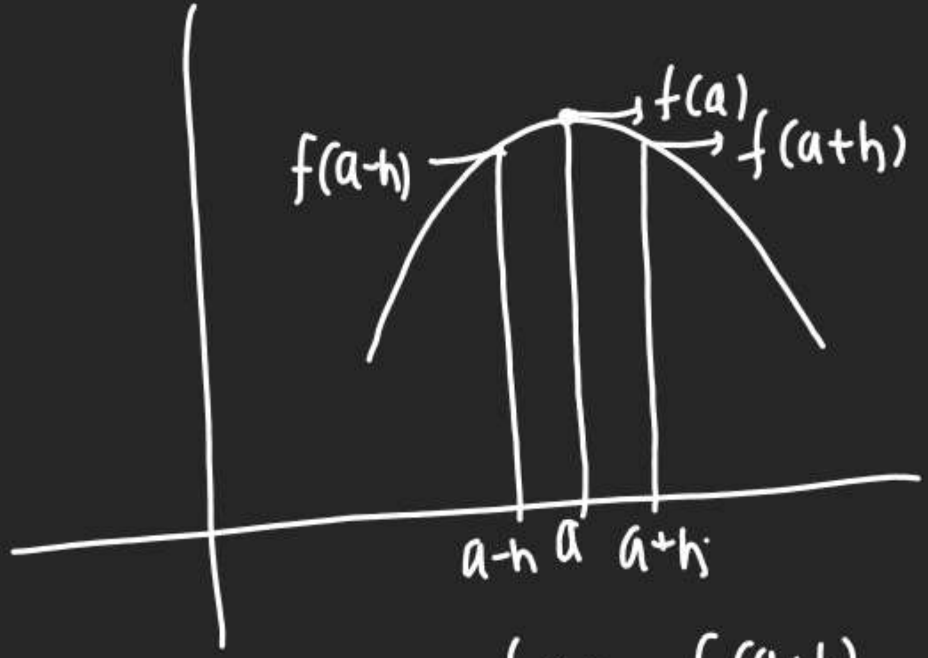
Nishant Jindal

Maxima & Minima (In ABD this most Qs generating chapter)

① Local Maxima = Relative Max.



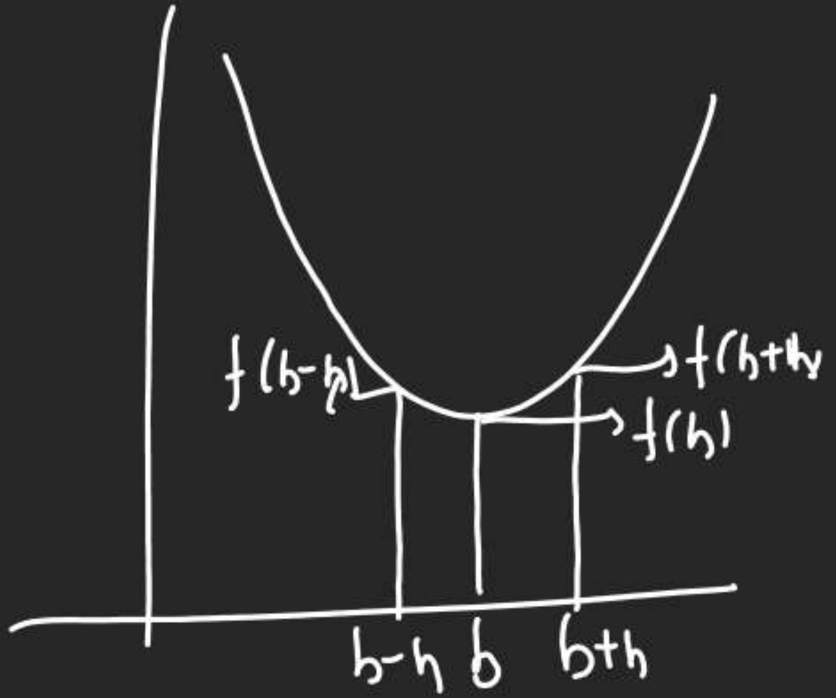
$f(a) > f(a+h)$

$f(a) > f(a-h)$

f(x) having max^{a} at

$x = a$

② Local Minima



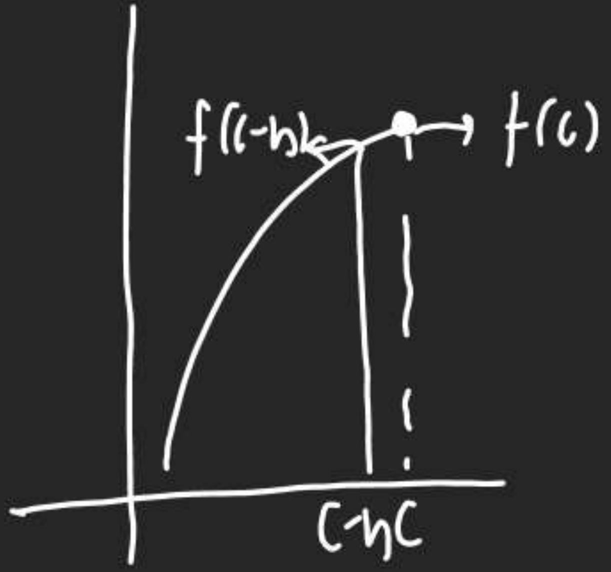
$f(b) < f(b-h)$

$f(b) < f(b+h)$

$\therefore x = b$ is L. Min

1) Maxima & Minima Pts are also known as Pt of Extrema

2) One Sided Extrema



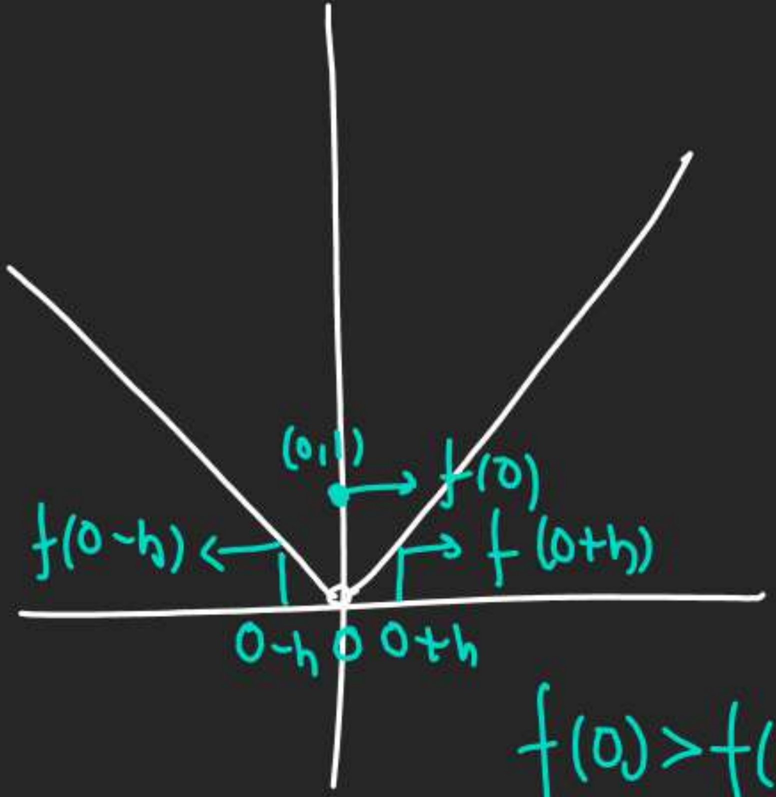
$f(c) > f(c-h)$

$\therefore x = c$ is Pt of Max^{a}.

Nishant Jindal

Q $f(x) = \begin{cases} |x| & x \neq 0 \\ 1 & x = 0 \end{cases}$

Check for extrema at $x=0$.



$f(0) > f(0+h)$

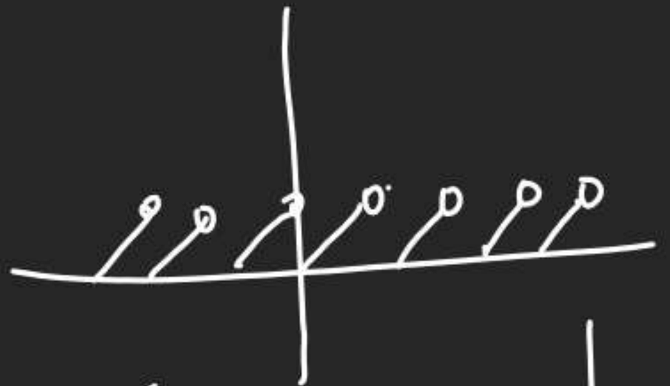
$f(0) > f(0-h)$

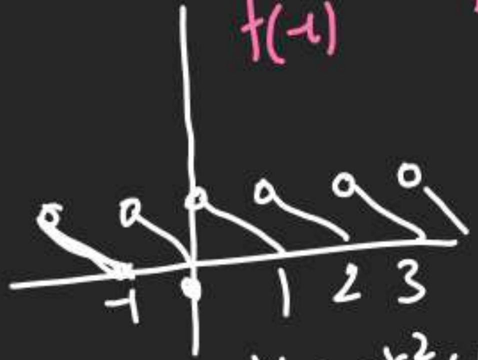
$\therefore$ $x=0$ is Pt of Max^m.

Q $f(x) = \begin{cases} \{-x\} & -1 \le x < 0 \\ 1 - x^2 & 0 \le x \le 1 \\ [x] & 1 < x \le 2 \end{cases}$

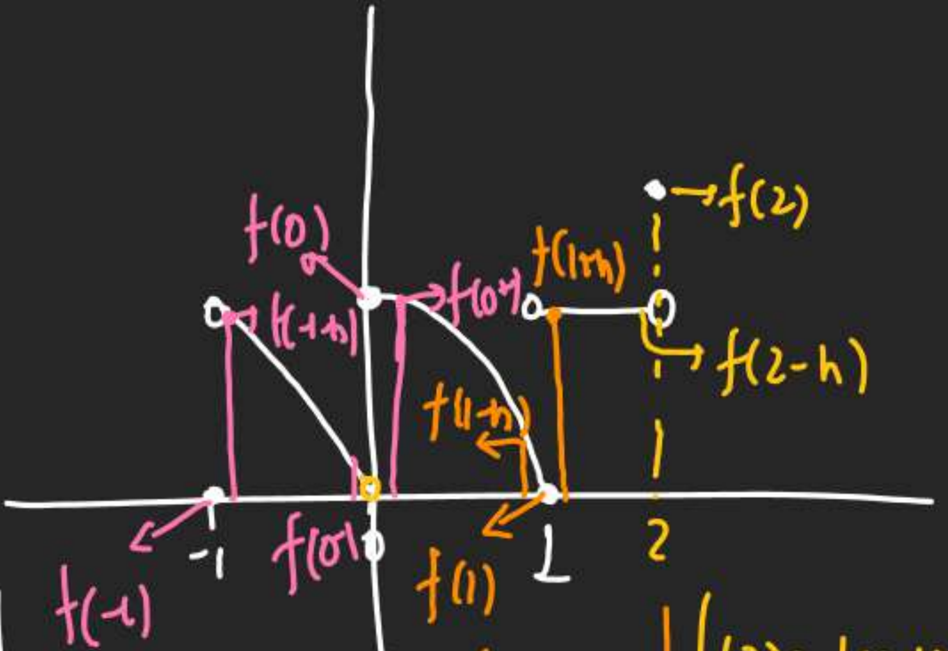
Find Pts of LMax, LMin.

① $y = \{x\} \rightarrow y = \{-x\}$





② $y = 1 - x^2$



$f(-1) < f(-1+h)$

$x = -1$ is LMin.

$f(0) > f(0+h)$

$f(0) > f(0-h)$

$x = 0$ is LMax.

$f(1) < f(1-h)$

$f(1) < f(1+h)$

$x = 1$ is LMin.

$f(2) > f(2-h)$

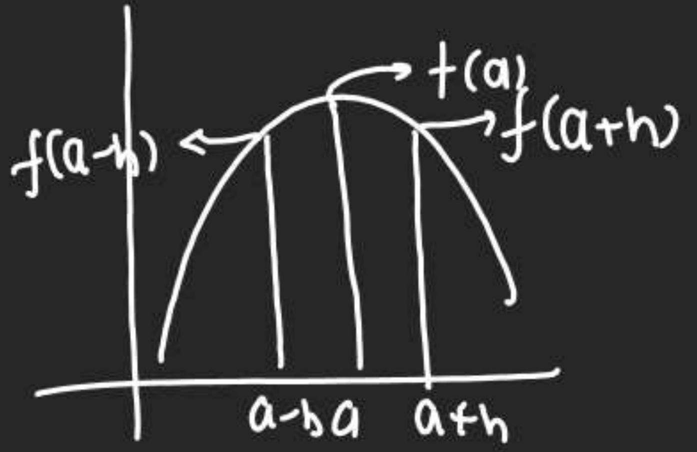
2 is LMax.

Max$\{0, 2\}$

Min$\{-1, 1\}$

Nishant Jindal

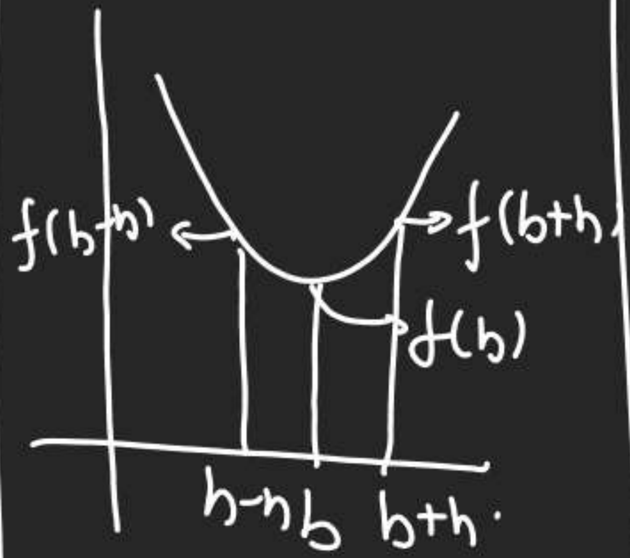
Max-Min



$f(a) > f(a+h)$

$f(a) > f(a-h)$

$\Rightarrow x = a$ is L.Max.



$f(b) < f(b+h)$

$f(b) < f(b-h)$

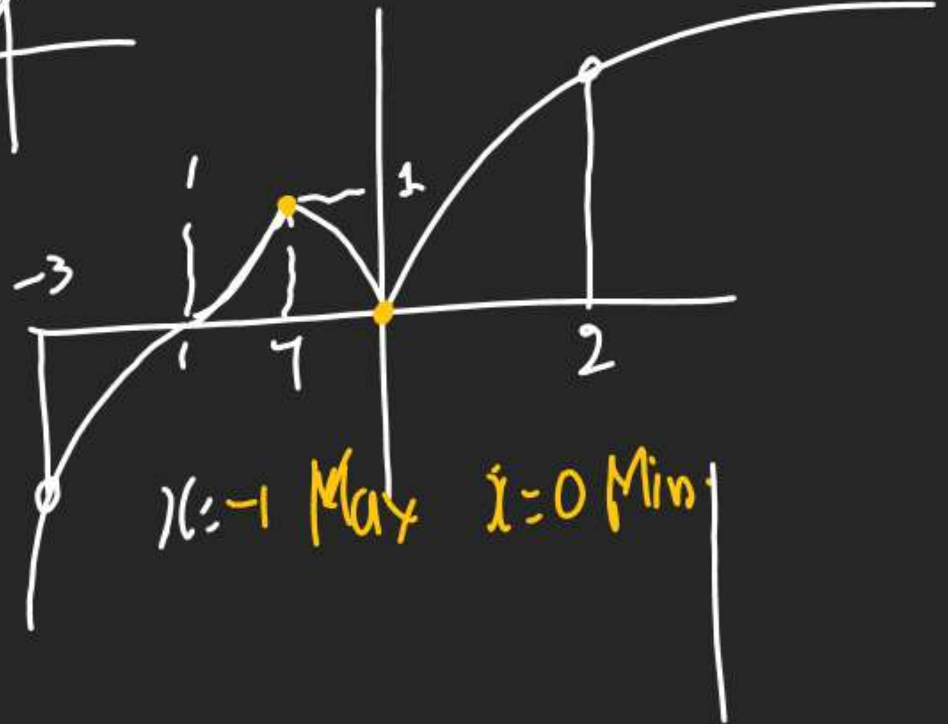
$\Rightarrow x = b$ is L Min.

Q find total No. of L Max.
L Min pts of $f(x)$

$$f(x) = \begin{cases} (2+x)^3 & -3 < x \le -1 \\ x^{2/3} & -1 < x < 2 \end{cases}$$

① $y = x^{2/3} = (x^{1/3})^2 =$ 

② $y = (x+2)^3$
$\hookrightarrow x = -2$ पर x^3



$x = -1$ Max $\quad x = 0$ Min

H.W.
Q $f(x) = |x^2 - 2|x||$ $x \in [-4, 6]$

Pts of L Max / L Min.

$-4, -1, 1, 6 =$ Max.

$-2, 0, 2 =$ Min.

Q 2 $f(x) = \underbrace{2|x|}_{h(x)} + \underbrace{|x+2|}_{m(x)} - \Big|\underbrace{|x+2|}_{m(x)} - \underbrace{2|x|}_{h(x)}\Big|$

then find P.t of L Max / Min of $f(x)$?

$\text{Min}\{f(x), g(x)\} = \begin{cases} f(x) & \text{if } f(x) \le g(x) \\ g(x) & \text{if } g(x) < f(x) \end{cases}$

① $m(x) \geq h(x) \Rightarrow$ as it is open

$f(x) = 2|x| + |x+2| - |x+2| + 2|x|$

$= 4|x| = 2h(x)$ if $h(x) \le m(x)$

② $m(x) \le h(x) \Rightarrow$ −ve के साथ open

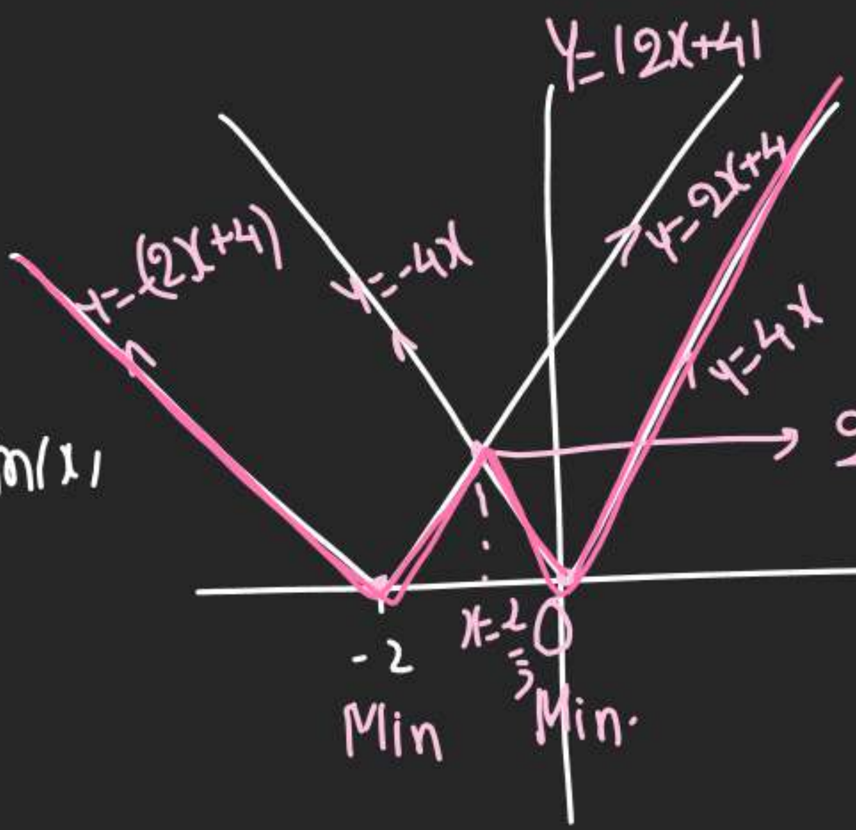
$f(x) = 2|x| + |x+2| + |x+2| - \cancel{2|x|}$

$f(x) = 2|x+2| = 2\,m(x)$ when $m(x) \le h(x)$

$f(x) = \text{Min}\{2h(x), 2m(x)\}$

$= \text{Min}\{4|x|, |2x+4|\}$ $\quad 2|x+2|$

$x = -2$

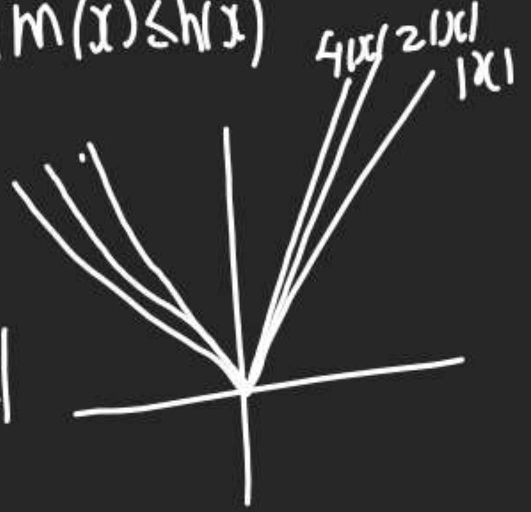


$2x+4 = -4x$

$6x = -4$

$x = -\frac{2}{3}$

Max.



Nishant Jindal

Q3 $f(x)=\begin{cases} -x^3+\dfrac{b^3-b^2+b-1}{b^2+3b+2} & 0\le x<1 \rightarrow f(1-h) \\ 2x-3 & \boxed{1\le x}\le 3 \end{cases}$

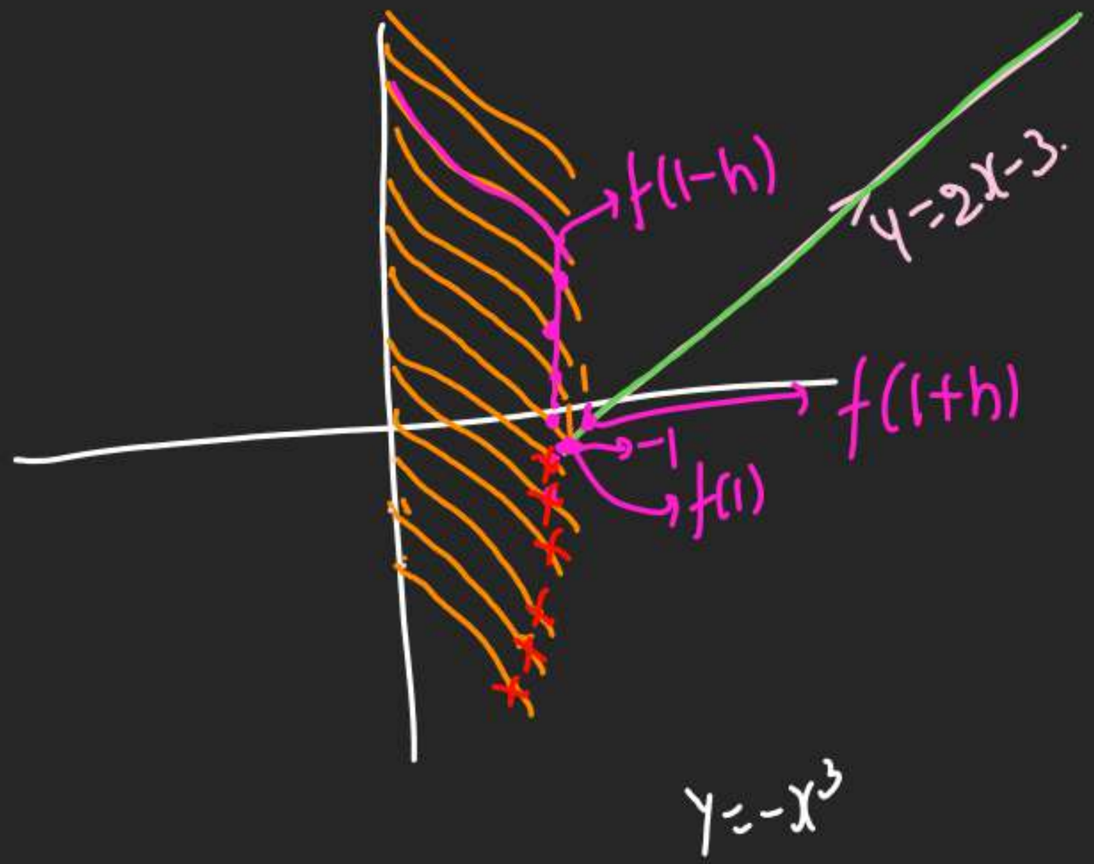
$\left.\begin{array}{l} f(1) \\ f(1^+) \end{array}\right\} f(1)=f(1+h)$

$b\in(-2,-1)\cup(1,\infty)$

find all Psbl values of b Such that f(x) has $\boxed{\text{smallest}}$ value at $x=1$.

Min

$f(x)=\begin{cases} -x^3+K & 0\le x<1 \\ 2x-3 & 1\le x\le 3 \end{cases}$



as fxn Requires Min at $x=1$

$f(1)<f(1-h)$

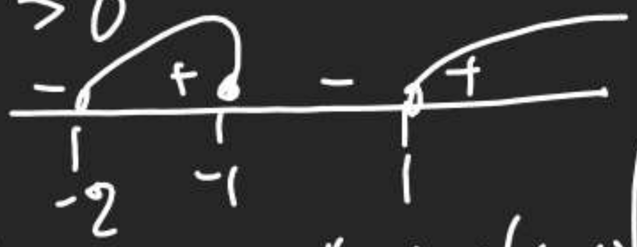
$2(1)-3<-(1-\cancel{h})^3+\dfrac{b^3-b^2+b-1}{b^2+3b+2}$

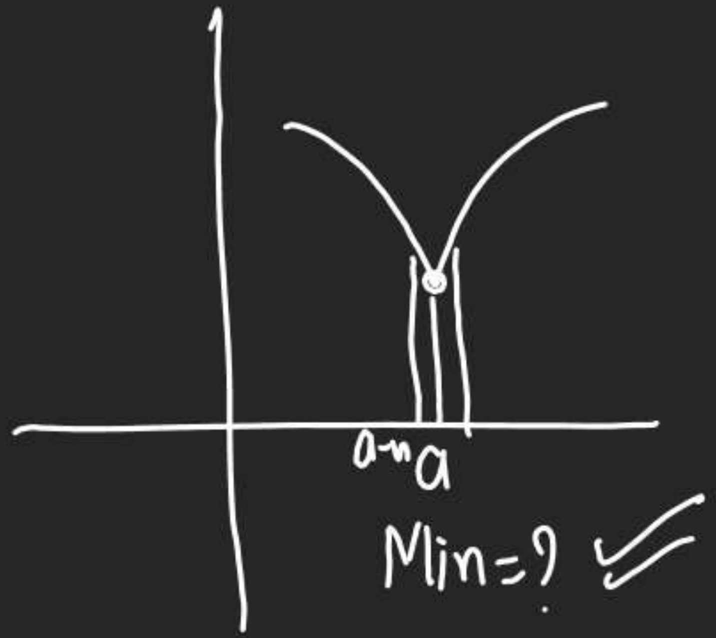
$-\cancel{1}<-\cancel{1}+\dfrac{b^3-b^2+b-1}{b^2+3b+2}$

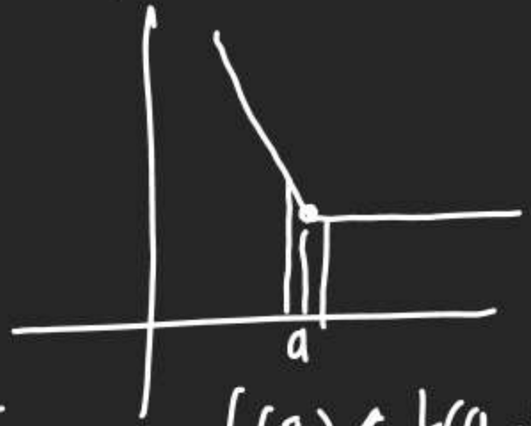
$\dfrac{b^2(b-1)+1(b-1)}{(b+1)(b+2)}>0$

$\dfrac{(b-1)(b^2+1)}{(b+1)(b+2)}>0$ +ve

$\dfrac{(b-1)}{(b+1)(b+2)}>0$





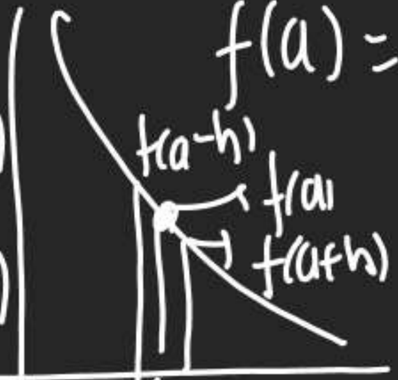


$f(a)<f(a-h)$

$f(a)=f(a+h)$

$f(a)<f(a-h)$

$f(a)>f(a+h)$



$x=a$ is niether Min Nor Max

Q 4 $f(x) = \begin{cases} 4x - x^3 + \ln(a^2 - 3a + 3) & 0 < \boxed{x < 3} \quad f(3-h) \\ x - 18 & \boxed{x \geq 3} \end{cases}$

$f(3) \neq f(3+h)$

$f(x)$ has Maxima value at $x = 3$ find a

Max.

$f(3) > f(3-h)$

$3 - 18 > 4(3) - 3^3 + \ln(a^2 - 3a + 3)$

$-\cancel{15} > -\cancel{15} + \ln(a^2 - 3a + 3)$

$\ln(a^2 - 3a + 3) < 0$

$a^2 - 3a + 3 < 1$

$a^2 - 3a + 2 < 0$

$(a-1)(a-2) < 0$

$1 < a < 2$

$a \in (1, 2)$

Q 5 $f(x) = \begin{cases} K - 2x & \boxed{x \leq -1} \quad f(-1) = f(-1-h) \\ 2x + 3 & \boxed{x > -1} \quad f(-1+h) \end{cases}$

$f(x)$ has Min at $x = -1$

then $K = ?$

$f(-1) < f(-1+h)$

$K - 2(-1) < 2(-1) + 3$

$K + 2 < 1$

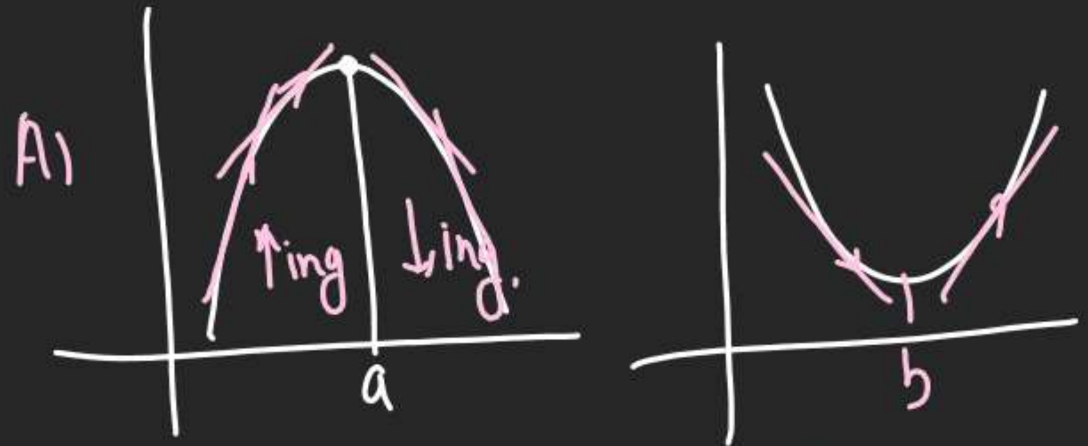
$K < -1$

$K \in (-\infty, -1)$

Nishant Jindal

First Derivative Test := FDT

A)



↑ then ↓ then $x=a$ is L. Max.

↓ then ↑ing $x=b$ is L Min.

② ① Find $\frac{dy}{dx}$ & get Cr. pt $x=a,b$

② Arrange Cr. pt on No Lines. Put +, − Sign.

$\frac{dy}{dx}$: ⊖ + at a: Min

$\frac{dy}{dx}$: ⊕ ⊖ at b: Max.

⊕ ⊕ at a: No

⊖ ⊖ at b: NMNM

Q $f(x)=x^3-6x^2+9x-8$ find Extrema pts.

$$\frac{dy}{dx}=3x^2-12x+9$$

$$=3(x^2-4x+3)$$

$$=3(x-1)(x-3)$$

$\frac{dy}{dx}$: + (1) − (3) +

1: Max. 3: Min

Max. at $x=1$

Min at $x=3$.

Q 6 $f(x)=(x)^2(x-2)^2$ find Extrema pts.

$$\frac{dy}{dx}=(x)(x-2)(2x-4+2x)$$

$$=(x)(x-2)(4x-4)$$

$$=4(x)(x-2)(x-1)$$

$\frac{dy}{dx}$: − (0) + (1) − (2) +

0: Min, 1: Max, 2: Min

Min at $x=0,2$

Max. at $x=1$

Nishant Jindal

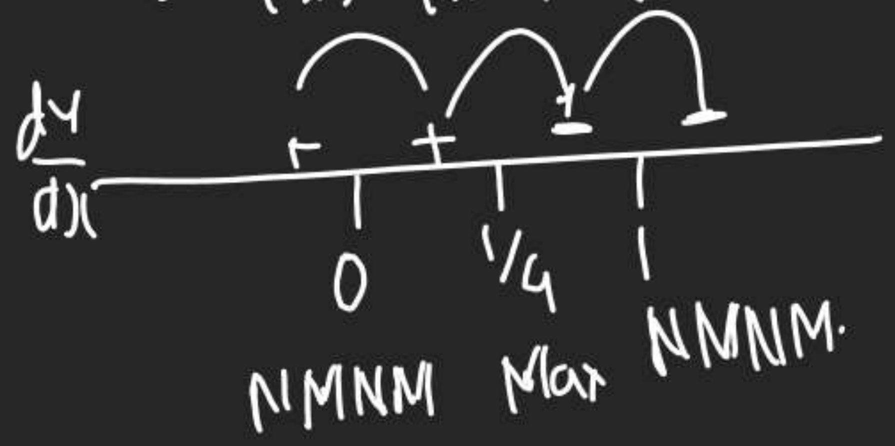
Q Find Pt of Extrema

7 for $f(x) = (x)^{25}(1-x)^{75}$

$f(x) = -(x)^{25}(x-1)^{75}$

$\frac{dy}{dx} = -(x)^{24}(x-1)^{74}(75x+25x-25)$

$= -(x)^{24}(x-1)^{74}(100x-25)$

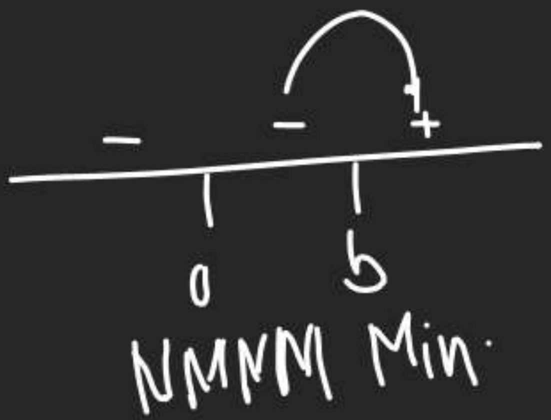


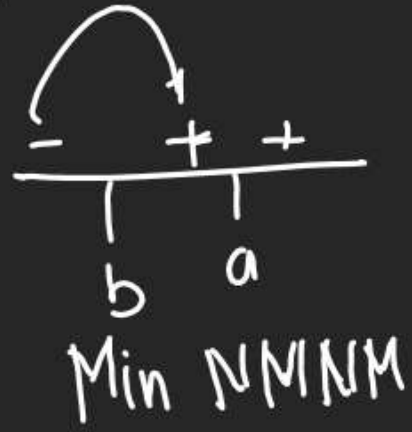
$x = 1/4$ is Pt of Max.

Q 8 $f'(x) = (x-a)^{2n}(x-b)^{2n+1}$

find Max / Min?

① $f'(x)$ is already given





$x = b$ is Min.

as we know that fxn is

↑ in $(3,\infty) \Rightarrow [4,5]$ also ↑ing

Max^m value at $x=5$

$f(5) = 250 - 375$

$+180 - 48$

$= 7$

Q find Max Value of

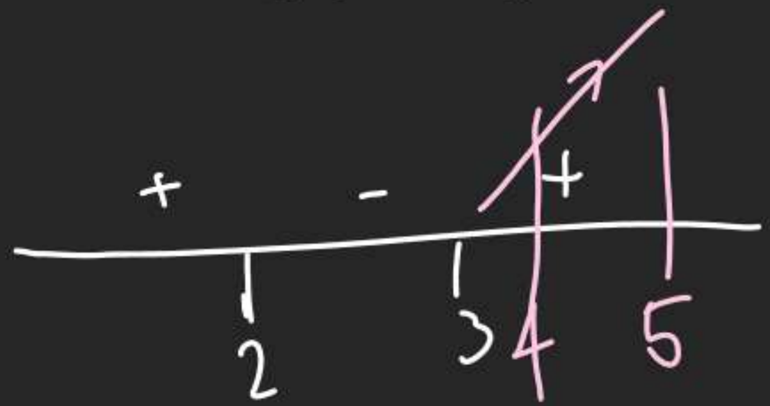
9 $f(x) = 2x^3 - 15x^2 + 36x - 48$

on Set $A = \{x \,/\, x^2 + 20 \le 9x\}$

$f'(x) = 6x^2 - 30x + 36$

$= 6(x^2 - 5x + 6)$

$= 6(x-2)(x-3)$



$A = \{x \,/\, x^2 - 9x + 20 \le 0\}$

$\{x \,/\, (x-5)(x-4) \le 0\}$

$\{x \,/\, 4 \le x \le 5\}$