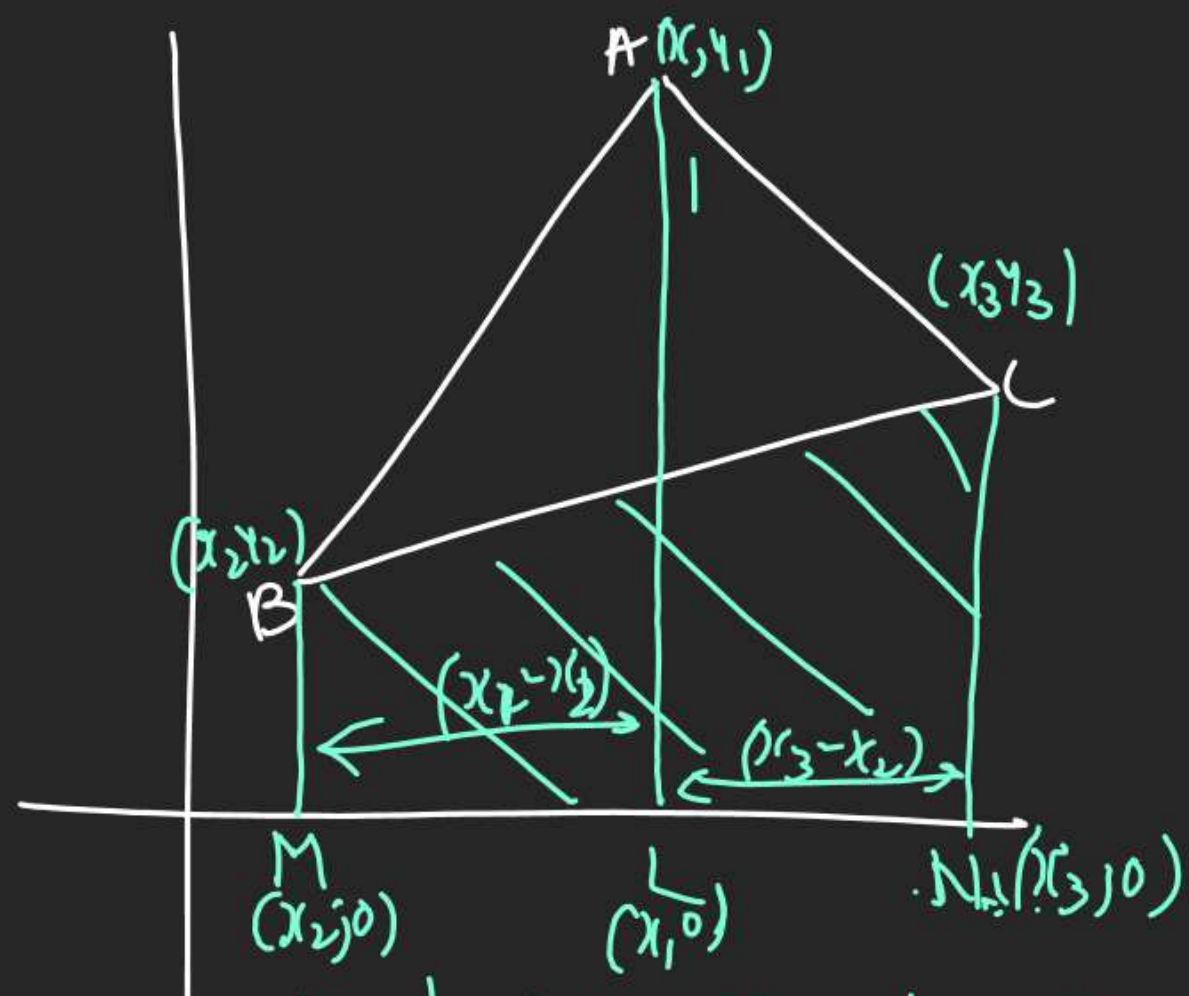


## Base of Area of $\Delta$ .

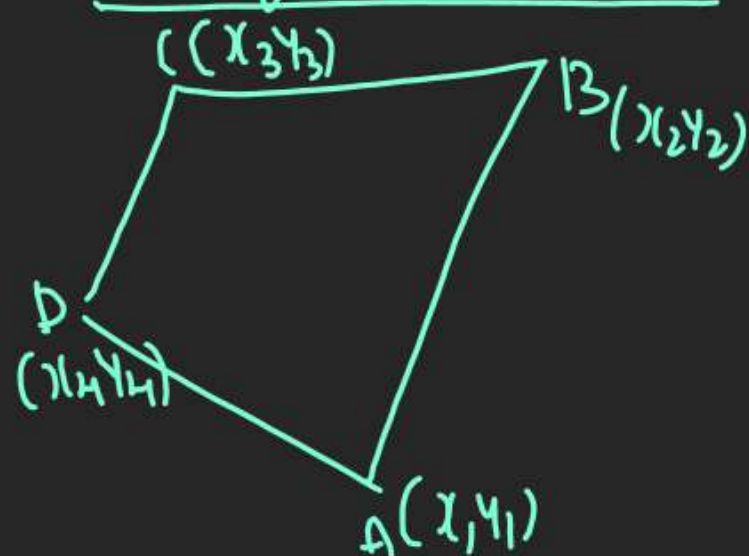


$$\Delta = \text{tr } MLAB + \text{tr } LNC - \text{tr } MNB$$

$$= \frac{1}{2} (y_2 + y_1)(x_1 - x_2) + \frac{1}{2} (y_1 + y_3)(x_3 - x_2) - \frac{1}{2} (y_2 + y_3)(x_3 - x_1)$$

$$= \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

## (2) Area of Quadrilateral.



$$\Delta = \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \\ x_4 & y_4 & 1 \end{vmatrix}$$

## (3) Area of Polygon.

$$\Delta = \frac{1}{2} \begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \\ \vdots \\ x_n & y_n \end{vmatrix} \text{ \& Solve.}$$

Q Area of  $\Delta$  having vertices  $A(3, 2)$ ,  $B(11, 8)$ ,  $C(18, 12)$

$$\Delta = \frac{1}{2} \begin{vmatrix} 3 & 2 \\ 11 & 8 \\ 18 & 12 \end{vmatrix} = 25$$



①  $\Delta$  whose vertices are.

$$(t, t-2), (t+2, t+2), (t+3, t)$$

is Independent of  $t$ . [T/F]

$$\Delta = \frac{1}{2} \begin{vmatrix} t & t-2 \\ t+2 & t+2 \\ t+3 & t \\ t & t-2 \end{vmatrix}$$

$$= \frac{1}{2} \left[ (t^2 + 2t - t^2 + 4) + (t^2 + 2t - t^2 - 5t - 6) + (t^2 + t - 6 - t^2) \right]$$

$$\frac{1}{2} [2t + 4 - 3t - 6 + t - 6] = \left| \frac{8}{2} \right| = |-4| = 4$$

② If coordinates of 2 Pts A & B are  $(3, 4)$  &  $(5, -2)$  Find coord of P such that  $PA = PB$  Area of  $\Delta PAB$  is 10 unit

① Let  $P = (x, y)$ ,  $A = (3, 4)$ ,  $B = (5, -2)$

$$PA = PB$$

$$\sqrt{(x-3)^2 + (y-4)^2} = \sqrt{(x-5)^2 + (y+2)^2}$$

$$-6x - 8y + 25 = -10x + 4y + 29$$

$$4x - 12y = 4 \Rightarrow \boxed{x - 3y = 1}$$

② Area = 10 (given)

$$\frac{1}{2} \begin{vmatrix} x & y \\ 3 & 4 \\ 5 & -2 \\ x & y \end{vmatrix} = \pm 10 \Rightarrow (4x - 3y) + (-6 - 20) + (5y + 2x) = \pm 20$$

$$6x + 2y = 26 + 20 \Rightarrow 6x + 2y = 46$$

$$3x + y = 23$$

$$\begin{array}{r} x - 3y = 1 \\ 9x + 3y = 9 \\ \hline 10x = 10 \end{array}$$

$$10x = 10$$

$$x = 1, y = 0 \Rightarrow (1, 0) \equiv P$$

$$\begin{array}{r} 9x + 3y = 9 \\ x - 3y = 1 \\ \hline 10x = 10 \end{array}$$

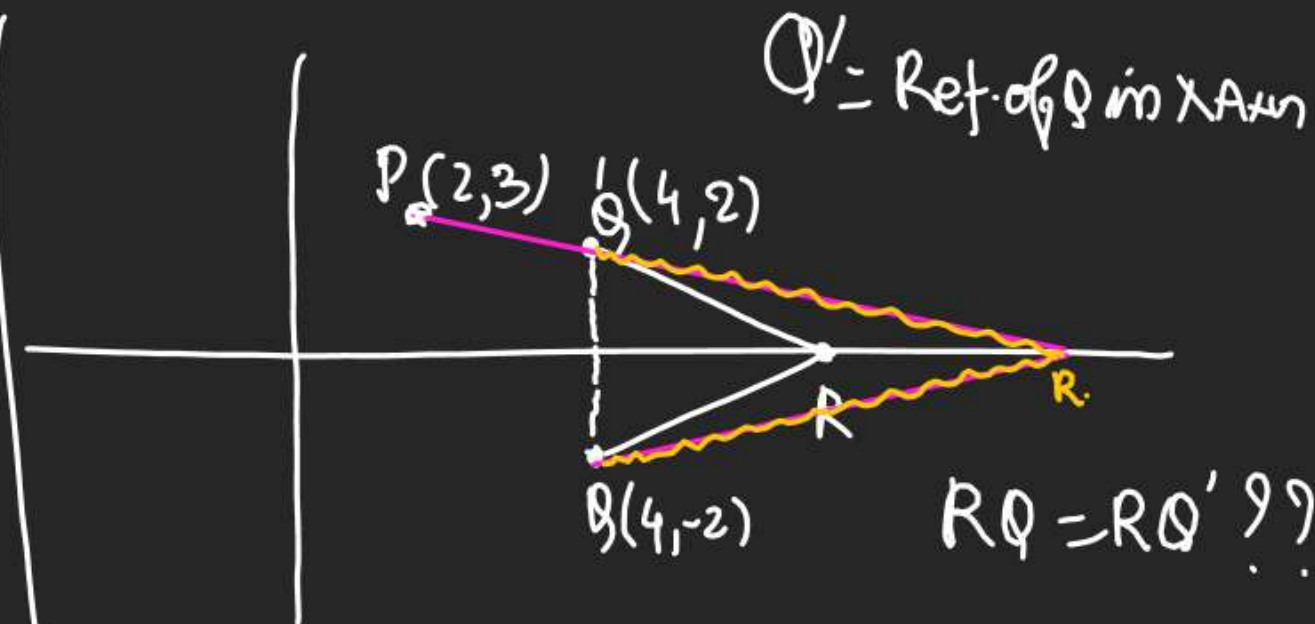
$$10x = 10$$

$$x = 1, y = 0 \Rightarrow P = (1, 0)$$



# A Very Imp / Complex Profile.

Q 3 hts  $P(2,3)$   $Q(4,-2)$  &  $R(x,0)$   $\rightarrow$   $\lambda$  Axis  
 is given. (A) Find value of  $\lambda$  if  $PR+RQ$  is Min.  
 (B) Find value of  $\lambda$  if  $|PR-RQ|$  is Max.



Qs change =  $|PR-RQ|$

$= |PR-RQ'| \leq PQ'$   
 Max.

Max distance  $PR-RQ' = PQ'$

$PR = PQ' + RQ'$

$P, Q', R$  s.t. line are

$\Delta PQ'R = 0$

$\Rightarrow \boxed{\lambda = 8}$

(A)  $PR+RQ$   $\rightarrow$   $\lambda$  axis  $\rightarrow$   $\lambda$  Axis

$PR+RQ$  will be min<sup>m</sup>

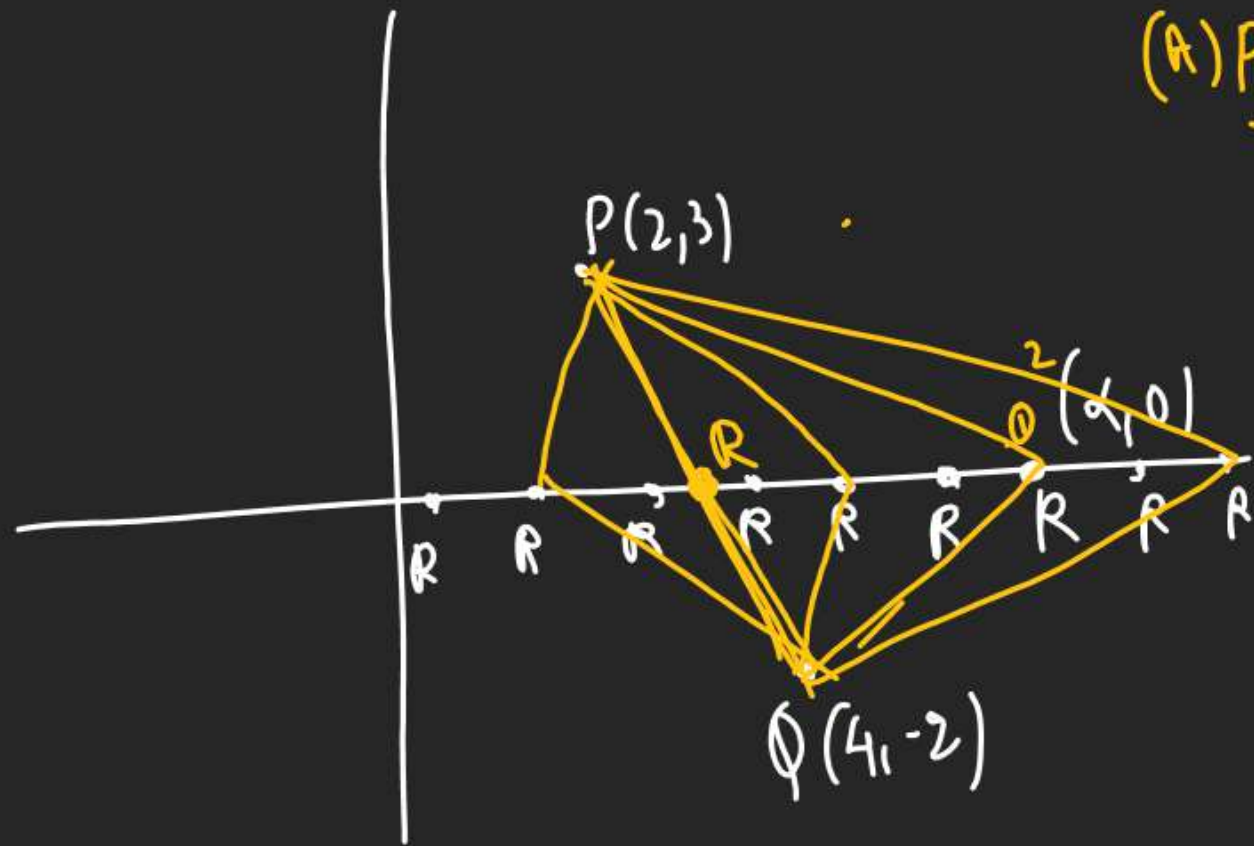
When  $P, R$  &  $RQ$  are on same st line

Area  $\Delta PRQ = 0$  (Collinear)

$$\frac{1}{2} \begin{vmatrix} 2 & 3 \\ 4 & -2 \\ x & 0 \end{vmatrix} = 0$$

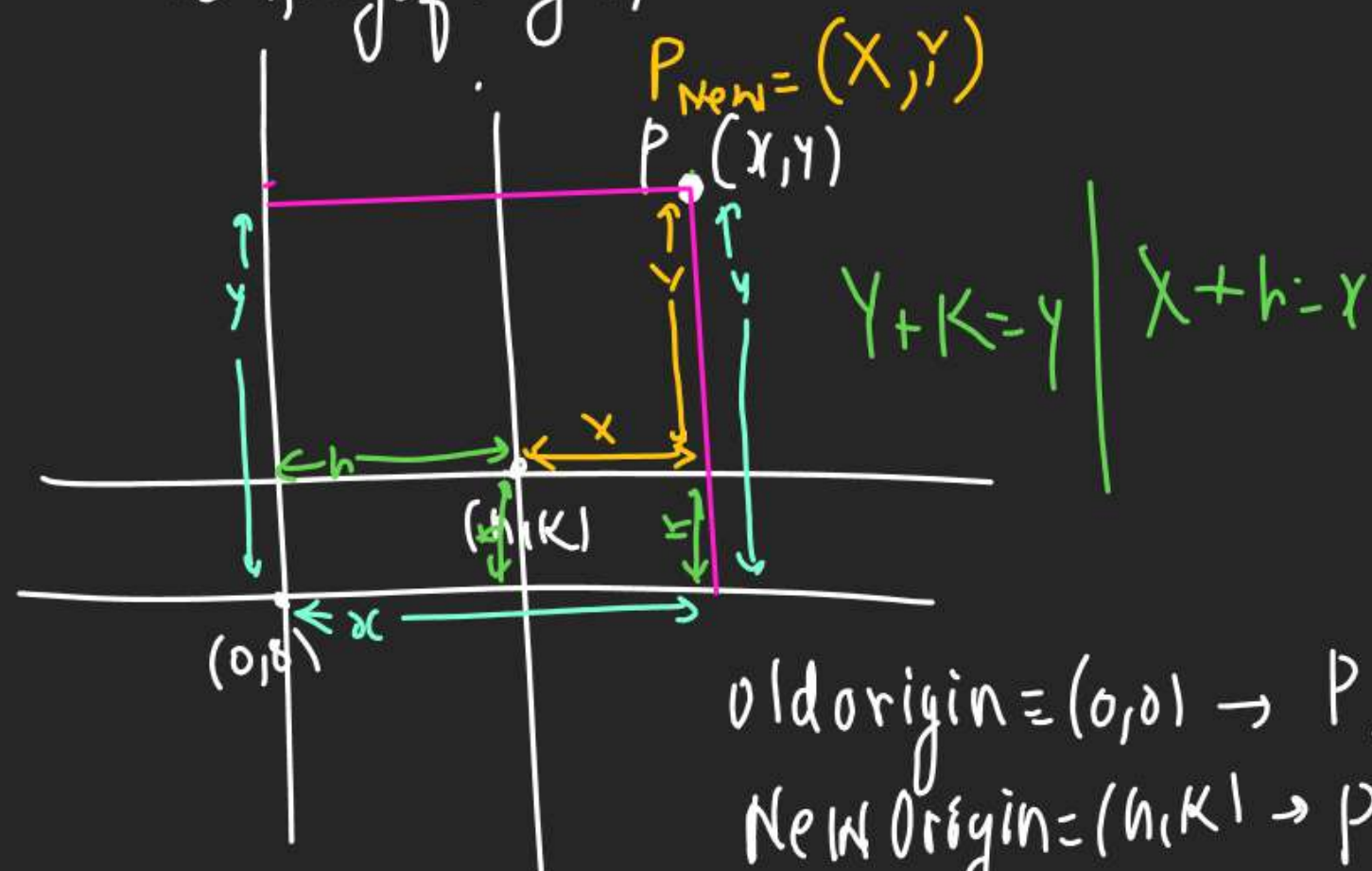
$$\frac{1}{2} [(-4-12) + (0+2x) + (3x-0)] = 0$$

$$5x = 16 \Rightarrow x = \frac{16}{5}$$



# Transformation of Axes.

(A) When Axes are shifted  $11^{\text{th}}$  to Axis  
(Shifting of Origin)



old origin = (0,0)  $\rightarrow$   $P = (x, y)$

New Origin = (h,k)  $\rightarrow$   $P_{\text{New}} = (X, Y)$

$\therefore (x-h, y-k)$

Q At what Pt. origin be shifted if coordinates

of P(4,5) becomes (-3,9).

(old) (New)

(x,y) (X,Y)

$$X = x - h, \quad Y = y - k$$

$$-3 = 4 - h, \quad 9 = 5 - k$$

$$h = 7$$

$$k = -4 \therefore \text{origin}(h,k) = (7, -4)$$



Q If origin is shifted to  $(h, k)$   
without Rotation of Axes find  
New eq<sup>n</sup> of following.

①  $2x^2 + y^2 - 4x + 4y = 0$  <sub>old.</sub>

②  $y^2 = 4x + 4y + 8 = 0$

New coord  $\rightarrow (X, Y)$

$X = x - h, Y = y - k$

$X = x - 1, Y = y + 2$

$X = x + 1, Y = y - 2$

$2(x+1)^2 + (y-2)^2 - 4(x+1) + 4(y-2) = 0$

$2x^2 + y^2 + 6 - 4 - 8 = 0$

$2x^2 + y^2 - 6 = 0$

New eq<sup>n</sup>

$Y = y - 2, X = x + 1$

$(Y+2)^2 - 4(X-1) + 4(X-2) + 8 = 0$

$Y^2 - 4X = 0$

Q Find origin such that eq<sup>n</sup>.

$Y^2 + 4Y + 8X - 2 = 0$  will not

contain a term of  $Y^2$  & constant term.

Let origin  $= (h, k)$

$X = x - h, Y = y - k$

$X = x + h, Y = y + k$

$(Y+k)^2 + 4(Y+k) + 8(X+h) - 2 = 0$

$Y^2 + Y(2k+4) + 8X + (k^2 + 4k + 8h - 2) = 0$

No  $Y^2$  term

$2k + 4 = 0$   
 $k = -2$

const. term  $= 0$

$k^2 + 4k + 8h - 2 = 0$

$4 - 8 + 8h - 2 = 0$

$8h = 6$

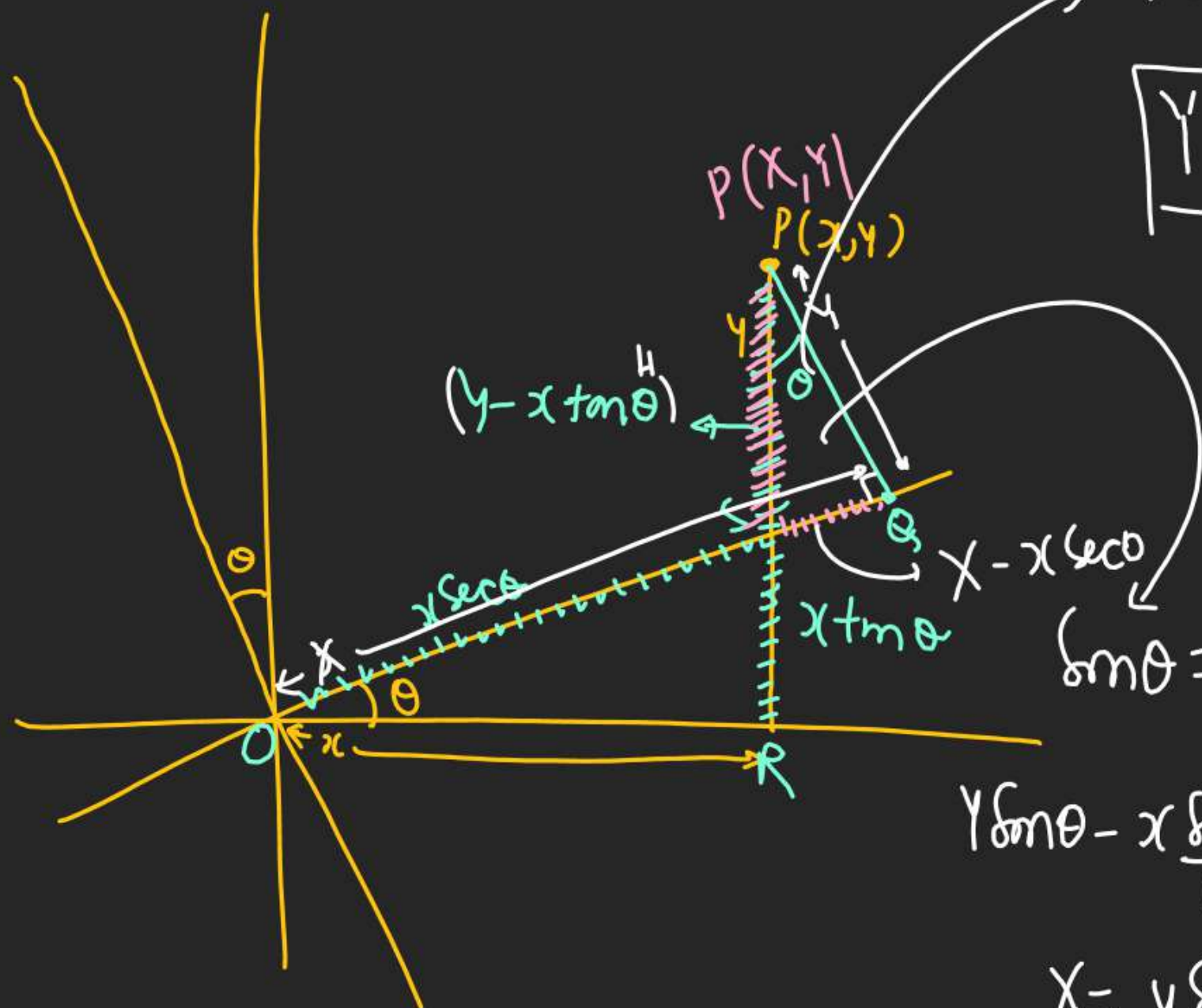
$h = \frac{3}{4}$

origin  $= (h, k) = (\frac{3}{4}, -2)$

→ constant term.



# Rotation of Axes about Origin.



$$\sec \theta = \frac{y}{y - x \tan \theta}$$

$$Y = y \sec \theta - x \tan \theta$$

$$\sec \theta = \frac{X - x \sec \theta}{y - x \tan \theta}$$

$$y \sec \theta - x \frac{\sec^2 \theta}{\sec \theta} = X - \frac{x}{\sec \theta}$$

$$X = y \sec \theta - \frac{x \sec^2 \theta}{\sec \theta} + \frac{x}{\sec \theta}$$

$$= y \sec \theta + \frac{x}{\sec \theta} (1 - \sec^2 \theta)$$

$$X = x \cos \theta + y \sec \theta$$

## Result

If Axes are Rotated at angle  $\theta$  on Origin. & Old Coordinates of Pt. P are  $(x, y)$  then New Coord  $(X, Y)$  will be

$$X = x \cos \theta + y \sec \theta$$

$$Y = -x \tan \theta + y \sec \theta$$

	$x$	$y$
$X$	$\cos \theta$	$\sec \theta$
$Y$	$-\tan \theta$	$\sec \theta$

$$X = x \cos \theta + y \sec \theta$$

Q Axes are rotated thru angle  $\frac{\pi}{3}$  in A.C.W direction. Find coord of  $(4, 2)$  in New system.

	4	2
x	$\cos \frac{\pi}{3}$	$\sin \frac{\pi}{3}$
y	$-\sin \frac{\pi}{3}$	$\cos \frac{\pi}{3}$

$$X = 4 \cos \frac{\pi}{3} + 2 \sin \frac{\pi}{3} \Rightarrow X = 2 + \sqrt{3}$$

$$Y = -4 \sin \frac{\pi}{3} + 2 \cos \frac{\pi}{3} \Rightarrow Y = -2\sqrt{3} + 1$$

$$\text{New} \Rightarrow (2 + \sqrt{3}, 1 - 2\sqrt{3})$$

Q  $\theta$  is an angle by which axes are rotated & eq<sup>n</sup>  $ax^2 + 2hxy + by^2$  does not contain any  $xy$  term in new system then P.T.  $\tan 2\theta = \frac{2h}{a-b}$

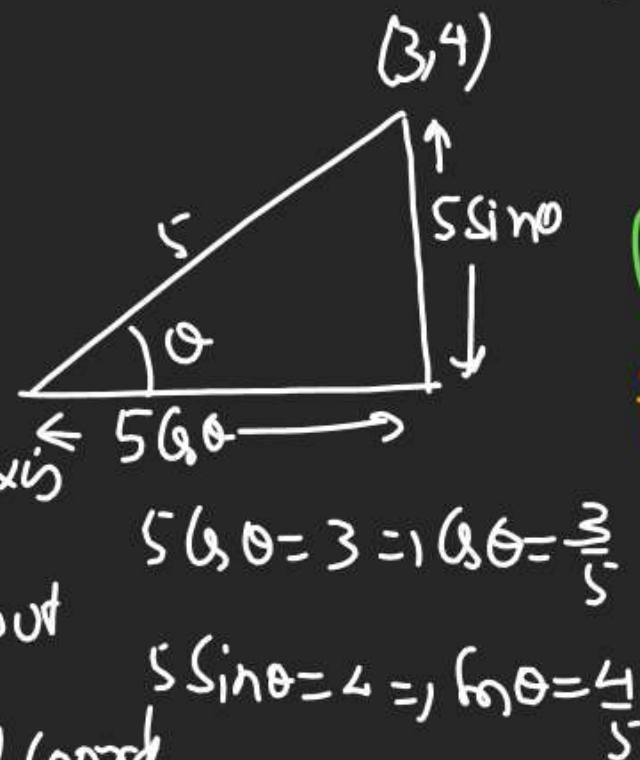
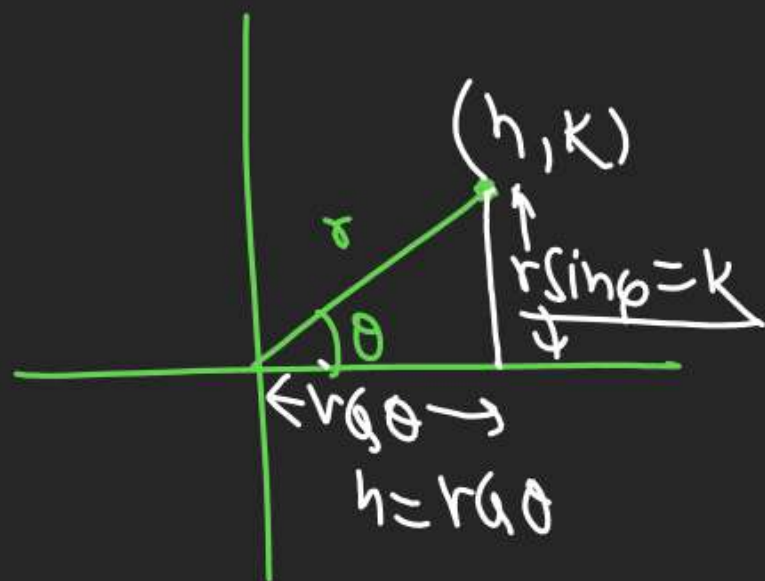


Q Pt. (4,1) Undergoes following transformation

① Reflection about  $y=x$  ✓

② Translation thru a dist. of 2 units along +ve direction of x-axis

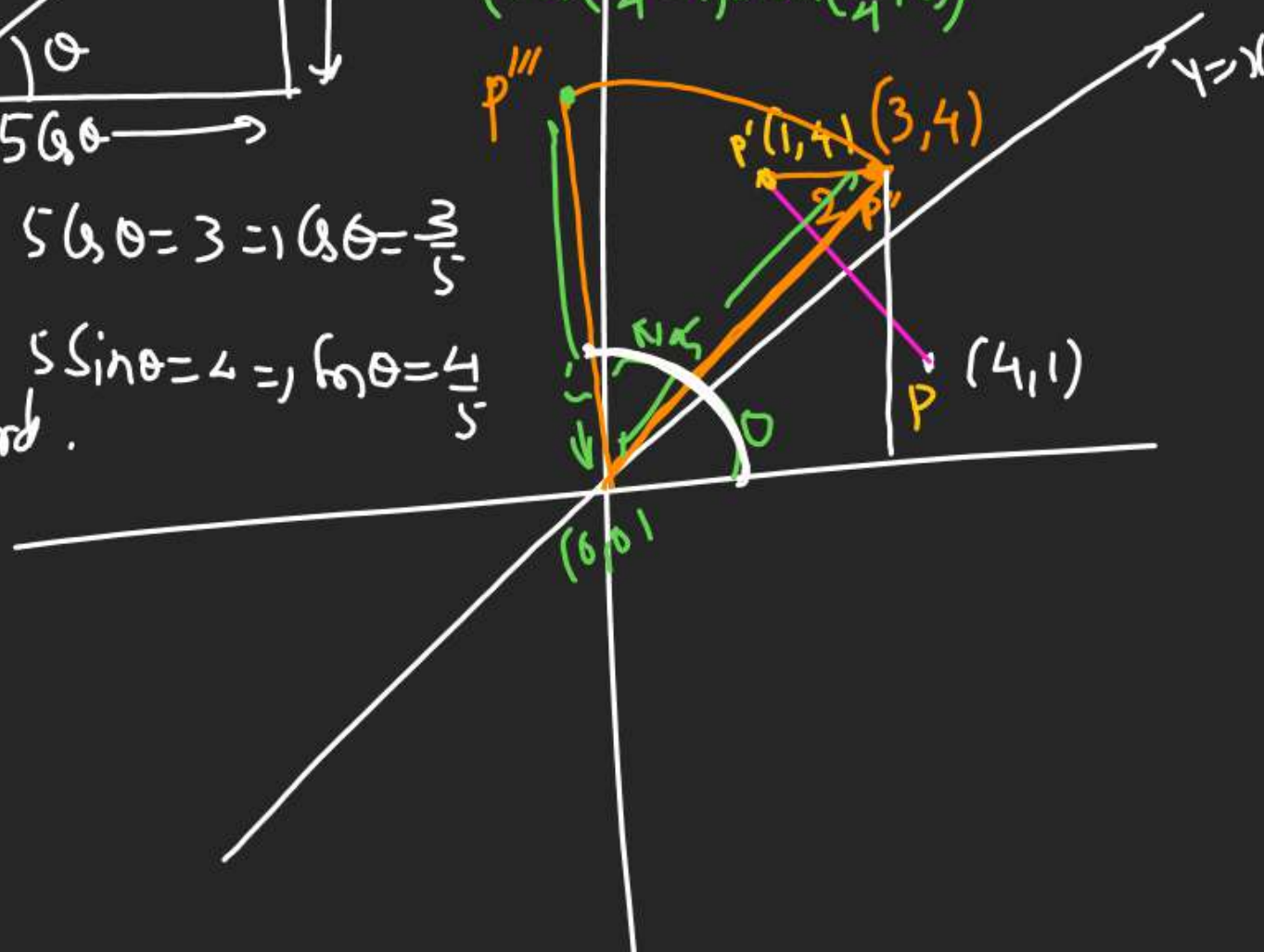
③ Rotation thru an angle of  $\frac{\pi}{4}$  about origin in ACW dir. find final coord.



$$5 \cos\left(\frac{\pi}{4} + \theta\right) = 5 \left( \cos \frac{\pi}{4} \cos \theta - \sin \frac{\pi}{4} \sin \theta \right)$$

$$= 4 \left( \frac{1}{\sqrt{2}} \times \frac{3}{5} - \frac{1}{\sqrt{2}} \times \frac{4}{5} \right)$$

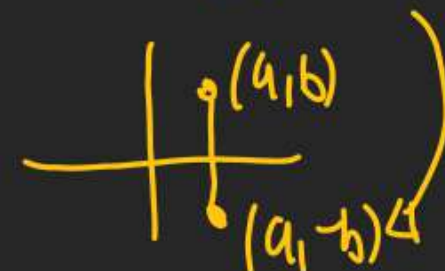
$$5 \cos\left(\frac{\pi}{4} + \theta\right), 5 \sin\left(\frac{\pi}{4} + \theta\right)$$



①  $y=x$  Ref.

$$(a,b) \rightarrow (b,a)$$

②  $(a,b)$  X Axis



③  $(a,b)$  Y Axis Ref.

