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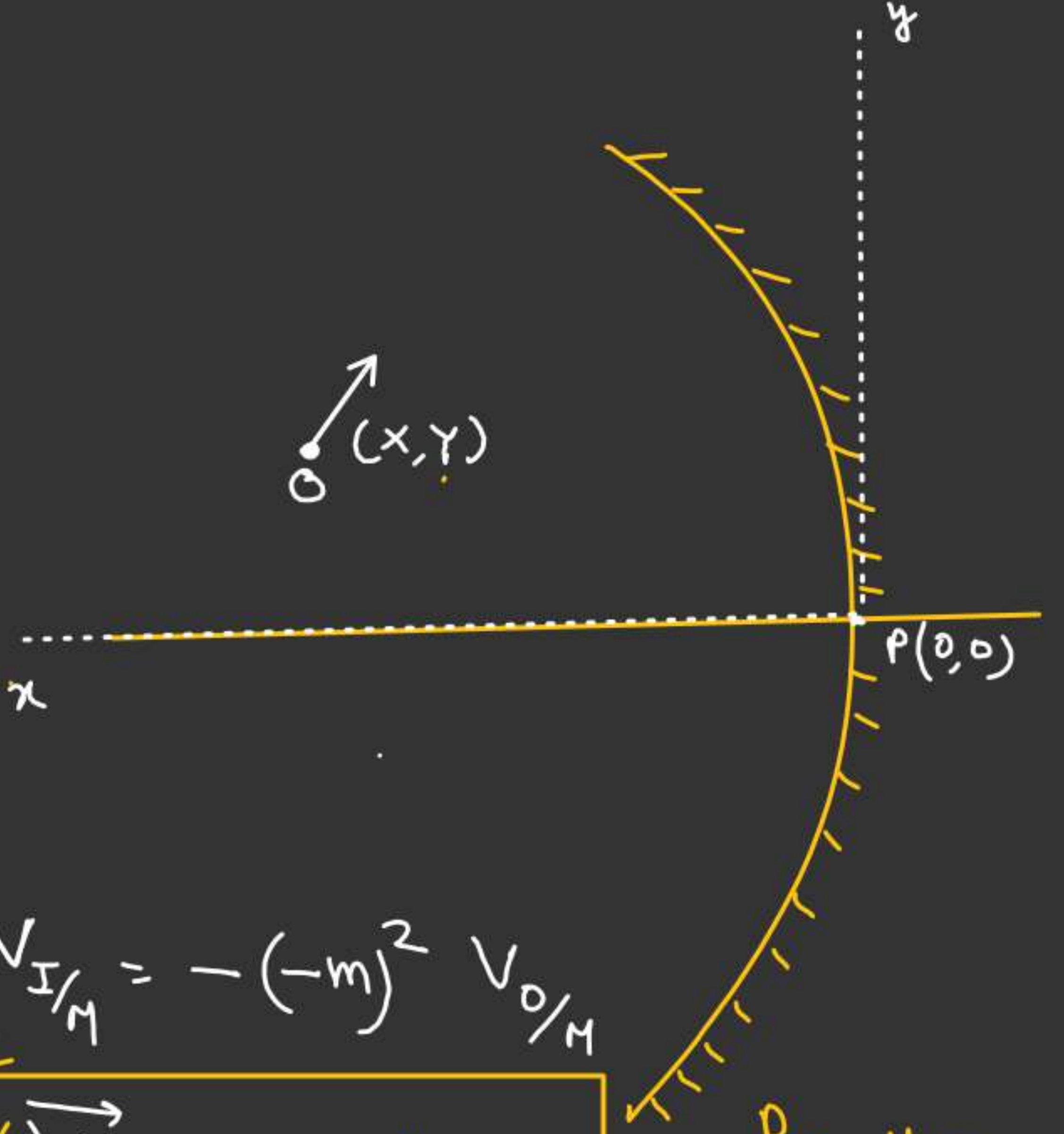
Velocity of Image in Case of Spherical Mirror

$X_{O/M} = X$ = x-Cordinate of object w.r.t Mirror.

$Y_{O/M} = Y$ = y-Co-ordinate of object w.r.t Mirror.



By Mirror formula. $u = X_{O/M}$

$v = X_{I/M}$

$$\frac{1}{V} + \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{X_{I/M}} + \frac{1}{X_{O/M}} = \frac{1}{f}$$

Differentiating w.r.t time.

$$\frac{-1}{(X_{I/M})^2} \cdot \frac{d}{dt}(X_{I/M}) - \frac{1}{(X_{O/M})^2} \frac{d}{dt}(X_{O/M}) = \frac{d}{dt}\left(\frac{1}{f}\right)$$

$$\frac{d}{dt}(X_{I/M}) = -\left(\frac{X_{I/M}}{X_{O/M}}\right)^2 \cdot \frac{d}{dt}(X_{O/M})$$

$$V_{I/M} = -(-m)^2 \, V_{O/M}$$

$$\boxed{(\vec{V}_{I/M})_x = -m^2 (\vec{V}_{O/M})_x}$$

→ Parallel to principal axis.

Velocity of image w.r.t Mirror.

Velocity of Object w.r.t Mirror.

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$$m = \left(-\frac{v}{u}\right)$$

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{v} = \frac{1}{f} - \frac{1}{u}$$

$$v = \left(\frac{uf}{u-f}\right)$$

$$m = \frac{-(\cancel{u}f)}{\cancel{u}(u-f)}$$

$$\boxed{m = \frac{f}{f-u}}$$

$$m = \frac{Y_{I/M}}{Y_{O/M}} = \frac{f}{(f - X_{O/M})}$$

$$\boxed{Y_{I/M} = \frac{f\,Y_{O/M}}{(f - X_{O/M})}}$$

Differentiating both side w.r.t time.

$$\frac{d}{dt}(Y_{I/M}) = f\left[Y_{O/M}\frac{d}{dt}\left(\frac{1}{f - X_{O/M}}\right) + \left(\frac{1}{f - X_{O/M}}\right)\frac{d}{dt}(Y_{O/M})\right]$$

$$(V_{I/M})_y = f\left[Y_{O/M}\left[\frac{-1}{(f - X_{O/M})^2}\right]\left[-\frac{d}{dt}(X_{O/M})\right] + \left(\frac{1}{f - X_{O/M}}\right)\frac{d}{dt}(Y_{O/M})\right]$$

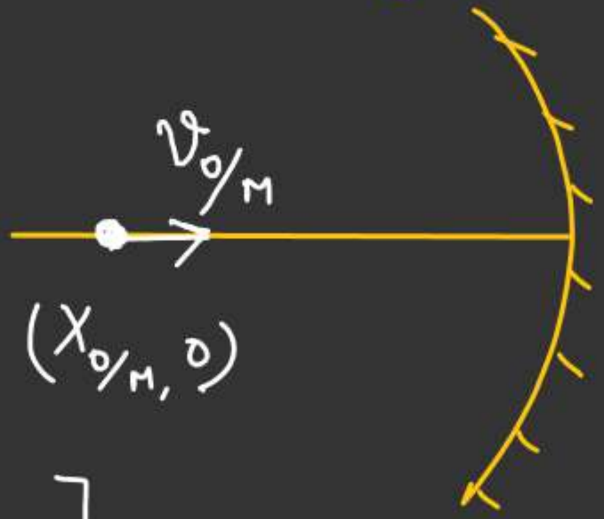
$$\boxed{(V_{I/M})_y = \frac{f\,Y_{O/M}}{(f - X_{O/M})^2}(V_{O/M})_x + \left(\frac{f}{f - X_{O/M}}\right)(V_{O/M})_y}$$

$$\boxed{(V_{I/M})_y = \frac{f\, Y_{o/M}}{(f - X_{o/M})^2}(V_{o/M})_x + \overbrace{\left(\frac{f}{f - X_{o/M}}\right)}^{\Rightarrow m}(V_{o/M})_y}$$

$$\boxed{(V_{I/M})_x = -m^2 (V_{o/M})_x}$$

Case-1

If object is on the principal axis and moving along the principal axis.



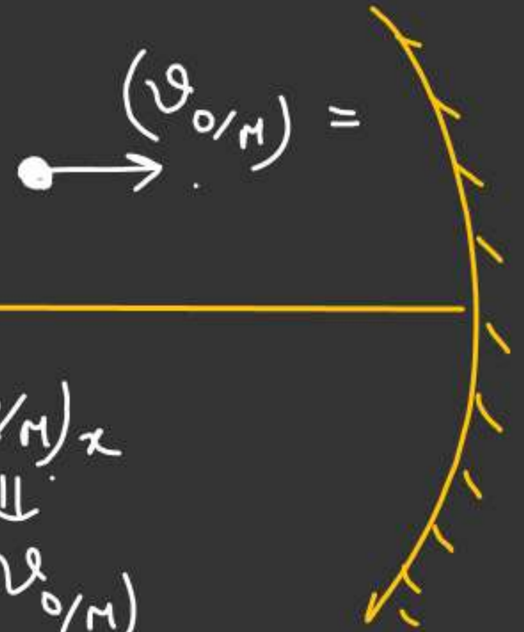
$$\left.\begin{aligned} Y_{o/M} &= 0 \\ (V_{o/M})_y &= 0 \end{aligned}\right] \Rightarrow (V_{I/M})_y = 0$$

$$\boxed{(V_{I/M})_x = -m^2 (\vec{V}_{o/M})_x}$$

Case-2

object velocity parallel to principal axis.

$(v_{o/M})_y = 0$



$$(\vec{v}_{I/M})_x = -m^2 \underbrace{(V_{o/M})_x}_{\Downarrow\ (v_{o/M})}$$

$$(\vec{v}_{I/M})_y = \left[\frac{f\, Y_{o/M}}{(f - X_{o/M})^2}\right](\vec{V}_{o/M})_x$$

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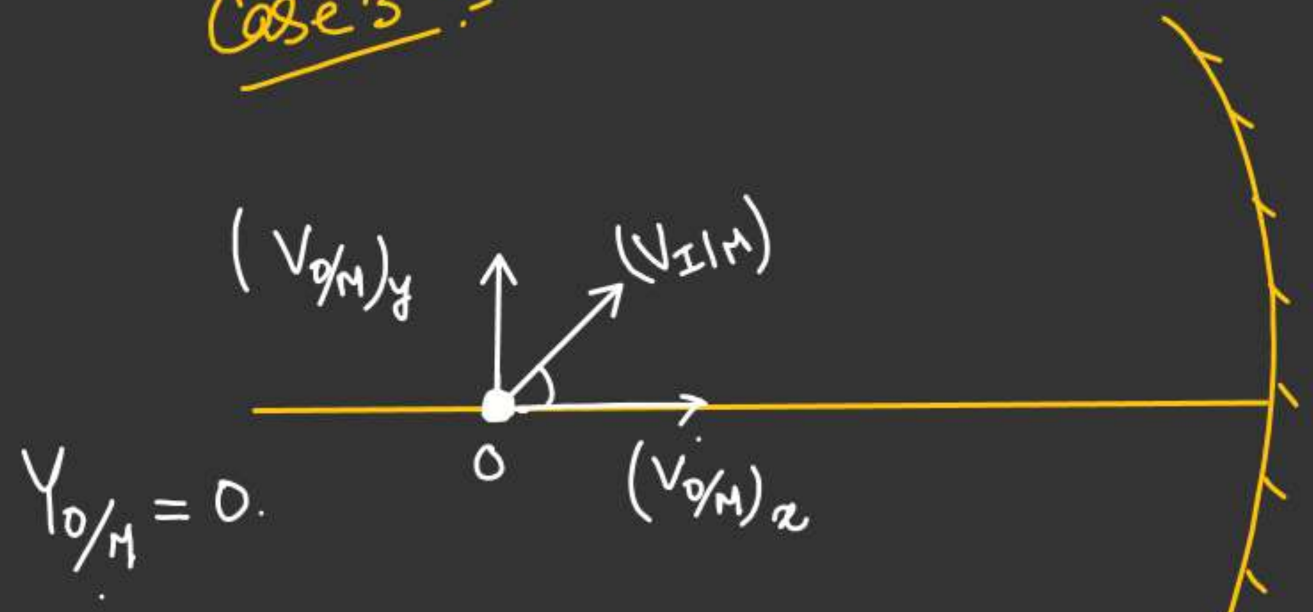
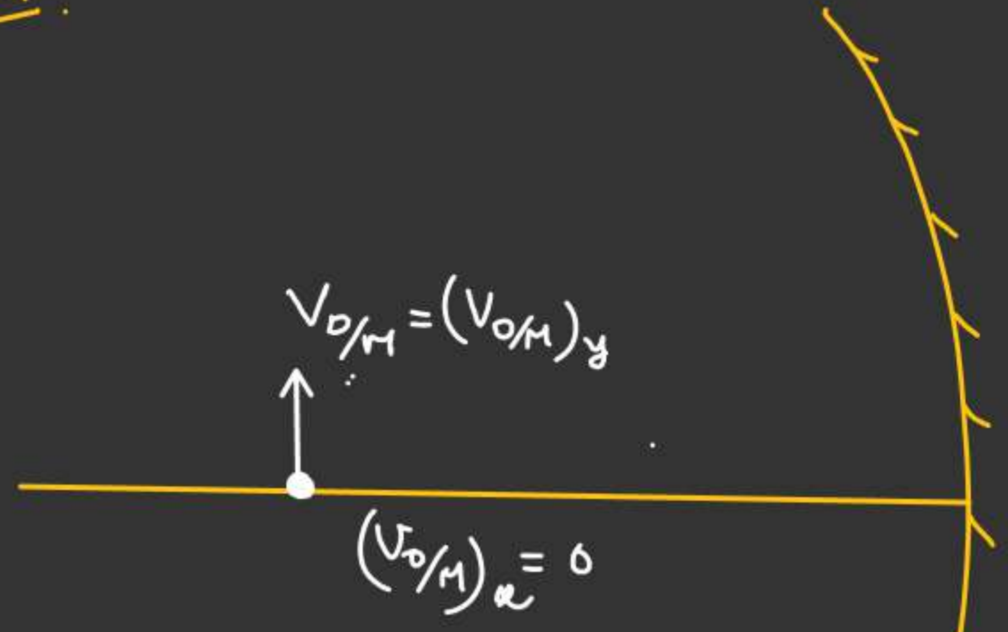
Case 3 :-



$$\boxed{\begin{aligned} (\vec{V}_{I/M})_x &= -m^2 (\vec{V}_{O/M})_x \\ (\vec{V}_{I/M})_y &= \left(\frac{f}{f - X_{O/M}}\right)(\vec{V}_{O/M})_y \end{aligned}}$$

Case-4.



$$\boxed{(\vec{V}_{I/M})_y = \left(\frac{f}{f - X_{O/M}}\right)(\vec{V}_{O/M})_y}$$

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Find velocity of image at that instant.

$$\vec{V}_{O/E} = 15\cos 53^\circ \hat{i} + 15\sin 53^\circ \hat{j}$$

$$= 15\times\left(\frac{3}{5}\right)\hat{i} + 15\left(\frac{4}{5}\right)\hat{j}$$

$$= 9\hat{i} + 12\hat{j}$$

$$\vec{V}_{M/E} = -2\hat{i}$$

$$\vec{V}_{O/M} = \vec{V}_{O/E} - \vec{V}_{M/E}$$

$$= 9\hat{i} + 12\hat{j} - (-2\hat{i})$$

$$= 11\hat{i} + 12\hat{j}$$

$$(\vec{V}_{O/M})_x = 11\hat{i}$$

$$(\vec{V}_{I/M})_x = -m^2(\vec{V}_{O/M})_x$$

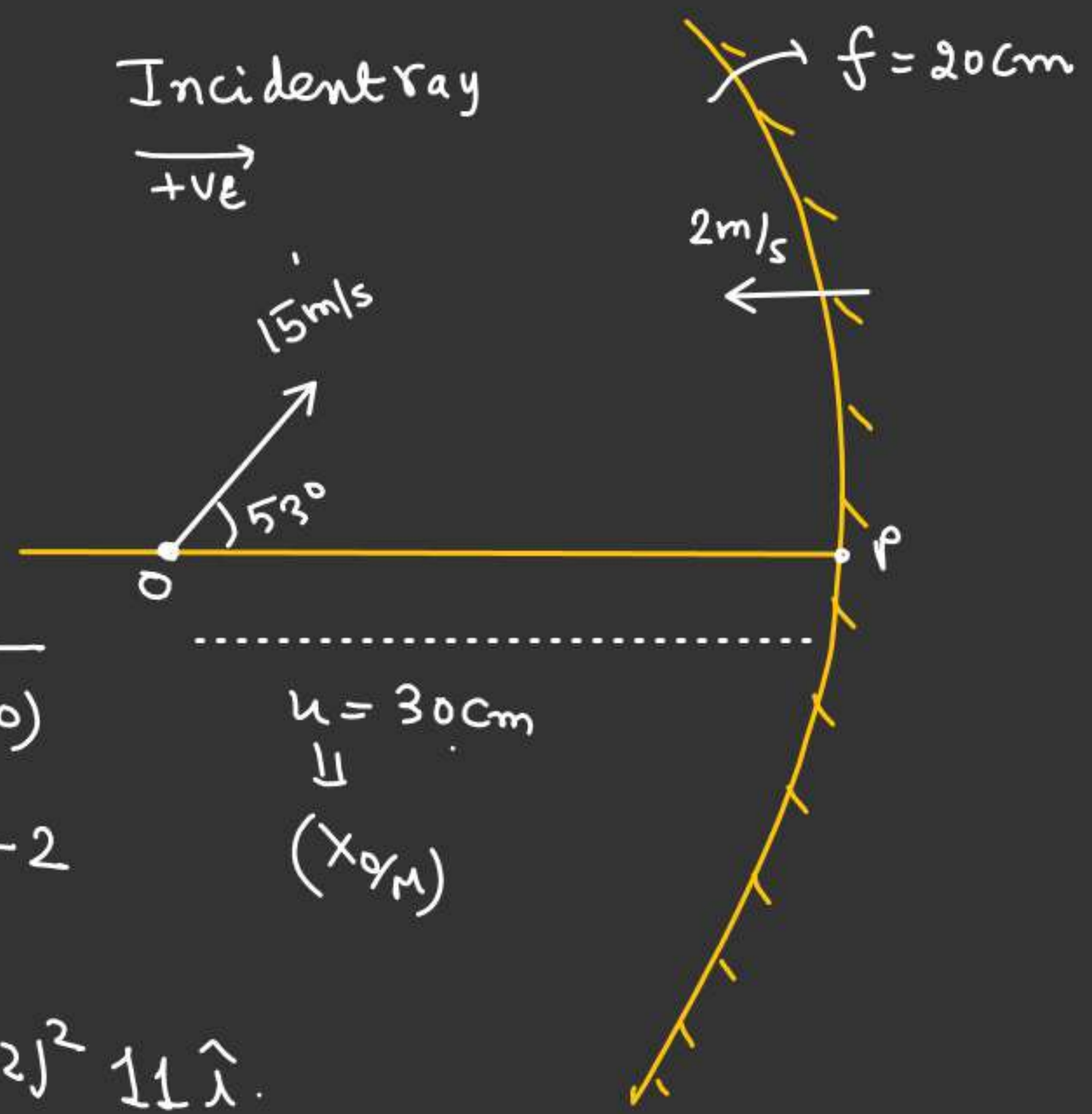
$$m = \left(\frac{f}{f-u}\right)$$

$$m = \frac{(-20)}{(-20)-(-30)}$$

$$m = \frac{-20}{10} = -2$$

$$(\vec{V}_{I/M})_x = -(-2)^2\, 11\hat{i}$$

$$= \underline{-44\hat{i}}$$



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$(V_{I/M})_y = ??$

$(\vec{V}_{O/M})_y = 12\hat{j}$

$(Y_{O/M})_y = 0$ ✓ $\Rightarrow m.$

$$(\vec{V}_{I/M})_y = \left(\frac{f}{f - x_{O/M}}\right)(\vec{V}_{O/M})_y.$$

$$= \left[\frac{-20}{-20-(-30)}\right](12\hat{j})$$

$$= -24\hat{j}$$

$$(\vec{V}_{I/M}) = (\vec{V}_{I/M})_x + (\vec{V}_{I/M})_y.$$

$$= -44\hat{i} - 24\hat{j}$$

$$\vec{V}_{I/E} = \vec{V}_{I/M} + \vec{V}_{M/E}$$

$$= -44\hat{i} - 24\hat{j} - 2\hat{i}$$

$$= -46\hat{i} - 24\hat{j}.$$

⟵ Ans

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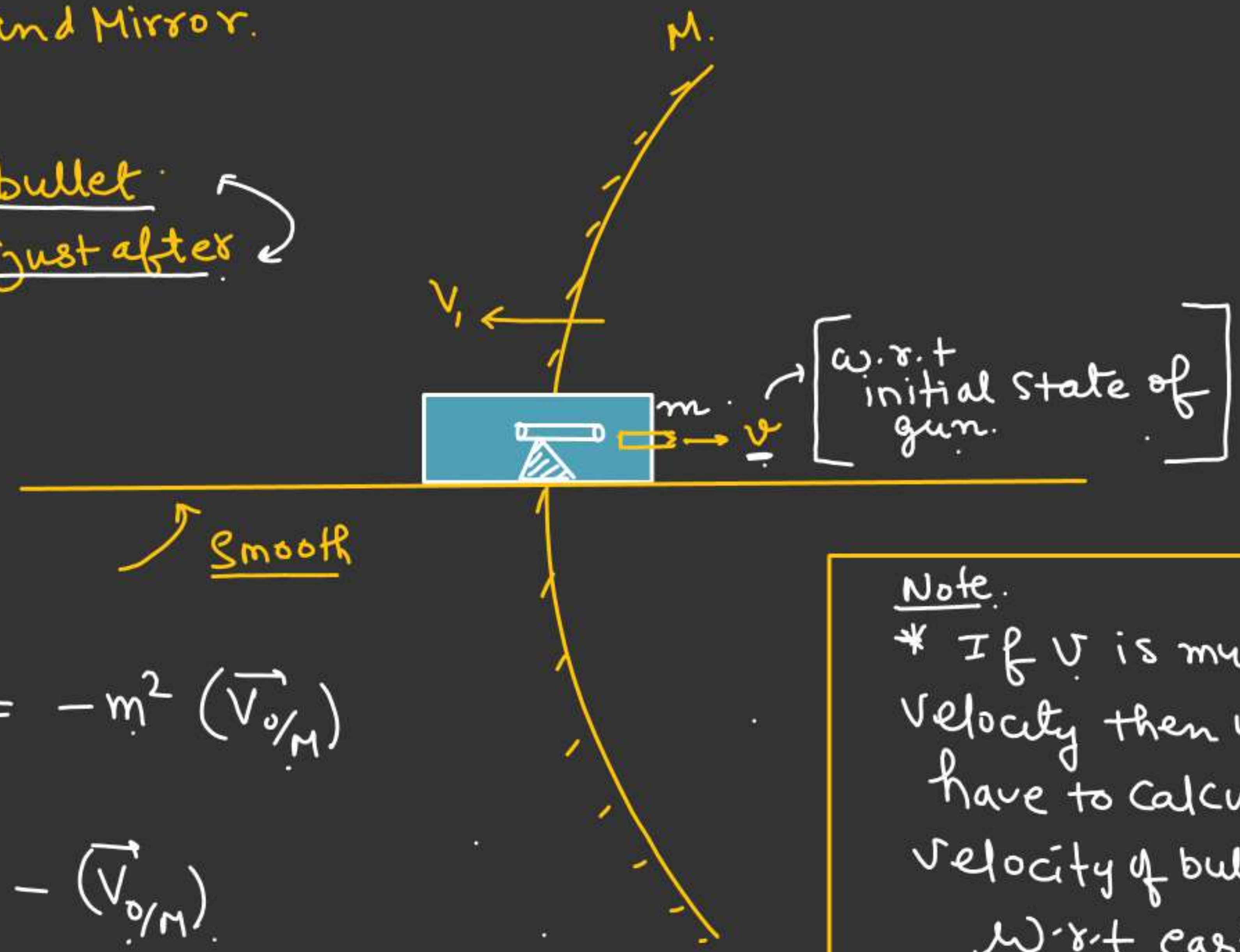
Ans: M = Mass of block, gun and Mirror.

m = Mass of bullet.

Find the speed of seperation of bullet with respect to its image just after firing.

V = velocity of bullet.



L.M.C in x-direction.

$$0 = mv - MV_1$$

$$V_1 = \left(\frac{mv}{M}\right) \checkmark$$

$$m = \left(\frac{f}{f-u}\right)$$

$$(u = 0, \quad m = 1)$$

$$\vec{V}_{I/M} = -m^2\left(\vec{V}_{O/M}\right)$$

$$\vec{V}_{I/M} = -\left(\vec{V}_{O/M}\right)$$

$$\vec{V}_{O/M} = \vec{V}_{O/E} - \vec{V}_{M/E}$$

$$= v\hat{i} - (-V_1)\hat{i}$$

$$= (v + V_1)\hat{i} = \left(v + \frac{mv}{M}\right)\hat{i} = \left(1 + \frac{m}{M}\right)v\hat{i} \checkmark$$

Note.

* If v is muzzle velocity then we have to calculate velocity of bullet w.r.t earth

$$\left(\vec{v}_{I/M}\right) = -\left(1+\frac{m}{M}\right) v\,\hat{i}$$

$$\left(\vec{v}_{bullet/M}\right) = \left(\vec{V}_{bullet/E} - \vec{V}_{Mirror/E}\right)$$

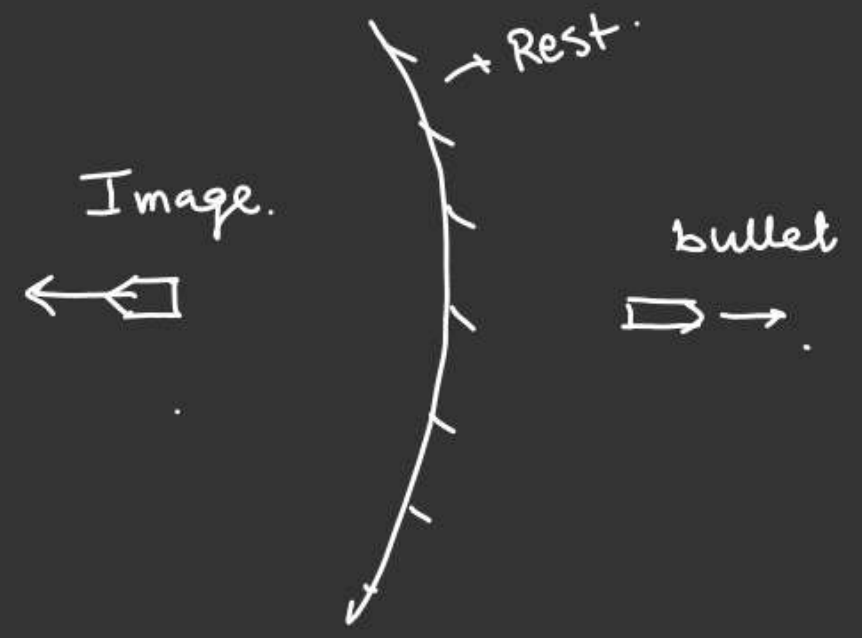
$$\Downarrow$$

$$\left(\vec{v}_{O/M}\right) = \left[v\hat{i} - (-V_1)\hat{i}\right]$$

$$= (v+V_1)\hat{i}$$

$$= \left(v + \frac{m}{M}v\right)\hat{i}$$

$$= v\left(1+\frac{m}{M}\right)\hat{i}$$



Relative speed of bullet w.r.t its image just after firing $= \underline{2v\left(1+\frac{m}{M}\right)}$ Ans

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<u>Case-2</u> if velocity of bullet is muzzle velocity.

muzzle velocity → (w.r.t gun)

(Relative velocity)

$$\vec{v}_{bullet/E} = \vec{v}_{bullet/gun} + \vec{v}_{gun/E}$$

$$= v\hat{i} - V_1\hat{i}$$

$$= (v - V_1)\hat{i}$$

<u>L.M.C</u>

$$p_i = p_f$$

$$0 = -MV_1 + m(v - V_1)$$

$$MV_1 + mV_1 = mv$$

$$V_1 = \left(\frac{mv}{M+m}\right)$$

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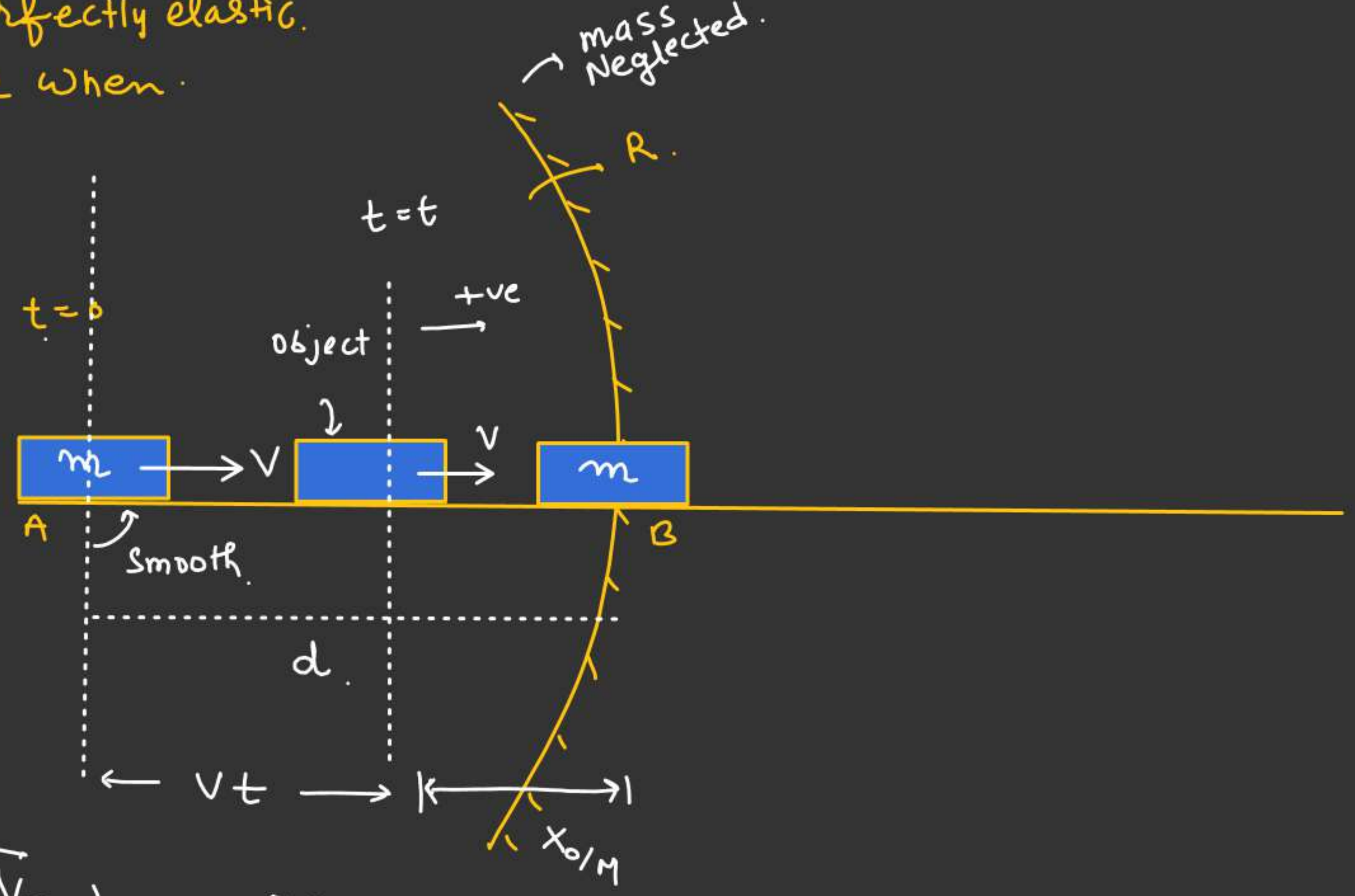
Collision b/w A & B perfectly elastic.
Find the velocity of image when.

a) $t < \frac{d}{v}$ b) $t > \frac{d}{v}$

Situation shown in fig at $t = 0$.



$$u = x_{O/M} = (d - vt)$$

$$V_{O/M} = V$$

$$\vec{V}_{I/M} = -m^2 (\vec{V}_{O/M})$$

$$m = \frac{f}{f-u}$$

$$m = \left[\frac{-R/2}{-\frac{R}{2} - [-(d-vt)]}\right]$$

$$m = \frac{-R/2}{-\frac{R}{2} + (d-vt)} = \left[\frac{R}{R - 2(d-vt)}\right]$$

$$(\vec{V}_{I/M}) = -\left[\frac{R}{R-2(d-vt)}\right]^2 V\hat{i}$$

$$|\vec{V}_{I/M}| = \frac{VR^2}{[R-2(d-vt)]^2} = \frac{VR^2}{[2(d-vt)-R]^2}$$

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$$\left(t > \frac{d}{V}\right)$$

$$t_1 + \frac{d}{V} = t$$

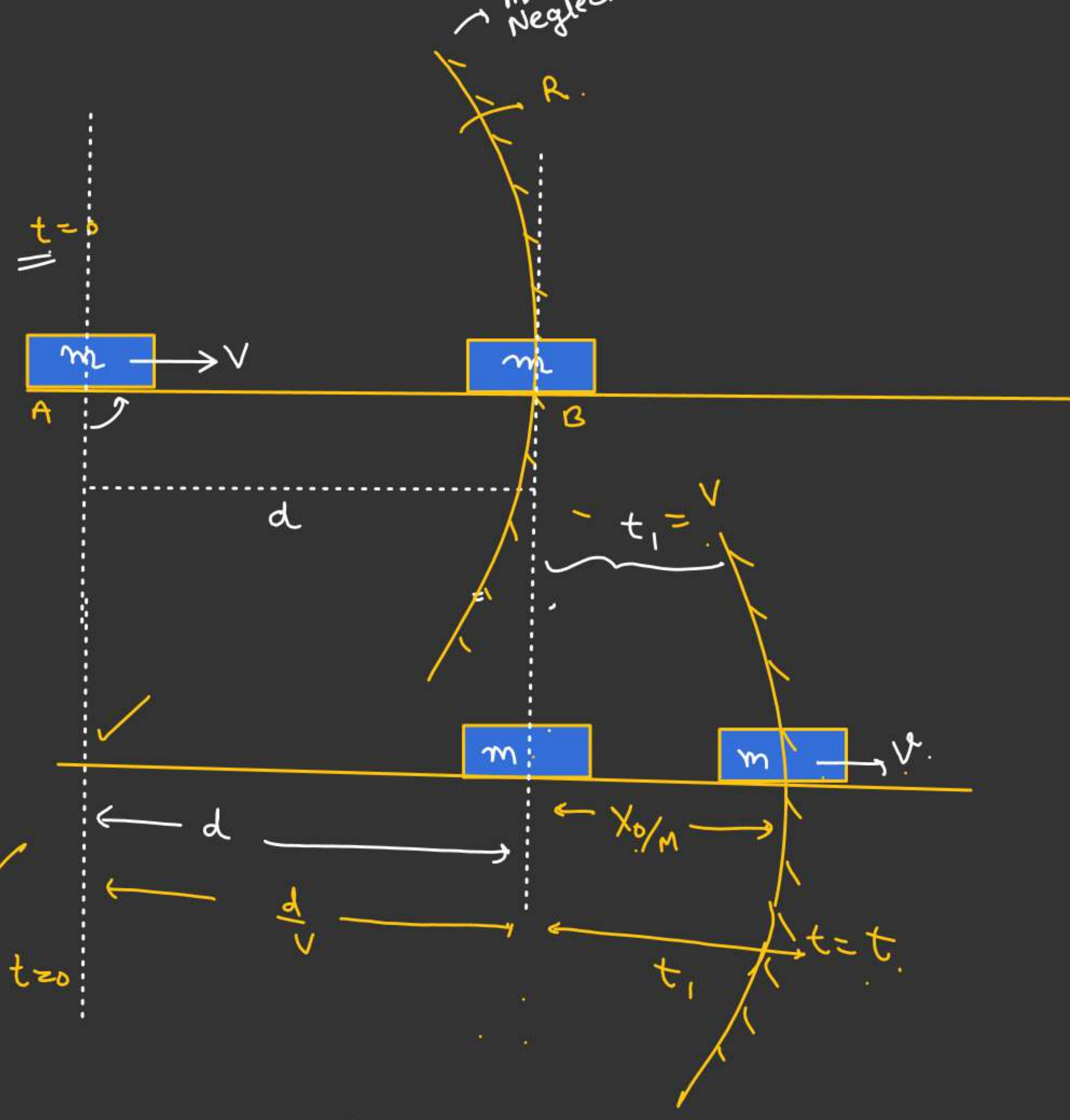
$$t_1 = \left(t - \frac{d}{v}\right)$$

$$X_{o/M} = V t_1$$

$$X_{o/M} = V\left(t - \frac{d}{V}\right)$$

$$m = \frac{-R/2}{-R/2 - (-X_{o/M})}$$

$$= \left(\frac{-R/2}{X_{o/M} - R/2}\right) = \left[\frac{-R/2}{V(t - d/v) - R/2}\right]$$



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$$\left(\vec{V}_{I/M}\right) = -\underline{m}^2\left(\vec{V}_{o/M}\right)$$

$$m = \frac{-R/2}{-R/2 + v(t - d/v)}$$

$$V_{o/M} = -v\hat{i}$$

$$m = \left[\frac{-R}{2(vt-d)-R}\right]$$

$$\vec{V}_{I/E} - \vec{V}_{M/E} = -m^2(-v\hat{i})$$

$$\vec{V}_{I/E} = \vec{V}_{M/E} + m^2 v\hat{i}$$

$$= v\hat{i} + m^2 v\hat{i}$$

$$= v(1+m^2)\hat{i}$$

$$\vec{V}_{I/E} = v\left(1 + \frac{R^2}{[2(vt-d)-R]^2}\right)\hat{i}$$

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H.W (Reflection)

Module

Ex 1 - ① → Q - 19.

Ex - 2 → 12, 13, 19.

Ex - 3 → 9, 10, 11,

Ex - 4 → 27,

Ex - 5 → 1, 2, 3, 7, 8, 14, 15, 19, 20.