Nishant Jindal

Note → (1) $ax^2+bx+c=0$, a, b, c are rational, $a \neq 0$, has rational roots if D is a perfect square of rational number.

$p+\sqrt{q}$, $p-\sqrt{q}$

$p, q \in \boxed{Q}$ $\alpha, \beta = \dfrac{-b \pm \sqrt{D}}{2a} = \dfrac{-b}{2a} \pm \dfrac{\sqrt{D}}{2a}$

$2+\sqrt{3}$, $2-\sqrt{3}$.

(2.) If $ax^2+bx+c=0$, a, b, c are rational, $a \neq 0$.

has irrational roots, then they exist in conjugate pair.

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③ If $ax^2+bx+c=0$, $a,b,c \in R$, $a \neq 0$ has imaginary roots, then they exist in conjugate pair.

$D<0$

$\boxed{p,q \in R}$

$p+iq$

$p-iq$

$$\alpha,\beta = \frac{-b}{2a} \pm \frac{\sqrt{D}}{2a}$$

$$= \frac{-b}{2a} \pm \frac{\sqrt{-(-D)}}{2a}$$

$$\frac{-b}{2a} \pm i\frac{\sqrt{-D}}{2a}$$

complex no.

iota

$\sqrt{-4}$

$\boxed{z = a+ib}$, $a,b \in R$, $i=\sqrt{-1}$

$i^2=-1$

* If $b=0$

z is purely real

* If $b \neq 0$

z is imaginary

z is complex no.

$a = Re(z)$

$b = Im(z)$

$-2 = -2 + i(0)$

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$a,b\in\mathbb{R}$, $\sqrt{ab}=\sqrt{a}\,\sqrt{b}$ if atleast one of a,b is non negative

$$1=\sqrt{1}=\sqrt{(-1)^2}\cancel{=}\sqrt{-1}\,\sqrt{-1}=i\,i=i^2=-1$$

$$\sqrt{-4}=\underbrace{\sqrt{-1}}_{i}\,\underbrace{\sqrt{4}}_{2}=2i$$

$-21i^2$

$x_1,x_2,y_1,y_2\in\mathbb{R}$.

$$(x_1+iy_1)+(x_2+iy_2)$$
$$=(x_1+x_2)+i(y_1+y_2)$$

$$(2+3i)(5-7i)=10-14i+15i+21$$
$$=31+i$$

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$$\frac{x_1+iy_1}{x_2+iy_2} = \frac{(x_1+iy_1)(x_2-iy_2)}{(x_2+iy_2)(x_2-iy_2)} = \frac{(x_1x_2+y_1y_2)+i(y_1x_2-x_1y_2)}{x_2^2+y_2^2}$$

$$= \left(\frac{x_1x_2+y_1y_2}{x_2^2+y_2^2}\right) + i\left(\frac{y_1x_2-x_1y_2}{x_2^2+y_2^2}\right)$$

Conjugate of $a+ib = a-ib$

Conj of $(-2-3i) = -2+3i$

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1. Obtain a quadratic equation with rational coefficients whose one root is $\cos 36^\circ$.

$$\alpha, \beta = \frac{\sqrt{5}+1}{4}, \frac{1-\sqrt{5}}{4} \qquad \alpha\beta = \frac{1-5}{16} = -\frac{1}{4}$$

$$x^2 - \frac{1}{2}x + \left(-\frac{1}{4}\right) = 0 \Rightarrow \boxed{4x^2 - 2x - 1 = 0}$$

2. Find 'a' for which $(a+4)x^2 - 2ax + 2a - 6 < 0$ $\forall x \in R$.

$$x = \frac{\sqrt{5}+1}{4} \Rightarrow 4x - 1 = \sqrt{5}$$

$$\boxed{4x^2-2x-1=0} \Leftarrow 16x^2 - 8x - 4 = 0 \Leftarrow (4x-1)^2 = 5$$

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$$(a+4)x^2-2ax+2a-6<0 \quad \forall x\in R.$$

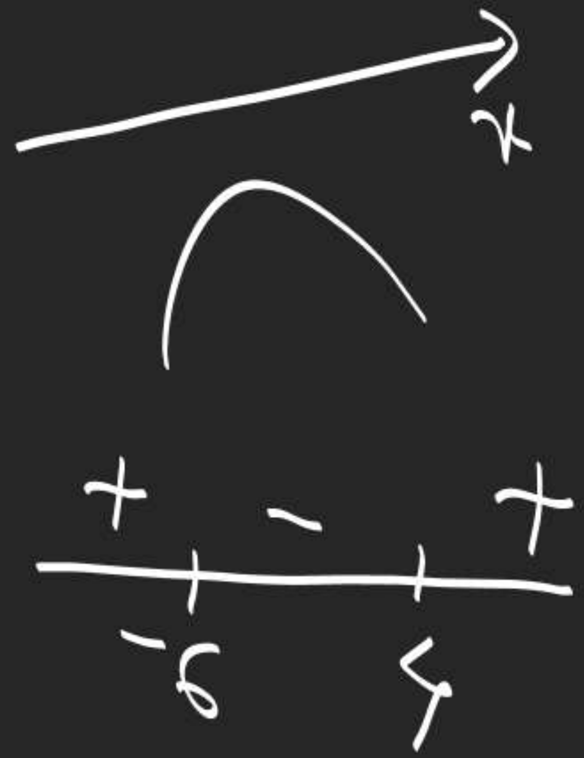
$$f(x)=(a+4)x^2-2ax+2a-6$$

If $a+4=0$ **OR** If $a+4\neq 0$

$8x-14<0 \quad \forall x\in R$

no

$a=-4$ (rejected)



$a+4<0 \Rightarrow \boxed{a\in(-\infty,-4)}$

&

$D<0$

$$\Rightarrow \cancel{4}a^2-\cancel{4}(a+4)(2a-6)<0$$

$$\Leftarrow a^2+2a-24>0$$

$$(a+6)(a-4)>0$$

$$\boxed{a\in(-\infty,-6)\cup(4,\infty)}$$

&

$$\boxed{a\in(-\infty,-6)} \Rightarrow \text{Ans.}$$

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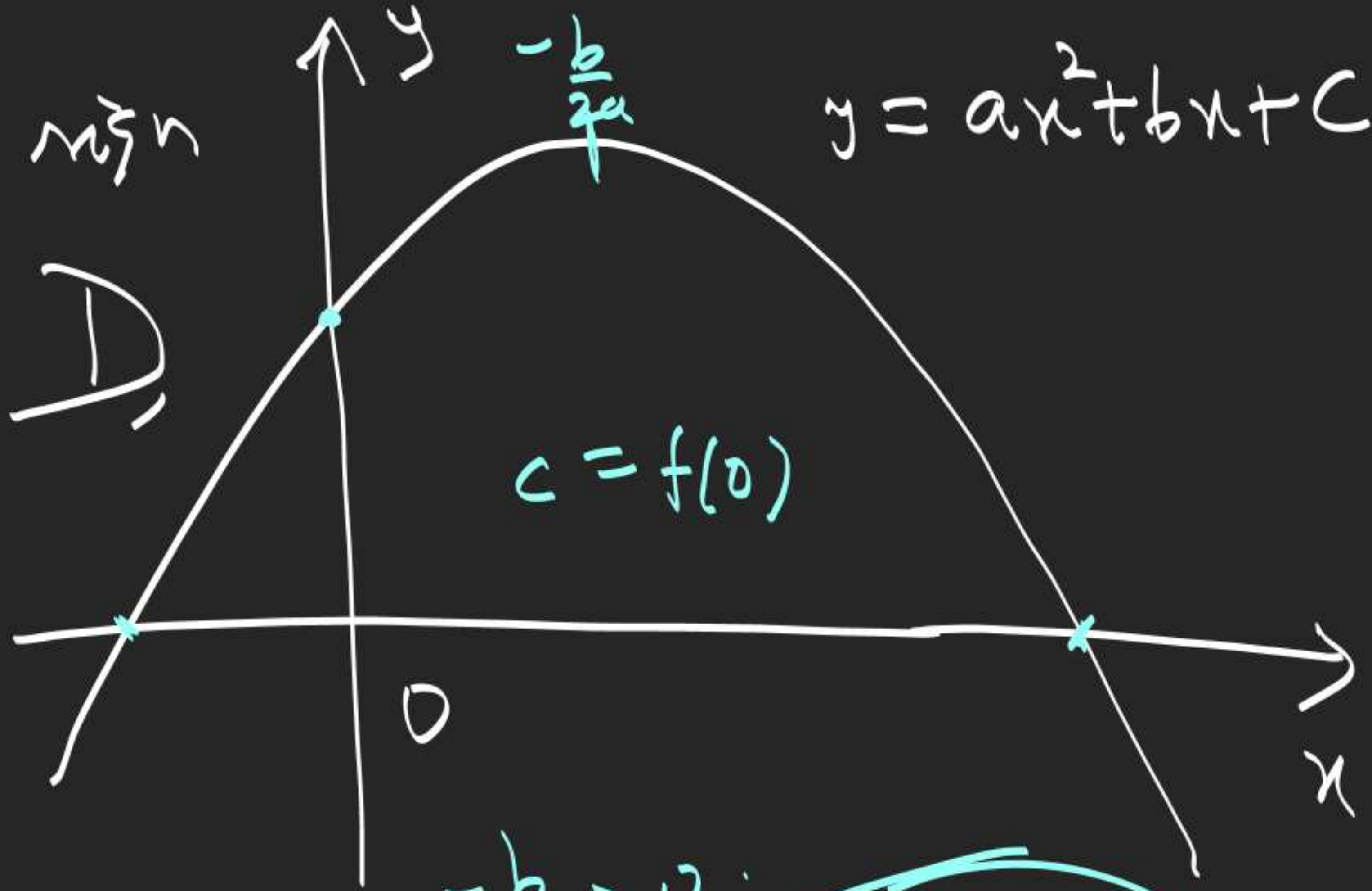
Comment upon the sign of $a, b, c, D,$ $\alpha+\beta, \alpha\beta$



$y = ax^2 + bx + C = f(x)$

$\alpha, \beta \rightarrow$ roots of $ax^2+bx+c=0$.

$-\frac{b}{2a} > 0$

$\alpha+\beta = -\frac{b}{a} > 0$

$\frac{c}{a} < 0$

$c > 0$

$\frac{b}{-a} > 0$

$b > 0$

$a < 0, b > 0, c > 0, D > 0, \alpha+\beta > 0$

$\alpha\beta < 0$

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4. Let the equation $ax^2+bx+c=0$, $a, b, c \in R$, $a \neq 0$ has imaginary solutions, then P.T.

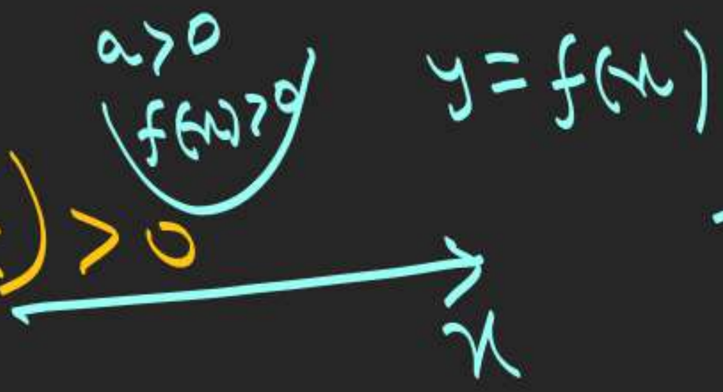
$f(x) = ax^2+bx+c$

(i) $c(a+b+c) > 0$

$c\left(\frac{a}{4} - \frac{b}{2} + c\right) > 0$

(ii) $(a+c)^2 > b^2$

$c(a-2b+4c) > 0$



(iii) $a(4a-2b+c) > 0$

(iv) $c(a-2b+4c) > 0$

(i) $f(0)f(1) > 0$

(ii) $f(1)f(-1) > 0$

(iii) $a\,f(-2) > 0$

(iv) $f(0)\,f\left(-\frac{1}{2}\right) > 0$

5. If α, β are the roots of quadratic equation $x^2-2x+5=0$, then form a quadratic equation whose roots are $\alpha^3+\alpha^2-\alpha+22$ and $\beta^3+4\beta^2-7\beta+35$

$$\alpha^3+\alpha^2-\alpha+22 = (\alpha^2-2\alpha+5)(\alpha+3)+7 = 0(\alpha+3)+7 = 7$$

$$\boxed{x^2-12x+35=0} \Rightarrow \underline{\text{Ans}}.$$

$$\alpha^2-2\alpha+5 \,\big)\, \overline{\alpha^3+\alpha^2-\alpha+22}$$

$$\beta^3+4\beta^2-7\beta+35 = \overbrace{(\beta^2-2\beta+5)}^{=0}(\beta+6)+5 = 5$$

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6. If $x = 3+\sqrt{5}$, find the value of

$$x^4 - 12x^3 + 44x^2 - 49x + 17.$$

$\boxed{\text{Prelipko Sec 1.4 (Page 25) Q. 26 to Q. 45}}$

43. $(\ )x^2 + (\ )x - 3(4^{a-1} - 2^{a-2}) = 0$

real roots $D \geq 0$

imaginary $D < 0$

equal real roots $D = 0$

2 distinct real roots $D > 0$