Nishant Jindal

(1) $\sin^{-1}\left(2x\sqrt{1-x^2}\right) = \begin{cases} -\pi - 2\sin^{-1}x & -1 \le x < -\frac{1}{2} \\ 2\sin^{-1}x & -\frac{1}{\sqrt{2}} \le x \le \frac{1}{\sqrt{2}} \\ \pi - 2\sin^{-1}x & \frac{1}{\sqrt{2}} < x \le 1 \end{cases}$

(2) $\cos^{-1}(2x^2-1) = \begin{cases} 2\cos^{-1}x & 0 \le x \le 1 \\ 2\pi - 2\cos^{-1}x & -1 \le x < 0 \end{cases}$

(3) $\tan^{-1}\frac{2x}{1-x^2} = \begin{cases} \pi + 2\tan^{-1}x & x \le -1 \\ 2\tan^{-1}x & -1 < x < 1 \\ -\pi + 2\tan^{-1}x & x \ge 1 \end{cases}$

(7) $\boxed{3\tan^{-1}x} = \tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)$

$\left(-\frac{\frac{\pi}{2}}{3}, \frac{\frac{\pi}{2}}{3}\right) = \left(-\frac{\pi}{6}, \frac{\pi}{6}\right)$

$x \in \left(-\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$

Trigo $\to$ $\sin 2\theta = 2\sin\theta\cos\theta$

$\cos 2\theta = 2\cos^2\theta - 1$

$\tan\theta = \frac{2\tan\theta}{1-\tan^2\theta}$

4) $\sin^{-1}\left(\frac{2x}{1+x^2}\right) = \begin{cases} -\pi - 2\tan^{-1}x & x \le -1 \\ 2\tan^{-1}x & -1 \le x \le 1 \\ \pi - 2\tan^{-1}x & x > 1 \end{cases}$

Direct (Without Int.)

$\boxed{20}$

1) $\sin^{-1}\left(2x\sqrt{1-x^2}\right) = \underline{2\sin^{-1}x}$

2) $\cos^{-1}(2x^2-1) = 2\cos^{-1}x$

3) $\tan^{-1}\left(\frac{2x}{1-x^2}\right) = 2\tan^{-1}x$

4) $\boxed{2\tan^{-1}x = \tan^{-1}\left(\frac{2x}{1-x^2}\right) = \sin^{-1}\left(\frac{2x}{1+x^2}\right) = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)}$

(5) $\sin^{-1}(3x-4x^3) = 3\sin^{-1}x$

(6) $\cos^{-1}(4x^3-3x) = 3\cos^{-1}x$

Nishant Jindal

Q $\sin^{-1}(3x-4x^3) = 3\boxed{\sin^{-1}x}$; $x = ?$

$$\left[-\frac{\pi}{6}, \frac{\pi}{6}\right] \Leftarrow \left[\frac{-\frac{\pi}{2}}{3}, \frac{\frac{\pi}{2}}{3}\right]$$

$$x \in \left[-\frac{1}{2}, \frac{1}{2}\right]$$

Q $\cos^{-1}(2x^2-1) = 2\boxed{\cos^{-1}x}$ for $x \in ?$

$$\left[\frac{0}{2}, \frac{\pi}{2}\right] \Rightarrow \left[0, \frac{\pi}{2}\right]$$

$$x \in [0, 1]$$

Dom $\rightarrow$ $-1 \le x \le 1$

$$x \in [-1, 0]$$

for Very Adv Students

$$\cos^{-1}(2x^2-1) = \boxed{2\pi} - 2\cos^{-1}x$$

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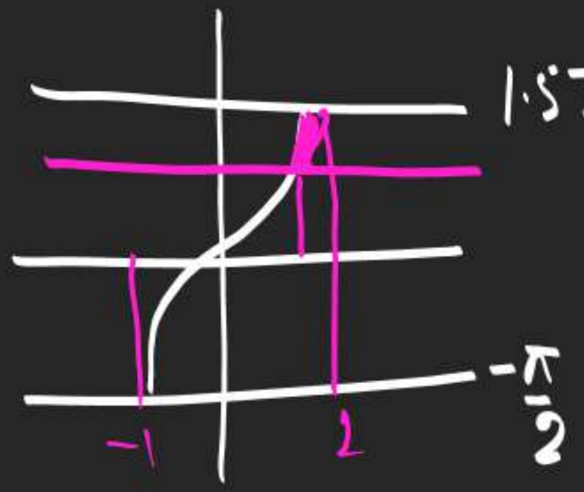
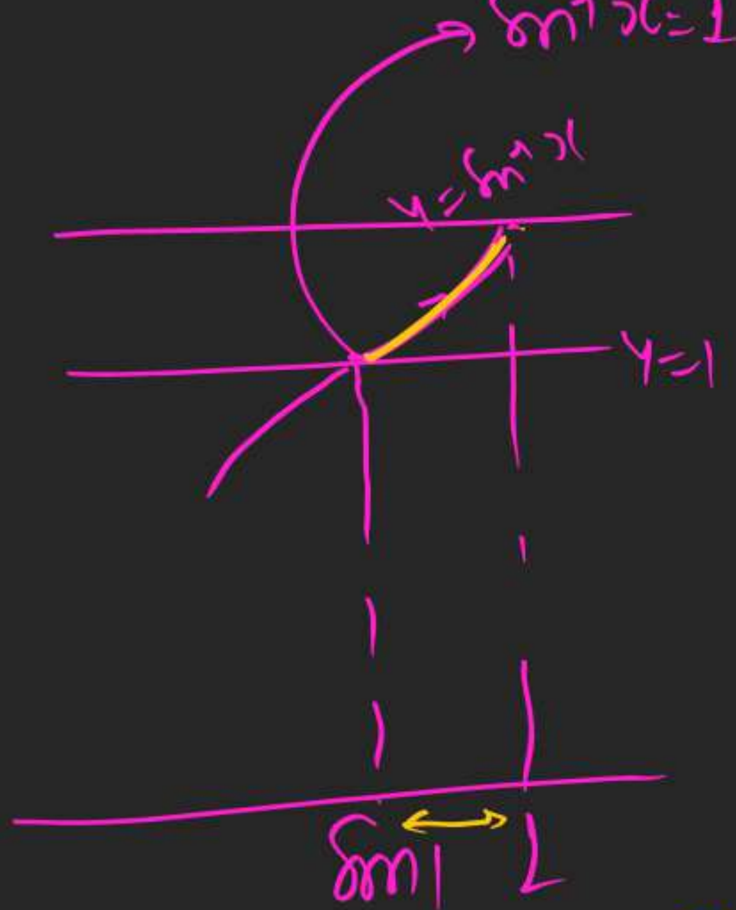
All about Inequality

Q $\sin^{-1} x > \boxed{1}$ then $x \in$?



$x = \sin 1$



$x \in [\sin 1, 1]$

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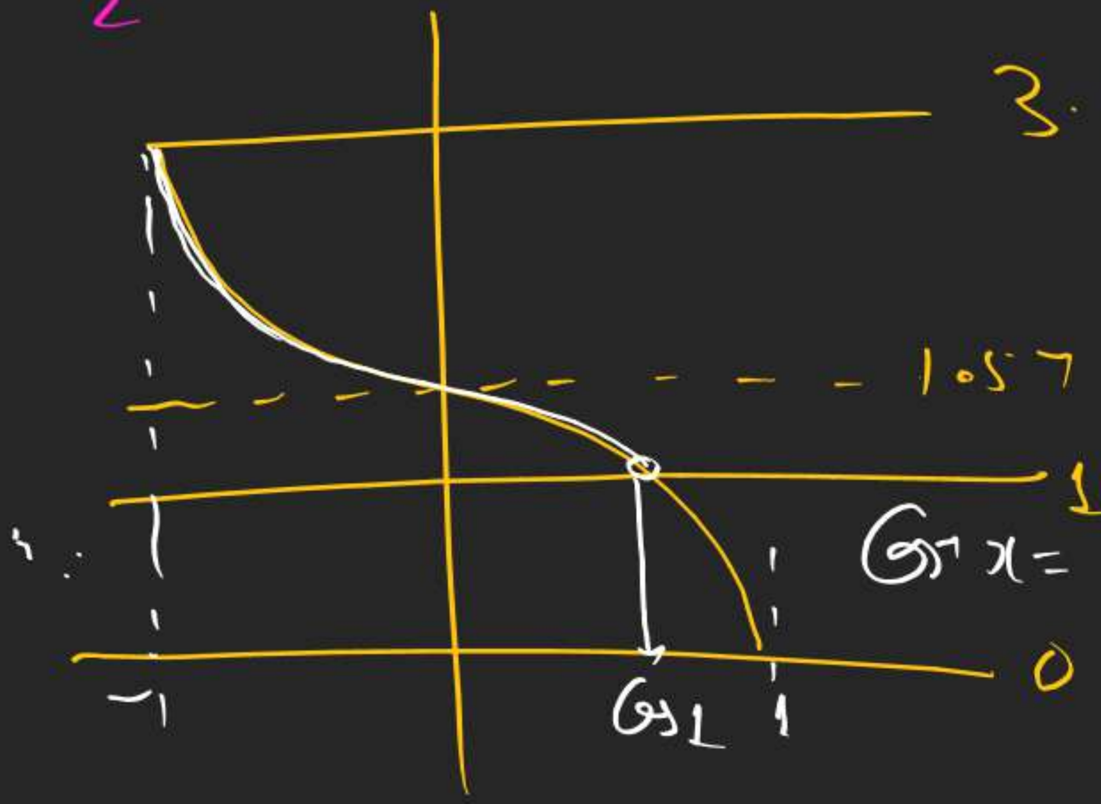
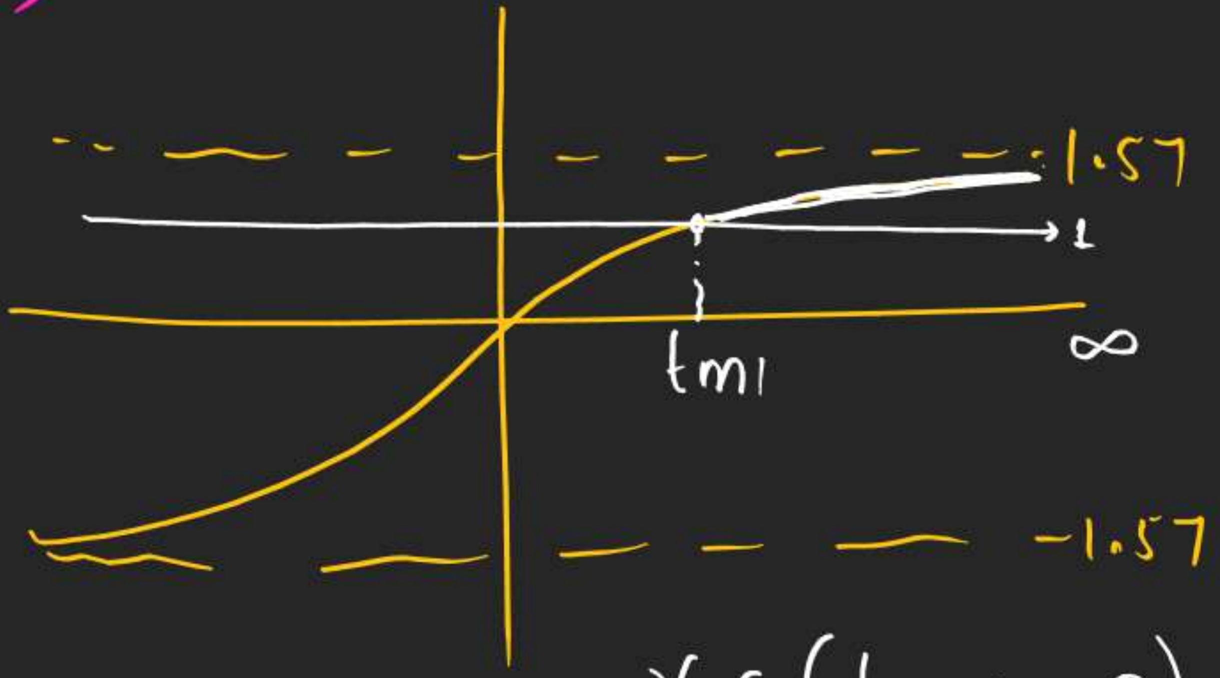
Q 2 $\cos^{-1} x > \boxed{1}$ then $x \in$

$[-1, \cos 1)$



$\cos^{-1} x = 1 \Rightarrow x = \cos 1$

Q 3 $\tan^{-1} x > 1$ then $x \in$?



$x \in (\tan 1, \infty)$

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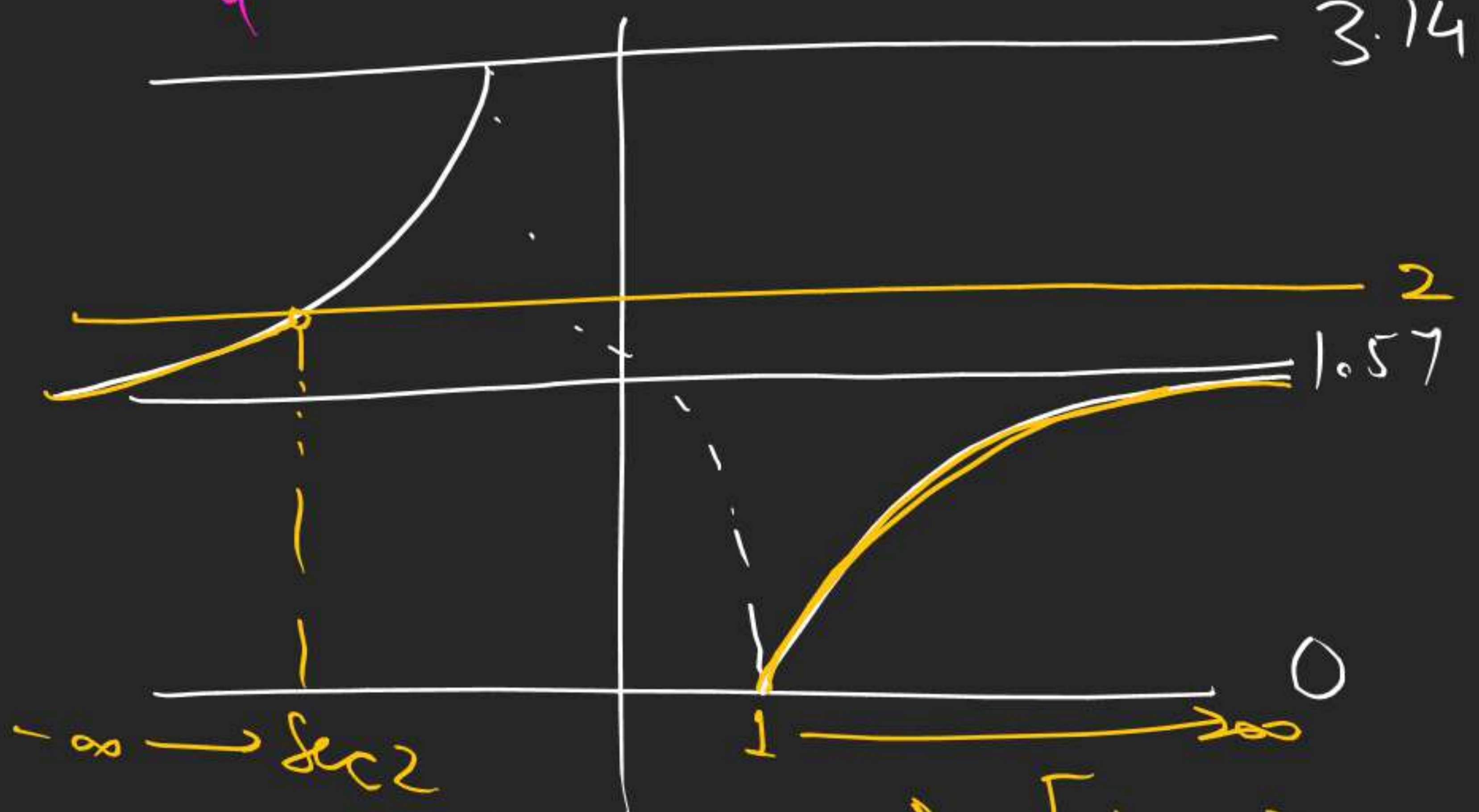
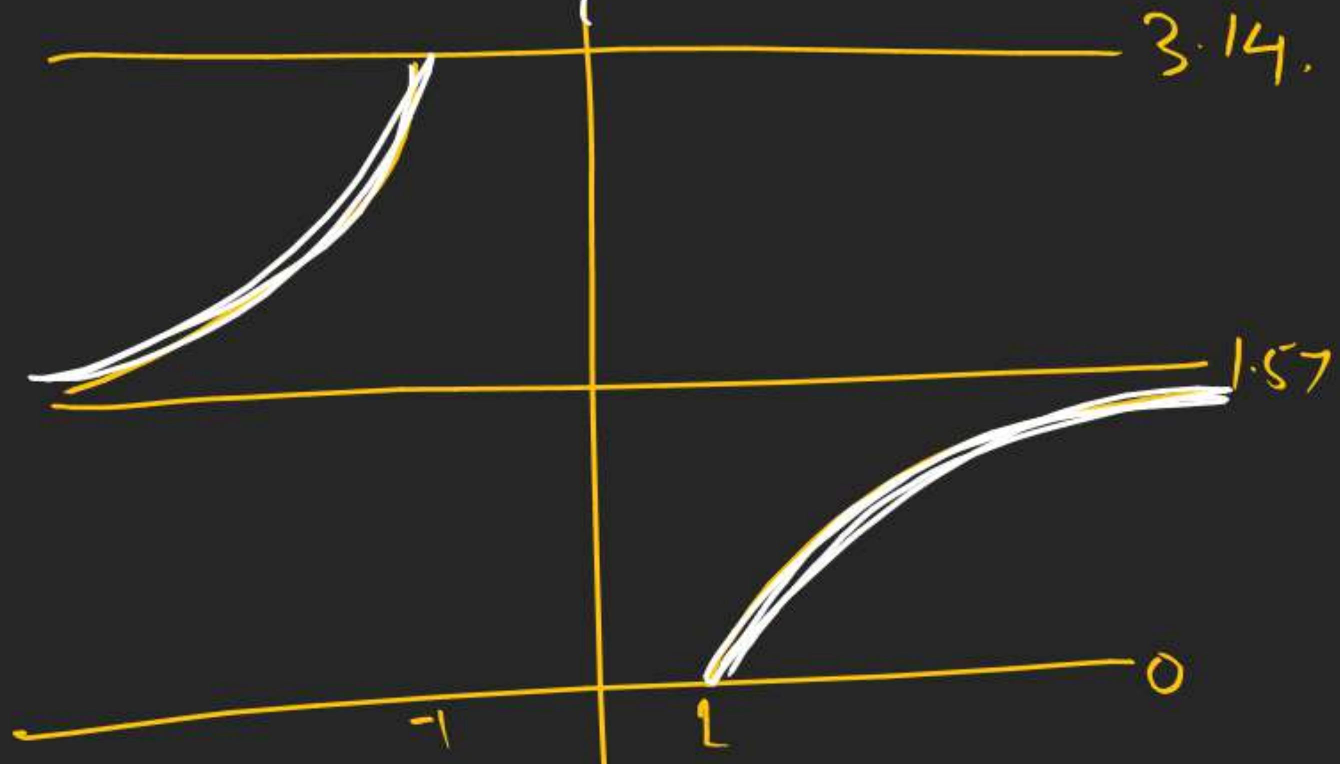
Q $\sec^{-1} x < 2$ then $x \in ?$



$x \in (-\infty, \sec 2) \cup [1, \infty)$

Q $\sec^{-1} x < 4$ then $x \in ?$

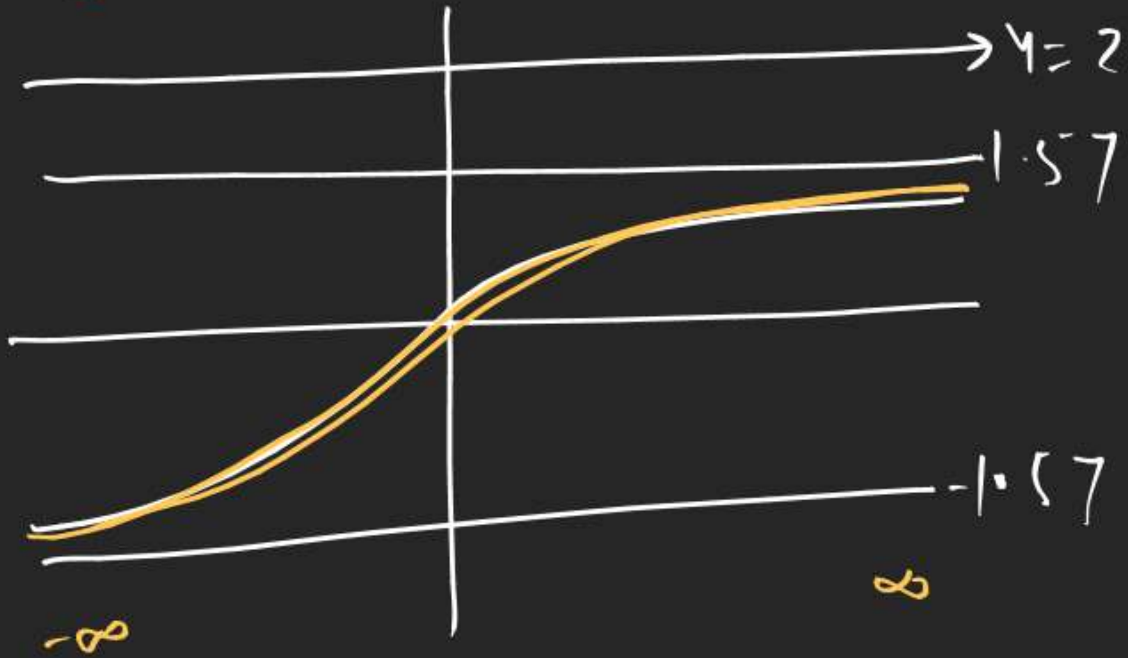


$x \in (-\infty, -1] \cup [1, \infty)$

$\cos 1 = \cos 57^\circ \approx \cos 60 = 1/2$

2 से कटेगा

Q6 $\tan^{-1}x < 2 \quad x \in ?$



$\sin 1$

$\sin 57^\circ$

$\approx \sin 60^\circ$

$= \frac{\sqrt{3}}{2}$

$= .85$

$x \in R$

Notice

$[\sin^{-1}x] = 1$

$1 \le \sin^{-1}x < \cancel{2}\ \frac{\pi}{2}$

$\sin 1 \le x \le 1$

$[\cos^{-1}x] = 0$

$0 \le \cos^{-1}x < 1$

$1 \ge x > \cos 1$

$\boxed{x \in [\sin 1, 1]}$

Q7 Tough

$[\sin^{-1}x] > [\cos^{-1}x] \quad x \in ?$

$\cancel{-2}, \cancel{-1}, \cancel{0}, \boxed{1} \rightarrow \boxed{0}, 1, 2, 3$

$[\sin^{-1}x] = 1$ & $[\cos^{-1}x] = 0$

$-\frac{\pi}{2} \le \sin^{-1}x \le \frac{\pi}{2}$

$[-1.57] \le [\sin^{-1}x] \le [1.57]$

$-2 \le [\quad] \le 1$ Pd'

$-2, -1, 0, 1$

$0 \le \cos^{-1}x \le \pi$

$[0] \le [\cos^{-1}x] \le [3.14]$

$0 \le [\cos^{-1}x] \le 3 \Rightarrow \underline{0, 1, 2, 3}$

Nishant Jindal

Q8 $\sin^{-1}x > \sin^{-1}x^2$

① $\sin^{-1}$ Removing ↑ fxn ~~it~~

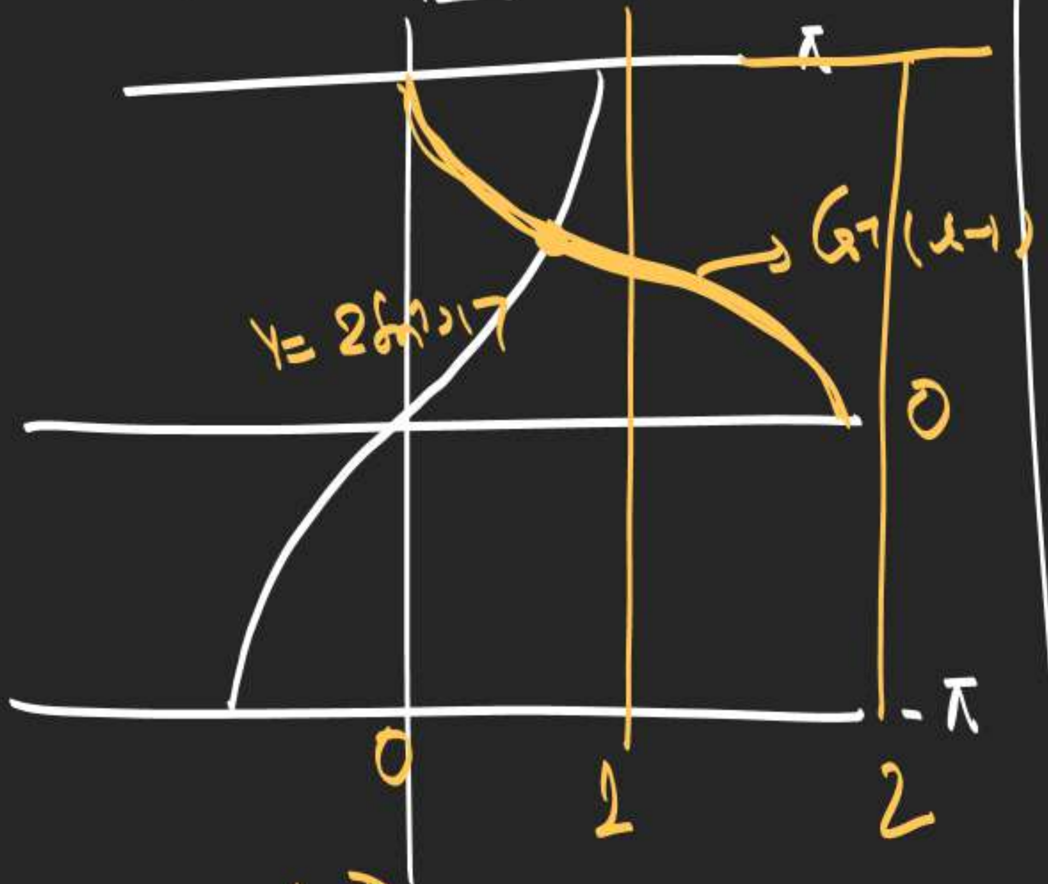
$x > x^2$	$-1 \le x \le 1$	$-1 \le x^2 \le 1$
$x^2 - x < 0$		$0 \le x^2 \le 1$
$(x)(x-1) < 0$		$0 \le \sqrt{x^2} \le 1$
$0 < x < 1$		$0 \le \lvert x \rvert \le 1$
		$-1 \le x \le 1$

$x \in (0,1)$

Nishant Jindal

Q9 No. of Sol. of $\underline{2\sin^{-1}x} = \pi - \underline{\cos^{-1}(1-x)}$

LHS $= \boxed{2}\boxed{\sin^{-1}x}$



1 Intersection
$\Rightarrow$ 1 Sol.

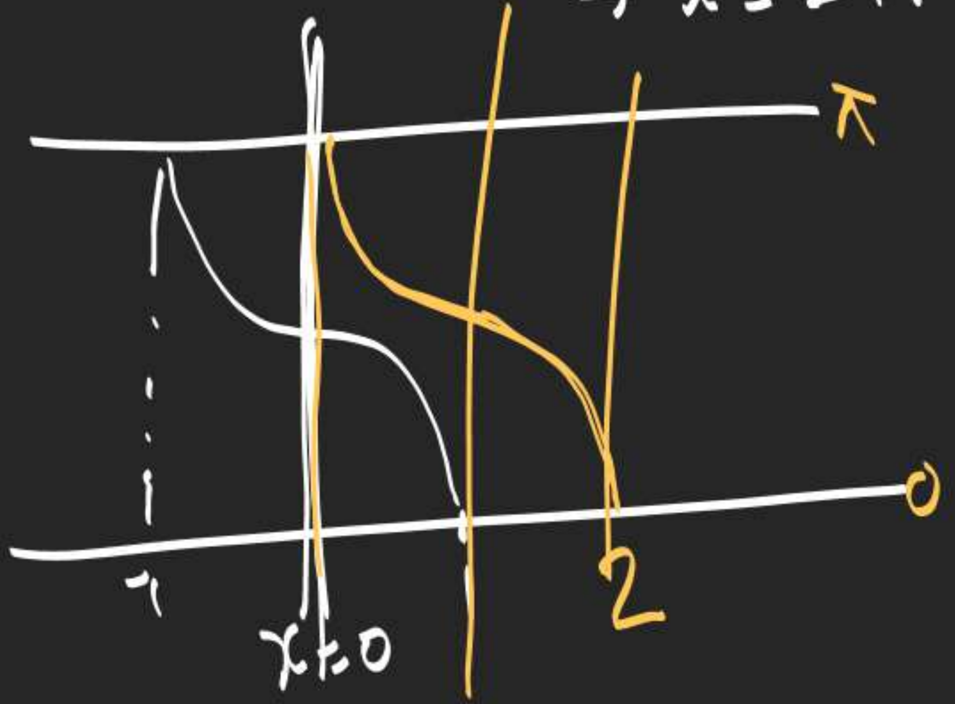
RHS $= \pi - \cos^{-1}(1-x)$

$= \pi - (\cos^{-1}(-(x-1))$

$= \cancel{\pi} - (\cancel{\pi} - \cos^{-1}(x-1))$

$= \cos^{-1}(x-1)$

$\hookrightarrow x = 1$ Pr $\cos^{-1}x$



Nishant Jindal

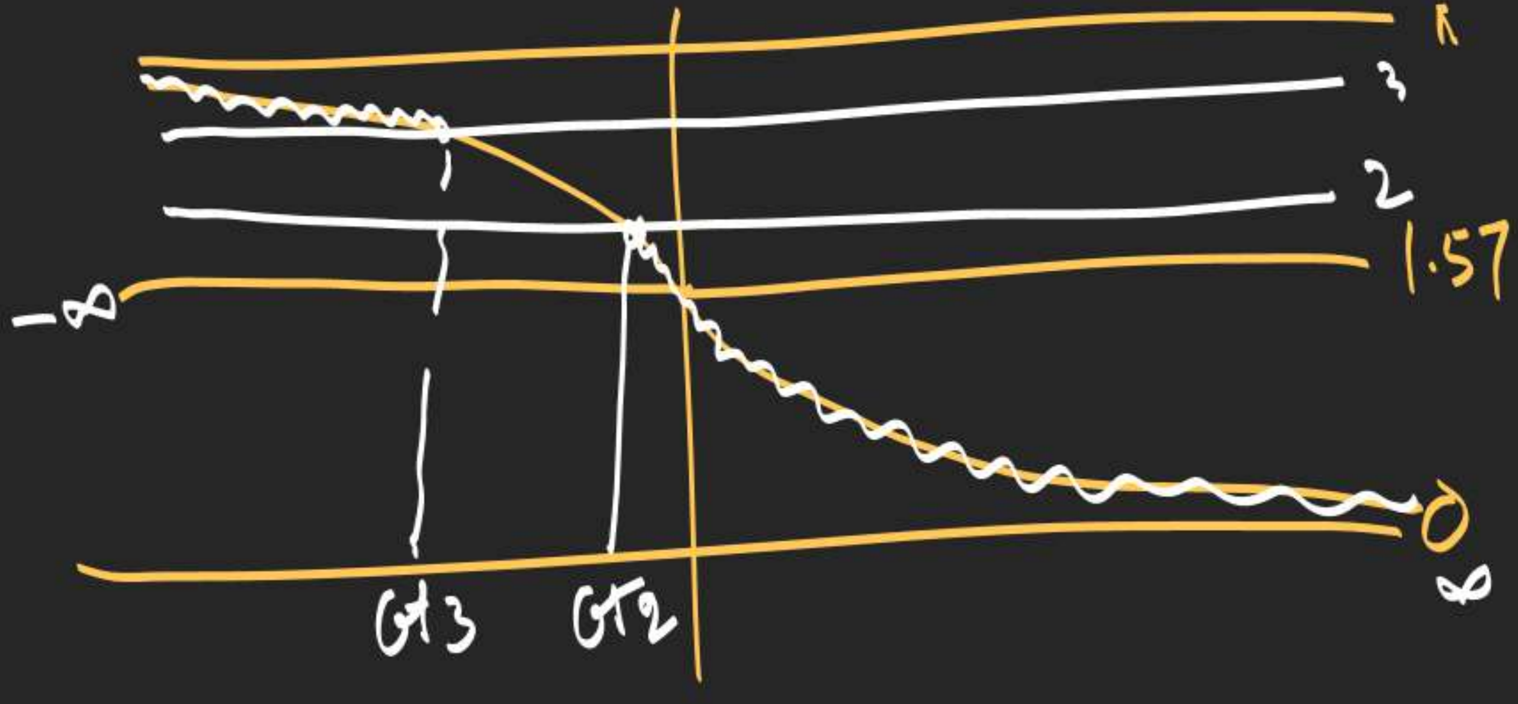
Q 10 $(\cot^{-1}x)^2 - 5\cot^{-1}x + 6 > 0$ find $x \in$?

1) $t^2 - 5t + 6 > 0$

$(t-2)(t-3) > 0$

$t < 2 \cup t > 3.$

2) $\cot^{-1}x < 2 \cup \cot^{-1}x > 3$



$x \in (-\infty, \cot 3) \cup (\cot 2, \infty)$

Q. 11 $(\sec^{-1}x)^2 - 6\sec^{-1}x + 8 > 0$ find x?

$t^2 - 6t + 8 > 0$

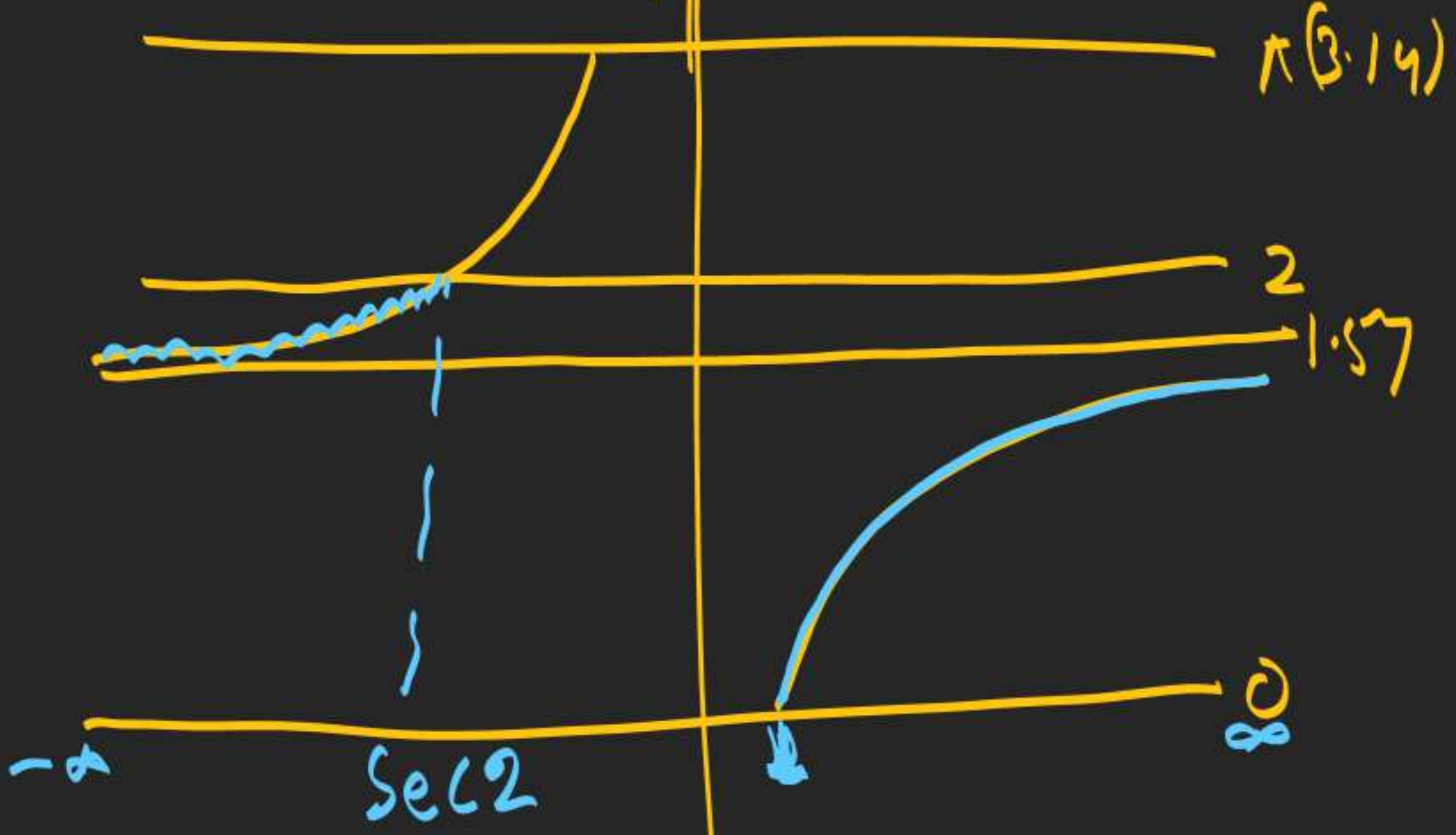
$(t-2)(t-4) > 0$

$t < 2 \cup t > 4$

$y = 4$

4 से Upar No Graph

$\sec^{-1}x < 2 \cup \boxed{\sec^{-1}x > 4}$



$x \in (-\infty, \sec 2) \cup [1, \infty)$

Nishant Jindal

Q 12 $\tan^2(\boxed{\sin^{-1}x}) > 1$ ($\sin^{-1}x \to \theta$)

$\tan^2\theta > 1$

$(\tan^2\theta - 1) > 0$

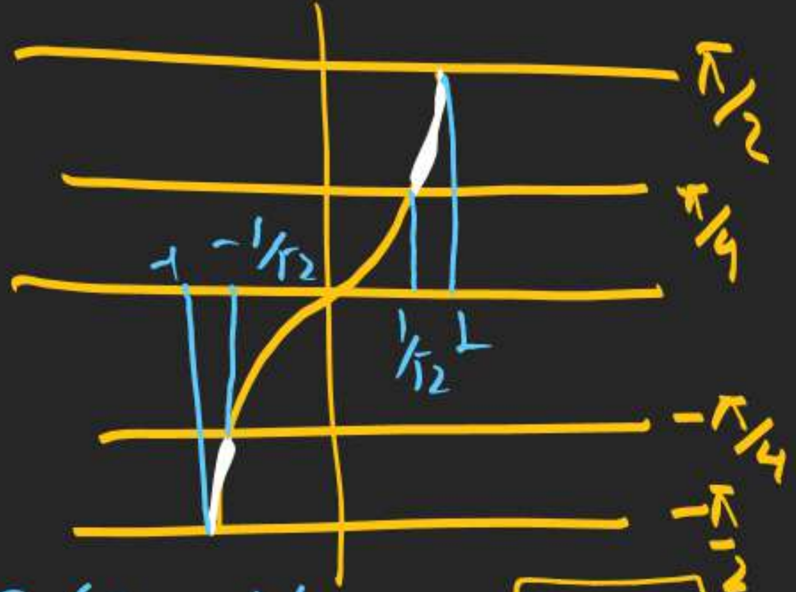
$(\tan\theta - 1)(\tan\theta + 1) > 0$

$\tan\theta < -1 \cup \tan\theta > 1$

$\tan\theta < \tan\left(-\frac{\pi}{4}\right) \cup \tan\theta > \tan\frac{\pi}{4}.$

$\theta < -\frac{\pi}{4} \cup \theta > \frac{\pi}{4}$

$\sin^{-1}x < -\frac{\pi}{4} \cup \sin^{-1}x > \frac{\pi}{4}$



$x \in \left[-1, -\frac{1}{\sqrt{2}}\right) \cup \left(\frac{1}{\sqrt{2}}, 1\right]$

Q 13 $(\cot^{-1}x)(\tan^{-1}x) + \boxed{\left(2 - \frac{\pi}{2}\right)}\cot^{-1}x - 3\tan^{-1}x - 3\boxed{\left(2 - \frac{\pi}{2}\right)} > 0$ find x?

$\tan^{-1}x\left((\cot^{-1}x - 3)\right) + \left(2 - \frac{\pi}{2}\right)\left((\cot^{-1}x - 3)\right) > 0$

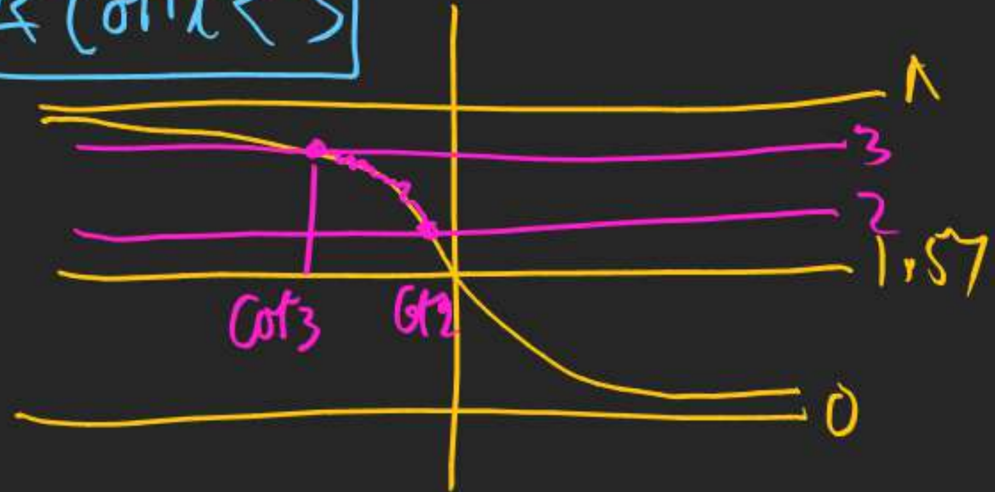
$(\cot^{-1}x - 3)\left(\tan^{-1}x + 2 - \frac{\pi}{2}\right) > 0 \longrightarrow$ Notice

$(\cot^{-1}x - 3)\left(2 - \left(\frac{\pi}{2} - \tan^{-1}x\right)\right) > 0$

$(\cot^{-1}x - 3)(2 - \cot^{-1}x) > 0 \Rightarrow (\cot^{-1}x - 3)(\cot^{-1}x - 2) < 0$

$\boxed{2 < \cot^{-1}x < 3}$

$x \in (\cot 3, \cot 2)$



Nishant Jindal

All about Series

$$\tan^{-1}\left(\frac{x-y}{1+xy}\right)=\tan^{-1}x-\tan^{-1}y$$

$$1<\tan^{-1}x<2$$

1) 90%

$$\tan^{-1}\frac{1}{x^2+x+1}+\tan^{-1}\frac{1}{\boxed{x^2+3x+3}}+\tan^{-1}\frac{1}{x^2+5x+7}+\tan^{-1}\frac{1}{x^2+7x+13}+\cdots\underbrace{\infty}_{n\text{ terms}}$$

$$\tan^{-1}\frac{1}{1+(x^2+x)}+\tan^{-1}\frac{1}{1+(x^2+3x+2)}+\tan^{-1}\frac{1}{1+(x^2+5x+6)}+\tan^{-1}\frac{1}{1+(x^2+7x+12)}+\cdots$$

$$\tan^{-1}\frac{(x+1)-x}{1+(x)(x+1)}+\tan^{-1}\frac{(x+2)-(x+1)}{1+(x+1)(x+2)}+\tan^{-1}\frac{(x+3)-(x+2)}{1+(x+2)(x+3)}+\tan^{-1}\frac{(x+4)-(x+3)}{1+(x+3)(x+4)}+\cdots$$

$$=\left(\cancel{\tan^{-1}(x+1)}-\tan^{-1}x\right)+\left(\cancel{\tan^{-1}(x+2)}-\cancel{\tan^{-1}(x+1)}\right)+\left(\cancel{\tan^{-1}(x+3)}-\cancel{\tan^{-1}(x+2)}\right)$$

$$+\left(\cancel{\tan^{-1}(x+4)}-\cancel{\tan^{-1}(x+3)}\right)+\cdots+\tan^{-1}(x+n)-\cancel{\tan^{-1}(x+n-1)}$$

$$\lim_{n\to\infty}\underbrace{\tan^{-1}(x+n)-\tan^{-1}x}_{\text{for } n\text{ terms}}=\tan^{-1}(\infty)-\tan^{-1}x=\underbrace{\boxed{\frac{\pi}{2}-\tan^{-1}x}}_{\infty\text{ term}}$$

Nishant Jindal

$$\tan^{-1}\left(\frac{a-b}{1+a\cdot b}\right)=\tan^{-1}a-\tan^{-1}b$$

Q $$S_n=\sum_{n=1}^{n}\tan^{-1}\left(\frac{4n}{n^4-2n^2+2}\right)=?$$

$$=\sum\tan^{-1}\left(\frac{4n}{1+(n^4-2n^2+1)}\right)=\sum\tan^{-1}\left(\frac{4n}{1+(n^2-1)^2}\right)$$

$$=\sum\tan^{-1}\left(\frac{(n+1)^2-(n-1)^2}{1+(n+1)^2(n-1)^2}\right)=\sum_{n=1}^{n}\tan^{-1}(n+1)^2-\tan^{-1}(n-1)^2$$

$$=\left\{\begin{array}{l}\tan^{-1}\cancel{(2)^2}-\tan^{-1}0^2\\+\tan^{-1}\cancel{3^2}-\tan^{-1}1^2\\+\tan^{-1}\cancel{4^2}-\tan^{-1}\cancel{2^2}\\+\tan^{-1}\cancel{5^2}-\tan^{-1}\cancel{3^2}\\ \vdots\\ \tan^{-1}(n+1)^2-\tan^{-1}\cancel{(n-1)}^2\end{array}\right.=\tan^{-1}(n+1)^2+\tan^{-1}n^2-\left(\underline{\tan^{-1}0^2}+\tan^{-1}1^2\right)$$

$$=\tan^{-1}(n+1)^2+\tan^{-1}n^2-\cancel{0}-\frac{\pi}{4}$$

Nishant Jindal

Q $\cot^{-1} 2 + \cot^{-1} 8 + \cot^{-1} 18 + \cot^{-1} 32 + \dots$ n terms

$\frac{1}{8} = \frac{1}{2\cdot 2^2}$

$\frac{1}{18} = \frac{1}{2\cdot 3^2}$, $\frac{1}{32} \Rightarrow \frac{1}{2\cdot 4^2}$

yaad

$\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{8} + \tan^{-1}\frac{1}{18} + \tan^{-1}\frac{1}{32} + \dots$ n terms

$\boxed{\text{Master step}}$

$$\sum_{n=1}^{n} \tan^{-1}\left(\frac{1}{2\cdot n^2}\right) \xRightarrow[\text{Turning Pt}]{} \sum \tan^{-1}\left(\frac{2}{4n^2}\right) = \sum \tan^{-1}\left(\frac{2}{1+(4n^2-1)}\right)$$

$$= \sum \tan^{-1}\left(\frac{(2n+1)-(2n-1)}{1+(2n-1)(2n+1)}\right) = \sum_{n=1}^{n} \tan^{-1}(2n+1) - \tan^{-1}(2n-1)$$

$$= \left\{ \begin{array}{l} \cancel{\tan^{-1}(3)} - \tan^{-1} 1 \\ + \cancel{\tan^{-1}(5)} - \cancel{\tan^{-1}(3)} \\ + \cancel{\tan^{-1}(7)} - \cancel{\tan^{-1}(5)} \\ + \tan^{-1} 9 - \tan^{-1} 7 \\ + \tan^{-1}(2n+1) - \cancel{\tan^{-1}(2n-1)} \end{array} \right. = \tan^{-1}(2n+1) - \tan^{-1} 1$$

$$= \tan^{-1}(2n+1) - \frac{\pi}{4}$$

Nishant Jindal

T_3

Q $\tan^{-1}\left(\frac{2}{2+1^2+1^4}\right)+\tan^{-1}\left(\frac{4}{2+2^2+2^4}\right)+\tan^{-1}\left(\frac{6}{2+3^2+3^4}\right)+\cdots n$ terms

Shanti

$$\sum \tan^{-1}\left(\frac{2n}{2+n^2+n^4}\right)=\sum \tan^{-1}\left(\frac{2n}{1+(n^4+n^2+1)}\right)=\sum \tan^{-1}\left(\frac{(n^2+n+1)-(n^2-n+1)}{1+(n^2+n+1)(n^2-n+1)}\right)$$

$$=\sum \tan^{-1}(n^2+n+1)-\tan^{-1}(n^2-n+1))$$

$$=\tan^{-1}(\cancel{3})-\tan^{-1}(1)$$
$$+\tan^{-1}(7)-\cancel{\tan^{-1}(3)}$$

$$\tan^{-1}(n^2+n+1)-\frac{\pi}{4}$$

Nishant Jindal

Q $\tan^{-1}\frac{1}{3}+\tan^{-1}\frac{2}{9}+\dots+\boxed{\tan^{-1}\left(\frac{2^{n-1}}{1+2^{2n-1}}\right)} \rightarrow n^{th}$ term

Diya hai

$$\sum \tan^{-1}\left(\frac{2^{n-1}}{1+2^{2n-1}}\right)=\sum \tan^{-1}\left(\frac{2^{n}-2^{n-1}}{1+2^{n}\cdot 2^{n-1}}\right)$$

$$=\sum \tan^{-1}(2^{n})-\tan^{-1}(2^{n-1})$$ DY

$2^{n}\cdot 2^{n-1}=2^{n+n-1}=2^{2n-1}$

$2^{n}-2^{n-1}=2^{n-1}(2-1)$

$=2^{n-1}$