Nishant Jindal

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ELECTRIC POTENTIAL

Q. Three charges $q_1 = 1\mu C, q_2 = -2\mu C$, and $q_3 = -1\mu C$ are placed at $A(0, 0, 0), B(-1, 2, 3,)$ and $C(2, -1, 1)$. Find the potential of the system of three charges at $P(1, -2, -1)$.

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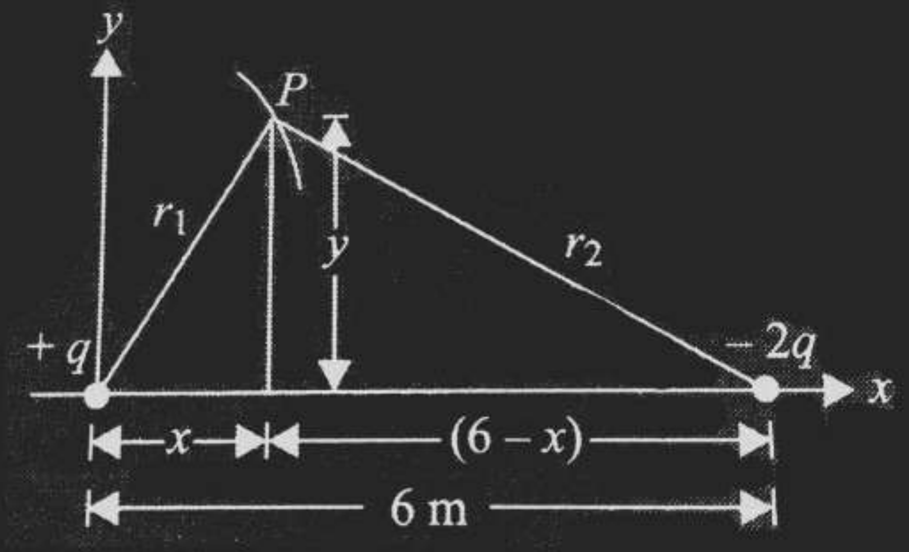
ELECTRIC POTENTIAL

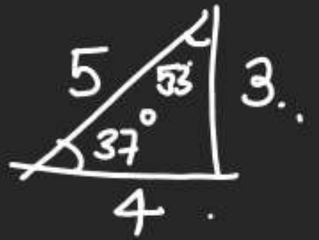
Q. Two electric charges q and $-2q$ are placed at a distance $6\ \text{m}$ apart on a horizontal plane. Find the locus of point on this plane where the potential has a value zero.



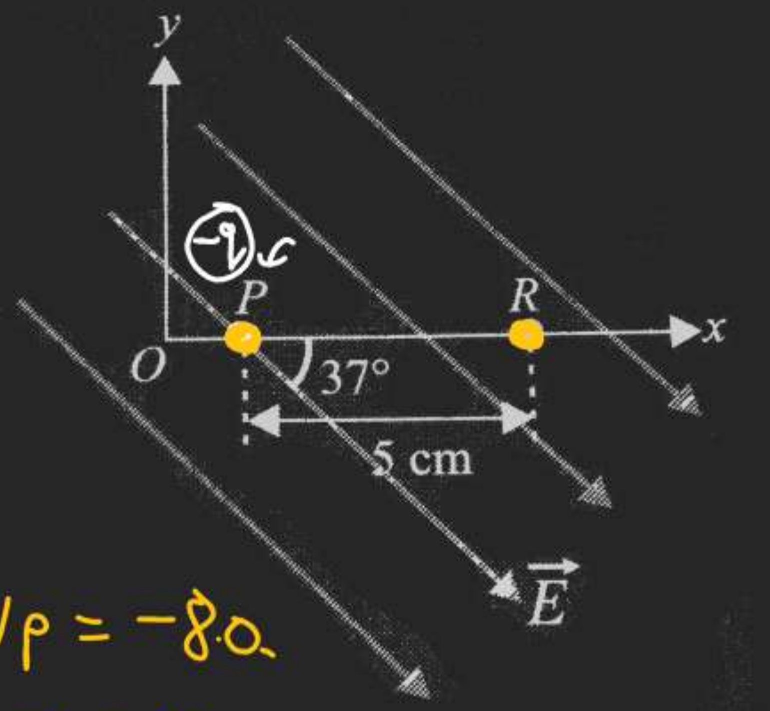
ELECTRIC POTENTIAL

Q. A uniform field of magnitude $\vec{E} = 2000\,\mathrm{NC^{-1}}$ is directed $\theta = 37^\circ$ below the horizontal Fig.

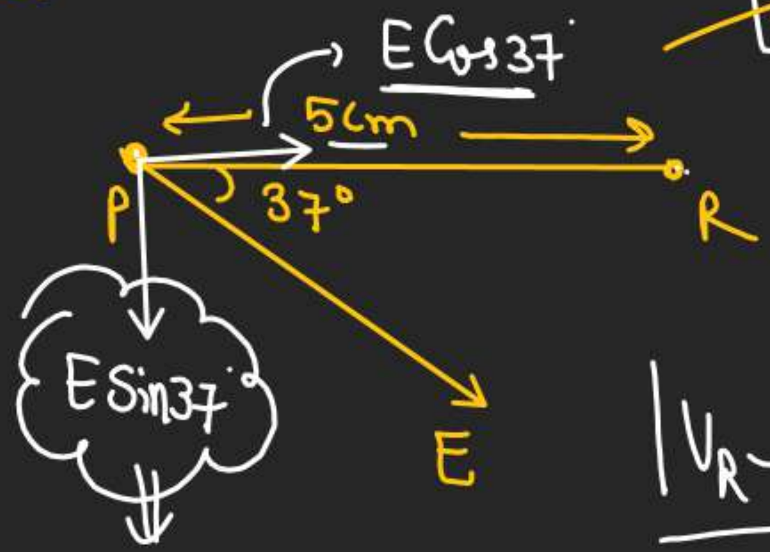
(i) Find the potential difference between P and R.



(ii) If we define the reference level of potential so that potential at R is $V_R = 500\ \mathrm{V}$, what is the potential at P ?



Sol^n Method- 1



$$\boxed{V_R - V_P} = -E r \cos 37^\circ$$

$$= -\frac{4}{5} \times E \times r$$

$$= -\frac{4}{\cancel{5}} \times 2\times10^{3} \times \cancel{5}\times10^{-2}$$

$$|V_R - V_P| = 80\,V = -80 \text{ Volt}$$

(b) $V_R - V_P = -80$

$V_P = V_R + 80$

$= 500 + 80$

$= 580$

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ELECTRIC POTENTIAL

$\int d(x) = x$

$\int d(xy) = xy + c$

Initial

Q. Find the potential difference V_{AB} between $A(0, 0, 0)$ and $B(1\ m, 1\ m, 1\ m)$ in an electric field:

Conservative field

(i) $\vec{E} = (y\hat{i} + x\hat{j})Vm^{-1}$

(ii) $\vec{E} = (3x^2y\hat{i} + x^3\hat{j})Vm^{-1}$

$V_{AB} = (V_B - V_A) = ??$

Sol^n $d\vec{r} = dx\hat{i} + dy\hat{j}$

$dV = -\vec{E}\cdot d\vec{r}$

$dV = -(y\hat{i} + x\hat{j})\cdot(dx\hat{i} + dy\hat{j})$

$\int_{V_A}^{V_B} dV = -\int_{(0,0)}^{(1,1)} [y\,dx + x\,dy] = -\left[\int_0^1 y\,dx + \int_0^1 x\,dy\right]$

$y \to f(x)$; $x \to f(y)$

$y_1 = \boxed{(xy)}$

Differentiating both Side w.r.t x

$\frac{dy_1}{dx} = \frac{d}{dx}(xy)$

$= x\left(\frac{dy}{dx}\right) + y\cdot\left(\frac{dx}{dx}\right)$

$\frac{dy_1}{dx} = x\frac{dy}{dx} + y$

$dy_1 = x\,dy + y\,dx$

$V_B - V_A = -\int_{(0,0)}^{(1,1)} d(y_1) = -\int_{(0,0)}^{(1,1)} d(xy) = -[xy]_{(0,0)}^{(1,1)} = -(1 - 0) = -1\text{ Volt}$

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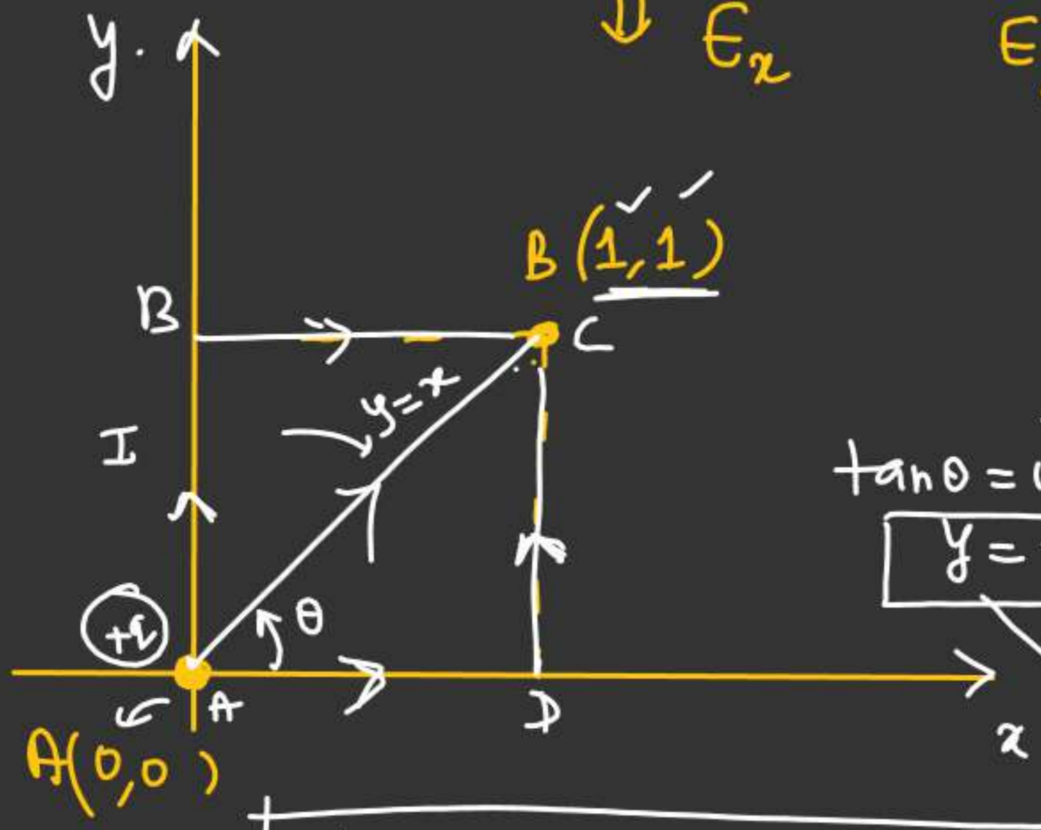
(ii) $\vec{E} = \left((3x^2y)\hat{i} + x^3\hat{j}\right)$

$\Downarrow E_x \qquad \Downarrow E_y$

$A(0,0,0) \rightarrow B(1,1,1)$



$$\boxed{\frac{W}{q} = \frac{\Delta U}{q} = \Delta V}$$

For AC path

$\tan\theta = 45$

$\boxed{y = x} \Rightarrow \vec{E} = 3x^2(x)\,\hat{i} + y^3\hat{j}$

$\vec{E} = 3x^3\hat{i} + y^3\hat{j}$

$d\vec{r} = dx\,\hat{i} + dy\,\hat{j}$

$dV = -\vec{E}\cdot d\vec{r}$

$$\int_{V_A}^{V_B} dV = -\left[3\int_0^1 x^3\,dx + \int_0^1 y^3\,dy\right]$$

$$V_B - V_A = -\left[3\left(\frac{x^4}{4}\right)_0^1 + \left[\frac{y^4}{4}\right]_0^1\right]$$

$$V_B - V_A = -\left[\frac{3}{4} + \frac{1}{4}\right]$$

$$= -1\,\text{Volt}$$

$$\boxed{W_{ADC} = W_{ABC} = W_{AC}}$$

ELECTRIC POTENTIAL

Q. The potential at any point is given by $V = x(y^2 - 4x^2)$. Calculate the Cartesian components of the electric field at the point.

Sol^n

$$V = (xy^2 - 4x^3)$$

$$\vec{E} = -\left[\frac{\partial V}{\partial x}\hat{i} + \frac{\partial V}{\partial y}\hat{j}\right]$$

$$\frac{\partial V}{\partial x} = \frac{\partial}{\partial x}(xy^2 - 4x^3)$$

$(y = c)$

$$= y^2\frac{\partial}{\partial x}(x) - 4\frac{\partial}{\partial x}(x^3)$$

$$= (y^2 - 12x^2)$$

$$\frac{\partial V}{\partial y} = \frac{\partial}{\partial y}(xy^2 - 4x^3)$$

$x = c$

$$= x\frac{\partial}{\partial y}(y^2) - \frac{\partial}{\partial y}(4x^3)$$

0

$$= (2xy)$$

$$\vec{E} = -\left[\frac{\partial V}{\partial x}\hat{i} + \frac{\partial V}{\partial y}\hat{j}\right]$$

$$= -\left[\underbrace{(y^2 - 12x^2)}_{E_x}\hat{i} + \underbrace{(2xy)}_{E_y}\hat{j}\right]$$

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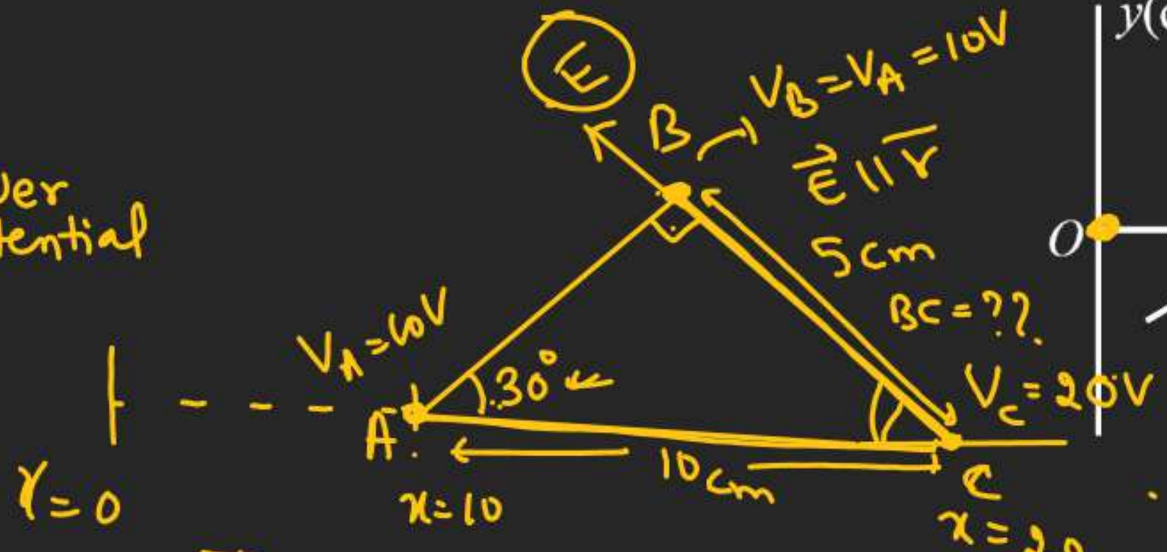
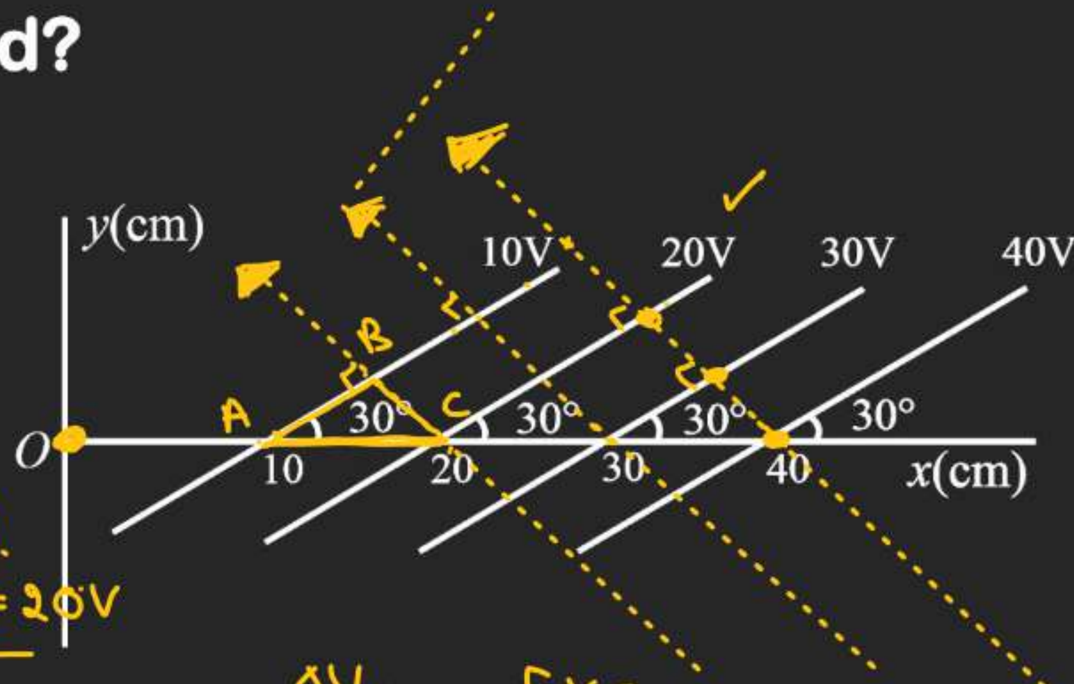
ELECTRIC POTENTIAL

Q. Some equipotential surfaces are shown in Fig. What can you say about the magnitude and the direction of the electric field?

(+q) → (−q)

Higher potential   lower potential





In △ACB

$\sin 30° = \frac{BC}{AC}$

$BC = AC \sin 30°$

$= 10 \times \frac{1}{2} = 5$

$\Delta V = -E r \cos\theta$

$V_B - V_C = -E(5\times10^{-2})\cos 0°$

$(10-20) = -(5\times10^{-2})E$

$\cancel{-}10 = \cancel{-}(5\times10^{-2})E$

$E = \frac{10}{5\times10^{-2}} = \frac{1000}{5} = 200\ V/m$

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ELECTRIC POTENTIAL

H.W.:

Q. Determine the electric field strength vector if the potential of this field depends upon x - and y-coordinates as:

(i) $V = a(x^2 - y^2)$ and

(ii) $V = axy$

ELECTRIC POTENTIAL

Q. A, B, C, D, P, and Q are points in a uniform electric field. The potentials at these points are $V(A) = 2\ V$. $V(P) = V(B) = V(D) = 5\ V$, and $V(C) = 8\ V$. Find the electric field at P.

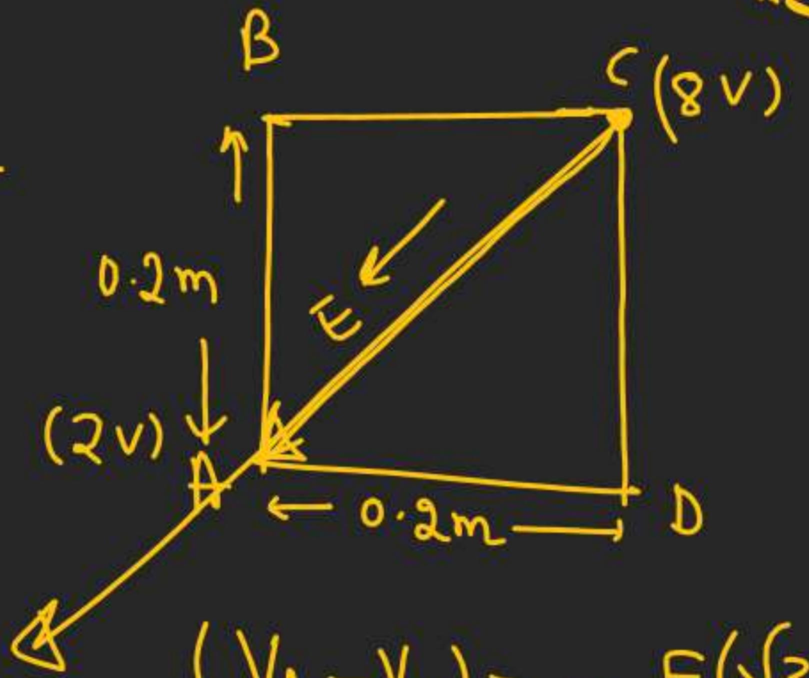
$E_p = ??$

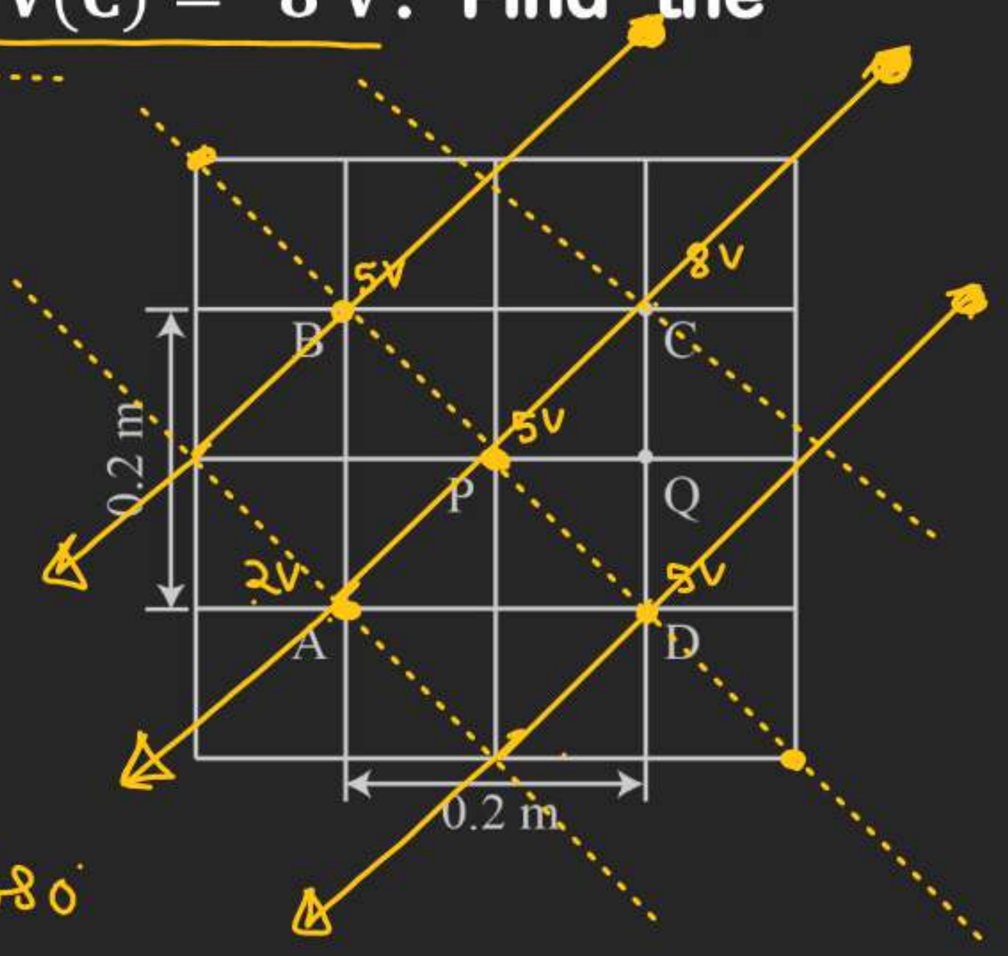
$\vec{E} = E\hat{n}$

$AC = \sqrt{(0.2)^2 + (0.2)^2}$

$= (0.2)\sqrt{2}$

$= \left(\frac{\sqrt{2}}{5}\right)$ ✓





$(V_A - V_c) = -E\left(\frac{\sqrt{2}}{5}\right)\cos 0^\circ$

$\cancel{-}6 = \cancel{-}\frac{\sqrt{2}E}{5}$

$E = \frac{30}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$

$\boxed{E = 15\sqrt{2}\ V/m}$

H.W

Q. Charge Q is given a displacement $\vec{r} = ai + bj$ in an electric field $\vec{E} = E_1\hat{i} + E_2\hat{j}$. The work done is

(A) $Q(E_1a \backsim E_2b)$

(B) $Q\sqrt{(E_1a)^2 + (E_2b)^2}$

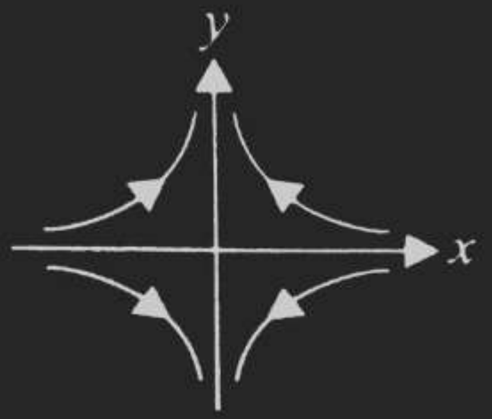
(C) $Q(E_1 + E_2)\sqrt{a^2 + b^2}$

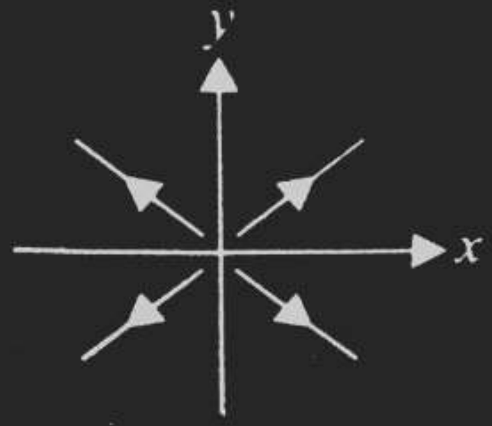
(D) $Q\sqrt{\left(E_1^2 + E_2^2\right)^2}\sqrt{a^2 + b^2}$

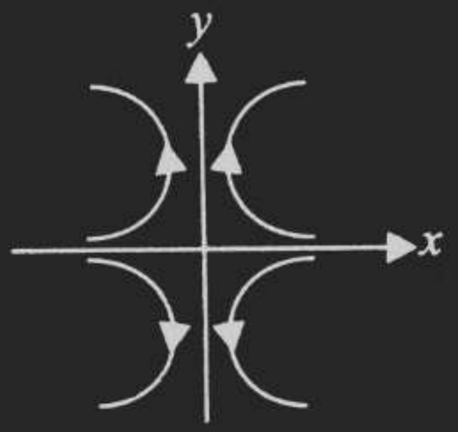
ELECTRIC POTENTIAL

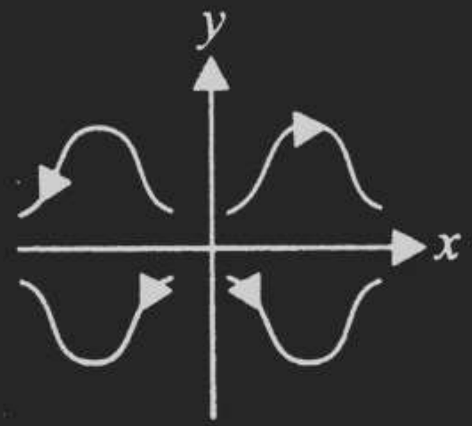
H-w.

Q. The potential field depends on the x - and y-coordinates as $V = x^2 - y^2$. The corresponding electric field lines in xy plane are as

(A) 

(B) 

(C) 

(D) 

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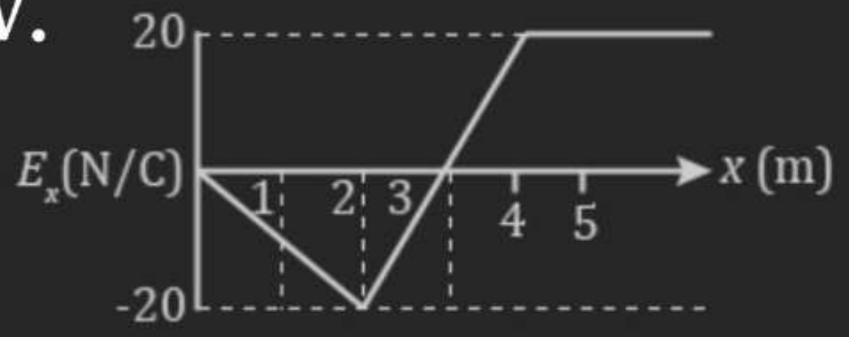
ELECTRIC POTENTIAL

H-W

A graph of the x-component of the electric field as a function of x in a region of space is shown in the figure. The y and z-components of the electric field are zero in this region. The electric potential at the origin is 10 V.



Q. The electric potential at $x = 2$ m is

(A) 10 V

(B) 20 V

(C) 30 V

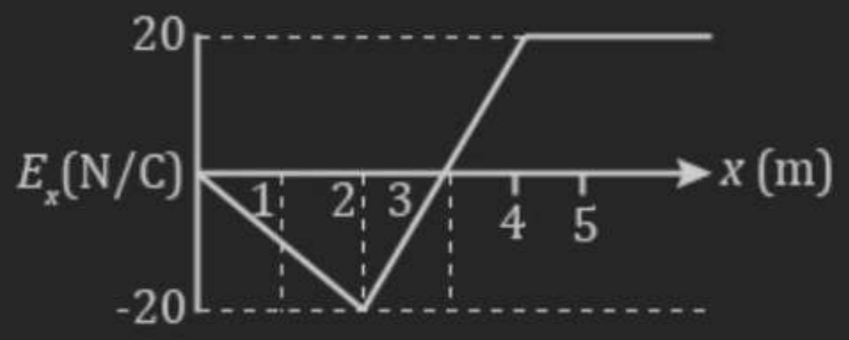
(D) 40 V

ELECTRIC POTENTIAL

H.W.

A graph of the x-component of the electric field as a function of x in a region of space is shown in the figure. The y and z-components of the electric field are zero in this region. The electric potential at the origin is $10\ V$.

Q. The greatest positive value of electric potential for points of the x-axis for which $0 \le x \le 6\ m$ is



(A) $10\ V$

(B) $20\ V$

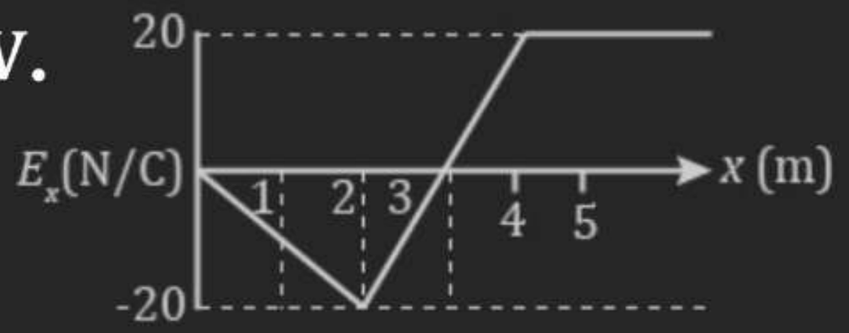
(C) $30\ V$

(D) $40\ V$

ELECTRIC POTENTIAL

HW:

A graph of the x-component of the electric field as a function of x in a region of space is shown in the figure. The y and z-components of the electric field are zero in this region. The electric potential at the origin is $10\ \text{V}$.



Q. The value of x for which potential is zero is

(A) $2\ \text{m}$

(B) $3\ \text{m}$

(C) $4\ \text{m}$

(D) $5.5\ \text{m}$

ELECTRIC POTENTIAL

H-W

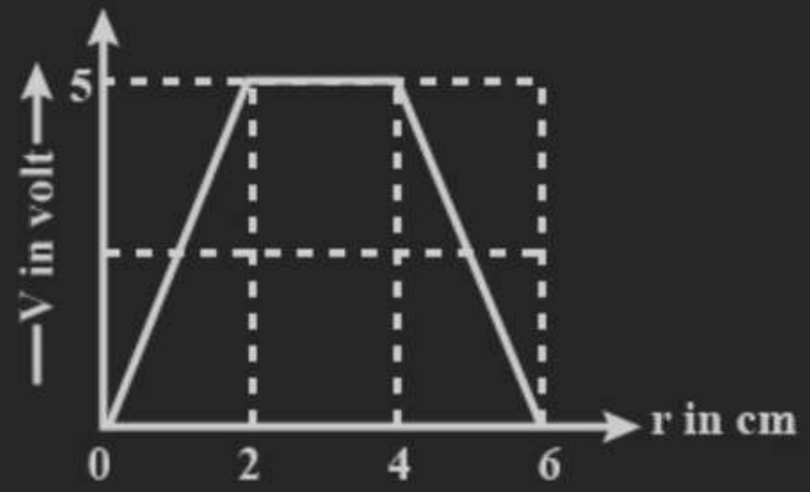
Q. The variation of potential with distance r from a fixed point is shown in figure. The electric field at $r = 3$ cm and $r = 5$ cm are, respectively,

(A) $0, 2\ \mathrm{V/cm}$

(B) $2\ \mathrm{V/cm}, -2\ \mathrm{V/cm}$

(C) $0, -2\ \mathrm{V/cm}$

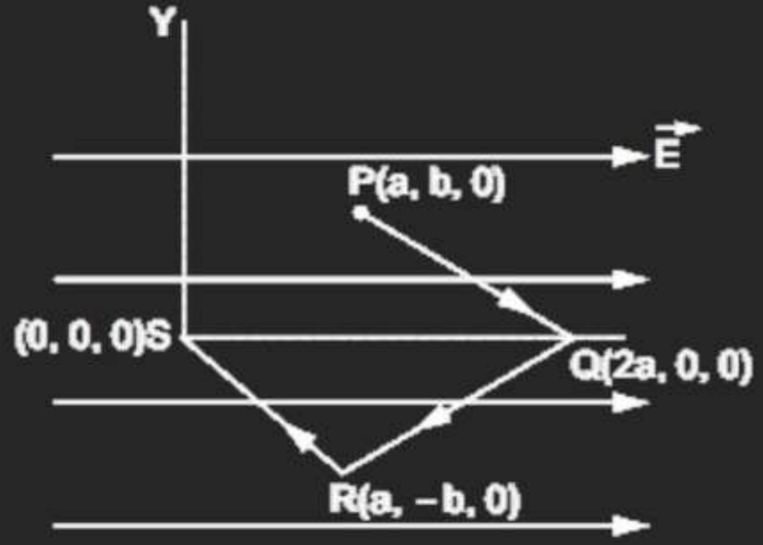
(D) $2\ \mathrm{V/cm}, 0$



ELECTRIC POTENTIAL

H.W.

Q. A point charge q moves from point P to point S along the path $PQRS$ in a uniform electric field $\vec{E}$ pointing parallel to the positive direction of the x-axis. The coordinate of the points P, Q, R and S are $(a, b, 0), (2a, 0, 0)$, $(a, -b, 0)$ and $(0, 0, 0)$ respectively. The work done by the field in the above process is given by the expression



(A) qaE

(B) $-qaE$

(C) $q\left(\sqrt{a^2 + b^2}\right)E$

(D) $3qE\sqrt{a^2 + b^2}$

ELECTRIC POTENTIAL

H-W

Q. Uniform electric field of magnitude $100\ \mathrm{V/m}$ in space is directed along the line $y = 3 + x$. Find the potential difference between point $A(3, 1)$ & $B(1, 3)$

(A) $100\ \mathrm{V}$

(B) $200\sqrt{2}\ \mathrm{V}$

(C) $200\ \mathrm{V}$

(D) zero

ELECTROSTATIC POTENTIAL AND POTENTIAL ENERGY

Potential difference in uniform electric field

$$\int_{V_i}^{V_f} dV = \ominus \int_{r_i}^{r_f} E\,dr$$

$$\left[V_f - V_i = \ominus \left(\text{Area under curve}\right) \right]$$

$(-)$ slope of $\left(\frac{dV}{dr}\right) = E$

$$\Delta U = (U_f - U_i) = q(V_f - V_i)$$

with sign

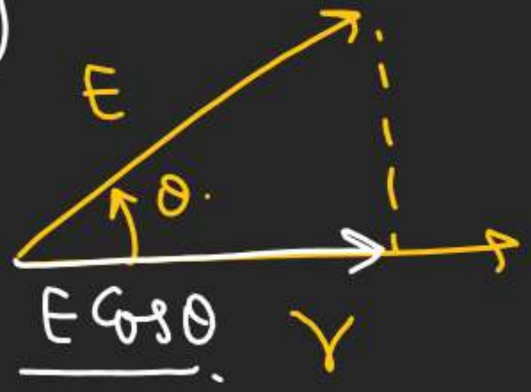
$$\left[\begin{array}{l} W_{system} = -(\Delta U) \\ W_{ext\,agent} = (+\Delta U) \end{array} \right]$$

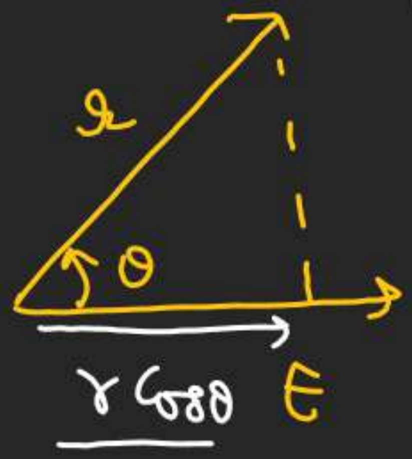
For uniform Electric field.

$$\Delta V = -\left[\vec{E} \cdot \vec{r}\right]$$

$$\Downarrow$$

$$(V_f - V_i) = -E r \cos\theta$$





ELECTROSTATIC POTENTIAL AND POTENTIAL ENERGY

Equipotential surface :-

Equal

↳ (Surface having same potential at every point)

↳ Potential difference b/w any two points on equipotential surface is always zero.

↳ Electric field lines always perpendicular to the equipotential surface.

$$dV = -\vec{E}\cdot\vec{dr}$$

$$\vec{E} \perp \vec{dr}, \quad \boxed{dV = 0}$$

↳ 

If $V_A = V_B$

$$\Delta U = q(\Delta V) = 0$$

⇒ [No work done in moving a charge on equipotential surface]

$$(W_{AB})_{I} = (W_{AB})_{II} = (W_{AB})_{III} = 0$$

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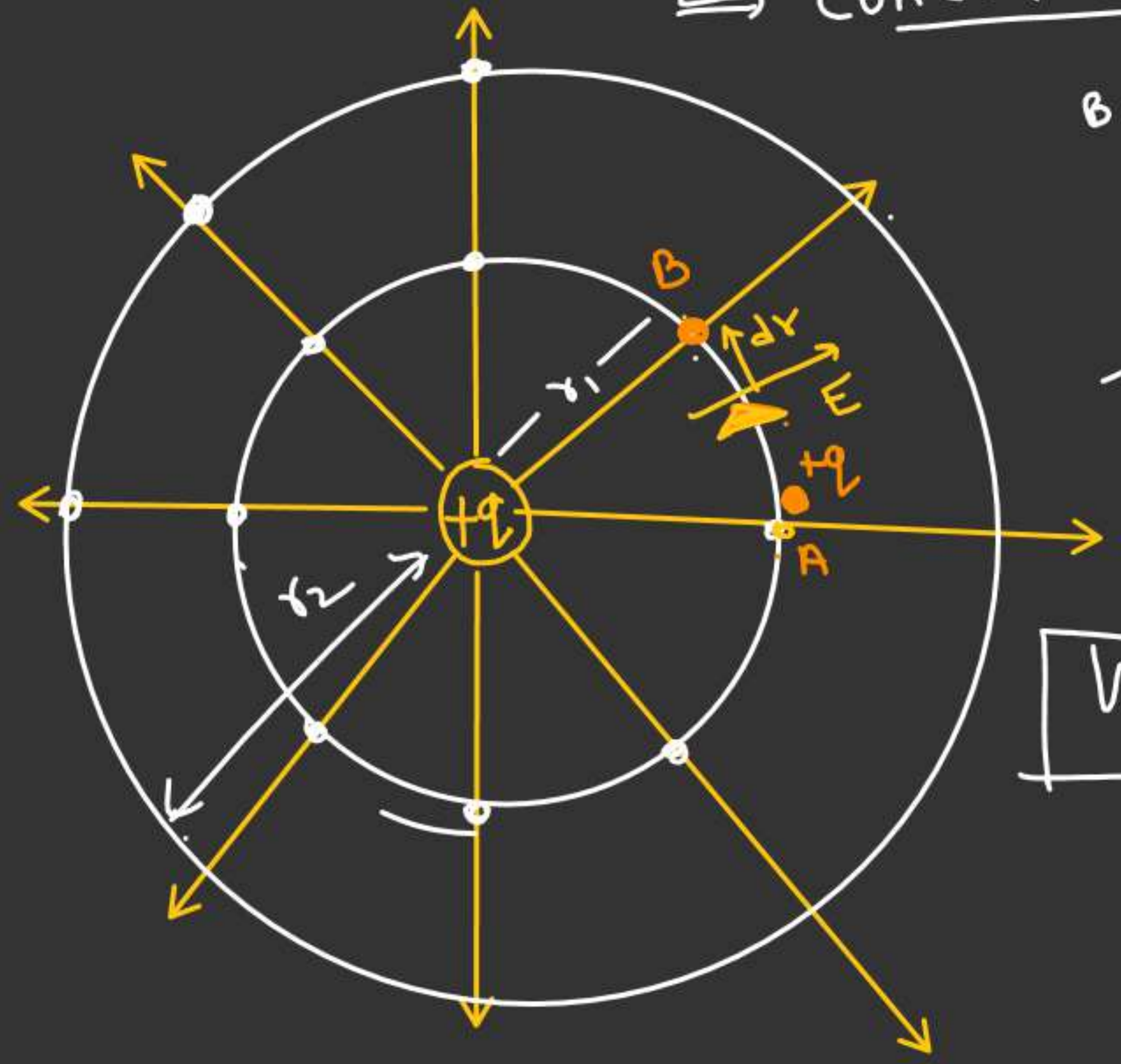
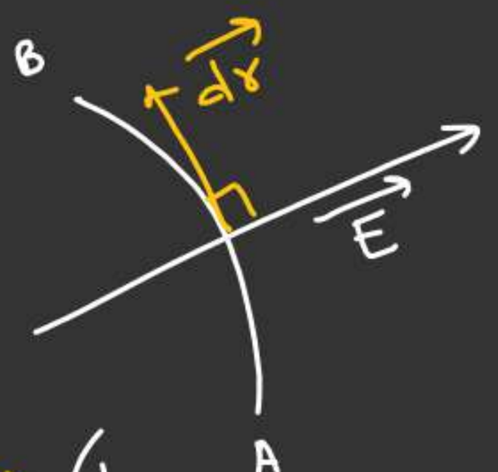
Equipotential Surface

→ For point Charge :-

↳ Concentric Spheres





$(V_A = V_B)$

$\boxed{W_{AB} = 0}$

For infinite line charge Equipotential surfaces are Concentric cylinder.