

L. Find the eqn. of common tangent to
 $y^2 = 8x$ and $3x^2 - y^2 = 3$

$$\begin{aligned}ty &= x + 2t^2 \\y &= \frac{1}{t}x + 2t\end{aligned}$$

$$4t^2 = 1 \left(\frac{1}{t^2}\right) - 3$$

$$\begin{aligned}4t^4 + 3t^2 - 1 &= 0 \\(4t^2 - 1)(t^2 + 1) &= 0 \\t &= \pm \frac{1}{2}\end{aligned}$$

$$y = 2x + 1, \quad y = -2x - 1$$

2. Find eqn. of tangent to $\frac{x^2}{36} - \frac{y^2}{9} = 1$ passing
thru (0, 4) :

$$y = mx \pm \sqrt{m^2(36) - 9}$$

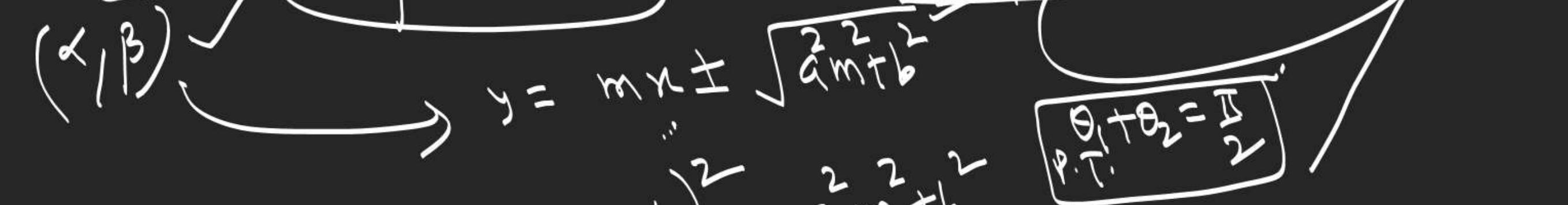
$$16 = 36m^2 - 9$$

$$m = \pm \frac{5}{6}$$

$$y - 4 = \frac{5}{6}x$$

$$y - 4 = -\frac{5}{6}x$$

3. P.T. two tangents drawn from any point on the hyperbola $x^2 - y^2 = a^2 - b^2$ to ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ make complementary angle with x-axis.



$$y = mx \pm \sqrt{a^2 m^2 + b^2}$$

$$(\beta - md)^2 = a^2 m^2 + b^2$$

$$m^2(\lambda^2 - a^2) - 2\lambda\beta m + \beta^2 - b^2 = 0$$

$$m_1 m_2 = \frac{\beta^2 - b^2}{\lambda^2 - a^2} = \frac{\lambda^2 - a^2 + b^2 - b^2}{\lambda^2 - a^2} = 1$$

Q. Find the eqn. and length of common tangent to hyperbolae

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \quad \& \quad \frac{y^2}{a^2} - \frac{x^2}{b^2} = 1 \quad \rightarrow \quad \left(\frac{x^2}{b^2}\right) - \left(\frac{y^2}{a^2}\right) = 1$$

$$PQ = \frac{(a^2 + b^2)}{\sqrt{a^2 - b^2}} \sqrt{2}$$

$$\left(\frac{x_1^2}{b^2}, \frac{y_1^2}{a^2}\right) = \left(\frac{x_2^2}{b^2}, \frac{y_2^2}{a^2}\right)$$

$$\frac{x_2^2}{b^2} = \frac{y_2^2}{a^2} = -\frac{x_1^2}{a^2} = \frac{y_1^2}{b^2} = \frac{1}{b^2 - a^2}$$

$$P(x_1, y_1) = \left(\frac{-a^2}{\sqrt{a^2 - b^2}}, \frac{-b^2}{\sqrt{a^2 - b^2}}\right)$$

$$y = mx \pm \sqrt{a^2 m^2 - b^2}$$

$$a^2 m^2 - b^2 = -b^2 m^2 - (-a^2)$$

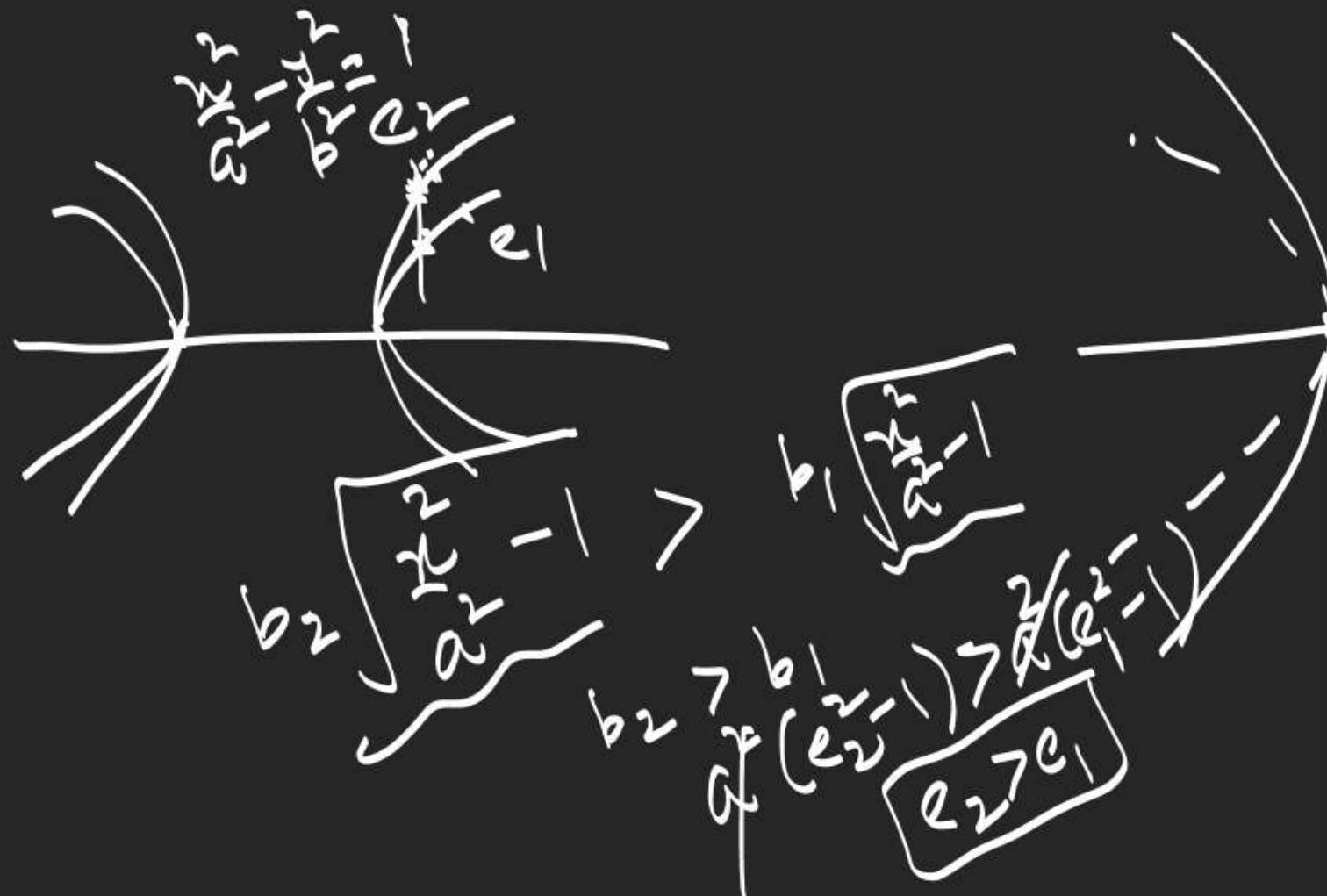
$$m = \pm 1$$

$$y - x = \sqrt{a^2 - b^2}$$

$$\frac{y_1 y_2}{a^2} - \frac{y_1 y_2}{b^2} = 1$$

$$\frac{y_2 y_2}{a^2} - \frac{x_2 x_2}{b^2} = 1$$

5. If one axis of varying hyperbola be fixed in magnitude and position. P.T. locus of point of contact of a tangent drawn to it from a fixed point on other axis is a parabola.



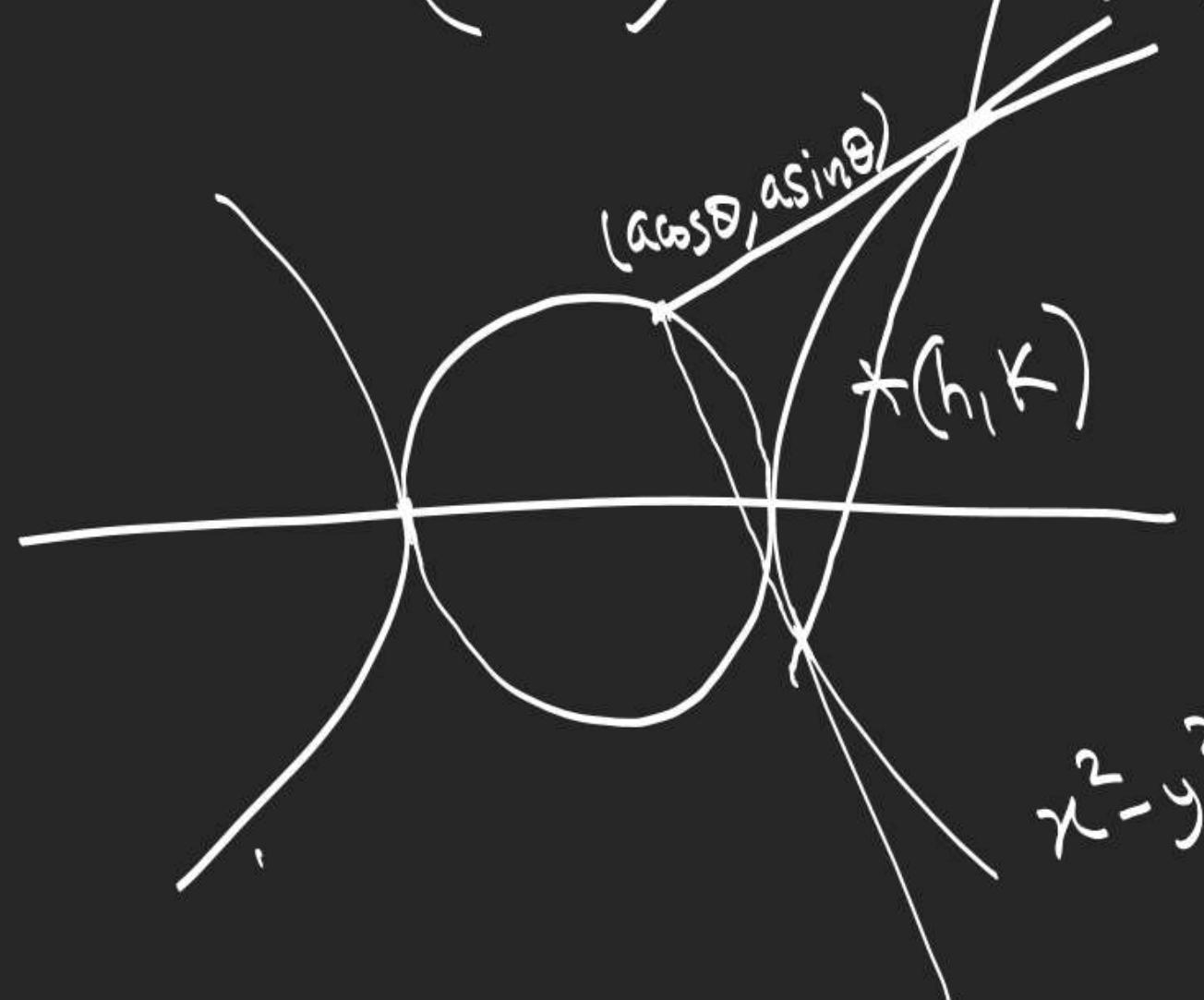
$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$$

$$(0, c), \quad -\frac{kc}{b^2} = 1$$

$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$$

$$\frac{y^2}{a^2} - \frac{x^2}{b^2 + k^2/c^2} = 1$$

6. From points on circle $x^2+y^2=a^2$, tangents are drawn to hyperbola $x^2-y^2=a^2$. P.T. locus of middle points of chord of contact is the curve $(x^2-y^2)^2 = a^2(x^2+y^2)$.



$$x \cos \theta - y \sin \theta = a$$

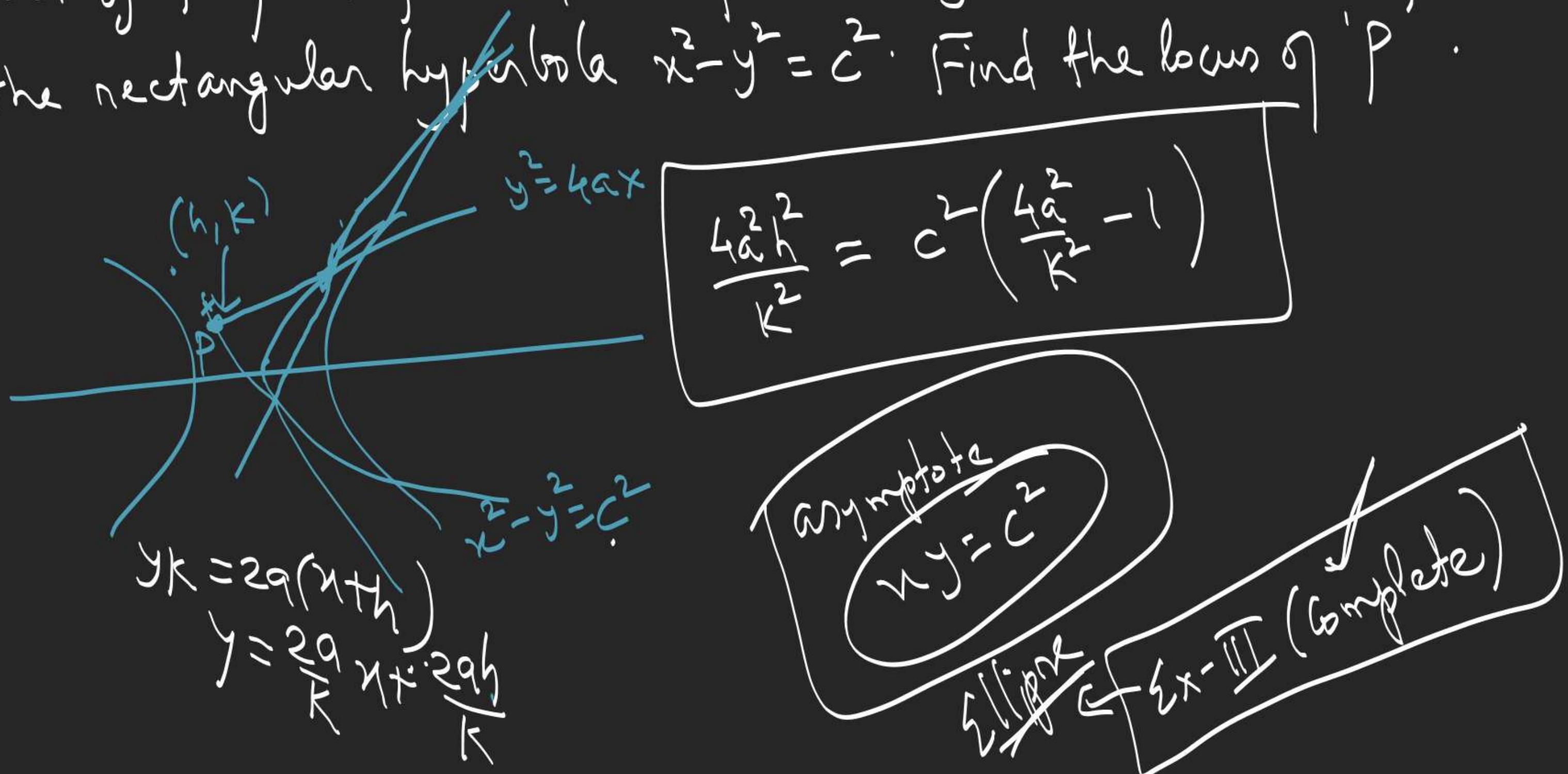
$$xh - yk = h - k^2$$

$$\frac{\cos \theta}{h} = \frac{\sin \theta}{k} = \frac{a}{h-k^2}$$

$$x^2 - y^2 = a^2$$

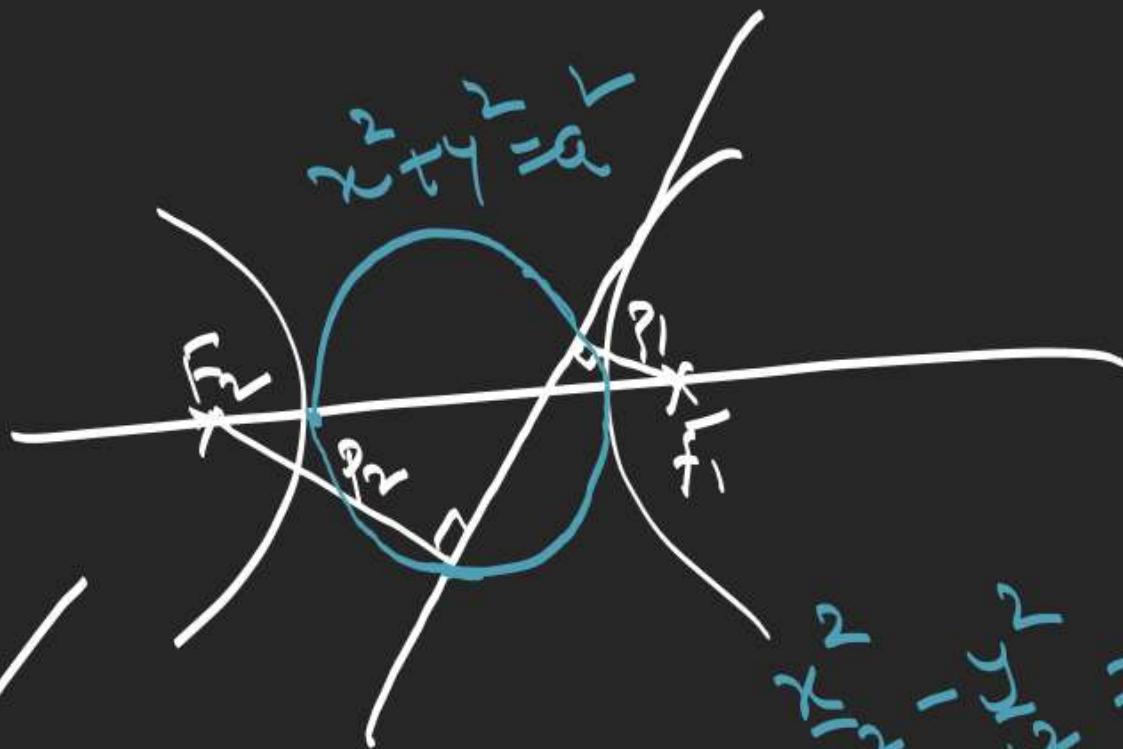
$$\left[\frac{a^2}{(h-k^2)^2} (h^2 + k^2) = 1 \right]$$

7. A point 'P' moves such that chord of contact of pair of tangents from P on parabola $y^2 = 4ax$ touches the rectangular hyperbola $x^2 - y^2 = c^2$. Find the locus of 'P'.

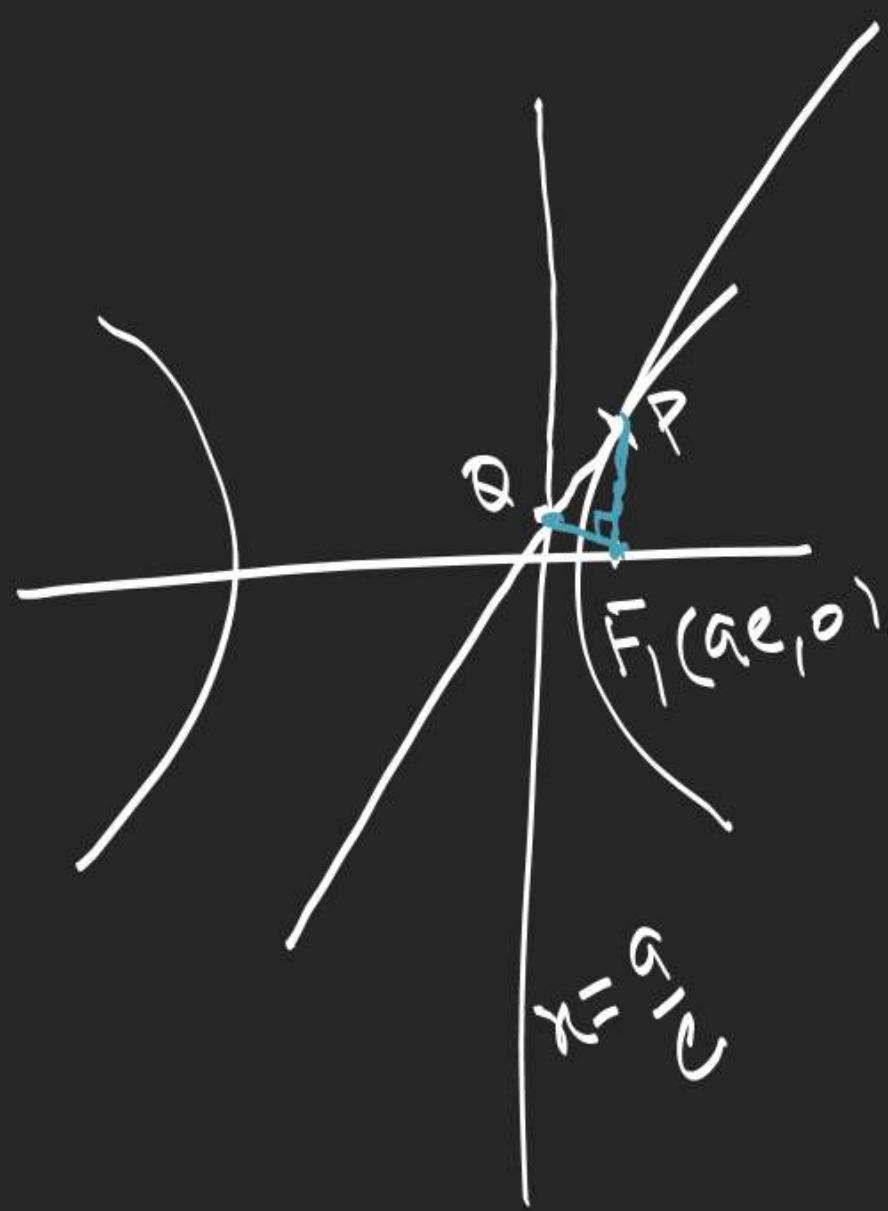


Note \rightarrow ①

$$P_1 P_2 = (\text{semi } A)^2$$



②



③ Reflection Prop



Conjugal Ellipse & Hyperbola are
orthogonal