Nishant Jindal

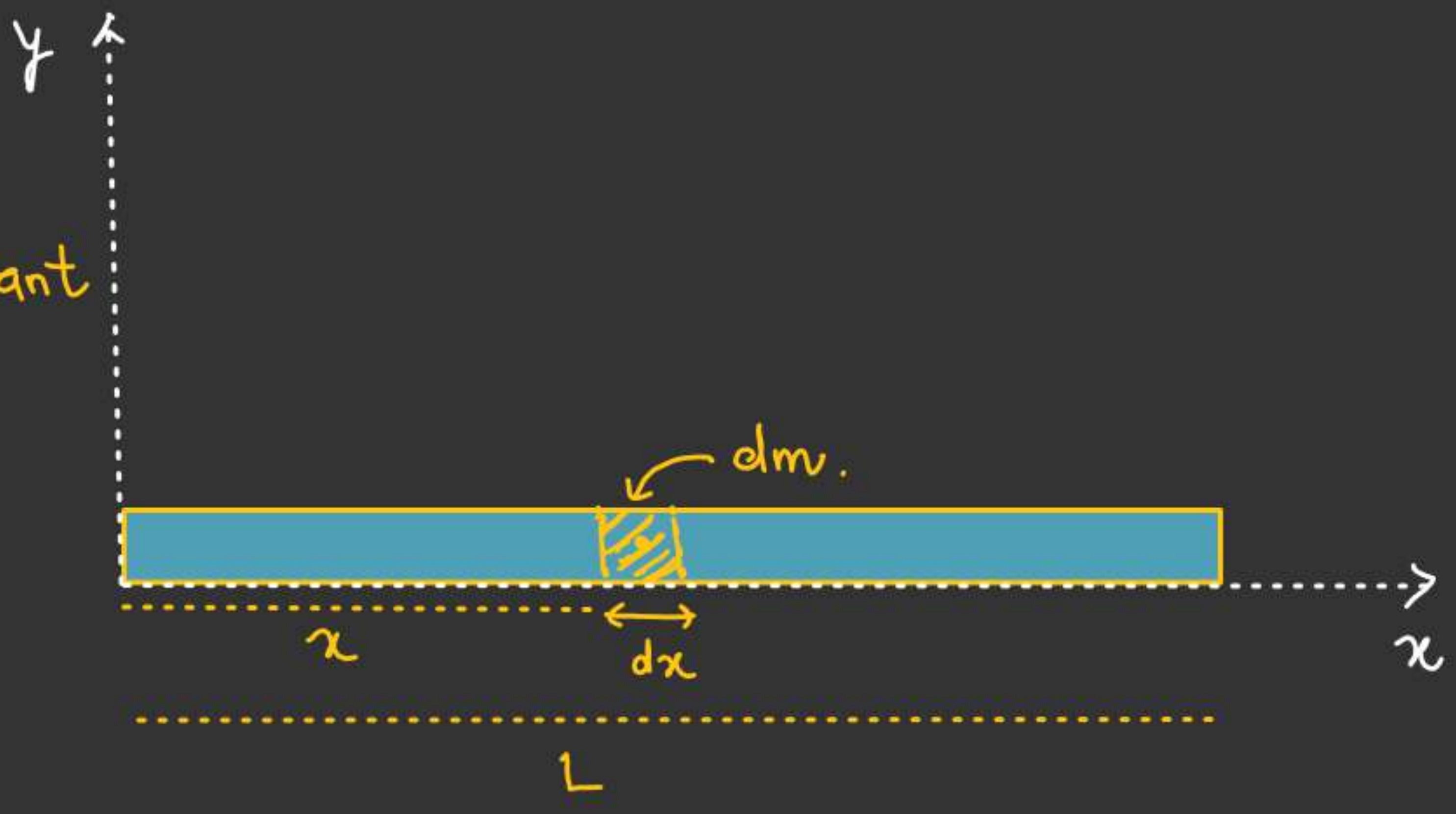
COM

*4. COM of Uniform Rod

$$X_{com} = \frac{\int_0^L dm\, x}{\left[\int_0^L dm\right]}$$

$$\lambda = \frac{M}{L} = \text{Constant}$$

$$dm = \frac{M}{L} dx.$$



$$X_{com} = \frac{\frac{M}{L}\int_0^L x\,dx}{\frac{M}{L}\int_0^L dx} = \frac{\frac{L^2}{2}}{L} = \frac{L}{2}$$

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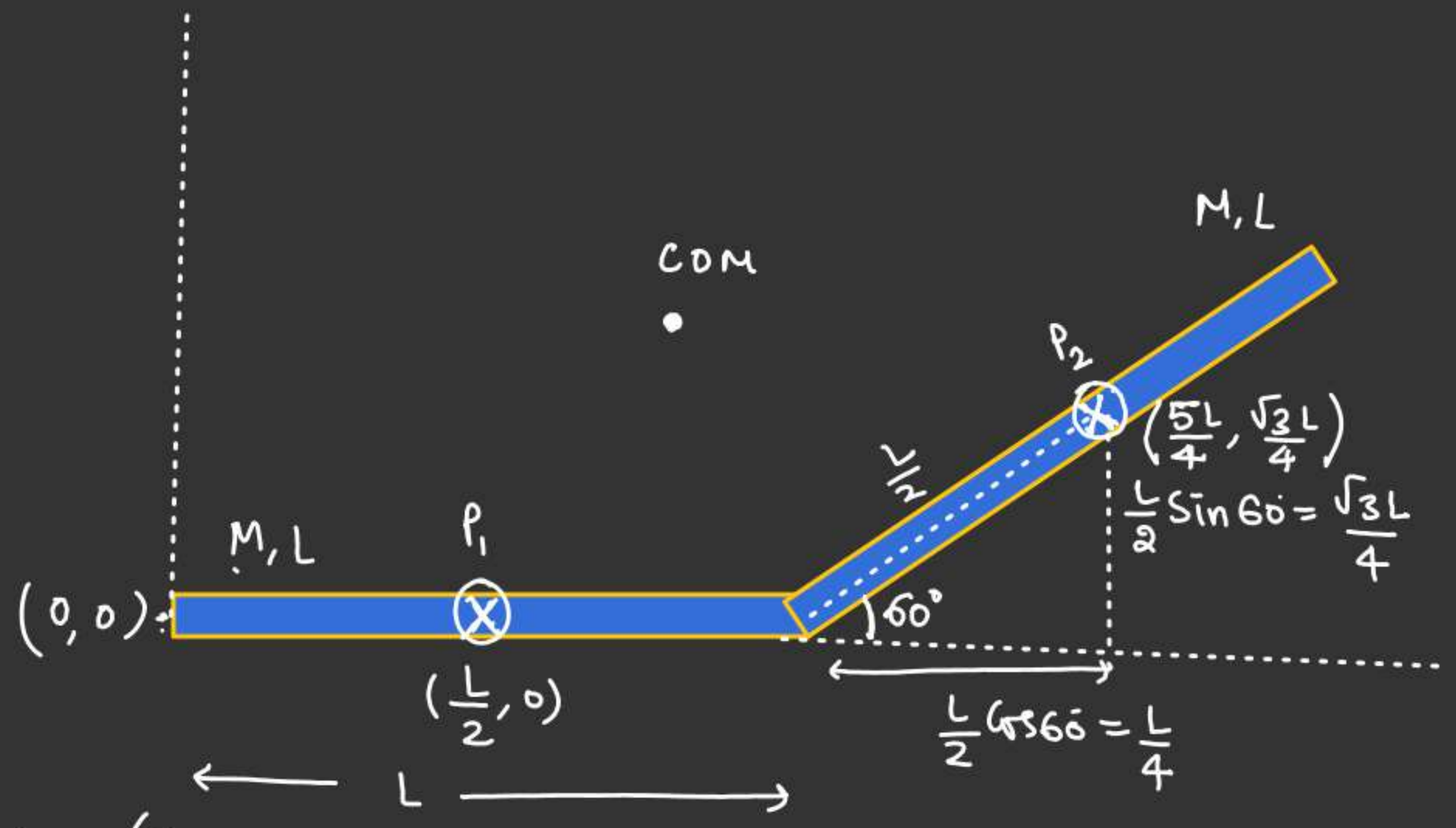
Find COM of both the Rods.
Both the rods are uniform having mass M and length L.



$$X_{com} = \frac{M\left(\frac{L}{2}\right) + M\left(\frac{5L}{4}\right)}{M+M}$$

$$X_{com} = \frac{2ML + 5ML}{4 \times 2M}$$

$$= \left(\frac{7L}{8}\right) \checkmark$$

$$Y_{com} = \frac{M(0) + M\left(\frac{\sqrt{3}L}{4}\right)}{M+M}$$

$$Y_{com} = \left(\frac{\sqrt{3}L}{8}\right) \checkmark$$

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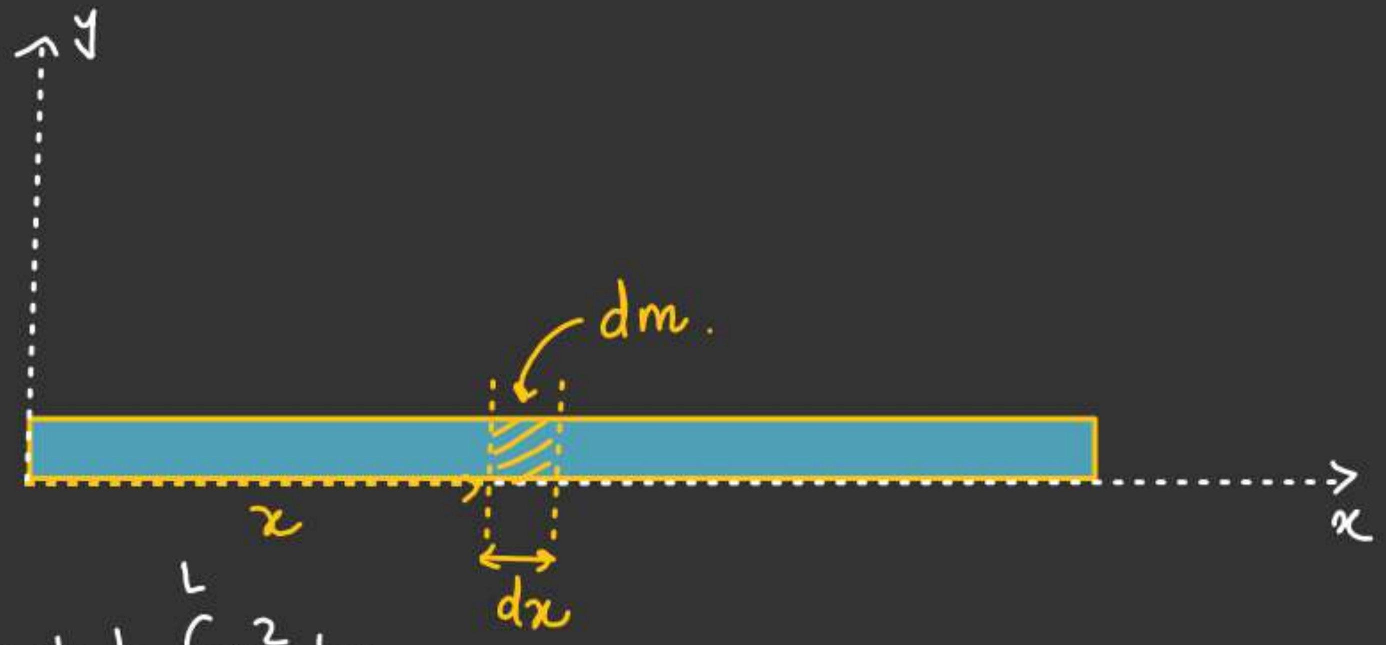
If Rod in Non-Uniform.

$\lambda = a + bx$ where a & b are Constant.



$$dm = \lambda \cdot dx$$

$$dm = (a+bx)\,dx$$

$$X_{com} = \frac{\int_0^L dm\, x}{\int_0^L dm}$$

$$X_{com} = \frac{\int_0^L (a+bx)\,x\,dx}{\int_0^L (a+bx)\,dx}$$

$$X_{com} = \frac{a\int_0^L x\,dx + b\int_0^L x^2\,dx}{a\int_0^L dx + b\int_0^L x\,dx}$$

$$X_{com} = \left(\frac{\frac{aL^2}{2} + \frac{bL^3}{3}}{aL + \frac{bL^2}{2}}\right) = \frac{3aL^2 + 2bL^3}{3(2aL + bL^2)} = \left(\frac{3aL + 2bL^2}{6a + 3bL}\right)$$

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COM

COM of an uniform arc

$L = R\theta$

$dl = R\,d\phi$ ✓

$x = R\sin\phi, \quad y = R\cos\phi$

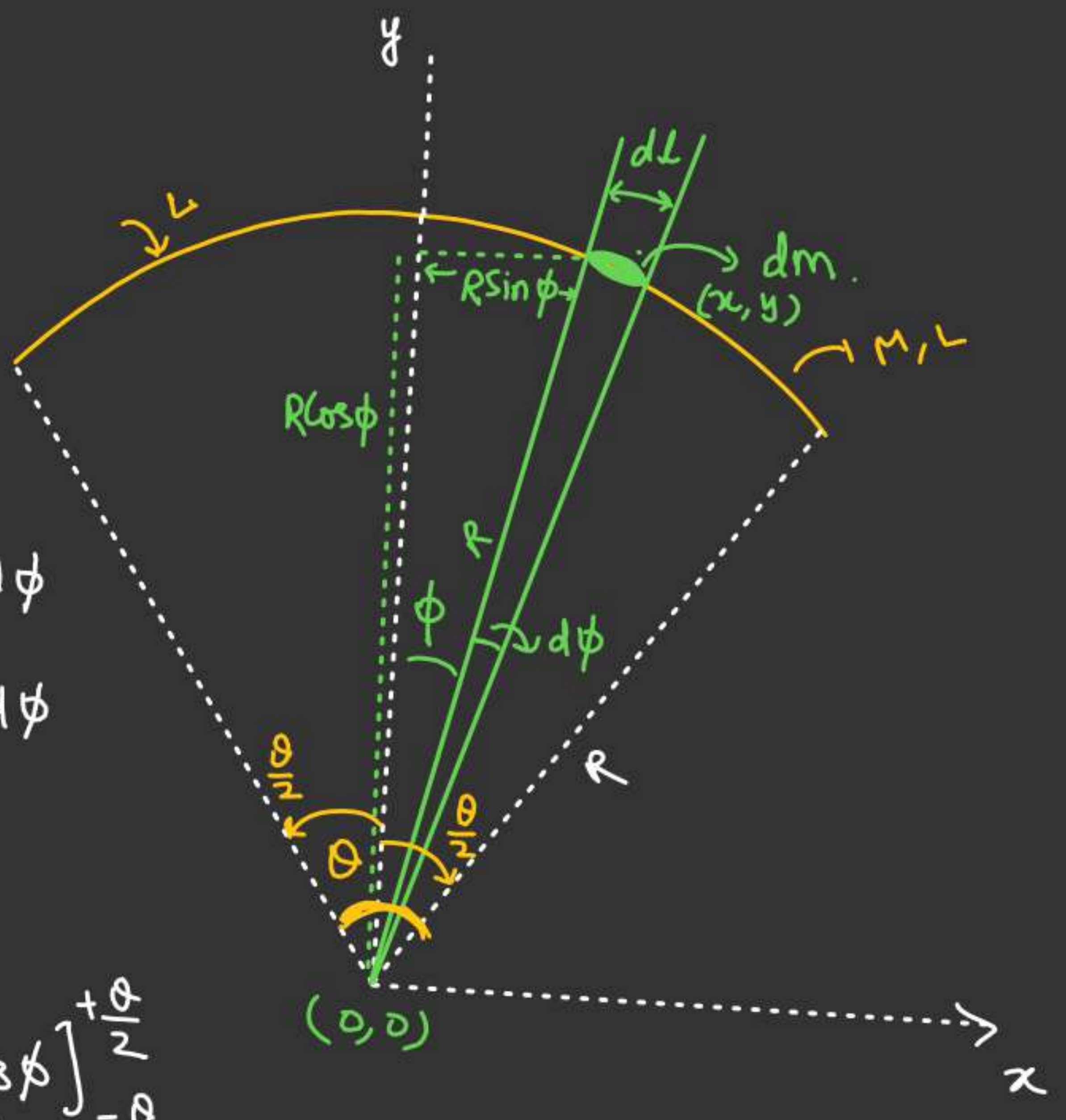
$\lambda = \frac{M}{L} = \frac{M}{R\theta}$

$dm = \lambda \cdot dl$

$= \frac{M}{R\theta} \times R\,d\phi$

$dm = \left(\frac{M}{\theta}\right) d\phi$

$$X_{com} = \frac{\int dm \cdot x}{\int dm} =$$



$$X_{com} = \frac{\int_{-\theta/2}^{+\theta/2} \frac{M}{\theta}(d\phi)\cdot(R\sin\phi)}{\int_{-\theta/2}^{+\theta/2} \frac{M}{\theta}\,d\phi} = \frac{R\int_{-\theta/2}^{+\theta/2} \sin\phi\, d\phi}{\int_{-\theta/2}^{+\theta/2} d\phi} = \frac{R\left[-\cos\phi\right]_{-\theta/2}^{+\theta/2}}{\left[\theta\right]_{-\theta/2}^{+\theta/2}} = 0$$

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COM

$$y_{com} = \frac{\int dm \cdot y}{\int dm} = \frac{\int_{-\frac{\theta}{2}}^{+\theta/2} \left(\frac{M}{\theta} \cdot d\phi\right) \cdot R\cos\phi}{M}$$

$\int dm \Downarrow M$ (uniform)

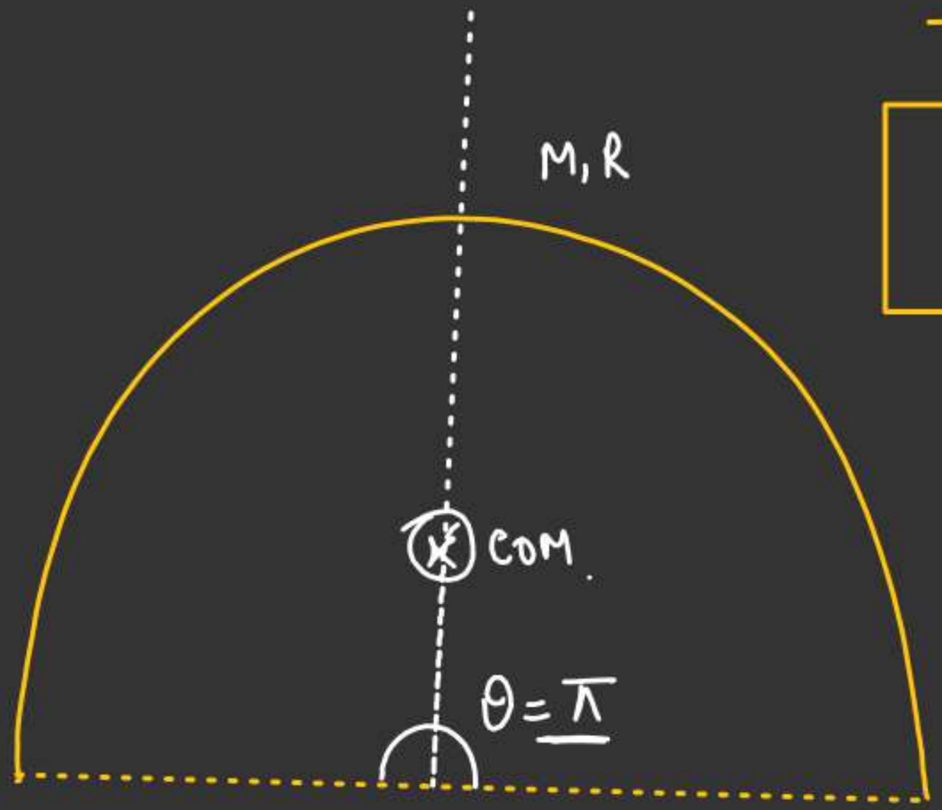
$$y_{com} = \frac{R}{\theta} \int_{-\frac{\theta}{2}}^{+\theta/2} \cos\phi \cdot d\phi$$

$$y_{com} = \frac{R}{\theta}\left[\sin\phi\right]_{-\frac{\theta}{2}}^{+\frac{\theta}{2}}$$

$$y_{com} = \frac{R}{\theta}\left[\sin\frac{\theta}{2} - \sin\left(-\frac{\theta}{2}\right)\right]$$

$$\boxed{y_{com} = 2R\left[\frac{\sin(\theta/2)}{\theta}\right]}$$

COM



$$\boxed{y_{com} = \frac{2R}{\pi}}$$

H.W. Non-Uniform arc

$$\lambda = \lambda_0 \cos\theta$$

$$y_{com} = 2R\left[\frac{\sin(\theta/2)}{\theta}\right]$$

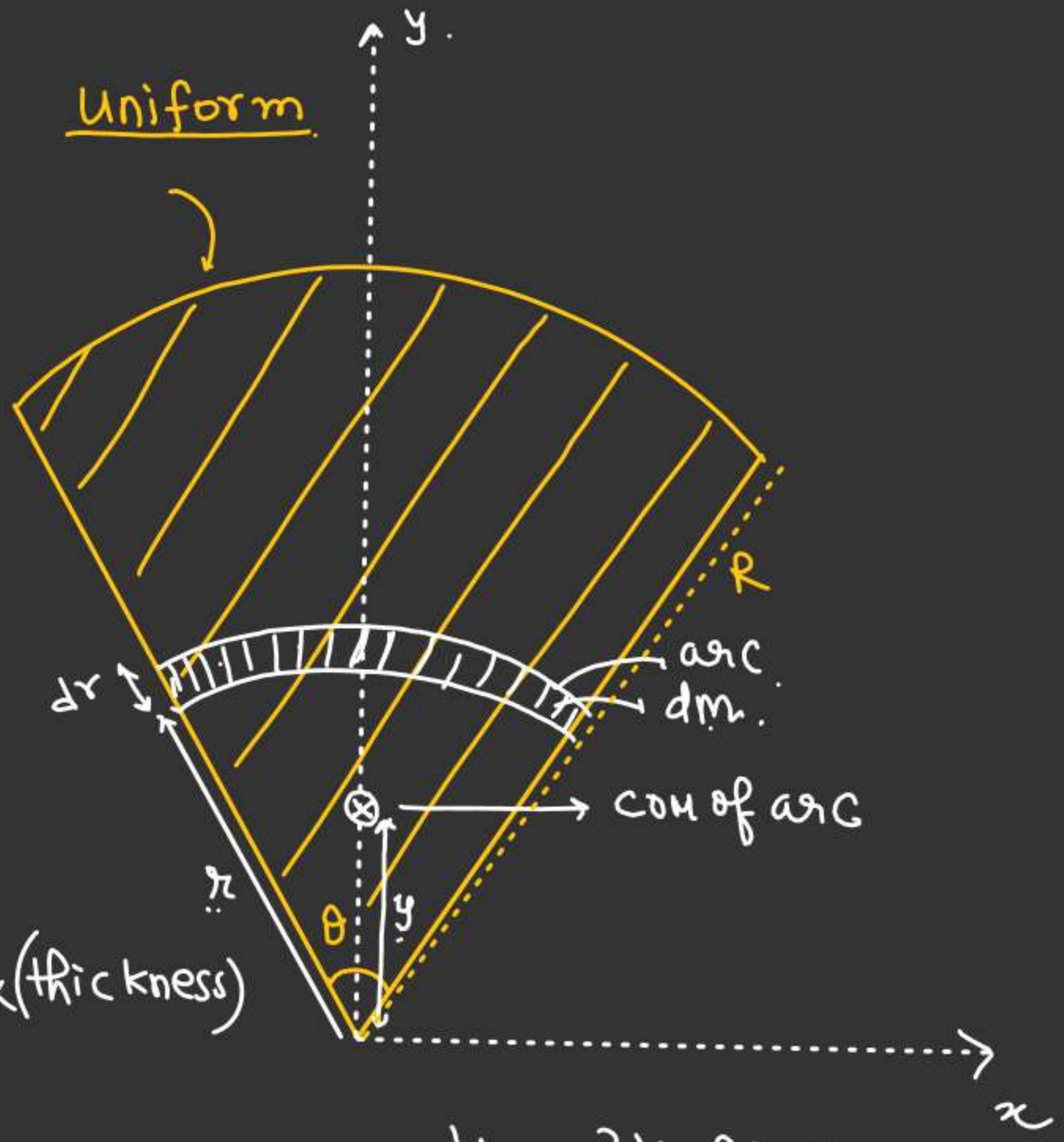
Put $\theta = \pi$

COM

Q.

COM of a sector having mass M & Radius R

Uniform



$$dm = \frac{M}{A} \times \underline{\underline{dA}}$$

Area of Sector $= \left(\frac{R^2\theta}{2}\right)$

$$dm = \frac{M}{\left(\frac{R^2\theta}{2}\right)} \times (r\theta)\,dr$$

$$dm = \left(\frac{2M}{R^2}\right) r\,dr.$$

$$dA = \begin{pmatrix}\text{length of}\\ \text{differential}\\ \text{element}\end{pmatrix} \times (\text{thickness})$$

$$dA = \underline{(r\theta) \times dr}.$$

COM of arc

$$y = \frac{2r}{\theta}.\sin\left(\frac{\theta}{2}\right)$$

$$y = \frac{2r}{\theta}.\sin\left(\theta/2\right)$$

COM of an arc

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COM

$$y_{com} = \frac{\int dm \cdot y}{\int dm} = \frac{\int_0^R \underbrace{\left(\frac{2M}{R^2} r\, dr\right)}_{dm} \underbrace{\left(\frac{2r}{\theta} \sin\frac{\theta}{2}\right)}_{y}}{M}$$

$\int dm = M$ (For uniform)

$$= \frac{4\cancel{M}}{\cancel{M}R^2} \frac{\sin(\theta/2)}{\theta} \int_0^R r^2\, dr$$

$$= \frac{4}{\cancel{R^2}} \frac{\sin(\theta/2)}{\theta} \times \frac{\cancel{R^3}}{3}$$

$$\boxed{Y_{com} = \left(\frac{4R}{3}\right)\left(\frac{\sin(\theta/2)}{\theta}\right)}$$

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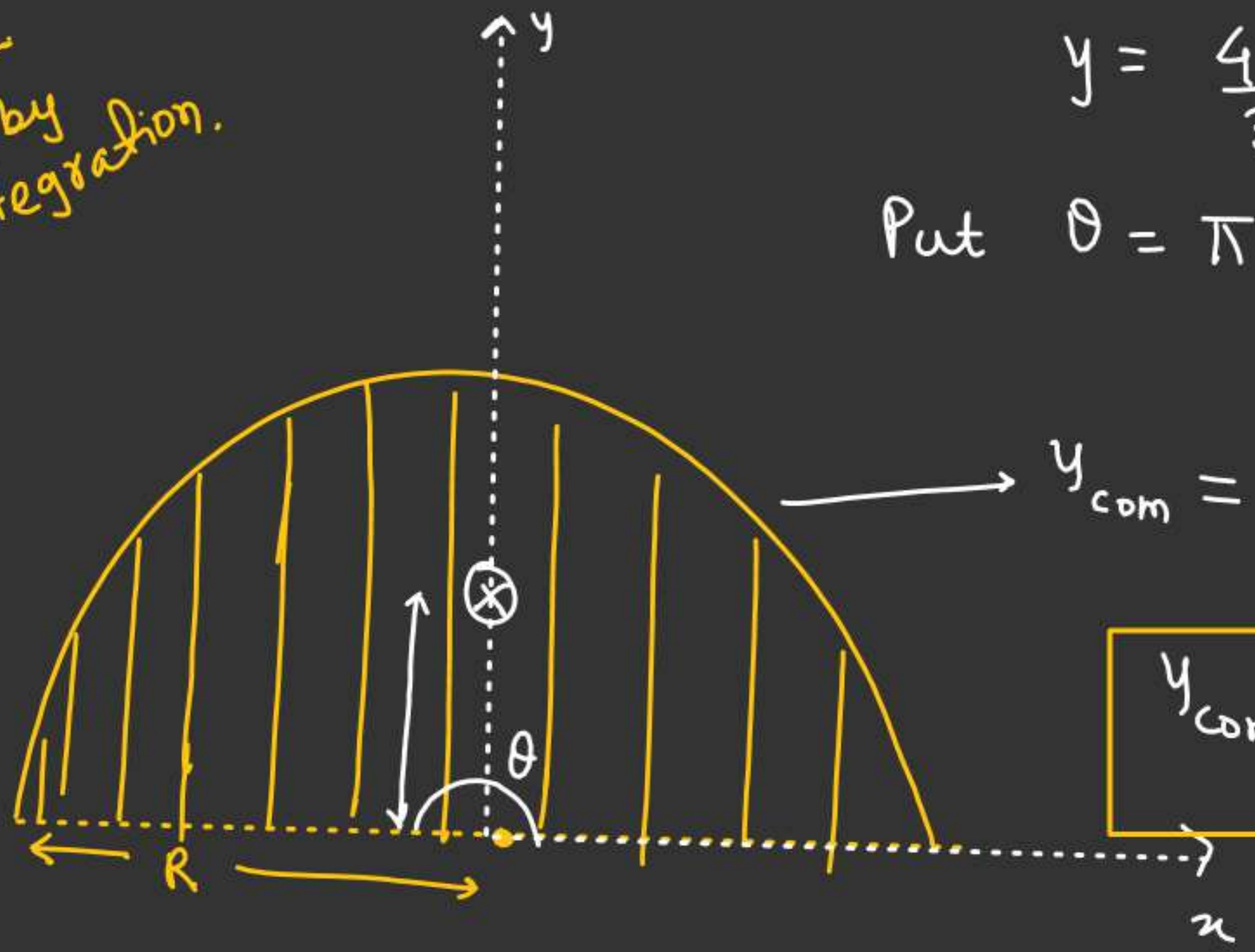
COM

COM of Semi Circular disc



H.W
Solve by Integration.

For Sector

$$y = \frac{4R}{3}\frac{\sin(\theta/2)}{\theta}$$

Put $\theta = \pi$

$$y_{com} = \frac{4R}{3}\frac{\sin(\pi/2)}{\pi}$$

$$\boxed{y_{com} = \frac{4R}{3\pi}}$$

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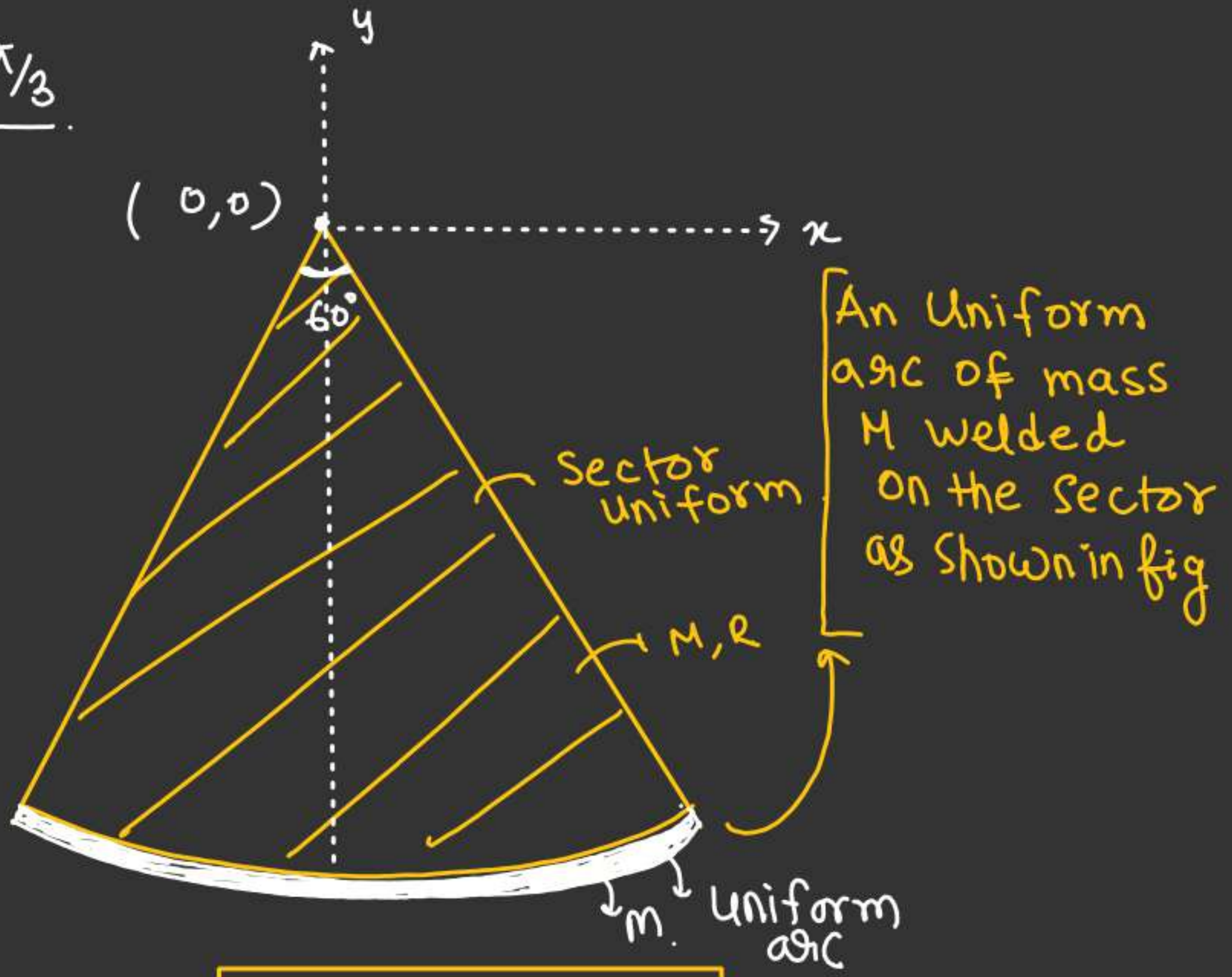
COM

Find COM of the System.

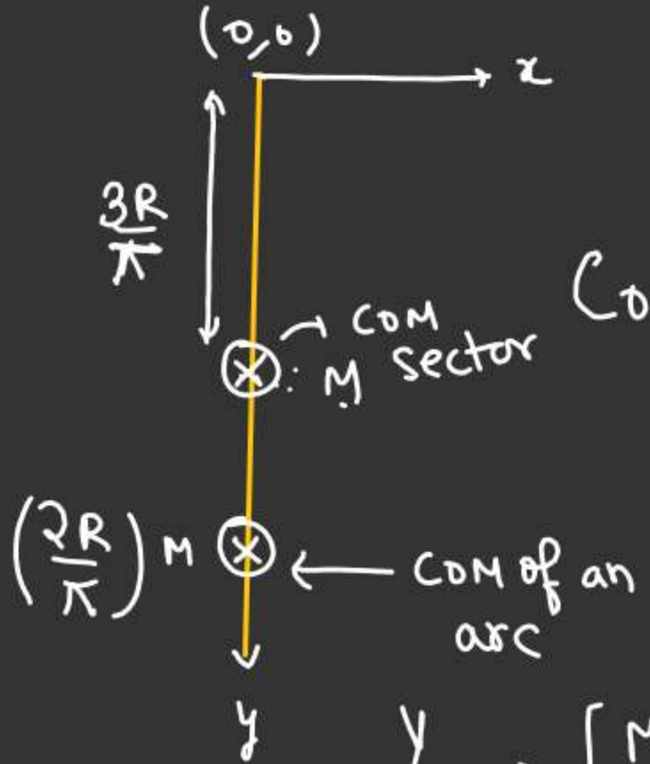
$\theta = \pi/3$



$$\text{COM of arc} = \frac{2R}{\pi/3} \cdot \sin(\pi/6)$$

$$= \frac{6R}{\pi} \times \frac{1}{2}$$

$$= \left(\frac{3R}{\pi}\right) \checkmark$$



$$\text{COM of sector} = \frac{4R}{3} \frac{\sin(\theta/2)}{(\theta)}$$

$$= \frac{4R}{3\left(\frac{\pi}{3}\right)} \cdot \sin(\pi/6)$$

$$y_{com} = \left[\frac{M \cdot \left(-\frac{3R}{\pi}\right) + M\left(-\frac{2R}{\pi}\right)}{2M}\right] \qquad = \frac{4R}{\pi} \times \frac{1}{2} = \left(\frac{2R}{\pi}\right) \checkmark$$

$$\boxed{y_{com} = \left(-\frac{5R}{2\pi}\right)}$$