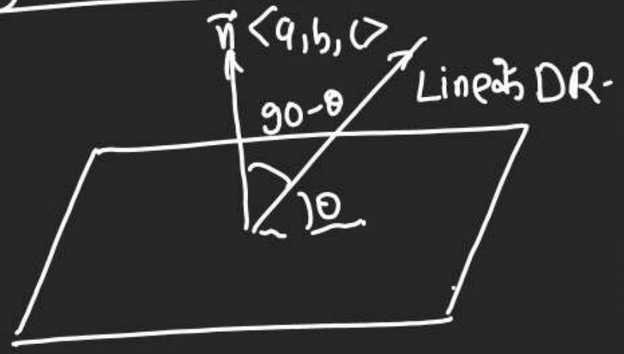
Nishant Jindal

Angle betⁿ Plane & Line.



Line: $\vec{r} = \vec{a} + \lambda \vec{p}$

$$\cos(90-\theta) = \frac{\vec{n}\cdot\vec{p}}{|\vec{n}||\vec{p}|}$$

$$\sin\theta = \frac{\vec{n}\cdot\vec{p}}{|\vec{n}||\vec{p}|}$$

Q Plane $P_1: x-y-z=4$ is Rotated thru 90° about line of Intersection with Plane $P_2: x+y+2z=4$. Find its Eqⁿ in new Position.

New Plane is also Family of Plane

$P: P_1 + \lambda P_2 = 0$

$P: x(1+\lambda) + y(-1+\lambda) + z(-1+2\lambda) - 4 - 4\lambda = 0$

P is ⊥ʳ to $P_1 \Rightarrow n_P \cdot n_{P_1} = 0$

$\langle 1+\lambda, -1+\lambda, -1+2\lambda \rangle \cdot \langle 1, -1, -1 \rangle = 0$

$1+\lambda+1-\lambda+1-2\lambda = 0 \Rightarrow 2\lambda = 3$

$\lambda = \frac{3}{2} \therefore P: \frac{5x}{2} + \frac{y}{2} + 2z - 10 = 0$

STRAIGHT LINE

A) Vector form of Line

$\vec{r} = \vec{a} + \lambda \vec{p}$

$\vec{a}$: Fix Pt, $\vec{p}$: DR

(B) Symmetrical form of Line.

If Line is P.T. $\langle a, b, c \rangle$ & having DR $= \langle l, m, n \rangle$

Line $\rightarrow \frac{x-a}{l} = \frac{y-b}{m} = \frac{z-c}{n} = K$

(C) General Pt: $\langle Kl+a, mK+b, nK+c \rangle$

Nishant Jindal

Q Express EOL in vector form.

$L: \frac{5x-3}{7} = \frac{2+2y}{3} = \frac{1-2z}{1}$

1) $\frac{x-\frac{3}{5}}{\frac{7}{5}} = \frac{y+1}{\frac{3}{2}} = \frac{z-\frac{1}{2}}{\left(\frac{-1}{2}\right)}$

2) Vector form

$\vec{r} = \langle \frac{3}{5}, -1, \frac{1}{2} \rangle + \lambda \langle \frac{7}{5}, \frac{3}{2}, \frac{-1}{2} \rangle$

Q Eqn of x Axis?

↳ Fixpt $\langle 0,0,0 \rangle$

$\frac{x-0}{1} = \frac{y-0}{0} = \frac{z-0}{0}$

(3) Unsymmetrical form of Line

here we get eqn of 2 Planes Simultaneously & Line in Line of Intersection of Both Plane.

$P_1: a_1x+b_1y+c_1z+d_1=0 = a_2x+b_2y+c_2z+d_2 : P_2$

Q Write EOL $x-y+2z=0 = 3x+y+z$ in Symm. form.

① DR $= \vec{n} = \begin{vmatrix} i & j & k \\ 1 & -1 & 2 \\ 3 & 1 & 1 \end{vmatrix} = \langle -3, +5, 4 \rangle$

(2) $z=0$ Put $\left.\begin{matrix} x-y=0 \\ 3x+y=0 \end{matrix}\right\} x=0, y=0$

Pt $\langle 0,0,0 \rangle$

Line $\frac{x-0}{-3} = \frac{y-0}{5} = \frac{z-0}{4}$

Nishant Jindal

Q Find Line P.T. $\langle 1, 4, -2 \rangle$ & $\perp^r$ to plane

$P_1: 6x+2y+2z+3=0$

$P_2: x+2y-6z+4=0$

(1) $\vec{n} = \begin{vmatrix} i & j & k \\ 6 & 2 & 2 \\ 1 & 2 & -6 \end{vmatrix}$

$= \langle -16, +38, 10 \rangle$

$= \langle -8, 19, 5 \rangle$

(2) $\frac{x-1}{-8} = \frac{y-4}{19} = \frac{z+2}{5}$

Q Angle bet^n Lines

$L_1: 3x+2y+z-5=0 = x+y-2z-3$

$L_2: 8x-4y-4z=0 = 7x+10y-8z$

$n_1 = \begin{vmatrix} i & j & k \\ 3 & 2 & 1 \\ 1 & 1 & -2 \end{vmatrix}$

$= \langle -5, +7, 1 \rangle$

$n_2 = \begin{vmatrix} i & j & k \\ 2 & -1 & -1 \\ 7 & 10 & -8 \end{vmatrix}$

$= \langle 18, +9, 27 \rangle$

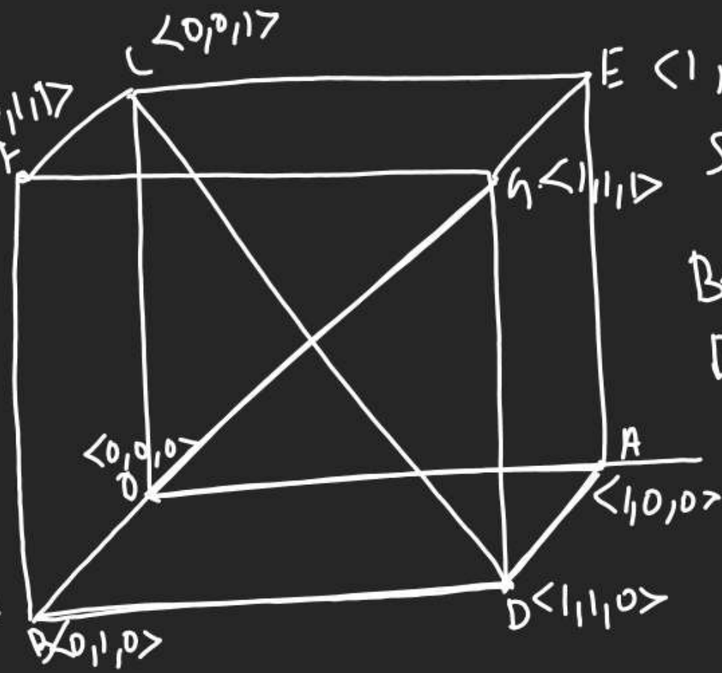
$\cos\theta = \frac{-90+63+27}{\sqrt{\quad}\sqrt{\quad}} = 0$

$L_1 \perp L_2$

Nishant Jindal

Q Let Q be the cube with set of Adv Board vertices $\{(x_1,x_2,x_3) \in R^3, x_1x_2x_3 \in \{0,1\}\}$

Let F be the set of all 12 Lines containing diagonal of Six faces of Cube Q. Let S be the set of all 4 Lines containing main diagonal of Cube Q. For Instance the Line Passing thru the vertices $\langle 0,0,0\rangle$ & $\langle 1,1,1\rangle$ is in S. For Lines l_1 & l_2, let $d\langle l_1, l_2\rangle$ denotes S.D. betⁿ them. Then max.ᵐ Value of $d\langle l_1, l_2\rangle$ as l_1 varies over F & l_2 varies over S, is?



$SD = \left|\frac{(FP-FP)\cdot(DR\times DR)}{|DR\times DR|}\right|$

Body Diag = $\{\overrightarrow{OG}, \overrightarrow{CD}, \overrightarrow{BE}, \overrightarrow{AF}\}$

$\overrightarrow{OG} \to \frac{x-0}{1} = \frac{y-0}{1} = \frac{z-0}{1}$

$\overrightarrow{CD} = \frac{x-0}{1} = \frac{y-0}{1} = \frac{z-1}{-1}$

$DR\times DR = \begin{vmatrix} i & j & k \\ 1 & 1 & -1 \\ 1 & -1 & 1 \end{vmatrix} = \begin{vmatrix} i & j & k \\ 1 & 1 & 1 \\ 1 & 1 & -1 \end{vmatrix}$

$= \langle 0,-2,-2\rangle \quad = \langle -2,+2,0\rangle$

$SD = \left|\frac{\langle 0,0,-1\rangle \cdot \langle -2,2,0\rangle}{\sqrt{4+4+0}}\right| = 0$

$\overrightarrow{BE} = \frac{x-0}{1} = \frac{y-1}{-1} = \frac{z-0}{1}$

$= 2\left|\frac{\langle 0,-1,1\rangle \langle 2,-2,2\rangle}{\sqrt{4+4+4}}\right| = 0+2-2$

$DR\times DR = \begin{vmatrix} i & j & k \\ 1 & 1 & -1 \\ 1 & 0 & 1 \end{vmatrix} = \langle 1,-2,-1\rangle$

$\overrightarrow{OE} = \frac{x-0}{1} = \frac{y-0}{0} = \frac{z-0}{1}$

$SD = \left|\frac{\langle 0,0,1\rangle \cdot \langle 1,-2,-1\rangle}{\sqrt{1+4+1}}\right| = \frac{1}{\sqrt{6}}$