Nishant Jindal

KINEMATICS

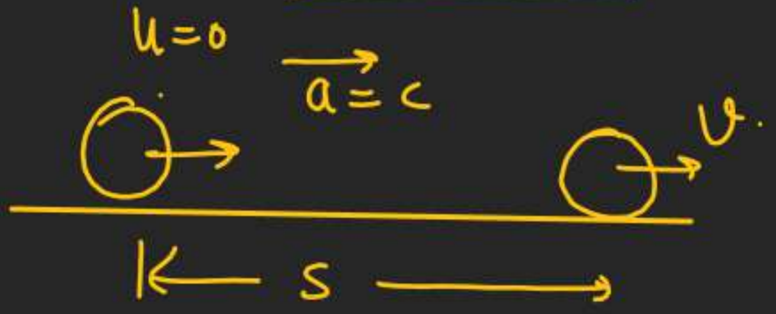
IRODOV

$u = 0$

Q. A point moves from rest with a uniform acceleration. Show that the space average of the velocity is $4/3$ times of the time average in any given interval.

Sol^n: → Space avg → (w.r.t displacement)

$$\boxed{v \rightarrow f(s)}$$



$$v^2 = 2as$$

$$v = \sqrt{2as}$$

$$(V_{avg}) = \frac{\int_0^s v \cdot ds}{\int_0^s ds} = \frac{\sqrt{2a}\int_0^s \sqrt{s} \cdot ds}{\int_0^s ds}$$

$$(V_{avg}) = \frac{\sqrt{2a}\,\left[s^{3/2}\right]_0^s}{\frac{3}{2} \times s}$$

$$= \left(\frac{2\sqrt{2as}}{3}\right)$$

(Space avg)

$$y = f(x).$$

$$y_{avg} = \frac{\int_{x_i}^{x_f} y \cdot dx}{\int_{x_i}^{x_f} dx}$$

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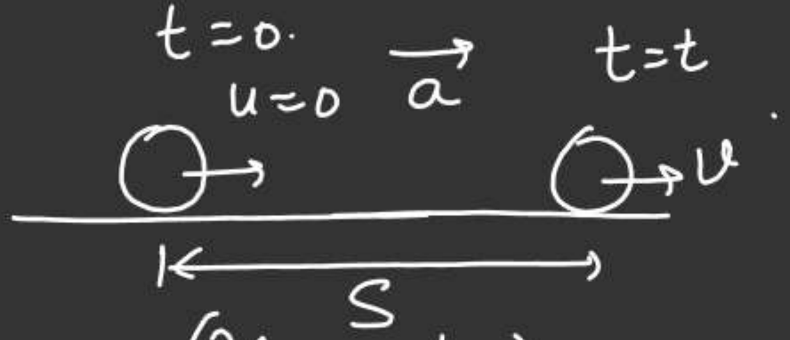
Time avg



$(v = at)$

$$v_{avg} = \frac{\int_0^t v\,dt}{\int_0^t dt}$$

$$\left[\; s = \frac{1}{2}at^2 \qquad t = \sqrt{\frac{2s}{a}} \;\right]$$

$$V_{avg} = \frac{a\int_0^t t\,dt}{\int_0^t dt} = \frac{at^2}{2t} = \left(\frac{a}{2}t\right)$$

$$V_{avg} = \frac{a}{2}\sqrt{\frac{2s}{a}} = \sqrt{\frac{as}{2}}$$

$\Downarrow$

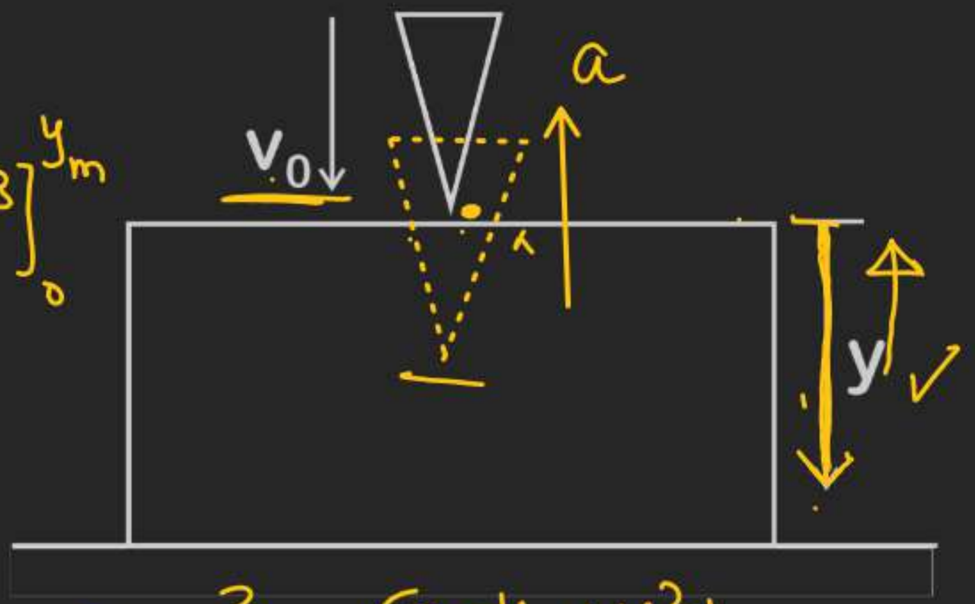
Time avg

$$\frac{\frac{2}{3}\sqrt{2as}}{\sqrt{\frac{as}{2}}} = \frac{v_{space\ avg}}{v_{time\ avg}}$$

$$\boxed{(v_{space\ avg}) = \frac{4}{3}(v_{time\ avg})}$$ ✓

KINEMATICS

Q. The cone falling with a speed v_0 strikes and penetrates the block of packing material (figure). The acceleration of the cone after impact is $\boxed{a = g - cy^2}$ where ' c ' is a positive constant and ' y ' is the penetration distance. If the maximum penetration depth is observed to be y_m, determine the constant ' c '. = ??



$a \rightarrow f(y) \longrightarrow \quad v \rightarrow f(y)$

(With sign) $\Leftarrow a = g - cy^2$

$$v\frac{dv}{dy} = (g - cy^2)$$

$$\int_{v_0}^{0} v\,dv = \int_0^{y_m} (g - cy^2)\,dy$$

$$-\frac{v_0^2}{2} = g[y]_0^{y_m} - \frac{c}{3}[y^3]_0^{y_m}$$

$$-\frac{v_0^2}{2} = gy_m - \frac{cy_m^3}{3}$$

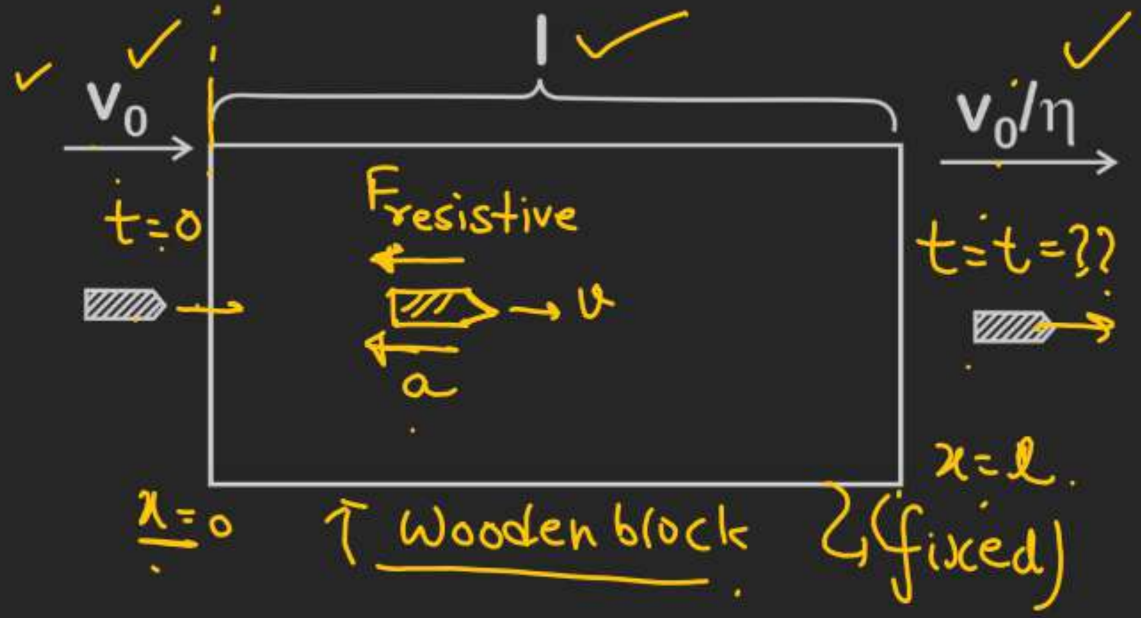
$$\frac{cy_m^3}{3} = \frac{2gy_m + v_0^2}{2} \Rightarrow c = \frac{3}{2y_m^3}(2gy_m + v_0^2)$$

$$c = \frac{3}{2}\left(\frac{2g}{y_m^2} + \frac{v_0^2}{y_m^3}\right)$$

$\int \frac{dx}{x} = \ln(x)$

Q. A bullet is fired horizontally on a fixed wooden block of length ' l ' as shown in the figure. It penetrates the block and emerges from its back face with velocity $[(v_0/\eta)(\eta > 1)]$. Resistance offered by the block against penetration is proprtional to the square of instantaneous velocity of the bullet. Find the time of penetration.

$(\eta > 1)$



$a \propto v^2$ (K → proportionality constant)

$\boxed{a = -Kv^2}$

(−) → (Retardation)

$\cancel{v}\frac{dv}{ds} = -Kv^{\cancel{2}}$

$\int_{v_0}^{v_0/\eta} \frac{dv}{v} = -K\int_0^l ds$

$\ln[v]_{v_0}^{v_0/\eta} = -K[s]_0^l$

$\ln\left(\frac{v_0}{\eta}\right) - \ln(v_0) = -Kl$

$\ln\left(\frac{v_0/\eta}{v_0}\right) = (-)Kl \Rightarrow$

$K = \frac{1}{l}\ln\left(\frac{v_0}{v_0/\eta}\right) = \frac{1}{l}\ln(\eta)$

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$$a = -Kv^2$$

$$\Downarrow$$

$$\frac{dv}{dt} = -Kv^2$$

$$\int_{v_0}^{\frac{v_0}{\eta}} \frac{dv}{v^2} = -K\int_0^t dt$$

$$\left[-\frac{1}{v}\right]_{v_0}^{\frac{v_0}{\eta}} = -Kt$$

$$\left(-\frac{\eta}{v_0} + \frac{1}{v_0}\right) = -Kt$$

$$\int_{v_0}^{\frac{v_0}{\eta}} v^{-2}\,dv = \left[\frac{v^{-1}}{-1}\right]_{v_0}^{\frac{v_0}{\eta}} = \left[-\frac{1}{v_0}\right]_{v_0}^{\frac{v_0}{\eta}}$$

$$t = -\frac{1}{K}\left(-\frac{\eta}{v_0} + \frac{1}{v_0}\right)$$

$$t = \frac{(\eta-1)}{Kv_0}$$

$$t = \frac{(\eta-1)}{v_0 \times \frac{1}{l}\ln\eta}$$

$$\boxed{t = \frac{l}{v_0}\frac{(\eta-1)}{\ln\eta}}$$ Ans

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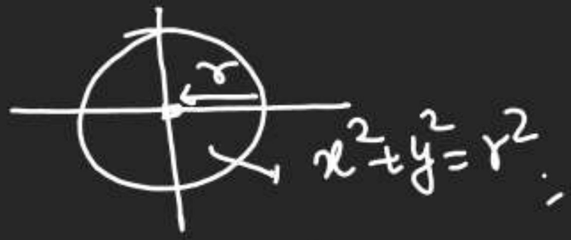
KINEMATICS



Q. A point moves in the $(x-y)$ plane according to the law $x = a\sin\omega t, y = a(1-\cos\omega t)$, where ' a ' and ω are positive constant. Find :

(a) the distance' s ' traversed by the point during the time ' t '.

(b) the angle between the point's velocity and acceleration vectors.

At $t=0$

$x=0,\ y=0$

$\vec{A}\cdot\vec{B} = |A||B|\cos\theta$

$\cos\theta = \dfrac{\vec{A}\cdot\vec{B}}{|A||B|}$

Locus: $\to$ $\boxed{y \to f(x)}$ $\leftarrow$

$x = a\sin\omega t, \quad y = a(1-\cos\omega t)$

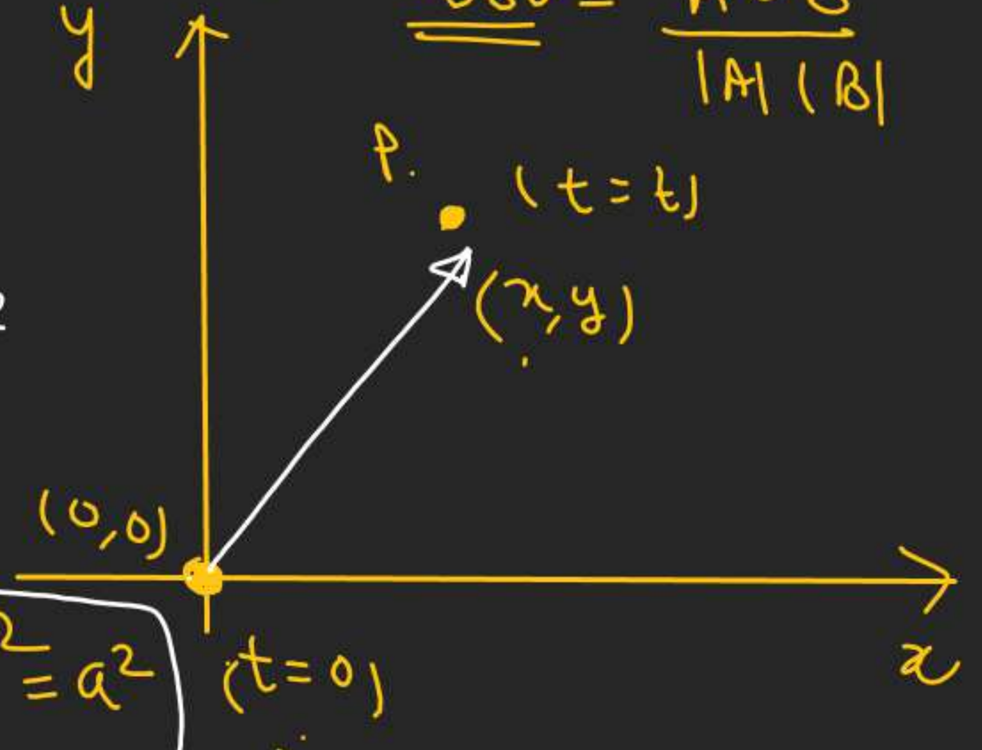
$\sin\omega t = \dfrac{x}{a}, \quad 1-\cos\omega t = \dfrac{y}{a}$

$\cos\omega t = (1 - y/a)$

$\sin^2\omega t + \cos^2\omega t = 1$

$\dfrac{x^2}{a^2} + (1 - y/a)^2 = 1 \Rightarrow \dfrac{x^2}{a^2} + \dfrac{(a-y)^2}{a^2} = 1 \Rightarrow \boxed{x^2 + (y-a)^2 = a^2}$

Equation of Circle



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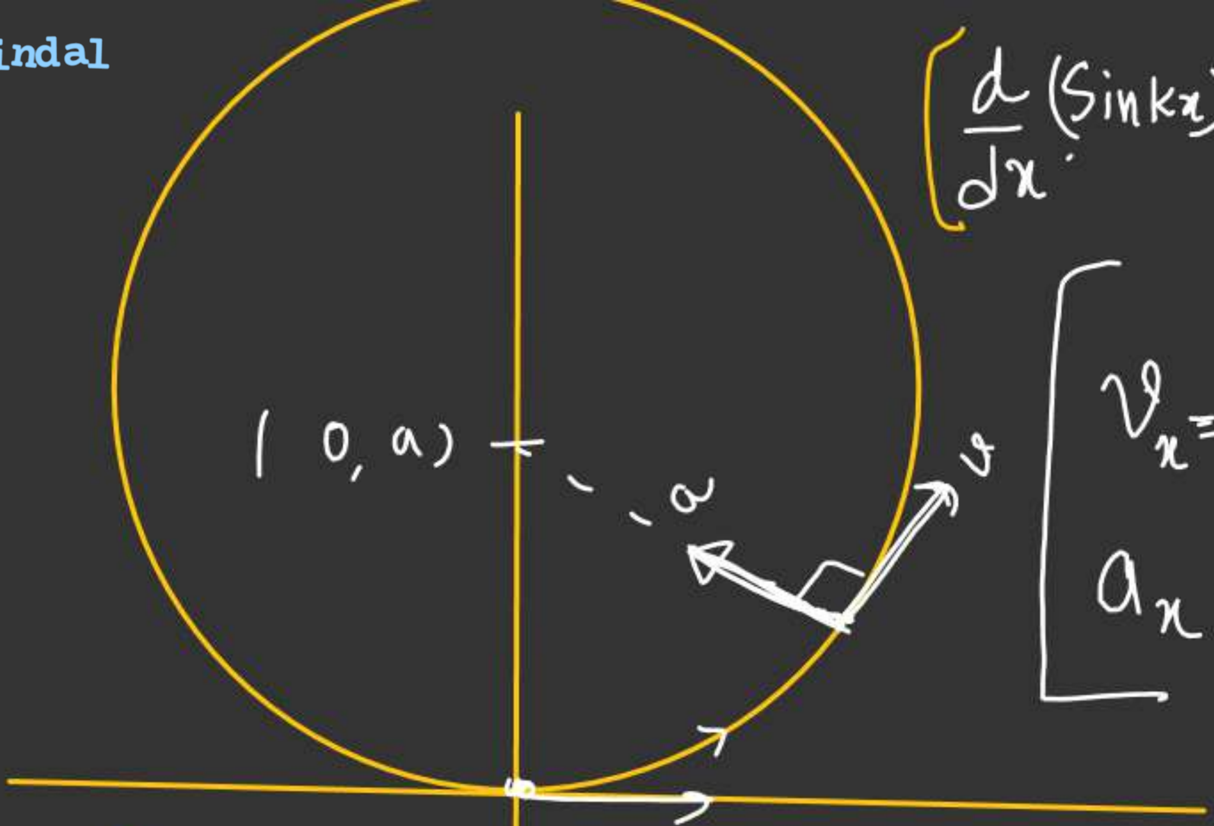
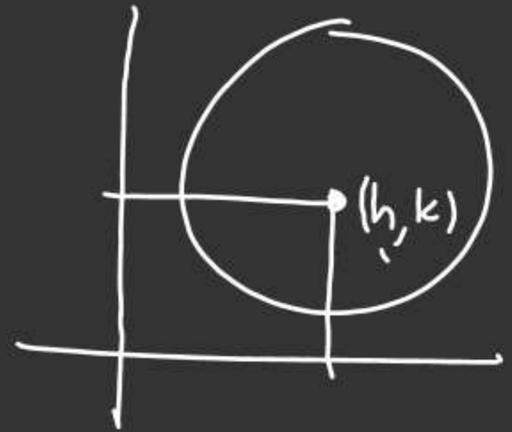


$$\boxed{\frac{d}{dx}(\sin kx) = k\cos kx}$$

$$x^2 + (y-a)^2 = a^2$$

Eq^n of Circle.

$$\boxed{(x-h)^2 + (y-k)^2 = r^2}$$



$$x = a\sin\omega t$$

$$v_x = \frac{dx}{dt} = a\omega\cos\omega t$$

$$a_x = \frac{dv_x}{dt} = -a\omega^2\sin\omega t$$

$$y = a(1-\cos\omega t)$$

$$v_y = \frac{dy}{dt} = +a\omega\sin\omega t$$

$$a_y = \frac{dv_y}{dt} = a\omega^2\cos\omega t$$

$$\vec{V} = V_x\hat{i} + V_y\hat{j}$$

$$\boxed{\vec{V} = a\omega\cos\omega t\,\hat{i} + a\omega\sin\omega t\,\hat{j}}$$

$$\vec{v} = a\omega(\cos\omega t\,\hat{i} + \sin\omega t\,\hat{j})$$

$$\vec{a} = a\omega^2(-\sin\omega t\,\hat{i} + \cos\omega t\,\hat{j})$$

$$\frac{ds}{dt} = |v|$$

$$\frac{ds}{dt} = \sqrt{a^2\omega^2(\sin^2\omega t + \cos^2\omega t)}$$

$$\vec{v}\cdot\vec{a} = 0 \qquad [\vec{v}\perp\vec{a}] \qquad \theta = 90$$

$$\frac{ds}{dt} = a\omega$$

$$\int_0^s ds = a\omega\int_0^t dt \Rightarrow \boxed{S = a\omega t}$$

KINEMATICS

Q.3 At the moment $t = 0$ a particle leaves the origin and moves in the positive direction of the x axis. Its velocity varies with time as $\boxed{v = v_0(1 - t/\tau)}$, where v_0 is the initial velocity vector whose modulus equals $v_0 = 10.0\ \text{cm/s}$; $\tau = 5.0\ \text{s}$. Find:

[Irodov]

(a) the x coordinate of the particle at the moments of time 6.0 , 10 , and $20\ \text{s}$;

(b) the moments of time when the particle is at the distance $10.0\ \text{cm}$ from the origin;

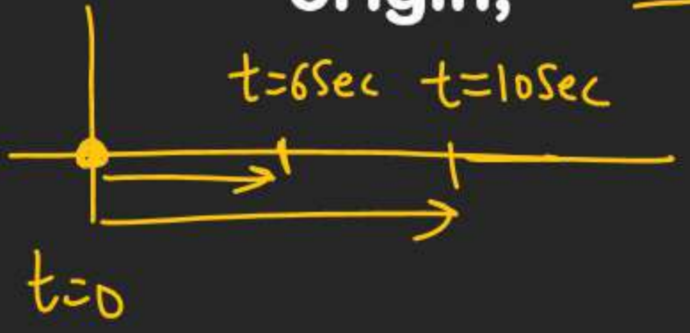
Sol^n

$$v = v_0 - \frac{v_0}{\tau}\cdot t = 10 - \frac{10}{5}t = (10-2t)$$

$$\boxed{v = (10-2t)}$$

$$\frac{dx}{dt} = (10-2t) \rightarrow \int_0^x dx = \int_0^t (10-2t)\,dt \Rightarrow x = 10\int_0^t dt - 2\int_0^t t\,dt$$

$$\Rightarrow \boxed{x = (10t - t^2)}$$



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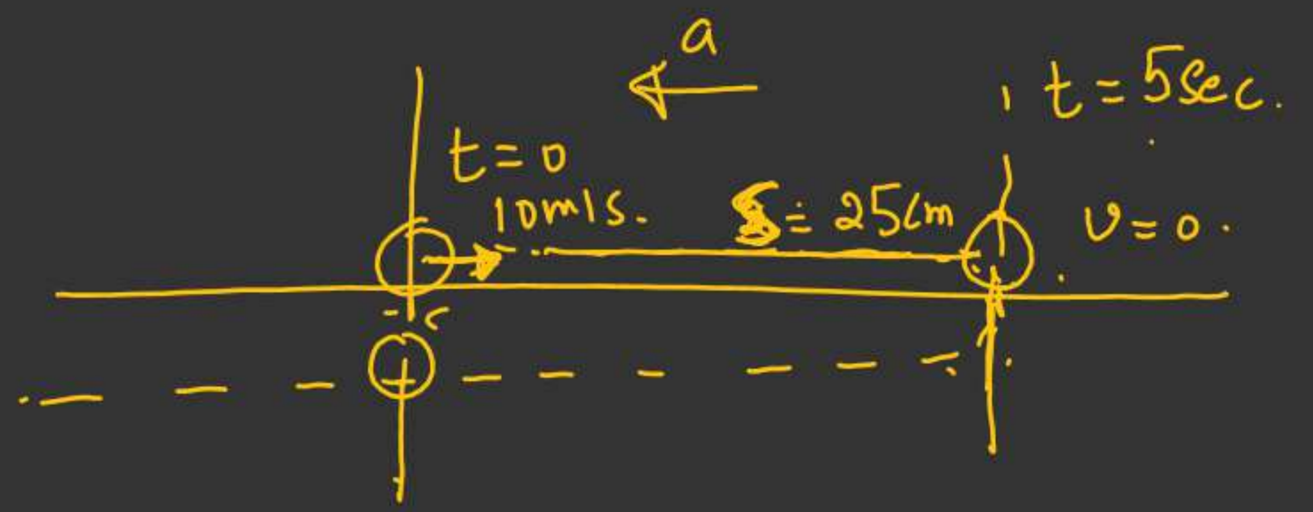
$a = \frac{dv}{dt} = (-2t) \leftarrow \quad v = (10 - 2t)$

$v = 0, \quad 10 - 2t = 0$

$\boxed{t = 5\,\text{sec}}$

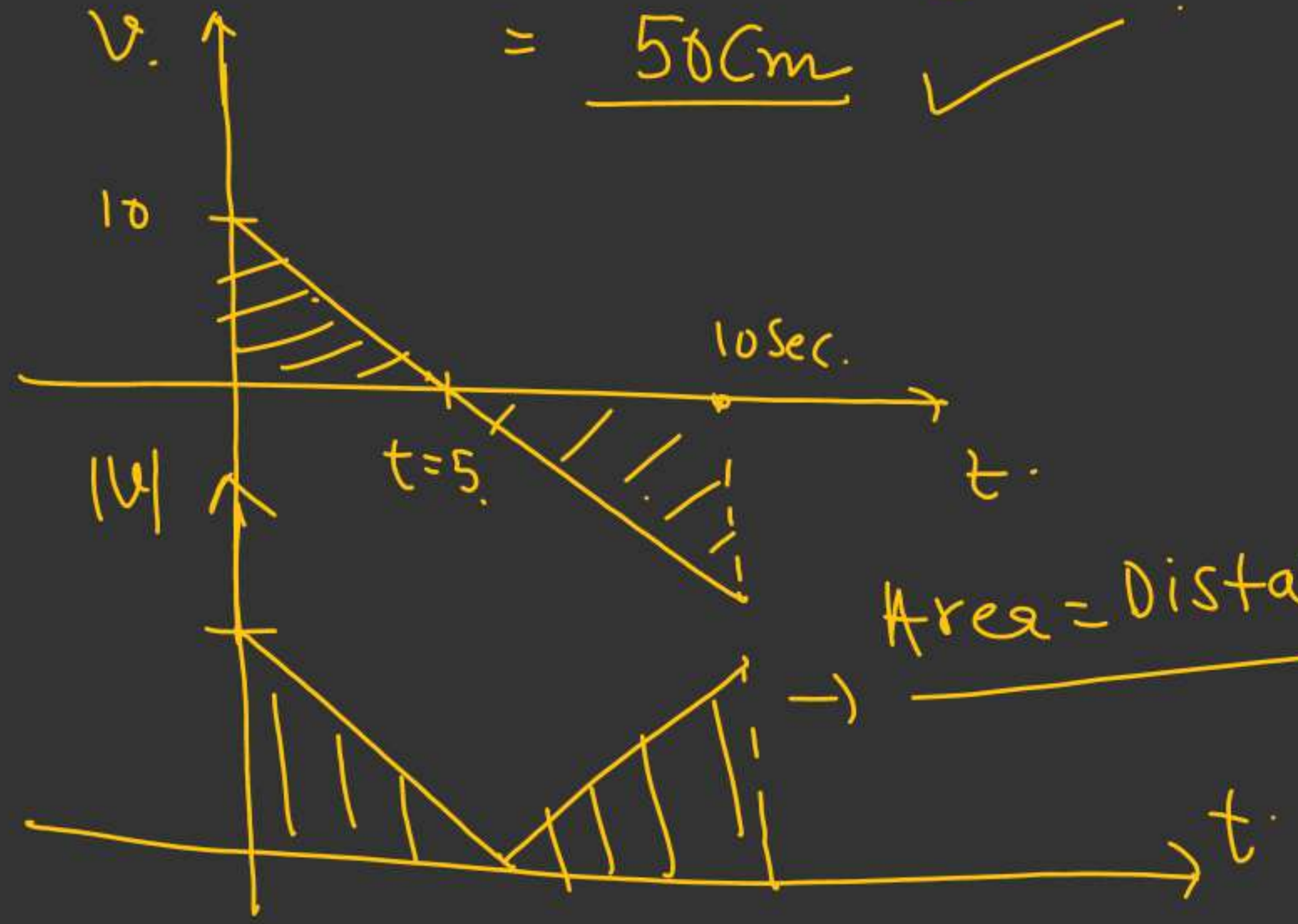


$x = 10t - t^2$

$x_{t=5\,\text{sec}} = (10 \times 5) - 25$
$= 25\,\text{cm}$

$x_{t=10\,\text{sec}} = (100 - 100) = 0.$

$\text{Distance} = [2 \times 25\,\text{cm}]$
$= 50\,\text{Cm}$



Area = Distance