

# Quadratic Equations

## Linear function

$$f(x) = ax + b, \quad a, b \in \mathbb{R},$$

## Quadratic fn

$$f(x) = ax^2 + bx + c \quad a, b, c \in \mathbb{R}, \quad a \neq 0.$$

## Cubic function

$$f(x) = ax^3 + bx^2 + cx + d \quad a, b, c, d \in \mathbb{R}, \quad a \neq 0.$$

## Biquadratic function

$$f(x) = ax^4 + bx^3 + cx^2 + dx + e, \quad a, b, c, d, e \in \mathbb{R}$$

$a \neq 0.$

Polynomial fn. or degree 'n'

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$$

$a_0, a_1, a_2, \dots, a_n \in \mathbb{R}$ .

$a_n \neq 0$ .

Leading coefficient =  $a_n$

Constant term =  $a_0$

Monic polynomial

leading coefficient = 1

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \dots + a_2 x^2 + a_1 x + a_0$$

$$a_0 = f(0)$$

$$a_r = \frac{f^{(r)}(0)}{1 \cdot 2 \cdot 3 \cdots r} = \frac{f^{(r)}(0)}{r!}$$

$$f'(x) = n a_n x^{n-1} + \dots + 3 a_3 x^2 + 2 a_2 x + a_1$$

$$\frac{f'(0)}{1} = a_1$$

$$a_3 = \frac{f'''(0)}{1 \cdot 2 \cdot 3}$$

$$f''(x) = n(n-1) a_n x^{n-2} + \dots + 3 \cdot 2 \cdot a_3 x^1 + 2 \cdot 1 \cdot a_2$$

$$\frac{f''(0)}{1 \cdot 2} = a_2$$

$$f'''(x) = n(n-1)(n-2) a_n x^{n-3} + \dots + 3 \cdot 2 \cdot 1 \cdot a_3$$

Factorial

$$L_n = \begin{cases} n! = n(n-1)! & , n \in \mathbb{N} \\ 0! = 1 \\ n = n^{(n-1)} \end{cases}$$

$$n! = 1 \cdot 2 \cdot 3 \cdots n$$

$$(-1)! \times$$

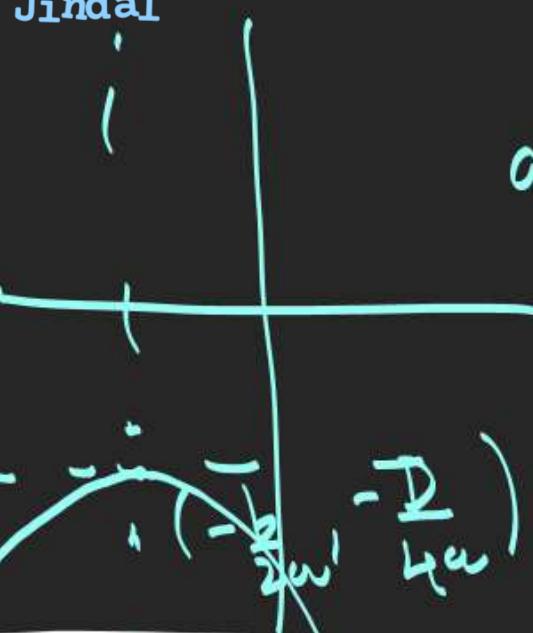
$$0! = 1$$

$$1! = 1 \quad 0! = 1$$

$$2! = 2 \cdot 1! = 2 \cdot 1$$

$$3! = 3 \cdot 2! = 3 \cdot 2 \cdot 1$$

# Quadratic Function

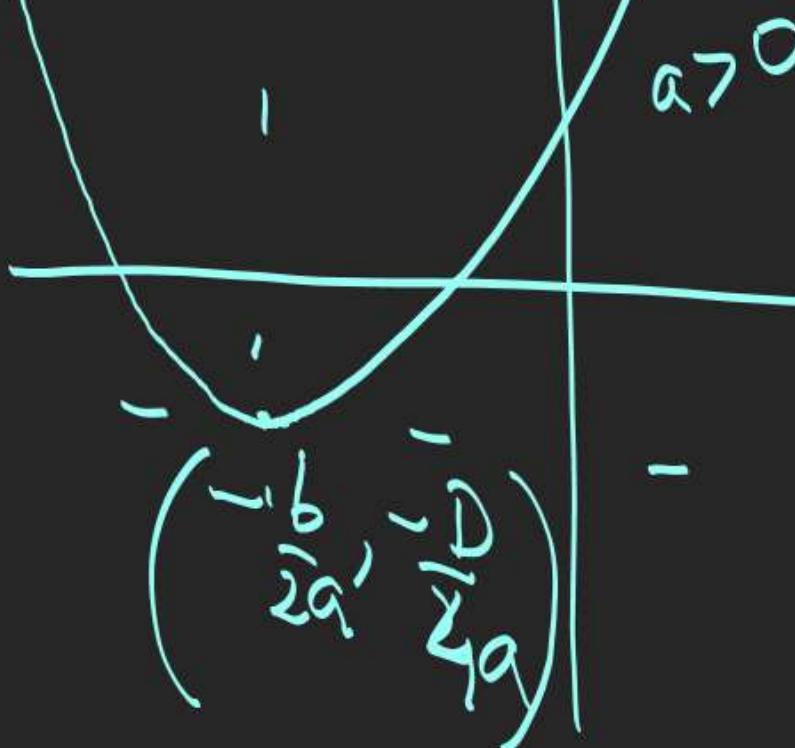
 $a < 0$ 

$$\therefore f(x) = ax^2 + bx + c$$

$$= a\left(x^2 + \frac{b}{a}x\right) + c$$

$$D = b^2 - 4ac \quad \text{Discriminant}$$

$$= a\left(\left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a^2}\right) + c$$



$$= a\left(x + \frac{b}{2a}\right)^2 + c - \frac{b^2}{4a}$$

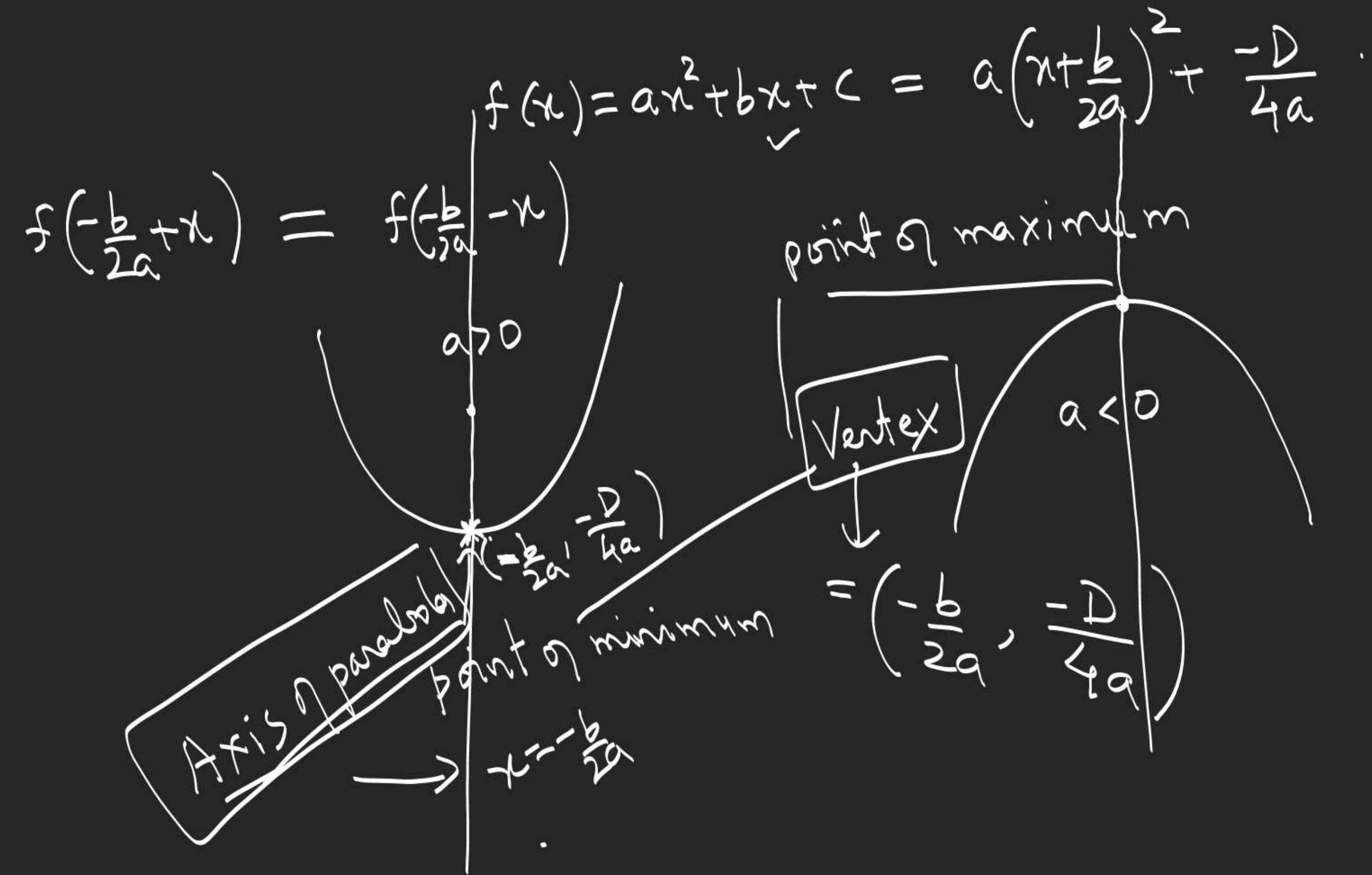
$$= a\left(x + \frac{b}{2a}\right)^2 - \frac{b^2 - 4ac}{4a}$$

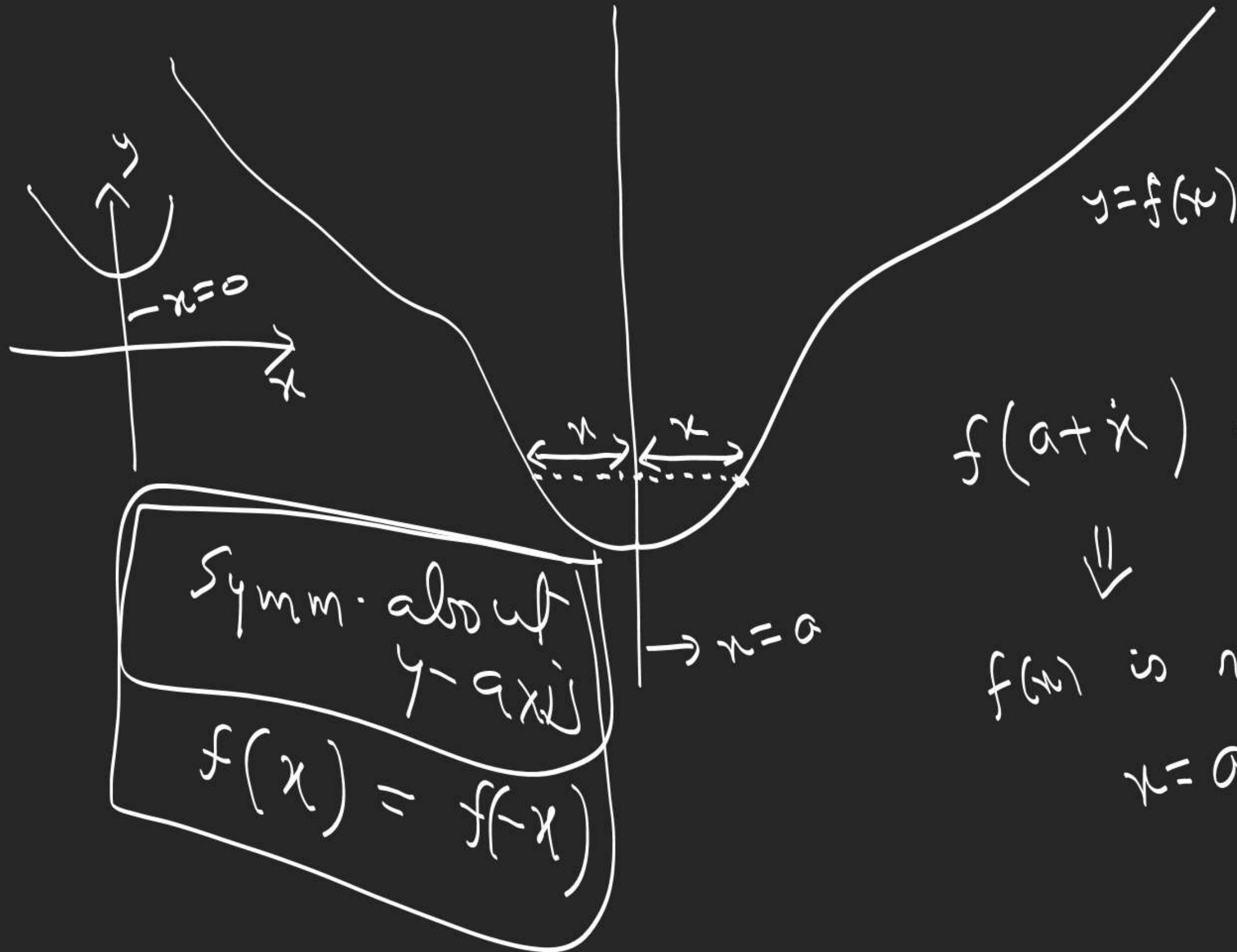
$a \neq 0, a, b, c \in \mathbb{R}$ .

$$y = f(x) = a\left(x + \frac{b}{2a}\right)^2 + \frac{-D}{4a}$$

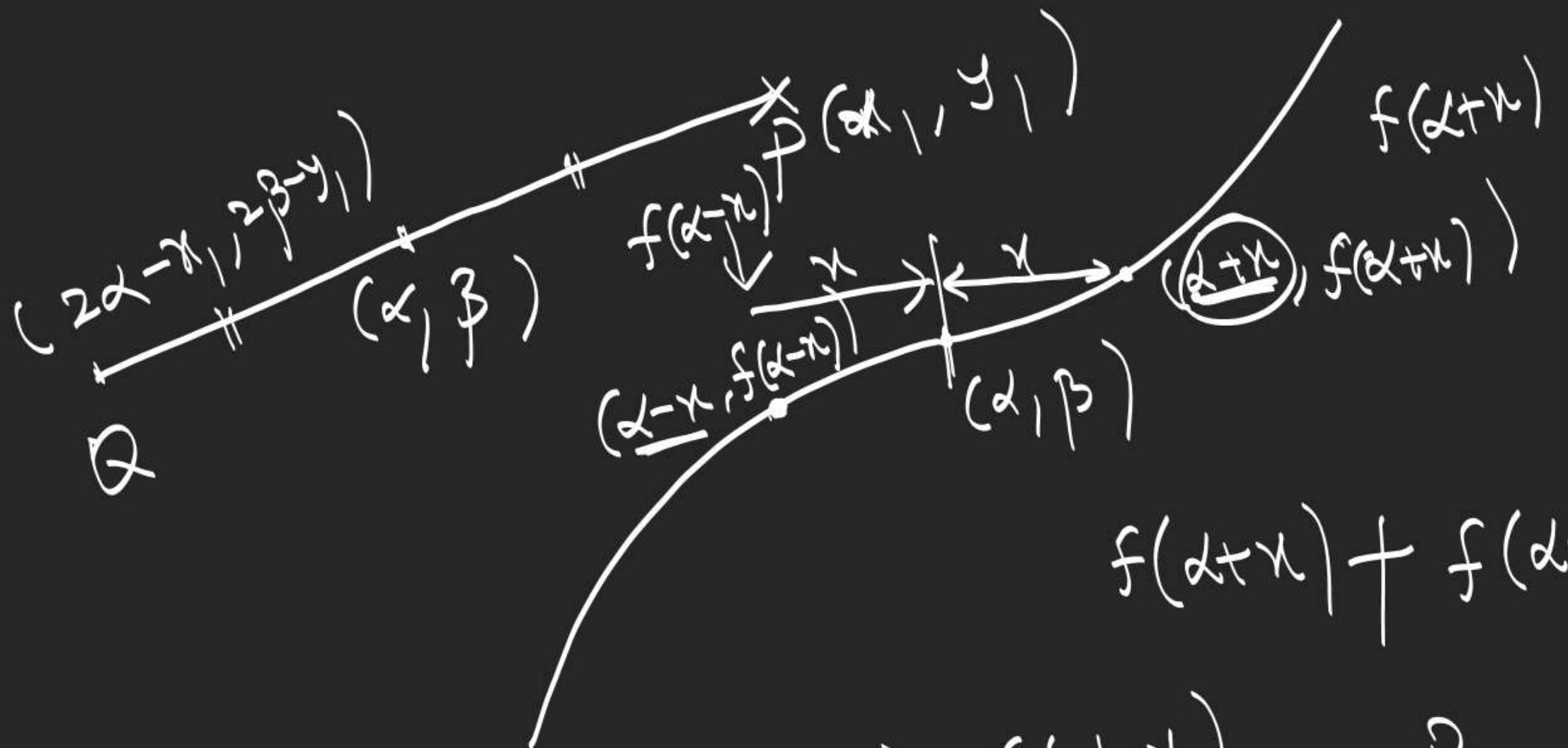
$$y + \frac{D}{4a} = a\left(x + \frac{b}{2a}\right)^2$$

$$y = aX^2$$





Symmetric about point  $(\alpha, \beta)$



$$f(\alpha+x) + f(\alpha-x) = 2\beta$$

$$\frac{f(\alpha+x) + f(\alpha-x)}{2} = \beta$$

Symmetric  
about  $(0,0)$

$$f(x) + f(-x) = 0$$

$f(x) = ax^2 + bx + c \checkmark$

Quadratic Equation

Roots of function

$f(x) = 0$   
point where graph of  $f(x)$  meet  $x$ -axis

$$x^2 = 9$$

$$x = \pm 3 \Rightarrow x = \pm \sqrt{9}$$

$$ax^2 + bx + c = 0$$

Roots / Zeros / Solutions of Quadratic Eqn.

$$\alpha, \beta = \frac{-b \pm \sqrt{D}}{2a}$$

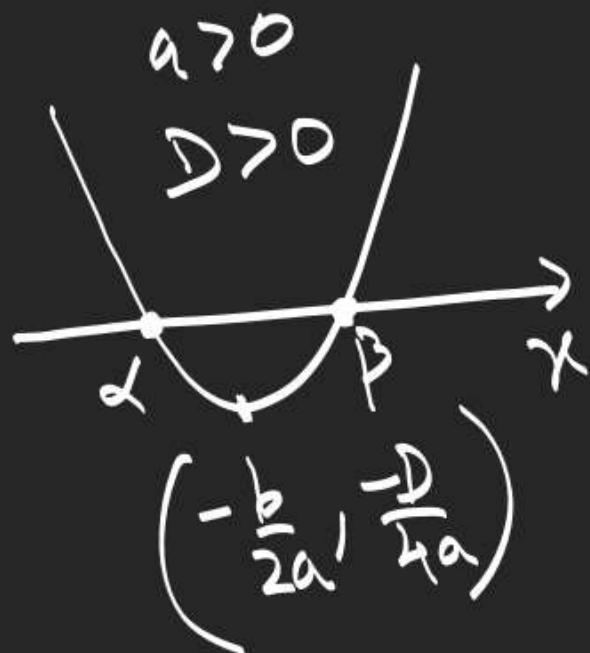
$$ax^2 + bx + c = 0$$

$$a\left(x + \frac{b}{2a}\right)^2 - \frac{D}{4a} = 0 \Rightarrow \left(x + \frac{b}{2a}\right)^2 = \frac{D}{4a^2}$$

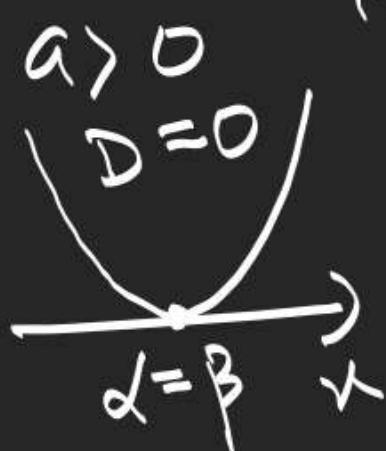
$$x + \frac{b}{2a} = \pm \frac{\sqrt{D}}{2a}$$

$$f(x) = ax^2 + bx + c$$

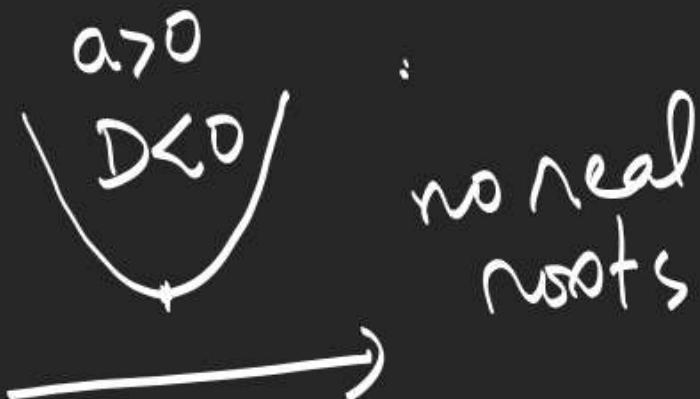
2 distinct real roots



2 equal real roots

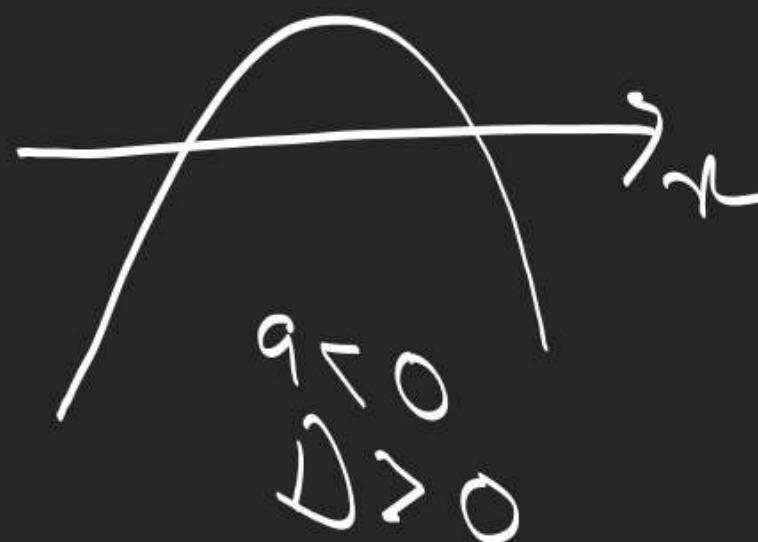


$$f(x) > 0$$



$$\frac{D}{4a} > 0$$

$D > 0$



$D > 0 \Rightarrow$  2 distinct real roots  
 $D = 0 \Rightarrow$  2 equal real roots  
 $D < 0 \Rightarrow$  no real roots

Note

$$f(x) = ax^2 + bx + c, \quad a, b, c \in \mathbb{R}, \quad a \neq 0$$

$$f(x) > 0 \quad \forall x \in \mathbb{R}$$

$$\Rightarrow a > 0 \text{ & } D < 0$$

②

$$\begin{aligned} \text{if } f(x) < 0 \quad \forall x \in \mathbb{R} \\ \Rightarrow a < 0 \text{ & } D < 0 \end{aligned}$$



③