Nishant Jindal

CONTINUITY

$[x]$ & $\{x\}$ Based Qs

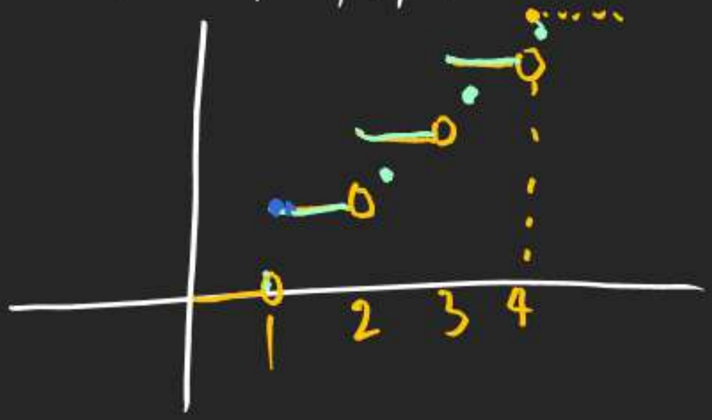
Basic → 1) $[x]$, $\{x\}$ are D.C. at every Int.

2) $[f(x)]$ or $\{f(x)\}$ should be checked for its cont^y at every x where $f(x)$ is giving Integer

Q $f(x) = [x]$ check Cont^y in $x \in [1, 4]$?

$f(1) = f(1^+)$

Sol. $x = 1, 2, 3, 4$ are Probable pts of D.C



D.C. at $x = 2, 3, 4$ only

$x = \overset{\times}{1}, \underbrace{2, 3, 4}_{DC}$

Q $f(x) = [x^3]$ in $x \in [1, 2]$

$x \in [1, 2] \to x^3 \in [1, 8]$

$x \in 1, 2$

$x^3 \in 1, \underbrace{2, 3, 4, 5, 6, 7, 8}_{7 \text{ Pts of D.C}}$

Q $f(x) = [2x - 4]$ in D.C. at $x \in [1, 3]$

$f(x) = [2x] - 4$

$x \in [1, 3)$

$2x \in [2, 6)$

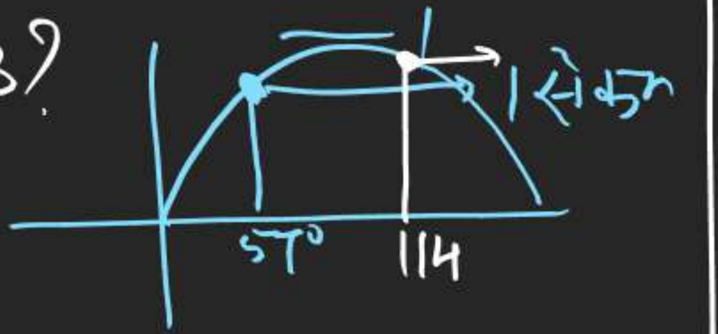
$2x \in 2, \underbrace{3, 4, 5, 6}$ → 4 Pts of D.C.

$x \in \frac{3}{2}, \frac{4}{2}, \frac{5}{2}, \frac{6}{2}$

CONTINUITY

Q $f(x)=[\sin[x]]$ Check D.C. at $x=1,2,3$?

$x=1$ पर check करना है तो है।



$f(1)=[\sin[1]]=[\sin 1]=[\sin 57^\circ]=[\text{less than } 1]=0$

$f(1^+)=[\sin[1+h]]=[\sin 1]=[\sin 57^\circ]=[\text{less than } 1]=0$

$f(1^-)=[\sin[1-h]]=[\sin 0]=[0]=0$

at $x=1$ $y=[\sin[x]]$ is cont^s

$f(2)=[\sin[2]]=[\sin 2]=[\text{less than } 1]=0$

$f(2^+)=[\sin[2+h]]=[\sin 2]=[\;"\;]=0$

$f(2^-)=[\sin[2-h]]=[\sin 1]=[\;"\;]=0$

Cont^s at $x=2$

Adv 2017

Q $f(x)=x\cdot\cos(\pi(x+[x]))$

Check Cont^y at $x=-1,0,1$ $\quad x=\boxed{n}$ 0 पर अलग से

$f(1)=1\cdot\cos(\pi(1+[1]))=\cos 2\pi=1$

$f(1^+)=(1+h)\cos(\pi(1+h+[1+h]))$

$=1\cdot\cos(\pi(1+1))=1$

$f(1^-)=(1-h)\cdot\cos(\pi(1-h+[1-h]))$

$=1\cdot\cos(\pi(1+0))=-1$ $\Big\}$ D.C.

$f(0)=0\cdot\cos(\ldots)=0$

$f(0^+)=h\cdot\cos(\pi\cdot(h+\underbrace{[h]}_{=0}))=h\cdot\cos\pi h=0$

$f(0^-)=-h\cos(\pi(-h+\underbrace{[-h]}_{=-1}))=-h\cos(\pi(-1-h))$

$=0$

Cont^s at $x=0$

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CONTINUITY

T_2

Q $f(x) = \begin{cases} \lim_{x\to 0} x^2\left[\frac{1}{x^2}\right] & x \neq 0 \\ 0 & x = 0 \end{cases}$

Check Cont^y at $x = 0, 1, 2$?

Funda at $x \to \infty$

$[x] = x$

Checking Cont^y at $x = 0$

L.V. $= \lim_{x\to 0} x^2\left[\frac{1}{x^2}\right] =$ Aakasham $1 \neq f(0)$

LHL $x = 0 - h$

$(-h)^2\left[\frac{1}{(-h)^2}\right]$

$\lim_{h\to 0} h^2 \times \left[\frac{1}{h^2}\right]$ $\to \infty$ Banjayega.

$h^2 \times \frac{1}{h^2} = 1$

RHL $x = 0 + h$ D.C.

$h^2\left[\frac{1}{h^2}\right]$

$h^2\left[\frac{1}{h^2}\right]$

$h \times \frac{1}{h^2} = 1$

$x = 1$ पर $f(x) = x^2 \times \left[\frac{1}{x^2}\right]$ ← only Yehi Use करेंगे

$f(1) = 1^2 \times \left[\frac{1}{1^2}\right] = 1 \times 1 = 1$

$\frac{1}{1 \text{ से Bde}} =$ less than 1

$f(1^+) = (1+h)^2\left[\frac{1}{(1+h)^2}\right] = (1+h)^2 \times 0 = 0$

$f(1^-) = (1-h)^2\left[\frac{1}{(1-h)^2}\right] = (1-h)^2 \times 1 = 1$

$\left[\frac{1}{1 \text{ से Chhota}}\right] = [1 \text{ से Bda}] = 1$

D.C.

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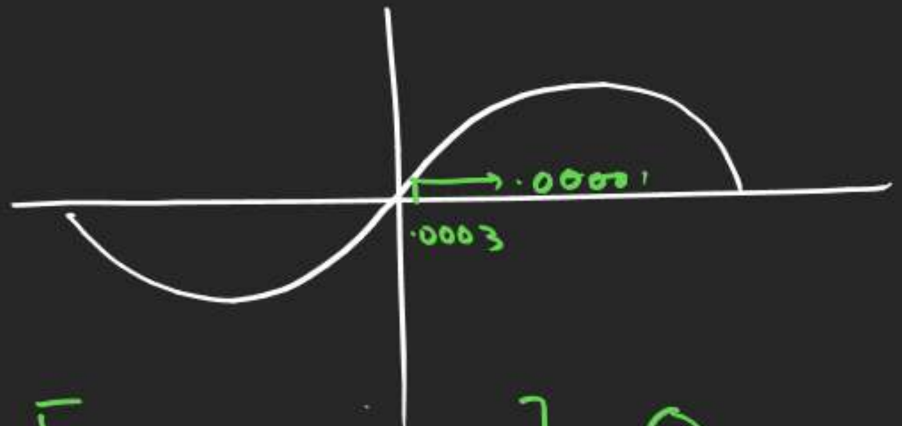
CONTINUITY

Q Check cont^y of $y = [x \cdot \sin \pi x]$ at $x = 0$?

$f(0) = [0 \cdot \sin \pi \times 0] = [0] = 0$

$f(0^+) = [h \sin \pi h] = [\cdot 0001 \times \sin(\cdot 0003)] = [\cdot 0001 \times \cdot 00001] = 0$

$f(0^-) = [-h \sin(-\pi h)] = [h \cdot \sin \pi h] = [\cdot 0001 \times \sin(\cdot 0003)] = 0$

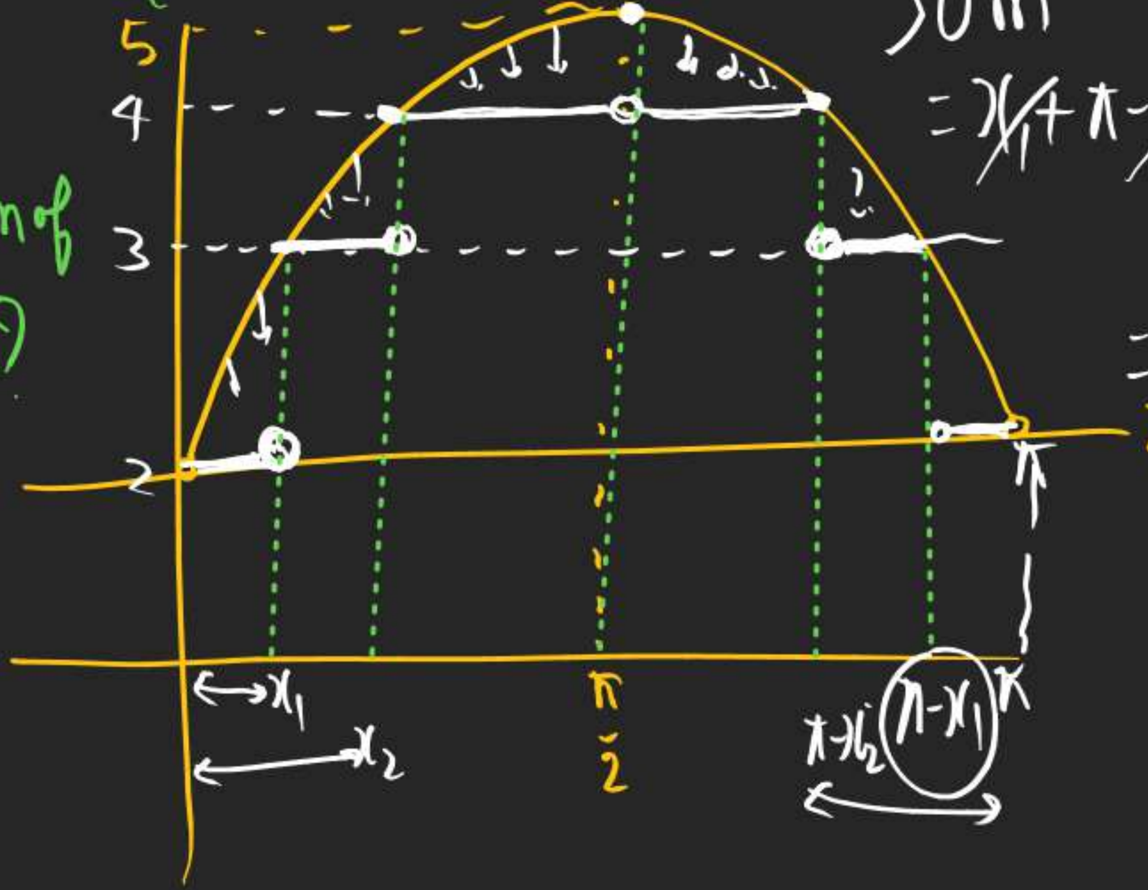


Q $f(x) = [2 + 3\sin x]$ $x \in (0, \pi)$ find sum of all values of x where fxn is D.C.?

Cont^s

$[0, 3] + 2$

$[2, 5]$



Sum

$= \cancel{x_1} + \pi - \cancel{x_1} + \cancel{x_2} + \pi - \cancel{x_2} + \frac{\pi}{2}$

$= \frac{5\pi}{2}$

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CONTINUITY

Q T2 $f(x) = \begin{cases} \lim\limits_{x\to 0} \dfrac{[\{x\}]e^{x^2}\{x+[x]\}}{(e^{1/x^2}-1)\,\mathrm{Sgn}(\sin x)} & x \neq 0 \\ 0 & x = 0 \end{cases}$

$[\{x\}] = 0$, $\{x+[x]\} = 0$

Check Conti at $x=0$?

$L.V_n = 0 = f(0)$

Conts

Pt. of D.C to check Based Qs

1) Sometime We need to create Pt of D.C. ourself.
2) For this if fxn in [ ] or { } then think about value of x where fxn can give Integer values.

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CONTINUITY

Q Pt. of D.C. of fxn $f(x) = \left[\frac{6x}{\pi}\right] \cos\left[\frac{3x}{\pi}\right]$

Adv

1) $\frac{\pi}{6}$ 2) $\frac{\pi}{3}$ 3) $\frac{\pi}{2}, \pi$ (4) all above.

$\left[\frac{6x}{\pi}\right]$ Can become D.C.

$\left[\frac{3x}{\pi}\right]$ will give it values to Cos. I.I can not give Impact to fxn directly.

When $\frac{6x}{\pi} = n$

$x = \frac{n\pi}{6}$

$x = \frac{\pi}{6}, \frac{\pi}{3}, \frac{\pi}{2}, \frac{2\pi}{3}, \frac{5\pi}{6}, \pi$ — — .

$f\left(\frac{\pi}{2}\right) = [3] \cos \sim = 3\cos \sim = 3$

$f\left(\frac{\pi}{2}^-\right) = \left[\frac{6}{\pi}\left(\frac{\pi}{2} - h\right)\right] \cos\left[\ \sim\ \right] = \left[3 - \frac{6h}{\pi}\right] \cos \sim = 2\cos \sim$

D.C.

Pt. of D.C. to check Based Qs.

1). Sometime We need to create Pt. of D.C. ourself.

2) For this if fxn in [ ] or { } then think about value of x where fxn can give Integer values.

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CONTINUITY

Q $f(x) = |[x-1](x-1)|;\ 0 \le x \le 3$ L.B. → R.H.

[ ] is the only threat

It can become D.C. at $x = 0, 1, 2, 3$

$x = 0$ $f(0) = |[0-1](0-1)| = |-1 \times -1| = 1$

$f(0^+) = |[-1+h](-1+h)| = |-1 \times -1| = 1$

$f(3) = |[3-1](3-1)| = |2 \times 2| = 4$

$f(3^-) = |[3-h-1](3-h-1)|$

$= |[2-h](2-h)| = |1 \times 2-h|$

$= 2$

$x = 3$ D.C.

$f(1) = |[1-1](1-1)| = 0$

$f(1^+) = |[\cancel{1}+h-\cancel{1}](\cancel{1}+h-\cancel{1})| = 0$

$f(1^-) = |[\cancel{1}-h-\cancel{1}](\cancel{1}-h-\cancel{1})| = |-1 \times -h| = 0$

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CONTINUITY

Q $f(x)=\begin{cases} |4x-5|[x] & x>1 \\ [\cos \pi x] & x\le 1 \end{cases}$

Pt. of D.C. → $x = 0, \frac{1}{2}, 2$

Check Cont^y in $x \in [0,2]$

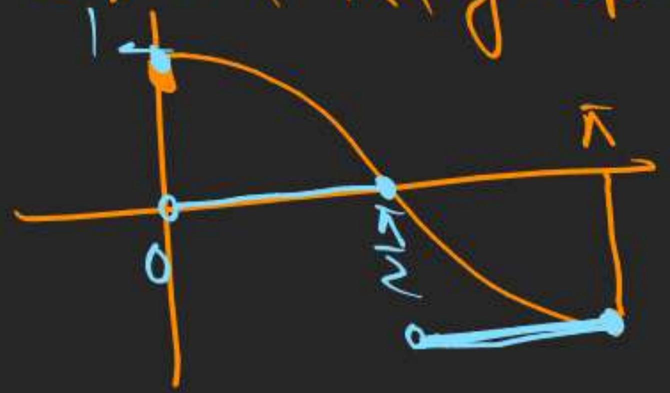
$[\cos \pi x]$ $\quad |4x-5|[x]$ → threat at $x=2$

RB

$x \in [0,1]$

$\pi x \in [0,\pi]$

$y = \cos \pi x$ Ka graph



$f(2) = |4\times 2-5|[2] = 6$

$f(2^-) = |4\times(2-h)-5|[2-h] = 3\times 1 = 3.$

$\Big\}$ D.C.

$[\cos \pi x]$ is D.C. at $\pi x = 0, \frac{\pi}{2}$ $\quad$ D.C.

$x = \frac{0}{\pi}, \frac{\pi}{2\pi} \Rightarrow \boxed{x = 0, \frac{1}{2}}$

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Q $f(x) = [5x] + \{3x\}$ is D.C. at $x \in [0,1]$? LB R

$\therefore$ No of Pt. of D.C = 6 $\to$ $\frac{1}{3}, \frac{2}{3}$; $\frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}$

Pahle Boundaries chhod Kar check Kr Rahe hain.

$x \in (0,1)$

$y = [5x]$ $\quad x \in (0,1)$

$5x \in (0,5)$

$5x \in \quad 1, 2, 3, 4$

D.C $\to x \in \left(\frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}\right)$

$y = \{3x\}$ $\quad x \in (0,1)$

$3x \in (0,3)$

$3x \in 1, 2$

$x \in \frac{1}{3}, \frac{2}{3}$

$x = 0$ $\quad f(x) = [5x] + \{3x\}$

$f(0) = [5\times 0] + \{3\times 0\} = 0$

$f(0^+) = [5\times h] + \{3h\} = 0 + 3h = 0$

Cont

$x = 1$ $\quad f(x) = [5x] + \{3x\}$

$f(1) = [5\times 1] + \{3\times 1\} = 5 + \{3\} = 5 + 0 = 5$

$f(1^-) = [5\times(1-h)] + \{3\times(1-h)\}$

$= [5-5h] + \{3-3h\}$

$= 4 + \{-3h\} = 4 + 1 - 3h = 5 - \cancel{3h} = 5$

Conts

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CONTINUITY

Sgn fxn is D.C. Where $f(x)=0$

Cont^y of Composite fxn.

A fxn $y = f(\boxed{g(x)})$ is said to be Cont^s at $x=a$ if $g(x)$ is Cont^s at $x=a$ & $f(x)$ is Cont^s at $g(a)$

Q $f(x) = \text{Sgn } x$, $g(x) = x(x^2-1)$ Check D.C of A) $fog(x)$ B) $gof(x)$

(A) $fog(x) = f(g(x)) = f(x(x^2-1))$

$= \text{Sgn}(x(x^2-1))$

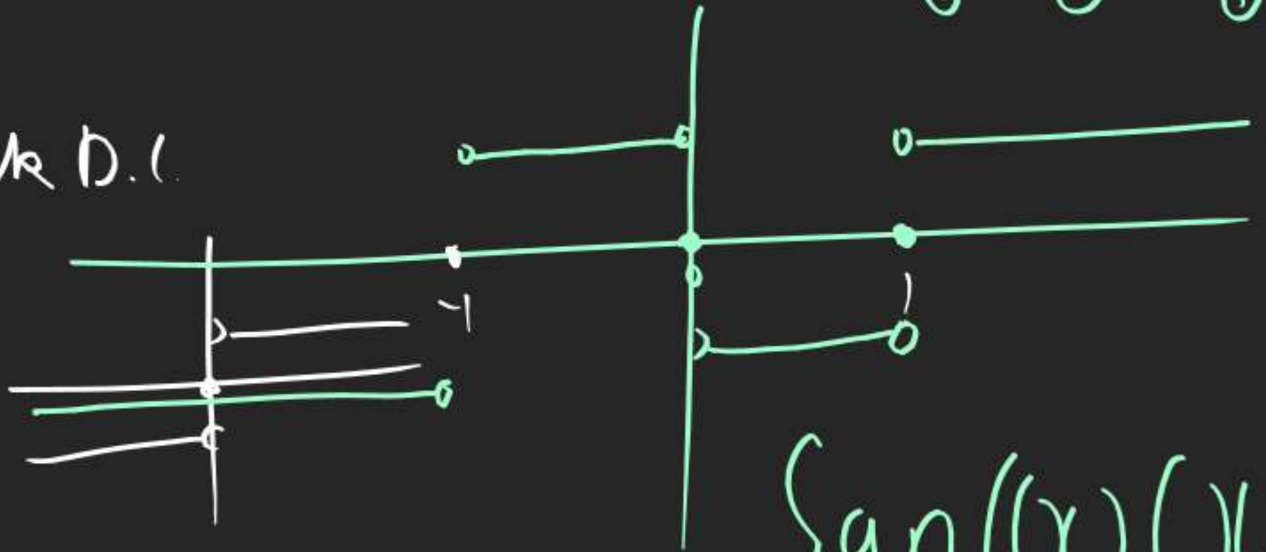
$fog(x) = \text{Sgn}((x)(x-1)(x+1))$ Cont. be D.C

$(-0.5)\ (-1.5)\ (-0.5+1)$

$\oplus \quad \ominus \quad \oplus$

Where $(x)(x-1)(x+1) = 0$

$x = 0, 1, -1$



$\text{Sgn}((x)(x-1)(x+1))$

$x = 2 \quad 2\ (1)(3)$

(B) $gof(x) = g(f(x)) = g(\text{Sgn } x)$

$gof(x) = \text{Sgn } x(\text{Sgn } x - 1)(\text{Sgn } x + 1) \quad \forall x \in R$

$x=0 \quad = 0(0-1)(0+1) = 0$

$x = +ve \quad = 1(1-1)(1+1) = 0$

$x = -ve \quad = (-1)(-1-1)(-1+1) = 0$

Cont^s $\forall x \in R$