

$$① \int \frac{1}{x} dx = \ln|x| + C$$

$$2) \int \frac{1}{x^2} dx = -\frac{1}{x} + C$$

$$3) \int \frac{1}{\sqrt{x}} dx = 2\sqrt{x} + C$$

$$4) \int \sqrt{x} dx = \frac{2}{3} x^{3/2}$$

$$5) \int \sin x = -\cos x$$

$$6) \int \cos x dx = \sin x$$

$$7) \int \tan x dx = \ln|\sec x|$$

$$8) \int \cot x dx = \ln|\sin x|$$

$$9) \int \sec x dx = \ln|\sec x + \tan x| = \ln\left|\tan\left(\frac{\pi}{4} + \frac{x}{2}\right)\right| + C$$

$$10) \int \csc x dx = \ln|\csc x - \cot x| = \ln\left|\tan\left(\frac{x}{2}\right)\right| + C$$

$$11) \int \sec^2 x dx = \tan x + C$$

$$12) \int \csc^2 x dx = -\cot x + C$$

$$13) \int \sec x \tan x dx = \sec x + C$$

$$14) \int \csc x \cot x dx = -\csc x + C$$

$$\int \frac{dx}{x^2 + a^2} = \frac{1}{a} \tan^{-1} \frac{x}{a}$$

$$\int \frac{dx}{\sqrt{x^2 + a^2}} = \ln$$

$$\int \sqrt{x^2 + a^2} dx = \frac{x}{2} \sqrt{\quad} + \frac{a^2}{2} \ln$$

$$\int \frac{dx}{4x^2 + 9}$$

$$\int \frac{dx}{(2x)^2 + 3^2} = \frac{1}{3 \times 2} \tan^{-1} \frac{2x}{3}$$

$$\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln \left| \frac{x-a}{x+a} \right|$$

$$\int \frac{dx}{\sqrt{x^2 - a^2}} = \ln$$

$$\int \sqrt{x^2 - a^2} dx = \frac{x}{2} \sqrt{\quad} - \frac{a^2}{2} \ln$$

$$\int \frac{dx}{\sqrt{4x^2 - 9}} = \int \frac{dx}{\sqrt{(2x)^2 - 3^2}}$$

$$= \frac{1}{2} \ln |2x + \sqrt{4x^2 - 9}|$$

$$\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \ln \left| \frac{a+x}{a-x} \right|$$

$$\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin$$

$$\int \sqrt{a^2 - x^2} dx = \frac{x}{2} \sqrt{\quad} + \frac{a^2}{2} \ln$$

$$\sin^2 A - \sin^2 B$$

$$= \sin(A+B) \cdot \sin(A-B)$$

$$\int \sin\left(\frac{9\pi}{8} + \frac{x}{4} + \frac{\pi}{8} + \frac{x}{4}\right) + \sin\left(\frac{9\pi}{8} + \frac{x}{4} - \frac{7\pi}{8} - \frac{x}{4}\right)$$

$$\frac{1}{12} \int \sin\left(2\pi + \frac{x}{2}\right)$$

$$Q_3 \quad \begin{array}{l} g'(x^2) = x^3 \quad x^2 = t \\ g'(t) = (x^2)^{3/2} = t^{3/2} \end{array}$$

$$g'(x) = x^{3/2}$$

$$g(x) = \int x^{3/2} dx = \frac{2}{5} x^{5/2} + C = \frac{2}{5} x^{5/2} + \frac{3}{5}$$

$$g(4) = \frac{2}{5} (4)^{5/2} + \frac{3}{5} = \frac{64}{5} + \frac{3}{5}$$

$$\begin{aligned} & \int \sin x \cdot \sin(x-x) dx + \frac{1}{2} \int \sin^2\left(\frac{x}{2} - x\right) dx \\ & \sin x \int \sin(x-x) dx + \frac{1}{2} \int \frac{1}{2} - \frac{\cos(2(\frac{x}{2} - x))}{2} dx \\ & = \sin x \cdot \frac{\cos(x-x)}{1} + \frac{1}{2} \left[\frac{x}{2} - \frac{1}{4} x \cos\left(\frac{x-x}{1}\right) \right] + C \end{aligned}$$

$$\begin{aligned} g(1) &= 1 \\ &= \frac{2}{5} (1)^{5/2} + C = 1 \\ C &= 1 - \frac{2}{5} = \frac{3}{5} \end{aligned}$$

$$6 \int \frac{(\sec^2(2x) - 1)}{2 \sec 2x} - \frac{\sec 8x}{\tan 4x} dx$$

$$\int \left(\frac{1 - \tan^2 2x}{2 \tan 2x} \right) - \frac{\sec 8x}{\tan 4x} dx$$

$$\int \frac{1}{\tan 4x} - \frac{\sec 8x}{\tan 4x} dx$$

$$\int \frac{1}{\tan 4x} (1 - \sec 8x) dx$$

$$\int \frac{1}{\tan 4x} \times 2 \tan^2(4x) = \int \frac{2 \tan^2 4x \times \sec 4x}{\tan 4x} dx$$

$$\int \tan 8x dx$$

7 ✓, 8 ✓, 9, 10 ✓, 11 ✓, 12 hold, 13 hold

$$(15) \int \frac{(1 + \tan 2x) - (1 + \tan 2K)}{\tan 4x}$$

$$\int \frac{(\sec x + \tan x)^2 - (\sec K + \tan K)^2}{\tan 4x}$$

$$\int \frac{(\sec x + \tan x + \sec K + \tan K)(\sec x + \tan x - \sec K + \tan K)}{\tan 4x}$$

$$Q 16 \int \frac{x^2+1}{x^6(x^2+1)} + 2 \frac{dx}{x^6(x^2+1)}$$

$$+ 2 \int \frac{x^4 - x^2 + 1}{x^6(x^2+1)} - 2 \int \frac{x^4 \cdot dx}{x^6(x^2+1)}$$

$$\int \frac{dx}{x^6} + 2 \int x^{-2} - x^{-4} + x^{-6} dx - 2 \tan^{-1} x$$

$$18) \int x^x \ln(ex) \cdot dx$$

$$\int x^x (\ln e + \ln x)$$

$$\int x^x (1 + \ln x) dx$$

$$\int dt = t + C = \underline{x^x + C}$$

$$\int \frac{x^x}{x^x} (1 + \ln x) dx = \int dt$$

def.

$$21) \frac{\sin x}{\sin(x-a)} = A + \frac{B \cdot \cos(x-a)}{\sin(x-a)}$$

$$\sin x = A \cdot \sin(x-a) + B \cos(x-a)$$

$$= \boxed{\cos a} \sin(x-a) + \boxed{\sin a} \cdot \cos(x-a)$$

$$\sin x = \sin(x-a+a)$$

$$29) \int 1 + \tan x \cdot \tan(x+\alpha) dx$$

$$\frac{1}{\tan \alpha} \int \tan(x+\alpha) - \tan x \cdot dx$$

$$30) \int \frac{a+x-(a-x)}{\sqrt{a^2-x^2}} dx$$

$$2 \int \frac{x dx}{\sqrt{a^2-x^2}}$$

$$\boxed{0} \quad \boxed{\int u^4} \quad \boxed{3}$$

$$-2 \sqrt{a^2-x^2} + C$$

$$100 \leftarrow 11,900 \rightarrow 12,000$$

$$\frac{1}{\tan \alpha} \int 1 + \tan A \cdot \tan B \rightarrow \tan(A+B)$$

$$\tan(x+(x+\alpha)) = \frac{\tan x + \tan(x+\alpha)}{1 - \tan x \cdot \tan(x+\alpha)}$$

$$1 + \tan x \cdot \tan(x+\alpha) = \frac{\tan x + \tan(x+\alpha)}{-\tan \alpha}$$

$$33) \int \frac{x dx}{(x^2)^2 + 1} \quad x^2 = t$$

$$34) \int \frac{x dx}{\sqrt{(a)^2 - (x^2)^2}} \quad x^2 = t$$

$$35) \int \frac{x^2 dx}{(x^3)^2 + 4} \quad x^3 = t$$

$$36) \int \frac{x^3 dx}{\sqrt{1 - (x^4)^2}} \quad x^4 = t$$

$$37) \int \frac{e^x}{(e^x)^2 + 2^2} \quad e^x = t$$

$$38) \int \frac{\ln x dx}{(a)^2 + (\ln x)^2} \quad \ln x = t$$

$$39) \text{ open}$$

Q1, Mod
(2)

$$40) \int \frac{1}{\sqrt{1-x^2}} + \int \frac{x dx}{\sqrt{1-x^2}} \quad 1-x^2=t$$

$$41) \int \frac{2x dx}{x^2+9} + \frac{1}{2} \int \frac{2(-2) \cdot dx}{x^2+9} \quad (8)$$

\downarrow
 $x^2+9=t$

$$+ \frac{1}{2} \int \frac{2x}{x^2+9} dx - \int \frac{dx}{x^2+3^2}$$

$$42) \text{ Rat \& Split} \quad 43) \int \frac{1-\sin x}{1-\sin^2 x} dx$$

$$12) \int \frac{e^x(1+x)}{x^2(x \cdot e^x)} dx \quad x, e^x \text{ Todiv} = t$$

$\underbrace{x \cdot e^x = t}$

$$\int \frac{dt}{t^2} \quad x \cdot e^x + e^x \cdot dx = dt$$

$$e^x(x+1) dx = dt$$

$$2(1) \int \left(x + \frac{1}{x}\right)^{\frac{1}{2}} \left(1 - \frac{1}{x^2}\right) dx$$

$x + \frac{1}{x} = t$

Rad
Q3 $\int \frac{\cos x \cdot dx}{\sin x - \cos x}$

$$Q6 \log(x + \sqrt{x^2+1}) = t$$

$$\Rightarrow \int \frac{2x}{\sqrt{1-x^2}} \Bigg| - \int \frac{\frac{1}{\sqrt{1-x^2}}}{\sqrt{1-x^2}} \quad \sin x = t$$

\downarrow
 $1-x^2=t$

$$* \quad \int \frac{e^x (\cos x - \sin x) dx}{(e^x + \sin x)^2}$$

$$\int \frac{e^{-x} (\cos x - \sin x) dx}{e^{2x} (1 + e^{-x} \sin x)^2}$$

$$\int \frac{dt}{t^2}$$

$$= -\frac{1}{t} + C$$

$$1 + e^{-x} \sin x = t$$

$$(e^{-x} \cos x - \sin x \cdot e^{-x}) dx = dt$$

$$e^{-x} (\cos x - \sin x) dx = dt$$

$$\int \frac{\sec x dx}{\sqrt{b^2 (a^2 x - a^2 \sec^2 x)}}$$

$$\int \frac{\sec x \cdot \tan x dx}{\sqrt{b^2 - a^2 \cdot \frac{\sec^2 x}{\cos^2 x}}}$$

$$\frac{\frac{1}{\cos x}}{\frac{\cos x}{\cos^2 x}}$$

$$\int \frac{\sec x \cdot \tan x dx}{\sqrt{b^2 - a^2 \sec^2 x}}$$

$$\int \frac{\sec x \cdot \tan x}{\sqrt{(b)^2 - (a \sec x)^2}}$$

$$a \sec x = t$$

$$a \sec x \tan x dx = dt$$

$$\frac{1}{a} \int \frac{dt}{\sqrt{(b)^2 - t^2}} = \frac{1}{a} \sin^{-1} \frac{t}{b} + C$$

$$Q \int \frac{1+x \cos x \cdot dx}{x(1-x^2 e^{2\sin x})}$$

$$x^2 e^{2\sin x} = 1-t$$

$$1-x^2 e^{2\sin x} = t$$

$$\Rightarrow \int \frac{dt}{(t-1) \cdot (t)}$$

$$= \frac{1}{2} \times \frac{1}{1} \ln \frac{t}{t-1} + C$$

$$= \frac{1}{2} \times \ln \frac{1-x^2 e^{2\sin x}}{-x^2 e^{2\sin x}} + C$$

$$0 - x^2 e^{2\sin x} \cdot 2 \cos x - e^{2\sin x} \cdot 2x dx = dt$$

$$-2x \cdot e^{2\sin x} (\underbrace{x \cos x + 1}) dx = dt$$

$$\frac{(x \cos x + 1) dx}{x} = \frac{dt}{-2(x^2 e^{2\sin x})}$$

$$= \frac{-dt}{2(1-t)}$$

$$Q \int (x^{3m} + x^{2m} + x^m) (2x^{2m} + 3x^m + 6)^{\frac{1}{m}} dx \quad \text{DPP4 Q}_8$$

$\xrightarrow{x^{6m}}$

$$\int (\underbrace{x^{3m-1} + x^{2m-1} + x^{m-1}}_{x^{6m}}) \cdot (\underbrace{2x^{3m} + 3x^{2m} + 6x^m}_{t})^{\frac{1}{m}} dx$$

$$\frac{1}{6m} \int t^{\frac{1}{m}} dt$$

$$\frac{1}{6m} \times \frac{t^{\frac{1}{m}+1}}{\frac{1}{m}+1} + C$$

$$2x^{3m} + 3x^{2m} + 6x^m = t$$

$$6mx^{3m-1} + 6mx^{2m-1} + 6mx^{m-1} dx = dt$$

$$x^{3m-1} + x^{2m-1} + x^{m-1} dx = \frac{dt}{6m}$$

$$\textcircled{*} \int \left(\left(\frac{x}{e} \right)^x + \left(\frac{e}{x} \right)^x \right) \ln x \cdot dx$$

$$(f^g)' = f^g \cdot \left(\frac{d}{dx} g \cdot \log f \right)$$

$$\int \left(t + \frac{1}{t} \right) \frac{dt}{t}$$

$$\int \left(1 + \frac{1}{t^2} \right) dt$$

$$t - \frac{1}{t} + C$$

$$\left(\frac{x}{e} \right)^x - \left(\frac{e}{x} \right)^x + C$$

$$\left(\frac{x}{e} \right)^x = t$$

$$\left(\frac{x}{e} \right)^x \cdot \left(\frac{d}{dx} x \cdot \log \left(\frac{x}{e} \right) \right) dx = dt$$

$$\left(\frac{x}{e} \right)^x \left\{ x \cdot \frac{1}{\left(\frac{x}{e} \right)} \cdot \frac{1}{e} + \log \frac{x}{e} \cdot x \right\} dx = dt$$

$$\left(\frac{x}{e} \right)^x \{ x + \log x - \log e \} dx = dt$$

$$\left(\frac{x}{e} \right)^x \cdot \ln x \cdot dx = dt$$

$$\ln x \cdot dx = \frac{dt}{\left(\frac{x}{e} \right)^x} = \frac{dt}{t}$$

$$\star \text{ Q } \int \frac{\ln 5x + \ln 4x}{1 - 2\ln 3x} \cdot dx \quad \Rightarrow - \int 2 \ln \frac{3x}{2} \cdot \ln \frac{x}{2} \cdot dx \quad \text{Prod} = \text{Sum.}$$

$$\int \frac{2 \ln\left(\frac{3x}{2}\right) \ln\left(\frac{x}{2}\right) \sin 3x}{1 - 2\ln 3x} \cdot dx$$

$$\Rightarrow - \int \ln(2x) + \ln(x) \, dx$$

$$\Rightarrow - \frac{\sin 2x}{2} - \sin x + C$$

$$\int \frac{2 \ln\left(\frac{3x}{2}\right) \cdot \ln \frac{x}{2} \cdot \sin 3x}{\sin 3x - \sin 6x}$$

$$\int \frac{x \ln\left(\frac{3x}{2}\right) \cdot \ln \frac{x}{2} \cdot \sin 3x \cdot dx}{2 \ln\left(\frac{3x}{2}\right) \cdot \sin\left(-\frac{3x}{2}\right)}$$

$$= \int \frac{\ln \frac{x}{2} \times 2 \sin\left(\frac{3x}{2}\right) \cdot \ln\left(\frac{3x}{2}\right) dx}{\sin\left(\frac{3x}{2}\right)}$$

$(x^2 + \frac{1}{x^2})$ type Qs

$$1) (x + \frac{1}{x})' = 1 - \frac{1}{x^2}$$

$$2) (x - \frac{1}{x})' = 1 + \frac{1}{x^2}$$

$$3) (x + \frac{1}{x})^2 = x^2 + \frac{1}{x^2} + 2$$

$$4) (x - \frac{1}{x})^2 = x^2 + \frac{1}{x^2} - 2$$

$$Q \int \frac{x^2+1 \cdot dx}{x\sqrt{x^4+1}} \quad \text{---} \quad x^2 \text{ com.}$$

$$\int \frac{x^2+1 \cdot dx}{x^2 \sqrt{x^2 + \frac{1}{x^2}}} \quad \text{ye Kiska diff hai??}$$

$$\int \frac{(1 + \frac{1}{x^2}) dx}{\sqrt{(x^2 + \frac{1}{x^2} - 2) + 2}} \quad (x - \frac{1}{x})'$$

$$\int \frac{(1 + \frac{1}{x^2}) dx}{\sqrt{(x - \frac{1}{x})^2 + (\sqrt{2})^2}} \quad \begin{matrix} x - \frac{1}{x} = t \\ (1 + \frac{1}{x^2}) dx = dt \end{matrix}$$

$$\int \frac{dt}{\sqrt{t^2 + 1^2}} = \ln | \quad | + C$$

$$Q \int \frac{e^{\ln(1+1/x^2)} dx}{(x^2 + 1/x^2)}$$

$$\int \frac{(1+1/x^2) dx}{(x^2 + 1/x^2 - 2) + 2} \quad \text{Kiska de } (x - 1/x)'$$

$$\int \frac{1+1/x^2 dx}{(x - 1/x)^2 + (\sqrt{2})^2} \quad x - \frac{1}{x} = t \quad (1+1/x^2) dx = dt$$

$$\int \frac{dt}{t^2 + (\sqrt{2})^2} = \frac{1}{\sqrt{2}} \tan^{-1} \frac{t}{\sqrt{2}} + C$$

$$Q. \int \frac{(ax^2 - b) dx}{x \sqrt{c^2 x^2 - (ax^2 + b)^2}}$$

$\underbrace{c^2 x^2 - (ax^2 + b)^2}_{x^2(\text{com.})}$

$$\int \frac{ax^2 - b \cdot dx}{x^2 \sqrt{c^2 - (ax + \frac{b}{x})^2}}$$

$$\int \frac{(a - \frac{b}{x^2}) dx}{\sqrt{c^2 - (ax + \frac{b}{x})^2}} \quad ax + \frac{b}{x} = t \quad (a - \frac{b}{x^2}) dx = dt$$

$$\int \frac{dt}{\sqrt{c^2 - t^2}} = \sin^{-1} \frac{t}{c} + C$$

$$Q \int \frac{(x^2-1)dx}{(x^4+3x^2+1) \tan^{-1}\left(\frac{x^2+1}{x}\right)}$$

$x^2 \text{ term} \rightarrow$ $\frac{(1-\frac{1}{x^2})dx}{(x^2+\frac{1}{x^2}+3) \tan^{-1}\left(\frac{1+\frac{1}{x^2}}{x+\frac{1}{x}}\right)}$ $\xrightarrow{\text{Kishkaduff}}$

$$\int \frac{(1-\frac{1}{x^2})dx}{((x^2+\frac{1}{x^2}+2)+1) \tan^{-1}\left(\frac{1+\frac{1}{x^2}}{x+\frac{1}{x}}\right)}$$

$$\int \frac{(1-\frac{1}{x^2})dx}{((x+\frac{1}{x})^2+1) \tan^{-1}\left(1+\frac{1}{x^2}\right)}$$

$$\int \frac{dt}{t} = \ln|\tan^{-1}(x+\frac{1}{x})| + C$$

$$\tan^{-1}\left(x+\frac{1}{x}\right) = t$$

$$\frac{1}{1+(x+\frac{1}{x})^2} \times (1-\frac{1}{x^2})dx = dt$$