KEY CONCEPTS

THINGS TO REMEMBER :

RESULT – 1

(i) ***SAMPLE–SPACE* :** The set of all possible outcomes of an experiment is called the **SAMPLE–SPACE(S)**.

(ii) ***EVENT* :** A sub set of sample–space is called an **EVENT**.

(iii) ***COMPLEMENT OF AN EVENT A* :** The set of all out comes which are in S but not in A is called the **COMPLEMENT OF THE EVENT A DENOTED BY** $\overline{A}$ **OR** A^c.

(iv) ***COMPOUND EVENT* :** If A & B are two given events then $A \cap B$ is called **COMPOUND EVENT** and is denoted by $A \cap B$ or AB or A & B .

(v) ***MUTUALLY EXCLUSIVE EVENTS* :** Two events are said to be **MUTUALLY EXCLUSIVE** (or disjoint or incompatible) if the occurence of one precludes (rules out) the simultaneous occurence of the other . If A & B are two mutually exclusive events then P (A & B) = 0.

(vi) ***EQUALLY LIKELY EVENTS* :** Events are said to be **EQUALLY LIKELY** when each event is as likely to occur as any other event.

(vii) ***EXHAUSTIVE EVENTS* :** Events A,B,C .... L are said to be **EXHAUSTIVE EVENTS** if no event outside this set can result as an outcome of an experiment . For example, if A & B are two events defined on a sample space S, then A & B are exhaustive $\Rightarrow A \cup B = S \Rightarrow P(A \cup B) = 1$.

(viii) ***CLASSICAL DEF. OF PROBABILITY* :** If n represents the total number of equally likely , mutually exclusive and exhaustive outcomes of an experiment and m of them are favourable to the happening of the event A, then the probability of happening of the event A is given by P(A) = m/n .

Note : **(1)** $0 \le P(A) \le 1$

(2) $P(A) + P(\overline{A}) = 1$, Where $\overline{A}$ = Not A .

(3) If x cases are favourable to A & y cases are favourable to $\overline{A}$ then $P(A) = \frac{x}{(x+y)}$ and $P(\overline{A}) = \frac{y}{(x+y)}$ We say that **ODDS IN FAVOUR OF A** are x: y & odds against A are y : x

Comparative study of Equally likely , Mutually Exclusive and Exhaustive events.

Experiment	Events	E/L	M/E	Exhaustive
1. Throwing of a die	A : throwing an odd face {1, 3, 5} B : throwing a composite face {4,. 6}	No	Yes	No
2. A ball is drawn from an urn containing 2W, 3R and 4G balls	E_1 : getting a W ball E_2 : getting a R ball E_3 : getting a G ball	No	Yes	Yes
3. Throwing a pair of dice	A : throwing a doublet {11, 22, 33, 44, 55, 66} B : throwing a total of 10 or more {46, 64, 55, 56, 65, 66}	Yes	No	No
4. From a well shuffled pack of cards a card is drawn	E_1 : getting a heart E_2 : getting a spade E_3 : getting a diamond E_4 : getting a club	Yes	Yes	Yes
5. From a well shuffled pack of cards a card is drawn	A = getting a heart B = getting a face card	No	No	No

RESULT – 2

AUB = A+ B = A or B denotes occurence of at least A or B. For 2 events A & B : **(See fig.1)**

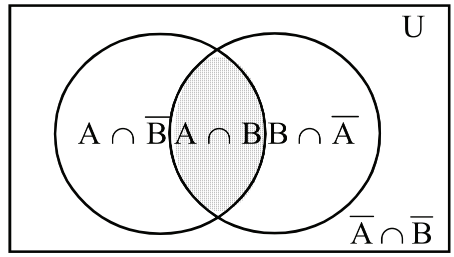

Fig . 1

(i) $P(A \cup B) = P(A) + P(B) - P(A \cap B) =$

$P(A.\overline{B}) + P(\overline{A}.B) + P(A.B) = 1 - P(\overline{A}.\overline{B})$

(ii) Opposite of' "atleast A or B" is NIETHER **A** NOR **B**

i.e. $\overline{A+B}$ = 1-(A or B) = $\overline{A} \cap \overline{B}$

Note that $P(A+B) + P(\overline{A} \cap \overline{B}) = 1$.

(iii) If A & B are mutually exclusive then $P(A \cup B) = P(A) + P(B)$.

(iv) For any two events A & B, P(exactly one of A , B occurs)

$= P(A \cap \overline{B}) + P(B \cap \overline{A}) = P(A) + P(B) - 2P(A \cap B)$

$= P(A \cup B) - P(A \cap B) = P(A^c \cup B^c) - P(A^c \cap B^c)$

(v) If A & B are any two events $P(A \cap B) = P(A).P(B/A) = P(B).P(A/B)$, Where P(B/A) means conditional probability of B given A & P(A/B) means conditional probability of A given B. (This can be easily seen from the figure)

(vi) **De Morgan's Law** : – If A & B are two subsets of a universal set U , then

(a) $(A \cup B)^c = A^c \cap B^c$ & **(b)** $(A \cap B)^c = A^c \cup B^c$

(vii) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ & $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

RESULT – 3

For any three events A,B and C we have **(See Fig. 2)**

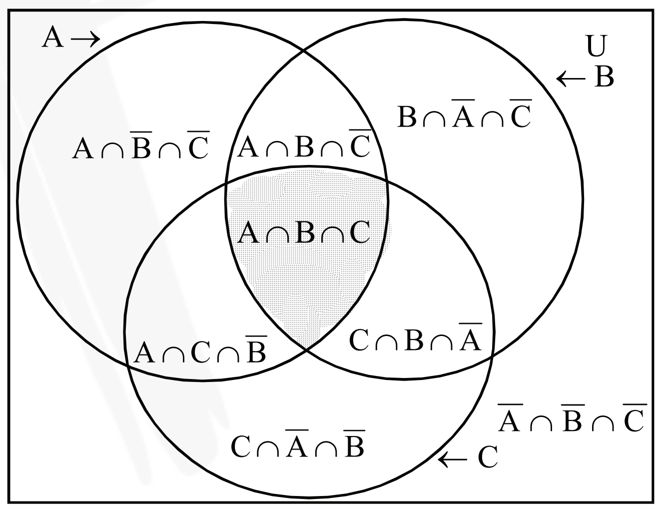

Fig. 2

(i) P(A or B or C) = P(A) + P(B) + P(C) – $P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$

(ii) P (at least two of A,B,C occur) = $P(B \cap C) + P(C \cap A) + P(A \cap B) - 2P(A \cap B \cap C)$

(iii) P(exactly two of A,B,C occur) = $P(B \cap C) + P(C \cap A) + P(A \cap B) - 3P(A \cap B \cap C)$

(iv) P(exactly one of A,B,C occurs) = $P(A) + P(B) + P(C) - 2P(B \cap C) - 2P(C \cap A) - 2P(A \cap B) + 3P(A \cap B \cap C)$

Note :

If three events A, B and C are pair wise mutually exclusive then they must be mutually exclusive. i.e $P(A \cap B) = P(B \cap C) = P(C \cap A) = 0 \Rightarrow P(A \cap B \cap C) = 0$. However the converse of this is not true.

RESULT – 4

Independent Events : Two events A & B are said to be independent if occurence or non occurence of one does not effect the probability of the occurence or non occurence of other.

(i) If the occurence of one event affects the probability of the occurence of the other event then the events are said to be **Dependent** or **Contingent** . For two independent events A and B : $P(A \cap B) = P(A). P(B)$. Often this is taken as the definition of independent events.

(ii) Three events A , B & C are independent if & only if all the following conditions hold ;

$P(A\cap B) = P(A) . P(B)$; $P(B\cap C) = P(B) . P(C)$

$P(C\cap A) = P(C) . P(A)$ & $P(A\cap B\cap C) = P(A) . P(B) . P(C)$

i.e. they must be pairwise as well as mutually independent .

Similarly for n events $A_1 , A_2 , A_3 , \ldots\ldots A_n$ to be independent , the number of these conditions is equal to ${}^nc_2 + {}^nc_3 + \ldots.. + {}^nc_n = 2^n - n - 1$.

(iii) The probability of getting exactly r success in n independent trials is given by

$P(r) = {}^nC_r\, p^r q^{n-r}$ where : p = probability of success in a single trial .

q = probability of failure in a single trial. note : p + q = 1 .

Note : Independent events are not in general mutually exclusive & vice versa.

Mutually exclusiveness can be used when the events are taken from the same experiment & independence can be used when the events are taken from different experiments .

RESULT – 5 : BAYE'S THEOREM OR TOTAL PROBABILITY THEOREM :

If an event A can occur only with one of the n mutually exclusive and exhaustive events $B_1, B_2, \ldots. B_n$ & the probabilities $P(A/B_1) , P(A/B_2) \ldots\ldots. P(A/B_n)$ are known then,

$$P(B_1/A) = \frac{P(B_i).P(A/B_i)}{\sum_{i=1}^{n} P(B_i).P(A/B_i)}$$

PROOF :

The events A occurs with one of the n mutually exclusive & exhaustive events $B_1,B_2,B_3,\ldots\ldots..B_n$

$A = AB_1 + AB_2 + AB_3 + \ldots\ldots. + AB_n$

$$P(A) = P(AB_1) + P(AB_2) + \ldots\ldots.+ P(AB_n) = \sum_{i=1}^{n} P(AB_i)$$

NOTE : A ≡ event what we have ; B_1 ≡ event what we want ;

$B_2, B_3, \ldots.B_n$ are alternative event .

Now,

$P(AB_i) = P(A) . P(B_i/A) = P(B_i) . P(A/B_i)$

$$P(B_i/A) = \frac{P(B_i) . P(A/B_i)}{P(A)} = \frac{P(B_i) . P(A/B_i)}{\sum_{i=1}^{n} P(AB_i)}$$

$$\boxed{P(B_i/A) = \frac{P(B_i) . P(A/B_i)}{\sum P(B_i) . P(A/B_i)}}$$

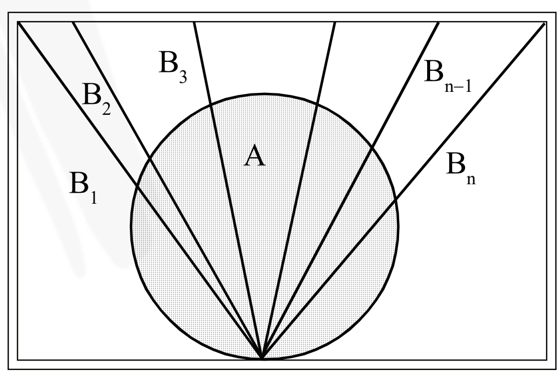

Fig . 3

RESULT – 6 : GEOMETRICAL APPLICATIONS :

The following statements are axiomatic :

(i) If a point is taken at random on a given staright line AB, the chance that it falls on a particular segment PQ of the line is PQ/AB .

(ii) If a point is taken at random on the area S which includes an area σ , the chance that the point falls on σ is σ/S .

EXERCISE-I

1. In a box , there are 8 alphabets cards with the letters: S, S, A, A,A, H, H, H. Find the probability that the word 'ASH' will form if:
(i) the three cards are drawn one by one & placed on the table in the same order that they are drawn.
(ii) the three cards are drawn simultaneously .

2. Numbers are selected at random , one at a time, from the two digit numbers 00, 01, 02, ... , 99 with replacement. An event E occurs if & only if the product of the two digits of a selected number is 18. If four numbers are selected, find the probability that the event E occurs at least 3 times.

3. To pass a test a child has to perform successfully in two consecutive tasks, one easy and one hard task. The easy task he can perform successfully with probability 'e' and the hard task he can perfrom successfully with probability 'h', where h < e. He is allowed 3 attempts, either in the order (Easy, Hard, Easy) (option A) or in the order (Hard, Easy, Hard) (option B) whatever may be the order, he must be successful twice in a row. Assuming that his attempts are independent, in what order he choses to take the tasks, in order to maximise his probability of passing the test.

4. There are 2 groups of subjects one of which consists of 5 science subjects & 3 engg. subjects & other consists of 3 science & 5 engg. subjects. An unbiased die is cast . If the number 3 or 5 turns up a subject is selected at random from first group, otherwise the subject is selected from 2^{nd} group . Find the probability that an engg. subject is selected.

5. A pair of fair dice is tossed. Find the probability that the maximum of the two numbers is greater than 4.

6. A covered basket of flowers has some lilies and roses. In search of rose, Sweety and Shweta alternately pick up a flower from the basket but puts it back if it is not a rose. Sweety is 3 times more likely to be the first one to pick a rose. If sweety begin this 'rose hunt' and if there are 60 lilies in the basket, find the number of roses in the basket.

7. The probability that an archer hits the target when it is windy is 0.4; when it is not windy, her probability of hitting the target is 0.7. On any shot, the probability of a gust of wind is 0.3. Find the probability that
(a) She hit the target on first shot
(b) Hits the target exactly once in two shots

8. There are 4 urns. The first urn contains 1 white & 1 black ball, the second urn contains 2 white & 3 black balls, the third urn contains 3 white & 5 black balls & the fourth urn contains 4 white & 7 black balls. The selection of each urn is not equally likely. The probability of selecting i^{th} urn is $\frac{i^2+1}{34}$ (i = 1, 2, 3, 4). If we randomly select one of the urns & draw a ball, then the probability of ball being white is p/q where p and q $\in$ N are in their lowest form. Find (p + q).

9. A room has three electric lamps . From a collection of 10 electric bulbs of which 6 are good 3 are selected at random & put in the lamps. Find the probability that the room is lighted.

10. A bomber wants to destroy a bridge. Two bombs are sufficient to destroy it. If four bombs are dropped, what is the probability that it is destroyed, if the chance of a bomb hitting the target is 0.4.

11. A box contains 5 radio tubes of which 2 are defective . The tubes are tested one after the other until the 2 defective tubes are discovered . Find the probability that the process stopped on the
(i) Second test; **(ii)** Third test. If the process stopped on the third test , find the probability that the first tube is non defective.

12. Anand plays with Karpov 3 games of chess. The probability that he wins a game is 0.5, looses with probability 0.3 and ties with probability 0.2. If he plays 3 games then find the probability that he wins atleast two games.

13. An aircraft gun can take a maximum of four shots at an enemy's plane moving away from it. The probability of hitting the plane at first, second, third & fourth shots are 0.4, 0.3, 0.2 & 0.1 respectively. What is the probability that the gun hits the plane .

14. In a batch of 10 articles, 4 articles are defective. 6 articles are taken from the batch for inspection. If more than 2 articles in this batch are defective , the whole batch is rejected Find the probability that the batch will be rejected.

15. A game is played with a special fair cubic die which has one red side, two blue sides, and three green sides. The result is the colour of the top side after the die has been rolled. If the die is rolled repeatedly, the probability that the second blue result occurs on or before the tenth roll, can be expressed in the form $\frac{3^p - 2^q}{3^r}$ where p, q, r are positive integers, find the value of $p^2 + q^2 + r^2$.

16. One hundred management students who read at least one of the three business magazines are surveyed to study the readership pattern. It is found that 80 read Business India, 50 read Business world and 30 read Business Today. Five students read all the three magazines. A student was selected randomly. Find the probability that he reads exactly two magazines.

17. An author writes a good book with a probability of 1/2. If it is good it is published with a probability of 2/3. If it is not, it is published with a probability of 1/4. Find the probability that he will get atleast one book published if he writes two.

18. A uniform unbised die is constructed in the shape of a regular tetrahedron with faces numbered 2, 2, 3 and 4 and the score is taken from the face on which the die lands. If two such dice are thrown together, find the probability of scoring.
(i) exactly 6 on each of 3 successive throws. (ii) more than 4 on at least one of the three successive throws.

19. Two cards are drawn from a well shuffled pack of 52 cards. Find the probability that one of them is a red card & the other is a queen.

20. A cube with all six faces coloured is cut into 64 cubical blocks of the same size which are thoroughly mixed. Find the probability that the 2 randomly chosen blocks have 2 coloured faces each.

21. Consider the following events for a family with children
A = {of both the genders}; B = {at most one boy}
In which of the following (are/is) the events A and B are independent.
(a) if a family has 3 children (b) if a family has 2 children
Assume that the birth of a boy or a girl is equally likely mutually exclusive and exhaustive.

22. A player tosses an unbiased coin and is to score two points for every head turned up and one point for every tail turned up. If P_n denotes the probability that his score is exactly n points, prove that

$$P_n - P_{n-1} = \frac{1}{2}(P_{n-2} - P_{n-1}) \qquad n \geq 3$$

Also compute P_1 and P_2 and hence deduce the pr that he scores exactly 4.

23. Each of the 'n' passengers sitting in a bus may get down from it at the next stop with probability p . Moreover , at the next stop either no passenger or exactly one passenger boards the bus . The probability of no passenger boarding the bus at the next stop being p_o . Find the probability that when the bus continues on its way after the stop , there will again be 'n' passengers in the bus.

24. A student bunks the class of probability and equally likely to choose one of the four regions : (Chambal Garden (R-I); Gumanpura (R-II); Jawahar Nagar (R-III); Rajeev Nagar (R-IV) to reach away from the eye of teacher. If he chooses R-I he is successful with probability 1/6 and for R-II; R-III; R-IV this is 1/8, 1/10, 1/12 respectively. If the student is successful, then the probability that he chooses the region

Column I	Column II
(A) I	(P) 12/57
(B) II	(Q) 15/57
(C) III	(R) 20/57
(D) IV	(S) 10/57
	(T) 9/57

25. 6 fair 6-sided dice are rolled. Then the probability that the sum of the values on the top faces of the dice is divisible by 7 can be expressed as $\frac{\lambda}{7776}$ then find the sum of the digits of λ. ($\lambda \in N$)

26. 16 players take part in a tennis tournament. The order of the matches is chosen at random. There is always a player better than another one, the better wins. Find

(a) The probability that all the 4 best players reach the semifinals.

(b) The probability that the sixth best reaches the semifinals.

EXERCISE-II

1. The probabilities that three men hit a target are, respectively, 0.3, 0.5 and 0.4. Each fires once at the target. (As usual, assume that the three events that each hits the target are independent)

(a) Find the probability that they all : **(i)** hit the target ; **(ii)** miss the target

(b) Find the probability that the target is hit : **(i)** at least once, **(ii)** exactly once.

(c) If only one hits the target, what is the probability that it was the first man?

2. Three shots are fired independently at a target in succession. The probabilities that the target is hit in the first shot is 1/2 , in the second 2/3 and in the third shot is 3/4. In case of exactly one hit , the probability of destroying the target is 1/3 and in the case of exactly two hits, 7/11 and in the case of three hits is 1.0. Find the probability of destroying the target in three shots.

3. In a game of chance each player throws two unbiased dice and scores the difference between the larger and smaller number which arise . Two players compete and one or the other wins if and only if he scores atleast 4 more than his opponent . Find the probability that neither player wins .

4. A train consists of n carriages , each of which may have a defect with probability p. All the carriages are inspected , independently of one another , by two inspectors ; the first detects defects (if any) with probability p_1 , & the second with probability p_2 . If none of the carriages is found to have a defect, the train departs . Find the probability of the event ; **"THE TRAIN DEPARTS WITH ATLEAST ONE DEFECTIVE CARRIAGE ".**

5. A is a set containing n distinct elements. A non-zero subset P of A is chosen at random. The set A is reconstructed by replacing the elements of P. A non-zero subset Q of A is again chosen at random. Find the probability that P & Q have no common elements.

6. During a power blackout, 100 persons are arrested on suspect of looting. Each is given a polygraph test. From past experience it is know that the polygraph is 90% reliable when administered to a guilty person and 98% reliable when given to some one who is innocent. Suppose that of the 100 persons taken into custody, only 12 were actually involved in any wrong doing. If the probability that a given suspect is innocent given that the photograph says he is guilty is a/b where *a* and *b* are relatively prime, find the value of (a + b).

7. n people are asked a question successively in a random order & exactly 2 of the n people know the answer :
(a) If $n > 5$, find the probability that the first four of those asked do not know the answer.
(b) Show that the probability that the r^{th} person asked is the first person to know the answer is : $\left[\frac{2(n-r)}{n(n-1)}\right]$, if $1 < r < n$.

8. A box contains three coins two of them are fair and one two – headed. A coin is selected at random and tossed. If the head appears the coin is tossed again, if a tail appears, then another coin is selected from the remaining coins and tossed.
(i) Find the probability that head appears twice.
(ii) If the same coin is tossed twice, find the probability that it is two headed coin.
(iii) Find the probability that tail appears twice.

9. The ratio of the number of trucks along a highway, on which a petrol pump is located, to the number of cars running along the same highway is 3 : 2. It is known that an average of one truck in thirty trucks and two cars in fifty cars stop at the petrol pump to be filled up with the fuel. If a vehicle stops at the petrol pump to be filled up with the fuel, find the probability that it is a car.

10. A batch of fifty radio sets was purchased from three different companies A, B and C. Eighteen of them were manufactured by A, twenty of them by B and the rest were manufactured by C.
The companies A and C produce excellent quality radio sets with probability equal to 0.9 ; B produces the same with the probability equal to 0.6.
What is the probability of the event that the excellent quality radio set chosen at random is manufactured by the company B?

11. Integers a, b, c and d not necessarily distinct, are chosen independently and at random from the set $S = \{0, 1, 2, 3, ... 2006, 2007\}$. If the probability that $|ad - bc|$ is even, is $\frac{p}{q}$ where p and q are relatively prime the find the value of $(p + q)$.

12. Suppose that there are 5 red points and 4 blue points on a circle. Let $\frac{m}{n}$ be the probability that a convex polygon whose vertices are among the 9 points has at least one blue vertex where *m* and *n* are relatively prime. Find $(m + n)$.

13. Two cards are randomly drawn from a well shuffled pack of 52 playing cards, without replacement. Let x be the first number and y be the second number.
Suppose that Ace is denoted by the number 1; Jack is denoted by the number 11 ; Queen is denoted by the number 12 ; King is denoted by the number 13.
Find the probability that x and y satisfy $\log_3(x + y) - \log_3 x - \log_3 y + 1 = 0$.

14. (a) Two numbers x & y are chosen at random from the set $\{1,2,3,4,....3n\}$. Find the probability that $x^2 - y^2$ is divisible by 3 .
(b) If two whole numbers x and y are randomly selected from the set of natural numbers, then find the probability that $x^3 + y^3$ is divisible by 8.

15. A hunter's chance of shooting an animal at a distance r is $\frac{a^2}{r^2}$ $(r > a)$. He fires when $r = 2a$ & if he misses he reloads & fires when $r = 3a, 4a, ...$ If he misses at a distance 'na', the animal escapes. Find the odds against the hunter.

16. A hotel packed breakfast for each of the three guests. Each breakfast should have consisted of three types of rolls, one each of nut, cheese and fruit rolls. The preparer wrapped each of the nine rolls and once warpped, the rolls were indistinguishable from one another. She then randomly put three rolls in a bag for each of the guests. If the probability that each guset got one roll of each type is m/n where *m* and *n* are relatively prime integers, find the value of (m + n).

17. A coin is tossed (m + n) times (m>n). Show that the probability of at least m consecutive heads is $\frac{n+2}{2^{m+1}}$

18. There are two lots of identical articles with different amount of standard and defective articles. There are N articles in the first lot, n of which are defective and M articles in the second lot, m of which are defective. K articles are selected from the first lot and L articles from the second and a new lot results. Find the probability that an article selected at random from the new lot is defective.

19. *m* red socks and *n* blue socks (m > n) in a cupboard are well mixed up, where $m + n \le 101$. If two socks are taken out at random, the chance that they have the same colour is 1/2. Find the largest value of *m*.

20. With respect to a particular question on a multiple choice test (having 4 alternatives with only 1 correct) a student knows the answer and therefore can eliminate 3 of the 4 choices from consideration with probability 2/3, can eliminate 2 of the 4 choices from consideration with probability 1/6, can eliminate 1 choice from consideration with probability 1/9, and can eliminate none with probability 1/18. If the student knows the answer, he answers correctly, otherwise he guesses from among the choices not eliminated.
If the answer given by the student was found correct, then the probability that he knew the answer is $\frac{a}{b}$ where a and b are relatively prime. Find the value of (a + b).

21. A match between two players A and B is won by the player who first wins two games. A's chance of winning, drawing or losing any particular games are 1/2, 1/6 or 1/3 respectively. If the probability of A's winning the match can be expressed in the form p/q, find (p + q).

EXERCISE-III

1. (a) If the integers m and n are chosen at random from 1 to 100, then the probability that a number of the form $7^m + 7^n$ is divisible by 5 equals

(A) $\frac{1}{4}$ (B) $\frac{1}{7}$ (C) $\frac{1}{8}$ (D) $\frac{1}{49}$

(b) The probability that a student passes in Mathematics, Physics and Chemistry are m, p and c respectively. Of these subjects, the student has a 75% chance of passing in at least one, a 50% chance of passing in at least two, and a 40% chance of passing in exactly two, which of the following relations are true?

(A) $p + m + c = \frac{19}{20}$ (B) $p + m + c = \frac{27}{20}$ (C) $pmc = \frac{1}{10}$ (D) $pmc = \frac{1}{4}$

(c) Eight players $P_1, P_2, P_3, \ldots\ldots\ldots\ldots P_8$ play a knock-out tournament. It is known that whenever the players P_i and P_j play, the player P_i will win if i < j. Assuming that the players are paired at random in each round, what is the probability that the player P_4 reaches the final. **[JEE ' 99, 2 + 3 + 10 (out of 200)]**

2. Four cards are drawn from a pack of 52 playing cards. Find the probability of drawing exactly one pair. **[REE'99, 6]**

3. A coin has probability 'p' of showing head when tossed. It is tossed 'n' times. Let p_n denote the probability that no two (or more) consecutive heads occur. Prove that, **[JEE ' 2000 (Mains), 5]**

$$p_1 = 1\ ,\ p_2 = 1 - p^2\ \&\ p_n = (1 - p)\,p_{n-1} + p\,(1 - p)\,p_{n-2}\ ,\ \text{for all } n \ge 3.$$

4. A and B are two independent events. The probability that both occur simultaneously is 1/6 and the probability that neither occurs is 1/3. Find the probabilities of occurance of the events A and B separately.
[REE ' 2000 (Mains), 3]

5. Two cards are drawn at random from a pack of playing cards. Find the probability that one card is a heart and the other is an ace. **[REE ' 2001 (Mains), 3]**

6. (a) An urn contains 'm' white and 'n' black balls. A ball is drawn at random and is put back into the urn along with K additional balls of the same colour as that of the ball drawn. A ball is again drawn at random. What is the probability that the ball drawn now is white.

(b) An unbiased die, with faces numbered 1, 2, 3, 4, 5, 6 is thrown n times and the list of n numbers showing up is noted. What is the probability that among the numbers 1, 2, 3, 4, 5, 6, only three numbers appear in the list. **[JEE ' 2001 (Mains), 5 + 5]**

7. A box contains N coins, m of which are fair and the rest are biased. The probability of getting a head when a fair coin is tossed is 1/2, while it is 2/3 when a biased coin is tossed. A coin is drawn from the box at random and is tossed twice. The first time it shows head and the second time it shows tail. What is the probability that the coin drawn is fair? **[JEE ' 2002 (mains)]**

8. (a) A person takes three tests in succession. The probability of his passing the first test is p, that of his passing each successive test is p or p/2 according as he passes or fails in the preceding one. He gets selected provided he passes at least two tests. Determine the probability that the person is selected.

(b) In a combat, A targets B, and both B and C target A. The probabilities of A, B, C hitting their targets are 2/3 , 1/2 and 1/3 respectively. They shoot simultaneously and A is hit. Find the probability that B hits his target whereas C does not. **[JEE' 2003, Mains-2 + 2 out of 60]**

9. (a) Three distinct numbers are selected from first 100 natural numbers. The probability that all the three numbers are divisible by 2 and 3 is

(A) $\frac{4}{25}$ (B) $\frac{4}{35}$ (C) $\frac{4}{55}$ (D) $\frac{4}{1155}$

(b) If A and B are independent events, prove that $P(A \cup B) \cdot P(A' \cap B') \le P(C)$, where C is an event defined that exactly one of A or B occurs.

(c) A bag contains 12 red balls and 6 white balls. Six balls are drawn one by one without replacement of which atleast 4 balls are white. Find the probability that in the next two draws exactly one white ball is drawn (leave the answer in terms of ${}^nC_r$). **[JEE 2004, 3 + 2 + 4]**

10. (a) A six faced fair dice is thrown until 1 comes, then the probability that 1 comes in even number of trials is

(A) 5/11 (B) 5/6 (C) 6/11 (D) 1/6

[JEE 2005 (Scr)]

(b) A person goes to office either by car, scooter, bus or train probability of which being $\frac{1}{7}, \frac{3}{7}, \frac{2}{7}$ and $\frac{1}{7}$ respectively. Probability that he reaches office late, if he takes car, scooter, bus or train is $\frac{2}{9}, \frac{1}{9}, \frac{4}{9}$ and $\frac{1}{9}$ respectively. Given that he reached office in time, then what is the probability that he travelled by a car.
[JEE 2005 (Mains), 2]

Comprehension (3 questions)

There are n urns each containing $n + 1$ balls such that the i^{th} urn contains i white balls and $(n + 1 – i)$ red balls. Let u_i be the event of selecting i^{th} urn, $i = 1, 2, 3, ..., n$ and w denotes the event of getting a white ball.

11. (a) If $P(u_i) \propto i$ where i = 1, 2, 3,....., n then $\lim\limits_{n\to\infty} P(w)$ is equal to

(A) 1 (B) 2/3 (C) 3/4 (D) 1/4

(b) If $P(u_i) = c$, where c is a constant then $P(u_n/w)$ is equal to

(A) $\frac{2}{n+1}$ (B) $\frac{1}{n+1}$ (C) $\frac{n}{n+1}$ (D) $\frac{1}{2}$

(c) If n is even and E denotes the event of choosing even numbered urn ($P(u_i) = \frac{1}{n}$), then the value of $P(w/E)$, is **[JEE 2006, 5 marks each]**

(A) $\frac{n+2}{2n+1}$ (B) $\frac{n+2}{2(n+1)}$ (C) $\frac{n}{n+1}$ (B) $\frac{1}{n+1}$

12. (a) One Indian and four American men and their wives are to be seated randomly around a circular table. Then the conditional probability that the Indian man is seated adjacent to his wife given that each American man is seated adjacent to his wife is

(A) 1/2 (B) 1/3 (C) 2/5 (D) 1/5

(b) Let E^c denote the complement of an event E. Let E, F, G be pairwise independent events with P(G) > 0 and $P(E \cap F \cap G) = 0$. Then $P(E^c \cap F^c \mid G)$ equals

(A) $P(E^c) + P(F^c)$ (B) $P(E^c) – P(F^c)$ (C) $P(E^c) – P(F)$ (D) $P(E) – P(F^c)$

(c) Let $H_1, H_2, ... , H_n$ be mutually exclusive and exhaustive events with $P(H_i) > 0$, i = 1, 2, ..., n. Let E be any other event with 0 < P(E) < 1.

Statement-1: $P(H_i / E) > P(E / H_i) \cdot P(H_i)$ for i = 1, 2, ..., n.

because

Statement-2: $\sum_{i=1}^{n} P(H_i) = 1$

(A) Statement-1 is true, statement-2 is true; statement-2 is a correct explanation for statement-1.
(B) Statement-1 is true, statement-2 is true; statement-2 is NOT a correct explanation for statement-1.
(C) Statement-1 is true, statement-2 is false.
(D) Statement-1 is false, statement-2 is true. **[JEE 2007, 3+3+3]**

13. (a) An experiment has 10 equally likely outcomes. Let A and B be two non-empty events of the experiment. If A consists of 4 outcomes, the number of outcomes that B must have so that A and B are independent, is

(A) 2, 4 or 8 (B) 3, 6, or 9 (C) 4 or 8 (D) 5 or 10

(b) Consider the system of equations

$ax + by = 0$, $cx + dy = 0$, where $a, b, c, d \in \{0, 1\}$.

STATEMENT-1 : The probability that the system of equations has a unique solution is $\frac{3}{8}$.

and

STATEMENT-2 : The probability that the system of equations has a solution is 1.

(A) Statement-1 is True, Statement-2 is True ; statement-2 is a correct explanation for statement-1
(B) Statement-1 is True, Statement-2 is True ; statement-2 is NOT a correct explanation for statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True **[JEE 2008, 3+3]**

Pragraph for Question Nos. 14 to 16 **[JEE 2009]**

A fair die is tossed repeatedly until a six is obtained. Let X denote the number of tosses required.

14. The probability that x = 3 equals

(A) $\frac{25}{216}$ (B) $\frac{25}{36}$ (C) $\frac{5}{36}$ (D) $\frac{125}{216}$

15. The probability that $X \geq 3$ equals

(A) $\frac{125}{216}$ (B) $\frac{25}{36}$ (C) $\frac{5}{36}$ (D) $\frac{5}{216}$

16. The conditional probability that $X \geq 6$ given $X > 3$ equals

(A) $\frac{125}{216}$ (B) $\frac{25}{216}$ (C) $\frac{5}{36}$ (D) $\frac{25}{36}$

17. Let ω be a complex cube root of unity with $\omega \neq 1$. A fair die is thrown three times. If r_1, r_2 and r_3 are the numbers obtained on the die, then the probability that $\omega^{r_1} + \omega^{r_2} + \omega^{r_3} = 0$ is **[JEE 2010]**

(A) $\frac{1}{18}$ (B) $\frac{1}{9}$ (C) $\frac{2}{9}$ (D) $\frac{1}{36}$

18. A signal which can be green or red with probability $\frac{4}{5}$ and $\frac{1}{5}$ respectively, is received by station A and then transmitted to station B. The probability of each station receiving the signal correctly is 3/4. If the signal received at station B is green, then the probability that the original signal was green is **[JEE 2010]**

(A) $\frac{3}{5}$ (B) $\frac{6}{7}$ (C) $\frac{20}{23}$ (D) $\frac{9}{20}$

Paragraph for question nos. 19 to 20 **[JEE 2011]**

Let U_1 and U_2 be two urns such that U_1 contains 3 white and 2 red balls, and U_2 contains only 1 white ball. A fair coin is tossed. If head appears then 1 ball is drawn at random from U_1 and put into U_2. However, if tail appears then 2 balls are drawn at random from U_1 and put into U_2. Now 1 ball is drawn at random from U_2.

19. The probability of the drawn ball from U_2 being white is

(A) $\frac{13}{30}$ (B) $\frac{23}{30}$ (C) $\frac{19}{30}$ (D) $\frac{11}{30}$

20. Given that the drawn ball from U_2 is white, the probability that head appeared on the coin is

(A) $\frac{17}{23}$ (B) $\frac{11}{23}$ (C) $\frac{15}{23}$ (D) $\frac{12}{23}$

21. Let E and F be two independent events. The probability that exactly one of them occurs is $\frac{11}{25}$ and the probability of none of them occurring is $\frac{2}{25}$. If P(T) denotes the probability of occurrence of the event T, then **[JEE 2011]**

(A) $P(E) = \frac{4}{5}, P(F) = \frac{3}{5}$ (B) $P(E) = \frac{1}{5}, P(F) = \frac{2}{5}$ (C) $P(E) = \frac{2}{5}, P(F) = \frac{1}{5}$ (D) $P(E) = \frac{3}{5}, P(F) = \frac{4}{5}$

22. A ship is fitted with three engines E_1, E_2 and E_3. The engines function independently of each other with respective probabilities $\frac{1}{2}, \frac{1}{4}$ and $\frac{1}{4}$. For the ship to be operational at least two of its engines must function. Let X denote the event that the ship is operational and let X_1, X_2 and X_3 denote respectively the events that the engines E_1, E_2 and E_3 are functioning. Which of the following is (are) true ? **[JEE 2012]**

(A) $P\left[X_1^c \middle| X\right] = \frac{3}{16}$ (B) P (Exactly two engines of the ship are functioning $|X) = \frac{7}{8}$

(C) $P[X|X_2] = \frac{5}{16}$ (D) $P[X|X_1] = \frac{7}{16}$

23. Four fair dice D_1, D_2, D_3 and D_4 each having six faces numbered 1, 2, 3, 4, 5, and 6, are rolled simultaneously. The probability that D_4 shows a number appearing on D_1, D_2 and D_3 is : **[JEE 2012]**

(A) $\frac{91}{216}$ (B) $\frac{108}{216}$ (C) $\frac{125}{216}$ (D) $\frac{127}{216}$

24. Let X and Y be two events such that $P(X \mid Y) = \frac{1}{2}$, $P(Y \mid X) = \frac{1}{3}$ and $P(X \cap Y) = \frac{1}{6}$. Which of the following is (are) correct? **[JEE 2012]**

(A) $P(X \cup Y) = \frac{2}{3}$ (B) X and Y are independent

(C) x and Y are not independent (D) $P(X^C \cap Y) = \frac{1}{3}$

25. A multiple choice examinaton has 5 questions. Each question has three alternative answers of which exactly one is correct. The probability that a student will get 4 or more correct answers just by guessing is:

(A) $\frac{10}{3^5}$ (B) $\frac{17}{3^5}$ (C) $\frac{13}{3^5}$ (D) $\frac{11}{3^5}$ **[IIT JEE Main 2013]**

26. Four persons independently solve a certain problem correctly with probabilities $\frac{1}{2}, \frac{3}{4}, \frac{1}{4}, \frac{1}{8}$. Then the probability that the problem is solved correctly by at least one of them is : **[JEE 2013]**

(A) $\frac{235}{256}$ (B) $\frac{21}{256}$ (C) $\frac{3}{256}$ (D) $\frac{253}{256}$

Comprehension (Q.27 to Q.28) **[JEE 2013]**

A box B_1 contains 1 white ball, 3 red balls and 2 black balls. Another box B_2 contains 2 white balls, 3 red ball and 4 black balls. A third box B_3 contains 3 white balls, 4 red balls and 5 black balls.

27. If 2 balls are drawn (without replacement) from a randomly selected box and one of the balls is white and the other ball is red, the probability that these 2 balls are drawn from box B_2 is

(A) $\frac{116}{181}$ (B) $\frac{126}{181}$ (C) $\frac{65}{181}$ (D) $\frac{55}{181}$

28. If 1 ball is drawn from each of the boxes B_1, B_2 and B_3, the probability that all 3 drawn balls are of the same colour is

(A) $\frac{82}{648}$ (B) $\frac{90}{648}$ (C) $\frac{558}{648}$ (D) $\frac{566}{648}$

29. Of the three independent events E_1, E_2 and E_3, the probability that only E_1 occurs is α, only E_2 occurs is β and only E_3 occurs is γ. Let the probability p that none of events E_1, E_2 or E_3 occurs satisfy the equations $(\alpha - 2\beta)\,p = \alpha\beta$ and $(\beta - 3\gamma)\,p = 2\beta\gamma$. All the given probabilities are assumed to lie in the interval (0, 1)

Then $\frac{\text{Probability of occurrence of } E_1}{\text{Probability of occurrence of } E_3} =$ **[JEE 2013]**

30. Let A and B be two events such that $P(\overline{A \cup B}) = \frac{1}{6}, P(A \cap B) = \frac{1}{4}$ and $P(\overline{A}) = \frac{1}{4}$, where $\overline{A}$ stands for the complement of the event A. Then the events A and B are **[IIT JEE Main 2014]**

(A) Independent and equally likely. (B) Mutually exclusive and independent.

(C) Equally likely but not independent. (D) Independent but not equally likely.

31. Three boys and two girls stand in a queue. The probability, that the number of boys ahead of every girl is at least one more than the number of girls ahead of her, is : **[IIT JEE Advance 2014]**

(A) $\frac{1}{2}$ (B) $\frac{1}{3}$ (C) $\frac{2}{3}$ (D) $\frac{3}{4}$

Comprehension (Q.32 to Q.33) **[IIT JEE Advance 2014]**

Box 1 contains three cards bearing numbers 1, 2, 3 ; box 2 contains five cards bearing numbers 1, 2, 3, 4, 5 ; and box 3 contains seven cards bearing numbers 1, 2, 3, 4, 5, 6, 7. A card is drawn from each of the boxes. Let x_1 be the number on the card drawn from the i^{th} box, i = 1, 2, 3.

32. The probability that $x_1 + x_2 + x_3$ is odd, is :

(A) $\frac{29}{105}$ (B) $\frac{53}{105}$ (C) $\frac{57}{105}$ (D) $\frac{1}{2}$

33. The probability that x_1, x_2, x_3 are in an arithmetic progression, is :

(A) $\frac{9}{105}$ (B) $\frac{10}{105}$ (C) $\frac{11}{105}$ (D) $\frac{7}{105}$

34. If 12 identical balls are to be placed in 3 identical boxes, then the probability that one of the boxes contains exactly 3 balls is : **[IIT JEE Main 2015]**

(A) $22\left(\frac{1}{3}\right)^{11}$ (B) $\frac{55}{3}\left(\frac{2}{3}\right)^{11}$ (C) $55\left(\frac{2}{3}\right)^{10}$ (D) $220\left(\frac{1}{3}\right)^{12}$

35. The minimum number of times a fair coin needs to be tossed, so that the probability of getting at least two heads is at least 0.96, is **[IIT JEE Advance 2015]**

Paragraph for question no. 36 to 37 **[IIT JEE Advance 2015]**

Let n_1 and n_2 be the number of red and black balls, respectively, in box I. Let n_3 and n_4 be the number of red and black balls, respectively, in box II.

36. One of the two boxes, box I and box II, was selected at random and a ball was drawn randomly out of this box. The ball was found to be red. If the probability that this red ball was drawn from box II is $\frac{1}{3}$, then the correct option(s) with the possible values of n_1, n_2, n_3 and n_4 is(are)

(A) $n_1 = 3, n_2 = 3, n_3 = 5, n_4 = 15$ (B) $n_1 = 3, n_2 = 6, n_3 = 10, n_4 = 50$
(C) $n_1 = 8, n_2 = 6, n_3 = 5, n_4 = 20$ (D) $n_1 = 6, n_2 = 12, n_3 = 5, n_4 = 20$

37. A ball is drawn at random from box I and transferred to box II. If the probability of drawing a red ball from box I, after this transfer, is $\frac{1}{3}$, then the correct option(s) with the possible values of n_1 and n_2 is(are)

(A) $n_1 = 4$ and $n_2 = 6$ (B) $n_1 = 2$ and $n_2 = 3$ (C) $n_1 = 10$ and $n_2 = 20$ (D) $n_1 = 3$ and $n_2 = 6$

38. Let two fair six-faced dice A and B be thrown simultaneously. If E_1 is the event that die A shows up four, E_2 is the event that die B shows up two and E_3 is the event that the sum of numbers on both dice is odd, then which of the following statements is NOT true ? **[IIT JEE Main 2016]**

(A) E_1 and E_2 are independent (B) E_2 and E_3 are independent
(C) E_1 and E_3 are independent (D) E_1, E_2 and E_3 are independent

39. A box contains 15 green and 10 yellow balls. If 10 balls are randomly drawn, one-by-one, with replacement, then the variance of the number of green balls drawn is : **[IIT JEE Main 2017]**

(A) 4 (B) $\frac{6}{25}$ (C) $\frac{12}{5}$ (D) 6

40. If two different numbers are taken from the set {0, 1, 2, 3, ..., 10} ; then the probability that their sum as well as absolute difference are both multiple of 4, is : **[IIT JEE Main 2017]**

(A) $\frac{14}{45}$ (B) $\frac{7}{55}$ (C) $\frac{6}{55}$ (D) $\frac{12}{55}$

41. For three events A, B and C, P(Exactly one of A or B occurs) = P(Exactly one of B or C occurs) = P(Exactly one of C or A occurs) = $\frac{1}{4}$ and P(All the three events occur simultaneously) = $\frac{1}{16}$.
Then the probability that at least one of the events occurs, is : **[IIT JEE Main 2017]**

(A) $\frac{7}{64}$ (B) $\frac{3}{16}$ (C) $\frac{7}{32}$ (D) $\frac{7}{16}$

42. Let X and Y be two events such that $P(X)=\frac{1}{3}, P(X|Y)=\frac{1}{2}$ and $P(Y|X)=\frac{2}{5}$. Then **[IIT JEE Advance 2017]**

(A) $P(Y)=\frac{4}{15}$ (B) $P(X'|Y)=\frac{1}{2}$ (C) $P(X\cap Y)=\frac{1}{5}$ (D) $P(X\cup Y)=\frac{2}{5}$

43. Three randomly chosen nonnegative integers x, y and z are found to satisfy the equation x + y + z = 10. Then the probability that z is even, is : **[IIT JEE Advance 2017]**

(A) $\frac{36}{55}$ (B) $\frac{6}{11}$ (C) $\frac{5}{11}$ (D) $\frac{1}{2}$

44. A bag contains 4 red and 6 black balls. A ball is drawn at random from the bag, its colour is observed and this ball along with two additional balls of the same colour are returned to the bag. If now a ball is drawn at random from the bag, then the probability that this drawn ball is red, is : **[JEE Main 2018]**

(A) $\frac{3}{4}$ (B) $\frac{3}{10}$ (C) $\frac{2}{5}$ (D) $\frac{1}{5}$

Paragraph 'A' for Question Nos. 45 to 46

There are five students S_1, S_2, S_3, S_4 and S_5 in a music class and for them there are five seats R_1, R_2, R_3, R_4 and R_5 arranged in a row, where initially the seat R_i is allotted to the student S_i , i = 1, 2, 3, 4, 5. But, on the examination day, the five students are randomly allotted the five seats. **[JEE Advanced 2018]**

45. The probability that, on the examination day, the student S_1 gets the previously allotted seat R_1, and **NONE** of the remaining students gets the seat previously allotted to him/her is

(A) $\frac{3}{40}$ (B) $\frac{1}{8}$ (C) $\frac{7}{40}$ (D) $\frac{1}{5}$

46. For i = 1, 2, 3, 4 let T_i denote the event that the students S_i and S_{i+1} do **NOT** sit adjacent to each other on the day of the examination. Then, the probability of the event $T_1 \cap T_2 \cap T_3 \cap T_4$ is

(A) $\frac{1}{15}$ (B) $\frac{1}{10}$ (C) $\frac{7}{60}$ (D) $\frac{1}{5}$

47. There are three bags B_1, B_2 and B_3. The bag B_1 contains 5 red and 5 green balls, B_2 contains 3 red and 5 green balls, and B_3 contains 5 red and 3 green balls. Bags B_1, B_2 and B_3 have probabilities $\frac{3}{10}, \frac{3}{10}$ and $\frac{4}{10}$ respectively of being chosen. A bag is selected at random and a ball is chosen at random from the bag. Then which of the following options is/are correct? **[JEE Advanced 2019]**

(A) Probability that the chosen ball is green, given that the selected bag is B_3, equals $\frac{3}{8}$

(B) Probability that the selected bag is B_3, given that the chosen ball is green equals $\frac{5}{13}$

(C) Probability that the selected bag is B_3 and the chosen ball is green equals $\frac{3}{10}$

(D) probability that the chosen ball is green equals $\frac{39}{80}$

48. Let S be the sample space of all 3 × 3 matrices with entries from the set {0, 1}. Let the events E_1 and E_2 be given by

$E_1 = \{A \in S : \det A = 0\}$ and

$E_2 = \{A \in S : \text{sum of entries of A is } 7\}$. **[JEE Advanced 2019]**

If a matrix is chosen at random from S, then the conditional probability $P(E_1 | E_2)$ equals____

49. Let |X| denote the number of elements in a set X. Let S = {1, 2, 3, 4, 5, 6} be a sample sapce, where each element is equally likely to occur. If A and B are independent events associated with S, then the number of ordered pairs (A, B) such that $1 \le |B| < |A|$, equals___. **[JEE Advanced 2019]**

50. The probability that a missile hits a target successfully is 0.75. In order to destroy the target completely, at least three successful hits are required. Then the minimum number of missiles that have to be fired so that the probability of completely destroying the target is **NOT** less than 0.95, is _____ **[JEE Advanced 2020]**

51. Two fair dice, each with faces numbered 1, 2, 3, 4, 5 and 6, are rolled together and the sum of the numbers on the faces is observed. This process is repeated till the sum is either a prime number or a perfect square. Suppose the sum turns out to be a perfect square before it turns out to be a prime number. If p is the probability that this perfect square is an odd number, then the value of 14p is _____ **[JEE Advanced 2020]**

52. Consider three sets $E_1 = \{1, 2, 3\}$, $F_1 = \{1, 3, 4\}$ and $G_1 = \{2, 3, 4, 5\}$. Two elements are chosen at random, without replacement, from the set E_1, and let S_1 denote the set of these chosen elements. Let $E_2 = E_1 - S_1$ and $F_2 = F_1 \cup S_1$. Now two elements are chosen at random, without replacement, from the set F_2 and let S_2 denote the set of these chosen elements.

Let $G_2 = G_1 \cup S_2$. Finally, two elements are chosen at random, without replacement, from the set G_2 and let S_3 denote the set of these chosen elements.

Let $E_3 = E_2 \cup S_3$. Given that $E_1 = E_3$, let p be the conditional probability of the event $S_1 = \{1, 2\}$. Then the value of p is **[JEE Advanced 2021]**

(A) $\frac{1}{5}$ (B) $\frac{3}{5}$ (C) $\frac{1}{2}$ (D) $\frac{2}{5}$

53. Let E, F and G be three events having probabilities

$P(E) = \frac{1}{8}$, $P(F) = \frac{1}{6}$ and $P(G) = \frac{1}{4}$, and let $P(E \cap F \cap G) = \frac{1}{10}$.

For any event H, if H^c denotes its complement, then which of the following statements is (are) **TRUE** ?

[JEE Advanced 2021]

(A) $P(E \cap F \cap G^c) \le \frac{1}{40}$

(B) $P(E^c \cap F \cap G) \le \frac{1}{15}$

(C) $P(E \cup F \cup G) \le \frac{13}{24}$

(D) $P(E^c \cap F^c \cap G^c) \le \frac{5}{12}$

Question Stem for Question Nos. 54 and 55

Question Stem **[JEE Advanced 2021]**

Three numbers are chosen at random, one after another with replacement, from the set $S = \{1, 2, 3, \ldots, 100\}$. Let p_1 be the probability that the maximum of chosen numbers is at least 81 and p_2 be the probability that the minimum of chosen numbers is at most 40.

54. The value of $\frac{625}{4} p_1$ is_____.

55. The value of $\frac{125}{4} p_2$ is_____.

56. A number is chosen at random from the set {1, 2, 3, … , 2000}. Let p be the probability that the chosen number is a multiple of 3 or a multiple of 7. Then the value of 500p is ___ . **[JEE Advanced 2021]**

57. Suppose that **[JEE Advanced 2022]**

Box-I contains 8 red, 3 blue and 5 green balls,
Box-II contains 24 red, 9 blue and 15 green balls,
Box-III contains 1 blue, 12 green and 3 yellow balls,
box-IV contains 10 green, 16 orange and 6 white balls.

A ball is chosen randomly from Box-I ; call this ball b. If b is red then a ball is chosen randomly from Box-II, if b is blue then a ball is chosen randomly from Box-III , and if b is green then a ball is chosen randomly from Box-IV. The conditional probability of the even 'one of the chosen ball is white' given that the event 'at least one of the chosen balls is green' has happened, is equal to :

(A) $\frac{15}{256}$ (B) $\frac{3}{16}$ (C) $\frac{5}{52}$ (D) $\frac{1}{8}$

58. Two players, P_1 and P_2, play a game against each other. In every round of the game, each player rolls a fair die once, where the six faces of the die have six distinct numbers. Let x and y denote the readings on the die rolled by P_1 and P_2, respectively. If $x > y$, then P_1 scores 5 points and P_2 scores 0 point. If $x = y$, then each player scores 2 points. If $x < y$, then P_1 scores 0 point and P_2 scores 5 points. Let X_i and Y_i be the total scores of P_1 and P_2, respectively, after playing the i^{th} round. **[JEE Advanced 2022]**

List-I	**List-II**
(I) Probability of $(X_2 \geq Y_2)$ is	(P) $\frac{3}{8}$
(II) Probability of $(X_2 > Y_2)$ is	(Q) $\frac{11}{16}$
(III) Probability of $(X_3 = Y_3)$ is	(R) $\frac{5}{16}$
(IV) Probability of $(X_3 > Y_3)$ is	(S) $\frac{355}{864}$
	(T) $\frac{77}{432}$

The correct option is :

(A) (I) $\rightarrow$ (Q); (II) $\rightarrow$ (R); (III) $\rightarrow$ (T); (IV) $\rightarrow$ (S)
(B) (I) $\rightarrow$ (Q); (II) $\rightarrow$ (R); (III) $\rightarrow$ (T); (IV) $\rightarrow$ (T)
(C) (I) $\rightarrow$ (P); (II) $\rightarrow$ (R); (III) $\rightarrow$ (Q); (IV) $\rightarrow$ (S)
(D) (I) $\rightarrow$ (P); (II) $\rightarrow$ (R); (III) $\rightarrow$ (Q); (IV) $\rightarrow$ (T)

59. In a study about a pandemic, data of 900 persons was collected. It was found that

190 persons had symptom of fever,
220 persons had symptom of cough,
220 persons had symptom of breathing problem,
330 persons had symptom of fever or cough or both,
350 persons had symptom of cough or breathing problem or both,
340 persons had symptom of fever or breathing problem or both,
30 persons had all three symptoms (fever, cough and breathing problem).

If a person is chosen randomly from these 900 persons, then the probability that the person has at most one symptom is ________. **[JEE Advanced 2022]**

ANSWER KEY
EXERCISE-I

1. **(i)** 3/56 **(ii)** 9/28 **2.** $97/(25)^4$ **3.** Option B **4.** 13/24 **5.** 5/9

6. 120 **7.** **(a)** 0.61; **(b)** 0.4758 **8.** 2065 **9.** $\frac{29}{30}$

10. $\frac{328}{625}$ **11.** **(i)** 1/10, **(ii)** 3/10, **(iii)** 2/3 **12.** 1/2

13. 0.6976 **14.** 19/42 **15.** 283 **16.** 1/2

17. 407/576 **18.** (i) $\frac{125}{16^3}$; (ii) $\frac{63}{64}$ **19.** 101/1326 **20.** $\frac{{}^{24}C_2}{{}^{64}C_2}$ or $\frac{23}{168}$

21. Independent in (a) and not independent in (b) **22.** $P_1 = 1/2$, $P_2 = 3/4$

23. $(1-p)^{n-1} . [\, p_o (1-p) + np(1-p_0)\,]$ **24.** $A \to R$; $B \to Q$; $C \to P$; $D \to S$;

25. 4 **26.** (a) $\frac{64}{455}$; (b) $\frac{24}{91}$

EXERCISE-II

1. (a) 6%, 21% ; (b) 79%, 44%, (c) 9/44 ≈ 20.45% **2.** 5/8 **3.** 74/81

4. $1-[\,1-p\,(1-p_1)\,(1-p_2)]^n$ **5.** $(3^n-2^{n+1}+1)/(4^n-2^{n+1}+1)$ **6.** 179 **7.** **(a)** $\frac{(n-4)\,(n-5)}{n((n-1)}$

8. 1/2, 1/2, 1/12 **9.** $\frac{4}{9}$ **10.** $\frac{4}{13}$ **11.** 13

12. 458 **13.** $\frac{11}{663}$ **14.** **(a)** $\frac{(5n-3)}{(9n-3)}$ **(b)** $\frac{5}{16}$

15. n + 1 : n – 1 **16.** 79 **18.** $\frac{KnM + LmN}{MN(K+L)}$ **19.** 55

20. 317 **21.** 206

EXERCISE-III

1. **(a)** A **(b)** B, C **(c)** 4/35 **2.** 0.304 **4.** $\frac{1}{2}$ & $\frac{1}{3}$ or $\frac{1}{3}$ & $\frac{1}{2}$

5. $\frac{1}{26}$ **6.** **(a)** $\frac{m}{m+n}$; **(b)** $\frac{{}^6C_3\left(3^n - 3.2^n + 3\right)}{6^n}$ **7.** $\frac{9m}{m+8N}$

8. (a) $p^2(2-p)$; (b) 1/2 **9.** **(a)** D , **(c)** $\frac{{}^{12}C_2\,{}^6C_4\,{}^{10}C_1\,{}^2C_1 + {}^{12}C_1\,{}^6C_5\,{}^{11}C_1\,{}^1C_1}{{}^{12}C_2\left({}^{12}C_2\,{}^6C_4 + {}^{12}C_1\,{}^6C_5 + {}^{12}C_0\,{}^6C_6\right)}$

10. **(a)** A, **(b)** 1/7 **11.** (a) B, (b) A, (c) B **12.** **(a)** C; **(b)** C; **(c)** D

13. **(a)** D, **(b)** B **14.** A **15.** B **16.** D **17.** C

18. C **19.** B **20.** D **21.** AD

22. B, D **23.** A **24.** A, B **25.** D **26.** A

27. D **28.** A **29.** 6 **30.** D **31.** A

32. B **33.** C **34.** B **35.** 8 **36.** A, B **37.** C, D **38.** D

39. C **40.** C **41.** D **42.** AB **43.** B **44.** C **45.** A

46. C **47.** A D **48.** 0.50 **49.** 422.00 **50.** 6 **51.** 8.00

52. A **53.** A,B,C **54.** 76.25 **55.** 24.50 **56.** 214.00 **57.** C **58.** A

59. 0.8