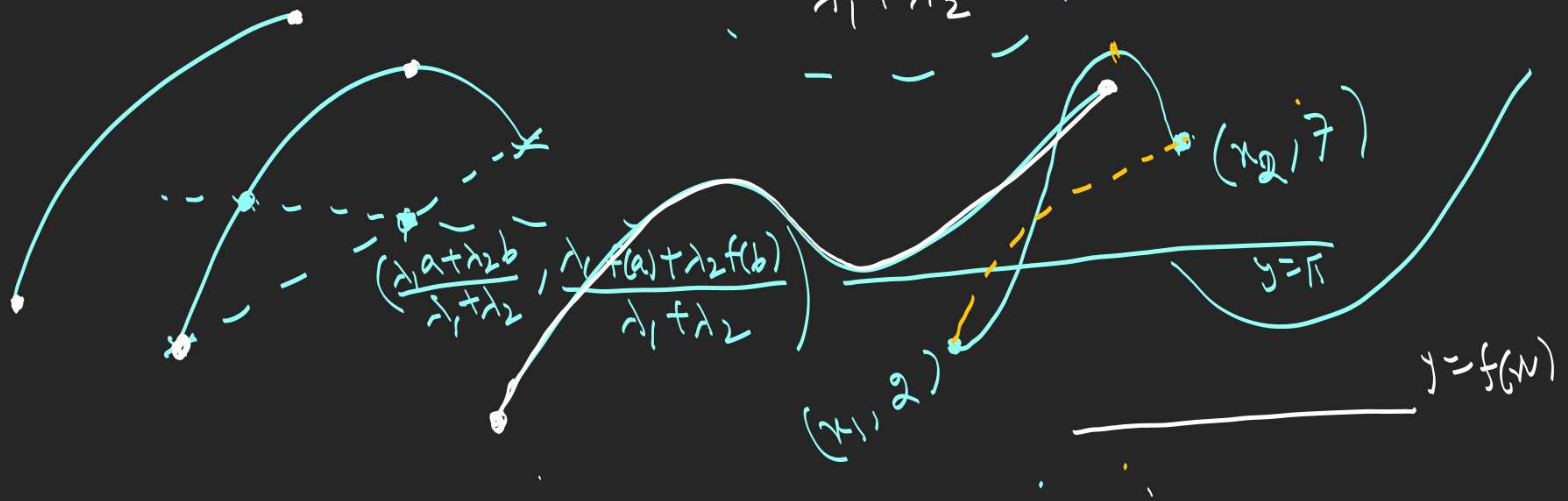
Nishant Jindal

Intermediate Value Theorem

there exists

Let $f(x)$ is continuous in $[a, b]$, then $\exists\, c \in [a,b]$ such that $f(c) = \dfrac{\lambda_1 f(a) + \lambda_2 f(b)}{\lambda_1 + \lambda_2}$, where $\lambda_1, \lambda_2 > 0$.



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Let f is cont. in $[a,b]$, then in interval $[a,b]$

- f will attain a maximum & minimum value.
- Range of $f(x)$ is closed interval $[m, M]$ or $\{m\}$

$m = \min(f(x))$

$M = \max(f(x))$

- I) &

Then $\exists\ c \in [a,b]$, $f(c) = \dfrac{\lambda_1 m + \lambda_2 M}{\lambda_1 + \lambda_2}$, $\lambda_1 > 0, \lambda_2 > 0$.

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1: Show that function $f(x) = (x-a)^2(x-b)^2 + x$ takes the value $\frac{a+b}{2}$ for some $x \in [a,b]$

$f(x)$ is continuous :

$f(a) = a$

$f(b) = b$

$\exists c \in [a,b]$, $f(c) = \frac{f(a)+f(b)}{2} = \frac{a+b}{2}$

(Using IVT)

$\Rightarrow g(c) = 0$ for some $c \in (0,1)$

$g(0) = -1$

$g(1) = \underline{0+2-1} = 1$

2: Let $f(x)$ is continuous in $[0,1]$ and $f(0)=0$, $f(1)=0$, then P.T. $f(c) = 1-2c^2$ for some $c \in [0,1)$.

$f(c)+2c^2-1$

$g(x) = \underline{f(x)} + (2x^2-1)$

g is cont. in $[0,1]$

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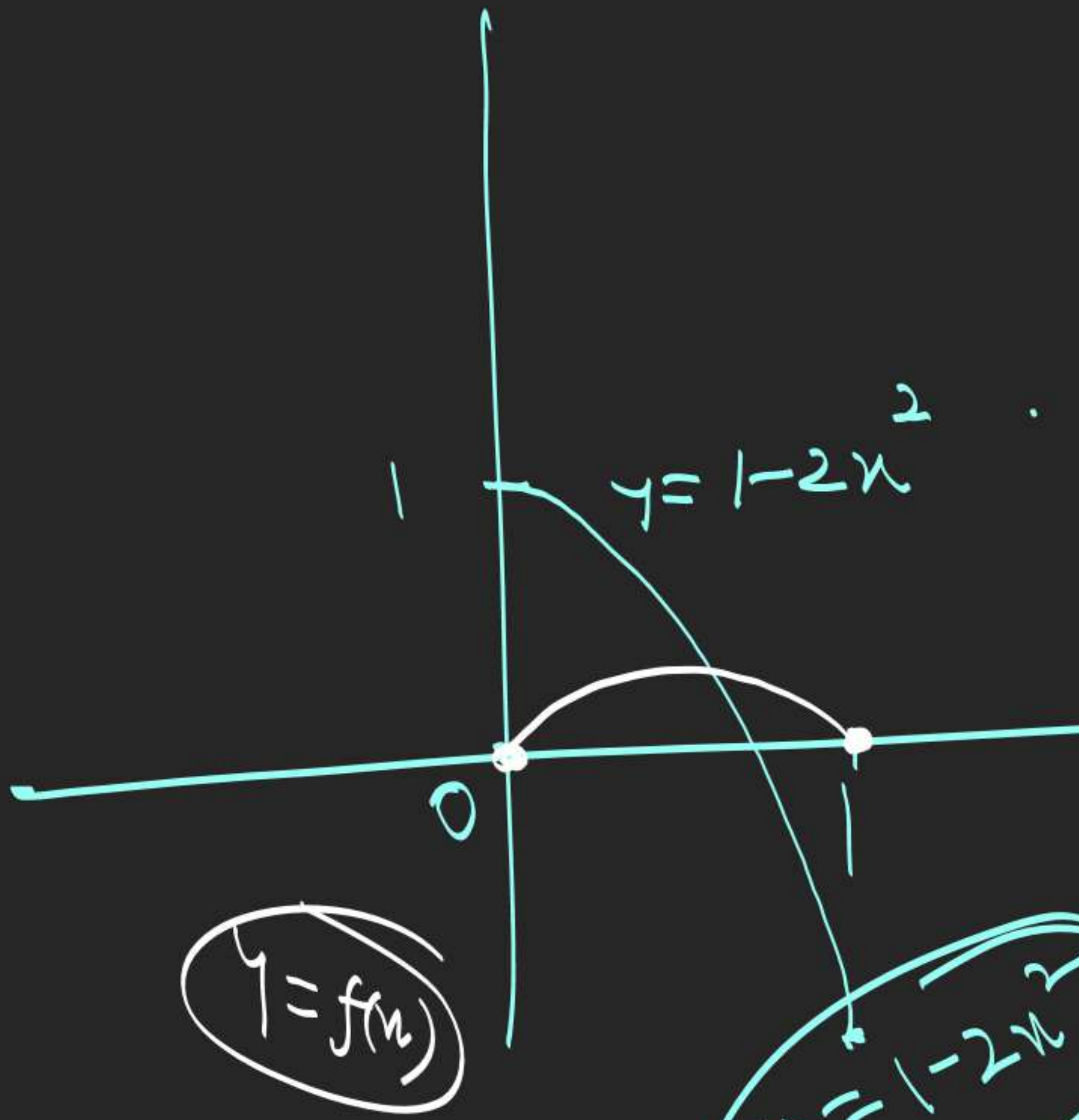

$f(x) =$

$g(x) =$

$(x_2, 7)$

f is cont. in $[x_1, x_2]$

$(x_1, 2)$

$y = 1 - 2x^2$

$c \in (x_1, x_2)$, $f(c) = \pi$.

$f(c) = 1 - 2c^2$

3. Let $f : [0,1] \to [0,1]$ be continuous and onto function. P.T. $\exists\, c \in [0,1]$ such that

$$f(c) = 1 - c$$

$g(x) = f(x) + x - 1$ g is cont. in $[0,1]$

$g(0) = f(0) - 1 \le 0$

$g(1) = f(1) \ge 0$

$g(c) = 0$ for some $c \in [0,1]$

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4. Show that $x = a\sin x + b$ where $0<a<1$, $b>0$ has atleast one positive root which doesn't exceed $b+a$.

$f(x) = x - a\sin x - b$ f is cont. $\forall x \in R$.

$f(0) = -b < 0$

$f(a+b) = (a+\cancel{b}) - a\sin(a+b) - \cancel{b} = \underset{>0}{a}\underbrace{\left(1-\sin(a+b)\right)}_{\geq 0}$

$\exists\ c \in (0, a+b]$, $f(c) = 0$.

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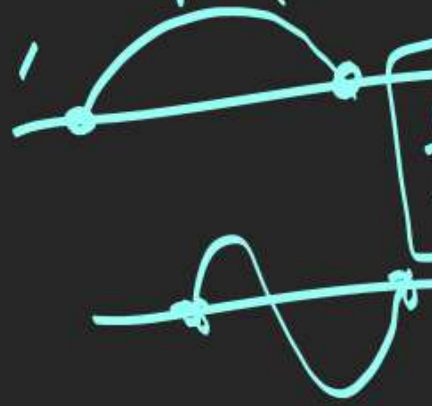
5. Let $f:[0,2]\to R$ be continuous and $f(0)=f(2)$.

P.T. there exists x_1, x_2 in $[0,2]$ such that

$\boxed{x_2-x_1=1}$ and $f(x_2)=f(x_1)$.

$\exists\ c\in[0,1]$, $g(c)=0$



$\boxed{f(1+c)=f(c)}$

$$f(1+x_1)=f(x_1)$$

$$g(x)=f(x+1)-f(x)$$

g is continuous in $[0,1]$

$$g(0)=f(1)-f(0)$$

$$g(1)=f(2)-f(1)=f(0)-f(1)$$

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Continuity of Composite Function

Cont → Proficiency Test.

Limits → Ex-5 (1 – 13)

- Let $f(x)$ is continuous at $x = a$.

& • $y = g(x)$ is continuous at $x = f(a)$

$\Rightarrow$ $g(f(x))$ is continuous at $x = a$

$$\lim_{x\to a} g(\underbrace{f(x)}) = g(f(a))$$

$f(a)^-$ $\quad$ $= f(a)$ $\quad$ $f(a)^+$