Nishant Jindal

KINEMATICS

Concept of Avg velocity and Instantanious velocity.

Avg velocity :- $\left[\frac{\text{Total Displacement}}{\text{Total time taken}}\right]$

$$\vec{V}_{avg} = \left(\frac{\Delta\vec{r}}{\Delta t}\right) = \left[\frac{\vec{r}_f - \vec{r}_i}{\Delta t}\right]$$

✓ **Avg Speed :-** $\left[\frac{\text{Total Distance}}{\text{Total time taken}}\right]$

$$\lim_{\Delta x \to 0}\left(\frac{\Delta y}{\Delta x}\right) \to \frac{dy}{dx}$$

$V_{inst} \Rightarrow$ [Rate of Change of displacement w.r.t time]

$$\vec{v}_{int} = \frac{d\vec{r}}{dt}$$

$$v_{inst} = \frac{dr}{dt} \Leftarrow$$ Instantanious

↳ At a particular point of time.

$$v_{inst} = \lim_{\Delta t \to 0}\left(\frac{\Delta r}{\Delta t}\right) = \frac{dr}{dt}$$

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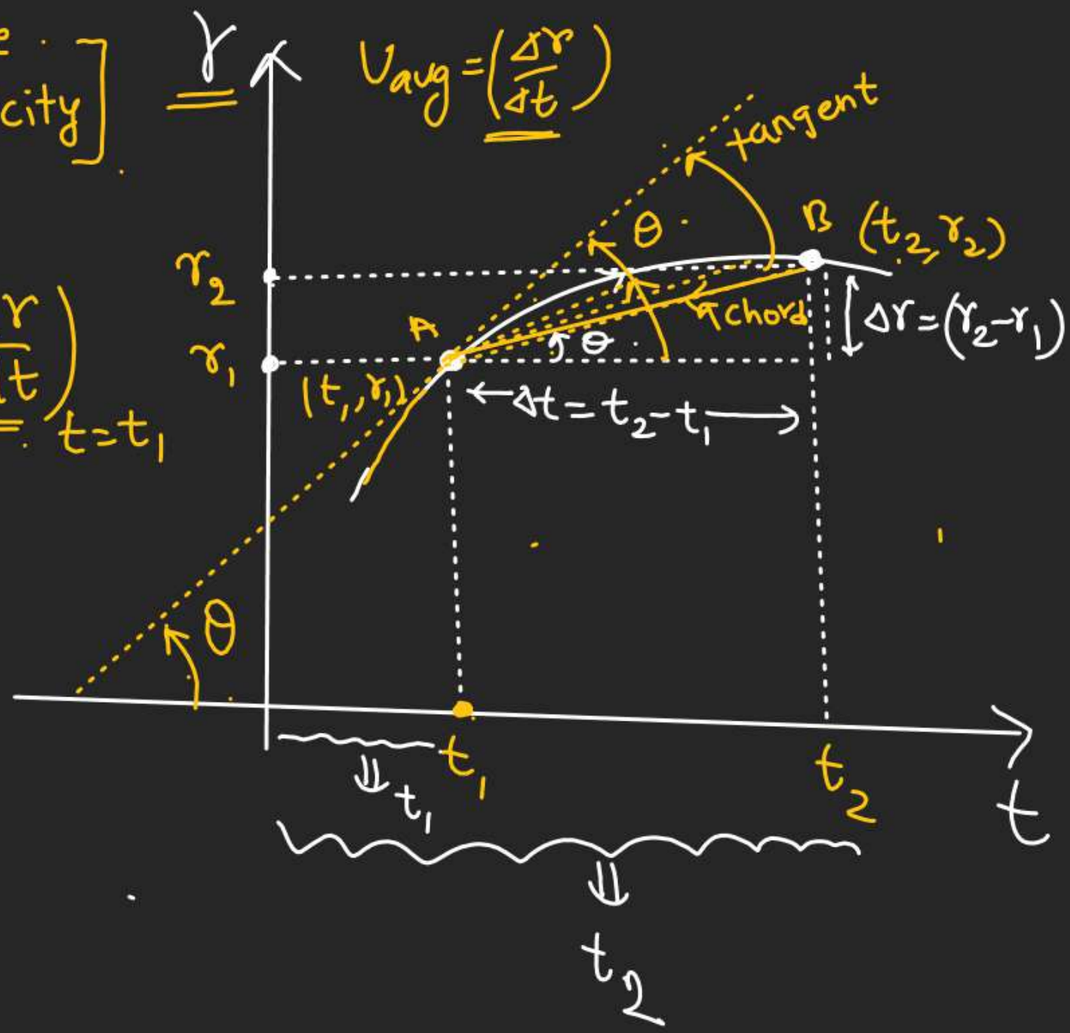
Note :- [Slope of displacement vs time graph gives instantanious velocity]

$$v_{\text{inst velocity}} = m = \tan\theta = \left(\frac{dr}{dt}\right)_{t=t_1}$$

$$\boxed{\lim_{\Delta t \to 0} \frac{\Delta r}{\Delta t} = \left(\frac{dr}{dt}\right)}$$



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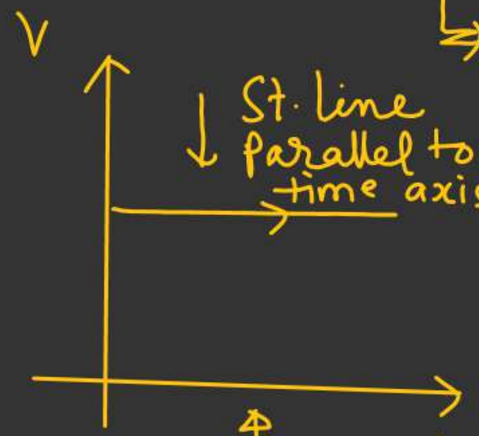
(★)

Uniform velocity & Non-uniform velocity:-

1 **Uniform velocity:-** "Equal displacement Covered in equal interval of time.

↳ For uniform velocity Motion displacement Vs time graph

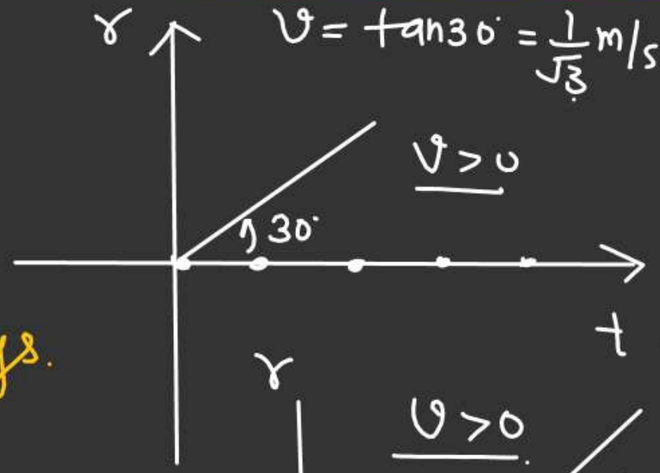


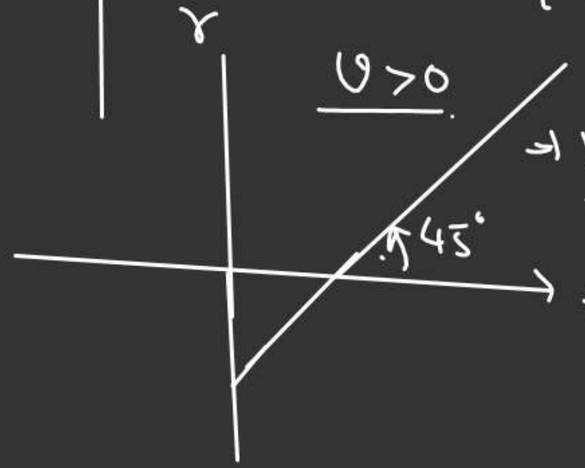
$V = C$

$\left(\frac{dx}{dt}\right) = C$

⇓

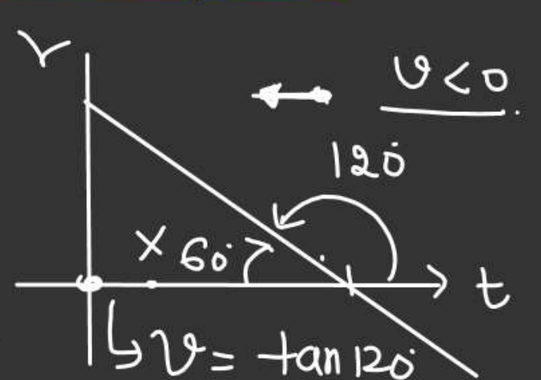
Slope of displacement Vs time curve is always Constant.





→ $v = \tan 45$

$v = 1\ m/s$



↳ $v = \tan 120$

$= \tan(180 - 60)$

$= -\tan 60$

$= -\sqrt{3}\ m/s$

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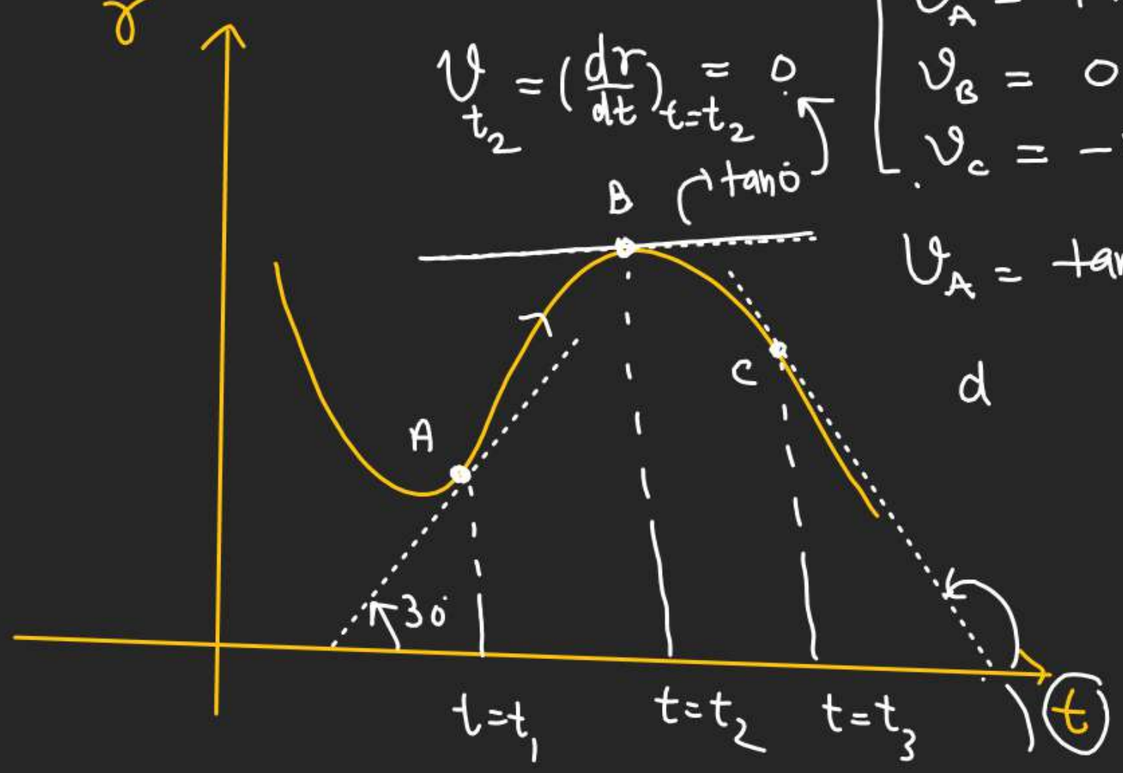
Non-Uniform velocity

"Unequal displacement cover in equal interval of time".

$$v_{inst} = \frac{dr}{dt}$$



v_A = +ve.
v_B = 0
v_C = −ve

$v_A = \tan 30° = \frac{1}{\sqrt{3}}$ m/s.

d

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KINEMATICS

Displacement of a particle as a function of time is given as.

$$\underline{r = (2t^2 + t)} \leftarrow \text{(Parabola)}$$

Find. a) velocity of particle at $t = 3\,\text{sec}$ ←

b) Find avg velocity in the interval $t = 1\,\text{sec}$ to $t = 3\,\text{sec}$.

$$v_{inst} = \left(\frac{dr}{dt}\right)$$

$$v_{inst} = \frac{d}{dt}(2t^2 + t)$$

$$v_{inst} = 2\frac{d}{dt}(t^2) + \frac{d}{dt}(t)$$

$$\boxed{v_{inst} = 4t + 1} \Leftarrow \text{St. line}$$

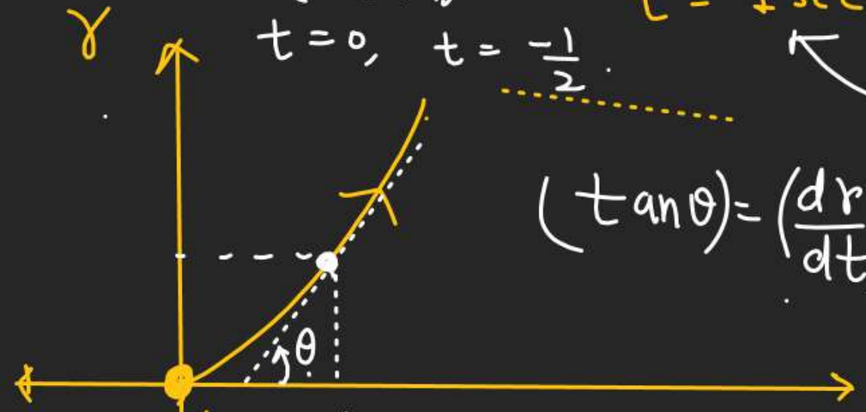
$y = mx + c$

$r = (2t^2 + t)$

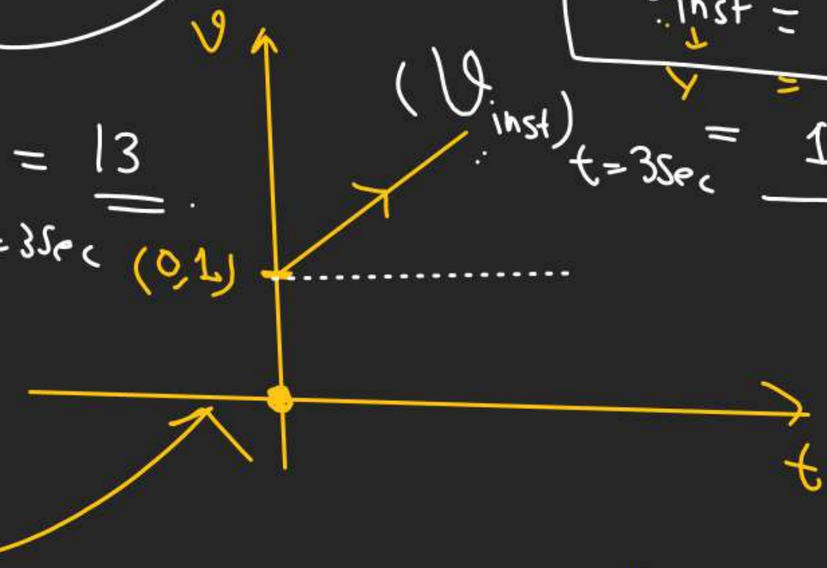
roots. $r = 0$.

$t(2t + 1) = 0$

$t = 0,\ t = -\frac{1}{2}$.



$$(\tan\theta) = \left(\frac{dr}{dt}\right)_{t=3\,\text{sec}} = 13$$



$(v_{inst})_{t=3\,\text{sec}} = 13\ \text{m/s}$. St. line.

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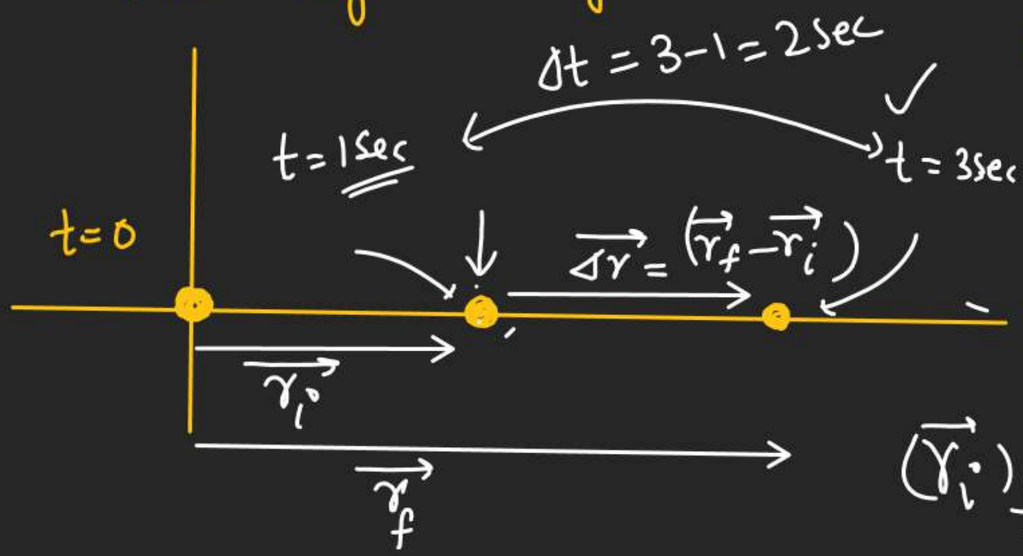
(b) $\vec{r} = (2t^2 + t)$

Find avg velocity at $t = 1\text{sec}$ to $t = 3\text{sec}$.



At $t = 0$, $\vec{r} = 0$, $\therefore r_i = 0$.

$\vec{\Delta r} = \vec{r}_f - \vec{r}_i$

$(\vec{r}_i)_{t=1\text{sec}} = [2(1)^2 + 1]\hat{\imath}$

$= 3\hat{\imath}$

$(\vec{r}_f)_{t=3\text{sec}} = [2(3)^2 + 3]\hat{\imath}$

$= 21\hat{\imath}$

$\vec{\Delta r} = (\vec{r}_f - \vec{r}_i)$

$= (21 - 3)\hat{\imath}$

$= 18\hat{\imath}$

$\Delta t = (3 - 1)$

$= 2\text{sec}$

$\vec{V}_{avg} = \frac{\vec{\Delta r}}{\Delta t} = \frac{18}{2}\hat{\imath}$

$V_{avg} = 9\hat{\imath}$

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KINEMATICS

A particle moving in x-y plane whose position vector is given as.

$$\vec{r} = t^2\hat{i} + 2t\hat{j} \quad \rightarrow \quad \vec{r} = x\hat{i} + y\hat{j}$$

Find:- i) Instantanious velocity of the particle at $t = 3\,Sec$

ii) Avg velocity in the interval $t = 1\,Sec$ to $t = 3\,Sec$.

1st Method

$x = t^2$; $y = 2t$

$v_x = \frac{dx}{dt} = 2t$; $v_y = \frac{dy}{dt} = 2$

$\vec{v} = v_x\hat{i} + v_y\hat{j}$

$\boxed{\vec{v} = (2t)\hat{i} + 2\hat{j}}$

$\vec{v}_{t=3Sec} = (6\hat{i} + 2\hat{j})$

$|\vec{v}_{t=3Sec}| = \sqrt{36+4}$

$= \sqrt{40} = 2\sqrt{10}\ m/s$

2nd method

$\vec{v} = \frac{d\vec{r}}{dt} = \frac{d}{dt}(t^2\hat{i} + 2t\hat{j})$

$\vec{v} = \left[\frac{d}{dt}(t^2)\right]\hat{i} + \left[\frac{d}{dt}(2t)\right]\hat{j}$

$\vec{v} = (2t)\hat{i} + 2\hat{j}$

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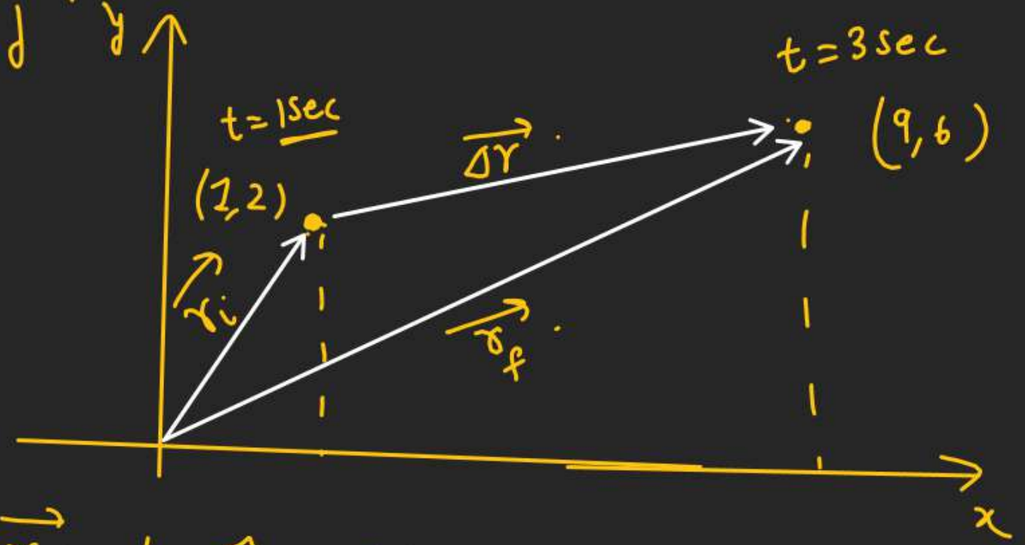
(b) Avg velocity, $t = 1\,sec$ to $t = 3\,sec$

$$\vec{r} = t^2\hat{i} + 2t\hat{j}$$

$$\vec{r}_{t=1sec} = (1)\hat{i} + 2\hat{j}$$

$$\vec{r}_{t=3sec} = \underset{\Downarrow x}{9}\hat{i} + \underset{\Downarrow y}{6}\hat{j}$$



$$\vec{\Delta r} = \vec{\Delta x} = \vec{r}_f - \vec{r}_i = (9\hat{i} + 6\hat{j}) - (\hat{i} + 2\hat{j})$$

$$\vec{\Delta x} = (8\hat{i} + 4\hat{j}), \quad \Delta t = 2\,sec$$

$$\left(\vec{v}_{avg}\right) = \frac{\vec{\Delta r}}{\Delta t} = \frac{8\hat{i} + 4\hat{j}}{2} = (4\hat{i} + 2\hat{j})$$

$$|\vec{v}_{avg}| = \sqrt{16 + 4} = \sqrt{20}$$

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KINEMATICS

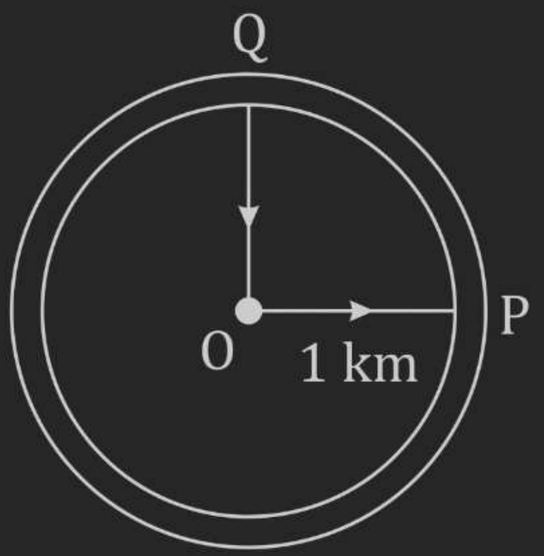
H.W.

Q. A cyclist travels from centre O of a circular park of radius 1 km and reaches point P. After cycling 1/4th of the circumference along PQ, he returns to the centre of the park QO. If the total time taken is 10 minute, calculate

(i) net displacement Fig.

(ii) average velocity and

(iii) average speed of the cyclist.



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KINEMATICS

Q. A particle moves in a semicircular path of radius R from O to A (Fig.) Then it moves parallel to z – axis covering a distance R upto B. Finally it moves along BC parallel to y – axis through a distance 2 R. Find the ratio of D/s.

Displacement / Distance

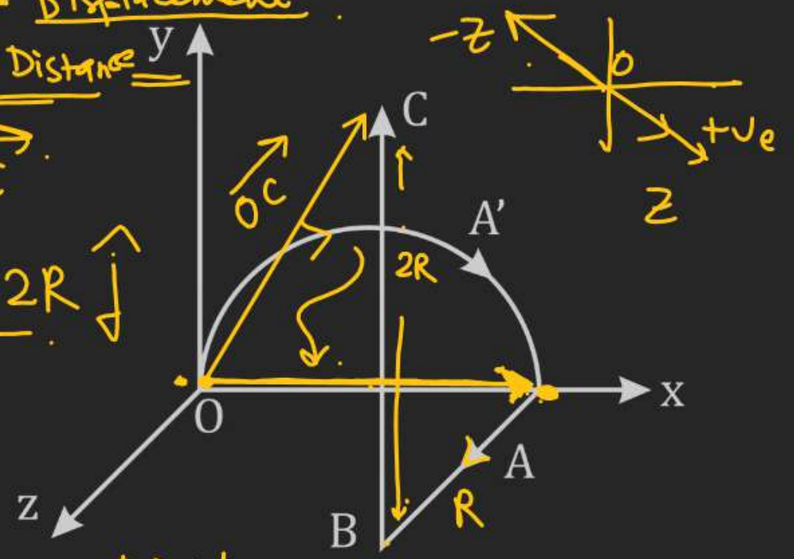
Sol^n:– $\underline{\text{Displacement}} = \vec{OA} + \vec{AB} + \vec{BC}$

$\Rightarrow \vec{OC} = (2R)\hat{i} + R\hat{k} + 2R\hat{j}$

$|\vec{OC}| = \sqrt{4R^2 + R^2 + 4R^2}$

Distance:– = Actual path length $= \sqrt{9R^2} = 3R$.

$= (\pi R + R + 2R) = (3R + \pi R) = (\pi + 3)R$



$\frac{\text{Displacement}}{\text{Distance}} = \frac{3R}{(\pi+3)R}$

$= \left(\frac{3}{\pi+3}\right)$ Ans

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Q. A point traversed half the distance with a velocity v_0. The remaining part of the distance was covered with velocity v_1 for half the time, and with velocity v_2 for the other half of the time. Find the mean velocity of the point averaged over the whole time of motion.

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KINEMATICS

H.W.

Q. A man walks on a straight road from his home to a market 2.5 km away with a speed of 5 km/h. Finding market closed, he instantly turns and walks back home with a speed of 7.5 km/h. What is the

(a) magnitude of average velocity,

(b) average speed of the man over the interval of time

(i) 0 to 30 min., (ii) 0 to 50 min., (iii) 0 to 40 min. ?