Nishant Jindal

Greatest term

$T_{r+1} \rightarrow$ greatest

$$\boxed{\begin{gathered} T_{r+1} \geq T_r \\ \& \\ T_{r+1} \geq T_{r+2} \end{gathered}}$$

$^nC_r \geq {}^nC_{r-1} \Rightarrow \frac{1}{r} \geq \frac{1}{n-r+1}$

$\& \quad \Rightarrow r \leq n-r+1$

$^nC_r \geq {}^nC_{r+1}$

$r \leq \frac{n+1}{2}$

$r+1 \geq \frac{n+1}{2}$

$$\boxed{\frac{n-1}{2} \leq r \leq \frac{n+1}{2}}$$

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1. Find the greatest term in $(1+4x)^8$ if $x=\frac{1}{3}$.

$T_{r+1} \to$ greatest $\quad T$

$T_{r+1} \geq T_r \Rightarrow {}^8C_r\left(\frac{4}{3}\right)^r \geq {}^8C_{r-1}\left(\frac{4}{3}\right)^{r-1} \Rightarrow \frac{1}{r}\frac{4}{3} \geq \frac{1}{9-r}$

$4(9-r) \geq 3r \Rightarrow r \leq \frac{36}{7}$.

$T_6 \to {}^8C_5\left(\frac{4}{3}\right)^5$

$T_{r+1} \geq T_{r+2} \Rightarrow r+1 \geq \frac{36}{7} \Rightarrow r \geq \frac{29}{7}$.

$\boxed{r=5}$

$\frac{29}{7} \leq r \leq \frac{36}{7}$.

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2. Find the numerically greatest coefficient in the expansion of $(3-2x)^9$.

$\underline{T_{r+1}}$.

$${}^9C_r\, 3^{9-r}\, 2^r \geq {}^9C_{r-1}\, 3^{10-r}\, 2^{r-1} \Rightarrow \frac{2}{r} \geq \frac{3}{10-r}$$

$$20-2r \geq 3r \Rightarrow \boxed{r \leq 4}$$

$$r+1 \geq 4 \Rightarrow \boxed{r \geq 3} \qquad r = 3, 4$$

$$3 \leq r \leq 4$$

$$T_4, T_5$$

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3. Given that only 4^{th} term in expansion of $\left(2+\frac{3x}{8}\right)^{10}$ has maximum numerical value, find x.

$$|T_{r+1}| > |T_r| \Rightarrow {}^{10}C_r 2^{10-r}\left(\frac{3}{8}|x|\right)^r > {}^{10}C_{r-1} 2^{11-r}\left(\frac{3}{8}|x|\right)^{r-1}$$

$$\Rightarrow \frac{1}{r}\frac{3}{8}|x| > \frac{2}{11-r}$$

$$|x| > \frac{16r}{3(11-r)}$$

$$|T_{r+1}| > |T_{r+2}|$$

$$|x| < \frac{16(r+1)}{3(10-r)}$$

$$\Rightarrow \frac{16\times 3}{3(8)} < |x| < \frac{16\times 4}{3\times 7}$$

$$2 < |x| < \frac{64}{21}$$

$$\boxed{x \in \left(-\frac{64}{21}, -2\right) \cup \left(2, \frac{64}{21}\right)}$$

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4. Let $x = (7+4\sqrt{3})^n$, $n \in N$ $\qquad x N = (7+4\sqrt{3})^n (7-4\sqrt{3})^n = 1$

P.T. (i) $[x]$ is odd $\qquad$ (ii) $x(1-\{x\}) = 1$

$[\cdot]$ = Greatest Integer function, $\{\cdot\}$ = Fraction part function.

$$\boxed{\{x\} + N = 1}$$

$$[x] + \{x\} = (7+4\sqrt{3})^n$$

$0 \le \{x\} < 1$

$0 < N < 1$

$0 < \{x\} + N < 2$

$$(7+4\sqrt{3})^n + \underbrace{(7-4\sqrt{3})^n}_{N} = 2\left[{}^nC_0 7^n + {}^nC_2 7^{n-2}(4\sqrt{3})^2 + {}^nC_4 7^{n-4}(4\sqrt{3})^4 + \dots\right]$$

$$[x] + (\{x\} + N) = 2K, \quad K \in I$$

$\{x\} + N \in I$ ✓

$[x] = 2K - 1$

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$x = [x] + \{x\}$

$[x] \to$ G.I.F

$\{x\} \to$ FPF

$0 \leq \{x\} < 1$

$x = -3.657 = \underbrace{-4}_{[x]} + \underbrace{0.343}_{\{x\}}$

$-4 = \underbrace{-4}_{[x]} + \underbrace{0}_{\{x\}}$

$-3.99998 = \underbrace{-4}_{[x]} + \underbrace{0.00002}_{\{x\}}$

$-3 = \underbrace{-3}_{[x]} + \underbrace{0}_{\{x\}}$

$-3.001 = -4 + 0.999 \to \{x\}$

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1. Let $x = (6\sqrt{6}+14)^{2n+1}$, $n \in N$.

P.T. $x\{x\} = (20)^{2n+1}$

$\{\cdot\} = FPF$.

2. Let $x = (\sqrt{3}+1)^{2n}$, $n \in N$. P.T.

$[x]+1$ is divisible by 2^{n+1}.

$\boxed{\{x\text{-}2\ (1\text{-}15)\}}$