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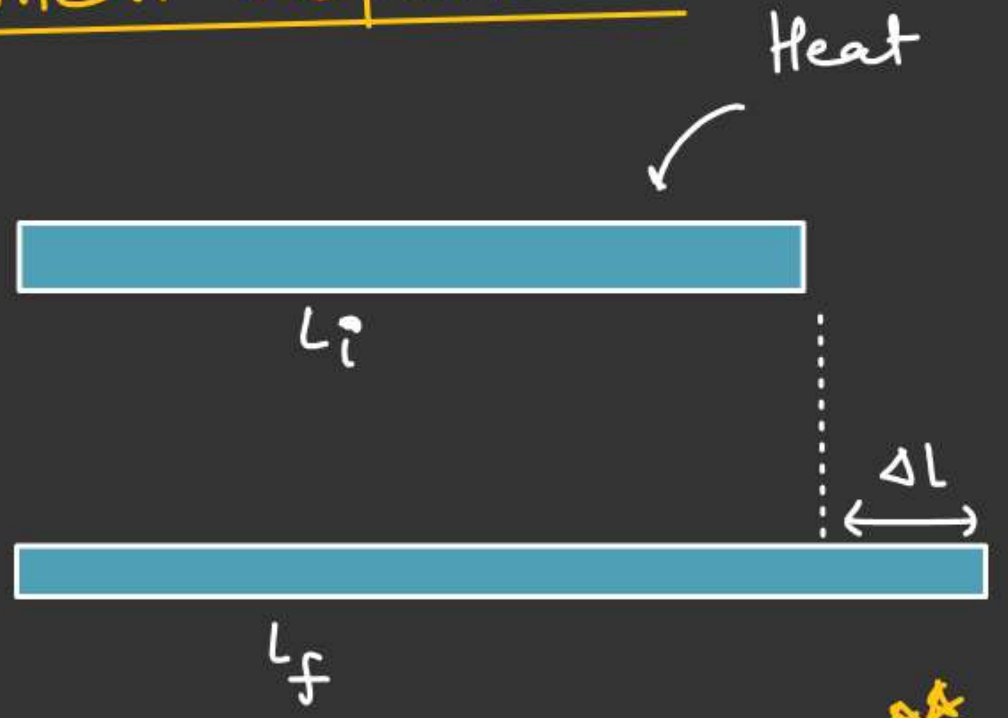
Heat & Thermodynamics

- → Thermal Expansion.
- → Calorimetry
- → Heat transfer
- → Thermodynamics

Thermal Expansion

Linear Expansion



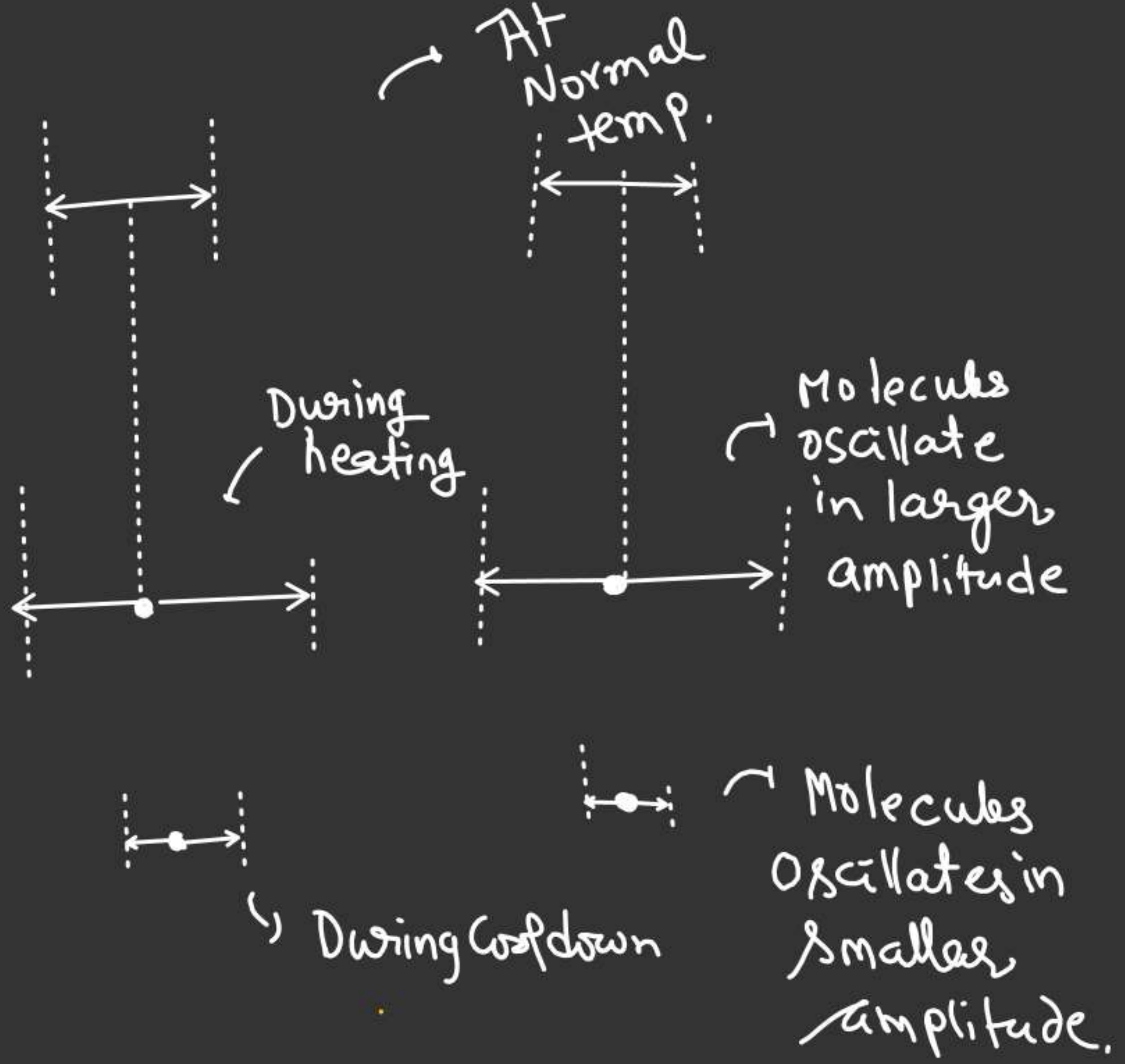
$$\boxed{L_f = L_i(1 + \alpha \Delta T)}$$

ΔT = Change in temp.

α = Cofficient of linear expansion.

L_f = Final length

L_i = Initial length



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$$L_f = L_i(1+\alpha\Delta T)$$

$$L_f = L_i + L_i\alpha\Delta T$$

$$\underbrace{L_f - L_i}_{} = L_i\alpha\Delta T$$

$$\Downarrow$$

$$\boxed{\Delta L = L_i\alpha\Delta T}$$

$$\boxed{\frac{dl}{l} = \alpha\, dT}$$

$$\frac{\Delta L}{L_i} = \text{Fractional Change}$$

$$\frac{\Delta L}{L_i}\times 100 = \text{Percentage Change}$$

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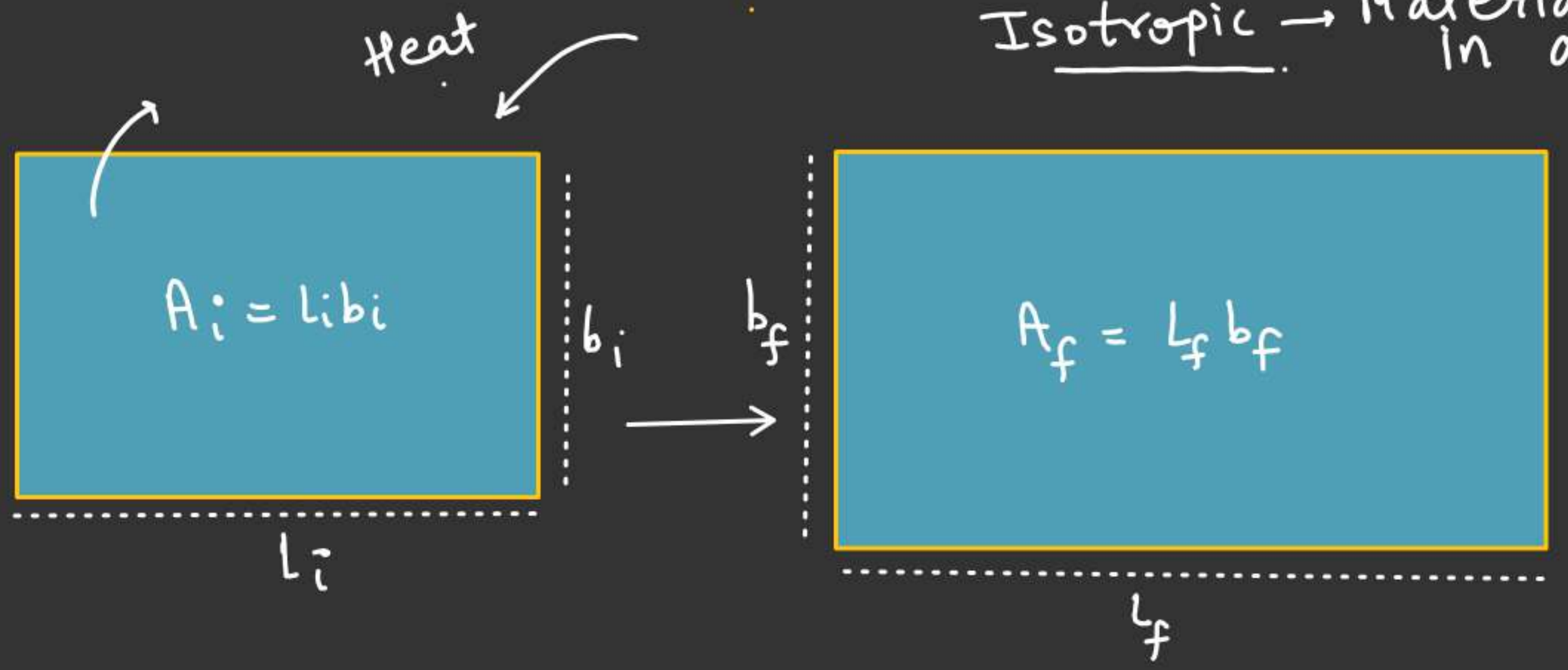
Areal Expansion (2-Dimensional)

Isotropic → Material whose expansion in all direction is same.



$(\alpha \Delta T << 1)$

$x << 1$

$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2$

$\frac{n(n-1)}{2!}x^2 \to 0$

$$l_f = l_i(1+\alpha\Delta T)$$

$$b_f = b_i(1+\alpha\Delta T)$$

$$l_f b_f = l_i b_i (1+\alpha\Delta T)^2$$

$$\Downarrow$$

$$A_f = A_i(1+\alpha\Delta T)^2$$

$$A_f = A_i(1+2\alpha\Delta T)$$

$2\alpha \to \beta$

$$\boxed{A_f = A_i(1+\beta\Delta T)}$$

β = coff^n of Areal expansion

$\beta = 2\alpha$

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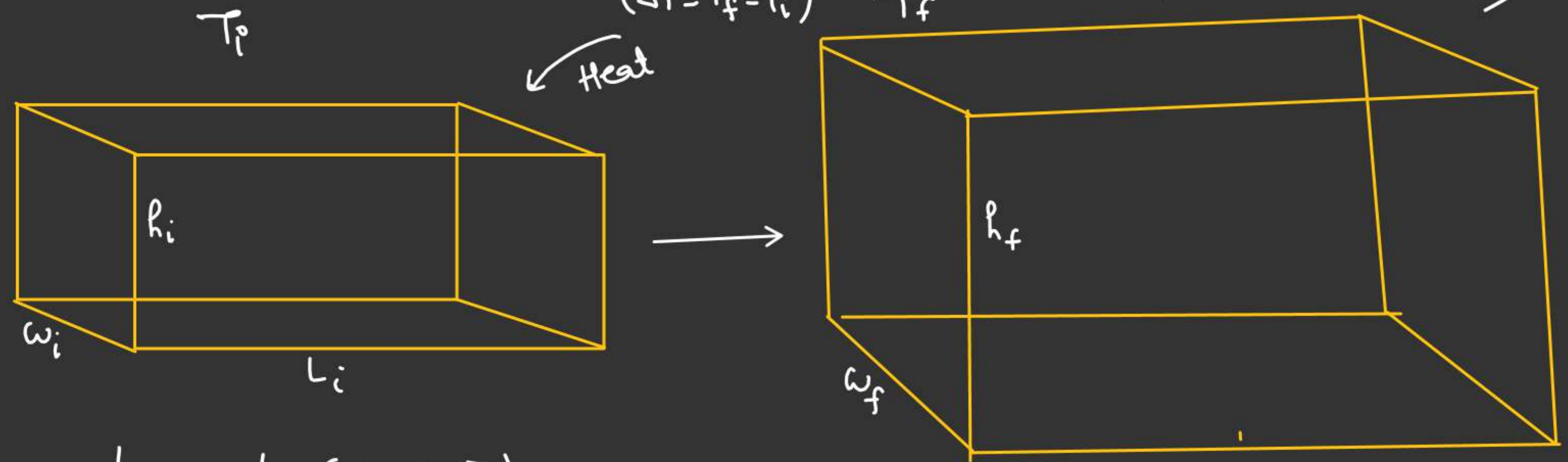
Volume Expansion (3-D expansion) (Isotropic i.e α in all direction is same)

($\Delta T = T_f - T_i$)



$L_f = L_i(1+\alpha\Delta T)$

$w_f = w_i(1+\alpha\Delta T)$

$h_f = h_i(1+\alpha\Delta T)$

$L_f h_f w_f = l_i w_i h_i (1+\alpha\Delta T)^3$

$\Downarrow$

$V_f = V_i(1+3\alpha\Delta T)$

$\alpha\Delta T \ll 1$

$3\alpha \to \gamma$

γ = coffn of volume expansion

$$\boxed{V_f = V_i(1+\gamma\Delta T)}$$

$$\boxed{\gamma = 3\alpha}$$

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$$A_f = A_i(1+\beta\Delta T)$$

$$\frac{A_f - A_i}{A_i} = \beta\Delta T$$

$$\boxed{\frac{dA}{A} = \beta\, dT}$$

$$V_f = V_i(1+\gamma\Delta T)$$

$$\frac{V_f - V_i}{V_i} = \gamma\Delta T$$

$$\boxed{\frac{dV}{V} = \gamma\, dT}$$

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#

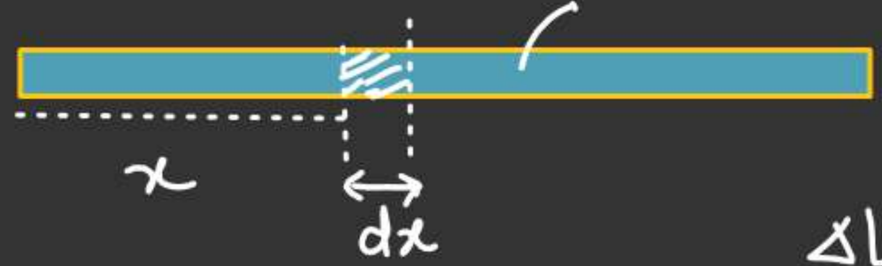
$\alpha = (a + bx)$ where a & b are constant.

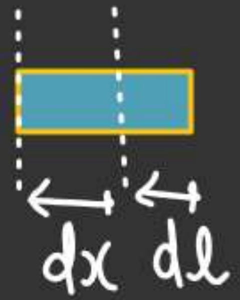
Find total elongation when temp of rod changes by ΔT°C

L = Initial length. (x from A)

Let, dl be elongation in 'dx' length of the rod.

$(\Delta L = L_i \alpha \Delta T)$





$$dl = dx\, \underline{\alpha_x}\, \Delta T$$

$$L_f = (L + \Delta L)$$

$$\int_0^{\Delta L} dl = \Delta T \int_0^L (a + bx)\, dx.$$

$$\Delta L = \Delta T \left[a \int_0^L dx + b \int_0^L x\, dx \right]$$

$$\Delta L = \Delta T \left[aL + \frac{bL^2}{2} \right].$$

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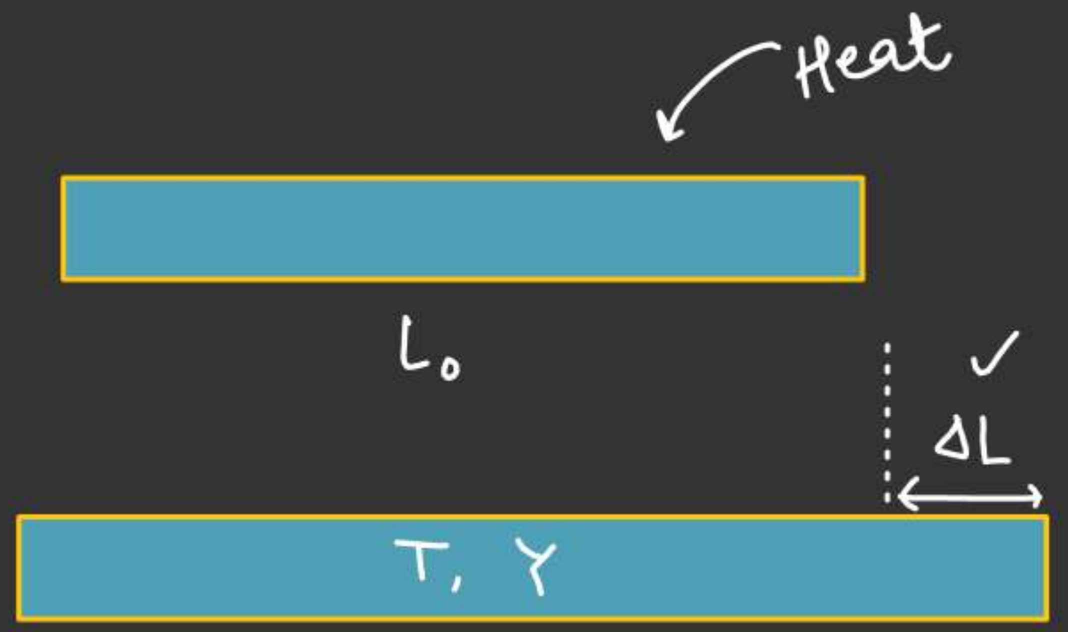
** THERMAL STRESS

$$\text{Thermal Strain} = \left(\frac{\text{Unachived length}}{\text{Initial length}}\right)$$

$$\frac{\text{Stress}}{\text{Strain}} = Y \qquad \text{Stress} = F/A \qquad \text{Strain} = \frac{\Delta L}{L}$$



Here, rod is free to elongate so no thermal stress & thermal strain.

WRONG ??

$$\Delta L = L\alpha\Delta T$$

$$\frac{\Delta L}{L} = \alpha\Delta T$$

$\Downarrow$ Strain.

$$\text{Stress} = Y\,\text{Strain} = Y\alpha\Delta T$$

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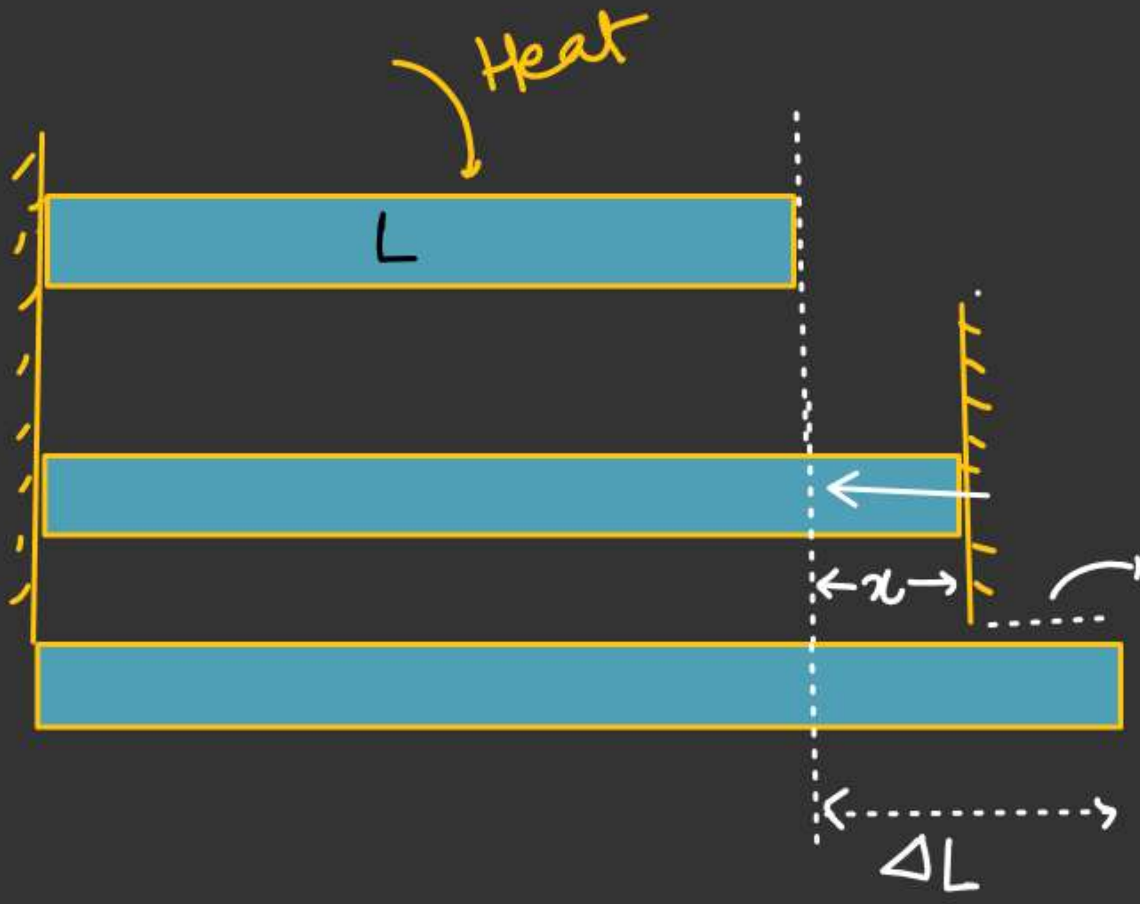
Thermal Stress

$$\frac{\text{Stress}}{\text{Strain}} = Y \qquad \text{Stress} = F/A \qquad \text{Strain} = \frac{\Delta L}{L}$$



unachive elongation $= (\Delta L - x)$

when free to elongate

$$\boxed{\text{Thermal Strain} = \left(\frac{\Delta L - x}{L}\right)}$$

$$\boxed{\text{Thermal Stress} = Y\left(\frac{\Delta L - x}{L}\right)}$$

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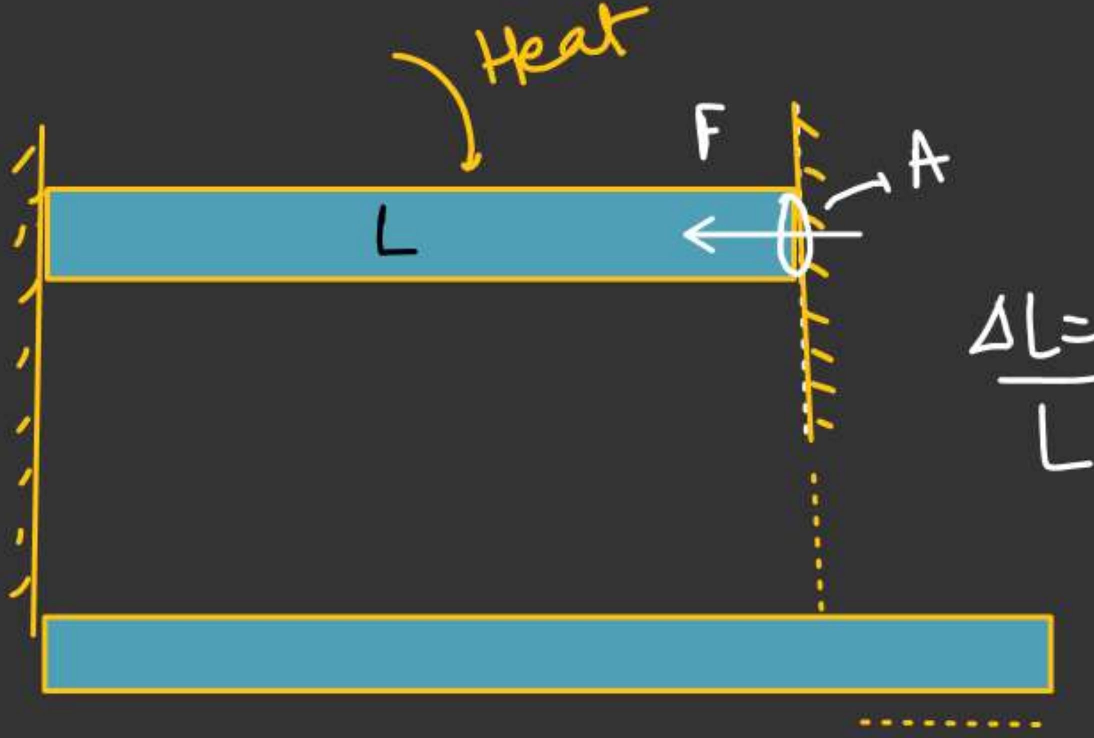
THERMAL STRESS



$\frac{\Delta L}{L} = \alpha \Delta T$

$\Delta L \rightarrow$ unachived

$\text{Strain} = \frac{\Delta L}{L}$

$\text{Stress} = \left(Y \frac{\Delta L}{L} \right)$

$\frac{F}{A} = Y \frac{\Delta L}{L}$

$\frac{F}{A} = Y \alpha \Delta T$

$$\boxed{F = Y A \alpha \Delta T}$$

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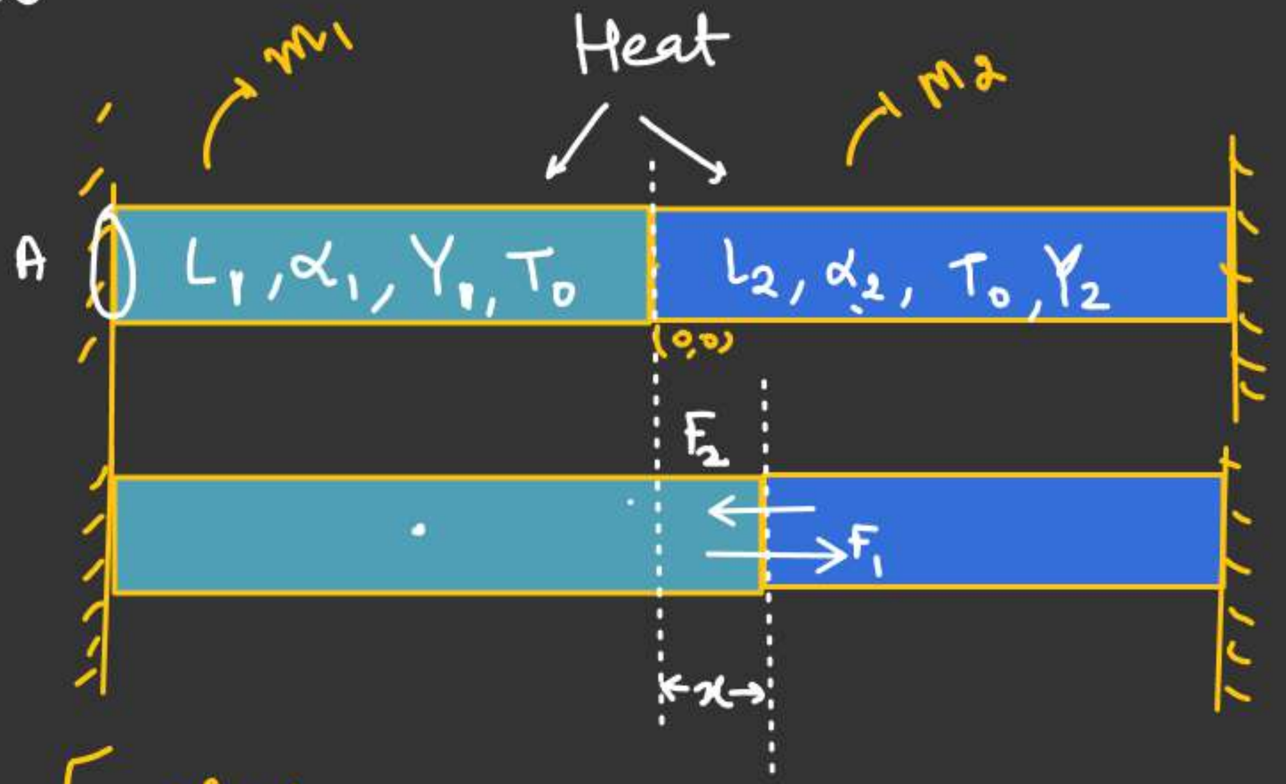
Both the rod fixed at their end with rigid support after heating find the shift of junction.

A = Crosectional Area



Sol^n:- $x \to$ Shifting of junction

Shifting of junction stop when

$$F_1 = F_2.$$

$$\left(\frac{F_1}{A}\right) = \left(\frac{F_2}{A}\right) = \text{Thermal stress}$$

$$Y_1\begin{pmatrix}\text{Thermal} \\ \text{strain of} \\ \text{Rod-1}\end{pmatrix} = Y_2\begin{pmatrix}\text{Thermal Strain of} \\ \text{Rod-2}\end{pmatrix}$$

H.W.

[Calculate shifting of COM]

$$\Delta X_{com} = (X_{com})_f - (X_{com})_i$$

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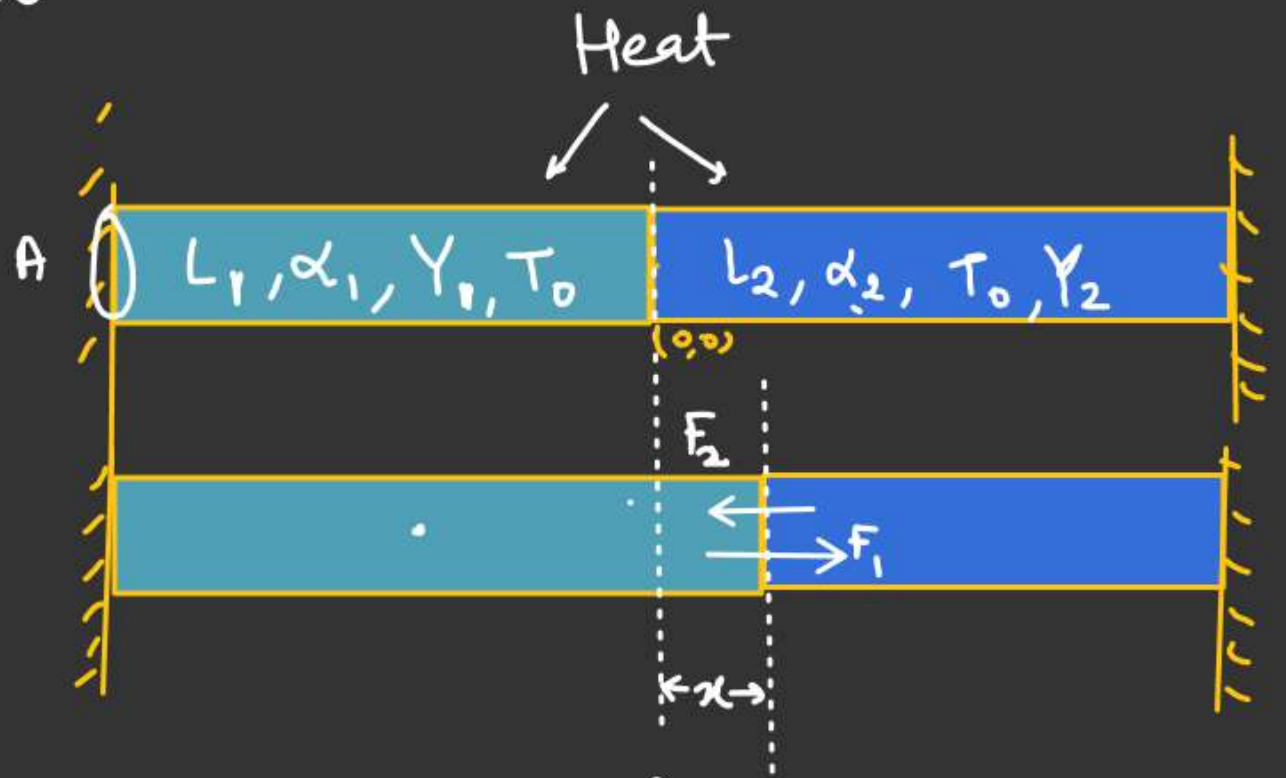
$Y_1\left(\text{Thermal strain of Rod-1}\right) = Y_2\left(\text{Thermal Strain of Rod-2}\right)$

A = Crosectional Area

$\frac{\Delta L_1}{L_1} = \alpha_1 \Delta T$

$\frac{\Delta L_2}{L_2} = \alpha_2 \Delta T$



$$Y_1 \frac{(\Delta L_1 - x)}{L_1} = \frac{Y_2 (\Delta l_2 + x)}{L_2}$$

$$Y_1\left(\frac{\Delta L_1}{L_1}\right) - Y_2\left(\frac{\Delta l_2}{L_2}\right) = \left(\frac{Y_1}{L_1} + \frac{Y_2}{L_2}\right)x$$

ΔL_1 = Elongation when rod-1 free to expand

Δl_2 = Elongation when rod-2 free to expand.

$$\frac{Y_1\alpha_1\Delta T - Y_2\alpha_2\Delta T}{\left(\frac{Y_1}{L_1} + \frac{Y_2}{L_2}\right)} = x$$

if $Y_1\alpha_1 > Y_2\alpha_2$

$x > 0$

if $Y_1\alpha_1 < Y_2\alpha_2$

$x < 0$

$$\boxed{\left(\frac{Y_1\alpha_1 - Y_2\alpha_2}{\frac{Y_1}{L_1} + \frac{Y_2}{L_2}}\right)\Delta T = x}$$

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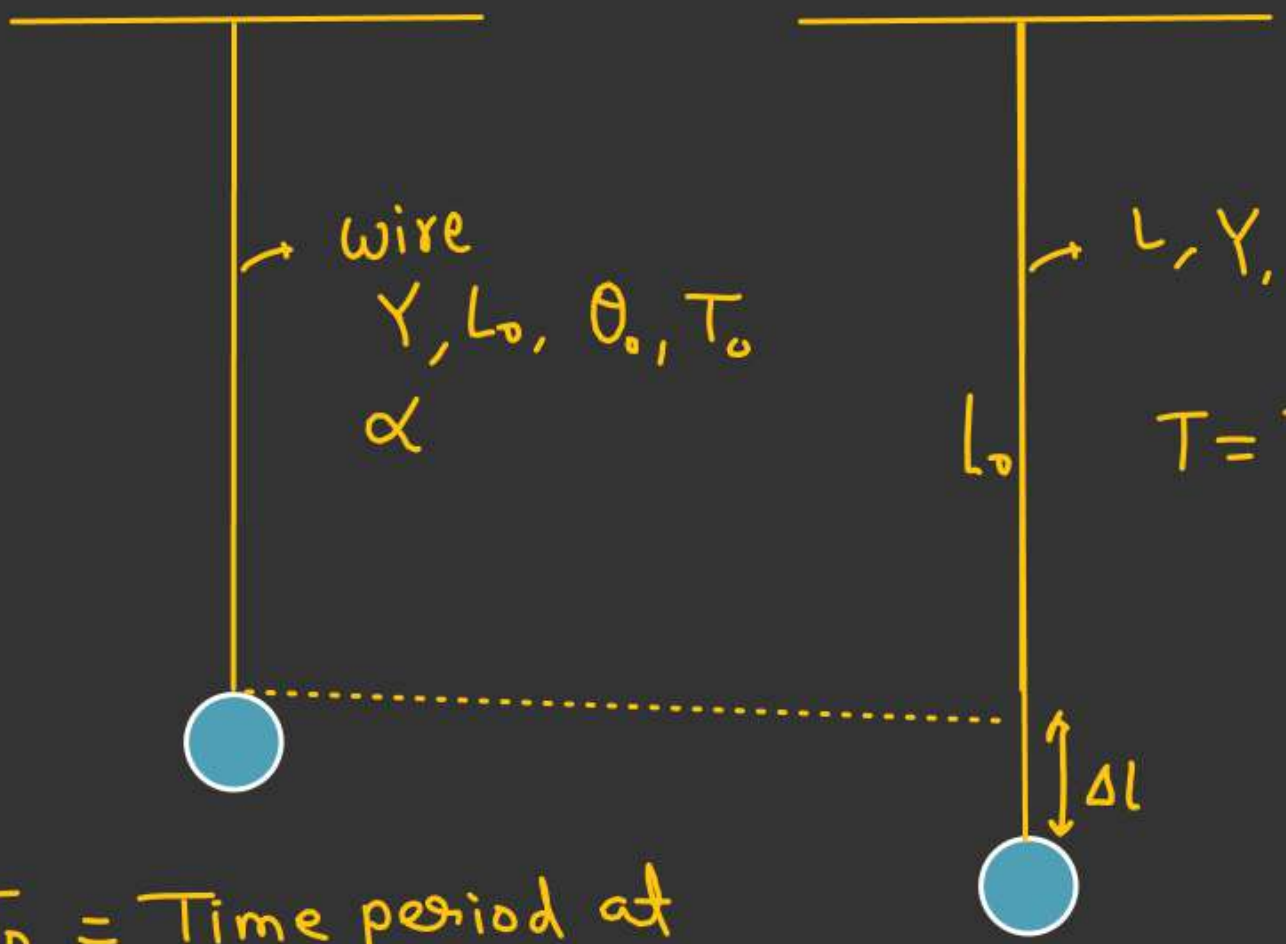
$\Delta\theta$:

$$T = 2\pi\sqrt{L/g}$$

$\Delta\theta$ = Change in temp.

$$L = L_0(1+\alpha(\theta - \theta^\circ))$$

$$L = L_0(1+\alpha\Delta\theta)$$



T = Time period at $\theta^\circ C$

$$T = 2\pi\sqrt{\frac{L}{g}}$$

$$T = 2\pi\sqrt{\frac{L_0(1+\alpha\Delta\theta)}{g}}$$

T_0 = Time period at $\theta_0^\circ C$

$$T_0 = 2\pi\sqrt{\frac{L_0}{g}}$$

$$T = \underbrace{2\pi\sqrt{\frac{L_0}{g}}}_{T_0}(1+\alpha\Delta\theta)^{1/2}$$

$\alpha\Delta\theta << 1$

$(1+x)^n \simeq 1+nx$

$n = \frac{1}{2}$

$x = \alpha\Delta\theta$

$$\boxed{T = T_0\left(1+\frac{\alpha\Delta\theta}{2}\right)}$$

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$$T = T_0\left(1 + \frac{\alpha\Delta\theta}{2}\right)$$

$$T = T_0 + \frac{T_0\alpha\Delta\theta}{2}$$

$$\left(\frac{T - T_0}{T_0}\right) = \frac{\alpha\Delta\theta}{2}$$

$$\left(\frac{\Delta T}{T_0}\right) = \left(\frac{\alpha\Delta\theta}{2}\right)$$

$$\left(\frac{\Delta T}{T_0} = \frac{\Delta t}{t}\right)$$

$\Delta t \rightarrow$ Change in time

$t \rightarrow$ total time

$$\frac{\Delta t}{t} = \frac{\alpha\Delta\theta}{2}$$

$$\boxed{\Delta t = \left(\frac{\alpha\Delta\theta}{2}\right) \times t}$$

Delay or fast of clock in 't' time.