Nishant Jindal

# VERTICAL CIRCULAR MOTION

**Case-3 :-** $\boxed{\sqrt{2gR} < u < \sqrt{5gR}}$

↳ Bob doesn't complete the Vertical Circular Motion.

At any θ angle string become Slack $(T=0)$.

$\theta = ??$

Energy Conservation

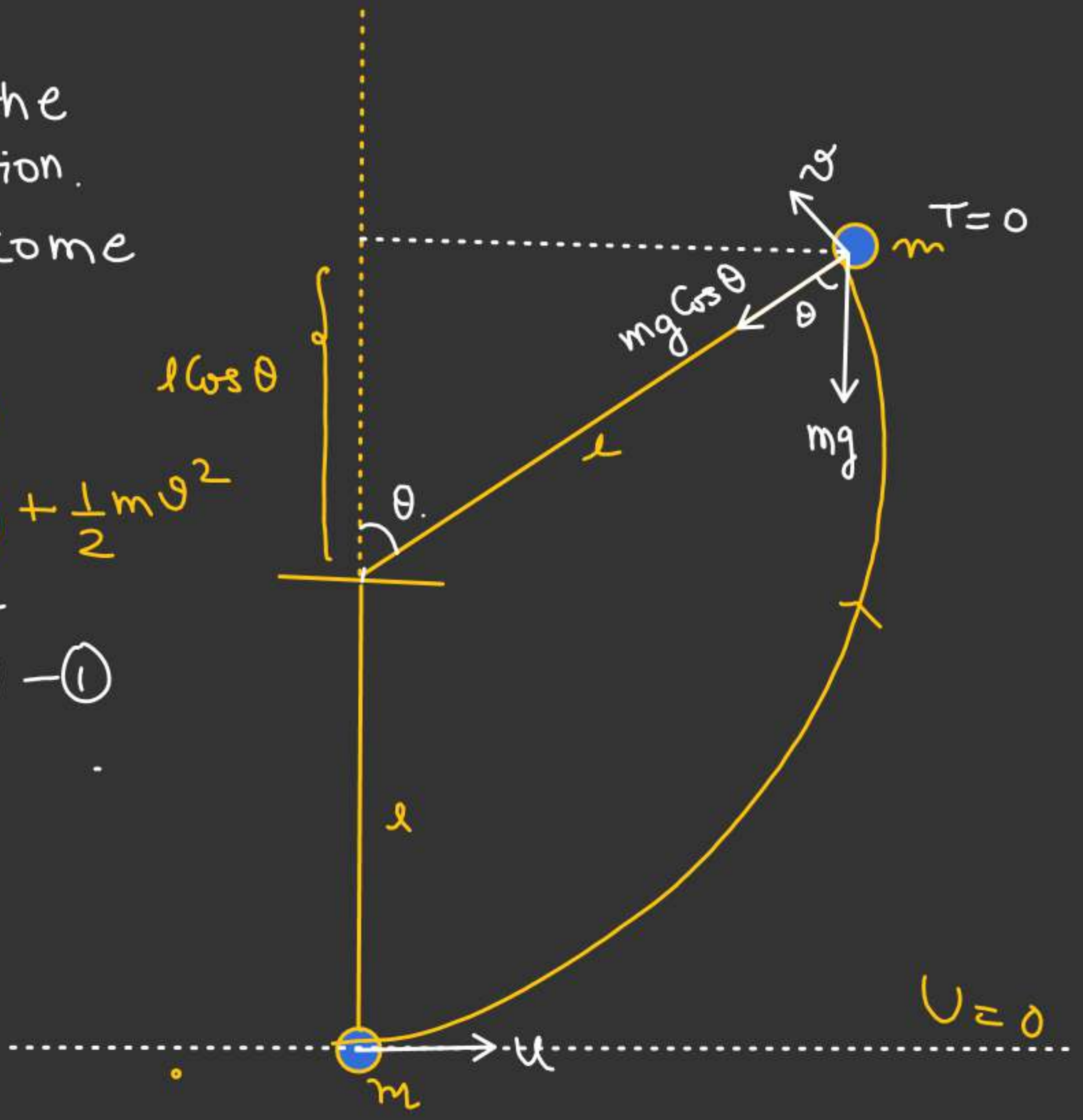
$$\frac{1}{2}mu^2 = mgl(1+\cos\theta) + \frac{1}{2}mv^2$$

$$v = \sqrt{u^2 - 2gl(1+\cos\theta)} \quad \text{—(1)}$$

If string slag at θ, $T=0$.

$$mg\cos\theta = \frac{mv^2}{l}$$

$$gl\cos\theta = v^2 \quad \text{—(2)}$$



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# VERTICAL CIRCULAR MOTION

$$v = \sqrt{u^2 - 2gl(1+\cos\theta)} \quad \text{—(1)}$$

If string slag at $\theta$, $T=0$.

$$mg\cos\theta = \frac{mv^2}{l}$$

$$gl\cos\theta = v^2 \quad \text{—(2)}$$

$$gl\cos\theta = u^2 - 2gl(1+\cos\theta)$$

$$u^2 = 2gl + 3gl\cos\theta. \Rightarrow 3gl\cos\theta = u^2 - 2gl.$$

$$u = \sqrt{gl(2+3\cos\theta)}$$

$$\cos\theta = \left(\frac{u^2-2gl}{3gl}\right)$$

$$\theta = \cos^{-1}\left(\frac{u^2-2gl}{3gl}\right)$$

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# VERTICAL CIRCULAR MOTION

※ If string slag at an angle $\theta$ such that bob passes through point of suspension. for this find $\theta = ??$

**Sol$^n$**

$$u = \sqrt{g\ell\,(2+3\cos\theta)}, \quad v = \sqrt{g\ell\cos\theta}$$

In x-direction

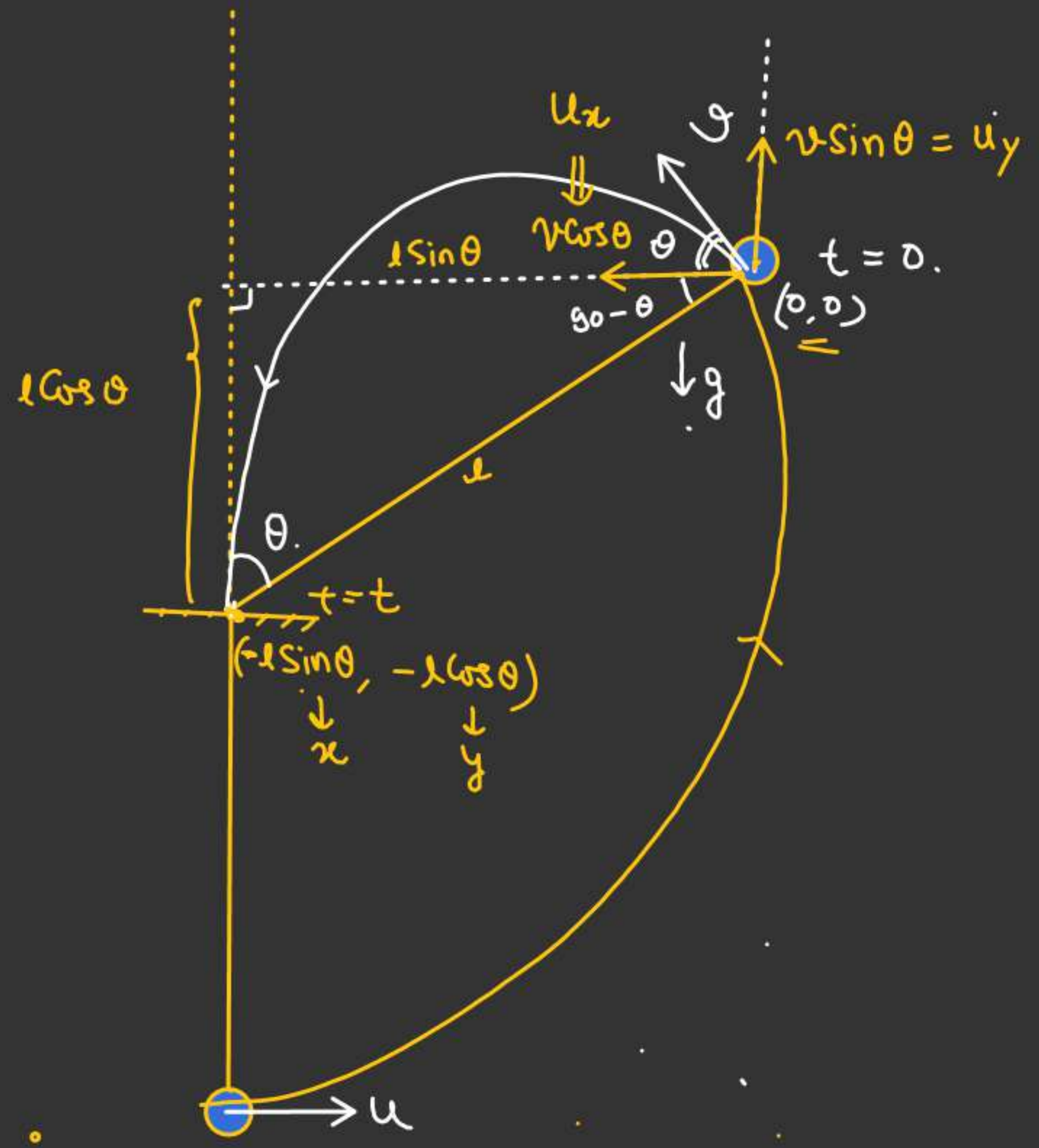
$$-\ell\sin\theta = -v\cos\theta \times t$$

$$t = \left(\frac{\ell\sin\theta}{v\cos\theta}\right) \quad \text{---} (1)$$

In y-direction

$$-\ell\cos\theta = u_y t - \frac{1}{2}gt^2$$

$$-\ell\cos\theta = (v\sin\theta)t - \frac{1}{2}gt^2 \quad \text{---} (2)$$



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In x-direction

$-l\sin\theta = -v\cos\theta \times t$

$t = \left(\frac{l\sin\theta}{v\cos\theta}\right)$ — 1

In y-direction

$-l\cos\theta = u_y t - \frac{1}{2}gt^2$

$-l\cos\theta = (v\sin\theta)t - \frac{1}{2}gt^2$ — 2

$v = \sqrt{gl\cos\theta}$ — 3

$-l\cos\theta = (\cancel{v}\sin\theta)\left(\frac{l\sin\theta}{\cancel{v}\cos\theta}\right) - \frac{1}{2}g\left(\frac{l\sin\theta}{v\cos\theta}\right)^2$

$-l\cos\theta = \frac{l\sin^2\theta}{\cos\theta} - \frac{\cancel{g}\cancel{l^2}\sin^2\theta}{2\cos^2\theta\,(\cancel{gl}\cos\theta)}$ $(v^2 = gl\cos\theta)$

$-\cos\theta = \left[\frac{\sin^2\theta}{\cos\theta} - \frac{\sin^2\theta}{2\cos^3\theta}\right] = \frac{1}{\cos\theta}\left[\sin^2\theta - \frac{\sin^2\theta}{2\cos^2\theta}\right]$

$-\cos^2\theta = \sin^2\theta - \frac{\tan^2\theta}{2}$ ✓

$\frac{\tan^2\theta}{2} = \sin^2\theta + \cos^2\theta$

$\tan^2\theta = 2$

$\tan\theta = \sqrt{2}$

$\boxed{\theta = \tan^{-1}(\sqrt{2})\ \text{Ans}}$

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# VERTICAL CIRCULAR MOTION

Block to reach at C

For $v \to v_{min} \to N = 0$.

For $u \to u_{min}$, $v \to v_{min}$.

$$\frac{1}{2}mu^2 = mg2R + \frac{1}{2}mv^2 \quad \text{— (1)}$$

$$Mg = \frac{mv^2}{R} \quad \text{— (2)}$$

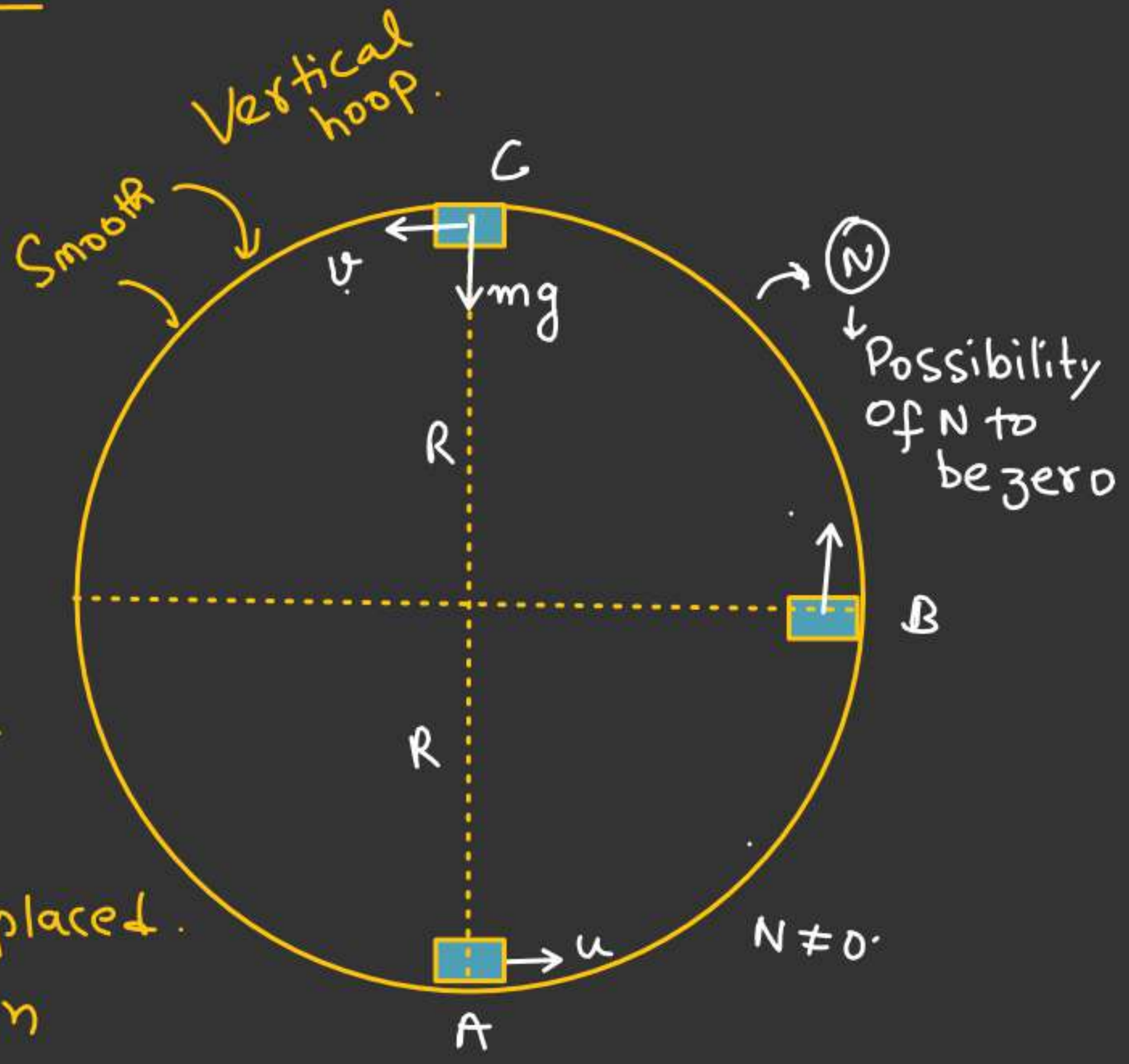
$$\boxed{u = \sqrt{5gR}}$$

Block to reach at B

$$\boxed{v = \sqrt{2gR}}$$

Same as string bob system.

Tension in string replaced by Normal reaction



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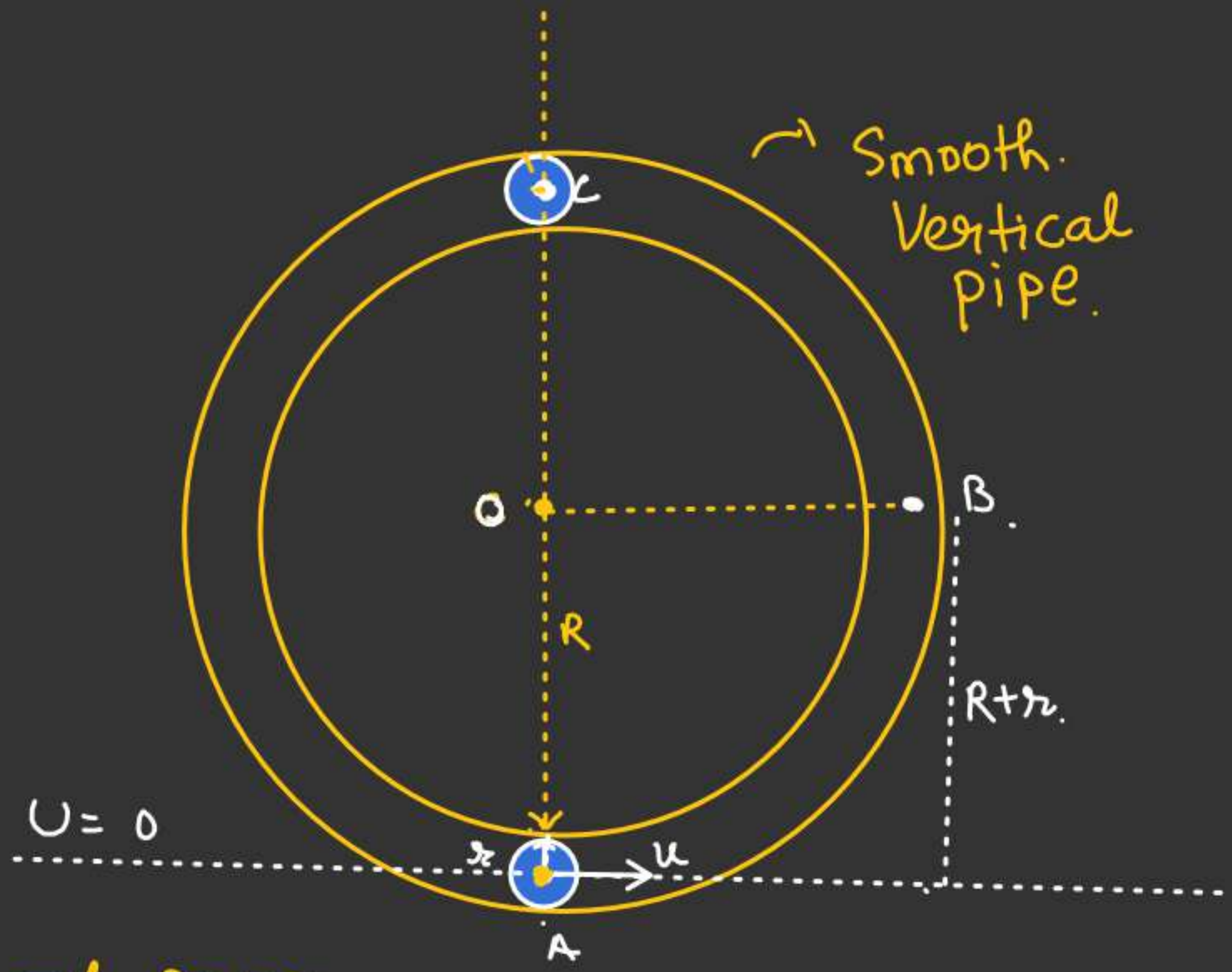
# VERTICAL CIRCULAR MOTION

$r$ = radius of ball.

$R$ = Inner Radius of pipe.




$u_{min}$ to reach at B

For $u_{min}$, $v_B = 0$

$$\frac{1}{2} m u_{min}^2 = mg(R+r)$$

$$u_{min} = \sqrt{2g(R+r)}$$

If $\underline{r << R}$, $\left(u_{min} = \sqrt{2gR}\right)$

At C

For $u_{min}$, $v$ at c will be zero

Energy Conservation from A to C.

$$\frac{1}{2} m u_{min}^2 = mg\,2(R+r)$$

$$\boxed{u_{min} = \sqrt{4g(R+r)}}$$

If $\underline{R >> r}$

$\left(u_{min} = \sqrt{4gR}\right)$ ✓

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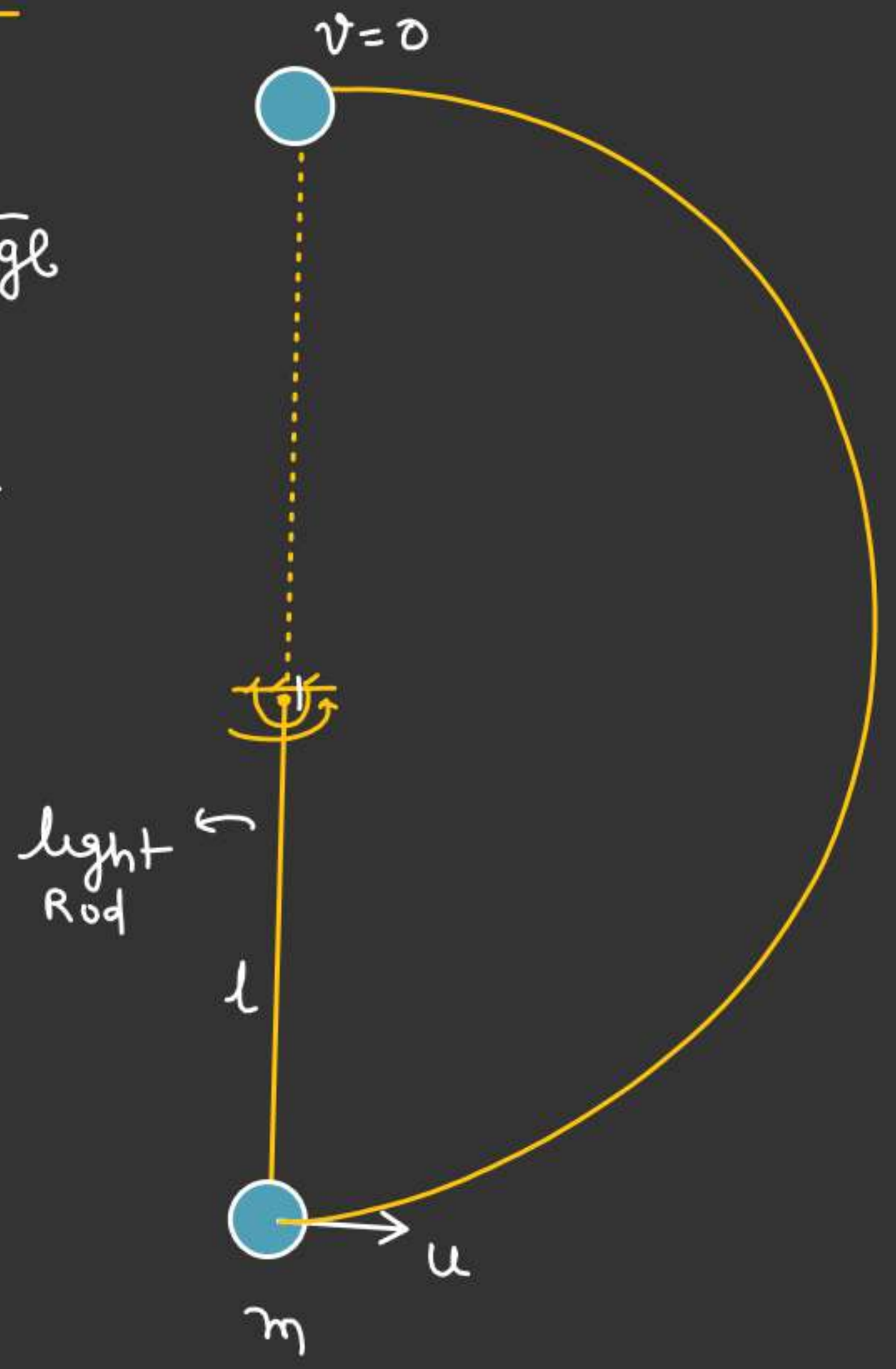
# VERTICAL CIRCULAR MOTION

Find $u_{min}$

a) For ball to reach at quater Circle. $= \sqrt{2gl}$

b) For ball to Complete the Vertical Circle.

$= \sqrt{4gl}$.



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# VERTICAL CIRCULAR MOTION

\# $U_{min}$ So that bob Complete the Vertical Circle.


F.B.D w.r.t NIF.

Work-Energy theorem.

$$W_{gravity} + W_{pseudo} = \Delta K.E$$

$$-mg(2l) - ma(2l) = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$$

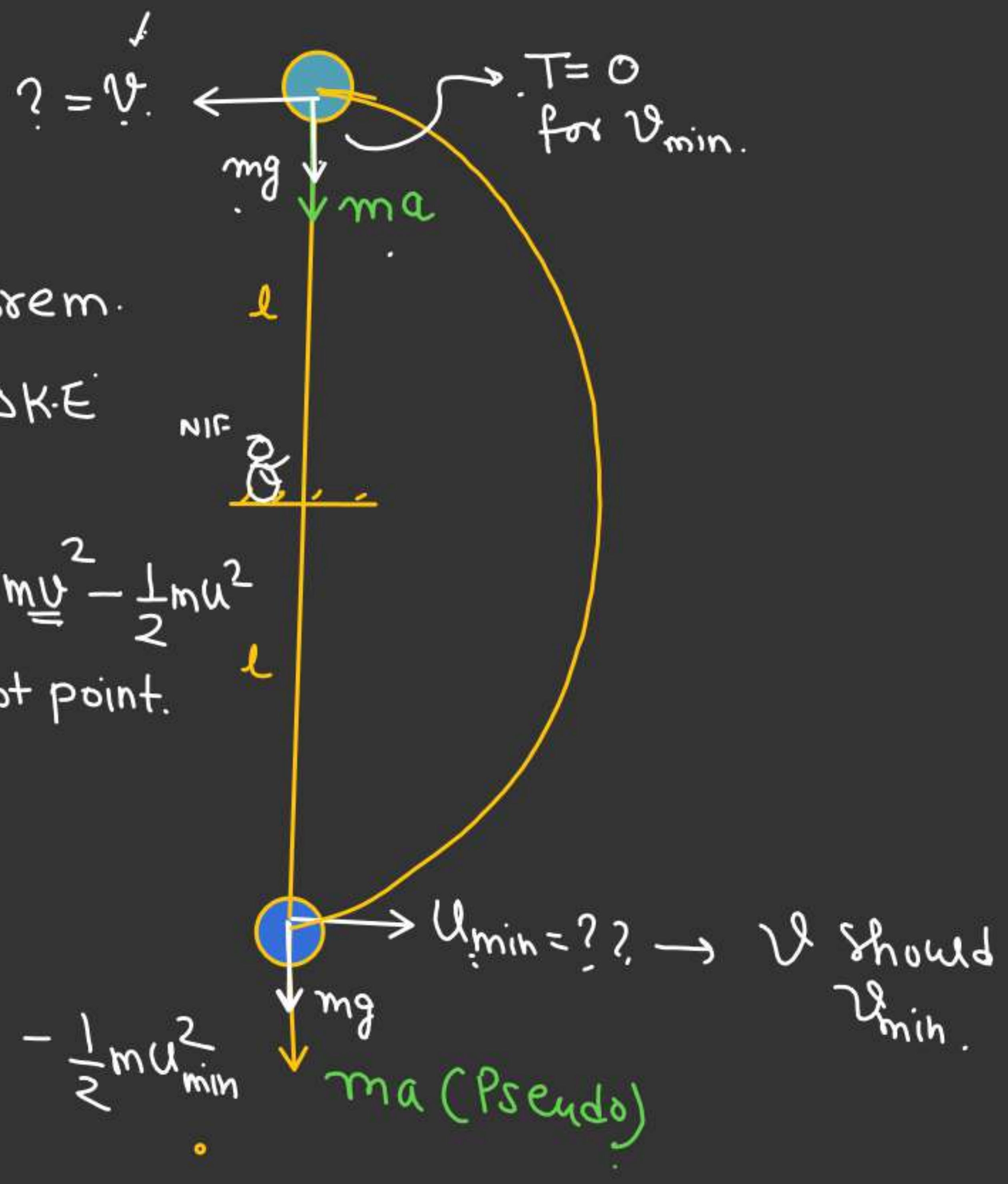
Net Centripetal at the highest point.

$$m(g+a) = \frac{mv^2}{l}$$

$$mv^2 = ml(g+a)$$

$$-2ml(g+a) = \frac{ml(g+a)}{2} - \frac{1}{2}mu_{min}^2$$

$$\boxed{U_{min} = \sqrt{5(g+a)l}}$$



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# VERTICAL CIRCULAR MOTION

## ON INCLINED PLANE

**TRICK**

Valid when.

$(a \uparrow\uparrow g)$ or $(a \uparrow\downarrow g)$

$\downarrow$ $\qquad\qquad$ $\downarrow$

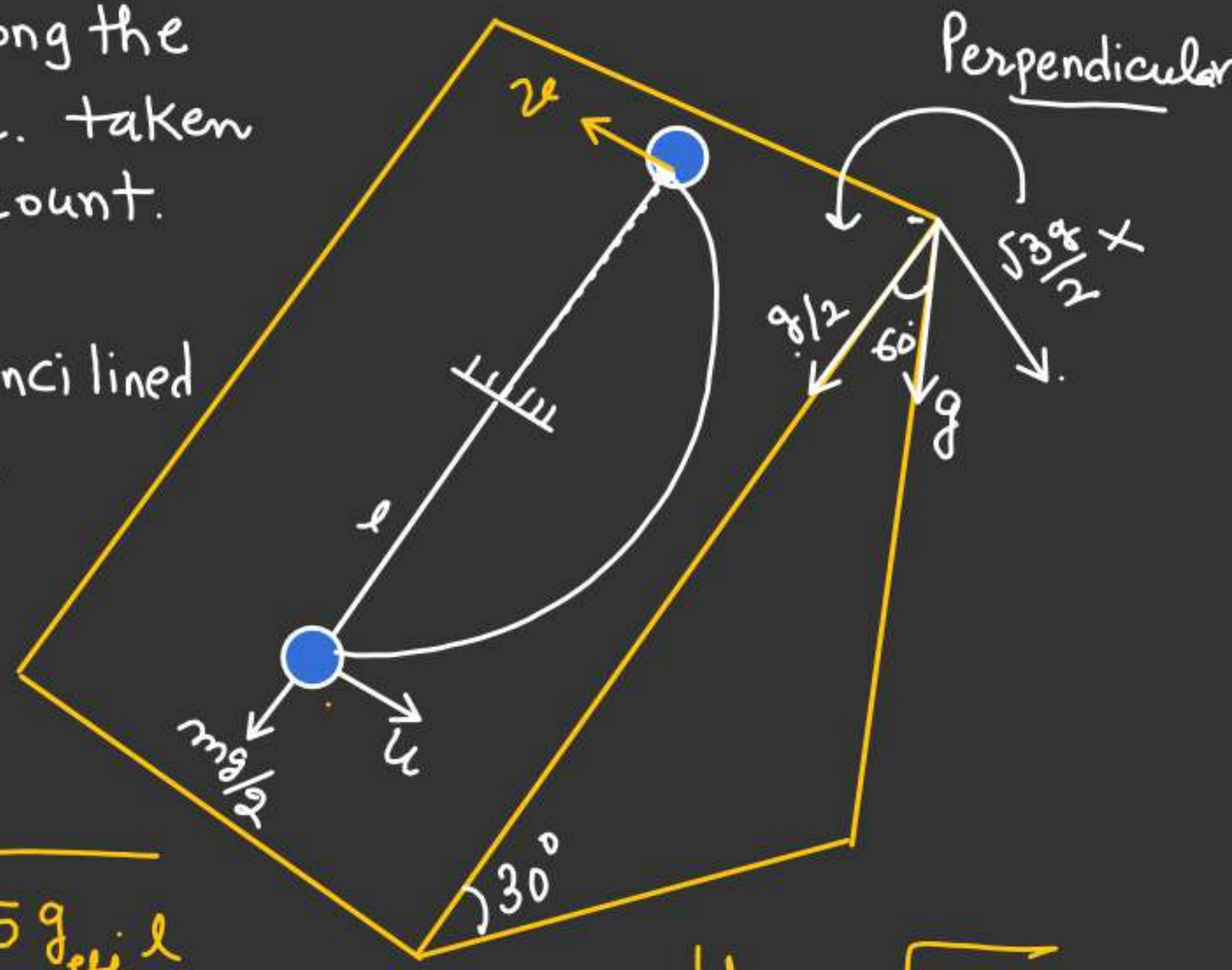
$g_{eff} = (g+a)$ $\qquad\qquad$ $(g-a) = g_{eff}$.

$U_{min} = \sqrt{2 g_{eff} \cdot l}$ (For quater circle)

$U_{min} = \sqrt{5 g_{eff} \cdot l}$ (To Complete the Circle)

$(W_{mg})$ along the Inclined. taken into account.

$(W_{mg})_{\perp}$ to incilined plane = 0.




$U_{min} = \sqrt{5 g_{eff} \cdot l}$

$g_{eff}$ = along the inclined plane.

To Complete the circle.

$U_{min} = \sqrt{2 g_{eff} \cdot l}$

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# VERTICAL CIRCULAR MOTION

H.W.

Find Min. u

① To Comple the Vertical Circle.

String.