PROBLEM SET-01

SOLUTION

Link to View Video Solution: Click Here

Q.1 There are 6 roads between A & B and 4 roads between B & C.

(i) In how many ways can one drive from A to C by way of B?

(ii) In how many ways can one drive from A to C and back to A, passing through B on both trips?

(iii) In how many ways can one drive the circular trip described in (ii) without using the same road more than once.

Ans. (i) 24; (ii) 576; (iii) 360

Sol. (i) $6 \times 4 = 24$

(ii) $24 \times 24 = 576$

(iii) A to $C \Rightarrow 24$

C to $A \Rightarrow 3 \times 5 = 15$

Circular $\Rightarrow 24 \times 15$
Trip $= 360$

Q.2 If the letters of the word "VARUN" are written in all possible ways and then are arranged as in a dictionary, then the rank of the word VARUN is:

(A) 98 (B) 99 (C) 100 (D) 101

Ans. (C)

Sol. VARUN

$4.4! + 0 \cdot 3! + 1 \cdot 2! + 1 \cdot 1! + 0!$
$96 + 2 + 1 + 1 = 100$

Q.3 How many natural numbers are their from 1 to 1000 which have none of their digits repeated.

Ans. (738)

Sol. $SD = 9; DD = 9 \cdot 9 = 81; TD = 9 \cdot 9 \cdot 8 = 648$

Q.4 A family is being arranged in a line for a group photograph. If the family consists of a mother, a father, a baby and five children, the number of arrangements that begin and end with a parent is.

(A) 720 (B) 1440 (C) 5040 (D) 40320

Ans. (B)

Link to View Video Solution: Click Here

Sol. $M, F, B, C_1, C_2, C_3, C_4, C_5$

M	-	-	-	-	-	-	F

$\Rightarrow 2 \times 6!$
$= 2 \times 720$
$= 1440$

Q.5 (i) Find the number of four letter word that can be formed from the letters of the word HISTORY. (each letter to be used at most once)

(ii) How many of them contain only consonants?

(iii) How many of them begin & end in a consonant?

(iv) How many of them begin with a vowel?

(v) How many contain the letters Y ?

(vi) How many begin with T& end in a vowel?

(vii) How many begin with T& also contain S ?

(viii) How many contain both vowels?

Ans. (i) 840; (ii) 120; (iii) 400; (iv) 240; (v) 480; (vi) 40; (vii) 60; (vii) 240

Sol. (i) $7 \cdot 6 \cdot 5 \cdot 4 = 42 \times 20 = 840$; (ii) $5 \cdot 4 \cdot 3 \cdot 2 = 120$; (iii) $5 \cdot 4 \cdot 4 \cdot 4 = 400$; (iv) 240 ; (v) 480 (vi) 40 ; (vii) 60 ; (vii) 240]

Q.6 If repetitions are not permitted

(i) How many 3 digit numbers can be formed from the six digits 2, 3, 5, 6, 7 & 9?

(ii) How many of these are less than 400?

(iii) How many are even?

(iv) How many are odd?

(v) How many are multiples of 5?

Ans. (i) 120; (ii) 40; (iii) 40; (iv) 80; (v) 20

Sol. (i)

6	5	4

$\Rightarrow 120$

(ii) $2.5.4 = 40$

(iii) $5.\ 4.\ 2 = 40$

(iv) Total-Even $= 120 - 40 = 80$

(v) $5 \cdot 4 \cdot 1 = 20$

Link to View Video Solution: Click Here

Q.7 Every telephone number consists of 7 digits. How many telephone numbers are there which do not include any other digits but 2,3,5 & 7?

Ans. (4^7)

Sol. $4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 = 4^7$

Q.8 How many of the arrangements of the letter of the word "LOGARITHM" begin with a vowel and end with a consonant?

Ans. (90720)

Sol. $3 \cdot 6 \cdot 7! = 9 \times 5040 = 90720$

Q.9 Anew flag is to be designed with six vertical strips using some or all of the colours yellow, green, blue and red. Then, the number of ways this can be done such that no two adjacent strips have the same colour is

(A) 12×81 (B) 16×192 (C) 20×125 (D) 24×216

Ans. (A)

Sol. 1^{st} place can be filled in 4 ways
2^{nd} place can be filled in 3 ways
3^{rd} place can be filled in 3 ways and ‖ 1ly4^{th}, 5^{th} and 6^{th} each can be filled in 3 ways.
hence total ways $= 4 \times 3^5 = 12 \times 81$

Q.10 How many four digit numbers are there which are divisible by 2.

Ans. (4500)

Sol.

9	10	10	5

=4500

Q.11 In a telephone system four different letter P, R, S, T and the four digits 3, 5, 7, 8 are used. Find the maximum number of "telephone numbers" the system can have if each consists of a letter followed by a four-digit number in which the digit may be repeated.

Ans. (1024)

Sol. $4 \cdot 4^4 = 2^2 \cdot 2^8 = 2^{10} = 1024$

Q.12 Find the number of 5 lettered palindromes which can be formed using the letters from the English alphabets.

Ans. (26^3)

Sol. $26 \cdot 26 \cdot 26 \cdot 1 \cdot 1 = 26^3$

Link to View Video Solution: Click Here

Q.13 Number of ways in which 7 different colours in a rainbow can be arranged if green is always in the middle.

Ans. (720)

Sol. "VI B GYOR"

x

$\Rightarrow 6! = 720$

Q.14 Find the number of ways in which 4 boys and 3 girls can be seated in a line if the terminal seats are occupied by the boys.

Ans. (1440)

Sol. $B_1\ B_2\ B_3\ B_4 G_1 G_2 G_3$ $\times\times\times\times\times\times\times$

${}^4C_2 \cdot 2! \cdot 5! = (12)(120) = 1440$ Ans

Q.15 Numbers of words which can be formed using all the letters of the word "AKSHI", if each word begins with vowel or terminates in vowel.

Ans. (84)

Sol. $2 \cdot 24 + 2 \cdot 24 - 2 \cdot 6 = 84$

Q.16 A letter lock consists of three rings each marked with 10 different letters. Find the number of ways in which it is possible to make an unsuccessful attempts to open the lock.

Ans. (999)

Sol. $10^3 - 1 = 999$

Q.17 How many 10 digit numbers can be made with odd digits so that no two consecutive digits are same.

Ans. $(5 \cdot 4^9)$

Sol. $1/3/5/7/9 \rightarrow 5 \cdot 4^9$

Q.18 It is required to seat 5 men and 4 women in a row so that the women occupy the even places. How many such arrangements are possible?

Ans. (2880)

Sol. $\times\ \times\ \times\ \times\ \times\ \times\ \times\ \times\ \times = 4!\ 5!$

$\uparrow$ W $\uparrow$ W $\uparrow$ W $\uparrow$ W

Q.19 Find the number of three digits or four digit even numbers that can be formed from the number 2,3,5,6,7 (without repetition).

Ans. (72)

Link to View Video Solution: Click Here

Sol. $4 \cdot 3 \cdot 2 \quad + \quad 4 \cdot 3 \cdot 2$

$24 + 48 = 72$

Q.20 How many natural numbers are there with the property that they can be expressed as the sum of the cubes of two natural numbers in two different ways.

Ans. (Infinitely many)

Sol. Answer is Infinitely many (Ramanujan Numbers)