Nishant Jindal

Q Normal to Curve

$$x^2+2xy-3y^2=0 \text{ at } (1,1)$$

1) meet Curve again in 1st Q

2) ______ in 4th Q.

3) does not meet again

4) meet again in 2nd Q.

① $2x^2 + 2x\frac{dy}{dx} + 2y - 6y\frac{dy}{dx} = 0$

$x=1$
$y=1$

$2 + 2\frac{dy}{dx} + 2 - 6\frac{dy}{dx} = 0$

$4\frac{dy}{dx} = 4 \Rightarrow \frac{dy}{dx} = 1$

EON $\rightarrow (y-1) = \frac{-1}{1}(x-1)$

$x+y=2$

② Normal meets curve again

$y=2-x$ in $x^2+2xy-3y^2=0$

$x^2+2x(2-x)+3(2-x)^2=0$

$x^2+4x-2x^2+6-12x+3x^2=0$

$2x^2-8x+6=0$

$x^2-4x+3=0$

$x=3 \;,,\; x=1$

$y=2-3 \quad y=2-1$

$(3,-1) \quad (1,1)$

4th 1st

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① The intercepts on x Axis made by tangents to curve $y = \int_0^x |t| dt$, $x \in R$, which are $\parallel^{el}$ to the line $y = 2x$ are equal to

1) ± 2 2) ± 3 (3) ± 4 (4) ± 1

② tangent $\parallel$ line $\Rightarrow (Sl)_T = (Sl)_L$

① $\frac{dy}{dx} = |x| = 2$

(3) $x = 2$

$y = \int_0^2 t\, dt$

$\left.\frac{t^2}{2}\right|_0^2$

$= 2$

$(2, 2)$

$x = -2$

$y = \int_0^{-2} -t\, dt$

$= -\left.\frac{t^2}{2}\right|_0^{-2}$

$= -2$

$(-2, -2)$

(4) EOT

$y - 2 = 2(x - 2)$

$2x - y = 2$

$y = 0$ $x = 1$

$(y + 2) = 2(x + 2)$

$2x - y = -2$

$x = -1$

$\therefore$ int. on X Axis

$= \pm 1$

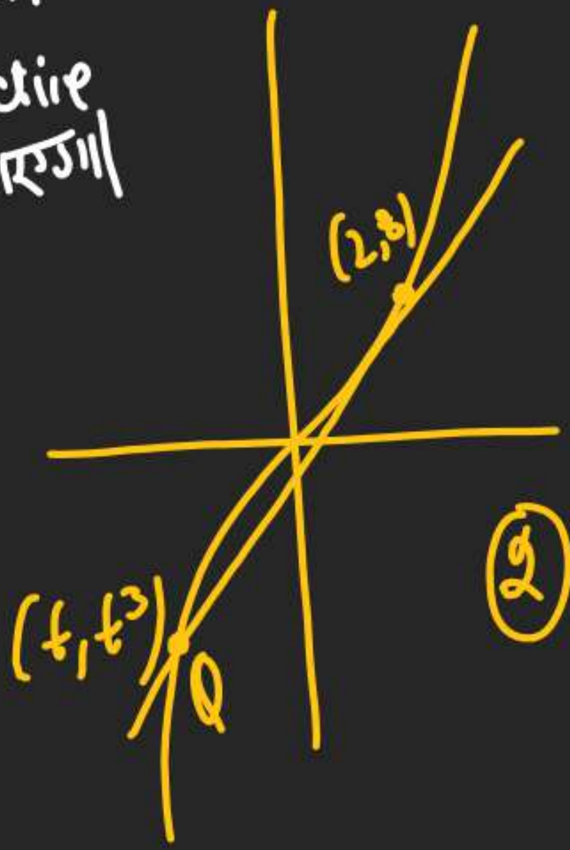
Nishant Jindal

Q If tangent at Pt. (2,8) on curve $Y = x^3$ meets curve again at Q. Then Coord. of Q = ?

Repeat

Check

नया tricky है Practice करवाएगा



$\left.\frac{dy}{dx}\right|_{2,8} = 3 \times 2^2 = 12$

① EOT at 2,8

$(Y-8) = 12(x-2)$

$12x - Y = 16$

② $Y = 12x - 16$ in $Y = x^3$

$12x - 16 = x^3$

$x^3 - 12x + 16 = 0$

(3)* Line is tangent at $x = 2$

$x = 2$ is Repeated Root here

$(x-2)^2(x-t) = x^3 - 12x + 16 \;\Big|\; \Rightarrow -4t = 16$

$t = -4$

$Q = (-4, -64)$

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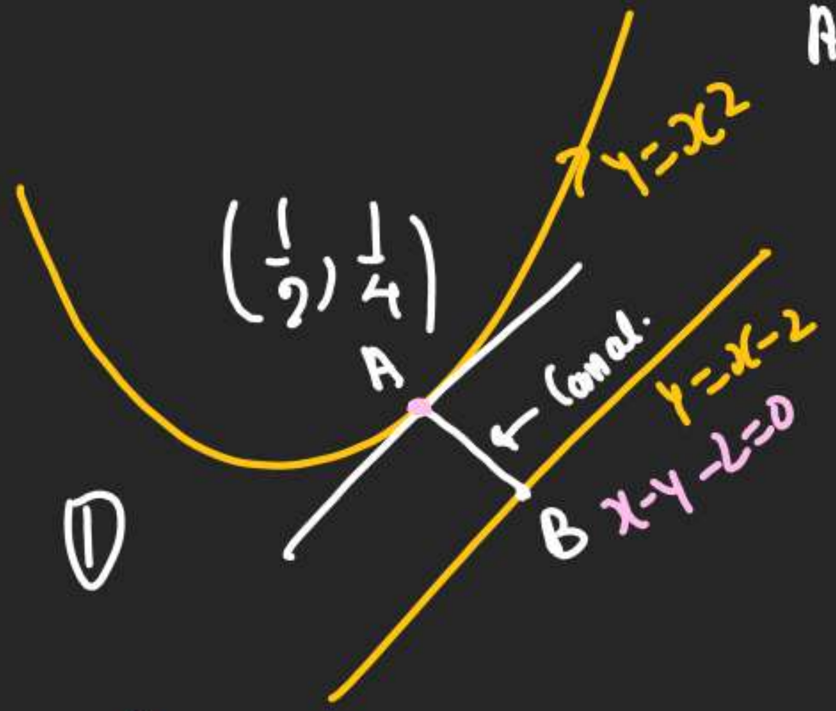
Shortest distance betⁿ 2 Curves.

Sh. distance at Pts where tangent to Both Curves are ||ʳᵉ.

A line & a Curve are Said to be Closest where both have Com. Normal.

Q Beds of 2 Rivers are in Shape of $Y=x^2$ & $Y=x-2$. These Rivers are to be Connected by a Straight Canal then find the Coord of the ends of shortest Canals



A & B find out

distance

$$d = \frac{\left|\frac{1}{2}-\frac{1}{4}-2\right|}{\sqrt{1^2+(-1)^2}} = \frac{7}{4\sqrt{2}}$$

① $(Sl)_{\text{Tangent at } A} = (Sl)_{y=x-2}$

$$2x = 1 \Rightarrow x = \frac{1}{2}$$

$$\therefore A = \left(\frac{1}{2},\frac{1}{4}\right)$$

② B is PoI of Normal at $\left(\frac{1}{2},\frac{1}{4}\right)$ & line $y=x-2$

Canal → $\left(y-\frac{1}{4}\right) = -1\left(x-\frac{1}{2}\right)$

$$y+x = \frac{3}{4}$$

Solving

$$\cancel{y}+x = \frac{3}{4}$$

$$x-\cancel{y} = 2$$

$$2x = \frac{11}{4}$$

$$x = \frac{11}{8}$$

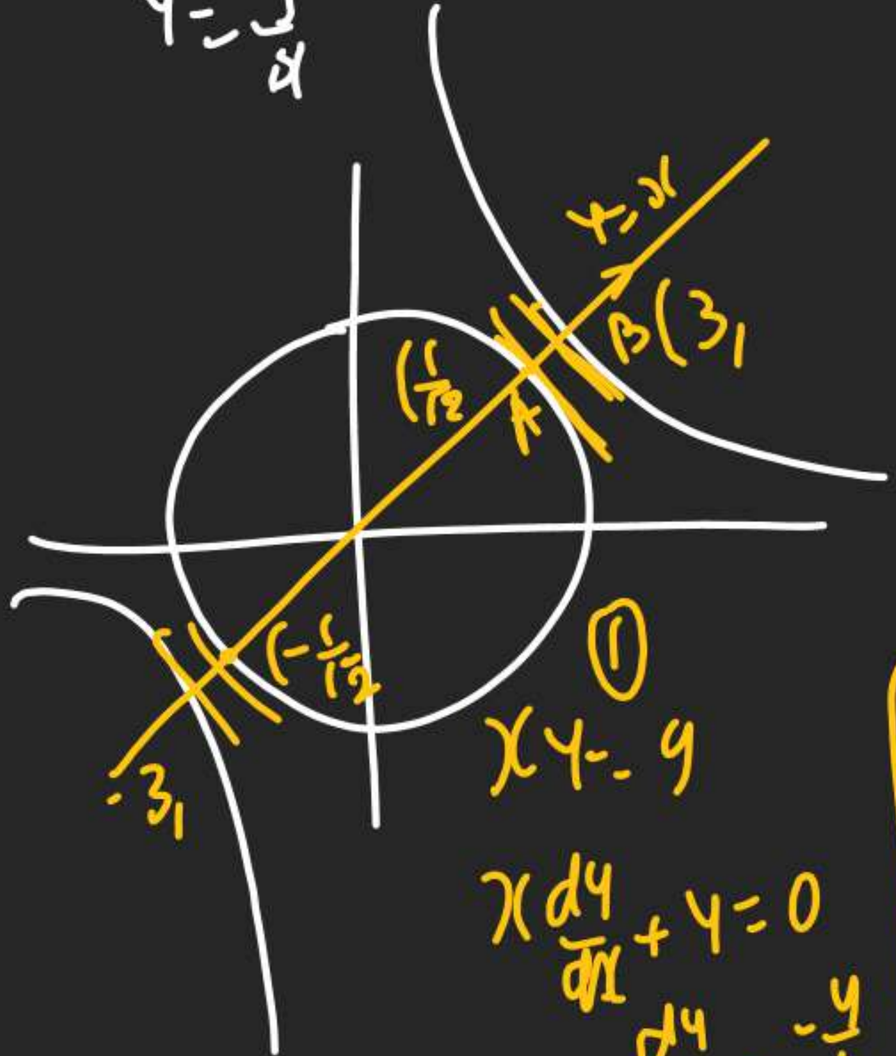
$$y = -\frac{5}{8}$$

$$B = \left(\frac{11}{8},-\frac{5}{8}\right)$$

Q Sh. dist. betn

$xy = 9$ & $x^2+y^2=1$

$y = \frac{9}{x}$



(1)

$xy = 9$

$x\frac{dy}{dx} + y = 0$

$\frac{dy}{dx} = -\frac{y}{x}$

$2x + 2y\frac{dy}{dx} = 0$

$\frac{dy}{dx} = -\frac{x}{y}$

Both are ||

$+\frac{y}{x} = +\frac{x}{y} \Rightarrow y^2 = x^2 \Rightarrow y = x \,/\, y = -x$

(2) Solving $y = x$ with Both

$xy = 9$ & $x^2+y^2=1$

$x^2 = 9$	$x^2 + x^2 = 1$
$x = 3, y = 3$	$x^2 = \frac{1}{2}$
$(3, 3)$	$x = \frac{1}{\sqrt{2}} \Rightarrow y = \frac{1}{\sqrt{2}}$
	$\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

$$\text{dist} = \sqrt{\left(3 - \frac{1}{\sqrt{2}}\right)^2 + \left(3 - \frac{1}{\sqrt{2}}\right)^2}$$

$$\left(3 - \frac{1}{\sqrt{2}}\right)\sqrt{2}$$

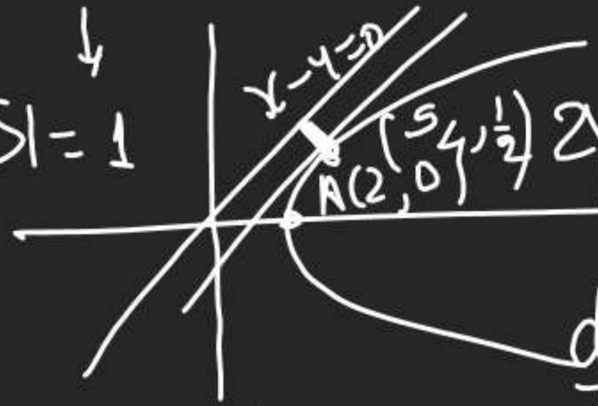
$$= 3\sqrt{2} - 1$$

Q. Sh. distance betn

$y = x$ & $y^2 = x - 2$

↓

$Sl = 1$



$2y\frac{dy}{dx} = 1$

$\frac{dy}{dx} = \frac{1}{2y}$

$\frac{1}{2y} = 1 \Rightarrow y = \frac{1}{2}, \; x = \frac{9}{4}$

Pt A $\left(\frac{9}{4}, \frac{1}{2}\right)$

$$d = \frac{\left|\frac{9}{4}\times 1 + \frac{1}{2}\times(-1) + 0\right|}{\sqrt{1^2+1^2}} = \frac{7}{4\sqrt{2}}$$

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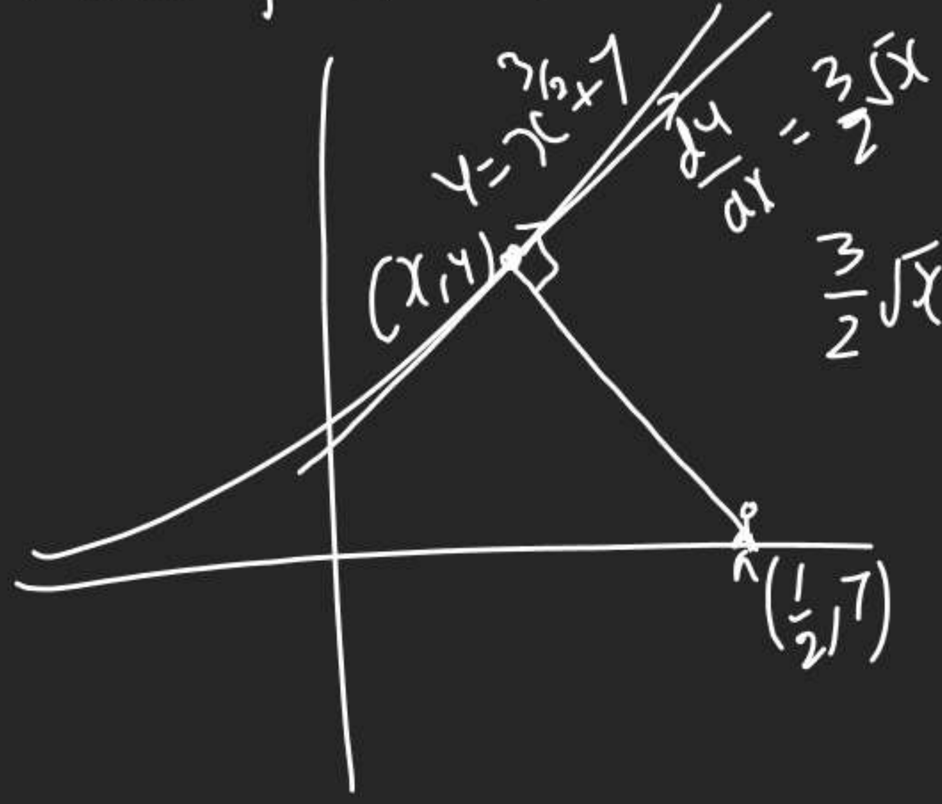
Sham Korector-25:45

Q A helicopter is flying along the curve is given by $y - x^{3/2} = 7$ $(x \geq 0)$. A soldier positioned at pt. $\left(\frac{1}{2}, 7\right)$ wants to shoot down the helicopter when it is nearest to him. Also find Nearest distance.



$\frac{dy}{dx} = \frac{3}{2}\sqrt{x}$

$\frac{3}{2}\sqrt{x} \times \left(\frac{7 - y}{\frac{1}{2} - x}\right) = -1$

$\frac{\sqrt{x}\left(+x^{3/2}\right)}{\left(\frac{1}{2} - x\right)} = +\frac{2}{3}$

$3x^2 = 2\left(\frac{1}{2} - x\right)$

$3x^2 + 2x - 1 = 0$

$(3x - 1)(x + 1) = 0$

$x = \frac{1}{3}, \ x = -1$ ⊗

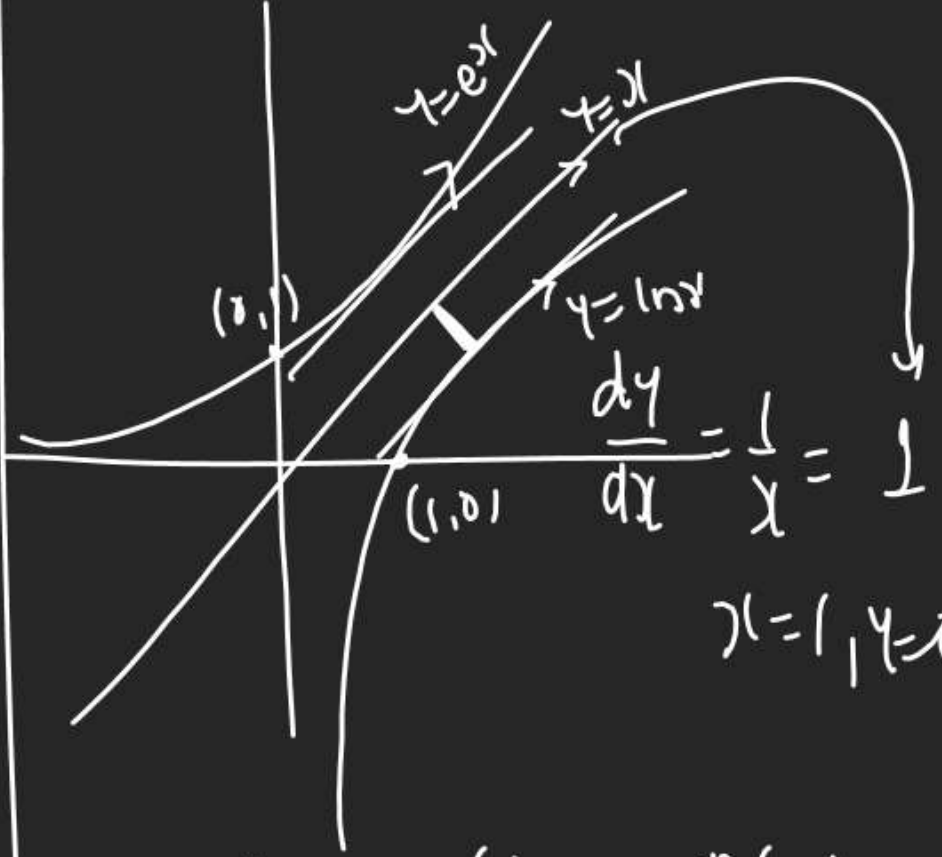
$\left(\frac{1}{3}, \frac{1}{3}\sqrt{\frac{1}{3}} + 7\right)$

$\text{dist} = \sqrt{\left(\frac{1}{3} - \frac{1}{2}\right)^2 + \left(\frac{1}{3\sqrt{3}} + \cancel{7} - \cancel{7}\right)^2}$

$= \sqrt{\frac{1}{36} + \frac{1}{27}}$

$= \sqrt{\frac{\cancel{63}\,7}{\underset{4}{\cancel{36}} \times 27}} = \frac{1}{6}\sqrt{\frac{7}{3}}$

Sh. dis betn $y = \ln x$, $y = e^x$



$\frac{dy}{dx} = \frac{1}{x} = 1$

$x = 1, y = 0$

Sh. dist $= 2\left(\text{dist. bet}^n\ (1,0)\ \&\ x - y = 0\right)$

$\frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$

$= 2\frac{|1 \times 1 + 0 \times -1 + 0|}{\sqrt{1^2 + 1^2}} = \sqrt{2}$

$\Rightarrow$ dist. betn (x_1, y_1) & line $ax + by + c = 0$

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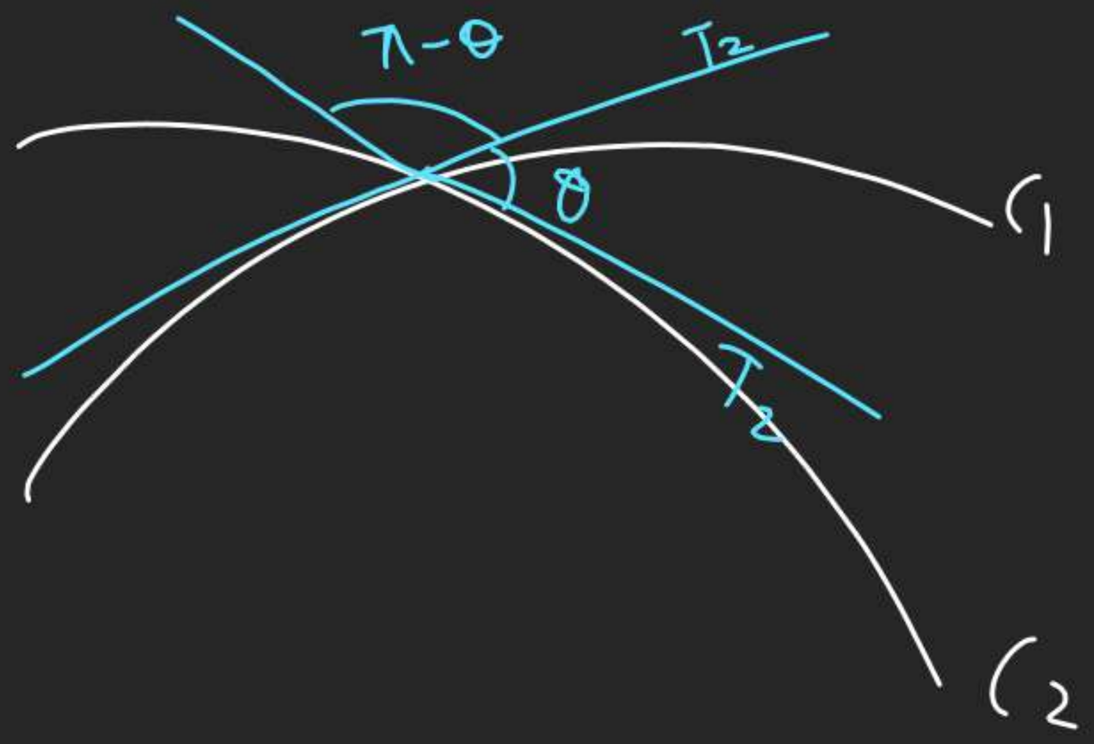
Angle of Intersection betn 2 Curves.



① find PoI

② find $\left.\frac{dy}{dx}\right|_{C_1}$ & $\left.\frac{dy}{dx}\right|_{C_2}$

③ Now use

$$\tan\theta = \left|\frac{m_1-m_2}{1+m_1m_2}\right|$$

(4) AOI = θ & $\pi-\theta$

RK

① If PoI in difficult to solve.

Try $m_1m_2=-1 \rightarrow$ Orthogonal

(2) If Curve in Exponantial or difficult Use hit & trial Method for PoI

(3) $m_1=m_2$ then Curve are touching eachother.

Nishant Jindal

Q Find AoI betⁿ $y^2 = 16x$ & $2x^2 + y^2 = 4$

① PoI → $2x^2 + 16x = 4$

$x^2 + 8x - 4 = 0$

$x = -8 \pm \sqrt{\ }$

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① Find AOI betn $y^2=16x$ & $x^2+y^2=4$

① PoI $2x^2+16x=4$

$x^2+8x-2=0$

$x=\frac{-8\pm\sqrt{64+8}}{2}$

$=\frac{-8\pm\sqrt{72}}{2}$ (handi की value y आंद Bekaar)

(2) $2y\frac{dy}{dx}=16$ | $4x+2y\frac{dy}{dx}=0$

$\frac{dy}{dx}=\frac{8}{y}$ | $\frac{dy}{dx}=-\frac{2x}{y}$

$m_1\times m_2=\frac{8}{y}\times-\frac{2x}{y}=\frac{-16x}{y^2}=-\frac{\cancel{16x}}{\cancel{16x}}=-1$

③ Curve are orthogonal. C_1+C_2

Q Find AOI betn $y=a^x$ & $y=b^x$.

① PoI → $x=0\to y=1,\ y=1$

PoI $=(0,1)$

② $\left.\frac{dy}{dx}\right|_{x=0}=a^x\ln a$ | $\left.\frac{dy}{dx}\right|_{x=0}=b^x\ln b$

$m_1=\ln a$ | $m_2=\ln b$.

(3) $\tan\theta=\left|\frac{\ln a-\ln b}{1+\ln a\ln b}\right|$

$\theta=\tan^{-1}\left|\frac{\ln a/b}{\ln ae/b}\right|$

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Q Find AOI betn

$y = 3^{x-1}\ln x$ & $y = x^x - 1$

①

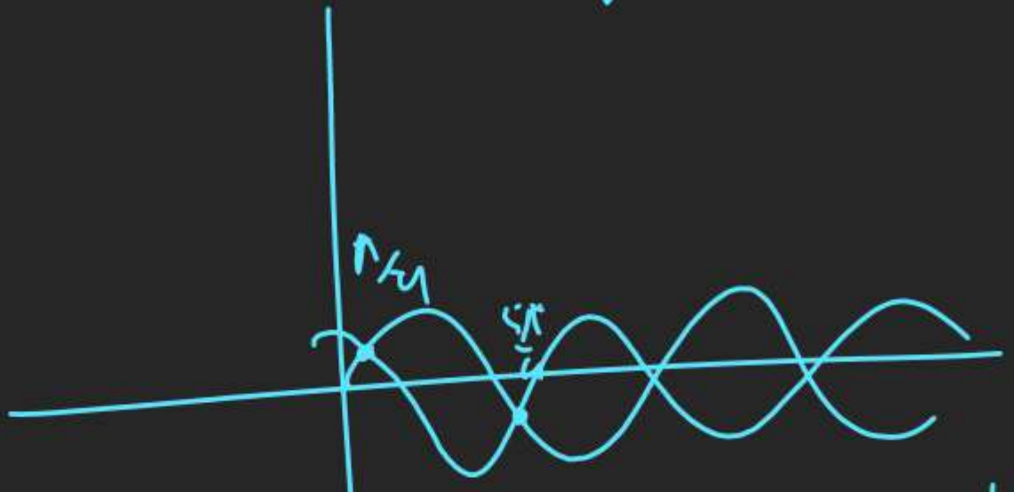
$y = 3^{x-1}\ln x$	$y = x^x - 1$
Tokku $x=0$: DNE	
$x=1$: $y = 3^0 \cdot \ln 1$, $y = 0$	$y = 1^1 - 1 = 0$

PoI $= (1,0)$

② $\left.\dfrac{dy}{dx}\right|_{x=1} = 3^{x-1}\cdot\dfrac{1}{x} + \ln x \cdot 3^{x-1}\ln 3 = \dfrac{1}{1} + 0 = 1 = m_1$ | $\left.\dfrac{dy}{dx}\right|_{x=1} = x^x(1+\ln x)$, $m_2 = 1$

Curve are touching each other

Q $y = \sin x$ & $y = \cos x$ are Intersecting at ∞ pts. Find angle betn them at one such pt. of Intersection.

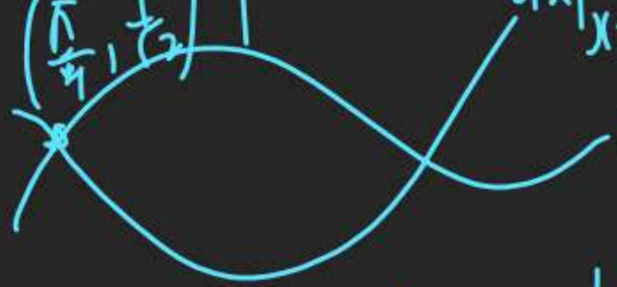


$\theta = \tan^{-1} 2\sqrt{2}$

या

$\pi - \tan^{-1} 2\sqrt{2}$

① PoI $\left(\dfrac{\pi}{4}, \dfrac{1}{\sqrt{2}}\right)$



② $\left.\dfrac{dy}{dx}\right|_{x=\frac{\pi}{4}} = \cos x$, $m_1 = \dfrac{1}{\sqrt{2}}$ | $\dfrac{dy}{dx} = -\sin x$, $m_2 = -\dfrac{1}{\sqrt{2}}$

$$\tan\theta = \left|\frac{\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}}{1 - \frac{1}{\sqrt{2}}\times\frac{1}{\sqrt{2}}}\right| = \left|\frac{\sqrt{2}}{\frac{1}{2}}\right| = 2\sqrt{2}$$

Nishant Jindal

Q If θ denotes acute angle betn curves $y = 10 - x^2$ & $y = 2 + x^2$ at their PoI then $|\tan\theta| = ?$

main

① $10 - x^2 = 2 + x^2$

$2x^2 = 8$

$x = 2$	$x = -2$
$y = 6$	$y = 6$

$(2, 6)$ & $(-2, 6)$

② $\left.\frac{dy}{dx}\right|_{x=2} = -2x$ | $\left.\frac{dy}{dx}\right|_{x=2} = 2x$

$m_1 = -4$ $\quad$ $m_2 = 4$

$$\tan\theta = \left|\frac{-4-4}{1+-4\times 4}\right| = \frac{8}{15}$$

$(-2, 6)$

$m_1 = -2x - 2$	$m_2 = 2x - 2$
$= 4$	$m_2 = -4$

$\tan\theta = \frac{8}{15}$

$\therefore |\tan\theta| = \frac{8}{15}$