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Triple Product:

$(\vec{a}\cdot\vec{b})\vec{c}$ ✓

$(\vec{a}\cdot\vec{b})\cdot\vec{c}$ ✗

$(\vec{a}\cdot\vec{b})\times\vec{c}$ ✗

$(\vec{a}\times\vec{b})\vec{c}$ ✗

$(\vec{a}\times\vec{b})\cdot\vec{c}$ ✓ $\longrightarrow$ Scalar Triple Product

$(\vec{a}\times\vec{b})\times\vec{c}$ ✓

$\downarrow$

vector Triple Product.

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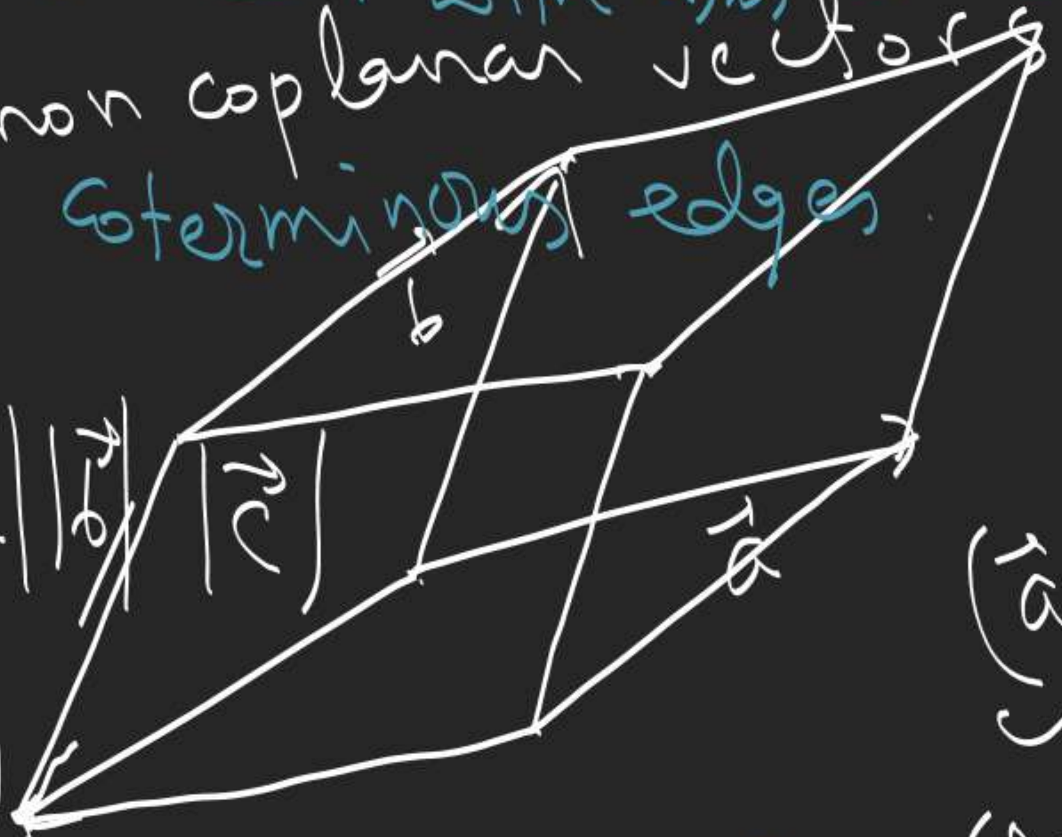
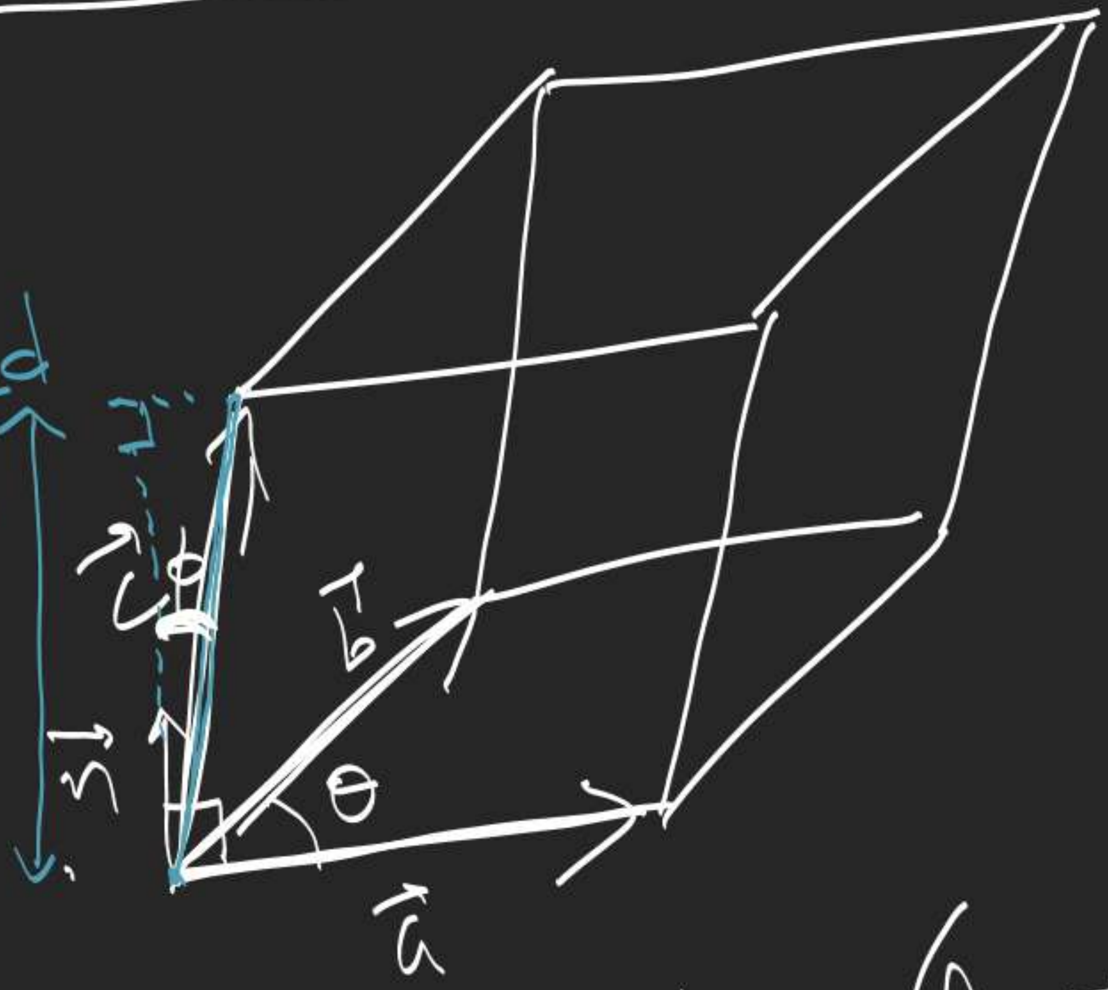
Scalar Triple Product (Box Product)

$[\vec{a}\ \vec{b}\ \vec{c}] = (\vec{a} \times \vec{b}) \cdot \vec{c} =$

$|[\vec{a}\ \vec{b}\ \vec{c}]|$ = Volume of ||piped with $\vec{a}, \vec{b}, \vec{c}$ as coterminous edges

Let $\vec{a}, \vec{b}, \vec{c}$ non coplanar vectors





$[\vec{a}\ \vec{b}\ \vec{c}] \le |\vec{a}||\vec{b}||\vec{c}|$

$[\vec{a}\ \vec{b}\ \vec{c}] = |\vec{a}||\vec{b}||\vec{c}|$ ↓ cuboid

$(\vec{a} \times \vec{b}) \cdot \vec{c} = |\vec{a}||\vec{b}|\sin\theta\,(\hat{n} \cdot \vec{c})$

(Area of ||gm)$(\hat{n} \cdot \vec{c})$ $= |\vec{a}||\vec{b}|\sin\theta\,|\vec{c}|\cos\phi$

(Base Area)(Height) = (Area of ||gm)(Proj. of $\vec{c}$ on $\hat{n}$)

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Note→ $\vec{a}, \vec{b}, \vec{c}$ are linearly dependent (coplanar)

iff $[\vec{a}\ \vec{b}\ \vec{c}] = 0$.

$$\vec{c} = x\vec{a} + y\vec{b}$$

$$\vec{c} + (-x)\vec{a} + (-y)\vec{b} = \vec{0}$$

$$\underbrace{(\vec{a} \times \vec{b})}_{\perp\text{ar to } \vec{c}} \cdot \vec{c} = 0$$

$$\phi = \frac{\pi}{2}$$

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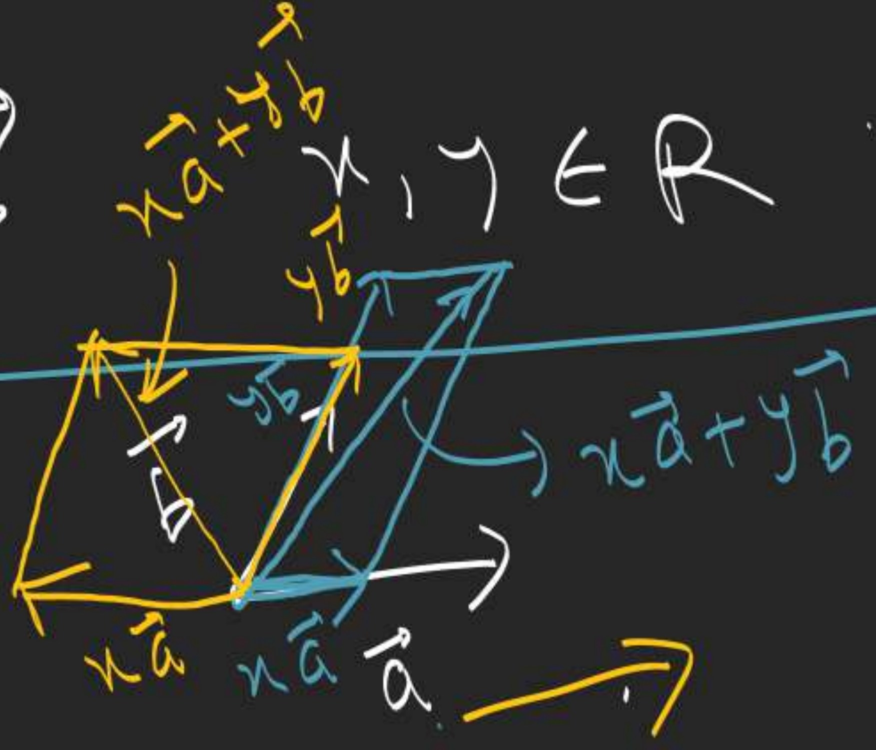
Theorem for Plane

If $\vec{a}, \vec{b}$ are non collinear vectors, then any vector $\vec{c}$ coplanar with $\vec{a}, \vec{b}$ can be expressed as their linear combination.

$\hat{i}, \hat{j}$

$$\vec{c} = x\vec{a} + y\vec{b} \quad x, y \in R$$



$\vec{v} = x\hat{i} + y\hat{j}$

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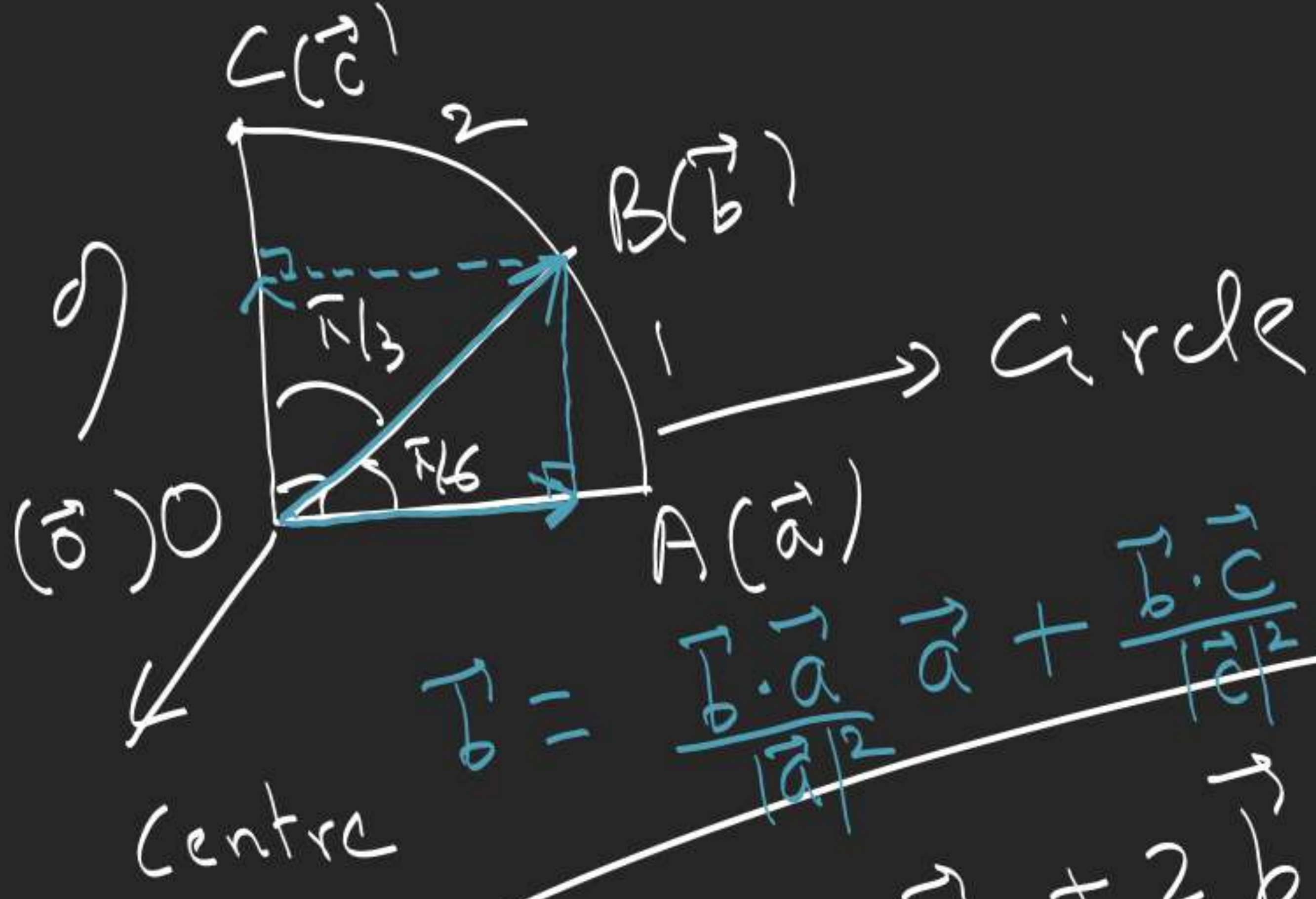
1.

Express $\vec{c}$ in terms of $\vec{a}$, $\vec{b}$.



$\vec{b} = \frac{\vec{b}\cdot\vec{a}}{|\vec{a}|^2}\vec{a} + \frac{\vec{b}\cdot\vec{c}}{|\vec{c}|^2}\vec{c}$ $\quad \vec{c} = \frac{\sqrt{3}}{2}\vec{a} + \frac{1}{2}\vec{c}$

$\vec{c} = x\vec{a} + y\vec{b}$

$\vec{c}\cdot\vec{a} = x\vec{a}\cdot\vec{a} + y\vec{b}\cdot\vec{a}$

$0 = x + y\frac{\sqrt{3}}{2}$ — (1)

$\vec{c}\cdot\vec{c} = x\,\vec{a}\cdot\vec{c} + y\,\vec{b}\cdot\vec{c}$

$1 = 0 + y\left(\frac{1}{2}\right)$

$y = 2$, $x = -\sqrt{3}$

$\boxed{\vec{c} = -\sqrt{3}\,\vec{a} + 2\vec{b}}$

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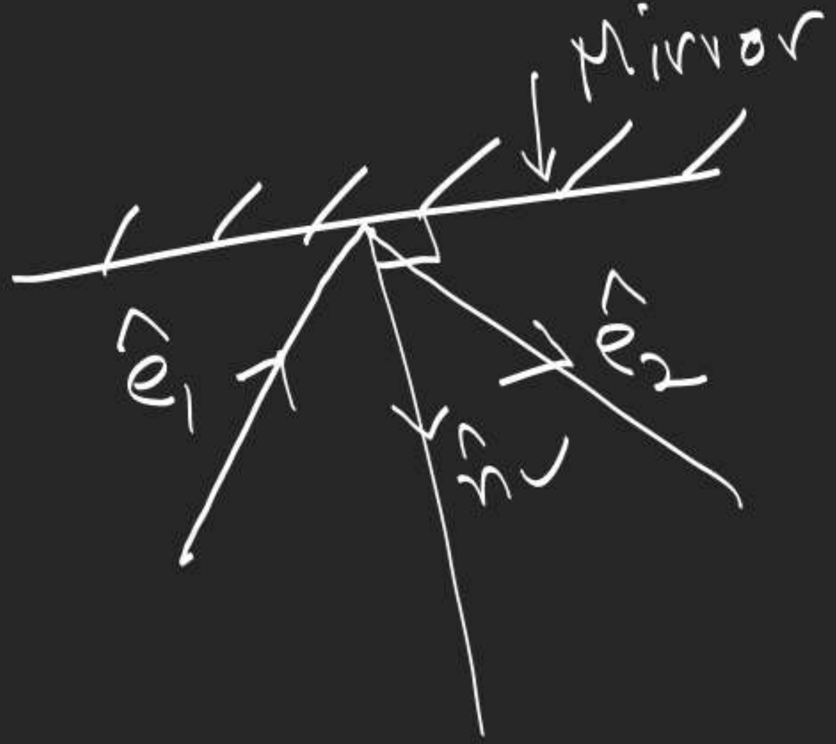

Express $\hat{e}_2$ in terms of $\hat{e}_1$ & $\hat{n}$.

$$\hat{e}_2 - \hat{e}_1 = 2\left(\frac{-\hat{e}_1 \cdot \hat{n}}{|\hat{n}|^2}\,\hat{n}\right)$$

$$\hat{e}_2 = \hat{e}_1 - 2(\hat{e}_1 \cdot \hat{n})\hat{n}.$$

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3 non coplanar vectors are always linearly independent

$\vec{a}, \vec{b}, \vec{c}$ non coplanar

$$x\vec{a} + y\vec{b} + z\vec{c} = \vec{0}$$

Let $z \neq 0$

$$\vec{c} = -\frac{x}{z}\vec{a} - \frac{y}{z}\vec{b}$$

Contradiction

$z = 0$

$\hat{i}, \hat{j}, \hat{k}$

$$x_1\hat{i} + y_1\hat{j} + z_1\hat{k} = x_2\hat{i} + y_2\hat{j} + z_2\hat{k}$$

$$(x_1 - x_2)\hat{i} + (y_1 - y_2)\hat{j} + (z_1 - z_2)\hat{k} = \vec{0}$$

$x_1 - x_2 = 0 \quad y_1 - y_2 = 0 \quad z_1 - z_2 = 0$

$$(\hat{i} \times \hat{j}) \cdot \hat{k} = \hat{k} \cdot \hat{k} = 1$$

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• $[\vec{a}\ \vec{b}\ \vec{c}]$

$\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

$\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

$\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$

$$(\vec{a}\times\vec{b})\cdot\vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_3 & b_3 \end{vmatrix} \cdot (c_1\hat{i} + c_2\hat{j} + c_3\hat{k})$$

$$(C_{11}\hat{i} + C_{12}\hat{j} + C_{13}\hat{k}) \cdot (c_1\hat{i} + c_2\hat{j} + c_3\hat{k})$$

$$= c_1 C_{11} + c_2 C_{12} + c_3 C_{13}$$

$$= \begin{vmatrix} c_1 & c_2 & c_3 \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

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- $[\vec{a}\ \vec{b}\ \vec{c}] = 0$

any two are collinear

$\vec{a}, \lambda\vec{a}, \vec{c}$