S.D.T.

Q Find Pt. of Extrema for

$f(x) = 2x^3 - 9x^2 + 12x + 6.$

(1) $\frac{dy}{dx} = 6x^2 - 18x + 12$

$= 6(x^2 - 3x + 2)$

$= 6(x-\ )(x-\ ) = 0$

Cr. pt $\Rightarrow x = 1, 2$

(2) $\frac{d^2y}{dx^2} = 2x - 3.$

$\left.\frac{d^2y}{dx^2}\right|_{x=1} = 2 - 3 = -1 =$ Max. at $x=1$

$\left.\frac{d^2y}{dx^2}\right|_{x=2} = 4 - 3 = +ve$ Min at $x=2$

Q $f(x) = (1+b^2)x^2 + 2bx + 1$

& let $m(b)$ be the min value of $f(x)$ then Range of $m(b)$

→ Min value of fxn

(1) $\frac{dy}{dx} = 2(1+b^2)x + 2b = 0$

$x = -\frac{b}{1+b^2}$

(2) $\left.\frac{d^2y}{dx^2}\right|_{x=-\frac{b}{1+b^2}} = 2(1+b^2)$ +ve Min.

(3) Min value $= f\left(-\frac{b}{1+b^2}\right) = \frac{\cancel{(1+b^2)}b^2}{(1+b^2)^2} + \frac{2b\cdot(-b)}{1+b^2} + 1$

$m(b) = \frac{b^2 - 2b^2 + 1 + b^2}{1+b^2} = \frac{1}{1+b^2}$

$0 \le b^2 < \infty$

$1 \le 1+b^2 < \infty$

$\frac{1}{1} \ge \frac{1}{1+b^2} > \frac{1}{\infty}$

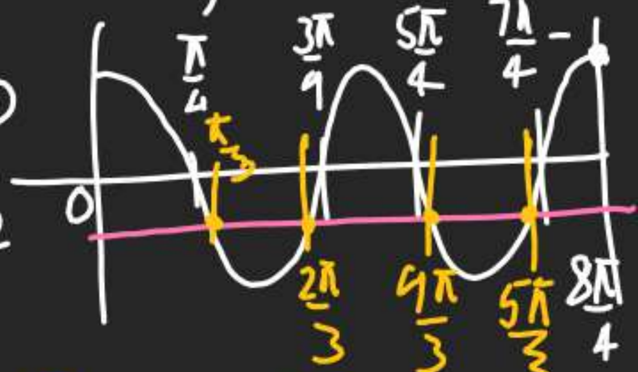
$1 \ge m(b) > 0 \Rightarrow R_f = (0, 1]$

Q Find Pt. of Extrema for

$f(x) = x + \sin 2x \,;\ x \in (0, 2\pi)$

(1) $\frac{dy}{dx} = 1 + 2\cos 2x = 0$

$\cos 2x = -\frac{1}{2}$



$2x = 2n\pi \pm \frac{2\pi}{3}$

$x = n\pi \pm \frac{\pi}{3}$

4 Pt of Extrema

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Q $f(x) = \begin{cases} e^x & 0 \le x \le 1 \\ 2 - e^{x-1} & 1 < x \le 2 \\ x - e & 2 < x \le 3 \end{cases}$

& $g(x) = \int_0^x f(t)\,dt \; ; \; x \in [0,3]$

then find L Max / L Min for $g(x)$.

① $g'(x) = f(x) = \begin{cases} e^x = 0 \; \times \\ 2 - e^{x-1} = 0 \quad x = 1 + \ln 2 \\ x - e = 0 \to x = e \end{cases}$

$e^{x-1} = 2$

$x - 1 = \ln 2$

$x = 1 + \ln 2$

② $g''(x) = f'(x) = \begin{cases} e^x & 0 \le x \le 1 \\ -e^{x-1} & 1 < x \le 2 \\ 1 & 2 < x \le 3 \end{cases}$

$g''(1+\ln 2) = -e^{\cancel{1}+\ln 2 - \cancel{1}} = -2$ Max.

$g''(e) = 1$ ⊕ Min.

L Max at $x = 1 + \ln 2$

L Min at $x = e$

Q find Pt of Extrema

for $f(x) = x^3 - px + q \; (p > 0)$

① $\frac{dy}{dx} = 3x^2 - p = 0 \Rightarrow x = \pm\sqrt{\frac{p}{3}}$

② $\frac{d^2y}{dx^2} = 6x \; - \begin{cases} \to \sqrt{\frac{p}{3}} = 6\sqrt{\frac{p}{3}} \; ⊕ \; \text{Min} \\ \to -\sqrt{\frac{p}{3}} = -6\sqrt{\frac{p}{3}} = \text{(-ve)} \; \text{Max} \end{cases}$

Q Let $f(x) = \frac{a}{x} + x^2$. find a if $f(x)$ attains max^m at $x = -3$.

① $\frac{dy}{dx} = -\frac{a}{x^2} + 2x \Rightarrow \left.\frac{dy}{dx}\right|_{x=-3} = 0$ आरम्भ

$-\frac{a}{9} - 6 = 0 \Rightarrow a = -54$

② $\left.\frac{d^2y}{dx^2}\right|_{x=-3} = +\frac{2a}{x^3} + 2 = \frac{2 \times \cancel{-54}^{\,2}}{\cancel{-27}} + 2 = 6$ ⊕ Min

$\Rightarrow a = \phi$

Q $f(x) = 1 + 2x^2 + 4x^4 + 6x^6 + \dots + 100x^{100}$

then $f(x)$ has

(A) NMNM (B) only one Max

(C) only one Min (D) One Max One Min.

① $\frac{dy}{dx} = 4x + 16x^3 + 36x^5 + \dots + (100)^2 x^{99}$

$= x(4 + 16x^2 + 36x^4 + \dots + (100)^2 x^{98}) = 0$

$x = 0$ is only C.P.t

$\left.\frac{d^2y}{dx^2}\right|_{x=0} = 4 + 48x^2 + \dots = 4$ ⊕ Min

Q

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$f(x) = axe^{bx^2}$ attain.

max^m value 1 at $x=2$ find a, b?

fxn's Max^m value $= 1$ at $x=2$

$f(2) = 1 \Rightarrow 2ae^{4b} = 1$

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fxn's Max^{ma} at $x=2$ $\left.\frac{dy}{dx}\right|_{x=2} = 0$

$f'(x) = a\left(x \cdot e^{bx^2} \cdot 2bx + e^{bx^2}\right)$

$f'(2) = a\left(8be^{4b} + e^{4b}\right) = 0$

$e^{4b}(1+8b) = 0 \Rightarrow b = -\frac{1}{8}$

$-\quad +$

2

$2a \cdot e^{-1/2} = 1 \Rightarrow \frac{2a}{\sqrt{e}} = 1$

$a = \frac{\sqrt{e}}{2}$

Maxmap. Check Urself

$a = \frac{\sqrt{e}}{2}$ & $b = -1/8$

Q If $P(x)$ be a Real Poly. of deg 3 Satisfying

$P(-1) = 10$, $P(1) = -6$

& $P(x)$ has max^{ma} at $x = -1$

& $P'(x)$ has Min at $x = 1$

Find distance betn L Max & L Min of Curve

1) $P(x) = ax^3 + bx^2 + cx + d$

$P(-1) = -a + b - c + d = 10$

$P(1) = \underline{a + b + c + d = -6}$

$-(a+c) = 8 \qquad b + d = 2$

$a + c = -8$

(2) $P'(-1) = 0$

$P'(x) = 3ax^2 + 2bx + c$

$P'(-1) = 3a - 2b + c = 0$

(3) $P'(x)$ Min $x = 1 \Rightarrow P''(1) = 0$

$P''(x) = 6ax + 2b \Rightarrow 6a + 2b = 0$

$\boxed{b = -3a}$

$3a + 6a + c = 0 \Rightarrow \boxed{c = -9a}$ & $\boxed{d = 2 + 3a}$

$a + c = -8 \Rightarrow -8a = -8 \quad a = 1$

$a = 1, b = -3, c = -9, d = 5$

$f(x) = x^3 - 3x^2 - 9x + 5$ Max^m / Min fns

Q If $f(x)$ is a Cubic Poly. In which has L Max at $x = -1$, $f(2) = 18$, $f(1) = -1$ & $f'(x)$ has L Min at $x = 0$. A) $f(0) = 5$

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$d = \frac{17}{2}$

① $f(x) = ax^3 + bx^2 + cx + d \Rightarrow f'(-1) = 0$

$f'(x) = 3ax^2 + 2bx + c \Rightarrow f'(-1) = 3a - 2b + c = 0$

② $8a + 4b + 2c + d = 18$

$\underline{a + b + c + d = -1}$

$7a + 3b + c = 19$

$7a + c = 19 \Rightarrow c = 19 - \frac{7 \times 19}{4}$

$c = -\frac{57}{4}$

$\underline{7a + 3b + c = 19}$

$-4a - 5b = -19$

$a = \frac{19}{4}$

③ $f''(0) = 0$

$f''(x) = 6ax + 2b$

$f''(0) = 2b = 0 \Rightarrow b = 0$

Nishant Jindal

Q Let $P(x)$ be a Real Poly.

JEE of deg 4 having extremum at $x=1,2$ & $\lim_{x\to 0}\left(1+\frac{P(x)}{x^2}\right)=2$

find $P(12)=?$

Let $P(x)=ax^4+bx^3+cx^2$

$P'(x)=4ax^3+3bx^2+2cx$

$P'(1)=4a+3b+2c=0$

$P'(2)=32a+12b+4c=0$

$\underline{8a+3b+c=0}$

$-4a+c=0 \Rightarrow c=4a$

$\lim_{x\to 0}\left(1+\cancel{ax^2}+\cancel{bx}+c\right)=2$

$1+c=2$

$\boxed{c=1}$, $a=\frac{1}{4}$ $b=-1$

$f(x)=\frac{x^4}{4}-x^3+x^2$

Q Let $P(x)$ be a real Poly.

JEE of least deg. which has L.Max. at $x=1$ & L.Min at $x=3$. If $P(1)=6$, $P(3)=2$

find $P'(0)=?$

① $P'(x)=K(x-1)(x-3)=K(x^2-4x+3)$

$P(x)=K\left(\frac{x^3}{3}-\frac{4x^2}{2}+3x\right)+\lambda$

2) $P(1)=K\left(\frac{1}{3}-2+3\right)+\lambda=6$

$\lambda+\frac{4K}{3}=6$

$P(3)=K(9-18+9)+\lambda=2$

$\lambda=2$ $K=3$

$P(x)=3\left(\frac{x^3}{3}-2x^2+3x\right)+2$

Q $f(x)$ is 4 deg Poly having Extrema at $1,0,-1$ $S=\{x\in R, f(x)=f(0)\}$ Contain Exactly

A) 4 Rational No. (B) 2 Irr. 2 Rational No

(C) 4 Irr. No (D) 2 Irr. & 1 Rational No

① $f'(x)=K(x-1)(x-0)(x+1)$

$=Kx(x^2-1)=K(x^3-x)$

$f(x)=K\left[\frac{x^4}{4}-\frac{x^2}{2}\right)+\lambda$

② $f(x)=f(0)\Rightarrow K\left[\frac{x^4}{4}-\frac{x^2}{2}\right]+\cancel{\lambda}=\cancel{\lambda}$

$\frac{x^4}{4}-\frac{x^2}{2}=0$

$x^4-2x^2=0\Rightarrow x^2(x^2-2)=0$

$x=0,0,\sqrt{2},-\sqrt{2}$