HOMEWORK–2

SOLUTION

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1. DIFFERENTIATION OF IMPLICIT FUNCTION

1. If $y=\sqrt{\sin x+y}$, then $\frac{dy}{dx}=$

(A) $\frac{\sin x}{2y-1}$ (B) $\frac{\sin x}{1-2y}$ (C) $\frac{\cos x}{1-2y}$ (D) $\frac{\cos x}{2y-1}$

Ans. **(D)**

Sol. $y=\sqrt{\sin x+y}$

squaring both side

$y^2=\sin x+y$

differentiating w.r.t. to x

$2yy'=\cos x+y'$

$y'=\frac{\cos x}{2y-1}$

2. If $ax^2+2hxy+by^2=0$, then $\frac{dy}{dx}$ equals-

(A) $\frac{ax+hy}{hx+by}$ (B) $-\frac{ax+hy}{hx+by}$ (C) $\frac{hx+by}{ax+hy}$ (D) $-\frac{hx+by}{ax+hy}$

Ans. **(B)**

Sol. $ax^2+2hxy+by^2=0$

$$2ax+2hx\frac{dy}{dx}+2hy\cdot 1+2by\frac{dy}{dx}=0$$

$$\frac{dy}{dx}(hx+by)=-(ax+hy)$$

$$\frac{dy}{dx}=-\left(\frac{ax+hy}{hx+by}\right)$$

3. If $x\sqrt{y}+y\sqrt{x}=1$, then $\frac{dy}{dx}$ equals-

(A) $-\frac{y+2\sqrt{xy}}{x+2\sqrt{xy}}$ (B) $-\sqrt{\frac{x}{y}}\left(\frac{y+2\sqrt{xy}}{x+2\sqrt{xy}}\right)$ (C) $-\sqrt{\frac{y}{x}}\left(\frac{y+2\sqrt{xy}}{x+2\sqrt{xy}}\right)$ (D) $\frac{x}{y}$

Ans. **(C)**

Sol. $x\sqrt{y}+y\sqrt{x}=1$

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$$x \cdot \frac{1}{2\sqrt{y}}\frac{dy}{dx} + \sqrt{y} \cdot 1 + y\frac{1}{2\sqrt{x}} + \sqrt{x}\frac{dy}{dx} = 0$$

$$\frac{dy}{dx}\left[\frac{x}{2\sqrt{y}} + \sqrt{x}\right] = -\left(\frac{y}{2\sqrt{x}} + \sqrt{y}\right)$$

$$\frac{dy}{dx}\left(\frac{x + 2\sqrt{xy}}{2\sqrt{y}}\right) = -\left(\frac{y + 2\sqrt{xy}}{2\sqrt{x}}\right)$$

$$\frac{dy}{dx} = -\frac{\sqrt{y}}{\sqrt{x}}\left(\frac{y + 2\sqrt{xy}}{x + 2\sqrt{xy}}\right)$$

4. If $e^x \sin y - e^y \cos x = 1$, then $\frac{dy}{dx}$ equals-

(A) $\frac{e^x \sin y + e^y \sin x}{e^y \cos x - e^x \cos y}$
(B) $\frac{e^x \sin y + e^y \sin x}{e^y \cos x + e^x \cos y}$
(C) $\frac{e^x \sin y - e^y \sin x}{e^y \cos x - e^x \cos y}$
(D) $\frac{e^x}{e^y}$

Ans. **(A)**

Sol. $e^x \sin y - e^y \cos x = 1$

$$e^x \cdot \cos y\frac{dy}{dx} + \sin y e^x - e^y(-\sin x) - \cos x.$$

$$e^y \cdot \frac{dy}{dx} = 0$$

$$\frac{dy}{dx}(e^x \cos y - e^y \cos x) = -(e^x \sin y + e^y \sin x)$$

$$\frac{dy}{dx} = -\left(\frac{e^x \sin y + e^y \sin x}{e^x \cos y - e^y \cos x}\right)$$

$$= \frac{e^x \sin y + e^y \sin x}{e^y \cos x - e^x \cos y}$$

5. If $\sqrt{1 - x^2} + \sqrt{1 - y^2} = a(x - y)$, then the value of dy/dx is -

(A) $\frac{\sqrt{1-x^2}}{\sqrt{1-y^2}}$
(B) $\frac{\sqrt{1-y^2}}{\sqrt{1-x^2}}$
(C) $-\frac{\sqrt{1-x^2}}{\sqrt{1-y^2}}$
(D) $-\frac{\sqrt{1-y^2}}{\sqrt{1-x^2}}$

Ans. **(B)**

Sol. $\frac{\sqrt{1-y^2}}{\sqrt{1-x^2}}$

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Substituting $x = \sin\theta$ and $y = \sin\phi$ in the given equation, we get

$\cos\theta + \cos\phi = a(\sin\theta - \sin\phi)$

$$\Rightarrow 2\cos\frac{\theta+\phi}{2}\cdot\cos\frac{\theta-\phi}{2} = 2a\cos\frac{\theta+\phi}{2}\cdot\sin\frac{\theta-\phi}{2}$$

$$\Rightarrow \cot\frac{\theta-\phi}{2} = a \Rightarrow \theta - \phi = 2\cot^{-1}a$$

$$\Rightarrow \sin^{-1}x - \sin^{-1}y = 2\cot^{-1}a$$

Differentiating with respect to x, we get

$$\frac{1}{\sqrt{1-x^2}} - \frac{1}{\sqrt{1-y^2}}\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = \frac{\sqrt{1-y^2}}{\sqrt{1-x^2}}$$

6. If $x^3\cos(xy) + y^3\sin(xy) + 1 = 0$, then $\frac{dy}{dx}$ equals

(A) $\frac{x^3y\tan(xy)-(3x^2+y^4)}{xy^3+(3y^2-x^4)\tan xy}$ (B) $\frac{x^3y\tan(xy)+(3x^2+y^4)}{xy^3-(3y^2-x^4)\tan xy}$

(C) $\frac{x^3y-(3x^2+y^4)\tan(xy)}{xy^3\tan(xy)+(3y^2-x^4)}$ (D) $x^2y\tan(xy) + 3x^2$

Ans. **(A)**

Sol. $f(x) = x^3\cos xy + y^3\sin xy + 1$

$$\frac{\partial f}{\partial y} = x^3\cdot(-\sin xy)\cdot y + \cos xy\cdot 3x^2 + y^3\cdot\cos xy\cdot y$$

$$= -x^3y\sin xy + 3x^2\cos xy + y^4\cos xy$$

$$\&\frac{\partial f}{\partial y} = x^3(-\sin xy)\cdot x + y\cos xy\cdot x + \sin xy\cdot 3y^2$$

$$= -x^4\sin xy + y^3x\cos xy + 3y^2\sin xy$$

$$\therefore \frac{dy}{dx} = -\frac{\partial f/\partial x}{\partial f/\partial y}$$

$$= -\left(\frac{-x^3\sin xy + 3x^2\cos xy + y^4\cos xy}{-x^4\sin xy + y^3x\cos xy + 3y^2\sin y}\right)$$

$$= -\left(\frac{-x^3y\tan xy + 3x^2 + y^4}{-x^4\tan xy + y^3x + 3y^2\tan xy}\right)$$

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$$= \frac{x^3 y\tan xy - (3x^2 + y^4)}{y^3 x + (3y^2 - x^4)\tan xy}$$

7. If $y = 2^{\log_2 x^{2x}} + \left(\tan \frac{\pi x}{4}\right)^{\frac{4}{\pi x}}$ then $\left.\frac{dy}{dx}\right]_{x=1}$ is

(A) 4 (B) 1 (C) 0 (D) Not defined

Ans. **(A)**

Sol. $y = x^{2x} + \left(\tan\left(\frac{\pi x}{4}\right)\right)^{4/\pi x}$

$y = u + v$

$$\Rightarrow \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$$

$$u = x^{2x}, v = \left(\tan\left(\frac{\pi x}{4}\right)\right)^{4/\pi x}$$

2. DIFFERENTIATION OF LOGARITHMIC FUNCTION

8. If $x^m \cdot y^n = (x+y)^{m+n}$, then $\frac{dy}{dx}$ is

(A) $\frac{x+y}{xy}$ (B) xy (C) $\frac{x}{y}$ (D) $\frac{y}{x}$

Ans. **(D)**

Sol. Given that, $x^m y^n (x+y)^{m+n}$
Taking log on both sides, we get
$m.\log x + n\log y = (m+n)\log(x+y)$
On differentiating w.r.t. x, we get

$$\frac{m}{x} + \frac{n}{y}\frac{dy}{dx} = \frac{(m+n)}{(x+y)}\left(1 + \frac{dy}{dx}\right)$$

$$\Rightarrow \frac{dy}{dx}\left(\frac{m+n}{x+y} - \frac{n}{y}\right) = \frac{m}{x} - \frac{m+n}{x+y}$$

$$\Rightarrow \frac{dy}{dx}\left(\frac{my+ny-nx-ny}{y(x+y)}\right)$$

$$= \frac{mx+my-mx-nx}{x(x+y)} \Rightarrow \frac{dy}{dx} = \frac{y}{x}$$

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9. If $f(x) = |x|^{|\sin x|}$ then $f'(\pi/4)$ equals

(A) $\left(\frac{\pi}{4}\right)^{1/\sqrt{2}}\left(\frac{\sqrt{2}}{2}\ln\frac{4}{\pi}-\frac{2\sqrt{2}}{\pi}\right)$ (B) $\left(\frac{\pi}{4}\right)^{1/\sqrt{2}}\left(\frac{\sqrt{2}}{2}\ln\frac{4}{\pi}+\frac{2\sqrt{2}}{\pi}\right)$

(C) $\left(\frac{\pi}{4}\right)^{1/\sqrt{2}}\left(\frac{\sqrt{2}}{2}\ln\frac{\pi}{4}-\frac{2\sqrt{2}}{\pi}\right)$ (D) $\left(\frac{\pi}{4}\right)^{1/\sqrt{2}}\left(\frac{\sqrt{2}}{2}\ln\frac{\pi}{4}+\frac{2\sqrt{2}}{\pi}\right)$

Ans. **(D)**

Sol. $f(x) = |x|^{|\sin x|}$

$x > 0, f(x) = x^{\sin x} \Rightarrow y = x^{\sin x}$

$\ell ny = \sin x \ln x$

$$y' = x^{\sin x}\left\{\frac{\sin x}{x} + \cos x \ln x\right\}$$

$$y|_{x=\pi/4} = \left(\frac{\pi}{4}\right)^{\frac{1}{\sqrt{2}}}\left\{\frac{1}{\sqrt{2}} \times \frac{4}{\pi} + \frac{1}{\sqrt{2}}\ln\frac{\pi}{4}\right\}$$

$x < 0\ y = -x^{-\sin x}$

$\ell ny = -\sin x \ell n(-x)$

10. If $y = (1+x)(1+x^2)(1+x^4)\ldots.(1+x^{2n})$, then $\frac{dy}{dx}$ at $x = 0$ is

(A) -1 (B) 1 (C) 0 (D) 2^n

Ans. **(B)**

Sol. $y = (1+x)(1+x^2)\ldots(1+x^{2n})$

$$y = \frac{(1-x^2)(1+x^2)(1+x^4)\ldots(1+x^{2n})}{(1-x)}$$

$$y = \frac{1-x^{4n}}{1-x}$$

$$\frac{dy}{dx} = \frac{(1-x)(-4nx^{4n-1}) + (1-x^{4n})}{(1-x)^2}$$

$$\frac{dy}{dx} = \frac{-4nx^{4n-1} + 4nx^{4n} + 1 - x^{4n}}{(1-x)^2}$$

$$\left.\frac{dy}{dx}\right|_{x=0} = \frac{-4n \times 0 + 0 + 1 - 0}{1} = 1$$

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3. DIFFERENTIATION OF INFINITE SERIES

11. If $x = e^{y+e^{y+\cdots \text{upto} \infty}}, x > 0$, then $\frac{dy}{dx}$

(A) $\frac{x}{1+x}$ (B) $\frac{1}{x}$ (C) $\frac{1-x}{x}$ (D) $\frac{1+x}{x}$

Ans. **(C)**

Sol. $x = e^{y} + e^{y+e^{y+\cdots\infty}}$

$x = e^{y+x}$

$x = e^{(y+x)}\left\{\frac{dy}{dx} + 1\right\}$

$$\frac{dy}{dx} = \frac{1-e^{x+y}}{e^{x+y}} = \frac{1-x}{x}$$

12. If $y = \sqrt{x}^{\sqrt{x}^{\sqrt{x}\ldots\ldots\infty}}$, then the value of $\frac{dy}{dx}$ is

(A) $\frac{xy^2}{2-y\log x}$ (B) $\frac{x^2}{y(2-y\log x)}$ (C) $\frac{y^2}{x(2-y\log x)}$ (D) $\frac{y^2}{x(2+y\log x)}$

Ans. **(C)**

Sol. $y = \sqrt{x}^{\sqrt{x}^{\sqrt{x}^{\sqrt{x}\ldots\ldots\ldots\infty}}}$

$y = (\sqrt{x})^{y}$

$\log y = y\log\sqrt{x}$ ....(i)

$$\frac{1}{y}\frac{dy}{dx} = y \cdot \frac{1}{\sqrt{x}} \times \frac{1}{2\sqrt{x}} + \log\sqrt{x} \cdot \frac{dy}{dx}$$

$$\left(\frac{1}{y} - \log\sqrt{x}\right)\frac{dy}{dx} = \frac{y}{2x}$$

$$\left(\frac{1}{y} - \frac{\log y}{y}\right)\frac{dy}{dx} = \frac{y}{2x} \quad \text{(from (1))}$$

$$\frac{dy}{dx} = \frac{y^2}{2x(1-\log y)}$$

$$\text{or } \frac{dy}{dx} = \frac{y^2}{2x(1-y\log x^{1/2})}$$

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$$= \frac{y^2}{2x\left(1 - \frac{1}{2}y\log x\right)} = \frac{y^2}{x(2 - y\log x)}$$

13. If $y = \sqrt{\log x + \sqrt{\log x + \sqrt{\log x + \ldots\ldots\ldots}}}$, then $\frac{dy}{dx}$ equals-

(A) $x/(2y+1)$ (B) $1/x(2y-1)$ (C) $(2y-1)/x$ (D) $x(2y-1)$

Ans. **(B)**

Sol. $y = \sqrt{\log x + \sqrt{\log x + \sqrt{\log x + \cdots\ldots\ldots}}}$

$y = \sqrt{\log x + y}$

$y^2 = \log x + y$

$$2y\frac{dy}{dx} = \frac{1}{x} + \frac{dy}{dx}$$

$$\frac{dy}{dx}(2y-1) = \frac{1}{x}$$

$$\frac{dy}{dx} = \frac{1}{x(2y-1)}$$

14. Let $f(x) = x + \cfrac{1}{2x + \cfrac{1}{2x + \cfrac{1}{2x + \ldots\ldots\infty}}}$. Compute the value of $f(100).f'(100)$

Ans. **100**

Sol. $2y = \frac{1}{2x+(y-x)} + y - x + 2x$

$$y = \frac{1}{y+x} + x$$

$y^2 - x^2 = 0 \Rightarrow yy^1 - x = 0$

$f(100) \cdot f^1(100) = 100$

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15. If $y = x + \cfrac{1}{x+\cfrac{1}{x+\cfrac{1}{x+\ldots\ldots\ldots}}}$, prove that $\frac{dy}{dx} = \cfrac{1}{2-\cfrac{x}{x+\cfrac{1}{x+\cfrac{1}{x+\ldots\ldots\ldots}}}}$

Sol. $y = x + \cfrac{1}{x+\cfrac{1}{x+\cfrac{1}{x+\cdots}}}$

$$y = x + \frac{1}{y}$$

$$y^2 = xy + 1$$

$$2yy' = xy' + y \Rightarrow y' = \frac{y}{2y - x}$$

$$y' = \frac{1}{2 - \frac{x}{y}}\ y' = \cfrac{1}{2 - \cfrac{x}{x + \cfrac{1}{x + \cdots}}}$$

16. If $y = \tan^{-1}\frac{1}{x^2+x+1} + \tan^{-1}\frac{1}{x^2+3x+3} + \tan^{-1}\frac{1}{x^2+5x+7} + \tan^{-1}\frac{1}{x^2+7x+13} + \ldots\ldots$ to n terms.

Find dy/dx, expressing your answer in 2 terms,

Ans. $\frac{1}{1+(x+n)^2} - \frac{1}{1+x^2}$

Sol. $y = \tan^{-1}(x+1) - \tan^{-1}x$

$+\tan^{-1}(x+2) - \tan^{-1}(x+1)$

$+\tan^{-1}(x+n) - \tan^{-1}(x+(n-1))$

$y = \tan^{-1}(x+n) - \tan^{-1}x$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{1+(x+n)^2} - \frac{1}{1+x^2}$$

4. DIFFERENTIATION OF DETERMINANT

17. If $f(x), g(x), h(x)$ are polynomials in x of degree 2

and $F(x) = \begin{vmatrix} f & g & h \\ f' & g' & h' \\ f'' & g'' & h'' \end{vmatrix}$, then $F'(x)$ is equal to

(A) 1 (B) 0 (C) -1 (D) $f(x) \cdot g(x) \cdot h(x)$

Ans. **(B)**

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Sol. $F'(x) = \begin{vmatrix} f' & g' & h' \\ f' & g' & h' \\ f'' & g'' & h'' \end{vmatrix} + \begin{vmatrix} f & g & h \\ f'' & g'' & h'' \\ f'' & g'' & h'' \end{vmatrix} + \begin{vmatrix} f & g & h \\ g' & g' & h' \\ f''' & g''' & h''' \end{vmatrix} = 0$

18. If $y = \sin mx$ then the value of $\begin{vmatrix} y & y_1 & y_2 \\ y_3 & y_4 & y_5 \\ y_6 & y_7 & y_8 \end{vmatrix}$

(where subscripts of y shows the order of derivative) is

(A) independent of x but dependent on m (B) dependent of x but independent of m

(C) dependent on both m & x (D) independent of m & x

Ans. **(D)**

Sol. $\begin{vmatrix} \sin mx & m\cos mx & -m^2\sin mx \\ -m^3\cos mx & m^4\sin mx & m^5\cos mx \\ -m^6\sin mx & -m^7\cos mx & m^8\sin mx \end{vmatrix}$

by expanding $= 0$

19. If $f(x) = \begin{vmatrix} \cos(x+x^2) & \sin(x+x^2) & -\cos(x+x^2) \\ \sin(x-x^2) & \cos(x-x^2) & \sin(x-x^2) \\ \sin 2x & 0 & \sin 2x^2 \end{vmatrix}$ then

(A) $f(-2) = 0$ (B) $f'(-1/2) = 0$

(C) $f'(-1) = -2$ (D) $f''(0) = 4$

Ans. **(B, C, D)**

Sol. $f(x) = \begin{vmatrix} \cos(x+x^2) & \sin(x+x^2) & -\cos(x+x^2) \\ \sin(x-x^2) & \cos(x-x^2) & \sin(x-x^2) \\ \sin 2x & 0 & \sin 2x^2 \end{vmatrix}$

5. HIGHER ORDER DERIVATIVE

20. If $y = a\cos(\ln x) + b\sin(\ln x)$, then $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx}$

(A) 0 (B) y (C) $-y$ (D) xy

Ans. **(C)**

Sol. $y = a\cos\ell nx + b\sin\ell nx$

differentiating w.r.t. to x

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$$y' = -\frac{a}{x}\sin \ell nx + \frac{b}{x}\cos \ell nx$$

$$xy' = -a\sin \ell nx + b\cos \ell nx$$

$$xy'' + y' = -\frac{a\cos \ell nx}{x} - \frac{b\sin \ell nx}{x}$$

$$x^2y'' + xy' = -y$$

21. $\frac{d^2x}{dy^2}$ equals :

(A) $\left(\frac{d^2y}{dx}\right)^{-1}$

(B) $-\left(\frac{d^2y}{dx^2}\right)^{-1}\left(\frac{dy}{dx}\right)^{-3}$

(C) $\left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right)^{-2}$

(D) $-\left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right)^{-3}$

Ans. **(D)**

Sol. Here, $\frac{dy}{dx} = \left(\frac{dy}{dx}\right)^{-1}$

Differentiating both sides w.r.t. y, we get

$$\frac{d^2x}{dy^2} = \left(\frac{dy}{dx}\right)^{-2} \cdot \frac{d}{dy} \cdot \left(\frac{dy}{dx}\right)$$

$$= -\left(\frac{dy}{dx}\right)^{-2} \cdot \frac{d}{dy}\left(\frac{dy}{dx}\right) \cdot \frac{dx}{dx}$$

$$= -\left(\frac{dy}{dx}\right)^{-2} \cdot \frac{d}{dx} \cdot \left(\frac{dy}{dx}\right) \cdot \frac{dx}{dy}$$

$$= -\left(\frac{dy}{dx}\right)^{-2} \cdot \frac{d^2y}{dx^2} \cdot \left(\frac{dy}{dx}\right)^{-1}$$

$$= -\left(\frac{dy}{dx}\right)^{-3} \cdot \left(\frac{d^2y}{dx^2}\right)$$

6.TRIGONOMETRIC SUBSTITUTIONS

22. The derivative of $\sec^{-1}\left(\frac{1}{2x^2-1}\right)$ w.r.t. $\sqrt{1-x^2}$ at $x = \frac{1}{2}$ is

(A) 4 (B) 1/4 (C) 1 (D) 7

Ans. **(A)**

Sol. Let $u = \sec^{-1}\left(\frac{1}{2x^2-1}\right)$

$= \cos^{-1}(2x^2 - 1)$ Let $x = \cos\theta$

$= \cos^{-1}(2\cos^2\theta - 1)$

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$= \cos^{-1}(\cos 2\theta)$

$u = 2\theta \qquad \Rightarrow u = 2\cos^{-1}x$

$$\Rightarrow \frac{du}{dx} = 2 \times \frac{-1}{\sqrt{1-x^2}} = \frac{-2}{\sqrt{1-x^2}}$$

Let $v = \sqrt{1-x^2}$

$$\Rightarrow \frac{dv}{dx} = \frac{1}{2\sqrt{1-x^2}} x - 2x = \frac{-x}{\sqrt{1-x^2}}$$

$$\therefore \frac{du/dx}{dv/dx} = \frac{2/\sqrt{1-x^2}}{x/\sqrt{1-x^2}} = \frac{2}{x}$$

$$\Rightarrow \frac{du}{dv} = \frac{2}{x} \Rightarrow \left(\frac{du}{dv}\right) = \frac{2}{1/2} = 2 \times 2 = 4$$

$$\therefore \left(\frac{du}{dv}\right)_{x=1/2} = 4$$

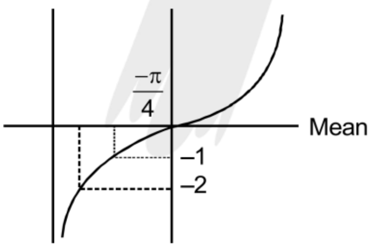
23. If $y = \sin^{-1}\frac{2x}{1+x^2}$ then $\left.\frac{dy}{dx}\right]_{x=-2}$ is

(A) $\frac{2}{5}$ (B) $\frac{2}{\sqrt{5}}$ (C) $-\frac{2}{5}$ (D) $\frac{\sqrt{5}}{2}$

Ans. **(C)**

Sol. $y = \sin^{-1}\left(\frac{2x}{1+x^2}\right) \cdot \left.\frac{dy}{dx}\right|_{x=-2}$

$x = \tan\theta \ \Rightarrow \ y = \sin^{-1}(\sin 2\theta)$



$$\theta < -\frac{-\pi}{4}$$

$$2\theta < -\frac{-\pi}{2}$$

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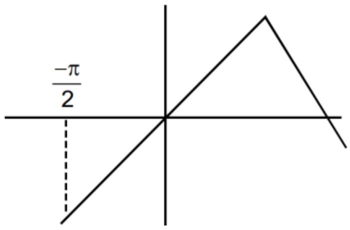

$y = \pi - 2\theta = \pi - 2\tan^{-1}x$

$\left.\frac{dy}{dx}\right|_{x=-2} = \frac{-2}{(1+x^2)} = \frac{-2}{5}$

24. $d/dx\left[\tan^{-1}\left(\frac{\sqrt{x^2+a^2}+x}{\sqrt{x^2+a^2}-x}\right)^{1/2}\right]$, is equal to -

(A) $\frac{a}{2(x^2+a^2)}$ (B) $\frac{a}{x^2+a^2}$

(C) ½ (D) None of these

Ans. **(A)**

Sol. $y = \tan^{-1}\left(\frac{\sqrt{a^2+x^2}+x}{\sqrt{a^2+x^2}-x}\right)^{1/2}$

put $x = a\tan\theta$

$= \tan^{-1}\left(\frac{\sqrt{a^2 + a^2\tan^2\theta} + a\tan\theta}{\sqrt{a^2 + a^2\tan^2\theta} - a\tan\theta}\right)^{1/2}$

$= \tan^{-1}\left(\frac{\sec\theta + \tan\theta}{\sec\theta - \tan\theta}\right)^{1/2}$

$= \tan^{-1}\left(\frac{1 + \sin\theta}{1 - \sin\theta}\right)^{1/2}$

$= \tan^{-1}\left\{\frac{\cos\frac{\theta}{2}+\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}-\sin\frac{\theta}{2}}\right\}^{-1/2}$

$= \tan^{-1}\left(\frac{1+\tan\theta/2}{1-\tan\theta/2}\right) = \tan^{-1}\tan(\pi/4 + \theta/2)$

$= \pi/4 + \theta/2$

$y = \frac{\pi}{4} + \frac{1}{2}\tan^{-1}\frac{x}{a}$

$\frac{dy}{dx} = 0 + \frac{1}{2} \times \frac{1}{1+\frac{x^2}{a^2}} \times \frac{1}{a} = \frac{fa^2}{2(x^2+a^2)}$

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25. $d/dx\left[\sin^2\cot^{-1}\frac{1}{\sqrt{\frac{1+x}{1-x}}}\right]$, is equal to -

(A) 0 (B) 1/2 (C) $-1/2$ (D) -1]

Ans. **(B)**

Sol. $y=\sin^2\cot^{-1}\frac{1}{\sqrt{\frac{1+x}{1-x}}}$

$$=\sin^2\tan^{-1}\sqrt{\frac{1+x}{1-x}}\ \left\{\because \tan^{-1}x=\sin^{-1}\frac{x}{\sqrt{1+x}}\right\}$$

$$=\sin^2\sin^{-1}\frac{\sqrt{\frac{\sqrt{1+x}}{1-x}}}{\sqrt{1+x}}=\sin^2\sin^{-1}\frac{\sqrt{1+x}}{\sqrt{2}}$$

$$=\left(\sin^{-1}\sin^{-1}\frac{\sqrt{1+x}}{\sqrt{2}}\right)^2=\left(\frac{\sqrt{1+x}}{\sqrt{2}}\right)^2$$

$$\frac{dy}{dx}=0+\frac{1}{2}$$

26. If $y=\tan^{-1}\frac{u}{\sqrt{1-u^2}}$ & $x=\sec^{-1}\frac{1}{2u^2-1}$

$u\in\left(0,\frac{1}{\sqrt{2}}\right)\cup\left(\frac{1}{\sqrt{2}},1\right)$ prove that $2\frac{dy}{dx}+1=0$

Sol. $y=\tan^{-1}\frac{u}{\sqrt{1-u^2}}$ &$x=\sec^{-1}\frac{1}{\sqrt{2u^2-1}}$

$$u\in\left(0,\frac{1}{\sqrt{2}}\right)\cup\left(\frac{1}{\sqrt{2}},1\right)$$

$u=\sin\theta$ $\quad 0<u<\frac{1}{\sqrt{2}}$ OR $\frac{1}{\sqrt{2}}<u<1$

$\left(\frac{\pi}{4},\frac{\pi}{2}\right)$ OR $\left(0,\frac{\pi}{4}\right)\to 1^{st}\ Q^{nt}$

$$y=\tan^{-1}\left(\frac{\sin\theta}{\cos\theta}\right)=\theta$$

$$x=\sec^{-1}=\sec^{-1}\left(-\frac{1}{\cos2\theta}\right)$$

$$\Rightarrow \sec^{-1}(-\sec2\theta)$$

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$x = \pi - 2\theta$

$$\frac{dy}{dq} = 1, \frac{dx}{dq} = -2$$

$\frac{dy}{dx} = -\frac{1}{2} \Rightarrow 2 \cdot \frac{dy}{dx} + 1 = 0$

7. MIXED PROBLEMS

27. If $y = \sec(\tan^{-1}x)$, then $\frac{dy}{dx}$ at $x = 1$ is equal to: **[JEE Main 2013]**

(A) 1 (B) $\sqrt{2}$ (C) $\frac{1}{\sqrt{2}}$ (D) $\frac{1}{2}$

Ans. **(C)**

Sol. $y^{-1} = \sec(\tan^{-1}x)\tan(\tan^{-1}x) \times \frac{1}{1+x^2}$

at $x = 1$

$$y' = \sqrt{2} \times \frac{1}{2} = \frac{1}{\sqrt{2}}$$

28. For $x \in R$, $f(x) = |\log 2 - \sin x|$ and $g(x) = f(f(x))$, then: **[JEE MAIN 2016]**

(A) $g'(0) = \cos(\log 2)$

(B) $g'(0) = -\cos(\log 2)$

(C) g is differentiable at $x = 0$ and $g'(0) = -\sin(\log 2)$

(D) g is not differentiable at $x = 0$

Ans. **(A)**

Sol. We have, $f(x) = |\log 2 - \sin x|$
and $g(x) = f(f(x)), x \in R$
Note that, for $x \to 0, \log 2 > \sin x$

$f(x) = \log 2 - \sin x$

$\Rightarrow g(x) = \log 2 - \sin(f(x))$

$= \log 2 - \sin(\log 2 - \sin x)$

Clearly, $g(x)$ is differentiable at $x = 0$ as $\sin x$ is differentiable.

Now, $g'(x) = -\cos(\log 2 - \sin x)(-\cos x)$

$= \cos x \cdot \cos(\log 2 - \sin x)$

$\Rightarrow g'(0) = 1 \cdot \cos(\log 2)$

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29. If for $x \in \left(0, \frac{1}{4}\right)$, then derivative of $\tan^{-1}\left(\frac{6x\sqrt{x}}{1-9x^3}\right)$ is $\sqrt{x} \cdot g(x)$, then $g(x)$ equals **[JEE MAIN 2017]**

(A) $\frac{9}{1+9x^3}$ (B) $\frac{3x\sqrt{x}}{1-9x^3}$

(C) $\frac{3x}{1-9x^3}$ (D) $\frac{3x}{1+9x^3}$

Ans. **(A)**

Sol. Let $y = \tan^{-1}\left(\frac{6x\sqrt{x}}{1-9x^3}\right) = \tan^{-1}\left[\frac{2\cdot(3x^{3/2})}{1-(3x^{3/2})^2}\right]$

$= 2\tan^{-1}(3x^{3/2}) \left[\because 2\tan^{-1}x = \tan^{-1}\frac{2x}{1-x^2}\right]$

$\frac{dy}{dx} = 2 \cdot \frac{1}{1+(3x^{3/2})^2} \cdot 3 \times \frac{3}{2}(x)^{1/2}$

$= \frac{9}{1+9x^3} \cdot \sqrt{x}$

$\therefore\ g(x) = \frac{9}{1+9x^3}$

30. Let $f(\theta) = \sin\left(\tan^{-1}\left(\frac{\sin\theta}{\sqrt{\cos 2\theta}}\right)\right)$ where $-\frac{\pi}{4} < \theta < \frac{\pi}{4}$, Then the value of $\frac{d}{d(\tan\theta)}(f(\theta))$ is

[JEE 2011]

Ans. **1**

Sol. Given $f(\theta) = \sin\left(\tan^{-1}\left(\frac{\sin\theta}{\sqrt{\cos 2\theta}}\right)\right)$

Let $\tan^{-1}\left(\frac{\sin\theta}{\sqrt{\cos 2\theta}}\right) = y$

$\Rightarrow \tan y = \frac{\sin\theta}{\sqrt{2\cos^2\theta - 1}}$

Refer to the figure below,

$\Rightarrow \sin y = \frac{\sin\theta}{\cos\theta}$

$f(\theta) = \sin y = \frac{\sin\theta}{\cos\theta} = \tan\theta$

$\Rightarrow \frac{d}{d(\tan\theta)}(f(\theta)) = 1$