Nishant Jindal

(Q) Find work done by F & mg when bob displaced from A to B.

Normal Method

$$W = \int \vec{F} \cdot d\vec{s}$$

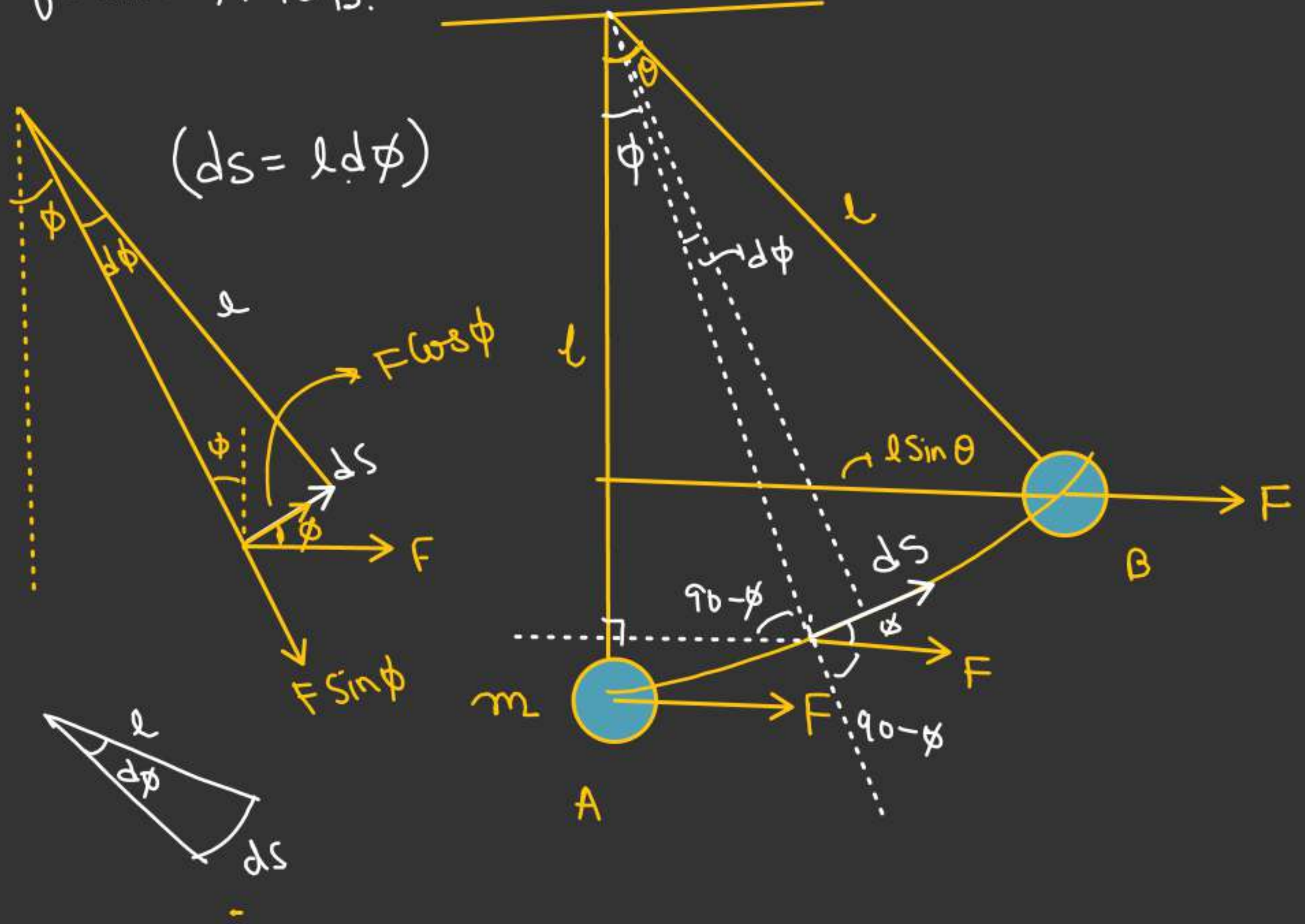
$(ds = l\,d\phi)$

$$\int_0^W dW = F\int ds \cos\phi$$

$$W = Fl\int_0^\theta \cos\phi \, d\phi$$

$$W = Fl[\sin\phi]_0^\theta$$

$$\boxed{W = Fl\sin\theta}$$



Nishant Jindal

Work done by gravity for $\theta = 0$ to $\theta = \theta$

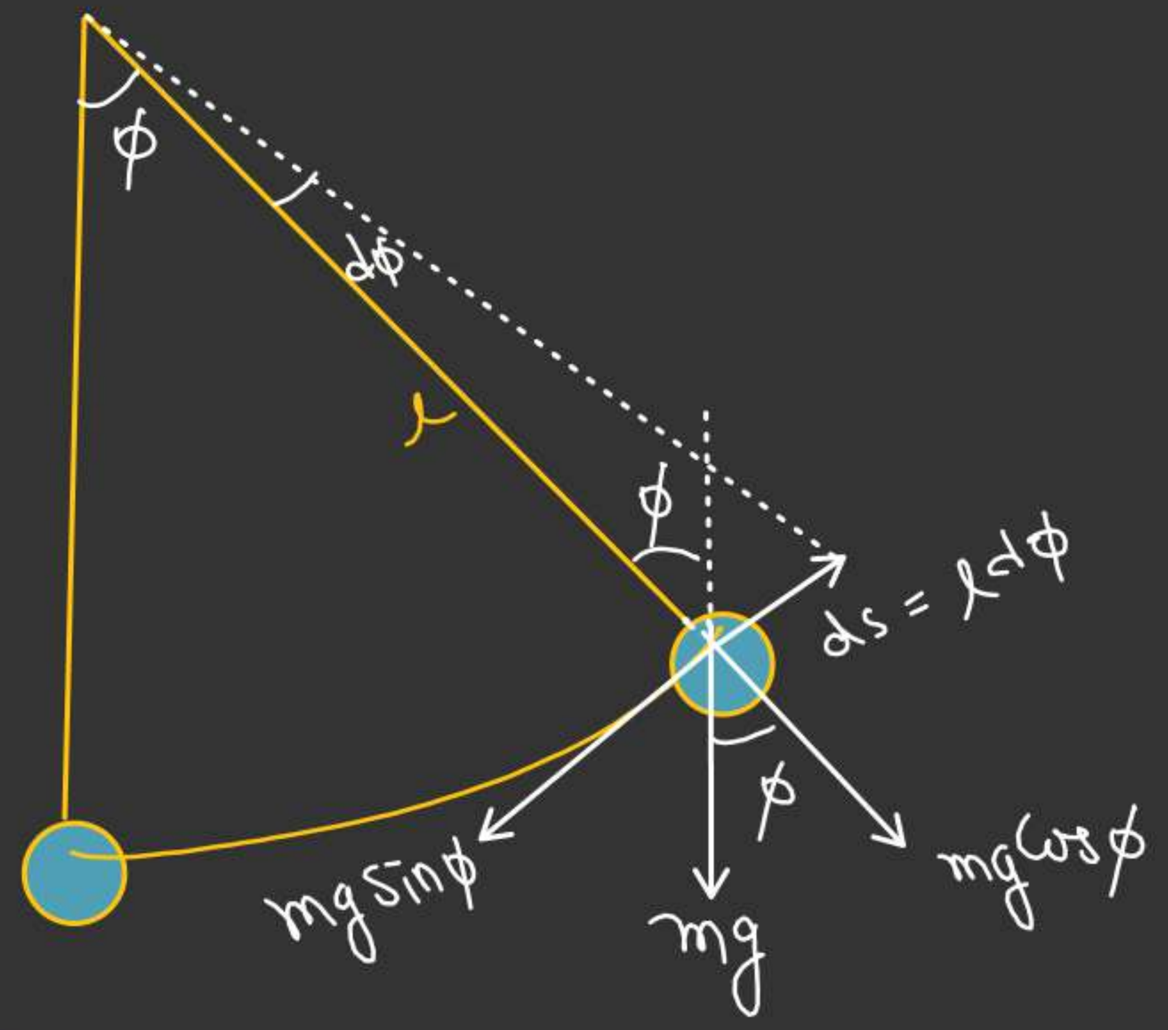
$$dW = -mg\sin\phi \, ds$$

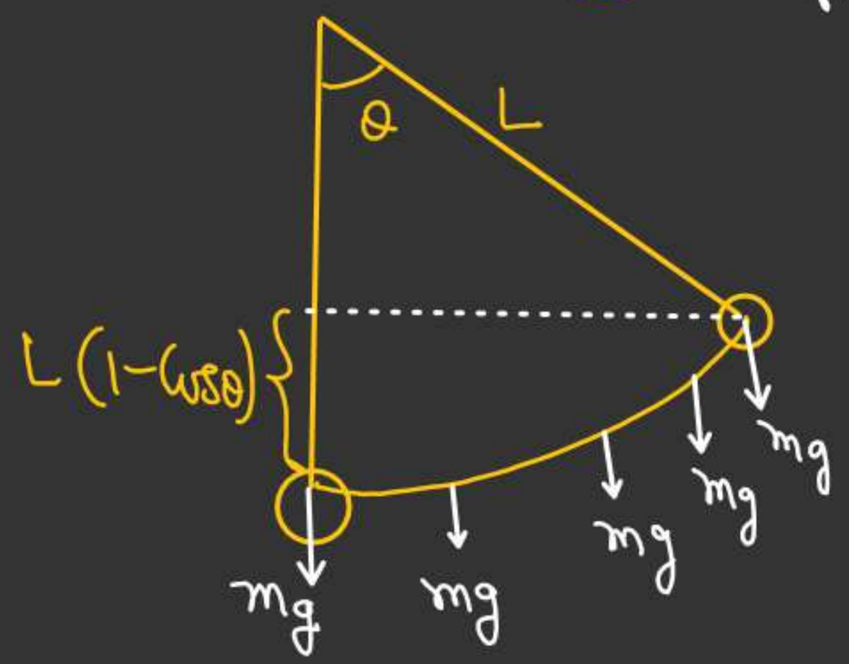
$$\int_0^W dW = -mgl \int_0^\theta \sin\phi \, d\phi$$

$$W = -mgl\left[-\cos\phi\right]_0^\theta$$

$$W = -mgl\left[-\cos\theta + 1\right]$$

$$\boxed{W = -mgl\left[1 - \cos\theta\right]}$$

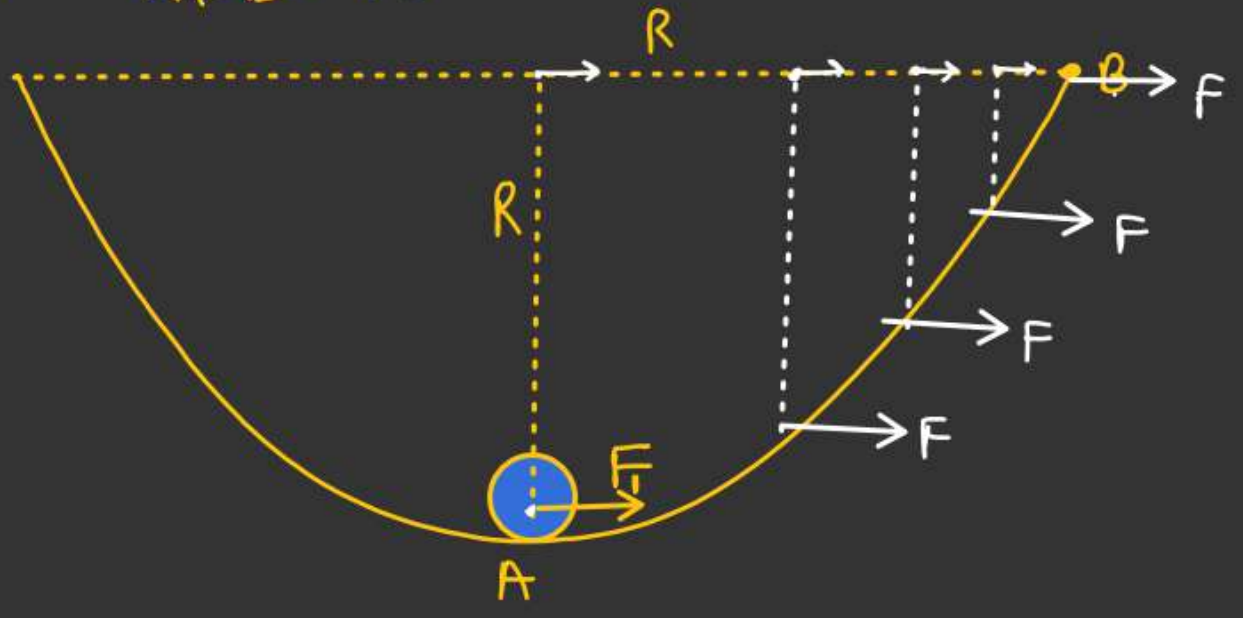




Nishant Jindal

F_1 = Always acts horizontally and constant force.

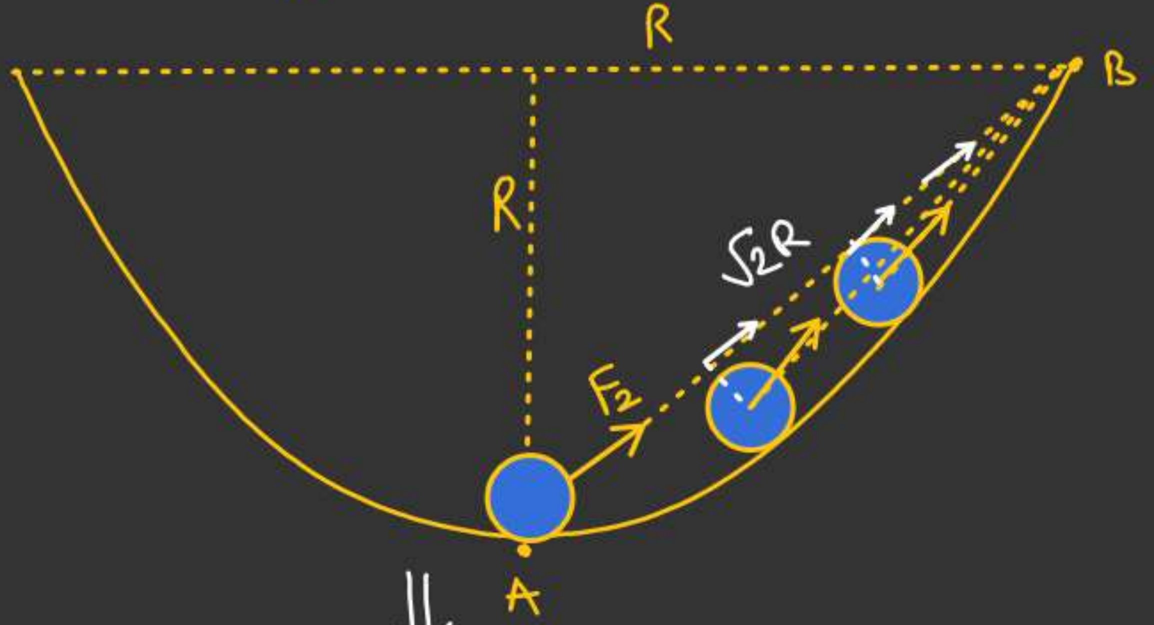
$(W_{F_1})_{A-B} = ??$



$(W_{F_1}) = (F_1 R)$

$F_2 \Rightarrow$ Always directed towards point B. & Constant force.

$(W_{F_2})_{A-B} =$

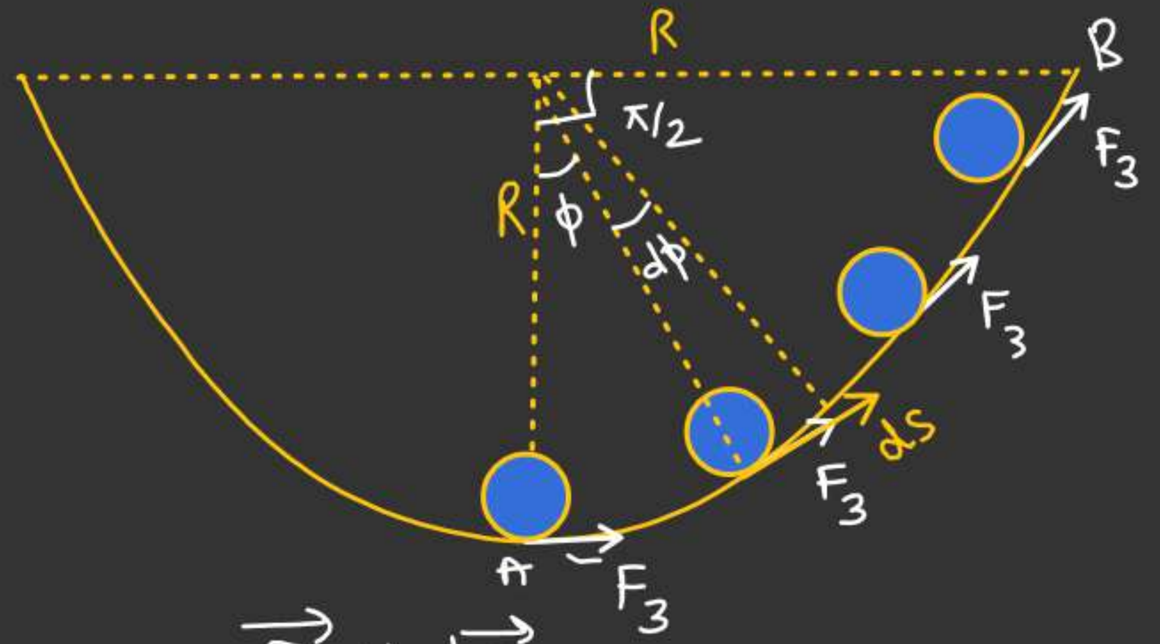


$\Downarrow$

$W_{F_2} = (F_2\sqrt{2}R)$ J

Nishant Jindal

$(W_{F_3})_{A-B} = ??$ $\quad F_3 =$ A constant force always acts tangentially.



$\vec{F} \parallel \vec{ds}$

$$dW = F_3 \cdot ds \cos 0$$

$$\int_0^W dW = F_3 R \int_0^{\pi/2} d\phi$$

$$W = F_3 R \frac{\pi}{2} \text{ J}$$

$$W_{F_3} = F_3 (\text{Arc length } AB)$$

$$W_{F_3} = \left(F_3 \frac{R\pi}{2}\right) \text{J}$$

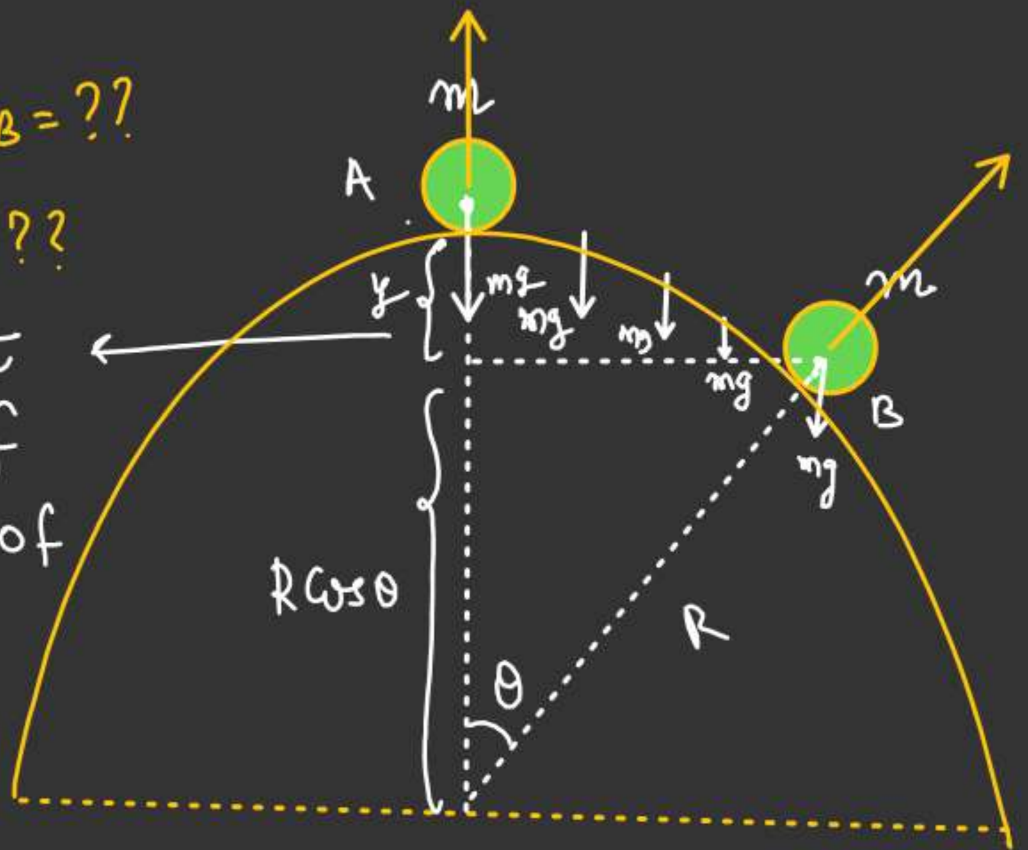
Nishant Jindal

Q:

$(W_{mg})_{A-B} = ??$

$(W_N) = ??$

Displacement of point of application of force or Displacement of body along the direction of applied force

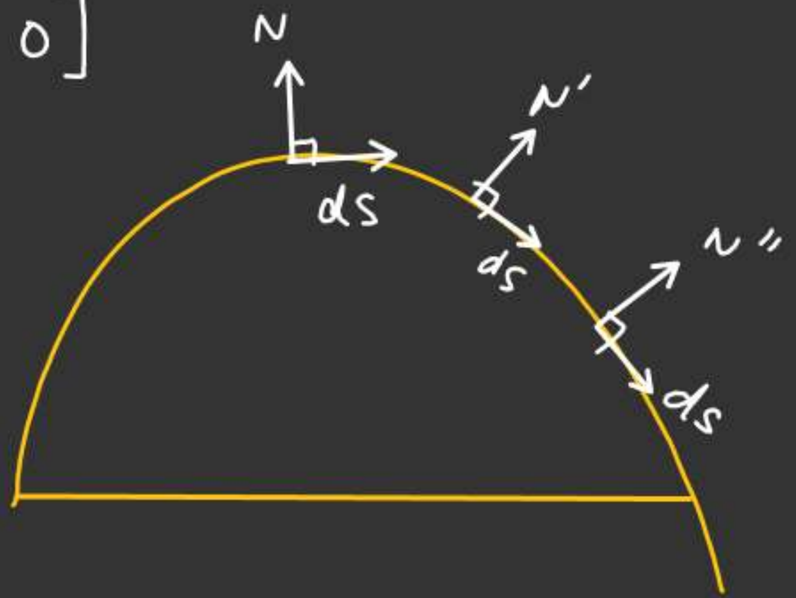


$y = R - R\cos\theta$

$y = R(1-\cos\theta)$

$W_{mg} = +\,mgR(1-\cos\theta)$

$[W_N = 0]$



Nishant Jindal

$$x = r\cos\theta \Rightarrow \cos\theta = \frac{x}{r}$$

$$y = r\sin\theta \Rightarrow \sin\theta = y/r$$

$$\hat{\theta} \perp \hat{r}$$

$$|r| = \sqrt{x^2+y^2}$$

$$\hat{r} = \cos\theta\,\hat{i} + \sin\theta\,\hat{j}$$

$$\boxed{\hat{r} = \frac{x\hat{i}+y\hat{j}}{r} = \frac{x\hat{i}+y\hat{j}}{\sqrt{x^2+y^2}}}$$

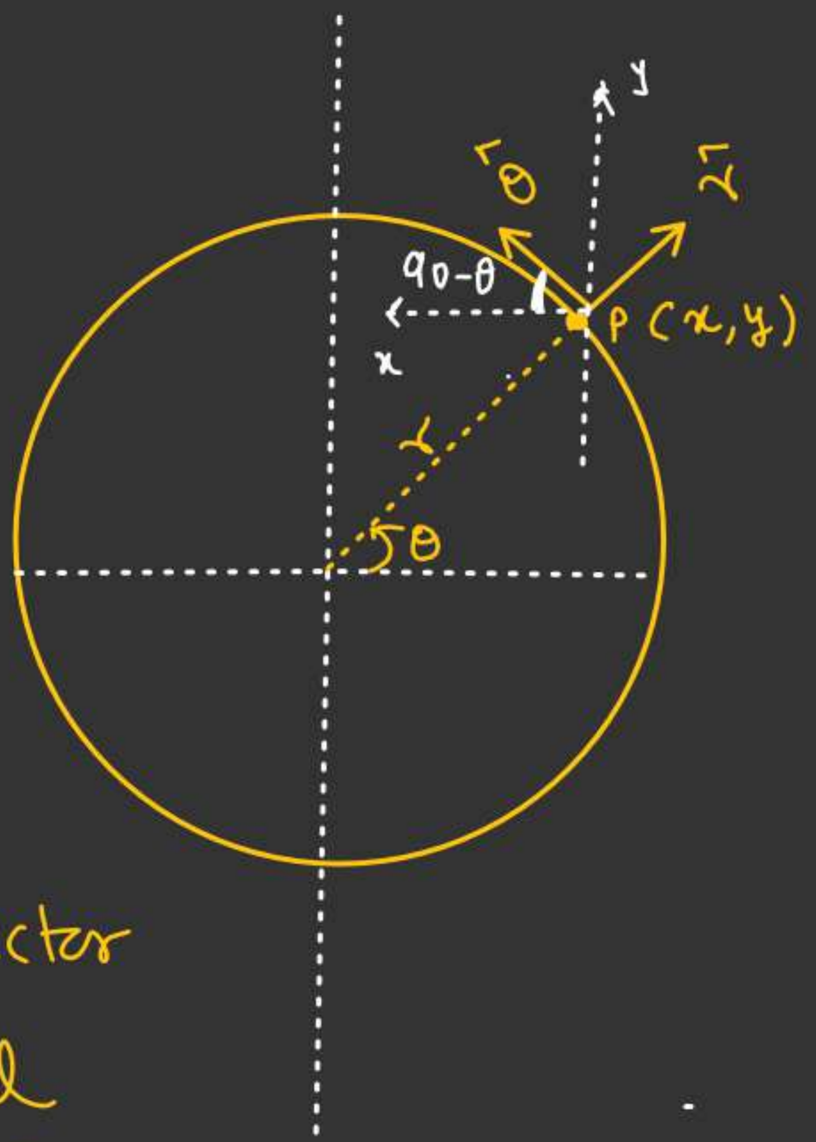
$$\hat{\theta} = -\sin\theta\,\hat{i} + \cos\theta\,\hat{j}$$

$$\hat{\theta} = -\frac{y}{r}\hat{i} + \frac{x}{r}\hat{j}$$

$$\boxed{\hat{\theta} = \frac{-y\hat{i}+x\hat{j}}{\sqrt{x^2+y^2}}}$$

$\hat{r} \rightarrow$ radial Unit vector or Normal Unit Vector

$\hat{\theta} \rightarrow$ Tangential Unit vector



Nishant Jindal

\# Find work done by a force F for one complete rotation in a Circle of radius 'r'. (K = +ve Constant)

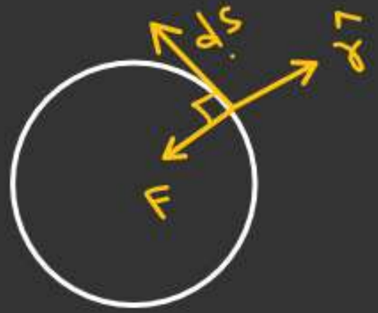
a) $\vec{F} = -K\left(\frac{x\hat{i} + y\hat{j}}{x^2+y^2}\right)$

$\Downarrow$

$\vec{F} = -K\left(\frac{x\hat{i}+y\hat{j}}{\sqrt{x^2+y^2}}\right)\left(\frac{1}{\sqrt{x^2+y^2}}\right)$

$\Downarrow$

$\vec{F} = \left(\frac{K}{r}\right)(-\hat{r})$



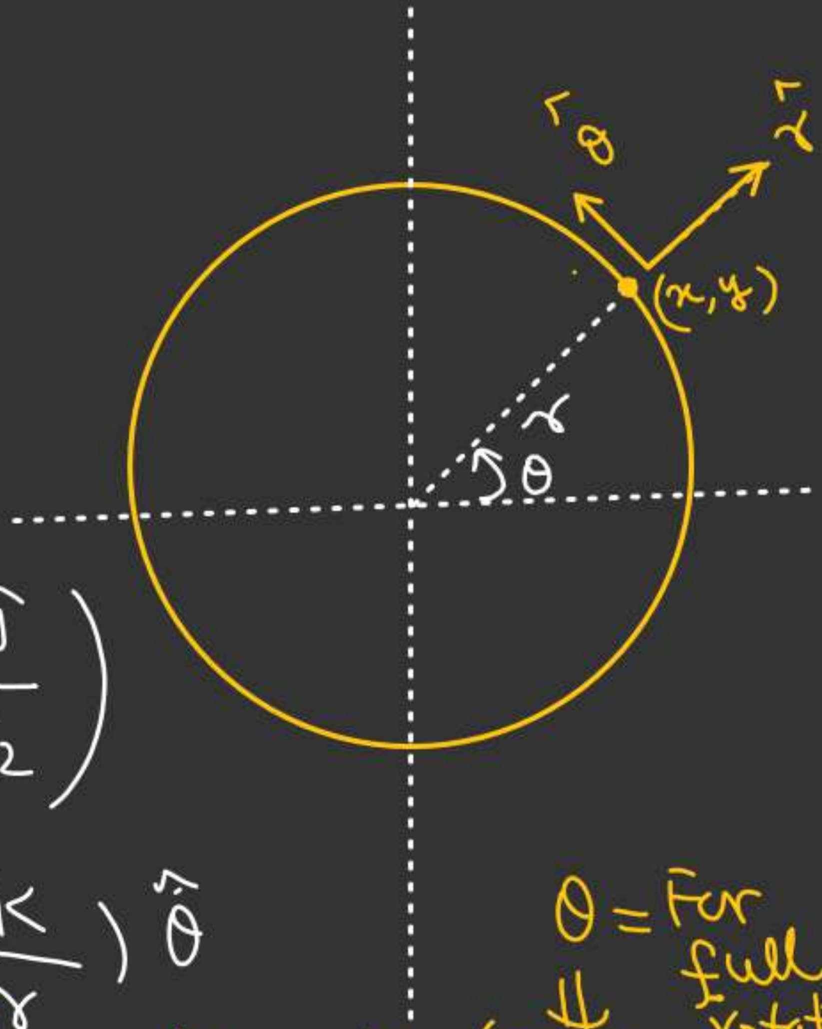
$\boxed{W_F = 0}$ (Radial force)

b) $F = K\left(\frac{-y\hat{i}+x\hat{j}}{x^2+y^2}\right)$

$r = \sqrt{x^2+y^2}$

$\vec{F} = \frac{K}{\sqrt{x^2+y^2}}\left(\frac{-y\hat{i}+x\hat{j}}{\sqrt{x^2+y^2}}\right)$

$\vec{F} = \left(\frac{K}{\sqrt{x^2+y^2}}\right)\hat{\theta} = \left(\frac{K}{r}\right)\hat{\theta}$



$\theta = 2\pi$ for full rotation

$W = \frac{K}{r} \times (2\pi r)$

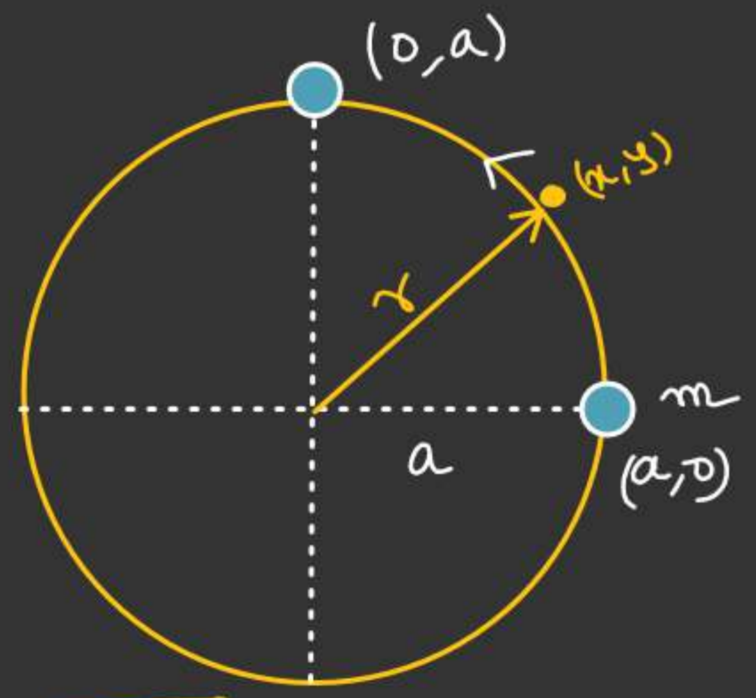
$W = (K \cdot 2\pi)\ J$

Nishant Jindal

JEE Adv

$$F = K\left[\frac{x}{(x^2+y^2)^{3/2}}\hat{i} + \frac{y}{(x^2+y^2)^{3/2}}\hat{j}\right]$$

Find work done by Force F on a particle of mass m when particle moves along a circle of radius a from (a, 0) to (0, a)



$$|\vec{r}| = \sqrt{x^2+y^2}$$

$$r^2 = x^2+y^2$$

$$\vec{F} = \frac{K}{(x^2+y^2)}\left[\frac{x}{\sqrt{x^2+y^2}}\hat{i} + \frac{y}{\sqrt{x^2+y^2}}\hat{j}\right]$$

$$\vec{F} = \frac{K}{x^2+y^2}\underbrace{\left(\frac{x\hat{i}+y\hat{j}}{\sqrt{x^2+y^2}}\right)}_{\hat{r}} \Rightarrow \vec{F} = \left(\frac{K}{r^2}\right)\hat{r} \Rightarrow (W_F = 0)$$

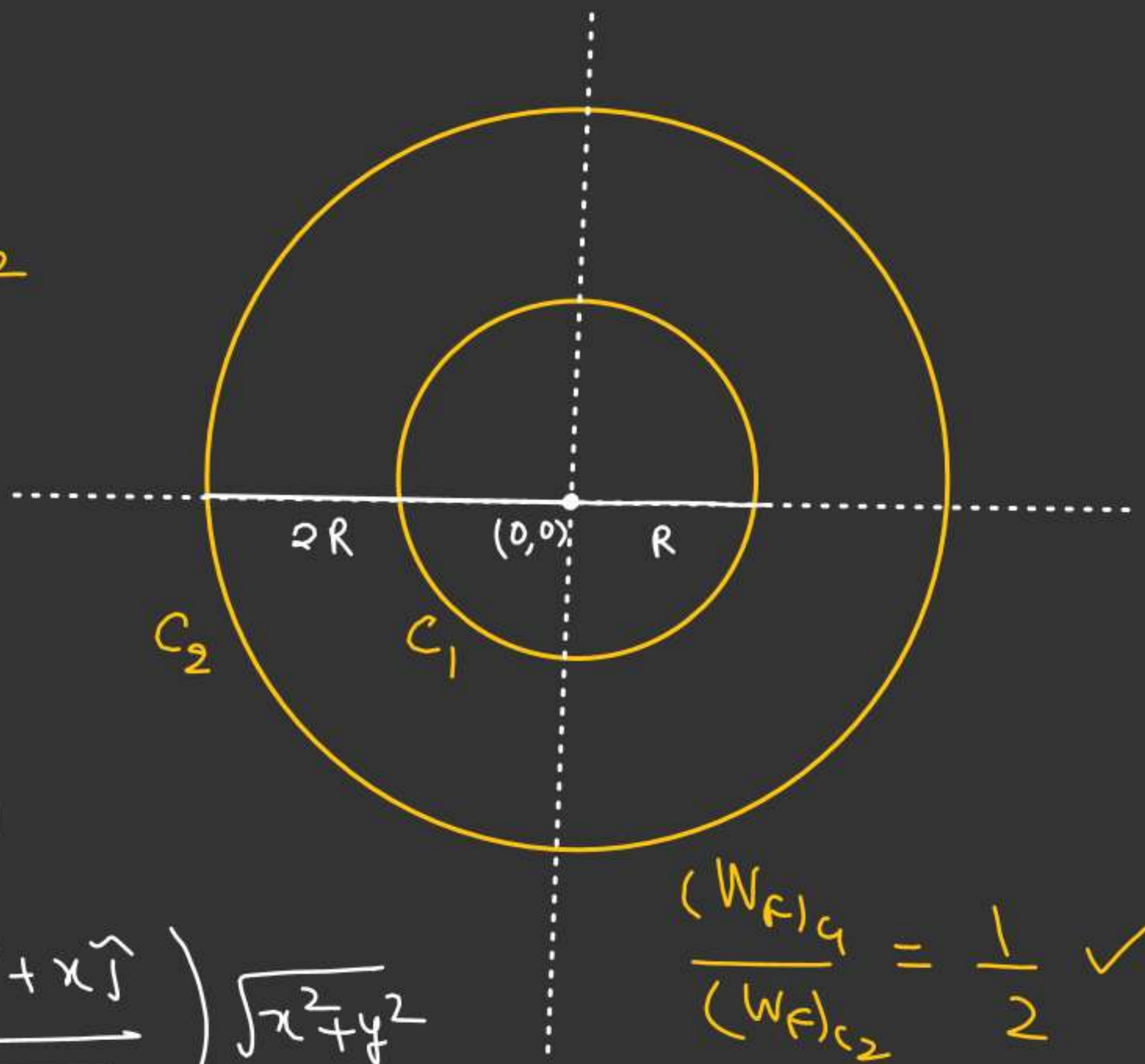
$C_1 \Rightarrow x^2+y^2=R^2$

$C_2 \Rightarrow x^2+y^2=4R^2$

A particle is moving either in C_1 or C_2 under the influence of force F

$$\vec{F} = (px-qy)\hat{i} + (qx+py)\hat{j}$$

Find ratio of work done by F in C_1 and C_2 for one complete rotation.



Sol^n:-

$$\vec{F} = p(x\hat{i}+y\hat{j}) + q(-y\hat{i}+x\hat{j})$$

$$\vec{F} = p\left(\sqrt{x^2+y^2}\right)\underbrace{\left(\frac{x\hat{i}+y\hat{j}}{\sqrt{x^2+y^2}}\right)}_{\hat{r}} + q\underbrace{\left(\frac{-y\hat{i}+x\hat{j}}{\sqrt{x^2+y^2}}\right)}_{\hat{\theta}}\sqrt{x^2+y^2}$$

$$\vec{F} = \underbrace{(pr)\hat{r}}_{\text{Radial component}} + \underbrace{(qr)\hat{\theta}}_{\text{Tangential}}$$

$(W_F)_{C_1} = qR \times (2\pi)$

$(W_F)_{C_2} = q(2R) \times 2\pi$

$$\frac{(W_F)_{C_1}}{(W_F)_{C_2}} = \frac{1}{2} \checkmark$$

Nishant Jindal

H.W

D.P.P - 1

H.C.V :- 4, 5, 6, 7, 8, 9

15, 16, 17