

$$\begin{aligned}
& \left(2R \sin \frac{\Delta}{n} \right)^2 + \left(2R \sin 2\frac{\Delta}{n} \right)^2 + \left(2R \sin 3\frac{\Delta}{n} \right)^2 + \dots + \left(2R \sin \frac{(n-1)\Delta}{n} \right)^2 \\
&= 2R^2 \left(\left(1 - \cos 2\frac{\Delta}{n} \right) + \left(1 - \cos 4\frac{\Delta}{n} \right) + \left(1 - \cos 6\frac{\Delta}{n} \right) + \dots + \left(1 - \cos 2\frac{(n-1)\Delta}{n} \right) \right) \\
&= 2R^2 \left((n-1) - \frac{\sin \frac{(n-1)\Delta}{n}}{\sin \frac{\Delta}{n}} \cos \Delta \right) \\
&= 2nR^2.
\end{aligned}$$

13.

$$\frac{\cos^2 x (1 - \sin^2 x)}{\cos^2 y} + \frac{\sin^2 x (1 - \cos^2 x)}{\sin^2 y} = 1$$

$$\frac{\cos^2 x}{\cos^2 y} + \frac{\sin^2 x}{\sin^2 y} - \sin^2 x \cos^2 x \left(\frac{\sin^2 y + \cos^2 y}{\cos^2 y \sin^2 y} \right) = 1$$

$$\underbrace{\cos^2 x \sin^2 y + \sin^2 x \cos^2 y} - \underbrace{\sin^2 x \cos^2 x - \sin^2 y \cos^2 y} = 0$$

$$(\cos^2 x - \cos^2 y)(\sin^2 y - \sin^2 x) = 0$$

$$\cos^2 y = \cos^2 x$$

$$\boxed{\sin^2 y = \sin^2 x}$$

$$\frac{\cos^4 y}{\cos^2 x} + \frac{\sin^4 y}{\sin^2 x} = \frac{\cos^4 x}{\cos^2 x} + \frac{\sin^4 x}{\sin^2 x}$$

$$= 1$$

$$\frac{b}{a} \sin^4 \alpha + \sin^4 \alpha + \frac{a}{b} \cos^4 \alpha + \cos^4 \alpha = 1$$

$$0 = \left(\sqrt{\frac{b}{a}} \sin^2 \alpha - \sqrt{\frac{a}{b}} \cos^2 \alpha \right)^2 + \frac{\tan^4 \alpha}{a} + \frac{1}{b} = \frac{(1 + \tan^2 \alpha)^2}{a + b}$$

$$\frac{b}{a} \sin^4 \alpha + \frac{a}{b} \cos^4 \alpha - 2 \sin^2 \alpha \cos^2 \alpha = 0$$

$$\cancel{\tan^4 \alpha} + \frac{b}{a} \tan^4 \alpha + \cancel{1} + \frac{a}{b} = \cancel{1 + \tan^4 \alpha} + 2 \tan^2 \alpha$$

$$\frac{b}{a} \tan^4 \alpha - 2 \tan^2 \alpha + \frac{a}{b} = 0$$

$$\boxed{\tan^2 \alpha = \frac{a}{b}} \leftarrow \left(\sqrt{\frac{b}{a}} \tan^2 \alpha - \sqrt{\frac{a}{b}} \right)^2 = 0$$

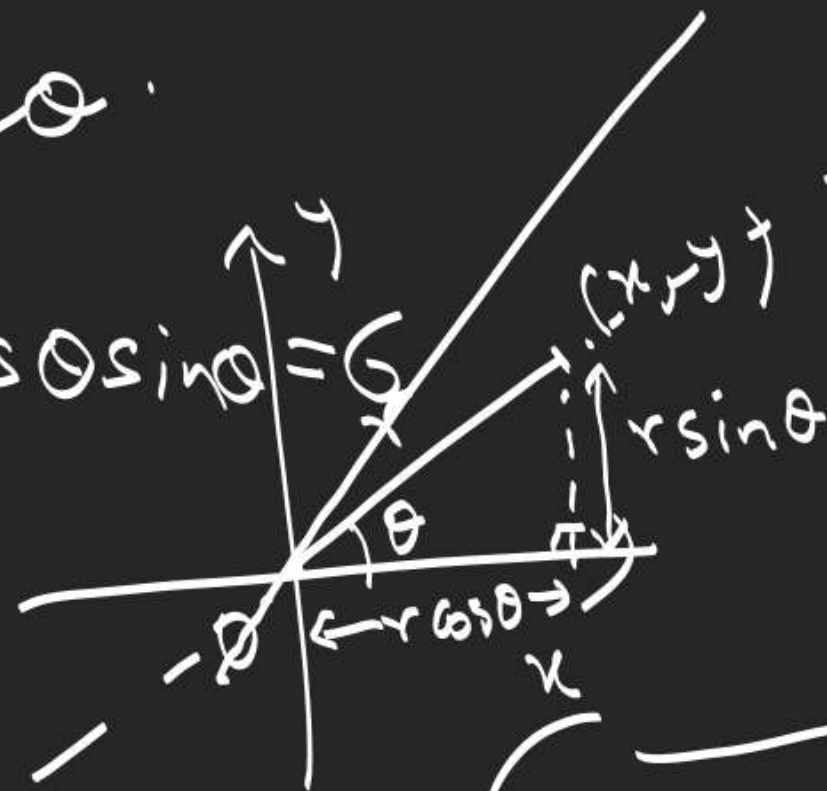
15.

$$\underline{x^2 + 2xy - y^2 = 6}$$

$$x = r \cos \theta, \quad y = r \sin \theta$$

$$r^2 (\cos^2 \theta - \sin^2 \theta) + 2r^2 \cos \theta \sin \theta = 6$$

$$r^2 = \frac{6}{\cos 2\theta + \sin 2\theta} \geq \frac{6}{\sqrt{2}}$$



$$(x, y) = (r \cos \theta, r \sin \theta)$$

r, θ varies

$$\underline{(x^2 + y^2)^2}_{\min} = r_{\min}^4$$

$$(x^2 + y^2)^2 = (x^2 - y^2)^2 + 4x^2y^2$$

$$= (6 - 2xy)^2 + 4x^2y^2 = 8x^2y^2 - 24xy + 36$$

$$= 8 \left(xy - \frac{3}{2} \right)^2 - \frac{9}{4} + 36 = 8 \left(xy - \frac{3}{2} \right)^2 + 18 \geq \boxed{18}$$

$$\therefore 5^{1+\log_4 x} + 5^{\frac{(\log_4 x)-1}{4}} = \frac{26}{5}$$

$$5^{1+\log_4 x} + 5^{\frac{-\log_4 x - 1}{4}} = \frac{26}{5}$$

$$t + \frac{1}{t} = \frac{26}{5}$$

$$5t^2 + 5 - 26t = 0$$

$$5t^2 - 25t - t + 5 = 0$$

$$(5t - 1)(t - 5) = 0$$

$$5^{1+\log_4 x} = 5^{-1}$$

$$5^1 \Rightarrow 1 + \log_4 x = -1 \text{ or } 1$$

$$\log_4 x = -2 \text{ or } 0$$

Ans

$$x = \frac{1}{16}, 1$$

$$\underline{2.} \quad \log_5 \left(5^{\frac{1}{2x}} + 125 \right) = \log_5(6) + 1 + \frac{1}{2x}$$

$$\log_5 \left(5^{\frac{1}{2x}} + 125 \right) = \log_5 6 + \log_5 5 + \log_5 \left(5^{\frac{1}{2x}} \right) \quad 5^{\frac{1}{2x}} = 5^1 \text{ or } 5^2$$

$$\log_5 \left(5^{\frac{1}{2x}} + 125 \right) = \log_5 \left(30 \cdot \left(5^{\frac{1}{2x}} \right) \right)$$

$$\frac{1}{2x} = 1 \text{ or } 2$$

$$5^{\frac{1}{2x}} = t \Rightarrow \boxed{x = \frac{1}{2}, \frac{1}{4}}$$

$$\log_a x_1 = \log_a x_2$$

$$\Rightarrow x_1 = x_2$$

$$\Rightarrow$$

$$5^{\frac{1}{2x}} + 125 =$$

$$a = 5^N$$

$$t^2 - 30t + 125 = 0$$

$$(t - 5)(t - 25) = 0$$

$$5^{\log_5(6) + 1 + \frac{1}{2x}} = 5^{\log_5 6} \cdot 5^1 \cdot 5^{\frac{1}{2x}}$$

$$= 6 \cdot 5 \cdot 5^{\frac{1}{2x}}$$

$$\Rightarrow 5^{\frac{1}{2x}} + 125 = 30 \cdot \left(5^{\frac{1}{2x}} \right)$$

$$\underline{3.} \quad \log_4 \left(2 \log_3 (1 + \log_2 (1 + 3 \log_2 x)) \right) = \frac{1}{2}$$

$$2 \log_3 (1 + \log_2 (1 + 3 \log_2 x)) = 4^{\frac{1}{2}} = 2$$

$$\log_3 (1 + \log_2 (1 + 3 \log_2 x)) = 1$$

$$1 + \log_2 (1 + 3 \log_2 x) = 3$$

$$\log_2 (1 + 3 \log_2 x) = 2$$

$$1 + 3 \log_2 x = 2^2 \Rightarrow \boxed{\log_2 x = 1}$$

$$\Rightarrow x = 2$$

$$\log_a b = x$$

$$b = a^x$$

$$\log_4 x = \frac{1}{2}$$

$$\Rightarrow x = 4^{\frac{1}{2}}$$

4. $(x+1)^{\log_{10}(x+1)} = 100(x+1)$

$(\log_a b)^2 = \log_a^2 b$ $\Rightarrow \log_{10} \left((x+1)^{\log_{10}(x+1)} \right) = \log_{10}(100(x+1))$

$x_1 = x_2$

$\log_a x_1 = \log_a x_2$

$\Rightarrow \left(\log_{10}(x+1) \right) \log_{10}(x+1) = \log_{10} 100 + \log_{10}(x+1)$

$\left(\log_{10}(x+1) \right)^2 = 2 + \log_{10}(x+1)$

$x = 99, -\frac{2}{10} \Rightarrow \underline{\underline{\text{Ans}}}$

$x+1 = 10 \text{ or } 10^{-1} \Leftrightarrow \log_{10}(x+1) = 2 \text{ or } -1 \Leftrightarrow$

$t^2 - t - 2 = (t-2)(t+1) = 0$

$$\underline{5.} \quad 3^{\log_3^2 x} + x^{\log_3 x} = 162$$

$$\log_3^2 x = (\log_3 x)^2 = (\log_3 x)(\log_3 x)$$

$$\left(3^{\log_3 x}\right)^{\log_3 x} + x^{\log_3 x} = 162$$

$$x^{\log_3 x} + x^{\log_3 x} = 162$$

$$x^{\log_3 x} = 81 \Rightarrow \log_3 (x^{\log_3 x}) = \log_3 81$$

$$\boxed{x=9, \frac{1}{9}} \Leftarrow \log_3 x = 2, -2 \Leftarrow (\log_3 x)^2 = 4 \Leftarrow (\log_3 x)(\log_3 x) = 4$$

6.

$$x^{\log_{\sqrt{x}}(x-2)} = 9$$

$$x^{2 \log_x(x-2)} = 9$$

$$x = 5$$

$$\left(x^{\log_x(x-2)} \right)^2 = 9$$

$$(x-2)^2 = 9 \Rightarrow x-2 = 3 \text{ or } -3$$

$$x = 5, -1$$

$x = -1$ rejected

Sec 1.9

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NCERT

Natural logarithm $(x-2)^{\log}$

$$\log x = \log_e x = \ln x$$

$e = \text{irrational no.} \approx 2.71828$

Preface $\rightarrow \log x = \log_{10} x$

$$10^2 \log(x-2)$$

$$2 \log(x-2)$$

$$10$$

Sec 1.9

125-140

Ex-II (Q. 16, Q. 17)