DPP – 2

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1. $u = 0$

final velocity is v

W = Area of F – x graph

$W = 2 \times 2 + 1 \times 2 + \frac{1}{2} \times 1 \times 1$

$W = 6 + 0.5 = 6.5$ Joule.

$W = \frac{1}{2} m(v^2 - u^2)$

$6.5 = \frac{1}{2} \times mv^2$

$K_f = 6.5J = 3.25\ K$

$k = 2$

2. $K =$ spring constant $= 800\ N/m$

$x = 5 \times 10^{-2}\ m$

$W = \frac{1}{2} k(x_f^2 - x_i^2)$

$= \frac{1}{2} \times 8 \times 10^2 [225 \times 10^{-4} - 25 \times 10^{-4}]$

$= 4 \times 10^2 \times 10^{-4} [200] = 8\ J$

$W = 2N = 8\ J$

$N = 4$

3. $U = 2x^2 + 3y^3 + 2z$

$\vec{F} = -\frac{\partial U}{\partial x}\hat{i} - \frac{\partial U}{\partial y}\hat{j} - \frac{\partial U}{\partial z}\hat{k}$

$\vec{F_x} = -\frac{\partial U}{\partial x}\hat{i}$

$\vec{F_x} = -4x\hat{i}$

$x = 1$

$\vec{F} = -4\hat{i} = |\vec{F_x}| = 4 = 0.5K$

$K = 8$

4. $\vec{F} = 3\hat{i} + 4\hat{j}$

$\vec{S}_{Op} = a\hat{i} + a\hat{j}$

$W_1 = \vec{F} \cdot \vec{s} = 3a + 4a = 7a$

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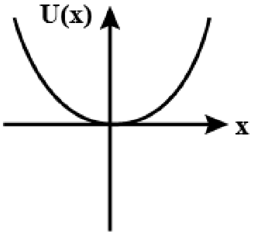
$W_2 = \vec{F}\cdot\vec{S}_{0Qp} = (3i + 4\hat{j})(ai + a\hat{j}) = 7a$

$W_1 = W_2$

5. $F = kx$

$\text{Wext} = \int F dx = \frac{kx^2}{2}$

$\Delta U = \text{Wext} = \frac{kx^2}{2}$



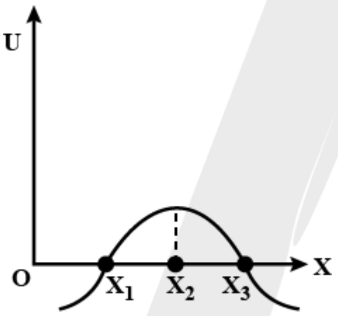
6. $\Delta U = -\text{Wconservative.}$

W conservative = +ive

$\Delta U \downarrow$ ses

option – B

7.



For equilibrium $F = 0$

For stable equilibrium $\frac{d^2y}{\partial^2x} < 0$

$\vec{F} = -\frac{\partial u}{\partial x}\hat{x}$

At $x_2 \Rightarrow \frac{\partial u}{\partial x} = 0$

but at $x_2 = U_{max}$

option D is correct

8. Work = Force x displacement of point of Application indir^n of force.

$W_3 = F_3 \cdot \frac{\pi R}{2} = \frac{15 \times \pi \times 6}{2}$

$W_3 = 45\pi$ Joule

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$W_2 = F_2 \times 6 = 30 \times 6 = 180\ \text{J}$

$W_1 = \int \overline{F_1}\, \overline{dr}$

$= F_1(P_1P_2)$

$= 20(6\sqrt{2})$

$= 120\sqrt{2}$

[So second option is correct (B, C, D)]

9. The correct options are

A $\vec{F} = 2r^3\hat{r}$

B $\vec{F} = -\dfrac{5}{r}$

C $\vec{F} = \dfrac{3(x\hat{i} + y\hat{j})}{(x^2 + y^2)^{\frac{3}{2}}}$

Since: $W = \int \vec{F} \cdot \overrightarrow{dr}$

Clearly for forces (A) and (B) the integration do not require any information of the path taken it depends only on initial and final positions.

For (C):

$$W_c = \int \frac{3(x\hat{i} + y\hat{j})}{(x^2 + y^2)^{\frac{3}{2}}}(dx\hat{i} + dy\hat{j}) = 3\int \frac{xdx + ydy}{(x^2 + y^2)^{\frac{3}{2}}}$$

Taking : $x^2 + y^2 = t$

$2xdx + 2ydy = dt$

$\Rightarrow xdx + ydy = \dfrac{dt}{2}$

$$\Rightarrow W_c = 3\int \frac{\frac{dt}{2}}{t^{\frac{3}{2}}} = \frac{3}{2}\int \frac{dt}{t^{\frac{3}{2}}} = \frac{-3}{\sqrt{x^2 + y^2}} + C$$

Work done is independent of the path. Hence (A), (B)and (C) are conservative forces.

For (D):

$$W_d = \int \frac{3(y\hat{i} + x\hat{j})}{(x^2 + y^2)^{\frac{3}{2}}}(dx\hat{i} + dy\hat{j}) = 3\int \frac{ydx + xdy}{(x^2 + y^2)^{\frac{3}{2}}}$$

(D) requires some more information on path followed by particle to calculate work done. Hence non-conservative in nature.

10. A,C,D

11. $m = 1\ \text{kg}$

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$F = x(3x - 2)$

$F = 3x^2 - 2x$

For equilibrium

$F = 0$

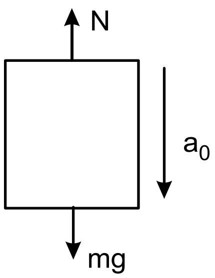
$3x^2 - 2x = 0$

$x(3x - 2) = 0$

$x = 0, \ x = \frac{2}{3}$

A & C

12. According to observer B



$mg - N = ma_0$

$N = m(g - a_0)$

$S = \frac{1}{2}a_0t_0^2$

$W = mg.\frac{1}{2}a_0t_0^2$

13. $W_{\text{Normal}} = -m(g - a_0)\cdot\frac{1}{2}a_0t_0^2$

$= -\frac{ma_0}{2}(g - a_0)t_0^2$

$= -\frac{N}{2}a_0 + t_0^2$

14. $U(x) = \frac{a}{x^{12}} - \frac{b}{x^6}$

$x = \infty \ U_\infty = 0$

$F = -\frac{\partial U}{\partial x} = \frac{\partial}{\partial x}(ax^{-12} - bx^{-6})$

$F = -12ax^{-13} + 6bx^{-7}$

$0 = -12ax^{-13} + 6bx^{-7}$

$\cancel{6}bx^{-7} = \cancel{12}ax^{-13}$

$x^6 = \frac{2a}{b}$

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$x = \left(\frac{2a}{b}\right)^{1/6}$

15. The correct option is **D 0**

$K\left[\frac{x}{(x^2+y^2)^{3/2}}\hat{\imath} + \frac{y}{(x^2+y^2)^{3/2}}\right]$

$dW = F, d\bar{x}$

$= dW = K\left[\frac{xdx + ydy}{(x^2+y^2)^{3/2}}\right]$

Let $x^2 + y^2 = r^2$

$xdx + ydy = r^2$

$\Rightarrow dW = \frac{Krdr}{r^3} = \frac{K}{r^2}dr$

$W = \left[\frac{-K}{r}\right]_{r_1}^{r_2}$

Now, $r_1 = a, r_2 = a \Rightarrow W = 0$