(MATHEMATICS)

COMPOUND ANGLE

KEY CONCEPTS

1. BASIC TRIGONOMETRIC IDENTITIES :

(a) $\sin^2\theta + \cos^2\theta = 1$; $\quad -1 \le \sin\theta \le 1$; $\; -1 \le \cos\theta \le 1\ \forall\ \theta \in R$

(b) $\sec^2\theta - \tan^2\theta = 1$; $\quad |\sec\theta| \ge 1\ \forall\ \theta \in R$

(c) $\text{cosec}^2\theta - \cot^2\theta = 1$; $\quad |\text{cosec}\,\theta| \ge 1\ \forall\ \theta \in R$

2. IMPORTANT 'T' RATIOS:

(a) $\sin n\pi = 0$; $\quad \cos n\pi = (-1)^n$; $\quad \tan n\pi = 0 \quad$ where $n \in I$

(b) $\sin\frac{(2n+1)\pi}{2} = (-1)^n$ & $\cos\frac{(2n+1)\pi}{2} = 0$ where $n \in I$

(c) $\sin 15^\circ$ or $\sin\frac{\pi}{12} = \frac{\sqrt{3}-1}{2\sqrt{2}} = \cos 75^\circ$ or $\cos\frac{5\pi}{12}$;

$\cos 15^\circ$ or $\cos\frac{\pi}{12} = \frac{\sqrt{3}+1}{2\sqrt{2}} = \sin 75^\circ$ or $\sin\frac{5\pi}{12}$

$\tan 15^\circ = \frac{\sqrt{3}-1}{\sqrt{3}+1} = 2-\sqrt{3} = \cot 75^\circ$; $\tan 75^\circ = \frac{\sqrt{3}+1}{\sqrt{3}-1} = 2+\sqrt{3} = \cot 15^\circ$

(d) $\sin\frac{\pi}{8} = \frac{\sqrt{2-\sqrt{2}}}{2}$; $\cos\frac{\pi}{8} = \frac{\sqrt{2+\sqrt{2}}}{2}$; $\tan\frac{\pi}{8} = \sqrt{2}-1$; $\tan\frac{3\pi}{8} = \sqrt{2}+1$

(e) $\sin\frac{\pi}{10}$ or $\sin 18^\circ = \frac{\sqrt{5}-1}{4}$ & $\cos 36^\circ$ or $\cos\frac{\pi}{5} = \frac{\sqrt{5}+1}{4}$

3. TRIGONOMETRIC FUNCTIONS OF ALLIED ANGLES :

If θ is any angle, then $-\theta, 90 \pm \theta, 180 \pm \theta, 270 \pm \theta, 360 \pm \theta$ etc. are called Allied Angles.

(a) $\sin(-\theta) = -\sin\theta$; $\cos(-\theta) = \cos\theta$

(b) $\sin(90^\circ - \theta) = \cos\theta$; $\cos(90^\circ - \theta) = \sin\theta$

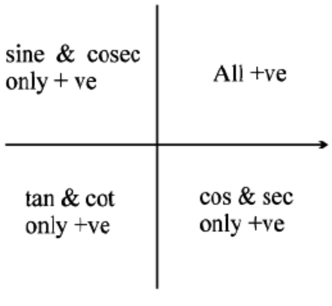
(c) $\sin(90^\circ + \theta) = \cos\theta$; $\cos(90^\circ + \theta) = -\sin\theta$

(d) $\sin(180^\circ - \theta) = \sin\theta$; $\cos(180^\circ - \theta) = -\cos\theta$

(e) $\sin(180^\circ + \theta) = -\sin\theta$; $\cos(180^\circ + \theta) = -\cos\theta$

(f) $\sin(270^\circ - \theta) = -\cos\theta$; $\cos(270^\circ - \theta) = -\sin\theta$

(g) $\sin(270^\circ + \theta) = -\cos\theta$; $\cos(270^\circ + \theta) = \sin\theta$



4. TRIGONOMETRIC FUNCTIONS OF SUM OR DIFFERENCE OF TWO ANGLES :

(a) $\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

(b) $\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

(c) $\sin^2 A - \sin^2 B = \cos^2 B - \cos^2 A = \sin(A+B) \cdot \sin(A-B)$

(d) $\cos^2 A - \sin^2 B = \cos^2 B - \sin^2 A = \cos(A+B) \cdot \cos(A-B)$

(e) $\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$ (f) $\cot(A \pm B) = \frac{\cot A \cot B \mp 1}{\cot B \pm \cot A}$

5. FACTORISATION OF THE SUM OR DIFFERENCE OF TWO SINES OR COSINES :

(a) $\sin C + \sin D = 2\sin\frac{C+D}{2}\cos\frac{C-D}{2}$

(b) $\sin C - \sin D = 2\cos\frac{C+D}{2}\sin\frac{C-D}{2}$

(c) $\cos C + \cos D = 2\cos\frac{C+D}{2}\cos\frac{C-D}{2}$

(d) $\cos C - \cos D = -2\sin\frac{C+D}{2}\sin\frac{C-D}{2}$

6. TRANSFORMATION OF PRODUCTS INTO SUM OR DIFFERENCE OF SINES & COSINES :

(a) $2\sin A\cos B = \sin(A+B) + \sin(A-B)$

(b) $2\cos A\sin B = \sin(A+B) - \sin(A-B)$

(c) $2\cos A\cos B = \cos(A+B) + \cos(A-B)$

(d) $2\sin A\sin B = \cos(A-B) - \cos(A+B)$

7. MULTIPLE ANGLES AND HALF ANGLES :

(a) $\sin 2A = 2\sin A\cos A$; $\sin\theta = 2\sin\frac{\theta}{2}\cos\frac{\theta}{2}$

(b) $\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$;

$\cos\theta = \cos^2\frac{\theta}{2} - \sin^2\frac{\theta}{2} = 2\cos^2\frac{\theta}{2} - 1 = 1 - 2\sin^2\frac{\theta}{2}$.

$2\cos^2 A = 1 + \cos 2A$, $2\sin^2 A = 1 - \cos 2A$; $\tan^2 A = \frac{1-\cos 2A}{1+\cos 2A}$

$2\cos^2\frac{\theta}{2} = 1 + \cos\theta$, $2\sin^2\frac{\theta}{2} = 1 - \cos\theta$

(c) $\tan 2A = \frac{2\tan A}{1-\tan^2 A}$; $\tan\theta = \frac{2\tan(\theta/2)}{1-\tan^2(\theta/2)}$

(d) $\sin 2A = \frac{2\tan A}{1+\tan^2 A}$, $\cos 2A = \frac{1-\tan^2 A}{1+\tan^2 A}$

(e) $\sin 3A = 3\sin A - 4\sin^3 A$

(f) $\cos 3A = 4\cos^3 A - 3\cos A$

(g) $\tan 3A = \frac{3\tan A - \tan^3 A}{1-3\tan^2 A}$

8. THREE ANGLES:

(a) $\tan(A+B+C) = \frac{\tan A + \tan B + \tan C - \tan A\tan B\tan C}{1 - \tan A\tan B - \tan B\tan C - \tan C\tan A}$

Note If : (i) $A + B + C = \pi$ then $\tan A + \tan B + \tan C = \tan A\tan B\tan C$

(ii) $A + B + C = \frac{\pi}{2}$ then $\tan A \tan B + \tan B \tan C + \tan C \tan A = 1$

(b) If $A + B + C = \pi$ then : (i) $\sin 2A + \sin 2B + \sin 2C = 4\sin A \sin B \sin C$

(ii) $\sin A + \sin B + \sin C = 4\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}$

9. MAXIMUM & MINIMUM VALUES OF TRIGONOMETRIC FUNCTIONS:

(a) Min. value of $a^2\tan^2\theta + b^2\cot^2\theta = 2ab$ where $\theta \in R$

(b) Max. and Min. value of $a\cos\theta + b\sin\theta$ are $\sqrt{a^2+b^2}$ and $-\sqrt{a^2+b^2}$

(c) If $f(\theta) = a\cos(\alpha+\theta) + b\cos(\beta+\theta)$ where a, b, α and β are known quantities then

$$-\sqrt{a^2+b^2+2ab\cos(\alpha-\beta)} \le f(\theta) \le \sqrt{a^2+b^2+2ab\cos(\alpha-\beta)}$$

(d) If A, B, C are the angles of a triangle then maximum value of $\sin A + \sin B + \sin C$ and $\sin A \sin B \sin C$ occurs when $A = B = C = 60°$

(e) In case a quadratic in $\sin\theta$ or $\cos\theta$ is given then the maximum or minimum values can be interpreted by making a perfect square.

10. Sum of sines or cosines of n angles,

$$\sin\alpha + \sin(\alpha+\beta) + \sin(\alpha+2\beta) + \cdots \ldots + \sin(\alpha + \overline{n-1}\beta) = \frac{\sin\frac{n\beta}{2}}{\sin\frac{\beta}{2}}\sin\left(\alpha + \frac{n-1}{2}\beta\right)$$

$$\cos\alpha + \cos(\alpha+\beta) + \cos(\alpha+2\beta) + \cdots \ldots + \cos(\alpha + \overline{n-1}\beta) = \frac{\sin\frac{n\beta}{2}}{\sin\frac{\beta}{2}}\cos\left(\alpha + \frac{n-1}{2}\beta\right)$$

PROFICIENCY TEST-1

1(a). If $x = r\sin\theta\cos\phi, y = r\sin\theta\sin\phi, z = r\cos\theta$. Prove that $x^2 + y^2 + z^2 = r^2$.

(b) $\frac{(\sin^2\alpha\sin^2\beta+\sin^2\alpha\cos^2\beta+\cos^2\alpha)+(\sin^2(2\pi+\alpha)+\cos^2(6\pi-\alpha)+1)}{\left(\sin^2\left(\frac{\pi}{3}-4\pi\right)+\cos^2\left(8\pi+\frac{\pi}{3}\right)+2\right)}$

(A) $\frac{4}{3}$ (B) $\frac{5}{3}$ (C) $\frac{1}{3}$ (D) 1

2. Find the value of each of the following :

(i) $\cos 210°$ (ii) $\sin 315°$ (iii) $\tan(-1125°)$ (iv) $\cos 510°$ (v) $\sin(-330°)$

(vi) $\tan\frac{11\pi}{6}$ (vii) $\sin\frac{5\pi}{3}$ (viii) $\sec 150°$ (ix) $\text{cosec}\,660°$ (x) $\cot 225°$

3. Find values of :

(i) $\tan 225°\cot 405° + \tan 765°\cot 675°$

(ii) $\tan 720° - \cos 270° - \sin 150°\cos 120°$

(iii) $\sin 600°\cos 390° + \cos 480°\sin 150°$

(iv) $\cos 24° + \cos 55° + \cos 125° + \cos 204° + \cos 300°$

(v) $\tan\frac{11\pi}{3} - 2\sin\frac{2\pi}{3} - \frac{3}{4}\text{cosec}^2\frac{\pi}{4} + 4\cos^2\frac{17\pi}{6}$

(vi) $\sin(1560°) + \cos(-3030°) + \tan(-1260°)$

4. If $2\sin^2 x + 3\sin x + 1 = 0$ then the sum of all the values of x lying in $[0,2\pi]$ is $\frac{k\pi}{2}$ where k is

(A) 3 (B) 5 (C) 7 (D) 9

5. Which of the following relations is (are) possible?

(A) $\sin\theta = \frac{\pi}{2}$ (B) $\tan\theta = 2016$

C) $\cos\theta = \frac{1+t^2}{1-t^2}(t \neq 0, \pm 1)$ (D) $\sec\theta = \frac{3}{4}$

6. Show that $2(\sin^6 x + \cos^6 x) - 3(\sin^4 x + \cos^4 x) + 1 = 0$.

7. Prove that $(1 + \cot\theta - \text{cosec}\,\theta)(1 + \tan\theta + \sec\theta) = 2$.

8. Prove that $2\sec^2\theta - \sec^4\theta - 2\text{cosec}^2\theta + \text{cosec}^4\theta = \frac{1-\tan^8\theta}{\tan^4\theta}$.

9. The value of $(\sec\theta - \cos\theta)(\text{cosec}\,\theta - \sin\theta)(\cot\theta + \tan\theta)$[wherever defined] is equal to

(A) 1 (B) -1 (C) 2 (D) -2

10. Which one of the following number is largest in value?

(A) $\tan\left(\frac{5\pi}{4}\right)$ (B) $\sin^2\left(\frac{5\pi}{4}\right)$ (C) $\log_2\left(\frac{5\pi}{4}\right)$ (D) $\ln\left(\frac{5\pi}{4}\right)$

PROFICIENCY TEST-2

1. Prove that $\frac{\sin(A+B+C)}{\cos A \cos B \cos C} = \tan A + \tan B + \tan C - \tan A \tan B \tan C$

2. Prove that $\frac{\sin 5A - \sin 3A}{\cos 5A + \cos 3A} = \tan A$

3. Prove that $\frac{\cos 4x + \cos 3x + \cos 2x}{\sin 4x + \sin 3x + \sin 2x} = \cot 3x$

4. Prove that $\frac{\cos 8A \cos 5A - \cos 12A \cos 9A}{\sin 8A \cos 5A + \cos 12A \sin 9A} = \tan 4A$

5. Prove that $\frac{\sin(A-C) + 2\sin A + \sin(A+C)}{\sin(B-C) + 2\sin B + \sin(B+C)} = \frac{\sin A}{\sin B}$

6. Prove that $1 + \cos 2x + \cos 4x + \cos 6x = 4\cos x \cos 2x \cos 3x$

7. If $\sin A = \frac{3}{5}, \cos B = \frac{-12}{13}$, where $\frac{\pi}{2} < A < \pi$ and $\frac{\pi}{2} < B < \pi$, then $\sin(A+B)$ equals

(A) $\frac{56}{65}$ (B) $\frac{-56}{65}$ (C) $\frac{33}{65}$ (D) $\frac{-33}{65}$

8. If $\tan A - \tan B = x$ and $\cot A - \cot B = y$, prove that $\cot(A-B) = \frac{1}{x} - \frac{1}{y}$

9. If $\theta = \frac{\pi}{12}$ then the value of the expression $E = \frac{\sqrt{1+\tan^2\theta}}{\tan\theta} - \frac{\sqrt{1+\cot^2\theta}}{\cot\theta}$ is equal to

(A) $\sqrt{4}$ (B) $\sqrt{8}$ (C) $\sqrt{12}$ (D) $\sqrt{24}$

10. The value of expression $\frac{\cos 68^\circ}{\sin 56^\circ \cdot \sin 34^\circ \cdot \tan 22^\circ}$ is equal to

(A) 1 (B) 2 (C) 3 (D) 4

11. Prove that $\cos^4\frac{\pi}{8} + \cos^4\frac{3\pi}{8} + \cos^4\frac{5\pi}{8} + \cos^4\frac{7\pi}{8} = \frac{3}{2}$

12. If $\tan x = \frac{3}{4}, \pi < x < \frac{3\pi}{2}$, then $\tan\frac{x}{2}$ is equal to

(A) -3 (B) $\frac{1}{3}$ (C) $\frac{-1}{2}$ (D) $\frac{-1}{3}$

EXERCISE–I

1. (a) If $y = 10\cos^2 x - 6\sin x.\cos x + 2\sin^2 x$, then find the greatest & least value of y.
(b) If $y = 1 + 2\sin x + 3\cos^2 x$, find the maximum & minimum values of $y \ \forall\, x \in R$.
(c) If $a \le 3\cos\left(\theta + \frac{\pi}{3}\right) + 5\cos\theta + 3 \le b$, find a and b.

2. If the expression $\cos^2\frac{\pi}{11} + \cos^2\frac{2\pi}{11} + \cos^2\frac{3\pi}{11} + \cos^2\frac{4\pi}{11} + \cos^2\frac{5\pi}{11}$ has the value equal to $\frac{p}{q}$ in its lowest form ; then find $(p + q)$.

3. Prove that $\sqrt{\frac{1-\sin\theta}{1+\sin\theta}} = \begin{cases} \sec\theta - \tan\theta & ;\ \text{if } -\frac{\pi}{2} < \theta < \frac{\pi}{2} \\ -\sec\theta + \tan\theta & ;\ \text{if } \frac{\pi}{2} < \theta < \frac{3\pi}{2} \end{cases}$

4. Prove that $\cos 6A = 32\cos^6 A - 48\cos^4 A + 18\cos^2 A - 1$

5. If $\cos\alpha + \cos\beta + \cos\gamma = 0$; then prove that $\cos 3\alpha + \cos 3\beta + \cos 3\gamma = 12\cos\alpha\cos\beta\cos\gamma$

6. Prove that: $\cos^2\alpha + \cos^2(\alpha+\beta) - 2\cos\alpha \cdot \cos\beta\cos(\alpha+\beta) = \sin^2\beta$

7. Prove that: $\cos 2\alpha = 2\sin^2\beta + 4\cos(\alpha+\beta)\sin\alpha\sin\beta + \cos 2(\alpha+\beta)$

8. Prove that: $\tan\alpha + 2\tan 2\alpha + 4\tan 4\alpha + 8\cot 8\alpha = \cot\alpha$.

9. Prove that:

(a) $\tan 20^\circ \cdot \tan 40^\circ \cdot \tan 60^\circ \cdot \tan 80^\circ = 3$

(b) $\tan 9^\circ - \tan 27^\circ - \tan 63^\circ + \tan 81^\circ = 4$.

(c) $\sin^4\frac{\pi}{16} + \sin^4\frac{3\pi}{16} + \sin^4\frac{5\pi}{16} + \sin^4\frac{7\pi}{16} = \frac{3}{2}$

10. If $X = \sin\left(\theta + \frac{7\pi}{12}\right) + \sin\left(\theta - \frac{\pi}{12}\right) + \sin\left(\theta + \frac{3\pi}{12}\right)$, $Y = \cos\left(\theta + \frac{7\pi}{12}\right) + \cos\left(\theta - \frac{\pi}{12}\right) + \cos\left(\theta + \frac{3\pi}{12}\right)$ then prove that $\frac{X}{Y} - \frac{Y}{X} = 2\tan 2\theta$.

11. Find the positive integers p, q, r, s satisfying $\tan\frac{\pi}{24} = (\sqrt{p} - \sqrt{q})(\sqrt{r} - s)$.

12. If the value of the expression $\sin 25^\circ \cdot \sin 35^\circ \cdot \sin 85^\circ$ can be expressed as $\frac{\sqrt{a}+\sqrt{b}}{c}$ where $a, b, c \in N$ and are in their lowest form, find the value of $(a + b + c)$.

13. Prove that $(4\cos^2 9^\circ - 3)(4\cos^2 27^\circ - 3) = \tan 9^\circ$.

14. If $A + B + C = \pi$, prove that $\sum\left(\frac{\tan A}{\tan B \cdot \tan C}\right) = \sum(\tan A) - 2\sum(\cot A)$.

15. If $\alpha + \beta = \gamma$, prove that $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1 + 2\cos\alpha\cos\beta\cos\gamma$.

16. Calculate without using trigonometric tables:

(a) $4\cos 20^\circ - \sqrt{3}\cot 20^\circ$ (b) $\frac{2\cos 40^\circ - \cos 20^\circ}{\sin 20^\circ}$

(c) $\cos^6\frac{\pi}{16} + \cos^6\frac{3\pi}{16} + \cos^6\frac{5\pi}{16} + \cos^6\frac{7\pi}{16}$ (d) $\tan 10^\circ - \tan 50^\circ + \tan 70^\circ$

17. Given that $(1 + \tan 1^\circ)(1 + \tan 2^\circ) \ldots\ldots (1 + \tan 45^\circ) = 2^n$, find n.

18. If A, B, C denote the angles of a triangle ABC then prove that the triangle is right angled if and only if $\sin 4A + \sin 4B + \sin 4C = 0$

19. Let $A_1, A_2, \ldots\ldots, A_n$ be the vertices of an n-sided regular polygon such that;

$\frac{1}{A_1 A_2} = \frac{1}{A_1 A_3} + \frac{1}{A_1 A_4}$. Find the value of n.

20. In a kite ABCD, $AB = AD$ and $CB = CD$. If $\angle A = 108^\circ$ and $\angle C = 36^\circ$ then the ratio of the area of $\triangle ABD$ to the area of $\triangle CBD$ can be written in the form $\frac{a - b\tan^2 36^\circ}{c}$ where a, b and c are relatively prime positive integers. Determine the ordered triple (a, b, c). Also find the numerical value of this ratio.

EXERCISE–II

1. If $m\tan(\theta-30^\circ)=n\tan(\theta+120^\circ)$, show that $\cos 2\theta=\frac{m+n}{2(m-n)}$.

2. Prove that $4\cos\frac{2\pi}{7}\cdot\cos\frac{\pi}{7}-1=2\cos\frac{2\pi}{7}$

3. In a right angled triangle, acute angles A and B satisfy $\tan A+\tan B+\tan^2 A+\tan^2 B+\tan^3 A+\tan^3 B=70$ find the angle A and B in radians.

4. If the product $(\sin1^\circ)(\sin3^\circ)(\sin5^\circ)(\sin7^\circ)\ (\sin89^\circ)=\frac{1}{2^n}$, then find the value of n

5. $\tan\alpha=p/q$ where $\alpha=6\beta$, α being an acute angle, prove that: $\frac{1}{2}(p\,\mathrm{cosec}2\beta-q\sec2\beta)=\sqrt{p^2+q^2}$.

6. Determine the smallest positive value of x (in degrees) for which $\tan(x+100^\circ)=\tan(x+50^\circ)\tan x\tan(x-50^\circ)$

7. Let a,b,c,d be real number such that $a^2+b^2=9, c^2+d^2=4$ and $ad-bc=6$. Find the maximum value of ac.

8. Prove that: $\frac{\cos3\theta+\cos3\phi}{2\cos(\theta-\phi)-1}=(\cos\theta+\cos\phi)\cos(\theta+\phi)-(\sin\theta+\sin\phi)\sin(\theta+\phi)$

9. Let $x_1=\prod_{r=1}^{5}\cos\frac{r\pi}{11}$ and $x_2=\sum_{r=1}^{5}\cos\frac{r\pi}{11}$, then show that $x_1\cdot x_2=\frac{1}{64}\left(\mathrm{cosec}\frac{\pi}{22}-1\right)$, where Π denotes the continued product.

10. If $(1+\sin t)(1+\cos t)=\frac{5}{4}$. Find the value of $(1-\sin t)(1-\cos t)$

11. Let $A_1, A_2, A_3 \dots\dots\dots A_n$ are the vertices of a regular n sided polygon inscribed in a circle of radius R. If $(A_1A_2)^2+(A_1A_3)^2+\cdots\dots\dots+(A_1A_n)^2=14R^2$, find the number of sides in the polygon.

12. Let $k=1^\circ$, then prove that $\sum_{n=0}^{88}\frac{1}{\cos nk\cdot\cos(n+1)k}=\frac{\cos k}{\sin^2 k}$

13. If $\frac{\cos^4 x}{\cos^2 y}+\frac{\sin^4 x}{\sin^2 y}=1$, then prove that $\frac{\cos^4 y}{\cos^2 x}+\frac{\sin^4 y}{\sin^2 x}=1$.

14. Prove that from the equality $\frac{\sin^4\alpha}{a}+\frac{\cos^4\alpha}{b}=\frac{1}{a+b}$ follows the relation; $\frac{\sin^8\alpha}{a^3}+\frac{\cos^8\alpha}{b^3}=\frac{1}{(a+b)^3}$.

15. If x and y are real number such that $x^2+2xy-y^2=6$, find the minimum value of $(x^2+y^2)^2$.

16. If ' θ ' is eliminated from the equations $\cos\theta-\sin\theta=b$ and $\cos3\theta+\sin3\theta=a$, find the eliminant.

17. Show that elliminating x & y from the equations, $\sin x + \sin y = a$; $\cos x + \cos y = b$ & $\tan x + \tan y = c$ gives $\frac{8ab}{(a^2+b^2)^2-4a^2} = c$

18. Given that $3\sin x + 4\cos x = 5$ where $x \in (0, \pi/2)$. Find the value of $2\sin x + \cos x + 4\tan x$

19. Show that $\frac{3+\cos x}{\sin x}$ $\forall x \in R$ can not have any value between $-2\sqrt{2}$ and $2\sqrt{2}$. What inference can you draw about the values of $\frac{\sin x}{3+\cos x}$?

20. (a) If $A + B + C = \pi$; prove that $\tan^2\frac{A}{2} + \tan^2\frac{B}{2} + \tan^2\frac{C}{2} \geq 1$.
(b) Prove that the triangle ABC is equilateral iff , $\cot A + \cot B + \cot C = \sqrt{3}$.

EXERCISE–III

1. The value of $\tan 9^\circ + \tan 36^\circ + \tan 9^\circ \cdot \tan 36^\circ$ is equal to

(A) 2 (B) 1 (C) $\tan 60^\circ$ (D) $\tan 30^\circ$

2. If $T_n = (\sin^n \theta + \cos^n \theta)$, then $\frac{T_5 - T_3}{T_7 - T_5}$ is equal to

(A) $\frac{T_1}{T_3}$ (B) $\frac{T_2}{T_4}$ (C) $\frac{T_5}{T_7}$ (D) $\frac{T_3}{T_7}$

3. The sum of all possible values of $\cot x$ for which $9\sin x + 2\cos x = 6$, is

(A) $\frac{-5}{4}$ (B) $\frac{-9}{8}$ (C) $\frac{4}{5}$ (D) $\frac{9}{8}$

4. The smallest positive value of x (in radians) satisfyinig the equation $(\sin x)(\cos^3 x) - (\cos x)(\sin^3 x) = \frac{1}{4}$, is

(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{8}$ (C) $\frac{\pi}{12}$ (D) $\frac{\pi}{15}$

5. If $3\tan\left(\frac{x+y}{2}\right) = 5\tan\left(\frac{x-y}{2}\right)$ then $\frac{\sin x}{\sin y}$ is equal to

(A) 2 (B) 3 (C) 4 (D) 5

6. The value of $\left(1 + \cos\frac{\pi}{9}\right)\left(1 + \cos\frac{3\pi}{9}\right)\left(1 + \cos\frac{5\pi}{9}\right)\left(1 + \cos\frac{7\pi}{9}\right)$ is

(A) $\frac{9}{16}$ (B) $\frac{10}{16}$ (C) $\frac{12}{16}$ (D) $\frac{5}{16}$

7. Let $f(\theta) = 2\cos\theta - \cos^2\theta, \forall\theta \in R$ then which one of the following relation is true ?

(A) $-2 \le f(\theta) \le 1$ (B) $\frac{1}{4} \le f(\theta) \le 1$

(C) $-3 \le f(\theta) \le 1$ (D) $-3 \le f(\theta) \le 0$

8. The minimum value of the expression $\frac{9x^2\sin^2 x + 4}{x\sin x}$ for $x \in (0, \pi)$ is

(A) $\frac{16}{3}$ (B) 6 (C) 12 (D) $\frac{8}{3}$

9. Calculate the value of $\tan 20^\circ + 4\sin 20^\circ$

(A) $\frac{3}{2}$ (B) $\sqrt{3}$ (C) $2\sqrt{3}$ (D) 2

10. If $y = 9\sec^2 x + 16\operatorname{cosec}^2 x$, find the minimum value of $y \ \forall \ x \in R$.

(A) 25 (B) 36 (C) 42 (D) 49

11. If $4\sin x \cdot \cos y + 2\sin x + 2\cos y + 1 = 0$ where $x, y \in [0, 2\pi]$ find the largest possible value of the sum $(x + y)$

(A) $\frac{23\pi}{6}$ (B) $\frac{7\pi}{2}$ (C) $\frac{19\pi}{6}$ (D) 3π

12. If $p(x) = \sin x\,(\sin^3 x + 3) + \cos x\,(\cos^3 x + 4) + \frac{1}{2}\sin^2 2x + 5$, then find the sum of the least and greatest values of $p(x)$.

(A) 8 (B) 12 (C) 16 (D) 20

13. Find the exact value of $\tan^2\frac{\pi}{16} + \tan^2\frac{3\pi}{16} + \tan^2\frac{5\pi}{16} + \tan^2\frac{7\pi}{16}$

(A) 21 (B) 24 (C) 28 (D) 36

14. If $\sec\theta + \tan\theta = 12$, then $\sec\theta = \frac{p}{q}$, $(p, q \in N$ in lowest from), and $p + q = k^2$, find $|k|$

(A) 6 (B) 8 (C) 10 (D) 13

15. If $\cot\left(7\frac{1}{2}\right)^{\circ} = \sqrt{p} + \sqrt{q} + \sqrt{r} + \sqrt{s}$, where $p, q, r, s \in N$, such that, $p < q < r < s$, then $(p + s) - (q + r)$ equals:

(A) -1 (B) 0 (C) 1 (D) 2

EXERCISE–IV

1. The sum of the radii of inscribed and circumscribed circles for an n sided regular polygon of side a, is **[AIEEE 2003]**

(A) $\frac{a}{4}\cot\left(\frac{\pi}{2n}\right)$ (B) $a\cot\left(\frac{\pi}{n}\right)$ (C) $\frac{a}{2}\cot\left(\frac{\pi}{2n}\right)$ (D) $a\cot\left(\frac{\pi}{2n}\right)$

2. A person standing on the bank of a river observes that the angle of elevation of the top a tree on the opposite bank of the river is 60° and when he retires 40 meters away from the tree the angle of elevation becomes 30°. The breadth of the river is **[AIEEE 2004]**

(A) 60 m (B) 30 m (C) 40 m (D) 20 m

3. Let α and β be such that $\pi < \alpha - \beta < 3\pi$. If $\sin\alpha + \sin\beta = -21/65$ and $\cos\alpha + \cos\beta = -27/65$, then the value of $\cos[(\alpha - \beta)/2]$ is **[AIEEE 2004]**

(A) $-\frac{3}{\sqrt{130}}$ (B) $\frac{3}{\sqrt{130}}$ (C) $\frac{6}{65}$ (D) $-\frac{6}{65}$

4. If $u = \sqrt{a^2\cos^2\theta + b^2\sin^2\theta} + \sqrt{a^2\sin^2\theta + b^2\cos^2\theta}$, then the difference between the maximum and minimum values of u^2 is given by **[AIEEE 2004]**

(A) $2(a^2 + b^2)$ (B) $2\sqrt{a^2 + b^2}$ (C) $(a + b)^2$ (D) $(a - b)^2$

5. If $0 < x < \pi$ and $\cos x + \sin x = 1/2$, then $\tan x$ is **[AIEEE 2006]**

(A) $\frac{(1-\sqrt{7})}{4}$ (B) $\frac{(4-\sqrt{7})}{3}$ (C) $-\frac{(4+\sqrt{7})}{3}$ (D) $\frac{(1+\sqrt{7})}{4}$

6. A tower stands at the centre of a circular park. A and B are two points on the boundary of the park such that AB (= a) subtends an angle of 60° at the foot of the tower, and the angle of elevation of the top of the tower from A or B is 30°. The height of the tower is **[AIEEE 2007]**

(A) $\frac{a}{\sqrt{3}}$ (B) $a\sqrt{3}$ (C) $\frac{2a}{\sqrt{3}}$ (D) $2a\sqrt{3}$

7. AB is a vertical pole with B at the ground level and A at the top. A man finds that the angle of elevation of the point A from a certain point C on the ground is 60°. He moves away from the pole along the line BC to a point D such that CD = 7 m. From D the angle of elevation of the point A is 45°. Then the height of the pole is **[AIEEE 2008]**

(A) $\frac{7\sqrt{3}}{2}\frac{1}{\sqrt{3}-1}$m (B) $\frac{7\sqrt{3}}{2}(\sqrt{3}+1)$m (C) $\frac{7\sqrt{3}}{2}(\sqrt{3}-1)$m (D) $\frac{7\sqrt{3}}{2}\frac{1}{\sqrt{3}+1}$m

8. Let A and B denote the statements

A: $\cos\alpha + \cos\beta + \cos\gamma = 0$ B : $\sin\alpha + \sin\beta + \sin\gamma = 0$

If $\cos(\beta - \gamma) + \cos(\gamma - \alpha) + \cos(\alpha - \beta) = -\frac{3}{2}$, then **[AIEEE 2009]**

(A) A is true and B is false (B) A is false and B is true

(C) Both A and B are true (D) Both A and B are false

9. For a regular polygon, let r and R be the radii of the inscribed and the circumscribed circles. A false statement among the following is **[AIEEE 2010]**

(A) There is a regular polygon with $\frac{r}{R}=\frac{1}{\sqrt{2}}$

(B) There is a regular polygon with $\frac{r}{R}=\frac{2}{3}$

(C) There is a regular polygon with $\frac{r}{R}=\frac{\sqrt{3}}{2}$

(D) There is a regular polygon with $\frac{r}{R}=\frac{1}{2}$

10. Let $\cos(\alpha+\beta)=4/5$ and let $\sin(\alpha-\beta)=5/13$, where $0\le\alpha,\beta\le\pi/4$. Then $\tan 2\alpha$ is equal to **[AIEEE 2010]**

(A) $\frac{20}{7}$ (B) $\frac{25}{16}$ (C) $\frac{56}{33}$ (D) $\frac{19}{12}$

11. If $A=\sin^2 x+\cos^4 x$, then for all real x **[AIEEE 2011]**

(A) $\frac{3}{4}\le A\le\frac{13}{16}$ (B) $\frac{3}{4}\le A\le 1$ (C) $\frac{13}{16}\le A\le 1$ (D) $1\le A\le 2$

12. In a $\triangle PQR$, if $3\sin P+4\cos Q=6$ and $4\sin Q+3\cos P=1$, then the angle R is equal to **[AIEEE 2012]**

(A) $\frac{5\pi}{6}$ (B) $\frac{\pi}{6}$ (C) $\frac{\pi}{4}$ (D) $\frac{3\pi}{4}$

13. The expression $\frac{\tan A}{1-\cot A}+\frac{\cot A}{1-\tan A}$ can be written as : **[JEE-Mains 2013]**

(A) $\sec A+\operatorname{cosec} A$ (B) $\sin A\cos A+1$

(C) $\sec A\operatorname{cosec} A+1$ (D) $\tan A+\cot A$

14. Let $f_k(x)=\frac{1}{k}\left(\sin^k x+\cos^k x\right)$ where $x\in R$ and $k\ge 1$. Then $f_4(x)-f_6(x)$ equals **[JEE-Mains 2014]**

(A) $\frac{1}{12}$ (B) $\frac{1}{6}$ (C) $\frac{1}{3}$ (D) $\frac{1}{4}$

15. A bird is sitting on the top of a vertical pole 20 m high and its elevation from a point O on the ground is 45°. It flies off hoizontally straight away from the point O. After one second, the elevation of the bird from O is reduced to 30°. Then the speed (in m/s) of the bird is **[JEE Main-2014]**

(A) $20\sqrt{2}$ (B) $20(\sqrt{3}-1)$ (C) $40(\sqrt{2}-1)$ (D) $40(\sqrt{3}-\sqrt{2})$

16. If the angles of elevation of the top of a tower from three collinear points A, B and C on a line leading to the foot of the tower, are 30°, 45° and 60° respectively, then the ratio, AB: BC, is **[JEE Main-2015]**

(A) $1:\sqrt{3}$ (B) $2:3$ (C) $\sqrt{3}:1$ (D) $\sqrt{3}:\sqrt{2}$

17. Let a vertical tower AB have its end A on the level ground. Let C be the mid-point of AB and P be a point on the ground such that AP = 2AB. If $\angle BPC=\beta$, then $\tan\beta$ is equal to : **[JEE-Mains 2017]**

(A) $\frac{2}{9}$ (B) $\frac{4}{9}$ (C) $\frac{6}{7}$ (D) $\frac{1}{4}$

18. If $5(\tan^2 x-\cos^2 x)=2\cos 2x+9$, then the value of $\cos 4x$ is : **[JEE-Mains 2017]**

(A) $\frac{2}{9}$ (B) $-\frac{7}{9}$ (C) $-\frac{3}{5}$ (D) $\frac{1}{3}$

EXERCISE–V

1. For a positive integer n, let $f_n(\theta) = \left(\tan\frac{\theta}{2}\right)(1+\sec\theta)(1+\sec 2\theta)(1+\sec 4\theta)\ldots(1+\sec 2^n\theta)$ Then **[JEE '99,3]**

(A) $f_2\left(\frac{\pi}{16}\right) = 1$ (B) $f_3\left(\frac{\pi}{32}\right) = 1$ (C) $f_4\left(\frac{\pi}{64}\right) = 1$ (D) $f_5\left(\frac{\pi}{128}\right) = 1$

2. (a) Let $f(\theta) = \sin\theta(\sin\theta + \sin 3\theta)$. Then $f(\theta)$: **[JEE 2000 Screening. 1 out of 35]**

(A) ≥ 0 only when $\theta \geq 0$ (B) ≤ 0 for all real θ

(C) ≥ 0 for all real θ (D) ≤ 0 only when $\theta \leq 0$.

(b) In any triangle ABC, prove that, $\cot\frac{A}{2} + \cot\frac{B}{2} + \cot\frac{C}{2} = \cot\frac{A}{2}\cot\frac{B}{2}\cot\frac{C}{2}$.

3. (a) Find the maximum and minimum values of $27^{\cos 2x} \cdot 81^{\sin 2x}$.

(b) Find the smallest positive values of x & y satisfying, $x - y = \frac{\pi}{4}$, $\cot x + \cot y = 2$.

[REE 2000,3]

4. If $\alpha + \beta = \frac{\pi}{2}$ and $\beta + \gamma = \alpha$ then $\tan\alpha$ equals **[JEE 2001 (Screening), 1 out of 35]**

(A) $2(\tan\beta + \tan\gamma)$ (B) $\tan\beta + \tan\gamma$

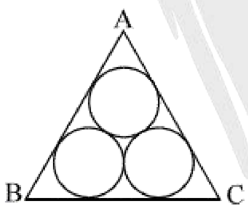
(C) $\tan\beta + 2\tan\gamma$ (D) $2\tan\beta + \tan\gamma$

5. If θ and ϕ are acute angles satisfying $\sin\theta = \frac{1}{2}$, $\cos\phi = \frac{1}{3}$, then $\theta + \phi \in$

[JEE 2004 (Screening)]

(A) $\left(\frac{\pi}{3}, \frac{\pi}{2}\right]$ (B) $\left(\frac{\pi}{2}, \frac{2\pi}{3}\right)$ (C) $\left(\frac{2\pi}{3}, \frac{5\pi}{6}\right)$ (D) $\left(\frac{5\pi}{6}, \pi\right)$

6. In an equilateral triangle, 3 coins of radii 1 unit each are kept so that they touch each other and also the sides of the triangle. Area of the triangle is



[JEE 2005 (Screening)]

(A) $4 + 2\sqrt{3}$ (B) $6 + 4\sqrt{3}$ (C) $12 + \frac{7\sqrt{3}}{4}$ (D) $3 + \frac{7\sqrt{3}}{4}$

7. Let $\theta \in (0, \pi/4)$ and $t_1 = (\tan\theta)^{\tan\theta}$, $t_2 = (\tan\theta)^{\cot\theta}$, $t_3 = (\cot\theta)^{\tan\theta}$, $t_4 = (\cot\theta)^{\cot\theta}$, then **[JEE 2006, 3]**

(A) $t_1 > t_2 > t_3 > t_4$ (B) $t_4 > t_3 > t_1 > t_2$

(C) $t_3 > t_1 > t_2 > t_4$ (D) $t_2 > t_3 > t_1 > t_4$

8. If $\frac{\sin^4 x}{2} + \frac{\cos^4 x}{3} = \frac{1}{5}$, then **[JEE 2009, 4]**

(A) $\tan^2 x = \frac{2}{3}$ (B) $\frac{\sin^8 x}{8} + \frac{\cos^8 x}{27} = \frac{1}{125}$ (C) $\tan^2 x = \frac{1}{3}$ (D) $\frac{\sin^8 x}{8} + \frac{\cos^8 x}{27} = \frac{2}{125}$

9. The maximum value of the expression $\frac{1}{\sin^2\theta+3\sin\theta\cos\theta+5\cos^2\theta}$ is **[JEE 2010]**

10. Two parallel chords of a circle of radius 2 are at a distance $\sqrt{3}+1$ apart. If the chords subtend at the centre, angles of $\frac{\pi}{k}$ and $\frac{2\pi}{k}$, where $k > 0$, then the value of $[k]$ is [Note : $[k]$ denotes the largest integer less than or equal to k]. **[JEE 2010]**

11. The positive integer value of $n > 3$ satisfying the equation $\frac{1}{\sin\left(\frac{\pi}{n}\right)} = \frac{1}{\sin\left(\frac{2\pi}{n}\right)} + \frac{1}{\sin\left(\frac{3\pi}{n}\right)}$ is

[JEE 2011]

12. The value of $\sum_{k=1}^{13} \frac{1}{\sin\left(\frac{\pi}{4}+\frac{(k-1)\pi}{6}\right)\sin\left(\frac{\pi}{4}+\frac{k\pi}{6}\right)}$ is equal to **[JEE Advanced 2016]**

(A) $3-\sqrt{3}$ (B) $2(3-\sqrt{3})$ (C) $2(\sqrt{3}-1)$ (D) $2(2+\sqrt{3})$

13. Let α and β be nonzero real numbers such that $2(\cos\beta-\cos\alpha)+\cos\alpha\cos\beta=1$. Then which of the following is/are true? **[JEE Advanced 2017]**

(A) $\tan\left(\frac{\alpha}{2}\right)-\sqrt{3}\tan\left(\frac{\beta}{2}\right)=0$ (B) $\sqrt{3}\tan\left(\frac{\alpha}{2}\right)-\tan\left(\frac{\beta}{2}\right)=0$

(C) $\tan\left(\frac{\alpha}{2}\right)+\sqrt{3}\tan\left(\frac{\beta}{2}\right)=0$ (D) $\sqrt{3}\tan\left(\frac{\alpha}{2}\right)+\tan\left(\frac{\beta}{2}\right)=0$

14. For non-negative integers n, let **[JEE Advanced 2019]**

$$f(n)=\frac{\sum_{k=0}^{n}\sin\left(\frac{k+1}{n+2}\pi\right)\sin\left(\frac{k+2}{n+2}\pi\right)}{\sum_{k=0}^{n}\sin^2\left(\frac{k+1}{n+2}\pi\right)}$$

Assuming $\cos^{-1}x$ takes values in $[0,\pi]$, which of the following options is/are correct?

(A) $f(4)=\frac{\sqrt{3}}{2}$

(B) $\sin(7\cos^{-1}f(5))=0$

(C)If $\alpha=\tan(\cos^{-1}f(6))$, then $\alpha^2+2\alpha-1=0$

(D) $\lim_{n\to\infty}f(n)=\frac{1}{2}$

15. The value of $\sec^{-1}\left(\frac{1}{4}\sum_{k=0}^{10}\sec\left(\frac{7\pi}{12}+\frac{k\pi}{2}\right)\sec\left(\frac{7\pi}{12}+\frac{(k+1)\pi}{2}\right)\right)$ in the interval $\left[-\frac{\pi}{4},\frac{3\pi}{4}\right]$ equals

[JEE Advanced 2019]

ANSWER SHEET

PROFICIENCY TEST–1

1. (b) D

2. (i) $-\frac{\sqrt{3}}{2}$ (ii) $-\frac{1}{\sqrt{2}}$ (iii) -1 (iv) $-\frac{\sqrt{3}}{2}$ (v) $\frac{1}{2}$ (vi) $-\frac{1}{\sqrt{3}}$ (vii) $-\frac{\sqrt{3}}{2}$
(viii) $-\frac{2}{\sqrt{3}}$ (ix) $-\frac{2}{\sqrt{3}}$ (x) 1

3. (i) 0 (ii) $\frac{1}{4}$ (iii)-1 (iv) $\frac{1}{2}$ (v) $\frac{3-4\sqrt{3}}{2}$ (vi) 0

4. D **5.** B **9.** A **10.** C

PROFICIENCY TEST–2

7. B **9.** B **10.** B **12.** A

EXERCISE–I

1. (a) $y_{max} = 11, y_{min} = 1$; (b) $y_{max} = \frac{13}{3}, y_{min} = -1$; (c) $a = -4$ & $b = 10$

2. 13

11. $p = 3, q = 2, r = 2, s = 1$ **12.** 24 **16.** (a) -1, (b) $\sqrt{3}$, (c) $\frac{5}{4}$, (d) $\sqrt{3}$

17. 23 **19.** 7 **20.** (1,1,2); $\sqrt{5} - 2$

EXERCISE–II

3. $\frac{\pi}{12}$ and $\frac{5\pi}{12}$ **4.** $\frac{89}{2}$ **6.** 30° **7.** 3 **10.** $\frac{13}{4} - \sqrt{10}$ **11.** 7

15. 18 **16.** $a = 3b - 2b^3$ **18.** 5 **19.** $\left[-\frac{1}{2\sqrt{2}}, \frac{1}{2\sqrt{2}}\right]$

EXERCISE–III

1. B **2.** A **3.** D **4.** B **5.** C **6.** A **7.** C

8. C **9.** B **10.** D **11.** A **12.** B **13.** C **14.** D

15. C

EXERCISE–V

1. C **2.** D **3.** A **4.** D **5.** C **6.** A **7.** B

8. C **9.** B **10.** C **11.** B **12.** B **13.** C **14.** A

15. B **16.** C **17.** A **18.** B

EXERCISE–V

1.	A, B, C, D	**2.(a)** C	**3.**	(a) max. $= 3^5$ & min. $= 3^{-5}$; (b) $x = \frac{5\pi}{12}; y = \frac{\pi}{6}$									
4.	C	**5.**	B	**6.**	B	**7.**	B	**8.**	A, B	**9.**	2	**10.**	3
11.	7	**12.**	C	13.	A, C	**14.**	A,B,C	**15.**	0.00				