## HOMEWORK–3

### DIFFRENTIATION OF A FUNCTION RESPECT TO ANOTHER FUNCTION

**1.** If $x = e^{\sin^{-1}t}, y = \tan^{-1}t$, then $\frac{dy}{dx} =$

(A) $\frac{1}{1+t^2}e^{-\sin^{-1}t}\sqrt{1-t^2}$ (B) $\frac{1}{1+t^2}e^{-\sin^{-1}t}$

(C) $(1+t^2)e^{-\sin^{-1}t}\sqrt{1-t^2}$ (D) None of these

**2.** Find derivative of f(tanx) w.r.t. g(secx) at $x = \frac{\pi}{4}$

where $f'(1) = 2$ and $g'(\sqrt{2}) = 4$ is

(A) 3 (B) -4 (C) $\frac{2}{19}$ (D) $\frac{1}{\sqrt{2}}$

**3.** Differential coefficient of $\sin^{-1}x$ with respect to $3.\ \text{Sin}^{-1}(3x-4x^3)$ is

(A) $\frac{1}{3}$ if $\frac{-\pi}{8} < x < \frac{\pi}{8}$ (B) 3 if $\frac{-\pi}{8} < x < \frac{\pi}{8}$

(C) $\frac{1}{3}$ if $\frac{-\pi}{9} < x < \frac{\pi}{9}$ (D) $\frac{1}{3}$ if $\frac{-1}{2} < x < \frac{1}{2}$

**4.** The differential coefficient of $\sin^{-1}\frac{t}{\sqrt{1+t^2}}$ w.r.t. $\cos^{-1}\frac{1}{\sqrt{1+t^2}}$ is

(A) $1\ \forall\ t > 0$ (B) $-1\ \forall\ t < 0$ (C) $1\ \forall\ t \in R$ (D) $2\ \forall\ t > 0$

### LOGARITHMIC FUNCTION/TRIGONOMETRIC SUBSTITUTIONS

**5.** $y = \cos^{-1}\sqrt{\frac{\sqrt{1+x^2}+1}{2\sqrt{1+x^2}}}$ then $\frac{dy}{dx}$ is

(A) $\frac{1}{2(1+x^2)}, x \in R$ (B) $\frac{1}{2(1+x^2)}, x > 0$

(C) $\frac{-1}{2(1+x^2)}, x < 0$ (D) $\frac{1}{2(1+x^2)} < 0$

### INFINITE SERIES

**6.** If $y = \sqrt{x + \sqrt{y + \sqrt{x + \cdots}}}$, then $\frac{dy}{dx} =$

(A) $\frac{x-y^2}{2y^3-2xy+1}$ (B) $\frac{x-y^2}{2y^3-2xy-1}$ (C) $\frac{x+y^2}{2y^3-2xy-1}$ (D) $\frac{y^2-x}{(y^2-x)^3-1}$

### DIFFERENTION OF PARAMETRIC EQUATIONS

**7.** If $x = \frac{1+t}{t^3}, y = \frac{3}{2t^2} + \frac{2}{t}$ then, $x\left(\frac{dy}{dx}\right)^3 - \frac{dy}{dx} =$

(A) 0 (B) -1 (C) 1 (D) 2

**8.** If $\sin x = \frac{2t}{1+t^2}$ and $\cot y = \frac{1-t^2}{2t}$. Then value of $\frac{d^2x}{dy^2}$ is equal to

(A) 0 (B) 1 (C) -1 (D) $\frac{1}{2}$

**MIXED PROBLEMS**

**9.** If $y = \tan^{-1}\left(\frac{\ln \frac{e}{x^2}}{\ln ex^2}\right) + \tan^{-1}\frac{3+2\ln x}{1-6\ln x}$ then

(A) $\frac{dy}{dx} = 0$ (B) $\frac{d^2y}{dx^2} = 0$ (C) $\frac{dy}{dx} = \frac{2}{x(1+\ln^2 x)}$ (D) $\frac{dy}{dx} = 1$

## JEE MAIN

**10.** If $x^m \cdot y^n = (x+y)^{m+n}$, then $\frac{dy}{dx}$ is - **[AIEEE-2006]**

(A) $\frac{x+y}{xy}$ (B) $xy$ (C) $\frac{x}{y}$ (D) $\frac{y}{x}$

**11.** Let y be an implicit function of x defined by $x^{2x} - 2x^x \cot y - 1 = 0$. then $y'(1)$ equals :

(A) log2 (B) $-$log2 (C) -1 (D) 1 **[AIEEE-2009]**

**12.** Let $f: (-1,1) \to R$ be a differentiable function with $f(0) = -1$ and $f'(0) = 1$.

Let $g(x) = [f(2f(x)+2)]^2$. Then $g'(0)$: $-$ **[AIEEE-2010]**

(A) 4 (B) -4 (C) 0 (D) -2

**13.** $\frac{d^2x}{dy}$ equals :- **[AIEEE-2011]**

(A) $\left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right)^{-2}$ (B) $-\left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right)^{-3}$

(C) $\left(\frac{d^2y}{dx^2}\right)^{-1}$ (D) $-\left(\frac{d^2y}{dx^2}\right)^{-1}\left(\frac{dy}{dx}\right)^{-3}$

**14.** If $y = \sec(\tan^{-1}x)$, then $\frac{dy}{dx}$ at $x = 1$ is equal to : **[JEE-MAIN-2013]**

(A) $\frac{1}{\sqrt{2}}$ (B) $\frac{1}{2}$ (C) 1 (D) $\sqrt{2}$

**15.** If g is the inverse of a function f and $f'(x) = \frac{1}{1+x^5}$, then $g'(x)$ is equal to : **[JEE-MAIN-2014]**

(A) $1 + x^5$ (B) $5x^4$ (C) $\frac{1}{1+\{g(x)\}^5}$ (D) $1 + \{g(x)\}^5$

**16.** If for $x \in R$, $f(x) = |\log 2 - \sin x|$ and $g(x) = f(f(x))$, then : **[JEE(Main)-2016]**

(A) g is differentiable at $x = 0$ and $g'(0) = -\sin(\log 2)$

(B) g is not differentiable at $x = 0$

(C) $g'(0) = \cos(\log 2)$

(D) $g'(0) = -\cos(\log 2)$

**17.** If for $x \in \left(0, \frac{1}{4}\right)$, the derivative of $\tan^{-1}\left(\frac{6x\sqrt{x}}{1-9x^3}\right)$ is $\sqrt{x}.g(x)$ then g(x) equals : **[JEE (Main)2017]**

(A) $\frac{3}{1+9x^3}$ (B) $\frac{9}{1+9x^3}$ (C) $\frac{3x\sqrt{x}}{1-9x^3}$ (D) $\frac{3x}{1-9x^3}$

**18.** If $x = 3\tan t$ and $y = 3\sec t$, then the value of $\frac{d^2y}{dx^2}$ at $t = \frac{\pi}{4}$, is : **[JEE (Main)2019]**

(A) $\frac{3}{2\sqrt{2}}$ (B) $\frac{1}{6\sqrt{2}}$ (C) $\frac{1}{3\sqrt{2}}$ (D) $\frac{1}{6}$

**19.** Let $f: R \to R$ be a function such that $f(x) = x^3 + x^2f'(A) + xf''(B) + ff'''(C), x \in R$. Then $f(B)$ equals: **[JEE (Main)2019]**

(A) 8 (B) 30 (C) -4 (D) -2

**20.** If $x\log_e(\log_e x) - x^2 + y^2 = 4 (y > 0)$, then $\frac{dy}{dx}$ at $x = e$ is equal to : **[JEE (Main)2019]**

(A) $\frac{e}{\sqrt{4+e^2}}$ (B) $\frac{(2e-1)}{2\sqrt{4+e^2}}$ (C) $\frac{(1+2e)}{2\sqrt{4+e^2}}$ (D) $\frac{(1+2e)}{\sqrt{4+e^2}}$

**21.** For $x > 1$, if $(2x)^{2y} = 4e^{2x-2y}$, then $(1 + \log_e 2x)^2 \frac{dx}{dy}$ is equal to : **[JEE (Main)2019]**

(A) $\frac{x\log_e 2x - \log_e 2}{x}$ (B) $x\log_e 2x$

(C) $\log_e 2x$ (D) $\frac{x\log_e 2x + \log_e 2}{x}$

**22.** If $2y = \left(\cot^{-1}\left(\frac{\sqrt{3}\cos x + \sin x}{\cos x - \sqrt{3}\sin x}\right)\right)^2, x \in \left(0, \frac{\pi}{2}\right)$, then $\frac{dy}{dx}$ is equal to **[JEE (Main)2019]**

(A) $\frac{\pi}{6} - x$ (B) $x - \frac{\pi}{6}$ (C) $2x - \frac{\pi}{3}$ (D) $\frac{\pi}{3} - x$

**23.** The derivative of $\tan^{-1}\left(\frac{\sin x - \cos x}{\sin x + \cos x}\right)$, with respect to $\frac{x}{2}$, where $\left(x \in \left(0, \frac{\pi}{2}\right)\right)$ is :

(A) 2 (B) $\frac{2}{3}$ (C) 1 (D) $\frac{1}{2}$ **[JEE (Main)2019]**

**24.** Let $(x)^k + (y)^k = (a)^k$ where $a, k > 0$ and $\frac{dy}{dx} + \left(\frac{y}{x}\right)^{\frac{1}{3}} = 0$, then find k - **[JEE (Main)2020]**

(A) $\frac{1}{3}$ (B) $\frac{2}{3}$ (C) $\frac{4}{3}$ (D) 2

**25.** If $y^{14} + y^{-14} = 2x$, and $(x^2 - 1)\frac{d^2y}{dx^2} + \alpha x\frac{dy}{dx} + \beta y = 0$ then $|\alpha - \beta|$ is equal to .

**[JEE (Main)2021]**

**26.** If $y(x) = \left(x^{x^x}\right), x > 0$ then $\frac{d^2x}{dy^2} + 20$ at $x = 1$ is equal to: **[JEE (Main)2022]**

**27.** Let $y(x) = (1 + x)(1 + x^2)(1 + x^4)(1 + x^8)(1 + x^{16})$ **[JEE (Main)2023]**

Then $y' - y''$ at $x = -1$ is equal to

(A) 976 (B) 464 (C) 496 (D) 944

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### SECTION-1

**28.** For $x > 0, \lim_{x\to 0}\left((\sin x)^{1/x} + (1/x)^{\sin x}\right)$ is :- **[JEE 2006, 3]**

(A) 0 (B) -1 (C) 1 (D) 2

**29.** $\frac{d^2x}{dy^2}$ equals :-

(A) $\left(\frac{d^2y}{dx^2}\right)^{-1}$ (B) $-\left(\frac{d^2y}{dx^2}\right)^{-1}\left(\frac{dy}{dx}\right)^{-3}$ (C) $\left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right)^{-2}$ (D) $-\left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right)^{-3}$

**30.** **(a)** Let $g(x) = \ln f(x)$ where $f(x)$ is a twice differentiable positive function on $(0,\infty)$ such that $f(x+1) = xf(x)$. then for $N = 1,2,3, \ldots\ldots\ldots.., g''\left(N+\frac{1}{2}\right) - g''\left(\frac{1}{2}\right) =$

(A) $-4\left\{1+\frac{1}{9}+\frac{1}{25}+\cdots\cdot\frac{1}{(2N-1)^2}\right\}$ (B) $4\left\{1+\frac{1}{9}+\frac{1}{25}+\cdots\ldots\frac{1}{(2N-1)^2}\right\}$

(C) $-4\left\{1+\frac{1}{9}+\frac{1}{25}+\cdots\ldots.\frac{1}{(2N+1)^2}\right\}$ (D) $4\left\{1+\frac{1}{9}+\frac{1}{25}+\cdots\ldots\cdot\frac{1}{(2N+1)^2}\right\}$

**(b)** Let f and g be real valued functions defined on interval $(-1,1)$ such that $g(x)$ is continuous, $g(0) \neq 0$, $g'(0) = 0, g''(0) \neq 0$, and $f(x) = g(x)\sin x$.

**Statement-1 :** $\lim_{x\to 0}[g(x)\cot x - g(0)\operatorname{cosec} x] = f''(0)$

And

**Statement-2 :** $f'(0) = g(0)$

(A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation of statement-1.

(B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1

(C) Statement-1 is true, statement-2 is false.

(D) Statement-1 is false, statement-2 is true. **[JEE 2008, 3 + 3]**

**31.** If the function $f(x) = x^3 + e^{\frac{x}{2}}$ and $g(x) = f^{-1}(x)$, then the value of $g'(1)$ is **[JEE 2009, 4]**

**32.** Let $f(\theta) = \sin\left(\tan^{-1}\left(\frac{\sin\theta}{\sqrt{\cos 2\theta}}\right)\right)$, where $-\frac{\pi}{4} < \theta < \frac{\pi}{4}$. then the value of $\frac{d}{d(\tan\theta)}(f(\theta))$ is

**[JEE 2011, 4]**

**33.** The slope of the tangent to the curve $(y - x^5)^2 = x(1+x^2)^2$ at the point $(1,3)$ is

**[JEE(Advanced)-2014, 3]**

### SECTION-2

**34.** Let $f: R \to R, g: R \to R$ and $h: R \to R$ be differentiable functions such that $f(x) = x^3 + 3x + 2, g(f(x)) = x$ and $h(g(g(x))) = x$ for all $x \in R$. Then-

(A) $g'(2) = \frac{1}{15}$ (B) $h'(1) = 666$

(C) $h(0) = 16$ (D) $h(g(3)) = 36$ **[JEE(Advanced)-2016, 4(-2)]**

**35.** For any positive integer n, define $f_n : (0,\infty) \to R$ as

**[JEE(Advanced)-2018, 4(0)]**

$f_n(x) = \sum_{j=1}^{n} \tan^{-1}\left(\frac{1}{1+(x+j)(x+j-1)}\right)$ for all $x \in (0,\infty)$

(Here, the inverse trigonometric function $\tan^{-1} x$ assumes values in $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$ )

(A) $\sum_{j=1}^{5} \tan^2\left(f_j(0)\right) = 55$

(B) $\sum_{j=1}^{10} \left(1 + f_j'(0)\right)\sec^2\left(f_j(0)\right) = 10$

(C) For any fixed positive integer n, $\lim_{x\to\infty} \tan\left(f_n(x)\right) = \frac{1}{n}$

(D) For any fixed positive integer n, $\lim_{x\to\infty} \sec^2\left(f_n(x)\right) = 1$

## ANSWER KEY

### DIFFRENTIATION OF A FUNCTION RESPECT TO ANOTHER FUNCTION

**1.** (A) **2.** (D) **3.** (A,C,D) **4.** (A,B)

### LOGARITHMIC FUNCTION/TRIGONOMETRIC SUBSTITUTIONS

**5.** (B,C)

### INFINITE SERIES

**6.** (D)

### DIFFERENTION OF PARAMETRIC EQUATIONS

**7.** (C) **8.** (A)

### MIXED PROBLEMS

**9.** (A,B)

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|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **10.** | (D) | **11.** | (C) | **12.** | (B) | **13.** | (B) | **14.** | (A) | **15.** | (D) | **16.** | (C) |
| **17.** | (B) | **18.** | (B) | **19.** | (D) | **20.** | (B) | **21.** | (A) | **22.** | (B) | **23.** | (A) |
| **24.** | (B) | **25.** | (17) | **26.** | (16) | **27.** | (C) | | | | | | |

## JEE ADVANCED

### SECTION-1

**28.** (C) **29.** (D) **30.** **(a)** A ; **(b)** A **31.** (2) **32.** (1) **33.** (8)

### SECTION-2

**34.** (B,C) **35.** (D)