Nishant Jindal

Q If system of L. Eqn

Main 2020

$x+y+z=6$

$x+2y+3z=10$

$3x+2y+\lambda z=\mu$

has $\boxed{\text{more than 2 Sol}}$ then $\mu-\lambda^2=?$

uq / ∞ Sol.

$D\cdot x=D_1,\ D\cdot y=D_2,\ D\cdot z=D_3$

$0\cdot x=0,\ 0\cdot y=0,\ 0\cdot z=0$

$D=0=D_1=D_2=D_3$

$D=0,\ D_3=0$

$D=0$

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 3 & 2 & \lambda \end{vmatrix}=0$$

$(2\lambda+9+2)-(6+6+\lambda)=0$

$\boxed{\lambda=1}$

$D_3=0$

$$\begin{vmatrix} 1 & 1 & 6 \\ 1 & 2 & 10 \\ 3 & 2 & \mu \end{vmatrix}=0$$

$(2\mu+30+12)-(36+20+\mu)=0$

$\mu=14$

$\mu-\lambda^2$

$14-1=13$

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Q for a Real No. α if the System.

Adv 2017

$$\begin{bmatrix} 1 & \alpha & \alpha^2 \\ \alpha & 1 & \alpha \\ \alpha^2 & \alpha & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

of L eqn has ∞ many sol. then $1+\alpha+\alpha^2 = ?$

$x+\alpha y+\alpha^2 z = 1$ $\quad D = 0$

$\alpha x + y + \alpha z = -1$

$\alpha^2 x + \alpha y + z = 1$

$$\begin{vmatrix} 1 & \alpha & \alpha^2 \\ \alpha & 1 & \alpha \\ \alpha^2 & \alpha & 1 \end{vmatrix} = 0 \Rightarrow (1+\alpha^4+\cancel{\alpha^4}) - (\cancel{\alpha^4}+\alpha^2+\alpha^2) = 0$$

$\alpha^4 - 2\alpha^2 + 1 = 0$

$\alpha^2 - 1 = 0$

$\alpha = \pm 1$

$1+1+1 = 3$ ×

OR

$1-1+1 = 1$

$\alpha = 1$

$x+y+z=1$ ✓

$x+y+z=-1$ ×

$x+y+z=1$ ✓

$\alpha = -1$

$x-y+z=1$

$-x+y-z=-1$

$x-y+z=1$ ✓✓

$\therefore$ 1 is Rt. answer.

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Homogeneous Eqn of 3 Variable.

$x+y+z=\boxed{3}$ ← Constant ← Non Hom Eqn.

$x-2y+z=\boxed{0}$ ← Zero → Hom Eqn

$$\left.\begin{aligned}2x-y&=0\\x+3y&=0\end{aligned}\right\}$$ Hom. System of 2 variable

$$\left.\begin{aligned}a_1x+b_1y+c_1z&=0\\a_2x+b_2y+c_2z&=0\\a_3x+b_3y+c_3z&=0\end{aligned}\right\}$$ Hom system of 3 variable

for Hom. eqn $D_1=\begin{vmatrix}d_1 & b_1 & c_1\\d_2 & b_2 & c_2\\d_3 & b_3 & c_3\end{vmatrix}=\begin{vmatrix}0 & b_1 & c_1\\0 & b_2 & c_2\\0 & b_3 & c_3\end{vmatrix}=0$

$D_1=D_2=D_3=0$

$D\cdot x=D_1,\ D\cdot y=D_2,\ D\cdot z=D_3.$

$D\,x=0,\ D\cdot y=0,\ D\cdot z=0$

95% Q's $\boxed{D=0}$

$0\cdot x=0$

$0\,y=0$

$0\cdot z=0$

∞ Sol.

(Non Trivial)

$D\neq 0$

$x=\frac{0}{D},\ y=\frac{0}{D},\ z=\frac{0}{D}$

$x=y=z=0$

$(0,0,0)=(x,y,z)$

Unq. Sol

Trivial Sol

System is always Consistent.

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Yad

Q If $ax+y+z=0$, $x+by+z=0$, $x+y+cz=0$

$\boxed{x, y, z \text{ not all zero}}$ then $\frac{1}{1-a}+\frac{1}{1-b}+\frac{1}{1-c}=?$ (1)

→ Non Trivial Sol.

$D=0$

$$\begin{vmatrix} a & 1 & 1 \\ 1 & b & 1 \\ 1 & 1 & c \end{vmatrix}=0$$

$C_1 \to C_1 - C_3$

$C_2 \to C_2 - C_3$

$$\begin{vmatrix} a-1 & 0 & 1 \\ 0 & b-1 & 1 \\ 1-c & 1-c & c \end{vmatrix}=0$$

$((a-1)(b-1)c+0+0)-((1-c)(b-1)+(a-1)(1-c)+0)=0$

$(1-a)(1-b)c+(1-b)(1-c)+(1-a)(1-c)=0$

$\div (1-a)(1-b)(1-c)$

$\frac{c}{1-c}+\frac{1}{1-a}+\frac{1}{1-b}=0$

$\frac{1-(1-c)}{(1-c)}+\frac{1}{1-a}+\frac{1}{1-b}=0 \Rightarrow \underbrace{\frac{1}{1-c}+\frac{1}{1-a}+\frac{1}{1-b}}=1$

Q If $\lambda x + (\cos\theta)y + (\cos\theta)\cdot z = 0$

$x + (\cos\theta)\cdot y + (\sin\theta) z = 0$

$-x + (\sin\theta)\cdot y - (\cos\theta)\cdot z = 0$

has Non Trivial sol. Find λ in terms of θ

find Range of λ & $\theta \in R$

$D = 0$

$$\Rightarrow \begin{vmatrix} \lambda & \cos\theta & \cos\theta \\ 1 & \cos\theta & \sin\theta \\ -1 & \sin\theta & -\cos\theta \end{vmatrix} = 0$$

$$(-\cos^2\theta\cdot\lambda - \sin\theta\cdot\cancel{\cos\theta} + \cancel{\sin\theta}\cdot\cos\theta) - (-\cos^2\theta + \lambda\sin^2\theta - \cos^2\theta) = 0$$

$$\lambda(-\cos^2\theta - \sin^2\theta) + 2\cos^2\theta = 0 \Rightarrow \lambda = 2\cos^2\theta$$

(2) R_f : $2\cos^2\theta \in [0, 2]$

$\therefore \lambda \in [0, 2]$ $\forall \theta \in R$

Bagula Method

$$D = \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} \qquad \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}$$

$$D = (aei + bfg + dhc) - (gec + ahf + dbi)$$

(2) $\lambda = 1$, $\theta \in 1^{st}$ Quad then Solve system

$2\cos^2\theta = 1$

$\cos^2\theta = \frac{1}{2}$

$\boxed{\cos\theta = \frac{1}{\sqrt{2}}}$, $\cancel{-\frac{1}{\sqrt{2}}}$

$x + \frac{y}{\sqrt{2}} + \frac{z}{\sqrt{2}} = 0$

$x + \frac{y}{\sqrt{2}} + \frac{z}{\sqrt{2}} = 0$

$-x + \frac{y}{\sqrt{2}} - \frac{z}{\sqrt{2}} = 0$

$\sqrt{2}y = 0 \Rightarrow y = 0$

Non Triv. = ∞ Sol.

(x, y, z)

$= \left(-\frac{z}{\sqrt{2}}, 0, z\right)$

$= \left(-\frac{t}{\sqrt{2}}, 0, t\right)$

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Q Find value of p & q such that system has
① unique sol. ② Infinite sol. ③ No sol.

for $2x+py+6z=8,\ x+2y+qz=5;\ x+y+3z=4$

<u>Non Hom. Sys.</u>

$$D=\begin{vmatrix} 2 & p & 6 \\ 1 & 2 & q \\ 1 & 1 & 3 \end{vmatrix} = (\cancel{12}+pq+6)-(\cancel{12}+2q+3p)$$

$$pq-2q+6-3p$$

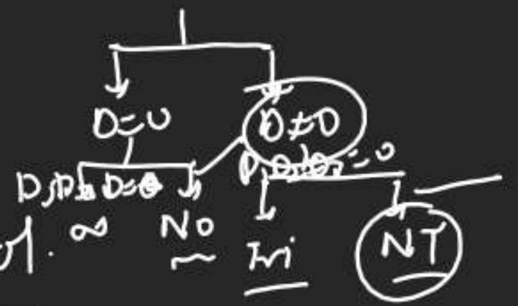
$$q(p-2)+3(2-p)$$

$$(p-2)(q-3)$$

$D_1=(p-2)(4q-15)$

$D_2=0.$

$D_3=(p-2).$



① <u>unq sol.</u>

$D\neq 0 \Rightarrow (p-2)(q-3)\neq 0$

$p\neq 2,\ q\neq 3$ ✓

$p\in R-\{2\},\ q\in R-\{3\}$

② <u>∞ Sol.</u>, $D=0$

$D_1=D_2=D_3=0$

$\boxed{p=2}$ & $q\in R$

(3) No sol. ($D=0$, $D_1\,D_2\,D_3$ Baghi)

$\boxed{q=3}$ But $p\neq 2$

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Adjoint of Matrix

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$$\operatorname{adj} A = \begin{bmatrix} C_{11} & C_{21} & C_{31} \\ C_{12} & C_{22} & C_{32} \\ C_{13} & C_{23} & C_{33} \end{bmatrix}$$

$$A \cdot \operatorname{adj} A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \cdot \begin{bmatrix} C_{11} & C_{21} & C_{31} \\ C_{12} & C_{22} & C_{32} \\ C_{13} & C_{23} & C_{33} \end{bmatrix} = \begin{bmatrix} |A| & 0 & 0 \\ 0 & |A| & 0 \\ 0 & 0 & |A| \end{bmatrix} = |A| \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\boxed{A \operatorname{adj} A = |A| \cdot I}$$

(1) $A \cdot \operatorname{adj} A = |A| I$

(2) $|A \cdot \operatorname{adj} A| = ||A| I|$

$\cancel{|A|}\, |\operatorname{adj} A| = \overset{|A|^{n-1}}{\cancel{|A|^{n}}} \cdot |I|$

$$\boxed{|\operatorname{adj} A| = |A|^{n-1}}$$

(3) $A \operatorname{adj} A = |A| I$

$\underbrace{\operatorname{adj} A} \cdot \operatorname{adj}(\operatorname{adj} A) = \underbrace{|\operatorname{adj} A|} \cdot I$

$(A \cdot \operatorname{adj} A) \operatorname{adj}(\operatorname{adj} A) = |A|^{n-1} A \cdot I$

$\cancel{|A|} \cdot I \operatorname{adj}(\operatorname{adj} A) = \underset{|A|^{n-2}}{\cancel{|A|^{n-1}}} A\, I \Rightarrow$ $\boxed{\operatorname{adj}(\operatorname{adj} A) = |A|^{n-2} \cdot A}$

(4) $|\operatorname{adj}(\operatorname{adj} A)| = |\underbrace{|A|^{n-2}} A|$

$= (|A|^{n-2})^{n} \cdot |A|$

$= |A|^{n^2-2n} \cdot |A|^{1}$

$= |A|^{n^2-2n+1}$

$$\boxed{|\operatorname{adj}(\operatorname{adj} A)| = |A|^{(n-1)^2}}$$

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(1) $A \cdot \text{adj}\, A = |A| \cdot I$

(2) $|\text{adj}\, A| = |A|^{n-1}$

(3) $|\text{adj}(\text{adj}\, A)| = |A|^{(n-1)^2}$

(4) $\text{adj}(\text{adj}\, A) = |A|^{n-2} \cdot A$

(5) $\text{adj}(K \cdot A) = K^{n-1} \cdot \text{adj}\, A$

(6) $\text{adj}(AB) = \text{adj}\, B \cdot \text{adj}\, A$

Q If $P = \begin{bmatrix} 1 & \alpha & 3 \\ 1 & 3 & 3 \\ 2 & 4 & 4 \end{bmatrix}$ is adjoint of a 3×3 Matrix A

Mains

& $|A| = 4$ then $\alpha = ?$

$|P| = |\text{adj}\, A| = \begin{vmatrix} 1 & \alpha & 3 \\ 1 & 3 & 3 \\ 2 & 4 & 4 \end{vmatrix} = |A|^{n-1} = (4)^{3-1} = 16$

$(12 + 6\alpha + \cancel{12}) - (18 + \cancel{12} + 4\alpha) = 16$

$2\alpha - 6 = 16$

$\alpha = 11$

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Q If A is sq^n matrix of order n.

then $|adj(adj(\cancel{adj}\,A))| = ?$

$= |A|^{(n-1)^2} = |adj\,A|^{(n-1)^2}$

$= \left(|A|^{(n-1)}\right)^{(n-1)^2}$

$= |A|^{(n-1)^3}$

A

$|adj\,A| = |A|^{n-1}$

Q If Matrix $A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 3 & 4 \\ 1 & -1 & 3 \end{bmatrix}$ & $B = adj(A)$ & $C = 3A$

2020 Main

then $\frac{|adj\,B|}{|C|} = ?$

$\frac{|adj(adj\,A)|}{|3A|}$

$= \frac{|A|^{(3-1)^2}}{3^3\,|A|} = \frac{(6)^{\cancel{4}\,3}}{3^3 \times \cancel{6}}$

$= \frac{6 \times 6 \times 6}{3 \times 3 \times 3}$

$= 8$

$|A| = \begin{vmatrix} 1 & 1 & 2 \\ 1 & 3 & 4 \\ 1 & -1 & 3 \end{vmatrix}$

$= (9+4+-2) - (6-4+3)$

$= 11 - 5$

$= 6$

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Q $A = \begin{bmatrix} 0 & 1 & 2 \\ -1 & 0 & 2 \\ 3 & 1 & 4 \end{bmatrix}$

(1) $|A| = (0+6+-2)-(0+0+-4) = 8.$

(2) $|3A| = 3^3|A| = 27\times 8$

(3) $|\text{adj}\,A| = |A|^{n-1} = |A|^{3-1} = 8^2 = 64$

(4) $|4\cdot\text{adj}A| = 4^3|\text{adj}\,A| = 4^3\times|A|^{3-1}$
$= 4^3\times(8)^2$

(5) $|3\,\text{adj}(\text{adj}A)| = ?$ $\quad 3^3|\text{adj}(\text{adj}A)|$
$= 3^3|A|^{(n-1)^2} = 3^3\times(8)^{2^2}$
$= \underline{3^3\times 8^4}$

Q A, B, C is $n\times n$ matrix such that

$\det(A) = 2$, $\det(B) = 3$, $\det(C) = 5$

Find $\det(A^2B^{-1}C) = |A^2B^{-1}C|$

$= \dfrac{|A|^2\cdot|C|}{|B|}$

$= \dfrac{4\times 5}{3} = \dfrac{20}{3}$

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Det Ex2, Ex1

$f(x) = |\log 2 - \sin x|$, $g(x) = f(f(x))$

$g(x) = |\log 2 - \sin(|\underbrace{\log 2 - \sin x}_{+ve}|)|$

$x = 0$

$|\log 2 - \sin(\log 2 - \sin x)|$

B − C (+)

$x \Rightarrow 0$

$\sin x < x$

$\sin x + \sin(\log 2 - \sin x) < \log 2 - \cancel{\sin x}$

$g(x) = \log 2 - \sin(\log 2 - \sin x)$

10 (m, b, 7, 9

$x\ln(\ln x) - x^2 + y^2 = 4$ $(y > 0)$ $\frac{dy}{dx}\Big|_{x=e}$

$e \cdot \cancel{\ln(\ln e)} - e^2 + y^2 = 4$

$y = \sqrt{4 - e^2}$

$\frac{x}{x \cdot \ln x} + \ln(\ln x) - 2x + 2y \cdot y' = 0$

$\frac{1}{\ln e} + \cancel{\ln(\ln e)} - 2e + 2\sqrt{4-e^2} \cdot y' = 0$

$\frac{dy}{dx} = \frac{2e-1}{2\sqrt{4-e^2}}$

$\tan^{-1}\left(\frac{3x\sqrt{x} + 3x\sqrt{x}}{1 - 3x\sqrt{x} \times 3x\sqrt{x}}\right)$

$\tan^{-1}(\quad) + \tan^{-1}(\quad)$