KEY CONCEPTS

1. The symbol $\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$ is called the determinant of order two. Its value is given by :

$D = a_1b_2 - a_2b_1$

2. The symbol $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ is called the determinant of order three.

Its value can be found as : $D = a_1\begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} - a_2\begin{vmatrix} b_1 & c_1 \\ b_3 & c_3 \end{vmatrix} + a_3\begin{vmatrix} b_1 & c_1 \\ b_2 & c_2 \end{vmatrix}$ OR

$$D = a_1\begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} - b_1\begin{vmatrix} a_2 & c_2 \\ a_3 & c_3 \end{vmatrix} + c_1\begin{vmatrix} a_2 & b_2 \\ a_3 & b_3 \end{vmatrix} \ldots\ldots \text{ and so on .}$$

In this manner we can expand a determinant in 6 ways using elements of ; R_1, R_2, R_3 or C_1, C_2, C_3.

3. Following examples of short hand writing large expressions are:

(i) The lines :

$$a_1x + b_1y + c_1 = 0 \ldots\ldots (1)$$
$$a_2x + b_2y + c_2 = 0 \ldots\ldots (2)$$
$$a_3x + b_3y + c_3 = 0 \ldots\ldots (3)$$

are concurrent if, $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0.$

Condition for the consistency of three simultaneous linear equations in 2 variables.

(ii) $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represents a pair of straight lines if :

$abc + 2fgh - af^2 - bg^2 - ch^2 = 0 = \begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix}$

(iii) Area of a triangle whose vertices are (x_r, y_r); $r = 1,2,3$ is :

$D = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$ If $D = 0$ then the three points are collinear.

(iv) Equation of a straight line passing through (x_1, y_1)&(x_2, y_2) is $\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0$

4. **MINORS :**

The minor of a given element of a determinant is the determinant of the elements which remain after deleting the row & the column in which the given element stands. For example, the minor of a_1 in (Key Concept 2)

is $\begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix}$ & the minor of b_2 is $\begin{vmatrix} a_1 & c_1 \\ a_3 & c_3 \end{vmatrix}$.

Hence a determinant of order two will have " 4 minors" & a determinant of order three will have "9 minors".

5. COFACTOR :

If M_{ij} represents the minor of some typical element then the cofactor is defined as :

$C_{ij} = (-1)^{i+j}.M_{ij}$; Where i & j denotes the row & column in which the particular element lies.

Note that the value of a determinant of order three in terms of 'Minor' & 'Cofactor' can be written as :

$D = a_{11}M_{11} - a_{12}M_{12} + a_{13}M_{13}$ or $D = a_{11}C_{11} + a_{12}C_{12} + a_{13}C_{13}$ & so on ....

6. PROPERTIES OF DETERMINANTS :

P-1: The value of a determinant remains unaltered, if the rows & columns are inter changed.

e.g. if $D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = D'$

D & D' are transpose of each other . If $D' = -D$ then it is **Skew Symmetric** determinant but

$D' = D \Rightarrow 2D = 0 \Rightarrow D = 0 \Rightarrow$ Skew symmetric determinant of third order has the value zero.

P-2: If any two rows (or columns) of a determinant be interchanged , the value of determinant is changed in sign only. e.g.

Let $D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ & $D' = \begin{vmatrix} a_2 & b_2 & c_2 \\ a_1 & b_1 & c_1 \\ a_3 & b_3 & c_3 \end{vmatrix}$ Then $D' = -D$.

P-3: If a determinant has any two rows (or columns) identical , then its value is zero.

e.g. Let $D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_1 & b_1 & c_1 \\ a_3 & b_3 & c_3 \end{vmatrix}$ then it can be verified that $D = 0$.

P-4: If all the elements of any row (or column) be multiplied by the same number, then the determinant is multiplied by that number.

e.g. If $D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ and $D' = \begin{vmatrix} Ka_1 & Kb_1 & Kc_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ Then $D' = KD$

P-5: If each element of any row (or column) can be expressed as a sum of two terms then the determinant can be expressed as the sum of two determinants. e.g.

$$\begin{vmatrix} a_1 + x & b_1 + y & c_1 + z \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} + \begin{vmatrix} x & y & z \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

P- 6: The value of a determinant is not altered by adding to the elements of any row (or column) the same multiples of the corresponding elements of any other row

(or column). e.g. Let $D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ and

$$D' = \begin{vmatrix} a_1 + ma_2 & b_1 + m\,b_2 & c_1 + mc_2 \\ a_2 & b_2 & c_2 \\ a_3 + na_2 & b_3 + n\,b_2 & c_3 + nc_2 \end{vmatrix}. \text{ Then } D' = D$$

Note : that while applying this property **ATLEAST ONE ROW (OR COLUMN)** must remain unchanged .

P- 7: If by putting $x = a$ the value of a determinant vanishes then $(x - a)$ is a factor of the determinant.

7. MULTIPLICATION OF TWO DETERMINANTS :

(i) $$\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} \times \begin{vmatrix} l_1 & m_1 \\ l_2 & m_2 \end{vmatrix} = \begin{vmatrix} a_1l_1 + b_1l_2 & a_1m_1 + b_1m_2 \\ a_2l_1 + b_2l_2 & a_2m_1 + b_2m_2 \end{vmatrix}$$

Similarly two determinants of order three are multiplied.

(ii) If $D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \neq 0$ then , $D^2 = \begin{vmatrix} A_1 & B_1 & C_1 \\ A_2 & B_2 & C_2 \\ A_3 & B_3 & C_3 \end{vmatrix}$ where A_i, B_i, C_i are cofactors

Proof : Consider $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \times \begin{vmatrix} A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \\ C_1 & C_2 & C_3 \end{vmatrix} = \begin{vmatrix} D & 0 & 0 \\ 0 & D & 0 \\ 0 & 0 & D \end{vmatrix}$

Note: $a_1A_2 + b_1B_2 + c_1C_2 = 0$ etc.

Therefore , D x $\begin{vmatrix} A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \\ C_1 & C_2 & C_3 \end{vmatrix} = D^3 \Rightarrow \begin{vmatrix} A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \\ C_1 & C_2 & C_3 \end{vmatrix} = D^2$ or $\begin{vmatrix} A_1 & B_1 & C_1 \\ A_2 & B_2 & C_2 \\ A_3 & B_3 & C_3 \end{vmatrix} = D^2$

8. SYSTEM OF LINEAR EQUATION (IN TWO VARIABLES) :

(i) Consistent Equations : Definite & unique solution . [intersecting lines]

(ii) Inconsistent Equation : No solution . [Parallel line]

(iii) Dependent equation : Infinite solutions . [Identical lines]

Let $a_1x + b_1y + c_1 = 0$ & $a_2x + b_2y + c_2 = 0$ then :

$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2} \Rightarrow$ Given equations are inconsistent

& $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \Rightarrow$ Given equations are dependent

9. CRAMER'S RULE : [Simultaneous Equations Involving Three Unknowns]

Let , $a_1x + b_1y + c_1z = d_1$ ... (I); $a_2x + b_2y + c_2z = d_2$ ... (II) ; $a_3x + b_3y + c_3z = d_3$ ... (III)

Then, $x = \frac{D_1}{D}, Y = \frac{D_2}{D}, Z = \frac{D_3}{D}$.

Where $D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}; D_1 = \begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix}; D_2 = \begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix}$ & $D_3 = \begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix}$

NOTE :

(a) If $D \neq 0$ and alteast one of $D_1, D_2, D_3 \neq 0$, then the given system of equations are consistent and have unique non trivial solution .

(b) If $D \neq 0$ & $D_1 = D_2 = D_3 = 0$, then the given system of equations are consistent and have trivial solution only .

(c) If $D = D_1 = D_2 = D_3 = 0$, then the given system of equations are consistent and have infinite solutions.

In case $\left.\begin{matrix} a_1x + b_1y + c_1z = d_1 \\ a_2x + b_2y + c_2z = d_2 \\ a_3x + b_3y + c_3z = d_3 \end{matrix}\right\}$ represents these parallel planes then also

$D = D_1 = D_2 = D_3 = 0$ but the system is inconsistent.

(d) If $D = 0$ but atleast one of D_1, D_2, D_3 is not zero then the equations are inconsistent and have no solution.

If x, y, z are not all zero, the condition for $a_1x + b_1y + c_1z = 0$; $a_2x + b_2y + c_2z = 0$ &

$a_3x + b_3y + c_3z = 0$ to be consistent in x, y, z is that $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0.$

Remember that if a given system of linear equations have **Only Zero** Solution for all its variables then the given equations are said to have **TRIVIAL SOLUTION.**

PROFICIENCYTEST-01

1. $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} =$

2. $\begin{vmatrix} 1 & 5 & \pi \\ \log_e e & 5 & \sqrt{5} \\ \log_{10} 10 & 5 & e \end{vmatrix} =$

3. $\begin{vmatrix} 19 & 17 & 15 \\ 9 & 8 & 7 \\ 1 & 1 & 1 \end{vmatrix} =$

4. The value of the determinant $\begin{vmatrix} 4 & -6 & 1 \\ -1 & -1 & 1 \\ -4 & 11 & -1 \end{vmatrix}$ is :

5. The value of the determinant $\begin{vmatrix} 31 & 37 & 92 \\ 31 & 58 & 71 \\ 31 & 105 & 24 \end{vmatrix}$ is :

6. $\begin{vmatrix} a-b & b-c & c-a \\ x-y & y-z & z-x \\ p-q & q-r & r-p \end{vmatrix} =$

7. $\begin{vmatrix} 1 & a & a^2-bc \\ 1 & b & b^2-ac \\ 1 & c & c^2-ab \end{vmatrix} =$

8. $\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+x & 1 \\ 1 & 1 & 1+y \end{vmatrix} =$

9. The roots of the equation $\begin{vmatrix} 1 & 4 & 20 \\ 1 & -2 & 5 \\ 1 & 2x & 5x^2 \end{vmatrix} = 0$ are

10. If $a \neq b \neq c$, the value of x (independent of a, b, c) which satisfies the equation

$\begin{vmatrix} 0 & x-a & x-b \\ x+a & 0 & x-c \\ x+b & x+c & 0 \end{vmatrix} = 0$, is:

11. If $a+b+c=0$, then the solution of the equation $\begin{vmatrix} a-x & c & b \\ c & b-x & a \\ b & a & c-x \end{vmatrix} = 0$ is :

12. If $\begin{vmatrix} x+1 & 3 & 5 \\ 2 & x+2 & 5 \\ 2 & 3 & x+4 \end{vmatrix} = 0$, then x =

13. $\begin{vmatrix} 1 & a & b \\ -a & 1 & c \\ -b & -c & 1 \end{vmatrix} =$

14. $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix} =$

15. $\begin{vmatrix} 0 & a & -b \\ -a & 0 & c \\ b & -c & 0 \end{vmatrix} =$

PROFICIENCY TEST-02

1. $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} =$

2. $\begin{vmatrix} a+b & a+2b & a+3b \\ a+2b & a+3b & a+4b \\ a+4b & a+5b & a+6b \end{vmatrix} =$

3. $\begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix} =$

4. The roots of the equation $\begin{vmatrix} 1+x & 1 & 1 \\ 1 & 1+x & 1 \\ 1 & 1 & 1+x \end{vmatrix} = 0$ are

5. One of the roots of the given equation $\begin{vmatrix} x+a & b & c \\ b & x+c & a \\ c & a & x+b \end{vmatrix} = 0$ is :

(A) $a+b+c$
(B) $-(a+b+c)$
(C) $a^2+b^2+c^2$
(D) $-(a^2+b^2+c^2)$

6. $\begin{vmatrix} x+1 & x+2 & x+4 \\ x+3 & x+5 & x+8 \\ x+7 & x+10 & x+14 \end{vmatrix} =$

7. $\begin{vmatrix} 1/a & a^2 & bc \\ 1/b & b^2 & ca \\ 1/c & c^2 & ab \end{vmatrix} =$

8. $\begin{vmatrix} b^2+c^2 & a^2 & a^2 \\ b^2 & c^2+a^2 & b^2 \\ c^2 & c^2 & a^2+b^2 \end{vmatrix} =$

9. $\begin{vmatrix} 1+x & 1 & 1 \\ 1 & 1+y & 1 \\ 1 & 1 & 1+z \end{vmatrix} =$

10. If $\begin{vmatrix} y+z & x & y \\ z+x & z & x \\ x+y & y & z \end{vmatrix} = k(x+y+z)(x-z)^2$, then $k =$

11. If -9 is a root of the equation $\begin{vmatrix} x & 3 & 7 \\ 2 & x & 2 \\ 7 & 6 & x \end{vmatrix} = 0$ then the other two roots are:

12. If a, b, c are unequal what is the condition that the value of the following determinant is zero

$\Delta = \begin{vmatrix} a & a^2 & a^3+1 \\ b & b^2 & b^3+1 \\ c & c^2 & c^3+1 \end{vmatrix}$

13. The value of the determinant $\begin{vmatrix} 1 & a & b+c \\ 1 & b & c+a \\ 1 & c & a+b \end{vmatrix}$ is :

14. If a, b and c are non zero numbers , then $\Delta = \begin{vmatrix} b^2c^2 & bc & b+c \\ c^2a^2 & ca & c+a \\ a^2b^2 & ab & a+b \end{vmatrix}$ is equal to :

15. If $\begin{vmatrix} 1 & k & 3 \\ 3 & k & -2 \\ 2 & 3 & -1 \end{vmatrix} = 0$, then the value of k is :

PROFICIENCY TEST-03

1. If $\Delta = \begin{vmatrix} a & b & c \\ x & y & z \\ p & q & r \end{vmatrix}$, then $\begin{vmatrix} ka & kb & kc \\ kx & ky & kz \\ kp & kq & kr \end{vmatrix} =$

2. If $\begin{vmatrix} x^2+x & x+1 & x-2 \\ 2x^2+3x-1 & 3x & 3x-3 \\ x^2+2x+3 & 2x-1 & 2x-1 \end{vmatrix} = Ax - 12$, then the value of A is :

3. The roots of the equation $\begin{vmatrix} 3-x & -6 & 3 \\ -6 & 3-x & 3 \\ 3 & 3 & -6-x \end{vmatrix} = 0$ are :

4. $\begin{vmatrix} \sin^2 x & \cos^2 x & 1 \\ \cos^2 x & \sin^2 x & 1 \\ -10 & 12 & 2 \end{vmatrix} =$

5. If $D_P = \begin{vmatrix} p & 15 & 8 \\ p^2 & 35 & 9 \\ p^3 & 25 & 10 \end{vmatrix}$, then $D_1 + D_2 + D_3 + D_4 + D_5 =$

6. If $\begin{vmatrix} 4 & 1 \\ 2 & 1 \end{vmatrix}^2 = \begin{vmatrix} 3 & 2 \\ 1 & x \end{vmatrix} - \begin{vmatrix} x & 3 \\ -2 & 1 \end{vmatrix}$, then x =

7. If a, b, c are in A.P., then the value of $\begin{vmatrix} x+2 & x+3 & x+a \\ x+4 & x+5 & x+b \\ x+6 & x+7 & x+c \end{vmatrix}$ is :

8. If $\Delta = \begin{vmatrix} x & y & z \\ p & q & r \\ a & b & c \end{vmatrix}$, then $\begin{vmatrix} x & 2y & z \\ 2p & 4q & 2r \\ a & 2b & c \end{vmatrix}$ equals

9. If $\begin{vmatrix} a & b & c \\ m & n & p \\ x & y & z \end{vmatrix} = k$, then $\begin{vmatrix} 6a & 2b & 2c \\ 3m & n & p \\ 3x & y & z \end{vmatrix} =$

10. If $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 5$; then the value of $\begin{vmatrix} b_2c_3 - b_3c_2 & c_2a_3 - c_3a_2 & a_2b_3 - a_3b_2 \\ b_3c_1 - b_1c_3 & c_3a_1 - c_1a_3 & a_3b_1 - a_1b_3 \\ b_1c_2 - b_2c_1 & c_1a_2 - c_2a_1 & a_1b_2 - a_2b_1 \end{vmatrix}$ is:

11. If $\begin{vmatrix} (b+c)^2 & a^2 & a^2 \\ b^2 & (c+a)^2 & b^2 \\ c^2 & c^2 & (a+b)^2 \end{vmatrix} = kabc(a+b+c)^3$, then the value of k is :

12. If A, B, C be the angles of a triangle, then $\begin{vmatrix} -1 & \cos C & \cos B \\ \cos C & -1 & \cos A \\ \cos B & \cos A & -1 \end{vmatrix} =$

13. $\begin{vmatrix} (a^x + a^{-x})^2 & (a^x - a^{-x})^2 & 1 \\ (b^x + b^{-x})^2 & (b^x - b^{-x})^2 & 1 \\ (c^x + c^{-x})^2 & (c^x - c^{-x})^2 & 1 \end{vmatrix} =$

14. The value of the determinant $\begin{vmatrix} 1 & \cos(\alpha - \beta) & \cos\alpha \\ \cos(\alpha - \beta) & 1 & \cos\beta \\ \cos\alpha & \cos\beta & 1 \end{vmatrix}$ is :

15. If $\begin{vmatrix} y+z & x-z & x-y \\ y-z & z+x & y-x \\ z-y & z-x & x+y \end{vmatrix} = kxyz$, then the value of k is :

EXERCISE–I

1. (a) Prove that the value of the determinant $\begin{vmatrix} -7 & 5+3i & \frac{2}{3}-4i \\ 5-3i & 8 & 4+5i \\ \frac{2}{3}+4i & 4-5i & 9 \end{vmatrix}$ is real.

(b) Prove that the value of the determinant $\begin{vmatrix} 1 & a & a^2-bc \\ 1 & b & b^2-ca \\ 1 & c & c^2-ab \end{vmatrix} = 0$

(c) On which one of the parameter out of a, p, d or x, the value of the determinant

$\begin{vmatrix} 1 & a & a^2 \\ \cos(p-d)x & \cos px & \cos(p+d)x \\ \sin(p-d)x & \sin px & \sin(p+d)x \end{vmatrix}$ does not depend

2. Without expanding as far as possible, prove that

(a) $\begin{vmatrix} a^2+2a & 2a+1 & 1 \\ 2a+1 & a+2 & 1 \\ 3 & 3 & 1 \end{vmatrix} = (a-1)^3$, (b) $\begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ x^3 & y^3 & z^3 \end{vmatrix} = [(x-y)(y-z)(z-x)(x+y+z)]$

3. If $\begin{vmatrix} x^3+1 & x^2 & x \\ y^3+1 & y^2 & y \\ z^3+1 & z^2 & z \end{vmatrix} = 0$ and x , y, z are all different then , prove that $xyz = -1$.

4. Using properties of determinants or otherwise evaluate $\begin{vmatrix} 18 & 40 & 89 \\ 40 & 89 & 198 \\ 89 & 198 & 440 \end{vmatrix}$.

5. Prove that $\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^3$.

6. If $D = \begin{vmatrix} a & b & c \\ c & a & b \\ b & c & a \end{vmatrix}$ and $D' = \begin{vmatrix} b+c & c+a & a+b \\ a+b & b+c & c+a \\ c+a & a+b & b+c \end{vmatrix}$ then prove that $D' = 2D$.

7. Prove that $\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} = (1+a^2+b^2)^3$.

8. Prove that $\begin{vmatrix} a & b-c & c+b \\ a+c & b & c-a \\ a-b & b+a & c \end{vmatrix} = (a+b+c)(a^2+b^2+c^2)$.

9. Show that the value of the determinant $\begin{vmatrix} \tan(A+P) & \tan(B+P) & \tan(C+P) \\ \tan(A+Q) & \tan(B+Q) & \tan(C+Q) \\ \tan(A+R) & \tan(B+R) & \tan(C+R) \end{vmatrix}$ vanishes for all values of A, B, C, P, Q & R where $A+B+C+P+Q+R = 0$

10. Factorise the determinant $\begin{vmatrix} bc & bc'+b'c & b'c' \\ ca & ca'+c'a & c'a' \\ ab & ab'+a'b & a'b' \end{vmatrix}$.

11. Prove that $\begin{vmatrix} (\beta+\gamma-\alpha-\delta)^4 & (\beta+\gamma-\alpha-\delta)^2 & 1 \\ (\gamma+\alpha-\beta-\delta)^4 & (\gamma+\alpha-\beta-\delta)^2 & 1 \\ (\alpha+\beta-\gamma-\delta)^4 & (\alpha+\beta-\gamma-\delta)^2 & 1 \end{vmatrix} = -64(\alpha-\beta)(\alpha-\gamma)(\alpha-\delta)(\beta-\gamma)(\beta-\delta)(\gamma-\delta)$

12. For a fixed positive integer n, if $D = \begin{vmatrix} n! & (n+1)! & (n+2)! \\ (n+1)! & (n+2)! & (n+3)! \\ (n+2)! & (n+3)! & (n+4)! \end{vmatrix}$ then show that $\left[\frac{D}{(n!)^3} - 4\right]$ is divisible by n.

13. Solve for x $\begin{vmatrix} x+2 & 2x+3 & 3x+4 \\ 2x+3 & 3x+4 & 4x+5 \\ 3x+5 & 5x+8 & 10x+17 \end{vmatrix} = 0$

14. If $p+q+r=0$, prove that $\begin{vmatrix} pa & qb & rc \\ qc & ra & pb \\ rb & pc & qa \end{vmatrix} = pqr \begin{vmatrix} a & b & c \\ c & a & b \\ b & c & a \end{vmatrix}$.

15. If a, b, c are all different & $\begin{vmatrix} a & a^3 & a^4-1 \\ b & b^3 & b^4-1 \\ c & c^3 & c^4-1 \end{vmatrix} = 0$, then prove that, $abc(ab+bc+ca) = a+b+c$.

16. Show that, $\begin{vmatrix} a^2+\lambda & ab & ac \\ ab & b^2+\lambda & bc \\ ac & bc & c^2+\lambda \end{vmatrix}$ is divisible by λ^2 and find the other factor.

17. (a) Without expanding prove that $\begin{vmatrix} bc & a & a^2 \\ ca & b & b^2 \\ ab & c & c^2 \end{vmatrix} = \begin{vmatrix} 1 & a^2 & a^3 \\ 1 & b^2 & b^3 \\ 1 & c^2 & c^3 \end{vmatrix}$.

(b) $\begin{vmatrix} a^2 & b^2 & c^2 \\ (a+1)^2 & (b+1)^2 & (c+1)^2 \\ (a-1)^2 & (b-1)^2 & (c-1)^2 \end{vmatrix} = 4 \begin{vmatrix} a^2 & b^2 & c^2 \\ a & b & c \\ 1 & 1 & 1 \end{vmatrix}$.

18. Solve for x : $\begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ x-4 & 2x-9 & 3x-16 \\ x-8 & 2x-27 & 3x-64 \end{vmatrix} = 0$

19. If $D_r = \begin{vmatrix} 2^{r-1} & 2(3^{r-1}) & 4(5^{r-1}) \\ x & y & z \\ 2^n-1 & 3^n-1 & 5^n-1 \end{vmatrix}$ then prove that $\sum_{r=1}^{n} D_r = 0$

20. In a $\triangle$ ABC, determine condition under which $\begin{vmatrix} \cot\frac{A}{2} & \cot\frac{B}{2} & \cot\frac{C}{2} \\ \tan\frac{B}{2}+\tan\frac{C}{2} & \tan\frac{C}{2}+\tan\frac{A}{2} & \tan\frac{A}{2}+\tan\frac{B}{2} \\ 1 & 1 & 1 \end{vmatrix} = 0$

21. Prove that: $\begin{vmatrix} (a-p)^2 & (a-q)^2 & (a-r)^2 \\ (b-p)^2 & (b-q)^2 & (b-r)^2 \\ (c-p)^2 & (c-q)^2 & (c-r)^2 \end{vmatrix} = \begin{vmatrix} (1+ap)^2 & (1+aq)^2 & (1+ar)^2 \\ (1+bp)^2 & (1+bq)^2 & (1+br)^2 \\ (1+cp)^2 & (1+cq)^2 & (1+cr)^2 \end{vmatrix}$

22. Prove that $\begin{vmatrix} (a_1-b_1)^2 & (a_1-b_2)^2 & (a_1-b_3)^2 \\ (a_2-b_1)^2 & (a_2-b_2)^2 & (a_2-b_3)^2 \\ (a_3-b_1)^2 & (a_3-b_2)^2 & (a_3-b_3)^2 \end{vmatrix} = 2(a_1-a_2)(a_2-a_3)(a_3-a_1)(b_1-b_2)(b_2-b_3)(b_3-b_1)$

23. If $ax_1^2+by_1^2+cz_1^2 = ax_2^2+by_2^2+cz_2^2 = ax_3^2+by_3^2+cz_3^2 = d$

and $ax_2x_3+by_2y_3+cz_2z_3 = ax_3x_1+by_3y_1+cz_3z_1 = ax_1x_2+by_1y_2+cz_1z_2 = f$,

then prove that $\begin{vmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{vmatrix} = (d-f)\left[\frac{d+2f}{abc}\right]^{1/2}$ $(a,b,c \neq 0)$

24. If $S_r = \alpha^r + \beta^r + \gamma^r$ then show that $\begin{vmatrix} S_0 & S_1 & S_2 \\ S_1 & S_2 & S_3 \\ S_2 & S_3 & S_4 \end{vmatrix} = (\alpha - \beta)^2(\beta - \gamma)^2(\gamma - \alpha)^2$.

25. If $u = ax^2 + 2bxy + cy^2, u' = a'x^2 + 2b'xy + c'y^2$. Prove that

$$\begin{vmatrix} y^2 & -xy & x^2 \\ a & b & c \\ a' & b' & c' \end{vmatrix} = \begin{vmatrix} ax + by & bx + cy \\ a'x + b'y & b'x + c'y \end{vmatrix} = -\frac{1}{y}\begin{vmatrix} u & u' \\ ax + by & a'x + b'y \end{vmatrix}.$$

EXERCISE–II

1. Solve using Cramer's rule : $\frac{4}{x+5}+\frac{3}{y+7}=-1$ & $\frac{6}{x+5}-\frac{6}{y+7}=-5$.

2. Solve the following using Cramer's rule and state whether consistent or not.

(a) $x+y+z-6=0$

$2x+y-z-1=0$

$x+y-2z+3=0$

(b) $7x-7y+5z=3$

$3x+y+5z=7$

$2x+3y+5z=5$

3. Solve the system of equations ; $\left.\begin{aligned} z+ay+a^2x+a^3 &= 0\\ z+by+b^2x+b^3 &= 0\\ z+cy+c^2x+c^3 &= 0\end{aligned}\right]$

4. For what value of K do the following system of equations possess a non trivial (i.e. not all zero) solution over the set of rationals Q ?

$x+Ky+3z=0$, $3x+Ky-2z=0$, $2x+3y-4z=0$

For that value of K , find all the solutions of the system.

5. Given $x=cy+bz$; $y=az+cx$; $z=bx+ay$ where x , y , z are not all zero , prove that $a^2+b^2+c^2+2abc=1$

6. Given $a=\frac{x}{y-z}$; $b=\frac{y}{z-x}$; $c=\frac{z}{x-y}$ where x, y, z are not all zero, prove that : $1+ab+bc+ca=0$.

7. If $\sin q \neq \cos q$ and x , y , z satisfy the equations

$x\cos p - y\sin p + z = \cos q + 1$

$x\sin p + y\cos p + z = 1 - \sin q$

$x\cos(p+q) - y\sin(p+q) + z = 2$

then find the value of $x^2+y^2+z^2$.

8. Investigate for what values of λ , μ the simultaneous equations $x+y+z=6$; $x+2y+3z=10$ & $x+2y+\lambda z=\mu$ have ;

(a) A unique solution.

(b) An infinite number of solutions.

(c) No solution.

9. For what values of p , the equations : $x+y+z=1$; $x+2y+4z=p$ & $x+4y+10z=p^2$ have a solution ? Solve them completely in each case.

10. Solve the equations : $Kx+2y-2z=1$, $4x+2Ky-z=2$, $6x+6y+Kz=3$ considering specially the case when $K=2$.

11. Let a, b, c, d are distinct numbers to be chosen from the set $\{1,2,3,4,5\}$. If the least possible positive solution for x to the system of equations $\left.\begin{matrix} ax + by = 1 \\ cx + dy = 2 \end{matrix}\right\}$ can be expressed in the form $\frac{p}{q}$ where p and q are relatively prime, then find the value of $(p + q)$.

12. If $bc + qr = ca + rp = ab + pq = -1$ show that $\begin{vmatrix} ap & a & p \\ bq & b & q \\ cr & c & r \end{vmatrix} = 0$.

13. If the following system of equations $(a - t)x + by + cz = 0$, $bx + (c - t)y + az = 0$ and $cx + ay + (b - t)z = 0$ has non-trivial solutions for different values of t, then show that we can express product of these values of t in the form of determinant.

14. Show that the system of equations

$3x - y + 4z = 3$, $x + 2y - 3z = -2$ and $6x + 5y + \lambda z = -3$

has atleast one solution for any real number λ. Find the set of solutions of $\lambda = -5$.

EXERCISE–III

1. If α, β, γ are the roots of $x^3 + px^2 + q = 0$, where $q \neq 0$, then $\Delta = \begin{bmatrix} 1/\alpha & 1/\beta & 1/\gamma \\ 1/\beta & 1/\gamma & 1/\alpha \\ 1/\gamma & 1/\alpha & 1/\beta \end{bmatrix}$ equals

(A) p/q (B) q/p (C) pq (D) 0

2. If $\Delta = \begin{vmatrix} \sqrt{6} & 2i & 3+\sqrt{6} \\ \sqrt{12} & \sqrt{3}+\sqrt{8}i & 3\sqrt{2}+\sqrt{6}i \\ \sqrt{18} & \sqrt{2}+\sqrt{12}i & \sqrt{27}+2i \end{vmatrix}$ then Δ is $(i^2 = -1)$

(A) a negative integer (B) a natural number

(C) an irrational number (D) an imaginary number

3. If x, y, z are different from zero and

$\Delta = \begin{vmatrix} a & b-y & c-z \\ a-x & b & c-z \\ a-x & b-y & c \end{vmatrix} = 0$, then the value of the expression $\frac{a}{x} + \frac{b}{y} + \frac{c}{z}$ is

(A) 0 (B) -1 (C) 1 (D) 2

4. If $p + q + r = a + b + c = 0$, then the determinant $\Delta = \begin{vmatrix} pa & qb & rc \\ qc & ra & pb \\ rb & pc & qa \end{vmatrix}$ equals

(A) 0 (B) 1 (C) $pa + qb + rc$ (D) abcpqr

5. If $p \neq a, q \neq b, r \neq c$ and the system of equations

$px + ay + az = 0$

$bx + qy + bz = 0$

$cx + cy + rz = 0$

has a non-trivial solution, then the value of $\frac{p}{p-a} + \frac{q}{q-b} + \frac{r}{r-c}$ is

(A) -1 (B) 0 (C) 1 (D) 2

6. If the system of linear equations $x + y + z = 6$, $x + 2y + 3z = 14$ and $2x + 5y + \lambda z = \mu$, $(\lambda, \mu \in R)$ has more than one solution, then

(A) $\lambda \neq 8, \mu \in R$ (B) $\lambda = 8, \mu \neq 36$

(C) $\lambda = 8, \mu = 36$ (D) $\lambda = a, \mu \in R$

7. If a, b, c are positive and not all equal, then the value of the determinant $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$ is always

(A) > 0 (B) 0 (C) < 0 (D) none of these

8. Let $px^4 + qx^3 + rx^2 + sx + t =$

$\begin{vmatrix} x^2+3x & x-1 & x+3 \\ x+1 & -2x & x-4 \\ x-3 & x+4 & 3x \end{vmatrix}$ be an identity, where p, q, r, s, t are constants. Then t =

(A) 0 (B) 1 (C) 2 (D) -1

9. If $\begin{vmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{vmatrix} = \lambda \begin{vmatrix} a^2 & b^2 & c^2 \\ a & b & c \\ 1 & 1 & 1 \end{vmatrix}$, then $\lambda =$

(A) 1 (B) -1 (C) 2 (D) -2

10. The number of real values of λ for which the system of equations $\lambda x + y + z = 0$, $x - \lambda y - z = 0$, $x + y - \lambda z = 0$ will have nontrivial solution is

(A) 0 (B) 1 (C) 2 (D) 3

11. If x, y, z be non-unity positive numbers, then $\begin{vmatrix} 1 & \log_x y & \log_x z \\ \log_y x & 1 & \log_y z \\ \log_z x & \log_z y & 1 \end{vmatrix} =$

(A) 0 (B) 1 (C) -1 (D) value depends on x, y, z

12. $\begin{vmatrix} xp+y & x & y \\ yp+z & y & z \\ 0 & xp+y & yp+z \end{vmatrix} = 0$ if

(A) x, y, z are in A.P (B) x, y, z are in G.P

(C) x, y, z are in H.P (D) xy, yz, zx are in A.P

13. The number of distinct real roots of the equation $\begin{vmatrix} \sin x & \cos x & \cos x \\ \cos x & \sin x & \cos x \\ \cos x & \cos x & \sin x \end{vmatrix} = 0$ in $\left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$ is

(A) 0 (B) 1 (C) 2 (D) 3

14. The value of λ such that the system $x - 2y + z = -4$, $2x - y + 2z = 2$, $x + y + \lambda z = 4$ has infinitely many solutions is

(A) 0 (B) 1 (C) -1 (D) none of these

15. If the equations : $x + ay - z = 0, 2x - y + az = 0, ax + y + 2z = 0$; have nontrivial solutions , then a can't be

(A) 2 (B) -2 (C) $1 + \sqrt{3}$ (D) $1 - \sqrt{3}$

EXERCISE–IV

1. If $a > 0$ and discriminant of $ax^2 + 2bx + c$ is $-$ve, then **[AIEEE 2002]**

$\begin{vmatrix} a & b & ax+b \\ b & c & bx+c \\ ax+b & bx+c & 0 \end{vmatrix}$ is equal to

(A) $+$ ve (B) $(ac - b^2)(ax^2 + 2bx + c)$

(C) $-$ve (D) 0

2. If the system of linear equations **[AIEEE 2003]**

$x + 2ay + az = 0$; $x + 3by + bz = 0$; $x + 4cy + cz = 0$;; has a non-zero solution, then a, b, c.

(A) Satisfy $a + 2b + 3c = 0$ (B) Are in A.P.

(C) Are in G.P. (D) Are in H.P.

3. If $a_1, a_2, a_3, \dots\dots, a_n, \dots\dots$ are in G.P., then the value of the determinant **[AIEEE 2004]**

$\begin{vmatrix} \log a_n & \log a_{n+1} & \log a_{n+2} \\ \log a_{n+3} & \log a_{n+4} & \log a_{n+5} \\ \log a_{n+6} & \log a_{n+7} & \log a_{n+8} \end{vmatrix}$

(A) -2 (B) 1 (C) 2 (D) 0

4. The system of equations **[AIEEE 2005]**

$\alpha x + y + z = \alpha - 1$

$x + \alpha y + z = \alpha - 1$

$x + y + \alpha z = \alpha - 1$

has infinite solutions, if α is

(A) -2 (B) Either -2 or 1 (C) not -2 (D) 1

5. If $a^2 + b^2 + c^2 = -2$ and $f(x) = \begin{vmatrix} 1+a^2x & (1+b^2)x & (1+c^2)x \\ (1+a^2)x & 1+b^2x & (1+c^2)x \\ (1+a^2)x & (1+b^2)x & 1+c^2x \end{vmatrix}$, then f(x) is a polynomial of degree **[AIEEE 2005]**

(A) 1 (B) 0 (C) 3 (D) 2

6. If $a_1, a_2, a_3, \dots\dots\dots, a_n$, are in G.P., then the determinant **[AIEEE 2005]**

$\Delta = \begin{vmatrix} \log a_n & \log a_{n+1} & \log a_{n+2} \\ \log a_{n+3} & \log a_{n+4} & \log a_{n+5} \\ \log a_{n+6} & \log a_{n+7} & \log a_{n+8} \end{vmatrix}$ is equal to

(A) 1 (B) 0 (C) 4 (D) 2

7. If $D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+x & 1 \\ 1 & 1 & 1+y \end{vmatrix}$ for $x \neq 0, y \neq 0$, then D is **[AIEEE 2007]**

(A) Divisible by x but not y (B) Divisible by y but not x

(C) Divisible by neither x nor y (D) Divisible by both x and y

8. Let $A = \begin{vmatrix} 5 & 5\alpha & \alpha \\ 0 & \alpha & 5\alpha \\ 0 & 0 & 5 \end{vmatrix}$. If $|A^2| = 25$, then $|\alpha|$ equals. **[AIEEE 2007]**

(A) 1/5 (B) 5 (C) 5^2 (D) 1

9. Let a, b, c be any real numbers. Suppose that there are real numbers x, y, z not all zero such that $x = cy + bz$, $y = az + cx$, and $z = bx + ay$. Then $a^2 + b^2 + c^2 + 2abc$ is equal to **[AIEEE 2008]**

(A) 2 (B) -1 (C) 0 (D) 1

10. Let a, b, c be such that $b(a + c) \neq 0$ if **[AIEEE 2009]**

$$\begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix} + \begin{vmatrix} a+1 & b+1 & c-1 \\ a-1 & b-1 & c+1 \\ (-1)^{n+2}a & (-1)^{n+1}b & (-1)^{n}c \end{vmatrix} = 0$$, then the value of n is :

(A) Any even integer (B) Any odd integer

(C) Any integer (D) Zero

11. Consider the system of linear equations : **[AIEEE 2010]**

$x_1 + 2x_2 + x_3 = 3$

$2x_1 + 3x_2 + x_3 = 3$

$3x_1 + 5x_2 + 2x_3 = 1$

Then system has

(A) Exactly 3 solutions (B) A unique solution

(C) No solution (D) Infinite number of solutions

12. The number of values of k for which the linear equation $4x + ky + 2z = 0, kx + 4y + z = 0$ and $2x + 2y + z = 0$ posses a non-zero solution is **[AIEEE 2011]**

(A) 2 (B) 1 (C) Zero (D) 3

13. The number of values of k, for which the system of equations : **[JEE Main 2013]**

$(k + 1)x + 8y = 4k$

$kx + (k + 3)y = 3k - 1$

has no solution, is :

(A) 3 (B) infinite (C) 1 (D) 2

14. If $\alpha, \beta \neq 0$, and $f(n) = \alpha^n + \beta^n$ and **[JEE Main 2014]**

$$\begin{vmatrix} 3 & 1+f(1) & 1+f(2) \\ 1+f(1) & 1+f(2) & 1+f(3) \\ 1+f(2) & 1+f(3) & 1+f(4) \end{vmatrix} = K(1-\alpha)^2(1-\beta)^2(\alpha-\beta)^2$$, then K is equal to :

(A) -1 (B) $\alpha\beta$ (C) $\frac{1}{\alpha\beta}$ (D) 1

15. The set of all values of λ for which the system of linear equations : **[JEE Main 2015]**

$2x_1 - 2x_2 + x_3 = \lambda x_1$

$2x_1 - 3x_2 + 2x_3 = \lambda x_2$

$-x_1 + 2x_2 = \lambda x_3$

has a non-trivial solution,

(A) contains more than two elements (B) is an empty set

(C) is a singleton (D) contains two elements

16. The system of linear equations **[JEE Main 2016]**

$x + \lambda y - z = 0$

$\lambda x - y - z = 0$

$x + y - \lambda z = 0$

has a non-trivial solution for :

(A) infinitely many values of λ. (B) exactly one value of λ

(C) exactly two values of λ (D) exactly three values of λ

17. If S is the set of distinct values of 'b' for which the following system of linear equations

$x + y + z = 1$ **[JEE Main 2017]**

$x + ay + z = 1$

$ax + by + z = 0$

has no solution, then S is :

(A) a finite set containing two or more elements (B) a singleton

(C) an empty set (D) an infinite set

18. If the system of linear equations **[JEE Main 2018]**

$x + ky + 3z = 0$

$3x + ky - 2z = 0$

$2x + 4y - 3z = 0$

has a non-zero solution (x, y, z), then $\frac{xz}{y^2}$ is equal to

(A) 30 (B) -10 (C) 10 (D) -30

19. If $\begin{vmatrix} x-4 & 2x & 2x \\ 2x & x-4 & 2x \\ 2x & 2x & x-4 \end{vmatrix} = (A + Bx)(x - A)^2$, then the ordered pair (A, B) is equal to

(A) $(4,5)$ (B) $(-4,-5)$ (C) $(-4,3)$ (D) $(-4,5)$ **[JEE Main 2018]**

20. Let S be the set of all real values of k for which the system of linear equations

$x + y + z = 2$

$2x + y - z = 3$

$3x + 2y + kz = 4$

has a unique solution. Then S is : **[JEE Main 2018]**

(A) an empty set (B) equal to $\{0\}$ (C) equal to R (D) equal to $R - \{0\}$

21. If the system of linear equations

$x + ay + z = 3$

$x + 2y + 2z = 6$

$x + 5y + 3z = b$

has no solution, then : **[JEE Main 2018]**

(A) $a = -1, b = 9$ (B) $a = -1, b \neq 9$

(C) $a \neq -1, b = 9$ (D) $a = 1, b \neq 9$

EXERCISE–V

1. (a) If $f(x) = \begin{vmatrix} 1 & x & x+1 \\ 2x & x(x-1) & (x+1)x \\ 3x(x-1) & x(x-1)(x-2) & (x+1)x(x-1) \end{vmatrix}$ then f(100) is equal to:

(A) 0 (B) 1 (C) 100 (D) -100

(b) Let a, b, c, d be real numbers in G.P. If u, v, w satisfy the system of equations,

$u + 2v + 3w = 6$

$4u + 5v + 6w = 12$

$6u + 9v = 4$

then show that the roots of the equation,

$\left(\frac{1}{u}+\frac{1}{v}+\frac{1}{w}\right)x^2 + [(b-c)^2 + (c-a)^2 + (d-b)^2]x + u + v + w = 0$

and $20x^2 + 10(a-d)^2x - 9 = 0$

are reciprocals of each other. **[JEE ' 99, 2 + 10 out of 200]**

2. If the system of equations $x - Ky - z = 0$, $Kx - y - z = 0$ and $x + y - z = 0$ has a non zero solution, then the possible values of K are **[JEE 2000 (Screening)]**

(A) $-1, 2$ (B) 1, 2 (C) 0, 1 (D) $-1, 1$

3. Prove that for all values of θ , $\begin{vmatrix} \sin\theta & \cos\theta & \sin 2\theta \\ \sin\left(\theta+\frac{2\pi}{3}\right) & \cos\left(\theta+\frac{2\pi}{3}\right) & \sin\left(2\theta+\frac{4\pi}{3}\right) \\ \sin\left(\theta-\frac{2\pi}{3}\right) & \cos\left(\theta-\frac{2\pi}{3}\right) & \sin\left(2\theta-\frac{4\pi}{3}\right) \end{vmatrix} = 0$

[JEE 2000 (Mains)]

4. Find the real values of r for which the following system of linear equations has a non-trivial solution. Also find the non-trivial solutions : **[REE 2000 (Mains)]**

$2rx - 2y + 3z = 0$

$x + ry + 2z = 0$

$2x + rz = 0$

5. Solve for x the equation $\begin{vmatrix} a^2 & a & 1 \\ \sin(n+1)x & \sin nx & \sin(n-1)x \\ \cos(n+1)x & \cos nx & \cos(n-1)x \end{vmatrix} = 0$ **[REE 2001 (Mains)]**

6. Test the consistency and solve them when consistent, the following system of equations for all values of λ **[REE 2001 (Mains)]**

$x + y + z = 1$

$x + 3y - 2z = \lambda$

$3x + (\lambda+2)y - 3z = 2\lambda + 1$

7. Let a, b, c be real numbers with $a^2 + b^2 + c^2 = 1$. Show that the equation **[JEE 2001 (Mains)]**

$$\begin{vmatrix} ax - by - c & bx + ay & cx + a \\ bx + ay & -ax + by - c & cy + b \\ cx + a & cy + b & -ax - by + c \end{vmatrix} = 0$$ represents a straight line.

8. The number of values of k for which the system of equations **[JEE 2002 (Screening), 3]**

$(k + 1)x + 8y = 4k$

$kx + (k + 3)y = 3k - 1$

has infinitely many solutions is

(A) 0 (B) 1 C) 2 (D) inifinite

9. The value of λ for which the system of equations

$2x - y - z = 12\,, x - 2y + z = -4\,, x + y + \lambda z = 4$ has no solution is **[JEE 2004 (Scr.)]**

(A) 3 (B) -3 (C) 2 (D) -2

10. (a) Consider three points $P = (-\sin(\beta - \alpha), -\cos\beta), Q = (\cos(\beta - \alpha), \sin\beta)$ and $R = (\cos(\beta - \alpha + \theta), \sin(\beta - \theta))$, where $0 < \alpha, \beta, \theta < \pi/4$

(A) P lies on the line segment RQ (B) Q lies on the line segment PR

(C) R lies on the line segment QP (D) P, Q, R are non collinear

(b) Consider the system of equations

$x - 2y + 3z = -1$

$-x + y - 2z = k$

$x - 3y + 4z = 1$

STATEMENT-1 : The system of equations has no solution for $k \neq 3$.

STATEMENT-2 : The determinant $\begin{vmatrix} 1 & 3 & -1 \\ -1 & -2 & k \\ 1 & 4 & 1 \end{vmatrix} \neq 0$, for $k \neq 3$.

(A) Statement-1 is True, Statement-2 is True ; statement-2 is a correct explanation for statement-1

(B) Statement-1 is True, Statement-2 is True ; statement-2 is NOT a correct explanation for statement-1

(C) Statement-1 is True, Statement-2 is False

(D) Statement-1 is False, Statement-2 is True **[JEE 2008, 3 + 3]**

11. Which of the following values of α satisfy the equation **[IIT Advance - 2015]**

$$\begin{vmatrix} (1+\alpha)^2 & (1+2\alpha)^2 & (1+3\alpha)^2 \\ (2+\alpha)^2 & (2+2\alpha)^2 & (2+3\alpha)^2 \\ (3+\alpha)^2 & (3+2\alpha)^2 & (3+3\alpha)^2 \end{vmatrix} = -648\alpha\ ?$$

(A) -4 (B) 9 (C) -9 (D) 4

12. The total number of distinct $x \in R$ for which $\begin{vmatrix} x & x^2 & 1+x^3 \\ 2x & 4x^2 & 1+8x^3 \\ 3x & 9x^2 & 1+27x^3 \end{vmatrix} = 10$ is :

[IIT Advance - 2016]

13. Let $\alpha, \lambda, \mu \in R$. Consider the system of linear equations **[IIT Advance - 2016]**

$\alpha x + 2y = \lambda$

$3x - 2y = \mu$

Which of the following statement(s) is(are) correct?

(A) If $\alpha = -3$, then the system has infinitely many solutions for all values of λ and μ.

(B) If $\alpha \neq -3$, then the system has a unique solution for all values of λ and μ.

(C) If $\lambda + \mu = 0$, then the system has infinitely many solutions for $\alpha = -3$

(D) If $\lambda + \mu \neq 0$, then the system has no solution for $\alpha = -3$

14. If $f(x) = \begin{vmatrix} \cos(2x) & \cos(2x) & \sin(2x) \\ -\cos x & \cos x & -\sin x \\ \sin x & \sin x & \cos x \end{vmatrix}$, then **[IIT Advance - 2017]**

(A) $f'(x) = 0$ at exactly three points in $(-\pi, \pi)$

(B) $f(x)$ attains its maximum at $x = 0$

(C) $f(x)$ attains its minimum at $x = 0$

(D) $f'(x) = 0$ at more than three points in $(-\pi, \pi)$

ANSWER KEY

PROFICIENCY TEST-01

1. $(a-b)(b-c)(c-a)$ **2.** 0 **3.** 0 **4.** -25 **5.** 0

6. 0 **7.** 0 **8.** xy **9.** $x = 2, -1$

10. $x = 0$ **11.** $x = 0, \pm\sqrt{\frac{3}{2}(a^2+b^2+c^2)}$

12. $1, -9$ **13.** $1 + a^2 + b^2 + c^2$

14. $(a-b)(b-c)(c-a)(a+b+c)$ **15.** 0

PROFICIENCY TEST-02

1. $-(a^3+b^3+c^3-3abc)$ **2.** 0 **3.** 4abc **4.** $0, -3$

5. B 6. -2 **7.** 0 **8.** $4a^2b^2c^2$

9. $xyz\left(1+\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right)$ **10.** 1 **11.** 2, 7 **12.** $abc = -1$

13. 0 14. 0 **15.** $\frac{33}{8}$

PROFICIENCY TEST-03

1. $k^3\Delta$ **2.** 24 **3.** $\{-9, 0, 9\}$ **4.** 0 **5.** -28000 **6.** 6

7. 0 **8.** 4Δ **9.** 6k **10.** 25 **11.** 2 **12.** 0

13. 0 **14.** 0 **15.** 8

EXERCISE–I

1. (c) p **4.** -1 **10.** $(ab'-a'b)(bc'-b'c)(ca'-c'a)$

13. $x = -1$ or $x = -2$ **16.** $\lambda^2(a^2+b^2+c^2+\lambda)$ **18.** $x = 4$

20. Triangle ABC is isosceles.

EXERCISE–II

1. $x = -7, y = -4$ **2.** (a) $x = 1, y = 2, z = 3$; consistent (b) inconsistent

3. $x = -(a+b+c), y = ab+bc+ca,\ z = -abc$

4. $K = \frac{33}{2},\ x : y : z = -\frac{15}{2} : 1 : -3$ **7.** 2

8. (a) $\lambda \neq 3$ (b) $\lambda = 3, \mu = 10$ (c) $\lambda = 3, \mu \neq 10$

9. $x = 1 + 2K, y = -3K, z = K$, when $p = 1$; $x = 2K, y = 1 - 3K, z = K$ when $p = 2$; where $K \in R$

10. If $K \neq 2, \frac{x}{2(K+6)} = \frac{y}{2K+3} = \frac{z}{6(K-2)} = \frac{1}{2(K^2+2K+15)}$, If $K = 2$, then $x = \lambda, y = \frac{1-2\lambda}{2}$ and $z = 0$ where $\lambda \in R$

11. 19 **13.** $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$

14. If $\lambda \neq -5$ then $x = \frac{4}{7}$; $y = -\frac{9}{7}$ and $z = 0$;

If $\lambda = 5$ then $x = \frac{4-5K}{7}$; $y = \frac{13K-9}{7}$ and $z = K$ where $K \in R$

EXERCISE–III

1.	D	**2.**	A	**3.**	D	**4.**	A	**5.**	D	**6.**	C	**7.**	C
8.	A	**9.**	B	**10.**	B	**11.**	A	**12.**	B	**13.**	B	**14.**	D
15.	A												

EXERCISE–IV

1.	C	**2.**	D	**3.**	D	**4.**	D	**5.**	D	**6.**	B	**7.**	D
8.	A	**9.**	D	**10.**	B	**11.**	C	**12.**	A	**13.**	C	**14.**	D
15.	D	**16.**	D	**17.**	B	**18.**	C	**19.**	D	**20.**	D	**21.**	B

EXERCISE–V

1. (a) A **2.** D **4.** $r = 2$; $x = k$; $y = k/2$; $z = -k$ where $k \in R - \{0\}$

5. $x = n\pi, n \in I$

6. If $\lambda = 5$, system is consistent with infinite solution given by $z = K$,

$$y = \frac{1}{2}(3K + 4) \text{ and } x = -\frac{1}{2}(5K + 2) \text{ where } K \in R$$

If $\lambda \neq 5$, system is consistent with unique solution given by $z = \frac{1}{3}(1 - \lambda)$; $x = \frac{1}{3}(\lambda + 2)$ and $y = 0$.

8. B **9.** D **10.** (a) D ; (b) A **11.** BC **12.** 2 **13.** BCD

14. BD