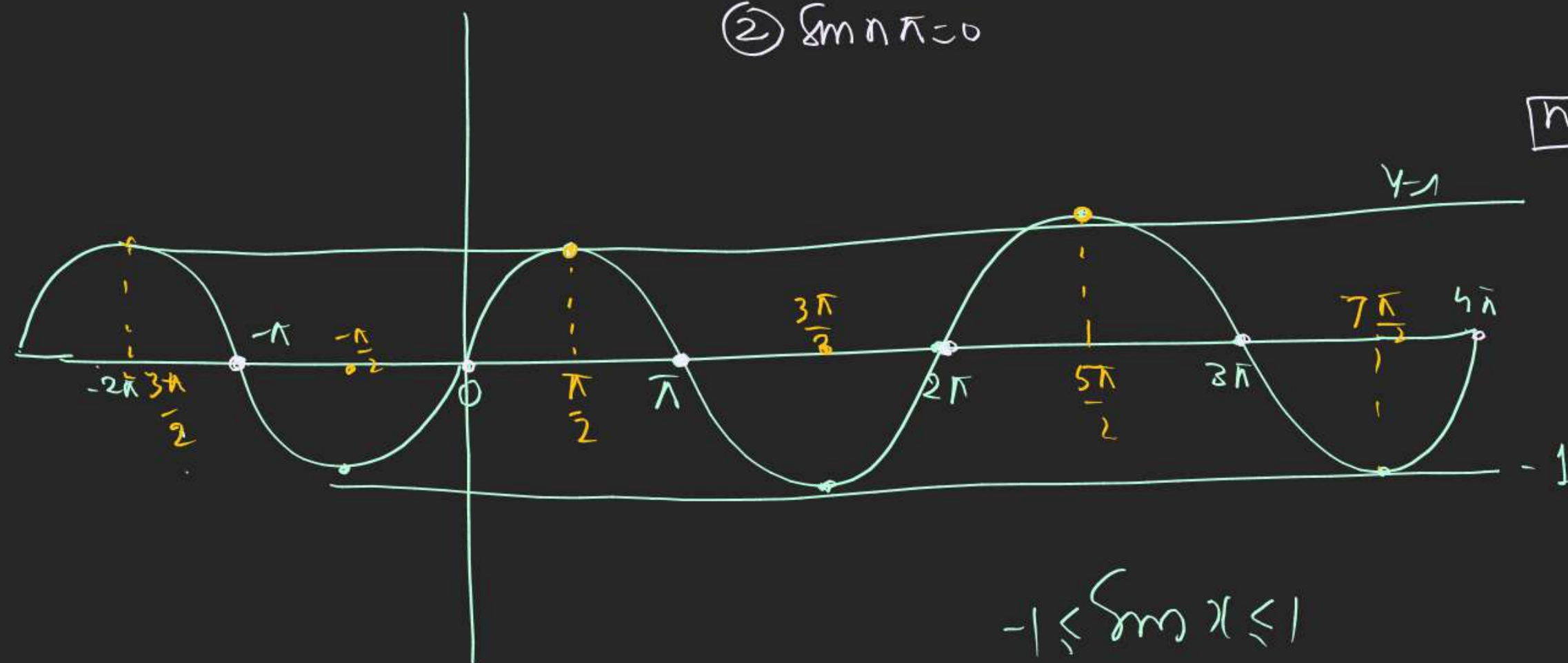


Trigonometry

① $y = \sin x$

① graph 2π

② $\sin n\pi = 0$



$$\sin x = 0 \Rightarrow x = n\pi$$

$$\sin 0 = 0$$

$$\sin \pi = 0$$

$$\sin 2\pi = 0$$

$$\sin (-3\pi) = 0$$

$$\sin (2023\pi) = 0$$

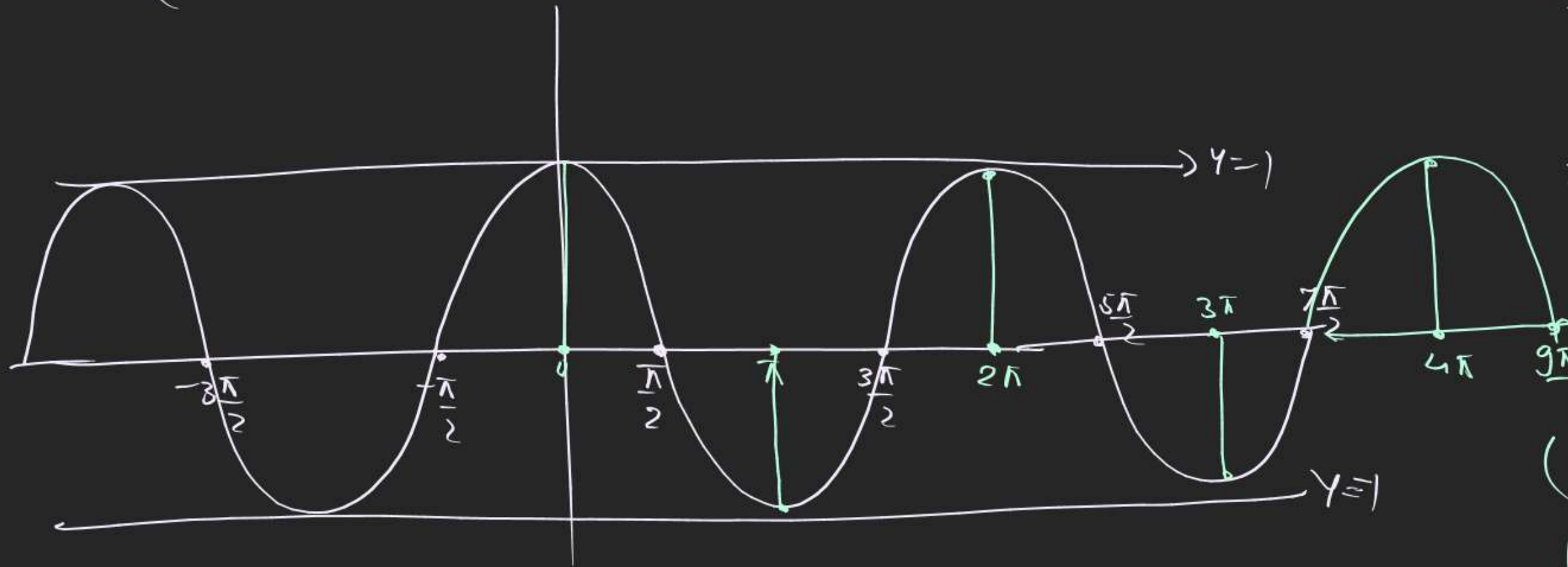
$$\left. \begin{array}{l} 2\pi \\ \sin n\pi = 0 \end{array} \right\}$$

① $\sin x = 1 \Rightarrow x = (4n+1)\frac{\pi}{2}$

② $\sin x = -1 \Rightarrow x = (4n+3)\frac{\pi}{2}$

Trigonometry

(2) $y = \cos x$



1) \cos even $\pi = 1$
 $\cos(2n\pi) = 1$

2) \cos odd $\pi = -1$
 $\cos(2n+1)\pi = -1$

(3) $\cos n\pi = \pm 1$

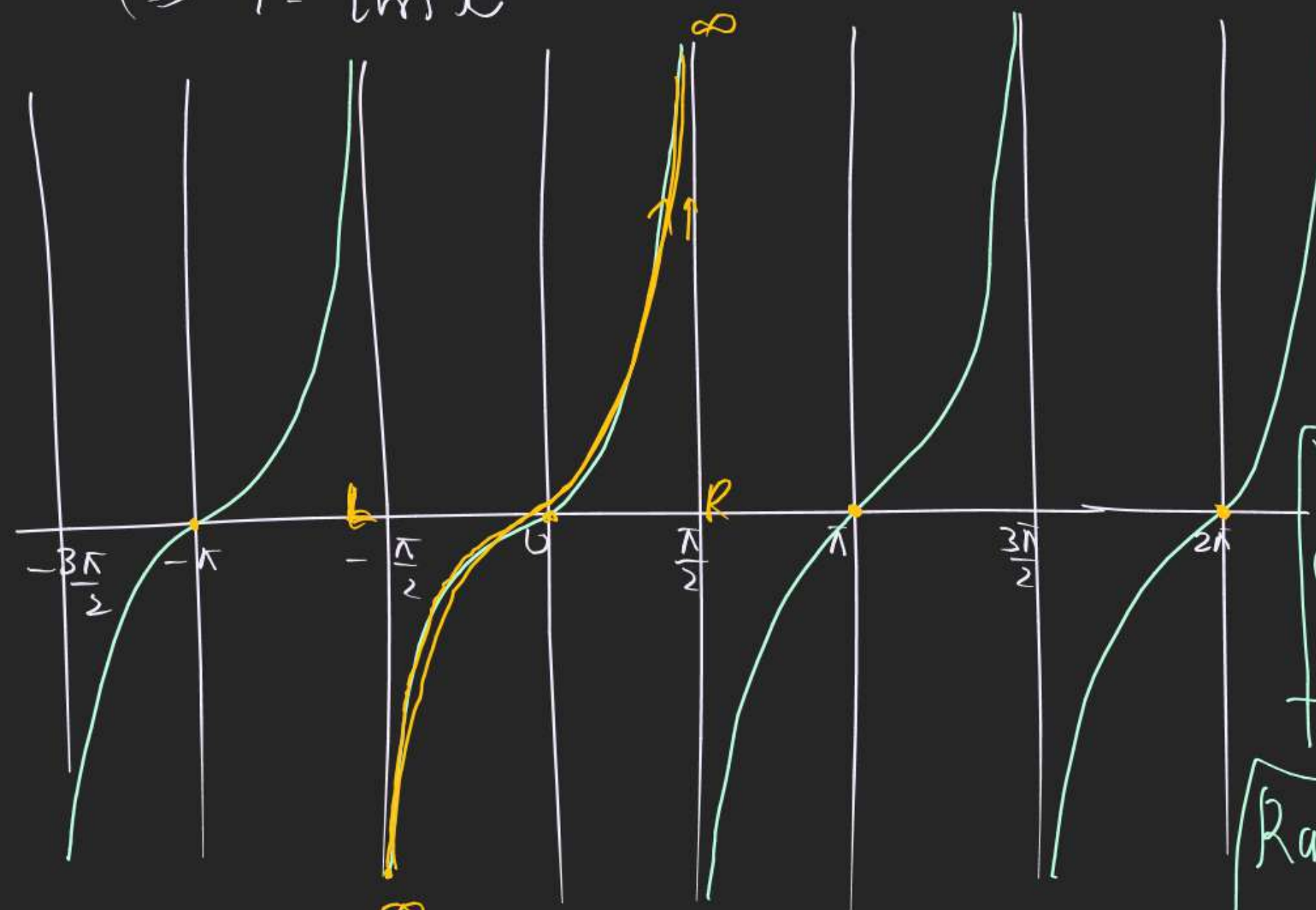
(4) $\cos x = 0$

When $x = (2n+1)\frac{\pi}{2}$

$$\tan \frac{\pi}{2} = \frac{\sin \frac{\pi}{2}}{\cos \frac{\pi}{2}} = \frac{1}{0}$$

$$(3) y = \tan x$$

Trigonometry



Trigo

Q Eqn / Seqn / princ / BT / Coord

Domain

Values of x
acceptable
by $f(x)$

(2)

for $\tan x$

Range = value of y

ht of graph

Domain

$$x \neq (2n+1)\frac{\pi}{2}$$

$$x \in (-\infty, \infty) - (2n+1)\frac{\pi}{2}$$

$$(3) -\infty < \tan x < \infty$$

Range

$$1) \tan x = 0$$

$$\tan 0 = 0$$

$$\tan \pi = 0$$

$$\tan 2\pi = 0$$

$$\tan 3\pi = 0$$

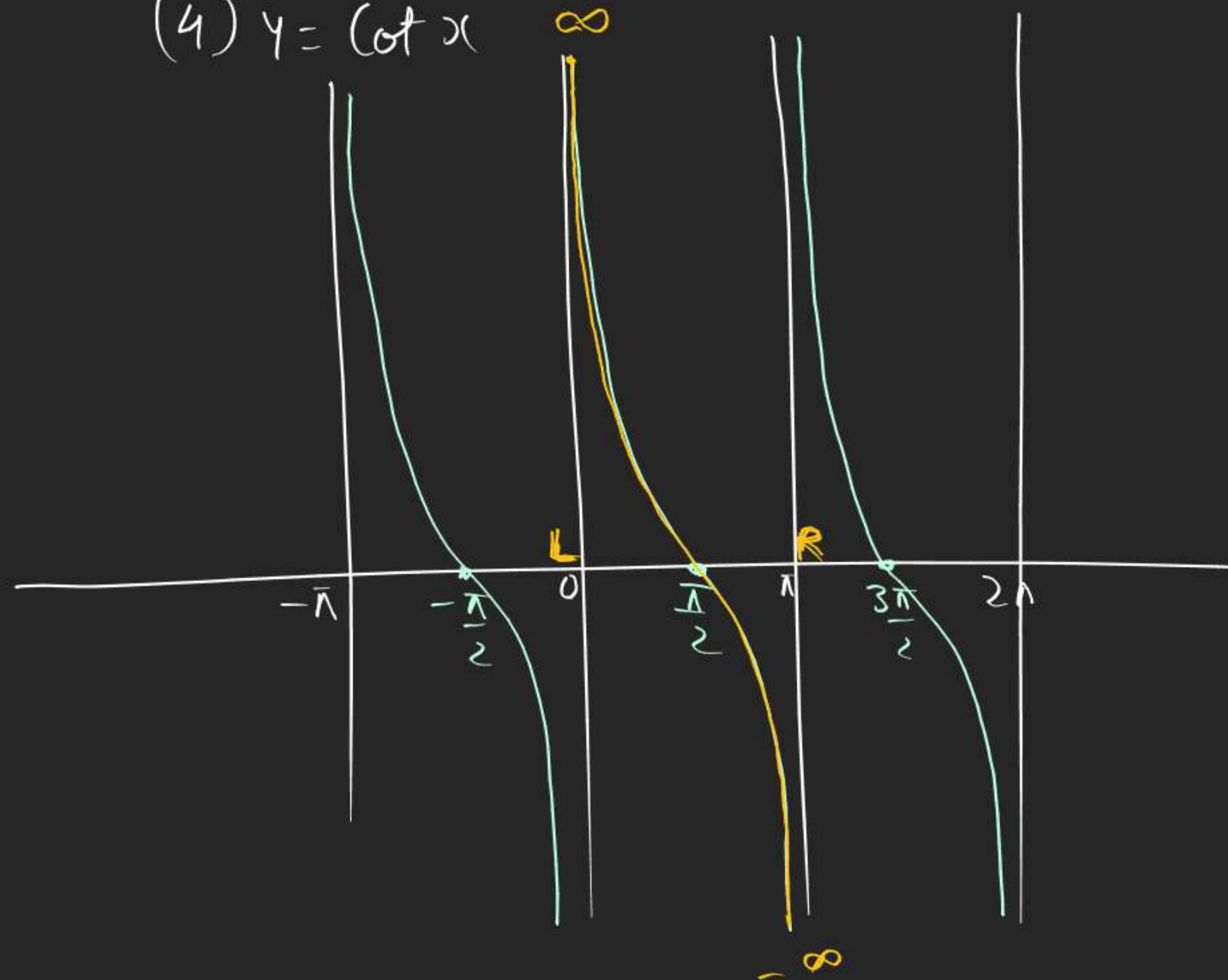
$$\tan(-2\pi) = 0$$

$$\tan n\pi = 0$$

Trigonometry

$\tan x$ ka diffⁿ always +ve \uparrow
 $\cot x$ ka diffⁿ always -ve \downarrow (graph Niche Kiaune Ayega)

(4) $y = \cot x$



(1) $\cot x = 0$ at
 $x = (2n+1)\frac{\pi}{2}$

(4) A) $y = \sin x$
 $\frac{dy}{dx} = \cos x$

(2) for $y = \tan x$

(B) $y = \cos x$
 $\frac{dy}{dx} = -\sin x$

Domain: $x \neq n\pi$
 $x \in (-\infty, \infty) - (n\pi)$

(C) $y = \tan x$

$\frac{dy}{dx} = \sec^2 x \geq 0$

(3) Range: $y \in (-\infty, \infty)$

(D) $y = \cot x$

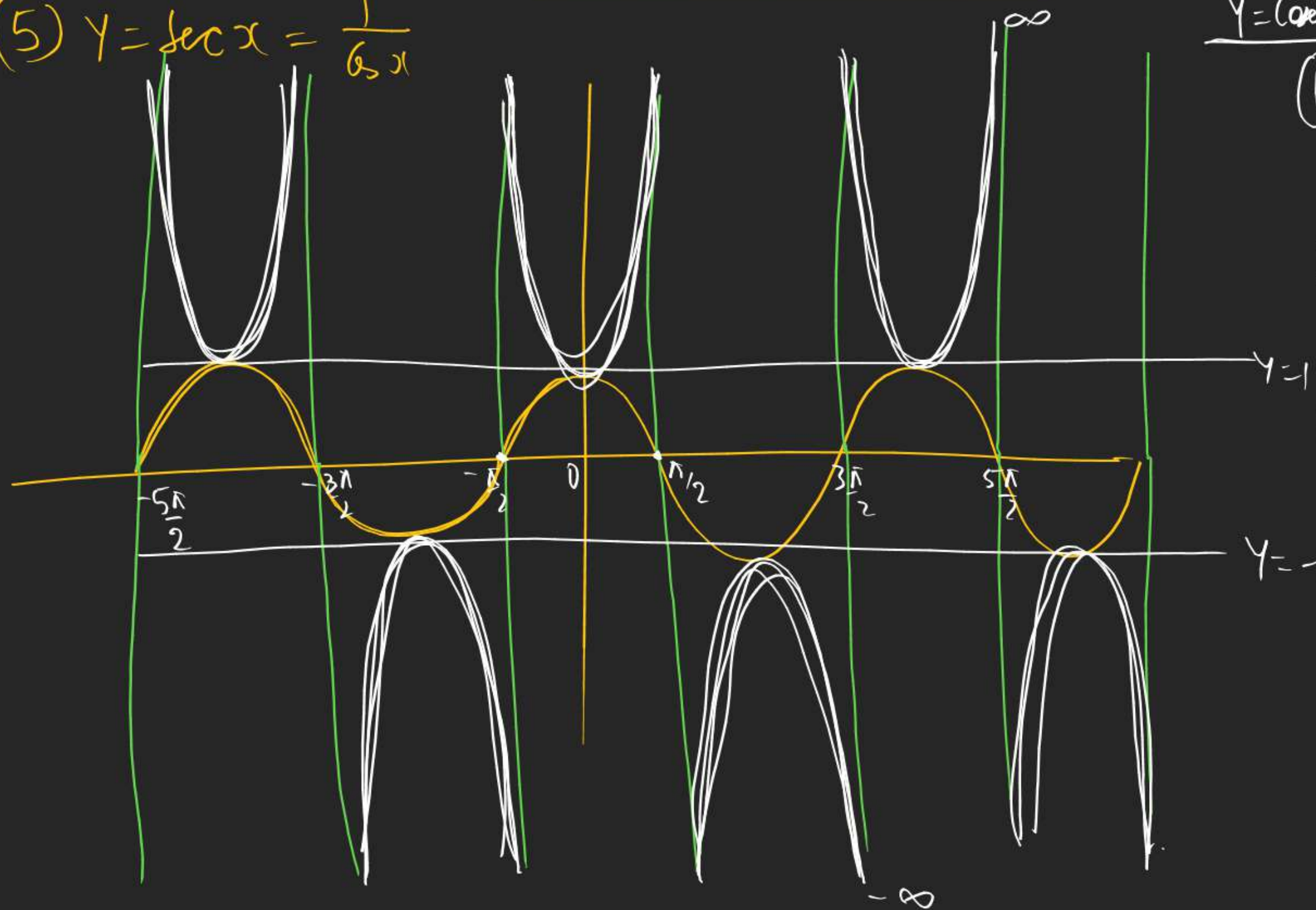
$\frac{dy}{dx} = -\csc^2 x \leq 0$

Trigonometry

HW → Search all Net

$y = \csc x \rightarrow \text{Copy}$

(5) $y = \sec x = \frac{1}{\cos x}$



① for $y = \sec x$

$$x \neq (2n+1)\frac{\pi}{2}$$

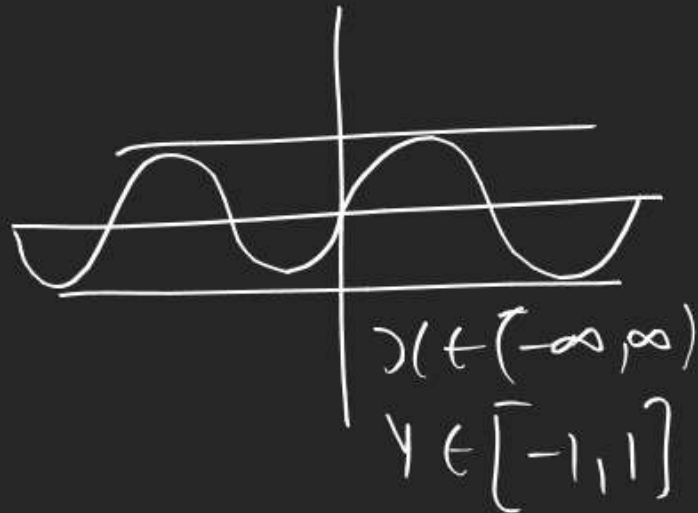
Dom — $x \in (-\infty, \infty) - (2n+1)\frac{\pi}{2}$

② $y \in (-\infty, -1] \cup [1, \infty)$

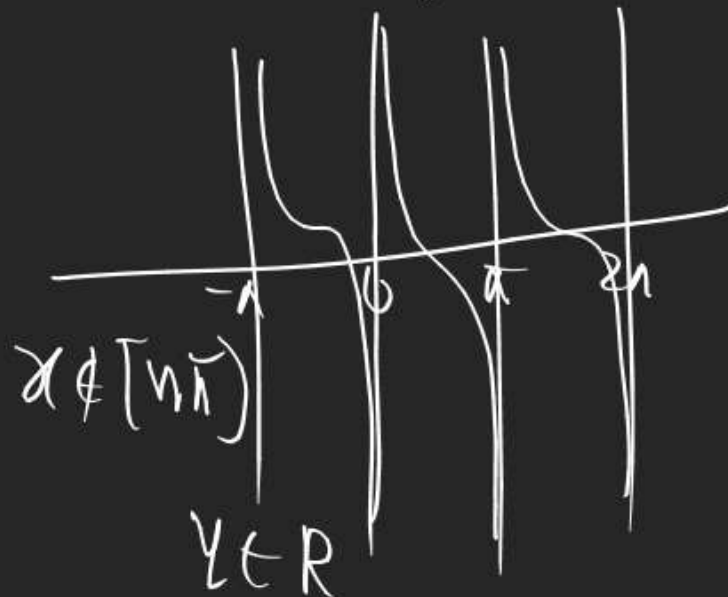
Range

Trigonometry

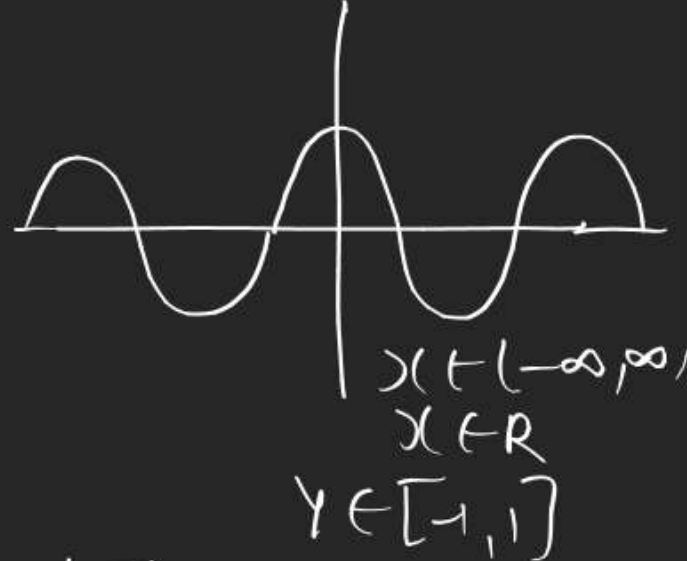
① $y = \sin x$



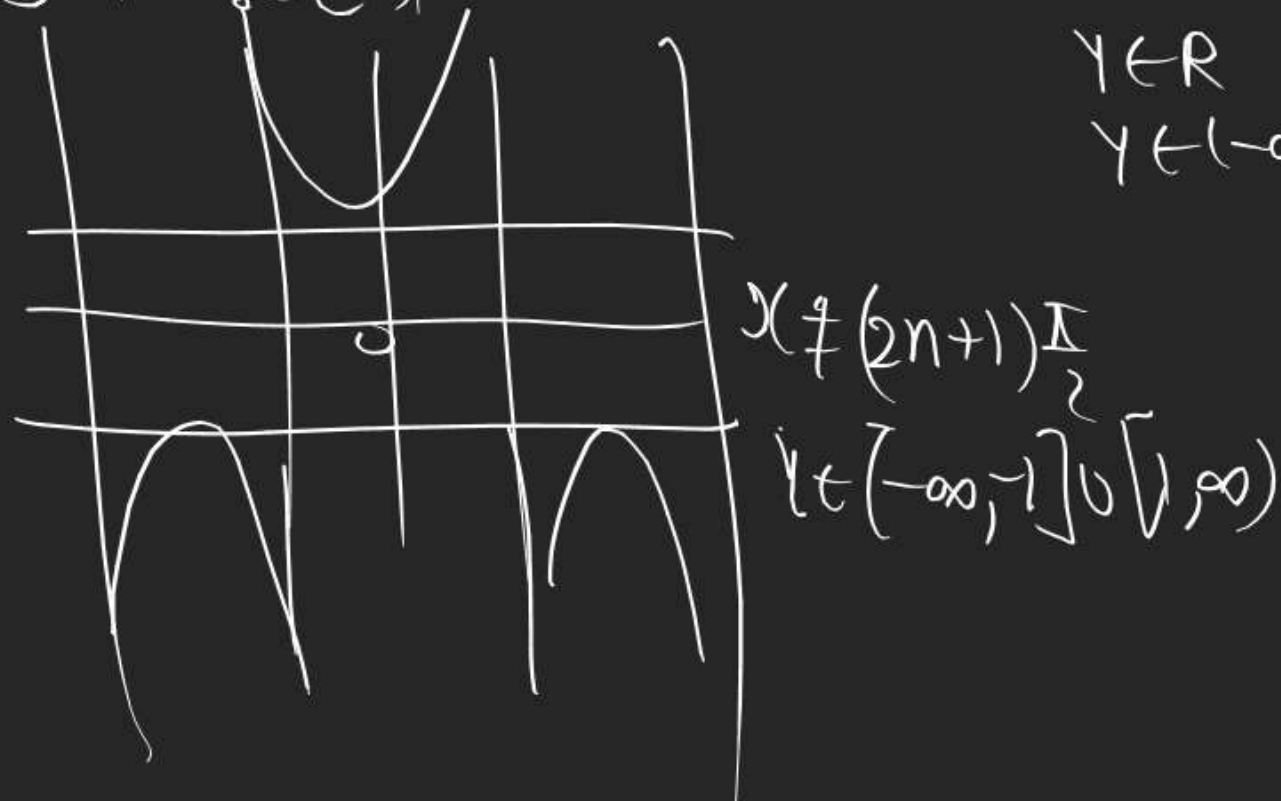
(4) $y = \cot x$



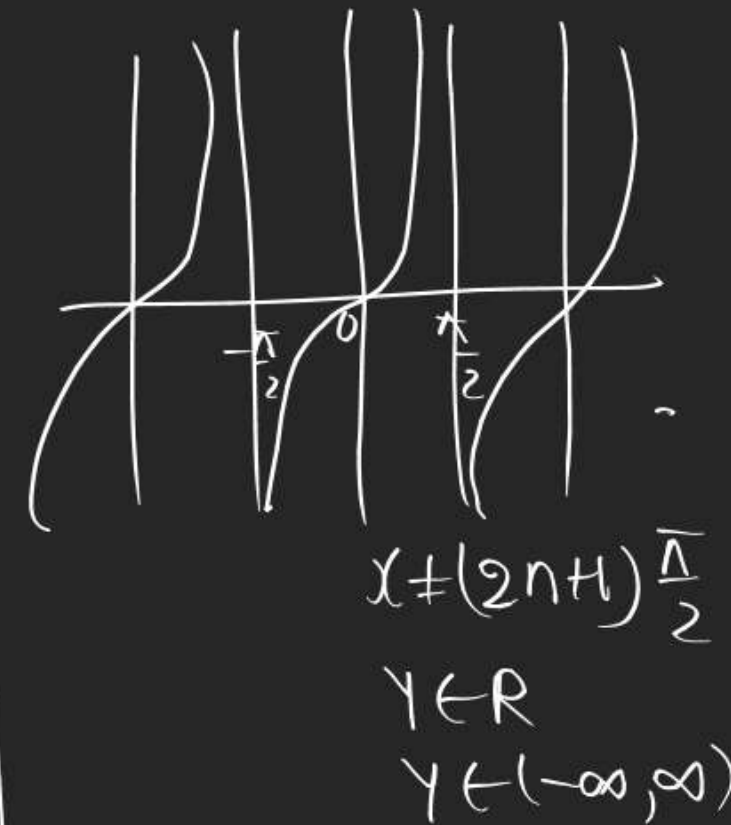
② $y = \cos x$



(5) $y = \sec x$



(3) $y = \tan x$



1) $\sin n\pi = 0$

2) $\cos n\pi = \pm 1$

3) $\cos(2n+1)\frac{\pi}{2} = 0$

Trigonometry

Adv → (concept)

$$y = \cos x = 0$$

$$x = (2n+1)\frac{\pi}{2}$$

$$y = \sin x = 0$$

$$x = n\pi$$

Q, $y = \sec x$'s Dm?

$$= \frac{1}{(\cos x)}$$

$$\cos x \neq 0$$

$$x \neq (2n+1)\frac{\pi}{2}$$

$$D_f: x \in \mathbb{R} - (2n+1)\frac{\pi}{2}$$

Q, $y = \tan x$ Domain

$$y = \frac{\sin x}{\cos x}$$

$$y = \sin x \times \frac{1}{(\cos x)} \rightarrow \cos x \neq 0$$

\mathbb{R}

$$x \neq (2n+1)\frac{\pi}{2}$$

$$x \in \mathbb{R} - (2n+1)\frac{\pi}{2}$$

$$\therefore x \in \mathbb{R} - (2n+1)\frac{\pi}{2}$$

Q, $y = \csc x$'s Dm?

$$= \frac{1}{\sin x}$$

$$\sin x \neq 0$$

$$x \neq n\pi$$

$$x = \mathbb{R} - (n\pi)$$

Dm

Trigonometry

Range of Trigo fxn.

1) $y = \sin x$'s R_f ?



$$y \in [-1, 1]$$

2) $y = \cos x$'s R_f ?



$$y \in [-1, 1]$$

3) $y = \tan x$'s R_f ?

$$y \in \mathbb{R}$$

$$y \in (-\infty, \infty)$$

Concept

$$x \rightarrow ax + b$$

Range Remains Same

(1) $y = \sin(3x)$'s R_f ?

$$y \in [-1, 1]$$

(2) $y = \sin(3x - 25)$'s R_f ?

$$y \in [-1, 1]$$

(3) $y = \sin\left(\frac{x}{2} + \frac{7}{19}\right)$'s R_f

$$y \in [-1, 1]$$

Trigonometry

Q₁₀ $y = \tan(3x-7)$ is \mathbb{R}_+ ?

Same as $y = \tan x$

$y \in \mathbb{R}$



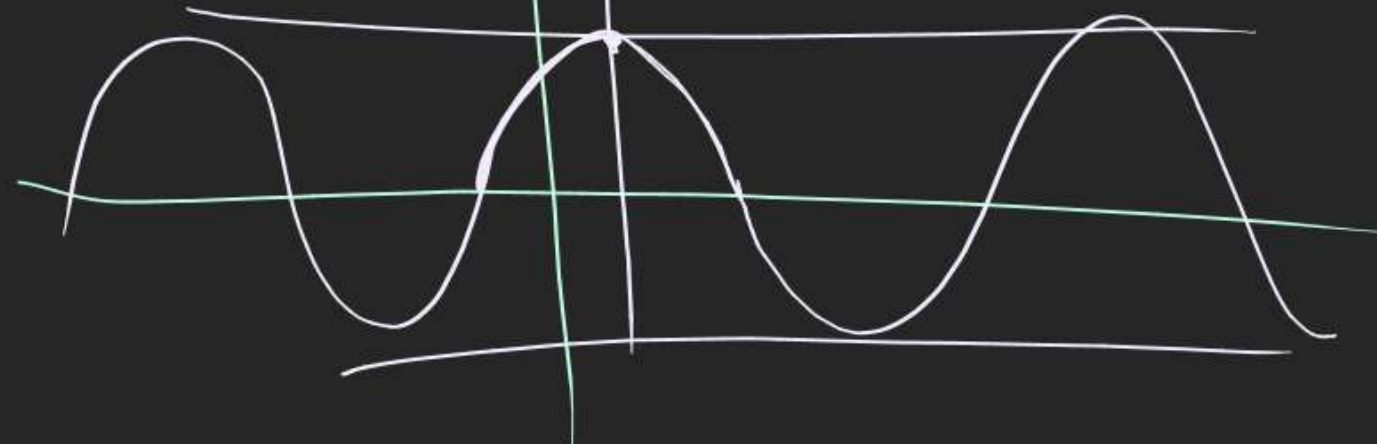
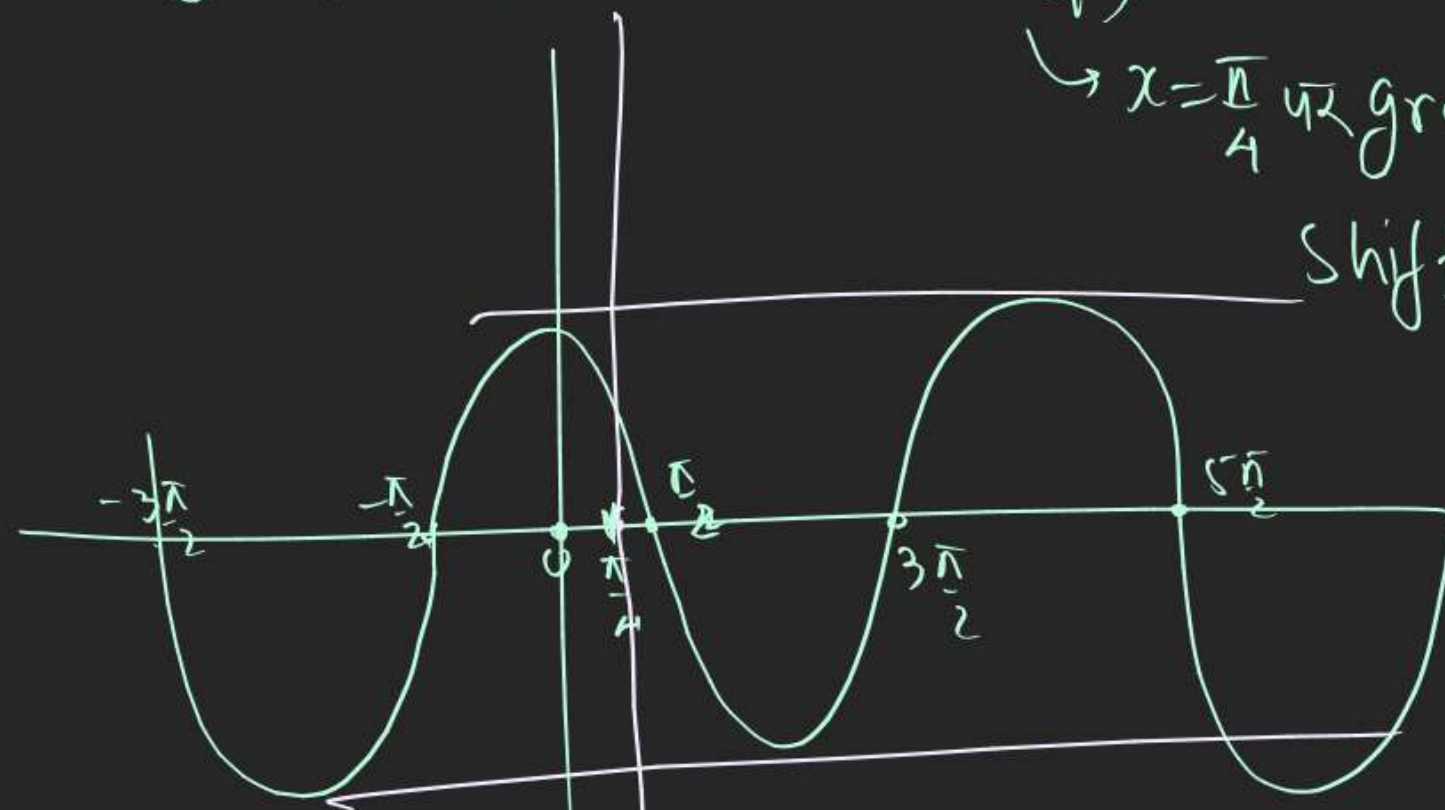
Q₁₁ $y = \cos(x - \frac{\pi}{4})$ is \mathbb{R}_+ ?

$y \in [-1, 1]$

① $y = \cos x$ & $y = \cos(x - \frac{\pi}{4})$

$\rightarrow x = \frac{\pi}{4}$ is graph

Shift



Trigonometry

Concept

$$1) ()^2 \geq 0$$

$$2) ()^2 = 0 \text{ Kab Banata hai?}$$

Ans. When fcn Inside Becomes Zero.

$$Q_{12} \quad y = \left(\sin x - \frac{3}{4} \right)^2 \text{ (can give value 0 or not?)}$$

\Downarrow

If $\sin x = \frac{3}{4}$ is PSBL then $\left(\sin x - \frac{3}{4} \right)^2$ will be 0.

$$-1 \leq \frac{3}{4} \leq 1 \text{ as } \frac{3}{4} \text{ comes bet}^n -1 \text{ to } +1 \text{ hence } \sin x = \frac{3}{4} \text{ Possible}$$

$$\Rightarrow \left(\sin x - \frac{3}{4} \right)^2 = 0 \text{ PSBL}$$



$$Q \quad y = (\cos x - 2)^2 \text{ (can give 0 or not?)}$$

$$\text{If } \cos x - 2 = 0 \text{ PSBL then it is PSBL.}$$

$$\Rightarrow \cos x = 2$$



$$-1 \leq 2 \leq 1$$

Not PSBL.



$$\Rightarrow (\cos x - 2)^2 = 0 \text{ Not PSBL}$$

Trigonometry

Q $(\tan x - 2)^2$ Can give 0 or not?

14

Is $\tan x = 2$ possible??

yes!!!

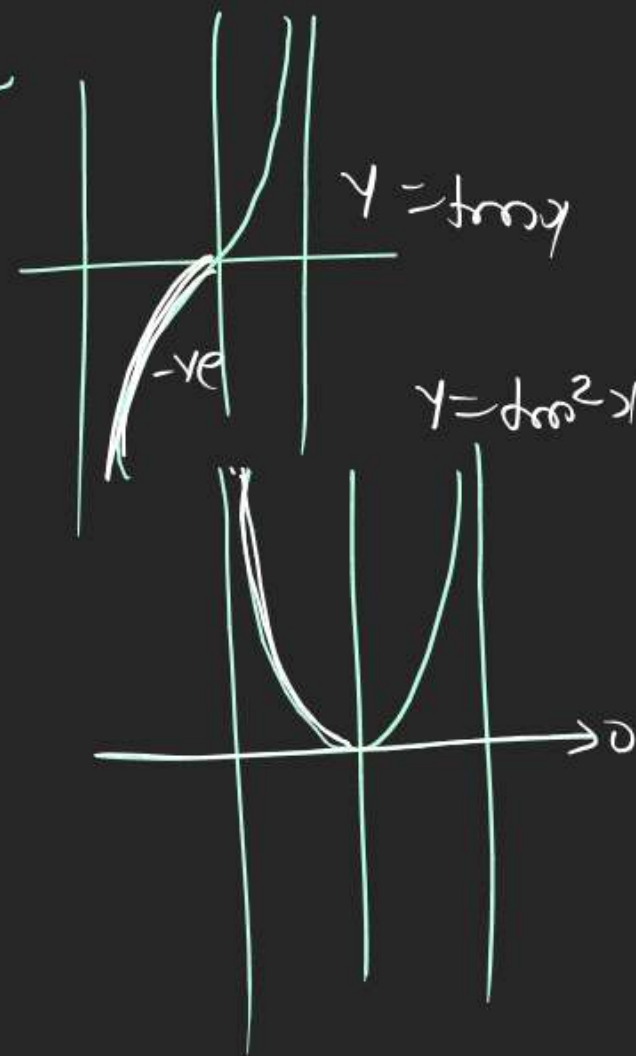
as $\tan x$'s Range $\in (-\infty, \infty)$



Q $y = \sin^2 x$'s Rf

16

$\infty > \sin^2 x \geq 0$



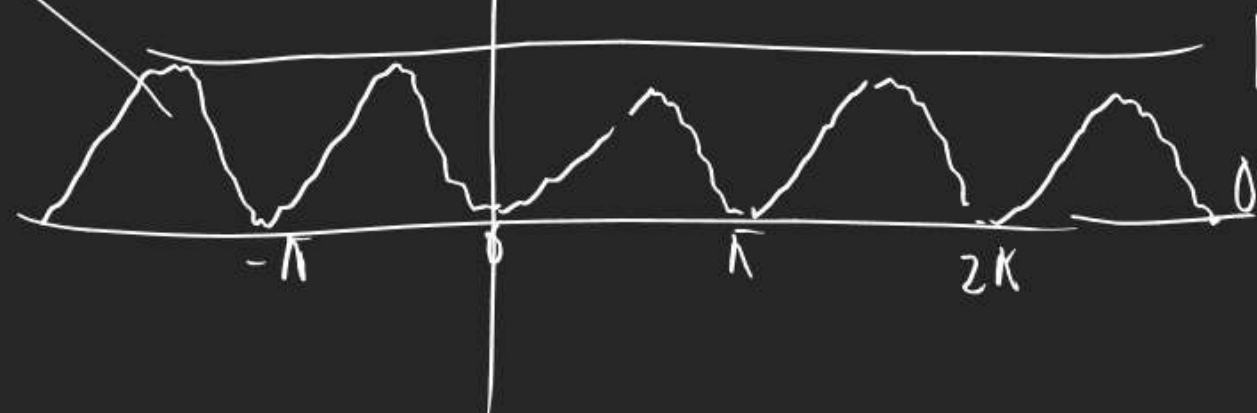
Q 15 $y = \sin^2 x$'s Rf?

① $\sin^2 x \geq 0$

② $-1 \leq \sin x \leq 1$

$\Rightarrow 0 \leq \sin^2 x \leq 1$

Adv $y = \sin^2 x$

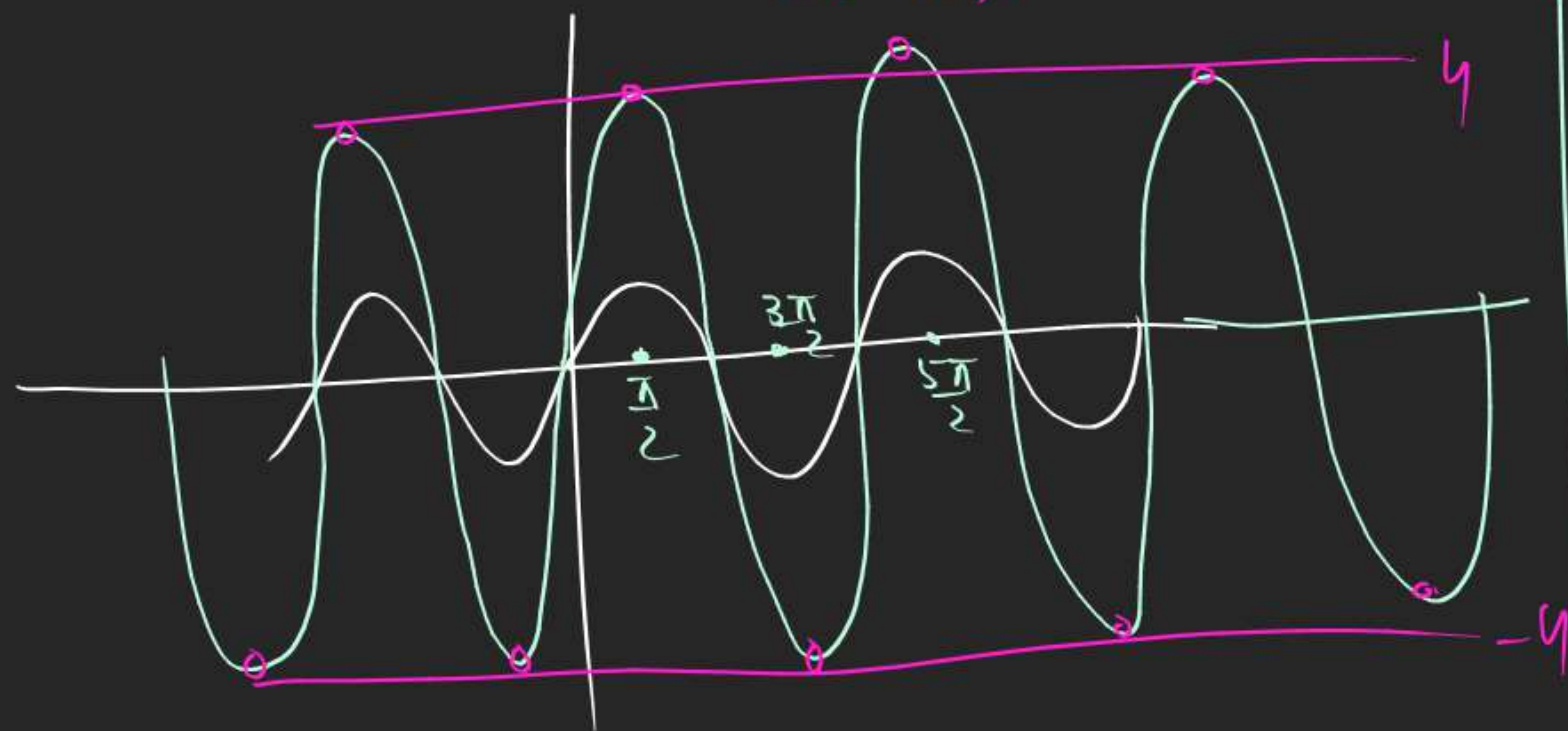


Trigonometry

Q. 17 $y = 4 + \tan x \cdot \cos x$'s R.f?

$$y = 4 \frac{\sin x}{\cos x} \times \cancel{\cos x} \quad (\cos x \neq 0)$$

$$y = 4 \sin x \quad (-4, 4) \quad x \neq (2n+1)\frac{\pi}{2}$$



Q. 18 R.f of $y = \cos^2(\frac{x}{4}) - \sin^2(\frac{x}{4})$

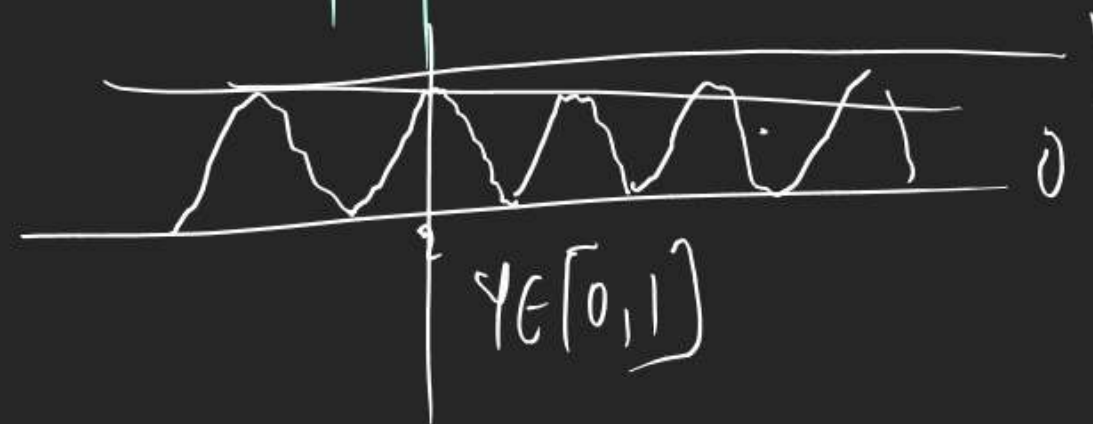
Yad Aap $\rightarrow \cos^2 \theta - \sin^2 \theta = \cos 2\theta$

$$y = \cos^2(\frac{x}{4}) - \sin^2(\frac{x}{4}) = \cos \frac{x}{2}$$

$$R.f \in [-1, 1]$$

Q. $y = \cos^2(x-2)$ R.f?

$\rightarrow x-2$ shift



$$y \in [0, 1]$$

Trigonometry

Q $y = \sqrt{\sin x}$'s \mathbb{R}_+ ?

(concept: $\sqrt{\quad}$ gives +ve quantities only)

$$\sqrt{\sin x} \in [0, 1]$$

Q $y = \sin \sqrt{x}$'s \mathbb{R}_+ ?

$y \in [-1, 1]$

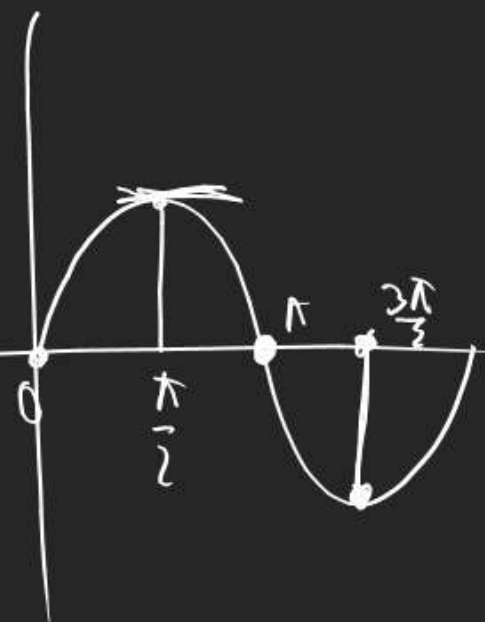
$$\begin{cases} x = \frac{\pi^2}{4} \rightarrow y = \sin \sqrt{\frac{\pi^2}{4}} = \sin \frac{\pi}{2} = 1 \\ x = \frac{9\pi^2}{4} \rightarrow y = \sin \sqrt{\frac{9\pi^2}{4}} = \sin \frac{3\pi}{2} = -1 \\ x = \pi^2 \rightarrow y = \sin \sqrt{\pi^2} = \sin \pi = 0 \end{cases}$$

Q $y = 2 \sin x$'s \mathbb{R}_+

$$-1 \leq \sin x \leq 1$$

$$-2 \leq 2 \sin x \leq 2$$

$$\therefore y \in [-2, 2]$$



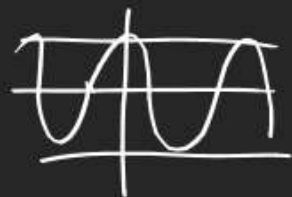
Trigonometry

Q $y = \frac{1}{\sqrt{2}} \cos x$'s Range.

$$-1 \leq \cos x \leq 1$$

$$-\frac{1}{\sqrt{2}} \leq \frac{1}{\sqrt{2}} \cos x \leq \frac{1}{\sqrt{2}}$$

$$\therefore y \in \left[-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right]$$



Q $y = \sin^2\left(\frac{15\pi}{8} - 4x\right) - \sin^2\left(\frac{17\pi}{8} - 4x\right)$'s Range?

This matches to $\sin^2 A - \sin^2 B$

$$= \sin(A+B) \sin(A-B)$$

$$\sin\left\{\left(\frac{15\pi}{8} - 4x\right) + \left(\frac{17\pi}{8} - 4x\right)\right\} \cdot \sin\left\{\left(\frac{15\pi}{8} - 4x\right) - \left(\frac{17\pi}{8} - 4x\right)\right\}$$

$$+ \sin\left(\frac{32\pi}{8} - 8x\right) \cdot \sin\left(-\frac{2\pi}{8}\right)$$

$$y = + \sin \frac{\pi}{4} \sin(8x) = \frac{1}{\sqrt{2}} \boxed{\sin 8x} \text{ 's Range}$$

$$-1 \leq \sin 8x \leq 1$$

$$-\frac{1}{\sqrt{2}} \leq \frac{1}{\sqrt{2}} \sin 8x \leq \frac{1}{\sqrt{2}} \therefore y \in \left[-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right]$$