Nishant Jindal

$L_1 : a_1x + b_1y + c_1 = 0$

$L_2 : a_2x + b_2y + c_2 = 0$

B_1, B_2 Bisector PSBL

A) $\dfrac{a_1x + b_1y + c_1}{\sqrt{a_1^2 + b_1^2}} = \pm \dfrac{(a_2x + b_2y + c_2)}{\sqrt{a_2^2 + b_2^2}}$

2 Lines $\rightarrow B_1$ & B_2

B) acute Angle Bisector / Obtuse A.B

c_1, c_2 +ve

Check $a_1a_2 + b_2b_1 = +ve$ ⊖ वाली Eqn Acute

$a_1a_2 + b_1b_2 = -ve$ ⊕ ,,

New (C) Eqn Containing Pt. (α, β)

$L_1(\alpha, \beta) \rightarrow a_1\alpha + b_1\beta + c_1$

$L_2(\alpha, \beta) \rightarrow a_2\alpha + b_2\beta + c_2$

If $a_1\alpha + b_1\beta + c_1$ & $a_2\alpha + b_2\beta + c_2$ Same Sign.

⊕ वाली Normal Eqn Carry (α, β)

If $a_1\alpha + b_1\beta + c_1$ & $a_2\alpha + b_2\beta + c_2$ Opp sign.

⊖ वाली Normal Eqn Carries (α, β)

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Q1 If Lines $4x+3y-6=0$ & $5x+12y+9=0$ then find EOL.

① Bisector of obtuse Angle.

② Bisector of Acute Angle.

$L_1: 4x+3y-6=0$

$L_2: 5x+12y+9=0$

Bisector's Eq

$$\frac{4x+3y-6}{5} = \pm \frac{5x+12y+9}{13}$$

① C_1, C_2 +ve

$$-4x-3y+\boxed{6}=0 \quad \text{+ve}$$

$$5x+12y+\boxed{9}$$

$a_1a_2+b_1b_2<0$

$\left.-4\times5+-3\times12<0\right\}$ -ve आया है

$\therefore$ ⊕ Sign will give Acute AB

A cute AB

$$\frac{-4x-3y+6}{5} = +\frac{(5x+12y+9)}{13}$$

$$-52x-39y+78=25x+60y+45$$

$$-77x-99y+33=0$$

$7x+9y-3=0 \rightarrow$ Acute Angle Bisector.

OB Line A.B.

$$\frac{-4x-3y+6}{5} = -\frac{5x+12y+9}{13}$$

This Method is normally Not in Use.

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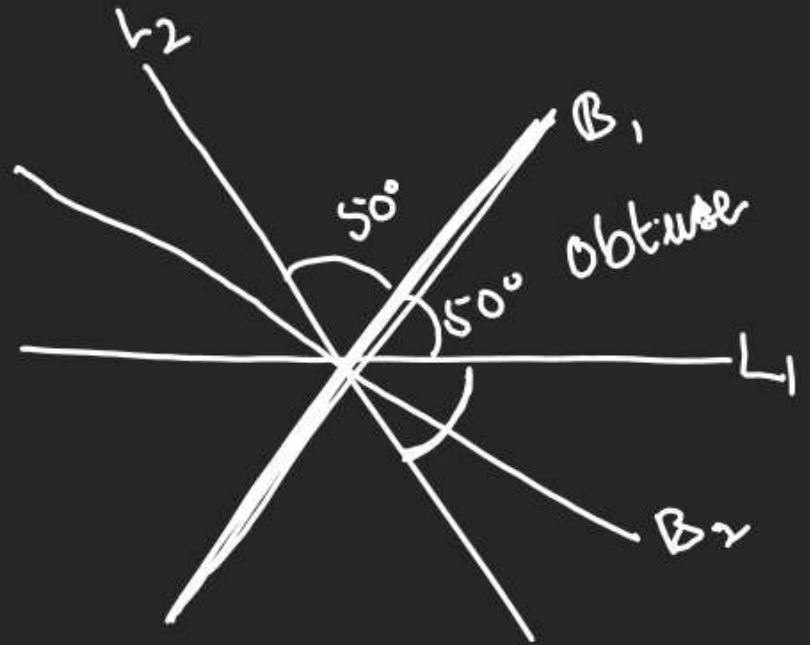
Method 2

$L_1 : 4x+3y-6=0 \rightarrow m_1 = -\frac{4}{3}$

$L_2 : 5x+12y+9=0 \rightarrow$



$$\frac{4x+3y-6}{5} = \pm \frac{5x+12y+9}{13}$$

(+) $\frac{4x+3y-6}{5} = \frac{5x+12y+9}{13}$

$52x+39y-78 = 25x+60y+45$

$27x-21y-123=0$

$9x-7y-41=0$

$m_2 = \frac{9}{7}$

(−) $\frac{4x+3y-6}{5} = -\frac{5x+12y+9}{13}$

$52x+39y-78 = -25x-60y-45$

$77x+99y-33=0$

$7x+9y-3=0$

$\therefore$ $9x-7y-41=0$ is obtuse Angle Bi

$7x+9y-3=0$ is acute A.B.

$$\tan\theta = \left|\frac{-\frac{4}{3}-\frac{9}{7}}{1+\left(-\frac{4}{3}\right)\times\frac{9}{7}}\right| = \left|\frac{-28-27}{21-36}\right| = \left|\frac{55}{15}\right| > 1$$

L_1, Bisector and Bi ch angle $> 45°$

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Q $L_1: 4x+3y-6=0$

$L_2: 5x+12y+9=0$

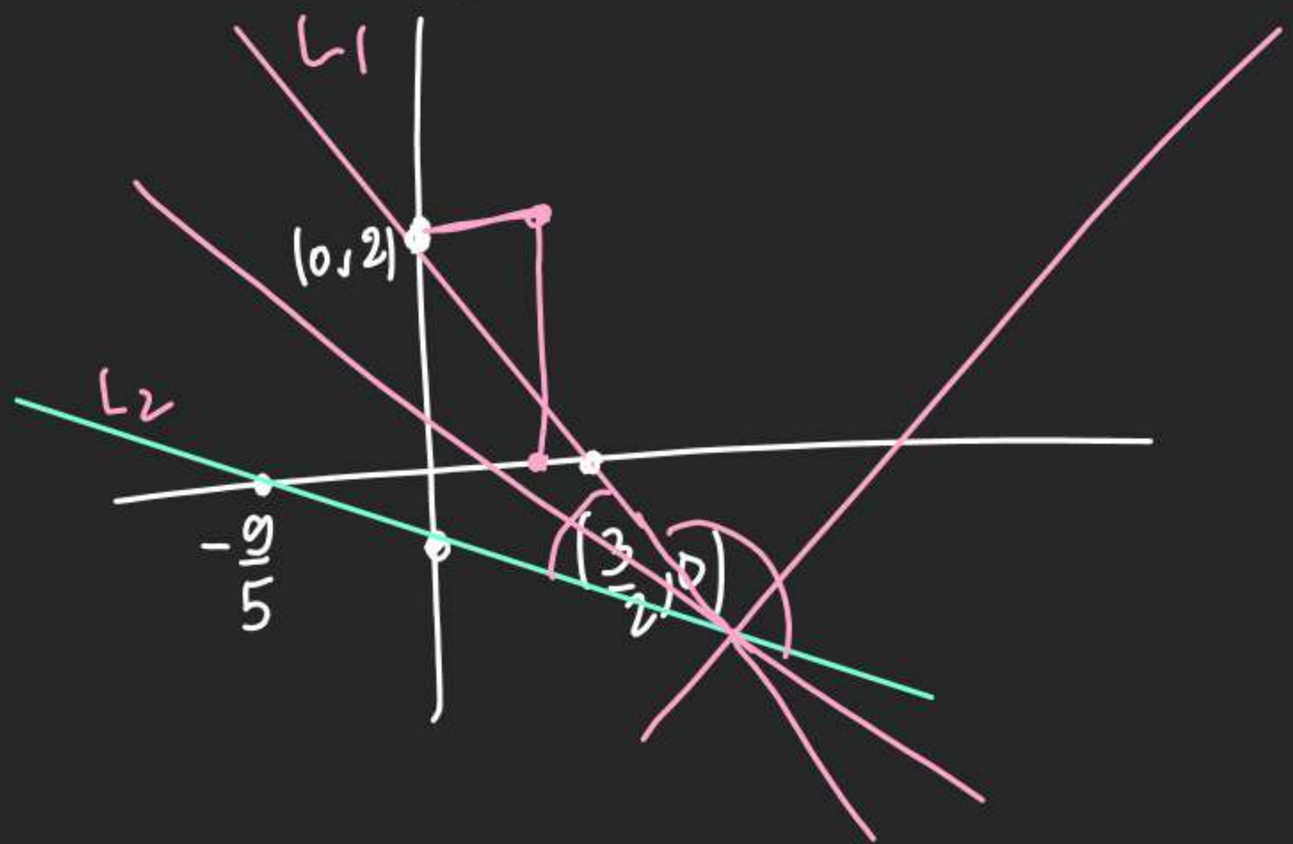
find Bisector's Eqn containing $(1,2)$?

$4x+3y=6$

$\frac{x}{3/2}+\frac{y}{2}=1$

$5x+12y=-9$

$\frac{x}{-9/5}+\frac{y}{-9/12}=1$



$L_1(1,2) = 4+6-6 > 0$

$L_2(1,2) = 5+24+9 > 0$

$$\frac{4x+3y-6}{\sqrt{4^2+3^2}} = + \frac{5x+12y+9}{\sqrt{5^2+12^2}}$$

$\Rightarrow 9x-7y-41=0$

$\Rightarrow$ obtuse angle Bisector.

$(1,2)$ will lie.

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Q_3 $L_1: x+y-1=0 \rightarrow m_1 = -1$

$L_2: x-y+3=0$ $\quad \frac{x}{-3}+\frac{y}{3}=0$

$\tan\theta = \left|\frac{-1-0}{1+-1\times 0}\right| = \left|\frac{-1}{1}\right|$

$\tan\theta = 1$

$\theta = 45^\circ$

find Angle Bisector.

Containing $(1,2)$

$L_1(1,2) = 1+2-1 > 0$
$L_2(1,2) = 1-2+3 > 0$ } Same Sign

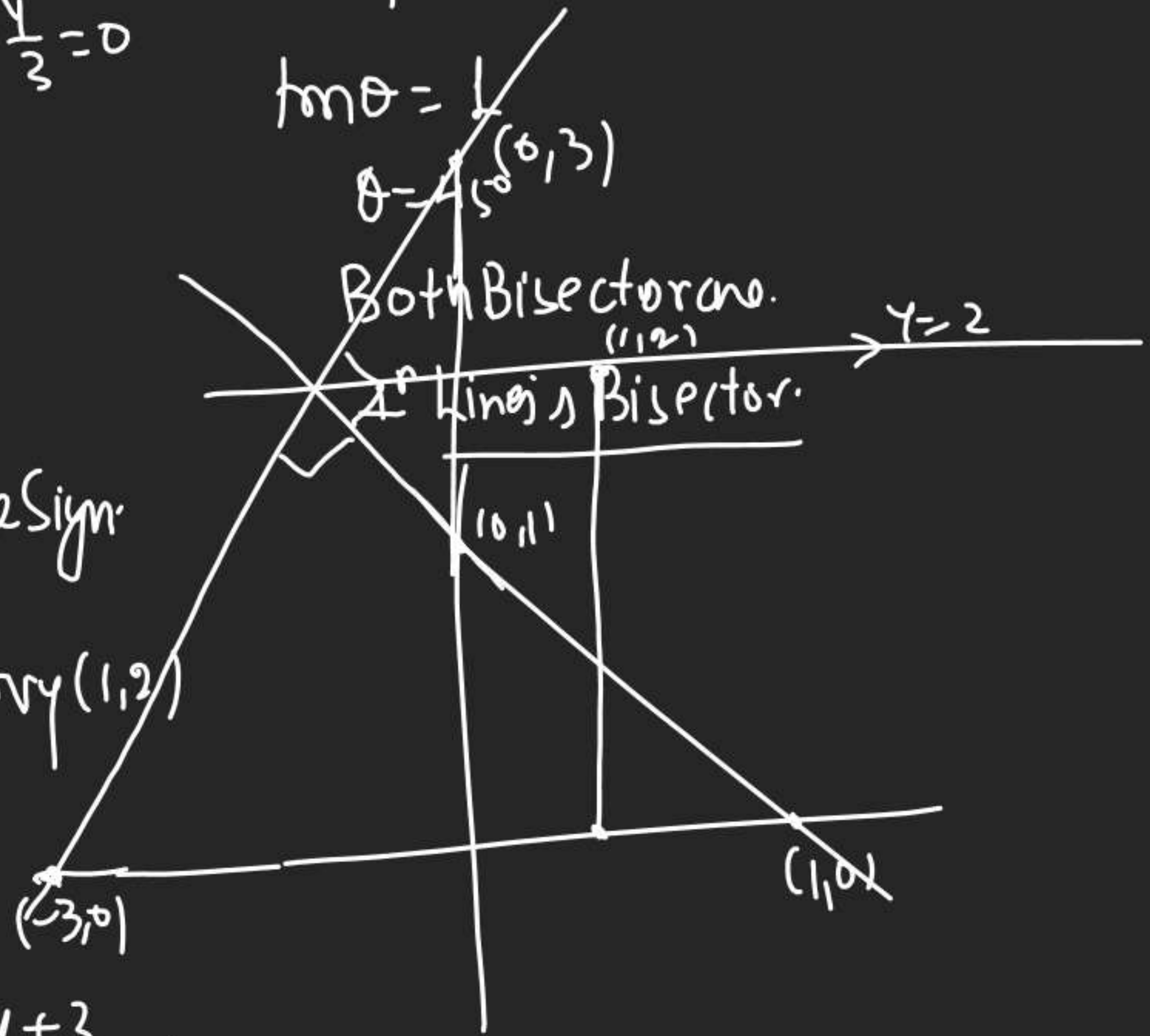
$\oplus$ ale AB will carry $(1,2)$

$$\frac{x+y-1}{\sqrt{2}} = + \frac{x-y+3}{\cancel{\sqrt{2}}}$$

$\cancel{x}+y-1 = \cancel{x}-y+3$

$2y = 4$

$y = 2$



$m_2 = 0$

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Q $L_1: x+2y+2=0 \rightarrow m_1=-\frac{1}{2}$

$L_2: 2x+y-3=0$

find Bisector Contaning Origin?

$L_1(0,0) = 2 > 0$

$L_2(0,0) = -3 < 0$ Different Sign

$\ominus$

$$\frac{x+2y+2}{\sqrt{5}} = -\frac{2x+y-3}{\cancel{\sqrt{5}}}$$

$x+2y+2 = -2x-y+3$

$3x+3y = +1$

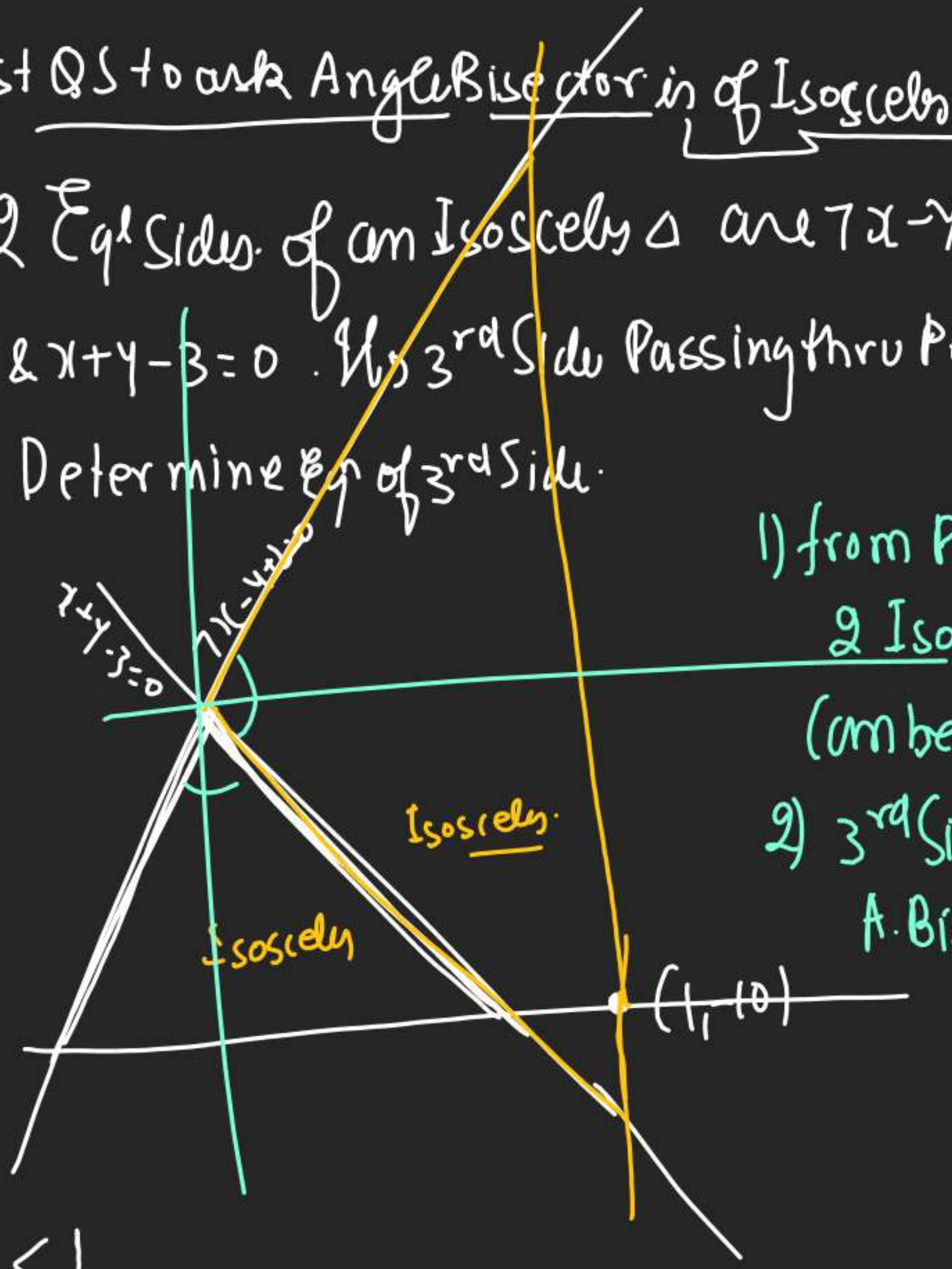
$m_2 = -1$

$$\tan\theta = \left|\frac{-\frac{1}{2}+1}{1+\left(+\frac{1}{2}\right)(+1)}\right| = \left|\frac{\frac{1}{2}}{\frac{3}{2}}\right| = \frac{1}{3} < 1$$

Acute A.B

Best QS to ask Angle Bisector in of Isosceles Δ.

Q5 2 Eq^l Sides of an Isosceles Δ are $7x-y+3=0$ & $x+y-3=0$. Its 3rd Side Passing thru Pt. $(1,-10)$ Determine Eq^n of 3rd Side.



1) from Pt. $(1,-10)$ 2 Isosceles Δ can be made.

2) 3rd Side will be $\perp^{r}$ to A. Bisectors.

Nishant Jindal

Best QS to ask Angle Bisector in of Isosceles △.

Q5 2 Eq^l Sides of an Isosceles △ are $7x-y+3=0$ & $x+y-3=0$. It's 3^rd Side Passing thru Pt. $(1,-10)$ Determine Eq^n of 3^rd Side.

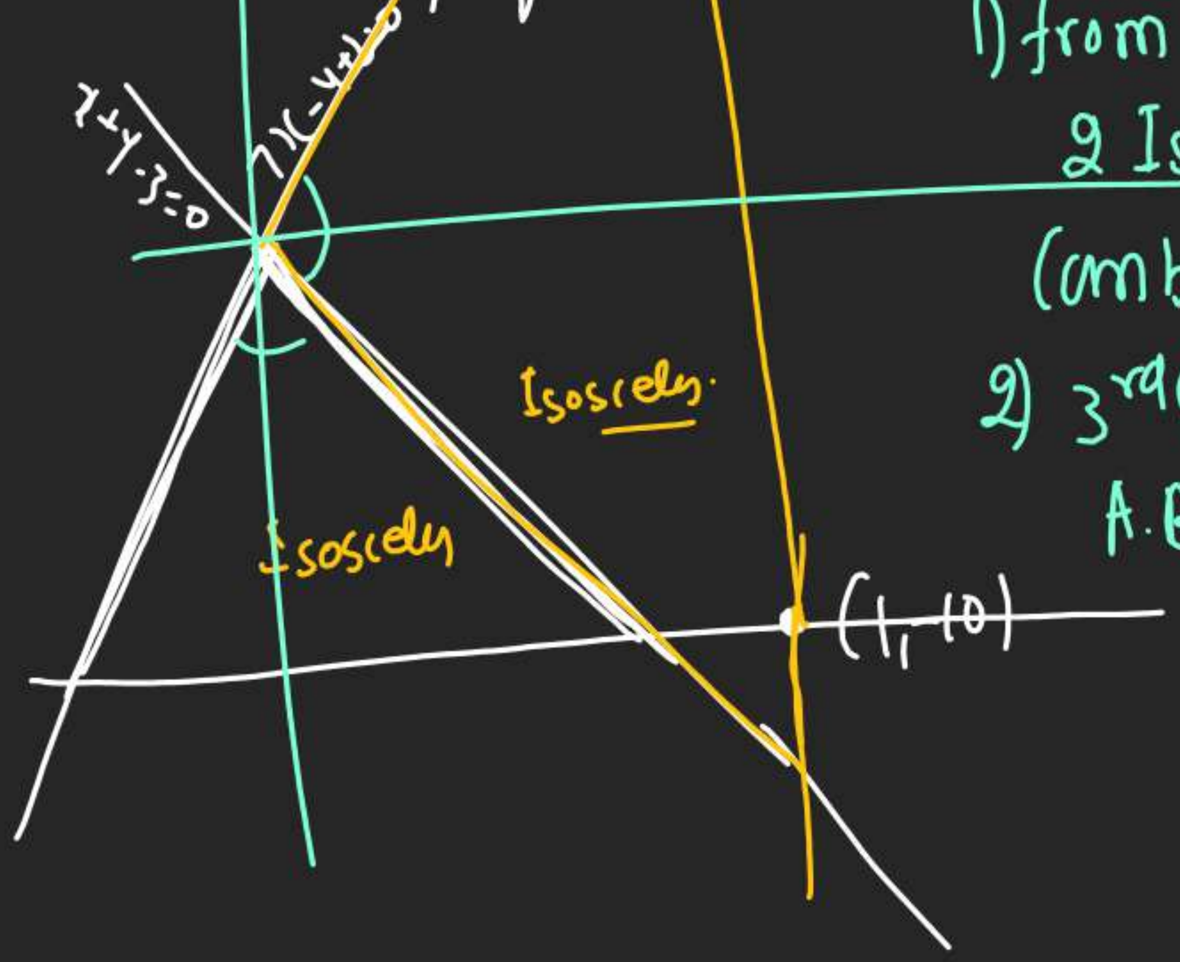
1) $\frac{7x-y+3}{5\sqrt{2}} = \pm \frac{x+y-3}{\sqrt{2}}$

⊕	⊖
$7x-y+3=5x+5y-15$	$7x-y+3=-5x-5y+15$
$2x-6y=-18$	$12x+4y=12$
$x-3y+9=0$	$3x+y-4=0$

2) 3^rd Side will be

$x-3y+k=0$	$3x+y+k=0 \quad (1,-10)$
$1+30+k=0$	$3-10+k=0$
$k=-31$	$k=7$

∴ Line $\boxed{x-3y=31}$ $\boxed{3x+y+7=0}$



1) from Pt. (1, -10) 2 Isosceles △ can be made.

2) 3^rd Side will be ⊥^r to A. Bisectors.

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Determinant

Solving Determinant of 3rd Order

Q Solve.

$$\Delta = \begin{vmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ -1 & 2 & 1 \end{vmatrix}$$

$$\Delta = \left|\begin{array}{ccc|cc} 1 & 2 & 3 & 1 & 2 \\ 0 & 1 & 0 & 0 & 1 \\ -1 & 2 & 1 & -1 & 2 \end{array}\right|$$

$$\{1+0+0\} - \{-3+0+0\}$$

$$= 4$$

$$\Delta = \begin{vmatrix} 1 & 1 & -1 \\ 2 & 1 & 1 \\ 0 & 3 & 4 \end{vmatrix} = ?$$

$$= \left|\begin{array}{ccc|cc} 1 & 1 & -1 & 1 & 1 \\ 2 & 1 & 1 & 2 & 1 \\ 0 & 3 & 4 & 0 & 3 \end{array}\right|$$

$$\{4+0+-6\} - \{0+3+8\}$$

$$= -2-11 = -13.$$

M2

$$\begin{vmatrix} 1 & 1 & -1 \\ 2 & 1 & 1 \\ 0 & 3 & 4 \end{vmatrix}$$

$$1\begin{vmatrix} 1 & 1 \\ 3 & 4 \end{vmatrix} - 1\begin{vmatrix} 2 & 1 \\ 0 & 4 \end{vmatrix} + (-1)\begin{vmatrix} 2 & 1 \\ 0 & 3 \end{vmatrix}$$

$$1(4-3) - 1(8-0) + -1(6-0)$$

$$1-8-6 = -13$$

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Concurrency of 3 Lines.

1) When 3 Lines P.T. same pt. then these are called Concurrent Lines.

(2) $L_1: a_1x+b_1y+c_1=0$
$L_2: a_2x+b_2y+c_2=0$ are Concurrent
$L_3: a_3x+b_3y+c_3=0$

When
$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}=0$$

(3) If L_3 is P.T. the Point of Intersection of L_1 & L_2 then Lines are concurrent.

(4) Area of Δ made by L_1, L_2, L_3 $=0$

(5) If $L_1=0$, $L_2=0$, $L_3=0$ are such that $L_1+L_2+L_3=0$ then Lines are concurrent (opposite is not true)

Q Find λ if
$3x-4y-13=0$
$8x-11y-33=0$
$2x-3y+\lambda=0$ are Concurrent?

$$\Delta=\begin{vmatrix} 3 & -4 & -13 \\ 8 & -11 & -33 \\ 2 & -3 & \lambda \end{vmatrix}=0$$

$$\left|\begin{array}{ccc|cc} 3 & -4 & -13 & 3 & -4 \\ 8 & -11 & -33 & 8 & -11 \\ 2 & -3 & \lambda & 2 & -3 \end{array}\right|$$

$-34+15$

$=\{-33\lambda+264+3(7)\}-\{286+297-32\lambda\}=0$

$-\lambda+7=0 \Rightarrow \lambda=7$

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Q If $\lambda \in R, \theta \in R$.

$L_1: \lambda x + (\sin\alpha)y + \cos\alpha = 0$

$L_2: x + (\cos\alpha)y + \sin\alpha = 0$

$L_3: -x + (\sin\alpha)y - \cos\alpha = 0$

are concurrent find $\lambda = ?$

$$\begin{vmatrix} \lambda & \sin\alpha & \cos\alpha \\ 1 & \cos\alpha & \sin\alpha \\ -1 & \sin\alpha & -\cos\alpha \end{vmatrix} = 0$$

$$\Rightarrow \lambda \begin{vmatrix} \cos\alpha & \sin\alpha \\ \sin\alpha & -\cos\alpha \end{vmatrix} - \sin\alpha \begin{vmatrix} 1 & \sin\alpha \\ -1 & -\cos\alpha \end{vmatrix} + \cos\alpha \cdot \begin{vmatrix} 1 & \cos\alpha \\ -1 & \sin\alpha \end{vmatrix}$$

$$\Rightarrow \lambda(-\cos^2\alpha - \sin^2\alpha) - \sin\alpha(-\cos\alpha + \sin\alpha) + \cos\alpha(\sin\alpha + \cos\alpha)$$

$$-\lambda + \sin\alpha\cdot\cos\alpha - \sin^2\alpha + \sin\alpha\cos\alpha + \cos^2\alpha = 0$$

$\lambda = 2\sin\alpha\cos\alpha + \cos^2\alpha - \sin^2\alpha$.

$\lambda = \sin 2\alpha + \cos 2\alpha$

Qs $\to$ Range of λ

$-\sqrt{1^2+1^2} \le \sin 2\alpha + \cos 2\alpha \le \sqrt{1^2+1^2}$

$-\sqrt{2} \le \lambda \le \sqrt{2}$

$\lambda \in [-\sqrt{2}, \sqrt{2}]$ Ans

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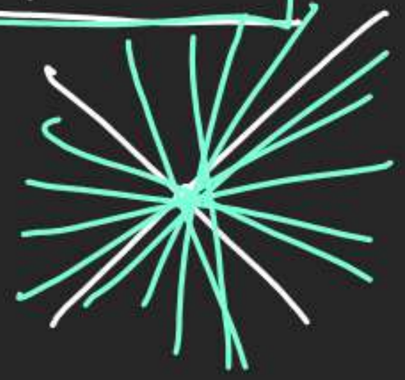
Family of Lines.

1) all Lines P.T. one pt are Family of Lines

2) all Lines P.T. Point of Intersection of L_1 & L_2 are Family of Lines for L_1 & L_2 & denoted by

$$\boxed{L_1 + \lambda L_2 = 0}$$



L_1 $x - y + 2 = 0$

L_2: $x + 2y + 1 = 0$

then Family of Lines

$(x - y + 2) + \lambda(x + 2y + 1) = 0$

Q Find Lines Passing thru origin & PoI of $x - y + 2 = 0$ & $x + 2y + 1 = 0$

$(x - y + 2) + \lambda(x + 2y + 1) = 0$ Satisfy by $(0, 0)$

$2 + \lambda(1) = 0$

$\lambda = -2$

$(x - y + 2) - 2(x + 2y + 1) = 0$

$-x - 5y = 0$

$x + 5y = 0$