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DPP-5

Solutions

1. Prove that $\cot\theta - \tan\theta = 2\cot 2\theta$.

Sol. $\cot\theta - \tan\theta = \frac{\cos\theta}{\sin\theta} - \frac{\sin\theta}{\cos\theta} = \frac{\cos^2\theta - \sin^2\theta}{\sin\theta\cos\theta} = \frac{2\cos 2\theta}{\sin 2\theta} = 2\cot 2\theta$.

2. Prove that $\frac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta} = \sec 2\theta - \tan 2\theta$.

Sol. $\frac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta}$

multiplying by $\frac{\cos\theta - \sin\theta}{\cos\theta - \sin\theta}$

$$= \frac{(\cos\theta - \sin\theta)^2}{\cos^2\theta - \sin^2\theta} = \frac{\cos^2\theta + \sin^2\theta - 2\sin\theta\cos\theta}{\cos 2\theta} = \frac{1 - \sin 2\theta}{\cos 2\theta} = \sec 2\theta - \tan 2\theta$$

3. Prove that $\tan\left(\frac{\pi}{4} + \theta\right) - \tan\left(\frac{\pi}{4} - \theta\right) = 2\tan 2\theta$.

Sol. $\tan\left(\frac{\pi}{4} + \theta\right) = \frac{\tan(\pi/4) + \tan\theta}{1 - \tan(\pi/4)\tan\theta} = \frac{1 + \tan\theta}{1 - \tan\theta}$

Similarly, $\tan\left(\frac{\pi}{4} - \theta\right) = \frac{1 + \tan\theta}{1 - \tan\theta}$

$$\text{LHS} = \frac{1 + \tan\theta}{1 - \tan\theta} - \frac{1 - \tan\theta}{1 + \tan\theta} = \frac{(1 + \tan\theta)^2 - (1 - \tan\theta)^2}{1 - \tan^2\theta} = \frac{4\tan\theta}{1 - \tan^2\theta} =$$

$$\frac{2\tan\theta}{1 - \tan^2\theta} = 2\tan 2\theta$$

4. Prove that $1 + \tan\theta\tan 2\theta = \sec 2\theta$

Sol. $1 + \tan 2\theta\tan\theta = 1 + \frac{(2\tan\theta\tan\theta)}{(1 - \tan^2\theta)} = \frac{1 - \tan^2\theta + (2\tan^2\theta)}{(1 - \tan^2\theta)} = \frac{1 + \tan^2\theta}{1 - \tan^2\theta} = \frac{1 + \frac{\sin^2\theta}{\cos^2\theta}}{1 - \frac{\sin^2\theta}{\cos^2\theta}}$

$$\cos^2\theta + \sin^2\theta = \frac{\frac{\cos^2\theta}{\cos^2\theta - \sin^2\theta}}{\cos^2\theta} = \frac{1}{\cos 2\theta} = \sec 2\theta$$

5. Prove that $\frac{1 + \sin 2A - \cos 2A}{1 + \sin 2A + \cos 2A} = \tan A$

Sol. $\frac{1 + \sin 2A - \cos 2A}{1 + \sin 2A + \cos 2A} = \frac{\sin 2A + 1 - (1 - 2\sin^2 A)}{\sin 2A + 1 + 2\cos^2 A - 1} = \frac{2\sin A\cos A + 2\sin^2 A}{2\sin A\cos A + 2\cos^2 A} = \frac{2\sin A(\sin A + \cos A)}{2\cos A(\cos A + \sin A)} = \frac{\sin A}{\cos A} = \tan A$.

6. Show that $\frac{1}{\sin 10^\circ} - \frac{\sqrt{3}}{\cos 10^\circ} = 4$

Sol. $\frac{1}{\sin 10^\circ} - \frac{\sqrt{3}}{\cos 10^\circ} = 2\left[\frac{1}{2\sin 10^\circ} - \frac{\sqrt{3}}{2\cos 10^\circ}\right] = 2\left[\frac{\sin 30^\circ}{\sin 10^\circ} - \frac{\cos 30^\circ}{\cos 10^\circ}\right] = 2\left[\frac{\sin 30^\circ\cos 10^\circ - \cos 30^\circ\sin 10^\circ}{\sin 10^\circ\cos 10^\circ}\right]$

$$= 2\left[\frac{\sin 20^\circ}{\sin 10^\circ\cos 10^\circ}\right] = 2 \times 2\left[\frac{\sin 10^\circ\cos 10^\circ}{\sin 10^\circ\cos 10^\circ}\right] = 4$$

7. Prove that $\operatorname{cosec}A - 2\cot 2A\cos A = 2\sin A$

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Sol. LHS $= \text{cosecA} - 2\cot 2A \cos A = \frac{1}{\sin A} - 2 \times \frac{\cos 2A}{\sin 2A} \times \cos A = \frac{1}{\sin A} - \frac{2(1-2\sin^2 A)}{2\sin A \cos A} \times \cos A$

$$= \frac{1}{\sin A} - \frac{(1-2\sin^2 A)}{\sin A} = \frac{1-1+2\sin^2 A}{\sin A} = 2\sin A$$

8. Prove that $\frac{1+\sin 2A}{\cos 2A} = \frac{\cos A + \sin A}{\cos A - \sin A} = \tan\left(\frac{\pi}{4} + A\right)$

Sol. $\frac{1+\sin 2A}{\cos 2A}$

using trigonometric identities to convert the expression in the form of tan

$$= \frac{1 + \frac{2\tan A}{1+\tan^2 A}}{\frac{1-\tan^2 A}{1+\tan^2 A}} = \frac{1+\tan^2 A + 2\tan A}{1-\tan^2 A} = \frac{(1+\tan A)^2}{1-\tan^2 A}$$

$$= \frac{1+\tan A}{1-\tan A} = \frac{1+\frac{\sin A}{\cos A}}{1-\frac{\sin A}{\cos A}} = \frac{\cos A + \sin A}{\cos A - \sin A} = \frac{1+\tan A}{1-\tan A} = \tan\left(\frac{\pi}{4} + A\right)$$

9. Prove that $\cos^3\theta \sin 3\theta + \sin^3\theta \cos 3\theta = \frac{3}{4}\sin 4\theta$

Sol. $\cos^3\theta \sin 3\theta + \sin^3\theta \cos 3\theta = \cos^3\theta (3\sin\theta - 4\sin^3\theta) + \sin^3\theta (4\cos^3\theta - 3\cos\theta)$

$$= 3\cos^3\theta \sin\theta - 4\sin^3\theta \cos^3\theta + 4\sin^3\theta \cos^3\theta - 3\sin^3\theta \cos\theta$$

$$= 3\cos\theta \sin\theta (\cos^2\theta - \sin^2\theta) = \frac{3}{2}\sin 2\theta \cos 2\theta = \frac{3}{4}\sin 4\theta$$

10. Prove that $\tan\theta + \tan(60° + \theta) + \tan(120° + \theta) = 3\tan 3\theta$

Sol. $\tan\theta + \tan(60° + \theta) + \tan(120° + \theta) = 3\tan 3\theta$

LHS $= \tan\theta + \tan(60° + \theta) + \tan(120° + \theta)$

$$= \tan\theta + \frac{(\tan 60° + \tan\theta)}{(1 - \tan 60° \cdot \tan\theta)} + \frac{(\tan 120° + \tan\theta)}{(1 - \tan 120° \cdot \tan\theta)} = \tan\theta + \frac{[\sqrt{3} + \tan\theta]}{[1 - \sqrt{3}\tan\theta]} + \frac{[\tan\theta - \sqrt{3}]}{[1 + \sqrt{3}\tan\theta]}$$

$$= \frac{\tan\theta + \left[\sqrt{3} + 3\tan\theta + \tan\theta + \sqrt{3}\tan^2\theta + \tan\theta - \sqrt{3} - \sqrt{3}\tan^2\theta + 3\tan\theta\right]}{[1 - 3\tan^2\theta]}$$

$$= \tan\theta + \frac{[8\tan\theta]}{[1 - 3\tan^2\theta]} = \frac{[9\tan\theta - 3\tan^3\theta]}{[1 - 3\tan^2\theta]} = 3\tan 3\theta = \text{RHS}$$

12. If $\cos\theta = \frac{\cos\alpha - \cos\beta}{1 - \cos\alpha\cos\beta}$, prove that one of the values of $\tan\frac{\theta}{2}$ is $\tan\frac{\alpha}{2}\cot\frac{\beta}{2}$

Sol. $\cos\theta = \frac{\cos\alpha - \cos\beta}{1 - \cos\alpha\cos\beta}$ or, $\frac{1-\tan^2\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}} = \frac{\cos\alpha - \cos\beta}{1 - \cos\alpha\cos\beta}$ or, $\frac{1+\tan^2\frac{\theta}{2}}{1-\tan^2\frac{\theta}{2}} = \frac{1 - \cos\alpha\cos\beta}{\cos\alpha - \cos\beta}$

or, $\tan^2\frac{\theta}{2} = \frac{1 - \cos\alpha\cos\beta - \cos\alpha + \cos\beta}{1 - \cos\alpha\cos\beta + \cos\alpha - \cos\beta}$ or, $\tan^2\frac{\theta}{2} = \frac{(1-\cos\alpha)(1+\cos\beta)}{(1+\cos\alpha)(1-\cos\beta)}$ or, $\tan^2\frac{\theta}{2} = \tan^2\frac{\alpha}{2}\cot^2\frac{\beta}{2}$

or, $\tan\frac{\theta}{2} = \pm\tan\frac{\alpha}{2}\cot\frac{\beta}{2}$

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13. If $\tan\theta \tan\phi = \sqrt{\frac{(a-b)}{(a+b)}}$, prove that $(a - b\cos 2\theta)(a - b\cos 2\phi)$ is independent of θ and ϕ.

Sol. Let us put $\tan\theta = t_1, \tan\phi = t_2$

$\therefore t_1^2 t_2^2 = \left(\frac{a-b}{a+b}\right)$ or $(a+b)t_1^2 t_2^2 = a - b \ldots (1)$

Also $\cos 2\theta = \frac{1-\tan^2\theta}{1+\tan^2\theta} = \frac{1-t_1^2}{1+t_1^2}$ etc.

Now $a - b\cos 2\theta = a - b\frac{(1-t_1^2)}{(1+t_1^2)} = \frac{(a-b)+(a+b)t_1^2}{(1+t_1^2)}$

Put for (a-b) from (1), $= \frac{a+b}{(1+t_1^2)}[t_1^2 t_2^2 + t_1^2] = \frac{(a+b)t_1^2(1+t_2^2)}{(1+t_1^2)}$

Similarly, $a - b\cos 2\phi = \frac{(a+b)}{1+t_2^2} t_2^2(1+t_1^2)$

$\therefore (a - b\cos 2\theta)(a - b\cos 2\phi) = (a+b)^2 t_1^2 t_2^2 = (a+b)^2\{(a-b)/(a+b)\} = a^2 - b^2$

which is independent of θ.

14. If θ is an acutc angle and $\sin\frac{\theta}{2} = \sqrt{\frac{x-1}{2x}}$, find $\tan\theta$ in terms of x.

Sol. $\sin\frac{\theta}{2} = \sqrt{\frac{x-1}{2x}} \Rightarrow \sin^2\frac{\theta}{2} = \frac{x-1}{2x} \Rightarrow 1 - \cos^2\frac{\theta}{2} = \frac{x-1}{2x} \Rightarrow \cos^2\frac{\theta}{2} = 1 - \frac{x-1}{2x}$

$\Rightarrow \cos^2\frac{\theta}{2} = \frac{2x-x+1}{2x} \Rightarrow \cos^2\frac{\theta}{2} = \frac{x+1}{2x} \Rightarrow \cos\frac{\theta}{2} = \sqrt{\frac{x+1}{2x}}$

$\Rightarrow \sin\frac{\theta}{2}\cdot\cos\frac{\theta}{2} = \sqrt{\frac{x-1}{2x}}\cdot\sqrt{\frac{x+1}{2x}} \Rightarrow \sin\frac{\theta}{2}\cdot\cos\frac{\theta}{2} = \frac{1}{2x}\sqrt{x^2-1}$

$\Rightarrow 2\sin\frac{\theta}{2}\cdot\cos\frac{\theta}{2} = \frac{1}{x}\sqrt{x^2-1} \Rightarrow \sin\theta = \frac{1}{x}\sqrt{x^2-1} \Rightarrow \sin^2\theta = \frac{x^2-1}{x^2} \Rightarrow 1 - \cos^2\theta = \frac{x^2-1}{x^2}$

$\Rightarrow \cos^2\theta = 1 - \frac{x^2-1}{x^2} \Rightarrow \cos^2\theta = 1 - \frac{x^2-1}{x^2} \Rightarrow \cos^2\theta = \frac{1}{x^2}$

$\Rightarrow \cos\theta = \frac{1}{x}$

$$\therefore \tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\frac{\sqrt{x^2-1}}{\frac{x}{x}}}{\frac{1}{x}} = \frac{\sqrt{x^2-1}}{x} \times \frac{x}{1} \Rightarrow \tan\theta = \sqrt{x^2-1}$$

Hence, the answer is $\sqrt{x^2-1}$.

15. Prove that $(1+\sec 2\theta)(1+\sec 4\theta)(1+\sec 8\theta) = \frac{\tan 8\theta}{\tan\theta}$

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Sol. $(1+\sec 2\theta)(1+\sec 4\theta)(1+\sec 8\theta)$

$= (1+\frac{1}{\cos 2\theta})(1+\frac{1}{\cos 4\theta})(1+\frac{1}{\cos 8\theta})$

$= \left(\frac{\cos 2\theta+1}{\cos 2\theta}\right)\left(\frac{\cos 4\theta+1}{\cos 4\theta}\right)\left(\frac{\cos 8\theta+1}{\cos 8\theta}\right)$

$= \left(\frac{2\cos^2\theta-1+1}{\cos 2\theta}\right)\left(\frac{2\cos^2 2\theta-1+1}{\cos 4\theta}\right)\left(\frac{2\cos^2 4\theta-1+1}{\cos 8\theta}\right)$

$= \frac{8\cos^2\theta}{\cos 8\theta}\cos 2\theta\cos 4\theta$

$= \frac{8\sin\theta\cos\theta\cos 2\theta\cos 4\theta\cos\theta}{\sin\theta\cos 8\theta}$

$= \frac{4\sin 2\theta\cos 2\theta\cos 4\theta\cos\theta}{\sin\theta\cos 8\theta}$

$= \frac{2\sin 4\theta\cos 4\theta}{\sin\theta\cos 8\theta}\cos\theta = \frac{\sin 8\theta\cos\theta}{\cos\theta\sin 8\theta} = \frac{\tan 8\theta}{\tan\theta}$

16. Prove that $\frac{\sin^2 3A}{\sin^2 A} - \frac{\cos^2 3A}{\cos^2 A} = 8\cos 2A$.

Sol. $\frac{\sin^2 3A}{\sin^2 A} - \frac{\cos^2 3A}{\cos^2 A} \Rightarrow \frac{\cos^2 A\sin^2 3A-\cos^2 3A\sin^2 A}{\sin^2 A\cos^2 A} \Rightarrow \frac{\cos^2 A(1-\cos^2 3A)-\cos^2 3A(1-\cos^2 A)}{\sin^2 A\cos^2 A}$

$\Rightarrow \frac{\cos^2 A-(\cos^2 A\cos^2 3A)-\cos^2 3A+(\cos^2 3A\cos^2 A)}{\sin^2 A\cos^2 A}$

$\Rightarrow \frac{\cos^2 A-\cos^2 3A}{\sin^2 A\cos^2 A} \Rightarrow -\frac{\sin(A-3A)\sin(A+3A)}{\sin^2 A\cos^2 A}$

$\Rightarrow \frac{4\sin 2A\sin 4A}{\sin^2 2A} \Rightarrow 4\frac{\sin 4A}{\sin 2A} = 8\frac{\sin 2A\cos 2A}{\sin 2A} = 8\cos 2A$

17. If $A = 110°$, then prove that $\frac{1+\sqrt{1+\tan^2 2A}}{\tan 2A} = -\tan A$.

Sol. Given $A = 110° \Rightarrow \sec 2A < 0$

So, $\frac{1+\sqrt{1+\tan^2 2A}}{\tan 2A} = \frac{1+\sqrt{\sec^2 2A}}{\tan 2A} = \frac{1+|\sec 2A|}{\tan 2A} = \frac{1-\sec 2A}{\tan 2A}$ $(\because \sec 2A < 0)$

$= \frac{1-\frac{1}{\cos 2A}}{\frac{\sin 2A}{\cos 2A}} = \frac{\cos 2A-1}{\sin 2A} = \frac{-2\sin^2 A}{2\sin A\cos A} = \frac{-\sin A}{\cos A} = -\tan A$

18. In triangle ABC, $a = 3, b = 4$ and $c = 5$. Then find the value of $\sin A + \sin 2B + \sin 3C$.

Sol. $\Delta = \sqrt{s(s-a)(s-b)(s-c)} = 6$, $\cos B = \frac{a^2+c^2-b^2}{2ac} = \frac{-16+9+25}{30} = \frac{18}{30}$

$\sin A = \frac{2\Delta}{bc} = \frac{3}{5}$

$\sin B = \frac{2\Delta}{ac} = \frac{12}{15} = \frac{4}{5}$

$\sin C = \frac{2\Delta}{ab} = \frac{12}{12} = 1$

Now : $\sin A + \sin 2B + \sin 3C = \sin A + 2\sin B\cos B + 3\sin C - 4\sin^3 C = \frac{14}{25}$