

$$\text{Q} \quad J = \int_0^{\pi/2} \frac{x \sin(x)}{\sin^4 x + \cos^4 x} dx$$

Ans

$$= \frac{\pi}{4} \int_0^{\pi/2} \frac{\sin x}{\sin^4 x + \cos^4 x} dx \quad \div (\cos^4 x)$$

$$= \frac{\pi}{4} \int_0^{\pi/2} \frac{\tan x \cdot \sec^2 x}{1 + (\tan^2 x)^2} dx$$

$$\begin{aligned} \tan^2 x &= t \\ 2 \tan x \cdot \sec^2 x dx &= dt \end{aligned}$$

$$\begin{aligned} &= \frac{\pi}{4} \int_0^{\infty} \frac{dt}{1+t^2} = \frac{\pi}{4} \left(\tan^{-1} t \right)_0^{\infty} \\ &= \frac{\pi}{4} \left(\frac{\pi}{2} - 0 \right) = \frac{\pi^2}{16} \end{aligned}$$

Achha.

$$\text{Q} \quad I := \int_0^{\pi/4} \frac{x dx}{1 + \sin 2x + \cos 2x} = \boxed{\frac{\pi \ln 2}{16}} \text{ fmdb?}$$

Removal of x.

$$= \frac{\pi}{8} \int_0^{\pi/4} \frac{dx}{2(\cos^2 x + 2 \sin x \cos x)}$$

$20 \rightarrow 390 + 0S$

$$= \frac{\pi}{8} \int_0^{\pi/4} \frac{dx}{2(\cos^2 x)(1 + \tan x)}$$

$$= \frac{\pi}{8} \int_0^{\pi/4} \frac{\sec^2 x dx}{1 + \tan x}$$

$$= \frac{\pi}{16} \int_1^2 \frac{dt}{t} = \frac{\pi}{16} \times \left[\ln t \right]_1^2 = \boxed{\frac{\ln 2 \cdot \pi}{16}}$$

 $b=16$

$$\text{Q. } \int_0^{\frac{\pi}{2}} \frac{x dx}{\sin x (\sin x + \cos x)} = ?$$

Pichhla Q.S.

Adv

$$\text{Q If } I_1 = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} x F(x(1-x)) dx$$

$$I_2 = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} F(x(1-x)) dx$$

$$\text{K then } \frac{I_1}{I_2} = ?$$

 I_1, I_2 has one difference "x"

$$F(x(1-x)) \xrightarrow{x} F((1-x)(1-(1-x)))$$

$$= F((1-x)(x))$$

Removal of x

$$I_1 = \frac{K+1-K}{2} \int_{-K}^{K} F(x(1-x)) dx$$

$$I_1 = \frac{1}{2} I_2 \Rightarrow \frac{I_1}{I_2} = \frac{1}{2}$$

Now

$$I_1 = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} x g(x(2-x)) dx$$

$$I_2 = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} g(x(2-x)) dx$$

$$\text{then } \frac{I_1}{I_2} = \frac{1+\sec^2 t - \tan^2 t}{2}$$

Mainly
Q Value of
 $\frac{2\pi}{2\pi}$

$$I = \int_0^{2\pi} [\sin 2x (1 + \cos 3x)] dx \rightarrow A$$

$$= \int_0^{2\pi} [\sin 2(2\pi - x) (1 + \cos 3(2\pi - x))] dx$$

$$= \int_0^{2\pi} [\sin(4\pi - 2x) (1 + \cos 3(4\pi - 3x))] dx$$

$$= \int_0^{2\pi} [-\sin 2x (1 + \cos 3x)] dx \rightarrow B$$

$$\frac{I_1 - I_2}{2} = \int_{-K}^{K} -1 dx$$

$$= (-x) \Big|_0^{2K} = -2K$$

$$I = -K$$

Q. Let $f: (0, 2) \rightarrow \mathbb{R}$.

$$\text{Held} \quad f(x) = \log_2 \left(1 + \tan \frac{\pi x}{4} \right)$$

$$\text{then } \lim_{n \rightarrow \infty} \frac{2}{n} \left(f\left(\frac{1}{n}\right) + f\left(\frac{2}{n}\right) + \dots + f\left(\frac{n}{n}\right) \right) = ?$$

Properties - Property Based on.

For Pkt
of Mains/Adv.

Even / odd fxn.

①

$$\int_a^b f(x) \cdot dx = \begin{cases} 0 & f(x) = \text{odd} \\ 2 \int_0^a f(x) dx & f(x) = \text{Even} \\ \int_0^a [f(x) + f(-x)] dx & f(x) = \text{None} \end{cases}$$



$$f(x) = \text{odd}$$

$$\int_a^b f(x) \cdot dx = A + -A = 0$$

Pehchan $\rightarrow \int_{-1/2}^{1/2} f(x) \cdot dx, \int_{-1}^1 f(x) \cdot dx, \int_{-a}^a f(x) \cdot dx$

$$\int_{-k/2}^{k/2} f(x) \cdot dx, \int_{-n/\lambda}^{n/\lambda} f(x) \cdot dx$$

②



$$f(x) = \text{Even}$$

$$\int_{-a}^a f(x) \cdot dx = 2 \int_0^a f(x) \cdot dx$$

$$\int_{-1}^1 \{x^2 + 1\} \{x+1\} dx$$

$$\int_{-1}^1 \{x^2\} \{x\} + \{x^2\} \{x^2\} dx$$

$$\frac{2}{3}$$

$$2 \int_{-1}^1 \{x^2\} \{x\} dx$$

N E N O $\int f(x)dx = \int_0^a f(x)dx + \int_{-a}^0 f(-x)dx$

$$2 \int_0^1 \{x^2\} \{x\} + \{(-x)^2\} \{-x\} dx$$

$$2 \int_0^1 \{x^2\} (\{x\} + \{-x\}) dx$$

$$\boxed{\{x\} + \{-x\} = \begin{cases} 1 & x=I \\ 0 & x \neq I \end{cases}}$$

$x \in (0, 1) \rightarrow x: \text{Non Integer} = \text{Decimal No}$

$$2 \int_0^1 x^2 dx = 2 \int_0^1 x^2 dx = \frac{2}{3}$$

$$\{.9\} = .9, \{.4\} = .4$$

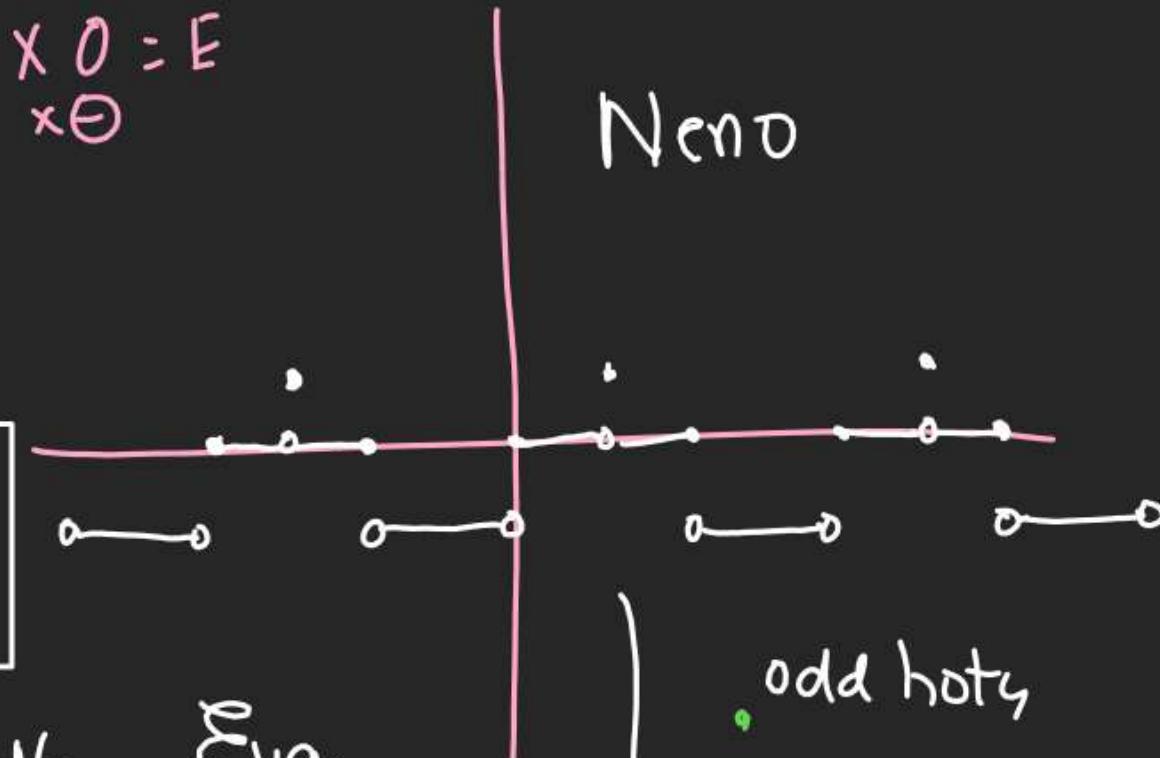
$$\int_{-1}^1 x^3 dx = ?$$

$$0 \quad 0(0)$$

$$0 \times 0 = E$$

$\bar{S} \sin x$

Nendo



Even
Hoty

odd hoty

$$\text{Q. } I = \int_{-\pi}^{\pi} [\sin x] dx$$

Noneo

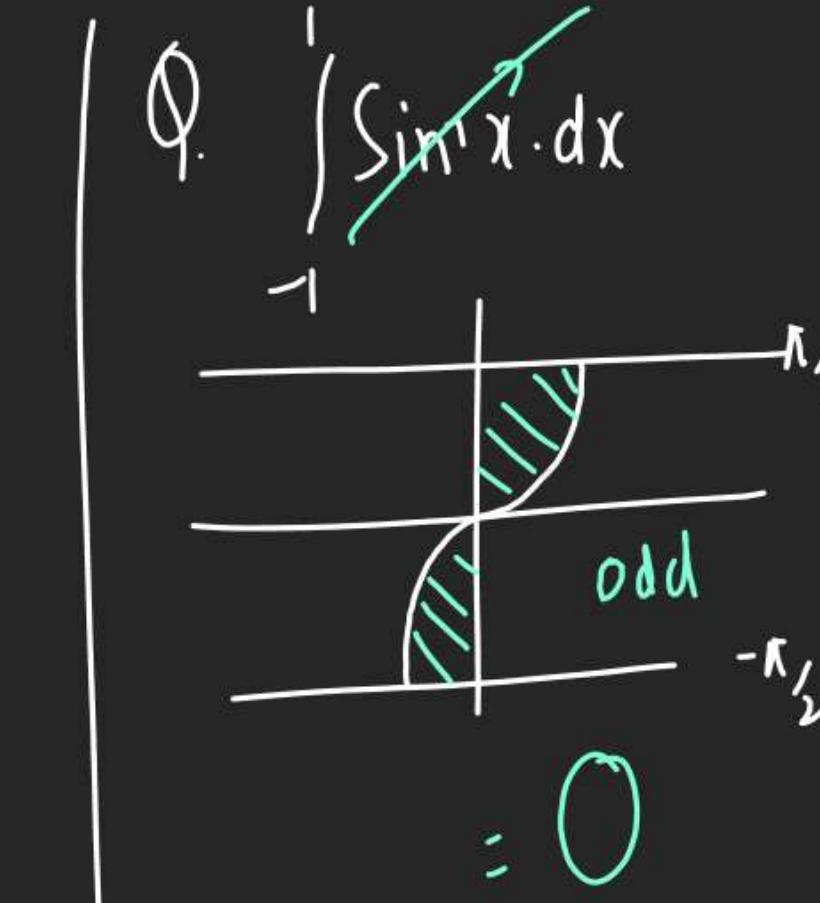
$$= \int [\sin x] + [\sin(-x)] dx$$

$$= \int [\sin x] + [-\sin(-x)] dx$$

$$I = \int -1 \cdot dx = -(\sin x) \Big|_0^\pi$$

$$I = -\pi$$

$$[\sin x] + [\sin(-x)] = \begin{cases} 0 & x \in \mathbb{I} \\ 0 & x = \mathbb{I} \end{cases}$$



$$\text{Q. } \int \sin\left(\frac{2x}{1+x^2}\right) dx$$

\mathcal{F}

$$\Rightarrow \int 2 \tan(x) dx$$

$$2 \int \tan(x) dx$$

\mathcal{F}

$\therefore 2 \times 0 = 0$

$$\text{Q. } \int_{-1/2}^{1/2} \sec x \cdot \ln\left(\frac{1-x}{1+x}\right) dx$$

$\textcircled{E} \times \text{odd} = \text{odd}$

$\textcircled{+} \times \textcircled{-} = \textcircled{-}$

$$= 0$$

$$Q \int_{-\pi}^{\pi} \frac{x^7 - 3x^5 + 3x^3 - x + 1}{(8^2)x} \cdot dx$$

$$\int_{-\pi}^{\pi} \frac{x^7}{(8^2)x} dx - \frac{3x^5}{(8^2)x} + \frac{3x^3}{(8^2)x} - \frac{x}{(8^2)x} + \frac{1}{(8^2)x} \cdot dx$$

$$= 2 \int_0^{\pi/4} \sec^2 x = 2 \times \tan x \Big|_0^{\pi/4} = 2$$

$$Q \int_{-\pi/6}^{\pi/2} \frac{8mx}{\text{Odd}} \cdot f(6x) dx = \text{Even}$$

$\text{Odd} \times \text{Even} = \Theta \times \Theta = \text{Odd}$

$$Q \int_{-1}^1 \frac{2x^{332} + x^{998} + 4x^{1668} \cdot 8m x^{691}}{(1+x^{666})} dx = ?$$

$x^{333} = t$

$$333 \int_{-1}^{333} \frac{dt}{1+t^2} + \frac{x^{333}}{333} \Big|_{-1}^1 + 4 \times 0$$

$$333 \cdot \tan t \Big|_1 + \left(\frac{1}{333} + \frac{1}{333} \right)$$

$$333 \left(\frac{\pi}{4} + \frac{\pi}{4} \right) + \frac{2}{333}$$

Prop 6

King
42 - 56 + 38 - 41

$$52 - 56 + 38 - 41$$

$\epsilon x = \text{odd}$
 $\epsilon x = \text{odd}$
 Even

$$\frac{\Theta \times \Theta}{\Theta} = \Theta$$

