DPP–02 (AREA UNDER THE CURVE)

SUBJECTIVE

1. Find the values of $m(m > 0)$ for which the area bounded by the line $y = mx + 2$ and the curve $x = 2y - y^2$ is, (i) $\frac{9}{2}$ square units and (ii) minimum. Also find the minimum area.

2. For what value of ' a ' is the area bounded by the curve $y = a^2x^2 + ax + 1$ and the straight line $y = 0, x = 0$ and $x = 1$ the least?

3. Let ' c ' be the constant number such that $c > 1$. If the least area of the figure given by the line passing through the point $(1, c)$ with gradient ' m ' and the parabola $y = x^2$ is 36 sq, units find the value of $(c^2 + m^2)$.

4. If $f(x)$ is monotonic in (a, b) then prove that the area bounded by the ordinates at $x = a; x = b$; $y = f(x)$ and $y = f(c), c \in (a, b)$ is minimum when $c = \frac{a+b}{2}$. Hence if the area bounded by the graph of $f(x) = \frac{x^3}{3} - x^2 + a$, the straight lines $x = 0, x = 2$ and the x-axis is minimum then find the value of ' a ',

5. For what values of $a \in [0,1]$ does the area of the figure bounded by the graph of the function $y = f(x)$ and the straight lines $x = 0, x = 1$ and $y = f(a)$ is at a minimum and for what values it is at a maximum if $f(x) = \sqrt{1 - x^2}$. Find also the maximum and the minimum areas.

6. A figure is bounded by the curves $y = \left|\sqrt{2}\sin \frac{\pi x}{4}\right|, y = 0, x = 2$ and $x = 4$. At what angles to the positive x-axis straight lines must be drawn through $(4,0)$ so that these lines partition the figure into three parts of the same size.

7. The line $3x + 2y = 13$ divides the area enclosed by the curve, $9x^2 + 4y^2 - 18x - 16y - 11 = 0$ into two parts. Find the ratio of the larger area to the smaller area.

8. Find the area bounded by the curve $y = xe^{-x^2}$, the x-axis and the line $x = c$ where $y(c)$ is maximum.

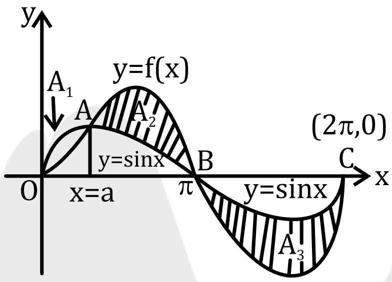
9. A polynomial function $f(x)$ satisfies the condition $f(x + 1) = f(x) + 2x + 1$. Find $f(x)$ if $f(0) = 1$. Find also the equations of the pair of tangents from the origin on the curve $y = f(x)$ and compute the area enclosed by the curve and the pair of tangents.

10. Find the equation of the line passing through the origin and dividing the curvilinear triangle with vertex at the origin, bounded by the curves $y = 2x - x^2$, $y = 0$ and $x = 1$ into two parts of equal area.

11. Consider the curve $y = x^n$ where $n > 1$ in the quadrant. If the area bounded by the curve, the x axis and the tangent line to the graph of $y = x^n$ at the point $(1,1)$ is maximum then find the value of n

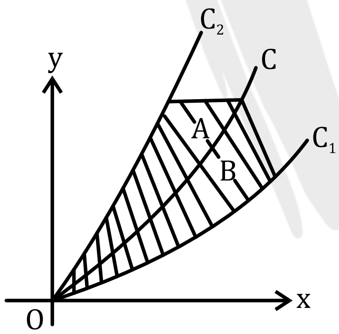
12. In the adjacent figure, graphs of two functions $y = f(x)$ and $y = \sin x$ are given. $y = \sin x$ intersects, $y = f(x)$ at $A(a, f(a))$; $B(\pi, 0)$ and $C(2\pi, 0)$. $A_i(i = 1, 2,3)$ is the area bounded by the curves $y = f(x)$ and $y = \sin x$ between $x = 0$ and $x = a$; $i = 1$, between $x = a$ and $x = \pi$; $i = 2$, between $x = \pi$ and $x = 2\pi$, $i = 3$. If $A_1 = 1 - \sin a + (a - 1)\cos a$, determine the furction $f(x)$. Hence determine ' a ' and A_1. Also calcu. late A_2 and A_3.

13. Let A_n be the area bounded by the curve $y = (\tan x)^2$ and the lines $x = 0, y = 0$ and $x = \frac{\pi}{4}$. Prove that for $n > 2$, $A_n + A_{n-2} = \frac{1}{(n-1)}$ and deduce that $\frac{1}{(2n+2)} < A_n < \frac{1}{(2n-2)}$



14. Find the whole area included between the curve $x^2y^2 = a^2(y^2 - x^2)$ and its asymptotes (asympt totes are the lines which meet the curve at infinity).

15. Let C_1 and C_2 be two curves passing through the origin as shown in the figure. A curve C is said to "bisect the area" the region between C_1 and C_2, if for each point P of C, the two shaded regions A and B shown in the figure have equal areas. Determine the upper curve C_2, given that the bisecting curve C has the equation $y = x^2$ and that the lower curve C_1 has the equation $y = \frac{x^2}{2}$.



PREVIOUS YEAR(JEE ADVANCED)

16. (a) The area of the region between the curves $y = \sqrt{\frac{1+\sin x}{\cos x}}$ and $y = \sqrt{\frac{1-\sin x}{\cos x}}$ bounded by the lines $x = 0$ and $x = \frac{\pi}{4}$ is **[JEE Adv. 2008]**

(A) $\int_0^{\sqrt{2}-1} \frac{t}{(1+t^2)\sqrt{1-t^2}} dt$ (B) $\int_0^{\sqrt{2}-1} \frac{4t}{(1+t^2)\sqrt{1-t^2}} dt$

(C) $\int_0^{\sqrt{2}-1} \frac{4t}{(1+t^2)\sqrt{1-t^2}} dt$ (D) $\int_0^{\sqrt{2}+1} \frac{t}{(1+t^2)\sqrt{1-t^2}} dt$

Comprehension (3 questions together):

Consider the functions defined implicitly by the equation $y^3 - 3y + x = 0$ on various intervals in the real line. If $x \in (-\infty, -2) \cup (2, \infty)$, the equation implicitly defines a unique real valued differentiable function $y = f(x)$. If $x \in (-2,2)$, the equation implicitly defines a unique real valued differentiable function $y = g(x)$ satisfying $g(0) = 0$

(i) If $f(-10\sqrt{2}) = 2\sqrt{2}$, then $f''(-10\sqrt{2}) =$

(A) $\frac{4\sqrt{2}}{7^3 3^2}$ (B) $-\frac{4\sqrt{2}}{7^3 3^2}$

(C) $\frac{4\sqrt{2}}{7^3 3}$ (D) $-\frac{4\sqrt{2}}{7^3 3}$

(ii) The area of the region bounded by the curve $y = f(x)$, the x-axis and the lines $x = a$ and $x = b$, where $-\infty < a < b < -2$, is

(A) $\int_a^b \frac{x}{3((f(x))^2-1)} dx + bf(b) = af(a)$ (B) $-\int_a^b \frac{x}{3((f(x))^2-1)} dx + bf(b) - af(a)$

(C) $\int_a^b \frac{x}{3((f(x))^2-1)} dx - bf(b) + af(a)$ (D) $-\int_a^b \frac{x}{3((f(x))^2-1)} dx - bf(b) + af(a)$

(iii) $\int_{-1}^{1} g'(x)dx$ equals

(A) $2\,g(-1)$ (B) 0 (C) $-2g(1)$ (D) $2\,g(1)$

17. Area of the region bounded by the curve $y = e^x$ and lines $x = 0$ and $y = e$ is **[JEE Adv. 2009]**

(A) $e - 1$ (B) $\int_1^c \ln(e + 1 \cdot y)dt$

(C) $e - \int_0^1 e^x dx$ (D) $\int_1^e \text{In}\, ydy$

18. Let the straight line $x = b$ divide the area enclosed by $y = (1 - x)^2, y = 0$, and $x = 0$ into two parts R_1 $(0 \le x \le b)$ and $R_2 (b \le x \le 1)$ such that $R_1 - R_2 = \frac{1}{4}$. Then b equals **[JEE Adv. 2011]**

(A) $\frac{3}{4}$ (B) $\frac{1}{2}$

(C) $\frac{1}{3}$ (D) $\frac{1}{4}$

19. Let $f: [-1,2] \to [0, \infty)$ be a continuous function such that $f(x) = f(1 - x)$ for all $x \in [-1,2]$. Let $R_1 = \int_{-1}^{2} xf(x)dx$, and R_2 be the area of the region bounded by $y = f(x), x = -1, x = 2$, and the x axis. Then **[JEE Adv. 2011]**

(A) $R_1 = 2R_2$ (B) $R_1 = 3R_2$

(C) $2R_1 = R_2$ (D) $3R_1 = R_2$

20. Let S be the area of the region enclosed by $y = e^{-x^2}, y = 0, x = 0$, and $x = 1$. Then

[JEE Adv. 2012]

(A) $s \geq \frac{1}{e}$
(B) $s \geq 1-\frac{1}{e}$
(C) $s \leq \frac{1}{4}\left(1+\frac{1}{\sqrt{e}}\right)$
(D) $s \leq \frac{1}{\sqrt{2}}+\frac{1}{\sqrt{e}}\left(1 \cdot \frac{1}{\sqrt{2}}\right)$

21. The area enclosed by the curves $y=\sin x+\cos x$ and $y=|\cos x-\sin x|$ over the interval $\left[0, \frac{\pi}{2}\right]$ is [JEE Adv. 2013]

(A) $4(\sqrt{2}-1)$
(B) $2\sqrt{2}(\sqrt{2}-1)$
(C) $2(\sqrt{2}+1)$
(D) $2\sqrt{2}(\sqrt{2}+1)$

22. Let $F(x)=\int_x^{x^2+\frac{\pi}{6}} 2\cos^2 t dt$ for all $x \in R$ and $f:\left[0, \frac{1}{2}\right] \rightarrow [0, \infty)$ be a continuous function. For $a \in \left[0, \frac{1}{2}\right]$, if $F'(a)+2$ is the area of the region bounded by $x=0, y=0, y=f(x)$ and $x=a$, then $f(0)$ is. [JEE Adv. 2015]

(A) 1
(B) 2
(C) 3
(D) 4

23. Area of the region $\left\{(x, y) \in R^2: y \geq \sqrt{|x+3|}, 5y \leq x+9 \leq 15\right\}$ is equal to [JEE Adv. 2016]

(A) $\frac{1}{6}$
(B) $\frac{4}{3}$
(C) $\frac{3}{2}$
(D) $\frac{5}{3}$

24. If the line $x=\alpha$ divides the area of region $R=\{(x, y) \in R^2: x^3 \leq y \leq x, 0 \leq x \leq 1\}$ into two equal parts, then [JEE Adv. 2017]

(A) $\alpha^4+4\alpha^2-1=0$
(B) $0<a \leq \frac{1}{2}$
(C) $2\alpha^4-4\alpha^2+1=0$
(D) $\frac{1}{2}<\alpha<1$

25. A farmer F_1 has a land in the shape of a triangle with vertices at $P(0,0)$, $Q(1,1)$ and $R(2,0)$. From this land, a neighbouring farmer F_2 takes away the region which lies between the side PQ and a curve of the form $y=x^n (n>1)$. If the area of the region taken away by the farmer F_2 is exactly 30% of the area of $\triangle$ PQR, then the value of n is [JEE Adv. 2018]

26. The area of region $\{(x, y): xy \leq 8, 1 \leq y \leq x^2\}$ is [JEE Adv. 2019]

(A) $16\log_c 2-\frac{14}{3}$
(B) $8\log_e 2-\frac{7}{3}$
(C) $8\log_c 2-\frac{14}{3}$
(D) $16\log_c 2-6$

27. Let the functions $f: R \rightarrow R$ and $g: R \rightarrow R$ be defined by $f(x)=e^{x-1}-e^{-|x-1|}$ and $g(x)=\frac{1}{2}\left(e^{x-1}+e^{1-x}\right)$. Then the area of the region in the first quadrant bounded by the curves $y=f(x)$, $y=g(x)$ and $x=0$ is. [JEE Adv. 2020]

(A) $(2-\sqrt{3})+\frac{1}{2}\left(e-e^{-1}\right)$
(B) $(2+\sqrt{3})+\frac{1}{2}\left(e-e^{-1}\right)$
(C) $(2-\sqrt{3})+\frac{1}{2}\left(e+e^{-1}\right)$
(D) $(2+\sqrt{3})+\frac{1}{2}\left(e+e^{-1}\right)$

28. The area of the region $\left\{(x, y): 0 \le x \le \frac{9}{4},\ 0 \le y \le 1,\ x \ge 3y,\ x + y \ge 2\right\}$ is **[JEE Adv. 2021]**

(A) $\frac{11}{32}$ (B) $\frac{35}{96}$ (C) $\frac{37}{96}$ (D) $\frac{13}{32}$

29. Consider the functions $f, g: R \to R$ defined by

$f(x) = x^2 + \frac{5}{12}$ and $g(x) = \begin{cases} 2\left(1 - 4\frac{|x|}{3}\right), & |x| \le \frac{3}{4}, \\ 0, & |x| > \frac{3}{4} \end{cases}$. If α is the area of the region

$\left\{(x, y) \in R \times R: |x| \le \frac{3}{4}, 0 \le y \le \min\{f(x), g(x)\}\right\}$, then the value of 9α is **[JEE Adv. 2022]**

ANSWER KEY

1. (i) $m = 1$, (ii) $m = \infty; A_{min} = 4/3$

2. $a = -\frac{3}{4}$

3. 104

4. $a = \frac{2}{3}$

5. $a = \frac{1}{2}$ gives minima, $A\left(\frac{1}{2}\right) = \frac{3\sqrt{3} - \pi}{12}$; $a = 0$ gives local maxima $A(0) = 1 - \frac{\pi}{4}$; $a = 1$ gives maximum value, $A(1) = \frac{\pi}{4}$

6. $\pi - \tan^{-1}\frac{2\sqrt{2}}{3\pi}; \pi - \tan^{-1}\frac{4\sqrt{2}}{3\pi}$

7. $\frac{3\pi + 2}{\pi - 2}$

8. $\frac{1}{2}\left(1 - e^{-1/2}\right)$

9. $f(x) = x^2 + 1; y = \pm 2x; A = \frac{2}{3}$ sq. units

10. $y = \frac{2x}{3}$

11. $\sqrt{2} + 1$

12. $f(x) = x\sin x, a = 1; A_1 = 1 - \sin 1; A_2 = \pi - 1 - \sin 1; A_3 = (3\pi - 2)$ sq. units

14. $14a^2$

15. $\left(\frac{16}{9}\right)x^2$

PREVIOUS YEAR(JEE ADVANCED)

16. **A.** (B);

B. (i) (B), (ii) (A), (iii) (D)

17.	(BCD)	**18.**	(B)	**19.**	(C)	**20.**	(ABD)	**21.**	(B)
22.	(C)	**23.**	(C)	**24.**	(CD)	**25.**	(4)	**26.**	(A)
27.	(A)	**28.**	(A)	**29.**	(6)				