PROBABILITY

Q. **For any two independent events E_1 & E_2, then $P([(E_1 \cup E_2 \cap (\bar{E}_1 \cap \bar{E}_2)]$ is**

Ans

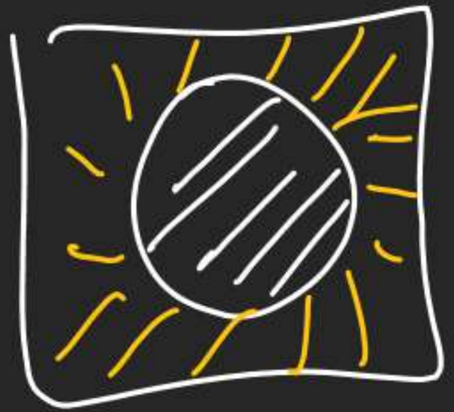
$\le \frac{1}{4}$ $\quad \ge \frac{1}{4}$ $\quad \ge \frac{1}{2}$ $\quad \le \frac{1}{2}$

$P((E_1 \cup E_2) \cap (\bar{E}_1 \cap \bar{E}_2))$ De Morgan.

$P((E_1 \cup E_2) \cap (\overline{E_1 \cup E_2}))$

$\bar{A} \cap \bar{B} = \overline{A \cup B}$

$= 0$



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Q If Prob. of Passing in P, C.

Q If Prob. of Passing in P, C, M is p, c, m

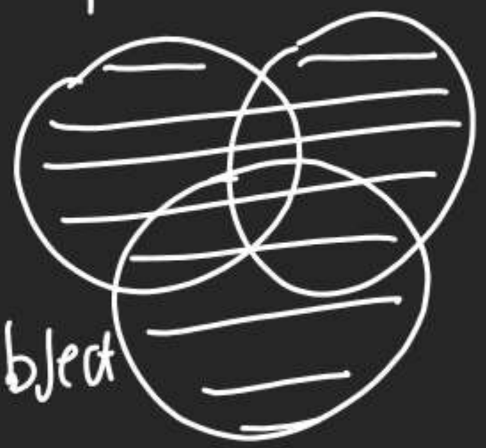
Adv

& 75% Prob. of Passing in Atleast one Subject, 50% Prob in Passing atleast 2 Subjects, 40% Prob in Passing exactly 2 Subject

then (A) $p+c+m = \frac{19}{20}$ (B) $p+c+m = \frac{27}{20}$

(C) $p \cdot c \cdot m = \frac{1}{4}$ (D) NOT.

(1)



Atleast one Subject

$$p+c+m-p\cdot c-cm-pm+p\cdot c\cdot m=\frac{3}{4}$$

(2)

$$PC+CM+PM-2PCM=\frac{1}{2}$$

(3)

$$P\cdot C+CM+PM-3PCM=\frac{2}{5}$$

$$PCM=\frac{1}{2}-\frac{2}{5}=\frac{1}{10}$$

in 2nd

$$PC+CM+PM-\frac{2}{10}=\frac{1}{2}$$

$$PC+CM+PM=\frac{7}{10}$$

1st

$$p+c+m-\left(\frac{7}{10}\right)+\frac{1}{10}=\frac{3}{4}$$

$$p+c+m=\frac{135}{100}=\frac{27}{20}$$

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Conditional Probability

1) It talks about happenning of A when B has already happenned

(2) A & B are Connected

⇒ They have something Common.

(3) So Conditional Prob is Rep by $P(A/B)$ or $P(B/A)$

(4) $P(A/B)$ is Read as → Prob. of A When B has occured

or

Prob of A When B is given

$P(B/A)$ = Prob of B When A has occurred

$$P\left(\frac{A}{B}\right) = \frac{n(A\cap B)}{n(B)} = \frac{\frac{n(A\cap B)}{n(S)}}{\frac{n(B)}{n(S)}}$$

$$\boxed{P\left(\frac{A}{B}\right) = \frac{P(A\cap B)}{P(B)}}$$

Sly $\boxed{P\left(\frac{B}{A}\right) = \frac{P(A\cap B)}{P(A)}}$

Q If $A = \{1, 2, 3, 4\}$

$B = \{3, 4, 5, 6\}$

$P(A/B) = ?$

$$P(A/B) = \frac{2}{4} = \frac{1}{2}$$

Q $A = \{1, 2, 3, 4\}$

$B = \{3, 4, 5, 6, 7\}$

$P(A/B)$ & $P(B/A) = ?$

$$P\left(\frac{A}{B}\right) = \frac{2}{5}$$

$$P(B/A) = \frac{2}{4}$$

Q A = Even No. of 1st Dice out of 2 dices.

B = (Sum = 8) find $P\left(\frac{A}{B}\right)$ & $P\left(\frac{B}{A}\right)$.

= (2,6)(6,2)(4,4)(3,5)(5,3)

A = (2,1)(2,2)(2,3)(2,4)(2,5)(2,6)
(4,1)(4,2)(4,3)(4,4)(4,5)(4,6)
(6,1)(6,2)(6,3)(6,4)(6,5)(6,6)

① $P\left(\frac{A}{B}\right) = \frac{\frac{3}{36}}{\frac{5}{36}} = \frac{3}{5}$ | ② $P\left(\frac{B}{A}\right) = \frac{\frac{3}{36}}{\frac{18}{36}} = \frac{1}{6}$

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Q If 4 comes at 1st dice in a throw of 2 dics find Prob. of coming sum 8 or more.

A= 4 at 1st Dice

$= (4,1)(4,2)(4,3)(4,4)(4,5)(4,6)$

B= Sum 8 or more. $= 8, 9, 10, 11, 12$

$$P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)} = \frac{3}{6} = \frac{1}{2}$$

Q 1 Card is drawn from 52 Cards If that card is face card find Prob. of it to Queen.

$$P(A) = \frac{\frac{4C_1}{52C_1}}{\frac{12C_1}{52C_1}} = \frac{4}{12} = \frac{1}{3}$$

Q A Class has 45% Student with Brown hair. & 25% Student has Brown Eyes, 15% has Brown hair & Brown eyes Both. If a Student is Randomly selected having Brown hair find Prob. of him having Brown eyes.

$$P\left(\frac{BE}{BH}\right) = \frac{P(BH \cap BE)}{P(BH)} = \frac{.15}{.45} = \frac{1}{3}.$$

Q. 2 Cards are drawn from 52 Cards one by one without Replacement. If 1st Card is King find Prob. of 2nd Card also being King?

$\frac{3}{51}$ Ans

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Generalised Multiplication Theorem.

GMT

(1) $P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$

$\Rightarrow P(A \cap B) = P(B) \times P\left(\frac{A}{B}\right)$

(2) $P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)}$

$\Rightarrow P(A \cap B) = P(A) \times P\left(\frac{B}{A}\right)$

(3) $P(A \cap B \cap C) = P(A) \times P\left(\frac{B}{A}\right) \times P\left(\frac{C}{A \cap B}\right)$

(4) $P(A \cap B \cap C \cap D) = P(A) \cdot P\left(\frac{B}{A}\right) \times P\left(\frac{C}{A \cap B}\right) \times P\left(\frac{D}{A \cap B \cap C}\right)$

Independent Event

1) When occurance of one event does not Impact to another than they are Indep. Events.

A, B Ind. Event.

(2) $P\left(\frac{A}{B}\right) = P(A)$

B is not effecting A So $P\left(\frac{A}{B}\right)$ is becoming Prob of A only.

(3) Now for Ind. Event GMT is changed

$P(A \cap B) = P(A) \cdot P\left(\frac{B}{A}\right)$

$\boxed{P(A \cap B) = P(A) \cdot P(B)}$

(4) $P(A \cap B \cap C)$

$= P(A) \cdot P\left(\frac{B}{A}\right) \cdot P\left(\frac{C}{A \cap B}\right)$

$= P(A)\, P(B) \cdot P(C)$

(5) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \cup B) = P(A) + P(B) - P(A) \cdot P(B)$

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(6) A & B Independent

happenning of A not make Impact on happenning of B.

A', B Ind

A, B' Ind

A', B' Ind.

(7) $A_1, A_2, A_3 - - - A_n$ Indepent Event

Prob. of happenning none of them.

$= P(\bar{A}_1 \cap \bar{A}_2 \cap \bar{A}_3 \cap \bar{A}_4 - - \bar{A}_n)$

$= P(\bar{A}_1) \cdot P(\bar{A}_2) \cdot P(\bar{A}_3) - - - P(\bar{A}_n)$

$= (1-P_1)(1-P_2)(1-P_3) - - (1-P_n)$

(8) Prob of happenning Atleast one of them.

$\cup$

$P(A_1 \cup A_2 \cup A_3 - - - \cup A_n)$

$= 1 - P(\bar{A}_1 \cap \bar{A}_2 \cap \bar{A}_3 - - \bar{A}_n)$

$= 1 - (1-P_1)(1-P_2)(1-P_3) - - (1-P_n)$

(9) Prob of happening of 1st Event & not happenning of Remaining

$= P_1 (1-P_2)(1-P_3) - - (1-P_n)$

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Q $P\left(\frac{A}{B}\right)+P\left(\frac{A'}{B}\right)=1$

$$\frac{P(A\cap B)}{P(B)}+\frac{P(A'\cap B)}{P(B)}$$

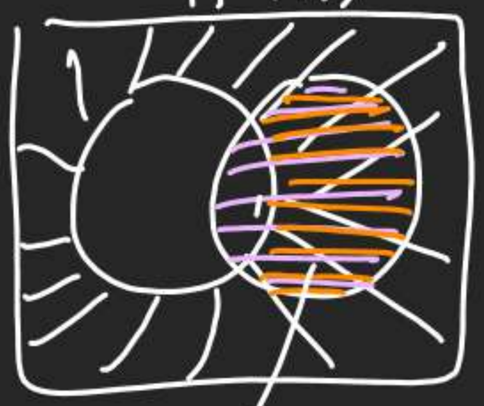
$$\frac{P(A\cap B)+P(A'\cap B)}{P(B)}$$

$$\frac{\cancel{P(A\cap B)}+P(B)-\cancel{P(A\cap B)}}{P(B)}$$

$$=1$$

Q

$A'\cap B$



only B

$P(B)-P(A\cap B)$

PROBABILITY

Q. Let E and F be two independent events. The probability that exactly one of them occurs is $\frac{11}{25}$ and the probability of none of them occurring is $\frac{2}{25}$. If $P(T)$ denotes the probability of occurrence of the event T, then $P(E), P(F) = ?$

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$P(E) = x,\ P(F) = y$

① $x(1-y) + y(1-x) = \frac{11}{25}$

$x + y - 2xy = \frac{11}{25}$

$x + y - xy = \frac{23}{25}$

$\underline{- \quad - \quad + \quad -}$

$+xy = +\frac{12}{25}$

② $(1-x)\cdot(1-y) = \frac{2}{25}$

$1 - x - y + xy = \frac{2}{25}$

$x + y - xy = \frac{23}{25}$

$x + y = \frac{23}{25} + \frac{12}{25} = \frac{35}{25} = \frac{7}{5}$

$x = \frac{3}{5},\ y = \frac{4}{5}$ या $x = \frac{4}{5},\ y = \frac{3}{5}$

PROBABILITY

Q. If X and Y are two events such that $P(X \mid Y) = \frac{1}{2}$, $P(Y \mid X) = \frac{1}{3}$ and $P(X \cap Y) = \frac{1}{6}$. Then, which of the following is/are correct?

(A) $P(X \cup Y) = \frac{2}{3}$ ✓✓

(B) X and Y are inde~~p~~endent

(C) X and Y are not independent

(D) $P(\bar{X} \cap Y) = \frac{1}{3}$ ×

$P(x \cup y) = P(x) + P(y) - P(x)\cdot P(y)$

$= \frac{1}{2} + \frac{1}{3} - \frac{1}{6} = \frac{3+2-1}{6} = \frac{4}{6} = \frac{2}{3}$

$P(\bar{x} \cap y) = P(\bar{x})\cdot P(y)$

$= \left(1 - \frac{1}{2}\right) \times \frac{1}{3} = \frac{1}{6}$

$P\left(\frac{x}{y}\right) = \frac{1}{2}$

$\frac{P(x \cap y)}{P(y)} = \frac{1}{2}$

$\frac{1/6}{P(y)} = \frac{1}{2}$

$P(y) = \frac{1}{3}$

$P\left(\frac{y}{x}\right) = \frac{1}{3}$

$\frac{P(x \cap y)}{P(x)} = \frac{1}{3}$

$\frac{\frac{1}{6}}{P(x)} = \frac{1}{3}$

$P(x) = \frac{1}{2}$

$P(x \cap y) = \frac{1}{6}$

True.

$P(x \cap y) = P(x)\cdot P(y)$

$\frac{1}{6} = \frac{1}{2} \times \frac{1}{3}$