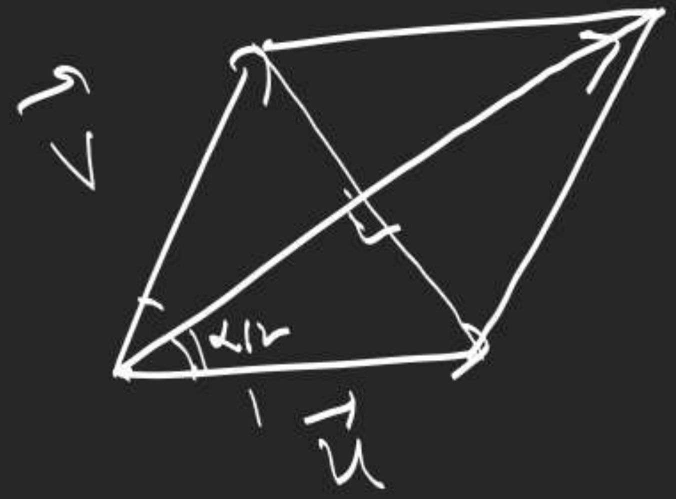
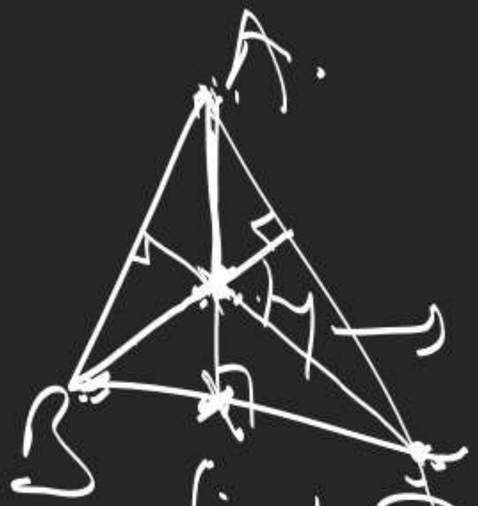
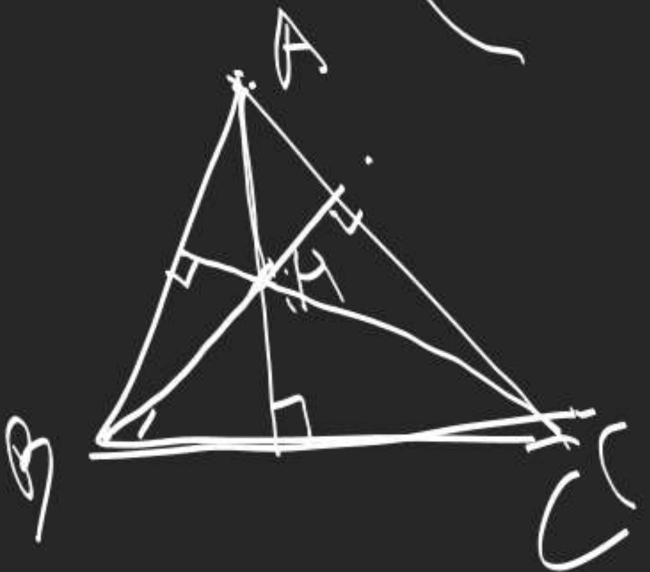
Nishant Jindal



$$\vec{x} = \frac{\vec{u}+\vec{v}}{|\vec{u}+\vec{v}|} = \frac{\vec{u}+\vec{v}}{2\cos\frac{\alpha}{2}}$$

$$\begin{bmatrix}\vec{x} & \vec{y} & \vec{z}\end{bmatrix}^2 = \left(\frac{1}{2\cos\frac{\alpha}{2}}\underbrace{\begin{bmatrix}\vec{u}+\vec{v} & \vec{v}+\vec{w} & \vec{w}+\vec{u}\end{bmatrix}}_{2\begin{bmatrix}\vec{u} & \vec{v} & \vec{w}\end{bmatrix}}\right)^2$$





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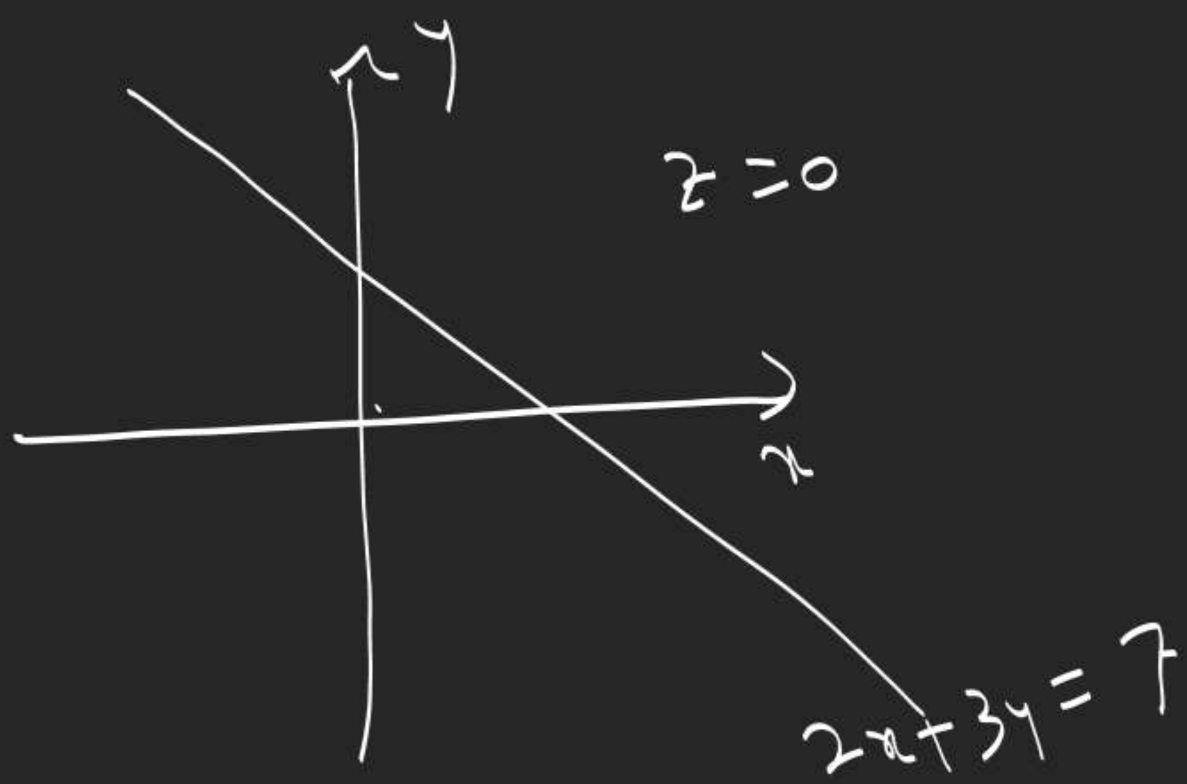
$2x+3y=7 \rightarrow$ Plane



$2x+3y=7$ & $z=0$

line

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$$\alpha x + \beta y + \gamma z = d \qquad \text{not all of } \alpha, \beta, \gamma = 0.$$

$\downarrow$

plane

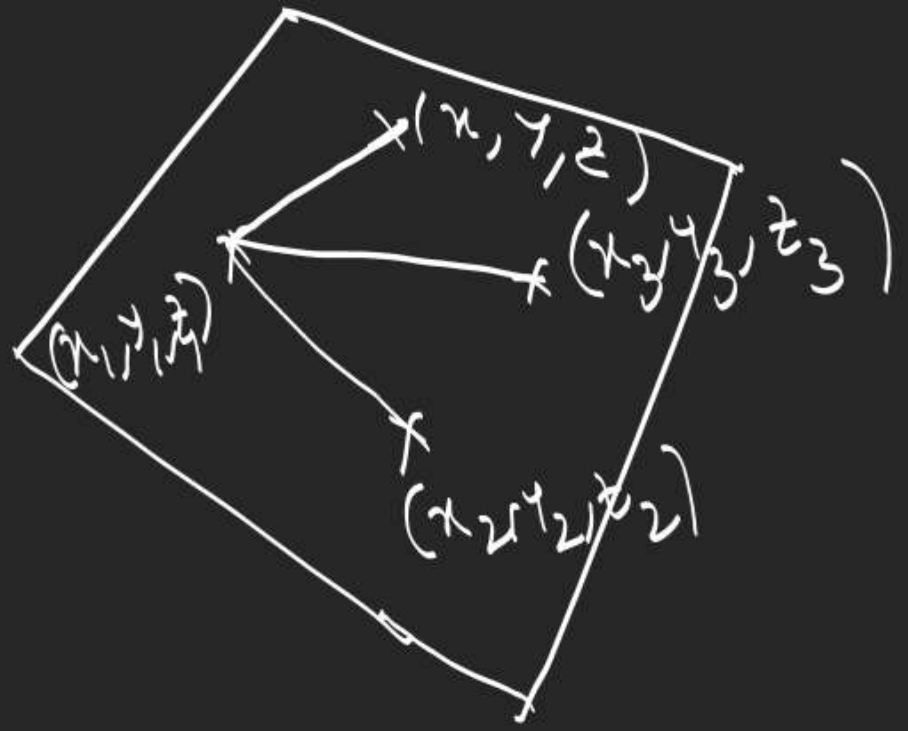
let $\alpha \neq 0$

$$x + \frac{\beta}{\alpha} y + \frac{\gamma}{\alpha} z = \frac{d}{\alpha}$$

$(x_i, y_i, z_i) \quad i = 1, 2, 3$

3 non collinear points lying on plane will determine a unique plane

$$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \end{vmatrix} = 0.$$



1. Find the eqn. of plane containing a point with p.v. $\vec{a}$ and $\parallel$ to two non collinear vectors $\vec{b}$ & $\vec{c}$.

Parametric form

$$\vec{r}-\vec{a} = \lambda\vec{b}+\mu\vec{c}$$

$$\boxed{\vec{r} = \vec{a}+\lambda\vec{b}+\mu\vec{c}}$$

$\lambda, \mu \in R$

$\vec{r}-\vec{a}$, $\vec{b}$, $\vec{c}$

$$\vec{n} = \vec{b}\times\vec{c}$$

$$(\vec{r}-\vec{a})\cdot(\vec{b}\times\vec{c}) = 0$$

$$\vec{r}\cdot(\vec{b}\times\vec{c}) = [\vec{a}\ \vec{b}\ \vec{c}]$$

$$\vec{r} = \hat{i}-\hat{j}+\lambda\hat{k}+\mu(\hat{i}-\hat{j}+\hat{k})$$

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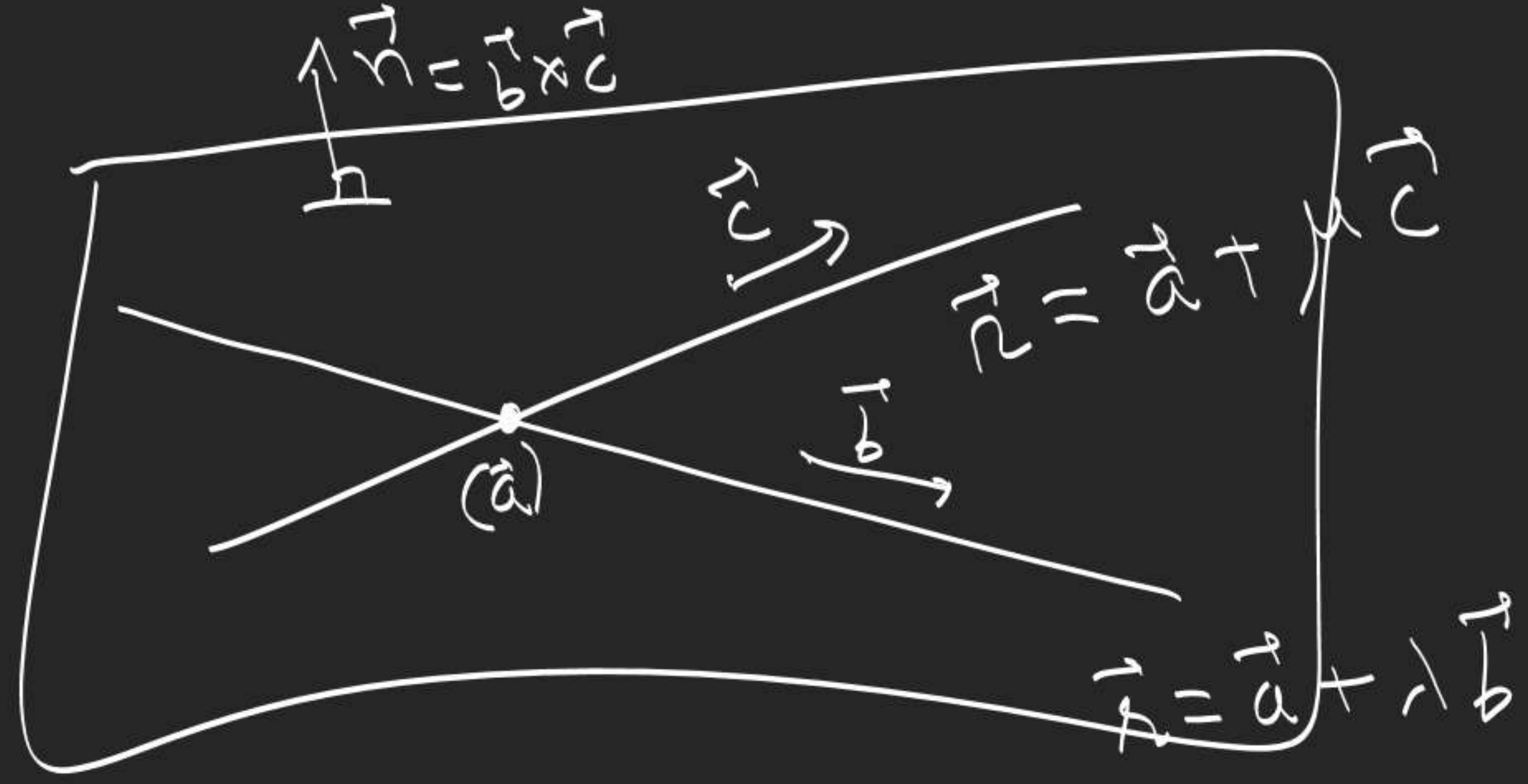
2.



Find eqn. of plane containing given lines.

$$(\vec{r} - \vec{a}) \cdot (\vec{b} \times \vec{c}) = 0$$

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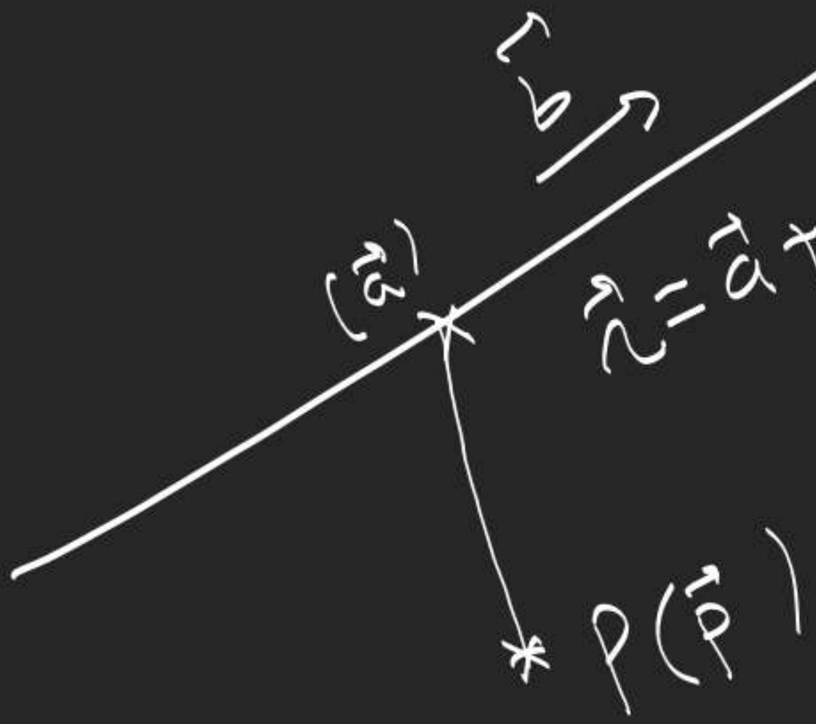

$$\vec{r} = \vec{a} + \lambda \vec{b}$$

$$(\vec{r} - \vec{a}) \cdot \left((\vec{p} - \vec{a}) \times \vec{b} \right) = 0$$

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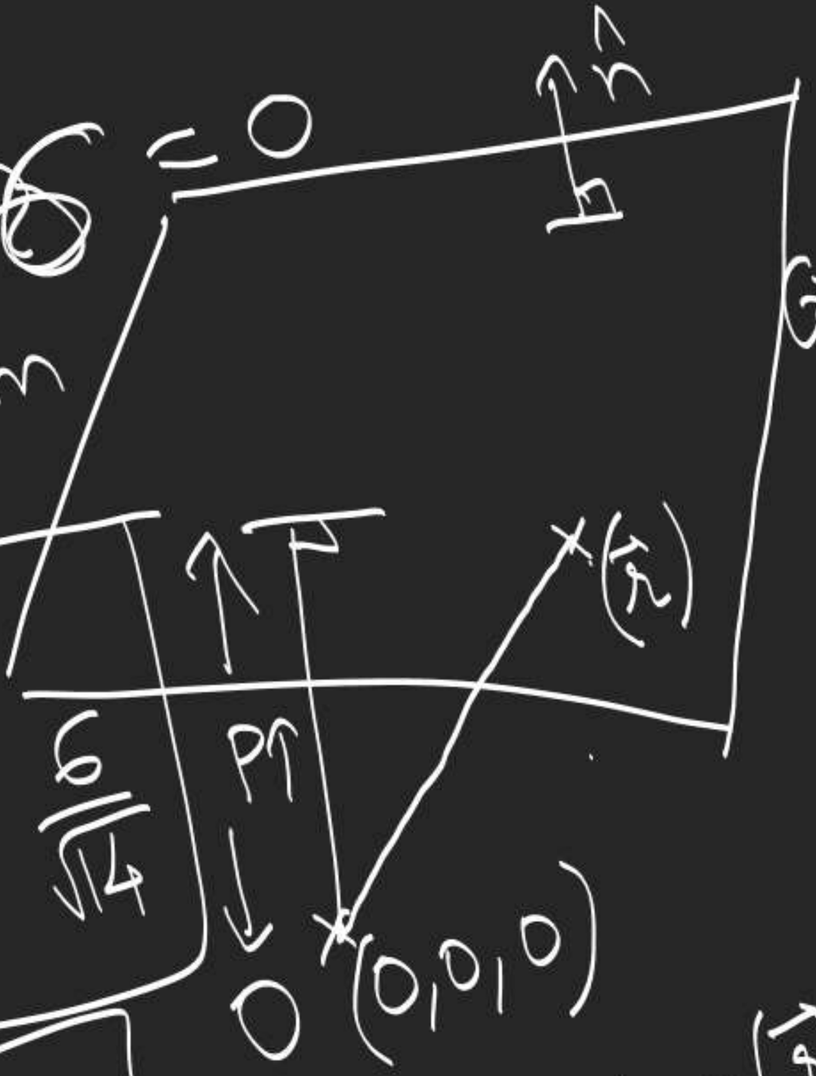
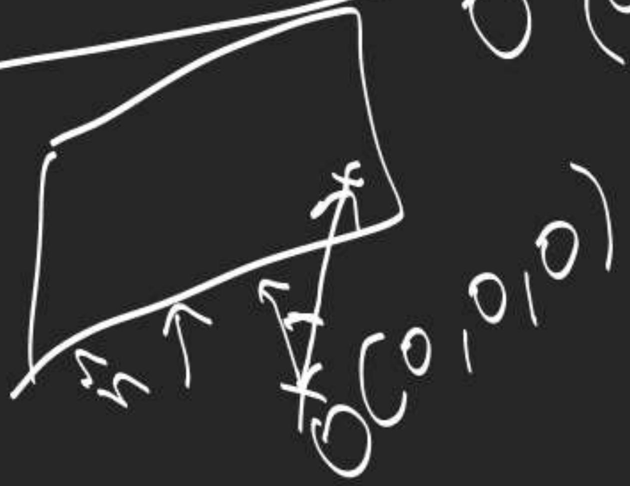
Normal form

convert into
$2x - 3y + z + 6 = 0$
normal form

$$\boxed{\vec{r}\cdot\left(\frac{-2\hat{i}+3\hat{j}-\hat{k}}{\sqrt{14}}\right) = \frac{6}{\sqrt{14}}}$$





Given: $\hat{n}$ unit vector $\perp$ar to plane & $\perp$ar distance of origin from plane.

Proj of $(\vec{r} - \vec{O})$ on $\hat{n} = p$.

$$\boxed{\vec{r}\cdot\hat{n} = p}$$

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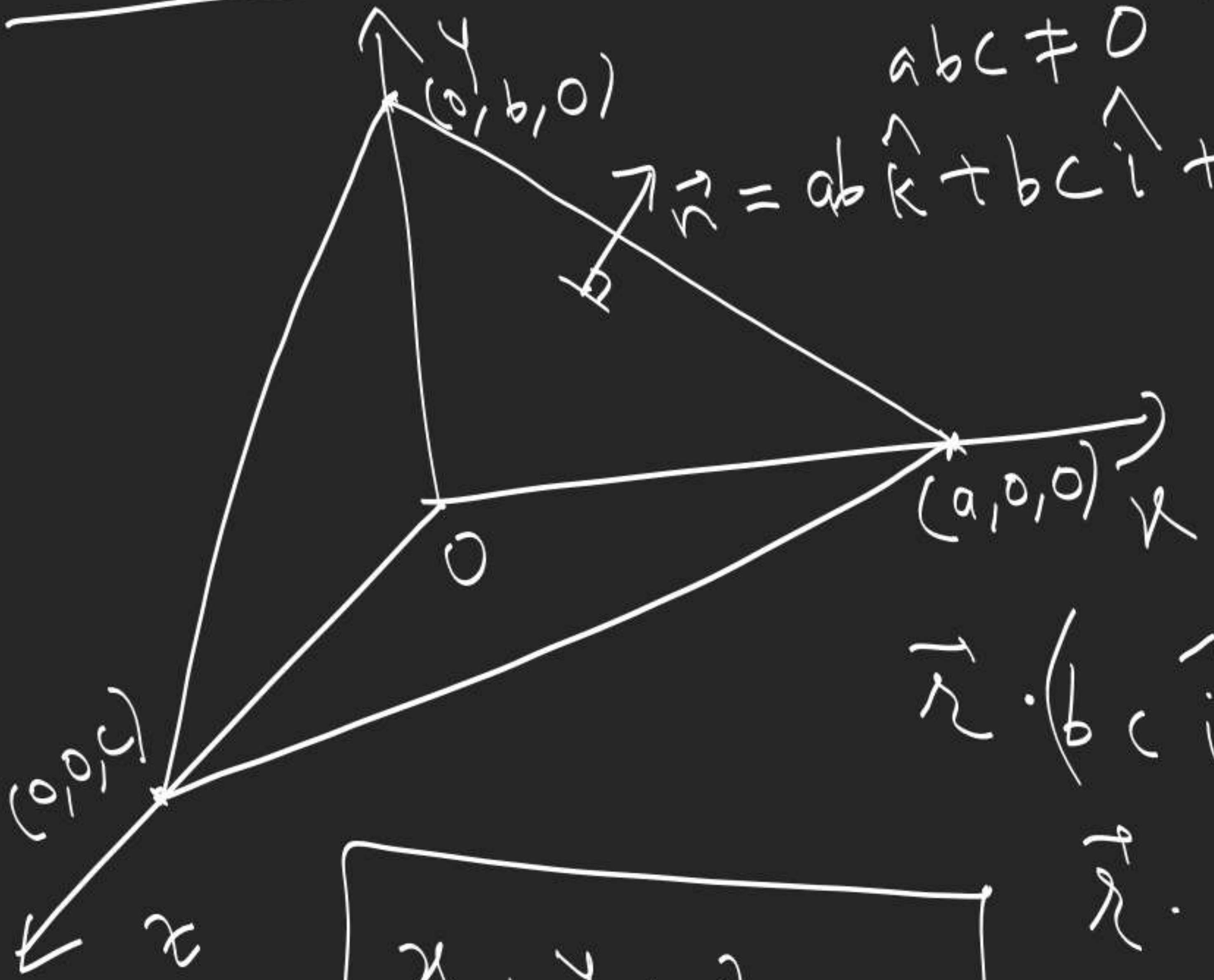
Intercept Form



$abc \neq 0$

$\vec{n} = ab\hat{k} + bc\hat{i} + ca\hat{j}$

$(\vec{r} - a\hat{i}) \cdot (bc\hat{i} + ca\hat{j} + ab\hat{k}) = 0$

$\vec{r} \cdot (bc\hat{i} + ca\hat{j} + ab\hat{k}) = abc$

$\vec{r} \cdot \left(\frac{1}{a}\hat{i} + \frac{1}{b}\hat{j} + \frac{1}{c}\hat{k}\right) = 1$

$$\boxed{\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1}$$

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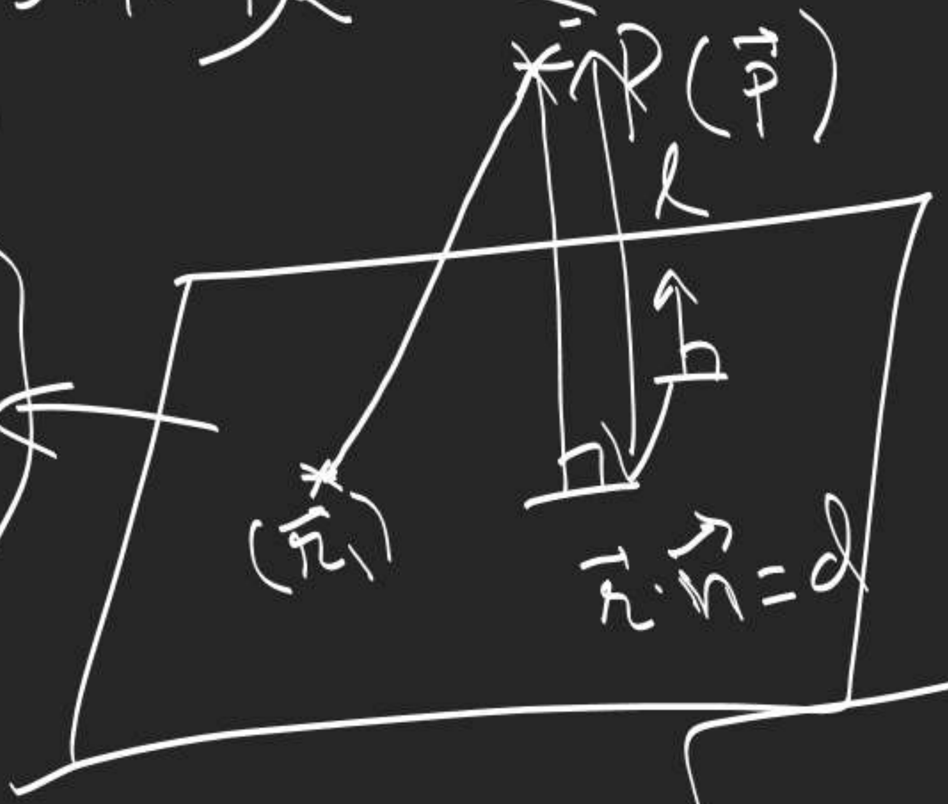
Ist Distance of Point from Plane

(x_1, y_1, z_1)



$\alpha x + \beta y + \gamma z = d$

$$l = \frac{|\alpha x_1 + \beta y_1 + \gamma z_1 - d|}{\sqrt{\alpha^2 + \beta^2 + \gamma^2}}$$

$$\left| \frac{(\vec r_1 - \vec p) \cdot \vec n}{|\vec n|} \right| = l$$

$$l = \left| \frac{d_1 - \vec p \cdot \vec n}{|\vec n|} \right|$$

$$\vec r \cdot \vec n - d$$

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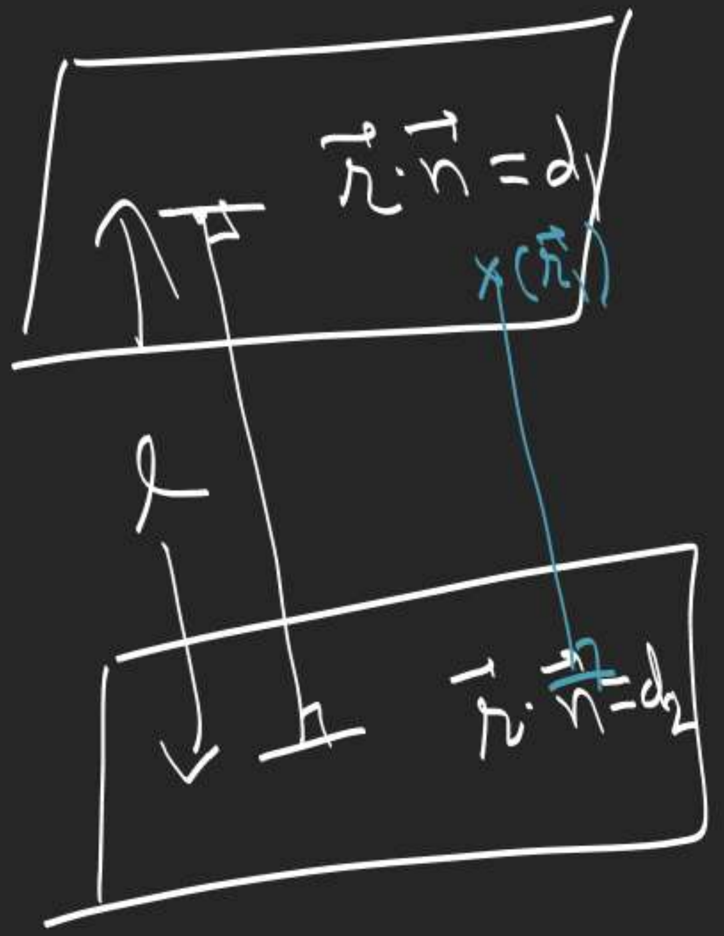
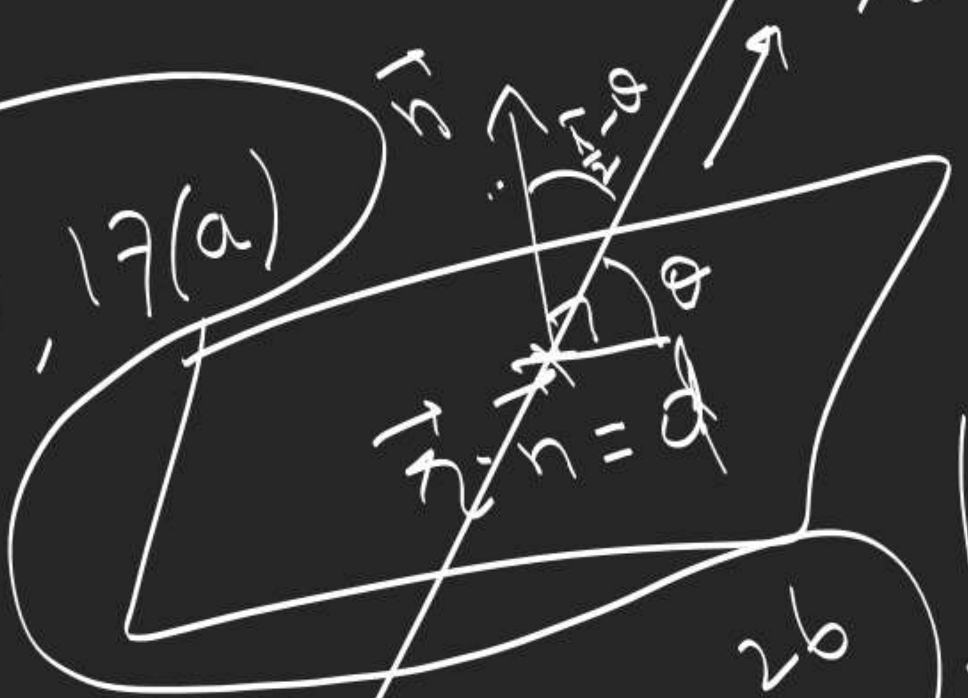


$$l = \left| \frac{d_1 - d_2}{|\vec{n}|} \right|$$

$$l = \left| \frac{\vec{r}_1 \cdot \vec{n} - d_2}{|\vec{n}|} \right| = \left| \frac{d_1 - d_2}{|\vec{n}|} \right|$$

Angle b/n a line & a plane

$$\vec{r} = \vec{a} + \lambda \vec{m}$$



$$\left| \frac{\vec{n}\cdot\vec{m}}{|\vec{n}|\,|\vec{m}|} \right| = \cos\left(\frac{\pi}{2}-\theta\right) = \sin\theta$$

$\boxed{15 - 30}$

15(b), 16(b), 17(a), 17(c), 17(e), 18(d), 19(c,d), 21, 22, 26, 28, 29