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Find principle argument ?

1. $z = 1 + \cos\frac{6\pi}{5} + i\sin\frac{6\pi}{5} = 2\underbrace{\cos\frac{3\pi}{5}}_{<0}\, e^{i\frac{3\pi}{5}}$

$\boxed{\theta = -\frac{2\pi}{5}}$

2. $z = \left(\frac{(8+i)(7+i)}{5-i}\right)^6 = \left(\frac{(55+15i)(5+i)}{26}\right)^6$

$\Leftarrow \left(10+5i\right)^6$

$\boxed{\theta = 6\tan^{-1}\frac{1}{2}}$

$\frac{4\pi}{3} + \frac{3\pi}{2}$

$\arg z$

$\arg(-2z) = \pi + \arg z$

3. $z = \frac{(2\sqrt{3}+2i)^8}{(1-i)^6} + \frac{(1+i)^6}{(2\sqrt{3}-2i)^8}$

$\boxed{\theta = \frac{5\pi}{6}}$

$8\frac{\pi}{6} + \frac{6\pi}{4}$

$= \frac{17\pi}{6} - 2\pi$

$= \left(1 + \frac{2^6}{16^8}\right)\frac{(2\sqrt{3}+2i)^8}{(1-i)^6}$

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Solve for z.

4. (i) $z^2 - i\bar{z} - 1 = 0$

$z = x + iy \Rightarrow x^2 - y^2 + 2ixy - i(x - iy) - 1 = 0$

$(x^2 - y^2 - y - 1) + i(2xy - x) = 0$

$x^2 - y^2 - y - 1 = 0$ & $(2y - 1)x = 0$

If $x = 0$, $y^2 + y + 1 = 0$ $\quad \phi$

$y = \frac{1}{2}$, $x^2 = \frac{7}{4}$

$$\boxed{z = \frac{\sqrt{7}}{2} + \frac{i}{2},\ -\frac{\sqrt{7}}{2} + \frac{i}{2}}$$

(ii) $z^2 - iz - 1 = 0$

$z = \frac{i \pm \sqrt{-1+4}}{2}$

$$\boxed{z = \frac{i \pm \sqrt{3}}{2}}$$

5. $z^3 - 2z + 1 = 0$

$(z-1)(z^2 + z - 1) = 0$

$z = 1, \frac{-1 \pm \sqrt{5}}{2}$

6. $z^3 + \frac{z^2}{2} + \frac{z}{2} - \frac{1}{2} = 0$

$\left(z - \frac{1}{2}\right)(z^2 + z + 1) = 0$

$z = \frac{1}{2}, \frac{-1 \pm i\sqrt{3}}{2}$

7. $iz^3 + z^2 - z + i = 0$

$(z - i)(iz^2 - 1) = 0$ $\quad z = i, \boxed{z^2 = -i}$

$x^2 - y^2 + 2ixy = -i$

$x^2 - y^2 = 0$ & $2xy = -1$ $\Rightarrow 2x^2 = 1$

$$\boxed{z = \frac{1}{\sqrt{2}} - \frac{i}{\sqrt{2}},\ -\frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}}$$

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$$a_nx^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \cdots + a_1x + a_0 = 0 = a_n(x-\alpha_1)(x-\alpha_2)\cdots(x-\alpha_n)$$

Hit & Trial

$0, \pm 1, \pm i, \pm 2, \pm 2i, \pm 3, \pm 3i, \ldots, \pm \frac{1}{2}, \pm \frac{1}{2}i,$

$\pm \frac{1}{3}, \pm \frac{1}{3}i$

$a, b \in \mathbb{R}$

$\sqrt{ab} = \sqrt{a}\sqrt{b}$ if atleast one of a, b is non negative

$x^2 + x + 1 = 0$

$$x = \frac{-1 \pm \sqrt{1-4}}{2} = \frac{-1 \pm \sqrt{-1}\sqrt{3}}{2}$$

$$= \frac{-1 \pm i\sqrt{3}}{2}$$

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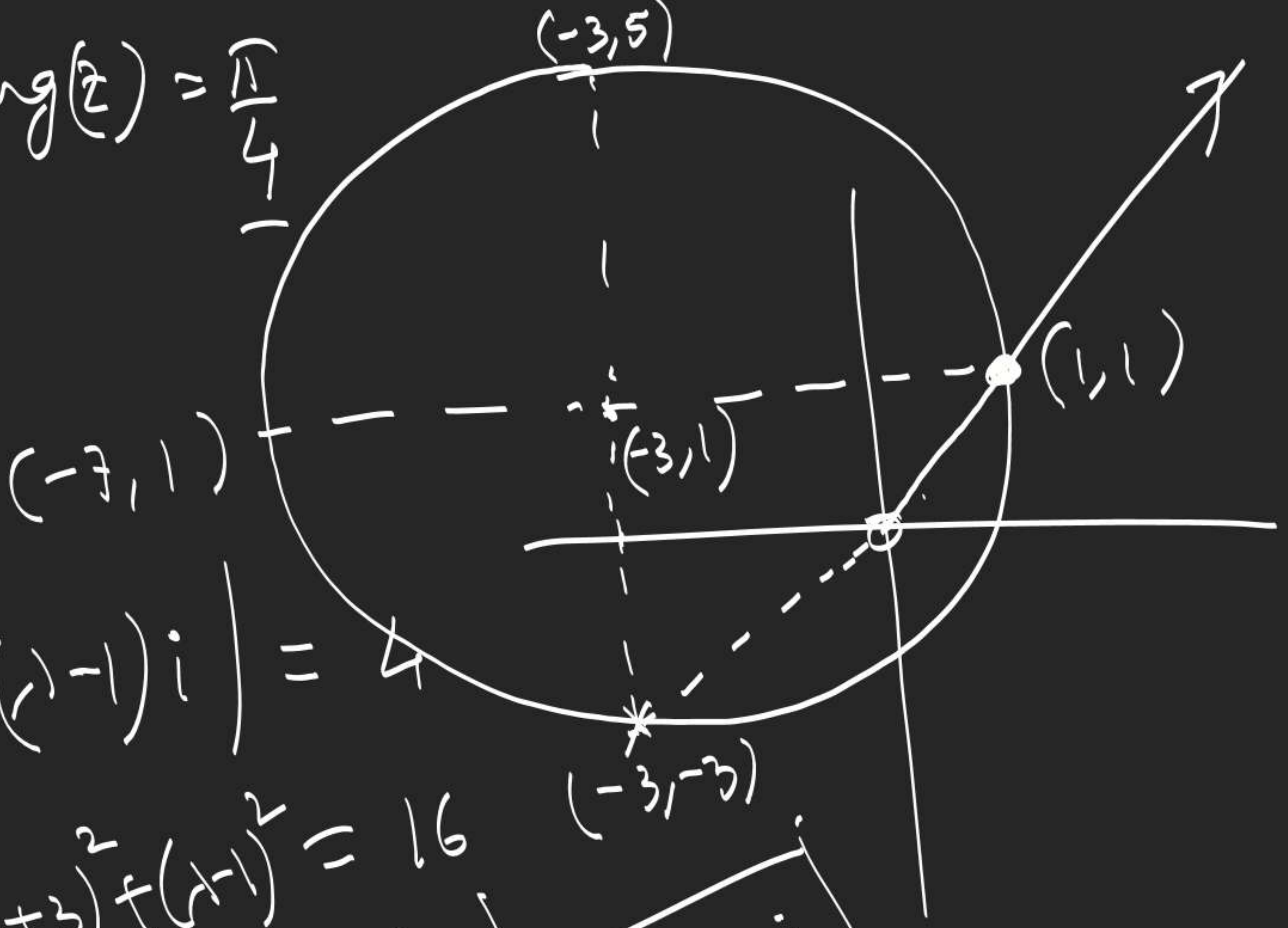
8. Find z if $\arg(z) = \frac{\pi}{4}$

$z = \lambda + \lambda i$, $\lambda > 0$



$$\left| \lambda + 3 + (\lambda - 1) i \right| = 4$$

$$(\lambda+3)^2 + (\lambda-1)^2 = 16$$

$$\lambda = \cancel{-3}, 1$$

$$\boxed{z = 1 + i}$$

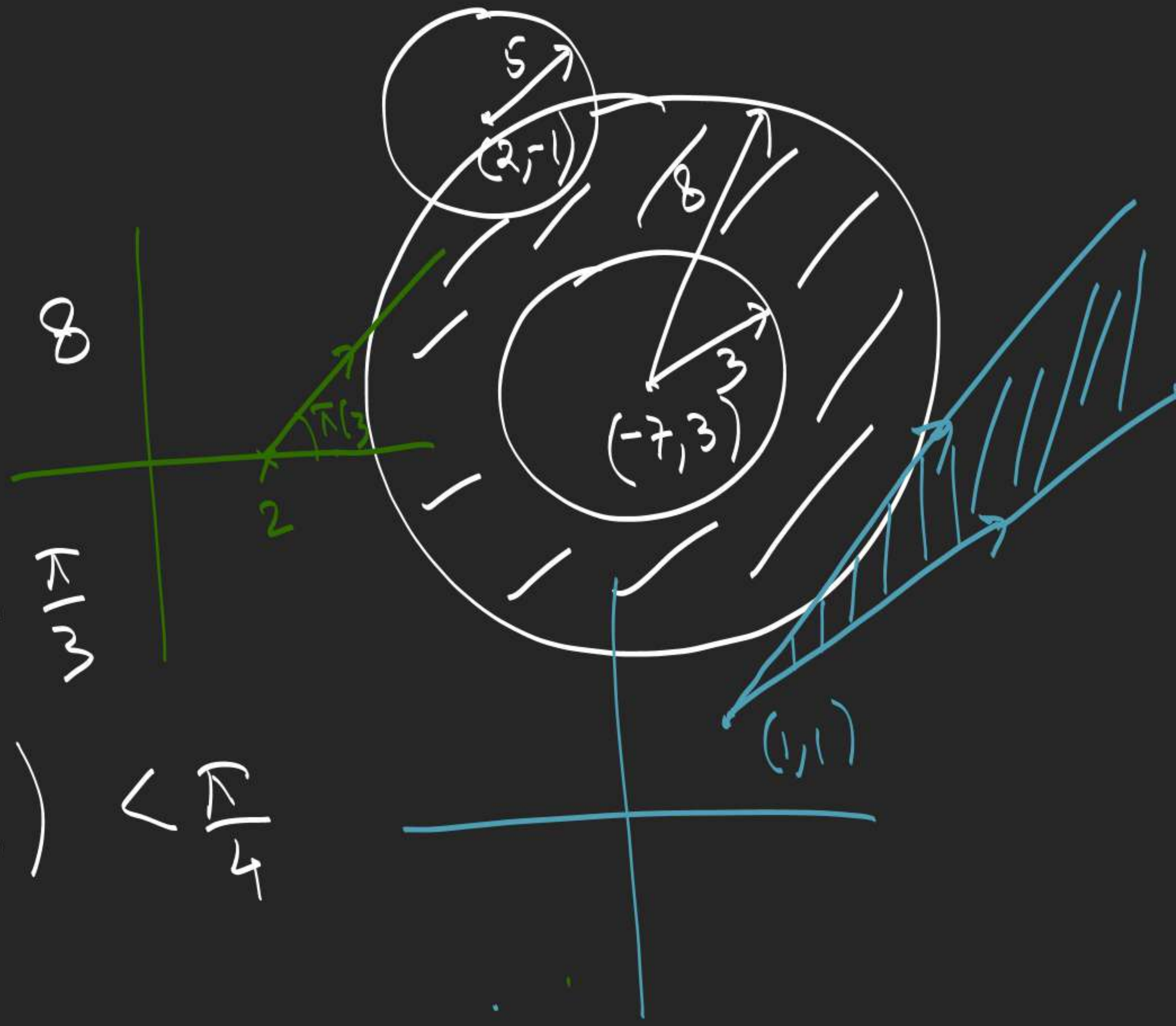
Represent z in argand plane satisfying.

9. $|z-2+i| = 5$

10. $3 \le |z+7-3i| < 8$

11. $\arg(z-2) = \frac{\pi}{3}$

12. $\frac{\pi}{6} < \arg(z-1-i) < \frac{\pi}{4}$



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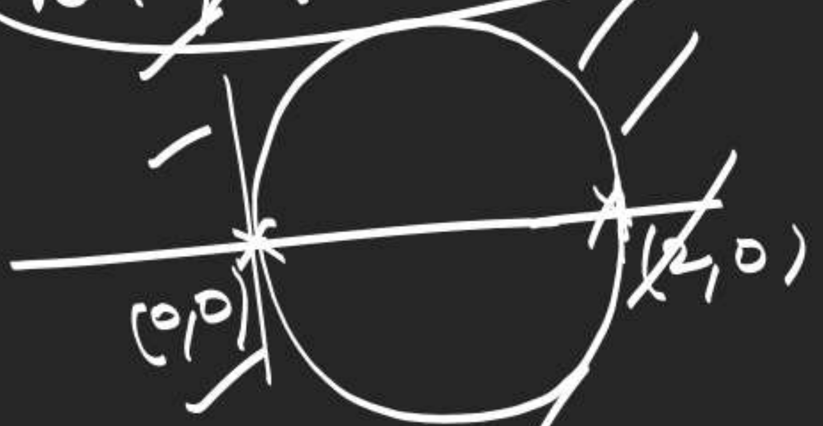
13. $Re\left(\frac{1}{z}\right) < \frac{1}{2}$ $\qquad \frac{1}{z} = \frac{1}{x+iy} = \frac{x-iy}{x^2+y^2}$

$\frac{x}{x^2+y^2} < \frac{1}{2} \Rightarrow \boxed{x^2+y^2 > 2x}$



14. $|z-1| < |z-2|$

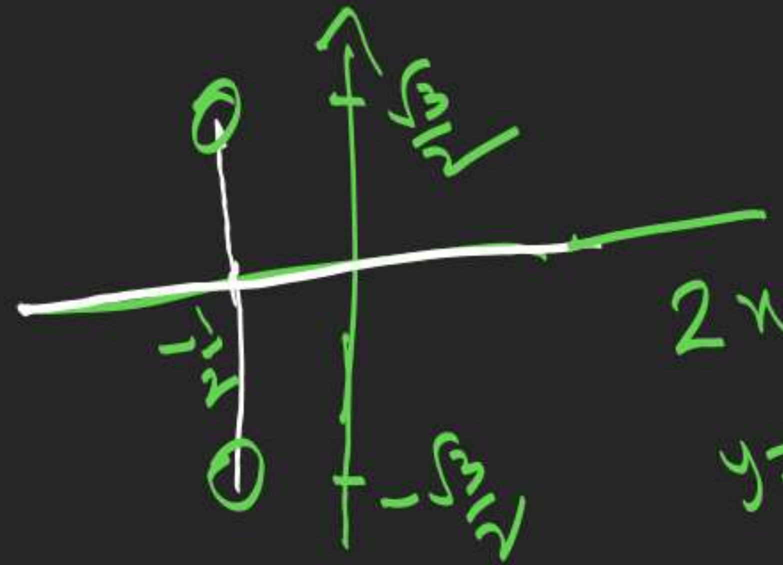
15. Find 'z' on complex plane for which z^2+z+1 is real and positive.

$x^2-y^2+2ixy+x+iy+1$

$2xy+y=0$ & $x^2-y^2+x+1>0$

$y=0$, $x^2+x+1>0$, $y^2 < \frac{3}{4}$

$x=-\frac{1}{2}$



$\boxed{32-46}$

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16. Find locus of point $P(w)$ denoting the complex number $z + \frac{1}{z}$ on complex plane w.r.t. $|z| = a$, where $a > 0$, $a \neq 1$ (a is const.)