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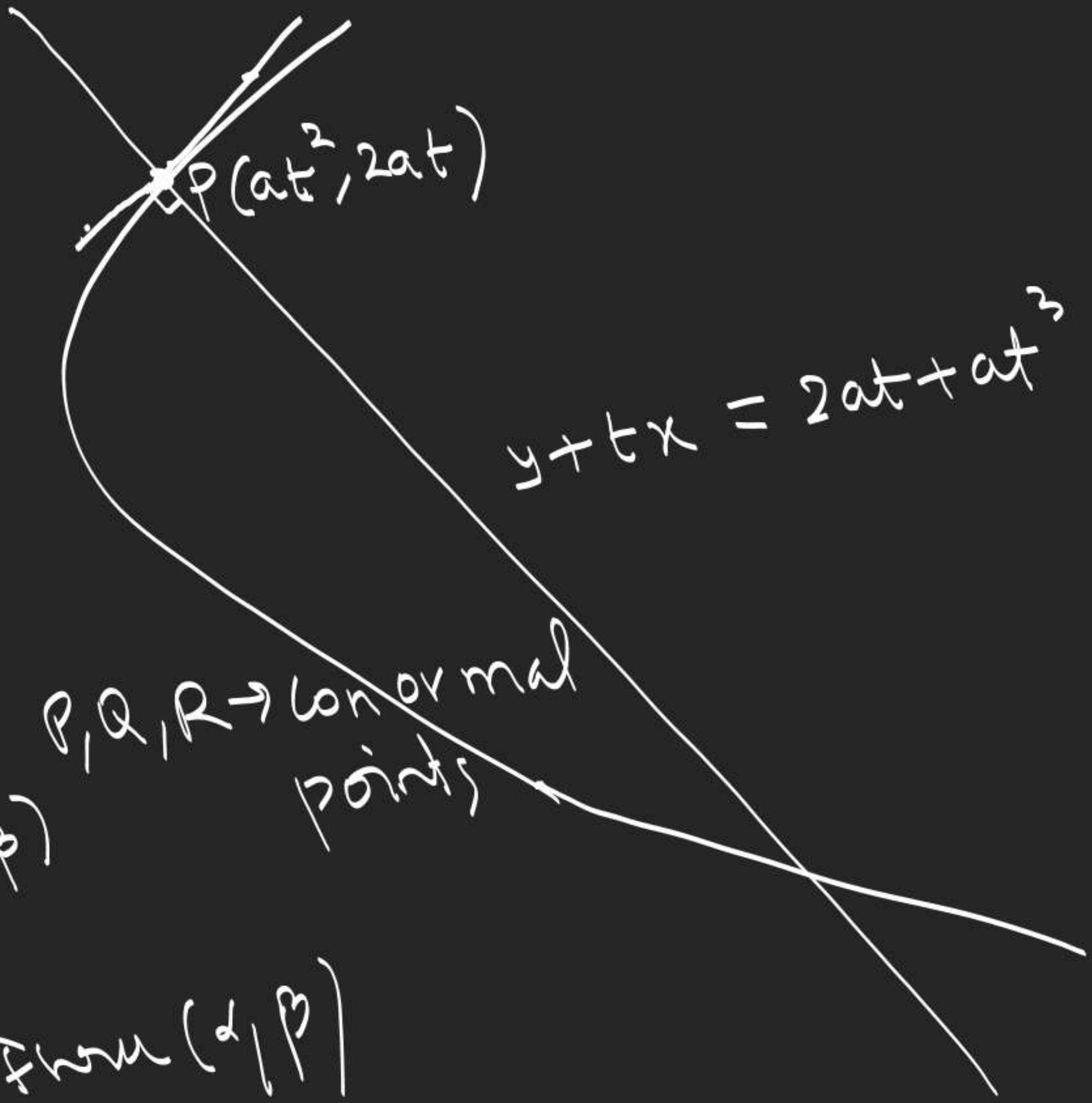
Normal

$y^2 = 4ax$

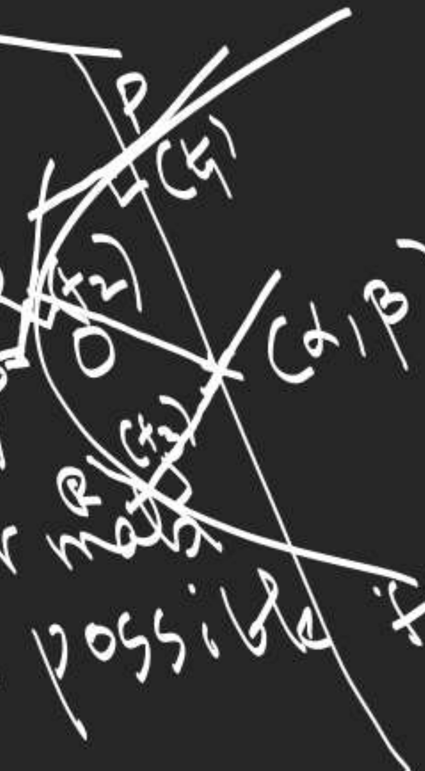


$y + tx = 2at + at^3$

Normal passing through a given point (α, β)

$$\frac{\beta - 2at}{\alpha - at^2} = -t$$

$$at^3 + (2a - \alpha)t - \beta = 0$$



at most 3 normals are possible from (α, β)

P, Q, R → conormal points

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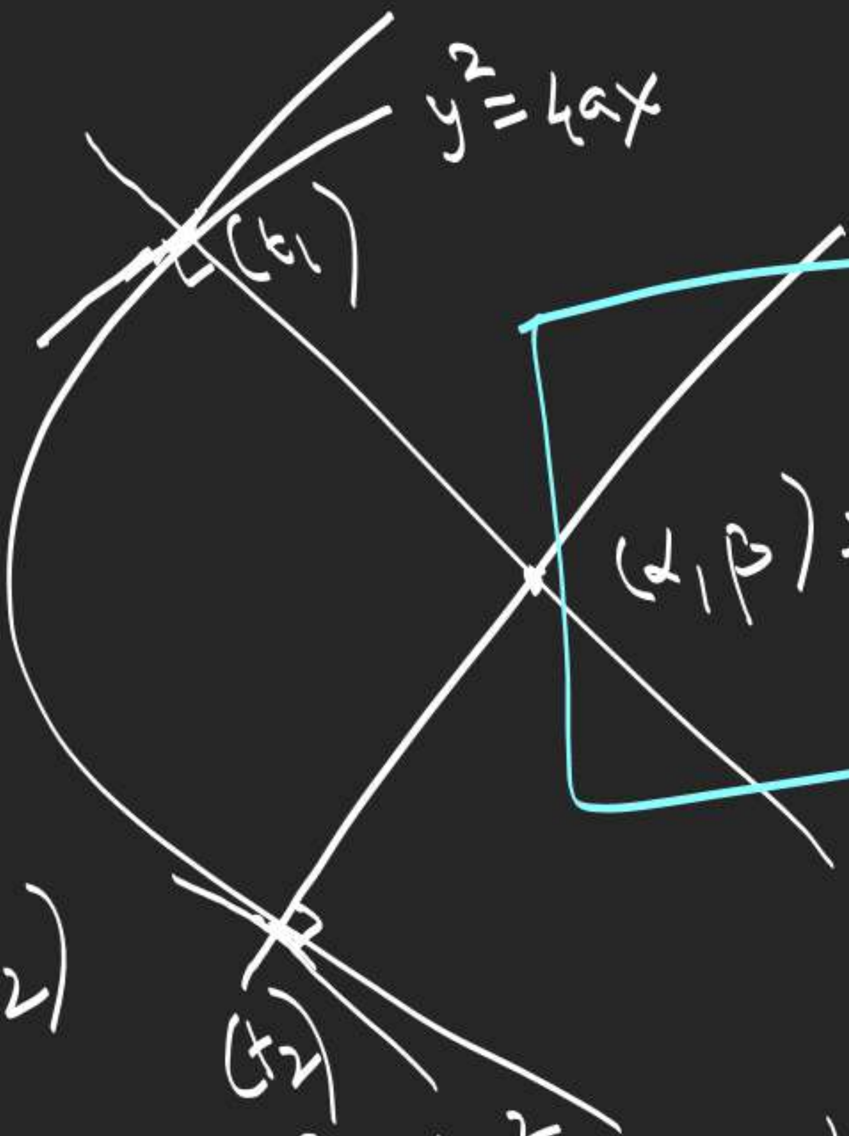
Intersection point of 2 normals to $y^2=4ax$



$$\frac{\beta-2at}{\alpha-at^2}=-t$$

$$at^3+(2a-\alpha)t-\beta=0 \quad \begin{matrix} t_1 \\ t_2 \\ t_3 \end{matrix}$$

$$t_1+t_2+t_3=0$$

$$\frac{\beta}{a}=t_1t_2t_3=-t_1t_2(t_1+t_2)$$

$$\beta=-at_1t_2(t_1+t_2)$$

$$\frac{2a-\alpha}{a}=t_1t_2+(t_1+t_2)t_3=t_1t_2-(t_1+t_2)^2=-t_1^2-t_2^2-t_1t_2$$

$$\alpha=a(2+t_1^2+t_2^2+t_1t_2)$$

$$\boxed{(\alpha,\beta)=\left(a(t_1^2+t_2^2+t_1t_2+2),\,-at_1t_2(t_1+t_2)\right)}$$

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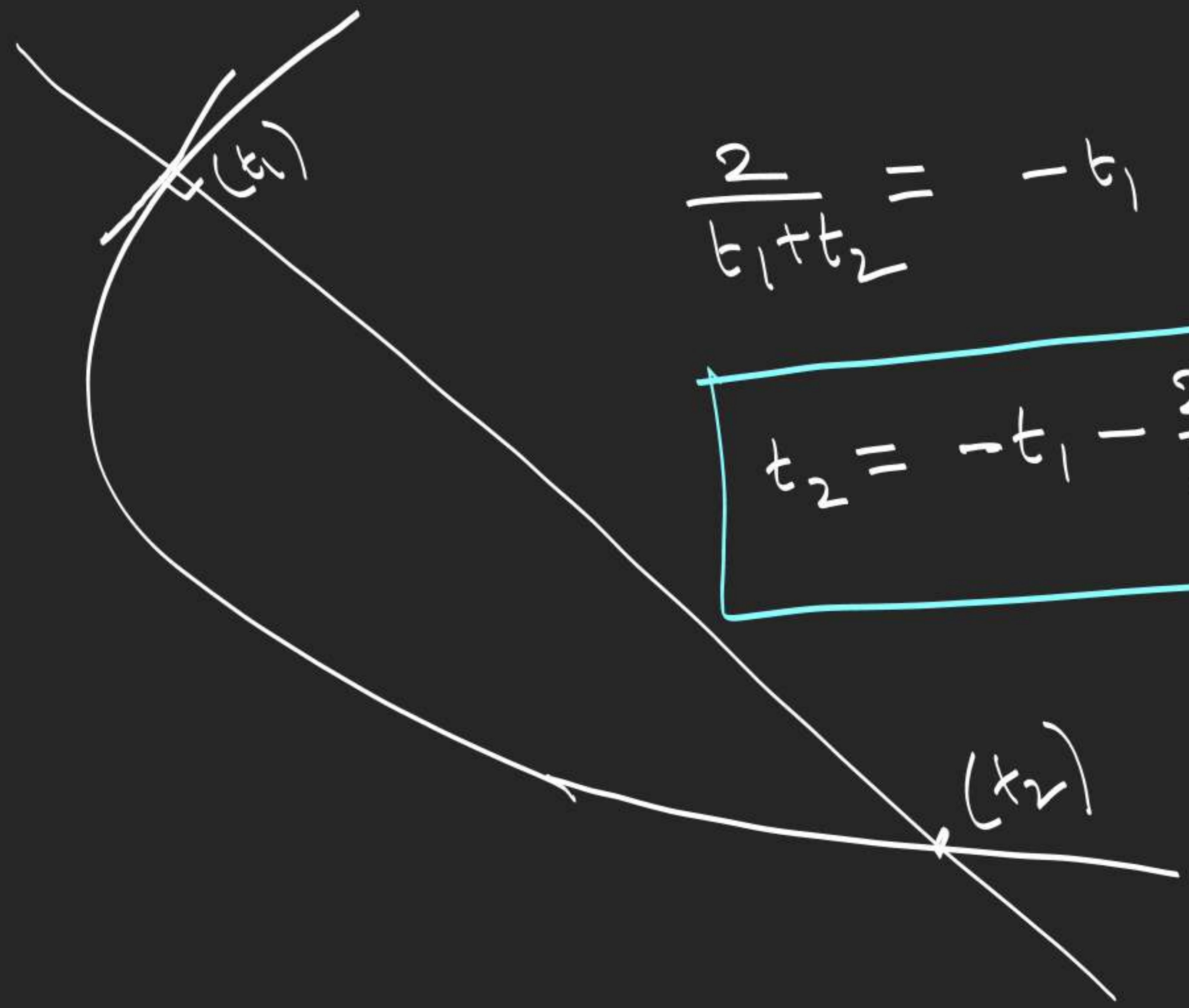
Normal Chord



$$\frac{2}{t_1+t_2} = -t_1$$

$$\boxed{t_2 = -t_1 - \frac{2}{t_1}}$$

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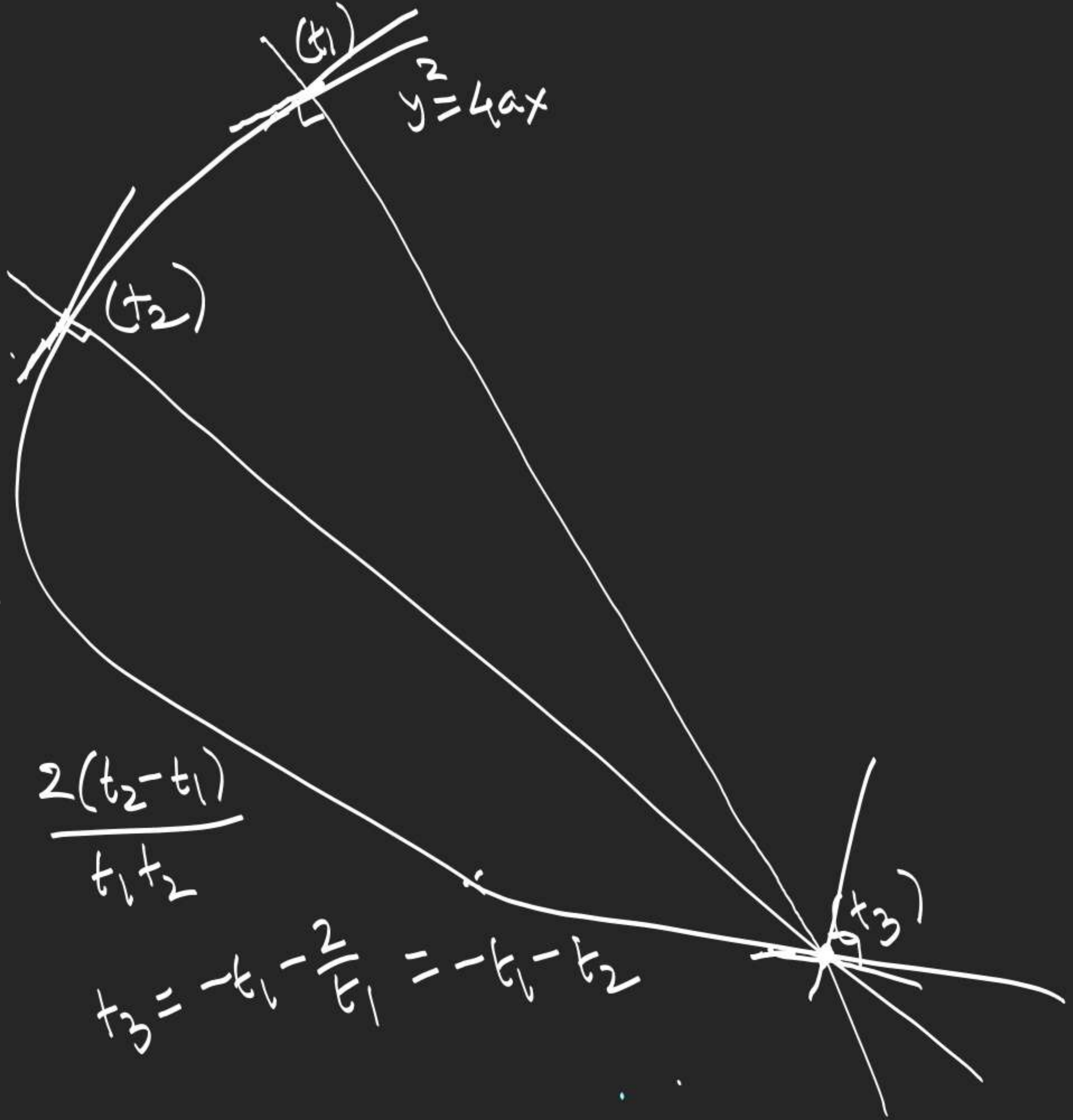
$$\boxed{\begin{aligned} t_1 t_2 &= 2 \\ t_1 + t_2 + t_3 &= 0 \end{aligned}}$$

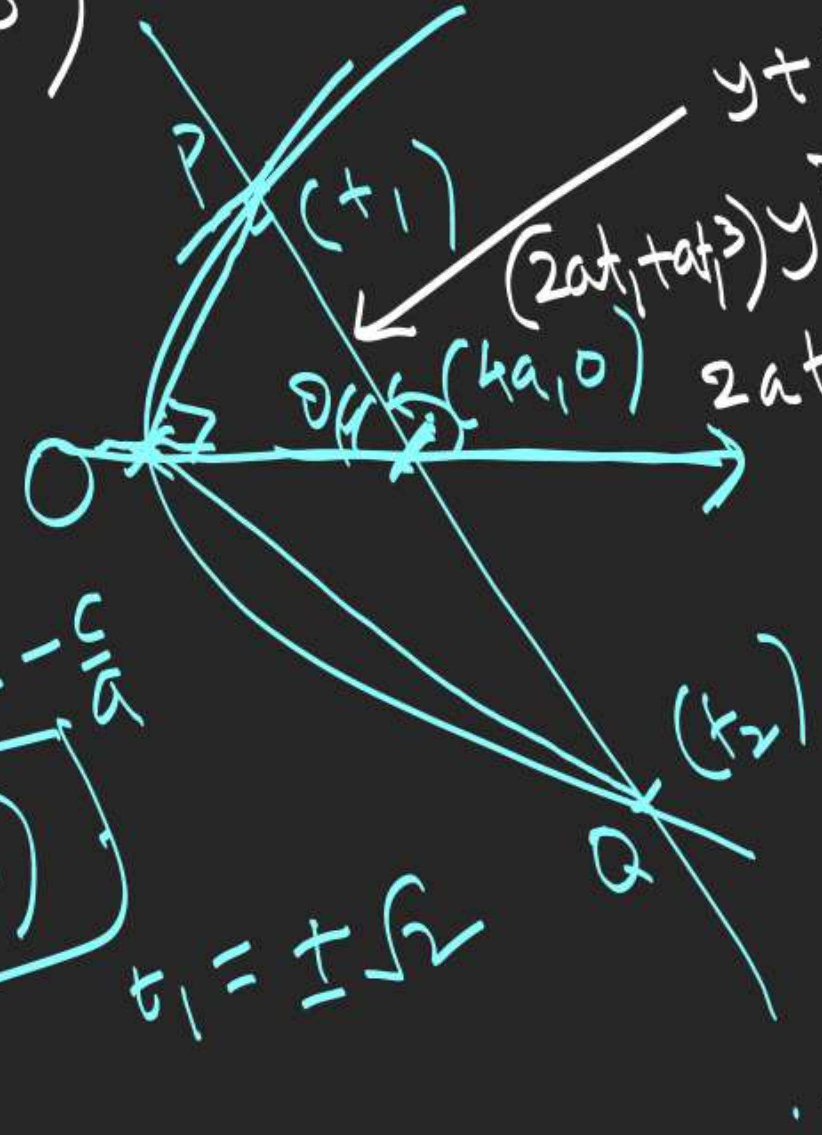


$$t_3 = -t_1 - \frac{2}{t_1} = -t_2 - \frac{2}{t_2}$$

$$t_2 - t_1 = \frac{2}{t_1} - \frac{2}{t_2} = \frac{2(t_2 - t_1)}{t_1 t_2}$$

$$t_1 t_2 = 2 \qquad t_3 = -t_1 - \frac{2}{t_1} = -t_1 - t_2$$

1. If a chord which is normal to parabola $y^2=4ax$ at one end subtend a right angle at vertex. P.T. it is inclined at angle $\tan^{-1}\sqrt{2}$ to axis and also P.T. normal chord passes thru $(4a,0)$



$y+t_1x=2at_1+at_1^3$

$(2at_1+at_1^3)y^2-4ax(y+t_1x)=0$

$2at_1+at_1^3-4at_1=0 \Rightarrow \boxed{t_1^2=2}$

$\boxed{\tan\theta=\sqrt{2}}$

$t_2=-t_1-\frac{2}{t_1}$

$\frac{2}{t_1}\frac{2}{t_2}=-1$

$t_1t_2=-4 \quad =-\frac{c}{a}$

$-\frac{4}{t_1}=-t_1-\frac{2}{t_1}$

$t_1=\frac{2}{t_1}$

$\boxed{(4a,0)}$

$t_1=\pm\sqrt{2}$

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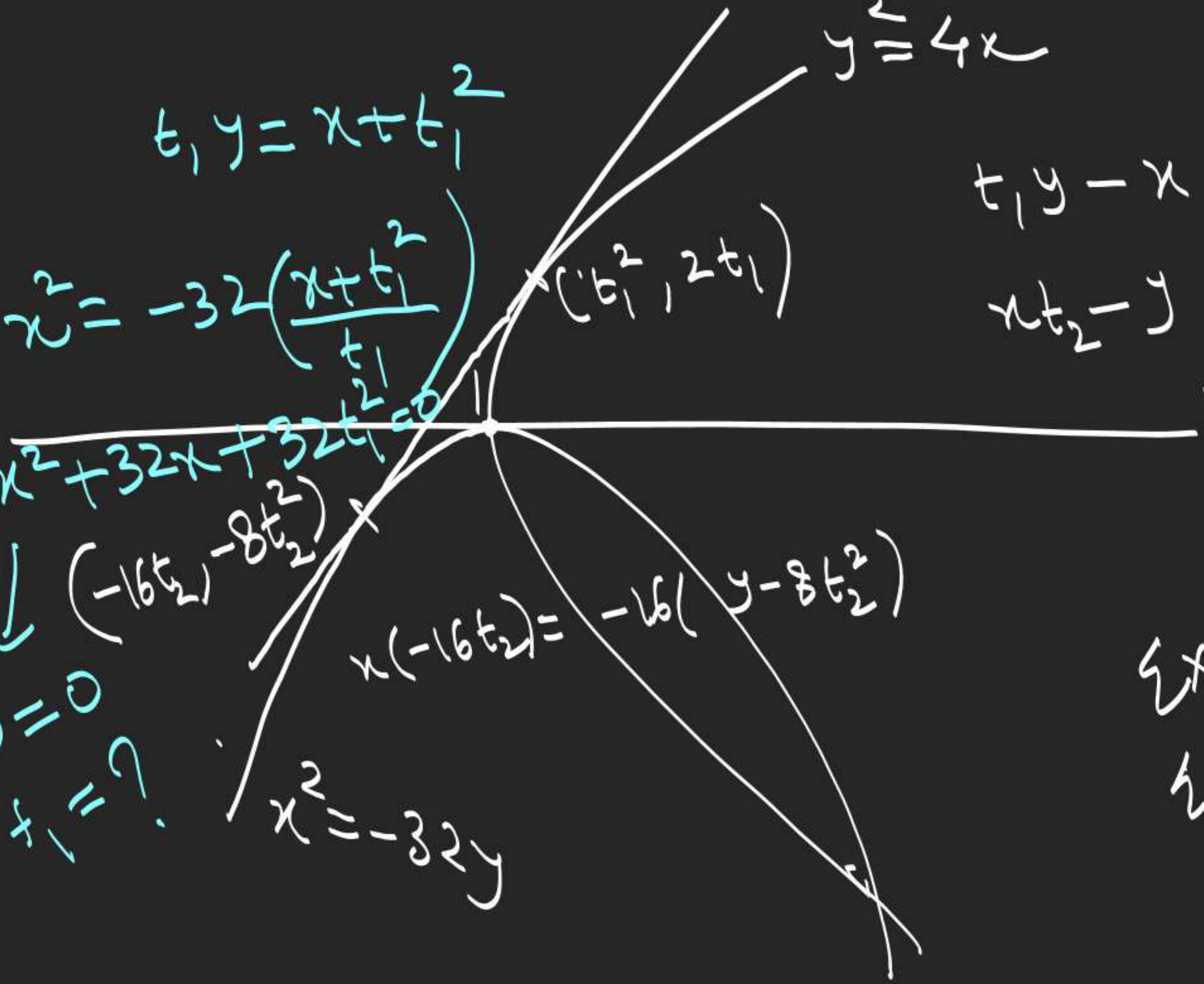
Q. Find the equation to the line touching both the parabolas $y^2 = 4x$ and $x^2 = -32y$.

$$\boxed{2y = x + 4}$$



$t_1 y = x + t_1^2$

$x^2 = -32\left(\frac{x + t_1^2}{t_1}\right)$

$t_1 x^2 + 32x + 32t_1^2 = 0$

$D = 0$

$t_1 = ?$

$t_1 y - x = t_1^2$

$x t_2 - y = -8t_2^2$

$\frac{t_1}{-1} = \frac{-1}{t_2} = \frac{t_1^2}{-8t_2^2}$

$t_2 = \frac{1}{t_1} \Rightarrow \frac{t_1^4}{-8} = -t_1$

$\cancel{t_1 = 0}$, $t_1^3 = 8$

$\boxed{t_1 = 2}$

Ex-25 (13, 6, 14, 19, 21)

Ex-26 (6, 5, 15, 17, 19, 22)

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3. Let the normal at any point P to parabola $y^2=4ax$ meet its axis in G and the tangent at vertex in Y. If A be the vertex and the rectangle GAYQ be completed, find the locus of Q.