

ENERGY CONSERVATION

- ① No non-conservative force.
- ② Defined initial & final state

$$U_i + K.E_i = U_f + K.E_f$$

- ③ Choose as reference potential as zero potential line.

If above zero potential line $U = +mgh$

If below zero potential line $U = -mgh$



Block is released from rest, Find maximum compression in the Spring.

$$U_i^o + K \cdot E_i^o = U_f + K \cdot E_f$$

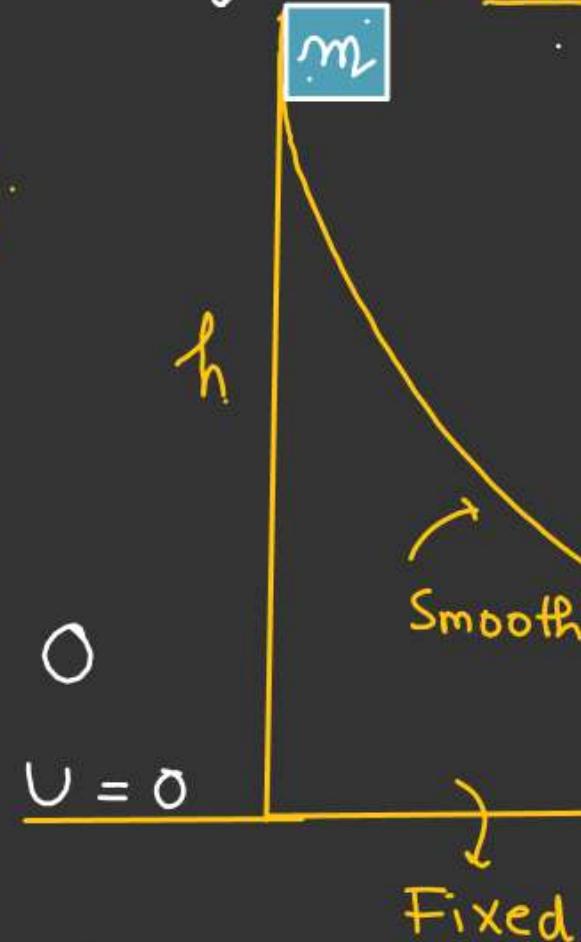
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$$mgh + 0 = [U_{\text{block}} + U_{\text{spring}}] + 0$$

$$mgh = 0 + \frac{1}{2} K x_{\max}^2$$

$$x_{\max} = \sqrt{\frac{2mgh}{K}}$$

Initial State → Released

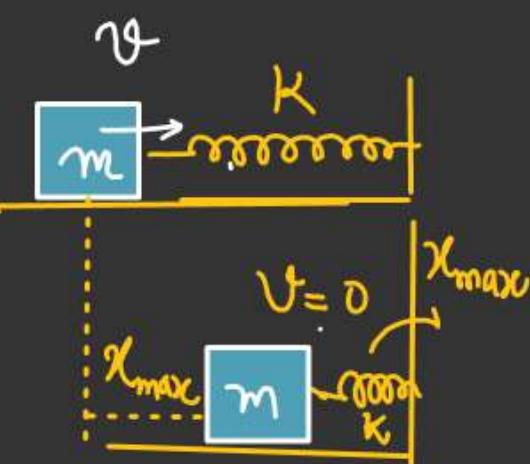


$$W_{mg} + W_N + W_{\text{spring force}} = \Delta K \cdot E$$

$$W_{mg} = -\Delta U = U_i^o - U_f$$

$$= mgh - 0$$

$$mgh + 0 - \frac{1}{2} K x_{\max}^2 = 0$$



Final State

$$\begin{aligned} W_{\text{spring force}} &= (-\Delta U) \\ &= U_i^o - U_f \\ &= 0 - \frac{1}{2} K x_{\max}^2 \\ &= -\frac{1}{2} K x_{\max}^2 \end{aligned}$$

⑥ Find velocity of block if compression in the Spring is half of its maximum compression

$$U_i + K \cdot E_i = \underline{U_f + K \cdot E_f}$$

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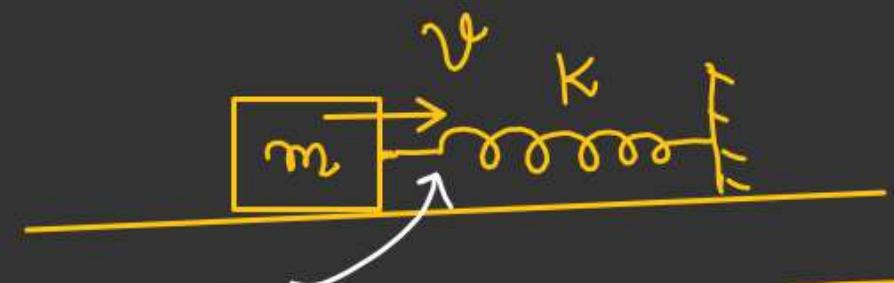
$$mgh + 0 = \frac{1}{2}Kx^2 + \frac{1}{2}mv^2$$

~~$$mgh = \frac{1}{2}K \times \frac{1}{4} \times \left(\frac{2mgh}{K} \right) + \frac{1}{2}mv^2$$~~

$$mgh - \frac{mgh}{4} = \frac{1}{2}mv^2$$

~~$$\frac{3}{4}mgh = \frac{1}{2}mv^2$$~~

$$v = \sqrt{\frac{3gh}{2}}$$



$$x = \frac{x_{\max}}{2} = \frac{1}{2} \sqrt{\frac{2mgh}{K}}$$

(compression in Spring
(given))

#

Block is released from rest
Find the value of h for
maximum range.

Energy Conservation from A to B.

$$U_i^o + K \cdot E_i^o = U_f + K \cdot E_f$$

$$mgH + 0 = mgh + \frac{1}{2}mv^2 \quad H$$

$$mg(H-h) = \frac{1}{2}mv^2$$

$$v = \sqrt{2g(H-h)}$$

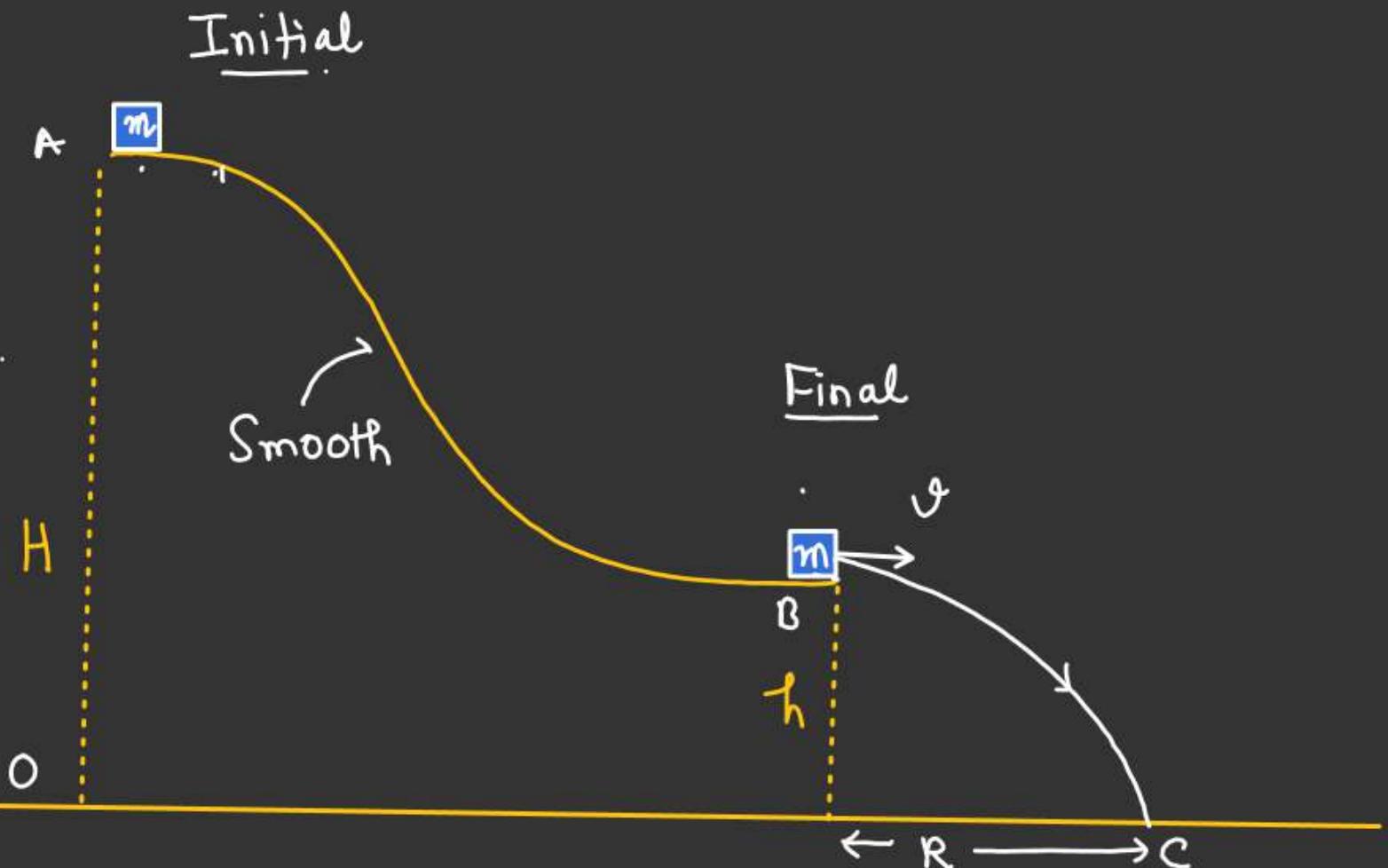
$$t_{BC} = \sqrt{\frac{2h}{g}}$$

$$h = \frac{1}{2}gt_{BC}^2$$

$$R = t_{BC} \cdot v$$

$$R = \sqrt{\frac{2h}{g}} \sqrt{2g(H-h)}$$

$$R = 2\sqrt{h(H-h)}$$



$$R = 2 \sqrt{h(H-h)}.$$

For R to be maximum, or minimum.

$$\frac{dR}{dh} = 0$$

$$\frac{H-2h}{2 \sqrt{Hh-h^2}} = 0$$

$$2 \frac{d}{dh} \left(\sqrt{Hh-h^2} \right) = 0$$

$$h = \frac{H}{2} \quad \underline{\text{Ans}}$$

put $Hh-h^2 = t$.

$$\frac{d \sqrt{t}}{dh} = \frac{d \sqrt{t}}{dt} \times \frac{dt}{dh}$$

$$= \frac{1}{2\sqrt{t}} (H-2h)$$

$$= \left(\frac{H-2h}{2\sqrt{Hh-h^2}} \right)$$

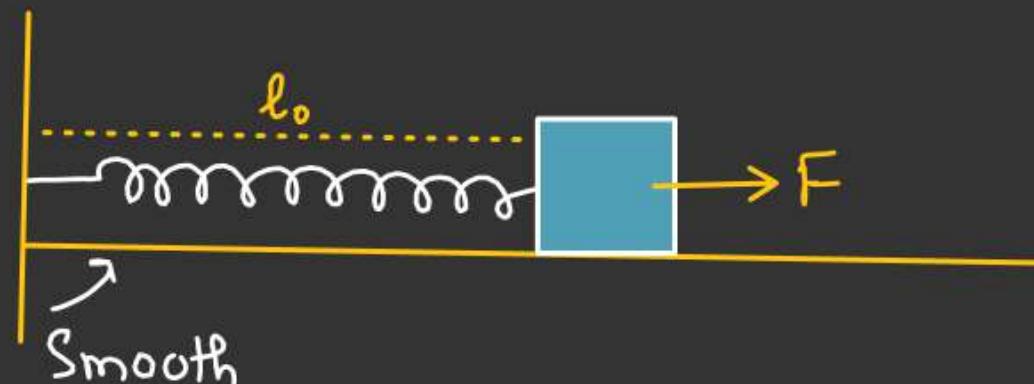
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l_0 = Natural length of the Spring

K = Spring Constant.

Block pulled by a constant force F . When Spring attains natural length.

- ① Find x_{\max} i.e. Maximum elongation in the spring.
- ② Find v_{\max} of the block.



M-1

By Force Concept
At Equilibrium.

$$F = kx$$

$$x = \left(-\frac{F}{k} \right)$$

$$x_{max} = 2x = \left(\frac{2F}{k} \right)$$

Work-Energy theorem.

$$\underline{W_F} + W_{spring \text{ force}} + \underline{W_N} + \underline{W_{mg}} = \Delta K \cdot E$$

For Maximum elongation block must be at rest at that moment

$$\Delta K \cdot E = K \cdot E_f - K \cdot E_i \\ = 0$$

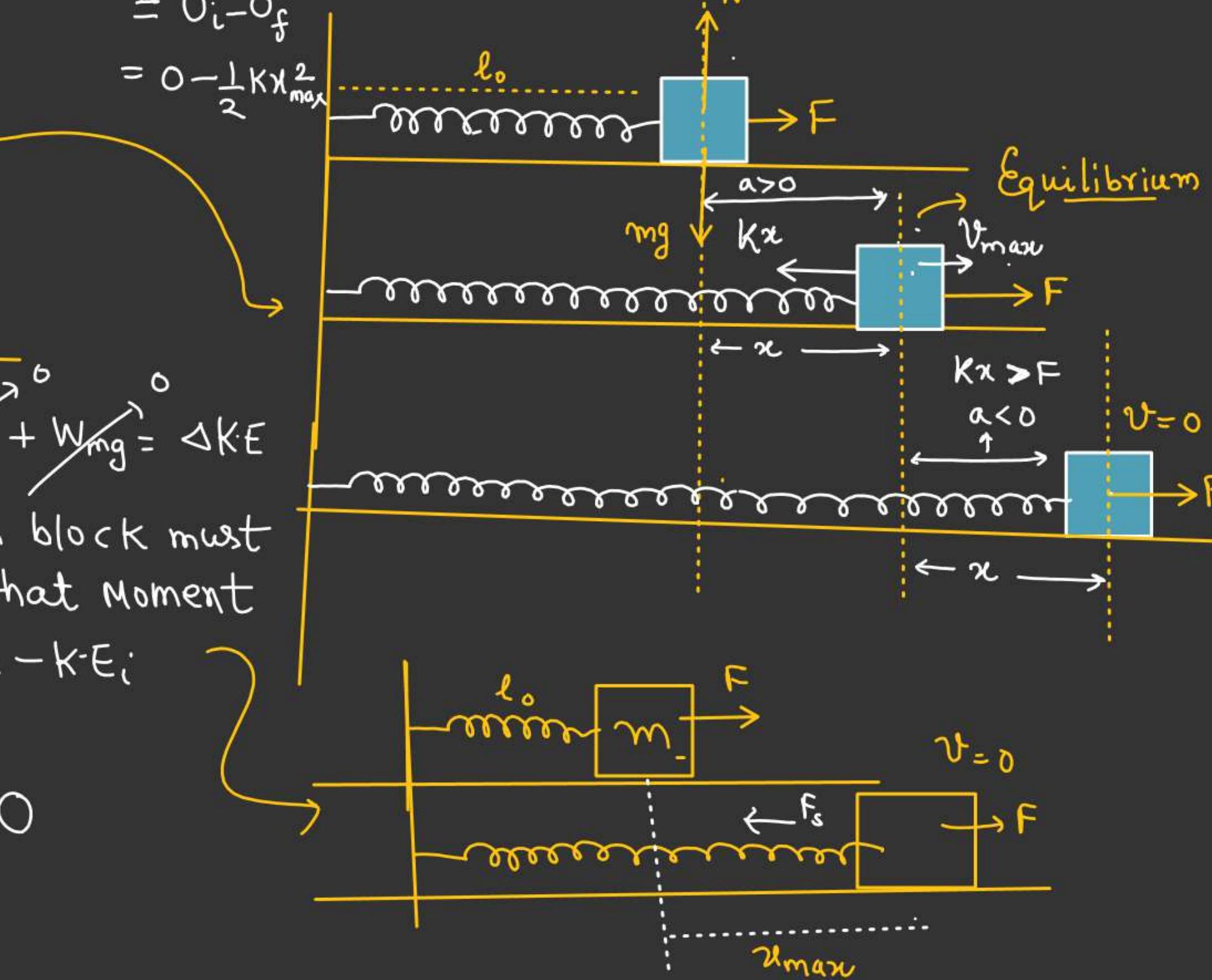
$$F x_{max} - \frac{1}{2} k x_{max}^2 = 0$$

$$x_{max} = \left(\frac{2F}{k} \right)$$

$$W_{spring} = -\Delta U$$

$$= U_i - U_f$$

$$= 0 - \frac{1}{2} k x_{max}^2$$



For $v_{max} = ??$

v_{max} always at Equilibrium position.

At $x = \left(\frac{F}{K}\right)$ Equilibrium.

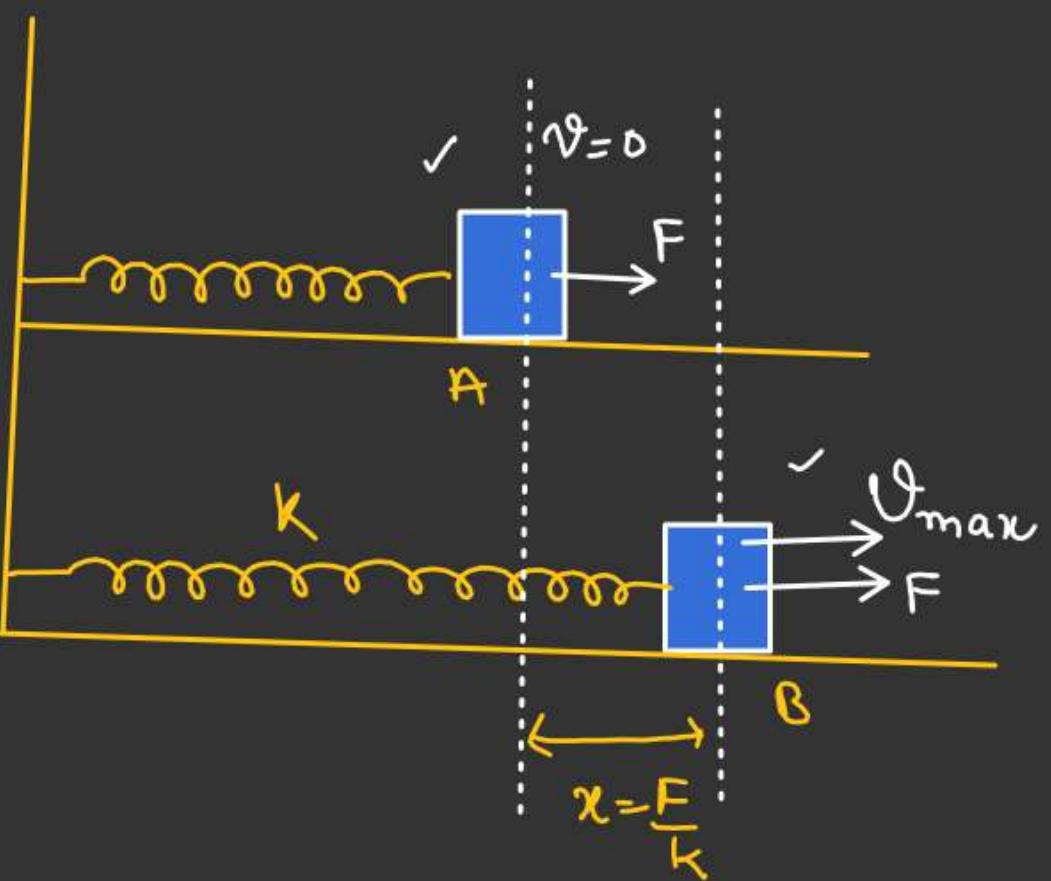
Work-Energy theorem from A to B.

$$W_F + W_{spring} = \Delta KE$$

$$F \cdot \left(\frac{F}{K}\right) - \frac{1}{2} K \left(\frac{F}{K}\right)^2 = \frac{1}{2} m v_{max}^2 - 0$$

$$\frac{F^2}{K} - \frac{F^2}{2K} = \frac{1}{2} m v_{max}^2$$

$$\frac{1}{2} m v_{max}^2 = \frac{F^2}{2K} \Rightarrow \left[v_{max} = \frac{F}{\sqrt{mK}} \right] \checkmark$$



~~★~~ Block is pulled by constant force F when spring at its natural length.

Find a) $x_{\max} = ? \Rightarrow$ Maximum elongation in the Spring.
 b) $v_{\max} = ?$

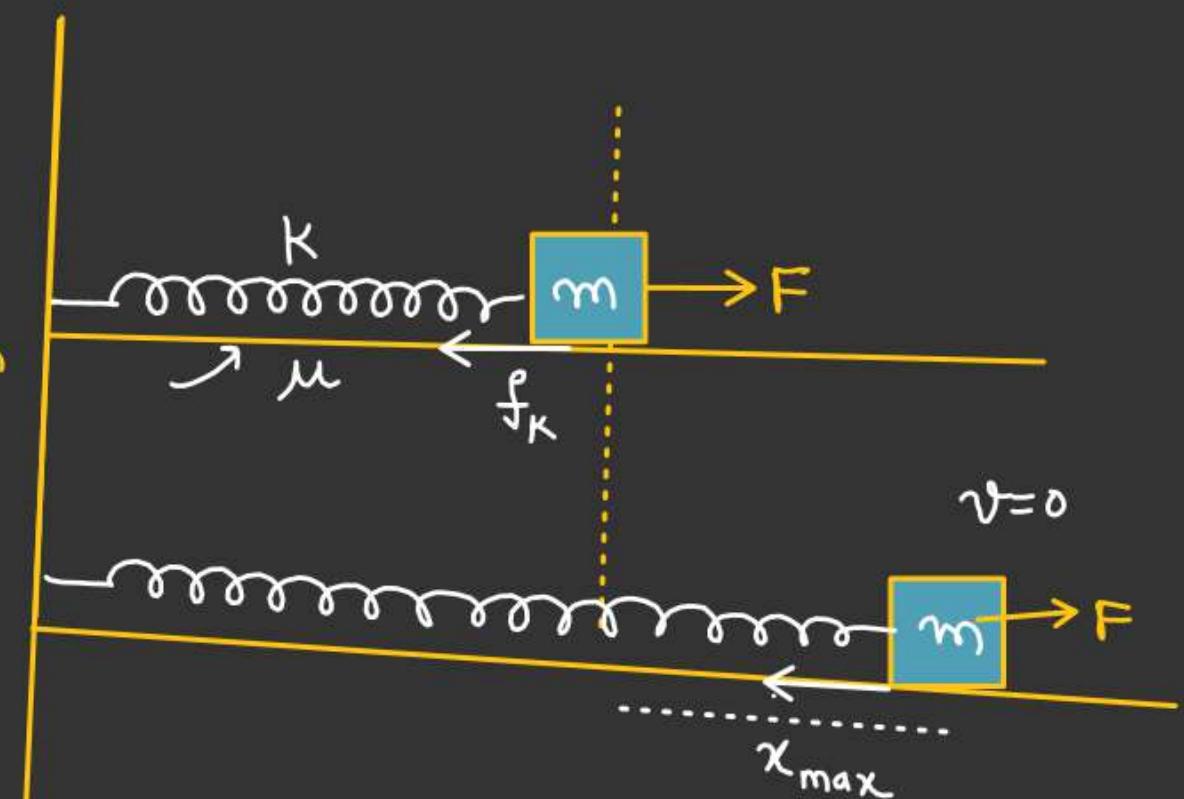
By Work-Energy theorem

$$W_F + W_{f_K} + W_{\text{Spring}} = \Delta K.E$$

$$F \cdot x_{\max} - \mu mg \cdot x_{\max} - \frac{1}{2} K x_{\max}^2 = 0$$

$$F - \mu mg = \frac{1}{2} K x_{\max}$$

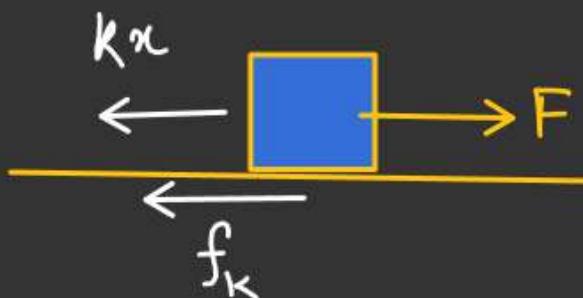
$$x_{\max} = \left[\frac{2(F - \mu mg)}{K} \right] \underline{\text{Ans}}$$



$$\begin{aligned} W_{\text{Spring}} &= -\Delta U \\ &= U_i - U_f \\ &= 0 - \frac{1}{2} K x_{\max}^2 \end{aligned}$$

For v_{\max}

At Equilibrium



$$F = Kx + f_k$$

$$F = Kx + \mu mg$$

$$\frac{F - \mu mg}{K} = \frac{x}{\text{II}}$$

elongation at
Equilibrium.

Work-Energy theorem

$$W_F + W_{\text{Spring}} + W_{f_K} = \Delta K.E$$

$$F \cdot x - \frac{1}{2} Kx^2 - \mu mg x = \frac{1}{2} m v_{\max}^2 - 0$$

$$F \left(\frac{F - \mu mg}{K} \right) - \frac{1}{2} K \frac{(F - \mu mg)^2}{K^2} - \mu mg \left(\frac{F - \mu mg}{K} \right)$$

$$\left(\frac{F - \mu mg}{K} \right) \left[F - \left(\frac{F - \mu mg}{2} \right) - \mu mg \right] = \frac{1}{2} m v_{\max}^2$$

$$\frac{(F - \mu mg)^2}{2K} = \frac{1}{2} m v_{\max}^2 \Rightarrow v_{\max} = \sqrt{\frac{(F - \mu mg)^2}{mK}}$$

H.W. H.C. Verma

Q. No (1 to 21)

Q. No (31 to 37)

Q. No (40 to 43)

Q. No (47, 48, 51)