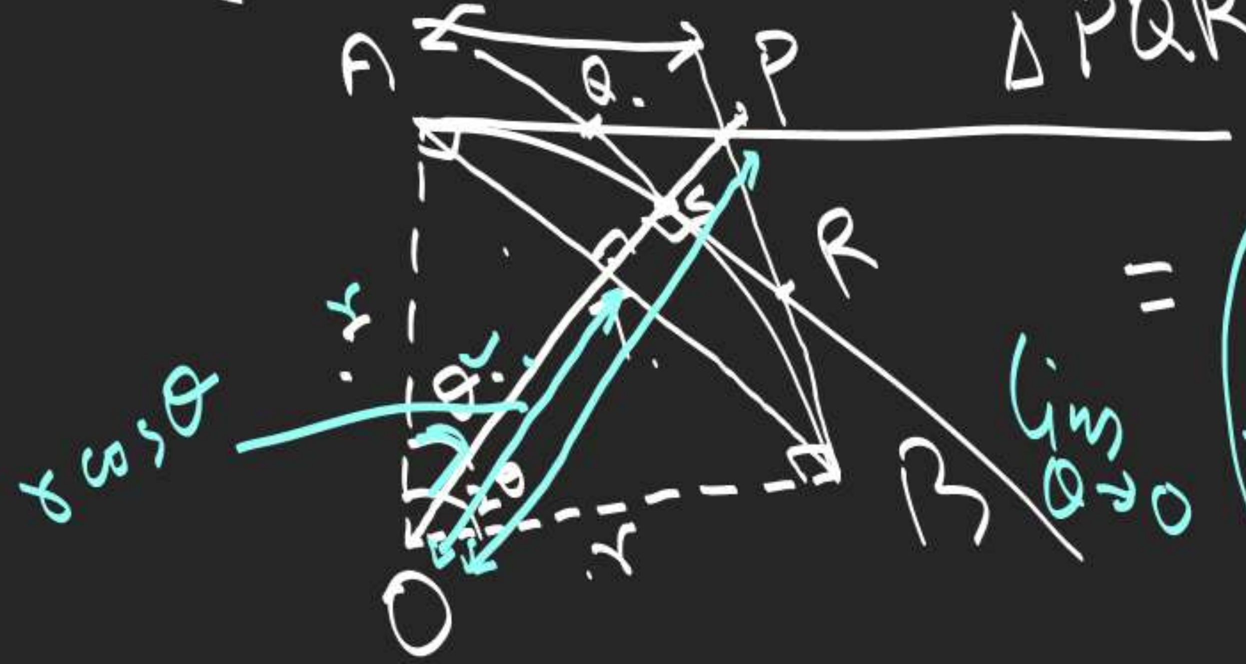
Nishant Jindal

14.

$$\lim_{x\to\infty}\left(\frac{e^{\frac{\pi}{x}}+e^{-\frac{\pi}{x}}}{2\cos\frac{\pi}{x}}\right)^{x^2}$$

$$\lim_{x\to\infty} x^2\,\frac{\left(e^{\frac{\pi}{x}}+e^{-\frac{\pi}{x}}-2\right)+\left(2-2\cos\frac{\pi}{x}\right)}{\left(\frac{\pi}{x}\right)^2\;2\cos\frac{\pi}{x}}$$

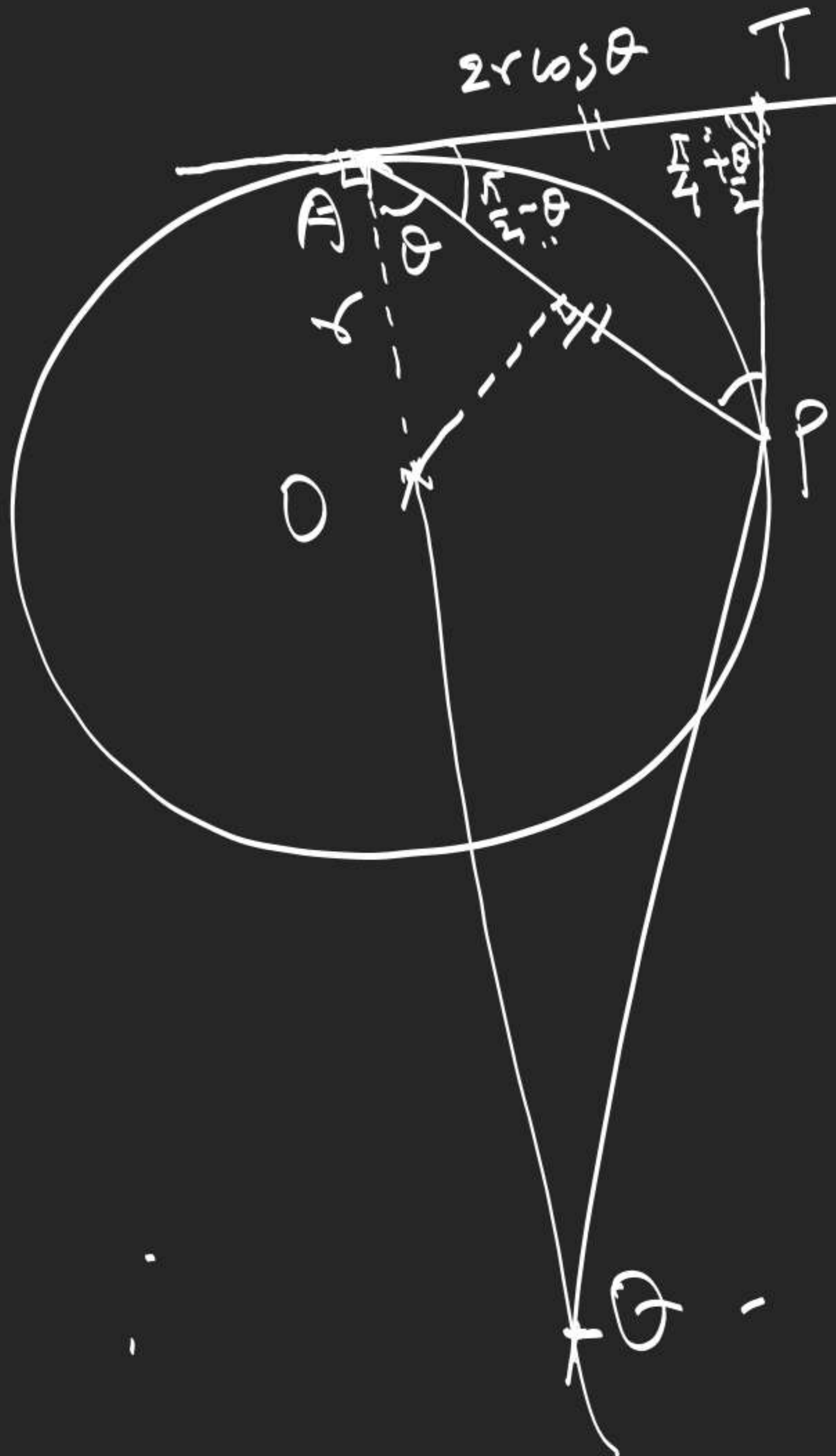
e



$$\frac{\Delta PAB}{\Delta PQR}=\left(\frac{PT}{PS}\right)^2$$

$$\lim_{\theta\to 0}=\left(\frac{\frac{r}{\cos\theta}-r\cos\theta}{\frac{r}{\cos\theta}-r}\right)^2$$

$$=(1+\cos\theta)^2=4$$



$\underline{\Delta ATQ}$

$$\lim_{\theta\to\frac{\pi}{2}} AQ = \lim_{\theta\to\frac{\pi}{2}} 2r\cos\theta\,\tan\left(\frac{\pi}{4}+\frac{\theta}{2}\right)$$

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Theorem over Continuity

$C \to$ Continuous

$D \to$ Discontinuous

f	g	$f+g$	$f-g$	fg	$\frac{f}{g}$
C	C	C	C	C	C , $g(a) \neq 0$
C	D	D	D	–	–
D	C	D	D	–	–
D	D	–	–	–	–

$f \to C,\ g \to D$

Let $f-g$ be cont. At $x=a$

$g(x) = f(x) - (f-g)(x)$

$\Rightarrow g(x)$ is cont.

$\boxed{\text{Contradiction}}$

$f \to C,\ g \to C$

$\lim_{x\to a} f(x) = f(a)$

$\lim_{x\to a} g(x) = g(a)$

$\lim_{x\to a} (fg)(x)$

$= \left(\lim_{x\to a} f(x)\right)\left(\lim_{x\to a} g(x)\right)$

$= f(a)\, g(a)$

$= (fg)(a)$

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1. $f(x) = x$, $g(x) = \{x\}$ → D

$f(x)$ → C

$\{\cdot\} = \text{FPF}$.

$f+g$ at $x = 2$ → Discontinuous

Jump $= 1$

$\lim_{x\to 2}\left(x + \{x\}\right)$ — LHL $= 3$, RHL $= 2$

Non removable finite exist.

$f(x) = \begin{cases} 1 & x \geq 0 \\ -1 & x < 0 \end{cases}$, $g(x) = \begin{cases} -1 & x \geq 0 \\ 1 & x < 0 \end{cases}$

$f(x)$ → D, $g(x)$ → D

$(f-g)(x) = \begin{cases} 2 & x \geq 0 \\ -2 & x < 0 \end{cases}$

RHL $= 2$
LHL $= -2$
$(f-g)(0) = 2$

$f+g$, $f-g$, fg , $\frac{f}{g}$ at $x = 0$ $x \in R$

$f+g$ → C, fg → C, $\frac{f}{g}$ → C

$f+g = 0$

$fg(x) = -1$, $\frac{f(x)}{g(x)} = -1$ $\forall x \in R$

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$f(x) = x$, $\quad g(x) = \begin{cases} \sin\frac{\pi}{x} & x \neq 0 \\ 0 & x = 0 \end{cases}$

$f \to C \qquad g \to D$

Discuss fg at $x = 0$

$$(fg)(x) = \begin{cases} x\sin\frac{\pi}{x} & x \neq 0 \\ 0 & x = 0 \end{cases}$$

$$\lim_{x\to 0} (fg)(x) = \lim_{x\to 0} x \underbrace{\sin\frac{\pi}{x}} = 0$$

$$fg(0) = 0$$

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1. If $f(x) = \begin{cases} x^2-1 & , x<3 \\ 2ax & , x \geq 3 \end{cases}$ is continuous $\forall x \in R$, find a.

Cont. at $x=3$

$LHL = \lim_{x \to 3^-} (x^2-1) = 8$

$RHL = \lim_{x \to 3^+} (2ax) = 6a$

$f(3) = 2a(3) = 6a$

$8 = 6a$

$\boxed{a = \frac{4}{3}}$

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2. Let $f(x) = \begin{cases} \left(1+|\sin x|\right)^{\frac{a}{|\sin x|}} & \text{for } -\frac{\pi}{6} < x < 0 \\ b & \text{for } x = 0 \\ e^{\frac{\tan 2x}{\tan 3x}} & \text{for } 0 < x < \frac{\pi}{6} \end{cases}$

is continuous at $x=0$, find a, b.

$$RHL = \lim_{x\to 0^+} e^{\frac{\tan 2x}{2x} \cdot \frac{3x}{\tan 3x} \cdot \frac{2}{3}} = e^{2/3}$$

$$LHL = \lim_{x\to 0^-} \left(1+|\sin x|\right)^{\frac{a}{|\sin x|}} = e^{\lim_{x\to 0^-}(a)} = e^{a}$$

$$e^{a} = e^{\frac{2}{3}} = b$$

$$\boxed{a = \frac{2}{3},\ b = e^{2/3}}$$

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3: Let $f(x) = \begin{cases} x + a\sqrt{2}\sin x & 0 \le x < \frac{\pi}{4} \\ 2x\cot x + b & \frac{\pi}{4} \le x \le \frac{\pi}{2} \\ a\cos 2x - b\sin x & \frac{\pi}{2} < x \le \pi \end{cases}$

is cont. in $[0, \pi]$, find a, b.

$\boxed{b = -\frac{\pi}{12},\ a = \frac{\pi}{6}}$

Cont at $x = \frac{\pi}{4}$

LHL = $\frac{\pi}{4} + a$

RHL = $\frac{\pi}{2} + b = f\left(\frac{\pi}{4}\right)$

$\frac{\pi}{4} + a = \frac{\pi}{2} + b$

$a - b = \frac{\pi}{4}$ — (1)

Cont. at $x = \frac{\pi}{2}$

LHL = $b = f\left(\frac{\pi}{2}\right)$

RHL = $-a - b$

$b = -a - b$

$2b = -a$ — (2)

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4. Let $f(x) = \begin{cases} \dfrac{(e^{2x}+1)-(x+1)(e^{x}+e^{-x})}{x(e^{x}-1)}, & \text{if } x \neq 0 \\ K & \text{if } x = 0 \end{cases}$

is continuous at $x = 0$, find K.

5. Let $f(x) = \dfrac{\sqrt{x^2+kx+1}}{x^2-k}$ be continuous $\forall x \in \mathbb{R}$,

find k.

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6. Let $f(x) = \operatorname{cosec}(2x) + \operatorname{cosec}(2^2x) + \operatorname{cosec}(2^3x) + \dots + \operatorname{cosec}(2^nx)$

$x \in \left(0, \frac{\pi}{2}\right)$.

and $g(x) = f(x) + \cot(2^n x)$

Limits $\boxed{\frac{\text{Ex-IV}}{}\ /\ \begin{matrix}\text{Ex-V} \\ (1-5)\end{matrix}}$

If $h(x) = \begin{cases} (\cos x)^{g(x)} + (\sec x)^{\operatorname{cosec} x} + 1 & \text{if } x > 0 \\ p & \text{if } x = 0 \\ \dfrac{e^x + e^{-x} - 2\cos x}{x \sin x} & \text{if } x < 0 \end{cases}$

Find 'p' if possible so that $h(x)$ is continuous at $x = 0$.