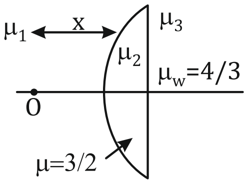
DPP 06

Lens

SOLUTION

1. $\frac{\mu_3}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R_1} + \frac{\mu_3 - \mu_2}{R_2}$

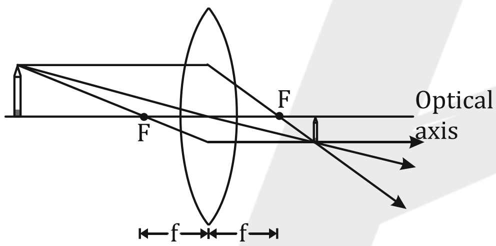


$$\frac{\mu_3}{\infty} - \frac{1}{x} = \frac{1.5 - 1}{10} + \frac{4/3 - 3/2}{\infty}$$

$$\frac{-1}{x} = \frac{0.5}{10}$$

$$x = -20 \text{ cm}$$

2.



$$\frac{1}{f} = \frac{n_2 - n_1}{n_1}\left(\frac{2}{R}\right)$$

$$= \frac{(3/2) - 1}{1}\left(\frac{2}{10}\right) = \frac{1}{10}$$

The given parameters are

$u = -30$ cm

$f = +10$ cm

Substituting these values in thin lens equation, we have

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{v} - \frac{1}{(-30)} = \frac{1}{10}$$

$$v = +15 \text{ cm.}$$

3. Power of lens $= 1.25 \text{ m}^{-1}$

Let refractive index of medium is μ_1.

Refractive index of lens, $\mu_2 = 1.5$

Radius of curvature, $R_1 = 20$ cm and $R_2 = 40$ cm

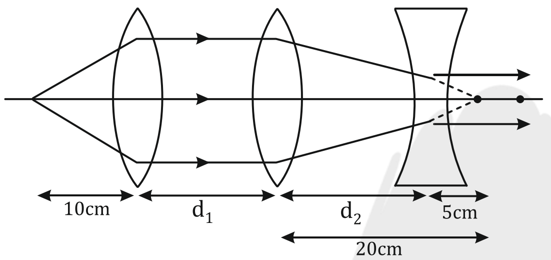
So, Power, $P = \frac{1}{f} = \left(\frac{\mu_2}{\mu_1} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

$\Rightarrow \ 1.25 = \left(\frac{1.5}{\mu_1} - 1\right)\left(\frac{1}{0.2} - \frac{1}{(-0.4)}\right)$

$\Rightarrow \ \mu_1 = \frac{9}{7}$

4. $d_2 = 20 - 5 = 15$cm

d_1 can take any value.



5. For first lens, $f_1 = 10$ cm and $u_1 = -30$ cm

From lens formula, $\frac{1}{f_1} = \frac{1}{v_1} - \frac{1}{u_1}$

or $\frac{1}{10} = \frac{1}{v_1} + \frac{1}{30}$ or $\frac{1}{v_1} = \frac{1}{10} - \frac{1}{30}$

or $v_1 = 15$ cm

This image will now act as virtual object for the second lens.

For second lens, $f_2 = -10$ cm and $u_2 = (15 - 5)$cm $= 10$ cm

$\therefore \frac{1}{f_2} = \frac{1}{v_2} - \frac{1}{u_2}$

or $\frac{1}{v_2} = \frac{1}{f_2} + \frac{1}{u_2} = -\frac{1}{10} + \frac{1}{10}$

or $v_2 = \infty$

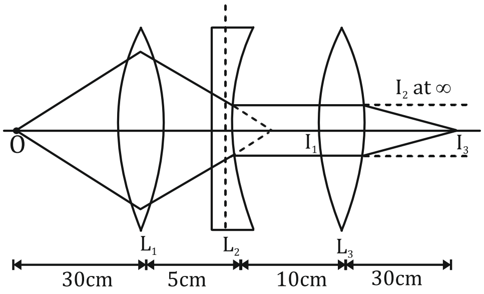
This image will now act as an object for the third lens.

For third lens, $f_3 = 30$ cm and $u_3 = \infty$

$\therefore \frac{1}{f_3} = \frac{1}{v_3} - \frac{1}{u_3}$

or $\frac{1}{v_3} = \frac{1}{f_3} + \frac{1}{u_3} = \frac{1}{30} + \frac{1}{\infty}$

or $v_3 = 30$ cm (Final image)



∴ Distance of final image from object O is 75 cm.

6. Let the focal lengths of planoconvex lens and plano-concave lens be F_1 and F_2 respectively.

$$\frac{1}{F_1} = (\mu_1 - 1)\left(\frac{1}{R}\right)$$

and $\dfrac{1}{F_2} = (\mu_2 - 1)\left(-\dfrac{1}{R}\right)$

Equivalent focal length of the combined lenses is,

$$\frac{1}{P} = \frac{1}{F_1} + \frac{1}{F_2}$$

or $\dfrac{1}{F} = \dfrac{(\mu_1 - 1)}{R} - \dfrac{(\mu_2 - 1)}{R}$

or $\dfrac{1}{F} = \dfrac{\mu_1 - \mu_2}{R}$ $\quad \therefore \dfrac{R}{F} = \mu_1 - \mu_2$

7. For lens L_1 :

$R_1 = 20$ cm; $R_2 = -20$ cm; $\mu = 1.5$

Let the focal length is f_1.

$$\frac{1}{f_1} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{1}{f_1} = (1.5 - 1)\left(\frac{1}{20} - \frac{1}{-20}\right) = 0.5 \times \frac{2}{20}$$

or $f_1 = 20$ cm

For lens L_2: $R_1 = -20$ cm; $R_2 = 20$ cm; $\mu = 1.5$ Let the focal length is f_2.

$$\frac{1}{f_2} = (1.5 - 1)\left(\frac{1}{(-20)} - \frac{1}{20}\right) = -0.5 \times \frac{2}{20}$$

or $f_2 = -20$ cm

For lens L_1: $u = -10$ cm; $f_1 = 20$ cm; $v = ?$

$$\frac{1}{v} = \frac{1}{20} - \frac{1}{10} = -\frac{1}{20}$$

$$v = -20\text{ cm}; m_1 = \frac{v}{u} = \frac{-20}{-10} = 2$$

For lens L_2 :

$u' = -30\text{ cm}; f_2 = -20\text{ cm}; v' = ?$

$$\frac{1}{-20} = \frac{1}{v'} + \frac{1}{30} \Rightarrow \frac{1}{v'} = \frac{-1}{20} - \frac{1}{30}$$

$$v' = -12\text{ cm}; m_2 = \frac{v'}{u'} = \frac{-12}{-30} = \frac{2}{5}$$

so, total magnification

$$m_1 m_2 = 2 \times \frac{2}{5} = \frac{4}{5} = 0.8$$

8. By using relation, $v = \frac{uf}{u+f}$ ....(i)

Case-I: If $v = u$

$\Rightarrow f + u = f$ (Using eq. (i))

$\Rightarrow u = f - f = 0$

Case-II: If $u = \infty$

then $v = f$ (Using eq. (i))

9. For the first lens,

$$\frac{1}{v_1} - \frac{1}{u_1} = \frac{1}{f_1}$$

$$\Rightarrow \frac{1}{v_1} + \frac{1}{20} = \frac{1}{5} \Rightarrow v_1 = \frac{20}{3} = 6.67\text{ cm}$$

Now, for the second lens,

$$u_2 = 6.67 - 2 = \frac{14}{3}\text{ cm}$$

$$\frac{1}{v_2} - \frac{1}{u_2} = \frac{1}{f_2} \Rightarrow \frac{1}{v_2} = \frac{1}{-5} + \frac{3}{14}$$

$v_2 = 70$ cm right of second lens.

10. From the lens equation,

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u} \dots .(i)$$

$$\Rightarrow \frac{1}{v} = \frac{1}{0.3} + \frac{1}{-20} = \frac{197}{60} \Rightarrow v = \frac{60}{197}\text{ m}$$

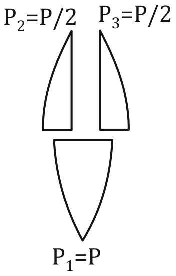
Differentiating eqn. (i),

$$0 = -\frac{1}{v^2}\frac{dv}{dt} + \frac{1}{u^2}\frac{du}{dt}$$

$$\Rightarrow \left(\frac{197}{60}\right)^2 \frac{dv}{dt} = \frac{1}{20^2}(5)$$

$$\Rightarrow \frac{dv}{dt} = 1.16 \times 10^{-3}\ \mathrm{m/s}$$

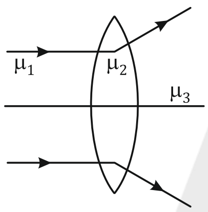
11. When the lens is cut horizontally, then the focal length of the lens does not change. Hence, no change in power but when the lens is cut vertically then focal length of the lens gets doubled and hence power of the lens, gets halved.



12. $\frac{\mu_3}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R_1} + \frac{\mu_3 - \mu_2}{R_2}$

$$\frac{\mu_3}{v} - \frac{\mu_1}{\infty} = \frac{\mu_2 - \mu_1}{R} + \frac{\mu_3 - \mu_2}{(-R)}$$

$$\frac{\mu_3}{v} = \frac{\mu_2 - \mu_1}{R} - \frac{(\mu_3 - \mu_2)}{R} < 0$$



For diverging nature $v < 0$

$$\frac{\mu_2 - \mu_1}{R} < \frac{\mu_3 - \mu_2}{R}$$

$$2\mu_2 < \mu_1 + \mu_3$$