Nishant Jindal

$$\lim_{x\to\infty}\left(\left(x^3+3x^2\right)^{\frac{1}{3}}-\left(x^2-2x\right)^{\frac{1}{2}}\right)=\lim_{x\to\infty}\left(\left(\left(x^3+3x^2\right)^{\frac{1}{3}}-x\right)+\left(x-\left(x^2-2x\right)^{\frac{1}{2}}\right)\right)$$

$$\left(x^3+3x^2\right)^{\frac{1}{3}} \downarrow x\left(1+\frac{3}{x}\right)^{\frac{1}{3}} \downarrow x$$

$$\left(x^2-2x\right)^{\frac{1}{2}} \downarrow \underbrace{x\left(1-\frac{2}{x}\right)^{\frac{1}{2}}} \downarrow x$$

$$=\lim_{x\to\infty}\left[\frac{\left(x^3+3x^2\right)-x^3}{\left(x^3+3x^2\right)^{2/3}+x^2+x\left(x^3+3x^2\right)^{1/3}}+\frac{\cancel{x^2}-\left(\cancel{x^2}-2x\right)}{x+\left(x^2-2x\right)^{1/2}}\right]$$

$$a^3-b^3=(a-b)\left(a^2+b^2+ab\right)$$

$$a-b=\frac{a^3-b^3}{a^2+b^2+ab}$$

$$=\frac{3\cancel{x^2}}{\cancel{x^2}\left(\left(1+\frac{3}{x}\right)^{2/3}+1+\left(1+\frac{3}{x}\right)^{1/3}\right)}+\frac{2\cancel{x}}{\cancel{x}\left(1+\left(1-\frac{2}{x}\right)^{1/2}\right)}$$

$$=1+1=2.$$

$n \in \mathbb{N}$, $a^n - b^n = (a-b)(a^{n-1} + a^{n-2}b + a^{n-3}b^2 + \cdots + b^{n-1})$

n is odd, $n \in \mathbb{N}$, $a^n + b^n = (a+b)(a^{n-1} - a^{n-2}b + a^{n-3}b^2 - a^{n-4}b^3 + \cdots - ab^{n-2} + b^{n-1})$

$(-b)^n + b^n$

$(-1)^n b^n + b^n$

$-b^n + b^n$

$= 0$

$a^3 + b^3$

$a^5 + b^5$

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$$\lim_{x\to\infty}\left(\underbrace{\left(x^3+3x^2\right)^{1/3}}_{a}-\underbrace{\left(x^2-2x\right)^{1/2}}_{b}\right)=\lim_{x\to\infty}\frac{\left(x^3+3x^2\right)^2-\left(x^2-2x\right)^3}{\left(x^3+3x^2\right)^{5/3}+\left(x^3+3x^2\right)^{4/3}\left(x^2-2x\right)^{1/2}+\left(x^3+3x^2\right)^{3/3}\left(x^2-2x\right)^{2/2}+\cdots-\left(x^2-2x\right)^{5/2}}$$

$$a-b=\frac{a^6-b^6}{a^5+a^4b+a^3b^2+a^2b^3+ab^4+b^5}$$

$$=\frac{12x^5+\cdots}{x^5\left(\left(1+\frac{3}{x}\right)^{5/3}+\left(1+\frac{3}{x}\right)^{4/3}\left(1-\frac{2}{x}\right)^{1/2}+\left(1+\frac{3}{x}\right)\left(1-\frac{2}{x}\right)+\cdots\left(1-\frac{2}{x}\right)^{5/2}\right)}$$

$$=\frac{12}{6}=2.$$

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$$\lim_{x\to\infty}\left(a_0+\underbrace{\frac{a_1}{x}}_{\to 0}+\underbrace{\frac{a_2}{x^2}}_{\to 0}+\underbrace{\frac{a_3}{x^3}}_{\to 0}+\cdots\infty\right)$$

$$= a_0$$

$0\times\infty$

$$\lim_{n\to\infty}\frac{1^2+2^2+\cdots+n^2}{n^3}$$

$$\frac{1^2}{n^3}+\frac{2^2}{n^3}+\frac{3^2}{n^3}+\cdots+\frac{n^2}{n^3}$$

$0\times\infty$

$$\lim_{t\to 0}\left(a_0+a_1t+a_2t^2+a_3t^3+\cdots\infty\right)$$

continuous at $x=0$.

$$f(x)=a_0+a_1x+a_2x^2$$

Continuous function

Cont at $x=a$

$$\boxed{\lim_{x\to a}f(x)=f(a)}$$

$$f(x)=a_0+a_1x+a_2x^2+a_3x^3+a_4x^4$$

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2.
$$\lim_{x\to\infty}\left(\left((x+a)(x+b)(x+c)\right)^{\frac{1}{3}}-x\right)$$

$$= \frac{(x+a)(x+b)(x+c) - x^3 = (a+b+c)x^2+\left(\sum ab\right)x+abc}{\left((x+a)(x+b)(x+c)\right)^{2/3} + x\left((x+a)(x+b)(x+c)\right)^{1/3} + x^2}$$

$$= \frac{\cancel{x^2}\left((a+b+c)+\frac{\sum ab}{x}+\frac{abc}{x^2}\right)}{\cancel{x^2}\left(\left(\left(1+\frac{a}{x}\right)\left(1+\frac{b}{x}\right)\left(1+\frac{c}{x}\right)\right)^{2/3}+\left(\left(1+\frac{a}{x}\right)\left(1+\frac{b}{x}\right)\left(1+\frac{c}{x}\right)\right)^{1/3}+1\right)}$$

$$= \frac{a+b+c}{3}$$

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3. $\lim_{x\to 5} \dfrac{x^2-9x+20}{\{x\}=x-[x]}$ $\{\cdot\}$ = FPF

not exist.

$$LHL = \lim_{x\to 5^-} \frac{(x-5)\cancel{(x-4)}}{\cancel{(x-4)}} = 0$$

$$RHL = \lim_{x\to 5^+} \frac{\cancel{(x-5)}(x-4)}{\cancel{(x-5)}} = 1$$

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4. $\lim_{x\to 1}\left(\frac{x^m-1}{x^n-1}\right)$, $m, n \in N$.

$$= \lim_{x\to 1} \frac{\cancel{(x-1)}(x^{m-1}+x^{m-2}+\cdots+x+1)}{\cancel{(x-1)}(x^{n-1}+x^{n-2}+x^{n-3}+\cdots+x+1)}$$

$= \frac{m}{n}$ $\quad = \quad \frac{m}{n}$

$x = 1+h$ $\quad \lim_{h\to 0} \frac{(1+h)^m-1}{(1+h)^n-1}$

$$= \lim_{h\to 0} \frac{\cancel{1}+{}^mC_1h+{}^mC_2h^2+\cdots{}^mC_mh^m-\cancel{1}}{\cancel{1}+{}^nC_1h+{}^nC_2h^2+\cdots+{}^nC_nh^n-\cancel{1}}$$

$$\lim_{h\to 0} \frac{\cancel{h}({}^mC_1+{}^mC_2h+\cdots+{}^mC_mh^{m-1})}{\cancel{h}({}^nC_1+{}^nC_2h+{}^nC_3h^2+\cdots+{}^nC_nh^{n-1})}$$

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$$\lim_{x\to -1}\left[x\left[\frac{1}{x}\right]\right]=1 \qquad [\cdot]=G.I.F.$$

$h>0$

$$L.H.L=\lim_{h\to 0}\left[(-1-h)\left[\frac{1}{-1-h}\right]\right]=\lim_{h\to 0}\left[(-1-h)(-1)\right]=\lim_{h\to 0}\left[1+h\right]=\underline{1}$$

$$RHL=\lim_{h\to 0}\left[(-1+h)\left[\frac{1}{-1+h}\right]\right]=\lim_{h\to 0}\left[(-1+h)(-2)\right]$$

$$=\lim_{h\to 0}\left[\underbrace{2-2h}_{2^-}\right]=\underline{1}$$

$$-\frac{1}{1-h}\to -1^-$$

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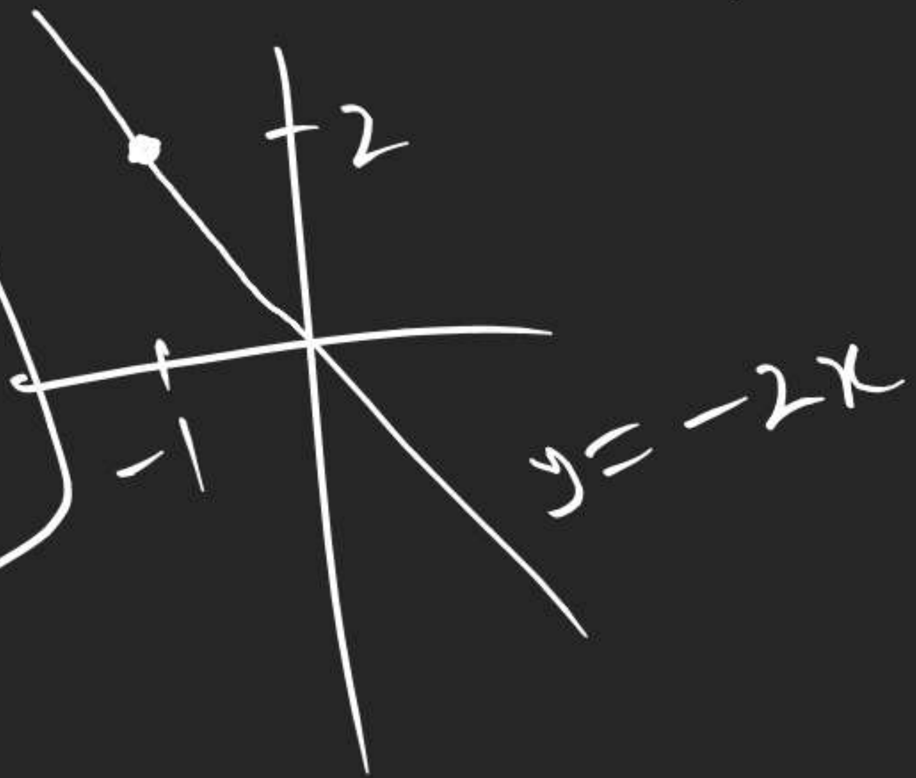
$$\lim_{x\to -1}\left[x\,\underbrace{\left[\frac{1}{x}\right]}_{-1}\right]$$

$$LHL = \lim_{x\to -1^-}\left[-x\right] = 1$$

$$RHL = \lim_{x\to -1^+}\left[-2x\right] = 1$$



H.W

G.N. Berman
245-305
Ex-IV (1-10)