Nishant Jindal

4113

Unit digit = 3 ✓

last 2 digits = 13 ✓

last 3 digits = 113. ✓

$$10\overline{)4113}(411$$
40
11
10
13
10
3

$$100\overline{)4113}(41$$
400
113
100
13

$$1000\overline{)4113}(4$$
4000
113

① Unit digit = 10K + 3.

last 2 digit = 100K + Remainder ← Ans.

last 3 digit = 1000K + Remainder = Ans

② for Unit digit we do not Require Binomial

we can Use Cyclicity.

2
$2^2 = 4$
$2^3 = 8$
$2^4 = 16$
$2^5 = 32$
$2^6 = 64$
$2^7 = 128$
256

$\boxed{2, 4, 8, 6}$ $\boxed{2, 4, 8, 6}$

$3^1 = 3$
$3^2 = 9$
$3^3 = 27$
$3^4 = 81$
$3^5 = 243$
$3^6 = 729$

$\boxed{3, 9, 7, 1}$ 3, 9, 7

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Cofficient of x^n

Q Find coff of x^2 in $\dfrac{\left(2x-\frac{1}{x}\right)^{10}}{x^4}$ → B.T कैसे? ← divide by x^4

Coff of x^6 in $\left(2x-\frac{1}{x}\right)^{10}$

$$r = \frac{10\times 1-6}{1+1} = \frac{n\alpha-m}{\alpha+\beta}$$

$r=2$

$$T_3 = {}^{10}C_2 \cdot (2x)^8\left(-\frac{1}{x}\right)^2$$

$$\text{Coff} = \frac{10\cdot 9}{1\cdot 2}\times 2^8$$

Coff $= 128\times 90$

Q2 Find Coff of x^0 in $(x+1)^m\left(1+\frac{1}{x}\right)^n$

Coff of x^0 in $(x+1)^m\left(\frac{x+1}{x}\right)^n$

x^0 in $\dfrac{(x+1)^{m+n}}{x^n}$ → BT

Actual Q's → Coff of x^n in $(x+1)^{m+n}$

$T_{r+1} = {}^{m+n}C_r\cdot(x)^{m+n-r}$ $m+n-r=n$

$\boxed{r=m}$

Coff $= \boxed{{}^{m+n}C_m}(x)^n$

A

Next Lvl

Q3 Find Coff of x^2 in

$$(1-x+2x^2)\cdot\left(1+\frac{1}{x}\right)^{10}$$

$$T_{r+1} = {}^{10}C_r\left(\frac{1}{x}\right)^r = {}^{10}C_r\cdot x^{-r}$$

${}^{10}C_r\cdot x^{-r}$	$-{}^{10}C_r\cdot x^{-r+1}$	$2\cdot{}^{10}C_r\cdot x^{-r+2}$
$-r=2$	$-r+1=2$	$-r+2=2$
$r=-2$	$r=-1$	$r=0$
~~${}^{10}C_{-2}$~~	~~$-{}^{10}C_{-1}$~~	Coff $=2\cdot{}^{10}C_0$
		$=2$

Direct W/T h(M bhi sahi Ans Ata hai

∴ $\boxed{\text{Coff of } x^2 \text{ is } 2}$

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ध्यान

Q Find Coff of x^{50} in $\left((1-x+x^2)^{100}\cdot(1+x)^{101}\right)$

Coff of x^{50} in $\underbrace{(1-x+x^2)^{100}\cdot(1+x)^{100}}\cdot(1+x)$

$$\left((1-x+x^2)(1+x)\right)^{100}\cdot(1+x)$$

Coff of x^{50} $(1+x^3)^{100}\cdot(1+x)$

$${}^{100}C_r\cdot(x^3)^r$$

${}^{100}C_r\cdot x^{3r}$	${}^{100}C_r\cdot x^{3r+1}$
$3r=50$	$3r+1=50$
	$3r=49$
X	X
	$r=\frac{49}{3}$ 16.33

$0+0$

Coff of $x^{50}=0$

Q5 Coff of x^4 in $\left((1+x)^4(1+x^2)^5\right)$

$${}^4C_{r_1}x^{r_1}\cdot{}^5C_{r_2}(x^2)^{r_2}$$

$${}^4C_{r_1}\cdot x^{r_1}\cdot{}^5C_{r_2}\cdot(x)^{2r_2}$$

$${}^4C_{r_1}\cdot{}^5C_{r_2}\cdot x^{r_1+2r_2}=4 \text{ (Demand)}$$

$0+2r_2=4$

$1+2r_2=4$

$2r_2=3$

$r_2=1.5$ ×

$2+2r_2=4$

$3+2r_2=4$

X $r_2=.5$

$4+2r_2=4$

r_1	r_2
0	2
1	×
2	1
3	×
4	0

$$\text{Coff}={}^4C_0\cdot{}^5C_2+{}^4C_2\cdot{}^5C_1+{}^4C_4\cdot{}^5C_0$$

$$=1\times10+6\times5+1\times1$$

$$=10+30+1=41$$

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Q Find Coff of x^4 in $((2-x)+3x^2)^6$.

6

$G.T. = {}^6C_{r_1}(2-x)^{6-r_1}\cdot(3x^2)^{r_1}$

$= {}^6C_{r_1}\cdot {}^{6-r_1}C_{r_2}(2)^{6-r_1-r_2}(-x)^{r_2}\cdot(3)^{r_1}\cdot(x)^{2r_1}$

$= {}^6C_{r_1}\,{}^{6-r_1}C_{r_2}\cdot(2)^{6-r_1-r_2}\cdot(3)^{r_1}(-1)^{r_2}\cdot(x)^{r_2+2r_1=4}$

Coff $= {}^6C_2\cdot{}^4C_0(2)^4\,3^2\times -1^0$

$+ {}^6C_1\cdot{}^5C_2(2)^3(3)^1(-1)^2$

$+ {}^6C_0\cdot{}^6C_4\cdot(2)^2(3)^0(-1)^4$

$= \cancel{15}^{30}\times 72 + 60\times 24$

$+ \underline{60}$

$r_2+2r_1=4$

r_1	r_2
2	0
1	2
0	4

Q Coff of x^7 in $(1-x-x^2+x^3)^6$

$((1-x)-x^2(1-x))^6$

Coff of x^7 in $(1-x)^6(1-x^2)^6$

${}^6C_{r_1}(-x)^{r_1}\times{}^6C_{r_2}\cdot(-x^2)^{r_2}$

${}^6C_{r_1}\,{}^6C_{r_2}(-1)^{r_1}\cdot(-1)^{r_2}(x)^{r_1+2r_2}$

Coff

$= {}^6C_1\cdot{}^6C_3\cdot(-1)^1\cdot(-1)^3$

$+ {}^6C_3\,{}^6C_2\cdot(-1)^3(-1)^2$

$+ {}^6C_5\cdot{}^6C_1(-1)^5(-1)^1$

$r_1+2r_2=7$

r_1	r_2
1	3
3	2
5	1
~~7~~	~~0~~

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Q $(1+x)^m(1-x)^n = a_0 + a_1x^1 + a_2x^2 + a_3x^3 + \cdots$

Repeat

If $a_1 = a_2 = 10$ find $m, n = ?$

$G.T. = {}^mC_{r_1}(x)^{r_1} \cdot {}^nC_{r_2}(-x)^{r_2}$

$= {}^mC_{r_1} \cdot {}^nC_{r_2}(x)^{r_1+r_2} \cdot (-1)^{r_2}$

for $a_1 \rightarrow a_1$ in coff of x^1

$r_1 + r_2 = 1$

$(1,0)$ या $(0,1)$

$a_1 = {}^mC_1 \cdot {}^nC_0(-1)^0 + {}^mC_0 \cdot {}^nC_1(-1)^1$

$= m \times 1 \times 1 - 1 \times n = \boxed{m - n = 10}$

for a_2

coff of x^2

$r_1 + r_2 = 2$

$(2,0)$ या $(1,1)$ या $(0,2)$

$a_2 = {}^mC_2 \cdot {}^nC_0(-1)^0 + {}^mC_1 \cdot {}^nC_1(-1)^1 + {}^mC_0 \cdot {}^nC_2(-1)^2 = 10$

$= \frac{(m)(m-1)}{1 \times 2} \cdot 1 \cdot 1 - m \times n + 1 \times \frac{n(n-1)}{1 \times 2} = 10$

$m(m-1) - 2mn + n(n-1) = 20$

$(m^2 - 2mn + n^2) - m - n = 20$

$(m-n)^2 - (m+n) = 20$

$100 - 20 = m + n$

$m + n = 80$ & $m - n = 10$

$m = 45, n = 35$

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Binomial Coefficients World

(1) $^nC_r = {^nC_{n-r}}$

(2) If $^nC_a = {^nC_b}$ then either $a=b$ OR $a+b=n$

(3) $^nC_r = \frac{n}{r} \cdot {^{n-1}C_{r-1}}$ (D U S)

(4) $\underbrace{^nC_r + {^nC_{r-1}}}_{\text{diff}=1} = {^{n+1}C_r}$ (Same n)

(5) $C_0 + C_1 + C_2 + C_3 + \ldots = 2^n$

(6) $C_0 + C_2 + C_4 + \ldots = 2^{n-1}$

(7) $C_1 + C_3 + C_5 + \ldots = 2^{n-1}$

(5) $(1+x)^n = {^nC_0}\cdot x^0 + {^nC_1}\cdot x^1 + {^nC_2}x^2 + {^nC_3}\cdot x^3 + {^nC_4}x^4 + \ldots + {^nC_n}\cdot x^n$

Put $\boxed{x=1}$

$$\boxed{2^n = {^nC_0} + {^nC_1} + {^nC_2} + {^nC_3} + \ldots {^nC_n}} \rightarrow \boxed{\sum_{r=0}^{n} {^nC_r} = 2^n}$$

Put $x=-1$

$$0 = {^nC_0} - {^nC_1} + {^nC_2} - {^nC_3} + {^nC_4} - {^nC_5} + \ldots$$

$$\boxed{\underbrace{^nC_0 + {^nC_2} + {^nC_4} + {^nC_6} + \ldots}_{x} = \underbrace{^nC_1 + {^nC_3} + {^nC_5} + \ldots}_{x} = 2^{n-1}}$$

Rewriting

$$2^n = {^nC_0} + {^nC_1} + {^nC_2} + {^nC_3} + {^nC_4} + {^nC_5} + {^nC_6} + \ldots$$

$$= \underbrace{({^nC_0} + {^nC_2} + {^nC_4} + \ldots)} + \underbrace{({^nC_1} + {^nC_3} + {^nC_5} + \ldots)}$$

$$2^n = x + x \Rightarrow 2x = 2^n \Rightarrow x = 2^{n-1}$$

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$$\boxed{(1+x)^n = {}^nC_0 + {}^nC_1 \cdot x + {}^nC_2 \cdot x^2 + {}^nC_3 \cdot x^3 + \cdots}$$

When G.P is Multiplied to Bin. Coff.

AK

① Starting ${}^nC_0$ से हो

② ${}^nC_1$ Should be multiplied to x

Q. ${}^nC_0 + {}^nC_1 \cdot 2^1 + {}^nC_2 \cdot 2^2 + {}^nC_3 \cdot 2^3 + \cdots = ?$

GP

$= (1+2)^n = 3^n$

Q ${}^nC_0 + {}^nC_1 \cdot 4^2 + {}^nC_2 \cdot 4^4 + \cdots = ?$

GP

$= (1+16)^n = 17^n$

83

~~88~~

82

68

60

57

55

40

41