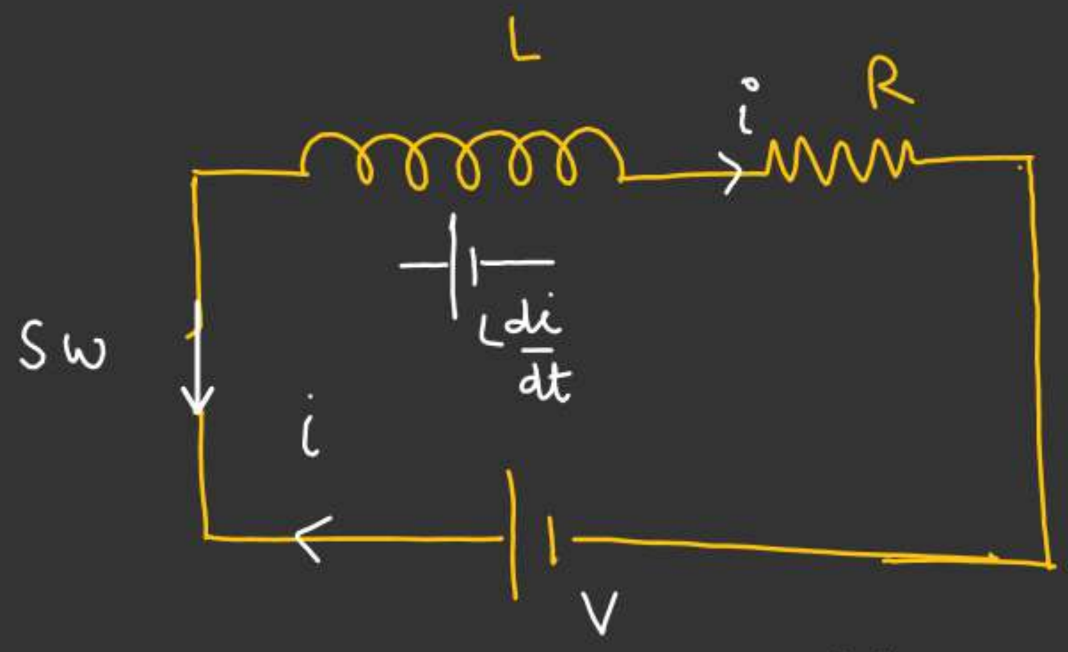
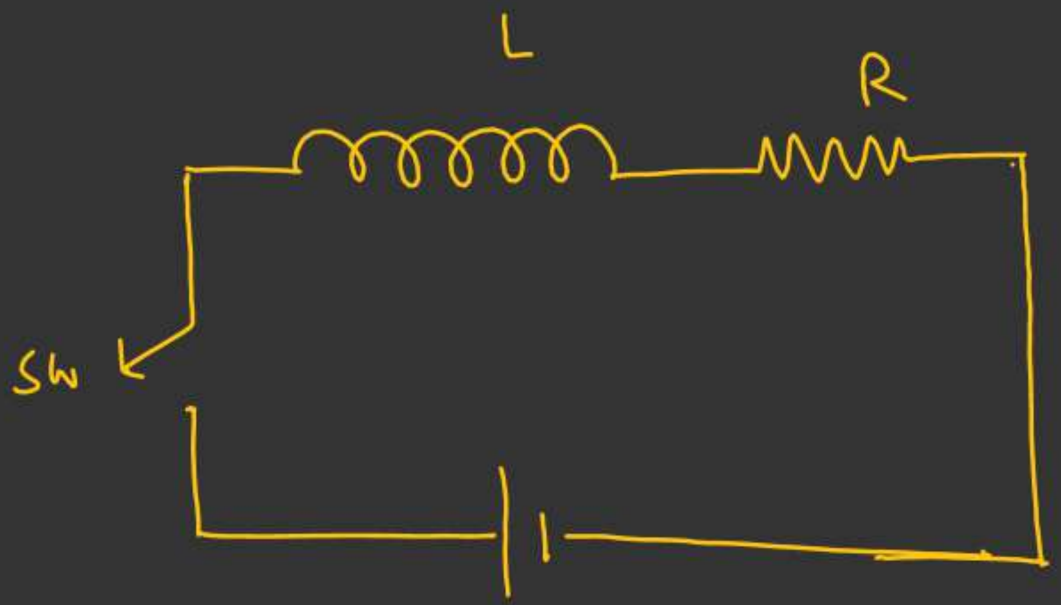
Nishant Jindal

L-C OSCILLATION

Energy Stored in Inductor





$$V - L\frac{di}{dt} - iR = 0$$

$$V = L\frac{di}{dt} + iR$$

$$Vi = L\,i\frac{di}{dt} + i^2R \quad \left[\text{Multiplying both side by } i\right]$$

$$\int_0^i Vi\,dt = L\int_0^i i\,di + \int_0^i i^2R\,dt$$

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$$\int_0^i Vi\,dt = \underbrace{L\int_0^i i\,di}_{} + \int_0^i i^2R\,dt$$

$$\frac{dW}{dt} = P = Vi$$

$$W_{battery} = \int_0^i Vi\,dt$$

$$\frac{dH}{dt} = i^2R$$

$$\int_0^H dH = \int_0^i i^2R\,dt$$

$$\Downarrow$$

Total heat dissipiated across resister.

$$L\int_0^i i\,di = U_L$$

$$\boxed{U_L = \frac{1}{2}Li^2}$$

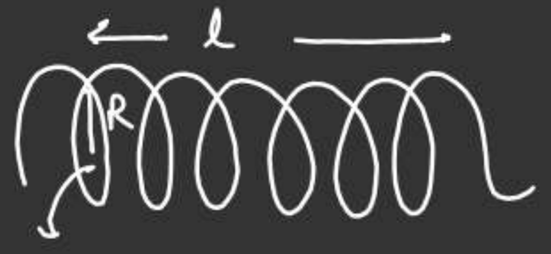
Self inductance of an Inductor

$$L = \mu_0 n^2 \underline{\pi R^2}\,\ell$$

$$\Downarrow$$

Volume = $A\ell$.

$$L = \mu_0 n^2 (A\ell)$$



$A = \pi R^2$

$$U_L = \frac{1}{2} \times \mu_0 n^2 (A\ell)\,\frac{B^2}{(\mu_0 n^2)}$$

$n = \frac{N}{\ell}$

$B = \mu_0 n i$

$$i = \left(\frac{B}{\mu_0 n}\right)$$

$$U_L = \left(\frac{B^2}{2\mu_0}\right)(A\ell)$$

$$\boxed{\text{Energy density} = \frac{U_L}{\text{Volume}} = \frac{B^2}{2\mu_0}}$$

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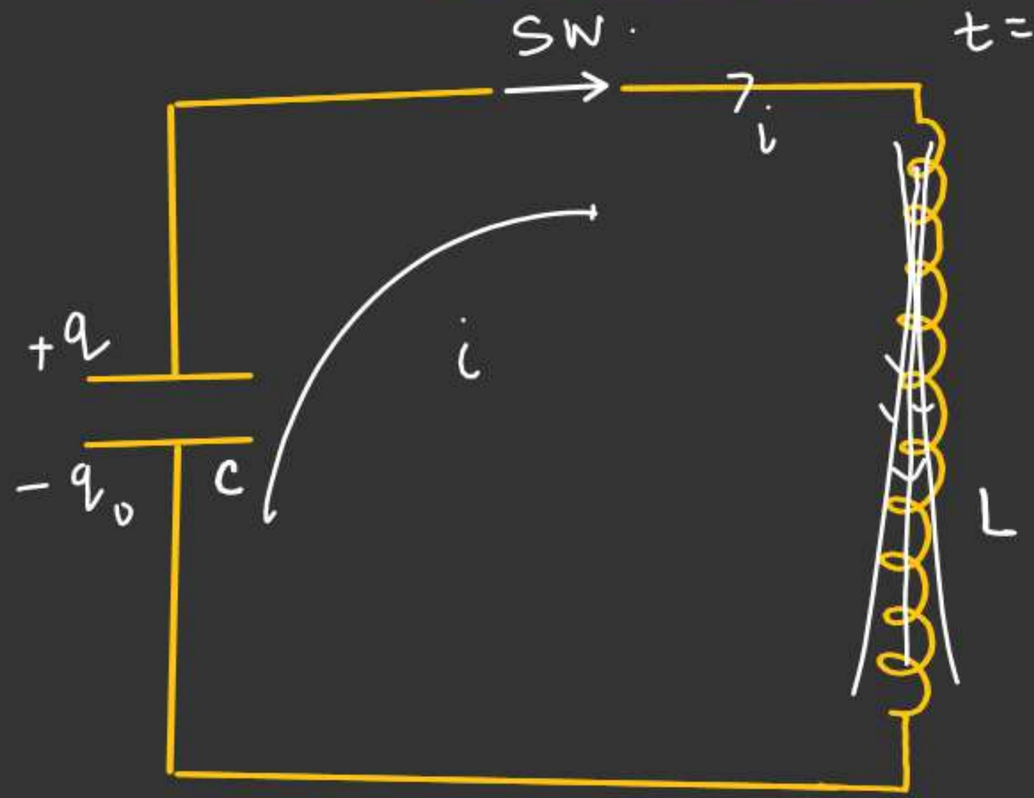
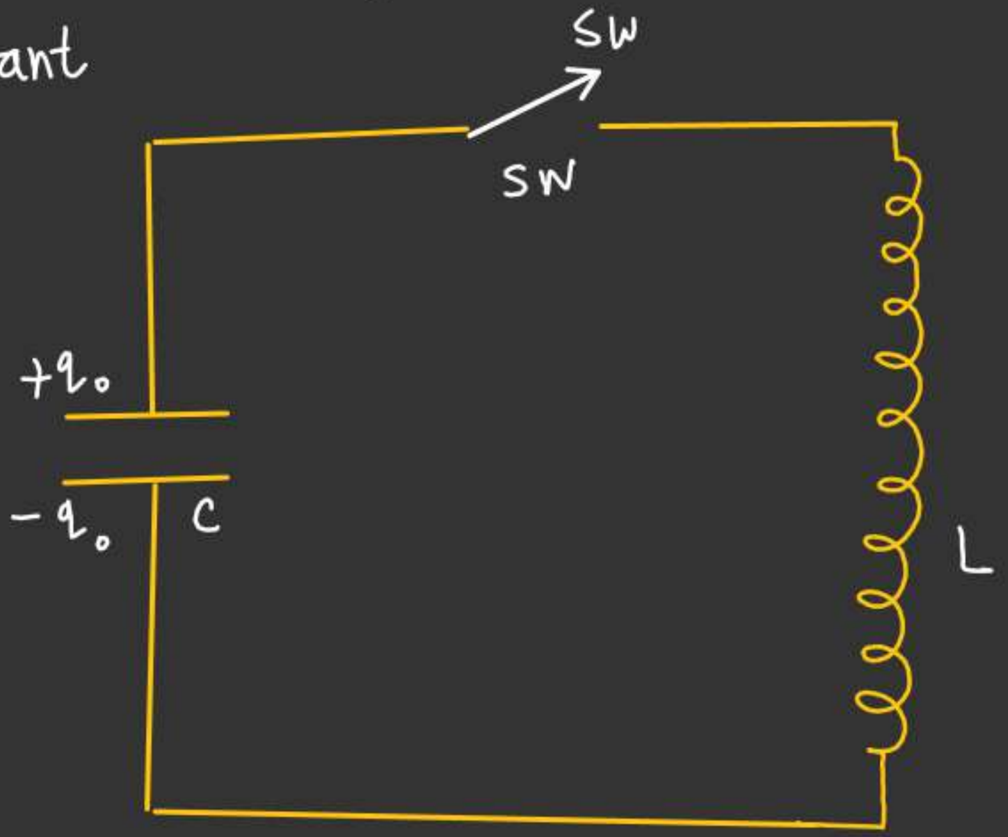
L-C Oscillation:-

At $t=0$, SW closed.



$t = t$.

$$U_T = \frac{q_0^2}{2C} = \text{Constant}$$



At $t = t$

$$U_T = \frac{q^2}{2C} + \frac{1}{2}Li^2$$

$$\left(i = \ominus \frac{dq}{dt} \right)$$

Charge on the Capacitor decreasing w.r.t time.

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$$U_T = \frac{q^2}{2C} + \frac{1}{2}Li^2$$

$$\frac{dU_T}{dt} = 0. \qquad [U_T = \text{Constant}]$$

$$0 = \frac{1}{2C}(2q)\left(\frac{dq}{dt}\right) + \frac{1}{2}L(2i)\left(\frac{di}{dt}\right)$$

$$0 = \frac{q}{C}\left(\frac{dq}{dt}\right) + Li\frac{d}{dt}\left(\frac{dq}{dt}\right)$$

$$-\frac{q}{C}\cancel{\left(\frac{dq}{dt}\right)} = L\cancel{i}\left(\frac{d^2q}{dt^2}\right)$$

$$\boxed{\frac{d^2q}{dt^2} = -\underbrace{\frac{1}{LC}}_{\omega^2}q} \longrightarrow \boxed{q = q_0 \sin(\omega t + \phi)}$$

$$\omega^2 = \frac{1}{LC}$$

$$\boxed{\omega = \frac{1}{\sqrt{LC}}}$$

$$2\pi f = \frac{1}{\sqrt{LC}}$$

$$\boxed{f = \frac{1}{2\pi}\frac{1}{\sqrt{LC}}}$$

$\underline{\phi = ?}$

At $t=0$, $q = q_0$

$$q_0 = q_0 \sin\phi$$

$$\sin\phi = 1$$

$$\phi = \pi/2.$$

$$q = q_0 \sin\left(\omega t + \frac{\pi}{2}\right)$$

$$\boxed{q = q_0 \cos\omega t} \ **$$

$$i = -dq/dt$$

$$i = q_0\omega \sin\omega t$$

$$\boxed{i = i_0 \sin\omega t} \leftarrow i = \frac{q_0}{\sqrt{LC}}\sin\omega t$$

Maximum current $\leftarrow i_0 = \frac{q_0}{\sqrt{LC}}$

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Energy in LC oscillation

$i_0 = \frac{q_0}{\sqrt{LC}}$

$U_L = \frac{1}{2} L i^2$

$U_L = \frac{1}{2} L i_0^2 \sin^2 \omega t$

$$\boxed{U_L = \frac{1}{2} \frac{q_0^2}{C} \sin^2 \omega t}$$

$U_C = \frac{q^2}{2C} = \frac{q_0^2 \cos^2 \omega t}{2C}$

$$\boxed{U_C = \frac{q_0^2}{2C} \cos^2 \omega t}$$

At $t = t$

$\rightarrow U_L + U_C = \frac{q_0^2}{2C}$

$T = \frac{2\pi}{\omega}$

$t = T/4 = \left(\frac{\pi}{2\omega}\right)$

Minimum time when

$U_L = U_C$

$\frac{q_0^2}{2C} \sin^2 \omega t = \frac{q_0^2}{2C} \cos^2 \omega t$

$\omega t = \frac{\pi}{4}$

$t = \frac{\pi}{4\omega} \Rightarrow t = \frac{\pi}{4}\sqrt{LC}$

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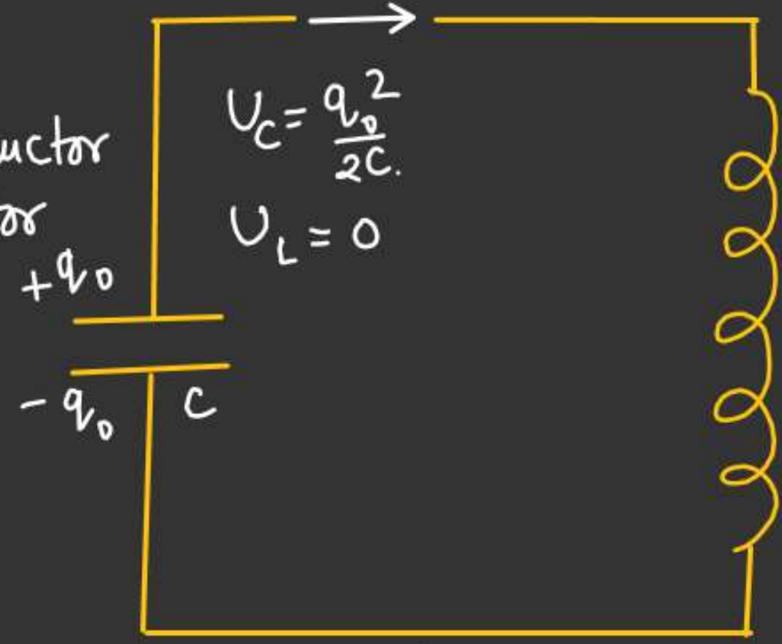
Block → Inductor
Spring → Capacitor

$t=0$



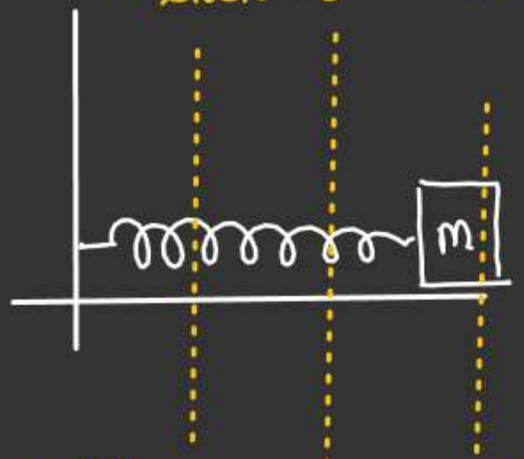
$U_C = \frac{q_0^2}{2C}$

$U_L = 0$

$U_S = \frac{1}{2}Kx_{max}^2$

$U_{block} = 0$

$t=0$



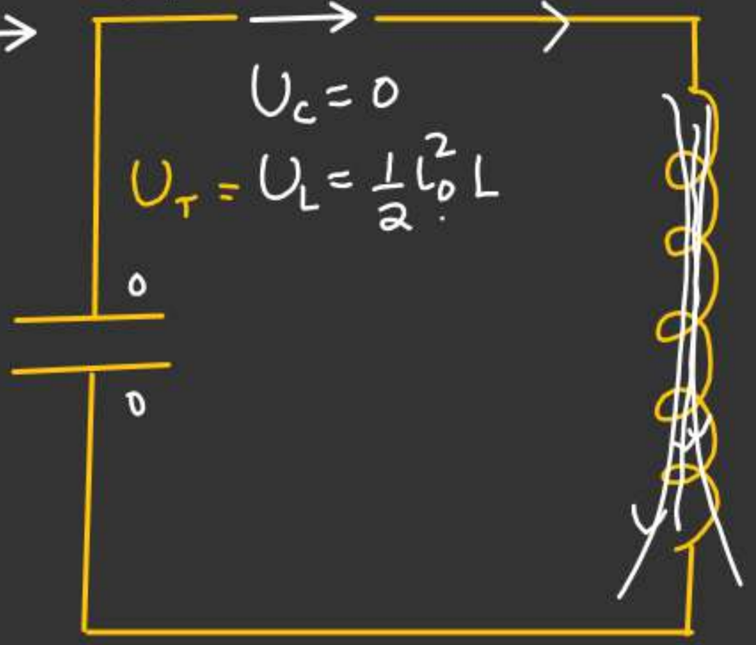
-ve extreme position

Mean position

+ve extreme position

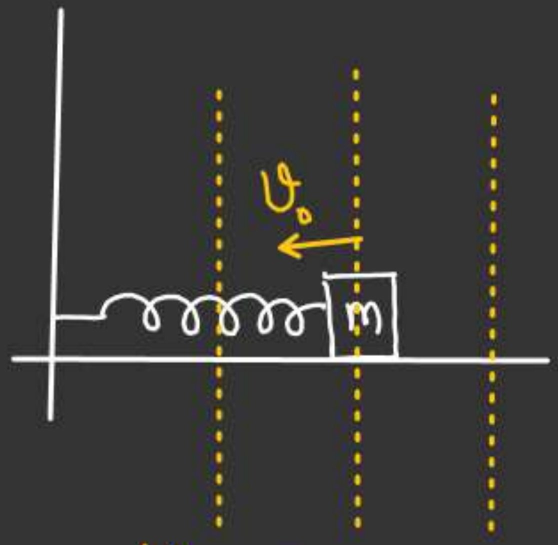
$t = T/4$

$L-0$



$U_C = 0$

$U_T = U_L = \frac{1}{2}i_0^2 L$

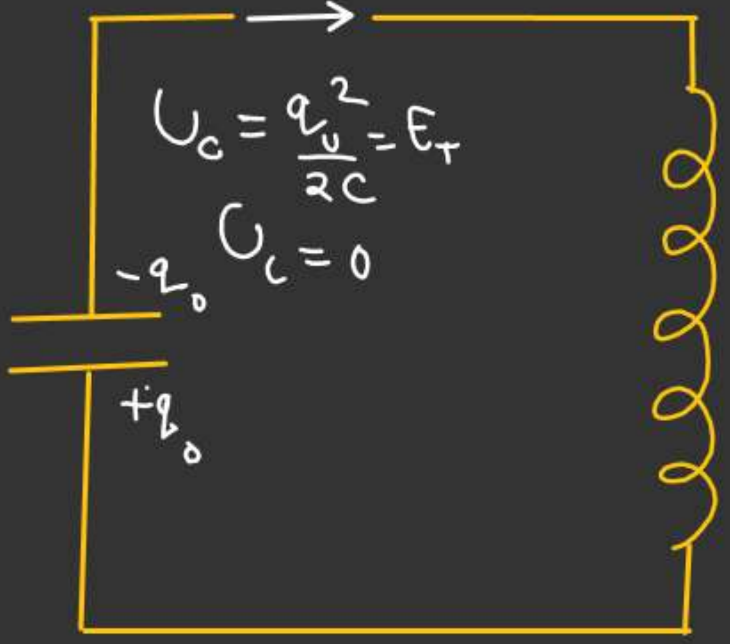


$U_S = 0$

$E_T = U_{block} = \frac{1}{2}mv_0^2$

$t=0$

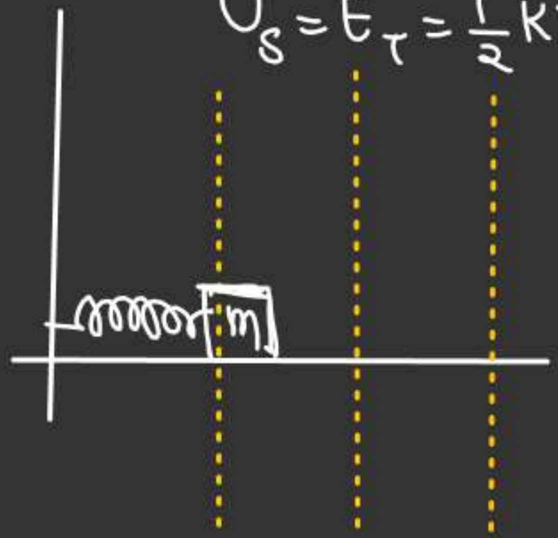
$t = \frac{T}{2}$



$U_C = \frac{q_0^2}{2C} = E_T$

$U_L = 0$

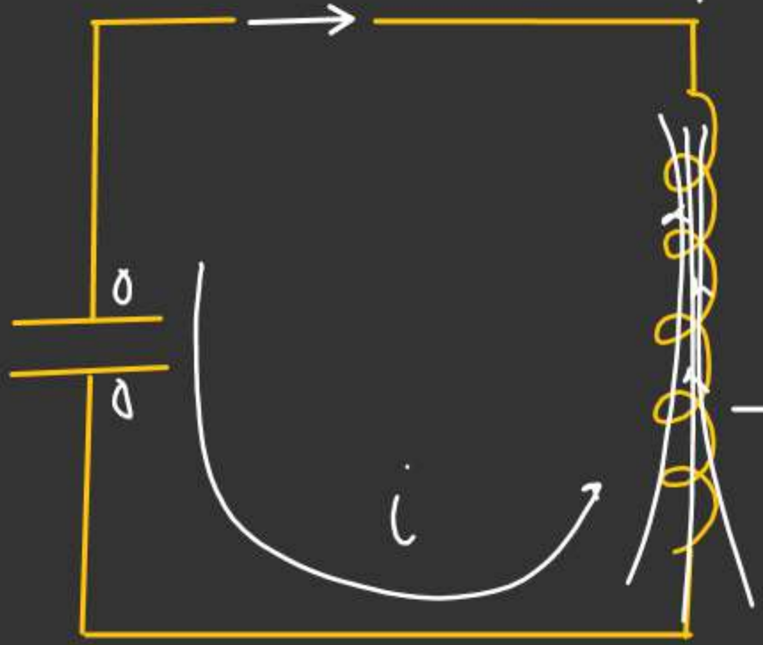
$U_S = E_T = \frac{1}{2}KX_{max}^2$



$t = \frac{3T}{4}$

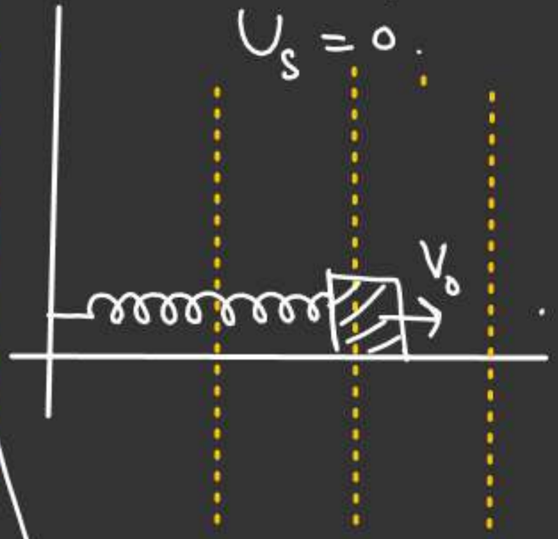
$t=0$

$t=T$



$E_T = \frac{1}{2}mv_0^2$

$U_S = 0$



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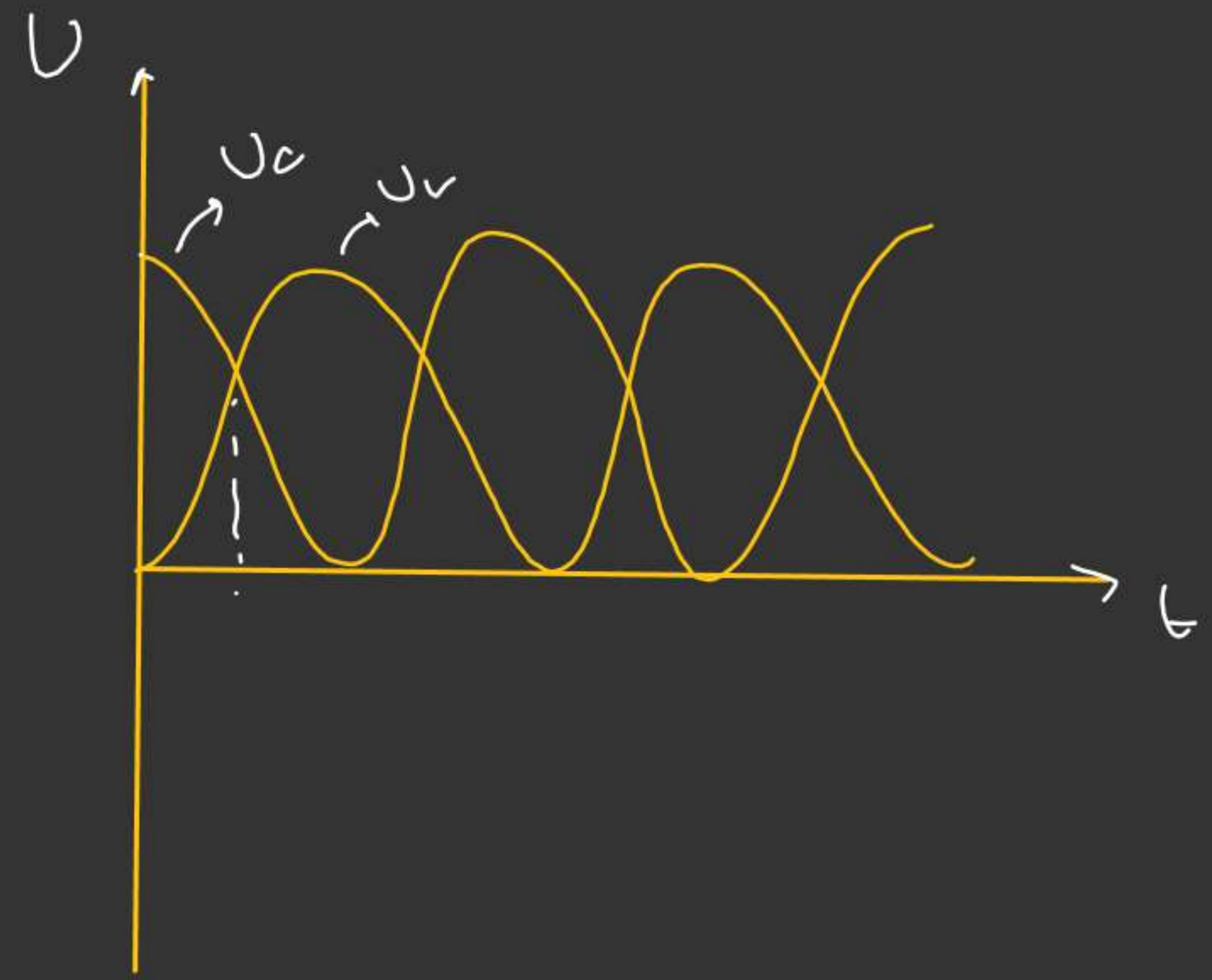

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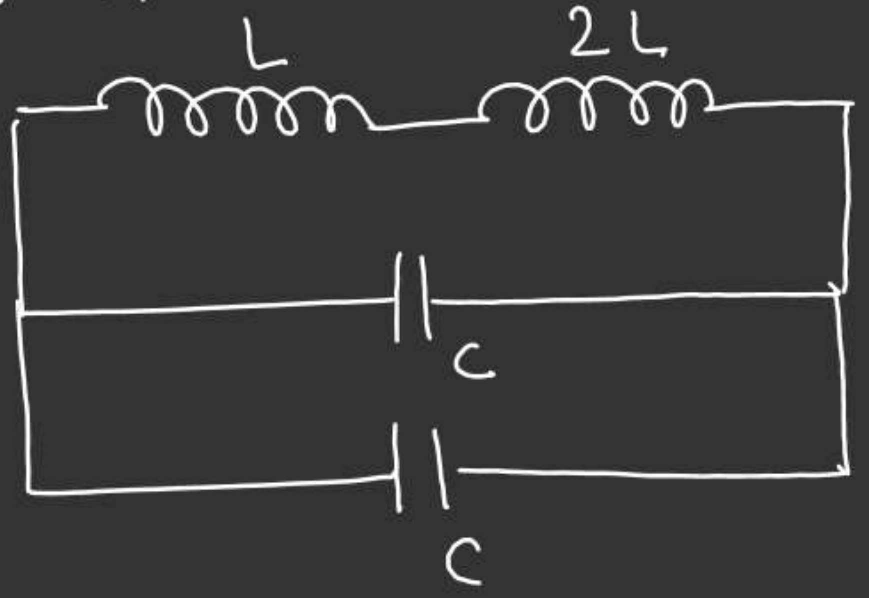
$f = ??$



$L_{eq} = 3L$

$C_{eq} = 2C$

$$\omega = \frac{1}{\sqrt{6LC}} \Rightarrow 2\pi f = \frac{1}{\sqrt{6LC}}$$

$$\Rightarrow f = \frac{1}{2\pi\sqrt{6LC}} \checkmark$$

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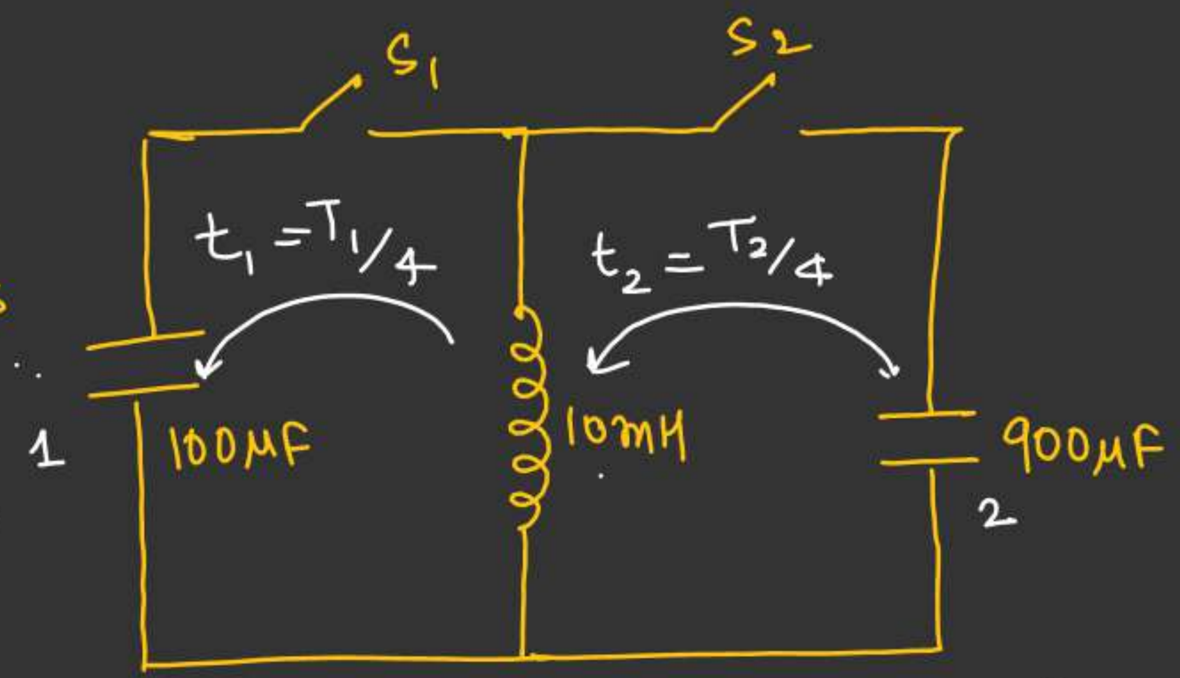
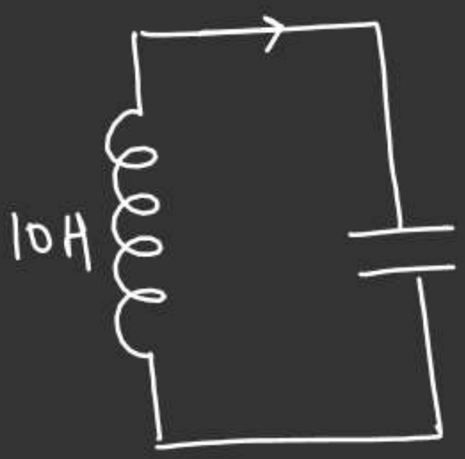
Initially 900 μF Capacitor charged to 100V battery and 100 μF is uncharged.
S_2 Closed for t_2 time, after which it is opened at the same time S_1 is Closed for t_1 time. & then opened.
It is found that Voltage across 100 μF is 300 V.
Find min. possible value of t_1 and t_2.



A. Charge on 900 μF $= 900 \times 10^{-6} \times 100$
$= 9 \times 10^{-2}$ C

$T_1 = \frac{2\pi}{\omega}$

$T_1 = 2\pi(\sqrt{LC})$



Energy in Capacitor 2
$= \frac{1}{2} \times 900 \times 10^{-6} \times (100)^2$
$= \frac{9}{2} = 4.5$ J

$U_{C_1} = \frac{1}{2} \times 100 \times 10^{-6} \times (300)^2$

$U_{C_1} = \frac{9}{2} = 4.5$ J

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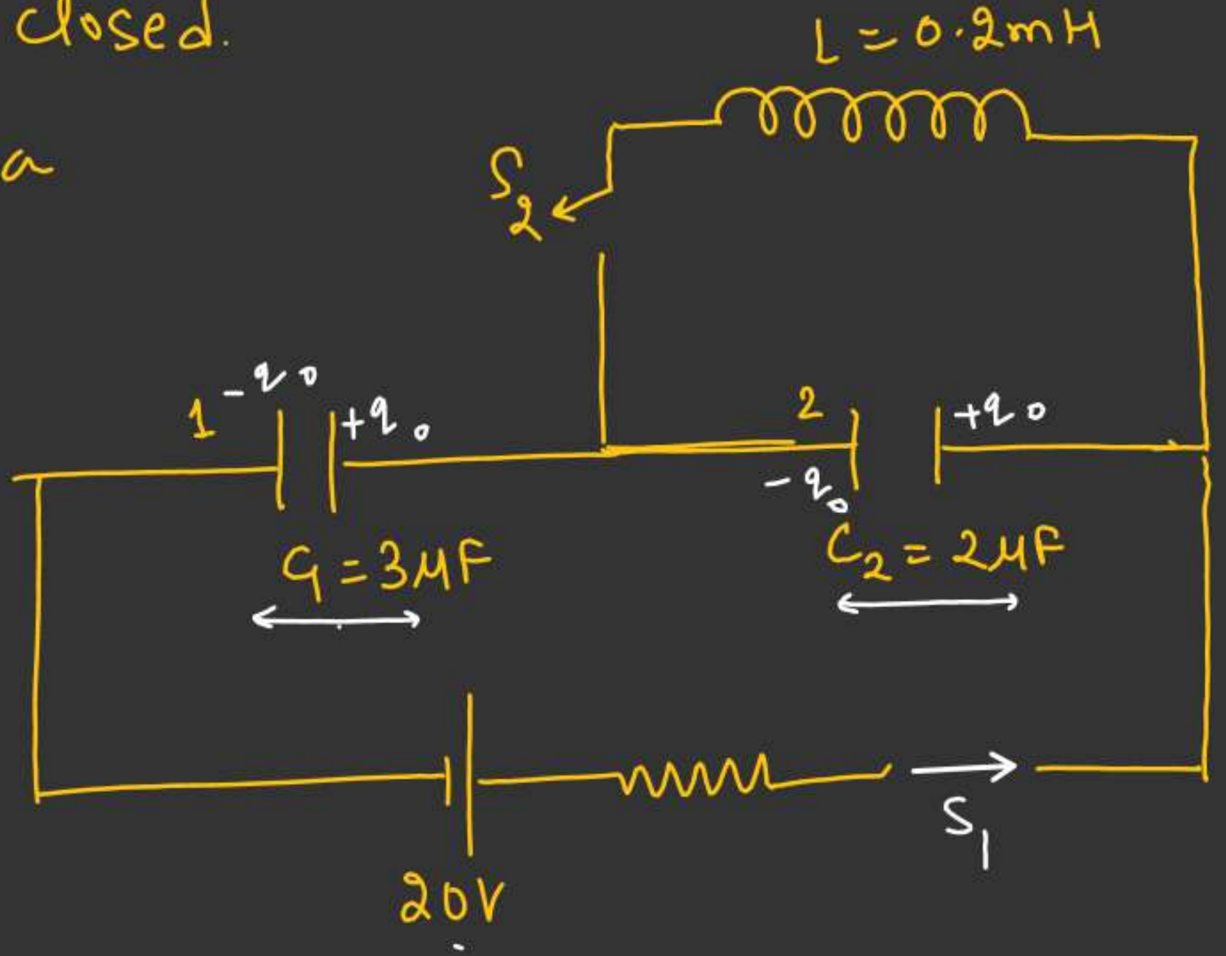
CRt Shown in fig in Steady state with S_1 Closed.

At $t=0$, S_1 is opened and S_2 is closed.

a) Derive charge on Capacitor as a function of time.

b) The frist instant 't' when energy of inductor become $\frac{1}{3}$rd of that in the Capacitor.



When S_1 Closed

$$20 - \frac{q_0}{2} - \frac{q_0}{3} = 0$$

$$20 = \frac{3q_0 + 2q_0}{6}$$

$$120 = 5q_0$$

$$q_0 = 24\mu C$$

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b) The frist instant 't' when energy of inductor become $\frac{1}{3}$rd of that in the Capacitor.

$$\omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{0.2\times10^{-3}\times2\times10^{-6}}}$$

$$\omega = \frac{1}{2\times10^{-5}} = 0.5\times10^{5} = \underline{5\times10^{4}}$$

$$q = q_0 \cos\omega t$$

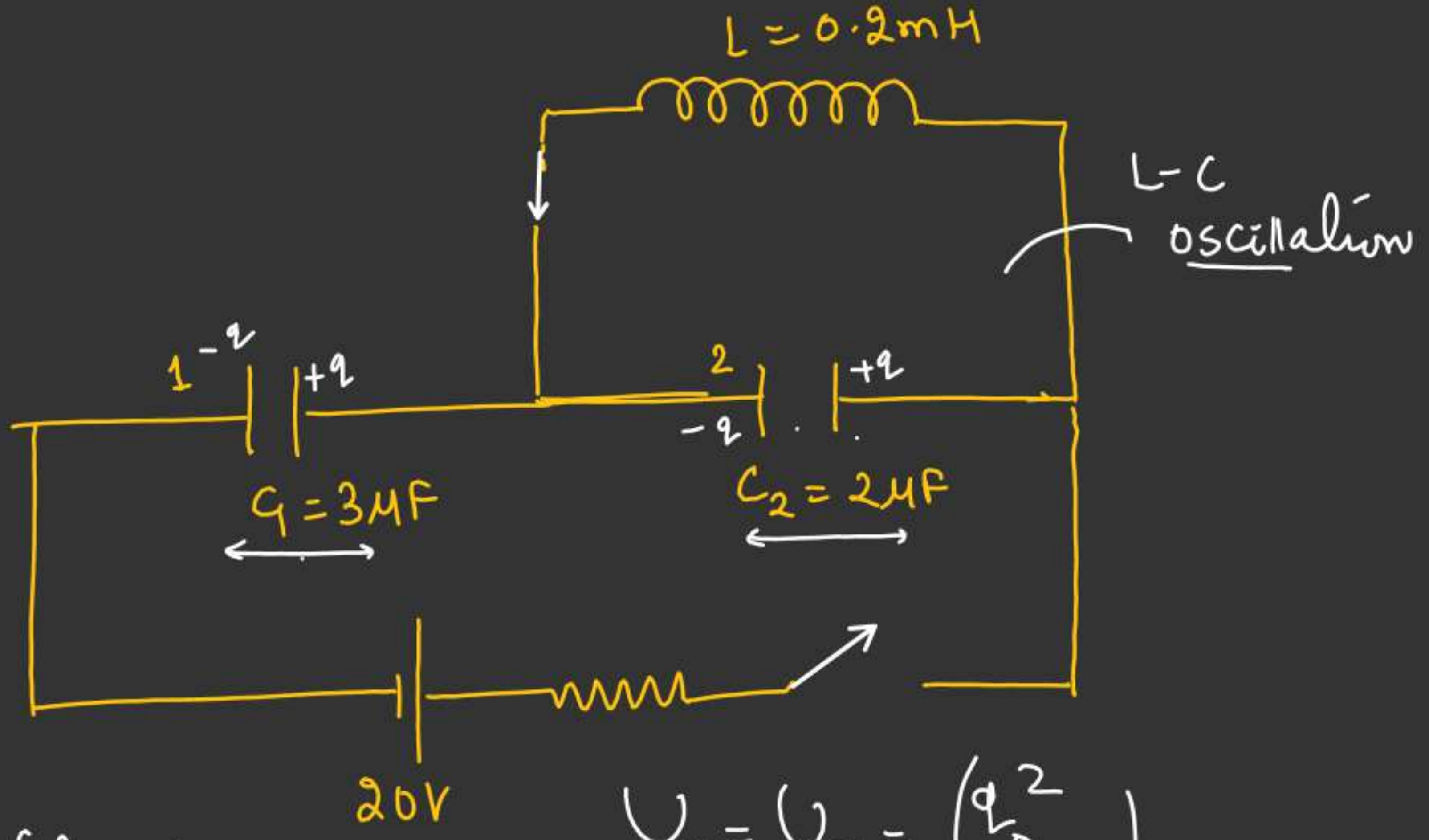
$$q = (24\times10^{-6})\cos[(5\times10^{4})t]$$

$$U_c = \frac{3}{4}\left(\frac{q_0^2}{2C}\right)$$

$$\Downarrow$$

$$\frac{q^2}{2C} = \frac{3}{4}\left(\frac{q_0^2}{2C}\right) \Rightarrow t = ??$$

$\underline{10.5\,\mu s \text{ Ans}}$



$$U_c = U_T = \left(\frac{q_0^2}{2C}\right)$$

$$U_L + U_c = \frac{q_0^2}{2C}$$

$$U_L = \left(\frac{U_c}{3}\right)$$

(According to question)

$$\frac{U_c}{3} + U_c = \frac{q_0^2}{2C} \Rightarrow \frac{4U_c}{3} = \frac{q_0^2}{2C}$$