KEY CONCEPTS

DEFINITION :

A sequence is a set of terms in a definite order with a rule for obtaining the terms. e.g. $1, 1/2, 1/3, \ldots\ldots, 1/n$ is a sequence.

AN ARITHMETIC PROGRESSION (AP) :

AP is a sequence whose terms increase or decrease by a fixed number. This fixed number is called the common difference. If a is the first term & d the common difference, then AP can be written as $a, a+d, a+2d, \ldots\ldots. a+(n-1)d, \ldots\ldots$

n^{th} term of this AP $t_n = a + (n-1)d$, where $d = a_n - a_{n-1}$.

The sum of the first n terms of the AP is given by; $S_n = \frac{n}{2}[2a + (n-1)d] = \frac{n}{2}[a + l]$. where l is the last term.

NOTES :

(i) If each term of an A.P. is increased, decreased, multiplied or divided by the same non zero number, then the resulting sequence is also an AP.

(ii) Three numbers in AP can be taken as a-d, a, a + d; four numbers in AP can be taken as a-3d, $a - d, a + d, a + 3d$; five numbers in AP are $a - 2d, a - d, a, a + d, a + 2d$ & six terms in AP are $a - 5d, a - 3d, a - d, a + d, a + 3d, a + 5d$ etc.

(iii) The common difference can be zero, positive or negative.

(iv) The sum of the two terms of an AP equidistant from the beginning & end is constant and equal to the sum of first & last terms.

(v) Any term of an AP (except the first) is equal to half the sum of terms which are equidistant from it.

(vi) $t_r = S_r - S_{r-1}$

(vii) If a, b, c are in AP $\Rightarrow 2b = a + c$.

GEOMETRIC PROGRESSION (GP) :

GP is a sequence of numbers whose first term is non zero & each of the succeeding terms is equal to the proceeding terms multiplied by a constant. Thus in a GP the ratio of successive terms is constant. This constant factor is called the **COMMON RATIO** of the series & is obtained by dividing any term by that which immediately proceeds it. Therefore $a, ar, ar^2, ar^3, ar^4, \ldots\ldots$ is a GP with a as the first term & r as common ratio.

(i) n^{th} term $= ar^{n-1}$

(ii) Sum of the 1^{st} n terms i.e. $S_n = \frac{a(r^n - 1)}{r-1}$, if $r \neq 1$.

(iii) Sum of an infinite GP when $|r| < 1$ when $n \to \infty$ $r^n \to 0$ if $|r| < 1$ therefore,

$S_\infty = \dfrac{a}{1-r}\,(|r| < 1).$

(iv) If each term of a GP be multiplied or divided by the same non-zero quantity, the resulting sequence is also a GP.

(v) Any 3 consecutive terms of a GP can be taken as a/r, a, ar ; any 4 consecutive terms of a GP can be taken as $a/r^3, a/r, ar, ar^3$ & so on.

(vi) If a, b, c are in GP $\Rightarrow b^2 = ac$.

HARMONIC PROGRESSION (HP) :

A sequence is said to HP if the reciprocals of its terms are in AP.

If the sequence $a_1, a_2, a_3, \ldots, a_n$ is an HP then $1/a_1, 1/a_2, \ldots .1/a_n$ is an AP & converse. Here we do not have the formula for the sum of the n terms of an HP. For HP whose first term is a & second term is b, the n^{th} term is $t_n = \frac{ab}{b+(n-1)(a-b)}$.

If a, b, c are in HP $\Rightarrow b = \frac{2ac}{a+c}$ or $\frac{a}{c} = \frac{a-b}{b-c}$.

MEANS

ARITHMETIC MEAN :

If three terms are in AP then the middle term is called the AM between the other two, so if a, b, c are in AP, b is AM of a & c.

AM for any n positive number $a_1, a_2, \ldots, a_n$ is ; $A = \frac{a_1+a_2+a_3+\cdots.+a_n}{n}$.

n-ARITHMETIC MEANS BETWEEN TWO NUMBERS :

If a, b are any two given numbers &a, $A_1, A_2, \ldots, A_n$, b are in AP then $A_1, A_2, \ldots A_n$ are the n AM's between a & b

$$A_1 = a + \frac{b-a}{n+1}, A_2 = a + \frac{2(b-a)}{n+1}, \ldots\ldots, A_n = a + \frac{n(b-a)}{n+1}$$

$= a + d, \ = a + 2d, \ldots\ldots, A_n = a + nd$, where $d = \frac{b-a}{n+1}$

NOTES :

Sum of n AM's inserted between a & b is equal to n times the single AM between a & b

i.e. $\sum_{r=1}^{n} A_r = nA$ where A is the single AM between a & b.

GEOMETRIC MEANS :

If a, b, c are in GP, b is the GM between a &c.

$b^2 = ac$, therefore $b = \sqrt{ac}; a > 0, c > 0$.

n-GEOMETRIC MEANS BETWEEN a, b :

If a, b are two given numbers & a, $G_1, G_2, \ldots., G_n$, b are in GP. Then $G_1, G_2, G_3, \ldots., G_n$ are n GMs between a & b.

$G_1 = a(b/a)^{1/n+1}, G_2 = a(b/a)^{2/n+1}, \ldots\ldots, G_n = a(b/a)^{n/n+1}$

$= ar, \quad = ar^2, \quad = ar^n$, where $r = (b/a)^{1/n+1}$

NOTES :

The product of n GMs between a & b is equal to the n^{th} power of the single GM between a & b i.e. $\prod_{r=1}^{n} G_r = (G)^n$ where G is the single GM between a & b.

HARMONIC MEAN :

If a, b, c are in HP, b is the HM between a & c, then $b = 2ac/[a + c]$.

THEOREM :

If A, G, H are respectively AM, GM, HM between a & b both being unequal & positive then,

(i) $G^2 = AH$

(ii) $A > G > H (G > 0)$. Note that A, G, H constitute a GP.

ARITHMETICO-GEOMETRIC SERIES :

A series each term of which is formed by multiplying the corresponding term of an AP & GP is called the **Arithmetico-Geometric Series.** e.g. $1 + 3x + 5x^2 + 7x^3 + \cdots \ldots$

Here 1,3,5, ... are in AP & $1, x, x^2, x^3$ ... are in GP.

Standart appearance of an Arithmetico-Geometric Series is

Let $S_n = a + (a + d)r + (a + 2d)r^2 + \cdots \ldots + [a + (n - 1)d]r^{n-1}$

SIGMA NOTATIONS

THEOREMS :

(i) $\sum_{r=1}^{n}(a_r \pm b_r) = \sum_{r=1}^{n} a_r \pm \sum_{r=1}^{n} b_r$.

(ii) $\sum_{r=1}^{n} ka_r = k\sum_{r=1}^{n} a_r$.

(iii) $\sum_{r=1}^{n} k = nk$; where k is a constant.

RESULTS

(i) $\sum_{r=1}^{n} r = \frac{n(n+1)}{2}$ (sum of the first n natural nos.)

(ii) $\sum_{r=1}^{n} r^2 = \frac{n(n+1)(2n+1)}{6}$ (sum of the squares of the first n natural numbers)

(iii) $\sum_{r=1}^{n} r^3 = \frac{n^2(n+1)^2}{4} [\sum_{r=1}^{n} r]^2$ (sum of the cubes of the first n natural numbers)

METHOD OF DIFFERENCE :

If $T_1, T_2, T_3, \ldots\ldots, T_n$ are the terms of a sequence then some times the terms $T_2 - T_1, T_3 - T_2, \ldots\ldots$ constitute an AP/GP. n^{th} term of the series is determined & the sum to n terms of the sequence can easily be obtained.

Remember that to find the sum of n terms of a series each term of which is composed of r factors in AP, the first factors of several terms being in the same AP, we "write down the nth term, affix the next factor at the end, divide by the number of factors thus increased and by the common difference and add a constant. Determine the value of the constant by applying the initial conditions".

PROFICIENCY TEST-01

1. A sequence is given by the formula of its nth term : $a_n = 10 - 3n$. prove that a_n is an arithmetic progression.

2. Let $a_n = n^2 + 1$ and b_n is defined $b_n = a_{n+1} - a_n$. Show that $\{b_n\}$ is an arithmetic sequence.

3. Prove that if the numbers $\log_k x$, $\log_m x$ and $\log_n x$ $(x \neq 1)$ form an arithmetic progression then $n^2 = (kn)^{\log_k m}$.

4. The sum of three numbers in A.P. is 27 and the sum of their squares is 293. Find the numbers.

5. Find four numbers in A.P. such that their sum is 50 and the greatest of them is 4 times the least.

6. How many terms are identical in the two arithmetic progressions 2,4,6,8, … … up to 100 terms and 3,6,9, ... up to 80 terms.

7. The interior angles of a polygon are in AP. The smallest angle is 120°& the common difference is 5°. Find the number of sides of the polygon.

8. Suppose $a_1, a_2, \ldots \ldots$ are in A.P. and S_k denotes the sum of the first k terms of this A.P. If $S_n/S_m = n^4/m^4$ for all m, n, ∈ N, then prove that

$$\frac{a_{m+1}}{a_{n+1}} = \frac{(2m+1)^3}{(2n+1)^3}$$

9. In an A.P. of 99 terms, the sum of all the odd numbered terms is 2550 . Then find the sum of all the 99 terms of the A.P.

10. Find the degree of the expression $(1+x)(1+x^6)(1+x^{11})\ldots\ldots\ldots\ldots(1+x^{101})$

PROFICIENCY TEST-02

1. Find the sum of all three-digit natural numbers, which are divisible by 7 .

2. Find the sum of first 24 terms of the A.P. $a_1, a_2, a_3, \ldots \ldots \ldots$, if it is known that $a_1 + a_5 + a_{10} + a_{15} + a_{20} + a_{24} = 225$.

3. If the arithmetic progression whose common difference is non-zero, the sum of first 3n terms is equal to the sum of next n terms. Then, find the ratio of the sum of the first 2n terms to the sum of next 2n terms.

4. Insert three arithmetic means between 3 and 19 .

5. If eleven A.M.'s are inserted between 28 and 10 , then find the number of integral A.M.'s.

6. Prove that the average of the numbers $n \sin n°$, $n = 2,4,6, \ldots \ldots,180$, is $\cot 1°$.

7. The ratio of the sums of m and n terms of an A.P. is $m^2 : n^2$. Show that the ratio of the m^{th} and n^{th} terms is $(2m-1):(2n-1)$

8. The sum of n terms of two arithmetic series are in the ratio of $(7n+1):(4n+27)$. Find the ratio of their n^{th} term.

9. Show that $\ln(4 \times 12 \times 36 \times 108 \times \text{up to n terms}) = 2n\ln 2 + \frac{n(n-1)}{2}\ln 3$

10. If the sum of n terms of a G.P. is $3 - \frac{3^{n+1}}{4^{2n}}$, then find the common ratio.

PROFICIENCY TEST-03

1. Fifth term of a G.P. is 2. Find the product of its first nine terms.

2. Three numbers are in G.P. If we double the middle term, we get an A.P. Then find the common ratio of the G.P.

3. Determine the number of terms in a G.P., if $a_1 = 3, a_n = 96$ and $S_n = 189$.

4. Prove that $6^{1/2} \times 6^{1/4} \times 6^{1/8} \ldots\ldots\infty = 6$.

5. Find $\sum_{i=1}^{6} 2.3^i$

6. Find the sum of n terms of the series $2 + 22 + 222 + \cdots \ldots\ldots$

7. If $x = 1 + a + a^2 + a^3 + \cdots \ldots \infty$ and $y = 1 + b + b^2 + b^3 + \ldots\ldots\infty$, show that $1 + ab + a^2b^2 + a^3b^3 + \ldots\ldots\infty = \frac{xy}{x+y-1}$, where $0 < a < 1$ and $0 < b < 1$

8. If G be the geometric mean of x and y, then prove that $\frac{1}{(G^2-x^2)} + \frac{1}{(G^2-y^2)} = \frac{1}{G^2}$.

9. Find two numbers whose arithmetic mean is 34 and the geometric mean is 16.

10. If a is the A.M. of b & c, and the two geometric means between b & c are G_1 and G_2, then prove that ${G_1}^3 + {G_2}^3 = 2abc$.

PROFICIENCY TEST-04

1. Let $P = \prod_{n=1}^{\infty}\left(10^{\left(\frac{1}{2^{(n-1)}}\right)}\right)$ then find $\log_{0.01}(P)$.

2. The A.M. between two positive numbers exceeds the G.M. by 5, and the G.M. exceeds the H.M. by 4. Find the numbers

3. If the sum to infinity of the series $3 + (3+d)\frac{1}{4} + (3+2d)\frac{1}{4^2} + \ldots\ldots\infty$ is $\frac{44}{9}$, then find d.

4. Find the sum to n terms of the series $1 + (1+2) + (1+2+3) + \ldots\ldots$

5. Evaluate $1 + 5 + 12 + 22 + 35 +$ (upto ' n ' terms)

6. Find the sum of n-terms $1 + 4 + 10 + 22 + \ldots\ldots$

7. If a, b, c, d are four positive real numbers such that $abcd = 1$, prove that $(1+a)(1+b)(1+c)(1+d) \geq 16$

8. If x, y, z be positive numbers, show that $(x+y+z)^3 \geq 27xyz$.

9. If H is the harmonic mean between P and Q, then find the value of $H/P + H/Q$.

10. If a, b, c and d are in H.P., then find the value of $\frac{a^{-2}-d^{-2}}{b^{-2}-c^{-2}}$

11. Find sum of the following series to n terms and to infinity : $\frac{1}{1.4.7}+\frac{1}{4.7.10}+\frac{1}{7.10.13}+\ldots\ldots$

12. Find sum of the following series to n terms: $\sum_{r=1}^{n} r(r+1)(r+2)(r+3)$

13. Find sum of the following series to n terms and to infinity : $\sum_{r=1}^{n} \frac{1}{4r^2-1}$

14. If the 10^{th} term of an HP is 21 and 21^{st} term of the same HP is 10 , then find the 210^{th} term.

15. Given that $a^x = b^y = c^z = d^u$ & a, b, c, d are in GP, show that x, y, z, u are in HP.

EXERCISE–I

1. In an AP of which ' a ' is the Ist term, if the sum of the Ist p terms is equal to zero, show that the sum of the next q terms is $-\left(\frac{aq(p+q)}{p-1}\right)$.

2. The first term of an arithmetic progression is 1 and the sum of the first nine terms equal to 369 . The first and the ninth term of a geometric progression coincide with the first and the ninth term of the arithmetic progression. Find the seventh term of the geometric progression.

3. In a set of four numbers, the first three are in GP & the last three are in AP, with common difference 6 . If the first number is the same as the fourth, find the four numbers.

4. The 1^{st}, 2^{nd} and 3^{rd} terms of an arithmetic series are a, b and a^2 where ' a ' is negative. The 1^{st}, 2^{nd} and 3^{rd} terms of a geometric series are a, a^2 and b find the

(a) value of a and b

(b) sum of infinite geometric series if it exists. If no then find the sum to n terms of the G.P.

(c) sum of the 40 term of the arithmetic series.

5. Find three numbers a, b, c ($\geq 2, \leq 18$) such that;

(i) their sum is 25

(ii) the numbers 2, a, b are consecutive terms of an AP &

(iii) the numbers b, c, 18 are consecutive terms of a GP.

6. If $S_1, S_2, S_3, \ldots S_n, \ldots$ are the sums of infinite geometric series whose first terms are 1,2,3, … n, …. and whose common ratios are $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots . \frac{1}{n+1}, \ldots$ respectively, then find the value of $\sum_{r=1}^{2n-1} S_r^2$.

7. Find the sum of the first n terms of the sequence :

$$1+2\left(1+\frac{1}{n}\right)+3\left(1+\frac{1}{n}\right)^2+4\left(1+\frac{1}{n}\right)^3+\ldots\ldots\ldots$$

8. Find the sum of the n terms of the sequence $\frac{1}{1+1^2+1^4}+\frac{2}{1+2^2+2^4}+\frac{3}{1+3^2+3^4}+\ldots\ldots\ldots$

9. Evaluate the sum $\sum_{n=1}^{\infty} \frac{n^2}{6^n}$

10. If the sum $\sqrt{1+\frac{1}{1^2}+\frac{1}{2^2}}+\sqrt{1+\frac{1}{2^2}+\frac{1}{3^2}}+\sqrt{1+\frac{1}{3^2}+\frac{1}{4^2}}+\ldots\ldots+\sqrt{1+\frac{1}{(1999)^2}+\frac{1}{(2000)^2}}$ equal to $n-\frac{1}{n}$ where $n \in N$. Find n.

11. The AM of two numbers exceeds their GM by 15 & HM by 27 . Find the numbers.

12. An AP & an HP have the same first term, the same last term & the same number of terms; prove that the product of the r^{th} term from the beginning in one series & the r^{th} term from the end in the other is independent of r.

13. The sequence $a_1, a_2, a_3, \ldots \ldots . a_{98}$ satisfies the relation $a_{n+1}=a_n+1$ for $n=1,2,3, .97$ and has the sum equal to 4949 . Evaluate $\sum_{k=1}^{49} a_{2k}$.

14. If the sum to infinity of the series $1 + 4x + 7x^2 + 10x^3 + \cdots ....$ is $\frac{35}{16}$ then find x.

15. In the equation $x^4 + px^3 + qx^2 + rx + 5 = 0$ has four positive real roots, then find the minimum value of pr.

16. Find the nth term and the sum to n terms of the sequence: $1 + 5 + 13 + 29 + 61 + ...$

17. Find the n.h term and the sum to n terms of the sequence: $6 + 13 + 22 + 33 + ...$

18. If $a, a_1, a_2, a_3, \ldots., a_{2n}, b$ are in A.P. and $a, g_1, g_2, g_3, \ldots. g_{2n}, b$ are in G.P. and h is the H.M. of a and b, then prove that

$$\frac{a_1 + a_{2n}}{g_1 g_{2n}} + \frac{a_2 + a_{2n-1}}{g_2 g_{2n-1}} + \ldots\ldots + \frac{a_n + a_{n+1}}{g_n g_{n+1}} = \frac{2n}{h}$$

19. Let f(x) denote the sum of the infinite trigonometric series, $f(x) = \sum_{n=1}^{\infty} \sin \frac{2x}{3^n} \sin \frac{x}{3^n}$.

Find f(x) (independent of n) also evaluate the sum of the solutions of the equation $f(x) = 0$ lying in the interval $(0, 629)$.

20. If the roots of $10x^3 - cx^2 - 54x - 27 = 0$ are in harmonic progression, then find c and all the roots.

EXERCISE–II

1. If $\sin x, \sin^2 2x$ and $\cos x \cdot \sin 4x$ form an increasing geometric sequence, find the numerical value of $\cos 2x$. Also find the common ratio of geometric sequence.

2. If the first 3 consecutive terms of a geometrical progression are the real roots of the equation $2x^3 - 19x^2 + 57x - 54 = 0$ find the sum to infinite number of terms of G.P.

3. Find the sum of the infinite series $\frac{1.3}{2} + \frac{3.5}{2^2} + \frac{5.7}{2^3} + \frac{7.9}{2^4} + \cdots \ldots \ldots$

4. One of the roots of the equation $2000x^6 + 100x^5 + 10x^3 + x - 2 = 0$ is of the form $\frac{m+\sqrt{n}}{r}$, where m is non zero integer and n and r are relatively prime natural numbers.
Find the value of $m + n + r$.

5. Find the condition that the roots of the equation $x^3 - px^2 + qx - r = 0$ are in A.P. and hence solve the equation $x^3 - 12x^2 + 39x - 28 = 0$

6. If a, b, c, d, e be 5 numbers such that a, b, c are in AP ; b, c, d are in GP & c, d, e are in HP then :
(i) Prove that a, c, e are in GP .
(ii) Prove that $e = (2b - a)^2/a$.
(iii) If $a = 2$ & $e = 18$, find all possible values of b, c, d.

7. A computer solved several problems in succession. The time it took the computer to solve each successive problem was the same number of times smaller than the time it took to solve the preceding problem. How many problems were suggested to the computer if it spent 63.5 min to solve all the problems except for the first, 127 min to solve all the problems except for the last one, and 31.5 min to solve all the problems except for the first two?

8. If n is a root of the equation $x^2(1 - ac) - x(a^2 + c^2) - (1 + ac) = 0$ & if n HM's are inserted between a and **c**, show that the difference between the first & the last mean is equal to ac $(a - c)$.

9. Given that the cubic $ax^3 - ax^2 + 9bx - b = 0 (a \neq 0)$ has all three positive roots. Find the harmonic mean of the roots independent of a and b, hence deduce that the root are all equal. Find also the minimum value of $(a + b)$ if a and $b \in N$.

10. Let a sequence whose n^{th} term is $\{a_n\}$ be defined as

$$a_1 = \frac{1}{2} \text{ and } (n-1)a_{n-1} = (n+1)a_n \text{ for } n \geq 2$$

find a_n and also find S_n and $\text{Lim}_{n\to\infty} S_n$.

11. In a right angled triangle, S_a and S_b denote the medians that belong to the legs of the triangle, the median belonging to the hypotenuse is S_c. Find the maximum value of the expression $\frac{S_a+S_b}{S_c}$.
(You may use the fact that R.M.S. $\geq$ A.M).

12. Find the sum the series to n terms and to infinity : $\frac{1}{4}+\frac{1.3}{4.6}+\frac{1.3.5}{4.6.8}+\cdots\ldots\ldots.$

13. (a) The value of $x+y+z$ is 15 if a, x, y, z, b are in AP while the value of ; $(1/x)+(1/y)+(1/z)$ is 5/3 if a, x, y, z, b are in HP. Find a & b.

(b) The values of xyz is 15/2 or 18/5 according as the series a, x, y, z, b is an AP or HP. Find the values of a & b assuming them to be positive integer.

14. If there are n quantities in GP with common ratio r & S_m denotes the sum of the first m terms, show that the sum of the products of these m terms taken two & two together is $\left[\frac{r}{r+1}\right][S_m][S_{m-1}]$.

15. Find the value of $\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\sum_{\substack{k=0 \\ (i\neq j\neq k)}}^{\infty}\frac{1}{3^i 3^j 3^k}$

EXERCISE–III

1. If $1, \log_9(3^{1-x}+2), \log_3(4.3^x-1)$ are in A.P. then x equals **[AIEEE-2002]**

(A) $\log_3 4$ (B) $1-\log_3 4$ (C) $1-\log_4 3$ (D) $\log_4 3$

2. The value of $2^{1/4}\cdot 4^{1/8}\cdot 8^{1/16}\ldots\ldots\infty$ is **[AIEEE- 2002]**

(A) 1 (B) 2 (C) 3/2 (D) 4

3. Fifth term of a GP is 2 , then the product of its 9 terms is **[AIEEE- 2002]**

(A) 256 (B) 512 (C) 1024 (D) None of these

4. Sum of infinite number of terms of GP is 20 and sum of their squares is 100. The common ratio of GP is **[AIEEE- 2002]**

(A) 5 (B) 3/5 (C) 8/5 (D) 1/5

5. $1^3-2^3+3^3-4^3+\ldots\ldots+9^3=$ **[AIEEE- 2002]**

(A) 425 (B) -425 (C) 475 (D) -475

6. Let T_r be the r^{th} term of an A.P. whose first term is a and common difference is d. If for some positive integers $m, n, m \neq n, T_m=\frac{1}{n}$ and $T_n=\frac{1}{m}$, then $a-d$ equals- **[AIEEE- 2004]**

(A) 0 (B) 1 (C) 1/mn (D) $\frac{1}{m}+\frac{1}{n}$

7. The sum of the first n terms of the series $1^2+2.2^2+3^2+2.4^2+5^2+2.6^2+\cdots$. is $\frac{n(n+1)^2}{2}$ when n is even. When n is odd the sum is- **[AIEEE- 2004]**

(A) $\frac{3n(n+1)}{2}$ (B) $\frac{n^2(n+1)}{2}$ (C) $\frac{n(n+1)^2}{4}$ (D) $\left[\frac{n(n+1)}{2}\right]^2$

8. If $x=\sum_{n=0}^{\infty}a^n, y=\sum_{n=0}^{\infty}b^n, z=\sum_{n=0}^{\infty}c^n$ where a, b, c are in A.P. and $|a|<1, |b|<1, |c|<1$ then x, y, z are in - **[AIEEE- 2005]**

(A) GP (B) AP

(C) Arithmetic - Geometric Progression (D) HP

9. Let $a_1, a_2, a_3, \ldots$ be terms of an A.P. If $\frac{a_1+a_2+\cdots+a_p}{a_1+a_2+\cdots+a_q}=\frac{p^2}{q^2}, p \neq q$ then $\frac{a_6}{a_{21}}$ equals- **[AIEEE- 2006]**

(A) $\frac{7}{2}$ (B) $\frac{2}{7}$ (C) $\frac{11}{41}$ (D) $\frac{41}{11}$

10. If $a_1, a_2, \ldots . a_n$ are in H.P., then the expression $a_1a_2+a_2a_3+\ldots\ldots+a_{n-1}a_n$ is equal to - **[AIEEE-2006]**

(A) $(n-1)(a_1-a_n)$ (B) na_1a_n

(C) $(n-1)a_1a_n$ (D) $n(a_1-a_n)$

11. In a geometric progression consisting of positive terms, each term equals the sum of the next two terms. Then the common ratio of this progression equals- **[AIEEE-2007]**

(A) $\frac{1}{2}(1-\sqrt{5})$ (B) $\sqrt{5}$ (C) $\frac{1}{2}\sqrt{5}$ (D) $\frac{1}{2}(\sqrt{5}-1)$

12. The first two terms of a geometric progression add up to 12. The sum of the third and the fourth terms is 48 . If the terms of the geometric progression are alternately positive and negative, then the first term is **[AIEEE-2008]**

(A) -4 (B) -12 (C) 12 (D) 4

13. The sum to infinity of the series $1+\frac{2}{3}+\frac{6}{3^2}+\frac{10}{3^3}+\frac{14}{3^4}+\ldots\ldots$ **[AIEEE 2009]**

(A) 2 (B) 3 (C) 4 (D) 6

14. A person is to count 4500 currency notes. Let a_n denote the number of notes he counts in the n^{th} minute. If $a_1 = a_2 = \cdots = a_{10} = 150$ and $a_{10}, a_{11}, \ldots$ are in an AP with common difference -2, then the time taken by him to count all notes is **[AIEEE 2010]**

(A) 34 minutes (B) 125 minutes

(C) 135 minutes (D) 24 minutes

15. A man saves 200 in each of the first three months of his service. In each of the subsequent months his saving increases by 40 more than the saving of immediately previous month. His total saving from the start of service will be ` 11040 after **[AIEEE 2011]**

(A) 19 months (B) 20 months (C) 21 months (D) 18 months

16. **Statement-1**: The sum of the series

$1+(1+2+4)+(4+6+9)+(9+12+16)+\ldots\ldots+(361+380+400)$ is 8000.

Statement-2 : $\sum_{k=1}^{n}(k^3-(k-1)^3)=n^3$, for any natural number n. **[AIEEE 2012]**

(A) Statement-1 is true, Statement-2 is true, Statement-2 is a correct explanation for Statement-1

(B) Statement-1 is true, Statement-2 is true, Statement-2 is a NOT the correct explanation for Statement-1

(C) Statement-1 is true, Statement-2 is false

(D) Statement-1 is false, Statement-2 is true

17. The sum of first 20 terms of the sequence 0.7,0.77,0.777, ..., is : **[JEE (Mains)2013]**

(A) $\frac{7}{9}(99+10^{-20})$ (B) $\frac{7}{81}(179-10^{-20})$

(C) $\frac{7}{9}(99-10^{-20})$ (D) $\frac{7}{81}(179+10^{-20})$

18. Let α and β be the roots of equation $px^2+qx+r=0, p\neq 0$. If p, q, r are in A.P. and $\frac{1}{\alpha}+\frac{1}{\beta}=4$, then the value of $|\alpha-\beta|$ is **[IIT Main 2014]**

(A) $\frac{2\sqrt{13}}{9}$ (B) $\frac{\sqrt{61}}{9}$ (C) $\frac{2\sqrt{17}}{9}$ (D) $\frac{\sqrt{34}}{9}$

19. Three positive numbers form an increasing G.P. If the middle term in this G.P. is doubled, the new numbers are in A.P. Then the common ratio of the G.P. is : **[IIT Main 2014]**

(A) $2+\sqrt{3}$ (B) $\sqrt{2}+\sqrt{3}$ (C) $3+\sqrt{2}$ (D) $2-\sqrt{3}$

20. If $(10)^9 + 2(11)^1(10)^8 + 3(11)^2(10)^7 + \ldots\ldots + 10(11)^9 = k(10)^9$, then k is equal to **[IIT Main 2014]**

(A) 110 (B) $\frac{121}{10}$ (C) $\frac{441}{100}$ (D) 100

21. The sum of first 9 terms of the series $\frac{1^3}{1} + \frac{1^3+2^3}{1+3} + \frac{1^3+2^3+3^3}{1+3+5} + \cdots$ is : **[JEE Mains-2015]**

(A) 192 (B) 71 (C) 96 (D) 142

22. If m is the A.M. of two distinct real numbers ℓ and $n(\ell, n > 1)$ and G_1, G_2 between ℓ and n, then $G_1^4 + 2G_2^4 + G_3^4$ equals.

(A) $4\ell^2m^2n^2$ (B) $4\ell^2mn$ (C) $4\ell m^2n$ (D) $4\ell mn^2$

23. If the 2^{nd}, 5^{th} and 9^{th} terms of a non-constant A.P. are in G.P., then the

(A) $\frac{8}{5}$ (B) $\frac{4}{3}$ (C) 1 (D) $\frac{7}{4}$

24. If the sum of the first ten terms of the series **[JEE Mains-2016]**

$\left(1\frac{3}{5}\right)^2 + \left(2\frac{2}{5}\right)^2 + \left(3\frac{1}{5}\right)^2 + 4^2 + \left(4\frac{4}{5}\right)^2 + \ldots\ldots$ is $\frac{16}{5}$ m, then m is equal to :

(A) 102 (B) 101 (C) 100 (D) 99

25. If, for a positive integer, n the quadratic equation, **[JEE Mains-2017]**

$x(x+1) + (x+1)(x+2) + \cdots \ldots + (x + \overline{n-1})(x+n) = 10n$

has two consecutive integral solutions, then n is equal to :

(A) 10 (B) 11 (C) 12 (D) 9

26. For any three positive real numbers a, b and c $9(25a^2 + b^2) + 25(c^2 - 3ac) = 15b(3a + c)$. Then **[JEE Mains-2017]**

(A) a, b and c are in A.P. (B) a, b and c are in G.P.

(C) b, c and a are in G.P. (D) b, c and a are in A.P.

27 Let A be the sum of the first 20 terms and B be the sum of the first 40 terms of the series

$1^2 + 2.2^2 + 3^2 + 2.4^2 + 5^2 + 2.6^2 + \ldots\ldots$

If $B - 2A = 100\lambda$, then λ is equal to **[JEE Main-2018]**

(A) 496 (B) 232 (C) 248 (D) 464

28. Let $a_1, a_2, a_3, \ldots \ldots a_{49}$ be in A.P. such that $\sum_{k=0}^{12} a_{4k+1} = 416$ and $a_9 + a_{43} = 66$.

If $a_1^2 + a_2^2 + \cdots + a_{17}^2 = 140m$, then m is equal to **[JEE Main-2018]**

(A) 33 (B) 66 (C) 68 (D) 34

EXERCISE–IV

1. (a) The harmonic mean of the roots of the equation $(5+\sqrt{2})x^2-(4+\sqrt{5})x+8+2\sqrt{5}=0$ is

(A) 2 (B) 4 (C) 6 (D) 8

(b) Let $a_1, a_2, \ldots, a_{10}$, be in A.P. & $h_1, h_2, \ldots, h_{10}$ be in H.P. If $a_1 = h_1 = 2 \& a_{10} = h_{10} = 3$ then $a_4 h_7$ is: **[JEE '99, 2 + 2 out of 200]**

(A) 2 (B) 3 (C) 5 (D) 6

2. The sum of an infinite geometric series is 162 and the sum of its first n terms is 160. If the inverse of its common ratio is an integer, find all possible values of the common ratio, n and the first terms of the series. **[REE '99, 6]**

3. (a) Consider an infinite geometric series with first term ' a ' and common ratio r. If the sum is 4 and the second term is 3/4, then :

(A) $a=\frac{7}{4}, r=\frac{3}{7}$ (B) $a=2, r=\frac{3}{8}$ (C) $a=\frac{3}{2}, r=\frac{1}{2}$ (D) $a=3, r=\frac{1}{4}$

(b) If a, b, c, d are positive real numbers such that $a+b+c+d=2$, then $M=(a+b)(c+d)$ satisfies the relation: **[JEE 2000, Screening, 1 + 1 out of 35]**

(A) $0 \le M \le 1$ (B) $1 \le M \le 2$ (C) $2 \le M \le 3$ (D) $3 \le M \le 4$

(c) The fourth power of the common difference of an arithmetic progression with integer entries added to the product of any four consecutive terms of it. Prove that the resulting sum is the square of an integer. **[JEE 2000, Mains, 4 out of 100]**

4. Given that α, γ are roots of the equation, $Ax^2-4x+1=0$ and β, δ the roots of the equation, $Bx^2-6x+1=0$, find values of A and B, such that α, β, γ & δ are in H.P.

[REE 2000, 5 out of 100]

5. The sum of roots of the equation $ax^2+bx+c=0$ is equal to the sum of squares of their reciprocals. Find whether bc^2, ca^2 and ab^2 in A.P., G.P. or H.P.? **[REE 2001, 3 out of 100]**

6. Solve the following equations for x and y **[REE 2001, 5 out of 100]**

$$\log_2 x+\log_4 x+\log_{16} x+\ldots\ldots=y$$
$$\frac{5+9+13+\cdots\ldots\ldots+(4y+1)}{1+3+5+\cdots\ldots\ldots+(2y-1)}=4\log_4 x$$

7. (a) Let α, β be the roots of $x^2-x+p=0$ and γ, δ be the roots of $x^2-4x+q=0$. If $\alpha, \beta, \gamma, \delta$ are in G.P., then the integral values of p and q respectively, are

(A) $-2,-32$ (B) $-2,3$ (C) $-6,3$ (D) $-6,-32$

(b) If the sum of the first 2 n terms of the A.P. 2,5,8, is equal to the sum of the first n terms of the A.P. 57,59,61 then n equals

(A) 10 (B) 12 (C) 11 (D) 13

(c) Let the positive numbers a, b, c, d be in A.P. Then abc, abd, acd, bcd are

(A) NOT in A.P./G.P./H.P. (B) in A.P.

(C) in G.P. (D) in H.P.

[JEE 2001, Screening, 1 + 1 + 1 out of 35]

(d) Let $a_1, a_2 \ldots \ldots \ldots$ be positive real numbers in G.P. For each n, let A_n, G_n, H_n, be respectively, the arithmetic mean, geometric mean and harmonic mean of $a_1, a_2, a_3, \ldots \ldots \ldots . a_n$. Find an expression for the G. M. of $G_1, G_2, \ldots \ldots \ldots G_n$ in terms of $A_1, A_2 \ldots \ldots \ldots \ldots A_n, H_1, H_2, \ldots \ldots \ldots H_n$

[JEE 2001 (Mains); 5]

8. (a) Suppose a, b, c are in A.P. and a^2, b^2, c^2 are in G.P. If $a < b < c$ and $a + b + c = \frac{3}{2}$, then the value of a is

(A) $\frac{1}{2\sqrt{2}}$ (B) $\frac{1}{2\sqrt{3}}$ (C) $\frac{1}{2} - \frac{1}{\sqrt{3}}$ (D) $\frac{1}{2} - \frac{1}{\sqrt{2}}$

[JEE 2002 (Screening), 3]

(b) Let a, b be positive real numbers. If a, A_1, A_2, b are in A.P.; a, G_1, G_2, b are in G. P. and a, H_1, H_2, b are in H.P. , show that

$$\frac{G_1 G_2}{H_1 H_2} = \frac{A_1 + A_2}{H_1 + H_2} = \frac{(2a+b)(a+2b)}{9ab}$$

[JEE 2002, Mains, 5 out of 60]

9. If a, b, c are in A.P., a^2, b^2, c^2 are in H.P. , then prove that either $a = b = c$ or $a, b, -\frac{c}{2}$ form a G.P.

[JEE-03, Mains-4 out of 60]

10. The first term of an infinite geometric progression is x and its sum is 5 . Then

(A) $0 \leq x \leq 10$ (B) $0 < x < 10$ (C) $-10 < x < 0$ (D) $x > 10$

[JEE 2004 (Screening)]

11. If a, b, c are positive real numbers, then prove that $[(1+a)(1+b)(1+c)]^7 > 7^7 a^4 b^4 c^4$.

[JEE 2004, 4 out of 60]

12. (a) In the quadratic equation $ax^2 + bx + c = 0$, if $\Delta = b^2 - 4ac$ and $\alpha + \beta, \alpha^2 + \beta^2, \alpha^3 + \beta^3$ are in G.P. where α, β are the roots of $ax^2 + bx + c = 0$, then

(A) $\Delta \neq 0$ (B) $b\Delta = 0$ (C) $c\Delta = 0$ (D) $\Delta = 0$

[JEE 2005 (Screening)]

(b) If total number of runs scored in n matches is $\left(\frac{n+1}{4}\right)(2^{n+1} - n - 2)$ where $n > 1$, and the runs scored in the $\mathbf{k}^{\text{th}}$ match are given by $k \cdot 2^{n+1-k}$, where $1 \leq k \leq n$. Find n.

[JEE 2005 (Mains), 2]

13. If $A_n = \left(\frac{3}{4}\right) - \left(\frac{3}{4}\right)^2 + \left(\frac{3}{4}\right)^3 + \cdots \ldots + (-1)^{n-1}\left(\frac{3}{4}\right)^n$ and $B_n = 1 - A_n$, then find the minimum natural number n_0 such that $B_n > A_n \cdot \forall n > n_0$. **[JEE 2006, 6]**

Comprehension (3 questions)

14. Let V_r denote the sum of the first ' **r** ' terms of an arithmetic progression (A.P.) whose first term is ' r ' and the common difference is $(2r-1)$.

Let $T_r = V_{r+1} - V_r - 2$ and $Q_r = T_{r+1} - T_r$ for $r = 1,2, \ldots$

(a) The sum $V_1 + V_2 + \cdots \ldots + V_n$ is

(A) $\frac{1}{12}n(n+1)(3n^2-n+1)$ (B) $\frac{1}{12}n(n+1)(3n^2+n+2)$

(C) $\frac{1}{2}n(2n^2-n+1)$ (D) $\frac{1}{3}(2n^3-2n+3)$

(b) T_r is always

(A) an odd number (B) an even number

(C) a prime number (D) a composite number

(c) Which one of the following is a correct statement?

(A) Q_1, Q_2, Q_3,..........are in A.P. with common difference 5.

(B) Q_1, Q_2, Q_3,......... are in A.P. with common difference 6.

(C) Q_1, Q_2, Q_3,..........are in A.P. with common difference 11.

(D) $Q_1 = Q_2 = Q_3 = \ldots\ldots$

[JEE 2007, 4 + 4 + 4]

Comprehension (3 questions)

[JEE 2007, 4 + 4 + 4]

15. Let A_1, G_1, H_1 denote the arithmetic, geometric and harmonic means, respectively, of two distinct positive numbers. For $n \geq 2$, let A_{n-1} and H_{n-1} have arithmetic, geometric and harmonic means as A_n, G_n, H_n respectively.

(a) Which one of the following statements is correct?

(A) $G_1 > G_2 > G_3 > \ldots\ldots\ldots$ (B) $G_1 < G_2 < G_3 < \ldots\ldots\ldots$

(C) $G_1 = G_2 = G_3 = \ldots\ldots\ldots$ (D) $G_1 < G_3 < G_5 < \ldots\ldots\ldots$ and $G_2 > G_4 > G_6 > \ldots\ldots\ldots$

(b) Which one of the following statements is correct?

(A) $A_1 > A_2 > A_3 > \ldots\ldots$

(B) $A_1 < A_2 < A_3 < \ldots\ldots$

(C) $A_1 > A_3 > A_5 > \ldots\ldots$ and $A_2 < A_4 < A_6 < \ldots\ldots$

(D) $A_1 < A_3 < A_5 < \ldots\ldots$ and $A_2 > A_4 > A_6 > \ldots\ldots$

(c) Which one of the following statements is correct?

(A) $H_1 > H_2 > H_3 > \ldots\ldots$

(B) $H_1 < H_2 < H_3 < \ldots\ldots$

(C) $H_1 > H_3 > H_5 > \ldots\ldots$ and $H_2 < H_4 < H_6 < \ldots\ldots.$

(D) $H_1 < H_3 < H_5 < \ldots\ldots$ and $H_2 > H_4 > H_6 > \ldots\ldots.$

16. (a) A straight line through the vertex P of a triangle PQR intersects the side QR at the point S and the circumcircle of the triangle PQR at the point T. If S is not the centre of the circumcircle, then **[JEE 2008, 4]**

(A) $\frac{1}{PS} + \frac{1}{ST} < \frac{2}{\sqrt{QS \times SR}}$ (B) $\frac{1}{PS} + \frac{1}{ST} > \frac{2}{\sqrt{QS \times SR}}$

(C) $\frac{1}{PS} + \frac{1}{ST} < \frac{4}{QR}$ (D) $\frac{1}{PS} + \frac{1}{ST} > \frac{4}{QR}$

ASSERTION & REASON :

(b) Suppose four distinct positive numbers a_1, a_2, a_3, a_4 are in G.P. Let $b_1 = a_1$, $b_2 = b_1 + a_2, b_3 = b_2 + a_3$ and $b_4 = b_3 + a_4$.

STATEMENT-1 : The numbers b_1, b_2, b_3, b_4 are neither in A.P. nor in G.P. and

STATEMENT-2 : The numbers b_1, b_2, b_3, b_4 are in H.P.

(A) Statement-1 is True, Statement- 2 is True; statement- 2 is a correct explanation for statement-1

(B) Statement- 1 is True, Statement- 2 is True; statement- 2 is NOT a correct explanation for statement-1

(C) Statement-1 is True, Statement-2 is False

(D) Statement-1 is False, Statement-2 is True **[JEE 2008, 3(-1)]**

17. Let S_k, $k = 1,2 \ldots ,100$, denote the sum of the infinite geometric series whose first term is $\frac{k-1}{k!}$ and the common ratio is $\frac{1}{k}$. Then he value of $\frac{100^2}{100!} + \sum_{k=1}^{100} |(k^2 - 3k + 1)s_k|$ is **[JEE 2010]**

18. Let $a_1, a_2, a_3, \ldots \ldots \ldots, a_{11}$ be real numbers satisfying $a_1 = 15,\ 27 - 2a_2 > 0$ and $a_k = 2a_{k-1} - a_{k-2}$ for $k = 3,4 \ldots \ldots 11$, If $\frac{a_1^2 + a_2^2 + \cdots + a_{11}^2}{11} = 90$, then the value of $\frac{a_1 + a_2 + \cdots + a_{11}}{11}$ is equal to **[JEE 2010]**

19. Let $a_1, a_2, a_3, \ldots, a_{100}$ be an arithmetic progression with $a_1 = 3$ and $S_p = \sum_{i=1}^{p} a_i, 1 \le p \le 100$. For any integer n with $1 \le n \le 20$, let $m = 5n$. If $\frac{S_m}{S_n}$ does not depend on n, then a_2 is **[JEE 2011]**

20. The minimum value of the sum of real numbers $a^{-5}, a^{-4}, 3a^{-3}, 1, a^8$ and a^{10} with $a > 0$ is

[JEE 2011]

21. Let $a_1, a_2, a_3, \ldots \ldots \ldots$ be in harmonic progression with $a_1 = 5$ and $a_{20} = 25$. The least positive integer n for which $a_n < 0$ is : **[JEE 2012]**

(A) 22 (B) 23 (C) 24 (D) 25

22. Let $S_n = \sum_{k=1}^{4n} (-1)^{\frac{k(k+1)}{2}} k^2$. Then S_n can take value(s) **[JEE Advance 2013]**

(A) 1056 (B) 1088 (C) 1120 (D) 1332

23. Let a, b, c be positive integers such that $\frac{b}{a}$ is an integer. If a, b, c are in geometric progression and the arithmetic mean of a, b, c is b + 2, then the value of $\frac{a^2+a-14}{a+1}$ is **[JEE Advance 2014]**

24. Suppose that all the terms of an arithmetic progression (A.P.) are natural numbers. If the ratio of the sum of the first seven terms to the sum of the first eleven terms is **6**: **11** and the seventh term lies in between 130 and 140 , then the common difference of this A.P. is

[JEE Advance 2015]

25. Let $b_i > 1$ for $i = 1,2, \ldots \ldots, 101$. Suppose $\log_e b_1, \log_e b_2, \ldots \ldots, \log_e b_{101}$ are in Arithmetic Progression (A.P.) with the common difference $\log_e 2$. Suppose $a_1, a_2, \ldots \ldots, a_{101}$ are in A.P. such that $a_1 = b_1$ and $a_{51} = b_{51}$. If $t = b_1 + b_2 + \cdots \ldots + b_{51}$ and $s = a_1 + a_2 + \cdots \ldots + a_{51}$, then

[JEE Advance 2016]

(A) $s > t$ and $a_{101} > b_{101}$

(B) $s > t$ and $a_{101} < b_{101}$

(C) $s < t$ and $a_{101} > b_{101}$

(D) $s < t$ and $a_{101} < b_{101}$

26. The sides of a right angled triangle are in arithmetic progression. If the triangle has area 24 , then what is the length of its smallest side? **[JEE Advance 2017]**

27. Let X be the set consisting of the first 2018 terms of the arithmetic progression 1,6,11, … …, and Y be the set consisting of the first 2018 terms of the arithmetic progression 9,16,23, … …. Then the number elements in the set $X \cup Y$ is **[JEE Advanced 2018]**

28. Let α and β be the roots of $x^2 - x - 1 = 0$ with $\alpha > \beta$. For all positive integers **n**, define

$$a_n = \frac{\alpha^n - \beta^n}{\alpha - \beta}, n \geq 1,$$

$b_1 = 1$ and $b_n = a_{n-1} + a_{n+1}, n \geq 2$.

Then which of the following options is/are correct? **[JEE Advanced 2019]**

(A) $\sum_{n=1}^{\infty} \frac{a_n}{10^n} = \frac{10}{89}$

(B) $a_1 + a_2 + a_3 + \cdots .. + a_n = a_{n+2} - 1$ for all $n \geq 1$

(C) $\sum_{n=1}^{\infty} \frac{b_n}{10^n} = \frac{8}{89}$

(D) $b_n = \alpha^n + \beta^n$ for all $n \geq 1$

29. Let AP(a; d) denote the set of all the terms of an infinite arithmetic progression with first term a and common difference $d > 0$. If

$AP(1; 3) \cap AP(2; 5) \cap AP(3; 7) = AP(a; d)$ then $a + d$ equals **[JEE Advanced 2019]**

30. Let a_1, a_2, a_3 ... be a sequence of positive integers in arithmetic progression with common difference 2. Also, let $b_1, b_2, b_3, \ldots$ be a sequence of positive integers in geometric progression with common ratio 2 . If $a_1 = b_1 = c$, then the number of all possible values of **c**, for which the equality $2(a_1 + a_2 + \cdots + a_n) = b_1 + b_2 + \cdots . + b_n$ holds for some positive integer **n**, is

[JEE Advanced 2020]

ANSWER SHEET

SEQUENCE & PROGRESSION

PROFICIENCY TEST-1

4. 14, 9, 4 or 4, 9, 14 **5.** 5, 10,15, 20 **6.** 33 **7.** 9 **9.** 5049

10. 1071

PROFICIENCY TEST-2

1. 70336 **2.** 900 **3.** 1/5 **4.** 7, 11, 15 **5.** 5

8. (14n – 6)/(8n + 23) **10.** 3/16

PROFICIENCY TEST-3

1. 512 **2.** $2 \pm \sqrt{3}$ **3.** 6 **5.** 2184 **6.** $\frac{2}{9}\left[\frac{10}{9}(10^n - 1) - n\right]$

9. 64 and 4

PROFICIENCY TEST-4

1. -1 **2.** 40,10 **3.** 2 **4.** $\frac{\mathbf{n(n+1)(n+2)}}{\mathbf{6}}$ **5.** $\frac{\mathbf{n^3+n^2}}{\mathbf{2}}$ **6.** $\mathbf{3.2^n - 2n - 3}$

9. 2 **10.** 3 11. $s_n = (1/24) - [1/\{6(3n+1)(3n+4)\}]; s_\infty = 1/24$

12. $(1/5)n(n+1)(n+2)(n+3)(n+4)$ 13. $n/(2n+1); s_\infty = 1/2$

14. 1

EXERCISE-I

2. 27 **3.** $(\mathbf{8, -4, 2, 8})$ **4.** (a) $\mathbf{a} = -\frac{1}{2}, \mathbf{b} = -\frac{1}{8}$; (b) $-\frac{1}{3}$ (c) $\frac{545}{2}$

5. $a = 5, b = 8, c = 12$ **6.** $\frac{\mathbf{n(2n+1)(4n+1)}}{\mathbf{3}} - \mathbf{1}$ **7.** n^2 **8.** $\frac{\mathbf{n(n+1)}}{\mathbf{2(n^2+n+1)}}$

9. $\mathbf{S} = \frac{42}{125}$ **10.** $n = 2000$ **11.** 120,30

13. 2499 **14.** $\mathbf{x = 1/5}$ **15.** 80

16. $2^{n+1} - 3; 2^{n+2} - 4 - 3n$

17. $n^2 + 4n + 1;\ (1/6)n(n+1)(2n+13) + n$

19. $f(x) = \frac{1}{2}[1 - \cos x]; S = 10100\pi$

20. $C = 9; (3, -3/2, -3/5)$

EXERCISE-II

1. $\frac{\sqrt{5}-1}{2}; \sqrt{2}$ **2.** $\frac{27}{2}$ **3.** 23 **4.** 200

5. $2p^3 - 9pq + 27r = 0$; roots are 1,4,7 (iii) $b = 4, c = 6, d = 9$ or $b = -2, c = -6, d = -18$

7. 8 problems, 127.5 minutes

9. 28

10. $a_n = \frac{1}{n(n+1)}, S_n = 1 - \frac{1}{(n+1)}, S_\infty = 1$

11. $\sqrt{10}$

12. $S_n = 2\left[\frac{1}{2} - \frac{1\cdot3.5.....(2n-1)(2n+1)}{2.4.6.......(2n)(2n+2)}\right]; S_\infty = 1$

13. (a) $a = 1, b = 9$ OR $b = 1, a = 9$; (b) $a = 1; b = 3$ or vice versa

15. $\frac{81}{208}$

EXERCISE–III

1.	B	**2.**	B	**3.**	B	**4.**	B	**5.**	A	**6.**	A	**7.**	B
8.	D	**9.**	C	**10.**	C	**11.**	D	**12.**	B	**13.**	B	**14.**	A
15.	C	**16.**	A	**17.**	D	**18.**	A	**19.**	A	**20.**	D	**21.**	C
22.	C	**23.**	B	**24.**	B	**25.**	B	**26.**	D	**27.**	C	**28.**	D

EXERCISE-IV

1. (a) B (b) D

2. $(r, n, a) \in \left\{\left(\frac{1}{3}, 4{,}108\right), \left(\frac{-1}{3}, 4{,}216\right), \left(\frac{1}{9}, 2{,}144\right), \left(\frac{-1}{9}, 2{,}180\right), \left(\frac{1}{81}, 1{,}160\right)\right\}$

3. (a) D (b) A

4. $A = 3; B = 8$

5. A.P.

6. $x = 2\sqrt{2}$ and $y = 3$

7. (a) A, (b) C, (c) D, (d) $[(A_1, A_2, \ldots\ldots\ldots.. A_n)(H_1, H_2, \ldots\ldots\ldots.. H_n)]^{\frac{1}{2n}}$

8. (a) D 10. B 12. (a) C, (b) $n = 7$

13.	$n_0 = 5$				
14.	(a) B; (b) D; (c) B	**15.**	(a) C; (b) A; (c) B	**16.**	(a) B, D; (b) C
17.	3	**18.**	0	**19.**	9 or 3
20.	8	**21.**	D	**22.**	A, D
23.	4	**24.**	9	**25.**	B
26.	6	**21.**	3748	**28.**	A, B, D
29.	157.00	**30.**	1.00		