Nishant Jindal

Q A.M. & G.M. of the Roots of a Q Eqn are 10 & 8 find Q Eqn.

let α, β are Q Eqn.

$\Rightarrow x^2 - (\alpha+\beta)x + \alpha\beta = 0$

Q S

$A = \frac{\alpha+\beta}{2} = 10$

$\boxed{\alpha+\beta = 2A}$

GM of α, β

$= \sqrt{\alpha\beta} = 8 = G$

$\alpha\beta = G^2$

$\boxed{x^2 - 2Ax + G^2 = 0}$ ← Q Eqn in form of AM & GM

$x^2 - 20x + 64 = 0$

Result ⇒ If AM & GM of Roots are A & G then Q Eqn will be $x^2 - 2Ax + G^2 = 0$

Q Find AM & GM of Roots of Q Eqn → $2x^2 - 5x + 3 = 0$

$x^2 - \frac{5}{2}x + \frac{3}{2} = 0$

$x^2 - 2Ax + G^2 = 0$

$2A = \frac{5}{2}, \quad G^2 = \frac{3}{2}$

$A = \frac{5}{4}, \quad G = \sqrt{\frac{3}{2}}$

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★ V.V.Imp.

Find Nos (α,β) whose AM is A & G.M is G.

$$x^2-2Ax+G^2=0$$

A = AM of α,β

G = GM of α,β.

Target → α,β

Roots are values of x

$$x=\frac{2A\pm\sqrt{4A^2-4G^2}}{2}$$

$$x=\underset{\alpha}{A+\sqrt{A^2-G^2}},\ \underset{\beta}{A-\sqrt{A^2-G^2}}$$

Check → $\frac{16+4}{2}=10=A$

$\sqrt{16\times4}=\sqrt{64}=8=G$.

Q. 10 & 8 are AM & GM of 2 No. find those No.

$A=10, G=8$

$\alpha=A+\sqrt{A^2-G^2}=10+\sqrt{10^2-8^2}=10+6=16$

$\beta=A-\sqrt{A^2-G^2}=10-\sqrt{10^2-8^2}=10-6=4$

∴ Nos No 16 & 4

Q. If AM of 2 No. is 2 Times of its GM then Ratio of the No. is

$\frac{1+\sqrt3}{1-\sqrt3}$ // $\frac{2+\sqrt3}{2-\sqrt3}$ $\frac{1+2\sqrt3}{1-2\sqrt3}$ NOT.

$A=2G$.

Ratio of No $\frac{\alpha}{\beta}=\frac{A+\sqrt{A^2-G^2}}{A-\sqrt{A^2-G^2}}=\frac{2G+\sqrt{4G^2-G^2}}{2G-\sqrt{4G^2-G^2}}$

$=\frac{2G+G\sqrt3}{2G-G\sqrt3}=\frac{\cancel{G}(2+\sqrt3)}{\cancel{G}(2-\sqrt3)}=\frac{2+\sqrt3}{2-\sqrt3}$.

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Q If p & q are GM betⁿ 2 No & A is one AM of same No. then $\frac{p^3+q^3}{Apq} = ?$

let No. are a, b.

p & q are 2 GM betⁿ a, b. $\rightarrow$ A is AM

$a, p, q, b \rightarrow GP$

We know

$p^2 = a \cdot q \quad \times p$

$q^2 = pb \quad \times q$

$A = \frac{a+b}{2}$

$a+b = 2A$

$p^3 = apq$

$q^3 = bpq$

$p^3+q^3 = pq(a+b)$

$p^3+q^3 = 2Apq \Rightarrow \frac{p^3+q^3}{Apq} = 2$ Ans

Q If r is One AM & p, q are 2 GM betⁿ 2 Given Nos then $p^3+q^3 = ?$

// $2pqr \quad 2p^2q^2r^2 \quad \frac{2pq}{r} \quad$ NOT.

last Qs $\rightarrow p^3+q^3 = 2Apq$

$p^3+q^3 = 2rpq$

Q let a, b, c be +ve Integers such that $\frac{b}{a}$ is an Integer. If a, b, c are in GP & AM of a, b, c is $b+2$ find value of $\frac{a^2+a-14}{a+1} = \frac{36+6-14}{7} = 4$

Jee Adv

1, 2, 4

$a = a, \ b = ar$

$\frac{b}{a} = r > 1$

AM of $a, b, c = b+2$

$\Rightarrow \frac{a+b+c}{3} = b+2 \Rightarrow a+b+c = 3b+6$

$a - 2b + c = 6 \Rightarrow a - 2ar + ar^2 = 6$

$\Rightarrow a(r-1)^2 = 6 \rightarrow r = 2, \ a = 6$

Nishant Jindal

Q If G_1 & G_2 be 2 GM & A is the AM bet^n 2 No. then $\frac{G_1^2}{G_2}+\frac{G_2^2}{G_1}=?$

$\frac{A}{2}$, A, 2A, A^2

let 2 No are a, b.

$a, G_1, G_2, b \rightarrow$ GP.

$r=\left(\frac{b}{a}\right)^{\frac{1}{n+1}}=\left(\frac{b}{a}\right)^{\frac{1}{2+1}}=\left(\frac{b}{a}\right)^{\frac{1}{3}}$.

$G_1=ar=a\cdot\left(\frac{b}{a}\right)^{1/3}=a^{2/3}b^{1/3}$

$G_2=ar^2=a\left(\frac{b}{a}\right)^{2/3}=a^{1/3}\cdot b^{2/3}$.

$\frac{G_1^2}{G_2}+\frac{G_2^2}{G_1}=\frac{a^{4/3}\cdot \cancel{b^{2/3}}}{a^{1/3}\cdot\cancel{b^{2/3}}}+\frac{\cancel{a^{2/3}}\cdot b^{4/3}}{\cancel{a^{2/3}}\cdot b^{1/3}}$

$=a+b=2A$

Jee mains

Q. If m is AM of 2 distinct No. l & n $(l,n>1) \rightarrow m=\frac{l+n}{2}$ & G_1, G_2, G_3 are 3 G.M. bet^n l & n then $G_1^4+2G_2^4+G_3^4=?$

$4l^2mn$ $\quad$ $4lm^2n$ $\quad$ $4lmn^2$ $\quad$ $4l^2m^2n^2$

① $l, G_1, G_2, G_3, n \rightarrow$ GP. ② $r=\left(\frac{n}{l}\right)^{\frac{1}{4}}$

$G_1=lr=l\cdot\left(\frac{n}{l}\right)^{1/4}=l^{3/4}\cdot n^{1/4}$

$G_2=lr^2=l\cdot\left(\frac{n}{l}\right)^{1/2}=l^{1/2}\cdot n^{1/2}$

$G_3=lr^3=l\cdot\left(\frac{n}{l}\right)^{3/4}=l^{1/4}\cdot n^{3/4}$

$G_1^4+2G_2^4+G_3^4$

$l^3\cdot n+2l^2n^2+l\cdot n^3$

$ln\{l^2+2ln+n^2\}$

$ln\{l+n\}^2=(2m)^2 ln$

$=4m^2ln$

Nishant Jindal

Harmonic Progression (H.P.)

1) A Series obtained by Reciprocal of an AP.

$2, 4, 6 \rightarrow AP$

$\frac{1}{2}, \frac{1}{4}, \frac{1}{6} \rightarrow HP$

2) nth term of HP.

then find nth term of AP then reciprocal it.

$\frac{1}{a}, \frac{1}{a+d}, \frac{1}{a+2d} - - \boxed{\frac{1}{a+(n-1)d}}$

$T_n = \frac{1}{a+(n-1)d}$

3) Sum of H.P is not defined.

Q. 7th & 12th term of HP are $\frac{1}{10}$ & $\frac{1}{49}$ find 20th term of HP.

$a+6d = 10$

$a+11d = 49$

$- \quad - \quad -$

$-5d = -39$

$d = \frac{39}{5}$

$a + 6 \times \frac{39}{5} = 10$

$a = 10 - \frac{234}{5}$

$a = -\frac{184}{5}$

Ans→

$a+19d = -\frac{184}{5} + 19 \times \frac{39}{5}$

$a+19d = \frac{557}{5}$

$HP \rightarrow \frac{5}{557}$ Ans

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Q. If p^{th} term of an HP is q & q^{th} term be p

then its $(p+q)^{th}$ term is

$\frac{1}{p+q}$ $\quad \frac{1}{p}+\frac{1}{q}$ $\quad \frac{pq}{p+q}$ // $\quad p+q$

AP

$T_p = \frac{1}{q} \Rightarrow a+(p-1)d = \frac{1}{q}$

$T_q = \frac{1}{p} \Rightarrow a+(q-1)d = \frac{1}{p}$

$d(\cancel{p-q}) = \frac{1}{q} - \frac{1}{p} = \frac{\cancel{p-q}}{pq}$

$d = \frac{1}{pq}$

$a+(p-1)\times\frac{1}{pq} = \frac{1}{q} \Rightarrow a = \frac{1}{q} - \frac{p-1}{pq} = \cancel{\frac{1}{q}} - \left(\cancel{\frac{1}{q}} - \frac{1}{pq}\right) = \frac{1}{pq}$

$T_{p+q} = a+(p+q-1)d = \frac{1}{pq} + (p+q-1)\cdot\frac{1}{pq}$

$= \frac{1}{pq}\{\cancel{1}+p+q-\cancel{1}\}$

$T_{p+q} = \frac{p+q}{pq}$ AP

$HP \to \frac{pq}{p+q}$

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Q Five No. a, b, c, d, e are such that a, b, c are in AP; b, c, d are in HP; c, d, e are in HP. If a = 2, e = 18 then b, c, d are

2, 6, 10 $\quad$ 4, 6, 9 ✓ $\quad$ 4, 6, 8 $\quad$, −2 −6, 18.

2, b, c, d, 18

$b = \frac{2+c}{2}$

$2b = 2 + c$

$c = 2b - 2$

$c^2 = bd$

$(2b-2)^2 = bd$

$d = \frac{4(b-1)^2}{b}$

c, d, 18 HP

$\frac{1}{c}, \frac{1}{d}, \frac{1}{18}$ AP

$\frac{2}{d} = \frac{1}{c} + \frac{1}{18}$

$\frac{2}{d} = \frac{18+c}{18 \cdot c} = \frac{18+2b-2}{18(2b-2)} = \frac{\cancel{2}(b+8)}{18\,\cancel{36}(b-1)}$

$d = \frac{\cancel{36}^{9}\cancel{(b-1)}}{(b+8)} = \frac{4(b-1)^{\cancel{2}}}{b}$

$9b = b^2 + 7b - 8$

$b^2 - 2b - 8 = 0$

$(b-4)(b+2) = 0$

$b = 4$ $\quad$ $b = -2$

4, 6, 9

$d = \frac{4(4-1)^2}{\cancel{4}} = 9$

$c = 2b - 2 = 2 \times 4 - 2 = 6$

Nishant Jindal

Harmonic Mean (HM)

(1) one HM betn a, b.

H is HM betn a, b.

$\Rightarrow a, H, b \to HP.$

$\Rightarrow \frac{1}{a}, \frac{1}{H}, \frac{1}{b} \to AP.$

$$\boxed{\frac{2}{H} = \frac{1}{a} + \frac{1}{b}.}$$

$$\frac{2}{H} = \frac{a+b}{ab}$$

$$\boxed{H = \frac{2ab}{a+b}}$$

(2) n HM betn a, b.

$a, H_1, H_2, H_3 \dots H_n, b \to HP.$

$\frac{1}{a}, \frac{1}{H_1}, \frac{1}{H_2}, \frac{1}{H_3} \dots \frac{1}{H_n}, \frac{1}{b} \to AP$

$\frac{1}{a} \quad A_1 \quad A_2 \quad A_3 \quad A_n, \frac{1}{b}.$

3^{rd} HM $\to 3^{rd}$ A.M Ka reciprocal

(1) $d = \frac{b-a}{n+1}$ AM $\to d = \frac{\frac{1}{b} - \frac{1}{a}}{n+1}$

$\boxed{A_3 = a + 3d}$

H_3 layenge.

HW3

(3) Sum of Reciprocal of n HM.

$\frac{n}{2}(a+b)$

$$\frac{1}{H_1} + \frac{1}{H_2} + \frac{1}{H_3} + \dots + \frac{1}{H_n} = \frac{n}{2}\left(\frac{1}{a} + \frac{1}{b}\right)$$

$$= n\left(\frac{a+b}{2ab}\right)$$

$$\boxed{\frac{1}{H_1} + \frac{1}{H_2} + \frac{1}{H_3} \dots \frac{1}{H_n} = \frac{n}{H}}$$

(4) Random NO's HM

HM of $a, b = \frac{2}{\frac{1}{a} + \frac{1}{b}}.$

HM of $a, b, c = \frac{3}{\frac{1}{a} + \frac{1}{b} + \frac{1}{c}}$