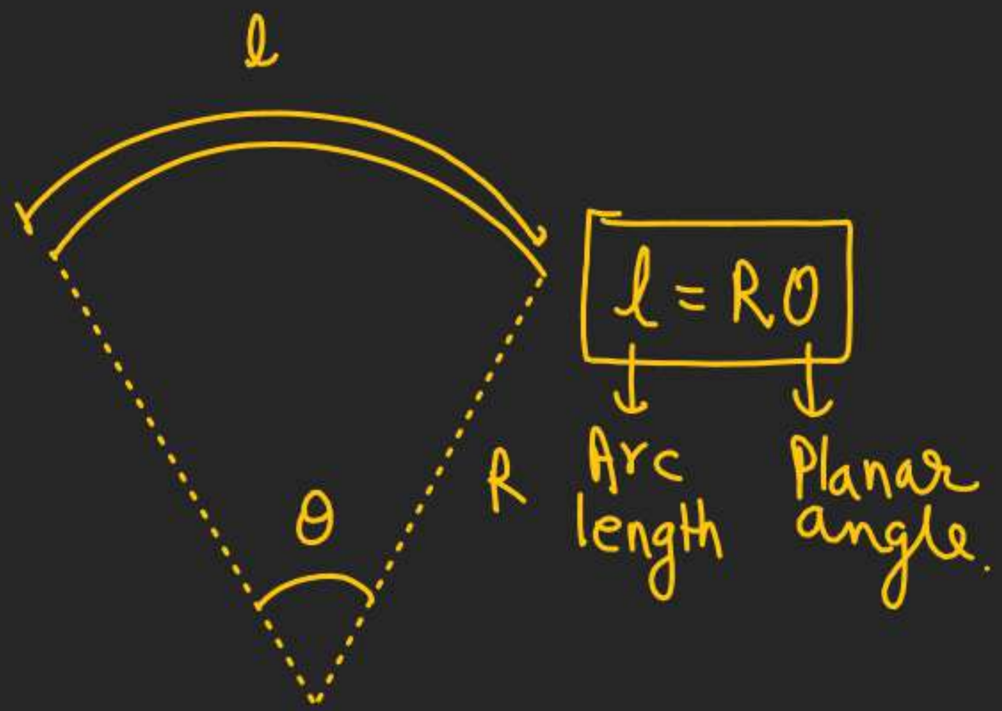
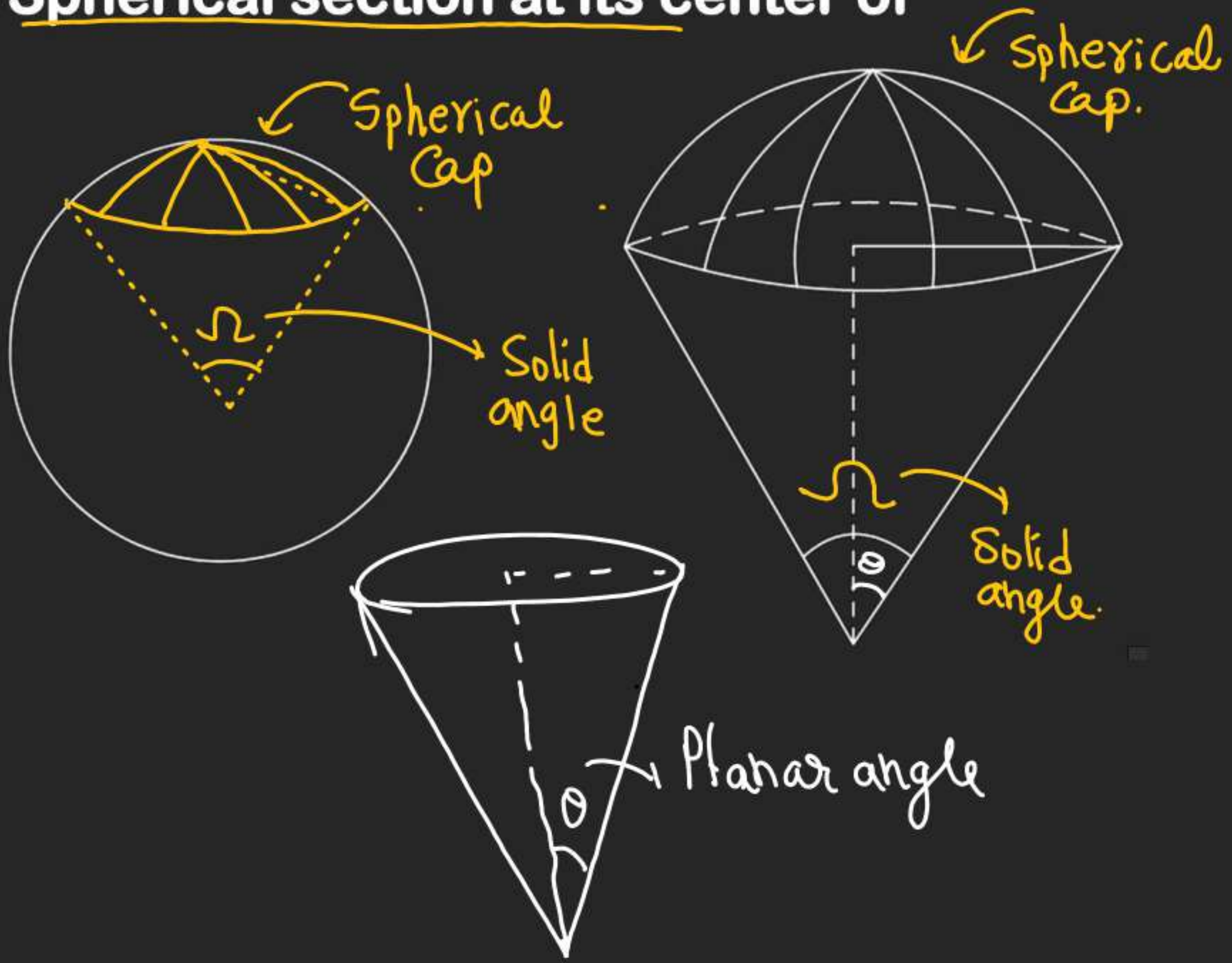
GAUSS'S LAW

Solid Angle

⇒ Three dimensional angle subtended by an Spherical section at its center of curvature.

S.I Unit" - "Steradian"





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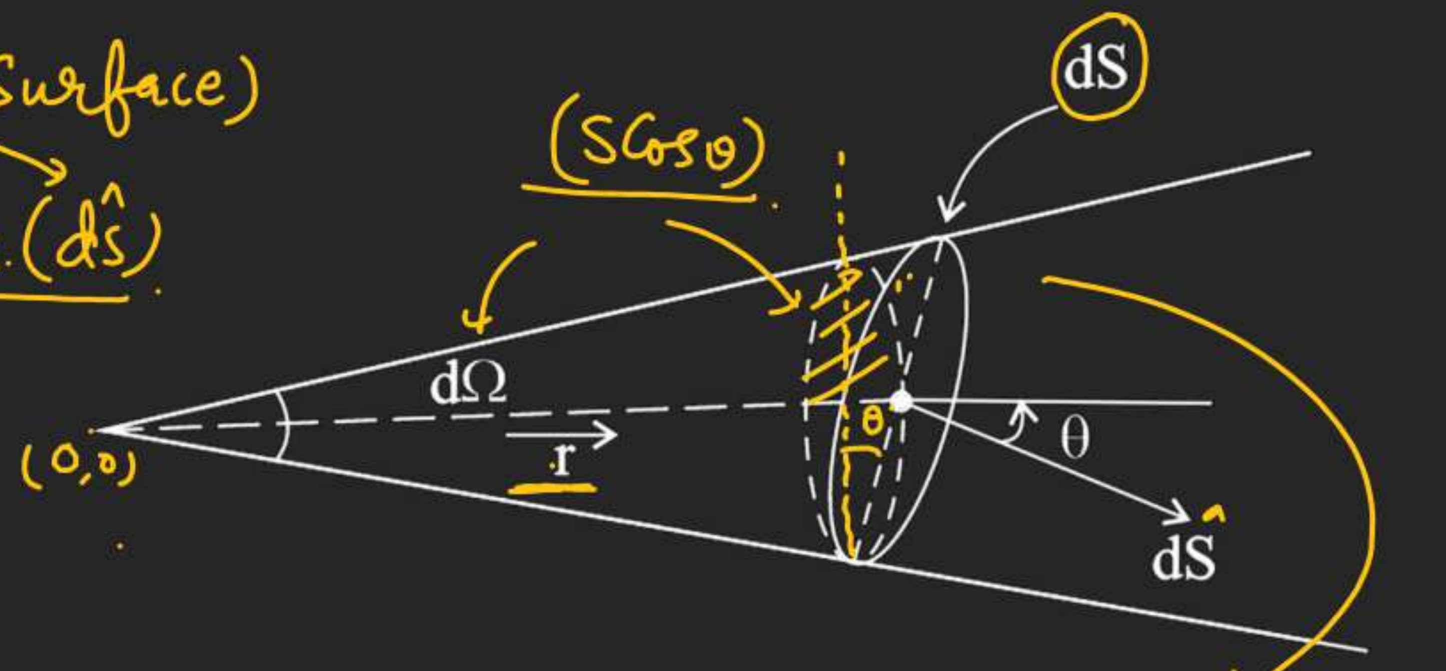
GAUSS'S LAW

General formula (for any 3-dimension surface)

$$\vec{ds} = ds\,(\hat{ds})$$

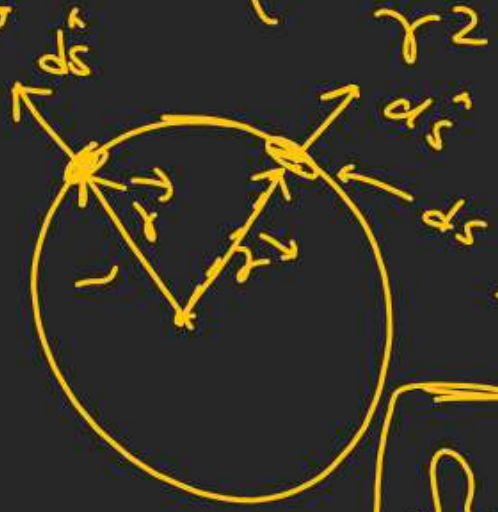


$$\boxed{\Omega = \frac{\vec{S}\cdot\hat{r}}{|\vec{r}|^2}}$$

$$\Omega = \frac{\vec{S}\cdot}{|\vec{r}|^2}\left(\frac{\vec{r}}{|\vec{r}|}\right) = \left(\frac{\vec{S}\cdot\vec{r}}{|\vec{r}|^3}\right) \Rightarrow \Omega = \frac{S\,r\cos\theta}{r^3} = \left(\frac{S\cos\theta}{r^2}\right)$$

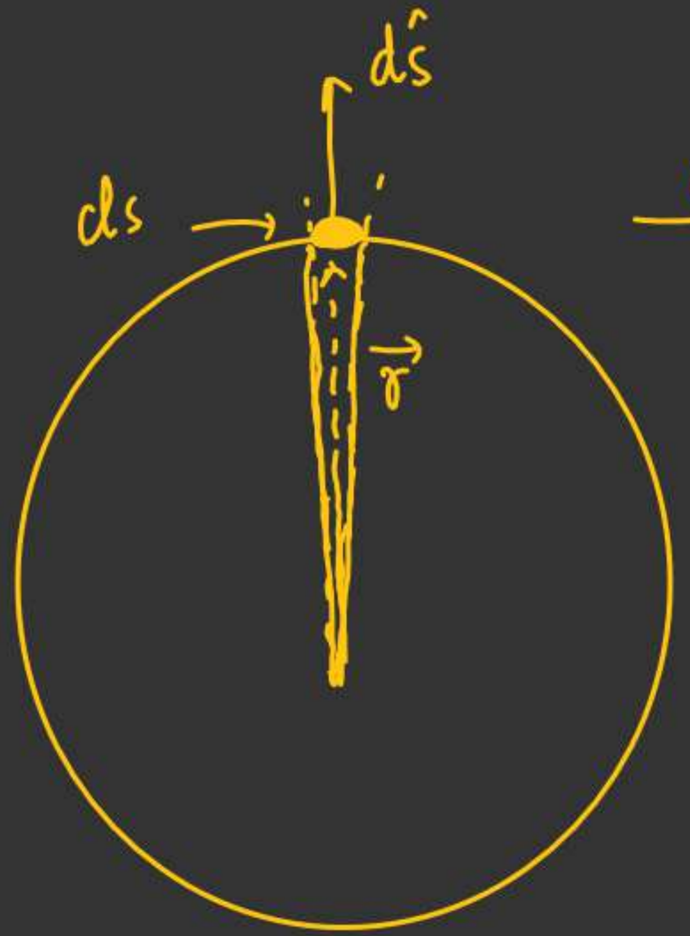
$$\boxed{(d\Omega) = \left(\frac{\vec{ds}\cdot\vec{r}}{|\vec{r}|^3}\right)}$$

For spherical



$$\vec{S} \parallel \vec{r}.$$

$$\boxed{\Omega = \frac{S}{r^2}}$$ ⊛



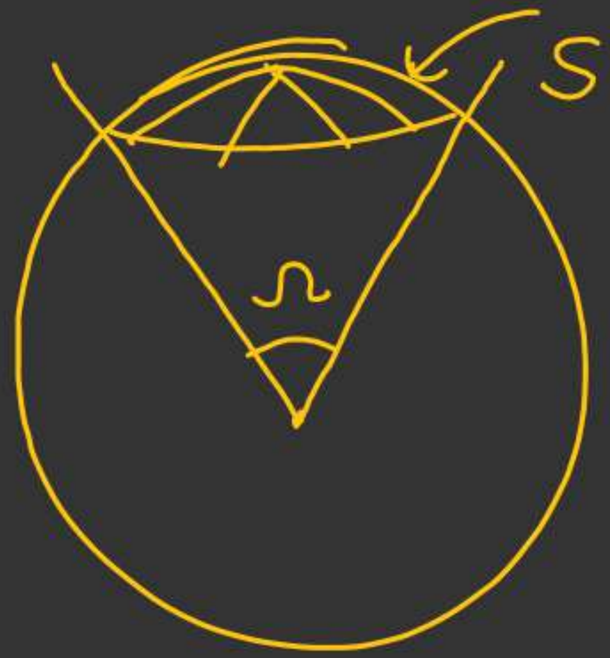
$d\hat{s} \parallel \vec{r}$

$$\Omega = \left(\frac{S\cos\theta}{r^2}\right)$$

For Spherical. $[\theta = 0]$

$$\cos\theta = (1)$$

$$\boxed{\Omega_{\text{spherical cap}} = \frac{S}{r^2}}$$



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GAUSS'S LAW

Solid Angle

$$\Omega_{sphere} = \left(\frac{S}{r^2}\right)$$

Area of Spherical Cap.

❖ **Relation between plane angle and Solid angle Subtended at the center of the sphere due to. the Spherical Cap of Sphere.**

ring

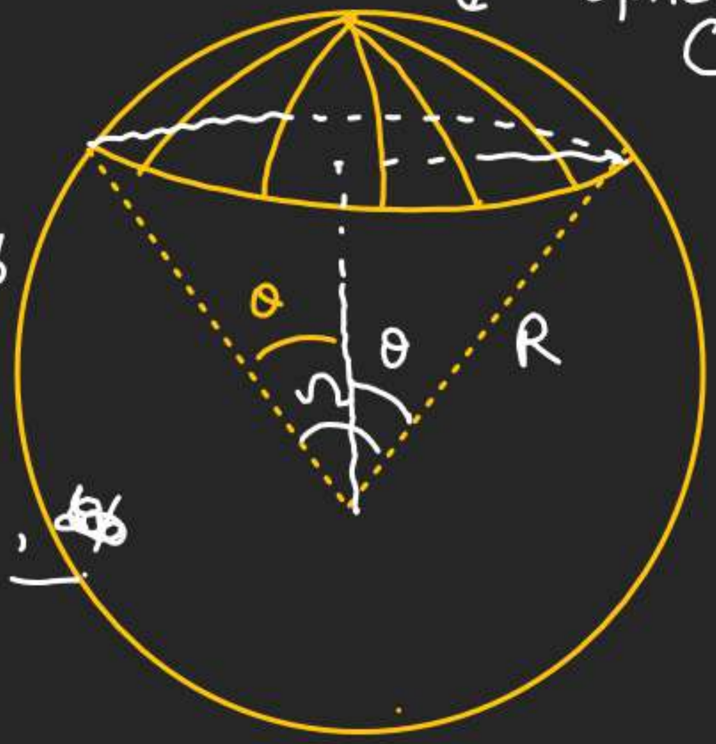
$\boxed{r = R\sin\phi}$

$$dA = (2\pi R\sin\phi)Rd\phi$$

$$dA = 2\pi R^2 \sin\phi\, d\phi$$

differential area of ring

Spherical Cap.



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(Area of Spherical Cap) $\rightarrow S = \int_0^\theta dA = 2\pi R^2 \int_0^\theta \sin\phi \, d\phi$

$S = 2\pi R^2 [-\cos\phi]_0^\theta$

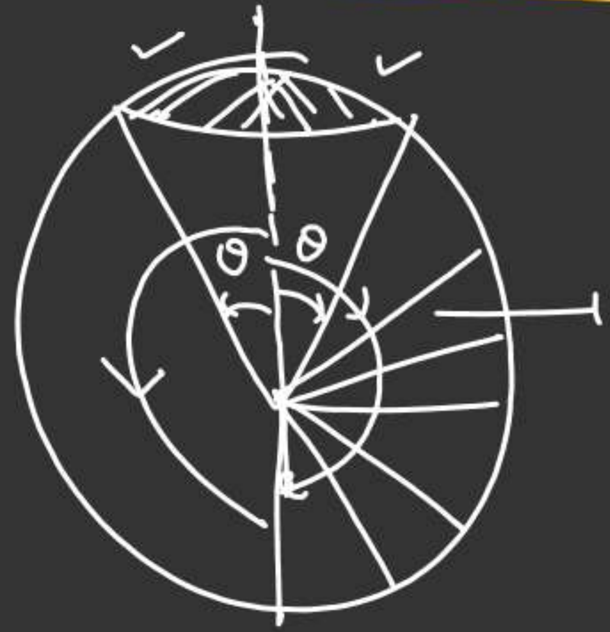
$S = 2\pi R^2 [-\cos\theta + \cos 0]$

$$\boxed{S = 2\pi R^2 (1 - \cos\theta)} \quad (**)$$

$\Omega_{\text{Spherical Cap}} = \frac{S}{R^2}$

$= \frac{2\pi R^2 (1 - \cos\theta)}{R^2}$

$$\boxed{\Omega_{\text{Spherical Cap}} = 2\pi (1 - \cos\theta)} \quad (**)$$



$\theta \rightarrow 0 \text{ to } \pi$

For Complete Sphere

$\theta = \pi$

$\Omega_{\text{Sphere}} = 2\pi (1 - \cos\pi)$

$$\boxed{\Omega_{\text{Sphere}} = 4\pi} \quad (*)(*)$$

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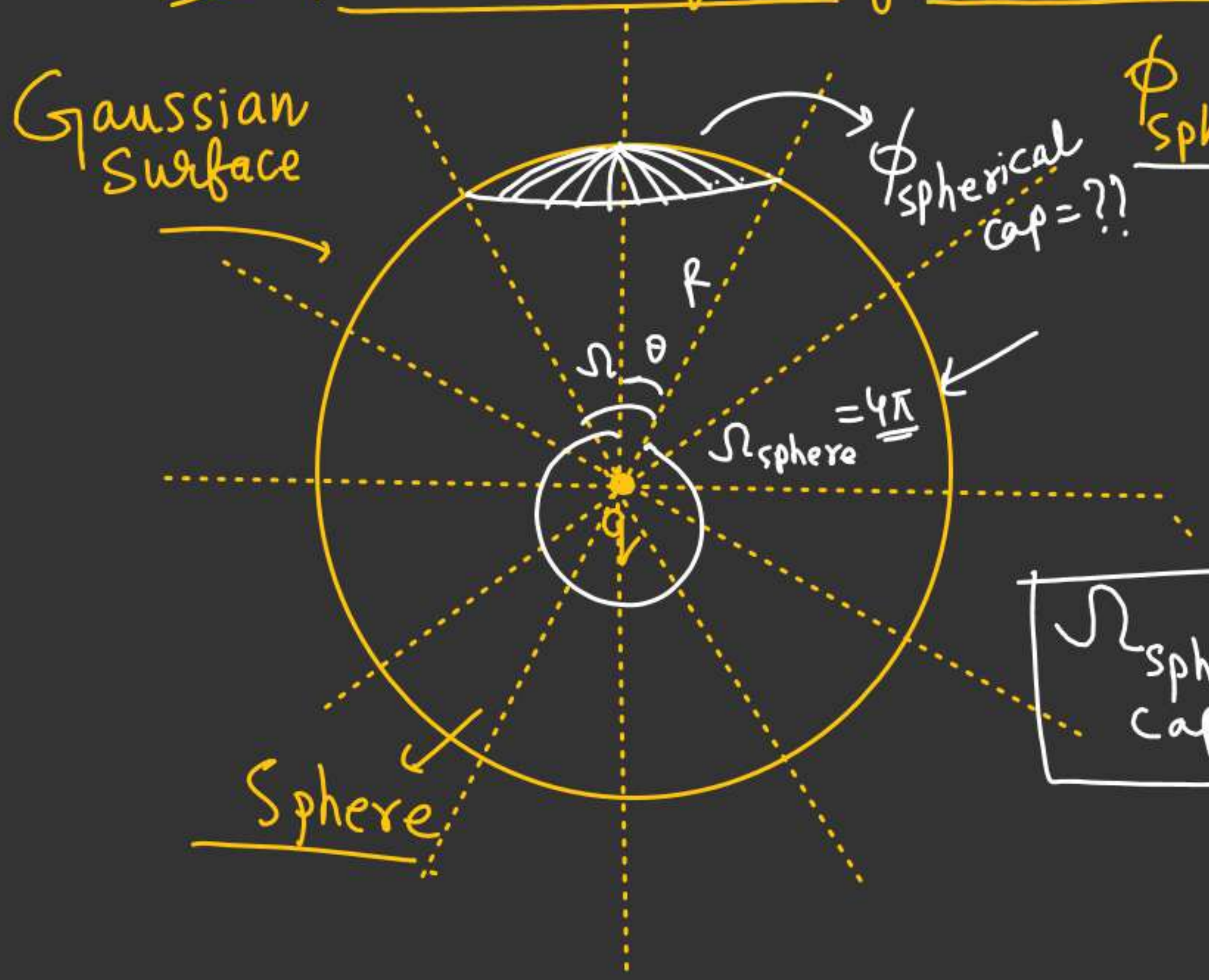
Flux through any Spherical Cap :-



$\phi_{sphere} = \left(\frac{q}{\varepsilon_0}\right)$

Flux per unit Solid angle $= \left(\frac{q/\varepsilon_0}{4\pi}\right)$

Flux throug the Spherical Cap having Solid angle 'Ω'.

$$\boxed{\Omega_{\substack{spherical \\ cap}} = 2\pi(1-\cos\theta)}$$

$$= \frac{q}{4\pi\varepsilon_0} \times (\Omega)_{\substack{spherical \\ cap}}$$

$$= \frac{q}{4\pi\varepsilon_0} \times 2\pi(1-\cos\theta)$$

$$\boxed{\phi_{\substack{spherical \\ cap}} = \frac{q}{2\varepsilon_0}(1-\cos\theta)}$$

**

GAUSS'S LAW

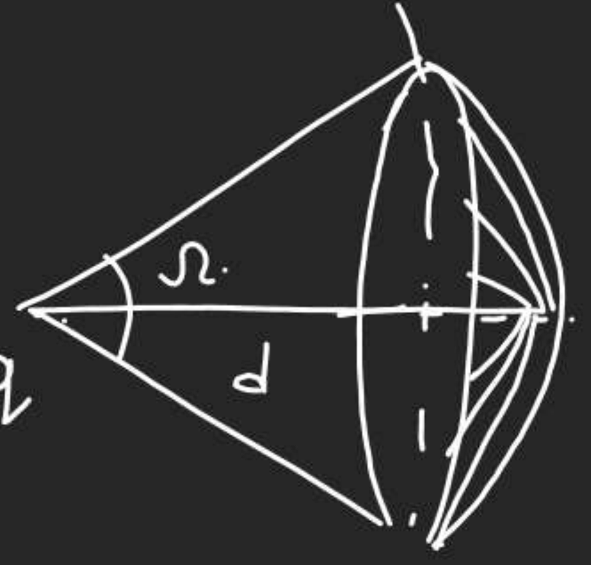
Solid Angle

❖ **Electric flux Calculation using solid angle:**

- **Flux through the disc**

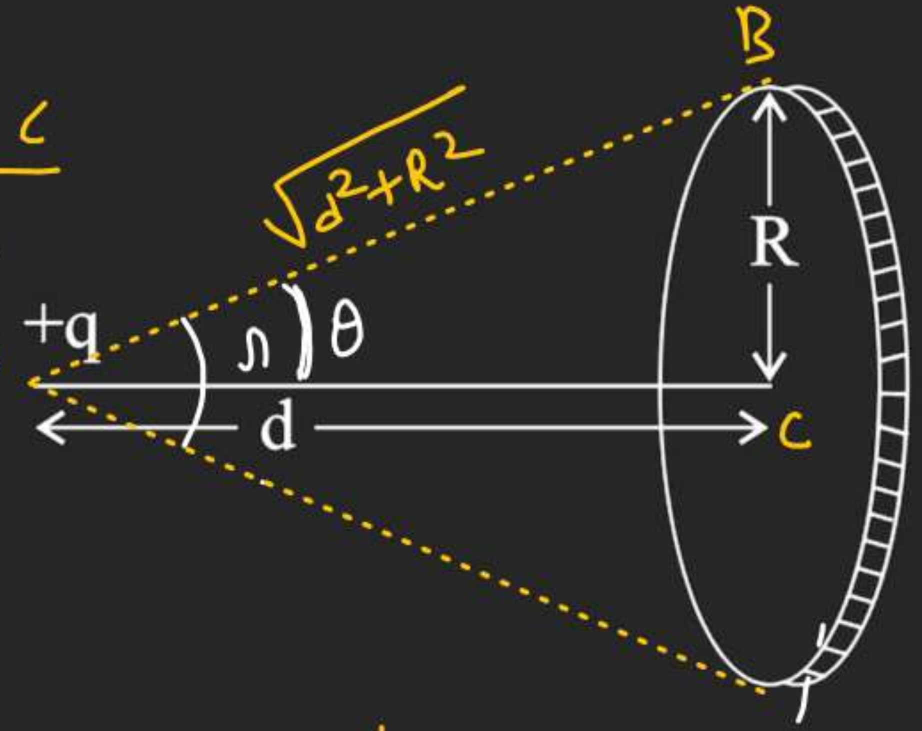
$$\phi_{\Omega} = \frac{q}{2\varepsilon_0}(1-\cos\theta)$$

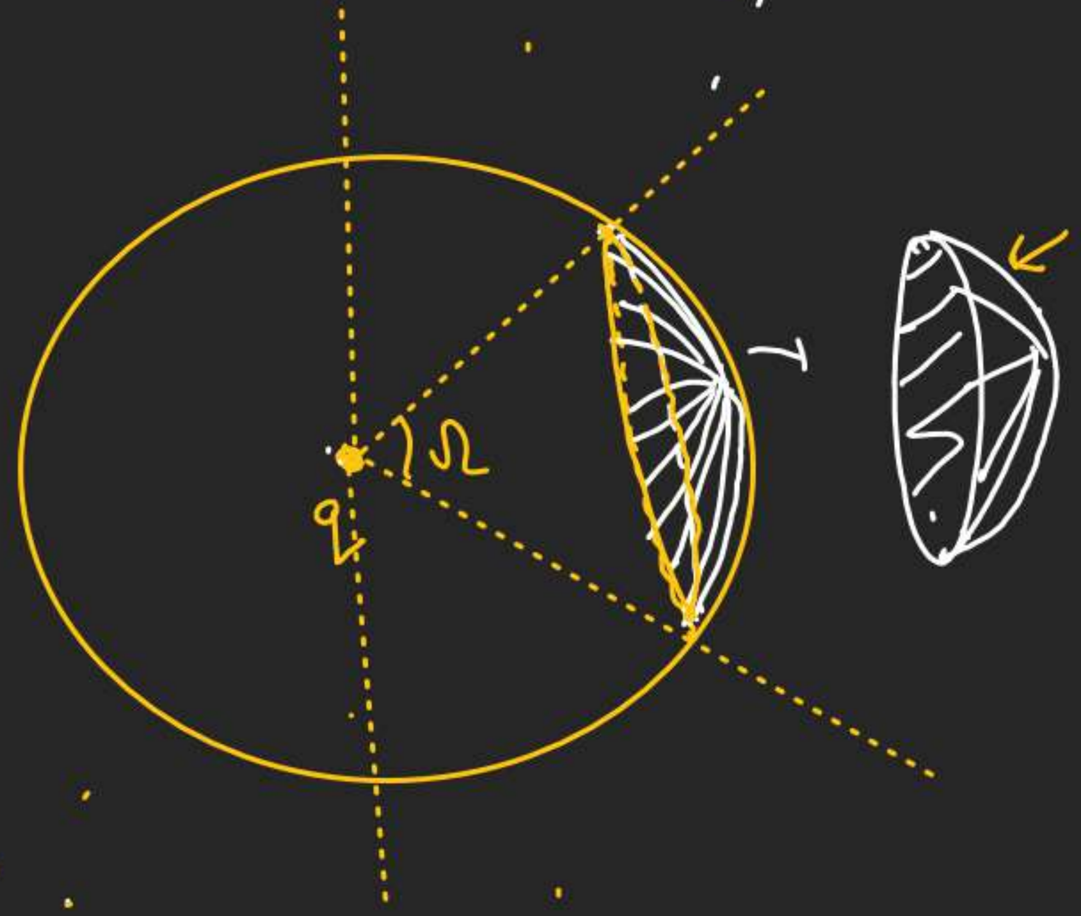
$$\boxed{\phi_{\Omega} = \frac{q}{2\varepsilon_0}\left(1-\frac{d}{\sqrt{d^2+R^2}}\right)}$$



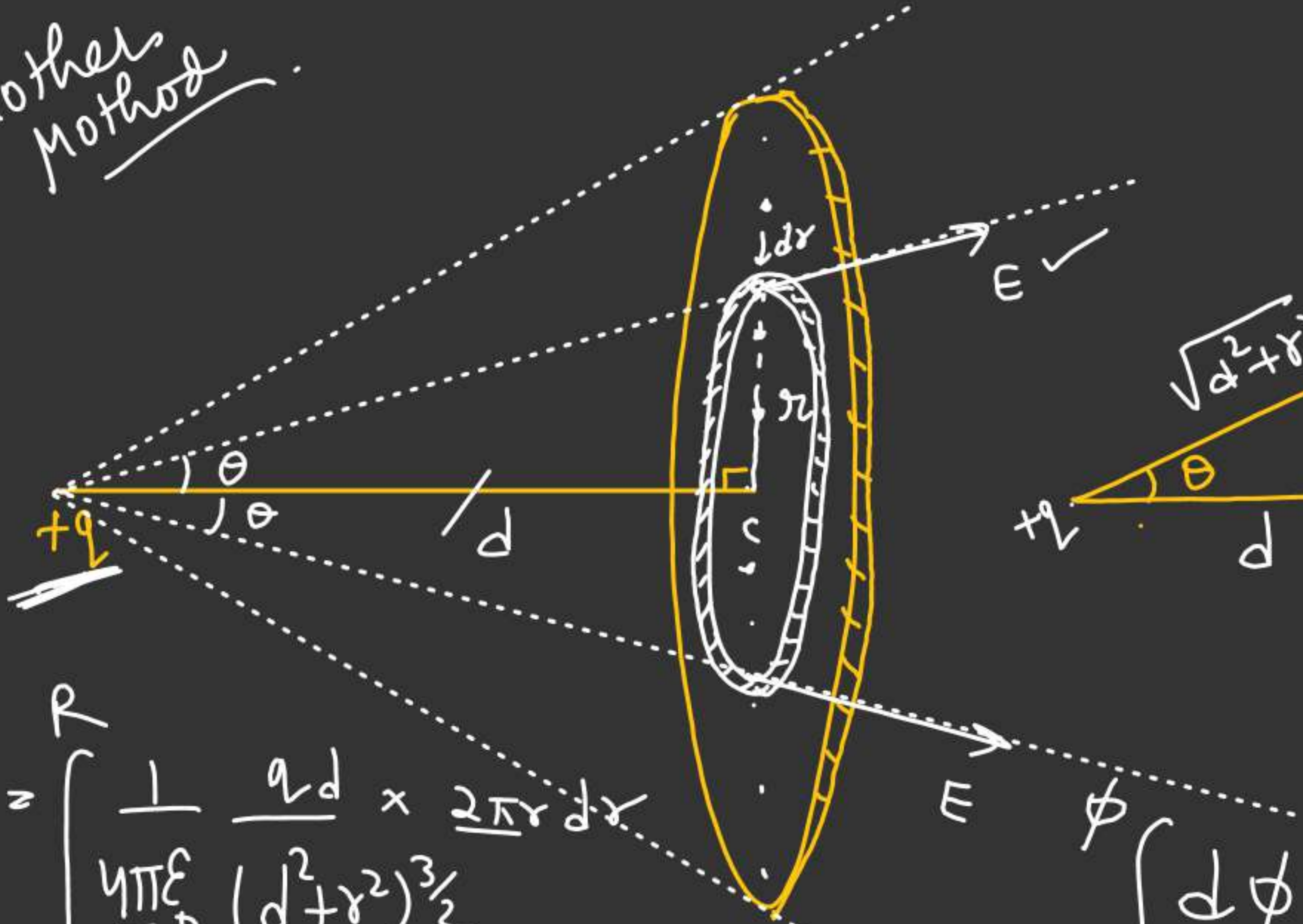
In ΔABC

$$\cos\theta = \frac{d}{\sqrt{d^2+R^2}}$$

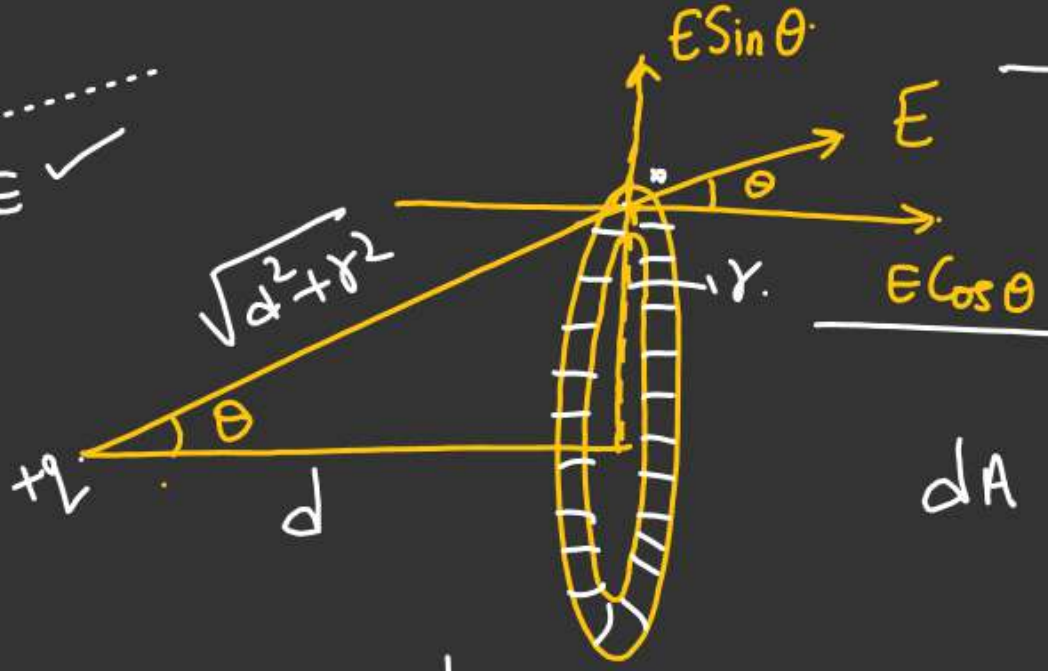




Another Method.



$$E = \frac{Kq}{r^2}$$



$$E = \frac{Kq}{(d^2+r^2)}$$

$$\cos\theta = \left(\frac{d}{\sqrt{d^2+r^2}}\right)$$

dA = Area of ring

$= (2\pi r)\,dr$.

$$d\phi = (E\cos\theta)\,dA$$

$$\int_0^{\phi} d\phi = \int_0^R \left(\frac{Kq}{d^2+r^2}\right) \times \frac{d}{\sqrt{d^2+r^2}} \times 2\pi r\,dr$$

$$\phi = \int_0^R \frac{1}{4\pi\varepsilon_0}\,\frac{qd}{(d^2+r^2)^{3/2}} \times 2\pi r\,dr$$

$$\phi = \frac{qd}{2\varepsilon_0}\int_0^R \frac{r\,dr}{(d^2+r^2)^{3/2}}$$

put $(d^2+r^2 = t)$.

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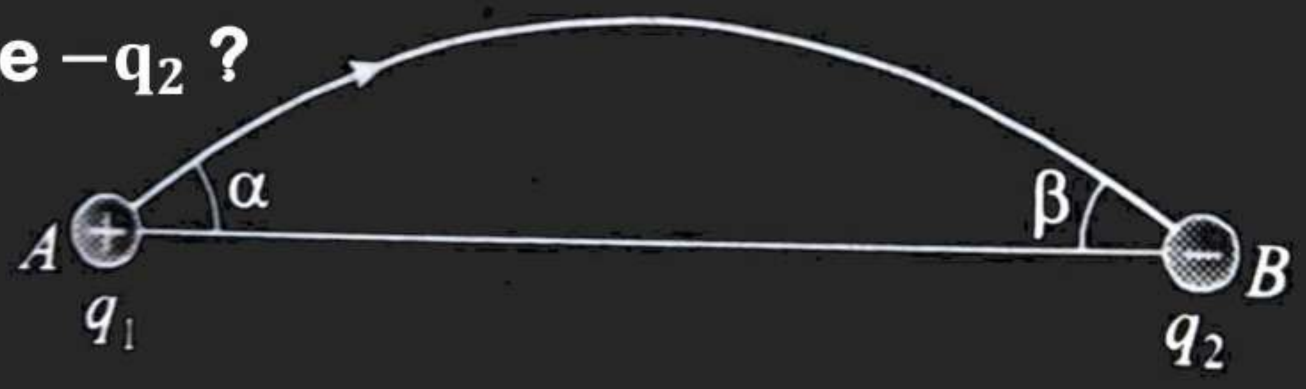
Solid Angle

Two charges $+q_1$ and $-q_2$ are placed at points A and B respectively. A line of force originates from the charge q_1 at an angle α with the line AB. Find at what angle this line will be terminating at the charge $-q_2$?



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GAUSS'S LAW

Q. Two point charges ' q ' and $-q$ are separated by the distance 2L. Then the electric flux across the area of circle of radius R, is;

(A) $\frac{q}{2\varepsilon_0}\left(1+\frac{1}{\sqrt{1-(R^2/L^2)}}\right)$

(B) $\frac{2q}{\varepsilon_0}\left(1-\frac{1}{\sqrt{1+(R^2/L^2)}}\right)$

(C) $\frac{q}{\varepsilon_0}\left(1+\frac{1}{\sqrt{1-(R^2/L^2)}}\right)$

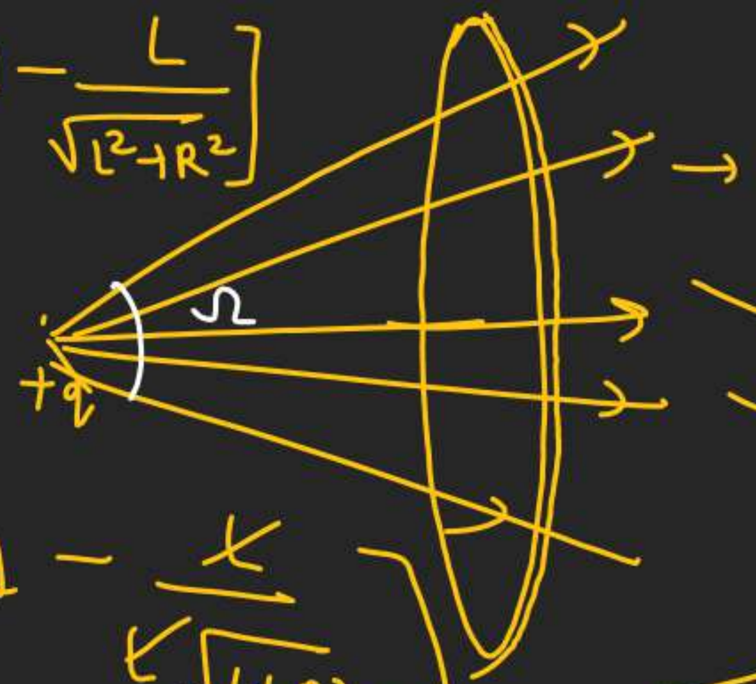
(D) $\frac{q}{\varepsilon_0}\left(1-\frac{1}{\sqrt{1+(R/L)^2}}\right)$

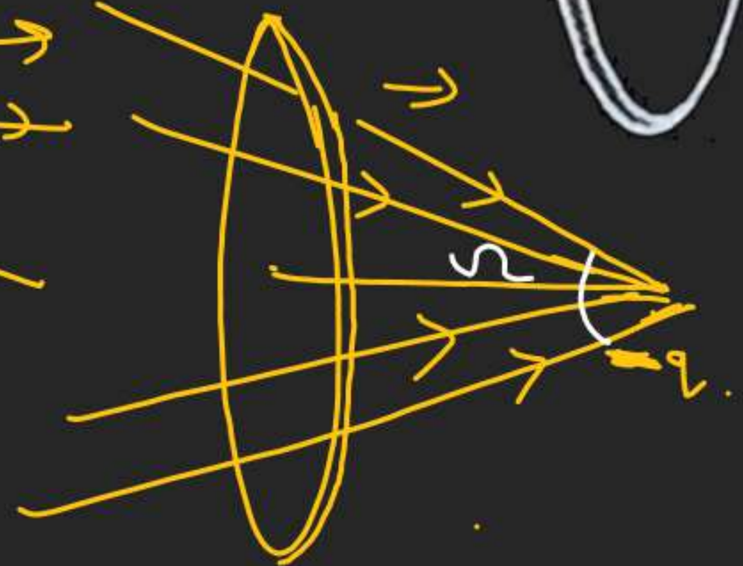
$$\phi_T = 2\times\frac{q}{2\varepsilon_0}\left[1-\frac{L}{\sqrt{L^2+R^2}}\right]$$

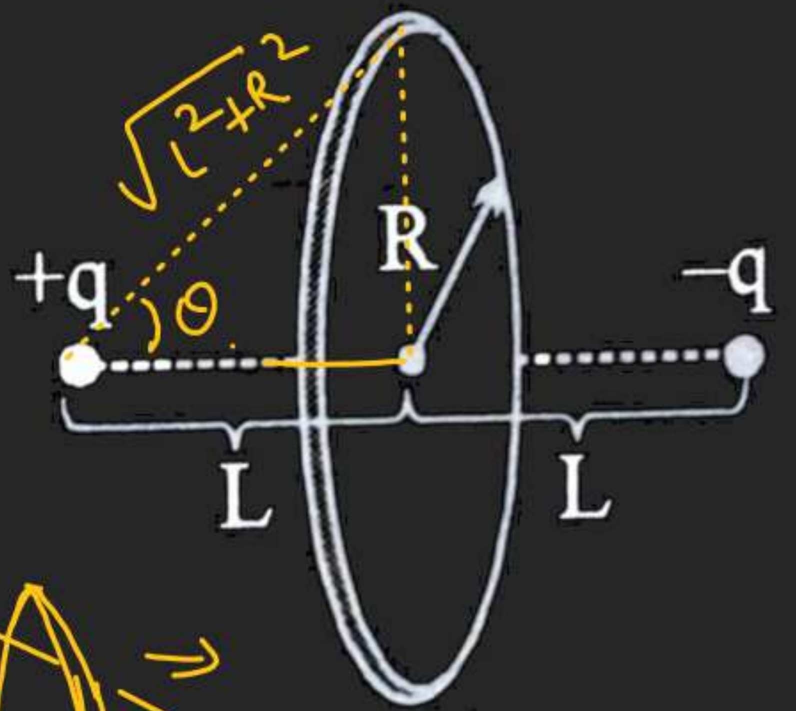
$$\phi_T = \frac{q}{\varepsilon_0}\left[1-\frac{L}{\sqrt{L^2+R^2}}\right]$$

$$\phi_T = \frac{q}{\varepsilon_0}\left[1-\frac{\cancel{L}}{\cancel{L}\sqrt{1+R^2/L^2}}\right]$$

$$\phi_T = \frac{q}{\varepsilon_0}\left[1-\frac{1}{\sqrt{1+R^2/L^2}}\right].$$







GAUSS'S LAW

Q. A point charge $+q$ is placed at the mid point on the axis of cylindrical shell as shown in the figure. Then the electric flux through the curved surface of cylinder will be;

(A) $\frac{2q}{\varepsilon_0}\left(\frac{L}{L^2+4r^2}\right)$

(B) $\frac{q}{\varepsilon_0}\left(\frac{L}{\sqrt{L^2+4r^2}}\right)$ ✓

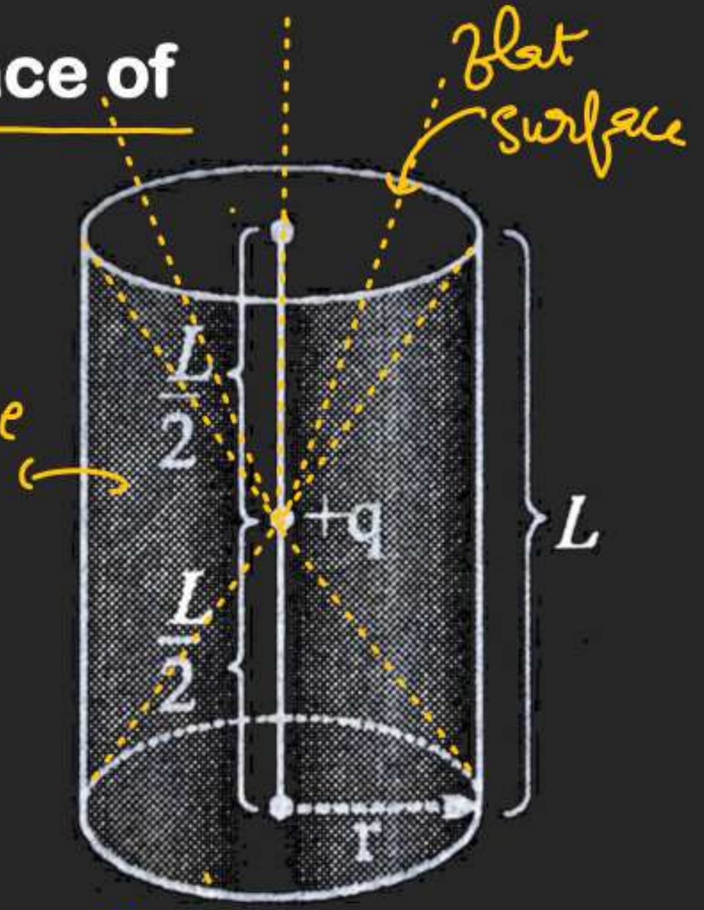
(C) $\frac{q}{2\varepsilon_0}\left(\frac{L}{\sqrt{L^2+4r^2}}\right)$

(D) none of these

$$(\phi_T)_{\text{closed cylinder}} = \frac{q}{\varepsilon_0}$$

$$\Downarrow$$

$$2(\phi_{\text{flat part}}) + \phi_{\text{curve part}} = \frac{q}{\varepsilon_0}$$



$$\phi_{\text{curve part}} = \frac{q}{\varepsilon_0} - 2\left[\phi_{\text{flat part}}\right]$$

$$= \frac{q}{\varepsilon_0} - \frac{q}{\varepsilon_0}\left[1 - \frac{L}{\sqrt{4r^2+L^2}}\right]$$

$$= \frac{q}{\varepsilon_0}\left[\frac{L}{\sqrt{4r^2+L^2}}\right]$$

$$\phi_{\text{flat part}} = \frac{q}{2\varepsilon_0}\left[1 - \frac{L}{2\sqrt{r^2+\frac{L^2}{4}}}\right]$$

$$\phi_{\text{flat part}} = \frac{q}{2\varepsilon_0}\left[1 - \frac{L}{\sqrt{4r^2+L^2}}\right]$$

GAUSS LAW

H.W.

Q. Gauss's law and Coulomb's law expressed in different forms, although are equivalent ways of describing the relation between charge and electric field in static conditions. Gauss's law is $\varepsilon_0 \phi = q_{enc}$ in which q_{end} is the net charge inside an imaginary closed surface called Gaussian surface and ϕ is the net flux of the electric field through the surface. $\phi = \oint \vec{E} d\vec{A}$ gives electric flux through Gaussian surface. The two equations hold only when the net charge is in vacuum or air.

Nishant Jindal

GAUSS LAW

H.Q.

Q. A Gaussian surface encloses two of the 4 positively charged particles. The particles which contribute to the electric field at point P on the surface are:

(A) q_1 and q_2

(B) q_2 and q_3

(C) q_4 and q_3

(D) q_1, q_2, q_3 and q_4

Nishant Jindal

GAUSS LAW

H.W

Q. The net flux of the electric field through the surface is:

(A) due to q_1 and q_2 only

(B) due to q_3 and q_4 only

(C) equal due to all the four charges

(D) cannot say

Nishant Jindal

GAUSS LAW

H-W

Q. The net flux of the electric field through the surface due to q_3 and q_4 is:

(A) zero

(B) positive

(C) negative

(D) can't say

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GAUSS LAW

H.W.

Q. If the charges q_3 and q_4 are displaced (always remaining outside the Gaussian surface), then consider the following two statements

A: Electric field at each point on Gaussian surface will remain same

B: The value of $\oint \vec{E} \cdot \vec{d}A$ for the Gaussian surface will remains same:

(A) Both A and B are true

(B) Both A and B are false

(C) A is true but B is false

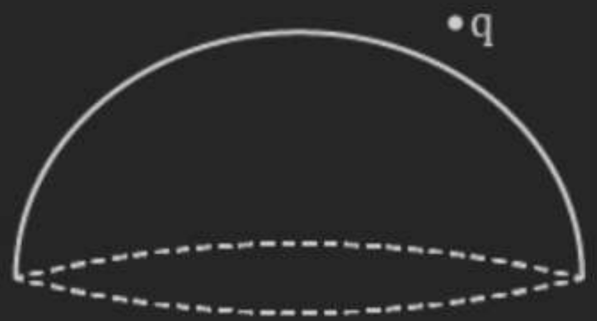
(D) B is true but A is false

GAUSS LAW

H.W.

Q. In which of the following cases, the flux crossing through the surface is zero ?

(A)

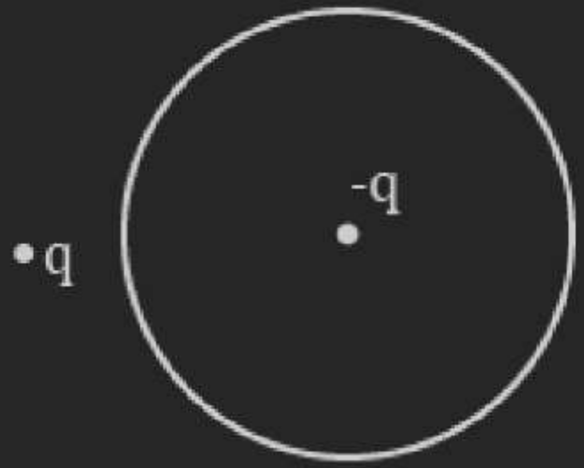


Hemispherical surface without base

(B)

Cylindrical surface with one end open

(C)



Spherical surface

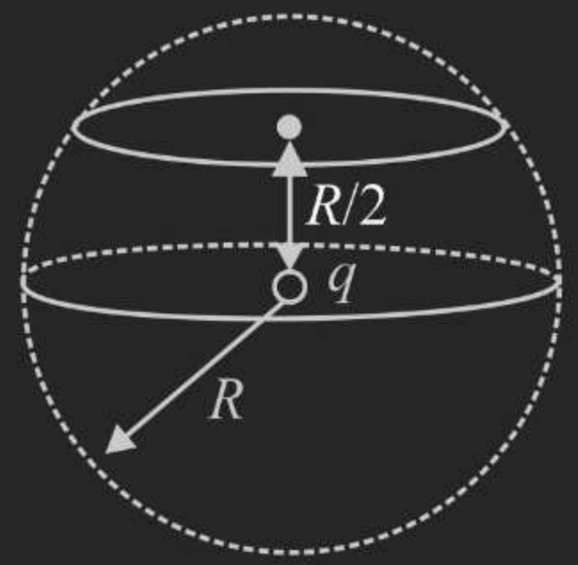
(D)

Hemispherical surface with base

GAUSS LAW

H-W

Q. Flux passing through the shaded surface of a sphere when a point charge q is placed at the center is (radius of the sphere is R)



(A) q/ε_0

(B) $q/2\varepsilon_0$

(C) $q/4\varepsilon_0$

(D) zero

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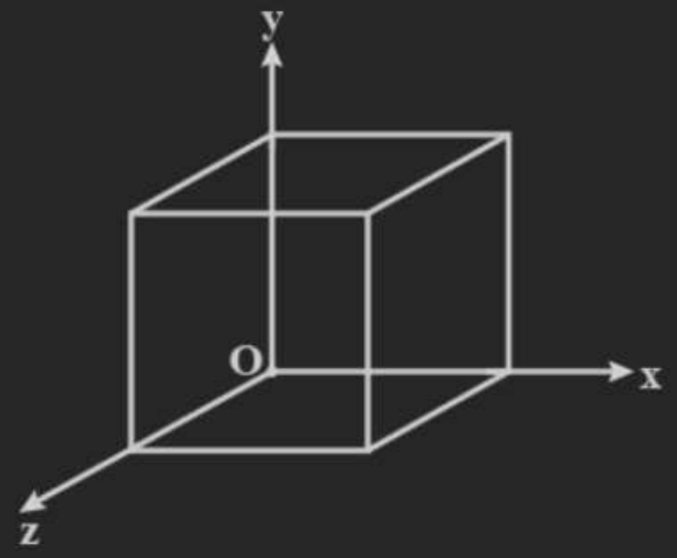
GAUSS LAW

H.W.

A cube of side a is placed such that the nearest face, which is parallel to the yz plane, is at a distance a from the origin. The electric field components are

$E_x = \alpha x^{1/2}, E_y = E_z = 0$



Q. The charge within the cube is

(A) $\sqrt{2}\alpha\varepsilon_0 a^{5/2}$

(B) $-\alpha\varepsilon_0 a^{5/2}$

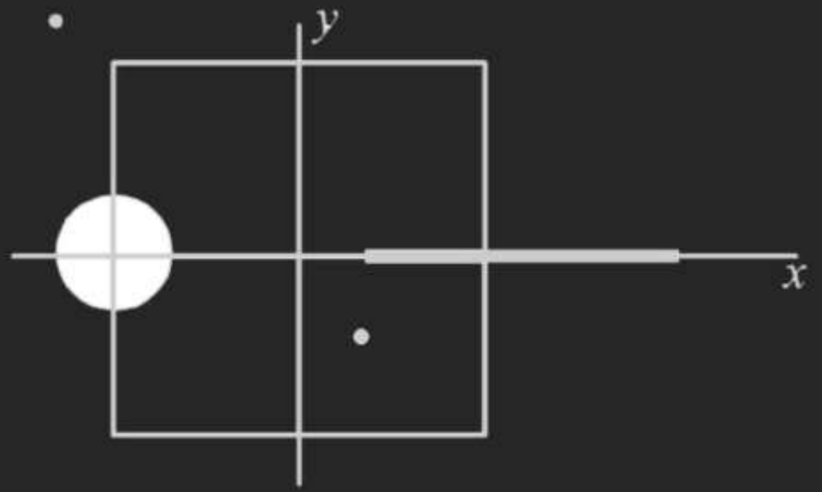
(C) $(\sqrt{2}-1)\alpha\varepsilon_0 a^{5/2}$

(D) zero

GAUSS LAW

H.W.

Q. A disk of radius $a/4$ having a uniformly distributed charge $6C$ is placed in the $x-y$ plane with its centre at $(-a/2, 0, 0)$. A rod of length a carrying a uniformly distributed charge $8C$ is placed on the x-axis from $x = a/4$ to $x = 5a/4$. Two point charges $-7C$ and $3C$ are placed at $(a/4, -a/4, 0)$ and $(-3a/4 3a/4, 0)$, respectively. Consider a cubical surface formed by six surfaces $x = \pm a/2, y = \pm a/2, z = \pm a/2$. The electric flux through this cubical surface is



(A) $\frac{-2C}{\varepsilon_0}$

(B) $\frac{2C}{\varepsilon_0}$

(C) $\frac{10C}{\varepsilon_0}$

(D) $\frac{12C}{\varepsilon_0}$

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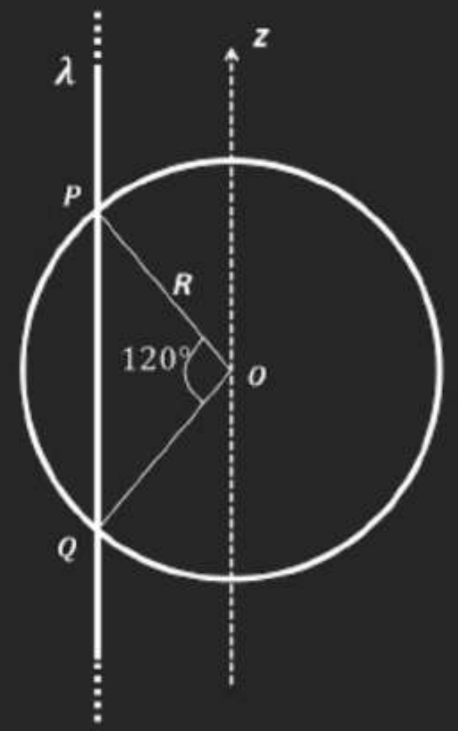
GAUSS LAW

H.W.

Q. An infinitely long thin non-conducting wire is parallel to the z-axis and carries a uniform line charge density λ. It pierces a thin non-conducting spherical shell of radius R in such a way that the arc PQ subtends an angle 120° at the centre O of the spherical shell, as shown in the figure. The permittivity of free space is ε_0. Which of the following statements is (are) true? **[JEE (Adv)-2018 (Paper-2)]**



(A) The electric flux through the shell is $\frac{\sqrt{3}R\lambda}{\varepsilon_0}$

(B) The z-component of the electric field is zero at all the points on the surface of the shell

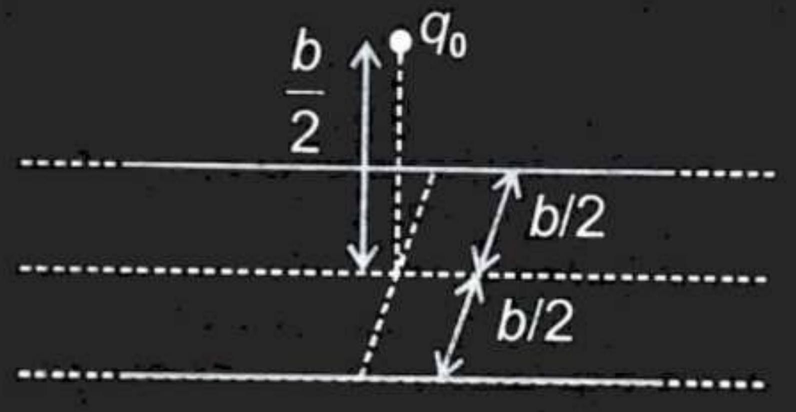
(C) The electric flux through the shell is $\frac{\sqrt{2}R\lambda}{\varepsilon_0}$

(D) The electric field is normal to the surface of the shell at all points

GAUSS LAW

H.W.

Q. An infinitely large plate of width b is placed in horizontal $\mathrm{x}-\mathrm{y}$ plane. A point charge $\mathrm{q_0}$ is placed symmetrically at distance of $\frac{\mathrm{b}}{2}$ perpendicular to plane of this plate as shown. Find electric flux passing through this plate.



(A) $\left(\frac{q_0}{\varepsilon_0}\right)$

(B) $\left(\frac{1}{2}\right)\left(\frac{q_0}{\varepsilon_0}\right)$

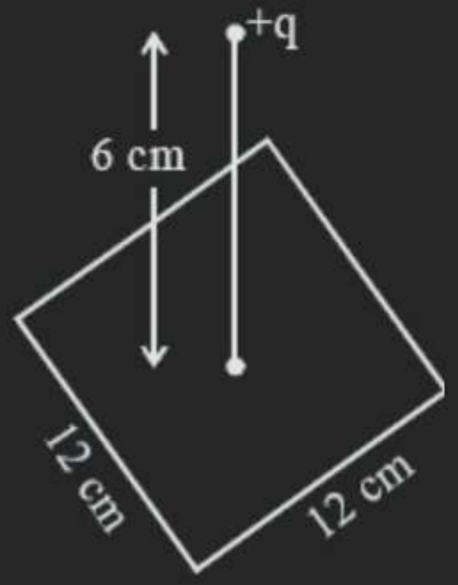
(C) Zero

(D) $\left(\frac{1}{4}\right)\left(\frac{q_0}{\varepsilon_0}\right)$

GAUSS LAW

H.W.

Q. The electric field in a region is given by $\vec{E} = \frac{2}{5}E_0\hat{i} + \frac{3}{5}E_0\hat{j}$ with $E_0 = 4.0 \times 10^3 \frac{N}{C}$. The flux of this field through a rectangular surface area $0.4\ m^2$ parallel to the $Y-Z$ plane is Nm^2C^{-1} **[JEE (Main)-2021]**



GAUSS LAW

H.W.

Q. A cube is placed inside an electric field, $\vec{E} = 150y^2\hat{j}$. The side of the cube is $0.5\ \text{m}$ and is placed in the field as shown in the given figure. The charge inside the cube is: [JEE (Main)-2021]

GAUSS LAW

H-W

Q. A cube is placed inside an electric field, $\vec{E} = 150y^2\hat{j}$. The side of the cube is 0.5 m and is placed in the field as shown in the given figure. The charge inside the cube is: **[JEE (Main)-2021]**

(A) 8.3×10^{-11}C

(B) 3.8×10^{-11}C

(C) 8.3×10^{-12}C

(D) 3.8×10^{-12}C