HOMEWORK–01 (Solution)

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1. If $y = \frac{a + bx^{\frac{3}{2}}}{x^{\frac{5}{4}}}$ & $\frac{dy}{dx}$ vanishes when $x = 5$ then $\frac{a}{b} =$

(A) $\sqrt{3}$ (B) 2 (C) $\sqrt{5}$ (D) 3

Ans. **(C)**

Sol. $y = \frac{a+bx^{3/2}}{x^{5/4}}$

$$\left.\frac{dy}{dx}\right|_{x=5} = 0 \Rightarrow \left.\left(\frac{-5ax^{-9/4}}{4} + \frac{b}{4}x^{-3/4}\right)\right|_{x=5} = 0$$

$$\left(\frac{5}{4} \cdot ax^{-9/4}\right)_{x=5} = \left(\frac{b}{4}x^{-3/4}\right)_{x=5}$$

$$\left(\frac{5}{4} \cdot ax^{-6/4}\right)_{x=5} = \frac{b}{4}$$

$$\Rightarrow\ 5a \cdot (5)^{-6/4} = b$$

$$\frac{a}{b} = \frac{1}{5^{1-6/4}} = \frac{1}{5^{-1/2}} = \sqrt{5}$$

2. If $\frac{d}{dx}\left(\frac{1 + x^2 + x^4}{1 + x + x^2}\right) = ax + b$ then the value of a and b are respectively

(A) 2 and 1 (B) –2 and 1 (C) 2 and –1 (D) 3 and 1

Ans. **(C)**

Sol. $\frac{d}{dx}\left(\frac{x^4+x^2+1}{x^2+x+1}\right) = ax + b$

$$\frac{d}{dx}\left\{\frac{(x^2 + x + 1)^2x - 2(x^3 + x^2 + x)}{(x^2 + x + 1)}\right\}$$

$$\frac{d}{dx}\{(x^2 + x + 1) - 2x\} = 2x - 1$$

3. If $y = x - x^2$, then the derivative of y^2 w.r.t. x^2 is

(A) $2x^2 + 3x - 1$ (B) $2x^2 - 3x + 1$ (C) $2x^2 + 3x + 1$ (D) $2x^2 + 5x + 1$

Ans. **(B)**

Sol. $y = x - x^2$

$y^2 = x^2 + x^4 - 2x^3$

Let $u=y^2$

$u = x^2 + x^4 - 2x^3$

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$\frac{du}{dx} = 2x + 4x^3 - 6x^2$

$v = x^2 \Rightarrow dv/dx = 2x$

$\frac{du}{dv} = 2x^2 - 3x + 1$

4. The differential coefficient of $a^{\sin^{-1}x}$ w.r.t. $\sin^{-1}x$ is –

(A) $a^{\sin^{-1}x} \log_e a$ (B) $a^{\sin^{-1}x}$ (C) $\frac{a^{\sin^{-1}x}}{\sqrt{(1-x^2)}}$ (D) $a^{\sin^{-1}x}\sqrt{(1-x^2)}$

Ans. **(A)**

Sol. $y = a^{\sin^{-1}x}$ & $z = \sin^{-1}x$

$\frac{dy}{dx} = a^{\sin^{-1}x} \cdot \log_e a \times \frac{-1}{\sqrt{1-x^2}}$

$\frac{dz}{dx} = \frac{1}{\sqrt{1-x^2}}$

$\therefore \frac{dy}{dz} = \frac{dy/dz}{dz/dx} = a^{\sin^{-1}x}\log_e a$

5. The value of derivative of $\tan^{-1}\left(\frac{2x\sqrt{1-x^2}}{1-2x^2}\right)$ w.r.t $\sec^{-1}\left(\frac{1}{2x^2-1}\right)$ at $x = 1/2$ equals-

(A) 1 (B) -1 (C) 0 (D) 2

Ans. **(B)**

Sol. $y = \tan^{-1}\left(\frac{2x\sqrt{1-x^2}}{1-2x^2}\right)$ & $z = \sec^{-1}\frac{1}{2x^2-1}$

put $x = \sin\theta$ & $z = \cos^{-1}(2x^2 - 1)$

$y = \tan^{-1}\left(\frac{2\sin\theta\cos\theta}{1-2\sin 2\theta}\right)$ & $\frac{dz}{dx} = \frac{-2}{\sqrt{1-x^2}}$

$y = 2\theta = 2\sin^{-1}x$

$\frac{dy}{dx} = \frac{2}{\sqrt{1-x^2}}$ $\therefore \frac{dy}{dz} = -1$

6. If $y = \cos^{-1}(\cos x)$ then $\frac{dy}{dt}$ at $x = \frac{5\pi}{4}$ is equal to

(A) 1 (B) -1 (C) $\frac{1}{\sqrt{2}}$ (D) $-\frac{1}{\sqrt{2}}$

Ans. **(B)**

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Sol. $y = \cos^{-1}(\cos x)$

$$y' = \frac{-1}{\sqrt{1-\cos^2 x}} x - \sin x = \frac{\sin x}{|\sin x|}$$

$$y'|_{x=\frac{5\pi}{4}=-1}$$

7. If $8f(x) + 6f\left(\frac{1}{x}\right) = x + 5$ and $y = x^2 f(x)$, then $\frac{dy}{dx}$ at $x = -1$ is equal to

(A) 0 (B) $\frac{1}{14}$ (C) $-\frac{1}{14}$ (D) 14

Ans. **(C)**

Sol. $8f(x) + 6f\left(\frac{1}{x}\right) = x + 5$

$y = x^2 f(x)$

$$\frac{dy}{dx} = x^2 \cdot f'(x) + 2xf(x)$$

$$8f'(x) - \frac{6}{x^2} f'\left(\frac{1}{x}\right) = 1$$

$$8f'(-1) - 6f'(-1) = 1$$

$$2f'(-1) = 1 \Rightarrow f'(-1) = \frac{1}{2}$$

$$f(-1) = \frac{4}{14}$$

$$\left.\frac{dy}{dx}\right|_{x=-1} = 1 \times \frac{1}{2} - 2 \times \frac{4}{14} = \frac{7-8}{14} = \frac{-1}{14}$$

8. If $f(x) = x^n$, then the value of $f(1) - \frac{f'(1)}{1!} + \frac{f''(1)}{2!} - \frac{f'''(1)}{3!} + \cdots .. + \frac{(-1)^n f^n(1)}{n!}$ is

(A) 2^n (B) 2^{n-1} (C) 0 (D) 1

Ans. **(C)**

Sol. $f(x) = x^n$

$$f(1) - \frac{f'(1)}{11} + \frac{f''(1)}{21} - \cdots . + (+)\frac{f^n(1)}{n!}$$

$$f'(x) = n \cdot x^{n-1} \Rightarrow f'(1) = n$$

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$f''(x) = n(n-1)x^{n-2} \Rightarrow f''(1) = n(n-1)$

$f''(x) = n(n-1)(n-2)x^{n-3}$

$\Rightarrow f''(1) = n(n-1)(n-2)$

$\vdots$

$$f(1-x)^n = 1 - nx + \frac{n(n-1)x^2}{2!} - \cdots$$

$$0 = 1 - n + \frac{n(n-1)}{2!} + \cdots$$

9. If f is differentiable in $(0,6)$ & $f'(4) = 5$ then $\mathrm{Lim}_{x\to 2} \frac{f(4) - f(x^2)}{2-x} =$

(A) 5 (B) $\frac{5}{4}$ (C) 10 (D) 20

Ans. **(D)**

Sol. $f'(4) = 5, \lim_{x\to 2} \frac{f(4)-f(x^2)}{2-x}$

$$f'(x)\lim_{x\to 2} 0 - \frac{f'(x^2)\cdot 2x}{-1}$$

$= f'(4).4 = 20$

10. If $u = ax + b$ then $\frac{d^n}{dx^n}(f(ax+b))$ is equal to

(A) $\frac{d^n}{du^n}(f(u))$ (B) $a\frac{d^n}{du^n}(f(u))$

(C) $a^n\frac{d^n}{du^n}(f(u))$ (D) $a^{-n}\frac{d^n}{du^n}(f(u))$

Ans. **(C)**

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Sol.

$$\text{Let} y = f(ax+b)$$
$$\frac{dy}{dx} = af'(ax+b)$$
$$\frac{d^2y}{dx^2} = a^2 f''(ax+b)$$
$$\frac{d^3y}{dx^3} = a^3 f'''(ax+b)$$
$$\vdots$$
$$\frac{d^ny}{dx^n} = a^n f^n(ax+b)$$
$$\Rightarrow \frac{d^n}{dx^n} f(ax+b) = a^n f^n(u)$$
$$= a^n \frac{d^n}{du^n} f(u)$$

11. Let f(x) be a polynomial in x. Then the second derivative of $f(e^x)$, is

(A) $f''(e^x) \cdot e^x + f'(e^x)$
(B) $f''(e^x) \cdot e^{2x} + f'(e^x) \cdot e^{2x}$
(C) $f'(e^x)e^{2x}$
(D) $f''(e^x) \cdot e^{2x} + f'(e^x) \cdot e^x$

Ans. (D)

Sol. $y = f(x) = f(e^x)$

$$\frac{dy}{dx} = f'(e^x) \cdot e^x \Rightarrow \frac{d^2y}{dx^2} = f''(e^x)e^{2x} + e^x f'(e^x)$$

12. If $y = f(x)$ is an odd differentiable function defined on $(-\infty, \infty)$ such that $f'(3) = -2$, then $f'(-3)$ equals

(A) 4 (B) 2 (C) -2 (D) 0

Ans. (C)

Sol. $y = f(x)$

$f(-x) = -f(x) \Rightarrow -f'(-x) = -f'(x)$

$f'(3) = f'(-3) = -2$

13. If g is inverse of f and $f'(x) = \frac{1}{1+x^n}$, then $g'(x)$ equals –

(A) $1 + x^n$ (B) $1 + (f(x))^n$ (C) $1 + (g(x))^n$ (D) $1 - x^n$

Ans. (C)

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Sol. $\because$ g is inverse of f(x)

$fog\ (x) = x$

$\Rightarrow f'(g(x)) \cdot g'(x) = 1$

$\Rightarrow g'(x) = \frac{1}{f'(g(x))}$

$= \frac{\frac{1}{1}}{1+(g(x))^n} = 1 + (g(x))^n$

14. Derivative of $\log_e(\log_e|\sin x|)$ with respect to x at $x = \frac{\pi}{6}$ is

(A) $-\frac{\sqrt{3}}{\log_e 2}$ (B) $\frac{\sqrt{3}}{\log_e 2}$ (C) $-\frac{\sqrt{3}}{2\log 2}$ (D) does not exist

Ans. **(D)**

Sol. for any real value of x, $|\sin x| \in [0,1]$

$\Rightarrow \log_e|\sin x| < 0$

$\Rightarrow \log_e(\log_e|\sin x|)$ does not exists at$x = \frac{\pi}{6}$ and so the derivative does not exist.

15. If $f(x) = f'(x) + f''(x) + f'''(x) + f''''(x) \ldots \ldots \ldots \infty$ also $f(0) = 1$ and f(x) is a differentiable function indefinitely then f(x) has the value

(A) e^x (C) $e^{x/2}$ (B) e^{2x} (D) e^{4x}

Ans. **(B)**

Sol. $f(x) = f'(x) + f''(x) + \cdots . \infty$

$f(0) = 1$

16. If $f(x) = |(x-4)(x-5)|$, then $f'(x)$ is

(A) $-2x + 9$, for all $x \in R$ (B) $2x - 9$ if $x > 5$

(C) $-2x + 9$ if $4 < x < 5$ (D) not defined for $x = 4, 5$

Ans. **(B, C, D)**

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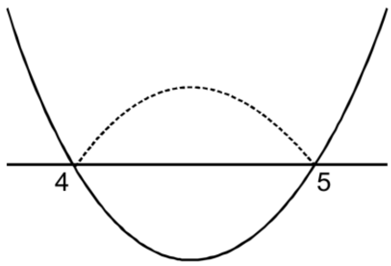
Sol. Not Differentiable the for $x = y$ & 5

$$f(x) = \begin{cases} (x-4)(x-5) & x < 4 \\ -(x-4)(x-5) & 4 < x < 5 \\ (x-4)(x-5) & x > 5 \end{cases}$$



$x > 5$

$f'(x) = 2x - 9$

$4 < x < 5$

$f'(x) = -2x + 9$

17. If f is twice differentiable such that $f''(x) = -f(x)$ and $f'(x) = g(x)$. If $h(x)$ is twice differentiable function such that $h'(x) = [f(x)]^2 + [g(x)]^2$. If $h(0) = 2$, $h(1) = 4$, then the equation $y = h(x)$ represents

(A) a curve of degree 2

(B) a curve passing through the origin

(C) a straight line with slope 2

(D) a straight line with y intercept equal to 2.

Ans. **(C, D)**

Sol. $f''(x) = -f(x), f'(x) = g(x)$

$h'(x) = (f(x))^2 + (g(x))^2$

$h''(x) = 2f(x)f'(x) + 2g(x) \cdot g'(x)$

$f'(x) = +g(x)$

$f''(x) = g'(x) = -f(x)$

$h''(x) = -2g'(x) \cdot g(x) + 2g(x)g'(x)$

$h'(x) = k$

$h(x) = kx + c$

$h(0) = 2 \quad \Rightarrow \quad h(1) = 4$

$c = 2 \quad \Rightarrow \quad k = 2$

$h(x) = 2x + 2$

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18. Two functions f & g have first & second derivatives at $x = 0$ satisfy the relations,

$f(0) = \frac{2}{g(0)}, f'(0) = 2\, g'(0) = 4g(0), g''(0) = 5f'(0) = g(0) = 3$ then

(A) if $h(x) = \frac{f(x)}{g(x)}$ then $h'(0) = \frac{32}{9}$

(B) if $k(x) = f(x) \cdot g(x)\sin x$ then $k'(0) = 2$

(C) $\text{Lim}_{x\to 0} \frac{g'(x)}{f'(x)} = \frac{1}{2}$

(D) $f'(x) = g'(x)$

Ans. **(A, B, C)**

Sol. $f, g, f(0) = \frac{2}{g(0)}$

$f'(0) = 2g'(0) = 4g(0), g''(0) = 5f''(0) = g(0) = 3$

(A) $h(x) = \frac{f(x)}{g(x)} \Rightarrow h'(x) = \frac{f'(g) - g'f}{g^2}$

$= h'(0) = \frac{f'(0)g(0) - g'(0)f(0)}{g^2(0)}$

$= h'(0) = \frac{4(g(0))^2 - 2g(0)\cdot\frac{2}{g(0)}}{9}$

$h'(0) = \frac{36-4}{9} = \frac{32}{5}$ (A)

$k(x) = f(x) \cdot g(x) \cdot \sin x$

$k'(x) = f(x)g(x)\sin x + f(x)g'(x)\sin x + f'(x)g(x)\sin x$

$k'(x) = 2$

$\text{Lim}_{x\to 0} \frac{g'(x)}{f'(x)} = \text{Lim}_{x\to 0} \frac{g'(x)}{f'(x)} = \frac{1}{2}$

19. Differentiate the following functions with respect to x.

(i) $x^{2/3} + 7e - \frac{5}{x} + 7\tan x$

Ans. $\frac{2}{3}x^{-\frac{1}{3}} + \frac{5}{x^2} + 7\sec^2 x$

Sol. $y = x^{2/3} + 7e - \frac{5}{x} + 7\tan x$

$\frac{dy}{dx} = \frac{2}{3}x^{-1/3} + \frac{5}{x^2} + 7\sec^2 x$

(ii) $\ln\tan\left(\frac{\pi}{4} + \frac{x}{2}\right)$

Ans. $\sec x$

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Sol. $y = \ell \tan\left(\frac{\pi}{4}+\frac{x}{2}\right)$

$$\frac{dy}{dx} = \frac{\sec^2\left(\frac{\pi}{4}+\frac{x}{2}\right)}{\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)} \times \frac{1}{2}$$

$$= \frac{1}{\sin\left(\frac{\pi}{2}+x\right)} \Rightarrow \sec x$$

(iii) $\frac{\sin x\ - x\cos x}{x\sin x + \cos x}$

Ans. $\frac{x^2}{(x\sin x + \cos x)^2}$

Sol. $y = \frac{\sin x - x\cos x}{x\sin x + \cos x}$

$$y = \frac{\tan x - x}{x\tan x + 1}$$

$$y' = \frac{(x\tan x + 1)(\sec^2 x - 1) - (\tan x - x)(x\sec^2 x + \tan x)}{(x + \tan x + 1)^2}$$

$x\tan x\sec^2 x - x\tan x + \sec^2 x - 1 - x\sec^2 x\tan x$

$$= \frac{-\tan^2 x + x^2\sec^2 x + x\tan x}{(x\tan x + 1)^2}$$

$$\frac{dy}{dx} = \frac{x^2\sec^2 x}{(x\tan x + 1)^2}$$

$$\frac{dy}{dx} = \frac{x^2}{(x\sin x + \cos x)^2}$$

(iv) $\tan\left(\tan^{-1}\sqrt{\frac{1-\cos x}{1+\cos x}}\right)$

Ans. $\frac{1}{2}\sec^2\left(\frac{x}{2}\right)$

Sol. $y = \tan\left(\tan^{-1}\sqrt{\frac{1-\cos x}{1+\cos x}}\right)$

$$y = \tan\left\{\tan^{-1}\sqrt{\frac{2\sin^2\frac{x}{2}}{2\cos^2\frac{x}{2}}}\right\}$$

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$y = \tan\left\{\tan^{-1}\left(\tan\frac{x}{2}\right)\right\}$

$\frac{dy}{dx} = \frac{1}{2}\sec^2\left(\frac{x}{2}\right)$

20. Differentiate $x^2 . \ln x . e^x$ with respect to x.

Ans. $x^2 \ln x \cdot e^x + xe^x + 2xe^x \ln x$

Sol. $y = x^2 \cdot \ln x \cdot e^x$

$\frac{dy}{dx} = x^2 \ln x \cdot e^x + xe^x + 2xe^x \ln x$

21. If $\cos\frac{x}{2} \cdot \cos\frac{x}{2^2} \cdot \cos\frac{x}{2^3} \dots \infty = \frac{\sin x}{x}$ then find the value of $\frac{1}{2^2}\sec^2\frac{x}{2} + \frac{1}{2^4}\sec^2\frac{x}{2^2} + \frac{1}{2^6}\sec^2 + \dots \infty$.

Ans. $\operatorname{cosec}^2 x - \frac{1}{x^2}$

Sol. Take In on both the side

$\ln\cos\frac{x}{2} + \ln\cos\frac{x}{2^2} + \cdots \dots . = \ln\sin x - \ln x$

Diff. w.r.t. x

$\frac{-1}{2}\tan\frac{x}{2} - \frac{1}{2^2}\tan\frac{x}{2^2} \dots \dots = \cot x - \frac{1}{x}$

Again diff. w.r.t. x

$\frac{-1}{2}\sec^2\frac{x}{2} - \frac{1}{2^4}\sec^2\frac{x}{2^2} + \cdots . = -\operatorname{cosec}^2 x + \frac{1}{x^2}$

$\frac{1}{2^2}\sec^2\frac{x}{2} + \frac{1}{2^4}\sec^2\frac{x}{2^2} + \cdots .. = \operatorname{cosec}^2 x - \frac{1}{x^2}$

22. Let f , g and h are differentiable functions. If $f(0) = 1; g(0) = 2; h(0) = 3$ and the derivatives of their pair wise products at $x = 0$ are $(fg)'(0) = 6$; $(gh)'(0) = 4$ and $(hf)'(0) = 5$ then compute the value of $(fgh)'(0)$.

Ans. 16

Sol. $f(0) = 1, g(0) = 2, h(0) = 3$

$(fg)'(0) = 6, (gh)'(0) = 4, (hf)'(0) = 5$

$(fgh)'(0) = \frac{h(0)(fg')(0)+f(0)(gh)'(0)+g(0)(hf)'(0)}{2}$

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$= \frac{3\times6+1\times4+2\times5}{2} = \frac{18+4+10}{2}$

$=16$

23. If $f : R \to R$ is a function such that $f(x) = x^3 + x^2f'(1) + xf''(2) + f'''(3)$ for all $x \in R$, then prove that $f(2) = f(1) - f(0)$.

Sol. $f(x) = x^3 + x^2f'(1) + xf''(2) + f'''(3)$

$$\left.\begin{array}{r} f'(x) = 3x^2 + 2xf'(1) + f''(2) \\ f''(x) = 6x + 2f'(1) \\ f'''(x) = 6 \end{array}\right\} \Rightarrow \begin{array}{l} f'(1) = -5 \\ f''(2) = 2 \\ f''(3) = 6 \end{array}$$

$f(2) = 8 + 4 \times 5 + 2 \times 2 + 6 = -2$

$f(1) = 1 + 1 \times -5 + 1 \times 2 + 6 = 4$

$f(0) = f'''(3) = 6$

$f(2) = -2, f(1) = f(0) = 4 - 6 = -2$

So, $f(2) = f(1) - f(0)$

Paragraph for Question Nos. 24 to 26

$f(x)$ is a polynomial function $f : R \to R$ such that $f(2x) = f'(x)f''(x)$.

24. The value of $f(3)$ is

(A) 4 (B) 12 (C) 15 (D) 11

Ans. **(B)**

Sol. $f(2x) = f'(x)f''(x)$

$\downarrow \quad \downarrow \quad \downarrow$

$(n-1)(n-2)$

$n = n - 1 + n - 2$

$n = 3$

$f(x) = ax^3 + bx^2 + cx + d$

$a.8x^3 + b \cdot 4x^2 + c(2x) + d$

$= (3ax^2 + 2bx + c)(6ax + 2b)$

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$8a = 18a^2 - a$

$2a(9a - 4) = 0$

$a = 4/9$

Simplifies by component we find $b = c = d = 0$

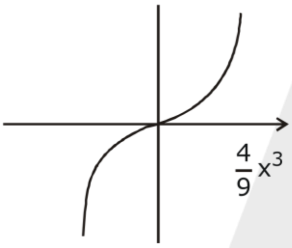
$f(x) = \frac{4}{9}x^3$

25. f(x) is:

(A) one-one and onto
(B) one-one and into
(C) many-one and onto
(D) many-one and into

Ans. **(A)**

Sol.



one-one + onto

26. Equation $f(x) = x$ has:

(A) three real and positive roots
(B) three real and negative roots
(C) one real root
(D) three real such that sum of roots is zero

Ans. **(D)**

Sol. $\frac{4}{9}x^3 = x \Rightarrow x = 0, +\frac{3}{2}$

Sum of roots $= 0$

27. Let $f : (-1,1) \to R$ be a differentiable function with $f(0) =- 1$ and $f'(0) = 1$.

Let $g(x) = [f(2f(x) + 2)]^2$, then $g'(0) =$

(A) 4
(C) 0
(B) -4
(D) -2

Ans. **(B)**

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Sol. We have, f: $(-1,1) \longrightarrow R$

$f(0) = -1$ $\quad\quad$ $f'(0) = 1$

$g(x) = [f(2f(x) + 2]^2$

$\Rightarrow g'(x) = 2[f(2f(x) + 2)] \times f'(2f(x) + 2) \times 2f'(x)$

$\Rightarrow g'(0) = 2[f(2f(0) + 2)] \times f'(2f(0) + 2) \times 2f'(0)$

$= 2[f(0)] \times f'(0) \times 2f'(0)$

$= 2 \times (-1) \times 1 \times 2 \times 1 = -4$

28. If $f(x) = \frac{2x-4}{x^2-1}$ and $f'(x) = \frac{p}{(x^2-1)^2}$, then p equals-

(A) $x^2 - 8x - 2$ (B) $-2x^2 + 8x + 2$

(C) $4x + 2$ (D) $-2x^2 + 8x - 2$

Ans. (D)

Sol. $f(x) = \frac{2x-4}{x^2-1}$

$$\Rightarrow f'(x) = \frac{(x^2-1)(2) - (2x-4)(2x)}{(x^2-1)^2}$$

$$= \frac{2x^2 - 2 - 4x^2 + 8x}{(x^2-1)^2} = \frac{-2x^2 + 8x - 2}{(x^2-1)^2}$$

$\therefore p = -2x^2 + 8x - 2$

29. If $y = \sqrt{\frac{1-\cos x}{1+\cos x}}$, $x \in (0, \pi)$ then $\frac{dy}{dx}$ equals-

(A) $\frac{1}{2}\sec^2 x/2$ (B) $\frac{1}{2}\operatorname{cosec}^2 x/2$

(C) $\sec^2 x/2$ (D) $\operatorname{cosec}^2 x/2$

Ans. (A)

Sol. $y = \sqrt{\frac{1-\cos x}{1+\cos x}}$

$$\Rightarrow y = \sqrt{\frac{1-\cos x}{1+\cos x}} \times \sqrt{\frac{1-\cos x}{1-\cos x}}$$

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$$=\frac{1-\cos x}{\sqrt{1-\cos^2 x}}=\frac{1-\cos x}{\sin x}=\csc x-\cot x$$

$$\therefore \frac{dy}{dx}=\tan\left(\frac{x}{2}\right)\csc x$$

$$=\frac{\sin\left(\frac{x}{2}\right)}{\cos\frac{x}{2}\left(2\sin\left(\frac{x}{2}\right)\cos\left(\frac{x}{2}\right)\right)}$$

$$=\frac{1}{2}\sec^2\left(\frac{x}{2}\right)$$

30. $\frac{d}{d\theta}\left\{\tan^{-1}\left(\frac{1-\cos\theta}{\sin\theta}\right)\right\}$ equals-

(A) 1/2 (B) 1 (C) $\sec\theta$ (D) $\mathrm{cosec}\theta$

Ans. **(A)**

Sol. Step 1: Use basic trigonometric formulas and make substitution

Let $u=\frac{1-\cos(\theta)}{\sin(\theta)}$

Then $u=\frac{2\sin^2\left(\frac{\theta}{2}\right)}{2\cos\left(\frac{\theta}{2}\right)\sin\left(\frac{\theta}{2}\right)}$

[Using formula $1-\cos 2\theta=2\sin^2\theta$ and $\sin 2\theta=2\sin\theta\cos\theta$]

$$\Rightarrow u=\frac{\sin\left(\frac{\theta}{2}\right)}{\cos\left(\frac{\theta}{2}\right)}$$

$$\Rightarrow u=\tan\left(\frac{\theta}{2}\right)$$

Step 2: Substitute and find the required differentiation

We can write

$$\tan^{-1}\left(\frac{1-\cos(\theta)}{\sin(\theta)}\right)=\tan^{-1}\left(\tan\left(\frac{\theta}{2}\right)\right)=\frac{\theta}{2}$$

$$\Rightarrow \frac{d}{d\theta}\left(\tan^{-1}\left(\tan\frac{\theta}{2}\right)\right)=\frac{d}{d\theta}\left(\frac{\theta}{2}\right)=\frac{1}{2}$$

31. d/dx(secx°) equals -

(A) secx tanx (B) secx°tanx°

(C) $\left(\frac{\pi}{180}\right)secx^\circ$tan$x^\circ$ (D) $\left(\frac{\pi}{180}\right)$secx tanx

Ans. **(C)**

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Sol. Give $y = \sec^{0}$

$$= \sec\frac{x\pi}{180^\circ}\left(\because x^\circ = \frac{x\pi}{180^\circ}\text{ radians}\right)$$

$$\therefore \frac{dy}{dx} = \frac{\pi}{180^\circ}\sec x^\circ \tan x^\circ$$

32. If $a\cos^2(x+y) = b$, then dy/dx equals-

(A) 2 (B) -2 (C) 1 (D) -1

Ans. **(D)**

Sol. Differentiating both sides w.r.t x

$$2a\cos(x+y)\cdot(-\sin(x+y))\left[1+\frac{dy}{dx}\right] = 0$$

$$-a\sin(2x+2y)\left[1+\frac{dy}{dx}\right] = 0$$

$$x+y = \frac{n\pi}{2}\text{ or }\frac{dy}{dx}+1 = 0$$

In Both cases $\frac{dy}{dx} = -1$

33. If $y = \log\sqrt{\frac{1-\sin x}{1+\sin x}}$, then dy/dx equals-

(A) $\sec x$ (B) $-\sec x$

(C) $\operatorname{cosec} x$ (D) $\sec x \tan x$

Ans. **(B)**

Sol. $y = \log\sqrt{\frac{1-\sin x}{1+\sin x}}$

$$y = \log\left(\frac{1-\sin x}{1+\sin x}\right)^{\frac{1}{2}}$$

$$y = \frac{1}{2}\log\left(\frac{1-\sin x}{1+\sin x}\right)$$

$$y = \frac{1}{2}[\log(1-\sin x) - \log(1+\sin x)]$$

Differentiate with respect to ' x '

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$$\frac{dy}{dx}=\frac{1}{2}\left[\frac{1}{1-\sin x}(-\cos x)-\frac{1}{1+\sin x}(\cos x)\right]$$

$$\frac{dy}{dx}=\frac{-\cos x}{2}\left[\frac{1}{1-\sin x}+\frac{1}{1+\sin x}\right]$$

$$\frac{dy}{dx}=\frac{-\cos x}{2}\left(\frac{1+\sin x+1-\sin x}{(1-\sin x)(1+\sin x)}\right)$$

$$\frac{dy}{dx}=\frac{-\cos x}{2}\left(\frac{2}{1-\sin^2 x}\right)$$

$$\frac{dy}{dx}=\frac{-\cos x}{2}\left(\frac{2}{\cos^2 x}\right)$$

$$\frac{dy}{dx}=\frac{-1}{\cos x}$$

$$\Longrightarrow \frac{dy}{dx}=-\sec x$$

34. If $y=\log_{10}(\sin x)$, then dy/dx equals-

(A) $\sin x \log_{10} e$ (B) $\cos x \log_{10} e$

(C) $\cot x \log_{10} e$ (D) $\cot x$

Ans. **(C)**

Sol. $y=\log_{10}\sin(x)$

$$\frac{dy}{dx}=\frac{1}{\sin x}\times\cos x$$

$\frac{dy}{dx}=\cot x \ldots\ldots\ldots\cdot \log 10=1.$

35. The derivative of $x|x|$ is-

(A) $2x$ (B) $-2x$ (C) $2|x|$ (D) Does not exist

Ans. **(C)**

Sol. $f(x)=x|^{x}|$

When $x<0$, then $|x|=-x$

$f(x)\ =x\times(-x)$

$=-x^2$

When $x>0$, then $|x|=x$ Therefore,

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$f(x) = x \times x$

$= x^2$

So, $f'(x) \ = \frac{d}{dx}(-x^2)$

$= -2x$, when $x < 0 \rightarrow (1)$

And, $f'(x) = \frac{d}{dx}(x^2)$

$= 2x$, when $x < 0 \rightarrow (2)$

Therefore; $f'(X) \ = 2 \times |X|$

$= 2|x|$

ANSWER KEY

1.	**(C)**	**2.**	**(C)**	**3.**	**(B)**	**4.**	**(A)**	**5.**	**(B)**	**6.**	**(B)**	**7.**	**(C)**
8.	**(C)**	**9.**	**(D)**	**10.**	**(C)**	**11.**	**(D)**	**12.**	**(C)**	**13.**	**(C)**	**14.**	**(D)**
15.	**(B)**	**16.**	**(BCD)**	**17.**	**(CD)**	**18.**	**(ABC)**						

19. **(i)** $\frac{2}{3}x^{-\frac{1}{3}} + \frac{5}{x^2} + 7\sec^2 x$ **(ii)** $\sec x$

(iii) $\frac{x^2}{(x\sin x + \cos x)^2}$ **(iv)** $\frac{1}{2}\sec^2\left(\frac{x}{2}\right)$

20. $x^2 \ln x \cdot e^x + xe^x + 2xe^x \ln x$ **21.** $\text{cosec}^2 x - \frac{1}{x^2}$

22.	16	**24.**	(B)	**25.**	(A)	**26.**	(D)	**27.**	(B)	**28.**	(D)	**29.**	(A)
30.	(A)	**31.**	(C)	**32.**	(D)	**33.**	(B)	**34.**	(C)	**35.**	(C)		