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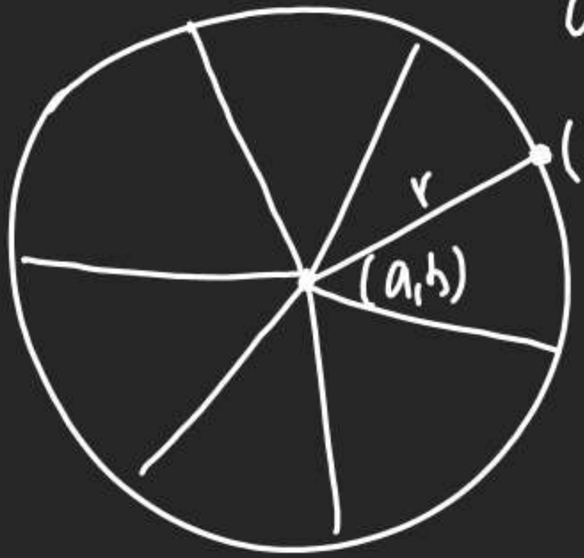
Circle.

① Basic Defination

A) Circle is Locus.

B) Circle is Locus of a pt. Which Remains at a Constant distance from a fixed Pt. & that distance is rad of Circle.



① $\sqrt{(x-a)^2+(y-b)^2} = r$

$\Rightarrow \boxed{(x-a)^2+(y-b)^2 = r^2}$ Centre Radius form

here (a,b) = Centre

r = Radius

Q Find EOC having Centre $(1,3)$ & Rad $= 2$

An EOC

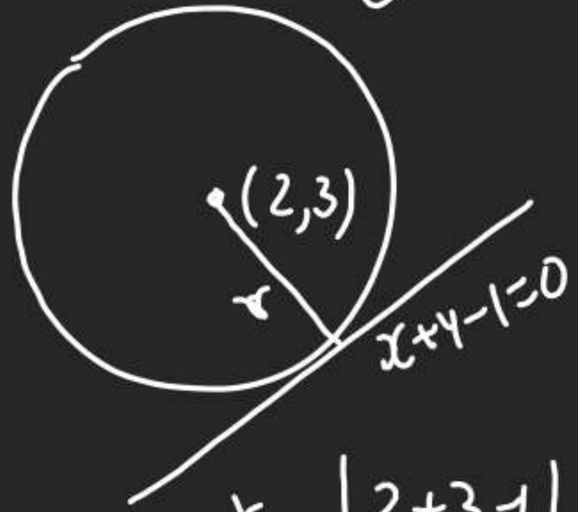
$(x-1)^2+(y-3)^2 = 2^2$ An

$x^2+y^2-2x-6y+1+9=4$

$x^2+y^2-2x-6y+6=0$ An

Q2 Find EOC having Centre $(2,3)$ & touching $x+y=1$

radius Missing



$r = \dfrac{|2+3-1|}{\sqrt{1^2+1^2}} = \dfrac{4}{\sqrt{2}}$

$r = 2\sqrt{2}$

$\therefore$ EOC $\Rightarrow (x-2)^2+(y-3)^2 = (2\sqrt{2})^2$

$(x-2)^2+(y-3)^2 = 8$

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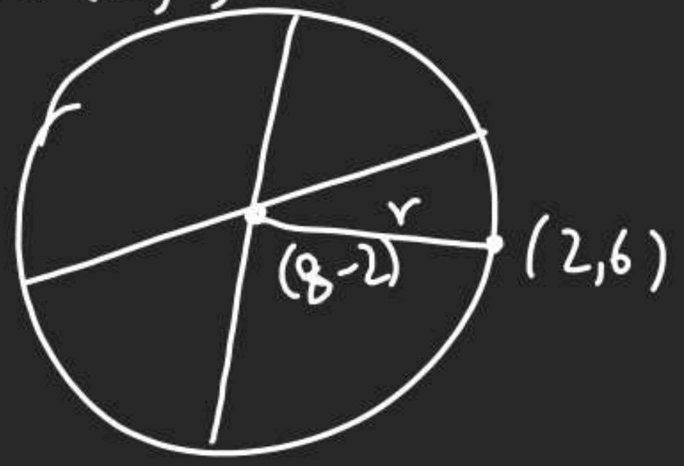
Q3 Find EOC having diametre $x+y=6$ & $x+2y=4$ & Passing thru $(2,6)$.



① PoI of diameter = Centre

$$\begin{aligned} x+y&=6 \\ x+2y&=4 \\ \hline -y&=2 \end{aligned}$$

$$\left.\begin{aligned} y&=-2 \\ x&=8 \end{aligned}\right\} \text{Centre}$$

② rad = dist. betn $(8,-2)$ & $(2,6)$

$$= \sqrt{(8-2)^2+(-2-6)^2} = \sqrt{36+64} = 10$$

$$(x-8)^2+(y+2)^2 = 10^2$$

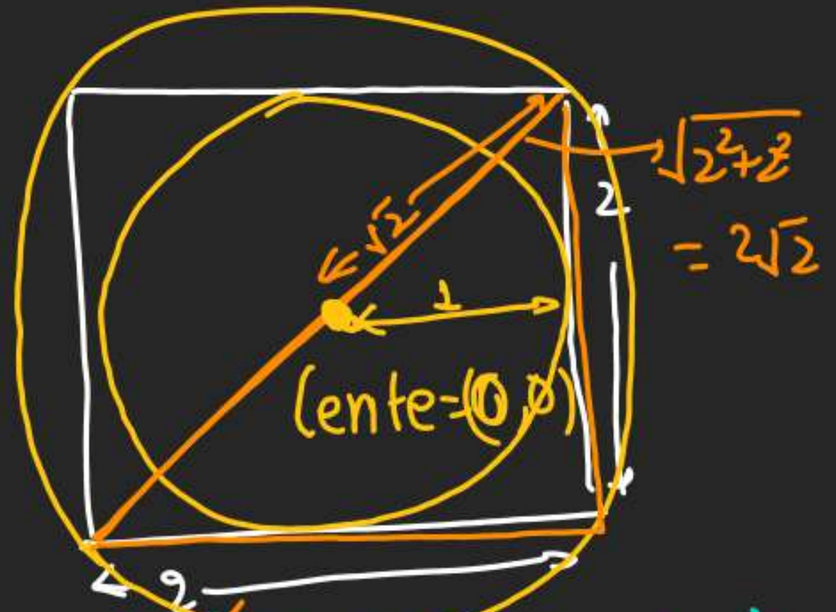
$$(x-8)^2+(y+2)^2 = 100$$

Q4 Consider a Quadrilateral formed by 4 Lines $3x+4y=5$, $4x-3y-5=0$, $3x+4y+5=0$, $4x-3y+5=0$. Find EOC Circumscribed & Inscribed in Quadrilateral.

1) $4x-3y-5=0$ $\parallel^{el}$ to $4x-3y+5=0$ $\Rightarrow d_1 = \dfrac{|-5-5|}{\sqrt{4^2+(-3)^2}} = \dfrac{10}{5} = 2$

2) $3x+4y-5=0$ $\parallel^{el}$ to $3x+4y+5=0$ $\rightarrow d_2 = \dfrac{|-5-5|}{\sqrt{3^2+4^2}} = \dfrac{10}{5} = 2$

(3)* So all 4 Lines are making sqr

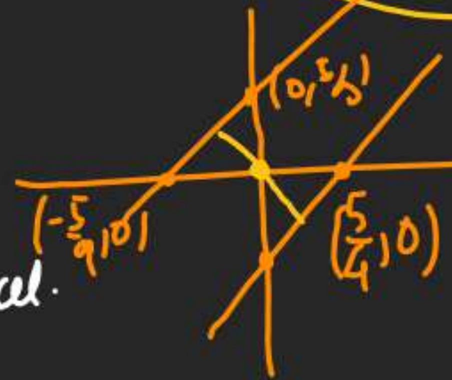


$(x-0)^2+(y-0)^2 = 1^2$

$\boxed{x^2+y^2=1}$

$(x-0)^2+(y-0)^2 = \sqrt{2}^2$

$\boxed{x^2+y^2=2}$



Clean diagram

$4x-3y=5$ & $4x-3y=-5$

$\dfrac{x}{5/4}+\dfrac{y}{-5/3}=1$ $\qquad$ $\dfrac{x}{-5/4}+\dfrac{y}{5/3}=1$

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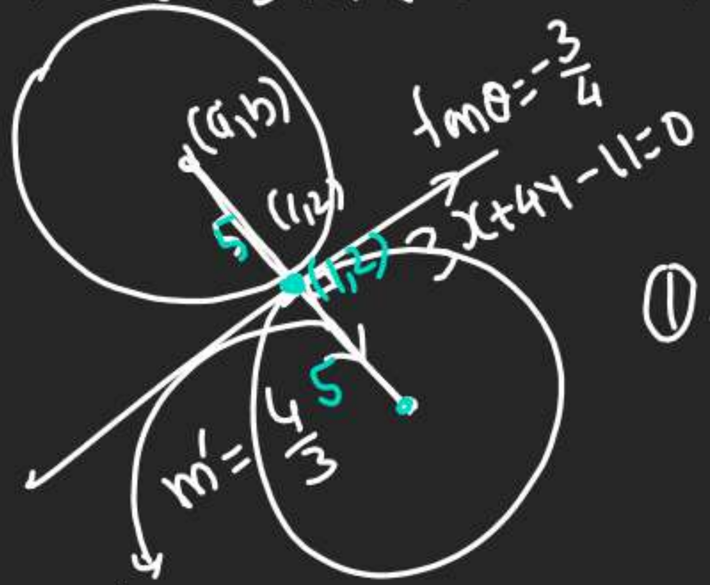
① Find EOC each having Radius 5 & touches line $3x+4y=11$ at $(1,2)$



① 2 Circle Possible

$\tan\theta = \frac{4}{3}$ $\sin\theta = \frac{4}{5}$, $\cos\theta = \frac{3}{5}$

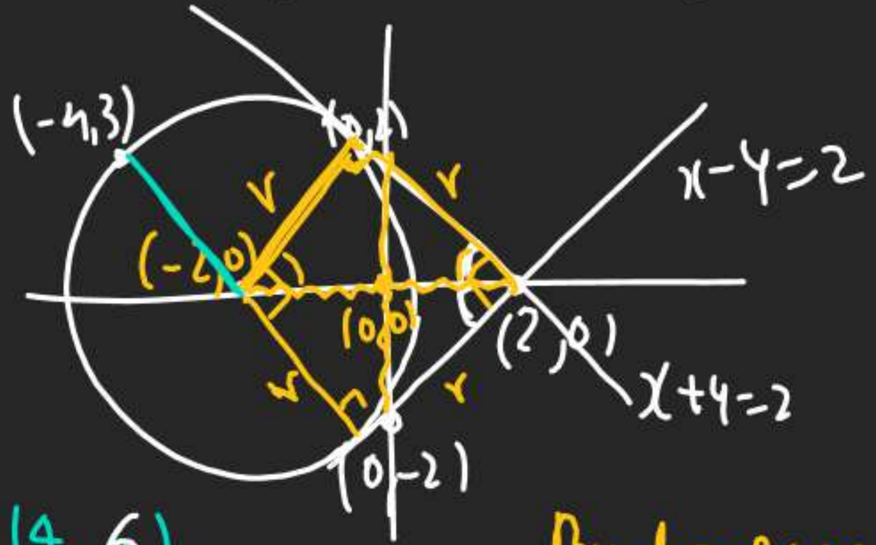
Centre $= \left(1+5\times\frac{3}{5}, 2+5\times\frac{4}{5}\right) = (4,6)$

$= \left(1-5\times\frac{3}{5}, 2-5\times\frac{4}{5}\right) = (-2,-2)$

Circle → $(x-4)^2+(y-6)^2 = 5^2$ & $(x+2)^2+(y+2)^2 = 5^2$

Q. 6 Find EOC P.T. Pt. $(-4,3)$ touching Lines $x+y=2$ & $x-y=2$

$\frac{x}{2}+\frac{y}{2}=1$ & $\frac{x}{2}+\frac{y}{-2}=1$



$(x+2)^2+(y-0)^2 = \sqrt{13}^2$

By diagram it is clear Centre will be $(-2,0)$

Rad $= \sqrt{(-4+2)^2+(3-0)^2}$

$= \sqrt{4+9} = \sqrt{13}$

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(2)* Non Homogeneous 2nd Deg Eqn.

1) $ax^2+by^2+2hxy+2gx+2fy+c=0$

$$\Delta = \begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} \begin{matrix} a & h \\ h & b \\ g & f \end{matrix}$$

$$\Delta = \{abc+fgh+fgh\}-\{bg^2+af^2+ch^2\}$$

discriminent $\boxed{\Delta = abc+2fgh-af^2-bg^2-ch^2}$

2) $ax^2+by^2+2hxy+2gx+2fy+c=0$

- $\Delta=0$: Pair of St. line
- $\Delta\neq 0$: Conic Section

(3) If $\Delta\neq 0$ & $a=b$ & $h=0$ then

$ax^2+by^2+2hxy+2gx+2fy+c=0$

becomes Circle.

$ax^2+ay^2+2gx+2fy+c=0$ is Circle.

for Standard form of Circle

$a=1$

$S: \boxed{x^2+y^2+2gx+2fy+c=0}$ is Standard form of Circle.

** Note :- If Circle $S: x^2+y^2+2gx+2fy+c=0$ is Satisfied by a pt (x_1, y_1)

$\therefore S_1: x_1^2+y_1^2+2gx_1+2fy_1+c=0$

$\therefore \boxed{S_1 \equiv EOC(Pt.)}$

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(3) Connection betn Central form & Standard form

(A) Central: $(x-a)^2+(y-b)^2=r^2 \rightarrow$ Centre(a,b), rad $=r$

Stm: $x^2+y^2+2gx+2fy+c=0$

$\hookrightarrow x^2+y^2-2ax-2by+a^2+b^2-r^2=0$

$S: x^2+y^2+2gx+2fy+c=0$

Compare $\Rightarrow a=-g$

$-2a=2g$

$-2b=2f$

$b=-f$

$c=a^2+b^2-r^2$

$c=g^2+f^2-r^2$

$r^2=g^2+f^2-c$

$\boxed{r=\sqrt{g^2+f^2-c}}$

Centre $=(a,b)$

$=(-g,-f)$

rad $=\sqrt{g^2+f^2-c}$

Q Centre & Rad of

$x^2+y^2-2x-4y+3=0$.

$x^2+y^2+2gx+2fy+c=0$

$g=-1,\ f=-2,\ c=3$.

Centre $=(-g,-f)=(1,2)$

rad $=\sqrt{g^2+f^2-c}=\sqrt{1^2+2^2-3}$

$=\sqrt{2}$

Q Centre & Rad of

$3x^2+3y^2-6x+9y+2=0$

$S: x^2+y^2-2x+3y+\frac{2}{3}=0$

$S: x^2+y^2+2gx+2fy+c=0$

$g=-1,\ f=\frac{3}{2},\ c=\frac{2}{3}$

Centre $=(-g,-f)=\left(1,-\frac{3}{2}\right)$

$r=\sqrt{1+\frac{9}{4}-\frac{2}{3}}=\sqrt{\frac{9}{4}+\frac{1}{3}}$

$=\frac{\sqrt{31}}{2}$

(B) Now Onwards.

$S: x^2+y^2+2gx+2fy+c=0$

Center $=(-g,-f)$, $r=\sqrt{g^2+f^2-c}$

$r=\sqrt{g^2+f^2-c}$

$g^2+f^2-c=0$	$g^2+f^2-c<0$	$g^2+f^2-c>0$
rad $=0$	$\sqrt{-ve}$	Real Circle.
Point Circle	Imaginary Circle	

Q. $Px^2+(2-q)xy+3y^2-6Px+30y+6q=0$

Rep. a Circle find $P, q=?$

$ax^2+2hxy+by^2+2gx+2fy+c=0$ Rep Circle.

$a=b$ | $h=0$

$P=3$ | $2-q=0 \Rightarrow q=2$

Q If $x^2+y^2-2x+2ay+a+3=0$ Rep. a Circle find $a\in$?

$x^2+y^2+2gx+2fy+c=0$

$a\in(-\infty,-1)\cup(2,\infty)$

$g=-1,\ f=a,\ c=a+3$

If it is a Real Circle $g^2+f^2-c>0$

$(-1)^2+a^2-(a+3)>0$

$a^2-a-2>0$

$(a-2)(a+1)>0$

$a<-1 \cup a>2$

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Q Circle $x^2+y^2+2\lambda x+2(1-\lambda)y-2=0$

will keep Integral Value of λ not for Radius more than 3.

① rad ≤ 3 ② Center: $(-\lambda, \lambda-1)$, $C=-2$

$$\sqrt{(-\lambda)^2+(\lambda-1)^2+2} \le 3$$

$$\lambda^2+\lambda^2-2\lambda+1+2 \le 9 \qquad \lambda=\frac{1\pm\sqrt{1+12}}{2}$$

$$2\lambda^2-2\lambda-6 \le 0$$

$$\lambda^2-\lambda-3 \le 0$$

$$\left(\lambda-\left(\frac{1+\sqrt{13}}{2}\right)\right)\left(\lambda-\left(\frac{1-\sqrt{13}}{2}\right)\right) \le 0$$

$$\frac{1-\sqrt{13}}{2} \le \lambda \le \frac{1+\sqrt{13}}{2} \quad \sqrt{13}\to 3.6$$

$$-1.3 \le \lambda \le 2.3 \Rightarrow \lambda \in \{-1,0,1,2\}$$

-1 0 1 2
-1.3 2.3

Q If 2 diameter of Circle $2x-3y=1$ & $x+3y=5$ & Area $=154\,m^2$ find EOC.

$$2x-3y=1$$
$$x+3y=5$$
$$x, y=(2,1)$$

$$\pi r^2=154$$
$$\frac{22}{7}r^2=154$$
$$r^2=\frac{\cancel{154}^{7}\times 7}{\cancel{22}}$$
$$r=7$$

Q Find Image of Circle $x^2+y^2-6x-8y=0$ in Line $3x+4y+25=0$

Center $=(3,4)$

Rad $=\sqrt{3^2+4^2-0}=5$

$3x+4y+25=0$

$=$ (Image of $(3,4)$ in $3x+4y+25=0$)

$$\frac{x-3}{3}=\frac{y-4}{4}=\frac{-2(9+16+25)}{3^2+4^2}$$

$$\frac{x-3}{3}=-4 \;\&\; \frac{y-4}{4}=-4$$

$$x=-9,\ y=-12 \quad (-9,-12)$$

$$EOC \Rightarrow (x+9)^2+(y+12)^2=5^2$$

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Q Find No of Circles touching Both axes & Line $4x+3y=6$

$(\oplus, \oplus) \to$ 1 Quad

+ve

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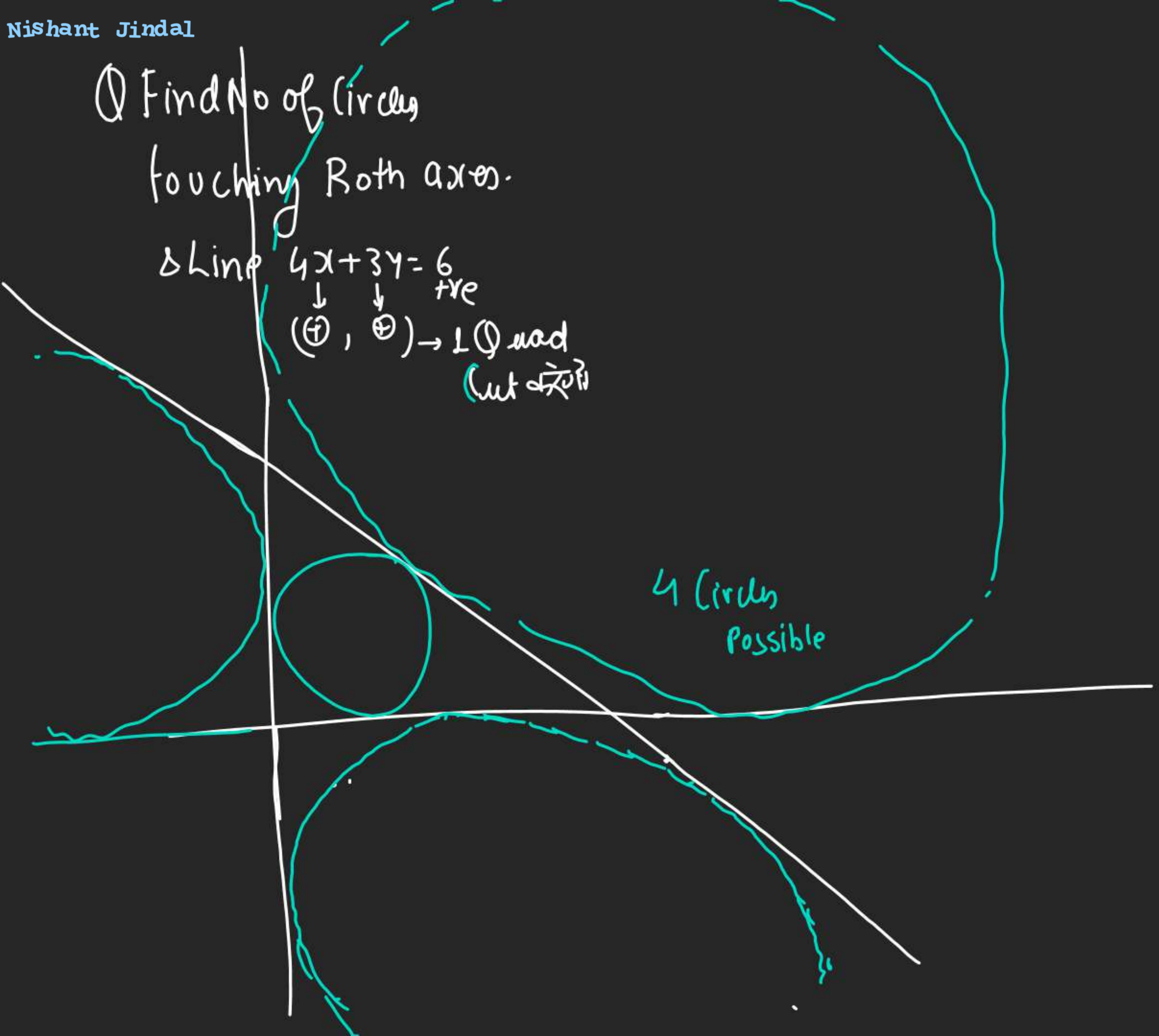

4 Circles possible