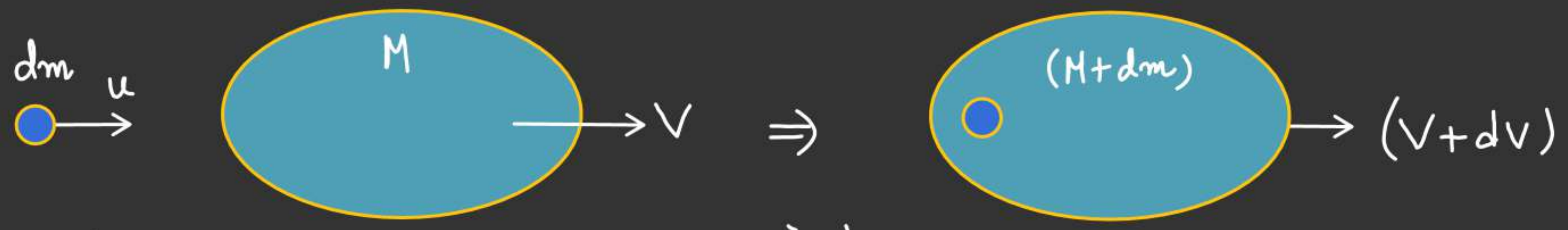
Nishant Jindal

VARIABLE MASS SYSTEM

$\underline{t=0}$. $\qquad\qquad$ $t = dt$.



dp = Change in momentum in 'dt' time

$$dp = p_f - p_i$$

$$= \left[(M+dm)(V+dV)\right] - \left[dm\,u + MV\right]$$

$$= \cancel{MV} + MdV + dmV + \underbrace{(dm\,dV)}_{\Downarrow\, 0} - dm\,u - \cancel{MV}$$

$$dp = MdV + dm(v-u)$$

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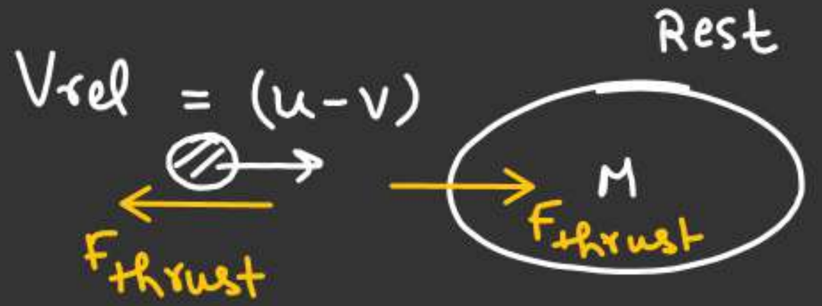
$$dp = MdV + (V-u)\,dm$$

$$dp = MdV - (u-V)\,dm$$



Dividing both Side by dt

$$\left(\frac{dp}{dt}\right) = M\left(\frac{dV}{dt}\right) - V_{rel}\left(\frac{dm}{dt}\right)$$

$\Downarrow$

$$\boxed{F_{ext} + \underbrace{V_{rel}\left(\frac{dm}{dt}\right)}_{\text{Thrust}} = M\left(\frac{dV}{dt}\right)}$$

**

$\Rightarrow$ Thrust always in the direction of V_{rel} if $\frac{dm}{dt}$ is increasing i.e when mass attached to the system

$\Rightarrow$ Thrust opposite to V_{rel} if $\frac{dm}{dt}$ is decreasing i.e mass leaving the system.

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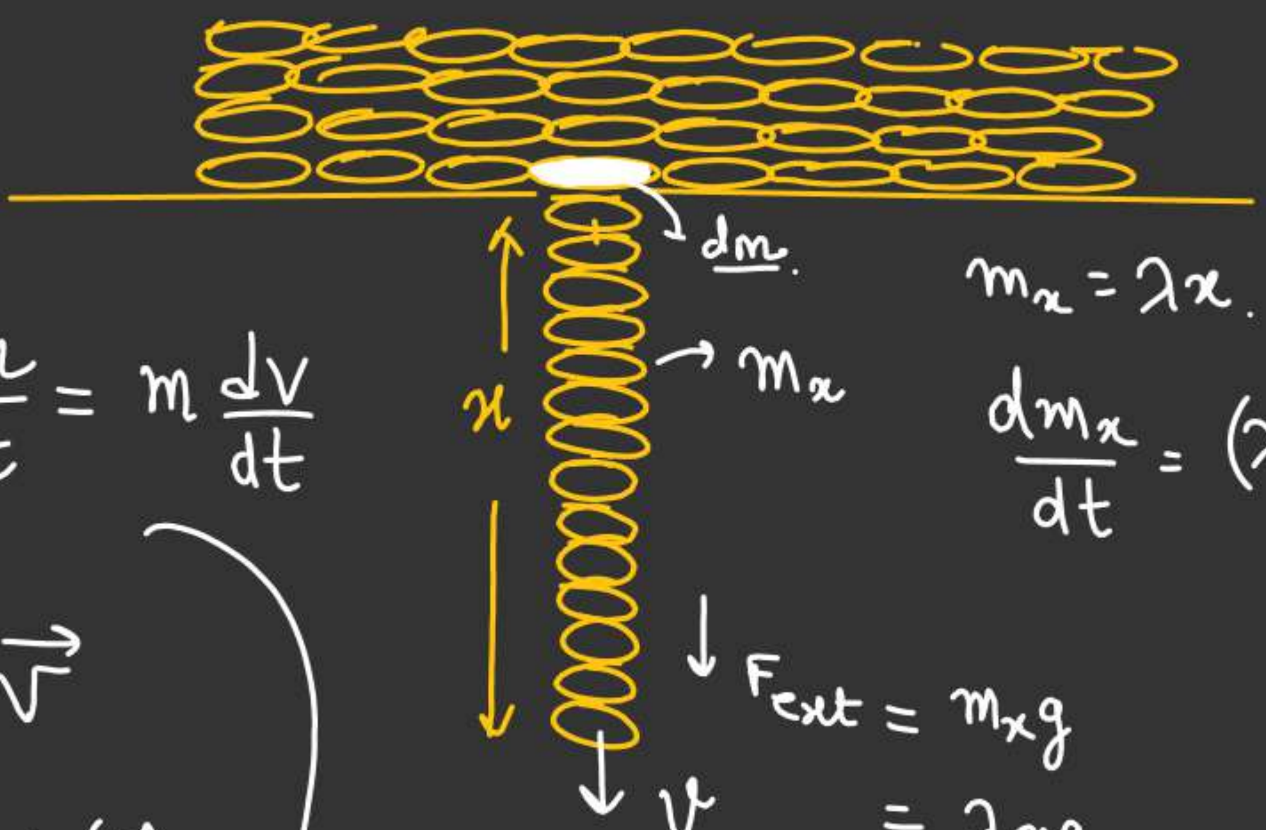
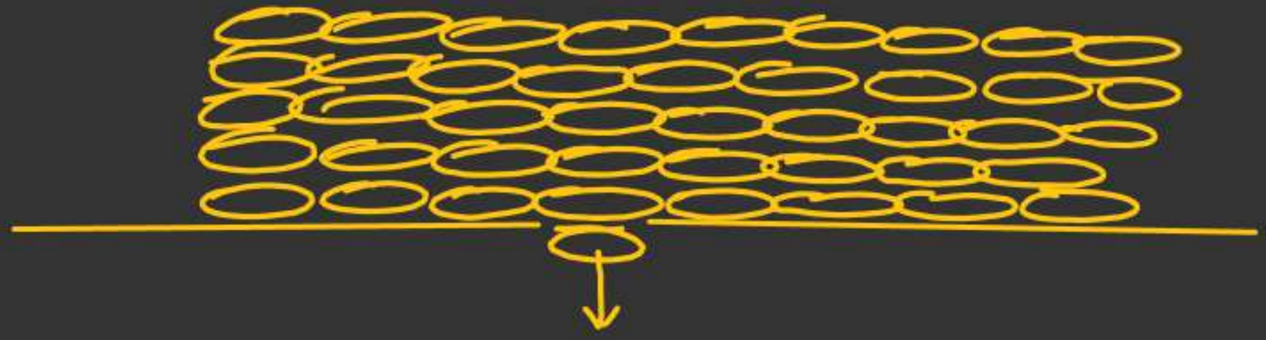
VARIABLE MASS SYSTEM

Case of uniform chain

$\lambda = $ (linear mass density)





$$F_{ext} + V_{rel}\frac{dm}{dt} = m\frac{dV}{dt}$$

$m_x = \lambda x$.

$$\frac{dm_x}{dt} = \left(\lambda \frac{dx}{dt}\right) = \lambda v$$

$F_{ext} = m_x g$

$= \lambda x g$

$V_{rel} = \vec{u} - \vec{v}$

$= 0 - v(-\hat{j})$

$V_{rel} = v\hat{j}$

$$\lambda x g + v(\lambda v) = m\frac{dV}{dt}$$

$$\boxed{(\lambda x g + \lambda v^2) = m\frac{dv}{dt}}$$

$$F_{thrust} = V_{rel}\frac{dm}{dt}$$

$= v\lambda v$

$= \lambda v^2$

$$\boxed{F_{thrust} = \lambda v^2}$$

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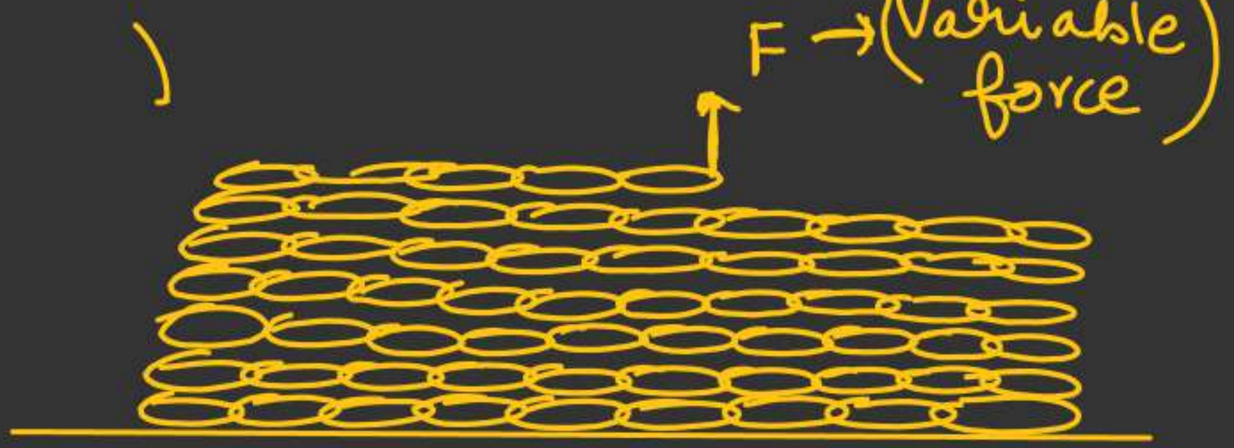
VARIABLE MASS SYSTEM

Uniform Chain piled up on a horizontal surface.



※ A Variable force lifting the Chain with Constant velocity

Find

a) $F = f(x)$ ✓

b) Energy lost during lifting

$$F_{ext} + V_{rel}\frac{dm}{dt} = m\left(\frac{dv}{dt}\right)$$

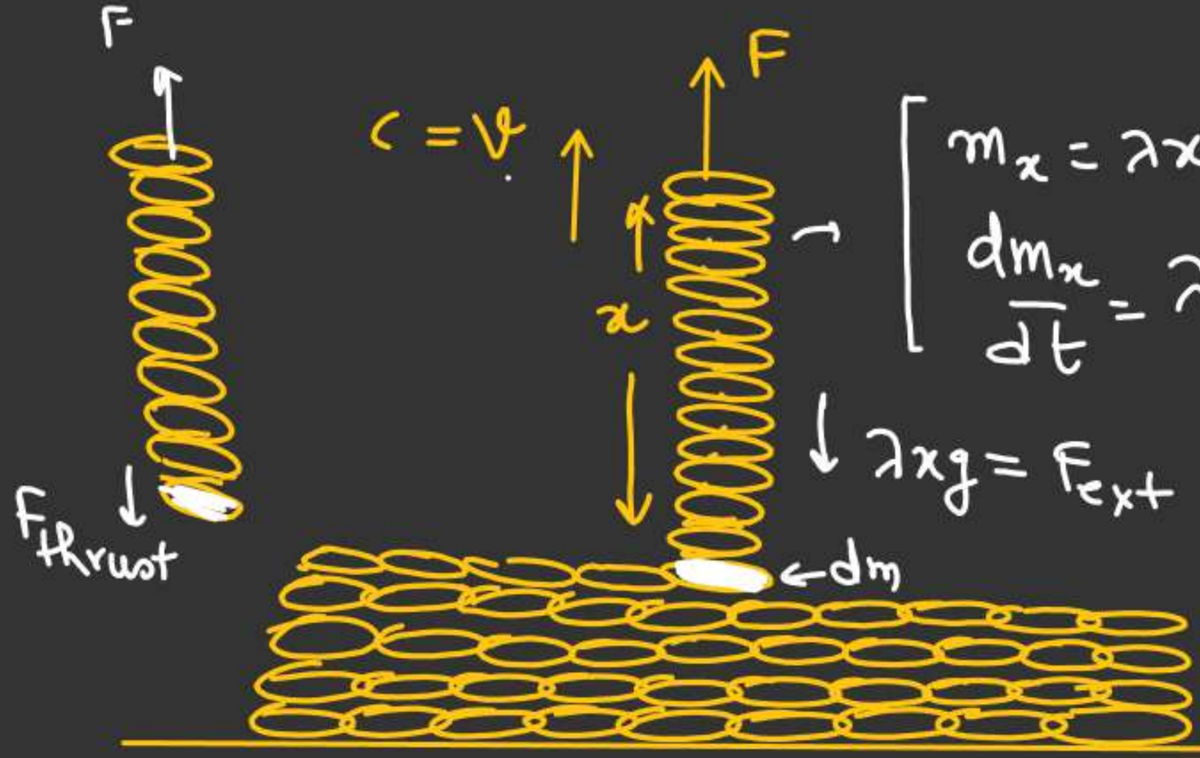
$$F_{ext} = (F - \lambda x g)$$

$$V_{rel} = \vec{u} - \vec{v} = 0 - v\hat{j} = -v\hat{j}$$

$$(F - \lambda x g) - \lambda v^2 = m\left(\frac{dv}{dt}\right)$$

According to question $\frac{dv}{dt} = 0$

a) $F = \lambda x g + \lambda v^2$



$$m_x = \lambda x$$

$$\frac{dm_x}{dt} = \lambda v$$

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VARIABLE MASS SYSTEM

Work done by F during lifting

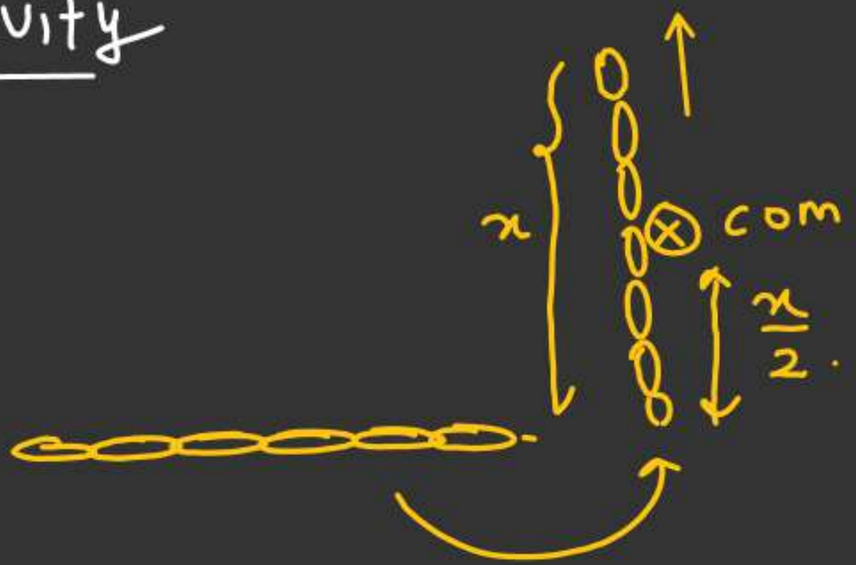
$$F = \left(\lambda x g + \lambda v^2\right)$$

$$\int_0^W dW = \int_0^x F \cdot dx$$

$$W = \lambda g \int_0^x x\,dx + \lambda v^2 \int_0^x dx$$

$$W = \left(\frac{\lambda g x^2}{2} + \lambda v^2 x\right)$$

Work done by gravity



$$W_{gravity} = -dU$$

$$= 0 - (\lambda x g)\frac{x}{2}$$

$$= \left(-\frac{\lambda g x^2}{2}\right)$$

$$\Delta K.E = \frac{1}{2}(\lambda x)v^2$$

By work-energy theorem

$$W_F + W_{gravity} = (\Delta K.E) + \text{Heat}$$

$$\left(\cancel{\frac{\lambda x^2 g}{2}} + \lambda v^2 x\right) - \cancel{\frac{\lambda g x^2}{2}} = \frac{\lambda x v^2}{2} + \text{heat}$$

$$\text{heat} = \lambda v^2 x - \frac{\lambda v^2 x}{2} = \left(\frac{\lambda v^2 x}{2}\right) J.$$

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VARIABLE MASS SYSTEM

M_0 = Mass of Car

F = Constant force.

Rate of loading of Sand particle is μ kg/sec

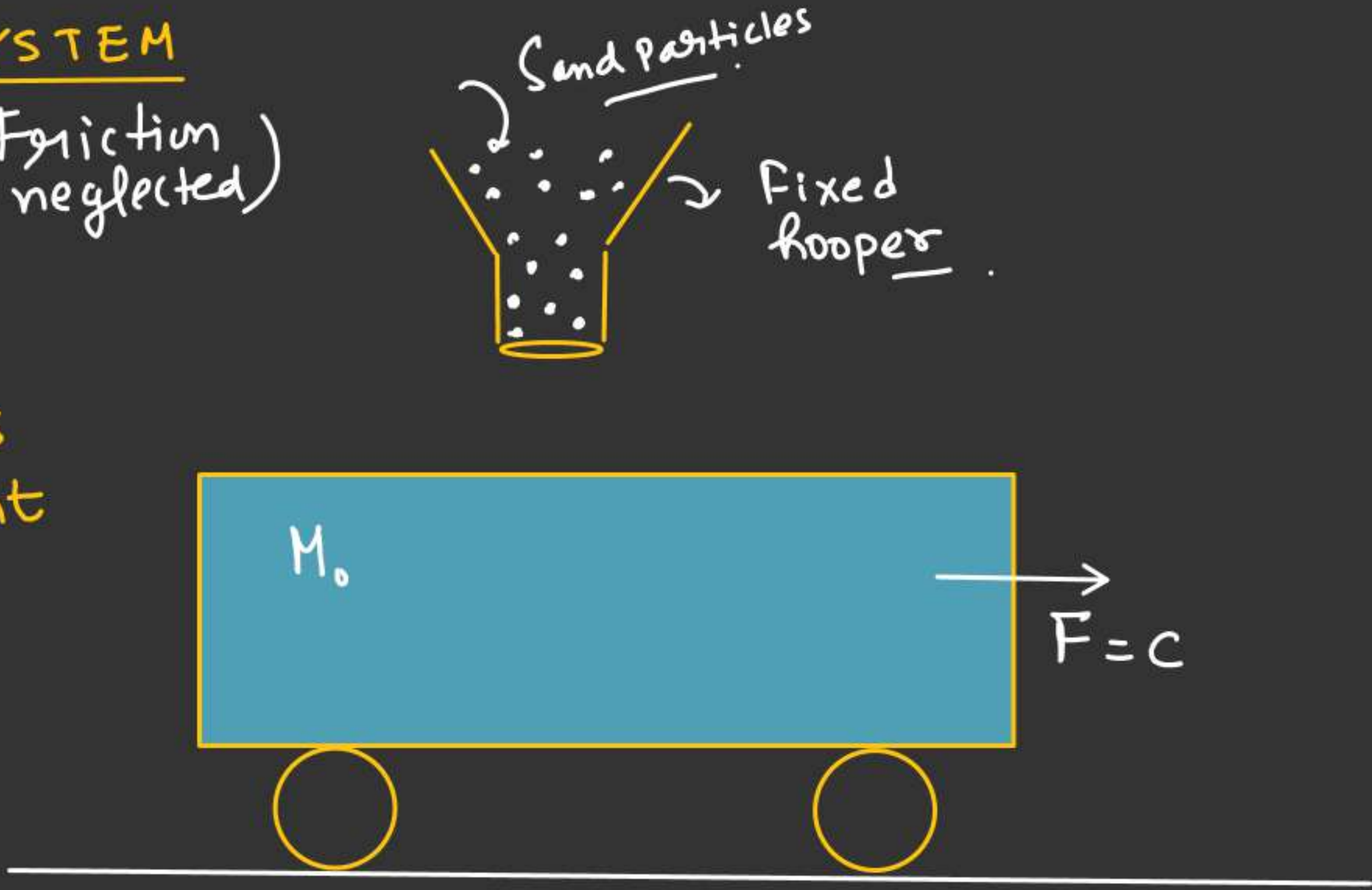
$$\frac{dm}{dt} = \mu \quad \text{(given)}$$

$$V_{car} = f(t)$$

(Friction neglected)

Initially trolley is pulled by Constant force F.

$v_i = 0$



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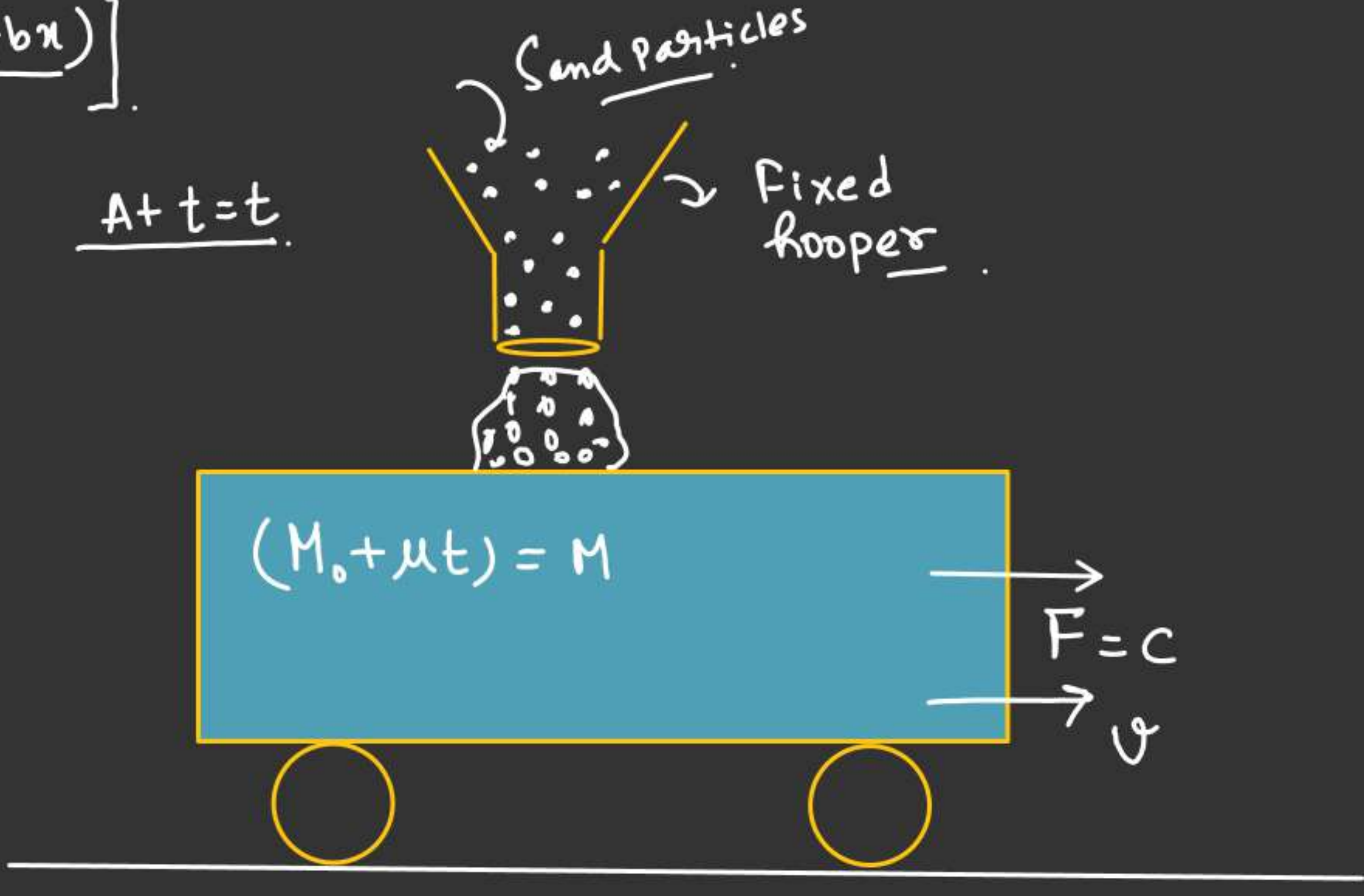
$M = (M_0 + \mu t)$ ✓

Mass of trolley at $t = t$.

$$\int \frac{dx}{a+bx} = \left[\frac{\ln(a+bx)}{b}\right]$$

At $t=t$.



$$F_{ext} + V_{rel}\frac{dm}{dt} = m\frac{dv}{dt}$$

$$(F - V\mu) = (M_0 + \mu t)\frac{dv}{dt}$$

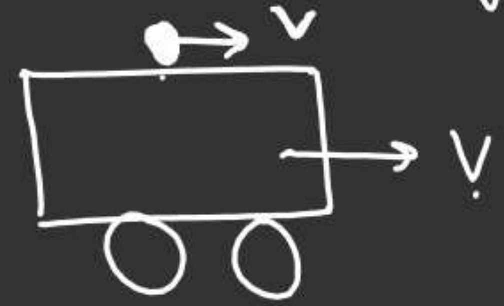
$$\int_0^v \left(\frac{dv}{F - \mu v}\right) = \int_0^t \frac{dt}{M_0 + \mu t}$$

$$\frac{\ln[F - \mu v]_0^v}{-\cancel{\mu}} = \frac{\ln[M_0 + \mu t]_0^t}{\cancel{\mu}}$$

$$-\ln\left(\frac{F - \mu v}{F}\right) = \ln\left(\frac{M_0 + \mu t}{M_0}\right)$$

$u = 0$

$V_{rel} = (-V)$



$V_{rel} = (u - v)$

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$$F_{ext} + v_{rel}\frac{dm}{dt} = m\frac{dv}{dt}$$

$$(F - v\mu) = (M_0 + \mu t)\frac{dv}{dt}$$

$$\frac{dv}{dt} \Downarrow A$$

$$\int_0^v \left(\frac{dv}{F-\mu v}\right) = \int_0^t \frac{dt}{M_0+\mu t}$$

$$\frac{\ln[F-\mu v]_0^v}{-\cancel{\mu}} = \frac{\ln[M_0+\mu t]_0^t}{\cancel{\mu}}$$

$$-\ln\left(\frac{F-\mu v}{F}\right) = \ln\left(\frac{M_0+\mu t}{M_0}\right)$$

$$\ln\left(\frac{F}{F-\mu v}\right) = \ln\left(\frac{M_0+\mu t}{M_0}\right)$$

$$\frac{F}{F-\mu v} = \left(\frac{M_0+\mu t}{M_0}\right)$$

$$\boxed{v = \left(\frac{Ft}{m_0+\mu t}\right)} \checkmark$$

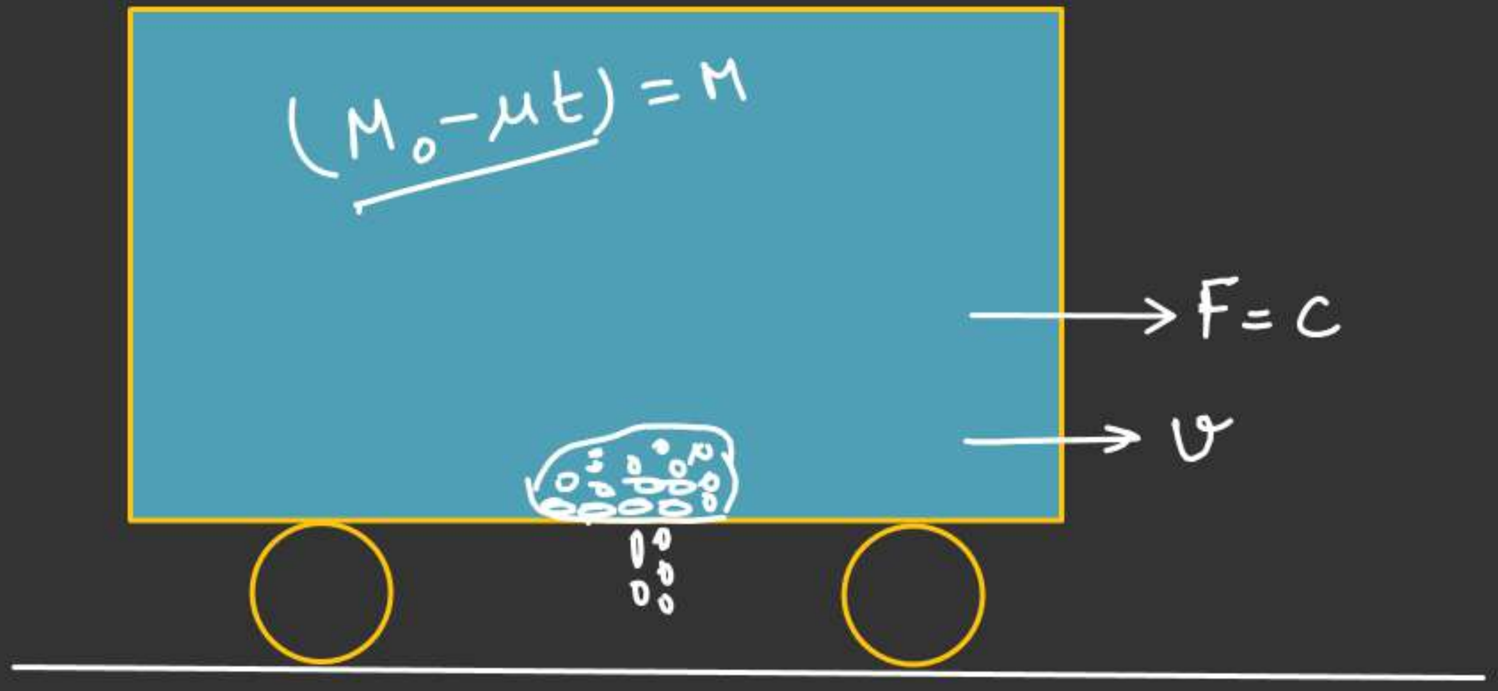
$$A = \left(\frac{F-\mu v}{M_0+\mu t}\right)$$

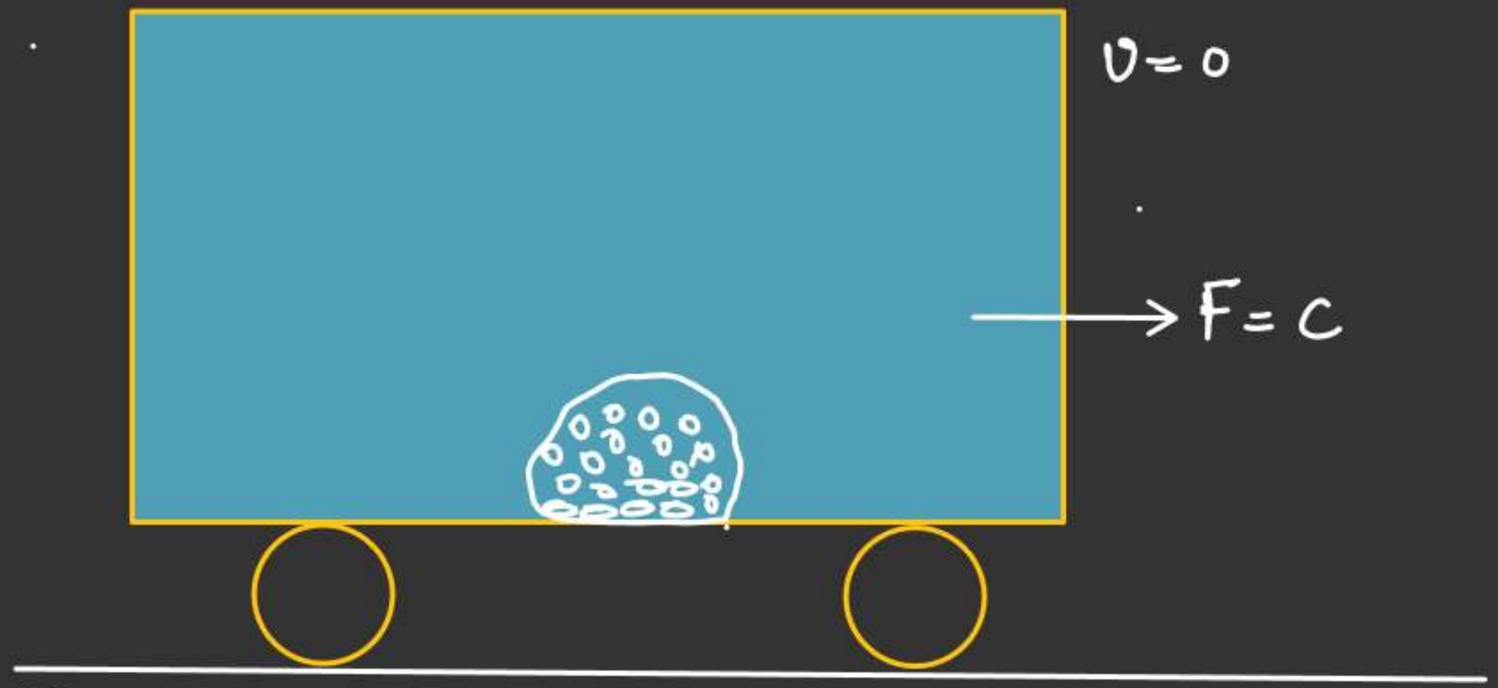
$$A = f(t) \checkmark$$

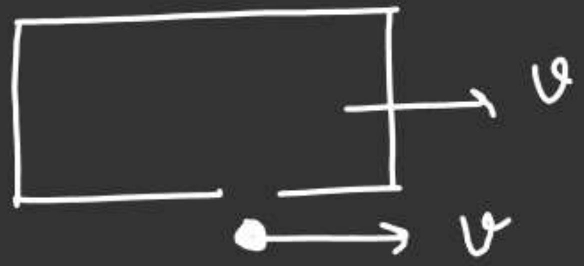
At $t = t$

$\left(\frac{dm}{dt} = \mu\, kg/sec\right)$ At $t = 0$

M_0 = Mass of trolley







$$\underline{V_{rel} = 0} \checkmark$$

$$F_{ext} + \underbrace{V_{rel}\frac{dm}{dt}}_{0} = M\frac{dv}{dt}$$

$$F = (M_0 - \mu t)\frac{dv}{dt}$$

$$\int_0^V dv = F\int_0^t \frac{dt}{M_0 - \mu t}$$

$$V = F\,\frac{\left[\ln(M_0 - \mu t)\right]_0^t}{(-\mu)}$$

$$V = \frac{-F}{\mu}\ln\left(\frac{M_0 - \mu t}{M_0}\right) = \frac{F}{\mu}\ln\left(\frac{M_0}{M_0 - \mu t}\right)$$

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VARIABLE MASS SYSTEM

Rocket propulsion

$$\vec{F}_{ext} + \vec{V}_{rel}\frac{dm}{dt} = M\left(\frac{d\vec{v}}{dt}\right)$$

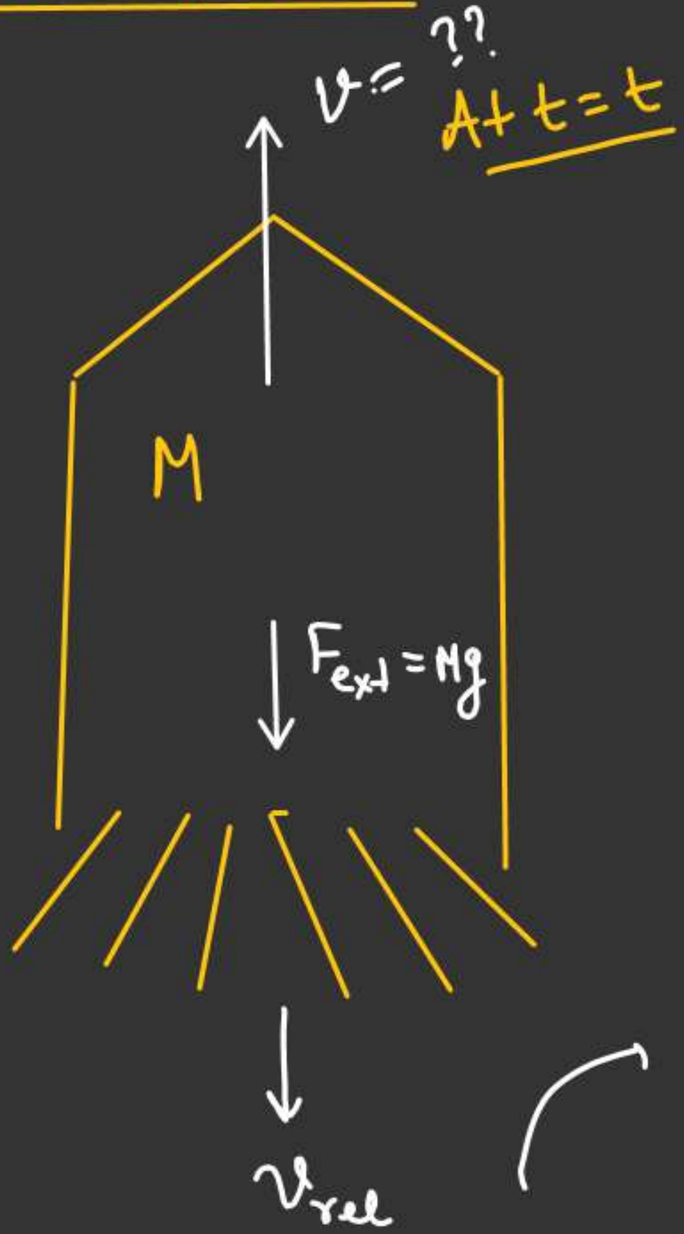
$$\Downarrow$$

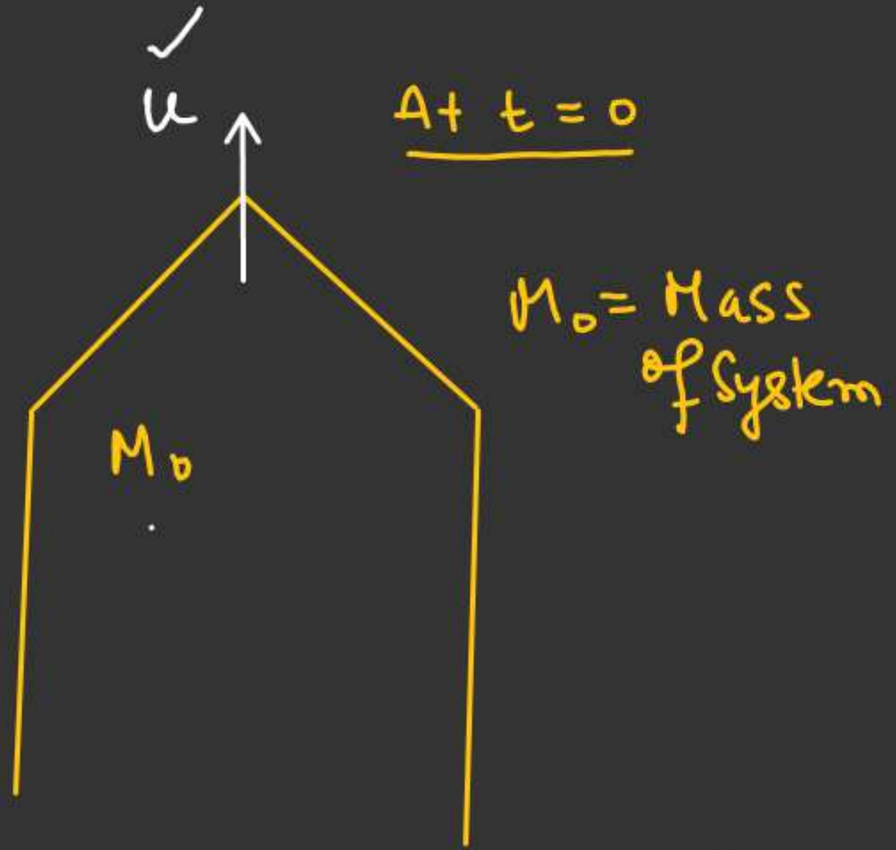
$$-Mg - V_{rel}\frac{dm}{dt} = M\left(\frac{dv}{dt}\right)$$

$$Mg + V_{rel}\frac{dm}{dt} = -M\left(\frac{dv}{dt}\right)$$

$$Mg\,dt + V_{rel}\,dm = -M\,dv$$

$$g\int_0^t dt + V_{rel}\int_{m_0}^{m}\frac{dm}{m} = -\int_u^v dv \Rightarrow gt + V_{rel}\ln\left(\frac{m}{m_0}\right) = -(v-u)$$





$$v = (u - gt) - V_{rel}\ln\left(\frac{m}{m_0}\right)$$

$$\boxed{v = (u-gt) + V_{rel}\ln\left(\frac{m_0}{m}\right)}$$

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$$\boxed{v = (u - gt) + v_{rel} \ln\left(\frac{m_0}{m}\right)}$$

if $u = 0$, g = neglected

$$\boxed{v = v_{rel} \ln\left(\frac{m_0}{m}\right)}$$