Nishant Jindal

① $\sin^{-1}(-x) = -\sin^{-1}x$

$\cos^{-1}(-x) = \pi - \cos^{-1}x$

$\tan^{-1}(-x) = -\tan^{-1}x$

$\cot^{-1}(-x) = \pi - \cot^{-1}x$

$\csc^{-1}(-x) = -\csc^{-1}x$

$\sec^{-1}(-x) = \pi - \sec^{-1}x$

② $\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$ $\quad |x| \le 1$

$\tan^{-1}x + \cot^{-1}x = \frac{\pi}{2}$ $\quad |x| \ge 0$

$\sec^{-1}x + \csc^{-1}x = \frac{\pi}{2}$ $\quad |x| \ge 1$

(3) $\left.\begin{array}{l}\sin(\sin^{-1}x) = x \\ \cos(\cos^{-1}x) = x\end{array}\right\} |x| \le 1$ graph.

$\left.\begin{array}{l}\tan(\tan^{-1}x) = x \\ \cot(\cot^{-1}x) = x\end{array}\right\} |x| \ge 0$

$\left.\begin{array}{l}\sec(\sec^{-1}x) = x \\ \csc(\csc^{-1}x) = x\end{array}\right\} |x| \ge 1$

(4) $\sin^{-1}\left(\frac{1}{x}\right) = \csc^{-1}(x) \rightarrow |x| \ge 1$

$\csc^{-1}\left(\frac{1}{x}\right) = \sin^{-1}(x) \rightarrow -1 \le x \le 1$

$\cos^{-1}\left(\frac{1}{x}\right) = \sec^{-1}(x) \rightarrow |x| \ge 1$

$\sec^{-1}\left(\frac{1}{x}\right) = \cos^{-1}(x) \rightarrow -1 \le x \le 1$

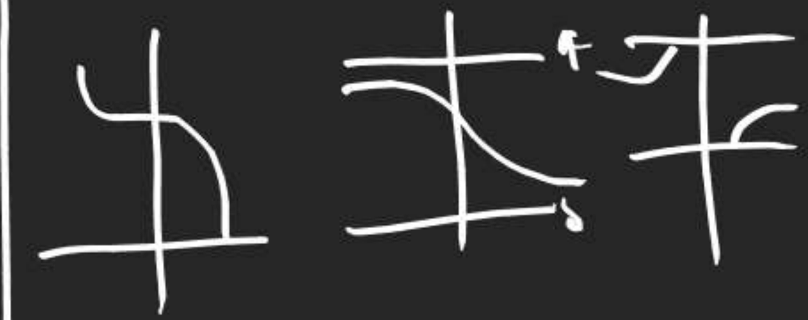
$\cot^{-1}(x) = \pi + \tan^{-1}\left(\frac{1}{x}\right) \rightarrow x < 0$

$= \tan^{-1}\left(\frac{1}{x}\right) \rightarrow x > 0$

(5) Conversion of one ITF into another is Possible Using $\triangle$

★ all ITF are ⊖

2, 4, 6 cos cot sec



1, 3, 5 sin tan cosec

Nishant Jindal

RK

$\sin^{-1}x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$\tan^{-1}x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

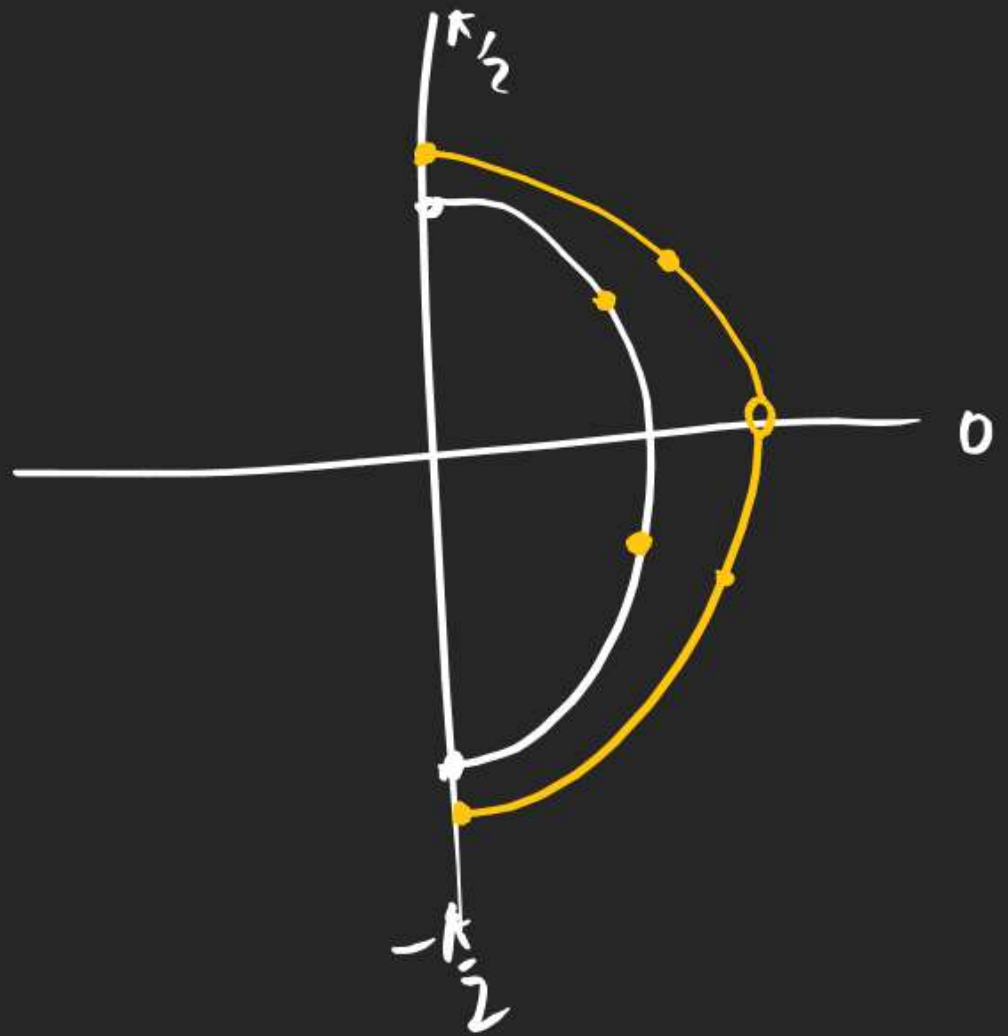
$\csc^{-1}x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$

$\cos^{-1}x \in [0, \pi]$

$\cot^{-1}x \in (0, \pi)$

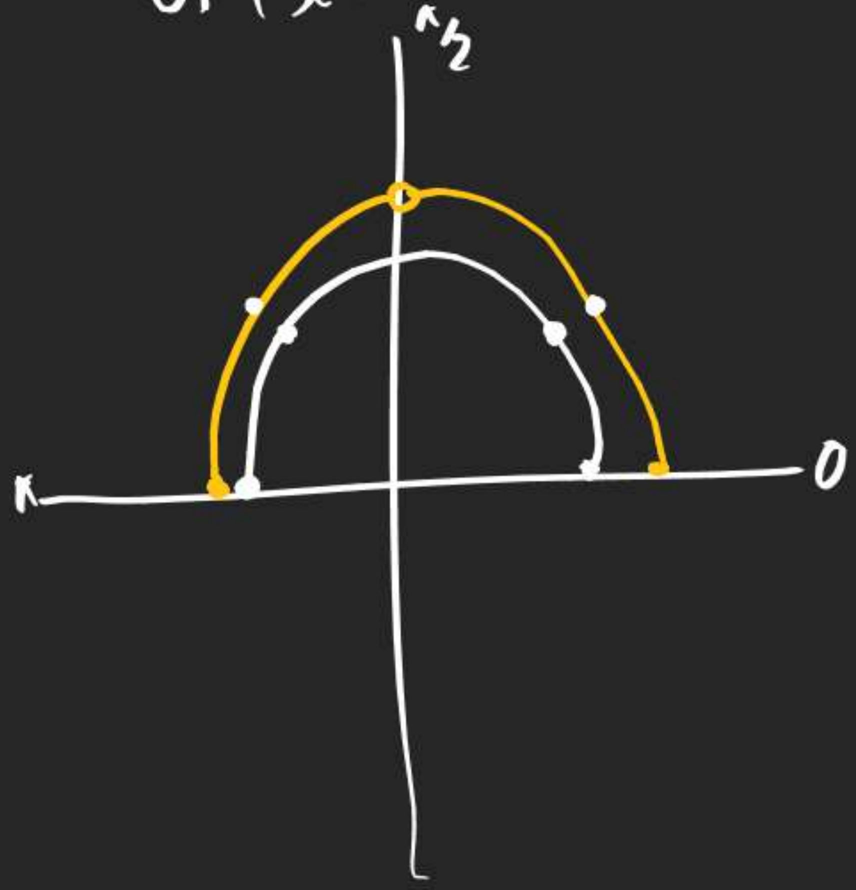
$\sec^{-1}x \in [0, \pi] - \left\{\frac{\pi}{2}\right\}$

$\sin^{-1} / \csc^{-1}$

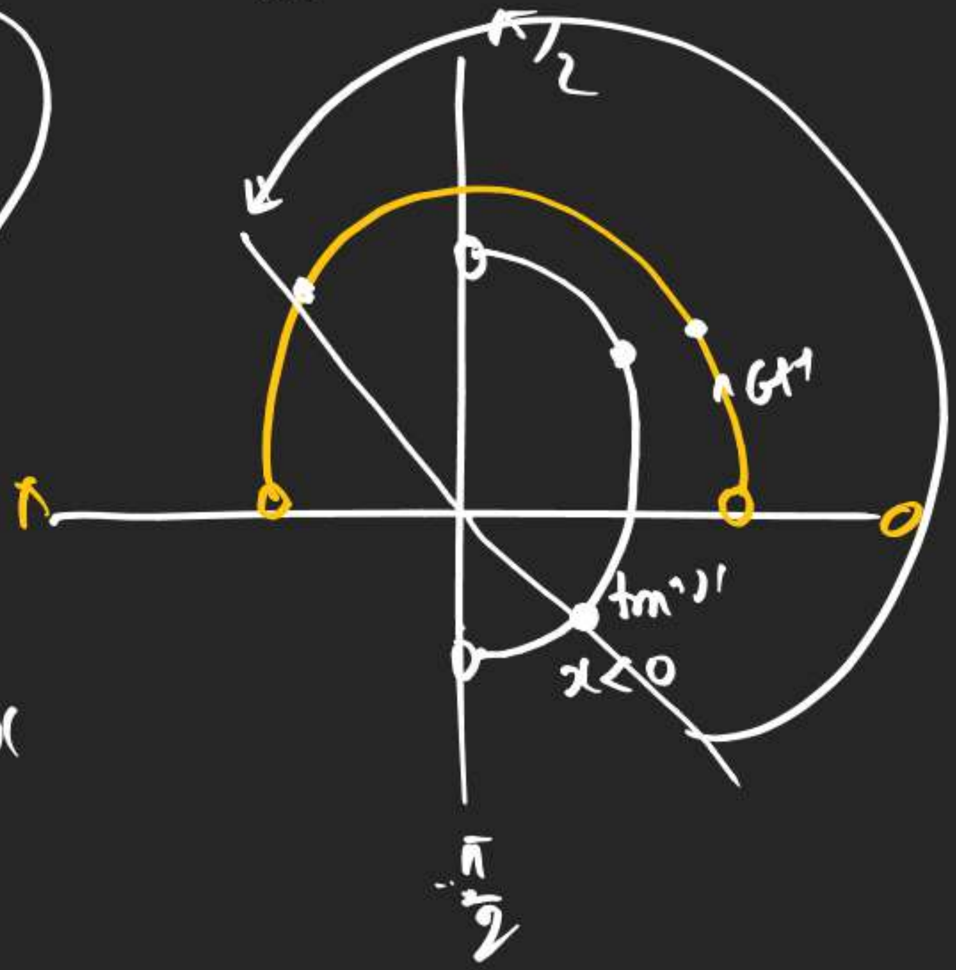


$\sin^{-1}\left(\frac{1}{x}\right) = \csc^{-1}x$

$\cos^{-1}\left(\frac{1}{x}\right) = \sec^{-1}x$



$\tan^{-1}\left(\frac{1}{x}\right) = \cot^{-1}x$



$\tan^{-1}\left(\frac{1}{x}\right) + \pi = \cot^{-1}(x)$

Proof of $\tan^{-1}\left(\frac{1}{x}\right)+\pi=\cot^{-1}(x)$ will be discussed later.

$\cot^{-1}(x)=\pi+\tan^{-1}\left(\frac{1}{x}\right)$ जब $x=-ve$

$\cot^{-1}(-\sqrt{2})=\pi+\tan^{-1}\left(\frac{1}{-\sqrt{2}}\right)$ $\quad x=-\sqrt{2}$

Q. Value of $\sec^{-1}(\sqrt{2})+\underbrace{\cot^{-1}(-\sqrt{2})}_{1,3,5\ \text{odd}}^{x=-ve}+\tan^{-1}\left(\frac{1}{\sqrt{2}}\right)=?$

$$\cos^{-1}\left(\frac{1}{\sqrt{2}}\right)+\pi+\tan^{-1}\left(\frac{1}{-\sqrt{2}}\right)+\tan^{-1}\left(\frac{1}{\sqrt{2}}\right)$$

$$\frac{\pi}{4}+\pi-\cancel{\tan^{-1}\left(\frac{1}{\sqrt{2}}\right)}+\cancel{\tan^{-1}\left(\frac{1}{\sqrt{2}}\right)}$$

$$=\frac{5\pi}{4}$$

Q. If $\sin^{-1}\left(\frac{x}{5}\right)+\operatorname{cosec}^{-1}\left(\frac{5}{4}\right)=\frac{\pi}{2}$ then value of x? $\rightarrow$ tempting

$\sin^{-1}\left(\frac{x}{5}\right)=\frac{\pi}{2}-\operatorname{cosec}^{-1}\frac{5}{4}$

$\sin^{-1}\left(\frac{x}{5}\right)=\sec^{-1}\frac{5}{4}$

$\sin^{-1}\left(\frac{x}{5}\right)=\cos^{-1}\left(\frac{4}{5}\right)$

$\sin^{-1}\left(\frac{x}{5}\right)=\sin^{-1}\left(\frac{3}{5}\right)\Rightarrow\boxed{x=3}$

Nishant Jindal

S202

Q3 If α is one of the root of $x^2+3x+2=0$

then find A) $\tan^{-1}\alpha + \tan^{-1}\left(\frac{1}{\alpha}\right)$ (B) $\cot^{-1}(\alpha)+\cot^{-1}\left(\frac{1}{\alpha}\right)$

$\tan^{-1}\left(\frac{1}{x}\right) = \cot^{-1}(x)$ $x>0$

$\tan^{-1}\left(\frac{1}{x}\right)+\pi = \cot^{-1}(x)$ $x<0$

$x^2+3x+2=0 \Rightarrow (x+1)(x+2)=0$

$x=-1$ & -2 (Roots) $\alpha=-1$ or -2 In Both Cases $\rightarrow \alpha = -ve$

$\tan^{-1}\left(\frac{1}{x}\right)+\pi$

$=\cot^{-1}(x)$

$x=-ve$

$\tan^{-1}\left(\frac{1}{x}\right)=-\pi+\cot^{-1}(x)$

A) $\tan^{-1}\alpha + \boxed{\tan^{-1}\left(\frac{1}{\alpha}\right)}$

$\tan^{-1}\alpha + -\pi + \cot^{-1}(\alpha)$

$-\pi+\frac{\pi}{2} = -\frac{\pi}{2}$

$\alpha = -ve$

(B) $\cot^{-1}(\alpha)+\cot^{-1}\left(\frac{1}{\alpha}\right)$

$\frac{\pi}{2}-\tan^{-1}(\alpha) + \frac{\pi}{2}-\tan^{-1}\left(\frac{1}{\alpha}\right)$

$\pi-\left(\tan^{-1}\alpha+\tan^{-1}\frac{1}{\alpha}\right)$

$\pi-\left(-\frac{\pi}{2}\right)=\frac{3\pi}{2}$

Nishant Jindal

S2C2

Profile 3

Q If x_1, x_2, x_3 are Roots of $x^3 - 6x^2 + 3Px - 2P = 0$ then find $\sin^{-1}\left(\frac{1}{x_1}+\frac{1}{x_2}\right) + \cos^{-1}\left(\frac{1}{x_1}+\frac{1}{x_2}\right) - \tan^{-1}\left(\frac{1}{x_1}+\frac{1}{x_2}\right) = ?$

Demand $= \sin^{-1}\left(\frac{1}{2}+\frac{1}{2}\right) + \cos^{-1}\left(\frac{1}{2}+\frac{1}{2}\right) - \tan^{-1}\left(\frac{1}{2}+\frac{1}{2}\right)$

$\sin^{-1}(1) + \cos^{-1}(1) - \tan^{-1} 1 = \frac{\pi}{2} + 0 - \frac{\pi}{4} = \frac{\pi}{4}$

$x^3 - 6x^2 + 3Px - 2P = 0 \; < x_1, x_2, x_3$

1) $x_1 + x_2 + x_3 = -\frac{b}{a} = -\frac{(-6)}{1} = 6$

2) $\Sigma x_1 x_2 = \frac{c}{a} = \frac{3P}{1} = 3P$

3) $x_1 x_2 x_3 = -\frac{d}{a} = -\frac{(2P)}{1} = 2P$

$\frac{K+K+K}{3} = 2 \Leftarrow AM = \frac{x_1+x_2+x_3}{3}$ $\quad HM = \frac{3}{\frac{1}{x_1}+\frac{1}{x_2}+\frac{1}{x_3}}$

$3K = 6 \Rightarrow K = 2$

$AM = \frac{6}{3} = 2$ | $HM = \frac{3(x_1 x_2 x_3)}{x_2 x_3 + x_1 x_3 + x_1 x_2} = \frac{3 \times 2P}{3P} = 2$

Sol

$AM = \frac{x_1+x_2+x_3}{3}$

$GM = (x_1 \cdot x_2 x_3)^{1/3}$

$HM = \frac{3}{\frac{1}{x_1}+\frac{1}{x_2}+\frac{1}{x_3}}$

$AM = HM = 2 \Rightarrow$ Concept $\Rightarrow$ If $AM = GM = HM$ then value of all elements is equal

$\Rightarrow x_1 = x_2 = x_3 = K = 2$

Nishant Jindal

Profile S-2 Q4

If $f(x) = \sin^{-1}x - x$, $g(x) = x^2+5x+6+\sin^{-1}x$

& $|f(x)| + |g(x)| = |f(x)+g(x)|$ then find Set of values of x?

$|a|+|b| = |a+b| \rightarrow a\cdot b \geq 0$ Use

$|2|+|3| = |2+3|$

$|-2|+|-3| = |-2+-3|$

$|-2|+|3| \neq |-2+3|$

$|2|+|-3| \neq |2+-3|$

$2+3 \neq |-1| = 1$

$-1 \leq x \leq 1$

$f(x)\cdot g(x) \geq 0$

$-1 \leq x \leq 1$

$(\boxed{\sin^{-1}x} - x)\cdot\underbrace{(x^2+5x+6+\boxed{\sin^{-1}x})}_{+ \quad \oplus} \geq 0$

$\sin^{-1}x - x \geq 0$

$\sin^{-1}x \geq x$

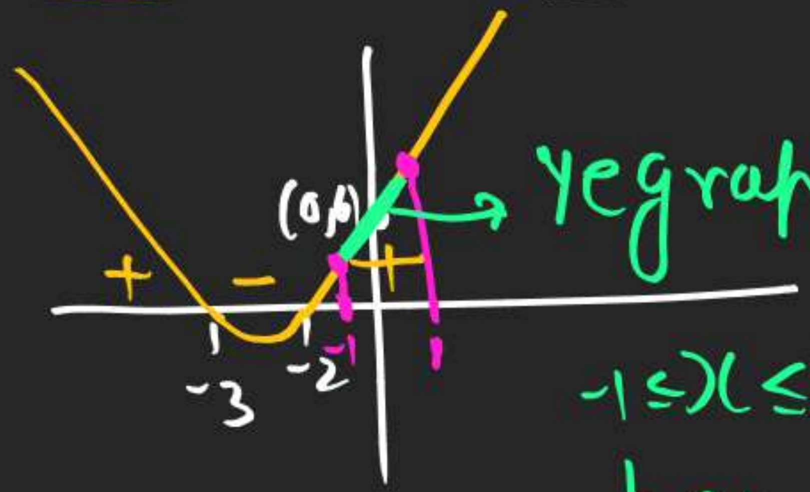
$\sin^{-1}x$ is Uncha to x Khaa?

$0 \leq x \leq 1$

$x=0$

$x=$

$x^2+5x+6 = (x+2)(x+3)$



ye graph $-1 \leq x \leq 1$ me hai

above X Axis

$\Rightarrow x^2+5x+6$ $\oplus$ hai

$-1 \leq x \leq 1$

Nishant Jindal

S_2O_2

Q Evaluate $\sum_{n=1}^{10}\sum_{m=1}^{10} \tan^{-1}\frac{m}{n}$

① Pahlem ki ek value Rkhenge.

$\sum_{n=1}^{10} \boxed{\tan^{-1}\frac{1}{n}} + \boxed{\tan^{-1}\frac{2}{n}} + \boxed{\tan^{-1}\frac{3}{n}} + \tan^{-1}\frac{4}{n} + \cdots + \boxed{\tan^{-1}\frac{10}{n}}$

$= \tan^{-1}\frac{1}{1} + \tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{4} + \cdots + \tan^{-1}\frac{1}{10}$

$+ \tan^{-1}\frac{2}{1} + \tan^{-1}\frac{2}{2} + \boxed{\tan^{-1}\frac{2}{3}} + \tan^{-1}\frac{2}{4} + \cdots + \tan^{-1}\frac{2}{10}$

$+ \tan^{-1}\frac{3}{1} + \boxed{\tan^{-1}\frac{3}{2}} + \tan^{-1}\frac{3}{3} + \tan^{-1}\frac{3}{4} \cdots + \tan^{-1}\frac{3}{10}$

$\vdots$

$+ \tan^{-1}\frac{10}{1} + \tan^{-1}\frac{10}{2} + \tan^{-1}\frac{10}{3} + \cdots + \tan^{-1}\frac{10}{10}$

$\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{2}{1}$

$\tan^{-1}\frac{1}{2} + \cot^{-1}\frac{1}{2}$

$= \frac{\pi}{2}$

$10 \times \tan^{-1} 1$

$+ 45 \times \frac{\pi}{2}$

$\cancel{10}^{5} \times \frac{\pi}{\cancel{4}_2} + \frac{45\pi}{2}$

$= \frac{50\pi}{2}$

$= 25\pi$

Nishant Jindal

S_2O_2

Q Value of $\operatorname{cosec}(\tan^{-1}(\cos(\cot^{-1}(\sec(\sin^{-1} a))))) = ?$

Adv

T.P → $\sec(\sin^{-1} a)$ पसन्द नहीं।

$\sec(\sec^{-1} a) = a$ पसन्द आता

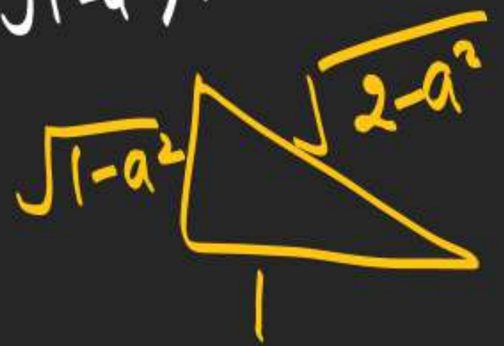
→ sec में Badlna है।

$\theta = \sin^{-1} a$

$\sin\theta = \frac{a}{1}$

$\cot^{-1}\left(\sec\left(\sec^{-1} \frac{1}{\sqrt{1-a^2}}\right)\right)$

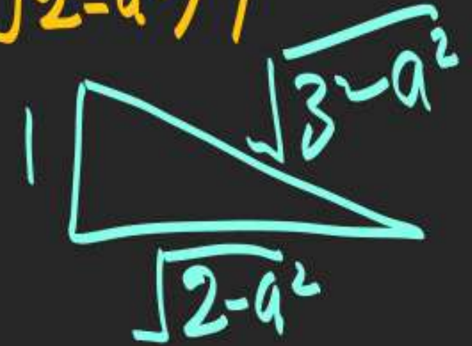
$\cos\left(\cot^{-1}\left(\frac{1}{\sqrt{1-a^2}}\right)\right)$ B/P



$\tan^{-1}\left(\cos\left(\cos^{-1} \frac{1}{\sqrt{2-a^2}}\right)\right)$ B/H

H/P

$\operatorname{cosec}\left(\tan^{-1}\left(\frac{1}{\sqrt{2-a^2}}\right)\right)$ P/B



$\operatorname{cosec}\left(\operatorname{cosec}^{-1} \frac{\sqrt{3-a^2}}{1}\right) = \sqrt{3-a^2}$

Nishant Jindal

$S_2 02$

Q Evaluate $\tan\left(\frac{\pi}{4}+\frac{1}{2}\cos^{-1}\frac{a}{b}\right)+\tan\left(\frac{\pi}{4}-\frac{1}{2}\cos^{-1}\frac{a}{b}\right)$ Raaz L ⇒

let $\frac{1}{2}\cos^{-1}\frac{a}{b}=\theta \Rightarrow \cos^{-1}\frac{a}{b}=2\theta \Rightarrow \cos 2\theta=\frac{a}{b}$.

Demand $\tan\left(\frac{\pi}{4}+\theta\right)+\tan\left(\frac{\pi}{4}-\theta\right)$

$$=\frac{1+\tan\theta}{1-\tan\theta}+\frac{1-\tan\theta}{1+\tan\theta}=\frac{(1+\tan\theta)^2+(1-\tan\theta)^2}{(1-\tan\theta)(1+\tan\theta)}=\frac{2(1+\tan^2\theta)}{1-\tan^2\theta}$$

Sochna

$$=\frac{2}{\frac{(1-\tan^2\theta)}{(1+\tan^2\theta)}}=\frac{2}{\cos 2\theta}$$

$$=\frac{2}{a/b}=\frac{2b}{a} \quad \text{Ans}$$

Nishant Jindal

Q find Range of $Y = \sin^{-1}(\cos(\sin^{-1}x)) + \cos^{-1}(\sin(\cos^{-1}x))$

Sol

$\cos^{-1}x = \theta$

$\cos\theta = \frac{x}{1}$

sin को cos⁻¹ Jantaa nahi
sin Me sin⁻¹ hota to Moja thu.

$$\sin^{-1}(\cos(\cos^{-1}\sqrt{1-x^2})) + \cos^{-1}\left(\sin\left(\sin^{-1}\frac{\sqrt{1-x^2}}{1}\right)\right)$$

$$\sin^{-1}(\sqrt{1-x^2}) + \cos^{-1}(\sqrt{1-x^2})$$

$$Y = \frac{\pi}{2} \qquad R_f \in \left\{\frac{\pi}{2}\right\}$$

Nishant Jindal

Q Let $h(x) = \tan\left\{\dfrac{\cos^{-1}(\sin(\cos^{-1}x)) + \sin^{-1}(\cos(\sin^{-1}x))}{2}\right\}$ find $\sum_{x=1}^{7} h\left(\frac{x}{8}\right) = ?$

Prev qs.

$h(x) = \tan\left(\frac{\frac{\pi}{2}}{2}\right) = \tan\left(\frac{\pi}{4}\right) = 1 \Rightarrow h(x) = 1 \leftarrow$ Constant

$h\left(\frac{x}{8}\right) = 1$

Demand $\sum h\left(\frac{x}{8}\right) = \sum_{x=1}^{7} 1 = 1+1+1+1+1+1+1$

$= 7$

Q Find value of $\sin\left(\sin^{-1}\frac{3}{5} - \cos^{-1}\frac{3}{5}\right) = ?$

Trigo (ITF (Constant)) → θ मानो

$\sin^{-1}\frac{3}{5} = \theta$

$\sin\theta = 3/5$

$\cos\theta = \sqrt{1-\left(\frac{3}{5}\right)^2} = \frac{4}{5}$

$\cos^{-1}\frac{3}{5} = \phi$

$\cos\phi = \frac{3}{5} \Rightarrow \sin\phi = \sqrt{1-\left(\frac{3}{5}\right)^2} = \frac{4}{5}$

Demand $= \sin(\theta - \phi)$

$= \sin\theta \cdot \cos\phi - \cos\theta \cdot \sin\phi$

$= \frac{3}{5}\cdot\frac{3}{5} - \frac{4}{5}\cdot\frac{4}{5} = \frac{9-16}{25} = \frac{-7}{25}$

$-\frac{3}{5} + \frac{3}{5} = 0$

Q. $\cos(2\boxed{\tan^{-1}2}) + \sin(2\tan^{-1}3) = ?$

$\tan^{-1}2 = \theta$

$\tan\theta = 2$

$\tan^{-1}3 = \phi$

$\tan\phi = \frac{3}{1}$

Demand $= \cos 2\theta + \sin 2\phi$

$= \frac{1-\tan^2\theta}{1+\tan^2\theta} + 2\sin\phi\cos\phi$

$= \frac{1-4}{1+4} + 2\cdot\frac{3}{\sqrt{10}} \times \frac{1}{\sqrt{10}}$

Nishant Jindal

Q Value of $\tan\left(\frac{1}{2}\boxed{\cos^{-1}\frac{\sqrt{5}}{3}}\right)$

$\cos^{-1}\frac{\sqrt{5}}{3} = \theta \Rightarrow \boxed{\cos\theta = \frac{\sqrt{5}}{3}} \Rightarrow \tan\theta = \frac{2}{\sqrt{5}} \Rightarrow \frac{2\tan\theta/2}{1-\tan^2\theta/2} = \frac{2}{\sqrt{5}}$

Demand: $\tan\frac{\theta}{2}$

$= \frac{3-\sqrt{5}}{2}$

$\Rightarrow 2\sqrt{5}\tan\frac{\theta}{2} = 2 - 2\tan^2\theta/2$

$\Rightarrow 2\tan^2\theta/2 + 2\sqrt{5}\tan\theta/2 - 2 = 0$

$\Rightarrow \tan^2\theta/2 + \sqrt{5}\tan\theta/2 - 1 = 0$

$\tan\theta/2 = \frac{-\sqrt{5} \pm \sqrt{5+4}}{2} = \frac{3-\sqrt{5}}{2},\ \frac{-3-\sqrt{5}}{2}$

Property 1 and Constant Property

Q.27 $6\left(\sin^{-1}x\right)^2 - \pi\sin^{-1}x \le 0$

$6t^2 - \pi t < 0$

$(t)(6t - \pi) < 0$

$0 < t < \frac{\pi}{6}$

$0 < \sin^{-1}x < \frac{\pi}{6}$

We know

$-\frac{\pi}{2} \le \sin^{-1}x \le \frac{\pi}{2}$

$0 < \sin^{-1}x < \frac{\pi}{6}$

$\sin 0 < x < \sin\frac{\pi}{6}$

$0 < x < 1/2$

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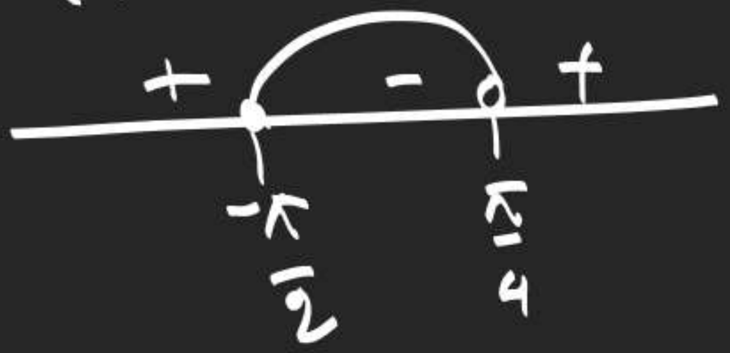
Property 1 and Constant Property

Q.28 $\dfrac{2\tan^{-1}x+\pi}{4\tan^{-1}x-\pi} \le 0$

$\dfrac{(2t+\pi)}{(4t-\pi)} \le 0$



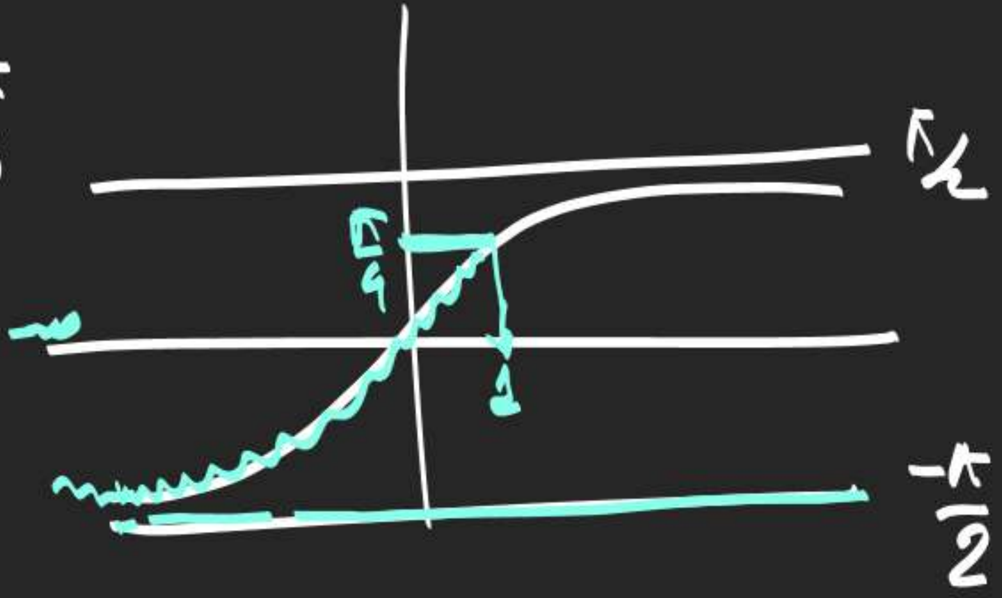
$-\frac{\pi}{2} \le \tan^{-1}x < \frac{\pi}{4}$

③ Imp

$-\frac{\pi}{2} < \tan^{-1}x < \frac{\pi}{2}$

$-\frac{\pi}{2} < \tan^{-1}x < \frac{\pi}{4}$

$-\infty < x < 1$



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Property 1 and Constant Property

Q.29 $\sin^{-1} x < \sin^{-1} x^2$

$x < x^2$

$x^2 - x > 0$

$(x)(x-1) > 0$

$x < 0 \cup x > 1$

We know.

$-1 \le x \le 1$

We know

$-1 \le x^2 \le 1$

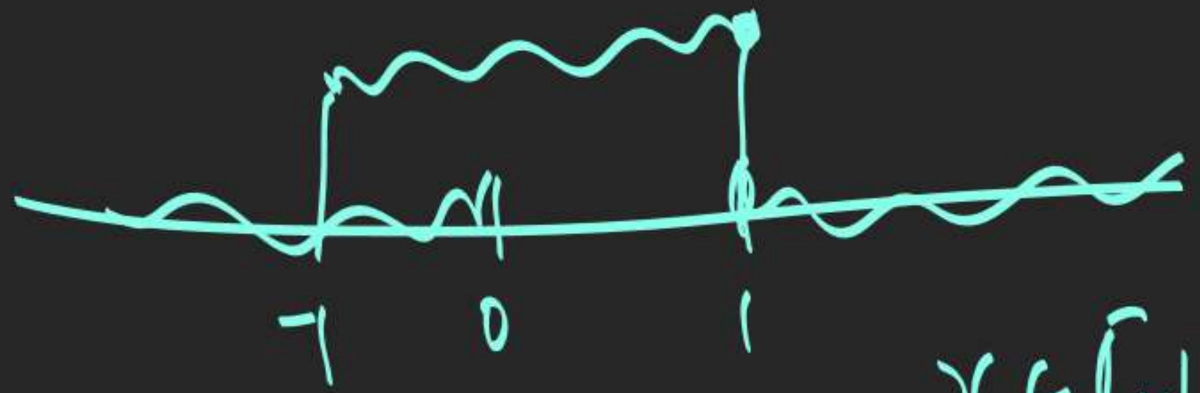
-ve ⊗

$0 \le x^2 \le 1$

$0 \le \sqrt{x^2} \le 1$

$0 \le |x| \le 1$ Ghoda

$x \in [-1, 1]$



$x \in [-1, 0)$