

$$\begin{aligned}
 k \sin \alpha &= \sin(2x - \alpha) \quad [-1 \leq k \sin \alpha \leq 1] \\
 \sin\left(\frac{\pi}{4} + \frac{\alpha}{2}\right) &= \sin x \quad x \in (0, \pi) \\
 \sin x &= 0 \quad \alpha = 2x - n\pi \\
 -n\pi < \alpha < 2\pi - n\pi \\
 (1+k)(\cos(\beta x - \alpha) + \cos(x - \alpha)) &= 2 \cos(x - \alpha) + \\
 &\quad k(\cos(\beta x - \alpha) + \cos(x + \alpha)) \\
 \Rightarrow k \left(\cos(x - \alpha) - \cos(x + \alpha) \right) &= \cos(x - \alpha) - \cos(\beta x - \alpha) \\
 2k \sin x \sin \alpha &= 2 \sin x \sin(kx - \alpha)
 \end{aligned}$$

$$(1-\cos 2x)^5 + (1+\cos 2x)^5 = 58 \\ = 29$$

$$\frac{\partial}{\partial n} f(x) = 0 \quad \forall n \in \mathbb{R} : \\ n \cos^3 y + 3n \cos y \sin^2 y = 14$$

$$x \sin^3 y + 3n \cos y \sin^2 y = 13$$

$$= \frac{(3+2\tan^2 \theta) - \tan x \tan \theta}{\tan^2 x - \tan^2 \theta} \cdot n (\cos y + \sin y)^3 = 27$$

~~$x - \cancel{x} - \cancel{x^2} + \cancel{x^2} + x^2 y + \cancel{x^2 y}$~~

~~$+ \cancel{x^2} + \cancel{x^2 y} + x^2 y + \cancel{x^2 y}$~~

$$\boxed{\tan^2 \theta = 3}$$

$$n (\cos y - \sin y)^3 = 1$$

$$\frac{3 \tan x + 2 \tan x \tan^2 \theta - \tan^3 \theta + \tan^2 x - \tan \theta}{(\tan^2 x - \tan^2 \theta) \tan x} = \frac{\frac{\tan^2 x + \tan x - \tan \theta}{1 + \tan x \tan \theta} + \tan x + \frac{\tan x + \tan \theta}{1 - \tan x \tan \theta}}{\frac{(\tan^2 x - \tan^2 \theta) \tan x}{(1 - \tan x \tan \theta)}}$$

$$\underline{19.} \quad \sin x \cos 2x = \sin 2x \cos 3x - \frac{1}{2} \sin 5x$$

$$\sin 3x - \cancel{\sin x} = \cancel{\sin 2x} - \sin x \Rightarrow \boxed{\sin 3x = 0}$$

$$\frac{n\pi}{3}$$

$$\sin x (3 - 4 \sin^2 x)$$

$$a \cos 2x + |a| \cos 4x = 1 - \cos 6x$$

$$= 2 \sin^2 3x$$

$\boxed{(a \in (-\infty, -1) \cup \{0\})}$

$$a + |a| = 0 \Rightarrow \boxed{a \leq 0}$$

$$2a \sin 3x \sin x = 2 \sin^2 3x$$

$$\boxed{[3 - 4, 3]}$$

$$a \sin x = \sin 3x$$

$$a = 3 - 4 \sin^2 x$$

$$\cos\left(\frac{\pi}{2} - \cos x\right) = \cos(\sin x)$$

$$\sin x = 2n\pi + \frac{\pi}{2} - \cos x \quad \text{or} \quad 2n\pi - \frac{\pi}{2} + \cos x$$

$$\begin{matrix} \sin x + \cos x = 2n\pi + \frac{\pi}{2} \\ -1 \quad -1 \end{matrix} \quad \text{or} \quad \begin{matrix} \sin x - \cos x = 2n\pi - \frac{\pi}{2} \\ -1 \quad -1 \end{matrix}$$

12. $\sin x \cos 2y = (q^2 - 1)^2 + 1 \geq 1$

$\sin x \cos 2y = 1$
 $\cos x \cos 2y = 0$

$q = -1$

$$\alpha + \beta = \tan \theta - 2$$

$$\alpha \beta = -1 - \tan \theta$$

$$\alpha + \beta + \alpha \beta = -3$$

$$\begin{array}{cc} (\alpha+1) & (\beta+1) \\ \frac{-1}{-1} & \frac{-1}{-1} \\ \frac{2}{2} & \frac{2}{2} \\ \frac{1}{1} & \frac{-1}{-1} \end{array} = -2$$

1. Simplify

$$\frac{(a^2 + b^2 + c^2)(a-b)(b-c)}{(c-a)(a+b+c)} \left| \begin{array}{l} 1 \\ 2\cos\theta \\ 4\cos^2\theta - 1 \\ = 3 - 4\sin^2\theta \end{array} \right.$$

$$(a^2 + b^2 + c^2) \left| \begin{array}{ccc} 1 & a & a \\ 1 & b & b^3 \\ 1 & c^2 & c^3 \end{array} \right.$$

(1)

$$(a^2 + b^2 + c^2) \left| \begin{array}{ccc} a & a^3 & 1 \\ b & b^3 & . \\ c & c^3 & . \end{array} \right.$$

Sin 3θ

$$\sin 6\theta \\ \sin 9\theta$$

Sin³ θ

$$\sin^3 2\theta \\ \sin^3 3\theta$$

$$C_2 \rightarrow C_2 - 3\sin\theta C_1 + 4C_3$$

II



$$\left| \begin{array}{ccc} 1 & a^2 & bc \\ 1 & b^2 & ca \\ 1 & c^2 & ab \end{array} \right| \xrightarrow[C_1 \rightarrow C_1 + C_2 - 2C_3]{(a^2 + b^2 + c^2)} \left| \begin{array}{ccc} 1 & a^2 & bc \\ 1 & b^2 & ca \\ 1 & c^2 & ab \end{array} \right|$$

$$(a^2 + b^2 + c^2) \left| \begin{array}{ccc} a & a^3 & abc \\ b & b^3 & abc \\ c & c^3 & abc \end{array} \right.$$

$$\Delta = \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} = (x-y)(y-z)(z-x)$$

$\xrightarrow{R_2 \rightarrow R_2 - R_1}$ $\xrightarrow{R_3 \rightarrow R_3 - R_1}$

$$(y-x)(z-x) \begin{vmatrix} 1 & x & x^2 \\ 0 & y-x & y^2-x^2 \\ 0 & z-x & z^2-x^2 \end{vmatrix} = (y-x)(z-x)(z-y)$$

$x=y$

$y=x$

$z=y$

$\boxed{\Delta=0}$

$$\Delta = \begin{vmatrix} 1 & x & x^3 \\ 1 & y & y^3 \\ 1 & z & z^3 \end{vmatrix} = (x-y)(y-z)(z-x)(x+y+z)$$

$$\Delta = \kappa (x-y)(y-z)(z-x)$$

$\kappa = 0, y=1, z=-1$

$$\kappa = 1 \quad \Leftarrow R(-1)(2)(-1) = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = 1$$

$$\begin{vmatrix} 1 & x^2 & x^3 \\ 1 & y^2 & y^3 \\ 1 & z^2 & z^3 \end{vmatrix} = (x-y)(y-z)(z-x)(xy+yz+zx)$$

Explain

$$\begin{vmatrix} a-b-c & & & \\ b-c-a & & & \\ c-a-b & & & \\ & & & \end{vmatrix} = 3abc - a^3 - b^3 - c^3$$

$$= a(bc-a^2) - b(b^2-ac) + c(ab-c^2)$$

$$= abc - a^3 - b^3 - c^3$$

$$\text{Ex: P.T.} \quad \left| \begin{array}{ccc|c} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{array} \right| = abc \left(1 + \frac{1}{a} + \frac{1}{b} + \frac{1}{c} \right)$$

$$= abc \left| \begin{array}{ccc|c} 1+\frac{1}{a} & \frac{1}{a} & \frac{1}{a} \\ \frac{1}{b} & 1+\frac{1}{b} & \frac{1}{b} \\ \frac{1}{c} & \frac{1}{c} & 1+\frac{1}{c} \end{array} \right| \xrightarrow{R_1 \rightarrow R_1 + R_2 + R_3} abc \left(1 + \frac{1}{a} + \frac{1}{b} + \frac{1}{c} \right) \left| \begin{array}{ccc|c} 1 & 1 & 1 \\ \frac{1}{b} & 1+\frac{1}{b} & \frac{1}{b} \\ \frac{1}{c} & \frac{1}{c} & 1+\frac{1}{c} \end{array} \right|$$

$$\boxed{abc \left(1 + \frac{1}{a} + \frac{1}{b} + \frac{1}{c} \right)} = abc \left(1 + \frac{1}{a} + \frac{1}{b} + \frac{1}{c} \right) \left| \begin{array}{ccc|c} 0 & 0 & 0 \\ \frac{1}{b} & 1 & 0 \\ \frac{1}{c} & 0 & 1 \end{array} \right|$$

$$\xrightarrow{C_2 \rightarrow C_2 - C_1} \xleftarrow{C_3 \rightarrow C_3 - C_1}$$

Trig. Egn

Ex-III, IV ✓