Nishant Jindal

WAVE

ENERGY DENSITY (i.e Energy per Unit volume)

$$\boxed{K.E \text{ per Unit Volume} = \frac{1}{2}\rho\left(\frac{\partial y}{\partial t}\right)^2}$$

$$\rho = \frac{m}{V}$$

$$K.E = \frac{1}{2} m v_p^2$$

$$= \frac{1}{2}\rho . V . v_p^2$$

$$\frac{K.E}{V} = \frac{1}{2}\rho v_p^2$$

$$v_p = \frac{\partial s}{\partial t} \text{ or } \frac{\partial y}{\partial t}$$

$$\boxed{P.E \text{ per unit Volume} = \frac{1}{2}\rho \underline{v^2\left(\frac{\partial y}{\partial x}\right)^2}}$$

$$\left(\frac{\partial y}{\partial t}\right)^2$$

$$\frac{\partial y}{\partial x} = \frac{-1}{v}\left(\frac{\partial y}{\partial t}\right)$$

$$v^2\left(\frac{\partial y}{\partial x}\right)^2 = \left(\frac{\partial y}{\partial t}\right)^2$$

$$\boxed{K.E \text{ per Unit volume} = P.E \text{ per Unit Volume}}$$

WAVE

Avg K.E per unit volume

$$y = A\sin(kx - \omega t)$$

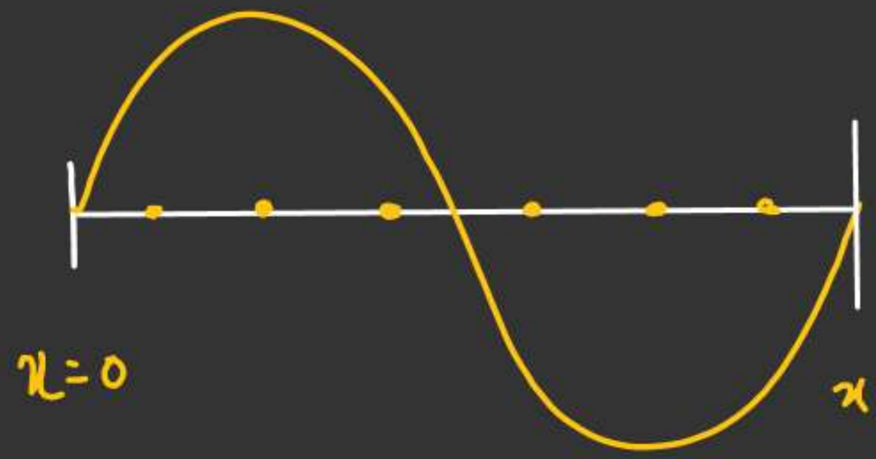
$$\frac{\partial y}{\partial t} = -A\omega\cos(kx - \omega t)$$

v_p

$$K.E = \frac{1}{2}\rho\left(\frac{\partial y}{\partial t}\right)^2$$

$$K.E = \frac{1}{2}\rho A^2\omega^2\cos^2(kx - \omega t)$$

Avg b/w $x = 0$ to $x = \lambda$ at $t = 0$.



$$K.E_{avg} = \frac{1}{2}\rho\omega^2 A^2 \frac{\int_0^{\lambda}\cos^2 kx\cdot dx}{\int_0^{\lambda} dx}$$

$\frac{\lambda}{2}$

$$\boxed{K.E_{avg} \text{ per unit volume} = \frac{1}{4}\rho\omega^2 A^2}$$

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$$\boxed{\begin{array}{l} P.E_{avg} \text{ per unit volume} = \dfrac{1}{4}\rho\omega^2 A^2 \\ (x=0 \text{ to } x=\lambda) \end{array}}$$

$$\boxed{(E_T)_{avg} \text{ per Unit volume} = \frac{1}{2}\rho\omega^2 A^2}$$

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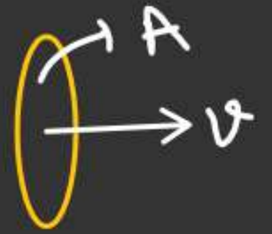
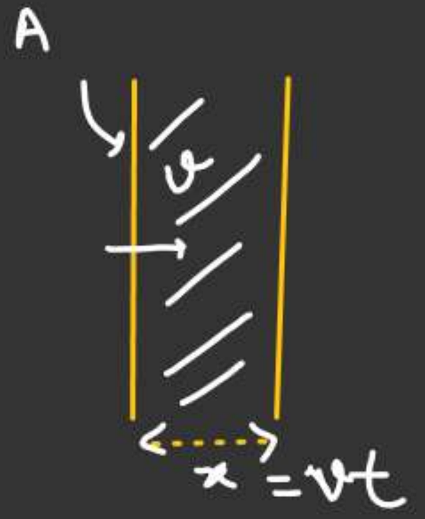
$A \to S_0$ For longitudinal

$$\text{Intensity} = \frac{\text{Avg. Energy}}{\text{Area} \times \text{Time}}$$

(Area always perpendicular to the direction of propagation)



$$= \left(\frac{\text{Avg. Energy}}{\text{Volume}}\right) \times \frac{\text{Volume}}{\text{Area} \times \text{time}}$$



$$= \frac{1}{2}\rho \omega^2 S_0^2 \times \frac{\cancel{\text{Volume}}}{\cancel{(\text{Area} \times \text{Distance})}}$$

speed

$\downarrow$

v

$A \times x = \text{Volume}$

$x \downarrow$ Distance

$$\boxed{I = \frac{1}{2}\rho \omega^2 S_0^2 v}$$

S_0 → Amplitude; v → Velocity of wave

$I \propto S_0^2$

$I \propto f^2$ $\quad \omega = 2\pi f$

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STANDING WAVE

[When two wave pulse of equal amplitude travelling in opposite direction interfere to form standing wave.

Important points

- Energy of wave confined b/w two points.
- Points which are always at rest position are called Nodes.
- Points which are at their maximum amplitude are called antiNodes
- Distance b/w any two consecutive nodes or antinodes is $\frac{\lambda}{2}$.
- Particle b/w two Nodes always oscillate in same phase.
- In standing wave amplitude of particle vary w.r.t distance but amplitude of travelling wave is same.

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$y_1 = A\sin(kx-\omega t)$ $y_2 = A\sin(kx+\omega t)$

$$y_1 = A\sin(kx-\omega t)$$

$$y_2 = A\sin(kx+\omega t)$$

By Superposition

$$y_R = y_1 + y_2$$

$$y_R = A\left[\sin(kx-\omega t) + \sin(kx+\omega t)\right]$$

$$y_R = A\left[2\sin\frac{(kx-\omega t)+(kx+\omega t)}{2} \cdot \cos\frac{(kx-\omega t)-(kx+\omega t)}{2}\right]$$

$$\boxed{y_R = 2A\sin kx\cos\omega t}$$

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$$\boxed{y = \underbrace{2A\sin kx}\cos\omega t}$$

$\Downarrow$

[Amplitude of standing wave]

Nodes $\rightarrow$ Amplitude $= 0$

$2A\sin kx = 0$

$\sin kx = 0$

$kx = n\pi$

$\downarrow$

$\frac{2\cancel{\pi}}{\lambda}\cdot x = n\cancel{\pi}$

$\left(x = \frac{n\lambda}{2}\right)$

$n = 0, 1, 2, 3, 4, 5 \ldots$

AntiNodes :-

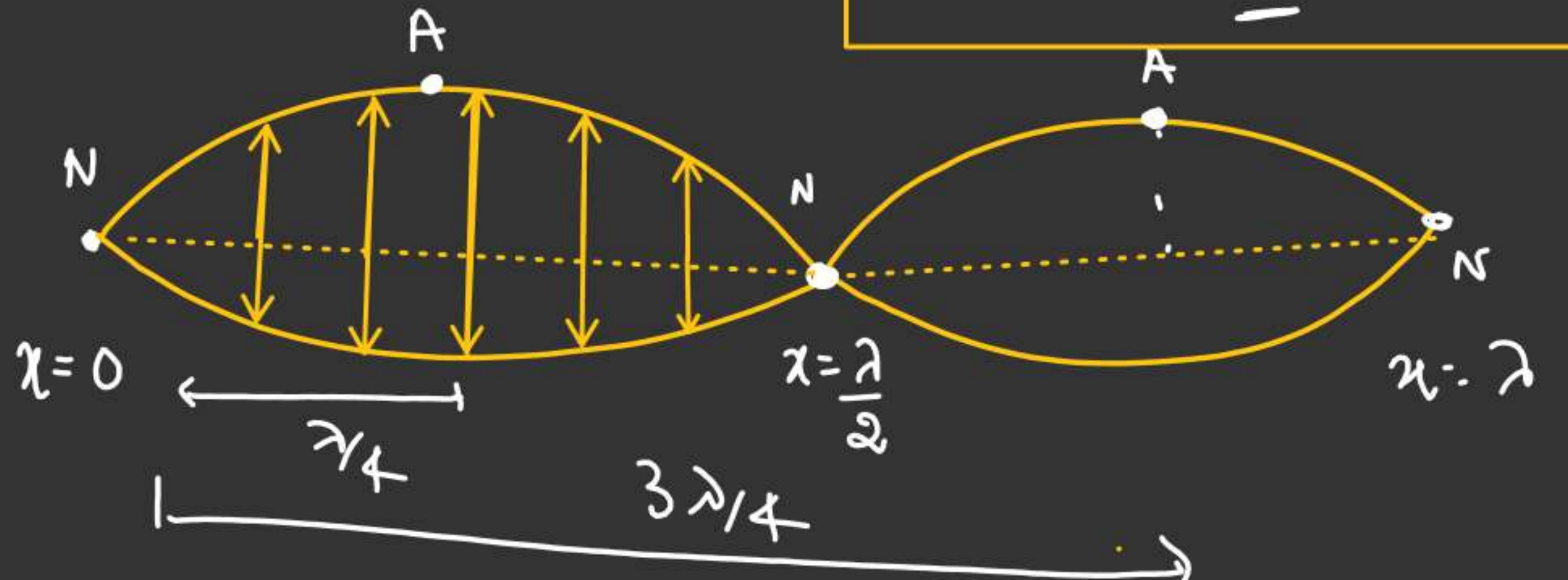
Amplitudes maximum.

$2A\sin kx \rightarrow$ Maximum.

$\sin kx = \pm 1$ $\quad n = 1, 2, 3 \ldots$ $\quad n = 0, 1, 2, 3.$

$kx = (2n-1)\frac{\pi}{2}$ or $(2n+1)\frac{\pi}{2}$.

$\frac{2\pi}{\lambda}x = (2n-1)\frac{\pi}{2} \Rightarrow \boxed{x = (2n-1)\frac{\lambda}{4} \text{ or } (2n+1)\frac{\lambda}{4}}$



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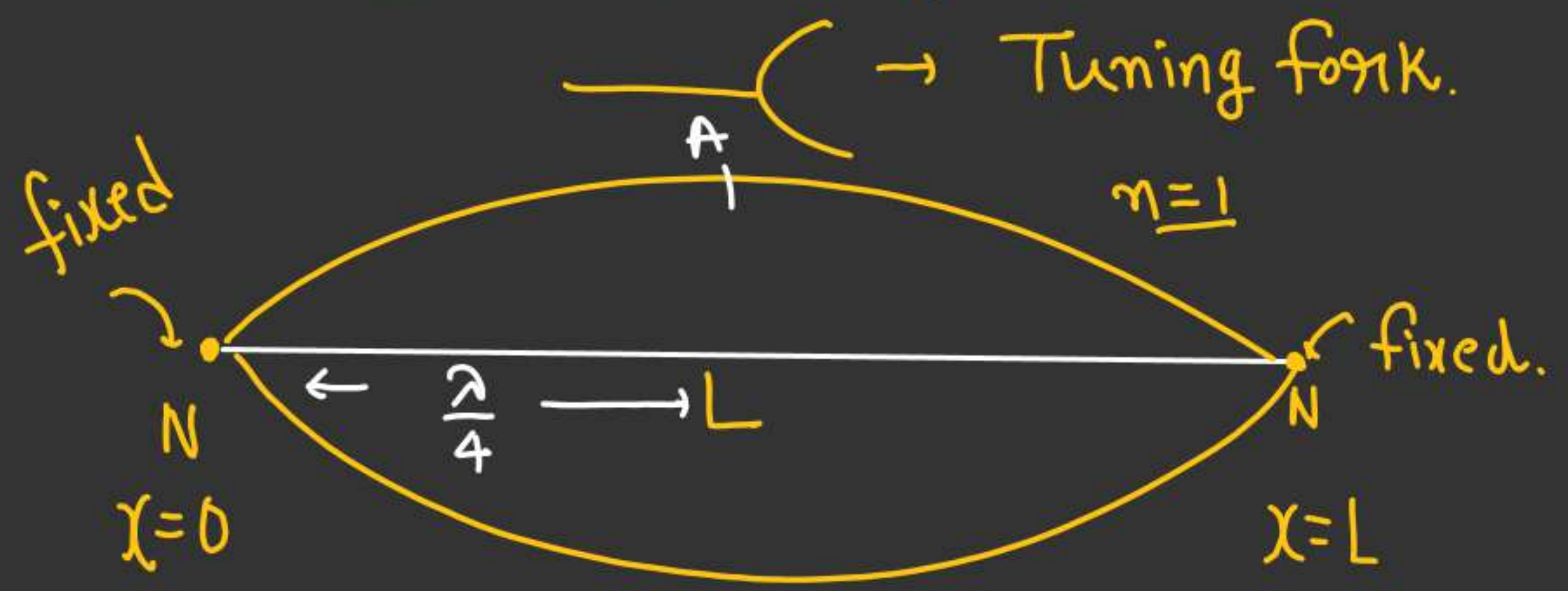
STANDING WAVE FORMATION IN STRING

Case :- 1 (String fixed at both end)



$x = \frac{n\lambda}{2}$

$x = 0,\ x = L$ Points of node

$L = \frac{n\lambda}{2} = \frac{n}{2}\frac{v}{f}$

$$\boxed{f = \frac{nv}{2L}}$$

At the resonating Condition

$$\left[f_{\text{tuning fork}} = f_{\text{wire}} \right]$$

For n = 1

$L = \frac{\lambda}{2}$

$\left(v = \frac{\lambda}{T} = \lambda f \right)$

$\lambda = \frac{v}{f}$

$L = \frac{v}{2f_0}$ $\quad f \to f_0,\ n = 1$

$$\boxed{f_0 = \frac{v}{2L}}$$

↓

Fundamental frequence or 1st harmonic

$\left(v = \sqrt{\frac{T}{\mu}} \right)$