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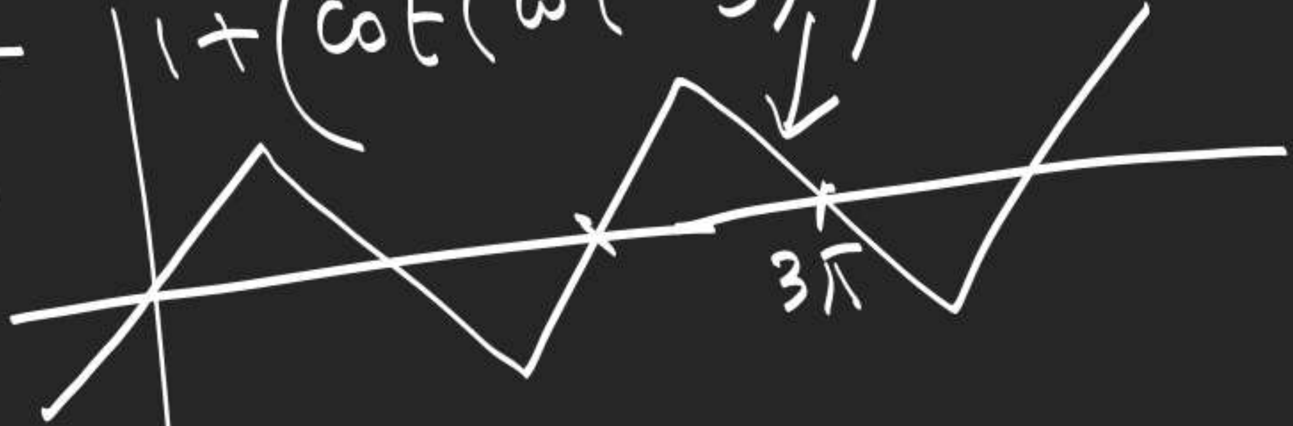
$$-\sin^{-1}\sin \underline{600^\circ} = -\left(3(180^\circ) - 600^\circ\right)$$

(X) $1+\tan^2\tan^{-1}2 + 1+\left(\cot(\cot^{-1}3)\right)^2$

$= 1+(2)^2 + 1 + 3^2$



4: $\boxed{\boxed{\sin^{-1}x} = 2\sin^{-1}a}$ 2: 3: $y \in \left(-\frac{\pi}{2}-\frac{\pi}{4}, \frac{\pi}{4}+\frac{\pi}{2}\right)$

$\sin^{-1}a \in \left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$ $\in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$\boxed{a \in \left[-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right]}$

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1.

$$\sec(\underbrace{\operatorname{cosec}^{-1}x}_{=\theta}) = \frac{\sqrt{x^2-1}}{x}$$

$$\operatorname{cosec}\left(\frac{\pi}{2} - \underbrace{\operatorname{cosec}^{-1}x}\right)$$

$$\theta \in \left[-\frac{\pi}{2}, 0\right) \cup \left(0, \frac{\pi}{2}\right]$$

$$\operatorname{cosec}(\sec^{-1}x)$$

$$\operatorname{cosec}\theta = x$$

3. $$\frac{\sqrt{1-x^2}}{x} = \frac{2}{\sqrt{5}}$$

π

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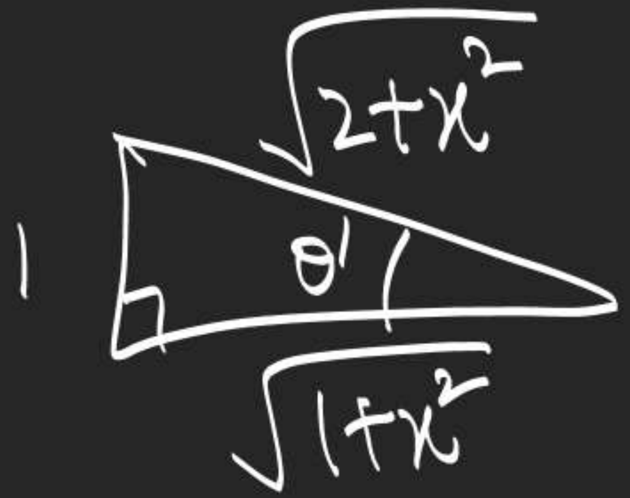
9. $$\cos\left(\tan^{-1}\left(\sin\left(\underbrace{\cot^{-1}x}_{\theta}\right)\right)\right) = \cos\underbrace{\tan^{-1}\left(\frac{1}{\sqrt{1+x^2}}\right)}_{\theta'}$$



$$= \frac{\sqrt{1+x^2}}{\sqrt{2+x^2}}$$

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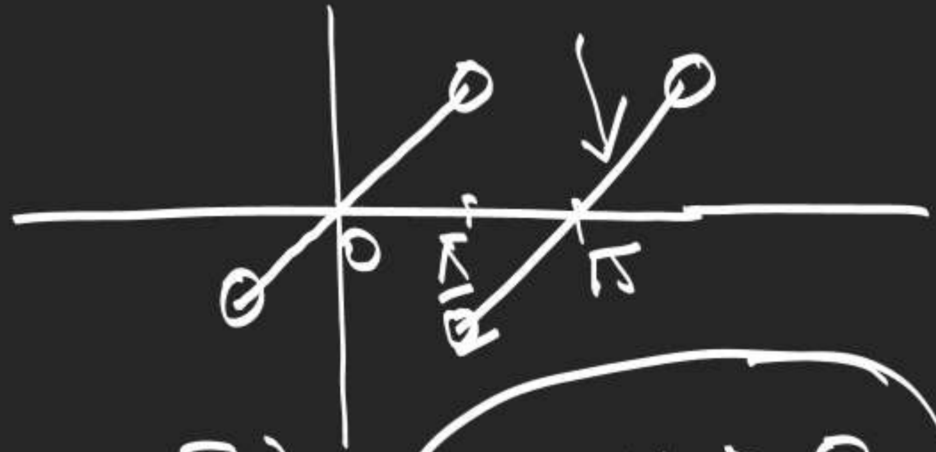
$$\boxed{\tan^{-1}x+\tan^{-1}y=\begin{cases}\tan^{-1}\left(\dfrac{x+y}{1-xy}\right), & x>0,\ y>0,\ xy<1\\ \pi+\tan^{-1}\left(\dfrac{x+y}{1-xy}\right), & x>0,\ y>0,\ xy>1\end{cases}}$$

$\tan^{-1}x=\theta_1$, $\quad \tan^{-1}y=\theta_2$ $\qquad \theta_1+\theta_2\in(0,\pi)$

$\theta_1\in\left(0,\frac{\pi}{2}\right)$ $\qquad \theta_2\in\left(0,\frac{\pi}{2}\right)$

$\tan\theta_1=x$ $\qquad \tan\theta_2=y$

$y=x-\pi$



$$\tan(\theta_1+\theta_2)=\frac{x+y}{1-xy} \quad x+y>0 \checkmark$$

$$\tan^{-1}\tan(\theta_1+\theta_2)=\tan^{-1}\left(\frac{x+y}{1-xy}\right)$$

$$\tan^{-1}\left(\frac{x+y}{1-xy}\right)=\begin{cases}\theta_1+\theta_2\\ \theta_1+\theta_2-\pi\end{cases}$$

$\theta_1+\theta_2\in\left(0,\frac{\pi}{2}\right)$, $\quad 1-xy>0$

$\theta_1+\theta_2\in\left(\frac{\pi}{2},\pi\right)$, $\quad 1-xy<0$

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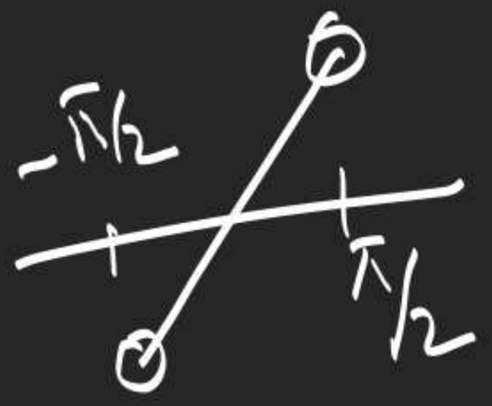
$$\boxed{\tan^{-1}x - \tan^{-1}y = \tan^{-1}\left(\frac{x-y}{1+xy}\right) \qquad x>0,\ y>0}$$

$= \theta \in \left(0, \frac{\pi}{2}\right)$ $\qquad \theta_2 \in \left(0, \frac{\pi}{2}\right) \Rightarrow -\theta_2 \in \left(-\frac{\pi}{2}, 0\right)$

$$\theta_1 - \theta_2 \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$$



$$\tan(\theta_1 - \theta_2) = \frac{x-y}{1+xy}$$

$$\tan^{-1}\tan(\theta_1 - \theta_2) = \tan^{-1}\left(\frac{x-y}{1+xy}\right) = \theta_1 - \theta_2 .$$

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$$\tan^{-1}7 + \tan^{-1}(-3) = \tan^{-1}7 - \tan^{-1}3$$

$$\tan^{-1}3 - \tan^{-1}(-2) = \tan^{-1}3 + \tan^{-1}2$$

$$= \pi + \tan^{-1}\left(\frac{3+2}{1-3(2)}\right)$$

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1. Solve for x satisfying

$$\cot^{-1}\left(\frac{x^2-1}{2x}\right)+\tan^{-1}\left(\frac{2x}{x^2-1}\right)=\frac{2\pi}{3}$$

$$\frac{x^2-1}{2x}>0 \Rightarrow \boxed{x\in(-1,0)\cup(1,\infty)}$$

$$\cancel{2}\tan^{-1}\left(\frac{2x}{x^2-1}\right)=\frac{\cancel{2}\pi}{3}$$

$$\frac{2x}{x^2-1}=\sqrt{3}$$

$$\sqrt{3}x^2-2x-\sqrt{3}=0$$

$$(x-\sqrt{3})(\sqrt{3}x+1)=0$$

$$x=\sqrt{3},\ -\frac{1}{\sqrt{3}}$$

$-\quad+\quad-\quad+$

$-1\quad 0\quad 1$

$$\frac{x^2-1}{2x}<0 \Rightarrow x\in(-\infty,-1)\cup(0,1)$$

$$\pi+2\tan^{-1}\left(\frac{2x}{x^2-1}\right)=\frac{2\pi}{3}$$

$$\frac{2x}{x^2-1}=-\frac{1}{\sqrt{3}}$$

$$x^2+2\sqrt{3}x-1=0$$

$$x=-\sqrt{3}\pm 2$$

$$\boxed{x=\sqrt{3},\ -\frac{1}{\sqrt{3}},\ -\sqrt{3}+2,\ -\sqrt{3}-2}$$

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Q: Simplify

(i) $\tan^{-1}1+\underbrace{\tan^{-1}2+\tan^{-1}3}=\cancel{\frac{\pi}{4}}+\pi+\tan^{-1}\left(\cancel{\frac{2+3}{1-2(3)}}\right)=\pi$

(ii) $\tan^{-1}5-\tan^{-1}3+\tan^{-1}\frac{7}{9}=\tan^{-1}\left(\frac{5-3}{1+5(3)}\right)+\tan^{-1}\frac{7}{9}$

$=\frac{1}{8}$

$=\tan^{-1}\left(\frac{\frac{1}{8}+\frac{7}{9}}{1-\frac{1}{8}\times\frac{7}{9}}\right)=\tan^{-1}1$

$=\frac{\pi}{4}$

(iii) $\tan^{-1}\frac{2}{11}+\cot^{-1}\frac{24}{7}+\tan^{-1}\frac{1}{3}$

$\tan^{-1}\left(\frac{\frac{2}{11}+\frac{1}{3}}{1-\frac{2}{11}\frac{1}{3}}\right)+\tan^{-1}\frac{7}{24}$

$17/31$

$=\tan^{-1}\left(\frac{\frac{17}{31}+\frac{7}{24}}{1-\frac{17}{31}\times\frac{7}{24}}\right)=\tan^{-1}1=\frac{\pi}{4}$

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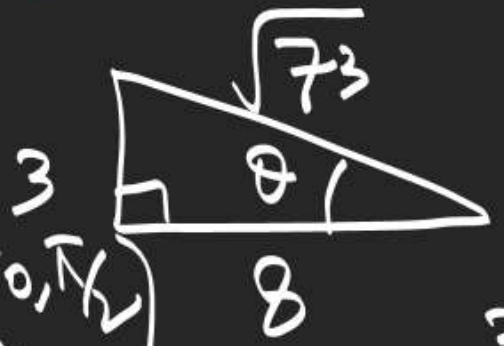
(iv) $\sin^{-1}\left(\frac{3}{\sqrt{73}}\right)+\cos^{-1}\left(\frac{11}{\sqrt{146}}\right)+\cot^{-1}\sqrt{3}$

(v) $\cos^{-1}\sqrt{\frac{2}{3}}-\cos^{-1}\left(\frac{\sqrt{6}+1}{2\sqrt{3}}\right)$



$\theta\in(0,\pi/2)$

$\tan\theta=\frac{3}{8}$

$\tan^{-1}\tan\theta=\tan^{-1}\frac{3}{8}$

$=\theta$

(iv) $\tan^{-1}\left(\frac{3}{8}\right)+\tan^{-1}\left(\frac{5}{11}\right)+\frac{\pi}{6}$

$=\tan^{-1}\left(\frac{\frac{3}{8}+\frac{5}{11}}{1-\frac{3}{8}\times\frac{5}{11}}\right)+\frac{\pi}{6}=\frac{\pi}{4}+\frac{\pi}{6}=\frac{5\pi}{12}$

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(v) $\cos^{-1}\sqrt{\frac{2}{3}} - \underbrace{\cos^{-1}\left(\frac{\sqrt{6}+1}{2\sqrt{3}}\right)}_{\theta \in \left(0, \frac{\pi}{2}\right)}$

$$\tan^{-1}\frac{1}{\sqrt{2}} - \tan^{-1}\left(\frac{\sqrt{3}-\sqrt{2}}{1+\sqrt{2}\sqrt{3}}\right)$$

$$\frac{\pi}{2}-\frac{\pi}{3} = \cot^{-1}\sqrt{2} - \left(\tan^{-1}\sqrt{3} - \tan^{-1}\sqrt{2}\right)$$

$$= \boxed{\frac{\pi}{6}}$$

$$\sqrt{3}-\sqrt{2} = \sqrt{5-2\sqrt{6}}$$

$$3+2-2\sqrt{3}\sqrt{2}$$

$$\tan\theta = \frac{\sqrt{3}-\sqrt{2}}{1+\sqrt{6}}$$

$$\boxed{\sqrt{x^2} = |x|}$$

3. If $\tan^{-1}4+\tan^{-1}5 = \cot^{-1}\lambda$

find λ

$$\boxed{\lambda = -\frac{19}{9}}$$

$$= \pi + \tan^{-1}\left(\frac{4+5}{1-4(5)}\right)$$

$$= \pi + \tan^{-1}\left(-\frac{9}{19}\right)$$

$$= \cot^{-1}\left(-\frac{19}{9}\right)$$

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4. Which is greater

$$\cos^{-1}\left(\frac{7}{25}\right) + \cos^{-1}\left(\frac{3}{5}\right) \quad \text{or} \quad \cot^{-1}(-1) = \frac{3\pi}{4}$$

$$\tan^{-1}\frac{24}{7} + \tan^{-1}\frac{4}{3}$$

$$= \pi + \tan^{-1}\left(\frac{\frac{24}{7} + \frac{4}{3}}{1 - \frac{24}{7}\cdot\frac{4}{3}}\right) = \pi + \tan^{-1}\left(-\frac{4}{3}\right)$$

$$= \pi - \underbrace{\tan^{-1}\left(\overset{>1}{\frac{4}{3}}\right)}_{>\frac{\pi}{4}} < \frac{3\pi}{4}$$

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5. If $\cos^{-1}x + \cos^{-1}y + \cos^{-1}z = \pi$, then

P.T. $x^2 + y^2 + z^2 + 2xyz = 1$

$\underbrace{\cos^{-1}x}_{A} + \underbrace{\cos^{-1}y}_{B} = \pi - \cos^{-1}z$

$\cos(\cos^{-1}x + \cos^{-1}y) = \cos(\pi - \cos^{-1}z)$

$= -\cos(\cos^{-1}z)$

H.W

PT-3

Ex-I (1-5)

$\cos^{-1}x = A \rightarrow [0, \pi]$

$\sin A = \sqrt{1-x^2}$

$\Rightarrow xy - \sqrt{1-x^2}\sqrt{1-y^2} = -z$

$\Rightarrow \boxed{(xy+z)^2 = (1-x^2)(1-y^2)}$

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$A+B+C=\pi$

$\cos^2 A+\cos^2 B+\cos^2 C+2\cos A\cos B\cos C=1$

$1+\underline{\cos^2 A-\sin^2 B}+\cos^2 C+2\cos A\cos B\cos C$

$1-\cos C\cos(A-B)+\cos^2 C+2\cos A\cos B\cos C$

$1-\cos C\left(\cos(A-B)+\cos(A+B)\right)+2\cos A\cos B\cos C$

$=1$