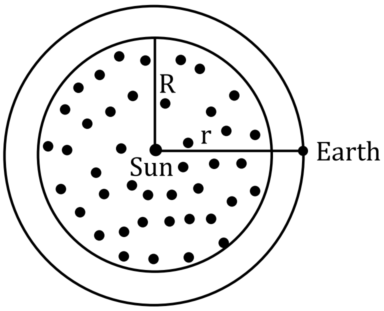
DPP 02

Solution

1. Energy radiated by sun, according to Stefan's law,



$E = \sigma T^4 \times (\text{ area } 4\pi R^2)$.

This energy is spread around sun in space, in a sphere of radius r.

Earth (E) in space receives part of this energy.

$$\frac{\text{Energy}}{\text{Area of envelope}} = \frac{\sigma T^4 \times 4\pi R^2 \times \text{ time}}{4\pi r^2}$$

$$\text{Energy incident per unit area on earth } = \frac{\sigma T^4 R^2 \times \text{ time}}{r^2}$$

$$\therefore \text{ Power incident per unit area on earth} = \left(\frac{R^2 \sigma T^4}{r^2}\right)$$

$$\therefore \text{ Power incident on earth} = \pi r_0^2 \times \frac{R^2 \sigma T^4}{r^2}$$

2. For conduction from inner sphere to outer one,

$$dQ = -KA\frac{dT}{dr} \times (\text{ time dt})$$

$$\frac{dQ}{dt} = -K \times (4\pi r^2)\frac{dT}{dr}$$

$$\therefore \text{ Radial rate of flow } Q = -4\pi K r^2 \frac{dT}{dr}$$

$$\therefore Q\int_{r_1}^{r_2} \frac{dr}{r^2} = -4\pi K \int_{T_1}^{T_2} dT$$

$$Q\left[\frac{r_1 - r_2}{r_1 r_2}\right] = 4\pi K[T_2 - T_1]$$

$$Q = \frac{4\pi K(T_1 - T_2)r_1 r_2}{(r_2 - r_1)}$$

$$\therefore \text{ Q is proportional to } \left(\frac{r_1 r_2}{r_2 - r_1}\right)$$

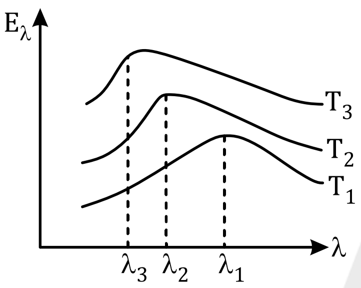
3. According to Stefan's law, energy radiated per second by a body of emissivity e at a temperature T is $e\sigma AT^4$;

$\therefore E = 0.6\sigma AT^4$.

4. Warming of glass of bulb due to filament is due to radiation mainly.

For convection process, a medium is required which can move between two points, to transfer heat. Bulb is almost evacuated to have no medium.

5. As per Wien's displacement law, λT = constant



From the graph given,

$\lambda_3 < \lambda_2 < \lambda_1 \;\therefore\; T_3 > T_2 > T_1$

Temperature of sun is higher than that of welding arc which is higher than that of tungsten filament lamp.

$\therefore$ Sun $= T_3$, Welding arc $= T_2$

Tungsten filament $= T_1$

6. According to Stefan's law,

Radiant energy $E = (\sigma T^4) \times$ area $\times$ time

$$\therefore \frac{E_2}{E_1} = \frac{\sigma(2T)^4 \times 4\pi(2R)^2 \times t}{\sigma T^4 \times (4\pi R)^2 \times t} = 16 \times 4$$

$$\therefore \frac{E_2}{E_1} = 64$$

7. According to Wien's displacement law,

$\lambda T = b =$ Wien's constant.

$$\therefore \lambda_A T_A = b \text{ or } T_A = \frac{b}{3 \times 10^{-7}}$$

$$T_A = \frac{b \times 10^7}{3} = \frac{z}{3} \text{ where } z = (b \times 10^7)$$

$$\text{Similarly } T_B = \frac{b \times 10^7}{4} = \frac{z}{4}$$

and $T_C = \frac{b \times 10^7}{5} = \frac{z}{5}$

Again, according to Stefan's law,

Q = Power radiated by black body $= A\sigma T^4$

where A = area of disc $= \pi R^2$.

$Q_A = (\pi R_A^2) \times \sigma \times (T_A)^4$

or $Q_A = \pi(2 \times 10^{-2})^2 \times \sigma \times \left(\frac{z}{3}\right)^4$

$Q_A = \pi\sigma \times 10^{-4} \times z^4 \times \frac{2^2}{3^4}$

or $Q_A = (\pi\sigma \times 10^{-4} \times z^4) \times \frac{4}{81}$

Put $\pi\sigma \times 10^{-4} \times z^4 = k$ = constant

or $Q_A = \frac{4k}{81} = 0.049k$

Similarly, $Q_B = \frac{k \times (4)^2}{(4)^4} = \frac{k}{16} = 0.062k$

$Q_C = \frac{k \times (6)^2}{(5)^4} = \frac{36k}{625} = 0.057k.$

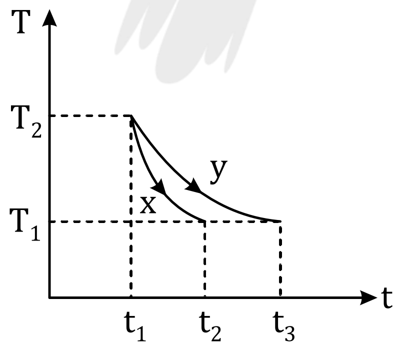
Hence Q_B is maximum.

8. According to Newton's law of cooling, rate of cooling is proportional to $\Delta\theta$.

$\therefore (\Delta\theta)^n = (\Delta\theta)$ or $n = 1$.

10. The two bodies x and y have same surface area. Emissivity (E) $\propto$ rate of cooling or $E \propto -\left(\frac{dT}{dt}\right)$

From graph, consider cooling of x and y from temperature



T_2 to T_1.

Time taken by x $= (t_2 - t_1)$

Time taken by y $= (t_3 - t_1)$

x takes less time and so $E_x > E_y$ ... (i)

According to Kirchhoff's law, a good emitter is also a good absorber.

$\therefore\ a_x > a_y$ ... (ii)

11. Energy radiated,

$E = \sigma T^4 \times (\text{ area } 4\pi R^2) \times \text{ time } \times e$

$$\frac{E_1}{E_2} = \frac{(4000)^4 \times (1)^2 \times 1 \times 4\pi\sigma e}{(2000)^4 \times (4)^2 \times 1 \times 4\pi\sigma e} = \frac{1}{1}$$

12. A good absorber is a good emitter but black holes do not emit all radiations.