Nishant Jindal

doable

(1) $\sec^{-1} t \Big|_0^x = \frac{\pi}{6}$

$\sec^{-1} x - 0 = \frac{\pi}{6}$

$x = \sec\frac{\pi}{6} = \frac{2}{\sqrt{3}}$

(2) $\int x^3 \cdot x^2 \cdot \sin x^3 \cdot dx$

$x^3 = t$

$\frac{1}{3}\int t \cdot \sin t\, dt$
(IBP)

(3)

(4) $\int (f(x) + f''(x)) \sin x \cdot dx = \int f(x)\sin x + \int f''(x) \cdot \sin x \cdot dx = -f(x)\cdot\cos x\Big|_0^\pi + \cancel{\int_0^\pi f'(x)\cos x \cdot dx} + \left\{ \sin x \cdot f'(x)\Big|_0^\pi - \cancel{\int_0^\pi \cos x \cdot f'(x) \cdot dx} \right\}$

Q5 $\int_0^1 e^{2x-[2x]} \cdot d(x - \cancel{[x]}) dx$

$\int_0^1 e^{2x-[2x]} dx$

$x \to 0 - 1$
$2x \to 0 - 2$
$2x \to 0 - 1 - 2$
$x \to 0 - \frac{1}{2} - 1$

$\int_0^{1/2} \quad + \int_{1/2}$

Q 6) $\frac{2}{3} e^x = t$

Q 7 → Prb1 → 1st Qs Jesa

Q8 (3) hold

Q13 $\int_0^{\pi/4} \tan^n x + \tan^{n-2} x \cdot d(x)$

hi hai

Q9 Prb-1-2 → Q 2nd वाला

Q10 $\frac{1}{c}\int_{ac}^{bc} f\left(\frac{x}{c}\right) dx \quad \frac{x}{c} = t$

Q11 $\int_{5/2}^{5} \frac{\sqrt{25 - x^2}}{x^4}$ $x = 5\sin\theta$ Use

Q12 (oby

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14) $I=\int_0^1 \frac{dx}{1+x^{\pi/2}}$

$$\boxed{\begin{matrix} x\in(0,1) \\ x^2 < x^{\pi/2} < x \end{matrix}}$$

Prop^g

$1+x^2 < 1+x^{\pi/2} < 1+x$

$\int_0^1 \frac{1}{1+x^2}dx > \int_0^1 \frac{1}{1+x^{\pi/2}}dx > \int_0^1 \frac{1}{1+x}dx$

$> \ln(1+x)\Big|_0^1$

$\tan^{-1}x\Big|_0^1 > I > \ln 2$

C
A $\frac{\pi}{4} > I > \ln 2$

4 times mains

Q16
Adv.

$I=\int_0^1 x(1-x)^n dx \to (A)$

P-4 $\downarrow$ $x\to 1-x$

$=\int_0^1 (1-x)(1-(1-x))^n dx$

$I=\int_0^1 (1-x)\cdot x^n dx \to (B)$

$I=\int_0^1 x^n - \int_0^1 x^{n+1}dx$

$=\frac{x^{n+1}}{n+1}\Big|_0^1 - \frac{x^{n+2}}{n+2}\Big|_0^1$

$=\left(\frac{1}{n+1}-\frac{1}{n+2}\right)$

Practice

18) Copy (19)✓ (20) Self (21) Copy

22) $\int_0^{\pi} |1+2\cos x|\,dx$ $\to$ T.P ⇒ $2\cos x+1=0$

$\cos x=-\frac{1}{2}$

$x=\frac{2\pi}{3}$.

27) Copy (28)

29)✓

$\int_0^{2\pi/3} 1+2\cos x - \int_{2\pi/3}^{\pi} 1+2\cos x$

$x+2\sin x\Big|_0^{2\pi/3} - (x+2\sin x)_{2\pi/3}^{\pi}$

23

24) Copy

25

26

$=9\times 2$

$=18$

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Set 1

$$\int_0^{\pi/2} \frac{\sin^n x}{\sin^n x+\cos^n x} = \frac{\pi}{4}$$

$$\int_0^{\pi/2} \frac{\cos^n x\, dx}{\sin^n x+\cos^n x} = \frac{\pi}{4}$$

$$\int_0^{\pi/2} \frac{dx}{1+\tan^n x} = \frac{\pi}{4}$$

$$\int_0^{\pi/2} \frac{dx}{1+\cot^n x} = \frac{\pi}{4}$$

$$\int_0^{\pi/2} \frac{\sec^n x}{\sec^n x+\csc^n x} = \frac{\pi}{4}$$

Set 2

1) $$\int_0^{\pi/2} \ln\tan\theta\, d\theta = \int_0^{\pi/2} \ln(\cot\theta)\, d\theta = 0$$

2) $$\int_0^{\pi/4} \ln(1+\tan\theta)\, d\theta = \frac{\pi}{8}\ln 2$$

Q $$\int_1^{\frac{1+\sqrt5}{2}} \frac{x^2+1}{x^4-x^2+1}\ln\left(1+x-\frac{1}{x}\right)dx$$

$$\Rightarrow \int_1^{\frac{1+\sqrt5}{2}} \frac{\left(1+\frac{1}{x^2}\right)\ln\left(1+x-\frac{1}{x}\right)dx}{x^2+\frac{1}{x^2}-1-2+2}$$

$$= \int_1^{\frac{1+\sqrt5}{2}} \frac{\left(1+\frac{1}{x^2}\right)\ln\left(1+x-\frac{1}{x}\right)dx}{\left(x-\frac{1}{x}\right)^2+1} = \int_1^{\pi/4} \frac{\cancel{\sec^2\theta}\,\ln(1+\tan\theta)\,d\theta}{\cancel{\tan^2\theta+1}}$$

$$x-\frac{1}{x} = \tan\theta$$

$$\left(1+\frac{1}{x^2}\right)dx = \sec^2\theta\, d\theta$$

x	θ
1	0
$\frac{1+\sqrt5}{2}$	$\frac{\pi}{4}$

$$= \int_0^{\pi/4} \ln(1+\tan\theta)\, d\theta = \frac{\pi}{8}\ln 2$$

Q (JEE Main) Let f & g be cont^s fxn on $[0,a]$ such that $f(x)=f(a-x)$ & $g(x)+g(a-x)=4$ then $\int_0^a f(x)\cdot g(x)\, dx = ?$

$4\int_0^a f(x)$ $\quad \int_0^a f(x)$ $\quad 2\int_0^a f(x)$ $\quad -3\int_0^a f(x)$

$$I = \int_0^a f(x)\cdot g(x)\,dx \;\to A \quad \xrightarrow[x\to a-x]{\text{Br 4}} \quad I = \int_0^a f(a-x)\,g(a-x)\,dx$$

$$I = \int_0^a f(x)\cdot g(a-x)\,dx \to B$$

Kind & Add A+B.

$$2I = \int_0^a f(x)\left(g(x)+g(a-x)\right)dx$$

$$2I = 4\int_0^a f(x)\,dx$$

$$I = 2\int_0^a f(x)\,dx$$

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Q $I = \int_0^{\pi} [\cot x]\,dx \to A$ $\qquad [x]+[-x] = -1$, $x \notin I$

Sol $\downarrow$ Pr(4) $(x \to \pi - x)$

Adv 2009

Qs

$= \int_0^{\pi} [-\cot x]\,dx \to B$ $\qquad A+B$

$2I = \int_0^{\pi} [\cot x] + [-\cot x]\,dx$

$= \int_0^{\pi} -1 \cdot dx = -(x)_0^{\pi}$

$2I = -\pi \Rightarrow I = -\frac{\pi}{2}$

Q $I = \int_a^b \frac{f(x)\,dx}{f(x)+f(a+b-x)} = ?$ (A)

$\int$ pr 4 $(x \to a+b-x)$

$I = \int_a^b \frac{f(a+b-x)\,dx}{f(a+b-x)+f(a+b-(a+b-x))}$

$= \int_a^b \frac{f(a+b-x)\,dx}{f(a+b-x)+f(x)} \to$ (B)

A+B

$2I = \int_a^b \frac{\cancel{f(x)+f(a+b-x)}}{\cancel{f(x)+f(a+b-x)}}\,dx$

$= (x)_a^b = b-a$

$I = \frac{b-a}{2}$

Set 3

$$\boxed{\int_a^b \frac{f(x)\,dx}{f(x)+f(a+b-x)} = \frac{b-a}{2}}$$

$$\int_a^b \frac{f(x)}{f(x)+f(a+b-x)} = \frac{U-L}{2}$$

Q $\int_1^3 \frac{e^x}{e^x+e^{4-x}}\,dx = \frac{3-1}{2} = 1$

Q $\int_1^3 \frac{\ln x \cdot dx}{\ln x + \ln(4-x)} = \frac{3-1}{2} = 1$

Q $\int_0^{\pi/2} \frac{\sin x}{\sin x + \cos x} = \frac{\pi}{4}$

Q $\displaystyle\int_2^4 \frac{\log x^2\,dx}{\log x^2+\log(36-12x+x^2)}$ Jeemain 2020

$$= \int_2^4 \frac{\log x^2}{\log x^2+\log(6-x)^2}$$

$$= \frac{4-2}{2} = 1$$

Q $\displaystyle\int_2^3 \frac{x^2\,dx}{2x^2-10x+25}$

$$\Rightarrow \int_2^3 \frac{x^2}{x^2+(5-x)^2} = \frac{3-2}{2} = \frac{1}{2}$$

Q Adv $\displaystyle\int_4^{10} \frac{[x^2]\,dx}{[x^2-28x+196]+[x^2]}$

$$= \int_4^{10} \frac{[x^2]}{[x^2]+[(14-x)^2]}$$

$$= \frac{10-4}{2} = 3.$$

Q $\displaystyle\int_{\sqrt{\ln 3}}^{\sqrt{\ln 4}} \frac{x\cdot\sin x^2\,dx}{\sin x^2+\sin(\ln 12-x^2)}$

$x^2 = t$

$x\,dx = \frac{dt}{2}$

x	t
$\sqrt{\ln 3}$	$\ln 3$
$\sqrt{\ln 4}$	$\ln 4$

$$= \frac{1}{2}\int_{\ln 3}^{\ln 4} \frac{\sin t\,dt}{\sin t+\sin(\ln 4+\ln 3-t)}$$

$$= \frac{1}{2}\times\frac{\ln 4-\ln 3}{2} = \frac{1}{4}\ln\frac{4}{3}$$

Imp → Set 4 (Removal of x)

Q Adv Mains

$$I = \int_0^{\pi} \frac{x\,\boxed{\sin x\,dx}}{\boxed{1+\cos^2 x}} \rightarrow (A) \qquad x = \frac{U+L}{2}$$

Pr 4 $(x \to \pi - x)$

Gb.sir.202312

$$= \int_0^{\pi} \frac{(\pi-x)\sin x\,dx}{1+(-\cos x)^2} = \int_0^{\pi} \frac{\pi\sin x - x\sin x\,dx}{1+\cos^2 x} \rightarrow (B)$$

A+B

$$2I = \int_0^{\pi} \frac{\cancel{x\sin x}+\pi\sin x-\cancel{x\sin x}}{1+\cos^2 x}$$

$$I = \frac{\pi}{2}\int_0^{\pi} \frac{\sin x\,dx}{1+\cos^2 x}$$

$\cos x = t$

$\sin x\,dx = -dt$

x	t
0	1
π	-1

$$= -\frac{\pi}{2}\int_1^{-1} \frac{dt}{1+t^2} = -\frac{\pi}{2}\left(\tan^{-1} t\right)_1^{-1}$$

$$I = -\frac{\pi}{2}\left(-\frac{\pi}{4}-\frac{\pi}{4}\right) = \frac{\pi^2}{4}$$

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Q $I = \int_0^{\pi/2} \frac{x \sin x \cos x}{\sin^4 x + \cos^4 x} \cdot dx$

Ans

$= \frac{\pi}{4} \int_0^{\pi/2} \frac{\sin x \cos x}{\sin^4 x + \cos^4 x} \cdot dx \quad \div \cos^4 x$

$20 \to 390 + Qs$

$= \frac{\pi}{4} \int_0^{\pi/2} \frac{\tan x \cdot \sec^2 x}{1 + (\tan^2 x)^2} dx$

$\tan^2 x = t$

$2 \tan x \cdot \sec^2 x \, dx = dt$

x	t
0	0
$\frac{\pi}{2}$	∞

$= \frac{\pi}{4 \times 2} \int_0^{\infty} \frac{dt}{1+t^2} = \frac{\pi}{8} \left(\tan^{-1} t \right)_0^{\infty}$

$= \frac{\pi}{8} \left(\frac{\pi}{2} - 0 \right) = \frac{\pi^2}{16}$