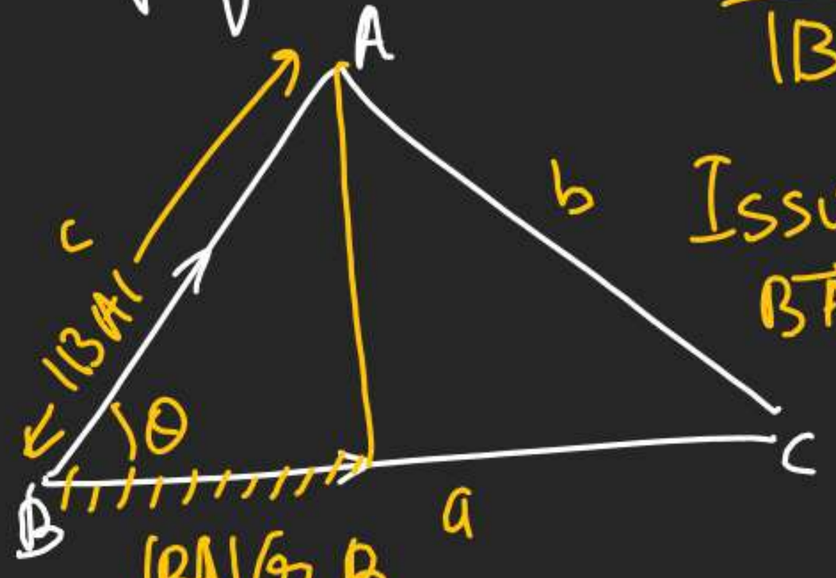
Nishant Jindal

Q In a $\triangle ABC$

$a = |BC| = 3$, $b = |CA| = 5$, $|BA| = 7 = c$

Prof. of $\vec{BA}$ on $\vec{BC}$ $= \dfrac{\vec{BA}\cdot\vec{BC}}{|BC|} = 7 \times \dfrac{\cancel{33}\,11}{2\times3\times7} = \dfrac{11}{2}$



Issue

$\vec{BA}, \vec{BC}$ Nahi hui

$|BA| \cos B$

$\Rightarrow \cos B = \dfrac{a^2+c^2-b^2}{2ac}$

$= \dfrac{9+49-25}{2\times3\times7}$

Q If $\vec{a}+3\vec{b}$ is $\perp^r$ to $7\vec{a}-5\vec{b}$

& $\vec{a}-4\vec{b}$ is $\perp^r$ to $(7\vec{a}-2\vec{b})$

find angle betn $\vec{a}$ & $\vec{b}$.

$(\vec{a}+3\vec{b})\cdot(7\vec{a}-5\vec{b})=0$, $(\vec{a}-4\vec{b})\cdot(7\vec{a}-2\vec{b})=0$

(1) $7|a|^2+16ab-15|b|^2=0$

(2) $7|a|^2-30ab+8|b|^2=0$

$23|b|^2 = \cancel{46}\,2\,ab$

$7|a|^2+8|b|^2-15|b|^2=0$

$|a|=|b|$

$\cos\theta = \dfrac{a\cdot b}{|a||b|} = \dfrac{|b|^2}{2|a||b|} = \dfrac{1}{2} \Rightarrow \theta = 60^\circ$

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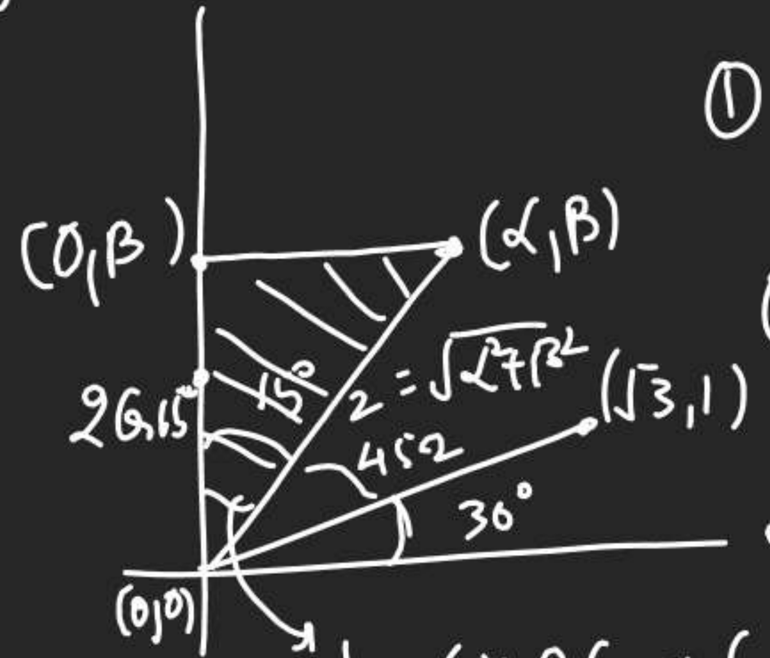
Q If a vector $\alpha\hat{i}+\beta\hat{j}$ is obtained by rotating vector $\sqrt{3}\hat{i}+\hat{j}$ by angle $\theta = 45^\circ$ in ACW direction. Then area of Δ having vertices (α,β), $(0,\beta)$, $(0,0)$ is?

$\sqrt{\alpha^2+\beta^2}$

$\sqrt{\sqrt{3}^2+1^2}=\sqrt{4}=2$



① length same $\sqrt{\alpha^2+\beta^2}$

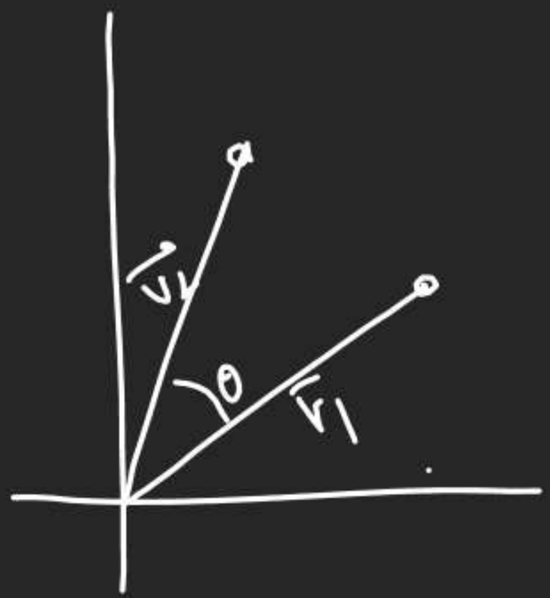
② $\alpha^2+\beta^2=4$

$\frac{1-0}{\sqrt{3}-0}=\tan\theta$

$\frac{1}{2}ab\sin\theta = \frac{1}{2}\times 2\times 2\cos 15^\circ\cdot\sin 15^\circ$

$= \sin 30^\circ = \frac{1}{2} = \frac{1}{2}$

Q If $(P>0)$, A vector $\vec{v}_2 = 2\hat{i}+(P+1)\hat{j}$ is obtained by $\vec{V}_1=\sqrt{3}P\hat{i}+\hat{j}$ by rotating by an angle θ in ACW dir. If $\tan\theta = \frac{\alpha\sqrt{3}-2}{4\sqrt{3}+3}$ than $\alpha = ?$



① length same $\Rightarrow |V_1|^2=|V_2|^2 = 2^2+(2+1)^2 = 13$

$2^2+(P+1)^2 = (\sqrt{3}P)^2+1^2$

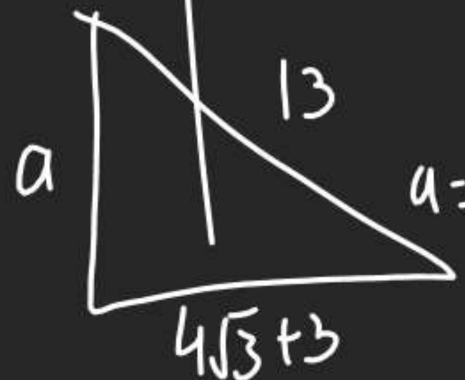
$P^2+2P+5 = 3P^2+1$

$\Rightarrow -2P^2+2P+4=0$

$\Rightarrow P^2-P-2=0 \Rightarrow P=2, -1$ (X)

$(P-2)(P+1)=0$

② $\cos\theta = \frac{V_1\cdot V_2}{|V_1||V_2|} = \frac{2\sqrt{3}P+(P+1)}{\sqrt{13}\sqrt{13}} = \frac{4\sqrt{3}+3}{13}$



$a=\sqrt{169-57-24\sqrt{3}}$

$\sqrt{112-24\sqrt{3}}$

$2\times 6\sqrt{3}\times 2$

3) $\tan\theta = \frac{\sqrt{112-24\sqrt{3}}}{4\sqrt{3}+3} = \frac{\sqrt{(6\sqrt{3}-2)^2}}{4\sqrt{3}+3}$

$= \frac{6\sqrt{3}-2}{4\sqrt{3}+3}$ $\alpha=6$

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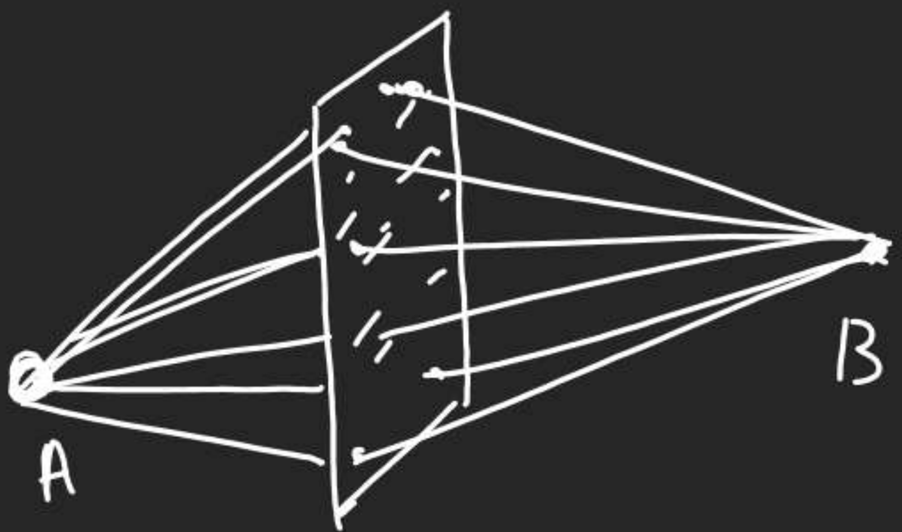
Plane

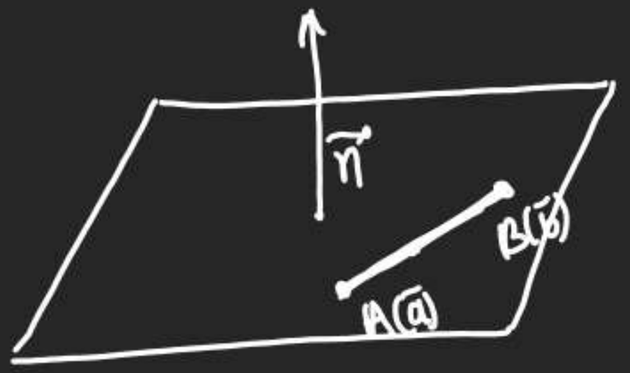


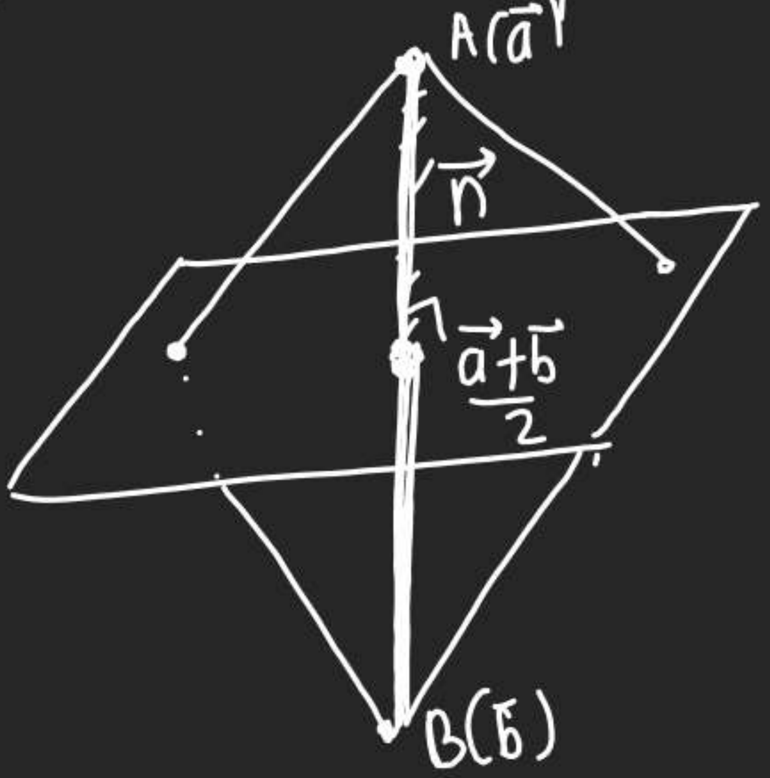
1) In 3D, Plane is Locus of all Pts equidistant from 2 fix Pts A & B

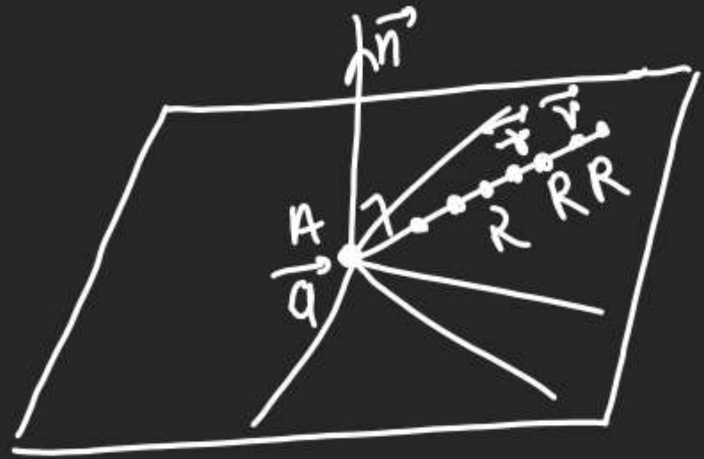
(2) Such Plane always P.T. $\frac{\vec{a}+\vec{b}}{2}$

(3) Line Joining $A(\vec{a})$ & $B(\vec{b})$ is always $\perp^r$ to Plane.

(4) Plane is Locus of Pts in which holds Line Joining 2 Pts such that it always Remains $\perp^r$ to a fixed Line & this fixed Line is known as Normal vector / Direction Vector of Plane & denoted by $\vec{n}$.







Plane $\vec{AR} \perp \vec{n}$

$$\boxed{(\vec{r}-\vec{a})\cdot\vec{n}=0}$$

$\Rightarrow \vec{r}\cdot\vec{n}-\vec{a}\cdot\vec{n}=0$

Variable   Fix Pt   Fix Nor.

$$\boxed{\vec{r}\cdot\vec{n}=d}$$

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Rem:-

> Eqⁿ of Plane
>
> $(\vec{r} - \text{Fix pt}) \cdot \text{normal} = 0$

(5) $\vec{r}$ = var. Pt = $x\hat{i} + y\hat{j} + z\hat{k}$

$\vec{a}$ = Fix pt = $a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$, $\vec{n}$ = normal vector = $a\hat{i} + b\hat{j} + c\hat{k}$

EOP → $\vec{r} \cdot \vec{n} = \vec{a} \cdot \vec{n}$

$ax + by + cz = a_1 a + a_2 b + a_3 c$

$\boxed{ax + by + cz = d}$ EOP in Cartesian form

Q. Find EOP when Plane is P.T. $(0, 1, 2)$ & ⊥ʳ to $3\hat{i} - \hat{j} - \hat{k}$

$(\vec{r} - \langle 0, 1, 2\rangle) \cdot \langle 3, -1, -1\rangle = 0$

$\vec{r} \cdot \langle 3, -1, -1\rangle = 0 + -1 - 2$

$\vec{r} \cdot \langle 3, -1, -1\rangle = -3$

$\langle x, y, z\rangle \cdot \langle 3, -1, -1\rangle = -3$

$3x - y - z = -3$

EOP

Q EOP P.T. $\langle 1, 2, 3\rangle$ & ⊥ʳ to $\langle 1, -1, 4\rangle$

① $(\vec{r} - \langle 1, 2, 3\rangle) \cdot \langle 1, -1, 4\rangle = 0$ Ans

$(r - a) \cdot \vec{n}$

② $\vec{r} \cdot \langle 1, -1, 4\rangle = 1 + -2 + 12$

$\vec{r} \cdot \langle 1, -1, 4\rangle = 11$ ans

(3) $x - y + 4z = 11$

Q EOP $2x - y + z = 4$ Vector form

$\vec{r} \langle 2, -1, 1\rangle = 4$ vector

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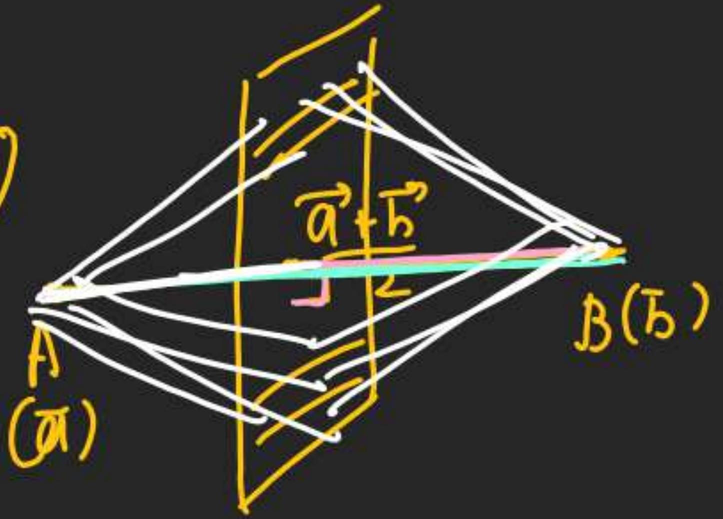
Q



EOP in above diagram is

1) $\left(r-\left(\frac{a+b}{2}\right)\right)\cdot\left(b-\frac{\bar{a}+\bar{b}}{2}\right)=0$

2) $\left(r-\left(\frac{\bar{a}+\bar{b}}{2}\right)\right)\cdot\left(\bar{b}-\bar{a}\right)=0$

3) $\left(r-\left(\frac{\bar{a}+\bar{b}}{2}\right)\right)\cdot\left(a-\frac{\bar{a}+\bar{b}}{2}\right)=0$

4) $|r-a|=|r-b|$

$(\bar{r}-\text{fixpt})\cdot(\text{normal})=0$

$\left(r-\left(\frac{a+b}{2}\right)\right)$

all correct

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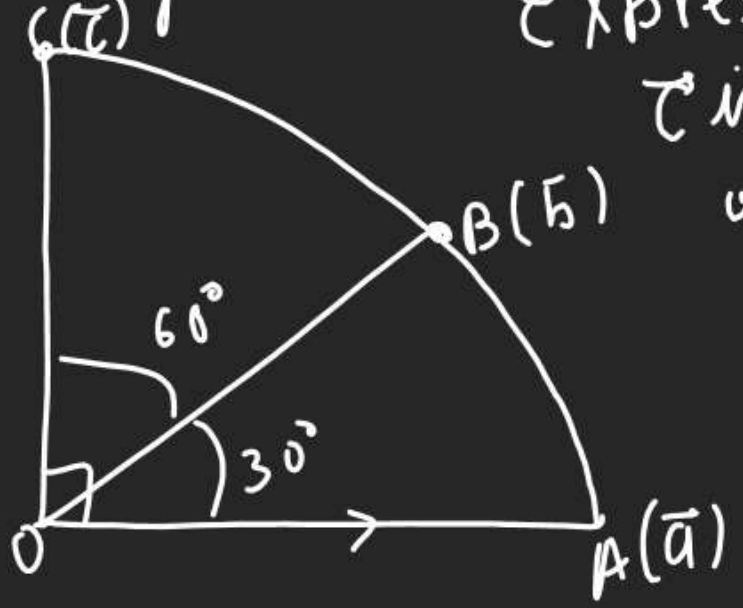
$\vec{r} = x\vec{a} + y\vec{b}$ Based Qs.

1) If 3 vectors $\vec{r}, \vec{a}, \vec{b}$ are Coplanner then any vector can be represented as Linear Combination of other two.

2) So $\vec{r}$ Can be Rep. as $\vec{r} = x\vec{a} + y\vec{b}$

Q. See diagram

Express $\vec{c}$ in terms of $\vec{a}$ & $\vec{b}$.



(1) $|\vec{a}| = |\vec{b}| = |\vec{c}| = K$

(2) $\vec{c}, \vec{a}, \vec{b}$ are in Same plane.

$$\underline{\vec{c} = x\vec{a} + y\vec{b}}$$

• With $\vec{a}$ • With $\vec{c}$

$0 = x|a|^2 + y\,\vec{a}\cdot\vec{b}$

$0 = x|a|^2 + y|a||b|\cos 30°$

$= xK^2 + yK^2\frac{\sqrt{3}}{2}$

$2x + \sqrt{3}y = 0$

$x = -\sqrt{3}$

$|c|^2 = x \times 0 + y\,\vec{b}\cdot\vec{c}$

$K^2 = y\,|b||c|\cos 60°$

$\cancel{K^2} = \frac{y\cancel{K^2}}{2}$

$y = 2$

$\therefore \vec{c} = -\sqrt{3}\vec{a} + 2\vec{b}$ Ans

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Q $\vec{a} = \hat{i} - \hat{j}$, $\vec{b} = \hat{i} + 2\hat{j}$ find 3rd Vector (coplanar with $\vec{a}$ & $\vec{b}$ & also $\perp^r$ to $\vec{b}$ with mag. Unity

① $\vec{r} = x\vec{a} + y\vec{b}$ ② $r \cdot \vec{b} = 0$ ③ $|\vec{r}| = 1$

$\cdot$ with $\vec{b}$

$0 = x \cdot \vec{a}\cdot\vec{b} + y|b|^2$

$= x((1-2)) + 5y$

$\boxed{x = 5y}$

$\vec{r} = x(\hat{i} - \hat{j}) + y(\hat{i} + 2\hat{j})$

$\vec{r} = (x+y)\hat{i} + (-x+2y)\hat{j}$

$\vec{r} = 6y\hat{i} - 3y\hat{j}$

$|r| = 1$

$\sqrt{36y^2 + 9y^2} = 1$

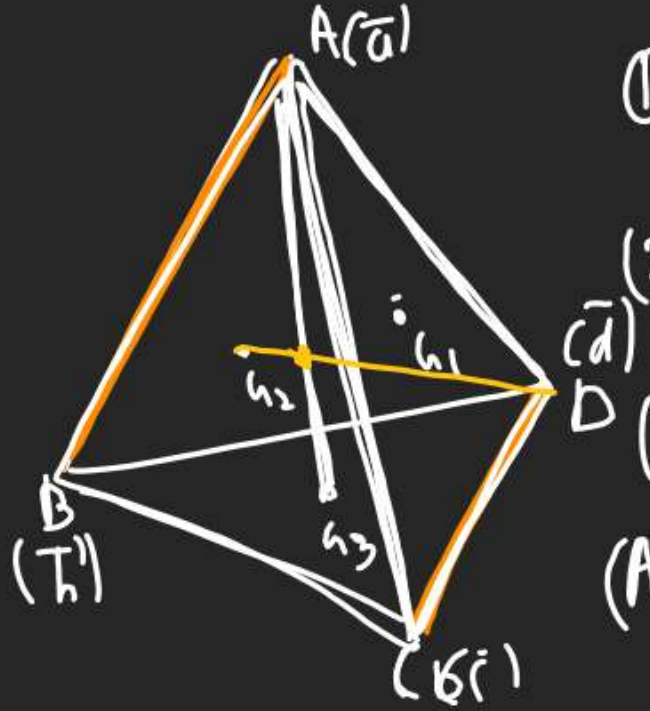
$45y^2 = 1$

$y^2 = \frac{1}{45}$

$y = \frac{1}{3\sqrt{5}}$ | $x = \frac{5\sqrt{5}}{3\sqrt{5}}$

$\therefore \vec{r} = \frac{\sqrt{5}}{3}\vec{a} + \frac{1}{3\sqrt{5}}\vec{b}$

Tetrahedron.



① It is a Pyramid whose Base is $\triangle$.

(2) It has 4 faces

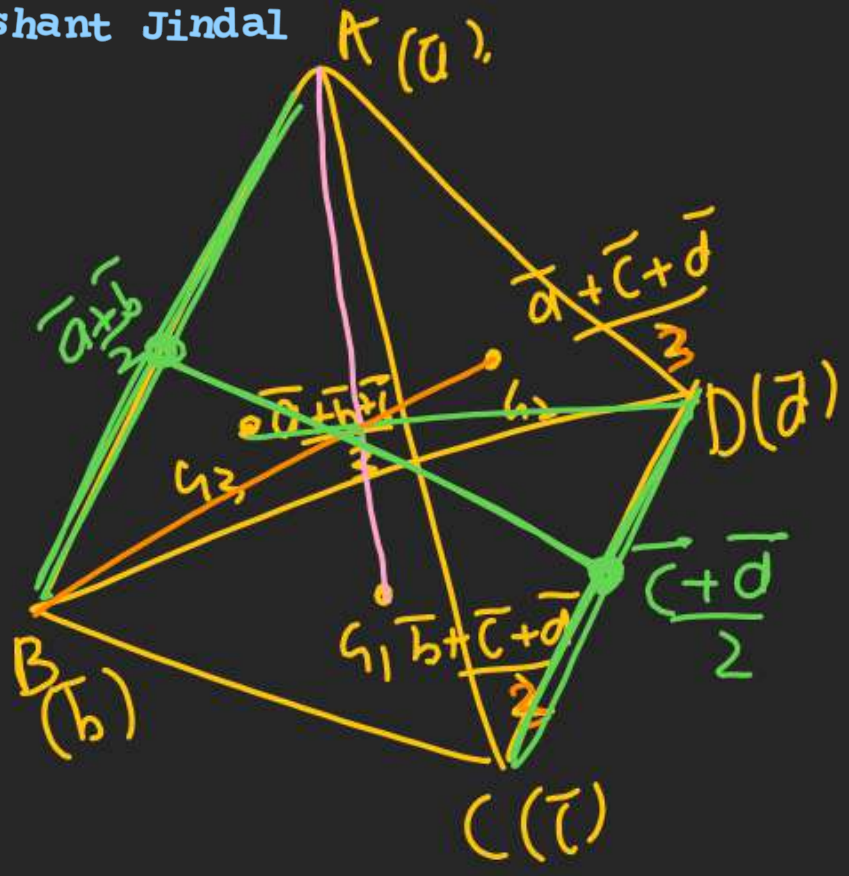
(3) It has opposite Edges

(AB, CD), (AD, BC) (AC, BD)

(4) It has 4 Centroids G_1, G_2, G_3, G_4

$G_1 = \frac{\vec{a} + \vec{c} + \vec{d}}{3}$, $\vec{G_2} = \frac{\vec{a} + \vec{b} + \vec{c}}{3}$

(5) Lines Joining Centroid to vertex opp to it Intersect at Center of Tetrahedron & Centre divide line joining G_2 & D in Ratio 1:3



$G_2\left(\frac{\vec{a}+\vec{c}+\vec{d}}{3}\right)$ —— $1:3$ —— G —— $B(\vec{b})$

$$\vec{G} = \frac{\vec{b}+\vec{a}+\vec{b}+\vec{c}}{4}$$

$G_1\left(\frac{\vec{b}+\vec{c}+\vec{d}}{3}\right)$ —— $1:3$ —— G —— $A(\vec{a})$

$$\vec{G} = \frac{\vec{b}+\vec{c}+\vec{d}+\vec{a}}{4}$$

$G_3\left(\frac{\vec{a}+\vec{b}+\vec{c}}{3}\right)$ —— $1:3$ —— G —— $D(\vec{d})$

$$\vec{G} = \frac{\vec{a}+\vec{b}+\vec{c}+\vec{d}}{4}$$

(5) Centre of tetrahedron is also Mid Pt of Line joining opp. edges.

$\frac{\vec{c}+\vec{d}}{2}$ —— A —— $\frac{\vec{a}+\vec{b}}{2}$

$$\frac{\frac{\vec{a}+\vec{b}}{2}+\frac{\vec{c}+\vec{d}}{2}}{2}$$

$$A = \frac{\vec{a}+\vec{b}+\vec{c}+\vec{d}}{4} = \vec{G}$$

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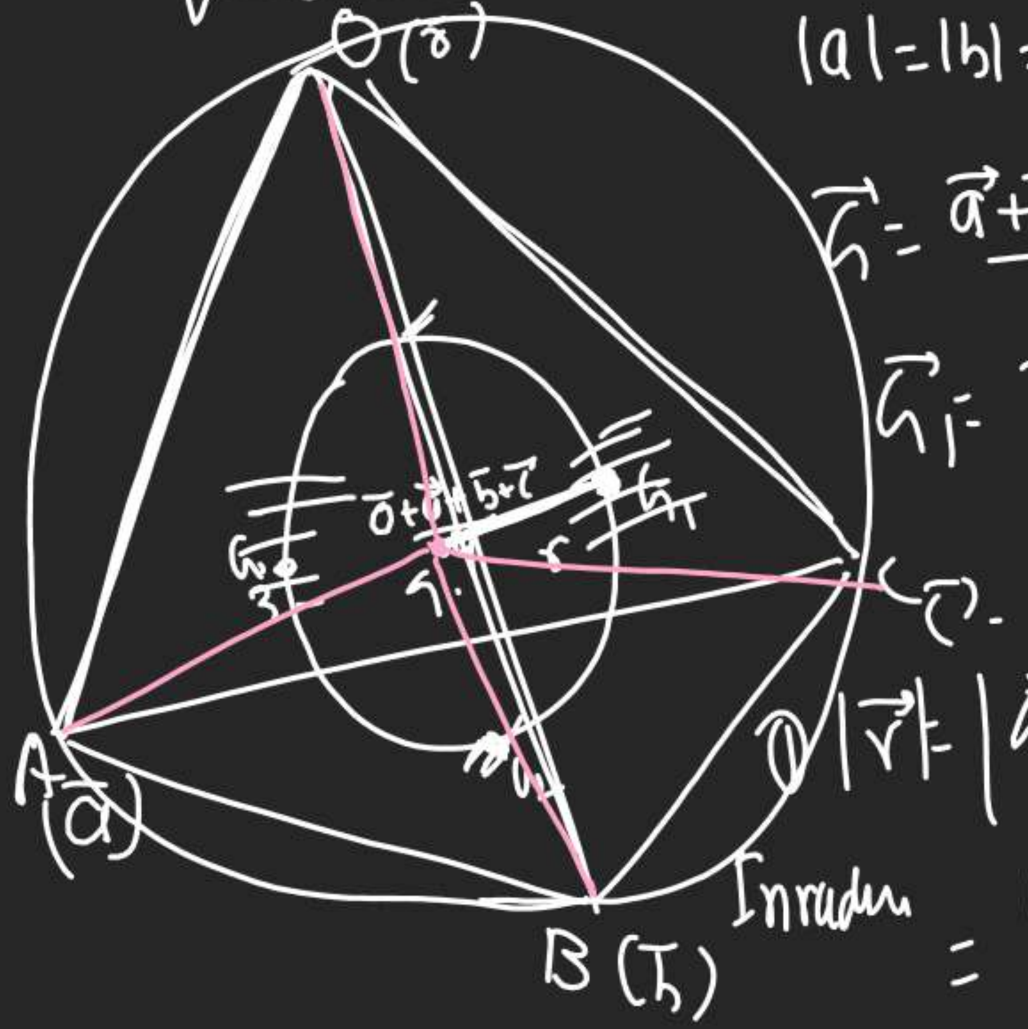
Q Find InRadius & Circumradius of Regular tetrahedron



$|a| = |b| = |c| = K$

$\vec{G} = \dfrac{\vec{a}+\vec{b}+\vec{c}}{4}$

$\vec{G_1} = \dfrac{\vec{b}+\vec{c}}{3}$

① Inradius

$|\vec{r}| = |\vec{G G_1}| = |\vec{G_1} - \vec{G}|$

$= \left| \dfrac{\vec{b}+\vec{c}}{3} - \dfrac{\vec{a}+\vec{b}+\vec{c}}{4} \right|$

$= \dfrac{1}{12} \left| 4\vec{b} + 4\vec{c} - 3\vec{a} - 3\vec{b} - 3\vec{c} \right|$

$= \dfrac{1}{12} \left| -3\vec{a} + \vec{b} + \vec{c} \right| = \dfrac{1}{12}\sqrt{9K^2 + K^2 + K^2 - \cancel{6}^{3} K\cdot K \cos 60^\circ - \cancel{6}^{3} K K \cos 60^\circ + \cancel{2} K K \cos 60^\circ}$

$= \dfrac{\sqrt{6K^2}}{12} = \dfrac{\sqrt{6}K}{12}$

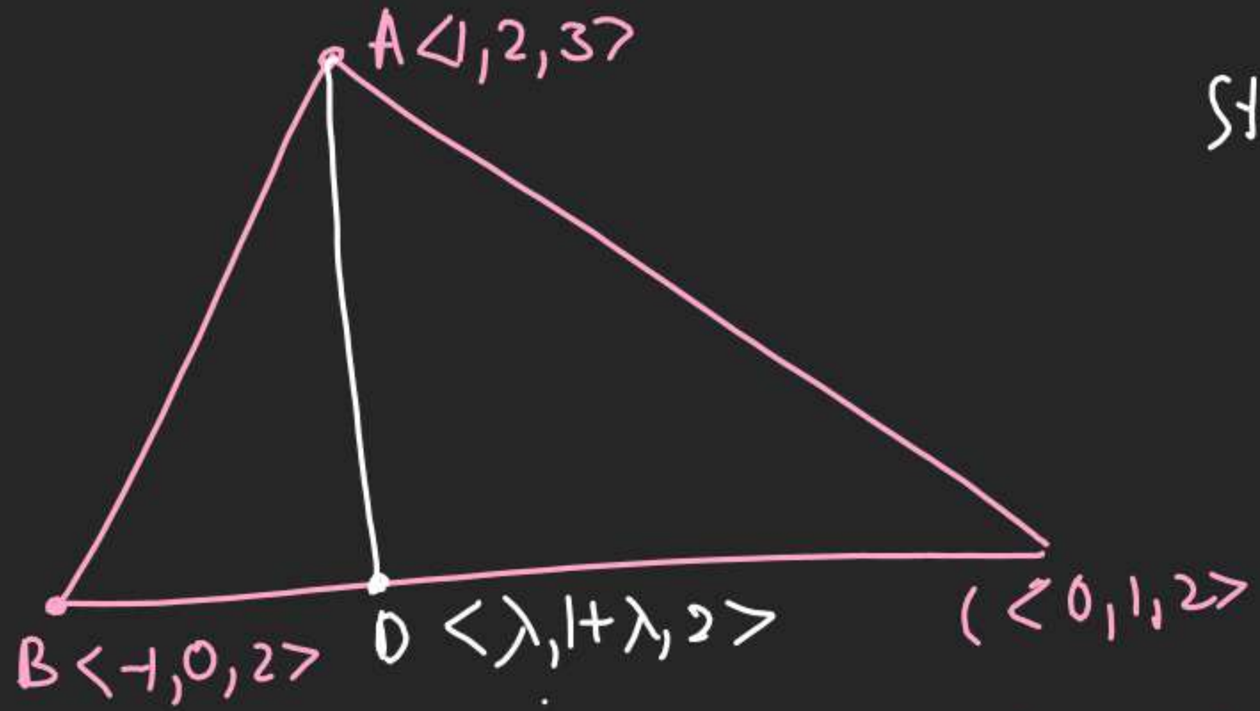
② Circum Radius $= \vec{OG} = \left| \dfrac{\vec{a}+\vec{b}+\vec{c}}{4} - 0 \right|$

$= \dfrac{1}{4}\sqrt{K^2 + K^2 + K^2 + K^2 + K^2 + K^2}$

$= \dfrac{\sqrt{6}K}{4}$

Nishant Jindal

Imp profile (IIT Adv.)



① Find Eqⁿ of Altitude from A to BC.

① $\vec{AD}$ Chahiye Qs Basically asking about Foot of ⊥r.

Step 1 Line BC $\vec{r} = \vec{a} + \lambda\vec{b}$

$\vec{r} = \langle 0,1,2\rangle + \lambda\langle 0+1, 1-0, 2-2\rangle$

$\vec{r} = \langle 0,1,2\rangle + \lambda\langle 1,1,0\rangle$ → BC के DR

Step 2 Gen. term BC = D

$\langle 0+\lambda, 1+\lambda, 2+0\rangle$

(घटा घटाकर निकालते हैं)

Step 3 → DR of $\vec{AD} = \langle \lambda-1, \lambda-1, -1\rangle$

Step 4 → $\vec{AD} \perp \vec{BC} \Rightarrow$ (DR of AD)·(DR of BC) = 0

$= \langle \lambda-1, \lambda-1, -1\rangle \cdot \langle 1,1,0\rangle = 0$

$\lambda-1+\lambda-1+0=0$

$\lambda = 1$

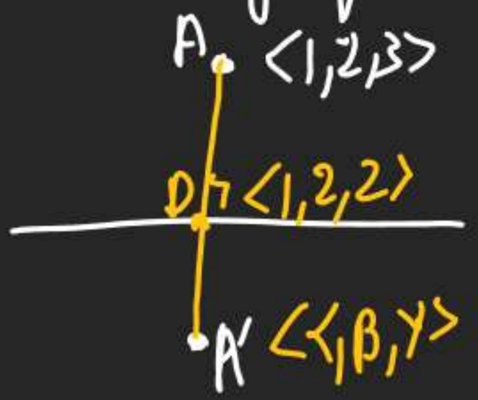
DR of AD = $\langle 0,0,-1\rangle$

Qs → Eqⁿ of AD ⇒ $\vec{r} = \langle 1,2,3\rangle + t\langle 0,0,-1\rangle$

(2) FOOT of ⊥r from A to BC ⇒ D बताना है

$D = \langle \lambda, 1+\lambda, 2\rangle = \langle 1,2,2\rangle$

(3) Image of <1,2,3> in BC



$A' = \langle \alpha, \beta, \gamma\rangle$ D is M.P of A & A'

$\frac{\alpha+1}{2} = 1$ | $\frac{\beta+2}{2} = 2$ | $\frac{\gamma+3}{2} = 2$

$\alpha = 1$ | $\beta = 2$ | $\gamma = 1$

$A' = \langle 1,2,1\rangle$

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Q(1) Foot of $\perp^r$ of Pt $\langle 1,1,0\rangle$ in Line

Joining $\langle 1,-1,2\rangle$ & $\langle 3,2,1\rangle$

(2) find Image of $\langle 1,1,0\rangle$ in Above Line.

Parallelopipe