Solution

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1. $x^4 - 5x^2 + 4 < 0.$

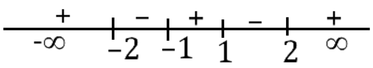
Sol. $(x^2)^2 - 5(x^2) + 4 < 0$

$(\text{ let } x^2 = t)$

$t^2 - 5t + 4 < 0 \Rightarrow (t-1)(t-4) < 0 \Rightarrow (x^2-1)(x^2-4) < 0 \Rightarrow (x^2-1^2)(x^2-2^2) < 0$

$\because a^2 - b^2 = (a+b)(a-b)$

$\therefore (x-1)(x+1)(x-2)(x+2) < 0$



$x \in (-2,-1) \cup (1,2)$

2. $x^4 - 2x^2 - 63 \leqslant 0.$

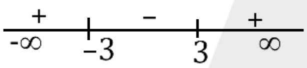
Sol. $(x^2)^2 - 2(x^2) - 63 \le 0$

$(\text{let } x^2 = t)$

$t^2 - 2t - 63 \le 0 \Rightarrow (t-9)(t+7) \le 0 \Rightarrow (x^2-9)(x^2+7) \le 0 \Rightarrow (x^2-3^2)(x^2+7) \le 0$

$(x+3)(x-3)(x^2+7) \le 0$ $(\because x^2 + 7 \text{ is always} > 0)$

$(x+3)(x-3) \le 0$



$x \in [-3,3]$

3. $\frac{x}{x-5} > \frac{1}{2}.$

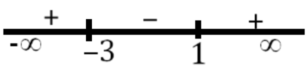
Sol. $\frac{2x}{x-5} > 1 \Rightarrow \frac{2x}{x-5} - 1 > 0 \Rightarrow \frac{2x-x+5}{x-5} > 0 \Rightarrow \frac{x+5}{x-5} > 0$

$x \in (-\infty,-5) \cup (5,\infty)$

4. $\frac{x-2}{x^2+1} < -\frac{1}{2}.$

Sol. $\frac{x-2}{x^2+1} + \frac{1}{2} < 0 \Rightarrow \frac{2x-4+x^2+1}{2(x^2+1)} < 0 \Rightarrow \frac{x^2+2x-3}{2(x^2+1)} < 0 \Rightarrow x^2 + 2x - 3 < 0$

$(x+3)(x-1) < 0$



$x \in (-3,1)$

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5. $\frac{x^4+x^2+1}{x^2-4x-5} < 0.$

Sol. $N^r \Rightarrow x^4 + x^2 + 1$

$(\text{ let } x^2 = t)$

$t^2 + t + 1$

$D = (1)^2 - 4(1)(1)$

$D = -3 < 0$

$\therefore\ x^4 + x^2 + 1 > 0(\text{always})$

$\frac{1}{x^2-4x-5} < 0 \Rightarrow \frac{1}{(x-5)(x+1)} < 0$

$-\infty \quad + \quad -1 \quad - \quad 5 \quad + \quad \infty$

$x \in (-1,5)$

6. $\frac{1+3x^2}{2x^2-21x+40} < 0.$

Sol. $(\because\ 1 + 3x^2 \text{ is always} > 0)$

$\frac{1}{2x^2-2x+40} < 0 \Rightarrow \frac{1}{2x^2-16x-5x+40} < 0 \Rightarrow \frac{1}{2x(x-8)-5(x-8)} < 0$

$\frac{1}{(9x-5)(x-8)} < 0$

$-\infty \quad + \quad 5/2 \quad - \quad 8 \quad + \quad \infty$

$x \in \left(\frac{5}{2}, 8\right)$

7. $\frac{x}{x^2-3x-4} > 0.$

Sol. $\frac{x}{(x-4)(x+1)} > 0$

$-\infty \quad - \quad -1 \quad + \quad 0 \quad - \quad 8 \quad + \quad \infty$

$x \in (-1,0) \cup (4, \infty)$

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8. $\frac{x^2-x-6}{x^2+6x} \geqslant 0.$

Sol. $\frac{(x-3)(x+2)}{x(x+6)} \geqslant 0$

$x \neq 0, -6$

$$\begin{array}{ccccccccccc} & + & & - & & + & & - & & + & \\ -\infty & & -6 & & -2 & & 0 & & 3 & & \infty \end{array}$$

$x \in (-\infty, -6) \cup [-2, 0) \cup [3, \infty)$

9. $\frac{x^2-8x+7}{4x^2-4x+1} < 0.$

Sol. $\frac{(x-7)(x-1)}{(2x-1)^2} \rightarrow$ aleays $+\infty$

but $2x - 1 \neq 0$

$\neq 1/2$

$\therefore (x-7)(x-1) < 0 \Rightarrow$

$$\begin{array}{ccccccc} & + & & - & & + & \\ -\infty & & 1 & & 7 & & \infty \end{array}$$

$x \in (1, 7)$

10. $\frac{x^2-36}{x^2-9x+18} < 0.$

Sol. $\frac{x^2-6^2}{(x-6)(x-3)} < 0 \Rightarrow \frac{(x+6)(x-6)}{(x-6)(x-3)} < 0 \begin{pmatrix} x - 6 \neq 0 \\ x \neq 6 \end{pmatrix}$

$\frac{(x+6)}{(x-3)<3} < 0 \Rightarrow$

$$\begin{array}{ccccccc} & + & & - & & + & \\ -\infty & & -6 & & 3 & & \infty \end{array}$$

$\Rightarrow x \in (-6, 3)$

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11. $\frac{4}{1+x}+\frac{2}{1-x}<1.$

Sol. $\frac{4}{1+x}+\frac{2}{1-x}-1<0 \Rightarrow \frac{4(1-x)+2(1+x)-(1+x)(1-x)}{(1+x)(1-x)}<0$

$\frac{4-4x+2+2x-1+x^2}{(1-x^2)}<0 \Rightarrow \frac{x^2-2x+7}{(1-x^2)}<0$

$$\frac{x^2-2x+7}{x^2-1}>0$$

$$\begin{bmatrix} \because x^2-2x+7 \\ D=(-2)^2-4(1)(7) \\ D<0 \end{bmatrix}$$

$\therefore x^2-2x+7>0$
always

$\frac{1}{x^2-1^2}>0 \Rightarrow \frac{1}{x^2-1^2}>0$

$$\frac{1}{(x+1)(x-1)}>0$$

$x \in (-\infty,-1) \cup (1,\infty)$

$-\infty \quad + \quad -6 \quad - \quad 1 \quad + \quad \infty$

12. $2+\frac{3}{x+1}>\frac{2}{x}.$

Sol. $\frac{2}{1}+\frac{3}{x+1}-\frac{2}{x}>0 \Rightarrow \frac{2x(x+1)+3x-2(x+1)}{x(x+1)}>0$

$\frac{2x^2+2x+3x-2x-2}{x(x+1)} \Rightarrow \frac{2x^2+3x-2}{x(x+1)}>0$

$\frac{2x^2+4x-x-2}{x(x+1)}>0 \Rightarrow \frac{(2x-1)(x+2)}{x(x+1)}>0$

$-\infty \quad + \quad -2 \quad - \quad -1 \quad + \quad 0 \quad - \quad 1/2 \quad + \quad \infty$

$x \in (-\infty,-2) \cup (-1,0) \cup \left(\frac{1}{2},\infty\right)$

13. $\frac{1}{x-2}+\frac{1}{x-1}>\frac{1}{x}.$

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Sol. $\frac{1}{x-2}+\frac{1}{x-1}-\frac{1}{x}>0$

$\frac{x(x-1)+x(x-2)-(x-1)(x-2)}{x(x-1)(x-2)}>0 \Rightarrow \frac{x^2-x+x^2-2x-(x^2-3x+2)}{x(x-1)(x-2)}>0$

$$\frac{(x^2-2)}{x(x-1)(x-2)}>0 \Rightarrow \frac{x^2-(\sqrt{2})^2}{x(x-1)(x-2)}>0 \Rightarrow \frac{(x+\sqrt{2})(x-\sqrt{2})}{x(x-4)(x-2)}>0$$

$x=-\sqrt{2},\sqrt{2},0,1,2$

$$\begin{array}{ccccccccccccc} & - & & + & & - & & + & & - & & + & \\ -\infty & & \sqrt{2} & & 0 & & 1 & & \sqrt{2} & & 2 & & \infty \end{array}$$

$x\in(-\sqrt{2},0)\cup(1,\sqrt{2})\cup(2,\infty)$

14. $\frac{7}{(x-2)(x-3)}+\frac{9}{x-3}+1<0.$

Sol. $\frac{7+9(x-2)+(x-2)(x-3)}{(x-2)(x-3)}<0 \Rightarrow \frac{7+9x-18+x^2-5x+6}{(x-2)(x-3)}<0$

$$\frac{x^2+4x-5}{(x-2)(x-3)}<0 \Rightarrow \frac{(x+5)(x-1)}{(x-2)(x-3)}<0$$

$x=-5,1,2,3$

$$\begin{array}{ccccccccccc} & + & & - & & + & & - & & + & \\ -\infty & & -5 & & 1 & & 2 & & 3 & & \infty \end{array}$$

$x\in(-5,1)\cup(2,3)$

15. $\frac{20}{(X-3)(x-4)}+\frac{10}{x-4}+1>0.$

Sol. $\frac{20+10(x-3)+(x-3)(x-4)}{(x-3)(x-4)}>0 \Rightarrow \frac{20+10x-30+x^2-7x+12}{(x-3)(x-4)}>0$

$\frac{x^2+3x+2}{(x-3)(x-4)}>0 \Rightarrow \frac{(x+1)(x+2)}{(x-3)(x-4)}>0$

$x=-2,-1,3,4$

$$\begin{array}{ccccccccccc} & + & & - & & + & & - & & + & \\ -\infty & & -2 & & -1 & & 3 & & 4 & & \infty \end{array}$$

$x\in(-\infty,-2)\cup(-1,3)\cup(4,\infty)$

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16. $\frac{(x-2)(x-4)(x-7)}{(x+2)(x+4)(x+7)} > 1.$

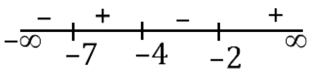
Sol. $\frac{(x-2)(x-4)(x-7)}{(x+2)(x+4)(x+7)} - 1 > 0$

$$\frac{(x^2-6x+8)(x-7)-(x^2+6x+8)(x+7)}{(x+2)(x+4)(x+7)} > 0$$

$$\frac{x[(x^2-6x+8)-(x^2+6x+8)]-7[(x^2-6x+8)+(x^2+6x+8)]}{(x+2)(x+4)(x+7)} > 0$$

$$\frac{x(-12x)-7(2x^2+16)}{(x+2)(x+4)(x+7)} > 0 \Rightarrow \frac{-26x^2-112}{(x+2)(x+4)(x+7)} > 0$$

$$\frac{(26x^2+112)}{(x+2)(x+4)(x+7)} < 0 \Rightarrow \frac{1}{(x+2)(x+4)(x+7)} < 0$$



$x \in (-\infty, -7) \cup (-4, -2)$

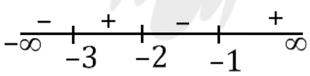
17. $\frac{(x-1)(x-2)(x-3)}{(x+1)(x+2)(x+3)} > 1.$

Sol. $\frac{(x^2-3x+2)(x-3)}{(x^2+3x+2)(x+3)} - 1 > 0 \Rightarrow \frac{(x^2-35+2)(x-3)-(x^2+3x+2)(x+3)}{(x+1)(x+2)(x+3)} > 0$

$$\frac{x[(x^2-3x+2)-(x^2+3x+2)]-3[(x^2-3/x+2)+(x^2+3x+2)]}{(x+1)(x+2)(x+3)} > 0 \Rightarrow \frac{x(-6x)-3(2x^2+4)}{(x+1)(x+2)(x+3)} > 0$$

$$\frac{-12x^2-12}{(x+1)(5+2)(x+3)} > 0 \Rightarrow \frac{(12x^2+12)}{(x+1)(x+2)(x+3)} < 0$$

$$\frac{1}{(x+1)(x+2)(x+3)} < 0$$

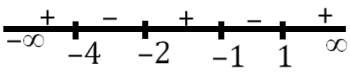


$x \in (-\infty, -3) \cup (-2, -1)$

18. $(x^2+3x+1)(x^2+3x-3) \geqslant 5.$

Sol. let $x^2+3x = t \Rightarrow \quad (t+1)(t-3)-5 \geqslant 0 \Rightarrow t^2-2t-8 \geqslant 0 \Rightarrow (t-4)(t+2) \geqslant 0$

$\Rightarrow (x^2+3x-4)(x^2+3x+2) \geqslant 0 \Rightarrow (x+4)(x-1)(x+1)(x+2) \geqslant 0$



$x \in (-\infty, -4] \cup [-2, -1] \cup [1, \infty)$

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19. $(x^2 - x - 1)(x^2 - x - 7) < -5.$

Sol. let $x^2 - x = t$

$(t-1)(t-7) + 5 < 0 \Rightarrow t^2 - 8t + 7 + 5 < 0 \Rightarrow t^2 - 8t + 12 < 0$

$(t-6)(t-2) < 0 \Rightarrow (x^2 - x - 6)(x^2 - x - 2) < 0$

$(x-3)(x+2)(x-2)(x+1) < 0$

$-\infty \;\; + \;\; -2 \;\; - \;\; -1 \;\; + \;\; 2 \;\; - \;\; 3 \;\; + \;\; \infty$

$x \in (-2,-1) \cup (2,3)$

20. $(x^2 - 2x)(2x-2) - 9\frac{2x-2}{x^2-2x} \leqslant 0.$

Sol. $(2x-2)\left[\frac{(x^2-2x)^2 - 9}{(x^2-2x)}\right] \leq 0 \Rightarrow \frac{2(x-1)\left[(x^2-2x)^2 - 3^2\right]}{x(x-2)} \leq 0$

$\frac{2(x-1)(x^2-2x+3)(x^2-2x-3)}{x(x-2)} \leq 0$

$(\because x^2 - 2x + 3$ is always +ve $\because D < 0)$

$(x \neq 0,2)$

$$\frac{2(x-1)(x-3)(x+1)}{x(x-2)} \leqslant 0$$

$-\infty \;\; + \;\; -1 \;\; - \;\; 0 \;\; + \;\; 1 \;\; - \;\; 2 \;\; 3 \;\; + \;\; \infty$

$x \in (-\infty, -1] \cup (0,1] \cup (2,3]$

21. $(x^2 + 3x)(2x+3) - 16\frac{2x+3}{x^2+3x} \geqslant 0.$

Sol. $(2x+3)\left[\frac{(x^2+3x)^2 - 4^2}{x^2+3x}\right] \geqslant 0 \Rightarrow \frac{(2x+3)(x^2+3x+4)(x^2+3x-4) \geqslant 0}{x(x+3)} \geqslant (x \neq 0, -3)$

$-\infty \;\; - \;\; -4 \;\; + \;\; -3 \;\; - \;\; -3/2 \;\; + \;\; 0 \;\; - \;\; 1 \;\; + \;\; \infty$

$\frac{(2x+3)(x+4)(x-1)}{x(x+3)} \geqslant 0 \Rightarrow x \in [-4,-3) \cup \left[-\frac{3}{2}, 0\right) \cup [1, \infty)$

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22. $\frac{x-1}{x^2-x-12} \leqslant 0.$

Sol. $\frac{(x-1)}{(x-4)(x+3)} \leq 0 \ (x \neq -3,4)$

$$\underset{-\infty}{} \ \overset{-}{} \ -3 \ \overset{+}{} \ 1 \ \overset{-}{} \ 4 \ \overset{+}{} \ \infty$$

$x \in (-\infty, -3) \cup [1,4)$

23. $1 < \frac{3x^2-7x+8}{x^2+1} \leqslant 2.$

Sol. (i)$\frac{3x^2-7x+8}{x^2+1} > 1 \Rightarrow \frac{3x^2-7x+8}{x^2+1} - 1 > 0 \Rightarrow \frac{3x^2-7x+8-x^2-1}{x^2+1} > 0$

$\frac{2x^2-7x+7}{x^2+1} > 0 \Rightarrow x \in R$

(ii) $\frac{3x^2-7x+8}{x^2+1} \leq 2 \Rightarrow \frac{3x^2-7x+8}{x^2+1} - 2 \leq 0 \Rightarrow \frac{3x^2-7x+8-2x^2-2}{x^2+1} \leqslant 0$

$\frac{x^2-7x+6}{x^2+1} \leq 0 \ \Rightarrow x^2 - 7x + 6 \leq 0$

$(x-1)(x-6) \leq 0 \Rightarrow \ x \in [1,6]$

Intersection of (i) & (ii) is $x \in [1,6]$