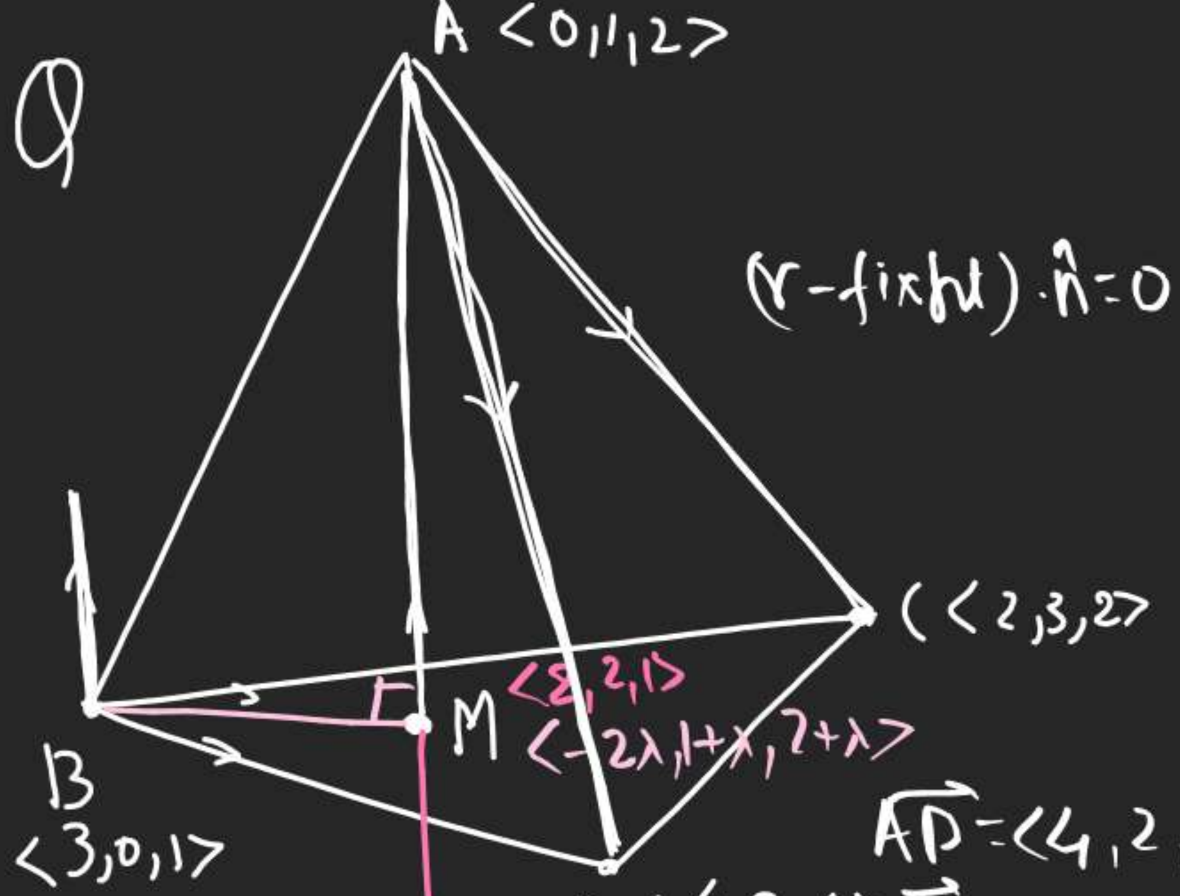
Nishant Jindal

Q



$(\vec{r}-\text{fixed pt})\cdot\hat{n}=0$

$\vec{AD} = \langle 4,2,4\rangle$

$\vec{AC} = \langle 2,2,0\rangle$

$\vec{AD}\times\vec{AC} =$

① Centre of tetrahedron:

$$\left\langle \frac{0+3+4+2}{4}, \frac{1+0+3+3}{4}, \frac{2+1+6+2}{4}\right\rangle$$

$A'\langle\alpha,\beta,\gamma\rangle$

$$\left\langle \frac{9}{4}, \frac{7}{4}, \frac{11}{4}\right\rangle$$

② Unit vector $\perp^r$ to plane BCD

$$\hat{n} = \frac{\vec{BD}\times\vec{BC}}{|\vec{BD}\times\vec{BC}|}$$

$\vec{BD} = \langle 1,3,5\rangle$

$\vec{BC} = \langle -1,3,1\rangle$

$$\vec{BD}\times\vec{BC} = \begin{vmatrix} i & j & k \\ 1 & 3 & 5 \\ -1 & 3 & 1 \end{vmatrix}$$

$$= \langle -12,-6,6\rangle$$

$$|BD\times BC| = \sqrt{144+36+36}$$

$$= \sqrt{216}$$

$$\hat{n} = \frac{\langle -12,-6,6\rangle}{6\sqrt{6}}$$

$$\boxed{\hat{n} = \frac{\langle -2,-1,1\rangle}{\sqrt{6}}}$$

(4.5) $\perp^r$ distance of A from BCD

$$\sqrt{(0-2)^2+(1-2)^2+(2-1)^2} = \sqrt{6}$$

(3) Eqn of line $\perp^r$ to BCD & P.T. A

Line's DR same as $\hat{n}$'s DR

EOL $\to \vec{r} = \langle 0,1,2\rangle + \lambda\langle -2,1,1\rangle$

④ Foot of $\perp^r$ from A to BCD.

↓

Pt. M lying on previous line

① Gen Pt $= \langle -2\lambda, 1-\lambda, 2+\lambda\rangle$

AM's DR $= \langle -2\lambda, -\lambda, \lambda\rangle$

BM's DR $= \langle -2\lambda-3, 1-\lambda, 1+\lambda\rangle$

$AM\perp BM \Rightarrow 4\lambda^2+6\lambda-\lambda+\lambda^2+\lambda^2+\lambda=0$

$6\lambda^2+6\lambda=0,\ \lambda=-1$

$M = \langle 2, 2, 1\rangle$

(5) Image of A in Plane BCD

$$\frac{\alpha+0}{2}=2 \ \Big|\ \frac{\beta+1}{2}=2 \ \Big|\ \frac{\gamma+2}{2}=1$$

$\alpha=4, \beta=3, \gamma=2 \Rightarrow \langle 4,3,0\rangle$

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(6) Eq^n of Plane ACD

$\vec{n} = \vec{AD} \times \vec{AC}$

$\langle 4,2,4\rangle \times \langle 2,2,0\rangle$

$$= \begin{vmatrix} i & j & k \\ 4 & 2 & 4 \\ 2 & 2 & 0 \end{vmatrix}$$

$= \langle -8, 8, 4\rangle \text{ or } \underline{\langle -2,2,1\rangle}$

$(r - \langle 0,1,2\rangle) \cdot \langle -2,2,1\rangle = 0$

$-2x + 2y + z = 0 + 2 + 2$

$-2x + 2y + z = 4$

(7) Angle bet^n Plane ACD & BCD.

Angle bet^n Planes can be find out by angle bet^n their Normal

$$\cos\theta = \frac{\vec{n}_{ACD} \cdot \vec{n}_{BCD}}{|n_1| \cdot |n_2|}$$

$$= \frac{\langle -2,2,1\rangle \cdot \langle -2,-1,1\rangle}{\sqrt{9}\ \sqrt{6}}$$

$$= \frac{4 + -2 + 1}{3 \times \sqrt{6}} = \frac{1}{\sqrt{6}}$$

$$\theta = \cos^{-1}\frac{1}{\sqrt{6}}$$

(1) Centre of tetrahedron

(2) Normal Vector $\perp^r$ to Plane BCD

(3) Normal Vector of ACD Plane.

(4) Angle bet^n 2 Plane ACD & BCD

(5) Eq^n of line P.T. A & $\perp^r$ to Plane BCD

(6) Foot of $\perp^r$ from given Pt A $\perp^r$ to Plane BCD

(7) $\perp^r$ distance of Pt A from Plane.

(8) Image of Pt. A in Plane BCD

(9) Area of Plane $\frac{1}{2}|BC \times BD|$

(10) Eq^n of Plane ACD.

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Q $P_1: \vec{r}\cdot\langle 2\hat{i}-\hat{j}+\hat{k}\rangle=6$

$P_2: \vec{r}\cdot\langle \hat{i}+2\hat{j}+\lambda\hat{k}\rangle=1$

are $\perp^r$ find $\lambda=?$

$\vec{n}_1 \perp \vec{n}_2$

$2-2+\lambda=0 \Rightarrow \lambda=0$

Q Angle betn $P_1: \vec{r}\cdot(2\hat{i}-\hat{j}+\hat{k})=6$

$P_2: \vec{r}\cdot(\hat{i}+\hat{j}+2\hat{k})=0$?

$$\cos\theta=\frac{\vec{n}_1\cdot\vec{n}_2}{|\vec{n}_1||\vec{n}_2|}=\frac{\langle 2,-1,1\rangle\cdot\langle 1,1,2\rangle}{\sqrt{6}\,\sqrt{6}}$$

$$=\frac{2-1+2}{6}=\frac{1}{2}$$

$\theta=60^\circ$

SCALAR TRIPLE PRODUCT

[STP]

1) dot Product of a vector $\vec{a}$ With cross Product of another 2 vector $(\vec{b}\times\vec{c})$ is STP.

$\vec{a}\cdot(\vec{b}\times\vec{c})$

$\vec{m}\cdot(\vec{n}\times\vec{p})$

(2) $\vec{a}\cdot(\vec{b}\times\vec{c})$ or $(\vec{b}\times\vec{c})\cdot\vec{a}$ is same, but for the Sake of Arrangement We take single vector at 1st Place

(3) $a\cdot(b\times c)=[a\ b\ c]$

$(a\times b)\cdot\vec{c}=?$

$\Rightarrow \vec{c}\cdot(\vec{a}\times\vec{b})=[c\ a\ b]$

(4) for 3 vector $\vec{a},\vec{b},\vec{c}$

$\vec{a}$ $\vec{b}$ $\vec{c}$

$[a\ b\ c]=[b\ c\ a]=[c\ a\ b]$

(5) $[\hat{i}\ \hat{j}\ \hat{k}]=?$

$\hat{i}\cdot(\hat{j}\times\hat{k})=\hat{i}\cdot\hat{i}=1$

Q $[2\hat{i} \;\; 3\hat{j}+\hat{k} \;\; \hat{i}] = ?$

$2\hat{i}\cdot((3\hat{j}+\hat{k})\times\hat{i}) = 0$

(6) $a\cdot(b\times c) = [a\ b\ c]$

Dot & Cross can be Exchanged

$= (a\times b)\cdot\vec{c}$

$= \vec{c}\cdot(\vec{a}\times\vec{b})$

$= [c\ a\ b]$

Q WOTF are Equivalent

(1) $\vec{U}\cdot(\vec{V}\times\vec{W}) = [U\ V\ W]$ ✓

(2) $(V\times W)\cdot U = U\cdot(V\times W) = [U\ V\ W]$

(3) $V\cdot(U\times W) = [V\ U\ W]$ ×

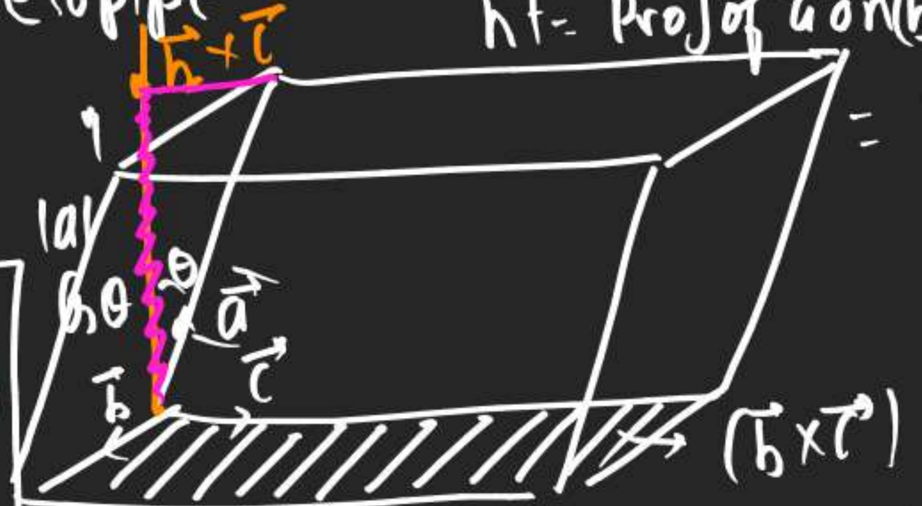
(4) $(U\times V)\cdot W = \vec{W}\cdot(\vec{U}\times\vec{V}) = [W\ U\ V]$

(1, 2, 4)

$[U\ V\ W] = [V\ W\ U] = [W\ U\ V]$

(7) Parallelopipe

ht = Proj of $\vec{a}$ on $(b\times c)$



Vol of ||pipe = (Base Area) × ht

$= |b\times c|\cdot|a|\cos\theta$

$= \vec{a}\cdot(\vec{b}\times\vec{c}) = [a\ b\ c]$

* Volume of ||pipe having Coterminous Edges

$\vec{a},\vec{b},\vec{c} = |a||b\times c|\cos\theta$

$= [a\ b\ c]$ — $\vec{a}$ & $\vec{b}\times\vec{c}$ के बीच Angle

*2 $V = |a||b||c|\sin\phi\cos\theta$

ϕ = Angle betn $\vec{b}$ & $\vec{c}$

θ = " " $\vec{a}$ & $(\vec{b}\times\vec{c})$

*3 Max. volume = 1, $\sin\phi = 1$ & $\cos\theta = 1$

$\phi = \frac{\pi}{2}$ & $\theta = 0$

||pipe = Cuboid

Q If $\vec{a}\cdot\hat{i} = 4$ then $(\vec{a}\times\hat{j})\cdot(2\hat{j}-3\hat{k}) = ?$

$|m|\cdot|n|\cos\theta = (\vec{m}\cdot\vec{n})$

$\vec{a}\cdot\vec{b} = |a||b|\cos\theta$

$(a\times\hat{j})\cdot(2\hat{j}-3\hat{k})$

$\vec{a}\cdot(\hat{j}\times(2\hat{j}-3\hat{k}))$

$\vec{a}\cdot(0-3\cdot\hat{i}) = -3\,\vec{a}\cdot\hat{i} = -12$

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*4 $\vec{a}\cdot(\vec{b}\times\vec{c})$ = Vector Volume of || piped

$$(a_1\hat{i}+a_2\hat{j}+a_3\hat{k})\cdot\begin{vmatrix} i & j & k \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

$$= a_1(b_2c_3-b_3c_2)+a_2(b_3c_1-b_1c_3)+a_3(b_1c_2-b_2c_1)$$

$$[abc] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

Best Mode

$$[\vec{a}\ \vec{b}\ \vec{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}[i\ j\ k]$$

8) $[abc]>0$ RHSystem

$[abc]<0$ LHSystem

9) $[\vec{a}\,\vec{b}\,\vec{c}]\cdot[\vec{l}\,\vec{m}\,\vec{n}]$

$$= \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}\times\begin{vmatrix} l_1 & l_2 & l_3 \\ m_1 & m_2 & m_3 \\ n_1 & n_2 & n_3 \end{vmatrix}$$

$a\cdot b = a_1b_1+a_2b_2+a_3b_3$

$$= \begin{vmatrix} a_1l_1+a_2l_2+a_3l_3 & a_1m_1+a_2m_2+a_3m_3 & a_1n_1+a_2n_2+a_3n_3 \\ b_1l_1+b_2l_2+b_3l_3 & b_1m_1+b_2m_2+b_3m_3 & b_1n_1+b_2n_2+b_3n_3 \\ c_1l_1+c_2l_2+c_3l_3 & c_1m_1+c_2m_2+c_3m_3 & c_1n_1+c_2n_2+c_3n_3 \end{vmatrix}$$

$$= \begin{vmatrix} \vec{a}\cdot\vec{l} & \vec{a}\cdot\vec{m} & \vec{a}\cdot\vec{n} \\ b\cdot l & b\cdot m & b\cdot n \\ \vec{c}\cdot l & c\cdot m & c\cdot n \end{vmatrix} \checkmark$$

Bhararth:- $[abc][abc] = [abc]^2 = \begin{vmatrix} a\cdot a & a\cdot b & a\cdot c \\ b\cdot a & b\cdot b & b\cdot c \\ c\cdot a & c\cdot b & c\cdot c \end{vmatrix}$

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10) Vol. of ||pipe

$= [a\,b\,c] = \sqrt{[a\,b\,c]^2}$

$= \sqrt{\begin{vmatrix} a\cdot a & a\cdot b & a\cdot c \\ b\cdot a & b\cdot b & b\cdot c \\ c\cdot a & c\cdot b & c\cdot c \end{vmatrix}}$

Q let $\vec{u}, \vec{v}, \vec{w}$ 3 vectors in space where $\vec{u}$ & $\vec{v}$ are unit vectors not $\perp^r$ to each other, $\vec{u}\cdot\vec{w} = 1$, $\vec{v}\cdot\vec{w} = 1$, $\vec{w}\cdot\vec{w} = 4$. If volume of ||pipe whose adjacent sides are represented by $\vec{u}, \vec{v}, \vec{w}$ is $\sqrt{2}$ then $|3\vec{u} + 5\vec{v}| = ?$

1) $[u\,v\,w]^2 = \begin{vmatrix} u\cdot u & u\cdot v & u\cdot w \\ v\cdot u & v\cdot v & v\cdot w \\ w\cdot u & w\cdot v & w\cdot w \end{vmatrix}$

$2 = \begin{vmatrix} 1 & x & 1 \\ x & 1 & 1 \\ 1 & 1 & 4 \end{vmatrix}$

$2 = \{4 + x + x\} - \{1 + 4x^2 + 1\}$

$4x^2 - 2x = 0 \Rightarrow x = 0$ or $\frac{1}{2}$ ✓

2) $|3\vec{u} + 5\vec{v}| = \sqrt{9u^2 + 25v^2 + 30\,u\cdot v}$

$= \sqrt{9\times 1 + 25\times 1 + 30\times\frac{1}{2}}$

$= 7$

Q Let $\vec{a}, \vec{b}, \vec{c}$ be 3 Non Coplanar unit vectors S.T. angle bet^n every pair of them is $\frac{\pi}{3}$. If $a\times b + b\times c = p\vec{a} + q\vec{b} + r\vec{c}$ where p, q, r are scalars then $\frac{p^2 + 2q^2 + r^2}{q^2} = ? = \frac{(-q)^2 + 2q^2 + (-q)^2}{q^2} = 4$

$b\cdot(a\times b)$ $\quad b\cdot(b\times c)$

$\vec{a}\times\vec{b} + \vec{b}\times\vec{c} = p\vec{a} + q\vec{b} + r\vec{c}\} \cdot \vec{a}, \vec{b}, \vec{c}$

(a) $0 + [a\,b\,c] = p + \frac{q}{2} + \frac{r}{2} = \frac{1}{\sqrt{2}}$

(b) $0 + 0 = \frac{p}{2} + q + \frac{r}{2} \Rightarrow p = r = -q$

(c) $[a\,b\,c] = \frac{p}{2} + \frac{q}{2} + r$

$[a\,b\,c]^2 = \begin{vmatrix} a\cdot a & a\cdot b & a\cdot c \\ b\cdot a & b\cdot b & b\cdot c \\ c\cdot a & c\cdot b & c\cdot c \end{vmatrix} = \begin{vmatrix} 1 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & 1 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & 1 \end{vmatrix}$

$= \left\{1 + \frac{1}{8} + \frac{1}{8}\right\} - \left\{\frac{1}{4} + \frac{1}{4} + \frac{1}{4}\right\} = \frac{1}{2}$

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11) $[a\ b\ b] = 0$

12) $[K\vec{a}\ \vec{b}\ \vec{c}] = ?$

$K[a\ b\ c]$

(13) $[\vec{a}+\vec{b}\ \ \vec{c}\ \ \vec{d}]$

$[\vec{a}\ \vec{c}\ \vec{d}] + [\vec{b}\ \vec{c}\ \vec{d}]$

Q $[\lambda a\ \ \lambda b\ \ u\vec{c}] = ?$

$\lambda^2 u[a\ b\ c]$ A

Q $[a+b\ \ b+c\ \ c+a]$ Cyclic Order

$= [a\ b\ c] + [b\ c\ a]$

$= 2[a\ b\ c]$

Q $[2a-b\ \ 2b-c\ \ 2c-a]$

$[2a\ \ 2b\ \ 2c] - [b\ c\ a]$

$8[a\ b\ c] - [a\ b\ c]$

$= 7[a\ b\ c]$

Q $[a\times b\ \ b\times c\ \ c\times a]$

$[a\ b\ c]\times[b\ c\ a]$

$[a\ b\ c]^2$

Q a, b, c are L.I vector then Non Coplanar $= [a\ b\ c] \neq 0$

WOTF is Coplanar → D bba = 0

A) $[a+b\ \ b+c\ \ c+a] = 2[a\ b\ c] \neq 0$ (NCP)

B) $[a\times b\ \ b\times c\ \ c\times a] = [a\ b\ c]^2 \neq 0$ (NCP)

c) $[a-b\ \ b-c\ \ c-a] = [a\ b\ c] - [b\ c\ a] = 0$ (Coplanar)