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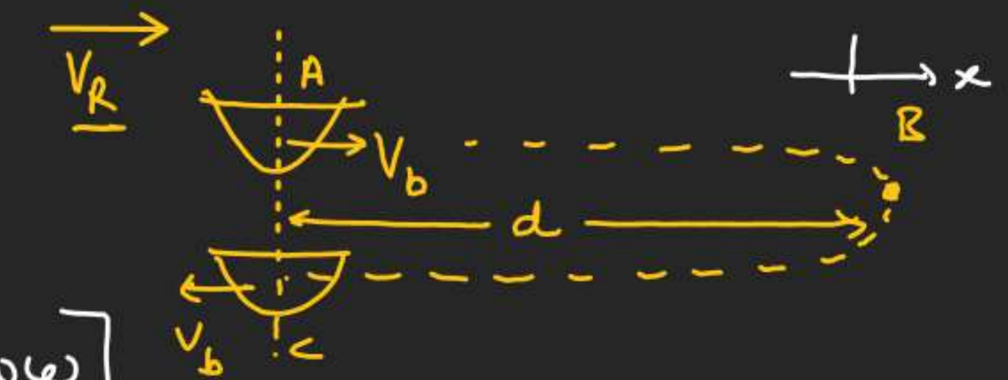
Relative velocity

Down stream and Up stream flow :→

$V_b = $ (w.r.t river flow)

Total time = ??



Down stream :- [In the direction of river flow]

Upstream :- [Opposite to the direction of river flow]

AB [Downstream motion]

$$\vec{V}_{b/E} = \vec{v}_{b/R} + \vec{v}_{R/E}$$

$$= V_b\hat{i} + V_R\hat{i}$$

$$= (V_b + V_R)\hat{i}$$

$$T_{AB} = \left(\frac{d}{V_b + V_R}\right)$$

Upstream (BC)

$$\vec{V}_{b/E} = \vec{V}_{b/R} + \vec{V}_{R/E}$$

$$= -V_b\hat{i} + V_R\hat{i}$$

$$= -(V_b - V_R)\hat{i}$$

$$\underbrace{(V_b - V_R)}_{\text{Speed}}$$

$$T_{BC} = \left(\frac{d}{V_b - V_R}\right)$$

$$T_{ABC} = T_{AB} + T_{BC}$$

$$= \left(\frac{d}{V_b + V_R}\right) + \left(\frac{d}{V_b - V_R}\right)$$

$$T_{ABC} = \frac{d(2V_b)}{V_b^2 - V_R^2} = \left(\frac{2V_b d}{V_b^2 - V_R^2}\right)$$

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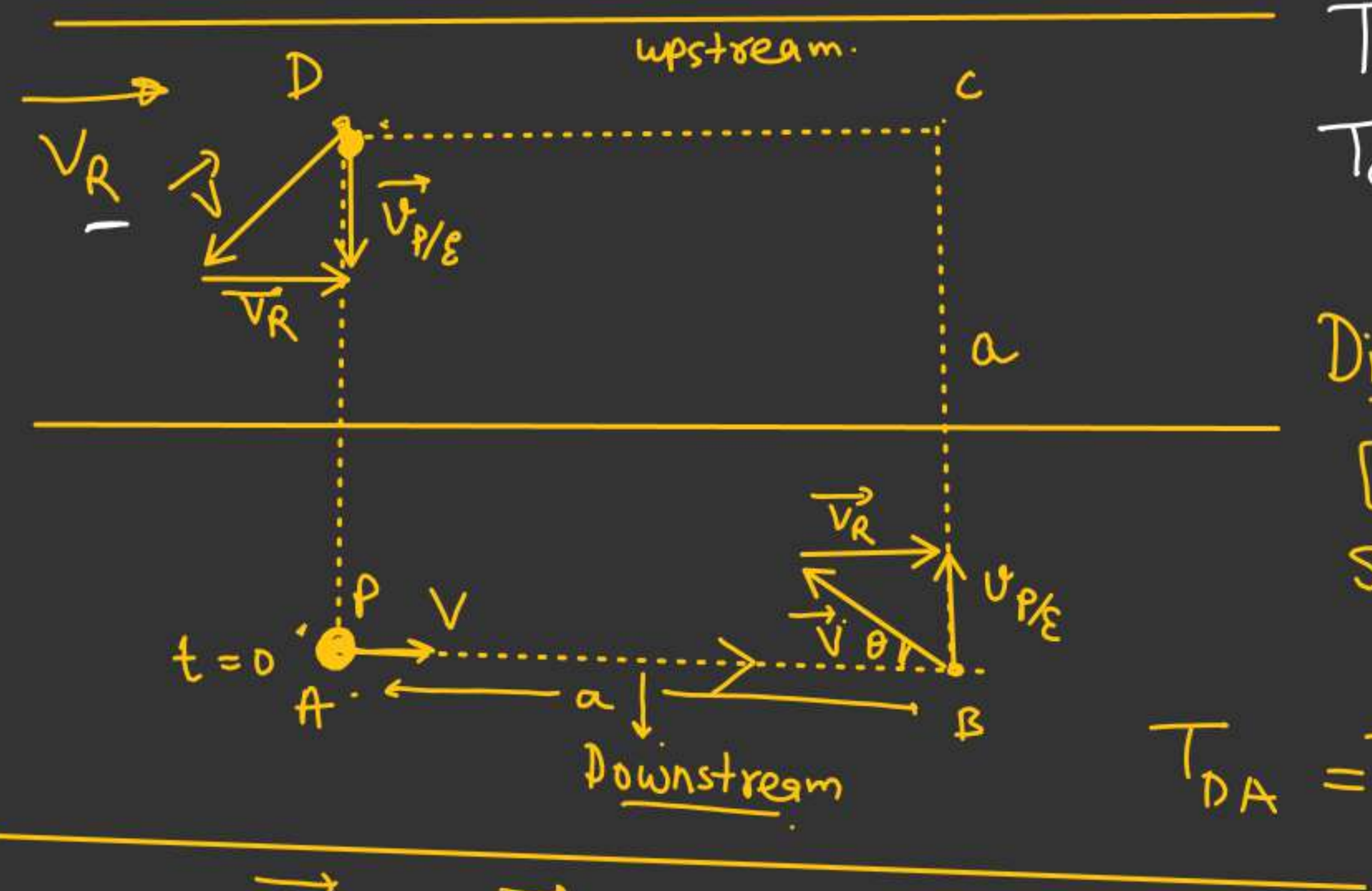
$V =$ w.r.t river



$$T = T_{AB} + T_{BC} + T_{CD} + T_{DA}$$

$$T_{AB} = \left(\frac{a}{V_R+V}\right)$$

$$T_{CD} = \left(\frac{a}{V-V_R}\right)$$

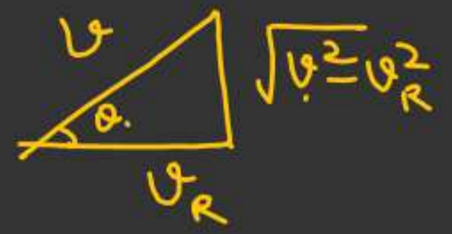
$$T = \left(\frac{a}{V_R+V} + \frac{a}{V-V_R}\right) + \left(\frac{2a}{\sqrt{V^2-V_R^2}}\right)$$

$$T = \left[\frac{2Va}{(V^2-V_R^2)} + \frac{2a}{\sqrt{V^2-V_R^2}}\right]$$

Directly opposite to the bank

$$\left[\cos\theta = \left(\frac{v_R}{v}\right)\right]$$

$$\sin\theta = \left[\frac{\sqrt{v^2-v_R^2}}{v}\right]$$



$$T_{DA} = T_{BC} = \left(\frac{a}{v\sin\theta}\right) = \frac{a}{\cancel{v}\frac{\sqrt{v^2-v_R^2}}{\cancel{v}}} = \left(\frac{a}{\sqrt{v^2-V_R^2}}\right)$$

For directly opposite to the bank

$$V_R = V\cos\theta$$

$$\cos\theta = (V_R/V)$$

$$\vec{V}_{P/E} = \vec{v}_{P/R} + \vec{v}_{R/E}$$

$$= -V\cos\theta\,\hat{i} + V\sin\theta\,\hat{j} + V_R\,\hat{i}$$

$$= \underbrace{(V_R - V\cos\theta)\,\hat{i}}_{\Downarrow} + V\sin\theta\,\hat{j}$$

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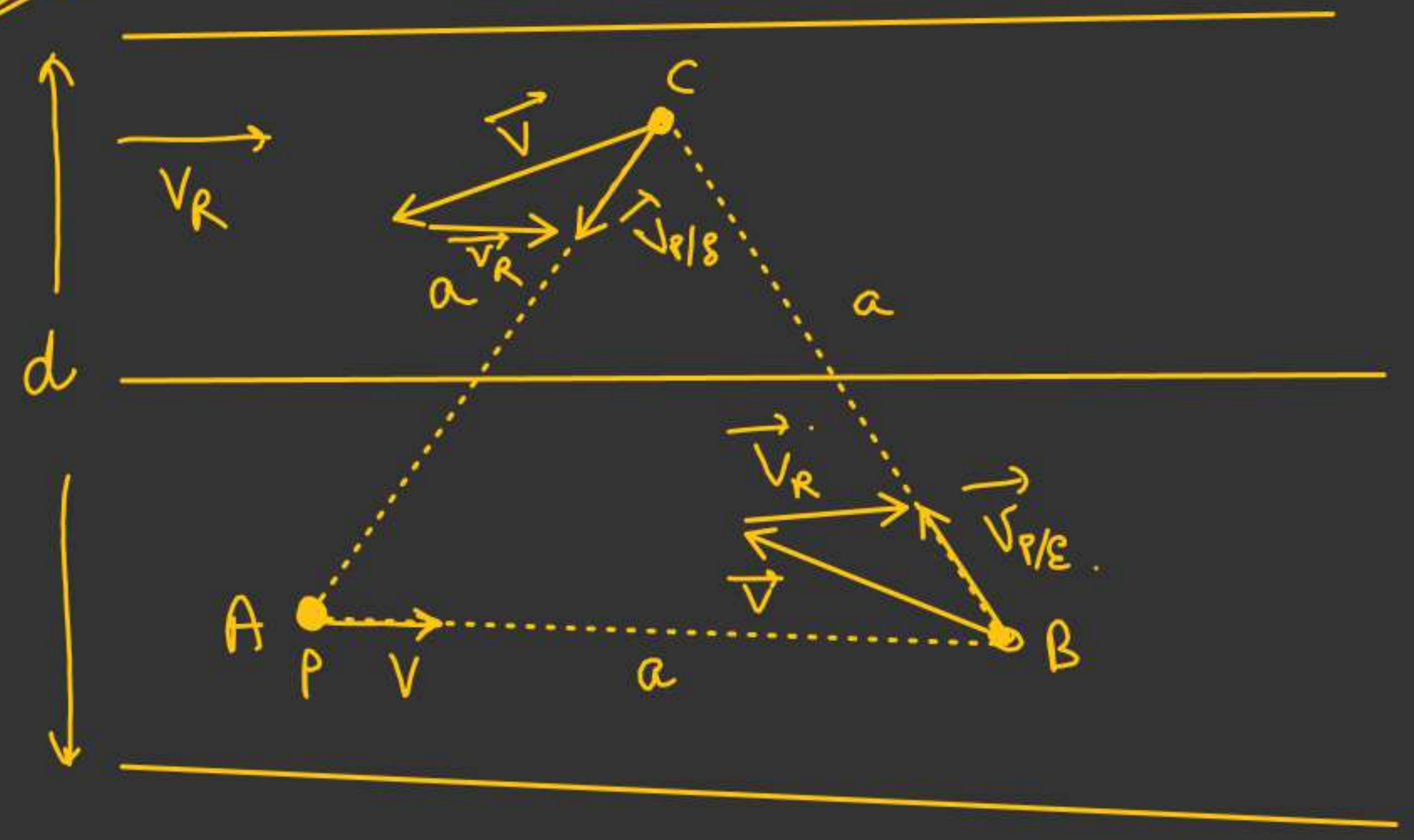
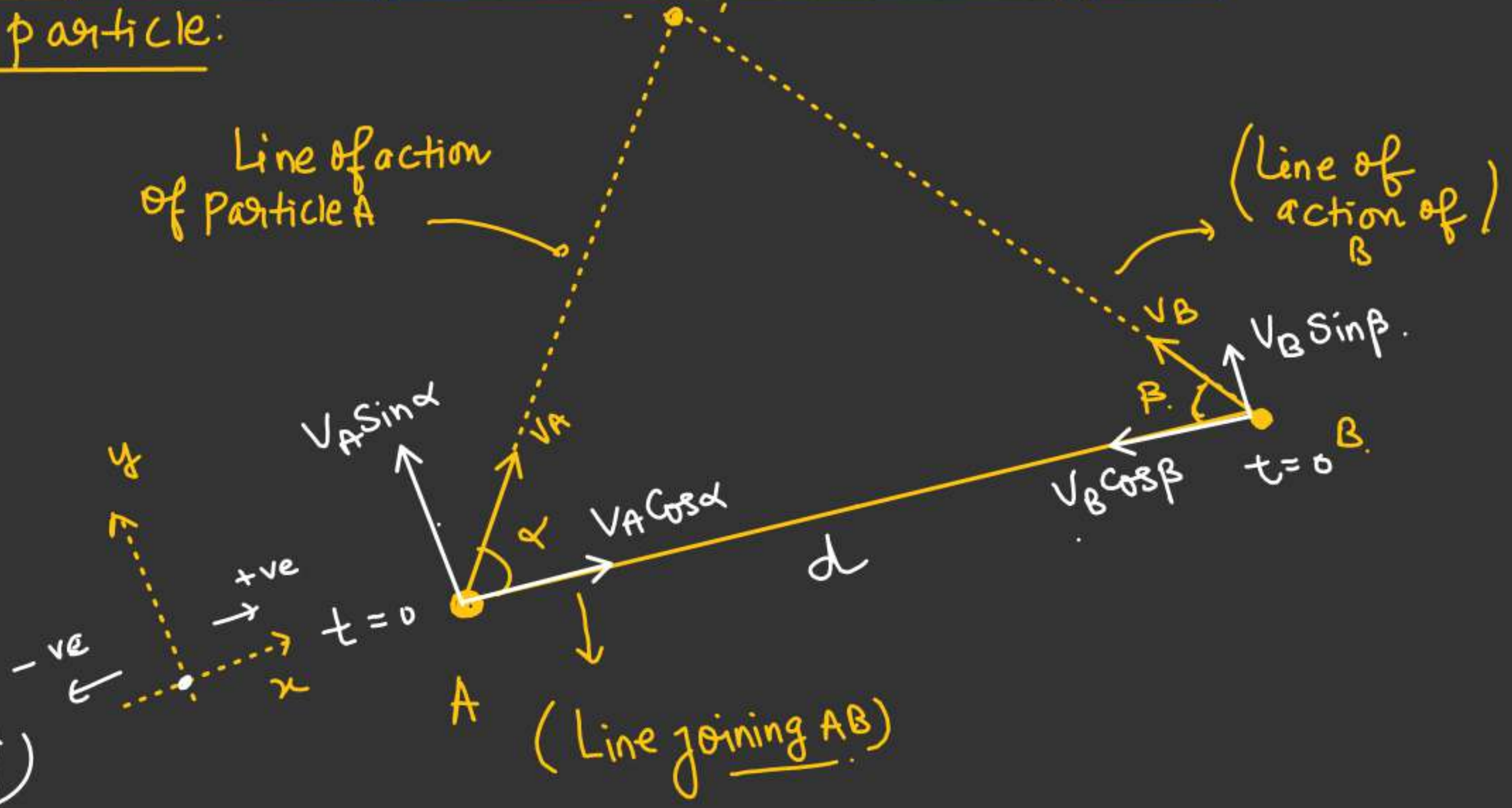
\# Total time taken by particle 'P' for the path ABC.

H.W.

V_R = River flow velocity. V = velocity of Particle w.r.t river flow.



(4). Relative velocity when line of action of two particles are different w.r.t line joining the two particle:



Condition for Collision:-

$$S_{rel} = 0.$$

$$\boxed{S_{rel} = (V_{rel})T}$$

$$\vec{V}_{B/A} = \vec{V}_{B/E} - \vec{V}_{A/E}$$

$$= \left(-V_B\cos\beta\,\hat{i} + V_B\sin\beta\,\hat{j}\right) - \left(V_A\cos\alpha\,\hat{i} + V_A\sin\alpha\,\hat{j}\right)$$

$$\vec{V}_{B/A} = \underbrace{-(V_A\cos\alpha + V_B\cos\beta)}_{(v_{B/A})_{\text{Along the line joining}}}\hat{i} + \underbrace{(V_B\sin\beta - V_A\sin\alpha)}_{(v_{B/A})_{\perp \text{ to line joining}}}\hat{j}$$

$$=$$

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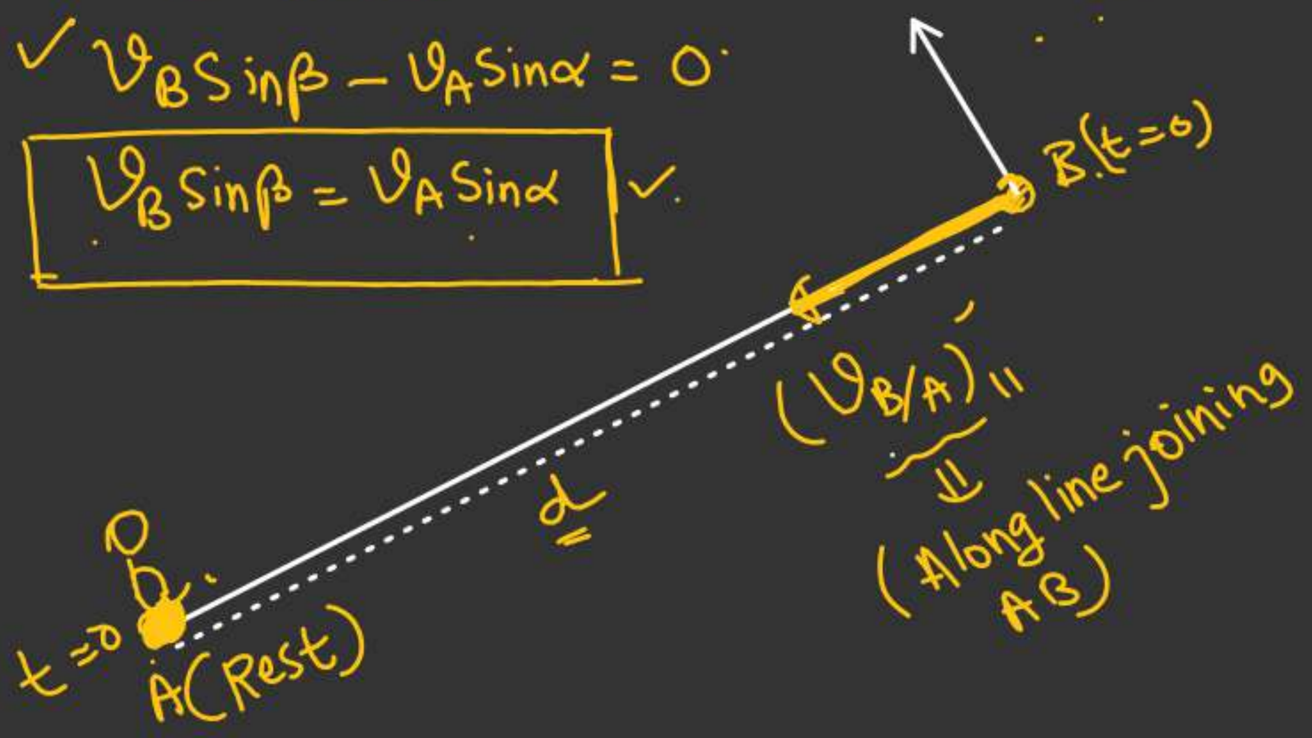
Condition for Collision

$$(v_{B/A})_\perp = 0$$

$$v_B \sin\beta - v_A \sin\alpha = 0$$

$$\boxed{v_B \sin\beta = v_A \sin\alpha}$$

$(v_B \sin\beta > v_A \sin\alpha)$



Time of Collision

$$T = \frac{d}{\left|(\vec{v}_{B/A})_{\parallel}\right|}$$

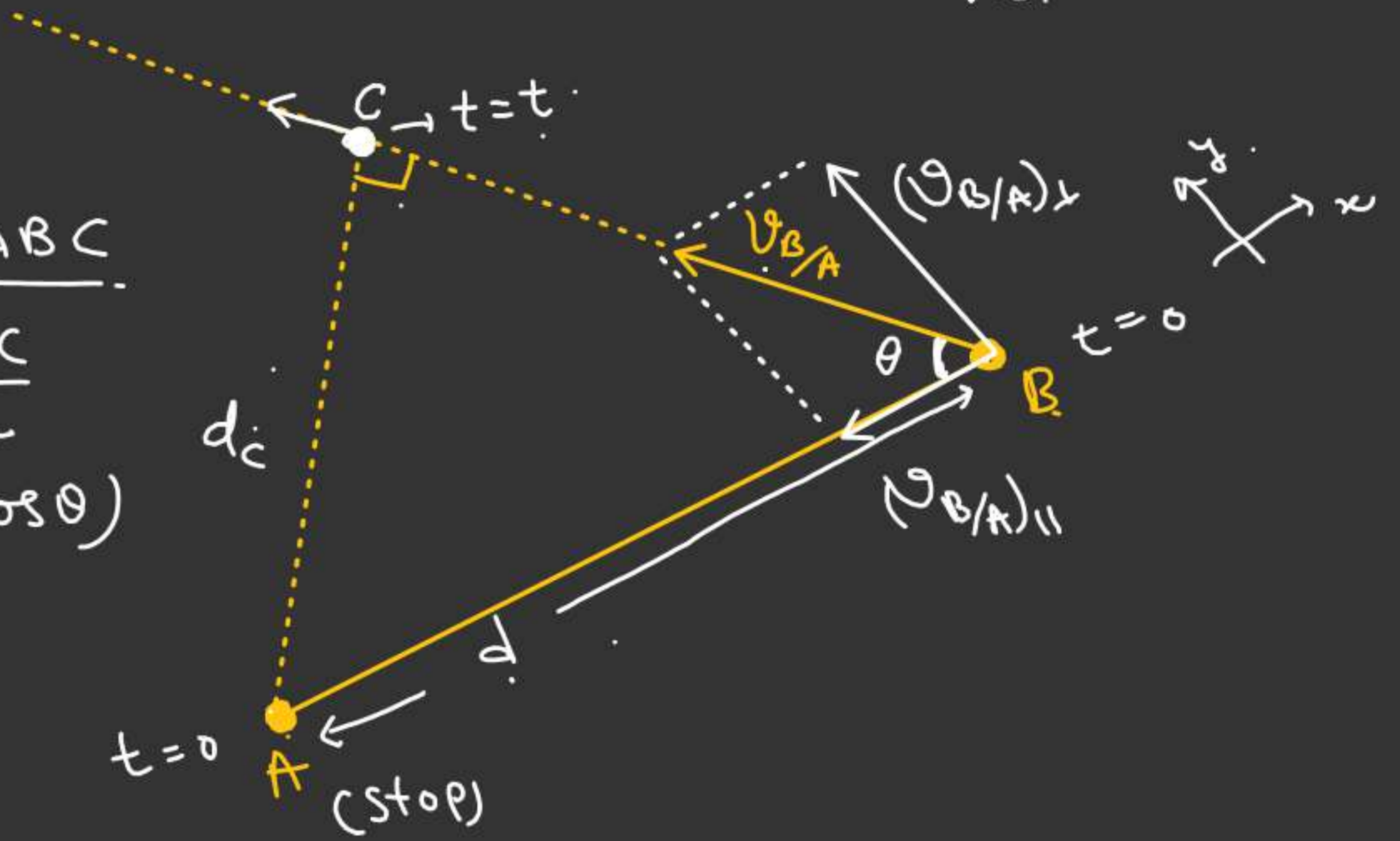
$$T = \left(\frac{d}{v_A \cos\alpha + v_B \cos\beta}\right)$$

Concept of Shortest distance of approach :→

$(v_{B/A})_\perp$ → Perpendicular to line joining AB.

$(v_{B/A})$ → parallel to line joining AB.

$$\tan\theta = \frac{|(v_{B/A})_\perp|}{|(v_{B/A})_{\parallel}|}$$



AC → (Shortest distance)

$$\sin\theta = \left(\frac{AC}{AB}\right)$$

$$AC = AB\sin\theta$$

$$d_c = [d\sin\theta]$$

In Δ ABC

$$\cos\theta = \frac{BC}{d}$$

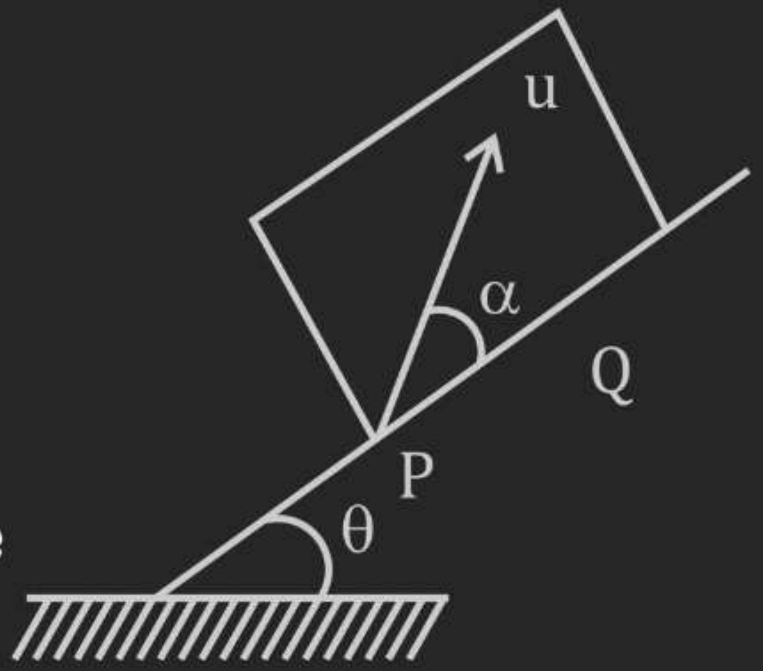
$$BC = (d\cos\theta)$$

Time taken for Closest distance

$$t = \frac{BC}{|\vec{v}_{B/A}|} = \left[\frac{d\cos\theta}{|\vec{v}_{B/A}|}\right]$$

Relative velocity

Q.4 A large heavy box is sliding without friction down a smooth inclined plane of inclination θ. From a point P on the bottom of the box a particle is projected inside the box, with speed u (relative to box) at angle α with the bottom of the box.



(a) Find the distance along the bottom of the box between the point of projection P and the point Q where the particle lands. The particle does not hit any other surface of the box. Neglect the air resistance.

(b) If horizontal displacement of the particle with respect to ground is zero. Find the speed of the box w.r.t. the ground at the moment when particle was projected. (1998)

Relative velocity

Q.7 A girl standing on road holds her umbrella at 45° with the vertical to keep the rain away. If she starts running without umbrella with a speed of $15\sqrt{2}\,\text{kmh}^{-1}$, the rain drops hit her head vertically. The speed of rain drops with respect to the moving girl is: [June 27, 2022 (I)]

(A) $30\ \text{kmh}^{-1}$

(B) $\frac{25}{\sqrt{2}}\ \text{kmh}^{-1}$

(C) $\frac{30}{\sqrt{2}}\ \text{kmh}^{-1}$

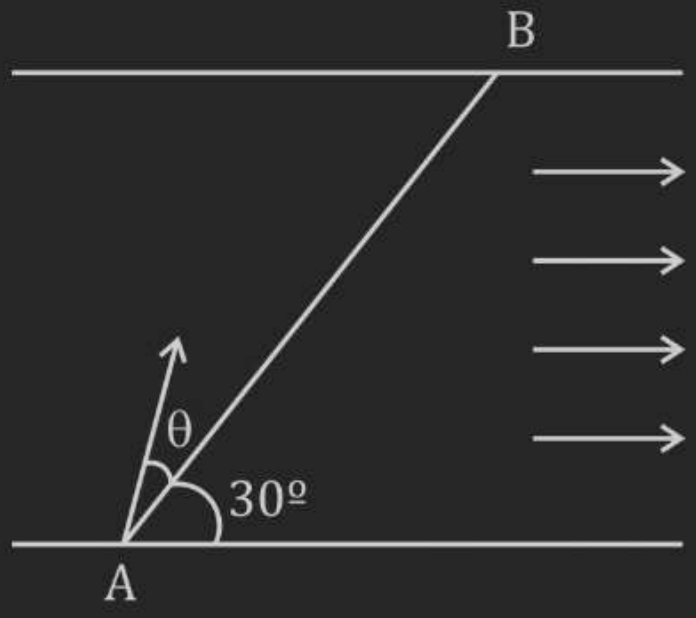
(D) $25\ \text{kmh}^{-1}$

Relative velocity

H.W.

Q.8 A swimmer wants to cross a river from point A to point B. Line AB makes an angle of 30° with the flow of river. Magnitude of velocity of the swimmer is same as that of the river. The angle θ with the line AB should be __ $^\circ$, so that the swimmer reaches point B.

[NA, July 27, 2021 (II)]



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Relative velocity

H.W

Q.9 A person is swimming with a speed of $10 \mathrm{~m/s}$ at an angle of 120° with the flow and reaches to a point directly opposite on the other side of the river. The speed of the flow is ' x ' m / s. The value of ' x ' to the nearest integer is

[March 18, 2021 (I)]

Relative velocity

H.W

Q.10 When a car is at rest, its driver sees raindrops falling on it vertically. When driving the car with speed v, he sees that raindrops are coming at an angle 60° from the horizontal. On further increasing the speed of the car to $(1+\beta)v$, this angle changes to 45°. The value of β is close to: [Sep. 06, 2020 (II)]

(A) 0.73

(B) 0.41

(C) 0.37

(D) 0.50

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Relative velocity

H.W

Q.11 Ship A is sailing towards north-east with velocity $\bar{v} = 30\hat{i} + 50\hat{j}$ km/hr where $\hat{i}$ points east and $\hat{j}$ north. Ship B is at a distance of 80 km east and 150 km north of Ship A and is sailing towards west at 10 km/hr. A will be at minimum distance from B in: [8 April 2019 I]

(A) 4.2hrs.

(B) 2.6hrs.

(C) 3.2hrs.

(D) 2.2hrs.

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H-W

Relative velocity

Q.12 Find the time an airplane take to fly around a square with side a with the wind blowing at a velocity u, in the two cases, (a) If direction of wind is along one side of the square; (b) the direction of wind is along one of the diagonal of the square?

(a) $2a\left(\frac{v+\sqrt{v^2-u^2}}{v^2-u^2}\right)$ (b) $2\sqrt{2}a\left(\frac{\sqrt{2v^2-u^2}}{v^2-u^2}\right)$]