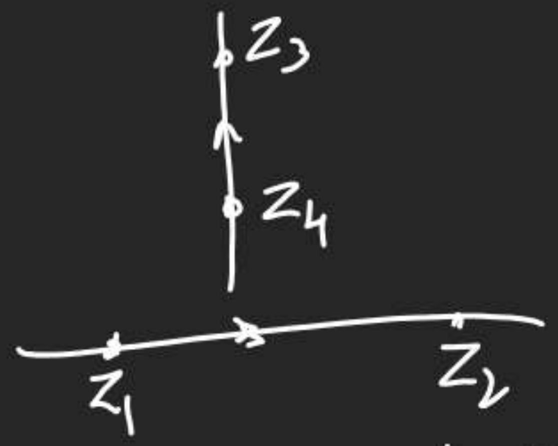
Nishant Jindal

RK :- If z_1, z_2, z_3 are Vertices of eq^l $\triangle$ then.

$$z_1^2 + z_2^2 + z_3^2 = z_1 z_2 + z_2 z_3 + z_3 z_1$$

Cases (Rem)

① When $L_1 \perp L_2$



$$\frac{z_2 - z_1}{|z_2 - z_1|} \times e^{i\frac{\pi}{2}} = \frac{z_3 - z_4}{|z_3 - z_4|}$$

$$K \cdot i = \frac{z_3 - z_4}{z_2 - z_1} = \text{img No}$$

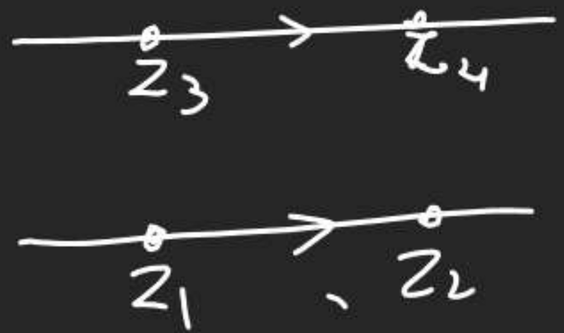
$z = $ img.
$z = -\bar{z}$

$$\frac{z_3 - z_4}{z_2 - z_1} + \frac{\bar{z}_3 - \bar{z}_4}{\bar{z}_2 - \bar{z}_1} = 0$$

$$z_3 - z_2 = i z_1$$

$$\frac{z_3 - z_2}{z_1} + \frac{\bar{z}_3 - \bar{z}_2}{\bar{z}_1} = 0$$

(2) When $L_1 \parallel L_2$



$$\frac{(z_2 - z_1)}{|\;\;|} \times e^{i0} = \frac{z_4 - z_3}{|\;\;|}$$

Real No. $= \frac{z_4 - z_3}{z_2 - z_1}$

$z = $ Real No $\Rightarrow z = \bar{z}$

$$\frac{z_4 - z_3}{z_2 - z_1} = \frac{\bar{z}_4 - \bar{z}_3}{\bar{z}_2 - \bar{z}_1}$$

Q If $\boxed{|z| = 2}$ & $\boxed{\frac{z_1 - z_3}{z_2 - z_3} = \frac{z - 2}{z + 2}}$ then P.T. z_1, z_2, z_3 are Vertices of Rt. angle $\triangle$.

Circle Centre $(0,0)$
Rad $= 2$

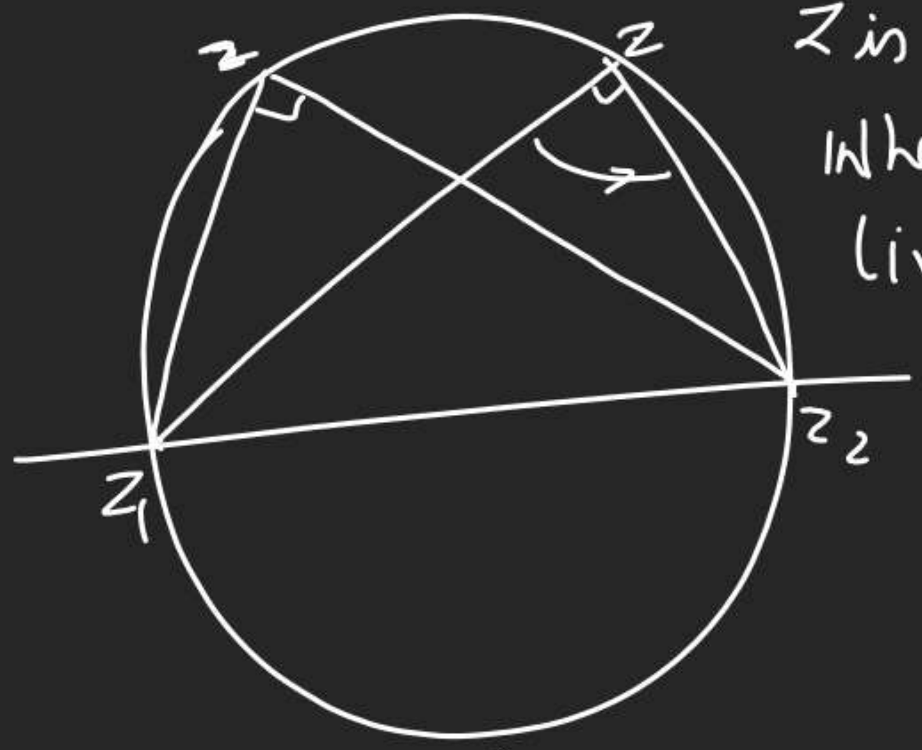
$$\text{Arg}\left(\frac{z - 2}{z + 2}\right) = \frac{\pi}{2}$$

$$\text{Re}\left(\frac{z_1 - z_3}{z_2 - z_3}\right) = 0 \qquad \text{Arg}\left(\frac{z_1 - z_3}{z_2 - z_3}\right) = \frac{\pi}{2}$$

$\Rightarrow \triangle z_1 z_2 z_3$ is also Rt. angle $\triangle$.

Q Find Eq^n of Circle in Diametri form where.

z_1, z_2 are end Pts of diameter of circle

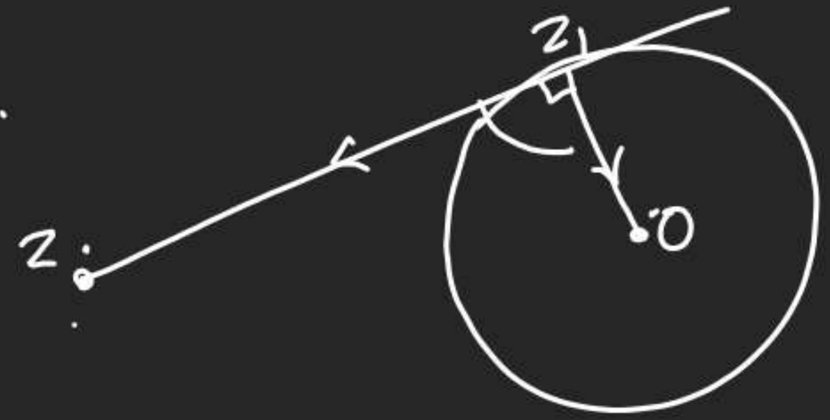


z is every where on circle.

$$\left(\frac{z-z_1}{z-z_2}\right) = \text{Purely Img.}$$

$$\frac{z-z_1}{z-z_2}+\frac{\bar{z}-\bar{z}_1}{\bar{z}-\bar{z}_2}=0 \text{ EOC}$$

Q.



$$\left(\frac{z-z_1}{-z_1}\right) = \text{Purely Img.}$$

$$\frac{z-z_1}{-z_1}+\frac{\bar{z}-\bar{z}_1}{-\bar{z}_1}=0$$

$$\frac{z-z_1}{-z_1} = -\frac{\bar{z}-\bar{z}_1}{-\bar{z}_1}$$

$$\frac{z-z_1}{\bar{z}-\bar{z}_1} = -\frac{z_1}{\bar{z}_1}$$

$$\frac{z-z_1}{\bar{z}-\bar{z}_1}+\frac{z_1}{\bar{z}_1}=0$$

Eqn of Circle

$x^2+y^2+2gx+2fy+c=0 \rightarrow$ Centre$(-g,-f)$

Rad $=\sqrt{g^2+f^2-c}$

$z\bar{z}+g(z+\bar{z})-if(z-\bar{z})+c=0$

Conjugate

$z\bar{z}+z(g-if)+\bar{z}(\boxed{g+if})+c=0 \rightarrow$ ये Centre के Kaam Ayega

$a=(g+if)=(g,f)$

$z\bar{z}+a\bar{z}+\bar{a}z+c=0 \rightarrow$ Eqn of Circle in Complex form

$|a|=\sqrt{g^2+f^2}$

① Centre $=-a=-$ Coff of $\bar{z}$

② Rad $=\sqrt{|a|^2-c}=\sqrt{a\bar{a}-c}$

Q $z\bar{z}+(2-i)z+(2+i)\bar{z}-4=0$ is Circle find Centre & Rad?

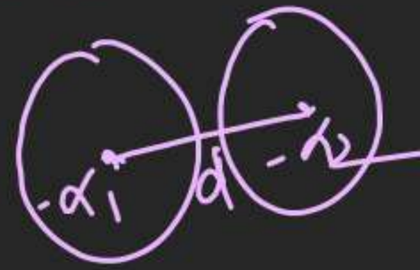
$(g,f)=(2,1) \Rightarrow \therefore$ Centre $=(-g,-f)=(-2,-1)$

Rad $=\sqrt{2^2+1^2+4}=3$

Q Angle of Intersection of 2 Circles.

$z\bar{z}+\bar{\alpha}_1 z+\boxed{\alpha_1}\bar{z}+\beta_1=0 \rightarrow$ Centre

$z\bar{z}+\bar{\alpha}_2 z+\boxed{\alpha_2}\bar{z}+\beta_2=0$



Sol.

AOI $=\cos\theta=\dfrac{r_1^2+r_2^2-d^2}{2r_1r_2}$

$r_1=\sqrt{|\alpha_1|^2-\beta_1}$

$r_2=\sqrt{|\alpha_2|^2-\beta_2}$

$$\cos\theta=\frac{(|\alpha_1|^2-\beta_1)+(|\alpha_2|^2-\beta_2)-|\alpha_1-\alpha_2|^2}{2\sqrt{|\alpha_1|^2-\beta_1}\sqrt{|\alpha_2|^2-\beta}}$$

Orthogonal Circles $\Rightarrow \cos\theta=0$

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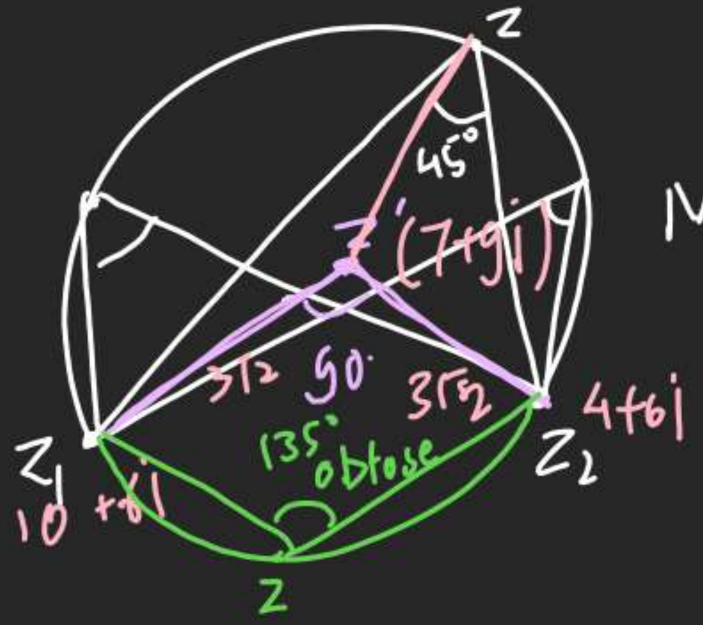
Q) $z_1 = 10+6i$, $z_2 = 4+6i$

Then P.T. all C.N. Satisfying

$\operatorname{Arg}\left(\frac{z-z_1}{z-z_2}\right) = \frac{\pi}{4}$ also Satisfying

$\boxed{|z-7-9i| = 3\sqrt{2}}$ → z dist (7,9) se dist. $= 3\sqrt{2}$ also.

→ z_1, z_2 fix pts, z = var. pt



Major Arc 45°

$$\frac{z'-z_1}{z'-z_2} = e^{i\frac{\pi}{2}}$$

$$\frac{z'-10-6i}{z'-4-6i} = i$$

$$z'-10-6i = z'i-4i+6$$

$$z'(1-i) = 16+2i$$

$$z' = \frac{16+2i}{1-i} \times \frac{1+i}{1+i}$$

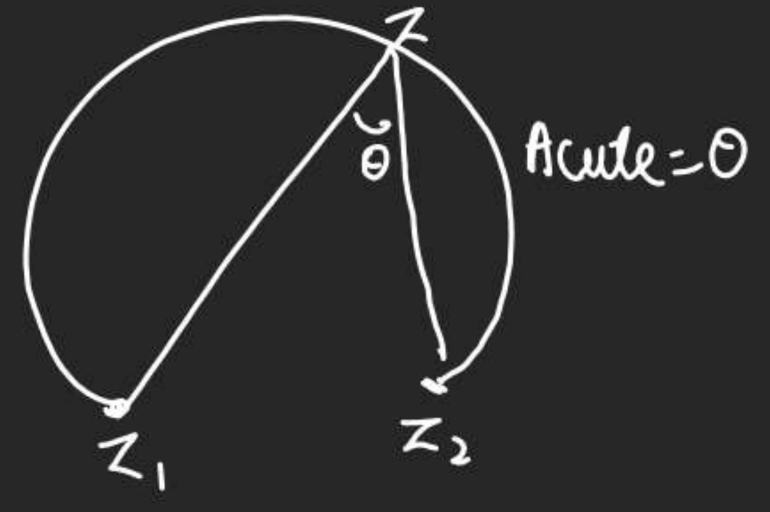
$$= \frac{16+2i+16i-2}{2}$$

$$z' = \frac{14+18i}{2} = 7+9i$$

Centre

BatseBat

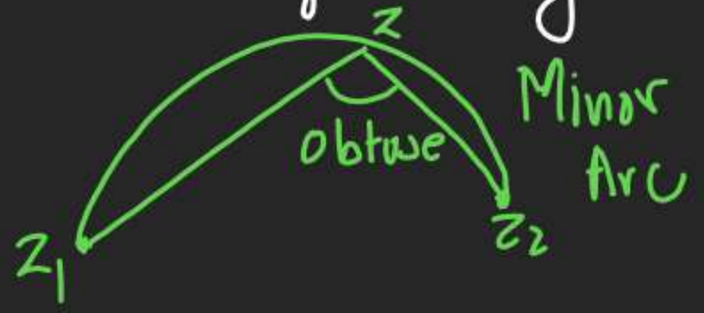
A)



$\operatorname{Arg}\left(\frac{z-z_1}{z-z_2}\right) = \theta$

Locus of z is Major Arc.

(B)



$z_1 z_2$ are making obtuse Angle with z

$\operatorname{Arg}\left(\frac{z-z_1}{z-z_2}\right)$ = obtuse → Minor Arc.

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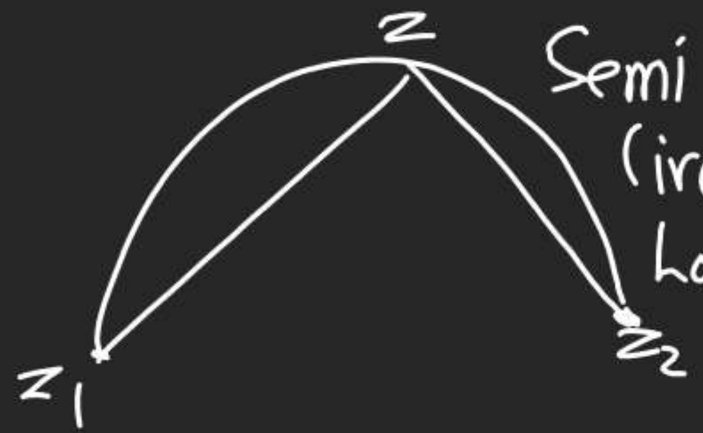
(C) When $z_1 z_2$ are making $\pm\frac{\pi}{2}$ with z

$$\operatorname{Arg}\left(\frac{z-z_1}{z-z_2}\right)=\pm\frac{\pi}{2}$$



Semi Circle Locus.

Q Locus of z if

$$\operatorname{Arg}\left\{\frac{3}{2}\left(\frac{2z^2-5z+3}{3z^2-z-2}\right)\right\}=\frac{2\pi}{3}.$$

$\rightarrow z=1$ Satisfy.

$\rightarrow$ Satisfy.

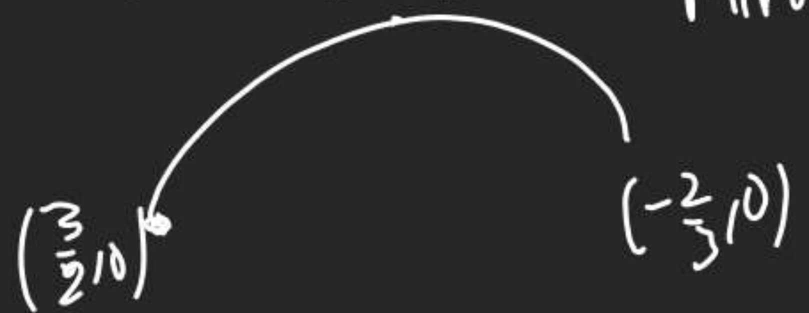
$$\operatorname{Arg}\left\{\frac{3}{2}\,\frac{(\cancel{z-1})(2z-3)}{(\cancel{z-1})(3z+2)}\right\}=\frac{2\pi}{3}.$$

$z\neq 1$

$$\operatorname{Arg}\left\{\frac{3}{2}\times\frac{2(z-3/2)}{3(z+2/3)}\right\}=\frac{2\pi}{3}.$$

$$\operatorname{Arg}\left(\frac{z-3/2}{z-(-2/3)}\right)=\frac{2\pi}{3}=\text{obtuse}$$

Minor A..



$$-\frac{(a+\bar{a})i}{(a-\bar{a})}$$

Basic Complex Eqn of St. line

$\cancel{x^2+y^2}+2gx+2fy+c=0\rightarrow$ EQ

Linear Eqn $\Rightarrow$ St. Line

$g(z+\bar{z})-if(z-\bar{z})+\beta=0$

$z(g-if)+\bar{z}(g+if)+\beta=0$

$a\bar{z}+\bar{a}z+\beta=0 \longrightarrow$ Eqn of Line

Slope of Line

$$(Sl)_{Line}=\frac{-2g}{2f}=+i\frac{(a+\bar{a})}{(a-\bar{a})}$$

$a=g+if$ $\quad a-\bar{a}=2if$

$\bar{a}=g-if$

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Q) find Slope of StL

$(1-i)\bar{z}+(1+i)z+5=0$

$g+if \qquad (g-if)$

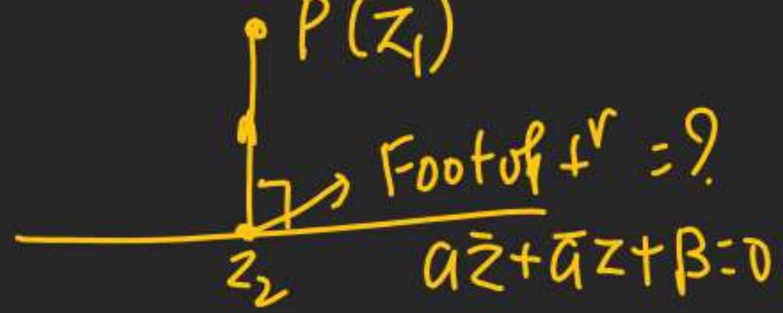
$(g,f)=(1,-1)$

$2gx+2fy+c=0$

$2x-2y+5=0$

$(Sl)=-\frac{2}{-2}=1$

(D)



$\frac{z_1-z_2}{\bar{z}_1-\bar{z}_2}+\frac{a}{\bar{a}}=0$

Q) If $\boxed{\alpha\bar{z}+\bar{\alpha}z+1}=0$ & $\beta\bar{z}+\bar{\beta}z-1=0$ → This STL is always $\perp^r$ to Coff of z

are 2 $\perp^r$ lines then.

A) $\alpha\bar{\alpha}+\beta\bar{\beta}=0$ (B) $\alpha\bar{\beta}+\beta\bar{\alpha}=0$

C) $\alpha\bar{\alpha}-\beta\bar{\beta}=0$ (D) $\alpha\bar{\beta}-\beta\bar{\alpha}=0$

$m_1=-i\left(\frac{\alpha+\bar{\alpha}}{\alpha-\bar{\alpha}}\right)$

$m_2=-i\left(\frac{\beta+\bar{\beta}}{\beta-\bar{\beta}}\right)$

$m_1\times m_2=-1$

$+1\left(\frac{\alpha+\bar{\alpha}}{\alpha-\bar{\alpha}}\right)\cdot\left(\frac{\beta+\bar{\beta}}{\beta-\bar{\beta}}\right)=-1$

$\alpha\cancel{\beta}+\alpha\bar{\beta}+\bar{\alpha}\beta+\cancel{\bar{\alpha}\bar{\beta}}=\cancel{\alpha\beta}-\alpha\bar{\beta}-\bar{\alpha}\beta+\cancel{\bar{\alpha}\bar{\beta}}$

$\alpha\bar{\beta}+\beta\bar{\alpha}=0$

(B) Parametric Eqn of STL

$\vec{AR}\parallel\vec{AB}$

$\vec{r}=\vec{a}+\lambda\vec{b}$ $\quad z-z_1\parallel z_2-z_1$

$z=z_1+\lambda(z_2-z_1)$ $\quad \frac{z-z_1}{z_2-z_1}=\lambda$ Re

(C) $L_1\perp L_2$

$Sl_{L_1}+Sl_{L_2}=0$

$L_1\parallel L_2$ $\quad Sl_{L_1}-Sl_{L_2}=0$

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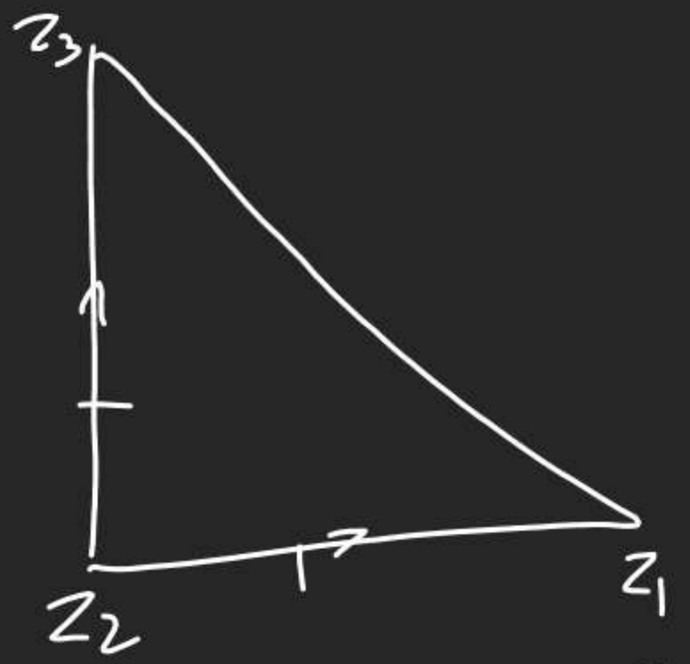
Q Z_1, Z_2, Z_3 Vertices of Rt. isosceles Δ. at z_2 then.

P.T.

$$z_1^2+2z_2^2+z_3^2=2z_2(z_1+z_3)$$



$$\frac{z_1-z_2}{\cancel{|z_1-z_2|}}\times e^{i\frac{\pi}{2}}=\frac{z_3-z_2}{\cancel{|z_3-z_2|}}$$

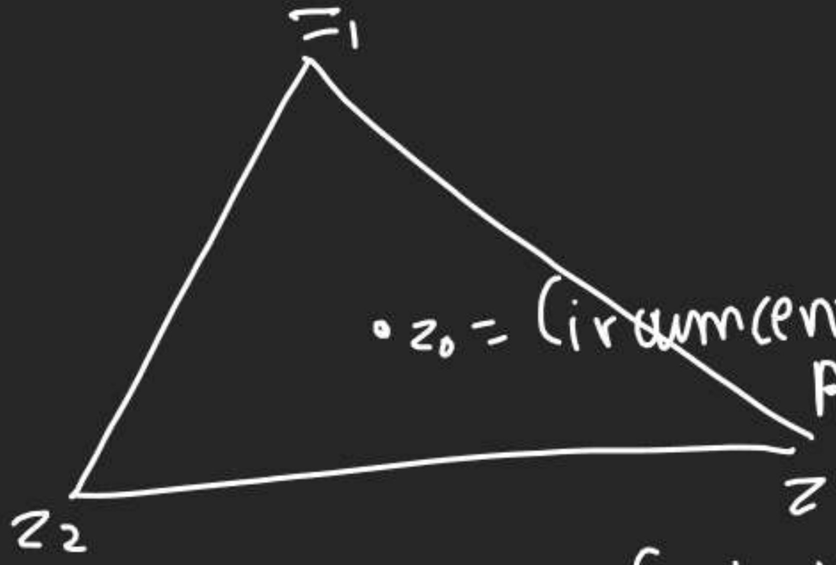
Sqr $(z_3-z_2)=(z_1-z_2)\cdot i$

Result Aayega

Q $z_1 z_2 z_3$ are vertices of eq^l Δ.

then P.T. $\sum z_1^2 = 3z_0^2$

Where z_0 = Circumcentre.



• z_0 = Circumcentre hai

But eq^l Δ

Centroid bhi hai

$$z_0=\frac{z_1+z_2+z_3}{3}$$

$$3z_0=z_1+z_2+z_3$$

Eq^l Δ

$$\sum z_1^2=\sum z_1 z_2$$

Sqr $9z_0^2=\sum z_1^2+2\sum z_1 z_2$

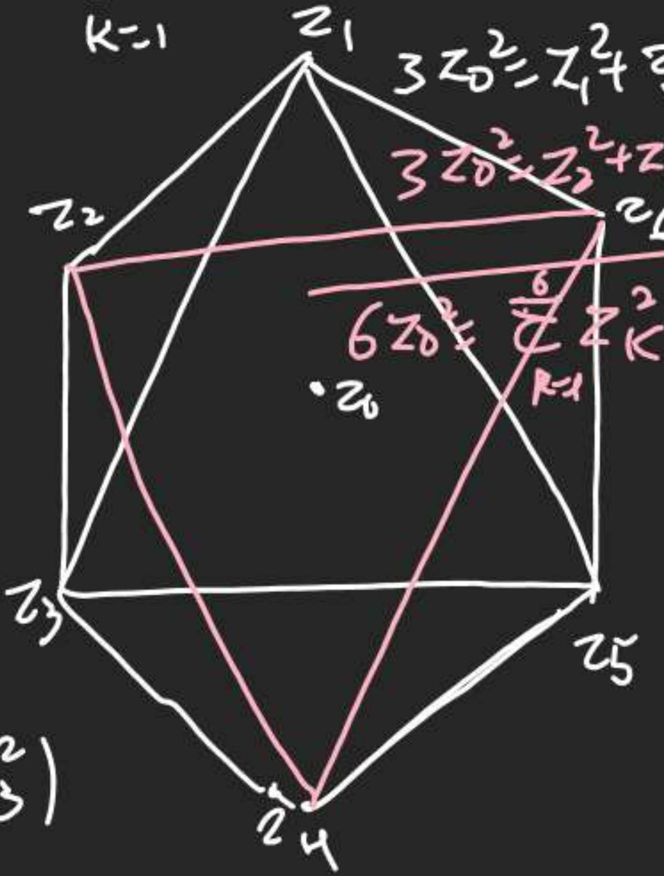
$$\cancel{9}z_0^2=\sum z_1^2+2\sum z_1^2=\cancel{3}(z_1^2+z_2^2+z_3^2)$$

$$z_1^2+z_2^2+z_3^2=3z_0^2$$

Q $z_1 z_2 z_3 \ldots z_6$ are vertices Regular hexagon with centre z_0 find λ if

$$\sum_{k=1}^{6} z_k^2=\lambda z_0^2$$

Prev qs



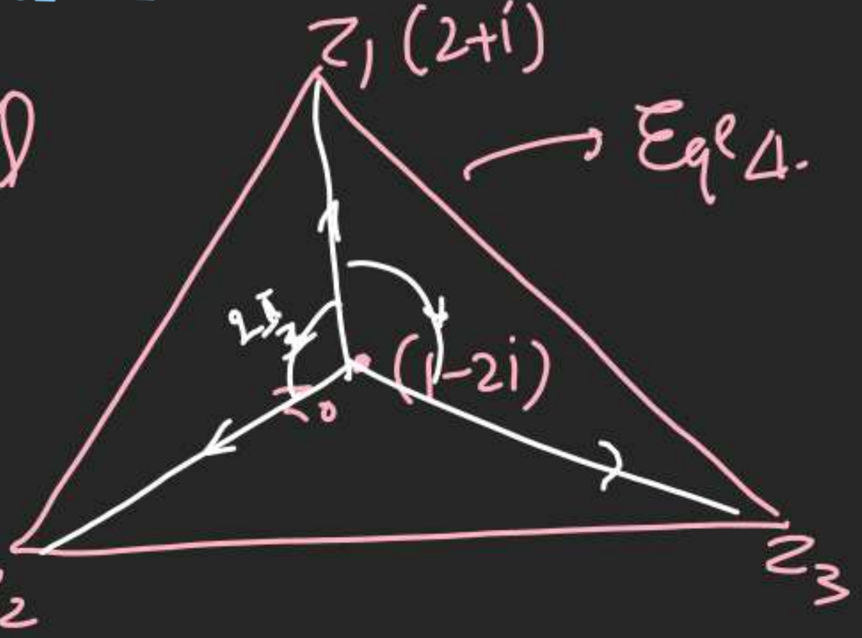
$$3z_0^2=z_1^2+z_3^2+z_5^2$$

$$3z_0^2=z_2^2+z_4^2+z_6^2$$

$$6z_0^2=\sum_{k=1}^{6} z_k^2$$

Nishant Jindal

Q

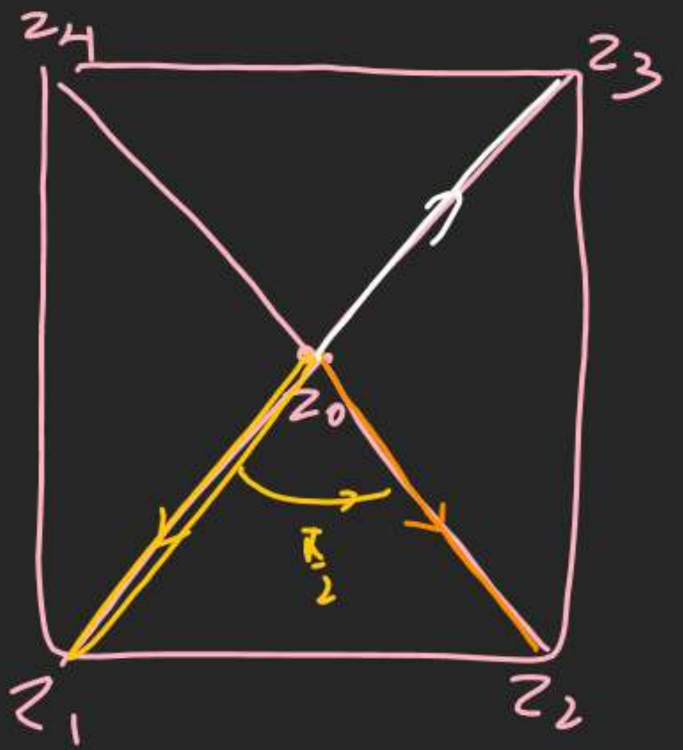


A1 $e^{i\frac{2\pi}{3}}(z_1 - z_0) = (z_2 - z_0)$

$\left(\cos\frac{2\pi}{3} + i\sin 2\frac{\pi}{3}\right)(1+3i) = z_2 - z_0$

$z_3 - z_0 = (z_1 - z_0)\cdot e^{-i\frac{2\pi}{3}}$

Q



Sq^r

Centre = z_0

Exl Dscs

find Value of

$(z_1-z_0)^2+(z_2-z_0)^2+(z_3-z_0)^2+(z_4-z_0)^2 = ?$

$z_2 - z_0 = (z_1 - z_0)\cdot i$

$z_3 - z_0 = (z_1 - z_0)\, i^2$

$z_4 - z_0 = (z_1 - z_0)\, i^3$

$|| \quad (z_1-z_0)^2 + (z_1-z_0)^2 i^2 + (z_1-z_0)^2 i^4 + (z_1-z_0)^2 i^6$

$(z_1-z_0)^2\{1+i^2+i^4+i^6\}$

$\{1-1+1-1\}$

$= 0$