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MAGNETIC FIELD

Magnetic Moment and Torque

Magnetic Moment

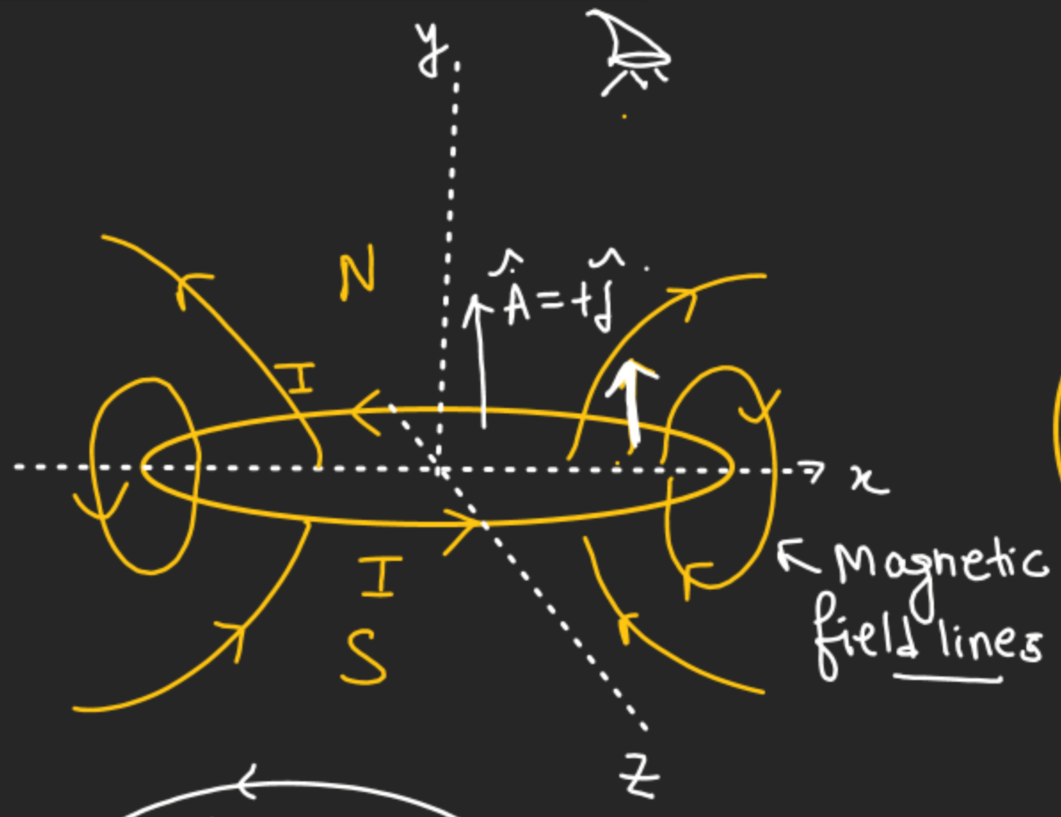
$$\vec{M} = [niA]\hat{A}$$

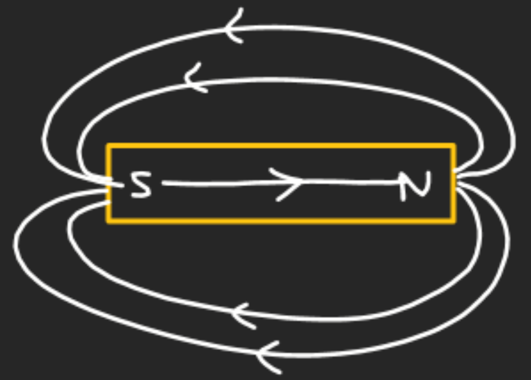
n = No of turns

i = Current

A = Area of loop

$\hat{A}$ = It is unit vector perpendicular to plane of the loop



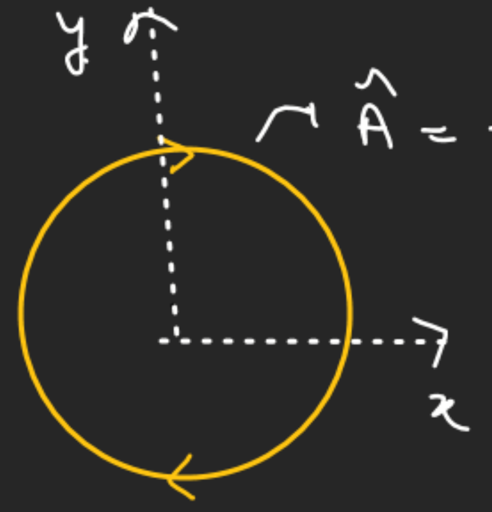


N → B (exit)

S → B (enter)

If current in anticlockwise $\hat{A}$ → outward

$\hat{A} = +\hat{k}$



$\hat{A} = -\hat{k}$

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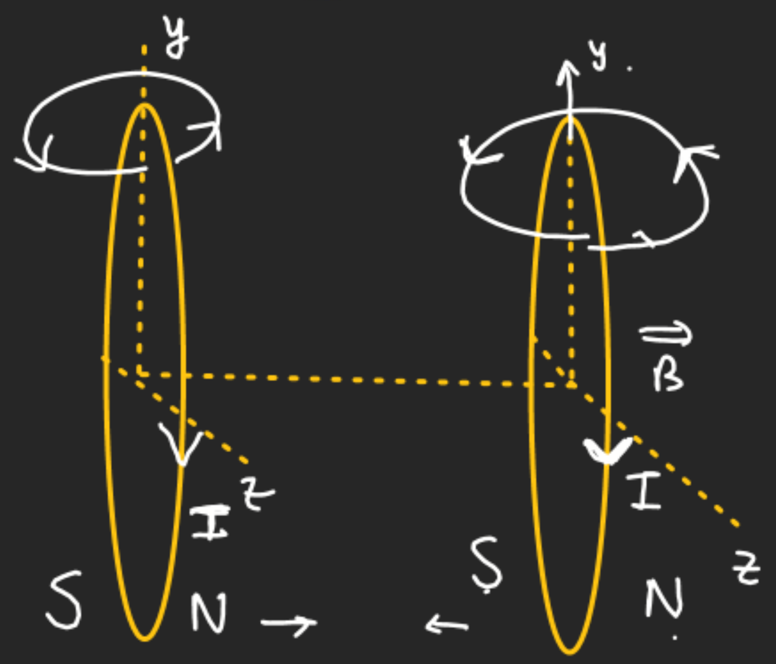
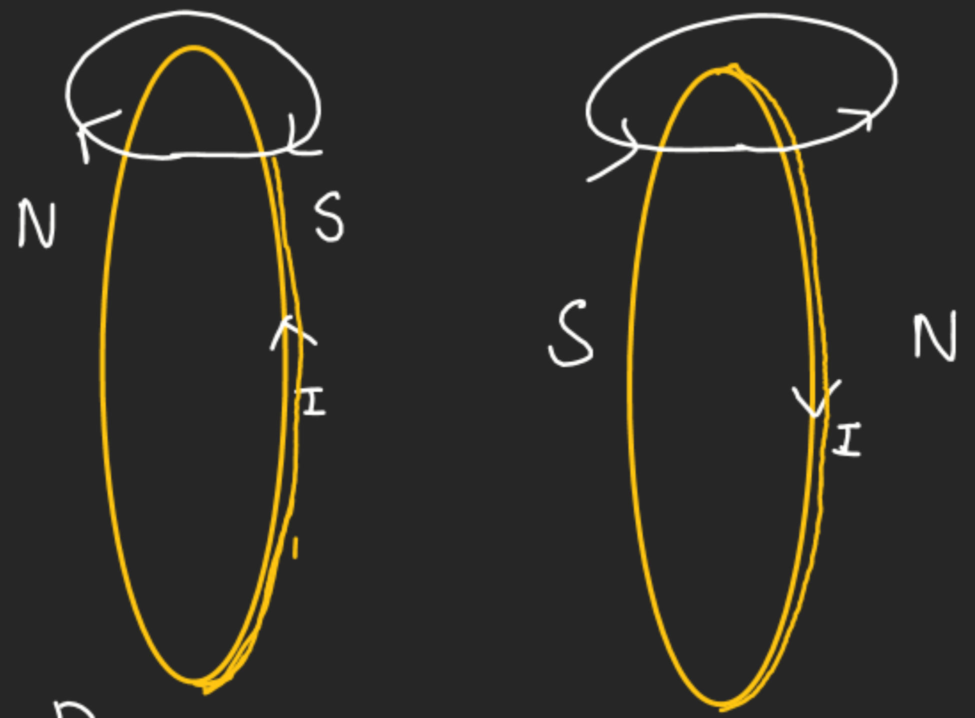
Magnetic Moment and Torque



"Loop attract each other"



Repel each other

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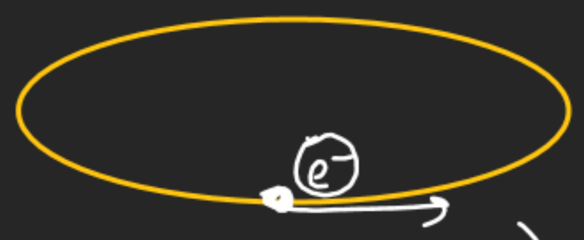
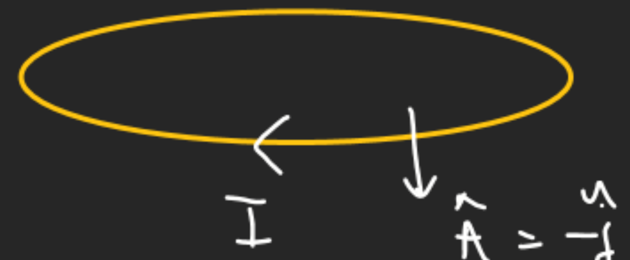
Magnetic Moment and Torque

Magnetic Moment of a rotating Charge :→

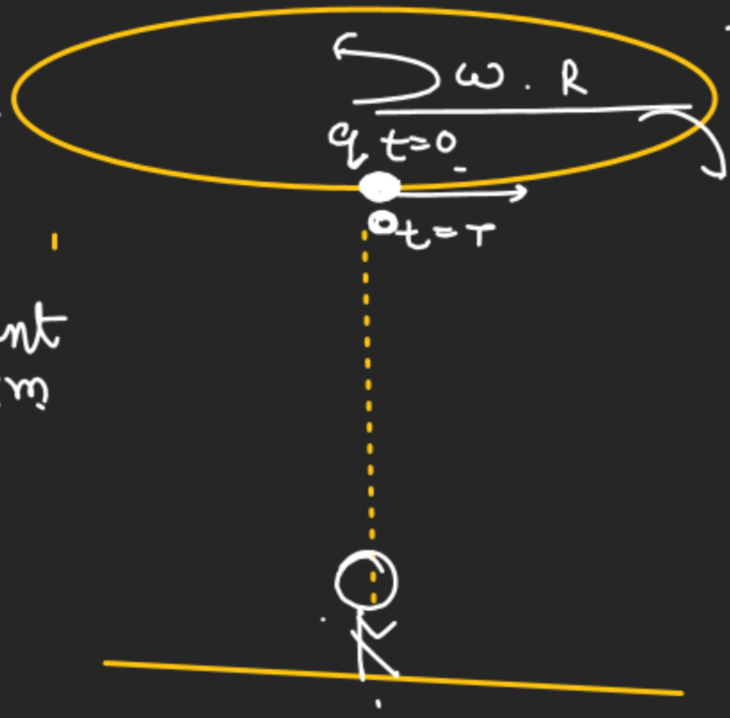


Equivalent system



$\hat{A} = -\hat{j}$

$$\vec{M} = \left(\frac{q\omega R^2}{2}\right)(-\hat{j})$$

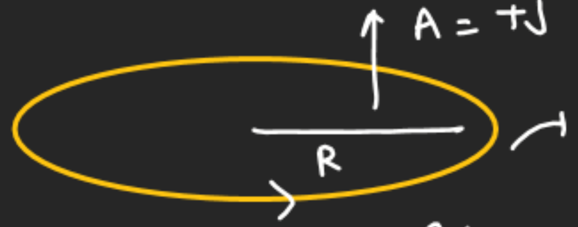


$$I = \frac{Q}{T}$$

$$I = \frac{q\omega}{2\pi}$$

Equivalent system ⇒ assume to be current carrying ring.

$\hat{A} = +\hat{j}$



$$\vec{M} = \left(\left(\frac{q\omega}{2\cancel{\pi}}\right) \times \cancel{\pi} R^2\right) + \hat{j}$$

$$I = \frac{q\omega}{2\pi}$$

$$\vec{M} = \left(\frac{q\omega R^2}{2}\right) + \hat{j}$$

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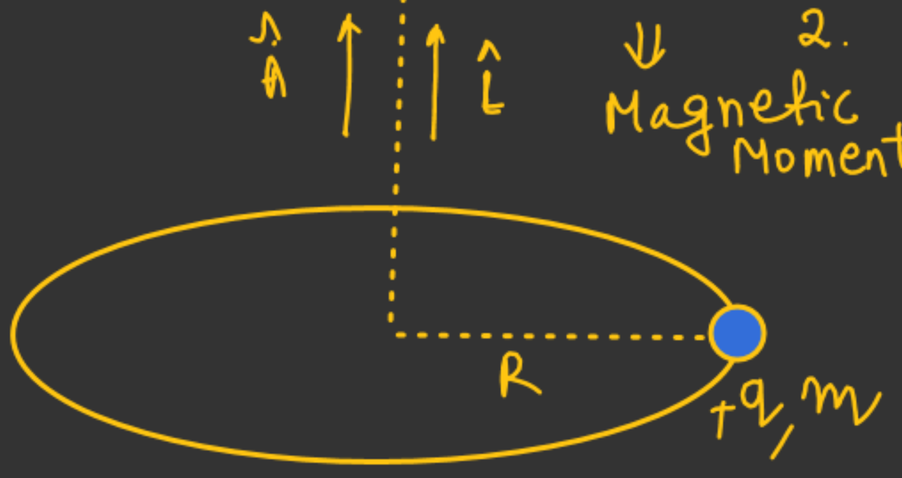
$$M = \frac{q\omega R^2}{2}$$
$\Downarrow$
Magnetic Moment

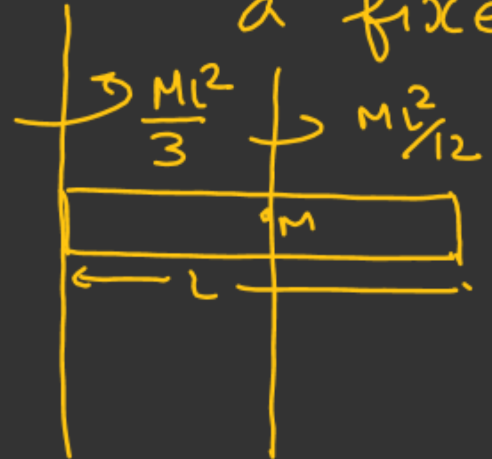


$$L = (MR^2)\omega$$
$\Downarrow$
Angular Momentum

$$\frac{M}{L} = \frac{q\omega R^2}{2 \times M\omega R^2}$$

$$\boxed{\frac{M}{L} = \frac{q}{2M}}$$ **

⇒ Angular Momentum in Case of rotation about a fixed axis = $I\omega$.



Ring / hollow Cylinder

Disc

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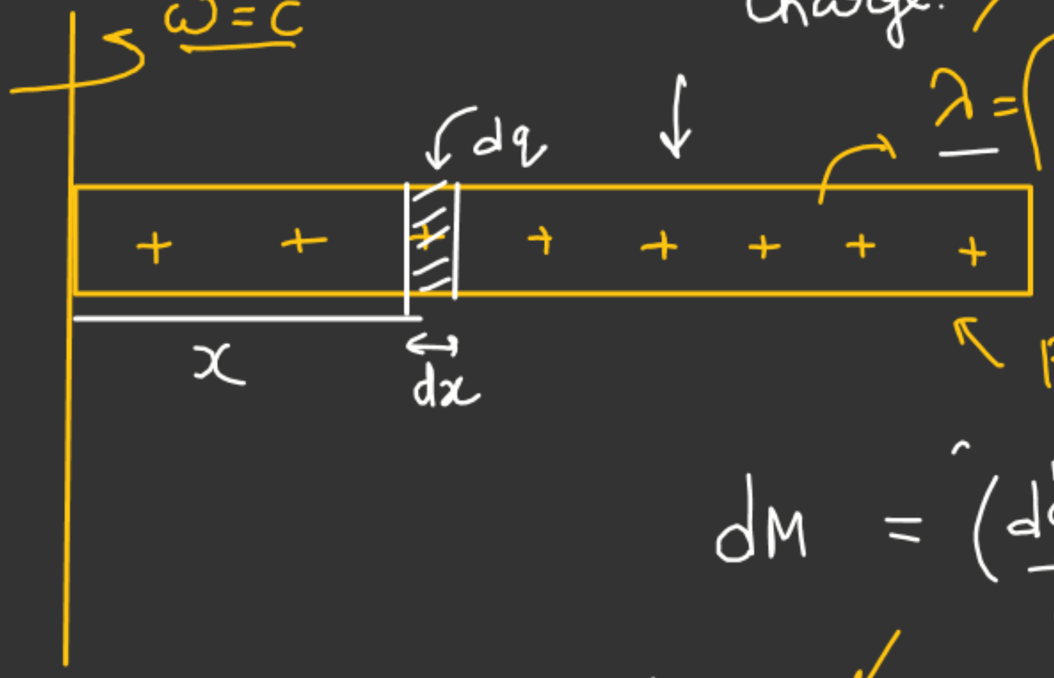
$$\underline{Q = \lambda L} \Rightarrow \frac{Q}{L} = \lambda$$

Q = Total charge.

Uniform.

λ = (Linear charge density.)



Rod is uniform

$$dM = \left(\frac{dq\,\omega}{2\cancel{\pi}}\right)\cancel{\pi}x^2$$

$$dq = \lambda dx$$

$$\int_0^M dM = \frac{\lambda\omega}{2}\int_0^L x^2 dx$$

$$M = \left(\frac{\lambda\omega L^3}{6}\right) = \left(\frac{Q\omega L^2}{6}\right) \text{ — (1)}$$

Angular Momentum

$$L = \frac{ML^2}{3}\omega \text{ — (2)}$$

$$(1) \div (2) \Rightarrow \frac{M}{L} = \frac{Q\omega L^2}{6} \times \frac{3}{ML^2\omega}$$

$$\left[\frac{M}{L} = \frac{Q}{2M}\right]$$

If λ is non uniform :

$$\lambda = \lambda_0 x^2$$

$$\int_0^Q dq = \lambda_0 \int_0^L x^2 dx$$

$$Q = \left(\frac{\lambda_0 L^3}{3}\right)$$

$$\int_0^M dM = \frac{\lambda_0\omega}{2}\int_0^L x^4 dx$$

$$M = \frac{\lambda_0\omega}{2}\left(\frac{L^5}{5}\right) = \left(\frac{\lambda_0\omega L^5}{10}\right)$$

$$= (\lambda_0 L^3)\left(\frac{\omega L^2}{10}\right)$$

$$= \frac{3Q\omega L^2}{10}$$

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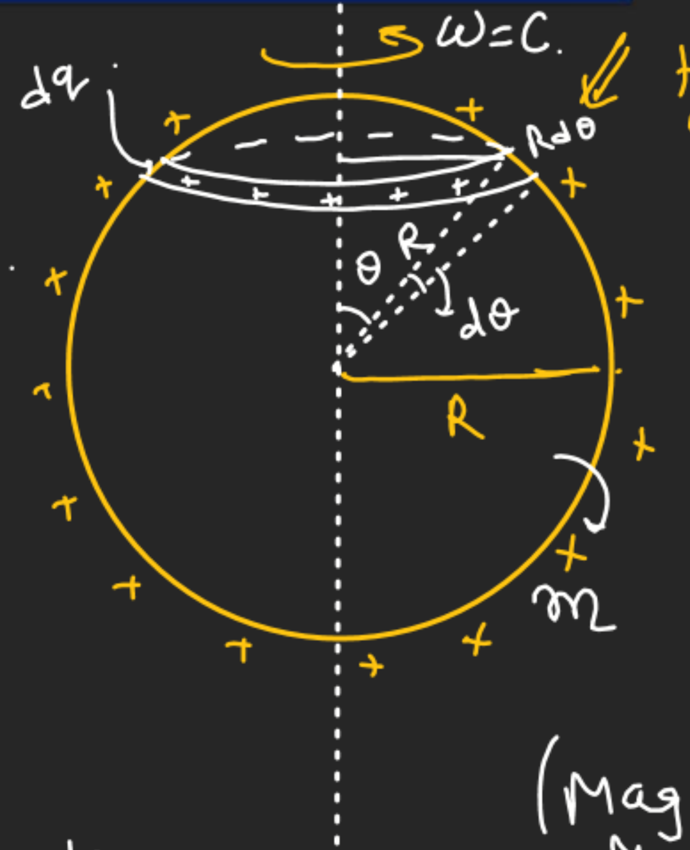
MAGNETIC FIELD

Magnetic Moment and Torque

$$\boxed{\vec{M} = \left(\frac{Q}{2m}\right)\vec{L}}$$

$\Downarrow$

(Valid for uniform Charge distribution)



Hollow conducting Sphere.

σ = surface charge density.

$L = I\omega$

$L = \frac{2}{3} m R^2 \omega$

$Q = (\sigma \cdot 4\pi R^2)$

Trick

$$\frac{M}{L} = \frac{Q}{2m}$$

(Magnetic Moment) $M = \frac{Q}{2m} \times L$

$$M = \frac{\sigma \times 4\pi R^2}{2\cancel{m}} \times \frac{\cancel{2}}{3}\cancel{m} R^2 \omega$$

$$M = \left[\frac{4\pi\sigma R^4 \omega}{3}\right] \checkmark$$

$dq = \sigma\, dA$

$dq = \sigma(2\pi R \sin\theta) R\, d\theta$

$$dI = \frac{dq}{T} = \left(\frac{dq\,\omega}{2\pi}\right)$$

$$\int_0^M dM = \int_0^\pi \frac{dq\,\omega}{2\pi} \times (\pi R \sin\theta) = ??$$

$$M = \frac{4\cancel{\pi} R^{\cancel{4}} \omega}{3} \times \frac{Q}{\cancel{4\pi R^2}}$$

$$\left(M = \frac{Q\omega R^2}{3}\right) \checkmark$$

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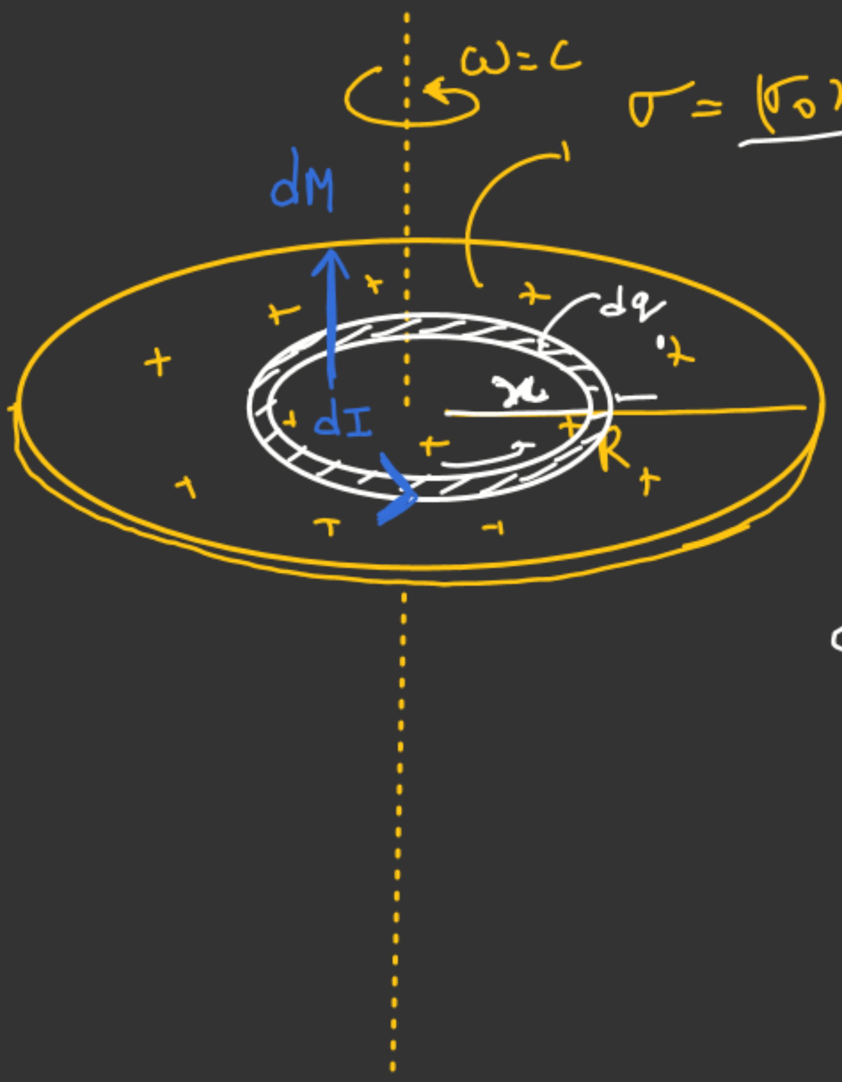
$\sigma_0 = \text{Constant}$

$x \Rightarrow$ Radial distance.



$\sigma = \underline{(\sigma_0 x)}$

Find Magnetic Moment of the disc = ??

$$dM = \frac{dq\,\omega}{2\cancel{\pi}} \times \cancel{\pi} x^2$$

$$dq = \sigma_x \cdot dA$$

$$= (\sigma_0 x)(2\pi x)\,dx$$

$$= \underline{\sigma_0 2\pi x^2 dx}$$

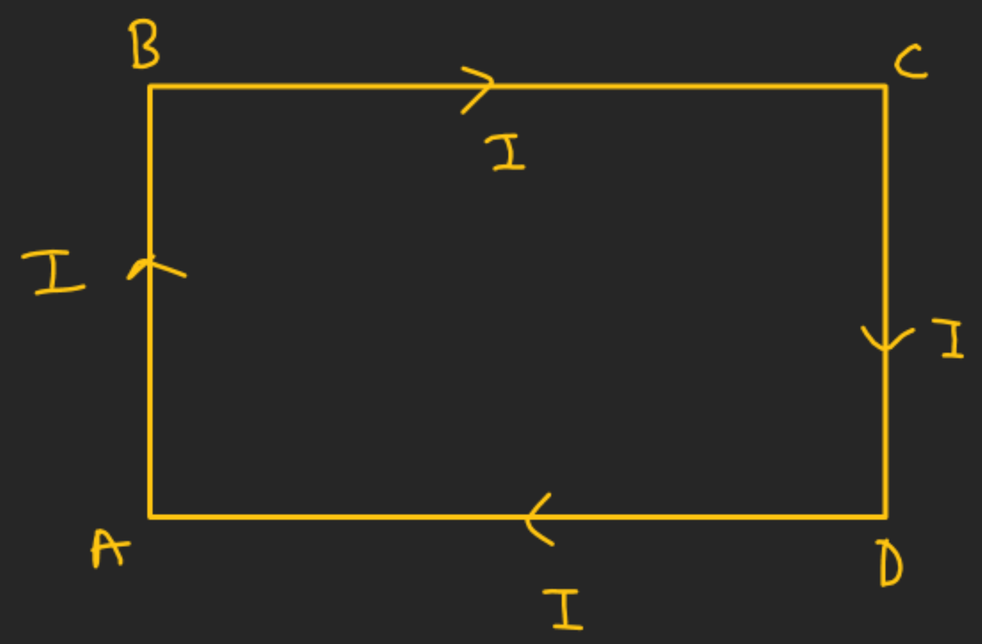
$$\int_0^M dM = \frac{\omega}{\cancel{2}} \sigma_0 \cancel{2}\pi \int_0^R x^4 dx$$

$$= \frac{\sigma_0 \omega \pi R^5}{5}$$

MAGNETIC FIELD

Magnetic Moment and Torque

Magnetic Moment of rectangular loop: →



$$\vec{M} = nI\vec{A}$$

$$\vec{A} = (\vec{AB} \times \vec{BC}) = (\vec{BC} \times \vec{CD})$$

$$= (\vec{CD} \times \vec{DA})$$

(Area vector) of loop ABCD.

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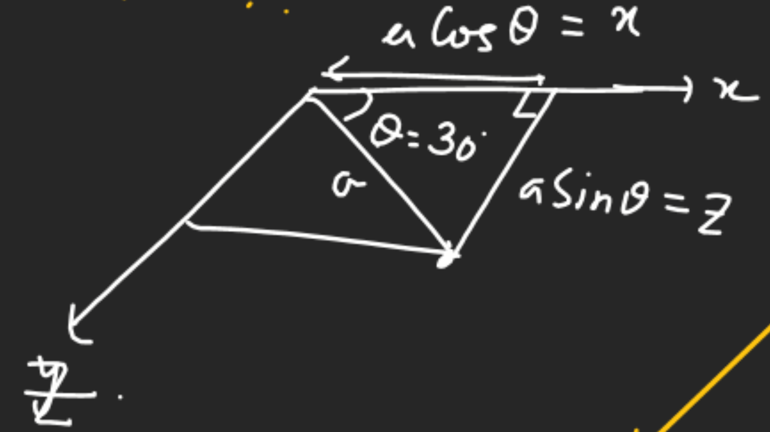
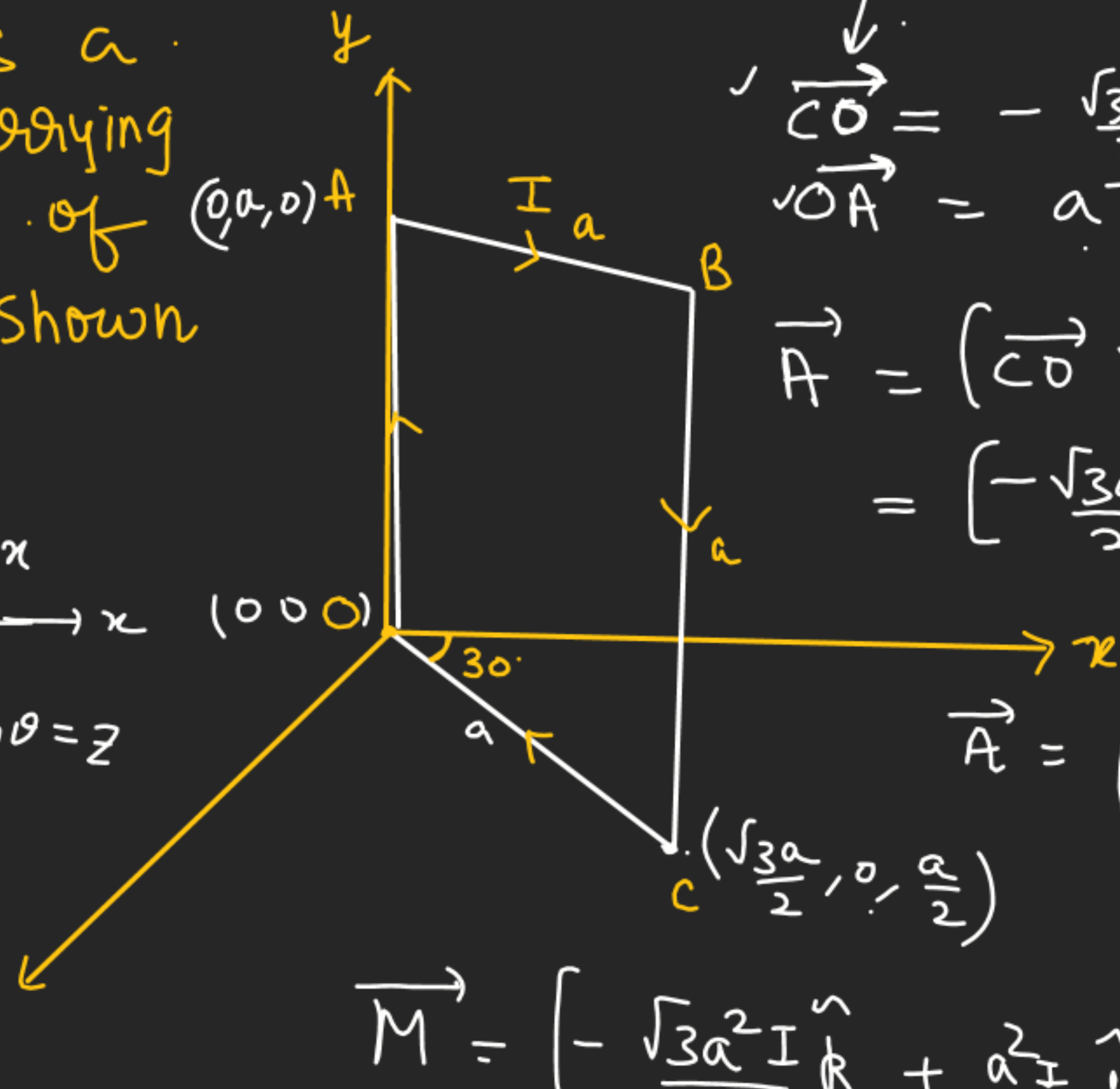
MAGNETIC FIELD

Magnetic Moment and Torque

OABC is a current carrying square loop of side a as shown in fig.

find $\vec{M} = ??$





$$\vec{CO} = -\frac{\sqrt{3}a}{2}\hat{i} - \frac{a}{2}\hat{k}$$

$$\vec{OA} = a\hat{j}$$

$$\vec{A} = \left[\vec{CO} \times \vec{OA}\right]$$

$$= \left[-\frac{\sqrt{3}a}{2}\hat{i} - \frac{a}{2}\hat{k}\right] \times a\hat{j}$$

$$\vec{A} = \left[-\frac{\sqrt{3}a^2}{2}\hat{k} + \frac{a^2}{2}\hat{i}\right]$$

$$\vec{M} = \left[-\frac{\sqrt{3}a^2 I}{2}\hat{k} + \frac{a^2 I}{2}\hat{i}\right]$$