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Continuity

$= \boxed{\frac{5\pi}{2} - \frac{5}{2}\left(\frac{3\pi}{4}\right) - \frac{5}{6}\tan^{-1}3}$ $\pi - \frac{\pi}{4}$

$\frac{5\pi}{2} - \frac{5}{2}\tan^{-1}2 - \frac{10}{3}\tan^{-1}3 = \frac{5\pi}{2} - \frac{5}{2}\left(\underbrace{\tan^{-1}2 + \tan^{-1}3}_{\geq 1}\right) - \frac{5}{6}\tan^{-1}3$

$1 \leq (\sin\theta + 1)^2 + 1 \leq 5$

$\underbrace{\sin^2\theta + 2\sin\theta + 2}_{\geq 1} = 4 \underbrace{\sec^2\phi + 1}_{\geq 5}$

$\boxed{\sin\theta = 1 \ \& \ \sec^2\phi = 1}$

$|B - A| = \left|2\cot^{-1}2 + 3\cot^{-1}3 - \frac{1}{2}\cot^{-1}\frac{1}{2} - \frac{1}{3}\cot^{-1}\frac{1}{3}\right|$

$\left|\frac{5\pi}{2} - 3\tan^{-1}3 - \frac{1}{2}\tan^{-1}2 - \frac{1}{3}\tan^{-1}3 - 2\tan^{-1}2\right|$

$\frac{5}{2} - \frac{10}{3}$

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x, y, z

$27 - z > z$

$z < \frac{27}{2}$

$\boxed{\leq x, y, z \leq 13}$

$x + y + z = 27$

$$\boxed{{}^{27-3+2}C_2 - {}^{3}C_1 \; {}^{27-16+2}C_2}$$

$A \to x > 13$

$B \to z$

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Continuity

Continuity of function at point $x=a$

If we can draw the graph of function $y=f(x)$ $\forall\, x \in (a-h, a+h)$ without raising pen (h is infinitesimally small)

Then $f(x)$ is said to be cont. at $x=a$.

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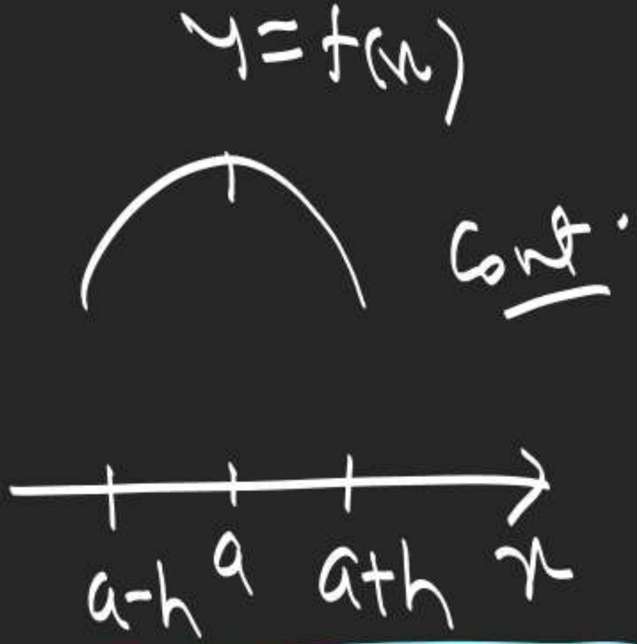
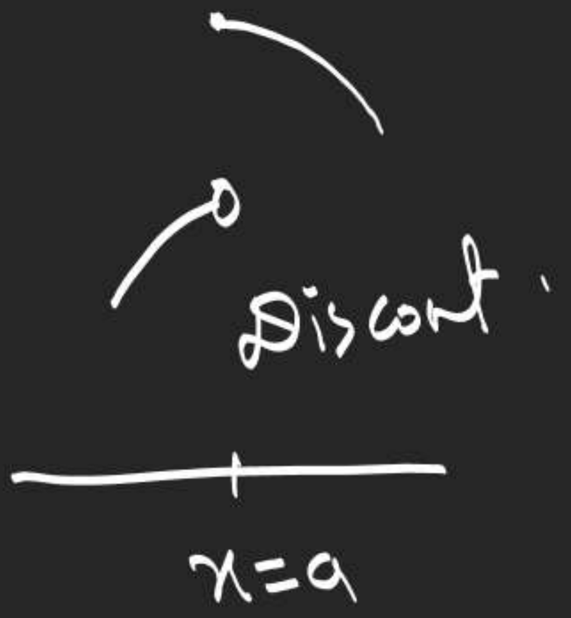
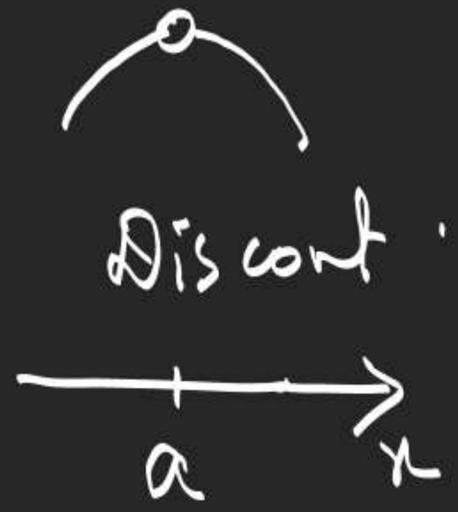
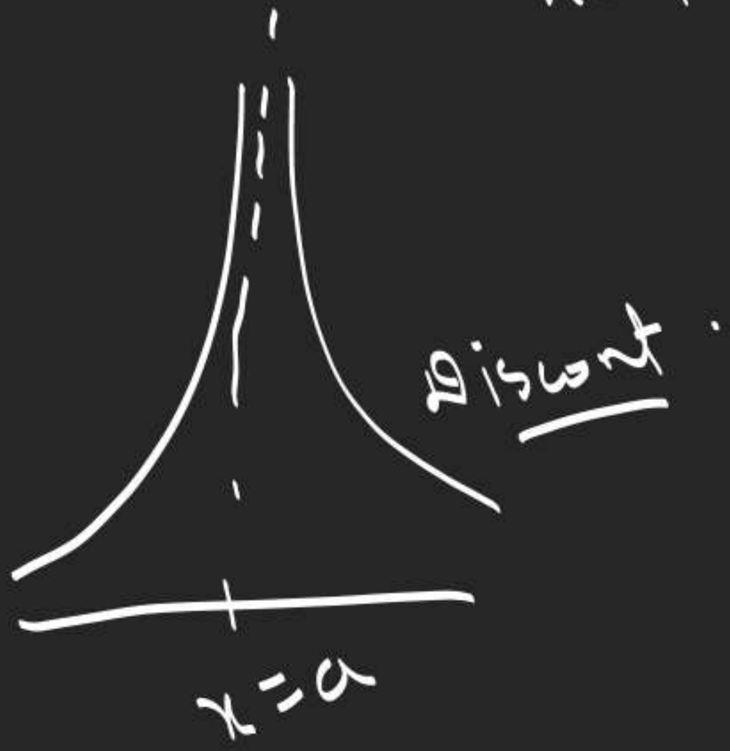
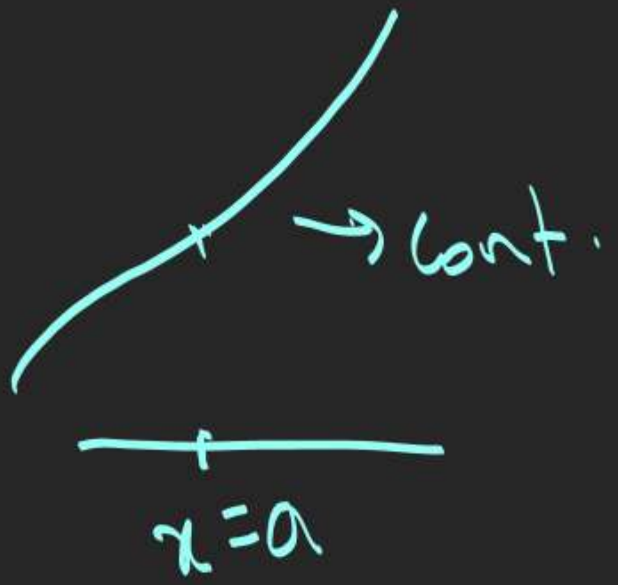






$$\boxed{\begin{array}{l} LHL = RHL = f(a) \\ \Rightarrow f(x) \text{ is continuous at } \\ x = a \end{array}}$$





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Continuity of function in $[a, b]$

- $\alpha \in (a, b)$, LHL = RHL = $f(\alpha)$
- $x = a$, RHL = $f(a)$
- $x = b$, LHL = $f(b)$

Cont. of $f(x)$ in (a, b)

- $\alpha \in (a, b)$

LHL = RHL = $f(\alpha)$

Cont. of $f(x)$ in $[a, b]$

- $\alpha \in (a, b)$, LHL = RHL = $f(\alpha)$
- $x = a$, RHL = $f(a)$

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Reasons of Discontinuity at $x = a$

- $f(a)$ is not defined.
- $\lim_{x \to a} f(x)$ not exist.
- $\lim_{x \to a} f(x)$ exists $\neq f(a)$

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$f(x) = \text{sgn}\{x\}$ at $x = 2$.

$\{\cdot\} = \text{FPF}$.

$$\boxed{\begin{gathered} LHL = 1 = RHL \\ f(2) = 0 \end{gathered}}$$

f is discont. at $x = 2$.

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Types of Discontinuity

- **Removable type** (If $\lim_{x \to a} f(x)$ exists)
  - Missing point type ($f(a)$ not defined)
  - Isolated point type ($\lim_{x \to a} f(x) \neq f(a)$)
- **Non removable type** (If $\lim_{x \to a} f(x)$ not exists)
  - Finite type (LHL & RHL both finite)
  - Infinite type: atleast one of LHL or RHL $\to \pm\infty$
  - Oscillatory type

Jump of function at $x = a$ $= |LHL - RHL|$

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① $f(x) = \frac{\sin x}{x}$ at $x=0$

removable, missing point.

$\lim_{x\to 0} \frac{\sin x}{x} = 1$

$\boxed{\text{Jump} = \pi}$

② $f(x) = \tan^{-1}\frac{1}{x}$ at $x=0$.

$LHL = -\frac{\pi}{2}$

$RHL = \frac{\pi}{2}$

$\boxed{\text{non removable, finite type}}$

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1 Limits: 6 - 17 (Ex-II)

Paper - 2 → 2

$f(x) = 2^{\tan x}$ at $x = \frac{\pi}{2}$

LHL = ∞

RHL = 0

Non removable, infinite type

$$f(x) = \left[\underbrace{1 + \frac{1}{3}\sin(\ln \underbrace{|x|}_{0^+})}_{\left[1-\frac{1}{3},\, 1+\frac{1}{3}\right]} \right] \quad \text{at } x = 0$$

$-\infty$

$[\cdot] = G \cdot I \cdot F$

↓

non removable, oscillatory type

$\{0, 1\}$