

1. At equilibrium, $F_{\text{net}} = 0$

\therefore Conservation of momentum,

$$mv = m_1v_1 + m_2v_2$$

$$mv = \frac{m}{2}v_1 + \frac{m}{2}v_2 \Rightarrow v = \frac{v_1 + v_2}{2} \dots (i)$$

By conservation of energy,

$$\frac{1}{2}mv^2 = \frac{1}{2}m\frac{v_1^2}{2} + \frac{1}{2}m\frac{v_2^2}{2} \text{ or } v^2 = \frac{v_1^2 + v_2^2}{2} \dots (ii)$$

By (i) and (ii)

$$2v^2 = v_1^2 + (2v - v_1)^2$$

$$2v^2 = v_1^2 + 4v^2 + v_1^2 - 4vv_1 \text{ or } 2v_1^2 - 4vv_1 + 2v^2 = 0$$

Solving, we get $v = v_1 = v_2 \dots (iii)$

According to the question, Initial amplitude = A, final amplitude = fA

From (iii), $v = v_1$ or $A\omega = fA\omega_f$

$$A\sqrt{\frac{k}{m}} = fA\sqrt{\frac{2k}{m}} \text{ or } f = \frac{1}{\sqrt{2}}.$$

2. According to the question

Potential energy = Kinetic energy

$$\frac{1}{2}m\omega^2x^2 = \frac{1}{2}m\omega^2(A^2 - x^2) \text{ or, } 2x^2 = A^2; x = \frac{A}{\sqrt{2}}$$

3. For SHM, speed, $v = \omega\sqrt{A^2 - x^2}$

Acceleration, $a = -\omega^2x$

As $|v| = |a|$

$$\omega\sqrt{A^2 - x^2} = \omega^2x$$

$$A^2 - x^2 = \omega^2x^2 \Rightarrow \omega^2 = \frac{A^2 - x^2}{x^2}$$

$$= \frac{5^2 - 4^2}{4^2} = \left(\frac{3}{4}\right)^2; \omega = \frac{3}{4} \Rightarrow T = \frac{2\pi}{\omega} = \frac{8\pi}{3}$$

4. Restoring force due to pressing the bottle with small amount x,

$$F = -(\rho Ax)g$$

$$a = -\left(\frac{\rho Ag}{m}\right)x \therefore \omega^2 = \frac{\rho Ag}{m} = \frac{\rho(\pi r^2)g}{m}$$

$$\omega = \sqrt{\frac{10^3 \times \pi \times (2.5 \times 10^{-2})^2 \times 10}{310 \times 10^{-3}}} \approx 7.95 \text{ rad/s}$$

5. Maximum K.E. of the particle $= \frac{1}{2} m(A^2 \omega^2)$

P.E. of the particle at any time $t = \frac{1}{2} m \omega^2 x^2$

By energy conservation

$$\Rightarrow \frac{KE}{PE} = \frac{KE_{\max}}{PE} - 1 \Rightarrow \frac{KE}{PE} = \frac{\frac{1}{2} m A^2 \omega^2}{\frac{1}{2} m \omega^2 x^2} - 1$$

$$= \frac{A^2}{A^2 \sin^2 \frac{\pi}{90} \times 210} - 1 = \frac{1}{\left[\sin \left(2\pi + \frac{\pi}{3}\right)\right]^2} - 1 = \frac{1}{3}$$

6. Here, $x = a \sin \omega t$

$$y = a \sin 2\omega t$$

$$y = 2 a \sin \omega t \cos \omega t$$

$$y = 2x \sqrt{1 - \frac{x^2}{a^2}} \text{ or } y = \frac{2}{a} x \sqrt{(a-x)(a+x)}$$

$$y = 0, \text{ at } x = 0, \pm a$$

7. $x_1 = A \sin (\omega t + \phi_1)$

$$x_2 = A \sin (\omega t + \phi_2)$$

$$x_1 - x_2 = A \left[2 \sin \left[\omega t + \frac{\phi_1 + \phi_2}{2} \right] \sin \left[\frac{\phi_1 - \phi_2}{2} \right] \right]$$

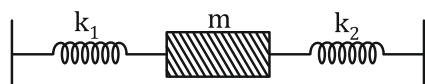
$$A = 2 A \sin \left(\frac{\phi_1 - \phi_2}{2} \right)$$

$$\frac{\phi_1 - \phi_2}{2} = \frac{\pi}{6}$$

$$\phi_1 = \frac{\pi}{3}$$

8. In given arrangement two springs are connected in parallel. So, the effective spring constant will be

$$k_{\text{eff}} = k_1 + k_2$$



Frequency of oscillation,

$$f = \frac{1}{2\pi} \sqrt{\frac{k_{\text{eff}}}{m}} = \frac{1}{2\pi} \sqrt{\frac{k_1 + k_2}{m}} \dots (i)$$

As k_1 and k_2 are increased four times

$$\text{New frequency, } f' = \frac{1}{2\pi} \sqrt{\frac{4(k_1 + k_2)}{m}} = 2f \quad (\text{using (i)})$$

9. For a particle to perform simple harmonic motion its displacement at any time t is given by

$$x(t) = a(\cos \omega t + \phi)$$

a = amplitude, ω = angular frequency, ϕ = phase constant.

Let us choose $\phi = 0 \therefore x(t) = a \cos \omega t$

$$\text{Velocity of a particle } v = \frac{dx}{dt} = -a\omega \sin \omega t$$

K.E. of a particle is

$$K = \frac{1}{2}mv^2 = \frac{1}{2}ma^2\omega^2 \sin^2 \omega t$$

$$\text{Average K. E. } \langle K \rangle = \langle \frac{1}{2}ma^2\omega^2 \sin^2 \omega t \rangle$$

$$= \frac{1}{2}m\omega^2 a^2 \langle \sin^2 \omega t \rangle$$

$$= \frac{1}{2}m\omega^2 a^2 \left(\frac{1}{2} \right) \left[\because \langle \sin^2 \theta \rangle = \frac{1}{2} \right]$$

$$= \frac{1}{4}ma^2(2\pi v)^2 \left[\because \omega = 2\pi v \right]$$

$$= \pi^2 ma^2 v^2$$

10. For a SHM, acceleration,

$$a = -\omega^2 x, \text{ where } \omega \text{ is a constant} = \frac{2\pi}{T}.$$

$$a = -\frac{4\pi^2}{T^2} \cdot x \Rightarrow \frac{aT^2}{x} = -\frac{4\pi^2}{T}$$

The period of oscillation T is a constant.

$$\therefore \frac{aT^2}{x} \text{ is a constant.}$$

