Nishant Jindal

1) $\frac{dy}{dx} = 0 \rightarrow x = 0$ —

$\downarrow$

y में Q ...

y — —

(~ ~)(~ ~)

2) $\frac{dy}{dx} = 3.$

3) ✓

4) ✓

(5) ✓ $y = ax^2 + bx + \frac{7}{2}$ $(1,2) \Rightarrow 1 = 4a + 2b + \frac{7}{2}$

(6) $\left.\frac{dy}{dx}\right| = 2ax + b$ $\left.\frac{dy}{dx}\right|_{x=-2} = 2x + 6 = 2$

$_{x=1} = 2a + b = -\frac{1}{2}$

(7) $3x^2 + 2x + 1 = 1$

Vector की Class is not today.
tommorow at 5:45

(8) $\left(\frac{x}{a}\right)^n + \left(\frac{y}{b}\right)^n = 2 \xrightarrow{x=a} 1 + \left(\frac{y}{b}\right)^n = 2$

$\left(\frac{y}{b}\right)^n = 1$

$\Rightarrow y = b.$

$n\left(\frac{x}{a}\right)^{n-1} \cdot \frac{1}{a} + \frac{n}{b}\left(\frac{y}{b}\right)^{n-1} \cdot \frac{dy}{dx} = 0$

$\left.\frac{dy}{dx}\right|_{x=a,\, y=b} = -\frac{b}{a}\left(\frac{x}{a}\right)^{n-1} \cdot \left(\frac{b}{y}\right)^{n-1}$

n = odd

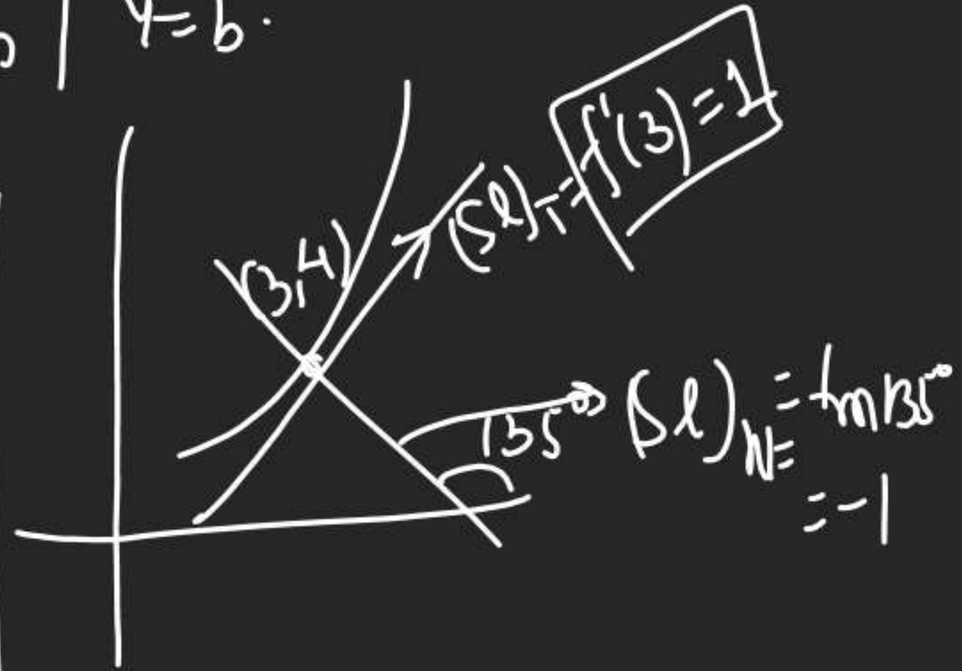
$= -\frac{b}{a}$

n = Even	n = odd
$y = \pm b$	$y = b.$

$(y - b) = \frac{a}{b}(x - a)$

$by - b^2 = ax - a^2$

$ax - by = a^2 - b^2$ (C)



Nishant Jindal

10) When tangent is making acute angle then $\frac{dy}{dx} > 0$

$3x^2+2\lambda x+1 > 0$

$D < 0$

$4\lambda^2 - 12 < 0$

$\lambda^2 - 3 < 0$

$-\sqrt{3} < \lambda < \sqrt{3}$

$y = e^{2x} + x^2 \Rightarrow y = e^0 + 0 = 1$

$\left.\frac{dy}{dx}\right|_{x=0} = 2e^{2x} + 2x$

$= 2$

$(y-1) = -\frac{1}{2}(x-0)$

$2y - 2 = -x$

$x + 2y + 2 = 0$ EON

$d = \frac{|2|}{\sqrt{1^2+2^2}} = \frac{2}{\sqrt{5}}$

12) ✓

13) ✓

14) ✓

15) ✓

16) ✓

Nishant Jindal

Vector का class को 5:45PM

Q find locus of Point of contact to $y^2 = 4a\left(x + a\cdot \sin\frac{x}{a}\right)$ at Which tangent is || to x Axis?

$\frac{dy}{dx} = 0$

$2y\frac{dy}{dx} = 4a + 4a^2 \cos\frac{x}{a} \times \frac{1}{a}$

$0 = 4a\left(1 + \cos\frac{x}{a}\right)$

$1 + \cos\frac{x}{a} = 0$

$\cos\frac{x}{a} = -1 \Rightarrow \sin\frac{x}{a} = 0$ तो Using Cond^n

$\therefore$ locus $y^2 = 4a(x + a\times 0)$

$y^2 = 4ax$ is Req locus

Q Line of tangent to Curve.

$y = \sin x - \tan x \quad x \in \left(0, \frac{\pi}{2}\right)$

A) Will lie below the Curve.

B) ——— above "

C) Crosses Curve.

D) Nothing Can be Said.

Concept → Concavity

$\frac{d^2y}{dx^2} > 0$ Con. up

tangent below Curve

Con down $\frac{d^2y}{dx^2} < 0$

tangent above Curve

$\frac{dy}{dx} = \cos x - \sec^2 x$

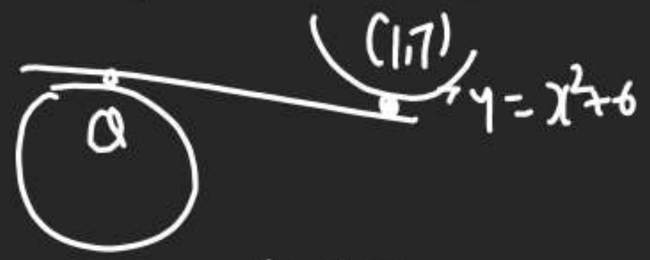
$\frac{d^2y}{dx^2} = -\underbrace{\sin x}_{-ve} - 2\underbrace{\sec^2 x \tan x}_{-ve} = -ve \,|\, \left(0, \frac{\pi}{2}\right)$

Con down

Qs of AOD Mantra do atleast 20 Qs in a line Urself.

Nishant Jindal

Q tangent to Curve $y = x^2 + 6$ at Pt $(1, 7)$ touches the Circle $x^2 + y^2 + 16x + 12y + c = 0$ at Q find Q ?



① $\left.\frac{dy}{dx}\right|_{x=1} = 2x = 2 \Rightarrow$ ② EOT

$(y - 7) = 2(x - 1)$

$2x - y = -5$

$y = 2x + 5$

Combine Eqⁿ

(3) $x^2 + (2x+5)^2 + 16x + 12(2x+5) + c = 0$

tangent/Line $5x^2 + 60x + 85 + c = 0$

दिया है + Curve दिया है & asking Where Line is touching /Crossing find combine eqⁿ Solve $D = 0$

Condⁿ of tangency $D = 0$ ** $(\alpha = \beta)$

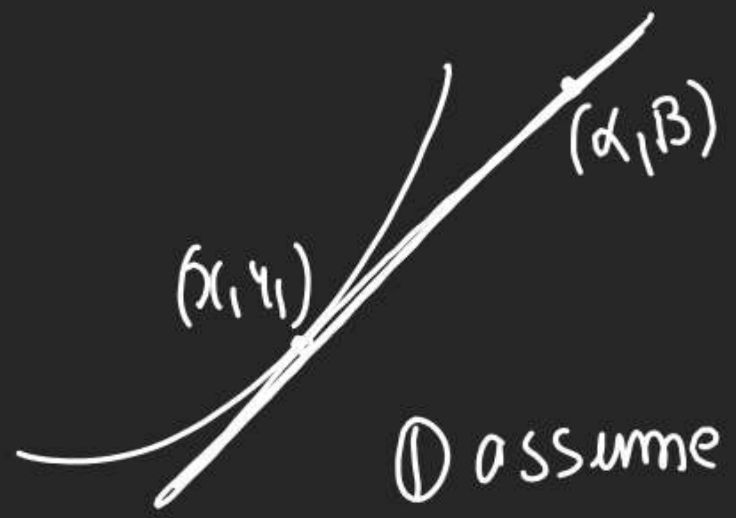
$\alpha + \beta = -12$

$2\alpha = -12 \Rightarrow \alpha = -6 = x$

$y = 2x - 6 + 5 = -7$ } $Q \equiv (-6, -7)$

When tangent is P.T. from External Pt.

When Pt. given in Qs is not satisfying Curve & EOT is asked



① assume a pt (x_1, y_1) on Curve.

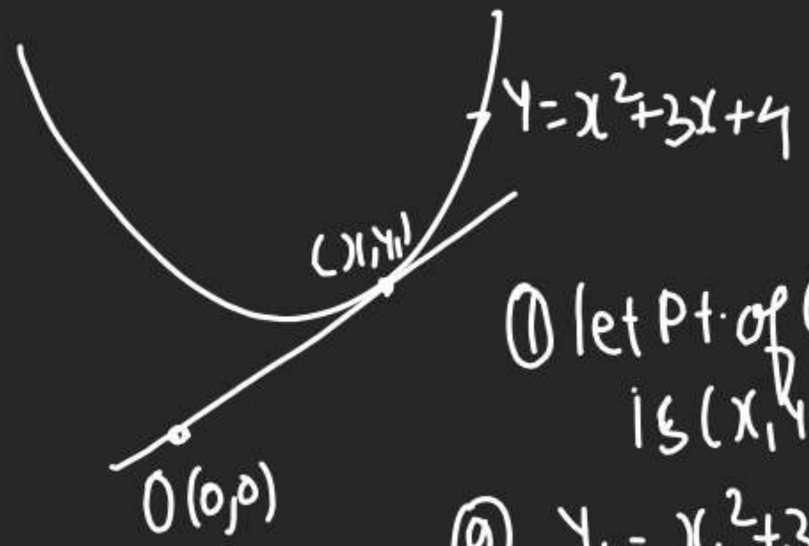
(2) Satisfy (x_1, y_1) on Curve → Eq①

(3) Now Use $\left.\frac{dy}{dx}\right|_{(x_1,y_1)} = \frac{\beta - y_1}{\alpha - x_1}$ & Solve → Eq②

④ Solve Eq① & Eq2

Nishant Jindal

Q Coord of Pt. on Curve $y = x^2+3x+4$ the tangent at which P.T. origin.



(1) let Pt. of Contact is (x_1,y_1)

(2) $y_1 = x_1^2+3x_1+4 \rightarrow (1)$

(3) $\left.\dfrac{dy}{dx}\right|_{x_1,y_1} = 2x+3 = 2x_1+3$

$2x_1+3 = \dfrac{y_1-0}{x_1-0}$

$y_1 = 2x_1^2+3x_1 \rightarrow (2)$

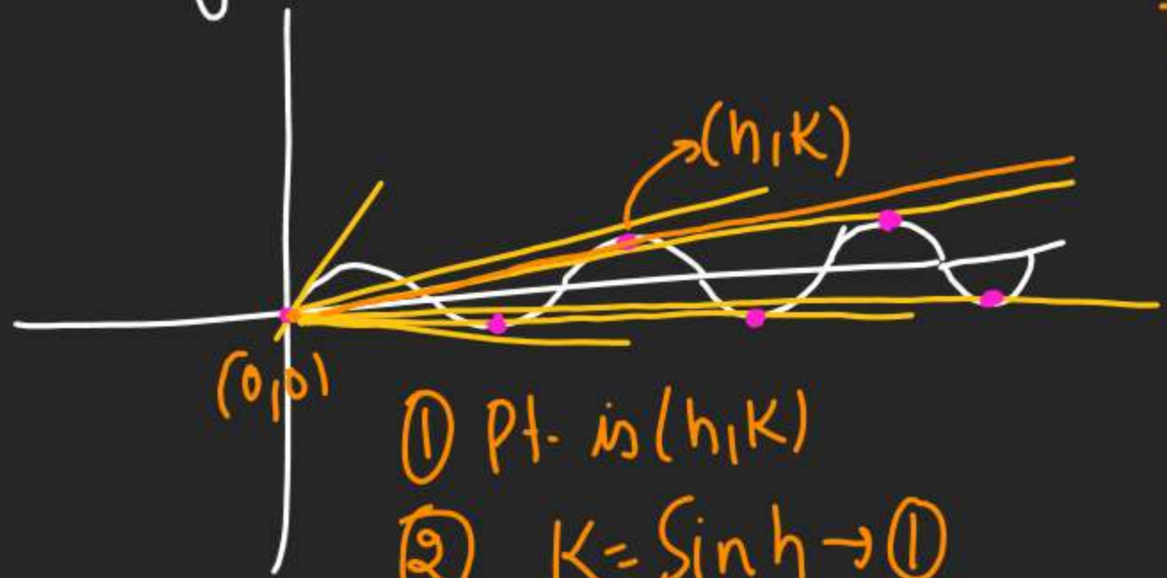
(4) $2x_1^2+\cancel{3x_1} = x_1^2+\cancel{3x_1}+4$

$x_1^2 = 4 \Rightarrow x_1 = \pm 2$

$x_1 = 2$ | $x = -2$
$y_1 = 4+6+4$ | $y_1 = 4-6+4$
$(2,14)$ | $(-2,2)$

Ans

Q tangents are drawn from $(0,0)$ to $\underline{y=\sin x}$ find the locus of Pt. of Contact lying on Curve



(1) Pt. is (h,k)

(2) $k = \sin h \rightarrow (1)$

(3) $\left.\dfrac{dy}{dx}\right|_{(h,k)} = \cos x = \underline{\cos h}$

$\cos h = \dfrac{k-0}{h-0} \rightarrow (1)$

for locus

$\sin^2 h+\cos^2 h = 1$

$k^2+\dfrac{k^2}{h^2} = 1$

$\boxed{x^2-y^2 = x^2y^2}$

Nishant Jindal

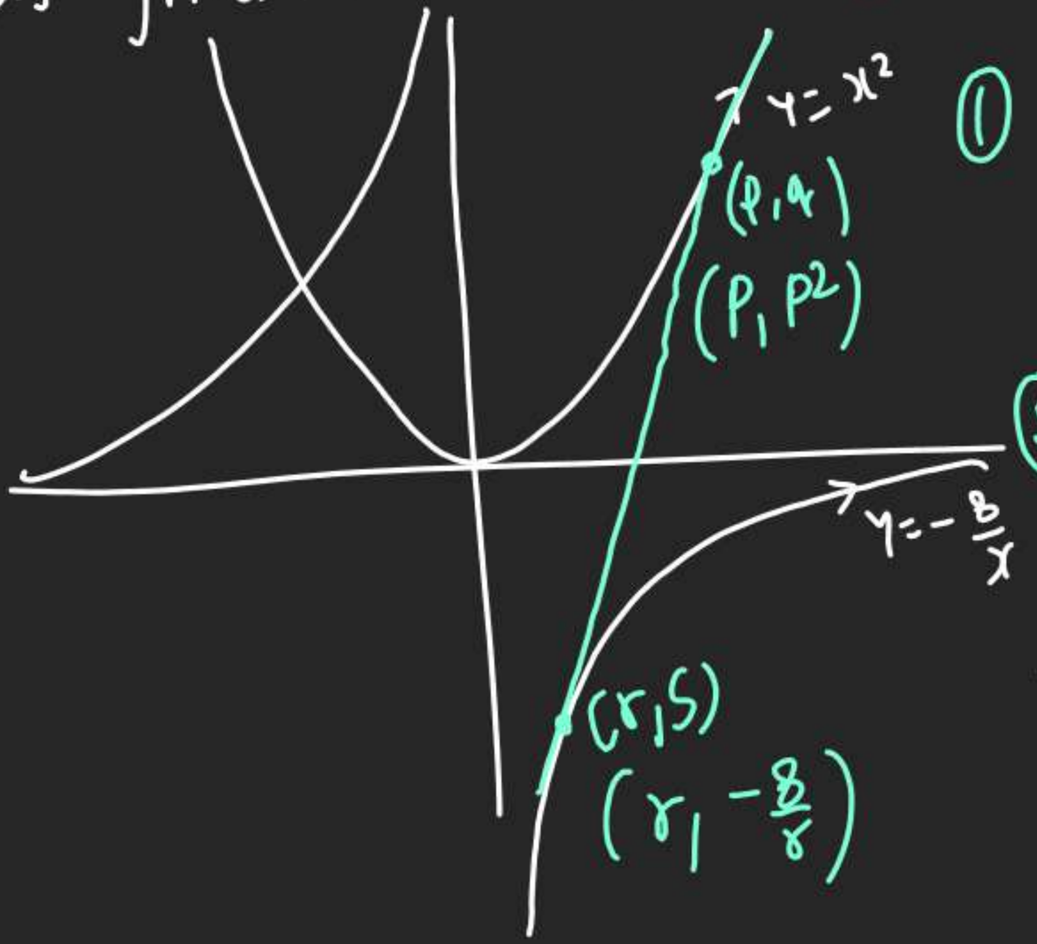
Q There is a pt. (p, q) on graph of $f(x) = x^2$

& pt. (r, s) on graph $y = -\frac{8}{x}$ $(p \geq 0, r \geq 0)$

If line thru (p, q) & (r, s) is also

tangent on both curves at these

pts find $p + r$. $= 4 + 1 = 5$ Ans



① $2p = \frac{8}{r^2}$

$pr^2 = 4 \longrightarrow p = 4, r = 1$

② $2p = \dfrac{-\frac{8}{r} - p^2}{r - p}$

$2pr - 2p^2 = -\frac{8}{r} - p^2$

$2pr = -\frac{8}{r} + p^2$

$2pr^2 = -8 + p^2 r \Rightarrow p^2 r = 16$

Parametric Coordinates

When we need to assume a pt.

on curve keeping prop. of curve inside.

1) $x^2 + y^2 = a^2$ $\qquad x = a\cos\theta, y = a\sin\theta$

2) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ $\qquad x = a\cos\theta, y = b\sin\theta$

3) $y^2 = 4ax$ $\qquad x = at^2, \; y = 2at$

(4) $x^{2/3} + y^{2/3} = a^{2/3}$ $\qquad x = a\cos^3\theta, \; y = a\sin^3\theta$

$(a\cos^3\theta)^{2/3} + (a\sin^3\theta)^{2/3}$

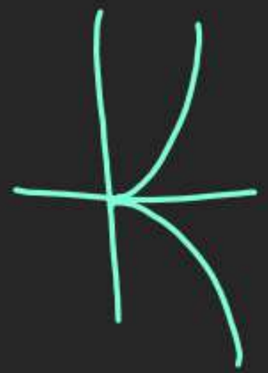
$a^{2/3}\cos^2\theta + a^{2/3}\sin^2\theta$

$a^{2/3}(\cos^2\theta + \sin^2\theta) = a^{2/3} = \text{RHS}$

Nishant Jindal

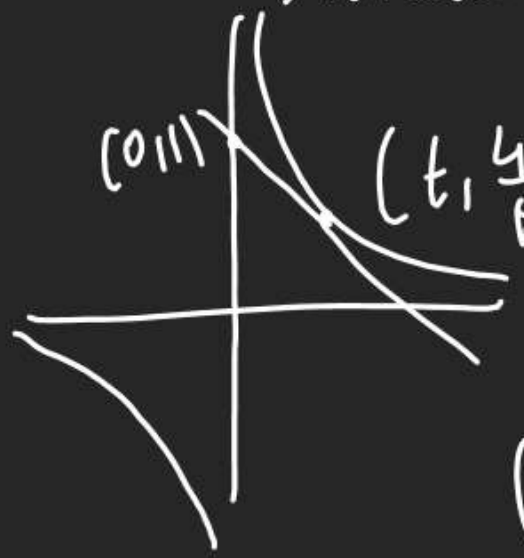
(5) $y = -\frac{8}{x}$ $\quad (t, -\frac{8}{t})$ $\quad (t_1, -\frac{8}{t_1})$ & $(t_2, -\frac{8}{t_2})$

(6) $y^2 = x^3$ $\quad (t^2, t^3)$



(1) EOT drawn to curve $xy = 4$ from $(0,1)$ $\to$ outside

$y = \frac{4}{x}$



$(t, \frac{4}{t})$

(1) $\frac{dy}{dx}\Big|_{(t, \frac{4}{t})} = -\frac{4}{x^2} = -\frac{4}{t^2} = -\frac{4}{64}$

(2) $-\frac{4}{t^2} = \frac{\frac{4}{t} - 1}{t - 0} \Rightarrow -\frac{4}{t} = \frac{4}{t} - 1 \Rightarrow \frac{8}{t} = 1 \Rightarrow t = 8$,

(3) Pt. of tangent $(t, \frac{4}{t})$

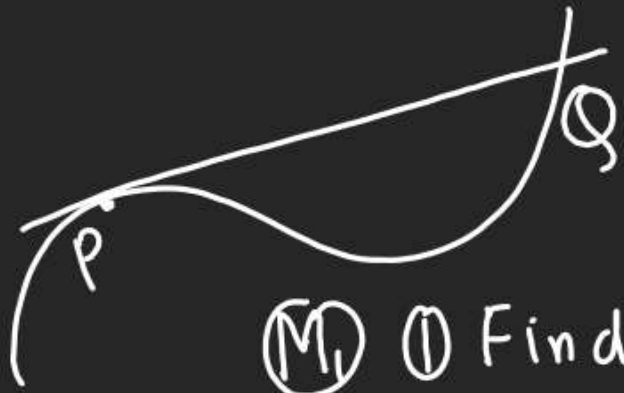
$(8, \frac{1}{2})$

(4) $y - \frac{1}{2} = -\frac{1}{16}(x - 8)$

$16y - 8 = -x + 8$

$x + 16y = 16$

Tangent meets Curve Again.



(M1) (1) Find EOT at P.

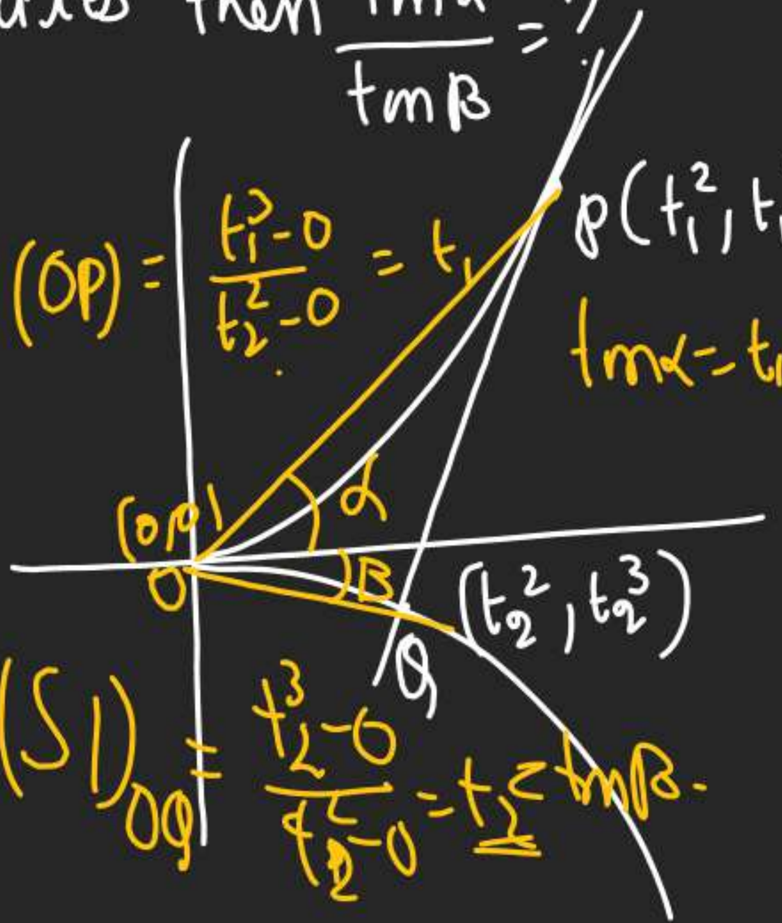
(2) Solve EOT with Curve & Get Q.

(M2) (1) assume P & Q (Parametrically) if not given

(2) Solve $\frac{dy}{dx} = \frac{y_2 - y_1}{x_2 - x_1}$ (See Qs)

Nishant Jindal

Q If tangent at P on curve $y^2 = x^3$ intersect the curve again at Q & st. line OP & OQ makes angle α, β with axes then $\frac{\tan\alpha}{\tan\beta} = ?$



$Sl_{(OP)} = \frac{t_1^3 - 0}{t_2^2 - 0} = t_1$

$\tan\alpha = t_1$

$(Sl)_{OQ} = \frac{t_2^3 - 0}{t_2^2 - 0} = t_2 = \tan\beta.$

① $y^2 = x^3$

$2y\frac{dy}{dx} = 3x^2$

$\left.\frac{dy}{dx}\right|_{(t_1^2, t_1^3)} = \frac{3x^2}{2y} = \frac{3\,t_1^4}{2\,t_1^3}$

$= \frac{3}{2}t_1$

② $\frac{3t_1}{2} = \frac{t_2^3 - t_1^3}{t_2^2 - t_1^2}$

$\frac{3t_1}{2} = \frac{t_2^2 + t_1^2 + t_1t_2}{t_2 + t_1}$

$3t_1t_2 + 3t_1^2 = 2t_2^2 + 2t_1^2 + 2t_1t_2$

$t_1^2 - 2t_2^2 + t_1t_2 = 0$

$\left(\frac{t_1}{t_2}\right)^2 + \frac{t_1}{t_2} - 2 = 0 \quad \div t_2^2$

$x^2 + x - 2 = 0$

$(x-2)(x+1) = 0$

$\frac{t_1}{t_2} = 2 \quad$ Or $\quad \frac{t_1}{t_2} = -1$

$\frac{\tan\alpha}{\tan\beta} = 2$ Or $\frac{\tan\alpha}{\tan\beta} = -1$

$\underline{\underline{40}}$

All Qs Whatever We have Completed