Nishant Jindal

CIRCULAR MOTION

Centripetal force

In vertical direction

$$T\cos\theta = mg \quad -①$$

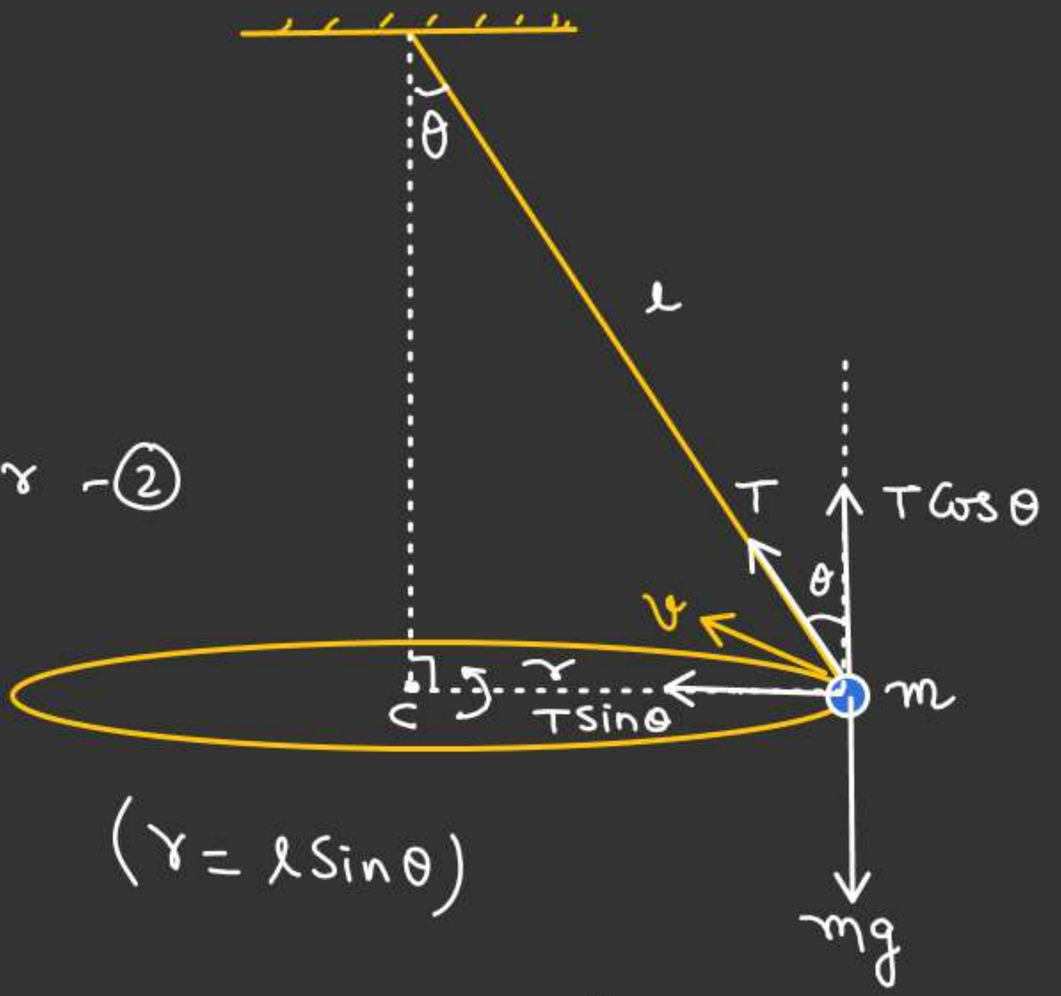
In horizontal direction

$$T\sin\theta = \frac{mv^2}{r} = m\omega^2 r \quad -②$$

$$T\sin\theta = \left(\frac{mv^2}{l\sin\theta}\right)$$

② ÷ ①

$$\tan\theta = \frac{\omega^2 r}{g}$$



$(r = l\sin\theta)$

$\omega = \left(\frac{v}{r}\right)$

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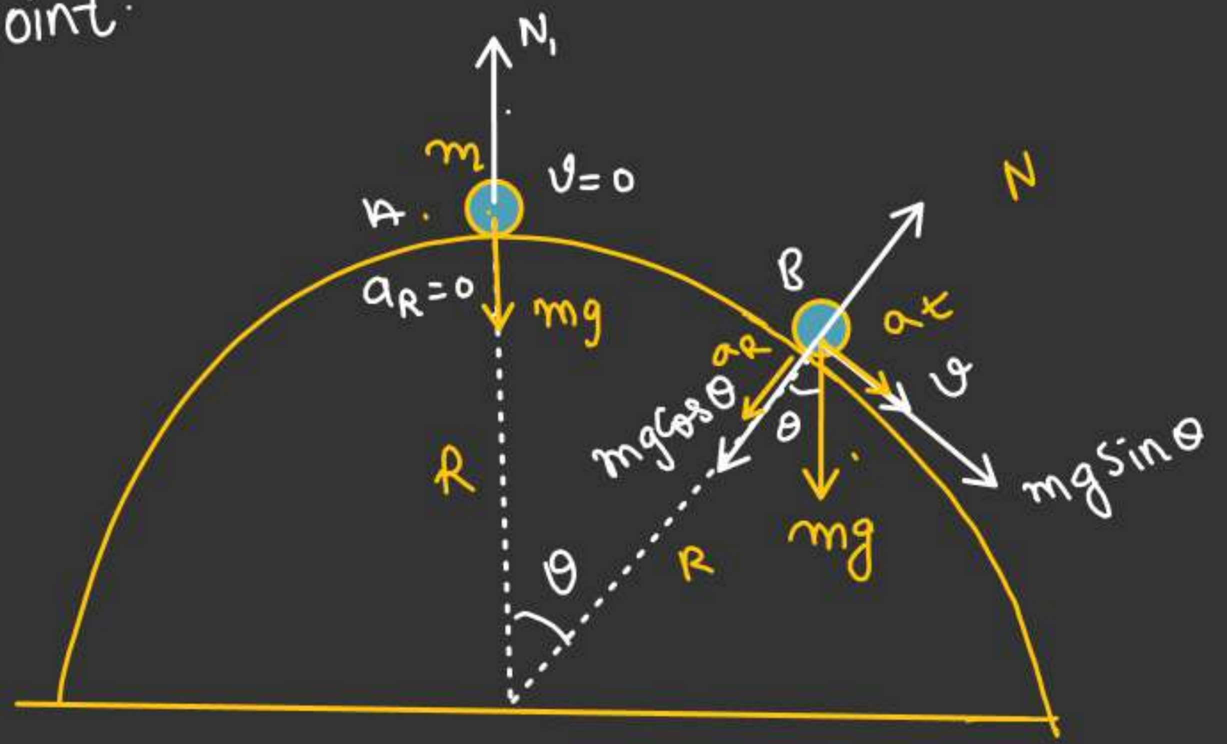
At top most point.

$N_1 = mg$.



At B.

$$mg\cos\theta - N = \frac{mv^2}{R}.$$

Tagnetial accleration.

$$mg\sin\theta = ma_t$$

$$a_t = g\sin\theta.$$

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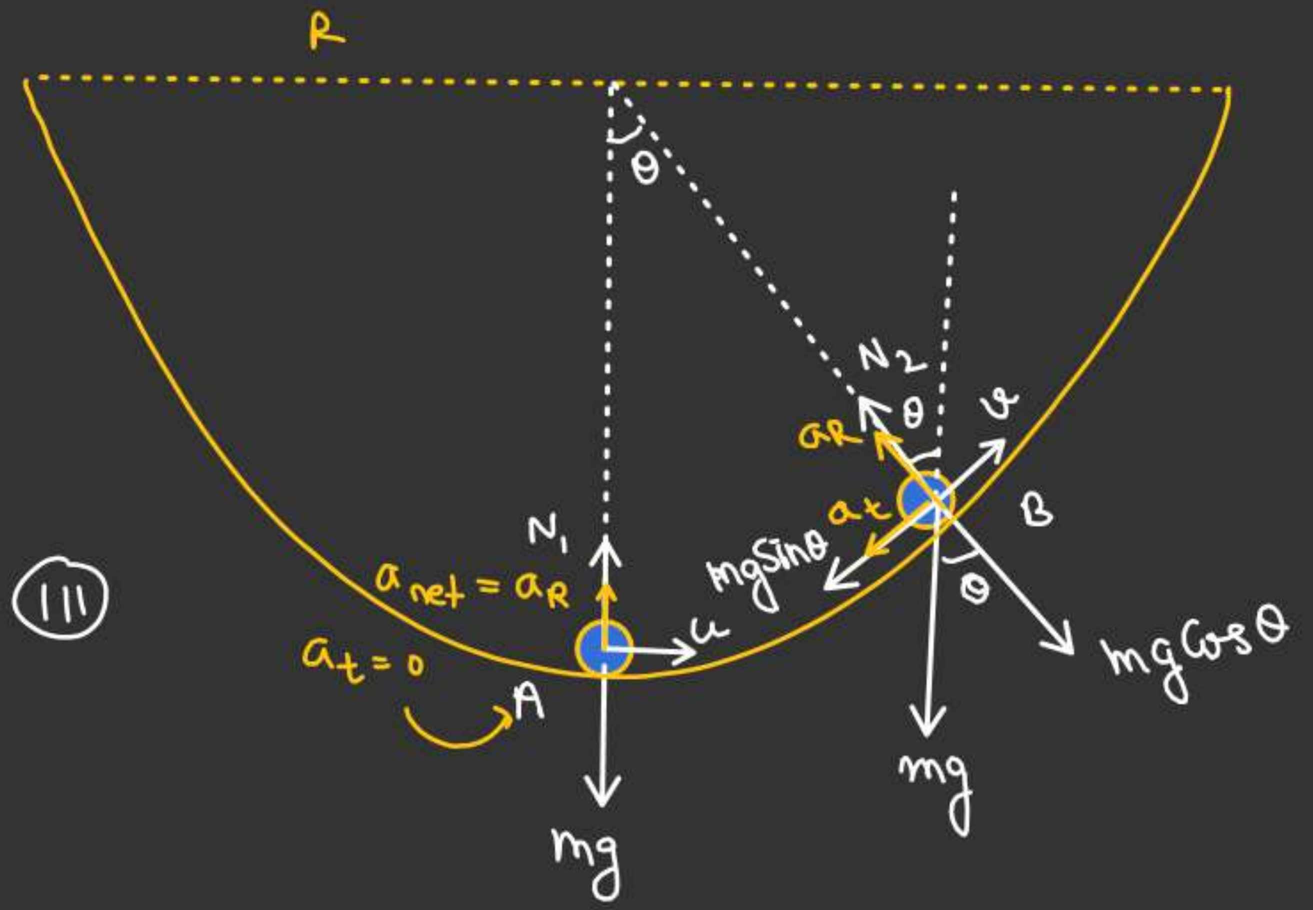
At A

$$N_1 - mg = \frac{mu^2}{R} \quad -(1)$$

At B

(Radial direction) $N_2 - mg\cos\theta = \frac{mv^2}{R}$ (II)

(Tangential direction) $\Rightarrow$ $mg\sin\theta = ma_t$ $-$ (III)



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Concept of Centrifugal force (Pseudo force in rotating frame)

↳ To make Newton's Law applicable in rotating frame we apply an imaginary force always radially outward from the center & whose magnitude is equal to centripetal force.

Block doesn't slip w.r.t turntable

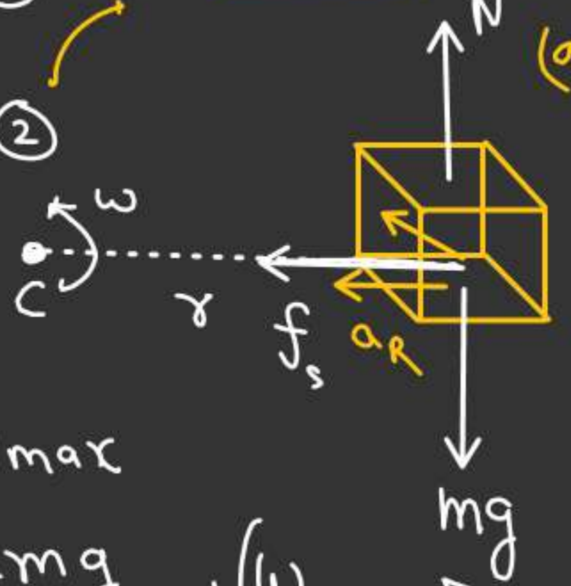
F.B.D of block w.r.t earth

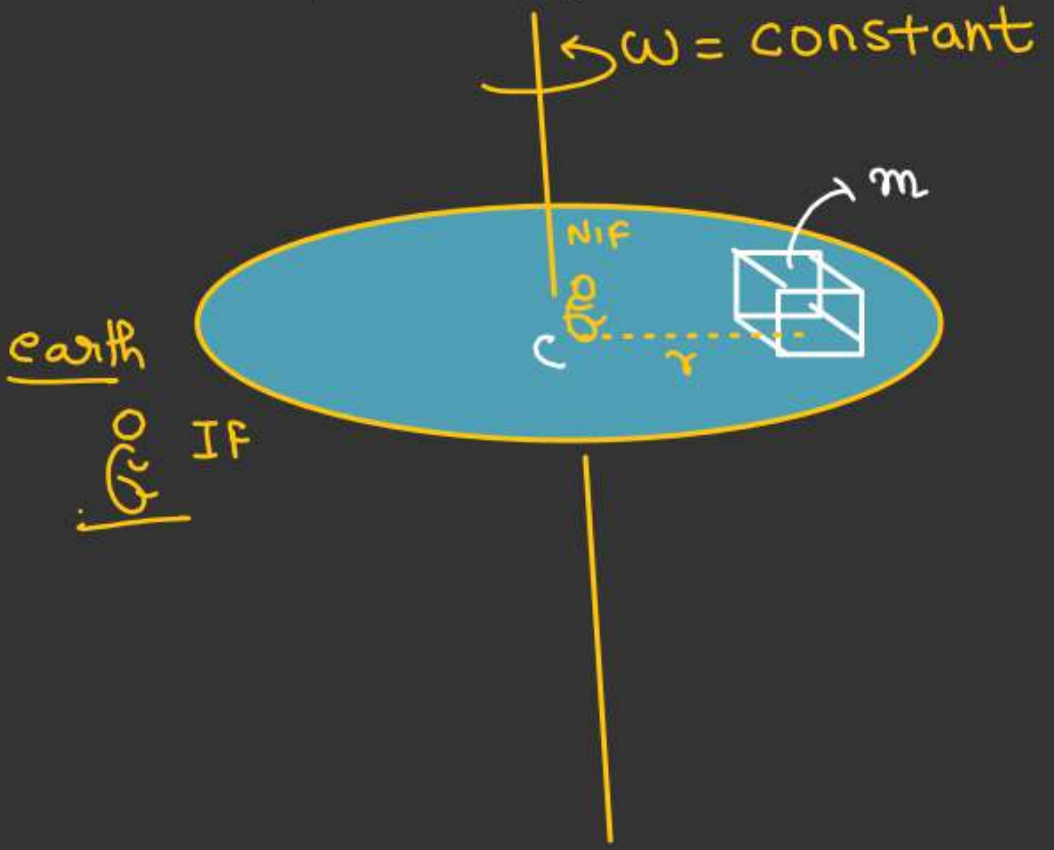
$N = mg$ — ①

$f_s = m\omega^2 r$ — ②

Newton's 2nd Law

$\left(a_R = \frac{v^2}{R} = \omega^2 r\right)$





For ω_{max} so that block doesn't slip.

$f_s \leq (f_s)_{max}$

$m\omega^2 r \leq \mu mg$

$\omega \leq \sqrt{\mu g / r}$

$\left(\omega_{min} = \sqrt{\mu g / r}\right)$

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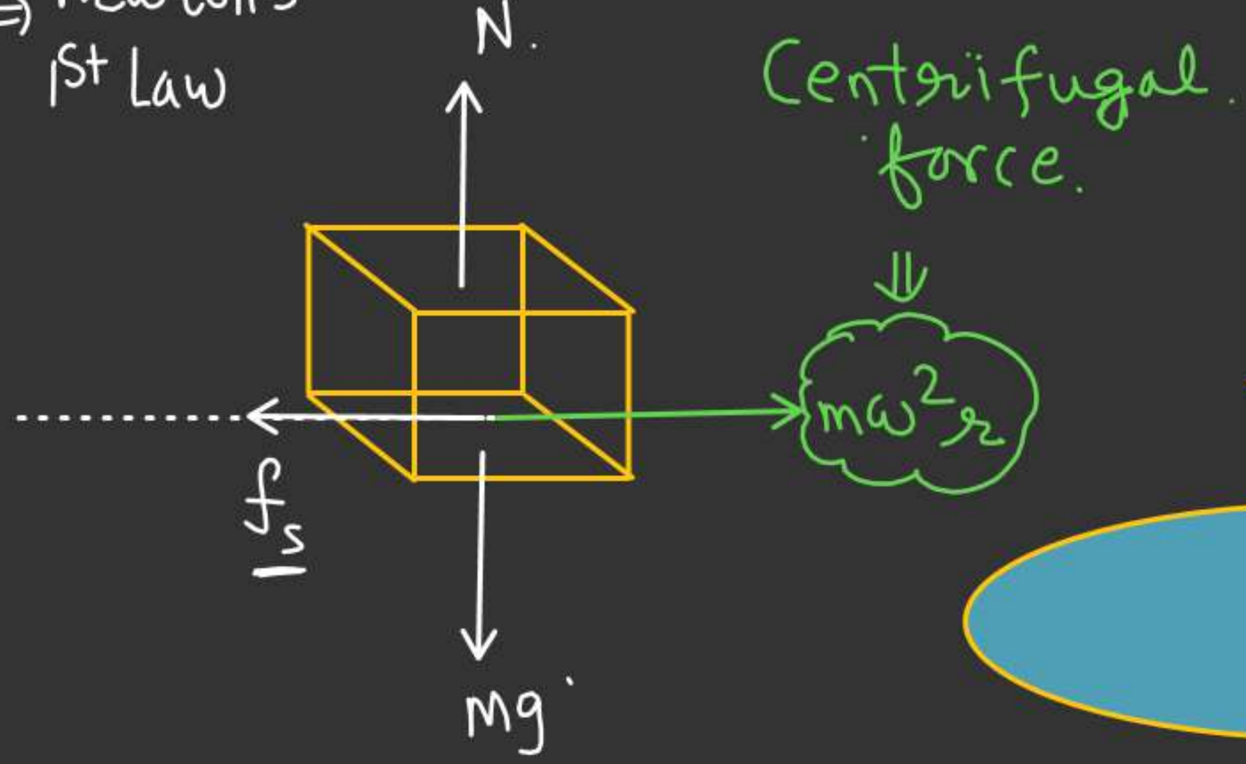
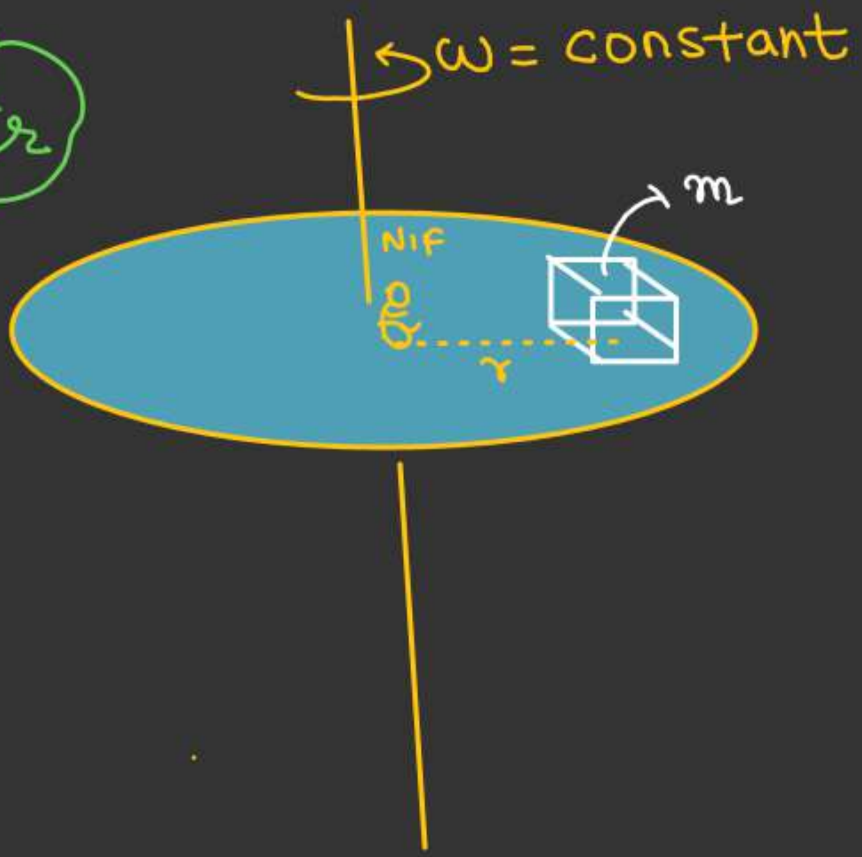
F·B·D of block in rotating frame

$$\left[\begin{array}{l} N = mg \\ f_s = m\omega^2 r \end{array}\right] \Rightarrow \text{Newton's } 1^{st} \text{ Law}$$





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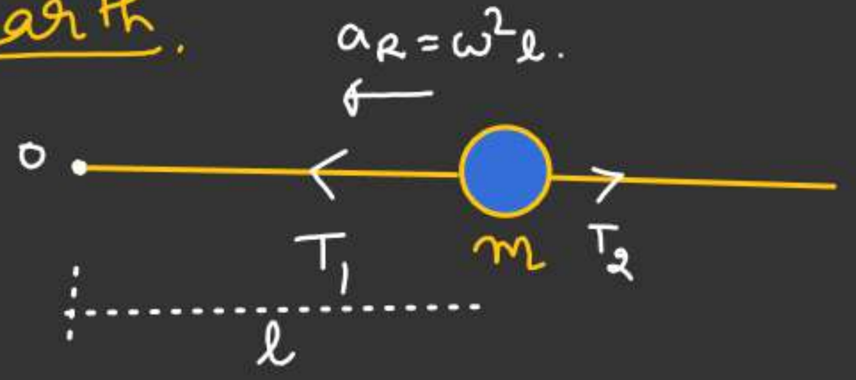
Find ratio of tension in all the strings.

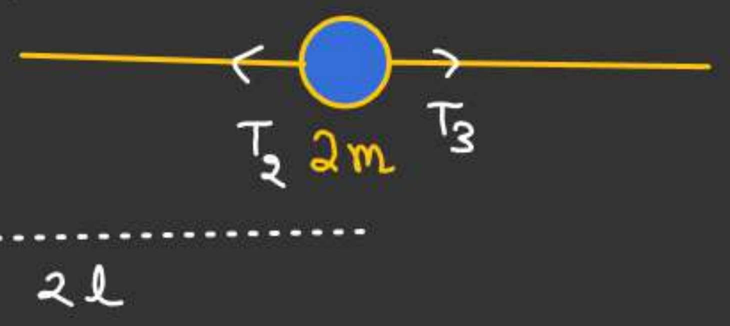
w.r.t Earth



$T_1 - T_2 = m\omega^2 l$ — ①

$T_1 = m\omega^2 l + T_2$

$T_1 = 14 m\omega^2 l$

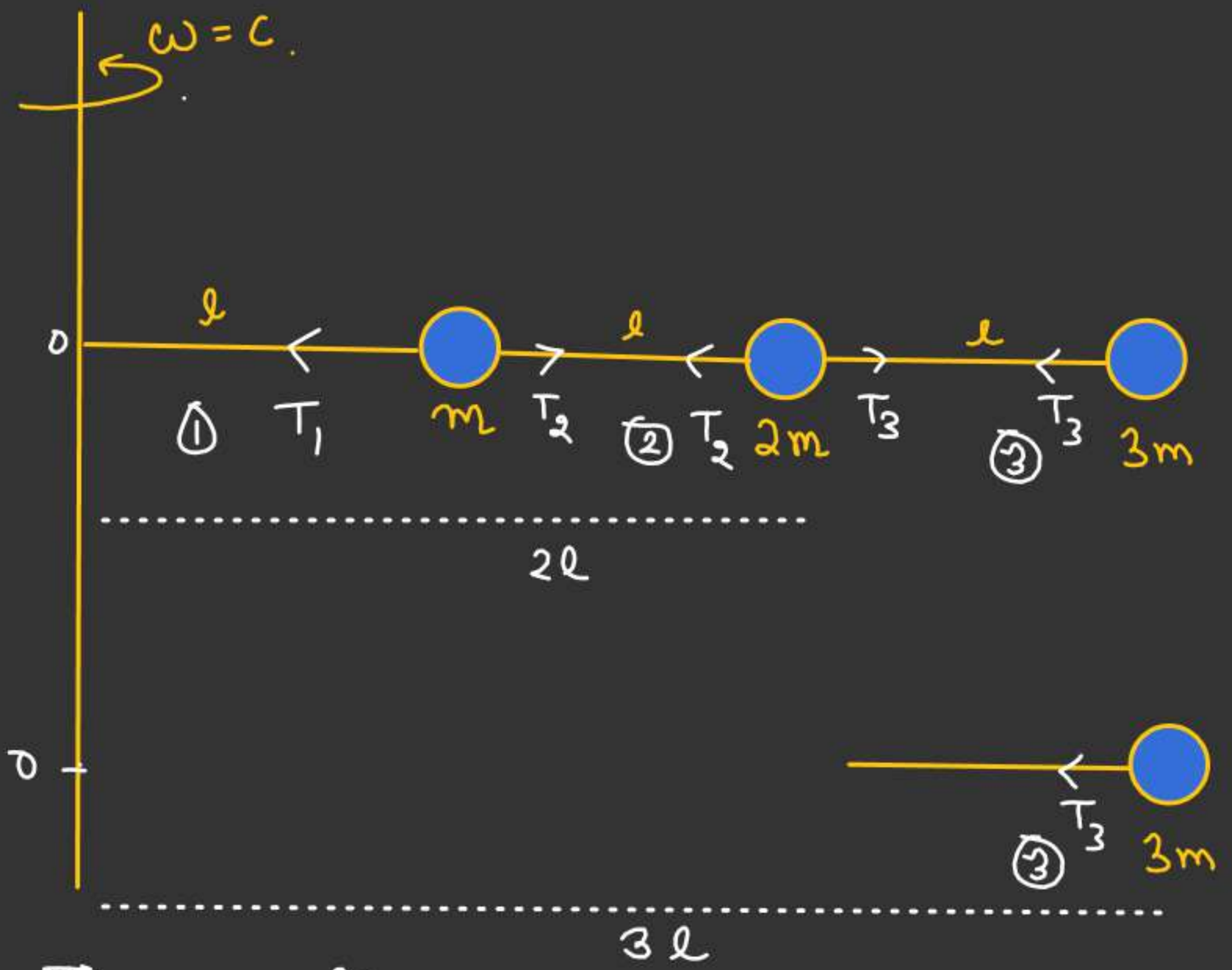


$T_2 - T_3 = 2m\omega^2(2l)$

$T_2 - T_3 = 4m\omega^2 l$ — ②

$T_2 = 4m\omega^2 l + T_3$

$T_2 = 13 m\omega^2 l$ ✓



$T_3 = 3m\omega^2(3l)$

$T_3 = 9m\omega^2 l$ ✓

$T_1 : T_2 : T_3 :: 14 : 13 : 9$ ✓

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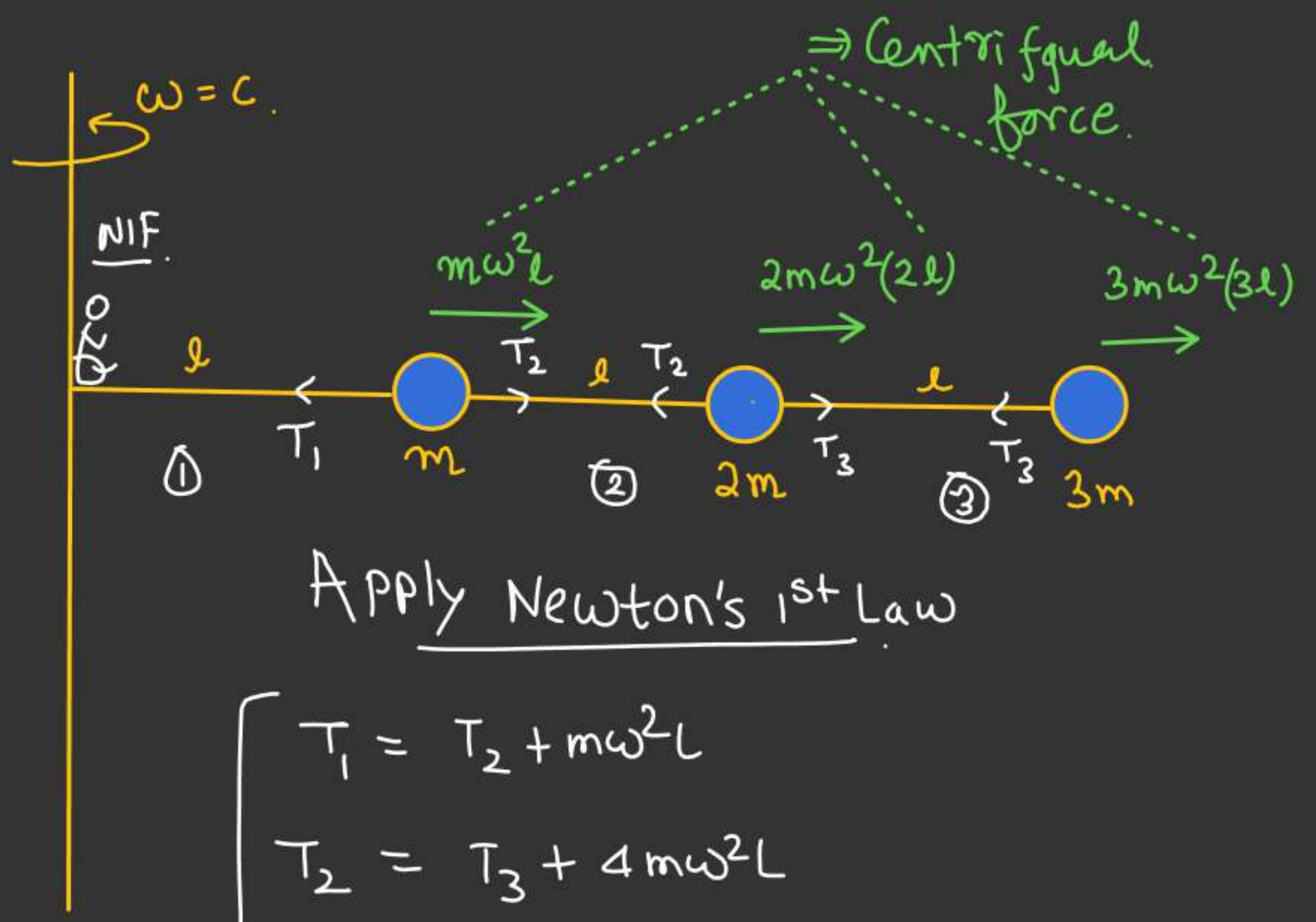
Find ratio of tension in all the strings.



Apply Newton's 1st Law

$$\begin{cases} T_1 = T_2 + m\omega^2 L \\ T_2 = T_3 + 4m\omega^2 L \\ T_3 = 9m\omega^2 L \end{cases}$$

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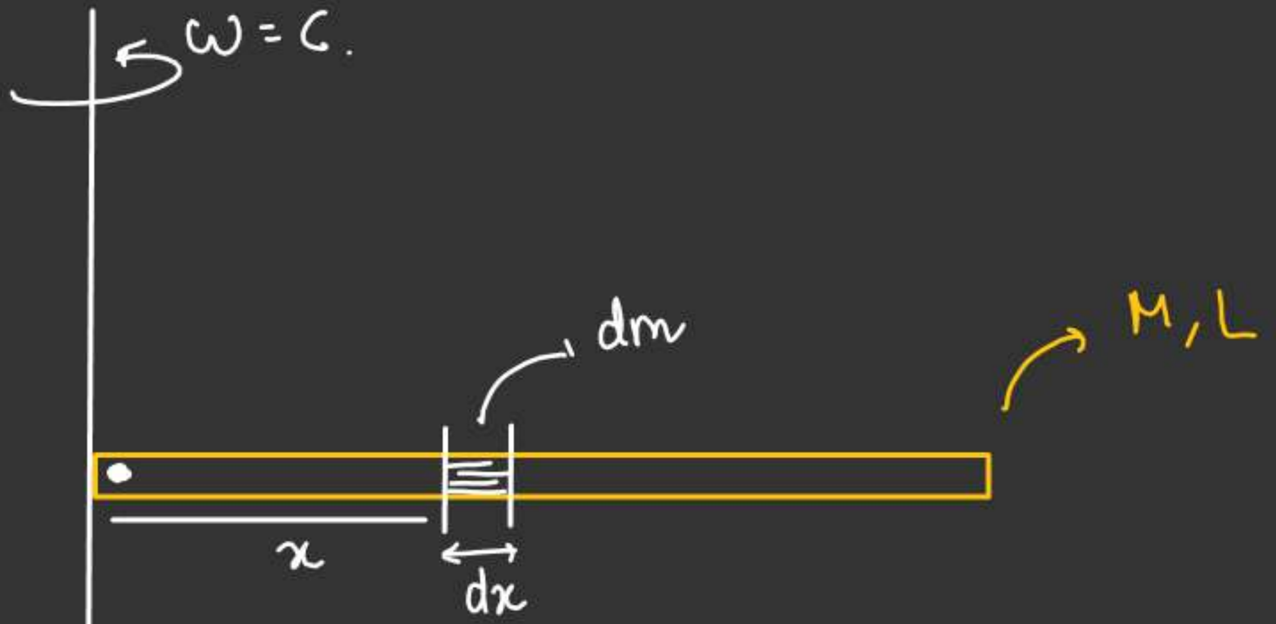
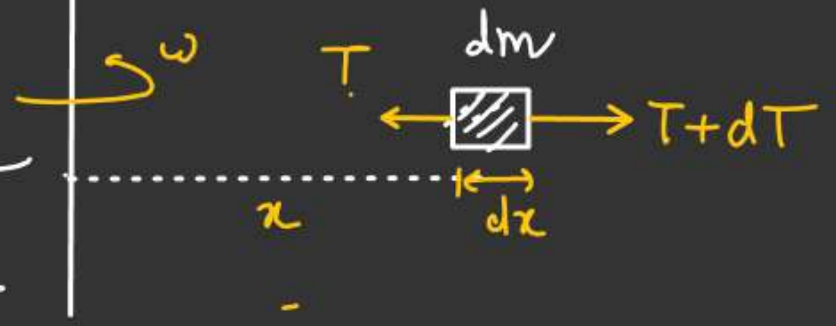
Tension is rotating Rod

Rod in uniform rotating with Constant angular velocity ω. The whole system is on a smooth horizontal plane



$$dm = \left(\frac{M}{L}dx\right)$$

$$T - (T+dT) = dm\,\omega^2 x$$



$$-dT = \left(\frac{M}{L}dx\right)\omega^2 x$$

$$-\int_{T_{max}}^{T_x} dT = \frac{M\omega^2}{L}\int_0^x x\,dx$$

$$-(T_x - T_{max}) = \frac{M\omega^2 x^2}{2L}$$

$$T_x = T_{max} - \frac{M\omega^2 x^2}{2L}$$

For T_{max}

$$-\int_{T_{max}}^{0} dT = \frac{M\omega^2}{L}\int_{0}^{L} x\,dx$$

$$-(0 - T_{max}) = \frac{M\omega^2}{L} \times \frac{L^2}{2}$$

$$\boxed{T_{max} = \frac{M\omega^2 L}{2}}$$

$$T_x = T_{max} - \frac{M\omega^2 x^2}{2L}$$

$$T_x = \frac{M\omega^2 L}{2} - \frac{M\omega^2}{2L}x^2$$

$$\boxed{T_x = \frac{M\omega^2}{2L}(L^2 - x^2)}$$

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H.W Tension in a Rotating ring :-

H.W

HCV Circular Motion

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Q. No - 1 to 14