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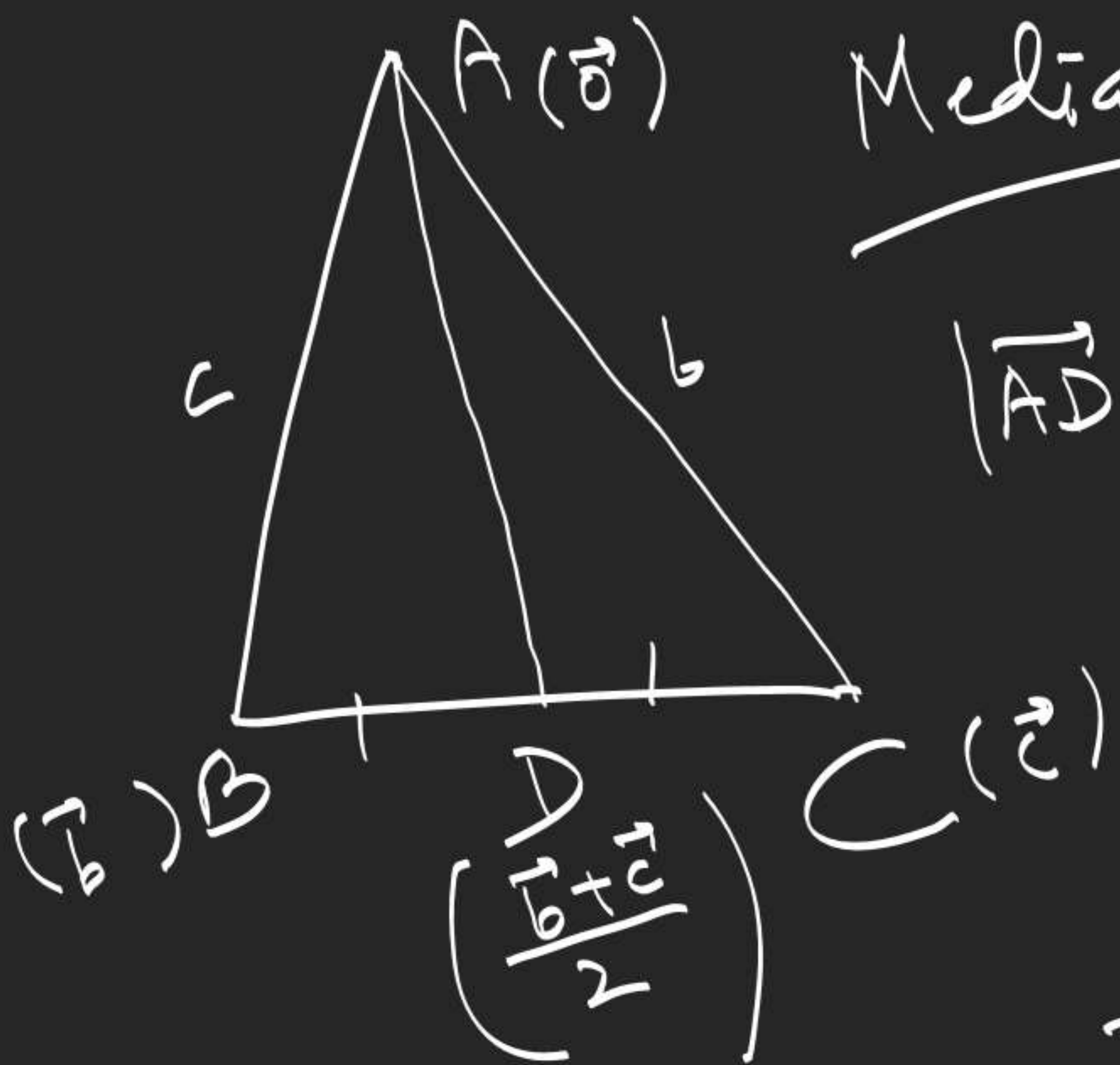


Median

$$|\vec{AD}| = \frac{1}{2}|\vec{b}+\vec{c}|$$

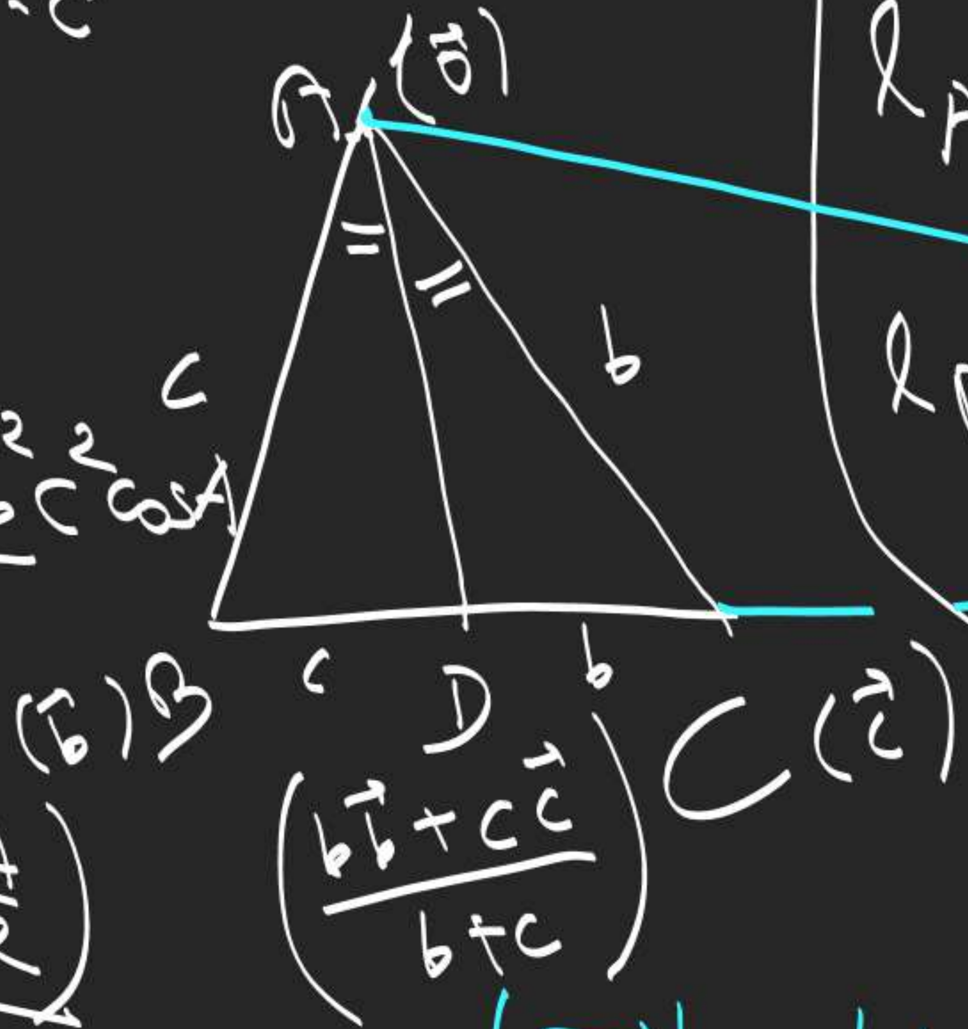
$$AD^2 = \frac{1}{4}\left[c^2 + b^2 + \underline{2cb\cos A}\right]$$

$$= \frac{1}{4}\left(c^2 + b^2 + b^2 + c^2 - a^2\right)$$

Angle Bisector

$b^2|\vec b|^2 + c^2|\vec c|^2 + 2bc\,\vec b\cdot\vec c$

$$|\vec{AD}| = \left|\frac{b\vec b + c\vec c}{b+c}\right|$$



$$AD^2 = \frac{b^2c^2 + c^2b^2 + 2b^2c^2\cos A}{(b+c)^2}$$

$$= \frac{2b^2c^2\left(2\cos^2\frac{A}{2}\right)}{(b+c)^2}$$

$$AD = \frac{2bc}{(b+c)}\cos\frac{A}{2}$$

$$l_A = \frac{2bc}{b+c}\cos\frac{A}{2}$$

$$l_B = \frac{2ca}{c+a}\cos\frac{B}{2}$$

$$l_C = \frac{2ab}{a+b}\cos\frac{C}{2}$$

$$|\vec{AE}| = \left|\frac{b\vec b - c\vec c}{b-c}\right| \qquad \frac{b\vec b - c\vec c}{b-c}$$

$$= \frac{2bc\sin\frac{A}{2}}{|b-c|}$$

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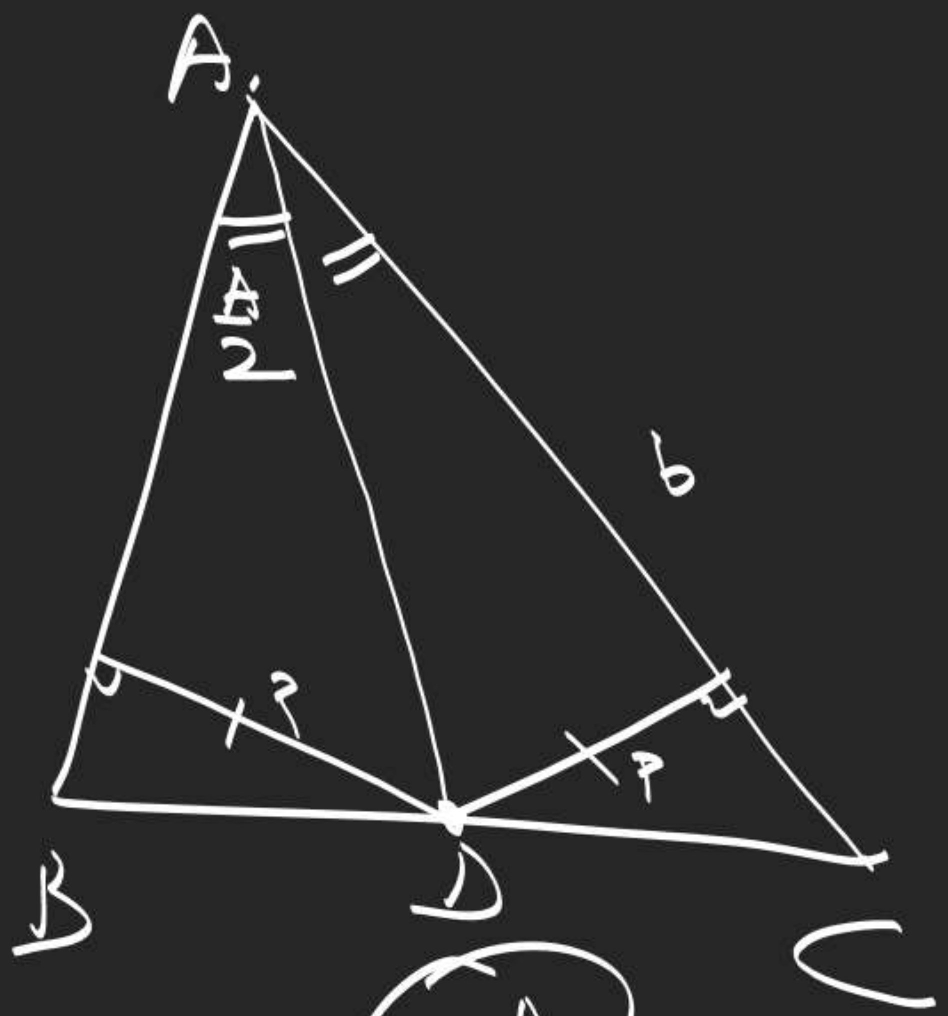
ADB + ADC = ABC

$$\frac{1}{2} p(c+b) = \Delta$$

$$p = \frac{2\Delta}{b+c}$$



$$AD = \frac{p}{\sin\frac{A}{2}}$$

$$= \frac{2\Delta}{(b+c)\sin\frac{A}{2}} = \frac{bc\sin A}{(b+c)\sin\frac{A}{2}} = \frac{2bc\cos\frac{A}{2}}{(b+c)}$$

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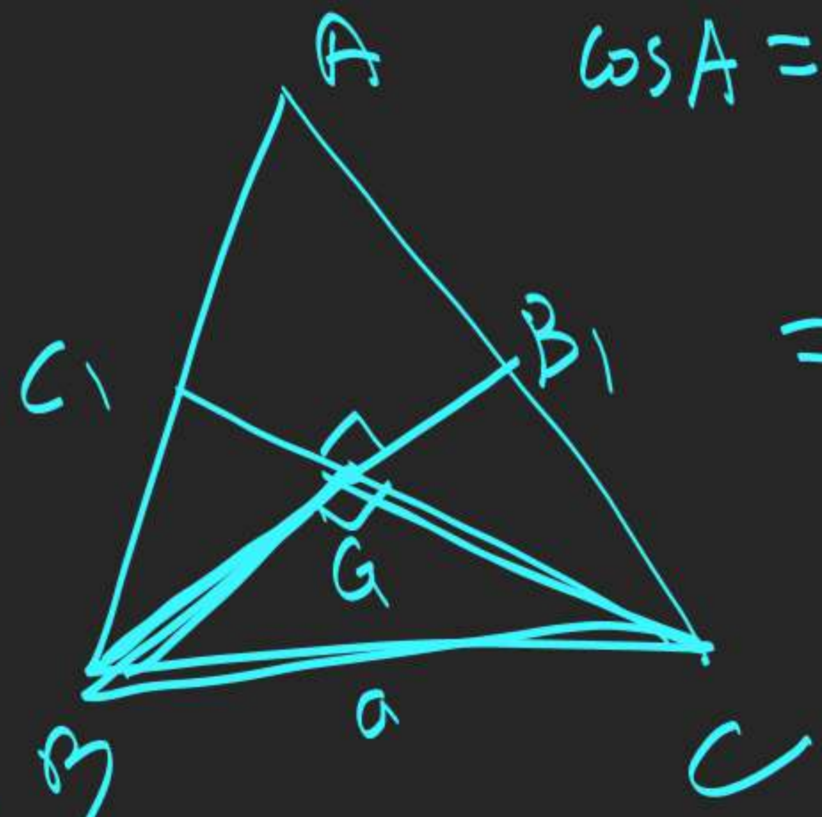
1. Let medians BB_1 and CC_1 of ΔABC are $\perp$ar to eachother. Then P.T.

$$\cos A = \frac{2a^2}{bc}$$



$$\cos A = \frac{b^2+c^2-a^2}{2bc} = \frac{4a^2}{2bc} = \frac{2a^2}{bc}.$$

$$a^2 = \left(\frac{2}{3}BB_1\right)^2 + \left(\frac{2}{3}CC_1\right)^2$$

$$a^2 = \frac{2(c^2+a^2)-b^2}{9} + \frac{2(a^2+b^2)-c^2}{9}$$

$$5a^2 = \underline{c^2+b^2}$$

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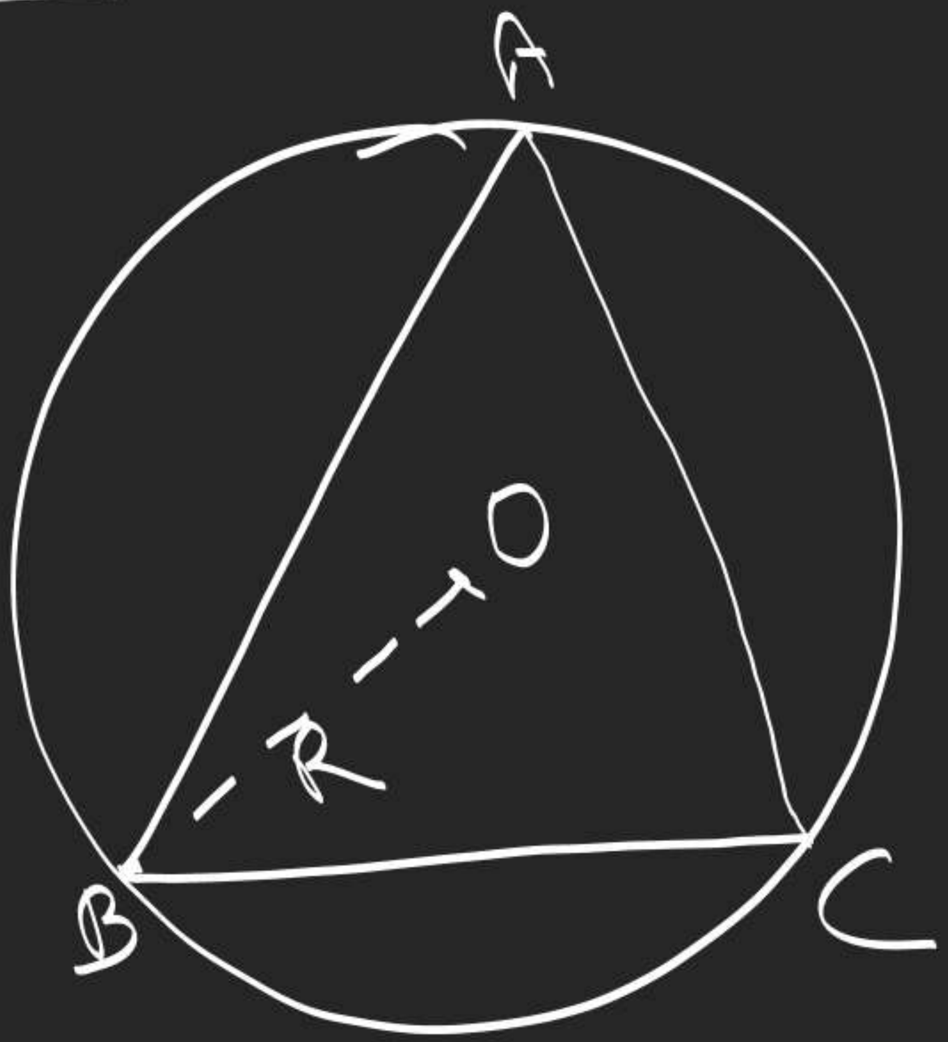
Circumcircle

$$\boxed{\begin{gathered}\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R \\ \frac{abc}{4R} = \Delta\end{gathered}}$$



$$\Delta = \frac{1}{2} bc \sin A = \frac{1}{2} bc\left(\frac{a}{2R}\right) = \frac{abc}{4R}$$

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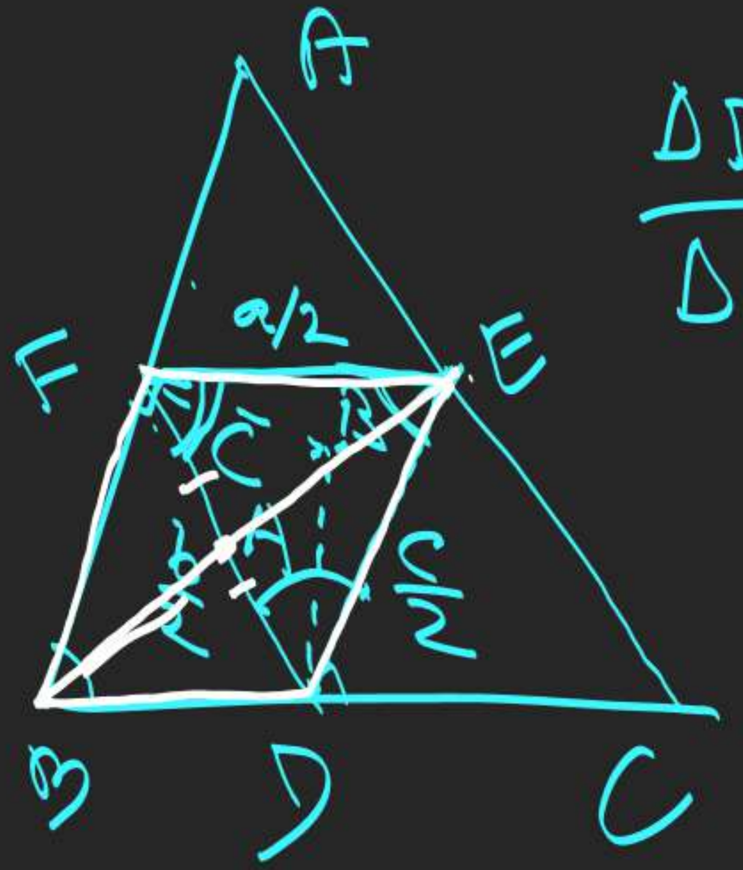
1. If the foot of $\perp$ ars from circumcentre of ΔABC on sides BC, CA, AB are D, E, F respectively. Solve ΔDEF.



$$\frac{\Delta DEF}{\Delta ABC} = \left(\frac{FE}{BC}\right)^2 = \frac{1}{4}.$$

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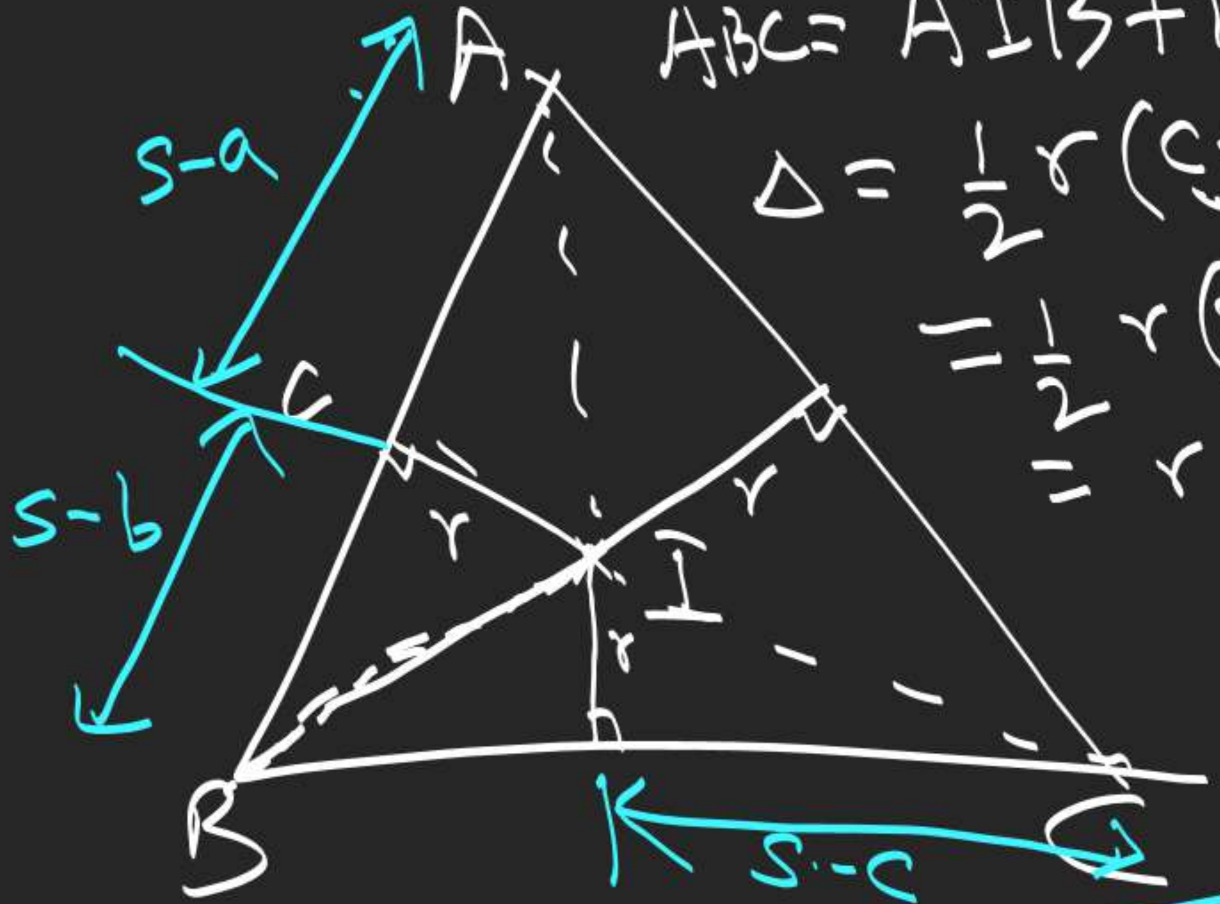
Incircle



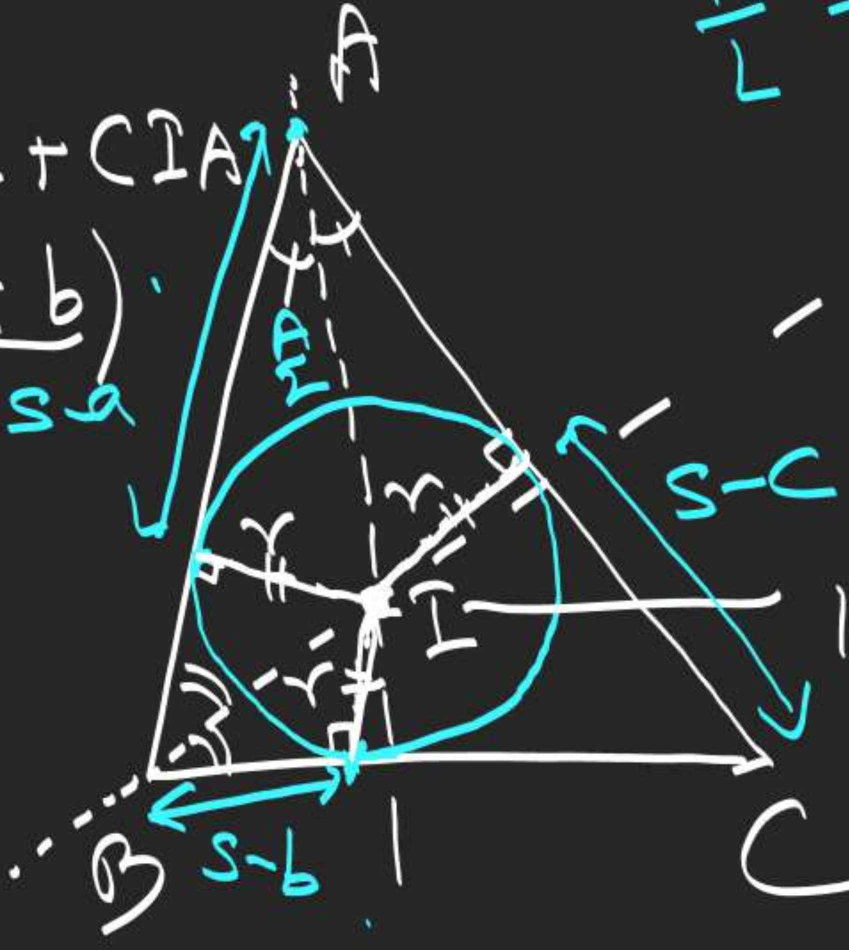
$ABC = AIB + BIC + CIA$

$\Delta = \frac{1}{2} r(c+a+b)$

$= \frac{1}{2} r(2s)$

$= rs$

$$\boxed{\frac{\Delta}{s} = r}$$



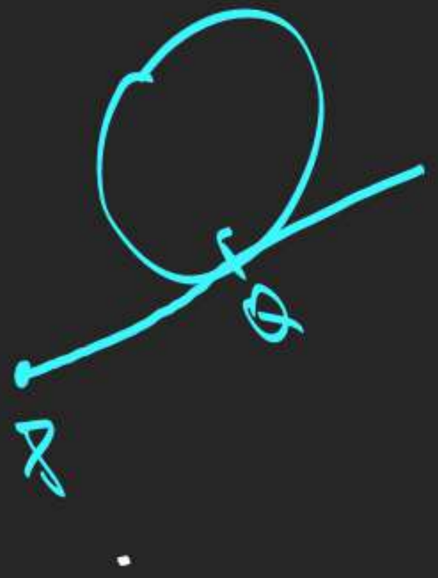
$\frac{r}{L} = \tan\frac{A}{2}$

$L = r\cot\frac{A}{2}$

$= r\,\frac{s(s-a)}{\Delta}$

$= \frac{\cancel{\Delta}(s-a)}{\cancel{\Delta}}$

$= s-a.$



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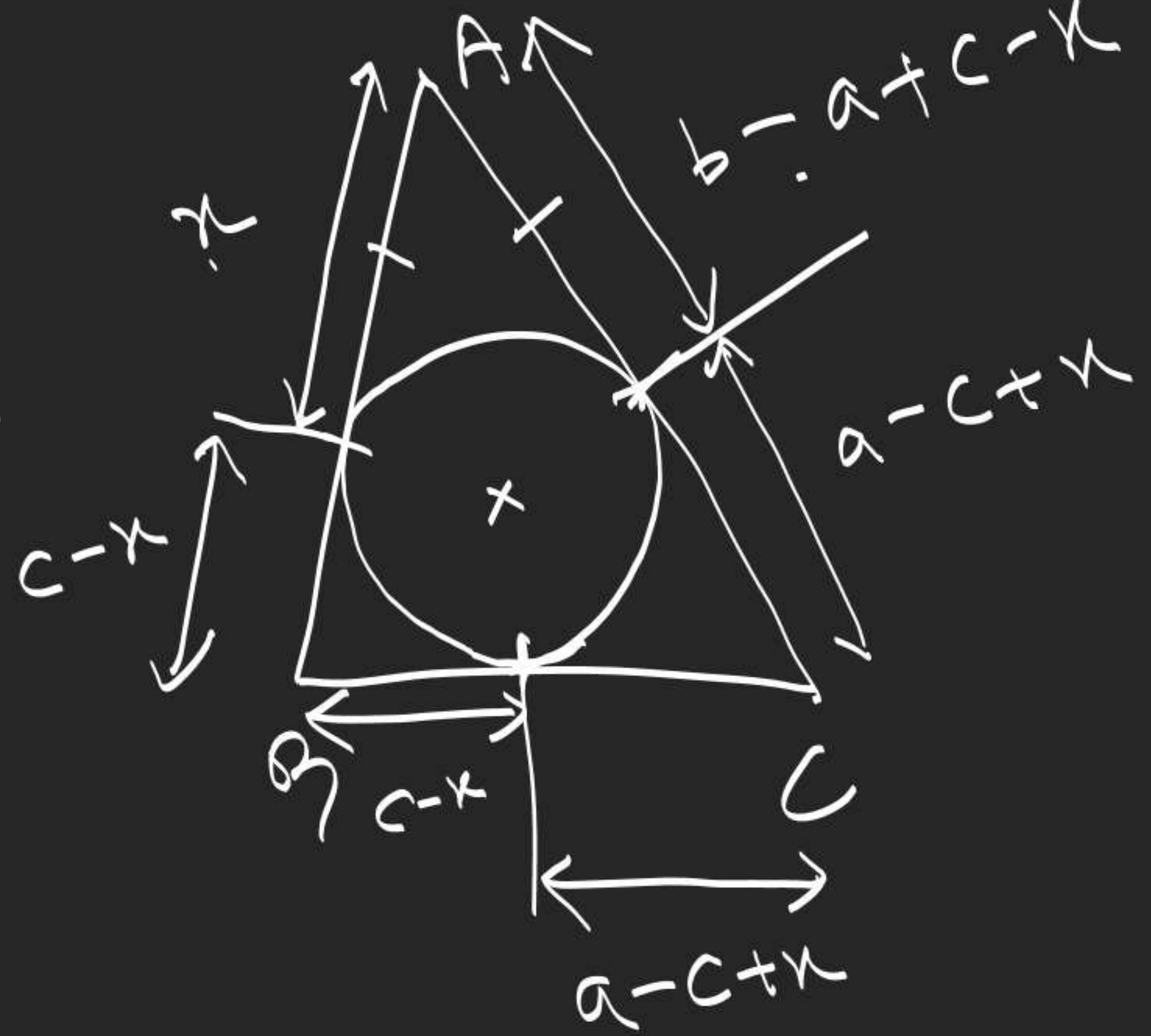
$$x = \underline{b - a + c - x}$$

$$2x = 2s - 2a$$

$$\boxed{x = s - a}$$



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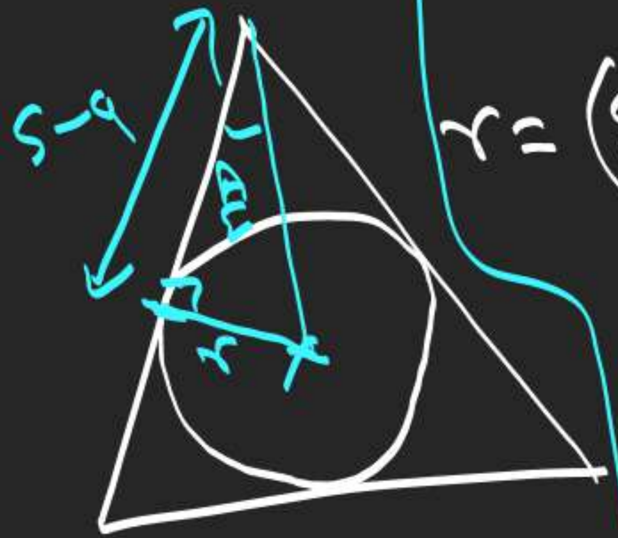
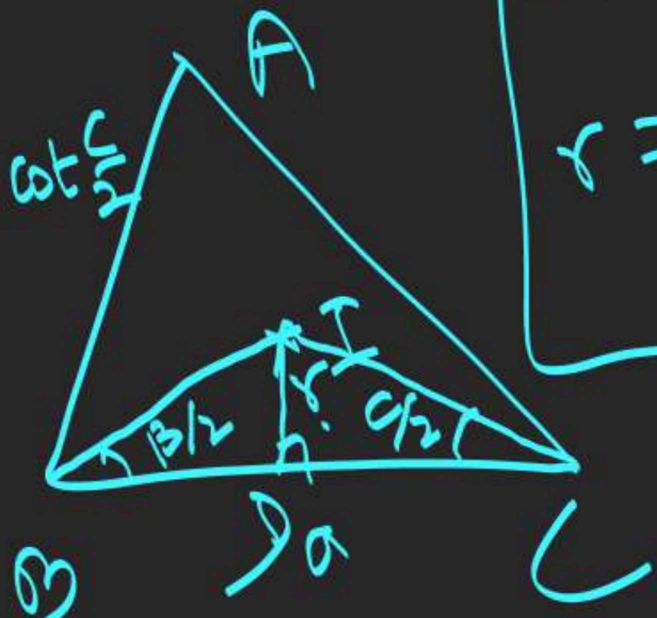


$$\boxed{\begin{aligned} r &= (s-a)\tan\frac{A}{2} \\ &= (s-b)\tan\frac{B}{2} \\ &= (s-c)\tan\frac{C}{2} \end{aligned}}$$

$$\boxed{\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2} \le \frac{1}{8}}$$

$\boxed{R, r}$



$$\boxed{r = 4R\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}}$$

$$2R\sin A = a = BD + DC = r\cot\frac{B}{2} + r\cot\frac{C}{2}$$

$$4R\sin\frac{A}{2}\cancel{\cos\frac{A}{2}} = \frac{r\,\cancel{\sin\left(\frac{B+C}{2}\right)}}{\sin\frac{B}{2}\sin\frac{C}{2}}$$

$$\boxed{\frac{r}{R} \le \frac{1}{2}}$$

$\frac{r}{R} = \frac{1}{2}$ if
Δ is equilateral

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Seq.

Ex-4 $\rightarrow$ (Complete.)

Ex-2 (11-15)