Nishant Jindal

$\boxed{1-11} \rightarrow Ex-I \cdot (TE)$

P.T.

1. $a\sin(B-C) + b\sin(C-A) + c\sin(A-B) = 0$.

$\pi-(B+C)$

$= \sum 2R\sin A\sin(B-C) = 2R\sum \sin(B+C)\sin(B-C) = 2R\sum(\sin^2 B - \sin^2 C)$

$= 0$.

2. $\sin(B-C) = \frac{b^2-c^2}{a^2}\sin A$.

$\frac{b^2-c^2}{a^2} = \frac{\sin^2 B - \sin^2 C}{\sin^2 A} = \frac{\sin(B-C)\sin(B+C)}{\sin^2 A}$

3. $a\cos\frac{B-C}{2} = (b+c)\sin\frac{A}{2}$

$\frac{a}{b+c} = \frac{\sin A}{\sin B + \sin C} = \frac{2\sin\frac{A}{2}\cancel{\cos\frac{A}{2}}}{2\cancel{\sin\frac{B+C}{2}}\cos\frac{B-C}{2}} = \frac{\sin(B-C)}{\sin A}$

$= \frac{\sin\frac{A}{2}}{\cos\left(\frac{B-C}{2}\right)}$.

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4. $(b^2-c^2)\cot A+(c^2-a^2)\cot B+(a^2-b^2)\cot C=0$

$4R^2\sum(\sin^2B-\sin^2C)\frac{\cos A}{\sin A}=4R^2\sum\sin(B-C)\cancel{\sin(B+C)}\frac{\cos A}{\cancel{\sin A}}$ $\cos A \to \pi-(B+C)$ $=-2R^2\sum 2\sin(B-C)\cos(B+C)$

$=-2R^2\sum(\sin 2B-\sin 2C)$

$=0.$

5. If a^2, b^2, c^2 are in A.P., then P.T. $\cot A, \cot B, \cot C$ are in A.P.

$$\sum\frac{(b^2-c^2)(b^2+c^2-a^2)}{2\boxed{bc\sin A}}=\frac{1}{4\Delta}\sum\left((b^4-c^4)-a^2(b^2-c^2)\right)$$

$=0.$

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$$\sin^2 B - \sin^2 A = \sin^2 C - \sin^2 B$$

$$\sin(B-A)\sin(B+A) = \sin(C-B)\sin(C+B)$$

$$\Rightarrow \frac{\sin(B-A)}{\sin A \sin B} = \frac{\sin(C-B)}{\sin C \sin B}$$

$$\cot A - \cot B = \cot B - \cot C$$

$$\Rightarrow$$

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6. In ΔABC, if $a\cos^2\frac{C}{2} + c\cos^2\frac{A}{2} = \frac{3b}{2}$, then P.T. a, b, c are in A.P.

$$\boxed{a+c=2b}$$

$$\frac{a}{2}(1+\cos C) + \frac{c}{2}(1+\cos A) = \frac{3b}{2}$$

$$\frac{a+c+b}{2} = \frac{3b}{2} \Leftarrow \frac{a}{2} + \frac{c}{2} + \frac{1}{2}(a\cos C + c\cos A) = \frac{3b}{2}$$

7. If $b=\sqrt{3}$, $c=1$, $\boxed{A=30^\circ}$. Solve the triangle ABC

$$\frac{\sqrt{3}}{2} = \cos A = \frac{3+1-a^2}{2\sqrt{3}(1)}$$

$$4-a^2 = 3$$

$$\boxed{a=1}$$

$$C=30^\circ$$

$$\tan\frac{B-C}{2} = \frac{b-c}{b+c}\cot\frac{A}{2} = \left(\frac{\sqrt{3}-1}{\sqrt{3}+1}\right)(2+\sqrt{3}) = 1$$

$$\boxed{a=1} \quad C=30^\circ, \ B=120^\circ \Leftarrow \begin{aligned} B-C &= 90^\circ \\ B+C &= 150^\circ \end{aligned}$$