Nishant Jindal

1) V S Method.

2) Reducible to V S. $f(x+y)$ $\hookrightarrow t$

3) Polar Sub $\rightarrow x\,dy + y\cdot dx$, $x\,dy - y\,dx$

4) Exact diff^l.

5) $\frac{dy}{dx} = \frac{a_1 x + b_1 y + c_1}{a_2 x + b_2 y + c_2}$

(6) H.D.E.

Homogeneous D.E.

$$\boxed{\frac{dy}{dx} = \frac{f_1(x,y)}{f_2(x,y)} \Big/ \frac{dy}{dx} = f\left(\frac{y}{x}\right)}$$

When putting $y = vx$

No term of x left then

DE is HDE

Q $\frac{dy}{dx} = \frac{y-x}{y+x}$

Check

$y = vx$

$\frac{dy}{dx} = \frac{vx - x}{vx + x}$

$= \frac{v-1}{v+1}$

Method to solve.

Sol.

Q $\frac{dy}{dx} = \frac{y-x}{y+x}$?

let

① $y = vx$ v = var. x = var.

② $\frac{dy}{dx} = v + x\cdot\frac{dv}{dx}$

$$v + x\cdot\frac{dv}{dx} = \frac{vx - x}{vx + x} = \frac{v-1}{v+1}$$

$$x\cdot\frac{dv}{dx} = \frac{v-1}{v+1} - v = \frac{\cancel{v} - 1 - v^2 - \cancel{v}}{v+1}$$

$$\int \frac{1+v}{1+v^2}\,dv = -\int \frac{dx}{x}$$

$$\Rightarrow \int \frac{dv}{1+v^2} + \frac{1}{2}\int \frac{2v\,dv}{1+v^2} = -\int \frac{dx}{x}$$

$$\tan^{-1} v + \frac{1}{2}\ln(1+v^2) = -\ln x + C$$

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Solve

Q $\left(x\frac{dy}{dx}-y\right)\left(\tan^{-1}\frac{y}{x}\right)=x$ $\quad y|_{x=1}=0$

$\left(\frac{dy}{dx}-\frac{y}{x}\right)\left(\tan^{-1}\frac{y}{x}\right)=1$

① $y=vx$ (let) $\quad \frac{y}{x}=v$

② $\frac{dy}{dx}=v+x\cdot\frac{dv}{dx}$

③ $\left(\cancel{v}+x\frac{dv}{dx}-\cancel{v}\right)\cdot\tan^{-1}(v)=1$

$\int\tan^{-1}v\cdot dv=\int\frac{dx}{x}$

$\tan^{-1}v(v)-\int\frac{v}{1+v^2}dv=\ln x+c$

$v\tan^{-1}v-\frac{1}{2}\ln(1+v^2)=\ln x+c$

$\frac{y}{x}\tan^{-1}\left(\frac{y}{x}\right)=\ln\frac{x}{\sqrt{1+\left(\frac{y}{x}\right)^2}}+c$

$x=1$, $y=0$: $\quad 0=\ln 1+c\Rightarrow c=0$

$\frac{y}{x}\cdot\tan^{-1}\left(\frac{y}{x}\right)=\ln\frac{x}{\sqrt{1+\left(\frac{y}{x}\right)^2}}$ A.

Q A Curve P.T. $\left(1,\frac{\pi}{6}\right)$ & let Slope of Curve at each pt (x,y) be $\frac{y}{x}+\sec\left(\frac{y}{x}\right)$; $x>0$ then Eqn of Curves?

Adv 2013

$\frac{dy}{dx}=\frac{y}{x}+\sec\left(\frac{y}{x}\right)$

$\cancel{v}+x\frac{dv}{dx}=\cancel{v}+\sec v\Rightarrow\int\cos v\cdot dv=\boxed{\int\frac{dx}{x}}$

$\sin v=\ln x+c\Rightarrow\sin\left(\frac{y}{x}\right)=\ln x+c$

P.T. $\left(1,\frac{\pi}{6}\right)$

$\sin\left(\frac{\pi}{6}\right)=c\Rightarrow c=\frac{1}{2}$

$\sin\left(\frac{y}{x}\right)=\ln x+\frac{1}{2}$

Solve.

Q $\left[x\cos\frac{y}{x}+y\sin\frac{y}{x}\right]y=\left[y\sin\frac{y}{x}-x\cos\frac{y}{x}\right]x\frac{dy}{dx}$

$\frac{dy}{dx}=\frac{\left(x\cos\frac{y}{x}+y\sin\frac{y}{x}\right)y}{\left(y\sin\frac{y}{x}-x\cos\frac{y}{x}\right)\cdot x}=\frac{\left(\cos\frac{y}{x}+\frac{y}{x}\sin\frac{y}{x}\right)\frac{y}{1}}{\left(\frac{y}{x}\sin\frac{y}{x}-\cos\frac{y}{x}\right)x}$

$v+x\frac{dv}{dx}=\frac{v(\cos v+v\sin v)}{(v\sin v-\cos v)}$

$\Rightarrow x\frac{dv}{dx}=\frac{v\cos v+\cancel{v^2\sin v}-\cancel{v^2\sin v}+v\cos v}{v\sin v-\cos v}\Rightarrow\frac{1}{2}\int\frac{v\sin v-\cos v}{v\cos v}dv=\int\frac{dx}{x}$

$\Rightarrow-\frac{1}{2}\ln(v\cos v)=\ln(x$

$\frac{1}{\sqrt{v\cos v}}=Cx$

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Q A Curve P.T. (1,1) has a Property that the $\perp^r$ distance of origin from Normal at any pt. P of the curve is equal to distance of P from X Axis. Det. Eq^n of curve.

EON at Pt. (x,y)

$$Y-y=-\frac{dx}{dy}(X-x)$$

Origin se Normal Ki dist. = Pt. $\overset{(x,y)}{P}$ का X Axis से dist

$$\frac{\left|y+x\cdot\frac{dy}{dx}\right|}{\sqrt{1+\left(\frac{dy}{dx}\right)^2}}=|y|$$

$$\cancel{y^2}+x^2\left(\frac{dy}{dx}\right)^2+2xy\cdot\frac{dy}{dx}=\cancel{y^2}+y^2\left(\frac{dy}{dx}\right)^2$$

$$\left(\frac{dy}{dx}\right)^2(x^2-y^2)=-2xy\frac{dy}{dx}$$

$$\frac{dy}{dx}\left\{\left(\frac{dy}{dx}\right)(x^2-y^2)+2xy\right\}=0$$

$$\frac{dy}{dx}=0$$

$$y=\text{Const.}$$

$$\frac{dy}{dx}=\frac{2xy}{y^2-x^2}$$

HDE

S.Y

$$2\sqrt{x^2+y^2}=\pm 2x+C$$

Q $y\cdot\left(\frac{dy}{dx}\right)^2+2x\frac{dy}{dx}-y=0$ & $y\Big|_{x=0}=\sqrt{5}$ Solve.

$$\frac{dy}{dx}=\frac{-x\pm\sqrt{x^2+y^2}}{y}$$

M1

$$\frac{dy}{dx}=\frac{-1\pm\sqrt{1+(y/x)^2}}{\frac{y}{x}}$$

$$v+x\cdot\frac{dv}{dx}=\frac{-1\pm\sqrt{1+v^2}}{v}\quad\text{(HDE)}$$

M2

$$y\,dy=-x\,dx\pm\sqrt{x^2+y^2}\cdot dx$$

Exact

$$\frac{2x\,dx+2y\,dy}{\sqrt{x^2+y^2}}=\pm 2dx$$

$$\int\frac{d(x^2+y^2)}{\sqrt{x^2+y^2}}=\pm\int 2dx$$

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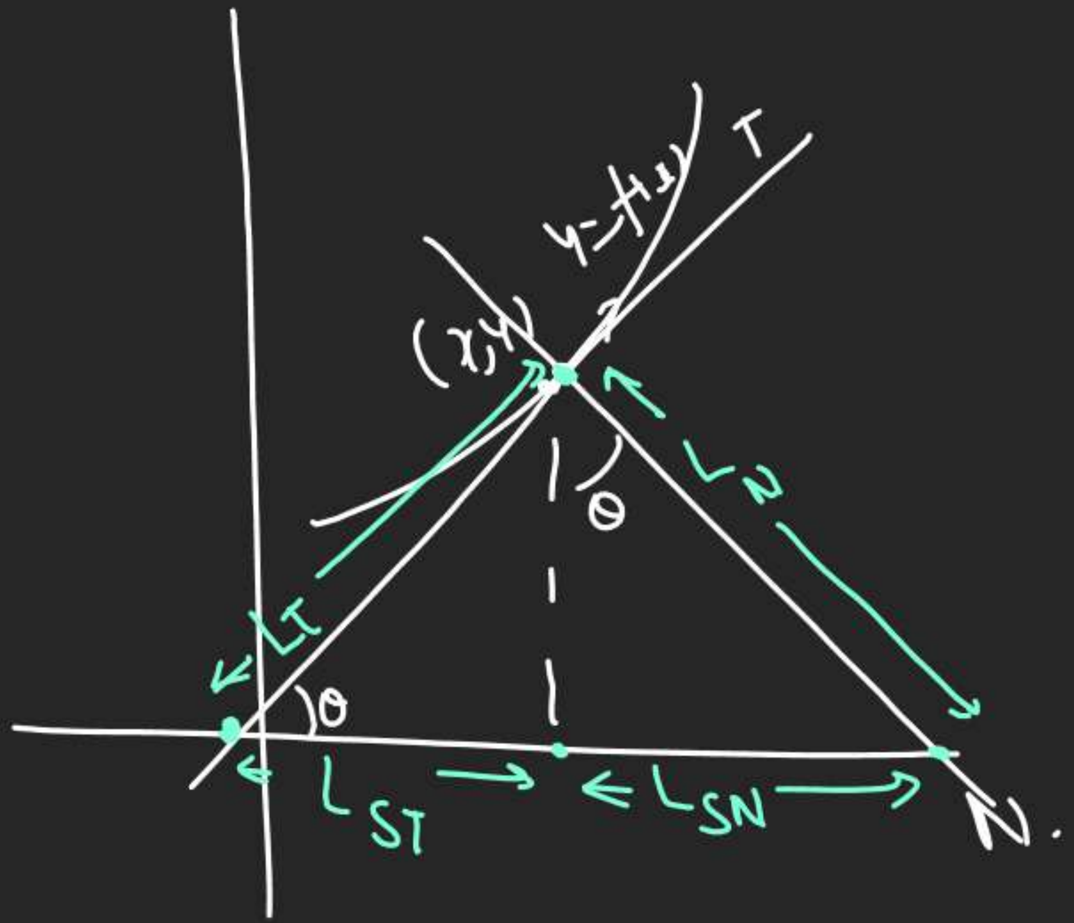
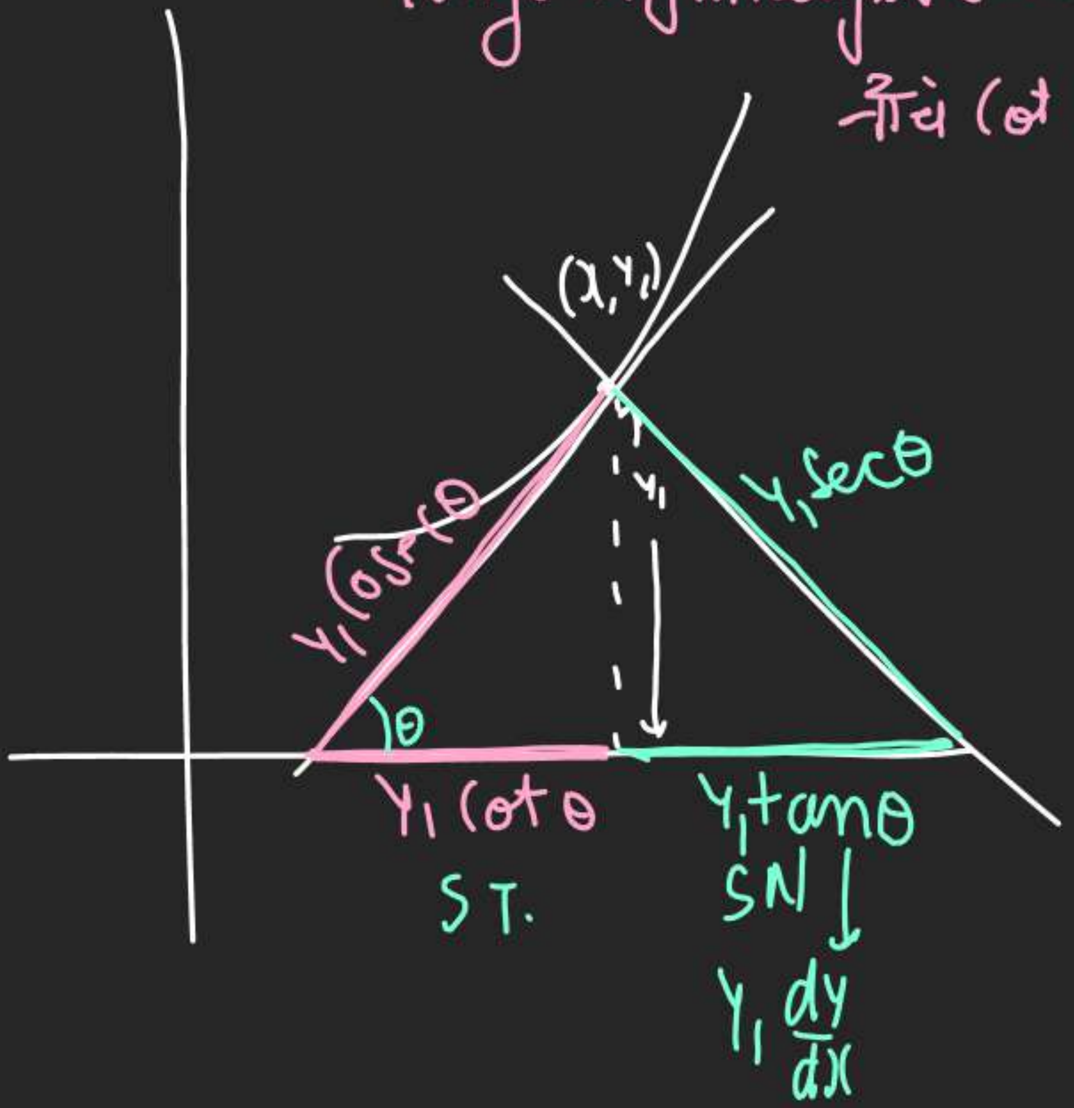
Subtangent / Sub Normal.



tangent family is Co Co family.

नीचे Cot ऊपर Cosec



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Q Find Eqⁿ of Curve intersecting X Axis at $x=1$ & satisfying the Prop. that length of SN at any Pt. of the Curve is equal to mean of coordinates of that Pt.

Pt. (x, y)

$$L_{SN} = y\tan\theta = \frac{x+y}{2}$$

$$y\frac{dy}{dx} = \frac{x+y}{2}$$

$$\frac{dy}{dx} = \frac{\left(1+\frac{y}{x}\right)}{2\left(\frac{y}{x}\right)}$$

$$v + x\cdot\frac{dv}{dx} = \frac{1+v}{2v}$$

$$x\frac{dv}{dx} = \frac{1+v}{2v} - v$$

$$x\frac{dv}{dx} = \frac{1+v-2v^2}{2v}$$

$$\frac{2v\,dv}{1+v-2v^2} = \frac{dx}{x}$$

$$-\frac{1}{2}\int \frac{-4v+1-1}{1+v-2v^2}\,dv = \int\frac{dx}{x}$$

$$-\frac{1}{2}\int\frac{-4v+1}{-2v^2+v+1}\,dv - \frac{1}{2}\int\frac{dv}{2v^2-v-1} = \ln x + c$$

$$= -\frac{1}{2}\ln|1+v-2v^2| - \frac{1}{2}\int\frac{dv}{(2v+1)(v-1)}$$

$$-\frac{1}{4}\times\frac{1}{3/2}\ln\left|\frac{v-1}{v+1/2}\right| + c$$

40%. LDE = Linear D.E

1) Any D.E whose deg of y & Diff. Coff $\left(\frac{dy}{dx}, \frac{d^2y}{dx^2}, \ldots\right)$ is 1, known as LDE

Ex: $\left(\frac{d^3y}{dx^3}\right)^2 + \left(\frac{dy}{dx}\right) + y = 0$ is LDE?

Not LDE

Ex: $\frac{dy}{dx} + \frac{y}{\left(\frac{dy}{dx}\right)} = 1$ is LDE?

$\left(\frac{dy}{dx}\right)$'s Deg $= 2$ (Not LDE)

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Q $\frac{d^2y}{dx^2}+\frac{dy}{dx}+xy^2=0$ is LDE?

Not LDE

Q $y\cdot\frac{dy}{dx}=1$ is LDE?

overall deg = 2

Not LDE

Q $x\left(\frac{d^2y}{dx^2}\right)^1+2\left(\frac{dy}{dx}\right)^1+y^1=0$ is LDE?

Yes it is LDE

* Our syllabus has opportunity 1st order, 1st deg LDE

LDE

LDE

$\frac{dy}{dx}+Py=Q$

P, Q are fxn of x

① $IF=e^{\int P\,dx}$

② $y\cdot(IF)=\int Q\cdot IF$

LBE

$\frac{dx}{dy}+Px=Q$

P, Q are fxn of y

① $IF=e^{\int P\,dy}$

② $x\cdot IF=\int Q\,IF$

BDE

$\frac{dy}{dx}+Py=Qy^n$

P, Q are fxn of x

① y की top deg से divide

② $\frac{dy}{dx}$ के पास वाले y को t मानते हैं

③ $\frac{dt}{dx}+Pt=Q$ Convert

④ Solve like LDE

* IF = Integrating factor

** $\frac{dy}{dx}+Py=Q$ is not Integratable directly as a fxn from RHS & LHS has been cancelled

*** finding that fxn is finding Integrating factor.

*4) I.F makes LHS = $(u\cdot v)'$ form.

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Solve. LDE $\Rightarrow \boxed{\frac{dy}{dx}} + Py = Q$

Q $(1+x^2)\frac{dy}{dx} + 2xy = 4x^2$

$\frac{dy}{dx}$ Akela

① $\frac{dy}{dx} + \boxed{\frac{2x}{1+x^2}}y = \boxed{\frac{4x^2}{1+x^2}}$ (P, Q)

② $IF = e^{\int P\cdot dx} = e^{\int \frac{2x}{1+x^2}dx} = e^{\ln(1+x^2)} = 1+x^2$

(3) $y\cdot(IF) = \int Q\, IF$

$\Rightarrow y\cdot(1+x^2) = \int \frac{4x^2}{\cancel{1+x^2}} \times \cancel{(1+x^2)}\,dx$

$y(1+x^2) = \frac{4x^3}{3} + C$ Ans

Solve.

Q $x\ln x\cdot\frac{dy}{dx} + y = 2\ln x$

① $\frac{dy}{dx} + \frac{y}{x\ln x} = \frac{2}{x}$

② $P = \frac{1}{x\ln x}$, $Q = \frac{2}{x}$

$\ln x = t$

$\frac{dx}{x} = dt$

③ $IF = e^{\int P\cdot dx} = e^{\int \frac{1}{x\ln x}dx}$

$= e^{\int \frac{dt}{t}} = e^{\ln(\ln x)} = \ln x$

④ $y\cdot(IF) = \int Q\, IF$

$\ln x = t$

$\Rightarrow y\cdot\ln x = \int \frac{2}{x}\ln x\, dx$

$= 2\int \frac{dt}{t} = 2\ln(\ln x) + C$

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Q $(x+2y^3)\dfrac{dy}{dx}=y$ Solve. $\boxed{\dfrac{x}{y}=y^2+C}$

① $\dfrac{dy}{dx}=\dfrac{y}{x+2y^3}$

$$\frac{dx}{dy}=\frac{x+2y^3}{y}$$

$$\frac{dx}{dy}=\frac{x}{y}+2y^2$$

$$\frac{dx}{dx}-\frac{x}{y}=2y^2$$

② $P=-\dfrac{1}{y}$ $\quad Q=2y^2$

③ $IF=e^{-\int\frac{1}{y}dy}=e^{-\ln y}=e^{\ln\frac{1}{y}}=\dfrac{1}{y}.$

④ $x\cdot IF=\int Q\,IF$

$\Rightarrow \dfrac{x}{y}=\int 2y^2\cdot\dfrac{1}{y}dy=y^2+C$

Def Int

Area, DE

Sheet done