MATRICES

USEFUL IN STUDY OF SCIENCE, ECONOMICS AND ENGINEERING

1. Definition : Rectangular array of mn numbers. Unlike determinants it has no value.

$$A = \begin{bmatrix} a_{11} & a_{12} & \dots\dots & a_{1n} \\ a_{21} & a_{22} & \dots\dots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ a_{m1} & a_{m2} & \dots\dots & a_{mn} \end{bmatrix} \text{ or } \begin{pmatrix} a_{11} & a_{12} & \dots\dots & a_{1n} \\ a_{21} & a_{22} & \dots\dots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ a_{m1} & a_{m2} & \dots\dots & a_{mn} \end{pmatrix}$$

Abbreviated as : $A = [a_{ij}]$ $1 \le i \le m; 1 \le j \le n$, i denotes the row and j denotes the column is called a matrix of order $m \times n$.

2. Special Type Of Matrices :

(a) Row Matrix : $A = [a_{11}, a_{12}, \dots\dots a_{1n}]$ having one row . $(1 \times n)$ matrix. (or row vectors)

(b) Column Matrix : (or column vectors) $A = \begin{bmatrix} a_{11} \\ a_{21} \\ \vdots \\ a_{m1} \end{bmatrix}$ having one column. $(m \times 1)$ matrix

(c) Zero or Null Matrix: $(A = O_{m \times n})$ An $m \times n$ matrix all whose entries are zero.

$A = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$ is a 3×2 null matrix & $B = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ is 3×3 null matrix

(d) Horizontal Matrix : A matrix of order $m \times n$ is a horizontal matrix if $n > m$.

(e) Verical Matrix : A matrix of order $m \times n$ is a vertical matrix if $m > n$. $\begin{bmatrix} 2 & 5 \\ 1 & 1 \\ 3 & 6 \\ 2 & 4 \end{bmatrix}$

(f) Square Matrix : (Order n)

If number of row = number of column $\Rightarrow$ a square matrix.

Note (i) In a square matrix the pair of elements a_{ij}&a_{ji} are called Conjugate Elements. e.g.

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$$

(ii) The elements $a_{11}, a_{22}, a_{33}, \dots\dots a_{nn}$ are called Diagonal Elements. The line along which the diagonal elements lie is called "Principal or Leading " diagonal.

The qty $\sum a_{ii}$ = trace of the matrice written as, i.e. $t_r A$

Square Matrix

- Triangular Matrix
  - $A = \begin{pmatrix} 1 & 3 & -2 \\ 0 & 2 & 4 \\ 0 & 0 & 5 \end{pmatrix}$ Upper Triangular $a_{ij} = 0 \ \forall \ i > j$
  - $B = \begin{pmatrix} 1 & 0 & 0 \\ 2 & -3 & 0 \\ 4 & 3 & 3 \end{pmatrix}$ Lower Triangular $a_{ij} = 0 \ \forall \ i < j$
  - **Note that :** Minimum number of zeros in a triangular matrix of order $n = n(n-1)/2$
- Diagonal Matrix denote as $d_{dia}(d_1, d_2, \dots\dots, d_n)$ all elements except the leading diagonal are zero
  - diagonal Matrix: $\begin{bmatrix} d_1 & 0 & 0 \\ 0 & d_2 & 0 \\ 0 & 0 & d_3 \end{bmatrix}$
  - Unit or Identity Matrix: $a_{ij} = \begin{bmatrix} 1 & \text{if} & i = j \\ 0 & \text{if} & i \ne j \end{bmatrix}$
  - Note: (1) If $d_1 = d_2 = d_3 = a$ Scalar Matrix
    (2) If $d_1 = d_2 = d_3 = 1$ Unit Matrix

Note: Min. number of zeros in a diagonal matrix of order $n = n(n-1)$

"It is to be noted that with square matrix there is a corresponding determinant formed by the elements of A in the same order."

3. Equality Of Matrices:

Let $A = [a_{ij}]$ & $B = [b_{ij}]$ are equal if

(i) both have the same order. **(ii)** $a_{ij} = b_{ij}$ for each pair of i&j.

4. Algebra Of Matrices:

Addition : $A + B = [a_{ij} + b_{ij}]$ where A&B are of the same type. (same order)

(a) Addition of matrices is commutative.

i.e. $A + B = B + A$ $A = m \times n$; $B = m \times n$

(b) Matrix addition is associative.

Note: A, B&C are of the same type.

(c) Additive inverse.

5. Multiplication Of A Matrix By A Scalar:

If $A = \begin{bmatrix} a & b & c \\ b & c & a \\ c & a & b \end{bmatrix}$; $kA = \begin{bmatrix} ka & kb & kc \\ kb & kc & ka \\ kc & ka & kb \end{bmatrix}$

6. Multiplication Of Matrices : (Row by Column)

AB exists, but BA does not $\Rightarrow AB \neq BA$

Note: In the product AB, $\begin{cases} A = & \text{prefactor} \\ B = & \text{post factor} \end{cases}$

$$A = (a_1, a_2, \ldots\ldots a_n) \ \& \ B = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix}$$

$1 \times n$ $\qquad n \times 1$

$$AB = [a_1 b_1 + a_2 b_2 + \cdots .. + a_n b_n]$$

$A = [a_{ij}]m \times n$ & $B = [b_{ij}]$ $n \times p$ matrix, then $(AB)_{ij} = \sum_{r=1}^{n} a_{ir} \cdot b_{rj}$

Properties Of Matrix Multiplication :

1. Matrix multiplication is not commutative.

$A = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$; $B = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$; $AB = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$; $BA = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \Rightarrow AB \neq BA$ (in general)

2. $AB = \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \Rightarrow AB = 0 \Rightarrow A = 0$ or $B = 0$

Note: If A and B are two non- zero matrices such that $AB = O$ then A and B are called the divisors of zero.

Also if $[AB] = 0 \Rightarrow |AB| \Rightarrow |A||B| = 0 \Rightarrow |A| = 0$ or $|B| = 0$ but not the converse.

If A and B are two matrices such that

(i) $AB = BA \Rightarrow$ A and B commute each other

(ii) $AB = -BA \Rightarrow$ A and B anti commute each other

3. Matrix Multiplication Is Associative:

If A, B & C are conformable for the product AB&BC, then $(A.B) \cdot C = A \cdot (B.C)$

4. Distributivity :

$$\left.\begin{array}{l} A(B+C) = AB + AC \\ (A+B)C = AC + BC \end{array}\right] \text{Provided A, B\&C are conformable for respective products}$$

5. POSITIVE INTEGRAL POWERS OF A SQUARE MATRIX :

For a square matrix A, $A^2A = (AA)A = A(AA) = A^3$.

Note that for a unit matrix I of any order, $I^m = I$ for all $m \in N$.

6. MATRIX POLYNOMIAL:

If $f(x) = a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \cdots \dots \dots + a_nx^0$ then we define a matrix polynomial

$f(A) = a_0A^n + a_1A^{n-1} + a_2A^{n-2} + \cdots . + a_nn^n$

where A is the given square matrix. If f(A) is the null matrix then A is called the zero or root of the polynomial f(x).

DEFINITIONS

(a) Idempotent Matrix : A square matrix is idempotent provided $A^2 = A$.

Note that $A^n = A \forall n \geq 2, n \in N$.

(b) Nilpotent Matrix: A square matrix is said to be nilpotent matrix of order m, $m \in N$, if

$A^m = 0, A^{m-1} \neq 0$.

(c) Periodic Matrix : A square matrix is which satisfies the relation $A^{K+1} = A$, for some positive integer K, is a periodic matrix. The period of the matrix is the least value of K for which this holds true.

Note that period of an idempotent matrix is 1.

(d) Involutary Matrix : If $A^2 = 1$, the matrix is said to be an involutary matrix.

Note that $A = A^{-1}$ for an involutary matrix.

7. The Transpose Of A Matrix : (Changing rows & columns)

Let A be any matrix. Then, $A = a_{ij}$ of order $m \times n$

$\Rightarrow A^T$ or $A' = [a_{ji}]$ for $1 \leq i \leq n$&$1 \leq j \leq m$ of order $n \times m$

Properties of Transpose: If A^T&B^T denote the transpose of A and B,

(a) $(A \pm B)^T = A^T \pm B^T$; note that A&B have the same order.

IMP. (b) $(AB)^T = B^T A^T$ A&B are conformable for matrix product AB.

(c) $(A^T)^T = A$

(d) $(kA)^T = kA^T$ k is a scalar .

General : $(A_1, A_2, \ldots\ldots A_n)^T = A_n^T, \ldots\ldots A_2^T, A_1^T$ (reversal law for transpose)

8. Symmetric & Skew Symmetric Matrix :

A square matrix $A = [a_{ij}]$ is said to be , symmetric if ,

$a_{ij} = a_{ji} \forall i \& j$ (conjugate elements are equal) **(Note $A = A^T$)**

Note: Max. number of distinct entries in a symmetric matrix of order n is $\frac{n(n+1)}{2}$. and skew symmetric if,

$a_{ij} = -a_{ji}\ \forall i \& j$ (the pair of conjugate elements are additive inverse of each other)

(Note $A = -A^T$)

Hence If A is skew symmetric, then $a_{ii} = -a_{ii} \Rightarrow a_{ii} = 0\ \forall\ i$

Thus the digaonal elements of a skew symmetric matrix are all zero, but not the converse .

Properties Of Symmetric & Skew Matrix :

P − 1 A is symmetric if $A^T = A$

A is skew symmetric if $A^T = -A$

P−2 $2A + A^T$ is a symmetric matrix

$A - A^T$ is a skew symmetric matrix .

Consider $(A + A^T)^T = A^T + (A^T)^T = A^T + A = A + A^T$

$A + A^T$ is symmetric .

Similarly we can prove that $A - A^T$ is skew symmetric.

P − 3 The sum of two symmetric matrix is a symmetric matrix and the sum of two skew symmetric matrix is a skew symmetric matrix .

Let $A^T = A$; $B^T = B$ where A&B have the same order.

$(A + B)^T = A + B$

Similarly we can prove the other

P − 4 If A&B are symmetric matrices then,

(a) $AB + BA$ is a symmetric matrix

(b) $AB - BA$ is a skew symmetric matrix .

P - 5 Every square matrix can be uniquely expressed as a sum of a symmetric and a skew symmetric matrix.

$$A = \underbrace{\frac{1}{2}(A + A^T)}_{\substack{P \\ \text{Symmetric}}} + \underbrace{\frac{1}{2}(A - A^T)}_{\substack{Q \\ \text{Skew Symmetric}}}$$

9. Adjoint Of A Square Matrix :

Let $A = [a_{ij}] = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$ be a square matrix and let the matrix formed by the cofactors of $[a_{ij}]$ in determinant $|A|$ is $= \begin{pmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{pmatrix}$.

Then $(\text{adj } A) = \begin{pmatrix} C_{11} & C_{21} & C_{31} \\ C_{12} & C_{22} & C_{32} \\ C_{13} & C_{23} & C_{33} \end{pmatrix}$

V. Imp. Theorem : $A(\text{ adj. } A) = (\text{adj. } A) \cdot A = |A|I_n$, If A be a square matrix of order n.

Note: If A and B are non singular square matrices of same order, then

(i) $|\text{adj } A| = |A|^{n-1}$

(ii) $\text{adj } (AB) = (\text{adj } B)(\text{adj } A)$

(iii) $\text{adj } (KA) = K^{n-1}(\text{adj } A)$, K is a scalar

Inverse Of A Matrix (Reciprocal Matrix):

A square matrix A said to be invertible (non singular) if there exists a matrix B such that,

$$AB = 1 = BA$$

B is called the inverse (reciprocal) of A and is denoted by A^{-1}. Thus

$$A^{-1} = B \Leftrightarrow AB = 1 = BA.$$

We have, $A \cdot (\text{adj } A) = |A|I_n$

$$A^{-1}A(\text{adj } A) = A^{-1}I_n|A|$$
$$I_n(\text{adj } A) = A^{-1}|A|I_n$$
$$\therefore \quad A^{-1} = \frac{(\text{adj } A)}{|A|}$$

Note: The necessary and sufficient condition for a square matrix A to be invertible is that $|A| \neq$

Imp. Theorem : If A&B are invertible matrices ofthe same order, then $(AB)^{-1} = B^{-1}A^{-1}$. This is reversal law for inverse.

Note:

(i) If A be an invertible matrix, then A^T is also invertible & $(A^T)^{-1} = (A^{-1})^T$.

(ii) If A is invertible, **(a)** $(A^{-1})^{-1} = A$; **(b)** $(A^k)^{-1} = (A^{-1})^k = A^{-k}$, $k \in N$

(iii) If A is an Orthogonal Matrix. $A^T = I = A^T A$

(iv) A square matrix is said to be **orthogonal** if, $A^{-1} = A^{T}$.

(v) $|A^{-1}| = \frac{1}{|A|}$

SYSTEM OF EQUATION & CRITERIAN FOR CONSISTENCY

GAUSS - JORDAN METHOD

$$x + y + z = 6$$

$$x - y + z = 2$$

$$2x + y - z = 1$$

$$\begin{pmatrix} x + y + z \\ x - y + z \\ 2x + y - z \end{pmatrix} = \begin{pmatrix} 6 \\ 2 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 2 & 1 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 6 \\ 2 \\ 1 \end{pmatrix}$$

$$AX = B \Rightarrow A^{-1}AX = A^{-1}B$$

$$X = A^{-1}B = \frac{(\text{adj} \cdot A) \cdot B}{|A|}.$$

Note :

(1) If $|A| \neq 0$, system is consistent having unique solution

(2) If $|A| \neq 0$ & $(\text{adj } A).B \neq O$ (Null matrix), system is consistent having unique non-trivial solution .

(3) If $|A| \neq 0$ & $(\text{adj } A).B = 0$ (Null matrix) system is consistent having trivial solution .

(4) If $|A| = 0$, **matrix method fails**

If (adj A) . B = null matrix **= O** → Consistent (Infinite solutions)

If (adj A) . B ≠ **O** → Inconsistent (no solution)

PROFICIENCY TEST-01

1. In the following, upper triangular matrix is

(A) $\begin{bmatrix}1 & 0 & 0\\0 & 2 & 0\\3 & 0 & 3\end{bmatrix}$ (B) $\begin{bmatrix}5 & 4 & 2\\0 & 0 & 3\\0 & 0 & 1\end{bmatrix}$ (C) $\begin{bmatrix}0 & 2 & 3\\0 & 0 & 4\end{bmatrix}$ (D) $\begin{bmatrix}2 & 1\\0 & 3\\0 & 0\end{bmatrix}$

2. If $A = \begin{bmatrix}5 & 2\\1 & 0\end{bmatrix}$ and $B = \begin{bmatrix}2 & 3\\5 & -1\end{bmatrix}$, then $|2A - 3B|$ equals

(A) 77 (B) -53 (C) 53 (D) -77

3. For a square matrix $A = [a_{ij}], a_{ij} = 0$, when $i \neq j$, then A is

(A) unit matrix (B) scalar matrix (C) diagonal matrix (D) None of these

4. If A and B are matrices of order $m \times n$ and $n \times n$ respectively, then which of the following are defined

(A) AB, BA (B) AB, A^2 (C) A^2, B^2 (D) AB, B^2

5. If $A = \begin{bmatrix}-1 & 0 & 2\\3 & 1 & 2\end{bmatrix}$ and $B = \begin{bmatrix}-1 & 5\\2 & 7\\3 & 10\end{bmatrix}$, then

(A) AB and BA both exist (B) AB exists but not BA

(C) BA exists but not AB (D) Both AB and BA do not exist

6. If A is a matrix of order 3×4, then both AB^T and B^TA are defined if order of B is

(A) 3×3 (B) 4×4 (C) 4×3 (D) 3×4

7. Matrix $\begin{bmatrix}0 & 5 & -7\\-5 & 0 & 11\\7 & -11 & 0\end{bmatrix}$ is a

(A) Diagonal matrix (B) Upper triangular matrix

(C) Skew-symmetric matrix (D) Symmetric matrix

8. If A is symmetric as well as skew symmetric matrix, then

(A) A is a diagonal matrix (B) A is a null matrix

(C) A is a unit matrix (D) A is a triangular matrix

9. If A is symmetric matrix and B is a skew-symmetric matrix, then for $n \in N$, false statement is

(A) A^n is symmetric when n is odd (B) A^n is symmetric only when n is even

(C) B^n is skew symmetric when n is odd (D) B^n is symmetric when n is even

10. Let A be a square matrix. Then which of the following is not a symmetric matrix

(A) $A + A^T$ (B) AA^T (C) A^TA (D) $A - A^T$

11. If $A = \begin{bmatrix}1 & 1\\1 & 1\end{bmatrix}$ and $n \in N$, then A^n is equal to

(A) 2^nA (B) $2^{n-1}A$ (B) nA (D) None of these

12. If $A = [a_{ij}]$ is scalar matrix of order $n \times n$ such that $a_{ii} = k$ for all i, then $|A|$ equals

(A) nk (B) $n + k$ (C) n^k (D) k^n

13. If $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$, then for every positive integer n, A^n is equal to

(A) $\begin{bmatrix} 1+2n & 4n-8 \\ n & 1-2n \end{bmatrix}$ (B) $\begin{bmatrix} 1+2n & -4n \\ n & 1-2n \end{bmatrix}$

(C) $\begin{bmatrix} 1-2n & -4n \\ n & n-2 \end{bmatrix}$ (D) None of these

14. If A is any skew-symmetric matrix of odd orders, then $|A|$ equals

(A) -1 (B) 0 (C) 1 (D) None of these

15. If $A = \begin{bmatrix} 0 & c & -b \\ -c & 0 & a \\ b & -a & 0 \end{bmatrix}$ and $B = \begin{bmatrix} a^2 & ab & ac \\ ab & b^2 & bc \\ ac & bc & c^2 \end{bmatrix}$ then AB is equal to

(A) A (B) B

(C) an Identity matrix (D) a Null matrix

PROFICIENCY TEST-02

1. The root of the equation $\begin{bmatrix} x & 1 & 2 \end{bmatrix}\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}\begin{bmatrix} x \\ -1 \\ 1 \end{bmatrix} = 0$ is

(A) $\frac{1}{3}$ (B) $-\frac{1}{3}$ (C) 0 (D) 1

2. For square matrices A and B, AB = O, then {O : null matrix }

(A) A = O or B = O
(B) A = O and B = O
(C) It is not necessary that A = O and/or B = O
(D) None of these

3. If A and B are matrices of order $m \times n$ and $n \times m$ respectively, then the order of matrix $B^T(A^T)^T$ is

(A) $m \times n$ (B) $m \times m$ (C) $n \times n$ (D) Not defined

4. If $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 0 & 0 & 2 \end{bmatrix}$, then the value of adj (adj A) is

(A) $4A^2$ (B) $-2A$ (C) $2A$ (D) A^2

5. If $A = \begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}$ and A.(adj A) $= k\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then k equals

(A) sinx cosx (B) 1 (C) sin2x (D) -1

6. If $A = \begin{bmatrix} 1 & -2 & 3 \\ 4 & 0 & -1 \\ -3 & 1 & 5 \end{bmatrix}$, then $(\text{adj}A)_{23} =$

(i.e., the element of (adjA) which belongs to second row and third column)

(A) 13 (B) -13 (C) 5 (D) -5

7. $(\text{adj}A^T) - (\text{adj}A)^T$ equals

(A) $|A|I$ (B) $2|A|I$ (C) Null matrix (D) Unit matrix

8. If $A = \begin{bmatrix} 2 & 3 \\ 1 & 3 \end{bmatrix}, B = \begin{bmatrix} 4 & 6 \\ 2 & 3 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then which of these matrices are invertible ?

(A)A and B (B) B and C (C)A and C (D)All

9. Which of the following matrices is inverse of itself

(A) $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$
(B) $\begin{bmatrix} 3 & -2 \\ 4 & -3 \end{bmatrix}$
(C) $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 1 \end{bmatrix}$
(D) $\begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

10. If D is a diagonal matrix with diagonal elements as $\{d_1, d_2, d_2 \ldots, d_n\}$ in order, then we may represent it as $D = \text{diag}(d_1, d_2, \ldots \ldots, d_n)$. Then D' equals

(A) D
(B) $\text{diag}\left({d_1}^{n-1}, {d_2}^{n-1}, \ldots, d_n^{n-1}\right)$
(C) $\text{diag}\left(d_1^n, d_2^n, \ldots, d_n^n\right)$
(D) None of these

11. If $A = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$, then

(A) $\text{adj}A = A$
(B) $\text{adj}A = A^{-1}$
(C) $A^{-1} = -A$
(D) None of these

12. If A is invertible matrix, then det (A^{-1}) equals {where, det (B) means determinant of matrix B}

(A) det (A)
(B) $\frac{1}{\det(A)}$
(C) 1
(D) None of these

13. If A and B are non-zero square matrices of the same order such that $AB = O$, then {O : null matrix}

(A) Either adj $A = O$ or adj $B = O$
(B) adj $A = O$ and adj $B = O$
(C) Either $|A| = 0$ or $|B| = 0$
(D) $|A| = 0$ and $|B| = 0$

14. Let A be an idempotent square matrix, then $(I + A)^4$ is :

(A) $1 - A$
(B) $I + A$
(C) $I + 15A$
(D) I

15. If A and B are two square matrices such that $B = -A^{-1}BA$, then $(A + B)^2 =$

(A) $A^2 + 2BA + B^2$
(B) $A^2 + B^2$
(C) $A^2 + 2AB + B^2$
(D) $A^2 - B^2$

EXERCISE- I

1. Find the number of 2×2 matrix satisfying
(i) a_{ij} is 1 or -1 (ii) $a_{11}^2 + a_{12}^2 = a_{21}^2 + a_{22}^2 = 2$; (iii) $a_{11}a_{21} + a_{12}a_{22} = 0$

2. Find the value of x and y that satisfy the equations.

$$\begin{bmatrix} 3 & -2 \\ 3 & 0 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} y & y \\ x & x \end{bmatrix} = \begin{bmatrix} 3 & 3 \\ 3y & 3y \\ 10 & 10 \end{bmatrix}$$

3. Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and $B = \begin{bmatrix} p \\ q \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \end{bmatrix}$. Such that $AB = B$ and $a + d = 5050$. Find the value of $(ad - bc)$

4. Define $A = \begin{bmatrix} 0 & 1 \\ 3 & 0 \end{bmatrix}$. Find a vertical vector V such that $(A^8 + A^6 + A^4 + A^2 + I)V = \begin{bmatrix} 0 \\ 11 \end{bmatrix}$ (where I is the 2×2 identity matrix).

5. If, $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$, then show that the madrix A is a root of the polynomial $f(x) = x^3 - 6x^2 + 7x + 2$.

6. If the matrices $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ (a, b, c, d not all simultaneously zero) commute, find the value of $\frac{d-b}{a+c-b}$. Also show that the matrix which commutes with A is of the form $\begin{bmatrix} \alpha - \beta & 2\beta/3 \\ \beta & \alpha \end{bmatrix}$

7. If $\begin{bmatrix} a & b \\ c & 1-a \end{bmatrix}$ is an idempotent matrix. Find the value of f(a), where $f(x) = x - x^2$, when $bc = 1/4$. Hence otherwise evaluate a.

8. If the matrix A is involutary, show that $\frac{1}{2}(I + A)$ and $\frac{1}{2}(I - A)$ are idempotent and $\frac{1}{2}(I + A) \cdot \frac{1}{2}(I - A) = O$

9. Show that the matrix $A = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$ can be decomposed as a sum of a unit and a nilpotent marix. Hence evaluate the matrix $\begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}^{2007}$.

10. Given matrices $A = \begin{bmatrix} 1 & x & 1 \\ x & 2 & y \\ 1 & y & 3 \end{bmatrix}$; $B = \begin{bmatrix} 3 & -3 & z \\ -3 & 2 & -3 \\ z & -3 & 1 \end{bmatrix}$
Obtain x, y and z if the matrix AB is symmetric.

11. Let X be the solution set of the equation $A^x = 1$, where $A = \begin{bmatrix} 0 & 1 & -1 \\ 4 & -3 & 4 \\ 3 & -3 & 4 \end{bmatrix}$ and I is the corresponding unit matrix and $x \subseteq \mathbb{N}$ then find the minimum value of $\sum(\cos^x\theta + \sin^x\theta), \theta \in \mathbb{R}$.

12. $A = \begin{pmatrix} 3 & a & -1 \\ 2 & 5 & c \\ b & 8 & 2 \end{pmatrix}$ is Symmetric and $B = \begin{pmatrix} d & 3 & a \\ b-a & e & -2b-c \\ -2 & 6 & -f \end{pmatrix}$ is Skew Symmetric, then find AB. Is AB a symmetric, Skew Symmetric of neither of them. Justify your answer.

13. A is a square matrix of order n.

I = maximum number of distinct entries if A is a triangular matrix

m = maximum number of distinct entries if A is a diagonal matrix

p = minimum number of zeroes if A is a triangular matrix

If $I + 5 = p + 2m$, find the order of the matrix.

14. If A is an idempotent non zero matrix and I is an identity matrix of the same order, find the value of n, $n \in N$, such that $(A + I)^n = I + 127\,A$.

15. Consider the two matrices A and B where $A = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}$; $B = \begin{bmatrix} 5 \\ -3 \end{bmatrix}$. If n(A) denotes the number of elements in A such that $n(XY) = 0$, when the two matrices X and Y are not conformable for multiplication. If $C = (AB)(B'A)$: $D = (B'A)(AB)$ then, find the value of $\left(\frac{n(C)\left(|D|^2 + n(D)\right)}{n(A) - n(B)}\right)$

EXERCISE- II

1. $A_{3\times3}$ is a matrix such that $|A| = a, B = (\text{adj } A)$ such that $|B| = b$. Find the value of $(ab^2 + a^2b + 1)S$ where $\frac{1}{2}$ $S = \frac{a}{b} + \frac{a^2}{b^3} + \frac{a^3}{b^5} + \cdots$. up to ∞, and $a = 3$.

2. For the matrix $A = \begin{bmatrix} 4 & -4 & 5 \\ -2 & 3 & -3 \\ 3 & -3 & 4 \end{bmatrix}$ find A^{-2}.

3. Given $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 4 & 1 \\ 2 & 3 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 3 \\ 3 & 4 \end{bmatrix}$. Find P such that $BPA = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

4. Given the matrix $A = \begin{bmatrix} -1 & 3 & 5 \\ 1 & -3 & -5 \\ -1 & 3 & 5 \end{bmatrix}$ and X be the solution set of the equation $A^x = A$, where $x \in N - \{1\}$. Evaluate $\prod\left(\frac{x^3+1}{x^3-1}\right)$ where the continued product extends $\forall x \in X$

5. If $F(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$ then show that $F(x).F(y) = F(x+y)$

Hence prove that $[F(x)]^{-1} = F(-x)$.

6. Use matrix to solve the following system of equations.

(i) $\begin{matrix} x+y+z=3 \\ x+2y+3z=4 \\ 2x+3y+9z=6 \end{matrix}$ (ii) $\begin{matrix} x+y+z=3 \\ x+2y+3z=4 \\ 2x+3y+4z=7 \end{matrix}$ (Iii) $\begin{matrix} x+y+z=3 \\ x+2y+3z=4 \\ 2x+3y+4z=9 \end{matrix}$

7. Let A be a 3×3 matrix such that $a_{11} = a_{33} = 2$ and all the other $a_{ij} = 1$. Let $A^{-1} = xA^2 + yA + zI$ then find the value of $(x + y + z)$ where I is a unit matrix of order 3 .

8. Find the matrix A satisfying the matrix equation, $\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} . A . \begin{bmatrix} 3 & 2 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ 3 & -1 \end{bmatrix}$.

9. If $A = \begin{bmatrix} k & m \\ l & n \end{bmatrix}$ and $kn \neq lm$; then show that $A^2 - (k+n)A + (kn - lm)I = 0$.

Hence find A^{-1}.

10. Given $A = \begin{bmatrix} 2 & 1 \\ 2 & 1 \end{bmatrix}$; $B = \begin{bmatrix} 9 & 3 \\ 3 & 1 \end{bmatrix}$. I is a unit matrix of order 2. Find all possible matrix X in the following cases.

(i) $AX = A$ (ii) $XA = I$ (iii) $XB = O$ but $BX \neq O$.

11. If $A = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$ then, find a non-zero square matrix X of order 2 such that $AX = O$. Is $XA = O$. If $A = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$, is it possible to find a square matrix X such that $AX = O$. Give reasons for it.

12. Determine the values of a and b for which the system $\begin{bmatrix} 3 & -2 & 1 \\ 5 & -8 & 9 \\ 2 & 1 & a \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} b \\ 3 \\ -1 \end{bmatrix}$

(i) has a unique solution ; (ii) has no solution and (iii) has infinilely many solutions

13. If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$; $B = \begin{bmatrix} 3 & 1 \\ 1 & 0 \end{bmatrix}$; $C = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$ and $X = \begin{bmatrix} x_1 & x_2 \\ x_3 & x_4 \end{bmatrix}$ then solve the following matrix equation.

(a) $AX = B - I$

(b) $(B - I)X = IC$

(c) $CX = A$

14. If A is an orthogonal matrix and $B = AP$ where P is a non singular matrix then show that the matrix PB^{-1} is also orthogonal.

15. Consider the matrices $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} a & b \\ 0 & 1 \end{bmatrix}$ and let P be any orthogonal matrix and $Q = PAP^T$ ard $R = P^TQ^KP$ also $S = PBP^T$ and $T = P^TS^KP$

	Column I		**Column II**
(A)	If we vary K from 1 to n then the first row first column elements at R will form	(P)	G.P. with common ratio a
(B)	If we vary K from 1 to then the 2^{-d} row 2^{nd} column elements at R will form	(Q)	A.P. with common difference 2
(C)	If we vary K from 1 to n then the first row first column elements of T will form	(R)	G.P. with common ratio b
(D)	I we vary K from 3 to n then the first row 2^{nd} Elements of T will represent the sum of	(S)	A.P. with common difference -2.

EXERCISE- III

1. If $\begin{bmatrix}\alpha & \beta \\ \gamma & -\alpha\end{bmatrix}$ is square root of I_2(2×2 Identity matrix), then α, β and γ will satisfy the relation

(A) $1+\alpha^2+\beta\gamma=0$ (B) $1-\alpha^2+\beta\gamma=0$

(C) $1+\alpha^2-\beta\gamma=0$ (D) $-1+\alpha^2+\beta\gamma=0$

2. If $A_\alpha = \begin{bmatrix}\cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha\end{bmatrix}$, then which of following stalement is true

(A) $A_\alpha \cdot A_\beta = A_{\alpha\beta}$ and $(A_\alpha)^n = \begin{bmatrix}\cos^n\alpha & \sin^n\alpha \\ -\sin^n\alpha & \cos^n\alpha\end{bmatrix}$

(B) $A_\alpha \cdot A_\beta = A_{\alpha\beta}$ and $(A_\alpha)^n = \begin{bmatrix}\cos n\alpha & \sin n\alpha \\ -\sin n\alpha & \cos n\alpha\end{bmatrix}$

(C) $A_\alpha \cdot A_\beta = A_{\alpha+\beta}$ and $(A_\alpha)^n = \begin{bmatrix}\cos^n\alpha & \sin^n\alpha \\ -\sin^n\alpha & \cos^n\alpha\end{bmatrix}$

(D) $A_\alpha \cdot A_\beta = A_{\alpha+\beta}$ and $(A_\alpha)^n = \begin{bmatrix}\cos n\alpha & \sin n\alpha \\ -\sin n\alpha & \cos n\alpha\end{bmatrix}$

3. If $M = \begin{bmatrix}1 & 2 \\ 2 & 3\end{bmatrix}$ and $M^2 - \lambda M - I = 0$, then λ equals

(A) -2 (B) 2 (C) -4 (D) 4

4. If A be a matrix such that imverse of 7A is the matrix $\begin{bmatrix}-1 & 2 \\ 4 & -7\end{bmatrix}$, then A equals

(A) $\begin{bmatrix}1 & 2 \\ 4 & 1\end{bmatrix}$ (B) $\begin{bmatrix}1 & 4/7 \\ 2/7 & 1/7\end{bmatrix}$

(C) $\begin{bmatrix}1 & 4 \\ 2 & 1\end{bmatrix}$ (D) $\begin{bmatrix}1 & 2/7 \\ 4/7 & 1/7\end{bmatrix}$

5. If $A = \begin{bmatrix}0 & 1 \\ -1 & 0\end{bmatrix}$ and $(aI + bA)^2 = A, (a > 0)$, then

(A) $a = b = \sqrt{2}$ (B) $a = b = \frac{1}{\sqrt{2}}$

(C) $a = b = \sqrt{3}$ (D) $a = b = \frac{1}{\sqrt{3}}$

6. If A and B are square matrices such that $AB = B$ and $BA = A$, then $A^2 + B^2$ is equal to

(A) 2AB (B) 2BA (C) A + B (D) None of these

7. If $\begin{bmatrix}1 & -\tan\theta \\ \tan\theta & 1\end{bmatrix}\begin{bmatrix}1 & \tan\theta \\ -\tan\theta & 1\end{bmatrix}^{-1} = \begin{bmatrix}a & -b \\ b & a\end{bmatrix}$, then

(A) $a = \sin 2\theta, b = -\cos 2\theta$ (B) $a = \cos 2\theta, b = \sin 2\theta$

(C) $a = \sin 2\theta, b = \cos 2\theta$ (D) $a = \cos 2\theta, b = -\sin 2\theta$

8. Let the matrices A and B be defined as $A = \begin{bmatrix}1 & 2 \\ 2 & 3\end{bmatrix}$ and $B = \begin{bmatrix}3 & 7 \\ 1 & 3\end{bmatrix}$, then the value of determinant of matrix $(2\,A^7B^{-1})$, is :

(A) 2 (B) 1 (C) -1 (D) -2

9. There are two possible values of A in the solution of the matrix equation

$\begin{bmatrix} 2A+1 & -5 \\ -4 & A \end{bmatrix}^{-1} \begin{bmatrix} A-5 & B \\ 2A-2 & C \end{bmatrix} - \begin{bmatrix} 14 & D \\ E & F \end{bmatrix}$, where A, B, C, D, E, F are real numbers.

The absolute value of the difference of these two solutions, is:

(A) $\frac{13}{3}$ (B) $\frac{11}{3}$ (C) $\frac{17}{3}$ (D) $\frac{19}{3}$

10. If A is a square matrix, and B is a singular matrix of same order, then for a positive integer n, $(A^{-1}BA)^n$ equals

(A) $A^{-n}B^nA^n$ (B) $A^n B^n A^{-n}$ (C) $A^{-1}B^nA$ (D) $n(A^{-1}BA)$

EXERCISE-IV

1. If $A = \begin{bmatrix} a & b \\ b & a \end{bmatrix}$ and $A^2 = \begin{bmatrix} \alpha & \beta \\ \beta & \alpha \end{bmatrix}$, then **[AIEEE 2003]**

(A) $\alpha = a^2 + b^2, \beta = ab$ (B) $\alpha = a^2 + b^2, \beta = 2ab$

(C) $\alpha = a^2 + b^2, \beta = a^2 - b^2$ (D) $\alpha = 2ab, \beta = a^2 + b^2$

2. Let $A = \begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$. The only correct statement about the matrix A is : **[AIEEE 2004]**

(A) A is a zero matrix (B) $A^2 = I$

(C) A^{-1} does not exist (D) $A = -I$, where I is a unit matrix

3. Let $A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}$ and $10B = \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 3 \end{bmatrix}$. If B is the imverse of A, then α is:

[AIEEE 2004]

(A) -2 (B) 5 (C) 2 (D) -1

4. If $A^2 - A + I = O$, then the inverse of A is :

[AIEEE 2005]

(A) $A + I$ (B) A (C) A-I (D) $I - A$

5. If $A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then which one of the following holds for all $n \geq 1$, by the principle of mathemalical induction **[AIEEE 2005]**

(A) $A^n = nA - (n-1)I$ (B) $A^n = 2^{n-1}A - (n-1)I$

(C) $A^n = nA + (n-1)I$ (D) $A^n = 2^{n-1}A + (n-1)I$

6. If A and B are square matrices of size $n \times n$ such that $A^2 - B^2 = (A - B)(A + B)$, then which of the following will be always true? **[AIEEE 2006]**

(A) $A = B$ (B) $AB = BA$

(C) Either A or B is a zero matrix (D) Either A or B is an identity matrix

7. Let $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$, $a, b \in N$. Then **[AIEEE 2006]**

(A) there cannot exists any B such that $AB = BA$.

(B) there exists more than one but finite number of B's such that $AB = BA$.

(C) there exists exactly one B such that $AB = BA$.

(D) there exists infinitely marry B 's such that $AB = BA$.

8. Let $A = \begin{bmatrix} 5 & 5\alpha & \alpha \\ 0 & \alpha & 5\alpha \\ 0 & 0 & 5 \end{bmatrix} \cdot$ If $|A^2| = 25$, then $|\alpha|$ equals : **[AIEEE 2007]**

(A) 5^2 (B) 1 (C) 1/5 (D) 5

9. Let A be a 2×2 matrix with real entries. Let I be the 2×2 identity matrix. Denote by tr(A), the sum of diagonal entires of A. Assume that $A^2 = 1$. **[AIEEE 2008]**

Statement 1 : If $A \neq I$ and $A \neq -I$, then $\det A = -1$.

Statement 2: If $A \neq I$ and $A \neq -I$, then tr $(A) \neq 0$.

(A) Statement 1 is false, statement 2 is true.

(B) Statement 1 is true, statement 2 is true; statement 2 is a correct explanation for statement 1.

(C) Statement 1 is true, statement 2 is true, statement 2 is not a correct explanation for statement 1.

(D) Statement 1 is true, statement 2 is false.

10. Let A be a 2×2 matrix. **[AIEEE 2009]**

Statement 1: adj.(adj A) $= A$

Statement 2: $|\text{adj } A| = |A|$

(A) Statement 1 is true, statement 2 is true; statement 2 is a correct explanation for statement 1.

(B) Statement 1 is true, statement 2 is true; statement 2 is not a correct explanation for statement 1.

(C) Statement 1 is true, statement 2 is false.

(D) Statement 1 is false, statement 2 is true.

11. The number of 3×3 non-singular matrices with four entries as 1 and all other entries as 0 is: **[AIEEE 2010]**

(A) at least 7 (B) less than 4 (C) 5 (D)

12. Let A be a 2×2 matrix with non-zero entries and let $A^2 = I$, where I is a 2×2 identity matrix. Define Tr $(A) =$ sum of diagonal elements of A and $|A| =$ determinant of matrix A. **[AIEEE 2010]**

Statement 1: Tr $(A) = 0$

Statement 2: $|A| = 1$

(A) Statement 1 is false, statement 2 is true.

(B) Statement 1 is true, statement 2 is true; statement 2 is a correct explanation for statement 1.

(C) Statement 1 is true, statement 2 is true; statement 2 is not a correct explanation for statement 1.

(D) Statement 1 is true, statement 2 is false.

13. Let A and B two symmetric matrices of order 3. **[AIEEE 2011]**

Statement 1 : A(BA) and (AB)A are symmetric matrices

Statement 2: AB is symmetric matrix if matrix multiplication of A with B is commutative.

(A) Statement 1 is false, statement 2 is true.

(B) Statement 1 is true, statement 2 is true; statement 2 is a correct explanation for statement 1.

(C) Statement 1 is true, statement 2 is true; statement 2 is not a correct explanation for statement 1.

(D) Statement 1 is true, statement 2 is false.

14. Let $A = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{pmatrix}$. If u_1, and u_2 are column matrices such that $Au_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ and $Au_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$, then $u_1 + u_2$ is equal to: **[AIEEE 2012]**

(A) $\begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$ (B) $\begin{pmatrix} -1 \\ 1 \\ -1 \end{pmatrix}$ (C) $\begin{pmatrix} -1 \\ -1 \\ 0 \end{pmatrix}$ (D) $\begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix}$

15. Let P and Q be 3×3 matrices $P \neq Q$. If $P^3 = Q^3$ and $P^2Q = Q^2P$, then determinant of $(P^2 + Q^2)$ is equal io : **[AIEEE 2012]**

(A) -2 (B) 1 (C) 0 (D) -1

16 If $P = \begin{bmatrix} 1 & \alpha & 3 \\ 1 & 3 & 3 \\ 2 & 4 & 4 \end{bmatrix}$ is the adjoint of 3×3 matrix A and $|A| = 4$, then α is equal to:

[JEE Main - 2013]

(A) 0 (B) 4 (C) 11 (D) 5

17. If A is an 3×3 non-singular matrix such that $AA' = A'A$ and $B = A^{-1} A'$, then BB' equals

[JEE Main - 2014]

(A) $(B^{-1})'$ (B) $I + B$ (C) I (D) B^{-1}

18. If $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ a & 2 & b \end{bmatrix}$ is a matrix satisfying the equation $AA^T = 9I$, where I is 3×3 identity matrix, then the ordered pair (a, b) is equal to: **[JEE Main = 2015]**

(A) $(-2,-1)$ (B) $(2,-1)$ (C) $(-2,1)$ (D) $(2,1)$

19. If $A = \begin{bmatrix} 5a & -b \\ 3 & 2 \end{bmatrix}$ and $A \text{ adj } A = AA^T$, then $5a + b$ is equal to: **[JEE Main-2016]**

(A) -1 (B) 5 (C) 4 (D) 13

20. If $A = \begin{bmatrix} 2 & -3 \\ -4 & 1 \end{bmatrix}$, then adj $(3A^2 + 12A)$ is equal to: **[JEE Main 2017]**

(A) $\begin{bmatrix} 51 & 84 \\ 63 & 72 \end{bmatrix}$ (B) $\begin{bmatrix} 72 & -63 \\ -84 & 51 \end{bmatrix}$ (C) $\begin{bmatrix} 72 & -84 \\ -63 & 51 \end{bmatrix}$ (D) $\begin{bmatrix} 51 & 63 \\ 84 & 72 \end{bmatrix}$

21. Let A be a matrix such that $A \cdot \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$ is a scalar matrix and $|3A| = 108$. Then A^2 equals :

[JEE Main - 2018]

(A) $\begin{bmatrix} 4 & -32 \\ 0 & 36 \end{bmatrix}$

(B) $\begin{bmatrix} 36 & 0 \\ -32 & 4 \end{bmatrix}$

(C) $\begin{bmatrix} 4 & 0 \\ -32 & 36 \end{bmatrix}$

(D) $\begin{bmatrix} 36 & -32 \\ 0 & 4 \end{bmatrix}$

22. Suppose A is any 3×3 non-singular matrix and $(A - 3I)(A - 5I) = 0$, where $I = I_3$ and $O = O_3$. If $\alpha A + \beta A^{-1} = 4I$, then $\alpha + \beta$ is equal to : **[JEE Main - 2018]**

(A) 8 (B) 7 (C) 13 (D) 12

23 Let $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$ and $B = A^{20}$. Then the sum of the elements of the first column of B is

[JEE Main - 2018]

(A) 211 (B) 210 (C) 281 (D) 251

EXERCISE-V

1. If matrix $A = \begin{bmatrix} a & b & c \\ b & c & a \\ c & a & b \end{bmatrix}$ where a, b, c are real positive numbers, $abc = 1$ and $A^T A = I$, then find the value of $a^3 + b^3 + c^3$ **[JEE 2003, Mains-2 out of 60]**

2. If $A = \begin{bmatrix} \alpha & 2 \\ 2 & \alpha \end{bmatrix}$ and $|A^3| = 125$, then $\alpha =$

(A) ± 3 (B) ± 2 (C) ± 5 (D) 0

[JEE 2004(Ser)]

3. If M is a 3×3 matrix, where $M^T M = I$ and det (M) = I, then prove that det $(M - I) = 0$.

[JEE 2004, 2 out of 60]

4. $A = \begin{bmatrix} a & 1 & 0 \\ 1 & b & d \\ 1 & b & c \end{bmatrix}, B = \begin{bmatrix} a & 1 & 1 \\ 0 & d & c \\ f & g & h \end{bmatrix}, U = \begin{bmatrix} f \\ g \\ h \end{bmatrix}, V = \begin{bmatrix} a^2 \\ 0 \\ 0 \end{bmatrix}, x = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

If AX = U has infinitely marry solution, then prove that BX = V cannot have a unique solution If further afd $\neq 0$, then prove that BX = V has no solution. **[JEE 2004, 4 out of 60]**

5. $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & -2 & 4 \end{bmatrix}, I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $A^{-1} = \left[\frac{1}{6}(A^2 + cA + dI)\right]$, then the value of c and d are

(A) $-6, -11$ (B) 6, 11 (C) $-6, 11$ (D) 6, -11

[JEE 2005(Ser)]

6. If $P = \begin{bmatrix} \sqrt{3}/2 & 1/2 \\ -1/2 & \sqrt{3}/2 \end{bmatrix}, A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ and $Q = PAP^T$ and $x = P^T Q^{2005} P$, then x is equal to

(A) $\begin{bmatrix} 1 & 2005 \\ 0 & 1 \end{bmatrix}$ (B) $\begin{bmatrix} 4 + 2005\sqrt{3} & 6015 \\ 2005 & 4 - 2005\sqrt{3} \end{bmatrix}$

(C) $\frac{1}{4}\begin{bmatrix} 2 + \sqrt{3} & 1 \\ -1 & 2 - \sqrt{3} \end{bmatrix}$ (D) $\frac{1}{4}\begin{bmatrix} 2005 & 2 - \sqrt{3} \\ 2 + \sqrt{3} & 2005 \end{bmatrix}$

[JEE 2005 (Screening)]

Comprehension (3 questions)

[JEE 2006, 5 marks each]

7. $A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{bmatrix}$, U_1, U_2 and U_3 are columns matrices satisfying. $AU_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$; $AU_2 = \begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix}$, and

$AU_3 = \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$

U is 3×3 matrix whose columns are U_1, U_2, U_3 then answer the following questions

(a) The value of | U | is

(A) 3 (B) -3 (C) 3/2 (D) 2

(b) The sum of elements of U^{-1} is

(A) -1 (B) 0 (C) 1 (D) 3

(c) The value of $[3 \quad 2 \quad 0]U\begin{bmatrix}3\\2\\0\end{bmatrix}$ is

(A) 5 (B) 5/2 (C) 4 (D) 3/2

8. Match the statements / Expression in **Column-I** with the statements /Expressions in **Column-II** and indicate your answer by darkening the appropriate bubbles in the 4×4 matrix given in OMR.

	Column-I		**Column-II**
(A)	The minimum value of $\frac{x^2+2x+4}{x+2}$ is	(P)	0
(B)	Let A and B be 3×3 matrices of real numbers, where A is symmetric, B is skew-symmetric, and $(A+B)(A-B) = (A-B)(A+B)$. If $(AB)^t = (-1)^k AB$, where $(AB)^t$ is the transpose of the matrix AB, then the possible values of k are	(Q)	1
			[JEE 2008, 6]
(C)	Let $a = \log_3\log_3 2$. An integer k satisfying $1 < 2^{(-k+3^{-a})} < 2$, must be less than	(R)	2
(D)	If $\sin\theta = \cos\phi$, then the possible values of $\frac{1}{\pi}\left(\theta \pm \phi - \frac{\pi}{2}\right)$ are	(5)	3

Paragraph for Question Nos. 9 to 11

Let A be the set of all 3×3 symmetric matrices all of whose entries are either 0 or 1. Five of these entries are 1 and four of them are 0. **[JEE-2009]**

9. The number of matrices in A is

(A) 12 (B) 6 (C) 9 (D) 3

10 The number of matrices A in A for which the system of linear equations $A\begin{bmatrix}x\\y\\z\end{bmatrix} = \begin{bmatrix}1\\0\\0\end{bmatrix}$ has a unique solution, is

(A) less than 4 (B) at least 4 but less than 7

(C) at least 7 but less than 10 (D) at least 10

11. The number of matrices A in A for which the system of linear equations $A\begin{bmatrix}x\\y\\z\end{bmatrix} = \begin{bmatrix}1\\0\\0\end{bmatrix}$ is inconsistent, is

(A) 0 (B) more than 2 (C) 2 (D) 1

12 The number of 3×3 matrices A whose entries are either 0 or 1 and for which the system

$A\begin{bmatrix} x \\ y \\ z \end{bmatrix} - \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ has exactly two distinct solutions, is **[JEE-2010]**

(A) 0 (B) $2^9 - 1$ (C) 168 (D) 2

Paragraph for Questions 13 to 15

Let P be an odd prime number and T_p be the following set of 2×2 matrices :

$$T_p = \left\{A = \begin{bmatrix} a & b \\ c & a \end{bmatrix}; a, b, c = \{0,1,2 \dots, P - 11\}\right.$$

13. The number of A in T_p such that A is either symmetric of skew-symmetric or both, and det(A) divisible by p is

(A) $(p-1)^2$ (B) $2(p-1)$ (C) $(p-1)^2 + 1$ (D) $2p - 1$

14 The number of A in T_p such that the trace of A is not divisible by p but det (A) is divisible by p is [Note : The trace of a matrix is the sum of its dagonal ertries.]

(A) $(p-1)(p^2 - p + 1)$ (B) $p^3 - (p-1)^2$

(C) $(p-1)^2$ (D) $(p-1)(p^2 - 2)$

15. The number of A in T_p such that det (A) is not divisible by p is **[JEE-2010]**

(A) $2p^2$ (B) $p^3 - 5p$ (C) $p^3 - 3p$ (D) $p^3 - p^2$

Paragraph for question nos. 16 to 18

Let a, b and c be three real numbers satisfying $[a\ b\ c]\begin{bmatrix} 1 & 9 & 7 \\ 8 & 2 & 7 \\ 7 & 3 & 7 \end{bmatrix} = [0\ 0\ 0]$ .... (E)

[JEE-2011]

16. If the point P(a, b, c), with reference to (E), lies on the plane $2x + y + z = 1$, then the value of $7a + b + c$ is

(A) 0 (B) 12 (C) 7 (D) 6

17. Let ω be a solution of $x^3 - 1 = 0$ with Im (ω) > 0. If a = 2 with b and c satisfying (E), then the value of $\frac{3}{\omega^a} + \frac{1}{\omega^b} + \frac{3}{\omega^c}$ is equal to

(A) − 2 (B) 2 (C) 3 (D) -3

18 Let b = 6, with a and c satisfying (E). If α and β are the roots of the quadratic equation $ax^2 + bx + c = 0$, then $\sum_{n=0}^{\infty}\left(\frac{1}{\alpha} + \frac{1}{\beta}\right)^n$ is

(A) 6 (B) 7 (C) 6/7 (D)∞

19. Let M and N be two 3×3 non-singular skew symmetric matrices such that MN = NM. If P^T denotes the transpose of P, then $M^2N^2(M^TN)^{-1}(MN^{-1})^T$ is equal to

(A) M^2 (B) $-N^2$ (C) $-M^2$ (D) MN

20. Let $\omega \neq 1$ be a cube root of unity and S be the set of all non-singular matrices of the form $\begin{bmatrix} 1 & a & b \\ \omega & 1 & c \\ \omega^2 & \omega & 1 \end{bmatrix}$, where each of a, b, and c is either ω or ω^2. Then the number of distinct matrices in the set S is **[JEE-2011]**

(A) 2 (B) 6 (C) 4 (D) 8

21. Let M be a 3 × 3 matrix satisfying **[JEE-2011]**

$$M\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}, M\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}, \text{and } M\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 12 \end{bmatrix}.$$

Then the sum of the diagonal entries of M is:

22. Let $P = [a_{ij}]$ be a 3 × 3 matrix and let $Q = [b_{ij}]$ where $b_{ij} = 2^{i+j}a_{ij}$, for $1 \leq i, j \leq 3$. If the determinant of P is 2 , then the determinant of the matrix Q is **[JEE-2012]**

(A) 2^{10} (B) 2^{11} (C) 2^{12} (D) 2^{13}

23. P is 3 × 3 matrix such that $P^T = 2P + I$, where P^T is the transposes of P and I is the 3 × 3 identity matrix, then there exists a column matrix $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ such that **[JEE-2012]**

(A) $PX = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ (B) $PX = X$

(C) $PX = 2X$ (D) $PX = -X$

24. If the adjoint of 3 × 3 matrix P is $\begin{bmatrix} 1 & 4 & 4 \\ 2 & 1 & 7 \\ 1 & 1 & 3 \end{bmatrix}$, then the possible value(s) of the determinant of P is (are) **[JEE-2012]**

(A) −2 (B) -1 (C) 1 (D) 2

25. For 3 × 3 matrices M and N, which of the following statement(s) is (are) NOT correct? **[JEE Advanced - 2013]**

(A) N^TMN is symmetric or skew symmetric, according as M is symmetric or skew symmetric

(B) MN − NM is skew symmetric for all symmetric matrices M and N

(C) MN is symmeric for all symmetric matrices M and N

(D) (adj M) (adj N) = adj (MN) for all invertible matrices M and N

26. Let M be a 2 × 2 symmetric matrix with integer entries. Then M is invertible if

(A) the first column of M is the transpose of the second row of M **[JEE Advanced 2014]**

(B) the second row of M is the transpose of the first column of M

(C) M is a diagonal matrix with non-zero entries in the main diagonal

(D) the product of entries in the main diagonal of M is not the square of an integer

27. Let M and N be two 3×3 matrices such that $MN = NM$. Further, if $M \neq N^2$ and $M^2 = N^4$, then

(A) determinant of $(M^2 + MN^2)$ is 0 **[JEE Advanced 2014]**

(B) there is a 3×3 non-zero matrix v such that $(M^2 + MN^2)U$ is the zero matrix

(C) determinant of $(M^2 + MN^2) \geq 1$

(D) for a 3×3 matrix U, if $(M^2 + MN^2)U$ euqals the zero matrix then U is the zero matrix

28. Let X and Y be two arbitrary, 3×3, non-zero, skew-symmelric matrices and Z be an arbitrary 3×3, non-zero, symmetric matrix. Then which of the following matrices is(are) skew symmetric? **[JEE Advanced 2015]**

(A) $Y^3Z^4 - Z^4Y^3$ (B) $X^{44} + Y^{44}$

(C) $X^4Z^3 - Z^3X^4$ (D) $X^{23} + Y^{23}$

29. Let $P = \begin{bmatrix} 3 & -1 & -2 \\ 2 & 0 & \alpha \\ 3 & -5 & 0 \end{bmatrix}$, where $\alpha \in R$. Suppose $Q = [q_{ij}]$ is a matrix such that $PQ = kI$, where $k \in R, k \neq 0$ and I is the identity matrix of order 3. If $q_{23} = -\frac{k}{8}$ and $\det(Q) = \frac{k^2}{2}$, then **[JEE Advanced 2016]**

(A) $\alpha = 0, k = 8$ (B) $4\alpha - k + B = 0$

(C) $\det(P\,\text{adj}(Q)) = 2^9$ (D) $\det(Q\,\text{adj}(P)) = 2^{13}$

30. Let $P = \begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 16 & 4 & 1 \end{bmatrix}$ and I be the identity matrix of order 3 . If $Q = [q_{ij}]$ is a matrix such that $P^{50} - Q = I$, then $\frac{q_{31}+q_{32}}{q_{21}}$ equals **[JEE Advanced 2016]**

(A) 52 (B) 103 (C) 201 (D) 205

31. For a real number α, if the system $\begin{bmatrix} 1 & \alpha & \alpha^2 \\ \alpha & 1 & \alpha \\ \alpha^2 & \alpha & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$ of linear equations, has infinitely many solutions, then $1 + \alpha + \alpha^2 =$ **[JEE Advanced 2017]**

32. Which of the following is (are) NOT the square of a 3×3 matrix with real entries ? **[JEE-Advanced-2017]**

(A) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ (B) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ (C) $\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ (D) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

33. Let S be the set of all column matrices $\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$ such that $b_1, b_2, b_2 \in R$ and the system of equations (in real variables) **[JEE Advanced 2018]**

$-x + 2y + 5z = b_1$, $2x - 4y + 3z = b_2$, $x - 2y + 2z = b_3$

has at least one solution. Then, which of the following system(s) (in real variables) has (have) at least one solution for each $\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} \in S$?

(A) $x + 2y + 3z = b_1, 4y + 5z = b_2$ and $x + 2y + 6z = b_3$

(B) $x + y + 3z = b_1, 5x + 2y + 6z = b_2$ and $-2x - y - 3z = b_3$

(C) $-x + 2y - 5z = b_1, 2x - 4y + 10z = b_2$ and $x - 2y + 5z = b_3$

(D) $x + 2y + 5z = b_1, 2x + 3z = b_2$ and $x + 4y - 5z = b_3$

34. Let P be a matrix of order 3×3 such that all the entries in P are from the set $\{-1,0,1\}$. Then, the maximum possible value of the determinant of P is **[JEE Advanced 2018]**

35. Let $M = \begin{bmatrix} \sin^4\theta & -1-\sin^2\theta \\ 1+\cos^2\theta & \cos^4\theta \end{bmatrix} = \alpha I + \beta M^{-1}$

Where $\alpha = \alpha(\theta)$ and $\beta = \beta(\theta)$ are real numbers, and I is the 2×2 identity matrix. If

α^* is the minimum of the set $\{\alpha(\theta): \theta \in [0,2\pi)\}$ and

β^* is the minimum of the set $\{\beta(\theta): \theta \in [0,2\pi]$,

then the value of $\alpha^* + \beta^*$ is: **[JEE Advanced 2019]**

(A) $-\frac{29}{16}$ (B) $-\frac{37}{16}$ (C) $-\frac{17}{16}$ (D) $-\frac{31}{16}$

36. Let $M = \begin{bmatrix} 0 & 1 & a \\ 1 & 2 & 3 \\ 3 & b & 1 \end{bmatrix}$ and adj $M = \begin{bmatrix} -1 & 1 & -1 \\ B & -6 & 2 \\ -5 & 3 & -1 \end{bmatrix}$

Where a and b are real numbers. Which of the following options is/are correct?

[JEE Advanced 2019]

(A) $(\text{adj } M)^{-1} + \text{adj } M^{-1} = -M$

(B) If $M\begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, then $\alpha - \beta + \gamma = 3$

(C) $\det(\text{adj } M^2) = 81$

(D) $a + b = 3$

37. Let $x \in R$ and let **[JEE Advanced 2019]**

$$P = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 2 \\ 0 & 0 & 3 \end{bmatrix}, Q = \begin{bmatrix} 2 & x & x \\ 0 & 4 & 0 \\ x & x & 6 \end{bmatrix} \text{ and } R = PQP^{-1}.$$

Then which of the following options is/are correct?

(A) There exists a real number x such that $PQ = QP$

(B) $\det R = \det \begin{bmatrix} 2 & x & x \\ 0 & 4 & 0 \\ x & x & 5 \end{bmatrix} + 8$, for all $x \in R$

(C) For $x = 0$, if $R\begin{bmatrix} 1 \\ a \\ b \end{bmatrix} = 6\begin{bmatrix} 1 \\ a \\ b \end{bmatrix}$, then $a + b = 5$

(D) For $x = 1$, there exists a unit vector $\alpha\hat{i} + \beta\hat{j} + \gamma\hat{k}$ for which $R\begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

38. Let **[JEE Advanced 2019]**

$$P_1 = I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, P_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}, P_3 = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

$$P_4 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}, P_5 = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, P_6 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \text{ and } X = \sum_{k=1}^{6} P_k \begin{bmatrix} 2 & 1 & 3 \\ 1 & 0 & 2 \\ 3 & 2 & 1 \end{bmatrix} P_k^T$$

Where P_k^T denotes the transpose of the matrix P_k. Then which of the following options is/are correct?

(A) $X - 30I$ is an invertible matrix

(B) X is a symmetric matrix.

(C) if $X\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \alpha \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$, then $\alpha = 30$

(D) The sum of diagonal entries of X is 18

39. Let M be a 3×3 invertible matrix with real entries and let I denote the 3×3 identity matrix. If $M^{-1} = \text{adj}(\text{adj } M)$, then which of the following statements is/are ALWAYS TRUE?

[JEE Advanced 2020]

(A) $M = I$

(B) $\det\ M = I$

(C) $M^2 = I$

(D) $(\text{adj}\ \ M)^2 = I$

40. The trace of a square matrix is defined to be the sum of its diagonal entries. If A is a 2×2 matrix such that the trace of A is 3 and the trace of A^3 is -18 , then the value of the determinant of A is **[JEE Advanced 2020]**

41. For any 3×3 matrix M, let $|M|$ denote the determinant of M. Let

$$E = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 8 & 13 & 18 \end{bmatrix}, P = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \text{ and } F = \begin{bmatrix} 1 & 3 & 2 \\ 8 & 18 & 13 \\ 2 & 4 & 3 \end{bmatrix}$$

If Q is a nonsingular matrix of order 3×3, then which of the following statements is (are) TRUE? **[JEE Advanced 2021]**

(A) $F = PEP$ and $P^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(B) $|EQ + PFQ^{-1}| = |EQ| + |PFQ^{-1}|$

(C) $|(EF)^3| > |EF|^2$

(D) Sum of the diagonal entries of $P^{-1}EP + F$ is equal to the sum of diagonal entries of $E + P^{-1}FP$

42. For any 3×3 matrix M, let $|M|$ denote the determinant of M. Let I be the 3×3 identity matrix. Let E and F be two 3×3 matrices such that $(I - EF)$ is invertble. If $G = (I - EF)^{-1}$, then which of the following statements is (are) TRUE? **[JEE Advanced 2021]**

(A) $|FE| = |I - FE||FGE|$

(B) $(I - FE)(I + FGE) = I$

(C) $EFG = GEF$

(D) $(I - FE)(I - FGE) = I$

ANSWER KEY

PROFICIENCY TEST-01

1.	B	**2.**	B	**3.**	C	**4.**	D	**5.**	A	**6.**	D	**7.**	C
8.	B	**9.**	B	**10.**	D	**11.**	B	**12.**	D	**13.**	B	**14.**	B
15.	D												

PROFICIENCY TEST-02

1.	A	**2.**	C	**3.**	D	**4.**	B	**5.**	B	**6.**	A	**7.**	C
8.	C	**9.**	B	**10.**	C	**11.**	B	**12.**	B	**13.**	D	**14.**	C
15.	B												

EXERCISE-I

1. 8 **2.** $x = \frac{3}{2}, y = 2$ **3.** 5049 **4.** $V = \begin{bmatrix} 0 \\ \frac{1}{11} \end{bmatrix}$ **6.** 1

7. $f(a) = 1/4, a = 1/2$ **9.** $\begin{bmatrix} 1 & 0 \\ 4014 & 1 \end{bmatrix}$

10. $\left(-\frac{4\sqrt{2}}{3}, \frac{2}{3}, 2\sqrt{2}\right), \left(\frac{4\sqrt{2}}{3}, \frac{2}{3}, -2\sqrt{2}\right), (3,3,-1)$ **11.** 2

12. AB is neither symmetric nor skew symmetric **13.** 4 **14.** $n = 7$

15. 650

EXERCISE-II

1. 225 **2.** $\begin{bmatrix} 17 & 4 & -19 \\ -10 & 0 & 13 \\ -21 & -3 & 25 \end{bmatrix}$ **3.** $\begin{bmatrix} -4 & 7 & -7 \\ 3 & -5 & 5 \end{bmatrix}$ **4.** 3/2

6. (i) $x = 2, y = 1, z = 0$;

(ii) $x = 2 + k, y = 1 - 2k, z = k$ where $k \in R$;

(iii) inconsistent, hence no solution

7. 1 **8.** $\frac{1}{19}\begin{bmatrix} 48 & -25 \\ -70 & 42 \end{bmatrix}$ **9.** $\frac{1}{kn - \ell m}\begin{bmatrix} n & -m \\ -\ell & k \end{bmatrix}$

10. (i) $X = \begin{bmatrix} a & b \\ 2 - 2a & 1 - 2b \end{bmatrix}$ for $a, b \in R$; (ii) X does not exist ;

(iii) $X = \begin{bmatrix} a & -3a \\ c & -3c \end{bmatrix}$ $a, c \in R$ and $3a + c \neq 0; 3b + d \neq 0$

11. $X = \begin{bmatrix} -2c & -2d \\ c & d \end{bmatrix}$, where $c, d \in R - \{0\}$, NO

12. (i) $a \neq -3, b \in R$; (ii) $a = -3$ and $b \neq 1/3$; (iii) $a = -3, b = 1/3$

13. (a) $X = \begin{bmatrix} -3 & -3 \\ 5/2 & 2 \end{bmatrix}$, (b) $X = \begin{bmatrix} 1 & 2 \\ -1 & -2 \end{bmatrix}$, (c) no solution

15. (A) Q; (B) S; (C) P; (D) P

EXERCISE-III

1.	D	**2.**	D	**3.**	D	**4.**	D	**5.**	B	**6.**	C	**7.**	B
8.	D	**9.**	D	**10.**	C								

EXERCISE-IV

1.	B	**2.**	B	**3.**	B	**4.**	D	**5.**	A	**6.**	B	**7.**	D
8.	C	**9.**	D	**10.**	B	**11.**	A	**12.**	D	**13.**	C	**14.**	D
15.	C	**16.**	C	**17.**	C	**18.**	A	**19.**	B	**20.**	D	**21.**	D
22.	A	**23.**	C										

EXERCISE-V

1.	4	**2.**	A	**5.**	C	**6.**	A						

7. (a) A, (b) B, (c) A **8.** (A) R (B) Q, S (C) R, S (D) P, R

9.	A	**10.**	B	**11.**	B	**12.**	A	**13.**	D	**14.**	C	**15.**	D
16.	D	**17.**	A	**18.**	B	**19.**	Bonus	**20.**	A	**21.**	9	**22.**	D
23.	D	**24.**	A, D	**25.**	C, D	**26.**	C, D	**27.**	A, B	**28.**	C, D	**29.**	B, C
30.	B	**31.**	1	**32.**	A, C	**33.**	A, D	**34.**	4	**35.**	A	**36.**	ABD
37.	BC	**38.**	BCD	**39.**	BCD	**40.**	5.00	**41.**	A, B, D	**42.**	A, B, C		