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$$A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 5 \\ -2 & 7 & 1 \end{bmatrix}$$

$$\operatorname{adj} A = \begin{bmatrix} -32 & 1 & -5 \\ -12 & 1 & -5 \\ 20 & -5 & 5 \end{bmatrix}$$

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1: If A is non singular matrix of order 'n', then P.T. (i) $adj(adj A) = |A|^{n-2} A$

(ii) $|adj(adj A)| = |A|^{(n-1)^2}$.

$= |adj A|^{n-1} = \left(|A|^{n-1}\right)^{n-1} = |A|^{(n-1)^2}$

$|adj(adj A)| = \left||A|^{n-2} A\right|$

$= \left(|A|^{n-2}\right)^n |A|$

$= |A|^{n^2-2n+1}$

$= |A|^{(n-1)^2}$

$adj(A)\, adj(adj A) = |adj A| I = |A|^{n-1} I$.

$(\underbrace{A\, adj A}_{|A| I})\, adj(adj A) = |A|^{n-1} A$

$|A|\, adj(adj A) = |A|^{n-1} A$

$adj(adj A) = |A|^{n-2} A$

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Inverse of matrix

If A is non singular, then there exists a unique matrix B such that $\boxed{AB = BA = I}$. Then A & B are said to mutually inverse to each other.

$\boxed{AA^{-1} = A^{-1}A = I}$

$A = \boxed{B^{-1}}$ or $B = A^{-1}$

$\boxed{A^{-1} = \frac{\operatorname{adj} A}{|A|}}$

$IB = CI \Leftarrow CAB = CI \Leftarrow AB = I$

$B = C$

$A \operatorname{adj} A = (\operatorname{adj} A)A = |A| I$

$A\left(\frac{\operatorname{adj} A}{|A|}\right) = \left(\frac{\operatorname{adj} A}{|A|}\right)A = I$

$AB = BA = I$

$AC = CA = I$

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Properties

- $(A^{-1})^{-1} = A$

$(A_1 A_2 \cdots A_n)^{-1} = A_n^{-1} A_{n-1}^{-1} \cdots A_2^{-1} A_1^{-1}$

- $(AB)^{-1} = B^{-1}A^{-1}$

- $(A^n)^{-1} = (A^{-1})^n = A^{-n}, n \in N.$

- $(A^T)^{-1} = (A^{-1})^T$ ✓

$(A_1 A_2 \cdots A_n)^{-1}$

$= (A_2 A_3 \cdots A_n)^{-1} A_1^{-1}$

$= (A_3 \cdots A_n)^{-1} A_2^{-1} A_1^{-1}$

$A^{-3} = (A^{-1})^3$

$= (A^3)^{-1}$

$A A^{-1} = A^{-1}A = I.$

$\underbrace{(A^T)^{-1}}_{P} = \underbrace{(A^{-1})^T}_{Q}$

$PQ = A^T (A^{-1})^T$

$= (A^{-1}A)^T$

$= I^T$

$\Rightarrow \boxed{P^{-1} = Q}$ ✓

$\boxed{\therefore PQ = I = QP.}$

$PQ = (AB)(B^{-1}A^{-1}) = I.$

$= A(BB^{-1})A^{-1} = AA^{-1}$

$= I.$

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$\text{adj}\, A = \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$ $|A| = 1$

1. Show that matrix $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$ satisfies the eqn $A^2 - 4A + I = 0$. Using this find A^{-1}. 2

$A^{-1} = \dfrac{\text{adj}\, A}{|A|}$

$|A - \lambda I| = \begin{vmatrix} 2-\lambda & 3 \\ 1 & 2-\lambda \end{vmatrix} = 0 \Rightarrow \lambda^2 - 4\lambda + 4 - 3 = 0$

$\boxed{A^2 - 4A + I = 0} \Rightarrow A^2 A^{-1} - 4AA^{-1} + A^{-1} = 0$

$A - 4I + A^{-1} = 0$

$= \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix} = A^{-1} = 4I - A$

2. Let $F(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $G(y) = \begin{bmatrix} \cos y & 0 & \sin y \\ 0 & 1 & 0 \\ -\sin y & 0 & \cos y \end{bmatrix}$

P.T. (i) $(F(x))^{-1} = F(-x)$ (ii) $(G(y))^{-1} = G(-y)$

(iii) $(F(x)G(y))^{-1} = G(-y)F(-x)$.

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$$F(x)\,F(-x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos x & \sin x & 0 \\ -\sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I.$$

$$G(y)\,G(-y) = I$$

$$\left(F(x)\,G(y)\right)^{-1} = \left(G(y)\right)^{-1}\left(F(x)\right)^{-1}$$

$$= G(-y)\,F(-x)$$

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3. Find matrix A satisfying the eqn.

$$\underset{\downarrow P}{\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}} A \underset{\downarrow Q}{\begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix}} = \underset{B}{\begin{bmatrix} -2 & 4 \\ 3 & -1 \end{bmatrix}}$$

$$PAQ = B$$

$$AQ = P^{-1}P\underline{AQ} = P^{-1}B$$

$$AQQ^{-1} = P^{-1}BQ^{-1}$$

$$A = P^{-1}BQ^{-1}$$

$$A = \overbrace{\begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix}\begin{bmatrix} -2 & 4 \\ 3 & -1 \end{bmatrix}}\begin{bmatrix} 3 & 2 \\ 5 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 24 & 13 \\ -34 & -18 \end{bmatrix} = \begin{bmatrix} -7 & 9 \\ 12 & -14 \end{bmatrix}\begin{bmatrix} 3 & 2 \\ 5 & 3 \end{bmatrix}$$

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System of Equations

$$a_1x + b_1y + c_1z = d_1$$
$$a_2x + b_2y + c_2z = d_2$$
$$a_3x + b_3y + c_3z = d_3$$

$$\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$$

$$AX = B.$$

Matrices → PT-2, Ex-I ✓

Determinants → Ex-IV (15 to 21)

Ex-V → Complete.