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CIRCULAR MOTION

Projectile is projected horizontally with u m/s. Find

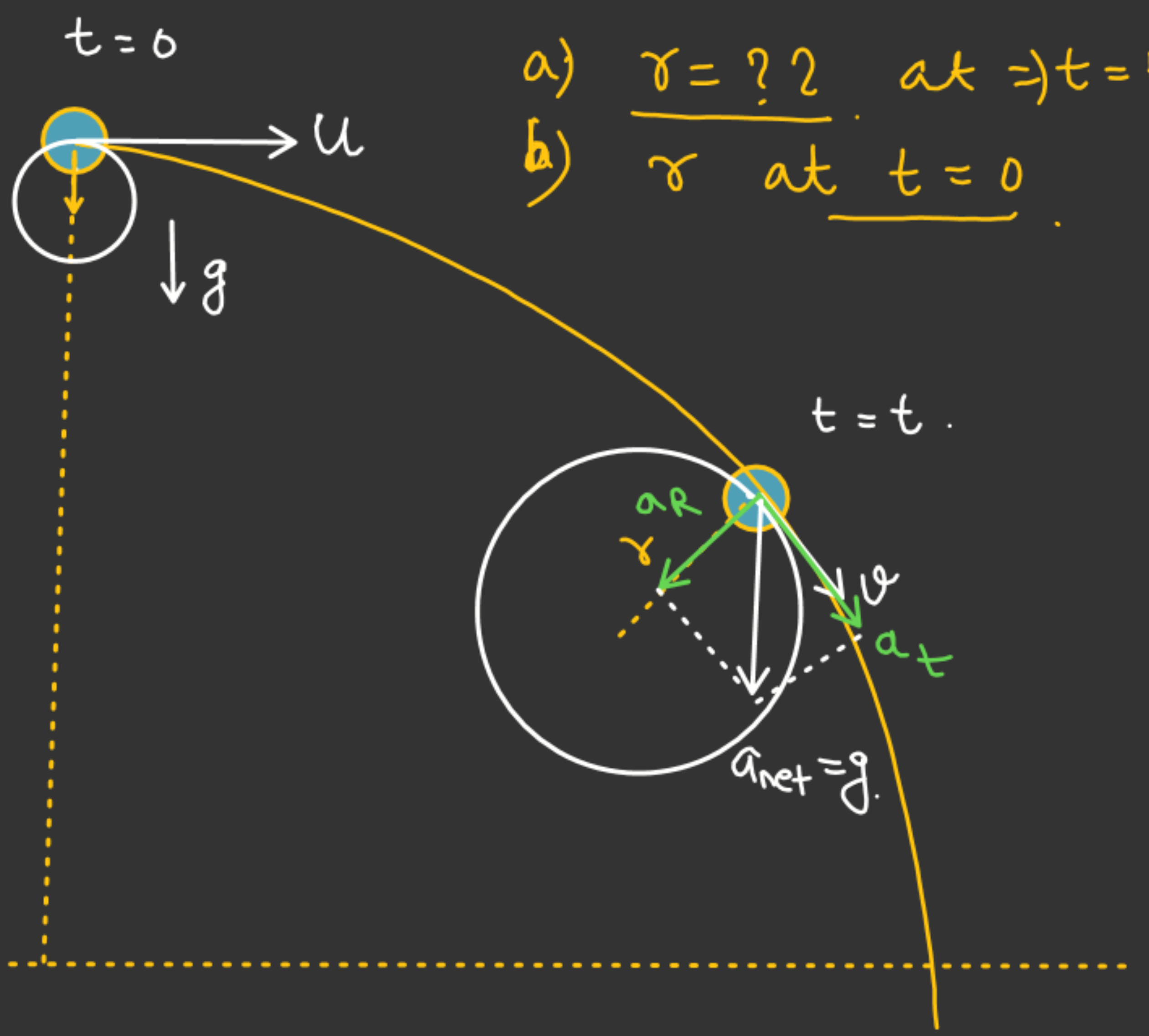
a_t, a_R, & a_{net} at $t = t$

a) $r = ??$ at $\Rightarrow t = t$

b) r at $t = 0$.

$$\vec{v} = v_x \hat{i} + v_y \hat{j}$$
$$= u\hat{i} - gt\hat{j}$$

$$v_y = \cancel{u_y}^{\,0} + gt$$
$$v_y = gt$$



$$\vec{a}_{net} = \frac{d\vec{v}}{dt} = \frac{d}{dt}(u)\hat{i} - \frac{d}{dt}(gt)\hat{j}$$
$$= 0 - g\hat{j}$$

$$|\vec{a}_{net}| = g.$$

At $t = 0$

$a_t = 0$, $a_{net} = g = a_R$

r at $t = 0$

$$\frac{u^2}{r} = a_R = g. \Rightarrow r = \left(\frac{u^2}{g}\right)$$

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$$\vec{v} = u\hat{i} - gt\hat{j}$$

$$|\vec{v}| = \sqrt{u^2 + g^2t^2}$$

$\Downarrow$

Speed

$$a_t = \frac{d}{dt}|\vec{v}|$$

$$a_t = \frac{d}{dt}\left(\sqrt{u^2 + g^2t^2}\right)$$

put $u^2 + g^2t^2$ x

$$a_t = \frac{d}{dx}(\sqrt{x}) \times \frac{d}{dt}(x)$$

$$a_t = \frac{1}{2\sqrt{x}} \frac{d}{dt}(u^2 + g^2t^2)$$

$$a_t = \frac{1}{2\left(\sqrt{u^2 + g^2t^2}\right)} \times (g^2)(2t)$$

$$a_t = \frac{g^2t}{\sqrt{u^2 + g^2t^2}}$$ Ans.

$$a_{net}^2 = a_t^2 + a_R^2$$

$$a_R^2 = (a_{net}^2 - a_t^2) = g^2 - \frac{g^4t^2}{(u^2 + g^2t^2)}$$

$$a_R^2 = \left(\frac{u^2g^2}{u^2 + g^2t^2}\right) \Rightarrow a_R = \frac{ug}{\sqrt{u^2 + g^2t^2}}$$ Ans.

* Radius of Curvature at $t = t$

$$a_R = \frac{v^2}{r}$$

$$r = \frac{v^2}{a_R}$$

$$r = \frac{u^2 + g^2t^2}{\frac{ug}{\sqrt{u^2 + g^2t^2}}}$$

$$r = \frac{(u^2 + g^2t^2)^{3/2}}{ug}$$ ✓

$$\vec{r} = |\vec{r}|\,\hat{r}$$

$$|\hat{r}| = |\hat{\theta}| = 1.$$

$$\hat{r} = \cos\theta\,\hat{i} + \sin\theta\,\hat{j}$$

$$\hat{\theta} = -\sin\theta\,\hat{i} + \cos\theta\,\hat{j}$$

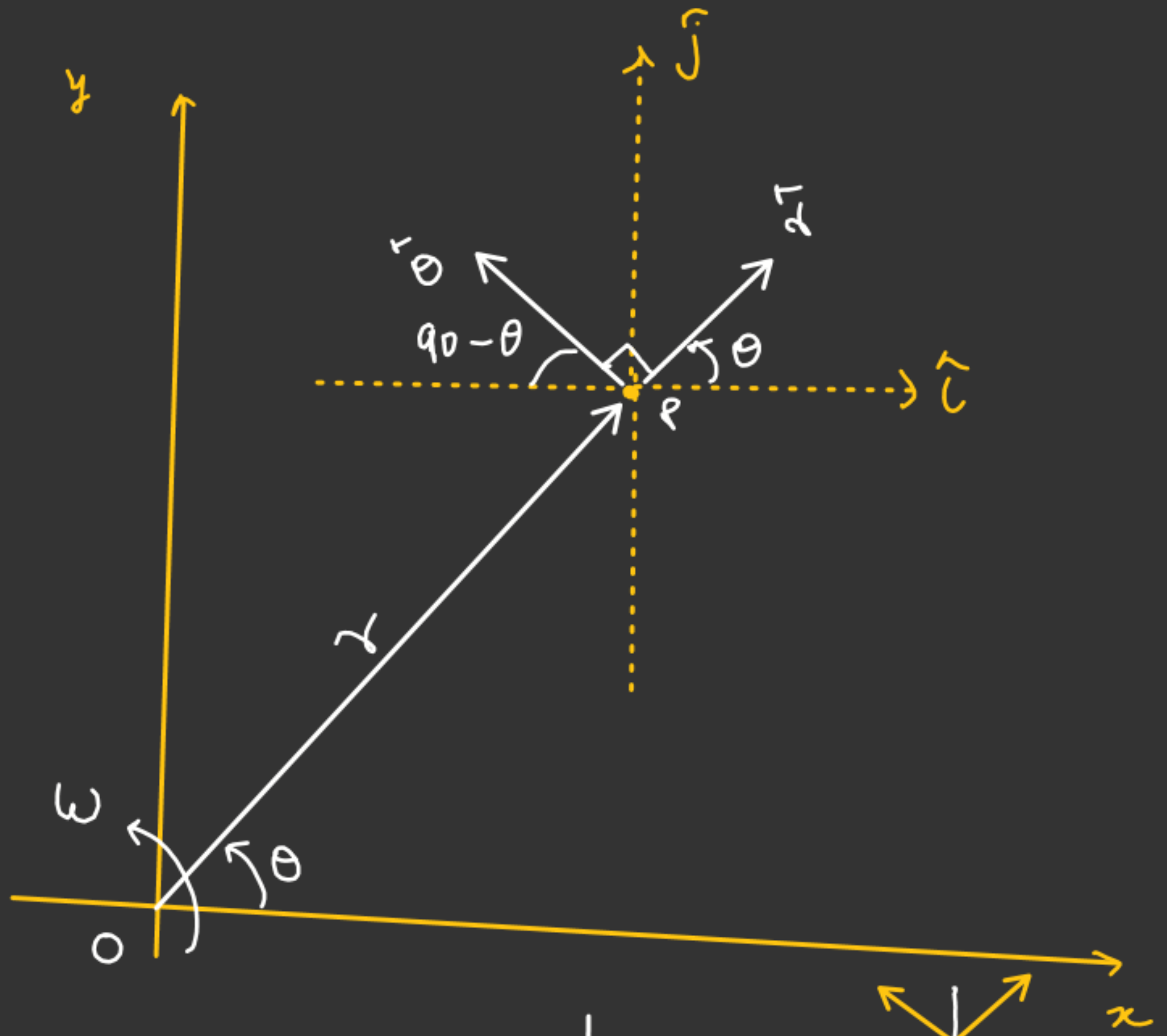
$$\frac{d\hat{r}}{dt} = \frac{d}{dt}(\cos\theta)\,\hat{i} + \frac{d}{dt}(\sin\theta)\,\hat{j}$$

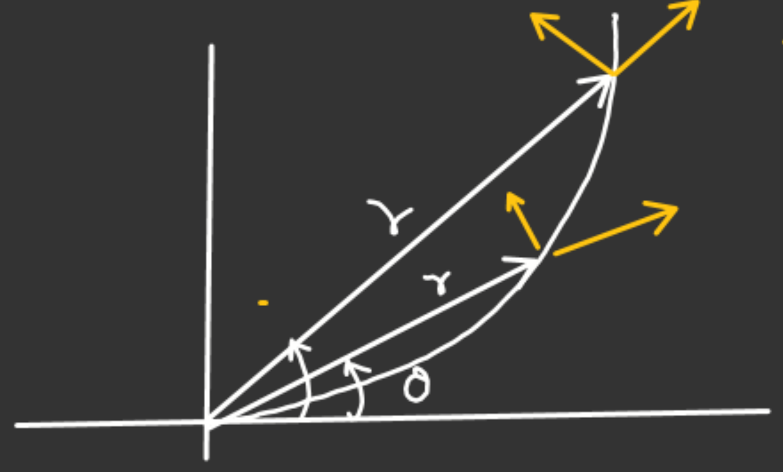
$$= \left[\frac{d}{d\theta}(\cos\theta) \times \left(\frac{d\theta}{dt}\right)\right]\hat{i} + \left[\frac{d}{d\theta}(\sin\theta)\left(\frac{d\theta}{dt}\right)\right]\hat{j}$$

$$= -(\sin\theta)\left(\frac{d\theta}{dt}\right)\hat{i} + (\cos\theta)\left(\frac{d\theta}{dt}\right)\hat{j}$$

$$\frac{d\hat{r}}{dt} = \left[-\sin\theta\,\hat{i} + \cos\theta\,\hat{j}\right]\left(\frac{d\theta}{dt}\right)$$

$$\boxed{\frac{d\hat{r}}{dt} = \omega\,\hat{\theta}}$$





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$$\hat{\theta} = -\sin\theta\,\hat{i} + \cos\theta\,\hat{j}$$

$$\frac{d\hat{\theta}}{dt} = \left[-\frac{d}{d\theta}(\sin\theta) \times \left(\frac{d\theta}{dt}\right)\right]\hat{i} + \left[\frac{d}{d\theta}(\cos\theta) \times \left(\frac{d\theta}{dt}\right)\right]\hat{j}$$

$$\frac{d\hat{\theta}}{dt} = \left[-\cos\theta\,\hat{i} - \sin\theta\,\hat{j}\right]\left(\frac{d\theta}{dt}\right)$$

$$\Downarrow$$

$$\frac{d\hat{\theta}}{dt} = -\left(\cos\theta\,\hat{i} + \sin\theta\,\hat{j}\right)\left(\frac{d\theta}{dt}\right)$$

$$\boxed{\frac{d\hat{\theta}}{dt} = -\omega\,\hat{r}}$$

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For Circular motion $|\vec{r}| = \text{Constant}$.

$$\vec{r} = (r)\hat{r}$$

$$\frac{d\vec{r}}{dt} = r\left(\frac{d\hat{r}}{dt}\right)$$

$$\frac{d\vec{r}}{dt} = r(\omega\hat{\theta}) = (r\omega)\hat{\theta}$$

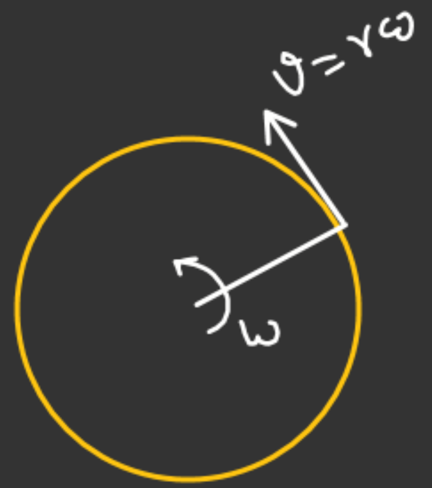
$$\vec{v} = (r\omega)\hat{\theta}$$

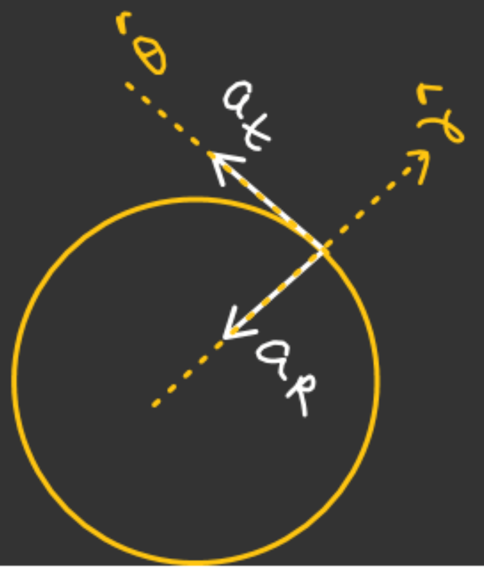
$$\frac{d\vec{v}}{dt} = r\frac{d}{dt}(\omega\hat{\theta})$$

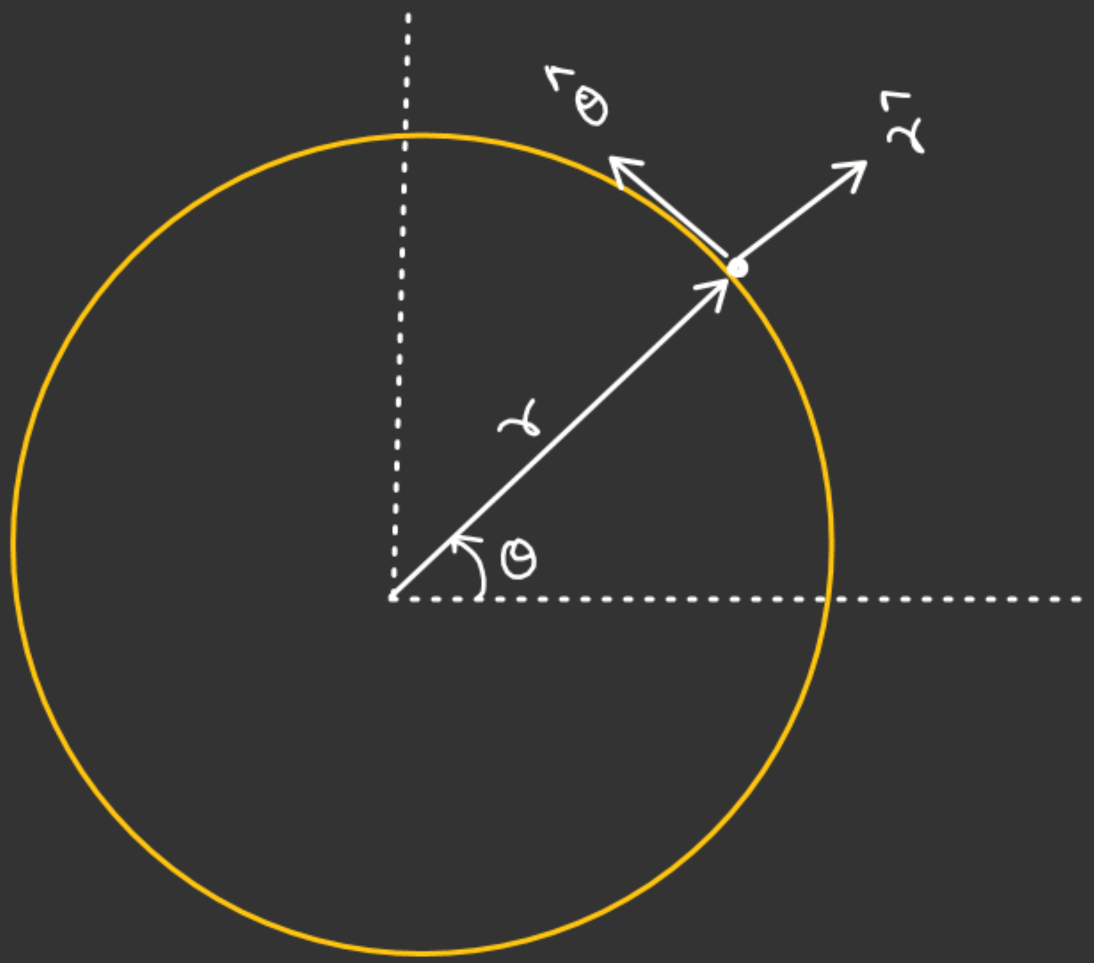
$$\vec{a} = r\left[\omega\left(\frac{d\hat{\theta}}{dt}\right) + \hat{\theta}\left(\frac{d\omega}{dt}\right)\right]$$

$$\vec{a} = r\omega(-\omega\hat{r}) + (r\alpha)\hat{\theta}$$

$$\boxed{\vec{a} = -\underbrace{(\omega^2 r)}_{a_R}\hat{r} + \underbrace{(r\alpha)}_{a_t}\hat{\theta}}$$







$$\boxed{\begin{aligned} a_R &= \omega^2 r = \frac{v^2}{r} \\ a_t &= r\alpha = \frac{d|v|}{dt} \end{aligned}}$$

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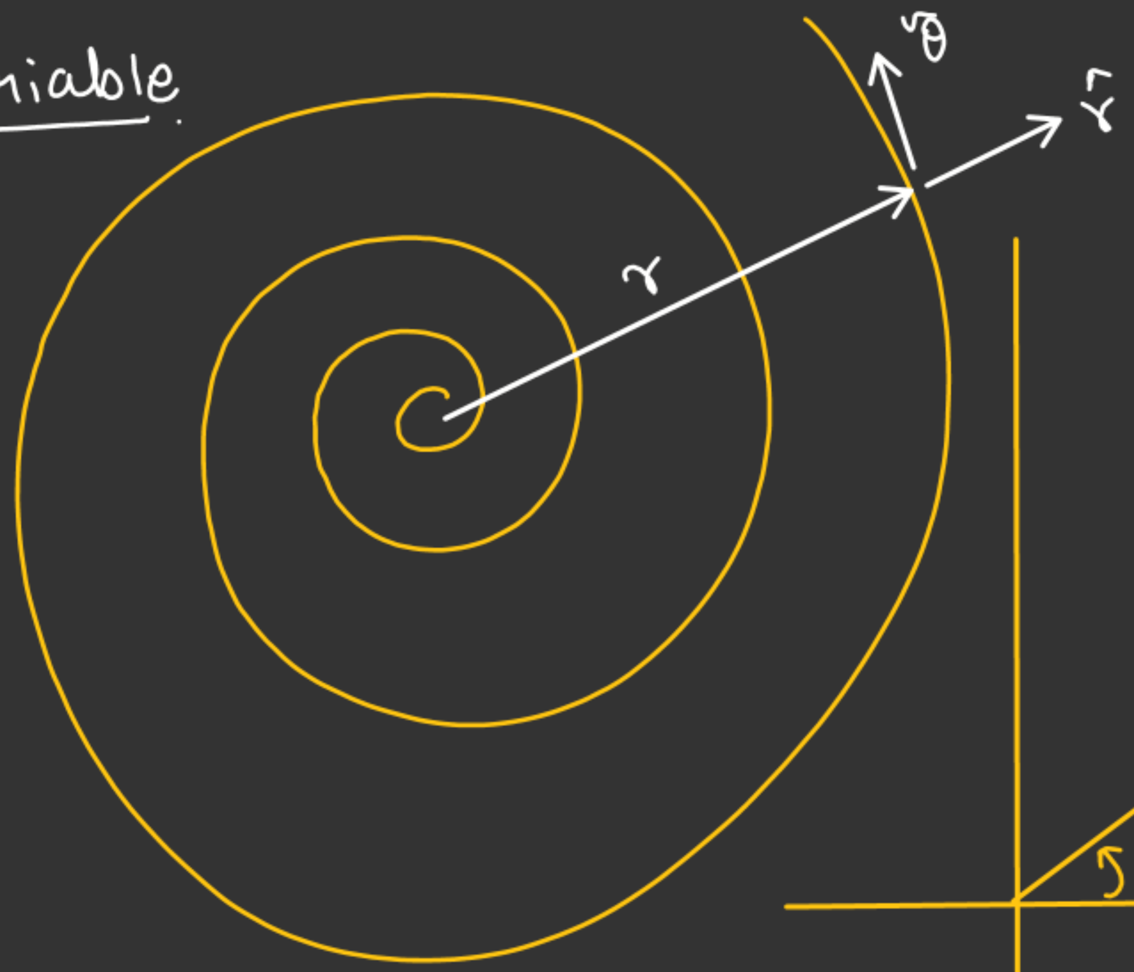
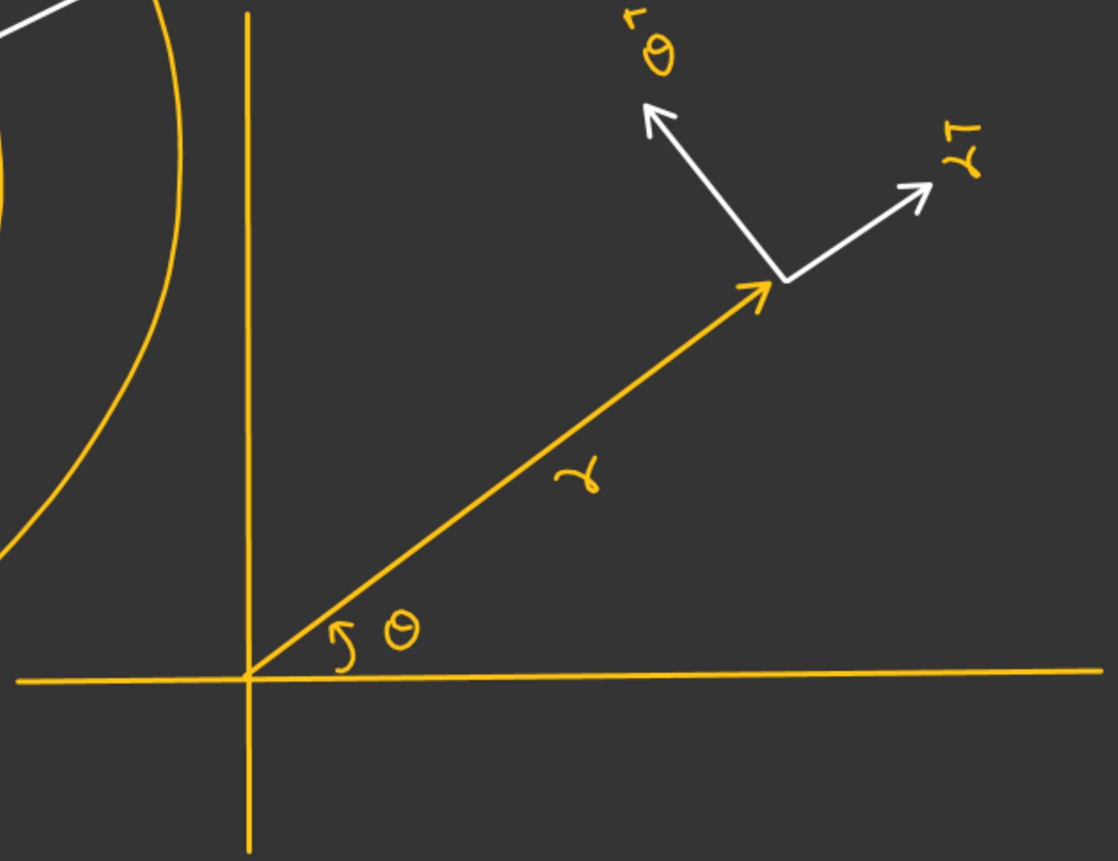
H.W Optional ✓ $r \to$ Variable

$\vec{r} = r(\hat{r})$





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Concept of Centripetal force

" Centripetal force is not a new type of force.
Some forces or their resultant like gravitational, tension, normal reaction play the role of centripetal force "