1. Find x for which 4th term in expansion

of $\left(\sqrt{x^{(1+\log_{10}x)^{-1}}} + \sqrt[12]{10}\right)^6$ is equal to 200.

$$\left(x^{\frac{1}{1+\log_{10}x}}\right)^{1/2}$$

$$\binom{6}{3}\left(x^{\frac{1}{2(1+\log_{10}x)}}\right)^3 \left(10^{\frac{1}{12}}\right)^3 = \cancel{200}^{10}$$

$$\frac{3}{2(1+\log_{10}x)} = 10^{3/4}$$

$$x^{\frac{3}{2(1+\log_{10}x)}} = 10^{3/4}$$

$$\frac{3}{2(1+\log_{10}x)}\log_{10}x = \frac{3}{4}$$

$$\log_{10}x = 1 \qquad \boxed{x = 10}$$

2: Find the term which is independent of x in the expansion of $\left(x^2 + \dfrac{1}{x}\right)^{12}$.

$$T_{r+1} = \,^{12}C_r \left(x^2\right)^{12-r} \left(\dfrac{1}{x}\right)^r = \,^{12}C_r \, x^{24-3r}$$

$$24 - 3r = 0$$
$$r = 8$$

$$T_9 = \,^{12}C_8 = \boxed{495}$$

3. Find the term independent of x in

(i) $\left(\dfrac{x+1}{-x^{\frac{1}{3}}+x^{\frac{2}{3}}+1} - \dfrac{x - \overbrace{\dfrac{x-1}{\sqrt{x}(\sqrt{x}-1)}}^{\frac{\sqrt{x}+1}{\sqrt{x}}}}{x - x^{\frac{1}{2}}} \right)^{10}$ $= \left(\left(x^{\frac{1}{3}}+1\right) - \left(1 + x^{-\frac{1}{2}}\right) \right)^{10}$

$= \left(x^{\frac{1}{3}} - x^{-\frac{1}{2}} \right)^{10}$

(ii) $\left(1 + x + 2x^3\right)\left(\dfrac{3}{2}x^2 - \dfrac{1}{3x}\right)^9$

$T_{r+1} = {}^9C_r \left(\dfrac{3}{2}\right)^{9-r}\left(\dfrac{-1}{3}\right)^r x^{18-3r}$

$\overline{T_{r+1}} = {}^{10}C_r \left(x^{\frac{1}{3}}\right)^{10-r}\left(-x^{-\frac{1}{2}}\right)^r$

$= {}^{10}C_r (-1)^r \, x^{\frac{20-5r}{6}}$

$\left[x \quad {}^9C_6 \left(\dfrac{3}{2}\right)^3 \left(\dfrac{-1}{3}\right)^6 + 2 \times {}^9C_7 \left(\dfrac{3}{2}\right)^2 \left(\dfrac{-1}{3}\right)^7 \right.$

$T_5 = {}^{10}C_4 (-1)^4 = \boxed{210}$ $\boxed{r = 4}$

4. Find the coeff $\cdot$ of (i) x^{50} (ii) x^{49} in

$$(1+x)^{41}(1-x+x^2)^{40}$$

$$(1+x)\left(1+x^3\right)^{40}$$

$$\downarrow$$

$$^{40}C_r \, x^{3r}$$

$$^{40}C_{16}$$

5. Find the coeff. of x^4 in

(i) $(1+x+x^2+x^3)^n$, $n > 4$

(ii) $(2-x+3x^2)^6$

$(1+x)^n (1+x^2)^n$

		x^2	x^3
1	x		
	1	0	1
$n-2$			
$n-2$	0	2	0
$n-3$	2	1	0
$n-4$	4	0	0

$\frac{n!}{(n-2)!}$ $^nC_0 \, ^nC_2 + \, ^nC_2 \, ^nC_1 + \, ^nC_4 \, ^nC_0$

$\longrightarrow \frac{n!}{(n-2)! \, 2!}$

$\longrightarrow \frac{n!}{(n-3)! \, 2!}$

$\longrightarrow \frac{n!}{(n-4)! \, 4!}$

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6. Find the last 3 digits in

(i) $(17)^{256}$

(ii) 3^{100}

$(10-1)^{50} = (\quad) + {}^{50}C_{48}10^2$

$- {}^{50}C_{49}10 + 1$

$\boxed{001}$

$\boxed{\begin{array}{l} Ex-I(21-30) \\ Ex-II(1-7) \end{array}}$

$\left(10K \pm 1\right)^n$

$(290-1)^{128} = \left({}^{128}C_0(290)^{128} - \cdots - {}^{128}C_{125}(290)^3 + {}^{128}C_{126}(290)^2 - {}^{128}C_{127}290\right)$

$\cdots ---000$

$+1$

$\boxed{681}$

$\dfrac{128 \times 127}{2} \times (290)^2 - 128 \times 290 + 1$

$$\left(2 - x + 3x^2\right)^6 = \left(2 + x(3x-1)\right)^6 = \qquad {}^6C_2$$

2	$(-x)$	$3x^2$
4	0	2
3	2	1
2	4	0

$$\frac{6!}{4!\,2!}(2)^4(3)^2$$
$$+$$
$$\frac{6!}{3!\,2!}\,2^3(-1)^2(3)^1$$
$$+$$
$$\frac{6!}{2!\,4!}(2)^2(-1)^4$$

$$\boxed{3660}$$

$$\ldots + {}^6C_2\, 2^4\, x^2(3x-1)^2 + {}^6C_3\, 2^3\, x^3(3x-1)^3 + {}^6C_4\, 2^2\, x^4(3x-1)^4 + \ldots$$

$${}^6C_2\, 2^4\, 3^2 + {}^6C_3\, 2^3\, {}^3C_2\, 3^1(-1) + {}^6C_4\, 2^2\, {}^4C_4(-1)^4$$