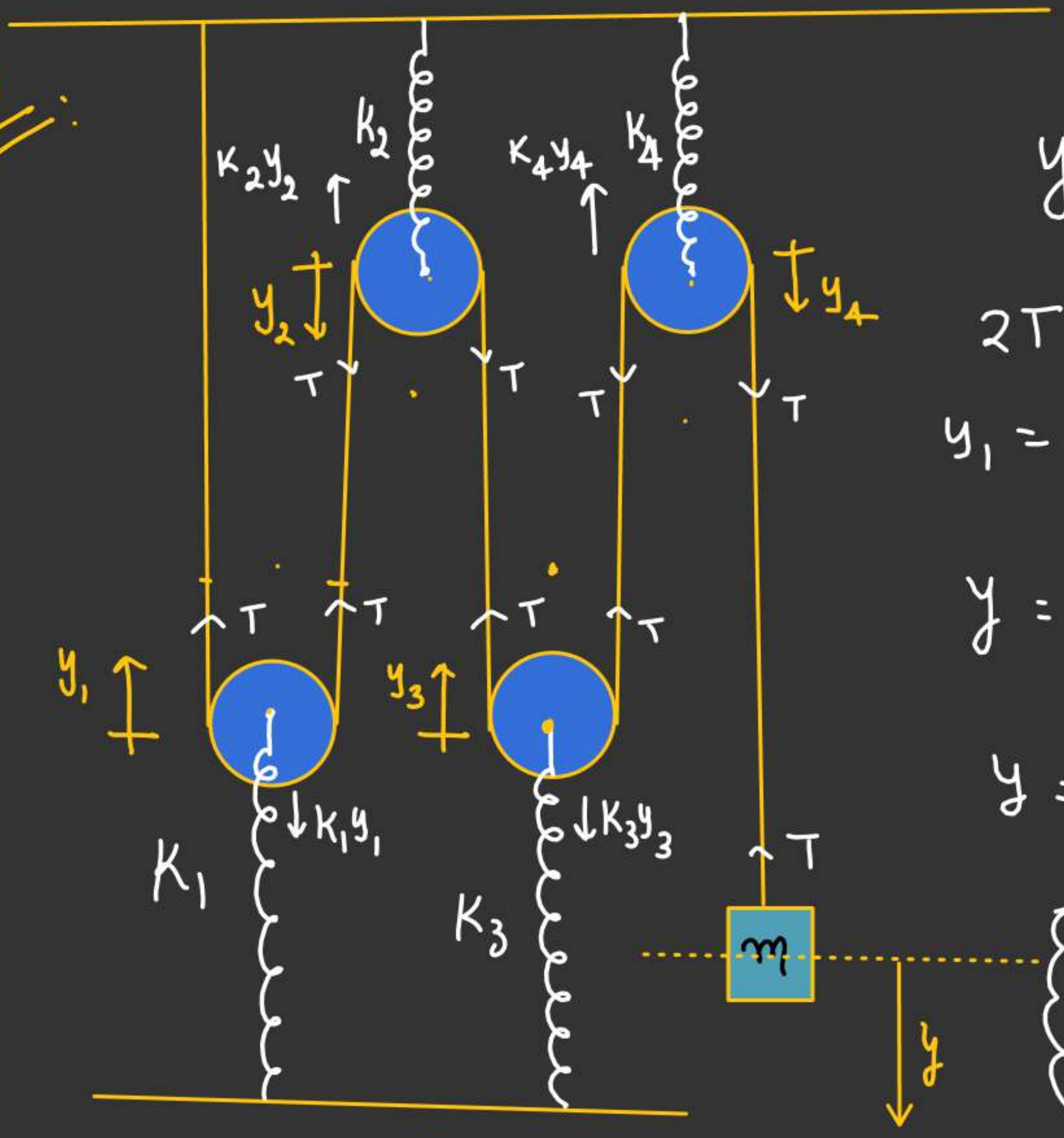
Nishant Jindal

H.W :



Time period of block = ??

$$y = 2(y_1 + y_2 + y_3 + y_4)$$

$$2T = K_1 y_1 = K_2 y_2 = K_3 y_3 = K_4 y_4$$

$$y_1 = \frac{2T}{K_1}, \quad y_2 = \frac{2T}{K_2}, \quad y_3 = \frac{2T}{K_3}, \quad y_4 = \frac{2T}{K_4}$$

$$y = 2\left[\frac{2T}{K_1} + \frac{2T}{K_2} + \frac{2T}{K_3} + \frac{2T}{K_4}\right]$$

$$y = 4\left[\frac{1}{K_1} + \frac{1}{K_2} + \frac{1}{K_3} + \frac{1}{K_4}\right] T$$

$$\frac{1}{4\left[\frac{1}{K_1} + \frac{1}{K_2} + \frac{1}{K_3} + \frac{1}{K_4}\right]} y = T \Rightarrow K.$$

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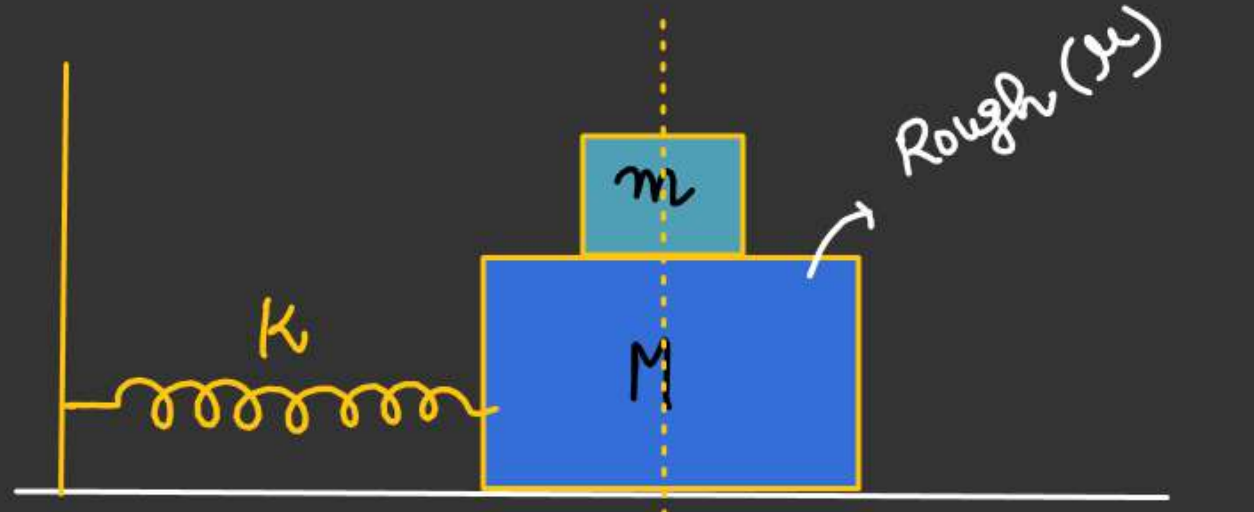
Q: Find A_{max} so that both the blocks oscillate together.



$a = -\omega^2 x.$ $\omega = \sqrt{\frac{K}{M+m}}$.

when x becomes maximum i.e $x = A$.

$a_{max} = \omega^2 A.$

$f_s = ma$

$f_s = (m\omega^2 A)$

$f_s \le (f_s)_{max}$

$\cancel{m}\omega^2 A \le \mu \cancel{m} g$

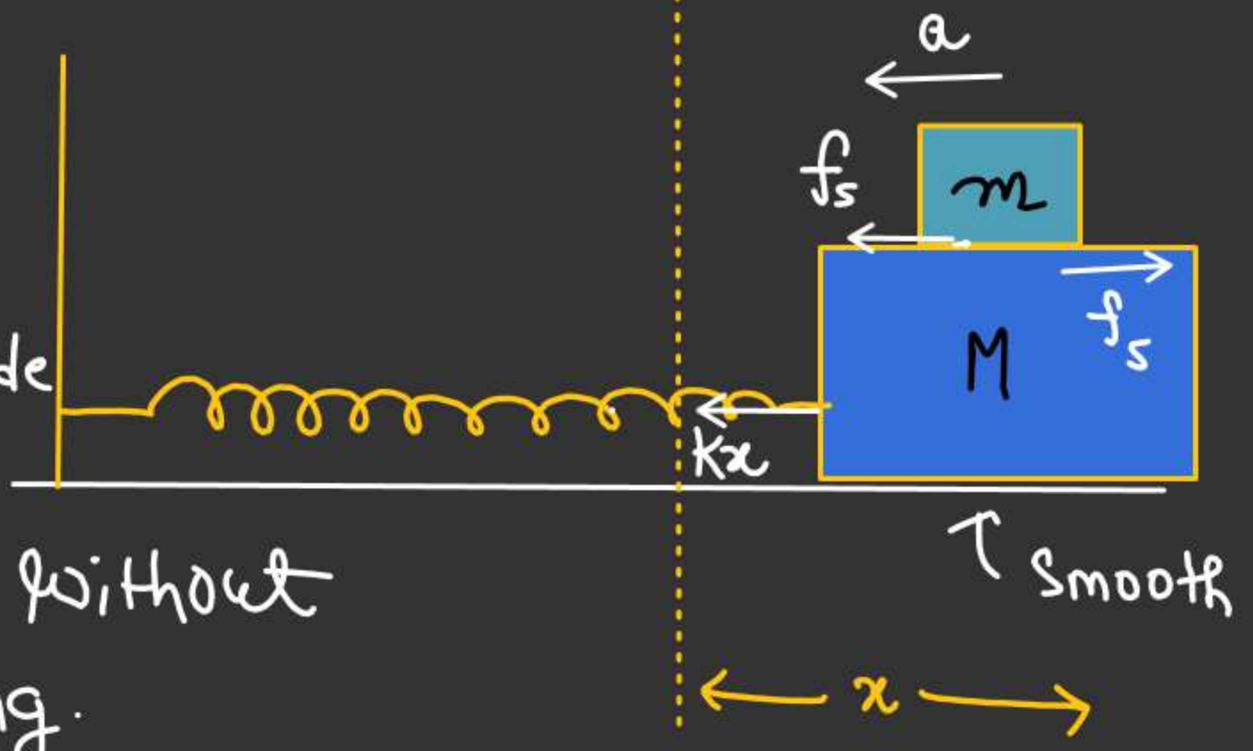
$A \le \frac{\mu g}{\omega^2}$

$A \le \frac{\mu g (M+m)}{K}$

$A_{max} = \frac{\mu g (M+m)}{K}$

$\Downarrow$

Maximum Amplitude through which both the blocks without relative slipping.



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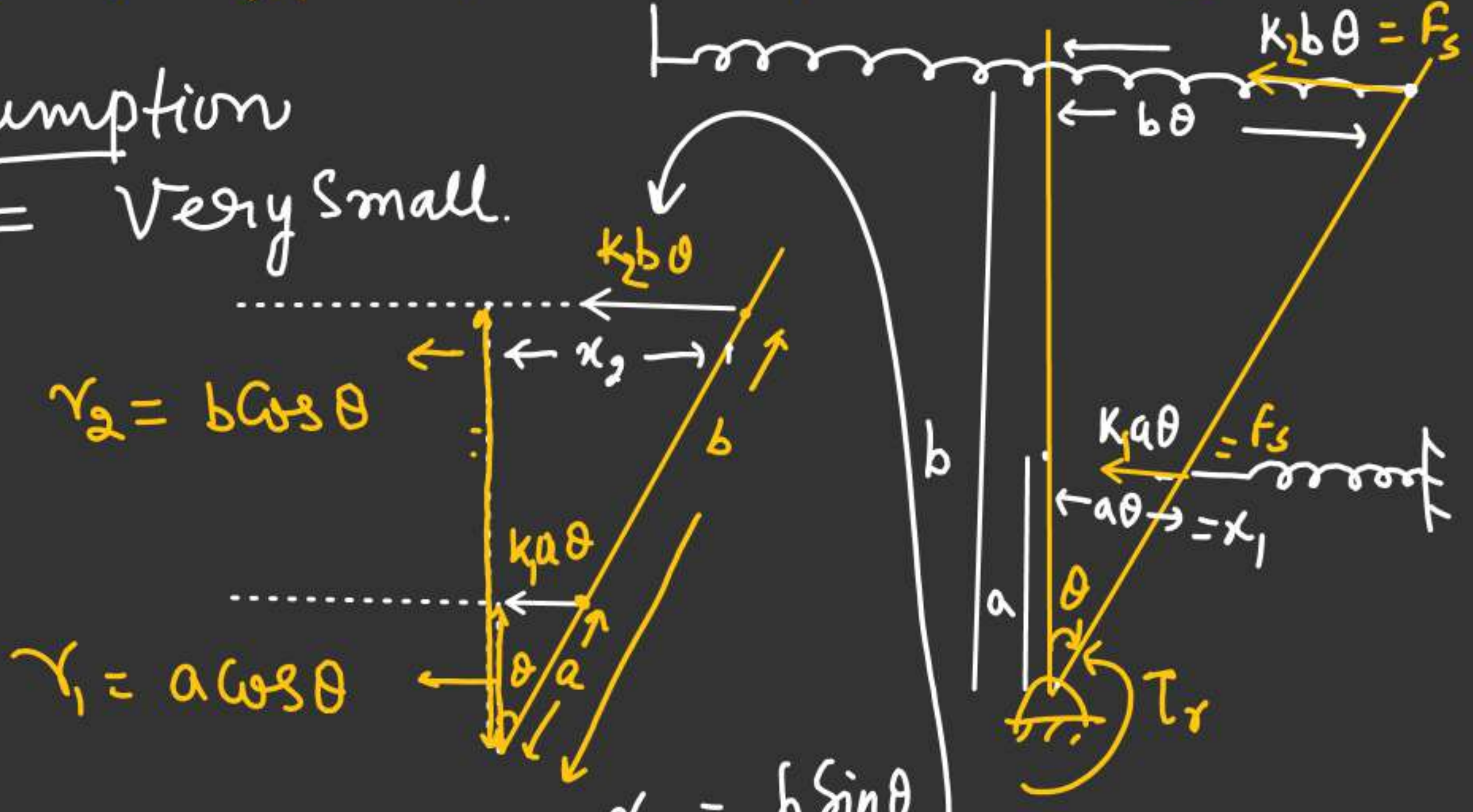
The whole System is on a horizontal plane. Initially rod is vertical and Springs at its natural length. If rod is slightly displaced & released find $T = ??$ M = Mass of Rod, L = length of Rod.

Assumption

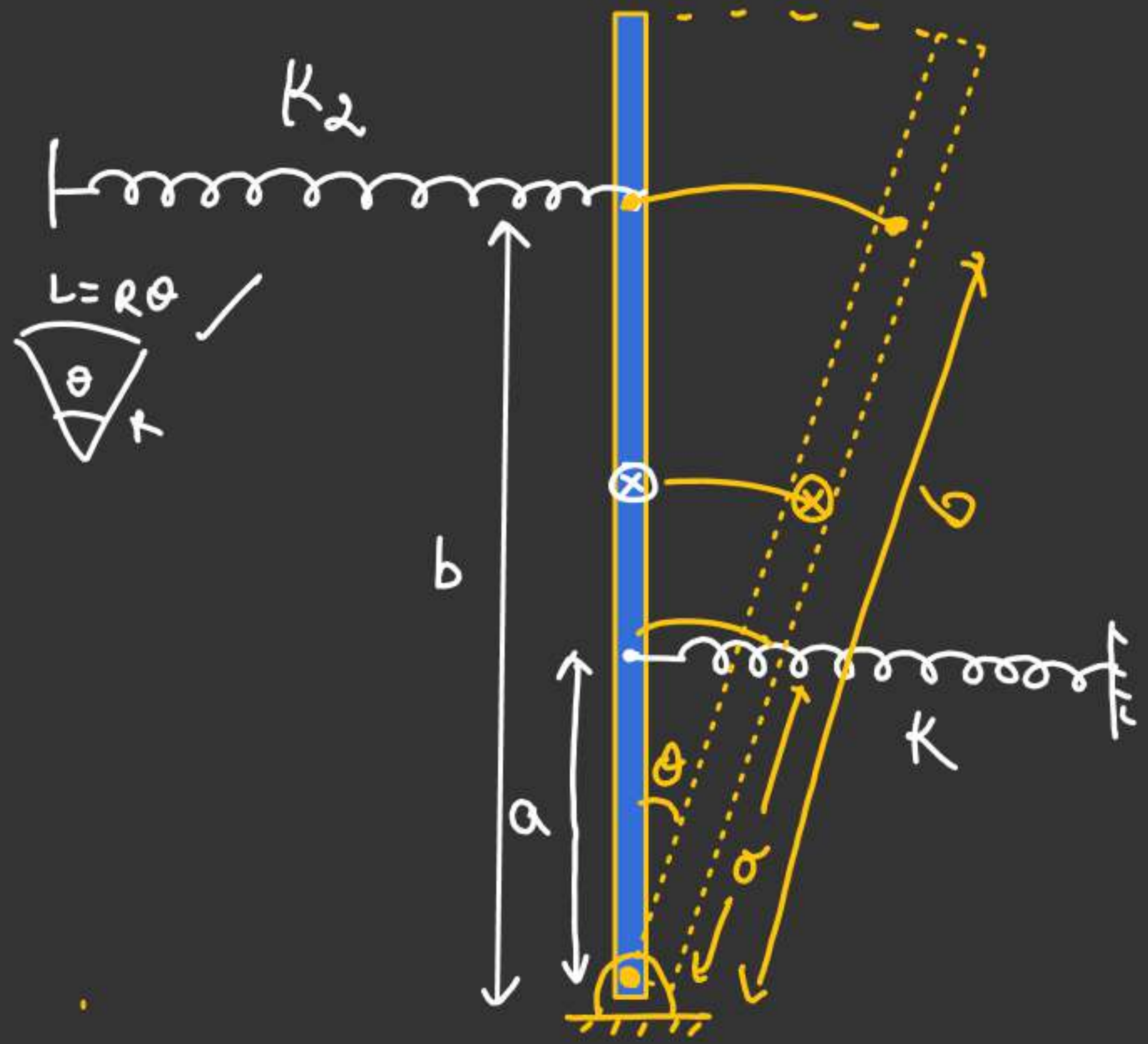
θ = Very Small.



$x_2 = b\sin\theta$

$\sin\theta \simeq \theta$

$x_2 = b\theta$



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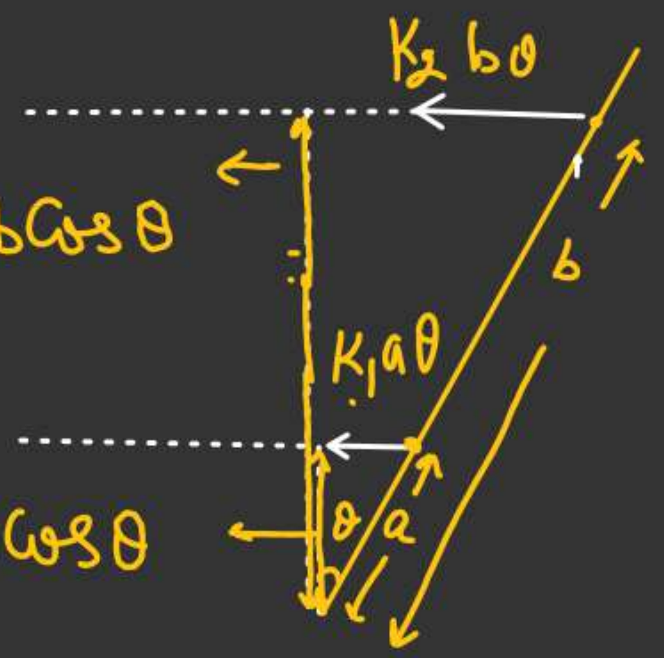

Since θ is very small so

$a\cos\theta \simeq a \qquad \cos\theta \to 1$

$b\cos\theta \simeq b \qquad \theta \to 0$

$$\tau_r = -\left[K_1 a\theta\, \underline{r_1} + K_2 b\theta \cdot \underline{r_2}\right]$$

$$\tau_r = -\left[K_1 a\theta (a) + K_2 b\theta (b)\right]$$

$$\tau_r = -\left[K_1 a^2 + K_2 b^2\right]\theta.$$

$$\alpha = \frac{\tau_r}{I} = -\frac{\left[K_1 a^2 + K_2 b^2\right]\theta}{\frac{ML^2}{3}}$$

$$\alpha = -\left\{\frac{3(K_1 a^2 + K_2 b^2)}{ML^2}\right\}\theta$$

$$\alpha = -\omega^2 \theta$$

$$\omega = \sqrt{\frac{3(K_1 a^2 + K_2 b^2)}{ML^2}}$$

$$T = 2\pi\sqrt{\frac{ML^2}{3(K_1 a^2 + K_2 b^2)}} \checkmark$$

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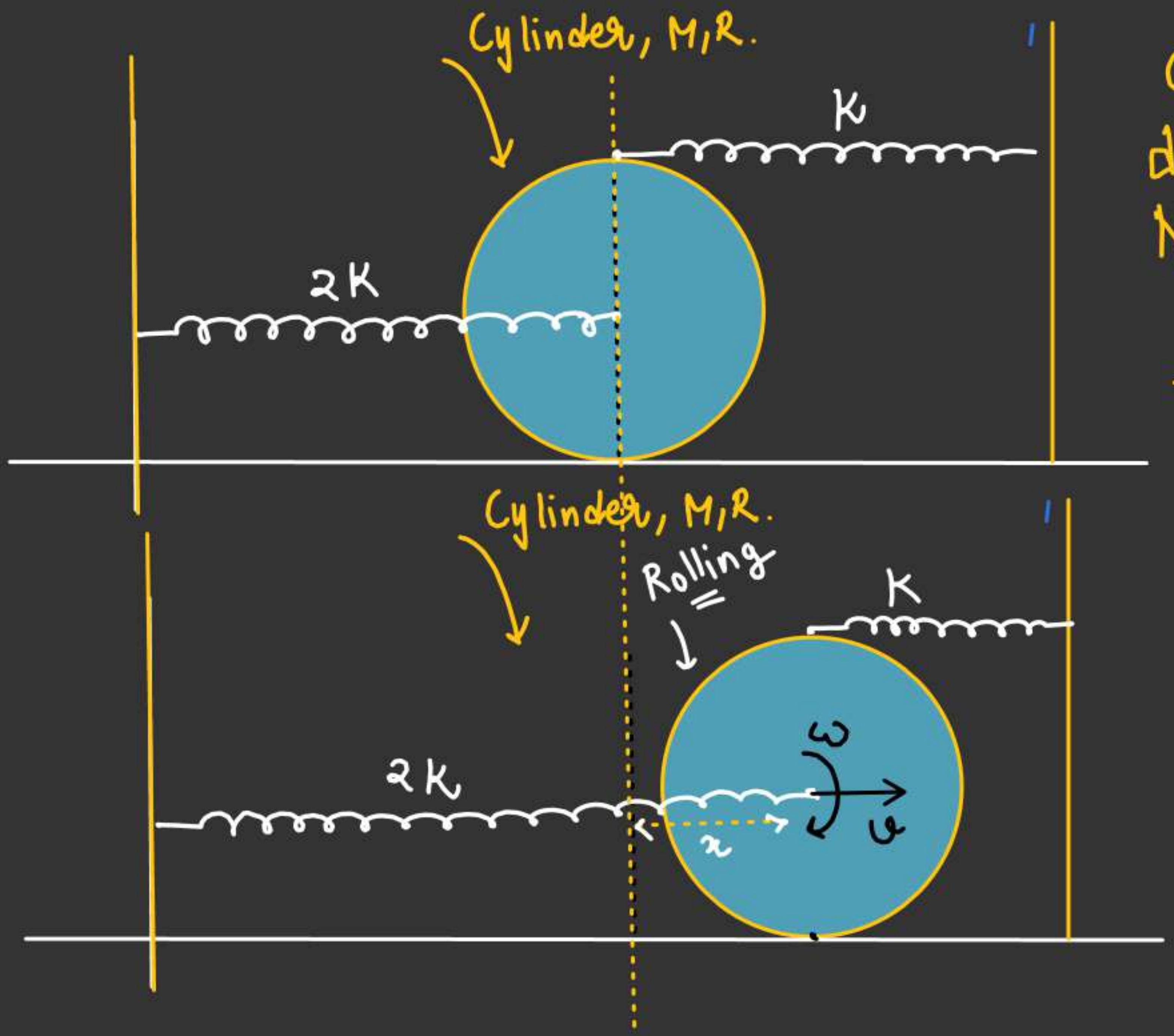

Cylinder slightly displaced and released.
No relative slipping of Cylinder on the ground.

Initially spring at its natural length.

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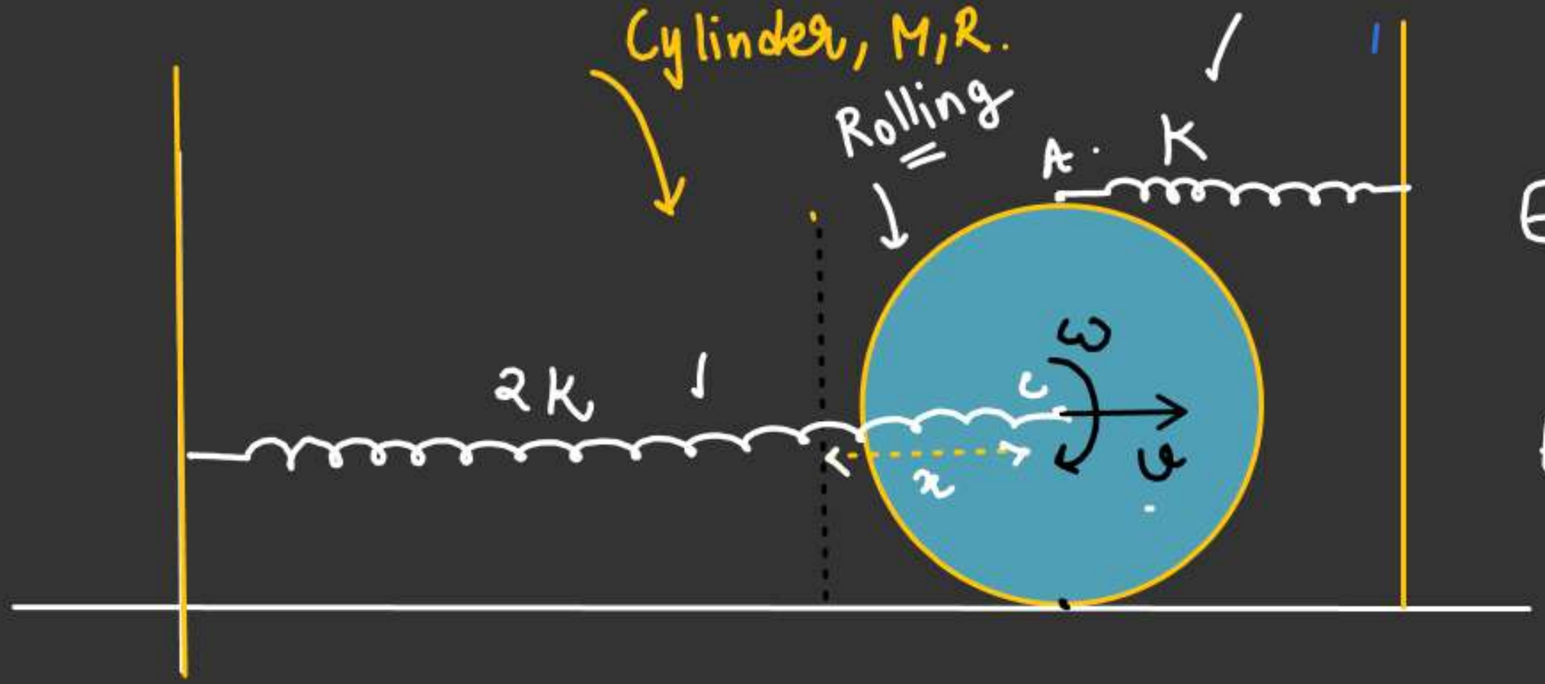

$$x_{A/E} = x_{A/COM} + x_{COM/E}$$

$$= R\theta + x = 2x$$

For pure rolling

$$x = R\theta, \quad v = R\omega$$

$$E_T = (K.E)_T + (K.E)_{Rotational} + P.E_{spring}$$

$$E_T = \frac{1}{2}Mv^2 + \frac{1}{2}\left(\frac{MR^2}{2}\right)\omega^2 + \frac{1}{2}K(2x)^2 + \frac{1}{2}(2K)x^2$$

$$E_T = \frac{Mv^2}{2} + \frac{M\cancel{R^2}}{4} \times \frac{v^2}{\cancel{R^2}} + 2Kx^2 + Kx^2$$

$$E_T = \frac{3Mv^2}{4} + 3Kx^2$$

Differentiating both side w.r.t time.

$$\frac{dE_T}{dt} = \frac{3}{4}M\frac{d}{dt}(v^2) + 3K\frac{d}{dt}(x^2)$$

$$0 = \frac{3M}{4}\left[\frac{d}{dv}(v^2) \times \frac{dv}{dt}\right] + 3K\frac{d}{dx}(x^2) \times \left(\frac{dx}{dt}\right)$$

$$0 = \frac{3M}{4} \times 2v\frac{dv}{dt} + (6Kx)\cdot\frac{dx}{dt}.$$

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$$0 = \frac{3M}{4} \times 2v \frac{dv}{dt} + (6kx) \cdot \frac{dx}{dt}$$

$$\frac{3M}{2}\left(\frac{dv}{dt}\right) \times v = -(6kx)\left(\frac{dx}{dt}\right)$$

$$a = -\frac{12K}{3M}x$$

$$a = -\frac{4K}{M}x$$

$$a = -\omega^2 x$$

(Angular frequency)

$$\omega = \sqrt{\frac{4K}{M}} \checkmark$$

$$T = \left(2\pi\sqrt{\frac{M}{4K}}\right) \checkmark$$

$$f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{4K}{M}}$$

frequency

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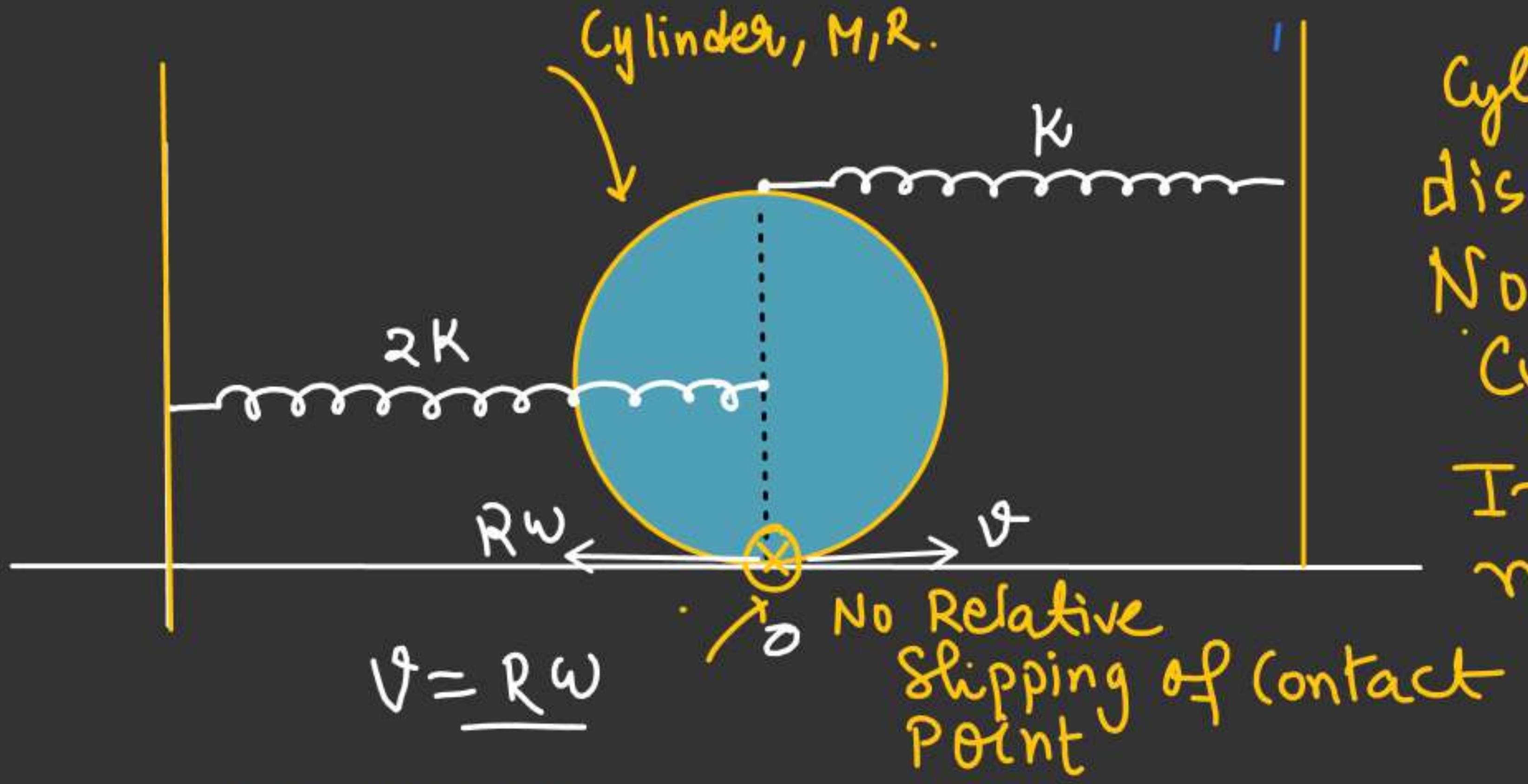
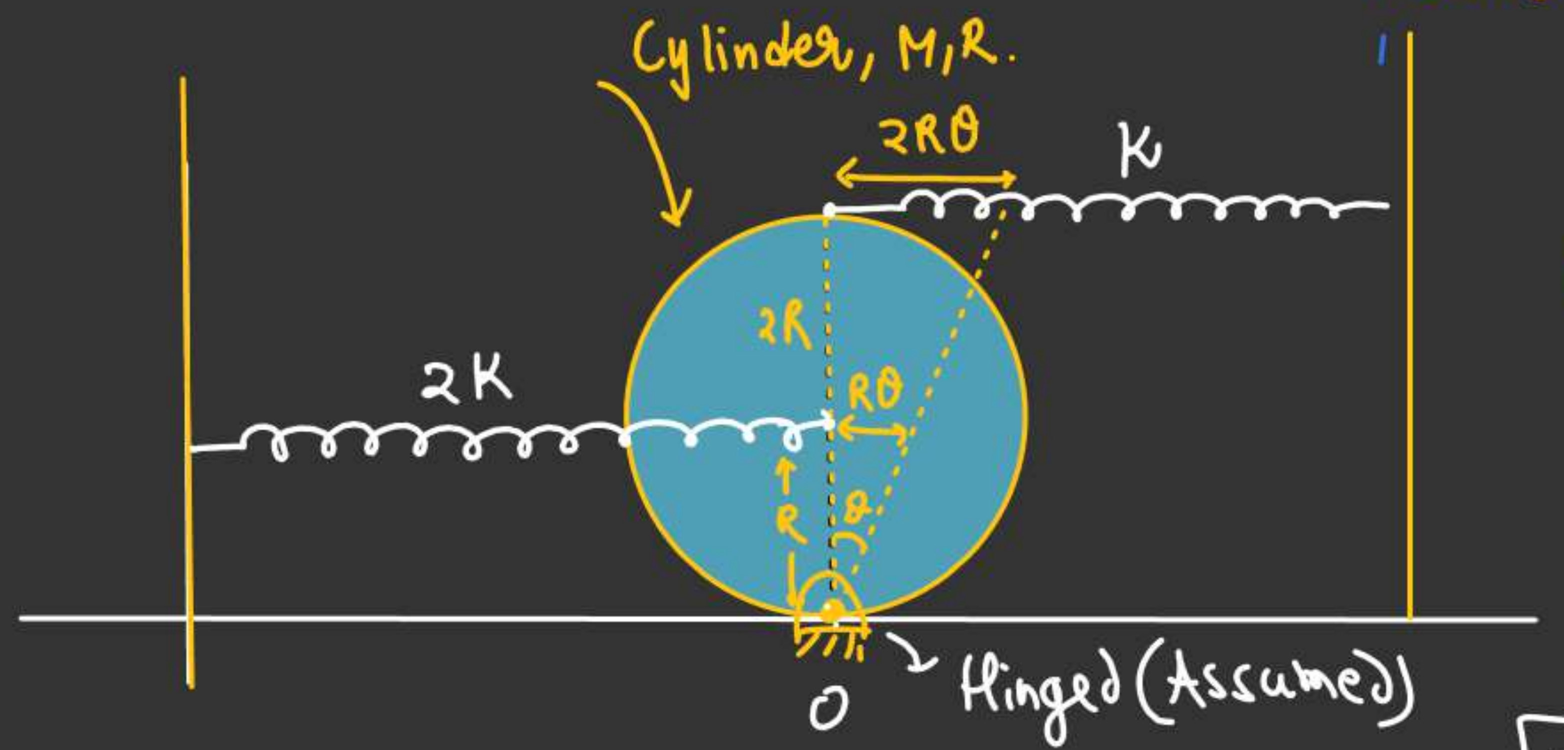
M-2 (Another Method)

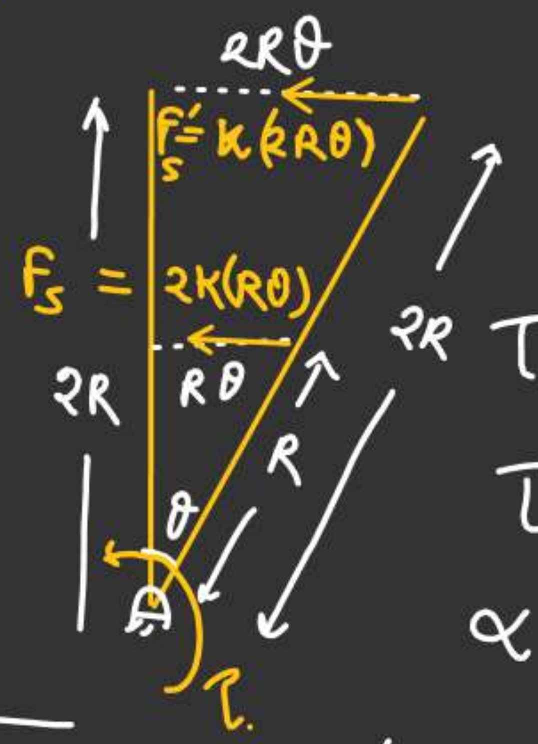


$v = R\omega$

Cylinder slightly displaced and released. No relative slipping of Cylinder on the ground.

Initially spring at its natural length.





$$\tau_r = -\left[K(2R\theta)\,2R\underbrace{\cos\theta}_{1} + (2KR\theta)\,R\underbrace{\cos\theta}_{1}\right]$$

$$\tau_r = -\left[4KR^2\theta + 2KR^2\theta\right]$$

$$\tau_r = -6KR^2\theta$$

$$\alpha = \frac{\tau_r}{I} = \frac{-6KR^2\theta}{\left(\frac{MR^2}{2} + MR^2\right)}$$

$$\alpha = -\frac{\cancel{12}KR^2\theta}{\cancel{3}MR^2} = \left(-\frac{4K}{M}\theta\right)$$

$$T = 2\pi\sqrt{\frac{M}{4K}}$$

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Simple pendulum

$$\tau_r = -(Mg\sin\theta)L$$

$$\sin\theta \simeq \theta$$

$$\tau_r = -(MgL)\theta$$

$$\alpha = -\frac{\cancel{M}gL}{\cancel{M}L^{\cancel{2}}}\theta$$

$$\alpha = -\frac{g}{L}\theta$$

$$\alpha = -\omega^2\theta$$

$$\omega = \sqrt{\frac{g}{L}}$$

$$\boxed{T = 2\pi\sqrt{\frac{L}{g}}}$$