DPP-1(Roots of Quadratic Equation)

SOLUTION

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Subjective :

1. If α and β are the roots of $ax^2 + bx + c = 0$, find the value of $(a\alpha + b)^{-2} + (a\beta + b)^{-2}$.

Ans. $\frac{b^2-2ac}{a^2c^2}$

Sol. We know that $\alpha + \beta = -\frac{b}{a}$ & $\alpha\beta = \frac{c}{a}$ $\quad (a\alpha + b)^{-2} + (a\beta + b)^{-2} = \frac{1}{(a\alpha+b)^2} + \frac{1}{(a\beta+b)^2}$

$$= \frac{a^2\beta^2+b^2+2ab\beta+a^2\alpha^2+b^2+2ab\alpha}{(a^2\alpha\beta+ba\beta+ba\alpha+b^2)^2} = \frac{a^2(\alpha^2+\beta^2)+2ab(\alpha+\beta)+2b^2}{(a^2\alpha\beta+ab(\alpha+\beta)+b^2)^2}$$

($\alpha^2 + \beta^2$ can always be written as $(\alpha + \beta)^2 - 2\alpha\beta$)

$$= \frac{a^2[(\alpha+\beta)^2 - 2\alpha\beta] + 2ab(\alpha+\beta) + 2b^2}{(a^2\alpha\beta + ab(\alpha+\beta) + b^2)^2} = \frac{a^2\left[\frac{b^2-2ac}{a^2}\right] + 2ab\left(-\frac{b}{a}\right) + 2b^2}{\left(a^2\frac{c}{a} + ab\left(-\frac{b}{a}\right) + b^2\right)^2} = \frac{b^2-2ac}{a^2c^2}$$

2. If the coefficient of the quadratic equation are rational & the coefficient of x^2 is 1 , then find the equation one of whose roots is $\tan\frac{\pi}{8}$.

Ans. $x^2 + 2x - 1$

Sol. We know that $\tan\frac{\pi}{8} = \sqrt{2} - 1$

Irrational roots always occur in conjugational pairs.

Hence if one root is $(-1 + \sqrt{2})$, the other root will be $(-1 - \sqrt{2})$. Equation is

$(x - (-1 + \sqrt{2}))(x - (-1 - \sqrt{2})) = 0 \Rightarrow x^2 + 2x - 1 = 0$

3. If equation $\frac{x^2-bx}{ax-c} = \frac{k-1}{k+1}$ has roots equal in magnitude & opposite in sign, then find the value of k

Ans. $k = \frac{a-b}{a+b}$

Sol. Let the roots are α & $-\alpha$. Given equation is $(x^2 - bx)(k + 1) = (k - 1)(ax - c)$

{Considering, $x \neq \frac{c}{a}$ & $k \neq -1$} $\quad \Rightarrow x^2(k + 1) - bx(k + 1) = ax(k - 1) - c(k - 1)$

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$\Rightarrow\ x^2(k+1) - bx(k+1) - ax(k-1) + c(k-1) = 0$

Now sum of roots $= 0\ (\because \alpha - \alpha = 0)$ $\quad\therefore\ b(k+1) + a(k-1) = 0 \Rightarrow k = \frac{a-b}{a+b}$

4. The coefficient of x in the quadratic equation $x^2 + px + q = 0$ was taken as 17 in place of 13 , its roots were found to be -2 and -15 . Find the roots of the original equation.

Ans. $-10, -3$

Sol. Here $q = (-2) \times (-15) = 30$, correct value of $p = 13$. Hence original equation is

$x^2 + 13x + 30 = 0$ as $(x+10)(x+3) = 0$ $\quad\therefore$ roots are $-10, -3$

5. If the equation $(\lambda^2 - 5\lambda + 6)x^2 + (\lambda^2 - 3\lambda + 2)x + (\lambda^2 - 4) = 0$ has more than two roots, then find the value of λ ?

Ans. $\lambda = 2$

Sol. As the equation has more than two roots so it becomes an identity. Hence

$\lambda^2 - 5\lambda + 6 = 0 \quad \Rightarrow \quad \lambda = 2,3$

and $\lambda^2 - 3\lambda + 2 = 0 \quad \Rightarrow \quad \lambda = 1,2$

and $\lambda^2 - 4 = 0 \quad \Rightarrow \quad \lambda = 2, -2$

So $\lambda = 2$

6. If the roots of the equation $(x-a)(x-b) - k = 0$ be c and d, then prove that the roots of the equation $(x-c)(x-d) + k = 0$, are a and b.

Sol. By given condition

$(x-a)(x-b) - k = (x-c)(x-d)$ or $(x-c)(x-d) + k = (x-a)(x-b)$

Above shows that the roots of $(x-c)(x-d) + k = 0$ are a and b.

Single correct answer type :

7. The roots of the quadratic equation $(a+b-2c)x^2 - (2a-b-c)x + (a-2b+c) = 0$ are -

(A) $a+b+c$ & $a-b+c$ (B) $1/2$ & $a-2b+c$

(C) $a-2b+c$ & $1/(a+b-2c)$ (D) none of these

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Ans. **(D)**

Sol. Since sum of coefficients $= 0$

$\therefore$ It's one root is 1 and other root is $\frac{a-2b+c}{a+b-2c}$

8. For the equation $3x^2 + px + 3 = 0, p > 0$ if one of the roots is square of the other, then p is equal to:

(A) 1/3 (B) 1 (C) 3 (D) 2/3

Ans. **(C)**

Sol. Let α, α^2 be the roots of $3x^2 + px + 3 = 0$

Now, $S = \alpha + \alpha^2 = -\frac{p}{3}$,

$P = \alpha^3 = 1 \Rightarrow \alpha = 1, \omega, \omega^2$

Now, $\alpha + \alpha^2 = \frac{p}{3}$ $\quad \Rightarrow \omega + \omega^2 = -\frac{p}{3} \Rightarrow -1 = -\frac{p}{3} \Rightarrow p = 3$

9. If α, β are the roots of quadratic equation $x^2 + px + q = 0$ and γ, δ are the roots of $x^2 + px - r = 0$, then $(\alpha - \gamma) \cdot (\alpha - \delta)$ is equal to :

(A) $q + r$ (B) $q - r$ (C) $-(q + r)$ (D) $-(p + q + r)$

Ans. **(C)**

Sol. Given that,

α, β are the roots of the equation $x^2 + px + q = 0$

$\Rightarrow \alpha + \beta = \frac{-b}{a} = -p$ and $\alpha\beta = \frac{c}{a} = q$

γ, δ are the roots of the equation $x^2 + px + q = 0 \Rightarrow \gamma + \delta = \frac{-b}{a} = -p$ and $\gamma\tilde{o} = \frac{c}{a} = -r$

$\therefore (\alpha - \gamma)(\alpha - \delta) = \alpha^2 - \alpha(\gamma + \delta) + \gamma\delta \quad = \alpha^2 + p\alpha - r \quad = -q - r$

$= -(q + r)(\text{ Ans })$

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10. If sinα & cosα are the roots of the equation $ax^2 + bx + c = 0$ then -

(A) $a^2 - b^2 + 2ac = 0$ (B) $a^2 + b^2 + 2ac = 0$

(C) $a^2 - b^2 - 2ac = 0$ (D) $a^2 + b^2 - 2ac = 0$

Ans. **(A)**

Sol. sinα and cosα are the roots of $ax^2 + bx + c = 0$

$\Rightarrow \sin\alpha + \cos\alpha = \frac{-b}{a}$ and $\sin\alpha\cos\alpha = \frac{c}{a}$

$(\sin\alpha + \cos\alpha)^2 = \sin^2\alpha + \cos^2\alpha + 2\sin\alpha\cos\alpha \qquad \Rightarrow \frac{b^2}{a^2} = 1 + \frac{2c}{a}$

$\Rightarrow b^2 = a^2 + 2ac \qquad \therefore a^2 - b^2 + 2ac = 0$

11. If α, β are the roots of the equation $x^2 - 3x + 1 = 0$, then the equation with roots $\frac{1}{\alpha-2}, \frac{1}{\beta-2}$ will be

(A) $x^2 - x - 1 = 0$ (B) $x^2 + x - 1 = 0$

(C) $x^2 + x + 2 = 0$ (D) none of these

Ans. **(A)**

Sol. α, β are the roots of the equation $x^2 - 3x + 1 = 0$

$\Rightarrow \alpha^2 - 3\alpha + 1 = 0$..........(1)

Let $\frac{1}{\alpha-2} = y$

$\Rightarrow \alpha = 2 + \frac{1}{y}$

From (1), we get

$$\left(2 + \frac{1}{y}\right)^2 - 3\left(2 + \frac{1}{y}\right) + 1 = 0$$

$$\Rightarrow \frac{(2y+1)^2}{y^2} - \frac{3(2y+1)}{y} + 1 = 0$$

$\Rightarrow y^2 - y - 1 = 0$

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$\therefore$ The equation with roots $\frac{1}{\alpha-2}, \frac{1}{\beta-2}$ is $x^2 - x - 1 = 0$

Hence, option A.

12. Let α, β, γ be the roots of $(x-a)(x-b)(x-c) = d, d \neq 0$, then the roots of the equation $(x-\alpha)(x-\beta)(x-\gamma) + d = 0$ are :

(A) $a+1, b+1, c+1$

(B) a, b, c

(C) $a-1, b-1, c-1$

(D) $\frac{a}{b}, \frac{b}{c}, \frac{c}{a}$

Ans. **(B)**

Sol. Clearly $(x-a)(x-b)(x-c) = -(x-\alpha)(x-\beta)(x-\gamma)$

$\therefore$ if α, β, γ are the roots of given equation

then $(x-\alpha)(x-\beta)(x-\gamma) + d = 0$ will have roots a, b, c.

13. Let two numbers have arithmetic mean 9 and geometric mean 4 . Then these numbers are the roots of the quadratic equation-

(A) $x^2 + 18x - 16 = 0$

(B) $x^2 - 18x + 16 = 0$

(C) $x^2 + 18x + 16 = 0$

(D) $x^2 - 18x - 16 = 0$

Ans. **(B)**

Sol. $x^2 - 18x + 16 = 0$

Explanation of the correct option:

Finding the quadratic equation:

Given the arithmetic mean as 9 and the geometric mean as 4 .

Let a and b be the two numbers in arithmetic and geometric sequences We know that the arithmetic mean is $\frac{a+b}{2}$,

$\Rightarrow \frac{a+b}{2} = 9 \qquad \Rightarrow a + b = 2 \times 9 \qquad = 18$

Also, The geometric mean is $\sqrt{ab}$, $\Rightarrow \sqrt{ab} = 4$

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$ab = 4^2 = 16$

Using the obtained root the quadratic equation can be defined as $x^2 - (a+b)x + ab = 0$

Now substitute a + b as 18 and ab as 16 in $x^2 - (a+b)x + ab = 0$. $\Rightarrow x^2 - 18x + 16 = 0$

Therefore, the equation is obtained as $x^2 - 18x + 16 = 0$.

14. If $(1-p)$ is a root of quadratic equation $x^2 + px + (1-p) = 0$ then its roots are

(A) $0, -1$ (B) - 1, 1 (C) 0,1 (D) $-1,2$

Ans. **(A)**

Sol. $x^2 + px + (1-p) = 0$

$(1-p)^2 + p(1-p) + (1-p) = 0 \quad \Rightarrow \quad (1-p)(1-p+p+1) = 0$

$p = 1 \quad \Rightarrow \quad x^2 + x = 0 \Rightarrow x = 0, -1$

15. If one root of the equation $x^2 + px + 12 = 0$ is 4 , while the equation $x^2 + px + q = 0$ has equal roots, then the value of ' q ' is-

(A) 3 (B) 12 (C) 49/4 (D) 4

Ans. **(C)**

Sol. $x^2 + px + 12 = 0 \quad \Rightarrow \quad 16 + 4p + 12 = 0$

because 4 is root $p = -7$

$x^2 + px + q = 0$ has equal root $\Rightarrow \quad p^2 = 4q \Rightarrow 49 = 4q \Rightarrow q = \frac{49}{4}$

16. The value of a for which the sum of the squares of the roots of the equation $x^2 - (a-2)x - a - 1 = 0$ assume the least value is-

(A) 2 (B) 3 (C) 0 (D) 1

Ans. **(D)**

Sol. $x^2 - (a-2)x - a - 1 = 0 \quad [\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta]$

$(a-2)^2 + 2a + 2 \quad \Rightarrow \quad a^2 - 2a + 6$

$(a-1)^2 + 5$ is min. at $a = 1$

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17. If the roots of the equation $x^2 - bx + c = 0$ be two consecutive integers, then $b^2 - 4c$ equals-

(A) 1 (B) 2 (C) 3 (D) -2

Ans. (A)

Sol. For consecutive integers roots

$|\alpha - \beta| = 1 \quad \Rightarrow \quad b^2 - 4c = 1$

18. If the roots of the quadratic equation $x^2 + px + q = 0$ are $\tan 30^\circ$ and $\tan 15^\circ$, respectively then the value of $2 + q - p$ is-

(A) 0 (B) 1 (C) 2 (D) 3

Ans. (D)

Sol. Step 1: Use the concept of Sum and Product of roots of a Quadratic equation We know that if α, β are the roots of the quadratic equation $ax^2 + bx + c = 0$ then

$\alpha + \beta = -\frac{b}{a}$ and $\alpha\beta = \frac{c}{a}$

Therefore, for the given equation $x^2 + px + q = 0$,

$\tan(30^\circ) + \tan(15^\circ) = -p \quad \Rightarrow \quad \tan(30^\circ) \cdot \tan(15^\circ) = q$

Step 2: Apply the compound angle formula of tangent

$\tan(30^\circ + 15^\circ) = \frac{\tan(30^\circ) + \tan(15^\circ)}{1 - \tan(30^\circ) \cdot \tan(15^\circ)}$

$\tan(45^\circ) = \frac{-p}{1 - q}$

$$
\begin{aligned}
\Rightarrow \quad 1 &= \frac{-p}{1-q} \\
\Rightarrow \quad 1 - q &= -p \\
\Rightarrow \quad 1 &= q - p \\
\Rightarrow \quad 1 + 2 &= q - p + 2 \\
\Rightarrow \quad 3 &= 2 + q - p \\
\Rightarrow \quad 2 + q - p &= 3
\end{aligned}
$$

19. If the difference between the roots of the equation $x^2 + ax + 1 = 0$ is less than $\sqrt{5}$, then the set of possible values of a is

(A) $(-3, \infty)$ (B) $(3, \infty)$ (C) $(-\infty, -3)$ (D) $(-3, 3)$

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Ans. **(D)**

Sol. Given, the difference between the roots of the equation $x^2 + ax + 1 = 0$ is less than $\sqrt{5}$

Let α, β are the roots of the equation then: $\Rightarrow$ The sum of the roots is: $\alpha + \beta = -a$

The product of the roots is: $\alpha\beta = 1$.

$|\alpha - \beta| < \sqrt{5}$ $\Rightarrow$ $\sqrt{(\alpha+\beta)^2 - 4\alpha\beta} < \sqrt{5}$

$\sqrt{a^2 - 4} < \sqrt{5}$ $\Rightarrow$ $a^2 - 4 < 5$ [Squaring both sides]

$a^2 < 9$ $\Rightarrow$ $-3 < a < 3$

i.e; $a \in (-3,3)$

20. If α and β are the roots of the equation $x^2 - x + 1 = 0$, then $\alpha^{2009} + \beta^{2009} =$

(A) -2 (B) -1 (C) 1 (D) 2

Ans. **(C)**

Sol. $x^3 + 1 = (x+1)(x^2 - x + 1)$

So, since α is a root of $x^2 - x + 1 = 0$ we have,

$\alpha^3 + 1 = (\alpha+1)(\alpha^2 - \alpha + 1) = (\alpha+1)(0) = 0$ $\Rightarrow$ So, $\alpha^3 = -1$

Similarly, $\beta^2 = -1$

Also, $x^2 - x + 1 = (x-\alpha)(x-\beta) = x^2 - (\alpha+\beta)x + \alpha\beta$

so, $\begin{cases} \alpha + \beta = 1 \\ \alpha\beta = 1 \end{cases}$

$\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta = 1 - 2 = -1$.........(i)

so, $\alpha^{2009} + \beta^{2009} = \alpha^{3.669+2} + \beta^{3.669+2}$ $= (\alpha^3)^{669} \cdot \alpha^2 + (\beta^3)^{669} \cdot \beta^2$

$= (-1)^{669} \cdot (\alpha^2 + \beta^2)$ [From (i)]

$= (-1)(-1)$ =1

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21. Let α and β be the roots of equation $x^2 - 6x - 2 = 0$. If $a_n = \alpha^n - \beta^n$, for $n \geq 1$, then the value of $\frac{a_{10}-2a_8}{2a_9}$ is equal to:

(A) 3 (B) -3 (C) 6 (D) -6

Ans. **(A)**

Sol. α and β are the roots of the equation

$x^2 - 6x - 2 = 0 \quad \Rightarrow \quad \alpha^2 = 6\alpha + 2$

$\alpha^{10} = 6\alpha^9 + 2\alpha^8 \ldots\ldots\ldots\ldots (i)$

$\beta^{10} = 6\beta^9 + 2\beta^8 \ldots\ldots\ldots\ldots (ii)$

Subtracting equation (ii) from (i)

$\Rightarrow \alpha^{10} - \beta^{10} = 6(\alpha^9 - \beta^9) + 2(\alpha^8 - \beta^9) \quad \Rightarrow a_{10} = 6a_9 + 2a_8 (\because a_n = a^n - \beta^n)$

$\Rightarrow a_{10} - 2a_8 = 6a_9$

$$\Rightarrow \frac{a_{10} - 2a_8}{2a_9} = 3$$

22. The sum of all real values of x satisfying the equation $(x^2 - 5x + 5)^{x^2+4x-60} = 1$ is

(A) -4 (B) 6 (C) 5 (D) 3

Ans. **(D)**

Sol. $(x^2 - 5x + 5)^{x^2+4x-60} = 1$

First possibility is $x^2 + 4x - 60 = 0 \quad \Rightarrow \quad x = -10, 6$

Seond possibility is $x^2 - 5x + 5 = 1 \quad \Rightarrow \quad x = 1, 4$

Third Possibility $\quad \Rightarrow \quad x^2 - 5x + 5 = -1$

$x = 2, 3$

substitute $x = 3$ it will give odd number in $x^2 + 4x - 60$

So $x = 2$ is only acceptable.

Sum of the integers $= -10 + 4 + 1 + 6 + 2 = 3$

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More than one answer type :

23. If α is a root of the equation $2x(2x+1)=1$, then the other root is –

(A) $3\alpha^3-4\alpha$ (B) $-2\alpha(\alpha+1)$ (C) $4\alpha^3-3\alpha$ (D) none of these

Ans. **(BC)**

Sol. α and β are the roots of the equation $4x^2+2x-1=0$

$4\alpha^2+2\alpha=1 \Rightarrow \frac{1}{2}=2\alpha^2+\alpha \ldots \quad \Rightarrow \quad \beta=\frac{-1}{2}-\alpha$

using equation (1)

$\beta=-(2\alpha^2+\alpha)-\alpha \Rightarrow \quad \beta=-2\alpha^2-2\alpha \quad \Rightarrow \quad \beta=-2\alpha(\alpha+1) \quad =\alpha(-2\alpha-2)$

$\alpha(4\alpha^2-3)[\because 4\alpha^2+2\alpha-1=0 \Rightarrow -2\alpha=4\alpha^2-1 \Rightarrow -2\alpha-2=4\alpha^2-3$

$=4\alpha^2-3\alpha$

Option $-$ C

24. If a, b are non-zero real numbers and α,β the roots of $x^2+ax+b=0$, then

(A) α^2,β^2 are the roots of $x^2-(2b-a^2)x+a^2=0$

(B) $\frac{1}{\alpha},\frac{1}{\beta}$ are the roots of $bx^2+ax+1=0$

(C) $\frac{\alpha}{\beta},\frac{\beta}{\alpha}$ are the roots of $bx^2+(2b-a^2)x+b=0$

(D) $(\alpha-1),(\beta-1)$ are the roots of the equation $x^2+x(a+2)+1+a+b=0$

Ans. **(BCD)**

Answer Key

1.	$\frac{b^2-2ac}{a^2c^2}$	**2.**	x^2+2x-1			**3.**	$k=\frac{a-b}{a+b}$	**4.**	$-10,-3$	**5.**	$\lambda=2$	**7.**	D
8.	C	**9.**	C	**10.**	A	**11.**	A	**12.**	B	**13.**	B	**14.**	A
15.	C	**16.**	D	**17.**	A	**18.**	D	**19.**	D	**20.**	C	**21.**	A
22.	D	**23.**	BC	**24.**	BCD								