DPP 01

SOLUTION

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1. $\alpha = \frac{\omega d\omega}{d\theta}$

$\int \omega d\omega = \int \alpha d\theta$

$\frac{\omega^2}{2} = $ Area under α vs θ graph $= \frac{1}{2}(9 \times 4)$

$\omega = \sqrt{36} = 6\text{rad/s}$

2. $a_{net} = \sqrt{a_t^2 + a_c^2}$

$\omega^2 = \omega_0^2 + 2\alpha\theta$

$\because \omega_0 = 0$

so, $\omega^2 = 2\alpha\theta$

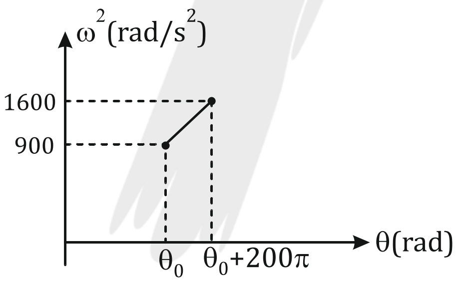
$\omega^2 R = 2(\alpha R\theta)$

$a_c = \omega^2 R = 2a_t\theta$

$1 = \sqrt{0.36 + (1.2 \times \theta)^2}$

$\Rightarrow 1 - 0.36 = (1.2\theta)^2 \Rightarrow \frac{0.8}{1.2} = \theta \Rightarrow \theta = \frac{2}{3}\text{rad}$

3. $100\text{rev} = 200\pi$ rad



Slope of graph $= \frac{d}{d\theta}(\omega^2) = \frac{2\omega d\omega}{d\theta}$

$\Rightarrow \frac{1600 - 900}{200\pi} = 2\alpha$

$\Rightarrow \alpha = \frac{7}{4\pi}\frac{\text{rad}}{s^2}$

$\omega = \omega_0 + \alpha t$

$\Rightarrow 40 = 30 + \frac{7t}{4\pi} \Rightarrow t = \frac{40\pi}{7}$ s

4. Both changes in direction although their magnitudes remains constant.

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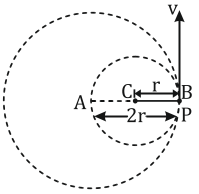
5. Change in velocity $= 2v\sin\left(\frac{\theta}{2}\right) = 2v\sin 20°$

6. Angular velocity of particle P about point A,



$\omega_A = \frac{v}{r_{AB}} = \frac{v}{2r}$

Angular velocity of particle P about point C,

$\omega_C = \frac{v}{r_{BC}} = \frac{v}{r}$

Ratio $\frac{\omega_A}{\omega_C} = \frac{v/2r}{v/r} = \frac{1}{2}$

7. $\vec{v} = \vec{\omega} \times \vec{r} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -4 & 1 \\ 5 & -6 & 6 \end{vmatrix} = -18\hat{i} - 13\hat{j} + 2\hat{k}$

8. By using equation $\omega^2 = \omega_0^2 - 2\alpha\theta$

$\left(\frac{\omega_0}{2}\right)^2 = \omega_0^2 - 2\alpha(2\pi n) \Rightarrow \alpha = \frac{3}{4}\frac{\omega_0^2}{4\pi\times 36}, (n = 36) \ldots (i)$

Now let fan completes total n′ revolution from the starting to come to rest

$0 = \omega_0^2 - 2\alpha(2\pi n') \Rightarrow n' = \frac{\omega_0^2}{4\alpha\pi}$

Substituting the value of α from equation (i)

$n' = \frac{\omega_0^2}{4\pi}\frac{4\times 4\pi\times 36}{3\omega_0^2} = 48$ revolutions

Number of rotation $= 48 - 36 = 12$

9. $h = \frac{1}{2}gt^2$

$\therefore t = \sqrt{\frac{2h}{g}}$

Let n be the number of revolutions made.

Then $n(2\pi R) = v_0 t$

or $n = \frac{v_0}{2\pi R}.t$

or $n = \frac{v_0}{2\pi R}\sqrt{\frac{2h}{g}}$

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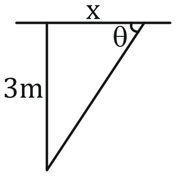
10. $\frac{d\theta}{dt} = -\omega$



$\tan\ \theta = \frac{3}{x}$

$\sec^2\theta \frac{d\theta}{dt} = -\frac{3}{x^2}\frac{dx}{dt}$

For $\theta = 45°, \sec^2\theta = 2; x = 3\text{ m}, \omega = 0.1\text{rad s}^{-1}$

$\frac{dx}{dt} = 6 \times 0.1 = 0.6\text{ m s}^{-1}$

11. Net acceleration: $a = \sqrt{a_c^2 + a_t^2}$

$= \sqrt{\left(\frac{v^2}{R}\right)^2 + a_t^2}$

As v increases, a also increases.

So size of arrow should be increasing and angle between velocity and acceleration should be acute.

12. $a_{resultant} = \sqrt{a_{radial}^2 + a_{tangential}^2} = \sqrt{\frac{v^4}{r^2} + a^2}$

13. Given $v = 1.5t^2 + 2t$

Linear acceleration a

$= \frac{dv}{dt} = 3t + 2$

This is the linear acceleration at time t

Now angular acceleration at time t

$\alpha = \frac{a}{r}$

$\Rightarrow \alpha = \frac{3t+2}{2 \times 10^{-2}}$

Angular acceleration at $t = 2\text{sec}$

$(\alpha)_{at} t = 2\text{ s} = \frac{3 \times 2 + 2}{2 \times 10^{-2}} = \frac{8}{2} \times 10^2$

$= 4 \times 10^2 = 400\text{rad/sec}^2$