Nishant Jindal

Range of α so that ball complete the track.

AB = Range of projectile

$$AB = \frac{v^2 \sin 2\alpha}{g}$$

Energy Conservation from D to A.

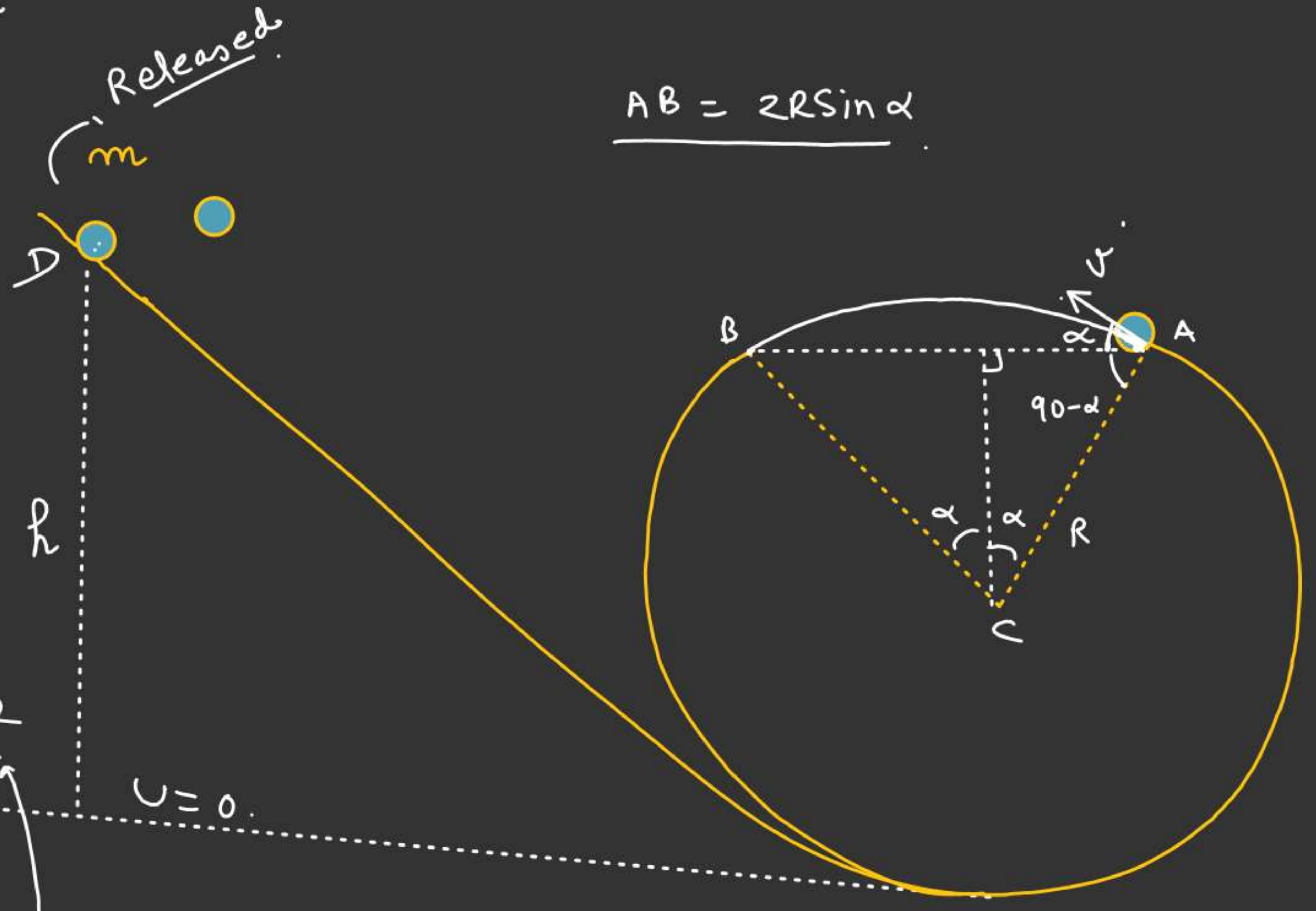
$$mgh = mg(R + R\cos\alpha) + \frac{1}{2}mv^2$$

$$AB = 2R\sin\alpha$$

$$\cancel{2}R\cancel{\sin}\alpha = \frac{v^2}{g}\cancel{2}\cancel{\sin}\alpha \cdot \cos\alpha$$

$$v^2 = \left(\frac{gR}{\cos\alpha}\right)$$

$$AB = 2R\sin\alpha$$



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$$mgh = mg(R+R\cos\alpha)+\frac{1}{2}mv^2$$

$$AB = 2R\sin\alpha$$

$$\cancel{2}R\cancel{\sin}\alpha = \frac{v^2}{g}\cancel{2}\cancel{\sin}\alpha\cdot\cos\alpha$$

$$v^2 = \left(\frac{gR}{\cos\alpha}\right)$$

$$mgh = mgR + mgR\cos\alpha + \frac{mgR}{2\cos\alpha}$$

$$\frac{h}{R} = 1+\cos\alpha+\frac{1}{2\cos\alpha}$$

$$\frac{h}{R} = \frac{2\cos\alpha+2\cos^2\alpha+1}{2\cos\alpha}$$

$$\Downarrow$$

$$K > 1 \checkmark$$

$$2K\cos\alpha = 2\cos\alpha + 2\cos^2\alpha + 1$$

$$2\cos^2\alpha + 2(1-K)\cos\alpha + 1 = 0.$$

$$\cos^2\alpha + (1-K)\cos\alpha + \frac{1}{2} = 0.$$

$$\boxed{\cos^2\alpha - (K-1)\cos\alpha + \frac{1}{2} = 0}$$

$$D \geq 0.$$

$$(K-1)^2 - 4\times\frac{1}{2} \geq 0$$

$$(K-1)^2 \geq 2$$

$$(K-1) \geq \pm\sqrt{2}$$

$$(K-1) \geq +\sqrt{2}$$

$$K \geq (\sqrt{2}+1) \quad (1)$$

$$\cos\alpha = \frac{(K-1)\pm\sqrt{(K-1)^2-2}}{2}$$

$$\cos\alpha \leq 1$$

$$(K-1)\pm\sqrt{(K-1)^2-2} \leq 2$$

$$\sqrt{(K-1)^2-2} \leq 2-(K-1)$$

$$\cancel{(K-1)^2}-2 \leq 4+\cancel{(K-1)^2}-4(K-1)$$

$$-6 \leq -4(K-1)$$

$$\frac{3}{2} \geq (K-1) \Rightarrow K \leq 5/2 \quad (2)$$

Nishant Jindal

from ① & ②

$$(\sqrt{2}+1) \le \underline{K} \le \underline{2.5}$$

$$(\sqrt{2}+1) \le \frac{h}{R} \le 2.5$$

$$\boxed{R(\sqrt{2}+1) \le \underline{h} \le (2.5R)}$$

$$\boxed{45^\circ \le \alpha \le 60^\circ}$$

$$\cos\alpha = \frac{(K-1) \pm \sqrt{(K-1)^2 - 2}}{2}$$

for $K = (\sqrt{2}+1)$

$$\cos\alpha = \frac{\sqrt{2} \pm \sqrt{0}}{2} = \frac{1}{\sqrt{2}}$$

$$\underline{\alpha_{min} = 45^\circ}$$ ✓

$$\underline{K = 2.5}$$

$$\cos\alpha = \frac{1.5 \pm \sqrt{(1.5)^2 - 2}}{2}$$

$$\cos\alpha = \left(\frac{1}{2}\right)$$ ✓

$$\boxed{\alpha = 60^\circ}$$

Nishant Jindal

Power

↳ Avg power = $\frac{\text{Work}}{\text{Time}} = \frac{\text{Energy}}{\text{Time}}$

$$\boxed{P_{avg} = \frac{W}{t}} \qquad P_{avg} = \left(\frac{\Delta W}{\Delta t}\right)$$

Inst. Power

$$P_{inst} = \lim_{\Delta t \to 0}\left(\frac{\Delta W}{\Delta t}\right) = \left(\frac{dW}{dt}\right)$$

$$P_{inst} = \frac{\vec{F}\cdot d\vec{s}}{dt} = \vec{F}\cdot\frac{d\vec{s}}{dt}$$

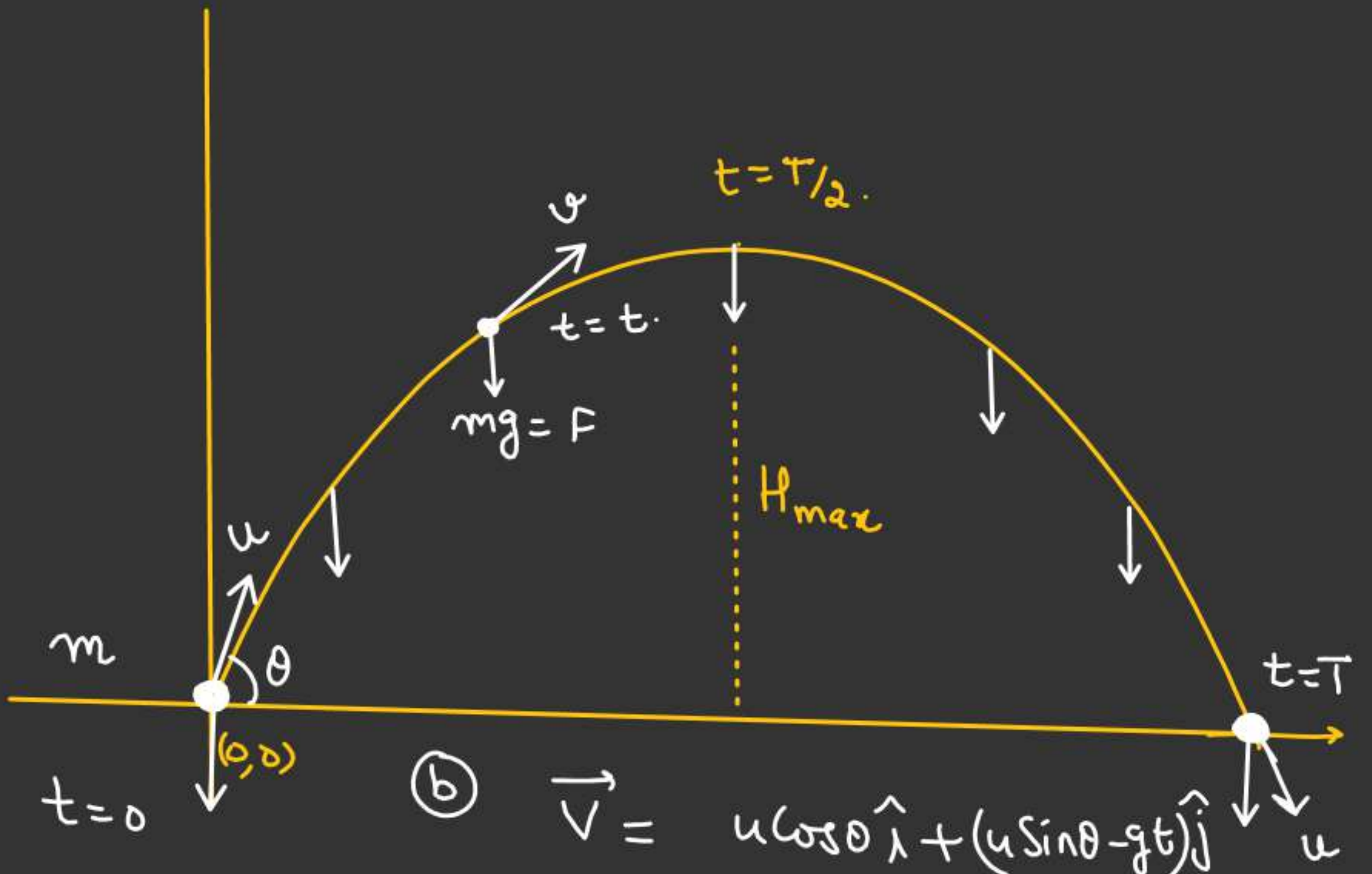
$$\boxed{P_{inst} = \vec{F}\cdot\vec{v}}$$

Q. (a) Find Avg power due to gravity in the interval

I) $t=0$ to $t=T$.

II) $t=0$ to $t=T/2$.

(b) Find Inst. power due to gravity at any instant $t=t$.



a)

i) $(P_{mg})_{avg} = \frac{W_{mg}}{T} = 0$ $\quad W_{mg} = 0$

II) $(P_{mg})_{avg} = \frac{W_{mg}}{T} = \frac{-mg H_{max}}{(T/2)}$

$= -mg \cdot \frac{u^2 \sin^2\theta}{2g \times \left(\frac{u\sin\theta}{g}\right)}$

$= -\frac{mgu\sin\theta}{2}$ ✓.

(b) $\vec{V} = u\cos\theta\,\hat{i} + (u\sin\theta - gt)\hat{j}$

$\vec{F} = -mg\,\hat{j}$

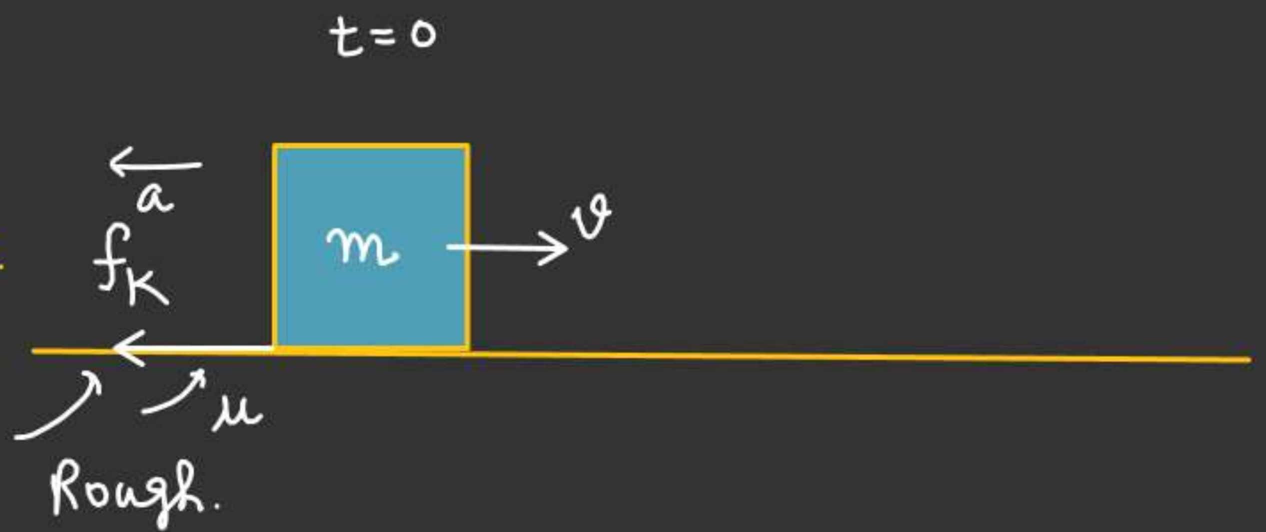
$P_{inst} = \vec{F}\cdot\vec{V}$

$= -mg(u\sin\theta - gt)$

Nishant Jindal

a) Find. Avg power delevered by friction force on the block.

b) If $\mu = kx$ when k is a constant. Find Maximum Instantaneous power of friction.



a) $$a = \frac{f_k}{m} = \frac{\mu m g}{m} = \mu g.$$

let, t be the time when block stop.

$$0 = v - \mu g t \Rightarrow t = \frac{v}{\mu g}.$$

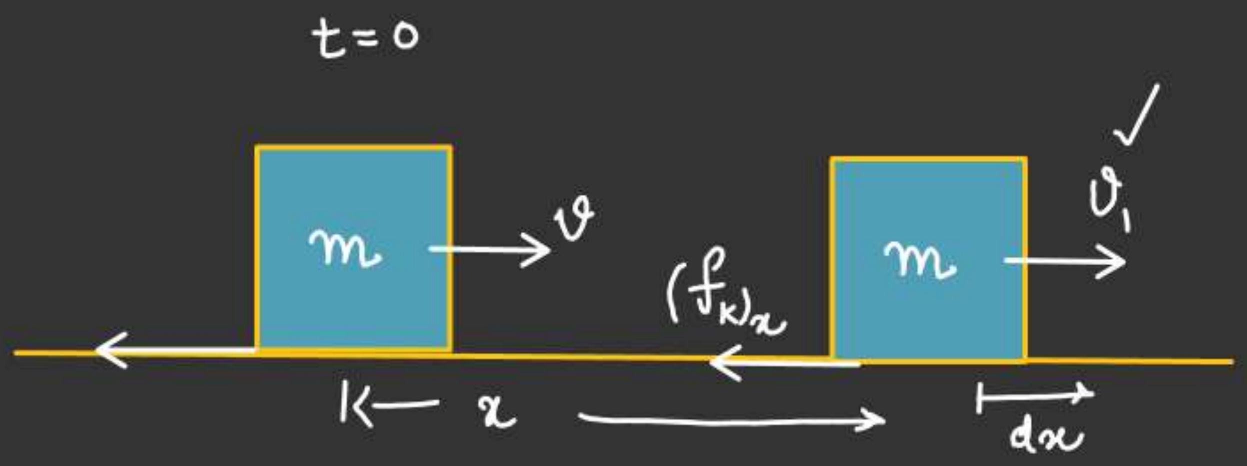
$$W_{f_k} = (\Delta K.E)_{block} = K.E_f - K.E_i$$

$$W_{f_k} = 0 - \frac{1}{2}mv^2$$

$$|P_{avg}| = \frac{W_{f_k}}{t} = \frac{\frac{1}{2}mv^2}{v/\mu g} = \left(\frac{\mu m g v}{2}\right)$$

Nishant Jindal

b) If $\mu = Kx$ when K is a constant. Find Maximum Instantaneous Power of friction.



$(f_k)_x$ = Kinetic friction at a distance x.

$$(f_k)_x = \mu_x mg = Kmgx$$

$$a = -Kgx$$

$$\frac{v\,dv}{dx} = -Kg\,x$$

$$\int_{v}^{v_1} v\,dv = -Kg\int_{0}^{x} x\,dx$$

$$\frac{v_1^2 - v^2}{\cancel{2}} = \frac{-Kgx^2}{\cancel{2}}$$

$$v_1^2 = v^2 - Kgx^2$$

$$\left(v_1 = \sqrt{v^2 - Kgx^2}\right)$$

$$P_{inst} = \overrightarrow{(f_k)_x} \cdot \overrightarrow{v_1}$$

$$= -Kmgx\sqrt{v^2 - Kgx^2}$$

Nishant Jindal

$$P_{inst} = \vec{(f_k)_x} \cdot \vec{v_1}$$

$$= -Kmgx\sqrt{v^2 - Kgx^2} = \left(-Kmg\sqrt{x^2v^2 - Kgx^4}\right)$$

put $x = \frac{v}{\sqrt{2Kg}}$

For maxima

$$\frac{dP_{inst}}{dx} = 0$$

$$-Kmg\frac{d}{dx}\left(\sqrt{\underbrace{x^2v^2 - Kgx^4}_{t}}\right) = 0$$

$$\frac{d(\sqrt{t})}{dt} \times \frac{dt}{dx} = 0$$

$$\frac{1}{2\sqrt{x^2v^2 - Kgx^4}} \times \frac{d}{dx}\left(x^2v^2 - Kgx^4\right) = 0$$

$$v^2(2x) - 4Kgx^3 = 0$$

$$x\left(2v^2 - 4Kgx^2\right) = 0$$

$$x = 0, \quad x = \sqrt{\frac{2v^2}{4Kg}}$$

$$x = \frac{v}{\sqrt{2Kg}}$$

$$(P_{inst})_{max} = ??$$

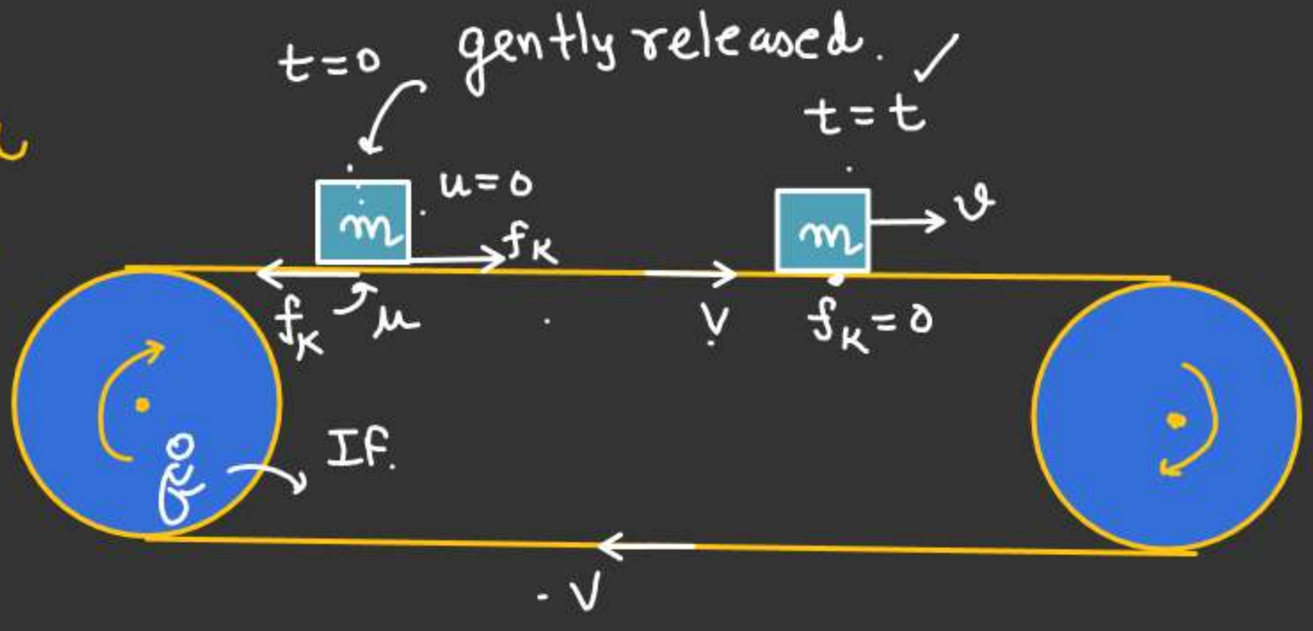
$$= -\sqrt{\alpha g}\left(\frac{mv^2}{2}\right)$$

Nishant Jindal

Conveyor belt is very long.

a) Avg. power due to friction.

b) Distance travelled by block and Conveyor before relative motion Stop.



$a = \mu g$

$v = 0 + \mu g t$

$t = \left(\frac{v}{\mu g}\right)$

$W_{f_K} = \Delta K.E$

$= +\frac{1}{2}mv^2 - 0$

$= +\frac{1}{2}mv^2$

V = constant

↳ velocity of Conveyor belt.

$S_{block} = \frac{1}{2}at^2$

$= \frac{1}{2} \times \mu g \times \left(\frac{v}{\mu g}\right)^2$

$= \frac{v^2}{2\mu g}$

$(P_{avg}) = \frac{W_{f_K}}{t} = \frac{\frac{mv^2}{2}}{v/\mu g} = \left(+\frac{\mu m g v}{2}\right)$

$S_{conveyor\ belt} = v \times \frac{v}{\mu g}$

$= \frac{v^2}{\mu g}$

Nishant Jindal

AA :

Power delevered by pump.

$$W_{mg} + W_{pump} = (\Delta K.E)$$

$$dW_{mg} = -dmgh$$

$$dW_{pump} = dmgh + \frac{1}{2}dm\,v^2$$

$$\frac{dW_{pump}}{dt} = gh\left(\frac{dm}{dt}\right) + \frac{1}{2}v^2\left(\frac{dm}{dt}\right)$$

$$\Downarrow$$

$$\boxed{P_{pump} = \frac{dm}{dt}\left(gh + \frac{v^2}{2}\right)}$$

$$\frac{d}{dt}\left(\frac{m}{\rho}\right) = Av$$

$$\frac{1}{\rho}\frac{dm}{dt} = Av$$

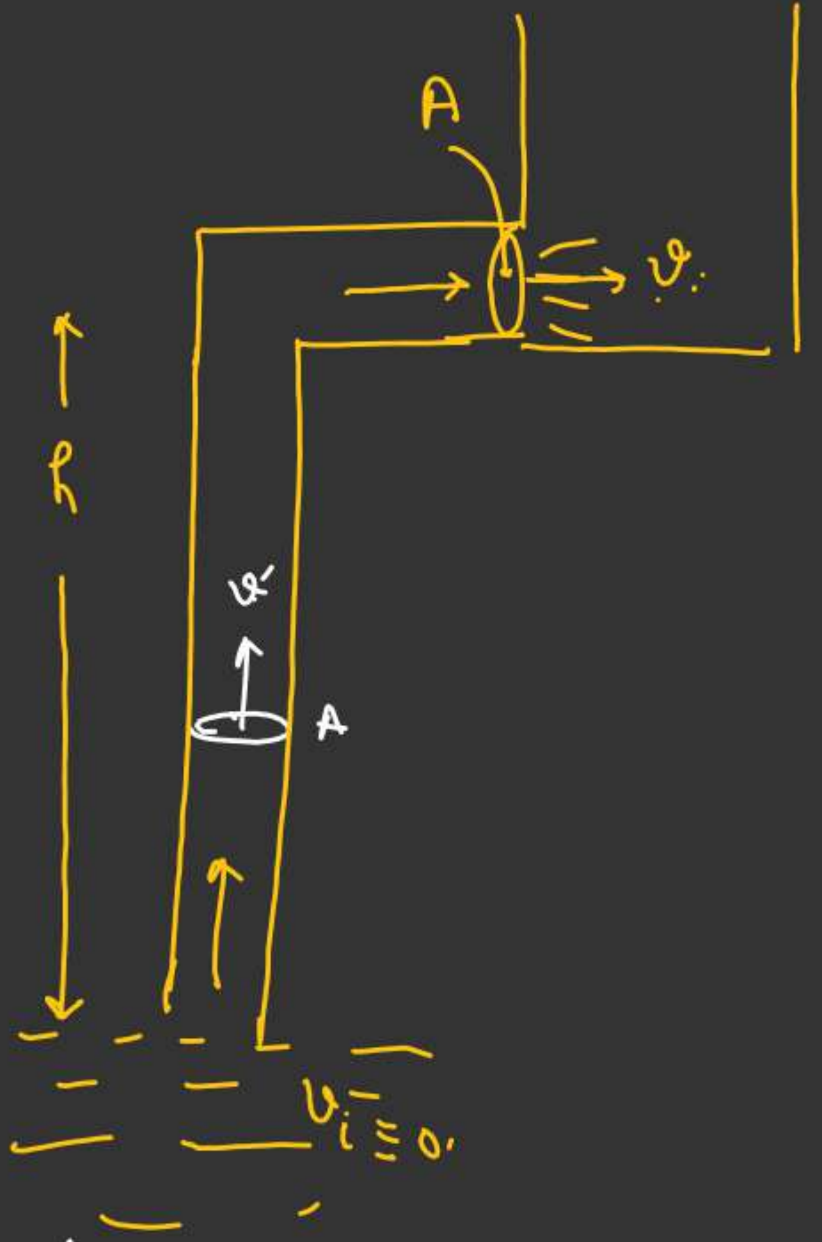
$$\boxed{\frac{dm}{dt} = \rho A v}$$

$$\rho = \frac{m}{V}$$

$$V = \frac{m}{\rho}$$

$\frac{dV}{dt}$ = Volume flowrate

$$\boxed{\frac{dV}{dt} = Av}$$



A = Crosectional area of pipe.
v = Velocity of liquid.

WORK POWER ENERGY

Q.1 A small body of mass m lies on a horizontal plane. The body is given a velocity v_0, along the plane.

(a) Find the mean power developed by the friction during the whole time of motion, if friction coefficient is $\mu = 0.3; m = 2.0\ \text{kg}$ and $v_0 = 3\ \text{m/s}$.

(b) Find the maximum instantaneous power developed by the friction force, if the friction coefficient varies as $\mu = \alpha x$, where α is a constant and x is distance from the starting point.

Q.2 A particle of mass m is moving in a circular path of constant radius r such that its centripetal acceleration a_c is varying with time t as $a_c = k^2rt^2$, where k is a constant. Calculate the power delivered to the particle by the force acting on it.

$a_c = \frac{v^2}{r}$

$a_c = k^2rt^2$

$\frac{v^2}{r} = k^2rt^2$

$v^2 = k^2r^2t^2$

$v = (krt)$

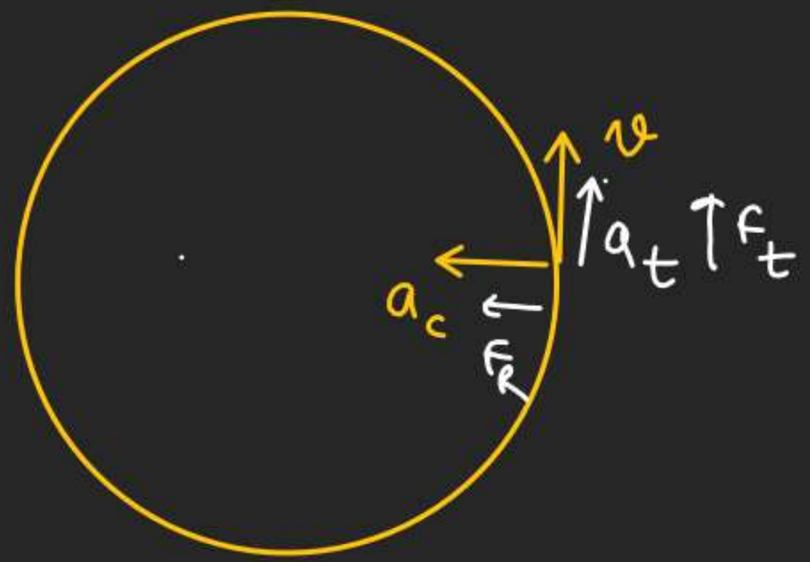
$a_t = \frac{d}{dt}|v|$

$a_t = \frac{d}{dt}(krt) = kr$

$F_t = ma_t = (mkr)$

$P = \vec{F_t}\cdot\vec{v} = F_t \cdot v = (mkr)(krt)$

$P = (mk^2r^2)t$



Q.3 An automobile of mass m accelerates, starting from rest. The engine supplies constant power P; show that

(a) The velocity is given as a function of time by

$$v = (2Pt/m)^{1/2}$$

(b) The position is given as function of time by

$$s = (8P/9m)^{1/2} t^{3/2}$$

Sol^n

$P = K$

$Fv = K$

$mav = K$

$mv\frac{dv}{dt} = K = P$

$mv\frac{dv}{dt} = P$

$\int_0^v v\,dv = \frac{P}{m}\int_0^t dt$

$\frac{v^2}{2} = \frac{Pt}{m}$

$v = \left(\frac{2Pt}{m}\right)^{1/2}$

$\frac{ds}{dt} = \sqrt{\frac{2P}{m}} \cdot t^{1/2}$

$\int_0^s ds = \sqrt{\frac{2P}{m}} \int_0^t t^{1/2}\,dt$

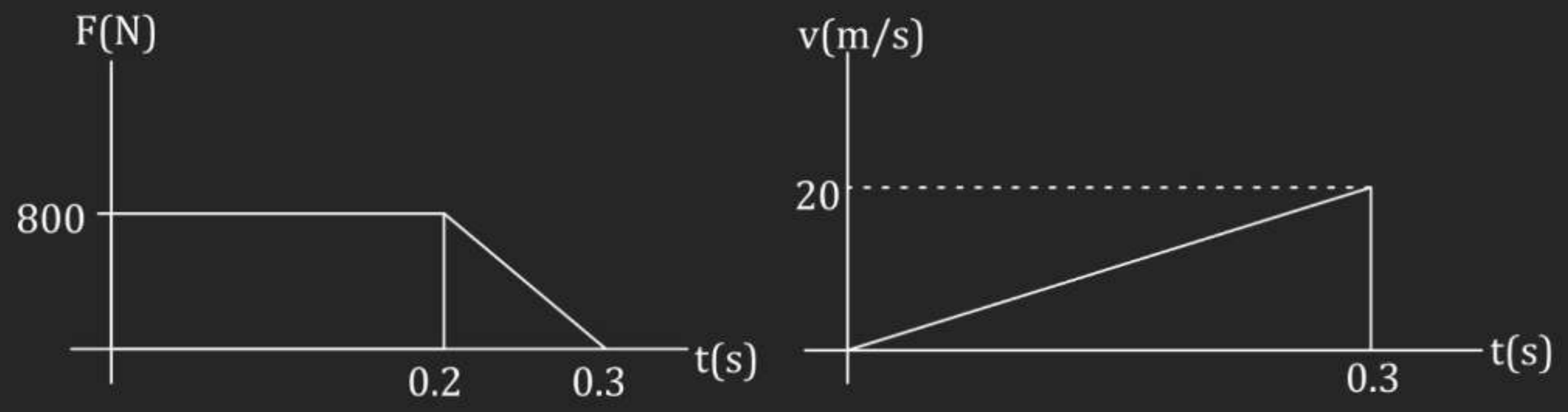
$s = \sqrt{\frac{2P}{m}} \frac{t^{3/2}}{(3/2)}$

$s = \frac{2}{3}\sqrt{\frac{2P}{m}} \cdot t^{3/2}$

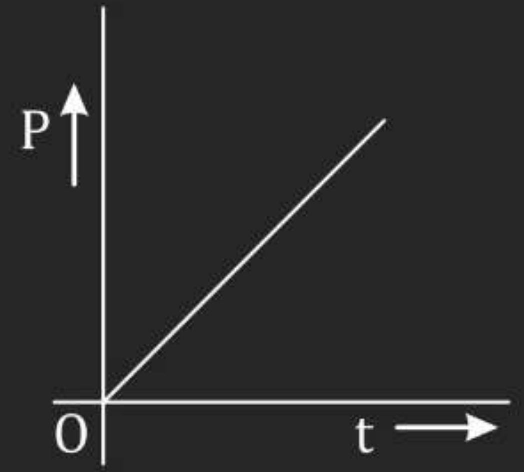
$s = \sqrt{\frac{8P}{9m}} \cdot t^{3/2}$

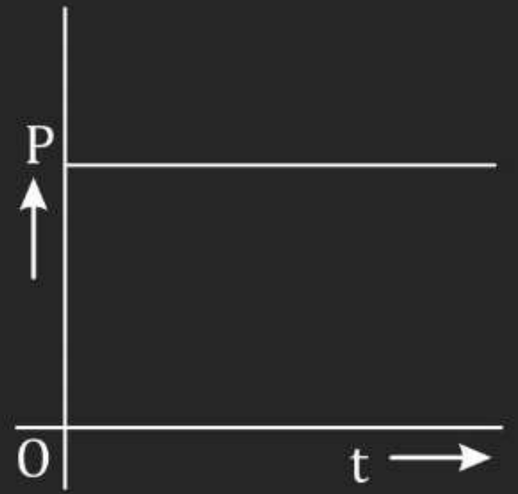
WORK POWER ENERGY

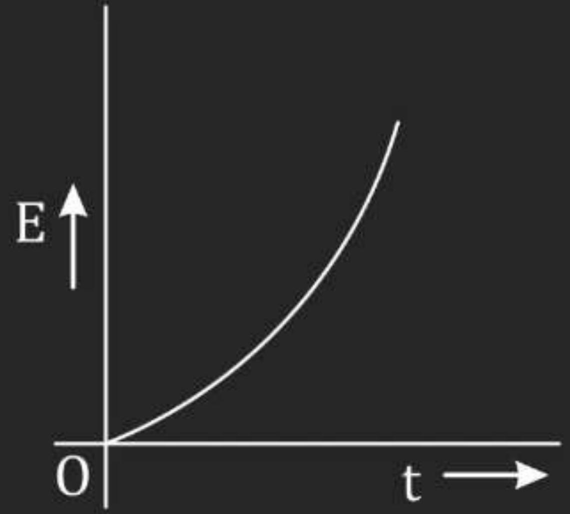
Q.4 A baseball having a mass of $0.4\ \mathrm{kg}$ is thrown such that the force acting on it varies with time as shown in the first graph. Also, the velocity of the ball acting in the same direction as the force varies with time as shown in the second graph. Determine the power applied as a function of time and the work done in $t = 0.3\ \mathrm{s}$.

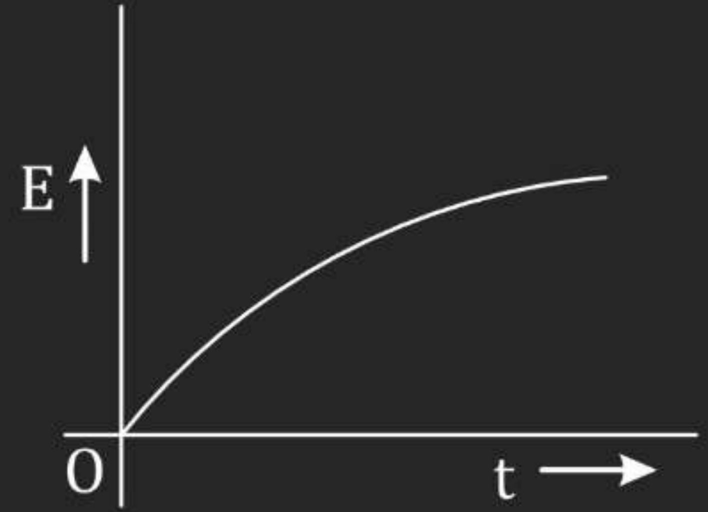


Q.5 A vehicle is driven along a straight horizontal track by a motor which exerts a constant driving force. The vehicle starts from rest at $t = 0$ and the effects of friction and air resistance are negligible. If kinetic energy of vehicle at time t is E and power delivered by the motor is P, which of the following graphs is/are correct:

(A)


(B)


(C)


(D)


Q.6 Water is pumped from a depth of $10 \mathrm{~m}$ and delivered through a pipe of cross-section $10^{-2} \mathrm{~m}^2$. If it is needed to deliver a volume of $10^{-1} \mathrm{~m}^3$ per second the power required will be :

(A) $10 \mathrm{~kW}$

(B) $9.8 \mathrm{~kW}$

(C) $15 \mathrm{~kW}$

(D) $4.9 \mathrm{~kW}$

WORK POWER ENERGY

Q.7 A particle of mass m moves along a circle of radius R with a normal acceleration varying with time as $a_n = kt^2$, where k is a constant. If the power developed by all the forces acting on it is P and the mean value of this power averaged over the first t second is $\bar{p}_t$ after the beginning of the motion. Then:

(A) $P = mkRT$

(B) $P = \frac{mkRt}{2}$

(C) $\bar{p}_t = 0$

(D) $\bar{p}_t = \frac{mkRt}{2}$

Q.8 A particle of mass m moves along a circle of radius R with a normal acceleration varying with time as $a_n = kt^2$, where k is a constant. If the power developed by all the forces acting on it is P and the mean value of this power averaged over the first t second is $\bar{p}_t$ after the beginning of the motion. Then:

(A) $P = mkRT$

(B) $P = \frac{mkRt}{2}$

(C) $\bar{p}_t = 0$

(D) $\bar{p}_t = \frac{mkRt}{2}$

WORK POWER ENERGY

Q.8 An engine supplies a constant power ' P ' to an automobile of mass m starting from rest. At an instant of time t :

(A) Velocity is proportional to $\sqrt{P}$

(B) Velocity is proportional to $\sqrt{t}$

(C) Displacement is proportional to $\sqrt{\frac{1}{m}}$

(D) Displacement is proportional to $t^{3/2}$