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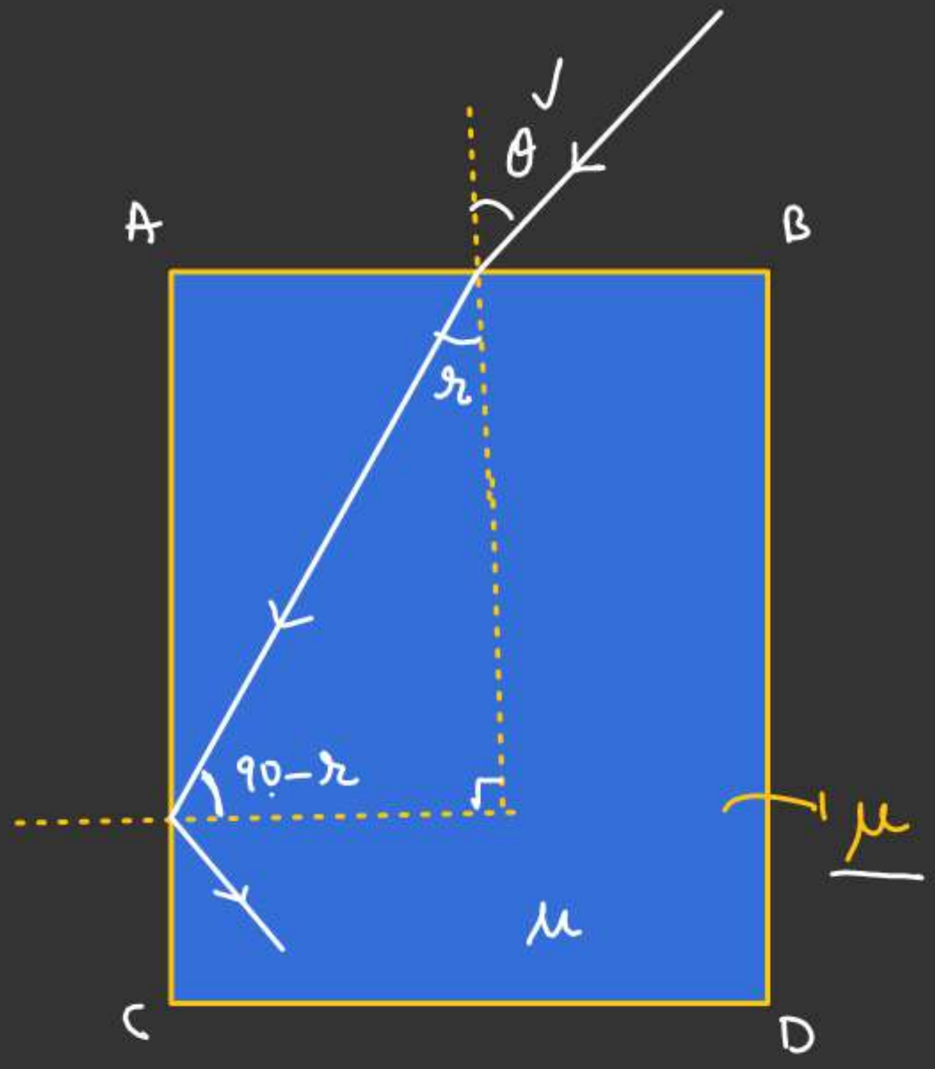
Find max θ so that light doesn't come out from the Vertical face AC.



For TIR to take's place

$$(90-r) \geq \theta_c$$

$$\sin(90^\circ-r) \geq \sin\theta_c$$

$$\cos r \geq \sin\theta_c$$

$$\cos r \geq \frac{1}{\mu} \quad -(1)$$

Snell's law at AB interface

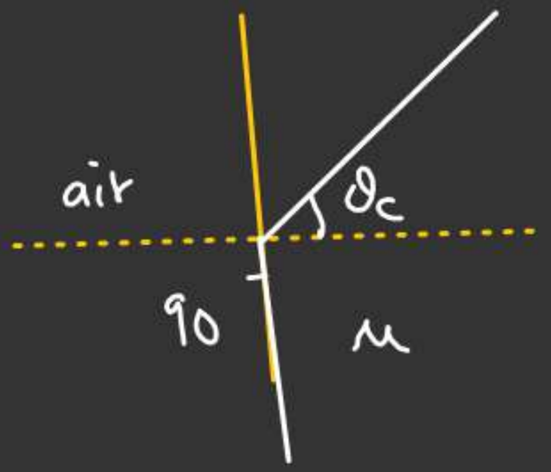
$$1\cdot\sin\theta = \mu\cdot\sin r.$$

$$\sin r = \left(\frac{\sin\theta}{\mu}\right) \quad -(2)$$

From (1)

$$\sqrt{1-\sin^2 r} \geq \frac{1}{\mu}$$

$$\sqrt{1-\frac{\sin^2\theta}{\mu^2}} \geq \frac{1}{\mu}$$



$$\sin\theta_c = \frac{1}{\mu}$$

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$$\sqrt{1-\frac{\sin^2\theta}{\mu^2}} \geq \frac{1}{\mu}$$

$$1-\frac{\sin^2\theta}{\mu^2} \geq \frac{1}{\mu^2}$$

$$1-\frac{1}{\mu^2} \geq \frac{\sin^2\theta}{\mu^2}$$

$$\mu^2-1 \geq \sin^2\theta$$

$$\sin\theta \leq \sqrt{\mu^2-1}$$

$$\boxed{\theta \leq \sin^{-1}\left(\sqrt{\mu^2-1}\right)}$$

$$\theta_{max} = \sin^{-1}\left(\sqrt{\mu^2-1}\right)$$

Find μ_{min} for TIR to take place at the face AC.

$$\sin^2\theta \leq \mu^2-1$$

$$\mu^2 \geq (\sin^2\theta+1)$$

↳ This inequality holds true if it is true for maximum value of $\sin\theta$.

$\sin\theta = +1$

$$\mu^2 \geq 2$$

$$\mu \geq \sqrt{2}$$

$$[\mu_{min} = \sqrt{2}]$$

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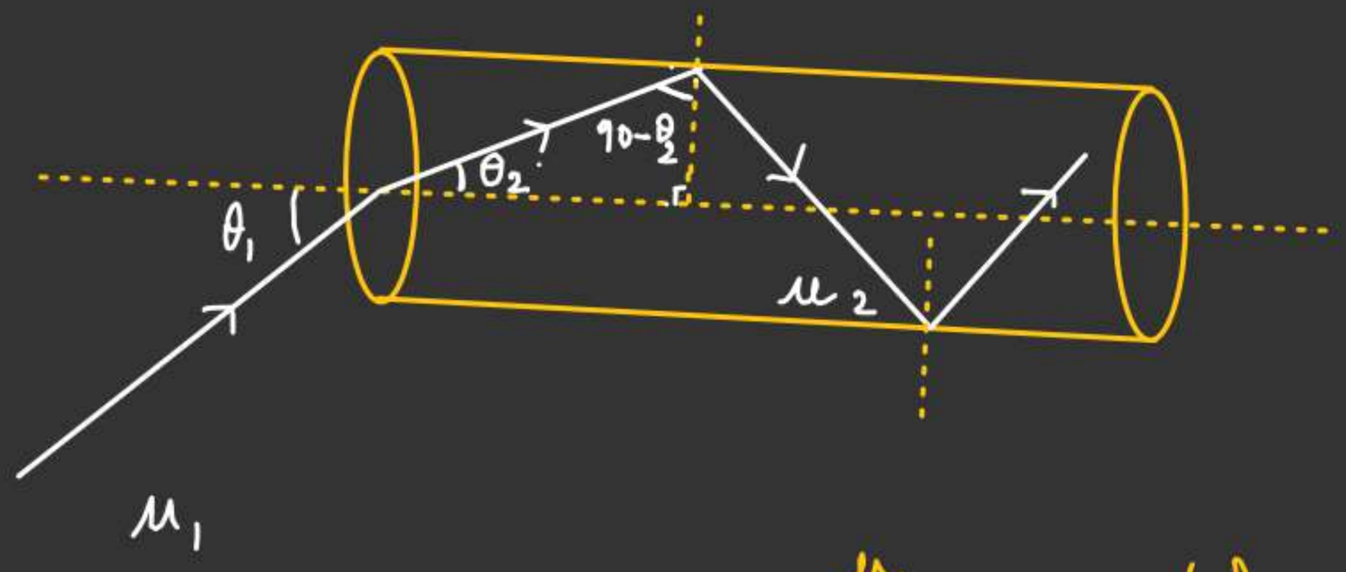
#. Find $(\theta_1)_{max}$ so that TIR always take's plance from the Curved Surface



For TIR to take place at the Curved Surface.

$$(90-\theta_2) \geq \theta_c$$

$$\sin(90-\theta_2) \geq \sin\theta_c$$

$$\cos\theta_2 \geq \sin\theta_c$$

$$\sqrt{1-\sin^2\theta_2} \geq \sin\theta_c$$

$$\sqrt{1-\frac{\mu_1^2}{\mu_2^2}\sin^2\theta_1} \geq \frac{\mu_1}{\mu_2}$$

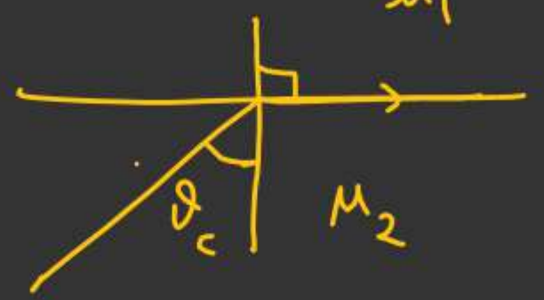
$$\left(1-\frac{\mu_1^2}{\mu_2^2}\right) \geq \frac{\mu_1^2}{\mu_2^2}\sin^2\theta_1 \Rightarrow \sqrt{\left(\frac{\mu_2^2}{\mu_1^2}-1\right)} \geq \sin\theta_1$$

$$\Rightarrow \theta_1 \leq \sin^{-1}\sqrt{\left(\frac{\mu_2^2}{\mu_1^2}-1\right)}$$

By Snell's law

$$\mu_1\sin\theta_1 = \mu_2\sin\theta_2$$

$$\sin\theta_2 = \left(\frac{\mu_1}{\mu_2}\sin\theta_1\right)$$



$$\mu_2\sin\theta_c = \mu_1\sin 90^\circ$$

$$\sin\theta_c = \left(\frac{\mu_1}{\mu_2}\right)$$

$$(\theta_1)_{max} = \left[\sin^{-1}\sqrt{\frac{\mu_2^2}{\mu_1^2}-1}\right]$$

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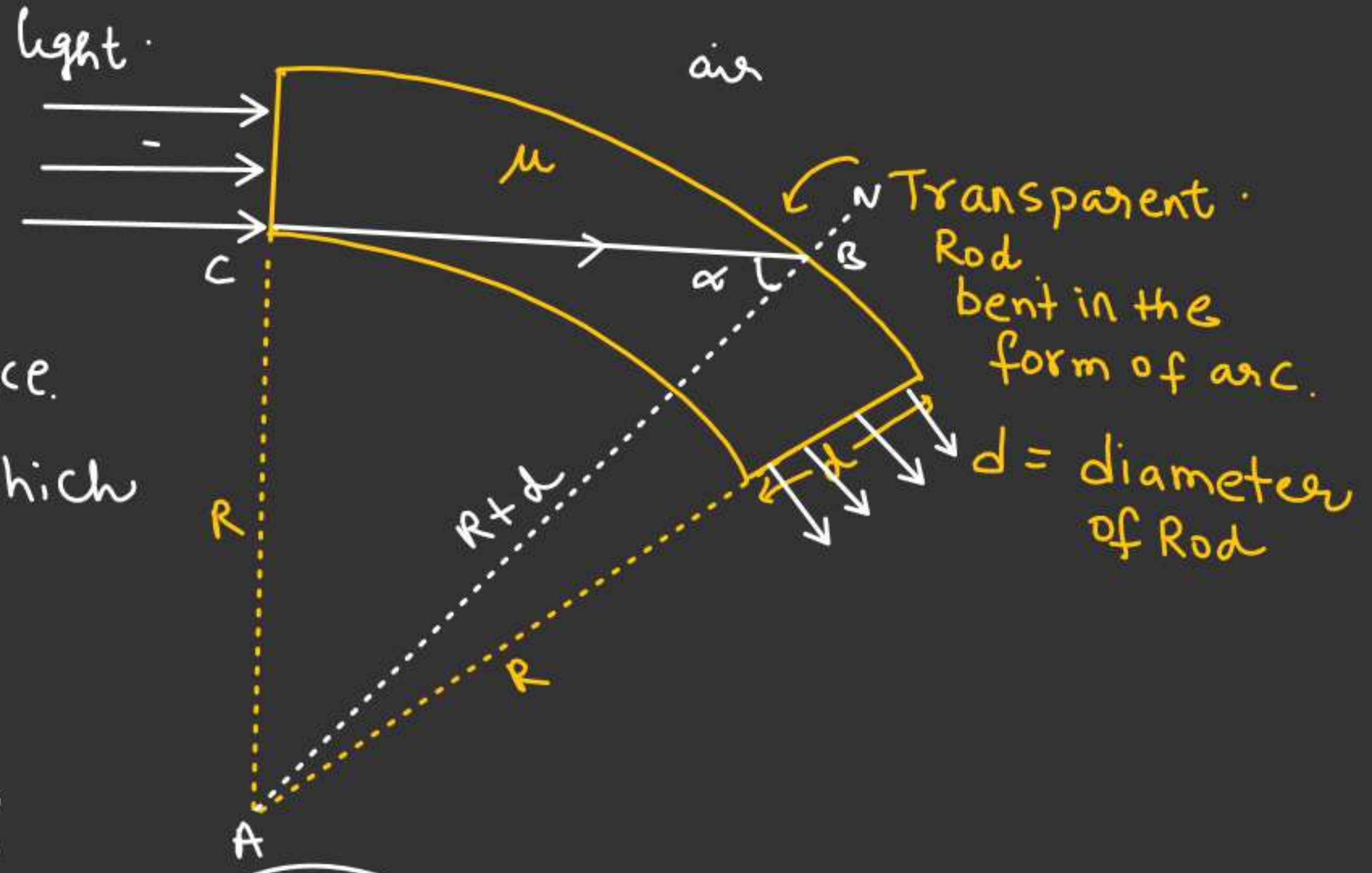
Find R_{min} so that light beam passes around the glass slab.



If all light ray passes through the tube then no refraction of light ray from the curve surface.

Min angle of incidence for the ray which grazes the inner curved surface.

$$\alpha \geq \theta_c$$

$$\sin\alpha \geq \sin\theta_c$$

In $\triangle ABC$

$$\frac{R}{R+d} \geq \frac{1}{\mu}$$

$$\mu R \geq R+d$$

$$(\mu-1)R \geq d$$

$$R \geq \left(\frac{d}{\mu-1}\right)$$

$$R_{min} = \frac{d}{\mu-1}$$ Ans

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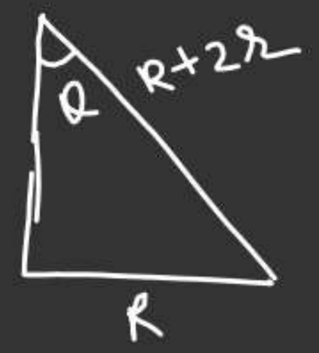
Find μ so that all the light ray entering the tube will exist from the other side of the tube.

$$\theta \geq \theta_c$$

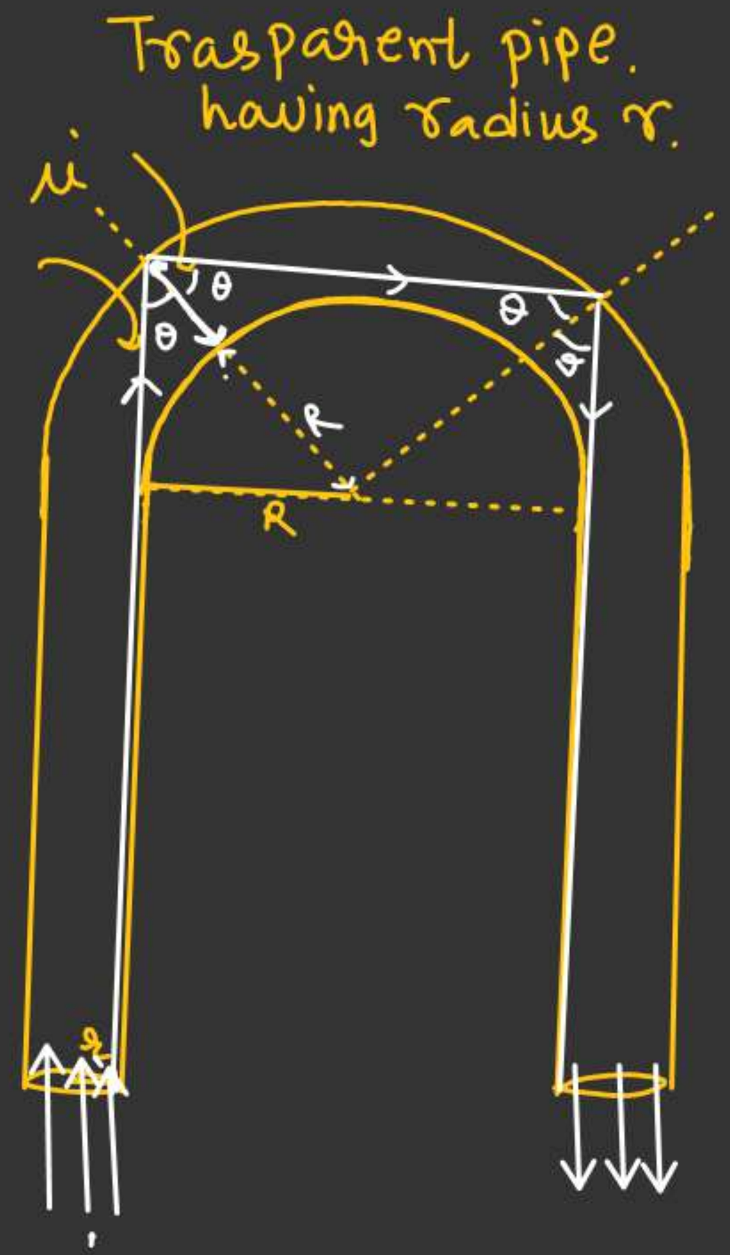
$$\sin\theta \geq \sin\theta_c$$

$$\frac{R}{R+2r} \geq \frac{1}{\mu}$$



$$\frac{R+2r}{R} \leq \mu$$

$$\mu_{min} = \left(\frac{R+2r}{R}\right)$$



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Area of illumenance

$$\sin\theta_c = \frac{1}{\mu}$$

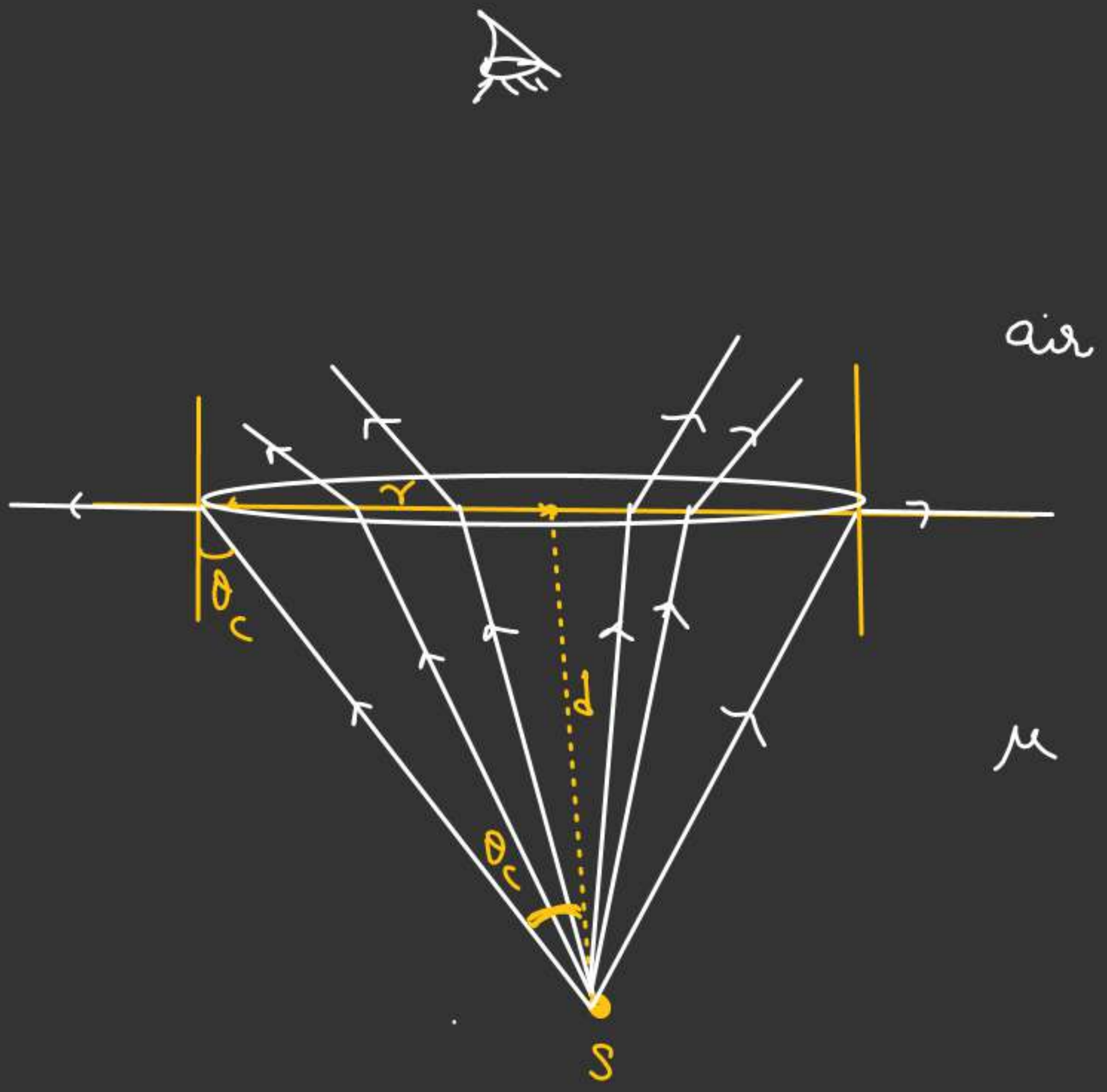
$$\frac{r}{\sqrt{d^2+r^2}} = \frac{1}{\mu}$$

$$\frac{r^2}{d^2+r^2} = \frac{1}{\mu^2}$$

$$\mu^2 r^2 = d^2+r^2$$

$$(\mu^2-1)r^2 = d^2$$

$$\boxed{r = \left(\frac{d}{\sqrt{\mu^2-1}}\right)}$$



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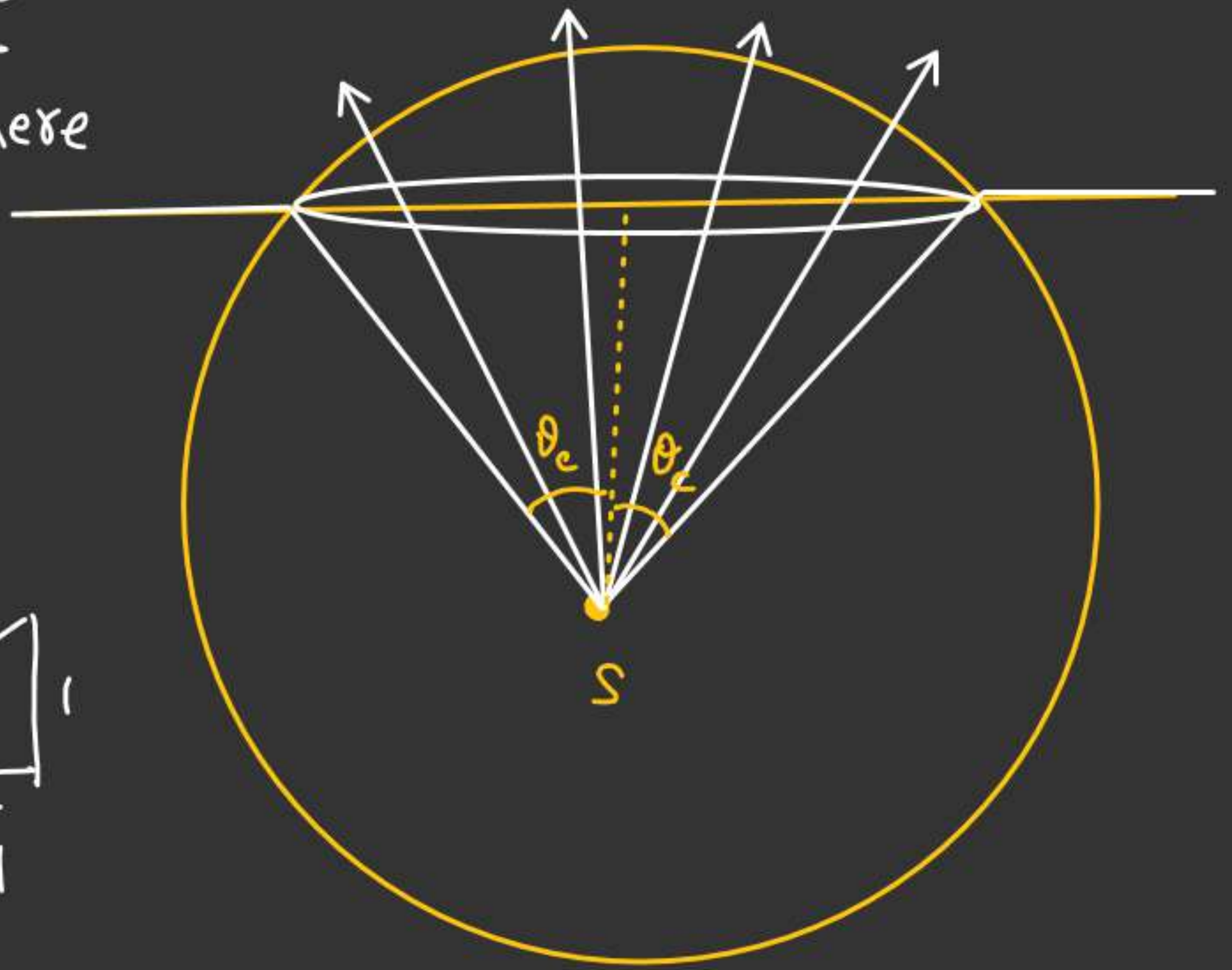
Fraction of light Comming out

$$\text{fraction of light Comming out} = \frac{\text{Area of Spherical Cap}}{\text{Total Area of Sphere}}$$

$$= \frac{2\pi R^2(1-\cos\theta_c)}{4\pi R^2}$$

$$= \frac{1}{2}\left[1-\cos\theta_c\right]$$

$$= \frac{1}{2}\left[1-\frac{\sqrt{\mu^2-1}}{\mu}\right]$$



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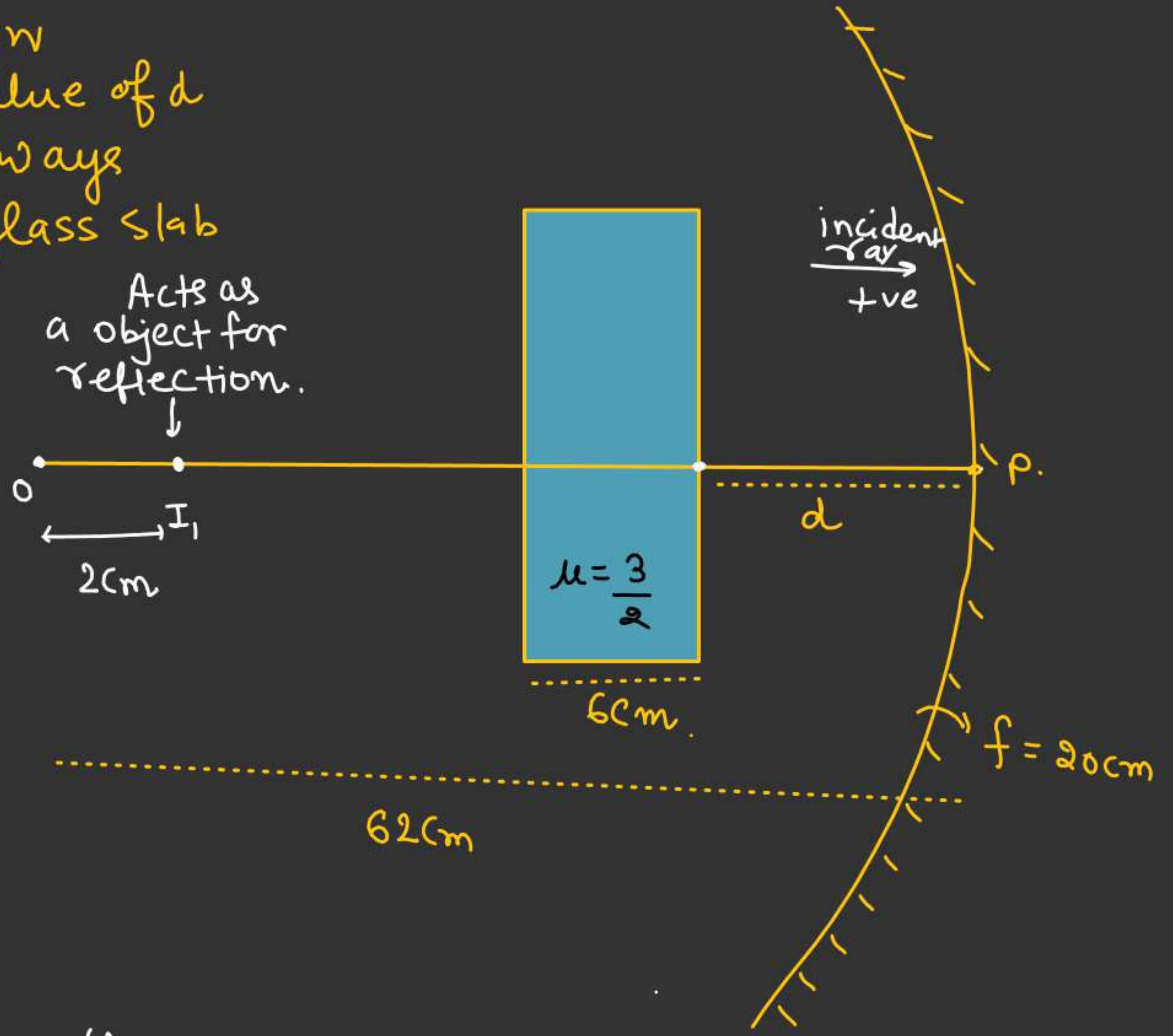
Considering all refraction and reflection. Find value of d so that final image always formed within the glass slab



1st Refraction from glass slab.

$$\text{Shift} = t\left(1-\frac{1}{\mu}\right)$$

$$= 6\left(1-\frac{2}{3}\right)$$

$$= 2\,cm$$

Reflection

$$u = -60\,cm$$

$$f = -20\,cm$$

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$$

$$v = \frac{uf}{u-f} = \frac{(-60)(-20)}{(-60+20)} = -\frac{1200}{40} = -30\,cm.$$

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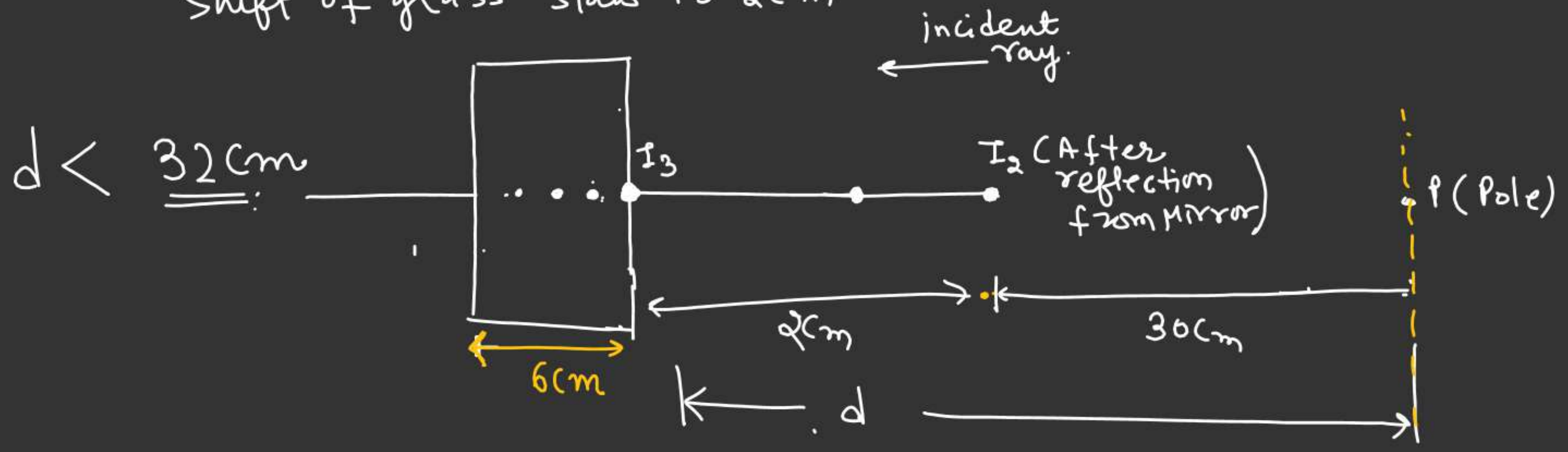
After reflection from Mirror light ray again refracted from glass slab.

Shift of glass slab is 2cm



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Range

$$\boxed{26\,cm < d < 32\,cm}$$