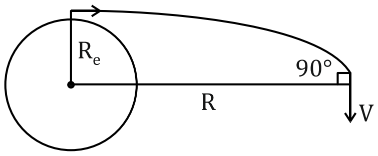
DPP 02

SOLUTION

1. $V_0\sqrt{\frac{GM}{R_e}} \times \sqrt{\frac{3}{2}}$

$Mv_0R_e = mVR$



$$\sqrt{\frac{3}{2}}\sqrt{\frac{GM}{R_e}}R_e = VR$$

$$-\frac{GMm}{R_e} + \frac{1}{2}mv_0^2 = -\frac{GMm}{R} + \frac{1}{2}mv^2$$

$$-\frac{GMm}{R_e} + \frac{1}{2}m\left(\frac{3}{2}\right)\frac{GM}{R_e} = -\frac{GMm}{R}$$

$$+\frac{1}{2}m\frac{3}{2}\frac{GM}{R_e}\frac{R_e^2}{R^2}$$

$$-\frac{1}{R_e} + \frac{3}{4R_e} = -\frac{1}{R} + \frac{3}{4}\frac{R_e}{R^2}$$

$$-\frac{1}{4R_e} = -\frac{1}{R} + \frac{3}{4}\frac{R_e}{R^2}$$

By further calculating $R = 3R_e$

2. Given: mass of planet = M, radius of planet = R, radius of orbit = R

Orbital velocity, $v_0 = \sqrt{\frac{GM_E}{R_E}} = \sqrt{\frac{GM_E}{R}}$ .... (i)

Escape velocity, $v_e = \sqrt{\frac{2GM_E}{R}}$ .... (ii)

From (i) and (ii), we get $\frac{v_0}{v_e} = \frac{1}{\sqrt{2}}$

3. The escape velocity from any planet's surface,

$$v_e = \sqrt{\frac{2GM}{R}}$$

$$\text{So,} \frac{v_A}{v_B} = \sqrt{\frac{M_A/R_A}{M_B/R_B}} \text{ or } \frac{v_A}{v_B} = \sqrt{\frac{M_A}{R_A} \Big/ \frac{\frac{1}{2}M_A}{\frac{1}{2}R_A}}$$

$$\text{or } \frac{v_A}{v_B} = 1 \Rightarrow n = 4 \left[\because \frac{v_A}{v_B} = \frac{n}{4}\right]$$

4. Time period of the spaceship,

$$T = 2\pi\sqrt{\frac{(R_e+h)^3}{GM_e}} = 2\pi\sqrt{\frac{(2\times10^6 + 2\times10^4)^3}{6.67\times10^{-11}\times8\times10^{22}}}$$

$= 7805.1 \text{ s} = 2.17 \text{ h}$

So, the number of complete revolutions in 24 hours,

$$n = \frac{24}{2.17} = 11.07 \approx 11$$

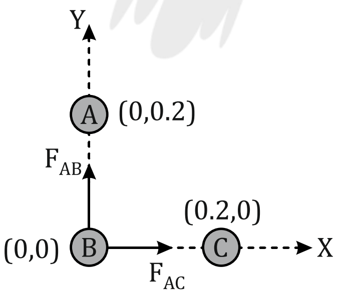
5. According to Kepler's law, $T^2 \propto r^3$

$$\therefore \left(\frac{T_1}{T_2}\right)^2 = \left(\frac{r_1}{r_2}\right)^3 = \left(\frac{1}{4}\right)^3 = \frac{1}{64} \text{ or } \frac{T_1}{T_2} = \frac{1}{8}$$

or $T_2 = 8T_1 = 8 \times 5 = 40$ hour

6. The centripetal and centrifugal forces disappear, the satellite has the tangential velocity and it will move in a straight line.

7. Let particle A lies at origin, particles B and C on y and X-axis, respectively.



$$\vec{F}_{AC} = \frac{Gm_Am_B}{r_{AB}^2}\hat{\imath} = \frac{6.67\times10^{-11}\times1\times1}{(0.2)^2}\hat{\imath} = 1.67\times10^{-9}\hat{\imath}\text{ N}$$

Similarly $\vec{F}_{AB} = 1.67 \times 10^{-9}\hat{\jmath}$ N

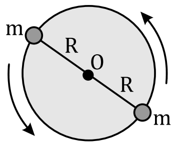
$\therefore$ Net force on particle A

$$\vec{F} = \vec{F}_{AC} + \vec{F}_{AB} = 1.67\times10^{-9}(\hat{\imath}+\hat{\jmath})\text{N}$$

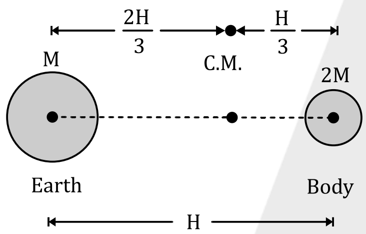
8. As observed from the earth, the sun appears to move in an approximate circular orbit. The gravitational force of attraction between the earth and the sun always follows inverse square law. Due to relative motion between the earth and mercury, the orbit of mercury, as observed from the earth, will not be approximately circular, since the major gravitational force on mercury is due to the sun.

9. Centripetal force provided by the gravitational force of attraction between two particles i.e.

$$\frac{mv^2}{R} = \frac{Gm \times m}{(2R)^2} \Rightarrow v = \frac{1}{2}\sqrt{\frac{Gm}{R}}$$



10. As the masses of the body and the earth are comparable, they will move towards their centre of mass, which remains stationary.



Hence the body of mass 2m move through distance $\frac{H}{3}$. and time to reach the earth surface

$$= \sqrt{\frac{2h}{g}}$$

$$= \sqrt{\frac{2H/3}{g}} = \sqrt{\frac{2H}{3g}}$$

11. We will be thrown into space, if weight mg is equal to gravitational force due to the planet. If y is the closest distance,

$$\frac{GMm}{R^2} = mg = \frac{G(KM)m}{(K'R + y)^2}$$

$$(K'R + y)^2 = \frac{KGM}{g}$$

$$y = \left(\frac{KGM}{g}\right)^{1/2} - K'R$$

12. At P gravitational pull is same

$$\frac{GMm}{(R + x)^2} = \frac{GMm}{g_1\left(\frac{R}{4} + y\right)^2}$$

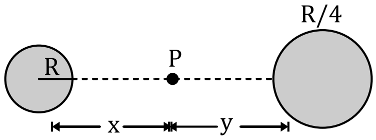
$$(R+x)^2 = 81\left(\frac{R}{4}+y\right)^2 = \left[9\left(\frac{R}{4}+y\right)\right]^2$$

$$R+x = 9\left(\frac{R}{4}+y\right) \ldots \text{(i)}$$

Let $x+y=r$

From (i)

$$R+(r-y) = 9\left(\frac{R}{4}+y\right)$$



$$y = \frac{1}{10}\left[r-\frac{5}{4}R\right]$$

Since $y>0, r>\frac{5}{4}R$ (at least)