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6.

$y = 1$

$$(f(x))^2 = x(f(1))^2$$

$x = 2.$

$$6^2 = 2(f(1))^2$$

$$18 = (f(1))^2$$

$$\boxed{(f(x))^2 = 18x}$$

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7. (i) $y = 2$

$$f(x) = x + 2$$

$$\frac{2\pi}{\left(\sum_{r=1}^{n} r^2\right)} = \frac{\pi}{7}$$

$$\frac{n(n+1)(2n+1)}{} = 3 \times 4 \times 7$$

$$n = 3$$

11.

(i) $f(x) = \begin{cases} e^{-\sqrt{|\ln\{x\}|}} - e^{\frac{1}{\sqrt{|\ln\{x\}|}}}, & x \notin I \\ e^{-\sqrt{\ln\{x\}}} - e^{-\sqrt{|\ln\{x\}|}} = 0, & x \in I \end{cases}$

$\ln\{x\} = -|\ln\{x\}|$

$\{x\} = 0$

$(f(x))^{g(x)} = e^{g(x)\ln f(x)}$

$\{x\}^{\sqrt{\frac{1}{|\ln\{x\}|}}} = \frac{x}{\sqrt{x}} = \sqrt{x}$

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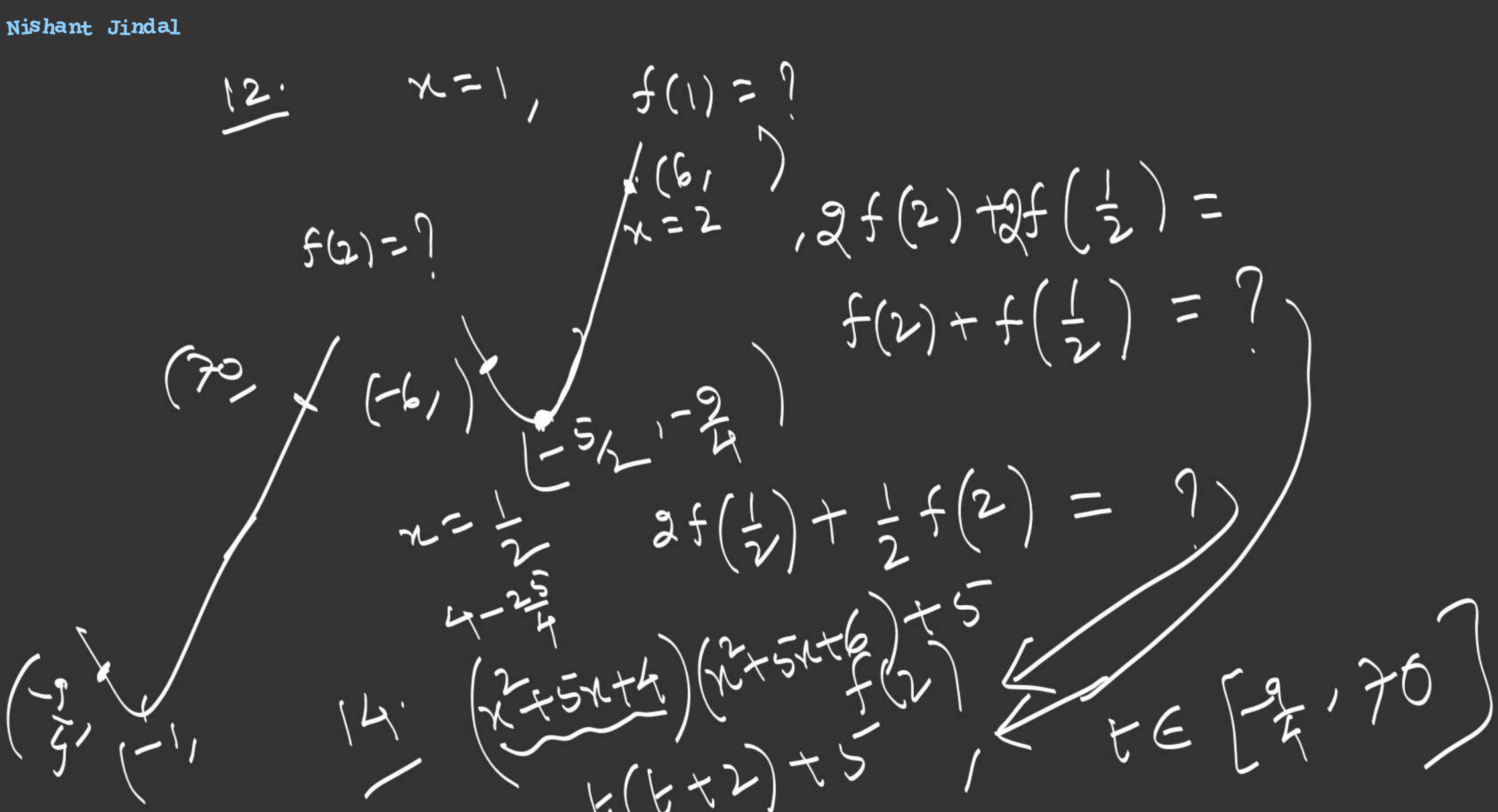
12. $x=1$, $f(1)=?$



$x=\frac{1}{2}$

$4-2\frac{5}{4}$

$2f(2)+2f\left(\frac{1}{2}\right)=$

$f(2)+f\left(\frac{1}{2}\right)=?$

$2f\left(\frac{1}{2}\right)+\frac{1}{2}f(2)=?$

14. $\left(x^2+5x+4\right)\left(x^2+5x+6\right)f(2)+5$

$t(t+2)+5$

$t \in \left[-\frac{9}{4}, 70\right]$

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15. $f(a+x) = b + \left((b-f(x))^3 + 1 \right)^{1/3}$ $\Rightarrow (b-f(a+x))^3$

$f(x+2a) = b + \left((b-f(x+a))^3 + 1 \right)^{1/3}$ $= -\left((b-f(x))^3 + 1 \right)$

$= f(x)$

$\swarrow\!\!\swarrow$

$1 + (b-f(x+a))^3 = -(b-f(x))^3$

$\left(1 + (b-f(x+a))^3 \right)^{1/3} = -(b-f(x))$

$\left(1 + (b-f(a+x))^3 \right)^{1/3} + b = f(x)$

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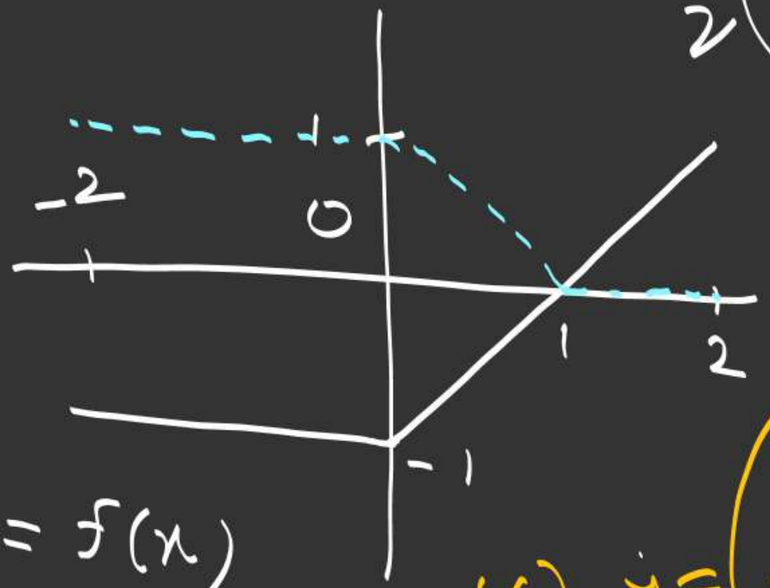
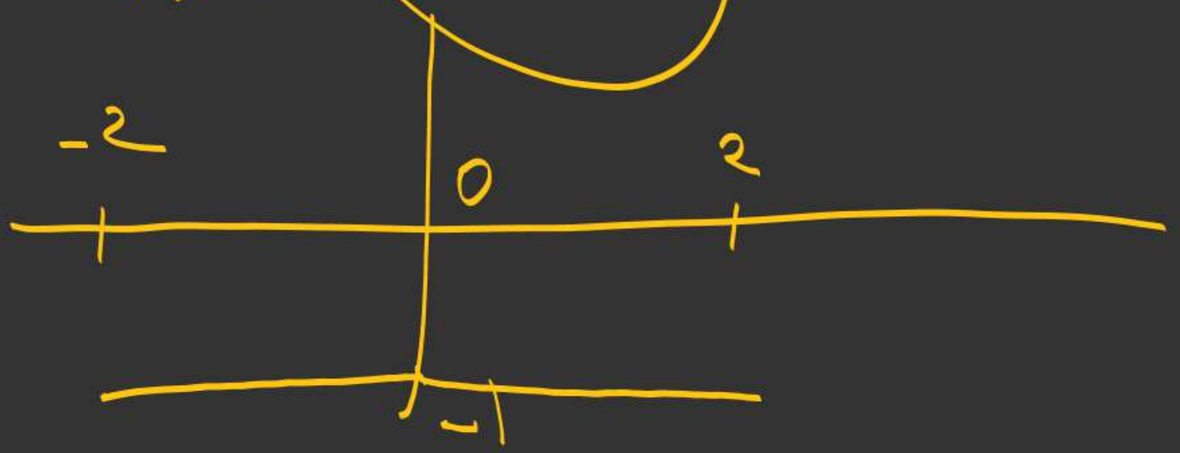
16. (d)

$$\frac{1}{2}\left(|f(x)| - f(x)\right) \in \begin{cases} 0 & x \in [1,2] \\ -f(x) & x \in [-2,1] \end{cases}$$



$y = f(x)$

(c) $y = f(-|x|) = f(x)$, $x \le 0$ → Even



17. (d) $\sqrt{x^2+12} > 2x$

$x < 0$ or $x \geq 0$ ✓

$x^2 + 12 > 4x^2$

$x^2 < 4$

$(-2, 2)$

$[0, 2)$

$x \in (-\infty, 0)$

$D_f = (-\infty, 2)$

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2. $3 < \log_4 9 + \log_9 28 < 4$

$\log_4 8 < \log_4 9 < \log_4 16 \Rightarrow \frac{3}{2} < \log_4 9 < 2$

$\log_9 27 < \log_9 28 < \log_9 81 \Rightarrow \frac{3}{2} < \log_9 28 < 2$

3. (a) $\left(f(1)+\cdots+f(n-1)\right)+f(n) = n^2 f(n)$

$$(n-1)^2 f(n-1) + f(n) = n^2 f(n)$$

$$\left(\frac{n-1}{n+1}\right) f(n-1) = f(n)$$

$$f(2004) = \frac{2003}{2005} f(2003) = \frac{2003}{2005} \times \frac{2002}{2004} f(2002)$$

$$= \frac{2003}{2005} \times \frac{2002}{2004} \times \frac{2001}{2003} f(2001)$$

$$\vdots$$

$$= \frac{2003}{2005} \times \frac{2002}{2004} \times \frac{2001}{2003} \times \frac{2000}{2002} \times \cdots \times \frac{2}{4} \times \frac{1}{3} f(1)$$

(b) $x>0$, $a, b>0$

$$\sqrt{x^2+ax}-\sqrt{x^2+bx}=\frac{(a-b)x}{\sqrt{x^2+ax}+\sqrt{x^2+bx}}$$

$$=\frac{a-b}{\left(\sqrt{1+\frac{a}{x}}+\sqrt{1+\frac{b}{x}}\right)}<\frac{a-b}{2}=1$$

$\sqrt{1+\frac{a}{x}}>1$, $\sqrt{1+\frac{b}{x}}>1$

$\sqrt{x^2}=|x|=x \quad x>0$

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Inverse Trigonometric Functions

$f(x-1) + f(x+1) = \sqrt{3}\, f(x)$ ✓

$f(x-2) + f(x) = \sqrt{3}\, f(x-1)$ — (1)

$f(x) + f(x+2) = \sqrt{3}\, f(x+1)$ — (2)

(1) + (2)

$f(x-2) + f(x+2) + 2f(x) = 3f(x)$

$f(x-2) + f(x+2) = f(x)$

$T = 12$

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$\sin^{-1}x = \arcsin(\sin x)$ is the angle in interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ whose sin value is x.

$$\theta = \sin^{-1}x \quad , \quad \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

$$\sin\theta = x$$

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$\arccos x = \cos^{-1} x = \theta \quad , \quad \theta \in [0, \pi]$

$\cos\theta = x$

$\tan^{-1} x = \theta \quad , \quad \theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$\cot^{-1} x = \theta \quad , \quad \theta \in (0, \pi)$

$\operatorname{cosec}^{-1} x = \theta \quad , \quad \theta \in \left[-\frac{\pi}{2}, 0\right) \cup \left(0, \frac{\pi}{2}\right]$

$\sec^{-1} x = \theta \quad , \quad \theta \in \left[0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \pi\right]$

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$$\sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$$

$$\theta = \sin^{-1}\frac{1}{2}$$

$$\theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

$$\sin\theta = \frac{1}{2}$$

$$\cot^{-1}(-1) = \frac{3\pi}{4}$$

$$\cos^{-1}\left(-\frac{1}{2}\right) = 2\pi/3$$

$$\sec^{-1}\left(-\frac{1}{2}\right) = \text{not defined}$$

$$\sec\theta = -\frac{1}{2} \times$$

$$\tan^{-1}(\sqrt{2}-1) = \pi/8$$

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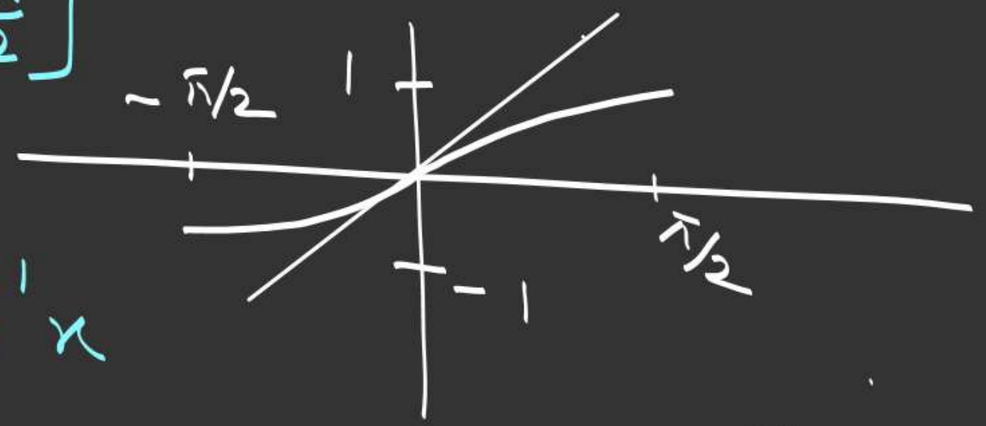
$f: \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \to [-1, 1]$, $f(x) = \sin x$

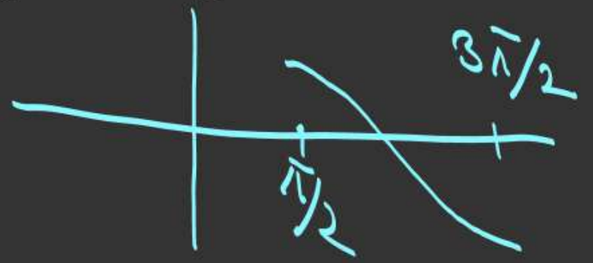
$f^{-1}: [-1, 1] \to \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$f^{-1}(x) = \sin^{-1} x$



$f: \left[\frac{\pi}{2}, \frac{3\pi}{2}\right] \to [-1, 1]$, $f(x) = \sin x$

$f^{-1}(x) = ?$



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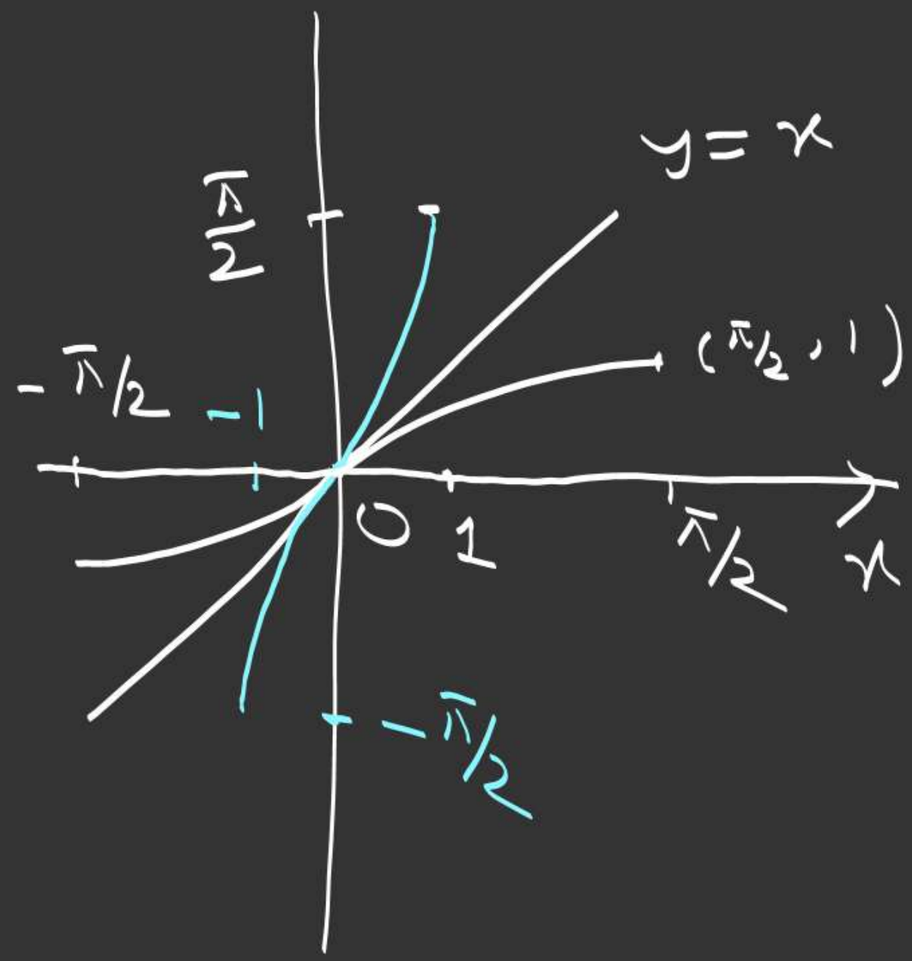

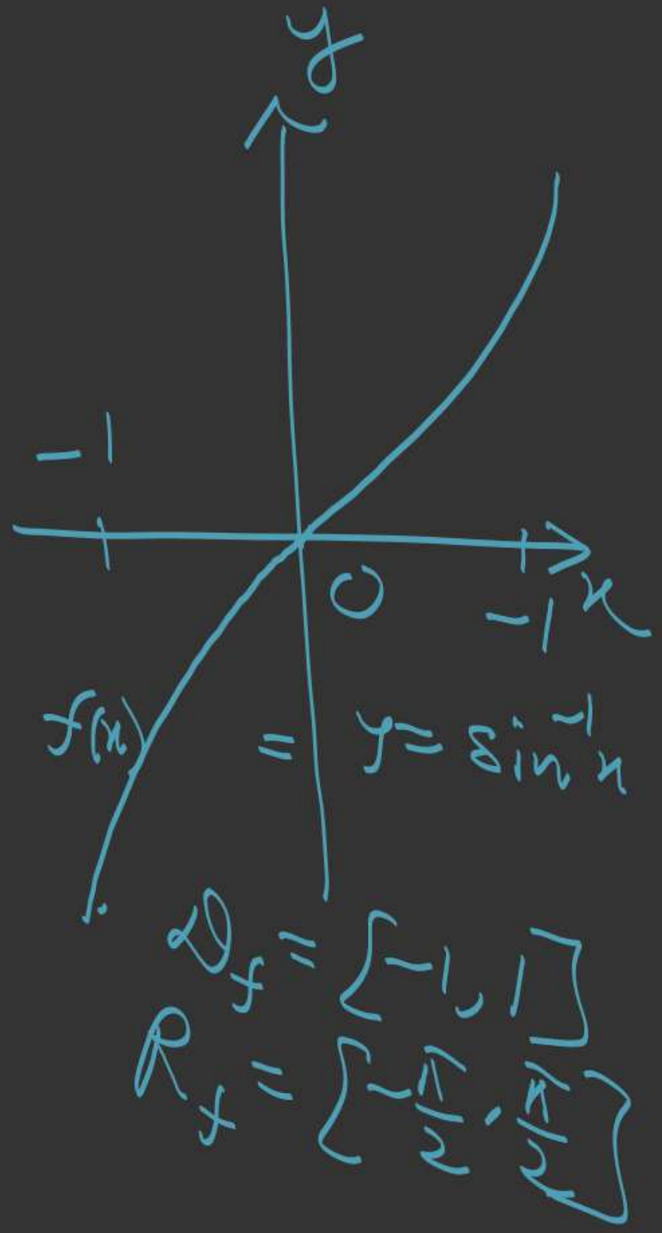

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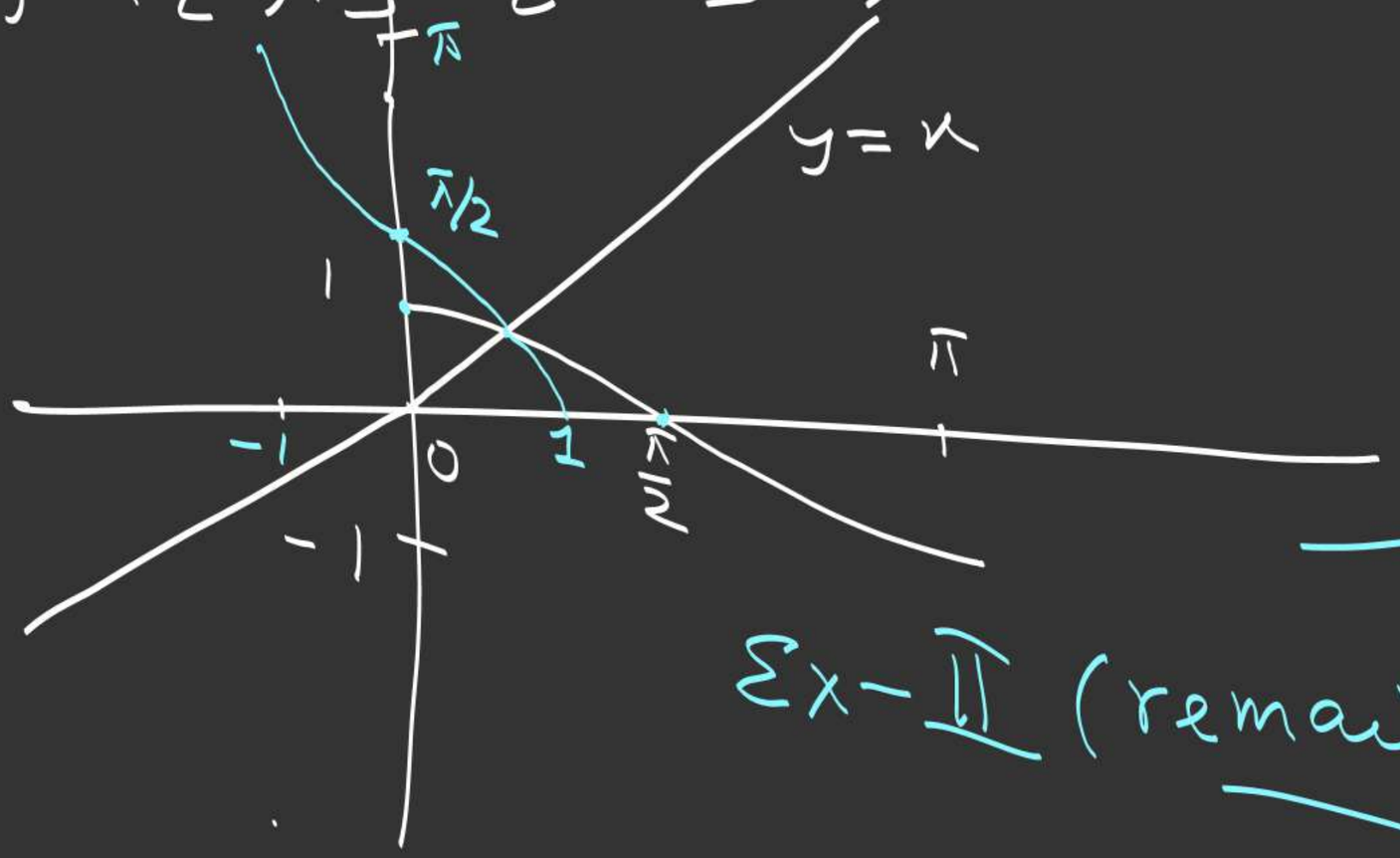
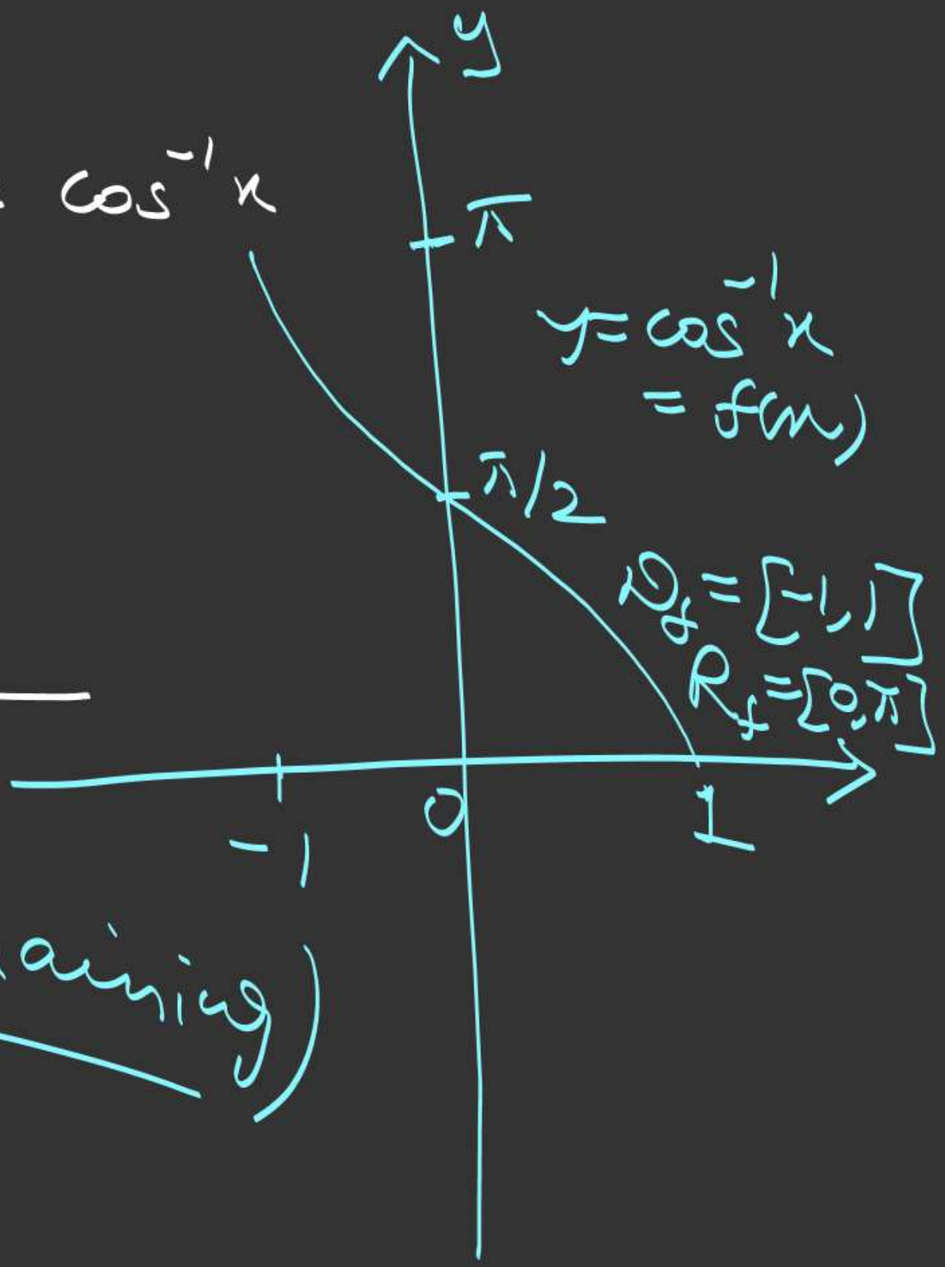
$f: [0, \pi] \to [-1, 1]$, $f(x) = \cos x$

$f^{-1}: [-1, 1] \to [0, \pi]$, $f^{-1}(x) = \cos^{-1} x$





Ex - II (remaining)