Q.9 (ii) Two rings O and O ' are put on two vertical stationary rods AB and A′B′ respectively as shown in figure. An inextensible string is fixed at point A′ and on ring O and is passed through O′. Assuming that ring O′ moves downwards at a constant speed v, find the velocity of the ring O in terms of α.

$$\left[\frac{v(1-\cos\alpha)}{\cos\alpha}\right]$$

M.1

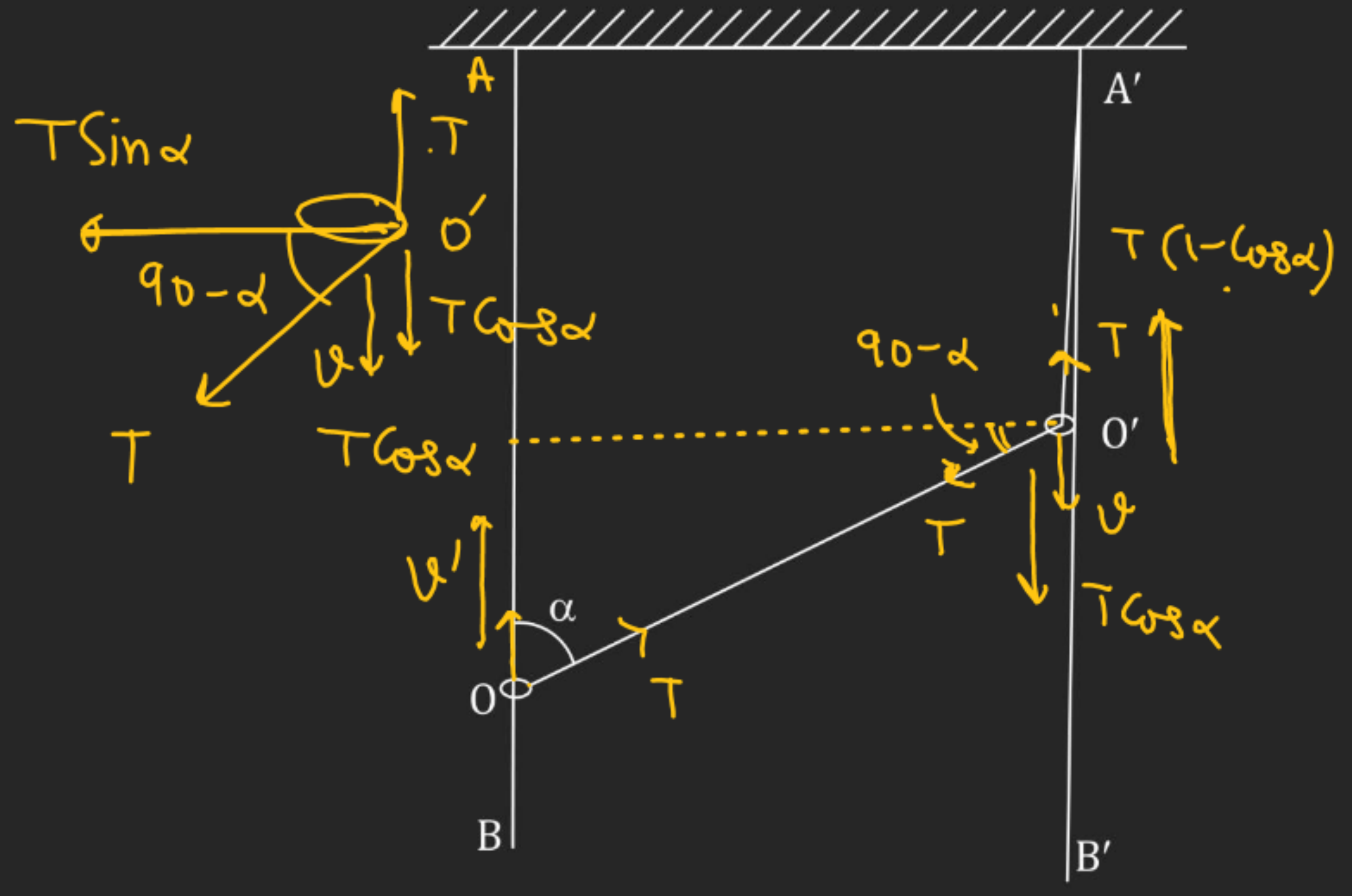
$$\sum \vec{T}\cdot\vec{v} = 0$$

$$\Downarrow$$

$$-T(1-\cos\alpha)v + T\cos\alpha\, v'$$

$$v'\cos\alpha = v(1-\cos\alpha)$$

$$\left[v' = \frac{v(1-\cos\alpha)}{\cos\alpha}\right]$$



Rigid body constrain

Law of Motion

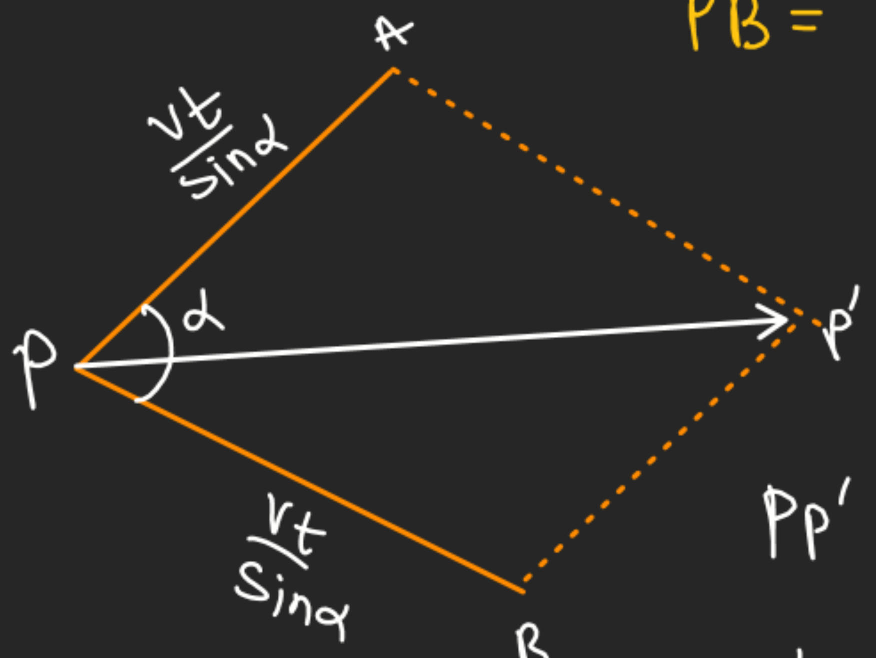
$$\cos 2\alpha = 2\cos^2\alpha - 1 = 1 - 2\sin^2\alpha$$

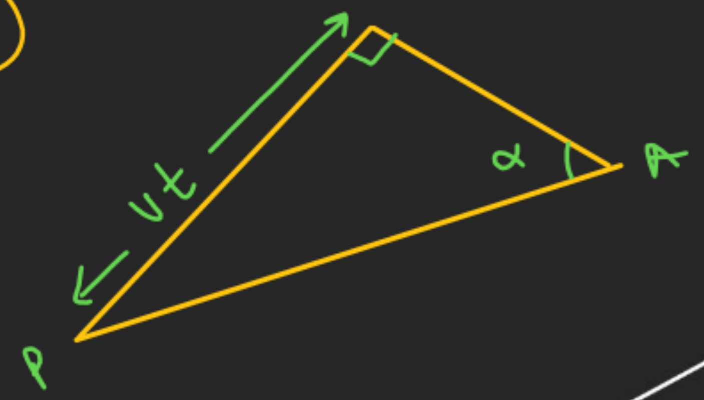
Q.11 Two lines AB and CD intersect at O at an inclination α, as shown in figure If they move out parallel to themselves with the speed v, find the speed of O.

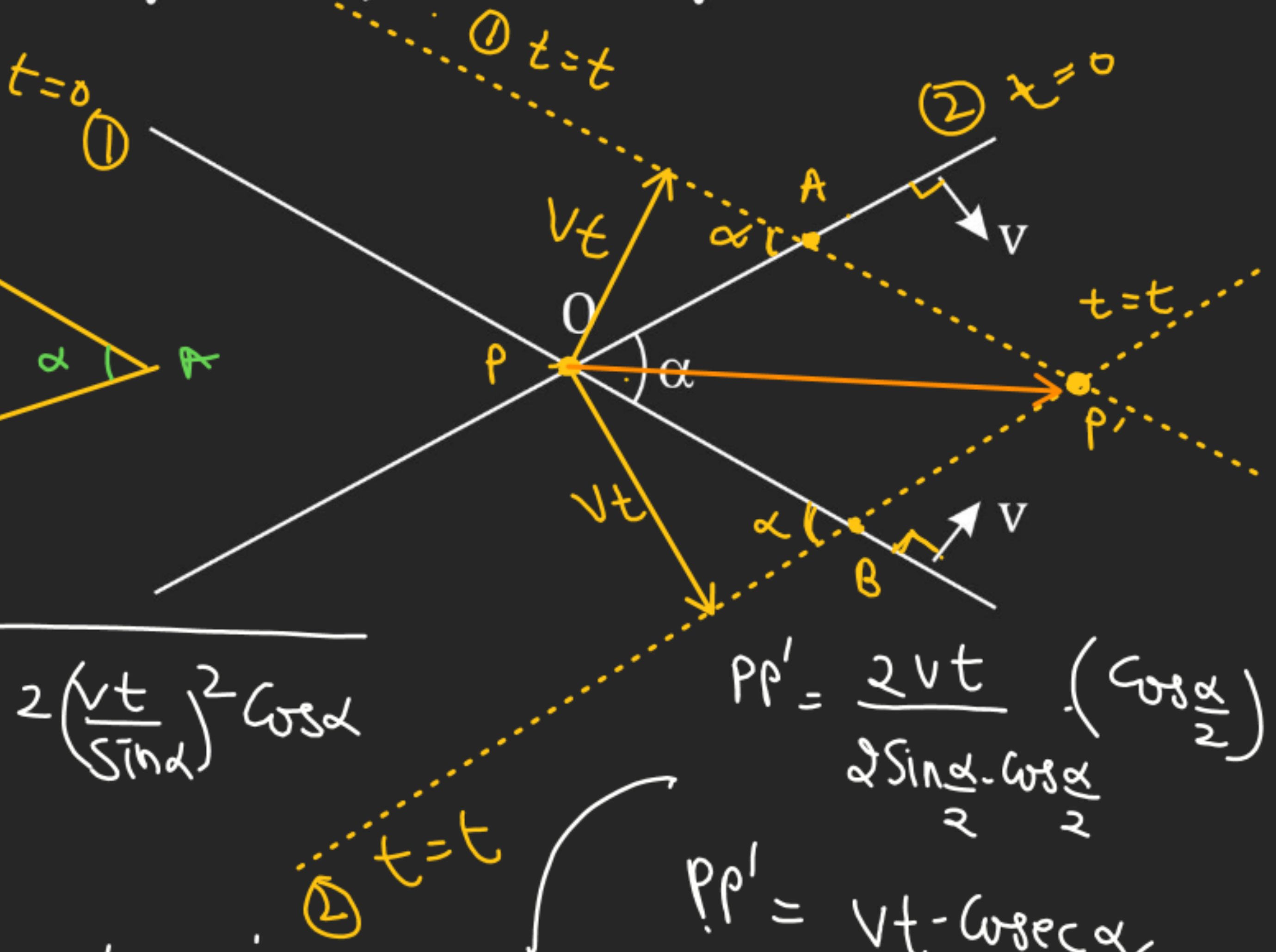
$$\sin\alpha = \frac{vt}{PA}$$

$$PA = \left(\frac{vt}{\sin\alpha}\right)$$

$$PB = \left(\frac{vt}{\sin\alpha}\right)$$







$$PP' = \sqrt{\left(\frac{vt}{\sin\alpha}\right)^2 + \left(\frac{vt}{\sin\alpha}\right)^2 + 2\left(\frac{vt}{\sin\alpha}\right)^2\cos\alpha}$$

$$PP' = \frac{vt}{\sin\alpha}\sqrt{2(1+\cos\alpha)}$$

$$PP' = \frac{vt}{\sin\alpha}\sqrt{2\left(1+2\cos^2\frac{\alpha}{2}-1\right)} \Rightarrow PP' = \frac{2vt}{\sin\alpha}\cdot\left(\cos\alpha/2\right)$$

$$PP' = \frac{2vt}{2\sin\frac{\alpha}{2}\cdot\cos\frac{\alpha}{2}}\left(\cos\frac{\alpha}{2}\right)$$

$$PP' = vt\cdot\operatorname{cosec}\alpha/2$$

$$\frac{PP'}{t} = v_0 = v\operatorname{cosec}\alpha/2$$

Q.12 A hinged construction consists of three rhombs with the ratio of sides 3:2:1 Vertex A_3 moves in the horizontal direction at a velocity v. Determine the velocities of vertices A_1, A_2, and B_2 at the instant when the angles of the construction are 90°.

$\theta = 90° \Rightarrow$ rhombus become Squar.
Diagonals also in the ratio 3:2:1

let, $A_2A_3 = x$, $A_1A_2 = 2x$, $A_0A_1 = 3x$

$X_{A_3} = 6x$ given $\frac{d}{dt}(X_{A_3}) = v$

$X_{A_2} = 5x$

$X_{A_1} = 3x$

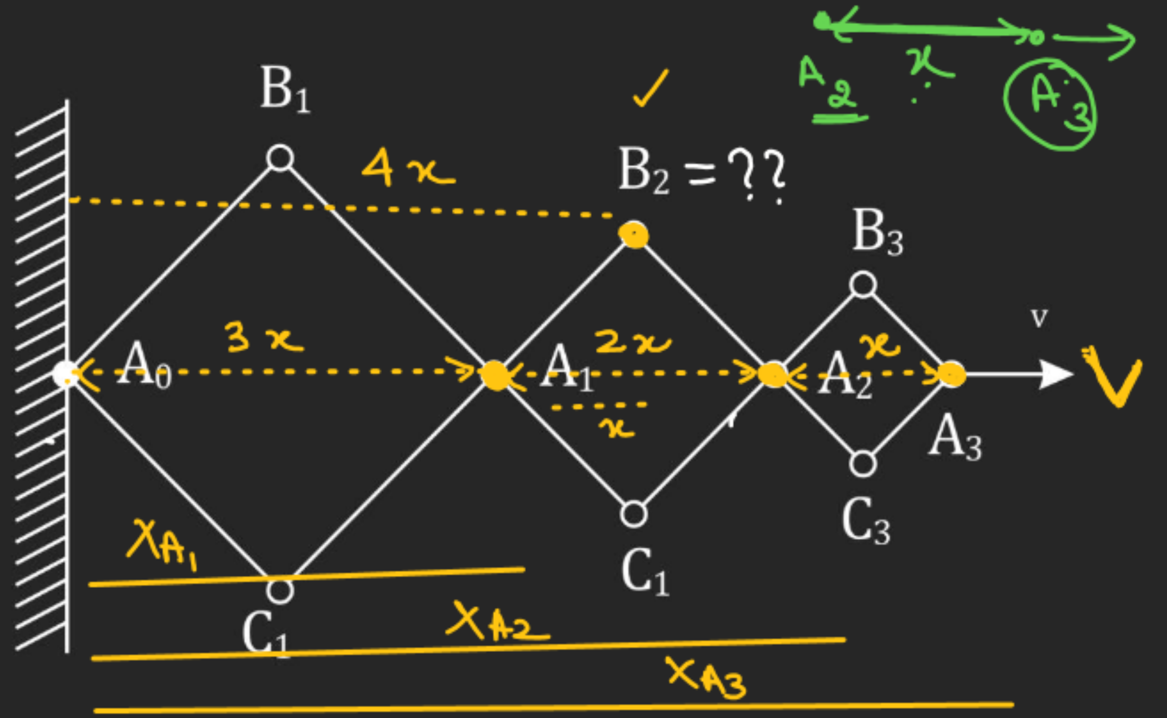
$x = \frac{1}{6}(x_{A_3})$

$\frac{dx}{dt} = \frac{1}{6}\frac{d(x_{A_3})}{dt} = v/6$

$\left(\frac{dx_{A_2}}{dt}\right) = 5\left(\frac{dx}{dt}\right) = \left(\frac{5v}{6}\right)$ Ans

$\Downarrow$

v_{A_2}



$\frac{dx_{A_1}}{dt} = 3\left(\frac{dx}{dt}\right) = 3\left(\frac{v}{6}\right) = \left(\frac{v}{2}\right)$

$\Downarrow$

$v_{A_1} = v/2$ Ans.

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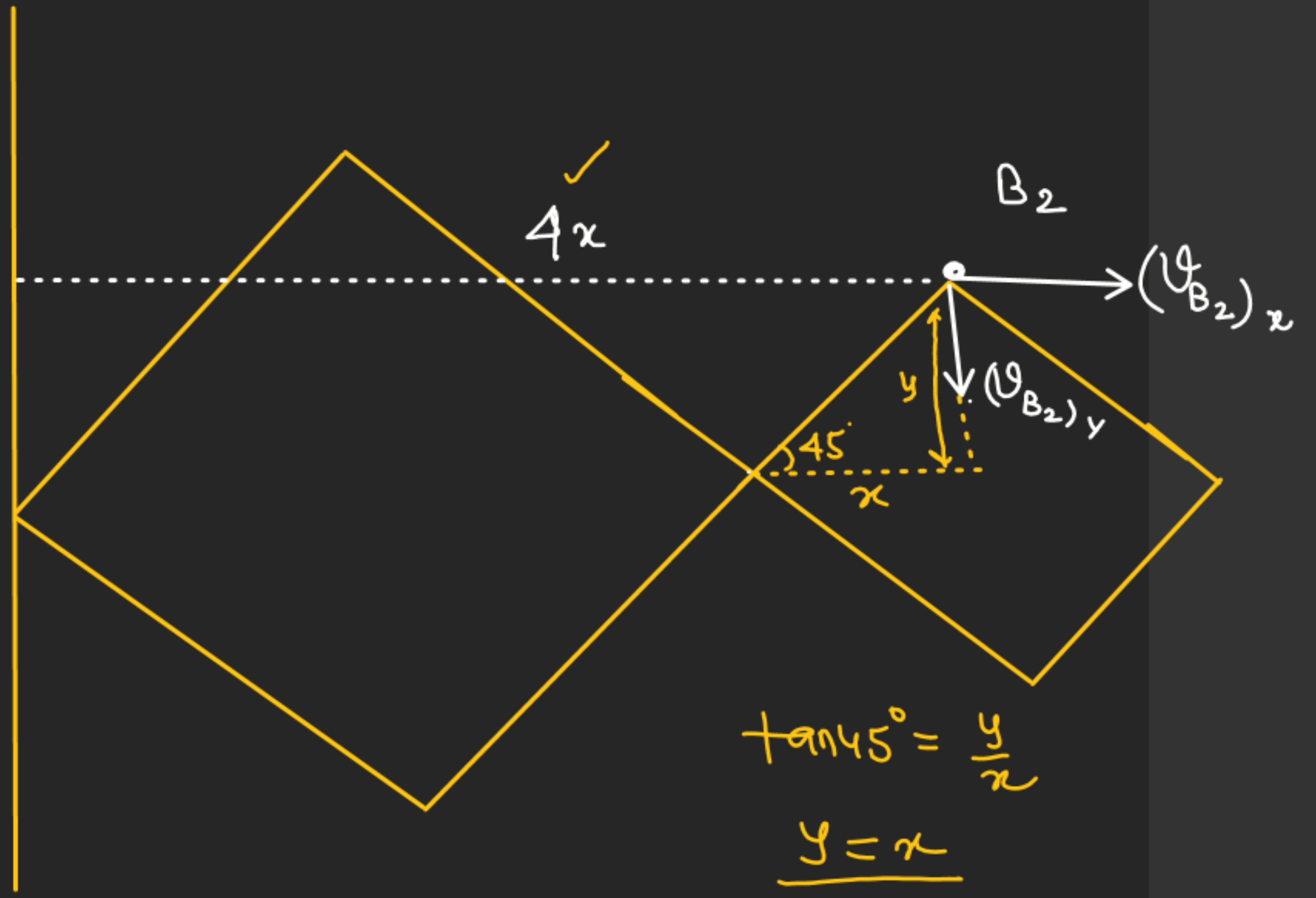
Law of Motion

$$(v_{B_2})_y = \left(\frac{dy}{dt}\right) = \left(\frac{dx}{dt}\right) = \left(\frac{v}{6}\right)$$

$$(v_{B_2})_x = \frac{d}{dt}(4x) = 4\left(\frac{dx}{dt}\right) = 4\left(\frac{v}{6}\right) = \left(\frac{2v}{3}\right)$$

$$(v_{B_2}) = \sqrt{(v_{B_2})_x^2 + (v_{B_2})_y^2} = \sqrt{\frac{4v^2}{9} + \frac{v^2}{36}} = \frac{v}{3}\sqrt{4 + \frac{1}{4}} \Rightarrow \frac{\sqrt{17}\,v}{6} \text{ Ans}$$

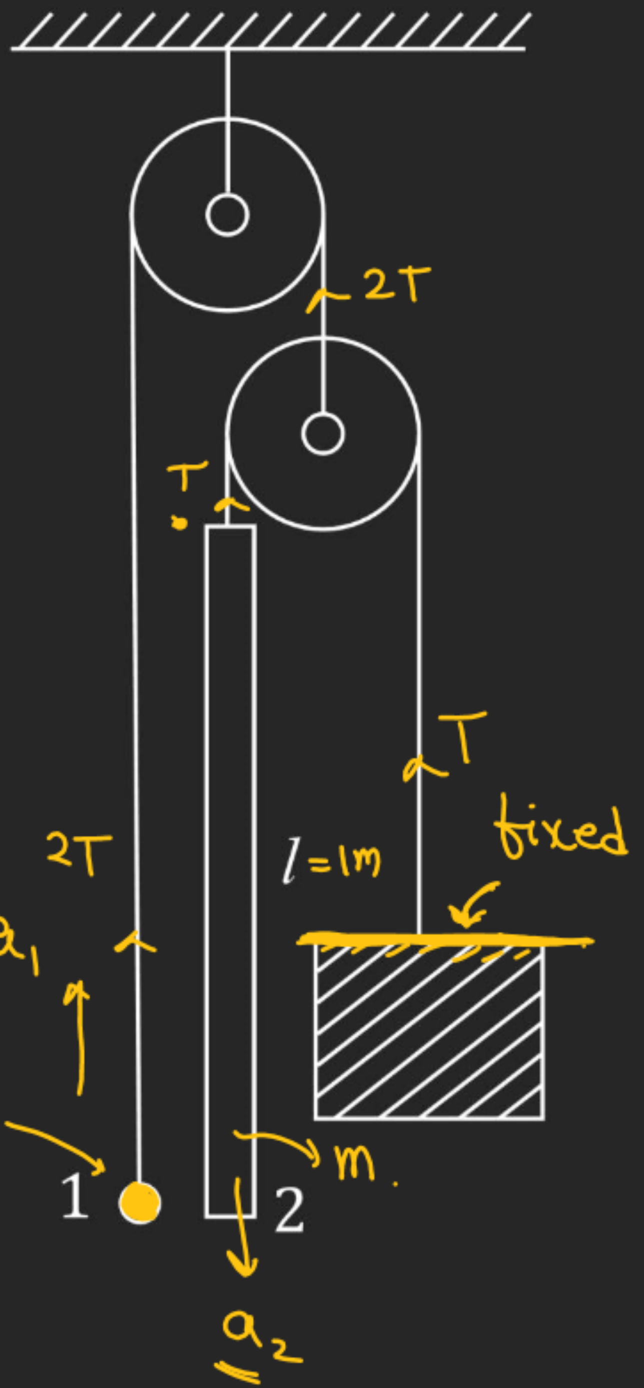


$$\tan 45^\circ = \frac{y}{x}$$

$$y = x$$

Law of Motion

Q.5 In the arrangement shown in Fig the mass of ball 1 is $\eta = 1.8$ times as great as that of rod 2 . The length of the latter is $l = 100$ cm. The masses of the pulleys and the threads, as well as the friction, are negligible. The ball is set on the same level as the lower end of the rod and then released. How soon will the ball be opposite the upper end of the rod? **[Irodov]**

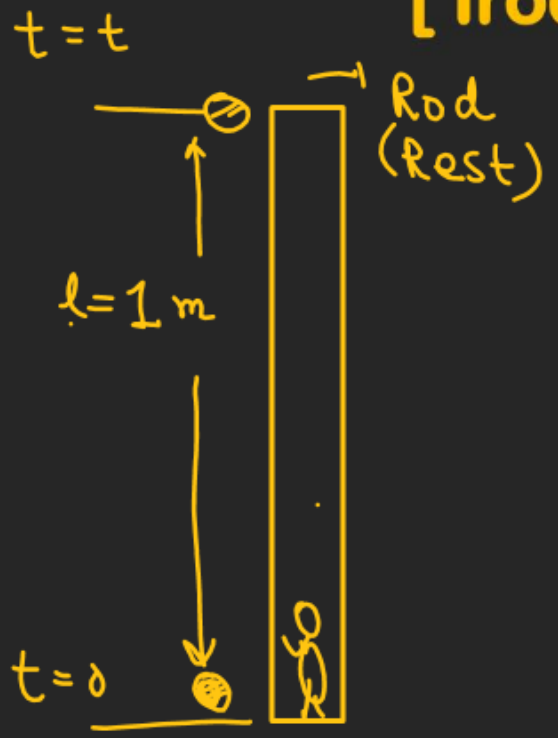


$$\vec{a}_{rel} = \vec{a}_{ball/Rod}$$

$$\vec{a}_{ball/Rod} = \vec{a}_{ball/E} - \vec{a}_{Rod/E}$$

$$l = \frac{1}{2} a_{rel} \cdot t^2$$

$$t = \sqrt{\frac{2l}{a_{rel}}}$$



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Law of Motion

Constrain relation

$$\sum \vec{T}\cdot\vec{a} = 0$$

$$2Ta_1 - Ta_2 = 0$$

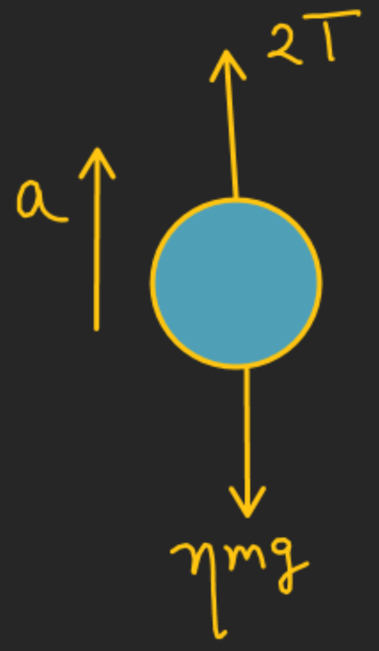
$$a_2 = 2a_1$$

If $\begin{cases} a_1 = a \\ a_2 = 2a \end{cases}$

$$a_r = 3a = 3g\left(\frac{2-\eta}{4+\eta}\right)$$

$$t = \sqrt{\frac{2L(4+\eta)}{3g(2-\eta)}}$$

F.B.D of ball



$$2T - \eta mg = \eta ma$$

$$2T = \eta m(g+a)$$

$$T = \frac{\eta m(g+a)}{2} \quad \text{—(1)}$$

F.B.D of Rod

$2a = a_2$

$$mg - T = 2ma \quad (2)$$

From (1) & (2)

$$mg - \frac{\eta m(g+a)}{2} = 2ma$$

$$mg - \frac{\eta mg}{2} = 2ma + \frac{\eta ma}{2}$$

$$\frac{\cancel{m}g(2-\eta)}{\cancel{2}} = \frac{(4+\eta)\cancel{m}a}{\cancel{2}}$$

$$a = \left(\frac{2-\eta}{4+\eta}\right)g$$

$a > 0 \Rightarrow$ assumed direction is correct

Law of Motion

Q.6 In the arrangement shown in Fig the mass of body 1 is $\eta = 4.0$ times as great as that of body 2 . The height $h = 20$ cm. The masses of the pulleys and the threads, as well as the friction, are negligible. At a certain moment body 1 is released and the arrangement set in motion. What is the maximum height that body 2 will go up to?

[Irodov]

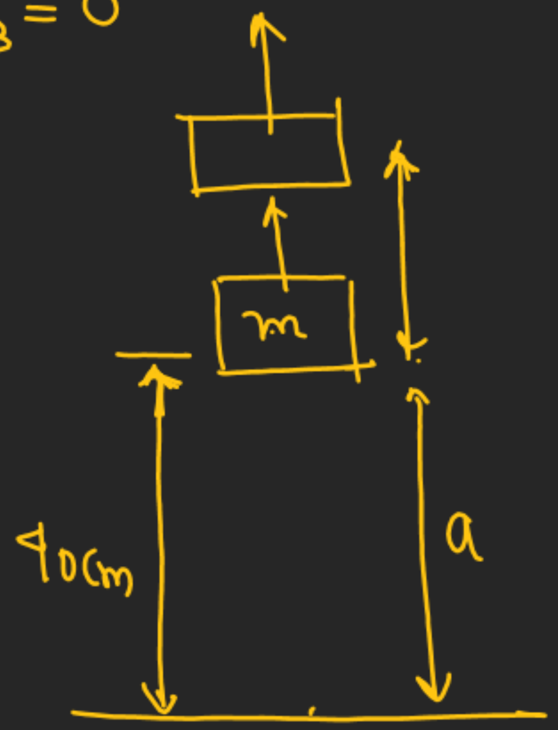
$$\sum \vec{T}\cdot\vec{x} = 0$$

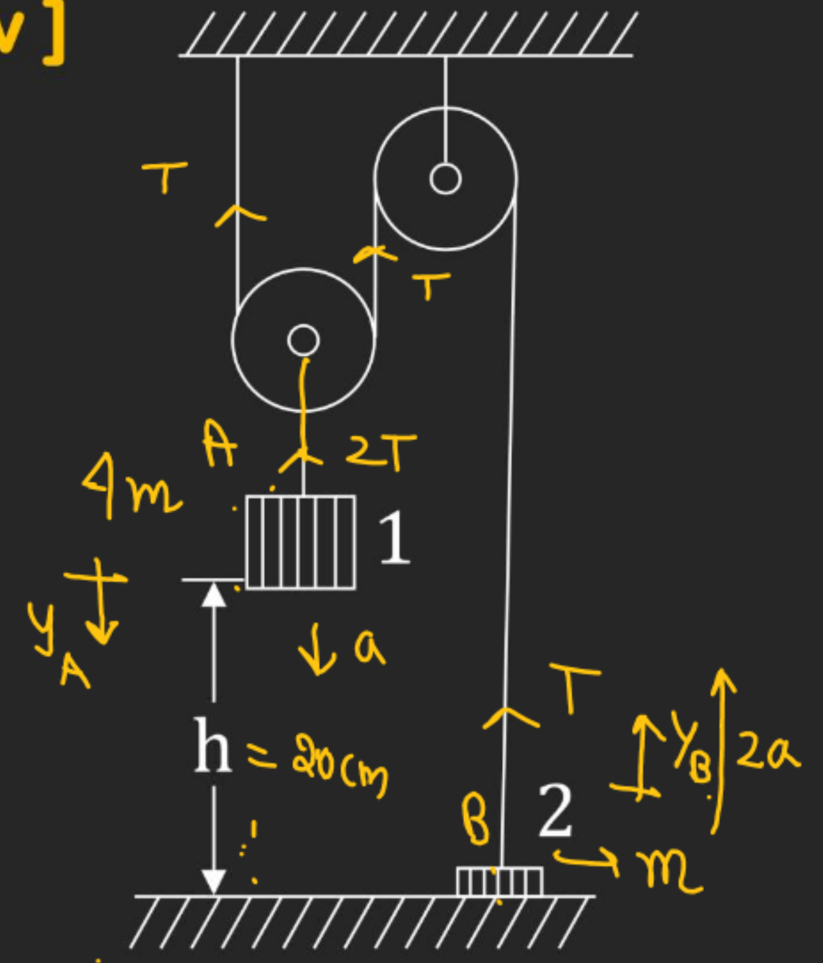
$$-2T y_A + T y_B = 0$$

$$y_B = 2y_A \Rightarrow a_B = 2a_A$$

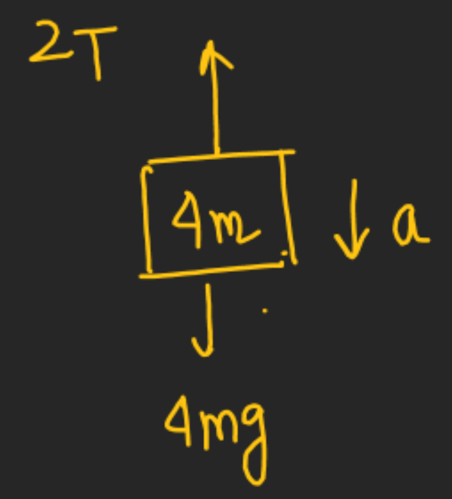
if $y_A = 20\,cm$, $y_B = 40\,cm$

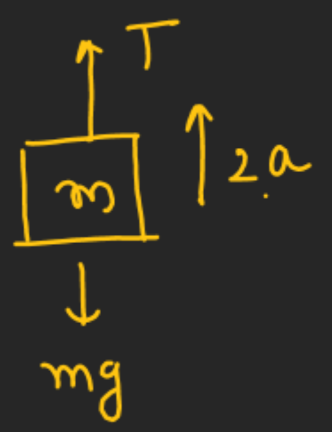
if $a_A = a$, $a_B = 2a$

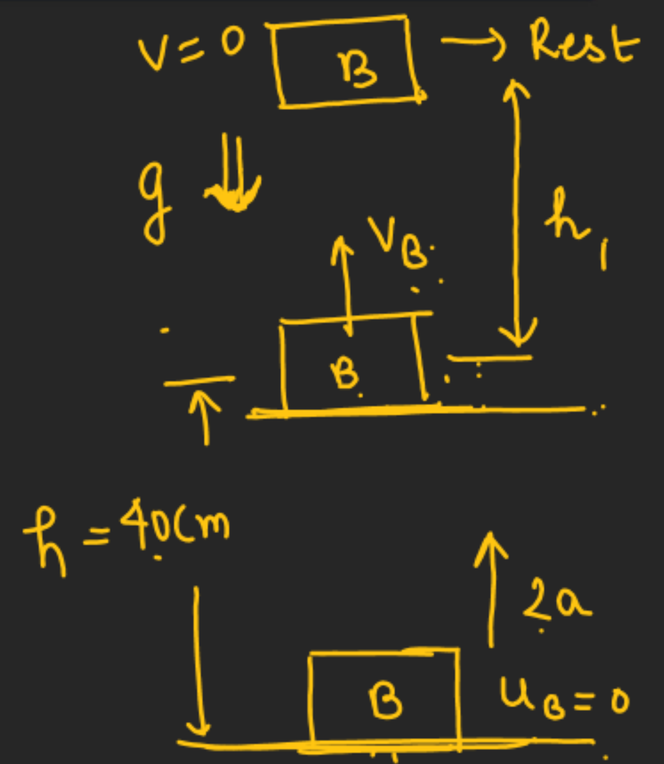




Law of Motion







$4mg - 2T = 4ma$ — ①

$\boxed{T - mg = 2ma}$ — ②

$2\times$② + ①

$2T - 2mg = 4ma$

$\Rightarrow 2\cancel{m}g = 8\cancel{m}a$

$a = \frac{g}{4}$

$a = \frac{10}{4} = \frac{5}{2}\, ms^{-2}$

$V_B^2 = 2\times 2a \times h$

$V_B = 2\sqrt{ah}$

$V_B = 2\sqrt{\frac{5}{2}\times 40\times 10^{-2}}$

$\underline{V_B = 2\, m/s}$

$\Rightarrow 0 = V_B^2 - 2gh_1$

$h_1 = \frac{V_B^2}{2g}$

$h_1 = \frac{4}{2\times 10} = \frac{2}{10} = \frac{1}{5} = \underline{0.2m}$

$h_1 = \underline{20cm}$

Maximum height from the ground

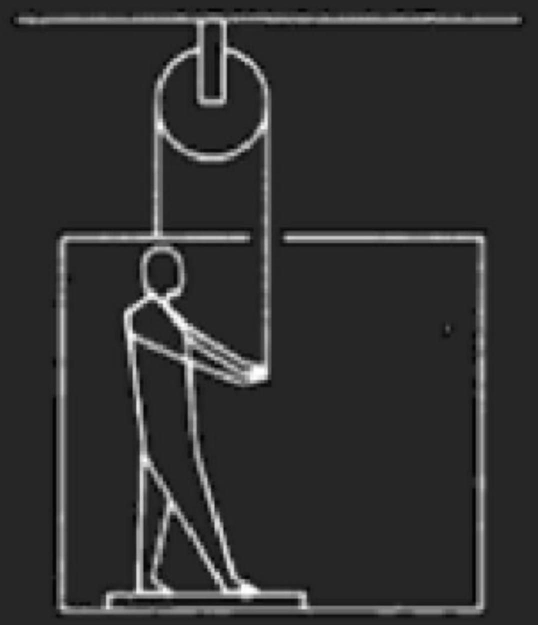
$= \underline{60cm}$

H.W

Law of Motion

Q.7 Figure (5-E21) shows a man of mass $60\,\mathrm{kg}$ standing on a light weighing machine kept in a box of mass $30\,\mathrm{kg}$. The box is hanging from a pulley fixed to the ceiling through a light rope, the other end of which is held by the man himself. If the man manages to keep the box at rest, what is the weight shown by the machine ? What force should he exert on the rope to get his correct weight on the machine?

[H.C.V.]



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H-W

Law of Motion

Q.8 The monkey B shown in figure is holding on to the tail of the monkey A which is climbing up a rope. The masses of the monkeys A and B are $5\ \mathrm{kg}$ and $2\ \mathrm{kg}$ respectively. If A can tolerate a tension of $30\ \mathrm{N}$ in its tail, what force should it apply on the rope in order to carry the monkey B with it? Take $g = 10\ \mathrm{m/s^2}$. [H.C.V.]