NEWTONS LAWS OF MOTION, FRICTION & UNIFORM CIRCULAR MOTION

THEORY –PROBLEMS & KEY-

Inertia:

- Inertia is the property of a body that prevents it from changing its state of rest or uniform motion, as well as its direction, by itself. It is the inability of a body to change its state of rest or of uniform motion or its direction by itself.
- Mass is a measure of inertia in translatory motion
- Heavier the mass, the larger the inertia & vice–versa.

Types of inertia: There are three types of inertia.

(i) Inertia of rest (ii) Inertia of motion and (iii) Inertia of direction.

Inertia of rest: It is the inability of a body to change its state of rest by itself.

Ex: When a bus is at rest and starts suddenly moving forward the passengers inside it will fall back.

Inertia of motion: It is the inability of a body to change its state of uniform motion by itself.

Ex: Passengers in a moving bus fall forward, when brakes are applied suddenly.

Inertia of direction: It is the inability of a body to change its direction of motion by itself.

Ex: When a bus takes a turn, passengers in it experience an outward force.

- When a person sitting in a moving train throws a coin vertically upwards, different outcomes are observed depending on the motion of the train:

  (i) If the train is accelerating, the coin falls behind the person.

  (ii) If the train is decelerating, the coin falls in front of the person.

  (iii) If the train is moving at a uniform velocity, the coin falls back into the person's hand. iv) If the train is at rest, the coin also falls back into the person's hand.

Newton's First Law (Law of Inertia)

- Every object remains at rest or in uniform motion in a straight line unless acted upon by an external force, according to Newton's first law.
- This law introduces the concepts of inertia, force, and mechanical equilibrium.
- If the net external force acting on an object is zero, the object's acceleration is also zero.

Linear momentum:

- Linear momentum is the product of the mass of a body and its velocity.

  $\vec{p} = m\vec{v}$

- Linear momentum is a vector. It has the same direction as the direction of velocity of the body.

  SI unit: kg ms^{-1}, CGS unit: g cm s^{-1}

- Dimensional Formula : MLT^{-1}

Change in momentum of a body in different cases

- Consider a body of mass *m* moving with velocity $\vec{v}_1$ and momentum $\vec{P}_i$. Due to a collision (or) due to the action of a force on it suppose its velocity changes to $\vec{v}_f$ and momentum changes to $\vec{P}_f$ in a small time interval Δt.

  Change in momentum of body = $\Delta\vec{P} = \vec{P}_f - \vec{P}_i$

  Where P_i = initial momentum

  P_f = final momentum

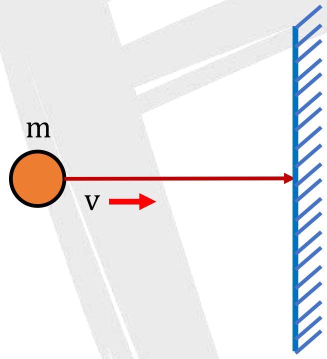
  $\Delta P = m\overline{v}_f - m\overline{v}_i$

  $$\left|\Delta\vec{P}\right| = \left|\vec{P}_f - \vec{P}_i\right| = \sqrt{P_f^2 + P_i^2 - 2P_fP_i\cos\theta}$$

  where = angle between $\vec{P}_f$ and $\vec{P}_i$

Consider a body of mass 'm' moving with velocity $'\vec{v}'$ along a straight line

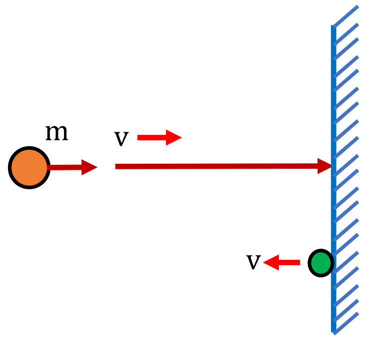
- **Case (i):** If it hits a wall and comes to rest, change in momentum of the body



$$\Delta\vec{P} = \vec{P}_f - \vec{P}_i = 0 - (m\vec{v})\hat{i}$$

$= -m\vec{v}\hat{i}$; $\left|\Delta\vec{P}\right| = mv$, along the normal and away from the wall.

- **Case (ii):** If the body rebounds with same speed 'v' then = 180^0

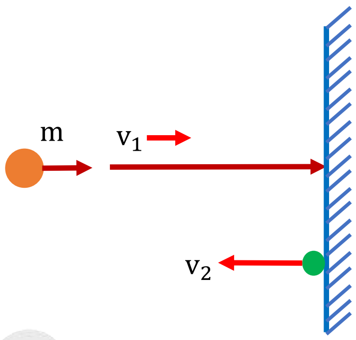


$$\Delta\vec{P} = \vec{P}_f - \vec{P}_i = \left\lfloor -(m\vec{v})\hat{i} \right\rfloor - \left\lfloor (m\vec{v})\hat{i} \right\rfloor = -(2m\vec{v})\hat{i}$$

$\therefore \left|\Delta\vec{P}\right| = 2mv$, along the normal and away from the wall.

- **Case (iii) :** If the body hits a rigid wall normally with speed v_1 and rebounds with speed v_2 then

  $\theta = 180^0$,



$\Delta\vec{P} = \vec{P}_f - \vec{P}_i$

$= \left\lfloor -(m\vec{v}_2)\hat{i} \right\rfloor - \left\lfloor (m\vec{v}_1)\hat{i} \right\rfloor$,

$\left|\Delta\vec{P}\right| = m(v_2 + v_1)$, along the normal and away from the wall.

- **Case (iv):** A body of mass 'm' moving the speed 'v' hits a rigid wall at an angle of incidence and rebounds with same speed 'v'

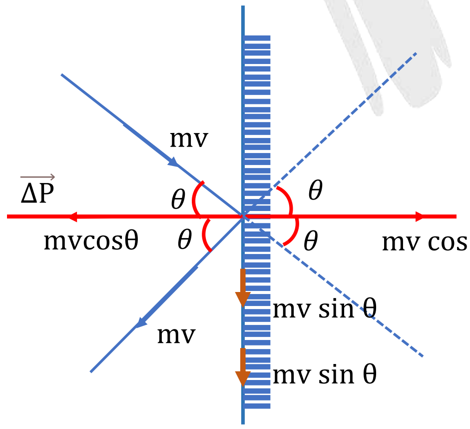
  $\Delta\vec{P}$ is along the normal, away from the wall

  $\Delta\vec{P}_x = -mv\cos\theta i - mv\cos\theta i$

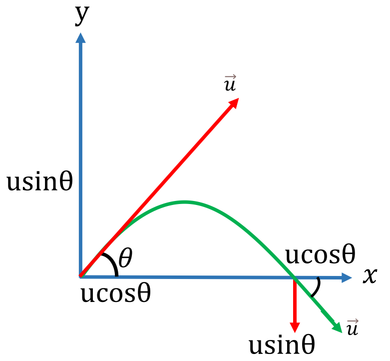
  $\Delta\vec{P}_y = mv\sin\theta\, j - mv\sin\theta\, j$

  $\Delta\vec{P} = \Delta\vec{P}_x + \Delta\vec{P}_y = 2mv\cos\theta(-i)$

  $\left|\Delta\vec{P}\right| = 2mv\cos$



- **Case (v):** In the above case if is the angle made with wall then $\left|\Delta\vec{P}\right| = 2mv\sin$, along the normal and away from the wall.
- **Case (vi): Projectile motion:**



(a) In case of projectile motion the change in momentum of a body between highest point and point of projection is

$$\vec{P}_i = (mu\cos\theta)\hat{i} + (mu\sin\theta)\hat{j}$$

$$\vec{P}_f = (mu\cos\theta)\hat{i} + 0,\ \overrightarrow{\Delta P} = -(mu\sin\theta)\hat{j}$$

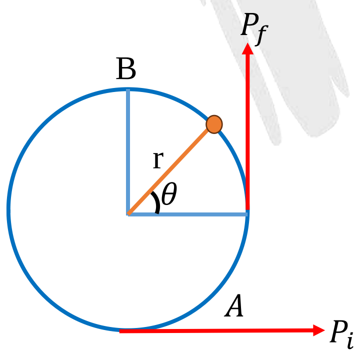
(b) The change in momentum of the projectile between the striking point and point of projection is

$$\vec{P}_i = (mu\cos\theta)\hat{i} + (mu\sin\theta)\hat{j}$$

$$\vec{P}_f = (mu\cos\theta)\hat{i} - (mu\sin\theta)\hat{j}$$

$$\overline{\Delta P} = -(2mu\sin\theta)\hat{j}$$

➢ A particle of mass 'm' is moving uniformly with a speed ' along a circular path of radius 'r'. As it moves from a point A to another point B, such that the arc AB subtends an angle θ at the centre, then the magnitude of change in momentum is 2 mv sin(θ /2) and is directed towards the centre of the circle.



Newton's second law:

- The rate of change of momentum of a body is directly proportional to the resultant (or) net external force acting on the body and takes place along the direction of force.

$$\vec{F}_{net} = \frac{d\vec{p}}{dt} \text{ (or) } \vec{F}_{net} = \frac{d(m\vec{v})}{dt}$$

- In a system if only velocity changes and mass remain constant, $\vec{F}_{net} = m\frac{d\vec{v}}{dt} = m\vec{a}$
- In a system, if only mass changes and velocity remains constant $\vec{F}_{net} = \vec{v}\frac{dm}{dt}$
- Force is a vector and the acceleration produced in the body is in the direction of net force,
- SI unit: newton (N). CGS unit: dyne.
  One newton = 10^5 dyne
- Dimensional Formula : MLT^{-2}

Gravitational units of force:

Kilogram weight (kg wt) and gram weight (gm wt); 1 kg.wt = 9.8N, 1 gm.wt= 980 dyne.

➢ A metallic plate of mass 'M' is kept held in mid air by n' bullets in 't' seconds each of mass 'm' with a velocity 'v' from below.

(a) If the bullet falls dead after hitting the plate, then $\frac{mnv}{t} = Mg$

(b) If the bullet rebounds after hitting the plate with same velocity, then $\frac{2mnv}{t} = Mg$

Applications of variable mass:

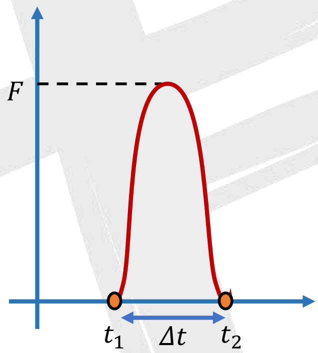
- When a machine gun fires *'n'* bullets each of mass *'m'* with a velocity *v* in a time interval *'t'* then force needed to hold the gun steadily is $F = \frac{nmv}{t}$
- When a jet of liquid coming out of a pipe strikes a wall normally and falls dead, then force exerted by
  the jet of liquid on the wall is $F=Adv^2$.
  A = Area of cross section of the pipe, v = Velocity of jet, d = density of the liquid.
- If the liquid bounces back with the same velocity, then the force exerted by the liquid on the wall is $F = 2Adv^2$
- If the liquid bounces back with velocity v' then the force exerted on the wall is $F = Adv(v+v')$
- When a jet of liquid strikes a wall by making an angle 'q' with the wall with a velocity 'v' and rebounds with same speed then force exerted by the water jet on wall is $F = 2Adv^2\sin\theta$
- If gravel is dropped on a conveyor belt at the rate of $\frac{dm}{dt}$, extra force required to keep the belt moving
  with constant velocity 'u' is $F = u\left(\frac{dm}{dt}\right)$

Impulse $(\vec{J})$:

- It is the product of impulsive force and time of action that produces a finite change in momentum of body.
- J=Ft = m(v-u) = change in momentum.

  SI unit: Ns **(or)**; $kg-ms^{-1}$; Dimensional Formula : MLT^{-1}
- It is a vector directed along the force
- change in momentum and Impulse are always in the same direction.
- For constant force, J=Ft,
- If impulsive force is a variable, then

  $$\vec{F}=\frac{d\vec{p}}{dt},\ J=\int_{t_1}^{t_2} Fdt$$
- The area bounded by the force-time graph measures Impulse.



Application of Impulse:

(a) shock absorbers are used in vehicles to reduce the magnitude of impulsive force.

(b) A cricketer lowers his hands, while catching the ball to reduce the impulsive force.

Equilibrium: The necessary and sufficient conditions for the translational equilibrium of the rigid body.

$\Sigma F=0; \sum F_y=0, \Sigma F_z=0$ For rotational equilibrium

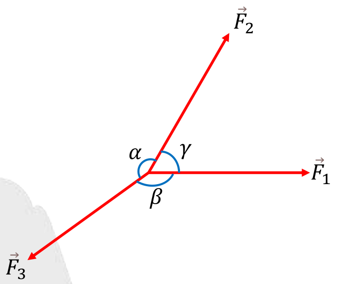
$\Sigma\tau=0; \sum\tau_x=0, \Sigma\tau_y=0, \Sigma\tau_z=0$

- As for, $\vec{F}=0\ m\vec{a}=0$ (or) $m(dv/dt)=0$ as $m\neq 0, \frac{d\vec{v}}{dt}=0$ (or) $\vec{v}=$ constant or zero
- A body in translatory equilibrium will either be stationary or in uniform motion. When it is stationary, the equilibrium is referred to as static, whereas if it is in motion, it is called dynamic equilibrium.

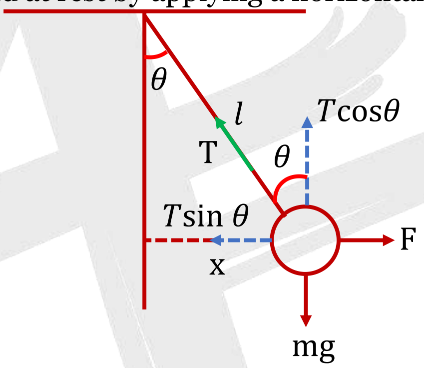
- If ' n 'coplanar forces of equal magnitudes acting simultaneously on a particle at a point, with the angle between any two adjacent forces is 'q' and keep it in equilibrium, then $\theta = \frac{360}{n}$

Lami's Theorem:

- If an object O is in equilibrium under three concurrent forces $\vec{F}_1, \vec{F}_2$ and $\vec{F}_3$ as shown in figure. Then,
  $\frac{F_1}{\sin\alpha} = \frac{F_2}{\sin\beta} = \frac{F_3}{\sin\gamma}$



- If the bob is simple pendulum is held at rest by applying a horizontal force 'F' as shown in fig.



If body is in equilibrium

$T\sin\theta = F, \qquad T\cos\theta = mg,$

$F = mg\tan\theta, \qquad \sqrt{F^2 + (mg)^2} = T$

$\frac{x}{F} = \frac{\ell}{T} = \sqrt{\frac{\ell^2 - x^2}{mg}}$

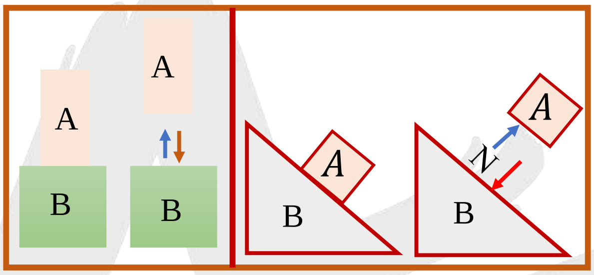
Newton's third law:

- Every action has an equal and opposite reaction.
- The action and reaction forces act on different bodies, not on the same body, at the same instant of time.
- Action and reaction forces, forming a pair of forces, have equal magnitudes but opposite directions. They do not cancel each other out.
- Newton's third law of motion is not applicable to pseudo forces.
- Newton's third law of motion defines the nature of forces and provides the law of conservation of linear momentum.

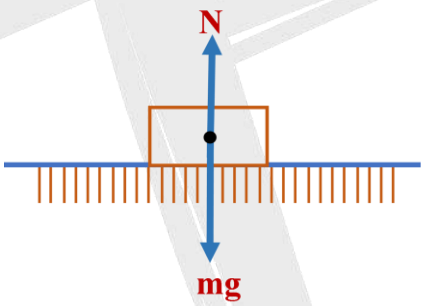
Limitations of newton's third law:-

- Newton's third law is not strictly applicable for the interaction between two bodies separated by large distances, such as astronomical units.
- It does not apply strictly when the objects involved in the interaction are moving at velocities close to the speed of light.
- Newton's third law does not apply in situations where the gravitational field is strong, such as near massive objects like black holes.

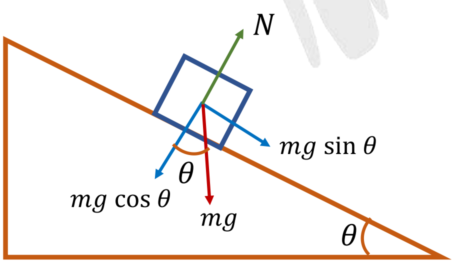
Normal reaction/force: The normal force is a contact force that acts perpendicular to the surfaces in contact. It occurs when one body exerts pressure on the surface of another body, causing the second body to push back in the opposite direction.



- When the body lies on a horizontal surface N = mg



- When the body lies on an inclined surface N = mg cos θ



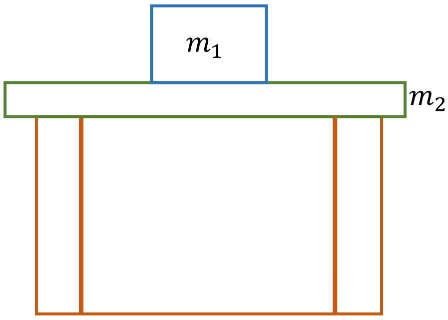
Free Body Diagram: -

A Free Body Diagram (FBD) is a visual representation of a body isolated from its surroundings, where all the forces acting on the body are considered and depicted. It includes forces from strings, springs, and contact surfaces. By focusing on the individual body, the FBD provides a clear illustration of the forces at play.

Some examples:

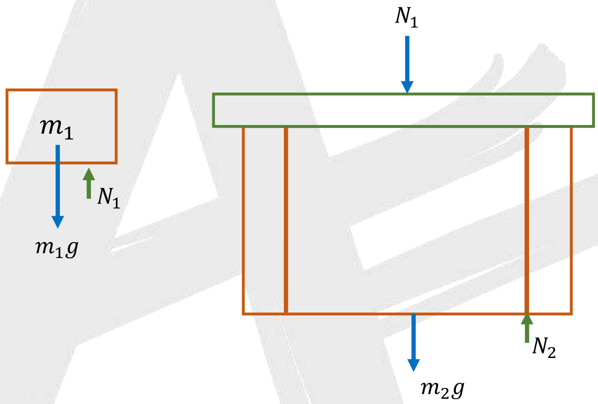
(i) A block is placed on a table and the table is kept on earth. Assuming no other body in the universe

exerts any force on the system, make the FBD of block and table.

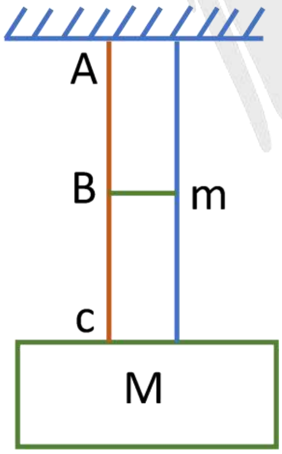


FDB of block, $N_1 = m_1 g$

FBD of table $N_2 = N_1 + m_2 g = m_1 g + m_2 g = (m_1 + m_2) g$



(ii) A block of mass *M* is suspended from the ceiling by means of a uniform string of mass *m*. Determine the tension in the string at points A, B and C. B is the midpoint of string. Also determine the tensions at A, B and C if the mass of string is negligible or it is massless.

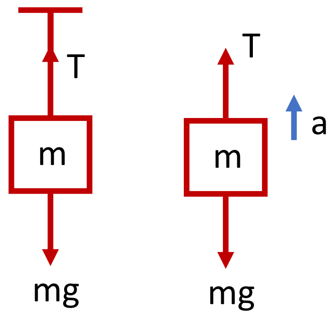


Tension at any point will be weight of the part below it.

So, $T_A = (M + m) g, T_B = \left(\frac{m}{2} + M \right) g, T_C = Mg$

Now if the string is massless: m=0 then $T_A = T_B = T_C = Mg$. So in a massless string, tension is the same at every point.

(iii) Find the tension in the massless string connected to the block accelerating upward.



Net force:

$F_{net} = T - mg$

Now apply $F_{net} = ma$

$\Rightarrow T - mg = ma \quad \Rightarrow \quad T = mg + ma = m(g + a)$

Note: If '*a*' is downward, then replace *a* with -*a*; we get $T = m(g - a)$

In free fall *a*=g then T=0.

Frames of Reference:

A frame of reference is a coordinate system used to determine the position of a particle or event in two- or three-dimensional space. Two types of frames of reference exist:

(a) Inertial or unaccelerated frames of reference

(b) Non-inertial or accelerated frames of reference

Inertial frames of reference:

(a) Frames of reference in which Newton's Laws of motion are applicable are called inertial frame.

(b) Inertial frames of reference are either at rest or move with uniform velocity with respect to a fixed imaginary axis.

(c) In inertial frame, acceleration of a body is caused by real forces.

(d) Equation of motion of mass '*m*' moving with acceleration '*a*' relative to an observer in an inertial frame is $\sum \vec{F}_{real} = ma$

Examples:

(1) A lift at rest,

(2) Lift moving up(or)down with constant velocity,

(3) Car moving with constant velocity on a straight road.

Real Force: Force acting on an object due to its interaction with another object is called a real force.

Ex: Normal force, Tension, weight, spring force, muscular force etc.

(a) All fundamental forces of nature are real.

(b) Real forces form action, reaction pair.

Non-Inertial frames:

(a) Non-inertial frames are frames of reference in which Newton's Laws are not applicable.

(B) Accelerated frames can have either uniform or non-uniform acceleration.

(c) All accelerated and rotating frames are examples of non-inertial frames of reference.

Examples:

(1) Accelerating car on a road.

(2) Merry go round.

(3) Artificial satellite around the earth.

Pseudo force:

(a) In non-inertial frames, Newton's second law is not applicable. To account for the effects of acceleration in these frames, a pseudo force is introduced. This pseudo force allows us to apply Newton's second law and analyze the motion in the non-inertial frame.

(b) If $\vec{a}$ is the acceleration of a non-inertial frame, the pseudo force acting on an object of mass m, as measured by an observer in the given non-inertial frame is $\vec{F}_{Pseudo} = -m\vec{a}$ i.e. Pseudo force acts on an object opposite to the direction of acceleration of the non-inertial frame.

(c) Pseudo forces are specific to observers in non-inertial frames and do not have any existence relative to an inertial frame. They are introduced to account for the effects of acceleration in the non-inertial frame and are not actual physical forces in themselves.

(d) Equation of motion relative to non-inertial frame is $\sum\left(\vec{F}_{real} + \vec{F}_{Pseudo}\right) = m\vec{a}'$

where a' is the acceleration of body as measured in non-inertial frame.

(e) Earth is an inertial frame for an observer on the earth but it is an accelerated frame for an observer at centre of earth (or) in a satellite.

Examples:

(i) Centrifugal force and deflection of pendulum relative to accelerating car.

(ii) Gain or loss of weight experienced in an accelerating elevator.

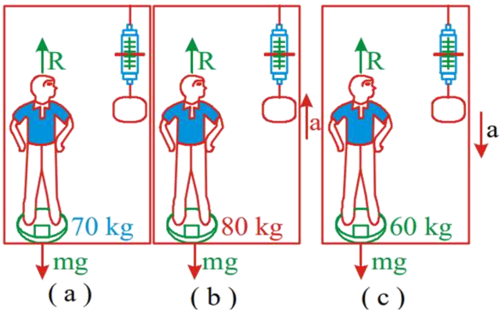
Apparent weight of a body in a moving elevator weight of a body on a surface comes due to the reaction of a supporting surface, i.e., apparent weight of a body in a lift.

W_{app} = Reaction of supporting surface. Consider a person standing on a spring balance, or in a lift. The following situations are possible:

Case (i): If lift is at rest or moving with constant velocity then the person will be in translatory equilibrium. So, R=mg

$\therefore W_{app} = mg \qquad [\text{as } W_{app} = R]$

or $\quad W_{app} = W_0 \;\; [\text{as } W_0 = mg = \text{true weight}]$



i.e., apparent weight (reading of balance) will be equal to true weight.

Case (ii) : If lift is accelerated up or retarding down with acceleration a from Newton's II law we have

$R - mg = ma$ or $R = m(g+a)$

or $W_{app} = m(g+a)$

$= mg\left[1+\frac{a}{g}\right] = W_0\left[1+\frac{a}{g}\right]$ or $W_{app} > W_0$

i.e., apparent weight (reading of balance) will be more than true weight.

Case (iii) : If lift is accelerated down or retarding up with acceleration 'a' mg-R = ma i.e.,

$R = m(g-a)$

or $W_{app} = m(g-a)$ $[\text{as } W_{app} = R]$ = $mg\left[1-\frac{a}{g}\right]$

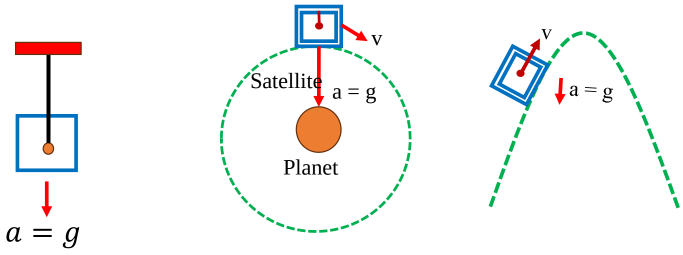
i.e., $W_{app} = W_0\left[1-\frac{a}{g}\right]$ $W_{app} < W_0$

i.e, apparent weight (reading of balance) will be lesser than true weight.

Note: If $a > g, W_{app}$ will be negative; negative weight will mean that the body is pressed against the roof of the lift instead of floor (as lift falls more faster than the body) and so the reaction will be downwards, the direction of apparent weight will be upwards.

Case (iv) : If lift is in freely falling, Then a = g,

So mg-R=mg i.e., R=0. So, $W_{app} = 0$



(a) (b) (c)

(a) Freely falling lift

(b) Satellite motion

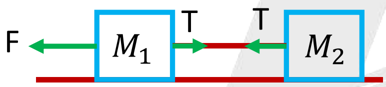
(c) Projectile motion

i.e., apparent weight of a freely falling body is zero. This is why the apparent weight of a body is zero, or body is weightless if it is in a (i) lift whose cable has broken, (ii) orbiting satellite.

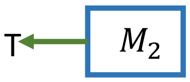
Connecting Bodies:

- If masses are connected by strings then acceleration of system and tension in the strings on smooth horizontal surface are

(a)

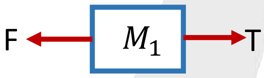


Free body diagram for M_2



$$T = M_2 a \qquad \text{... (1)}$$

Free body diagram for M_1

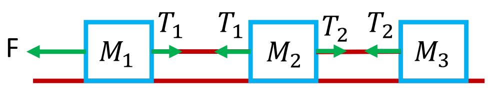


$$F - T = M_1 a \qquad \text{... (2)}$$

from (1) and (2)

$$a = \frac{F}{(M_1 + M_2)} \text{ and } T = \frac{M_2 F}{(M_1 + M_2)}$$

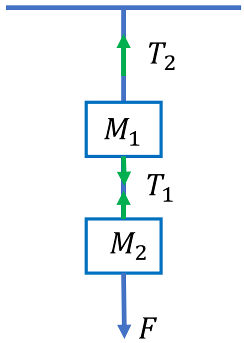
(b)



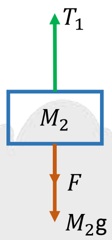
$$a = \frac{F}{M_1 + M_2 + M_3}; T_1 = \frac{(M_2 + M_3)F}{(M_1 + M_2 + M_3)}$$

$$T_2 = \frac{M_3 F}{(M_1 + M_2 + M_3)}$$

- If masses are connected by a string and suspended from a support then tension in the string when force F is applied downwards as shown in the figure

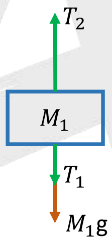


Free body diagram for M_2



$T_1 = F + M_2g$ ...... (1)

Free body diagram for M_1



$T_2 = T_1 + M_1g$ .... (2)

From (1) and (2), $T_2 = F + (M_1 + M_2)g$

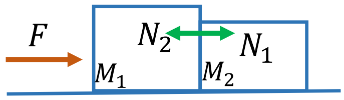
Contact Forces:

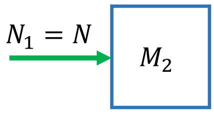
When two objects come into contact with each other, the molecules at the interface interact, giving rise to a net force known as the contact force. The contact force can be divided into two components:

(a) Normal force (N): This component of the contact force acts perpendicular to the interface. The normal force is independent of the nature of the surfaces in contact.

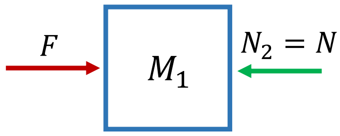
(b) Friction (f): This component of the contact force acts tangentially along the interface. Friction depends on the roughness of the surfaces in contact. The magnitude of friction can be reduced by polishing the surfaces.

- Masses are in contact on a smooth horizontal surface:



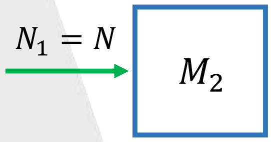


contact force $N_1 = N_2 = N = M_2a$ free body diagram for M_1



$F - N = M_1a$ .... (1)

free body diagram for M_2

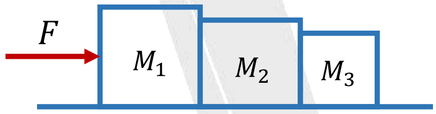


$N = M_2a$ .... (2)

From (1) and (2)

$a = \dfrac{F}{(M_1 + M_2)}$; contact force, $N = \dfrac{M_2F}{M_1 + M_2}$

- Contact forces are as shown in the figure



(a) Acceleration of system,

$a = \dfrac{F}{(M_1 + M_2 + M_3)}$

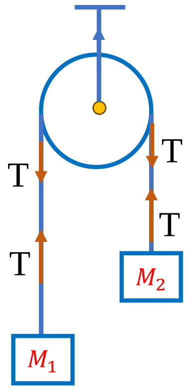
(b) Contact force between M_1 and M_2

$N = (M_2 + M_3)a$

(c) Contact force between M_2 and M_3, $N' = M_3a$

Atwood's Machine:

- Masses M_1 and M_2 $(M_1 > M_2)$ are tied to a string, which passes over a frictionless light pulley. The string is light and inextensible.



Acceleration of the system, $a = \dfrac{(M_1 - M_2)g}{M_1 + M_2}$

Tension in the string, $T = \left(\dfrac{2M_1M_2}{M_1 + M_2}\right) g$

Thrust on the pulley, $2T = \left(\dfrac{4M_1M_2}{M_1 + M_2}\right) g$

- If the pulley begins to move with acceleration $\vec{a}$ then

  (i) If the pulley accelerates upward, then

  $a_{net} = \left(\dfrac{M_1 - M_2}{M_1 + M_2}\right)(g + a)$ and $T_{net} = \left(\dfrac{2M_1M_2}{M_1 + M_2}\right)(g + a)$

  (ii) If the pulley accelerates downward, then

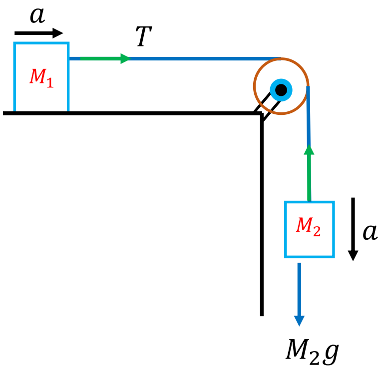
  $a_{net} = \left(\dfrac{M_1 - M_2}{M_1 + M_2}\right)(g - a)$ and

  $T_{net} = \left(\dfrac{2M_1M_2}{M_1 + M_2}\right)(g - a)$

➢ Thrust on the pulley when it comes downward with acceleration 'a' is

$$T = \frac{4M_1M_2}{(M_1 + M_2)}(g - a)$$

➢ Two blocks are connected by a string passing over a pulley fixed at the edge of a horizontal table then the acceleration of system and tension in the string $(M_2 > M_1)$

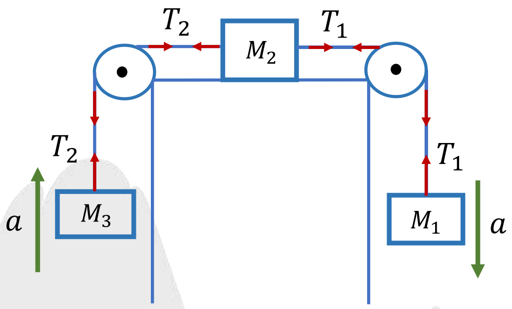


$$M_2g - T = M_2a \text{ and } T = M_1a$$

$$\Rightarrow a = \frac{M_2g}{(M_1 + M_2)}$$

$$T = M_1a = \frac{M_1M_2g}{(M_1 + M_2)}$$

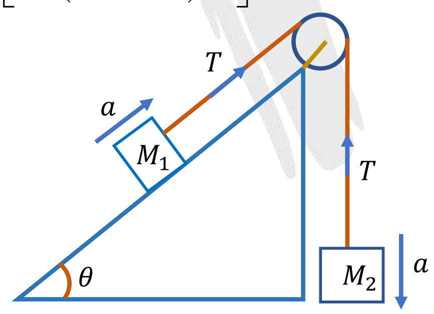
➢ Acceleration and Tension in the string when bodies are connected as shown in the figure if $M_1 > M_3$.



$$M_1g - T_1 = M_1a$$
$$T_1 - T_2 = M_2a$$
$$T_2 - M_3g = M_3a$$
$$\Rightarrow a = \frac{(M_1 - M_3)g}{(M_1 + M_2 + M_3)}$$
$$T_2 = \frac{M_3g(2M_1 + M_2)}{M_1 + M_2 + M_3}; \quad T_1 = \frac{M_1g(2M_3 + M_2)}{M_1 + M_2 + M_3}$$

➢ Masses are attached to a string passing through the pulley attached to the edge of an inclined plane, acceleration of system and tension in the string if M_2 moves down

$$a = \left(\frac{M_2 - M_1 \sin\theta}{M_1 + M_2}\right)g; \quad T = \left[\frac{M_1M_2(1 + \sin\theta)}{(M_1 + M_2)}\right]g$$



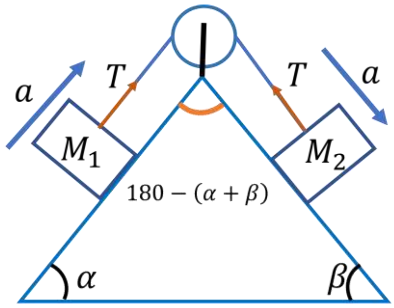
Thrust on the pulley :

Resultant Tension

$$T_g = \sqrt{T^2 + T^2 + 2T^2\cos(90 - \theta)}$$

$$T_g = \sqrt{2T^2(1 + \sin\theta)} = T\sqrt{2(1 + \sin\theta)}$$

➢ If position of masses is interchanged, then the tension in the string and acceleration remains unchanged. Acceleration and Tension in the string when bodies are connected as shown in the figure.



If M_2 slides down then M_1 moves up on smooth inclined planes then the acceleration of system and

tension in the string are given by, acceleration,

$$a = \left(\frac{M_2 \sin\beta - M_1 \sin\alpha}{M_1 + M_2}\right) g$$

$$\text{Tension } T = \frac{M_1 M_2 g}{(M_1 + M_2)}(\sin\alpha + \sin\beta)$$

Resultant Tension

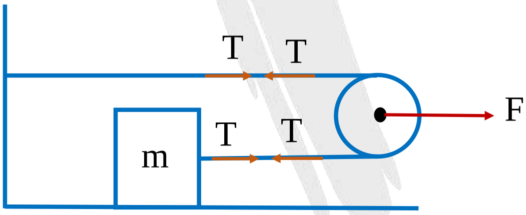
$$T_R = \sqrt{T^2 + T^2 + 2T^2 \cos\left[180 - (\alpha+\beta)\right]}$$

$$= \sqrt{2T^2\left[1 - \cos(\alpha+\beta)\right]}$$

Note: - If $M_2 \sin\beta = M_1 \sin\alpha \Rightarrow a = 0$

$\Rightarrow$ System does not accelerate

- A force F is applied on the massless pulley as shown in the figure and string is connected to the block on smooth horizontal surface. Then



$F = 2T$ and $T = ma_{block}$

- If the block moves a distance 'x' the pulley moves x/2 (Total length of the string remains constant)

Therefore, acceleration of the pulley = $\frac{a_{block}}{2}$

$$= \frac{T}{2m} = \frac{F/2}{2m} = \frac{F}{4m}$$

Constrained Motion:

(a) Constraint: Restriction to the free motion of body in any direction is called constraint.

(b) Constrained Body: A body, whose displacement in space is restricted by other bodies, either connected to or in contact with it, is called a constrained body.

(c) Kinematic Constraints: These are equations that relate the motion of two or more particles.

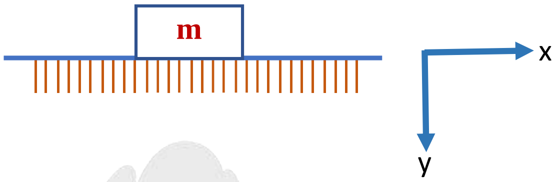
(d) Types of Constraints: i) General constraints ii) Pulley constraints

iii) Wedge constraints iv) Mixed constraints

General Constraints:

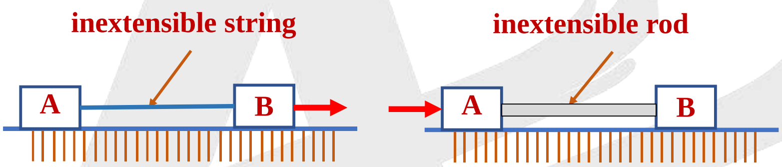
(i) A body placed on floor: The floor acting as a constraint restricts the kinematical quantities in

the downward direction such that



$y = 0$; $v_y = 0$ and $a_y = 0$ for the body placed on the floor.

(ii) Two bodies connected with a string or rod.



The string / rod is inextensible.

\ Displacements of A and B are equal in horizontal direction $\Rightarrow s_A = s_B$

Differentiating w.r.t time,

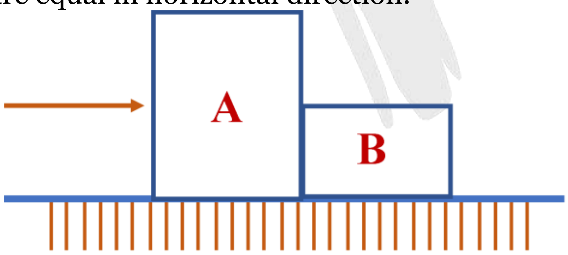
$$\frac{ds_A}{dt} = \frac{ds_B}{dt} \Rightarrow v_A = v_B$$

Again differentiating

$$\frac{dv_A}{dt} = \frac{dv_B}{dt} \Rightarrow a_A = a_B$$

(iii)Two bodies in contact with each other

- Displacement of A and B are equal in horizontal direction.



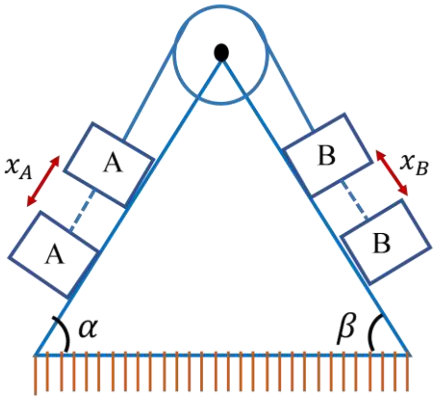
$\Rightarrow s_A = s_B$

By differentiating, we will get

$v_A = v_B$ and $a_A = a_B$ in horizontal direction

Pulley Constraints:

- For example, the motion of block A is downwards along the inclined plane in fig. will cause a corresponding motion of block B up the other inclined plane. Assuming string AB length is inextensible, i.e., length of AB is constant.

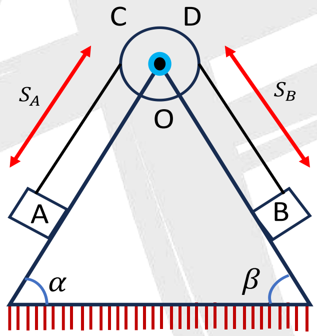


$\therefore$ The displacements of $A(x_A)$ and $B(x_B)$ are equal

$\therefore x_A = x_B$

Differentiating w.r.t. time, $\Rightarrow v_A = v_B$ i.e., if one body (A) moves down the inclined plane with certain acceleration, then the other body will move up inclined plane with an equal acceleration (magnitude).

Alternate Method: First specify the location of the blocks using position co-ordinates S_A and S_B.



From the fig. the position co-ordinates are related by the equations $s_A + \ell_{CD} + s_B = L$

where ℓ_{CD} = the length of the string over arc

CD = constant L= total length of the string = constant Differentiating w.r.t time, we get

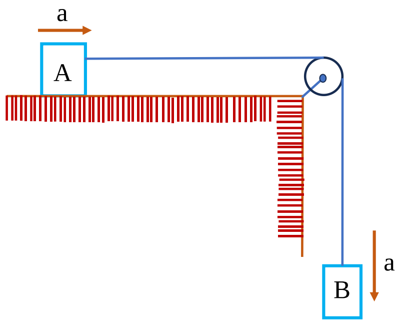
$$\frac{ds_A}{dt} + \frac{ds_B}{dt} = 0 \quad \Rightarrow \quad v_B = -v_A$$

The negative sign indicates that when block A has a velocity downward, i.e., in the direction of positive s_A ,it causes a corresponding upward velocity of block B, i.e., B moves in the negative s_B direction. Again differentiating w.r.t. time,

$$\frac{dv_B}{dt} = -\frac{dv_A}{dt} \Rightarrow a_B = -a_A$$

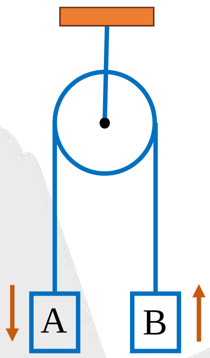
Similarly

(1)



$x_A = x_B \Rightarrow v_A = v_B \Rightarrow a_A = a_B$

2)



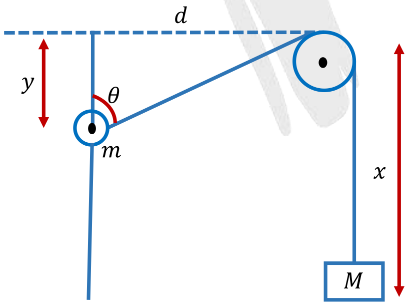
$x_A = x_B \Rightarrow v_A = v_B \Rightarrow a_A = a_B$

Mixed constraints:

Ring sliding on a smooth rod:

- Consider a ring of mass m connected through a string of length L with a block of mass M. If the ring is moving up with acceleration a_m and a_M is the acceleration of block. As the length of the string is constant,

  $L = \sqrt{d^2 + y^2} + x$



Since, L is constant, differentiating with respect to time t, we get

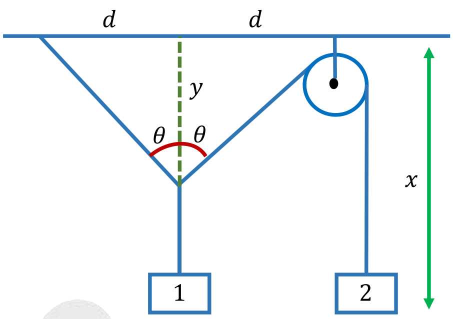
$$\frac{dL}{dt} = \frac{1}{2}\frac{2y}{\left(d^2+y^2\right)^{\frac{1}{2}}}\left(\frac{dy}{dt}\right) + \frac{dx}{dt} = 0$$

Since $\frac{dy}{dt} = v_m$ and $\frac{dx}{dt} = v_M$ and

$\cos\theta = \frac{y}{\sqrt{d^2+y^2}}$ so $v_M = -v_m\cos\theta$.

By differentiating, relation between a_m and a_M can be obtained, however, while doing so remember that $\cos\theta$ is not constant, but it is variable.

Two blocks connected with pulley: If the blocks are connected as shown in fig, then the length of the string is



$$L = 2\sqrt{d^2 + y^2} + x$$

Since, L is constant, differentiating with respect to time t, we get

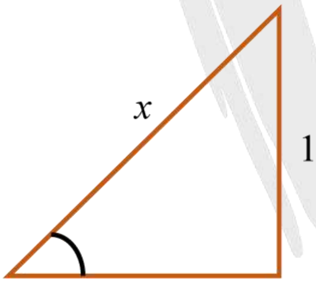
$$\frac{dL}{dt} = \frac{2 \times 2y}{2\left(d^2 + y^2\right)^{\frac{1}{2}}}\left(\frac{dy}{dt}\right) + \frac{dx}{dt} = 0$$

$$\Rightarrow 2v_1\cos\theta + v_2 = 0;\; v_2 = -2v_1\cos\theta$$

Note: If inclination is given as 1 in x, $\sin\theta = \frac{1}{x}$

$$\tan = \frac{1}{\sqrt{x^2 - 1}}$$

$$\Rightarrow \text{ Acceleration } a = g\tan = \frac{g}{\sqrt{x^2 - 1}}$$



Law of Conservation of Momentum:

- When the net external force acting on a system is zero, the total momentum of the system remains constant. This is known as the "law of conservation of linear momentum."
- The law of conservation of linear momentum is derived from Newton's third law of motion.
- Various phenomena such as walking, running, swimming, jet propulsion, rocket motion, rowing a boat, and gun recoil can be explained using Newton's third law of motion.
- Explosions, nuclear disintegration, gun recoil, and collisions can be understood based on the principle of conservation of linear momentum.

  Applications:

- When a shot is fired from a gun, while the shot moves forwards, the gun moves backwards. This motion of gun is called **recoil of the gun**. When a gun of mass 'M' fires a bullet of mass 'm' with a muzzle velocity 'v', the gun recoils with a velocity 'V' given by V = mv/M.
- When a bullet of mass 'm' moving with a velocity 'v' gets embedded into a block of mass M at rest and free to move on a smooth horizontal surface, then their common velocity` $V = mv/(M+m)$
- A boy of mass 'm' walks a distance 's' on a boat of mass 'M' that is floating on water and initially at rest. If the boat is free to move, it moves back a distance $d = ms/(M+m)$.

Explosion of Bomb

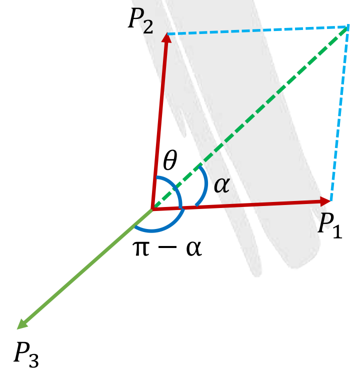
- A shell of mass 'M' at rest explodes into two fragments and one of masses 'm' moves out with a velocity 'v' the other piece of mass (M– m) moves in opposite direction with a velocity of V = mv / (M – m).
- Suppose a shell of mass *m* at rest explodes into three pieces of mass m_1, m_2 and m_3, moving with velocities $\vec{v}_1, \vec{v}_2$ and $\vec{v}_3$ respectively.

  $m_1\vec{v}_1 = \vec{p}_1; m_2\vec{v}_2 = \vec{p}_2; m_3\vec{v}_3 = \vec{p}_3$

  $\vec{p}_1 + \vec{p}_2 + \vec{p}_3 = m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3 = 0$

  (as shell is at rest initially) $\therefore \vec{p}_3 = -(\vec{p}_1 + \vec{p}_2)$

  So the third piece moves with the same magnitude of the resultant momentum of the other two pieces but in opposite direction.



$P_3 = \sqrt{P_1^2 + P_2^2 + 2P_1P_2\cos\theta}$

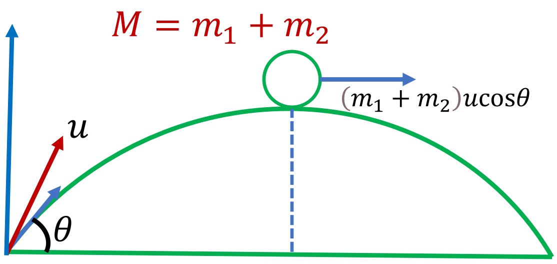
θ = angle between $\vec{P}_1, \vec{P}_2$

$(\pi - \alpha)$ = angle between $\vec{P}_3, \vec{P}_1$

$\tan\alpha = \dfrac{P_2 \sin\theta}{P_1 + P_2\cos\theta}$

Explosion of a shell travelling in a parabolic path at its highest point: (into two fragments)

- Consider a shell of mass M as a projectile with velocity u and angle of projection θ. Suppose the shell breaks into two fragments at maximum height and their initial velocities are $\vec{v}_1$ and $\vec{v}_2$.



Total momentum of the two parts is constant just before and just after the explosion.

$$[m_1 + m_2]u\cos\theta\vec{i} = m_1\vec{v}_1 + m_2\vec{v}_2$$

Case: (i) If the fragments travel in opposite direction after explosion, then

$$(m_1 + m_2)u\cos\theta\vec{i} = m_1 v_1\vec{i} - m_2 v_2\vec{i}$$

Case: (ii) If one fragment retraces its path and falls at the point of projection

$$(m_1 + m_2)u\cos\theta\vec{i} = -m_1 u\cos\theta\vec{i} + m_2\vec{v}_2$$

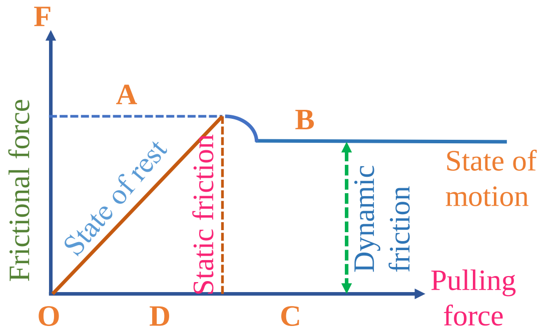
Case: (ii) If one fragment falls freely after explosion

$$(m_1 + m_2)u\cos\theta\vec{i} = m_1 0 + m_2\vec{v}_2$$

$$(m_1 + m_2)u\cos\theta\vec{i} = m_2\vec{v}_2$$

Friction:

- Friction is the resistance experienced when attempting to slide a body over a surface, caused by the bonding between the body and the surface.
- The force of friction acts parallel to the contact surfaces and opposes the intended or relative motion.
- There are three types of frictional forces:
  1. static friction
  2. dynamic friction, and
  3. rolling friction.
- If a body is at rest and no external force is applied, the force of friction on it is zero.
- Static friction comes into play when an applied force is unable to move the body, and it is equal in magnitude and opposite in direction to the applied force (static friction is a self-adjusting force).
- Dynamic or kinetic friction occurs when a force is applied and the body is in motion.
- Rolling friction is the friction that occurs when a body rolls on the surface of another body.
- Friction is caused by deformation at the point of contact and depends on the area of contact.



Note-i: When walking due east, the friction on the feet acts in the eastward direction, while the friction on the surface acts in the westward direction.

Note-ii: In a car where the engine is connected to the rear wheels, when the car is accelerated, the frictional force on the rear wheels will be in the direction of motion, while the frictional force on the front wheels will be in the opposite direction of motion.

Note-iii: In cycling, the force exerted by the rear wheel on the ground causes the force of friction to act on it in the forward direction. The front wheel, moving by itself, experiences the force of friction in the backward direction.

Note-iv: If a pedalling cycle is accelerating on a horizontal surface, the forward friction on the rear wheel is greater than the backward friction on the front wheel.

Note-v: When pedalling is stopped, the frictional force acts in the backward direction for both the wheels.

Laws of Friction:

- Friction is directly proportional to the normal reaction acting on the body.
- The law of static friction may thus be written as

  $f_s \alpha N \Rightarrow (f_s)_{max} = \mu_s N = f_\ell$

  Generally $0 \le \text{static friction} \le f_\ell$

  Where the dimensionless constant μ_s is called the coefficient of static friction and N is the magnitude of the normal force.

  $(f_s)_{max} = f_\ell = \mu_s N; f_\ell =$ Limiting friction
- Coefficient of static friction (μ_s) depends on the nature of the two surfaces in contact and is independent of the area of contact
- Static friction is independent of the area of contact between the two surfaces
- Coefficient of kinetic friction $(\mu_k) = \frac{f_k}{N}$

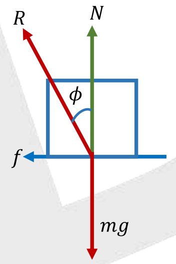
  It is independent of velocity of the body.

- Coefficient of rolling friction $(\mu_R) = \frac{f_R}{N}$
- Rolling friction depends on the area of the surfaces in contact.

  Note: $\mu_S > \mu_K > \mu_R$
- Friction depends on the nature of the two surfaces in contact i.e., nature of materials, surface finish,
  temperature of the two surfaces etc.

Angle of Friction:

- Angle made by the resultant of f and N with the normal reaction N is called angle of friction.
- Friction is parallel component of contact force to the surfaces.
- Normal force is perpendicular component of contact force to the surfaces.



$R = \sqrt{f^2 + N^2}$

- When the block is static $\tan\phi = \frac{f}{N}; \phi \le \phi_s$
- When the block is in impending state, $\tan\phi_s = \frac{\mu_s N}{N} = \mu_s$
- Where $\phi_s \to$ maximum angle of friction.
- When block is sliding, $\tan\phi_k = \frac{\mu_k N}{N} = \mu_k$

  Since $\mu_s > \mu_k$ it follows that $\phi_s > \phi_k$.

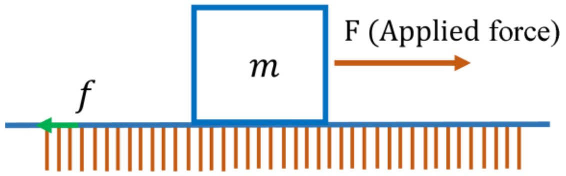
$$F_R = \sqrt{f_\ell^2 + N^2} = \sqrt{(\mu_2 N)^2 + N^2} = N\sqrt{\mu_s^2 + 1}$$

$$F_R = mg\sqrt{\tan^2\phi_s + 1} \qquad (\therefore \mu_s = \tan\phi_s)$$

$$F_R = mg\sec\phi_s$$

Motion on a horizontal rough surface: Consider a block of mass ‘m’ placed on a horizontal surface with normal reaction N.

Case (i): If applied force F = 0, then the force of friction is also zero.



Case (ii): If applied force $F<(f_s)_{max}$, the block does not move and the force of friction is $\boxed{f_s=F}$

Case (iii): If applied force $F=(f_s)_{max}$ block just ready to slide and frictional force $\boxed{(f_s)_{max}=f_\ell=\mu_s N}$

$F=\mu_s mg \quad (\because N=mg);$ (at time t=0)

Case(iv): If the above applied force continues to act ($t>0$) the body gets motion, static friction converts as kinetic friction and body possesses acceleration

$$a=\frac{F_{ext}-F_k}{m}=\frac{f_1-f_k}{m}=(\mu_s-\mu_k)g$$

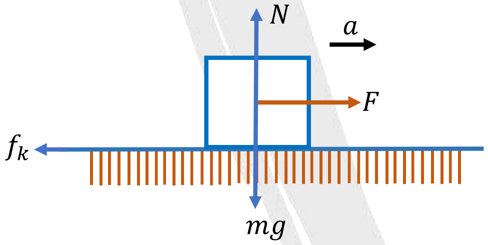
Case (v): If the applied force is greater than limiting friction the body starts moving and gets acceleration

$$a=\frac{F^1{}_{ext}-f_k}{m} \quad \textit{Here } F^1_{ext}>f_\ell$$

- If the block slides with an acceleration 'a' under the influence of applied force 'F',

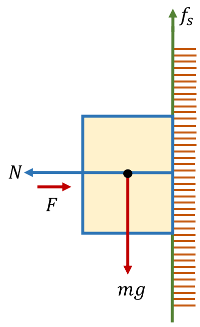
$F_R=F-f_k$; $ma=F-f_k$

$$\therefore a=\frac{F-f_k}{m}=\frac{F-\mu_k mg}{m}(f_k=\mu_k N=\mu_k mg)$$



Bodies in contact with vertical surfaces:

- A block of mass m is pressed against a wall without falling, by applying minimum horizontal force F. Then $\boxed{F=\frac{mg}{\mu_s}}$

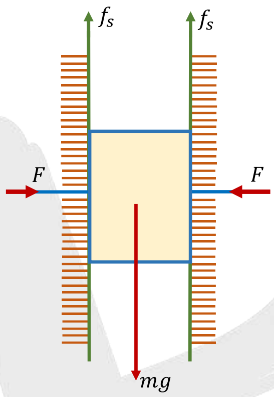


As the body is in vertical equilibrium

$f_s = mg;\ \mu_s N = mg$

$\mu_s F = mg\left(\because N = F\right) \Rightarrow F = \dfrac{mg}{\mu_s}$

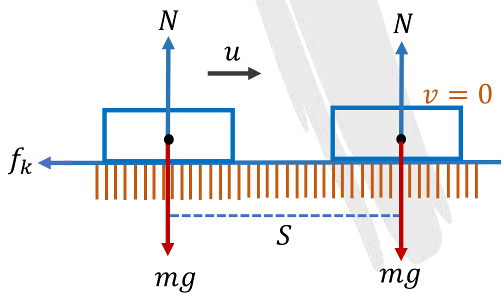
- A block is pressed between two hands without falling, by applying minimum horizontal force 'F' by each hand. Then $\boxed{F = \dfrac{mg}{2\mu_s}}$



$W = 2f_s;\ mg = 2\mu_s F \Rightarrow F = \dfrac{mg}{2\mu_s}$

Note: Here in the above two cases, by applying any amount of horizontal force the frictional force f_s can never be greater than 'mg'

Sliding block on a horizontal rough surface coming to rest:

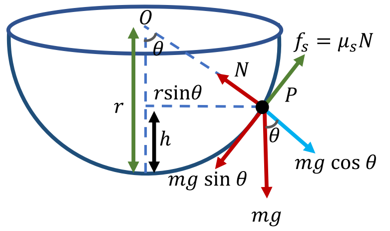


(a) The acceleration of the block is $\boxed{a = -\mu_k g}$

(b) Distance travelled by the block before coming to rest is $\boxed{S = \dfrac{u^2}{2\mu_k g}}$

(c) Time taken by the block to come to rest is $\boxed{t = \dfrac{u}{\mu_k g}}$

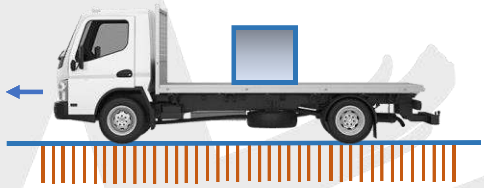
- An insect is crawling in a hemispherical bowl of radius 'r'. Maximum height up to which it can crawl is



$$h = r(1-\cos\theta) = r\left(1-\frac{1}{\sqrt{\mu_s^2+1}}\right)$$

Maximum angular displacement up to which it can crawl is '?'. Then $\mu_s = \tan\theta$

- A block is placed on rear horizontal surface of a truck moving along the horizontal with an acceleration 'a'.



Then

(1) The maximum acceleration of the truck for which block does not slide on the floor of the truck is $a = \mu_s g$

(2) If $a < \mu_s g$ block does not slide and frictional force on the block is f=ma.

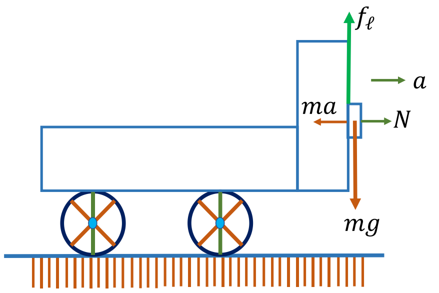
(3) If a > *m* g block slips or slides on the floor and the acceleration of the block relative to the truck is k $a' = a - \mu_k g$

(4) If ℓ is the distance of the block from rear side of the truck, time taken by the block to cover distance ℓ. $t = \sqrt{\frac{2\ell}{a-\mu_k g}}$

(5) Acceleration of the block relative to ground is $a'' = \mu_k g$

Body placed in contact with the front surface of accelerated truck:

- When a block of mass 'm' is placed in contact with the front face of the vehicle moving with acceleration 'a' then a pseudo force 'F_{pf}' acts on the block in a direction opposite to the direction of motion of the vehicle

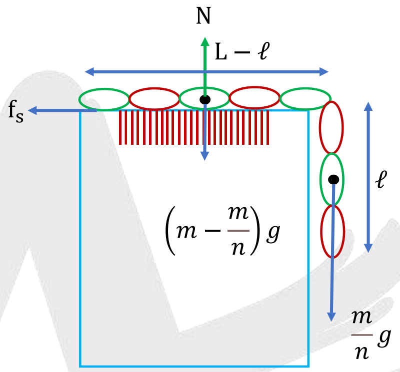


Under equilibrium, $f_{\ell} = mg; N = ma$

$$\mu_s N = mg \Rightarrow \mu_s ma = mg \Rightarrow a_{min} = \frac{g}{\mu_s}$$

Sliding of a chain on a horizontal table:

- Consider a uniform chain of mass "m" and length "L" lying on a horizontal table of coefficient of friction "μ_s". When $1/n^{th}$ of its length is hanging from the edge of the table, the chain is found to be about to slide from the table. Weight of the hanging part of the chain = $\frac{mg}{n}$



Weight of the chain lying on the table = $mg - \frac{mg}{n} = mg\left(1 - \frac{1}{n}\right)$

When the chain is about to slide from edge of the table, The weight of the hanging part of the chain = frictional force between the chain and the table surface.

$$\frac{mg}{n} = \mu_s mg\left(1 - \frac{1}{n}\right)$$

$$\Rightarrow \frac{mg}{n} = \mu_s mg\left(\frac{n-1}{n}\right) \qquad \therefore \mu_s = \frac{1}{(n-1)}$$

If l is the length of the hanging part, then $n = \frac{L}{\ell}$ Substituting this in the above expression we get,

$$\mu_s = \frac{\ell}{L-\ell} (or) n = \frac{L}{\ell} = \frac{\mu_s + 1}{\mu_s}$$

$\therefore$ The maximum fractional length of chain hanging from the edge of the table in equilibrium is

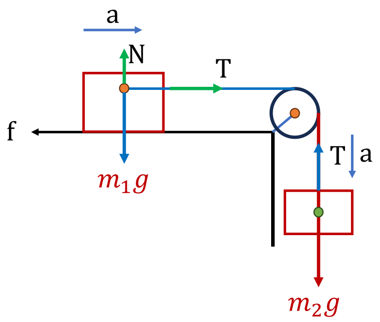
$$\frac{\ell}{L} = \frac{\mu_s}{\mu_s + 1}$$

- Fractional length of chain on the table

$$\frac{L-\ell}{L} = \frac{1}{\mu_s + 1}$$

Connected Bodies :

- A block of mass m_1 placed on a rough horizontal surface, is connected to a block of mass m_2 by a string which passes over a smooth pulley. The coefficient of friction between m_1 and the table is μ.



For body of mass m_2

$m_2g - T = m_2a$ .... (i)

For body of mass m_1

$T - f_k = m_1a \Rightarrow T - \mu_k N = m_1a$ .... (ii)

Solving Eqs (i) and (ii), we get

$$a = \left(\frac{m_2 - \mu_k m_1}{m_1 + m_2}\right) g; T = \frac{m_1 m_2 g}{m_1 + m_2}(1+\mu)$$

Motion of a body on an inclined plane:

Case (i): Body sliding down on a smooth inclined plane:

Let us consider a body of mass 'm' kept on a smooth inclined plane.

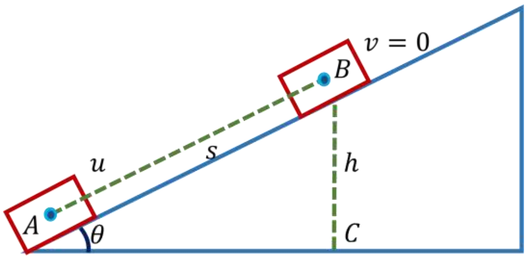
- Normal reaction $N = mg\cos\theta$
- Acceleration of sliding block $a = g\sin\theta$
- If *l* is the length of the inclined plane and h is the height. The time taken to slide down starting from rest from the top is $t = \sqrt{\frac{2\ell}{g\sin\theta}} = \frac{1}{\sin\theta}\sqrt{\frac{2h}{g}}$

$\left(\because \ell = \frac{h}{\sin\theta}\right)$

- Sliding block takes more time to reach the bottom than to fall freely in air from the top of the inclined plane to the ground.
- Speed of the block at the bottom of the inclined plane is same as the speed attained if block falls freely from the top of the inclined plane.

$V = \sqrt{2g\ell\sin\theta} = \sqrt{2gh}$

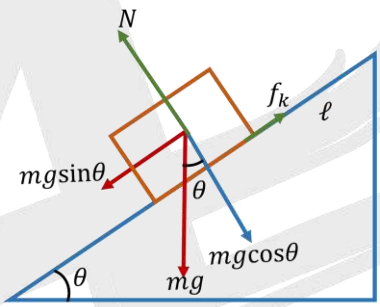
Case(ii): Body projected up on a smooth inclined plane:



- If a block is projected up the plane with a velocity u, the acceleration of the block is $a = -g\sin\theta$
- Distance travelled by the block up the plane before its velocity becomes zero is $S = \dfrac{u^2}{2g\sin\theta}$
- Time of ascent $t = \dfrac{u}{g\sin\theta}$

Case (iii) Motion of a body down the rough inclined plane:

- Let a body of mass 'm' be sliding down a rough inclined plane of angle of inclination θ and coefficient of kinetic friction μ_k.



Angle of Repose (α) : Angle of repose is the minimum angle of the rough inclined plane for which body placed on it may just start sliding down. It is numerically equal to the angle of friction.

- Let θ be the angle of inclination of a rough inclined plane, α be the angle of repose, m be the mass of the body and μ be the coefficient of friction. At limiting equilibrium (about to slide)

$$mg\sin\alpha = \mu_s mg\cos\alpha \Rightarrow \tan\alpha = \mu_s \Rightarrow \alpha = \tan^{-1}(\mu_s)$$

1. When $\theta_1 < \alpha$; the block remains at rest on the inclined plane. Frictional force $mg\sin\theta_1$ (self adjusting), acceleration a=0
2. When $\theta_2 = \alpha$; the block remains at rest on inclined plane or impending state of motion is achieved. $mg\sin\theta_2 = \mu_s mg\cos\theta_2$ (at time t=0) Here $\theta_2 > \theta_1$ and $f_s = f_1$ acceleration a=0
3. When $\theta_2 = \alpha$ and (t > 0) the same inclination is continued the block moves downwards with acceleration a.

$mg\sin\theta_2 > \mu_k mg\cos\theta_2$ acceleration

$$a = \frac{mg\sin\theta_2 - \mu_k mg\cos\theta_2}{m}$$

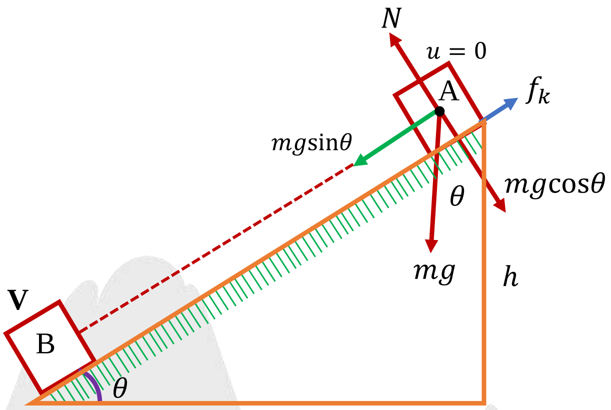
$$a = \frac{\mu_s mg\cos\theta_2 - \mu_k mg\cos\theta_2}{m} = g\cos\theta_2(\mu_s - \mu_k)$$

4. When $\theta > \alpha$, the body slides $f_k = \mu_k mg\cos\theta$

The resultant force acting on the body down the plane is $F_R = mg\sin\theta - f_K$,

$F_R = mg(\sin\theta - \mu_k\cos\theta)$

The acceleration of the body $a = g(\sin\theta - \mu_k\cos\theta)$



- Velocity of the body at the bottom of the plane $V = \sqrt{2g(\sin\theta - \mu_k\cos\theta)\ell}$
- If 't' is the time taken to travel the distance '*l*' with initial velocity u = 0 at the top of the plane,

$$t = \sqrt{\frac{2\ell}{g(\sin\theta - \mu_k\cos\theta)}}$$

- The time taken by a body to slide down on a rough inclined plane is 'n' times the time taken by it to

slide down on a smooth inclined plane of same inclination and length, then coefficient of friction μ_k is

$$n = \frac{t_{rough}}{t_{smooth}} = \frac{\sqrt{\frac{2\ell}{g(\sin\theta - \mu_k\cos\theta)}}}{\sqrt{\frac{2\ell}{g\sin\theta}}}$$

$$n^2 = \frac{\sin\theta}{\sin\theta - \mu_k\cos\theta}$$

$$\Rightarrow n^2\sin\theta - n^2\mu_k\cos\theta = \sin\theta$$

$$\Rightarrow \mu_k = \text{Tan}\theta\left[1 - \frac{1}{n^2}\right]$$

Body projected up a rough inclined plane:

If a body is projected with an initial velocity 'u' to slide up the plane, the kinetic frictional force acts down the plane and the body suffers retardation due to a resultant force

$F_R = (mg\sin\theta + f_k)$

- Acceleration $a = -g(\sin\theta + \mu_k\cos\theta)$
- Time taken to stop after travelling a distance l along the plane,

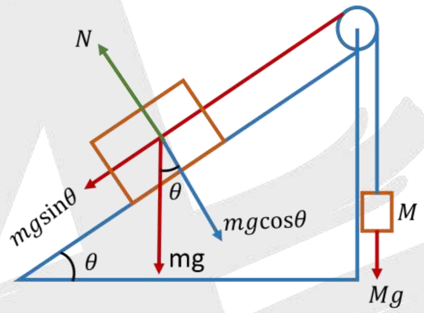
$$t = \sqrt{\frac{2\ell}{g(\sin\theta + \mu_k\cos\theta)}}$$

- Force required to drag with an acceleration 'a' is

$F = (\mu_k mg\cos\theta + mg\sin\theta + ma)$

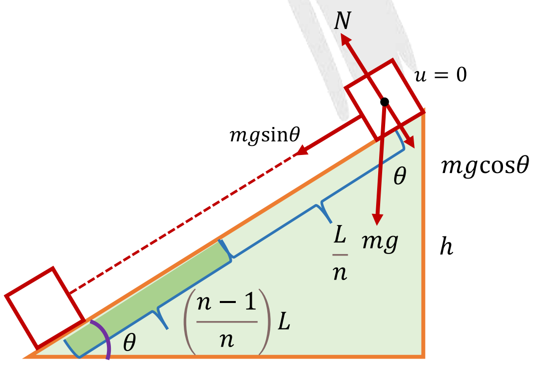
- Two blocks of mass m and M are placed on a rough inclined plane as shown, when $(\theta > \alpha)$

(i) Minimum value of M for which m slides upwards is $M = m(\sin\theta + \mu\cos\theta)$



(ii) Maximum value of M for which m slides downwards: $M = m(\sin\theta - \mu_s\cos\theta)$

- A body is released from rest from the top of an inclined plane of length 'L' and angle of inclination 'θ'. The top of plane of length $\frac{L}{n}(n>1)$ is smooth and the remaining part is rough. If the body comes to rest on reaching the bottom of the plane then find the value of coefficient of friction of rough surface.



For smooth part:

Using $v^2 - u^2 = 2as;\ V^2 = 2a_1\frac{L}{n}$,

$a_1 = g\sin\theta, a_2 = g(\sin\theta - \mu\cos\theta)$

For rough part $0 - V^2 = 2a_2\left(\frac{n-1}{n}\right)L$

$$2a_1\frac{L}{n} = -2a_2\left(\frac{n-1}{n}\right)L$$

$$g\sin\theta = -g[\sin\theta - \mu\cos\theta](n-1)$$

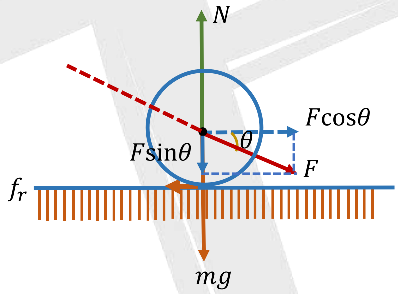
$$\mu = \mathrm{Tan}\theta\left[\frac{n}{n-1}\right]$$

- A body is pushed down with velocity 'u' from the top of an inclined plane of length 'L' and angle of inclination 'θ'. The top of plane of length $\frac{L}{n}(n>1)$ is rough and the remaining part is smooth. If the body reaches the bottom of the plane with a velocity equal to the initial velocity 'u', then the value of coefficient of friction of rough plane is $\boxed{\mu_K = n(\tan\theta)}$

Note: If the top surface is smooth and the remaining is rough, then $\mu_k = \tan\theta\left(\frac{n-2}{n-1}\right)$

Pushing & Pulling of a Lawn Roller:

(i) A Roller on Horizontal Surface Pushed by an Inclined Force:

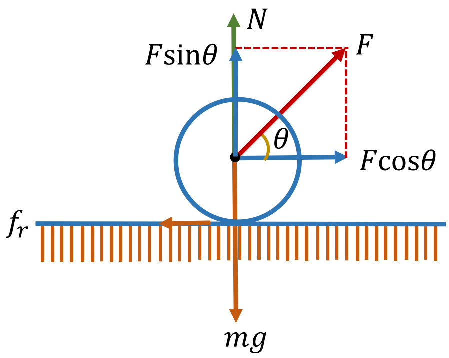


- When a lawn roller is pushed by a force 'F', which makes an angle q with the horizontal, then
- Normal reaction N = mg + F sin q .
- Frictional force $f_r = \mu_r N = \mu_r(mg + F\sin\theta)$

$\therefore$ The net horizontal pushing force is given by

$$F_1 = F(\cos\theta - \mu_r\sin\theta) - \mu_r mg$$

(ii) A Lawn Roller on a Horizontal Surface Pulled by an Inclined Force

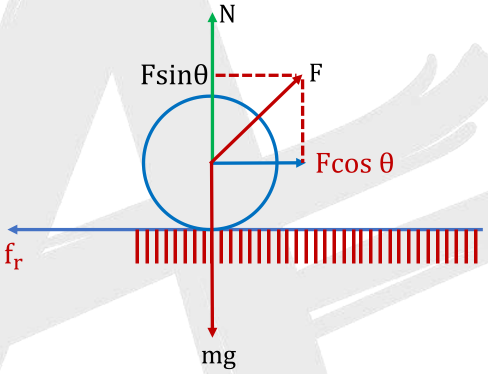


- Let a lawn roller be pulled on a horizontal road by a force 'F', which makes an angle *q* with the horizontal.
- Normal reaction N =mg – F sin *q*
- Frictional force $f_r = \mu_r N = \mu_r(mg - F\sin\theta)$

The net horizontal pulling force is $F_2 = F(\cos\theta + \mu_r \sin\theta) - \mu_r mg$ Pulling is easier than pushing.

Applying an Inclined Pulling Force:

Let an inclined force F be applied on the body so as to pull it on the horizontal surface as shown in the figure.



The body is in contact with the surface, and just ready to move

$N + F\sin\theta = mg \Rightarrow N = mg - F\sin\theta$

Frictional force $f_r = F\cos\theta$

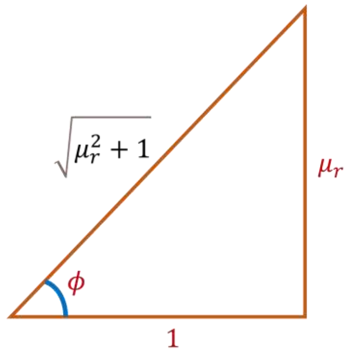
$F\cos\theta = \mu_r N, F\cos\theta = \mu_r(mg - F\sin\theta)$

$$F = \frac{\mu_r mg}{(\cos\theta + \mu_r \sin\theta)}$$

$$\Rightarrow F = \frac{mg\sin\phi}{\cos(\theta - \phi)} (\because \text{Tan}\phi = \mu_r)$$

For F to be minimum $\cos(\theta - \phi)$ should be maximum $\Rightarrow$ $\cos(\theta - \phi) = 1 \Rightarrow \theta - \phi = 0, \theta = \phi$

ϕ = angle of friction.



$\therefore F_{min} = mg\sin\theta = mg\sin\phi$, From the figure,

$$\sin\phi = \frac{\mu_r}{\sqrt{\mu_r^2+1}}, F_{min} = \frac{\mu_r mg}{\sqrt{\mu_r^2+1}}$$

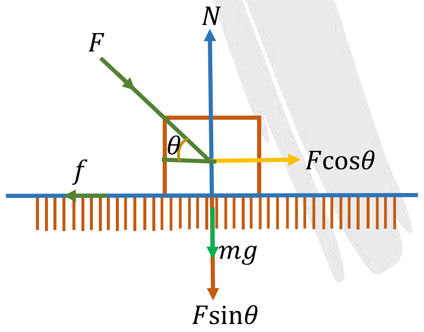
Minimum horizontal pulling force, when $\theta = 0$

$$\cos(0-\phi) = \cos\phi$$

$$F = \frac{mg\sin\phi}{\cos\phi} = mg\tan\phi$$

Applying an Inclined Pushing Force:

Let an inclined force F is applied on the body so as to push it on the horizontal surface as shown in the figure.



The body is in contact with the surface, and just ready to move, N= mg + F sin *q*

frictional force $f_\ell = F\cos\theta$

$$F\cos\theta = \mu_s N \Rightarrow F\cos\theta = \mu_s(mg + F\sin\theta)$$

$$F = \frac{\mu_s mg}{(\cos\theta - \mu_s\sin\theta)}$$

$$F = \frac{mg\sin\phi}{\cos(\theta+\phi)} \quad (\because \text{Tan}\phi = \mu_s)$$

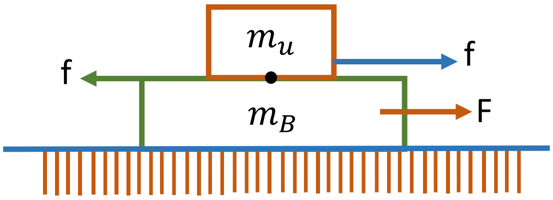
For F to be minimum *q* = 0

$$\therefore F_{min} = \frac{mg\sin\phi}{\cos\phi} = mg\tan\phi = \mu_s mg$$

(Since $\mu_s = \tan\phi$)

Block on Block:

Case I: Bottom block is pulled and there is no friction between bottom block and the horizontal surface.

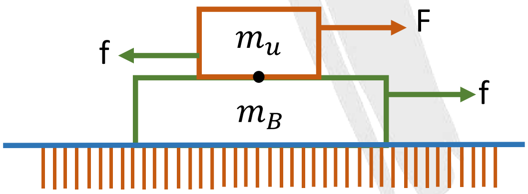


When the bottom block is pulled upper block is accelerated by the force of friction acting upon it.

The maximum acceleration of the system of two blocks to move together without slipping is $a_{max} = \mu_s g$, where μ_s is the coefficient of static friction between the two blocks. The maximum applied force for which both the blocks move together is $F_{max} = \mu_s g(m_u + m_B)$

- If $a < \mu_s g$ blocks move together and applied force $F = (m_B + m_u)a$. In this case frictional force between the two block $f = m_u a$
- If $a > \mu_s g$, blocks slip relative to each other and have different accelerations. The acceleration of the upper block is $a_u = \mu_k g$ and that for the bottom block is $a_B = \dfrac{F - \mu_k m_u g}{m_B}$

Case - II: Upper block pulled and there is no friction between bottom block and the horizontal surface.



- When the upper block is pulled, bottom block is accelerated by the force of friction acting on it.
- The maximum acceleration of the system of two blocks to move together without slipping is $a_{max} = \mu_s \dfrac{m_u}{m_B} g$ where, $\mu_s =$ coefficient of static friction between the two blocks.

  The maximum force for which both blocks move together is $F_{max} = \mu_s \dfrac{m_u}{m_B} g(m_u + m_B)$
- If $a < a_{max}$, blocks move together and frictional force between the two blocks is $f = m_B a$ The applied force on the upper block is $F = (m_B + m_u)a$

- If $a > a_{max}$, blocks slide relative to each other and hence they have different accelerations. The acceleration of the bottom block is $a_B = \mu_K \frac{m_u}{m_B} g$ and the acceleration of the upper block is $a_u = \frac{F - \mu_k m_u g}{m_u}$
- A number of blocks of identical masses m each are placed one above the other. Force required to pull out N^{th} block from the top is $F = (2N-1)\mu mg$

Uniform circular motion

- When a particle moves in a circular path with constant speed then it is said to be in uniform circular motion. In this case the acceleration of the particle is $a = v(\omega) = \frac{v^2}{r} = r\omega^2$,where v r, & ω are linear velocity, radius and angular velocity of the particle. In uniform circular motion (a) magnitude of linear velocity does not change
  (b) direction of linear velocity changes
  (c) Linear velocity changes
  (d) Angular velocity is constant
  (e) Linear momentum changes
  (f) Angular momentum w.r.t to centre does not change

Centripetal force (F_C):

- It is the force required to keep the body in uniform circular motion. This force changes the direction of linear velocity but not its magnitude

$$F_C = \frac{mv^2}{r} = mr\omega^2 = mv\omega \qquad (v = r\omega)$$

- The direction of the centripetal force is always perpendicular to the direction of the linear velocity. This force is responsible for keeping an object moving in a curved path. The work done by the centripetal force is always zero because it acts perpendicular to both the velocity and the instantaneous displacement of the object.
  Ex1: When an electron moves around the nucleus in a circular orbit, the electrostatic force of attraction between the electron and nucleus is the centripetal force.
  Ex2: If an electron of mass m and charge e moves around the nucleus of atomic number Z in a circular orbit of radius r, centripetal force on it is $\frac{mv^2}{r} = \frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{r^2}$

Ex3: When planet of mass m moves around the sun in a circular orbit of radius r, centripetal force on it is $\frac{mv^2}{r} = \frac{GMm}{r^2}$

Ex4: When a stone is whirled round in horizontal circle by attaching it at the end of string, tension in the string provides the centripetal force.

$$\frac{mv^2}{r} = T$$

Centrifugal force:

- The centrifugal force is a pseudo force that acts radially outward on a body moving along a circle. While the centripetal force and the centrifugal force have equal magnitudes and opposite directions, they do not form an action-reaction pair because they act on the same body in two different frames of reference.

Circular turning on roads:

The necessary centripetal force while taking a circular turn is being provided to a vehicle by follow in three ways

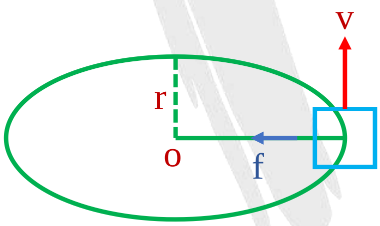
(1) by friction only

(2) by banking of roads only

(3) by both friction and banking of roads

Friction only:

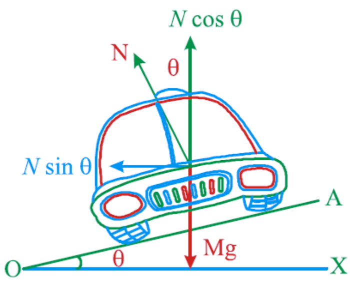
In this case the necessary centripetal force is pro video by static friction



$v \le \sqrt{\mu_s gr}$; $\quad v_{max} = \sqrt{\mu_s gr}$

- For a given radius of curvature and coefficient of friction, the safe maximum velocity of the vehicle is given by $v_{max} = \sqrt{\mu_s rg}$

- **Banking of roads only:**



- Let θ be the angle through which the outer edge is raised relative to the inner edge. This angle is also called "angle of banking". The normal reaction N exerted by the road on the vehicle is directed nor map to the surface as shown in the figure.

$N\cos\theta$ balances the weight of the vehicle.

$N\cos\theta = mg$

$N\sin\theta$ is directed towards the centre of the circular path. Which provides the centripetal force.

$$N\sin\theta = \frac{mv^2}{r}$$

From the above equations, we get

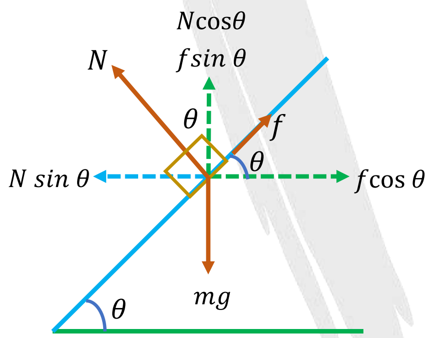
$$\tan\theta = \frac{v^2}{rg} \text{ (or) } \sqrt{rg\tan\theta}$$

Motion of a vehicle on a rough banked road:

If friction is present between the road and the tyres, the components of friction and normal reach-

to provide the centripetal force.

Case-I: If $N\sin\theta > \frac{mv^2}{r}$ the vehicle possesses the tendency to slip down the plane. The minimum speed for avoiding slipping down the plane can be obtained by taking friction up the plane.



$fig-(i)$

To find minimum speed we can use fig (i),

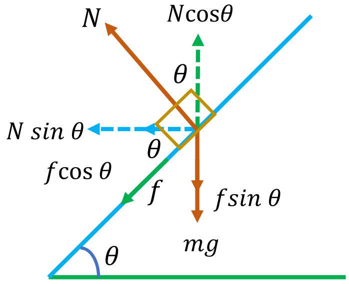
$$N\sin\theta - f\cos\theta = \frac{mv_{min}^2}{r} \text{ ..... (1)}$$

$$N\cos\theta + f\sin\theta = mg \text{ ..... (2) } (f = \mu N)$$

From (1) and (2) we get

$$v_{min} = \sqrt{\frac{rg(\sin\theta - \mu\cos\theta)}{(\cos\theta + \mu\sin\theta)}} = \sqrt{\frac{rg(\tan\theta - \mu)}{(1+\mu\tan\theta)}}$$

Case – II: If $N\sin\theta < \frac{mv^2}{r}$, the vehicle possesses the tendency to skid up the plane. The safe maximum speed for avoiding skidding can be obtained by taking friction acting down the plane.



$fig-(ii)$

To find maximum safe speed, we have to consider figure (ii)

$$N\sin\theta + f\cos\theta = \frac{mv_{max}^2}{r} \quad \text{..... (1)}$$

$$N\cos\theta - f\sin\theta = mg \quad \text{...... (2)}$$

From (1) and (2) we get

$$v_{max} = \sqrt{\frac{rg(\sin\theta + \mu\cos\theta)}{(\cos\theta - \mu\sin\theta)}} = \sqrt{\frac{rg(\tan\theta + \mu)}{(1-\mu\tan\theta)}}$$

- A cyclist is taking a turn of radius *r* with speed v then he should bend through an angle θ with the vertical such that $\tan\theta = \frac{v^2}{rg}$

Conical pendulum:

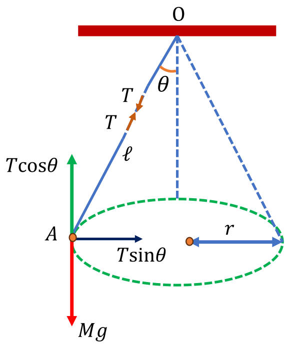
A bob of mass Mis given a horizontal push a little through angular displacement θ and arranged such that the bob describes a horizontal circle of radius 'r' with uniform angular velocity ω in such a way that the string always makes an angle θ with the vertical and T is the tension in the string.

Suppose the body is in rotational equilibrium,then

$$T\cos\theta = Mg \quad \text{...... (1)}$$

$$T\sin\theta = Mr\omega^2 \quad \text{..... (2)}$$

From (1) and (2) $\tan\theta = \frac{r\omega^2}{g}$

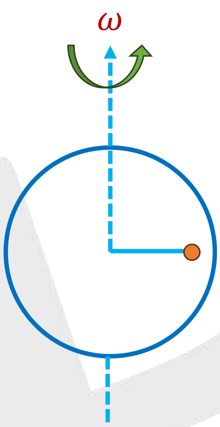


$\omega = \sqrt{\frac{g \tan\theta}{r}}$

but $r = \ell \sin\theta$ and $\omega = \frac{2\pi}{T_0}$ (T_0 is the time period of pendulum)

Time period of the pendulum is $T_0 = 2\pi\sqrt{\frac{\ell\cos\theta}{g}}$

➢ For the coin not to fly off on the turn table, the condition is



$\Rightarrow \frac{mv^2}{r} \le \mu_s mg$

Motion of a Cyclist in a Death Well:

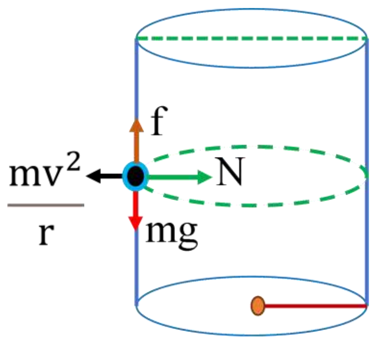
For equilibrium of cyclist in a death well, as shown in the figure, the normal reaction N provides the

centripetal force needed and the force of friction balances his weight mg.

Thus, $N = \frac{mv^2}{r}$

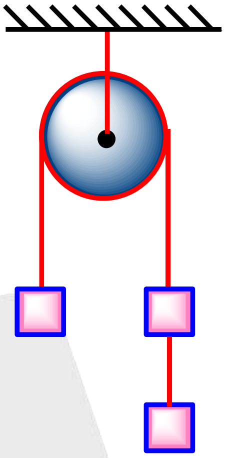
and $f = \mu N = mg$

$\Rightarrow \qquad V_{max} = \sqrt{\frac{rg}{\mu}}$



EXERCISE – 1

1. Three equal weights A, B and C of mass 2 kg each are hanging on a string passing over a fixed frictionless pulley as shown in the figure. The tension in the string connecting weights B and C is :



(A) zero (B) $\frac{4g}{3}$ (C) $\frac{2g}{3}$ (D) $\frac{g}{3}$

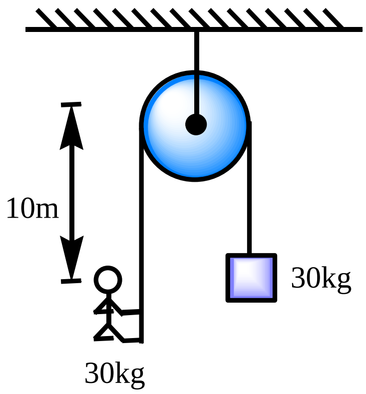
2. When a bird of weight W sits on a stretched wire, the tension T in the wire is always :

(A) $> \frac{W}{2}$ (B) $= W$ (C) $< W$ (D) None of these

3. A man is standing in a elevator which goes up and comes down with the same constant acceleration. If the ratio of the apparent weights in the two cases is 2 : 1, then the acceleration of the elevator is :

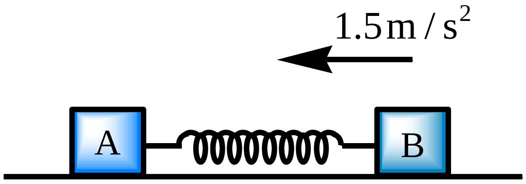
(A) $\frac{g}{2}$ (B) $\frac{g}{3}$ (C) $\frac{g}{4}$ (D) g

4. In the figure shown man is balanced by counter weight of same mass. He starts to climb the rope with an accelerator of $4\,m/s^2$ w.r.t. rope. The time after which he reaches the pulley will be



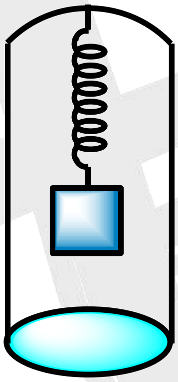
(A) $\sqrt{10}$ sec (B) $2\sqrt{5}$ sec (C) $\sqrt{20}$ sec (D) $\sqrt{5}$ sec

5. Two blocks A and B with mass 8kg and 10kg respectively are connected by a stretched spring of negligible mass as in figure. When the two blocks are released simultaneously the initial acceleration of B is $4m/sec^2$ westward. The acceleration of A is



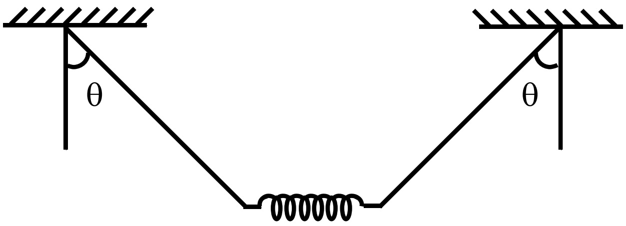
(A) $1m/sec^2$ westward
(B) $5m/sec^2$ eastward
(C) $1m/sec^2$ eastward
(D) $2.75m/sec^2$ westward

6. A steel block hangs at rest in equilibrium by a spring from the inside of an upside – down Teflon beaker, as seen in the figure below. Just after the beaker system is released, the weight will



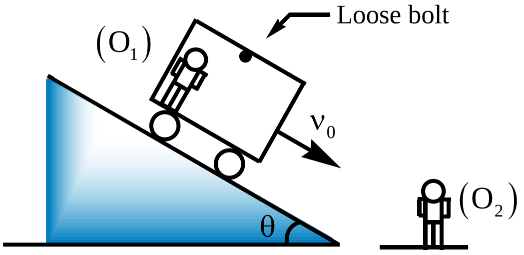
(A) Extend the spring further
(B) Move closer to upper surface of beaker
(C) Remain in the same configuration as it falls
(D) Depends on the value of spring constant

7. A stiff horizontal spring of weight W and force constant k is suspended in a horizontal position by two light strings attached to its two ends. Each string makes an angle θ with the vertical. The extension of the spring is



(A) $\left(\frac{W}{4k}\right)\tan\theta$ (B) $\left(\frac{W}{2k}\right)\tan\theta$ (C) $\left(\frac{W}{4k}\right)\sin\theta$ (D) zero

8. A cart is sliding on a smooth incline. An observer (O_1) is fixed to cart and another observer fixed on ground (O_2) observe, a loose bolt that is released from ceiling. At the instant of release cart has velocity v_0 as seen by O_2. Mark the correct option(s).



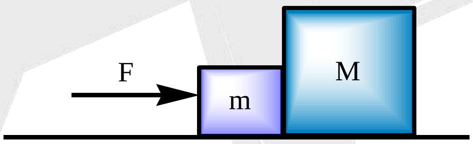
(A) Trajectory of bolt for O_1 is parabola

(B) Trajectory of bolt for O_2 is straight line inclined at an angle θ with vertical

(C) Trajectory of both for O_2 is a straight line perpendicular to ceiling of cart

(D) Trajectory of bolt for O_1 is straight line

9. Two blocks are arranged as shown on a smooth horizontal surface. A force F is exerted on the smooth block m. which of the following statement(s) is/are true?



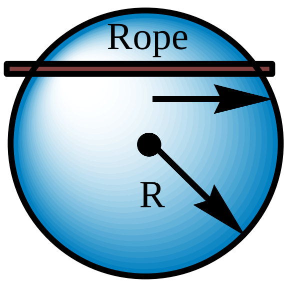
(A) The normal force exerted by m on M is equal to F

(B) The normal force exerted by m on M is smaller than F

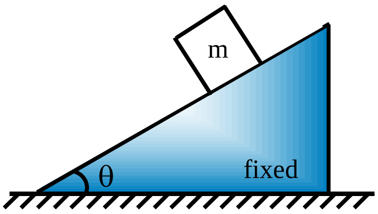
(C) The normal force exerted by m on M is larger than the normal force exerted by M on m

(D) The normal force exerted by m on M is smaller than the normal force exerted by M on m

10. A circular rope of weight W and radius $r = \frac{R}{2}$ is resting on a smooth sphere of radius R. the tension in rope is $\frac{w}{\pi\sqrt{\alpha}}$. Find α ?



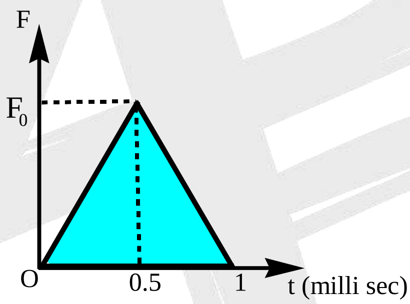
11. Choose the correct equation for normal reaction N between block and fixed smooth wedge, where a is acceleration of block along the incline.



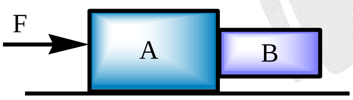
(A) $N\cot\theta = mg$ (B) $N = mg\cos\theta$

(C) $N\cos\theta + mg = ma\sin\theta$ (D) $N\cos\theta = mg$

12. A body of mass 1kg moving with velocity 1 m/s makes an elastic one-dimensional collision with an identical stationary body. They are in contact for time 1 milli sec. Their force of interaction increases from zero to F_0 linearly in time 0.5 milli sec and decreases linearly to zero in further time 0.5 milli sec as shown in figure. The magnitude of force F_0 (in kN) is



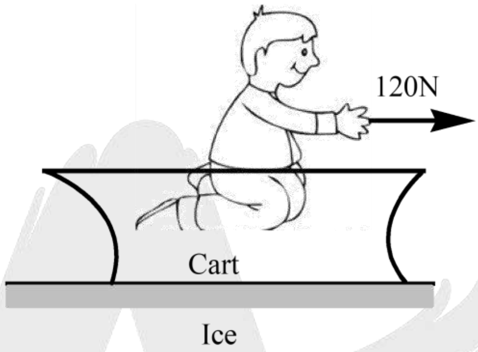
13. For the system consisting of the two blocks shown in the figure, the minimum horizontal force F is applied so that block B does not fall under the influence of gravity. The masses of A and B are 16.0kilograms and 4.00 kilograms, respectively. The horizontal surface is frictionless and the coefficient of friction between the two blocks is 0.50. The magnitude of F (in N) is



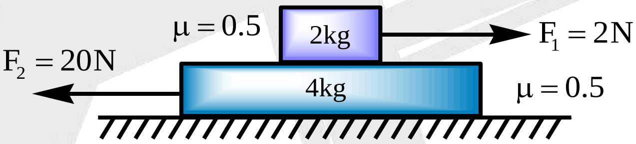
14. A small chain of length ℓ hangs from a table. It starts falling when the apart that sticks out from the table has length $\ell_1 (\ell_1 < \ell)$. The coefficient of static friction between the table and the chain is given by

(A) $\frac{\ell-\ell_1}{\ell_1}$ (B) $\frac{\ell_1}{\ell}$ (C) $\frac{\ell_1}{\ell-\ell_1}$ (D) None of these

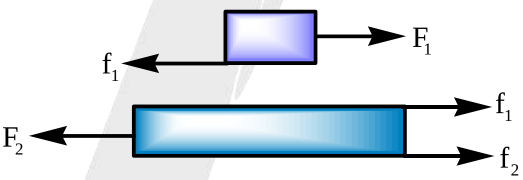
15. A child of mass 30kg was sitting on a snow cart of mass 10kg. the friction between the cart and the snow was negligible. If a force of 120N was applied to the child, as shown in the figure. The minimum coeffiecient of friction required between the child and cart to keep the child from slipping off is $\mu=\frac{1}{n}$. Find n?



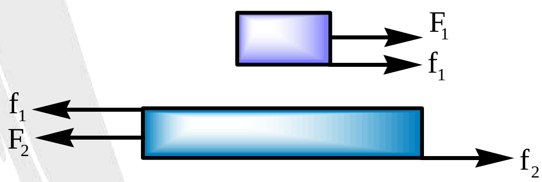
16. The friction force acting between surfaces in contact in the adjoining figure is best represented by



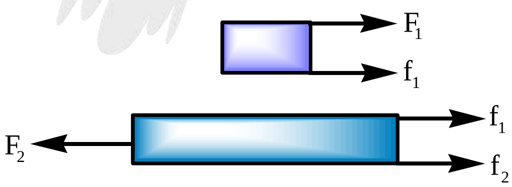
(A)



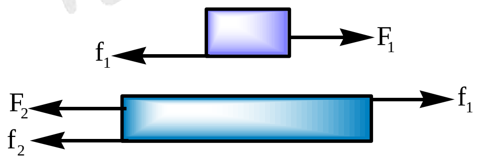
(B)



(C)



(D)

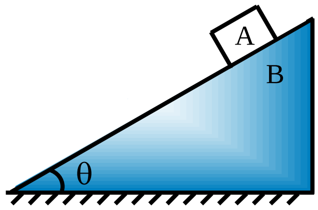


17. If the external forces acting on a body have zero resultant, the body

(A) must not move (B) may move (C) must not accelerate (D) may accelerate

Ans: BC

18. In the figure shown, A and B are free to move. All the surfaces are smooth. $(0<\theta<90^\circ)$



(A) The acceleration of A will be more than $g\sin\theta$

(B) The acceleration of A will be less than $g\sin\theta$

(C) Normal force A due to B will be more than $mg\cos\theta$

(D) Normal force on A due to B will be less than $mg\cos\theta$

19. Each of the system shown below is initially at rest. Pulleys are massless and frictionless.

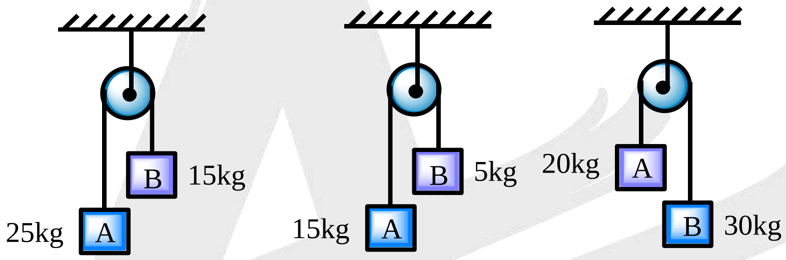


Figure - 1 Figure - 2 Figure - 3

(A) Acceleration of block A in figure 1 is same as that in figure 2

(B) Acceleration of block A in figure 1 is less than that in figure 2 and greater than that in figure 3

(C) The ratio of velocity of block A in figure 1 to that in figure 2 after it has moved through 1m is 1

(D) The ratio of velocity of block A in figure 1 to that in figure 2 after it has moved through 1m is $\frac{1}{\sqrt{2}}$

20. A block of weight 9.8N is placed on a table. The table surface exerts an upward force of 20N on the block. Assume $g = 9.8 m/s^2$

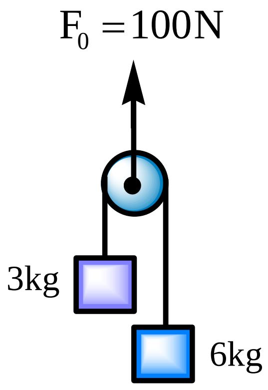
(A) The block exerts a force of 20N on the table

(B) The block exerts a force of 19.8N on the table

(C) The block exerts a force of 9.8N on the table

(D) The block has an upward acceleration

21. Two blocks of masses $m_1 = 3kg$ and $m_2 = 6kg$ hang over a massless pulley as shown in the figure. A force $F_0 = 100N$ acting at the axis of the pulley accelerates the system upwards. Then



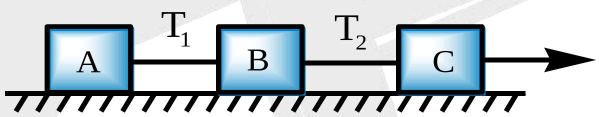
(A) Acceleration of 3kg mass is $\frac{20}{3} m/s^2$

(B) Acceleration of 6kg mass is $\frac{10}{6} m/s^2$

(C) Acceleration of both the masses is same

(D) Acceleration of both the masses is upward

22. Three blocks are connected by strings and pulley by a force $F = 120N$ as shown in figure. If $m_A = 10kg$, $m_B = 20kg$ and $m_C = 30kg$, then



(A) Acceleration of the system is $2m/s^2$ (B) $T_1 = 20N$

(C) $T_2 = 60N$ (D) $T_1 = 20N$ and $T_2 = 40N$

23. Consider the following two situations

(P) You stand still on a weighing machine

(Q) You stand on a weighing machine in an elevator that is accelerating upward at $2.0 m/s^2$

(A) In case (P) scale reading gives us the value of your weight

(B) In case (P) scale is measuring the force of the scale pushing up on your feet

(C) In case (Q) scale is measuring the force of the scale pushing up on your feet

(D) In case (Q) reading of scale corresponds to the weight of a person fatter than you

24. My weight is $w = mg$, but when I weigh myself in an elevator with a normal weighing machine I am surprised to see that it appears to be a different amount w_{ap}. If the coordinate y is measured vertically upward as positive, which on of the following statement(s) is true if acceleration of elevator is a_y.

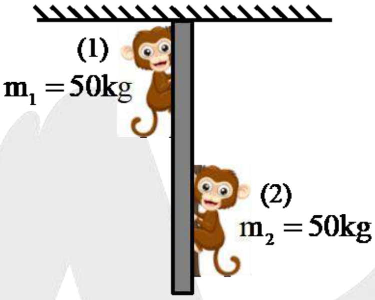
(A) If w_{ap} is greater than w, then the elevator is certainly travelling upward

(B) If a_v is positive, then w_{ap} will be smaller than w

(C) If $\vec{a}_v = \vec{g}$, then $w_{ap} = 0$

(D) If the elevator is moving upward, then w_{ap} could be greater w, but it could alos be smaller

25. Monkey (1) is climbing up a light rope with acceleration of $a_1 = 2m/s^2$ and monkey (2) us climbing upto the same rope with acceleration of $a_2\ m/s^2$



Choose incorrect options

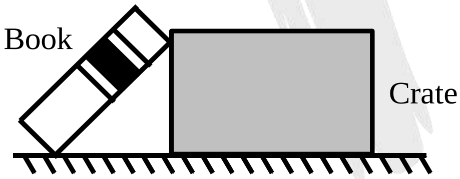
(A) Tension is same in all parts of rope

(B) a_1 must be equal to a_2

(C) a_2 must be zero

(D) Two monkeys can have different velocities

26. A book leans against a crate on a table. Neither is moving. Which of the following statements concerning this situation is/are incorrect?



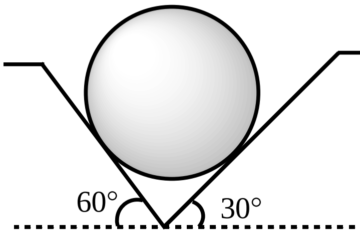
(A) The force of the book on the crate is less than that of crate on the book

(B) Although there is no friction acting on the crate, there must be friction acting on the book or else it will fall

(C) The net force acting on the book is zero

(D) The direction of the frictional force acting on the book is in the same direction as the frictional force acting on the crate.

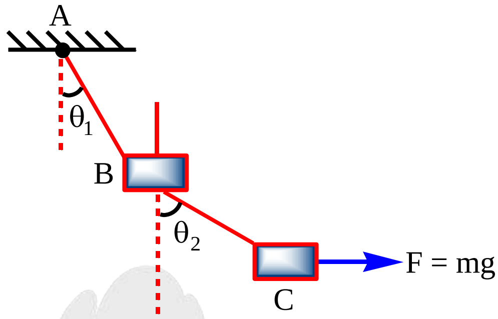
27. A cylinder of mass $\frac{1}{\sqrt{3}}$ kg is placed on the corner of two inclined planes as shown in the figure. Find the normal reaction in newtons at contact point of cylinder with the slope of inclination $30°$. $\left[g = 10 m/s^2\right]$



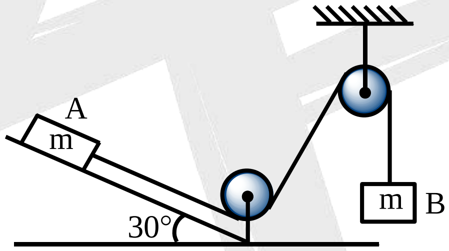
(A) 2N (B) 3N (C) 4N (D) 5N

EXERCISE – 2

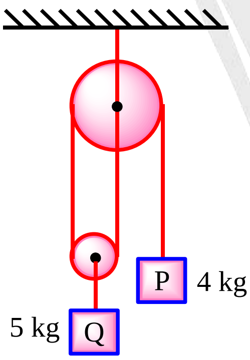
1. The blocks B and C in the figure have mass m each. The strings AB and BC are light, having tension T_1 and T_2, respectively. The system is in equilibrium with a constant horizontal force mg acting on C. Then, $T_1 = \sqrt{n}\ mg$. Find n?



2. Two blocks A and B of same mass are connected through a string and arranged as shown in figure. When the system is released from rest and there is no friction, then, tension in the string is $\frac{\alpha mg}{\beta}$. Find $\alpha + \beta$?



3. The acceleration of the blocks (P) and (Q) respectively in situation shown in the figure is : (pulleys and strings are massless)



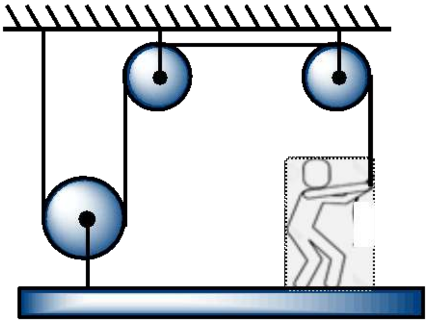
(A) $\frac{2g}{7}$ downward, $\frac{g}{7}$ upward

(B) $\frac{2g}{3}$ downward, $\frac{g}{3}$ upward

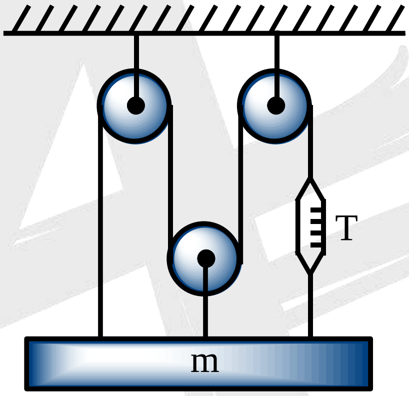
(C) $\frac{10g}{13}$ downward, $\frac{5g}{13}$ upward

(D) None of the above

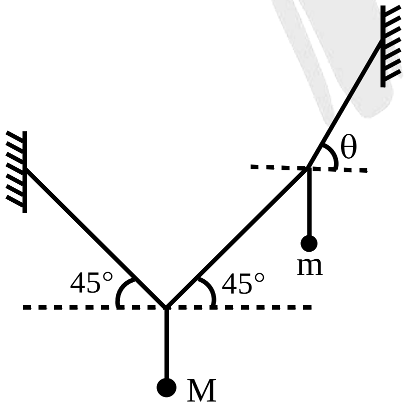
4. A 50 kg person stands on a 25 kg platform. He pulls on the rope which is attached to the platform via the frictionless pulleys as shown in the figure. The platform moves upwards at a steady rate if the force with which the person pulls the rope (in N) is



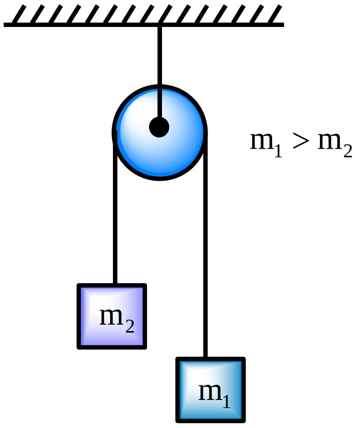
5. A spring scale indicates 10 N in the given situation as shown in figure. Neglecting the mass of the pulley and ignoring friction between the cable and pulley, the mass m (in kg) is

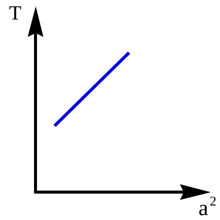


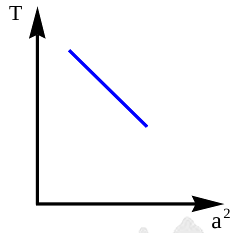
6. Two masses m and M are attached to strings as shown in the figure. In equilibrium $\tan\theta = \left(\alpha + \frac{\beta m}{M}\right)$. Find $\alpha + \beta$?

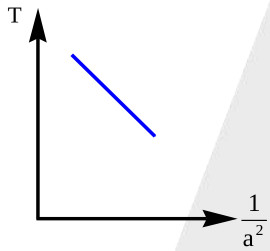


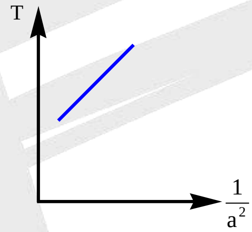
7. For the arrangement shown in the figure let a and T be the acceleration of the blocks and tension in the string respectively. The string and the pulley are frictionless and massless. Which of the graphs show the correct relationship between a and T for the system in which sum of the two masses m_1 and m_2 is constant.



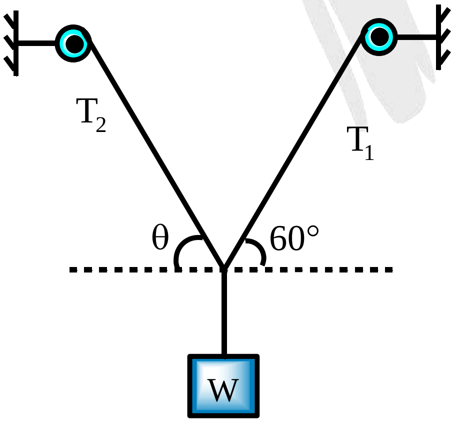
(A) 

(B) 

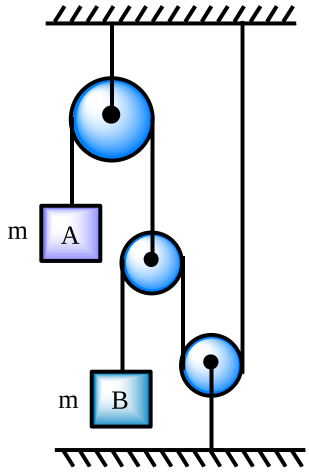
(C) 

(D) 

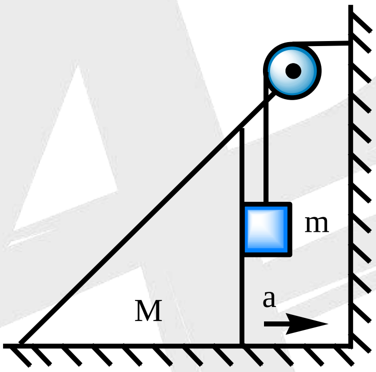
8. A weight W is supported by two cables as shown. The tension in the cable making angle θ with horizontal will be the minimum when the value of θ (in degree) is



9. In the arrangement shown, the pulleys and the strings are ideal. The acceleration of block B is $a = \frac{\alpha g}{\beta}$. Find $\alpha + \beta$?



10. If wedge is moving with acceleration a as shown in the figure and friction coefficient between two blocks is μ then value of net force on m is $F_{net} = \sqrt{\alpha}ma$. Find α ?



11. How could a 10kg object be lowered down from a height using a cord with a breakup strenght of 80N, without breaking the cord

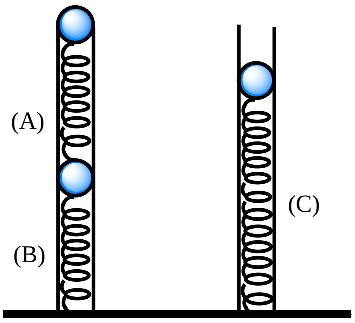
(A) Lowering the object very slowly

(B) Lowering it with an acceleration less than $2m/sec^2$

(C) Lowering it with an acceleration greater than $2m/sec^2$

(D) Object cannot be lowered down without breaking the cord.

12. Three springs, labelled (A), (B) and (C) are arranged as shown with three identical balls all the springs are parts cut from a single long spring such that A and B have equal natural length ℓ but C has natural length 2ℓ. Which spring is compressible by the smallest amount in equilibrium?



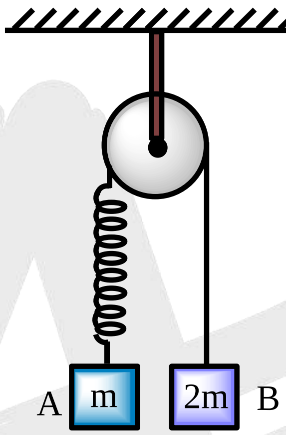
(A) A

(B) B

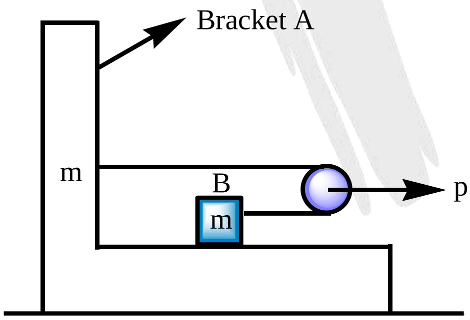
(C) C

(D) Cannot be determined with given information

13. In the figure, block 'A' of mass 'm' is attached to one end of a light spring and the outer end of the spring is connected to another block 'B' of mass 2m through a light string. A is held and B is at rest in equilibrium. Now A is released. The acceleration of A just after that instant is a. The same thing is repeated for B. In that case the acceleration of B is b, then value of a/b is



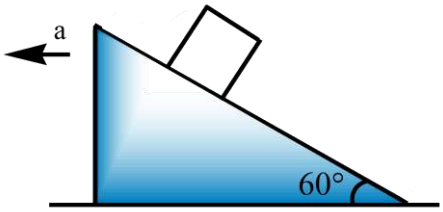
14. In the figure shown a block of mass m rests on a bracket of equal mass m. all the surfaces are smooth. The acceleration of pulley in ground frame when a force P is applied on it is $\frac{\alpha p}{\beta m}$. Find $\alpha + \beta$?



15. Three solid hemispheres of radii R each are placed in contact with each other, with their flat faces on a rough horizontal surface. A sphere of mass M and radius R is placed symmetrically on top of them. The normal reaction between the top sphere and any hemisphere, assuming the system to be static, is

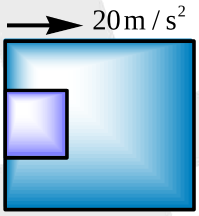
(A) $\frac{Mg}{3}$ (B) $\frac{2Mg}{\sqrt{6}}$ (C) $\frac{Mg}{\sqrt{3}}$ (D) $\frac{\sqrt{6}Mg}{6}$

16. The wedge is moved with constant horizontal acceleration a. for which of the following values of a will the acceleration of the block be different from its acceleration in other three cases.



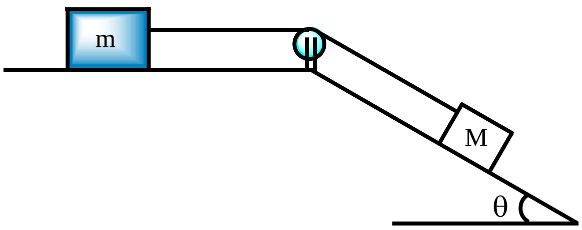
(A) 2g (B) $\sqrt{3}g$ (C) g (D) $\frac{g}{10}$

17. A box is accelerating with acceleration $= 20\,m/s^2$. A block of mass 10kg placed inside the box and is in contact with the vertical wall as shown. The friction coefficient between the block and the wall is $\mu = 0.6$ and take $g = 10\,m/s^2$. The force (in N) acting on the block will be



18. A rectangular body is held at rest by pressing it against a vertical wall for which $\mu < 1$. Which of the following is generally true?
(A) It will be easier to hold the body if the surfaces in contact are smooth
(B) Pressing force required is smaller than weight mg of the body
(C) Pressing force required is greater than weight mg of the body
(D) The required pressing force is independent of coefficient of friction between surfaces in contact

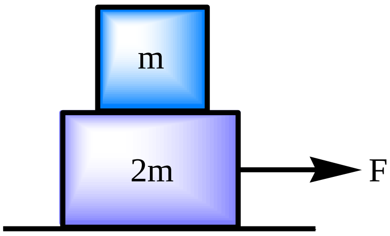
19. Find the maximum value of (M/m) in the situation shown in figure so that the system remains at rest. Friction coefficient of both the contacts is μ, string is massless and pulley is frictionless.



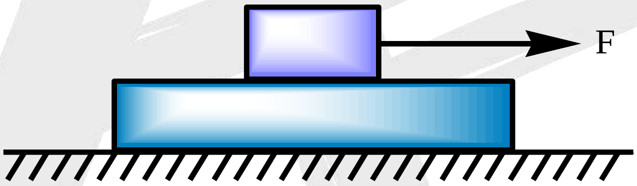
(A) $\frac{2\cos\theta}{\sin\theta + \mu\cos\theta}$ (B) $\frac{\cos\theta}{2\mu\sin\theta + \cos\theta}$

(C) $\frac{\mu\cos\theta}{\sin\theta - \mu\cos\theta}$ (D) $\frac{\mu}{\sin\theta - \mu\cos\theta}$

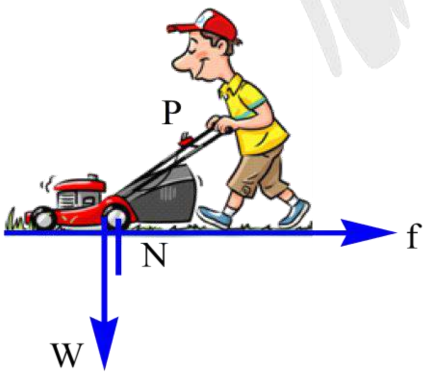
20. A small box of mass m is placed on top of larger box of mass 2m as shown in the diagram. When a force F is applied to the larger box, both boxes accelerate to the right with the same acceleration. If the coefficient of friction between all surfaces is μ, the net force accelerating the smaller mass is $\frac{\alpha F}{\beta} - mg\mu$. Find $\alpha + \beta$?



21. In the figure shown, the friction coefficient between the block of mass 1kg and the plank of mass 2kg is 0.4 while that between the plank and floor is 0.1. A constant force 'F' starts acting horizontally on the upper 1kg block. The acceleration of plank if F = 10N is $\frac{5}{n} m/s^2$. Find n?

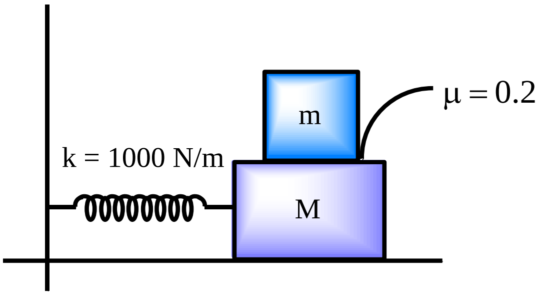


22. A homeowner pushes a lawn mower of across a horizontal patch of grass with a constant speed by applying a force P. The arrows in the diagram correctly indicate the directions but not necessarily the magnitudes of the various forces on the lawn mower. Which of the following relations among the various force magnitudes, W, f, N, P is CORRECT?



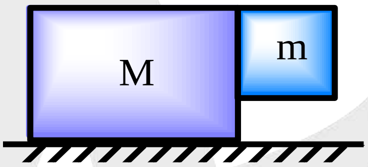
(A) P > f and N > W
(B) P < f and N < W
(C) P > f and N < W
(D) P = 2f and N < W

23. A block of mass M = 8kg is kept on a smooth horizontal plane. A bar of mass m = 2kg is kept on it. They are connected to a spring as shown and the spring is compressed. Then what is the maximum compression in the spring for which the bar will not slip on the block when released if coefficient of friction between them is 0.2, k = 1000 N/m



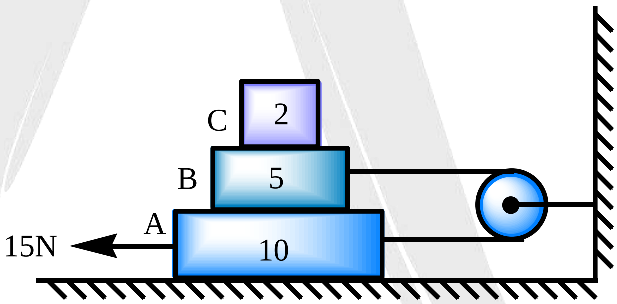
(A) 1cm (B) 1m (C) 1.25cm (D) 2cm

24. With what minimum acceleration mass M must be moved on frictionless surface so that m remains stick to it as shown. the coefficient of friction between M and m is μ



(A) $\frac{\mu g}{M-m}$ (B) $\frac{g}{\mu}$ (C) $\frac{\mu mg}{M+m}$ (D) $\frac{2\mu mg}{M+m}$

25. All surfaces are smooth, string is light and pulley is frictionless, mark the correct statement(s)



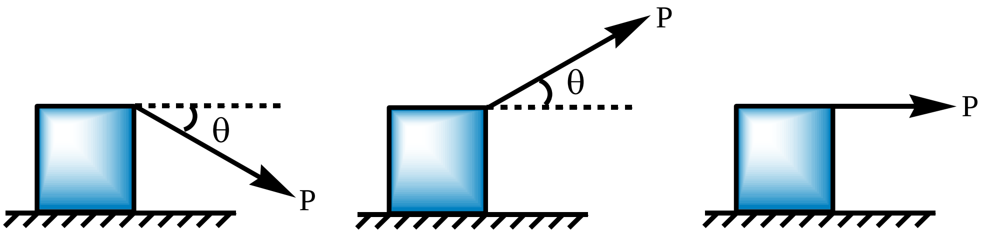
(A) Acceleration of block A is $1 m/s^2$

(B) Acceleration of block A and B will be same

(C) Acceleration of block C is zero

(D) Tension in string between pulley and wall is 20N

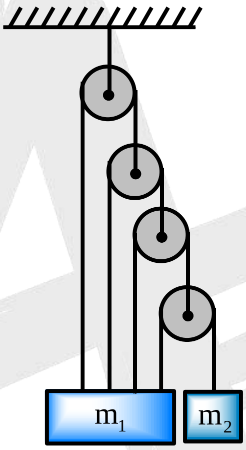
26. Figure shows three blocks on a rough surface under influence of a force P of same magnitude in all the three cases. Coefficient of friction is same between each block and ground. What possible relation holds between magnitudes of normal reaction and friction forces. (Assume that blocks do not overturn about edge). Here f_A, f_B and f_C are frictional forces and N_A, N_B and N_C are reactions



(A) $N_A < N_B > N_C$ (B) $f_A > f_C > f_B$

(C) $f_C > f_A = f_B$ (D) $N_C > N_A = N_B$

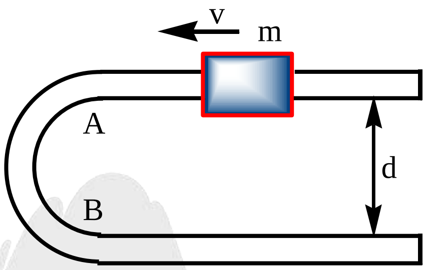
27. If system is in equilibrium then find ratio of $\frac{m_2}{m_1}$ is



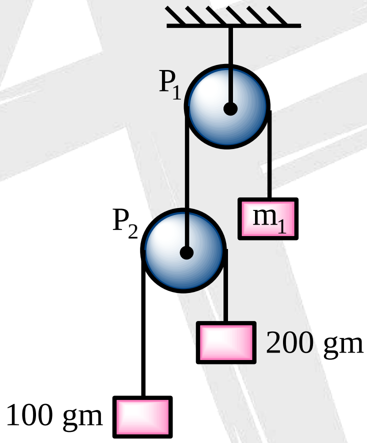
(A) 5 (B) 10 (C) 15 (D) 20

EXERCISE – 3

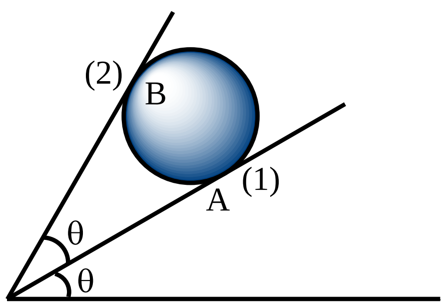
1. A U-shaped smooth wire has a semi-circular bending between A and b as shown in the figure. A bead of mass 'm' moving with uniform speed v through the wire enters the semicircular bend at A and leaves at B. The average force exerted by the bead on the part AB of the wire is $\frac{\alpha m v^{\beta}}{\pi d}$. Find $\alpha+\beta$?



2. In the system of pulleys shown what should be the value of m_1 such that 100 gm remains at rest w.r.t. ground (in gm) is

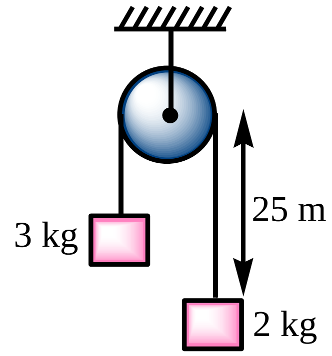


3. A sphere of mass m is kept between two inclined walls, as shown in the figure. If the coefficient of friction between each wall and the sphere is zero, then the ratio of normal reaction (N_1 / N_2) offered by the walls 1 and 2 on the sphere will be :

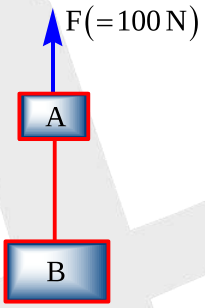


(A) $\cot\theta$ (B) $\cot 2\theta$ (C) $2\cos\theta$ (D) $\sin 2\theta$

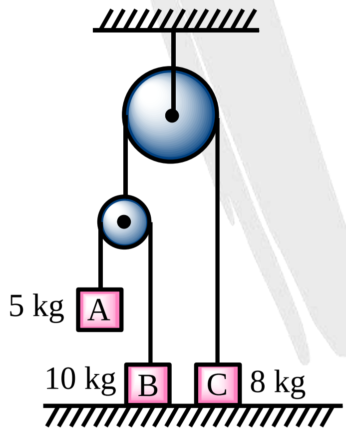
4. The time in which 2 kg mass will strikes the pulley after being released from rest (in sec) is



5. Consider the shown arrangement where the blocks A and B connected by means of a uniform string is being moved vertically up by the force F. Each block weighs 2kg while the mass of string is 1000 gm. The tension (in N) at bottom of the string is



6. In the following arrangement the system is initially at rest. The 5 kg block is now released. Assuming the pulleys and string to be massless and smooth, the acceleration of block C will be:



(A) zero (B) $2.5\ m/s^2$ (C) $10/7\ m/s^2$ (D) $5/7\ m/s^2$

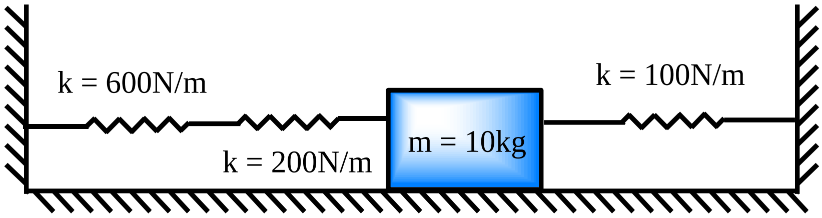
Ans: D

7. A load attached to the end of a spring and in equilibrium produces 9cm extension of spring. If the spring is cut into three equal parts and one end of each is fixed at 'O' and other ends are attached to the same load, the extension in cm of the combination in equilibrium now is

Ans: 1

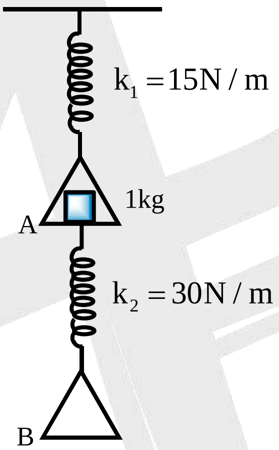
8. A 10kg mass is stationary in the position shown. Each of the three springs are relaxed. Each spring has a characteristic stiffness constant, k, as shown in the diagram. If the mass is now

displaced 40mm to the right, what will be the net force, in newtons, exerted by the three springs on the mass? Ignore gravity



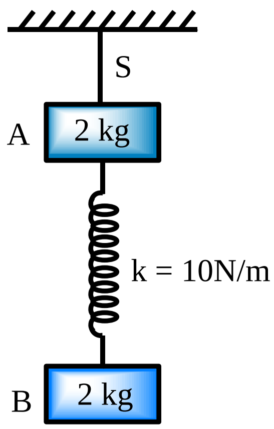
(A) 10N (B) 5N (C) 14N (D) 18N

9. Two ideal springs with light pans are hanging vertically as shown. 1kg mass is placed in pan A, the descent of pan B in equilibrium is x_1. When the same mass is instead placed in pan B the descent of pan B in equilibrium is x_2. Then x_2 / x_1



(A) $\frac{3}{2}$ (B) $\frac{2}{3}$ (C) $\frac{1}{2}$ (D) 1

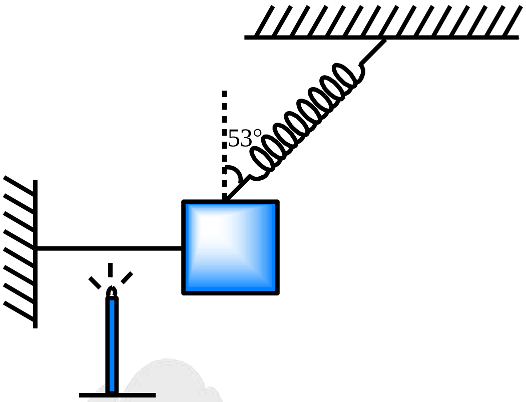
10. The system shown in figure is in equilibrium. The spring is light, the acceleration of both the blocks (in m/s^2) just after the string S is cut is



(A) $a_A = 10, a_B = 0$ (B) $a_A = 0, a_B = 10$

(C) $a_A = 20, a_B = 0$ (D) $a_A = 0, a_B = 20$

11. The block shown in the figure is equilibrium. The acceleration of the block just after the string burns is $\frac{\alpha g}{\beta}$. Find $\alpha+\beta$?



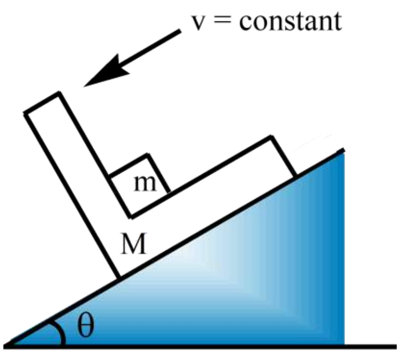
12. Two identical particles A and B, each of mass m, are interconnected by a spring of stiffness k and placed on horizontal ground. If the particle B experiences an external force F and the elongation of the spring is x, the magnitude of relative acceleration between the particles is equal to

(A) $\frac{F}{2m}$ (B) $\frac{F+kx}{2m}$ (C) $\frac{F-2kx}{m}$ (D) $\frac{3kx}{2m}$

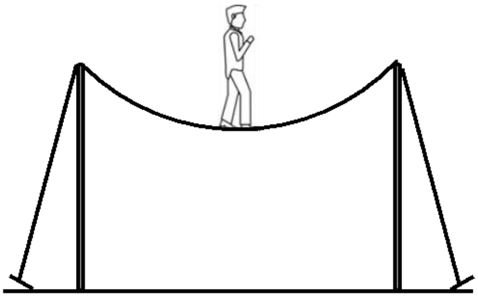
13. A perfectly straight portion of a uniform rope has mass M and length L. At end A of the segment, the tension in the rope is T_A; at end B it is $T_B\left(>T_A\right)$. The tension in the rope at a distance L/5 from end A is

(A) $\frac{T_B-T_A}{4}$ (B) $\frac{2\left(T_A+T_B\right)}{5}$ (C) $\frac{4T_A+T_B}{5}$ (D) $\frac{T_B-T_A}{5}$

14. Figure shows a block of mass m placed on a bracket of mass M. Bracket block system is moved downward with constant velocity on an incline. The magnitude of total force of bracket on block is $\frac{\alpha mg}{\beta}$. Find $\alpha+\beta$?



15. A circus performer of weight W is walking along a "high wire" as shown. Assume that wire is very tight. Tension in the wire



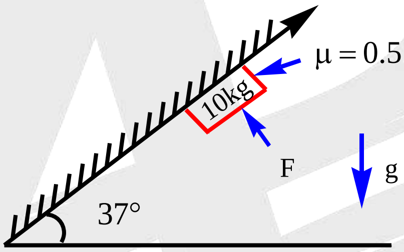
(A) is approximately W

(B) is much less than W

(C) is much more than W

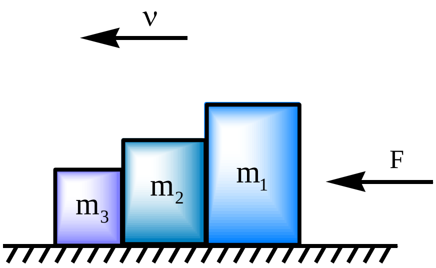
(D) depends on whether he stands on one foot or two feet

16. In the figure shown, the minimum force F (in N) to be applied perpendicular to the incline so that the block does not slide is



17. A given object taken m times as much time to slide down a 45° rough incline as it takes to slide down a perfectly smooth 45° incline. The coefficient of kinetic friction between the object and the incline is $\mu_k = \alpha - \frac{\beta}{m^\gamma}$. Find $\alpha + \beta + \gamma$?

18. Three blocks (m_1, m_2, m_3) are slid at constant velocity across a rough surface as shown. The coefficient of kinetic friction between each block and the surface is μ. Find force applied by m_1 on m_2?



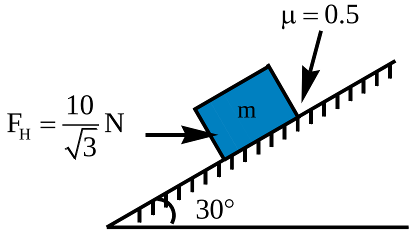
(A) $(m_1 - m_3)\mu g$

(B) $(m_2 + m_3)g\mu$

(C) $(m_1 - m_2 - m_3)\mu g$

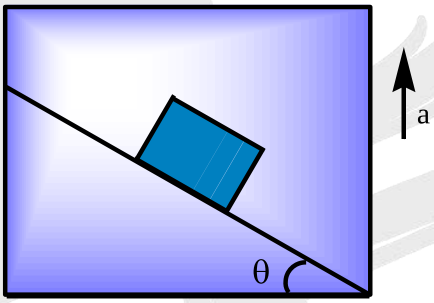
(D) $(m_1 - m_2 + m_3)\mu g$

19. If the mass of block is 2kg and a force of $\frac{10}{\sqrt{3}}$ N is applied horizontally on the block as shown in the figure. The frictional force acting on the block is



(A) zero (B) $\frac{10}{\sqrt{3}}$ N (C) $\frac{20}{\sqrt{3}}$ N (D) 5N

20. A block of mass m is at rest with respect to a rough incline kept in elevator moving up with acceleration a. which of following statement is correct?



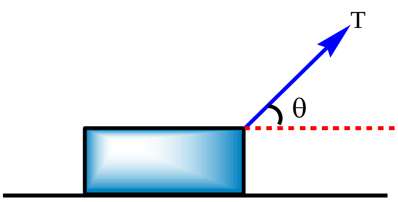
(A) The contact force between block and incline is parallel to the incline

(B) The contact force between block and incline is of magnitude $m(g+a)$

(C) The contact force between block and incline is perpendicular to the incline

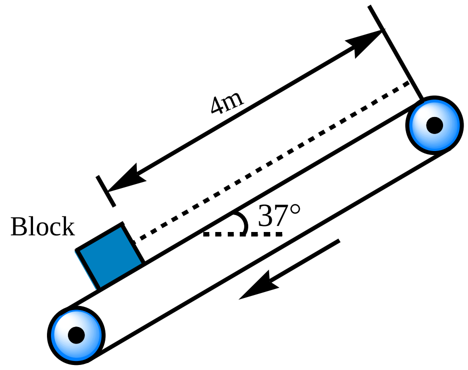
(D) The contact force is of magnitude $mg\cos\theta$

21. A block of mass m is pulled in the direction shown in the figure on a rough horizontal ground with a constant acceleration of magnitude 'a'. The magnitude of the frictional force is



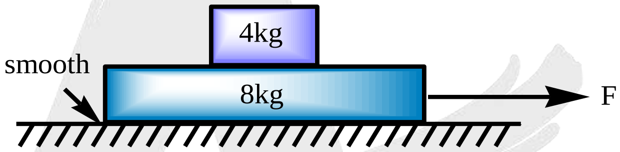
(A) $T\cos\theta - ma$ (B) $(T\cos\theta - 2ma)$ (C) $\mu_k(T - mg)$ (D) $(T\cos\theta + ma)$

22. The following figure shows an accelerating conveyor belt inclined at an angle 37° above horizontal. The coefficient of friction between the belt and block is '1'. The least time (in sec) in which block can reach the top, starting from rest at the bottom is

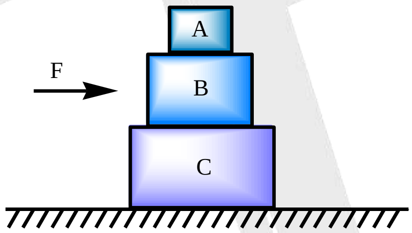


23. A block of 4kg is placed on a plank having mass 8kg. A force F = 20N is applied on plank. Then the friction force between 4 kg and 8kg block is $\frac{10\alpha}{3}$ N. Find α ?

(Here coefficient of friction between 4kg and 8kg is $\mu = 0.4$, floor is smooth).



24. Force 'F' is applied on block 'B' and the system remains at rest. All surfaces are rough. Which of the following statements is/are correct?



(A) Friction force on C does not depend upon force applied F

(B) Friction on C due to ground is equal to friction force on B due to C

(C) Friction force on A must be zero

(D) Friction force on A does not depend upon force applied F

25. A lorry is carrying a box of mass 400kg. The coefficient of static friction between the box and lorry is 0.37 and $\mu_k = 0.36$. Lorry begins to accelerate at a constant rate such that the box just begins to slide. Choose the correct statement

(A) The acceleration of lorry is $3.6 m/s^2$

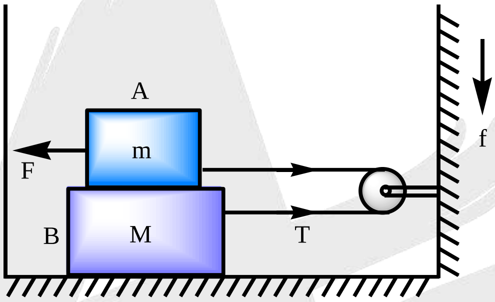
(B) The acceleration of box is $3.6 m/s^2$ in a direction of lorry

(C) The acceleration of lorry is $3.7 m/s^2$

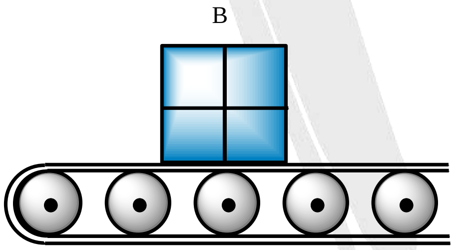
(D) The acceleration of box is $7.3 m/s^2$ in a direction same as that of the lorry

26. The rear side of a truck is open and a box of mass 20kg is placed on the truck 4meters away from the open end $\mu = 0.15$ and $g = 10 m/s^2$. The truck starts from rest with an acceleration of $2 m/s^2$ on a straight road. The box will fall off the truck when truck is at a distance (in meters) from the starting point equal to

27. Two blocks A and B of masses m and M are placed in a platform as shown in the figure. The whole arrangement is placed inside an elevator which is coming down with an accelerator f $(f < g)$. The maximum horizontal force F can be applied to A without disturbing the equilibrium of the system is $\alpha\mu m(g - f)$. Find α ? (Here μ is the coefficient of friction between blocks. Floor of lift is smooth)

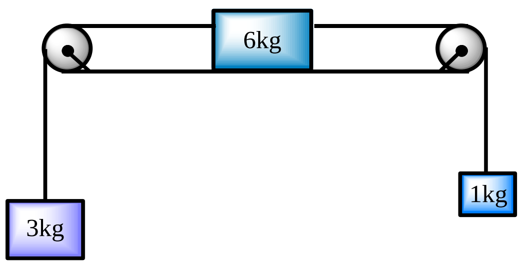


28. The conveyor belt is moving at 8m/s. The coefficient of static friction between the conveyor belt and the 10kg package B is $\mu_s = 0.4$. Determine the shortest time in which the belt can be stopped so that the package does not slide on the belt.



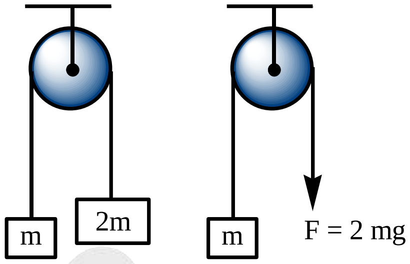
(A) 1s (B) 2s (C) 4s (D) 8s

29. Three blocks of masses 3kg, 6kg and 1kg are connected by a string passing over two smooth pulleys attached at the two ends of the horizontal surface. If the coefficient of friction between the block and the surface is 0.6, the frictional force (in N) acting on the block of mass 6kg is



EXERCISE – 4

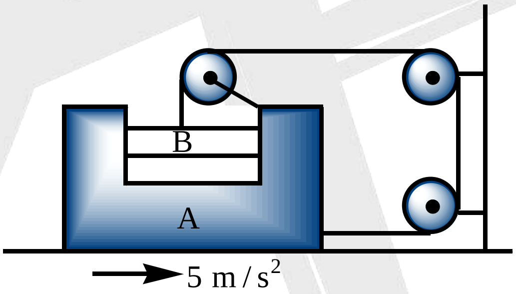
1. Figure shows two pulley arrangements for lifting a mass m. In (a) the mass is lifted by attaching a mass 2 m while in (b) the mass is lifted by pulling the other end with a downward force 2 mg, if f_a and f_b are the accelerations of the two masses then $f_a = \frac{\alpha f_b}{\beta}$. Find $\alpha + \beta$?



2. Two blocks w_1 and w_2 are suspended from the ends of a light string passing over a smooth pulley. If the pulley is pulled up at an acceleration g, the tension in the string will be:

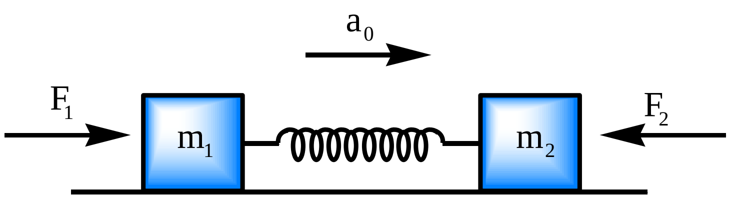
(A) $\frac{4w_1w_2}{w_1+w_2}$ (B) $\frac{2w_1w_2}{w_1-w_2}$ (C) $\frac{w_1+w_2}{w_1-w_2}$ (D) $\frac{w_1w_2}{2(w_1-w_2)}$

3. If blocks A is moving with an acceleration of $5\,m/s^2$, the acceleration of B w.r.t. ground is :



(A) $5\,m/s^2$ (B) $5\sqrt{2}\,m/s^2$ (C) $5\sqrt{5}\,m/s^2$ (D) $10\,m/s^2$

4. Two blocks m_1 and m_2 are connected with a compressed spring and placed on a smooth horizontal surface as shown in figure.



Force constant of spring is k. under the influence of forces F_1 and F_2, at an instant blocks move with common acceleration a_0. At that instant force F_2 is suddenly withdrawn. Mark correct option(s).

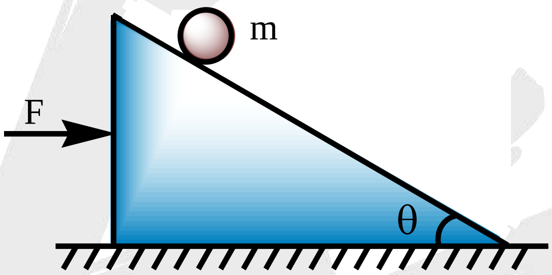
(A) Instantaneous acceleration of m_1 is $a_0 - \frac{F_1}{m_1}$

(B) Instantaneous acceleration of m_2 is $a_2 = a_0 + \frac{F_2}{m_2}$

(C) Instantaneous acceleration of m_1 is $a_1 = 0$

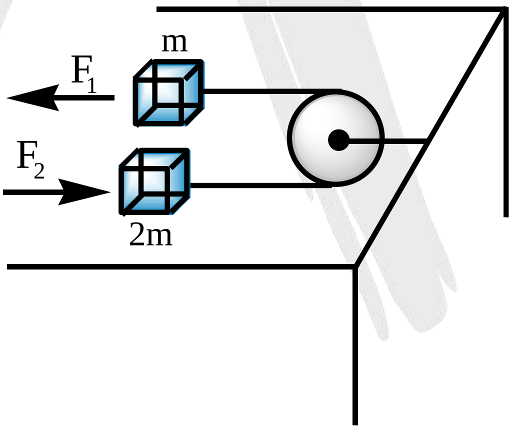
(D) Instantaneous acceleration of m_2 is $a_2 = 0$

5. In the given figure, the wedge is acted upon by a constant horizontal force F. The wedge is moving on a smooth horizontal surface. A ball of mass 'm' is at rest relative to the wedge. The ratio of forces exerted on 'm' by the wedge when F is acting, and when F is withdrawn, assuming no friction between the wedge and the ball, the ratio is equal to



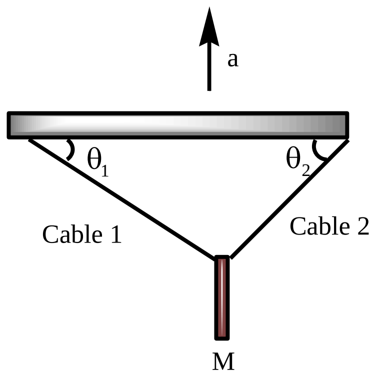
(A) $\sec 2\theta$ (B) $\cos 2\theta$ (C) 1 (D) $\sin 2\theta$

6. In which case will the string slack (the table is smooth)?



(A) $F_1 = F_2$ (B) $F_1 = 0.75F_2$ (C) $F_1 = 0.25F_2$ (D) Direction of F_2 is reversed

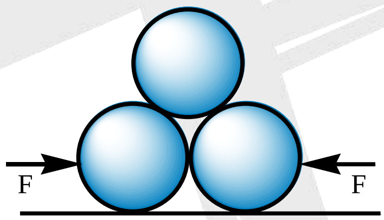
7. A steel block of mass M is being lifted by a helicopter, which is accelerating upward with an acceleration of magnitude a. The block is attached to a horizontal bar by two cables, which make angles of θ_1 and θ_2 w.r.t. horizontal as shown in the figure. The tension in the cable 1 in terms of M, θ_1, θ_2, g and a is



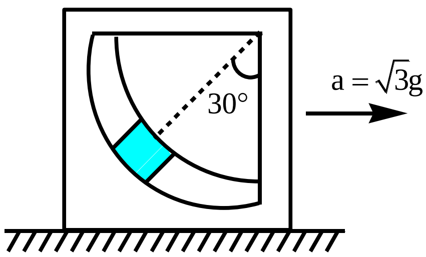
(A) $\dfrac{M(g+a)\cos\theta_1}{\sin(\theta_1+\theta_2)}$ (B) $\dfrac{M(g+a)\sin\theta}{\cos(\theta_1+\theta_2)}$

(C) $\dfrac{M(g+a)\sin\theta_2}{\cos(\theta_1+\theta_2)}$ (D) $\dfrac{M(g+a)\cos\theta_2}{\sin(\theta_1+\theta_2)}$

8. Two smooth cylindrical bars weighing W N each lie next to each other in contact. A similar third bar is placed over the two bars as shown in figure. Neglecting friction, the minimum horizontal force on each lower bar necessary to keep them together is $\dfrac{\alpha W}{2\sqrt{\beta}}$. Find $\alpha+\beta$?

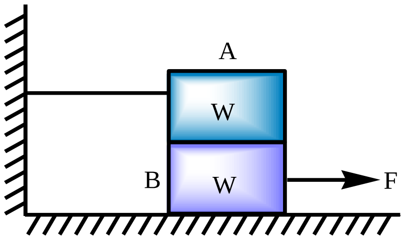


9. A wedge with a rough groove in the shape of a quarter of a circle is kept on a smooth table (see figure). A disc is placed in the groove with a small clearance. Friction exists between groove and disc. The wedge is moved with an acceleration $\sqrt{3}g$. If disc is to remain stationary relative to groove, the coefficient of friction required can be



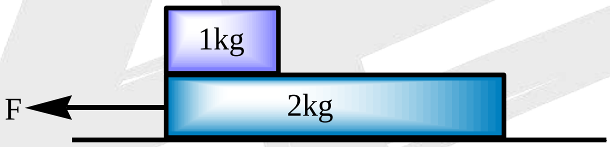
(A) $\dfrac{1}{6}$ (B) $\dfrac{2}{7}$ (C) $\dfrac{1}{8}$ (D) $\dfrac{9}{10}$

10. Two identical blocks of weight W are placed one on top of the other as shown in figure. The upper block is tied to the wall. The coefficient of static friction between B and ground is μ and friction between A and B is absent when $F = \mu W$ force is applied on the lower block as shown. the tension in the string will be

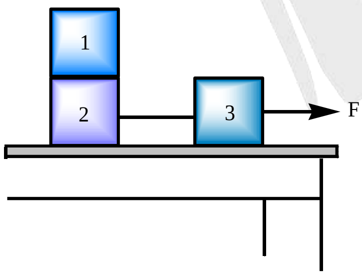


(A) $\frac{\mu W}{3}$ (B) $\frac{\mu W}{2}$ (C) 0 (D) $2\mu W$

11. A small block of mass 1kg is placed over one end of a plank of mass 2kg. The length of the plank is 2m. Coefficient of friction between the block and the plank is 0.5 and the ground is frictionless. A constant force F = 30N is applied on the plank in horizontal direction as shown in the figure. The time after which the block will separate from plank $\left(g = 10\,m/s^2\right)$ is $\frac{73}{n}$ sec. Find n?



12. Block 1 is stacked on top of block 2, block 2 is connected by a light cord to block 3, which is pulled along a frictionless surface with a force F as shown in the diagram. Block 1 is accelerated at the same rate as block 2 because of the frictional force between the two blocks. If all three blocks have the same mass m, which of the following cannot be coefficient of static friction between block 1 and block 2?

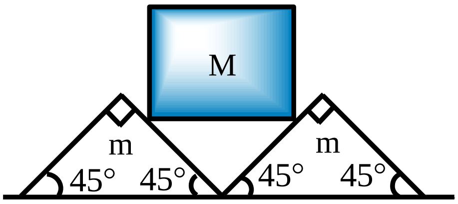


(A) $\frac{2F}{mg}$ (B) $\frac{2F}{3mg}$ (C) $\frac{3F}{mg}$ (D) $\frac{F}{4mg}$

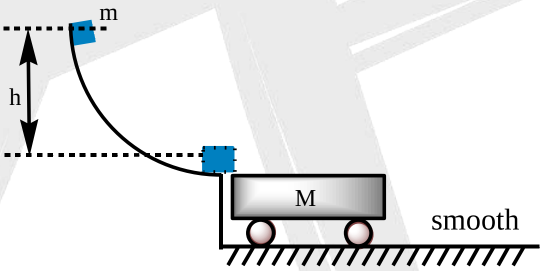
13. A horizontal force acting on a box of mass 1kg has a magnitude of $F = t^2 N$, where t is in seconds. If the box starts from rest, determine its speed at t = 4sec. The coefficient of static and kinetic friction between the box and horizontal floor are $\mu_s = 0.4$ and $\mu_k = 0.3$

(A) 19 m/s (B) 1.3 m/s (C) 3.33 m/s (D) 9.3 m/s

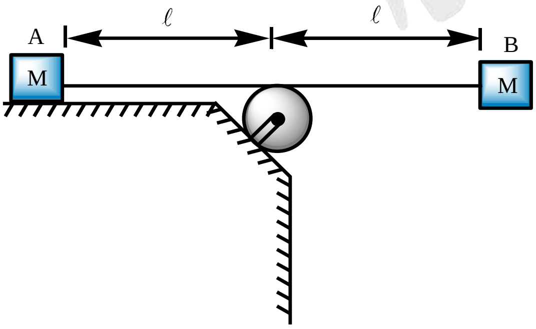
14. Two wedges, each of mass m, are placed next to each other on a flat floor. A cube of mass M is balanced on the wedges as shown. Assume no friction between the cube and the wedges, but a coefficient of static friction $\mu < 1$ between the wedges and the floor. The largest M that can be balanced as shown without motion of the wedges is $\frac{\alpha \mu m}{\beta - \mu}$. Find $\alpha + \beta$?



15. A carriage of mass M and length ℓ is joined to the end of a slope as shown in the figure. A block of mass m is released from the slope from height h. It slides till end of the carriage (The friction between the body and the slope and also friction between carriage and horizontal floor is negligible). Coefficient of friction between block and carriage is μ. The minimum h in terms of given parameters is $\mu\left(\alpha + \frac{\beta m}{M}\right)\ell$. Find $\alpha + \beta$?

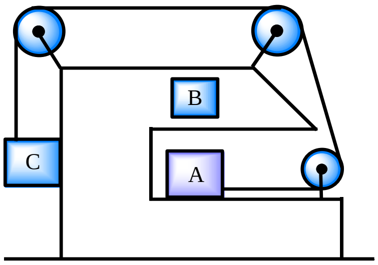


16. The system shown in figure is released from rest at t = 0. Block A hits pulley at $t = t_1$ and B hits the vertical wall at $t = t_2$. If the pulley is light and friction is absent, then



(A) $t_1 > t_2$ (B) $t_1 < t_2$ (C) $t_1 = 3t_2$ (D) $2t_1 = t_2$

17. For given figure m_A is 30kg, $m_B = m_C = 5\text{kg}$ respectively. All contact surfaces are smooth. $\left(g = 10\,m/s^2\right)$. Select the correct statement(s)



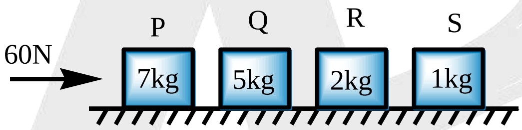
(A) Acceleration of block A, B and C are $1m/s^2$, $3m/s^2$ and $5m/s^2$ respectively

(B) Contact force between B and Ground is 350N

(C) Tension in string is 40N

(D) Contact force between 'B' and 'C' is 15N

18. Four blocks are kept in a row on a smooth horizontal table with their centres of mass collinear as shown in the figure. An external force of 60N is applied from left on the 7kg block to push all of them along the table. The forces exerted by them are

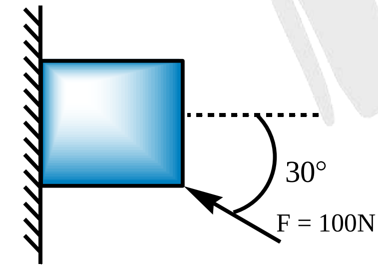


(A) 32N by P on Q (B) 28N by Q on P (C) 12N by Q on R (D) 8N by S on R

19. A block of mass 5kg is kept on a rough horizontal surface $(\mu = 0.5)$. The block can be pulled horizontally by an unknown force. The contact force between the surface and block can be

(A) 50N (B) 45N (C) 65N (D) 52N

20. A force of 100N is applied on a stationary block of mass 3kg as shown in figure. If the coefficient of friction between the surface and the block is 0.25 then



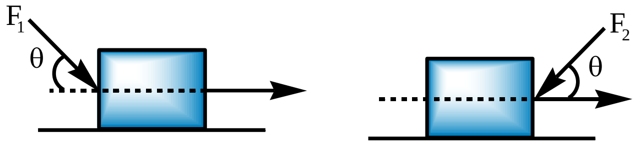
(A) The frictional force acting on the block is 30N downwards

(B) The friction force acting on the block is $\frac{25\sqrt{3}}{2}$N

(C) The normal reaction on the block is $50\sqrt{3}$

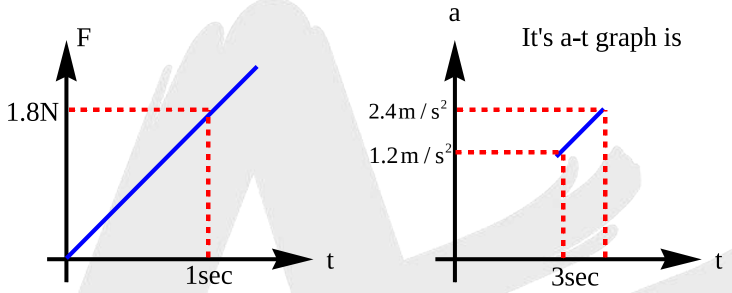
(D) If coefficient of friction is changed to 0.35 then the friction force acting on the block is again 20N downwards

21. In the two cases shown below, the coefficient of kinetic friction between the block and the surface is the same, and both the identical blocks are moving with the same uniform speed. Then



(A) $F_1 = F_2$ (B) $F_1 < F_2$ (C) $F_1 > F_2$ (D) $F_1 = 2F_2$ if $\sin\theta = \frac{Mg}{4F_2}$

22. 1.5kg box kept on horizontal rough ground is pushed by a horizontal force whose F – t graph is shown.



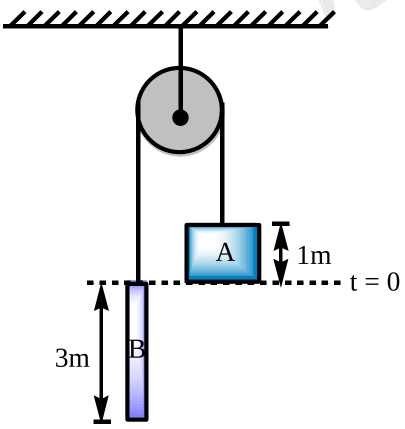
(A) Coefficient of static friction is 0.36

(B) Coefficient of static friction is 0.42

(C) Time when acceleration is $2.4 m/s^2$ is 5sec

(D) Coefficient of kinetic friction is 0.24

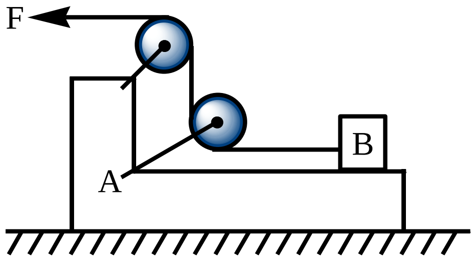
23. At t = 0, the lower end of the bar A is just above the upper end of bar B (mass of bar A = 3kg, mass of bar B = $\frac{11}{3}$ kg). Find time when upper end of block A just crosses the lower end of B. (Assume the system was released at t = 0). $\left[g = 10 m/s^2\right]$



(A) 4sec (B) 3sec (C) 2sec (D) 1sec

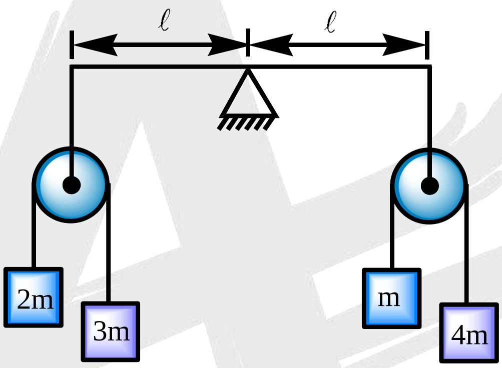
EXERCISE – 5

1. All surfaces are smooth and pulleys are ideal. The string is pulled with force F. mass of A = mass of B = m.



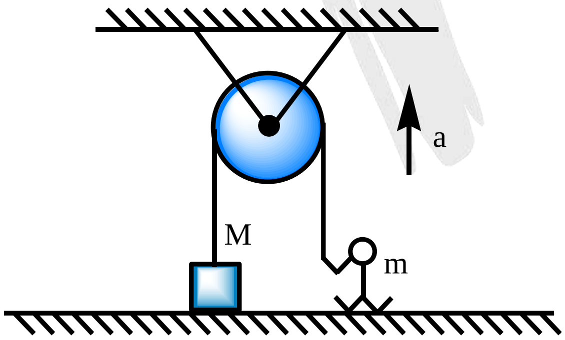
(A) $a_A = a_B$ (B) $a_A = 0, a_B \neq 0$ (C) $a_B = 0, a_A \neq 0$ (D) $a_A = 2a_B$

2. Find the accelerations in given diagram.



(A) $T_1 = \frac{12mg}{5}$ (B) $T_1 = \frac{5mg}{12}$ (C) $T_2 = \frac{5mg}{8}$ (D) $T_2 = \frac{8mg}{5}$

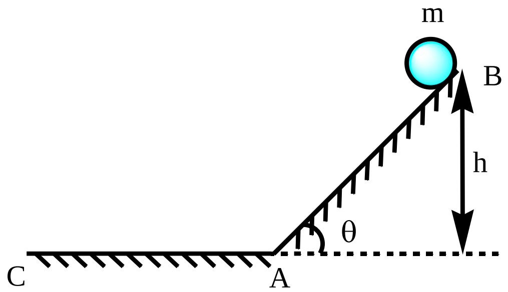
3. In the figure the block of mass M is at rest on the floor. The acceleration with which should a boy of mass m climb along the rope of negligible mass so as to lift the block from the floor, is



(A) Equal to $\left(\frac{M}{m}+1\right)g$ (B) Greater than $\left(\frac{M}{m}-1\right)g$

(C) Equal to $\frac{M}{m}g$ (D) Greater than $\frac{M}{m}g$

4. A particle of mass 'm' slides down on a smooth inclined plane from a point B at a height of h from rest. The magnitude of change in momentum of the particle between the positions A and C (assuming the angle of inclination of the plane as θ) with respect to the horizontal is (Consider A is lowest point of incline)



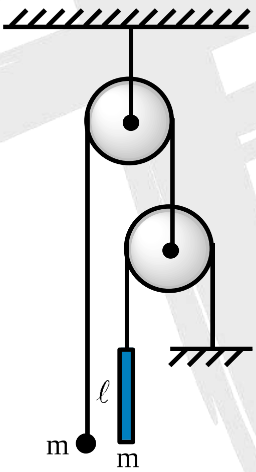
(A) 0

(B) $2m\sqrt{2gh}\sin\theta$

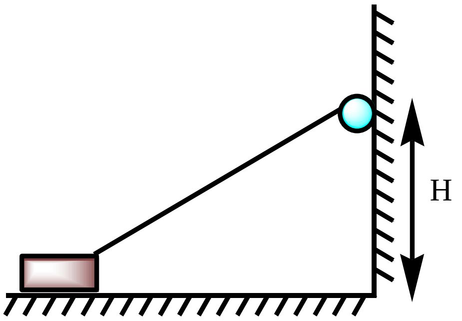
(C) $2m\sqrt{2gh}\sin\left(\frac{\theta}{2}\right)$

(D) $2m\sqrt{2gh}$

5. In the figure shown, all pulleys are massless and frictionless. The time taken by the ball to reach the upper end of the rod is $t=\sqrt{\frac{\alpha\ell}{\beta g}}$. Find $\alpha+\beta$?



6. A small engine, fixed to a vertical wall at a height of H, winds up a piece of thread with a constant speed of v_0. At the other end of the thread there is a small body which moves along the horizontal ground (friction is not negligible). How far is the small body from the wall when it rises from the ground? (Given: H = 10cm, $v_0 = 4\,m/s$)



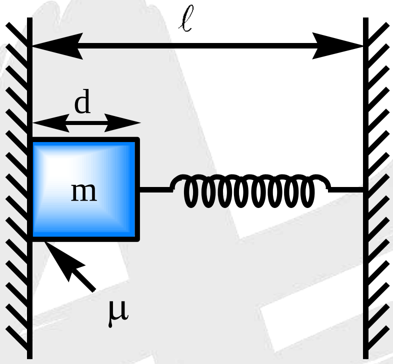
(A) 0.1m (B) 0.2m (C) 0.4m (D) 0.6m

7. A particle of mass m is made to move with a uniform speed v_0 along the perimeter of a regular hexagon. The magnitude of impulse applied at each corner of the hexagon is

(A) $2mv_0 \sin\left(\frac{\pi}{6}\right)$ (B) $mv_0 \cos\left(\frac{\pi}{3}\right)$

(C) $mv_0 \cos\left(\frac{\pi}{6}\right)$ (D) $2mv_0 \cos\left(\frac{\pi}{3}\right)$

8. A block of mass m is pressed against a vertical surface by a spring of unstretched length ℓ. If the coefficient of friction between the block and the surface is μ. Choose the correct statement



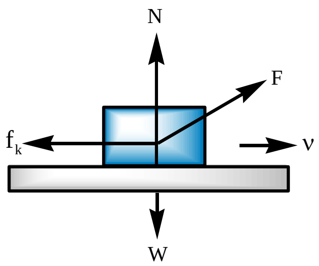
(A) If spring constant $k = \frac{2mg}{\mu d}$, block will not be in equilibrium

(B) If spring constant is $k = \frac{2mg}{\mu d}$, the normal reaction is $\frac{mg}{\mu}$

(C) In the part (B) force of friction is 2mg

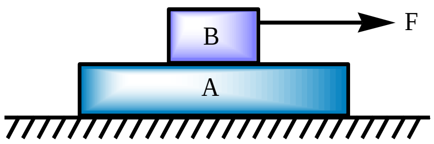
(D) Minimum spring consta k_{min} to keep the block of mass M in equilibrium is $\frac{mg}{\mu d}$

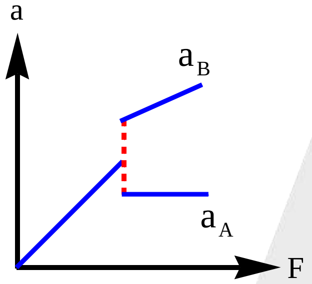
9. A person pulls a block across a rough horizontal surface at a constant speed by applying a force F. The arrows in the figure correctly indicate the directions, but not necessarily the magnitudes of the various forces on the block. Which of the following relations among the force magnitudes W, f_k, N and F must be true (symbols have their usual meaning)?



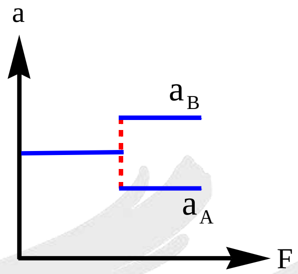
(A) $F = f_k$ (B) $F > f_k$ (C) $N = W$ (D) $N > W$

10. A long wooden plank A of mass M is placed on a frictionless horizontal surface. On top of A, another block B of equal mass M is placed. A horizontal force of magnitude F is applied to B. Force 'F' increases proportionally with time starting from value zero. The graphs below show the dependence of acceleration of the two bodies on the force F. Which of the graph is correct? Given that coefficient of static and kinetic friction between A and B are μ_S and μ_K respectively.

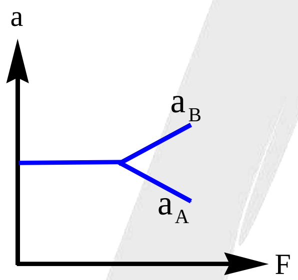




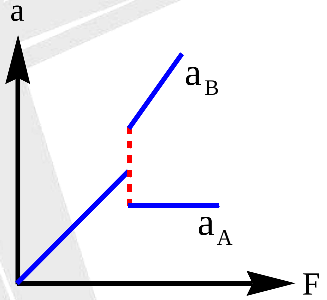
(A)



(B)

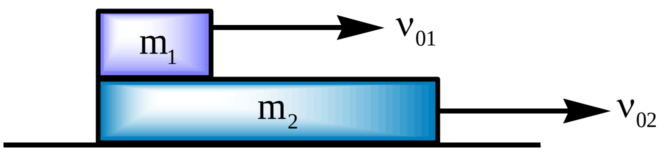


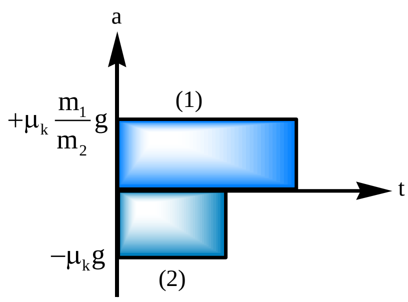
(C)

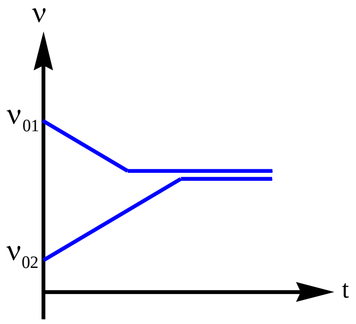


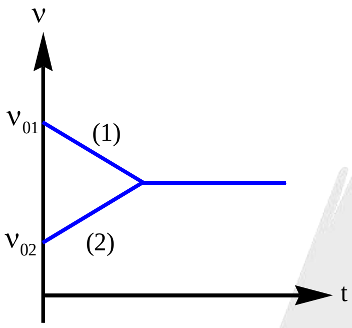
(D)

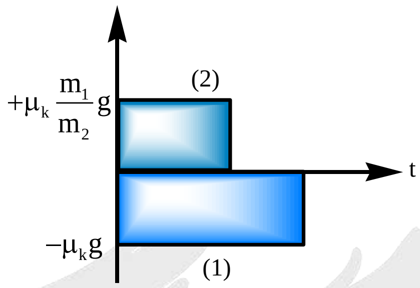
11. Block m_1 is projected on a long plank of mass m_2. Plank is placed on a smooth horizontal surface. There is friction between block and plank, coefficient of friction is μ. Block m_1 has initial velocity v_{01} and plank has initial velocity v_{02} with $(v_{01} > v_{02})$. Which of the following graphs are correct?



(A) 

(B) 

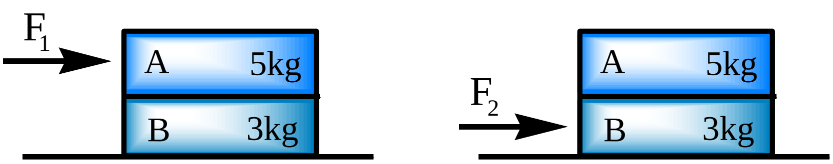
(C) 

(D) 

12. A body of mass m is initially at rest on a horizontal surface. A constant horizontal force of mg/2 is applied to the body directed to the right. The coefficient of friction of the surface changes with the distance pushed as $\mu = \mu_0 x$ where x is the distance from the initial location. For what distance is the body pushed until it comes to rest again?

(A) $\frac{2}{3\mu_0}$ (B) $\frac{3}{2\mu_0}$ (C) $\frac{1}{\mu_0}$ (D) $\frac{1}{4\mu_0}$

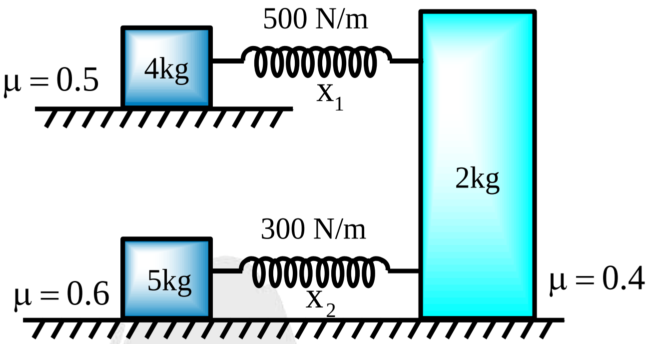
13. A block A (5kg) rests over another block B (3kg) placed over a smooth horizontal surface. There is friction between A and B. A horizontal force F_1 gradually increasing from zero to a maximum is applied to A so that the blocks move together without having motion relative to each other. Instead of this, another horizontal force F_2 gradually increasing from zero to a maximum is applied to B so that the blocks move together without relative motion. The magnitudes of friction between the blocks in the two cases are f_1 f_2 respectively during the variation of F_1 and F_2 respectively. Then



(A) $f_{1max} > f_{2max}$ (B) $F_{1max} : F_{2max} = 3:5$

(C) $F_{1max} : F_{2max} = 5:3$ (D) $f_1 < F_1$

14. The system shown is in equilibrium. Positive value of x_1 and x_2 denote extensions of the springs, while negative values denote compressions. Which of the following is not possible?



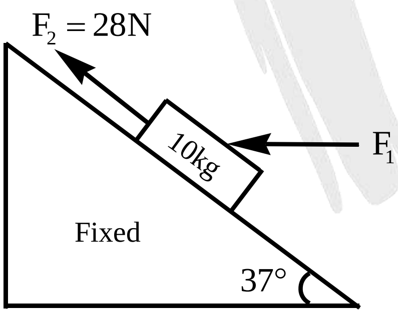
(A) $x_1 = 13cm,\ x_2 = 2cm$ (B) $x_1 = 8cm,\ x_2 = 3cm$

(C) $x_1 = 1cm,\ x_2 = -8cm$ (D) $x_1 = -1cm,\ x_2 = -2cm$

15. Two blocks of mass 2kg and 3kg having coefficient of static friction 0.3 and 0.4 respectively are joined by a spring of constant 100N/m and kept on level ground. What can be a possible extension in the spring so that both the blocks are in equilibrium?

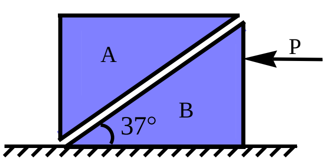
(A) 3cm (B) 6cm (C) 4cm (D) 15cm

16. Diagram shows a block at rest in equilibrium on a fixed inclined plane. If for the given case F_1 is horizontal and F_2 is along the incline. Find F_1 ?



(A) 40N (B) 30N (C) 20N (D) 10N

17. Blocks A and B each have same mass m = 1kg. Determine the largest horizontal force P which can be applied to B so that A will not slip up on B. Neglect any friction.

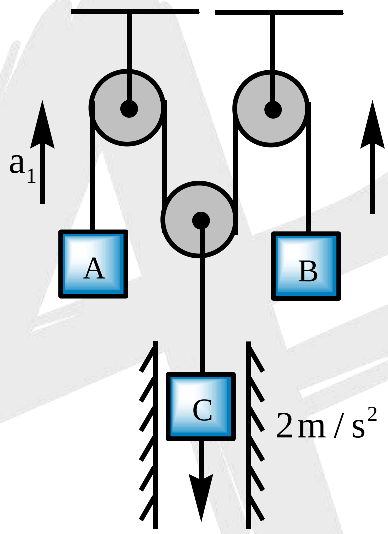


(A) 5N (B) 10N (C) 15N (D) 20N

18. A block of weight W produces an extension of 9cm when it is hung by an elastic spring of length 60cm and is in equilibrium. The spring is cut into two parts, one of length 40cm and the other of length 20cm. The same load W hangs in equilibrium supported by both parts. Find the extension in the spring.

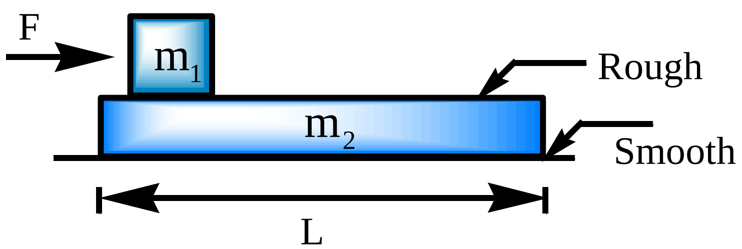
(A) 1cm (B) 2cm (C) 3cm (D) 4cm

19. The block C shown in the figure is descending with an acceleration $2m/s^2$ by means of some external mechanism, not shown here. The acceleration of the bodies A and B of masses 10kg and 5kg are a_1 and a_2 respectively, assuming pulleys and stings are massless and friction is absent everywhere then find the value of $|a_1|+|a_2|$.

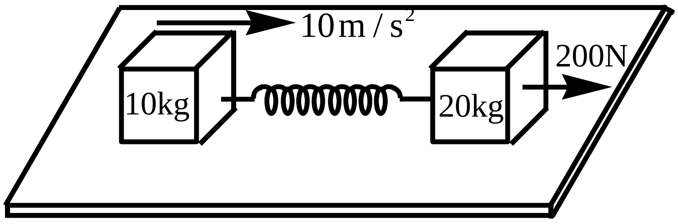


(A) $2m/s^2$ (B) $4m/s^2$ (C) $6m/s^2$ (D) $8m/s^2$

20. In the figure shown a plank of length L = 16m, mass $m_2 = 5kg$ rests on a smooth surface. Upper surface of plank is rough with coefficient of kinetic friction μ_k = 0.5 and static friction $\mu_s = 0.6$. A small block of mass $m_1 = 2kg$ is placed over it. A force F of magnitude 30N is applied on block m_1. What is displacement of plank till the small block falls over from plank?

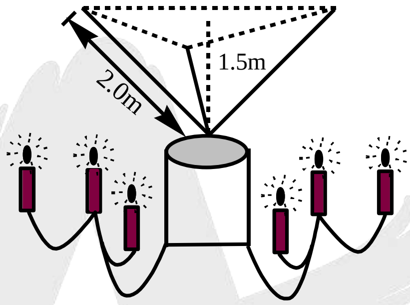


21. The masses of 10kg and 20kg respectively are connected by a massless spring as shown in figure. A force of 200N acts on the 20kg mass. At the instant shown, the 10kg mass has acceleration $10m/sec^2$ then calculate the acceleration of 20kg mass.



(A) $2.5\,m/sec^2$ (B) $5\,m/sec^2$ (C) $7.5\,m/sec^2$ (D) $10\,m/sec^2$

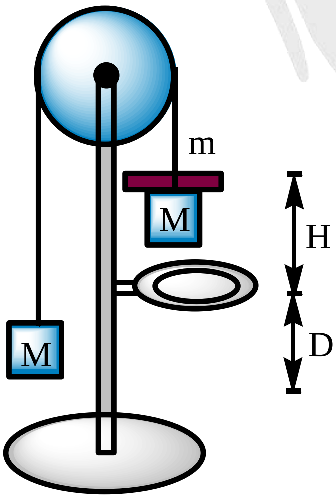
22. A 45kg chandelier is suspended 1.5m below a ceiling by three identical wires, each of which has the same tension and the same length of 2.0m (see the drawing). Find the tension in each wire.



(A) 50N (B) 100N (C) 150N (D) 200N

23. Atwood's machine was an apparatus that allowed a direct verification of Newton's second law. It could also be used to measure g. Two equal blocks of mass M hang at either side of a pulley, see figure. A small square rider of mass m is placed on one block. When the block is released, it accelerated for a distance H till the rider is caught by a ring that allows the block to pass. From then on the system moves at a constant speed which is measured by timing the fall through a distance D. t is the time moved at constant speed. Find value of 'g'.

[Given: $M = 4kg$, $m = 1kg$, $D = 2m$, $H = 1m$, $t = 1s$]



(A) $3m/s^2$ (B) $6m/s^2$ (C) $9m/s^2$ (D) $12m/s^2$

EXERICES – (ALP)

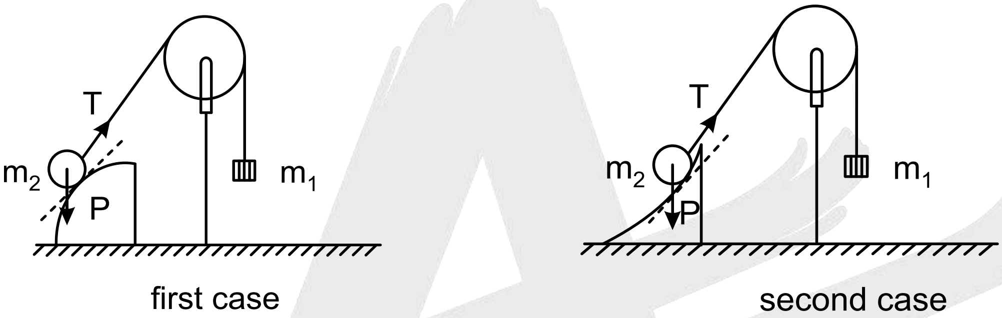
1. In both the cases shown in figure mass m_1 is so chosen that mass m_2 which rests on smooth surface, is in equilibrium. (for simplification we shall suppose that the pulley is sufficiently far away, and that therefore the line of the rope from mass m_2 is parallel to the line of the tangent to the surface)

(A) first case comes under unstable equilibrium condition

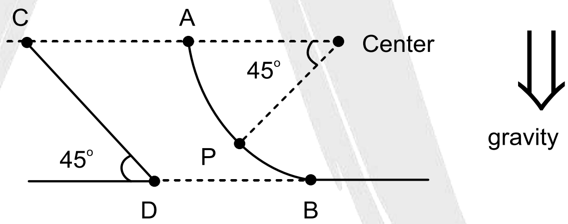
(B) second case comes under stable equilibrium condition

(C) first case comes under stable equilibrium condition

(D) second case comes under unstable equilibrium condition



2. CD is an inclined plane and AB is a circular portion. Two blocks of same mass are sliding on two tracks with same constant speed with the help of external force along the path. Coefficient of kinetic friction is same on both tracks for the block. Choose the incorrect alternatives.



(A) At point P in the track AB, friction has same magnitude as on track CD.

(B) Friction on AB track, first increases and then decreases

(C) Friction on AB track, first decreases and then increases

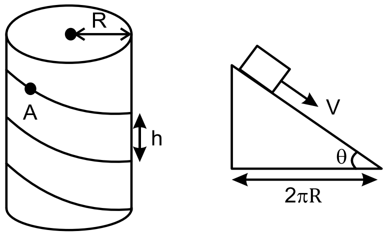
(D) None of above is correct

3. A small bead starts sliding down a rigid helical wire frame. The axis of the helix is vertical, radius R and pitch is h. The motion of the bead is a combination of horizontal motion and vertical motion. In one complete rotation , it travels $2\pi R$ horizontal distance and h vertical distance . The total distance travelled can be given by a right angled traiangle as shown in figure. The height of the triangle is h.

Consider

Case-a: friction less helical wire frame

Case-b: bead slides down the helical wire frame with a constant speed v such that the turns in the helix are tilted by angle θ with horizontal



Then identify the correct statements from the following:

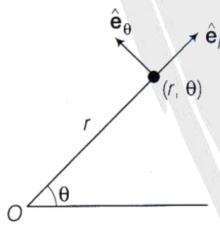
(A)if the total height descended by the bead is H, it takes a time $\sqrt{\frac{2H(4\pi^2R^2+h^2)}{gh^2}}$ in case-a

(B)if the total height descended by the bead is H, it takes a time $\sqrt{\frac{2H(4\pi^2R^2+h^2)}{gh^2}}$ in case-b

(C) in case-b coefficient of kinetic friction between the wire and the bead is $\frac{Rg\tan\theta}{\sqrt{g^2R^2+v^4\cos^2\theta}}$

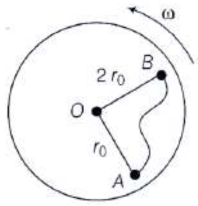
(D)in case-a, if the total height descended by the bead is H , the speed after descended by H is $\sqrt{\frac{2ghH}{\sqrt{(h^2+4\pi^2R^2)}}}$

4. A force $\left(F=\frac{b\cos\theta}{r^3}\hat{e}_r+\frac{b\sin\theta}{r^3}\hat{e}_\theta\right)$ N is applied on a particle. $\hat{e}_r$ is a unit vector along r from origin O of coordinate system and $\hat{e}_\theta$ is a unit vector along increasing angle θ. Find the work done by the force F when the particle is taken from point $A(a,0)$ to $\left(a,\frac{\pi}{2}\right)$ on a circular path of radius a.



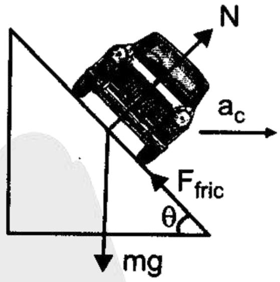
(A) $\frac{b}{a}$ (B) $\frac{b}{a^2}$ (C) $-\frac{b}{a^2}$ (D) $-\frac{2b}{a^2}$

5. A circular platform is rotating with constant angular velocity 10 rad/s. A bug starts from point A and reaches at point B through an irregular shape path. During motion of bug, the angular velocity of platform remains constant. Find work done by centrifugal force on the bug.(take, $mr_0^2=\frac{1}{2}kgm^2$)



(A) 5 J (B) 20 J (C) 75 J (D) None of these

6. A section of a hilly highways is a circle with radius r = 30m the banking angle θ of the roadbed so that cars travelling at v m/s. (g = 10m/sec²)



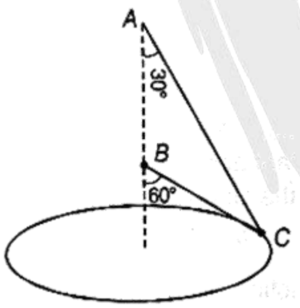
(A) If friction force between tyre and road is zero, then $v = \sqrt{gr\tan\theta}$

(B) If $\mu_s = \frac{1}{2}$ and $\mu_k = \frac{1}{3}$, $\theta = 45^0$, then maximum value of v is 30m/sec

(C) If $\mu_s = \frac{1}{2}$ and $\mu_k = \frac{1}{3}$, $\theta = 45^0$, then minimum value of v is 10m/sec

(D) If road is rough then friction force acting between tyre and road may be zero

7. A single wire ACB passes through a ring C of mass 'm', which revolves at a constant speed in the horizontal circle of radius 'r'?

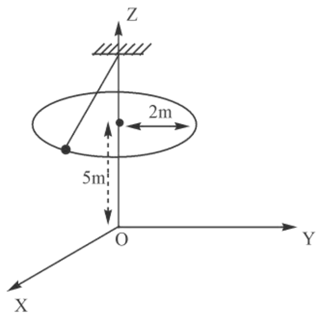


(A) The angular speed of revolution is $\sqrt{\frac{g}{r}}$ (B) The tension in the wire is $\left(\frac{2mg}{\sqrt{3}+1}\right)$

(C) The tension in the wire may be zero (D) In vertical direction, net force is zero

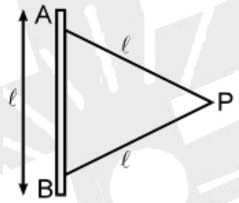
8. The upper end of the ideal string is fixed to vertical Z-axis and the other end of the string is tie with a particle of mass m, and set in motion such that the particle moves along a horizontal circular path of radius 2m, parallel to the x – y plane, 5m above the origin. The particle has a speed of 3m/sec. The string breaks when the particle is vertically above the x-axis, and it may lands on the x – y plane at a point

(x meter, y meter): $\left(g = 10\,m/\sec^2\right)$



(A) $x = 2$ (B) $x > 2$ (C) $y = 3$ (D) $y < 0$

9. A particle P of mass m is attached to a vertical axis by two strings AP and BP of length l each. The separation AB = l. P rotates around the axis with an angular velocity ω. The tensions in the upper and lower strings respectively are T_1 and T_2. Choose the correct option(s).



(A) $T_1 = T_2$

(B) $T_1 + T_2 = m\omega 2l$

(C) $T_1 - T_2 = 2mg$

(D) BP will remain taut only if $\omega \geq \sqrt{\frac{2g}{l}}$

10. Anushka is sitting on the edge of a horizontal "Merry go-around" that has a radius of 6m and is rotating steadily. Virat is standing still on the ground at a point that is 12m from the centre of the "Merry-go-around". Assume their line of sight is horizontal. At a particular instant, Virat observes Anushka moving directly towards him with a speed of 1 m/s and observes there is no component of velocity perpendicular to the line joining them. At the same moment, Anushka observes Virat to be moving at a speed $\sqrt{n}$ m/s, where n is a non-negative integer. At the same moment, the rate at which Anushka observes the distance between them is decreasing to be $\sqrt{m}$ m/s, where m is a positive integer. At the same moment, Anushka observes Virat's velocity perpendicular to the line joining them to be $\sqrt{p}$ m/s where p is non-negative integer. Choose the **CORRECT** option(s).

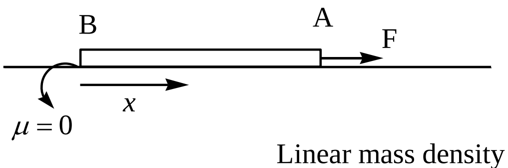
(A) m = 1

(B) n = 4

(C) p = 3

(D) Acceleration of Anushka as observed by Virat is a constant

11. A non-uniform rope of length L is placed on the ground and Rope is pulled by a constant force F as shown in the diagram, there is no friction anywhere, linear mass density of rope is given by $\lambda_x = \lambda_0\left(1+\frac{x}{L}\right)$, where x is measured from the end B. If tension at a distance of x from B is T_x, total mass of the rope is M and acceleration of rope is a_0. Then Choose the **CORRECT** option(s).



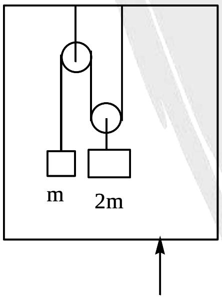
(A) $\int_O^L T_x.dx = \frac{4F_0L}{9}$

(B) $M = \frac{3\lambda_0 L}{2}$

(C) $a_0 = \frac{2F_0}{3\lambda_0 L}$

(D) $T_x = \frac{2F_0}{3L}\left[x+\frac{x^2}{2L}\right]$

12. An arrangement inside a lift is shown in figure. The pulleys and threads are ideal and acceleration due to gravity is g. The acceleration of the lift for which the thread remains taut and both blocks accelerate in the same direction relative to the ground is a_1 and a_2 respectively for lift accelerating upwards and downwards.



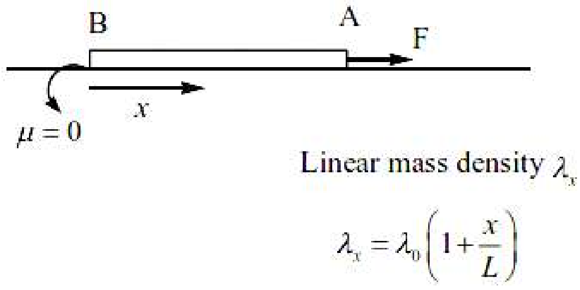
(A) $a_1 > \frac{g}{6}$

(B) $a_2 > \frac{g}{6}$

(C) There is upper limit on a_1

(D) There is upper limit on a_2

13. A non-uniform rope of length L is placed on the ground and rope is pulled by a constant force F as shown in the diagram, there is no friction anywhere, linear mass density of rope is given by $\lambda_x = \lambda_0\left(1+\frac{x}{L}\right)$, where x is measured from the end B. If tension at a distance of x from b is T_x, total mass of the rope is M and acceleration of rope is a_0. Then choose the **CORRECT** option(s).



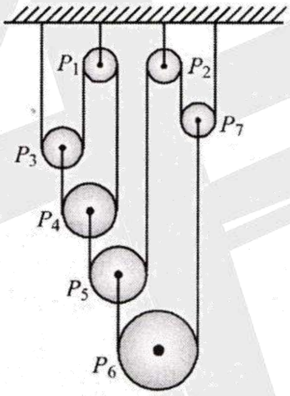
(A) $\int_0^L T_x \cdot dx = \frac{4F_0L}{9}$

(B) $M = \frac{3\lambda_0 L}{2}$

(C) $a_0 = \frac{2F_0}{3\lambda_0 L}$

(D) $T_x = \frac{2F_0}{3L}\left[x + \frac{x^2}{2L}\right]$

14. Seven pulleys are connected with the help of three light strings as shown in the figure below. Consider P_3, P_4, P_5 as light pulleys and pulleys P_6 & P_7 have masses m each. For this arrangement, mark the correct statement(s).



(A) Tension in the string connecting P_1, P_3 & P_4 is zero

(B) Tension in the string connecting P_1, P_3 & P_4 is mg/3.

(C) Tensions in all the three strings are same and equal to zero.

(D) Acceleration of P_6 is g downwards and that of P_7 is g upwards

15. A person is running on a horizontal surface with increasing speeds.

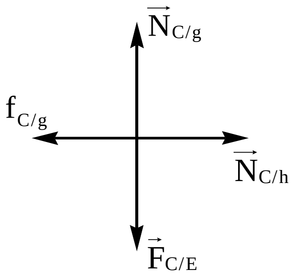
(A) His speed is increased due to internal forces in the person

(B) His speed is increases due to friction on the person

(C) His kinetic energy is increased due to internal forces

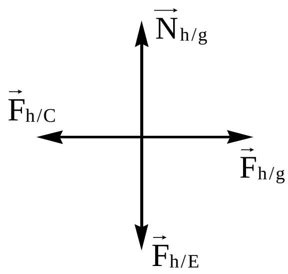
(D) His kinetic energy is increased due to friction

16. Consider a cart being pulled by a horse with constant velocity. The horse exerts force $\vec{F}_{C/h}$ on the cart, (The subscript indicate the force on the cart due to horse). The first subscript denotes the body on which force acts and second due to which it acts.



Free body diagram of cart

$C \rightarrow$ Cart
$g \rightarrow$ Ground
$h \rightarrow$ Horse
$E \rightarrow$ Earth



Free body diagram of horse

Choose the correct statement(s).

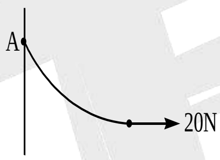
(A) $\vec{f}_{C/g}, \vec{N}_{C/g}, \vec{N}_{h/g}$ are external forces on a system consisting of horse and cart

(B) $\vec{F}_{h/g} + \vec{f}_{C/g} = 0$

(C) $\vec{N}_{C/g}$ and $\vec{F}_{C/E}$ are action reaction pairs

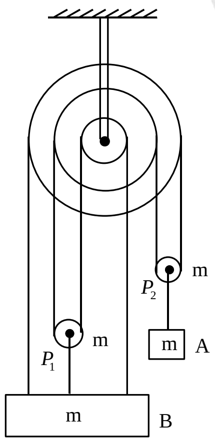
(D) $\vec{F}_{C/h}$ and $\vec{F}_{h/C}$ are action reaction pairs

17. One end of rope is fixed to a vertical wall and other end is pulled by a horizontal force of 20N.The shape of flexible rope is shown in fig. The angle made by tangent at A (fixed point of rope) with downward vertical is 30^0.



(A) Tension in the string at A is 40 N.
(B) mass of the rope 3.5 kg.
(C) The mass of rope is 1.5 kg.
(D) Tension in the rope is 20N at fixed point A

18. In the figure shown the three step pulley is fixed and smooth. All strings are light and the other two pulleys are smooth and has mass m each. Masses of blocks A and B are same and same as that of pulleys. The system is released from rest $\left(g = 10\, m/s^2\right)$



(A) The magnitude of net force acting on block B during the motion is $\frac{mg}{5}$

(B) The magnitude of net force acting on block A during the motion is $\frac{2mg}{5}$

(C) Displacement of pulley P_1 in 2 sec is 4m

(D) Speed acquired by pulley P_2 during its displacement of 2m is 4 m/s

19. Which of the following statement is/are correct about reference frames?

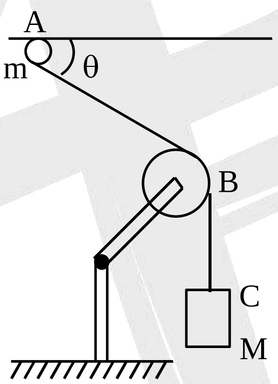
(A) In a non-inertial reference frame, an isolated particle does not retain a constant velocity

(B) A reference frame travelling with acceleration relative to an inertial reference is a non-inertial reference frame

(C) In an inertial reference frame, velocity vector of an isolated particle changes neither in direction nor in magnitude, with time

(D) If a block is stationary in an elevator then reference frame fixed to elevator must be inertial

20. A smooth ring of mass m can slide on a fixed horizontal rod. A string tied to the ring pass over a fixed pulley B and carries a block C of mass 2m as shown below. As the ring starts sliding



(A) The acceleration of the ring is $\frac{2g\cos\theta}{1+2\cos^2\theta}$

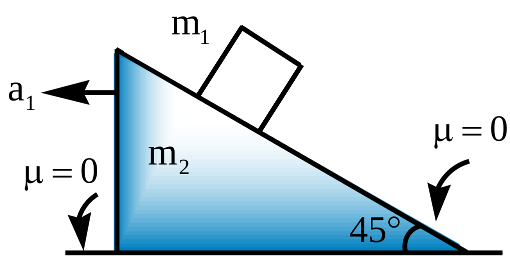
(B) The acceleration of the block is $\frac{2g}{1+2\cos^2\theta}$

(C) The tension in the string is $\frac{2mg}{1+2\cos^2\theta}$

(D) If the block descends with velocity υ then the ring slides with velocity $\upsilon\cos\theta$

PROFICIENCY TEST – 1

1. The triangular wedge shown in the figure is pulled towards left with an acceleration of 1.5g. what is the acceleration of m_1?



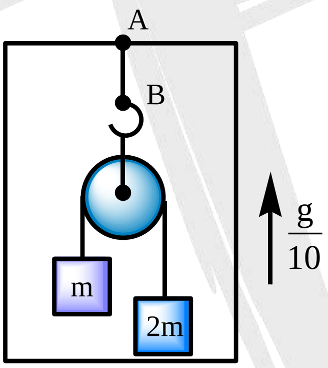
(A) g downwards

(B) 1.5g towards right

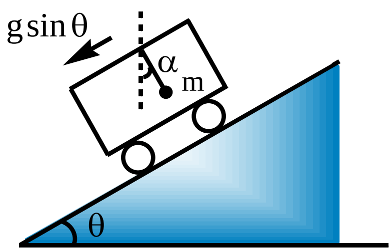
(C) 1.5g in vertically downwards direction

(D) None of the above

2. In the arrangement shown in the figure, the elevator is going up with an acceleration of g/10. If the pulley and the string are light and the pulley is smooth, the tension in the string AB is $\frac{\alpha}{\beta}mg$. Find $\alpha + \beta$?



3. A trolley is accelerating down an incline of angle θ with acceleration $g\sin\theta$. Which of the following is correct. (α is the angle made by the string with vertical).



(A) $\alpha = \theta$

(B) $\alpha = 0°$

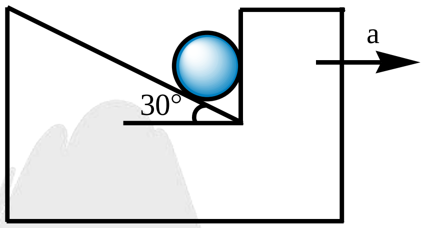
(C) Tension in the string, $T = mg$

(D) Tension in the string, $T = mg\sec\theta$

4. A man is standing on a weighing machine with a block in his hand. The machine records w. when he takes the block upwards with some acceleration the machine records w_1. When he takes the block down with some acceleration, the machine records w_2. Then choose correct option(s)

(A) $w_1 = w = w_2$ (B) $w_1 < w < w_2$ (C) $w_2 < w < w_1$ (D) $w_2 = w_1 > w$

5. A heavy spherical ball is constrained in a frame as in figure. The inclined surface is smooth. The maximum acceleration with which the frame can move without causing the ball to leave the frame



(A) $\frac{g}{2}$ (B) $g\sqrt{3}$ (C) $\frac{g}{\sqrt{3}}$ (D) $\frac{g}{\sqrt{2}}$

6. A physics textbook of mass m rests flat on a horizontal table of mass M placed on the ground. Let $N_{a\to b}$ be the contact force exerted by body "a" on body "b". according to Newton's 3rd Law, which of the following is an action – reaction pair of forces?

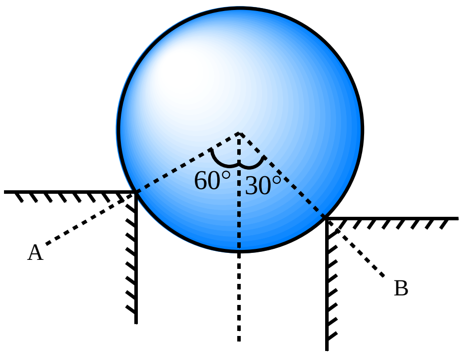
(A) mg and $N_{table\to book}$

(B) $(m+M)g$ and $N_{table\to book}$

(C) $N_{ground\to table}$ and Mg + $N_{book\to table}$

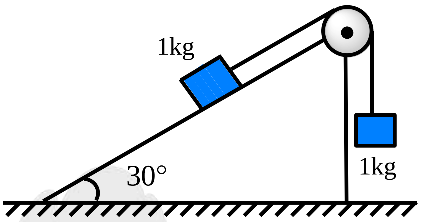
(D) $N_{ground\to table}$ and $N_{table\to ground}$

7. A smooth cylinder of mass M and radius R is resting on two corner edges A and B as shown in figure. The normal reaction at the edges A and B are $N_B = \sqrt{\alpha}N_A$. Find α ?

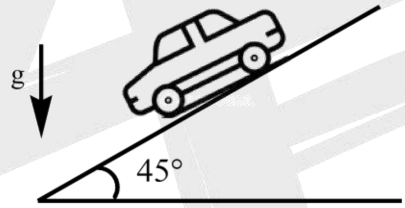


8. A block rests on a rough inclined plane making an angle of 30° with the horizontal. The coefficient of static friction between the block and the plane is 0.8. If the frictional force on the block is 10N, the mass of the block (in kg) is (take $g = 10 m/s^2$)

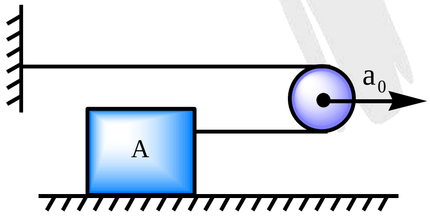
9. In the following figure, The minimum coefficient of friction needed between the block and fixed incline so that the system does not move is $\frac{1}{\sqrt{\alpha}}$. Find α ?



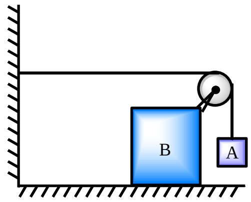
10. If the coefficients of static and kinetic friction between the tyres and the road are μ_s and μ_k, the maximum acceleration that a car can have going up a 45° slope is $\frac{g}{\sqrt{\alpha}}(\mu_s - \beta)$. Find $\alpha + \beta$?



11. The pulley is given an acceleration $a_0 = 2 m/sec^2$ starting from rest. A cable is connected to a block A of mass 50kg as shown. Neglect the mass of the pulley. If $\mu = 0.3$ between the block and the floor, then the tension (in N) in the cable connected to block A is



12. In the system shown in the diagram all surfaces are smooth, pulley and strings are ideal. If $\vec{a}_A$ and $\vec{a}_B$ are the accelerations of the two blocks, then just after the system is released from rest, then choose correct statement(s)



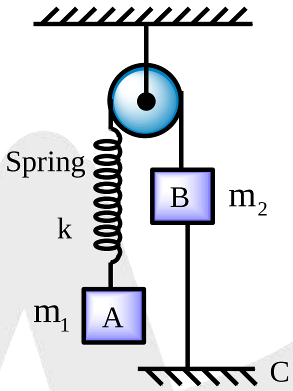
(A) $\left|\vec{a}_A\right| = \left|\vec{a}_B\right|$

(B) $\vec{a}_A \perp \vec{a}_B$

(C) Acceleration of A relative to B is vertically downwards

(D) Normal force exerted by B on A is 2mg

13. In the system shown in the figure $m_1 > m_2$. System is held at rest by thread BC. Just after the thread BC is brunt



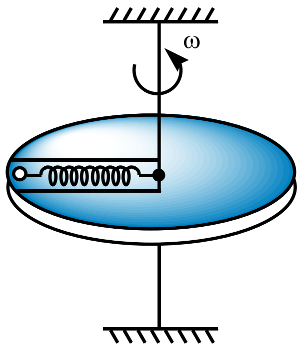
(A) Initial acceleration of m_2 will be upwards

(B) Magnitude of initial acceleration of both blocks will be equal to $\left(\frac{m_1 - m_2}{m_1 + m_2}\right)g$

(C) Initial acceleration of m_1 will be equal to zero

(D) Magnitude of initial acceleration of two blocks is $\left(\frac{m_1 + m_2}{m_2}\right)g$

14. A ball is fixed to spring attached to fixed axle at the centre of a rotating wheel. The ball and spring are constrained to move in the groove shown in figure. Two observers one in rotating frame on the ball and one on the ground at rest observe its motion. Platform is rotating with constant angular velocity and spring has attained its stable configuration. Choose the correct statement(s)



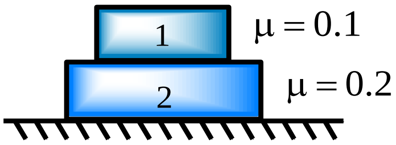
(A) Grounds observers sees the ball being pulled radially inward by spring and he attributes it as centripetal force

(B) Observer on platform sees the ball being pulled outward by centrifugal force that is being balanced by spring force resulting in constant non zero speed of ball

(C) Newton's first law is applicable on ball with respect to observer standing on platform

(D) Newton's second law is applicable with respect to both the observers

15. For the two blocks initially at rest under constant external forces of mass 1kg and 2kg shown, $a_1 = 5m/s^2 \rightarrow$, $a_2 = 2m/s^2 \leftarrow$. Which of the following is correct?



(A) Force of friction on 1 due to 2 is 1N to right

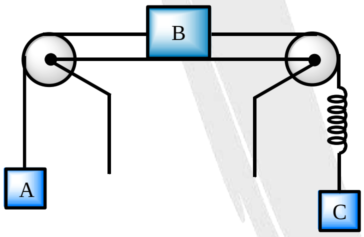
(B) Force of friction on 2 due to 1 is 1N to right

(C) Force of friction on 2 due to ground is 4N to right

(D) Force of friction on 2 due to ground is 6N to right

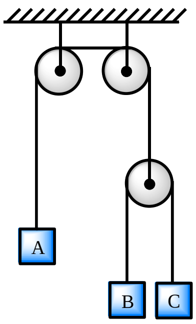
16. In figure all contacts are smooth and string and spring are light. Blocks A, B and C have same mass of 1kg. initially A is held by someone and B and C are at rest. determine acceleration of block C just after block A is released.

(Take $g = 10m/s^2$)



(A) $1m/s^2$ (B) $2m/s^2$ (C) Zero (D) $3m/s^2$

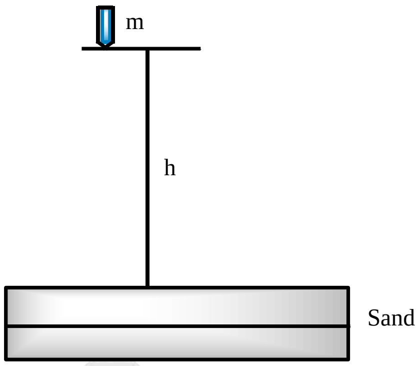
17. In the system shown, block A is of mass 4.0kg and blocks B and C are equal mass each of 3kg. find the acceleration in m/s^2 of block C, if the system is set free. $\left(g = 10m/s^2\right)$



(A) $1m/s^2$ (B) $2m/s^2$ (C) $3m/s^2$ (D) $4m/s^2$

PROFICIENCY TEST – 2

1. An iron nail is dropped from a height h on the level of a sand bed. If it penetrates through a distance x in the sand before coming to rest, the average force exerted by the sand on the nail is



(A) $mg\left(\frac{h}{x}+1\right)$ (B) $mg\left(\frac{x}{h}+1\right)$ (C) $mg\left(\frac{h}{x}-1\right)$ (D) $mg\left(\frac{x}{h}-1\right)$

2. A 10kg monkey is climbing a massless rope attached to a 15kg mass over a tree limb. The mass is lying on the ground. In order to raise the mass from the ground he must climb with

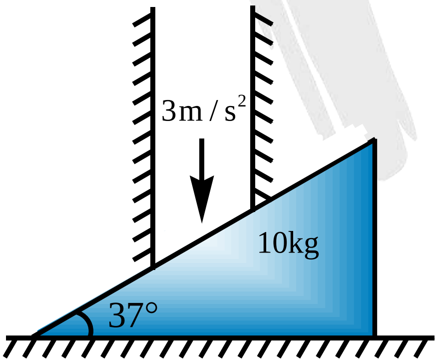
(A) Uniform acceleration greater than $5m/sec^2$

(B) Uniform acceleration greater than $2.5m/sec^2$

(C) High speed

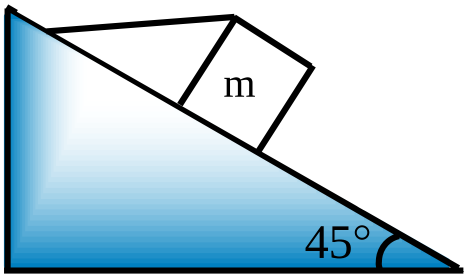
(D) Uniform acceleration greater than $10m/sec^2$

3. System is shown in figure. All the surfaces are smooth. Rod is moved by external agent with acceleration $3m/s^2$ vertically downwards. Force exerted on rod by the wedge will be

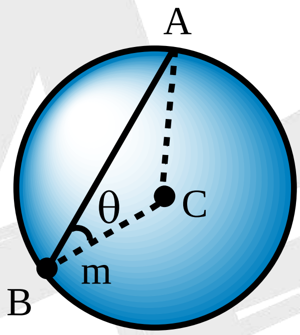


(A) 120N (B) 200N (C) $\frac{135}{2}N$ (D) $\frac{200}{3}N$

4. A block is placed on a smooth incline. It is fixed to incline by a horizontal string. The normal reaction by the wedge on the block is $N=\sqrt{\alpha}mg$. Find α ?

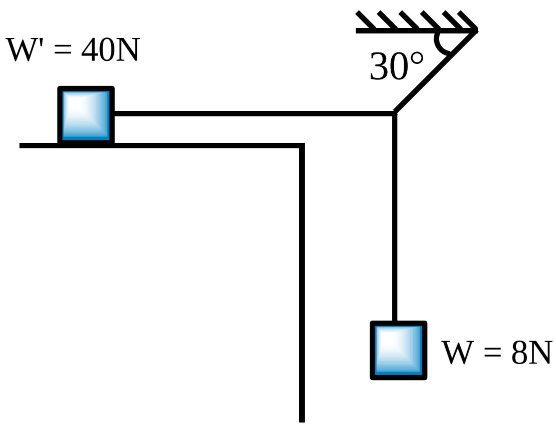


5. AB is a string whose one end A is fixed at the highest point of a ring placed in vertical plane. C is the centre of the ring. At other end B of the string, is attached a bead of mass m, which can slide along the ring. If the bead is in equilibrium at B and string is taut then the tension in the string is $\frac{20}{n} mg\cos\theta$. Find n?

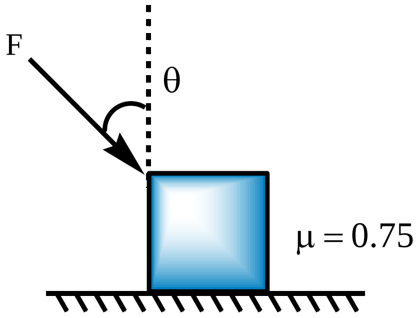


6. A rubber band (two parallel strands of elastic material forming a closed loop) has an effective spring constant of 10 N/m. If the band is cut in one place such that it now forms a single long strand of elastic material, The new effective spring constant is $\frac{25}{n} N/m$. Find n?

7. The system shown is just on the verge of slipping. The co-efficient of static friction between the block and the table top is $\mu = \frac{35}{n}$. Find 'n'?

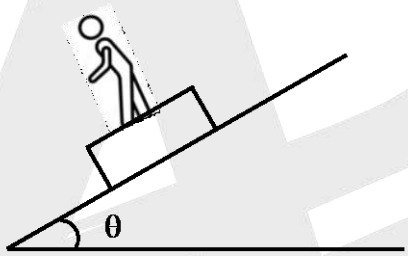


8. A 10kg block is being pushed from behind by a force of 100N at an angle of θ to vertical. At what angle(s) θ can the force be applied so as to move the block



(A) 75° (B) 85° (C) 40° (D) 37°

9. Ram is running down with some acceleration on a plank kept on a fixed inclined plane. Which of following cases are possible.



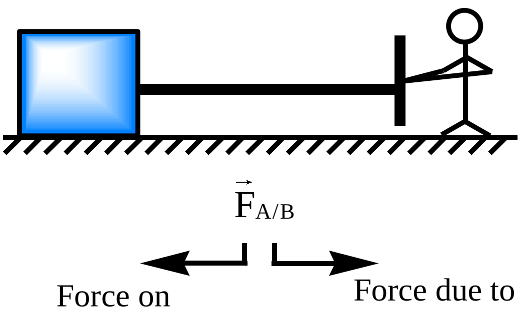
(A) Plank may accelerate down

(B) Plank may accelerate up

(C) Plank may be at rest

(D) Friction on plank due to Ram may be down the plane

10. Figure shows a man pulling on a string attached to a block kept on a rough surface. Man is heavier than block. Coefficient of friction is same for both block and man. Subscripts R, B and M denote rope block and man respectively, taking string to be light. Mark the correct statement(s)



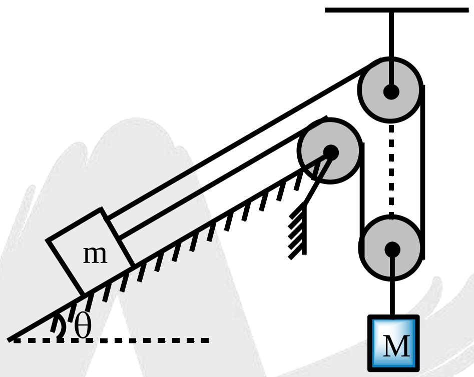
(A) $\overline{F}_{R/B} \neq \overline{F}_{M/R}$ always

(B) Block begins to move earlier than man if man continues to increase his pull from zero

(C) $\overline{F}_{R/B}$ and $\overline{F}_{M/R}$ can be action reaction pairs

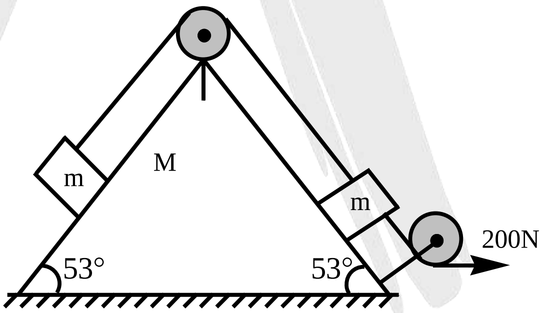
(D) Block can remain static if man walks with constant speed while letting string to pass through his hand

11. In the figure shown all strings and pulleys are massless. If all surfaces are smooth and mass of the blocks are M = 15kg and m = 10kg then find the acceleration of mass m (Given angle of incline $\theta = 30°$)



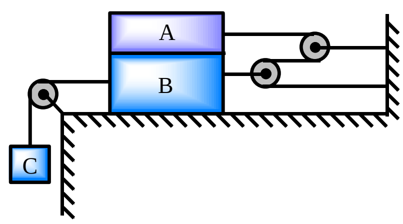
(A) $2\,m/s^2$ (B) $4\,m/s^2$ (C) $6\,m/s^2$ (D) m/s^2

12. A wedge of mass M kept on a rough surface. Two blocks of mass m each attached with string are kept on the two sides of wedge and a horizontal force of 200N is applied as shown in figure. Find the value of friction force acting on the wedge if it does not move.



(A) 20N (B) 40N (C) 60N (D) 80N

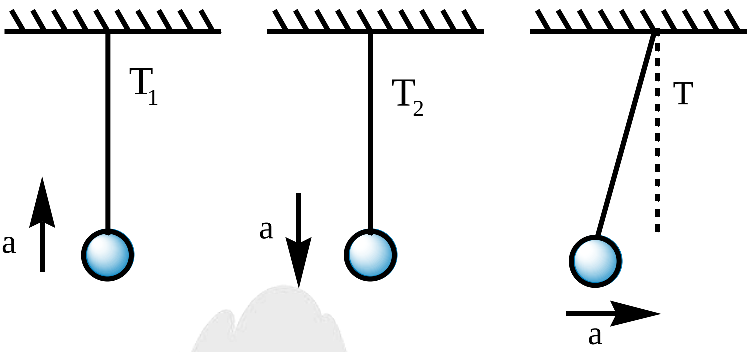
13. What is the largest load that can be suspended without moving blocks A and B? The static coefficient of friction for all plane surfaces of contact is 0.3. Block A weighs 5N and block B weighs 10N. Neglect friction in the pulley system.



(A) 3N (B) 6N (C) 9N (D) 12N

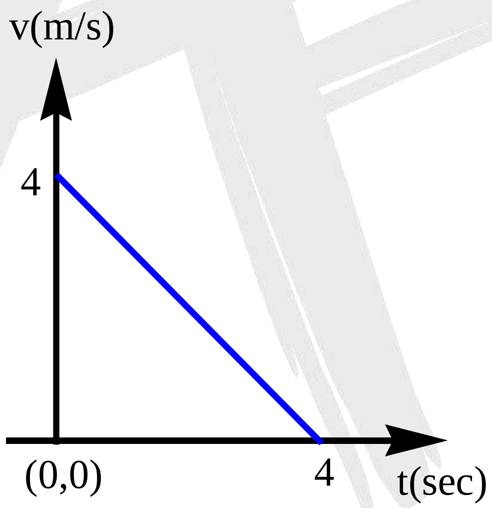
PROFICIENCY TEST – 3

1. A pendulum is hanging from the ceiling of a cage. If the cage moves up with constant acceleration a, its tension is T_1 and if it moves down with same acceleration, the corresponding tension is T_2. The tension in the string if the cage moves horizontally with same acceleration a is



(A) $\frac{\sqrt{T_1^2+T_2^2}}{2}$ (B) $\sqrt{\frac{T_1^2+T_2^2}{4}}$ (C) $\sqrt{\frac{T_1^2+T_2^2}{2}}$ (D) $\frac{\sqrt{T_1^2-T_2^2}}{4}$

2. The velocity time graph of the figure, shows the motion of a wooden block of mass 1kg which is given an initial push t = 0, along a horizontal table.



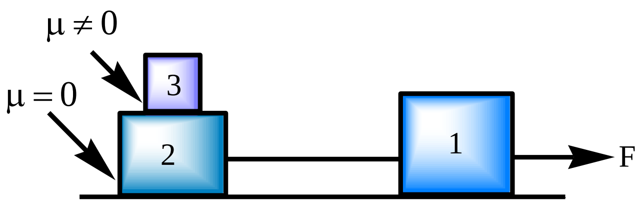
(A) The coefficient of friction between the blocks and the table is 0.1

(B) The coefficient of friction between the blocks and the table is 0.2

(C) If the table was half of its present roughness, the time taken by the block to complete the journey is 4sec

(D) If the table was half of its present roughness, the time taken by the block to complete the journey is 8sec

3. Three boxes of equal mass M are initially at rest on smooth floor. A force $\vec{F}$ is applied to the system so that the three blocks are to move together.



Mark the correct options

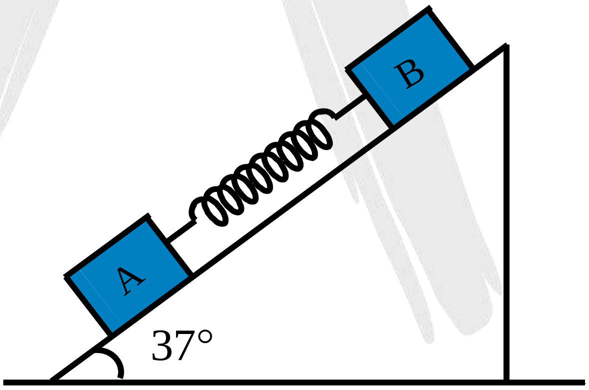
(A) The minimum coefficient of friction required is $\frac{F}{3mg}$

(B) The minimum coefficient of friction required is $\frac{F}{2mg}$

(C) Tension in string is $\frac{2F}{3}$

(D) Tension in string is $\frac{3F}{2}$

4. Two blocks A and B of mass 4kg and 2kg respectively connected by a spring of force constant 100N/m are placed on an fixed inclined plane of inclination $37°$ as shown in figure. If the system is released from rest with spring at natural length, which one of the following statement(s) is/are correct?



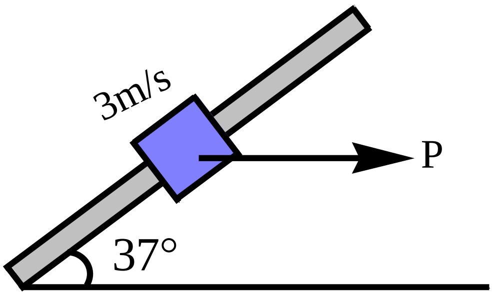
(A) There will be no compression/elongation in the spring if all surfaces are smooth

(B) There will be some extension in the spring if $\mu_A = 0.8$ and $\mu_B = 0.9$

(C) There will be some compression in the spring if $\mu_A = 0.8$ and $\mu_B = 0.7$

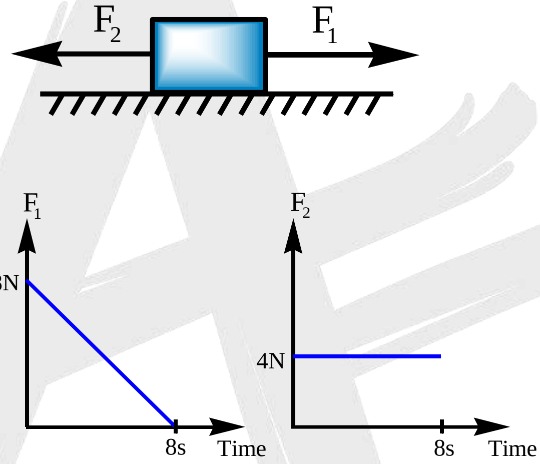
(D) There will be some compression in the spring if $\mu_A = 0.1$ and $\mu_B = 0.1$

5. A collar of mass 8kg is moving down the rod with a velocity of 3m/s. when a force P is applied to the collar, the magnitude of the force P if the collar stopped after moving 1m more down the rod, is [assuming negligible friction between the collar and the rod]



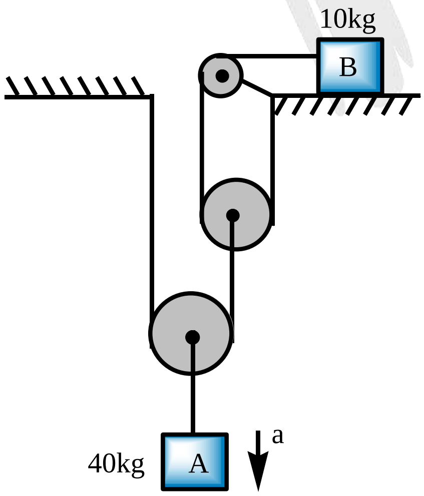
(A) 65N (B) 95N (C) 105N (D) 75N

6. Two forces F_1 and F_2 are applied simultaneously on a 2kg block resting on a frictionless horizontal ground. The forces vary with time according to graphs shown and cease to act after 8s. what maximum speed does the block acquire?



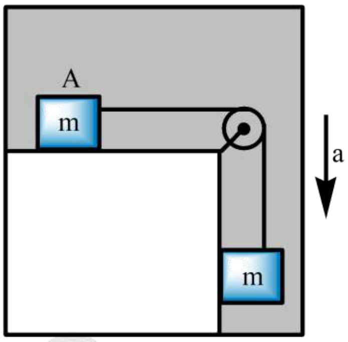
(A) 2m/s (B) 4m/s (C) 6m/s (D) 8m/s

7. In given figure A and B are connected by ideal pulley, find acceleration of A. (all surfaces are smooth).



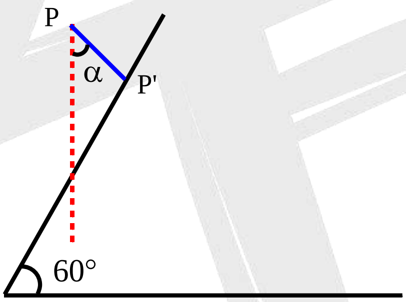
(A) $2\,m/s^2$ (B) $3\,m/s^2$ (C) $1\,m/s^2$ (D) $4\,m/s^2$

8. An arrangement of two blocks of same mass m kept inside a lift which is moving with acceleration a in vertical downward direction. If acceleration of block A with respect to earth is minimum, find the value of a. $\left(g = 10 m/s^2\right)$



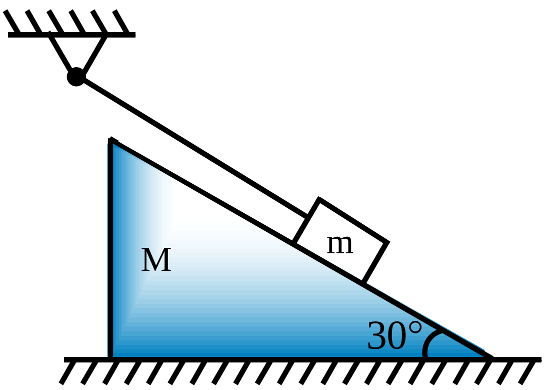
(A) $1\,m/s^2$ (B) $2\,m/s^2$ (C) $3\,m/s^2$ (D) $4\,m/s^2$

9. Point P is located above an inclined plane. It is possible to reach the plane by sliding under gravity down a straight frictionless wire joining P to some point P' on the plane. Find out angle (α) of PP' from vertical in degrees so that it reaches to inclined plane in minimum time.



(A) $30°$ (B) $45°$ (C) $60°$ (D) $15°$

10. A block of mass m = 1kg is connected with an ideal string as shown in figure. If the string is parallel to incline at the instant of release, find the ratio of acceleration of m to that of M = 2kg instantaneously. Consider all the surfaces to be smooth.



(A) 2 (B) 3 (C) 4 (D) 5

ANSWER KEY

EXERCISE – 1 – KEYs

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
B	A	B	A	B	B	B	D	B	12
11	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
B	2	400	CD	10	A	BC	AD	BD	AD
21	**22**	**23**	**24**	**25**	**26**	**27**			
AB	ABC	ABCD	CD	ABC	ABD	D			

EXERCISE – 2 – KEY

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
5	5	A	250	4	3	B	30	7	2
11	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
C	A	2	3	D	D	100	C	D	4
21	**22**	**23**	**24**	**25**	**26**	**27**			
10	A	D	B	AC	BC	C			

EXERCISE – 3 – KEY

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
6	160	C	5	40	D	1	A	D	C
11	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
7	C	C	2	C	200	4	B	D	B
21	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	
A	2	2	B	BC	16	2	B	20	

EXERCISE – 4 – KEY

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
4	A	C	B	A	C	D	4	D	C
11	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
100	D	D	3	2	B	AD	AC	AD	CD
21	**22**	**23**							
CD	AD	C							

EXERCISE – 5 – KEY

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
B	AD	B	C	13	B	A	D	B	D
11	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
C	C	CD	ABCD	ABC	A	C	B	D	4
21	**22**	**23**							
B	**D**	**C**							

EXERCISE – (ALP)

1	2	3	4	5	6	7	8	9	10
A,B	A,B,C	A,C	A,B	C,D	A,B,C,D	A,B,D	A,C,D	B,C,D	A,B,C
11	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
ABCD	AD	ABCD	AC	BC	ABD	AB	ABCD	ABC	AC

PROFICIENCY TEST – 1 – KEY

1	2	3	4	5	6	7	8	9	10
A	59	A	C	C	D	3	2	3	3
11	**12**	**13**	**14**	**15**	**16**	**17**			
350	AB	AC	AB	BD	C	B			

PROFICIENCY TEST – 2 – KEY

1	2	3	4	5	6	7	8	9	10
A	A	D	2	10	10	10	AB	ABCD	BD
11	**12**	**13**							
B	D	C							

PROFICIENCY TEST – 3 – KEY

1	2	3	4	5	6	7	8	9	10
C	A	AC	AC	C	B	A	B	A	A