

Mains

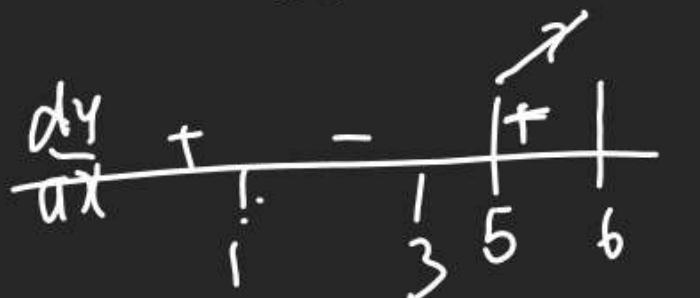
Q.  $f(x) = 3x^3 - 18x^2 + 27x - 40$

On the Set  $S = \{x \in R, x^2 + 30 \leq 11x\}$

Find Max<sup>m</sup> Value.

$$\begin{aligned} f'(x) &= 9x^2 - 36x + 27 \\ &= 9(x^2 - 4x + 3) \\ &= 9(x-1)(x-3) \end{aligned}$$

$x^2 - 11x + 30 \leq 0$   
 $(x-5)(x-6) \leq 0$   
 $5 \leq x \leq 6$



Max<sup>m</sup> at  $x=6$

$$\begin{aligned} f(6) &= 3 \times 216 - 18 \times 36 \\ &\quad + 27 \times 6 - 40 \\ &= 122 \end{aligned}$$

Q. If  $f$  be a fn defined on  $R$  such that

$$f(x) = 2010(x-2009)(x-2010)^2(x-2011)^3(x-2012)^4$$

$\forall x \in R$ . If  $g$  is a fn defined on  $R$  with values  $(0, \infty)$  such that  $f(x) = \ln g(x)$

The No of Pts at which  $g$  has L. Max?

$$g(x) = e^{f(x)}$$

$$g'(x) = e^{f(x)} \times f'(x)$$

$$g'(x) = e^{f(x)} \times 2010(x-2009)(x-2010)^2(x-2011)^3(x-2012)^4$$



2009 2010 2011 2012  
 Max NMNM Min NMNM

Only 1 pt of Max.  $x=2009$

$$\text{Q) } f(x) = \begin{cases} 3x^2 + 12x - 1 & -1 \leq x \leq 2 \\ 37 - x & 2 < x \leq 3 \end{cases}$$

then WOTF is correct.

A) (Ans)  $[-1, 3]$  (B) Max<sup>m</sup>  $x=2$

(C) Duff in  $[-1, 3]$  (D) NOT

A) Cont's  $\rightarrow x=2$  UR

$$3(2)^2 + 12 \times 2 - 1 = 37 - 2 \\ 35 = 35$$

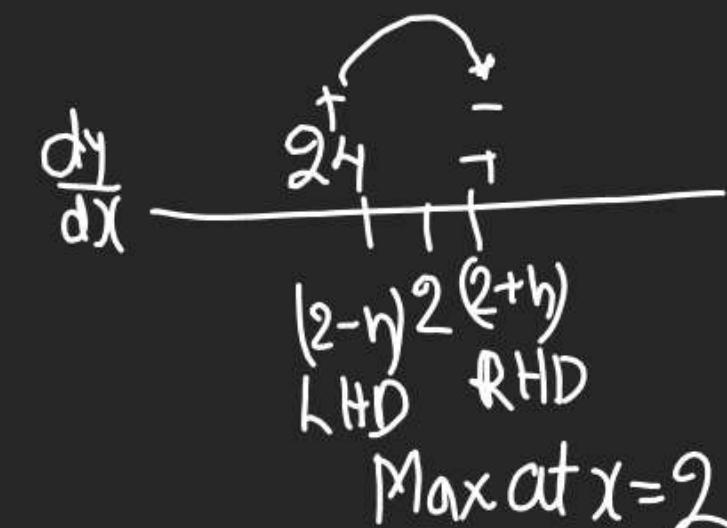
B) Duff

$$f(x) = \begin{cases} 6x + 12 & -1 \leq x \leq 2 \\ -1 & 2 < x \leq 3 \end{cases}$$

$$x=2 \quad LHD = 6 \times 2 + 12 = 24$$

$$RHD = -1$$

$LHD \neq RHD \Rightarrow$  Not duff



Max at  $x=2$

Now  $f(x)$  &  $f'(x)$  are sets of  
Min & Max of  
 $f(x) = g(x^4 + 12x^3 - 36x^2 + 25)$   $\forall x \in \mathbb{R}$

then  $S_1 = \{-\}$ ,  $S_2 = \{-\}$

$$S_1 = -2, +1, \quad S_2 = 0$$

Q Set of all Real values of  $\lambda$  for which  
Main  $f(x) = (1 - e^x)(\lambda + \sin x)$   
 $\lambda \in (-\frac{\pi}{2}, \frac{\pi}{2})$  has only one Max &  
one Min in  $\dots$ ?

$$f(x) = 8m^2 x (\lambda + 8m \sin x) = \lambda (8m^2 x + 8m^3 \sin x)$$

$$f'(x) = 2 \lambda (6m)(6x + 38m^2 x \cdot 6 \cos x)$$

$$= 8mx(6x(2\lambda + 38m \cos x))$$

$$\lambda \in \left(-\frac{3}{2}, \frac{3}{2}\right) \quad \begin{matrix} 0 \\ -50 \end{matrix} \quad \begin{matrix} 0 \\ 0 \end{matrix} \quad \begin{matrix} 0 \\ 0 \end{matrix} \quad 0(8m) = -\frac{2\lambda}{3} > 0$$

$$-\frac{2\lambda}{3} < 1$$

$$\begin{matrix} 1 \\ 0 \end{matrix} \quad \begin{matrix} \text{Min} \\ 8m \end{matrix} \quad \begin{matrix} 3 \\ 2 \end{matrix} > \lambda > -\frac{3}{2}$$

Q f: R → R be defined as

$$f(x) = |x| + |x^2 - 1|. \text{ Find}$$

total No of Pts at which

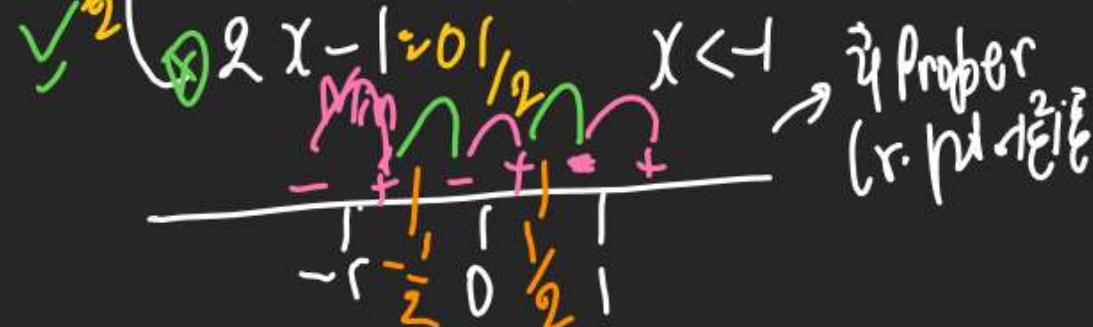
f(x) attains either L Max

$$\text{or. L Min} = 5$$

$$f(x) = |x| + |x^2 - 1|$$

$$f'(x) = \frac{|x|}{x} + \frac{|x^2 - 1|}{(x^2 - 1)} \times 2x$$

$$f'(x) = \begin{cases} 2x+1=0 & x > 1 \\ 1-2x=0 & 0 < x < 1 \\ -1-2x=0 & -1 < x < 0 \\ 2x-1=0 & x < -1 \end{cases}$$



Q Find all Possible values of

Max a for which f(x) =

$$x^3 + 3(a-1)x^2 + 3(a^2-a)x - 1$$

has +ve Pt. of Max<sup>ma</sup> = Origin<sup>2</sup>

Rem:— This is Indirect Q of LOR. R.S. Side

$$\begin{aligned} \text{① } -\frac{B}{A} > 0 \Rightarrow \frac{B}{A} < 0 \\ \text{② } \frac{6(a-1)}{3} < 0 \end{aligned}$$

$$a < 1$$

$$\frac{C}{A} > 0$$

$$\frac{3(a^2-a)}{3} > 0$$

$$a < -3 \cup a > 3$$

$$\text{④ } D > 0$$

$$\begin{aligned} 36(a-1)^2 - 36(a^2-a) &> 0 \\ a^2 - 14a + 49 - a^2 + a &> 0 \end{aligned}$$

$$14a < 58$$

$$a < \frac{29}{7}$$

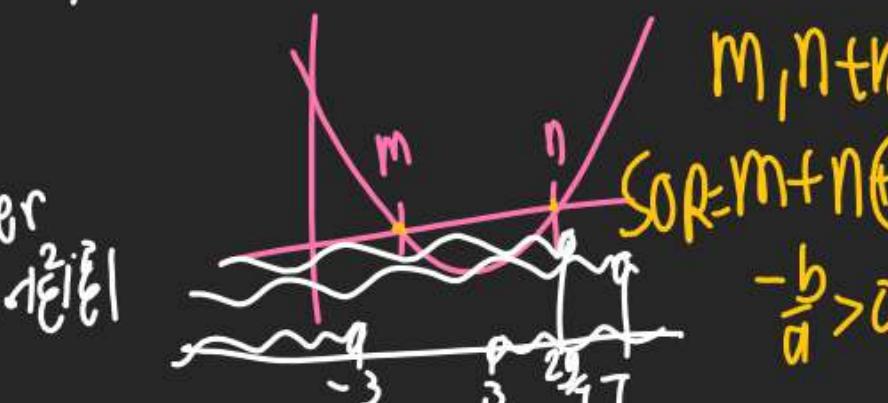
$$a \in (-\infty, 3) \cup (3, \frac{29}{7})$$

$$a < -3 \cup a > 3$$

$$\frac{3(a^2-a)}{(a-3)(a+3)} > 0$$



$$f'(x) = 3x^2 + 6(a-1)x + 3(a^2-a)$$



$$\begin{aligned} -\frac{b}{a} > 0 \mid \text{POR} \mid f(0) > 0 \\ \frac{C}{A} > 0 \end{aligned}$$

$$\begin{aligned} \frac{3(a^2-a)}{3} > 0 \\ (a-3)(a+3) > 0 \end{aligned}$$

M2

$$\begin{aligned} \text{① } D > 0 \mid \text{② } \frac{-b}{a} < \eta \mid \text{③ } f(0) > 0 \end{aligned}$$

M3 KKK

Pt of Max<sup>ma</sup> +

If it is clear that  
Max/Min Both +ve/-ve

$$\text{① } 0 < a < \beta \text{ Min}$$

$$\text{② } a = \frac{-6(a-1) + \sqrt{36(a-1)^2 - 36(a^2-a)}}{6} > 0$$

$$a < \frac{29}{7}$$

$$a \in (-\infty, 3) \cup (3, \frac{29}{7})$$

# Board S.D.T. (Second Derivative Test).

Basic  $\rightarrow$  Concave up

$$\text{Min} \quad \frac{d^2y}{dx^2} > 0$$

(on down Max.)



$$\frac{d^2y}{dx^2} < 0$$

$$\textcircled{1} \quad f'(x) = 0 \rightarrow \text{crit. pt. } (a, b)$$

$$\textcircled{2} \quad \left. \frac{d^2y}{dx^2} \right|_{x=a} = \begin{cases} + & x=a \text{ in Min pt} \\ - & x=b \text{ in Max pt} \end{cases}$$

$$\left. \frac{d^2y}{dx^2} \right|_{x=b} = \begin{cases} + & x=b \text{ in Max pt} \\ - & x=a \text{ in Min pt} \end{cases}$$

$$\textcircled{3} \quad \left. \frac{d^2y}{dx^2} \right|_{x=a} = 0 \quad \begin{array}{l} (\text{at point of inflection}) \\ \text{शाफ्ट} \end{array}$$

$$\textcircled{4} \quad \left. \frac{d^3y}{dx^3} \right|_{x=a} \neq 0 \rightarrow \begin{array}{l} \text{Concave up} \\ \text{PDI} \end{array}$$

$$\textcircled{5} \quad \text{But if } \left. \frac{d^3y}{dx^3} \right|_{x=a} = 0 \quad \begin{array}{l} (\text{उत्तराधिकारी}) \\ \text{उत्तराधिकारी} \end{array}$$

$$\textcircled{6} \quad \left. \frac{d^4y}{dx^4} \right|_{x=a} = \begin{array}{ll} + & \text{Min} \\ - & \text{Max.} \end{array} = 0 \quad \begin{array}{l} (\text{उत्तराधिकारी}) \\ (\text{उत्तराधिकारी}) \end{array}$$

Even derivative  $\oplus$  Min  
 $\ominus$  Max.  
 $= 0$   $\nexists$

$$1) \frac{dy}{dx} = 3x^2 - 0 \quad x=0$$

$$2) \frac{d^2y}{dx^2} = 6x$$

$$3) \left. \frac{d^3y}{dx^3} \right|_{x=0} = 6 \neq 0 \quad \begin{array}{l} (\text{उत्तराधिकारी}) \\ x=0 \text{ in PDI} \end{array}$$

$$4) \left. \frac{d^4y}{dx^4} \right|_{x=0} = 24 \quad \begin{array}{l} (\text{उत्तराधिकारी}) \\ x=0 \text{ in PDI} \end{array}$$

$$5) \frac{dy}{dx} = 4x^3 = 0 \quad x=0$$

$$6) \frac{d^2y}{dx^2} = 12x^2$$

$$7) \left. \frac{d^3y}{dx^3} \right|_{x=0} = (24)^2$$

$$8) \left. \frac{d^4y}{dx^4} \right|_{x=0} = 24x^0$$

$$9) \left. \frac{d^5y}{dx^5} \right|_{x=0} = 96 \quad \begin{array}{l} (\text{उत्तराधिकारी}) \\ x=0 \text{ in N(in y=0)} \end{array}$$