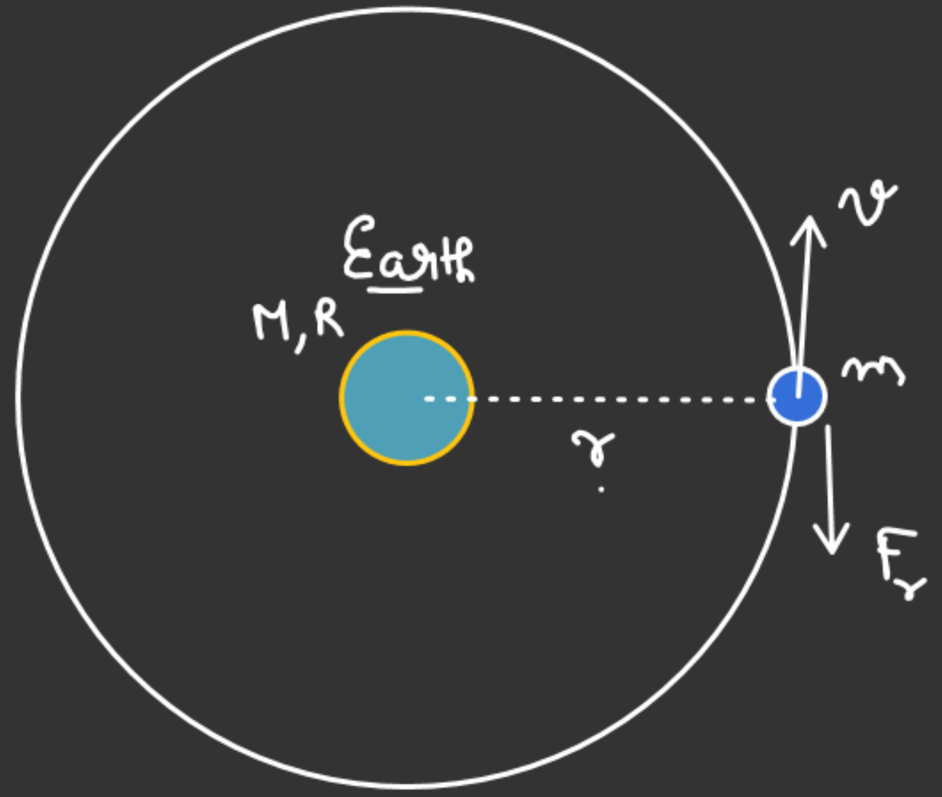
Nishant Jindal

GRAVITATION

A Satellite moving in a Circular orbit of radius ($r = nR$). around earth. Resistive force acting on the Satallite $F_r = \alpha v^2$.
How long it takes for Satellite to collide with earth.



Total Energy of Satellite at an orbital radius r is $E_T = \left(-\frac{GMm}{2r}\right)$

$$\left(\text{Rate of loss of Energy of Satellite}\right) = -(F \cdot v)$$

$$-\left(\frac{dE_T}{dt}\right) = -Fv$$

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$$\left(\frac{dE_T}{dt}\right) = -\underline{Fv} \checkmark$$

$$\frac{d}{dt}\left(-\frac{GMm}{2r}\right) = -(\alpha v^2)v$$

$$+\frac{GMm}{2}\left(\frac{-1}{r^2}\right)\left(\frac{dr}{dt}\right) = \alpha v^3$$

$$-\frac{\cancel{GM}m}{2r^2}\left(\frac{dr}{dt}\right) = \alpha\left(\overset{v^2}{\frac{\cancel{GM}}{r}} \cdot \overset{v}{\sqrt{\frac{GM}{r}}}\right)$$

$$-\frac{1}{2}\sqrt{\frac{1}{GM}}\int_{nR}^{R} r^{-1/2}\,dr = \frac{\alpha}{m}\int_0^t dt$$

$$t = \frac{m}{\alpha}\sqrt{\frac{R}{GM}}\left(\sqrt{n}-1\right)$$

Ans ✓

$$v_r = \sqrt{\frac{GM}{r}}$$

orbital velocity of Satellite at any radial distance r

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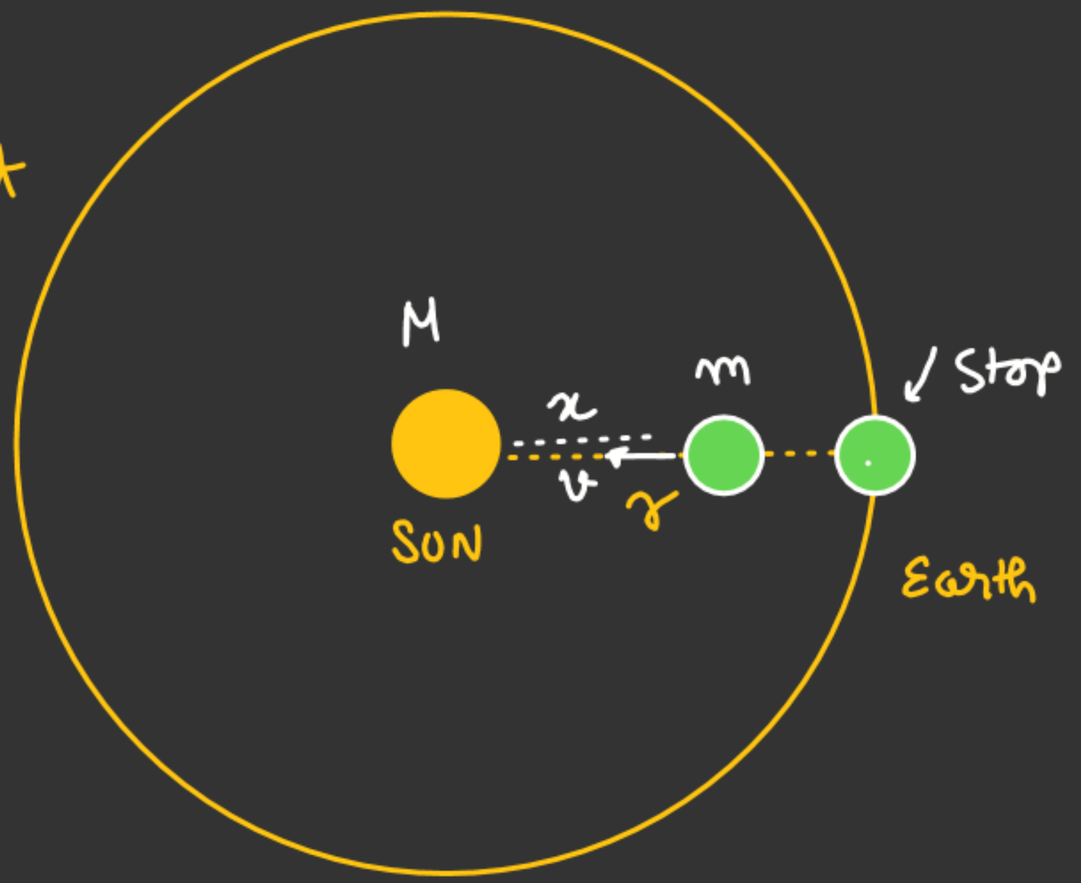
\# Assume Circular orbit of Earth around Sun. Assume Earth Suddenly stop rotating. Find time when earth Collapse in Sun. $T=$ (Time period of Earth about Sun)



$$-\frac{GMm}{r} = -\frac{GMm}{x} + \frac{1}{2}mv^2$$

$$\frac{1}{2}\cancel{m}v^2 = GM\cancel{m}\left(\frac{1}{x}-\frac{1}{r}\right)$$

$$v^2 = \frac{2GM}{r}\left(\frac{r-x}{x}\right)$$

$$v = \sqrt{\frac{2GM}{r}}\sqrt{\frac{r-x}{x}}$$

$$-\frac{dx}{dt} = \sqrt{\frac{2GM}{r}}\sqrt{\frac{r-x}{x}}$$

$$\int_r^0 \sqrt{\frac{x}{r-x}}\,dx = -\sqrt{\frac{2GM}{r}}\int_0^t dt$$

put

$$x = r\sin^2\theta$$

$$\left(t = \frac{T}{2\sqrt{8}}\right)$$

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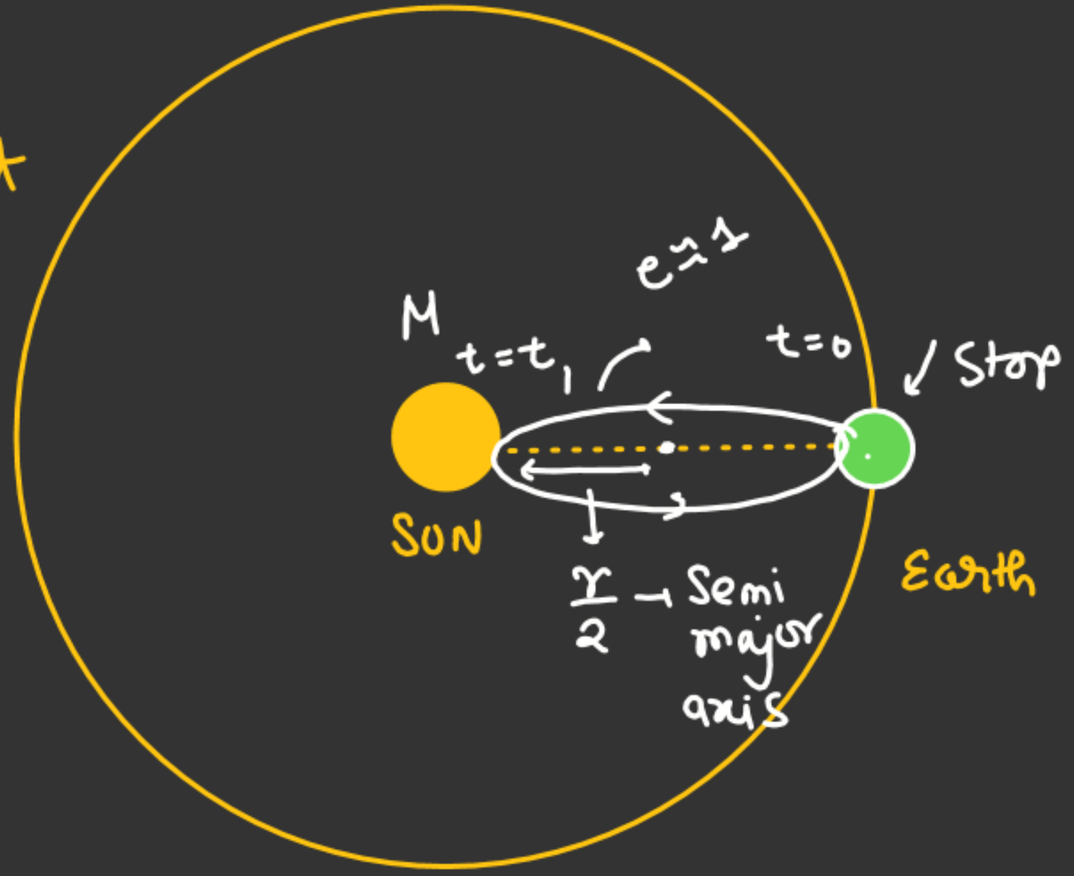

Assume motion of Earth as elliptical let, T' be its time period and $\frac{r}{2}$ be the Semimajor axis.

By Kepler's 3rd law.

$$\frac{(T')^2}{(T)^2} = \frac{(r/2)^3}{(r)^3}$$

$$\frac{(T')^2}{T^2} = \frac{1}{8} \Rightarrow T' = \frac{T}{\sqrt{8}}$$

$$t_1 = T'/2$$

$$t_1 = \frac{T}{2\sqrt{8}}$$

Ans.

$b \to 0 \quad e \to 1$

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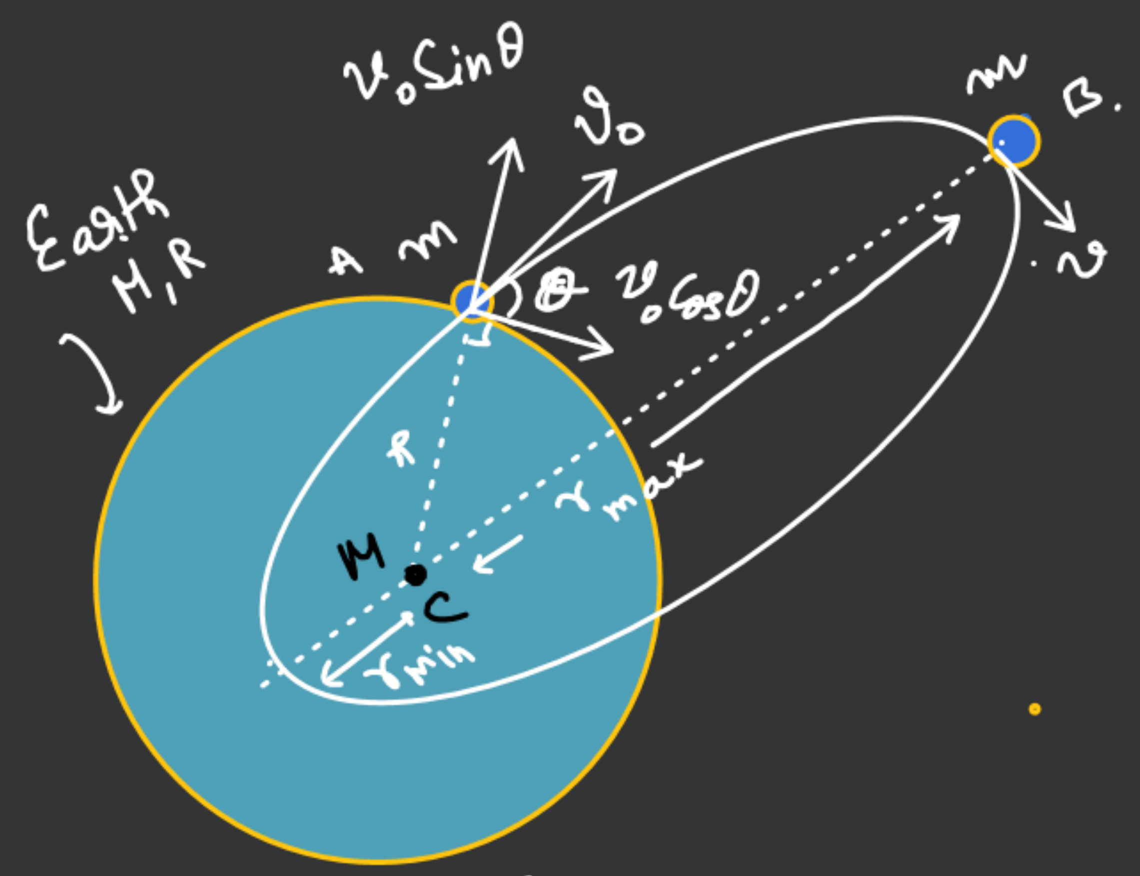
Q. A Body is projected at an angle θ from horizontal. from the Surface of earth with velocity v_0
Show that the semi-major axis of ellipse in independent of θ



Let, r be either min or maximum distance of the body.

A.M.C about C from A to B.

$$(mv_0\cos\theta)R = mvr \quad \text{—} \quad (1)$$

Energy Conservation from A to B.

$$-\frac{GMm}{R} + \frac{1}{2}mv_0^2 = -\frac{GMm}{r} + \frac{1}{2}mv^2 \quad \text{—} \quad (2)$$

From (1) $v = \left(Rv_0\cos\theta\right)$

Semimajor axis $= \left(\frac{r_{max} + r_{min}}{2}\right)$

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GRAVITATION

$$(mv_0\cos\theta)R = mvr \quad \text{— ①}$$

Energy Conservation from A to B.

$$-\frac{GMm}{R}+\frac{1}{2}mv_0^2 = -\frac{GMm}{r}+\frac{1}{2}mv^2 \quad \text{— ②}$$

From ①

$$v=\left(\frac{Rv_0\cos\theta}{r}\right)$$

$$-\frac{GMm}{R}+\frac{mv_0^2}{2} = -\frac{GMm}{r}+\frac{m}{2}\left(\frac{R^2v_0^2\cos^2\theta}{r^2}\right)$$

$$GM\cancel{m}\left(\frac{1}{r}-\frac{1}{R}\right) = \frac{\cancel{m}v_0^2}{2}\left(\frac{R^2\cos^2\theta}{r^2}-1\right)$$

$\Downarrow$

$r \rightarrow$ quadratic

$r_{max}+r_{min}$ = Sum of root

Semi major axis $=\left(\frac{r_{max}+r_{min}}{2}\right)$

Check

$$=\left(\frac{gR^2}{2gR-v_0}\right) \text{ Ans}$$

GRAVITATION

A Satellite revolving around earth in circular orbit of radius r with velocity v_0.
A particle is projected from the Satellite in forward direction with velocity $\left(\sqrt{\frac{5}{4}}-1\right)v_0$.
Find max and min distance of the particle w.r.t earth during its subsequent Motion

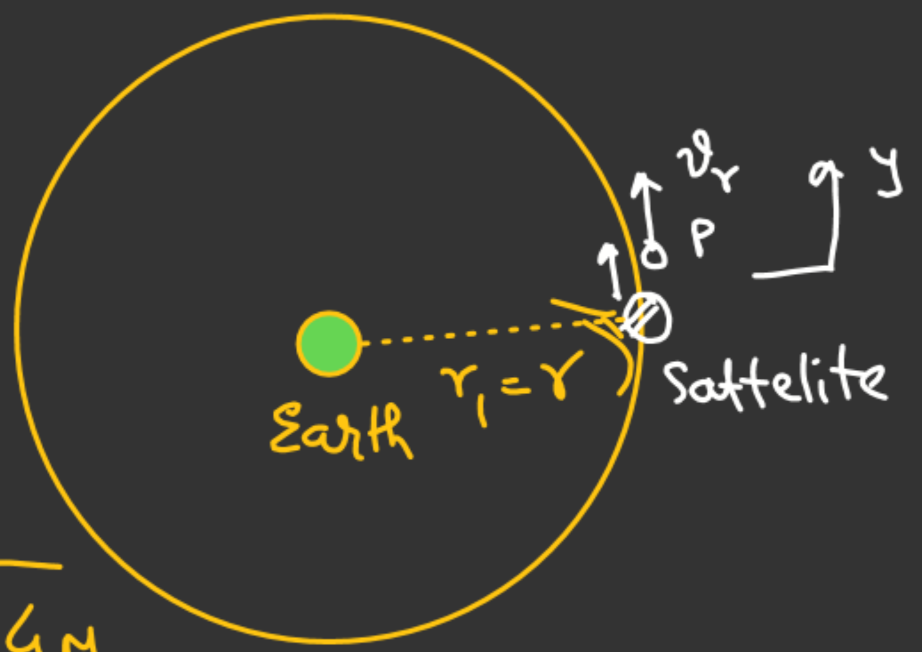
Sol^n Absolute velocity of particle

$$\vec{v}_{P/E} = \vec{v}_{P/\text{satellite}} + \vec{v}_{\text{satellite}/E}$$

$$= \left(\left(\sqrt{\frac{5}{4}}-1\right)v_0 + v_0\right)\hat{j}$$

$$= \left(\sqrt{\frac{5}{4}}\,v_0\right)$$

$v_p > v_0 \Rightarrow$ (Trajectory is ellipse) $\left(v_0 = \sqrt{\frac{GM}{r}}\right)$ ∴



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GRAVITATION

A.M.C

$v_0 = \sqrt{\frac{GM}{r}}$ ✓

$\sqrt{\frac{5}{4}}v_0 (r) = v_2 r_2 \quad -(1)$

r_1

$-\frac{GMm}{r} + \frac{1}{2}m\left(\sqrt{\frac{5}{4}}v_0\right)^2 = \frac{-GMm}{r_2} + \frac{1}{2}mv_2^2 \quad -(2)$

$\Downarrow$

$r_2 = \left(\frac{5r}{3}\right)$, $r_1 = r$