

# ELECTROSTATIC POTENTIAL AND POTENTIAL ENERGY

~~H.W.~~

## Total Electrostatic Energy

$\hookrightarrow (\text{Self Energy}) + (\text{Mutual P.E})$

$\hookrightarrow (U_{\text{self}})_{\text{conducting Sphere}} + (U_{\text{self}})_{\text{non conducting Sphere}} + (\text{Mutual P.E})$

$$U_T = \left[ \frac{KQ^2}{2R} + \frac{3}{5} \left( \frac{KQ^2}{R} \right) + \frac{KQQ}{r} \right]$$

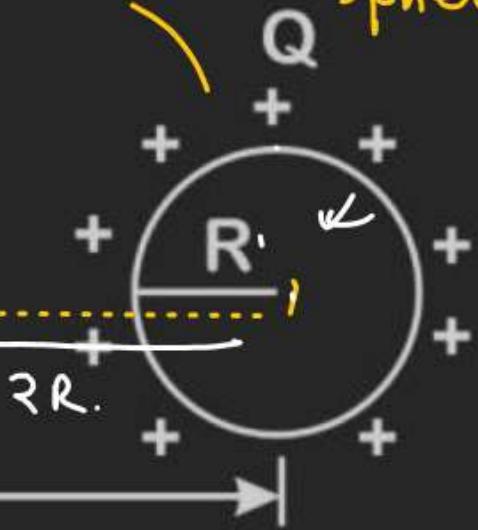
$$= \left[ \frac{3KQ^2}{4R} + \frac{3KQ^2}{5R} \right] = \frac{3KQ^2}{R} \left( \frac{1}{4} + \frac{1}{5} \right) = \frac{3KQ^2}{R} \left[ \frac{9}{20} \right] = \boxed{\frac{27KQ^2}{20R}}$$

Uniformly  
Charged non-Conduc.  
Sphere



$$\frac{\infty}{K} [U=0] \quad U = qV_\infty^0$$

Conducting  
Sphere.



# Find ( $V_0$ )<sub>min</sub> so that it can escape from the field of both spheres.

$$U_i + K \cdot E_i = U_f + K \cdot E_f$$

$$\frac{-KQq_0}{2R} \times 2 + \frac{1}{2} m V_0^2 = 0$$

$$V_0 = \sqrt{\frac{2KQq_0}{mR}}$$

# POTENTIAL ENERGY

$$\text{Impulse} \cdot F \Delta t = \Delta p$$

H.W.

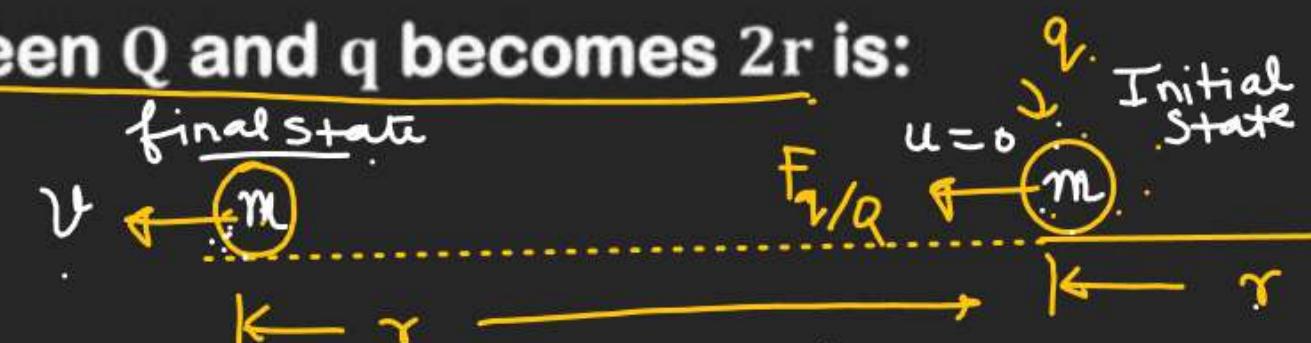
Q. A charged particle of charge  $Q$  is held fixed and another charged particle of mass  $m$  and charge  $q$  (of the same sign) is released from a distance  $r$ . The impulse of the force exerted by the external agent on the fixed charge by the time distance between  $Q$  and  $q$  becomes  $2r$  is:

(A)  $\sqrt{\frac{Qq}{4\pi\epsilon_0 mr}}$

(B)  $\sqrt{\frac{Qqm}{4\pi\epsilon_0 r}}$

(C)  $\sqrt{\frac{Qqm}{\pi\epsilon_0 r}}$

(D)  $\sqrt{\frac{Qqm}{2\pi\epsilon_0 r}}$



Energy Conservation

$$\frac{KqQ}{r} + 0 = \frac{KqQ}{2r} + \frac{1}{2}mv^2$$

$$\frac{KqQ}{r} - \frac{KqQ}{2r} = \frac{1}{2}mv^2 \Rightarrow v' = \sqrt{\frac{KqQ}{mr}}$$

$$\begin{aligned} \Delta p &= mv' \\ &= m \sqrt{\frac{KqQ}{mr}} \\ &\Rightarrow \sqrt{\frac{KqQm}{r}} = \sqrt{\frac{Qqm}{4\pi\epsilon_0 r}} \\ J_F &= J_{\text{ext agent}} \end{aligned}$$

# POTENTIAL ENERGY

H.W.

Q. A unit positive point charge of mass  $m$  is projected with a velocity  $v$  inside the tunnel as shown. The tunnel has been made inside a uniformly charged non-conducting sphere. The minimum velocity with which the point charge should be projected such it can reach the opposite end of the tunnel, is equal to:

(A)  $[\sigma R^2 / 4m\epsilon_0]^{1/2}$

(B)  $[\sigma R^2 / 24m\epsilon_0]^{1/2}$

(C)  $[\sigma R^2 / 6m\epsilon_0]^{1/2}$

(D) zero because the initial and the final points are at same potential

$$U = \frac{KQ}{2R^3} (3R^2 - r^2)$$

$$U_A \leftarrow r = R$$

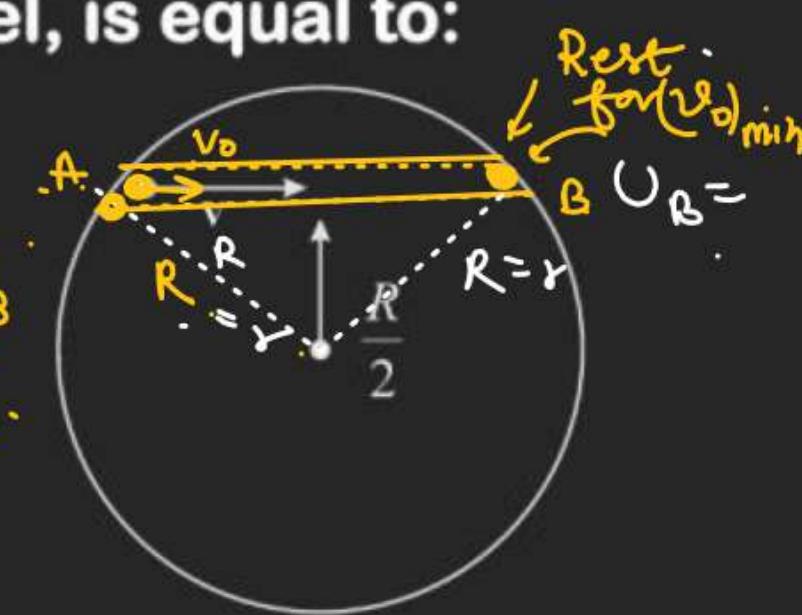
$$U_B \leftarrow r = R$$

$$U_A = U_B$$

$$U_A + (K.E)_A = U_B + (K.E)_B$$

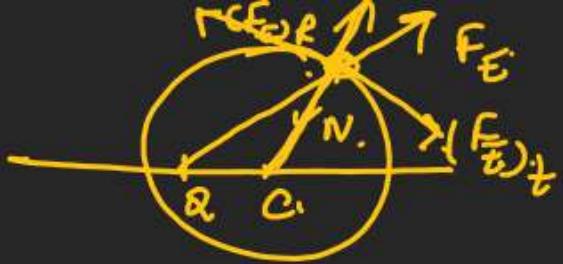
$$(K.E)_A = (U_B - U_A)$$

$$(K.E)_A = \sigma \Rightarrow$$



# POTENTIAL ENERGY

*H.W.*



Q. The diagram shows a small bead of mass  $m$  carrying charge  $q$ . The bead can freely move on the smooth fixed ring placed on a smooth horizontal plane. In the same plane a charge  $+Q$  has also been fixed as shown. The potential at the point  $P$  due to  $+Q$  is  $V$ . The velocity with which the bead should projected from the point  $P$  so that it can complete a circle should be greater than:

(A)  $\sqrt{\frac{6qV}{m}}$

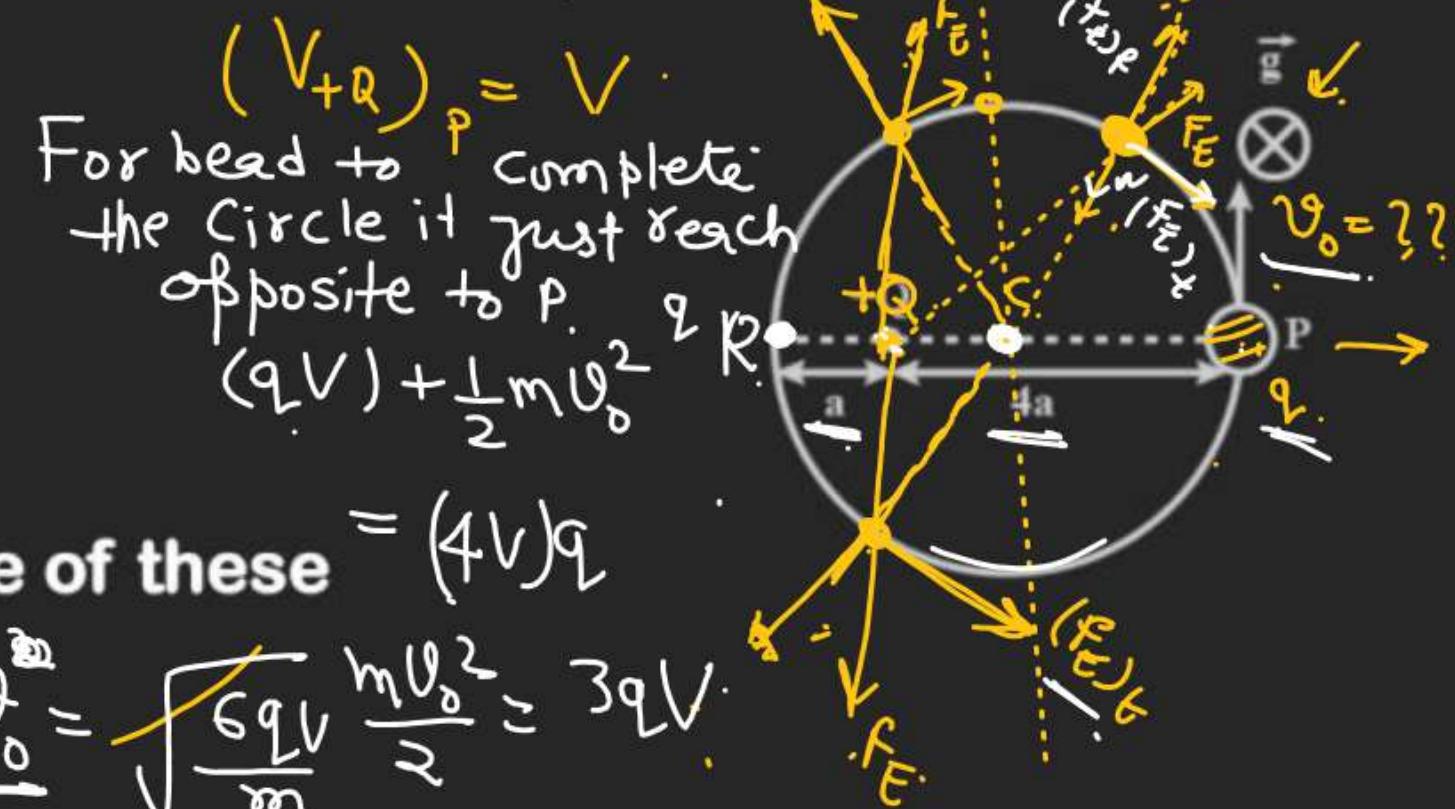
(C)  $\sqrt{\frac{3qV}{m}}$

$$\begin{aligned} \text{For } V_i &= V = \left( \frac{kQ}{4a} \right) \\ V_f &= \left( \frac{kQ}{a} \right) \approx 4V \end{aligned}$$

(B)  $\sqrt{\frac{qV}{m}}$

(D) none of these

$$V_0 = \sqrt{\frac{6qV}{m}} \frac{mV_0^2}{2} = 3qV$$



# POTENTIAL ENERGY

*H.W.*

**Q. A bullet of mass  $m$  and charge  $q$  is fired towards a solid uniformly charged sphere of radius  $R$  and total charge  $+q$ . If it strikes the surface of sphere with speed  $u$ , find the minimum speed  $u$  so that it can penetrate the sphere (to the center.) Neglect all resistance forces or friction acting on bullet except electrostatic forces.:**

(A)  $\frac{q}{\sqrt{2\pi\epsilon_0 m R}}$

(B)  $\frac{q}{\sqrt{4\pi\epsilon_0 m R}}$

(C)  $\frac{q}{\sqrt{2\pi\epsilon_0 m R}}$

(D)  $\frac{\sqrt{3}q}{\sqrt{4\pi\epsilon_0 m R}}$

$$U_A + K.E_A = U_C + (K.E)_C$$

$$U_A = \frac{kq^2}{R}$$

$$U_C = \frac{kq^2}{R}$$

$$K.E_A = \frac{1}{2}mu^2$$

$$K.E_C = \frac{3kq^2}{2R}$$

$$\frac{kq^2}{R} + \frac{1}{2}mu^2 = \frac{3kq^2}{2R} + 0$$

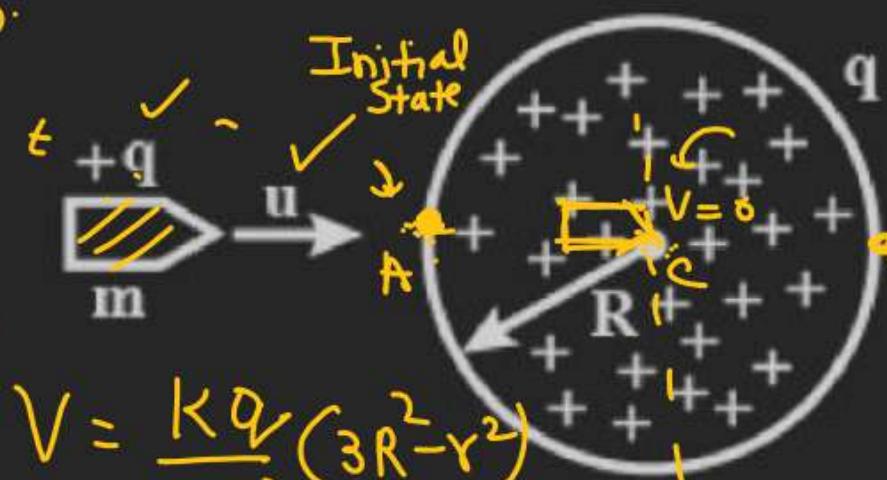
$$\frac{mu^2}{2} = \left( \frac{3kq^2}{2R} - \frac{kq^2}{R} \right)$$

$$\frac{mu^2}{2} = \frac{kq^2}{R} \left( \frac{3}{2} - 1 \right) = \frac{kq^2}{2R}$$

$$V = \frac{kq}{2R^3} (3R^2 - R^2)$$

$$V_C = \frac{(3kq)}{2R}$$

$$r = 0$$



# POTENTIAL ENERGY

$$\text{H}\ddot{\omega}$$

$$(K.E)_{\text{rotational}} = \left[ \frac{1}{2} I \omega^2 \right]$$

$$v = R\omega$$

**Q.** Fig. shows a ball having a charge  $q$  fixed at a point A. Two identical balls of mass  $m$  having charge  $+q$  and  $-q$  are attached to the end of a light rod of length  $2a$ . The system is released from the situation shown in fig. Find the angular velocity of the rod when the rod turns through  $90^\circ$ :

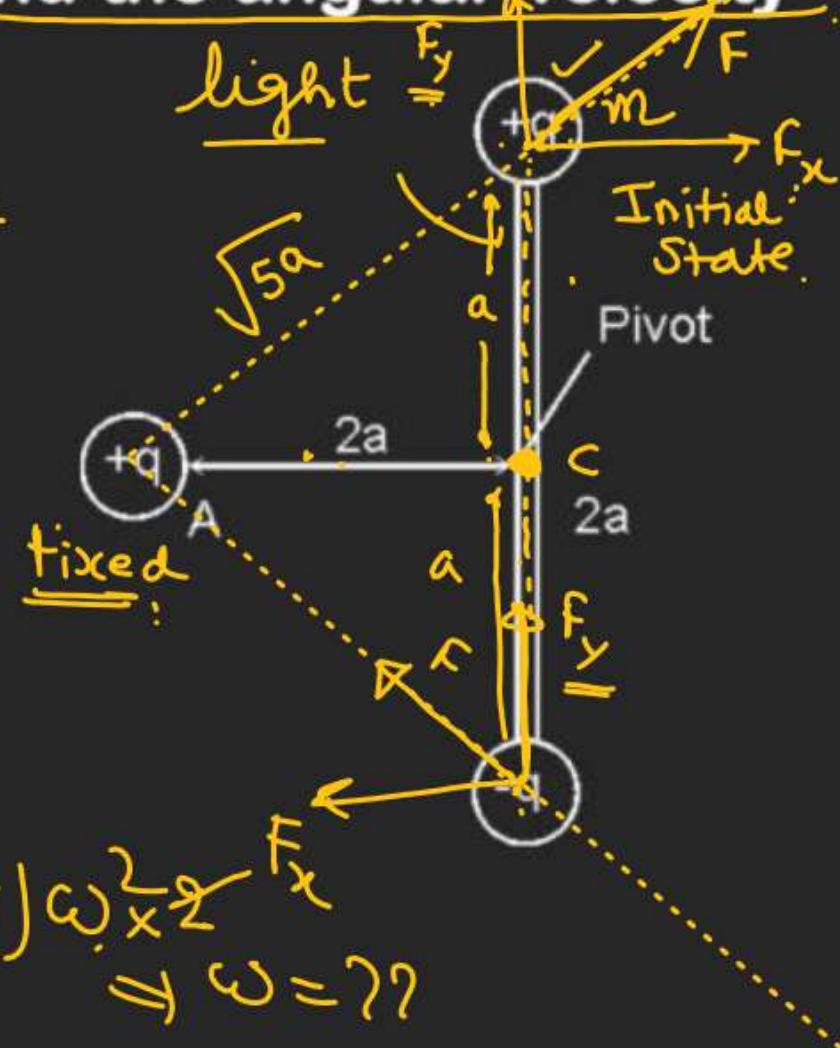
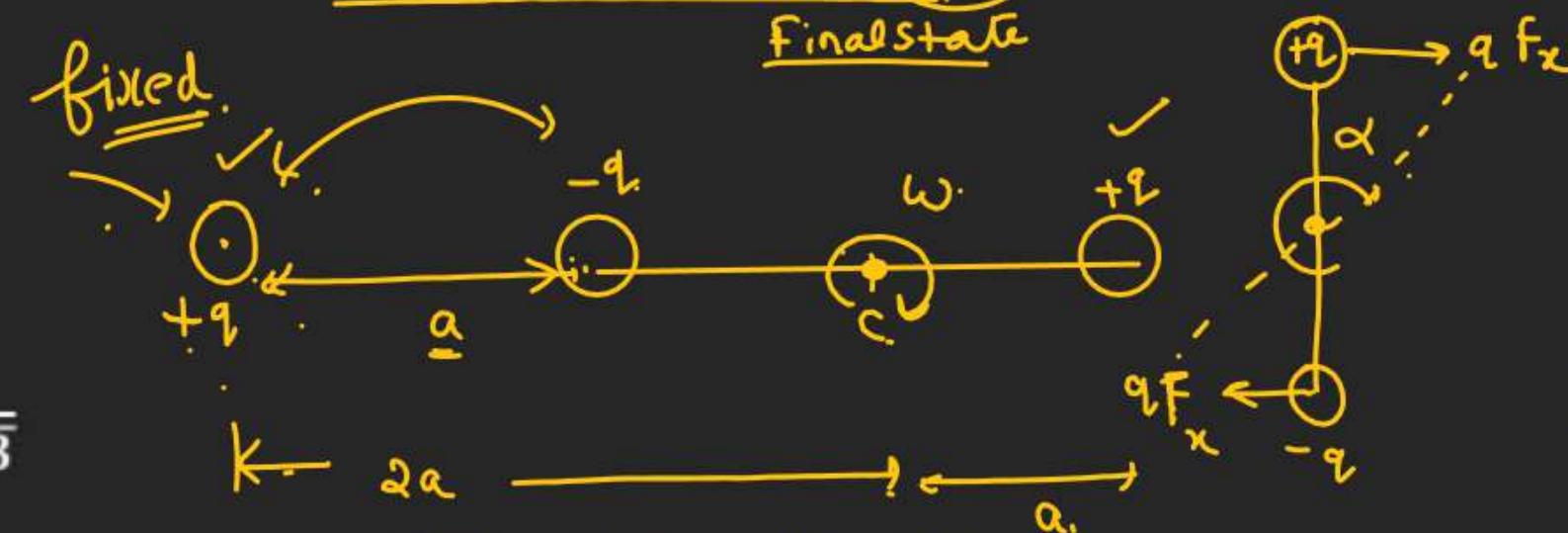
(A)  $\frac{\sqrt{2}q}{3\pi\epsilon_0 ma^3}$

(B)  $\frac{q}{\sqrt{3\pi\epsilon_0 ma^3}}$

(C)  $\frac{q}{\sqrt{6\pi\epsilon_0 ma^3}}$

(D)  $\frac{\sqrt{2}q}{4\pi\epsilon_0 ma^3}$

$$\tau = F \cdot r_\perp$$



$$U_i + K.E_i = U_f + K.E_f$$

$$\left( \frac{kq^2}{\sqrt{5a}} - \frac{kq^2}{\sqrt{5a}} \right) + 0 = \left( \frac{kq^2}{a} + \frac{kq^2}{3a} \right) + \frac{1}{2} (ma^2) \omega^2 \times 2 F_x \Rightarrow \omega = ??$$

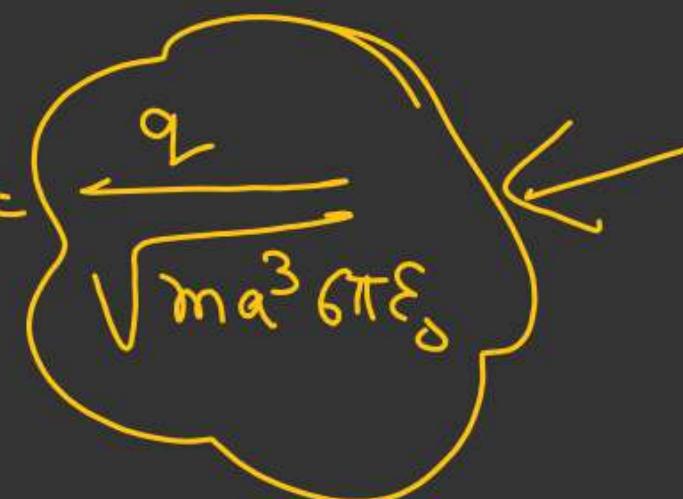
$$+\frac{Kq^2}{a} - \frac{Kq^2}{3a} = ma^2\omega^2$$

$$\overbrace{\frac{Kq^2}{a}} \left(1 - \frac{1}{3}\right) = ma^2\omega^2.$$

$$\frac{2}{3} \frac{Kq^2}{a} = ma^2\omega^2$$

$$\omega_0 = \sqrt{\frac{2Kq^2}{3ma^3}}$$

$$\omega = \sqrt{\frac{\frac{2}{3} \times 1 \times q^2}{2 \pi \epsilon_0 m a^3}} = \sqrt{\frac{q^2}{m a^3 6 \pi \epsilon_0}} = \sqrt{m a^3 6 \pi \epsilon_0}$$



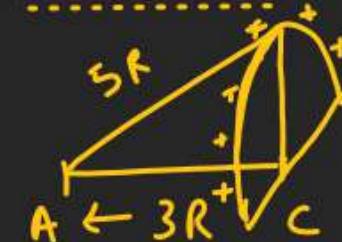
# POTENTIAL ENERGY

$$\frac{mu^2}{2} = \frac{3kq^2}{20R} \Rightarrow u = \sqrt{\frac{3kq^2}{10R}}$$

~~H.W.~~

Q. On a semicircular ring of radius =  $4R$ , charge  $+3q$  is distributed in such a way that on one quarter  $+q$  is uniformly distributed and on another quarter  $+2q$  is uniformly distributed. Along its axis a smooth non-conducting and uncharged pipe of length  $6R$  is fixed axially as shown. A small ball of mass  $m$  and charge  $+q$  is thrown from the other end of pipe. The ball can come out of the pipe if:

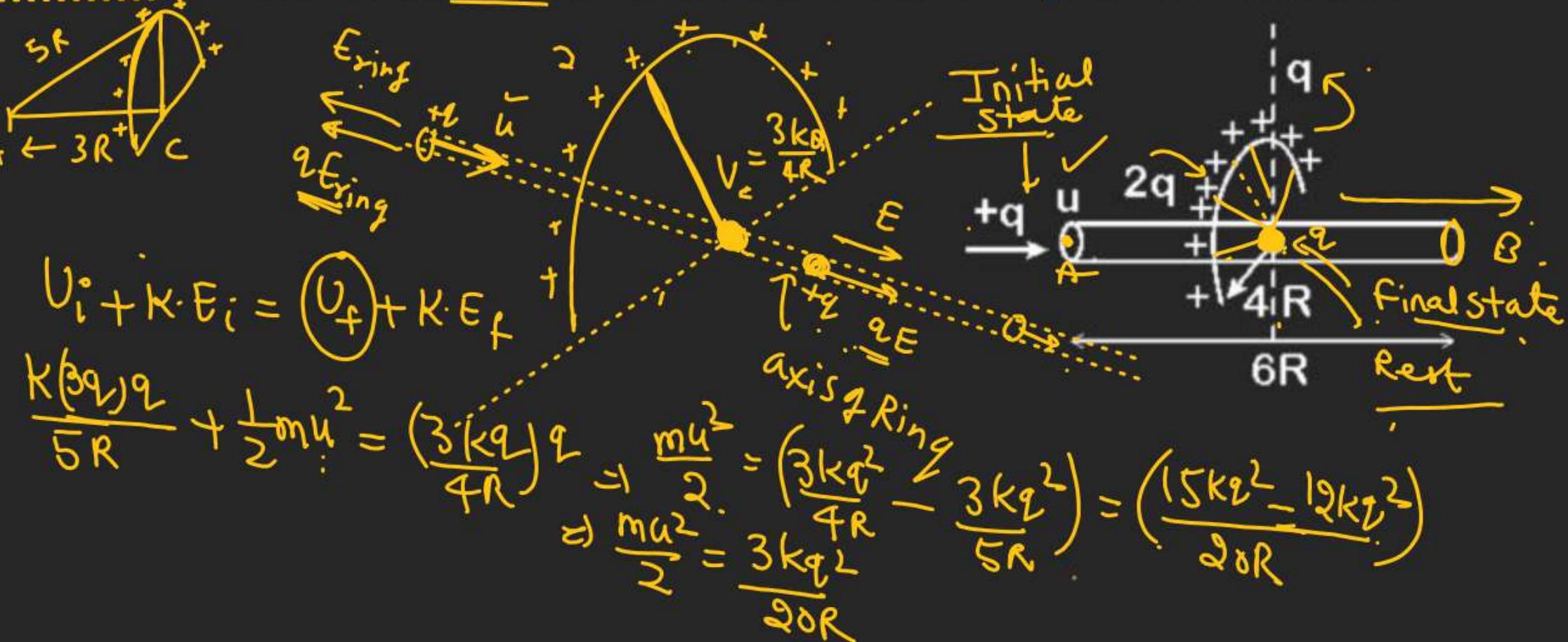
(A)  $u > \sqrt{\frac{7q^2}{40\pi\epsilon_0 Rm}}$



(B)  $u > \sqrt{\frac{3q^2}{40\pi\epsilon_0 Rm}}$

(C)  $u \geq \sqrt{\frac{3q^2}{40\pi\epsilon_0 Rm}}$

(D)  $u > \sqrt{\frac{9q^2}{40\pi\epsilon_0 Rm}}$



# ELECTRIC POTENTIAL

*H.W.*

**Q. Four equal point charges  $Q$  each are placed in the  $xy$  plane at  $(0, 2)$ ,  $(4, 2)$ ,  $(4, -2)$  and  $(0, -2)$ . The work required to put a fifth charge  $Q$  at the origin of the coordinate system will be**

**[JEE (Main)-2019]**

(A)  $\frac{Q^2}{2\sqrt{2}\pi\epsilon_0}$

(B)  $\frac{Q^2}{4\pi\epsilon_0} \left(1 + \frac{1}{\sqrt{3}}\right)$

(C)  $\frac{Q^2}{4\pi\epsilon_0}$

(D)  $\frac{Q^2}{4\pi\epsilon_0} \left(1 + \frac{1}{\sqrt{5}}\right)$

# ELECTRIC POTENTIAL

$\cancel{f\omega}$

Q. Three charges  $Q$ ,  $+q$  and  $+q$  are placed at the vertices of a right-angle isosceles triangles as shown below. The net electrostatic energy of the configuration is zero, if the value of  $Q$  is

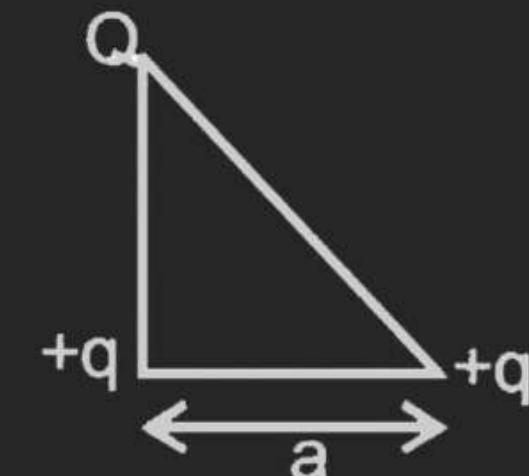
[JEE (Main)-2019]

(A)  $\frac{-\sqrt{2}q}{\sqrt{2}+1}$

(B)  $+q$

(C)  $-2q$

(D)  $\frac{-q}{1+\sqrt{2}}$



## ELECTRIC POTENTIAL

H.W

Q. The electric field in a region is given by  $\vec{E} = (Ax + B)\hat{i}$ , where E is in  $\text{NC}^{-1}$  and x is in meters. The values of constants are A = 20SI unit and B = 10SI unit. If the potential at  $x = 1$  is  $V_1$  and that at  $x = -5$  is  $V_2$ , then  $V_1 - V_2$  is :

[JEE (Main)-2019]

- (A) 180 V
- (B) -520 V
- (C) 320 V
- (D) -48 V

# ELECTRIC POTENTIAL

H.W.

Q. A positive point charge is released from rest at a distance  $r_0$  from a positive line charge with uniform density. The speed ( $v$ ) of the point charge, as a function of instantaneous distance  $r$  from line charge is proportional to :

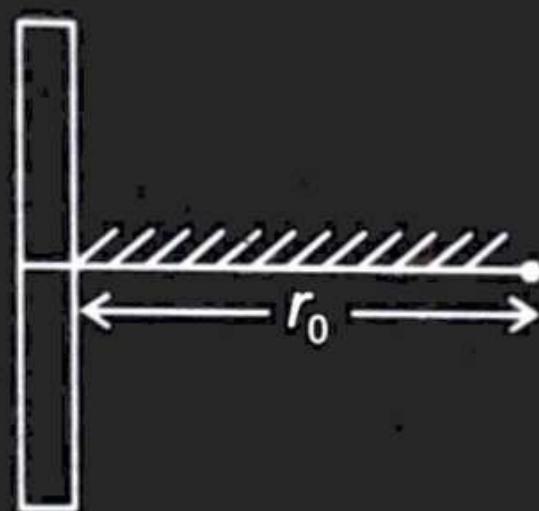
[JEE (Main)-2019]

(A)  $v \propto \sqrt{\ln\left(\frac{r}{r_0}\right)}$

(B)  $v \propto e^{+r/r_0}$

(C)  $v \propto \ln\left(\frac{r}{r_0}\right)$

(D)  $v \propto \left(\frac{r}{r_0}\right)$



## ELECTRIC POTENTIAL

H.W.

**Q. A uniformly charged ring of radius  $3a$  and total charge  $q$  is placed in  $xy$ -plane centred at origin. A point charge  $q$  is moving towards the ring along the  $z$ -axis and has speed  $v$  at  $z = 4a$ . The minimum value of  $v$  such that it crosses the origin is :**

**[JEE (Main)-2019]**

(A)  $\sqrt{\frac{2}{m}} \left( \frac{1}{15} \frac{q^2}{4\pi\epsilon_0 a} \right)^{1/2}$

(B)  $\sqrt{\frac{2}{m}} \left( \frac{1}{5} \frac{q^2}{4\pi\epsilon_0 a} \right)^{1/2}$

(C)  $\sqrt{\frac{2}{m}} \left( \frac{4}{15} \frac{q^2}{4\pi\epsilon_0 a} \right)^{1/2}$

(D)  $\sqrt{\frac{2}{m}} \left( \frac{2}{15} \frac{q^2}{4\pi\epsilon_0 a} \right)^{1/2}$

# ELECTRIC POTENTIAL

*H.W.*

**Q. A two point charges  $4q$  and  $-q$  are fixed on the x-axis at  $x = -\frac{d}{2}$  and  $x = \frac{d}{2}$ , respectively. If a third point charge '  $q$  ' is taken from the origin to  $x = d$  along the semicircle as shown in the figure the energy of the charge will**

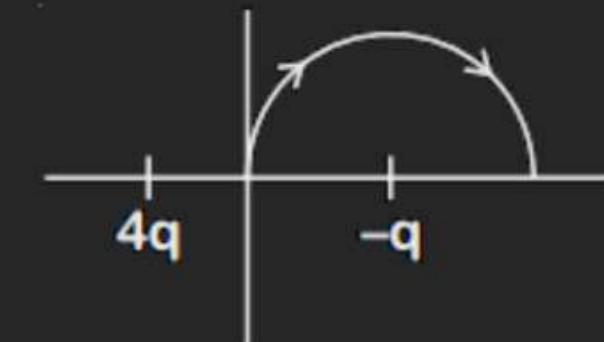
**[JEE (Main)-2020]**

**(A) Decrease by  $\frac{q^2}{4\pi\epsilon_0 d}$**

**(B) Decrease by  $\frac{4q^2}{3\pi\epsilon_0 d}$**

**(C) Increase by  $\frac{2q^2}{3\pi\epsilon_0 d}$**

**(D) Increase by  $\frac{3q^2}{4\pi\epsilon_0 d}$**



# ELECTRIC POTENTIAL

*H.W.*

**Q. A solid sphere of radius R carries a charge  $Q + q$  distributed uniformly over its volume. A very small point like piece of it of mass m gets detached from the bottom of the sphere and falls down vertically under gravity. This piece carries charge q. If it acquires a speed v when it has fallen through a vertical height y (see figure), then (assume the remaining portion to be spherical).**

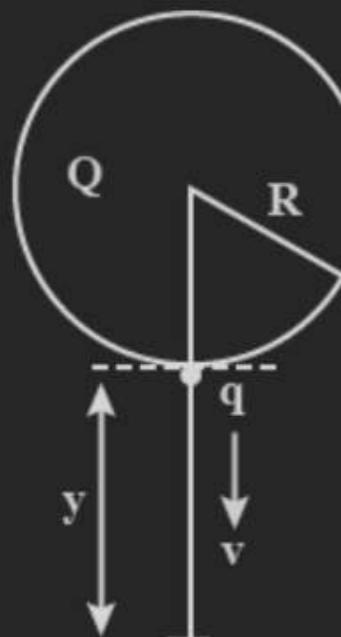
(A)  $v^2 = y \left[ \frac{qQ}{4\pi\epsilon_0 R(R+y)m} + g \right]$

**[JEE (Main)-2020]**

(B)  $v^2 = 2y \left[ \frac{qQR}{4\pi\epsilon_0 (R+y)^3 m} + g \right]$

(C)  $v^2 = y \left[ \frac{qQ}{4\pi\epsilon_0 R^2 y m} + g \right]$

(D)  $v^2 = 2y \left[ \frac{qQ}{4\pi\epsilon_0 R(R+y)m} + g \right]$



## ELECTRIC POTENTIAL

H.W.

**Q. Similar drops of mercury are maintained at 10 V each. All these spherical drops combine into a single big drop. The potential energy of the bigger drop is \_\_\_\_\_ times that of a smaller drop.**

**[JEE (Main)-2021]**