EXERCISE-I

SECTION - A : KINDS OF VECTORS

1. Consider points A, B, C and D with position vectors $7\hat{i} + 4\hat{j} + 7\hat{k}$, $\hat{i} - 6\hat{j} + 10\hat{k}$, $-\hat{i} - 3\hat{j} + 4\hat{k}$ and $5\hat{i} - \hat{j} + 5\hat{k}$ respectively. The ABCD is a

(A) square (B) rhombus
(C) rectangle (D) none of these

SECTION - B
ADDITION & SUBSTRACTION OF VECTORS

2. The vertices of a triangle are A(1, 1, 2), B(4, 3, 1) and C(2, 3, 5). A vector representing the internal bisector of the angle A is

(A) $\hat{i}+\hat{j}+2\hat{k}$ (B) $2\hat{i}-2\hat{j}+\hat{k}$
(C) $2\hat{i}+2\hat{j}-\hat{k}$ (D) $2\hat{i}+2\hat{j}+\hat{k}$

3. The vectors $\overline{AB} = 3\hat{i} + 4\hat{k}$ and $\overline{AC} = 5\hat{i} - 2\hat{j} + 4\hat{k}$ are the sides of a triangle ABC. The length of the median through A is

(A) $\sqrt{18}$ (B) $\sqrt{72}$
(C) $\sqrt{33}$ (D) $\sqrt{288}$

SECTION - C
COLLINEARITY OF THREE POINTS

4. If the vector $\vec{b}$ is collinear with the vector $\vec{a} = (2\sqrt{2}, -1, 4)$ and $|\vec{b}| = 10$, then

(A) $\vec{a} \pm \vec{b} = 0$ (B) $\vec{a} \pm 2\vec{b} = 0$
(C) $2\vec{a} \pm \vec{b} = 0$ (D) none of these

SECTION - D
RELATION BETWEEN TWO PARALLEL VECTORS

5. If $A(-\hat{i} + 3\hat{j} + 2\hat{k})$, $B(-4\hat{i} + 2\hat{j} - 2\hat{k})$ and $C(5\hat{i} + \lambda\hat{j} + \mu\hat{k})$ are collinear then

(A) $\lambda = 5, \mu = 10$ (B) $\lambda = 10, \mu = 5$
(C) $\lambda = -5, \mu = 10$ (D) $\lambda = 5, \mu = -10$

6. The vectors $2\hat{i}+3\hat{j}$, $5\hat{i}+6\hat{j}$ and $8\hat{j}+\lambda\hat{j}$ have their initial points at (1,1). Find the value of λ so that the vectors terminate on one straight line

(A) 9 (B) 8
(C) 7 (D) 6

7. If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are three non - zero vectors, no two of which are collinear, $\vec{a}+2\vec{b}$ is collinear with $\vec{c}$ and $\vec{b}+3\vec{c}$ is collinear with $\vec{a}$, then find the value of $\left|\vec{a}+2\vec{b}+6\vec{c}\right|$,

(A) 0 (B) 1
(C) 2 (D) 3

SECTION - E
COPLANAR AND NON - COPLANAR VECTORS

8. If $\vec{a}$, $\vec{b}$, $\vec{c}$ are non-coplanar vectors and λ is a real number, then the vectors $\vec{a} + 2\vec{b} + 3\vec{c}$, $\lambda\vec{b} + 4\vec{c}$ and $(2\lambda - 1)\vec{c}$ are non-coplanar for

(A) all values of λ
(B) all except one value of λ
(C) all except two values of λ
(D) non value of λ

9. The set of values of 'm' for which the vectors $\vec{a} = m\hat{i}+(m+1)\hat{j}+(m+8)\hat{k}$, $\vec{b} = (m+3)\hat{i}+(m+4)\hat{j}+(m+5)\hat{k}$ and $\vec{c} = (m+6)\hat{i}+(m+7)\hat{j}+(m+8)\hat{k}$ are non-coplanar is

(A) R (B) R – {1}
(C) R – {1, 2} (D) ϕ

SECTION - F
SCALAR OR DOT PRODUCT OF TWO VECTORS

10. If $|\vec{a}| = 5$, $|\vec{a} - \vec{b}| = 8$ and $|\vec{a} + \vec{b}| = 10$, then $|\vec{b}|$ is equal to

(A) 1 (B) $\sqrt{57}$
(C) 3 (D) none of these

11. Angle between diagonals of a parallelogram whose side are represented by $\vec{a} = 2\hat{i}+\hat{j}+\hat{k}$ and $\vec{b} = \hat{i}-\hat{j}-\hat{k}$

(A) $\cos^{-1}\left(\frac{1}{3}\right)$ (B) $\cos^{-1}\left(\frac{1}{2}\right)$
(C) $\cos^{-1}\left(\frac{4}{9}\right)$ (D) $\cos^{-1}\left(\frac{5}{9}\right)$

12. Let $\vec{a}$, $\vec{b}$, $\vec{c}$ be vectors of length 3, 4, 5 respectively. Let $\vec{a}$ be perpendicular to $\vec{b} + \vec{c}$, $\vec{b}$ to $\vec{c} + \vec{a}$ and $\vec{c}$ to $\vec{a} + \vec{b}$. Then $|\vec{a} + \vec{b} + \vec{c}|$

(A) $2\sqrt{5}$ (B) $2\sqrt{2}$

(C) $10\sqrt{5}$ (D) $5\sqrt{2}$

13. The value of a, for which the points A, B, C with position vectors $2\hat{i} - \hat{j} - \hat{k}$, $\hat{i} - 3\hat{j} - 5\hat{k}$ and $a\hat{i} - 3\hat{j} - \hat{k}$ respectively are the vertices of a right angled triangle with $C = \pi/2$ are

(A) –2 and –1 (B) –2 and 1

(C) 2 and –1 (D) 2 and 1

14. A particle is acted upon by constant forces $4\hat{i} + \hat{j} - 3\hat{k}$ and $3\hat{i} + \hat{j} - \hat{k}$ which displace it from a point $\hat{i} + 2\hat{j} + 3\hat{k}$ to the point $5\hat{i} + 4\hat{j} + \hat{k}$. The workdone in standard units by the force is given by

(A) 40 (B) 30

(C) 25 (D) 15

15. $\vec{a}$, $\vec{b}$, $\vec{c}$ are three vectors, such that $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, $|\vec{a}| = 1$, $|\vec{b}| = 2$, $|\vec{c}| = 3$ then $\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a}$ is equal to

(A) 0 (B) –7

(C) 7 (D) 1

16. If $\vec{a} = \hat{i} - \hat{j}$, $\vec{b} = \hat{i} + \hat{j}$, $\vec{c} = \hat{i} + 3\hat{j} + 5\hat{k}$ and $\vec{n}$ be a unit vector such that $\vec{b}.\vec{n} = 0$, $\vec{a}.\vec{n} = 0$ then value of $|\vec{c}.\vec{n}|$ is

(A) 1 (B) 3

(C) 5 (D) 2

17. Given the three vectors $\vec{a} = -2\hat{i} + \hat{j} + \hat{k}$, $\vec{b} = \hat{i} + 5\hat{j}$ and $\vec{c} = 4\hat{i} + 4\hat{j} - 2\hat{k}$. The projection of the vector $3\vec{a} - 2\vec{b}$ on the vector $\vec{c}$ is

(A) 11 (B) –11

(C) 13 (D) none of these

18. Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three units vectors that $3\vec{a} + 4\vec{b} + 5\vec{c} = 0$. Then which of the following statements is true ?

(A) $\vec{a}$ is parallel to $\vec{b}$

(B) $\vec{a}$ is perpendicular to $\vec{b}$

(C) $\vec{a}$ is neither parallel nor perdendicular to $\vec{b}$

(D) None of these

SECTION - G
VECTOR OR CROSS PRODUCT OF TWO VECTORS

19. Let $\vec{a} = \hat{i} + \hat{j}$ and $\vec{b} = 2\hat{i} - \hat{k}$. The point of intersection of the lines $\vec{r} \times \vec{a} = \vec{b} \times \vec{a}$ and $\vec{r} \times \vec{b} = \vec{a} \times \vec{b}$ is

(A) $-\hat{i} + \hat{j} + 2\hat{k}$ (B) $3\hat{i} - \hat{j} + \hat{k}$

(C) $3\hat{i} + \hat{j} - \hat{k}$ (D) $\hat{i} - \hat{j} - \hat{k}$

20. Vector $\vec{a}$ and $\vec{b}$ make an angle $\theta = \frac{2\pi}{3}$. if $|\vec{a}| = 1$, $|\vec{b}| = 2$, then $\{(\vec{a} + 3\vec{b}) \times (3\vec{a} - \vec{b})\}^2$ is equal to

(A) 225 (B) 250

(C) 275 (D) 300

21. Unit vector perpendicular to the plane of the triangle ABC with position vectors $\vec{a}$, $\vec{b}$, $\vec{c}$ of the vertices A, B, C is

(A) $\frac{(\vec{a}\times\vec{b} + \vec{b}\times\vec{c} + \vec{c}\times\vec{a})}{\Delta}$

(B) $\frac{(\vec{a}\times\vec{b} + \vec{b}\times\vec{c} + \vec{c}\times\vec{a})}{2\Delta}$

(C) $\frac{(\vec{a}\times\vec{b} + \vec{b}\times\vec{c} + \vec{c}\times\vec{a})}{4\Delta}$

(D) none of these

22. If $\vec{b}$ and $\vec{c}$ are two non-collinear vectors such that $\vec{a} \parallel (\vec{b} \times \vec{c})$, then $(\vec{a} \times \vec{b}).(\vec{a} \times \vec{c})$ is equal to

(A) $\vec{a}^2(\vec{b}.\vec{c})$ (B) $\vec{b}^2(\vec{a}.\vec{c})$

(C) $\vec{c}^2(\vec{a}.\vec{b})$ (D) none of these

23. Vector of length 3 unit which is perpendicular to $\hat{i} + \hat{j} + \hat{k}$ and lies in the plane of $\hat{i} + \hat{j} + \hat{k}$ and $2\hat{i} - 3\hat{j}$

(A) $\frac{3}{\sqrt{6}}(\hat{i} - 2\hat{j} + \hat{k})$ (B) $\frac{3}{\sqrt{6}}(2\hat{i} - \hat{j} - \hat{k})$

(C) $\frac{3}{\sqrt{114}}(8\hat{i} - 7\hat{j} - \hat{k})$ (D) $\frac{3}{\sqrt{114}}(-7\hat{i} + 8\hat{j} - \hat{k})$

24. Given the vertices A (2, 3, 1), B(4, 1, –2), C(6, 3, 7) & D(–5, –4, 8) of a tetrahedron. The length of the altitude drawn from the vertex D is

(A) 7 (B) 9

(C) 11 (D) none of these

25. For a non zero vector $\vec{A}$ If the equations $\vec{A}\cdot\vec{B}=\vec{A}\cdot\vec{C}$ and $\vec{A}\times\vec{B}=\vec{A}\times\vec{C}$ hold simultaneously, then

(A) $\vec{A}$ is perpendicular to $\vec{B}-\vec{C}$

(B) $\vec{A}=\vec{B}$

(C) $\vec{B}=\vec{C}$

(D) $\vec{C}=\vec{A}$

26. If u and v are unit vectors and θ is the acute angle between them, then 2u × 3v is a unit vector for

(A) Exactly two values of θ

(B) More than two values of θ

(C) No value of θ

(D) Exactly one value of θ

27. If $\vec{u}=\vec{a}-\vec{b}, \vec{v}=\vec{a}+\vec{b}$ and $|\vec{a}|=|\vec{b}|=2$, then $|\vec{u}\times\vec{v}|$ is equal to

(A) $\sqrt{2(16-(\vec{a}.\vec{b})^2)}$ (B) $2\sqrt{(16-(\vec{a}.\vec{b})^2)}$

(C) $2\sqrt{(4-(\vec{a}.\vec{b})^2)}$ (D) $\sqrt{2(4-(\vec{a}.\vec{b})^2)}$

28. If A(1, 1, 1), C(2, –1, 2), the vector equation of the line $\overline{AB}$ is $\vec{r}=(\hat{i}+\hat{j}+\hat{k})+t(6\hat{i}-3\hat{j}+2\hat{k})$ and d is the shortest distance of the point C from $\overline{AB}$, then

(A) B(6,–3,2) (B) B(5, –4, 1)

(C) d= $\sqrt{2}$ (D) d= $\sqrt{6}$

SECTION - H
SCALAR TRIPLE PRODUCT

29. The value of $[(\vec{a}+2\vec{b}-\vec{c}),(\vec{a}-\vec{b}),(\vec{a}-\vec{b}-\vec{c})]$ is equal to the box product

(A) $[\vec{a}\ \vec{b}\ \vec{c}]$ (B) $2[\vec{a}\ \vec{b}\ \vec{c}]$

(C) $3[\vec{a}\ \vec{b}\ \vec{c}]$ (D) $4[\vec{a}\ \vec{b}\ \vec{c}]$

30. The volume of the parallelopiped constructed on the diagonals of the faces of the given rectangular parallelopiped is m times the volume of the given parallelopiped. Then m is equal to

(A) 2 (B) 3

(C) 4 (D) none of these

31. If $\vec{u}$, $\vec{v}$ and $\vec{w}$ are three non-coplanar vectors, then $(\vec{u}+\vec{v}-\vec{w}).[(\vec{u}-\vec{v})\times(\vec{v}-\vec{w})]$ equals

(A) 0 (B) $\vec{u}\cdot\vec{v}\times\vec{w}$

(C) $\vec{u}\cdot\vec{w}\times\vec{v}$ (D) $3\vec{u}\cdot\vec{v}\times\vec{w}$

32. Let $\vec{a}=a_1\hat{i}+a_2\hat{j}+a_3\hat{k}, \vec{b}=b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$ and $\vec{c}=c_1\hat{i}+c_2\hat{j}+c_3\hat{k}$ be three non-zero vectors such that $\vec{c}$ is a unit vector perpendiuclar to both $\vec{a}$ and $\vec{b}$. If the angle between $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{6}$, then

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}^2$$ is equal to

(A) 0

(B) 1

(C) $\frac{1}{4}(a_1^2+a_2^2+a_3^2)(b_1^2+b_2^2+b_3^2)$

(D) $\frac{3}{4}(a_1^2+a_2^2+a_3^2)(b_1^2+b_2^2+b_3^2)(c_1^2+c_2^2+c_3^2)$

SECTION - I
VECTOR TRIPLE PRODUCT

33. Vector $\vec{x}$ satisfying the relation $\vec{A}.\vec{x}=c$ and $\vec{A}\times\vec{x}=\vec{B}$ is

(A) $\frac{c\vec{A}-(\vec{A}\times\vec{B})}{|\vec{A}|}$ (B) $\frac{c\vec{A}-(\vec{A}\times\vec{B})}{|\vec{A}|^2}$

(C) $\frac{c\vec{A}+(\vec{A}\times\vec{B})}{|\vec{A}|^2}$ (D) $\frac{c\vec{A}-2(\vec{A}\times\vec{B})}{|\vec{A}|^2}$

34. Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be non-zero non–collinear vectors such that $(\vec{a}\times\vec{b})\times\vec{c}=\frac{1}{3}|\vec{b}||\vec{c}|\vec{a}$, If θ is the angle between the vectors $\vec{b}$ and $\vec{c}$, then sinθ equals is

(A) $\frac{1}{3}$ (B) $\frac{\sqrt{2}}{3}$

(C) $\frac{2}{3}$ (D) $\frac{2\sqrt{2}}{3}$

SECTION - J / K
SCALAR / VECTOR PRODUCT OF 4 VECTORS

35. Let the pairs $\vec{a}$, $\vec{b}$ and $\vec{c}$, $\vec{d}$ each determine a plane. Then the planes are parallel if

(A) $(\vec{a}\times\vec{c})\times(\vec{b}\times\vec{d})=\vec{0}$

(B) $(\vec{a}\times\vec{c}).(\vec{b}\times\vec{d})=\vec{0}$

(C) $(\vec{a}\times\vec{b})\times(\vec{c}\times\vec{d})=\vec{0}$

(D) $(\vec{a}\times\vec{b}).(\vec{c}\times\vec{d})=\vec{0}$

36. let vectors $\vec{a}, \vec{b}, \vec{c}$ and $\vec{d}$ be such that $(\vec{a}\times\vec{b})\times(\vec{c}\times\vec{d}) = \vec{0}$. Let P_1 and P_2 be planes determined by the pairs of vectors $\vec{a}$, $\vec{b}$ and $\vec{c}, \vec{d}$, respectively. Then the angle between P_1 and P_2 is

(A) 0 (B) $\pi/4$

(C) $\pi/3$ (D) $\pi/2$

SECTION - L
MIXED PROBLEMS

37. If $\vec{a}$, $\vec{b}$, $\vec{c}$ are linearly independent vectors, then which one of the following set of vectors is linearly dependent ?

(A) $\vec{a}+\vec{b}, \vec{b}+\vec{c}, \vec{c}+\vec{a}$

(B) $\vec{a}-\vec{b}, \vec{b}-\vec{c}, \vec{c}-\vec{a}$

(C) $\vec{a}\times\vec{b}, \vec{b}\times\vec{c}, \vec{c}\times\vec{a}$

(D) none of these

38. A point taken on each median of a triangle divides the median in the ratio 1 : 3, reckoning from the vertex. Then the ratio of the area of the triangle with vertices at these points to that of the original triangle is

(A) 5 : 13 (B) 25 : 64

(C) 13 : 32 (D) none of these

39. If G is the centroid of a trinagle ABC, then $\overrightarrow{GA}+\overrightarrow{GB}+\overrightarrow{GC}$ is equal to

(A) $\vec{0}$ (B) $3\overrightarrow{GA}$

(C) $3\overrightarrow{GB}$ (D) $3\overrightarrow{GC}$

40. If vector $\overrightarrow{AB} = -3\hat{i}+4\hat{k}$ and $\overrightarrow{AC} = 5\hat{i}-2\hat{j}+4\hat{k}$ are the sides of a ΔABC, then the length of the median through A is

(A) $\sqrt{14}$ (B) $\sqrt{18}$

(C) $\sqrt{29}$ (D) 5

41. If the vectors $\vec{a}$ and $\vec{b}$ are linearly independent satisfying $(\sqrt{3}\tan\theta+1)\vec{a}+(\sqrt{3}\sec\theta-2)\vec{b}=0$, then the most general values of θ are

(A) $n\pi-\frac{\pi}{6}, n\in Z$ (B) $2n\pi\pm\frac{11\pi}{6}, n\in Z$

(C) $n\pi\pm\frac{\pi}{6}, n\in Z$ (D) $2n\pi+\frac{11\pi}{6}, n\in Z$

42. If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are three mutually perpendicular vectors, thent he vector which is equally inclined to these vectors is

(A) $\vec{a}+\vec{b}+\vec{c}$ (B) $\frac{\vec{a}}{|\vec{a}|}+\frac{\vec{b}}{|\vec{b}|}+\frac{\vec{c}}{|\vec{c}|}$

(C) $\frac{\vec{a}}{|\vec{a}|^2}+\frac{\vec{b}}{|\vec{b}|^2}+\frac{\vec{c}}{|\vec{c}|^2}$ (D) $|\vec{a}|\vec{a}-|\vec{b}|\vec{b}+|\vec{c}|\vec{c}$

EXER-I)

COPLANAR AND NON - COPLANAR VECTORS

1. Image of the point P with position vector $7\hat{i}-\hat{j}+2\hat{k}$ in the line whose vector equation is $\vec{r}=9\hat{i}+5\hat{j}+5\hat{k}+\lambda(\hat{i}+3\hat{j}+5\hat{k})$ has the position vector.

(A) (–9, 5, 2) (B) (9, 5, –2)
(C) (9, –5, –2) (D) none of these

SCALAR OR DOT PRODUCT OF TWO VECTORS

2. If a, b, c are pth, qth, rth terms of an H.P. and $\vec{u}=(q-r)\hat{i}+(r-p)\hat{j}+(p-q)\hat{k}$, $\vec{v}=\frac{\hat{i}}{a}+\frac{\hat{j}}{b}+\frac{\hat{k}}{c}$, then

(A) $\vec{u},\vec{v}$ are parallel vectors

(B) $\vec{u},\vec{v}$ are orthogonal vectors

(C) $\vec{u},\vec{v}=1$

(D) $\vec{u}\times\vec{v}=\hat{i}+\hat{j}+\hat{k}$

3. If the unit vectors $\vec{e}_1$ and $\vec{e}_2$ are inclined at an angle 2θ and $|\vec{e}_1-\vec{e}_2|<1$, then for $\theta\in[0,\pi]$, θ may lie in the interval

(A) $\left[0,\frac{\pi}{6}\right)$ (B) $\left[\frac{\pi}{6},\frac{\pi}{2}\right]$

(C) $\left(\frac{5\pi}{6},\pi\right]$ (D) $\left[\frac{\pi}{2},\frac{5\pi}{6}\right]$

4. Let $\vec{u},\vec{v},\vec{w}$ be such that $|\vec{u}|=1,|\vec{v}|=2,|\vec{w}|=3$. If the projection $\vec{v}$ along $\vec{u}$ is equal to that of $\vec{w}$ along $\vec{u}$ and $\vec{v},\vec{w}$ are perpendicular to each other, then $|\vec{u}-\vec{v}+\vec{w}|$ equals

(A) 2 (B) $\sqrt{7}$

(C) $\sqrt{14}$ (D) 14

5. Let $\vec{u}=\hat{i}+\hat{j}$, $\vec{v}=\hat{i}-\hat{j}$ and $\vec{w}=\hat{i}+2\hat{j}+3\hat{k}$. If $\hat{n}$ is a unit vector such that $\vec{u}\cdot\hat{n}=0$ and $\vec{v}\cdot\hat{n}=0$ then $|\vec{w}\cdot\hat{n}|$ is equal to

(A) 0 (B) 1
(C) 2 (D) 3

6. In a quadrilateral ABCD. $\vec{AC}$ is the bisector of $\vec{AB}$ and $\vec{AD}$, angle between $\vec{AB}$ and $\vec{AD}$ is 2π/3, $15|\vec{AC}|=3|\vec{AB}|=5|\vec{AD}|$. Then the angle between $\vec{BA}$ and $\vec{CD}$ is

(A) $\cos^{-1}\frac{\sqrt{14}}{7\sqrt{2}}$ (B) $\cos^{-1}\frac{\sqrt{21}}{7\sqrt{3}}$

(C) $\cos^{-1}\frac{2}{\sqrt{7}}$ (D) $\cos^{-1}\frac{2\sqrt{7}}{14}$

VECTOR OR CROSS PRODUCT OF TWO VECTORS

7. If $\vec{a}=\vec{b}+\vec{c}$, $\vec{b}\times\vec{d}=0$ and $\vec{c}\cdot\vec{d}=0$ then $\frac{\vec{d}\times(\vec{a}\times\vec{d})}{\vec{d}^2}$ is equal to

(A) $\vec{a}$ (B) $\vec{b}$
(C) $\vec{c}$ (D) $\vec{d}$

8. Consider a tetrahedron with faces f_1, f_2, f_3, f_4 Let $\vec{a}_1,\vec{a}_2,\vec{a}_3,\vec{a}_4$ be the vectors whose magnitudes are respectively equal to the areas of f_1, f_2, f_3, f_4 and whose directions are perpendicular to these faces in the outward direction. Then

(A) $\vec{a}_1+\vec{a}_2+\vec{a}_3+\vec{a}_4=0$
(B) $\vec{a}_1+\vec{a}_3=\vec{a}_2+\vec{a}_4$
(C) $\vec{a}_1+\vec{a}_2=\vec{a}_3+\vec{a}_4$
(D) none of these

9. Let $\vec{a}=\hat{i}+\hat{j}+\hat{k}$, $\vec{b}=\hat{i}-\hat{j}+2\hat{k}$ and $\vec{c}=x\hat{i}+(x-2)\hat{j}-\hat{k}$. If the vector $\vec{c}$ lies in the plane of $\vec{a}$ and $\vec{b}$, then x equals

(A) 0 (B) 1
(C) –4 (D) –2

10. Points L, M and N lie on the sides AB, BC and CA of the triangle ABC such that ℓ (AL) : ℓ (LB) = ℓ (BM) : ℓ (MC) = ℓ (CN) : ℓ (NA) = m : n, then the areas of the triangles LMN and ABC are in the ratio

(A) $\frac{m^2}{n^2}$ (B) $\frac{m^2-mn+n^2}{(m+n)^2}$

(C) $\frac{m^2-n^2}{m^2+n^2}$ (D) $\frac{m^2+n^2}{(m+n)^2}$

11. For any four points P, Q, R, S, $|\overline{PQ}\times\overline{RS}-\overline{QR}\times\overline{PS}+\overline{RP}\times\overline{QS}|$ is equal to 4 times the
area of the triangle

(A) PQR (B) QRS

(C) PRS (D) PQS

SCALAR TRIPLE PRODUCT

12. Given $\vec{a} = x\hat{i} + y\hat{j} + 2\hat{k}$, $\vec{b} = \hat{i} - \hat{j} + \hat{k}$, $\vec{c} = \hat{i} + 2\hat{j}$; $(\vec{a} \wedge \vec{b}) = \pi/2$, $\vec{a}\cdot\vec{c} = 4$, then

(A) $[\vec{a}\,\vec{b}\,\vec{c}]^2 = |\vec{a}|$ (B) $[\vec{a}\,\vec{b}\,\vec{c}] = |\vec{a}|$

(C) $[\vec{a}\,\vec{b}\,\vec{c}] = 0$ (D) $[\vec{a}\,\vec{b}\,\vec{c}] = |\vec{a}|^2$

13. Let $\vec{r}$ be a vector perpendicular to $\vec{a} + \vec{b} + \vec{c}$, where $[\vec{a}\,\vec{b}\,\vec{c}] = 2$. If $\vec{r} = \ell(\vec{b}\times\vec{c}) + m(\vec{c}\times\vec{a}) + n(\vec{a}\times\vec{b})$, then $(\ell + m + n)$ is equal to

(A) 2 (B) 1

(C) 0 (D) none of these

14. Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be non-coplanar unit vectors equally inclined to one another at an acute angle θ. Then $[\vec{a}\,\vec{b}\,\vec{c}]$ in terms of θ is equal to

(A) $(1 + \cos\theta)\sqrt{\cos 2\theta}$

(B) $(1 + \cos\theta)\sqrt{1-2\cos 2\theta}$

(C) $(1 - \cos\theta)\sqrt{1+2\cos 2\theta}$

(D) none of these

15. If the vectors $a\hat{i}+\hat{j}+\hat{k}$, $\hat{i}+b\hat{j}+\hat{k}$ and $\hat{i}+\hat{j}+c\hat{k}$ $(a \neq b \neq c \neq 1)$ are coplanar, then the value of $\frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c}$ is equal to

(A) 1 (B) –1

(C) 0 (D) none of these

VECTOR TIRPLE PRODUCT

16. $(\vec{d} + \vec{a}).(\vec{a} \times (\vec{b} \times (\vec{c} \times \vec{d})))$ simplifies to

(A) $(\vec{b}\cdot\vec{d})[\vec{a}\,\vec{c}\,\vec{d}]$ (B) $(\vec{b}\cdot\vec{c})[\vec{a}\,\vec{b}\,\vec{d}]$

(C) $(\vec{b}\cdot\vec{a})[\vec{a}\,\vec{b}\,\vec{d}]$ (D) none of these

17. $[(\vec{a}\times\vec{b})\times(\vec{b}\times\vec{c}), (\vec{b}\times\vec{c})\times(\vec{c}\times\vec{a}), (\vec{c}\times\vec{a})\times(\vec{a}\times\vec{b})]$ is equal to

(A) $[\vec{a}\vec{b}\vec{c}]^2$ (B) $[\vec{a}\vec{b}\vec{c}]^3$

(C) $[\vec{a}\vec{b}\vec{c}]^4$ (D) none of these

VECTOR PRODUCT OF 4 VECTORS

18. If $\vec{a}$, $\vec{b}$, $\vec{c}$ are three non-coplanar non-zero vectors and $\vec{r}$ is any vector in space, then $(\vec{a}\times\vec{b})\times(\vec{r}\times\vec{c}) + (\vec{b}\times\vec{c})\times(\vec{r}\times\vec{a}) + (\vec{c}\times\vec{a})\times(\vec{r}\times\vec{b})$ is equal to

(A) $2[\vec{a}, \vec{b}, \vec{c}]\vec{r}$ (B) $3[\vec{a}, \vec{b}, \vec{c}]\vec{r}$

(C) $[\vec{a}, \vec{b}, \vec{c}]\vec{r}$ (D) none of these

MIXED PROBLEMS

19. The vectors $\vec{a} = -4\hat{i} + 3\hat{k}, \vec{b} = 14\hat{i} + 2\hat{j} - 5\hat{k}$ are coinitial. The vector $\vec{d}$ which is bisecting the angle between the vectors $\vec{a}$ and $\vec{b}$ and is having the magnitude $\sqrt{6}$, is

(A) $\hat{i}+\hat{j}+2\hat{k}$ (B) $\hat{i}-\hat{j}+2\hat{k}$

(C) $\hat{i}+\hat{j}-2\hat{k}$ (D) none of these

20. If $\vec{b}$ and $\vec{c}$ are any two perpendicular unit vectors and $\vec{a}$ is any vector, then

$(\vec{a}\cdot\vec{b})\vec{b} + (\vec{a}\cdot\vec{c})\vec{c} + \frac{\vec{a}.(\vec{b}\times\vec{c})}{|\vec{b}\times\vec{c}|^2}(\vec{b}\times\vec{c})$ is equal to

(A) $\vec{a}$ (B) $\vec{b}$

(C) $\vec{c}$ (D) none of these

21. The vector $\hat{i}+x\hat{j}+3\hat{k}$ is rotated through an angle of $\cos^{-1}\frac{11}{14}$ and doubled in magnitude, then it becomes $4\hat{i}+(4x-2)\hat{j}+2\hat{k}$. The value of 'x' is

(A) $-\frac{2}{3}$ (B) $\frac{2}{3}$

(C) $\frac{1}{3}$ (D) 2

EXERCISE-II (LEVEL-II)

COLLINEARITY OF THREE POINTS

1. If a, b, c are different real numbers and $a\hat{i}+b\hat{j}+c\hat{k}, b\hat{i}+c\hat{j}+a\hat{k}$ and $c\hat{i}+a\hat{j}+b\hat{k}$ are position vectors of three non-collinear points A, B, and C, then

(A) centroid of triangle ABC is $\frac{a+b+c}{3}(\hat{i}+\hat{j}+\hat{k})$

(B) $\hat{i}+\hat{j}+\hat{k}$ is equally inclined to the three vectors

(C) perpendicular from the origin to the plane of triangle
ABC meet at centroid

(D) triangle ABC is an equilateral triangle.

RELATION BETWEEN TWO PARALLEL VECTORS

2. If a line has a vector equation $\vec{r}=2\hat{i}+6\hat{j}+\lambda(\hat{i}-3\hat{j})$, 2then which of the following statements hold goods ?

(A) the line is parallel to $2\hat{i}+6\hat{j}$

(B) the line passes through the point $2\hat{i}+6\hat{j}$

(C) the line passes through the point $\hat{i}+9\hat{j}$

(D) the line is parallel to XY-plane

3. The vector $\frac{1}{3}(2\hat{i}-2\hat{j}+\hat{k})$ is

(A) a unit vector

(B) makes an angle $\frac{\pi}{3}$ with the vector $2\hat{i}-4\hat{j}-3\hat{k}$

(C) parallel to the vector $-\hat{i}+\hat{j}-\frac{1}{2}\hat{k}$

(D) Perpendicular to the vector $3\hat{i}+2\hat{j}-2\hat{k}$

4. $\hat{a}$ and $\hat{b}$ are two given unit vectors at right angle. The unit vector equally inclined with $\hat{a}$, $\hat{b}$ and $\hat{a}\times\hat{b}$ will be

(A) $-\frac{1}{\sqrt{3}}(\hat{a}+\hat{b}+\hat{a}\times\hat{b})$

(B) $\frac{1}{\sqrt{3}}(\hat{a}+\hat{b}+\hat{a}\times\hat{b})$

(C) $\frac{1}{\sqrt{3}}(\hat{a}+\hat{b}-\hat{a}\times\hat{b})$

(D) $-\frac{1}{\sqrt{3}}(\hat{a}+\hat{b}-\hat{a}\times\hat{b})$

5. A line passes through a point A with position vector $3\hat{i}+\hat{j}-\hat{k}$ and parallel to the vector $2\hat{i}-\hat{j}+2\hat{k}$. If P is a point on this line such that AP = 15 units, then the position vector of the point P is/are

(A) $13\hat{i}+4\hat{j}-9\hat{k}$ (B) $13\hat{i}-4\hat{j}+9\hat{k}$

(C) $7\hat{i}-6\hat{j}+11\hat{k}$ (D) $-7\hat{i}+6\hat{j}-11\hat{k}$

SCALAR OR DOT PRODUCT OF TWO VECTORS

6. The vector $\vec{c}$, directed along the external bisector of the angle between the vectors $\vec{a}=7\hat{i}-4\hat{j}+4\hat{k}$ and $\vec{b}=2\hat{i}-\hat{j}+2\hat{k}$ with $|\vec{c}|=5\sqrt{6}$, is

(A) $(5\hat{i}-5\hat{j}-10\hat{k})$ (B) $\frac{5}{3}(\hat{i}+7\hat{j}-2\hat{k})$

(C) $-(5\hat{i}-5\hat{j}-10\hat{k})$ (D) $\frac{5}{3}(-\hat{i}+7\hat{j}+2\hat{k})$

7. If $\vec{z}_1=a\hat{i}+b\hat{j}$ and $\vec{z}_2=c\hat{i}+d\hat{j}$ are two vectors in $\hat{i}$ and $\hat{j}$ system, where $|\vec{z}_1|=|\vec{z}_2|=r$ and $\vec{z}_1.\vec{z}_2=0$, then $\vec{w}_1=a\hat{i}+c\hat{j}$ and $\vec{w}_2=b\hat{i}+d\hat{j}$ satisfy

(A) $|\vec{w}_1|=r$ (B) $|\vec{w}_2|=r$

(C) $\vec{w}_1.\vec{w}_2=0$ (D) none of these

8. If $\vec{a},\vec{b}$ are two non-collinear unit vectors and $\vec{a},\vec{b}, x\vec{a}-y\vec{b}$ form a triangle, then

(A) x = –1; y = 1 and $|\vec{a}+\vec{b}|=2\cos\left(\frac{\vec{a}\wedge\vec{b}}{2}\right)$

(B) x=–1; y=1 and $\cos(\vec{a}\wedge\vec{b})+|\vec{a}+\vec{b}|$
$\cos(\vec{a}\wedge-(\vec{a}+\vec{b}))=-1$

(C) $|\vec{a}+\vec{b}|=-2\cot\left(\frac{\vec{a}\wedge\vec{b}}{2}\right)\cos\left(\frac{\vec{a}\wedge\vec{b}}{2}\right)$ and x=–1, y=1

(D) none of these

9. The value(s) of $\alpha \in [0, 2\pi]$ for which vector $\vec{a} = \hat{i} + 3\hat{j} + (\sin 2\alpha)\hat{k}$ makes an obtuse angle with the z-axis and the vectors

$\vec{b} = (\tan\alpha)\hat{i} - \hat{j} + 2\sqrt{\sin\frac{\alpha}{2}}\hat{k}$ and

$\vec{c} = (\tan\alpha)\hat{i} + (\tan\alpha)\hat{j} - 3\sqrt{\operatorname{cosec}\frac{\alpha}{2}}\hat{k}$ are

orthogonal, is/are

(A) $\tan^{-1} 3$ (B) $\pi - \tan^{-1} 2$

(C) $\pi + \tan^{-1} 3$ (D) $2\pi - \tan^{-1} 2$

VECTOR OR CROSS PRODUCT OF TWO VECTORS

10. If $\vec{a} \times \vec{b} = \vec{c} \times \vec{d}$ and $\vec{a} \times \vec{c} = \vec{b} \times \vec{d}$, then the vectors $\vec{a} - \vec{d}$ and $\vec{b} - \vec{c}$ are

(A) collinear

(B) linearly independent

(C) perpendicular

(D) parallel

11. Unit vectors $\vec{a}, \vec{b}$ and $\vec{c}$ are coplanar. A unit vector $\vec{d}$ is perpendicular to them. If $(\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d}) = \frac{1}{6}\hat{i} - \frac{1}{3}\hat{j} + \frac{1}{3}\hat{k}$, and the angle between $\vec{a}$ and $\vec{b}$ is 30º, then $\vec{c}$ is

(A) $(\hat{i} - 2\hat{j} + 2\hat{k})/3$ (B) $(\hat{i} - 2\hat{j} + 2\hat{k})/3$

(C) $(-2\hat{i} - 2\hat{j} - \hat{k})/3$ (D) $(-\hat{i} + 2\hat{j} - 2\hat{k})/3$

SCALAR TRIPLE PRODUCT

12. The volume of a right triangular prism $ABCA_1B_1C_1$ is equal to 3. If the position vectors of the vertices of the base ABC are A(1, 0, 1), B(2, 0, 0) and C(0, 1, 0), then position vectors of the vertex A_1 can be

(A) (2, 2, 2) (B) (0, 2, 0)

(C) (0, –2, 2) (D) (0, –2, 0)

13. Let $\vec{p} = 2\hat{i} + 3\hat{j} - a\hat{k}$, $\vec{q} = b\hat{i} + 5\hat{j} - \hat{k}$ and $\vec{r} = \hat{i} + \hat{j} + 3\hat{k}$. If $\vec{p}, \vec{q}, \vec{r}$ are coplanar and $\vec{p}.\vec{q} = 20$, then a and b have the values

(A) –1, 3 (B) 9, 7

(C) 5, 5 (D) –13, 9

14. If $\vec{a}, \vec{b}, \vec{c}$ be three non-zero vectors satisfying the condition $\vec{a} \times \vec{b} = \vec{c}$ and $\vec{b} \times \vec{c} = \vec{a}$, then

(A) $\vec{a}, \vec{b}, \vec{c}$ are orthogonal is pairs

(B) $[\vec{a}, \vec{b}, \vec{c}] = [\vec{a}]^2$

(C) $[\vec{a}\,\vec{b}\,\vec{c}] = |\vec{c}|^2$

(D) $|\vec{b}| = |\vec{c}|$

VECTOR TRIPLE PRODUCT

15. If $\vec{a} = x\hat{i} + y\hat{j} + z\hat{k}, \vec{b} = y\hat{i} + z\hat{j} + x\hat{k}$ and $\vec{c} = z\hat{i} + x\hat{j} + y\hat{k}$, then $\vec{a} \times (\vec{b} \times \vec{c})$ is

(A) parallel to $(y - z)\hat{i} + (z - x)\hat{j} + (x - y)\hat{k}$

(B) orthogonal to $\hat{i} + \hat{j} + \hat{k}$

(C) orthogonal to $(y + z)\hat{i} + (z + x)\hat{j} + (x + y)\hat{k}$

(D) orthogonal to $x\hat{i} + y\hat{j} + z\hat{k}$

EXERCISE-III

1. The position vector of two points A and B are $6\vec{a}+2\vec{b}$ and $\vec{a}-3\vec{b}$. If a point C divides AB in the ratio 3 : 2 then show that the position vector of C is $3\vec{a}-\vec{b}$.

2. In a ΔOAB, E is the mid-point of OB and D is a point on AB such that AD : DB = 2 : 1. if OD and AE intersect at P, then determine the ratio OP : PD using vector methods.

3. Show that the points
$\vec{a}-2\vec{b}+3\vec{c}$; $2\vec{a}+3\vec{b}-4\vec{c}$ & $-7\vec{b}+10\vec{c}$
are collinear.

4. If $\vec{r}=(\hat{i}+2\hat{j}+3\hat{k})+\lambda(\hat{i}-\hat{j}+\hat{k})$ and $\vec{r}=(\hat{i}+2\hat{j}+3\hat{k})+\mu(\hat{i}+\hat{j}-\hat{k})$ are two lines, then find the equation of acute angle bisector of two lines.

5. If the three successive vertices of a parallelogram have the position vectors as,
A(–3, –2, 0); B(3, –3, 1) and C(5, 0, 2). Then find

(i) Position vector of the fourth vertex D

(ii) A vector having the same direction as that of $\overrightarrow{AB}$ but magnitude equal to $\overrightarrow{AC}$

(iii) The angle between $\overrightarrow{AC}$ and $\overrightarrow{BD}$.

6. (i) If $\hat{e}_1$ and $\hat{e}_2$ are two unit vectors such that $\vec{e}_1-\vec{e}_2$ is also a unit vector, then find the angle θ between $\hat{e}_1$ and $\hat{e}_2$.

(ii) Prove that $\left(\frac{\vec{a}}{\vec{a}^2}-\frac{\vec{b}}{\vec{b}^2}\right)^2=\left(\frac{\vec{a}-\vec{b}}{|\vec{a}||\vec{b}|}\right)^2$

7. Given that $\vec{x}+\frac{1}{\vec{p}^2}(\vec{p}.\vec{x})\vec{p}=\vec{q}$, then show that $\vec{p}.\vec{x}=\frac{1}{2}(\vec{p}.\vec{q})$ and find $\vec{x}$ in terms of $\vec{p}$ and $\vec{q}$.

8. Find out whether the following pairs of lines are parallel, non parallel; & intersecting, or non-parallel & non-intersecting.

(i) $\vec{r}_1=\hat{i}+\hat{j}+2\hat{k}+\lambda(3\hat{i}-2\hat{j}+4\hat{k})$
$\vec{r}_2=2\hat{i}+\hat{j}+3\hat{k}+\mu(-6\hat{i}+4\hat{j}-8\hat{k})$

(ii) $\vec{r}_1=\hat{i}-\hat{j}+3\hat{k}+\lambda(\hat{i}-\hat{j}+\hat{k})$
$\vec{r}_2=2\hat{i}+4\hat{j}+6\hat{k}+\mu(2\hat{i}+\hat{j}+3\hat{k})$

(iii) $\vec{r}_1=\hat{i}+\hat{k}+\lambda(\hat{i}+3\hat{j}+4\hat{k})$
$\vec{r}_2=2\hat{i}+3\hat{j}+\mu(4\hat{i}-\hat{j}+\hat{k})$

9. Let OACB be parallelogram with O at the origin & OC a diagonal. Let D be the mid point of OA. Using vector method prove that BD & CO intersect in the same ratio. Determine this ratio.

10. Find the shortest distance between the lines :
$\vec{r}=(4\hat{i}-\hat{j})+\lambda(\hat{i}+2\hat{j}-3\hat{k})$
and
$\vec{r}=(\hat{i}-\hat{j}+2\hat{k})+\mu(2\hat{i}+4\hat{j}-5\hat{k})$

11. (i) Let $\vec{A}=2\hat{i}+\hat{k}$, $\vec{B}=\hat{i}+\hat{j}+\hat{k}$ and $\vec{C}=4\hat{i}-3\hat{j}+7\hat{k}$. Determine a vector $\vec{R}$ satisfying $\vec{R}\times\vec{B}=\vec{C}\times\vec{B}$ and $\vec{R}.\vec{A}=0$

(ii) Find vector $\vec{v}$ which is coplanar with the vectors $\hat{i}+\hat{j}-2\hat{k}$ and $\hat{i}-2\hat{j}+\hat{k}$ and is orthogonal to the vector $-2\hat{i}+\hat{j}+\hat{k}$. It is given that the projection of $\vec{v}$ along the vector $\hat{i}-\hat{j}+\hat{k}$ is equal to $6\sqrt{3}$.

12. Find the point R in which the line AB cuts the plane CDE, where position vectors of points A, B, C, D, E are respectively
$\vec{a}=\hat{i}+2\hat{j}+\hat{k}$, $\vec{b}=2\hat{i}+\hat{j}+2\hat{k}$, $\vec{c}=-4\hat{j}+4\hat{k}$, $\vec{d}=2\hat{i}-2\hat{j}+2\hat{k}$ and $\vec{e}=4\hat{i}+\hat{j}+2\hat{k}$.

13. The position vectors of the angular points of a tetrahedron are $A(3\hat{i}-2\hat{j}+\hat{k})$, $B(3\hat{i}+\hat{j}+5\hat{k})$, $C(4\hat{i}+\hat{k})$ and $D(\hat{i})$. Then find the acute angle between the lateral faces ADC and the base ABC.

14. Examine for coplanarity of the following sets of points

(i) $4\hat{i}+8\hat{j}+12\hat{k},\ \ 2\hat{i}+4\hat{j}+6\hat{k},\ \ 3\hat{i}+5\hat{j}+4\hat{k},\ \ 5\hat{i}+8\hat{j}+5\hat{k}.$

(ii) $3\vec{a}+2\vec{b}-5\vec{c},\ \ 3\vec{a}+8\vec{b}+5\vec{c},\ \ -3\vec{a}+2\vec{b}+\vec{c},\ \ \vec{a}+4\vec{b}-3\vec{c}.$

15. The length of the edge of the regular tetrahedron DABC is 'a'. Point E and F are taken on the edges AD and BD respectively such that E divides $\overrightarrow{DA}$ and F divides $\overrightarrow{BD}$ in the ratio 2 : 1 each. Then find the area of triangle CEF.

16. The pv's of the four angular points of a tetrahedron are : $A(\hat{j}+2\hat{k});B(3\hat{i}+\hat{k});C(4\hat{i}+3\hat{j}+6\hat{k})$ & $D(2\hat{i}+3\hat{j}+2\hat{k})$. Find :

(i) The perpendicular distance from A to the line BC.

(ii) The volume of the tetrahedron ABCD.

(iii) The perpendicular distance from D to the plane ABC.

(iv) The shortest distance between the lines AB & CD.

17. ABCD is a tetrahedron with pv's of its angular points as A(–5, 22, 5); B(1, 2, 3); C(4, 3, 2) and D(–1, 2, –3). If the area of the triangle AEF where the quadrilaterals ABDE and ABCF are parallelograms is $\sqrt{S}$ then find the value of S.

18. Let $\vec{a}=\hat{i}+2\hat{j}+3\hat{k},\ \vec{b}=2\hat{i}-\hat{j}+\hat{k},\ \vec{c}=3\hat{i}+2\hat{j}+\hat{k}$ and $\vec{d}=3\hat{i}-\hat{j}-2\hat{k}$, then

(i) If $\vec{a}\times(\vec{b}\times\vec{c})=p\vec{a}+q\vec{b}+r\vec{c}$, then find value of p, q are r.

(ii) Find the value of $(\vec{a}\times\vec{b})\times(\vec{a}\times\vec{c}).\vec{d}$

19. Are the following set of vectors linearly independent?

(i) $\vec{a}=\hat{i}-2\hat{j}+3\hat{k},\vec{b}=3\hat{i}-6\hat{j}+9\hat{k}$

(ii) $\vec{a}=-2\hat{i}-4\hat{k},\vec{b}=\hat{i}-2\hat{j}-\hat{k},\vec{c}=\hat{i}-4\hat{j}+3\hat{k}$

20. The resultant of two vectors $\vec{a}\ \&\ \vec{b}$ is perpendicular to $\vec{a}$. If $|\vec{b}|=\sqrt{2}\,|\vec{a}|$ show that the resultant of $2\vec{a}\ \&\ \vec{b}$ is perpendicular to $\vec{b}$.

21. Given three points on the xy plane on O(0, 0), A(1, 0) and B(–1, 0). Point P is moving on the plane satisfying the condition $(\overrightarrow{PA}.\overrightarrow{PB})+3(\overrightarrow{OA}.\overrightarrow{OB})=0$. If the maximum and minimum values of $|\overrightarrow{PA}||\overrightarrow{PB}|$ are M and m respectively then find the value of M^2+m^2.

22. The vector $\overrightarrow{OP}=\hat{i}+2\hat{j}+2\hat{k}$ turns through a right angle, passing through the positive x-axis on the way. Find the vector in its new position.

23. If $p\vec{x}+(\vec{x}\times\vec{a})=\vec{b};(p\neq 0)$

prove that $\vec{x}=\dfrac{p^2\vec{b}+(\vec{b}.\vec{a})\vec{a}-p(\vec{b}\times\vec{a})}{p(p^2+a^2)}$

COMPREHENSION – 1

Let O, N, G and O' are the circumcentre, nine point centre, centroid and orthocentre of a ΔABC respectively. AL and BM are perpendiculars from A and B on sides BC and CA respectively. Let AD be the median and OD is perpendicular to side BC. Let R be the circum radius of ΔABC, then OA = OB = OC = R.

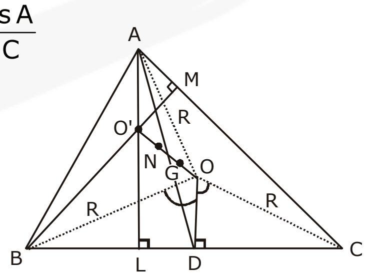
Now, in ΔOBD, OD = R cos A, in ΔAMB,

AO' = AM sec (90° – C) ($\because$ ∠O' AM = 90° – C)

$= \text{AM cosec C} = \dfrac{C\cos A}{\sin C}$

= 2R cos A

$\therefore$ AO' = 2OD



If S be any point in the plane of ΔABC and AP is the diameter of the circum circle.

24. $\overrightarrow{OA}+\overrightarrow{OB}+\overrightarrow{OC}$ is equal to

(A) $\overrightarrow{OO'}$ (B) $2\overrightarrow{O'O}$

(C) $2\overrightarrow{AO}$ (D) $\overrightarrow{ON}$

25. $\overrightarrow{O'A}+\overrightarrow{O'B}$ + O' C is equal to

(A) $\overrightarrow{O'B}$ (B) $2\overrightarrow{O'O}$

(C) $2\overrightarrow{AO'}$ (D) $2\overrightarrow{O'N}$

26. $\overrightarrow{AO'}+\overrightarrow{O'B}+\overrightarrow{O'C}$ is equal to

(A) $\overrightarrow{OO'}$ (B) $2\overrightarrow{O'O}$

(C) $2\overrightarrow{AO}$ (D) $3\overrightarrow{SG}$

Comprehension - 2

Given two orthogonal vectors $\vec{A}$ and $\vec{B}$ each of length unity. Let $\vec{P}$ be the vector satisfying the equation $\vec{P}\times\vec{B}=\vec{A}-\vec{P}$. Then

27. $(\vec{P}\times\vec{B})\times\vec{B}$ is equal to

(A) $\vec{P}$ (B) $-\vec{P}$

(C) $2\vec{B}$ (D) $\vec{A}$

28. $\vec{P}$ is equal to

(A) $\frac{\vec{A}}{2}+\frac{\vec{A}\times\vec{B}}{2}$ (B) $\frac{\vec{A}}{2}+\frac{\vec{B}\times\vec{A}}{2}$

(C) $\frac{\vec{A}\times\vec{B}}{2}-\frac{\vec{A}}{2}$ (D) $\vec{A}\times\vec{B}$

29. Which of the following statements is false ?

(A) vectors $\vec{P}$, $\vec{A}$ and $\vec{A}\times\vec{B}$ are linearly dependent

(B) vectors $\vec{P}$, $\vec{A}$ and $\vec{A}\times\vec{B}$ are linearly independent

(C) $\vec{P}$ is orthogonal to $\vec{B}$ and has length $1/\sqrt{2}$

(D) None of these

MATRIX MATCH TYPE

30. Observe the following columns :

Column - I	Column - I
(A) If V_1, V_2, V_3 are the volumes of parallelopiped, triangular prism and tetrahedron respectively. The three coterminus edges of all three figures are the vectors $\hat{i}-\hat{j}-6\hat{k}$, $\hat{i}-\hat{j}+4\hat{k}$ and $2\hat{i}-5\hat{j}+3\hat{k}$, then	(P) $2V_1+3V_3=5V_2$
(B) If V_1, V_2, V_3 are the volumes of parallelopiped, triangular prism and tetrahedron respectively. The three coterminus edges of all three figures are the vectors $-2\hat{i}+3\hat{j}-3\hat{k}$, $4\hat{i}+5\hat{j}-3\hat{k}$ and $6\hat{i}+2\hat{j}-3\hat{k}$, then	(Q) $V_1+V_2+V_3=60$
	(R) $V_1+3V_3=3V_2$
(C) If V_1, V_2, V_3 are the volumes of parallelopiped, triangular prism and tetrahedron respectively. The three coterminus edges of all three figures are the vectors $-3\hat{i}+\hat{j}+\hat{k}$, $4\hat{i}+2\hat{j}+4\hat{k}$ and $2\hat{i}+2\hat{j}$, then	(S) $V_1+V_2+V_3=50$
	(T) $V_1:V_2:V_3=6:3:1$

EXERCISE-IV (JEE-MAIN)

1. If $\vec{u}, \vec{v}, \vec{w}$ are non-coplanar vectors and p, q are real numbers, then the equality $[3\vec{u}\ p\vec{v}\ p\vec{w}] - [p\vec{v}\ \vec{w}\ q\vec{u}] - [2\vec{w}\ q\vec{v}\ q\vec{u}] = 0$ holds for : **[AIEEE 2009]**

(A) exactly two values of (p,q)
(B) more than two but not all values of (p, q)
(C) all values of (p, q)
(D) exactly one value of (p, q)

2. Let $\vec{a} = \hat{j} - \hat{k}$ and $\vec{c} = \hat{i} - \hat{j} - \hat{k}$. The the vector $\vec{b}$ satisfying $\vec{a} \times \vec{b} + \vec{c} = \vec{0}$ and $\vec{a} \cdot \vec{b} = 3$ is **[AIEEE 2010]**

(A) $-\hat{i} + \hat{j} - 2\hat{k}$ (B) $2\hat{i} - \hat{j} + 2\hat{k}$
(C) $\hat{i} - \hat{j} - 2\hat{k}$ (D) $\hat{i} + \hat{j} - 2\hat{k}$

3. If the vectors $\vec{a} = \hat{i} - \hat{j} + 2\hat{k}$, $\vec{b} = 2\hat{i} + 4\hat{j} + \hat{k}$ and $\vec{c} = \lambda\hat{i} + \hat{j} + \mu\hat{k}$ are mutually orthogonal, then $(\lambda, \mu) =$ **[AIEEE 2010]**

(A) (–3, 2) (B) (2, –3)
(C) (–2,3) (D) (3, –2)

4. The vectors $\vec{a}$ and $\vec{b}$ are not perpendicular and $\vec{c}$ and $\vec{d}$ are two vectors satisfying : $\vec{b} \times \vec{c} = \vec{b} \times \vec{d}$ and $\vec{a} \cdot \vec{d} = 0$. Then the vector $\vec{d}$ is equal to : **[AIEEE 2011]**

(A) $\vec{b} - \left(\dfrac{\vec{b}\cdot\vec{c}}{\vec{a}\cdot\vec{b}}\right)\vec{c}$ (B) $\vec{c} + \left(\dfrac{\vec{a}\cdot\vec{c}}{\vec{a}\cdot\vec{b}}\right)\vec{b}$

(C) $\vec{b} + \left(\dfrac{\vec{b}\cdot\vec{c}}{\vec{a}\cdot\vec{b}}\right)\vec{c}$ (D) $\vec{c} - \left(\dfrac{\vec{a}\cdot\vec{c}}{\vec{a}\cdot\vec{b}}\right)\vec{b}$

5. Let $\hat{a}$ and $\hat{b}$ be two unit vectors. If the vectors $\vec{c} = \hat{a} + 2\hat{b}$ and $\vec{d} = 5\hat{a} - 4\hat{b}$ are perpendicular to each other, then the angle between $\vec{a}$ and $\vec{b}$ is: **[AIEEE 2012]**

(A) $\frac{\pi}{3}$ (B) $\frac{\pi}{4}$

(C) $\frac{\pi}{6}$ (D) $\frac{\pi}{2}$

6. Let ABCD be a parallelogram such that $\overrightarrow{AB} = \vec{q}$, $\overrightarrow{AD} = \vec{p}$ and $\angle BAD$ be an acute angle. If $\vec{r}$ is the vector that coincides with the altitude directed from the vertex B to the side AD, then $\vec{r}$ is given by : **[AIEEE 2012]**

(A) $\vec{r} = \vec{q} - \left(\dfrac{\vec{p}\cdot\vec{q}}{\vec{p}\cdot\vec{p}}\right)\vec{p}$ (B) $\vec{r} = -3\vec{q} + \dfrac{3(\vec{p}\cdot\vec{q})}{(\vec{p}\cdot\vec{p})}\vec{p}$

(C) $\vec{r} = 3\vec{q} - \dfrac{3(\vec{p}\cdot\vec{q})}{(\vec{p}\cdot\vec{p})}\vec{p}$ (D) $\vec{r} = -\vec{q} + \left(\dfrac{\vec{p}\cdot\vec{q}}{\vec{p}\cdot\vec{p}}\right)\vec{p}$

7. If the vectors $\overrightarrow{AB} = 3\hat{i} + 4\hat{k}$ and $\overrightarrow{AC} = 5\hat{i} - 2\hat{j} + 4\hat{k}$ are the sides of a triangle ABC, then the length of the median through A is : **[JEE-MAIN 2013]**

(A) $\sqrt{33}$ (B) $\sqrt{45}$

(C) $\sqrt{18}$ (D) $\sqrt{72}$

8. If $\left[\vec{a}\times\vec{b}\ \ \vec{b}\times\vec{c}\ \ \vec{c}\times\vec{a}\right]=\lambda\left[\vec{a}\,\vec{b}\,\vec{c}\right]^2$ then λ is equal to : **[JEE-MAIN 2014]**

(A) 2 (B) 3
(C) 0 (D) 1

9. The angle between the lines whose direction cosines satisfy the equations $\ell + m + n = 0$ and $\ell^2 = m^2 + n^2$ is : **[JEE-MAIN 2014]**

(A) $\frac{\pi}{3}$ (B) $\frac{\pi}{4}$
(C) $\frac{\pi}{6}$ (D) $\frac{\pi}{2}$

10. Let $\vec{a},\vec{b}$ and $\vec{c}$ be three non- zero vectors such that no two of them are collinear and $(\vec{a}\times\vec{b})\times\vec{c} = \frac{1}{3}\left|\vec{b}\right|\left|\vec{c}\right|\vec{a}$. If θ is the angle between vectors $\vec{b}$ and $\vec{c}$, then a value of $\sin\theta$ is : **[JEE-MAIN 2015]**

(A) $\frac{2}{3}$ (B) $\frac{-2\sqrt{3}}{3}$
(C) $\frac{2\sqrt{2}}{3}$ (D) $\frac{-\sqrt{2}}{3}$

11. Let $\vec{a},\vec{b}$ and $\vec{c}$ be three unit vectors such that $\vec{a}\times\left(\vec{b}\times\vec{c}\right)=\frac{\sqrt{3}}{2}\left(\vec{b}+\vec{c}\right)$. If $\vec{b}$ is not parallel to $\vec{c}$, then the angle between $\vec{a}$ and $\vec{b}$ is: **[JEE – MAIN 2016]**

(A) $\frac{\pi}{2}$ (B) $\frac{2\pi}{3}$
(C) $\frac{5\pi}{6}$ (D) $\frac{3\pi}{4}$

12. Let $\vec{u}$ be a vector coplanar with the vectors $\vec{a}=2\hat{i}+3\hat{j}-\hat{k}$ and $\vec{b}=\hat{j}+\hat{k}$. if $\vec{u}$ is perpendicular to $\vec{a}$ and $\vec{u}\cdot\vec{b}=24$, then $|\vec{u}|^2$ is equal to : **[JEE-MAIN 2018]**

(A) 84 (B) 336
(C) 315 (D) 256

EXERCISE-IV (JEE-ADVANCED)

1. Let P, Q, R and S be the points on the plane with position vectors $-2\hat{i}-\hat{j}$, $4\hat{i}$, $3\hat{i}+3\hat{j}$ & $-3\hat{i}+2\hat{j}$ respectively. The quadrilateral PQRS must be a **[JEE 2010]**
(A) parallelogram, which is neither a rhombus nor a rectangle
(B) square
(C) rectangle, but not a square
(D) rhombus, but not a square

2. If $\vec{a}$ and $\vec{b}$ are vectors in space given by $\vec{a}=\dfrac{\hat{i}-2\hat{j}}{\sqrt{5}}$ and $\vec{b}=\dfrac{2\hat{i}+\hat{j}+3\hat{k}}{\sqrt{14}}$, then the value of $(2\vec{a}+\vec{b}).\left[(\vec{a}\times\vec{b})\times(\vec{a}-2\vec{b})\right]$ is **[JEE 2010]**

3. Let $\vec{a}=\hat{i}+\hat{j}+\hat{k}, \vec{b}=\hat{i}-\hat{j}+\hat{k}$ and $\vec{c}=\hat{i}-\hat{j}-\hat{k}$ be three vectors. A vector $\vec{v}$ in the plane of $\vec{a}$ and $\vec{b}$, whose projection on $\vec{c}$ is $\dfrac{1}{\sqrt{3}}$, is given by **[JEE 2011]**
(A) $\hat{i}-3\hat{j}+3\hat{k}$ (B) $-3\hat{i}-3\hat{j}-\hat{k}$
(C) $3\hat{i}-\hat{j}+3\hat{k}$ (D) $\hat{i}+3\hat{j}-3\hat{k}$

4. The vector(s) which is/are coplanar with vectors $\hat{i}+\hat{j}+2\hat{k}$ and $\hat{i}+2\hat{j}+\hat{k}$, and perpendicular to the vector $\hat{i}+\hat{j}+\hat{k}$ is/are **[JEE 2011]**
(A) $\hat{j}-\hat{k}$ (B) $-\hat{i}+\hat{j}$
(C) $\hat{i}-\hat{j}$ (D) $-\hat{j}+\hat{k}$

5. Let $\vec{a}=-\hat{i}-\hat{k}, \vec{b}=-\hat{i}+\hat{j}$ and $\vec{c}=\hat{i}+2\hat{j}+3\hat{k}$ be three given vectors. If $\vec{r}$ is a vector such that $\vec{r}\times\vec{b}=\vec{c}\times\vec{b}$ and $\vec{r}.\vec{a}=0$, then the value of $\vec{r}.\vec{b}$ is **[JEE 2011]**

6. Let $\overrightarrow{PR}=3\hat{i}+\hat{j}-2\hat{k}$ and $\overrightarrow{SQ}=\hat{i}-3\hat{j}-4\hat{k}$ determine diagonals of a parallelogram PQRS and $\overrightarrow{PT}=\hat{i}+2\hat{j}+3\hat{k}$ be another vector. Then the volume of the parallelepiped determined by the vectors $\overrightarrow{PT},\overrightarrow{PQ}$ and $\overrightarrow{PS}$ is **[JEE 2013]**
(A) 5 (B) 20
(C) 10 (D) 30

7. Consider the set of eight vectors $V=\{a\hat{i}+b\hat{j}+c\hat{k} : a, b, c \in \{-1, 1\}\}$. Three non-coplanar vectors can be chosen from V in 2^p ways. Then p is **[JEE 2013]**

8. Match List-I with List-II and select the correct answer using code given the lists **[JEE 2013]**

	List-I	List-II
P.	Volume of parallelepiped determined by vectors $\vec{a},\vec{b}$ and $\vec{c}$ is 2. then the volume of the parallelepiped determined by vectors $2(\vec{a}\times\vec{b}),3(\vec{b}\times\vec{c})$ and $(\vec{c}\times\vec{a})$ is	**1.** 100
Q.	Volume of parallelepiped determined by vectors $\vec{a},\vec{b}$ and $\vec{c}$ is 5. Then the volume of the parallelepioed determined by vectors $3(\vec{a}+\vec{b}),(\vec{b}+\vec{c})$ and $2(\vec{c}+\vec{a})$ is	**2.** 30
R.	Area of a triangel with adjacent sides determined by vectors $\vec{a}$ and $\vec{b}$ is 20. Then area of the triangle with adjacent sides determined by vectos $(2\vec{a}+3\vec{b})$ and $(\vec{a}-\vec{b})$ is	**3.** 24
S.	Area of a parallelogram with adjacent sides determined by vectors $\vec{a}$ and $\vec{b}$ is 30. Then the area of the parallelogram with adjacent sides determined by vectors $(\vec{a}+\vec{b})$ and $\vec{a}$ is	**4.** 60

Codes :

	P	Q	R	S
(A)	3	2	4	1
(B)	1	3	4	2
(C)	3	4	1	2
(D)	2	4	1	3

9. Let $\vec{x}$, $\vec{y}$ and $\vec{z}$ be three vectors each of magnitude $\sqrt{2}$ and the angle between each pair of them is $\frac{\pi}{3}$. If $\vec{a}$ is a nonzero vector perpendicular to $\vec{x}$ and $\vec{y} \times \vec{z}$ and $\vec{b}$ is a nonzero vector perpendicular to $\vec{y}$ and $\vec{z} \times \vec{x}$, then **[JEE 2014]**

(A) $\vec{b} = (\vec{b} \cdot \vec{z})(\vec{z} - \vec{x})$

(B) $\vec{a} = (\vec{a} \cdot \vec{y})(\vec{y} - \vec{z})$

(C) $\vec{a} \cdot \vec{b} = -(\vec{a} \cdot \vec{y})(\vec{b} \cdot \vec{z})$

(D) $\vec{a} = (\vec{a} \cdot \vec{y})(\vec{z} - \vec{y})$

10. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three non-coplanar unit vectors such that the angle between every pair of them is $\frac{\pi}{3}$. If $\vec{a} \times \vec{b} + \vec{b} \times \vec{c} = p\vec{a} + q\vec{b} + r\vec{c}$, where p, q and r are scalars, then the value of $\frac{p^2 + 2q^2 + r^2}{q^2}$ is

[JEE 2014]

11. Let ΔPQR be a triangle. Let $\vec{a} = \overline{QR}$, $\vec{b} = \overline{RP}$ and $\vec{c} = \overline{PQ}$. If $|\vec{a}| = 12$, $|\vec{b}| = 4\sqrt{3}$ and $\vec{b}.\vec{c} = 24$, then which of the following is (are) true?

[JEE 2015]

(A) $\frac{|\vec{c}|^2}{2} - |\vec{a}| = 12$

(B) $\frac{|\vec{c}|^2}{2} + |\vec{a}| = 30$

(C) $|\vec{a} \times \vec{b} + \vec{c} \times \vec{a}| = 48\sqrt{3}$

(D) $\vec{a}.\vec{b} = -72$

12. **Column I**

(A) In a triangle ΔXYZ, let a, b and c be the lengths of the sides opposite to the angles X, Y and Z, respectively. If $2(a^2 - b^2) = c^2$ and $\lambda = \frac{\sin(X - Y)}{\sin Z}$, then possible values of n for which $\cos(n\pi\lambda) = 0$ is (are)

(B) In a triangle ΔXYZ, let a, b and c the lengths of the sides opposite to the angles X, Y and Z, respectively. If $1 + \cos 2X - 2\cos 2Y = 2\sin X \sin Y$, then possible value(s) of $\frac{a}{b}$ is (are)

(C) In R^2, let $\sqrt{3}\hat{i} + \hat{j}$, $\hat{i} + \sqrt{3}\hat{j}$ and $\beta\hat{i} + (1 - \beta)\hat{j}$ be the position vectors of X, Y and Z with respect to the origin O, respectively. If the distance of Z from the bisector of the acute angle $\overline{OX}$ with $\overline{OY}$ is $\frac{3}{\sqrt{2}}$ then possible value(s) of $|\beta|$ is (are)

(D) Suppose that $F(\alpha)$ denotes the area of the region bounded by $x = 0$, $x = 2$, $y^2 = 4x$ and $y = |\alpha x - 1| + |\alpha x - 2| + \alpha x$, where $\alpha \in \{0, 1\}$. Then the value(s) of $F(\alpha) + \frac{8}{3}\sqrt{2}$, when $\alpha = 0$ and $\alpha = 1$, is (are)

Column II

(P) 1

(Q) 2

(R) 3

(S) 5

(T) 6

13. Let $\hat{u} = u_1\hat{i} + u_2\hat{j} + u_3\hat{k}$ be a unit vector in R^3 and $\hat{w} = \frac{1}{\sqrt{6}}(\hat{i} + \hat{j} + 2\hat{k})$. Given that there exists a vector $\vec{v}$ in R^3 such that $|\hat{u} \times \vec{v}| = 1$ and $\hat{w} \cdot (\hat{u} \times \vec{v}) = 1$. Which of the following statement(s) is(are) correct ?

(A) There is exactly one choice for such $\vec{v}$

(B) There are infinitely many choices for such $\vec{v}$

(C) If $\hat{u}$ lies in the xy-plane then $|u_1| = |u_2|$

(D) If $\hat{u}$ lies in the xz-plane then $2|u_1| = |u_3|$

[JEE 2016]

14. Let O be the origin and let PQR be an arbitrary triangle. The point S is such that $\overline{OP}.\overline{OQ} + \overline{OR}.\overline{OS} = \overline{OR}.\overline{OP} + \overline{OQ}.\overline{OS} = \overline{OQ}.\overline{OR} + \overline{OP}.\overline{OS}$ Then the triangle PQR has S as its **[JEE Adv. 2017]**

(A) circmcentre (B) Incentre

(C) Centroid (D) orthocenter

Paragraph 15 to 16

Let O be the origin, and $\overrightarrow{OX}$, $\overrightarrow{OY}$, $\overrightarrow{OZ}$ be three unit vectors in the directions of the sides $\overrightarrow{QR}$, $\overrightarrow{RP}$, $\overrightarrow{PQ}$, respectively, of a triangle PQR.

[JEE Adv. 2017]

15. If the triangle PQR varies, then the minimum value of cos(p + Q) + cos(Q + R) + Cos(R +P) is

(A) $\frac{3}{2}$ (B) $\frac{5}{3}$

(C) $-\frac{5}{3}$ (D) $-\frac{3}{2}$

16. $\left|\overrightarrow{OX} \times \overrightarrow{OY}\right| =$

(A) sin(P + R) (B) sin(Q + R)

(C) sin(P + Q) (D) sin2R

17. Let $\vec{a}$ and $\vec{b}$ be two unit vectors such that $\vec{a}.\vec{b} = 0$. For some x, y ∈ R, let $\vec{c} = x\vec{a} + y\vec{b} + (\vec{a} \times \vec{b})$. If $|\vec{c}| = 2$ and the vector $\vec{c}$ is inclined at the same angle α to both $\vec{a}$ and $\vec{b}$, then the value of $8\cos^2\alpha$ is ________ **[JEE Adv. 2018]**

18. Let $\vec{a} = 2\hat{i} + \hat{j} - \hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} + \hat{k}$ be two vectors. Consider a vector $\vec{c} = \alpha\vec{a} + \beta\vec{b}$. $\alpha, \beta, \in$. If the projection of $\vec{c}$ on the vector $(\vec{a} + \vec{b})$ is $3\sqrt{2}$, then the minimum value of $(\vec{c} - (\vec{a} \times \vec{b})) \cdot \vec{c}$ equals______ **[JEE Adv. 2019]**

ANSWER KEYS

EXERCISE - I

JEE Main

1.	D	2.	D	3.	C	4.	C	5.	A	6.	A	7.	A
8.	C	9.	A	10.	B	11.	A	12.	D	13.	D	14.	A
15.	B	16.	C	17.	B	18.	B	19.	C	20.	D	21.	B
22.	A	23.	D	24.	C	25.	C	26.	D	27.	B	28.	C
29.	C	30.	A	31.	B	32.	C	33.	B	34.	D	35.	C
36.	A	37.	B	38.	B	39.	A	40.	B	41.	D	42.	B

EXERCISE - II

(Level - I) Single correct Option - type Questions

1.	B	2.	B	3.	A	4.	C	5.	D	6.	C	7.	C
8.	A	9.	D	10.	B	11.	B	12.	D	13.	C	14.	C
15.	A	16.	A	17.	C	18.	A	19.	A	20.	A	21.	D

(Level - II) Multiple correct Option - type Questions

1.	A,B,C,D	2.	C,D	3.	A,C,D	4.	A,B	5.	B,D
6.	A,C	7.	A,B,C	8.	A,B	9.	B,D	10.	A,D
11.	A,D	12.	A,D	13.	A,D	14.	A,B,C	15.	A,B,C,D

EXERCISE - III

Subjective - type Questions

2. 3 : 2 **4.** $(1, 2, 3) + \lambda(\hat{j}-\hat{k})$ **5. (i)** D(–1, 1, 1) , **(ii)** $\frac{6}{\sqrt{19}}(6,-1,1)$, **(iii)** $\frac{2\pi}{3}$

6. (i) $\frac{\pi}{3}$ **7.** $\vec{x}=\vec{q}-\frac{(\vec{p}.\vec{q})\vec{p}}{2p^2}$ **8. (i)** parallel ; **(ii)** intersecting ; **(iii)** non intersecting

9. 2 : 1 **10.** $\frac{6}{\sqrt{5}}$ **11. (i)** $\vec{R}=(-1,-8-2)$; **(ii)** $\vec{v}=9(-\hat{j}+\hat{k})$

12. $\left(\frac{19}{8},\frac{11}{8},\frac{19}{8}\right)$ **13.** $\cos\theta = \frac{2}{\sqrt{89}\sqrt{41}}$ **15.** $\frac{5a^2}{12\sqrt{3}}$ sq. units

16. (i) $\frac{6}{7}\sqrt{14}$ **(ii)** 6 **(iii)** $\frac{3}{5}\sqrt{10}$ **(iv)** $\sqrt{6}$ **17.** 110

18. (i) p = 0; q = 10; r = –3 ; **(ii)** –100 **19. (i)** No ; **(ii)** No

21. 34 **22.** $\left(\frac{4}{\sqrt{2}},-\frac{1}{\sqrt{2}},-\frac{1}{\sqrt{2}}\right)$ **23.** $\pm\frac{1}{3\sqrt{3}}(-1, -5, 1)$

Comprehension - based Questions

24. A **25.** B **26.** C **27.** B **28.** B **29.** B

Matrix Match - type Questions

30. (A)–P,R,S,T; (B)–P,R,T; (C)–P,Q,R,T

EXERCISE - IV

Previous Year's Question

JEE Main

1.	D	**2.**	A	**3.**	A	**4.**	D	**5.**	A	**6.**	D	**7.**	A
8.	D	**9.**	A	**10.**	C	**11.**	C	**12.**	B				

JEE Advanced

1. A **2.** 5 **3.** C **4.** A,D **5.** 9 **6.** C

7. 5 **8.** C **9.** A,B,C **10.** 4 **11.** A,C,D

12. (A) $\rightarrow$ P,R,S (B) $\rightarrow$ P (C) $\rightarrow$ P,Q (D) $\rightarrow$ S,T **13.** **B,C** **14.** **D** **15.** **D**

16. **C** **17.** **3** **18.** **18**