## SCALAR OR DOT PRODUCT OF TWO VECTORS

**1.** If $|\vec{a}| = 5, |\vec{a} - \vec{b}| = 8$ and $|\vec{a} + \vec{b}| = 10$, then $|\vec{b}|$ is equal to

(A) 1 (B) $\sqrt{57}$ (C) 3 (D) none of these

**2.** Angle between diagonals of a parallelogram whose side are represented by $\vec{a} = 2\hat{\imath} + \hat{\jmath} + \hat{k}$ and $\vec{b} = \hat{\imath} - \hat{\jmath} - \hat{k}$

(A) $\cos^{-1}\left(\frac{1}{3}\right)$ (B) $\cos^{-1}\left(\frac{1}{2}\right)$ (C) $\cos^{-1}\left(\frac{4}{9}\right)$ (D) $\cos^{-1}\left(\frac{5}{9}\right)$

**3.** Let $\vec{a}, \vec{b}, \vec{c}$ be vectors of length 3,4,5 respectively. Let $\vec{a}$ be perpendicular to $\vec{b} + \vec{c}, \vec{b}$ to $\vec{c} + \vec{a}$ and $\vec{c}$ to $\vec{a} + \vec{b}$. Then $|\vec{a} + \vec{b} + \vec{c}|$

(A) $2\sqrt{5}$ (B) $2\sqrt{2}$ (C) $10\sqrt{5}$ (D) $5\sqrt{2}$

**4.** The value of a, for which the points A, B, C with position vectors $2\hat{\imath} - \hat{\jmath} - \hat{k}, \hat{\imath} - 3\hat{\jmath} - 5\hat{k}$ and $a\hat{\imath} - 3\hat{\jmath} - \hat{k}$ respectively are the vertices of a right-angled triangle with $C = \pi/2$ are

(A) -2 and -1 (B) -2 and 1 (C) 2 and -1 (D) 2 and 1

**5.** A particle is acted upon by constant forces $4\hat{\imath} + \hat{\jmath} - 3\hat{k}$ and $3\hat{\imath} + \hat{\jmath} - \hat{k}$ which displace it from a point $\hat{\imath} + 2\hat{\jmath} + 3\hat{k}$ to the point $5\hat{\imath} + 4\hat{\jmath} + \hat{k}$. The workdone in standard units by the force is given by

(A) 40 (B) 30 (C) 25 (D) 15

**6.** $\vec{a}, \vec{b}, \vec{c}$ are three vectors, such that $\vec{a} + \vec{b} + \vec{c} = \vec{0}, |\vec{a}| = 1, |\vec{b}| = 2, |\vec{c}| = 3$ then $\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$ is equal to

(A) 0 (B) -7 (C) 7 (D) 1

**7.** If $\vec{a} = \hat{\imath} - \hat{\jmath}, \vec{b} = \hat{\imath} + \hat{\jmath}, \vec{c} = \hat{\imath} + 3\hat{\jmath} + 5\hat{k}$ and $\vec{n}$ be a unit vector such that $\vec{b} \cdot \vec{n} = 0, \vec{a} \cdot \vec{n} = 0$ then value of $|\vec{c} \cdot \vec{n}|$ is

(A) 1 (B) 3 (C) 5 (D) 2

**8.** Given the three vectors $\vec{a} = -2\hat{\imath} + \hat{\jmath} + \hat{k}, \vec{b} = \hat{\imath} + 5\hat{\jmath}$ and $\vec{c} = 4\hat{\imath} + 4\hat{\jmath} - 2\hat{k}$. The projection of the vector $3\vec{a} - 2\vec{b}$ on the vector $\vec{c}$ is

(A) 11 (B) -11 (C) 13 (D) none of these

**9.** Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three units vectors that $3\vec{a} + 4\vec{b} + 5\vec{c} = 0$. Then which of the following statements is true?

(A) $\vec{a}$ is parallel to $\vec{b}$

(B) $\vec{a}$ is perpendicular to $\vec{b}$

(C) $\vec{a}$ is neither parallel nor perdendicular to $\vec{b}$

(D) None of these

**10.** If $a, b, c$ are pth, qth, rth terms of an H.P. and $\vec{u} = (q-r)\hat{\imath} + (r-p)\hat{\jmath} + (p-q)\hat{k}, \vec{v} = \frac{\hat{\imath}}{a} + \frac{\hat{\jmath}}{b} + \frac{\hat{k}}{c}$, then

(A) $\vec{u}, \vec{v}$ are parallel vectors
(B) $\vec{u}, \vec{v}$ are orthogonal vectors
(C) $\vec{u}, \vec{v} = 1$
(D) $\vec{u} \times \vec{v} = \hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{k}$

**11.** If the unit vectors $e_1$ and $\vec{e}_2$ are inclined at an angle $2\theta$ and $|\vec{e}_1 - \vec{e}_2| < 1$, then for $\theta \in [0, \pi], \theta$ may lie in the interval

(A) $\left[0, \frac{\pi}{6}\right)$
(B) $\left[\frac{\pi}{6}, \frac{\pi}{2}\right]$
(C) $\left(\frac{5\pi}{6}, \pi\right]$
(D) $\left[\frac{\pi}{2}, \frac{5\pi}{6}\right]$

**12.** Let $\vec{u}, \vec{v}, \vec{w}$ be such that $|\vec{u}| = 1, |\vec{v}| = 2, |\vec{w}| = 3$. If the projection $\vec{v}$ along $\vec{u}$ is equal to that of $\vec{w}$ along $\vec{u}$ and $\vec{v}, \vec{w}$ are perpendicular to each other, then $|\vec{u} - \vec{v} + \vec{w}|$ equals

(A) 2
(B) $\sqrt{7}$
(C) $\sqrt{14}$
(D) 14

**13.** Let $\vec{u} = \hat{\imath} + \hat{\jmath}, \vec{v} = \hat{\imath} - \hat{\jmath}$ and $\vec{w} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k}$. If $\hat{n}$ is a unit vector such that $\vec{u} \cdot \hat{n} = 0$ and $\vec{v} \cdot \hat{n} = 0$ then $|\vec{w} \cdot \hat{n}|$ is equal to

(A) 0
(B) 1
(C) 2
(D) 3

**14.** In a quadrilateral ABCD $\cdot$ $\overrightarrow{AC}$ is the bisector of $\overline{AB}$ and $\overline{AD}$, angle between $\overline{AB}$ and $\overline{AD}$ is $2\pi/3$, $15|\overrightarrow{AC}| = 3|\overrightarrow{AB}| = 5|\overline{AD}|$. Then the angle between $\overline{BA}$ and $\overline{CD}$ is

(A) $\cos^{-1} \frac{\sqrt{14}}{7\sqrt{2}}$
(B) $\cos^{-1} \frac{\sqrt{21}}{7\sqrt{3}}$
(C) $\cos^{-1} \frac{2}{\sqrt{7}}$
(D) $\cos^{-1} \frac{2\sqrt{7}}{14}$

**15.** The vector $\vec{c}$, directed along the external bisector of the angle between the vectors $\vec{a} = 7\hat{\imath} - 4\hat{\jmath} + 4\hat{k}$ and $\vec{b} = 2\hat{\imath} - \hat{\jmath} + 2\hat{k}$ with $|\vec{c}| = 5\sqrt{6}$, is

(A) $(5\hat{\imath} - 5\hat{\jmath} - 10\hat{k})$
(B) $\frac{5}{3}(\hat{\imath} + 7\hat{\jmath} - 2\hat{k})$
(C) $-(5\hat{\imath} - 5\hat{\jmath} - 10\hat{k})$
(D) $\frac{5}{3}(-\hat{\imath} + 7\hat{\jmath} + 2\hat{k})$

**16.** If $\vec{z}_1 = a\hat{\imath} + b\hat{\jmath}$ and $\vec{z}_2 = c\hat{\imath} + d\hat{\jmath}$ are two vectors in $\hat{\imath}$ and $\hat{\jmath}$ system, where $|\vec{z}_1| = |\vec{z}_2| = r$ and $\vec{z}_1, \vec{z}_2 = 0$, then $\vec{w}_1 = a\hat{\imath} + c\hat{\jmath}$ and $\vec{w}_2 = b\hat{\imath} + d\hat{\jmath}$ satisfy

(A) $|\vec{w}_1| = r$ (B) $|\vec{w}_2| = r$ (C) $\vec{w}_1 \cdot \vec{w}_2 = 0$ (D) none of these

**17.** If $\vec{a}\vec{b}$ are two non-collinear unit vectors and $\vec{a}, \vec{b}, x\vec{a} - y\vec{b}$ form a triangle, then

(A) $x = -1; y = 1$ and $|\vec{a} + \vec{b}| = 2\cos\left(\frac{\vec{a}\wedge\vec{b}}{2}\right)$

(B) $x = -1; y = 1$ and $\cos(\vec{a}\wedge\vec{b}) + |\vec{a} + \vec{b}| \cos\left(\vec{a}\wedge - (\vec{a} + \vec{b})\right) = -1$

(C) $|\vec{a} + \vec{b}| = 2\cot\left(\frac{\vec{a}\wedge\vec{b}}{2}\right)\cos\left(\frac{\vec{a}\wedge\vec{b}}{2}\right)$ and $x = 1, y = 1$

(D) none of these

**18.** The value(s) of $\alpha \in [0,2\pi]$ for which vector $\vec{a} = \hat{\imath} + 3\hat{\jmath} + (\sin\ 2\alpha)\hat{k}$ makes an obtuse angle with the z-axis and the vectors

$\vec{b} = (\tan\ \alpha)\hat{\imath} - \hat{\jmath} + 2\sqrt{\sin\frac{\alpha}{2}}\hat{k}$ and

$\vec{c} = (\tan\ \alpha)\hat{\imath} + (\tan\ \alpha)\hat{\jmath} - 3\sqrt{\operatorname{cosec}\frac{\alpha}{2}}\hat{k}$ are

orthogonal, is/are

(A) $\tan^{-1}\ 3$ (B) $\pi - \tan^{-1}\ 2$ (C) $\pi + \tan^{-1}\ 3$ (D) $2\pi - \tan^{-1}\ 2$

**19.** (ii) Prove that $\left(\frac{\vec{a}}{\vec{a}^2} - \frac{\vec{b}}{\vec{b}^2}\right)^2 = \left(\frac{\vec{a}-\vec{b}}{|\vec{a}||\vec{b}|}\right)^2$

**20.** Given that $\vec{x} + \frac{1}{\vec{p}^2}(\vec{p}\cdot\vec{x})\vec{p} = \vec{q}$, then show that $\vec{p}\cdot\bar{x} = \frac{1}{2}(\vec{p}\cdot\vec{q})$ and find $\vec{x}$ in terms of $\vec{p}$ and $\vec{q}$.

**21.** The position vectors of the angular points of a tetrahedron are $A(3\hat{\imath} - 2\hat{\jmath} + \hat{k})$, $B(3\hat{\mathbf{i}} + \hat{\mathbf{j}} + 5\hat{\mathbf{k}})$, $C(4\hat{\imath} + \hat{k})$ and $D(\hat{\imath})$. Then find the acute angle between the lateral faces ADC and the base ABC.

**22.** The resultant of two vectors $\vec{a}\&\vec{b}$ is perpendicular to $\vec{a}$. If $|\vec{b}| = \sqrt{2}|\vec{a}|$ show that the resultant of $2\vec{a}\&\vec{b}$ is perpendicular to $\vec{b}$.

**23.** (i) Let $\vec{A} = 2\hat{\imath} + \hat{k}, \vec{B} = \hat{\imath} + \hat{\jmath} + \hat{k}$ and $\vec{C} = 4\hat{\imath} - 3\hat{\jmath} + 7\hat{k}$.

Determine a vector $\vec{R}$ satisfying $\vec{R} \times \vec{B} = \vec{C} \times \vec{B}$ and $\vec{R}\cdot\vec{A} = 0$

(ii) Find vector $\vec{v}$ which is coplanar with the vectors $\hat{\imath}+\hat{\jmath}-2\hat{k}$ and $\hat{\imath}-2\hat{\jmath}+\hat{k}$ and is orthogonal to the vector $-2\hat{\imath}+\hat{\jmath}+\hat{k}$. It is given that the projection of $\vec{v}$ along the vector $\hat{\mathbf{i}}-\hat{\jmath}+\hat{k}$ is equal to $6\sqrt{3}$.

**24.** The length of the edge of the regular tetrahedron DABC is ' $a$ '. Point E and F are taken on the edges AD and BD respectively such that E divides $\overrightarrow{DA}$ and $F$ divides $\overrightarrow{BD}$ in the ratio 2: 1 each. Then find the area of triangle CEF.

**25.** ABCD is a tetrahedron with pv's of its angular points as $A(-5,22,5)$; $B(1,2,3)$; $C(4,3,2)$ and $D(-1,2,-3)$. If the area of the triangle AEF where the quadrilaterals ABDE and ABCF are parallelograms is $\sqrt{S}$ then find the value of S.

**26.** The pv's of the four angular points of a tetrahedron are : $A(\hat{\jmath}+2\hat{k})$; $B(3\hat{\imath}+\hat{k})$; $C(4\hat{\imath}+3\hat{\jmath}+6\hat{k})$& $D(2\hat{\imath}+3\hat{\jmath}+2\hat{k})$. Find :

(i) The perpendicular distance from A to the line BC.

(ii) The volume of the tetrahedron ABCD.

(iii) The perpendicular distance from D to the plane ABC.

(iv) The shortest distance between the lines AB&CD.

**MATRIX MATCH TYPE**

**27.** Observe the following columns:

| Column-I | Column-II |
|---|---|
| **(A)** If $V_1$, $V_2$, $V_3$ are the volumes of parallelopiped, triangular prism and tetrahedron respectively. The three coterminous edges of all three figures are the vectors $\hat{\imath}-\hat{\jmath}-6\hat{k}, \hat{\imath}-\hat{\jmath}+4\hat{k}$ and $2\hat{\imath}-5\hat{\jmath}+3\hat{k}$, then | **(P)** $2V_1+3V_3=5V_2$ |
| **(B)** If $V_1$, $V_2$, $V_3$ are the volumes of parallelopiped, triangular prism and tetrahedron respectively. The three coterminous edges of all three figures are the vectors $-2\hat{\imath}+3\hat{\jmath}-3\hat{k}, 4\hat{\imath}+5\hat{\jmath}-3\hat{k}$ and $6\hat{\imath}+2\hat{\jmath}-3\hat{k}$, then | **(Q)** $V_1+V_2+V_3=60$ |
| | **(R)** $V_1+3V_3=3V_2$ |

**(C)** If $V_1$, $V_2$, $V_3$ are the volumes of parallelopiped, triangular prism and tetrahedron respectively. The three coterminous edges of all three figures are the vectors $-3\hat{\imath}+\hat{\jmath}+\hat{k}, 4\hat{\imath}+2\hat{\jmath}+4\hat{k}$ and $2\hat{\imath}+2\hat{\jmath}$, then

**(S)** $V_1+V_2+V_3=50$

**(T)** $V_1 : V_2 : V_3 = 6:3:1$

**28.** Let $\vec{a}=\hat{\imath}+2\hat{\jmath}+3\hat{k}, \vec{b}=2\hat{\imath}-\hat{\jmath}+\hat{k}, \vec{c}=3\hat{\imath}+2\hat{\jmath}+\hat{k}$ and $\vec{d}=3\hat{\imath}-\hat{\jmath}-2\hat{k}$, then

(i) If $\vec{a}\times(\vec{b}\times\vec{c})=p\vec{a}+q\vec{b}+r\vec{c}$, then find value of $p, q$ are $r$.

(ii) Find the value of $(\vec{a}\times\vec{b})\times(\vec{a}\times\vec{c})\cdot\vec{d}$

**29.** The vector $\overrightarrow{OP}=\hat{\imath}+2\hat{\jmath}+2\hat{k}$ turns through a right angle, passing through the positive x-axis on the way. Find the vector in its new position.

**30.** If $p\vec{x}+(\vec{x}\times\vec{a})=\vec{b}; (p\neq 0)$

prove that $\vec{x}=\frac{p^2\vec{b}+(\vec{b}\cdot\vec{a})\vec{a}-p(\vec{b}\times\vec{a})}{p(p^2+a^2)}$

**VECTOR OR CROSS PRODUCT OF TWO VECTORS**

**31.** Let $\vec{a}=\hat{\imath}+\hat{\jmath}$ and $\vec{b}=2\hat{\imath}-\hat{k}$. The point of intersection of the lines $\vec{r}\times\vec{a}=\vec{b}\times\vec{a}$ and $\vec{r}\times\vec{b}=\vec{a}\times\vec{b}$ is

(A) $-\hat{\imath}+\hat{\jmath}+2\hat{k}$ (B) $3\hat{\imath}-\hat{\jmath}+\hat{k}$ (C) $3\hat{\imath}+\hat{\jmath}-\hat{k}$ (D) $\hat{\imath}-\hat{\jmath}-\hat{k}$

**32.** Vector $\vec{a}$ and $\vec{b}$ make an angle $\theta=\frac{2\pi}{3}$, if $|\vec{a}|=1, |\vec{b}|=2$, then $\{(\vec{a}+3\vec{b})\times(3\vec{a}-\vec{b})\}^2$ is equal to

(A) 225 (B) 250 (C) 275 (D) 300

**33.** Unit vector perpendicular to the plane of the triangle ABC with position vectors $\vec{a}, \vec{b}, \vec{c}$ of the vertices $A, B, C$ is

(A) $\frac{(\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a})}{\Delta}$ (B) $\frac{(\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a})}{2\Delta}$ (C) $\frac{(\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a})}{4\Delta}$ (D) none of these

**34.** If $\vec{b}$ and $\vec{c}$ are two non-collinear vectors such that $\vec{a}\parallel(\vec{b}\times\vec{c})$, then $(\vec{a}\times\vec{b})\cdot(\vec{a}\times\vec{c})$ is equal to

(A) $\vec{a}^2(\vec{b}\cdot\vec{c})$ (B) $\vec{b}^2(\vec{a}\cdot\vec{c})$ (C) $\vec{e}^2(\vec{a},\vec{b})$ (D) none of these

**35.** Vector of length 3 unit which is perpendicular to $\hat{\imath}+\hat{\jmath}+\hat{k}$ and lies in the plane of $\hat{\imath}+\hat{\jmath}+\hat{k}$ and $2\hat{\imath}-3\hat{\jmath}$

(A) $\frac{3}{\sqrt{6}}(\hat{\mathrm{i}}-2\hat{\mathrm{j}}+\hat{\mathrm{k}})$ (B) $\frac{3}{\sqrt{6}}(2\hat{\mathrm{i}}-\hat{\mathrm{j}}-\hat{\mathrm{k}})$

(C) $\frac{3}{\sqrt{114}}(8\hat{\mathrm{i}}-7\hat{\mathrm{j}}-\hat{\mathrm{k}})$ (D) $\frac{3}{\sqrt{114}}(-7\hat{\mathrm{i}}+8\hat{\mathrm{j}}-\hat{\mathrm{k}})$

**36.** Given the vertices $A(2,3,1), B(4,1,-2), C(6,3,7)$ & $\mathrm{D}(-5,-4,8)$ of a tetrahedron. The length of the altitude drawn from the vertex D is

(A) 7 (B) 9 (C) 11 (D) none of these

**37.** For a non zero vector $\vec{A}$ If the equations $\vec{A}\cdot\vec{B}=\vec{A}\cdot\vec{C}$ and $\vec{A}\times\vec{B}=\vec{A}\times\vec{C}$ hold simultaneously, then

(A) $\vec{A}$ is perpendicular to $\vec{B}-\vec{C}$ (B) $\vec{A}=\vec{B}$

(C) $\vec{B}=\vec{C}$ (D) $\vec{C}=\vec{A}$

**38.** If $u$ and $v$ are unit vectors and $\theta$ is the acute angle between them, then $2u\times 3v$ is a unit vector for

(A) Exactly two values of $\theta$ (B) More than two values of $\theta$

(C) No value of $\theta$ (D) Exactly one value of $\theta$

**39.** If $\vec{u}=\vec{a}-\vec{b}, \vec{v}=\vec{a}+\vec{b}$ and $|\vec{a}|=|\vec{b}|=2$, then $|\vec{u}\times\vec{v}|$ is equal to

(A) $\sqrt{2\left(16-(\vec{a}\cdot\vec{b})^2\right)}$ (B) $2\sqrt{\left(16-(\vec{a}\cdot\vec{b})^2\right)}$

(C) $2\sqrt{\left(4-(\vec{a}\cdot\vec{b})^2\right)}$ (D) $\sqrt{2\left(4-(\vec{a}\cdot\vec{b})^2\right)}$

**40.** If $A(1,1,1), C(2,-1,2)$, the vector equation of the line $\overline{AB}$ is $\vec{r}=(\hat{\mathrm{i}}+\hat{\mathrm{j}}+\hat{\mathrm{k}})+t(6\hat{\mathrm{i}}-3\hat{\mathrm{j}}+2\hat{\mathrm{k}})$ and di the shortest distance of the point C from $\overline{AB}$, then

(A) $B(6,-3,2)$ (B) $B(5,-4,1)$

(C) $d=\sqrt{2}$ (D) $d=\sqrt{6}$

**41.** If $\vec{a}=\vec{b}+\vec{c}, \vec{b}\times\vec{d}=0$ and $\vec{c}\cdot\vec{d}=0$ then $\frac{\vec{d}\times(\vec{a}\times\vec{d})}{d^2}$ is equal to

(A) a (B) $\vec{b}$ (C) c (D) $\vec{d}$

**42.** Consider a tetrahedron with faces $f_1, f_2, f_3, f_4$ Let $\vec{a}_1, \vec{a}_2, \vec{a}_3, \vec{a}_4$ be the vectors whose magnitudes are respectively equal to the areas of $f_1, f_2, f_3, f_4$ and whose directions are perpendicular to these faces in the outward direction. Then

(A) $\vec{a}_1 + \vec{a}_2 + \vec{a}_3 + \vec{a}_4 = 0$ (B) $\vec{a}_1 + \vec{a}_3 = \vec{a}_2 + \vec{a}_4$

(C) $\vec{a}_1 + \vec{a}_2 = \vec{a}_3 + \vec{a}_4$ (D) none of these

**43.** Let $\vec{a} = \hat{\imath} + \hat{\jmath} + \hat{k}, \vec{b} = \hat{\imath} - \hat{\jmath} + 2\hat{k}$ and $\vec{c} = x\hat{\imath} + (x-2)\hat{\jmath} - \hat{k}$. If the vector $\vec{c}$ lies in the plane of $\vec{a}$ and $\vec{b}$, then $x$ equals

(A) 0 (B) 1 (C) -4 (D) -2

**44.** Points L, M and N lie on the sides AB, BC and CA of the triangle ABC such that $\ell$ (AL) : $\ell$ (LB) = $\ell(\text{BM}) : \ell(\text{MC}) = \ell(\text{CN}) : \ell(\text{NA}) = \text{m} : \text{n}$, then the areas of the triangles LMN and ABC are in the ratio

(A) $\frac{m^2}{n^2}$ (B) $\frac{m^2 - mn + n^2}{(m+n)^2}$ (C) $\frac{m^2 - n^2}{m^2 + n^2}$ (D) $\frac{m^2 + n^2}{(m+n)^2}$

**45.** For any four points P, Q, R, S, $|\overline{PQ} \times \overline{RS} - \overline{QR} \times \overline{PS} + \overline{RP} \times \overline{QS}|$ is equal to 4 times the area of the triangle

(A) PQR (B) QRS (C) PRS (D) PQS

**SCALAR TRIPLE PRODUCT**

**46.** The value of $[(\vec{a} + 2\vec{b} - \vec{c}), (\vec{a} - \vec{b}), (\vec{a} - \vec{b} - \vec{c})]$ is equal to the box product

(A) $[\vec{a}\vec{b}\vec{c}]$ (B) $2[\vec{a}\vec{b}\vec{c}]$ (C) $3[\vec{a}\vec{b}\vec{c}]$ (D) $4[\vec{a}\vec{b}\vec{c}]$

**47.** The volume of the parallelopiped constructed on the diagonals of the faces of the given rectangular parallelopiped is m times the volume of the given parallelopiped. Then m is equal to

(A) 2 (B) 3 (C) 4 (D) none of these

**48.** If $\vec{u}, \vec{v}$ and $\vec{w}$ are three non-coplanar vectors, then $(\vec{u} + \vec{v} - \vec{w}) \cdot [(\vec{u} - \vec{v}) \times (\vec{v} - \vec{w})]$ equals

(A) 0 (B) $\vec{u} \cdot \vec{v} \times \vec{w}$ (C) $\vec{u} \cdot \vec{w} \times \vec{v}$ (D) $3\vec{u} \cdot \vec{v} \times \vec{w}$

**49.** Let $\vec{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}, \vec{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$ and $\vec{c} = c_1\hat{\imath} + c_2\hat{\jmath} + c_3\hat{k}$ be three non-zero vectors such that $\vec{c}$ is a unit vector perpendiuclar to both $\vec{a}$ and $\vec{b}$. If the angle between $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{6}$, then

$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}^2$ is equal to

(A) 0

(B) 1

(C) $\frac{1}{4}(a_1^2 + a_2^2 + a_3^2)(b_1^2 + b_2^2 + b_3^2)$

(D) $\frac{3}{4}(a_1^2 + a_2^2 + a_3^2)(b_1^2 + b_2^2 + b_3^2)(c_1^2 + c_2^2 + c_3^2)$

**50.** Given $\vec{a} = x\hat{\imath} + y\hat{\jmath} + 2\hat{k}, \vec{b} = \hat{\imath} - \hat{\jmath} + \hat{k}, \vec{c} = \hat{\imath} + 2\hat{\jmath}; (\vec{a}\,\hat{}\,\vec{b}) = \pi/2, \vec{a} \cdot \vec{c} = 4$, then

(A) $[\vec{a}\vec{b}\vec{c}]^2 = |\vec{a}|$

(B) $[\vec{a}\vec{b}\vec{c}] = |\vec{a}|$

(C) $[\vec{a}\vec{b}\vec{c}] = 0$

(D) $[\vec{a}\vec{b}\vec{c}] = |\vec{a}|^2$

**51.** Let $\vec{r}$ be a vector perpendicular to $\vec{a} + \vec{b} + \vec{c}$, where $[\vec{a}\vec{b}\vec{c}] = 2$. If $\vec{r} = \ell(\vec{b} \times \vec{c}) + m(\vec{c} \times \vec{a}) + n(\vec{a} \times \vec{b})$, then $(\ell + m + n)$ is equal to

(A) 2 (B) 1 (C) 0 (D) none of these

**52.** Let $\vec{a}, \vec{b}$ and $\vec{c}$ be non-coplanar unit vectors equally inclined to one another at an acute angle $\theta$. Then $[\vec{a}\vec{b}\vec{c}]$ in terms of $\theta$ is equal to

(A) $(1 + \cos\ \theta)\sqrt{\cos\ 2\theta}$

(B) $(1 + \cos\ \theta)\sqrt{1 - 2\cos\ 2\theta}$

(C) $(1 - \cos\ \theta)\sqrt{1 + 2\cos\ 2\theta}$

(D) none of these

**53.** If the vectors $a\hat{\imath} + \hat{\jmath} + \hat{k}, \hat{\imath} + b\hat{\jmath} + \hat{k}$ and $\hat{\imath} + \hat{\jmath} + c\hat{k}$ $(a \neq b \neq c \neq 1)$ are coplanar, then the value of $\frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c}$ is equal to

(A) 1 (B) -1 (C) 0 (D) none of these

**54.** The volume of a right triangular prism $ABCA_1\ B_1C_1$ is equal to 3 . If the position vectors of the vertices of the base ABC are A(1,0,1), B(2,0,0) and C(0,1,0), then position vectors of the vertex $A_1$ can be

(A) (2,2,2) (B) (0,2,0) (C) $(0, -2, 2)$ (D) $(0, -2, 0)$

**55.** Let $\vec{p} = 2\hat{\imath} + 3\hat{\jmath} - a\hat{k},\ \vec{q} = b\hat{\imath} + 5\hat{\jmath} - \hat{k}$ and $\vec{r} = \hat{\imath} + \hat{\jmath} + 3\hat{k}$. If $\vec{p}, \vec{q}, \vec{r}$ are coplanar and $\vec{p} \cdot \vec{q} = 20$, then $a$ and $b$ have the values

(A) $-1{,}3$ (B) 9,7 (C) 5,5 (D) $-13{,}9$

56. If $\vec{a}, \vec{b}, \vec{c}$ be three non-zero vectors satisfying the condition $\vec{a} \times \vec{b} = \vec{c}$ and $\vec{b} \times \vec{c} = \vec{a}$, then

(A) $\vec{a}, \vec{b}, \vec{c}$ are orthogonal is pairs
(B) $[\vec{a}, \vec{b}, \vec{c}] = [\vec{a}]^2$
(C) $[\vec{a}\vec{b}\vec{c}] = |\vec{c}|^2$
(D) $|\vec{b}| = |\vec{c}|$

**VECTOR TRIPLE PRODUCT**

57. Vector $\vec{x}$ satisfying the relation $\vec{A} \cdot \vec{x} = c$ and $\vec{A} \times \vec{x} = \vec{B}$ is

(A) $\frac{c\vec{A}-(\vec{A}\times\vec{B})}{|\vec{A}|}$ (B) $\frac{c\vec{A}-(\vec{A}\times\vec{B})}{|\vec{A}|^2}$ (C) $\frac{c\vec{A}+(\vec{A}\times\vec{B})}{|\vec{A}|^2}$ (D) $\frac{c\vec{A}-2(\vec{A}\times\vec{B})}{|\vec{A}|^2}$

58. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be non-zero non-collinear vectors such that $(\vec{a} \times \vec{b}) \times \vec{c} = \frac{1}{3}|\vec{b} \parallel \vec{c}|\vec{a}$, If $\theta$ is the angle between the vectors $\vec{b}$ and $\vec{c}$, then sin $\theta$ equals is

(A) $\frac{1}{3}$ (B) $\frac{\sqrt{2}}{3}$ (C) $\frac{2}{3}$ (D) $\frac{2\sqrt{2}}{3}$

59. If $\vec{a} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}, \vec{b} = y\hat{\imath} + z\hat{\jmath} + x\hat{k}$ and $\vec{c} = z\hat{\imath} + x\hat{\jmath} + y\hat{k}$, then $\vec{a} \times (\vec{b} \times \vec{c})$ is

(A) parallel to $(y - z)\hat{\imath} + (z - x)\hat{\jmath} + (x - y)\hat{k}$
(B) orthogonal to $\hat{\imath} + \hat{\jmath} + \hat{k}$
(C) orthogonal to $(y + z)\hat{\imath} + (z + x)\hat{\jmath} + (x + y)\hat{k}$
(D) orthogonal to $x\hat{\imath} + y\hat{\jmath} + z\hat{k}$

**SCALAR / VECTOR PRODUCT OF 4 VECTORS**

60. Let the pairs $\vec{a}, \vec{b}$ and $\vec{c}, \vec{d}$ each determine a plane. Then the planes are parallel if

(A) $(\vec{a} \times \vec{c}) \times (\vec{b} \times \vec{d}) = \vec{0}$
(B) $(\vec{a} \times \vec{c}) \cdot (\vec{b} \times \vec{d}) = \vec{0}$
(C) $(\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d}) = \vec{0}$
(D) $(\vec{a} \times \vec{b}) \cdot (\vec{c} \times \vec{d}) = \vec{0}$

61. let vectors $\vec{a}, \vec{b}, \vec{c}$ and $\vec{d}$ be such that $(\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d}) = \vec{0}$. Let $P_1$ and $P_2$ be planes determined by the pairs of vectors $\vec{a}, \vec{b}$ and $\vec{c}, \vec{d}$, respectively. Then the angle between $P_1$ and $P_2$ is

(A) 0 (B) $\pi/4$ (C) $\pi/3$ (D) $\pi/2$

**MIXED PROBLEMS**

62. A point taken on each median of a triangle divides the median in the ratio 1: 3, reckoning from the vertex. Then the ratio of the area of the triangle with vertices at these points to that of the original triangle is

(A) 5: 13 (B) 25: 64 (C) 13: 32 (D) none of these

**63.** If $\vec{a}, \vec{b}$ and $\vec{c}$ are three mutually perpendicular vectors, thent he vector which is equally inclined to these vectors is

(A) $\vec{a}+\vec{b}+\vec{c}$ (B) $\frac{\vec{a}}{|\vec{a}|}+\frac{\vec{b}}{|\vec{b}|}+\frac{\vec{c}}{|\vec{c}|}$

(C) $\frac{\vec{a}}{|\vec{a}|^2}+\frac{\vec{b}}{|\vec{b}|^2}+\frac{\vec{c}}{|\vec{c}|^2}$ (D) $|\vec{a}|\vec{a}-|\vec{b}|\vec{b}+|\vec{c}|\vec{c}$

**64.** If $\vec{b}$ and $\vec{c}$ are any two perpendicular unit vectors and $\vec{a}$ is any vector, then $(\vec{a}\cdot\vec{b})\vec{b}+(\vec{a}\cdot\vec{c})\vec{c}+\frac{\vec{a}\cdot(\vec{b}\times\vec{c})}{|\vec{b}\times\vec{c}|^2}(\vec{b}\times\vec{c})$ is equal to

(A) $\vec{a}$ (B) $\vec{b}$ (C) $\vec{C}$ (D) none of these

**VECTOR TIRPLE PRODUCT**

**65.** $(\vec{d}+\vec{a})\cdot(\vec{a}\times(\vec{b}\times(\vec{c}\times\vec{d})))$ simplifies to

(A) $(\vec{b}\cdot\vec{d})[\vec{a}\vec{c}\vec{d}]$ (B) $(\vec{b}\cdot\vec{c})[\vec{a}\vec{b}\vec{d}]$

(C) $(\vec{b}\cdot\vec{a})[\vec{a}\vec{b}\vec{d}]$ (D) none of these

**66.** $[(\vec{a}\times\vec{b})\times(\vec{b}\times\vec{c}),(\vec{b}\times\vec{c})\times(\vec{c}\times\vec{a}),(\vec{c}\times\vec{a})\times(\vec{a}\times\vec{b})]$ is equal to

(A) $[\vec{a}\vec{b}\vec{c}]^2$ (B) $[\vec{a}\vec{b}\vec{c}]^3$ (C) $[\vec{a}\vec{b}\vec{c}]^4$ (D) none of these

**VECTOR PRODUCT OF 4 VECTORS**

**67.** If $\vec{a}, \vec{b}, \vec{c}$ are three non-coplanar non-zero vectors and $\vec{r}$ is any vector in space, then $(\vec{a}\times\vec{b})\times(\vec{r}\times\vec{c})+(\vec{b}\times\vec{c})\times(\vec{r}\times\vec{a})+(\vec{c}\times\vec{a})\times(\vec{r}\times\vec{b})$ is equal to

(A) $2[\vec{a},\vec{b},\vec{c}]\vec{r}$ (B) $3[\vec{a},\vec{b},\vec{c}]\vec{r}$ (C) $[\vec{a},\vec{b},\vec{c}]\vec{r}$ (D) none of these

**COLLINEARITY OF THREE POINTS**

**68.** If a, b, c are different real numbers and $a\hat{\imath}+b\hat{\jmath}+c\hat{k}, b\hat{\imath}+c\hat{\jmath}+a\hat{k}$ and $c\hat{\imath}+a\hat{\jmath}+b\hat{k}$ are position vectors of three non-collinear points A, B, and C, then

(A) centroid of triangle $ABC$ is $\frac{a+b+c}{3}(\hat{\imath}+\hat{\jmath}+\hat{k})$

(B) $\hat{\imath}+\hat{\jmath}+\hat{k}$ is equally inclined to the three vectors

(C) perpendicular from the origin to the plane of triangle

ABC meet at centroid

(D) triangle ABC is an equilateral triangle.

**RELATION BETWEEN TWO PARALLEL VECTORS**

**69.** If a line has a vector equation $\vec{r} = 2\hat{\imath} + 6\hat{\jmath} + \lambda(\hat{\imath} - 3\hat{\jmath})$, 2 then which of the following statements hold goods?

(A) the line is parallel to $2\hat{\mathbf{i}} + 6\hat{\mathbf{j}}$

(B) the line passes through the point $2\hat{\imath} + 6\hat{\jmath}$

(C) the line passes through the point $\hat{\imath} + 9\hat{\jmath}$

(D) the line is parallel to XY-plane

**70.** The vector $\frac{1}{3}(2\hat{\imath} - 2\hat{\jmath} + \hat{k})$ is

(A) a unit vector

(B) makes an angle $\frac{\pi}{3}$ with the vector $2\hat{\imath} - 4\hat{\jmath} - 3\hat{k}$

(C) parallel to the vector $-\hat{\imath} + \hat{\jmath} - \frac{1}{2}\hat{k}$

(D) Perpendicular to the vector $3\hat{\imath} + 2\hat{\jmath} - 2\hat{k}$

**71.** $\hat{a}$ and $\hat{b}$ are two given unit vectors at right angle. The unit vector equally inclined with $\hat{a}, \hat{b}$ and $\hat{a} \times \hat{b}$ will be

(A) $-\frac{1}{\sqrt{3}}(\hat{a} + \hat{b} + \hat{a} \times \hat{b})$

(B) $\frac{1}{\sqrt{3}}(\hat{a} + \hat{b} + \hat{a} \times \hat{b})$

(C) $\frac{1}{\sqrt{3}}(\hat{a} + \hat{b} - \hat{a} \times \hat{b})$

(D) $-\frac{1}{\sqrt{3}}(\hat{a} + \hat{b} - \hat{a} \times \hat{b})$

**72.** A line passes through a point $A$ with position vector $3\hat{\imath} + \hat{\jmath} - \hat{k}$ and parallel to the vector $2\hat{\imath} - \hat{\jmath} + 2\hat{k}$. If $P$ is a point on this line such that AP $= 15$ units, then the position vector of the point P is/are

(A) $13\hat{\imath} + 4\hat{\jmath} - 9\hat{k}$

(B) $13\hat{\imath} - 4\hat{\jmath} + 9\hat{k}$

(C) $7\hat{\imath} - 6\hat{\jmath} + 11\hat{k}$

(D) $-7\hat{\imath} + 6\hat{\jmath} - 1\hat{k}$

**TECTOR OR CROSS PRODUCT OF TWO VECTORS**

**73.** If $\vec{a} \times \vec{b} = \vec{c} \times \vec{d}$ and $\vec{a} \times \vec{c} = \vec{b} \times \vec{d}$, then the vectors $\vec{a} - \vec{d}$ and $\vec{b} - \vec{c}$ are

(A) collinear

(B) linearly independent

(C) perpendicular

(D) parallel

**74.** Unit vectors $\vec{a}, \vec{b}$ and $\vec{c}$ are coplanar. $A$ unit vector $\vec{d}$ is perpendicular to them. If $(\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d}) = \frac{1}{6}\hat{\imath} - \frac{1}{3}\hat{\jmath} + \frac{1}{3}\hat{k}$, and the angle between $\vec{a}$ and $\vec{b}$ is $30^\circ$, then $\vec{C}$ is

(A) $(\hat{\imath} - 2\hat{\jmath} + 2\hat{k})/3$ (B) $(\hat{\imath} - 2\hat{\jmath} + 2\hat{k})/3$

(C) $(-2\hat{\mathbf{i}} - 2\hat{\mathbf{j}} - \hat{k})/3$ (D) $(-\hat{\imath} + 2\hat{\jmath} - 2\hat{k})/3$

**75.** If $\vec{u}, \vec{v}, \vec{w}$ are non-coplanar vectors and p, q are real numbers, then the equality $[3\vec{u} \;\; p\vec{v} \;\; p\vec{w}] - [p\vec{v} \;\; \vec{w} \;\; q\vec{u}] - [2\vec{w} \;\; q\vec{v} \;\; q\vec{u}] = 0$ holds for : **[AIEEE 2009]**

(A) exactly two values of $(p, q)$ (B) more than two but not all values of $(p, q)$

(C) all values of (p, q) (D) exactly one value of $(p, q)$

**76.** Let $\vec{a} = \hat{\jmath} - \hat{k}$ and $\vec{c} = \hat{\imath} - \hat{\jmath} - \hat{k}$. The the vector $\vec{b}$ satisfying $\vec{a} \times \vec{b} + \vec{c} = \vec{0}$ and $\vec{a} \cdot \vec{b} = 3$ is **[AIEEE 2010]**

(A) $-\hat{\imath} + \hat{\jmath} - 2\hat{k}$ (B) $2\hat{\mathbf{i}} - \hat{\jmath} + 2\hat{k}$

(C) $\hat{\imath} - \hat{\jmath} - 2\hat{k}$ (D) $\hat{\imath} + \hat{\jmath} - 2\hat{k}$

**77.** If the vectors $\vec{a} = \hat{\imath} - \hat{\jmath} + 2\hat{k}, \vec{b} = 2\hat{\imath} + 4\hat{\jmath} + \hat{k}$ and $\vec{c} = \lambda\hat{\imath} + \hat{\jmath} + \mu\hat{k}$ are mutually orthogonal, then $(\lambda, \mu) =$ **[AIEEE 2010]**

(A) $(-3,2)$ (B) $(2,-3)$ (C) $(-2,3)$ (D) $(3,-2)$

**78.** The vectors $\vec{a}$ and $\vec{b}$ are not perpendicular and $\vec{c}$ and $\vec{d}$ are two vectors satisfying: $\vec{b} \times \vec{c} = \vec{b} \times \vec{d}$ and $\vec{a} \cdot \vec{d} = 0$. Then the vector $\vec{d}$ is equal to : **[AIEEE 2011]**

(A) $\vec{b} - \left(\frac{\vec{b}\cdot\vec{c}}{\vec{a}\cdot\vec{b}}\right)\vec{c}$ (B) $\vec{c} + \left(\frac{\vec{a}\cdot\vec{c}}{\vec{a}\cdot\vec{b}}\right)\vec{b}$ (C) $\vec{b} + \left(\frac{\vec{b}\cdot\vec{c}}{\vec{a}\cdot\vec{b}}\right)\vec{c}$ (D) $\vec{c} - \left(\frac{\vec{a}\cdot\vec{c}}{\vec{a}\cdot\vec{b}}\right)\vec{b}$

**79.** Let $\hat{a}$ and $\hat{b}$ be two unit vectors. If the vectors $\vec{c} = \hat{a} + 2\hat{b}$ and $\vec{d} = 5\hat{a} - 4\hat{b}$ are perpendicular to each other, then the angle between $\vec{a}$ and $\vec{b}$ is: **[AIEEE 2012]**

(A) $\frac{\pi}{3}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{6}$ (D) $\frac{\pi}{2}$

**80.** Let $ABCD$ be a parallelogram such that $\overrightarrow{AB} = \vec{q}, \overrightarrow{AD} = \vec{p}$ and $\angle BAD$ be an acute angle. If $\vec{r}$ is the vector that coincides with the altitude directed from the vertex B to the side AD, then $\vec{r}$ is given by: **[AIEEE 2012]**

(A) $\vec{r} = \vec{q} - \left(\frac{\vec{p}\cdot\vec{q}}{\vec{p}\cdot\vec{p}}\right)\vec{p}$ (B) $\vec{r} = -3\vec{q} + \frac{3(\vec{p}\cdot\vec{q})}{(\vec{p}\cdot\vec{p})}\vec{p}$

(C) $\vec{r} = 3\vec{q} - \frac{3(\vec{p}\cdot\vec{q})}{(\vec{p}\cdot\vec{p})}\vec{p}$ (D) $\vec{r} = -\vec{q} + \left(\frac{\vec{p}\cdot\vec{q}}{\vec{p}\cdot\vec{p}}\right)\vec{p}$

**81.** If $[\vec{a}\times\vec{b}\;\vec{b}\times\vec{c}\;\vec{c}\times\vec{a}] = \lambda[\vec{a}\vec{b}\vec{c}]^2$ then $\lambda$ is equal to: **[JEE-MAIN 2014]**

(A) 2 (B) 3 (C) 0 (D) 1

**82.** The angle between the lines whose direction cosines satisfy the equations $\ell + \mathrm{m} + \mathrm{n} = 0$ and $\ell^2 = \mathrm{m}^2 + \mathrm{n}^2$ is: **[JEE-MAIN 2014]**

(A) $\frac{\pi}{3}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{6}$ (D) $\frac{\pi}{2}$

**83.** Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three non- zero vectors such that no two of them are collinear and $(\vec{a}\times\vec{b})\times\vec{c} = \frac{1}{3}|\vec{b}||\vec{c}|\vec{a}$. If $\theta$ is the angle between vectors $\vec{b}$ and $\vec{c}$, then a value of $\sin\ \theta$ is : **[JEE-MAIN 2015]**

(A) $\frac{2}{3}$ (B) $\frac{-2\sqrt{3}}{3}$ (C) $\frac{2\sqrt{2}}{3}$ (D) $\frac{-\sqrt{2}}{3}$

**84.** Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three unit vectors such that $\vec{a}\times(\vec{b}\times\vec{c}) = \frac{\sqrt{3}}{2}(\vec{b}+\vec{c})$. If $\vec{b}$ is not parallel to $\vec{c}$, then the angle between $\vec{a}$ and $\vec{b}$ is: **[JEE - MAIN 2016]**

(A) $\frac{\pi}{2}$ (B) $\frac{2\pi}{3}$ (C) $\frac{5\pi}{6}$ (D) $\frac{3\pi}{4}$

**85.** Let $\bar{u}$ be a vector coplanar with the vectors $\vec{a} = 2\hat{\imath} + 3\hat{\jmath} - \hat{k}$ and $\vec{b} = \hat{\jmath} + \hat{k}$. if $\vec{u}$ is perpendicular to $\vec{a}$ and $\vec{\mathrm{u}}\cdot\vec{\mathrm{b}} = 24$, then $|\vec{\mathrm{u}}|^2$ is equal to : **[JEE-MAIN 2018]**

(A) 84 (B) 336 (C) 315 (D) 256

**86.** Let P, Q, R and S be the points on the plane with position vectors $-2\hat{\imath} - \hat{\jmath}, 4\hat{\imath}, 3\hat{\imath} + 3\hat{\jmath} \& -3\hat{\imath} + 2\hat{\jmath}$ respectively. The quadrilateral PQRS must be a **[JEE 2010]**

(A) parallelogram, which is neither a rhombus nor a rectangle

(B) square

(C) rectangle, but not a square

(D) rhombus, but not a square

**87.** If $\vec{a}$ and $\vec{b}$ are vectors in space given by $\vec{a} = \frac{\hat{\imath}-2\hat{\jmath}}{\sqrt{5}}$ and $\vec{b} = \frac{2\hat{\imath}+\hat{\jmath}+3\hat{k}}{\sqrt{14}}$, then the value of $(2\vec{a}+\vec{b}) \cdot [(\vec{a}\times\vec{b})\times(\vec{a}-2\vec{b})]$ is **[JEE 2010]**

**88.** Let $\vec{a} = \hat{\imath}+\hat{\jmath}+\hat{k}, \vec{b} = \hat{\imath}-\hat{\jmath}+\hat{k}$ and $\vec{c} = \hat{\imath}-\hat{\jmath}-\hat{k}$ be three vectors. A vector $\vec{v}$ in the plane of $\vec{a}$ and $\vec{b}$, w hose projection on $\vec{c}$ is $\frac{1}{\sqrt{3}}$, is given by **[JEE 2011]**

(A) $\hat{\imath}-3\hat{\jmath}+3\hat{k}$ (B) $-3\hat{\imath}-3\hat{\jmath}-\hat{k}$

(C) $3\hat{\imath}-\hat{\jmath}+3\hat{k}$ (D) $\hat{\imath}+3\hat{\jmath}-3\hat{k}$

**89.** The vector(s) which is/are coplanar with vectors $\hat{\imath}+\hat{\jmath}+2\hat{k}$ and $\hat{\imath}+2\hat{\jmath}+\hat{k}$, and perpendicular to the vector $\hat{\mathbf{i}}+\hat{\mathbf{j}}+\hat{\mathrm{k}}$ is/are **[JEE 2011]**

(A) $\hat{\jmath}-\hat{k}$ (B) $-\hat{\imath}+\hat{\jmath}$ (C) $\hat{\imath}-\hat{\jmath}$ (D) $-\hat{\jmath}+\hat{k}$

**90.** Let $\vec{a} = -\hat{\imath}-\hat{k}, \vec{b} = -\hat{\imath}+\hat{\jmath}$ and $\vec{c} = \hat{\imath}+2\hat{\jmath}+3\hat{k}$ be three given vectors. If $\vec{r}$ is a vector such that $\vec{r}\times\vec{b} = \vec{c}\times\vec{b}$ and $\vec{r}\cdot\vec{a} = 0$, then the value of $\vec{r}\cdot\vec{b}$ is **[JEE 2011]**

**91.** Let $\overrightarrow{PR} = 3\hat{\imath}+\hat{\jmath}-2\hat{k}$ and $\overrightarrow{SQ} = \hat{\imath}-3\hat{\jmath}-4\hat{k}$ determine diagonals of a parallelogram PQRS and $\overrightarrow{\mathrm{PT}} = \hat{\mathrm{i}}+2\hat{\mathrm{j}}+3\hat{\mathrm{k}}$ be another vector. Then the volume of the parallelepiped determined by the vectors $\overrightarrow{\mathrm{PT}}, \overrightarrow{\mathrm{PQ}}$ and $\overrightarrow{\mathrm{PS}}$ is **[JEE 2013]**

(A) 5 (B) 20 (C) 10 (D) 30

**92.** Consider the set of eight vectors $V = \{a\hat{\imath}+b\hat{\jmath}+c\hat{k}: a,b,c \in \{-1,1\}\}$. Three noncoplanar vectors can be chosen from V in $2^p$ ways. Then $p$ is **[JEE 2013]**

**93.** Match List-I with List-II and select the correct answer using code given the lists **[JEE 2013]**

| **List-I** | **List-II** |
|---|---|
| **(P)** Volume of parallelepiped determined by vectors $\vec{a}, \vec{b}$ and $\vec{c}$ is 2 . then the volume of the parallelepiped determined by vectors $2(\vec{a}\times\vec{b}), 3(\vec{b}\times\vec{c})$ and $(\vec{C}\times\vec{a})$ is | **1.** 100 |
| **(Q)** Volume of parallelepiped determined by vectors $\vec{a}, \vec{b}$ and $\vec{c}$ is 5 . Then the volume of the parallelepiped | **2.** 30 |

determined by vectors $3(\vec{a}+\vec{b}), (\vec{b}+\vec{c})$ and $2(\vec{c}+\vec{a})$ is

**(R)** Area of a triangle with adjacent sides determined by vectors $\vec{a}$ and $\vec{b}$ is 20. Then area of the triangle with adjacent sides determined by vectos $(2\vec{a}+3\vec{b})$ and $(\vec{a}-\vec{b})$ is — **3.** 24

**(S)** Area of a parallelogram with adjacent sides determined by vectors $\vec{a}$ and $\vec{b}$ is 30 . Then the area of the parallelogram with adjacent sides determined by vectors $(\vec{a}+\vec{b})$ and $\vec{a}$ is — **4.** 60

**Codes:**

| | P | Q | R | S |
|---|---|---|---|---|
| (A) | 3 | 2 | 4 | 1 |
| (B) | 1 | 3 | 4 | 2 |
| (C) | 3 | 4 | 1 | 2 |
| (D) | 2 | 4 | 1 | 3 |

**94.** Let $\vec{x}, \vec{y}$ and $\vec{z}$ be three vectors each of magnitude $\sqrt{2}$ and the angle between each pair of them is $\frac{\pi}{3}$. If $\vec{a}$ is a nonzero vector perpendicular to $\vec{x}$ and $\vec{y}\times\vec{z}$ and $\vec{b}$ is a nonzero vector perpendicular to $\vec{y}$ and $\vec{z}\times\vec{x}$, then **[JEE 2014]**

(A) $\vec{b}=(\vec{b}\cdot\vec{z})(\vec{z}-\vec{x})$

(B) $\vec{a}=(\vec{a}\cdot\vec{y})(\vec{y}-\vec{z})$

(C) $\vec{a}\cdot\vec{b}=-(\vec{a}\cdot\vec{y})(\vec{b}\cdot\vec{z})$

(D) $\vec{a}=(\vec{a}\cdot\vec{y})(\vec{z}-\vec{y})$

**95.** Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three non-coplanar unit vectors such that the angle between every pair of them is $\frac{\pi}{3}$. If $\vec{a}\times\vec{b}+\vec{b}\times\vec{c}=p\vec{a}+q\vec{b}+r\vec{c}$, where $p, q$ and $r$ are scalars, then the value of $\frac{p^2+2q^2+r^2}{q^2}$ is **[JEE 2014]**

**96.** Let $\triangle$ PQR be a triangle. Let $\vec{a}=\overrightarrow{QR}, \vec{b}=\overrightarrow{RP}$ and $\vec{c}=\overrightarrow{PQ}$. If $|\vec{a}|=12, |\vec{b}|=4\sqrt{3}$ and $\vec{b}\cdot\vec{c}=24$, then which of the following is (are) true? **[JEE 2015]**

(A) $\frac{|\vec{c}|^2}{2}-|\vec{a}|=12$

(B) $\frac{|\vec{c}|^2}{2}+|\vec{a}|=30$

(C) $|\vec{a}\times\vec{b}+\vec{c}\times\vec{a}|=48\sqrt{3}$

(D) $\vec{a}\cdot\vec{b}=-72$

**97.** **Column-I** | **Column-II**

| Column-I | Column-II |
|---|---|
| (A) In a triangle $\triangle XYZ$, let a, b and $c$ be the lengths of the sides opposite to the angles X, Y and Z, respectively. If $2(a^2 - b^2) = c^2$ and $\lambda = \frac{\sin(X-Y)}{\sin Z}$, then possible values of $n$ for which $\cos(n\pi\lambda) = 0$ is (are) | **(P)** 1 |
| (B) In a triangle $\triangle$ XYZ, let a, b and c the lengths of the sides opposite to the angles X, Y and Z, respectively. If $1 + \cos 2X - 2\cos 2Y = 2\sin X \sin Y$, then possible value(s) of $\frac{a}{b}$ is (are) | **(Q)** 2 |
| (C) In $R^2$, let $\sqrt{3}\hat{\imath} + \hat{\jmath}, \hat{\imath} + \sqrt{3}\hat{\jmath}$ and $\beta\hat{\imath} + (1-\beta)\hat{\jmath}$ be the position vectors of X, Y and Z with respect to the origin O, respectively. If the distance of Z from the bisector of the acute angle $\overline{OX}$ with $\overline{OY}$ is $\frac{3}{\sqrt{2}}$ then possible value(s) of $\lvert\beta\rvert$ is (are) | **(R)** 3 |
| (D) Suppose that $F(\alpha)$ denotes the area of the region bounded by $x = 0, x = 2, y^2 = 4x$ and $y = \lvert\alpha x - 1\rvert + \lvert\alpha x - 2\rvert + \alpha x$, where $\alpha \in \{0,1\}$. Then the value(s) of $F(\alpha) + \frac{8}{3}\sqrt{2}$, when $\alpha = 0$ and $\alpha = 1$, is (are) | **(S)** 5 |
| | **(T)** 6 |

**98.** Let $\hat{u} = u_1\hat{\imath} + u_2\hat{\jmath} + u_3\hat{k}$ be a unit vector in $R^3$ and $\hat{w} = \frac{1}{\sqrt{6}}(\hat{\imath} + \hat{\jmath} + 2\hat{k})$. Given that there exists a vector $\vec{v}$ in $R^3$ such that $\lvert\hat{u} \times \vec{v}\rvert = 1$ and $\hat{w} \cdot (\hat{U} \times \vec{v}) = 1$. Which of the following statement (s) is(are) correct? **[JEE 2016]**

(A) There is exactly one choice for such $\bar{v}$

(B) There are infinitely many choices for such $\vec{v}$

(C) If $\hat{u}$ lies in the xy-plane then $\lvert u_1\rvert = \lvert u_2\rvert$

(D) If $\hat{u}$ lies in the $xz$-plane then $2\lvert u_1\rvert = \lvert u_3\rvert$

**99.** Let O be the origin and let PQR be an arbitrary triangle. The point $S$ is such that $\overrightarrow{OP} \cdot \overrightarrow{OQ} + \overrightarrow{OR} \cdot \overrightarrow{OS} = \overrightarrow{OR} \cdot \overrightarrow{OP} + \overrightarrow{OQ} \cdot \overrightarrow{OS} = \overrightarrow{OQ} \cdot \overrightarrow{OR} + \overrightarrow{OP} \cdot \overrightarrow{OS}$ Then the triangle PQR has S as its **[JEE Adv. 2017]**

(A) circumcenter (B) Incentre

(C) Centroid (D) orthocenter

**Paragraph 100 to 101**

Let O be the origin, and $\overrightarrow{OX}, \overrightarrow{OY}, \overrightarrow{OZ}$ be three-unit vectors in the directions of the sides $\overline{QR}$, $\overline{RP}, \overline{PQ}$, respectively, of a triangle $PQR$. **[JEE Adv. 2017]**

**100.** If the triangle $PQR$ varies, then the minimum value of cos (p + Q) + cos (Q + R) + Cos (R + P) is

(A) $\frac{3}{2}$ (B) $\frac{5}{3}$ (C) $-\frac{5}{3}$ (D) $-\frac{3}{2}$

**101.** $|\overrightarrow{OX} \times \overrightarrow{OY}| =$

(A) $\sin (P + R)$ (B) $\sin (Q + R)$ (C) $\sin (P + Q)$ (D) $\sin 2R$

**102.** Let $\vec{a}$ and $\vec{b}$ be two unit vectors such that $\vec{a} \cdot \vec{b} = 0$. For some $x, y \in R$, let $\vec{c} = x\vec{a} + y\vec{b} + (\vec{a} \times \vec{b})$. If $|\vec{c}| = 2$ and the vector $\vec{c}$ is inclined at the same angle $\alpha$ to both $\vec{a}$ and $\vec{b}$, then the value of $8\cos^2 \alpha$ is **[JEE Adv. 2018]**

**103.** Let $\vec{a} = 2\hat{\imath} + \hat{\jmath} - \hat{k}$ and $\vec{b} = \hat{\imath} + 2\hat{\jmath} + \hat{k}$ be two vectors. Consider a vector $\vec{c} = \alpha\vec{a} + \beta\vec{b}$. $\alpha, \beta, \in \square$. If the projection of $\vec{c}$ on the vector $(\vec{a} + \vec{b})$ is $3\sqrt{2}$, then the minimum value of $(\vec{c} - (\vec{a} \times \vec{b})) \cdot \vec{c}$ equals **[JEE Adv. 2019]**

## ANSWER KEY

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **1.** | **(B)** | **2.** | **(A)** | **3.** | **(D)** | **4.** | **(D)** | **5.** | **(A)** |
| **6.** | **(B)** | **7.** | **(C)** | **8.** | **(B)** | **9.** | **(B)** | **10.** | **(B)** |
| **11.** | **(A)** | **12.** | **(C)** | **13.** | **(D)** | **14.** | **(C)** | **15.** | **(AC)** |
| **16.** | **(ABC)** | **17.** | **(AB)** | **18.** | **(BD)** | **19.** | **()** | | |

**20.** ($\vec{x} = \vec{q} - \frac{(\vec{p}\cdot\vec{q})\vec{p}}{2p^2}$) **21.** ($\cos\ \theta = \frac{2}{\sqrt{89}\sqrt{41}}$)

**23.** ($\vec{R} = (-1, -8-2);$) (ii) ($\vec{v} = 9(-\hat{j} + \hat{k})$)

**24.** ($\frac{5a^2}{12\sqrt{3}}$ sq. units) **25.** **(110)**

**26.** (i) ($\frac{6}{7}\sqrt{14}$) (ii) **(6)** (iii) ($\frac{3}{5}\sqrt{10}$) (iv) ($\sqrt{6}$)

**27.** ((A)-P,R,S,T; (B)-P,R,T; (C)-P,Q,R,T)

**28.** (i) ($p = 0; q = 10; r = -3$) (ii) **(-100)**

**29.** ($\left(\frac{4}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$) **30.** ($\pm\frac{1}{3\sqrt{3}}(-1, -5, 1)$)

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **31.** | **(C)** | **32.** | **(D)** | **33.** | **(B)** | **34.** | **(A)** | **35.** | **(D)** |
| **36.** | **(C)** | **37.** | **(C)** | **38.** | **(D)** | **39.** | **(B)** | **40.** | **(C)** |
| **41.** | **(C)** | **42.** | **(A)** | **43.** | **(D)** | **44.** | **(B)** | **45.** | **(B)** |
| **46.** | **(C)** | **47.** | **(A)** | **48.** | **(B)** | **49.** | **(C)** | **50.** | **(D)** |
| **51.** | **(C)** | **52.** | **(C)** | **53.** | **(A)** | **54.** | **(AD)** | **55.** | **(AD)** |
| **56.** | **(ABC)** | **57.** | **(B)** | **58.** | **(D)** | **59.** | **(ABCD)** | **60.** | **(C)** |
| **61.** | **(A)** | **62.** | **(B)** | **63.** | **(B)** | **64.** | **(A)** | **65.** | **(A)** |
| **66.** | **(C)** | **67.** | **(A)** | **68.** | **(ABCD)** | **69.** | **(CD)** | **70.** | **(ACD)** |
| **71.** | **(AB)** | **72.** | **(BD)** | **73.** | **(AD)** | **74.** | **(AD)** | **75.** | **(D)** |
| **76.** | **(A)** | **77.** | **(A)** | **78.** | **(D)** | **79.** | **(A)** | **80.** | **(D)** |
| **81.** | **(D)** | **82.** | **(A)** | **83.** | **(C)** | **84.** | **(C)** | **85.** | **(B)** |
| **86.** | **(A)** | **87.** | **(5)** | **88.** | **(C)** | **89.** | **(AD)** | **90.** | **(9)** |
| **91.** | **(C)** | **92.** | **(5)** | **93.** | **(C)** | **94.** | **(ABC)** | **95.** | **(4)** |
| **96.** | **(ACD)** | **97.** | **((A)→P,R,S (B)→P (C) → P, Q (D) →S, T)** | | | | | **98.** | **(BC)** |
| **99.** | **(D)** | **100.** | **(D)** | **101.** | **(C)** | **102.** | **(3)** | **103.** | **(18)** |