Nishant Jindal

$$\boxed{\begin{array}{l} \text{EOT.} \\ \text{Slope form} \rightarrow y = mx \pm \sqrt{a^2m^2+b^2} \\ \text{Cart form} \xrightarrow{T=0} \dfrac{xx_1}{a^2} + \dfrac{yy_1}{b^2} = 1 \\ \text{Par. form.} \Rightarrow \dfrac{x\cos\theta}{a} + \dfrac{y\sin\theta}{b} = 1 \end{array}}$$

Pt. of tangency : $\left(\dfrac{\pm a^2 m}{\sqrt{a^2m^2+b^2}}, \dfrac{\pm b^2}{\sqrt{a^2m^2+b^2}} \right)$

$$\boxed{\begin{array}{l} m_1+m_2 = \dfrac{2kh}{h^2-a^2} \\ m_1m_2 = \dfrac{k^2-b^2}{h^2-a^2} \end{array}}$$

Q Find EOT. to an Ellipse

$9x^2+16y^2=144$ from $(2,3)$

$\dfrac{x^2}{16} + \dfrac{y^2}{9} = 1$

$(2,3)$'s Position.

$\left.\dfrac{4}{16} + \dfrac{9}{9} - 1 > 0\right\}$ outside.

$y = mx \pm \sqrt{a^2m^2+b^2}$

$3 = 2m \pm \sqrt{16m^2+9}$

$(3-2m)^2 = 16m^2+9$

$4m^2+9-12m = 16m^2+9$

$12m^2+12m$

$m = 0, -1$

$y-3 = 0(x-2)$

$y-3 = -1(x-2)$

Q Tangent to Ellipse make angle θ_1 & θ_2 with major Axes find Locus of their Pt. of Intersection if $\cot\theta_1 + \cot\theta_2 = \lambda^2$:

2 tangents $\rightarrow \theta_1, \theta_2 \Rightarrow m_1, m_2$

$m_1+m_2 = \dfrac{2kh}{h^2-a^2}$ | $m_1m_2 = \dfrac{k^2-b^2}{h^2-a^2}$

$\tan\theta_1 + \tan\theta_2 = \dfrac{2kh}{h^2-a^2}$ | $\tan\theta_1 \cdot \tan\theta_2 = \dfrac{k^2-b^2}{h^2-a^2}$

given $\dfrac{\tan\theta_1+\tan\theta_2}{\tan\theta_1 \cdot \tan\theta_2} = \lambda^2 \Rightarrow \dfrac{\frac{2kh}{\cancel{h^2-a^2}}}{\frac{k^2-b^2}{\cancel{h^2-a^2}}} = \lambda^2$

$y^2-b^2 = 2xy\lambda^2$

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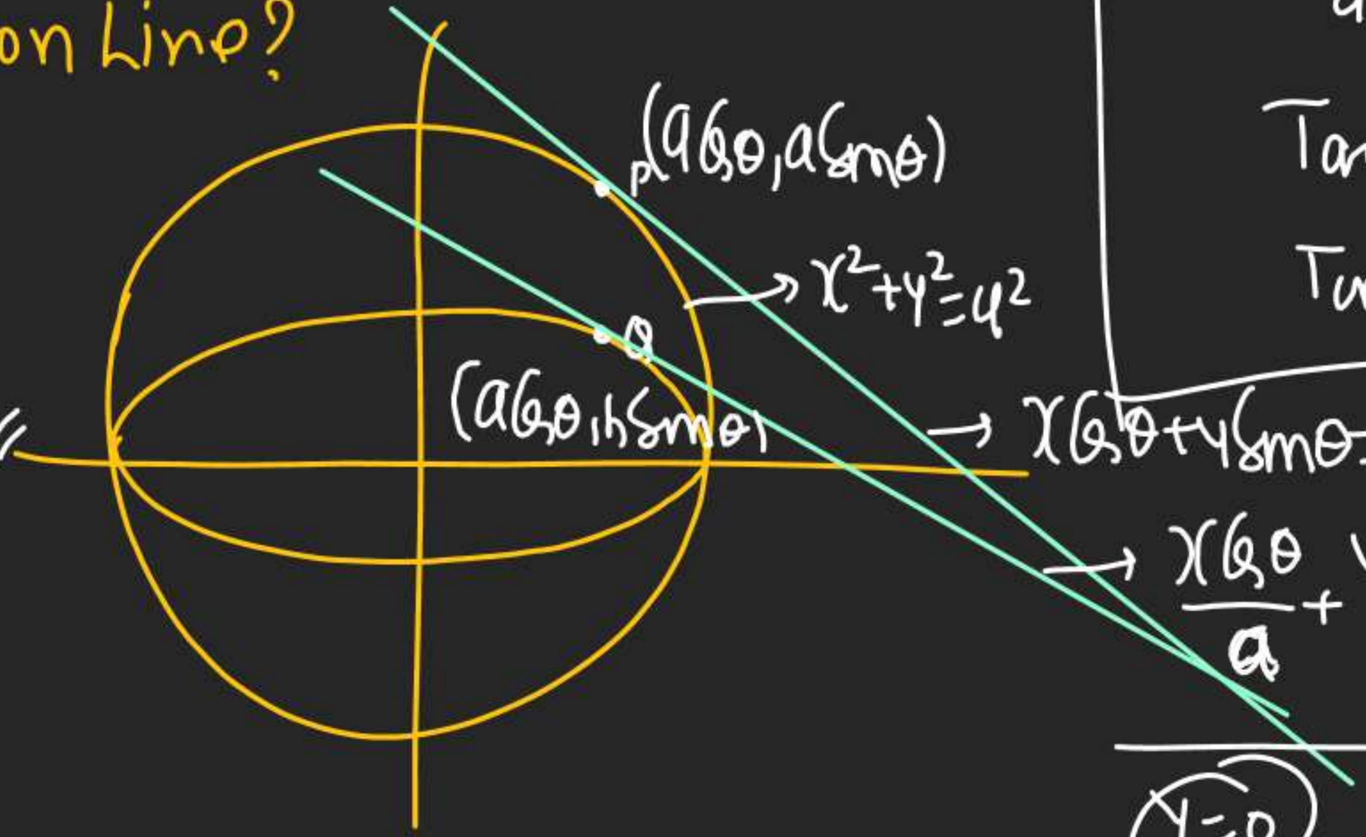
Q Pt. of Intersection of tangents at Pt. P on Ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ & its Corr. pt. Q on aux. Circle meet on Line?

$x=\frac{a}{e}$

$x=0$

$y=0$ //

None



$\rightarrow x\cos\theta+y\sin\theta=a \rightarrow ①$

$\rightarrow \frac{x\cos\theta}{a}+\frac{y\sin\theta}{b}=1 \rightarrow ②$

$y=0$

Q Find Com. tangent to Elli:

$$\frac{x^2}{a^2+b^2}+\frac{y^2}{b^2}=1 \ \&$$

$$\frac{x^2}{a^2}+\frac{y^2}{a^2+b^2}=1 \text{ is}$$

Tangent 1: $y=mx\pm\sqrt{(a^2+b^2)m^2+b^2}$

Tangent 2: $y=mx\pm\sqrt{a^2m^2+(a^2+b^2)}$

tangent 1 & tangent 2 are Same

$$(\cancel{a^2}+b^2)m^2+\cancel{b^2}=\cancel{a^2m^2}+(a^2+\cancel{b^2})$$

$$m^2=\frac{a^2}{b^2} \Rightarrow m=\pm\frac{a}{b}$$

$$\therefore \text{EOT } by=\pm ax\pm b\sqrt{(a^2+b^2)\times\frac{a^2}{b^2}+b^2}$$

$$\pm\sqrt{a^4+b^4+a^2b^2}$$

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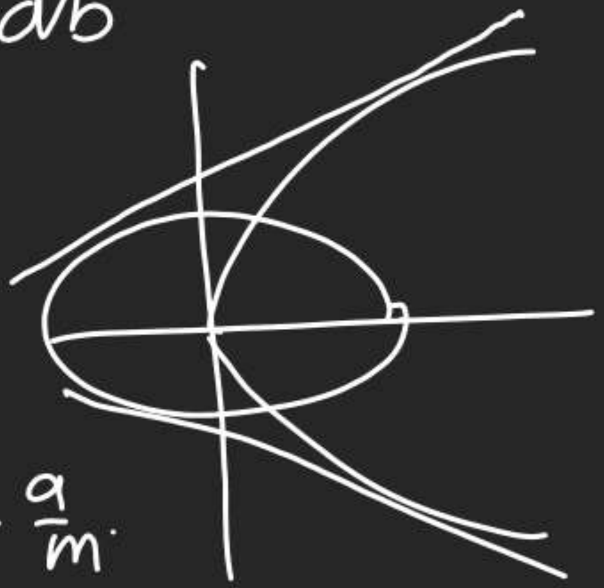
Q $x-2y+4=0$ is Com. tangent to $y^2=4x$ & $\frac{x^2}{4}+\frac{y^2}{b^2}=1$ find b & other Com. tangent?



$x-2y+4=0 \to m=\frac{1}{2}$, $c=2=\frac{a}{m}$

$c=\pm\sqrt{4\times\frac{1}{4}+b^2}$

$\sqrt{b^2+1}=\pm 2$

$b^2+1=4$

$b^2=3$

$b=\sqrt{3} \,/\, -\sqrt{3}$

$y^2=4x$

$\frac{x^2}{4}+\frac{y^2}{3}=1 \to y=mx\pm\sqrt{a^2m^2+b^2}$

$y=\frac{x}{2}\pm\sqrt{4\times\frac{1}{4}+3}$

$y=\frac{x}{2}+2 \Rightarrow x-2y+4=0$

$y=\frac{x}{2}-2 \Rightarrow x-2y-4=0$

$ax\sec\theta-by\csc\theta=a^2-b^2$

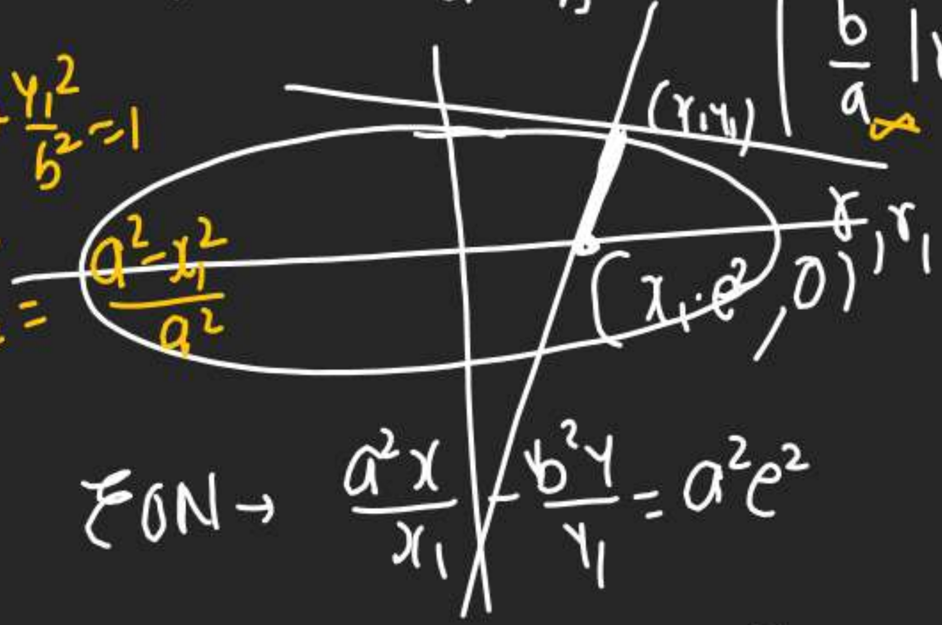
Q Length of Normal at (Terminated by Major Axis) Pt. of ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is

- $\frac{b}{a}(r+r_1)$
- $\frac{b}{a}|r-r_1|$
- $\frac{b}{a}\sqrt{r\cdot r_1}$ ✓
- Ind. of r, r_1

r, r_1 are Focal dist. of Pt.

$\frac{x_1^2}{a^2}+\frac{y_1^2}{b^2}=1$

$\frac{y_1^2}{b^2}=\frac{a^2-x_1^2}{a^2}$



EON $\to \frac{a^2x}{x_1}-\frac{b^2y}{y_1}=a^2e^2$

$L_N=\sqrt{(x_1e^2-x_1)^2+(0-y_1)^2}=\sqrt{x_1^2(e^2-1)^2+y_1^2}$

$=\sqrt{x_1^2(1-e^2)^2+\frac{b^2}{a^2}(a^2-x_1^2)}$

$=\sqrt{x_1^2(1-e^2)^2+(1-e^2)(a^2-x_1^2)}$

$=\sqrt{1-e^2}\sqrt{\cancel{x_1^2}-e^2x_1^2+a^2-\cancel{x_1^2}}$

$=\sqrt{1-e^2}\sqrt{(a-ex_1)(a+ex_1)}$

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Q Ecc. Angle of Pt. Where Line $5x-3y=8\sqrt{2}$ is a Normal to E: $\frac{x^2}{25}+\frac{y^2}{9}=1$

$\frac{\pi}{4}, \frac{3\pi}{4}, \tan^{-1}9, \frac{\pi}{6}$.

E O N

$5x-3y=8\sqrt{2}$ given (Hmari)

$5x\sec\theta-3y\csc\theta=16$ (Sarkari)

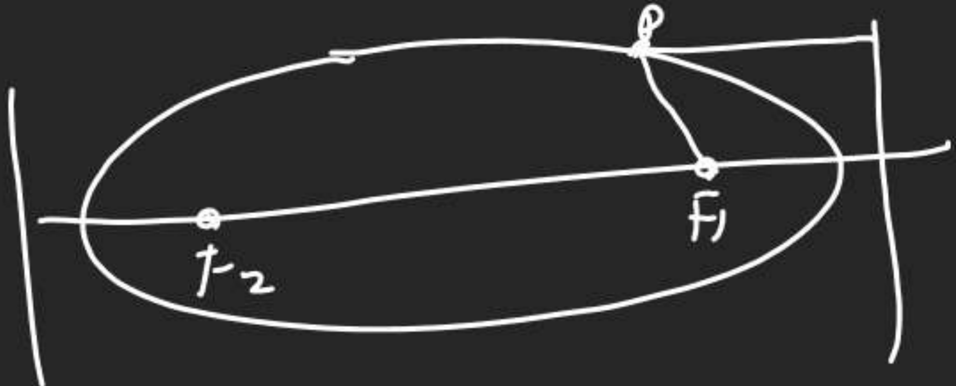
$\frac{\cancel{5}}{\cancel{5}\sec\theta}=\frac{\cancel{-3}}{\cancel{-3}\csc\theta}=\frac{8\sqrt{2}}{16}$

$\cos\theta=\sin\theta=\frac{1}{\sqrt{2}}$

$\theta=\frac{\pi}{4}$

Q Let d be ⊥r dist from Centre of $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ to tangent drawn at Pt. P on Ellipse if F_1, F_2 are 2 focii of ellipse then S.T.

$(PF_2-PF_1)^2=4a^2\left(1-\frac{b^2}{d^2}\right)$



$PF_1=a-ex_1$

$PF_2=a+ex_1$

$PF_1-PF_2=-2ex_1$

$(PF_1-PF_2)^2=4e^2x_1^2$

$=4x_1^2\left(1-\frac{b^2}{a^2}\right)$

$=4a^2\left(1-\frac{b^2}{d^2}\right)$

let $P\equiv(x_1,y_1)$

EOT: $\frac{xx_1}{a^2}+\frac{yy_1}{b^2}=1$

$\frac{x_1^2}{a^2}+\frac{y_1^2}{b^2}=1$

d = (0,0) से EOT की dist

$d=\frac{|-1|}{\sqrt{\frac{x_1^2}{a^4}+\frac{y_1^2}{b^4}}}\Rightarrow\frac{x_1^2}{a^4}+\frac{y_1^2}{b^4}=\frac{1}{d^2}$

$\Rightarrow\frac{b^2x_1^2}{a^4}+\frac{y_1^2}{b^2}=\frac{b^2}{d^2}$

$\frac{b^2x_1^2}{a^4}+1-\frac{x_1^2}{a^2}=\frac{b^2}{d^2}$

$a^2\left(1-\frac{b^2}{d^2}\right)=x_1^2\left(1-\frac{b^2}{a^2}\right)$