Translational Motion	Rotational Motion.
m	I
s	θ
v	ω
a	α
f	τ

$\left(\begin{array}{c}\text{Newton's}\\ \text{1st law}\end{array}\right)$ $\displaystyle\sum_{i=1}^{n} F_i = 0$ $\longrightarrow$ $\displaystyle\sum_{i=1}^{n} \vec{\tau}_i = 0$ (Newton's 1st law)

$$\vec{p} = m\vec{v} \longrightarrow \vec{L} = I\vec{\omega}$$

$$\vec{F} = \frac{d\vec{p}}{dt} \text{ (Newton's 2nd law)} \longrightarrow \vec{\tau}_{ext} = \frac{d\vec{L}}{dt} \text{ (Newton's 2nd Law)}$$

$$\vec{F}_{ext} = 0 \Rightarrow \vec{p}_i = \vec{p}_f \longrightarrow$$
Linear Momentum
Conservation

$$\vec{\tau}_{ext} = 0, \quad \frac{d\vec{L}}{dt} = 0 \quad \left(\begin{array}{c}\text{Angular Momentum}\\ \text{Conservation}\end{array}\right)$$
$$\vec{L}_i = \vec{L}_f$$

ANGULAR IMPULSE

$$\tau_{ext} = \frac{dL}{dt}$$

$$\tau_{ext} = F_{ext} \cdot r_\perp$$

$$\int_0^{\Delta t} \tau_{ext} \cdot dt = \int_{L_p}^{L_f} dL$$

$$\text{Angular Impulse} = \int_0^{\Delta t} (F_{ext}) \, r_\perp \cdot dt$$

$$= r_\perp \int_0^{\Delta t} F_{ext} \cdot dt \quad J$$

Angular impulse = Area under τ vs t curve.

$$\Delta t = (t_f - t_i)$$

$$\left(\begin{array}{c} \text{Angular} \\ \text{Impulse} \end{array} \right) = J \, r_\perp$$

"Rod kept on a Smooth horizontal table.

M, L

COM

α

$\xrightarrow{A}$

(COM)

$\dfrac{L}{2}$

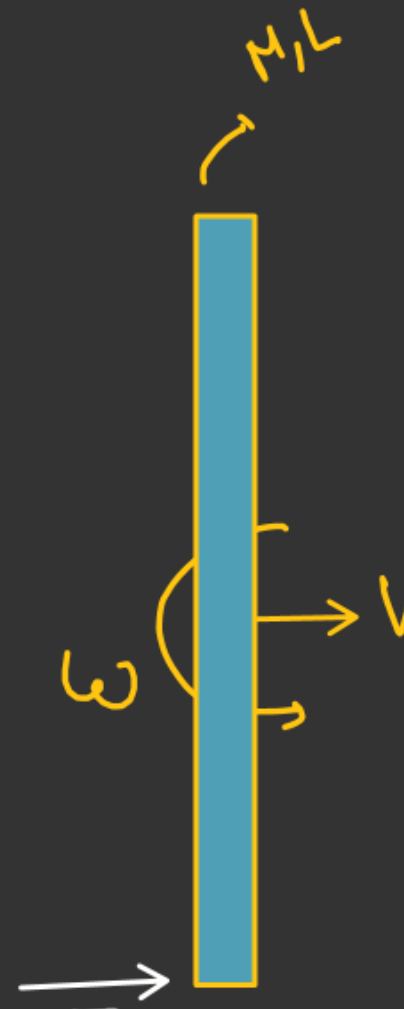$\xrightarrow{\hspace{2cm}} F$
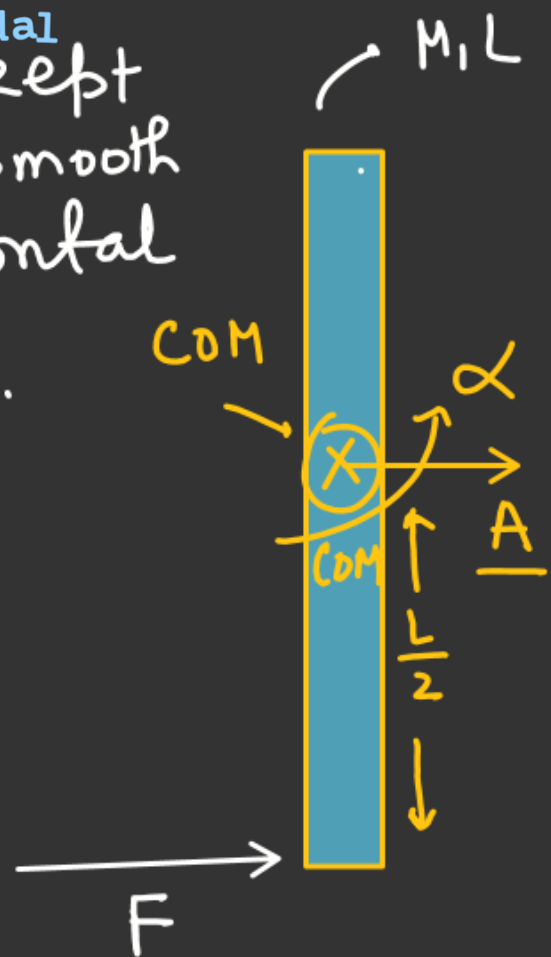
Just after F applied.

$$F = MA \quad (\text{Translational Motion})$$
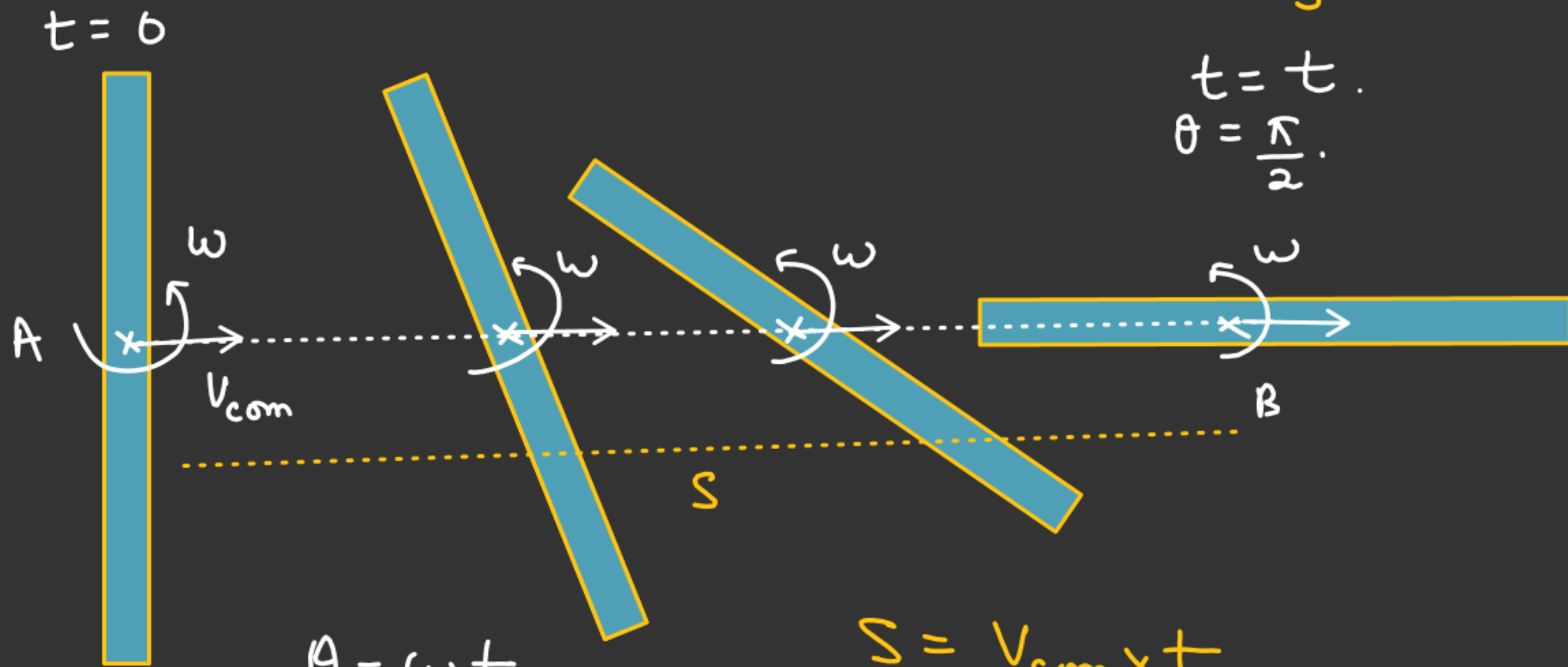
$$F\frac{L}{2} = \left(\frac{ML^2}{12}\right)\alpha$$

$$\alpha = \left(\frac{6F}{ML}\right), \quad \left(A = \frac{F}{M}\right)$$

$$\begin{cases} V_{com} = At \\ \omega = \alpha t \end{cases}$$

Rod kept on a Smooth horizontal ground ←

✓ Total distance covered by the com of the rod when rod become horizontal.

$J = MV.$

$$\boxed{V = \frac{J}{M} = \text{Const}}$$

$$J\frac{L}{2} = \frac{ML^2}{12}\cdot\omega.$$

$\rightarrow V_{com} = V$

$$J = \frac{ML}{6}\omega$$

$$\boxed{\omega = \left(\frac{6J}{ML}\right)}$$

↓

Constant.

J

$t = 0$

ω

V_{com}

A

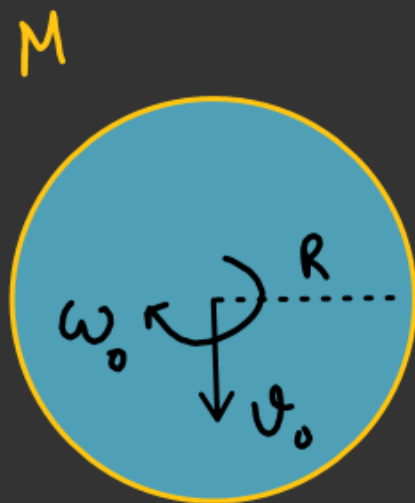B

S

$t = t.$
$\theta = \frac{\pi}{2}.$

ω

$\theta = \omega t$

$$t = \frac{\theta}{\omega} = \frac{\pi}{2\left(\frac{6J}{ML}\right)}$$

$$t = \left(\frac{\pi ML}{12J}\right)$$

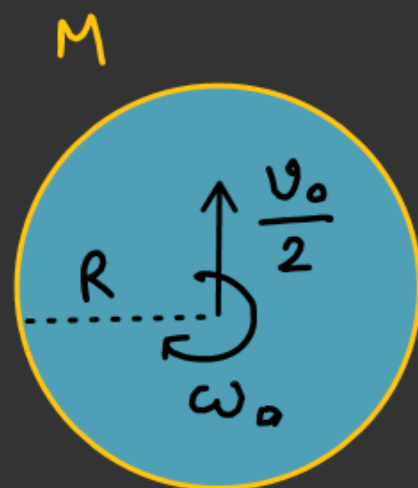$$S = V_{com} \times t$$
$$= \frac{J}{\cancel{M}} \times \frac{\pi \cancel{M} L}{12 \cancel{J}}$$
$$= \left(\frac{\pi L}{12}\right)\underline{\underline{m}} \checkmark$$

Case-1 (Ground Smooth)

M

Just
before
Collision

ω_0 ... R

$\downarrow v_0$

Just after
Collision

M

$\uparrow \dfrac{v_0}{2}$

R

ω_0
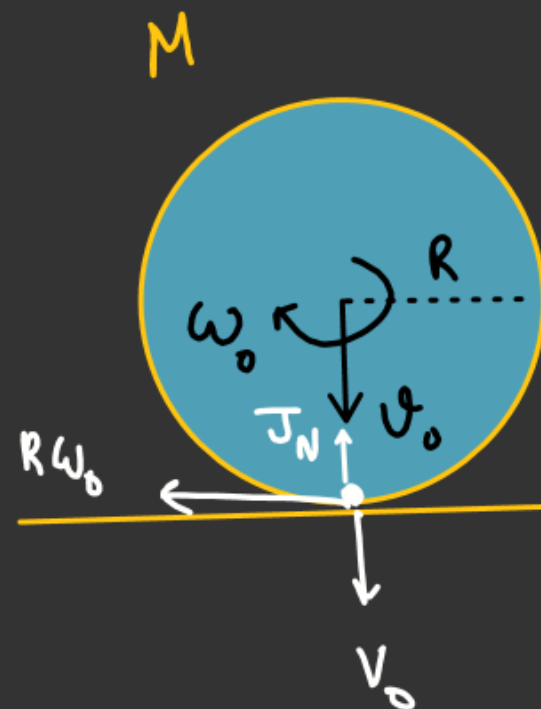
Just after Collision
disc move vertically upward
with velocity $\dfrac{v_0}{2}$.
Find angular velocity of
disc just after Collision.
and horizontal velocity of the
disc if

a) Ground is Smooth.

b) Ground is rough & $\mu = \dfrac{1}{2}$
be the coff^n of friction b/w
disc & ground.

M

ω_0 ... R

$R\omega_0 \longleftarrow$ $J_N \downarrow v_0$

$\downarrow v_0$

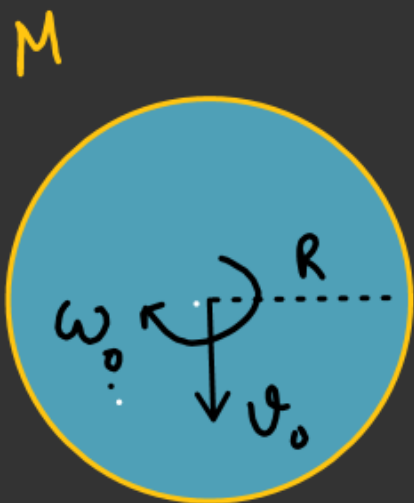$J_N = (N\Delta t) = (\Delta p)_y$

$J_N = m\dfrac{v_0}{2} - (-mv_0)$

$= \dfrac{3}{2} m v_0.$

Angular impulse due
to $J_N = 0$.
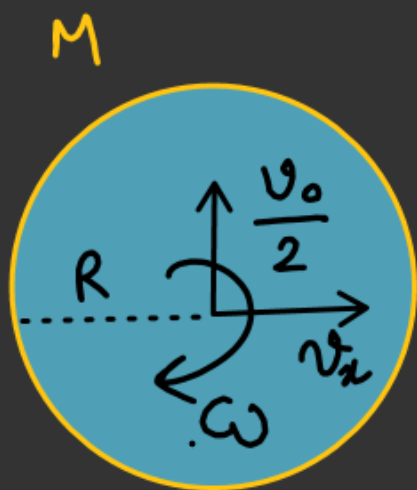So, ω_0 will not change

$e = \dfrac{\dfrac{v_0}{2}}{v_0}$

$\boxed{e = \dfrac{1}{2}}$

Case-1 (Ground Smooth)

Just before Collision

M

ω_0

R

v_0

$\omega_0 = \dfrac{v_0}{2R}$ (given)

Just after Collision ✓

M

R

$\dfrac{v_0}{2}$

v_x

ω

$J_N = \Delta p_y = \dfrac{3}{2} M v_0$

M

ω_0

R

J_N

v_0

J_{f_K}

$R\omega_0$

v_0
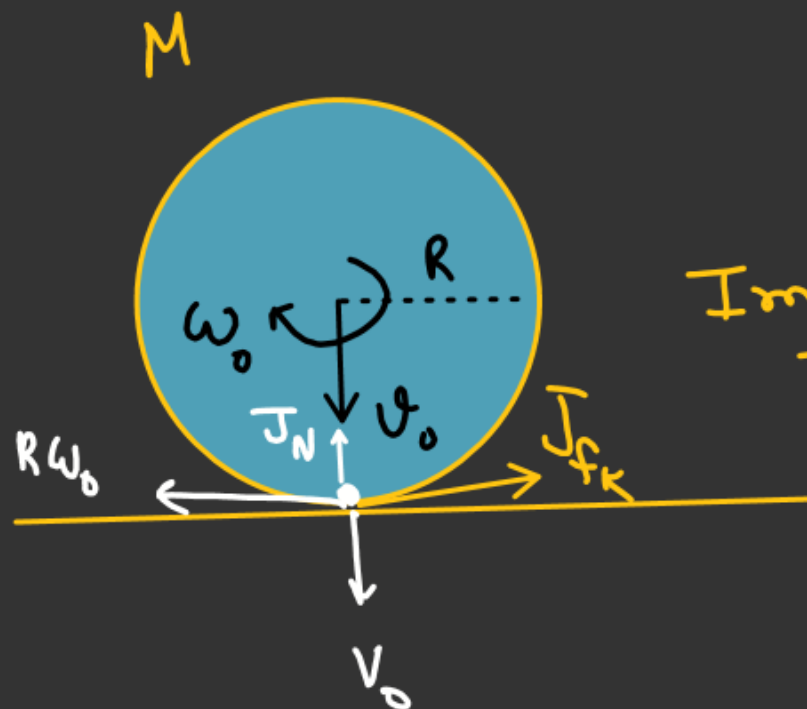
Impulsive
J_{f_K} = Angular impulse due to friction.

b) Ground is rough & $\mu = \dfrac{1}{2}$ be the coff^n of friction b/w disc & ground.

$J_{f_K} = (f_K \cdot \Delta t)$ ✓

$\tau_{J_{f_K}} = (f_K \cdot \Delta t) \cdot R$

$\quad = \mu (N \Delta t) R$

$\quad = (\mu J_N) R$

$\quad = \dfrac{1}{2} \times \dfrac{3}{2} M v_0 R$

$J_{f_K} = \mu J_N = \dfrac{3 M v_0}{4}$

$J_{f_K} = M v_x$

$v_x = \dfrac{J_{f_K}}{M} = \boxed{\dfrac{3 v_0}{4}}$ ←

$J_{f_K} \cdot r_\perp = \Delta L$

$\dfrac{3}{4} M v_0 R = I\omega - I\omega_0$

$\dfrac{3}{4} M v_0 R = I(\omega - \omega_0)$

$\dfrac{3}{4} \cancel{M} v_0 \cancel{R} = \dfrac{\cancel{M} R^{\cancel{2}}}{2} (\omega - \omega_0)$

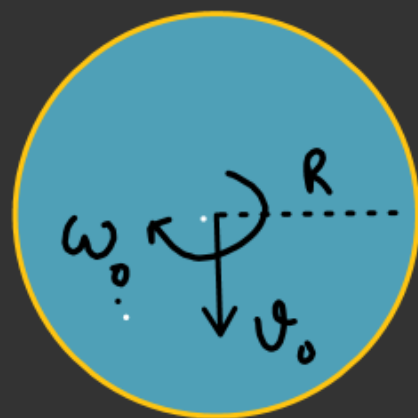$\boxed{\dfrac{3 v_0}{2R} + \omega_0 = \omega}$

$\omega = \dfrac{3 v_0}{2R} + \dfrac{v_0}{2R}$

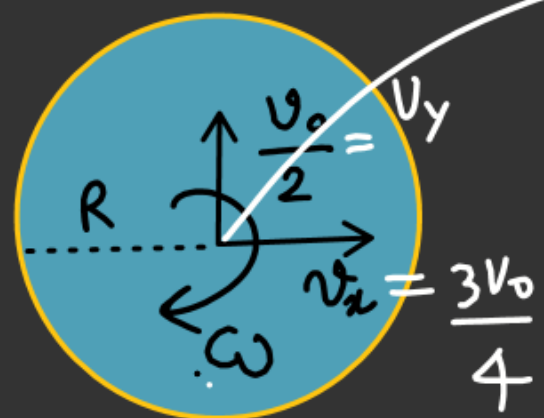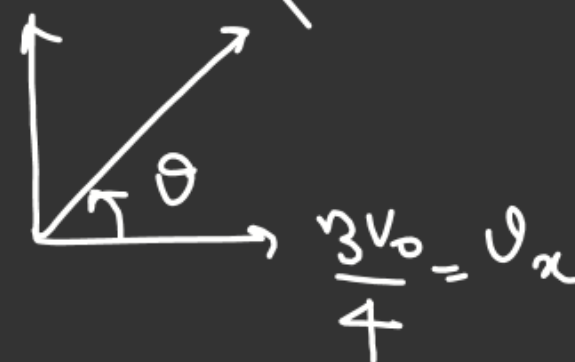$\omega = \left(\dfrac{2 v_0}{R}\right)$ ✓

Case-1 (Ground Smooth)

M

M

Just
before
Collision

$$\omega_0 = \frac{v_0}{2R} \text{ (given)}$$

$$R$$

$$\omega_0$$

$$v_0$$

$$\frac{v_0}{2} = v_y$$

$$R$$

$$v_x = \frac{3v_0}{4}$$

$$\omega$$

$$\frac{v_0}{2} = v_y$$

$$\frac{3v_0}{4} = v_x$$

$$\theta$$

$$v_x = \frac{3v_0}{4}$$

$$v_y = \frac{v_0}{2}$$

$$T = ?$$

$$H = ?$$

$$R = ?$$

$$\tan\theta = \frac{v_y}{v_x} = \left(\frac{v_0}{2} \times \frac{4}{3v_0}\right)$$

$$\tan\theta = \left(\frac{2}{3}\right)$$

$$\theta = \tan^{-1}\left(\frac{2}{3}\right) \checkmark$$