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$$2y = x^2 + x\sqrt{x^2+1} + \ln\left(x+\sqrt{x^2+1}\right)$$

$$2y' = 2x + \sqrt{x^2+1} + \frac{x^2}{\sqrt{x^2+1}} + \frac{1+\frac{x}{\sqrt{x^2+1}}}{x+\sqrt{x^2+1}} =$$

$$y' = x + \sqrt{x^2+1}$$

$$\frac{1}{x - \frac{1}{y'}} = y'$$

$$\frac{y}{2y-1} = y'$$

$$\frac{1}{\sqrt{x^2+1}}$$

$$\frac{1}{x + \frac{1}{x+\frac{1}{y'}}}$$

$$2y = xy' + \ln y'$$

$$y = x + \frac{1}{y'}$$

$$y' = 1 - \frac{y}{x}$$

$$y^2 = xy + 1$$

$$(2y - x)y' = y$$

$$y = \frac{1}{2\left(\frac{1-t^2}{1+t^2}\right)^2 - 1}$$

$$\sqrt{x^2+1}$$

$$\frac{x + x\sqrt{1-x^4}}{2x^2}$$

$$y' = \frac{y}{2y - x}$$

$$= \frac{1}{2 - \frac{x}{y}}$$

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$y = x + \frac{1}{y}$

$y = e^x \, y^x \, a^y \ln x$

$\frac{y'}{y} = 1 + \left(\frac{x}{y}y' + \ln y\right) + (\ln a)y' + \frac{1}{x\ln x}$

$y'\left(\frac{1}{y} - \frac{x}{y} - \ln a\right)$

$= \frac{1}{x \ln x} + \ln y + 1$

$\frac{1}{2}\sin^{-1}x + \sqrt{1-x^2}$

$\sin^{-1}x = \theta$

$\sin \le \tan\frac{\theta}{2}$

$\frac{\sin\theta}{1+\cos\theta}$

$x_1$

$f(x_1) = f(x_1)\sin(ax_1)$

$\boxed{\sin(ax_1) = 1}$

$m_1 = f'(x_1)$

$m_2 = f'(x_1)\sin(ax_1) + a\cos(ax_1) f(x_1)$

$\boxed{m_1 = m_2}$

$2 - \frac{x}{x + \frac{1}{x + \dots}}$

$\left(\frac{x}{1+\sqrt{1-x^2}}\right) + \frac{1}{2}\sin^{-1}x$

$\frac{2\sqrt{\frac{1-x}{1+x}}}{1+\frac{1-x}{1+x}}$

$\frac{2\sqrt{1-x^2}}{2}$

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$$\frac{\frac{A}{b}(a+bx) - \frac{Aa}{b}}{a+bx}$$

$$\frac{K_1}{x+1} + \frac{K_2}{2x-3}$$

$$-\int \frac{dx}{\left(x-\frac{1}{2}\right)^2 + \frac{11}{4}} = \frac{-2}{\sqrt{11}} \tan^{-1}\left(\frac{x-\frac{1}{2}}{\frac{\sqrt{11}}{2}}\right) + C$$

$$\frac{1}{4} - \frac{5}{2}$$

$$\frac{1+\sin^2 x+2\sin x}{\cos^2 x}$$

31. $$\int \operatorname{cosec}^6 x \, dx = -\int (1+\cot^4 x+2\cot^2 x)(\operatorname{cosec}^2 x)\, dx$$

$$(\cot^2 x+1)^2$$

$$= -\left(\cot x+\frac{\cot^5 x}{5}+\frac{2\cot^3 x}{3}\right)+C$$

$$\frac{1}{-3\sin^3 x}+\frac{1}{\sin x}+C = \frac{(1-\sin^2 x)\cos x \, dx}{\sin^4 x}.$$

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1. $$\int \frac{x^2\,dx}{\sqrt{a^6-x^6}} = \frac{1}{3}\int \frac{3x^2\,dx}{\sqrt{(a^3)^2-(x^3)^2}} = \frac{1}{3}\sin^{-1}\left(\frac{x^3}{a^3}\right)+C.$$

$$x^3 = a^3\sin\theta. \qquad \frac{x^3}{a^3} = \sin\theta$$

$$\frac{1}{3}\int \frac{a^3\cos\theta\,d\theta}{\sqrt{a^6\cos^2\theta}} = \frac{1}{3}\int d\theta = \frac{1}{3}\theta + C$$

$$= \frac{1}{3}\sin^{-1}\left(\frac{x^3}{a^3}\right)+C.$$

2. $$\int \frac{dx}{(x^2+4)\sqrt{4x^2+1}}$$

$x = \frac{1}{2}\tan\theta$

$\frac{1}{2}\sec^2\theta\, d\theta = dx$


$$= \int \frac{\frac{1}{2}\sec\theta\, d\theta}{\left(\frac{1}{4}\tan^2\theta + 4\right)} = 2\int \frac{\cos\theta\, d\theta}{\sin^2\theta + 16\cos^2\theta} = 2\int \frac{\cos\theta\, d\theta}{16 - 15\sin^2\theta}$$

$$\int \frac{dt}{\frac{16}{15} - t^2}$$

$$= \frac{2}{15}\int \frac{\cos\theta\, d\theta}{\frac{16}{15} - \sin^2\theta}$$

$$\frac{2}{15}\,\frac{1}{2\left(\frac{4}{\sqrt{15}}\right)}\ln\left|\frac{\frac{4}{\sqrt{15}} + \sin\theta}{\frac{4}{\sqrt{15}} - \sin\theta}\right| + c.$$

$$\sin\theta = \frac{2x}{\sqrt{1+4x^2}}$$

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3. $\displaystyle\int \frac{\sin 2x \, dx}{\sqrt{9-\sin^4 x}} = \sin^{-1}\left(\frac{\sin^2 x}{3}\right) + C$

$\sin^2 x = t$

$\displaystyle\int \frac{dt}{\sqrt{9-t^2}}$

4. $\displaystyle\int \frac{e^x \, dx}{\sqrt{e^{2x}-1}} = \ln\left|e^x + \sqrt{e^{2x}-1}\right| + C$

$e^x = t$

$\displaystyle\int \frac{dt}{\sqrt{t^2-1}} = \ln\left|t + \sqrt{t^2-1}\right| + C$

5. $\displaystyle\int \frac{e^x \, dx}{(e^{2x}+4)}$

$\parallel$

$\frac{1}{2}\tan^{-1}\left(\frac{e^x}{2}\right) + C$

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# Integrals of form

$$\int \frac{\overbrace{(ax+b)}^{N(x)}\,dx}{(\underbrace{px^2+qx+r}_{D(x)})} \quad , \quad \int \frac{(ax+b)\,dx}{\sqrt{px^2+qx+r}}$$

$$N(x) = K_1 D'(x) + K_2$$

1.

$$\int \frac{(4x+3)\,dx}{(3x^2+3x+1)} = \int \left( \frac{\frac{4}{6}(6x+3) + 1}{3x^2+3x+1} \right) dx$$

$$= \frac{2}{3} \int \frac{(6x+3)\,dx}{3x^2+3x+1} + \frac{1}{3} \int \frac{dx}{x^2+x+\frac{1}{3}}$$

$$= \frac{2}{3} \int \frac{(6x+3)\,dx}{3x^2+3x+1} + \frac{1}{3} \int \frac{dx}{\left(x+\frac{1}{2}\right)^2 + \frac{1}{12}}$$

$$= \frac{2}{3} \ln\left|3x^2+3x+1\right| + \frac{1}{3}\sqrt{12}\tan^{-1}\left(\frac{x+\frac{1}{2}}{\frac{1}{\sqrt{12}}}\right) + C.$$

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2: $\int \frac{(2x-1)\,dx}{\sqrt{4x^2+4x+2}} = \int \frac{\frac{1}{4}(8x+4) - 2}{\sqrt{4x^2+4x+2}}\,dx$

$= \frac{1}{4}\int \frac{(8x+4)\,dx}{\sqrt{4x^2+4x+2}} - \int \frac{dx}{\sqrt{x^2+x+\frac{1}{2}}} = \frac{1}{4} \times 2\sqrt{4x^2+4x+2}$

$(x+\frac{1}{2})^2+\frac{1}{4}$ $\quad - \ln\left|x+\frac{1}{2}+\sqrt{(x+\frac{1}{2})^2+\frac{1}{4}}\right|$

$+ C.$

3: $\int \frac{x\,dx}{(x^4+x^2+1)} = \frac{1}{2}\int \frac{2x\,dx}{(x^2+\frac{1}{2})^2+\frac{3}{4}}$

$x^2+\frac{1}{2} = t$

$\frac{1}{2}\int \frac{dt}{t^2+\frac{3}{4}}$

$\frac{1}{2} \times \frac{2}{\sqrt{3}} \tan^{-1}\left(\frac{x^2+\frac{1}{2}}{\frac{\sqrt{3}}{2}}\right) + C$

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$$\frac{(x^2+x+1)-(x^2-x+1)}{2}$$

$$\int \frac{x\,dx}{(x^2-x+1)(x^2+x+1)} = \frac{1}{2}\int\left(\frac{1}{x^2-x+1} - \frac{1}{x^2+x+1}\right)dx$$

$$= \frac{1}{2}\int\left(\frac{1}{\left(x-\frac{1}{2}\right)^2+\frac{3}{4}} - \frac{1}{\left(x+\frac{1}{2}\right)^2+\frac{3}{4}}\right)dx.$$

$$x^4+x^2+1 = (x^2-x+1)(x^2+x+1)$$

$$= \frac{1}{2}\frac{2}{\sqrt{3}}\left(\tan^{-1}\left(\frac{x-\frac{1}{2}}{\frac{\sqrt{3}}{2}}\right) - \tan^{-1}\left(\frac{x+\frac{1}{2}}{\frac{\sqrt{3}}{2}}\right)\right) + C$$

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# Integration By Parts

$$\boxed{\int (\mathrm{I}\ fn)(\mathrm{II}\ fn)\,dx = (\text{Integral of II } fn)(\mathrm{I}\ fn) - \int (\text{Integral of II } fn)(\text{Derivative of I } fn)\,dx}$$

$$\frac{d}{dx}\left(f(x)g(x)\right) = f'(x)g(x) + f(x)g'(x)$$

$$d(f(x)g(x)) = f'(x)g(x)dx + f(x)g'(x)dx$$

$$\int f(x)g'(x)\,dx = \int d(f(x)g(x)) - \int f'(x)g(x)\,dx.$$

$$\boxed{\int \underset{\mathrm{I}}{f(x)}\,\underset{\mathrm{II}}{g'(x)}\,dx = f(x)g(x) - \int f'(x)g(x)\,dx.}$$

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$$\int \underset{\mathrm{I}}{f(x)}\, \underset{\mathrm{II}}{g(x)}\,dx = f(x)(\phi(x)+c) - \int f'(x)(\phi(x)+c)\,dx$$

$$= f(x)\phi(x) + c\cancel{f(x)} - \int f'(x)\phi(x)\,dx$$

$$\cancel{- c\int f'(x)\,dx}$$

$$\int g(x)\,dx = \phi(x)+c$$

$$\int f(x)\,g(x)\,dx.$$

Inverse

Logarithm

Algebraic

Trig.

Exponential

ILATE → preference for I fn.