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$$\alpha, \beta = \frac{1 \pm \sqrt{5}}{2}$$

$$p\alpha^4 + q\beta^4 = p(3\alpha+2) + q(3\beta+2) = p\left(\frac{3+3\sqrt{5}}{2} + 2\right) + q\left(\frac{3-3\sqrt{5}}{2} + 2\right)$$

$$= \frac{7}{2}(p+q) + \frac{3\sqrt{5}}{2}(p-q) = \underline{28}.$$

$$\alpha^4 = (\alpha^2-\alpha-1)(\alpha^2+\alpha+2) + 3\alpha+2 = 3\alpha+2$$

$$\alpha^2 - \alpha - 1 = 0$$

$$p\alpha^{12} + q\beta^{12} = p\alpha^{11} + q\beta^{11} + p\alpha^{10} + q\beta^{10}$$

$$p - q = 0.$$

$$7p = 28$$

$$p = q = 4$$

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23.

$x, \underbrace{\ldots A_r \ldots}_{n}, 2y$

$A_r = x + r\left(\frac{2y-x}{n+1}\right) = \frac{(n+1-r)x + 2ry}{n+1}$

$2x, \underbrace{\ldots A_r' \ldots}_{n}, y$

$A_r' = 2x + r\left(\frac{y-2x}{n+1}\right) = \frac{(2(n+1)-2r)x + ry}{n+1}$

$\cancel{5}-3\beta, \cancel{5}-\beta, \cancel{5}+\beta, \cancel{5}+3\beta$

$\frac{25-9\beta^2}{25-\beta^2} = \frac{2}{3}$

$\beta = ?$

$\frac{p}{2}\left(2a+(p-1)d\right) = \frac{q}{2}\left(2a+(q-1)d\right)$

$a(p-q) + \frac{d}{2}\left((p^2-q^2) - (p-q)\right) = 0$

$2a + d(p+q-1) = 0.$

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$$\frac{\frac{m}{2}\left(2a+(m-1)d\right)}{\frac{n}{2}\left(2a+(n-1)d\right)} = \frac{m^2}{n^2} \Rightarrow \boxed{\frac{a+\left(\frac{m-1}{2}\right)d}{a+\left(\frac{n-1}{2}\right)d} = \frac{m}{n}}.$$

$$\frac{a+(m-1)d}{a+(n-1)d} = ?$$

$$\frac{m-1}{2} = M-1 \Rightarrow m = 2M-1$$

$$\frac{n-1}{2} = N-1 \Rightarrow n = 2N-1$$

$$\boxed{\frac{a+(M-1)d}{a+(N-1)d} = \frac{2M-1}{2N-1}}$$

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$$T_1, T_2, T_3, T_4, \dots T_n$$

$$T_1 + T_3 + T_5 + \dots + T_{n-1} = 24$$

$$T_2 + T_4 + T_6 + \dots + T_n = 30$$

$$6 = \frac{n}{2} d$$

$$\boxed{nd = 12}$$

$$n = ?$$

$$T_n - T_1 = \frac{21}{2}$$

$$\cancel{a} + (n-1)d - \cancel{a} = \frac{21}{2}$$

$$12 - d = \frac{21}{2}$$

$$d = ?$$

$$\boxed{S_n - S_{n-1} = T_n}$$

$$n(5n+3) - (n-1)(5n-8) = T_n$$

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AGP. $\rightarrow$ $a, (a+d)r, (a+2d)r^2, (a+3d)r^3, \ldots$

$35, 2^{35}$

① $S_n = a + (a+d)r + (a+2d)r^2 + (a+3d)r^3 + \ldots + (a+(n-1)d)r^{n-1}$

② $rS_n = \quad ar + (a+d)r^2 + (a+2d)r^3 + \ldots + (a+(n-2)d)r^{n-1} + (a+(n-1)d)r^n$

① − ② $(1-r)S_n = a + dr + dr^2 + dr^3 + \ldots dr^{n-1} - (a+(n-1)d)r^n$

$= a + \dfrac{dr(1-r^{n-1})}{(1-r)} - (a+(n-1)d)r^n$

$nr^n \xrightarrow{n\to\infty} 0$

$\dfrac{n}{2^n} \xrightarrow{n\to\infty} 0$

$n = 10, \quad 2^n = 2^{10}$

$S_\infty = \lim\limits_{n\to\infty}\left(\dfrac{a}{1-r} + \dfrac{dr - dr^n}{(1-r)^2} - \dfrac{ar^n + (n-1)r^n d}{1-r}\right)$

$= \dfrac{a}{1-r} + \dfrac{dr}{(1-r)^2}$ ✓

$\boxed{|r| < 1}$ ✓

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$$S = a + (a+d)r + (a+2d)r^2 + (a+3d)r^3 + \cdots \infty$$

$$rS = \quad ar + (a+d)r^2 + (a+2d)r^3 + \cdots \infty$$

$$(1-r)S = a + \underbrace{\left(dr + dr^2 + dr^3 + \cdots\right)}$$

$$S(1-r) = a + \frac{dr}{1-r}$$

$$S = \frac{a}{1-r} + \frac{dr}{(1-r)^2}, \quad |r| < 1$$

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1. If $|x|<1$, find

(i) $1+2x+3x^2+4x^3+5x^4+\cdots\infty = \sum_{r=1}^{\infty} r\,x^{r-1}$

$\boxed{x^r}$

$S = 1+2x+3x^2+4x^3+5x^4+\cdots\infty$

$xS = \quad x+2x^2+3x^3+4x^4+\cdots$

$(1-x)S = 1+x+x^2+x^3+x^4+\cdots = \frac{1}{1-x}$

$S = \frac{1}{(1-x)^2}$

$(x+1)^2 = x^2+2x+1$

$2(x+1) = 2x+2.$

$1+x+x^2+x^3+x^4+\cdots\infty = \frac{1}{1-x}$

$1+2x+3x^2+4x^3+\cdots\infty = \frac{-1}{(1-x)^2}(-1)$

$= \frac{1}{(1-x)^2}.$

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(ii) $1 + 3x + 6x^2 + 10x^3 + 15x^4 + \cdots \infty$

$$S = 1 + 3x + 6x^2 + 10x^3 + 15x^4 + \cdots \infty$$

$$xS = \quad x + 3x^2 + 6x^3 + 10x^4 + \cdots \infty$$

$$(1-x)S = 1 + 2x + 3x^2 + 4x^3 + 5x^4 + \cdots \infty$$

$$x(1-x)S = \quad x + 2x^2 + 3x^3 + 4x^4 + \cdots \infty$$

$$(1-x)(1-x)S = 1 + x + x^2 + x^3 + x^4 + \cdots \infty$$

$$= \frac{1}{1-x}$$

$$\boxed{S = \frac{1}{(1-x)^3}}$$

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2. Find the sum upto n terms & infinite terms if exist.

$$1+\frac{4}{5}+\frac{7}{5^2}+\frac{10}{5^3}+\frac{13}{5^4}+ - - -$$

$$S_n = 1+\frac{4}{5}+\frac{7}{5^2}+\frac{10}{5^3}+\frac{13}{5^4}+ - - - + \frac{3n-2}{5^{n-1}}$$

$$\frac{1}{5}S_n = \frac{1}{5}+\frac{4}{5^2}+\frac{7}{5^3}+\frac{10}{5^4}+ - - - + \frac{3n-5}{5^{n-1}} + \frac{3n-2}{5^n}$$

$$\frac{4}{5}S_n = 1+\left(\frac{3}{5}+\frac{3}{5^2}+\frac{3}{5^3}+ - - - + \frac{3}{5^{n-1}}\right) - \frac{3n-2}{5^n}$$

$$\frac{4}{5}S_n = 1+\frac{\frac{3}{5}\left(1-\frac{1}{5^{n-1}}\right)}{\left(1-\frac{1}{5}\right)} - \frac{3n-2}{5^n}$$

$$\frac{4}{5}S_\infty = 1+\frac{3}{5}+\frac{3}{5^2}+\frac{3}{5^3}+ - - -$$

$$= 1+\frac{\frac{3}{5}}{1-\frac{1}{5}} = 1+\frac{3}{4} = \frac{7}{4}$$

$$\boxed{S_\infty = \frac{35}{16}}$$

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$\boxed{\text{Ex-II(a)} \quad 21-28.}$ ✓

Ex-II(b) → 3, 4, 7, 9, 10, 15, $\boxed{17}$, 18, 19, 20, 21, 22, 23.