7. $\dfrac{(x-2)(x+4)^4(x-7)^7(x+2)^2}{(x-3)^9(x+3)^9\,x^8\,(2x-3)^4} > 0\ ,\ < 0\ ,\ \le 0$

(ii) $x \in (-3,2) \cup (3,7) - \{-2, 0, \frac{3}{2}\}$

(i) $x \in (-\infty,-4) \cup (-4,-3) \cup (2,3) \cup (7,\infty)$

(ii) $x \in (-3,-2) \cup (-2,0) \cup (0,\frac{3}{2}) \cup (\frac{3}{2},2) \cup (3,7)$

(iii) $x \in (-3,0) \cup (0,\frac{3}{2}) \cup (\frac{3}{2},2] \cup (3,7] \cup \{-4\}$

Nishant Jindal

1. $\dfrac{x+1}{x-1} \geqslant \dfrac{x+5}{x+1}$

$a \geqslant b \Rightarrow a-b \geqslant 0$

$$\dfrac{x+1}{x-1} - \dfrac{x+5}{x+1} \geqslant 0$$

$$\boxed{x \in (-\infty, -1) \cup (1, 3]}$$

$$\dfrac{(x+1)^2 - (x+5)(x-1)}{(x-1)(x+1)} \geqslant 0 \Rightarrow \dfrac{-2x+6}{(x-1)(x+1)} \geqslant 0$$

$$\Rightarrow \dfrac{(x-3)}{(x-1)(x+1)} \leqslant 0$$

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2. $$\frac{(x^2\overbrace{-x}^{-2x+x}-2)(x^2\overbrace{-3x}^{-2x-x}+2)}{(x^4-9x^2+8)(x-7)} \le 0.$$

$x^4-8x^2-x^2+8 \leftarrow (x^4-9x^2+8)$

$\frac{0}{0}$ not defind

$\frac{\ }{0} \rightarrow$ not defd

$$\frac{(x-2)\cancel{(x+1)}\cancel{(x-1)}(x-2)}{(x^2-8)\cancel{(x^2-1)}(x-7)} \le 0 \qquad x \ne -1, 1$$

$$\frac{(x-2)^2}{(x-2\sqrt{2})(x+2\sqrt{2})(x-7)} \le 0, \quad x \ne -1, 1$$

$0\overline{)2}$

$-$ $\quad -2\sqrt{2}$ $\quad +$ $\quad 2$ $\quad +$ $\quad 2\sqrt{2}$ $\quad -$ $\quad 7$ $\quad +$

$$\boxed{x \in (-\infty, -2\sqrt{2}) \cup (2\sqrt{2}, 7) \cup \{2\}}$$

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3. $x^2 + 2x + 1 > 0$

$(x+1)^2 > 0$

$+ \qquad\qquad +$

-1

$$\boxed{x \in (-\infty, -1) \cup (-1, \infty)}$$

$x \in R - \{-1\}$

$x \in (-\infty, \infty) - \{-1\}$

Nishant Jindal

4.

$$x^2 + x + 1 > 0 \Rightarrow \boxed{x \in R}$$

$$= x^2 + 2x\frac{1}{2} + 1$$

$$= \underbrace{x^2 + 2x\frac{1}{2} + \left(\frac{1}{2}\right)^2} + \frac{3}{4}$$

$$= \underbrace{\left(x + \frac{1}{2}\right)^2}_{\geq 0} + \frac{3}{4} \geq \frac{3}{4} > 0$$

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5.

$$x^2 - 3x + 7 \le 0$$

$$= \left(x - \frac{3}{2}\right)^2 + 7 - \frac{9}{4}$$

$$= \underbrace{\left(x - \frac{3}{2}\right)^2 + \frac{19}{4}}_{\geq \frac{19}{4}} \le 0$$

$x = \phi$

No real solution

$2 \le 0$ F

6.

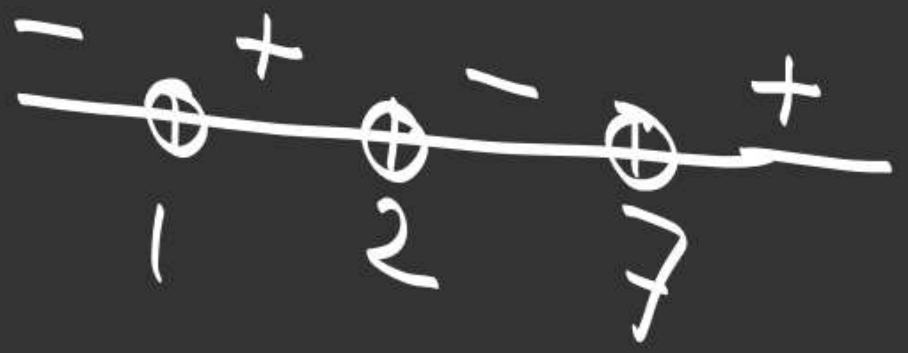
$$\frac{2(x-4)}{(x-1)(x-7)} \geq \frac{1}{x-2}$$

$$\frac{2(x-4)}{(x-1)(x-7)} - \frac{1}{x-2} \geq 0 \Rightarrow \frac{2(x^2-6x+8) - (x^2-8x+7)}{(x-1)(x-7)(x-2)} \geq 0$$

$$\Rightarrow \frac{(x^2-4x+9)}{(x-1)(x-7)(x-2)} \geq 0 \Rightarrow \frac{1}{(x-1)(x-7)(x-2)} \geq 0$$

$$x^2-4x+9 = (x-2)^2+5$$

$$\boxed{x \in (1,2) \cup (7,\infty)}$$



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7. $$1 < \frac{3x^2-7x+8}{x^2+1} \le 2$$

$$\frac{3x^2-7x+8}{x^2+1} > 1 \quad \text{and} \quad \frac{3x^2-7x+8}{x^2+1} \le 2$$

$$3x^2-7x+8 > x^2+1$$

$$\Rightarrow 2x^2-7x+7 > 0$$

$$x^2-\frac{7}{2}x+\frac{7}{2} > 0$$

$$\left(x-\frac{7}{4}\right)^2+\frac{7}{2}-\frac{49}{16} > 0$$

$$\left(x-\frac{7}{4}\right)^2+\frac{7}{16} > 0$$

$$\boxed{x \in R}$$

$$\Rightarrow 3x^2-7x+8 \le 2x^2+2$$

$$\Rightarrow x^2-7x+6 \le 0$$

$$\Rightarrow (x-1)(x-6) \le 0$$

+   −   +

1   6

$$\boxed{x \in [1,6]}$$

Ans

$$\boxed{x \in [1,6]}$$

8. $$(\underbrace{x^2+3x}+1)(\underbrace{x^2+3x}-3) \geq 5$$

$$(x^2+3x)^2 - 2(x^2+3x) - 8 \geq 0$$

$$(x^2+3x)^2 - 4(x^2+3x) + 2(x^2+3x) - 8 \geq 0$$

$$t^2 - 2t - 8 = (t-4)(t+2)$$

$$(x^2+3x-4)(x^2+3x+2) \geq 0$$

$$(x+4)(x-1)(x+1)(x+2) \geq 0$$

$+ \quad -4 \quad - \quad -2 \quad + \quad -1 \quad - \quad 1 \quad +$

$$\boxed{x \in (-\infty,-4] \cup [-2,-1] \cup [1,\infty)}$$

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Modulus / Absolute Value Function

$|x|$ = Numerical value of x.

$|3.7| = 3.7$

$|-4| = 4 = -(-4)$

$|0| = 0$

$$|x| = \begin{cases} x & , x \geq 0 \\ -x & , x < 0 \end{cases}$$

$|-13.86| = 13.86$
$= -(-13.86)$

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$$\sqrt{16} = 4$$

$$\sqrt{(4)^2} = 4$$

$$\sqrt{(-4)^2} = 4 = -(-4)$$

$$\sqrt{64} = \sqrt{8^2} = 8$$

$$\sqrt{(-8)^2} = -(-8) = 8$$

$$\sqrt{x^2} = \begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases}$$

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$$\boxed{\sqrt{x^2} = |x|}$$

$$\boxed{\sqrt{x^2} = |x| = \begin{cases} x & , \ x \geq 0 \\ -x & , \ x < 0 \end{cases}}$$

$$\boxed{|x|^2 = x^2}$$