Nishant Jindal

$$\boxed{(1+x)^n = {}^nC_0 + {}^nC_1\cdot x + {}^nC_2\cdot x^2 + {}^nC_3\cdot x^3 + \dots}$$

When G.P is Multiplied to Bin. Coff.

AK

(1) Starting ${}^nC_0$ से

(2) ${}^nC_1$ Should be multiplied to x

Q. ${}^nC_0 + {}^nC_1\cdot 2^1 + {}^nC_2\cdot 2^2 + {}^nC_3\cdot 2^3 + \dots = ?$

GP

$= (1+2)^n = 3^n$

GP

Q ${}^nC_0 + {}^nC_1\cdot 4^2 + {}^nC_2\cdot 4^4 + \dots = ?$

$= (1+16)^n = 17^n$

1) ${}^nC_0 + {}^nC_1 + {}^nC_2 + \dots + {}^nC_n = 2^n$

2) ${}^nC_0 + {}^nC_2 + {}^nC_4 + \dots = {}^nC_1 + {}^nC_3 + \dots = 2^{n-1}$

(3) ${}^nC_0 + {}^nC_1 x + {}^nC_2 x^2 + \dots = \boxed{(1+x)^n = \sum_{r=0}^{n} {}^nC_r\cdot x^r}$

Q ${}^{30}C_1 + {}^{30}C_2 + \dots + {}^{30}C_{29} = ?$

(1) n की value 30

(2) Starting ${}^{30}C_0$ se nahi

(3) End ${}^{30}C_{30}$ par nahi

$$\underbrace{\{{}^{30}C_0 + {}^{30}C_1 + {}^{30}C_2 + \dots + {}^{30}C_{29} + {}^{30}C_{30}\}}_{2^{30}} - ({}^{30}C_0 + {}^{30}C_{30})$$

$2^{30} - (1+1)$

$= 2^{30} - 2$

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Q $^{30}C_0 + {}^{30}C_1 + \dots + {}^{30}C_{14} = ?$

Mains की far.

$^{30}C_{14} = {}^{30}C_{16}$ | $^{30}C_{13} = {}^{30}C_{17}$

$^nC_r = {}^nC_{n-r}$

Same

$\underbrace{^{30}C_0 + {}^{30}C_1 + {}^{30}C_2 + \dots {}^{30}C_{13} + {}^{30}C_{14}}_{\text{Qs } (X)} + {}^{30}C_{15} + \underbrace{^{30}C_{16} + {}^{30}C_{17} + \dots + {}^{30}C_{30}}_{X} = 2^{30}$

$X + {}^{30}C_{15} + X = 2^{30}$

$2X = 2^{30} - {}^{30}C_{15}$

$X = \dfrac{2^{30} - {}^{30}C_{15}}{2}$

$X = 2^{29} - \frac{1}{2}\left({}^{30}C_{15}\right)$

Q3 $\underbrace{^{30}C_0 + {}^{30}C_1 + {}^{30}C_2 + \dots + {}^{30}C_{14}}_{X} + {}^{30}C_{15}$

$X + {}^{30}C_{15} = ??$

$= \left\{2^{29} - \frac{1}{2} \cdot {}^{30}C_{15}\right\} + {}^{30}C_{15}$

$= 2^{29} + {}^{30}C_{15}\left(1 - \frac{1}{2}\right)$

$= 2^{29} + \frac{1}{2} \cdot {}^{30}C_{15}$

Q

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Q3 $^{19}C_0 + {}^{19}C_1 + \dots + {}^{19}C_9 = ?$

$$\underbrace{{}^{19}C_0 + {}^{19}C_1 + \dots + {}^{19}C_9}_{x} + \underbrace{{}^{19}C_{10} + {}^{19}C_{11} + \dots + {}^{19}C_{18} + {}^{19}C_{19}}_{x} = 2^{19}$$

$$2x = 2^{19} \Rightarrow x = \frac{2^{19}}{2} = 2^{18}$$

Q5 $^{11}C_0 + {}^{11}C_1 + \dots + {}^{11}C_5 = 2^{10}$ [T/F]

$^{2n+1}C_0 + {}^{2n+1}C_1 + \dots + {}^{2n+1}C_n = 2^{2n}$, $n = 5 \to 2n+1 = 2\times5+1$

$$\underbrace{{}^{11}C_0 + {}^{11}C_1 + {}^{11}C_2 + \dots + {}^{11}C_5}_{x} + \underbrace{{}^{11}C_6 + {}^{11}C_7 + \dots + {}^{11}C_{10} + {}^{11}C_{11}}_{x} = 2^{11}$$ $= 11$

$$x = 2^{10}$$

Q4 $^{20}C_0 + {}^{20}C_1 + {}^{20}C_2 + \dots + {}^{20}C_9 = ?$

$$\underbrace{{}^{20}C_0 + {}^{20}C_1 + \dots + {}^{20}C_8 + {}^{20}C_9}_{x} + \boxed{{}^{20}C_{10}} + \underbrace{{}^{20}C_{11} + {}^{20}C_{12} + {}^{12}C_{13} + \dots + {}^{20}C_{20}}_{x} = 2^{20}$$

$$2x + {}^{20}C_{10} = 2^{20}$$

$$2x = 2^{20} - {}^{20}C_{10}$$

$$x = 2^{19} - \frac{1}{2}\,{}^{20}C_{10}$$

Q6 $^{10}C_1 + {}^{10}C_2 + \dots + {}^{10}C_9 = 512$? [T/F]

$$^{10}C_0 + {}^{10}C_1 + \dots + {}^{10}C_9 + {}^{10}C_{10} = 2^{10}$$

$$^{10}C_1 + {}^{10}C_2 + \dots + {}^{10}C_9 = 1024 - 1 - 1$$

$$= 1022$$

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Q1 $^{12}C_1 + {}^{12}C_2 + \ldots + {}^{12}C_{11} = 4094$ [T/F]?

$^{12}C_0 + \underbrace{{}^{12}C_1 + {}^{12}C_2 + \ldots + {}^{12}C_{11}}_{QS} + {}^{12}C_{12} = 2^{12} = 4096$

$^{12}C_1 + \ldots + {}^{12}C_{11} = 4096 - 1 - 1$

$= 4094$

Q8 $^{2n+1}C_0 + {}^{2n+1}C_1 + {}^{2n+1}C_2 + \ldots + {}^{2n+1}C_n = 2^{2n}$ [T/F]

$^{13}C_0 + {}^{13}C_1 + \ldots + {}^{13}C_6 = 2^{12}$

$\{{}^{13}C_0 + \ldots + {}^{13}C_6\} + \{{}^{13}C_7 + \ldots + {}^{13}C_{13}\} = 2^{13}$

$X + X = 2^{13} \Rightarrow 2X = 2^{13}$

$X = \frac{2^{13}}{2} = 2^{12}$

Q9 $^{41}C_0 + {}^{41}C_1 + \ldots + {}^{41}C_{20} = ?$ {Direct}

Ans $= 2^{40}$

Q10 $^{2n}C_0 + {}^{2n}C_1 + \ldots + {}^{2n}C_n = ?$

$^{2n}C_0 + {}^{2n}C_1 + \ldots + \boxed{{}^{2n}C_{n-2}} + \boxed{{}^{2n}C_{n-1}} + \boxed{{}^{2n}C_n} + \boxed{{}^{2n}C_{n+1}} + \boxed{{}^{2n}C_{n+2}} + \ldots + {}^{2n}C_{2n} = 2^{2n}$

$\leftarrow X \qquad \leftarrow X$

$X = \frac{2^{2n} - {}^{2n}C_n}{2}$

$X + {}^{2n}C_n = 2^{2n} - \frac{1}{2}\,{}^{2n}C_n + {}^{2n}C_n = 2^{2n} + {}^{2n}C_n\left(1 - \frac{1}{2}\right)$

$= 2^{2n} + \frac{1}{2}\,{}^{2n}C_n$

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Q_{11} $^{40}C_0 + {}^{40}C_1 + \cdots + {}^{40}C_{19}$ ← When Mid Bin Coff is Missing.

$= 2^{39} - \frac{1}{2}\,{}^{40}C_{20}$

Q_{12} $^{40}C_0 + {}^{40}C_1 + \cdots + {}^{40}C_{19} + \underline{{}^{40}C_{20}}$ ← When Mid Bi Coff is added.

$= 2^{39} + \frac{1}{2}\cdot{}^{40}C_{20}$

Q_{13} $^{39}C_0 + {}^{39}C_1 + \cdots + {}^{39}C_{19} = ?$

$= 2^{38}$

$^{2n+1}C_0 + \cdots + {}^{2n+1}C_n = 2^{2n}$ Use

Q Value of $({}^{21}C_1 - {}^{10}C_1) + ({}^{21}C_2 - {}^{10}C_2) + \cdots + ({}^{21}C_{10} - {}^{10}C_{10}) = ?$

$({}^{21}C_0 + {}^{21}C_1 + {}^{21}C_2 + \cdots + {}^{21}C_{10}) - ({}^{10}C_0 + {}^{10}C_1 + {}^{10}C_2 + {}^{10}C_3 \cdots {}^{10}C_{10})$

$(2^{20} - \cancel{1}) - (2^{10} - \cancel{1}) = 2^{20} - 2^{10}$

D.U.S = Dant Ukhado Scheme

(1) $^{n}C_r = \frac{n}{r}\cdot{}^{n-1}C_{r-1} = \frac{n}{r}\cdot\frac{n-1}{r-1}\,{}^{n-2}C_{r-2}$

D.J.S. = Dant Jodo Scheme

$\frac{n+1}{r+1} \times {}^{n}C_r = {}^{n+1}C_{r+1}$

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Q Find $\left(\frac{C_0+C_1}{C_0}\right)\cdot\left(\frac{C_1+C_2}{C_1}\right)\cdot\left(\frac{C_2+C_3}{C_2}\right)\times\cdots\times\left(\frac{C_{n-1}+C_n}{C_{n-1}}\right)=?$

Same

$$\left(\frac{{}^nC_0+{}^nC_1}{{}^nC_0}\right)\times\left(\frac{{}^nC_1+{}^nC_2}{{}^nC_1}\right)\times\left(\frac{{}^nC_2+{}^nC_3}{{}^nC_2}\right)\times\quad\times\left(\frac{{}^nC_{n-1}+{}^nC_n}{{}^nC_{n-1}}\right)$$

1st digit

DUS $\left(\frac{{}^{n+1}C_1}{{}^nC_0}\right)\times\left(\frac{{}^{n+1}C_2}{{}^nC_1}\right)\times\left(\frac{{}^{n+1}C_3}{{}^nC_2}\right)\times\cdots\times\left(\frac{{}^{n+1}C_n}{{}^nC_{n-1}}\right)$

$$\left(\frac{\left(\frac{n+1}{1}\right)\cdot\cancel{{}^nC_0}}{\cancel{{}^nC_0}}\right)\times\left(\frac{\frac{n+1}{2}\times\cancel{{}^nC_1}}{\cancel{{}^nC_1}}\right)\times\left(\frac{\frac{n+1}{3}\times\cancel{{}^nC_2}}{\cancel{{}^nC_2}}\right)\times\cdots\times\left(\frac{\frac{n+1}{n}\cdot\cancel{{}^nC_{n-1}}}{\cancel{{}^nC_{n-1}}}\right)$$

$$\underbrace{\frac{(n+1)}{1}\times\frac{(n+1)}{2}\times\frac{(n+1)}{3}\times\cdots\times\frac{(n+1)}{n}}_{\leftarrow n\text{ बार}}=\frac{(n+1)^n}{1\cdot2\cdot3\cdots n}=\frac{(n+1)^n}{\lfloor n}\ \text{Ans}$$

Q $(C_0+C_1)\cdot(C_1+C_2)\cdot(C_2+C_3)\cdots\times(C_{n-1}+C_n)=?$

$$({}^nC_0+{}^nC_1)({}^nC_1+{}^nC_2)({}^nC_2+{}^nC_3)\times\quad\times({}^nC_{n-1}+{}^nC_n)$$

$$({}^{n+1}C_1)\times({}^{n+1}C_2)\times({}^{n+1}C_3)\times\cdots\times({}^{n+1}C_n)$$

DUS

$$\left(\frac{n+1}{1}\right)\cdot{}^nC_0\times\left(\frac{n+1}{2}\right)\cdot{}^nC_1\times\left(\frac{n+1}{3}\right)\cdot{}^nC_2\times\cdots\times\left(\frac{n+1}{n}\right)\cdot{}^nC_{n-1}$$

$$\frac{(n+1)^n}{\lfloor n}\cdot(C_0\cdot C_1\cdot C_2\cdots C_{n-1})$$

Ans

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Q If P_n denotes Product of all coefficients in Exp of $(1+x)^n$; $n \in N$ then S.T. $\frac{P_{n+1}}{P_n} = \frac{(n+1)^n}{\lfloor n}$

1) Bi. Coff in Exp. of $(1+x)^n$

$$(1+x)^n = {}^nC_0 + {}^nC_1 x + {}^nC_2 x^2 + {}^nC_3 x^3 - - - + {}^nC_n x^n$$

2) $P_n = {}^nC_0 \cdot {}^nC_1 \cdot {}^nC_2 \cdot {}^nC_3 \cdot ---- \cdot {}^nC_n$

then $P_{n+1} = {}^{n+1}C_0 \cdot {}^{n+1}C_1 \cdot {}^{n+1}C_2 \cdot {}^{n+1}C_3 --- {}^{n+1}C_{n+1}$

half Qs done

Dus in Nr.

(3) $$\frac{P_{n+1}}{P_n} = \frac{{}^{n+1}C_0 \cdot {}^{n+1}C_1 \cdot {}^{n+1}C_2 \cdot {}^{n+1}C_3 \cdot --- \cdot {}^{n+1}C_n \cdot {}^{n+1}C_{n+1}}{{}^nC_0 \cdot {}^nC_1 \cdot {}^nC_2 \cdot {}^nC_3 --- {}^nC_n}$$

(Don't touch)

$$\frac{\frac{n+1}{1} \cdot \cancel{{}^nC_0} \times \frac{n+1}{2} \cdot \cancel{{}^nC_1} \times \frac{n+1}{3} \cdot \cancel{{}^nC_2} \times --- \times \frac{n+1}{n} \cdot \cancel{{}^nC_{n-1}} \cdot \cancel{\left(\frac{n+1}{n+1}\right)} \cdot \cancel{{}^nC_n}}{\cancel{{}^nC_0} \quad \cancel{{}^nC_1} \quad \cancel{{}^nC_2} \quad \cancel{{}^nC_{n-1}} \quad \cancel{{}^nC_n}} = \frac{(n+1)^n}{\lfloor n}$$

$${}^nC_0 + {}^nC_1 x + {}^nC_2 x^2 - - + {}^nC_n x^n = (1+x)^n$$

Q18. ${}^nC_0 + {}^nC_1 \cdot 3 + {}^nC_2 \cdot 3^2 + - - + {}^nC_n 3^n = ?$

G.P.

$$= (1+3)^n = 4^n$$

Q19 ${}^nC_0 - {}^nC_1 3 + {}^nC_2 3^2 - {}^nC_3 3^3 - - - + (-1)^n \cdot {}^nC_n 3^n = ?$

$$= (1-3)^n = (-2)^n$$

G.P.

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3) When A.P in Multiplied to Bin. Coff

$$^nC_0 \cdot a + {^nC_1}(a+d) + {^nC_2}(a+2d) + {^nC_3}(a+3d) \dots + {^nC_n}(a+nd)$$

$$= \left(\frac{1^{st} + L.T.}{2}\right) \cdot 2^n = \frac{(a + a + nd)}{2} \cdot 2^n$$

$$S = {^nC_0} \cdot a + {^nC_1}(a+d) + {^nC_2}(a+2d) \qquad + {^nC_n}(a+nd)$$

$$S = {^nC_n}(a+nd) + {^nC_{n-1}}(a+(n-1)d) + {^nC_{n-2}}(a+(n-2)d) \qquad + {^nC_0} \cdot a$$

$${^nC_n} = {^nC_0}$$

$$2S = {^nC_0}(2a+nd) + {^nC_1}(2a+nd) + {^nC_2}(2a+nd) + \dots + {^nC_n}(2a+nd)$$

$$= (2a+nd)({^nC_0} + {^nC_1} + \dots + {^nC_n})$$

$$S = \frac{2a+nd}{2} \cdot 2^n$$

Q $3 \cdot {^nC_0} + 5 \cdot {^nC_1} + 7 \cdot {^nC_2} + \dots + (2n+3) \cdot {^nC_n} = ?$

2 2 AP.

$$= \left(\frac{3 + 2n+3}{2}\right) \cdot 2^n = \left(\frac{2n+6}{2}\right) \cdot 2^n = (n+3) \cdot 2^n$$

Q $5 \cdot {^nC_1} + 9 \cdot {^nC_2} + 13 \cdot {^nC_3} + \dots + (4n+1) \cdot {^nC_n} = ?$

4 4

1^{st} term of ${^nC_0}$ is missing

$$\{1 \cdot {^nC_0} + 5 \cdot {^nC_1} + 9 \cdot {^nC_2} + 13 \cdot {^nC_3} + \dots + (4n+1) \cdot {^nC_n}\} - 1 \cdot {^nC_0}$$

$$\left(\frac{1 + 4n+1}{2}\right) \cdot 2^n - 1$$

$$(2n+1) \cdot 2^n - 1$$

(A)