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1. Find eqn. of tangent to ellipse $9x^2+16y^2=144$ passing thru $(2,3)$.

$$\frac{x^2}{16}+\frac{y^2}{9}=1$$

$$y = mx \pm \sqrt{16m^2+9}$$

Put $(2,3)$

$$(3-2m)^2 = 16m^2+9 \Rightarrow 12m^2+12m=0$$

$$\boxed{m=0,-1}$$

$$y-3=0(x-2)$$

$$y-3=-1(x-2)$$

$$\boxed{y=3,\ x+y=5}$$

2. Two tangents to ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ have inclinations θ_1, θ_2. Find the locus of their point of intersection if $\cot\theta_1+\cot\theta_2=\lambda$, λ is a given constant.

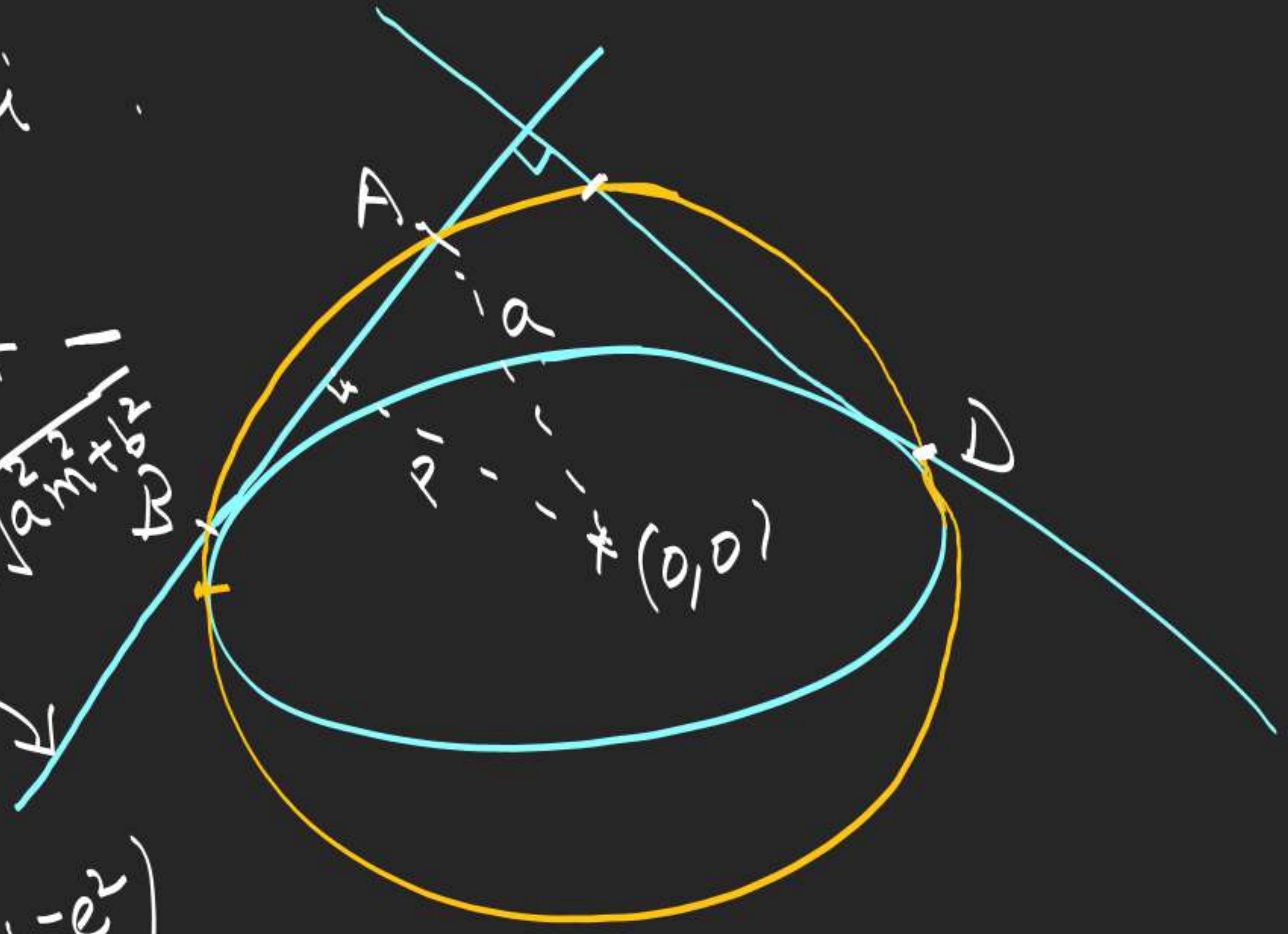
$$\frac{1}{m_1}+\frac{1}{m_2}=\lambda$$

$$\Rightarrow \lambda = \frac{2hk}{k^2-b^2}$$

$$y = mx \pm \sqrt{a^2m^2+b^2}$$

$$(k-mh)^2 = a^2m^2+b^2 \Rightarrow m^2(h^2-a^2)-2hkm+(k^2-b^2)=0$$

3. Two tangents to an ellipse intersect at right angles. P.T. the sum of square of chords which the auxiliary circle intersects on them is constant equal to square of distance between the foci.



$$AB^2 + CD^2 = 4\left(a^2 - \frac{a^2m^2+b^2}{1+m^2}\right) + -$$

$$= \frac{4(a^2-b^2)}{1+m^2} + \frac{4(a^2-b^2)}{1+\left(-\frac{1}{m}\right)^2}$$

$$y = mx \pm \sqrt{a^2m^2+b^2}$$

$$= 4(a^2-b^2) = 4a^2e^2 = (2ae)^2$$

$$b^2 = a^2(1-e^2)$$

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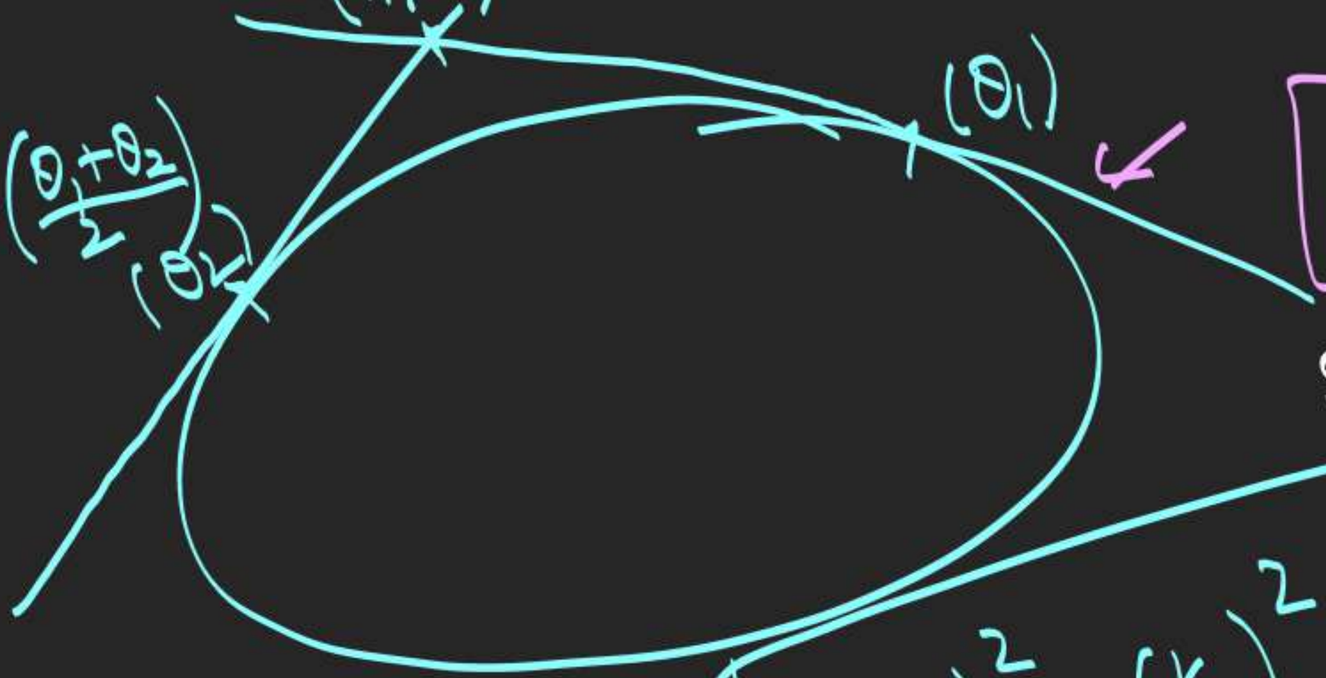
4. Find the locus of point of intersection of pair of tangents to ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ if the difference of eccentric angle of their point of contacts is $\frac{2\pi}{3}$.



(h,k)

(θ_1)

(θ_2)

$\frac{x}{a}\cos\theta+\frac{y}{b}\sin\theta=1$

$$\boxed{\frac{h}{a}\cos\theta+\frac{k}{b}\sin\theta=1} \quad \theta_1, \theta_2$$

$$h=\frac{a\cos\left(\frac{\theta_1+\theta_2}{2}\right)}{\cos\left(\frac{\theta_2-\theta_1}{2}\right)}=2a\cos\left(\frac{\theta_1+\theta_2}{2}\right)$$

$\frac{\pi}{3}$

$$k=\frac{b\sin\left(\frac{\theta_1+\theta_2}{2}\right)}{\cos\left(\frac{\theta_2-\theta_1}{2}\right)}=2b\sin\left(\frac{\theta_1+\theta_2}{2}\right)$$

$$\boxed{\left(\frac{h}{a}\right)^2+\left(\frac{k}{b}\right)^2=4}$$

$\sin\theta_1+\sin\theta_2=1$

$$1+\frac{k^2}{b^2}\sin^2\theta-\frac{2k}{b}\sin\theta=\frac{h^2}{a^2}(1-\sin^2\theta)$$

$$\left(\frac{h^2}{a^2}+\frac{k^2}{b^2}\right)\sin^2\theta-\frac{2k}{b}\sin\theta+1-\frac{h^2}{a^2}=0$$

$$\frac{\frac{2k}{b}}{\frac{h^2}{a^2}+\frac{k^2}{b^2}}=1$$

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5: Find the locus of point of intersection of two tangents to ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, if the sum of the ordinates of the points of contact is 'b'.

$$h = \frac{a\cos\left(\frac{\theta_1+\theta_2}{2}\right)}{\cos\left(\frac{\theta_1-\theta_2}{2}\right)} \quad -(1)$$

$$k = \frac{b\sin\left(\frac{\theta_1+\theta_2}{2}\right)}{\cos\left(\frac{\theta_1-\theta_2}{2}\right)} \quad -(2)$$

$$\sin\theta_1+\sin\theta_2 = 1 = 2\sin\left(\frac{\theta_1+\theta_2}{2}\right)\cos\left(\frac{\theta_1-\theta_2}{2}\right) \quad -(3)$$

$$\frac{(2)}{(3)} \quad \frac{k}{1} = \frac{b}{2\cos^2\left(\frac{\theta_1-\theta_2}{2}\right)}$$

$$\left(\frac{h}{a}\right)^2+\left(\frac{k}{b}\right)^2 = \frac{1}{\cos^2\left(\frac{\theta_1-\theta_2}{2}\right)}$$

$$\boxed{\frac{h^2}{a^2}+\frac{k^2}{b^2} = \frac{2k}{b}}$$

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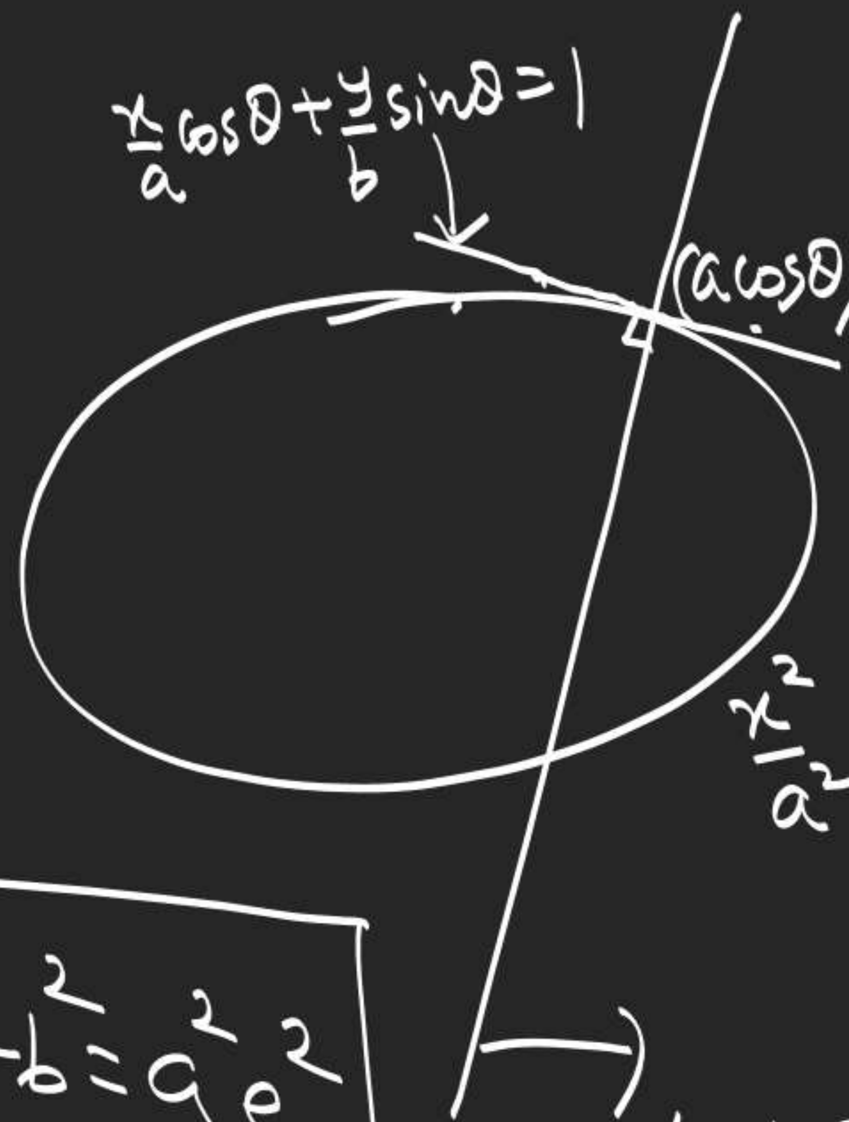
Normal

$\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta = 1$

$(a\cos\theta, b\sin\theta) = (x_1, y_1)$



$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\frac{x}{b}\sin\theta - \frac{y}{a}\cos\theta = \sin\theta\cos\theta\left(\frac{a}{b} - \frac{b}{a}\right)$$

$$\boxed{\frac{a^2 x}{x_1} - \frac{b^2 y}{y_1} = a^2 - b^2 = a^2 e^2}$$

$$\boxed{(a\sec\theta)x - (b\csc\theta)y = a^2 - b^2 = a^2 e^2}$$

Normal thru (α, β)

$\theta \in [0, 2\pi)$ $\qquad$ $\theta/2 \in [0, \pi)$

$$a x \sec\theta - b y \operatorname{cosec}\theta = a^2 e^2$$

$$\frac{a\alpha}{\cos\theta} - \frac{b\beta}{\sin\theta} = a^2e^2$$

$$\frac{a\alpha(1+t^2)}{1-t^2} - \frac{b\beta(1+t^2)}{2t} = a^2e^2$$

$$a\alpha(2t+2t^3) - b\beta(1-t^4) = a^2e^2(2t-2t^3)$$

$$f(t) = \underset{>0}{b\beta}\, t^4 + (2a\alpha + 2a^2e^2)t^3 + (2a\alpha - 2a^2e^2)t - b\beta = 0$$

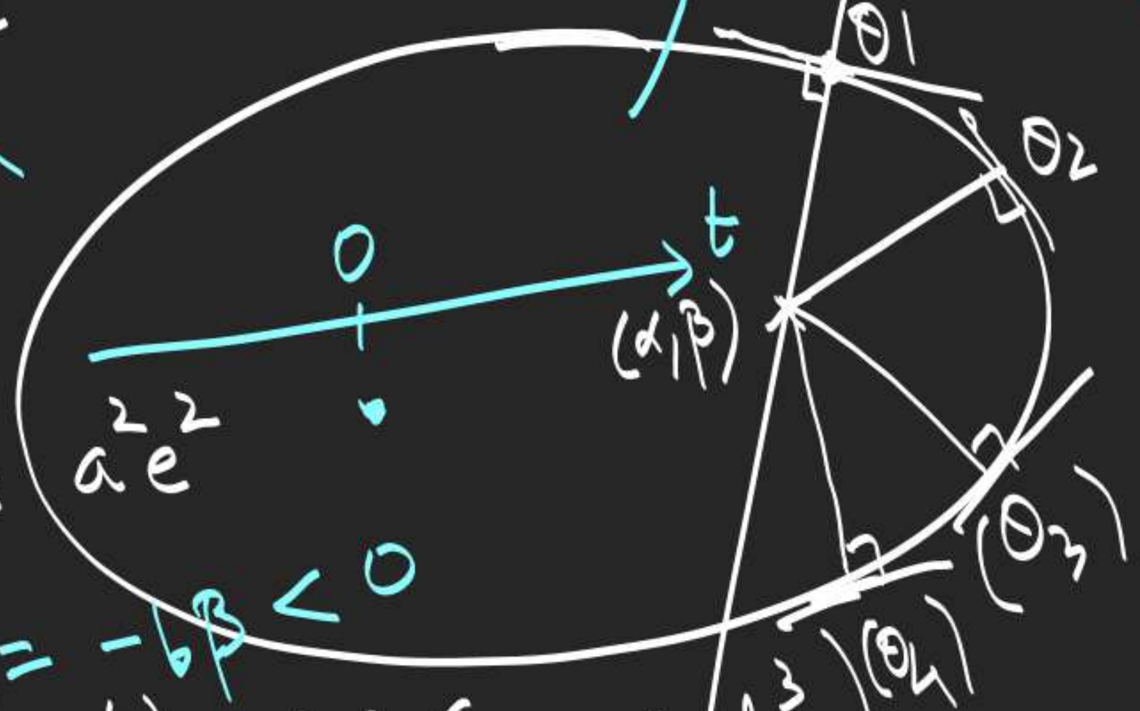
$f(0) = -b\beta < 0$

$t \to \infty$, $f(t) \to \infty$

$t \to -\infty$, $f(t) \to \infty$

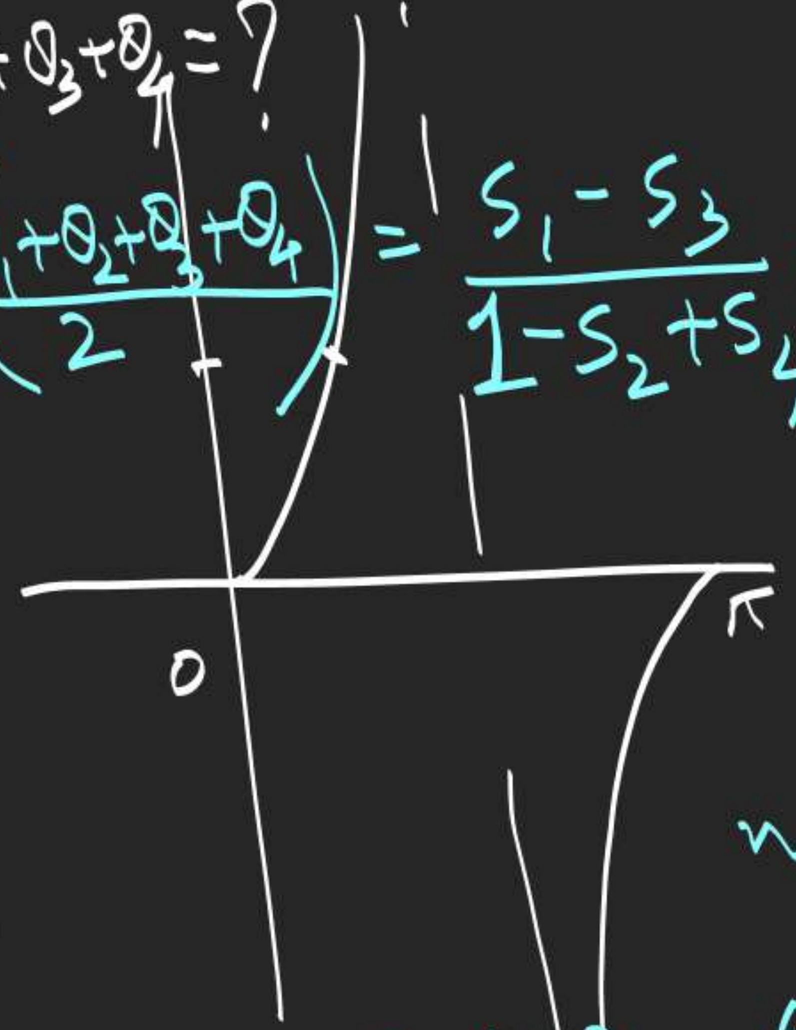
At most 4 normals are possible.

minimum 2



$\theta_1+\theta_2+\theta_3+\theta_4 = ?$

$$\tan\left(\frac{\theta_1+\theta_2+\theta_3+\theta_4}{2}\right) = \frac{S_1 - S_3}{1 - S_2 + S_4} = \frac{}{1-0+(-1)}$$

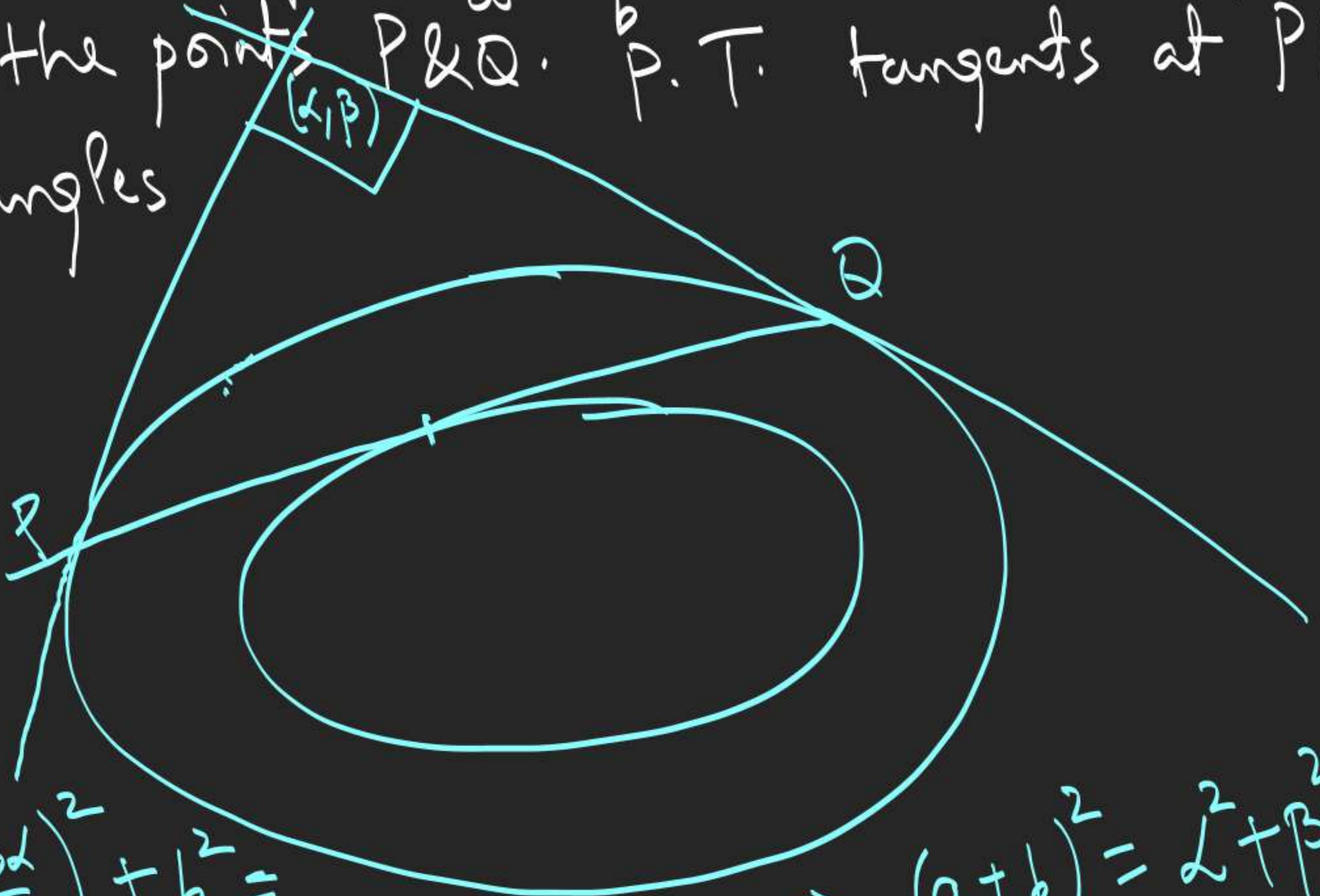


$$\frac{\theta_1+\theta_2+\theta_3+\theta_4}{2} = (2n+1)\frac{\pi}{2}$$

$n \in I$

$$\boxed{\Sigma\theta_i = (2n+1)\pi}$$

1. A tangent to ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ meets the ellipse $\boxed{\frac{x^2}{a}+\frac{y^2}{b}=a+b}$ in the points P & Q. P.T. tangents at P & Q are at right angles



$$\frac{x\alpha}{a}+\frac{y\beta}{b}=a+b$$

$$y=(a+b)\frac{b}{\beta}-\frac{x\alpha}{a}\frac{b}{\beta}$$

$$(a+b)^2\frac{b^2}{\beta^2}=a^2\left(-\frac{b\alpha}{a\beta}\right)^2+b^2=$$

$$\Rightarrow \frac{(a+b)^2}{\beta^2}=\frac{\alpha^2}{\beta^2}+1 \Rightarrow (a+b)^2=\alpha^2+\beta^2$$

$$\boxed{\alpha^2+\beta^2=a(a+b)+b(a+b)}$$

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Q. Find the locus of middle points of chords of ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

(i) which subtend right angle at centre.

(ii) the tangents at ends of which intersect at right angles.