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Family of lines passing thru intersection of 2 given lines

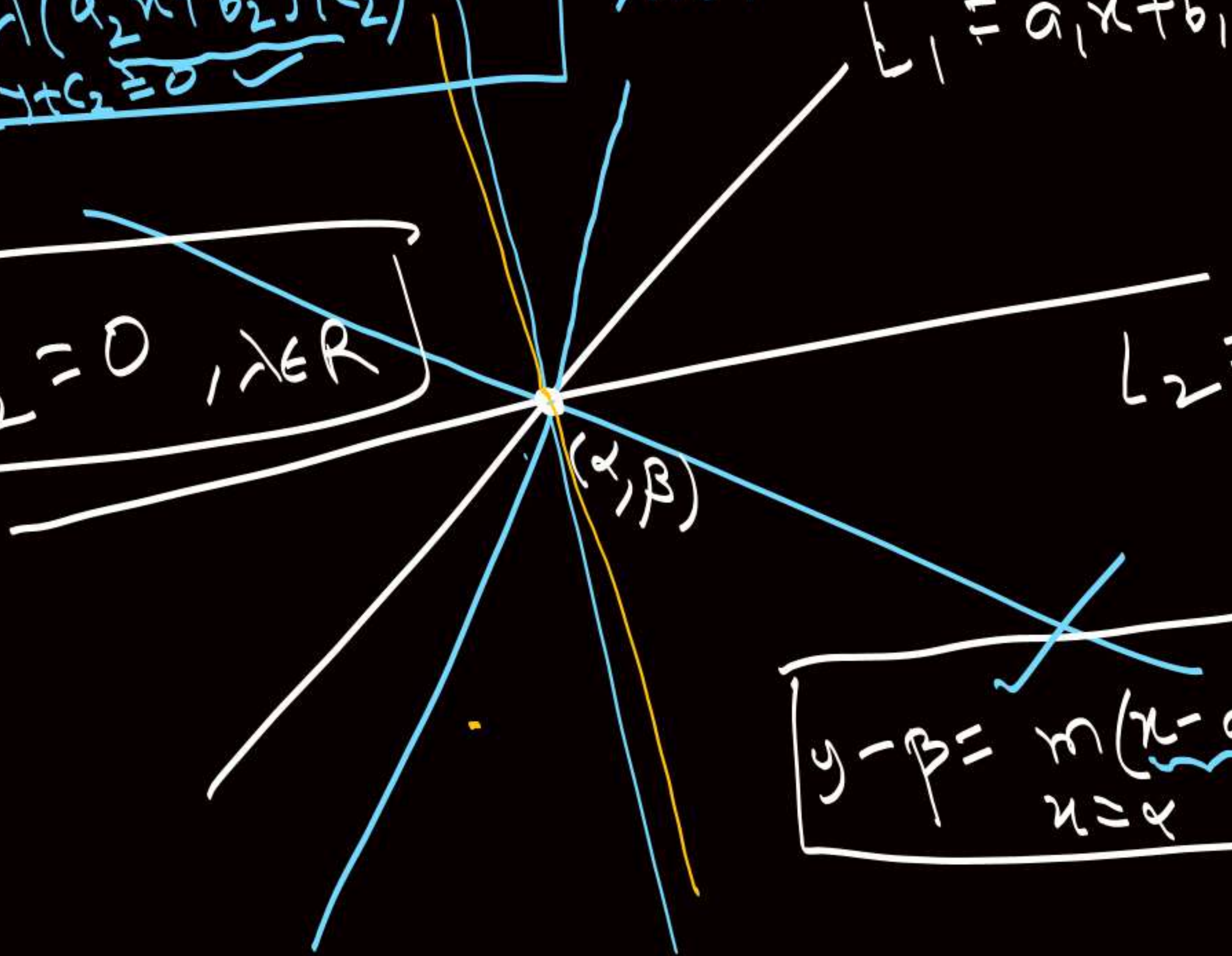
$$(a_1x+b_1y+c_1)+\lambda(a_2x+b_2y+c_2)=0 \quad ,\lambda\in R$$

$a_2x+b_2y+c_2=0$

$L_1 = a_1x+b_1y+c_1=0$

$L_2 = a_2x+b_2y+c_2=0$

$$L_1+\lambda L_2=0 \quad ,\lambda\in R$$



$$y-\beta = m(x-\alpha) \quad ,m\in R$$

$x=\alpha$

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Find the eqn. of line thru the intersection of lines $3x-4y+6=0$ and $x+y+2=0$ $\boxed{(-2,0)}$

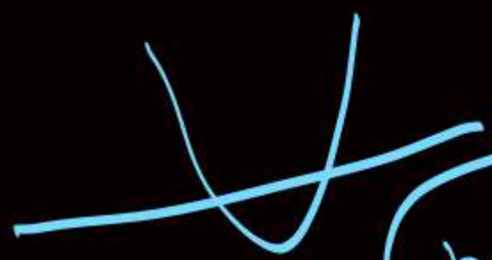
(i) which is $\perp$ar to $2x+y+7=0$ $\quad y-0=\frac{1}{2}(x+2)$

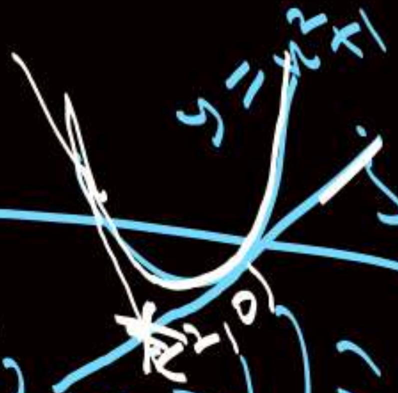
(ii) which has equal non zero interceptn on coordinate axes. $\quad y-0=-1(x+2)$

(iii) whose x-intercept is -3 $\quad (-3,0),(-2,0) \rightarrow y=0$

$\boxed{\frac{x}{-2}+\frac{y}{-3}=1}$ y-intercept is -3

(iv) which touches the curve $y=x^2+1$.





$y-0=m(x+2)$

$mx+2m=x^2+1$

$x^2-mx+1-2m=0$

$D=0 \Leftarrow$

$m^2+4(2m-1)=0$

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$3x-4y+6=0$ & $x+y+2=0$

$3x-4y+6+\lambda(x+y+2)=0$

(i) $-\left(\dfrac{3+\lambda}{-4+\lambda}\right)=\dfrac{1}{2}$

(ii) $\dfrac{3+\lambda}{4-\lambda}=-1 \Rightarrow 3=-4$ ✗

$\lambda+3=-4+\lambda$

$\lambda(0)=-7$

$\lambda \Rightarrow \infty$

$\boxed{x+y+2=0}$

(iii) y-intercept $=-3$ Put $(0,-3)$

$12+6+\lambda(-3+2)=0$

$\lambda=?$

(iv) $3x-4(x^2+1)+6+\lambda(x+x^2+3)$

$\lambda=?$ $(\ )x^2+(\ )x+(\ )=0$

$D=0$

$\Rightarrow \lambda=?$

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Fixed point Problems

$\boxed{(1, -2)}$

$L_1 + \lambda L_2 = 0$

$\boxed{(1, -2)}$

$a + c - 2b = 0$

1. If a, b, c are in A.P, then P.T. lines $\boxed{ax + by + c = 0}$ pass thru a fixed point.

$\boxed{a + c = 2b}$

$2ax + (a+c)y + 2c = 0 \Rightarrow (2x+y)a + c(y+2) = 0$

$(2x+y) + \frac{c}{a}(y+2) = 0$

$\boxed{(2x+y) + \lambda(y+2) = 0}$

$2x + y = 0$

&

$y + 2 = 0$

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Q. If $a^2+9b^2=6ab+4c^2$, then P.T. lines $ax+by+c=0$ pass thru one or the other of two fixed points.

$$(a-3b)^2-4c^2=0=(a-3b-2c)(a-3b+2c)$$

$$-\frac{a}{2}+\frac{3b}{2}+c=0 \quad \text{or} \quad \frac{a}{2}-\frac{3b}{2}+c=0$$

$$\left(-\frac{1}{2},\frac{3}{2}\right) \qquad \left(\frac{1}{2},-\frac{3}{2}\right)$$

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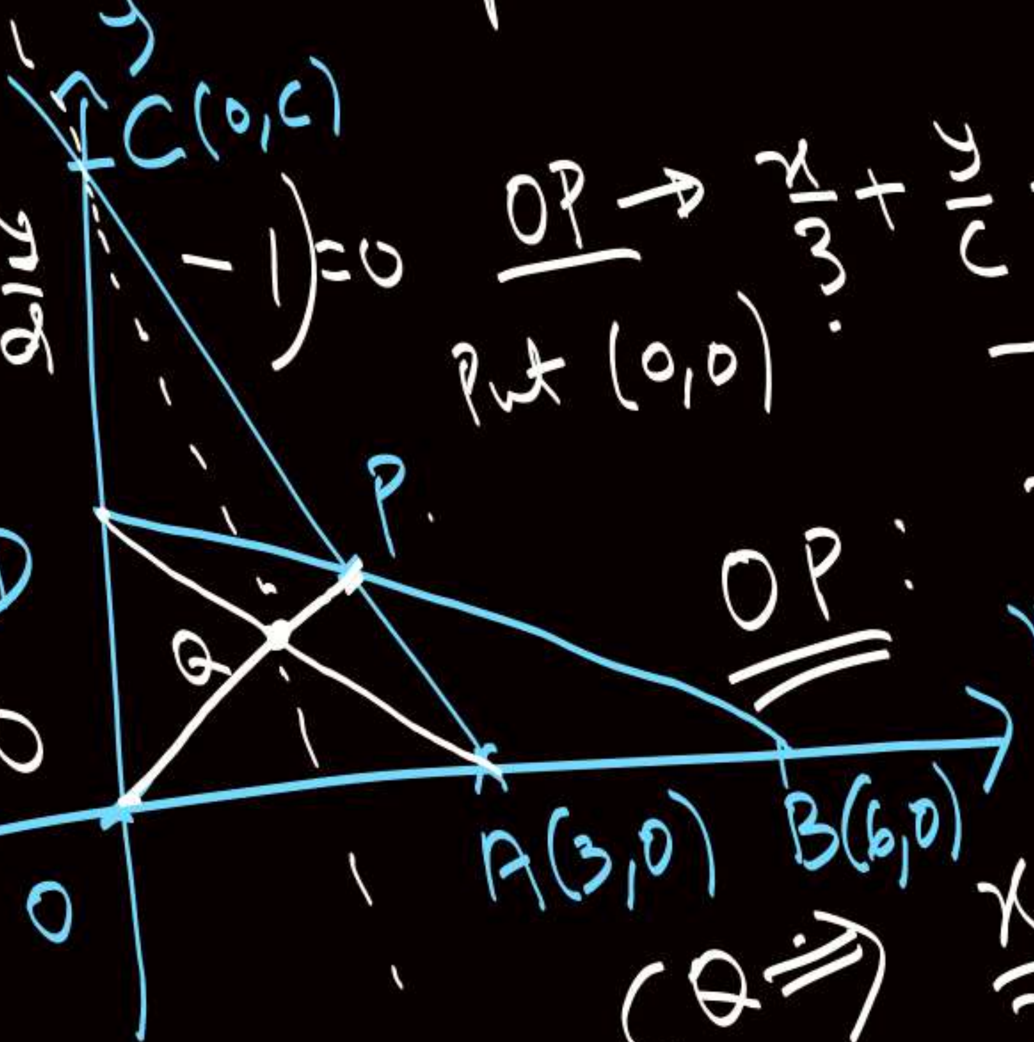
3. $A(3,0)$ and $B(6,0)$ are two fixed points and P is a variable point. Lines AP and BP meet y-axis at C and D respectively and AD meet OP at Q, where 'O' is origin. P.T. CQ passes thru a fixed point.



$\underline{OP} \rightarrow \frac{x}{3}+\frac{y}{c}-1+\lambda\left(\frac{x}{6}+\frac{y}{d}-1\right)=0$

Put $(0,0)$: $-1-\lambda=0 \Rightarrow \lambda=-1$

$\underline{\underline{OP}}: \frac{x}{6}+y\left(\frac{1}{c}-\frac{1}{d}\right)=0$

$\underline{CQ}: \frac{x}{6}+y\left(\frac{1}{c}-\frac{1}{d}\right)+\mu\left(\frac{x}{3}+\frac{y}{d}-1\right)=0$

Put $(0,c)$

$1-\frac{c}{d}+\mu\left(\frac{c}{d}-1\right)=0$

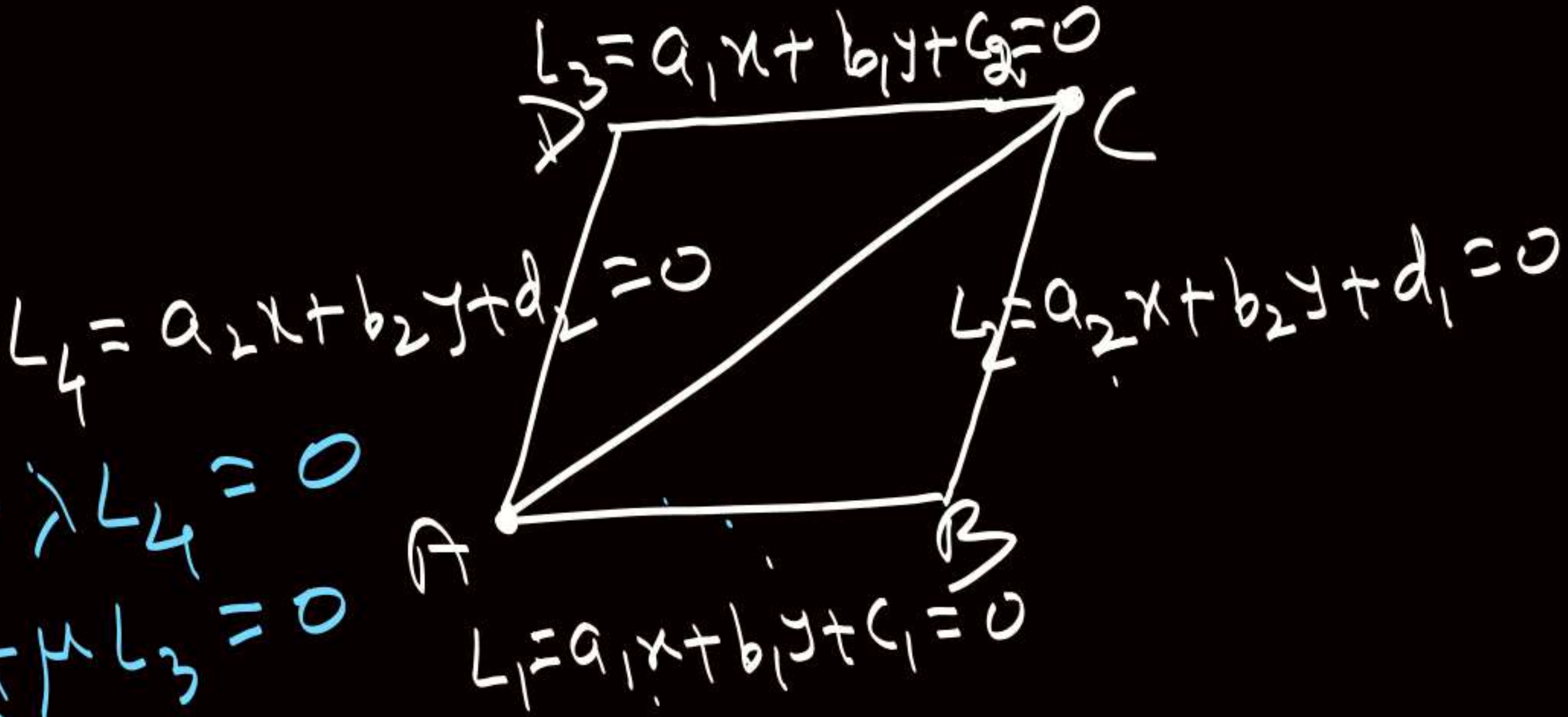
$\boxed{\mu=1}$

$CQ \Rightarrow \frac{x}{2}+\frac{y}{c}-1=0$

$\left(\frac{x}{2}-1\right)+\frac{1}{c}(y)=0$

$\boxed{(2,0)}$

4.



AC $\rightarrow$ $L_1 + \lambda L_4 = 0$

coincident

$L_2 + \mu L_3 = 0$

$\lambda, \mu = ?$

Write the eqn. of diagonals of the given parallelogram.

AC $\rightarrow$ $L_1 L_2 - L_3 L_4 = 0$

BD $\rightarrow$ $L_1 L_4 - L_2 L_3 = 0$

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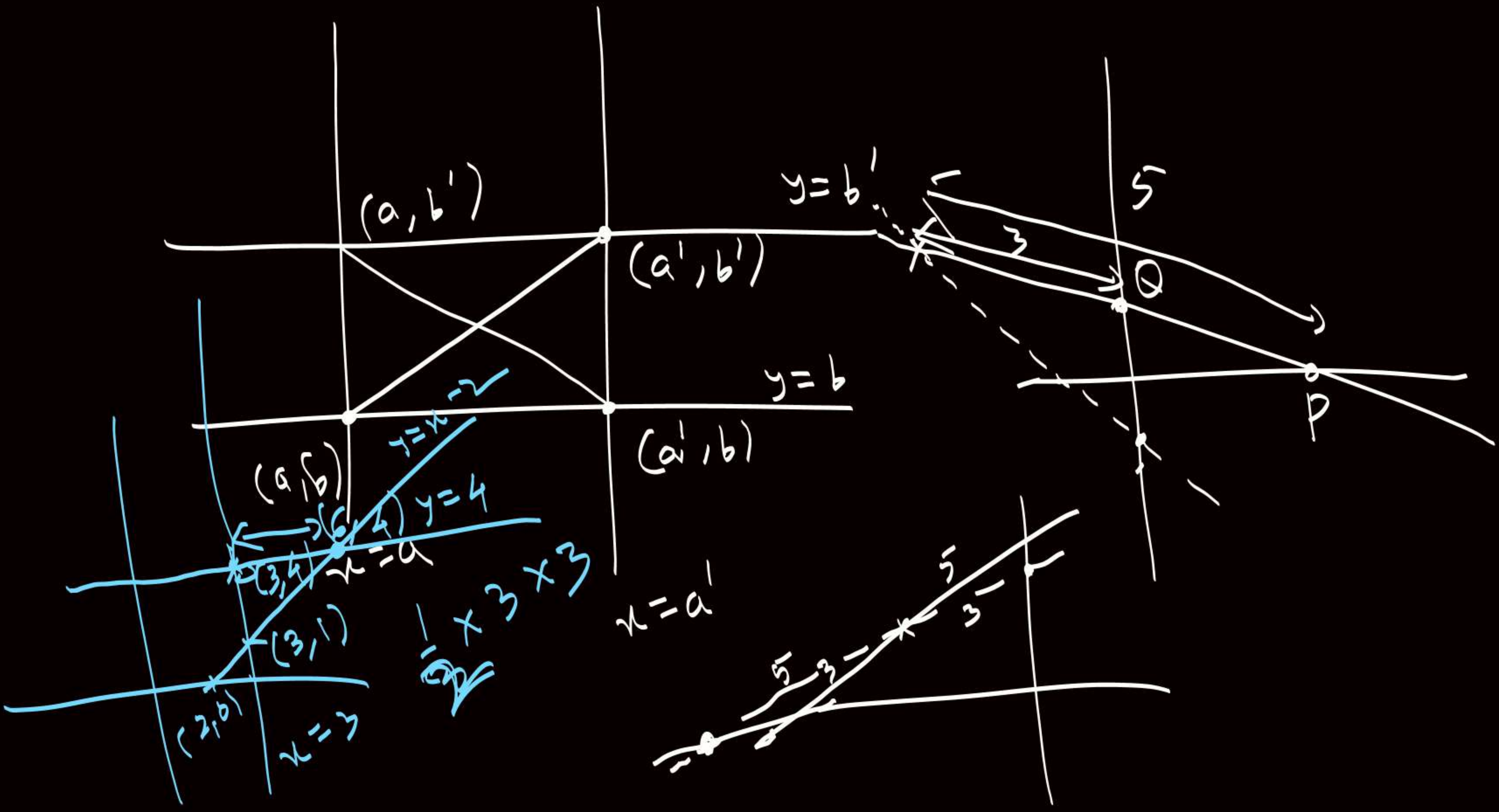

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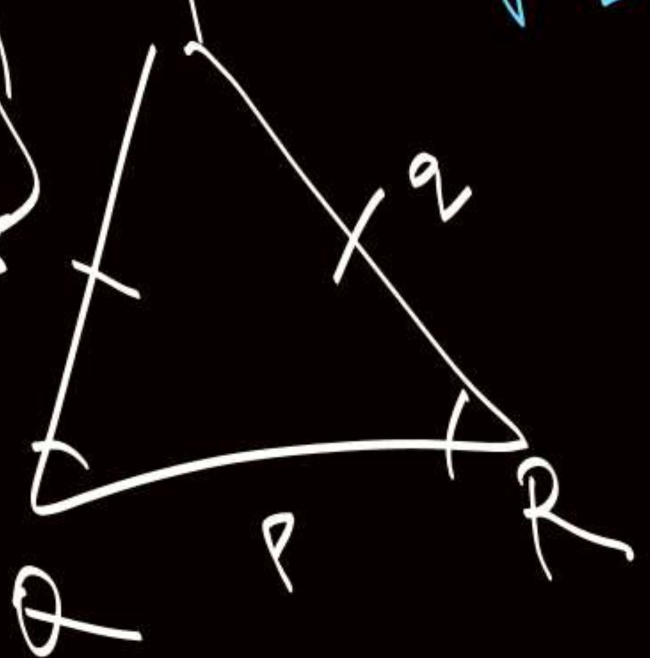
$$R_1 = r \quad = 2R_1 \sin 2R$$

$$\angle R = \frac{\pi}{6}$$



$$p = \frac{a}{\sqrt{\sec^2\theta + \operatorname{cosec}^2\theta}} = a\sin\theta\cos\theta = \frac{a}{2}\sin 2\theta$$

$$p' = \frac{|a\cos 2\theta|}{1}$$

$$(p')^2 + (2p)^2 = a^2$$

$$\tan\frac{C-A}{2} = \frac{c-a}{c+a}\cot\frac{B}{2}$$

$$c = 2a$$

$$\frac{\sin C}{\sin A} = 2 \Rightarrow \frac{\sin C - \sin A}{\sin C + \sin A} = \frac{2-1}{2+1} = \frac{1}{3}$$

$$\frac{2\sin\left(\frac{C-A}{2}\right)\sin\frac{B}{2}}{2\cos\frac{B}{2}\cos\frac{C-A}{2}}$$