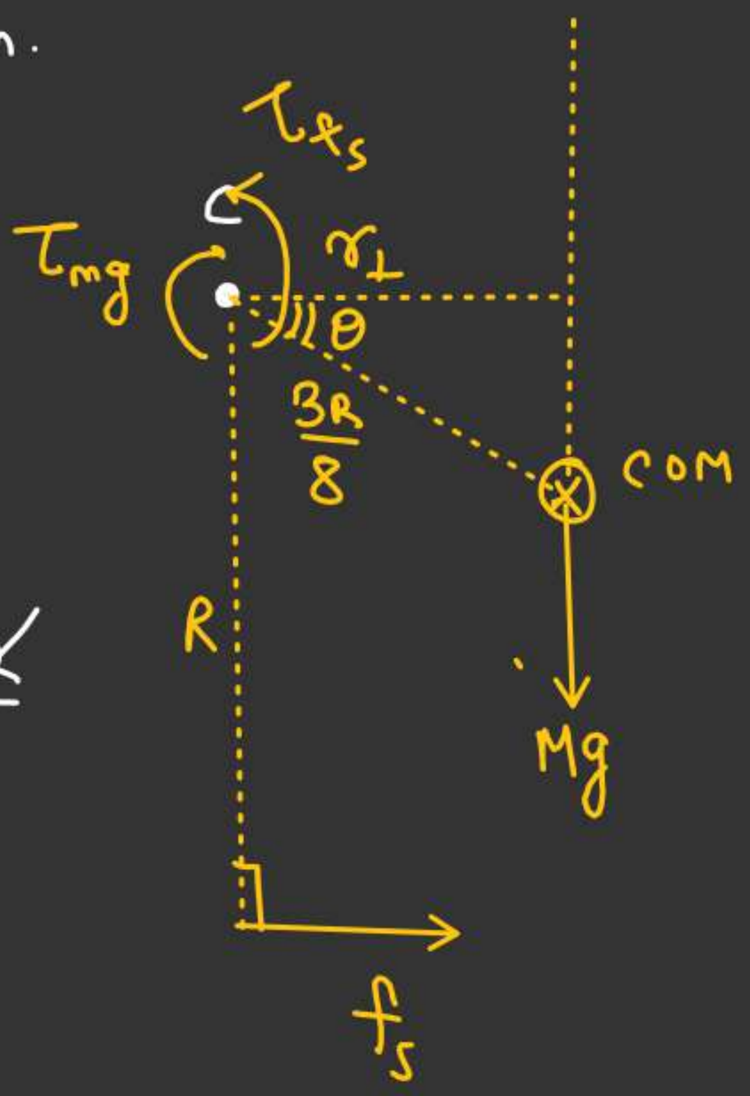
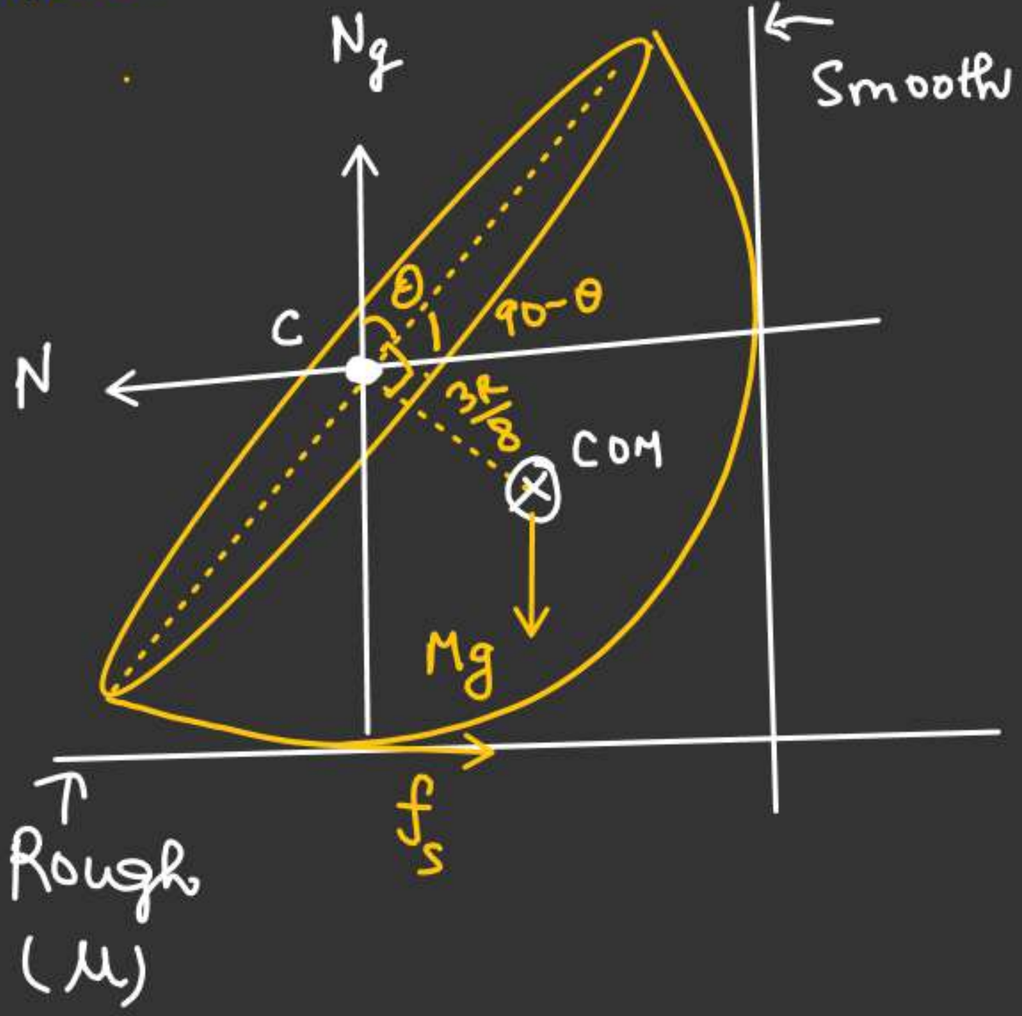
Nishant Jindal

Find μ_{min} for Equilibrium of Solid hemisphere.

$$\left.\begin{aligned} N_g &= Mg \\ N &= f_s \end{aligned}\right\} \text{Translational Equilibrium.}$$





For Rotational Equilibrium.

$$(\tau_{net})_c = 0$$

$$\Downarrow$$

$$-Mg\frac{3R}{8}\cos\theta\,\hat{k} + f_s R\,\hat{k} = 0$$

$$f_s \cancel{R} = (mg\cos\theta)\frac{3\cancel{R}}{8}$$

$$f_s = \frac{3}{8}mg\cos\theta$$

$$f_s \le (f_s)_{max}$$

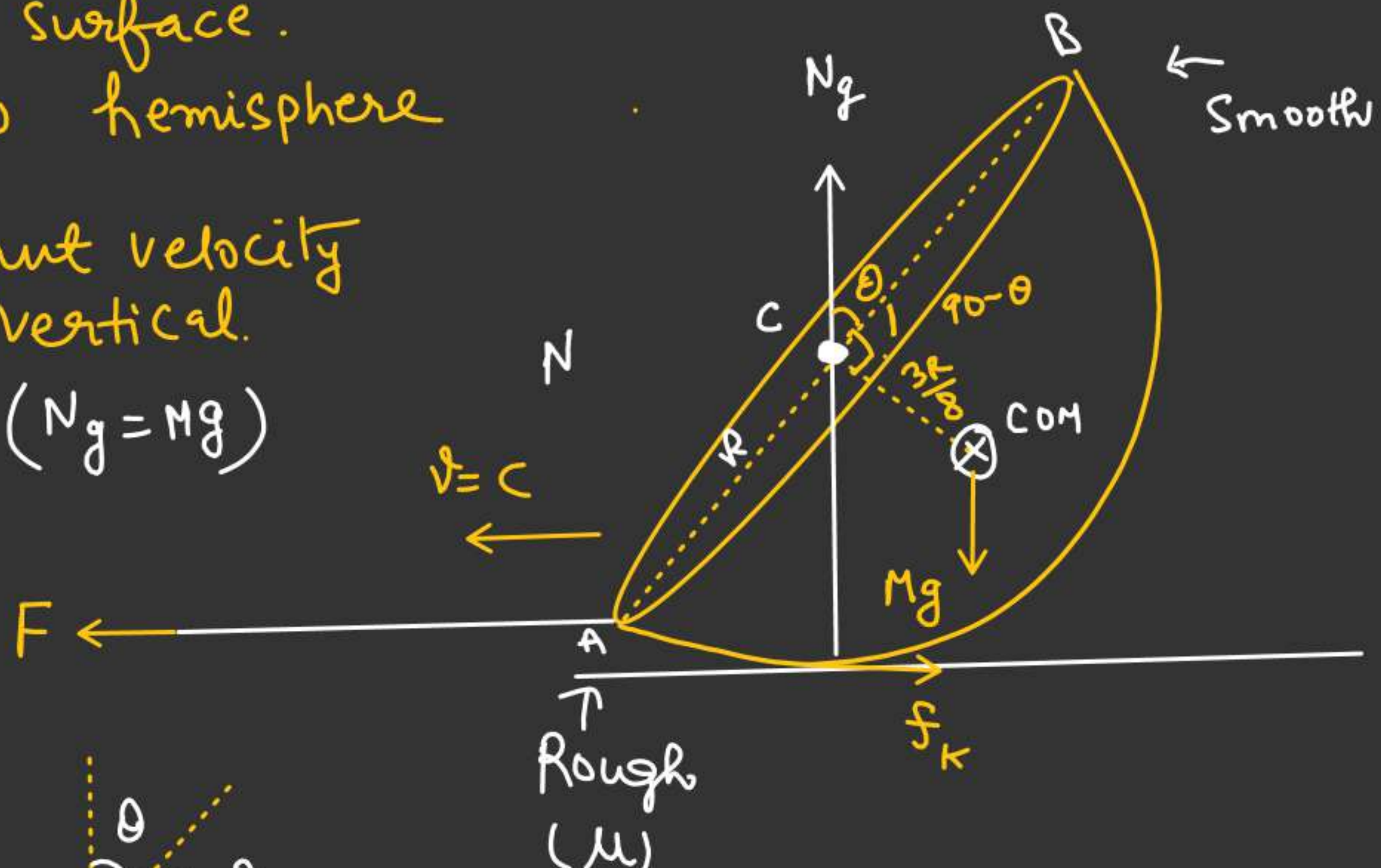
$$\frac{3}{8}mg\cos\theta \le \mu N_g \Rightarrow \mu \ge \frac{3}{8}\cos\theta$$

$$\frac{3}{8}\cancel{Mg}\cos\theta \le \mu(\cancel{mg}) \Rightarrow \mu_{min} = \frac{3}{8}\cos\theta \text{ Ans.}$$

Nishant Jindal

Hemisphere is pulled by constant force F. on a rough horizontal surface. μ be the coff^n of friction b/w hemisphere and ground.
Hemisphere moving with constant velocity making an angle 'θ' from vertical.

$(N_g = Mg)$



Sol^n. For constant velocity

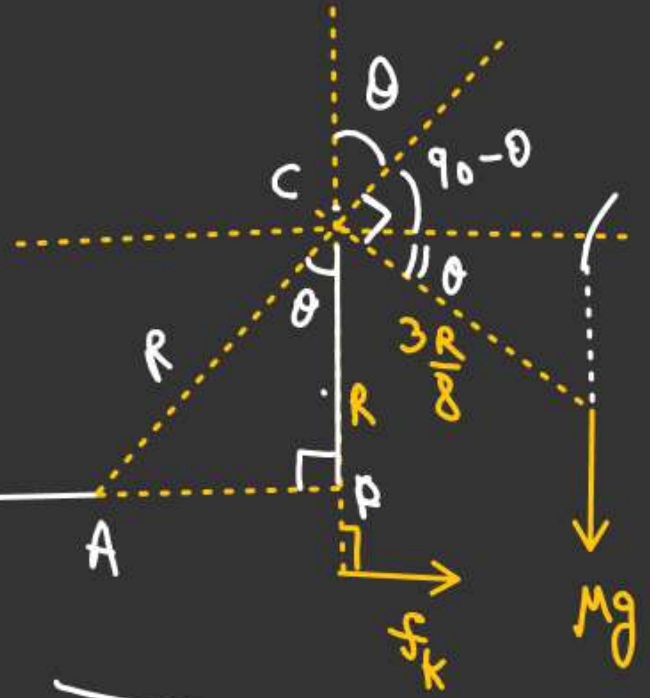
$$F = f_K = \mu mg.$$

For Rotational Equlibrium

$$(\vec{\tau}_{net})_c = 0$$

$$\vec{\tau}_F + \vec{\tau}_{Mg} + \vec{\tau}_{f_K} = 0$$

$$-FR\cos\theta\,\hat{k} + Mg\frac{3R\cos\theta}{8}(-\hat{k}) + \mu MgR\hat{k} = 0$$



$$\mu MgR = FR\cos\theta + \frac{3MgR\cos\theta}{8}$$

$$\mu MgR = \mu Mg\,R\cos\theta + \frac{3}{8}MgR\cos\theta$$

$$\mu = \left(\mu + \frac{3}{8}\right)\cos\theta$$

$$\cos\theta = \left(\frac{8\mu}{8\mu+3}\right)$$

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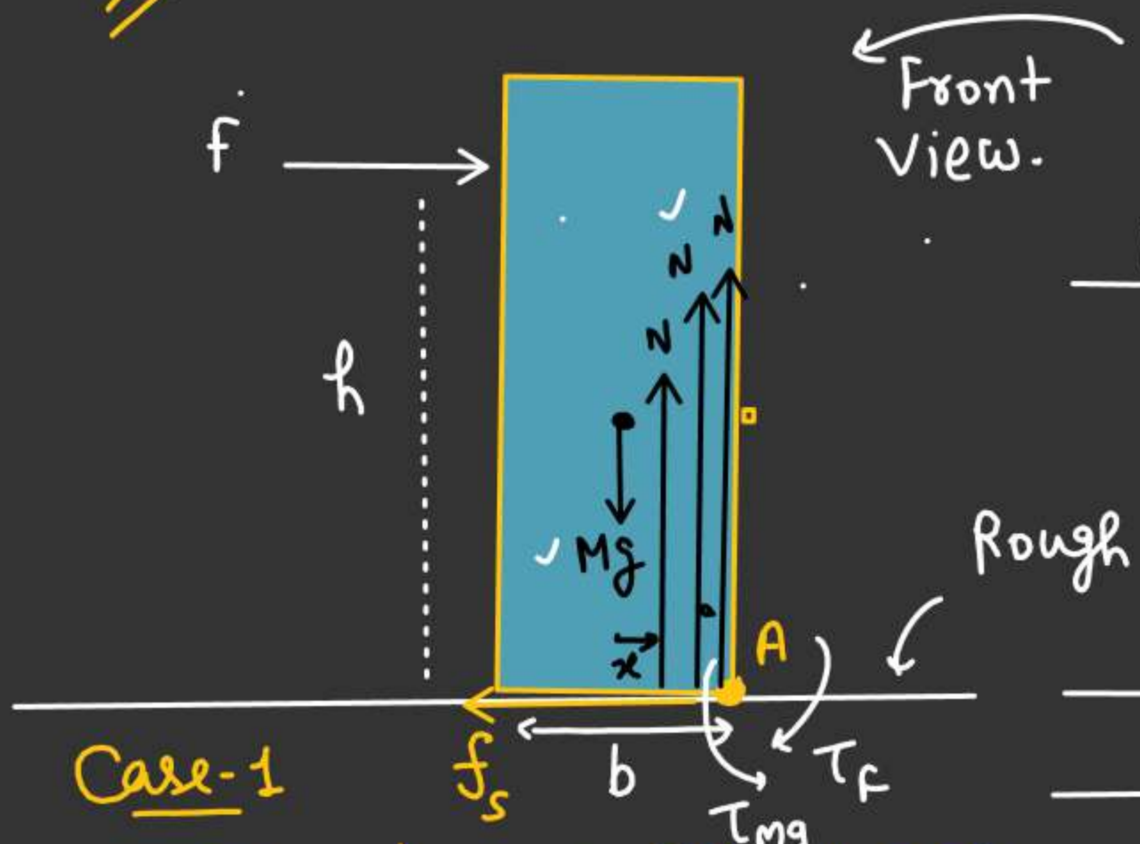
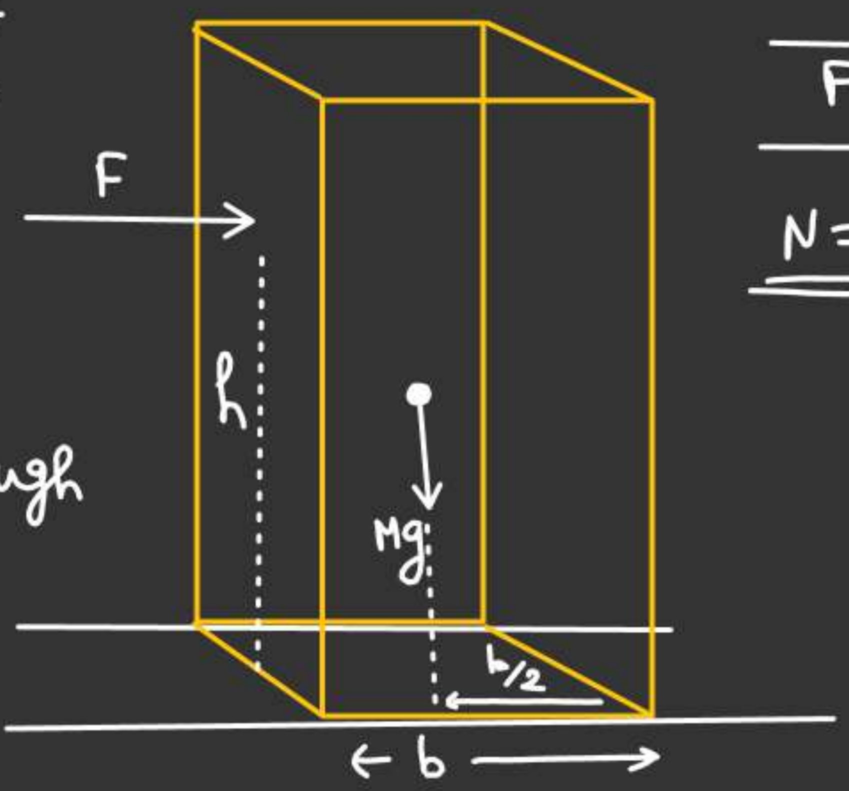
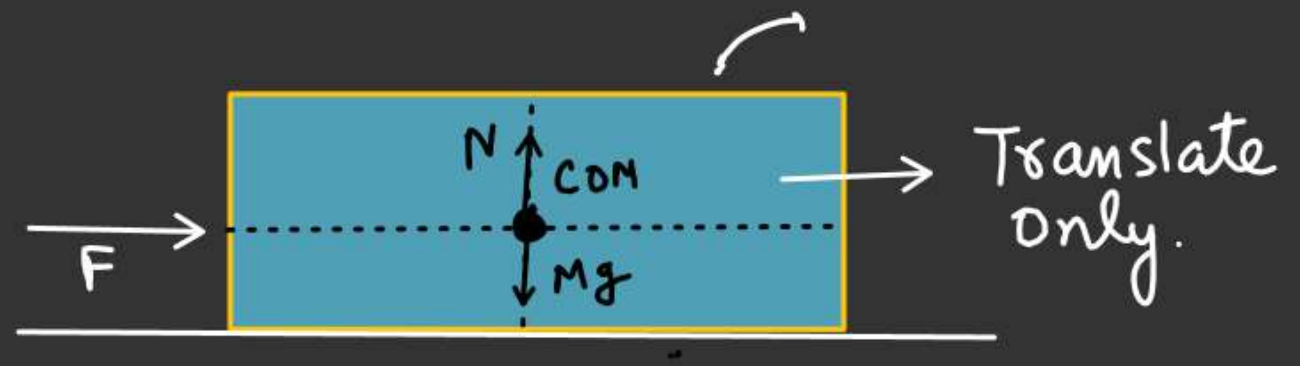
TOPPLING ($F = \text{Constant}$)



Case-1





$N = Mg$

Toppling without Sliding

For box not to Slide.

$$F = f_s \quad \text{—} \quad (1)$$

For box not to topple normal reaction shifted right side in order to balance the torque due to F and Mg

or w.r.t A

$$\tau_F \geq \tau_{Mg}$$

$$Fh \geq Mg\frac{b}{2}$$

In limiting condition

$$Fh = \frac{Mgb}{2} \quad \text{—} \quad (2)$$

$$f_s = F = \frac{Mgb}{2h} \quad \text{from (1) \& (2)}$$

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$$f_s = \frac{Mgb}{2h}.$$

$$f_s \leq (f_s)_{max}$$

$$\frac{\cancel{Mg}b}{2h} \leq \mu \cancel{Mg}.$$

$$\boxed{h \geq \frac{b}{2\mu}}$$

$$\left[h_{min} = \frac{b}{2\mu}\right]$$

$\Downarrow$

(For toppling)

$$\boxed{h \geq \frac{b}{2\mu}} \quad \boxed{F \leq \mu mg}$$

Case-1 $h < \frac{b}{2\mu}$, $F < \mu mg$

$h < \frac{b}{2\mu} \Downarrow$ No to topple. $F < \mu mg \downarrow$ Not to Slide

$\Rightarrow$ Box neither Slide nor topple.

Case-2 $h < \frac{b}{2\mu}$, $F > \mu mg$

$\Rightarrow$ Box not to topple but slide

Case-3 $h > \frac{b}{2\mu}$ $F < \mu mg$

Box topple without sliding

Case-4 $h = \frac{b}{2\mu}$ & $F = \mu mg$

Box have tendency of sliding & toppling

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Box topple without slide.

$Mg\sin\theta = f_s$ (For not to slide)

Box to topple

$[N = Mg\cos\theta]$

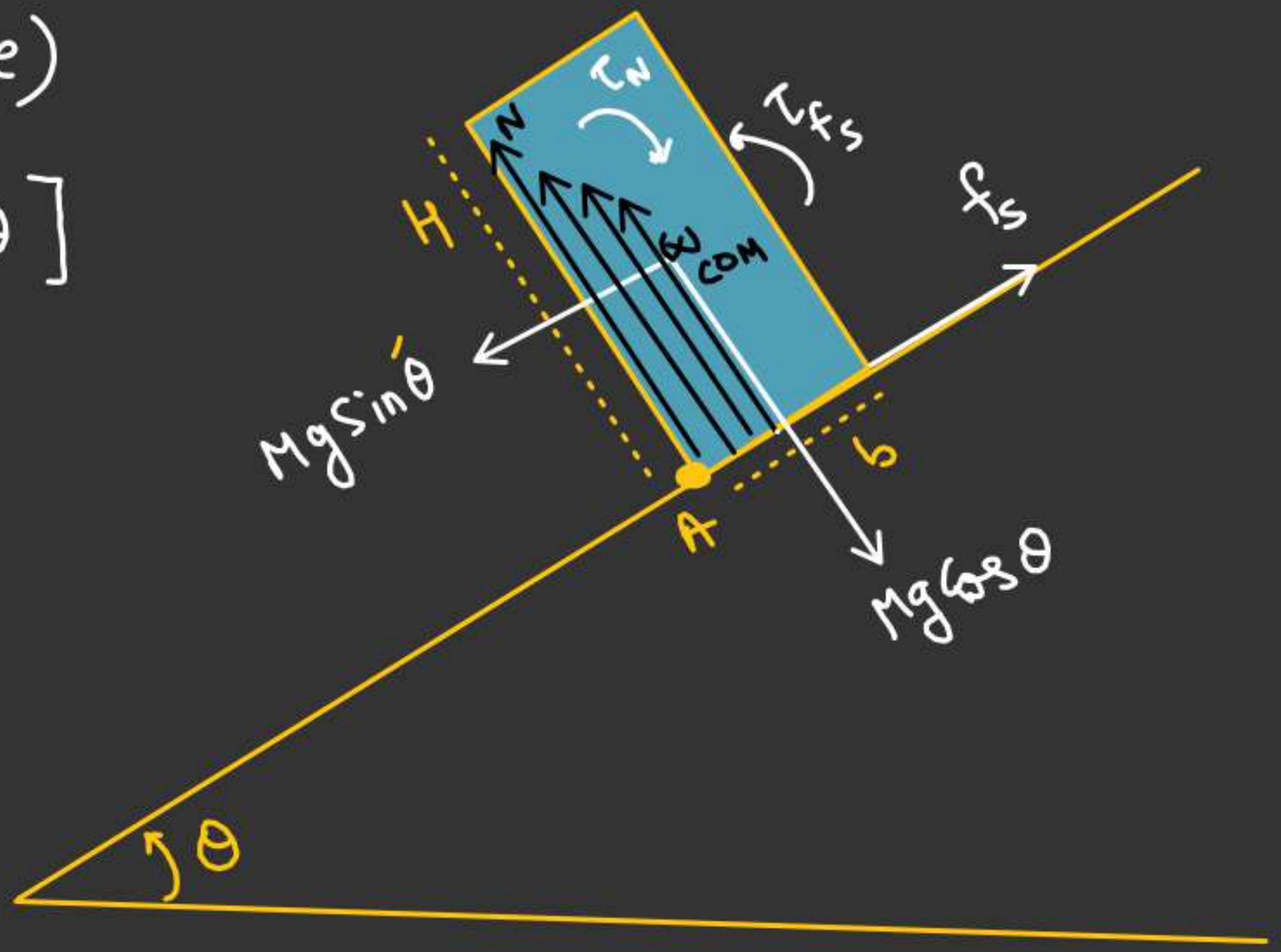
$\tau_{f_s} \geq \tau_N$ (In limiting condition)

$$f_s \frac{H}{2} = N \cdot \frac{b}{2} \checkmark$$

$$(Mg\sin\theta)\frac{H}{2} = (Mg\cos\theta)\frac{b}{2}$$

$$\tan\theta\, H = b$$

$$\tan\theta = \left(\frac{b}{H}\right) \quad -(1) \checkmark$$



$$f_s = Mg\sin\theta$$

$$f_s \leq (f_s)_{max}$$

$$mg\sin\theta \leq \mu mg\cos\theta$$

$$\tan\theta \leq \mu \quad -(2)$$

From (1) & (2)

$$\frac{b}{H} \leq \mu$$

$$\boxed{H \geq \frac{b}{\mu}}$$