Nishant Jindal

HW4

Q4 $\frac{1}{b+c}, \frac{1}{c+a}, \frac{1}{a+b}$ AP.

$\frac{(a+b)(\cancel{b+c})(c+a)}{(\cancel{b+c})}, \frac{(a+b)(b+c)(\cancel{c+a})}{(\cancel{c+a})}, \frac{(\cancel{a+b})(b+c)(c+a)}{(\cancel{a+b})}$ AP.

$a^2+(ac+bc+ab), b^2+(ab+ac+bc), c^2+(ab+bc+ca) \rightarrow$ AP

a^2, b^2, c^2 AP.

$\frac{1}{a^2}, \frac{1}{b^2}, \frac{1}{c^2}$ HP

Q2 $\log_2(x \cdot \sqrt{x} \cdot \sqrt[4]{x} \cdot \sqrt[8]{x} \cdots)$

$\log_2(x^{1+1/2+1/4+1/8 \cdots}) = 4$

$x^2 = 16 \Rightarrow x = 4$

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$a, b, c \to AP$

$\frac{a}{abc}, \frac{b}{abc}, \frac{c}{abc} \to AP$

$\frac{1}{bc}, \frac{1}{ac}, \frac{1}{ab}$ AP.

$\frac{ab+bc+ca}{bc}, \frac{ab+bc+ca}{ac}, \frac{ab+bc+ca}{ab} \to AP$

$\frac{ab+bc+ca}{bc}-1, \frac{ab+bc+ca}{ac}-1, \frac{ab+bc+ca}{ab}-1$ AP.

$\frac{ab+ca}{bc}, \frac{ab+bc}{ac}, \frac{bc+ca}{ab} \to AP$

$\frac{bc}{ab+ac}, \frac{ca}{ab+bc}, \frac{ab}{bc+ac} \to \underline{HP}$

Q1

$$a_n = \frac{\left(x^{\frac{1}{2^n}}+y^{\frac{1}{2^n}}\right)\left(x^{\frac{1}{2^n}}-y^{\frac{1}{2^n}}\right)}{\left(x^{\frac{1}{2^n}}-y^{\frac{1}{2^n}}\right)}$$

$$= \frac{\left(x^{\frac{1}{2^n}}\right)^2-\left(y^{\frac{1}{2^n}}\right)^2}{x^{\frac{1}{2^n}}-y^{\frac{1}{2^n}}} = \frac{x^{\frac{1}{2^{n-1}}}-y^{\frac{1}{2^{n-1}}}}{x^{\frac{1}{2^n}}-y^{\frac{1}{2^n}}}$$

$$a_n = \frac{b_{n-1}}{b_n}$$

$a_1 = \frac{b_0}{b_1}$

$a_2 = \frac{b_1}{b_2}$

$a_3 = \frac{b_2}{b_3}$

$$\to a_1 \cdot a_2 \cdot a_3 \ldots a_n = \frac{b_0}{\cancel{b_1}} \times \frac{\cancel{b_1}}{\cancel{b_2}} \times \frac{\cancel{b_2}}{b_3} \times \ldots \frac{\cancel{b_{n-1}}}{b_n}$$

$$= \frac{b_0}{b_n} = \frac{x^{\frac{1}{2^0}}-y^{\frac{1}{2^0}}}{x^{\frac{1}{2^n}}-y^{\frac{1}{2^n}}}$$

$$= \frac{x-y}{b_n}$$

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Q8 S.T. in an AP.

$a_2 - a_1 = d$

$a_1 - a_2 = -d$

$a_1, a_2, a_3 \ldots$

$$\underbrace{a_1^2 - a_2^2}_{\downarrow} + \underbrace{a_3^2 - a_4^2} + \cdots + \underbrace{a_{2K-1}^2 - a_{2K}^2} = \frac{K}{(2K-1)}(a_1^2 - a_{2K}^2)$$

K Pairs

$$(a_1-a_2)(a_1+a_2) + (a_3-a_4)(a_3+a_4) + \cdots + (a_{2K-1}-a_{2K})(a_{2K-1}+a_{2K})$$

$$-d(a_1+a_2) + (-d)(a_3+a_4) \quad + (-d)(a_{2K-1}+a_{2K})$$

$$-d\{\underbrace{a_1+a_2+a_3+a_4+\cdots+a_{2K-1}+a_{2K}}_{\text{Sum of } 2K \text{ terms}}]$$

$$-d \times \frac{\cancel{2}K}{\cancel{2}}[a_1 + a_{2K}]$$

$$\Rightarrow -Kd\,\frac{[a_1^2 - a_{2K}^2]}{(a_1 - a_{2K})} = \frac{+\cancel{K}d\,[a_1^2 - a_{2K}^2]}{\cancel{a_1} + (\cancel{a_1} + (2K-1)d)} = \frac{K}{2K-1}[a_1^2 - a_{2K}^2]$$

RHS

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Copy Add.

Q9 a, b, c +ve R No.

$9(25a^2+b^2)+25(c^2-3ac)=15b(3a+c)$

then a, b, c?

$225a^2+9b^2+25c^2-75ac-45ab-15bc=0$

$(15a)^2+(3b)^2+(5c)^2-75ac-45ab-15bc=0$

$\boxed{\text{feel} \to a^2+b^2+c^2-ab-bc-ca=0} \Rightarrow \frac{1}{2}\left\{(a-b)^2+(b-c)^2+(c-a)^2\right\}=0$

$\frac{1}{2}\left\{(15a-3b)^2+(3b-5c)^2+(5c-15a)^2\right\}=0$

$15a-3b=0$ & $3b-5c=0$ & $5c-15a=0$

$15a=3b=5c \quad \div 15$

$\frac{15a}{15}=\frac{3b}{15}=\frac{5c}{15}=K \Rightarrow a=\frac{b}{5}=\frac{c}{3}=K$

$\therefore \left.\begin{array}{l} a=K \\ b=5K \\ c=3K \end{array}\right\}$ a, c, b are in A.P.

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$$S = \underline{10^9} + \boxed{2}\,11^1\cdot 10^8 + \boxed{3}\,11^2\cdot 10^7 + \cdots + 10\cdot 11^9 = K\cdot 10^9 \text{ then } K.$$

GP $\frac{11}{10}$, $\frac{11}{10}$; AP

$$\frac{11S}{10} = \quad 11\cdot 10^8 + 2\cdot 11^2\cdot 10^7 + \quad + 9\cdot 11^9 + 11^{10}$$

$\leftarrow$ 10 terms GP $\rightarrow$

$$-\frac{S}{10} = \left\{10^9 + 11\cdot 10^8 + 11^2\cdot 10^7 + 11^3\cdot 10^6 + \cdots + 11^9\right\} - 11^{10}$$

$\frac{11}{10}$ $\frac{11}{10}$ GP

$$-\frac{S}{10} = \frac{10^9\cdot\left(\left(\frac{11}{10}\right)^{10} - 1\right)}{\left(\frac{11}{10} - 1\right)} - 11^{10}$$

$$+\frac{S}{10} = \cancel{10^{10}}\frac{\left(\cancel{11^{10}} + 10^{10}\right)}{\cancel{10^{10}}} - \cancel{11^{10}}$$

$$S = 10^{11}$$

Class off