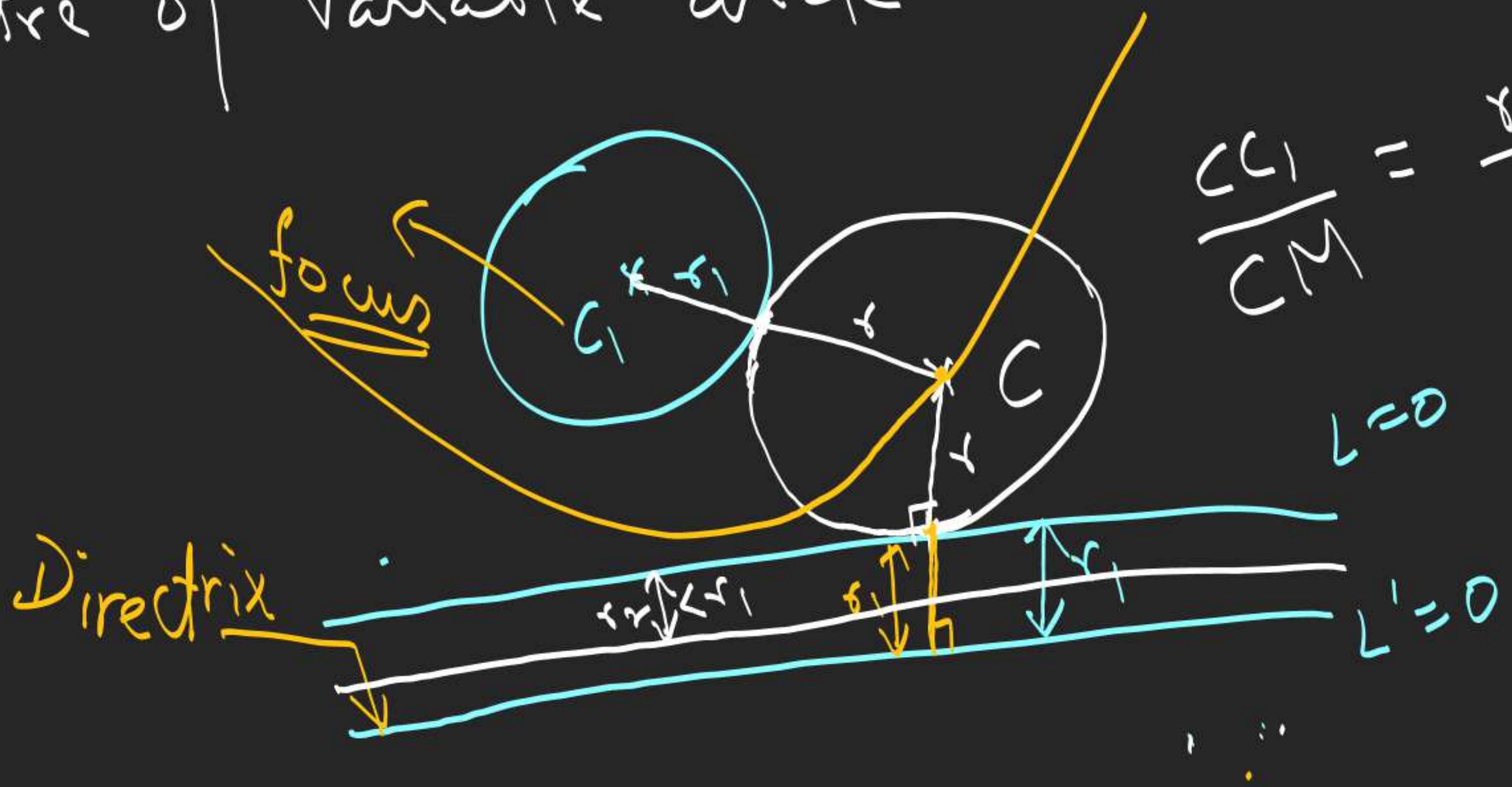
Nishant Jindal

1. If a variable circle touches a fixed circle (externally) and a fixed line (given circle & line are non intersecting). Then find the locus of centre of variable circle.



$$\frac{CC_1}{CM} = \frac{r+r_1}{r+r_2}$$

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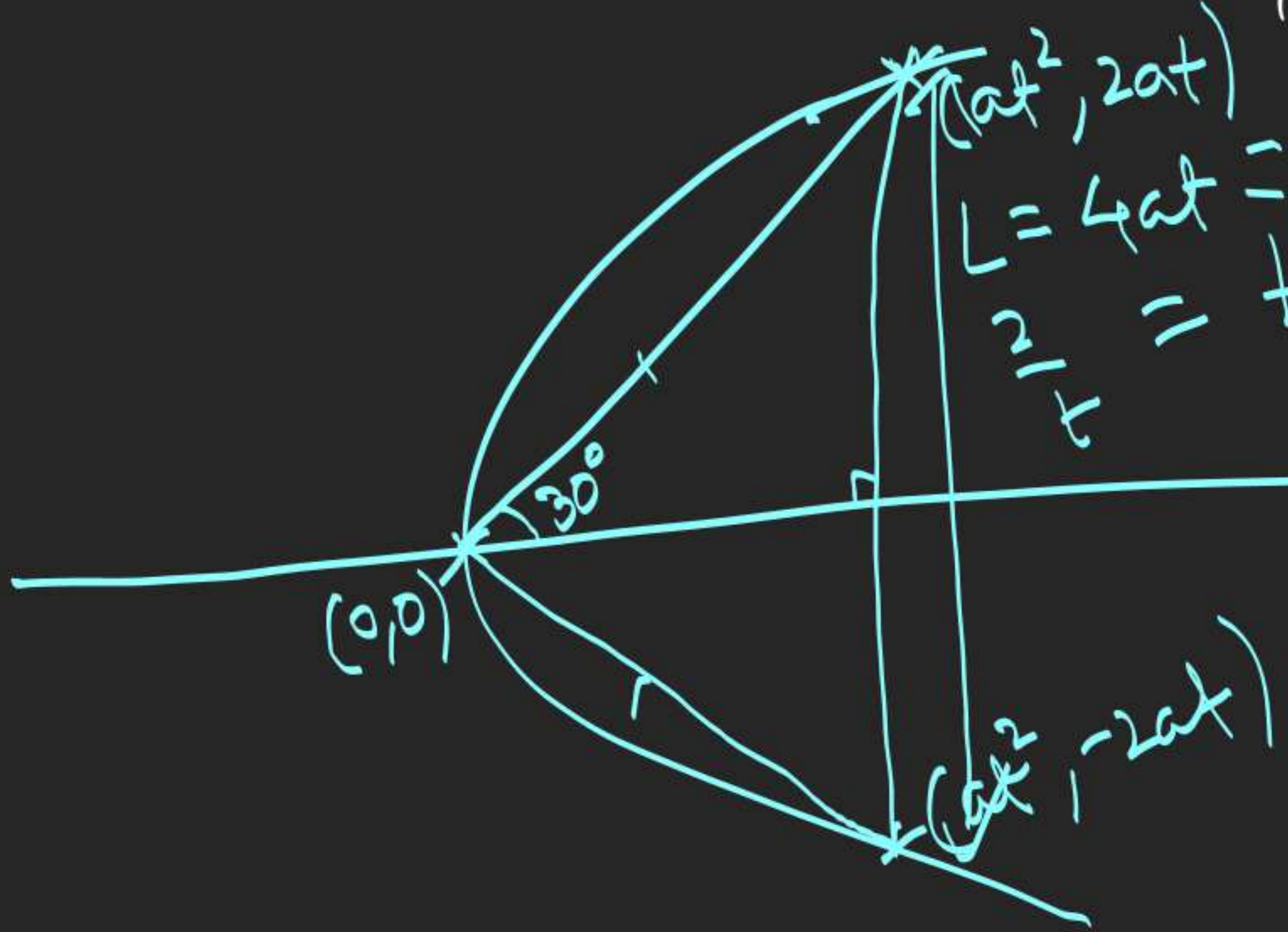
Q: Find the side of equilateral triangle inscribed in $y^2=4ax$ if one of its vertex coincides with the vertex of the parabola.



$$L = 4at = \boxed{8\sqrt{3}a}$$

$$\frac{2}{t} = \tan 30^\circ = \frac{1}{\sqrt{3}}$$

$$t = 2\sqrt{3}$$

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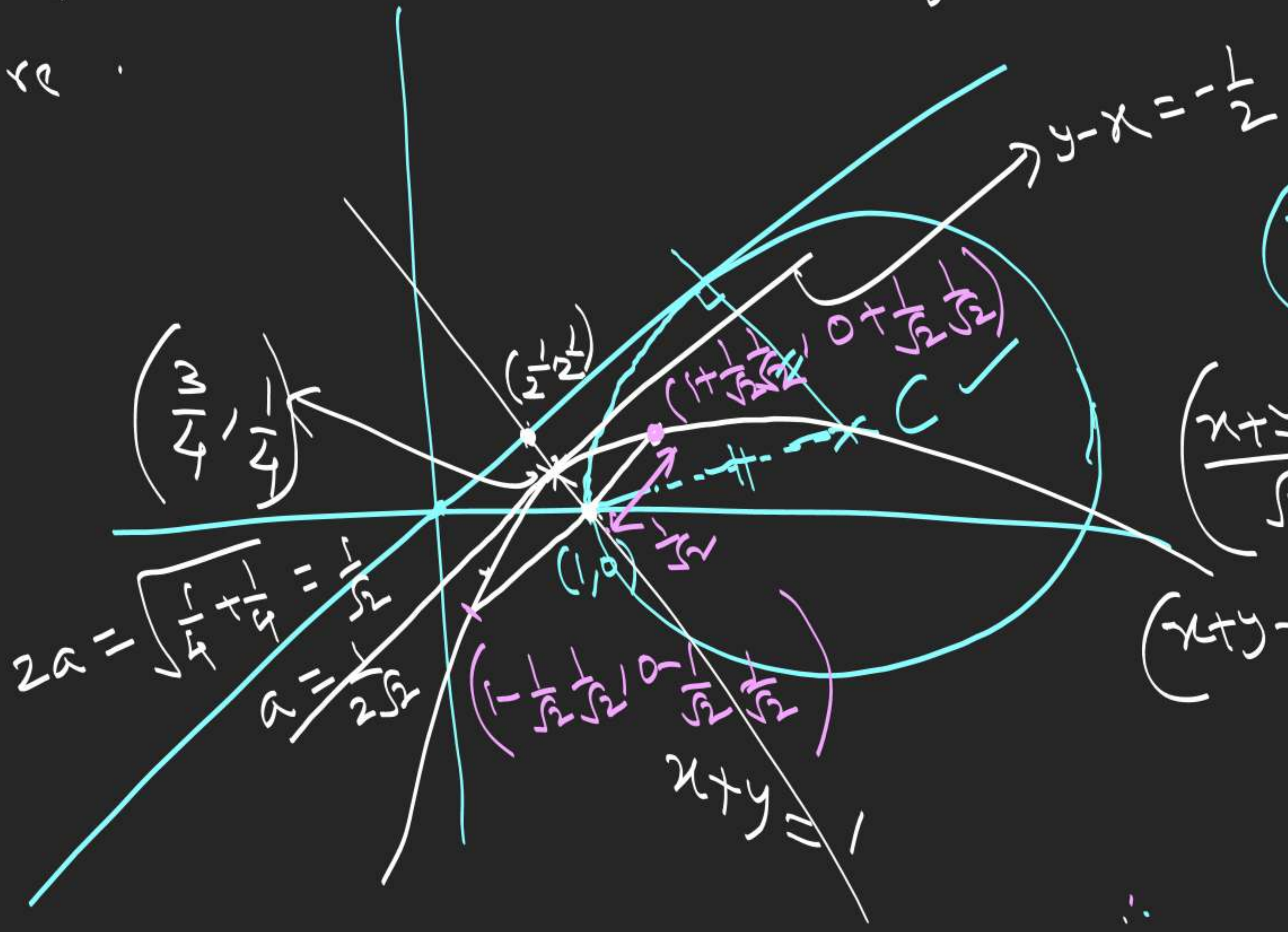
3. A variable circle always passes thru $(1,0)$ and touches line $y=x$. Find eqn. to locus of its centre.



$$\left(\frac{x-y}{\sqrt{2}}\right)^2 = (x-1)^2+y^2$$

$$\left(\frac{x+y-1}{\sqrt{2}}\right)^2 = \sqrt{2}\,\frac{\left|y-x+\frac{1}{2}\right|}{\sqrt{2}}$$

$$(x+y-1)^2 = 2\left(y-x+\frac{1}{2}\right)$$

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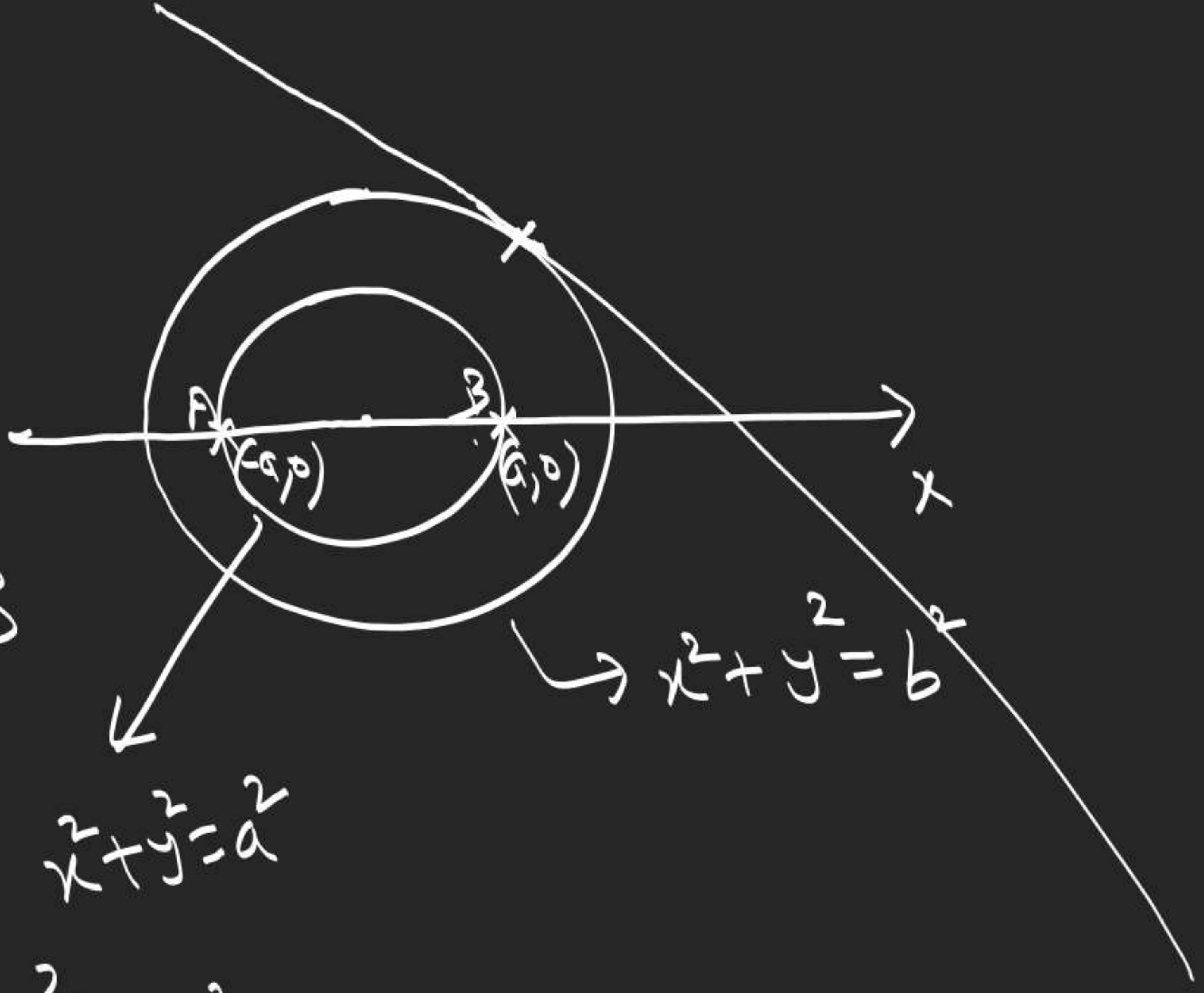
4.

Find the locus of focus of parabola passing thru A, B and having tangent to $x^2+y^2=b^2$ as directrix.



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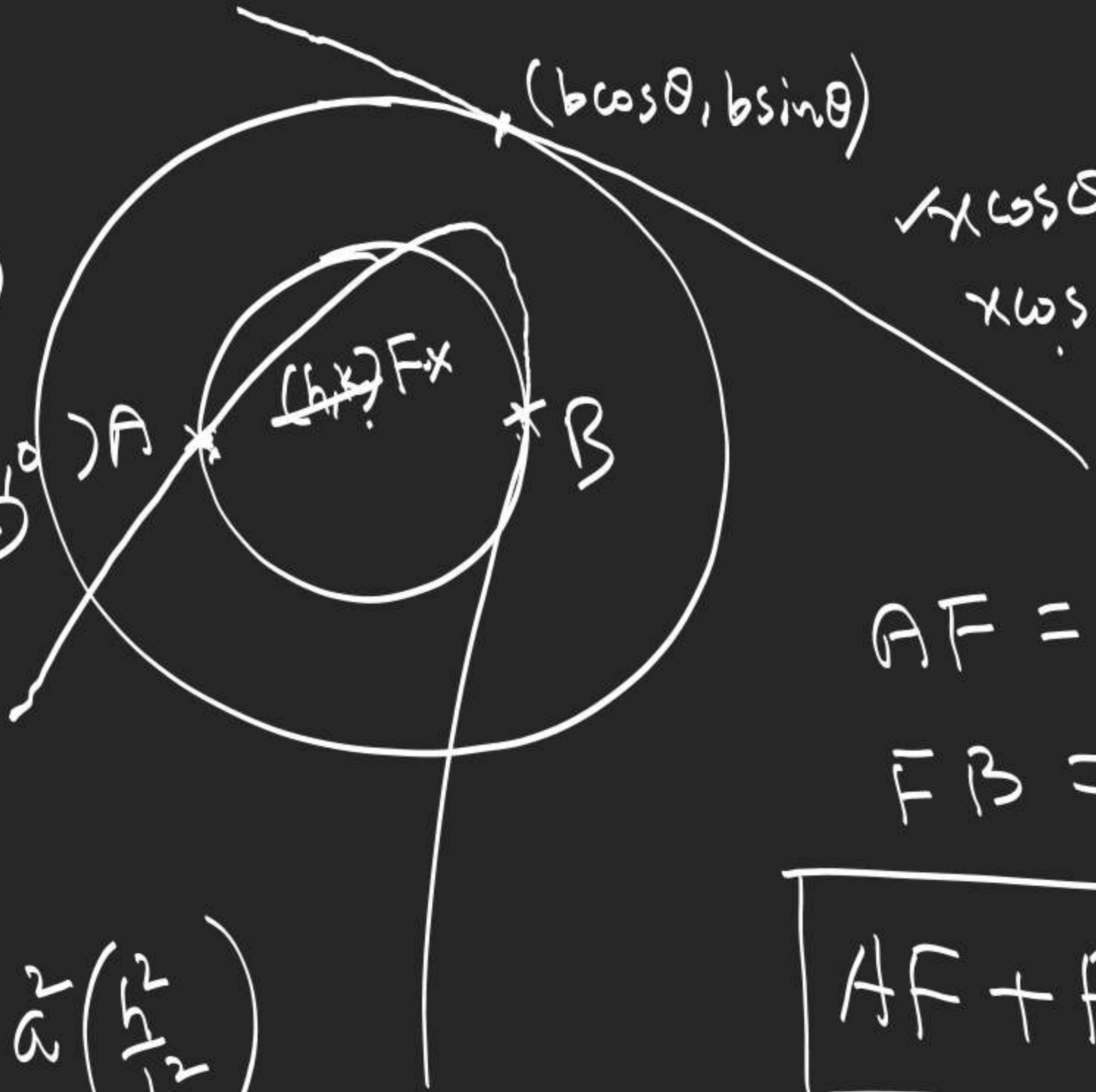


$x\cos\theta + y\sin\theta - b = 0$

$x\cos\theta + y\sin\theta = b$

$(h+a)^2 + k^2 = |b + a\cos\theta|^2$ ①

$(h-a)^2 + k^2 = |b - a\cos\theta|^2$ ②

$AF = |b + a\cos\theta| = a\cos\theta + b$

$FB = |b - a\cos\theta| = b - a\cos\theta$

$$\boxed{AF + FB = 2b}$$

$4ah = 4ab\cos\theta$

$\frac{h}{b} = \cos\theta$

$h^2 + a^2 + k^2 = b^2 + a^2\left(\frac{h^2}{b^2}\right)$

$x^2\left(1 - \frac{a^2}{b^2}\right) + y^2 = b^2 - a^2$

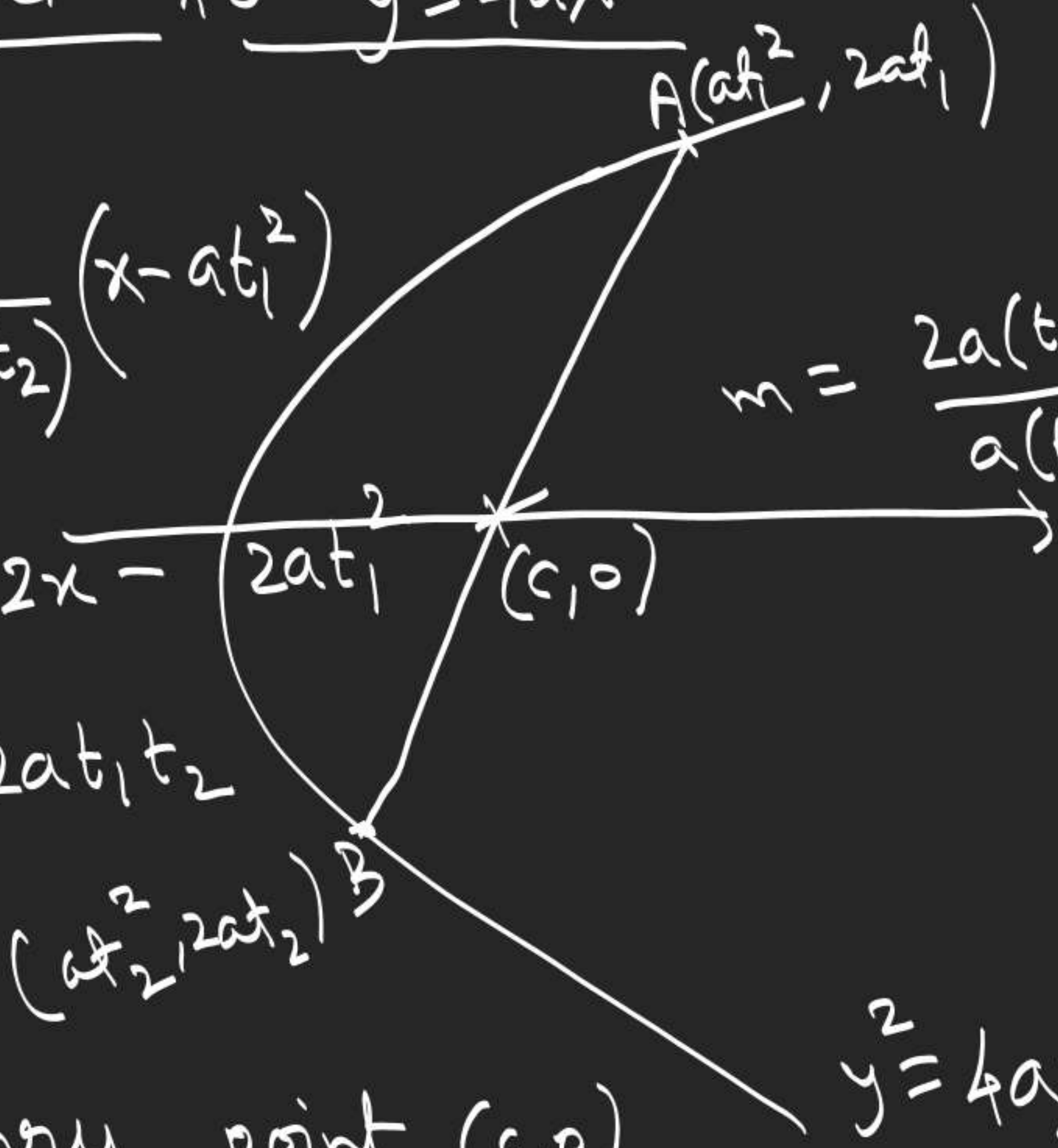
$\frac{x^2}{b^2} + \frac{y^2}{(b^2 - a^2)} = 1$

Chord to $y^2=4ax$

AB → $y-2at_1 = \frac{2}{(t_1+t_2)}(x-at_1^2)$

$$y(t_1+t_2)-2at_1(t_1+t_2) = 2x-2at_1^2$$

$$y(t_1+t_2) = 2x+2at_1t_2$$



$$m = \frac{2a(t_1-t_2)}{a(t_1^2-t_2^2)}$$

$$\boxed{m = \frac{2}{t_1+t_2}}$$

Focal chord

$t_1t_2=-1$

If AB passes thru point $(c,0)$

Put $(c,0)$ to AB $\quad 0 = 2c+2at_1t_2 \Rightarrow \boxed{-\frac{c}{a} = t_1t_2}$

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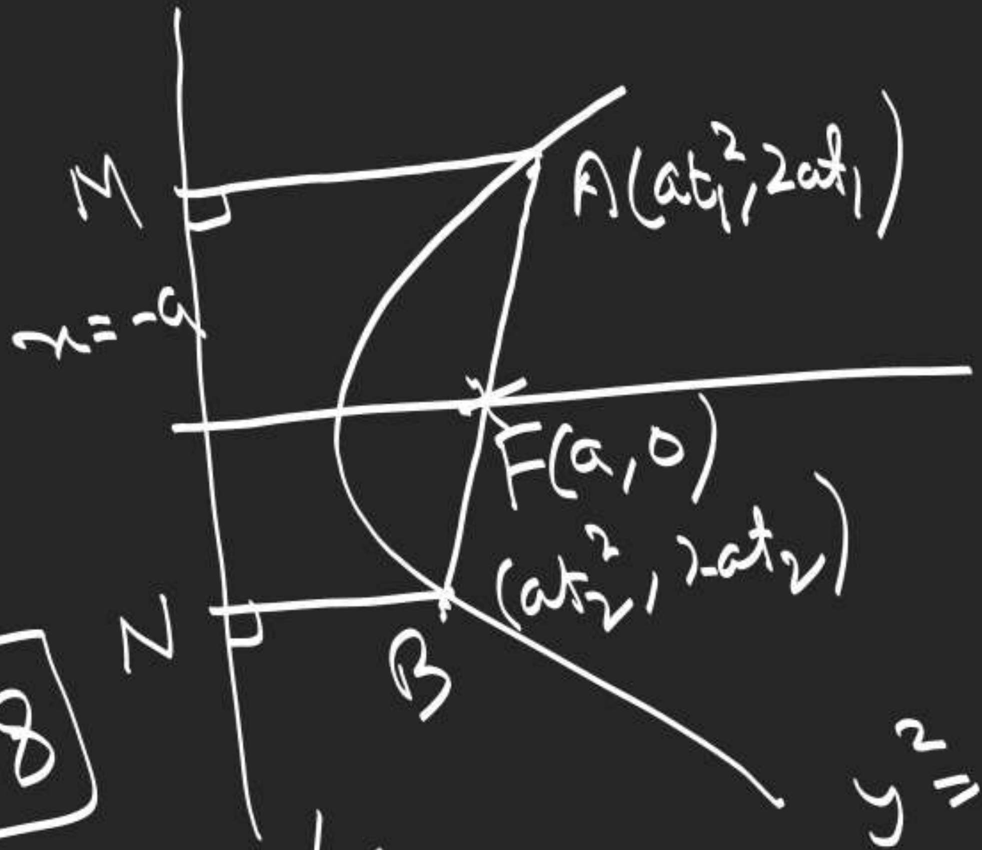
Note :- (1)



$\boxed{PT-2 \rightarrow 3}$

$PT-3 \rightarrow \boxed{6,7,8}$

$$AB = FA + FB$$
$$= AM + BN$$
$$= at_1^2 + a + at_2^2 + a$$
$$= 2a + a\left(\underbrace{t_1^2 + \frac{1}{t_1^2}}_{\geq 2}\right)$$
$$\geq 2a + 2a = 4a.$$

$(AB)_{min} = 4a$, LR

$t_1^2 = \frac{1}{t_1^2} \Rightarrow \boxed{t_1^2 = 1}$

(2)

$\boxed{FA,\ \text{semi LR},\ FB \text{ are in H.P.}}$

$$\frac{1}{FB} + \frac{1}{FA} = \frac{1}{a+at_1^2} + \frac{1}{a+at_2^2}$$
$$= \frac{1}{a+at_1^2} + \frac{1}{a+a\left(-\frac{1}{t_1}\right)^2}$$
$$= \frac{1}{a+at_1^2} + \frac{t_1^2}{a+at_1^2}$$
$$= \frac{1}{a} = \frac{2}{(2a)} \checkmark$$