1. CONIC SECTIONS :

A conic section, or conic is the locus of a point which moves in a plane so that its distance from a fixed point is in a constant ratio to its perpendicular distance from a fixed straight line.

☞ The fixed point is called the **Focus.**

☞ The fixed straight line is called the **Directrix.**

☞ The constant ratio is called the **Eccentricity** denoted by e.

☞ The line passing through the focus & perpendicular to the directrix is called the **Axis.**

☞ A point of intersection of a conic with its axis is called a **Vertex.**

2. GENERAL EQUATION OF A CONIC : FOCAL DIRECTRIX PROPERTY:

The general equation of a conic with focus (p, q) & directrix $lx + my + n = 0$ is:

$(l^2 + m^2) [(x - p)^2 + (y - q)^2] = e^2 (lx + my + n)^2 \equiv ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$

3. DISTINGUISHING BETWEEN THE CONIC:

The nature of the conic section depends upon the position of the focus S w.r.t. the directrix & also upon the value of the eccentricity e. Two different cases arise.

Case (I) : When The Focus Lies On The Directrix.

In this case $D \equiv abc + 2fgh - af^2 - bg^2 - ch^2 = 0$ & the general equation of a conic represents a pair of straight lines if:

$e > 1$ the lines will be real & distinct intersecting at S.

$e = 1$ the lines will coincident.

$e < 1$ the lines will be imaginary.

Case (II) : When The Focus Does Not Lie On Directrix.

a parabola	an ellipse	a hyperbola	rectangular hyperbola
$e = 1$; $D \neq 0$,	$0 < e < 1$; $D \neq 0$;	$e > 1$; $D \neq 0$;	$e > 1$; $D \neq 0$
$h^2 = ab$	$h^2 < ab$	$h^2 > ab$	$h^2 > ab$; $a + b = 0$

4. PARABOLA : DEFINITION :

A parabola is the locus of a point which moves in a plane, such that its distance from a fixed point (focus) is equal to its perpendicular distance from a fixed straight line (directrix).

Standard equation of a parabola is $y^2 = 4ax$. For this parabola:

(i) Vertex is (0, 0) (ii) focus is (a, 0) (iii) Axis is $y = 0$ (iv) Directrix is $x + a = 0$

FOCAL DISTANCE :

The distance of a point on the parabola from the focus is called the **Focal Distance Of The Point.**

FOCAL CHORD :

A chord of the parabola, which passes through the focus is called a **Focal Chord.**

DOUBLE ORDINATE :

A chord of the parabola perpendicular to the axis of the symmetry is called a **Double Ordinate.**

LATUS RECTUM :

A double ordinate passing through the focus or a focal chord perpendicular to the axis of parabola is called the **Latus Rectum.** For $y^2 = 4ax$.

- Length of the latus rectum = 4a.
- ends of the latus rectum are L(a, 2a) & L' (a, – 2a).

Note that: (i) Perpendicular distance from focus on directrix = half the latus rectum.
(ii) Vertex is middle point of the focus & the point of intersection of directrix & axis.
(iii) Two parabolas are laid to be equal if they have the same latus rectum.
Four standard forms of the parabola are $y^2 = 4ax$; $y^2 = -4ax$; $x^2 = 4ay$; $x^2 = -4ay$

5. POSITION OF A POINT RELATIVE TO A PARABOLA :

The point $(x_1\ y_1)$ lies outside, on or inside the parabola $y^2 = 4ax$ according as the expression $y_1^2 - 4ax_1$ is positive, zero or negative.

6. LINE & *A* PARABOLA :

The line $y = mx + c$ meets the parabola $y^2 = 4ax$ in two points real, coincident or imaginary according as $a \gtrless cm \Rightarrow$ condition of tangency is, $c = \frac{a}{m}$.

7. Length of the chord intercepted by the parabola on the line $y = mx + c$ is : $\left(\frac{4}{m^2}\right)\sqrt{a(1+m^2)(a-mc)}$.

Note: length of the focal chord making an angle α with the x– axis is $4a\text{Cosec}^2\alpha$.

8. PARAMETRIC REPRESENTATION :

The simplest & the best form of representing the co–ordinates of a point on the parabola is $(at^2, 2at)$. The equations $x = at^2$ & $y = 2at$ together represents the parabola $y^2 = 4ax$, t being the parameter. The equation of a chord joining t_1 & t_2 is $2x - (t_1 + t_2)y + 2at_1t_2 = 0$.

Note: If the chord joining t_1, t_2 & t_3, t_4 pass through a point $(c, 0)$ on the axis, then $t_1t_2 = t_3t_4 = -c/a$.

9. TANGENTS TO THE PARABOLA $y^2 = 4ax$:

(i) $yy_1 = 2a(x + x_1)$ at the point (x_1, y_1) ; **(ii)** $y = mx + \frac{a}{m}$ $(m \neq 0)$ at $\left(\frac{a}{m^2}, \frac{2a}{m}\right)$

(iii) $ty = x + at^2$ at $(at^2, 2at)$.

Note : Point of intersection of the tangents at the point t_1 & t_2 is $[\,at_1t_2, a(t_1 + t_2)\,]$.

10. NORMALS TO THE PARABOLA $y^2 = 4ax$:

(i) $y - y_1 = -\frac{y_1}{2a}(x - x_1)$ at (x_1, y_1) ; **(ii)** $y = mx - 2am - am^3$ at $(am^2, -2am)$

(iii) $y + tx = 2at + at^3$ at $(at^2, 2at)$.

Note : Point of intersection of normals at t_1 & t_2 are, $a(t_1^2 + t_2^2 + t_1t_2 + 2)$; $-at_1t_2(t_1 + t_2)$.

11. THREE VERY IMPORTANT RESULTS :

(a) If t_1 & t_2 are the ends of a focal chord of the parabola $y^2 = 4ax$ then $t_1t_2 = -1$. Hence the co-ordinates at the extremities of a focal chord can be taken as $(at^2, 2at)$ & $\left(\frac{a}{t^2}, -\frac{2a}{t}\right)$.

(b) If the normals to the parabola $y^2 = 4ax$ at the point t_1, meets the parabola again at the point t_2, then $t_2 = -\left(t_1 + \frac{2}{t_1}\right)$.

(c) If the normals to the parabola $y^2 = 4ax$ at the points t_1 & t_2 intersect again on the parabola at the point 't_3' then $t_1t_2 = 2$; $t_3 = -(t_1 + t_2)$ and the line joining t_1 & t_2 passes through a fixed point $(-2a, 0)$.

General Note :

(i) Length of subtangent at any point P(x, y) on the parabola $y^2 = 4ax$ equals twice the abscissa of the point P. Note that the subtangent is bisected at the vertex.

(ii) Length of subnormal is constant for all points on the parabola & is equal to the semi latus rectum.

(iii) If a family of straight lines can be represented by an equation $\lambda^2 P + \lambda Q + R = 0$ where λ is a parameter and P, Q, R are linear functions of x and y then the family of lines will be tangent to the curve $Q^2 = 4\,PR$.

12. The equation to the pair of tangents which can be drawn from any point (x_1, y_1) to the parabola $y^2 = 4ax$ is given by : $SS_1 = T^2$ where :
$S \equiv y^2 - 4ax$; $S_1 = y_1^2 - 4ax_1$; $T \equiv y\,y_1 - 2a(x + x_1)$.

13. DIRECTOR CIRCLE :
Locus of the point of intersection of the perpendicular tangents to the parabola $y^2 = 4ax$ is called the **Director Circle.** It's equation is $x + a = 0$ which is parabola's own directrix.

14. CHORD OF CONTACT :
Equation to the chord of contact of tangents drawn from a point $P(x_1, y_1)$ is $yy_1 = 2a\,(x + x_1)$.
Remember that the area of the triangle formed by the tangents from the point (x_1, y_1) & the chord of contact is $(y_1^2 - 4ax_1)^{3/2} \div 2a$. Also note that the chord of contact exists only if the point P is not inside.

15. POLAR & POLE :

(i) Equation of the Polar of the point $P(x_1, y_1)$ w.r.t. the parabola $y^2 = 4ax$ is
$y\,y_1 = 2a(x + x_1)$

(ii) The pole of the line $lx + my + n = 0$ w.r.t. the parabola $y^2 = 4ax$ is $\left(\frac{n}{l}, -\frac{2am}{l}\right)$.

Note:

(i) The polar of the focus of the parabola is the directrix.

(ii) When the point (x_1, y_1) lies without the parabola the equation to its polar is the same as the equation to the chord of contact of tangents drawn from (x_1, y_1) when (x_1, y_1) is on the parabola the polar is the same as the tangent at the point.

(iii) If the polar of a point P passes through the point Q, then the polar of Q goes through P.

(iv) Two straight lines are said to be conjugated to each other w.r.t. a parabola when the pole of one lies on the other.

(v) Polar of a given point P w.r.t. any Conic is the locus of the harmonic conjugate of P w.r.t. the two points is which any line through P cuts the conic.

16. CHORD WITH A GIVEN MIDDLE POINT :
Equation of the chord of the parabola $y^2 = 4ax$ whose middle point is
(x_1, y_1) is $y - y_1 = \frac{2a}{y_1}(x - x_1)$. This reduced to $T = S_1$
where $T = y\,y_1 - 2a\,(x + x_1)$ & $S_1 = y_1^2 - 4ax_1$.

17. DIAMETER :
The locus of the middle points of a system of parallel chords of a Parabola is called a **Diameter.** Equation to the diameter of a parabola is $y = 2a/m$, where m = slope of parallel chords.

Note:

(i) The tangent at the extremity of a diameter of a parabola is parallel to the system of chords it bisects.

(ii) The tangent at the ends of any chords of a parabola meet on the diameter which bisects the chord.

(iii) A line segment from a point P on the parabola and parallel to the system of parallel chords is called the ordinate to the diameter bisecting the system of parallel chords and the chords are called its double ordinate.

18. IMPORTANT HIGHLIGHTS :

(a) If the tangent & normal at any point 'P' of the parabola intersect the axis at T & G then ST = SG = SP where 'S' is the focus. In other words the tangent and the normal at a point P on the parabola are the bisectors of the angle between the focal radius SP & the perpendicular from P on the directrix. From this we conclude that all rays emanating from S will become parallel to the axis of the parabola after reflection.

(b) The portion of a tangent to a parabola cut off between the directrix & the curve subtends a right angle at the **focus.**

(c) The tangents at the extremities of a focal chord intersect at right angles on the **directrix**, and hence a circle on any focal chord as diameter touches the directrix. Also a circle on any focal radii of a point P $(at^2, 2at)$ as diameter touches the tangent at the vertex and intercepts a chord of length $a\sqrt{1+t^2}$ on a normal at the point P.

(d) Any tangent to a parabola & the perpendicular on it from the focus meet on the tangtent at the vertex.

(e) If the tangents at P and Q meet in T, then :

- TP and TQ subtend equal angles at the focus S.
- $ST^2 = SP.\ SQ$ &
- The triangles SPT and STQ are similar.

(f) Tangents and Normals at the extremities of the latus rectum of a parabola $y^2 = 4ax$ constitute a square, their points of intersection being $(-a, 0)$ & $(3a, 0)$.

(g) Semi latus rectum of the parabola $y^2 = 4ax$, is the harmonic mean between segments of any focal chord of the parabola is ; $2a = \frac{2bc}{b+c}$ **i.e.** $\frac{1}{b}+\frac{1}{c}=\frac{1}{a}$.

(h) The circle circumscribing the triangle formed by any three tangents to a parabola passes through the focus.

(i) The orthocentre of any triangle formed by three tangents to a parabola $y^2 = 4ax$ lies on the directrix & has the co-ordinates $-a,\ a(t_1 + t_2 + t_3 + t_1t_2t_3)$.

(j) The area of the triangle formed by three points on a parabola is twice the area of the triangle formed by the tangents at these points.

(k) If normal drawn to a parabola passes through a point P(h, k) then $k = mh - 2am - am^3$ i.e. $am^3 + m(2a - h) + k = 0$.

Then gives $m_1 + m_2 + m_3 = 0$; $m_1m_2 + m_2m_3 + m_3m_1 = \frac{2a-h}{a}$; $m_1\ m_2\ m_3 = -\frac{k}{a}$.

where m_1, m_2 & m_3 are the slopes of the three concurrent normals. Note that the algebraic sum of the:

- slopes of the three concurrent normals is zero.
- ordinates of the three conormal points on the parabola is zero.
- Centroid of the Δ formed by three co-normal points lies on the x-axis.

(*l*) A circle circumscribing the triangle formed by three co–normal points passes through the vertex of the parabola and its equation is, $2(x^2 + y^2) - 2(h + 2a)x - ky = 0$

***Suggested problems from Loney*:**

Exercise-25 (Q.5, 10, 13, 14, 18, 21, 22),

Exercise-26 (Important) (Q.4, 6, 7, 17, 22, 26, 27, 28, 34),

Exercise-27 (Q.4,), **Exercise-28** (Q.2, 7, 11, 14, 23),

Exercise-29 (Q.7, 8, 19, 21, 24, 27),

Exercise-30 (2, 3, 18, 20, 21, 22, 25, 26, 30)

***Note*:** Refer to the figure on Pg.175 if necessary.

EXERCISE - 1

1. Show that the normals at the points (4a, 4a) & at the upper end of the latus ractum of the parabola $y^2 = 4ax$ intersect on the same parabola.

2. Prove that the locus of the middle point of portion of a normal to $y^2 = 4ax$ intercepted between the curve & the axis is another parabola. Find the vertex & the latus rectum of the second parabola.

3. Find the equations of the tangents to the parabola $y^2 = 16x$, which are parallel & perpendicular respectively to the line $2x - y + 5 = 0$. Find also the coordinates of their points of contact.

4. A circle is described whose centre is the vertex and whose diameter is three-quarters of the latus rectum of a parabola $y^2 = 4ax$. Prove that the common chord of the circle and parabola bisects the distance between the vertex and the focus.

5. Find the equations of the tangents of the parabola $y^2 = 12x$, which passes through the point (2,5).

6. Through the vertex O of a parabola $y^2 = 4x$, chords OP & OQ are drawn at right angles to one another. Show that for all positions of P, PQ cuts the axis of the parabola at a fixed point. Also find the locus of the middle point of PQ.

7. Let S is the focus of the parabola $y^2 = 4ax$ and X the foot of the directrix, PP' is a double ordinate of the curve and PX meets the curve again in Q. Prove that P'Q passes through focus.

8. Three normals to $y^2 = 4x$ pass through the point (15, 12). Show that if one of the normals is given by $y = x - 3$ & find the equations of the others.

9. Find the equations of the chords of the parabola $y^2 = 4ax$ which pass through the point (–6a, 0) and which subtends an angle of 45° at the vertex.

10. 'O' is the vertex of the parabola $y^2 = 4ax$ & L is the upper end of the latus rectum. If LH is drawn perpendicular to OL meeting OX in H, prove that the length of the double ordinate through H is $4a\sqrt{5}$.

11. The normal at a point P to the parabola $y^2 = 4ax$ meets its axis at G. Q is another point on the parabola such that QG is perpendicular to the axis of the parabola. Prove that $QG^2 - PG^2$ = constant.

12. If the normal at P(18, 12) to the parabola $y^2 = 8x$ cuts it again at Q, show that $9PQ = 80\sqrt{10}$

13. Prove that, the normal to $y^2 = 12x$ at (3, 6) meets the parabola again in (27, –18) & circle on this normal chord as diameter is $x^2 + y^2 - 30x + 12y - 27 = 0$.

14. Find the equation of the circle which passes through the focus of the parabola $x^2 = 4y$ & touches it at the point (6, 9).

15. P & Q are the points of contact of the tangents drawn from the point T to the parabola $y^2 = 4ax$. If PQ be the normal to the parabola at P, prove that TP is bisected by the directrix.

16. From the point (–1, 2) tangent lines are drawn to the parabola $y^2 = 4x$. Find the equation of the chord of contact. Also find the area of the triangle formed by the chord of contact & the tangents.

17. From a point A common tangents are drawn to the circle $x^2 + y^2 = a^2/2$ & parabola $y^2 = 4ax$. Find the area of the quadrilateral formed by the common tangents, the chord of contact of the circle and the chord of contact of the parabola.

18. Show that the locus of a point, such that two of the three normals drawn from it to the parabola $y^2 = 4ax$ are perpendicular is $y^2 = a(x - 3a)$.

19. Prove that the two parabolas $y^2 = 4ax$ & $y^2 = 4c(x - b)$ cannot have a common normal, other than the axis, unless $\frac{b}{(a-c)} > 2$.

20. Find the condition on 'a' & 'b' so that the two tangents drawn to the parabola $y^2 = 4ax$ from a point are normals to the parabola $x^2 = 4by$.

EXERCISE - 2

1. In the parabola $y^2 = 4ax$, the tangent at the point P, whose abscissa is equal to the latus ractum meets the axis in T & the normal at P cuts the parabola again in Q. Prove that PT : PQ = 4 : 5.

2. Two tangents to the parabola $y^2 = 8x$ meet the tangent at its vertex in the points P & Q. If PQ = 4 units, prove that the locus of the point of the intersection of the two tangents is $y^2 = 8(x + 2)$.

3. A variable chord $t_1 t_2$ of the parabola $y^2 = 4ax$ subtends a right angle at a fixed point t_0 of the curve. Show that it passes through a fixed point. Also find the co–ordinates of the fixed point.

4. Two perpendicular straight lines through the focus of the parabola $y^2 = 4ax$ meet its directrix in T & T' respectively. Show that the tangents to the parabola parallel to the perpendicular lines intersect in the mid point of T T '.

5. Two straight lines one being a tangent to $y^2 = 4ax$ and the other to $x^2 = 4by$ are right angles. Find the locus of their point of intersection.

6. A variable chord PQ of the parabola $y^2 = 4x$ is drawn parallel to the line y = x. If the parameters of the points P & Q on the parabola are p & q respectively, show that p + q = 2. Also show that the locus of the point of intersection of the normals at P & Q is 2x – y = 12.

7. Show that an infinite number of triangles can be inscribed in either of the parabolas $y^2 = 4ax$ & $x^2 = 4by$ whose sides touch the other.

8. If (x_1, y_1), (x_2, y_2) and (x_3, y_3) be three points on the parabola $y^2 = 4ax$ and the normals at these points meet in a point then prove that $\frac{x_1 - x_2}{y_3} + \frac{x_2 - x_3}{y_1} + \frac{x_3 - x_1}{y_2} = 0$.

9. Show that the normals at two suitable distinct real points on the parabola $y^2 = 4ax$ ($a > 0$) intersect at a point on the parabola whose abscissa > 8a.

10. If Q (x_1, y_1) is an arbitrary point in the plane of a parabola $y^2 = 4ax$, show that there are three points on the parabola at which OQ subtends a right angle, where O is the origin. Show furhter that the normal at these three points are concurrent at a point R.,determine the coordinates of R in terms of those of Q.

11. A quadrilateral is inscribed in a parabola $y^2 = 4ax$ and three of its sides pass through fixed points on the axis. Show that the fourth side also passes through fixed point on the axis of the parabola.

12. Prove that the parabola $y^2 = 16x$ & the circle $x^2 + y^2 - 40x - 16y - 48 = 0$ meet at the point P(36, 24) & one other point Q. Prove that PQ is a diameter of the circle. Find Q.

13. A variable tangent to the parabola $y^2 = 4ax$ meets the circle $x^2 + y^2 = r^2$ at P & Q. Prove that the locus of the mid point of PQ is $x(x^2 + y^2) + ay^2 = 0$.

14. Find the locus of the foot of the perpendicular from the origin to chord of the parabola $y^2 = 4ax$ subtending an angle of 45^0 at the vertex.

15. Show that the locus of the centroids of equilateral triangles inscribed in the parabola $y^2 = 4ax$ is the parabola $9y^2 - 4ax + 32a^2 = 0$.

16. A fixed parabola $y^2 = 4ax$ touches a variable parabola. Find the equation to the locus of the vertex of the variable parabola. Assume that the two parabolas are equal and the axis of the variable parabola remains parallel to the x-axis.

17. Show that the circle through three points the normals at which to the parabola $y^2 = 4ax$ are concurrent at the point (h, k) is $2(x^2 + y^2) - 2(h + 2a)x - ky = 0$.

18. Prove that the locus of the centre of the circle, which passes through the vertex of the parabola $y^2 = 4ax$ & through its intersection with a normal chord is $2y^2 = ax - a^2$.

EXERCISE - 3 (JM)

1. Find the equations of the common tangents of the circle $x^2 + y^2 - 6y + 4 = 0$ and the parabola $y^2 = x$. [REE '99, 6]

2. (a) If the line $x - 1 = 0$ is the directrix of the parabola $y^2 - kx + 8 = 0$, then one of the values of ' k ' is
(A) 1/8 (B) 8 (C) 4 (D) 1/4

(b) If $x + y = k$ is normal to $y^2 = 12x$, then ' k ' is : [JEE'2000 (Scr), 1+1]
(A) 3 (B) 9 (C) – 9 (D) – 3

3. Find the locus of the points of intersection of tangents drawn at the ends of all normal chords of the parabola $y^2 = 8(x - 1)$. [REE '2001, 3]

4. (a) The equation of the common tangent touching the circle $(x - 3)^2 + y^2 = 9$ and the parabola $y^2 = 4x$ above the x – axis is
(A) $\sqrt{3}\,y = 3x + 1$ (B) $\sqrt{3}\,y = -(x + 3)$ (C) $\sqrt{3}\,y = x + 3$ (D) $\sqrt{3}\,y = -(3x + 1)$

(b) The equation of the directrix of the parabola, $y^2 + 4y + 4x + 2 = 0$ is
(A) $x = -1$ (B) $x = 1$ (C) $x = -3/2$ (D) $x = 3/2$
[JEE'2001(Scr), 1+1]

5. The locus of the mid-point of the line segment joining the focus to a moving point on the parabola $y^2 = 4ax$ is another parabola with directrix [JEE'2002 (Scr.), 3]
(A) $x = -a$ (B) $x = -a/2$ (C) $x = 0$ (D) $x = a/2$

6. The equation of the common tangent to the curves $y^2 = 8x$ and $xy = -1$ is [JEE'2002 (Scr), 3]
(A) $3y = 9x + 2$ (B) $y = 2x + 1$ (C) $2y = x + 8$ (D) $y = x + 2$

7. (a) The slope of the focal chords of the parabola $y^2 = 16x$ which are tangents to the circle $(x - 6)^2 + y^2 = 2$ are [JEE'2003, (Scr.)]
(A) ± 2 (B) – 1/2, 2 (C) ± 1 (D) – 2, 1/2

(b) Normals are drawn from the point 'P' with slopes m_1, m_2, m_3 to the parabola $y^2 = 4x$. If locus of P with $m_1 m_2 = \alpha$ is a part of the parabola itself then find α. [JEE 2003, 4 out of 60]

8. The angle between the tangents drawn from the point (1, 4) to the parabola $y^2 = 4x$ is
(A) $\pi/2$ (B) $\pi/3$ (C) $\pi/4$ (D) $\pi/6$ [JEE 2004, (Scr.)]

9. Let P be a point on the parabola $y^2 - 2y - 4x + 5 = 0$, such that the tangent on the parabola at P intersects the directrix at point Q. Let R be the point that divides the line segment PQ externally in the ratio $\frac{1}{2} : 1$. Find the locus of R. [JEE 2004, 4 out of 60]

10. (a) The axis of parabola is along the line y = x and the distance of vertex from origin is $\sqrt{2}$ and that of origin from its focus is $2\sqrt{2}$. If vertex and focus both lie in the 1st quadrant, then the equation of the parabola is

(A) $(x + y)^2 = (x - y - 2)$ (B) $(x - y)^2 = (x + y - 2)$
(C) $(x - y)^2 = 4(x + y - 2)$ (D) $(x - y)^2 = 8(x + y - 2)$ [JEE 2006, 3]

(b) The equations of common tangents to the parabola $y = x^2$ and $y = -(x - 2)^2$ is/are [JEE 2006, 5]

(A) $y = 4(x - 1)$ (B) $y = 0$ (C) $y = -4(x - 1)$ (D) $y = -30x - 50$

(c) **Match the following**

Normals are drawn at points P, Q and R lying on the parabola $y^2 = 4x$ which intersect at (3, 0). Then

(i)	Area of ΔPQR	(A) 2
(ii)	Radius of circumcircle of ΔPQR	(B) 5/2
(iii)	Centroid of ΔPQR	(C) (5/2, 0)
(iv)	Circumcentre of ΔPQR	(D) (2/3, 0)

[JEE 2006, 6]

11. Statement-1: The curve $y = \frac{-x^2}{2} + x + 1$ is symmetric with respect to the line x = 1.

because

Statement-2: A parabola is symmetric about its axis.

(A) Statement-1 is true, statement-2 is true; statement-2 is correct explanation for statement-1.
(B) Statement-1 is true, statement-2 is true; statement-2 is NOT a correct explanation for statement-1.
(C) Statement-1 is true, statement-2 is false.
(D) Statement-1 is false, statement-2 is true. [JEE 2007, 4]

Comprehension: (3 questions)

12. Consider the circle $x^2 + y^2 = 9$ and the parabola $y^2 = 8x$. They intersect at P and Q in the first and the fourth quadrants, respectively. Tangents to the circle at P and Q intersect the x-axis at R and tangents to the parabola at P and Q intersect the x-axis at S.

(a) The ratio of the areas of the triangles PQS and PQR is

(A) $1 : \sqrt{2}$ (B) 1 : 2 (C) 1 : 4 (D) 1 : 8

(b) The radius of the circumcircle of the triangle PRS is

(A) 5 (B) $3\sqrt{3}$ (C) $3\sqrt{2}$ (D) $2\sqrt{3}$

(c) The radius of the incircle of the triangle PQR is

(A) 4 (B) 3 (C) 8/3 (D) 2 [JEE 2007, 4+4+4]

13. Let $P(x_1, y_1)$ and $Q(x_2, y_2)$, $y_1 < 0$, $y_2 < 0$, be the end points of the latus rectum of the ellipse $x^2 + 4y^2 = 4$. The equations of parabolas with latus rectum PQ are

(A) $x^2 + 2\sqrt{3}\,y = 3 + \sqrt{3}$ (B) $x^2 - 2\sqrt{3}\,y = 3 + \sqrt{3}$
(C) $x^2 + 2\sqrt{3}\,y = 3 - \sqrt{3}$ (D) $x^2 - 2\sqrt{3}\,y = 3 - \sqrt{3}$ [JEE 2008, 4]

14. The tangent PT and the normal PN to the parabola $y^2 = 4ax$ at a point P on it meet its axis at points T and N, respectively. The locus of the centroid of the triangle PTN is a parabola whose **[JEE 2009]**

(A) vertex is $\left(\frac{2a}{3}, 0\right)$ (B) directrix is x = 0 (C) latus rectum is $\frac{2a}{3}$ (D) focus is (a, 0)

15. Let A and B be two distinct points on the parabola $y^2 = 4x$. If the axis of the parabola touches a circle of radius r having AB as its diameter, then the slope of the line joining A and B can be **[JEE - 2010]**

(A) – 1/r (B) 1/r (C) 2/r (D) –2/r

16. Let (x, y) be any point on the parabola $y^2 = 4x$. Let P be the point that divides the line segment from (0, 0) to (x, y) in the ratio 1 : 3. Then the locus of P is **[JEE 2011]**

(A) $x^2 = y$ (B) $y^2 = 2x$ (C) $y^2 = x$ (D) $x^2 = 2y$

17. Let L be a normal to the parabola $y^2 = 4x$. If L passes through the point (9, 6), then L is given by **[JEE 2011]**

(A) $y - x + 3 = 0$ (B) $y + 3x - 33 = 0$
(C) $y + x - 15 = 0$ (D) $y - 2x + 12 = 0$

18. Consider the parabola $y^2 = 8x$. Let Δ_1 be the area of the triangle formed by the end points of its latus rectum and the point $P\left(\frac{1}{2}, 2\right)$ on the parabola, and Δ_2 be the area of the triangle formed by drawing tangents at P and at the end points of the latus rectum. Then $\frac{\Delta_1}{\Delta_2}$ is **[JEE 2011]**

19. Let S be the foucs of the parabola $y^2 = 8x$ and let PQ be the comon chord of the circle $x^2 + y^2 - 2x - 4y = 0$ and the given parabola. The area of the triangle PQS is **[JEE 2012]**

20. Given : A circle, $2x^2 + 2y^2 = 5$ and a parabola, $y^2 = 4\sqrt{5}\,x$. **[IIT JEE Main - 2013]**

Statement-1 : An equation of a common tangent to these curves is $y = x + \sqrt{5}$.

Statement-2 : If the line, $y = mx + \frac{\sqrt{5}}{m}$ $(m \neq 0)$ is their common tangent, then m satisfies $m^4 - 3m^2 + 2 = 0$.

(A) Statement-1 is false ; Statement-2 is true.
(B) Statement-1 is true ; Statement-2 is true ; Statement-2 is a **correct** explanation for Statement-1
(C) Statement-1 is true ; Statement-2 is true ; Statement-2 is **not** a **correct** explanation for Statement-1
(D) Statement-1 is true ; Statement-2 is false

Comprehension (Q.21 to Q.22) **[IIT JEE Advance - 2013]**

Let PQ be a focal chord of the parabola $y^2 = 4ax$. The tangents to the parabola at P and Q meet at a point lying on the line $y = 2x + a$, $a > 0$.

21. If chord PQ subtends an angle θ at the vertex of $y^2 = 4ax$, then $\tan\theta$ =

(A) $\frac{2}{3}\sqrt{7}$ (B) $\frac{-2}{3}\sqrt{7}$ (C) $\frac{2}{3}\sqrt{5}$ (D) $\frac{-2}{3}\sqrt{5}$

22. Length of chord PQ is

(A) 7a (B) 5a (C) 2a (D) 3a

23. A line L : $y = mx + 3$ meets y-axis at E(0, 3) and the arc of the parabola $y^2 = 16x$, $0 \le y \le 6$ at the point $F(x_0, y_0)$. The tangent to the parabola at $F(x_0, y_0)$ intersects the y-axis at $G(0, y_1)$. The slope m of the line L is chosen such that the area of the triangle EFG has a local maximum.

Match List I with List-II and select the correct answer using the ocde given below the lists :

[IIT JEE Advance - 2013]

	List-I		List-II
(P)	m =	1.	1/2
(Q)	Maximum area of ΔEFG is	2.	4
(R)	y_0 =	3.	2
(S)	y_1 =	4.	1

(A) P $\to$ 4 ; Q $\to$ 1 ; R $\to$ 2 ; S $\to$ 3 (B) P $\to$ 3 ; Q $\to$ 4 ; R $\to$ 1 ; S $\to$ 2
(C) P $\to$ 1 ; Q $\to$ 3 ; R $\to$ 2 ; S $\to$ 4 (D) P $\to$ 1 ; Q $\to$ 3 ; R $\to$ 4 ; S $\to$ 2

24. The slope of the line touching both the parabolas $y^2 = 4x$ and $x^2 = -32y$ is **[JEE Main - 2014]**

(A) $\frac{2}{3}$ (B) $\frac{1}{2}$ (C) $\frac{3}{2}$ (D) $\frac{1}{8}$

25. The common tangents to the circle $x^2 + y^2 = 2$ and the parabola $y^2 = 8x$ touch the circle at the points P, Q and the parabola at the points R, S. Then the area of the quadrilateral PQRS is :

[IIT JEE Advance - 2014]

(A) 3 (B) 6 (C) 9 (D) 15

Comprehension (Q.26 to Q.27)

Let a, r, s, t be nonzero real numbers. Let $P(at^2, 2at)$, Q, $R(ar^2, 2ar)$ and $S(as^2, 2as)$ be distinct points on the parabola $y^2 = 4ax$. Suppose that PQ is the focal chord and lines QR and PK are parallel, where K is the point (2a, 0). **[IIT JEE Advance - 2014]**

26. The value of r is :

(A) $-\frac{1}{t}$ (B) $\frac{t^2+1}{t}$ (C) $\frac{1}{t}$ (D) $\frac{t^2-1}{t}$

27. If st = 1, then the tangent at P and the normal at S to the parabola meet at a point whose ordinate is

(A) $\frac{(t^2+1)^2}{2t^3}$ (B) $\frac{a(t^2+1)^2}{2t^3}$ (C) $\frac{a(t^2+1)^2}{t^3}$ (D) $\frac{a(t^2+2)^2}{t^3}$

28. Let O be the vertex and Q be any point on the parabola, $x^2 = 8y$. If the point P divides the line segment OQ internally in the ratio 1 : 3, then the locus of P is: **[JEE Main - 2015]**

(A) $x^2 = 2y$ (B) $x^2 = y$ (C) $y^2 = x$ (D) $y^2 = 2x$

29. Let P and Q be distinct points on the parabola $y^2 = 2x$ such that a circle with PQ as diameter passes through the vertex O of the parabola. If P lies in the first quadrant and the area of the triangle ΔOPQ is $3\sqrt{2}$, then which of the following is(are) the coordinates of P? **[IIT JEE Advance - 2015]**

(A) $(4, 2\sqrt{2})$ (B) $(9, 3\sqrt{2})$ (C) $\left(\frac{1}{4}, \frac{1}{\sqrt{2}}\right)$ (D) $(1, \sqrt{2})$

30. Let the curve C be the mirror image of the parabola $y^2 = 4x$ with rescpect to the line $x + y + 4 = 0$. If A and B are the points of intersection of C with the line $y = -5$, then the distance between A and B is

[IIT JEE Advance - 2015]

31. If the normals of the parabola $y^2 = 4x$ drawn at the end points of its latus rectum are tangents to the circle $(x - 3)^2 + (y + 2)^2 = r^2$, then the value of r^2 is : **[IIT JEE Advance - 2015]**

32. Let P be the point on the parabola, $y^2 = 8x$ which is at a minimum distance from the centre C of the circle, $x^2 + (y + 6)^2 = 1$. Then the equation of the circle, passing through C and having its centre at P is :

(A) $x^2 + y^2 - 4x + 8y + 12 = 0$ (B) $x^2 + y^2 - x + 4y - 12 = 0$ **[JEE Main - 2016]**

(C) $x^2 + y^2 - \frac{x}{4} + 2y - 24 = 0$ (D) $x^2 + y^2 - 4x + 9y + 18 = 0$

33. The circle $C_1 : x^2 + y^2 = 3$, with centre at O, intersects the parabola $x^2 = 2y$ at the point P in the first quadrant. Let the tangent to the circle C_1 at P touches other two circles C_2 and C_3 at R_2 and R_3, respectively. Suppose C_2 and C_3 have equal radii $2\sqrt{3}$ and centres Q_2 and Q_3, respectively. If Q_2 and Q_3 lie on the y-axis, then

(A) $Q_2Q_3 = 12$ (B) $R_2R_3 = 4\sqrt{6}$ **[IIT JEE Advanced - 2016]**

(C) area of the triangle OR_2R_3 is $6\sqrt{2}$ (D) area of the triangle PQ_2Q_3 is $4\sqrt{2}$

34. Let P be the point on the parabola $y^2 = 4x$ which is at the shortest distance from the center S of the circle $x^2 + y^2 - 4x - 16y + 64 = 0$. Let Q be the point on the circle dividing the line segment SP internally. Then

(A) $SP = 2\sqrt{5}$ **[IIT JEE Advanced - 2016]**

(B) $SQ : QP = (\sqrt{5}+1) : 2$

(C) The x-intercept of the normal to the parabola at P is 6

(D) The slope of the tangent to the circle at Q is 1/2

35. If a chord, which is not a tangent, of the parabola $y^2 = 16x$ has the equation $2x + y = p$, and midpoint (h, k), then which of the following is(are) possible value(s) of p, h and k? **[JEE Advanced - 2017]**

(A) $p = -2, h = 2, k = -4$
(B) $p = 5, h = 4, k = -3$
(C) $p = -1, h = 1, k = -3$
(D) $p = 2, h = 3, k = -4$

36. Tangent and normal are drawn at P(16, 16) on the parabola $y^2 = 16x$, which intersect the axis of the parabola at A and B, respectively. If C is the centre of the circle through the points P, A and B and $\angle CPB = \theta$, then a value of $\tan\theta$ is : **[JEE Main - 2018]**

(A) $\frac{4}{3}$
(B) $\frac{1}{2}$
(C) 2
(D) 3

37. Let E denote the parabola $y^2 = 8x$. Let $P = (-2, 4)$ and let Q and Q' be two distinct points on E such that the lines PQ and PQ' are tangents to E. Let F be the focus of E. Then which of the following statements is (are) **True** ? **[JEE Advance - 2021]**

(A) The triangle PFQ is a right-angled triangle
(B) The triangle QPQ' is a right-angled triangle
(C) The distance between P and F is $5\sqrt{2}$
(D) F lies on the line joining Q and Q'

Question stem for Quesntion No. 38 and 39

Question Stem **[JEE Advance - 2021]**

Consider the region $R = \{(x,y) \in R \times R : x \geq 0 \text{ and } y^2 \leq 4 - x\}$ .Let F be the family of all circles that are contained in R and have centres on the x-axis. Let C be the circle that has largest radius among the circles in F. Let (α, β) be a point where the circles C meets the curve $y^2 = 4-x$

38. The radius of the circle C is ________

39. The value of α is ________.

ANSWER KEY

PARABOLA

EXERCISE–I

2. (a, 0) ; a
3. $2x - y + 2 = 0$, (1, 4) ; $x + 2y + 16 = 0$, (16, –16)
5. $3x - 2y + 4 = 0$; $x - y + 3 = 0$
6. (4 , 0) ; $y^2 = 2a(x - 4a)$
8. $y = -4x + 72$, $y = 3x - 33$
9. $7y \pm 2(x + 6a) = 0$
14. $x^2 + y^2 + 18x - 28y + 27 = 0$
16. $x - y = 1$; $8\sqrt{2}$ sq. units
17. $15a^2/4$
20. $a^2 > 8b^2$

EXERCISE–II

3. $[a(t_o^2 + 4), -2at_o]$
5. $(ax + by)(x^2 + y^2) + (bx - ay)^2 = 0$
10. $\left((x_1 - 2a), 2y_1\right)$
12. Q(4, –8)
14. $(x^2 + y^2 - 4ax)^2 = 16a(x^3 + xy^2 + ay^2)$
16 $y^2 = 8ax$

EXERCISE–III

1. $x - 2y + 1 = 0$; $y = mx + \dfrac{1}{4m}$ where $m = \dfrac{-5 \pm \sqrt{30}}{10}$
2. (a) C ; **(b)** B
3. $(x + 3)y^2 + 32 = 0$
4. **(a)** C ; **(b)** D
5. C
6. D
7. **(a)** C ; **(b)** $\alpha = 2$
8. B
9. $2(y - 1)^2(x - 2) = (3x - 4)^2$
10. **(a)** D, **(b)** A, B, **(c)** (i) A, (ii) B, (iii) D, (iv) C
11. A
12. **(a)** C; **(b)** B; **(c)** D
13. B, C
14. A, D
15. C, D
16. C
17. ABD
18. 2
19. 4
20. C
21. D
22. B
23. A
24. B
25. D
26. D
27. B
28. A
29. A, D
30. 4
31. 2
32. A
33. A, B, C
34. A,C,D
35. D
36. C
37. A,B,D