PROBLEM SET-02

SOLUTION

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Q.1 A student solves the equation $^{n}C_2 = 10$ using the following steps, but finds the solution yields decimal answer and therefore he must not be correct which step did he make the mistake?

(A) Step-1: $\frac{n!}{(n-2)!} = 10$

(B) Step-2: $n! = 10(n-2)!$

(C) Step-3: $n(n-1)(n-2)! = 10(n-2)!$

(D) Step-4: $n(n-1) = 10 \Rightarrow n^2 - n - 10 = 0$

Ans. **(A)**

Sol. $^{n}c_2 = 10$

$$\frac{n!}{(n-2)!\,2!} = 10$$

Q.2 Number of natural numbers between 100 and 1000 such that at least one of their digits is 7 , is

(A) 225 (B) 243 (C) 252 (D) none

Ans. **(C)**

Sol. $9 \cdot 10 \cdot 10 = 900$

(including 100)

Total no. of numbers without 7 $= 81 \times 8 = 648$ (including 100)

$\therefore$ required number $= 900 - 648 = 252$

Q.3 For some natural N, the number of positive integral ' x ' satisfying the equation,

$1! + 2! + 3! + \cdots \ldots + (x!) = (N)^2$ is

(A) none (B) one (C) two (D) infinite

Ans. **(C)**

Sol. Hit 'n' Trial

$x = 1, 3$

Q.4 All possible three digits even numbers which can be formed with the condition that if 5 is one of the digit, then 7 is the next digit is:

(A) 5 (B) 325 (C) 345 (D) 365

Ans. **(D)**

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Sol. $5 + 8.9.5 = 365$; 1^{st} place is five + where 1^{st} place is not five $\boxed{5}\boxed{7}\boxed{5}$ + $\boxed{8}\boxed{9}\boxed{5}$

Q.5 How many of the 900 three digit numbers have at least one even digit?

(A)775 (B) 875 (C) 450 (D) 750

Ans. **(A)**

Sol. There are 900 three digit numbers and there are five odd digits. Thus, there are $5^3 = 125$ three digit numbers comprised of only odd digits. The other $900 - 125 = 775$ three digit numbers must contain at least one even digit.

Q.6 The number of different seven digit numbers that can be written using only three digits 1,2&3 under the condition that the digit 2 occurs exactly twice in each number is :

(A) 672 (B) 640 (C) 512 (D) none

Ans. **(A)**

Sol. Two blocks for filling 2 can be selected in $^7C_2$ ways and the digit 2 can be filled only in one way other 5 blocks can be filled in 2^5 ways.

Q.7 Out of seven consonants and four vowels, the number of words of six letters, formed by taking four consonants and two vowels is (Assume that each ordered group of letter is a word):

(A) 210 (B) 462 (C) 151200 (D) 332640

Ans. **(C)**

Sol. **7** consonants **4** Vowels

$^7c_1 \cdot {^4c_2} \cdot 6!$

Q.8 Find the number of natural numbers less than 1000 and divisible by 5 can be formed with the ten digits, each digit not occurring more than once in each number.

Ans. **(154)**

Sol. single digit = 1

two digit: $\boxed{}\boxed{5} = 8 + \boxed{}\boxed{0} = 9 \Rightarrow$ Total = 17

three digit: $\boxed{}\boxed{}\boxed{5} = 8 \cdot 8 = 64$; $\boxed{}\boxed{}\boxed{0} = 9 \cdot 8 = 72 \Rightarrow$ Total $= 136$

hence Single digit + two digit + three digit = 154

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Q.9 The set of values of r simultaneously satisfying the system of equations $P(5, r) = 2 \cdot P(6, r-1)$ and $5 \cdot P(4, r) = 6 \cdot P(5, r-1)$, is

(A) an empty set (B) a singleton set

(C) a set consisting of two elements (D) a set consisting of three elements.

Ans. **(B)**

Sol. ${}^5P_r = 2 \cdot {}^6P_{r-1}; \frac{5!}{(5-r)!} = 2 \cdot \frac{6!}{(7-r)!} = \frac{2 \cdot 6 \cdot 5!}{(7-r)(6-r)(5-r)!}$

$\therefore\ (6-r)(7-r) = 12 \Rightarrow 42 - 13r + r^2 = 12$

$r^2 - 13r + 30 = 0 \Rightarrow (r-10)(r-3) = 0 \Rightarrow r = 10$ (rejected) or 3

again $5 \cdot {}^4P_r = 6 \cdot {}^5P_{r-1}$

$$5 \cdot \frac{4!}{(4-r)!} = 6 \cdot \frac{5!}{(6-r)!} = \frac{6 \cdot 5 \cdot 4!}{(6-r)(5-r)(4-r)!}$$

$30 - 11r + r^2 = 6 \Rightarrow r^2 - 11r + 24 = 0 \Rightarrow (r-8)(r-3) = 0$

$r = 3$ or $r = 8$ (rejected)

Hence $r = 3$ satisfying both.

Q.10 Let P_n denotes the number of permutations of n distinct things taken all at a time and $x_n = {}^{n+5}C_4 - \left(\frac{143}{96}\right)\left(\frac{P_{n+5}}{P_{n+3}}\right)$ (where $n \in N$). The possible value of n for which x_n is negative, can be

(A)1 (B)2 (C) 3 (D) 4

Ans. **(A, B, C)**

Sol. $x_n = {}^{n+5}C_4 - \left(\frac{143}{96}\right)\left(\frac{P_{n+5}}{P_{n+3}}\right)$

given $x_n < 0$ hence ${}^{n+5}C_4 < \frac{143}{96} \cdot \frac{(n+5)!}{(n+3)!}$

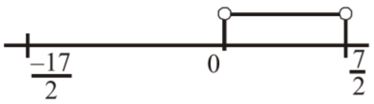
$$\frac{(n+5)!}{4!\,(n+1)!} < \frac{143}{96} \cdot \frac{(n+5)!}{(n+3)(n+2)(n+1)!}$$

$$\frac{1}{24} < \frac{143}{96} \cdot \frac{1}{(n+3)(n+2)}$$

$\Rightarrow\ 4(n+3)(n+2) < 143 \Rightarrow 4(n^2 + 5n + 6) - 143 < 0 \Rightarrow 4n^2 + 20n - 119 < 0$

$\Rightarrow\ 4n^2 + 34n - 14n - 119 < 0 \Rightarrow 2n(2n+17) - 7(2n+17) < 0$

$\Rightarrow\ (2n+17)(2n-7) < 0$



$n \in \{1,2,3\}$ Ans.

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Q.11 10 people are sitting around a circular table, each one shaking a hand with everyone else except from the people sitting on either side of him. Find the number of handshakers.

Ans. **(35)**

Sol. Each person shakes hand with 7 others.

$\therefore$ number of hand shakes $= 10 \cdot 7 = 70$.

but this is exactly twice (think!)

$\therefore$ Correct handshakes $= \frac{1}{2} \times 70 = 35$. Ans.