HOMEWORK-03

(H.P. & A.M. – G.M. – H. M.)

1. If $\mathbf{r}$ is one **AM** and $\mathbf{p}, \mathbf{q}$ are two GM's between two given numbers, then $\mathbf{p^3 + q^3}$ is equal to

(A) $\mathbf{2pqr}$ (B) $\mathbf{2p^2q^2r^2}$ (C) $\mathbf{2pq/r}$ (D) none of these

2. If $\mathbf{A_1, A_2; G_1, G_2}$ and $\mathbf{H_1, H_2}$ are respectively two AM's, two GM's and two HM's between two numbers, then $\frac{\mathbf{A_1+A_2}}{\mathbf{H_1+H_2}}$ equals

(A) $\frac{\mathbf{H_1H_2}}{\mathbf{G_1G_2}}$ (B) $\frac{\mathbf{G_1G_2}}{\mathbf{H_1H_2}}$ (C) $\frac{\mathbf{H_1H_2}}{\mathbf{A_1A_2}}$ (D) $\frac{\mathbf{G_1G_2}}{\mathbf{A_1A_2}}$

3. If $\mathbf{a_1, a_2, a_3, \dots, a_n}$ are positive real numbers such that their product is a fixed number $\mathbf{c}$, then minimum value of $\mathbf{a_1 + a_2 + a_3 + \cdots + 2a_n}$ is equal to

(A) $\mathbf{n(2c)^{1/n}}$ (B) $\mathbf{(n+1)c^{1/n}}$ (C) $\mathbf{2nc^{1/n}}$ (D) $\mathbf{(n+1)(2c)^{1/n}}$

4. If $\boldsymbol{\alpha} \in (\mathbf{0}, \boldsymbol{\pi}/\mathbf{2})$, then $\sqrt{\mathbf{x^2+x}} + \frac{\mathbf{\tan^2\alpha}}{\sqrt{\mathbf{x^2+x}}}$ is always greater than or equal to

(A) $\mathbf{2\tan\alpha}$ (B) 1 (C) 2 (D) $\mathbf{\sec^2\alpha}$

5. If **AM** and **GM** of two roots of a quadratic equation are 9 and 4 respectively, then this equation is

(A) $\mathbf{x^2 - 18x + 16 = 0}$ (B) $\mathbf{x^2 + 18x - 16 = 0}$

(C) $\mathbf{x^2 + 18x + 16 = 0}$ (D) $\mathbf{x^2 - 18x - 16 =}$

6. **Statement I:** For every natural number $\mathbf{n \geq 2}$ $\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \cdots + \frac{1}{\sqrt{n}} > \sqrt{\mathbf{n}}$.

Statement II : For every natural number $\mathbf{n \geq 2}$ $\sqrt{\mathbf{n(n+1)}} < \mathbf{n+1}$

For above statements :

(A) Statement I is true, Statement II is true, Statement II is a correct explanation for Statement I.

(B) Statement I is true, Statement II is true; Statement II is not a correct explanation for Statement I.

(C) Statement I is true, Statement II is false.

(D) Statement I is false, Statement II is true

7. Let $\mathbf{a, b, c}$ be positive ntegers such that $\mathbf{b/a}$ is an integer. If $\mathbf{a, b, c}$ are in geometric progression and the arithmetic mean of $\mathbf{a, b, c}$ is $\mathbf{b+2}$, then the value of $\frac{\mathbf{a^2+a-14}}{\mathbf{a+1}}$ is

8. If $\mathbf{G_1, G_2}$ be two GM's and $\mathbf{A}$ be one **AM** between two numbers, then $\frac{\mathbf{G_1^2}}{\mathbf{G_2}} + \frac{\mathbf{G_2^2}}{\mathbf{G_1}}$ is equal to

(A) $\mathbf{4/2}$ (B) $\mathbf{A}$ (C) $\mathbf{2A}$ (D) $\mathbf{A^2}$

9. If the **HM** and **GM** of two positive numbers are in the ratio $\mathbf{4:5}$, then the numbers are in the ratio

(A) $\mathbf{4:1}$ (B) $\mathbf{4:3}$ (C) $\mathbf{2:3}$ (D) $\mathbf{3:5}$

10. If the ratio between **AM** and **GM** of two numbers be $\mathbf{m}: \mathbf{n}$, then the ratio between these numbers is

(A) $\mathbf{m}+\sqrt{\mathbf{m}^2-\mathbf{n}^2}: \mathbf{m}-\sqrt{\mathbf{m}^2-\mathbf{n}^2}$
(B) $\mathbf{m}+\sqrt{\mathbf{m}^2+\mathbf{n}^2}: \mathbf{m}-\sqrt{\mathbf{m}^2+\mathbf{n}^2}$
(C) $\mathbf{m}-\sqrt{\mathbf{m}^2-\mathbf{n}^2}: \mathbf{m}+\sqrt{\mathbf{m}^2-\mathbf{n}^2}$
(D) none of these

11. If **AM** and **GM** of two numbers are $\mathbf{75/4}$ and 15 respectively, then the greater number is

(A) 30 (B) 25 (C) 15 (D) $\mathbf{15/2}$

12. Let $\mathbf{a_1}, \mathbf{a_2}, \mathbf{a_3}, \ldots$ be in harmonic progression with $\mathbf{a_1}=\mathbf{5}$ and $\mathbf{a_{20}}=\mathbf{25}$. The least positive integer $\mathbf{n}$ for which $\mathbf{a_n}<\mathbf{0}$ is

(A) 22 (B) 23 (C) 24 (D) 25

13. If $\mathbf{m}$ is the A.M. of two distinct real numbers $\mathbf{l}$ and $\mathbf{n}(\mathbf{l}, \mathbf{n}>1)$ and $\mathbf{G_1}, \mathbf{G_2}$ and $\mathbf{G_3}$ are three geometric means between $\mathbf{l}$ and $\mathbf{n}$, then $\mathbf{G_1^4}+\mathbf{2G_2^4}+\mathbf{G_3^4}$ equals.

(A) $\mathbf{4l^2mn}$ (B) $\mathbf{4lm^2n}$ (C) $\mathbf{4lmn^2}$ (D) $\mathbf{4l^2m^2n^2}$

14. If $\mathbf{H_1}, \mathbf{H_2}$ are two HM's between $\mathbf{a}$ and $\mathbf{b}$, then $\frac{\mathbf{H_1H_2}}{\mathbf{H_1+H_2}}$ is equal to

(A) $\frac{\mathbf{a+b}}{\mathbf{ab}}$ (B) $\frac{\mathbf{a+b}}{\mathbf{2ab}}$ (C) $\frac{\mathbf{2ab}}{\mathbf{a+b}}$ (D) $\frac{\mathbf{ab}}{\mathbf{a+b}}$

15. $\mathbf{n}$ AM's are inserted between 1 and 51 . if ratio between 4 th and 7th AM's is $\mathbf{3}: \mathbf{5}$, then $\mathbf{n}$ equals

(A) 48 (B) 42 (C) 36 (D) 24

16. If AM of two numbers $\mathbf{a}$ and $\mathbf{b}$ is twice of their GM, then $\mathbf{a}: \mathbf{b}$ is equal to

(A) $\sqrt{\mathbf{3}}+\mathbf{1}: \sqrt{\mathbf{3}}-\mathbf{1}$
(B) $\mathbf{2}+\sqrt{\mathbf{3}}: \mathbf{2}-\sqrt{\mathbf{3}}$
(C) $\mathbf{3}: \mathbf{2}$
(D) none of these

17. If $\mathbf{A}$ be one **AM** and $\mathbf{p}, \mathbf{q}$ be two GM's between two numbers, then $\mathbf{2A}$ is equal to

(A) $\frac{\mathbf{p^3+q^3}}{\mathbf{pq}}$ (B) $\frac{\mathbf{p^3-q^3}}{\mathbf{pq}}$ (C) $\frac{\mathbf{p^2+q^2}}{\mathbf{2}}$ (D) $\frac{\mathbf{pq}}{\mathbf{2}}$

18. If $\mathbf{a}, \mathbf{b}, \mathbf{c}$ are in GP and $\mathbf{x}, \mathbf{y}$ are AM's between $\mathbf{a}, \mathbf{b}$ and $\mathbf{b}, \mathbf{c}$ respectively, then

(A) $\frac{\mathbf{1}}{\mathbf{x}}+\frac{\mathbf{1}}{\mathbf{y}}=\mathbf{2}$ (B) $\frac{\mathbf{1}}{\mathbf{x}}+\frac{\mathbf{1}}{\mathbf{y}}=\frac{\mathbf{1}}{\mathbf{2}}$ (C) $\frac{\mathbf{1}}{\mathbf{x}}+\frac{\mathbf{1}}{\mathbf{y}}=\frac{\mathbf{2}}{\mathbf{a}}$ (D) $\frac{\mathbf{1}}{\mathbf{x}}+\frac{\mathbf{1}}{\mathbf{y}}=\frac{\mathbf{2}}{\mathbf{b}}$

19. If **AM** and **HM** of the roots of a quadratic equation are $\mathbf{3/2}$ and $\mathbf{4/3}$ respectively, then that equation is

(A) $\mathbf{x^2}+\mathbf{3x}+\mathbf{2}=\mathbf{0}$
(B) $\mathbf{x^2}-\mathbf{3x}+\mathbf{2}=\mathbf{0}$
(C) $\mathbf{x^2}-\mathbf{3x}-\mathbf{4}=\mathbf{0}$
(D) none of these

20. If $\mathbf{A}$ is one AM between two numbers $\mathbf{a}$ and $\mathbf{b}$, and the sum of $\mathbf{n}$ AM's between them is $\mathbf{S}$, then $\mathbf{S/A}$ depends on 3)

(A) $\mathbf{n}, \mathbf{a}, \mathbf{b}$ (B) $\mathbf{n}, \mathbf{b}$ (C) $\mathbf{n}, \mathbf{a}$ (D) $\mathbf{n}$

21. The AM of two numbers exceeds their GM by $\mathbf{15}$ & **HM** by 27 . Find the numbers.

22. The A.M. between two positive numbers exceeds the G.M. by 5 , and the G.M. exceeds the H.M. by 4. Find the numbers

23. If **G** be the geometric mean of **x** and **y**, then prove that $\frac{1}{(G^2-x^2)}+\frac{1}{(G^2-y^2)}=\frac{1}{G^2}$.

24. Find two numbers whose arithmetic mean is 34 and the geometric mean is 16.

25. If **a** is the A.M. of **b** & **c**, and the two geometric means between **b** & **c** are $\mathbf{G_1}$ and $\mathbf{G_2}$, then prove that $\mathbf{G_1}^3+\mathbf{G_2}^3=\mathbf{2abc}$.

26. Insert three arithmetic means between 3 and 19 .

27. If eleven A.M.'s are inserted between 28 and 10 , then find the number of integral A.M.'s.

H.P.

1. If pth term of a HP be $\mathbf{q}$ and qth term be $\mathbf{p}$, then its $'(\mathbf{p}+\mathbf{q})$ th term is

(A) $\frac{1}{p+q}$ (B) $\frac{1}{p}+\frac{1}{q}$ (C) $\frac{pq}{p+q}$ (D) $\mathbf{p}+\mathbf{q}$

2. Example 9. Five numbers $\mathbf{a}, \mathbf{b}, \mathbf{c}, \mathbf{d}$, e are such that $\mathbf{a}, \mathbf{b}, \mathbf{c}$ are in $\mathbf{AP}$; $\mathbf{b}, \mathbf{c}, \mathbf{d}$ are in GP and $\mathbf{c}, \mathbf{d}, \mathbf{e}$ are in HP. If $\mathbf{a}=\mathbf{2}, \mathbf{e}=\mathbf{18}$; then values of $\mathbf{b}, \mathbf{c}, \mathbf{d}$ are

(A) $\mathbf{2, 6, 18}$ (B) $\mathbf{4, 6, 9}$ (C) $\mathbf{4, 6, 8}$ (D) $\mathbf{-2, -6, 18}$

3. Example 23. If $\mathbf{a}, \mathbf{x}, \mathbf{y}, \mathbf{z}, \mathbf{b}$ are in $\mathbf{AP}$, then $\mathbf{x}+\mathbf{y}+\mathbf{z}=\mathbf{15}$ and if $\mathbf{a}, \mathbf{x}, \mathbf{y}, \mathbf{z}, \mathbf{b}$ are in HP, then $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=\frac{5}{3}$. Numbers $\mathbf{a}$ and $\mathbf{b}$ are

(A) 8,2 (B) 11,3 (C) 9,1 (D) none of these

4. If $\mathbf{a_1}, \mathbf{a_2}, \mathbf{a_3}, \ldots, \mathbf{a_n}$ are in $\mathbf{HP}$, then $\mathbf{a_1 a_2}+\mathbf{a_2 a_3}+\cdots+\mathbf{a_{n-1} a_n}$ is equal to

(A) $\mathbf{n a_1 a_n}$ (B) $(\mathbf{n}-\mathbf{1}) \mathbf{a_1 a_n}$ (C) $(\mathbf{n}+\mathbf{1}) \mathbf{a_1 a_n}$ (D) none of these

5. If $\mathbf{a}, \mathbf{b}, \mathbf{c}$ are in HP, then the value of $\left(\frac{1}{b}+\frac{1}{c}-\frac{1}{a}\right)\left(\frac{1}{c}+\frac{1}{a}-\frac{1}{b}\right)$ is

(A) $\mathbf{2/bc+1/b^2}$ (B) $\mathbf{3/c^2+2/ca}$

(C) $\mathbf{3/b^2-2/ab}$ (D) none of these

6. If $\mathbf{x}, \mathbf{1}, \mathbf{z}$ are in $\mathbf{AP}$ and $\mathbf{x}, \mathbf{2}, \mathbf{z}$ are in $\mathbf{GP}$, then $\mathbf{x}, \mathbf{4}, \mathbf{z}$ are in

(A) $\mathbf{AP}$ (B) GP (C) $\mathbf{HP}$ (D) none of these

7. If the roots of $\mathbf{10x^3-cx^2-54x-27=0}$ are in harmonic progression, then find $\mathbf{c}$ and all the roots.

8. An AP & an HP have the same first term, the same last term & the same number of terms; prove that the product of the $\mathbf{r}^{\text{th}}$ term from the beginning in one series & the $\mathbf{r}^{\text{th}}$ term from the end in the other is independent of $\mathbf{r}$.

9. If the $\mathbf{10}^{\text{th}}$ term of an HP is 21 and $\mathbf{21}^{\text{st}}$ term of the same HP is 10 , then find the $\mathbf{210}^{\text{th}}$ term.

10. Given that $\mathbf{a^x=b^y=c^z=d^u}$ & $\mathbf{a}, \mathbf{b}, \mathbf{c}, \mathbf{d}$ are in GP, show that $\mathbf{x}, \mathbf{y}, \mathbf{z}, \mathbf{u}$ are in HP.

11. If $\mathbf{a}, \mathbf{b}, \mathbf{c}$ and $\mathbf{d}$ are in H.P. , then find the value of $\frac{a^{-2}-d^{-2}}{b^{-2}-c^{-2}}$

ANSWER KEY

(H.P. & A.M. – G.M. – H. M.)

1.	(A)	**2.**	(B)	**3.**	(A)	**4.**	(A)	**5.**	(A)	**6.**	(B)	**7.**	4
8.	(C)	**9.**	(A)	**10.**	(A)	**11.**	(D)	**12.**	(D)	**13.**	(B)	**14.**	(D)
15.	(D)	**16.**	(B)	**17.**	(A)	**18.**	(D)	**19.**	(A)	**20.**	(D)	**21.**	120,30
22.	40,10	**24.**	64 and 4			**26.**	14, 9, 4 or 4, 9, 14			**27.**	5, 10,15, 20		

H.P.

1.	(C)	**2.**	(B)	**3.**	(C)	**4.**	(B)	**5.**	(C)	**6.**	(C)
7.	$\mathbf{C = 9}$; $(\mathbf{3, -3/2, -3/5})$					**9.**	1	**11.**	3		