

$$\frac{\cos \kappa - \sin \kappa}{\cos \kappa (1 - \sqrt{2} \sin \kappa)} = \frac{(\cos^2 \kappa - \sin^2 \kappa) (1 + \sqrt{2} \sin \kappa)}{\cos \kappa (\cos \kappa + \sin \kappa) (1 - 2 \sin^2 \kappa)}$$

2

$$\frac{\sqrt{1 - \sin 2x}}{\sqrt{1 + \sin 2x}} = \frac{\sqrt{2} \sin\left(x + \frac{\pi}{4}\right)}{\sqrt{1 + \sin 2x} \left(\frac{\pi}{4} - x\right)}$$

$$\lim_{n \rightarrow 0} \left( e^{\frac{\ln(\frac{\pi}{4} - x)}{n^2}} \right) = e^{-\frac{1}{2}}$$

$$\sqrt{x^2} = |x|$$

$$\lim_{x \rightarrow 0} \frac{2 \left[ \left( x + \frac{x^3}{3} + \frac{2}{15} x^5 + \dots \right) - \left( x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots \right) \right] - x^5}{x^5}$$

$$2 \left( \frac{2}{15} + \frac{1}{120} \right)$$

$$\frac{2 \sin x (1 - \cos x) - x^3 \cos x}{x^5 \cos x}$$

$$= \frac{8 \sin^3 \frac{x}{2} (\cos \frac{x}{2}) - x^3 (1 - 2 \sin^2 \frac{x}{2})}{x^5 \cos x}$$

$$= \frac{8 \left( \sin^3 \frac{x}{2} - \left( \frac{x}{2} \right)^3 \right)}{x^5 \cos x} + \frac{2 \sin^2 \frac{x}{2}}{x^2 \cos x} - \frac{16 \sin^3 \frac{x}{2}}{x^5 \cos x}$$

$$= \boxed{\ln a} \cdot \frac{(1 + x)(1 + x^2)(1 + x^4) \cdots (1 + x^{2^{n-1}})}{(1 - x)}$$

A hand-drawn diagram illustrating the proof of the inequality  $1 - x < \frac{1}{1-x}$ . The diagram shows two overlapping circles. The left circle has radius  $1-x$  and center at  $(1-x, 0)$ . The right circle has radius  $x/(1-x)$  and center at  $(x/(1-x), 0)$ . A horizontal line segment connects the centers. A dashed line segment connects the point  $(1-x, 0)$  on the left circle to the point  $(x/(1-x), 0)$  on the right circle. The intersection of the two circles is shaded gray. The region between the two circles is shaded white.

15:

$$\frac{2(\cos ax - (\cos(b+c)x + \cos(b-c)x))}{bc \sin^2 \frac{bx}{2} \sin \frac{cx}{2}}$$
$$\frac{2\left(-\frac{a^2}{2}\right) + \left(\frac{(b+c)^2}{2} + \frac{(b-c)^2}{2}\right)}{bc}$$

$$\lim_{x \rightarrow 0} \left\{ \frac{x^2}{\sin x \tan x} \right\} = 0.$$

$\checkmark$

$x^2$

$\sin x \tan x$

$x^2 (1 - \tan^2 \frac{x}{2})$

$\tan^2 \frac{x}{2}$

$\cos x \sec x$

$$\text{L.H.S. (ii)} \lim_{n \rightarrow \infty} \left( \sqrt{n^2 - n + 1} - a_n - b \right) = 0$$

$a \geq 0$

$$\frac{n^2 - n + 1 - (an + b)^2}{n^2(1-a^2) - (2ab+1)n + 1 - b^2}$$

$$\lim_{n \rightarrow \infty} \frac{\sqrt{n^2 - n + 1} + an + b}{\sqrt{n^2 - n + 1} + an + b}$$

$$1 - a^2 = 0$$

$$\frac{-1 - 2b}{2} =$$

~~$$\lim_{n \rightarrow \infty} \frac{n \left( - (2ab+1) + \frac{1-b^2}{n} \right)}{n \left( 1 - \frac{1}{n} + \frac{1}{n^2} + a + \frac{b}{n} \right)}$$~~

$$a = 1$$

Q-10 ( $\Sigma x - II$ )

Given  $\lim_{n \rightarrow 0} \left( \frac{f(n)}{n^2} \right) = -2$ , find

$$\textcircled{1} \quad \lim_{n \rightarrow 0} \frac{f(n)}{n^2} n^2 = 0$$

$$\textcircled{2} \quad \lim_{n \rightarrow 0} [f(n)] = -1$$

$$\textcircled{3} \quad \lim_{n \rightarrow 0} \{f(n)\} = 1$$

$$\textcircled{4} \quad \lim_{n \rightarrow 0} \left( \frac{f(n)}{n} \right) = 0$$

$$\textcircled{5} \quad \lim_{n \rightarrow 0} \left[ \frac{f(n)}{n} \right] \text{ not exist}$$

$$\textcircled{6} \quad \lim_{n \rightarrow 0} \left\{ \frac{f(n)}{n} \right\}$$

$$\frac{f(n)}{n} = \frac{f(n)}{n^2} n \cdot \frac{n}{0^-} \quad \begin{aligned} LHL &= 0 \\ RHL &= -1 \end{aligned}$$

$$f(n) = f(n) \left( \frac{n}{0^+} \right) \rightarrow 0^-$$

$$f(n) - \underbrace{[f(n)]}_{\text{not exist}} = 0 - (-1)$$

$$\begin{aligned} LM &= 0 \\ RML &= 1 \end{aligned}$$