**1. TRIGONOMETRIC EQUATION:**

An equation involving one or more trigonometrical ratios of unknown angles is called a trigonometrical equation.

**2. SOLUTION OF TRIGONOMETRIC EQUATION:**

A value of the unknown angle which satisfies the given equation is called a solution of the trigonometric equation.

**(a)** **Principal solution:** The solution of the trigonometric equation lying in the interval $(0, 2\pi)$

**(b)** **General solution:** Since all the trigonometric functions are many one & periodic, hence there are infinite values of $\theta$ for which trigonometric functions have the same value. All such possible values of $\theta$ for which the given trigonometric function is satisfied is given by a general formula. Such a general formula is called general solution of trigonometric equation.

**(c)** **Particular solution:** The solution of the trigonometric equation lying in the given interval.

**3. GENERAL SOLUTIONS OF SOME TRIGONOMETRIC EQUATIONS (TO BE REMEMBERED):**

**(a)** If $\sin\theta = 0$, then $\theta = n\pi, n \in I$ (set of integers)

**(b)** If $\cos\theta = 0$, then $\theta = (2n+1)\frac{\pi}{2}, n \in I$

**(c)** If $\tan\theta = 0$, then $\theta = n\pi, n \in I$

**(d)** If $\sin\theta = \sin\alpha$, then $\theta = n\pi + (-1)^n\alpha$, where $\alpha \in \left[\frac{-\pi}{2}, \frac{\pi}{2}\right], n \in I$

**(e)** If $\cos\theta = \cos\alpha$, then $\theta = 2n\pi \pm \alpha, n \in I, \alpha \in [0, \pi]$

**(f)** If $\tan\theta = \tan\alpha$, then $\theta = n\pi + \alpha, n \in I, \alpha \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$

**(g)** If $\sin\theta = 1$, then $\theta = 2n\pi + \frac{\pi}{2} = (4n+1)\frac{\pi}{2}, n \in I$

**(h)** If $\cos\theta = 1$, then $\theta = 2n\pi, n \in I$

**(i)** If $\sin^2\theta = \sin^2\alpha$ or $\cos^2\theta = \cos^2\alpha$ or $\tan^2\theta = \tan^2\alpha$, then $\theta = n\pi \pm \alpha, n \in I$

**(j)** For $n \in I$, $\sin n\pi = 0$ and $\cos n\pi = (-1)^n, n \in I$

$\sin(n\pi + \theta) = (-1)^n \sin\theta \cos(n\pi + \theta) = (-1)^n \cos\theta$

**(k)** $\cos n\pi = (-1)^n, n \in I$ If n is an odd integer, then $\sin\frac{n\pi}{2} = (-1)^{\frac{n-1}{2}}, \cos\frac{n\pi}{2} = 0,$

$\sin\left(\frac{n\pi}{2} + \theta\right) = (-1)^{\frac{n-1}{2}}\cos\theta$

$\cos\left(\frac{n\pi}{2} + \theta\right) = (-1)^{\frac{n+1}{2}}\sin\theta$

**Illustration 1:** Find the set of values of x for which $\frac{\tan 3x - \tan 2x}{1 + \tan 3x \cdot \tan 2x} = 1$

**Solution :** We have, $\frac{\tan 3x - \tan 2x}{1 + \tan 3x \cdot \tan 2x} = 1 \Rightarrow \tan(3x - 2x) = 1 \Rightarrow \tan x = 1$

$\Rightarrow \tan x = \tan\frac{\pi}{4} \Rightarrow x = n\pi + \frac{\pi}{4}, n \in I$ { using $\tan\theta = \tan\alpha \Leftrightarrow \theta = n\pi + \alpha$}

But for this value of x, $\tan 2x$ is not defined. Hence the solution set for x is $\phi$.**Ans.**

**Do yourself-1:**

**(i)** Find general solutions of the following equations:

(a) $\sin\theta = \frac{1}{2}$ (b) $\cos\left(\frac{3\theta}{2}\right) = 0$ (c) $\tan\left(\frac{3\theta}{4}\right) = 0$

(d) $\cos^2 2\theta = 1$ (e) $\sqrt{3}\sec 2\theta = 2$ (f) $\text{cosec}\left(\frac{\theta}{2}\right) = -1$

**4. IMPORTANT POINTS TO BE REMEMBERED WHILE SOLVING TRIGONOMETRIC EQUATIONS:**

**(a)** For equations of the type $\sin\theta = k$ or $\cos\theta = k$, one must check that $|k| \le 1$.

**(b)** Avoid squaring the equations, if possible, because it may lead to extraneous solutions. Reject extra solutions if they do not satisfy the given equation.

**(c)** Do not cancel the common variable factor from the two sides of the equations which are in a product because we may loose some solutions.

**(d)** The answer should not contain such values of $\theta$, which make any of the terms undefined or infinite.

**(i)** Check that denominator is not zero at any stage while solving equations.

**(ii)** If $\tan\theta$ or $\sec\theta$ is involved in the equations, $\theta$ should not be odd multiple of $\frac{\pi}{2}$.

**(iii)** If $\cot\theta$ or $\operatorname{cosec}\theta$ is involved in the equation, $\theta$ should not be multiple of $\pi$ or 0.

**5. DIFFERENT STRATEGIES FOR SOLVING TRIGONOMETRIC EQUATIONS:**

**(a) Solving trigonometric equations by factorization.**

e.g. $(2\sin x - \cos x)(1 + \cos x) = \sin^2 x$

$\therefore (2\sin x - \cos x)(1 + \cos x) - (1 - \cos^2 x) = 0$

$\therefore (1 + \cos x)(2\sin x - \cos x - 1 + \cos x) = 0 \therefore (1 + \cos x)(2\sin x - 1) = 0$

$\Rightarrow \cos x = -1$ or $\sin x = \frac{1}{2} \Rightarrow \cos x = -1 = \cos\pi \Rightarrow x = 2n\pi + \pi = (2n+1)\pi, n \in I$

or $\sin x = \frac{1}{2} = \sin\frac{\pi}{6} \Rightarrow x = k\pi + (-1)^k\frac{\pi}{6}, k \in I$ **Ans.**

**Illustration 2:** If $\frac{1}{6}\sin\theta, \cos\theta$ and $\tan\theta$ are in G.P. then the general solution for $\theta$ is-

(A) $2n\pi \pm \frac{\pi}{3}$ (B) $2n\pi \pm \frac{\pi}{6}$ (C) $n\pi \pm \frac{\pi}{3}$ (D) none of these

**Solution:** Since, $\frac{1}{6}\sin\theta, \cos\theta, \tan\theta$ are in G.P. $\Rightarrow \cos^2\theta = \frac{1}{6}\sin\theta \cdot \tan\theta \Rightarrow 6\cos^3\theta + \cos^2\theta - 1 = 0$

$\therefore (2\cos\theta - 1)(3\cos^2\theta + 2\cos\theta + 1) = 0$

$\Rightarrow \cos\theta = \frac{1}{2}$ (other values of $\cos\theta$ are imaginary)

$\Rightarrow \cos\theta = \cos\frac{\pi}{3} \Rightarrow \theta = 2n\pi \pm \frac{\pi}{3}, n \in I$. **Ans.**

**(b) Solving of trigonometric equation by reducing it to a quadratic equation.**

e.g. $6 - 10\cos x = 3\sin^2 x$

$\therefore 6 - 10\cos x = 3 - 3\cos^2 x \Rightarrow 3\cos^2 x - 10\cos x + 3 = 0$

$\Rightarrow (3\cos x - 1)(\cos x - 3) = 0 \Rightarrow \cos x = \frac{1}{3}$ or $\cos x = 3$

Since $\cos x = 3$ is not possible as $-1 \le \cos x \le 1$

$\therefore \cos x = \frac{1}{3} = \cos\left(\cos^{-1}\frac{1}{3}\right) \Rightarrow x = 2n\pi \pm \cos^{-1}\left(\frac{1}{3}\right), n \in I$ **Ans.**

**Illustration 3:** Solve $\sin^2\theta - \cos\theta = \frac{1}{4}$, for θ and write the values of θ in the interval $0 \le \theta \le 2\pi$.

**Solution:** The given equation can be written as $1 - \cos^2\theta - \cos\theta = \frac{1}{4} \Rightarrow \cos^2\theta + \cos\theta - 3/4 = 0$

$\Rightarrow 4\cos^2\theta + 4\cos\theta - 3 = 0 \Rightarrow (2\cos\theta - 1)(2\cos\theta + 3) = 0 \Rightarrow \cos\theta = \frac{1}{2}, -\frac{3}{2}$

Since, $\cos\theta = -3/2$ is not possible as $-1 \le \cos\theta \le 1$

$\therefore \cos\theta = \frac{1}{2} \Rightarrow \cos\theta = \cos\frac{\pi}{3} \Rightarrow \theta = 2n\pi \pm \frac{\pi}{3}, n \in I$

For the given interval, $n = 0$ and $n = 1. \Rightarrow \theta = \frac{\pi}{3}, \frac{5\pi}{3}$ **Ans.**

**Illustration 4:** Find the number of solutions of $\tan x + \sec x = 2\cos x$ in $[0, 2\pi]$.

**Solution:** Here, $\tan x + \sec x = 2\cos x \Rightarrow \sin x + 1 = 2\cos^2 x \Rightarrow 2\sin^2 x + \sin x - 1 = 0$

$\Rightarrow \sin x = \frac{1}{2}, -1$ But $\sin x = -1 \Rightarrow x = \frac{3\pi}{2}$ for which $\tan x + \sec x = 2\cos x$ is not defined.

Thus $\sin x = \frac{1}{2} \Rightarrow x = \frac{\pi}{6}, \frac{5\pi}{6} \Rightarrow$ number of solutions of $\tan x + \sec x = 2\cos x$ is 2. **Ans.**

**Illustration 5:** Solve the equation $5\sin^2 x - 7\sin x\cos x + 16\cos^2 x = 4$

**Solution:** To solve this equation we use the fundamental formula of trigonometric identities,

$\sin^2 x + \cos^2 x = 1$ writing the equation in the form.

$5\sin^2 x - 7\sin x \cdot \cos x + 16\cos^2 x = 4(\sin^2 x + \cos^2 x)$

$\Rightarrow \sin^2 x - 7\sin x\cos x + 12\cos^2 x = 0$

dividing by $\cos^2 x$ on both side we get, $\tan^2 x - 7\tan x + 12 = 0$

Now it can be factorized as : $(\tan x - 3)(\tan x - 4) = 0 \Rightarrow \tan x = 3, 4$

i.e., $\tan x = \tan(\tan^{-1}3)$ or $\tan x = \tan(\tan^{-1}4)$

$x = n\pi + \tan^{-1} 3$ or $x = n\pi + \tan^{-1} 4, n \in I$.**Ans.**

**Illustration 6:** If $x \ne \frac{n}{2}, n \in I$ and $(\cos x)^{\sin^2 x - 3\sin x + 2} = 1$, then find the general solutions of x.

**Solution :** As $x \ne \frac{n\pi}{2} \Rightarrow \cos x \ne 0, 1, -1$ So, $(\cos x)^{\sin^2 x - 3\sin x + 2} = 1$

$\Rightarrow \sin^2 x - 3\sin x + 2 = 0$

$\therefore (\sin x - 2)(\sin x - 1) = 0 \Rightarrow \overline{\sin x} = 1, 2$ where $\sin x = 2$ is not possible and

$\sin x = 1$ which is also not possible as $x \ne \frac{n\pi}{2}$ $\therefore$ no general solution is possible. **Ans.**

**Illustration 7:** Solve the equation $\sin^4 x + \cos^4 x = \frac{7}{2}\sin x \cdot \cos x$

**Solution :** $\sin^4 x + \cos^4 x = \frac{7}{2}\sin x \cdot \cos x \Rightarrow (\sin^2 x + \cos^2 x)^2 - 2\sin^2 x\cos^2 x = \frac{7}{2}\sin x \cdot \cos x$

$\Rightarrow 1 - \frac{1}{2}(\sin 2x)^2 = \frac{7}{4}(\sin 2x) \Rightarrow 2\sin^2 2x + 7\sin 2x - 4 = 0$

$\Rightarrow (2\sin 2x - 1)(\sin 2x + 4) = 0 \Rightarrow \sin 2x = \frac{1}{2}$ or $\sin 2x = -4$

(which is not possible) $\Rightarrow 2x = n\pi + (-1)^n\frac{\pi}{6}, n \in I$

i.e. $x = \frac{n\pi}{2} + (-1)^n\frac{\pi}{12}, n \in I$ **Ans.**

**Do yourself-2:**

**(i)** Solve the following equations:

(a) $3\sin x + 2\cos^2 x = 0$ (b) $\sec^2 2\alpha = 1 - \tan 2\alpha$

(c) $7\cos^2\theta + 3\sin^2\theta = 4$ (d) $4\cos\theta - 3\sec\theta = \tan\theta$

**(ii)** Solve the equation: $2\sin^2\theta + \sin^2 2\theta = 2$ for $\theta \in (-\pi, \pi)$.

**(c) Solving trigonometric equations by introducing an auxiliary argument.**

Consider, $a\sin\theta + b\cos\theta = c$ ............(i)

$$\therefore \frac{a}{\sqrt{a^2+b^2}}\sin\theta + \frac{b}{\sqrt{a^2+b^2}}\cos\theta = \frac{c}{\sqrt{a^2+b^2}}$$

equation (i) has a solution only if $|c| \le \sqrt{a^2+b^2}$

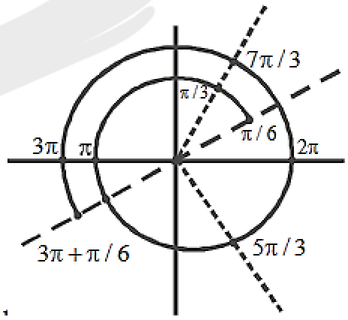
let $\frac{a}{\sqrt{a^2+b^2}} = \cos\phi, \frac{b}{\sqrt{a^2+b^2}} = \sin\phi \,\&\, \phi = \tan^{-1}\frac{b}{a}$

by introducing this auxiliary argument $\phi$, equation (i) reduces to

$\sin(\theta+\phi) = \frac{c}{\sqrt{a^2+b^2}}$ Now this equation can be solved easily.

**Illustration 8:** Find the number of distinct solutions of

$\sec x + \tan x = \sqrt{3}$, where $0 \le x \le 3\pi$.




**Solution:** Here, $\sec x + \tan x = \sqrt{3}$

$\Rightarrow 1 + \sin x = \sqrt{3}\cos x$ or $\sqrt{3}\cos x - \sin x = 1$

dividing both sides by $\sqrt{a^2+b^2}$ i.e. $\sqrt{4} = 2$ we get

$$\Rightarrow \frac{\sqrt{3}}{2}\cos x - \frac{1}{2}\sin x = \frac{1}{2} \Rightarrow \cos\frac{\pi}{6}\cos x - \sin\frac{\pi}{6}\sin x = \frac{1}{2} \Rightarrow$$

$$\cos\left(x+\frac{\pi}{6}\right) = \frac{1}{2}$$

As $0 \le x \le 3\pi$

$$\frac{\pi}{6} \le x + \frac{\pi}{6} \le 3\pi + \frac{\pi}{6} \Rightarrow x + \frac{\pi}{6} = \frac{\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{3} \Rightarrow x = \frac{\pi}{6}, \frac{3\pi}{2}, \frac{13\pi}{6}$$

But at $x = \frac{3\pi}{2}$, $\tan x$ and $\sec x$ is not defined $\therefore$ Total number of solutions are 2.**Ans.**

**Illustration 9:** Prove that the equation $k\cos x - 3\sin x = k+1$ possess a solution iff $k \in (-\infty, 4]$.

**Solution:** Here, $k\cos x - 3\sin x = k+1$, could be re-written as: $\frac{k}{\sqrt{k^2+9}}\cos x - \frac{3}{\sqrt{k^2+9}}\sin x = \frac{k+1}{\sqrt{k^2+9}}$

or $\cos(x+\phi) = \frac{k+1}{\sqrt{k^2+9}}$, where $\tan\phi = \frac{3}{k}$ which possess a solution only if $-1 \le \frac{k+1}{\sqrt{k^2+9}} \le 1$

i.e., $\left|\frac{k+1}{\sqrt{k^2+9}}\right| \le 1$ i.e., $(k+1)^2 \le k^2+9$ i.e., $k^2+2k+1 \le k^2+9$ or $k \le 4$

$\Rightarrow$ The interval of k for which the equation $(k\cos x - 3\sin x = k+1)$

has a solution is $(-\infty, 4]$. **Ans.**

**Do yourself - 3:**

**(i)** Solve the following equations:

(a) $\sin x + \sqrt{2} = \cos x.$ (b) $\operatorname{cosec} \theta = 1 + \cot \theta$

**(d) Solving trigonometric equations by transforming sum of trigonometric functions into product.**

e.g. $\cos 3x + \sin 2x - \sin 4x = 0$

$\cos 3x - 2\sin x\cos 3x = 0 \Rightarrow (\cos 3x)(1 - 2\sin x) = 0 \Rightarrow \cos 3x = 0$ or $\sin x = \frac{1}{2}$

$\Rightarrow \cos 3x = 0 = \cos \frac{\pi}{2}$ $\quad \sin x = \frac{1}{2} = \sin \frac{\pi}{6}$

$\Rightarrow 3x = 2n\pi \pm \frac{\pi}{2}$ $\quad x = m\pi + (-1)^m \frac{\pi}{6}; (n, m \in I) \Rightarrow x = \frac{2n\pi}{3} \pm \frac{\pi}{6}$ **Ans.**

**Illustration 10:** Solve: $\cos \theta + \cos 3\theta + \cos 5\theta + \cos 7\theta = 0$

**Solution:** We have $\cos \theta + \cos 7\theta + \cos 3\theta + \cos 5\theta = 0$

$\Rightarrow 2\cos 4\theta\cos 3\theta + 2\cos 4\theta\cos \theta = 0$

$\Rightarrow \cos 4\theta(\cos 3\theta + \cos \theta) = 0 \Rightarrow \cos 4\theta(2\cos 2\theta\cos \theta) = 0$

$\Rightarrow$Either $\cos \theta = 0 \Rightarrow \theta = (2n_1 + 1)\pi/2, n_1 \in I$

or $\cos 2\theta = 0 \Rightarrow \theta = (2n_2 + 1)\frac{\pi}{4}, n_2 \in I$

or $\cos 4\theta = 0 \Rightarrow \theta = (2n_3 + 1)\frac{\pi}{8}, n_3 \in I$ **Ans.**

**(e) Solving trigonometric equations by transforming a product into sum.**

e.g. $\sin 5x \cdot \cos 3x = \sin 6x \cdot \cos 2x$

$\sin 8x + \sin 2x = \sin 8x + \sin 4x \therefore 2\sin 2x \cdot \cos 2x - \sin 2x = 0$

$\Rightarrow \sin 2x(2\cos 2x - 1) = 0 \Rightarrow \sin 2x = 0$ or $\cos 2x = \frac{1}{2}$

$\Rightarrow \sin 2x = 0 = \sin 0$ $\quad \cos 2x = \frac{1}{2} = \cos \frac{\pi}{3}$

$\Rightarrow 2x = n\pi + (-1)^n \times 0, n \in I$ $\quad 2x = 2m\pi \pm \frac{\pi}{3}, m \in I$

$\Rightarrow x = \frac{n\pi}{2}, n \in I$ $\quad x = m\pi \pm \frac{\pi}{6}, m \in I$

**Illustration 11:** Solve: $\cos \theta\cos 2\theta\cos 3\theta = \frac{1}{4}$; where $0 \le \theta \le \pi$.

**Solution :** $\frac{1}{2}(2\cos \theta\cos 3\theta)\cos 2\theta = \frac{1}{4} \Rightarrow (\cos 2\theta + \cos 4\theta)\cos 2\theta = \frac{1}{2}$

$\Rightarrow \frac{1}{2}[2\cos^2 2\theta + 2\cos 4\theta\cos 2\theta] = \frac{1}{2} \Rightarrow 1 + \cos 4\theta + 2\cos 4\theta\cos 2\theta = 1$

$\therefore \cos 4\theta(1 + 2\cos 2\theta) = 0$

$\cos 4\theta = 0$ or $(1 + 2\cos 2\theta) = 0$

Now from the first equation: $2\cos 4\theta = 0 = \cos(\pi/2)$

$\therefore 4\theta = \left(n + \frac{1}{2}\right)\pi \Rightarrow \theta = (2n + 1)\frac{\pi}{8}, n \in I$

for $n=0, \theta=\frac{\pi}{8}; n=1, \theta=\frac{3\pi}{8}; n=2, \theta=\frac{5\pi}{8}; n=3, \theta=\frac{7\pi}{8} (\therefore 0 \le \theta \le \pi)$ and from the second equation:

$$\cos 2\theta=-\frac{1}{2}=-\cos(\pi/3)=\cos(\pi-\pi/3)=\cos(2\pi/3)$$

$$\therefore\ 2\theta=2k\pi \pm \frac{2\pi}{3} \Rightarrow \theta=k\pi \pm \frac{\pi}{3}, k \in I$$

again for $k=0, \theta=\frac{\pi}{3}; k=1, \theta=\frac{2\pi}{3} (\because 0 \le \theta \le \pi) \therefore\ \theta=\frac{\pi}{8},\frac{\pi}{3},\frac{3\pi}{8},\frac{5\pi}{8},\frac{2\pi}{3},\frac{7\pi}{8}$ **Ans.**

**Do yourself-4:**

**(i)** Solve $4\sin\theta\sin 2\theta\sin 4\theta=\sin 3\theta$ **(ii)** Solve for x: $\sin x+\sin 3x+\sin 5x=0$

**(f) Solving equations by a change of variable:**

**(i)** Equations of the form $P(\sin x \pm \cos x, \sin x \cdot \cos x)=0$, where $P(y,z)$ is a polynomial, can be solved by the substitution: $\cos x \pm \sin x=t \Rightarrow 1 \pm 2\sin x \cdot \cos x=t^2$

**Illustration 12:** Solve: $\sin x+\cos x=1+\sin x \cdot \cos x$.

**Solution :** put $\sin x+\cos x=t \Rightarrow \sin^2 x+\cos^2 x+2\sin x \cdot \cos x=t^2$

$$\Rightarrow 2\sin x\cos x=t^2-1\ (\because \sin^2 x+\cos^2 x=1) \Rightarrow \sin x\cos x=\left(\frac{t^2-1}{2}\right)$$

Substituting above result in given equation, we get:

$$t=1+\frac{t^2-1}{2} \Rightarrow 2t=t^2+1 \Rightarrow t^2-2t+1=0$$

$$\Rightarrow (t-1)^2=0 \Rightarrow t=1 \Rightarrow \sin x+\cos x=1$$

Dividing both sides by $\sqrt{1^2+1^2}$ i.e. $\sqrt{2}$. we get

$$\Rightarrow \frac{1}{\sqrt{2}}\sin x+\frac{1}{\sqrt{2}}\cos x=\frac{1}{\sqrt{2}} \Rightarrow \cos x\cos\frac{\pi}{4}+\sin x \cdot \sin\frac{\pi}{4}=\frac{1}{\sqrt{2}}$$

$$\Rightarrow \cos\left(x-\frac{\pi}{4}\right)=\cos\frac{\pi}{4} \Rightarrow x-\frac{\pi}{4}=2n\pi \pm \frac{\pi}{4}$$

$\Rightarrow x=2n\pi$ or $x=2n\pi+\frac{\pi}{2}=(4n+1)\frac{\pi}{2}, n \in I$ **Ans.**

**(ii)** Equations of the form of $a\sin x+b\cos x+d=0$, where a, b & d are real numbers can be solved by changing $\sin x$ & $\cos x$ into their corresponding tangent of half the angle.

**Illustration 13 :** Solve : $3\cos x+4\sin x=5$

**Solution :**

$$\Rightarrow 3\left(\frac{1-\tan^2 x/2}{1+\tan^2 x/2}\right)+4\left(\frac{2\tan x/2}{1+\tan^2 x/2}\right)=5 \Rightarrow \frac{3-3\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}+\frac{8\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}=5$$

$$\Rightarrow 3-3\tan^2\frac{x}{2}+8\tan\frac{x}{2}=5+5\tan^2\frac{x}{2} \Rightarrow 8\tan^2\frac{x}{2}-8\tan\frac{x}{2}+2=0$$

$$\Rightarrow 4\tan^2\frac{x}{2}-4\tan\frac{x}{2}+1=0 \Rightarrow \left(2\tan\frac{x}{2}-1\right)^2=0$$

$$\Rightarrow 2\tan\left(\frac{x}{2}\right)-1=0 \Rightarrow \tan\frac{x}{2}=\frac{1}{2}=\tan\left(\tan^{-1}\frac{1}{2}\right)$$

$$\Rightarrow \frac{x}{2}=n\pi+\tan^{-1}\left(\frac{1}{2}\right), n \in I \Rightarrow x=2n\pi+2\tan^{-1}\left(\frac{1}{2}\right), n \in I$$ **Ans.**

**(g) Solving trigonometric equations with the use of the boundness of the functions involved.**

**Illustration 14:** Solve the equation $(\sin x + \cos x)^{1+\sin 2x} = 2$, when $0 \le x \le \pi$

**Solution :** We know, $-\sqrt{a^2+b^2} \le a\sin\theta + b\cos\theta \le \sqrt{a^2+b^2}$ and $-1 \le \sin\theta \le 1$.

$\therefore$ $(\sin x + \cos x)$ admits the maximum value as $\sqrt{2}$ and $(1 + \sin 2x)$ admits the maximum value as 2. Also $(\sqrt{2})^2 = 2$.

$\therefore$ the equation could hold only when, $\sin x + \cos x = \sqrt{2}$ and $1 + \sin 2x = 2$

Now, $\sin x + \cos x = \sqrt{2}$

$\Rightarrow \cos\left(x - \frac{\pi}{4}\right) = 1 \Rightarrow x = 2n\pi + \pi/4, n \in I$ ..........(i)

and $1 + \sin 2x = 2 \Rightarrow \sin 2x = 1 = \sin\frac{\pi}{2}$

$\Rightarrow 2x = m\pi + (-1)^m\frac{\pi}{2}, m \in I \Rightarrow x = \frac{m\pi}{2} + (-1)^m\frac{\pi}{4}$ ..........(ii)

The value of x in $[0, \pi]$ satisfying equations (i) and (ii) is $x = \frac{\pi}{4}$

(when $n = 0$ & $m = 0$) **Ans.**

**Note:** $\sin x + \cos x = -\sqrt{2}$ and $1 + \sin 2x = 2$ also satisfies but as $x \ge 0$, this solution is not in domain.

**Illustration 15:** Solve for x and y: $2^{\frac{1}{\cos^2 x}}\sqrt{y^2 - y + 1/2} \le 1$

**Solution:** $2^{\frac{1}{\cos^2 x}}\sqrt{y^2 - y + 1/2} \le 1$ ...........(i)

$2^{\frac{1}{\cos^2 x}}\sqrt{\left(y-\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2} \le 1$

Minimum value of $2^{\frac{1}{\cos^2 x}} = 2 \Rightarrow$ Minimum value of $\sqrt{\left(y-\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2} = \frac{1}{2}$

$\Rightarrow$ Minimum value of $2^{\frac{1}{\cos^2 x}}\sqrt{y^2 - y + \frac{1}{2}}$ is $1 \Rightarrow$ (i) is possible

when $2^{\frac{1}{\cos^2 x}}\sqrt{\left(y-\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2} \le 1 \Rightarrow \cos^2 x = 1$ and $y = 1/2$

$\Rightarrow \cos x = \pm 1 \Rightarrow x = n\pi$, where $n \in I$ Hence $x = n\pi, n \in I$ and $y = 1/2$.**Ans.**

**Illustration 16:** The number of solution(s) of $2\cos^2\left(\frac{x}{2}\right)\sin^2 x = x^2 + \frac{1}{x^2}, 0 \le x \le \frac{\pi}{2}$ is/are-

(A) 0 (B) 1 (C) infinite (D) none of these

**Solution :** Let $y = 2\cos^2\left(\frac{x}{2}\right)\sin^2 x = x^2 + \frac{1}{x^2} \Rightarrow y = (1+\cos x)\sin^2 x$ and $y = x^2 + \frac{1}{x^2}$

when $y = (1+\cos x)\sin^2 x = (\text{ a number} < 2)(\text{ a number} \le 1) \Rightarrow y < 2$

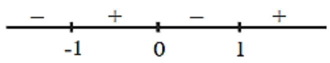
and when $y = x^2 + \frac{1}{x^2} = \left(x - \frac{1}{x}\right)^2 + 2 \ge 2 \Rightarrow y \ge 2$

No value of y can be obtained satisfying (i) and (ii), simultaneously

$\Rightarrow$ No real solution of the equation exists. **Ans.**

**Note:** If L.H.S. of the given trigonometric equation is always less than or equal to k and RHS is always greater than k, then no solution exists. If both the sides are equal to k for same value of θ, then solution exists and if they are equal for different values of θ, then solution does not exist.

**Do yourself-5 :**




**(i)** If $x^2 - 4x + 5 - \sin\ y = 0, y \in [0,2\pi)$, then –

(A) $x = 1, y = 0$ (B) $x = 1, y = \pi/2$

(C) $x = 2, y = 0$ (D) $x = 2, y = \pi/2$

**(ii)** If $\sin\ x + \cos\ x = \sqrt{y + \frac{1}{y}}, y > 0, x \in [0,\pi]$, then find the least positive value of x satisfying the given condition.

**6. TRIGONOMETRIC INEQUALITIES:**

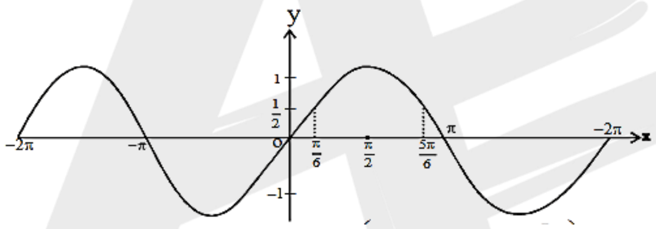
There is no general rule to solve trigonometric in equations and the same rules of algebra are valid here. The domain and range of trigonometric functions should be kept in mind.

**Illustration 17:** Find the solution set of inequality $\sin\ x > 1/2$.

**Solution:** When $\sin\ x = \frac{1}{2}$, the two values of x between 0 and 2π are π/6 and 5π/6.

From the graph of $y = \sin\ x$, it is obvious that between 0 and 2π, $\sin\ x > \frac{1}{2}$

for $\pi/6 < x < 5\pi/6$ Hence, $\sin\ x > 1/2 \Rightarrow 2n\pi + \pi/6 < x < 2n\pi + 5\pi/6, n \in I$




Thus, the required solution set is $\bigcup_{n\in I}\left(2n\pi + \frac{\pi}{6}, 2n\pi + \frac{5\pi}{6}\right)$ **Ans.**

**Illustration 18:** Find the values of α lying between 0 and π for which the inequality:

$\tan\alpha > \tan^3\ \alpha$ is valid.

**Solution :** We have : $\tan\ \alpha - \tan^3\ \alpha > 0 \Rightarrow \tan\ \alpha(1 - \tan^2\ \alpha) > 0$

$\Rightarrow (\tan\ \alpha)(\tan\ \alpha + 1)(\tan\ \alpha - 1) < 0$ So $\tan\ \alpha < -1, 0 < \tan\ \alpha < 1$

∴ Given inequality holds for $\alpha \in \left(0,\frac{\pi}{4}\right) \cup \left(\frac{\pi}{2},\frac{3\pi}{4}\right)$ **Ans.**

**Do yourself - 6:**

**(i)** Find the solution set of the inequality: $\cos\ x \geq -1/2$.

**(ii)** Find the values of x in the interval $[0,2\pi]$ for which $4\sin^2\ x - 8\sin\ x + 3 \leq 0$.

**Miscellaneous Illustration:**

**Illustration 19:** Solve the following equation: $\tan^2\ \theta + \sec^2\ \theta + 3 = 2(\sqrt{2}\sec\ \theta + \tan\ \theta)$

**Solution :** We have $\tan^2\ \theta + \sec^2\ \theta + 3 = 2\sqrt{2}\sec\ \theta + 2\tan\ \theta$

$\Rightarrow \tan^2\ \theta - 2\tan\ \theta + \sec^2\ \theta - 2\sqrt{2}\sec\ \theta + 3 = 0$

$\Rightarrow \tan^2\ \theta + 1 - 2\tan\ \theta + \sec^2\ \theta - 2\sqrt{2}\sec\ \theta + 2 = 0$

$\Rightarrow (\tan\ \theta - 1)^2 + (\sec\ \theta - \sqrt{2})^2 = 0 \Rightarrow\ \tan\ \theta = 1$ and $\sec\ \theta = \sqrt{2}$

As the periodicity of $\tan\ \theta$ and $\sec\ \theta$ are not same, we get

$\theta = 2n\pi + \frac{\pi}{4}, n \in I$ **Ans.**

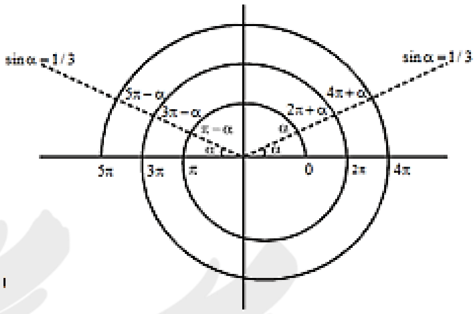
**Illustration 20 :** Find the solution set of equation $5^{(1+\log_5\ \cos\ x)} = 5/2$.

**Solution :** Taking log to base 5 on both sides in given equation :

$(1 + \log_5\ \cos\ x) \cdot \log_5\ 5 = \log_5\ (5/2) \Rightarrow \log_5\ 5 + \log_5\ \cos\ x = \log_5\ 5 - \log_5\ 2$

$\Rightarrow \log_5\ \cos\ x = -\log_5\ 2 \Rightarrow \cos\ x = 1/2 \Rightarrow\ x = 2n\pi \pm \pi/3, n \in I$ **Ans.**

**Illustration 21:** If the set of all values of x in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ satisfying $|4\sin\ x + \sqrt{2}| < \sqrt{6}$ is $\left(\frac{a\pi}{24}, \frac{b\pi}{24}\right)$ then find the value of $\left|\frac{a-b}{3}\right|$.




**Solution :** $|4\sin\ x + \sqrt{2}| < \sqrt{6}$

$\Rightarrow -\sqrt{6} < 4\sin\ x + \sqrt{2} < \sqrt{6}$

$\Rightarrow -\sqrt{6} - \sqrt{2} < 4\sin\ x < \sqrt{6} - \sqrt{2}$

$\Rightarrow \frac{-(\sqrt{6}+\sqrt{2})}{4} < \sin\ x < \frac{\sqrt{6}-\sqrt{2}}{4}$

$\Rightarrow -\frac{5\pi}{12} < x < \frac{\pi}{12}$ for $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Comparing with $\frac{a\pi}{24} < x < \frac{b\pi}{24}$, we get, $a = -10, b = 2$

$\therefore \left|\frac{a-b}{3}\right| = \left|\frac{-10-2}{3}\right| = 4$ **Ans.**

**Illustration 22 :** The number of values of x in the interval $[0,5\pi]$ satisfying the equation.

$3\sin^2\ x - 7\sin\ x + 2 = 0$ is- **[JEE 98]**

(A) 0 (B) 5 (C) 6 (D) 10

**Solution :** $3\sin^2\ x - 7\sin\ x + 2 = 0$

$\Rightarrow (3\sin\ x - 1)(\sin\ x - 2) = 0$

$\because \sin\ x \neq 2$

$\Rightarrow \sin\ x = \frac{1}{3} = \sin\ \alpha(\text{ say})$

where α is the least positive value of

x such that $\sin\alpha = \frac{1}{3}$.

Clearly $0 < \alpha < \frac{\pi}{2}$. We get the solution,

$x = \alpha, \pi - \alpha, 2\pi + \alpha, 3\pi - \alpha, 4\pi + \alpha$ and $5\pi - \alpha$.

Hence total six values in $[0,5\pi]$ **Ans.**

**ANSWERS FOR DO YOURSELF**

**1.** **(i)** (a) $\theta = n\pi + (-1)^n \frac{\pi}{6}, n \in I$ (b) $\theta = (2n+1)\frac{\pi}{3}, n \in I$

(c) $\theta = \frac{4n\pi}{3}, n \in I$ (d) $\theta = \frac{n\pi}{2}, n \in I$

(e) $\theta = n\pi \pm \frac{\pi}{12}, n \in I$ (f) $\theta = 2n\pi + (-1)^{n+1}\pi, n \in I$

**2 :** **(i)** (a) $x = n\pi + (-1)^{n+1}\frac{\pi}{6}, n \in I$ (b) $\alpha = \frac{n\pi}{2}$ or $\alpha = \frac{k\pi}{2} + \frac{3\pi}{8}, n, k \in I$

(c) $\theta = n\pi \pm \frac{\pi}{3}, n \in I$

(d) $\theta = n\pi + (-1)^n\alpha$, where $\alpha = \sin^{-1}\left(\frac{-1+\sqrt{17}}{8}\right)$ or $\sin^{-1}\left(\frac{-1-\sqrt{17}}{8}\right), n \in I$

**(ii)** $\theta = \left\{-\frac{\pi}{4}, \frac{-3\pi}{4}, \frac{-\pi}{2}, \frac{\pi}{4}, \frac{3\pi}{4}, \frac{\pi}{2}\right\}$

**3:** **(i)** (a) $x = 2n\pi - \frac{\pi}{4}, n \in I$ (b) $2\,m\pi + \frac{\pi}{2}, m \in I$

**4:** **(i)** $\theta = n\pi$ or $\theta = \frac{m\pi}{3} \pm \frac{\pi}{9}; n, m \in I$ **(ii)** $x = \frac{n\pi}{3}, n \in I$ and $k\pi \pm \frac{\pi}{3}, k \in I$

**5 :** **(i)** D **(ii)** $x = \frac{\pi}{4}$

**6:** **(i)** $\bigcup_{n \in I}\left[2n\pi - \frac{2\pi}{3}, 2n\pi + \frac{2\pi}{3}\right]$ **(ii)** $\left[\frac{\pi}{6}, \frac{5\pi}{6}\right]$

## EXERCISE 1

**1.** The number of solutions of the equation $\sin 2x - 2\cos x + 4\sin x = 4$ in the interval $[0, 5\pi]$ is-
(A) 6 (B) 4 (C) 3 (D) 5

**2.** Let $A = \{\theta : \sin(\theta) = \tan(\theta)\}$ and $B = \{\theta : \cos(\theta) = 1\}$ be two sets. Then -
(A) $A = B$
(B) $A \subset B$ and $B - A \neq \phi$
(C) $A \not\subset B$
(D) $B \not\subset A$

**3.** The complete solution set of the inequality $\tan^2 x - 2\sqrt{2}\tan x + 1 \leq 0$ is -
(A) $n\pi + \frac{\pi}{8} \leq x \leq \frac{3\pi}{8} + n\pi, n \in I$
(B) $n\pi + \frac{\pi}{4} \leq x \leq \frac{3\pi}{4} + n\pi, n \in I$
(C) $n\pi + \frac{\pi}{16} \leq x \leq \frac{3\pi}{8} + n\pi, n \in I$
(D) $n\pi + \frac{\pi}{3} \leq x \leq \frac{2\pi}{3} + n\pi, n \in I$

**4.** The general solution of the equation $\tan^2 \alpha + 2\sqrt{3}\tan \alpha = 1$ is given by-
(A) $\alpha = \frac{n\pi}{2} (n \in I)$
(B) $\alpha = (2n+1)\frac{\pi}{2} (n \in I)$
(C) $\alpha = (6n+1)\frac{\pi}{12} (n \in I)$
(D) $\alpha = \frac{n\pi}{12} (n \in I)$

**5.** If $2\tan^2 \theta = \sec^2 \theta$, then the general solution of $\theta$ -
(A) $n\pi + \frac{\pi}{4} (n \in I)$ (B) $n\pi - \frac{\pi}{4} (n \in I)$ (C) $n\pi \pm \frac{\pi}{4} (n \in I)$ (D) $2n\pi \pm \frac{\pi}{4} (n \in I)$

**6.** Number of principal solution(s) of the equation $4.16^{\sin^2 x} = 2^{6\sin x}$ is
(A) 1 (B) 2 (C) 3 (D) 4

**7.** The general solution of equation $4\cos^2 x + 6\sin^2 x = 5$ is-
(A) $x = n\pi \pm \frac{\pi}{2} (n \in I)$
(B) $x = n\pi \pm \frac{\pi}{4} (n \in I)$
(C) $x = n\pi \pm \frac{3\pi}{2} (n \in I)$
(D) None of these

**8.** If $\tan \theta + \tan 4\theta + \tan 7\theta = \tan \theta \tan 4\theta \tan 7\theta$, then $\theta =$
(A) $\frac{n\pi}{4}$ (B) $\frac{n\pi}{7}$ (C) $\frac{n\pi}{12}$ (D) $n\pi$
where $n \in I$, $\theta \neq (2n+1)\frac{\pi}{2}$, $\theta \neq (2n+1)\frac{\pi}{8}$, $\theta \neq (2n+1)\frac{\pi}{14}$

**9.** If $\frac{1-\cos 2\theta}{1+\cos 2\theta} = 3$, then the general solution of $\theta$ is-
(A) $2n\pi \pm \pi/6$ (B) $n\pi \pm \pi/6$ (C) $2n\pi \pm \pi/3$ (D) $n\pi \pm \pi/3$
where $n \in I$

**10.** The number of solutions of the equation $2\cos\left(\frac{x}{2}\right) = 3^x + 3^{-x}$ is-
(A) 1 (B) 2 (C) 3 (D) None

**11.** The number of real solutions of the equation $\sin(e^x) = 5^x + 5^{-x}$ is-
(A) 0 (B) 1 (C) 2 (D) infinitely many

**12.** If $x \in \left[-\frac{5\pi}{2}, \frac{5\pi}{2}\right]$, then the greatest positive solution of $1 + \sin^4 x = \cos^2 3x$ is-
(A) $\pi$ (B) $2\pi$ (C) $\frac{5\pi}{2}$ (D) none of these

**13.** The general value of $\theta$ satisfying $\sin^2 \theta + \sin \theta = 2$ is-
(A) $n\pi + (-1)^n\frac{\pi}{6}$ (B) $2n\pi + \frac{\pi}{4}$ (C) $n\pi + (-1)^n\frac{\pi}{2}$ (D) $n\pi + (-1)^n\frac{\pi}{3}$

**14.** The number of solutions of the equation $\tan^2 x - \sec^{10} x + 1 = 0$ in $(0,10)$ is-
(A) 3 (B) 6 (C) 10 (D) 11

**15.** The solution set of $(5+4\cos\theta)(2\cos\theta+1)=0$ in the interval $[0,2\pi]$ is

(A) $\left\{\frac{\pi}{3},\frac{2\pi}{3}\right\}$ (B) $\left\{\frac{\pi}{3},\pi\right\}$ (C) $\left\{\frac{2\pi}{3},\frac{4\pi}{3}\right\}$ (D) $\left\{\frac{2\pi}{3},\frac{5\pi}{3}\right\}$

**16.** The equation $\sin x\cos x=2$ has :

(A) one solution (B) two solutions (C) infinite solutions (D) no solution

**17.** If $\tan^2\theta-(1+\sqrt{3})\tan\theta+\sqrt{3}=0$, then the general value of $\theta$ is :

(A) $n\pi+\frac{\pi}{4}, n\pi+\frac{\pi}{3}$ (B) $n\pi-\frac{\pi}{4}, n\pi+\frac{\pi}{3}$

(C) $n\pi+\frac{\pi}{4}, n\pi-\frac{\pi}{3}$ (D) $n\pi-\frac{\pi}{4}, n\pi-\frac{\pi}{3}$ where $n\in I$

**18.** If $0\le x\le 3\pi, 0\le y\le 3\pi$ and $\cos x.\sin y=1$, then the possible number of values of the ordered pair $(x,y)$ is -

(A) 6 (B) 12 (C) 8 (D) 15

**19.** If $\frac{\tan 2\theta+\tan\theta}{1-\tan\theta\tan 2\theta}=0$, then the general value of $\theta$ is

(A) $n\pi; n\in I$ (B) $\frac{n\pi}{3}; n\in I$ (C) $\frac{n\pi}{4}$ (D) $\frac{n\pi}{6}; n\in I$

where $n\in I$

**20.** The most general values of x for which $\sin x+\cos x=\min_{a\in R}\{1,a^2-4a+6\}$ is given by

(A) $2n\pi$ (B) $2n\pi+\frac{\pi}{2}$

(C) $n\pi+(-1)^n\frac{\pi}{4}-\frac{\pi}{4}$ (D) None of these where $n\in I$

## EXERCISE 2 (JM)

**1.** The number of values of x in the interval $[0,3\pi]$ satisfying the equation

$2\sin^2 x + 5\sin x - 3 = 0$ is- **[AIEEE 2006]**

(1) 6 (2) 1 (3) 2 (4) 4

**2.** If $0 < x < \pi$, and $\cos x + \sin x = \frac{1}{2}$, then $\tan x$ is- **[AIEEE 2006]**

(1) $(4-\sqrt{7})/3$ (2) $-(4+\sqrt{7})/3$ (3) $(1+\sqrt{7})/4$ (4) $(1-\sqrt{7})/4$

**3.** Let A and B denote the statements

**A**: $\cos\alpha + \cos\beta + \cos\gamma = 0$, **B:** $\sin\alpha + \sin\beta + \sin\gamma = 0$

If $\cos(\beta-\gamma) + \cos(\gamma-\alpha) + \cos(\alpha-\beta) = -\frac{3}{2}$, then - **[AIEEE 2009]**

(1) Both **A** and **B** are true (2) Both **A** and **B** are false

(3) **A** is true and **B** is false (4) **A** is false and **B** is true

**4.** The possible values of $\theta \in (0,\pi)$ such that $\sin(\theta) + \sin(4\theta) + \sin(7\theta) = 0$ are **[AIEEE 2011]**

(1) $\frac{2\pi}{9}, \frac{\pi}{4}, \frac{4\pi}{9}, \frac{\pi}{2}, \frac{3\pi}{4}, \frac{8\pi}{9}$ (2) $\frac{\pi}{4}, \frac{5\pi}{12}, \frac{\pi}{2}, \frac{2\pi}{3}, \frac{3\pi}{4}, \frac{8\pi}{9}$

(3) $\frac{2\pi}{9}, \frac{\pi}{4}, \frac{\pi}{2}, \frac{2\pi}{3}, \frac{3\pi}{4}, \frac{35\pi}{36}$ (4) $\frac{2\pi}{9}, \frac{\pi}{4}, \frac{\pi}{2}, \frac{2\pi}{3}, \frac{3\pi}{4}, \frac{8\pi}{9}$

**5.** If $0 \le x < 2\pi$, then the number of real values of x, which satisfy the equation

$\cos x + \cos 2x + \cos 3x + \cos 4x = 0$, is:- **[JEE(Main) 2016]**

(1) 9 (2) 3 (3) 5 (4) 7

**6.** If $5(\tan^2 x - \cos^2 x) = 2\cos 2x + 9$, then the value of $\cos 4x$ is: **[JEE (Main)-2017]**

(1) $-\frac{7}{9}$ (2) $-\frac{3}{5}$ (3) $\frac{1}{3}$ (4) $\frac{2}{9}$

**7.** If sum of all the solutions of the equation $8\cos x \cdot \left(\cos\left(\frac{\pi}{6}+x\right)\cos\left(\frac{x}{6}-x\right) - \frac{1}{2}\right) = 1$ in $[0,\pi]$ is $k\pi$, then k is equal to **[JEE (Main)-2018]**

(1) $\frac{20}{9}$ (2) $\frac{2}{3}$ (3) $\frac{13}{9}$ (4) $\frac{8}{9}$

**8.** All the pairs (x, y) that satisfy the inequality $2\sqrt{\sin^2 x - 2\sin x + 5} \cdot \frac{1}{4\sin^2 y} \le 1$ also satisfy the equation: **[JEE (Main)-2019]**

(A) $2|\sin x| = 3\sin y$ (B) $2\sin x = \sin y$

(C) $\sin x = 2\sin y$ (D) $\sin x = |\sin y|$

**9.** If $\alpha$ and $\beta$ are the roots of equation $(k+1)\tan^2 x - \sqrt{2}\lambda\tan x = 1 - k$ and $\tan^2(\alpha+\beta) = 50$. Find value of $\lambda$. **[JEE (Main)-2020]**

(A) 10 (B) 5 (C) 7 (D) 12

**10.** The number of solutions of the equation $32^{\tan^2 x} + 32^{\sec^2 x} = 81, 0 \le x \le \frac{\pi}{4}$ is:

**[JEE (Main)-2021]**

(A) 3 (B) 1 (C) 0 (D) 2

**11.** The number of solutions of $|\cos x| = \sin x$, such that $-4\pi \le x \le 4\pi$ is: **[JEE (Main)-2022]**

(A) 4 (B) 6 (C) 8 (D) 12

**12.** If m and n respectively are the numbers of positive and negative value of $\theta$ in the interval $[-\pi,\pi]$ that satisfy the equation $\cos 2\theta\cos\frac{\theta}{2} = \cos 3\theta\cos\frac{9\theta}{2}$, then mn is equal to

## EXERCISE 3 (JA)

**1.** The number of integral values of k for which the equation $7\cos x + 5\sin x = 2k + 1$ has a solution is **[JEE 2002 (Screening), 3]**

(A) 4 (B) 8 (C) 10 (D) 12

**2.** $\cos(\alpha - \beta) = 1$ and $\cos(\alpha + \beta) = 1/e$, where $\alpha, \beta \in [-\pi, \pi]$, numbers of pairs of $\alpha, \beta$ which satisfy both the equations is **[JEE 2005 (Screening)]**

(A) 0 (B) 1 (C) 2 (D) 4

**3.** If $0 < \theta < 2\pi$, then the intervals of values of $\theta$ for which $2\sin^2\theta - 5\sin\theta + 2 > 0$, is

**[JEE-2006, 3]**

(A) $\left(0, \frac{\pi}{6}\right) \cup \left(\frac{5\pi}{6}, 2\pi\right)$ (B) $\left(\frac{\pi}{8}, \frac{5\pi}{6}\right)$ (C) $\left(0, \frac{\pi}{8}\right) \cup \left(\frac{\pi}{6}, \frac{5\pi}{6}\right)$ (D) $\left(\frac{41\pi}{48}, \pi\right)$

**4.** The number of solutions of the pair of equations **[JEE 2007, 3]**

$2\sin^2\theta - \cos 2\theta = 0$

$2\cos^2\theta - 3\sin\theta = 0$

in the interval $[0, 2\pi]$ is

(A) zero (B) one (C) two (D) four

**5.** The number of values of $\theta$ in the interval $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$ such that $\theta \neq \frac{n\pi}{5}$ for $n = 0, \pm 1, \pm 2$

and $\tan\theta = \cot 5\theta$ as well as $\sin 2\theta = \cos 4\theta$, is **[JEE 2010, 3]**

**6.** The positive integer value of $n > 3$ satisfying the equation **[JEE 2011, 4]**

$\frac{1}{\sin\left(\frac{\pi}{n}\right)} = \frac{1}{\sin\left(\frac{2\pi}{n}\right)} + \frac{1}{\sin\left(\frac{3\pi}{n}\right)}$ is

**7.** Let $\theta, \varphi \in [0, 2\pi]$ be such that $2\cos\theta(1 - \sin\varphi) = \sin^2\theta\left(\tan\frac{\theta}{2} + \cot\frac{\theta}{2}\right)\cos\varphi - 1$,

$\tan(2\pi - \theta) > 0$ and $-1 < \sin\theta < -\frac{\sqrt{3}}{2}$. Then $\varphi$ cannot satisfy –

(A) $0 < \varphi < \frac{\pi}{2}$ (B) $\frac{\pi}{2} < \varphi < \frac{4\pi}{3}$ (C) $\frac{4\pi}{3} < \varphi < \frac{3\pi}{2}$ (D) $\frac{3\pi}{2} < \varphi < 2\pi$.

**8.** For $x \in (0, \pi)$, the equation $\sin x + 2\sin 2x - \sin 3x = 3$ has **[JEE(Advanced)-2014, 3(-1)]**

(A) infinitely many solutions (B) three solutions

(C) one solution (D) no solution

**9.** The number of distinct solutions of equation $\frac{5}{4}\cos^2 2x + \cos^4 x + \sin^4 x + \cos^6 x + \sin^6 x$

$= 2$ in the interval $[0, 2\pi]$ is **[JEE 2015, 4M,-0M]**

**10.** Let a, b, c be three non-zero real numbers such that the equation

$\sqrt{3}a\cos x + 2b\sin x = c, x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ has two distinct real roots $\alpha$ and $\beta$ with $\alpha + \beta = \frac{\pi}{3}$. Then, the value $\frac{b}{a}$ is……… **[JEE (Advanced)-2018, 3(0), P- 1]**

**11.** Answer the following appropriately matching the list based on the information given in the paragraph.

Let $f(x) = \sin(\pi\cos x)$ and $g(x) = \cos(2\pi\sin x)$ be two functions defined for $x > 0$. Define the following sets whose elements are written in the increasing order.

$X = \{x : f(x) = 0\}$, $Y = \{x : f'(x) = 0\}$

$Z = \{x : g(x) = 0\}$, $W = \{x : g'(x) = 0\}$ **[JEE (Advanced)-2019]**

List I contains the sets X, Y, Z and W. List II contains some information regarding these sets.

| **List I** | **List II** |
|---|---|
| (I) X | (P) $\supseteq \left\{\frac{\pi}{2}, \frac{3\pi}{2}, 4\pi, 7\pi\right\}$ |
| (II) Y | (Q) an arithmetic progression |
| (III) Z | (R) NOT an arithmetic progression |
| (IV) W | (S) $\supseteq \left\{\frac{\pi}{6}, \frac{7\pi}{6}, \frac{13\pi}{6}\right\}$ |
| | (T) $\supseteq \left\{\frac{\pi}{3}, \frac{2\pi}{3}, \pi\right\}$ |
| | (U) $\supseteq \left\{\frac{\pi}{6}, \frac{3\pi}{4}\right\}$ |

Which of the following is the only CORRECT combination ?

(A) (I), (Q), (U)
(B) (II), (Q), (T)
(C) (I), (P), (R)
(D) (II), (R), (S)

**12.** Consider the following lists:

| **List – I** | **List - II** |
|---|---|
| (I) $\left\{x \in \left[-\frac{2\pi}{3}, \frac{2\pi}{3}\right] : \cos x + \sin x = 1\right\}$ | (P) has two elements |
| (II) $\left\{x \in \left[-\frac{5\pi}{18}, \frac{5\pi}{18}\right] : \sqrt{3}\tan 3x = 1\right\}$ | (Q) has three elements |
| (III) $\left\{x \in \left[-\frac{6\pi}{5}, \frac{6\pi}{5}\right] : 2\cos(2x) = \sqrt{3}\right\}$ | (R) has four elements |
| (IV) $\left\{x \in \left[-\frac{7\pi}{4}, \frac{7\pi}{4}\right] : \sin x - \cos x = 1\right\}$ | (S) has five elements |
| | (T) has six elements |

The correct option is: **[JEE (Advanced)-2022]**

(A) (I) → (P); (II) → (S); (III) → (P); (IV) → (S)
(B) (I) → (P); (II) → (P); (III) → (T); (IV) → (R)
(C) (I) → (Q); (II) → (P); (III) → (T); (IV) → (S)
(D) (I) → (Q); (II) → (S); (III) → (P); (IV) → (R)

## ANSWER KEY

## EXERCISE 1

| | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **1.** | (C) | **2.** | (C) | **3.** | (A) | **4.** | (C) | **5.** | (C) | **6.** | (C) | **7.** | (B) |
| **8.** | (C) | **9.** | (D) | **10.** | (A) | **11.** | (A) | **12.** | (B) | **13.** | (C) | **14.** | (A) |
| **15.** | (C) | **16.** | (D) | **17.** | (A) | **18.** | (A) | **19.** | (B) | **20.** | (C) | | |

## EXERCISE 2 (JM)

| | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **1.** | 4 | **2.** | 2 | **3.** | 1 | **4.** | 1 | **5.** | 4 | **6.** | 1 | **7.** | 3 |
| **8.** | D | **9.** | A | **10.** | B | **11.** | C | **12.** | 25 | | | | |

## EXERCISE 3 (JA)

| | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **1.** | B | **2.** | D | **3.** | A | **4.** | C | **5.** | 3 | **6.** | 7 | **7.** | A, C, D |
| **8.** | D | **9.** | 8 | **10.** | 0.50 | **11.** | B | **12.** | B | | | | |