

Q If $\vec{a}, \vec{b}, \vec{c}$ are Unitvectors.

$$\vec{a}^\wedge \vec{b} = \vec{b}^\wedge \vec{c} = (\vec{a} \cdot \vec{b}) - \frac{\pi}{3}$$

then $|\vec{a} + 2\vec{b} - 3\vec{c}| = ?$

$$|\vec{a} + 2\vec{b} - 3\vec{c}|^2 = |\vec{a}|^2 + 4|\vec{b}|^2 + 9|\vec{c}|^2 + 2(\vec{a} \cdot 2\vec{b} - 3\vec{c})$$

$$+ 2(\vec{a} \cdot 2\vec{b}) - 12(\vec{b} \cdot 3\vec{c})$$

$$- 6(\vec{a} \cdot 3\vec{c})$$

$$= 1 + 4 + 9 + 2 - 6 - 3$$

$$|\vec{a} + 2\vec{b} - 3\vec{c}| = \sqrt{7}$$

5 \vec{a}

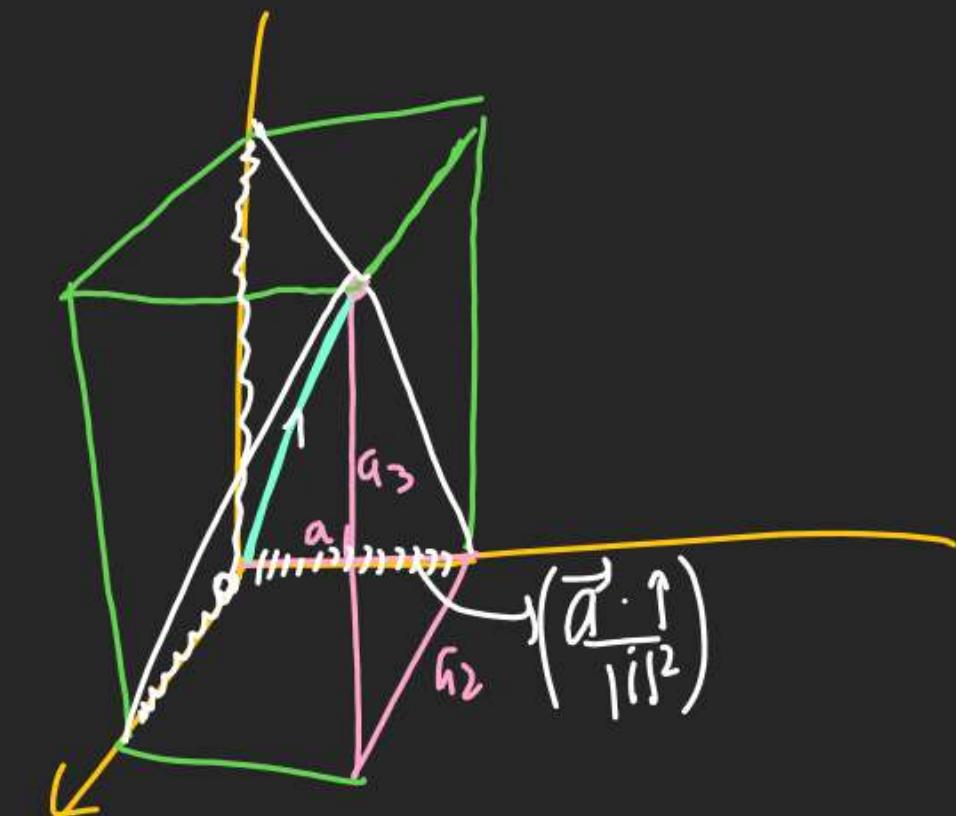
$$(\vec{a} \cdot \hat{i})\hat{i} + (\vec{a} \cdot \hat{j})\hat{j} + (\vec{a} \cdot \hat{k})\hat{k}$$

$$\left(\frac{\vec{a} \cdot \hat{i}}{|\vec{i}|^2} \right) \hat{i} + \left(\frac{\vec{a} \cdot \hat{j}}{|\vec{j}|^2} \right) \hat{j} + \left(\frac{\vec{a} \cdot \hat{k}}{|\vec{k}|^2} \right) \hat{k} = \vec{a}$$

Proj. of
 \vec{a} on
X Axis

Proj. of
 \vec{a}
on Y Axis

Proj. of
 \vec{a}
on Z Axis



1) Of Prod & Base

2) Repeating

then max. No of

Qs try

3) Bahut weak

जो पर तारे Examples

Revisations Self Tests

If you are feeling good
then try Adv. + Maths

Q If \vec{e}_1, \vec{e}_2 such that $|\vec{e}_1|=2, |\vec{e}_2|=1$

$\vec{e}_1 \cdot \vec{e}_2 = 60^\circ$ If angle betn $\vec{V}_1 = 2t\vec{e}_1 + 7\vec{e}_2$

$\vec{V}_2 = \vec{e}_1 + t \cdot \vec{e}_2$ lies betn $(\frac{\pi}{2}, \pi)$ find

Range of t ?

Obtuse Angle

$$\vec{V}_1 \cdot \vec{V}_2 < 0$$

$$\vec{V}_1 \cdot \vec{V}_2 = \vec{0} \quad \left((2t\vec{e}_1 + 7\vec{e}_2) \cdot (\vec{e}_1 + t\vec{e}_2) < 0 \right)$$

$$2t|\vec{e}_1|^2 + 7|\vec{e}_1||\vec{e}_2|(\cos 60^\circ) < 0$$

$$+ 2t^2|\vec{e}_1||\vec{e}_2|\cos 60^\circ + 7t|\vec{e}_2|^2 < 0$$

(K-10)

$$\vec{V}_1 \parallel \vec{V}_2 \quad \left(\frac{2t}{1} = \frac{7}{t} \right)$$

$$t^2 = \frac{7}{2}$$

$$t = \sqrt{\frac{7}{2}} - \sqrt{\frac{7}{2}}$$

$$8t_1 + 7 + 2t^2 + 7t < 0$$

$$2t^2 + 15t + 7 < 0$$

$$2t^2 + 14t + 1 - 7 < 0$$

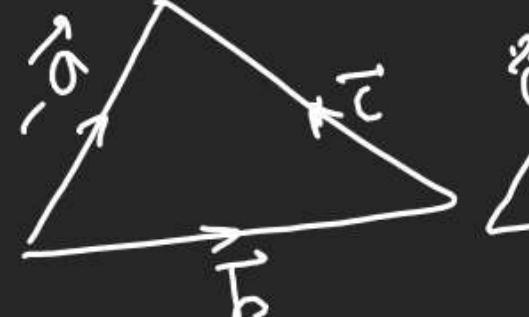
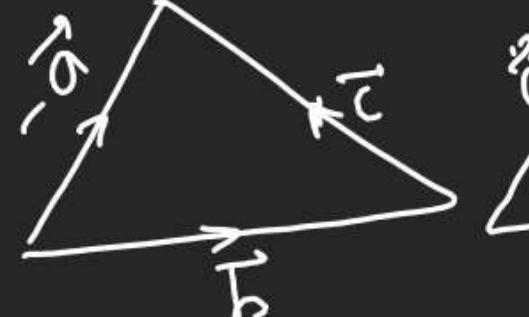
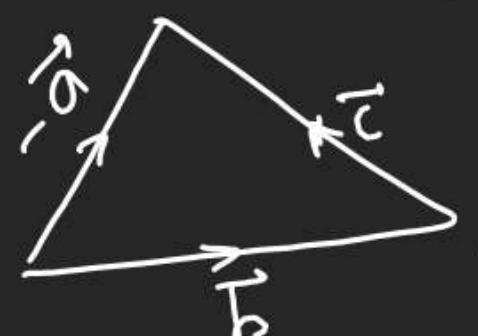
$$(2t+1)(t+7) < 0$$

$$t \in (-7, -\frac{1}{2}) - \{-\sqrt{\frac{7}{2}}\}$$

Q If $\vec{a} + \vec{b} + \vec{c} = 0$ then

angle betn \vec{b} & \vec{c}

$$\vec{b} + \vec{c} = -\vec{a}$$



Maths

$$\vec{b} + \vec{c} = -\vec{a}$$

$$(\vec{b} + \vec{c})^2 = (-\vec{a})^2$$

$$(|\vec{b}|^2 + |\vec{c}|^2 + 2|\vec{b}||\vec{c}| \cos 180^\circ) = |\vec{a}|^2$$

$$\text{or } \vec{a} = \frac{|\vec{a}|^2 - |\vec{b}|^2 - |\vec{c}|^2}{2|\vec{b}||\vec{c}|}$$

$$(\vec{a} + \vec{b})^2 = (-\vec{c})^2$$

$$(|\vec{a}|^2 + |\vec{b}|^2 + 2|\vec{a}||\vec{b}| \cos (\pi - \theta)) = |\vec{c}|^2$$

$$(\vec{a} + \vec{b})^2 = |\vec{c}|^2$$

$$(|\vec{a}|^2 + |\vec{b}|^2 - 2|\vec{a}||\vec{b}| \cos (\pi - \theta)) = |\vec{c}|^2$$

$$-2|\vec{a}||\vec{b}| \cos (\pi - \theta) = |\vec{c}|^2 - |\vec{a}|^2 - |\vec{b}|^2$$

$$\cos \theta = \frac{|\vec{a}|^2 + |\vec{b}|^2 - |\vec{c}|^2}{2|\vec{a}||\vec{b}|}$$

Q Proof of Cosine for



$$|\vec{a} + \vec{b} + \vec{c}| = 0$$

$$(\vec{a} + \vec{b})^2 = (-\vec{c})^2$$

$$(|\vec{a}|^2 + |\vec{b}|^2 + 2|\vec{a}||\vec{b}| \cos (\pi - \theta)) = |\vec{c}|^2$$

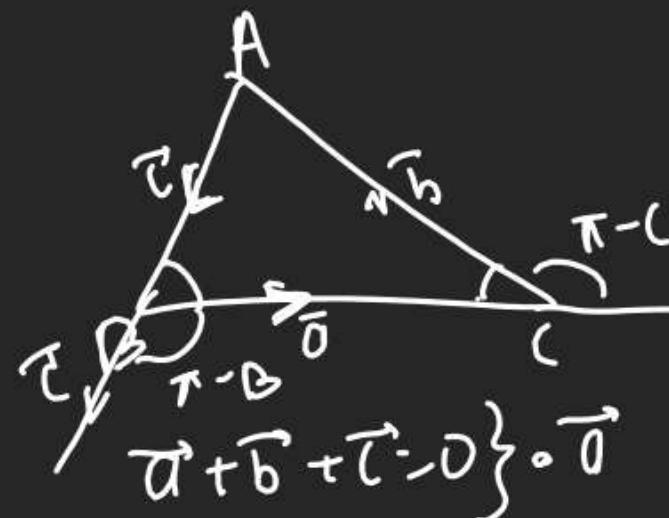
$$(|\vec{a}|^2 + |\vec{b}|^2 - 2|\vec{a}||\vec{b}| \cos (\pi - \theta)) = |\vec{c}|^2$$

$$-2|\vec{a}||\vec{b}| \cos (\pi - \theta) = |\vec{c}|^2 - |\vec{a}|^2 - |\vec{b}|^2$$

$$\cos \theta = \frac{|\vec{a}|^2 + |\vec{b}|^2 - |\vec{c}|^2}{2|\vec{a}||\vec{b}|}$$

Q Proof of Projection formula.

$$\vec{a} = b \cos C + c \cos B$$



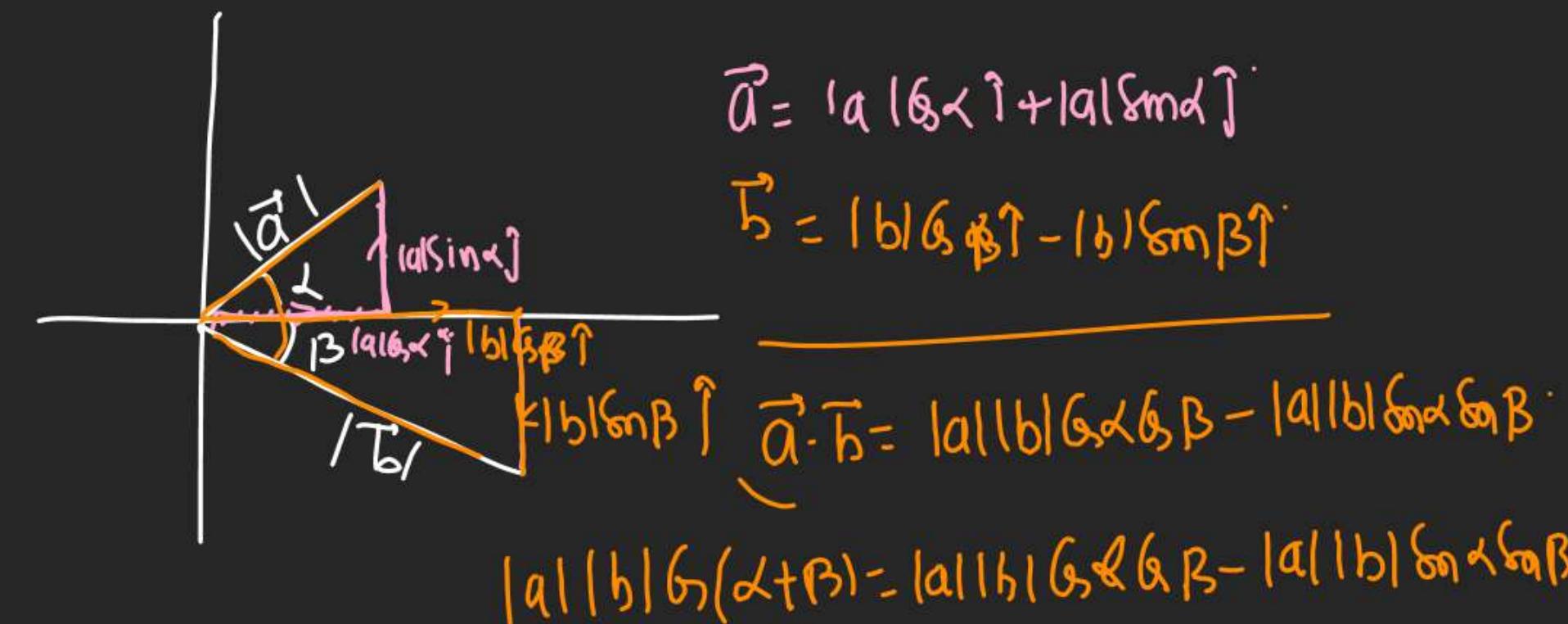
$$\vec{a} \cdot \vec{a} + \vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c} = 0$$

$$|\vec{a}|^2 + |\vec{a}| |\vec{b}| \cos(\pi - C) + |\vec{a}| |\vec{c}| \cos(\pi - B) = 0$$

$$|\vec{a}| - |\vec{b}| \cos C - |\vec{c}| \cos B = 0$$

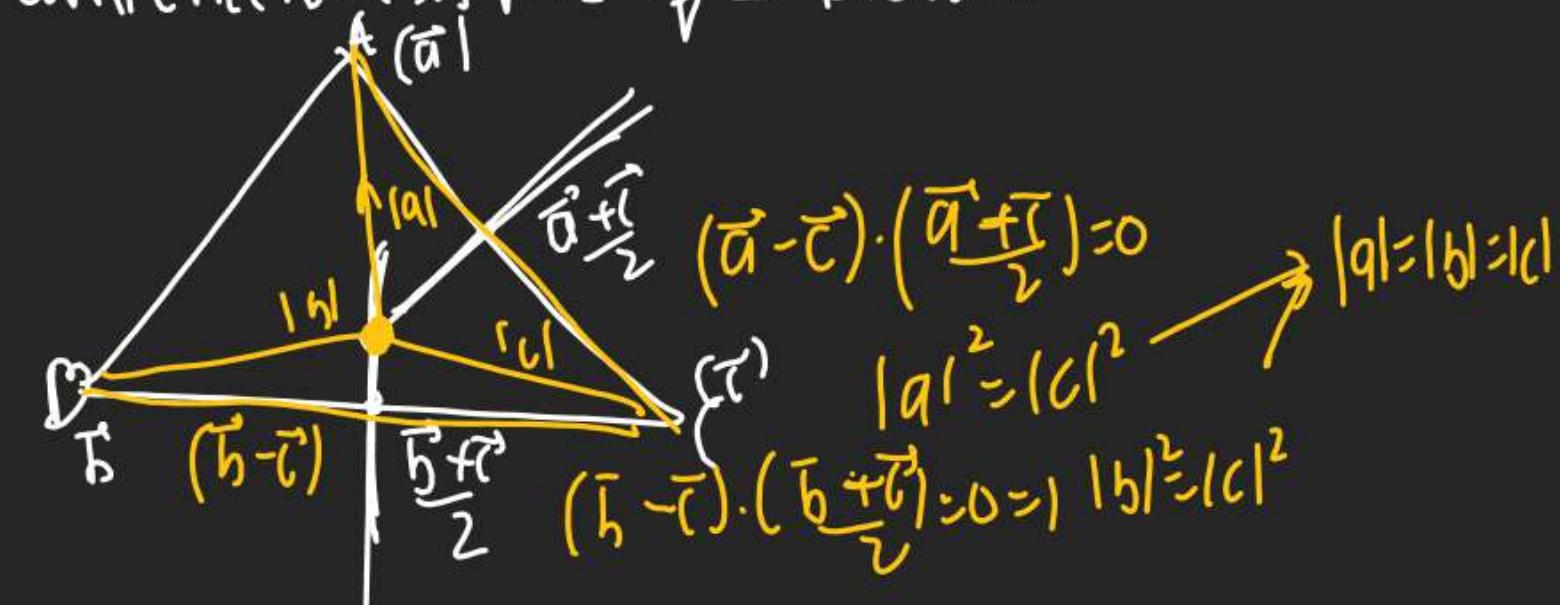
$$\vec{a} = b \cos C + c \cos B \quad (\text{H.P.})$$

Q Proof $\vec{a} \cdot (\vec{a} + \vec{b}) = \vec{a} \cdot \vec{a} + \vec{a} \cdot \vec{b}$

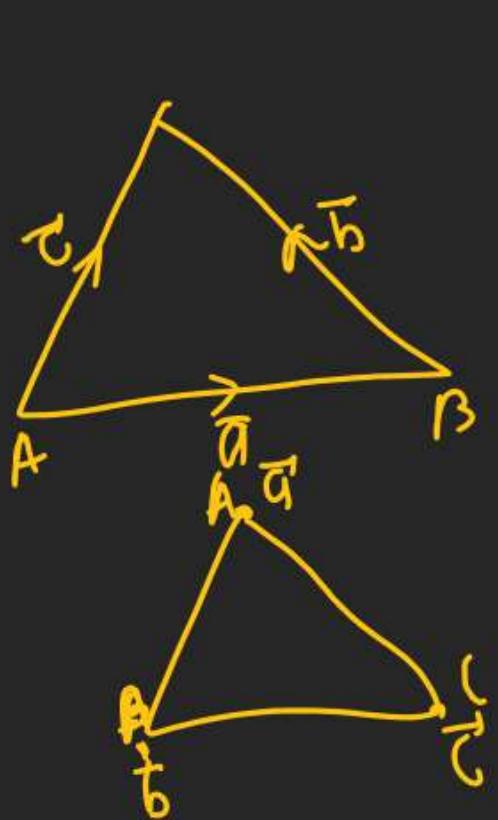
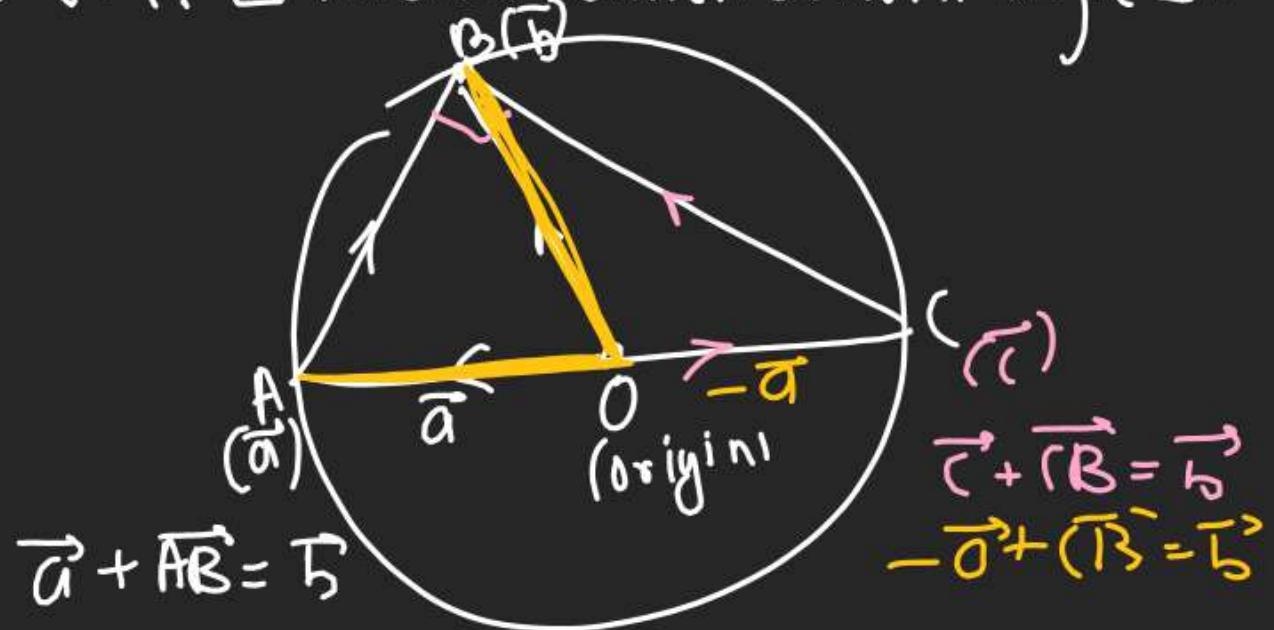


$$|\vec{a}| |\vec{b}| \cos(\alpha + \beta) = |\vec{a}| |\vec{b}| \cos \alpha \cos \beta - |\vec{a}| |\vec{b}| \sin \alpha \sin \beta \quad (\text{H.P.})$$

Q P.T. Circumcentre in P.I. of \perp^r Bisector.



Q. P. I. \triangle inside semicircle is rt. angle \triangle .



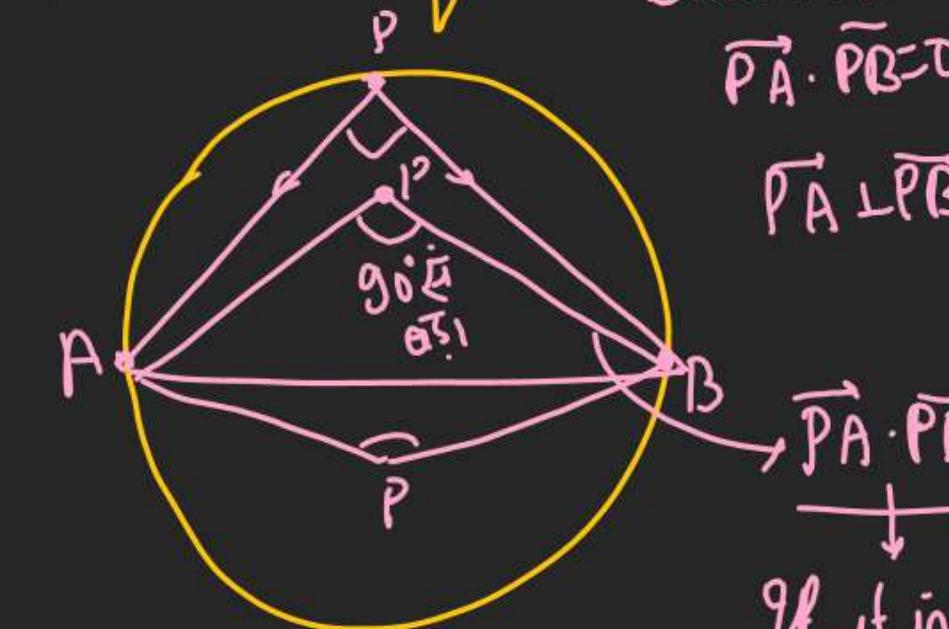
$$\begin{aligned}\vec{AB} &= \vec{b} - \vec{a} \\ \vec{CB} &= \vec{b} - \vec{c} \\ &= \vec{b} - (-\vec{a}) \\ &= \vec{b} + \vec{a}\end{aligned}$$

$$\begin{aligned}\vec{AB} \cdot \vec{CB} &= (\vec{b} + \vec{a})(\vec{b} - \vec{a}) \\ &= |\vec{b}|^2 - |\vec{a}|^2 \\ &= 0\end{aligned}$$

Q If P in ht in Space & $\vec{PA} \cdot \vec{PB} < 0$

Where A & B are 2 fixed Pt.

Find locus of P. \rightarrow Based on



$$\vec{PA} \cdot \vec{PB} = 0$$

$$\vec{PA} \perp \vec{PB}$$

$$\vec{PA} \cdot \vec{PB} < 0$$

If at given

line should understand
then P can lie inside
circle of diameter AP

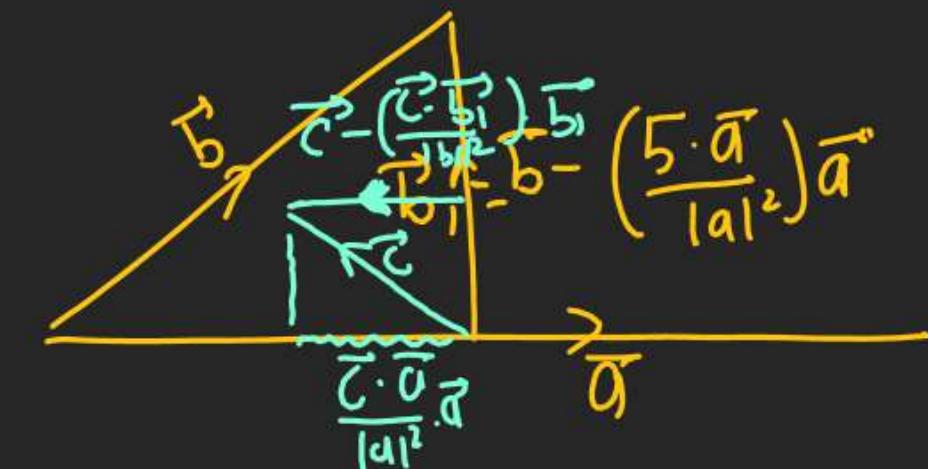
Q $\vec{b}_1 = \left(\vec{b} - \frac{\vec{b} \cdot \vec{a}}{|\vec{a}|^2} \cdot \vec{a} \right)$, \vec{a} are \perp

$$\vec{c}_2 = \vec{c} - \left(\frac{\vec{c} \cdot \vec{a}}{|\vec{a}|^2} \right) \vec{a} - \frac{\vec{c} \cdot \vec{b}_1}{|\vec{b}_1|^2} \cdot \vec{b}_1$$

$\vec{b}_1 \perp \vec{c}_2$ (T/F)?

$$\vec{b}_1 \cdot \vec{c}_2 = 0$$

$$\vec{b} \cdot \vec{c} - \left(\frac{\vec{c} \cdot \vec{a}}{|\vec{a}|^2} \right) \vec{a} \cdot \vec{b} - \left(\frac{\vec{c} \cdot \vec{b}_1}{|\vec{b}_1|^2} \right) \cdot \vec{b} \cdot \vec{b}_1 - \left(\frac{\vec{b} \cdot \vec{a}}{|\vec{a}|^2} \right) \cdot \left(\vec{c} \cdot \vec{a} \right) + \left(\frac{\vec{b} \cdot \vec{a}}{|\vec{a}|^2} \right) \left(\frac{\vec{c} \cdot \vec{a}}{|\vec{a}|^2} \right) |\vec{a}|^2 + \left(\frac{\vec{b} \cdot \vec{a}}{|\vec{a}|^2} \right)^2 \left(\frac{\vec{b} \cdot \vec{a}}{|\vec{a}|^2} \right) \vec{a} \cdot \vec{b}_1$$

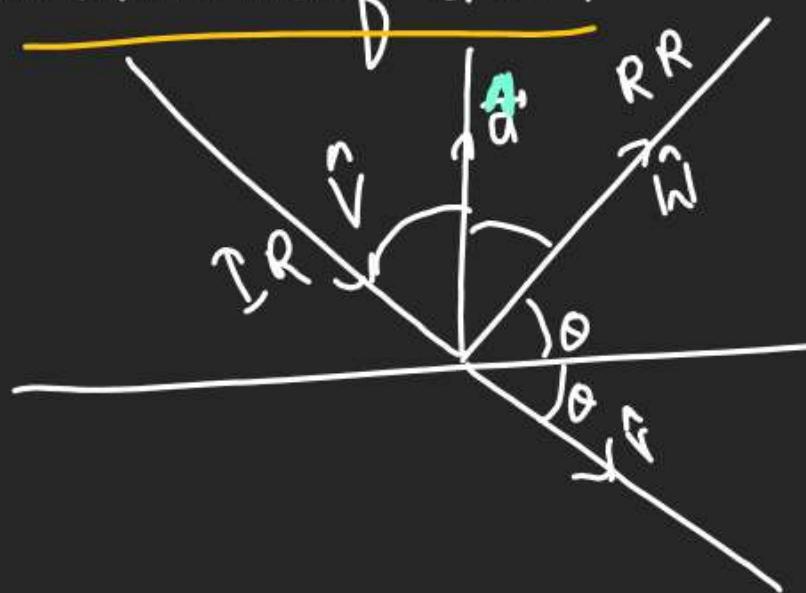


Solveकरें तो कैसे होता है

Q Incident Ray is along unit vector \hat{v} &
Reflected Ray along \hat{w} , Normal vector

along Unit vector \hat{a} outward. Express

in terms of \hat{a} & \hat{v} .



\vec{a} is External

Angle Bisector

$$\vec{u} = \lambda(\hat{v} - \hat{n})$$

$$t \vec{a} = \hat{v} - \hat{n}$$

$$(\hat{w})^2 = (\hat{v} - t \vec{a})^2$$

$$\hat{w} = \hat{v} - t \cdot \hat{a} \quad \leftarrow (\hat{w})^2 = (\hat{v} - t \vec{a})^2$$

$$\hat{w} = \hat{v} - 2(\hat{a} \cdot \hat{v}) \cdot \hat{a} \quad |\hat{w}|^2 = |\hat{v}|^2 + t^2 |\hat{a}|^2 - 2t(\hat{a} \cdot \hat{v})$$

$$0 = t(t - 2(\hat{a} \cdot \hat{v}))$$

$$t = 0, t = 2(\hat{a} \cdot \hat{v})$$

Max/Mint by Qs

$$Q \Rightarrow f^2(x) + g^2(x) + h^2(x) \leq 9$$

& $u(x) = 3f(x) + 4g(x) + 10h(x)$ find Max. Value of $u(x)$?

$$(1) \text{ Let } \vec{a} = 3\hat{i} + 4\hat{j} + 10\hat{k} \quad | \vec{b} = f(x)\hat{i} + g(x)\hat{j} + h(x)\hat{k}$$

$$|\vec{a}| = \sqrt{25 + 100} \\ = 5\sqrt{5}$$

$$(2) \quad \text{G.O: } \frac{\vec{a} \cdot \vec{b}}{|\vec{a}|(|\vec{b}|)} = \frac{u(x)}{5\sqrt{5} |\vec{b}|}$$

$$(3) \quad \text{G.O}^2 \theta = \frac{u^2(x)}{125 |\vec{b}|^2} \leq 1$$

$$\begin{aligned} u^2(x) &\leq 125(f^2(x) + g^2(x) + h^2(x)) \\ &\leq 125 \times 9 \\ u(x) &\leq 15\sqrt{5} \end{aligned}$$

Q If $a > 2$ A, B, C are Var. Angles of $\triangle ABC$

$$\text{S.T. } \sqrt{a^2 - 4} \tan A + a \tan B + \sqrt{a^2 + 4} \tan C = 6a$$

then find MIN value of $\sum \tan^2 A$.

$$\vec{L} = \sqrt{a^2 - 4} \hat{i} + a \hat{j} + \sqrt{a^2 + 4} \hat{k}$$

$$|\vec{L}| = \sqrt{a^2 - 4 + a^2 + a^2 + 4} = a\sqrt{3}$$

$$\vec{M} = \tan A \hat{i} + \tan B \hat{j} + \tan C \hat{k}$$

$$|\vec{M}| = \sqrt{\sum \tan^2 A}$$

$$\vec{L} \cdot \vec{M} = 6a \Rightarrow \cos \theta = \frac{\vec{L} \cdot \vec{M}}{(|\vec{L}| |\vec{M}|)}$$

$$\sum \tan^2 A > 12$$

$$\sum \tan^2 A = 12$$

$$\cos \theta = \frac{6a}{a\sqrt{3} \sqrt{\sum \tan^2 A}}$$

$$\cos \theta \leq 1 \Rightarrow \frac{12}{\sum \tan^2 A} \leq 1$$

let $x, y \in \mathbb{R}$ such that

$$28x^2 + 6xy + 36y^2 + 6(x + 6y) = 7 \quad \text{find } x^2 + 2xy + y^2 = ?$$