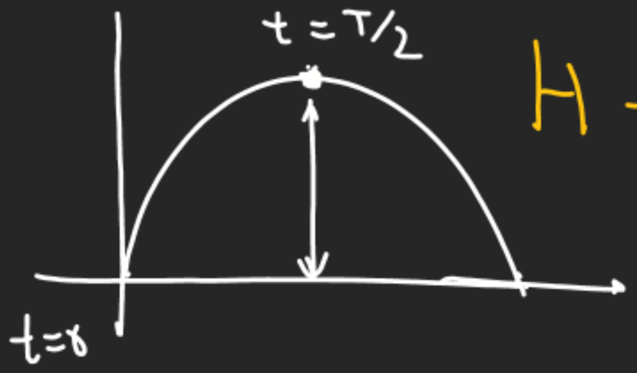
Nishant Jindal

Projectile Motion

$\rightarrow T = \frac{2u_y}{g}$

$\rightarrow H = \frac{u_y^2}{2g}$ ✓

$\rightarrow R = \frac{2u_x u_y}{g}$ ✓

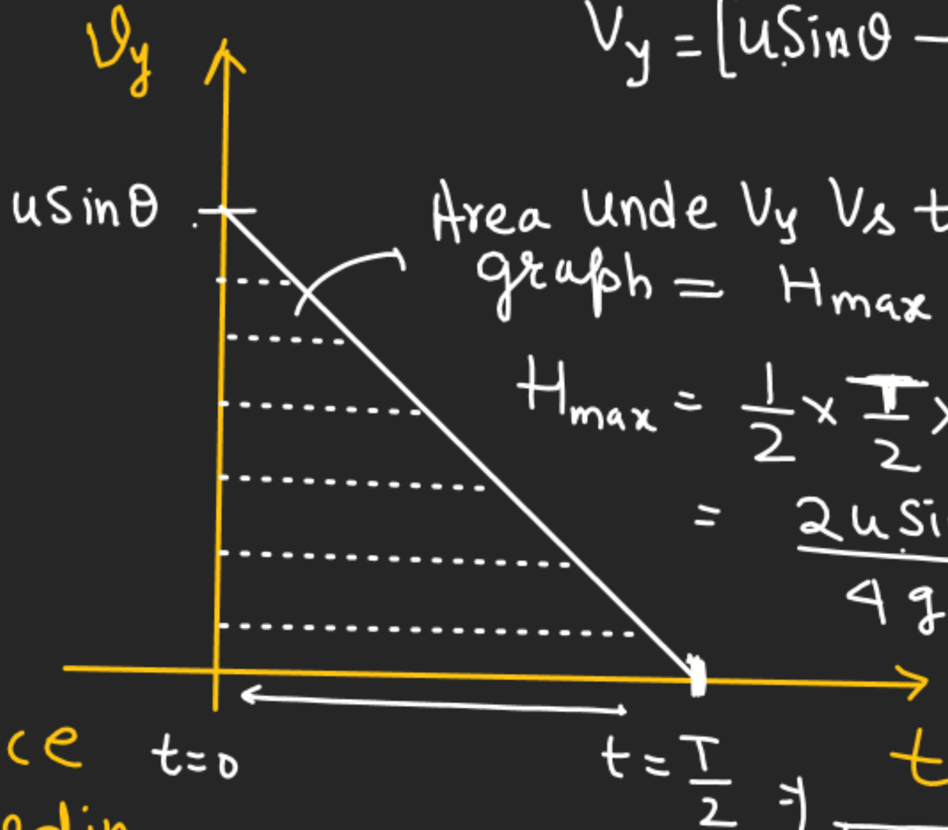


H → [Distance travelled in y-direction in $\frac{T}{2}$ time.

R → [Total distance in x-direction in T.

Graphical Method

$V_y = [u\sin\theta - gt]$

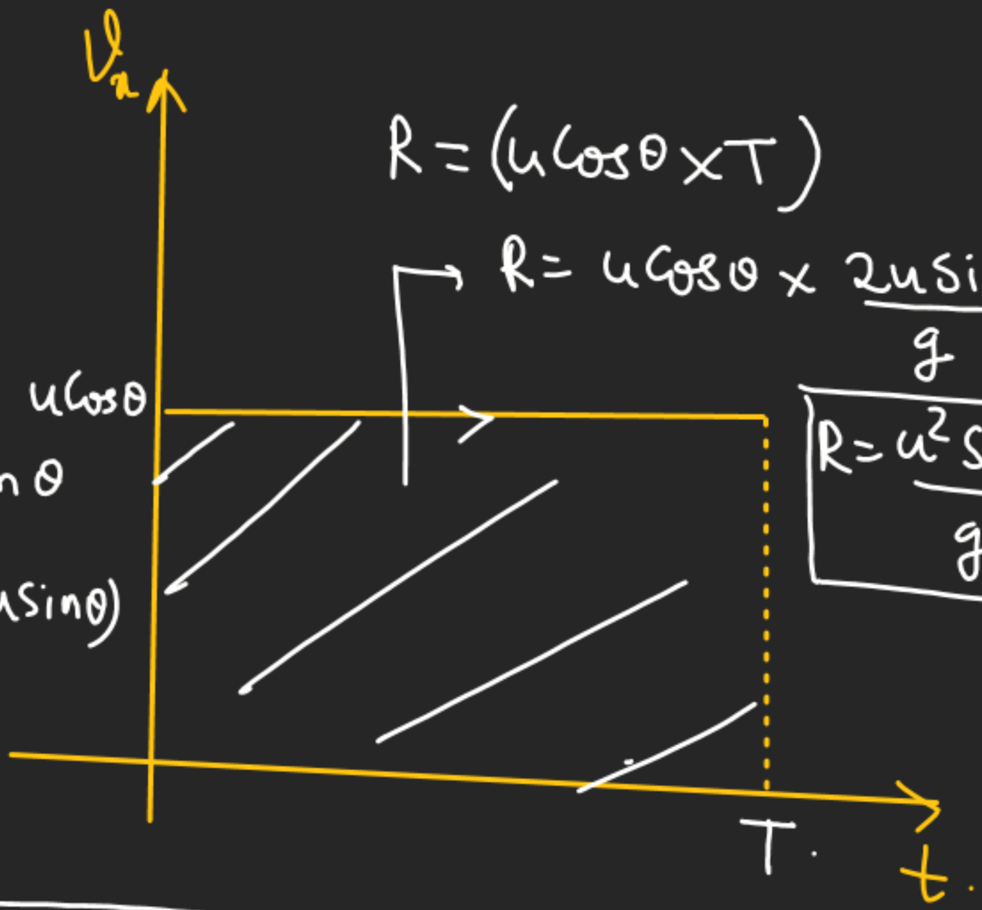


Area unde V_y Vs t graph = H_{max}.

$H_{max} = \frac{1}{2} \times \frac{T}{2} \times u\sin\theta$

$= \frac{2u\sin\theta}{4g} \times (u\sin\theta)$

$$\boxed{H_{max} = \frac{u^2\sin^2\theta}{2g}}$$



$R = (u\cos\theta \times T)$

$\rightarrow R = u\cos\theta \times \frac{2u\sin\theta}{g}$ ✓

$$\boxed{R = \frac{u^2\sin 2\theta}{g}}$$

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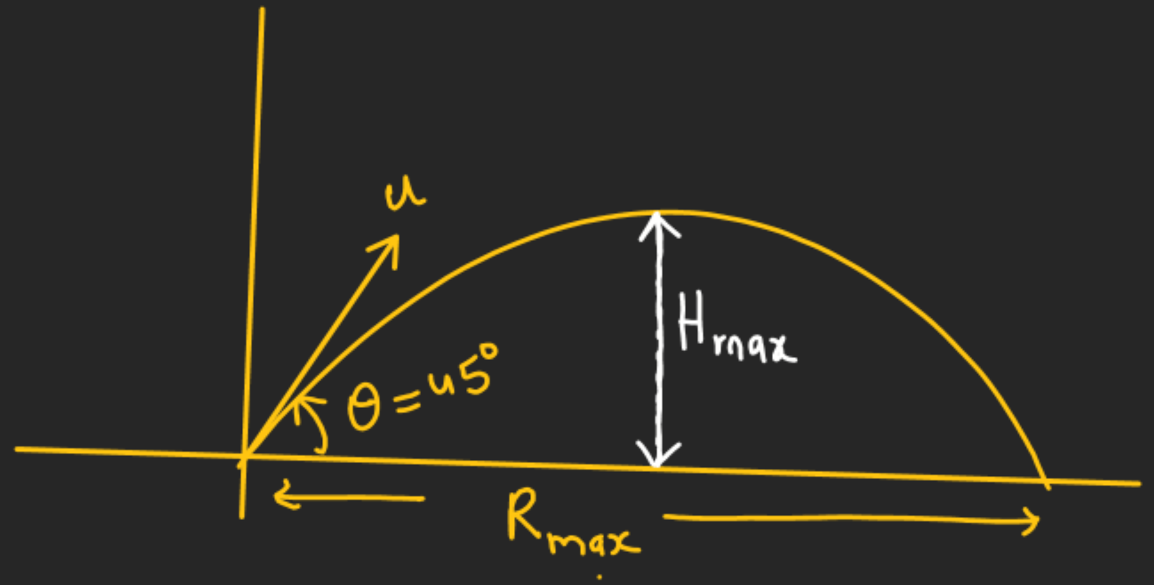
(*) Case of Maximum range :-



$$R = \frac{u^2 \sin 2\theta}{g}$$

For R to be maximum
$\sin 2\theta = 1$
$2\theta = 90$
$\theta = 45^\circ / \frac{\pi}{4}$

$$\boxed{R_{max} = \frac{u^2}{g}}$$

* H_{max} Corresponding to R_{max}.

$\theta = 45^\circ$

$$H_{max} = \frac{u^2 \sin^2\theta}{2g}$$

$$H_{max} = \frac{u^2}{2g} (\sin 45^\circ)^2$$

$$= \frac{u^2}{4g} \rightarrow R_{max}$$

At $\theta = 45^\circ$

$$\Rightarrow \boxed{R_{max} = 4 H_{max}}$$

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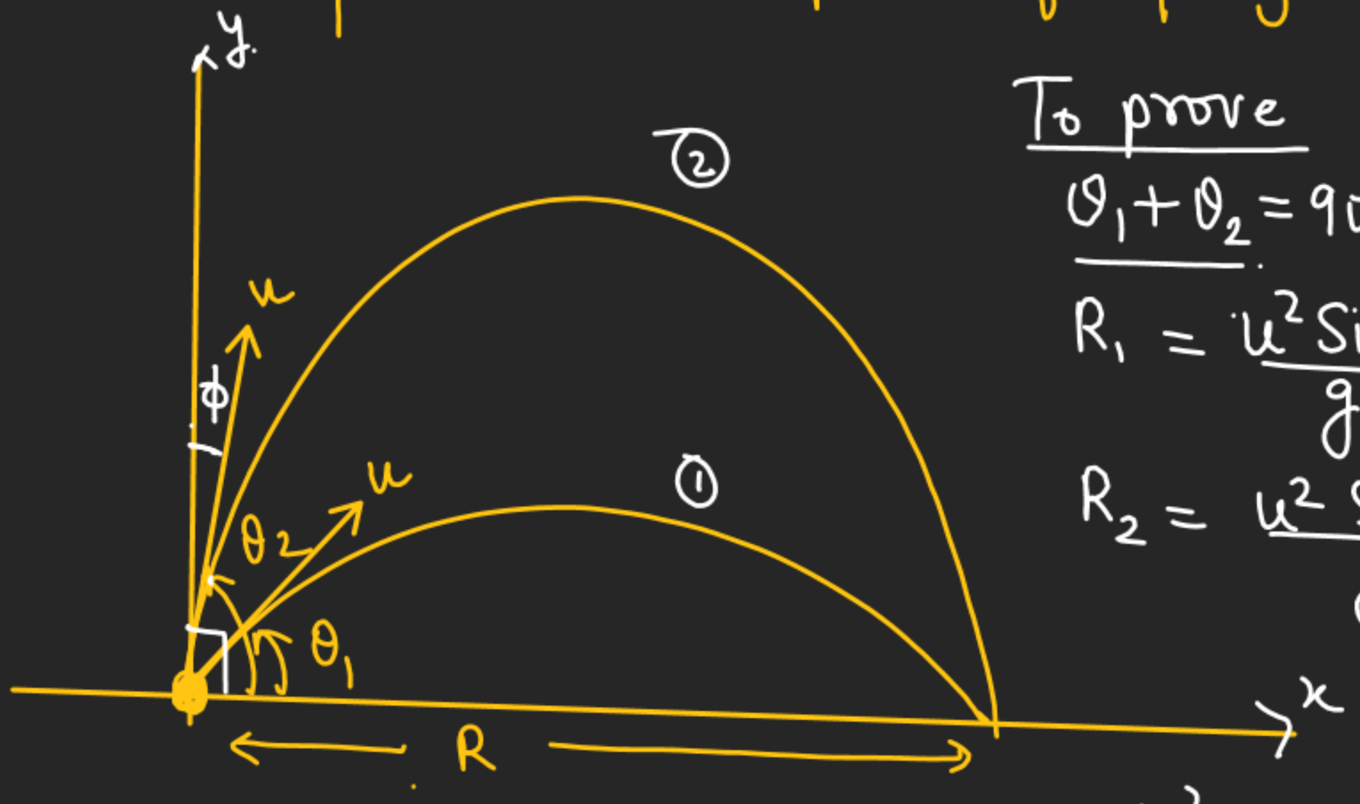
Projectile Motion

(*)(*) If angle of projections are Complementary with each other then range must be same provided speed of projection is same.

$\sin(\pi - \theta) = \sin\theta$



To prove

$\theta_1 + \theta_2 = 90^\circ$

$R_1 = \dfrac{u^2 \sin 2\theta_1}{g}$ — (1)

$R_2 = \dfrac{u^2 \sin 2\theta_2}{g}$

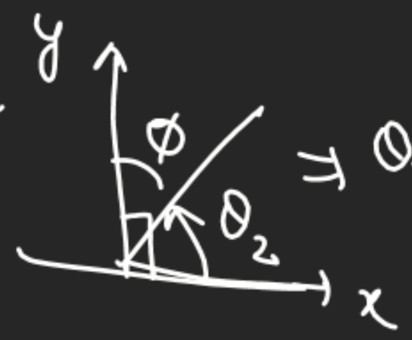
$\theta_2 + \phi = 90^\circ$

$\theta_2 = (90 - \phi)$

$R_2 = \dfrac{u^2 \sin 2(90-\phi)}{g}$

$R_2 = \dfrac{u^2}{g} \sin(180 - 2\phi)$

$R_2 = \dfrac{u^2}{g} \sin(2\phi)$ — (2)



$\Rightarrow \theta_2 + \phi = 90^\circ$

$R_1 = R_2$

$\dfrac{u^2 \sin 2\theta_1}{g} = \dfrac{u^2}{g} \sin 2\phi \Rightarrow \boxed{\theta_1 = \phi} \Rightarrow \boxed{\theta_2 + \theta_1 = 90^\circ}$

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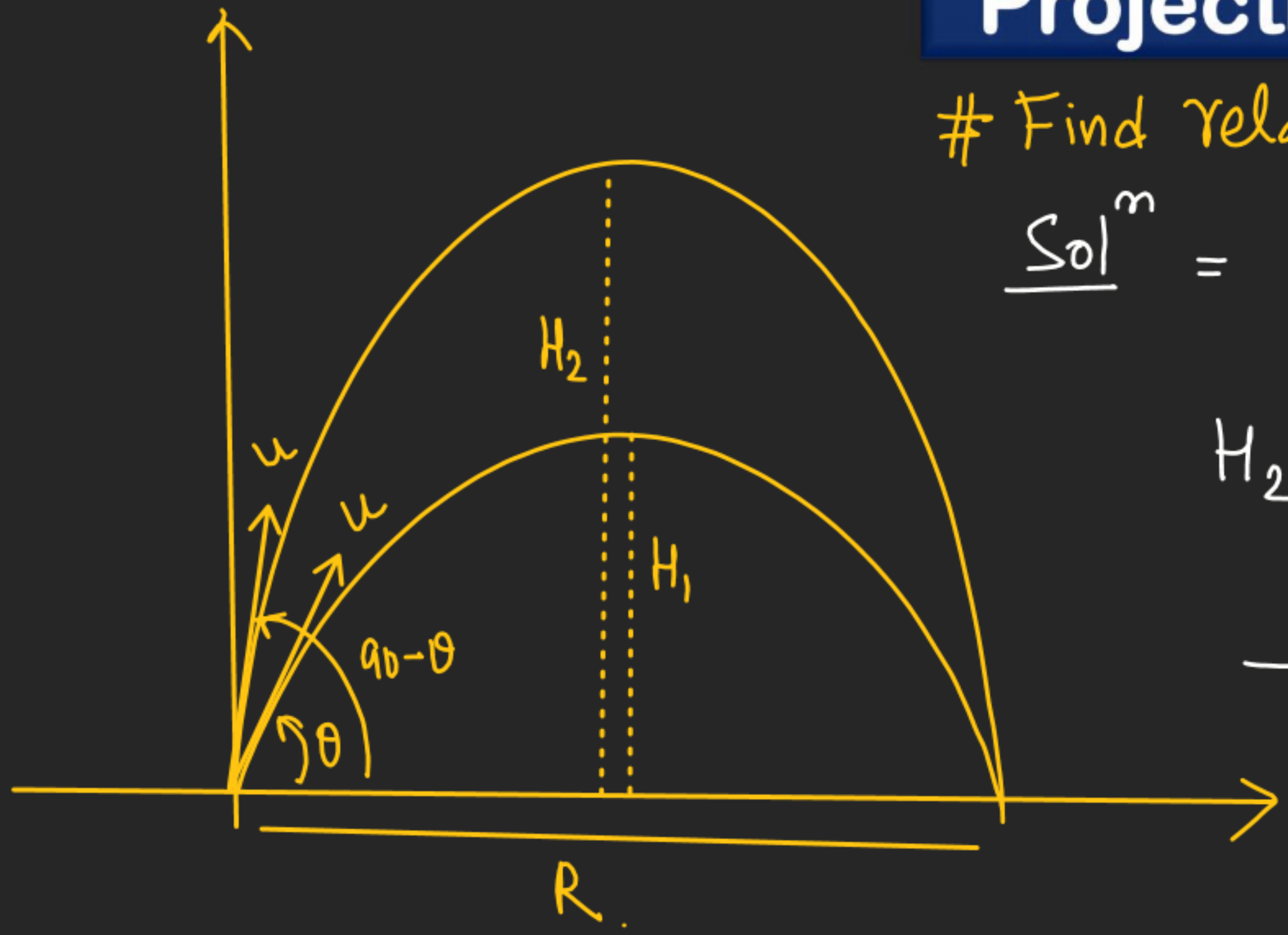
Projectile Motion



Find relation b/w R, H_1 & H_2

$$\left[\sin(90-\theta) = \cos\theta\right]$$

$$\sin 2\theta = 2\sin\theta\cos\theta$$

$$\left[R = \frac{u^2 \sin 2\theta}{g}\right]$$

Sol^n =

$$H_1 = \frac{u^2\sin^2\theta}{2g} \quad -(1)$$

$$H_2 = \frac{u^2\sin^2(90-\theta)}{2g} = \frac{u^2}{2g}\cos^2\theta \quad -(2)$$

(1) x (2)

$$H_1H_2 = \left(\frac{u^2}{2g}\right)^2 \sin^2\theta\cdot\cos^2\theta \times \left(\frac{4}{4}\right)$$

$$= \frac{1}{4}\times\Bigg[\underbrace{\frac{u^2}{g}\left(2\sin\theta\cdot\cos\theta\right)}_{R}\Bigg]^2 \times \frac{1}{4}$$

$$H_1H_2 = \frac{R^2}{16}$$

$$R = \sqrt{16\,H_1H_2}$$

$$\boxed{R = 4\sqrt{H_1H_2}}$$ Ans

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Projectile Motion

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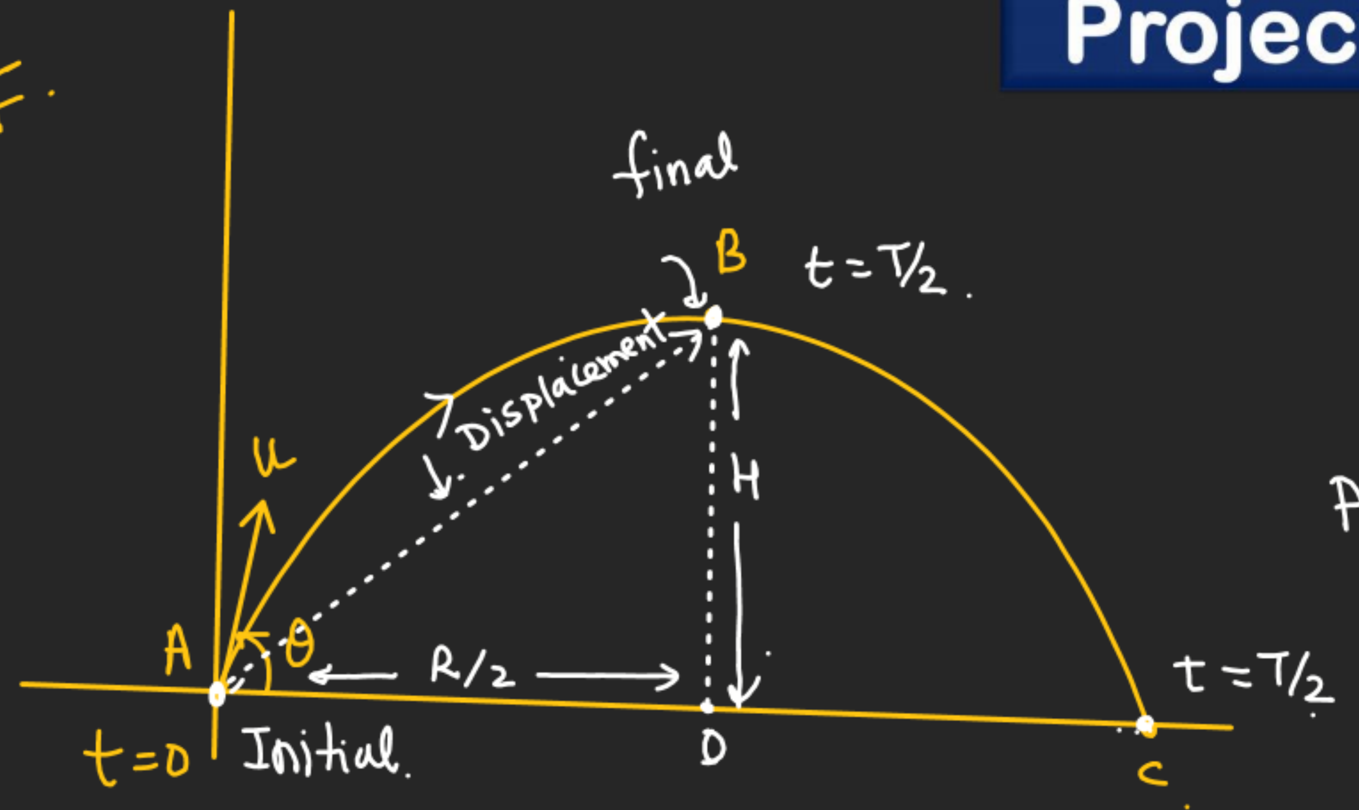

Find avg velocity and avg accleration of the particle in the interval.

a) $t = 0$ to $t = T/2$

b) $t = 0$ to $t = T$

Sol^n.

$$V_{avg} = \left(\frac{\text{Total displacement}}{\text{Total time taken}}\right)$$

$$AB = \sqrt{OA^2 + OB^2}$$

$$= \sqrt{\frac{R^2}{4} + H^2}$$

$$= \sqrt{\left(\frac{u^2\sin 2\theta}{g}\right)^2 \times \frac{1}{4} + \left(\frac{u^2\sin^2\theta}{2g}\right)^2}$$

$$= \sqrt{\frac{4u^4\sin^2\theta.\cos^2\theta}{g^2} \times \frac{1}{4} + \frac{u^4\sin^4\theta}{4g^2}}$$

$$= \frac{u^2\sin\theta}{2g}\sqrt{4\cos^2\theta + \sin^2\theta} = \frac{u^2\sin\theta}{2g}\sqrt{1+3\cos^2\theta}$$

$$V_{avg} = \frac{\frac{u^2\sin\theta}{2g}\sqrt{1+3\cos^2\theta}}{\left(\frac{u\sin\theta}{g}\right)} \rightarrow T/2$$

$$\boxed{V_{avg} = \frac{u}{2}\sqrt{1+3\cos^2\theta}}$$

Ans.

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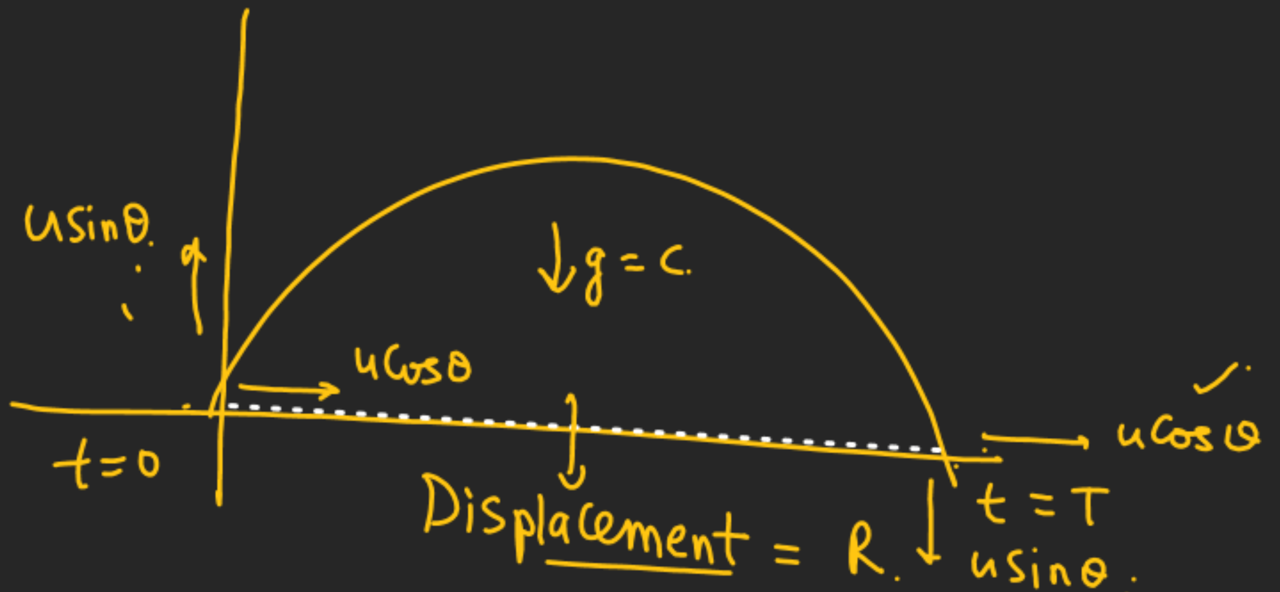
Projectile Motion

$\underline{V_{avg}}$ $[t=0 \text{ to } t=T]$

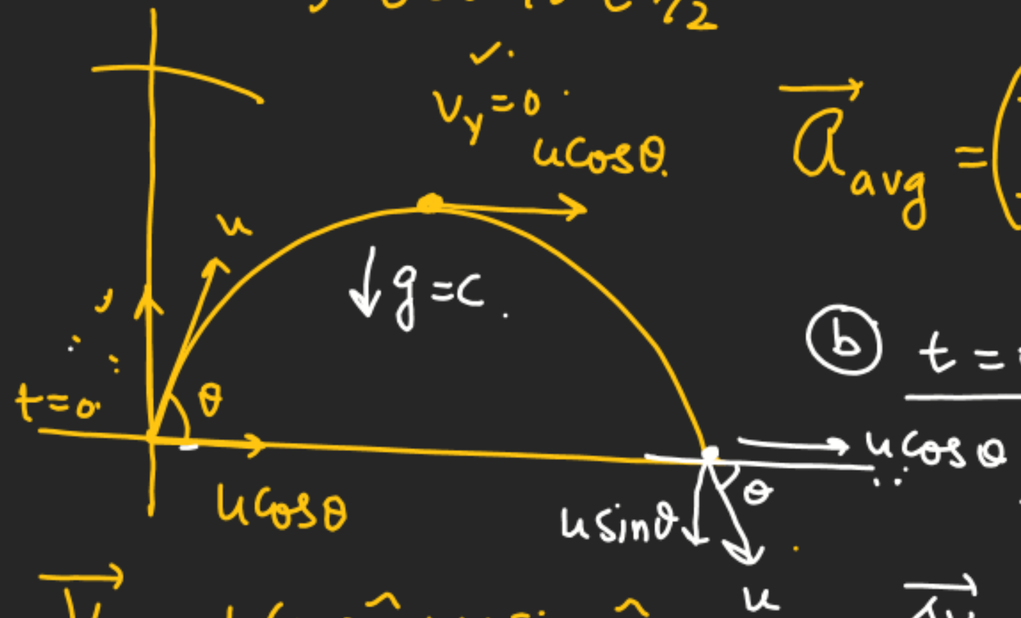


$$\underline{\underline{V_{avg}}} = \frac{R}{T} = \frac{u^2 \sin 2\theta}{g \times \left(\frac{2u\sin\theta}{g}\right)} = \underline{\underline{(u\cos\theta)}}$$

2nd Method

$$\left\{ \begin{array}{l} \dfrac{V_y + u_y}{2} = (V_{avg})_y \\ \dfrac{-u\sin\theta + u\sin\theta}{2} = 0 \end{array} \right\}$$

(a) $t=0$ to $t=T/2$



$$\vec{a}_{avg} = \left(\frac{\vec{V}_f - \vec{V}_i}{\Delta t}\right)$$

$$\vec{V}_i = u\cos\theta\,\hat{i} + u\sin\theta\,\hat{j}$$

$$\vec{V}_f = (u\cos\theta\,\hat{i})$$

$$\vec{\Delta V} = \vec{V}_f - \vec{V}_i = -(u\sin\theta)\hat{j}$$

$$\frac{T}{2} = \left(\frac{u\sin\theta}{g}\right) \qquad \underline{\vec{a}_{avg}} = \left(\frac{\vec{\Delta V}}{T/2}\right) = \left(-g\,\hat{j}\right)$$

(b) $t=0$ to $t=T$

$$\boxed{a_{avg} = g}$$

$$\vec{\Delta V} = -(2u\sin\theta)\hat{j}$$

$$T = \frac{2u\sin\theta}{g}$$

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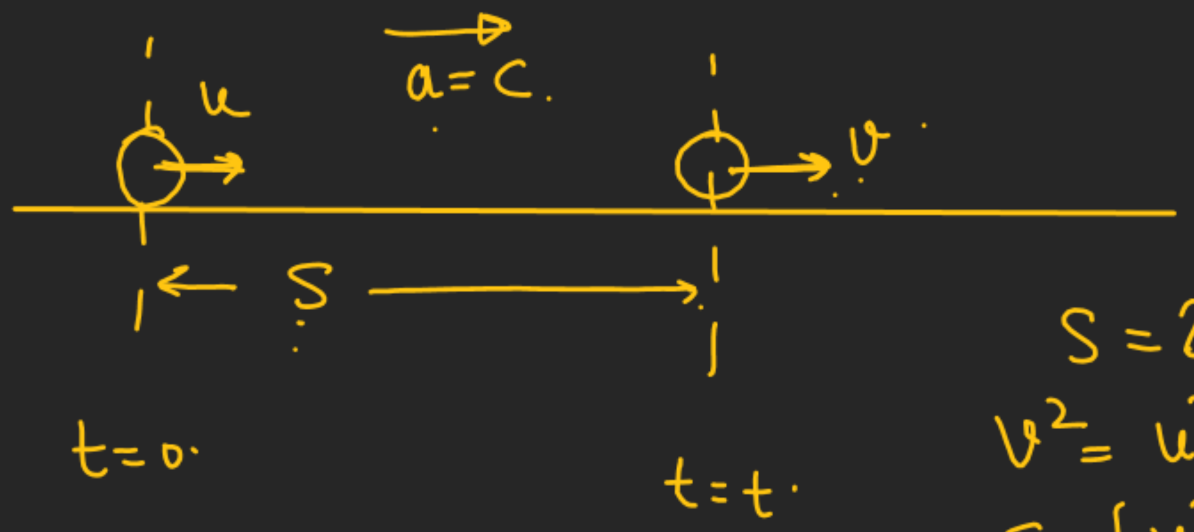
(*) **Avg velocity in Case of uniform acclerated Motion :-**



$$V_{avg} = \left(\frac{S}{t}\right)$$

$S = ?$

$v^2 = u^2 + 2aS$

$$S = \left[\frac{v^2 - u^2}{2a}\right]$$

$t = ?$

$V = u + at$

$$t = \left(\frac{V-u}{a}\right)$$

$$V_{avg} = \frac{\left[\frac{V^2 - u^2}{2a}\right]}{\left[\frac{V-u}{a}\right]} = \left(\frac{V+u}{2}\right)$$

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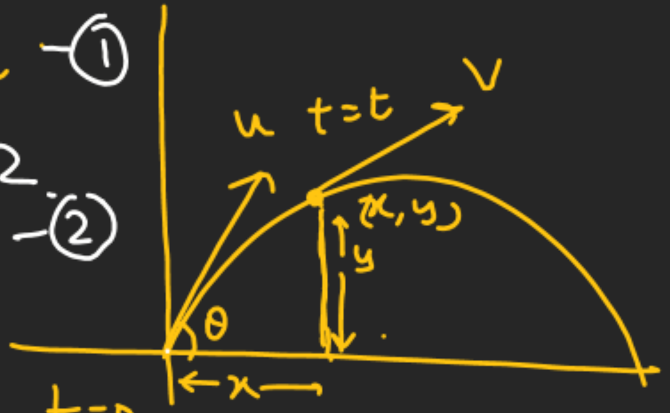
Projectile Motion

(*) **Trajectory of the projectile Motion :-**

$$X = (u\cos\theta)t \quad \text{—(1)}$$

$$y = (u\sin\theta)t - \frac{1}{2}gt^2 \quad \text{—(2)}$$



$$t = \left(\frac{x}{u\cos\theta}\right) \text{ put in (2)}$$

$$y = (u\sin\theta)\left(\frac{x}{u\cos\theta}\right) - \frac{1}{2}g\left(\frac{x}{u\cos\theta}\right)^2$$

**

$$\boxed{y = x\tan\theta - \left(\frac{g}{2u^2\cos^2\theta}\right)x^2} \Leftarrow \text{Equation of Trajectory}$$

$x\tan\theta \Downarrow a$, $\left(\frac{g}{2u^2\cos^2\theta}\right) \Downarrow b$

$$\boxed{y = ax - bx^2}$$

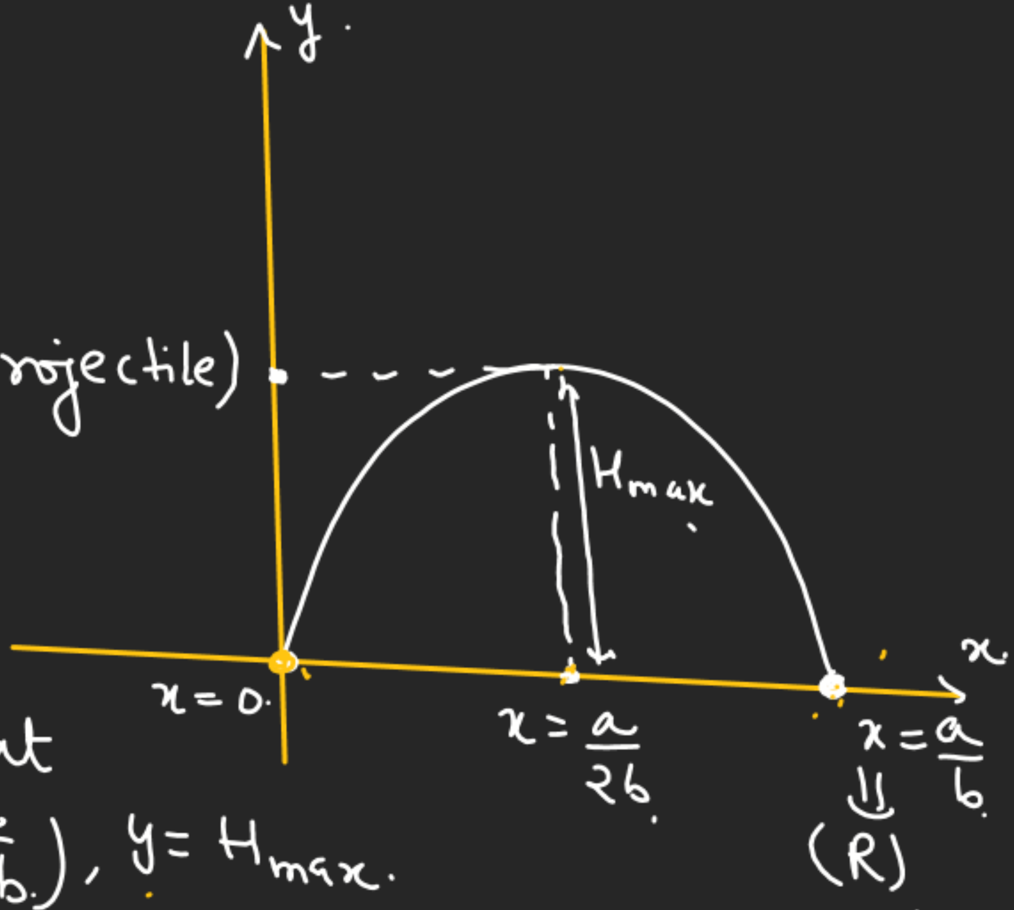
Roots, $y = 0$.

$x(a - bx) = 0$.

$x = 0$, $x = \left(\frac{a}{b}\right)$

$\Downarrow$

(Range of projectile)



Maximum height at $x = \left(\frac{a}{2b}\right)$, $y = H_{max}$.

$$H_{max} = a\left(\frac{a}{2b}\right) - b\left(\frac{a}{2b}\right)^2$$

$$= \frac{a^2}{2b} - \frac{a^2}{4b} = \left(\frac{a^2}{4b}\right)$$

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Projectile Motion

(x)

$$y = (2x - 8x^2)$$

$\Rightarrow$ $H_{max} = ?$

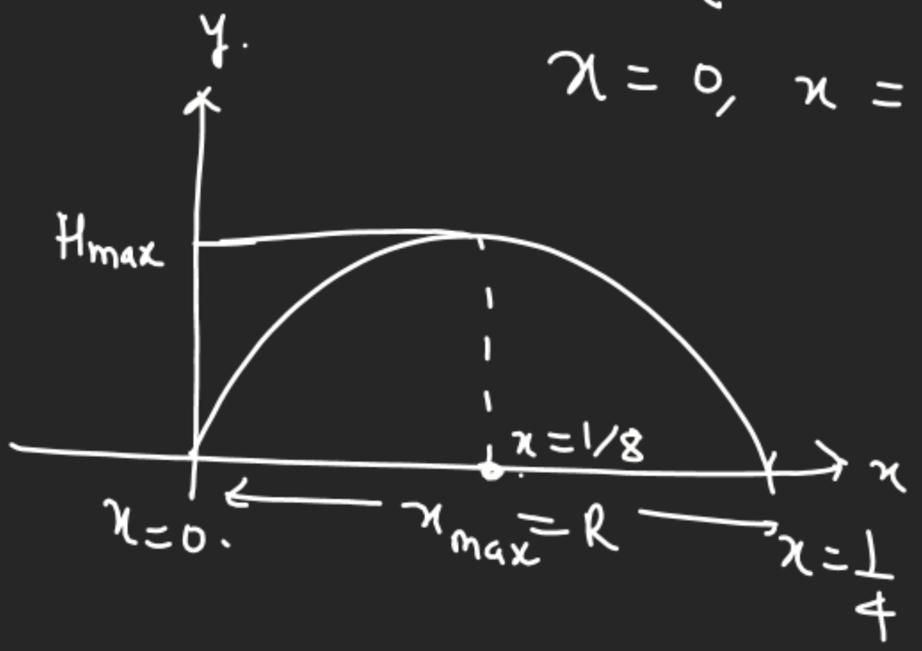
$R = ?$

Roots

$y = 0$.

$2x(1-4x) = 0$

$x = 0, \quad x = \frac{1}{4}$.



$\text{Rang} = \left(\frac{1}{4}\right)$.

H_{max} is value of y at $x = \frac{1}{8}$.

$$H_{max} = 2\times\left(\frac{1}{8}\right) - 8\times\left(\frac{1}{8}\right)^2$$

$$= \left(\frac{1}{4} - \frac{1}{8}\right) = \left(\frac{1}{8}\right) \checkmark$$