Nishant Jindal

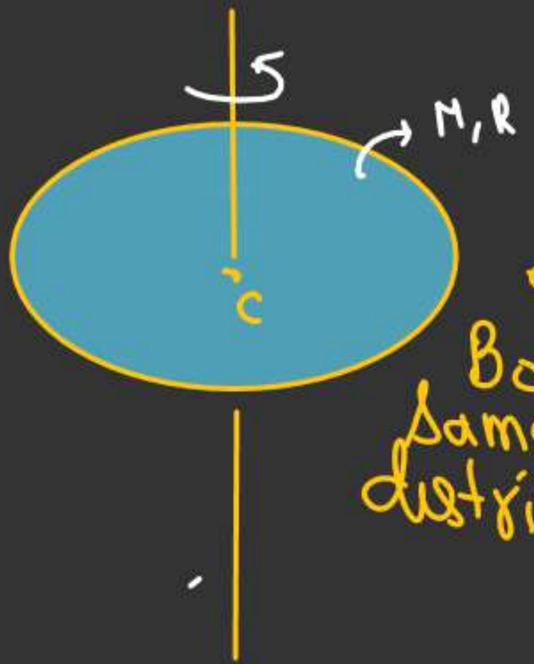
MOMENT OF INERTIA

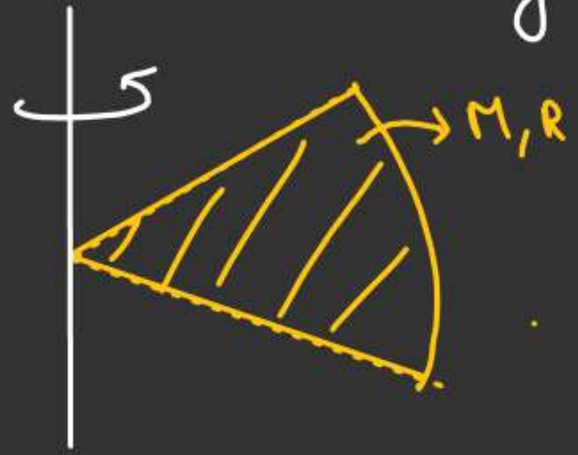
M·I → (I) → (Scalar quantity)

Defⁿ :- It is property of mass distribution of a body about any axis of rotation.

→ M·I of a body doesn't depend on the shape of a body

→ If two bodies of same mass and of different shape but their mass distribution about any fixed axis of rotation is identical then M·I of both the body same.





Both have same mass distribution about axis of Rotion so M·I Same.

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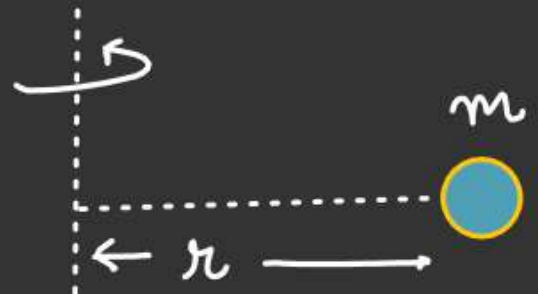
MOMENT OF INERTIA

★★ M.I of a point mass



$$\boxed{I = mr^2}$$

r = perpendicular distance of mass m about axis of rotation.

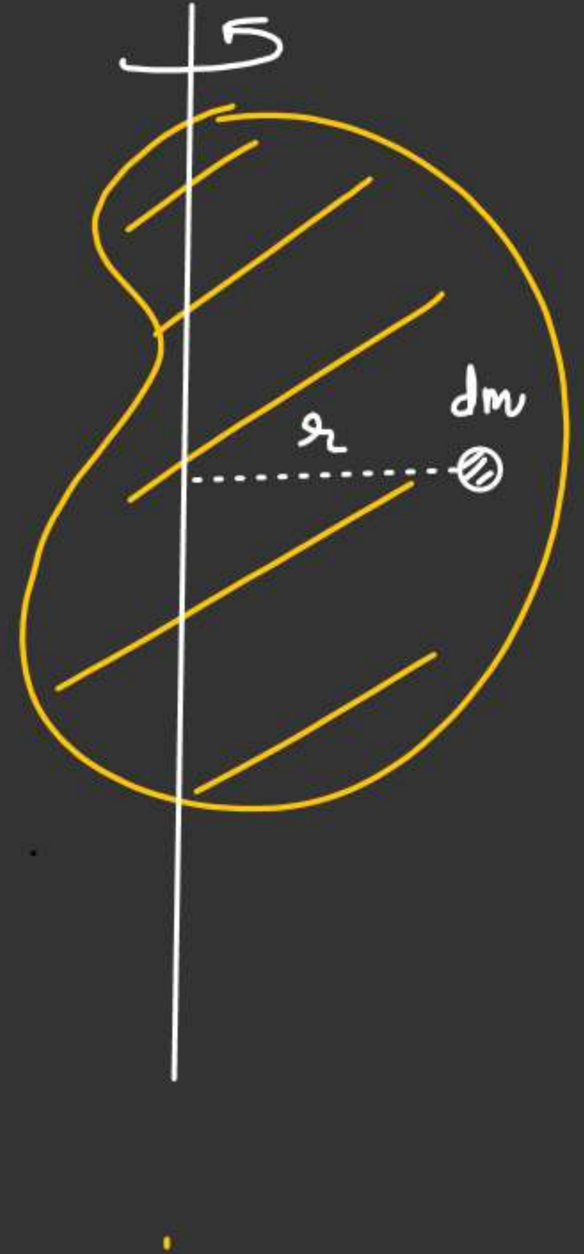
M.I of a Continious mass distribution

dI = due to dm mass

$$\int dI = \int dm\, r^2$$

$$\Downarrow$$

$$\boxed{(I_{body})_{\text{Axis of Ration}} = \int dm\, r^2}$$



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MOMENT OF INERTIA

** M.I of a Uniform Rod

Case-1 :- About axis perpendicular to the rod & passing through one of its end

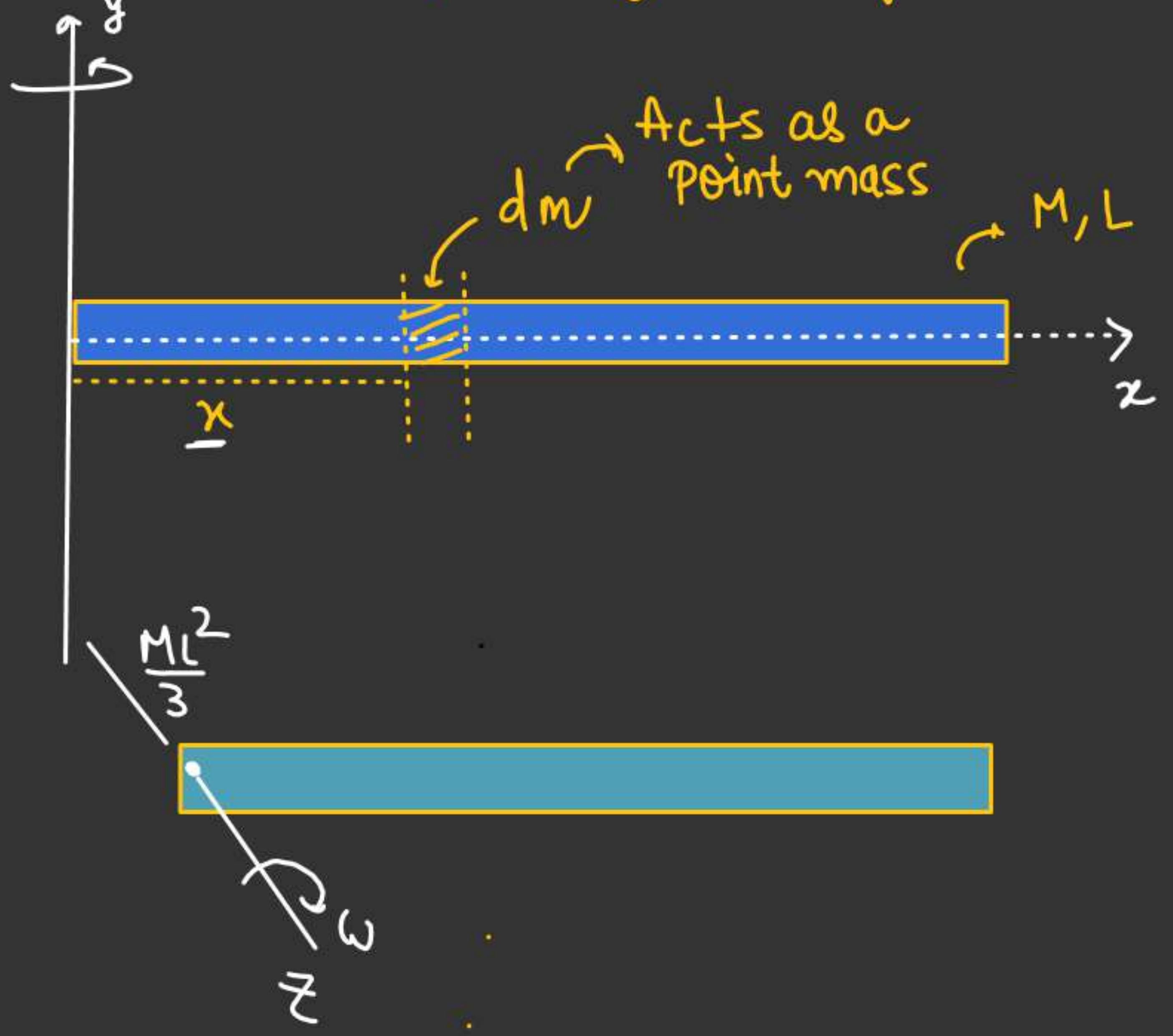
dI = M.I of dm mass about axis of rotation.

$$dm = \left(\frac{M}{L}dx\right)$$

$$\int_0^I dI = \int_0^L dm\, x^2$$

$$I = \frac{M}{L}\int_0^L x^2 dx = \left(\frac{ML^2}{3}\right)$$

$$\boxed{I = \frac{ML^2}{3}}$$



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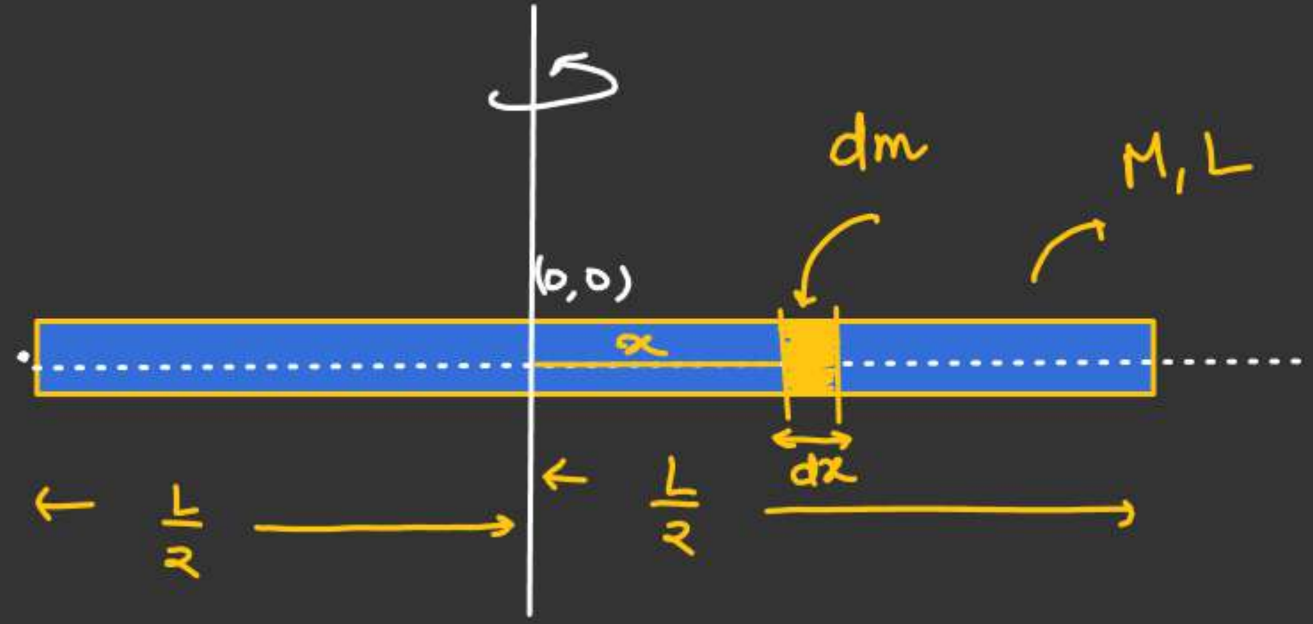
MOMENT OF INERTIA

** M.I of a Uniform Rod

Case-1 :- About axis perpendicular to the rod & passing through its Mid-point

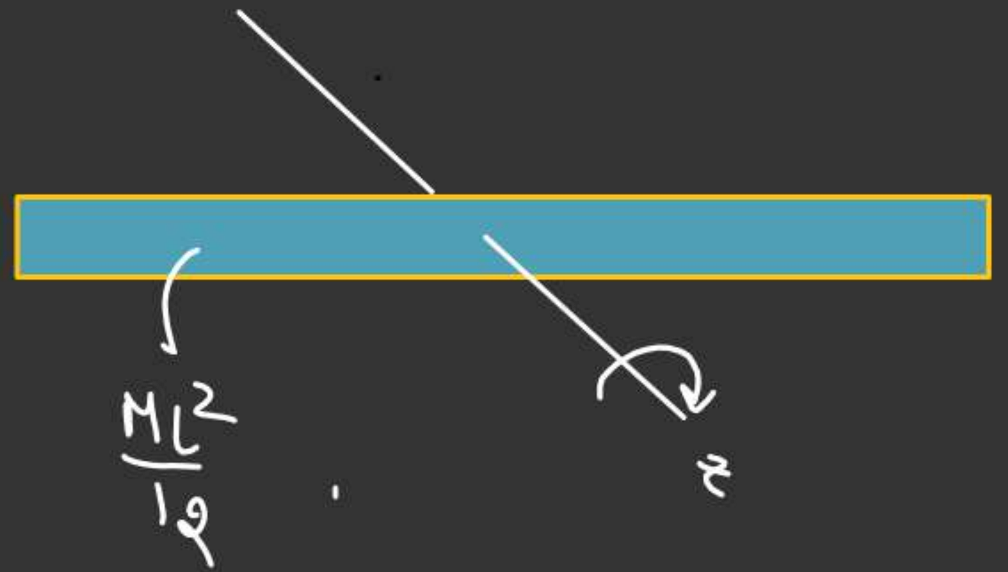
$$dI = dm\,x^2 \qquad dm = \frac{M}{L}dx$$



$$I = \int_{-l/2}^{+l/2} dm\,x^2$$

$$I = \frac{M}{L}\int_{-L/2}^{+L/2} x^2\,dx$$

$$I = \frac{M}{L}\left[\frac{L^3}{3}\right]_{-L/2}^{+L/2} = \boxed{I = \frac{ML^2}{12}}$$ **

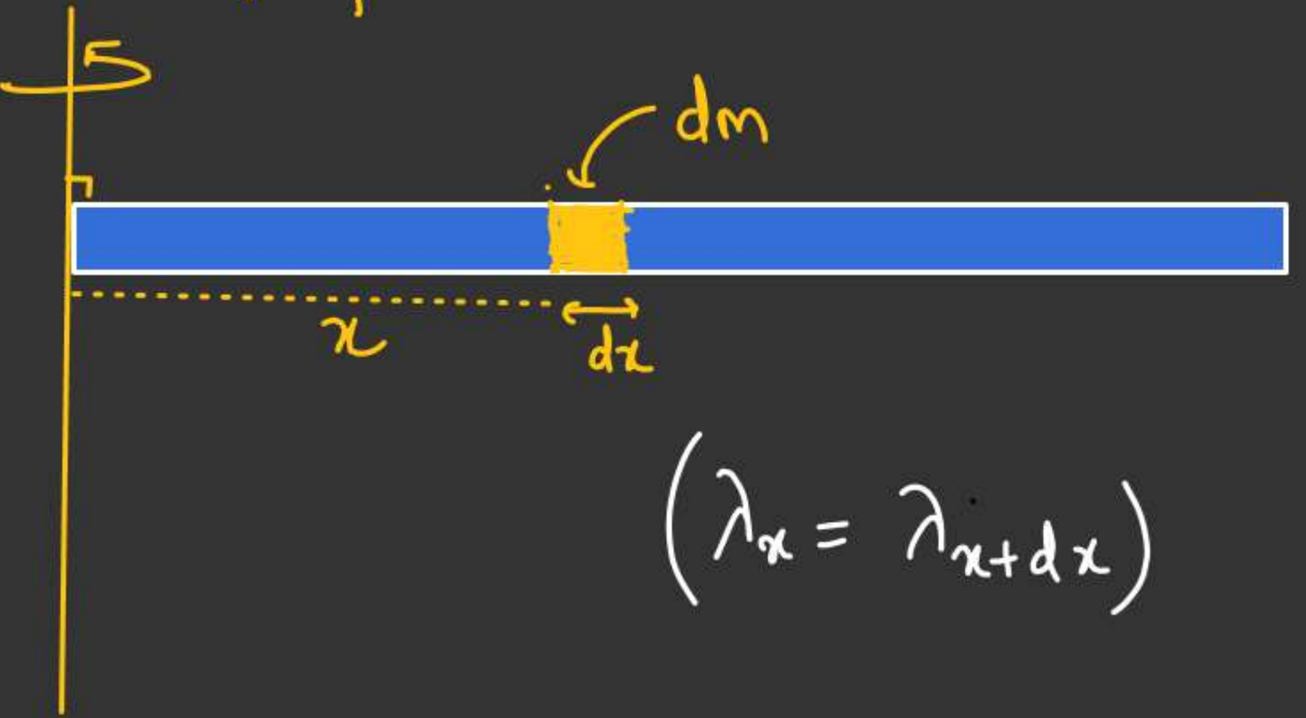


MOMENT OF INERTIA

M.I of a non-uniform rod about any axis perpendicular to the rod & passing through one of its end.

$\lambda = \lambda_0\left(1+\frac{x}{L}\right)$, λ = linear mass density of Rod.

L = length of Rod.



Sol^n For dx length λ assumed to be Constant

$$dm = \lambda_x \cdot dx$$

$$(\lambda_x = \lambda_{x+dx})$$

$$dm = \lambda_0\left(1+\frac{x}{L}\right)dx$$

$dI \rightarrow$ M.I due to dm mass

$$\int_0^I dI = \int_0^L dm\, x^2 \Rightarrow I = \lambda_0\int_0^L\left(1+\frac{x}{L}\right)x^2 dx = \lambda_0\left[\int_0^L x^2 dx + \frac{1}{L}\int_0^L x^3 dx\right]$$

$$= \lambda_0\left[\frac{L^3}{3}+\frac{L^3}{4}\right] = \frac{7\lambda_0 L^3}{12}$$

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MOMENT OF INERTIA

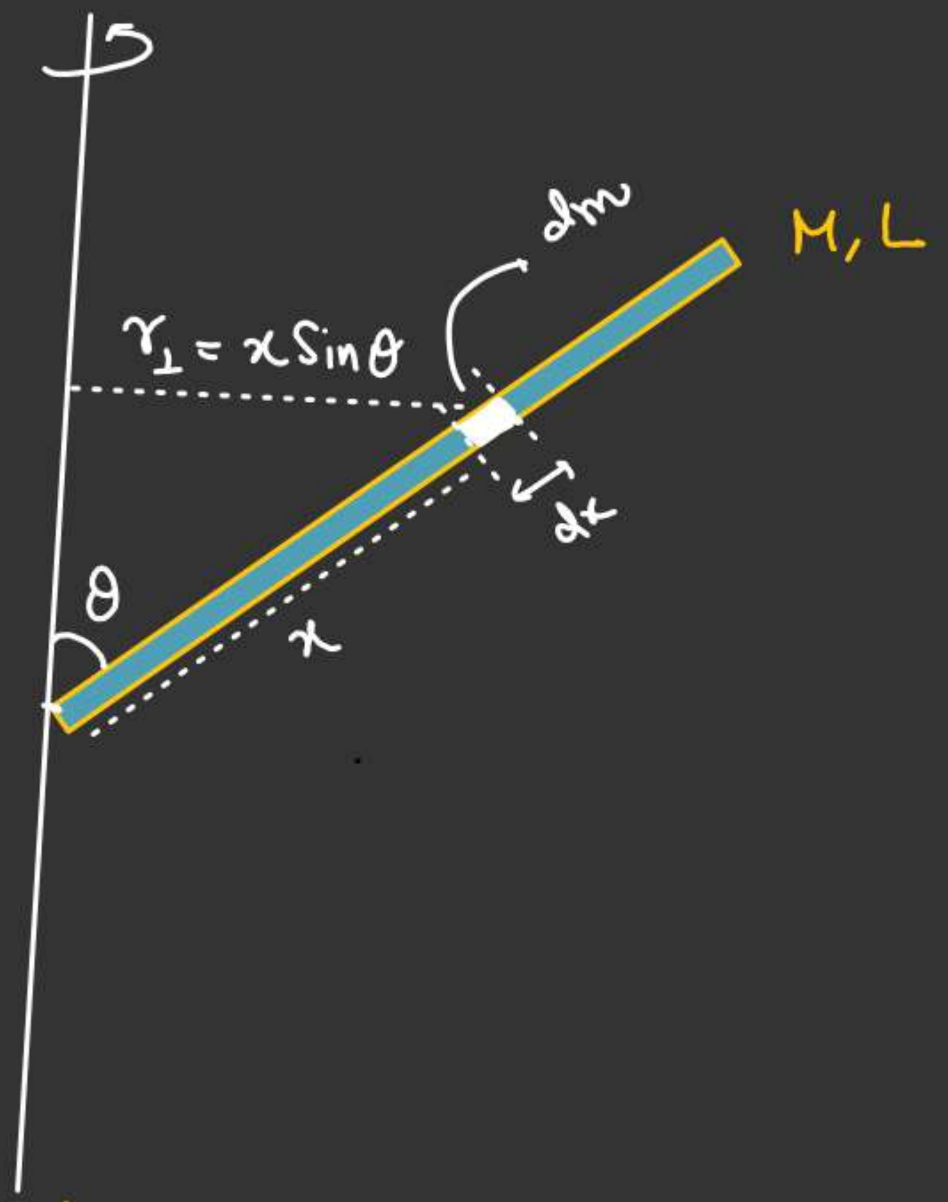
M.I of a uniform rod inclined about axis of Rotation

$$\int_0^I dI = \int_0^L dm\, r_\perp^2 \qquad dm = \frac{M}{L}dx$$

$$I = \frac{M}{L}\int_0^L x^2 \sin^2\theta \, dx$$

$$I = \frac{M\sin^2\theta}{L}\int_0^L x^2 dx$$

$$\boxed{I = \frac{ML^2}{3}\sin^2\theta}$$



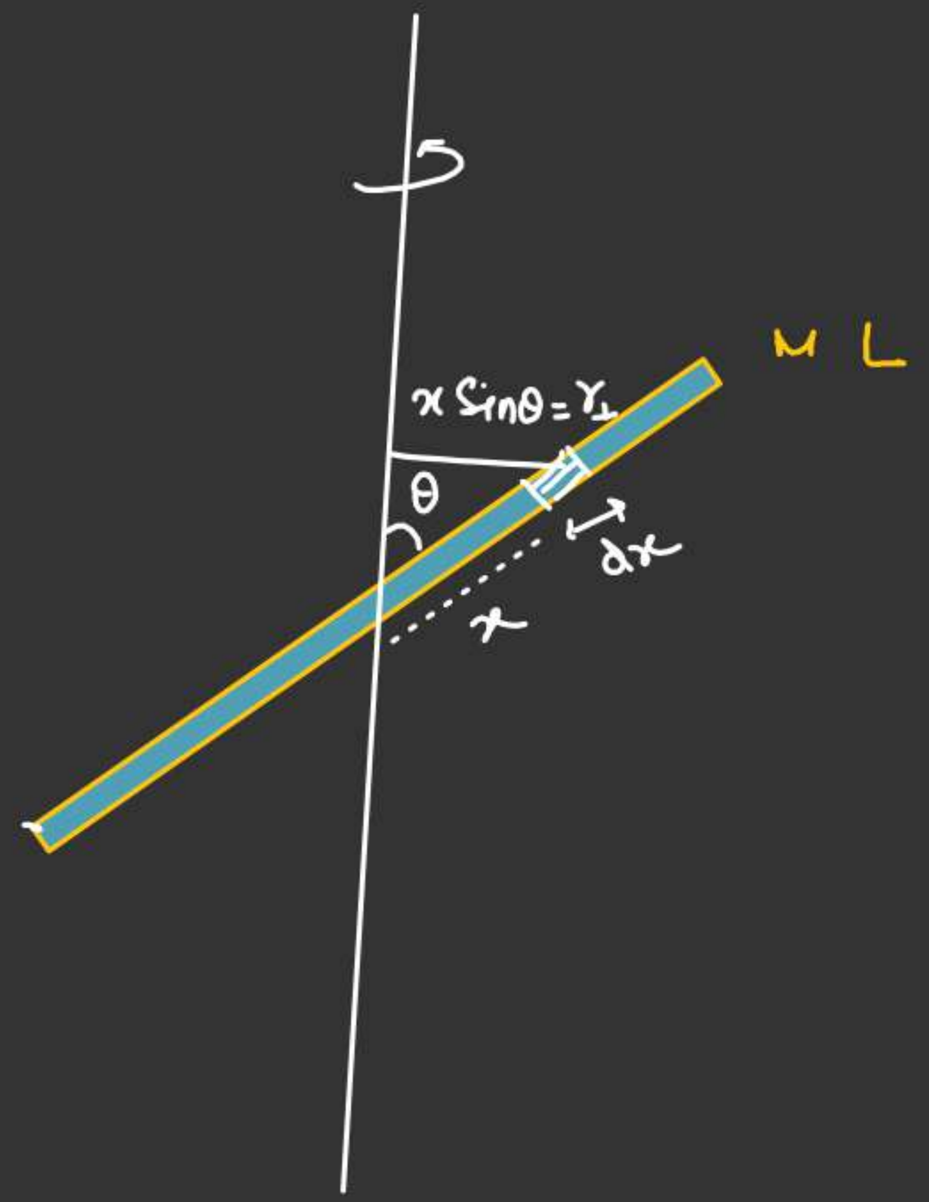
MOMENT OF INERTIA

Case-2

$$I = \int_{-l/2}^{+l/2} dm\, r_1^2$$

$$= \frac{M}{L} \int_{-l/2}^{+l/2} x^2 \sin^2\theta \cdot dx$$

$$\boxed{I = \frac{ML^2}{12} \sin^2\theta}$$



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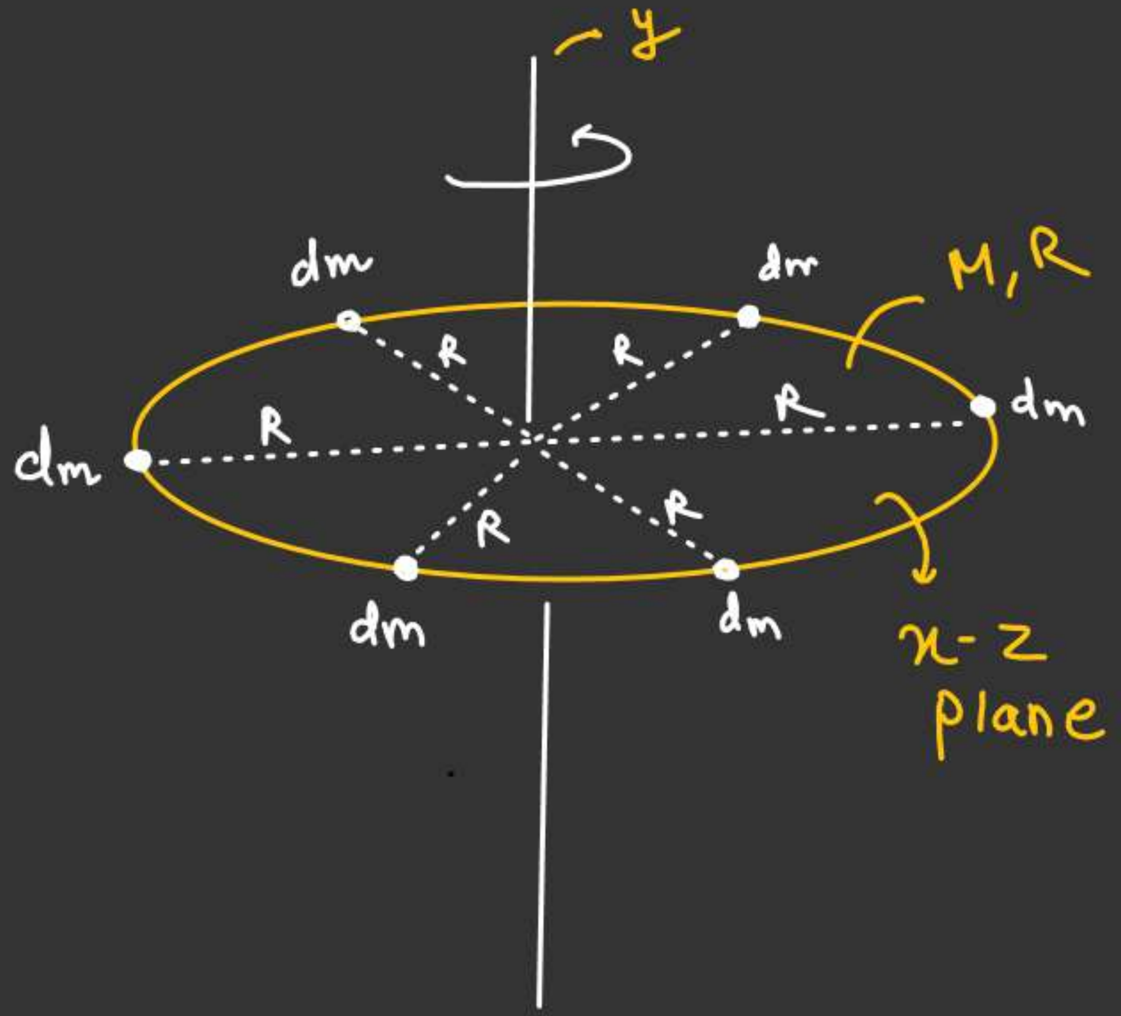
MOMENT OF INERTIA

**: M.I of a ring about an axis perpendicular to the plane of the ring and passing through its center

$$\int_0^I dI = \int dm\,R^2 = R^2 \int dm$$

$$\boxed{I = MR^2}$$ **



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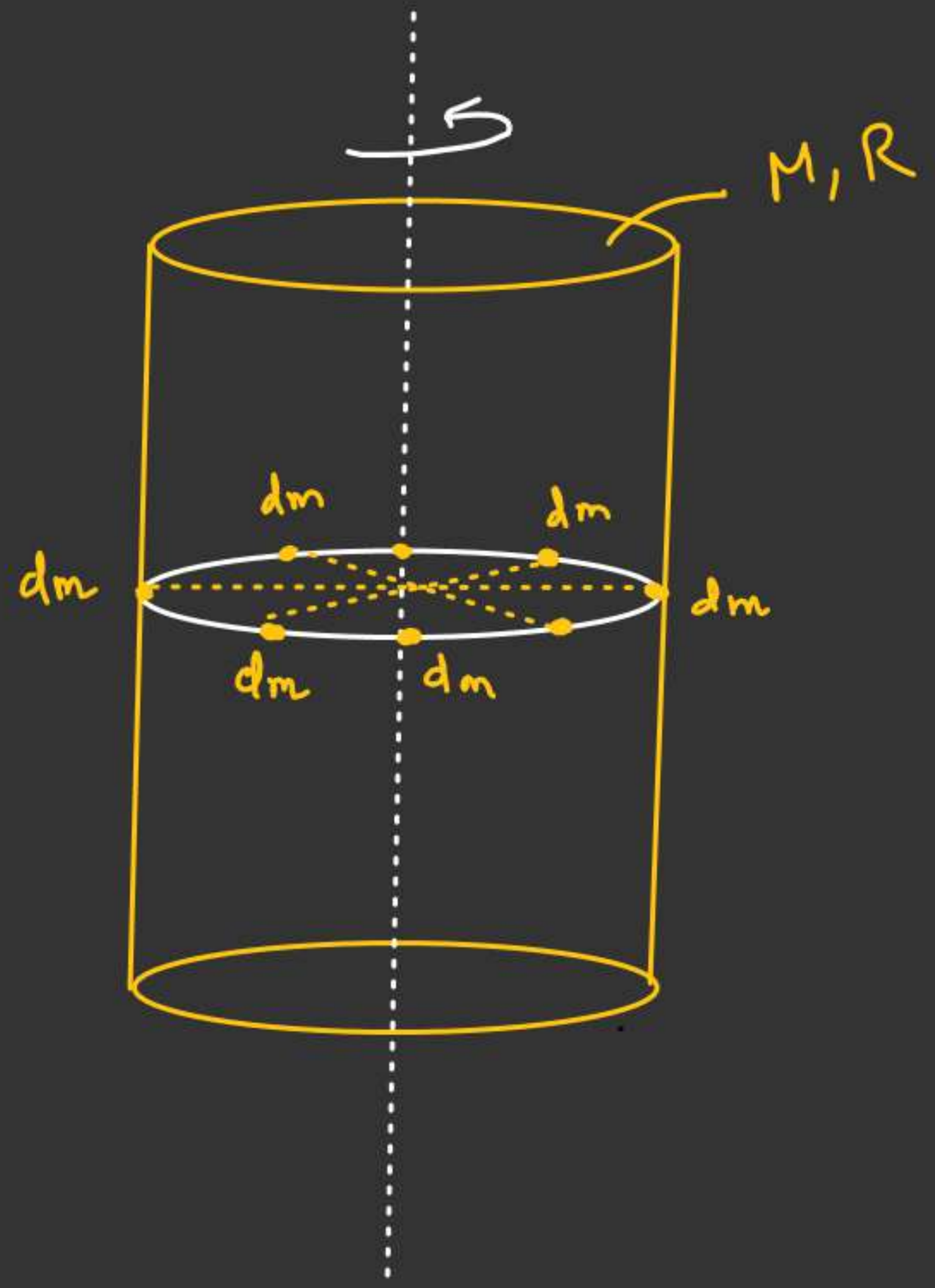
MOMENT OF INERTIA

** M.I of a hollow Cylinder about its axis

$$\int dI = \int dm\, R^2$$

$$I = R^2 \int dm = MR^2$$

$$\boxed{I = MR^2}$$



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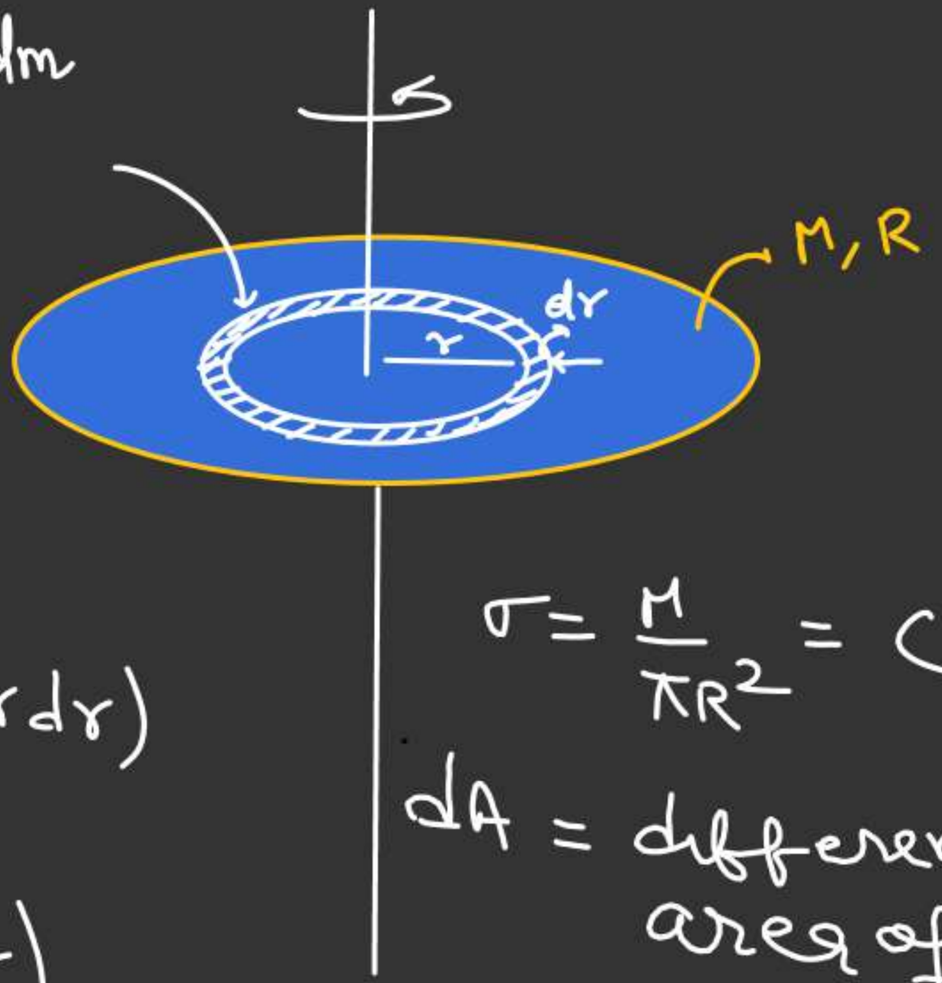
MOMENT OF INERTIA

★★ M.I of a Uniform disc about axis perpendicular to its plane and passing through its center

dI = M.I of ring about axis of rotation having radius r and mass dm



$dI = dm\, r^2$

$dm = \left(\frac{M}{\pi R^2}\right) \times dA$

$\sigma = \frac{M}{\pi R^2} = C$

dA = differential area of ring

$dm = \frac{M}{\cancel{\pi} R^2} \times (2\cancel{\pi} r\, dr)$

$\int_0^I dI = \frac{2M}{R^2} \int_0^R r^3\, dr$

$dm = \left(\frac{2M}{R^2} r\, dr\right)$

$I = \frac{2M}{R^2} \times \frac{R^4}{4} = \frac{MR^2}{2}$

$\boxed{I = \frac{MR^2}{2}}$

Nishant Jindal

MOMENT OF INERTIA

H.W Find M.I of a non-uniform disc whose areal mass density $\sigma = \sigma_0 r$ where r is radial distance & σ_0 is a constant.