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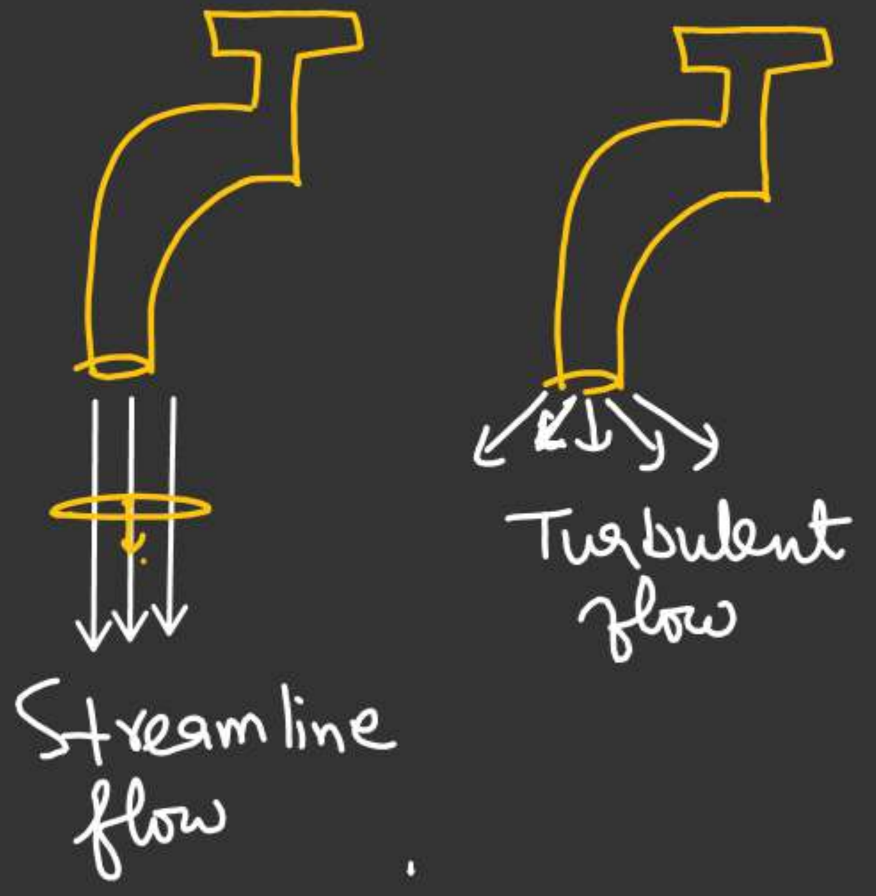
Fluid dynamics

Assumptions:-

① Fluid is Ideal → Incompressible
→ Non-Viscious.

② Stream line flow ✓

↳ All liquid layers have parallel flow.
No intruption of one layer with other
velocity of each particle passing through any point remain Constant with time.



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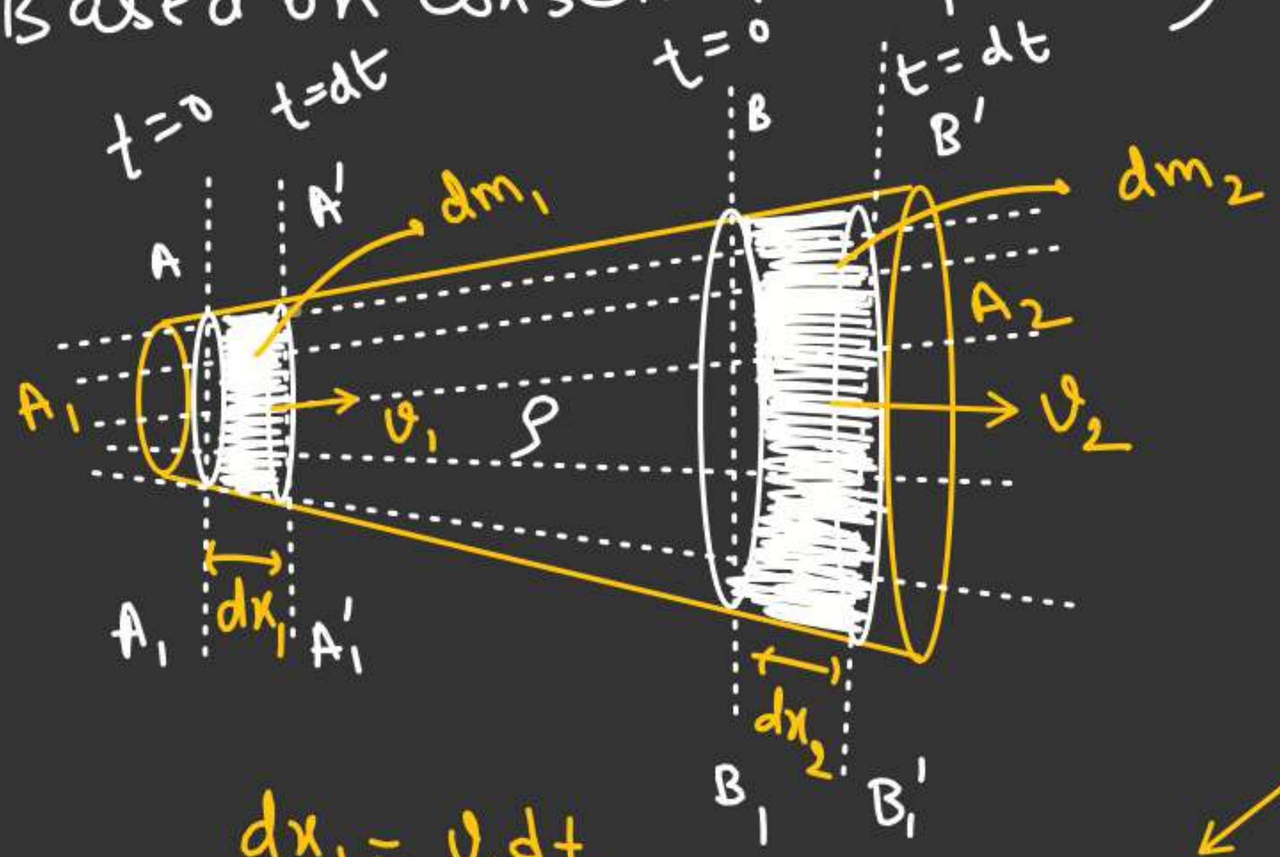
Law of Continuity

(Based on Conservation of mass)



$dx_1 = v_1 dt$

$dx_2 = v_2 dt$

$\rho = \dfrac{m}{V}$

$V = \dfrac{m}{\rho}$

By mass Conservation

$dm_1 = dm_2$

$\rho A_1 dx_1 = \rho A_2 dx_2$

$\cancel{\rho} A_1 v_1 d\cancel{t} = \cancel{\rho} A_2 v_2 d\cancel{t}$

$\boxed{A_1 v_1 = A_2 v_2}$

$m^2 \times \dfrac{m}{s} \rightarrow \underbrace{\dfrac{m^3}{s}}$

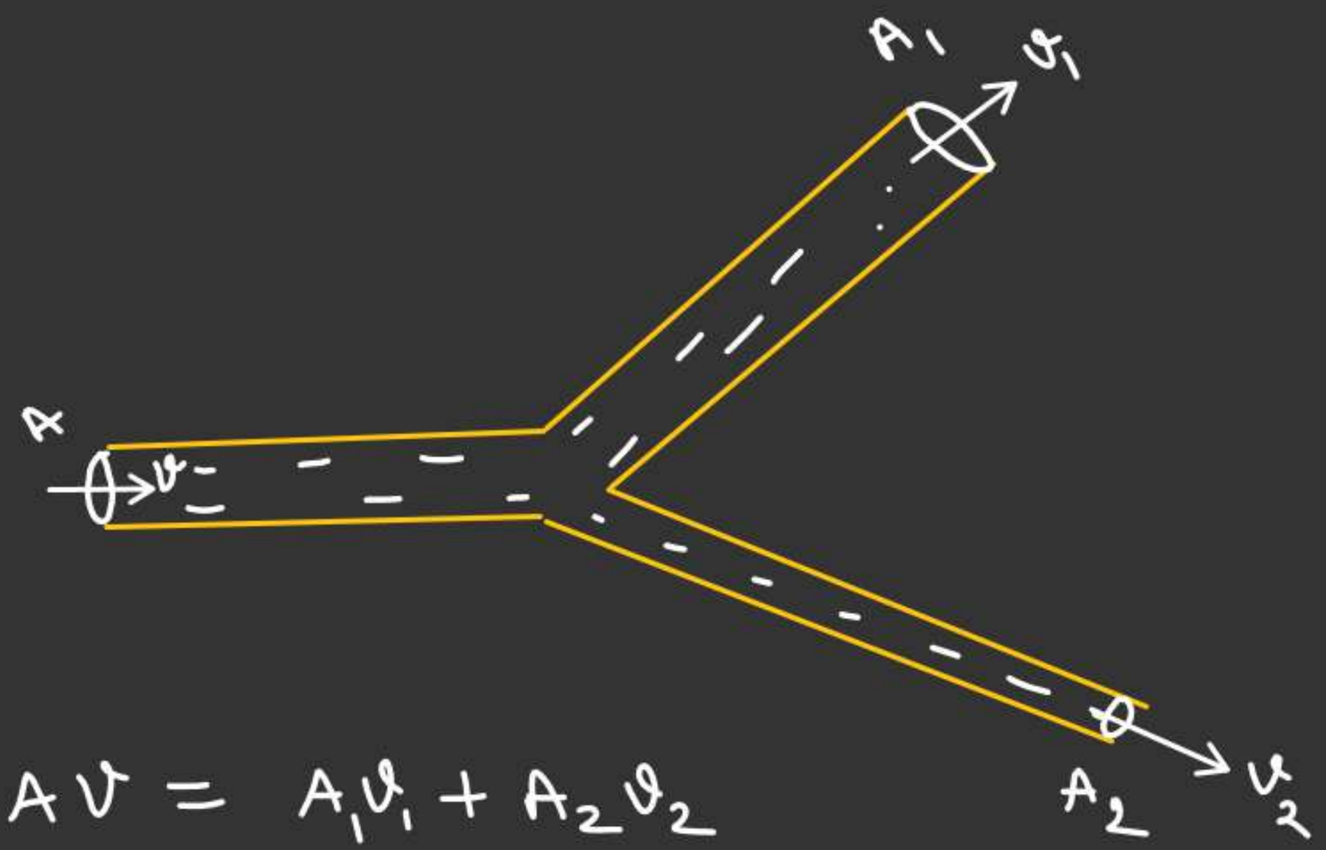
$\Downarrow$

Volume per Second.

$$\boxed{\frac{1}{\rho}\frac{dm}{dt} = \frac{dV}{dt} = A_1 v_1 = A_2 v_2}$$

$\Downarrow$

Volume flow rate



$$Av = A_1 v_1 + A_2 v_2$$

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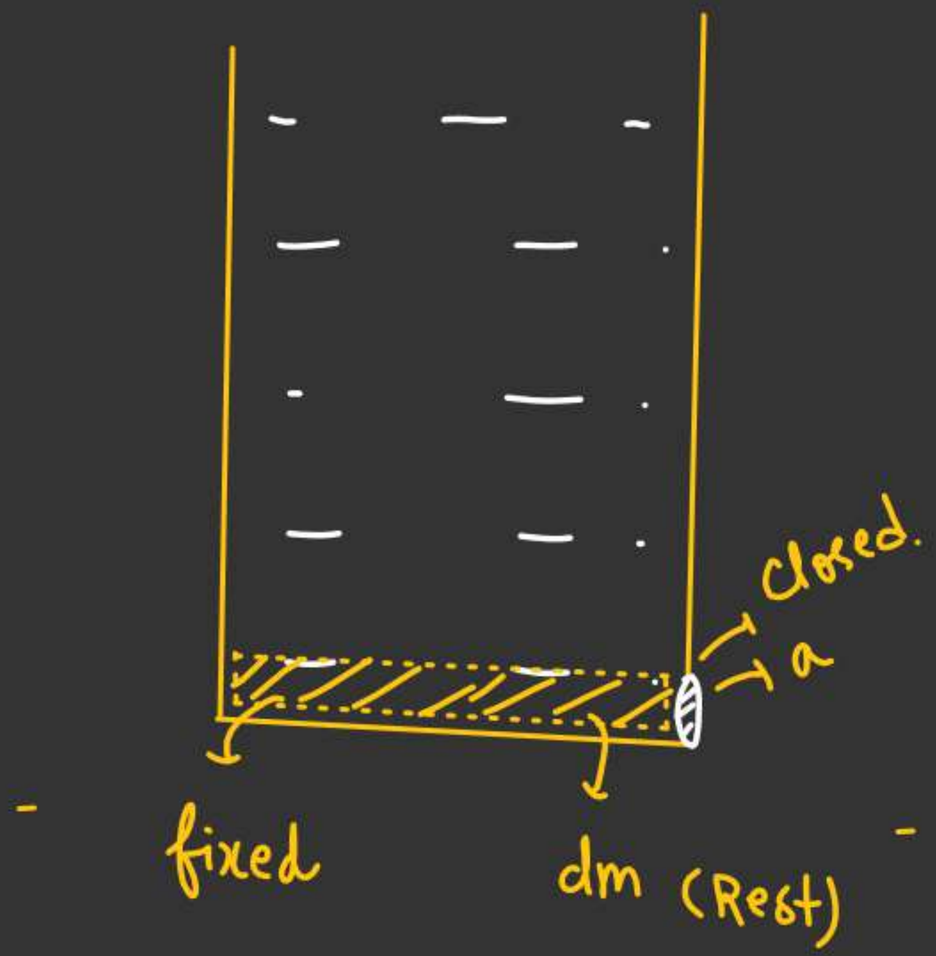
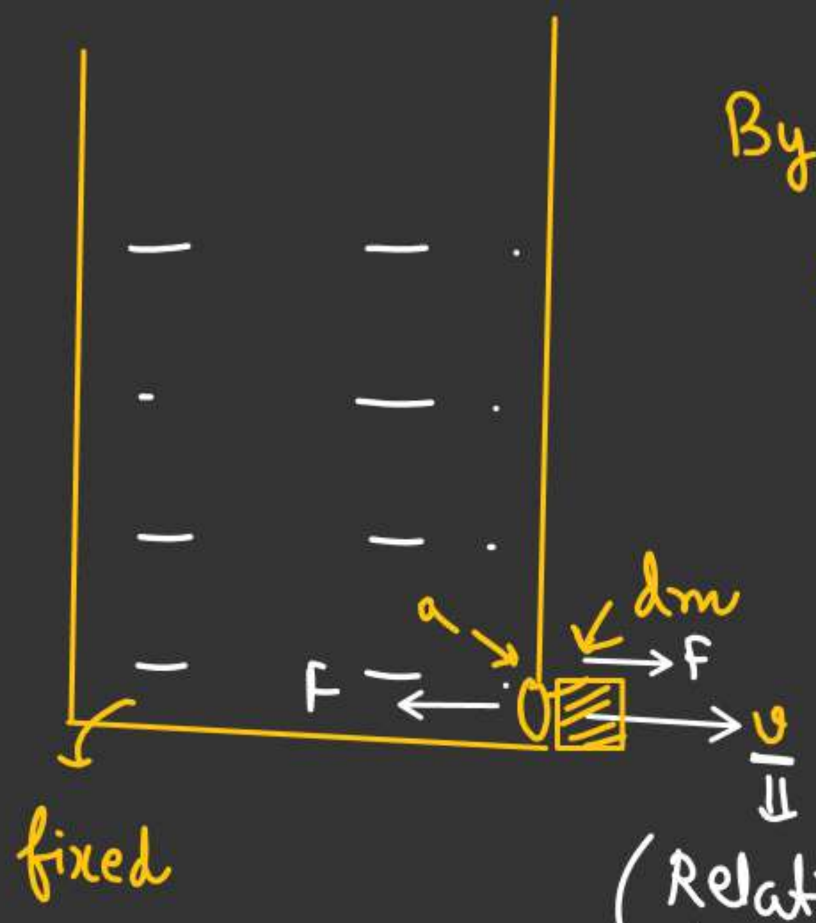
Q4:

Force acting on a vessel when liquid Coming out from a hole

$p = mv$





$$F = \frac{dp}{dt} = v\left(\frac{dm}{dt}\right)$$

By Continuity

$$\frac{dV}{dt} = \frac{1}{\rho}\left(\frac{dm}{dt}\right) = av$$

$$\frac{dm}{dt} = \rho a v$$

$$\boxed{F = \rho a v^2}$$

thrust

a = Crossection area of hole from where liquid exit

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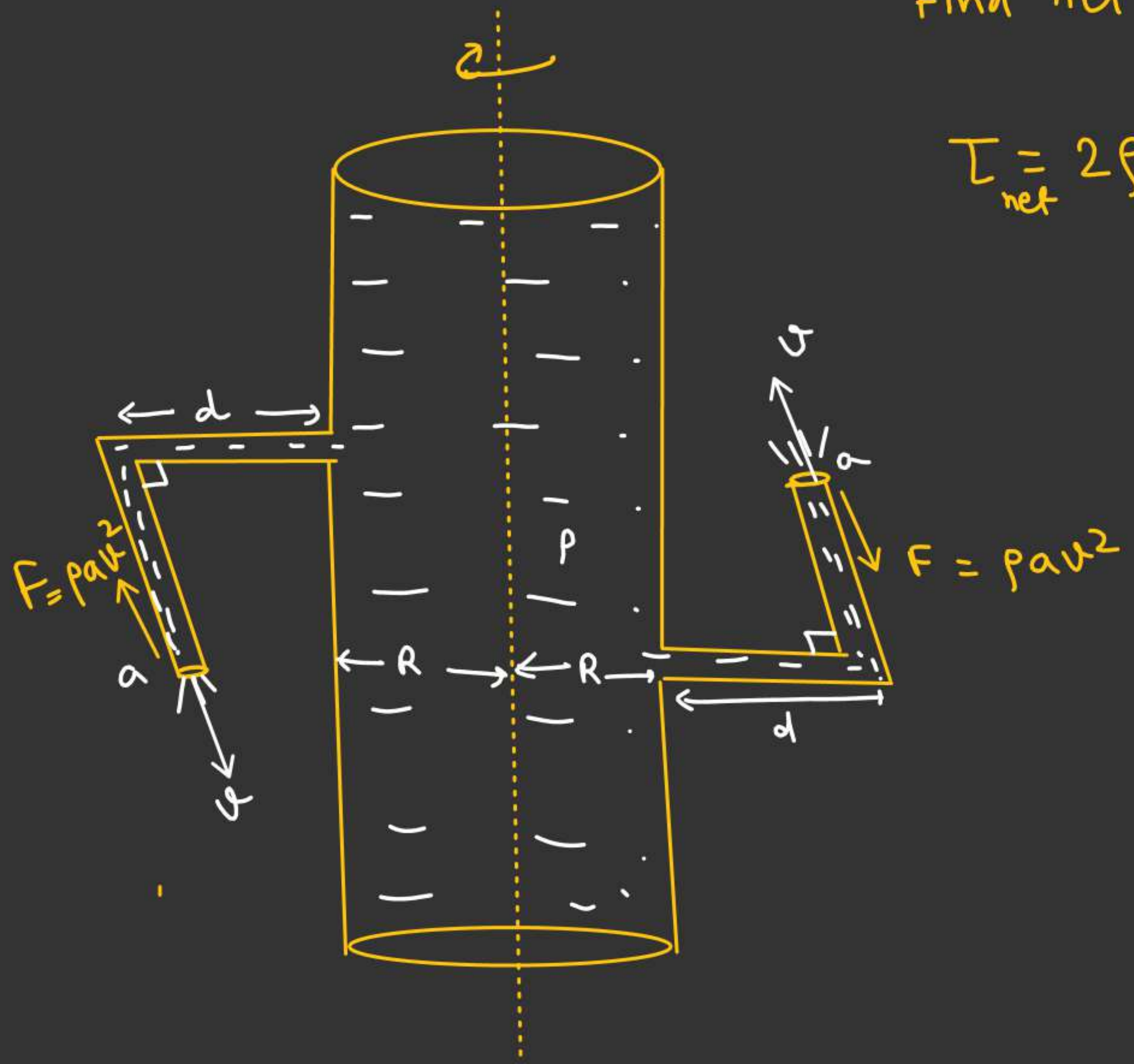
Find net torque on the vessel



$$\tau_{net} = 2\rho a v^2 (R+d)$$

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BERNOULLI'S EQUATION

(Based on Conservation of Energy)

By Work-Energy theorem.

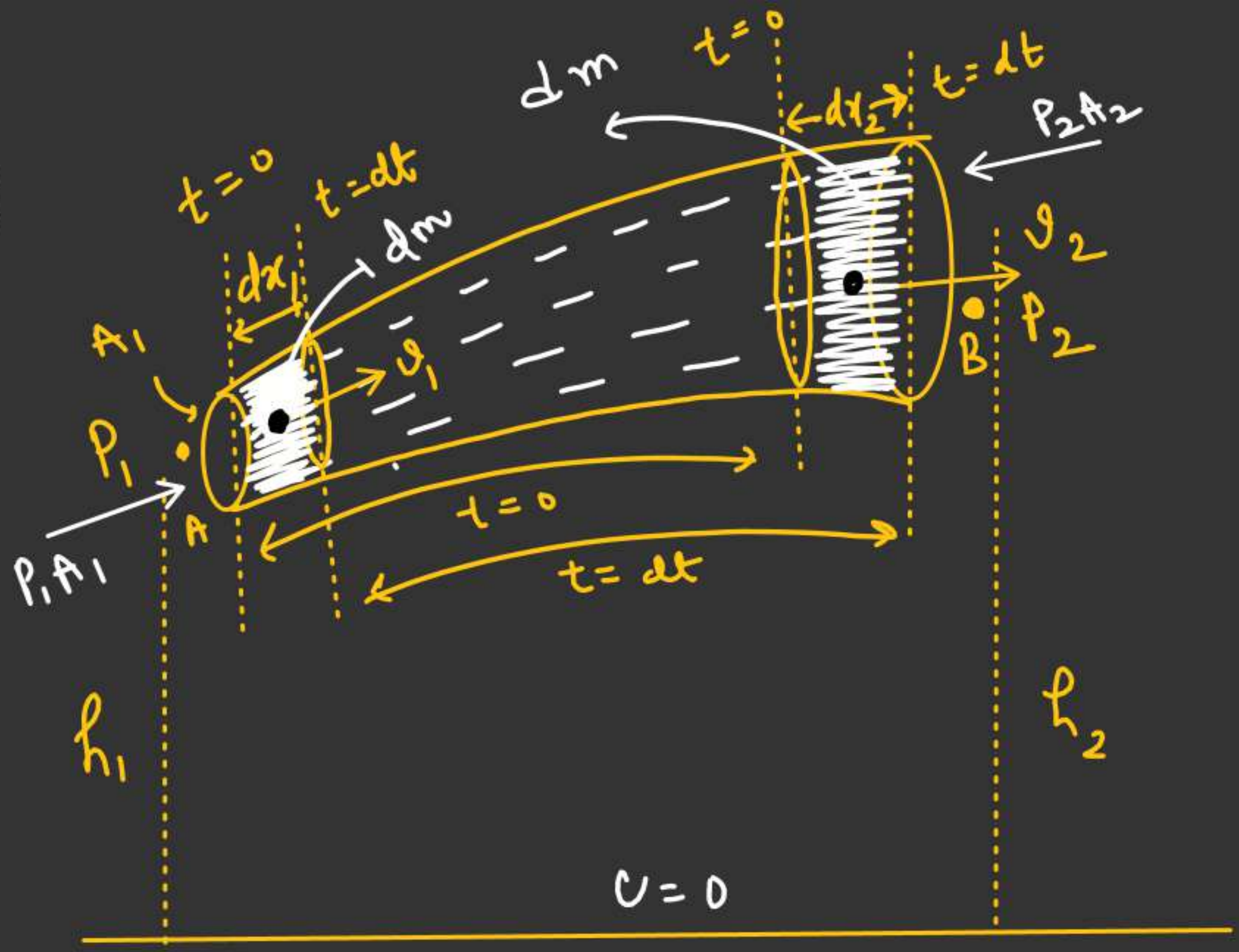
$$W_{\substack{\text{Pressure}\\ \text{force}}} + W_{\text{gravity}} = \Delta K.E$$

$$dW_{\substack{\text{pressure}\\ \text{fore}}} = P_1 A_1 dx_1 - P_2 A_2 dx_2$$

$$= P_1 \underline{A_1 v_1} dt - P_2 \underline{A_2 v_2} dt$$

$$\left(A_1 v_1 = A_2 v_2 = \frac{dV}{dt}\right)$$

$$= \underline{(P_1 - P_2) dV}$$



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$$dW_{gravity} = -dU$$
$$= dU_i - dU_f$$
$$= dmg(h_1 - h_2)$$

$$dm = (\rho dV)$$

$$dW_{gravity} = \rho g(h_1 - h_2)dV$$

$$\frac{dW_{gravity}}{dV} = \text{P.E per Unit volume}$$

$$d(K\cdot E) = \frac{1}{2}dm(v_2^2 - v_1^2)$$
$$= \frac{1}{2}\rho(v_2^2 - v_1^2)dV$$

$$\frac{d(K\cdot E)}{dV} = \frac{1}{2}\rho(v_2^2 - v_1^2)$$

$\Downarrow$

K·E per Unit volume

$$dW_{\substack{\text{pressure}\\\text{force}}} + W_{gravity} = dK\cdot E$$

$$(P_1 - P_2)dV + \rho g(h_1 - h_2)dV = \frac{1}{2}\rho(v_2^2 - v_1^2)dV$$

$$\boxed{P_1 + \rho g h_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho g h_2 + \frac{1}{2}\rho v_2^2}$$

P_1 & $P_2 \rightarrow$ Work done by pressure force per Unit Volume i.e Pressure energy per Unit Volume

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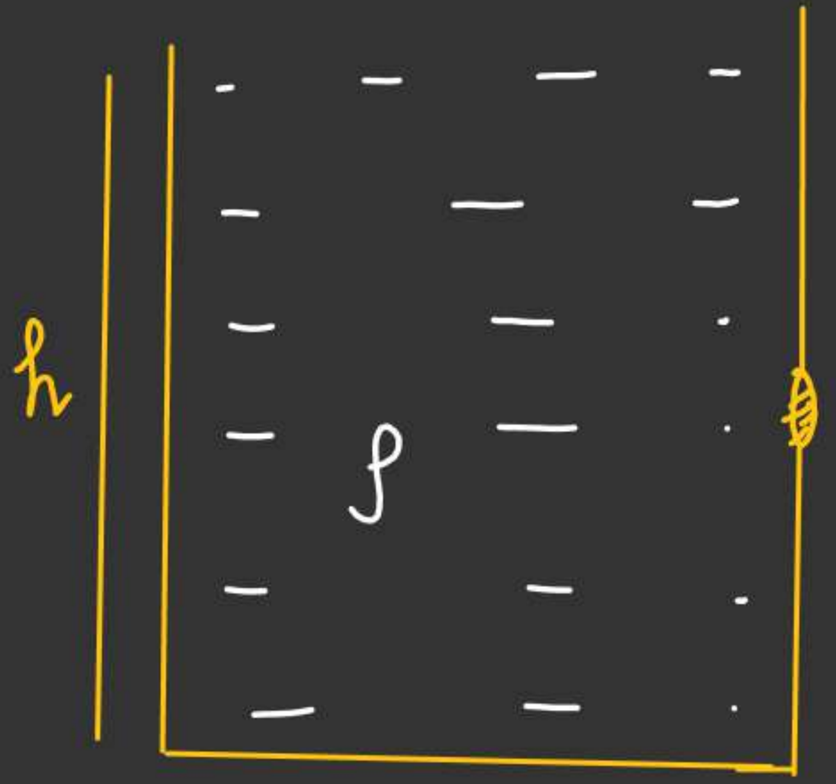
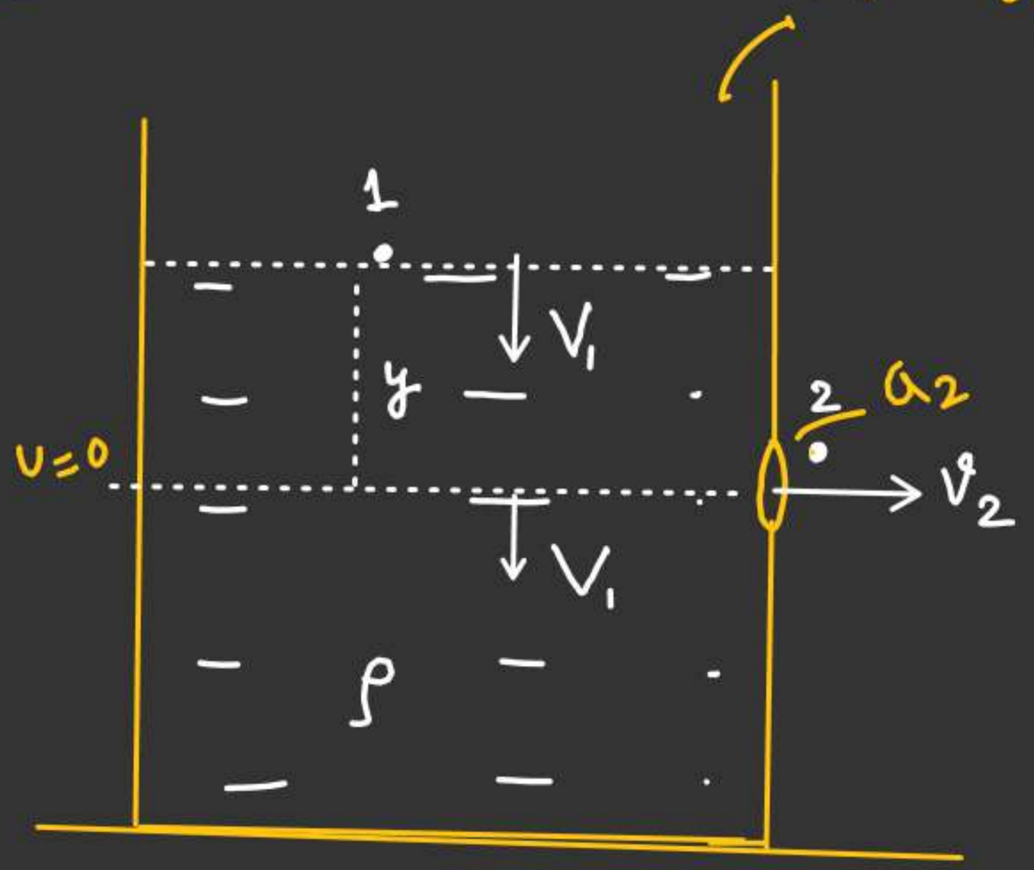
Velocity of Efflux





By Continuity

$$A_1 v_1 = a_2 v_2$$

$$v_1 = \left(\frac{a_2 v_2}{A_1}\right)$$

Bernoulli's Equation b/w 1 & 2.

$$P_1 = P_2 = \underline{P_{atm}}$$

$$\cancel{P_1} + \rho g y + \frac{1}{2}\rho v_1^2 = \cancel{P_2} + \frac{1}{2}\rho v_2^2$$

$$\frac{1}{2}\rho\left(v_2^2 - v_1^2\right) = \rho g y$$

$$v_2^2 - v_1^2 = 2gy$$

$$v_2^2\left(1 - \frac{a_2^2}{A_1^2}\right) = 2gy$$

$$v_2 = \sqrt{\frac{2gy}{\left(1 - \frac{a_2^2}{A_1^2}\right)}}$$

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Velocity of Efflux

$$v_2 = \sqrt{\frac{2gy}{\left(1 - \frac{a_2^2}{A_1^2}\right)}}$$

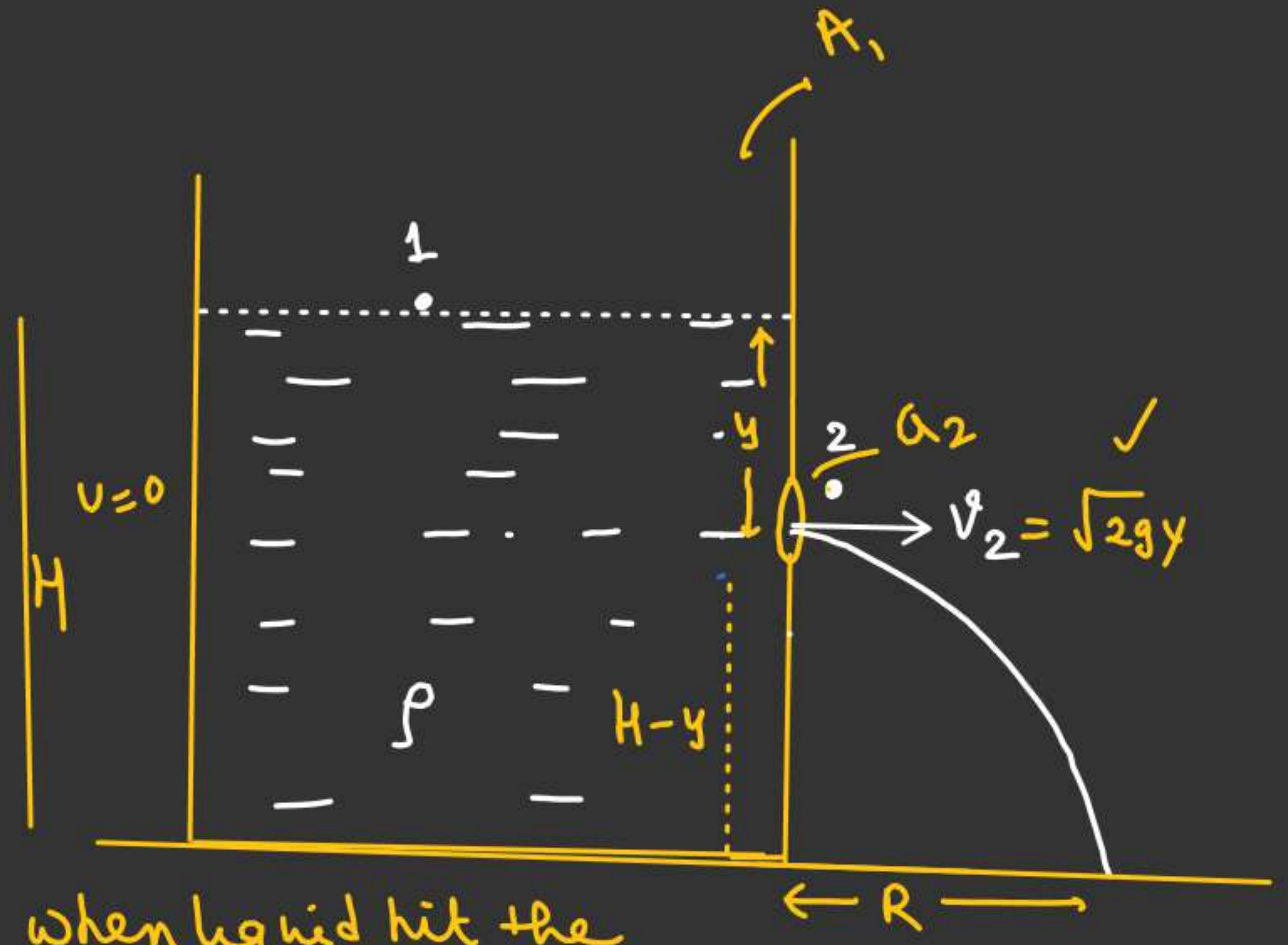
if $A_1 >> a_2$

$$\frac{a_2}{A_1} \to 0$$

$$\boxed{v_2 = \sqrt{2gy}}$$

velocity of efflux

y = depth of orifice (Small hole) from liquid surface



Time when liquid hit the ground $= \sqrt{\frac{2(H-y)}{g}}$

$$R = \sqrt{2gy} \times \sqrt{\frac{2(H-y)}{g}}$$

$$\boxed{R = 2\sqrt{y(H-y)}}$$

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For Range to be maximum.

$R = 2\sqrt{u}$

$\left(\frac{dR}{du} \times \frac{du}{dy}\right)$

$$R = 2\sqrt{\underbrace{y(H-y)}_{u}}$$

$$\frac{dR}{dy} = \cancel{2}\frac{1}{\cancel{2}\sqrt{y(H-y)}}\frac{d}{dy}(Hy-y^2)$$

$\downarrow$

$$0 = \frac{1}{\sqrt{y(H-y)}}(H-2y)$$

$$\boxed{y = \frac{H}{2}}$$

<u>Rmax</u>

$$R_{max} = 2\sqrt{\frac{H}{2}\left(H-\frac{H}{2}\right)}$$

$$\boxed{R_{max} = H}$$

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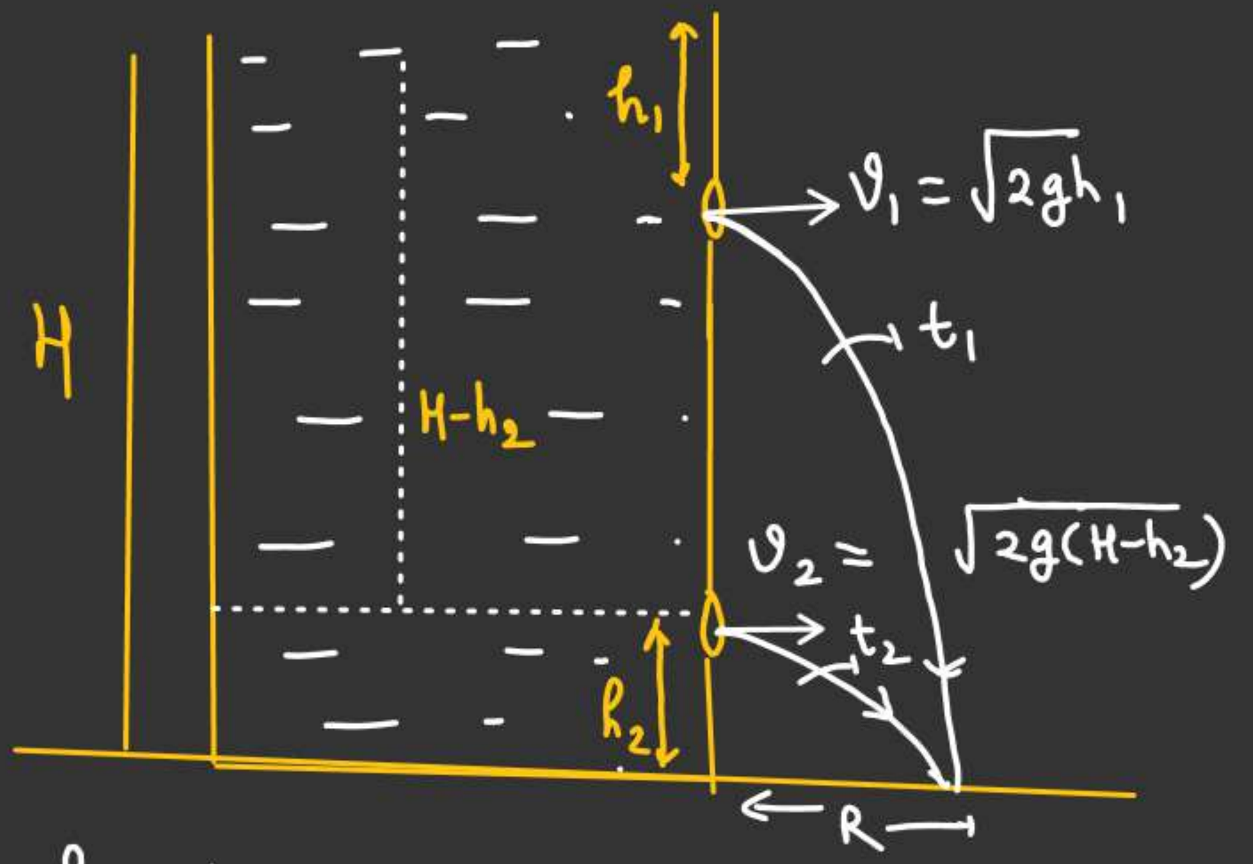
For R to be same find relation b/w h_1, h_2 & H



$$\sqrt{2gh_1} \times \sqrt{\frac{2(H-h_1)}{g}} = \sqrt{2g(H-h_2)} \times \sqrt{\frac{2h_2}{g}}$$

$$h_1(H-h_1) = h_2(H-h_2)$$

$$h_1 \cdot H - h_1^2 = h_2 H - h_2^2$$

$$(h_2^2 - h_1^2) - H(h_2 - h_1) = 0$$

$$(h_2 - h_1)\left[\underbrace{(h_1 + h_2) - H}_{\Downarrow}\right] = 0$$

Never be zero

$$h_2 - h_1 = 0$$

$$\boxed{h_2 = h_1}$$