Nishant Jindal

1. Find the no. of 5 letter words that can be formed using all letters of the word 'TOUGH'. Also find the rank of 'TOUGH' among them as arranged in dictionary.

G $\underline{4} \times \underline{3} \times \underline{2} \times \underline{1}$ → $4! = 24$

H _ _ _ _ → $4!$

O _ _ _ _ → $4!$

T G _ _ _ → $3!$

T H _ _ _ → $3!$

T O G _ _ → $2!$

T O H _ _ → $2!$

TOUGH ✓

$\boxed{89.}$

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2. Find the no. of words which can be formed using all letters of the word 'MACHINE' without repetition so that vowels may only occupy the

(i) odd position

(ii) even position

A, I, E

_ _ _ _ _ _ _

(i) $(4 \times 3 \times 2)\, 4!$

(ii) $(3 \times 2 \times 1)\, 4!$

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3: Find the no. of 4 letter words using only the letters from the word 'DAUGHTER' without repetition if each word is to include 'G'.

_ _ G _

$4 \times (7 \times 6 \times 5) = 840$

to fill remaining 3 places

no. of ways to fill 'G'

_ _ _ _

$8 \times 7 \times 6 \times 5 - 7 \times 6 \times 5 \times 4$

all 4 letter words.

no. of words not containing 'G'

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$$(100)! = 2^{m_1}\, 3^{m_2}\, 5^{m_3} \cdots$$

Find exponent of 2 $= \left[\frac{100}{2}\right] + \left[\frac{100}{2^2}\right] + \left[\frac{100}{2^3}\right] + \left[\frac{100}{2^4}\right] + \left[\frac{100}{2^5}\right] + \left[\frac{100}{2^6}\right]$

$$100 = 1\cdot 2\cdot 3\cdot 4 \cdots 98\cdot 99\cdot 100.$$

Greatest integer of x = $[x]$ = greatest integer which is less than or equal to x.

$[-34.99] = -35$

$[127] = 127.$

$[13.897] = 13$

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no. of multiples of 4

$$50+25+12+6+3+1 = \left[\frac{100}{2}\right] + \left[\frac{100}{2^2}\right] + \left[\frac{100}{2^3}\right] + \left[\frac{100}{2^4}\right] + \left[\frac{100}{2^5}\right] + \left[\frac{100}{2^6}\right] + \left[\frac{100}{2^7}\right] + \left[\frac{100}{2^8}\right] + \dots$$

no. of multiples of 2

$= 97$

$1 \cdot 2 \cdot 3 \cdot 4 \cdot - - - - \boxed{96} \cdot \cdot 100$

$96 = 3 \times 2^5$

$\left[\frac{101}{2}\right]$

2

$\left[\frac{100}{2^7}\right] = 0$

$101!$

$1 \cdot 2 - - 100$

$2(50)$

$$\frac{101}{2} = \frac{2(50)+1}{2} = 50 + 0.5$$

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$$n! = p_1^{a_1} \, p_2^{a_2} \, p_3^{a_3} \, p_4^{a_4} \cdots$$

$p_i \rightarrow$ prime no.

Find a_4

$$a_4 = \left[\frac{n}{p_4}\right] + \left[\frac{n}{p_4^2}\right] + \left[\frac{n}{p_4^3}\right] + \cdots \infty$$

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1. Find maximum value of t for which

(1) 9^t divides $(300)!$ $t \in \mathbb{N}$

$= \underline{\underline{3^{148}}}\, 2^{(\ )}\, 5^{(\ )} \ldots$

(2) 6^t divides $(300)!$

$$\left[\frac{300}{3}\right]+\left[\frac{300}{3^2}\right]+\left[\frac{300}{3^3}\right]+\left[\frac{300}{3^4}\right]+\left[\frac{300}{3^5}\right]$$

$= 100 + 33 + 11 + 3 + 1$

$= 148$

(1) $t = \boxed{74}$

(2) $\boxed{148}$

6×12

6^{148}

$\boxed{9} \times 8$

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2. Find no. of cyphers/zeroes in the end of $(1000)!$

$$\left[\frac{1000}{5}\right] + \left[\frac{1000}{5^2}\right] + \left[\frac{1000}{5^3}\right] + \left[\frac{1000}{5^4}\right]$$

$= 200 + 40 + 8 + 1$

$= 249$

Ex-3 (remaining)
↓
circles