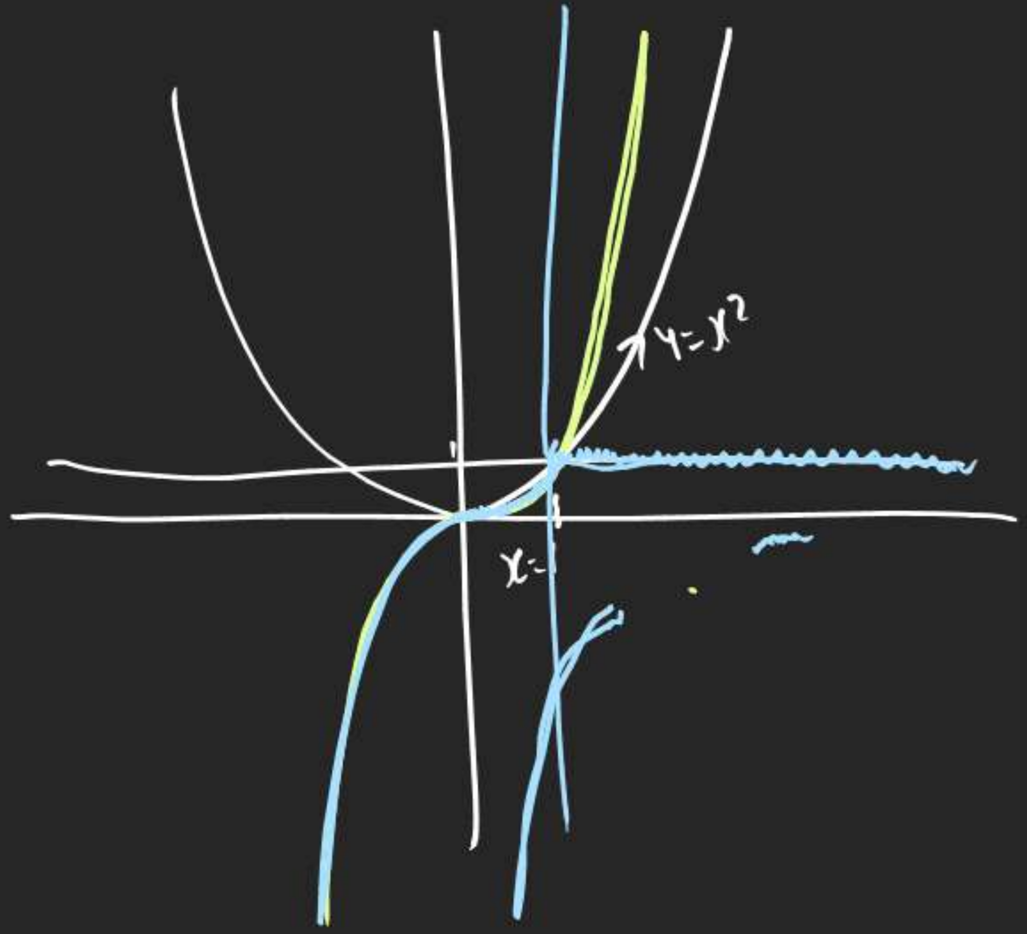
Nishant Jindal

Q $f(x) = \text{Min}(1, x^2, x^3)$

A) Cont^s ✓ B) $f'(x) > 0$ $x > 1$ ×

C) ND but Cont^s ✓

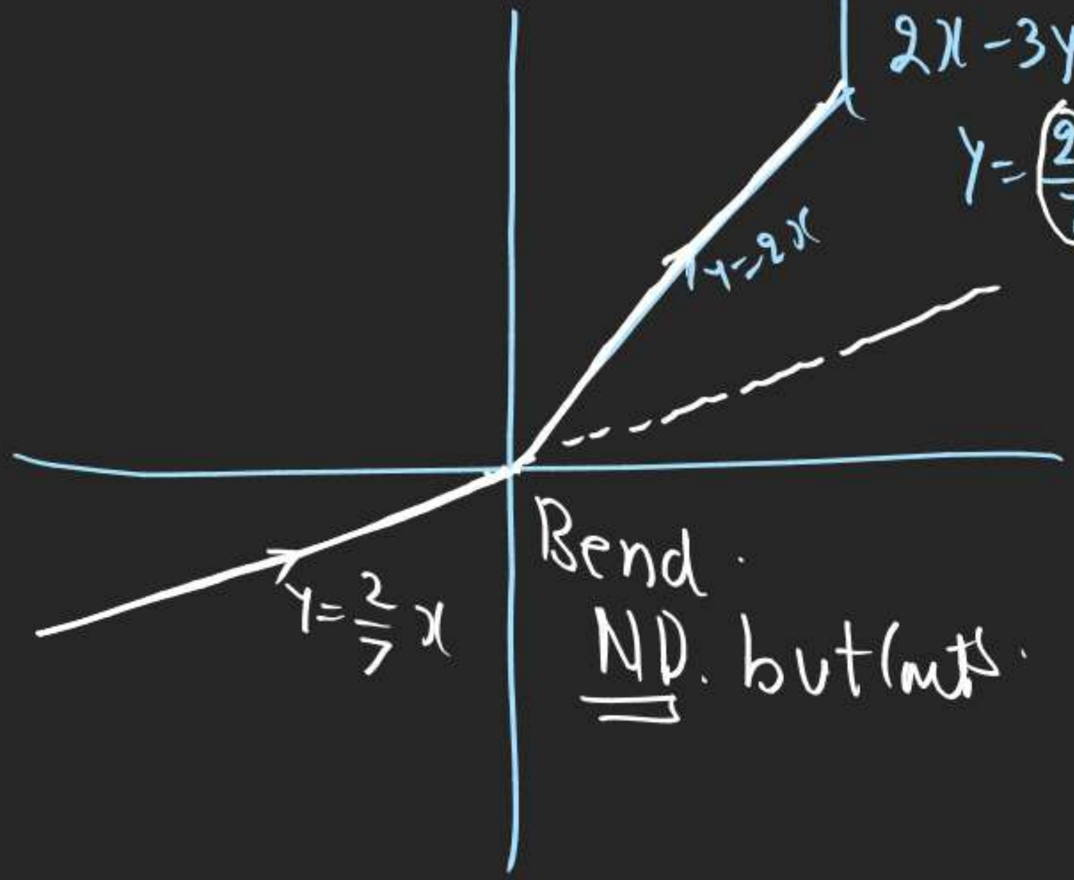
D) ND for 2 Values ×



Q $2x + 3|y| = 4y$ y is a fxn of x

(1) D.C. at 1 pt (2) ND at 1 pt 3) D.C & ND at same pt
(4) Cont^s & diff

$$2x + 3|y| = 4y \Rightarrow \begin{cases} 2x + 3y = 4y \Rightarrow \boxed{y = 2x} & y \geq 0 \\ 2x - 3y = 4y & y < 0 \\ y = \frac{2}{7}x \end{cases}$$



Bend.

ND. but Cont^s.

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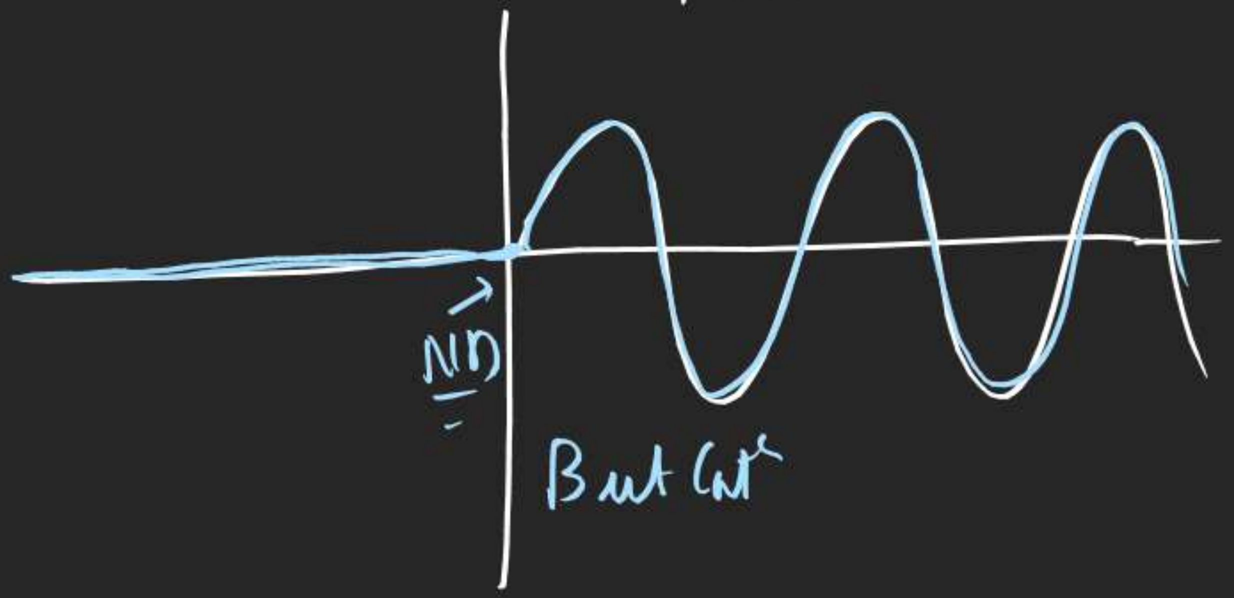
① $Y = \sin x + \sin|x|$ Draw graph.

& discuss Cont^y & diff^y.

$$Y = \sin x + \sin|x| = f(x) \begin{cases} Y = \sin x + \sin x & x \geq 0 \\ Y = 2\sin x & \\ Y = \cancel{\sin x} + \cancel{\sin(-x)} & x < 0 \\ Y = 0 & \end{cases}$$



$$f(x) = \begin{cases} 2\sin x & x \geq 0 \\ 0 & x < 0 \end{cases}$$

$x = 0$ 4R Cont^y

R L

$2\sin 0 = 0$

$0 = 0$ ✓

$$f'(x) = \begin{cases} 2\cos x & x \geq 0 \\ 0 & x < 0 \end{cases}$$

$x = 0$

RHD $= 2\cos(0)$

$= 2$

LHD $= 0$

ND at $x = 0$

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Q $f(x) = \begin{cases} 1 & -\infty < x < 0 \\ 1+|\sin x| & \boxed{0 \le x < \frac{\pi}{2}} \\ 2+(x-\frac{\pi}{2})^2 & \frac{\pi}{2} \le x < \infty \end{cases}$

$\rightarrow$ Quad^n = 1st

$\sin x = +ve$

$|\sin x| = \sin x$

(cnt^y & diff^y)

Actual $f(x) = \begin{cases} 1 & -\infty < x \le 0 \\ 1+\sin x & 0 \le x < \frac{\pi}{2} \\ 2+(x-\frac{\pi}{2})^2 & \frac{\pi}{2} \le x < \infty \end{cases}$

Cnt^y

$x = 0$ ✓

$1 = 1+\sin 0$

$1 = 1+0$

$\underline{1 = 1}$

$x = \frac{\pi}{2}$ ✓

$1+\sin\frac{\pi}{2} = 2+(\cancel{\frac{\pi}{2}}-\cancel{\frac{\pi}{2}})^2$

$1+1 = 2$

$2 = 2$

$f'(x) = \begin{cases} 0 & -\infty < x < 0 \\ \cos x & 0 \le x < \frac{\pi}{2} \\ 2(x-\frac{\pi}{2}) \times 1 & \frac{\pi}{2} \le x < \infty \end{cases}$

diff^y

$x = 0$

LHD $= 0$

RHD $= \cos(0) = 1$

$0 \ne 1$

ND at $x = 0$

$x = \frac{\pi}{2}$

LHD $= \cos\left(\frac{\pi}{2}\right) = 0$

RHD $= 2\left(\frac{\pi}{2}-\frac{\pi}{2}\right) = 0$

Diff^{ble} at $x = \frac{\pi}{2}$

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Q JIT Adv 2011

$$f(x)=\begin{cases} -x-\frac{\pi}{2} & x\le -\frac{\pi}{2} \\ \boxed{-\cos x} & -\frac{\pi}{2}<x\le 0 \\ x-1 & 0<x\le 1 \\ \ln x & x>1 \end{cases}$$

fxn is cont^s $x=-\frac{\pi}{2}$ ✓

✓ ND $x=0$

Diff $x=1$

✓ diff $x=-\frac{3}{2}=-1.5$

Cont^y

$x=-\frac{\pi}{2}$	$x=0$	$x=1$
$-\cancel{\left(-\frac{\pi}{2}\right)}-\cancel{\frac{\pi}{2}}=-\cos\left(-\frac{\pi}{2}\right)$	$-\cos(0)=(0)-1$	$(1)-1=\ln(1)$
$0=0$ ✓	$-1=-1$ ✓	$0=0$ ✓

diff^y

$$f'(x)=\begin{cases} -1 & x\le -\frac{\pi}{2} \\ \boxed{+\sin x} & \boxed{-\frac{\pi}{2}<x\le 0} \rightarrow \\ 1 & 0<x\le 1 \\ \frac{1}{x} & x>1 \end{cases}$$

$x=-\frac{\pi}{2}$	$x=0$	$x=1$
LHD $=-1$	LHD $=\sin(0)=0$	LHD $=1$
RHD $=\sin\left(-\frac{\pi}{2}\right)=-1$	RHD $=1=1$	RHD $=\frac{1}{(1)^2}=1$
Diff ✓	ND ✗	Diff ✓

Nishant Jindal

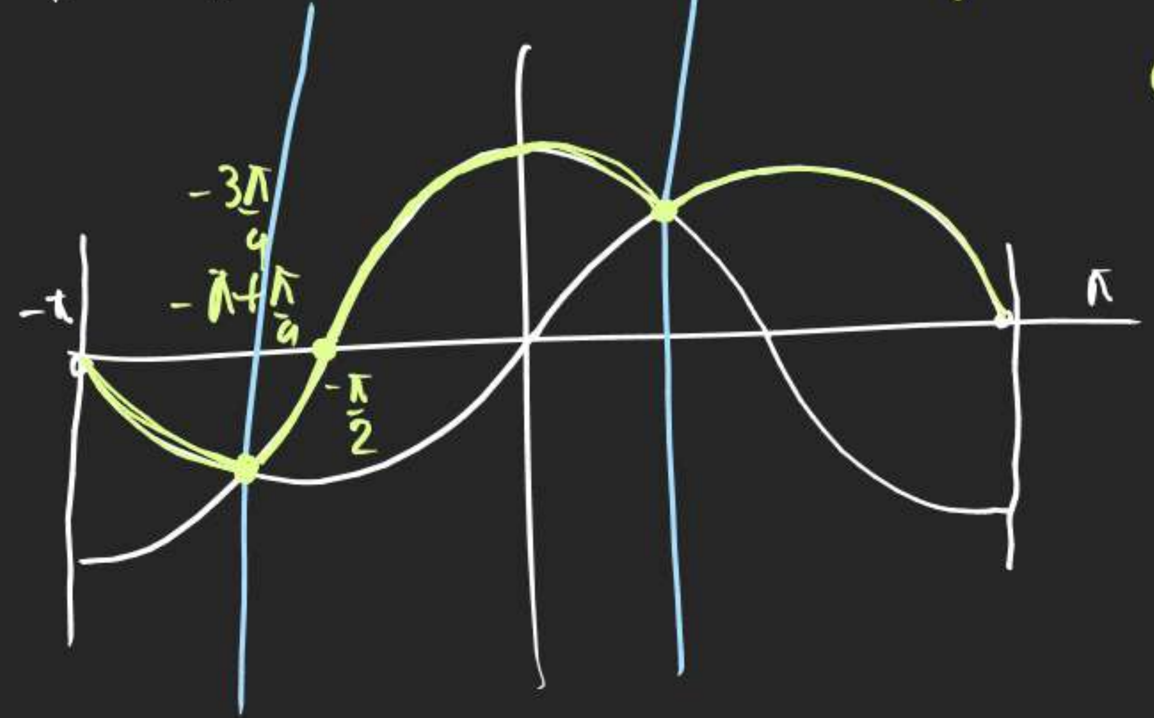
$f(f(1)) = f(|2-|1-3||) = f(0) = |2-|0-3|| = 1$ | $f(|2-|3-3||) = f(2) = |2-|2-3|| = 1$

Q let S be set of all Pts. in $(-\pi, \pi)$ at which fxn $f(x) = \text{Max}\{\sin x, \cos x\}$ is N.D. Thus S is Subset of which of the following

Mains

$\left\{-\frac{\pi}{2}, -\frac{\pi}{4}, \frac{\pi}{4}, \frac{\pi}{2}\right\}$ × $\left\{-\frac{\pi}{4}, 0, \frac{\pi}{4}\right\}$ ×

$\left\{-\frac{3\pi}{4}, -\frac{\pi}{4}, \frac{3\pi}{4}, \frac{\pi}{4}\right\}$ $\left\{-\frac{3\pi}{4}, -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{4}\right\}$ ×



$\sin x = \cos x$

$\tan x = 1$

$x = \frac{\pi}{4}, -\frac{3\pi}{4}$

Q $f(x) = ||2-|x-3||$ is N.D. in $x \in S$ then $\sum_{x \in S} f(f(x)) = ?$

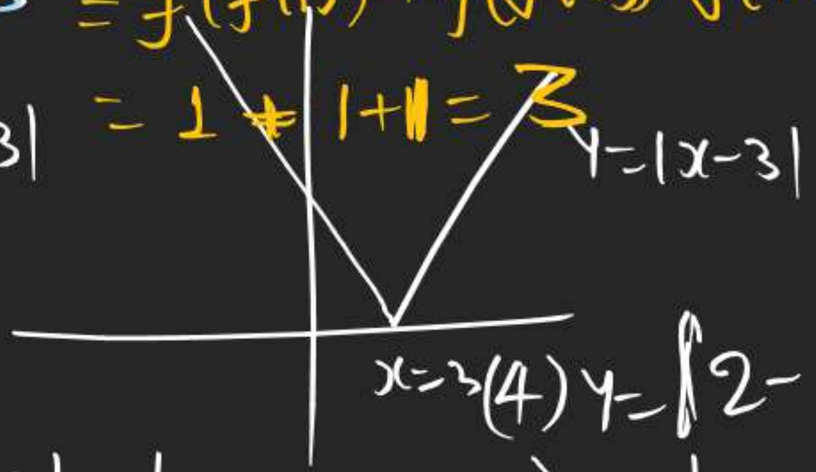
Mains 2021

$S = x \in \{1, 3, 5\}$

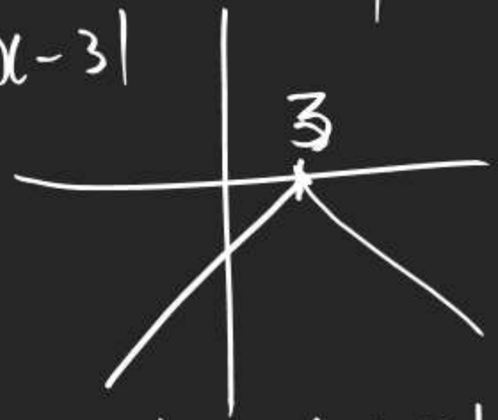
$= f(f(1)) + f(f(3)) + f(f(5))$

$= 1 + 1 + 1 = 3$

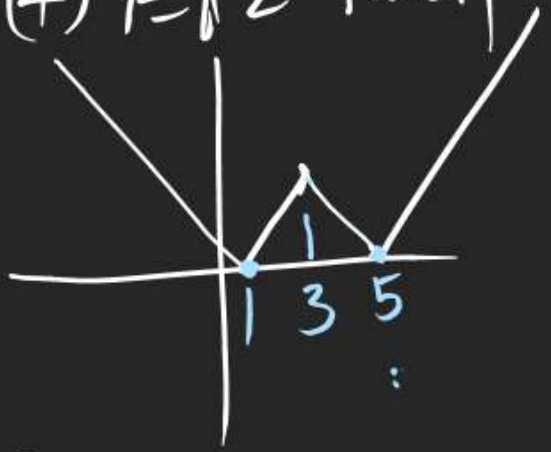
(1) $y = |x-3|$



(2) $y = -|x-3|$



(4) $y = ||2-|x-3||$



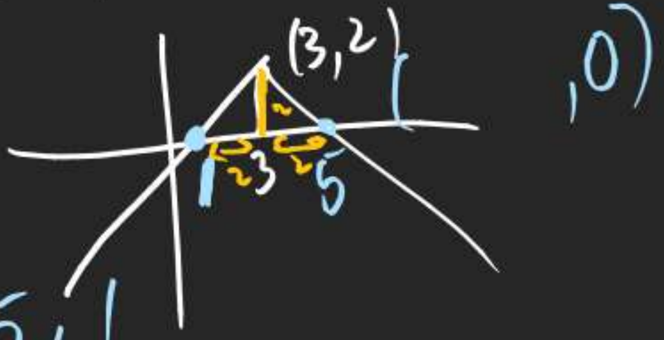
(3) $y = 2-|x-3| = -|x-3|+2$

$0 = 2-|x-3|$

$|x-3| = 2$

$x-3 = 2, -2$

$x = 5, 1$



Nishant Jindal

Q Let $f(x) = 15 - |x-10|;\ x \in R$ Then set of values of x at which fxn $f(f(x))$ is ND at?

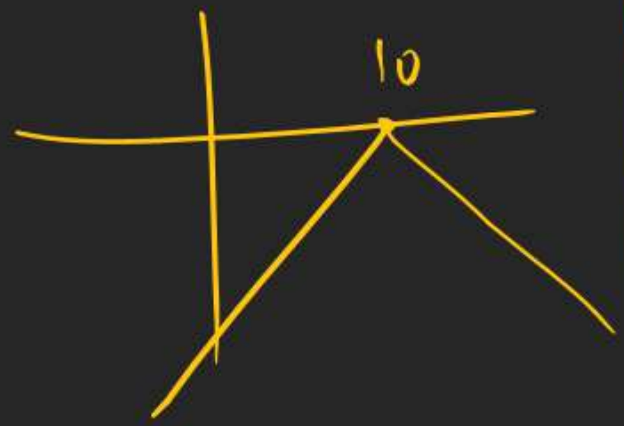
Mains

$$f(f(x)) = 15 - |f(x) - 10|$$

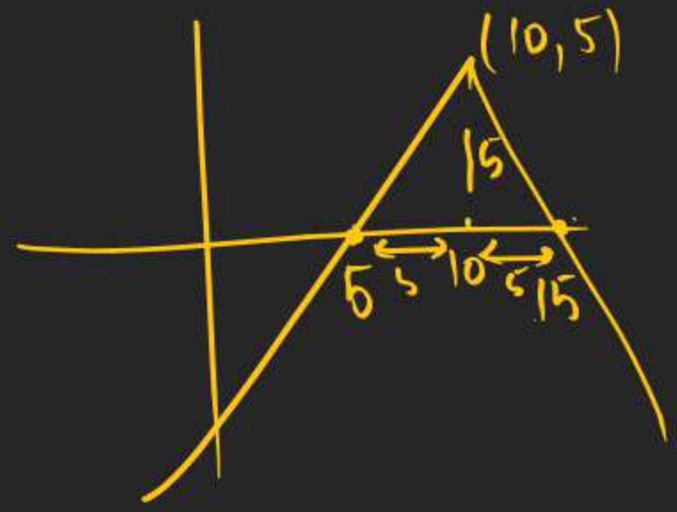
$$= 15 - |15 - |x-10| - 10|$$

$$f(f(x)) = 15 - |5 - |x-10||$$

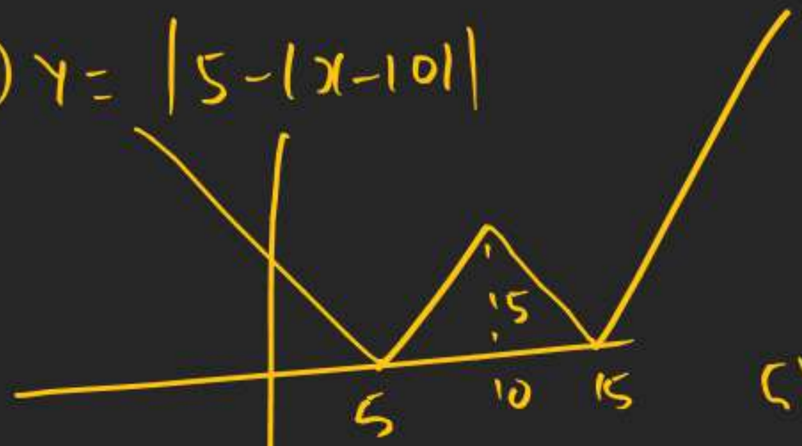
① $y = -|x-10|$



② $y = 5 - |x-10|$

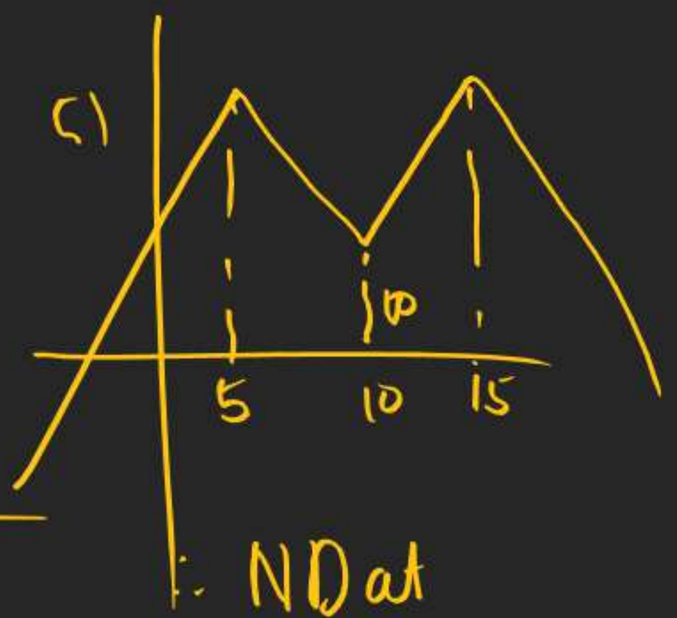


(3) $y = |5 - |x-10||$



(4) $y = -|5 - |x-10||$

5) $y = 15 - |5 - |x-10||$



$\therefore$ ND at $x = 5, 10, 15$

Nishant Jindal

Q let $f:(0,1)\to R$ $f(x)=[4x]$

then Pt. of N.D ?

this fxn is ND Where it is D.C

$x\in(0,1)$

$4x\in(0,4)$

$4x\in \cancel{0},1,2,3,\cancel{4}$

$\therefore$ ND. at $x=1,2,3$

$x=\frac{1}{4},\frac{2}{4},\frac{3}{4}$

$=\frac{1}{4},\frac{1}{2},\frac{3}{4}$

Q $f:(0,1)\to R$ $f(x)=[4x]\left(x-\frac{1}{4}\right)^2\left(x-\frac{1}{2}\right)$ then WOTF is true

2023 Adv

1) fxn f is D.C Exactly 1 pt in (0,1)

2) There is exactly one pt in (0,1) at which fxn is Conts But N.D.

3) fxn f is ND at more than 3 pt in (0,1)

Then is D.C at only 1 Pt. $x=3/4$

Critical Pt $\to x=\frac{1}{4},\frac{2}{4},\frac{3}{4}$

$f\left(\frac{1}{4}^+\right)=\underbrace{\left[4\left(\frac{1}{4}+h\right)\right]}_{[1+4h]}\left(\cancel{\frac{1}{4}}+h-\cancel{\frac{1}{4}}\right)^2\left(\frac{1}{4}+h-\frac{1}{2}\right)=0$

$f\left(\frac{1}{4}^-\right)=\underbrace{\left[4\left(\frac{1}{4}-h\right)\right]}_{[1-4h]}\left(\frac{1}{4}-h-\frac{1}{4}\right)^2\left(\frac{1}{4}-h-\frac{1}{2}\right)=0$

$f\left(\frac{1}{2}^+\right)=\underbrace{\left[4\left(\frac{1}{2}+h\right)\right]}_{[2+4h]}\left(\frac{1}{2}+h-\frac{1}{4}\right)^2\left(\cancel{\frac{1}{2}}+h-\cancel{\frac{1}{2}}\right)=0$

$f\left(\frac{1}{2}^-\right)=\underbrace{\left[4\left(\frac{1}{2}-h\right)\right]}_{[2-4h]}\left(\frac{1}{2}-h-\frac{1}{4}\right)^2\left(\cancel{\frac{1}{2}}-h-\cancel{\frac{1}{2}}\right)=0$

Nishant Jindal

Q let $f_1: R\to R$, $f_2:[0,\infty)\to R$, $f_3: R\to R$, $f_4: R\to[0,\infty)$

$$f_1(x)=\begin{cases}|x| & x<0\\ e^x & x\geq 0\end{cases} \qquad f_2(x)=x^2$$

$$f_3(x)=\begin{cases}\sin x & x<0\\ x & x\geq 0\end{cases}$$

$$f'=\begin{cases}\cos x\\ 1\end{cases}$$

$\cos 0=1$
$1=1$

$$f_4(x)=\begin{cases}f_2(f_1(x)) & x<0\\ f_2(f_1(x))-1 & x\geq 0\end{cases} \Rightarrow f_4(x)=\begin{cases}(|x|)^2 & x<0\\ (e^x)^2-1 & x\geq 0\end{cases}$$

P f_4 → 1) Onto but not 1-2-1

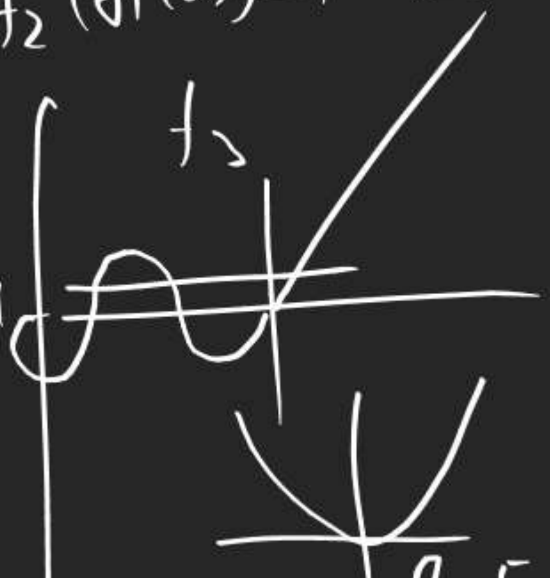
Q f_3 → 3)

R $f_2(f_1(x))$

S f_2 → 3)

1) Onto but not 1-2-1
× 2) Niether Cont Nor 1-2-1
3) Diff but not 1-2-1
4) Cont & 1-2-1

f_3

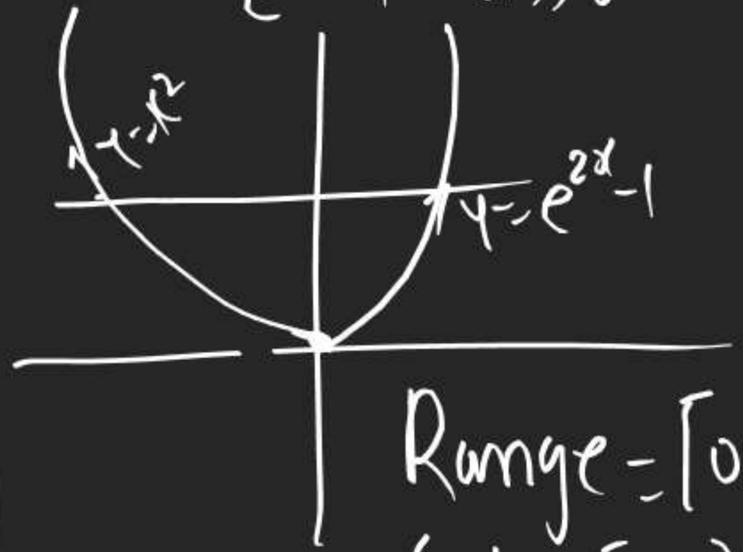


Cont, diff, M21

$R_f=[0,\infty)$
Cod $=R$
Into

$$f_4=\begin{cases}x^2 & x<0\\ e^{2x}-1 & x\geq 0\end{cases} \Rightarrow f_4'=\begin{cases}2x\\ 2e^{2x}\end{cases}$$

$2\times 0=0$
$2e^0=2$



Range $=[0,\infty)$ } M21, onto
Cod $=[0,\infty)$ } ND, Cont

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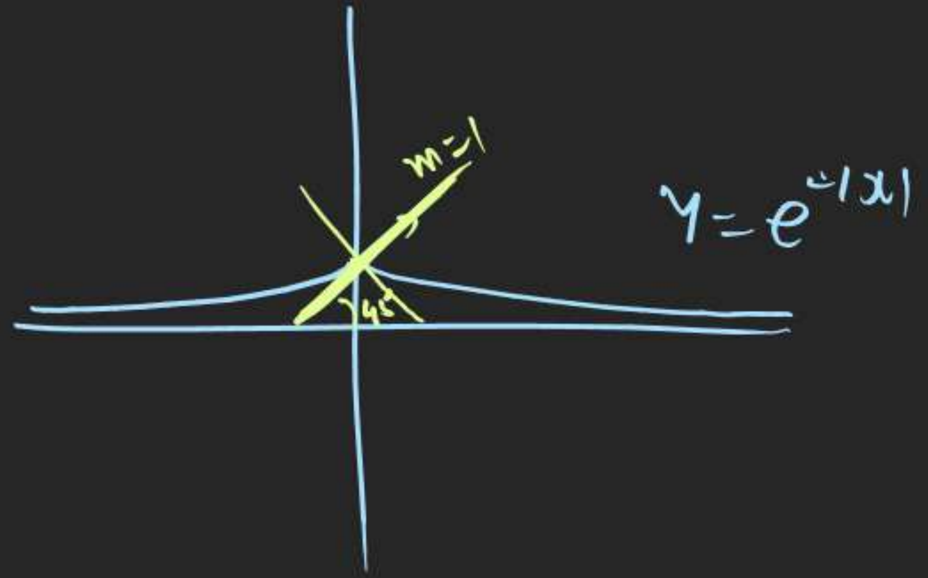
Q If $\lim_{x\to 0} \frac{1-\cos(1-\cos\frac{x}{2})}{2^{\lambda}x^{\mu}}$ is equal to LHD of

$y = e^{-|x|}$ at $x=0$ then $|\lambda+\mu| = ?$



$$f(x) = e^{-|x|} = \begin{cases} e^{-x} & x \geq 0 \\ e^{x} & x < 0 \end{cases}$$

$$f'(x) = \begin{cases} -e^{-x} & x \geq 0 \\ e^{x} & x < 0 \end{cases}$$

$x=0$

$LHD = e^{(0)} = 1$

acc. to Q,

$$\lim_{x\to 0} \frac{1-\cos(1-\cos\frac{x}{2})}{2^{\lambda}\cdot x^{\mu}} = 1$$

$$\lim_{x\to 0} \frac{1-\cos(1-\cos\frac{x}{2})}{(1-\cos\frac{x}{2})^2} \times \left(\frac{1-\cos\frac{x}{2}}{(\frac{x}{2})^2}\right)^2 \times \frac{(\frac{x}{2})^4}{2^{\lambda}\cdot x^{\mu}} = 1$$

$$\lim_{x\to 0} \frac{1}{2} \times \frac{1}{2^2} \times \frac{x^4}{2^4\cdot 2^{\lambda}\cdot x^{\mu}} = 1$$

$x^{(4-\mu)} \to 0 \qquad 4-\mu = 0 \Rightarrow \mu = 4$

$$\lim_{x\to 0} \frac{x^{(4-\mu)}}{2^{(7+\lambda)}} = 1$$

$2^{(7+\lambda)} \to 0$

$7+\lambda = 0$

$\lambda = -7$

Nishant Jindal

Unknown Values

Q $f(x) = \begin{cases} A + Bx^2 & x < 1 \\ 3Ax - B + 2 & x \geq 1 \end{cases}$

is $\boxed{\text{diff}^{ble}}$ at $x = 1$ find $(A, B) = ?$

Cont^s ऐ DI

$A + B(1)^2 = 3A(1) - B + 2$

$2A - 2B = -2$

$A - B = -1$

$A - \frac{3A}{2} = -1$

$-\frac{A}{2} = -1 \Rightarrow \boxed{A = 2}, \boxed{B = 3}$

$f'(x) = \begin{cases} 2Bx & x < 1 \\ 3A & x \geq 1 \end{cases}$

LHD = RHD at $x = 1$

$2B(1) = 3A$

$B = \frac{3A}{2}$

Ex 2

HW.

Main PQs

$f(x) = \begin{cases} a\sqrt{x+2} & 0 < x < 2 \\ bx + 2 & 2 \leq x < 5 \end{cases}$

$f(x)$ diff in $(0,5)$

$2a + b = ?$

Cont^s है

$a\sqrt{2+2} = 2b + 2$

$2a - 2b = 2$

$a - b = 1$

$f'(x) = \begin{cases} \frac{a}{2\sqrt{x+2}} & 0 < x < 2 \\ b & 2 \leq x < 5 \end{cases}$

$\frac{a}{2\sqrt{2+2}} = b$

$a = 4b$

$4b - b = 1$

$b = \frac{1}{3}$

$a = \frac{4}{3}$

$2a + b$

$\frac{8}{3} + \frac{1}{3}$

$= 3$