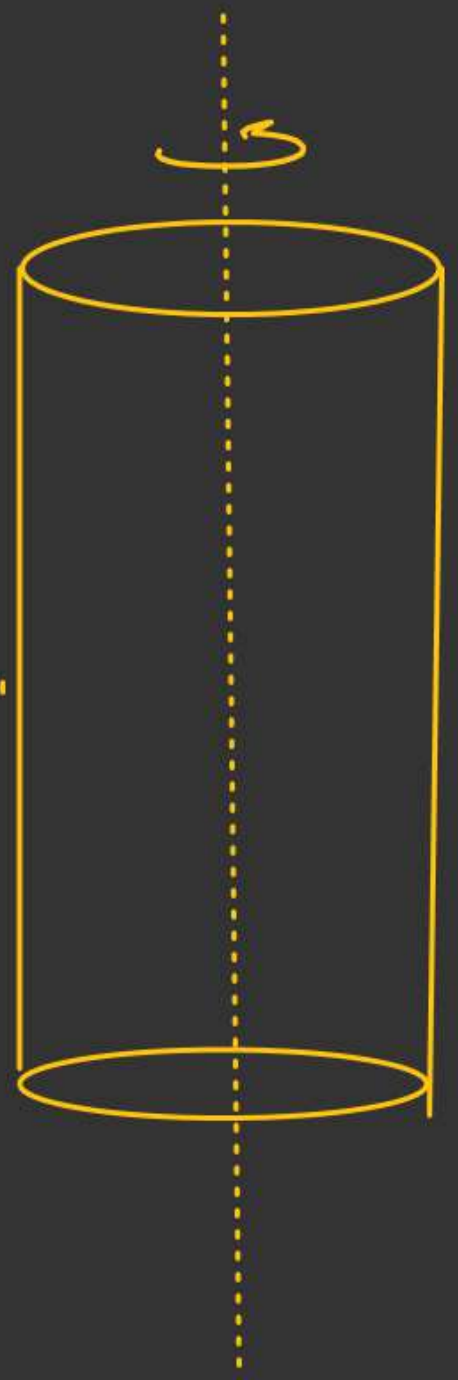
Nishant Jindal

M.I

-☆☆

M.I of a Solid Cylinder about an axis along the axis of the Cylinder

$$\boxed{I = \frac{MR^2}{2}}$$



M.I

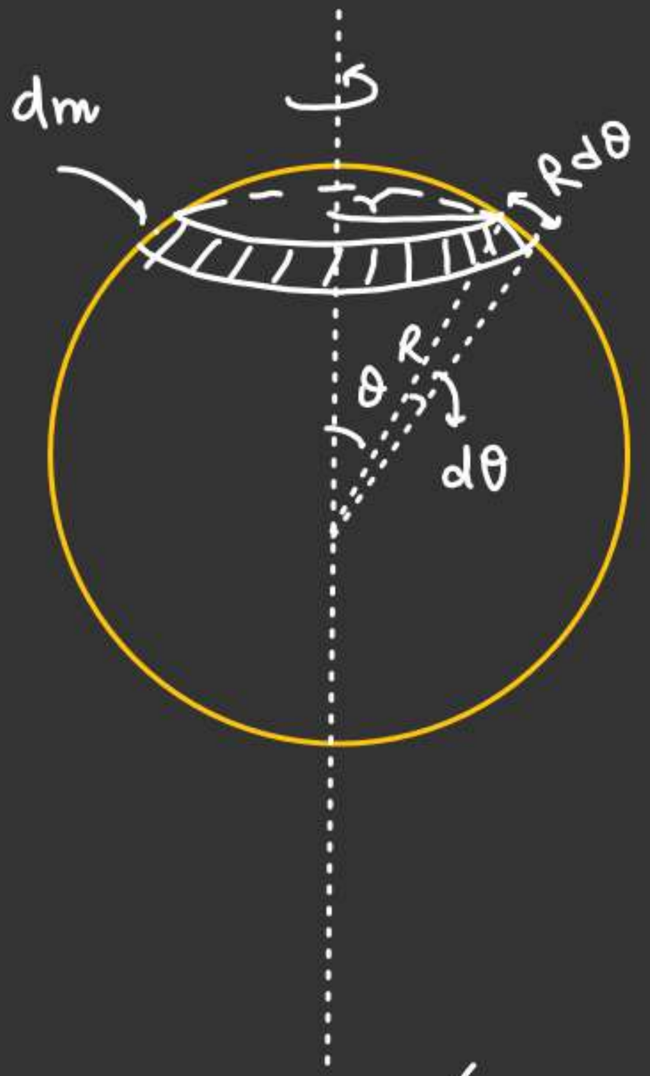
M.I of hollow Sphere or (Spherical Shell) about any diametrical axis

dI = M.I of ring having mass dm

$$dm = \left[\frac{M}{4\pi R^2} \times (2\pi r)(R\,d\theta)\right]$$

$$= \frac{M}{2\cancel{R}} \times \cancel{R}\sin\theta \cdot d\theta$$

$$= \left(\frac{M}{2}\sin\theta \cdot d\theta\right)$$



$$dI = dm\, r^2$$

$$\int_0^I dI = \int_0^\pi \left(\frac{M}{2}\sin\theta \cdot d\theta\right)(R^2 \sin^2\theta) \Rightarrow I = \frac{MR^2}{2}\int_0^\pi \sin^3\theta \cdot d\theta$$

$$\sin 3\theta = 3\sin\theta - 4\sin^3\theta \Rightarrow \sin^3\theta = \left(\frac{3\sin\theta - \sin 3\theta}{4}\right)$$

$$4\sin^3\theta = 3\sin\theta - \sin 3\theta$$

Nishant Jindal

M.I

M.I of hollow Sphere or (Spherical Shell) about any diametrical axis

$$I = \frac{MR^2}{2}\int_0^{\pi} \sin^3\theta \cdot d\theta$$

$$\sin 3\theta = 3\sin\theta - 4\sin^3\theta$$

$$4\sin^3\theta = 3\sin\theta - \sin 3\theta$$

$$\sin^3\theta = \left(\frac{3\sin\theta - \sin 3\theta}{4}\right)$$

$$\boxed{I = \frac{2}{3}MR^2}$$

$$I = \frac{MR^2}{2}\int_0^{\pi}\left(\frac{3\sin\theta - \sin 3\theta}{4}\right)d\theta$$

$$I = \frac{MR^2}{2}\left[\frac{3}{4}\int_0^{\pi}\sin\theta \cdot d\theta - \frac{1}{4}\int_0^{\pi}\sin 3\theta \cdot d\theta\right]$$

$$I = \frac{MR^2}{2}\left[\frac{3}{4}\left[-\cos\theta\right]_0^{\pi} - \frac{1}{4}\left[\frac{-\cos 3\theta}{3}\right]_0^{\pi}\right]$$

$$I = \frac{MR^2}{2}\left[\frac{3}{2} - \frac{1}{6}\right]$$

$$= \frac{MR^2}{2}\left[\frac{9-1}{6}\right] = \frac{4MR^2}{6} = \frac{2}{3}MR^2$$

$$\int \cos kx = \frac{\sin kx}{k}$$

$$\int \sin kx = -\frac{\cos kx}{k}$$

Nishant Jindal

M.I

M.I of a Solid Sphere about any diametrical axis

dm = Mass of hollow Sphere of radius r and thickness dr.

$$dm = \frac{M}{\frac{4}{3}\pi R^3}(dV)$$

$$= \frac{3M}{4\pi R^3} \times 4\pi r^2 dr$$

$$= \left(\frac{3M}{R^3} r^2 dr\right)$$

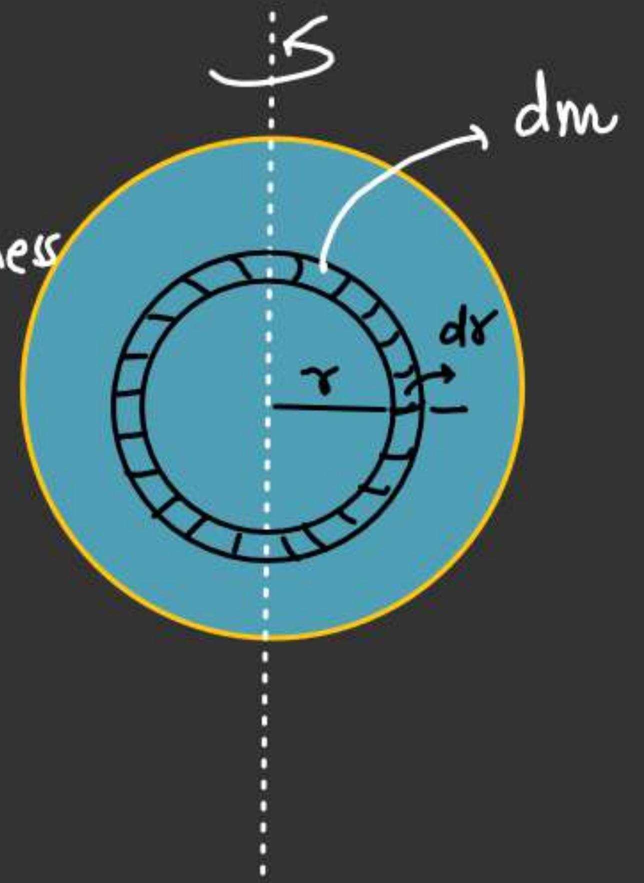
dV = (Area of differential element) × thickness

$$= (4\pi r^2) dr$$

$$V = \frac{4}{3}\pi r^3$$

$$\frac{dV}{dr} = 4\pi r^2$$

$$dV = (4\pi r^2 dr)$$



dI = M.I of hollow Sphere having mass dm

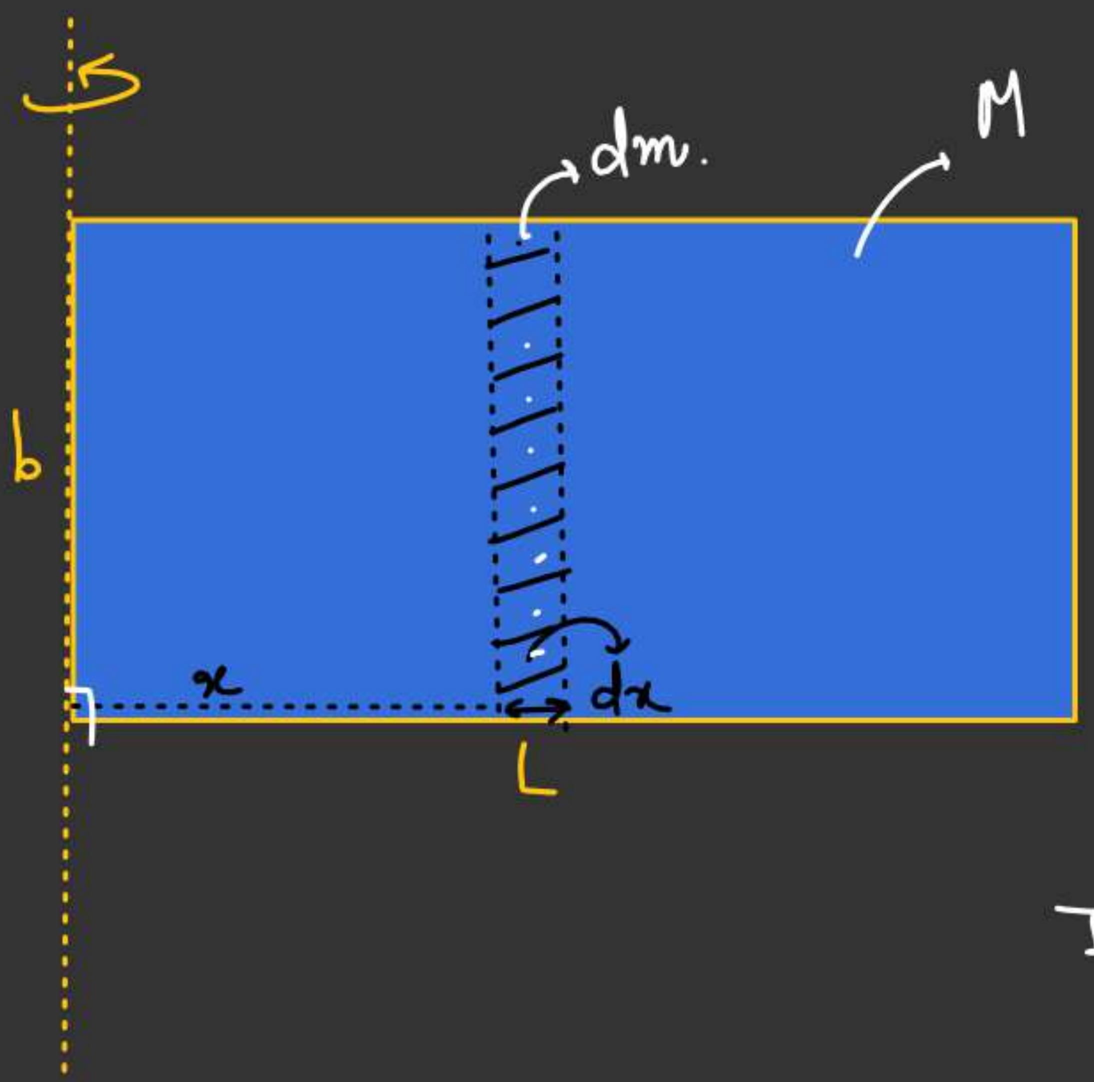
$$\int_0^I dI = \int_0^R \frac{2}{3}(dm) r^2 = \frac{2}{3} \times \frac{3M}{R^3}\int_0^R r^4 dr = \frac{2}{5}MR^2$$

$$\boxed{I = \frac{2}{5}MR^2}$$

Nishant Jindal

M.I

M.I of Rectangular Lamina (2-Dimensional body)



$dI =$ M.I of Rectangular strip about axis of rotation.

$$dI = dm\,x^2$$

$$dm = \left(\frac{M}{Lb}\right)(dA) \quad \text{Area of strip.}$$

$$= \frac{M}{L\cancel{b}} \times \cancel{b}\,dx$$

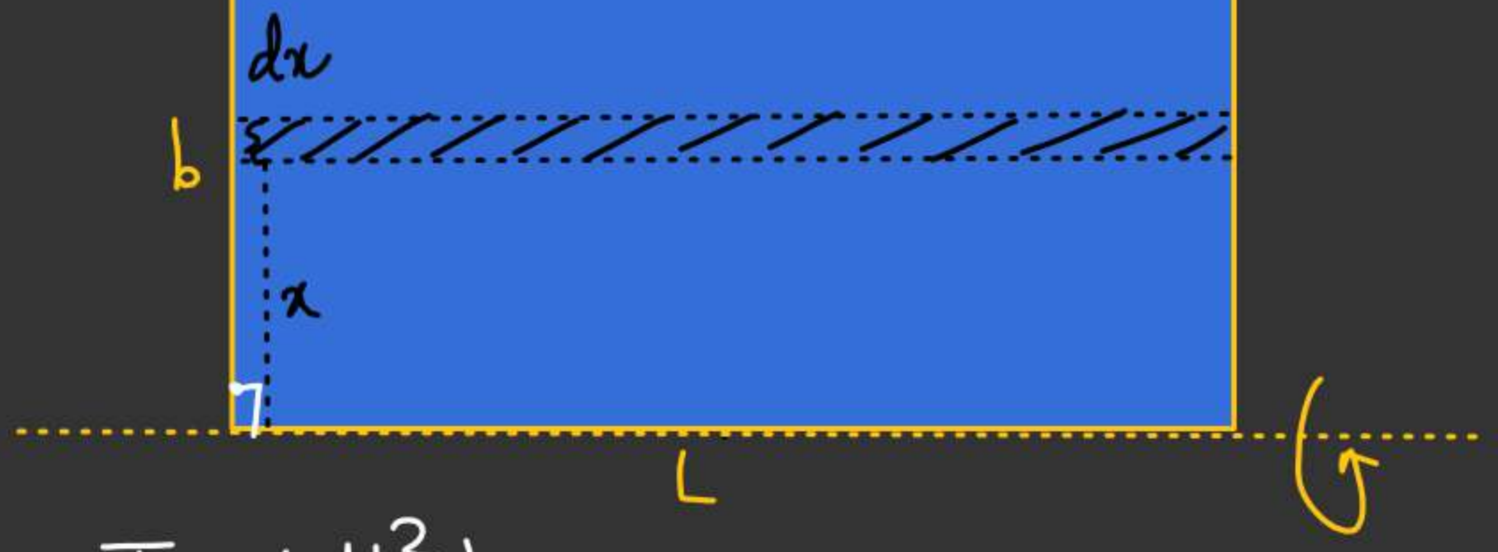
$$= \left(\frac{M}{L}dx\right)$$

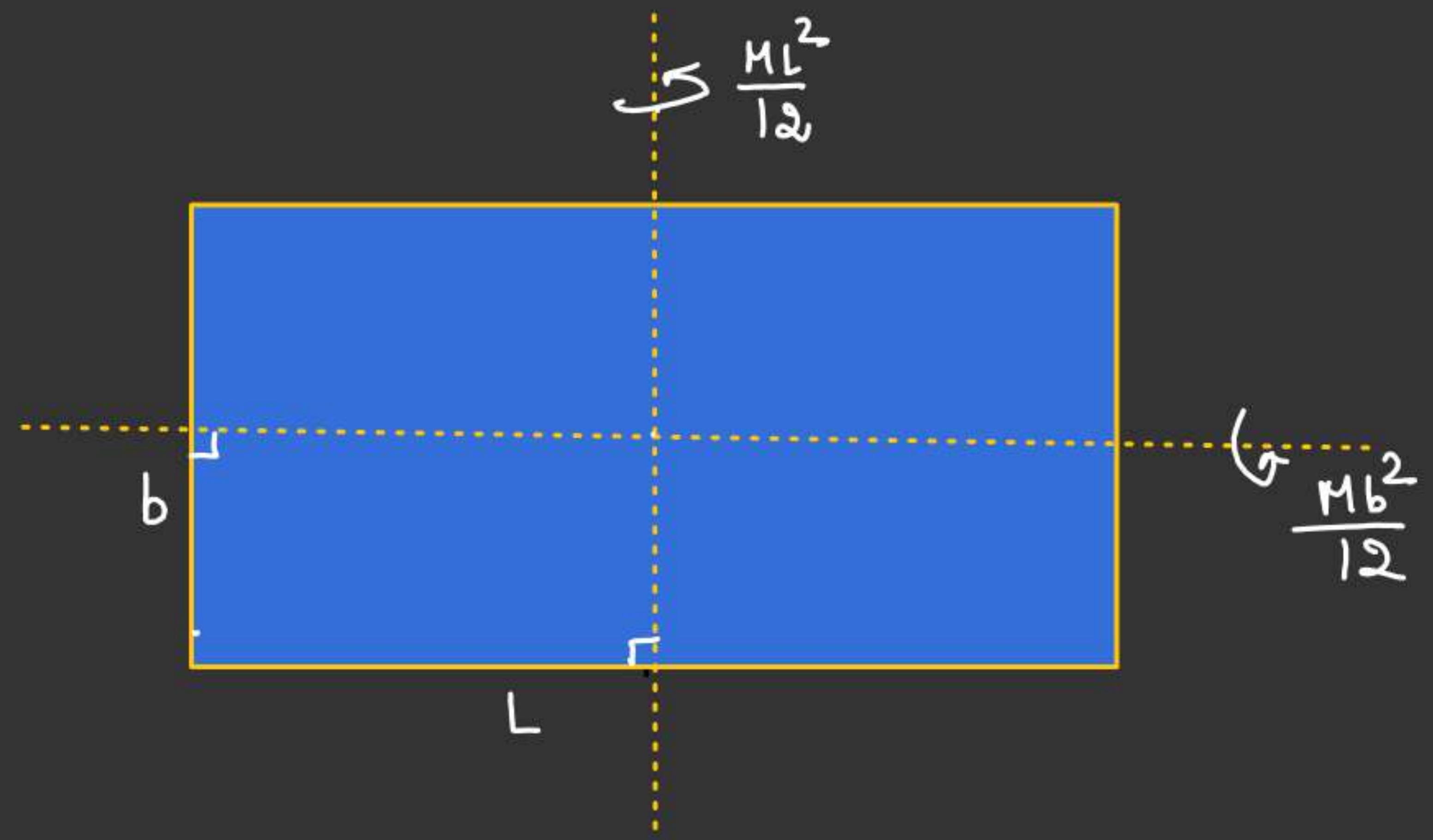
$$\int^{I} dI = \frac{M}{L}\int_0^L x^2\,dx$$

$$\boxed{I = \frac{ML^2}{3}}$$

M.I

M.I of Rectangular Lamina (2-Dimensional body)



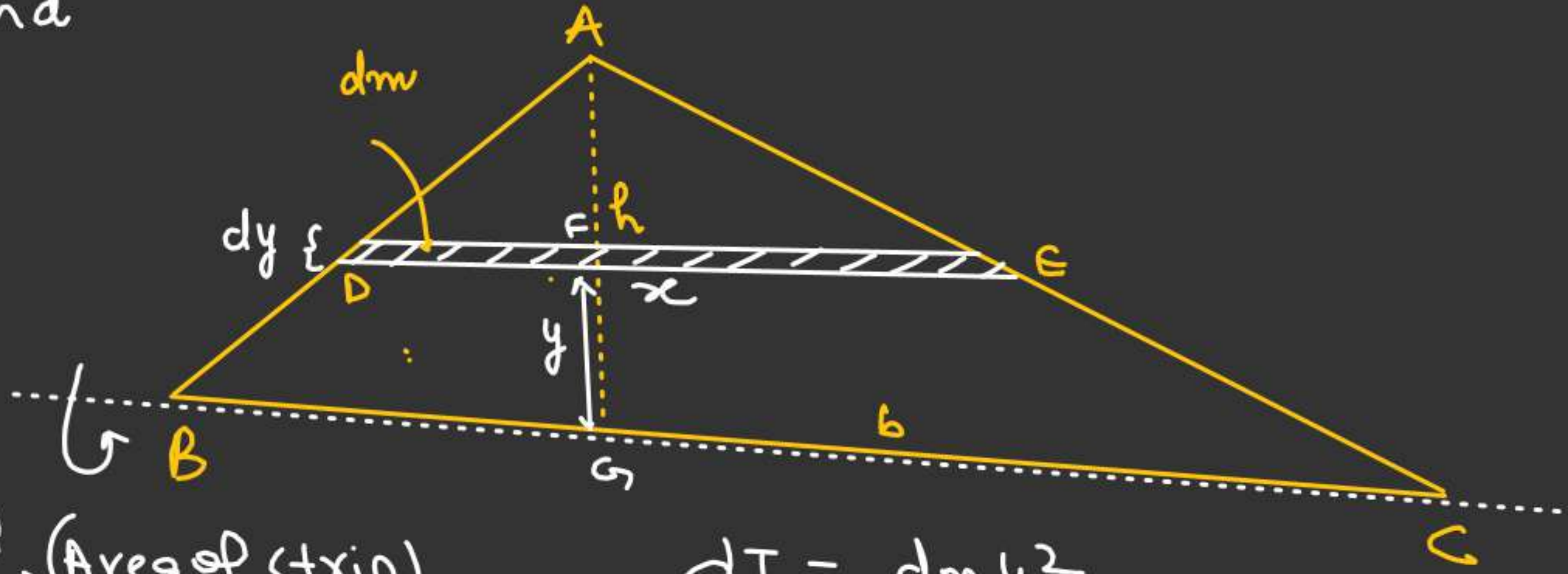


$$I = \left(\frac{Mb^2}{3}\right)$$

Nishant Jindal

M.I

M-I of a triangular lamina about any axis passing through its base



dm = Mass of strip of length x and thickness dy.

In ΔADE & ΔABC

$$\frac{DE}{BC} = \frac{AF}{AG}$$

$$\frac{x}{b} = \left(\frac{h-y}{h}\right)$$

$$x = \frac{b}{h}(h-y)$$

$$dm = \frac{M}{\frac{1}{2}bh} \times dA \text{ (Area of strip)}$$

$$= \frac{2M}{bh} \times x\, dy$$

$$= \frac{2M}{\cancel{b}h} \times \frac{\cancel{b}}{h}(h-y)dy$$

$$dm = \frac{2M}{h^2}(h-y)dy$$

$$dI = dm\, y^2$$

$$\int_0^I dI = \frac{2M}{h^2}\int_0^h (h-y)y^2\, dy$$

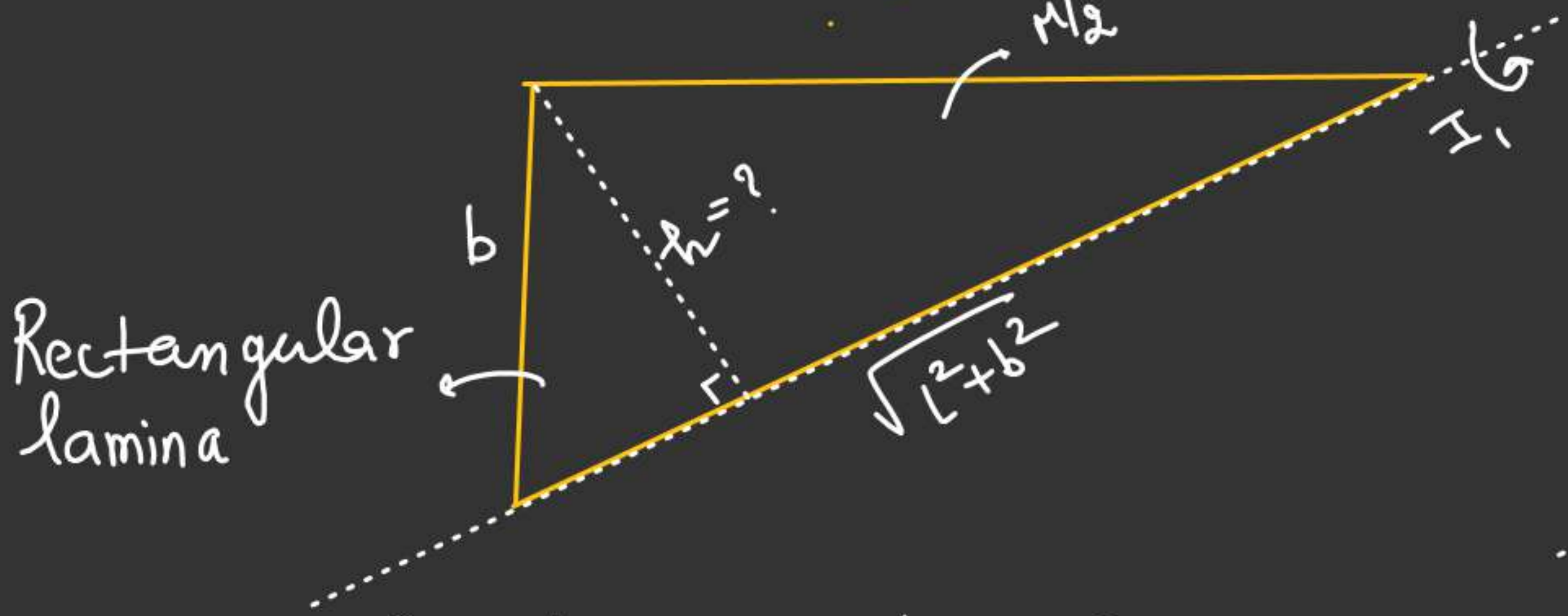
$$I = \frac{2M}{h^2}\left[h\int_0^h y^2 dy - \int_0^h y^3 dy\right]$$

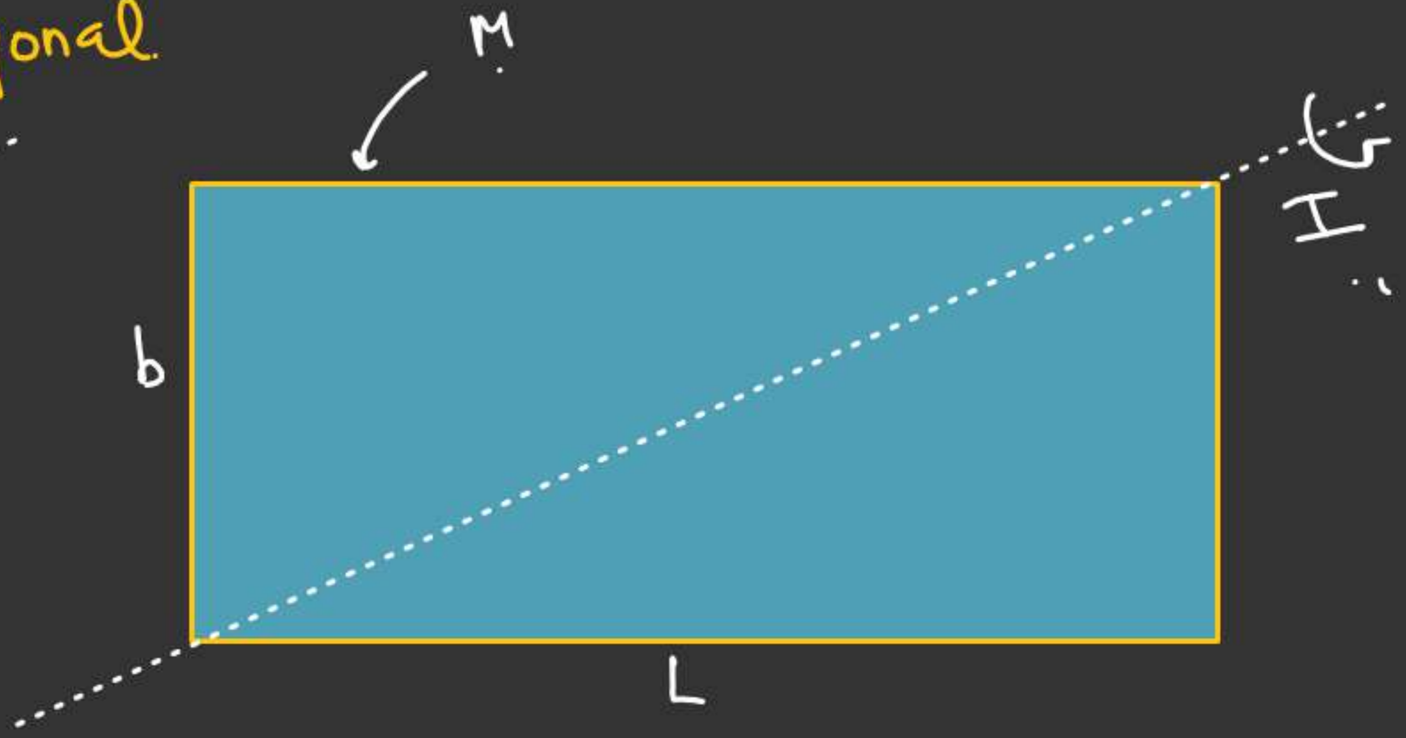
$$\boxed{I = \frac{Mh^2}{6}}$$

Nishant Jindal

M.I

M.I of Rectangular lamina about an axis passing through its diagonal.





Area of Δ Lamina $= \frac{1}{2}$(Area of Rectangular lamina)

$$\frac{1}{2} \times \sqrt{L^2+b^2} \times h = \frac{1}{2} \times Lb$$

$$h = \frac{Lb}{\sqrt{L^2+b^2}}$$

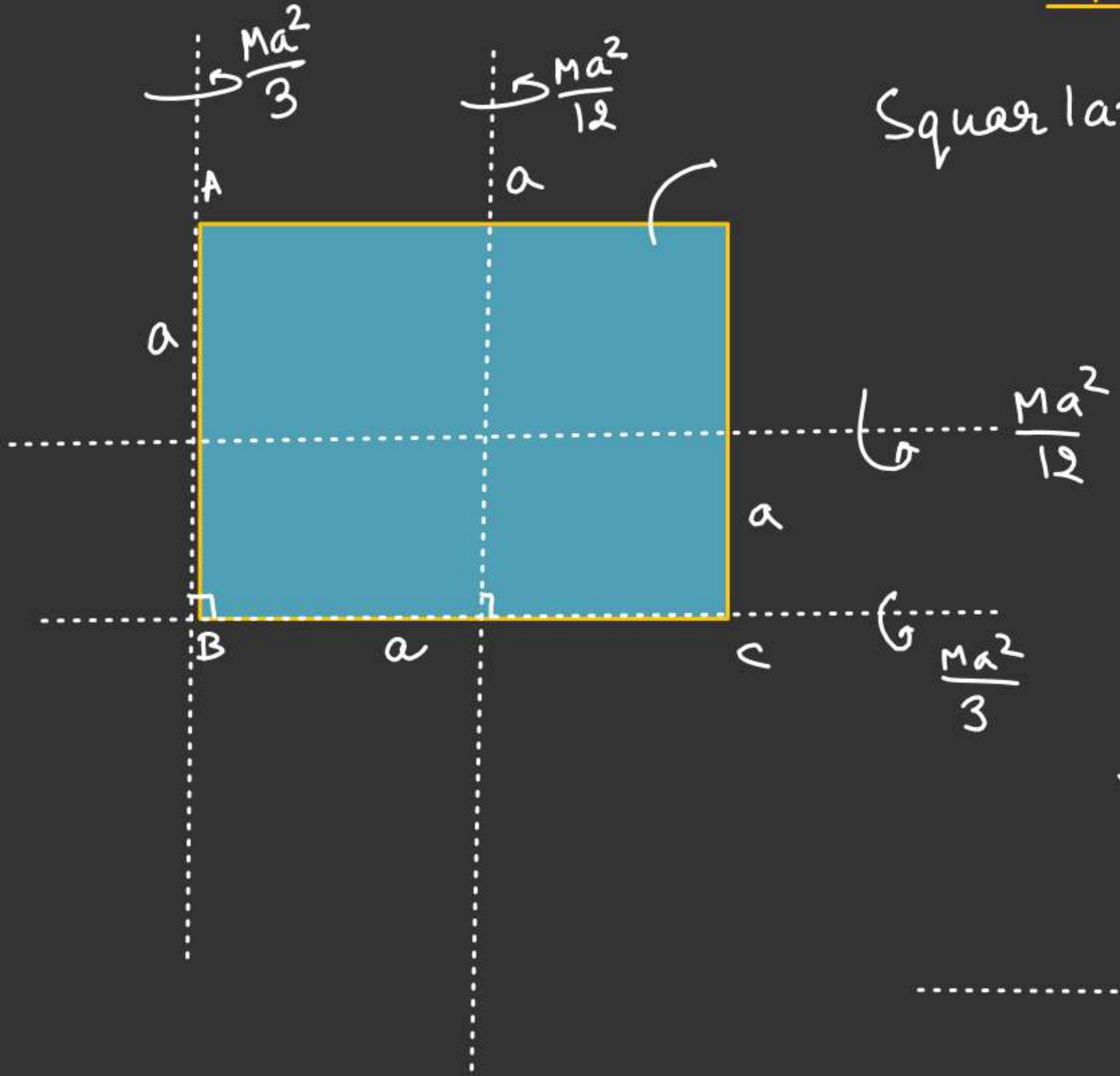
$$I_1 = \frac{M}{2} \times \left(\frac{Lb}{\sqrt{L^2+b^2}}\right)^2 \times \frac{1}{6}$$

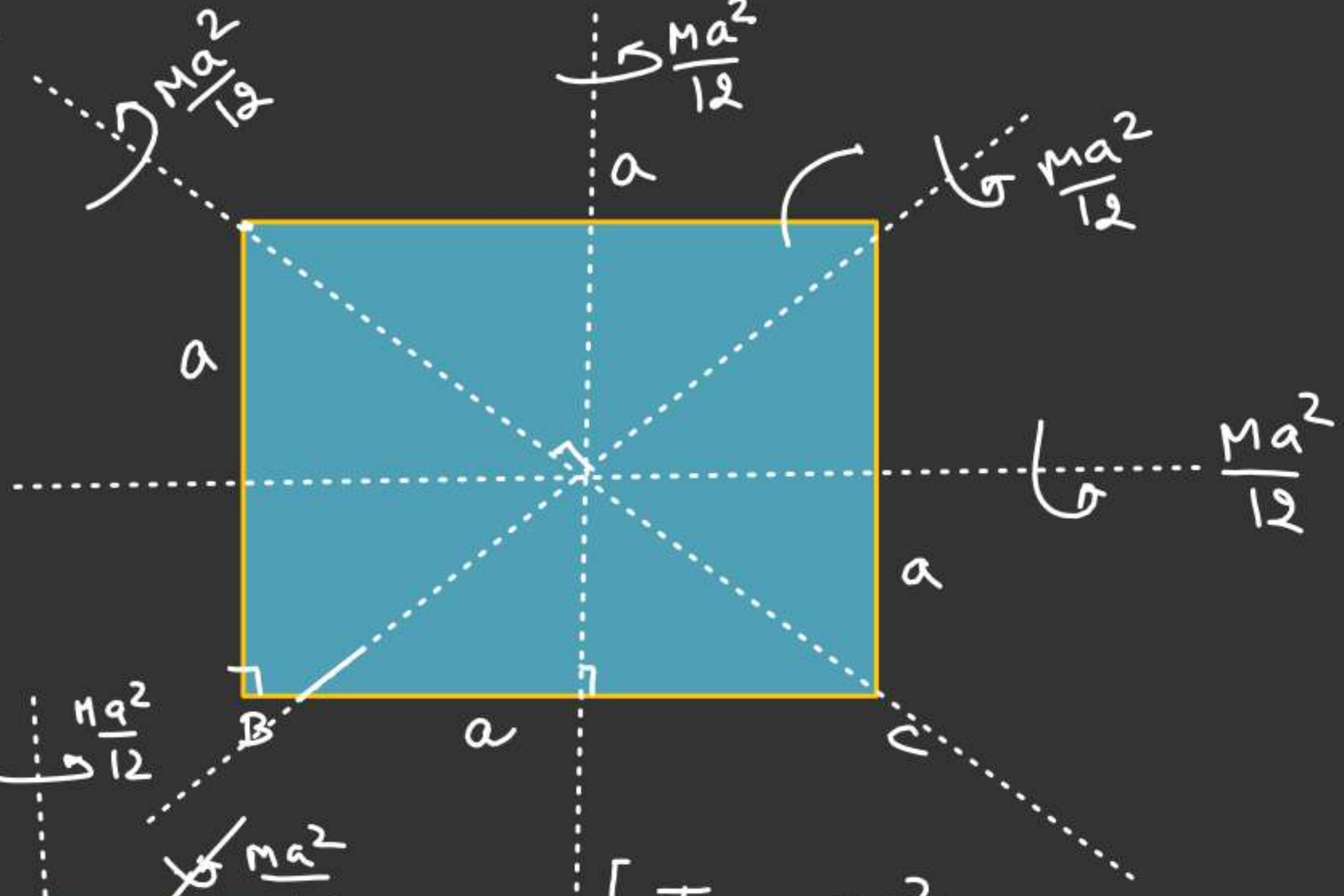
$$\boxed{I = 2I_1 = \frac{M}{6}\left[\frac{L^2b^2}{(L^2+b^2)}\right]}$$

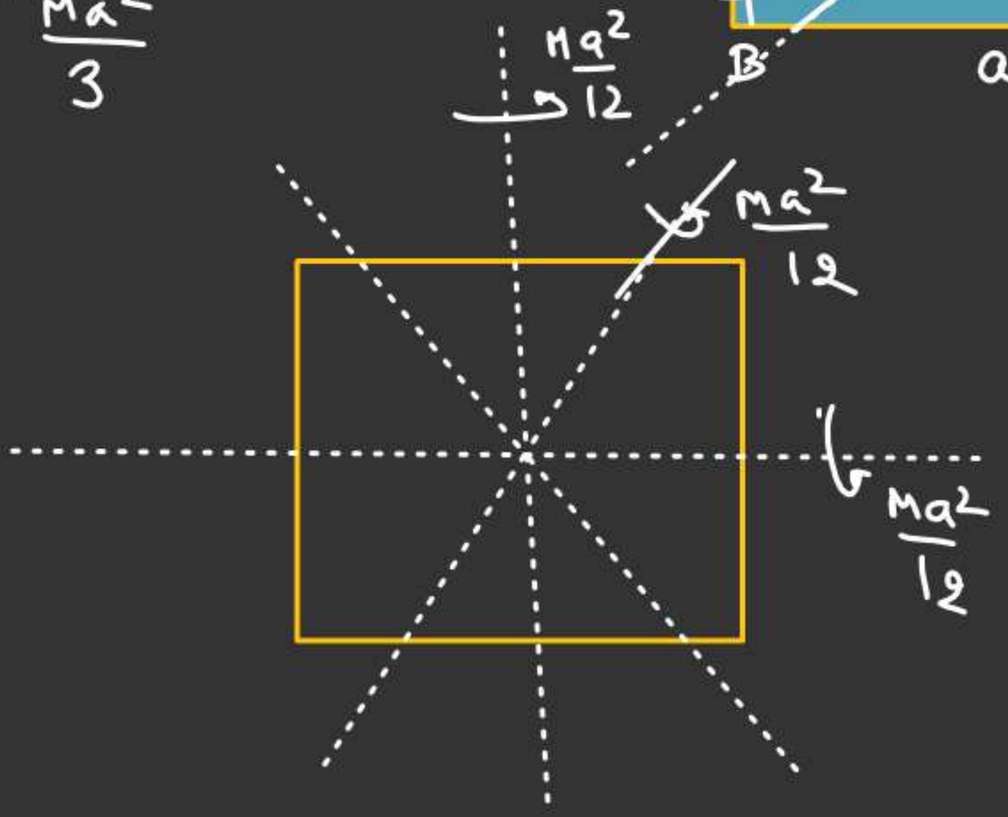
Nishant Jindal

M.I

Square lamina.





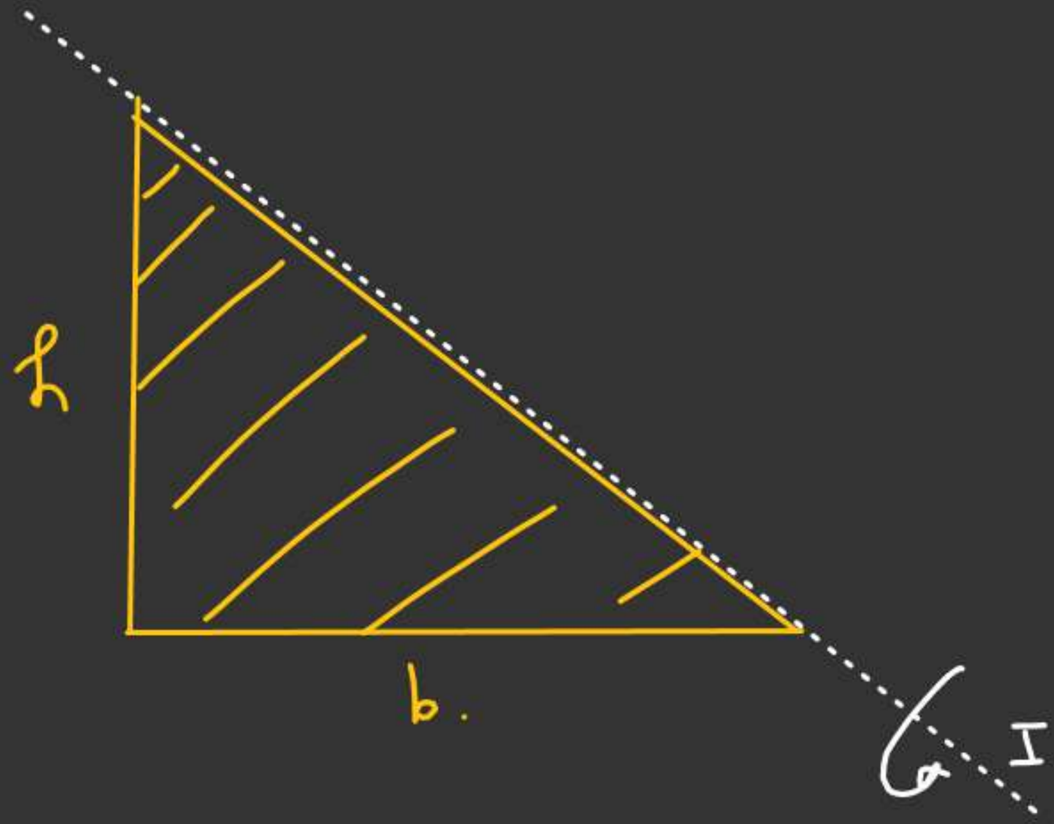


$$I = \frac{Ma^2}{12}$$

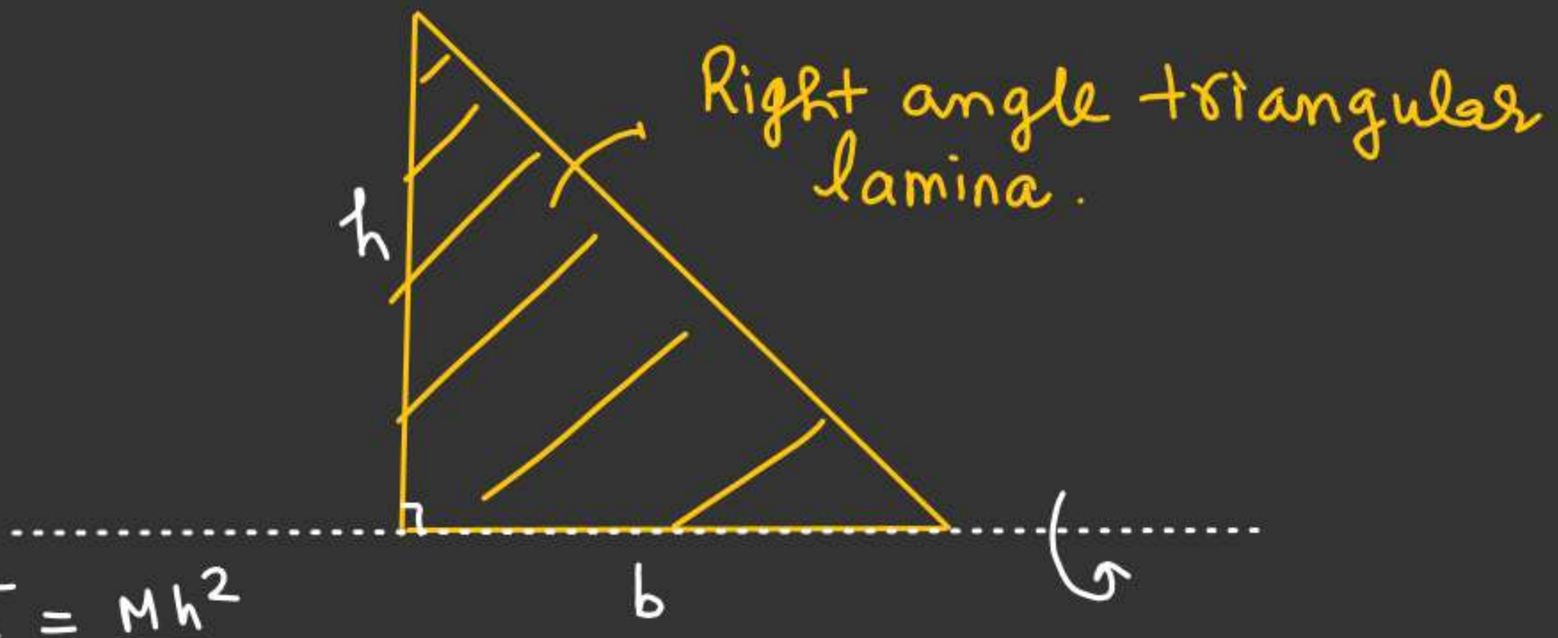
↳ About any axis in the plane of square lamina & passing through its center

Nishant Jindal

M.I



$I = ??$



$$I = \frac{Mh^2}{6}$$