Nishant Jindal

Consider the number $N = 75600$, find its

$N = 2^4 3^3 5^2 7^1$ $\quad 5\times4\times3\times2 = 120$

(i) number of divisors / proper divisors $= 120 - 1$ / even divisors $= 4\times4\times3\times2$ / odd divisors $1\times4\times3\times2$ /

divisors divisible by 10 $= 4\times4\times2\times2$

$0,1,2,3,4 \quad 0,1,2,3 \quad 0,1,2 \quad 0,1$

$div = 2^{m_1} 3^{m_2} 5^{m_3} 7^{m_4}$

$30\times40\times31\times8 \qquad 1\times40\times31\times8$

(ii) sum of all divisors / proper divisors / even divisors / odd divisors /

divisors divisible by 10 $= 30\times40\times30\times8$

$(2^0+2^1+2^2+2^3+2^4)(3^0+3^1+3^2+3^3)(5^0+5^1+5^2)(7^0+7^1) = \dfrac{(2^5-1)}{(2-1)}\dfrac{(3^4-1)}{(3-1)}\dfrac{(5^3-1)}{(5-1)}\,8$

(iii) number of ways in which N can be resolved as product of two divisors $= \dfrac{120}{2} = 60$

(iv) number of ways in which N can be resolved as product of two divisors which are relatively prime $= \boxed{8}$

$d_1 \times \dfrac{N}{d_1}$

$d_2 \times \dfrac{N}{d_2}$

$\vdots$

$\dfrac{N}{d_1} \times d_1$

$\dfrac{120}{2}$

Nishant Jindal

$N = p_1^{a_1} p_2^{a_2} p_3^{a_3} \cdots p_m^{a_m}$ ✓ (iv) 2^{m-1}

no. of divisors = d

$d = (a_1+1)(a_2+1)\cdots(a_m+1)$

$d_1 \quad \frac{N}{d_1}$

$d_2 \quad \frac{N}{d_2}$

$\vdots$

$\boxed{\sqrt{N} \cdot \sqrt{N}}$

$\vdots$

$\frac{N}{d_1} \quad d_1$

$$\frac{d-1}{2}+1 = \frac{\prod_{r=1}^{m}(a_r+1)+1}{2}, \text{ N is perfect square}$$

$$\frac{\prod_{r=1}^{m}(a_r+1)}{2}, \text{ N is not perfect square}$$

$N = 2^4 \, 3^3 \, 5^2 \, 7^1$

d_1

$\frac{N}{d_1}$

$\frac{2\times2\times2\times2}{2}$

d_1

2^4

$d_1 \; \boxed{2^4} \to \frac{N}{d_1} \; 3^3 5^2 7^1$

$\frac{N}{d_1} \; \boxed{3^3 5^2 7^1} \to d_1 \; 2^4$

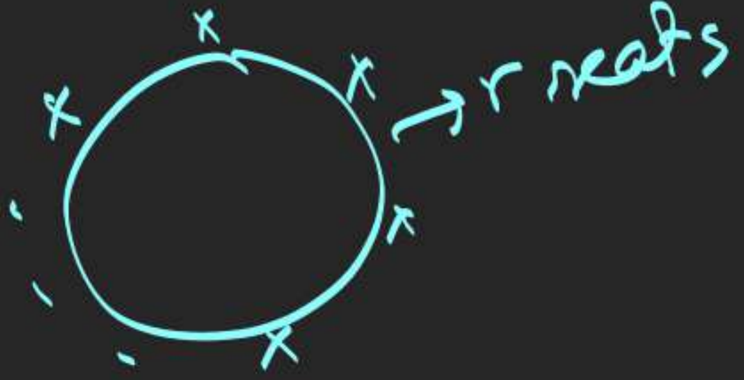
$2^3 3^2$

(iv) $\boxed{d_1} \quad \frac{N}{d_1}$

$2^1 3^1 5^2 7^1$

Nishant Jindal

Circular Permutations of 'n' distinct objects



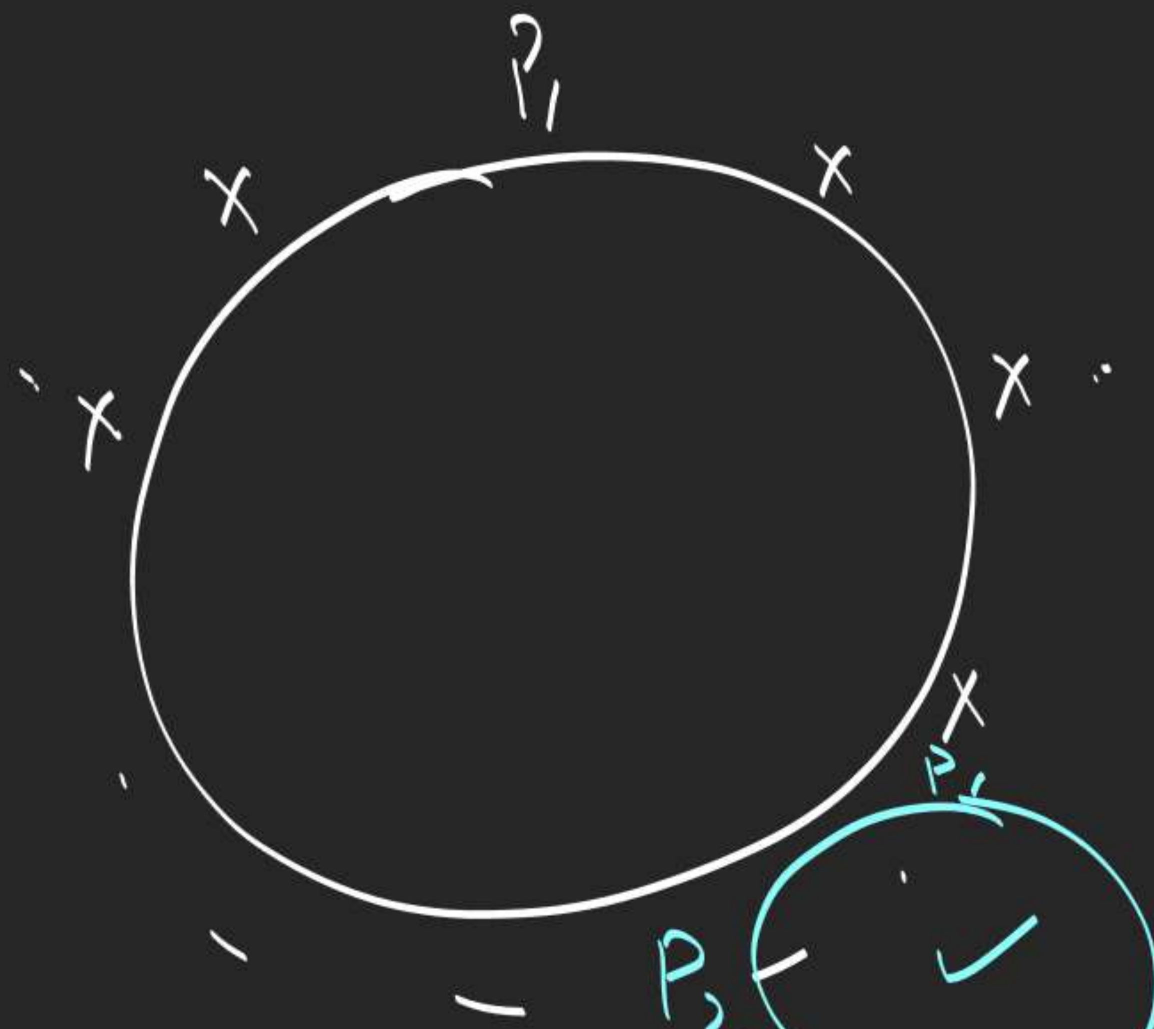
Taken all at a time

- Distinguishing anticlockwise and clockwise arrangements $= (n-1)!$
- not distinguishing anticlockwise and clockwise arrangements $= \frac{(n-1)!}{2}$

Taken some at a time (say 'r', $0 \le r \le n$)

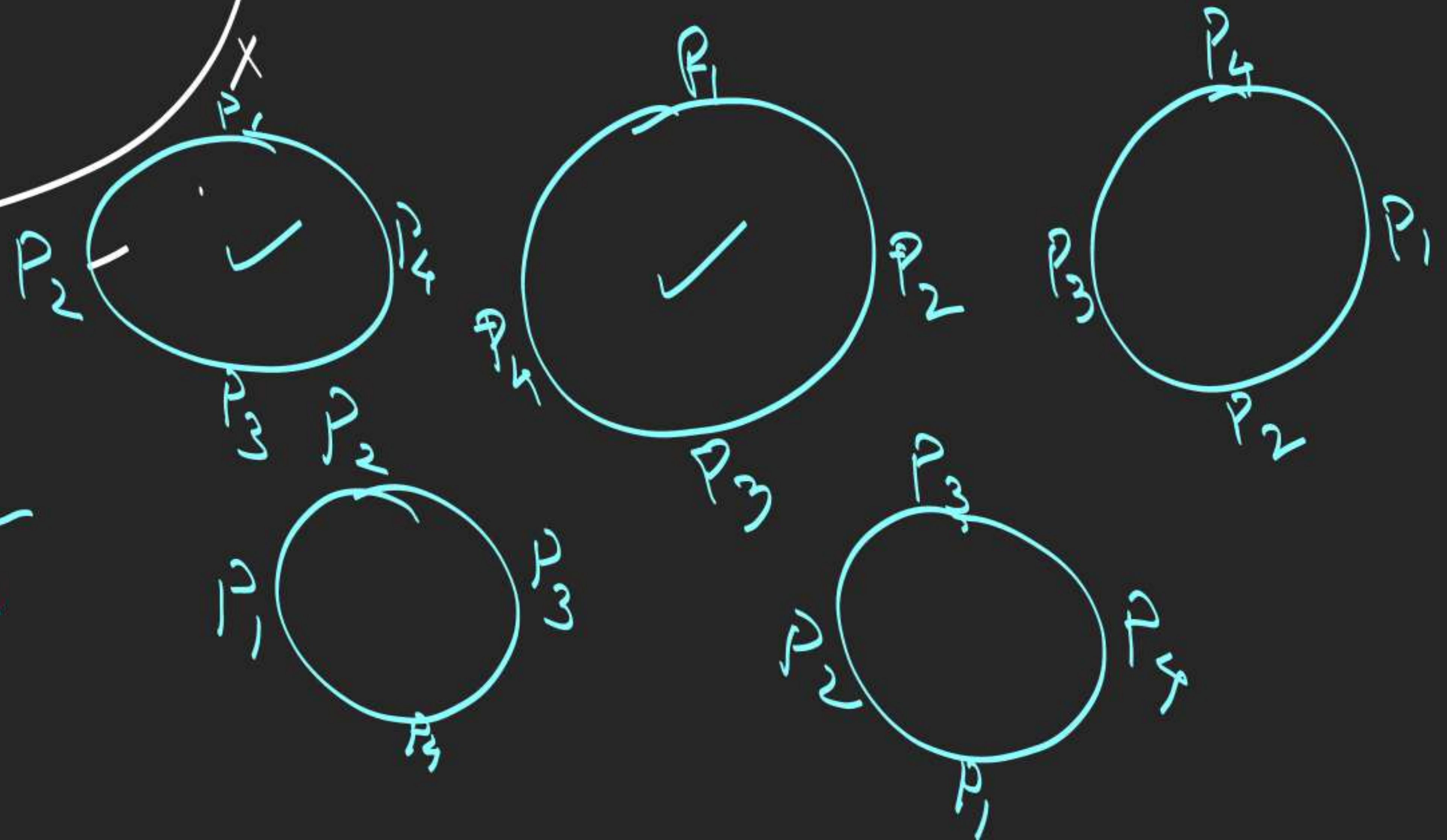
- Distinguishing anticlockwise and clockwise arrangements $= {}^{n}C_r \,(r-1)!$
- not distinguishing anticlockwise and clockwise arrangements $= \frac{{}^{n}C_r \,(r-1)!}{2}$

Nishant Jindal



P_1 sit $\rightarrow$ 1 way

$1 \times (n-1)!$



$\frac{4!}{4}$