DPP 06

Pure Rolling

1. velocity of C.M. of sphere be v. The velocity of the plank = 2v.

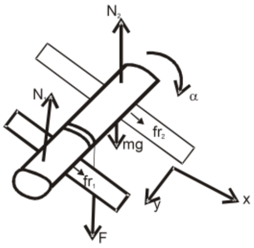
Kinetic energy of plank = $\frac{1}{2} \times m \times (2v)^2 = 2mv^2$

Kinetic energy of cylinder = $\frac{1}{2}mv^2 + \frac{1}{2} + \left(\frac{1}{2}mR^2\omega^2\right)$

$= \frac{1}{2}mv^2\left(1 + \frac{1}{2}\right) = \frac{3}{2}.\frac{1}{2}mv^2$

$\therefore \quad \frac{\text{K.E. of plank}}{\text{K.E. of sphere}} = \frac{2mv^2}{\frac{3}{4}mv^2} = \frac{8}{3}$

2.



Using Newton's second law in y and x dir.

$fr_1 + fr_2 = (mw_c)$ $\rightarrow$ x direction ...(i)

$N_1 + N_2 - mg - F = 0$ $\rightarrow$ y direction

$N_1 + N_2 = mg + F$ ...(ii)

Torque about axis of cylinder

$FR - (fr_1 + fr_2)R = \frac{mR^2}{2}\alpha = \frac{mR^2}{2}\left(\frac{w_c}{R}\right)$

pure rolling $\left(\frac{w_c}{R} = \alpha\right)$

$fr_1 + fr_2 \leq \mu(N_1 + N_2)$ ...(i)

from eq^1

$F \leq \left(\frac{3\mu gm}{2-3K}\right)$

$F_{max} = \left(\frac{3\mu gm}{2-3K}\right) = 10$ N

$w_{c\,max} = \frac{\mu(N_1+N_2)}{m}$

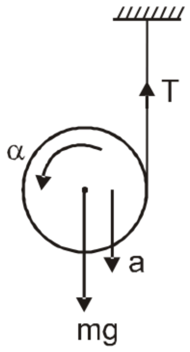
$= \frac{\mu}{m}(mg + F_{max}) = \frac{\mu}{m}\left(mg + \left(\frac{3\mu mg}{2-3K}\right)\right) = \left(\frac{2\mu g}{2-3\mu}\right) = \frac{20}{3}m/s^2$

3. For linear motion:

$mg - T = ma$ ...(i)

For angular motion:

$T.R. = \left(\frac{mR^2}{2}\right)\alpha$



$T = \frac{mR\alpha}{2}$ ...(ii)

For no slipping:

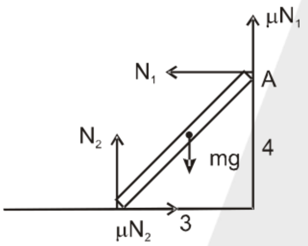
$a = R\alpha$ ...(iii)

From equation (i), (ii) & (iii)

$a = \frac{2}{3}g$

4. $N_1 = \mu N_2$,

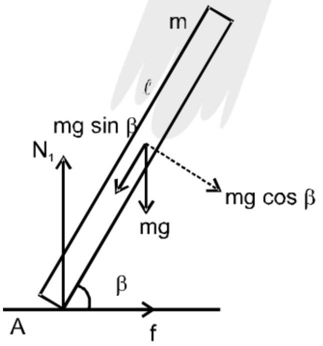
$\mu N_1 + N_2 = mg$, $\tau_A = 0 \Rightarrow$



$3 N_2 - 4 N_1 - \frac{3}{2} mg = 0$

Hence $\mu = \frac{1}{3}$

6.



Torque about A ds

$\tau = I\alpha$

(mssg $\cos\beta) \frac{\ell}{2} = \frac{m\ell^2}{3}\alpha$

$\alpha = \frac{3g\cos 60}{2\ell} = \left(\frac{3g}{4\ell}\right)$

using Newton 2^{nd} law

$mg - N = (ma_y)$

$N = (mg - ma_y)$ $\quad a_y: \alpha\left(\frac{\ell}{2}\cos\beta\right)$

$N = \left(mg - m\alpha\frac{\ell}{2}\cos\beta\right)$

$N = mg - m\ \frac{3g}{4\ell}\frac{\ell}{2}\ \cos 60 = mg - \frac{3mg}{6} = \left(\frac{13mg}{16}\right)$

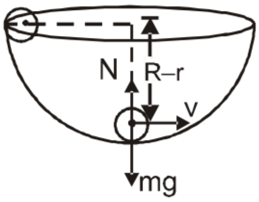
$f = (m\ a_x)$

$f = m\left(\alpha\frac{\ell}{2}\sin\beta\right)$

$f = m\left(\frac{3g}{4\ell}\right)\left(\frac{\ell}{2}\right)\ \sin 60$

$f = \frac{3mg}{4\ell}\frac{\ell}{2}\frac{\sqrt{3}}{2}$ $\quad \left(f = \frac{3\sqrt{3}}{16}mg\right)$

7.



Let R & r be the radii of hemispherical bowl & disc respectively

From energy conservation,

$mg\ (R - r) = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$

For pure rolling,

$v = r\omega$

$mg\ (R - r) = \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{1}{2}mr^2\right)\left(\frac{v}{r}\right)^2$

$mg\ (R - r) = \frac{3}{4}\ mv^2$ ...(i)

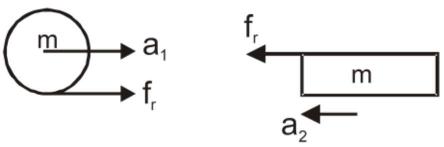
From FBD of bottom:

$N - mg = \frac{mv^2}{(R-r)}$ ...(ii)

From equ. (i) & (ii),

$N = \frac{7}{3}mg$

8. FBD for sphere & block



$a_1 = \frac{f_r}{m} = \frac{\mu mg}{m}$ $\quad a_2 = \frac{f_r}{m} = \frac{\mu mg}{m}$

$\vec{a}_1 = \mu g\hat{i}$ $\quad \vec{a}_2 = -\mu g\hat{i}$

$\vec{a}_{rel} = \vec{a}_1 - \vec{a}_2 = 2\mu g\hat{i}$

$a_{rel} = 2\mu g.$

Sol 9 to 11

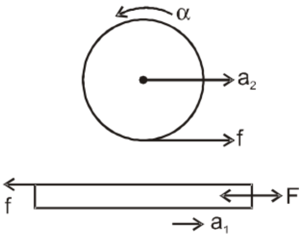
Sol. The free body diagram of plank and disc is

Applying Newton's second law

$F - f = Ma_1$ .... (1)

$f = Ma_2$ .... (2)

$FR = \frac{1}{2}MR^2\alpha$ .... (3)



from equation 2 and 3

$a_2 = \frac{R\alpha}{2}$

From constraint $a_1 = a_2 + R\alpha$

$\therefore a_1 = 3a_2$ .... (4)

Solving we get $a_1 = \frac{3F}{4M}$ and $\alpha = \frac{F}{2MR}$

If sphere moves by x the plank moves by L + x. The from equation (4)

$L + x = 3x$ or $x = \frac{L}{2}$