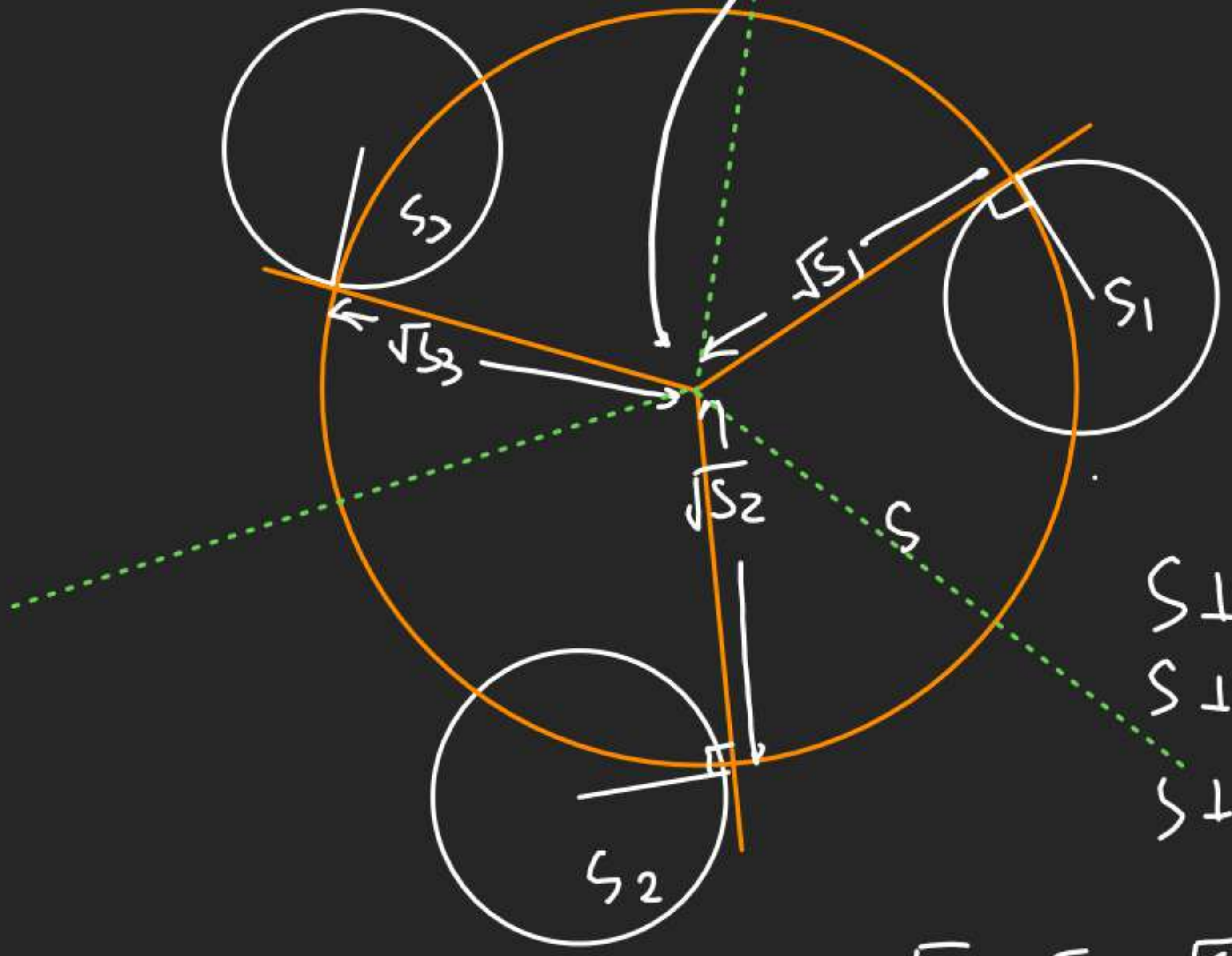
Nishant Jindal

Radical Centre



$S \perp S_1$
$S \perp S_2$
$S \perp S_3$

$\underbrace{\sqrt{S_1} = \sqrt{S_2}} = \sqrt{S_3}$

$S_1 - S_2 = 0 \rightarrow (1)$
$S_2 - S_3 = 0 \rightarrow (2)$ } R.C.

$S_1: x^2+y^2+2g_1x+2f_1y+c_1=0$

$S_2: x^2+y^2+2g_2x+2f_2y+c_2=0$

$S_3: x^2+y^2+2g_3x+2f_3y+c_3=0$

$RA_1: 2(g_1-g_2)x+2(f_1-f_2)y = c_2-c_1 \rightarrow (1)$

$RA_2 \Rightarrow 2(g_2-g_3)x+2(f_2-f_3)y = c_3-c_2 \rightarrow (2)$

(1) Solving 1, 2 will give (x, y) which is Rad. Centre.

(2) Now for Radius $= \sqrt{S_1}$

(3) Now Using (1) & (2) we can get a Circle which is orthogonal to all 3 Circle.

RK: Radical Centre of all 3 Circles described on the side of Δ as diameter is the orthocentre of Δ.

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Q If R.A of the Circles

$x^2+y^2+2gx+2fy+c=0$

& $2x^2+2y^2+3x+8y+2c=0$

touches the circle $x^2+y^2+2x+2y+1=0$

then find g, f?

RA:- $S_1: x^2+y^2+2gx+2fy+c=0$

$S_2: x^2+y^2+\frac{3}{2}x+4y+c=0$

$RA = x\left(2g-\frac{3}{2}\right)+y(2f-4)+ \quad =0$

touches $S_3: x^2+y^2+2x+2y+1=0$

$p=r \Rightarrow (-1,-1)\ r=\sqrt{1^2+1^2-1}=1$

$$p=\frac{|(-1)(4g-3)+(-1)(4f-8)|}{\sqrt{(4g-3)^2+(4f-8)^2}}=1$$

$$((4g-3)+(4f-8))^2=(4g-3)^2+(4f-8)^2$$

$(A+B)^2=A^2+B^2 \Rightarrow \boxed{2A \cdot B=0}$

$2(4g-3)(4f-8)=0$

$\Rightarrow g=\frac{3}{4}$ or $f=2$

Q Eqⁿ of 3 Circles are given

$x^2+y^2=1,\ x^2+y^2-8x+15=0$

$x^2+y^2+10y+24=0$

Det. the Pt. "P" Such that tangents drawn from it to the circles are eqᵘ in length

Sol: here P is Radical Centre.

$S_1: x^2+y^2-1=0$

$S_2: x^2+y^2-8x+15=0$

$S_3: x^2+y^2+10y+24=0$

① $S_1-S_2=0$

$8x-16=0 \Rightarrow x=2$

② $S_1-S_3=0$

$-10y-25=0 \Rightarrow \cancel{10}y=-\cancel{25}$

$y=-\frac{5}{2}$

$P: \left(2, -\frac{5}{2}\right)$ Ans

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Q Find EOC Which cuts 3 Circles

$S_1: x^2+y^2-3x-6y+14=0$

$S_2: x^2+y^2-x-4y+8=0$

$S_3: x^2+y^2+2x-6y-8=0$ orthogonally.

Sol.

$S_1-S_2 \Rightarrow -2x-2y+6=0$

$x+y=3 \rightarrow (A)$

$S_1-S_3 \Rightarrow -5x+0+22=0$

$x=\frac{22}{5}$

$y=3-\frac{22}{5}=-\frac{7}{5}$

$\therefore$ Centre $\left(\frac{22}{5},-\frac{7}{5}\right)$

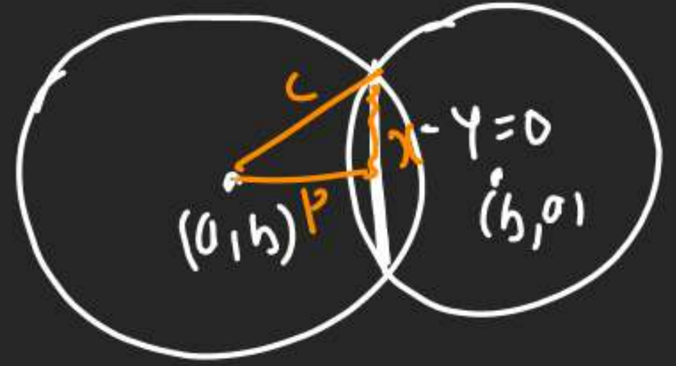
Rad $\rightarrow \sqrt{S_1}=\sqrt{\frac{484}{25}+\frac{49}{25}-\frac{66}{5}+\frac{42}{5}+14}$

$=\sqrt{\frac{533}{25}-\frac{120}{25}+\frac{350}{25}}=\sqrt{\frac{763}{25}}$

$\therefore S: \left(x-\frac{22}{5}\right)^2+\left(y+\frac{7}{5}\right)^2=\frac{763}{25}$

Q Length of Com. Chord of circles

$(x-a)^2+(y-b)^2=c^2$ & $(x-b)^2+(y-a)^2=c^2$ is?



Com. Chord $\rightarrow S_1-S_2=0$

$S_1: x^2+y^2-2ax-2by+a^2+b^2-c^2=0$

$S_2: x^2+y^2-2bx-2ay+a^2+b^2-c^2=0$

$+2(a-b)x+2(b-a)y=0$

$x-y=0$

$p=\frac{|a-b|}{\sqrt{1^2+1^2}}=\frac{|a-b|}{\sqrt{2}}$

$L_{chord}=2\sqrt{c^2-\frac{(a-b)^2}{4}}$

$=\sqrt{4c^2-(a-b)^2}$

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Chord Whose Mid Pt is given

1) If Mid Pt of chord is given (x_1, y_1) then Eqⁿ of chord will be $T = S_1$

2) If Circle $\to x^2+y^2=a^2$

$T \to xx_1 + yy_1 - a^2$

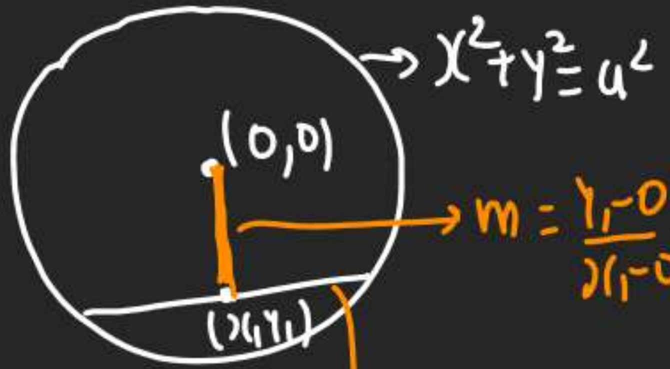
$S_1 \to x_1^2 + y_1^2 - a^2$

$\Rightarrow T = S_1$

$\Rightarrow \boxed{xx_1 + yy_1 - a^2 = x_1^2 + y_1^2 - a^2}$

$\to$ This is Eqⁿ of chord whose Mid Pt is (x_1, y_1)

(3) Proof



$m = \frac{y_1 - 0}{x_1 - 0} = \frac{y_1}{x_1}$

$m' = -\frac{x_1}{y_1}$

Eqⁿ of chord

$(y - y_1) = -\frac{x_1}{y_1}(x - x_1)$

$yy_1 - y_1^2 = -xx_1 + x_1^2$

$\underbrace{xx_1 + yy_1 - a^2}_{T} = \underbrace{x_1^2 + y_1^2 - a^2}_{= S_1}$

(4) RK There may be ∞ Lines can be Pass from (x_1, y_1)

But chord whose Mid Pt is (x_1, y_1) is smallest of all chords.

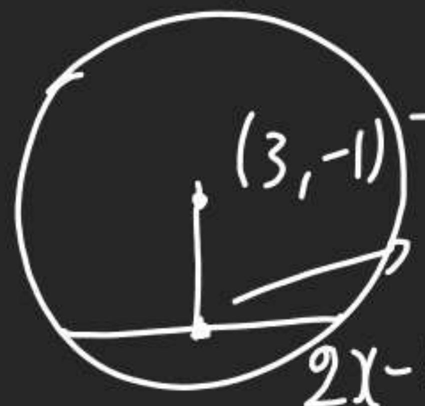
Q Find Eqⁿ of Chord having $(1,-2)$ as Mid Pt. for Circle $x^2+y^2=9$

$T = S_1$

$x\cdot 1 + y\cdot(-2) - 9 = 1^2+2^2-9$

$x - 2y = 5$ Ans

Q Find Mid Pt of chord $2x-5y+18=0$ for Circle $x^2+y^2-6x+2y-54=0$



$\rightarrow (y+1) = -\frac{5}{2}(x-3)$

Mid Pt is Foot of ⊥^{r}

$2x-5y+18=0$ $\times 2$

$m = \frac{2}{5}$

$2y+2 = -5x+15$

$5x+2y=13$ $\times 5$

$25x+10y=65$

$4x-10y=-36$

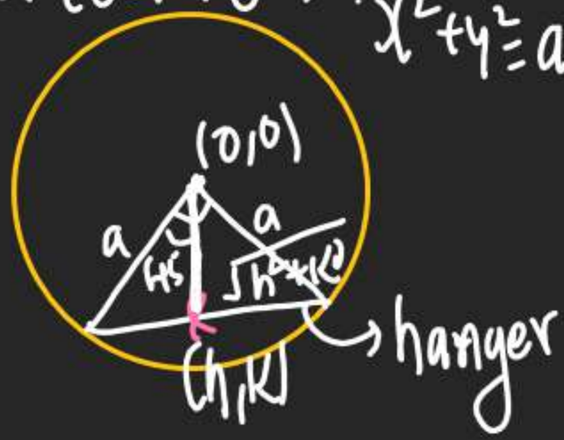
$29x = 29$

$x=1,\ y=4$

$\therefore$ Pt. $(1,4)$

Q Find Locus of Mid Pt of Chord which makes $90°$ at centre?

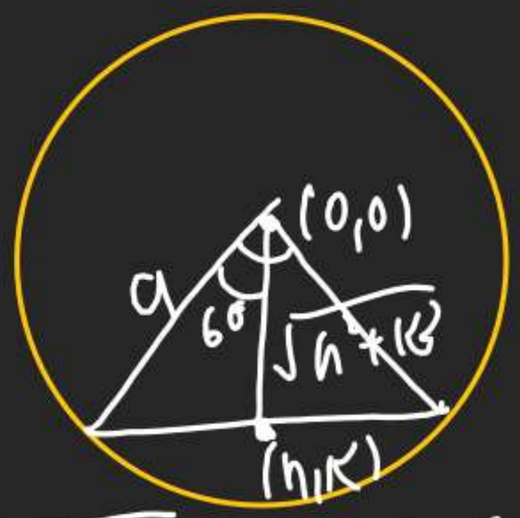
$x^2+y^2=a^2$



$\sqrt{h^2+k^2} = a\cos 45° = \frac{a}{\sqrt{2}}$

$x^2+y^2 = \frac{a^2}{2}$ in Reqᵈ

Q Find Locus of Mid Pt of Chords which make angle $\frac{2\pi}{3}$ at Centre



$\sqrt{h^2+k^2} = a\cos 60° = \frac{a}{2}$

$x^2+y^2 = \frac{a^2}{4}$

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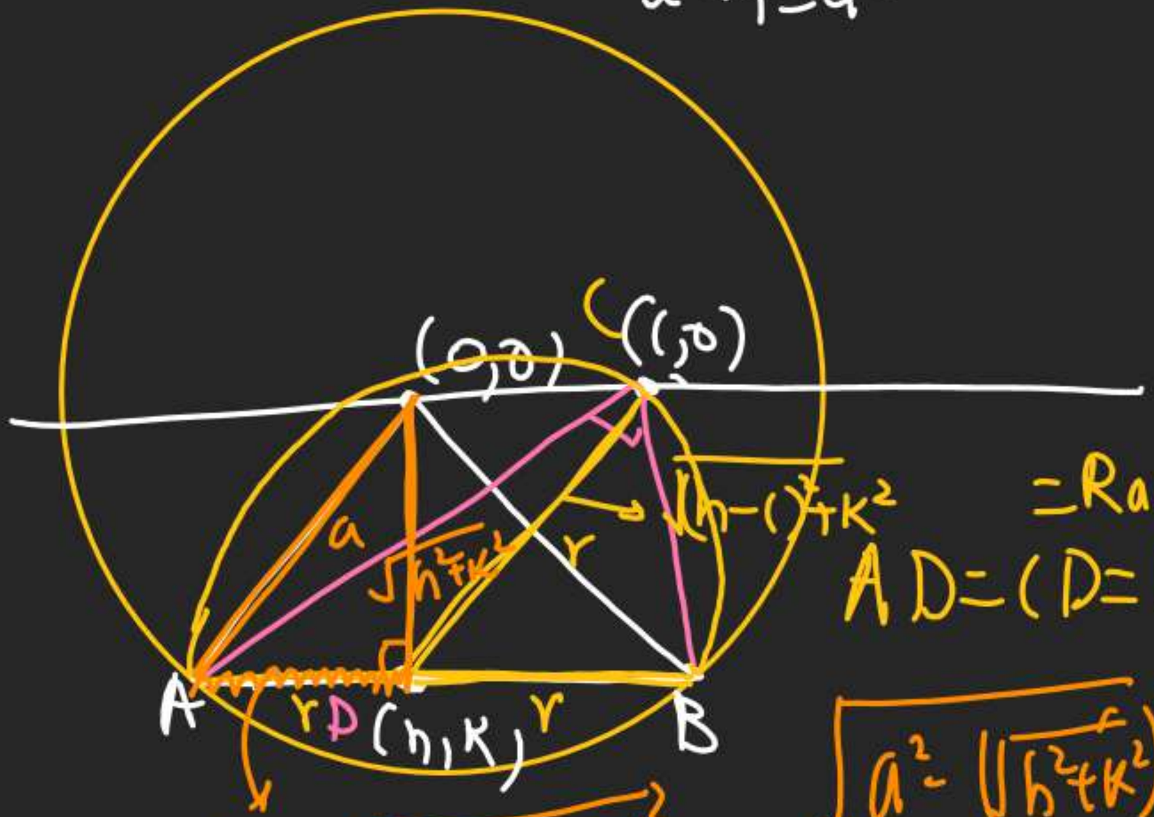
Q Find Locus of all Mid Pts of chords which makes 90° at Pt. $(c,0)$

$x^2+y^2=a^2$



$= \text{Rad}$

$AD = CD = BD$

$AD = \sqrt{a^2 - \left(\sqrt{h^2+k^2}\right)^2}$

$\sqrt{a^2 - \left(\sqrt{h^2+k^2}\right)^2} = \sqrt{(h-c)^2 + k^2}$

$a^2 - h^2 - k^2 = h^2 + k^2 + c^2 - 2ch$

$2x^2 + 2y^2 - 2cx + c^2 - a^2 = 0$

Family of Circles

4 Kind of Possibility

(1) Eqn of Circle P.T.

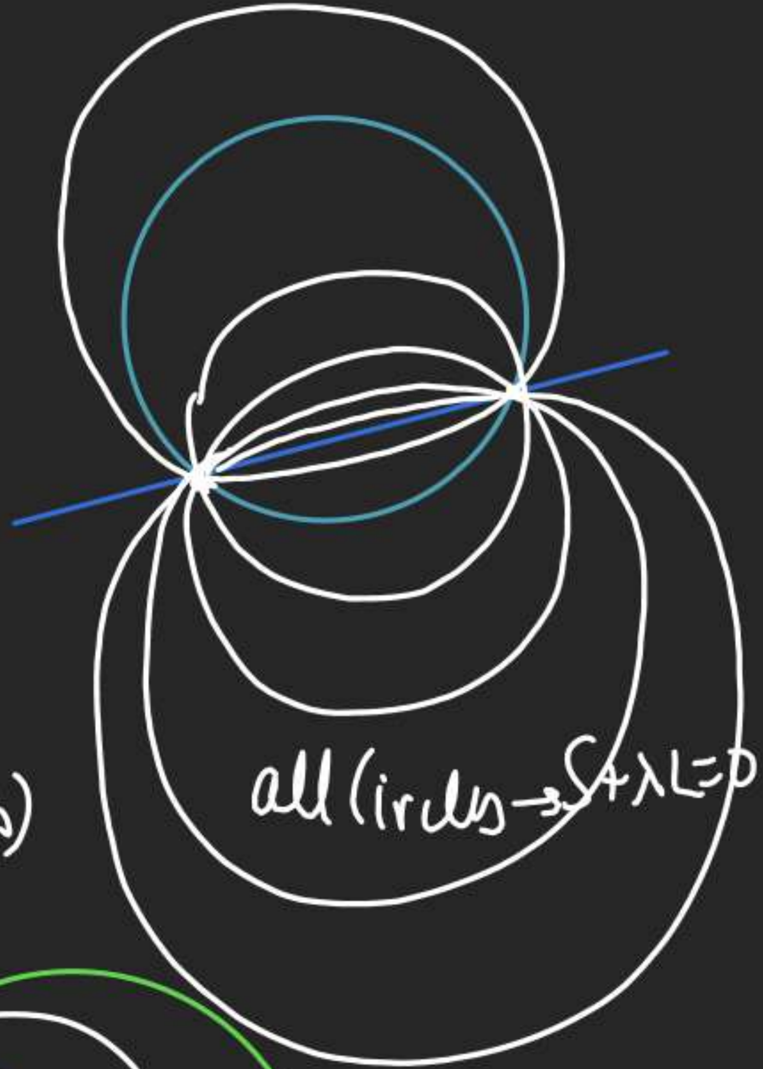
A) PoI of 2 Circle

B) PoI of A Circle & A Line

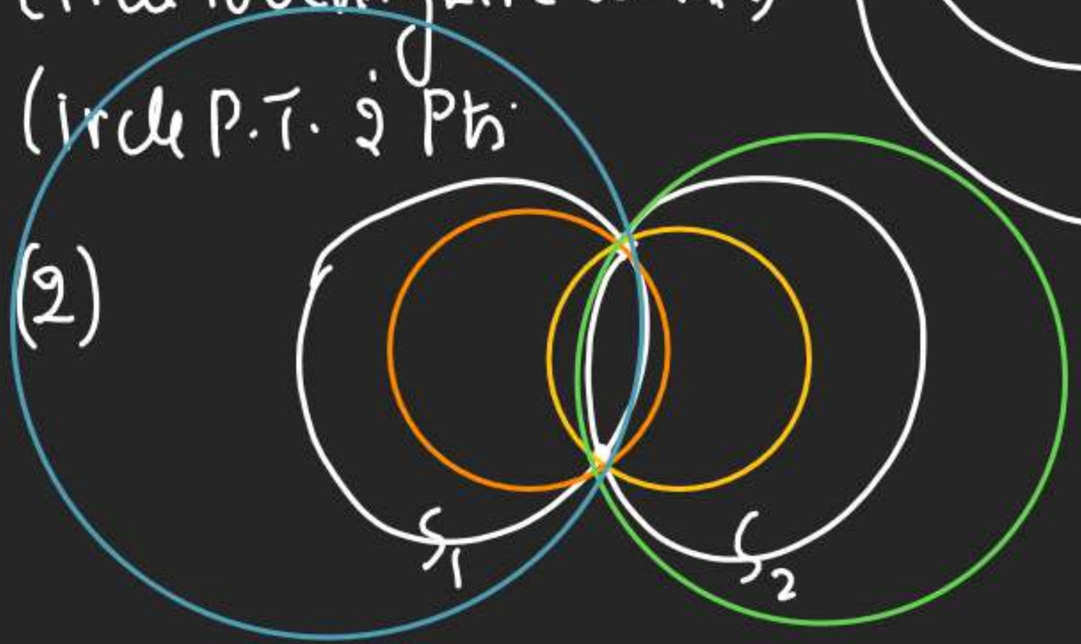
C) Circle touching Line at (a,b)

D) Circle P.T. 2 Pts

(3)

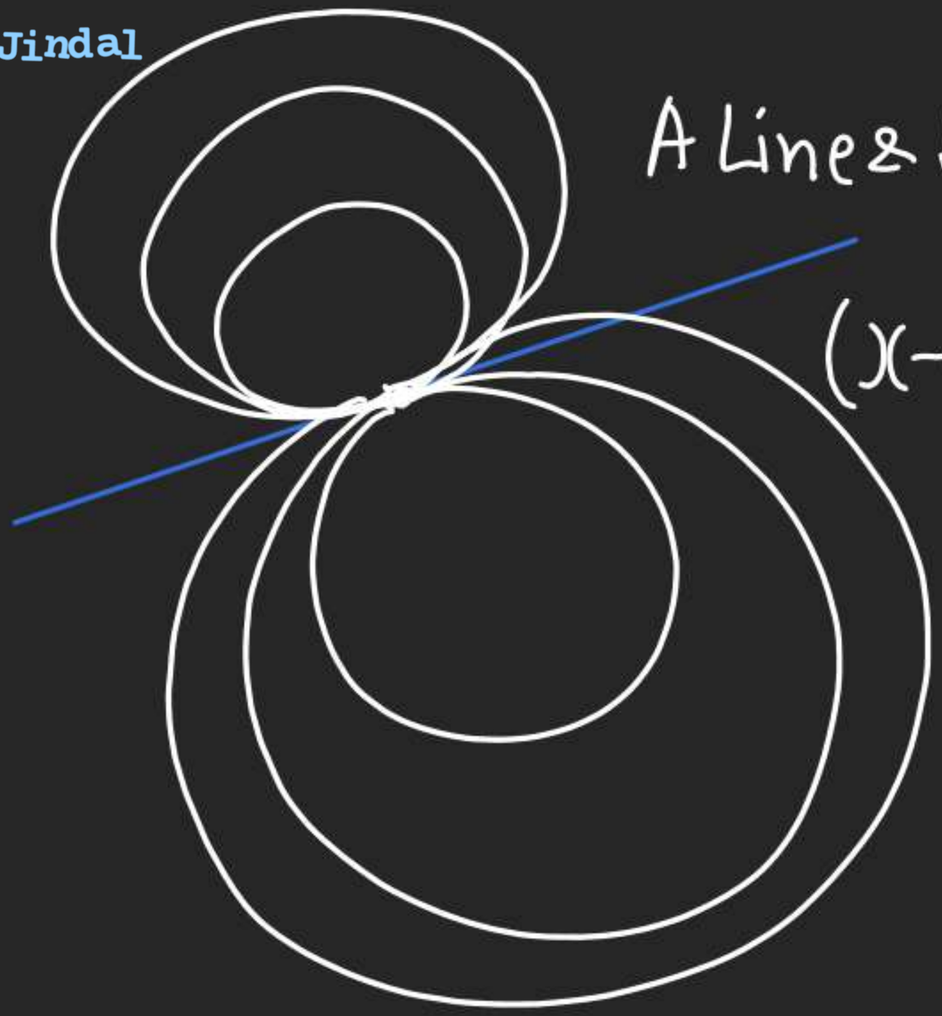


(2)



all ∞ Circles P.T. P.O.I. of 2 Circles are Family of Circle → $S_1 + \lambda S_2 = 0$

3) A Line & Pt. (a,b)



$$(x-a)^2+(y-b)^2+\lambda L=0$$

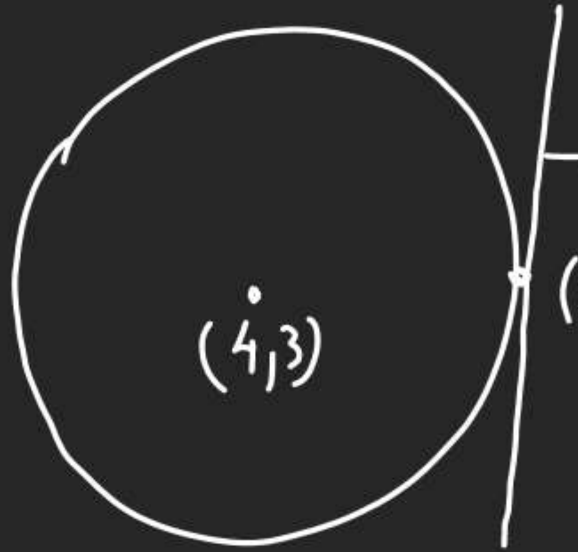
(4) Circle P.T. (a,b) & (c,d)

$$(x-a)(x-c)+(y-b)(y-d)+\lambda\begin{vmatrix} x & y & 1 \\ a & b & 1 \\ c & d & 1 \end{vmatrix}=0$$

Q Find EOC P.T. $(1,-2)$

& touches $S: x^2+y^2-8x-6y=0$

at $(0,0)$



$\rightarrow x\cdot 0+y\cdot 0-4(x+0)-3(y+0)=0$

$4x+3y=0$

Such Circle : $S+\lambda L=0$

New Circle $(x^2+y^2-8x-6y)+\lambda(4x+3y)=0$ P.T. $(1,-2)$

$$(1+4-8+12)+\lambda(4-6)=0$$

$$2\lambda=9 \Rightarrow \lambda=\frac{9}{2}$$

$$(x^2+y^2-8x-6y)+\frac{9}{2}(4x+3y)=0$$

$$2x^2+2y^2+20x+15y=0$$