

① Plane

$$(r - F.P) \cdot \vec{n} = 0$$

② $ax + by + cz + d = 0$

$(a, b, c) = \text{DR of Normal}$

(3) $a=0 \rightarrow$ Plane || x Axis

xy Plane || y axis Plane

$\Rightarrow \underline{z = d}$

(4) $P_1 \parallel P_2$

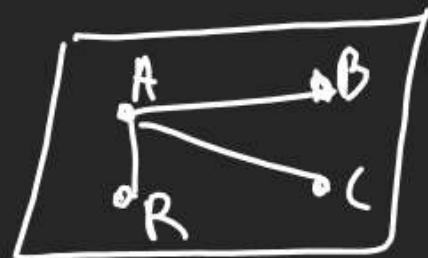
$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} = \frac{d_1}{d_2}$$

(5) $\cos \theta = \frac{n_{P_1} \cdot n_{P_2}}{|n_{P_1}| |n_{P_2}|}$

(6) $P_1 \perp P_2$

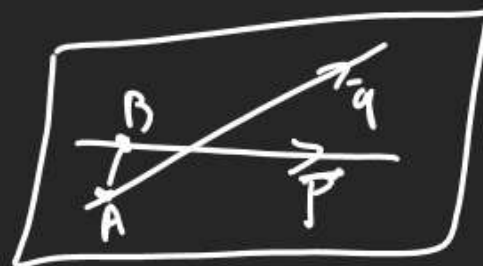
$a_1 a_2 + b_1 b_2 + c_1 c_2 = 0$

(7) 3 pts



$$[AR \ AB \ AC] = 0$$

(8) 2 Int. Lines



$$[\vec{AB} \ \vec{P} \ \vec{Q}] = 0$$

(9) $\vec{r} = \vec{a} + t\vec{p} + s\vec{q}$

Par. EOP $\vec{n} = \vec{p} \times \vec{q}$



Q Express EOP $\vec{r} = (\hat{i} - 2\hat{j}) + \lambda(2\hat{i} - \hat{j} + 3\hat{k}) + \mu(3\hat{i} + 4\hat{j} - \hat{k})$

① in Scalar Dot Prod. form.

(2) in Cart. form.

here $\vec{r} = \langle \hat{i} - 2\hat{j} \rangle + \lambda \langle 2\hat{i} - \hat{j} + 3\hat{k} \rangle + \mu \langle 3\hat{i} + 4\hat{j} - \hat{k} \rangle$
in Parametric form.

(2) normal vector $= \vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 3 \\ 3 & 4 & -1 \end{vmatrix}$

$= \langle -11, 11, 11 \rangle$

(3) $(r - \langle 1, -2, 0 \rangle) \cdot \langle -11, 11, 11 \rangle = 0$

$\vec{r} \cdot \langle -11, 11, 11 \rangle = -11 + 22 + 0$

$\vec{r} \cdot \langle -11, 11, 11 \rangle = -3$ SDP form

$[AR \ AB \ \vec{P}] = 0$

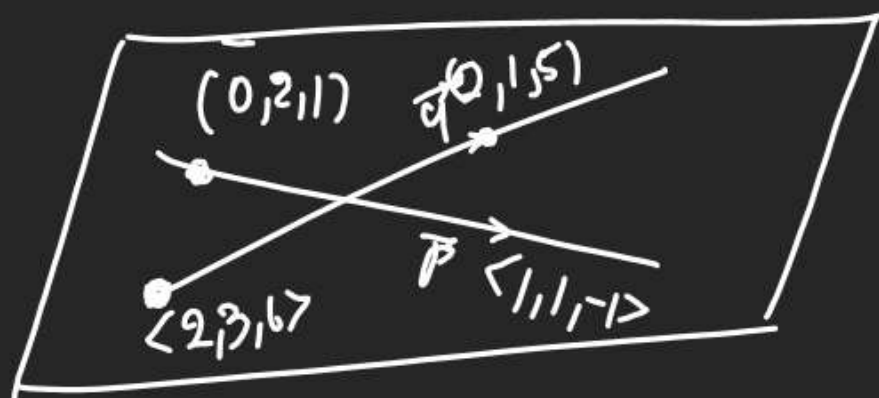
(4) Cart. form

$-x + y + z = -3$

Q Find EOP containing Lines

$$L_1: \vec{r} = \langle 0, 2, 1 \rangle + \lambda \langle 1, 1, 1 \rangle$$

$$L_2: \vec{r} = \langle 2, 3, 6 \rangle + \mu \langle 2, 1, 5 \rangle$$



$$(\vec{r} - \langle 2, 3, 6 \rangle) \cdot \langle 6, -7, -1 \rangle = 0$$

$$\vec{r} \cdot \langle 6, -7, -1 \rangle = 12 - 21 - 6$$

$$n = \vec{P} \times \vec{Q} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & -1 \\ 2 & 1 & 5 \end{vmatrix} = \langle 6, -7, -1 \rangle$$

$$6x - 7y - z = -15$$

Eqⁿ of Plane in Normal Form.

$$ax + by + cz = d \rightarrow \text{EOP}$$

$$\vec{n} = \langle a, b, c \rangle$$

If we convert $\langle a, b, c \rangle$ in $\langle l, m, n \rangle$
DR \vec{a} & DL

then EOP will be in Normal form.

$$p: ax + by + cz = d \div \sqrt{a^2 + b^2 + c^2}$$

or $\frac{d}{|\vec{n}|} + \text{ve sign}$

$$\left(\frac{a}{\sqrt{a^2 + b^2 + c^2}} \right) x + \left(\frac{b}{\sqrt{a^2 + b^2 + c^2}} \right) y + \left(\frac{c}{\sqrt{a^2 + b^2 + c^2}} \right) z = \left[\frac{d}{\sqrt{a^2 + b^2 + c^2}} \right] \rightarrow \text{This distance of Plane from origin}$$

$$lx + my + nz = p$$

$$3x + 4y - 5 = 0 \text{ or } (0, 0) \text{ is dis: } p = \frac{|0 + 0 - 5|}{\sqrt{3^2 + 4^2}} = \frac{1}{\sqrt{5}}$$

$$\vec{r} \cdot \vec{n} = d \quad (\text{Vector EOP}) \div |\vec{n}|$$

$$\vec{r} \cdot \hat{n} = \frac{d}{|\vec{n}|}$$

$$\vec{r} \cdot \hat{n} = p$$

Q Find Dir. Cosines of Normal & \pm distance of Plane $\vec{r} \cdot \langle 6\hat{i} - 3\hat{j} - 2\hat{k} \rangle + 1 = 0$ from $(0,0,0)$?

$$\vec{r} \cdot \langle 6\hat{i} - 3\hat{j} - 2\hat{k} \rangle = -1 \quad \leftarrow \oplus \text{ बनाया}$$

$$\vec{r} \cdot \langle -6\hat{i} + 3\hat{j} + 2\hat{k} \rangle = 1$$

$$\div \sqrt{6^2 + 3^2 + 2^2} = 1$$

$$\vec{r} \cdot \langle -\frac{6}{7}\hat{i} + \frac{3}{7}\hat{j} + \frac{2}{7}\hat{k} \rangle = \frac{1}{7}$$

D.C. of Normal $\langle -\frac{6}{7}, \frac{3}{7}, \frac{2}{7} \rangle$

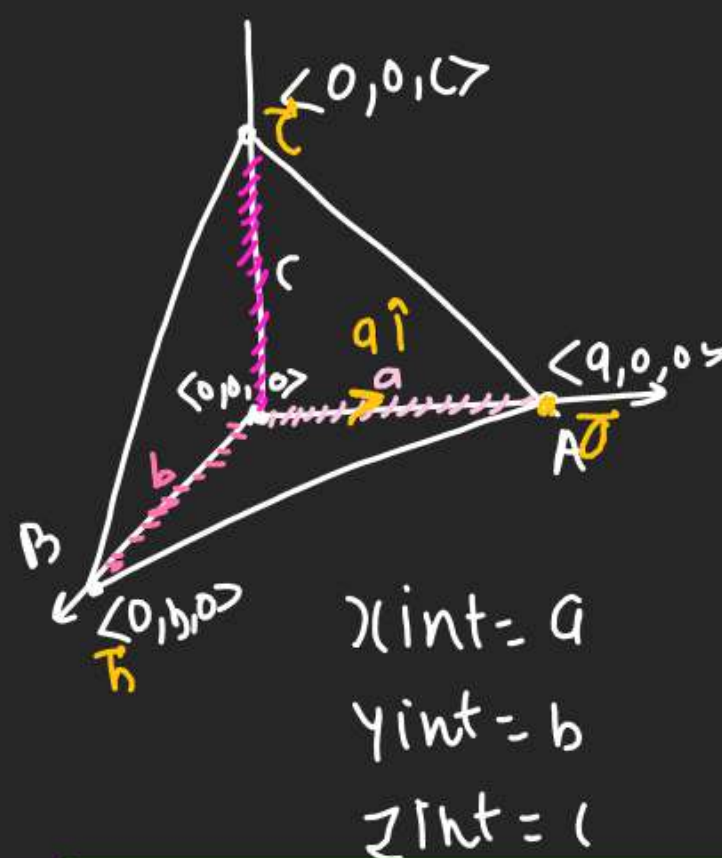
distance from origin = $\frac{1}{7}$

Rem: \rightarrow distance of $ax + by + cz + d = 0$ from $\langle 0,0,0 \rangle = \frac{|d|}{\sqrt{a^2 + b^2 + c^2}}$

Q Find \pm distance of $3x - 4y + z - 1 = 0$ from $\langle 0,0,0 \rangle$

$$d = \frac{|-1|}{\sqrt{9 + 16 + 1}} = \frac{1}{\sqrt{26}}$$

Intercept form of Plane.



$$(4) \Delta \text{ Area} = \frac{1}{2} |\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}|$$

$$= \frac{1}{2} |a\hat{i} \times b\hat{j} + b\hat{j} \times c\hat{k} + c\hat{k} \times a\hat{i}|$$

$$= \frac{1}{2} |ab\hat{k} + bc\hat{i} + ca\hat{j}|$$

$$A = \frac{1}{2} \sqrt{(ab)^2 + (bc)^2 + (ca)^2}$$

1) EOP $\rightarrow \frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$

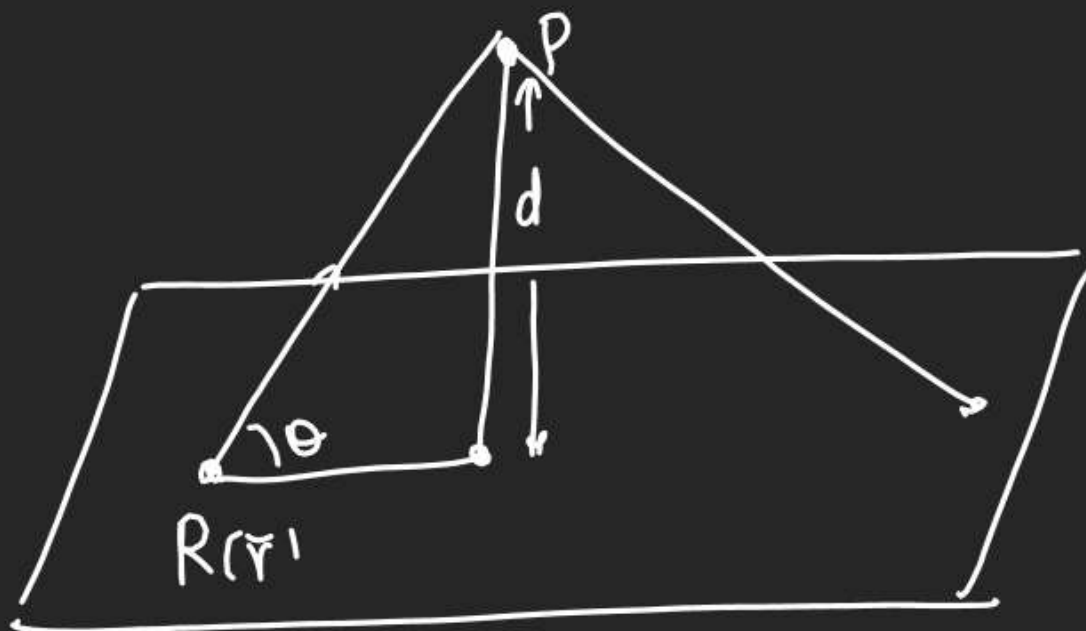
2) If Plane \parallel^r to Z Axis \rightarrow Off of Z = 0

$$\Rightarrow c \rightarrow \infty$$

$$\frac{x}{a} + \frac{y}{b} = 1$$

3) $ax + by + cz + d = 0$ is \parallel^r to X Axis $a = 0$

Per distance of a Pt. from a Plane.

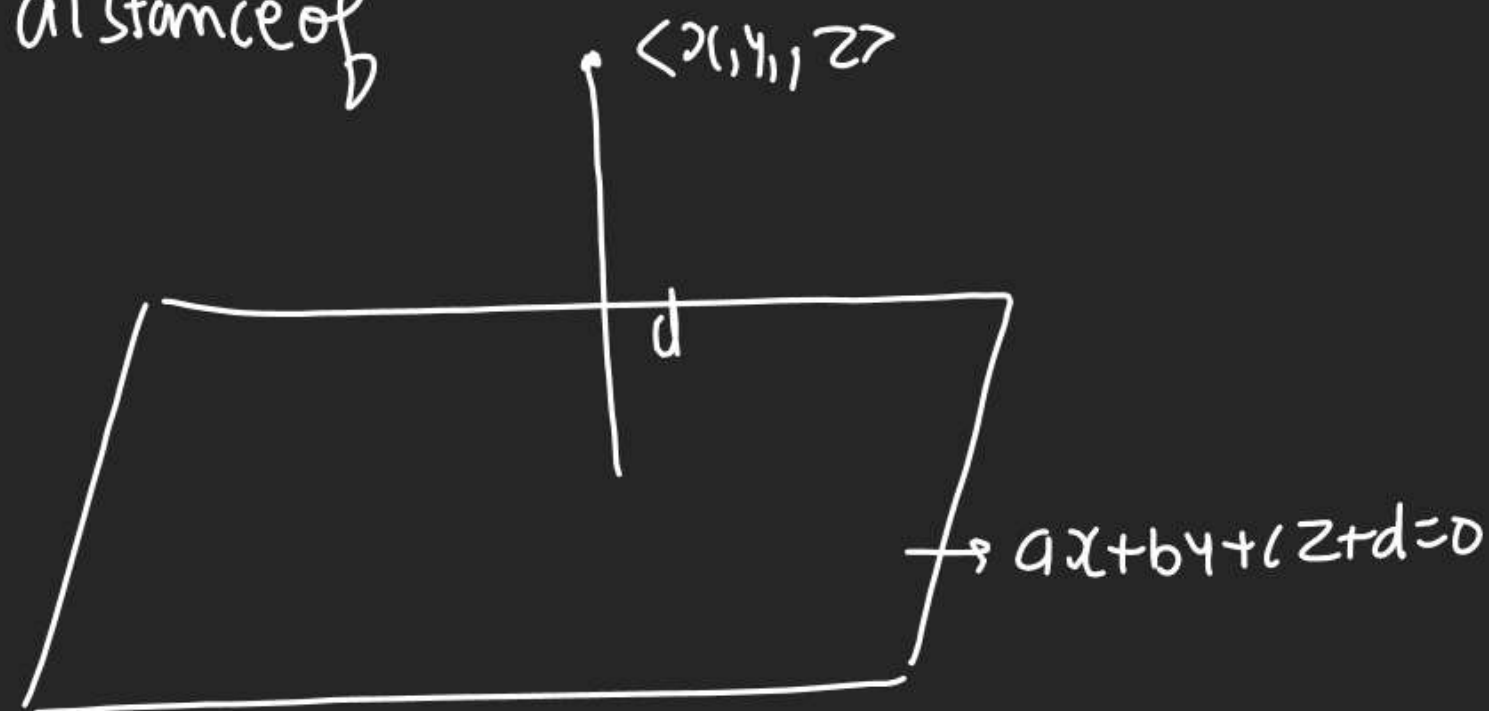


$$d = \text{proj of } \vec{RP} \text{ on } \vec{n}$$

$$= \left| \frac{\vec{RP} \cdot \vec{n}}{|\vec{n}|} \right| = \left| \frac{(\vec{r} - \vec{p}) \cdot \vec{n}}{|\vec{n}|} \right|$$

$$d = \left| \frac{\vec{r} \cdot \vec{n} - \vec{p} \cdot \vec{n}}{|\vec{n}|} \right|$$

distance of



$$\Rightarrow p = \frac{|ax_1 + by_1 + cz_1 + d|}{\sqrt{a^2 + b^2 + c^2}}$$

Q Per dist. of $\langle 1, -1, 1 \rangle$ from $(\vec{r} - \langle 6, 7, -1 \rangle) \cdot \langle 2, 0, 1 \rangle = 0$

$$d = \frac{|2 + 0 + 1 - 1|}{\sqrt{2^2 + 0^2 + 1^2}} \quad \begin{aligned} 2x + 0y + z &= 12 + 0 - 1 \\ 2x + z &= 11 \end{aligned}$$

$$= 8/\sqrt{5}$$

Q Find EOP which is a distance 4 from

origin & $\vec{n} = \langle 2, -1, 2 \rangle$

$$\vec{n} = \langle a, b, c \rangle = \langle 2, -1, 2 \rangle \rightarrow |\vec{n}| = 3$$

$$p = 4$$

$$\vec{r} \cdot \vec{n} = p$$

$$\hat{n} = \left\langle \frac{2}{3}, -\frac{1}{3}, \frac{2}{3} \right\rangle$$

$$\langle x, y, z \rangle \cdot \left\langle \frac{2}{3}, -\frac{1}{3}, \frac{2}{3} \right\rangle = 4$$

$$\frac{2x}{3} - \frac{y}{3} + \frac{2z}{3} = 4$$

$$\boxed{2x - y + 2z = 12}$$

Q Reduce $x - 3y + 5z + 35 = 0$ in

Intercept form.

$$x - 3y + 5z = -35$$

$$\frac{x}{-35} + \frac{y}{\frac{35}{3}} + \frac{z}{-7} = 1$$

$$\div (-35)$$

Q If Plane $2x + 3y - 4z = 12$

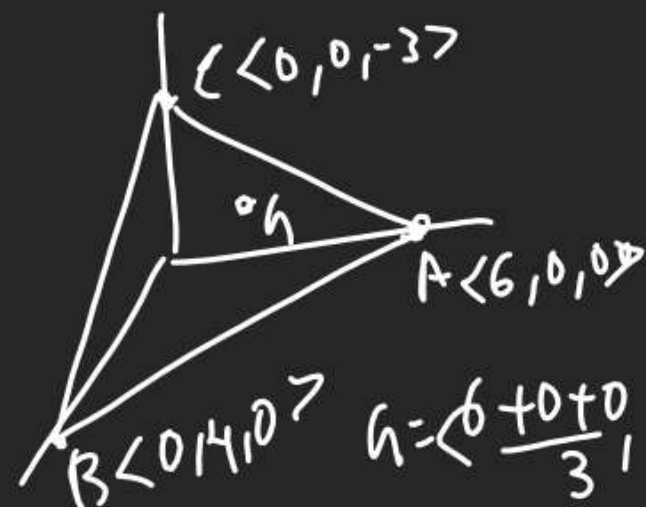
meet Coord Axes at A, B, C

find centroid of $\triangle ABC$

Intercept form

$$2x + 3y - 4z = 12 \quad \div 12$$

$$\frac{x}{6} + \frac{y}{4} - \frac{z}{3} = 1$$



$$G = \left\langle \frac{6+0+0}{3}, \frac{0+4+0}{3}, \frac{0+0+3}{3} \right\rangle$$

$$= \left\langle 2, \frac{4}{3}, 1 \right\rangle$$

Q Find EOP P.T. $\langle 2, -1, 3 \rangle$

& \vec{r} to vector

$$\vec{b} = 3\hat{i} - \hat{k}, \quad \vec{c} = -3\hat{i} + 2\hat{j} + 2\hat{k}$$

$$1) \vec{n} = \vec{b} \times \vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 0 & -1 \\ -3 & 2 & 2 \end{vmatrix}$$

$$= \langle 2, -3, 6 \rangle$$

$$= \left\langle \frac{2}{3}, -1, 2 \right\rangle$$

$$2) 2(x-2) - 3(y+1) + 6(z-3) = 0$$

$$2x - 3y + 6z = 4 + 3 + 18$$

$$2x - 3y + 6z = 25$$

1st distance betⁿ 2 11th Planes.

$$P_1: ax+by+cz+d_1=0$$

$$P_2: ax+by+cz+d_2=0$$

$$d = \frac{|d_1 - d_2|}{\sqrt{a^2 + b^2 + c^2}}$$

Q Find Plane 11th to $P_1: x+5y-4z+5=0$

& Sum of whose intercepts on axes is 19 then find distance betⁿ these 2 planes.

① plane 11th to $x+5y-4z+5=0$

$$P_2: x+5y-4z=\lambda$$

(2) Intercept form

$$\frac{x}{1} + \frac{y}{\left(\frac{\lambda}{5}\right)} + \frac{z}{\left(\frac{\lambda}{-4}\right)} = 1$$

$$3) \lambda + \frac{\lambda}{5} + \frac{\lambda}{-4} = 19$$

$$20\lambda + 4\lambda - 5\lambda = 380$$

$$19\lambda = 380$$

$$\lambda = 20$$

$$\therefore 11^{\text{th}} \text{ Plane } P_2: x+5y-4z=20$$

$$x+5y-4z-20=0$$

(4) dist. betⁿ P_1 & P_2

$$d = \frac{|5 - (-20)|}{\sqrt{1+25+16}} = \frac{25}{\sqrt{42}}$$

Q Find Plane 11th to $P_1: 2x-6y+3z=0$

at a distance of 2 units from

$$\text{Pt. } m: \langle 1, 2, -3 \rangle$$

$$11^{\text{th}} \text{ plane } \rightarrow P_2: 2x-6y+3z=\lambda$$

$$\text{dist. of } P_2 \text{ from } m \langle 1, 2, -3 \rangle = 2$$

$$\frac{|2-12-9-\lambda|}{\sqrt{4+36+9}} = 2$$

$$|\lambda+19|=14$$

$$\lambda+19=14$$

$$\lambda=-5$$

$$\lambda+19=-14$$

$$\lambda=-33$$

$$P_2: 2x-6y+3z=-5$$

$$2x-6y+3z=-33$$

Q A Plane which Remains at a constant distance p from origin cuts coord Axes at A, B, C. Find locus of

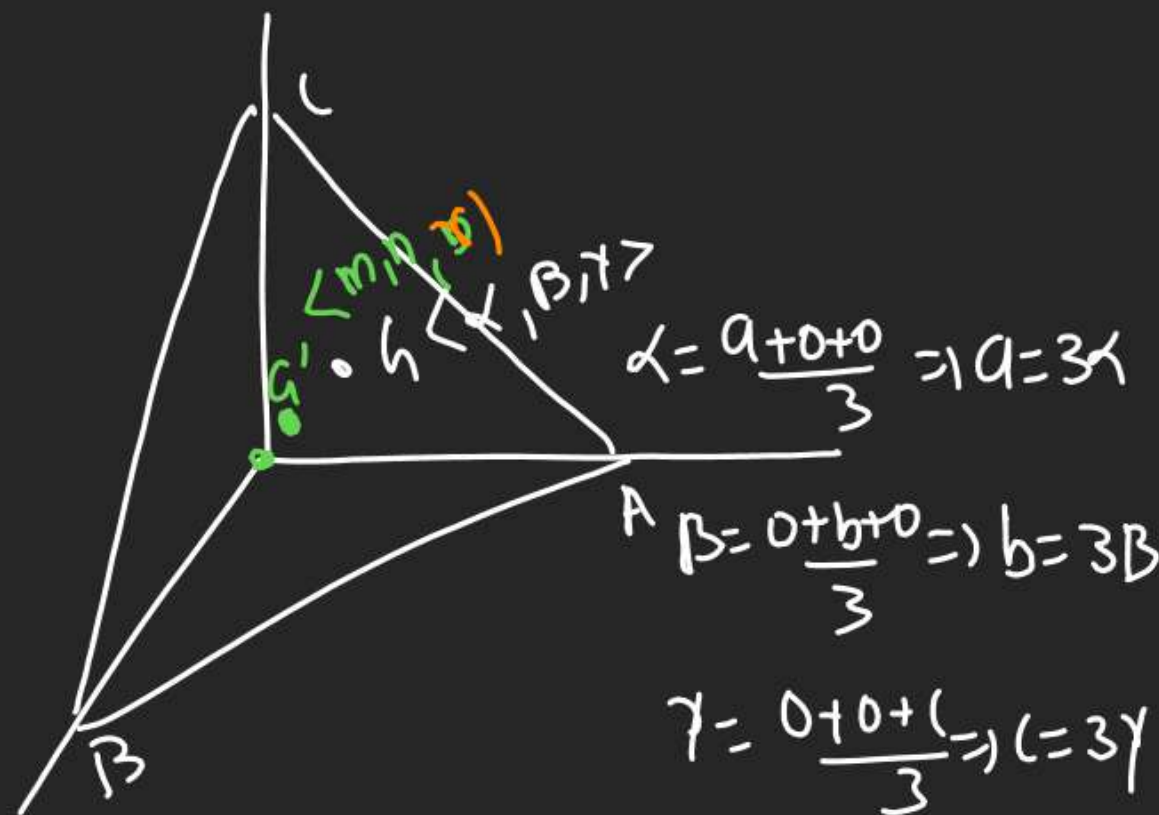
- Centroid of $\triangle ABC$
- Centre of tetrahedron OABC

1) Let Plane in $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$

$p = \text{dist. from } \langle 0, 0, 0 \rangle$

$$p = \frac{|0+0+0-1|}{\sqrt{\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2}}}$$

$$\sqrt{\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2}} = \frac{1}{p} \Rightarrow \boxed{\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2}}$$



$$x = \frac{a+0+0}{3} \Rightarrow a = 3x$$

$$y = \frac{0+b+0}{3} \Rightarrow b = 3y$$

$$z = \frac{0+0+c}{3} \Rightarrow c = 3z$$

$$\frac{1}{p^2} = \frac{1}{(3x)^2} + \frac{1}{(3y)^2} + \frac{1}{(3z)^2}$$

$$\boxed{\frac{1}{p^2} = x^2 + y^2 + z^2} \quad \text{Board}$$

(3) $G = \langle m, n, r \rangle$: (centre of tetrahedron)

$$m = \frac{0+a+0+0}{4}, n = \frac{0+0+b+0}{4}, r = \frac{0+0+0+c}{4}$$

$$a = 4m, b = 4n, c = 4r$$

$$\frac{1}{p^2} = \frac{1}{(4m)^2} + \frac{1}{(4n)^2} + \frac{1}{(4r)^2}$$

$$16p^2 = m^2 + n^2 + r^2$$

$$\boxed{16p^2 = x^2 + y^2 + z^2}$$