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6. $\frac{1}{2}\left(2\cos 2\theta \cos\frac{\theta}{2} - 2\cos 3\theta \cos\frac{9\theta}{2}\right)$

$= \frac{1}{2}\left(\cancel{\cos\frac{3\theta}{2}} + \underline{\cos\frac{5\theta}{2}} - \cancel{\cos\frac{3\theta}{2}} - \underline{\cos\frac{15\theta}{2}}\right)$

$= \frac{1}{2}\, 2\sin\frac{5\theta}{2}\sin 5\theta$

7. $\frac{1}{2}\left(\left(\cos 2B - \cos(\cancel{2A+2B})\right) - \left(\cos 2A - \cos(\cancel{2B+2A})\right)\right)$

$= \frac{1}{2}\left(\cos 2B - \cos 2A\right) = \sin(A-B)\sin(B+A)$

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10.

$$\frac{\cos A-\cancel{\cos 3A}+\cancel{\cos 3A}-\cancel{\cos 9A}+\cancel{\cos 9A}-\cos 17A}{\cancel{\sin 3A}-\sin A+\cancel{\sin 9A}-\cancel{\sin 3A}+\sin 17A-\cancel{\sin 9A}}$$

$$=\frac{\cos A-\cos 17A}{\sin 17A-\sin A}=\frac{2\cancel{\sin 8A}\sin 9A}{2\cancel{\sin 8A}\cos 9A}$$

$$=\tan 9A$$

12. $\frac{1}{2}\left(\cancel{\cos 72^\circ}+\cos 2A+\cancel{\cos 108^\circ}+\cos 2A\right)=\cos 2A.$

$180^\circ-72^\circ$

$-\cos 72^\circ$

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16.

$$\frac{1}{2}\Big(\sin(\beta-\gamma+\alpha-\delta)+\sin(\beta-\gamma-\alpha+\delta)+\sin(\gamma-\alpha+\beta-\delta)$$

$$+\sin(\gamma-\alpha-\beta+\delta)+\sin(\alpha-\beta+\gamma-\delta)+\sin(\alpha-\beta-\gamma+\delta)\Big)$$

$$=0$$

17.

$$\cos\frac{10\pi}{13}+\cos\frac{8\pi}{13}+\cos\frac{3\pi}{13}+\cos\frac{5\pi}{13}$$

$\pi-\frac{3\pi}{13}$ $\quad$ $\pi-\frac{5\pi}{13}$

$$=-\cos\frac{3\pi}{13}-\cos\frac{5\pi}{13}+\cos\frac{3\pi}{13}+\cos\frac{5\pi}{13}$$

$$=0$$

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7. $1+\tan A\tan\frac{A}{2} = \sec A$

$$= \frac{\tan A - \tan\frac{A}{2}}{\tan\frac{A}{2}} = \tan A\cot\frac{A}{2} - 1$$

$$\frac{\cos A\cos\frac{A}{2}+\sin A\sin\frac{A}{2}}{\cos A\cos\frac{A}{2}} = \frac{\cancel{\cos\left(A-\frac{A}{2}\right)}}{\cos A\cancel{\cos\frac{A}{2}}} = \sec A$$

$$\tan\left(A-\frac{A}{2}\right) = \frac{\tan A - \tan\frac{A}{2}}{1+\tan A\tan\frac{A}{2}}$$

$$= \frac{\tan A - \tan\frac{A}{2}}{\tan\frac{A}{2}}$$

$$= \frac{\cancel{\sin\left(A-\frac{A}{2}\right)}\ \cancel{\cos\frac{A}{2}}}{\cos A\cancel{\cos\frac{A}{2}}\ \cancel{\sin\frac{A}{2}}}$$

$$= \sec A.$$

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Multiple of angle

$2a^2 - 3ab + b^2 = 0$

$2A \quad 3A \quad \frac{a}{b} = t \quad 2\frac{a^2}{b^2} - \frac{3a}{b} + 1 = 0$

$2t^2 - 3t + 1 = 0 \Leftarrow$

$\sin 2A = \sin(A+A) = \sin A\cos A + \sin A\cos A$

$$\boxed{\sin 2A = 2\sin A\cos A = \frac{2\tan A}{1+\tan^2 A}}$$

$= 2\sin A\cos A = \left(\frac{2\sin A\cos A}{\cos^2 A}\right)\cos^2 A = \frac{2\tan A}{\sec^2 A} = \frac{2\tan A}{1+\tan^2 A}$

$= \frac{2\sin A\cos A}{\sin^2 A + \cos^2 A} = \frac{\left(\frac{2\sin A\cos A}{\cos^2 A}\right)}{\frac{\sin^2 A + \cos^2 A}{\cos^2 A}} = \frac{2\tan A}{1+\tan^2 A}$

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$$\cos 2A = \cos(A+A)$$

$$= \cos A \cos A - \sin A \sin A$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$= 1 - 2\sin^2 A$$

$$= 2\cos^2 A - 1$$

$$= \frac{1-\tan^2 A}{1+\tan^2 A}$$

$$= \cos^2 A\left(1 - \frac{\sin^2 A}{\cos^2 A}\right)$$

$$= \frac{1-\tan^2 A}{\sec^2 A} = \frac{1-\tan^2 A}{1+\tan^2 A}$$

$$\frac{\cos^2 A - \sin^2 A}{\cos^2 A + \sin^2 A} = \frac{(\cos^2 A - \sin^2 A)/\cos^2 A}{(\cos^2 A + \sin^2 A)/\cos^2 A}$$

$$= \frac{1-\tan^2 A}{1+\tan^2 A}$$

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$$\tan 2A = \tan(A+A) = \frac{\tan A + \tan A}{1 - \tan A \tan A}$$

$$\boxed{\tan 2A = \frac{2\tan A}{1-\tan^2 A}}$$

$$\cos 20^\circ = \cos^2 10^\circ - \sin^2 10^\circ$$

$$\sin 100^\circ = 2\sin 50^\circ \cos 50^\circ$$

$$\sin \frac{1}{2}^\circ = 2\sin \frac{1}{4}^\circ \cos \frac{1}{4}^\circ$$

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$$\boxed{\begin{aligned} 1+\cos 2A &= 2\cos^2 A \\ 1-\cos 2A &= 2\sin^2 A \end{aligned}}$$

$$\cos 2A = \cos^2 A - \sin^2 A$$
$$= 1 - 2\sin^2 A$$
$$= 2\cos^2 A - 1$$
$$= \frac{1-\tan^2 A}{1+\tan^2 A}$$

$$1+\sin 2A = \sin^2 A + \cos^2 A + 2\sin A\cos A$$

$$1-\sin 2A = \sin^2 A + \cos^2 A - 2\sin A\cos A$$

$$\boxed{\begin{aligned} 1+\sin 2A &= (\sin A+\cos A)^2 \\ 1-\sin 2A &= (\sin A-\cos A)^2 \end{aligned}}$$

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$$\sin 3A = \sin(2A + A) = \sin 2A \cos A + \cos 2A \sin A$$

$$= (2\sin A \cos A)\cos A + (1 - 2\sin^2 A)\sin A$$

$$= 2\sin A(1 - \sin^2 A) + \sin A(1 - 2\sin^2 A)$$

$$\sin 3A = 3\sin A - 4\sin^3 A \qquad \cos 3A = 4\cos^3 A - 3\cos A$$

$$A \to A + \frac{\pi}{2}$$

$$\sin\left(3A + \frac{3\pi}{2}\right) = 3\sin\left(\frac{\pi}{2} + A\right) - 4\sin^3\left(\frac{\pi}{2} + A\right)$$

$$-\cos 3A = 3\cos A - 4\cos^3 A$$

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$$\boxed{\begin{aligned} \sin 3A &= 3\sin A - 4\sin^3 A \\ \cos 3A &= 4\cos^3 A - 3\cos A \end{aligned}}$$

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P.T.

1. $(1+\sin 2A+\cos 2A)^2 = 4\cos^2 A(1+\sin 2A)$

$((1+\cos 2A)+\sin 2A)^2 = (2\cos^2 A+2\sin A\cos A)^2$

$=(2\cos A(\cos A+\sin A))^2$

$= 4\cos^2 A(\cos A+\sin A)^2$

$= 4\cos^2 A(1+\sin 2A)$

$((1+\sin 2A)+\cos 2A)^2$

$=((\cos A+\sin A)^2+\cos^2 A-\sin^2 A)^2$

$=((\cos A+\sin A)(2\cos A))^2$

2. $\dfrac{3-4\cos 2A+\cos 4A}{3+4\cos 2A+\cos 4A} = \tan^4 A$

$\dfrac{1+\cos 4A-4\cos 2A+2}{1+\cos 4A+4\cos 2A+2}$

$=\dfrac{2\cos^2 2A-4\cos 2A+2}{2\cos^2 2A+4\cos 2A+2}$

$\dfrac{(\cos 2A-1)^2}{(\cos 2A+1)^2} = \dfrac{(2\sin^2 A)^2}{(2\cos^2 A)^2} = \tan^4 A$

Nishant Jindal

Find the value of

3. $8\sin^3 40^\circ - 6\sin 40^\circ = 2(4\sin^3 40^\circ - 3\sin 40^\circ)$

$= -2(3\sin 40^\circ - 4\sin^3 40^\circ)$

$= -2\sin(120^\circ)$ $\to 180-60$

$= -2\sin 60$

$= -\sqrt{3}$

4. $\dfrac{\sin 3A}{\sin A} - \dfrac{\cos 3A}{\cos A}$

$\dfrac{3\sin A - 4\sin^3 A}{\sin A} - \dfrac{4\cos^3 A - 3\cos A}{\cos A}$

$3 - 4\sin^2 A - 4\cos^2 A + 3$

$= 6 - 4$

$= 2.$

$\dfrac{\sin 3A\cos A - \cos 3A\sin A}{\sin A\cos A}$

$= \dfrac{2\sin(3A - A)}{2\sin A\cos A} = \dfrac{2\sin 2A}{\sin 2A} = 2$

Ex-17

1, 2, 3, 4, 5, 6, 7, 8