DPP 01

1. Fundamental frequency of a stretched string,

$$v = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$$

As two wires are identical in all respect, so $v \propto \sqrt{T}$ or $T \propto v^2$

$$\therefore \frac{T_X}{T_Z} = \frac{v_X^2}{v_Z^2} \text{ or } \frac{T_X}{T_Z} = \left(\frac{450}{300}\right)^2 = 2.25$$

2. Given the distance between two crests is 5 m and between any crest and trough is 1.5 m.

3. $v_1 = v, T_1 = 2.06 \times 10^4$ N

$v_2 = v/2, T_2 = ?\ v \propto \sqrt{T}$

So, $\frac{v_2}{v_1} = \sqrt{\frac{T_2}{T_1}} \Rightarrow \frac{v}{2v} = \sqrt{\frac{T_2}{T_1}} \Rightarrow \frac{1}{4} = \frac{T_2}{T_1}$

$T_2 = \frac{2.06\times10^4}{4} = 0.515 \times 10^4\ T_2 = 5.15 \times$

$T_2 = 5.15 \times 10^3$ N

4. From the graph we observe, when $x = 0, y = 0$

$y(x,t) = A\sin(kx - \omega t + \phi)$

$\Rightarrow 0 = A\sin\ \phi \Rightarrow \sin\ \phi = 0 \Rightarrow \phi = 0$ or π

Slope of curve $= \frac{dy}{dx} = Ak\cos(kx - \omega t + \phi)$

At $x = 0, t = 0$, Slope $= Ak\cos\ \phi$

Since slope is negative at $x = 0$ so $\phi = \pi$

5. Given, audio output $= 2$ W

Intensity $I = 120$ dB

Reference intensity, $I_0 = 10^{-12}\ \text{W/m}^2$

$AB = 10\log\left(\frac{I}{I_0}\right) \Rightarrow 120 = 10\log\left(\frac{I}{I_0}\right) \Rightarrow I = 1\ \text{W/m}^2$

Final intensity, $I = \frac{P_{out}}{4\pi r^2}$

$$\therefore I = \frac{2}{4\pi r^2} \Rightarrow r = \sqrt{\frac{2}{4\pi}} = \sqrt{\frac{1}{2\pi}}\text{m} \approx 40 \text{ cm}$$

6. The given wave is $y = 0.03\sin(450t - 9x)$ Compare with standard wave equation

$y = a\sin(\omega t - kx)$

The velocity of travelling wave $v = \frac{\omega}{k}$

$\Rightarrow\ v = \dfrac{450}{9} = 50\ m/s$

velocity of wave on stretched string

$$v = \sqrt{\frac{T}{\mu}}$$

Using (i) and (ii), $\sqrt{\frac{T}{\mu}} = 50 \Rightarrow T = (50)^2 \times 5 \times 10^{-3}$

$\Rightarrow T = 12.5\ N$

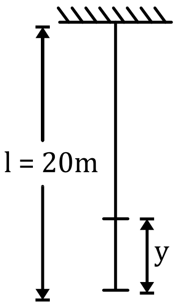
7. The given wave is $y(x, t) = 10^{-3}\sin(50t + 2x)$ Comparing with standard equation,

$y(x, t) = a\sin(\omega t + \beta x)$

Velocity $v = \frac{\omega}{\beta} = \frac{50}{2} = 25\ m\ s^{-1}$

In the given equation x is positive.

8. Speed of the wave pulse (wave) in the string,



$$v = \sqrt{\frac{T}{\mu}}$$

Here, $T = \frac{m}{l} \times y \times g$ and $\mu = \frac{m}{l}$

$$\therefore v = \sqrt{\frac{\frac{m}{l} \times y \times g}{m/l}} = \sqrt{gy}$$

Also, $v = \frac{dy}{dt} = \sqrt{gy}$ or, $\int_0^{20} \frac{dy}{\sqrt{y}} = \int_0^t \sqrt{g}dt$

or, $\left[\dfrac{y^{1/2}}{1/2}\right]_0^{20} = \sqrt{g}[t]_0^t \Rightarrow 2(\sqrt{20} - 0) = \sqrt{10} \times t$

$\therefore\ t = 2\sqrt{2}\ s$

9. $y(x, t) = e^{-(ax^2 + bt^2 + 2\sqrt{ab}xt)}$

$y(x, t) = e^{-(\sqrt{a}x + \sqrt{b}t)^2}$

Comparing equation (i) with standard equation

$y(x, t) = f(ax + bt)$

As there is positive sign between x and t terms, hence wave travel in $-x$ direction.

$$\text{Wave speed } = \frac{\text{Coefficient of t}}{\text{Coefficient of x}} = \sqrt{\frac{b}{a}}$$

10. Here, linear mass density $\mu = 0.04 \text{ kg m}^{-1}$

The given equation of a wave is

$$y = 0.02\sin\left[2\pi\left(\frac{t}{0.04} - \frac{x}{0.50}\right)\right]$$

$$y = A\sin(\omega t - kx)$$

we get, $\omega = \frac{2\pi}{0.04} \text{rads}^{-1}; k = \frac{2\pi}{0.5} \text{radm}^{-1}$

Wave velocity, $v = \frac{\omega}{k} = \frac{(2\pi/0.04)}{(2\pi/0.5)} \text{ms}^{-1}$

Also $v = \sqrt{\frac{T}{\mu}}$

Equating equations (i) and (ii), we get

$$\frac{\omega}{k} = \sqrt{\frac{T}{\mu}} \text{ or } T = \frac{\mu\omega^2}{k^2}$$

$$T = \frac{0.04 \times \left(\frac{2\pi}{0.04}\right)^2}{\left(\frac{2\pi}{0.05}\right)^2} = 6.25 \text{ N}$$