QUADRATIC EQUATIONS

KEY CONCEPTS

The general form of a quadratic equation in x is, $ax^2 + bx + c = 0$, where $a, b, c \in R \& a \neq 0$.

RESULTS:

1. The solution of the quadratic equation, $ax^2 + bx + c = 0$ is given by $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

The expression $b^2 - 4ac = D$ is called the discriminant of the quadratic equation.

2. If $\alpha \& \beta$ are the roots of the quadratic equation $ax^2 + bx + c = 0$, then;

(i) $\alpha + \beta = -b/a$ (ii) $\alpha\beta = c/a$ (iii) $\alpha - \beta = \sqrt{D}/a$.

3. **NATURE OF ROOTS:**

(A) Consider the quadratic equation $ax^2 + bx + c = 0$ where $a, b, c \in R \& a \neq 0$ then ;

(i) $D > 0 \Leftrightarrow$ roots are real & distinct (unequal).

(ii) $D = 0 \Leftrightarrow$ roots are real & coincident (equal).

(iii) $D < 0 \Leftrightarrow$ roots are imaginary .

(iv) If $p + iq$ is one root of a quadratic equation, then the other must be the conjugate $p - iq$ & vice versa. $(p, q \in R \& i = \sqrt{-1})$.

(B) Consider the quadratic equation $ax^2 + bx + c = 0$ where $a, b, c \in Q \& a \neq 0$ then;

(i) If $D > 0 \&$ is a perfect square, then roots are rational & unequal.

(ii) If $\alpha = p + \sqrt{q}$ is one root in this case, (where p is rational & $\sqrt{q}$ is a surd) then the other root must be the conjugate of it i.e. $\beta = p - \sqrt{q}$ & vice versa.

4. A quadratic equation whose roots are $\alpha \& \beta$ is $(x - \alpha)(x - \beta) = 0$ i.e.
$x^2 - (\alpha + \beta)x + \alpha\beta = 0$ i.e. $x^2 -$ (sum of roots) $x +$ product of roots $= 0$.

5. Remember that a quadratic equation cannot have three different roots & if it has, it becomes an identity.

6. **Consider the quadratic expression**, $y = ax^2 + bx + c, a \neq 0 \& a, b, c \in R$ then ;

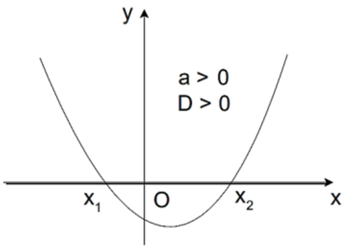
(i) The graph between x, y is always a parabola. If $a > 0$ then the shape of the parabola is concave upwards & if $a < 0$ then the shape of the parabola is concave downwards.

(ii) $\forall x \in R, y > 0$ only if $a > 0 \ \& \ b^2 - 4ac < 0$ (figure 3).

(iii) $\forall x \in R, y < 0$ only if $a < 0 \ \& \ b^2 - 4ac < 0$ (figure 6).

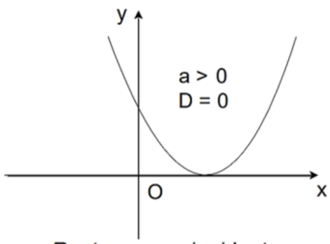
Carefully go through the 6 different shapes of the parabola given below.

Fig. 1



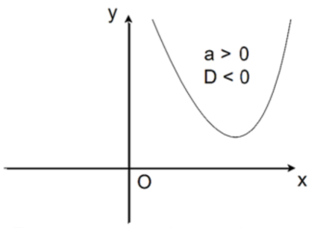
Roots are real & distinct

Fig. 2



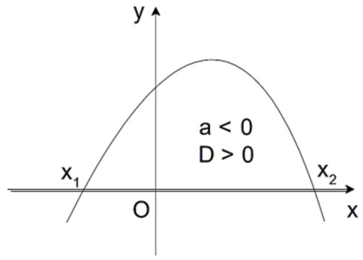
Roots are coincident

Fig. 3



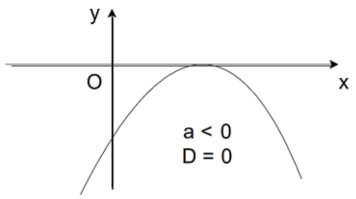
Roots are complex conjugate

Fig. 4



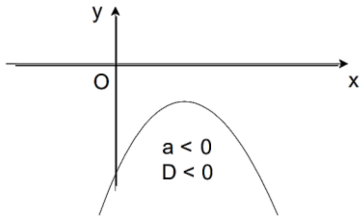
Roots are real & distinct

Fig. 5



Roots are coincident

Fig. 6



Roots are complex conjugate

7. SOLUTION OF QUADRATIC INEQUALITIES:

$ax^2 + bx + c > 0 (a \neq 0)$

(i) If $D > 0$, then the equation $ax^2 + bx + c = 0$ has two different roots $x_1 < x_2$.

Then $a > 0 \Rightarrow x \in (-\infty, x_1) \cup (x_2, \infty)$

$a < 0 \Rightarrow x \in (x_1, x_2)$

(ii) If $D = 0$, then roots are equal, i.e. $x_1 = x_2$.

In that case $a > 0 \Rightarrow x \in (-\infty, x_1) \cup (x_1, \infty)$

$a < 0 \Rightarrow x \in \phi$

(iii) Inequalities of the form $\frac{P(x)}{Q(x)} \lesseqgtr 0$ can be quickly solved using the method of intervals.

8. MAXIMUM & MINIMUM VALUE of $y = ax^2 + bx + c$ occurs at $x = -(b/2a)$ according as ;

$a < 0$ or $a > 0$. $y \in \left[\frac{4ac-b^2}{4a}, \infty\right)$ if $a > 0$ & $y \in \left(-\infty, \frac{4ac-b^2}{4a}\right]$ if $a < 0$.

9. COMMON ROOTS OF 2 QUADRATIC EQUATIONS [ONLY ONE COMMON ROOT] :

Let α be the common root of $ax^2 + bx + c = 0$ & $a'x^2 + b'x + c' = 0$. Therefore

$a\alpha^2 + b\alpha + c = 0; a'\alpha^2 + b'\alpha + c' = 0$. By Cramer's Rule $\frac{\alpha^2}{bc'-b'c} = \frac{\alpha}{a'c-ac'} = \frac{1}{ab'-a'b}$

Therefore, $\alpha = \frac{ca'-c'a}{ab'-a'b} = \frac{bc'-b'c}{a'c-ac'}$.

So the condition for a common root is $(ca' - c'a)^2 = (ab' - a'b)(bc' - b'c)$.

10. The condition that a quadratic function $f(x, y) = ax^2 + 2hxy + by^2 + 2gx + 2fy + c$ may be resolved into two linear factors is that ;

$$abc + 2fgh - af^2 - bg^2 - ch^2 = 0 \text{ OR } \begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = 0$$

11. THEORY OF EQUATIONS :

If $\alpha_1, \alpha_2, \alpha_3, \ldots \ldots \alpha_n$ are the roots of the equation;

$f(x) = a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \cdots + a_{n-1}x + a_n = 0$ where $a_0, a_1, \ldots . a_n$ are all real & $a_0 \neq 0$ then,

$$\sum \alpha_1 = -\frac{a_1}{a_0}, \sum \alpha_1\alpha_2 = +\frac{a_2}{a_0}, \sum \alpha_1\alpha_2\alpha_3 = -\frac{a_3}{a_0}, \ldots ., \alpha_1\alpha_2\alpha_3 \ldots \ldots . \alpha_n = (-1)^n \frac{a_n}{a_0}$$

Note: **(i)** If α is a root of the equation $f(x) = 0$, then the polynomial $f(x)$ is exactly divisible by $(x - \alpha)$ or $(x - \alpha)$ is a factor of $f(x)$ and conversely .

(ii) Every equation of nth degree $(n \geq 1)$ has exactly n roots & if the equation has more than n roots, it is an identity.

(iii) If the coefficients of the equation $f(x) = 0$ are all real and $\alpha + i\beta$ is its root, then $\alpha - i\beta$ is also a root. i.e. **imaginary roots occur in conjugate pairs.**

(iv) If the coefficients in the equation are all rational & $\alpha + \sqrt{\beta}$ is one of its roots, then $\alpha - \sqrt{\beta}$ is also a root where $\alpha, \beta \in Q$ & β is not a perfect square.

(v) If there be any two real numbers ' a ' & 'b' such that $f(a)$&$f(b)$ are of opposite signs, then $f(x) = 0$ must have atleast one real root between ' a ' and ' b ' .

(vi) Every equation $f(x) = 0$ of degree odd has atleast one real root of a sign opposite to that of its last term.

12. LOCATION OF ROOTS:

Let $f(x) = ax^2 + bx + c$, where $a > 0$ & $a, b, c \in R$.

(i) Conditions for both the roots of $f(x) = 0$ to be greater than a specified number 'd' are

$b^2 - 4ac \geq 0$; $f(d) > 0$ & $(-b/2a) > d$.

(ii) Conditions for both roots of $f(x) = 0$ to lie on either side of the number 'd' (in other words the number ' d ' lies between the roots of $f(x) = 0$) is $f(d) < 0$.

(ii) Conditions for exactly one root of $f(x) = 0$ to lie in the interval (d, e) i.e. $d < x < e$ are

$b^2 - 4ac > 0$ & $f(d) \cdot f(e) < 0$.

(iii) Conditions that both roots of $f(x) = 0$ to be confined between the numbers p & q are

$$(p < q).\ b^2 - 4ac \geq 0; f(p) > 0; f(q) > 0 \& p < (-b/2a) < q$$

13. LOGARITHMIC INEQUALITIES

(i) For $a > 1$ the inequality $0 < x < y$ & $\log_a x < \log_a y$ are equivalent.

(ii) For $0 < a < 1$ the inequality $0 < x < y$ & $\log_a x > \log_a y$ are equivalent.

(iii) If $a > 1$ then $\log_a x < p \Rightarrow 0 < x < a^p$

(iv) If $a > 1$ then $\log_a x > p \Rightarrow x > a^p$

(v) If $0 < a < 1$ then $\log_a x < p \Rightarrow x > a^p$

(vi) If $0 < a < 1$ then $\log_a x > p \Rightarrow 0 < x < a^p$

PROFICIENCY TEST-01

1. If α and β are the roots of equation $ax^2 + bx + c = 0$, find the value of following expressions.

(i) $\alpha^2 + \beta^2$ (ii) $\alpha^3 + \beta^3$ (iii) $\alpha^4 + \beta^4$ (iv) $(\alpha - \beta)^2$ (v) $\alpha^4 - \beta^4$

2. If α and β are the roots of equation $ax^2 + bx + c = 0$, form an equation whose roots are :

(i) $\alpha + \frac{1}{\beta}, \beta + \frac{1}{\alpha}$ (ii) $\frac{1}{\alpha+\beta}, \frac{1}{\alpha} + \frac{1}{\beta}$

3. What can you say about the roots of the following equations?

(i) $x^2 + 2(3a + 5)x + 2(9a^2 + 25) = 0$

(ii) $(y - a)(y - b) + (y - b)(y - c) + (y - c)(y - a) = 0$

4. In copying a quadratic equation of the form $x^2 + px + q = 0$, the coefficient of x was wrongly written as -10 in place of -11 and the roots were found to be 4 and 6 . Find the roots of the correct equation.

5. If the sum of the roots of quadratic equation $(a + 1)x^2 + (2a + 3)x + (3a + 4) = 0$ is -1 , then find the product of the roots.

6. Find the roots of the equation $(b - c)x^2 + (c - a)x + (a - b) = 0$.

7. If α and β are the roots of the equation $2x^2 + 2(a + b)x + a^2 + b^2 = 0$, then find the equation whose roots are $(\alpha + \beta)^2$ and $(\alpha - \beta)^2$

8. Find the values of m, for which the equation $5x^2 - 24x + 2 + m(4x^2 - 2x - 1) = 0$ has

(a) equal roots (b) the product of the roots is 2 (c) the sum of the roots is 6

9. For what value of a is the difference between the roots of the equation $(a - 2)x^2 - (a - 4)x - 2 = 0$ equal to 3 ?

10. If the difference of roots of the equation $2x^2 - (a + 1)x + a - 1 = 0$ is equal to their product, then prove that $a = 2$.

PROFICIENCY TEST-02

1. If α, β are roots of $x^2 - px + q = 0$ and $\alpha - 2, \beta + 2$ are roots of $x^2 - px + r = 0$, then prove that $16q + (r + 4 - q)^2 = 4p^2$

2. Show that if roots of equation $(a^2 - bc)x^2 + 2(b^2 - ac)x + c^2 - ab = 0$ are equal then either $b = 0$ or $a^3 + b^3 + c^3 = 3abc$

3. Show that the graph of the polynomial : $y = x^2 + kx - x + 9$ is above X-axis, if and only if $-5 < k < 7$.

4. The least integral value of ' k ' for which $(k - 2)x^2 + 8x + k + 4 > 0$ for all $x \in R$, is :

(A) 5 (B) 2 (C) 3 (D) None of these

5. If the quadratic polynomials defined on real coefficients $P(x) = a_1x^2 + 2b_1x + c_1$ and $Q(x) = a_2x^2 + 2b_2x + c_2$ take positive values $\forall\, x \in R$, then prove that $g(x) = a_1a_2x^2 + b_1b_2x + c_1c_2$ attains only positive values.

6. Find all values of a for which the inequality $(a - 1)x^2 - (a + 1)x + a + 1 > 0$ is satisfied for all real x.

7. Find all values of ' a ' for which the inequality $(a + 4)x^2 - 2ax + 2a - 6 < 0$ is satisfied for all $x \in R$.

8. Solve the equation

$(x^2 - 3x)^2 + 3x^2 - 9x - 28 = 0$ for real roots.

9. Solve the equation

$$\left(\frac{x-1}{x}\right)^2 - 3\left(\frac{x-1}{x}\right) - 4 = 0$$

10. For what integral k is the inequality $x^2 - 2(4k - 1)x + 15k^2 - 2k - 7 > 0$ valid for any real x ?

PROFICIENCY TEST-03

1. If $ax^2 + bx + c = 0$ has imaginary roots and $a + c < b$, then prove that $4a + c < 2b$. (where $a, b, c \in R$)

2. If $a, b, c \in R$, then prove that the roots of the equation $\frac{1}{x-a} + \frac{1}{x-b} + \frac{1}{x-c} = 0$ are always real and cannot have roots if $a = b = c$.

3. For what values of k the expression $kx^2 + (k+1)x + 2$ will be a perfect square of a linear polynomial.

4. If $2 + i\sqrt{3}$ is a root of the equation $x^2 + px + q = 0$, where $p, q \in R$, then find the ordered pair (p, q).

5. If one root of the equation $ax^2 + bx + c = 0$ is equal to n^{th} power of the other root, show that $(ac^n)^{1/(n+1)} + (a^n c)^{1/(n+1)} + b = 0$

6. Solve the equation $\frac{a^2(x-b)(x-c)}{(a-b)(a-c)} + \frac{b^2(x-c)(x-a)}{(b-c)(b-a)} = x^2$

7. If the roots of the equation $x^2 - bx + c = 0$ be two consecutive integers, then find the value of $b^2 - 4c$.

8. If the roots of the equation $12x^2 - mx + 5 = 0$ are in the ratio $2:3$, then find the value of m.

9. Show that $\frac{(x+b)(x+c)}{(b-a)(c-a)} + \frac{(x+c)(x+a)}{(c-b)(a-b)} + \frac{(x+a)(x+b)}{(a-c)(b-c)} = 1$, is an identity.

10. If the ratio of roots of the equation $x^2 + px + q = 0$ be equal to the ratio of roots of the equation $x^2 + bx + c = 0$, then prove that $p^2c = b^2q$.

PROFICIENCY TEST-04

1. If $\tan\theta$ and $\sec\theta$ are the roots of $ax^2 + bx + c = 0$, then prove that $a^4 = b^2(b^2 - 4ac)$.

2. Solve the inequality $\frac{(x-1)^4(x-3)^3}{(x-5)^2} \geq 0$.

3. Find interval of x satisfying the inequality given by $\frac{(2x+1)(x-1)}{(x^3-3x^2+2x)} \geq 0$.

4. Solve the inequality $\frac{(x^2-3x+2)(x^2+2x+2)}{(x-3)(x-1-2x^2)} \geq 0$.

5. Find the value of a for which the sum of the squares of the roots of the equation $x^2 - (a-2)x - a - 1 = 0$ assumes the least value.

6. If x_1 and x_2 are the roots of $x^2 + (\sin\theta - 1)x - 1/2\cos^2\theta = 0$, then find the maximum value of $x_1^2 + x_2^2$.

7. Find the range of $f(x) = 2x^2 - 3x + 2$ in $[0,2]$.

8. Find the difference between the least and greatest values of $y = -2x^2 + 3x - 2$ for $x \in [0,2]$.

9. Solve the inequality $x^3 - 3x^2 - x + 3 > 0$.

10. Solve the inequality $(x^2 - 3x + 2)(x^3 - 3x^2)(4 - x^2) \geq 0$.

PROFICIENCY TEST-05

1. Solve the equation $2|x-2|-3|x+4|=1$.

2. Solve the equation $|x-|4-x||-2x=4$.

3. Solve $|x^2+x|<2$.

4. Solve $\left|\frac{x-1}{x+2}\right|\geq 2$.

5. Solve the equation $\left|\frac{x^2-8x+12}{x^2-10x+21}\right|=-\frac{x^2-8x+12}{x^2-10x+21}$.

6. Solve the equation $|x^2-1|+|2-x^2|=1$.

7. Solve $|2x+3|=|3x+2|+|x-1|$.

8. Find the solution set of the inequality $\left|\frac{3|x|-2}{|x|-1}\right|\geq 2$.

9. Solve $|x^2-2x|+|x|\leq 2$.

10. Solve $|x^2-1|+\sqrt{x^2-3x+2}=0$.

PROFICIENCY TEST-06

1. Find the value of k, so that the equations $2x^2 + kx - 5 = 0$ and $x^2 - 3x - 4 = 0$ may have one root in common.

2. Find the value of ' a ' so that $x^2 - 11x + a = 0$ and $x^2 - 14x + 2a = 0$ have a common root.

3. If a, b, c, ∈ R and equations $ax^2 + bx + c = 0$ and $x^2 + 2x + 9 = 0$ have a common root, then find a : b : c.

4. Find the condition on a, b, c, d such that equations $2ax^3 + bx^2 + cx + d = 0$ and $2ax^2 + 3bx + 4c = 0$ have a common root.

5. If the equations $3x^2 + px + 1 = 0$ and $2x^2 + qx + 1 = 0$ have a common root, show that $2p^2 + 3q^2 - 5pq + 1 = 0$

6. If the equations $x^2 + abx + c = 0$ and $x^2 + acx + b = 0$ have a common root, prove that their other roots satisfy the equation $x^2 - a(b + c)x + a^2bc = 0$.

7. If the equation $x^2 - px + q = 0$ and $x^2 - ax + b = 0$ have a common root and the other root of the second equation is the reciprocal of the first, then prove that $(q - b)^2 - bq(p - a)^2 = 0$.

8. If the equations $x^2 - ax + b = 0$ and $x^2 - cx + d = 0$ have one root in common and second equation has equal roots, prove that $ac = 2(b + d)$.

9. If α, β and γ are the roots of $x^3 + 8 = 0$, then find the equation whose roots are α^2, β^2 and γ^2.

10. If α, β, γ are the roots of the equation $x^3 - px + q = 0$, then find the cubic equation whose roots are $\alpha/(1 + \alpha), \beta/(1 + \beta), \gamma/(1 + \gamma)$.

PROFICIENCY TEST-07

1. Determine the values of k for which the equation $\frac{x^2+x+2}{3x+1} = k$ has both roots real.

2. Find the range of

(i) $f(x) = \frac{x^2+34x-71}{x^2+2x-7}$ **(ii)** $f(x) = \frac{x^2-x+1}{x^2+x+1}$

3. For real values of x, prove that $\frac{11x^2+12x+6}{x^2+4x+2}$ cannot lie between -5 and 3.

4. If x is real, show that the expression $\frac{4x^2+36x+9}{12x^2+8x+1}$ can have any real value.

5. If x be real, show that the expression $\frac{x^2+2x-11}{x-3}$ can take all values which do not lie in the open interval $(4, 12)$.

6. Find the range of the expression $y = \frac{\tan^2\theta-2\tan\theta-8}{\tan^2\theta-4\tan\theta-5}$, for all permissible values of θ.

7. If x is real and $4y^2 + 4xy + x + 6 = 0$, then find the complete set of values of x for which y is real.

8. Find the range of real values of x & y satisfying the relation, $x^2 + y^2 = 6x - 8y$.

9. If x, y be real and $9x^2 + 2xy + y^2 - 92x - 20y + 244 = 0$, show that $x \in [3,6]$, $y \in [1,10]$.

10. If x, y and z are three real numbers such that $x + y + z = 4$ and $x^2 + y^2 + z^2 = 6$, then show that each of x, y and z lie in the closed interval $\left[\frac{2}{3}, 2\right]$

PROFICIENCY TEST-08

1. Find the values of a if $x^2 - 2(a-1)x + (2a+1) = 0$ has positive roots.

2. If the equation $(a-5)x^2 + 2(a-10)x + a + 10 = 0$ has roots of opposite sign, then find the values of a.

3. If both the roots of $x^2 - ax + a = 0$ are greater than 2 , then find the values of a.

4. If both the roots of $ax^2 + ax + 1 = 0$ are less than 1 , then find exhaustive range of values of a.

5. If both the roots of $x^2 + ax + 2 = 0$ lies in the interval $(0,3)$, then find exhaustive range of values of a.

6. If α, β are the roots of $x^2 - 3x + a = 0, a \in R$ and $\alpha < 1 < \beta$, then find the values of a.

7. Match the conditions in Column-I with the intervals in Column-II.

   Let $f(x) = x^2 - 2px + p^2 - 1$, then

Column-I		**Column-II.**	
(A)	both the roots of $f(x) = 0$ are less than 4 , if $p \in$	**(P)**	$(-1, \infty)$
(B)	both the roots of $f(x) = 0$ are greater than -2 if $p \in$	**(Q)**	$(-\infty, 3)$
(C)	exactly one root of $f(x) = 0$ lie in $(-2,4)$, if $p \in$	**(R)**	$(0,2)$
(D)	1 lies between the roots of $f(x) = 0$, if $p \in$	**(S)**	$(-3,-1] \cup [3,5)$

PROFICIENCY TEST-09

1. Solve the equation $\sqrt{3x+4}+\sqrt{x-4}=2\sqrt{x}$.

2. Solve the equation $\sqrt[3]{(2x-1)}+\sqrt[3]{(x-1)}=1$.

3. Solve $\sqrt{x^2+4x-5}>x-3$.

4. Solve the inequality $\sqrt{x^2+3x+2}<1+\sqrt{x^2-x+1}$.

5. Solve the inequality $2^{x+2}-2^{x+3}-2^{x+4}>5^{x+1}-5^{x+2}$.

6. Solve $2^{2x}-4.9^x\leq 3.6^x$

7. Solve $\log_2(x^2-1)<1$.

8. Solve $\log_{\frac{1}{4}}(2-x)>\log_{\frac{1}{4}}\left(\frac{2}{x+1}\right)$.

9. Solve $\log_{\frac{1}{3}}\left(\log_4(x^2-5)\right)>0$.

10. Solve $\log_x(2x-3/4)<2$.

EXERCISE- I

Q.1 A quadratic polynomial $f(x) = x^2 + ax + b$ is formed with one of its zeros being $\frac{4+3\sqrt{3}}{2+\sqrt{3}}$ where a and b are integers. Also $g(x) = x^4 + 2x^3 - 10x^2 + 4x - 10$ is a biquadratic polynomial such that $g\left(\frac{4+3\sqrt{3}}{2+\sqrt{3}}\right) = c\sqrt{3} + d$ where c and d are also integers. Find the values of a, b, c and d.

Q.2 Solve the inequality.

$$(\log_2 x)^4 - \left(\log_{\frac{1}{2}} \frac{x^5}{4}\right)^2 - 20\log_2 x + 148 < 0$$

Q.3 α, β are the roots of the equation $K(x^2 - x) + x + 5 = 0$. If K_1 & K_2 are the two values of K for which the roots α, β are connected by the relation $(\alpha/\beta) + (\beta/\alpha) = 4/5$. Find the value of $(K_1/K_2) + (K_2/K_1)$

Q.4 If one root of the equation $ax^2 + bx + c = 0$ be the square of the other, prove that $b^3 + a^2c + ac^2 = 3abc$

Q.5 If $ax^2 + bx + c = 0$ and $bx^2 + cx + a = 0$ have a common root and a, b, c are non-zero real numbers, then find the value of $\frac{a^3+b^3+c^3}{abc}$

Q.6 Find a quadratic equation whose sum and product of the roots are the values of the expressions $(\text{cosec } 10° - \sqrt{3}\sec 10°)$ and $(0.5\text{cosec } 10° - 2\sin 70°)$ respectively. Also express the roots of this quadratic in terms of tangent of an angle lying in $(0, \pi/2)$.

Q.7 If α be a root of the equation $4x^2 + 2x - 1 = 0$ then prove that $4\alpha^3 - 3\alpha$ is the other root.

Q.8 **(a)** If α, β are the roots of the quadratic equation $ax^2 + bx + c = 0$ then which of the following expressions in α, β will denote the symmetric functions of roots. Give proper reasoning.

(i) $f(\alpha, \beta) = \alpha^2 - \beta$ **(ii)** $f(\alpha, \beta) = \alpha^2\beta + \alpha\beta^2$

(iii) $f(\alpha, \beta) = \ln \frac{\alpha}{\beta}$ **(iv)** $f(\alpha, \beta) = \cos(\alpha - \beta)$

(b) If α, β are the roots of the equation $x^2 - px + q = 0$, then find the quadratic equation the roots of which are $(\alpha^2 - \beta^2)(\alpha^3 - \beta^3)$ & $\alpha^3\beta^2 + \alpha^2\beta^3$.

Q.9. Find the number of solutions of the equation $\sin\left(\frac{\pi x}{2}\right) = \frac{99x}{500}$

Q.10 If α, β are the roots of $x^2 - px + 1 = 0$ & γ, δ are the roots of $x^2 + qx + 1 = 0$, show that $(\alpha - \gamma)(\beta - \gamma)(\alpha + \delta)(\beta + \delta) = q^2 - p^2$

Q.11. Find all cubic polynomials $p(x)$ such that $(x - 1)^2$ is a factor of $p(x) + 2$ and $(x + 1)^2$ is a factor of $p(x) - 2$.

Q.12 If the roots of $x^2 - ax + b = 0$ are real & differ by a quantity which is less than $c(c > 0)$, prove that b lies between $(1/4)(a^2 - c^2)$ & $(1/4)a^2$.

Q.13 A quadratic polynomial $y = f(x)$ satisfies $f(x) = \left[\frac{f(x+1)-f(x-1)}{2}\right]^2$ for all real x. Find the leading coefficient of the quadratic polynomial and hence find the value of $[f(0) - f(-1)] + [f(0) - f(1)]$.

Q.14 If the quadratic equations $x^2 + bx + ca = 0$ & $x^2 + cx + ab = 0$ have a common root, prove that the equation containing their other root is $x^2 + ax + bc = 0$.

Q.15 If $ax^2 + bx + c = 0$ and $bx^2 + cx + a = 0$ have a root in common, find the relation between a, b and c.

Q.16 Find the value of m for which the quadratic equations $x^2 - 11x + m = 0$ and $x^2 - 14x + 2m = 0$ may have common root.

Q.17 Prove that $f(x) = x^{12} - x^9 + x^4 - x + 1$ is positive for all x.

Q.18 Find the values of ' a ' for which $-3 < [(x^2 + ax - 2)/(x^2 + x + 1)] < 2$ is valid for all real x.

Q.19 Find the minimum value of $\frac{\left(x+\frac{1}{x}\right)^6-\left(x^6+\frac{1}{x^6}\right)-2}{\left(x+\frac{1}{x}\right)^3+x^3+\frac{1}{x^3}}$ for $x > 0$.

Q.20 Find the product of the real roots of the equation,

$x^2 + 18x + 30 = 2\sqrt{x^2 + 18x + 45}$

Q.21 Find the set of values of 'y' for which the inequality, $2\log_{0.5}y^2 - 3 + 2x\log_{0.5}y^2 - x^2 > 0$ is valid for atleast one real value of 'x'.

Q.22 If λ be the smallest integral value of parameters 'a' for which the inequality $1 + \log_5(x^2 + 1) \le \log_5(ax^2 + 4x + a)$ is true for all $x \in R$. Find the value of 8λ.

Paragraph for Question Nos. 23 to 25

A polynomial p(x) satisfies the relation $(x - 16)\, p(2x) = 16(x - 1)\, p(x) \forall x \in R$.

Let $p(7) = 135$ then

Q.23 The value of $p(11)$ equals to

(A) -1145 (B) -1040 (C) -945 (D) -1045

Q.24 The sum of its roots equals to

(A) 10 (B) 20 (C) 30 (D) 40

Q.25 The product of its roots equals to

(A) 4^5 (B) 4^7 (C) 4^3 (D) 4^4

EXERCISE- II

Q.1 We call 'p' a good number if the inequality $\frac{2x^2+2x+3}{x^2+x+1} \leq p$ is satisfied for any real x. Find the smallest integral good number.

Q.2 Let a, b, c, d be distinct real numbers and a and b are the roots of quadratic equation $x^2 - 2cx - 5d = 0$. If c and d are the roots of the quadratic equation $x^2 - 2ax - 5b = 0$ then find the numerical value of $a + b + c + d$

Q.3 Let α, β and γ are the roots of the cubic $x^3 - 3x^2 + 1 = 0$. Find a cubic whose roots are $\frac{\alpha}{\alpha-2}, \frac{\beta}{\beta-2}$ and $\frac{\gamma}{\gamma-2}$. Hence or otherwise find the value of $(\alpha-2)(\beta-2)(\gamma-2)$.

Q.4 If α, β are the roots of the equation, $x^2 - 2x - a^2 + 1 = 0$ and γ, δ are the roots of the equation, $x^2 - 2(a+1)x + a(a-1) = 0$ such that $\alpha, \beta \in (\gamma, \delta)$ then find the values of 'a'.

Q.5 Two roots of a biquadratic $x^4 - 18x^3 + kx^2 + 200x - 1984 = 0$ have their product equal to (-32). Find the value of k.

Q.6 At what values of 'a' do all the zeroes of the function, $f(x) = (a-2)x^2 + 2ax + a + 3$ lie on the interval $(-2,1)$?

Q.7 Suppose a cubic polynomial $f(x) = x^3 + px^2 + qx + 72$ is divisible by both $x^2 + ax + b$ and $x^2 + bx + a$ (where a, b, p, q are constants and $a \neq b$). Find the sum of the squares of the roots of the cubic polynomial.

Q.8 Find the values of K so that the quadratic equation $x^2 + 2(K-1)x + K + 5 = 0$ has atleast one positive root.

Q.9 Find all the values of the parameter 'a' for which both roots of the quadratic equation $x^2 - ax + 2 = 0$ belong to the interval $(0,3)$.

Q.10 Solve the inequality $\left(\log_{|x+6|} 2\right) \cdot \log_2 (x^2 - x - 2) \geq 1$

Q.11 Find all the values of the parameter 'a' for which the inequality

a. $9^x + 4(a-1)3^x + a - 1 > 0$ is satisfied for all real values of x.

Q.12 Find the complete set of real values of ' a ' for which both roots of the quadratic equation $(a^2 - 6a + 5)x^2 - \sqrt{a^2 + 2a}\, x + (6a - a^2 - 8) = 0$ lie on either side of the origin.

Q.13 Given $x, y \in R, x^2 + y^2 > 0$. If the maximum and minimum value of the expression $E = \frac{x^2+y^2}{x^2+xy+4y^2}$ are M and m, and A denotes the average value of M and m, compute (2007)A.

Q.14 Let $P(x) = x^2 + bx + c$, where b and c are integer. If P(x) is a factor of both $x^4 + 6x^2 + 25$ and $3x^4 + 4x^2 + 28x + 5$, find the value of P(1)

Q.15 Given the cubic equation $x^3 - 2kx^2 - 4kx + k^2 = 0$. If one root of the equation is less than 1, other root is in the interval (1,4) and the 3^{rd} root is greater than 4, then the value of k lies in the interval $(a + \sqrt{b}, b(a + \sqrt{b}))$ where $a, b \in N$. Find the value of $(a + b)^3 + (ab + 2)^2$

Q.16 If $a < b < c < d$, then show that the quadratic equation $\mu(x - a)(x - c) + \lambda(x - b)(x - d) = 0$ has real roots for all real μ and λ.

Q.17 The polynomial $p(x)$ has integral coefficients and $p(x) = 7$ for four different integral values of x. Show that $p(x)$ never equals 14, for integral values of x.

Q.18 Show that if p, q, r & s are real numbers & $pr = 2(q + s)$, then at least one of the equations $x^2 + px + q = 0, x^2 + rx + s = 0$ has real roots.

Q.19 Find out the values of 'a' for which any solution of the inequality, $\frac{\log_3(x^2-3x+7)}{\log_3(3x+2)} < 1$ is also a solution of the inequality, $x^2 + (5 - 2a)x \leq 10a$

Q.20 Find all real numbers x such that, $\left(x - \frac{1}{x}\right)^{\frac{1}{2}} + \left(1 - \frac{1}{x}\right)^{\frac{1}{2}} = x.$

EXERCISE-III

JEE Main / AIEEE Questions :

Q.1 If $\alpha \neq \beta$ but $\alpha^2 = 5\alpha - 3$ and $\beta^2 = 5\beta - 3$ then the equation having α/β and β/α as its roots is **[AIEEE 2002]**

(A) $3x^2 - 19x + 3 = 0$ (B) $3x^2 + 19x - 3 = 0$

(C) $3x^2 - 19x - 3 = 0$ (D) $x^2 - 5x + 3 = 0$

Q.2 Difference between the corresponding roots of $x^2 + ax + b = 0$ and $x^2 + bx + a = 0$ is same and $a \neq b$, then **[AIEEE 2002]**

(A) $a + b + 4 = 0$ (B) $a + b - 4 = 0$

(C) $a - b - 4 = 0$ (D) $a - b + 4 = 0$

Q.3 If p and q are the roots of the equation $x^2 + px + q = 0$, then **[AIEEE 2002]**

(A) $p = 1, q = -2$ (B) $p = 0, q = 1$

(C) $p = -2, q = 0$ (D) $p = -2, q = 1$

Q.4 If a, b, c are distinct +ve real numbers and $a^2 + b^2 + c^2 = 1$ then $ab + bc + ca$ is **[AIEEE 2002]**

(A) less than 1 (B) equal to 1

(C) greater than 1 (D) any real no.

Q.5 The value of 'a' for which one root of the quadratic equation $(a^2 - 5a + 3)x^2 + (3a - 1)x + 2 = 0$ is twice as large as the other is **[AIEEE 2003]**

(A) $-\frac{1}{3}$ (B) $\frac{2}{3}$ (C) $-\frac{2}{3}$ (D) $\frac{1}{3}$

Q.6 The real positive number x when added to its inverse gives the minimum value of the sum at x equal to **[AIEEE2003]**

(A) -2 (B) 2 (C) 1 (D) -1

Q.7 If $(1 - p)$ is a root of quadratic equation $x^2 + px + (1 - p) = 0$ then its root are **[AIEEE 2004]**

(A) $-1,2$ (B) $-1,1$ (C) $0,-1$ (D) $0,1$

Q.8 If one root of the equation $x^2 + px + 12 = 0$ is 4 , while the equation $x^2 + px + q = 0$ has equal roots, then the value of ' q ' is **[AIEEE 2004]**

(A) 4 (B) 12 (C) 3 (D) $\frac{49}{4}$

Q.9 In a triangle PQR, $\angle R = \frac{\pi}{2}$. If $\tan\left(\frac{P}{2}\right)$ and $\tan\left(\frac{Q}{2}\right)$ are the roots of $ax^2 + bx + c = 0, a \neq 0$ then **[AIEEE 2005]**

(A) $a = b + c$ (B) $c = a + b$

(C) $b = c$ (D) $b = a + c$

Q.10 If both the roots of the quadratic equation $x^2 - 2kx + k^2 + k - 5 = 0$ are less than 5 , then k lies in the interval **[AIEEE 2005]**

(A) $(5,6]$ (B) $(6, \infty)$ (C) $(-\infty, 4)$ (D) $[4,5]$

Q.11 If the roots of the quadratic equation $x^2 + px + q = 0$ are $\tan 30°$ and $\tan 15°$, respectively, then the value of $2 + q - p$ is **[AIEEE 2006]**

(A) 2 (B) 3 (C) 0 (D) 1

Q.12 All the values of m for which both roots of the equation $x^2 - 2mx + m^2 - 1 = 0$ are greater than -2 but less than 4, lie in the interval **[AIEEE 2006]**

(A) $-2 < m < 0$ (B) $m > 3$ (C) $-1 < m < 3$ (D) $1 < m < 4$

Q.13 If x is real, the maximum value of $\frac{3x^2+9x+17}{3x^2+9x+7}$ is **[AIEEE 2006]**

(A) $\frac{1}{4}$ (B) 41 (C) 1 (D) $\frac{17}{7}$

Q.14 If the difference between the roots of the equation $x^2 + ax + 1 = 0$ is less than $\sqrt{5}$, then the set of possible values of a is **[AIEEE 2007]**

(A) $(3, \infty)$ (B) $(-\infty, -3)$ (C) $(-3, 3)$ (D) $(-3, \infty)$

Q.15 The quadratic equations $x^2 - 6x + a = 0$ and $x^2 - cx + 6 = 0$ have one root in common. The other roots of the first and second equations are integers in the ratio 4 : 3. Then the common root is **[AIEEE 2009]**

(A) 1 (B) 4 (C) 3 (D) 2

Q.16 If the roots of the equation $bx^2 + cx + a = 0$ be imaginary, then for all real values of x, the expression $3b^2x^2 + 6bcx + 2c^2$ is **[AIEEE 2009]**

(A) less than 4ab (B) greater than -4ab (C) less than -4ab (D) greater than 4ab

Q.17 The equation $e^{\sin x} - e^{-\sin x} - 4 = 0$ has **[AIEEE 2012]**

(A) infinite number of real roots (B) no real roots

(C) exactly one real root (D) exactly four real roots

Q.18 If the equations $x^2 + 2x + 3 = 0$ and $ax^2 + bx + c = 0$, $a, b, c \in R$, have a common root, then $a : b : c$ is : **[IIT Mains - 2013]**

(A) $3 : 1 : 2$ (B) $1 : 2 : 3$ (C) $3 : 2 : 1$ (D) $1 : 3 : 2$

Q.19 The real number k for which the equation, $2x^3 + 3x + k = 0$ has two distinct real roots in $[0,1]$

(A) does not exist (B) lies between 1 and 2 **[IIT Mains - 2013]**

(C) lies between 2 and 3 (D) lies between -1 and 0

Q.20 Let α and β be the roots of equation $x^2 - 6x - 2 = 0$. If $a_n = \alpha^n - \beta^n$, for $n \geq 1$, then the value of $\frac{a_{10} - 2a_8}{2a_9}$ is equal to : **[IIT Mains - 2015]**

(A) -3 (B) 6 (C) -6 (D) 3

21. Let $S = \{x \in R : x \geq 0 \text{ and } 2|\sqrt{x} - 3| + \sqrt{x}(\sqrt{x} - 6) + 6 = 0\}$. Then S **[IIT Mains - 2018]**

(A) contains exactly four elements (B) is an empty set

(C) contains exactly one element (D) contains exactly two elements

EXERCISE- IV

Q.1 Find the values of α & β, $0 < \alpha, \beta < \pi/2$, satisfying the following equation, **[REE '99, 6]**
$\cos\alpha \cdot \cos\beta \cos(\alpha + \beta) = -1/8$.

Q.2 If the roots of the equation $x^2 - 2ax + a^2 + a - 3 = 0$ are real & less than 3 then **[JEE'99, 2 + 2]**
(A) $a < 2$ (B) $2 \le a \le 3$ (C) $3 < a \le 4$ (D) $a > 4$

Q.3 If α, β are the roots of the equation, $(x - a)(x - b) + c = 0$, find the roots of the equation,
$(x - \alpha)(x - \beta) = c$. **[REE 2000 (Mains), 3]**

Q.4 **(a)** For the equation, $3x^2 + px + 3 = 0, p > 0$ if one of the roots is square of the other, then p is equal to:
(A) 1/3 (B) 1 (C) 3 (D) 2/3

(b) If α & $\beta(\alpha < \beta)$, are the roots of the equation, $x^2 + bx + c = 0$, where $c < 0 < b$, then
(A) $0 < \alpha < \beta$ (B) $\alpha < 0 < \beta < |\alpha|$
(C) $\alpha < \beta < 0$ (D) $\alpha < 0 < |\alpha| < \beta$

(c) If $b > a$, then the equation, $(x - a)(x - b) - 1 = 0$, has: **[JEE 2000 Screening, 1 + 1 + 1 out of 35]**
(A) both roots in $[a, b]$ (B) both roots in $(-\infty, a)$
(C) both roots in $[b, \infty)$ (D) one root in $(-\infty, a)$ & other in $(b, +\infty)$

(d) If α, β are the roots of $ax^2 + bx + c = 0$, $(a \neq 0)$ and $\alpha + \delta, \beta + \delta$, are the roots of,
$Ax^2 + Bx + C = 0$, $(A \neq 0)$ for some constant δ, then prove that, **[JEE 2000, Mains, 4 out of 100]**
$$\frac{b^2 - 4ac}{a^2} = \frac{B^2 - 4AC}{A^2}$$

Q.5 Let a, b, c be real numbers with $a \neq 0$ and let α, β be the roots of the equation
$ax^2 + bx + c = 0$. Express the roots of $a^3x^2 + abcx + c^3 = 0$ in terms of α, β.
[JEE 2001, Mains, 5 out of 100]

Q.6 The set of all real numbers x for which $x^2 - |x + 2| + x > 0$, is **[JEE 2002 (screening), 3]**
(A) $(-\infty, -2) \cup (2, \infty)$ (B) $(-\infty, -\sqrt{2}) \cup (\sqrt{2}, \infty)$
(C) $(-\infty, -1) \cup (1, \infty)$ (D) $(\sqrt{2}, \infty)$

Q.7 If $x^2 + (a - b)x + (1 - a - b) = 0$ where $a, b \in R$ then find the values of 'a' for which equation has unequal real roots for all values of 'b'. **[JEE 2003, Mains-4 out of 60]**

Q.8 **(a)** If one root of the equation $x^2 + px + q = 0$ is the square of the other, then
(A) $p^3 + q^2 - q(3p + 1) = 0$ (B) $p^3 + q^2 + q(1 + 3p) = 0$
(C) $p^3 + q^2 + q(3p - 1) = 0$ (D) $p^3 + q^2 + q(1 - 3p) = 0$

(b) If $x^2 + 2ax + 10 - 3a > 0$ for all $x \in R$, then **[JEE 2004 (Screening)]**
(A) $-5 < a < 2$ (B) $a < -5$
(C) $a > 5$ (D) $2 < a < 5$

Q.9 Find the range of values of t for which $2\sin t = \frac{1-2x+5x^2}{3x^2-2x-1}, t \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

[JEE 2005(Mains), 2]

Q.10 (a)Let a, b, c be the sides of a triangle. No two of them are equal and $\lambda \in R$. If the roots of the equation $x^2 + 2(a+b+c)x + 3\lambda(ab+bc+ca) = 0$ are real, then **[JEE 2006, 3]**

(A) $\lambda < \frac{4}{3}$ (B) $\lambda > \frac{5}{3}$ (C) $\lambda \in \left(\frac{1}{3}, \frac{5}{3}\right)$ (D) $\lambda \in \left(\frac{4}{3}, \frac{5}{3}\right)$

(b) If roots of the equation $x^2 - 10cx - 11d = 0$ are a, b and those of $x^2 - 10ax - 11b = 0$ are c, d, then find the value of $a+b+c+d$. (a, b, c and d are distinct numbers) **[JEE 2006, 6]**

Q.11 **(a)** Let α, β be the roots of the equation $x^2 - px + r = 0$ and $\alpha/2$, 2β be the roots of the equation $x^2 - qx + r = 0$. Then the value of ' r ' is

(A) $\frac{2}{9}(p-q)(2q-p)$ (B) $\frac{2}{9}(q-p)(2p-q)$

(C) $\frac{2}{9}(q-2p)(2q-p)$ (D) $\frac{2}{9}(2p-q)(2q-p)$

MATCH THE COLUMN:

(b) Let $f(x) = \frac{x^2-6x+5}{x^2-5x+6}$

Match the expressions / statements in Column I with expressions / statements in Column II.

Column I	Column II
(A) If $-1 < x < 1$, then f(x) satisfies	**(P)** $0 < f(x) < 1$
(B) If $1 < x < 2$, the f(x) satisfies	**(Q)** $f(x) < 0$
(C) If $3 < x < 5$, then f(x) satisfies	**(R)** $f(x) > 0$
(D) If $x > 5$, then f(x) satisfies	**(S)** $f(x) < 1$

Q.12 The smallest value of k, for which both the roots of the equation $x^2 - 8kx + 16(k^2 - k + 1) = 0$ are real, distinct and have values at least 4 , is **[JEE 2009]**

Q.13 Let p and q be real numbers such that $p \neq 0, p^3 \neq q$ and $p^3 \neq -q$. If α and β are nonzero complex numbers satisfying $\alpha + \beta = -p$ and $\alpha^3 + \beta^3 = q$, then a quadratic equation having $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$ as its roots is **[JEE 2010]**

(A) $(p^3+q)x^2 - (p^3+2q)x + (p^3+q) = 0$ (B) $(p^3+q)x^2 - (p^3-2q)x + (p^3+q) = 0$

(C) $(p^3-q)x^2 - (5p^3-2q)x + (p^3-q) = 0$ (D) $(p^3-q)x^2 - (5p^3+2q)x + (p^3-q) = 0$

Q.14 A value of b for which the equations

$x^2 + bx - 1 = 0$ **[JEE 2011]**

$x^2 + x + b = 0,$

have one root in common is

(A) $-\sqrt{2}$ (B) $-i\sqrt{3}$ (C) $i\sqrt{5}$ (D) $\sqrt{2}$

Q.15 Let S be the set of all non-zero real numbers α such that the quadratic equation $\alpha x^2 - x + \alpha = 0$ has two distinct real roots x_1 and x_2 satisfying the inequality $|x_1 - x_2| < 1$. Which of the following intervals is(are) a subset(s) of S? **[IIT Advance - 2015]**

(A) $\left(-\frac{1}{2}, -\frac{1}{\sqrt{5}}\right)$ (B) $\left(-\frac{1}{\sqrt{5}}, 0\right)$ (C) $\left(0, \frac{1}{\sqrt{5}}\right)$ (D) $\left(\frac{1}{\sqrt{5}}, \frac{1}{2}\right)$

Q.16. Let $-\frac{\pi}{6} < \theta < -\frac{\pi}{12}$. Suppose α_1 and β_1 are the roots of the equation $x^2 - 2x\sec\theta + 1 = 0$ and α_2 and β_2 are the roots of the equation $x^2 + 2x\tan\theta - 1 = 0$. If $\alpha_1 > \beta_1$ and $\alpha_2 > \beta_2$, then $\alpha_1 + \beta_2$ equals **[IIT Advance - 2016]**

(A) $2(\sec\theta - \tan\theta)$ (B) $2\sec\theta$ (C) $-2\tan\theta$ (D) 0

PARAGRAPH (Q.17 TO Q.18) **[IIT Advanced - 2017]**

Let p, q be integers and let α, β be the roots of the equation, $x^2 - x - 1 = 0$, where $\alpha \neq \beta$. For $n = 0,1,2 \ldots \ldots$, let $a_n = p\alpha^n + q\beta^n$.

Fact : If a and b are rational numbers and $a + b\sqrt{5} = 0$, then $a = 0 = b$.

Q.17 If $a_4 = 28$, then $p + 2q =$

(A) 14 (B) 7 (C) 12 (D) 21

Q.18 $a_{12} =$

(A) $2a_{11} + a_{10}$ (B) $a_{11} - a_{10}$ (C) $a_{11} + a_{10}$ (D) $a_{11} + 2a_{10}$

Q.19 Suppose a, b denote the distinct real roots of the quadratic polynomial $x^2 + 20x - 2020$ and suppose c, d denote the distinct complex roots of the quadratic polynomial $x^2 - 20x + 2020$. Then the value of $ac(a - c) + ad(a - d) + bc(b - c) + bd(b - d)$ **[IIT Advanced - 2020]**

(A) 0 (B) 8000 (C) 8080 (D) 16000

ANSWNER KEY

PROFICIENCY TEST-01

1. **(i)** $\frac{(b^2-2ac)}{a^2}$, **(ii)** $\frac{3abc-b^3}{a^3}$, **(iii)** $\frac{(b^2-2ac)^2-2c^2a^2}{a^4}$, **(iv)** $\frac{b^2-4ac}{a^2}$, **(v)** $\frac{\pm b}{a^4}(b^2-2ac)\sqrt{b^2-4ac}$
2. **(i)** $ac\cdot x^2+b(c+a)x+(c+a)^2=0$ **(ii)** $bcx^2+(ac+b^2)x+ab=0$
3. **(i)** roots are non-real if $a\neq 5/3$, and real iff $a=5/3$, **(ii)** roots are real
4. 8,3 **5.** 2 **6.** $1,\frac{a-b}{b-c}$ **7.** $x^2-4abx-(a^2-b^2)^2=0$
8. (a) ϕ (b) $-\frac{8}{9}$ (c) $\frac{-3}{11}$ **9.** $a=3/2,3$

PROFICIENCY TEST-02

4. A **6.** $a\in\left(\frac{5}{3},\infty\right)$ **7.** $a\in(-\infty,-6)$

8. $-1,4$ **9.** $-1/3,1/2$ **10.** 3

PROFICIENCY TEST-03

3. $3\pm 2\sqrt{2}$ **4.** $(-4,7)$ **6.** $\{a,b\}$ **7.** 1 **8.** $\pm 5\sqrt{10}$

PROFICIENCY TEST-04

2. $x\in\{1\}\cup[3,5)\cup(5,\infty)$ **3.** $x\in\left[-\frac{1}{2},0\right)\cup(2,\infty)$ **4.** $x\in(-\infty,1]\cup[2,3)$

5. 1 **6.** 4 **7.** $\left[\frac{7}{8},4\right]$ **8.** $\frac{25}{8}$

9. $x\in(-1,1)\cup(3,\infty)$ **10.** $x\in(-\infty,-2]\cup\{0\}\cup[1,3]$

PROFICIENCY TEST-05

1. $x=-15,-\frac{9}{5}$ **2.** $x=0$ **3.** $x\in(-2,1)$

4. $x\in[-5,-1]-\{-2\}$ **5.** $x\in[2,3)\cup[6,7)$ **6.** $x\in[-\sqrt{2},-1]\cup[1,\sqrt{2}]$

7. $x\in\left[-\frac{2}{3},1\right]$ **8.** $x\in(-\infty,-1)\cup\left(-1,\frac{-4}{5}\right]\cup\{0\}\cup\left[\frac{4}{5},1\right)\cup(1,\infty)$

9. $x\in\left[\frac{3-\sqrt{17}}{2},1\right]\cup\{2\}$ **10.** $x=1$

PROFICIENCY TEST-06

1. $k=-3,-\frac{27}{4}$ **2.** $a=0,24$ **3.** $1:2:9$

4. $(ad+4bc)^2=\frac{9}{2}(bd+4c^2)(b^2-ac)$ **9.** $y^3-64=0$

10. $(p+q-1)x^3-(2p+3q)x^2+(p+3q)x-q=0$

PROFICIENCY TEST-07

1. $k \le -7/9, k \ge 1$ **2.** (i) $R-(5,9)$(ii) $\left[\frac{1}{3},3\right]$ **6.** $y \in (-\infty,\infty)$

7. $(-\infty,-2] \cup [3,\infty)$ **8.** $x \in [-2,8], y \in [-9,1]$

PROFICIENCY TEST-08

1. $a \ge 4$ **2.** $(-10,5)$ **3.** $a \in \phi$ **4.** $\left(-\infty,-\frac{1}{2}\right) \cup [4,\infty)$

5. $\left(-\frac{11}{3},-2\sqrt{2}\right]$ **6.** $a < 2$ **7.** (A) Q, R;(B) P, R; (C) S; (D) R

PROFICIENCY TEST-09

1. 4 **2.** 1 **3.** $x \in (-\infty,-5] \cup [1,\infty)$

4. $x \in (-\infty,-2] \cup \left[-1,\frac{-1+\sqrt{13}}{6}\right)$ **5.** $x \in (0,\infty)$ **6.** $x \in \left[\log_{\frac{2}{3}} 4,\infty\right)$

7. $x \in (-\sqrt{3},-1) \cup (1,\sqrt{3})$ **8.** $x \in (-1,0) \cup (1,2)$ **9.** $x \in (-3,-\sqrt{6}) \cup (\sqrt{6},3)$

10. $x \in \left(\frac{1}{2},1\right) \cup \left(\frac{3}{2},\infty\right)$

EXERCISE-I

1. $a=2, b=-11, c=4, d=-1$ **2.** $x \in \left(\frac{1}{16},\frac{1}{8}\right) \cup (8,16)$ **3.** 254 **5.** 3

6. $x^2-4x+1=0; \alpha=\tan\left(\frac{\pi}{12}\right); \beta=\tan\left(\frac{5\pi}{12}\right)$

8. (a) (ii) and (iv); (b) $x^2-p(p^4-5p^2q+5q^2)x+p^2q^2(p^2-4q)(p^2-q)=0$

9. 7 **11.** x^3-3x

13. $a=\frac{1}{4}$ and $[f(0)-f(-1)]+[f(0)-f(1)]=-\frac{1}{2}$

15. $a=0$ or $a^3+b^3+c^3=3abc$

16. 0 or 24 **18.** $-2<a<1$ **19.** $y_{min}=6$

20. 20 **21.** $(-\infty,-2\sqrt{2}) \cup \left(-\frac{1}{\sqrt{2}},0\right) \cup \left(0,\frac{1}{\sqrt{2}}\right) \cup (2\sqrt{2},\infty)$

22. 56 **23.** C **24.** C **25.** A

EXERCISE-II

1. 4 **2.** 30 **3.** $3y^3-9y^2-3y+1=0; (\alpha-2)(\beta-2)(\gamma-2)=3$

4. $a \in \left(-\frac{1}{4},1\right)$ 5. $k=86$ **6.** $\left(-\infty,-\frac{1}{4}\right) \cup \{2\} \cup (5,6]$

7. 146 **8.** $K \le -1$ **9.** $2\sqrt{2} \le a < \frac{11}{3}$

10. $x<-7, -5<x\le -2, x\ge 4$

11 $[1,\infty)$ **12.** $(-\infty,-2] \cup [0,1) \cup (2,4) \cup (5,\infty)$ 13. 1338

13. 1338 **14.** $P(1)=4$ **15.** 2007 **19.** $a \ge \frac{5}{2}$ **20.** $x=\frac{\sqrt{5}+1}{2}$

EXERCISE-III

1.	A	**2.**	A	**3.**	A	**4.**	A	**5.**	B	**6.**	C	**7.**	C
8.	D	**9.**	B	**10.**	C	**11.**	B	**12.**	C	**13.**	B	**14.**	C
15.	D	**16.**	B	**17.**	B	**18.**	B	**19.**	A	**20**	D		

EXERCISE-IV

1. $\alpha = \beta = \pi/3,$ **2.** A **3.** (a, b) **4.** (a) C, (b) B, (c) D

5. $\gamma = \alpha^2\beta$ and $\delta = \alpha\beta^2$ or $\gamma = \alpha\beta^2$ and $\delta = \alpha^2\beta$

6. B **7.** $a > 1$ **8.** (a) D;; (b)A **9.** $\left[-\frac{\pi}{2}, -\frac{\pi}{10}\right] \cup \left[\frac{3\pi}{10}, \frac{\pi}{2}\right]$

10. (a) A , (b) 1210 **11.** (a) D , (b) (A)-P, R, S; (B)-Q, S; (C)-Q, S; (D)-P, R, S

12. $k = 2$ **13.** B **14.** B

15. A.D **16** C **17.** C

18. C **19.** D