

Excentral triangle

$$II_1 = 2(2R) \cos I_1 = 4R \sin A_2$$

$$II_2 = 4R \sin \frac{B}{2}$$

$$\angle I_1 = \frac{\pi}{2} - \frac{A}{2} \leq \pi - 2I_1 = A$$

$$\pi - 2I_2 = B$$

$$\pi - 2I_3 = C$$

$$I_2 = \frac{\pi}{2} - \frac{B}{2}$$

$$I_3 = \frac{\pi}{2} - \frac{C}{2}$$

$$\frac{R_{I_1 I_2 I_3}}{2} = R$$

$$R_{I_1 I_2 I_3} = 2R$$

$$I_2 I_3 = 4R \cos \frac{A}{2}$$

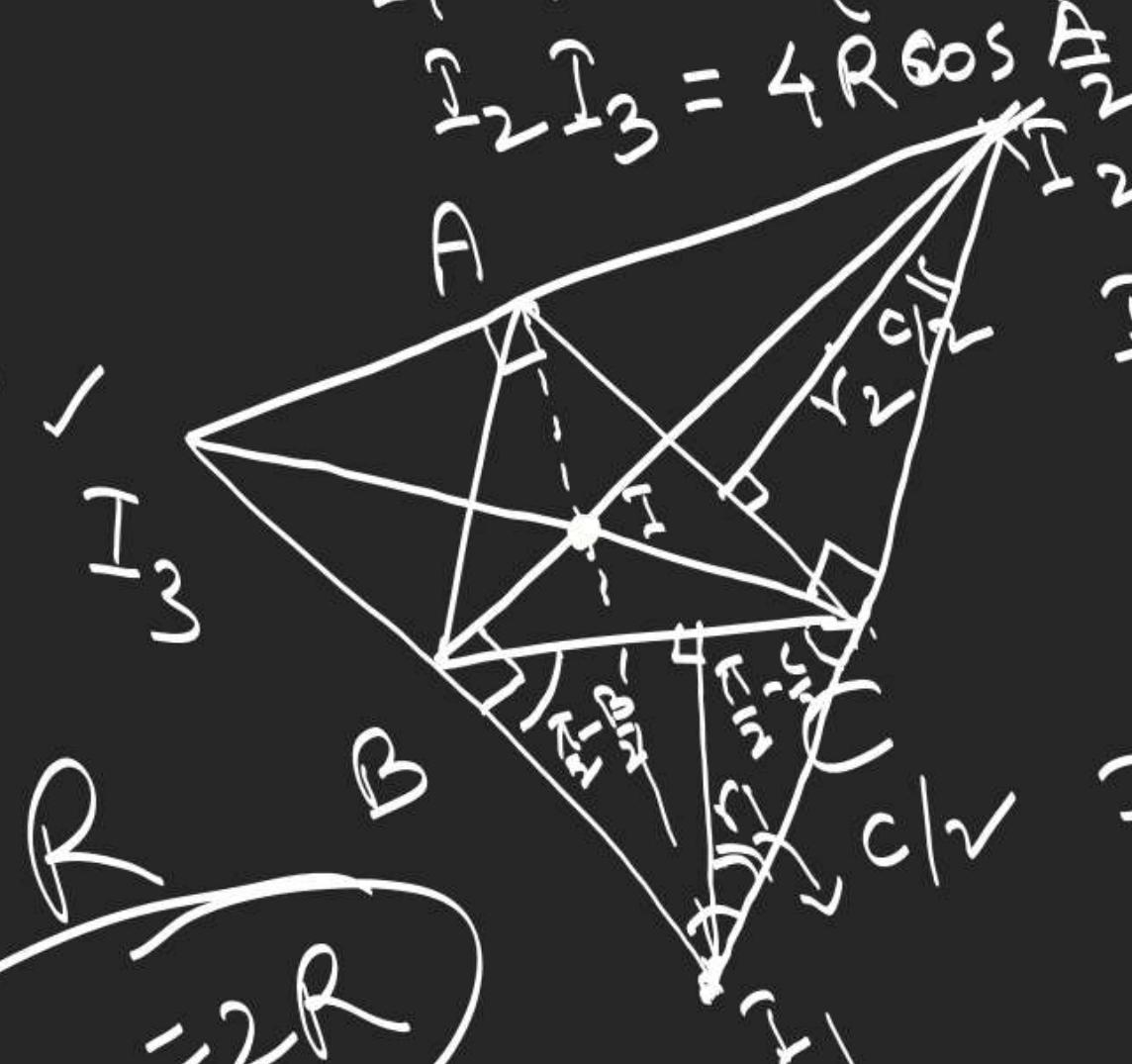
$$I_3 I_1 = 4R \cos \frac{B}{2}$$

$$I_1 I_2 = 4R \cos \frac{C}{2}$$

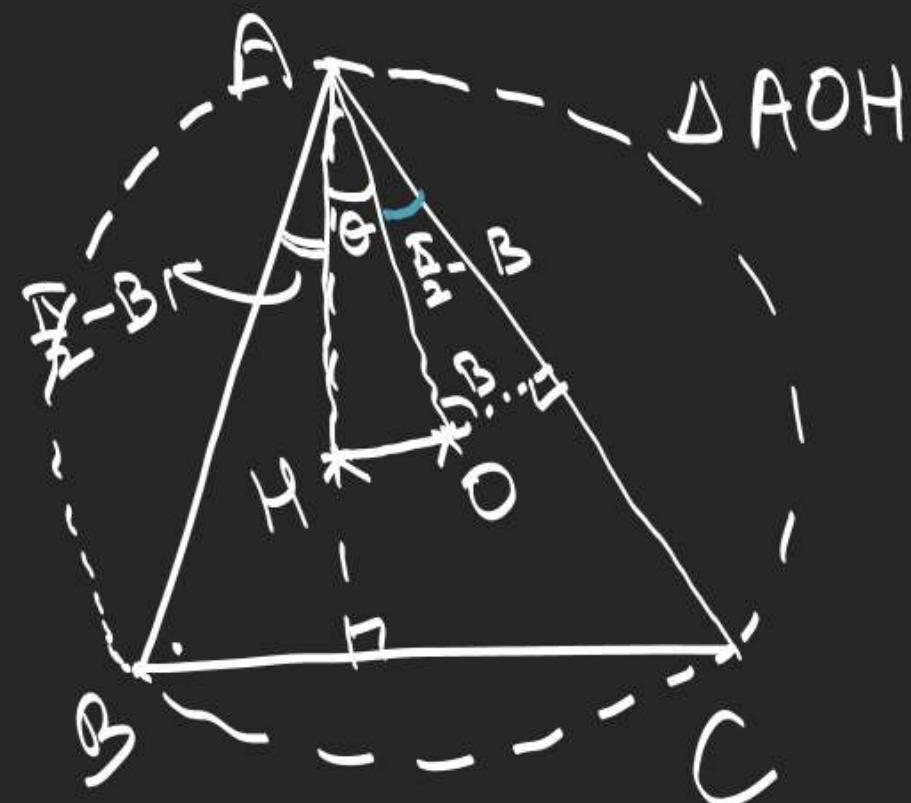
$$\pi - \left(\frac{\pi}{2} - \frac{B}{2} + \frac{\pi}{2} - \frac{C}{2} \right)$$

$$= \frac{B+C}{2}$$

$$= \frac{\pi}{2} - \frac{A}{2}$$



Distance between Orthocentre & Circumcentre



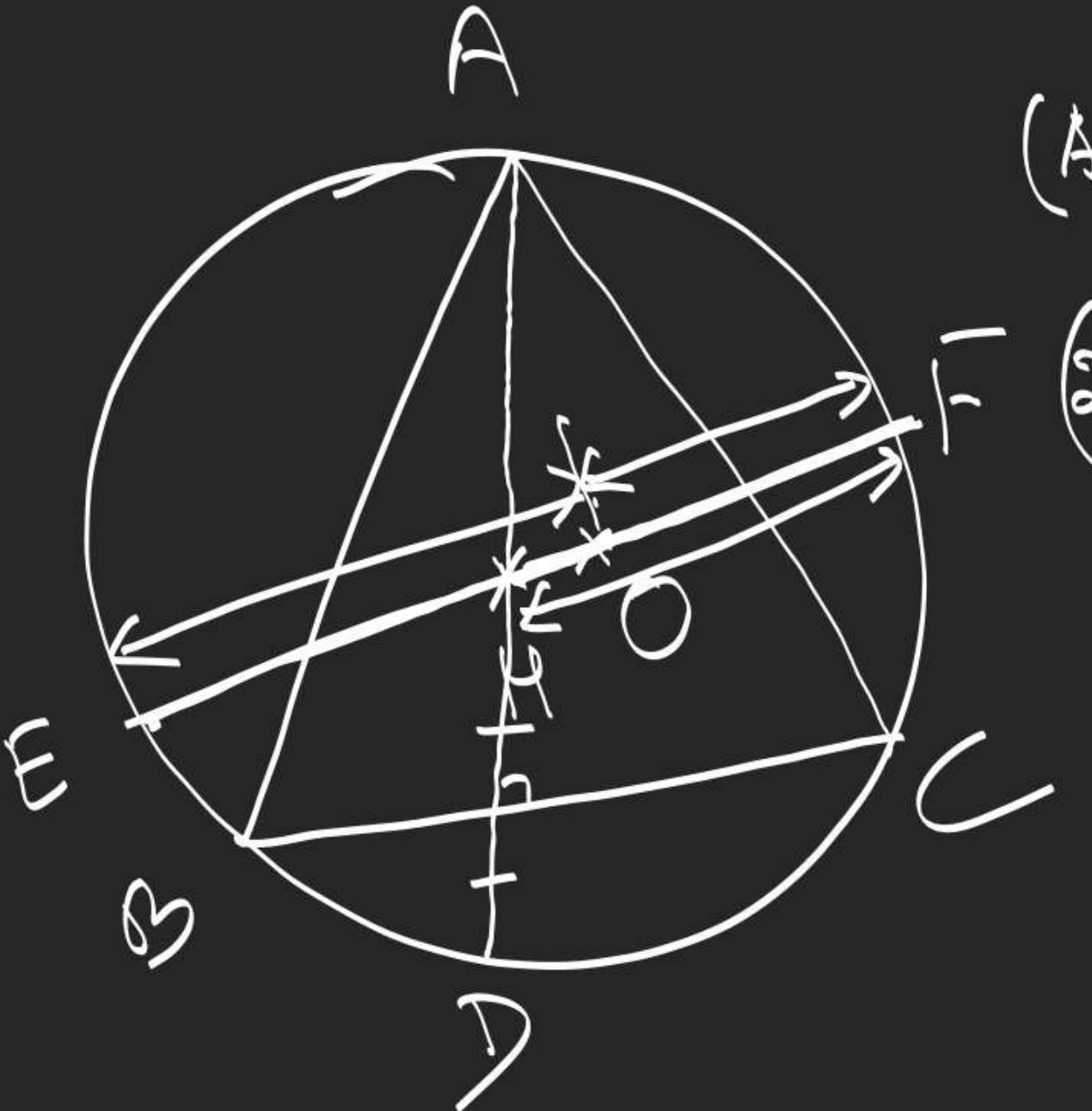
$$\begin{aligned}\theta &= A - (\pi - 2B) \\ &= A + 2B - (A + B + C) \\ &= B - C.\end{aligned}$$

$$\Rightarrow OH^2 = AM^2 + OA^2 - 2(AM)(OA) \cos \theta$$

$$\cos \theta = \frac{AM^2 + OA^2 - OH^2}{2(AM)(OA)}$$

$$R^2(1 - 8 \cos A \cos B \cos C) = R^2 + 4R^2 \cos^2 A - 4R^2 \cos A \cos(B-C)$$

$$= R^2 + 4R^2 \cos A (-\cos(B+C) - \cos(B-C))$$



$$\begin{aligned}
 (AH)(HD) &= (\underline{HE})(HF) \\
 (2R\cos A)(4R\cos B\cos C) &= (R-OH)(R+OH) \\
 &= R^2 - OH^2
 \end{aligned}$$

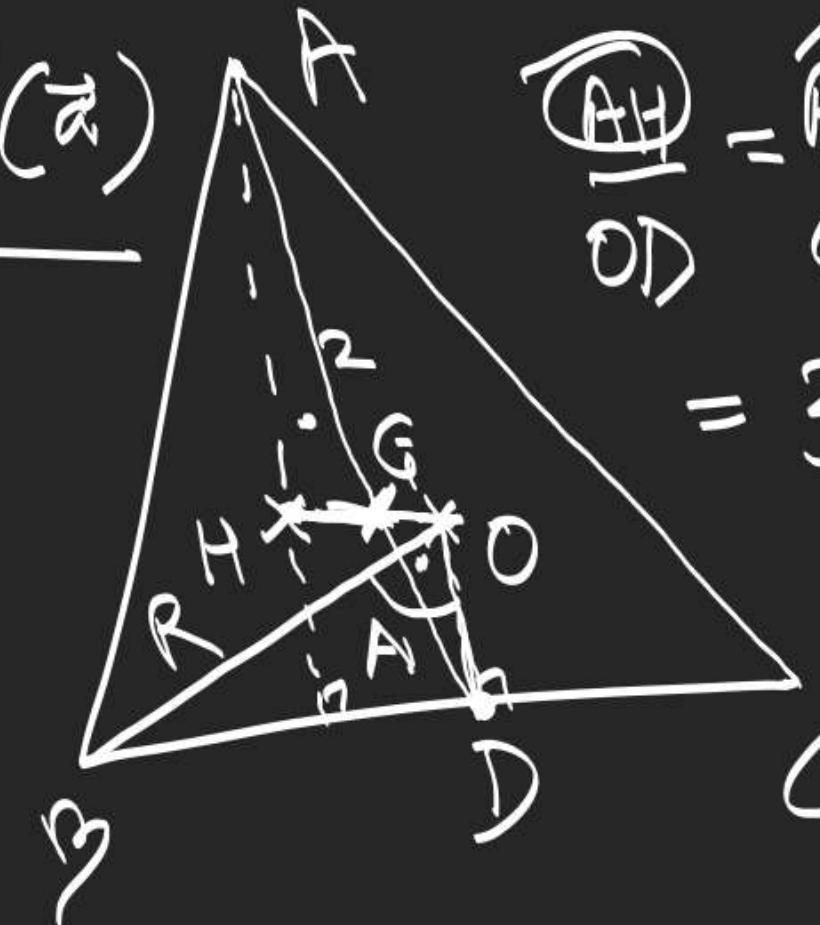
$$|\vec{OH}|^2 = |\vec{a} + \vec{b} + \vec{c}|^2 = 3R^2 + 2R^2(\cos 2A + \cos 2B + \cos 2C)$$



$$H = \frac{3G - 2O}{3 - 2}$$

$$= \boxed{\vec{a} + \vec{b} + \vec{c}}$$

$$\frac{2\left(\frac{\vec{b} + \vec{c}}{2}\right) + 1(\vec{a})}{2+1} = \frac{\vec{a} + \vec{b} + \vec{c}}{3}$$



$$\frac{GH}{OD} = \frac{AG}{GD} = \frac{HG}{GO}$$

$$= \frac{2R \cos A}{R \cos A}$$

$$c = \frac{2}{1}$$

Orthocentre $\xrightarrow{2} H \xrightarrow{1} G \xrightarrow{2} O \xrightarrow{1} \text{Circumcenter}$
 Centroid $\xrightarrow{1} G \xrightarrow{2} O \xrightarrow{1} \text{Circumcenter}$

Ex-37

6, 7, 11, 10, 19, 20, 27, 31, 32
34, 35, 36