S.H.M

Motion

↙ Periodic ↘ Non-periodic

Periodic ↓

Oscillatory :- A Type of periodic motion in which particle have to & fro motion about a fixed point Called (Mean position) under the influence of restoring force

↓

S.H.M → Restoring force if directly proportional to displacement of the particle from the mean position

$[F_r \propto x] \Rightarrow \boxed{F_r = -kx}$

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$F_r \propto x$

$F_r = -Kx$

(–) F_r always opposite to x

$x \rightarrow$ displacement from mean position

$a = -\frac{K}{m}x$

$\omega^2 = \frac{K}{m}$ $\omega =$ Angular frequency

$a = -\omega^2 x$

$\omega = \sqrt{K/m}$

$\omega = 2\pi f = \frac{2\pi}{T}$

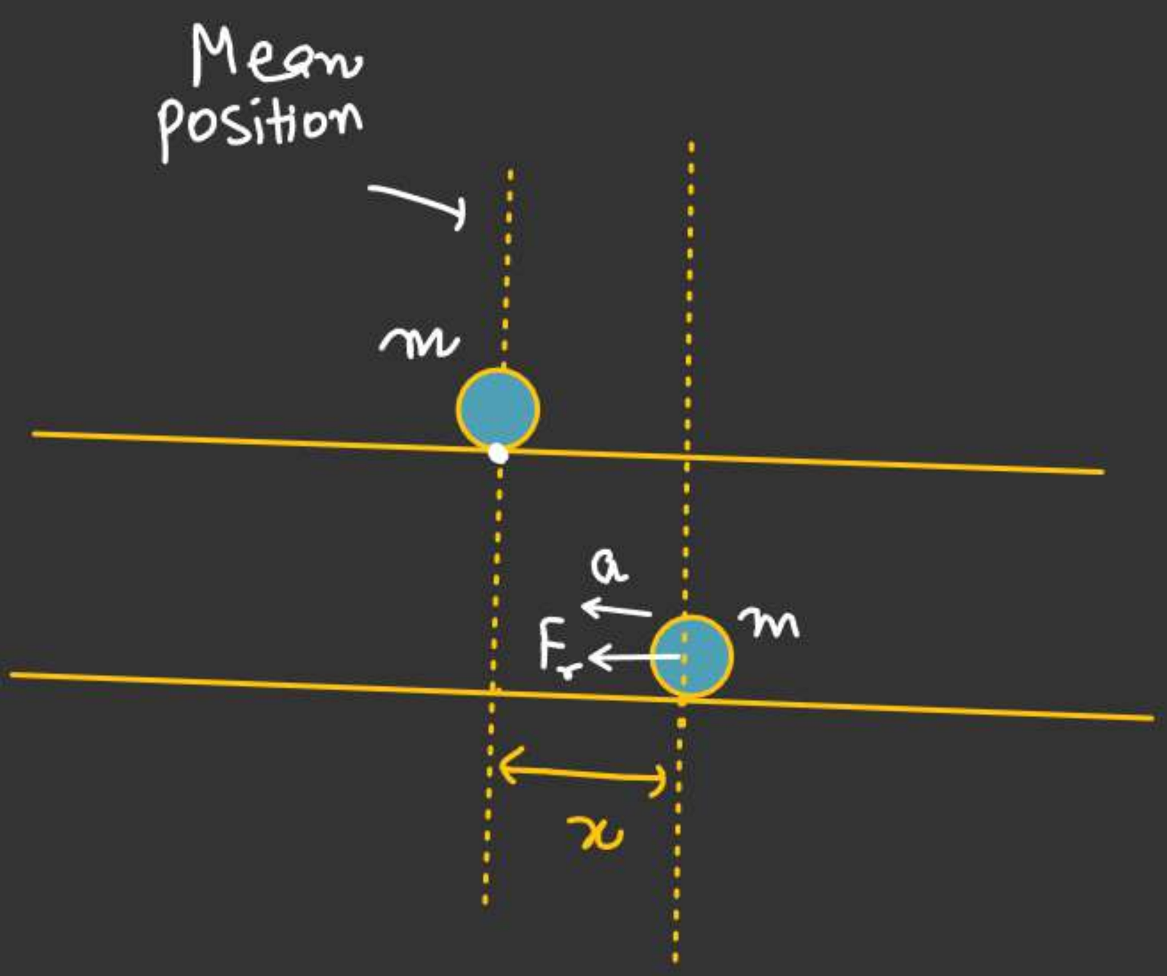
$\frac{d^2x}{dt^2} = -\omega^2 x$

$$\boxed{\frac{d^2x}{dt^2} + \omega^2 x = 0}$$

$$\boxed{x = \underbrace{A}_{\text{Amplitude}} \sin(\underbrace{\omega t + \phi}_{\text{Phase}})}$$

ϕ → Initial phase constant

Amplitude → (Maximum displacement of the particle from the mean position)



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$$a = -\omega^2 x$$

$$\downarrow$$

$$v\frac{dv}{dx} = -\omega^2 x$$

$$\int_{v_0}^{v} v\,dv = -\omega^2 \int_0^x x\,dx$$

$$\frac{v^2 - v_0^2}{\cancel{2}} = \frac{-\omega^2 x^2}{\cancel{2}}$$

$$v^2 = v_0^2 - \omega^2 x^2$$

$$\boxed{v = \sqrt{v_0^2 - \omega^2 x^2}}$$

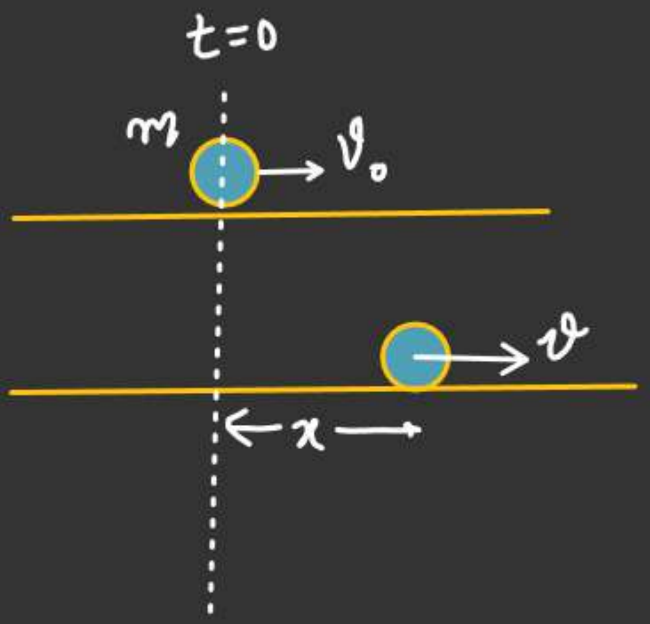
$$\frac{dx}{dt} = \sqrt{v_0^2 - \omega^2 x^2}$$

$$\int_0^x \frac{dx}{\sqrt{v_0^2 - \omega^2 x^2}} = \int_0^t dt$$

$$\int_0^x \frac{dx}{\omega\sqrt{\left(\frac{v_0}{\omega}\right)^2 - x^2}} = \int_0^t dt$$

$$\sin^{-1}\left[\frac{x}{\left(\frac{v_0}{\omega}\right)}\right]_0^x = \omega t$$

$$\frac{x}{\left(\frac{v_0}{\omega}\right)} = \sin\omega t$$



$$\frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\left(\frac{x}{a}\right)$$

$$x = \frac{v_0}{\omega}\sin\omega t$$

$$\Downarrow$$

$$\boxed{x = A\sin\omega t}$$

$$\boxed{x = A\sin(\omega t + \phi)}$$

$$\frac{v_0}{\omega} = A$$

$$\boxed{\begin{array}{l} v_0 = A\omega \\ v_{max} = A\omega \end{array}}$$

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$$\boxed{x = A\sin(\omega t + \phi)}$$

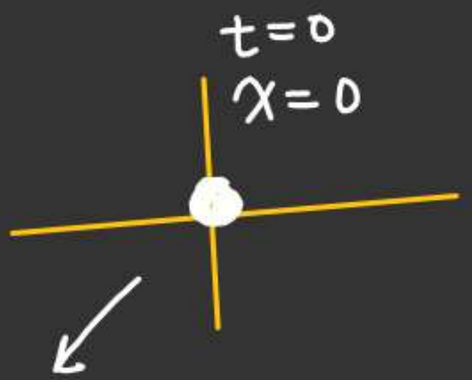
$\phi \rightarrow$ Initial phase Constant.

At $t = 0$, $x = 0$

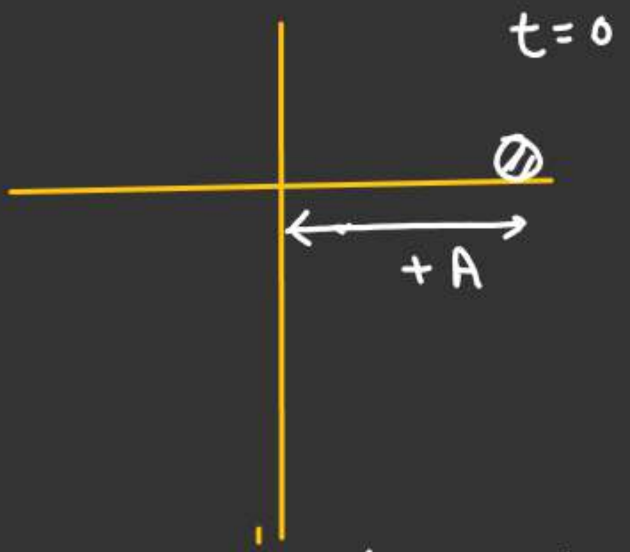
$x = A\sin\phi$

$0 = A\sin\phi$

$\phi = 0.$



$$\boxed{x = A\sin\omega t}$$



$x = A\sin(\omega t + \phi)$

At $t = 0$, $x = +A$

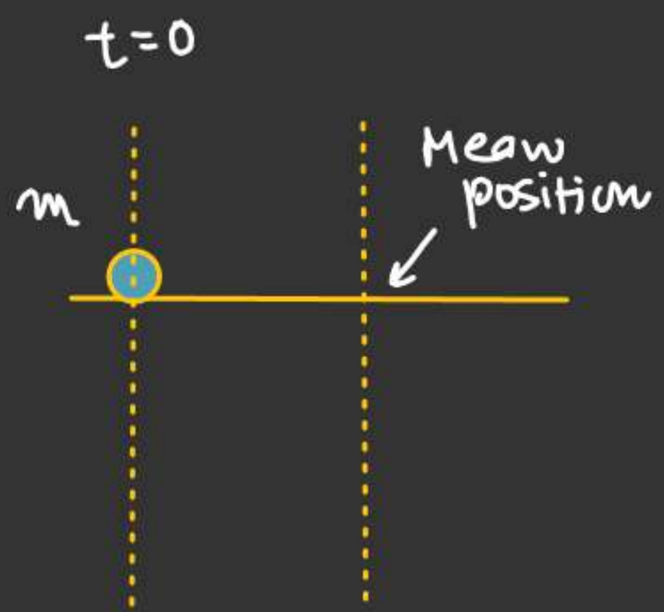
$+A = A\sin\phi$

$\sin\phi = +1$

$\phi = \frac{\pi}{2}$

$x = A\sin(\omega t + \frac{\pi}{2})$

$$\boxed{x = A\cos\omega t}$$



$x = -A$

$x = A\sin(\omega t + \phi)$

At $t = 0$, $x = -A$

$-A = A\sin\phi$

$\sin\phi = -1$

$\phi = \frac{3\pi}{2}$

$\underline{\underline{x}} = A\sin(\omega t + \frac{3\pi}{2})$

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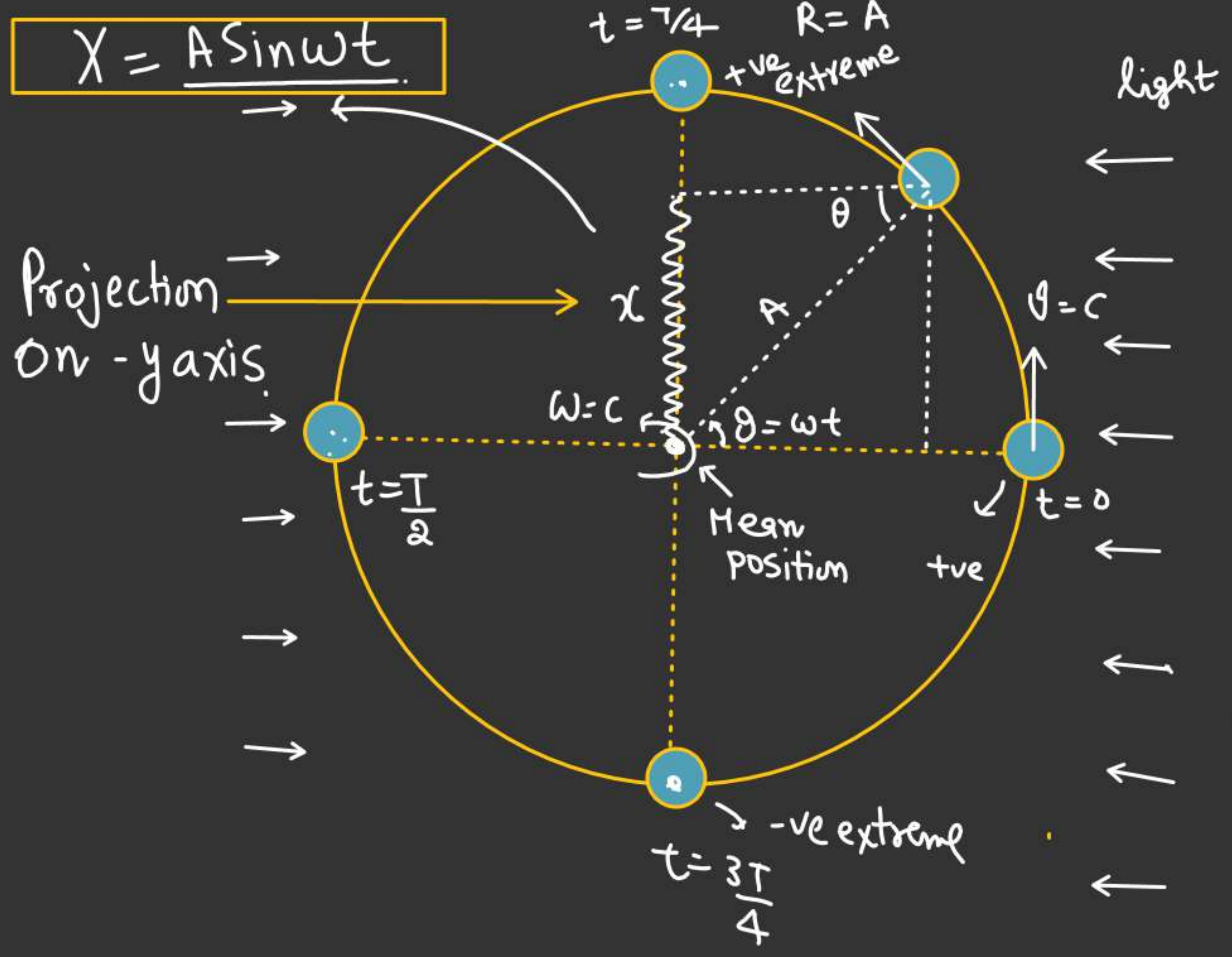
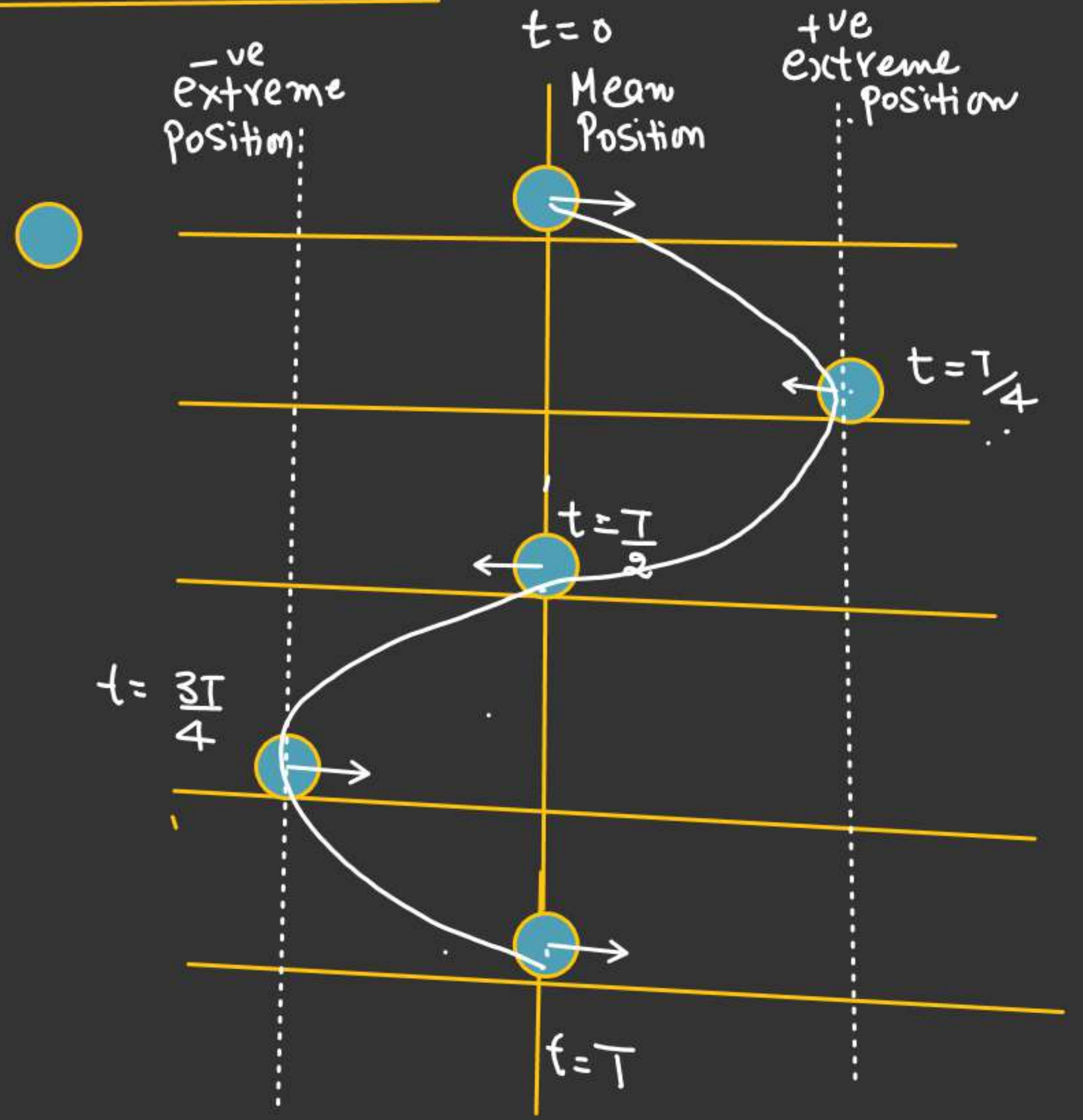
S.H.M as a projection of uniform Circular Motion

$$\boxed{X = A\sin\omega t}$$





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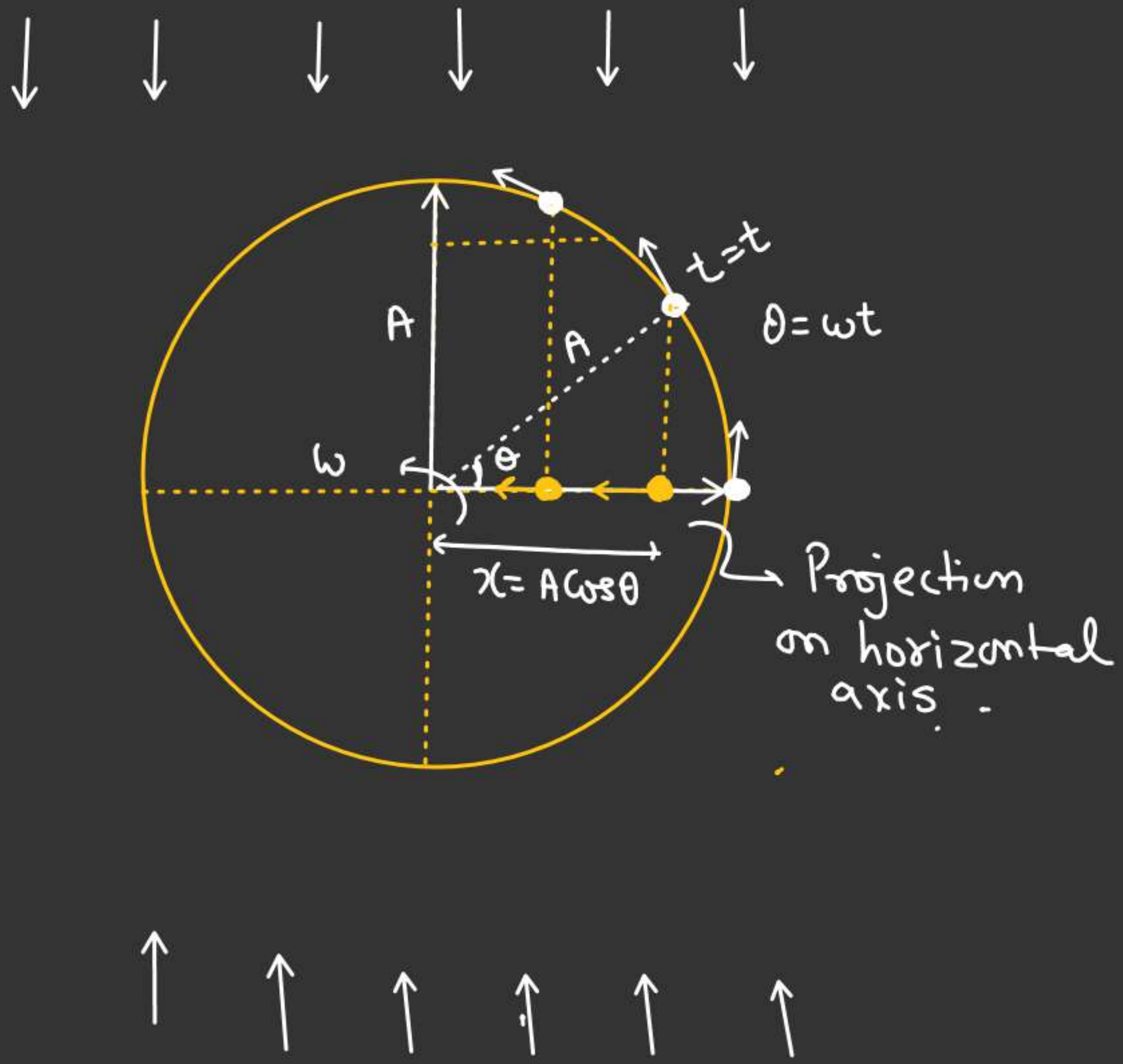
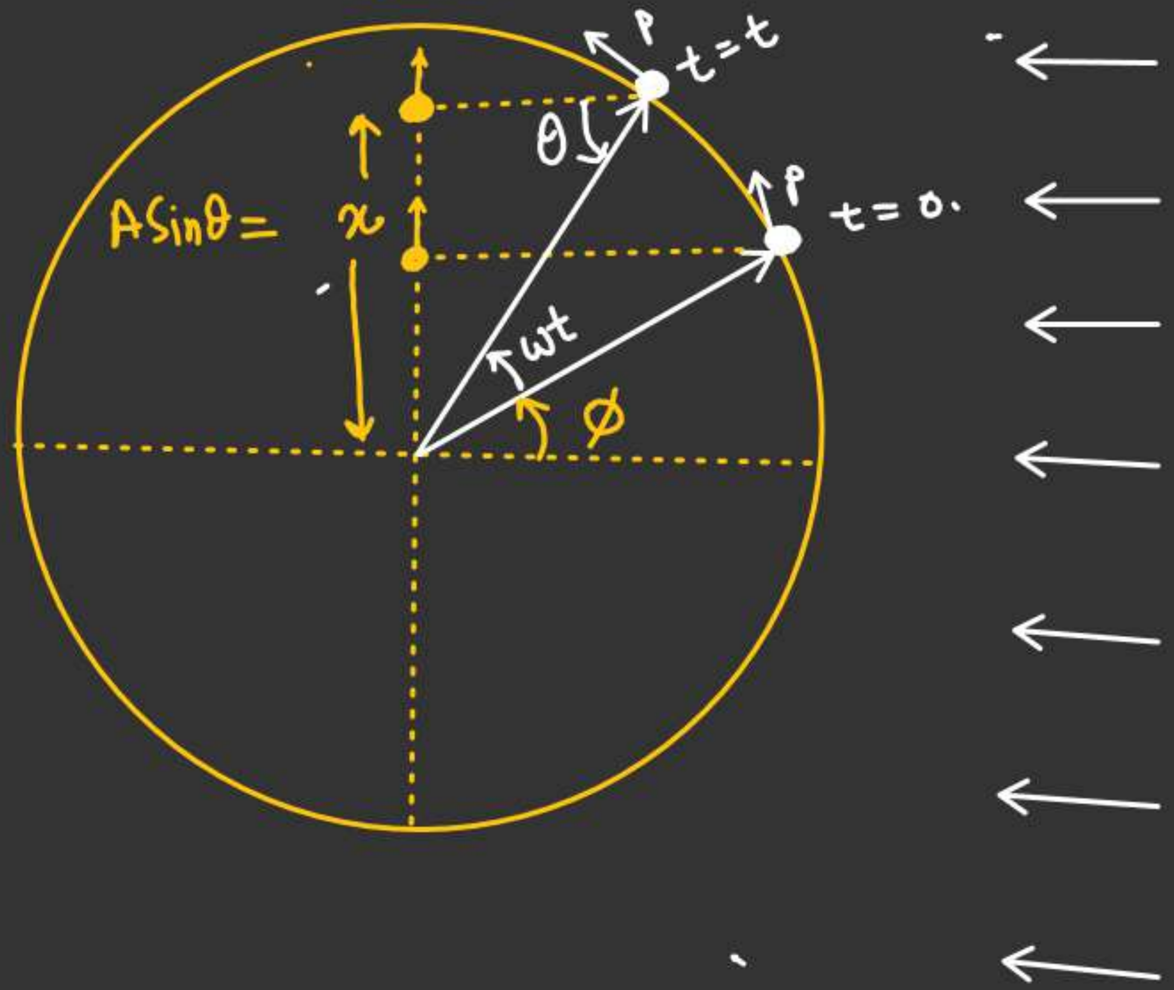
$x = A\cos\omega t = .$



$x = A\sin(\omega t+\phi)$ $\theta = (\omega t+\phi)$



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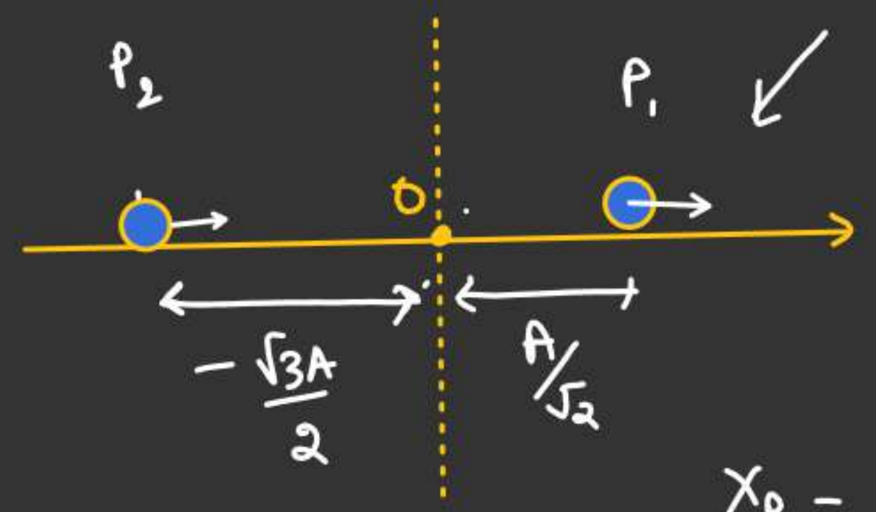
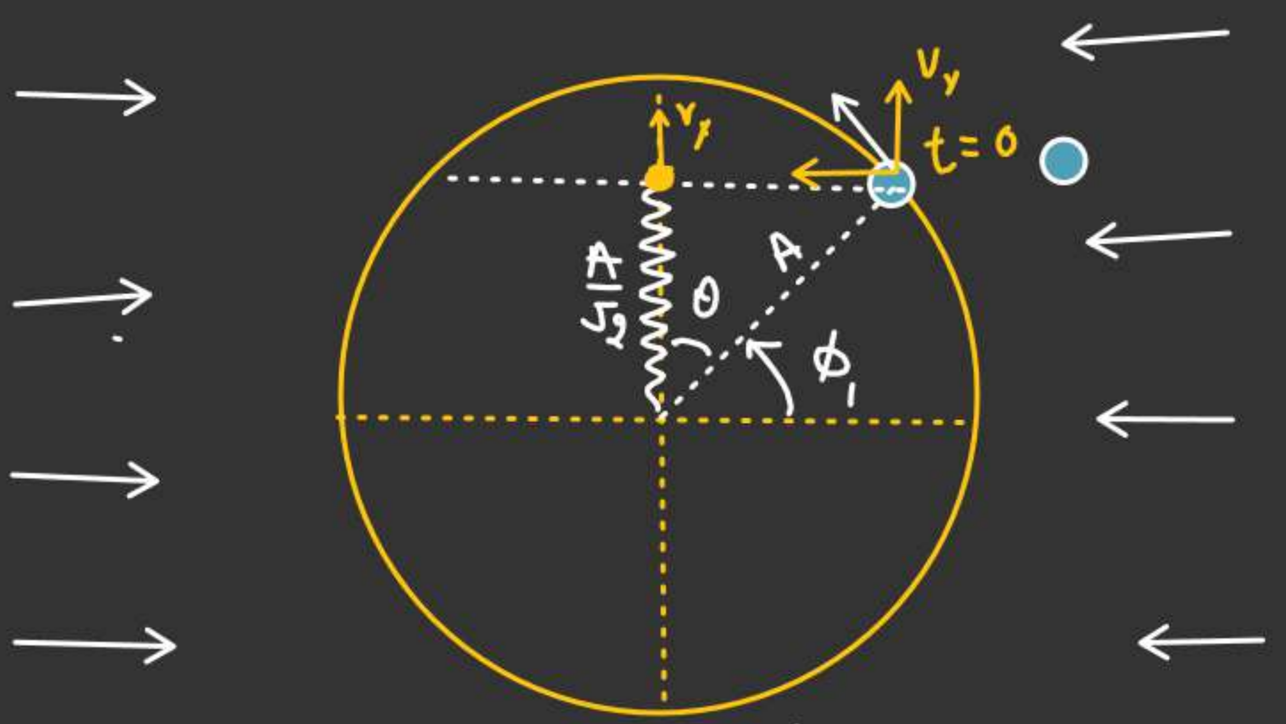
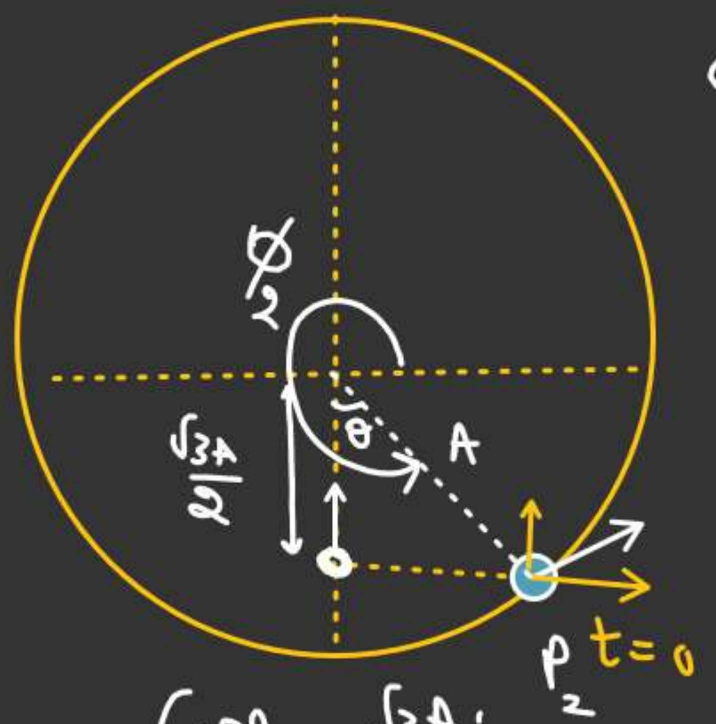


S.H.M

At $t=0$ position of P_1 and P_2 as shown in fig. Both have same amplitude and same angular frequency. Find the phase difference b/w P_1 and P_2

$$x_{P_1} = A\sin\left(\omega t + \frac{\pi}{4}\right) \qquad x_{P_2} = A\sin\left(\omega t + \frac{5\pi}{3}\right)$$





$$\cos\theta = \frac{A/\sqrt{2}}{A} = \frac{1}{\sqrt{2}}$$

$$\theta = \frac{\pi}{4} \Rightarrow \phi_1 = \frac{\pi}{4}$$

$$\cos\theta = \frac{\frac{\sqrt{3}A}{2}}{A} = \frac{\sqrt{3}}{2}$$

$$\theta = \pi/6$$

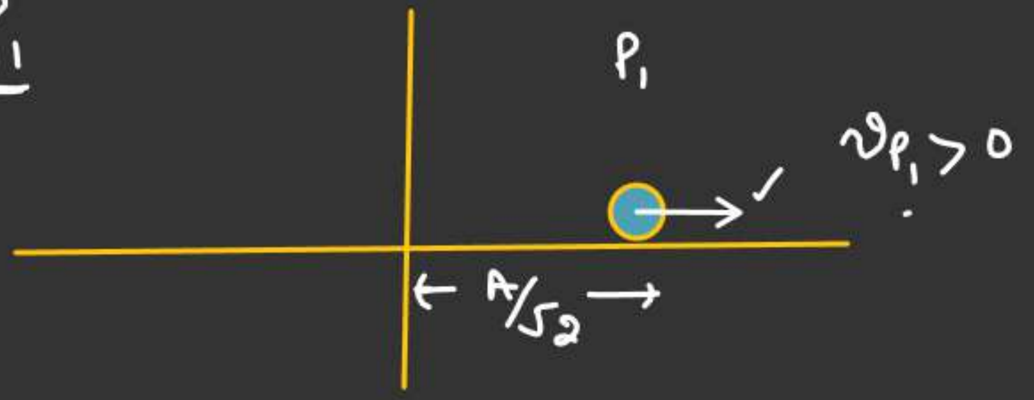
$$\phi_2 = \frac{3\pi}{2} + \theta = \frac{3\pi}{2} + \frac{\pi}{6} = \frac{9\pi + \pi}{6} = \frac{10\pi}{6} = \frac{5\pi}{3} \checkmark$$

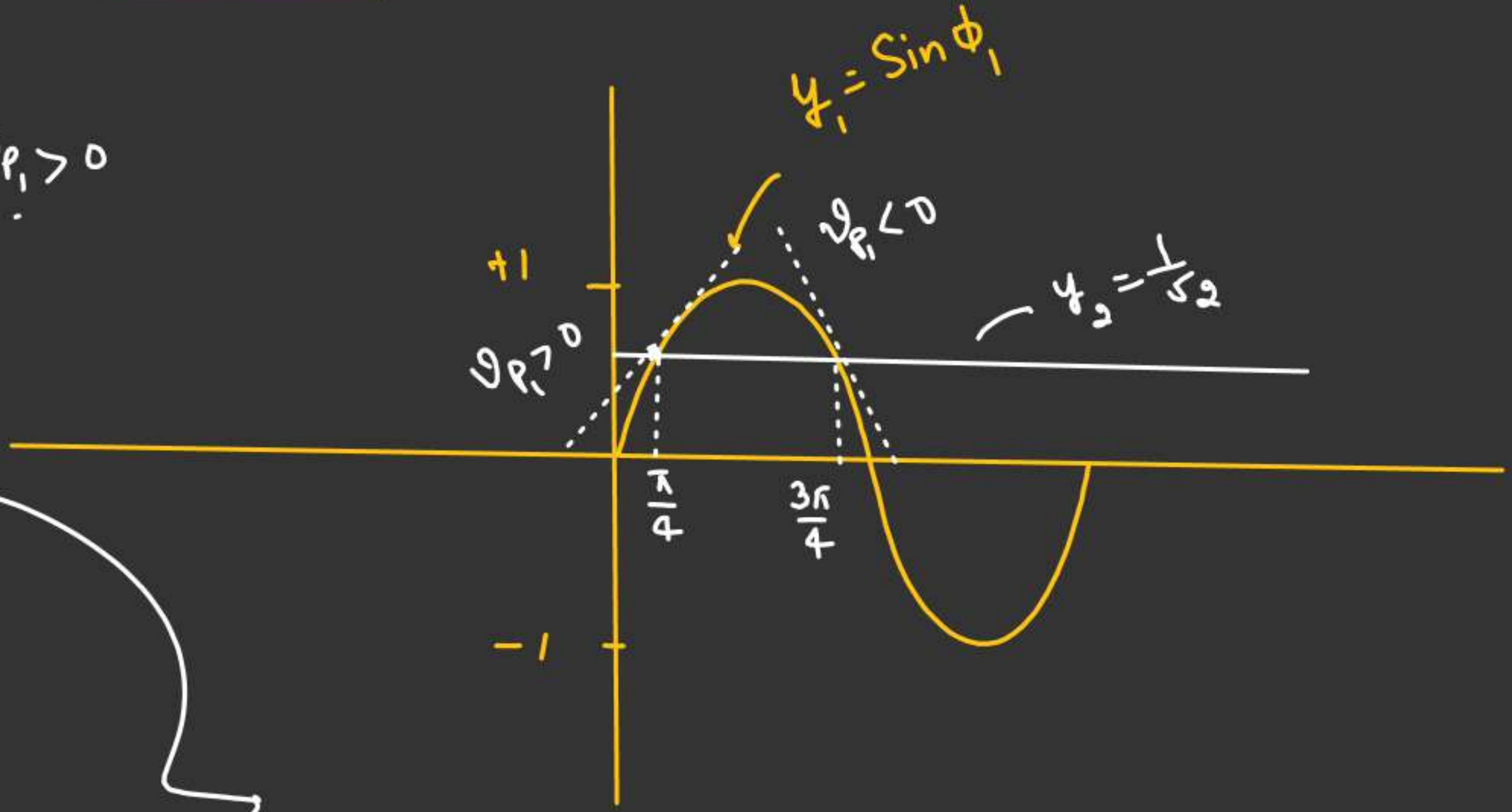
$$\Delta\phi = \phi_2 - \phi_1 = \frac{5\pi}{3} - \frac{\pi}{4} = \frac{20\pi - 3\pi}{12} = \left(\frac{17\pi}{12}\right) \checkmark$$

M-2

S.H.M

$\underline{P_1}$





$$X = A\sin(\omega t + \phi_1)$$

At $t = 0$, $x = +\frac{A}{\sqrt{2}}$

$$+\frac{A}{\sqrt{2}} = A\sin\phi_1$$

$$\sin\phi_1 = \frac{1}{\sqrt{2}} \rightarrow \phi_1 = \left(\frac{\pi}{4}, \frac{3\pi}{4}\right)$$

$y_1 = \sin\phi$

$y_2 = \frac{1}{\sqrt{2}}$

$$v_{P_1} = A\omega\cos(\omega t + \phi_1)$$

At $t = 0$

$$v_{P_1} = (A\omega\cos\phi_1)$$

$\phi_1 = \frac{\pi}{4}$ ans

v_{P_1} +ve at $\pi/4$

(v_{P_1}) at $\frac{\pi}{4}$ is +ve

(v_{P_1}) at $3\pi/4$ is −ve