

# Product of determinants

$$\begin{vmatrix} \overline{a_1} & \overline{a_2} & \overline{a_3} \\ b_1 & b_2 & b_3 \\ \underline{c_1} & \underline{c_2} & \underline{c_3} \end{vmatrix} \begin{vmatrix} \overline{d_1} & \overline{d_2} & \overline{d_3} \\ e_1 & e_2 & e_3 \\ \underline{f_1} & \underline{f_2} & \underline{f_3} \end{vmatrix} =$$

Row Column  
Row Row  
Column Row  
Column Column

$$\begin{vmatrix} \underline{a_1 d_1 + a_2 e_1 + a_3 f_1} & \underline{a_1 d_2 + a_2 e_2 + a_3 f_2} & \underline{a_1 d_3 + a_2 e_3 + a_3 f_3} \\ - & - & - \\ a_{ij} & - & - \\ - & \underline{c_1 d_2 + c_2 e_2 + c_3 f_2} & - \end{vmatrix}$$

1. Simplify

$$\begin{vmatrix} a_1l_1 + b_1m_1 & a_1l_2 + b_1m_2 & a_1l_3 + b_1m_3 \\ a_2l_1 + b_2m_1 & a_2l_2 + b_2m_2 & a_2l_3 + b_2m_3 \\ a_3l_1 + b_3m_1 & a_3l_2 + b_3m_2 & a_3l_3 + b_3m_3 \end{vmatrix}$$

$$= \begin{vmatrix} a_1 & b_1 & 0 \\ a_2 & b_2 & 0 \\ a_3 & b_3 & 0 \end{vmatrix} \begin{vmatrix} l_1 & l_2 & l_3 \\ m_1 & m_2 & m_3 \\ 0 & 0 & 0 \end{vmatrix} = 0$$

$$\underline{2.} \quad \begin{vmatrix} (a_1-b_1)^2 & (a_1-b_2)^2 & (a_1-b_3)^2 \\ (a_2-b_1)^2 & (a_2-b_2)^2 & (a_2-b_3)^2 \\ (a_3-b_1)^2 & (a_3-b_2)^2 & (a_3-b_3)^2 \end{vmatrix} = \begin{vmatrix} a_1^2 & -2a_1 & 1 \\ a_2^2 & -2a_2 & 1 \\ a_3^2 & -2a_3 & 1 \end{vmatrix} \begin{vmatrix} 1 & 1 & 1 \\ b_1 & b_2 & b_3 \\ b_1^2 & b_2^2 & b_3^2 \end{vmatrix}$$

$$a_1^2 - 2a_1b_1 + b_1^2 = 2 \begin{vmatrix} 1 & a_1 & a_1^2 \\ 1 & a_2 & a_2^2 \\ 1 & a_3 & a_3^2 \end{vmatrix} \begin{vmatrix} 1 & b_1 & b_1^2 \\ 1 & b_2 & b_2^2 \\ 1 & b_3 & b_3^2 \end{vmatrix}$$

$$= 2 (a_1-a_2)(a_2-a_3)(a_3-a_1)(b_1-b_2)(b_2-b_3)(b_3-b_1)$$

$$\begin{array}{c}
 \underline{3.} \\
 1 + a_1 b_1 + a_1^2 b_1^2
 \end{array}
 \left| \begin{array}{c}
 \frac{1 - a_1^3 b_1^3}{1 - a_1 b_1} \\
 \frac{1 - a_2^3 b_1^3}{1 - a_2 b_1} \\
 \frac{1 - a_3^3 b_1^3}{1 - a_3 b_1}
 \end{array} \right|
 \begin{array}{c}
 \frac{1 - a_1^3 b_2^3}{1 - a_1 b_2} \\
 \frac{1 - a_2^3 b_2^3}{1 - a_2 b_2} \\
 \frac{1 - a_3^3 b_2^3}{1 - a_3 b_2}
 \end{array}
 \begin{array}{c}
 \frac{1 - a_1^3 b_3^3}{1 - a_1 b_3} \\
 \frac{1 - a_2^3 b_3^3}{1 - a_2 b_3} \\
 \frac{1 - a_3^3 b_3^3}{1 - a_3 b_3}
 \end{array}
 \Bigg| = \left| \begin{array}{ccc}
 1 & a_1 & a_1^2 \\
 1 & a_2 & a_2^2 \\
 1 & a_3 & a_3^2
 \end{array} \right| \left| \begin{array}{ccc}
 1 & b_1 & 1 \\
 b_1 & b_2 & b_3 \\
 b_1^2 & b_2^2 & b_3^2
 \end{array} \right|$$

$$= (a_1 - a_2)(a_2 - a_3)(a_3 - a_1)(b_1 - b_2)(b_2 - b_3)(b_3 - b_1).$$

# System of Equations (Cramer's rule)

$$\Delta_3 = z \Delta$$

$$\Delta_2 = y \Delta$$

$$\Delta_1 = x \Delta$$

$$a_1x + b_1y + c_1z = d_1$$

$$a_2x + b_2y + c_2z = d_2$$

$$a_3x + b_3y + c_3z = d_3$$

$$\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

$$\Delta_1 = \begin{vmatrix} a_1x + b_1y + c_1z & b_1 & c_1 \\ a_2x + b_2y + c_2z & b_2 & c_2 \\ a_3x + b_3y + c_3z & b_3 & c_3 \end{vmatrix}$$

$$\Delta_1 =$$

$$\begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix}$$

$$\Delta_2 =$$

$$\begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix}$$

$$\Delta_3 = \begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix}$$

# System of equations

$$\begin{aligned}\Delta_1 &= x\Delta \\ \Delta_2 &= y\Delta \\ \Delta_3 &= z\Delta\end{aligned}$$

Unique solution  
 $\Delta \neq 0$

Infinite solution  
 $\Delta = 0 = \Delta_1 = \Delta_2 = \Delta_3$

No solution  
 $\Delta = 0$   
& at least one  
of  $\Delta_1, \Delta_2, \Delta_3$   
 $\neq 0$ .

$$(x, y, z) = \left( \frac{\Delta_1}{\Delta}, \frac{\Delta_2}{\Delta}, \frac{\Delta_3}{\Delta} \right)$$

$$\begin{vmatrix} 2 & -1 & -1 \\ 4 & -2 & -2 \\ 6 & -3 & -3 \end{vmatrix} = 0$$

$$2x - y - z = 1 \quad \checkmark$$

$$4x - 2y - 2z = 3$$

$$6x - 3y - 3z = 3$$

Exception

no solution

$$a_1x + a_2y + a_3z = d_1$$

$$(x, y, z) = (3k, 2k, 5)$$

$$k \in \mathbb{R}$$

$$\text{Let } z = k$$

$$a_1x + b_1y = d_1 - c_1k$$

$$a_2x + b_2y = d_2 - c_2k$$

$$x = f(k)$$

$$y = g(k)$$

Consistent  $\rightarrow$  A system of eqns is said to be consistent if it has atleast one solution.

Inconsistent  $\rightarrow$  No solution

# Homogeneous System of Equations

$$a_1x + b_1y + c_1z = 0$$

$$a_2x + b_2y + c_2z = 0$$

$$a_3x + b_3y + c_3z = 0$$

$$\underline{(0, 0, 0)}$$

Condition for given system to have

non trivial solutions

also

$$\Delta_1 = \Delta_2 = \Delta_3 = 0$$

$$\Delta = 0$$

$$\Delta \neq 0$$

Trivial Solution

$$(x, y, z) = (0, 0, 0)$$

Non Trivial Solution

At least one of  $x, y, z \neq 0$

$$\Delta = 0$$

1. Find  $p, q$  so that equations

$$2x + py + 6z = 8$$

$$x + 2y + qz = 5$$

$$x + y + 3z = 4$$

has (i) no solution

(ii) unique solution

(iii) infinite soln.

Ans  $\rightarrow p \in \mathbb{R} - \{2\}, q \in \mathbb{R} - \{3\}$

$$\Delta_3 = -p + 2$$

$$\Delta = (q - 3)(p - 2)$$

$$\Delta_1 = (p - 2)(4q - 15)$$

$$\Delta_2 = 0$$

$$\Delta = 0 \Rightarrow \underline{p = 2} \text{ or } q = 3$$

Ans  $\rightarrow p = 2, q \in \mathbb{R}$

$q = 3, p \in \mathbb{R} - \{2\}$ .

2. Find  $\theta$  for which system of eqns

$$(\sin 3\theta)x - y + z = 0$$

$$(\cos 2\theta)x + 4y + 3z = 0$$

has non trivial solutions

$$2x + 7y + 7z = 0$$

$$\theta = n\pi + (-1)^n \frac{\pi}{6}, \\ n\pi, n \in \mathbb{I}$$

$$\begin{pmatrix} \sin 3\theta \\ \cos 2\theta \\ 2 \end{pmatrix}$$

$$\begin{pmatrix} -1 & 1 \\ 4 & 3 \\ 7 & 7 \end{pmatrix}$$

$$= 0 = 7 \sin 3\theta + 14 \cos 2\theta - 14$$

$$3 \sin \theta - 4 \sin^3 \theta - 4 \sin^2 \theta = 0$$

$$4 \sin^2 \theta + 4 \sin \theta - 3 = 0 \\ -2 \sin \theta + 6 \sin \theta.$$

$$(2 \sin \theta + 3)(2 \sin \theta - 1) = 0$$

$$\sin \theta = 0$$

$$\sin \theta = \frac{1}{2}$$

3. I)  $A, B, C$  are angles of triangle, then P.T.

$$(\sin 2A)x + (\sin C)y + (\sin B)z = 0$$

$$(\sin C)x + (\sin 2B)y + (\sin A)z = 0$$

$$(\sin B)x + (\sin A)y + (\sin 2C)z = 0$$

possess non  
trivial solutions

$$\begin{vmatrix} \sin(A+A) & \sin(A+B) & \sin(A+C) \\ \sin(A+B) & \sin(B+B) & \sin(B+C) \\ \sin(C+A) & \sin(C+B) & \sin(C+C) \end{vmatrix} = \begin{vmatrix} \sin A & \cos A & 0 \\ \sin B & \cos B & 0 \\ \sin C & \cos C & 0 \end{vmatrix} \begin{vmatrix} \cos A & \cos B & \cos C \\ \sin A & \sin B & \sin C \\ 0 & 0 & 0 \end{vmatrix} = 0$$

PT-1, 2, 3