Nishant Jindal

Khushiyan hi Khushiyan. Seq^n & Prog

$$\frac{a(1-r^n)}{(1-r)}$$

Q If $x=\sum_{n=0}^{\infty} a^n$, $y=\sum_{n=0}^{\infty} b^n$, $z=\sum_{n=0}^{\infty} (ab)^n$

$a=\frac{x-1}{x}$ $\quad b=\frac{y-1}{y}$ $\quad ab=\frac{z-1}{z}$

$$\Rightarrow \left(\frac{x-1}{x}\right)\left(\frac{y-1}{y}\right)=\frac{z-1}{z}$$

$$\Rightarrow z(x-1)(y-1)=xy(z-1)$$

$$\Rightarrow z(xy-y-x+1)=xyz-xy$$

$$\Rightarrow \cancel{xyz}-yz-xz+z=\cancel{xyz}-xy$$

$$\boxed{xy+z=xz+yz} \text{ H.P.}$$

Q $\frac{1}{2}+\frac{3}{4}+\frac{7}{8}+\frac{15}{16}+\dots$ n terms

$$\left(1-\frac{1}{2}\right)+\left(1-\frac{1}{4}\right)+\left(1-\frac{1}{8}\right)+\left(1-\frac{1}{16}\right)+\dots$$

$$\Rightarrow n-\left(\frac{1}{2}+\frac{1}{2^2}+\frac{1}{2^3}+\dots n\text{ terms}\right) \quad r=\tfrac{1}{2}$$

$$=n-\frac{\frac{1}{2}\left(1-\left(\frac{1}{2}\right)^n\right)}{\cancel{\left(1-\frac{1}{2}\right)}}$$

$$=n-\left(1-\left(\tfrac{1}{2}\right)^n\right)$$

$$=n-1+\frac{1}{2^n}$$

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Q Sum of n terms of Series.

good

$$\frac{5}{2}+\frac{7}{4}+\frac{11}{8}+\frac{19}{16}+\cdots ?$$

$$\left(1+\frac{3}{2}\right)+\left(1+\frac{3}{4}\right)+\left(1+\frac{3}{8}\right)+\left(1+\frac{3}{16}\right)+\cdots$$

$$\underbrace{(1+1+\cdots)}_{\leftarrow n\text{ times}\rightarrow}+\left(\frac{3}{2}+\frac{3}{4}+\frac{3}{8}+\frac{3}{16}+\cdots\right)$$

$$n+3\left(\frac{1}{2}+\frac{1}{2^2}+\frac{1}{2^3}+\cdots n\text{ term}\right)$$

$$n+3\times\frac{\left(\frac{1}{2}\right)\left(1-\left(\frac{1}{2}\right)^n\right)}{\left(1-\frac{1}{2}\right)}$$

$$\Rightarrow n+3-3\left(\frac{1}{2^n}\right)$$

Q let α,β,γ are Roots of $2x^3+9x^2-27x-54=0$

If α,β,γ are in GP then find value of $\frac{2}{3}(|\alpha|+|\beta|+|\gamma|)$

$$2x^3+9x^2-27x-54=0 \to \alpha, \beta, \gamma$$

$ax^3+bx^2+cx+d=0$

$\alpha\cdot\beta\gamma=-\frac{d}{a}$

$$\alpha\cdot\beta\cdot\gamma=-\frac{(-54)}{2}=27$$

$$\frac{a}{r}\times a\times ar=27 \Rightarrow a^3=27\Rightarrow \boxed{a=3}=\beta$$

Roots are value of x

$\boxed{x=3}$

$$(x-3)(2x^2+15x+18)=0$$

$$(x-3)(2x+3)(x+6)=0$$

$-\frac{1}{2}$ $-\frac{1}{2}$

$$x=3,-\frac{3}{2},-6\to \underset{\alpha}{-6},\underset{\beta}{3},\underset{\gamma}{-\frac{3}{2}}$$

$$\frac{2}{3}\left(|-6|+|3|+\left|-\frac{3}{2}\right|\right)=\frac{2}{3}\left(6+3+\frac{3}{2}\right)=7$$

Nishant Jindal

Q If Sum of ∞ GP is 15 & the Sum of Series obtained on Sqring every term of this GP is 45. Find Series.

Series $= 5, \frac{10}{3}, \frac{20}{3^2}, \frac{40}{27}$ - - - —

① $a, ar, ar^2 - - - \infty$

$S_\infty = \frac{a}{1-r} = 15$

② $\overset{r^2}{\frown} \overset{r^2}{\frown}$ $a^2, a^2r^2, a^2r^4, a^2r^6 - - - \infty$

$S'_\infty = \frac{a^2}{1-r^2} = 45$

$$\frac{\left(\frac{\cancel{a^2}}{1-r^2}\right)}{\frac{\cancel{a^2}}{(1-r)^2}} = \frac{\cancel{45}}{\cancel{225}} \frac{1}{5} \Rightarrow \frac{(1-r)^2}{\cancel{1-r^2}\ 1+r} = \frac{1}{5}$$

$\Rightarrow 5 - 5r = 1 + r \Rightarrow 6r = 4 \Rightarrow \boxed{r = \frac{2}{3}}$

$\frac{a}{1-\frac{2}{3}} = 15$

$a = \frac{1}{\cancel{3}} \times \cancel{15}^{5}$

$\boxed{a = 5}$

Nishant Jindal

Q let T_n be the n^{th} term of a seq^n for $n=1,2,3,4---$

If $\boxed{4T_{n+1}=T_n}$ & $\boxed{T_5=\frac{1}{2560}}$ then value of $\sum_{n=1}^{\infty} T_{n+1}\cdot T_n = ?$

$$\boxed{\frac{T_{n+1}}{T_n}=\frac{1}{4}=r}$$

क्रमागत Consecutive Term का Ratio.

$$T_{n+1}=\frac{T_n}{4}$$

$$ar^4=\frac{1}{2560}$$

$$a\times\frac{1}{4^4}=\frac{1}{2560}$$

$$\frac{a}{\cancel{256}}=\frac{1}{\cancel{2560}}$$

$$\boxed{a=\frac{1}{10}}$$

$$\sum \frac{T_n}{4}\cdot T_n$$

$$\frac{1}{4}\sum_{n=1}^{\infty} T_n^2$$

$$\frac{1}{4}\{T_1^2+T_2^2+T_3^2+---\}$$

$$\frac{1}{4}\{a^2+a^2r^2+a^2r^4+a^2r^6+---\}$$

$$\frac{1}{4}\times\frac{a^2}{1-r^2}=\frac{1}{4}\times\frac{\frac{1}{100}}{1-\frac{1}{16}}=\frac{1}{\cancel{4}}\times\frac{1}{\cancel{100}_{25}}\times\frac{\cancel{16}^{4}}{15}$$

$$=\frac{1}{375}$$

Nishant Jindal

Q If $a = 66666\ldots n \text{ times}$, $b = \frac{8}{9}(10^n - 1)$, $c = \frac{4}{9}(10^{2n} - 1)$

Then P.T. $a^2 + b = c$?

Basic

666

$= 600 + 60 + 6$

$= 6\times10^2 + 6\times10 + 6$

$= 6(10^2 + 10 + 1)$

$a = 66666\ldots n \text{times}$

$= 6(10^{n-1} + 10^{n-2} \ldots 10^2 + 10 + 1)$

$= 6(1 + 10 + 10^2 + 10^3 \ldots 10^{n-1})$

← n पदों

$= 6 \times 1 \cdot \frac{(10^n - 1)}{(10 - 1)} = \frac{6}{9}(10^n - 1)$

$$\boxed{a = \frac{2}{3}(10^n - 1)}$$

Demand.

$a^2 + b$

$\frac{4}{9}(10^n - 1)^2 + \frac{8}{9}(10^n - 1)$

$\frac{4}{9}(10^n - 1)\{10^n - 1 + 2\}$

$\frac{4}{9}(10^n - 1)(10^n + 1)$

$\frac{4}{9}((10^n)^2 - 1^2)$

$\frac{4}{9}(10^{2n} - 1) = c$

H.P.

Nishant Jindal

$g_n = a\cdot r^{n-1}$

Q let g_n be the n^{th} term of G.P of +ve No. If

$\sum_{n=1}^{100} g_{2n} = \frac{10}{3}$ & $\sum_{n=1}^{100} g_{2n-1} = \frac{5}{9}$ then C.R = ?

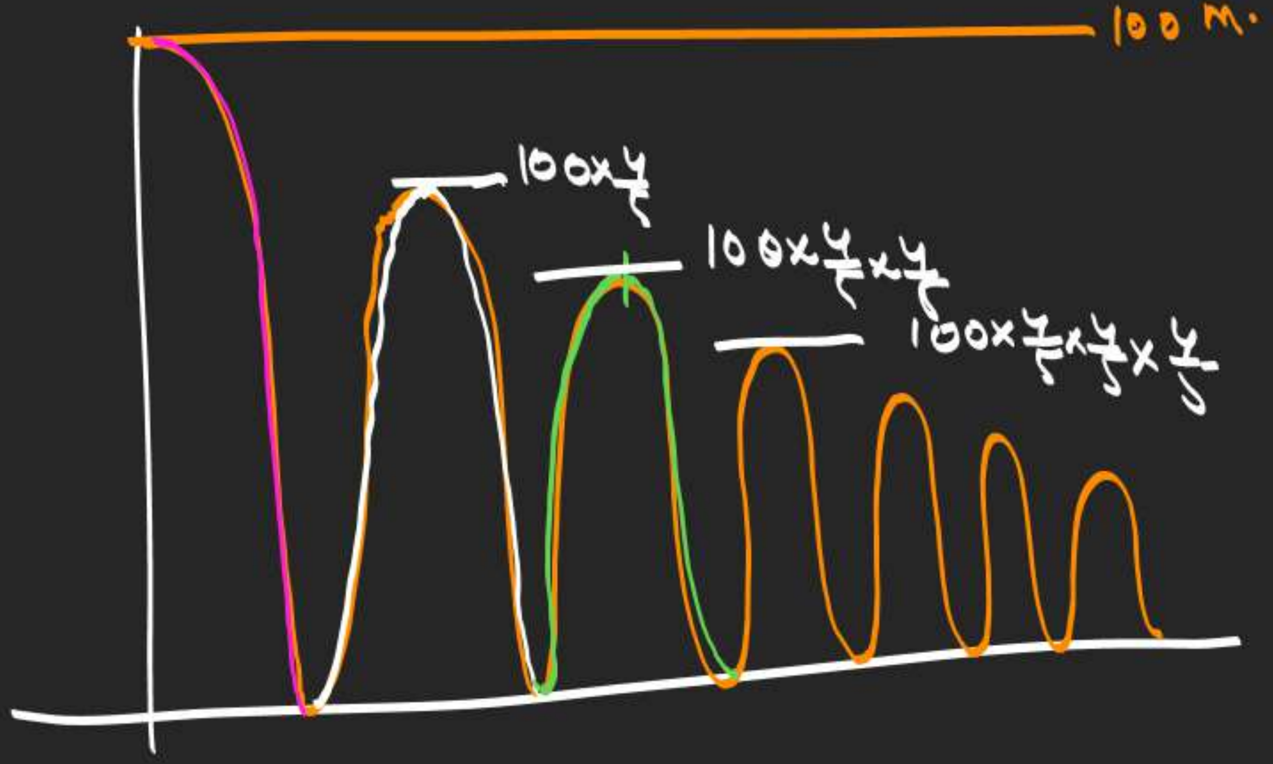
① $\sum_{n=1}^{100} g_{2n} = \sum_{n=1}^{100} a\cdot r^{2n-1} = a\cdot r^1 + ar^3 + ar^5 + ar^7 - — ar^{199} = \frac{10}{3}$

② $\sum_{n=1}^{100} g_{2n-1} = \sum a\cdot r^{2n-1-1} = \sum_{n=1}^{100} ar^{2n-2} = ar^0 + ar^2 + ar^4 + \dots + ar^{198} = \frac{5}{9}$

$\Rightarrow \dfrac{r\,(\cancel{a+ar^2+ar^4+ar^6 - — ar^{198}})}{(\cancel{a+ar^2+ar^4+ar^6 - — ar^{198}})} = \dfrac{10/3}{5/9}$

$r = \frac{\cancel{10}^2}{\cancel{3}} \times \frac{\cancel{9}^3}{\cancel{5}} = 6$

Q A rubber ball is dropped from a hieght of 100 m. It will Rebound $\frac{4}{5}$ times of its dropped ht. every time, then find the distance covered by the ball before coming to Rest



$$100 + \frac{4}{5}\times 100 \times 2 + \frac{4}{5}\times\frac{4}{5}\times 100\times 2 + \frac{4}{5}\times\left(\frac{4}{5}\times\frac{4}{5}\times 100\right)\times 2$$

$$100 + 200\left\{\frac{4}{5} + \left(\frac{4}{5}\right)^2 + \left(\frac{4}{5}\right)^3 + \cdots\right\}$$

$$100 + 200\times\frac{\frac{4}{5}}{1-\frac{4}{5}} = 100 + 200\times\frac{4}{\cancel{5}}\times\frac{\cancel{5}}{1}$$

$$= 900\text{ m}$$

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Nishant Jindal

Properties of GP

1) a, b, c in GP $\Rightarrow$ $b^2 = ac$.

2) If $a, b, c \rightarrow$ GP then $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ GP $\rightarrow$ $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}$ GP $\Rightarrow$ 2, 4, 8 GP.

3) a, b, c GP then $K \cdot a, Kb, Kc \rightarrow$ GP $\rightarrow$ 2, 4, 8 GP $\xrightarrow{\times 3}$ 6, 12, 24 GP

4) a, b, c GP then $\frac{a}{K}, \frac{b}{K}, \frac{c}{K} \rightarrow$ GP $\rightarrow$ 2, 4, 8 GP $\xrightarrow{\div 7}$ $\frac{2}{7}, \frac{4}{7}, \frac{8}{7}$ GP? $\left(\frac{4}{7}\right)^2 = \frac{8}{7} \times \frac{2}{7}$

$\frac{16}{49} = \frac{16}{49}$ ✓

5) a, b, c GP then a^K, b^K, c^K GP $\rightarrow$ 2, 4, 8 GP $\xrightarrow{1/2}$ $2^{1/2}, 4^{1/2}, 8^{1/2}$

$\Rightarrow \sqrt{2}, \boxed{2}, 2\sqrt{2}$

$(2)^2 = \sqrt{2} \times 2\sqrt{2}$

$4 = 4$ ✓

6) Multiply or Divide of 2 GP is also a GP.

(7) Product of equidistant term Remain Constant in GP.

2, 4, 8, 16, 32, 64, 128 $\Rightarrow 128 \times 2 = 4 \times 64 = 8 \times 32 = 16^2$

(8) a, b, c in GP then $\log a, \log b, \log c$ in AP

Nishant Jindal

$$T_n = \frac{(n)(n+1)}{2}$$

Q let $T_n = 1+2+3+\dots n$. If

$$\sum_{i=3}^{\infty}\sum_{K=1}^{\infty}\left(\frac{3}{T_i}\right)^K = \frac{p}{q}$$; when p, q are coprime

then find value of $(3q-p-8) = ?$ $\quad (150-137-8) = 5$

$$\sum \sum_{K=1}^{\infty}\left(\frac{3}{T_i}\right)^K = \sum\left\{\left(\frac{3}{T_i}\right)^1+\left(\frac{3}{T_i}\right)^2+\left(\frac{3}{T_i}\right)^3+\dots \infty\right\}$$ ∞ GP

$$= \sum_{i=3}^{\infty}\frac{\left(\frac{3}{T_i}\right)}{1-\left(\frac{3}{T_i}\right)} \Rightarrow \sum_{i=3}^{\infty}\frac{\frac{3\times 2}{(i)(i+1)}}{1-\frac{3\times 2}{(i)(i+1)}} = \sum \frac{6}{\cancel{(i)(i+1)}}\times\frac{\cancel{(i)(i+1)}}{(i)(i+1)-6} = 6\sum_{i=3}^{\infty}\frac{1}{i^2+i-6}$$

$$\boxed{40}$$

$$6\sum_{i=3}^{\infty}\frac{1}{(i+3)(i-2)} = \frac{6}{5}\sum_{i=3}^{\infty}\frac{1}{(i-2)}-\frac{1}{(i+3)}$$

$$\frac{6}{5}\left\{\left(\frac{1}{1}-\cancel{\frac{1}{6}}\right)+\left(\frac{1}{2}-\cancel{\frac{1}{7}}\right)+\left(\frac{1}{3}-\cancel{\frac{1}{8}}\right)+\left(\frac{1}{4}-\cancel{\frac{1}{9}}\right)+\left(\frac{1}{5}-\cancel{\frac{1}{10}}\right)+\left(\cancel{\frac{1}{6}}-\frac{1}{11}\right)-\dots\right.$$

$$\frac{6}{5}\times\left(1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}\right) = \frac{\cancel{6}}{5}\times\frac{60+30+20+15+12}{\cancel{60}\,10}$$

$$\frac{137}{50} = \frac{p}{q}$$

HW2 Complete