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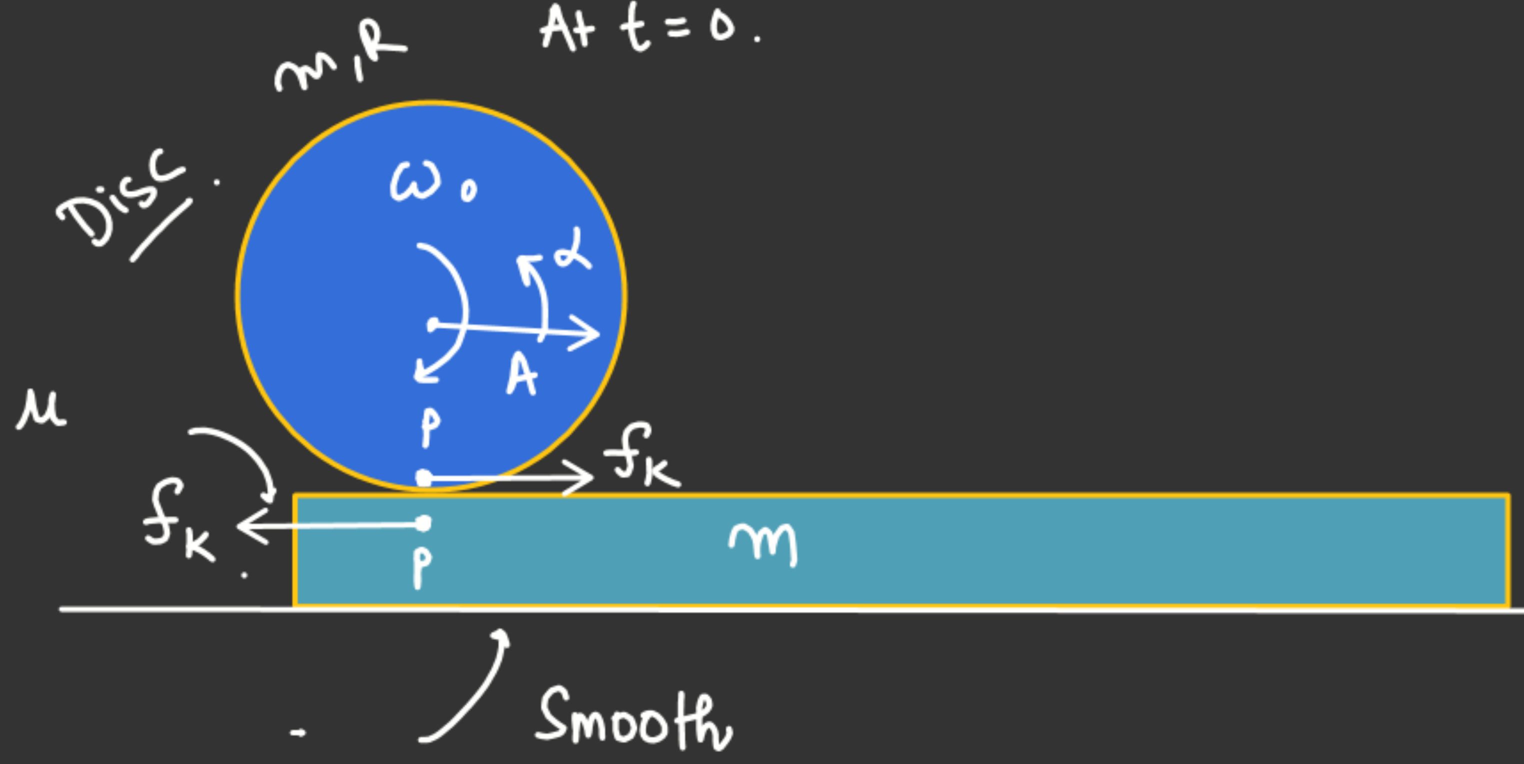
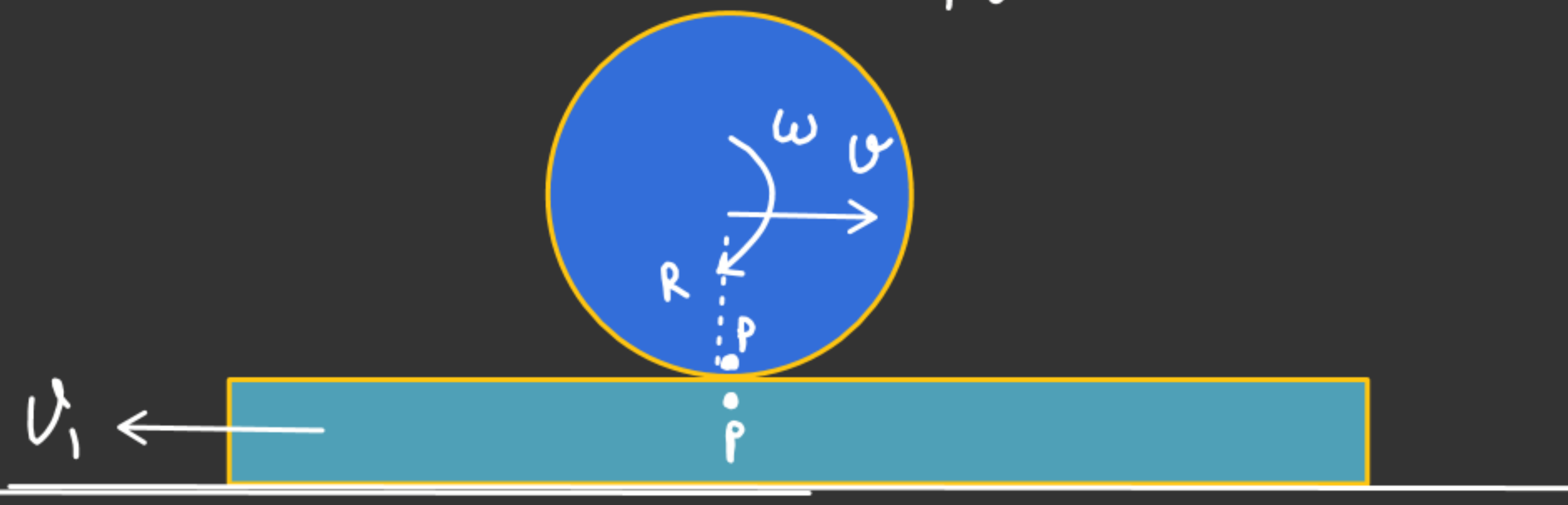
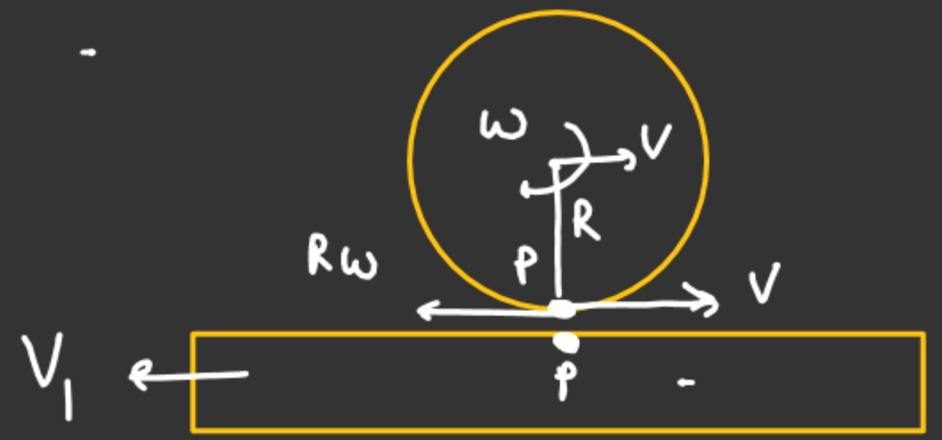
At $t = 0$.



At $t = t$ Pure Rolling of disc





1) Find time when disc starts pure rolling.

2) Distance travelled by disc from $t = 0$ to the time when it start pure rolling.

Condition of pure Rolling

$$R\omega - V = V_1 \quad \text{— (1)}$$

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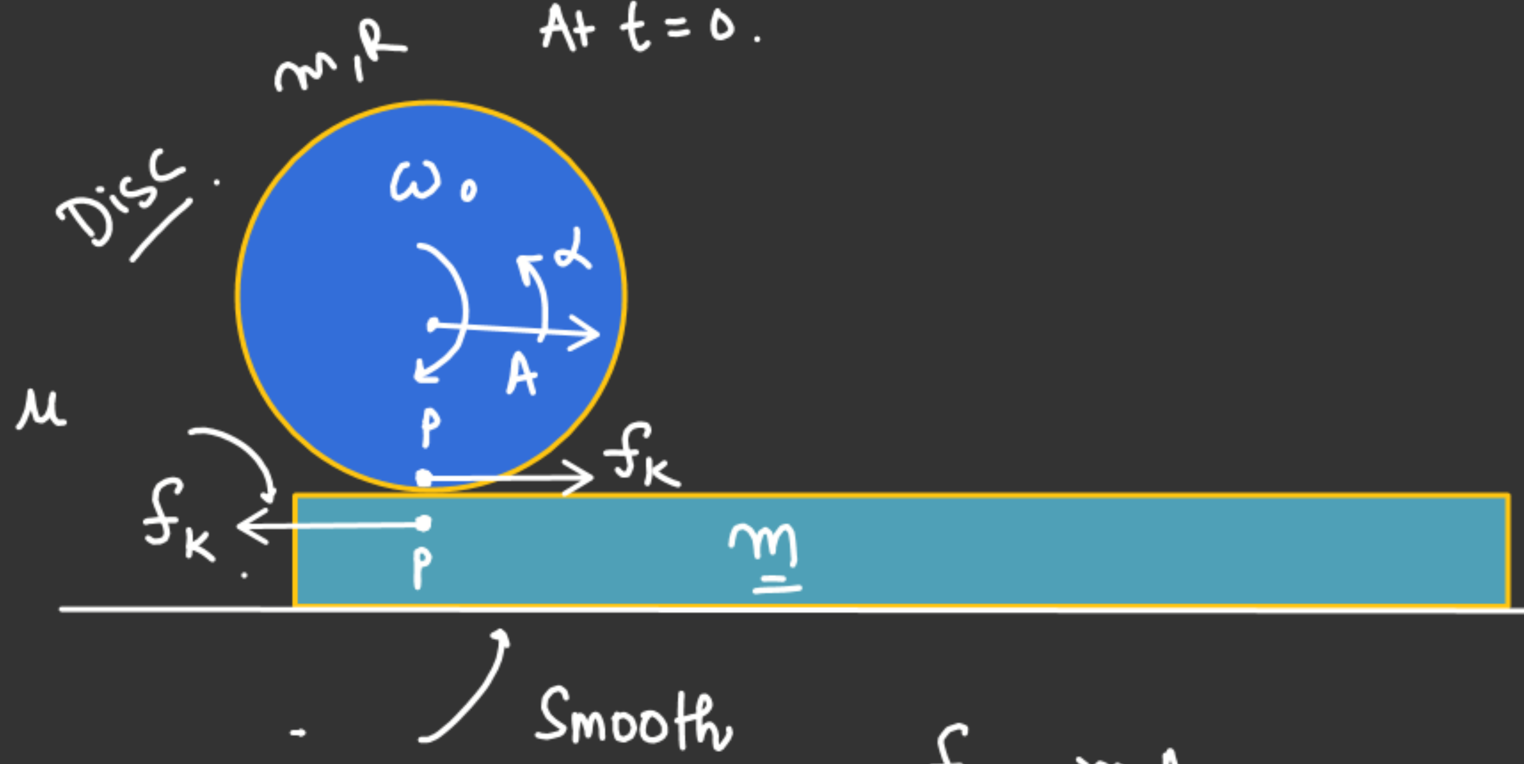
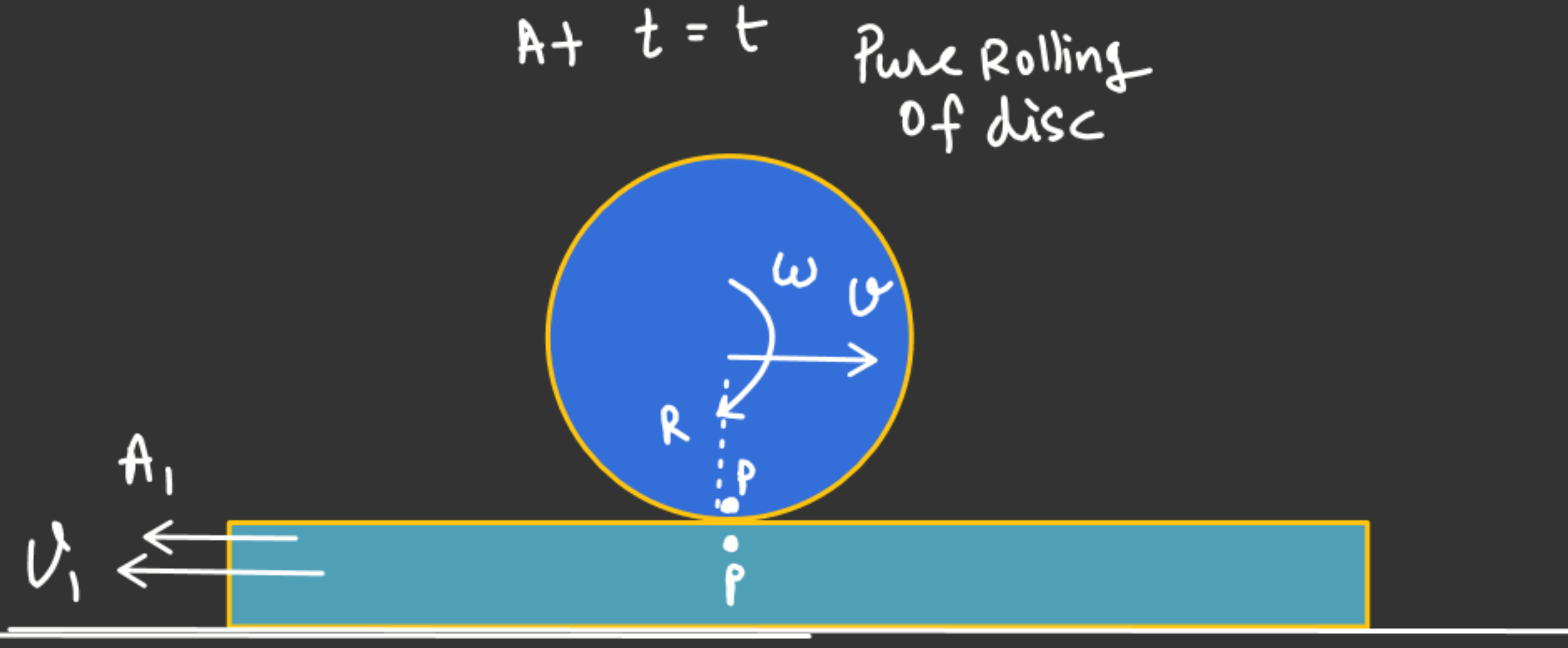




$f_K = mA_1$

$A_1 = \mu g$

$f_K = MA$

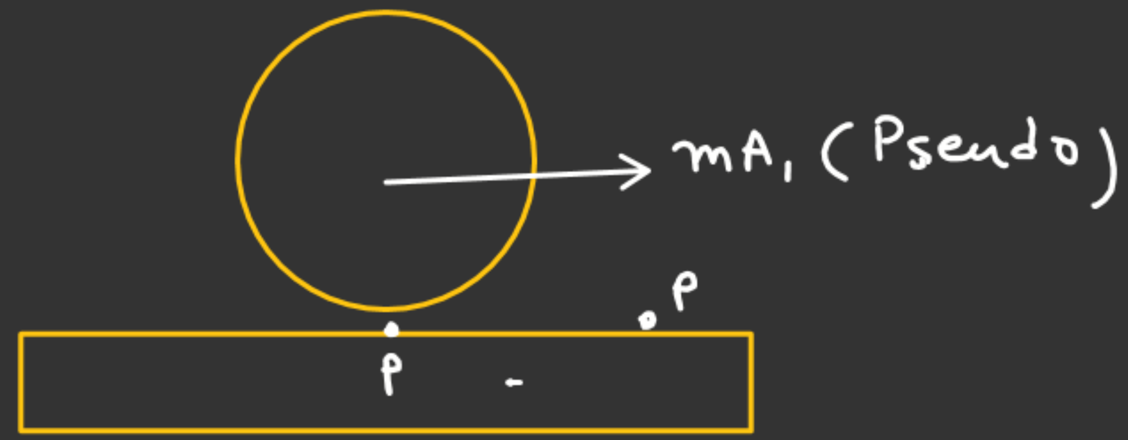
$\mu Mg = MA$

$A = \mu g.$

$\alpha = \dfrac{(f_K \cdot R)}{I} = \dfrac{\mu MgR}{\frac{MR^2}{2}}$

$\alpha = \left(\dfrac{2\mu g}{R}\right)$

$$\left[\begin{aligned} v &= At = \mu g t \\ \omega &= \omega_0 - \alpha t \\ &= \left(\omega_0 - \frac{2\mu g}{R} t\right) \\ v_1 &= A_1 t \\ &= \mu g t \end{aligned}\right.$$



We Cannot Conserve angular momentum of system about P as Torque of pseudo at P.

$(\tau_{net})_P \neq 0$ (w.r.t plank)

$$v = At = \mu g t$$

$$\omega = \omega_0 - \alpha t$$

$$= \left(\omega_0 - \frac{2\mu g}{R} t\right)$$

$$v_1 = A_1 t$$

$$= \mu g t$$

At the time of pure rolling

$$R\omega - v = v_1$$

$$R\left(\omega_0 - \frac{2\mu g}{R} t\right) - \mu g t = \mu g t$$

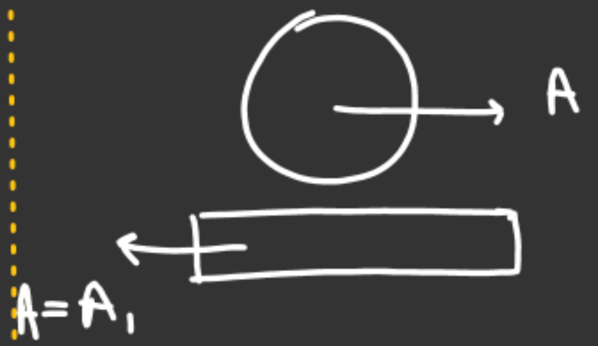
$$R\omega_0 - 2\mu g t - \mu g t = \mu g t$$

$$R\omega_0 = 4\mu g t$$

$$t = \left(\frac{R\omega_0}{4\mu g}\right) \checkmark$$

$$\vec{S}_{disc/g} = \vec{S}_{disc\,plank} + \vec{S}_{plank/g}.$$

$$S_{disc/plank} = ?? = \frac{1}{2}(a_{rel})t^2 = \frac{1}{2} \times 2\mu g \times \left(\frac{R\omega_0}{4\mu g}\right)^2$$



$$a_{rel} = 2A$$
$$= 2\mu g$$

$$= \left(\frac{R^2\omega_0^2}{16\mu g}\right)$$

$$S_{plank/g} = \frac{1}{2}A_1 t^2 = \frac{1}{2}\mu g\left(\frac{R\omega_0}{4\mu g}\right)^2 = \frac{R^2\omega_0^2}{32\mu g}$$

$$\vec{S}_{disc/g} = +\frac{R^2\omega_0^2}{16\mu g}\hat{i} - \frac{R^2\omega_0^2}{32\mu g}\hat{i}$$

$$= \left(\frac{R^2\omega_0^2}{32\mu g}\right)\hat{i}$$

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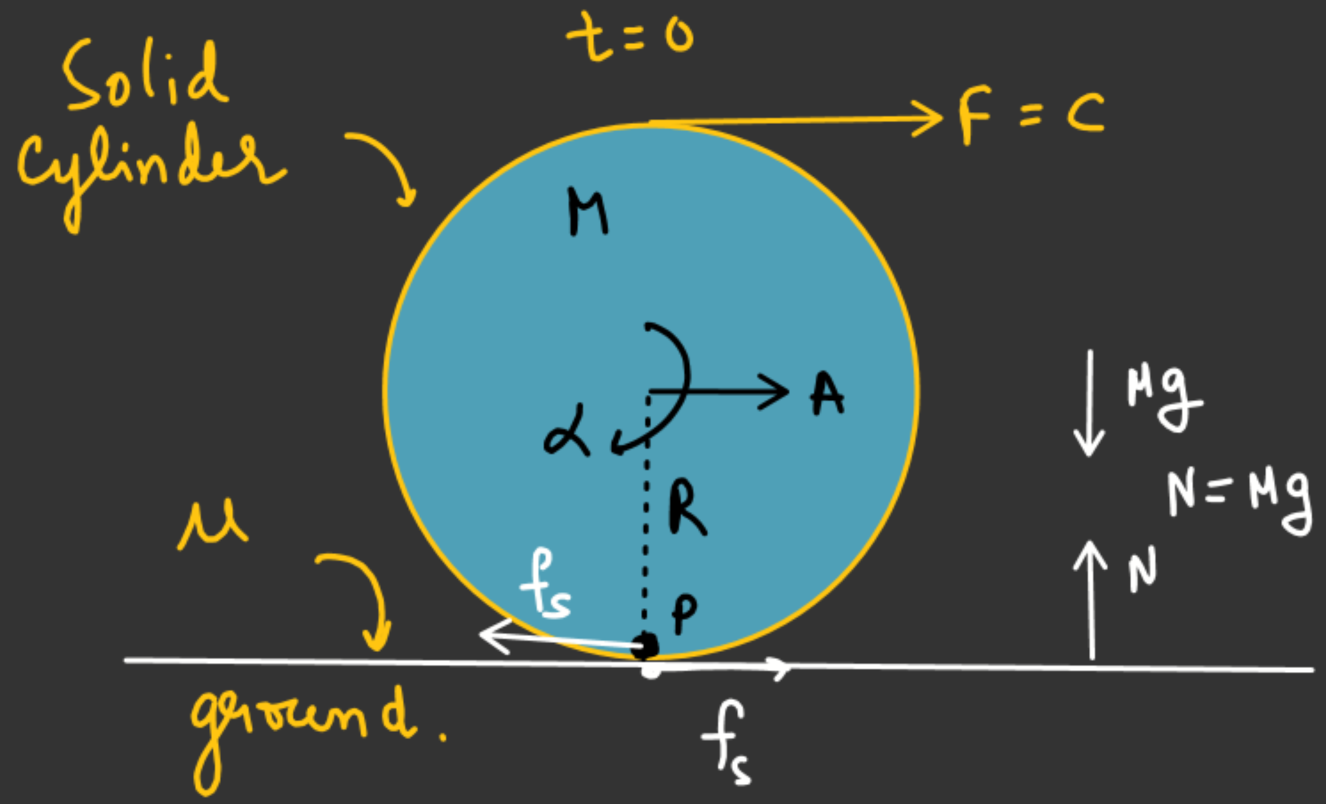
Rolling

Role of Static friction in pure Rolling Motion



Equation of translational Motion.

$$F - f_s = MA \quad \text{— (1)}$$

Equation for Rotational Motion

$$FR + f_s \cdot R = I\alpha = \frac{MR^2}{2}\alpha$$

$$F + f_s = \frac{M}{2} R\alpha \quad \text{— (2)}$$

Condition for pure Rolling

$$A = R\alpha \quad \text{— (3)}$$

$R\alpha \longleftarrow P \longrightarrow A$

From (2) & (3)

$$F + f_s = \frac{MA}{2} \quad \text{— (4)}$$

μ_{min} So that Cylinder starts pure rolling at t=0

A=? , α=? ,

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Equation of translational Motion.

$$F - f_s = MA \quad \text{— ①}$$

Equation for Rotational Motion

$$FR + f_s R = I\alpha = \frac{MR^2}{2}\alpha$$

$$F + f_s = \frac{M}{2} R\alpha \quad \text{— ②}$$

Condition for pure Rolling

$$A = R\alpha \quad \text{— ③}$$

S

$$R\alpha \longleftarrow \underset{P}{\bullet} \longrightarrow A$$

From ② & ③

$$F + f_s = \frac{MA}{2} \quad \text{— ④}$$

① + ④

$$2F = \frac{3}{2}MA$$

$$A = \left(\frac{4F}{3M}\right) \text{Ans.}$$

$$A = R\alpha$$

$$\alpha = \frac{4F}{3MR} \text{ Ans.}$$

$$f_s = F - MA$$

$$= F - \cancel{M}\left(\frac{4F}{3\cancel{M}}\right)$$

$$f_s = \left(-\frac{F}{3}\right)$$

Assumed direction is wrong.

$$f_s \leq (f_s)_{max}$$

$$\frac{F}{3} \leq \mu \cdot Mg.$$

$$\mu \geq \frac{F}{3Mg}$$

$$\mu_{min} = \frac{F}{3Mg} \text{ Ans.}$$

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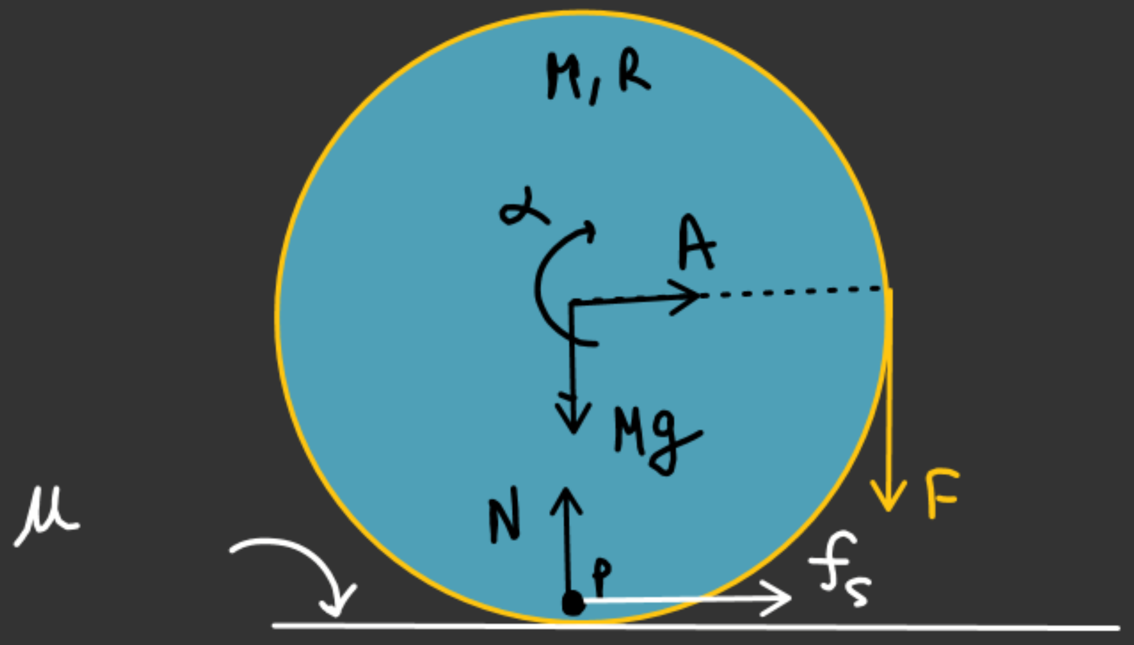
Rolling

★★ Cylinder starts pure rolling as soon as F acts.



$$\left(+f_s \cdot R\right) - \left(F \cdot R\right) = -I\alpha$$

For pure Rolling

$A = R\alpha$ — ①

$f_s = MA$ — ②

$F \cdot R - f_s \cdot R = \frac{MR^2}{2}\alpha$

$F - f_s = \frac{M}{2}R\alpha$ — ③

Find

$A = ?$

$\alpha = ?$

$\mu_{min} = ?$

$N = (F + Mg)$

① & ③

$F - f_s = \frac{M}{2}A$ — ④

② & ④

$F = \frac{MA}{2} + MA$

$F = \frac{3}{2}MA \quad \left(A = \frac{2F}{3M}\right)$

$f_s = MA = \left(\frac{2F}{3}\right)$ ✓

$f_s \leq (f_s)_{max}$

$\frac{2F}{3} \leq \mu(F + Mg)$

$\mu \geq \frac{2F}{3(F+Mg)}$

$\mu_{min} = \frac{2F}{3(F+Mg)}$ ✓

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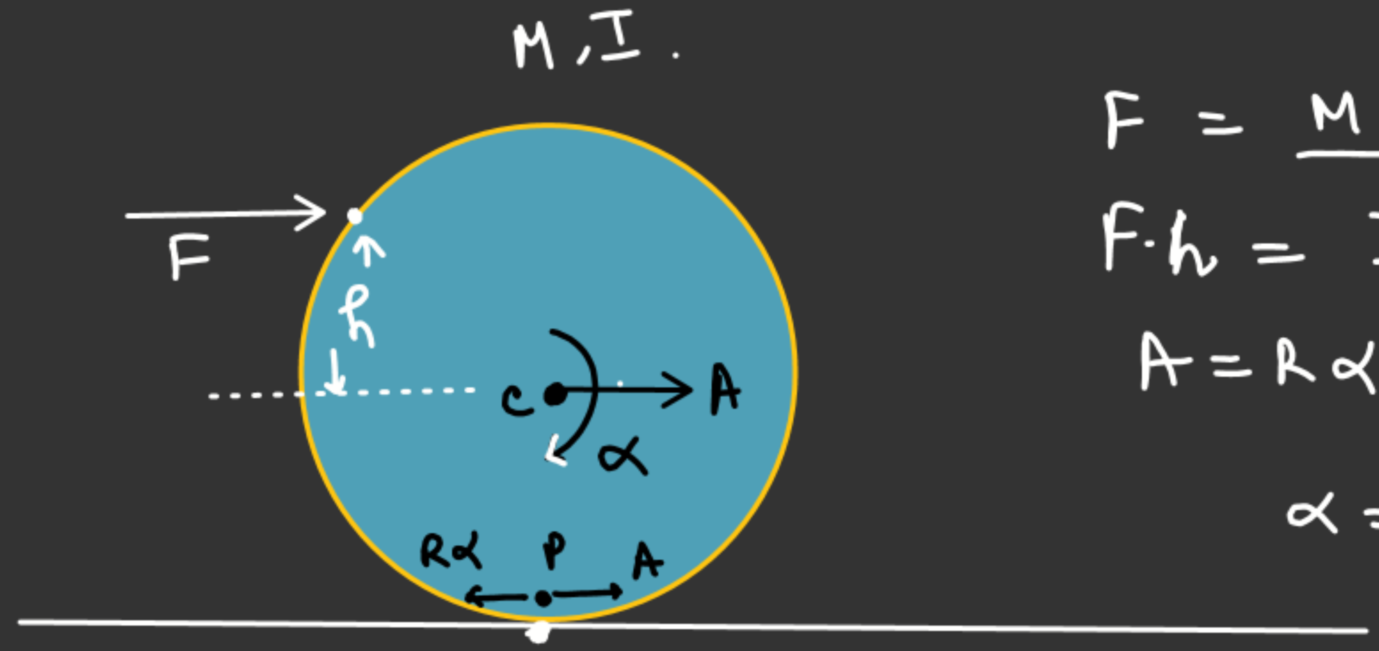
Rolling



$$F = MA \quad - (1)$$

$$F \cdot h = I\alpha \quad - (2)$$

$$A = R\alpha \quad - (3)$$

$\alpha = \left(\frac{A}{R}\right)$, put in (2)

$$Fh = \frac{IA}{R}$$

$$\cancel{F}h = \frac{I}{R}\left(\frac{\cancel{F}}{M}\right)$$

$$\boxed{h = \frac{I}{MR}}$$

If $(A = R\alpha)$
then no tendency of relative slipping of point P so, f_s doesn't act

Ex:-

For Ring, $I = MR^2$

$$h = \frac{MR^2}{MR} = R$$

For Disc, $I = \frac{MR^2}{2}$

$$h = \frac{R}{2}$$

For solid sphere

$$I = \frac{2}{5}MR^2$$

$$\left(h = \frac{2}{5}R\right)$$

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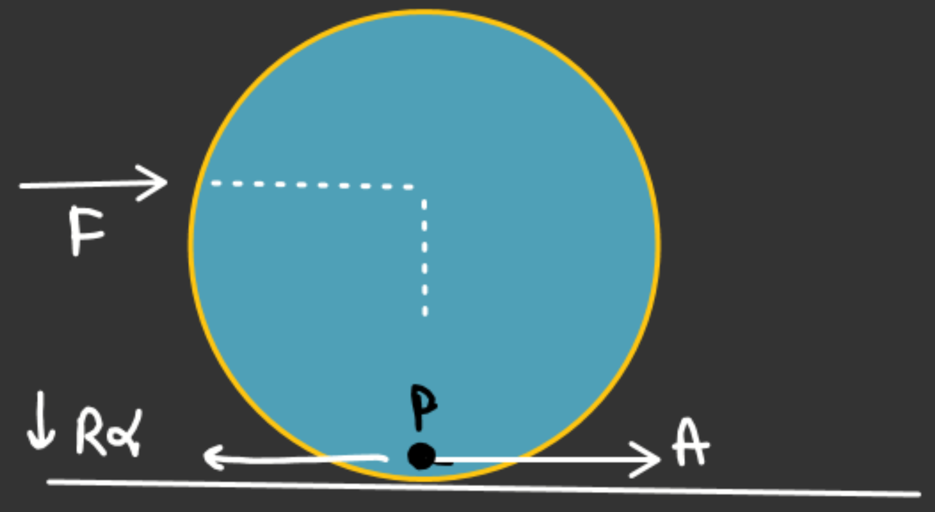
Rolling

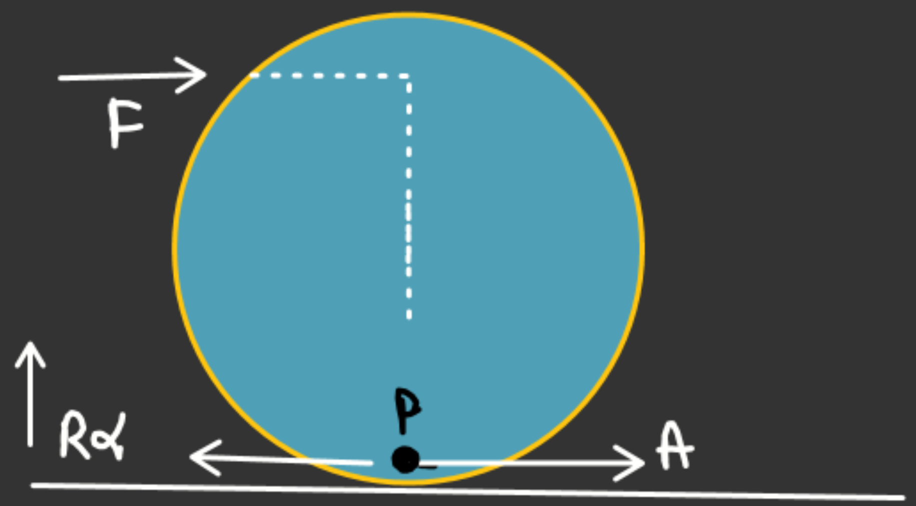
If $\left(h < \frac{I}{MR}\right)$



$A > R\alpha$

$\Rightarrow$ Point P has forward slipping so f_s acts backward.

$h > \frac{I}{MR}$



$R\alpha > A$

Point P has a tendency of backward slipping so f_s acts forward direction.

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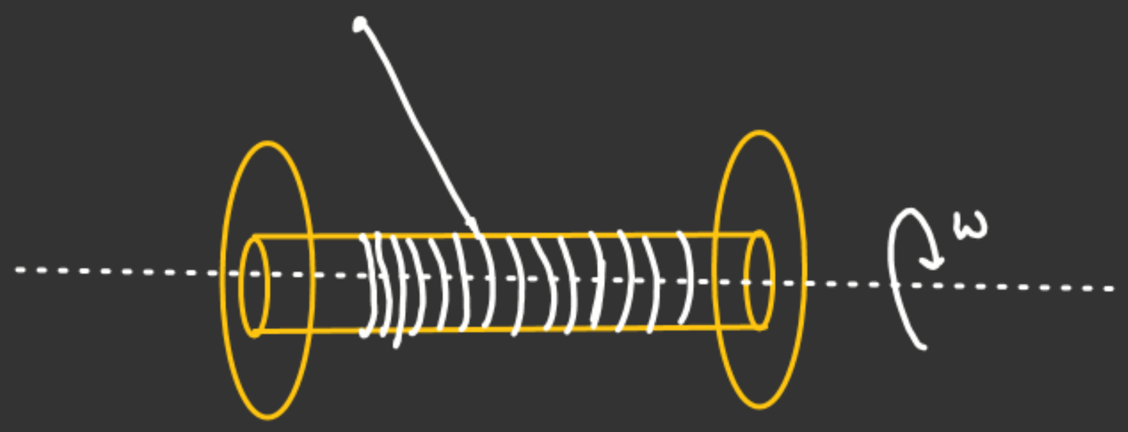
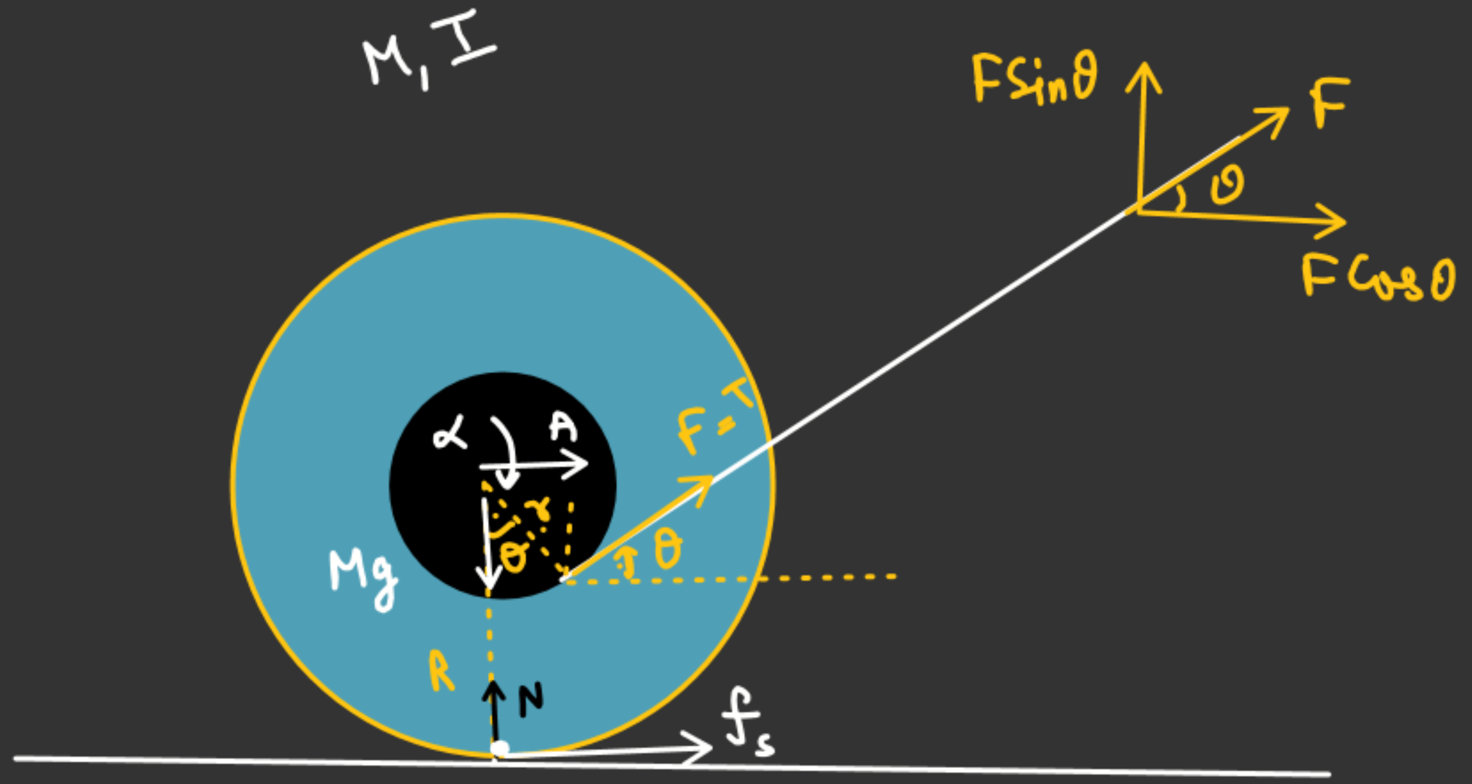
Rolling

Spool of thread : →





Condition for pure rolling

$$A = R\alpha \quad — (1)$$

Translational Motion

$$F\cos\theta + f_s = MA \quad — (2)$$

Rotational Motion

$$F.r + f_s.R = -I\alpha$$

$$Fr + f_s R = -\frac{IA}{R}$$

$$\frac{Fr}{R} + f_s = -\frac{IA}{R^2} \quad — (3)$$

(2) − (3)

$$F\cos\theta - \frac{Fr}{R} = MA + \frac{IA}{R^2}$$

$$F\left(\cos\theta - \frac{r}{R}\right) = MA\left(1 + \frac{I}{MR^2}\right)$$

$$\boxed{A = \frac{F}{M}\frac{\left(\cos\theta - r/R\right)}{\left(1 + \frac{I}{MR^2}\right)}}$$

$A > 0 \Rightarrow \cos\theta > \frac{r}{R}$

$A = 0 \Rightarrow \cos\theta = \frac{r}{R}$

$A < 0 \Rightarrow \cos\theta < r/R$

H.W $f_s = ??$