

Quadratic Equations

Linear function

$$f(x) = ax + b, \quad a, b \in \mathbb{R},$$

Quadratic fn

$$f(x) = ax^2 + bx + c, \quad a, b, c \in \mathbb{R}, \quad a \neq 0.$$

Cubic function

$$f(x) = ax^3 + bx^2 + cx + d, \quad a, b, c, d \in \mathbb{R}, \quad a \neq 0.$$

Biquadratic function

$$f(x) = ax^4 + bx^3 + cx^2 + dx + e, \quad a, b, c, d, e \in \mathbb{R}, \quad a \neq 0.$$

Polynomial fn. of degree 'n'

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$$

$$a_0, a_1, a_2, \dots, a_n \in \mathbb{R}.$$

$$a_n \neq 0.$$

$$\text{Leading coefficient} = a_n$$

$$\text{Constant term} = a_0$$

$$\frac{\text{Monic}}{\text{polynomial}} \text{ leading coefficient} = 1$$

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \dots + a_2 x^2 + a_1 x + a_0$$

$$a_0 = f(0)$$

$$a_r = \frac{f^{(r)}(0)}{1 \cdot 2 \cdot 3 \cdot \dots \cdot r} = \frac{f^{(r)}(0)}{r!}$$

$$f'(x) = n a_n x^{n-1} + \dots + 3 a_3 x^2 + 2 a_2 x + 1 \cdot a_1$$

$$\frac{f'(0)}{1} = a_1$$

$$a_3 = \frac{f'''(0)}{1 \cdot 2 \cdot 3}$$

$$f''(x) = n(n-1) a_n x^{n-2} + \dots + 3 \cdot 2 \cdot a_3 x + 2 \cdot 1 \cdot a_2$$

$$\frac{f''(0)}{1 \cdot 2} = a_2$$

$$f'''(x) = n(n-1)(n-2) a_n x^{n-3} + \dots + 3 \cdot 2 \cdot 1 \cdot a_3$$

Factorial

$$\boxed{n! = n(n-1)! \quad , n \in \mathbb{N}}$$

$$0! = 1$$

$$n = n \cdot (n-1)$$

$$n! = 1 \cdot 2 \cdot 3 \cdots n$$

$$(-1)! \quad \times$$

$$0! = 1$$

$$1! = 1 \cdot 0! = 1$$

$$2! = 2 \cdot 1! = 2 \cdot 1$$

$$3! = 3 \cdot 2! = 3 \cdot 2 \cdot 1$$

Quadratic Function

 $a < 0$

$$f(x) = ax^2 + bx + c$$

$$a \neq 0, a, b, c \in \mathbb{R}$$

$$= a\left(x^2 + \frac{b}{a}x\right) + c$$

$$D = b^2 - 4ac = \text{Discriminant}$$

$$= a\left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a^2} + c$$

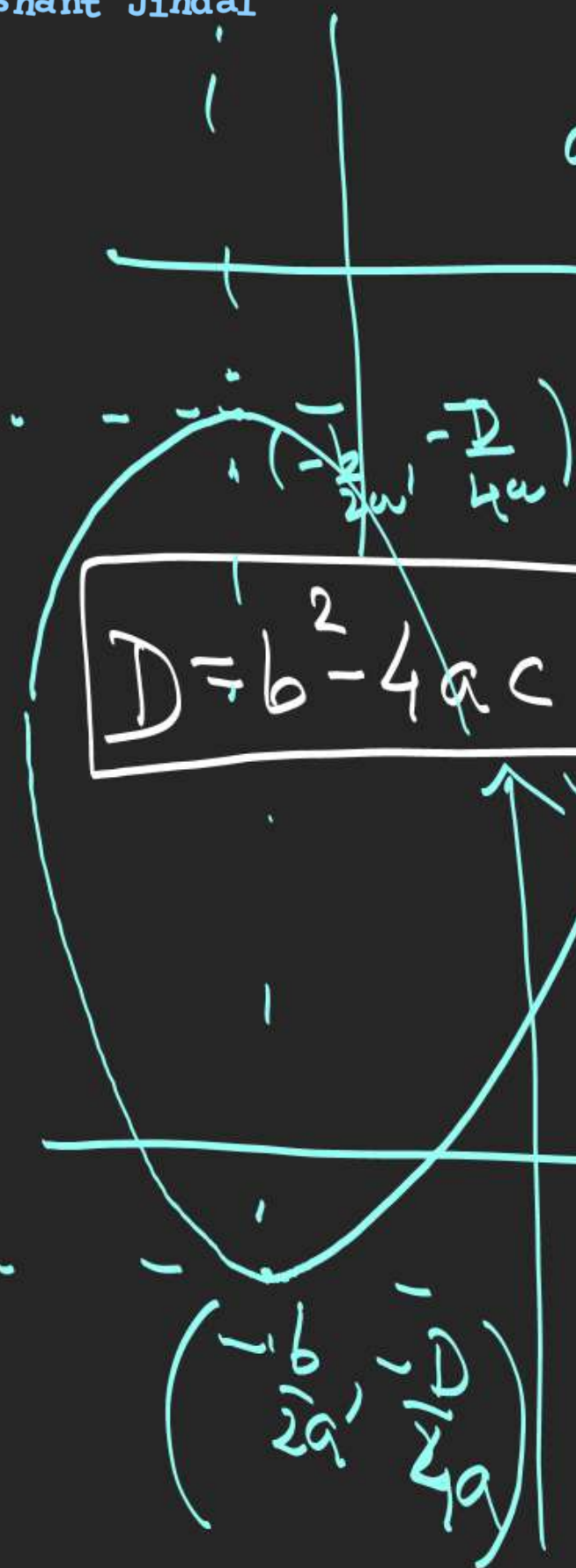
$$y = f(x) = a\left(x + \frac{b}{2a}\right)^2 + \left(\frac{-D}{4a}\right)$$

$$= a\left(x + \frac{b}{2a}\right)^2 + c - \frac{b^2}{4a}$$

$$= a\left(x + \frac{b}{2a}\right)^2 - \frac{b^2 - 4ac}{4a}$$

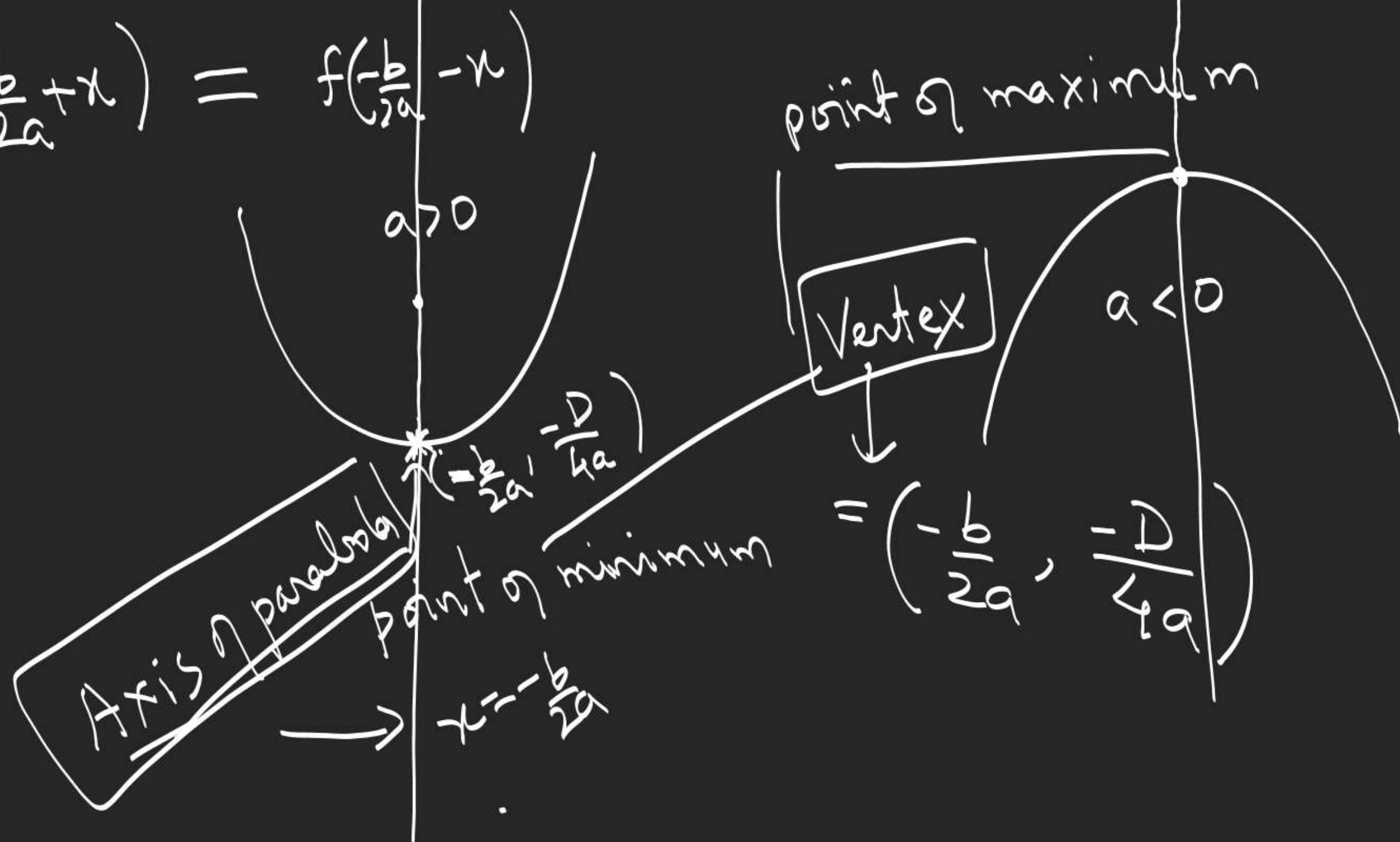
$$y + \frac{D}{4a} = a\left(x + \frac{b}{2a}\right)^2$$

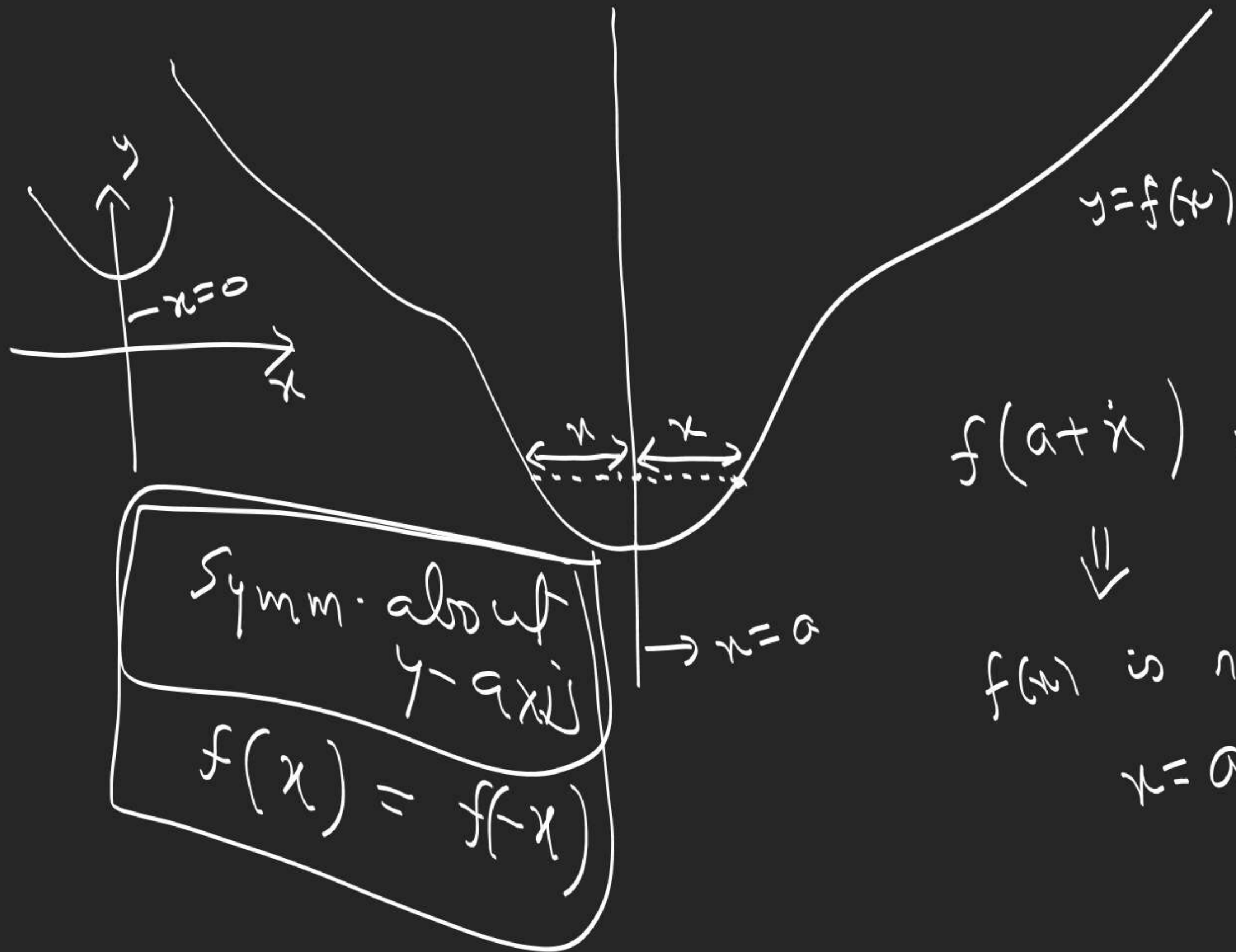
$$y = ax^2$$



$$f(x) = ax^2 + bx + c = a\left(x + \frac{b}{2a}\right)^2 + \frac{-D}{4a}$$

$$f\left(-\frac{b}{2a} + x\right) = f\left(-\frac{b}{2a} - x\right)$$

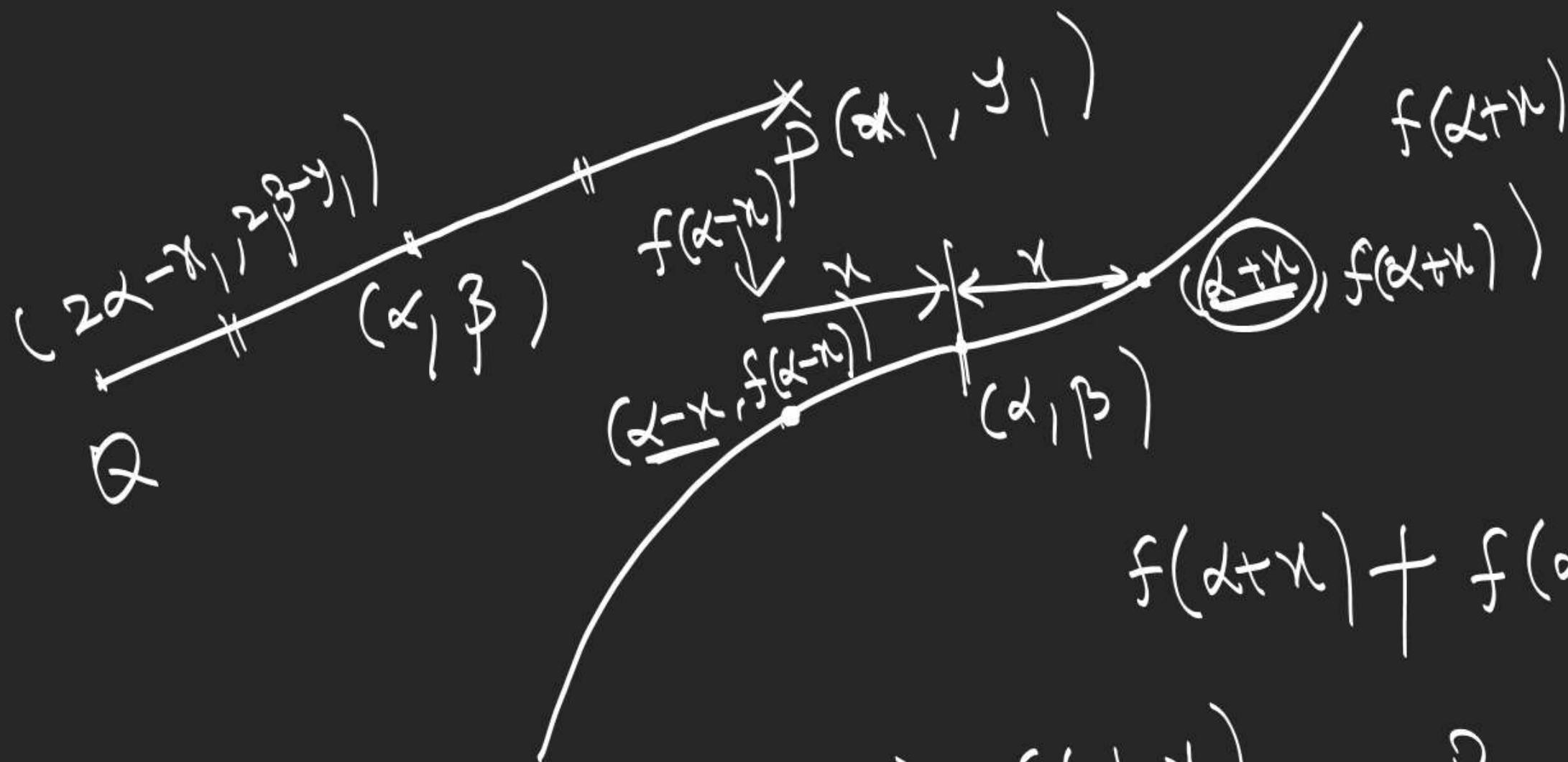




$$f(a+x) = f(a-x)$$

\Downarrow

$f(x)$ is symmetric about
 $x=a$ line.



$$f(x+\eta) + f(x-\eta) = 2\beta$$

$$\frac{1 \cdot f(x+h) + 1 \cdot f(x-h)}{2} = \beta$$

Symmetric
about $(0,0)$

$$f(x) + f(-x) = 0$$

$$f(x) = ax^2 + bx + c \checkmark$$

Quadratic Equation

$$ax^2 + bx + c = 0$$

Roots of function

$f(x) = 0$
point where graph of $f(x)$
meet x -axis

$$x^2 = 9$$

$$x = \pm 3 \Rightarrow x = \pm \sqrt{9}$$

Roots / Zeros / Solutions of Quadratic Eqn.

$$\alpha, \beta = \frac{-b \pm \sqrt{D}}{2a}$$

$$ax^2 + bx + c = 0 \quad \begin{matrix} \alpha \\ \beta \end{matrix}$$

$$a\left(x + \frac{b}{2a}\right)^2 - \frac{D}{4a} = 0 \Rightarrow \left(x + \frac{b}{2a}\right)^2 = \frac{D}{4a^2}$$

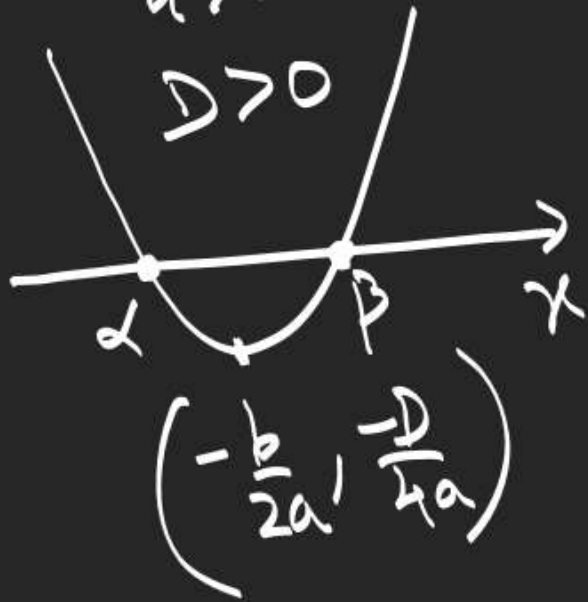
$$x + \frac{b}{2a} = \pm \frac{\sqrt{D}}{2a}$$

$$f(x) = ax^2 + bx + c$$

2 distinct real roots

$$a > 0$$

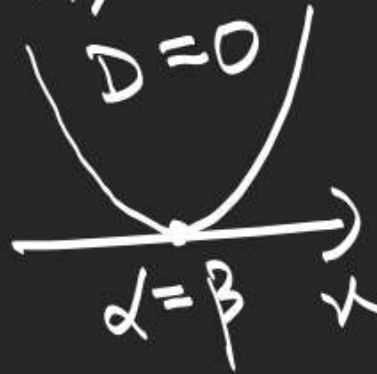
$$D > 0$$



2 equal real roots

$$a > 0$$

$$D = 0$$



$$f(x) > 0$$

$$a > 0$$

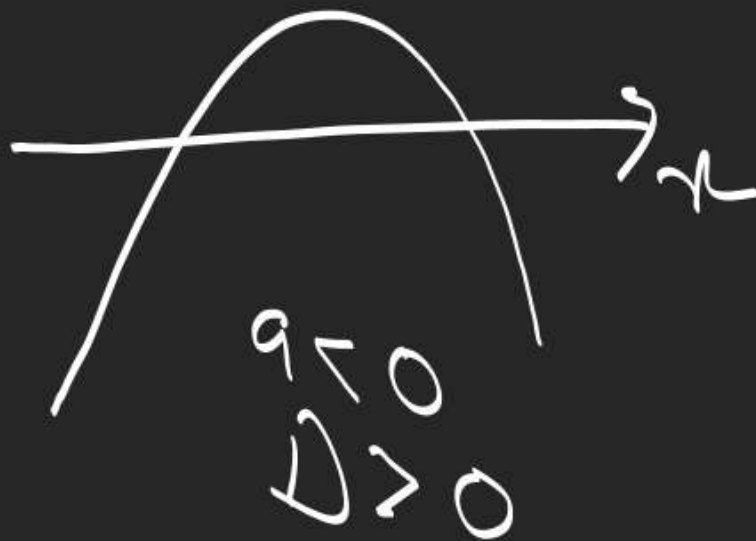
$$D < 0$$



no real roots

$$\frac{D}{4a} > 0$$

$$D > 0$$



$$a < 0$$

$$D > 0$$



$$a < 0$$

$$D = 0$$



$$a < 0$$

$$D < 0$$

$D > 0 \Rightarrow 2$ distinct real roots

$D = 0 \Rightarrow 2$ equal real roots

$D < 0 \Rightarrow$ no real roots

Note
① If

$$f(x) = ax^2 + bx + c, \quad a, b, c \in \mathbb{R}, a \neq 0$$

$$f(x) > 0 \quad \forall x \in \mathbb{R}$$

$$\Rightarrow a > 0 \text{ \& } D < 0$$

②

$$\text{If } f(x) < 0 \quad \forall x \in \mathbb{R}$$

$$\Rightarrow a < 0 \text{ \& } D < 0$$



③