

$$① \int \frac{1}{x} dx = \ln|x| + C$$

$$2) \int \frac{1}{x^2} dx = -\frac{1}{x} + C$$

$$3) \int \frac{1}{\sqrt{x}} dx = 2\sqrt{x} + C$$

$$4) \int \sqrt{x} dx = \frac{2}{3} x^{3/2}$$

$$5) \int \csc x dx = -\ln|\csc x + \cot x| + C$$

$$6) \int \sec x dx = \ln|\sec x + \tan x| + C$$

$$7) \int \tan x dx = \ln|\sec x| + C$$

$$8) \int \csc x dx = \ln|\csc x + \cot x| + C$$

$$9) \int \sec x dx = \ln|\sec x + \tan x| = \ln|\tan(\frac{\pi}{4} + \frac{x}{2})| + C$$

$$10) \int \csc x dx = \ln|\csc x - \cot x| = \ln|\tan(\frac{\pi}{2} - x)| + C$$

$$11) \int \sec^2 x dx = \tan x + C$$

$$12) \int \sec x \tan x dx = -\sec x + C$$

$$13) \int \sec x \tan x dx = \sec x + C$$

$$14) \int \sec x \csc x dx = -\csc x + C$$

$$\int \frac{dx}{x^2+a^2} = \frac{1}{a} \tan^{-1} \frac{x}{a}.$$

$$\int \frac{dx}{\sqrt{x^2+a^2}} = \ln$$

$$\int \sqrt{x^2+a^2} dx = \frac{1}{2} \sqrt{x^2 + a^2} - \frac{a^2}{2} \ln$$

$$\int \frac{dx}{4x^2+y}$$

$$\int \frac{dx}{(2x^2+3)^2} = \frac{1}{3} \tan^{-1} \frac{2x}{3}$$

$$\int \frac{dx}{x^2-a^2} = \frac{1}{2a} \ln \left| \frac{x-a}{x+a} \right|$$

$$\int \frac{dx}{\sqrt{x^2-a^2}} = \ln$$

$$\int \sqrt{x^2-a^2} dx = \frac{x}{2} \sqrt{x^2 - a^2} - \frac{a^2}{2} \ln$$

$$\int \frac{dx}{a^2-x^2} = \frac{1}{2a} \ln \left| \frac{a+x}{a-x} \right|$$

$$\int \frac{dx}{\sqrt{a^2-x^2}} = \sin$$

$$\int \sqrt{a^2-x^2} dx = \frac{x}{2} \sqrt{a^2-x^2} + \frac{a^2}{2} \ln$$

$$\int \frac{dx}{\sqrt{4x^2-y}} = \int \frac{dx}{\sqrt{(2x)^2-3^2}}$$

$$= \frac{1}{2} \ln \left| 2x + \sqrt{4x^2-y} \right|$$

$$\int m^2 A - \delta m^2 B.$$

$$= \delta m (A+B) \cdot m (A-B)$$

$$\int \delta m \left( \frac{9\pi}{8} + \frac{x}{4} + \frac{\pi}{8} + \frac{21}{4} \right) + \delta m \left( \frac{9\pi}{8} + \frac{x}{4} - \frac{7\pi}{8} - \frac{21}{4} \right)$$

$$\frac{1}{12} \int \delta m \left( 2\pi + \frac{x}{2} \right)$$

$$Q_{\frac{3}{2}}$$

$$g'(x^2) = x^3 \quad x^2 = t$$

$$g'(t) = (t^2)^{3/2} = t^{3/2}$$

$$g'(x) = x^{3/2}$$

$$g(1) = \int x^{3/2} dx = \frac{2}{5} x^{5/2} + C = \frac{2}{5} x^{5/2} + \frac{3}{5}$$

$$g(4) = \frac{2}{5}(4)^{5/2} + \frac{3}{5} = \frac{64}{5} + \frac{3}{5}$$

$$\begin{aligned} & \int \delta m \alpha \cdot \delta m (x-\alpha) dx + \frac{1}{2} \int 2 \delta m^2 \left( \frac{x}{2} - \alpha \right) dx \\ & \delta m \alpha \int \delta m (x-\alpha) dx + \frac{1}{2} \int \frac{1}{2} - \delta m \frac{2(x-\alpha)}{2} dx \\ & - \delta m \alpha \cdot \frac{\delta m (x-\alpha)}{2} + \frac{1}{2} \left[ \frac{x}{2} - \frac{1}{4} x \delta m (x-2\alpha) \right] \end{aligned}$$

$$6 \int \frac{(\phi^2(2x)-1)}{2(\phi 2x)} - \frac{G_2 g_{2x}}{\tan 4x} dx$$

$$\int \left( \frac{(1-\tan^2 2x)}{2 \tan 2x} \right) - \frac{G_2 g_{2x}}{\tan 4x} dx$$

$$\int \frac{1}{\tan 4x} - \frac{G_2 g_{2x}}{\tan 4x} dx$$

$$\int \frac{1}{\tan 4x} (1 - G_2 g_{2x}) dx$$

$$\int \frac{1}{\tan 4x} \times 2 \delta n^2(4x) = \int \frac{2 \delta n^2 4x \times g_{4x}}{\tan 4x}$$

$$\int \delta n g_{4x} dx$$

$$(15) \quad \int \frac{(1 + \delta n 2x) - (1 + \delta n 2K)}{(G_2 x + \delta n x)^2 - (G_2 K - \delta n K)^2}$$

$$\int \frac{(G_2 x + \delta n x + G_2 K - \delta n K)(G_2 x + \delta n x - G_2 K + \delta n K)}{(G_2 x + \delta n x)^2 - (G_2 K - \delta n K)^2}$$

7 ✓ 18 ÷ 9, CPM. 10 ✓, 11 ✓, 12 hold, 13 hold

$$\text{Q) } 16 \int \frac{x^2+1}{x^6(x^2+1)} + \int \frac{dx}{x^6(x^2+1)}$$

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$$+ 2 \int \frac{x^4-x^2+1}{x^6(x^2+1)} - 2 \int \frac{x^4 \cdot dx}{x^6(x^2+1)}$$

$$\int \frac{dx}{x^6} + 2 \int x^{-2} - x^{-4} + x^{-6} dx - 2 \tan^{-1} x$$

$$\text{Q) } \int x^k \ln(e^x) \cdot dx \\ \int x^k (\ln e + \ln x)$$

$$\int \boxed{x^k} (1 + \ln x) dy \\ \int dy = f + C = \underline{x^k + C} \quad \underline{\int x^k (1 + \ln x) dx - dk}$$

Ans.

$$21 \quad \frac{\delta m(x)}{\delta n(x-\alpha)} = A + \frac{B \cdot G_s(x-\alpha)}{\delta m(x-\alpha)}$$

$$\delta m(x) = A \cdot \delta n(x-\alpha) + B G_s(x-\alpha)$$

$$- \boxed{G_s(\alpha)} \delta n(x-\alpha) + \boxed{\delta n(\alpha)} \cdot G_s(x-\alpha)$$

$$\delta n(x) = \delta n(x-\alpha+\alpha) -$$

$$2y) \int | + \tan x \cdot \tan(x+\alpha) dx \\ \frac{1}{\tan \alpha} \int |\tan(x+\alpha) - \tan x| dx$$

$\tan A + \tan B = \tan(A+B)$

$$\tan(x + (x+\alpha)) = \frac{\tan x + \tan(x+\alpha)}{1 + \tan x \cdot \tan(x+\alpha)}$$

$$| + \tan x \cdot \tan(x+\alpha) = \frac{\tan x - \tan(x+\alpha)}{-\tan \alpha}$$

$$30) \int \frac{(x+1)(-(\cancel{x}-x)) dx}{\sqrt{a^2-x^2}}$$

$$2 \int \frac{x dx}{\sqrt{a^2-x^2}} \\ -2 \int \frac{dx}{\sqrt{a^2-x^2}} + 1 \\ 100 \xrightarrow{11,000} 12,000$$

$$33) \int \frac{x dx}{(a^2)^2 + 1} \quad x^2 = t$$

$$34) \int \frac{x dx}{\sqrt{(a^2)-(x^2)^2}} \quad x^2 = t$$

$$35) \int \frac{x^2 dx}{(x^3)^2 + 4} \quad x^3 = t$$

$$36) \int \frac{x^3 dx}{\sqrt{1-(x^4)^2}} \quad x^4 = t$$

$$37) \int \frac{e^x dx}{(\cancel{e^x})^2 + 2^2} \quad e^x = t$$

$$38) \int \frac{6x dx}{(a)^2 + (\ln x)^2} \quad \ln x = t$$

$$39) \text{ solve}$$



$$40) \int \frac{1}{\sqrt{1-x^2}} + \int \frac{dx}{\sqrt{1-x^2}} \quad | -x^2 = t$$

$$41) \int \frac{2x(dx)}{x^2+9} + \frac{1}{2} \int \frac{2(x-2) \cdot dx}{x^2+9} \quad (8)$$

$\downarrow$

$$x^2+9=t \quad \frac{1}{2} \int \frac{2x}{x^2+9} dx - \int \frac{dx}{x^2+3^2}$$

$$42) \text{ Rat & Split} \quad 43) \int \frac{1-\ln|x|}{1-\ln^2 x} dx$$

$$44) \int \frac{e^x(1+x)}{(\ln^2 x \cdot e^x)} dx \quad \ln, e^x \text{ Jodhi} = t$$

$\downarrow$

$$\int \frac{dt}{t^2} \quad x \cdot e^x + e^x \cdot dx = dt$$

$$e^x(x+1) dx = dt$$

$$2(1) \int \left(x + \frac{1}{x}\right)^{\frac{1}{2}} \left(1 - \frac{1}{x^2}\right) dx \quad x + \frac{1}{x} = t$$

$$Q_3 \int \frac{6x \cdot dx}{\ln x - \ln x}$$

$$Q_4 \log(x + \sqrt{x^2+1}) = t$$

$$7) \int \frac{2x}{\sqrt{1-x^2}} \quad \left| \begin{array}{l} -\int \frac{\ln x}{\sqrt{1-x^2}} \\ \downarrow \\ (-x^2=t) \end{array} \right. \quad \ln x = t$$

$$\int \frac{e^x (bx - \sin x) dx}{(e^x + \sin x)^2}$$

$$\int \frac{e^{-x} (bx - \sin x) dx}{e^{2x} (1 + e^{-x} \sin x)^2}$$

$$1 + e^{-x} \sin x = t$$

$$\int \frac{dt}{t^2}$$

$$-\frac{1}{t} + C$$

$$(e^{-x} (bx - \sin x \cdot e^{-x}) dx = dt)$$

$$e^{-x} (bx - \sin x) dx = dt$$

$$\int \frac{\sec x dx}{\sqrt{b^2(\cot^2 x - a^2 \sec^2 x)}}$$

$$\int \frac{\sec x \cdot \tan x dx}{\sqrt{b^2 - a^2 \cdot \frac{\sec^2 x}{(\cot^2 x)}}}$$

$$\frac{\frac{1}{2} \sec x}{\frac{a^2}{b^2}}$$

$$\int \frac{\sec x \cdot \tan x dx}{\sqrt{b^2 - a^2 \sec^2 x}}$$

$$\int \frac{\sec x \cdot \tan x}{\sqrt{(b)^2 - (a \sec x)^2}}$$

$$a \sec x = f$$

$$a \sec x \tan x dx = df$$

$$\frac{1}{a} \int \frac{dt}{\sqrt{(b)^2 - t^2}} = \frac{1}{a} \sin^{-1} \frac{t}{b} + C$$

$$\begin{aligned}
 & \int \frac{1 + x G_x \cdot dx}{x(1 - x^2 e^{2\sin x})} \\
 & \quad \xrightarrow{x^2 e^{2\sin x} = 1-t} \\
 & \quad 1 - x^2 e^{2\sin x} = t \\
 & \Rightarrow \frac{1}{2} \int \frac{dt}{(t-1) \cdot (t)} \\
 & = \frac{1}{2} \times \frac{1}{1} \ln \frac{t}{t-1} + C \\
 & = \frac{1}{2} x \ln \frac{1 - x^2 e^{2\sin x}}{-x^2 e^{2\sin x}} + C
 \end{aligned}$$

$$\begin{aligned}
 & 0 - x^2 \cdot e^{2\sin x} \cdot 2G_x - e^{2\sin x} \cdot 2x \cdot dx = du \\
 & -2x \cdot e^{2\sin x} (\underbrace{xG_x + 1}_{\frac{(xG_x + 1)dx}{x}}) dx = du \\
 & \frac{(xG_x + 1)dx}{x} = \frac{dt}{-2(1 \cdot e^{2\sin x})} \\
 & = -\frac{dt}{2(1-t)}
 \end{aligned}$$

$$Q \int (x^{3m} + x^{2m} + x^m) (2x^{2m} + 3x^m + 6)^{\frac{1}{m}} dx \quad DPP4 Q_8$$

$x^{(om)}$

$$\int (\underbrace{x^{3m-1} + x^{2m-1} + x^{m-1}}_{-} \cdot \underbrace{(2x^{3m} + 3x^{2m} + 6x^m)^{\frac{1}{m}}}_{+} dx$$

$$2x^{3m} + 3x^{2m} + 6x^m = t$$

$$6mx^{3m-1} + 6mx^{2m-1} + 6mx^{m-1} \cdot dx = dt$$

$$\frac{1}{6m} \int t^{\frac{1}{m}} dt$$

$$\frac{1}{6m} \times \frac{t^{\frac{1}{m}+1}}{\frac{1}{m}+1} + C$$

$$x^{3m-1} + x^{2m-1} + x^{m-1} dx = \frac{dt}{6m}$$

$$\oint \left( \left(\frac{x}{e}\right)^x + \left(\frac{e}{x}\right)^x \right) \ln x \cdot dx$$

$(f \cdot g)' = f' \cdot g + f \cdot \left( \frac{d}{dx} g \cdot \log f \right)$

$$\int \left( t + \frac{1}{t} \right) \frac{dt}{t}$$

$$\int \left( 1 + \frac{1}{t^2} \right) dt$$

$$t - \frac{1}{t} + C$$

$$\left(\frac{x}{e}\right)^x - \left(\frac{e}{x}\right)^x + C$$

$$\begin{aligned} & \left( \frac{x}{e} \right)^x \cdot \left( \frac{d}{dx} x \cdot \log \left( \frac{x}{e} \right) \right) dx = dt \\ & \left( \frac{x}{e} \right)^x \left\{ x \cdot \frac{1}{\left( \frac{x}{e} \right)} \cdot \frac{1}{e} + \log \frac{x}{e} \times 1 \right\} dx = dt \\ & \left( \frac{x}{e} \right)^x \left\{ x + \log x - \log e \right\} dx = dt \\ & \left( \frac{x}{e} \right)^x \cdot \ln x \cdot dx = dt \\ & \ln x \cdot dx = \frac{dt}{\left( \frac{x}{e} \right)^x} = \frac{dt}{t} \end{aligned}$$

$$\oint \frac{(G5x + G4x) \cdot dx}{1 - 2G3x} \quad \Rightarrow \quad - \int 2G\frac{3x}{2} \cdot G\frac{x}{2} \cdot dx \xrightarrow{\text{Prod} = \text{Sum}}$$

$$\int \frac{2G\left(\frac{3x}{2}\right)G\left(\frac{x}{2}\right)}{1 - 2G3x} \times \frac{\sin 3x \cdot dx}{\sin 3x} \quad \Rightarrow \quad - \int G(2x) + G(x) \, dy \\ \Rightarrow - \frac{\sin 2x}{2} - \sin x + C$$

$$\int \frac{2G\left(\frac{3x}{2}\right) \cdot G\left(\frac{x}{2}\right) \cdot \sin 3x}{\sin 3x - \sin 6x}$$

$$\int \frac{2G\left(\frac{3x}{2}\right) \cdot G\left(\frac{x}{2}\right) \cdot \sin 3x \cdot dx}{2G\left(\frac{3x}{2}\right) - \sin\left(-\frac{3x}{2}\right)}$$

$$- \int \frac{G\frac{x}{2} \times 2\sin\left(\frac{3x}{2}\right) \cdot G\left(\frac{3x}{2}\right) dx}{\sin\left(\frac{3x}{2}\right)}$$

$(x^2 + \frac{1}{x^2})$  hyperbolas

$$1) \left( x + \frac{1}{x} \right)' = 1 - \frac{1}{x^2}$$

$$2) \left( x - \frac{1}{x} \right)' = 1 + \frac{1}{x^2}$$

$$3) \left( x + \frac{1}{x} \right)^2 = x^2 + \frac{1}{x^2} + 2$$

$$4) \left( x - \frac{1}{x} \right)^2 = x^2 + \frac{1}{x^2} - 2$$

$$\int \frac{x^2 + 1 \cdot dx}{x \sqrt{x^4 + 1}} \quad \text{Ansatz}$$

$$\int \frac{x^2 + 1 \cdot dx}{x^2 \sqrt{x^2 + \frac{1}{x^2}}} \quad \text{ye Kiska diff hai ??}$$

$$\int \frac{\left( 1 + \frac{1}{x^2} \right) dx}{\sqrt{(x^2 + \frac{1}{x^2} - 2) + 2}}$$

$$\int \frac{\left( 1 + \frac{1}{x^2} \right) dx}{\sqrt{(x - \frac{1}{x})^2 + (\sqrt{2})^2}} \quad x - \frac{1}{x} = t$$

$$\left( 1 + \frac{1}{x^2} \right) dx = dt$$

$$\int \frac{dt}{\sqrt{t^2 + 1^2}} = \ln |t + \sqrt{t^2 + 1}|$$

$$Q \int \frac{e^{\ln(1+\frac{1}{x^2}) dx}}{(x^2 + \frac{1}{x^2})}$$

$$\int \frac{(1+\frac{1}{x^2}) dx}{(x^2 + \frac{1}{x^2} - 2) + 2} \quad \text{Kiskader}$$

$$\int \frac{1+\frac{1}{x^2} dx}{((-\frac{1}{x})^2 + (\sqrt{2})^2)} \quad x - \frac{1}{x} = t \\ (1+\frac{1}{x^2}) dx = du$$

$$\int \frac{dt}{t^2 + (\sqrt{2})^2} = \frac{1}{\sqrt{2}} \tan^{-1} \frac{t}{\sqrt{2}} + C$$

$$Q. \int \frac{(ax^2 - b) dx}{x \sqrt{x^2 - (ax^2 + b)^2}}$$

$\underbrace{x^2 - (ax^2 + b)^2}_{x^2 \text{ com.}}$

$$\int \frac{ax^2 - b \cdot dx}{x^2 \sqrt{(1 - (ax + \frac{b}{x}))^2}}$$

$$\int \frac{\left(a - \frac{b}{x^2}\right) dx}{\sqrt{(1 - (ax + \frac{b}{x}))^2}} \quad ax + \frac{b}{x} = t \\ (a - \frac{b}{x^2}) dx = du$$

$$\int \frac{dt}{\sqrt{1-t^2}} = \ln \frac{1+t}{1-t} + C$$

$$\int \frac{(x^2-1)dx}{(x^4+3x^2+1) \tan\left(\frac{x^2+1}{x}\right)}$$

$x^2 \text{ term 4}$

$$\int \frac{\left(1 - \frac{1}{x^2}\right) dx}{\left(x^2 + \frac{1}{x^2} + 3\right) \tan\left(\frac{x^2+1}{x}\right)}$$

$$\int \frac{\left(1 - \frac{1}{x^2}\right) dx}{\left((x^2 + \frac{1}{x^2} + 2) + 1\right) \tan\left(\frac{x^2+1}{x}\right)}$$

$\boxed{\int \frac{\left(1 - \frac{1}{x^2}\right) dx}{\left((x^2 + \frac{1}{x^2})^2 + 1\right) \tan\left(1 + \frac{1}{x^2}\right)}}$

$$\tan\left(1 + \frac{1}{x}\right) = t$$

$$\frac{1}{1 + (x^2 + \frac{1}{x^2})^2} \times \left(1 - \frac{1}{x^2}\right) dx = dt$$

$$\int \frac{dt}{t} = \ln(\tan(x + \frac{1}{x})) + C$$