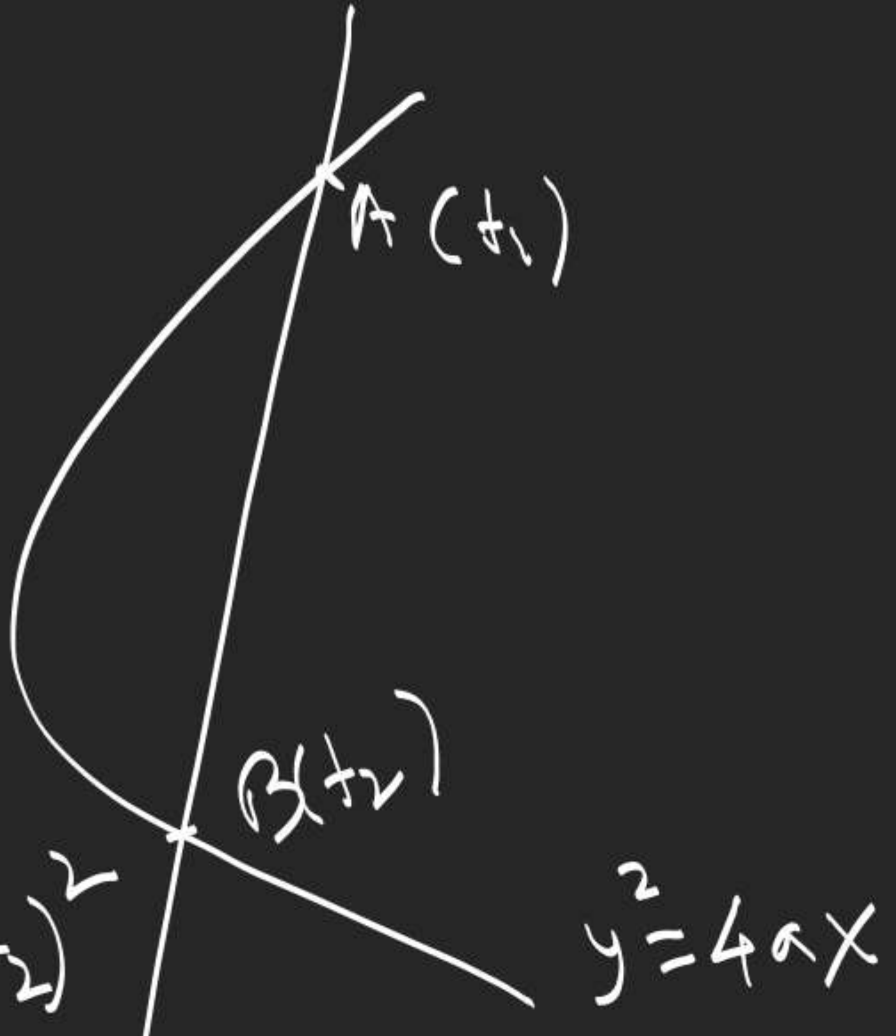
Nishant Jindal

1.

Find AB.



$$AB^2 = a^2(t_1^2 - t_2^2)^2 + 4a^2(t_1 - t_2)^2$$

$$= a^2\left((t_1+t_2)^2 - 4t_1t_2\right)\left((t_1+t_2)^2 + 4\right)$$

$y = mx + c$

$(at^2, 2at)$

$$= a^2\left(\frac{4}{m^2} - 4\frac{c}{am}\right)\left(\frac{4}{m^2} + 4\right)$$

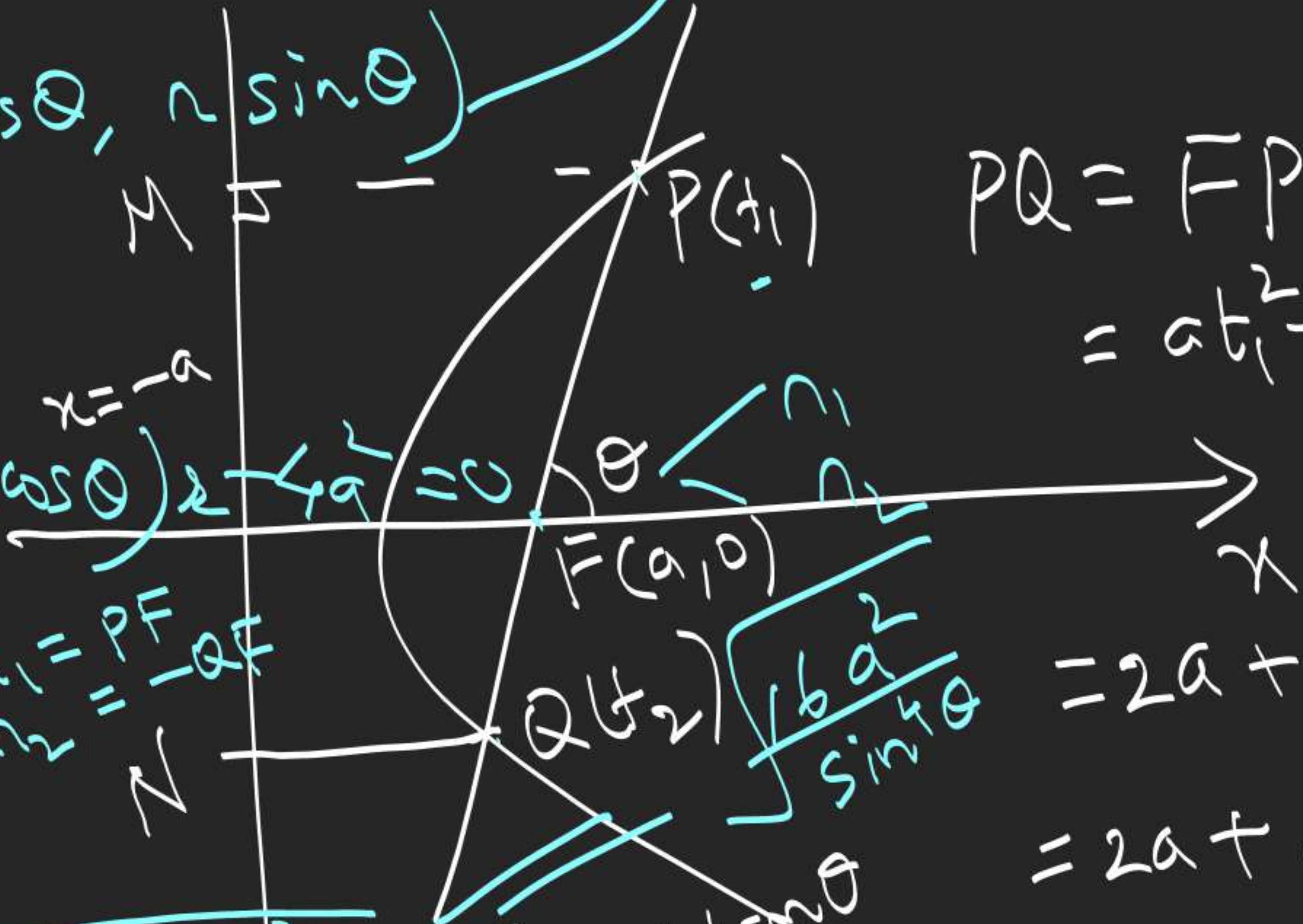
$2at = mat^2 + c$

$mat^2 - 2at + c = 0 \quad \begin{matrix} t_1 \\ t_2 \end{matrix}$

Nishant Jindal

Q2: P.T. length of focal chord to $y^2=4ax$ making angle 'θ' with its axis is $4a\operatorname{cosec}^2\theta$. $(a>0)$

$(a+r\cos\theta,\ r\sin\theta)$



$r^2\sin^2\theta-(4a\cos\theta)r-4a^2=0$

$(r_1-r_2)^2=(r_1+r_2)^2-4r_1r_2$

$r_1=PF$

$r_2=-QF$

$PQ=|r_1-r_2|$

$=\sqrt{\frac{16a^2\cos^2\theta}{\sin^4\theta}+\frac{16a^2}{\sin^2\theta}}=\sqrt{\frac{16a^2}{\sin^4\theta}}$

$\frac{2}{t_1+t_2}=\tan\theta$

$PQ=FP+FQ$

$=at_1^2+a+at_2^2+a$

$=2a+a(t_1^2+t_2^2)$

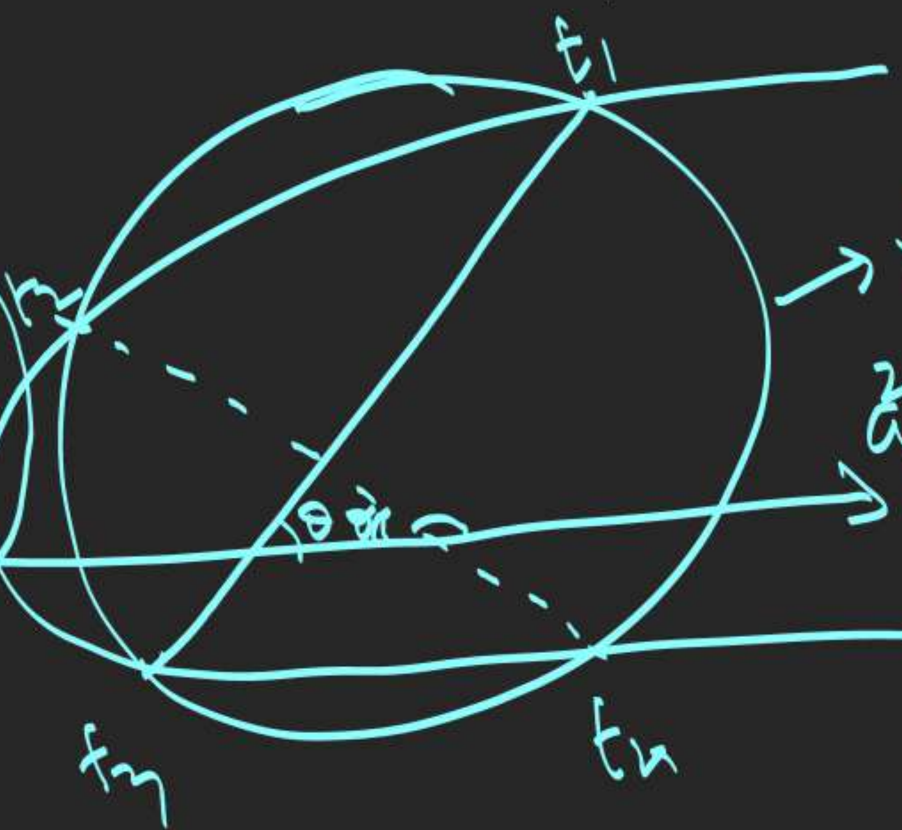
$=2a+a((t_1+t_2)^2-2t_1t_2)$, $t_1t_2=-1$

$=4a+4a\cot^2\theta$

$=4a\operatorname{cosec}^2\theta$

Nishant Jindal

3. A circle and a parabola $y^2=4ax$ intersect in four points. P.T. sum of ordinates of four points is zero. Also P.T. line joining one pair of these four points and line joining the other pair are equally inclined to the axis.



$$x^2+y^2+2gx+2fy+c=0 \quad t_1 \; t_2 \; t_3 \; t_4$$

$$a^2t^4+(4a^2+2ga)t^2+4fat+c=0$$

$$\boxed{t_1+t_2+t_3+t_4=0}$$

$$\sum y_i = 0$$

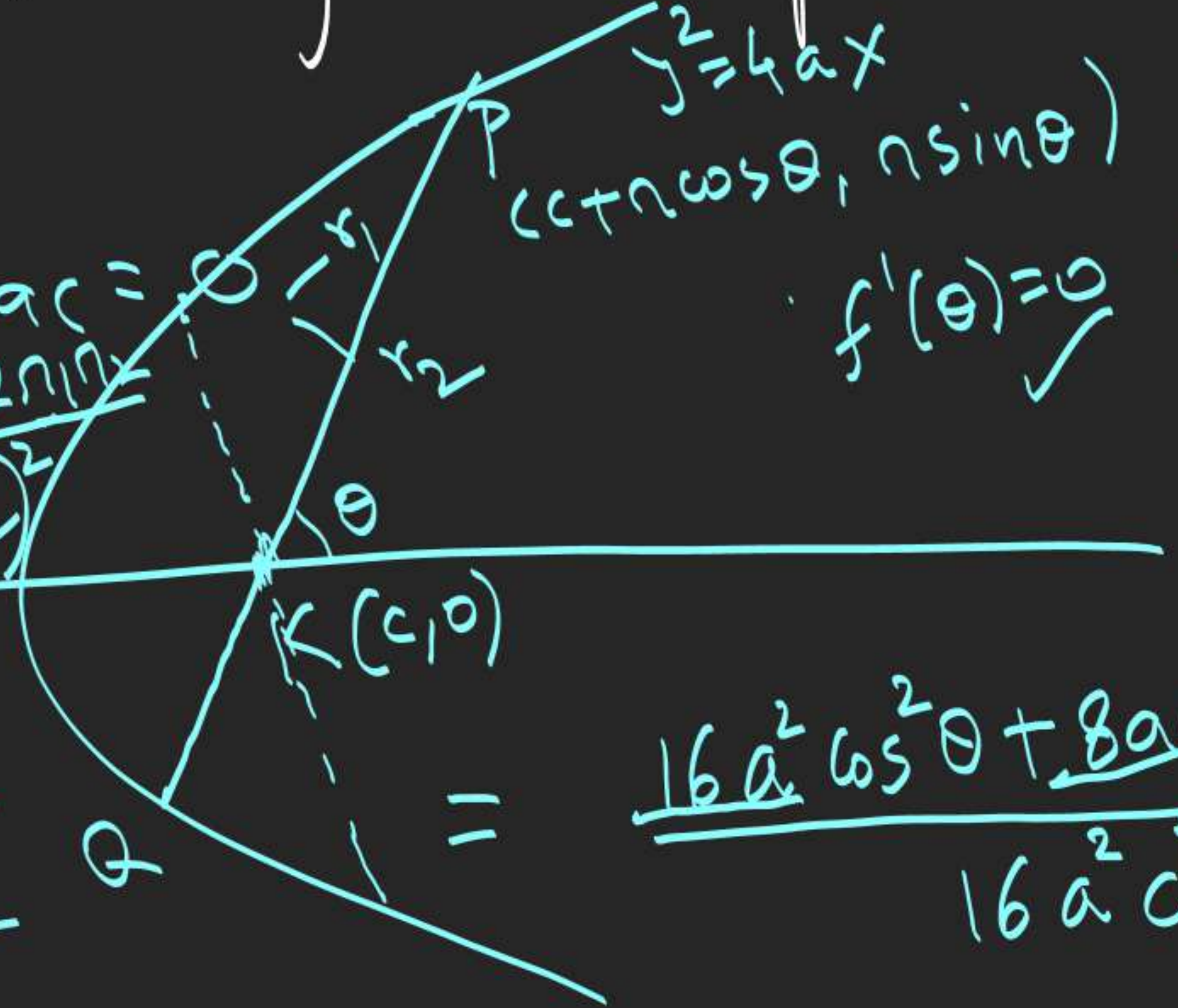
$$m_1=\frac{2}{t_1+t_3}=\frac{2}{-t_2-t_4}$$

$$=-m_2$$

Nishant Jindal

4. P.T. on axis of any parabola, there is a certain point K which has the property that, if a chord PQ of parabola be drawn through it, then

$\frac{1}{PK^2}+\frac{1}{QK^2}$ is the same for all positions of the chord.



$$r^2\sin^2\theta-4a\cos\theta\, r-4ac=0$$

$$\frac{1}{r_1^2}+\frac{1}{(-r_2)^2}=\frac{(r_1+r_2)^2-2r_1r_2}{(r_1r_2)^2}$$

$$=\frac{\frac{16a^2\cos^2\theta}{\sin^4\theta}+\frac{8ac}{\sin^2\theta}}{\frac{16a^2c^2}{\sin^4\theta}}=\frac{16a^2\cos^2\theta+8ac\sin^2\theta}{16a^2c^2}$$

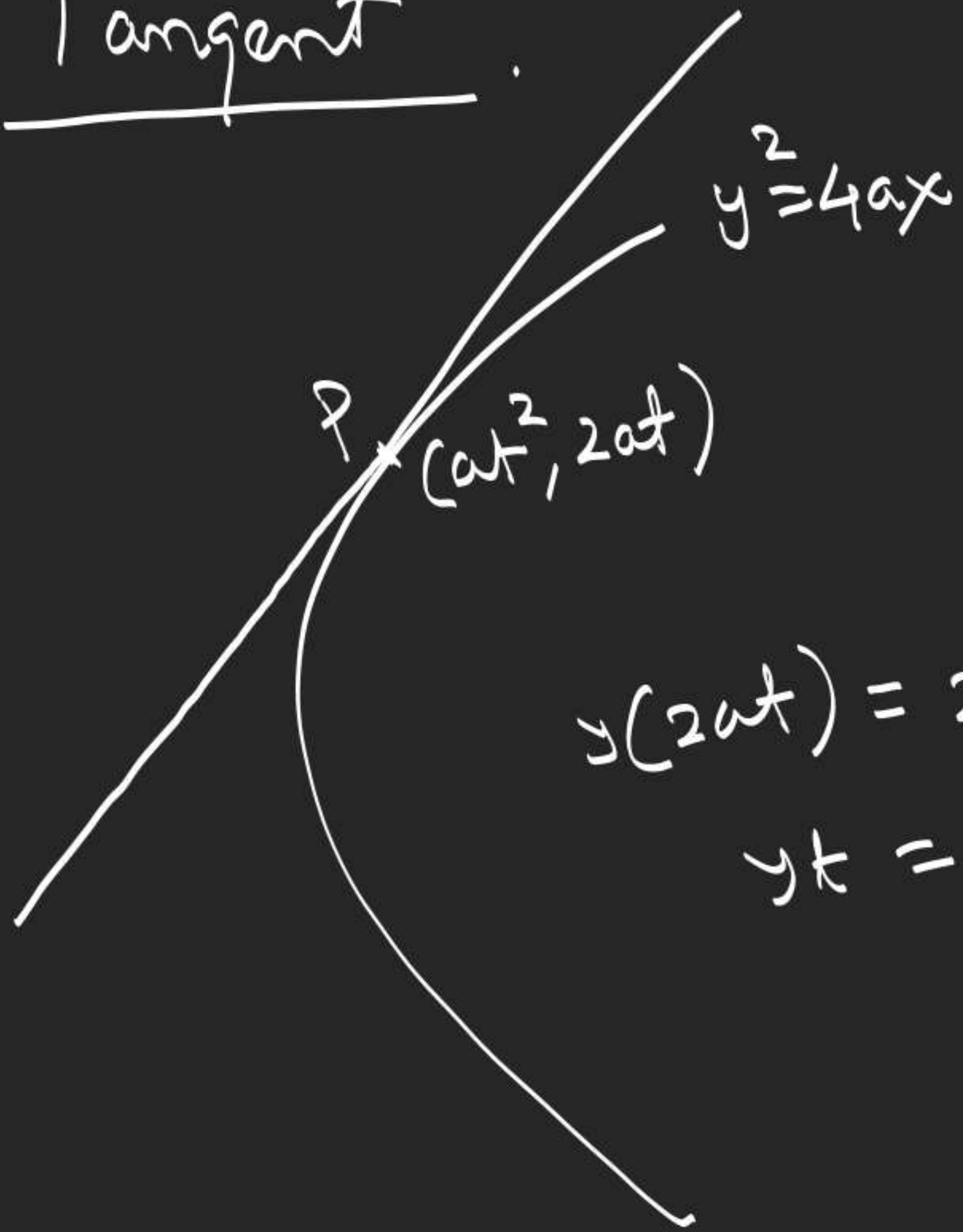
$f'(\theta)=0$ ✓ $\forall\theta\in R$

$16a^2=8ac$

$$\boxed{c=2a}$$

Nishant Jindal

Tangent

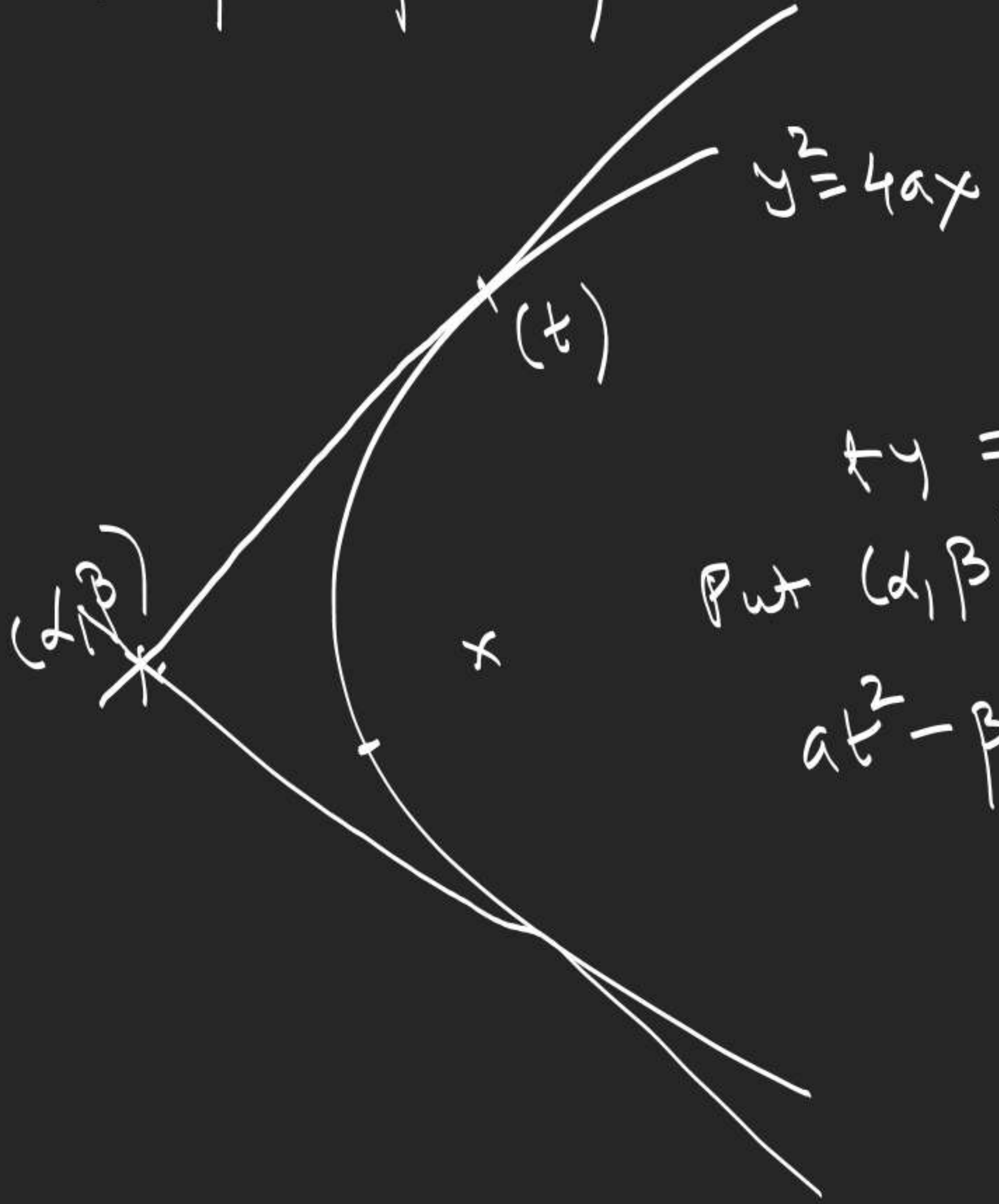


$$y(2at) = 2a(x+at^2)$$

$$yt = x+at^2$$

Nishant Jindal

Tangent passing thru (α, β)



$ty = x + at^2$

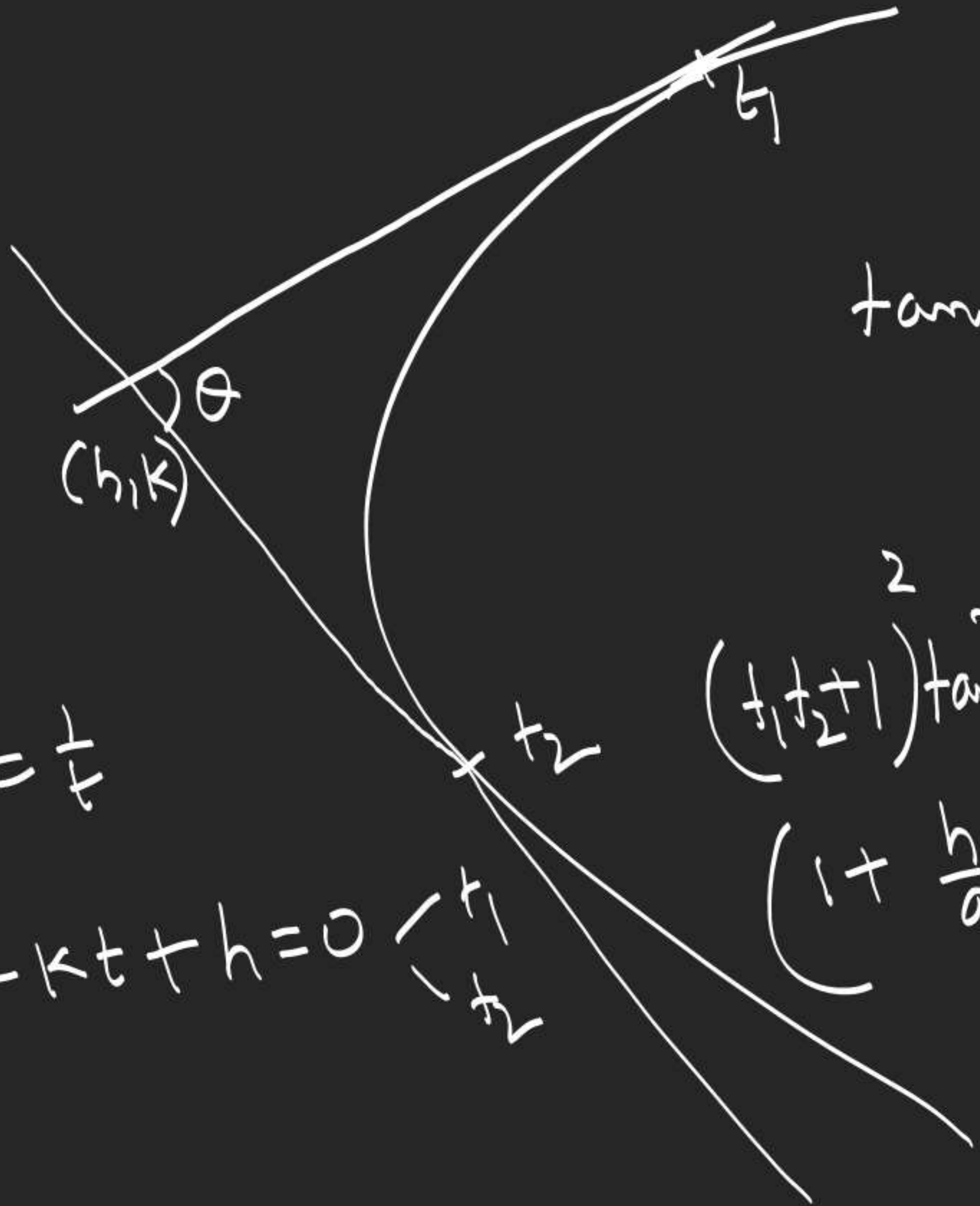
Put (α, β) ✓

$at^2 - \beta t + \alpha = 0$

$\downarrow$

$D = \beta^2 - 4a\alpha$

Nishant Jindal



$$\tan\theta = \left| \frac{\frac{1}{t_1} - \frac{1}{t_2}}{1 + \frac{1}{t_1}\frac{1}{t_2}} \right|$$

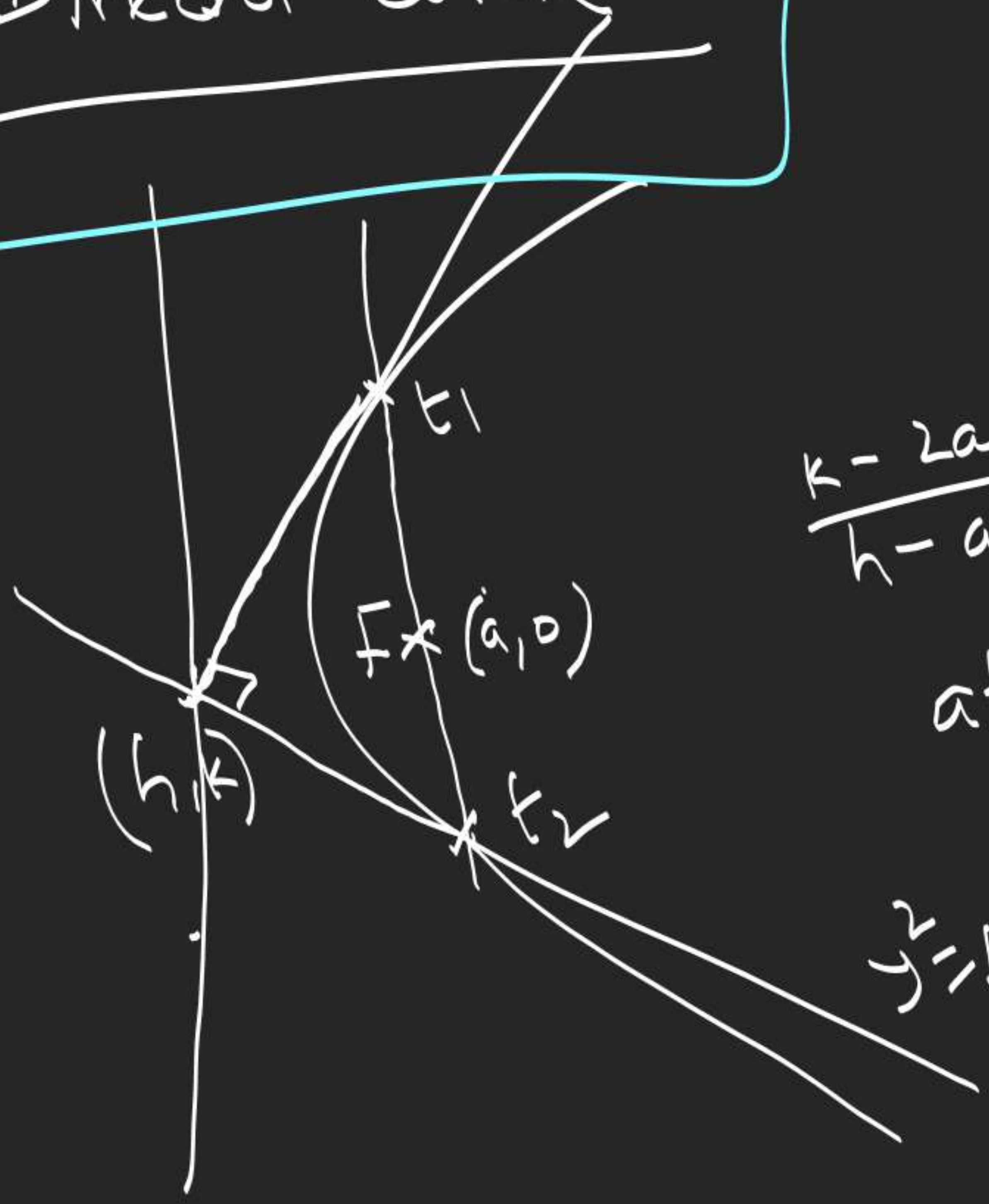
$$(t_1 t_2 + 1)^2 \tan^2\theta = (t_1 + t_2)^2 - 4 t_1 t_2$$

$$\left(1 + \frac{h}{a}\right)^2 \tan^2\theta = \frac{k^2}{a^2} - \frac{4h}{a}$$

$$\frac{k - 2at}{h - at^2} = \frac{1}{t}$$

$$at^2 - kt + h = 0 \; \begin{cases} t_1 \\ t_2 \end{cases}$$

Director Circle



$$\frac{k-2at}{h-at^2} = \frac{1}{t}$$

$$at^2 - kt + h = 0 \quad \begin{matrix} t_1 \\ t_2 \end{matrix}$$

$$\frac{1}{t_1}\cdot\frac{1}{t_2} = -1$$

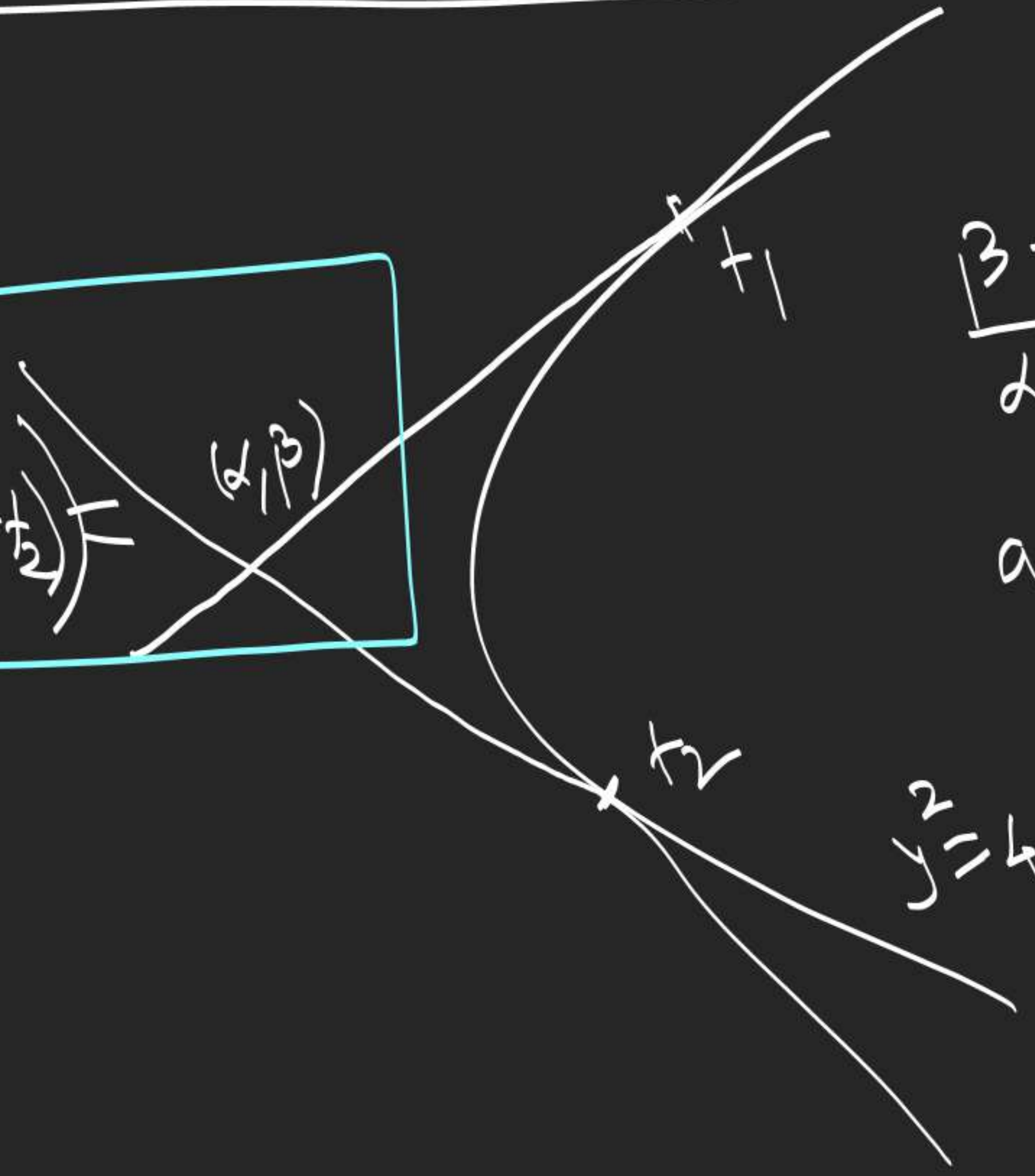
$$t_1 t_2 = -1$$

$$\frac{h}{a} = -1$$

$$x = -a$$

Intersection of 2 tangents to $y^2=4ax$

$$\boxed{\left(at_1t_2,\ a(t_1+t_2)\right)} =$$



$$\frac{\beta-2at}{\alpha-at^2}=\frac{1}{t}$$

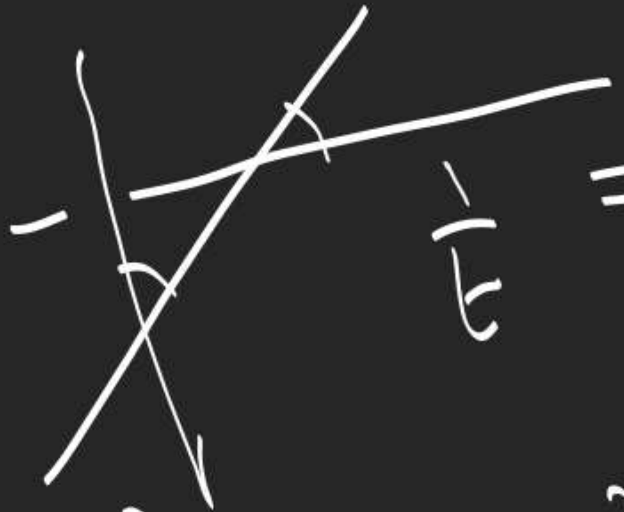
$$at^2-\beta t+\alpha=0 \begin{cases} t_1 \\ t_2 \end{cases}$$

$$\frac{\beta}{a}=t_1+t_2$$

$$\frac{\alpha}{a}=t_1t_2$$

Nishant Jindal

1. A tangent to parabola $y^2 = 8x$ makes an angle of 45° with the line $y = 3x + 5$ (θ). Find its eqn. and also its point of contact.



$$\frac{1}{t} = \tan(\theta \pm 45^\circ) = \frac{3+1}{1-3}, \frac{3-1}{1+3}$$

$$= -2, \frac{1}{2}$$

$$t = -\frac{1}{2}, \cancel{2}$$

$$\boxed{\cancel{\xi x - 5}}$$

$$ty = x + 2t^2$$

$$-\frac{y}{2} = x + \frac{1}{2}$$

$$2y = x + 8$$

$$(2t^2, 4t)$$

$$\left(\frac{1}{2}, -2\right)$$

$$(8, 8)$$