Nishant Jindal

Q. For $a>b>c>0$ then distance betⁿ
IIT Adv. $(1,1)$ & POI of lines $ax+by+c=0$ & $bx+ay+c=0$ is less than $2\sqrt{2}$ then

(A) $a+b-c>0$ (B) $a-b+c<0$

(C) $a-b+c>0$ (D) $a+b-c<0$

$$ax+by+c=0 \quad \times b$$
$$bx+ay+c=0 \quad \times a$$

$$\cancel{ab}x+b^2y+cb=0$$
$$\cancel{ab}x+a^2y+ac=0$$

$$y(b^2-a^2)=c(a-b)$$

$$y=\frac{-c}{a+b} \text{ then } x=\frac{-c}{a+b}$$

dist $<2\sqrt{2}$

$$\text{POI}=\left(\frac{-c}{a+b},\frac{-c}{a+b}\right)\longleftrightarrow(1,1)$$

$$\text{dist}=\sqrt{\left(1+\frac{c}{a+b}\right)^2+\left(1+\frac{c}{a+b}\right)^2}$$

$$=\left(1+\frac{c}{a+b}\right)\cancel{\sqrt{2}}<2\cancel{\sqrt{2}}$$

$$\frac{c}{a+b}-1<0$$

$$\frac{c-a-b}{\underset{\oplus}{a+b}}<0$$

① $\Rightarrow c-a-b<0$

$\Rightarrow a+b-c>0$

② $a>b>c>0$ Ⓒ option is correct

$a-b+c>0$

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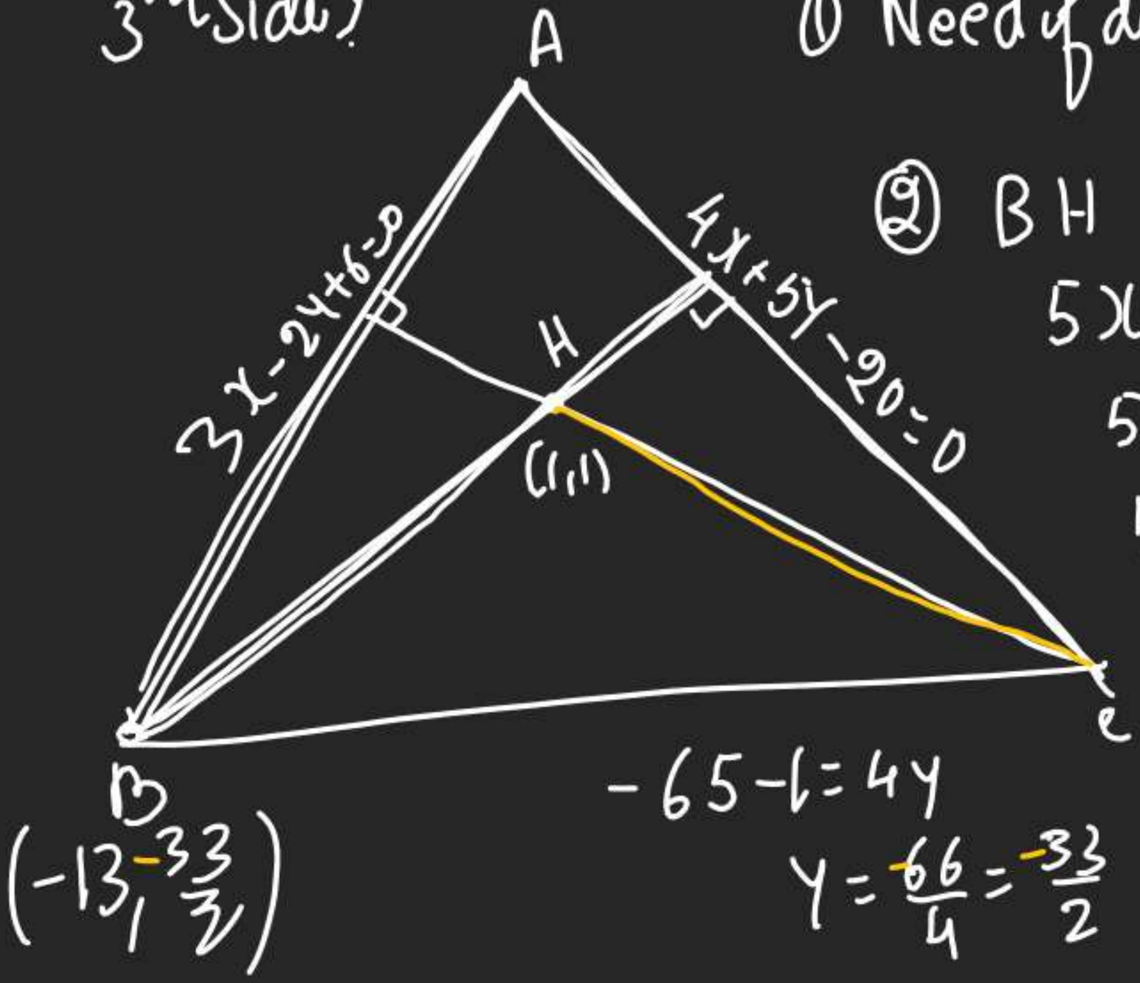
Q2 Adv IIT

Let Eqⁿ of 2 sides of △ be

$3x-2y+6=0$ & $4x+5y-20=0$

If orthocentre of △ is (1,1) then Eqⁿ of 3rd Side?



① Need of diagram.

② BH

$5x-4y+k=0$ (1,1)

$5-4+k=0 \Rightarrow k=-1$

$5x-4y-1=0$

③ Coord B are POI

$5x-4y-1=0$

$3x-2y+6=0 \times 2$

$5x-4y-1=0$

$6x-4y+12=0$

$-x=13 \Rightarrow x=-13$

$-65-1=4y$

$y=\frac{-66}{4}=\frac{-33}{2}$

④ CH

$2x+3y+k=0$ (1,1)

$k=-5$

$2x+3y-5=0 \times 2$

$4x+5y-20=0$

$4x+6y-10=0$

$4x+5y-20=0$

$y=-10$

$x=\frac{35}{2}$

$\left(\frac{35}{2},-10\right)$

(5) B $\left(-13,-\frac{33}{2}\right)$ C $\left(\frac{35}{2},-10\right)$

Find BC

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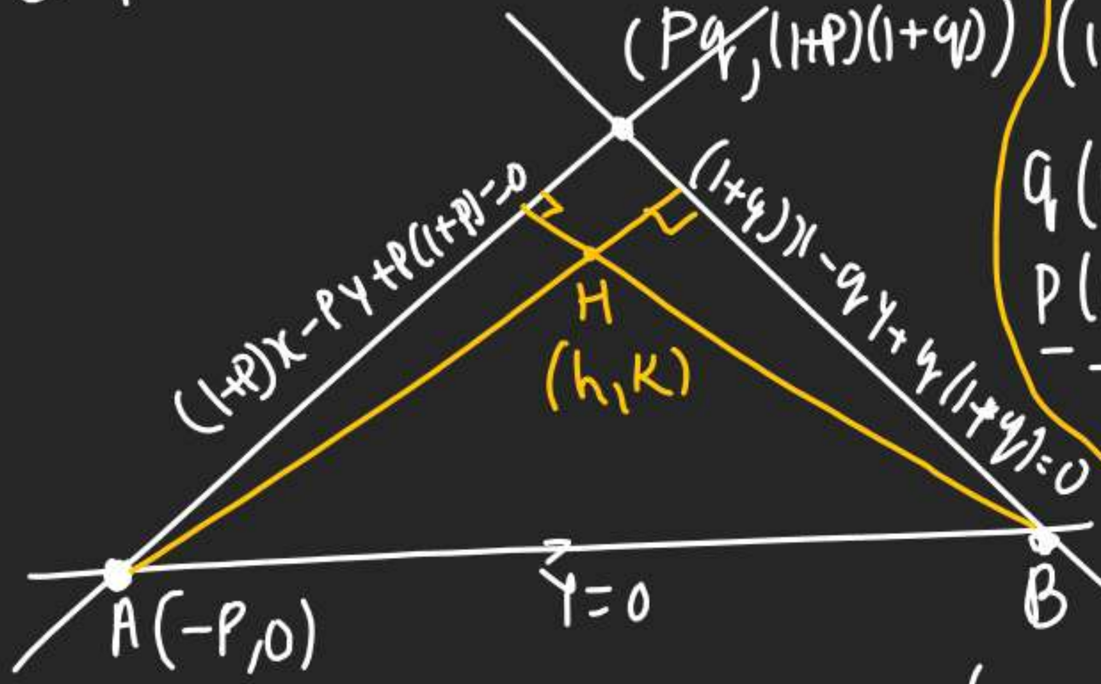
Q 3 IIT Adv

Locus of orthocentre of Δ formed by Lines $(1+p)x - py + p(1+p) = 0$ & $(1+q)x - qy + q(1+q) = 0$ & $y = 0$?



$(-q, 0)$

$(\cancel{1+p})x = -p(\cancel{1+p})$

$(\cancel{1+q})x = -q(\cancel{1+q})$

$(1+p)x - py + p(1+p) = 0 \quad \times q$

$(1+q)x - qy + q(1+q) = 0 \quad \times p$

$q(1+p)x - pqy + pq(1+p) = 0$

$\underline{p(1+q)x - pqy + pq(1+q) = 0}$
$\;\; - \quad\;\; + \quad\;\; \pm$

USELESS

$x(q-p) = q^2p - p^2q \Rightarrow x(\cancel{q-p}) = qp(\cancel{q-p})$

$x = qp$

$\cancel{p}q(1+p) + \cancel{p}(1+p) = \cancel{p}y$

$y = (1+p)(1+q)$

BH $\quad px + (1+p)y + k = 0 \quad (-q, 0)$

$+pq = +k$

$ph + (1+p)k + pq = 0$

AH $\quad qx + (1+q)y + k = 0 \quad (-p, 0)$

$+pq = +k$

$\underline{qh + (1+q)k + pq = 0}$

$ph + (1+p)k = qh + (1+q)k$

$h(p-q) = k(\cancel{1} + q - \cancel{1} - p)$

$h = -k \Rightarrow x + y = 0$

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When 2 Lines are ll$^{\text{re}}$ or Coincident

$L_1: a_1x+b_1y+c_1=0$

$L_2: a_2x+b_2y+c_2=0$

$L_1 \parallel L_2$

$\Rightarrow \dfrac{a_1}{a_2} = \dfrac{b_1}{b_2} \neq \dfrac{c_1}{c_2}$

If L_1 & L_2 Coincident:

$\dfrac{a_1}{a_2} = \dfrac{b_1}{b_2} = \dfrac{c_1}{c_2}$

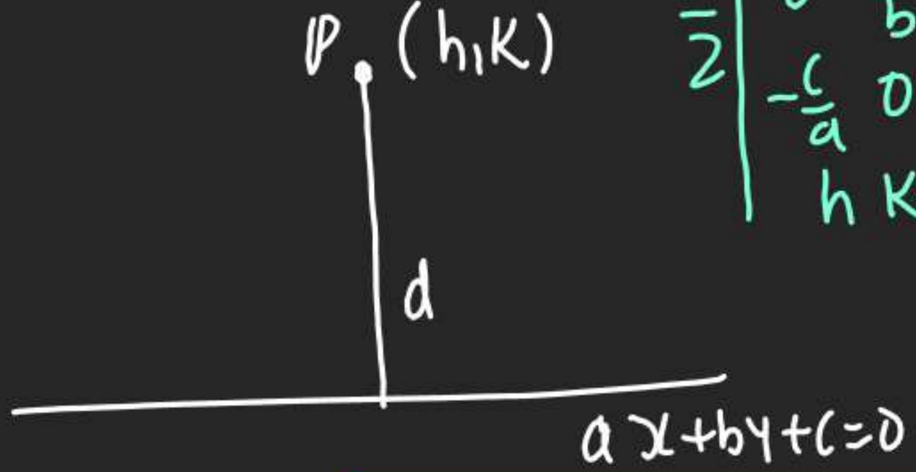
Q Find λ if

$2x+3y+1=0$ & $\lambda x+6y+2=0$ is ll$^{\text{rel}}$?

$\dfrac{\lambda}{2} = \dfrac{6}{3}\,2 = \dfrac{2}{1} \Rightarrow \lambda = 4$ &

Lines are Coincident as $\dfrac{b_1}{b_2} = \dfrac{c_1}{c_2}$

$\perp^{\text{r}}$ distance of a Pt. from a Line



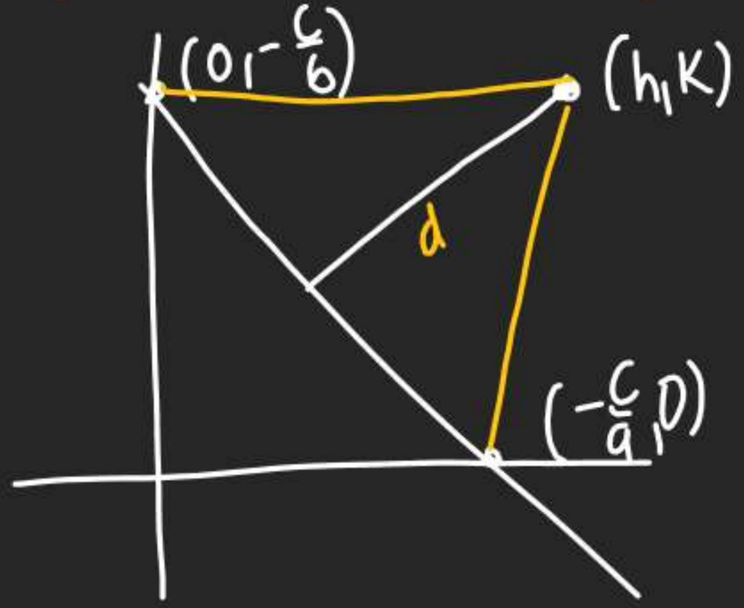
$$\frac{1}{2}\begin{vmatrix} h & K \\ 0 & -\frac{c}{b} \\ -\frac{c}{a} & 0 \\ h & K \end{vmatrix} = \frac{1}{2}\sqrt{\left(-\frac{c}{a}-0\right)^2+\left(0+\frac{c}{b}\right)^2} \times d$$

से d प्राप्त होगा

distance of P from Line

$$\boxed{d = \frac{|ah+bk+c|}{\sqrt{a^2+b^2}}}$$

Proof



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Q distance of $(1,-2)$ from $x-2y+3=0$

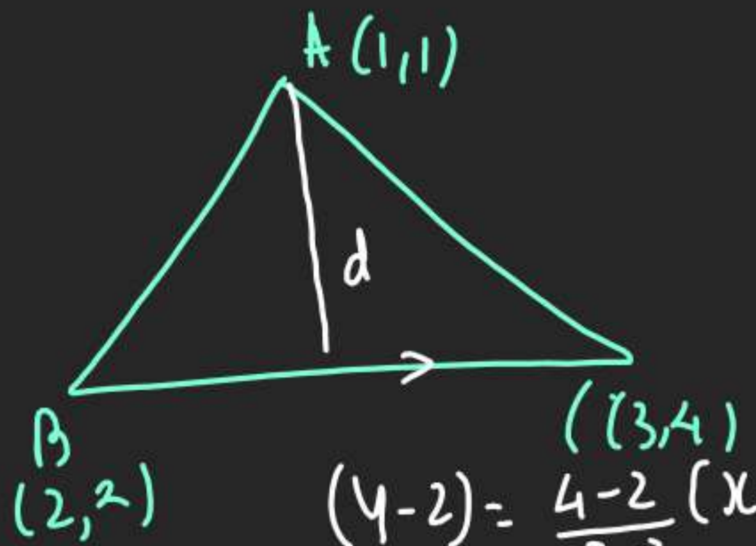
$$d = \frac{|1-2(-2)+3|}{\sqrt{1^2+(-2)^2}}$$

$$= \frac{8}{\sqrt{5}}$$

Q distance of origin $(0,0)$ from $px+qy+pq=0$

$$d = \frac{|0+0+pq|}{\sqrt{p^2+q^2}} = \frac{|pq|}{\sqrt{p^2+q^2}}$$

Q find distance of vertex A from BC if $A(1,1)$, $B(2,2)$, $C(3,4)$



$$(y-2) = \frac{4-2}{3-2}(x-2)$$

$$y-2 = 2x-4$$

$$2x-y-2=0$$

$$d = \frac{|2-1-2|}{\sqrt{2^2+(-1)^2}} = \frac{1}{\sqrt{5}}$$

1) When $\perp^r$ distance is asked always Use Modulus to make answer +ve
2) If $\perp^r$ distance is given always put $\pm$ sign

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distance betn $\|^r$ Lines

पता कैसे चलेगा that given Lines are $\|^{re}$?

Ans Photo से → $ax+by+c_1=0$

से Dono → $ax+by+c_2=0$

Part will be Same.

$ax+by+c_1=0$

$$d=\left|\frac{c_1-c_2}{\sqrt{a^2+b^2}}\right|$$

$ax+by+c_2=0$

Q Find distance betn $3x-2y+5=0$ & $3x-2y-2=0$

$$d=\frac{|5-(-2)|}{\sqrt{3^2+(-2)^2}}=\frac{7}{\sqrt{13}}$$

Q Find Least & gr. distance of Pt $(\cos\theta,\sin\theta)$ from

$3x-4y+10=0$

① find $\perp^r$ distance

$$d=\frac{|3\cos\theta-4\sin\theta+10|}{\sqrt{3^2+(-4)^2}}$$

$$d=\frac{3\cos\theta-4\sin\theta+10}{5}$$

(2) We know

$$-\sqrt{3^2+(-4)^2}\le 3\cos\theta-4\sin\theta\le\sqrt{3^2+(-4)^2}$$

$$-5\le 3\cos\theta-4\sin\theta\le 5$$

$$\frac{5}{5}\le\frac{3\cos\theta-4\sin\theta+10}{5}\le\frac{15}{5}$$

$$1\le d\le 3$$

Gr. Value of dist $=3$

least value of dist $=1$

1. (Gr. & Least distance is no term in Coord Geometry)

2) $\perp^r$ distance is always Min distance.

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Q 10 Find Product of length of $\perp^r$ from $(-\sqrt{a^2-b^2},0)$ & $(\sqrt{a^2-b^2},0)$ to Line

$$\frac{x}{a}\cos\theta+\frac{y}{b}\sin\theta=1$$

$(\sqrt{a^2-b^2},0)$

$$d_1=\frac{\left|\frac{\sqrt{a^2-b^2}}{a}\cdot\cos\theta+\cancel{\frac{0}{b}\sin\theta}-1\right|}{\sqrt{\frac{\cos^2\theta}{a^2}+\frac{\sin^2\theta}{b^2}}}$$

$$=\frac{\left|\frac{\sqrt{a^2+b^2}}{a}\cdot\cos\theta-1\right|}{\sqrt{\frac{c^2}{a^2}+\frac{s^2}{b^2}}}$$

$(-\sqrt{a^2-b^2},0)$

$$d_2=\frac{\left|-\frac{\sqrt{a^2-b^2}}{a}\cos\theta+0\cdot\frac{\sin\theta}{b}-1\right|}{\sqrt{\frac{\cos^2\theta}{a^2}+\frac{\sin^2\theta}{b^2}}}$$

$$=\frac{\left|\frac{\sqrt{a^2-b^2}}{a}\cos\theta+1\right|}{\sqrt{\frac{c^2}{a^2}+\frac{s^2}{b^2}}}$$

$$\text{Prod}=d_1d_2=\frac{\left|\frac{a^2-b^2}{a^2}\cos^2\theta-1\right|}{\left(\frac{c^2}{a^2}+\frac{s^2}{b^2}\right)}=\frac{(a^2\cos^2\theta-b^2\cos^2\theta-a^2)\,a^2b^2}{a^2(b^2\cos^2\theta+a^2\sin^2\theta)}$$

$$=\frac{\left|-\cancel{(b^2\cos^2\theta+a^2\sin^2\theta)}\right|\times\cancel{a^2}b^2}{\cancel{a^2}\cancel{(b^2\cos^2\theta+a^2\sin^2\theta)}}=|-b^2|=b^2$$

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Q 2 Sides of sq^n on Lines.

$x+y=1$ & $x+y+2=0$

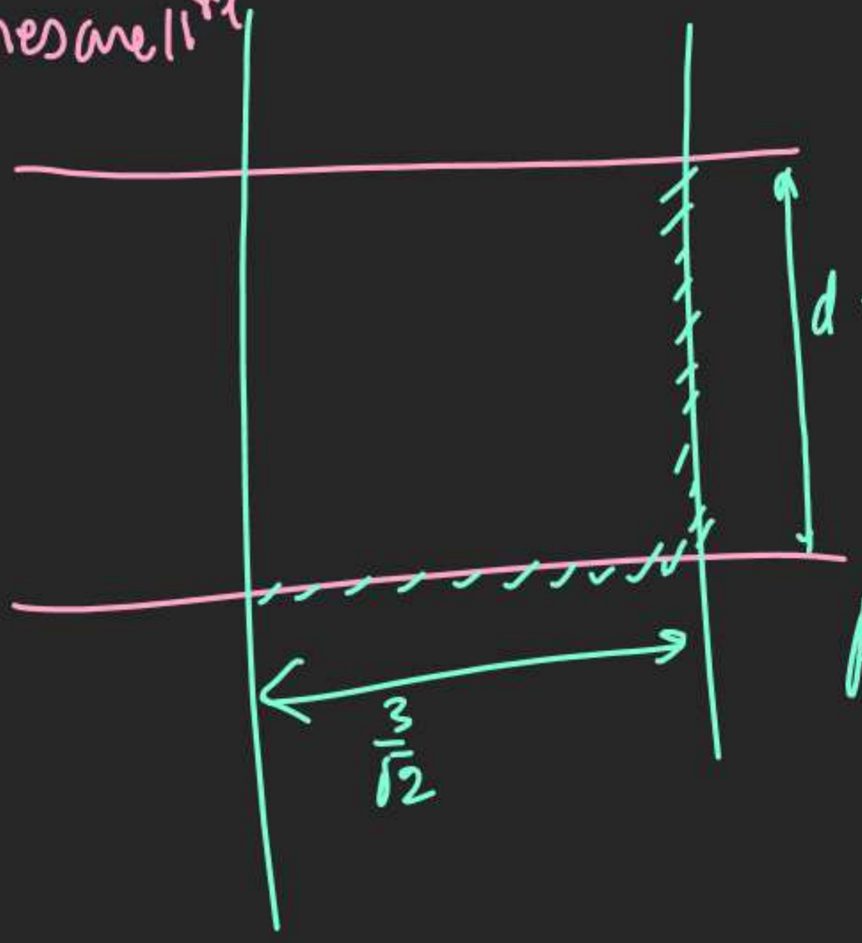
Find area of sq^n.

$x+y=1$

x, y Part Same

$x+y=-2$

$\Rightarrow$ Lines are $\parallel^{el}$



$$d = \frac{|1-(-2)|}{\sqrt{1^2+1^2}} = \frac{3}{\sqrt{2}}$$

$$\text{Area} = \left(\frac{3}{\sqrt{2}}\right)^2 = \frac{9}{2}$$

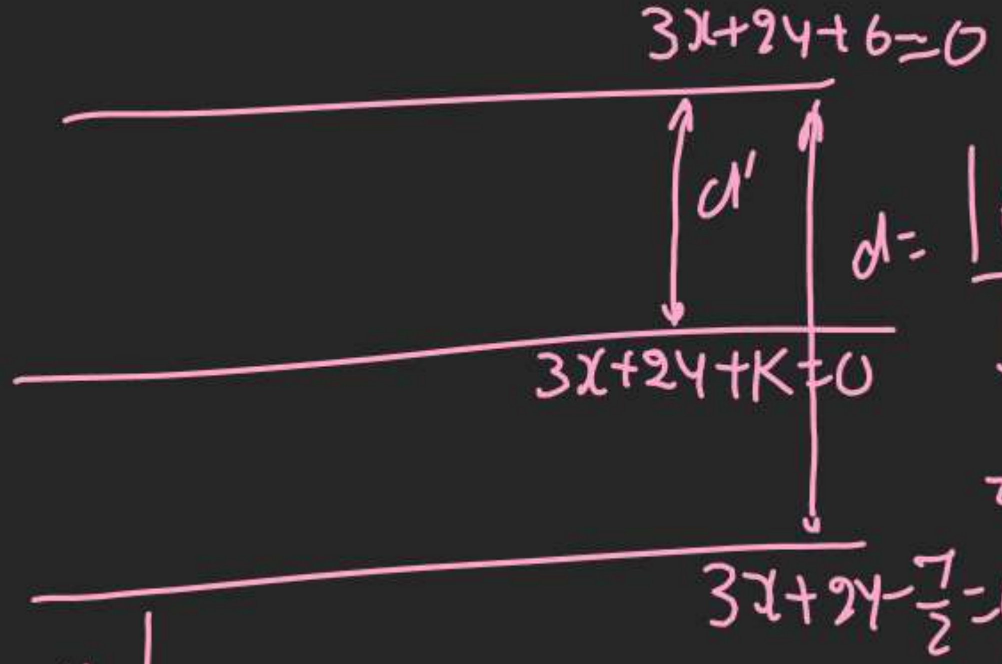
Q Lines Equidistant from $\parallel$ lines

$9x+6y-7=0$ & $3x+2y+6=0$

$3x+2y-\frac{7}{3}=0$

x, y वाला Part Same

$3x+2y+6=0$



$$d = \frac{\left|6-\left(-\frac{7}{3}\right)\right|}{\sqrt{9+4}} = \frac{25}{3\sqrt{13}}$$

$$d' = \frac{|6-K|}{\sqrt{13}} = \frac{25}{6\sqrt{13}}$$

$$|6-K| = \frac{25}{6}$$

$6-K = \frac{25}{6} \Rightarrow K = \frac{11}{6}$

$6-K = -\frac{25}{6} \rightarrow K = \frac{61}{6}$

L: $3x+2y+\frac{11}{6}=0$

L_1': $3x+2y+\frac{61}{6}=0$

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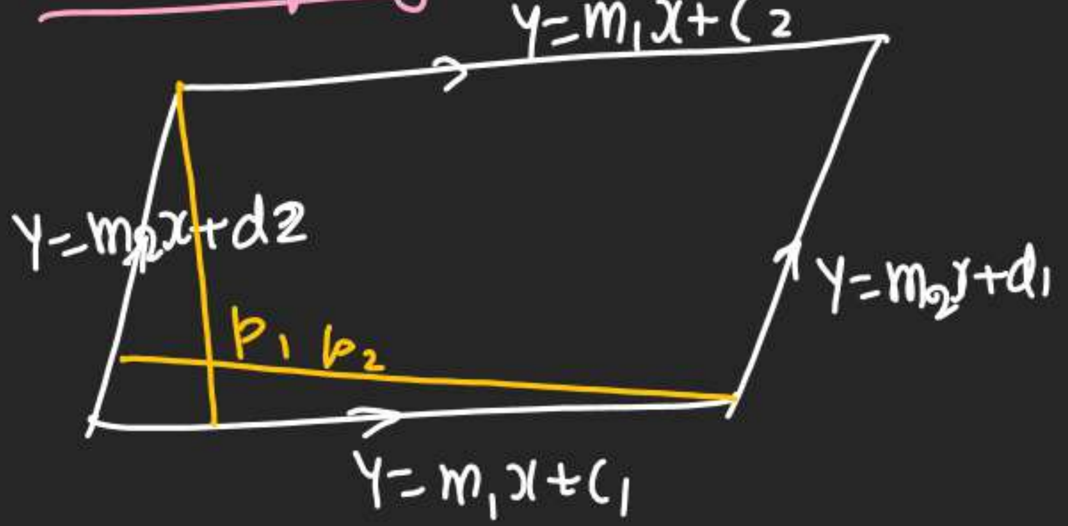
Area of llgm.



$$\Delta = \frac{|(c_1-c_2)(d_1-d_2)|}{(m_1-m_2)}$$

$$\Delta = p_1p_2\,\mathrm{cosec}\,\theta$$

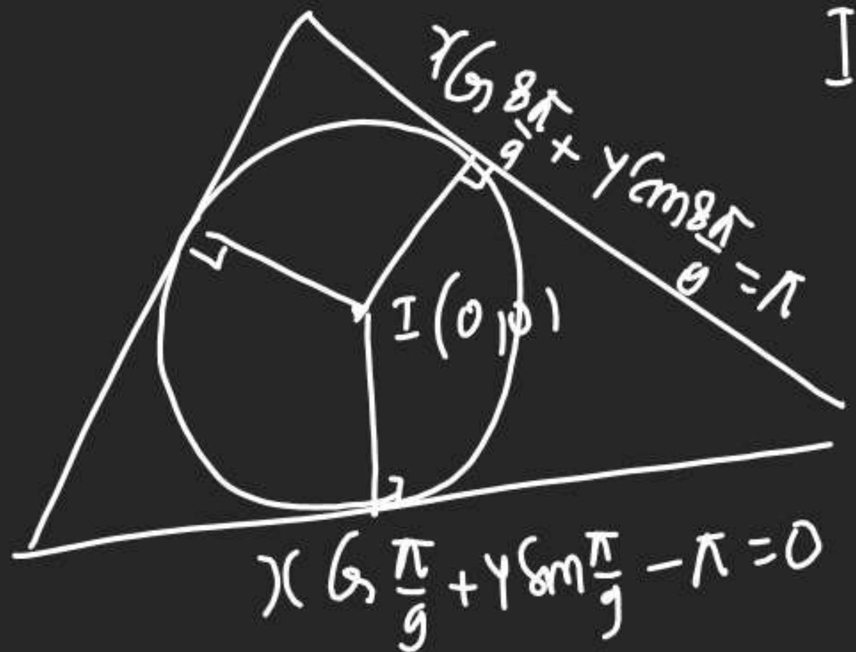
$$= p_1p_2\sqrt{1+\frac{1}{\tan^2\theta}}$$

$$= \frac{|c_1-c_2|}{\sqrt{m_1^2+1}} \times \frac{|d_1-d_2|}{\sqrt{m_2^2+1}} \times \sqrt{1+\frac{(1+m_1m_2)^2}{(m_1-m_2)^2}} = \left|\frac{(c_1-c_2)(d_1-d_2)}{\cancel{\sqrt{(1+m_1^2)(1+m_2^2)}}} \times \sqrt{\frac{m_1^2+m_2^2-\cancel{2m_1m_2}+1+m_1^2m_2^2+\cancel{2m_1m_2}}{(m_1-m_2)^2}}\right|$$

Q Incentre of Δ formed by Lines

13. $x\cos\frac{\pi}{9}+y\sin\frac{\pi}{9}=\pi$, $x\cos\frac{8\pi}{9}+y\sin\frac{8\pi}{9}=\pi$

$x\cos\frac{13\pi}{9}+y\sin\frac{13\pi}{9}=\pi$



Concept

Incentre is always at same distance from sides of Δ.

1) $(0,0)$ से L_1 dist $=\pi$
2) $(0,0)$ से L_2 से dis. $=\pi$
3) $(0,0)$ से L_3 से dist $=\pi$

Incentre is $(0,0)$