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2. $(x^2-1)\sin x \frac{dy}{dx} + \left(2x\sin x + (x^2-1)\cos x\right) y = (x^2-1)\cos x$

$\frac{d}{dx}\left(y(x^2-1)\sin x\right) = (x^2-1)\cos x$

$\frac{dy}{dx} + \left(\frac{2x\sin x + (x^2-1)\cos x}{(x^2-1)\sin x}\right) y = \cot x$

I.F $= (x^2-1)\sin x$

$y(x^2-1)\sin x = \int (x^2-1)\cos x\, dx$

$= (x^2-1)\sin x - \int 2x\sin x\, dx$

$= (x^2-1)\sin x + 2x\cos x - \int 2\cos x\, dx$

$y(x^2-1)\sin x = (x^2-1)\sin x + 2x\cos x - 2\sin x + C$

$4x^{-4}$ 50

$9x^{-3}$ (remaining)

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1.

$$\frac{dy}{dx} + \left(\frac{x^2-1}{x(x^2+1)}\right)y = \frac{x^2 \ln x}{x(x^2+1)}.$$

$$I.F = e^{\int \left(\frac{1-\frac{1}{x^2}}{x+\frac{1}{x}}\right)dx} = x + \frac{1}{x}$$

$$y\left(x+\frac{1}{x}\right) = \int \ln x \, dx$$

$$\boxed{y\left(x+\frac{1}{x}\right) = x\ln x - x + C.}$$

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3.

$$\frac{dy}{dx} = \frac{1}{x\cos y + \sin 2y}$$

$$\frac{dx}{dy} - x\cos y = \sin 2y$$

$$I.F = e^{\int -\cos y \, dy} = e^{-\sin y}$$

$$x e^{-\sin y} = \int 2e^{-\sin y} \sin y \cos y \, dy$$

$$-\sin y = t$$

$$= 2\int e^{t}\, t \; dt = 2\, e^{t}(t-1) + C$$

$$\boxed{x e^{-\sin y} = 2e^{-\sin y}(-\sin y - 1) + C}$$

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4. Let $f(x)$ is defined for $x \geq 2$ and K is a constant then P.T. if $\frac{d}{dx}\left(x f(x)\right) \leq -K f(x)$, then $f(x) \leq A x^{-1-K}$, where A is independent of x.

$$f'(x) + \frac{(1+K)}{x} f(x) \leq 0$$

$$\text{I.F.} = e^{(1+K)\ln x} = x^{1+K}$$

$$x^{1+K}\left(f'(x) + \frac{1+K}{x} f(x)\right) \leq 0$$

$$\frac{d}{dx}\left(x^{1+K} f(x)\right) \leq 0$$

$$g(x) = x^{1+K} f(x)$$

$$g(x) \leq g(2)$$

$$x^{1+K} f(x) \leq 2^{1+K} f(2)$$

$$\boxed{f(x) \leq \left(2^{1+K} f(2)\right) x^{-1-K}}$$

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5.

$$\frac{dy}{dx} = \frac{y}{2y\ln y + y - x}$$

$$2y\ln y\,dy + y\,dy - \underline{x\,dy} = \underline{y\,dx}$$

$$\int \left(\underbrace{2y\ln y + y}\right) dy = \int y\,dx + x\,dy$$

$$y^2 \ln y \qquad \boxed{y^2 \ln y = xy + C}$$

$$\frac{dx}{dy} + \frac{x}{y} = 2\ln y + 1$$

$$I.F = y$$

$$xy = \int y(2\ln y + 1)\,dy$$

$$= y^2 \ln y + C$$

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6. The function $y(x)$ satisfies the eqn.

$$y(x) + 2x\int_0^x \frac{y(u)\,du}{1+u^2} = 3x^2+2x+1.$$

S.T. the substitution

$$z(x) = \int_0^x \frac{y(u)\,du}{1+u^2}$$

converts the eqn. into first order linear DE for $z(x)$, Solve for $z(x)$ and hence solve for $y(x)$.

$\boxed{z(0)=0}$

$$\frac{dz}{dx} = \frac{y(x)}{1+x^2}$$

$$\frac{dz}{dx}(1+x^2) + 2xz = 3x^2+2x+1$$

$$\frac{dz}{dx} + \frac{2x}{1+x^2}z = \frac{3x^2+2x+1}{1+x^2}$$

$$I.F = 1+x^2$$

$$\Rightarrow z(1+x^2) = \int(3x^2+2x+1)\,dx$$

$$z(1+x^2) = x^3+x^2+x+C$$

$$z(0)=0 \Rightarrow \boxed{C=0}$$

$$\boxed{z(x) = \frac{x^3+x^2+x}{x^2+1}} = x+1-\frac{1}{x^2+1}$$

$$(x^2+1)(x+1)-1$$

$$y(x) = \left(1+\frac{2x}{(x^2+1)^2}\right)(x^2+1)$$

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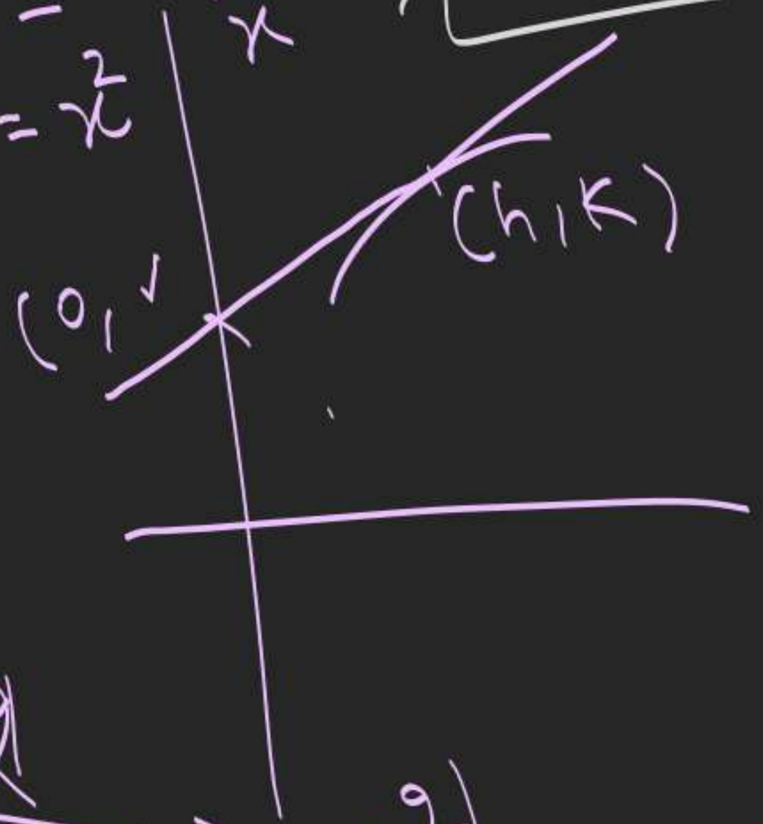
7. Find the curve s.t. y-intercept cut off by tangent on any arbitrary point on curve is proportional to cube of ordinate of point of tangency.

$$\frac{y-K}{-h} = \left(\frac{dy}{dx}\right)_{(h,K)}$$

$\boxed{\text{Ex}-2}$ → JEE Mains



$$y - x\frac{dy}{dx} = \lambda y^3$$

$$\frac{dy}{dx} - \frac{y}{x} = -\frac{\lambda y^3}{x}$$

$$\frac{2}{y^3}\frac{dy}{dx} - \frac{2}{xy^2} = -\frac{2\lambda}{x}$$

$$-\frac{1}{y^2} = t$$

$$\frac{2}{y^3}\frac{dy}{dx} = \frac{dt}{dx}$$

$$\frac{dt}{dx} + \frac{2t}{x} = -\frac{2\lambda}{x} \Rightarrow \boxed{x^2 t = -\lambda x^2 + C}$$

$$\text{I.F.} = x^2$$