cos x - 2 \leq -1$$

4. $f(x) = \cos 2x + 3 \sin x$

$$\Rightarrow 1 \leq (\cos x - 2)^2 \leq 9$$

$$1+9 \leq (\cos x - 2)^2 + 9 \leq 9+9$$

$$-1 < x < 2 \Rightarrow 0 \leq x^2 < 4$$

$$-3 < x < -1$$

$$-3 < x < 2$$

$$x^2 \in [0, 9)$$

$$1 < x < 3$$

$$\Rightarrow 1 < x^2 < 9$$

$$1 < x^2 < 9$$

$$\mathcal{D}_f = \mathbb{R}$$

$$f(x) = \cos 2x + 3 \sin x$$

$$= 1 - 2 \sin^2 x + 3 \sin x$$

$$= 1 - 2 \left(\sin^2 x - \frac{3}{2} \sin x \right)$$

$$= 1 - 2 \left(\left(\sin x - \frac{3}{4} \right)^2 - \frac{9}{16} \right) = \frac{17}{8} - 2 \left(\sin x - \frac{3}{4} \right)^2$$

$$-\frac{7}{4} = -1 - \frac{3}{4} \leq \sin x - \frac{3}{4} \leq 1 - \frac{3}{4} = \frac{1}{4}$$

$$0 \leq \left(\sin x - \frac{3}{4} \right)^2 \leq \frac{49}{16}$$

$$-\frac{49}{8} \leq -2 \left(\sin x - \frac{3}{4} \right)^2 \leq 0$$

$$R_f = \left[\frac{17}{8} - \frac{49}{8}, \frac{17}{8} + 0 \right]$$

$$= \left[-4, \frac{17}{8} \right]$$

Ex-I (Complete)