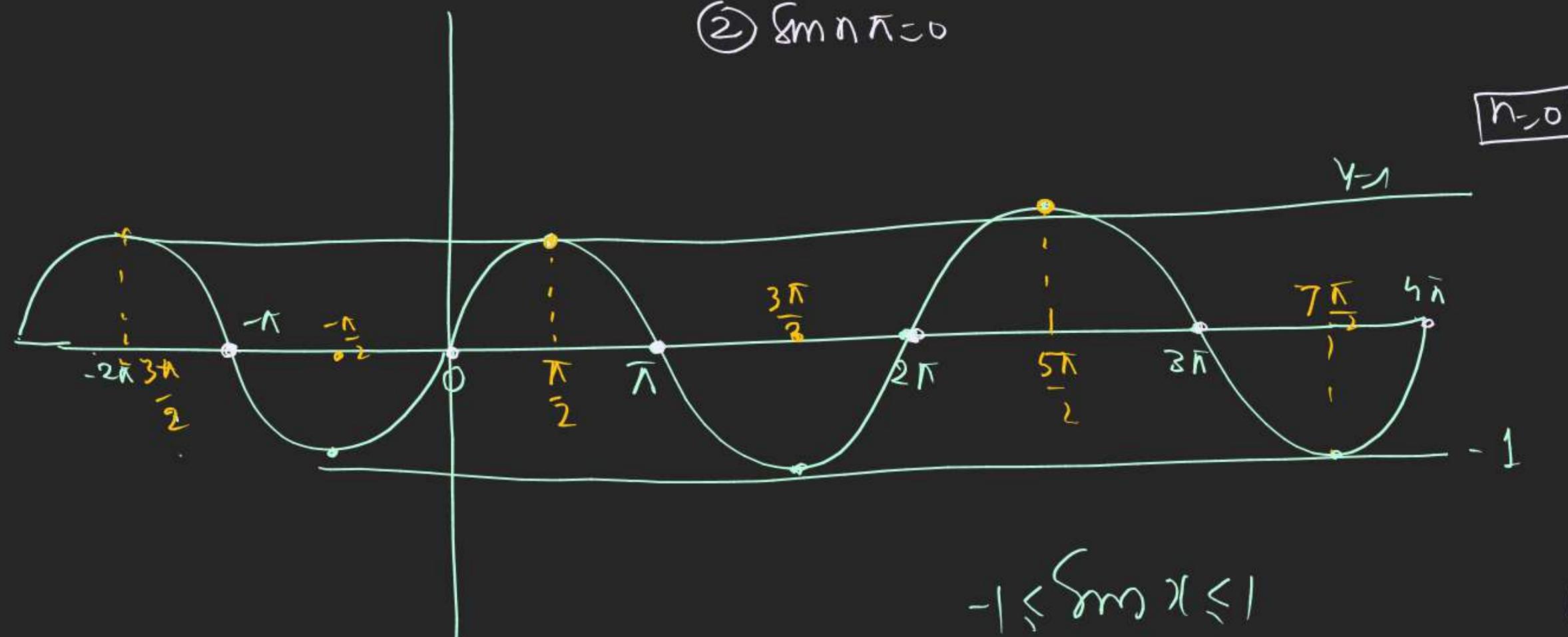


Trigonometry

$$\textcircled{1} \quad y = \sin x.$$

① graph $\sin x$

$$\textcircled{2} \quad \sin n\pi = 0$$



$$\sin x = 0 \Rightarrow x = n\pi$$

$$\sin 0 = 0$$

$$\sin \pi = 0$$

$$\sin 2\pi = 0$$

$$\sin(-3\pi) = 0$$

$$\sin(2023\pi) = 0$$

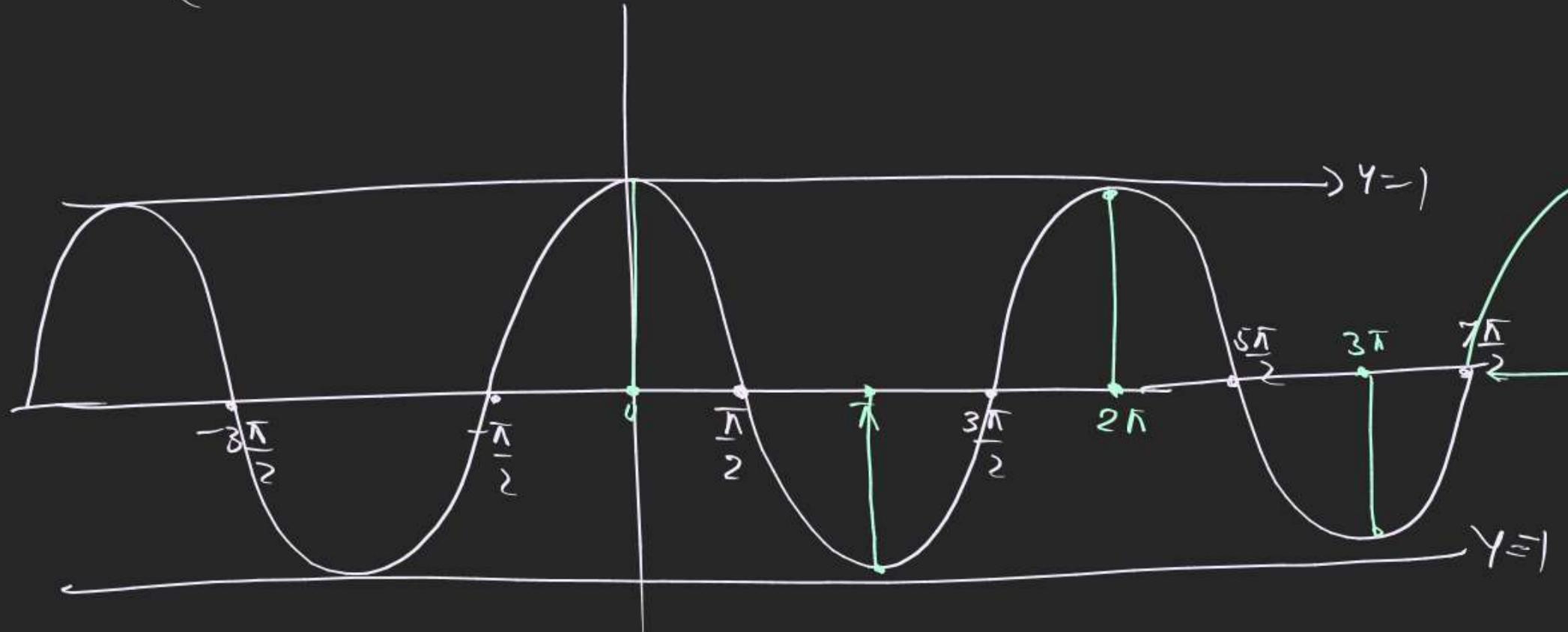
$$\left. \begin{array}{l} \sin 0 = 0 \\ \sin \pi = 0 \\ \sin 2\pi = 0 \\ \sin(-3\pi) = 0 \\ \sin(2023\pi) = 0 \end{array} \right\} \text{for } n \in \mathbb{Z}$$

$$\textcircled{1} \quad \sin x = 1 \Rightarrow x = (4n+1)\frac{\pi}{2}$$

$$\textcircled{2} \quad \sin x = -1 \Rightarrow x = (4n+3)\frac{\pi}{2}$$

Trigonometry

$$(2) \quad y = \cos x$$

 $\frac{4\pi}{2}$

1) $\cos \text{even } k = 1$

$\cos(2n\pi) = 1$

2) $\cos \text{odd } k = -1$

$\cos((2n+1)\pi) = -1$

(3) $\cos n\pi = \pm 1$

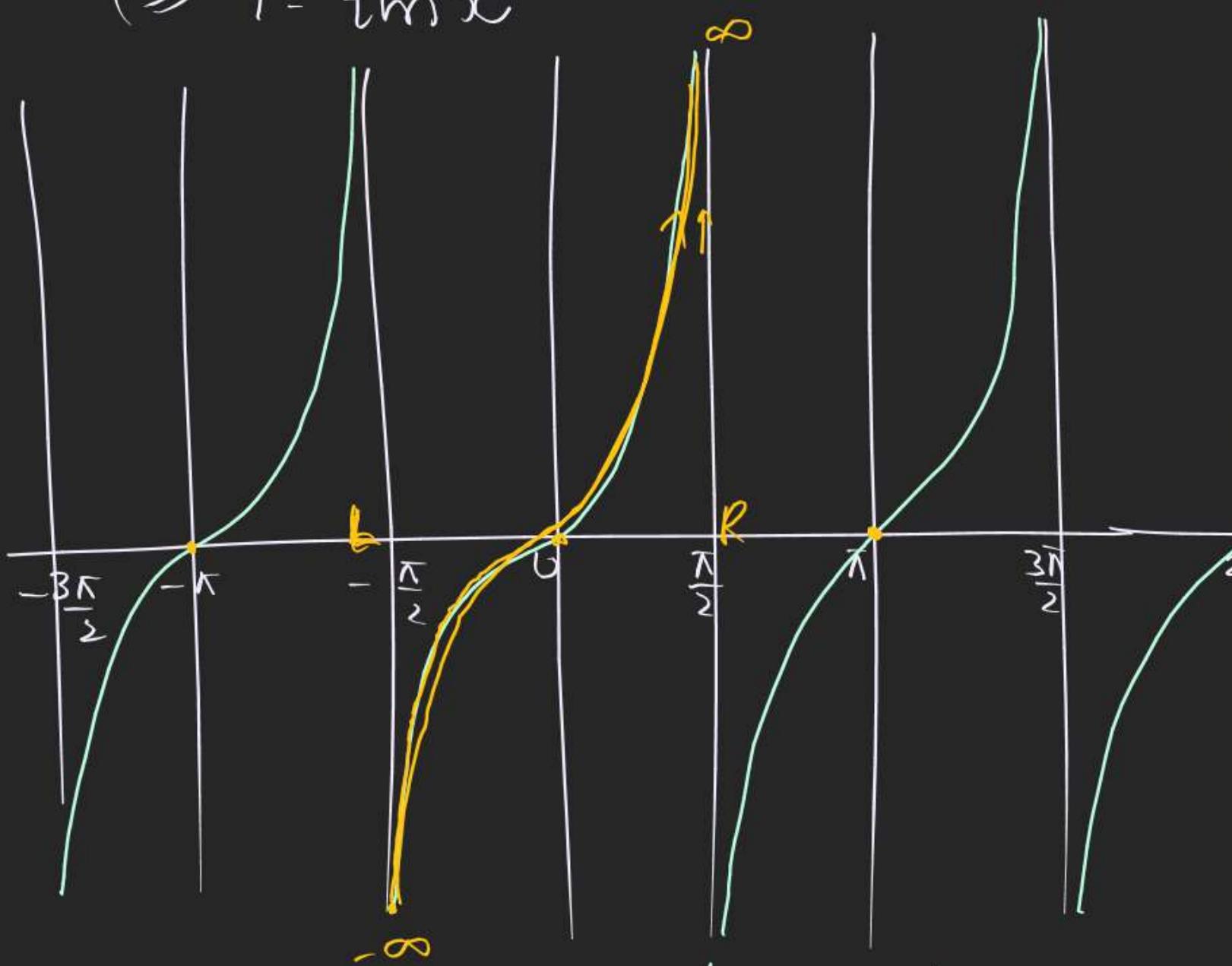
(4) $\cos x = 0$

When $x = (2n+1)\frac{\pi}{2}$

$$\tan \frac{\pi}{3} = \frac{\sin \frac{\pi}{2}}{\cos \frac{\pi}{2}} = \frac{1}{0}$$

(3) $y = \tan x$

Trigonometry



Trigo

Q Egn / Segn / Prn (|BT|) coord

Domain
Values of x
acceptable
by f(x)

(2) for $\tan x$

Range = value of y
ht of graph

$x + (2n+1)\frac{\pi}{2}$

Domain $x \in (-\infty, \infty) - (2n+1)\frac{\pi}{2}$

(3) $-\infty < f(x) < \infty$
Range

(1) $\tan x = 0$

$\tan 0 = 0$

$\tan \pi = 0$

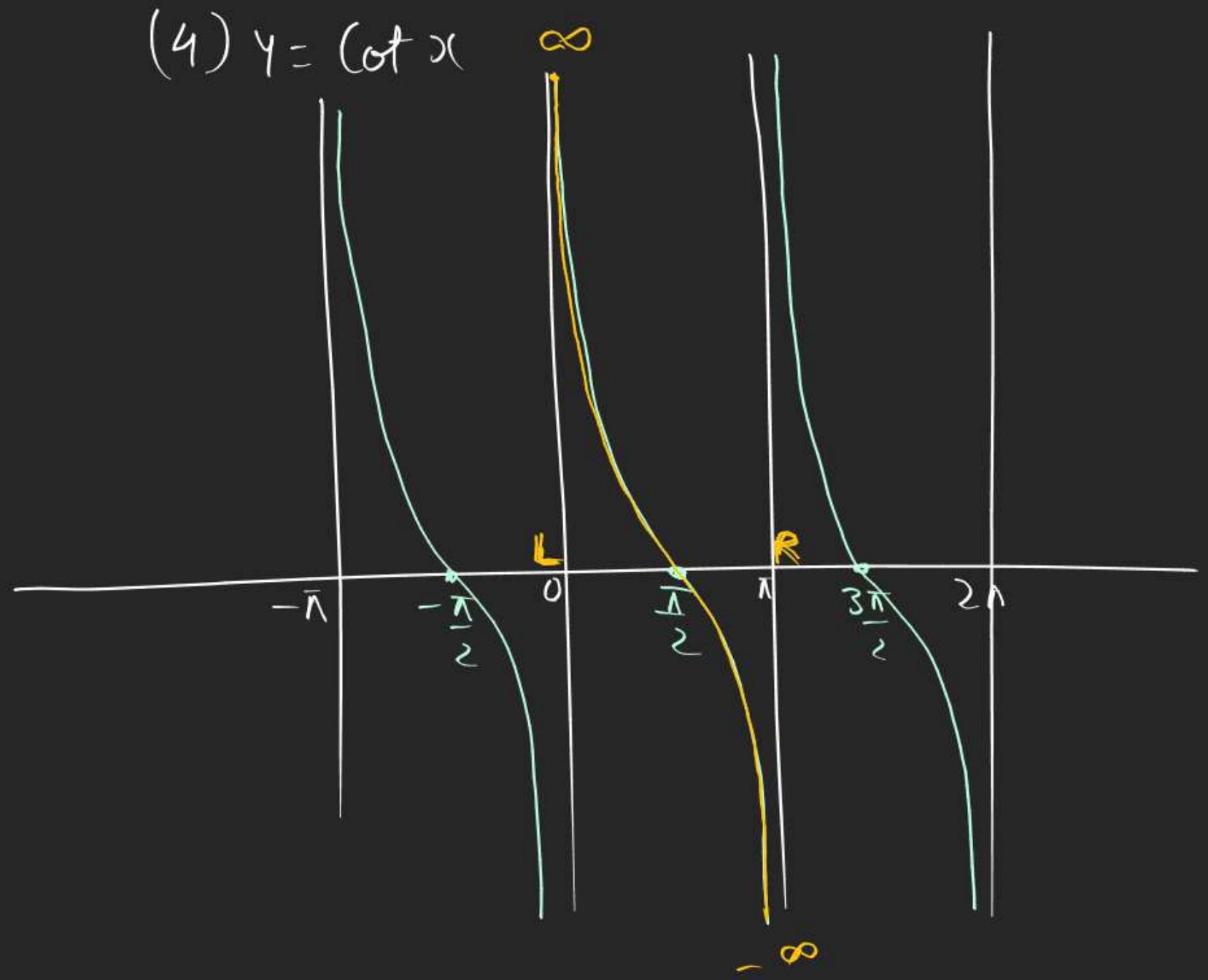
$\tan 2\pi = 0$

$\tan 3\pi = 0$

$\tan (-2\pi) = 0$

$\tan n\pi = 0$

Trigonometry



+ mx Kadaff always +ve ↑
(or) Kadaff always -ve ↓ (graph Niche Kiaune Ayege)

① $\cot x = 0$ at

$$x = (2n+1) \frac{\pi}{2}$$

(4) $A) y = \sin x$

$$\frac{dy}{dx} = \cos x$$

(2) $\int 07 y = \sin x$

(B) $y = \sin x$

$$\frac{dy}{dx} = \cos x$$

Domain $x \neq n\pi$

$$\boxed{x \in (-\infty, \infty) - (n\pi)}$$

(C) $y = \tan x$

$$\frac{dy}{dx} = \sec^2 x \geq 0$$

(3)

Range - $y \in (-\infty, \infty)$

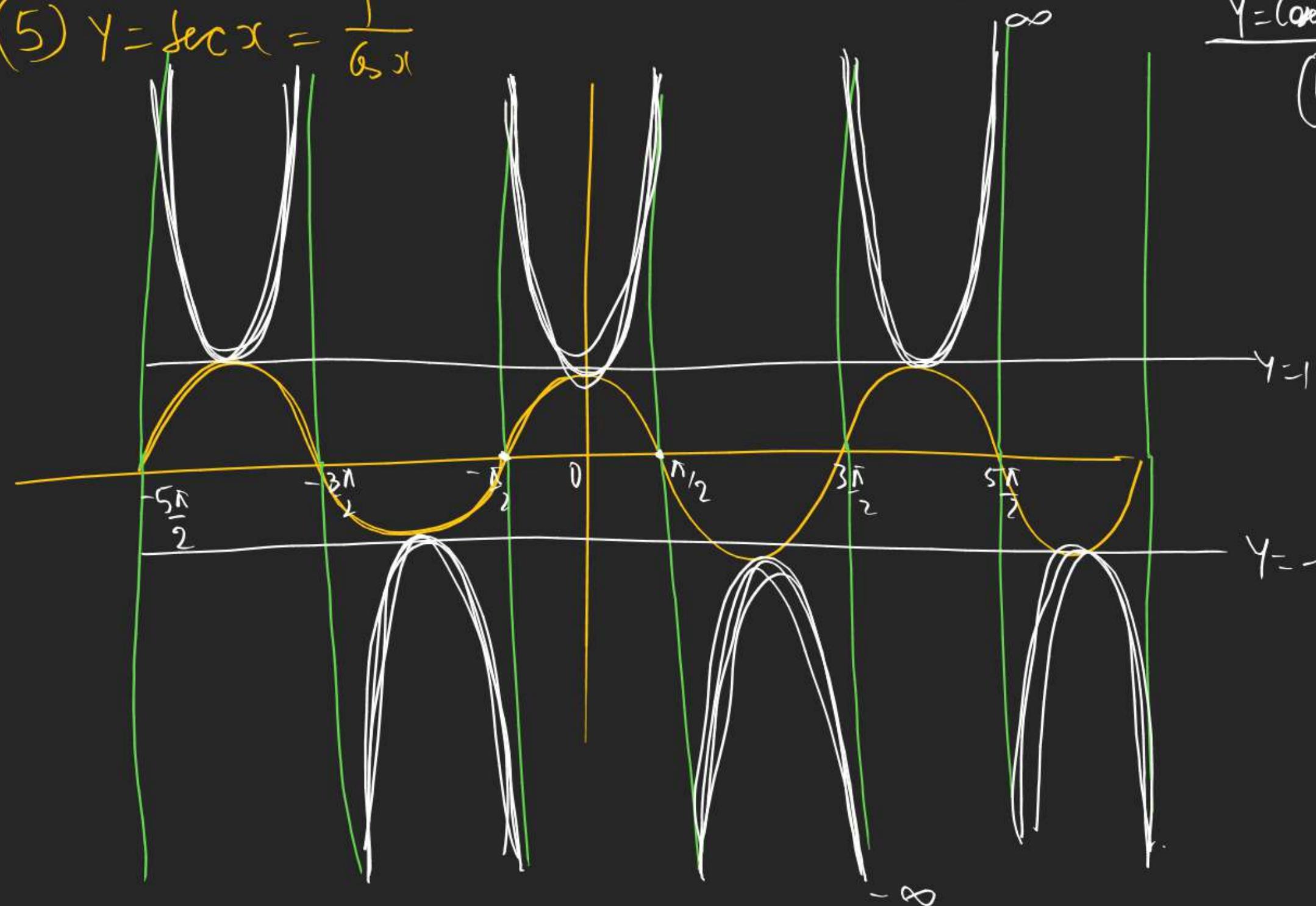
(D) $y = \cot x$

$$\frac{dy}{dx} = -\operatorname{cosec}^2 x \leq 0$$

Trigonometry

HW → Search all Net

$$(5) Y = \sec x = \frac{1}{\cos x}$$



$$Y = \sec x \rightarrow \text{Copy}$$

① for $y = \sec x$

$$\pi \notin (2n+1)\frac{\pi}{2}$$

$$\text{Dom } x \in (-\infty, \infty) - (2n+1)\frac{\pi}{2}$$

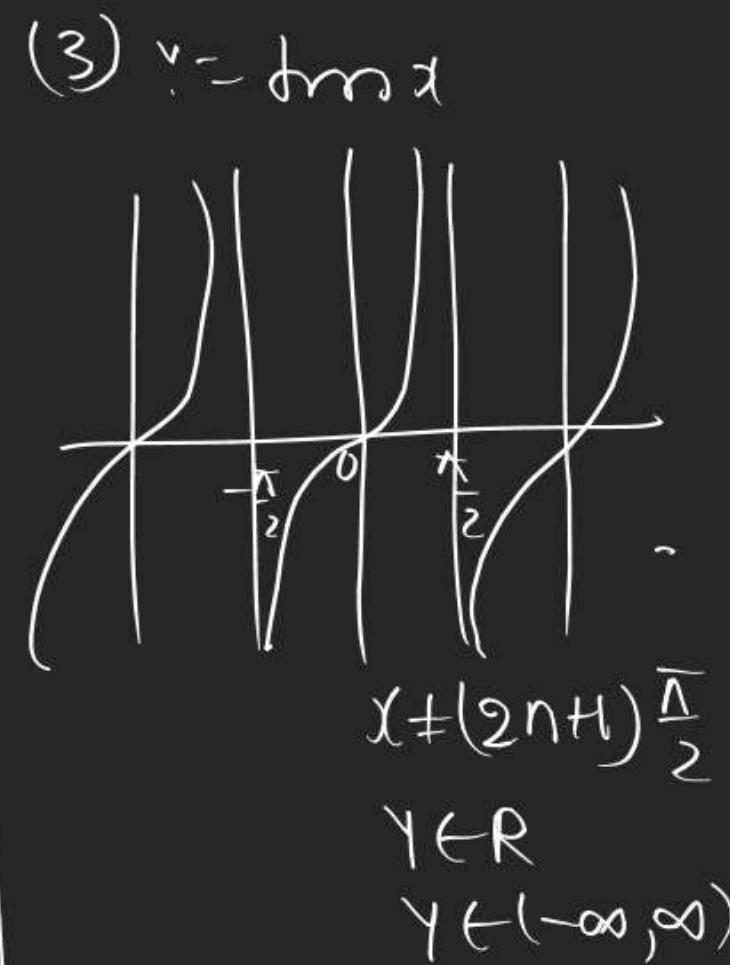
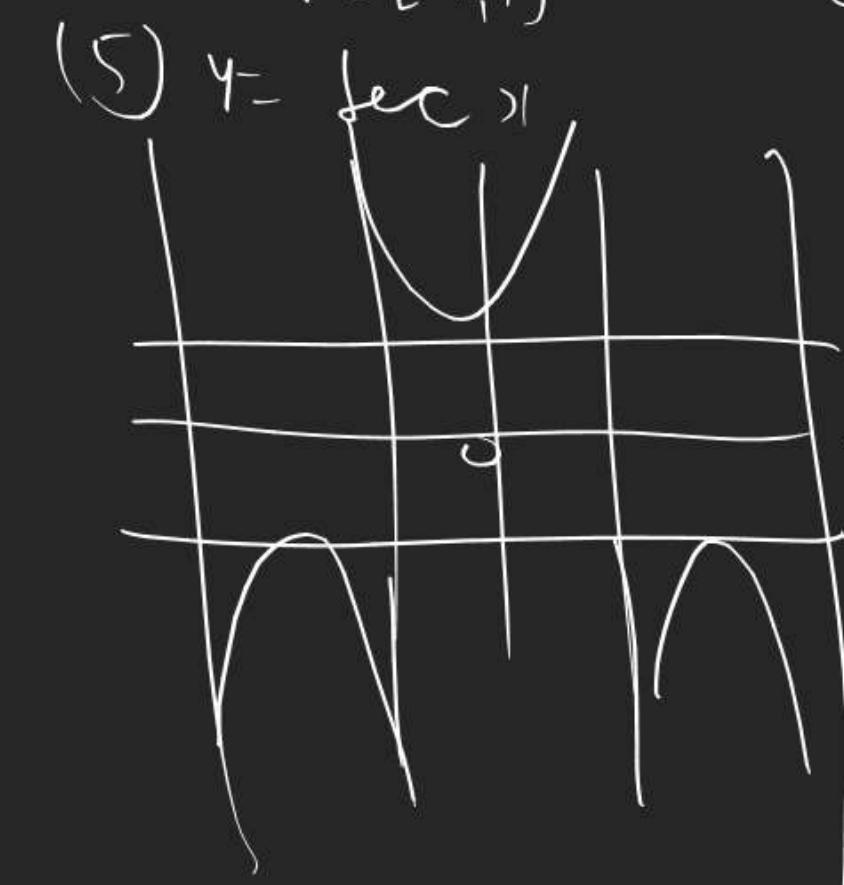
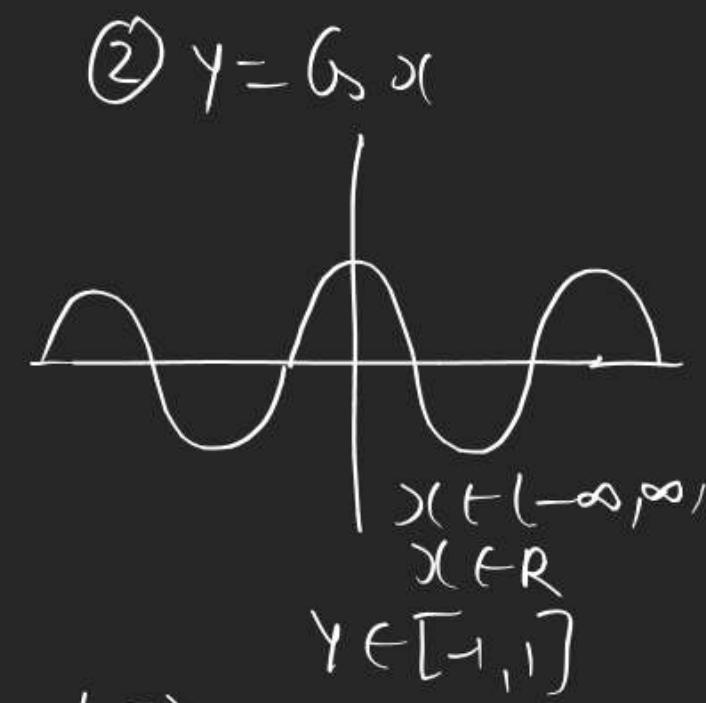
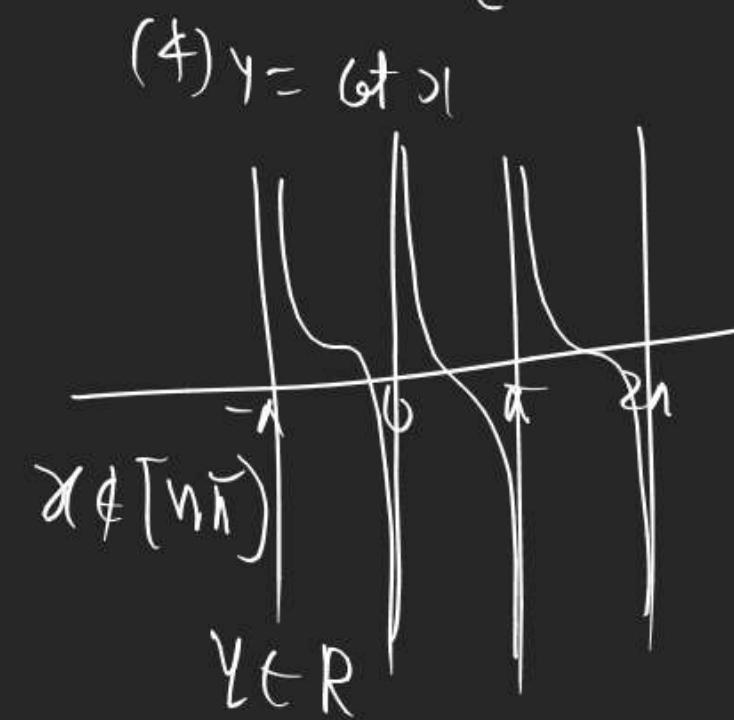
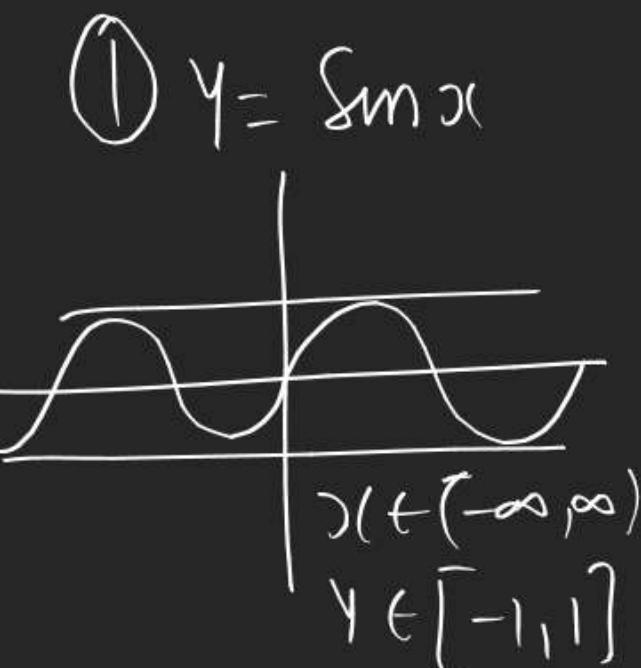
$$y=1$$

②

$$y \in (-\infty, -1] \cup [1, \infty)$$

Range

Trigonometry



$$\begin{aligned} 1) \lim_{x \rightarrow \pi^-} \tan x &= \infty \\ 2) \lim_{x \rightarrow \pi^+} \tan x &= -\infty \\ 3) \lim_{x \rightarrow (2n+1)\frac{\pi}{2}} \tan x &= 0 \end{aligned}$$

Trigonometry

$$\text{Ans} \rightarrow \boxed{\begin{array}{l} \text{concent} \\ y = \sec x = 0 \\ x = (2n+1)\frac{\pi}{2} \\ y = \sec x = 0 \\ x = n\pi \end{array}}$$

$$\begin{aligned} &Q, y = \sec x \text{ Dom?} \\ &= \frac{1}{(\sec x)} \\ &\sec x \neq 0 \\ &x + (2n+1)\frac{\pi}{2} \\ &\text{Def: } x \in R - (2n+1)\frac{\pi}{2} \end{aligned}$$

$$Q, y = \tan x \text{ Dom?}$$

$$y = \frac{\sin x}{\cos x}$$

$$y = \frac{\sin x}{\cos x} \xrightarrow[\cos x \neq 0]{R}$$

$$\begin{aligned} &x + (2n+1)\frac{\pi}{2} \\ &x \in R - (2n+1)\frac{\pi}{2} \end{aligned}$$

$$\therefore x \in R - (2n+1)\frac{\pi}{2}$$

$$\begin{aligned} &Q, y = \csc x \text{ Dom?} \\ &= \frac{1}{(\csc x)} \\ &\csc x \neq 0 \\ &x \neq n\pi \end{aligned}$$

$$x = R - (n\pi)$$

Dom

Trigonometry

Range of Trigo-fxn.

1) $y = \sin x$ Rf ? 
 $y \in [-1, 1]$

2) $y = (\sin x)^2$ Rf ? 
 $y \in [0, 1]$

3) $y = \tan x$ Rf ?
 $y \in \mathbb{R}$
 $y \in (-\infty, \infty)$

Concept

$$y \rightarrow ax + b$$

Range Remains Same

① $y = \sin(3x)$ Rf ?

$$y \in [-1, 1]$$

(2) $y = \sin(3x - 25)$ Rf ?

$$y \in [-1, 1]$$

(3) $y = \sin\left(\frac{x}{2} + \frac{\pi}{19}\right)$ Rf

$$y \in [-1, 1]$$

Trigonometry

Q) $y = \tan(3x - \pi/2) \forall R_f$

Same as $y = \tan x$
 $y \in R$

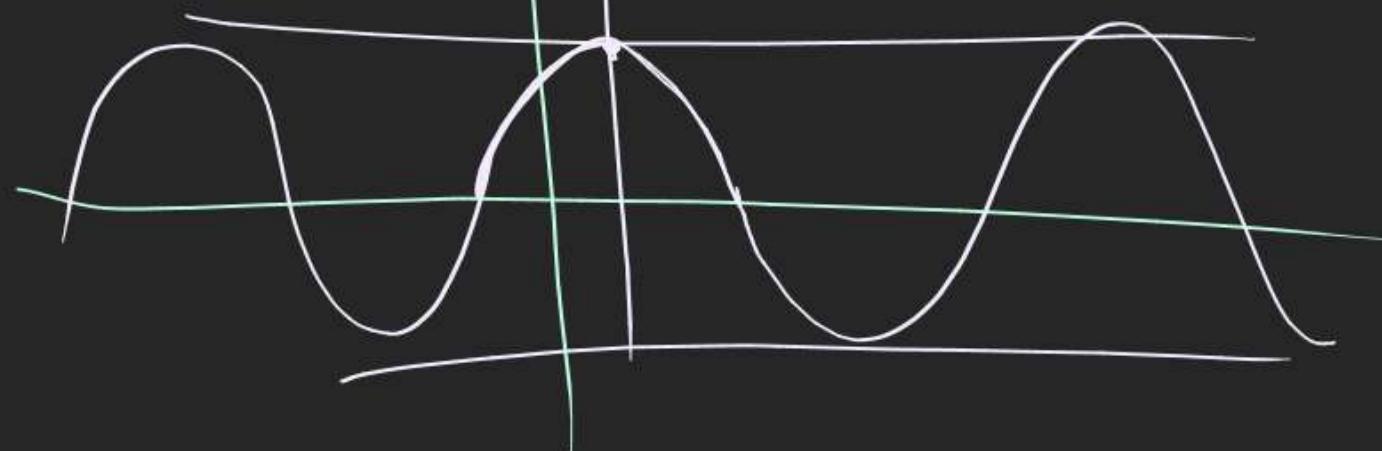
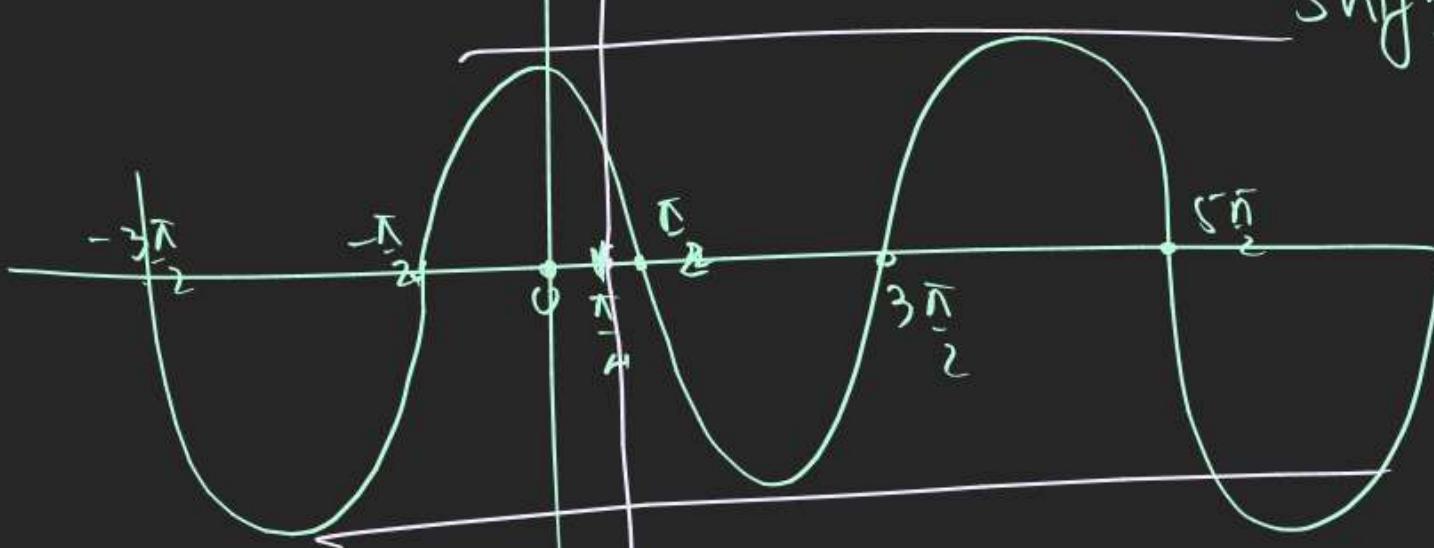


Q) $y = \cos(x - \pi/4) \forall R_f$

$y \in [-1, 1]$

① $y = G(x)$ & $y = G(x - \pi/4)$

$x = \pi/4$ w.r.t graph
Shift



Trigonometry

Concept

1) $(\quad)^2 \geq 0$

2) $(\quad)^2 = 0$ Kab Banata h?

Ans. When fn Inside becomes zero

Q12 $y = (\sin x - \frac{3}{4})^2$ (can give value 0 or not?)

If $\sin x - \frac{3}{4}$ is PSBL then $(\sin x - \frac{3}{4})^2$ will be 0.

$-1 \leq \frac{3}{4} \leq 1$ as $\frac{3}{4}$ comes betn -1 to +1 hence $\sin x = \frac{3}{4}$ Possible

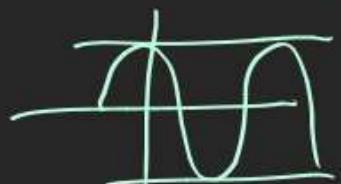
$\Rightarrow (\sin x - \frac{3}{4})^2 \geq 0$ PSBL



Q $y = (6x - 2)^2$ (can give 0 or not)

If $6x - 2 = 0$ PSBL then it is PSBL.

$$\Rightarrow \sqrt{6x} = 2$$



$-1 \leq 2 \leq 1$ (0.0)
Not PSBL.

$\Rightarrow (6x - 2)^2 = 0$ Not PSBL

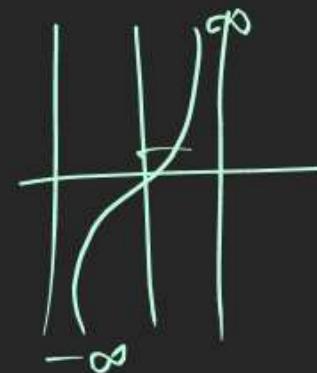
Trigonometry

Q $(mx-2)^2$ can give 0 or not

12

Is fm x-2 PSb1??

Yes!!!



as $\lim x$'s Range $\in (-\infty, \infty)$

Q Is $y = \underline{\sin^2 x} \circ R_f$?

$$\textcircled{1} \quad \delta m^2 x \geq 0$$

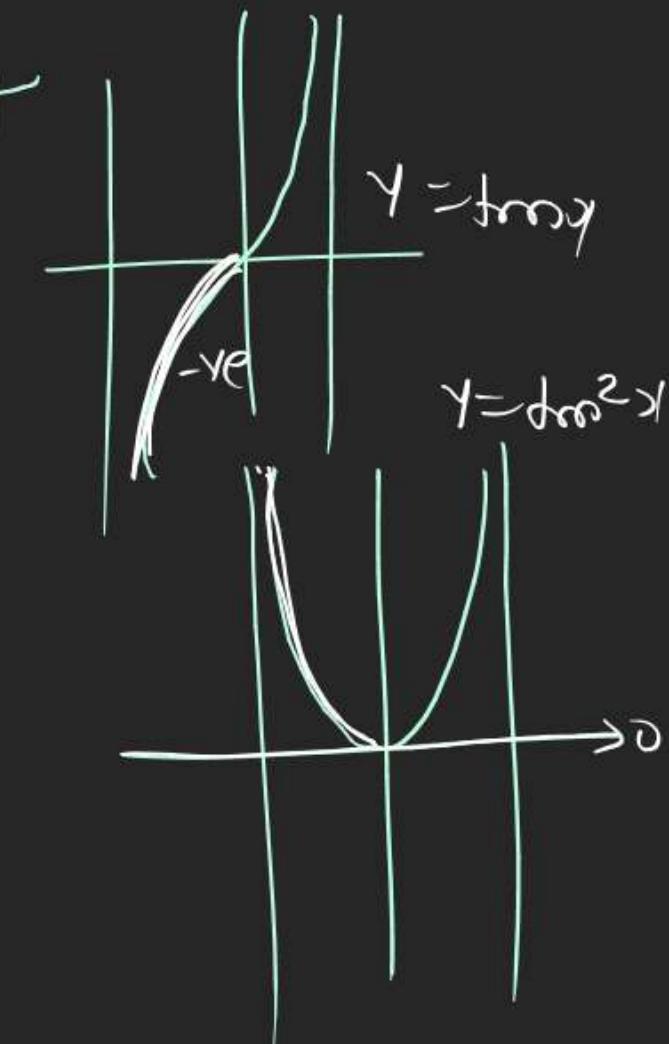
$$(2) -1 \leq \lim x \leq 1$$

$$\Rightarrow 0 \leq \delta_{\text{WR}}(x) \leq 1$$

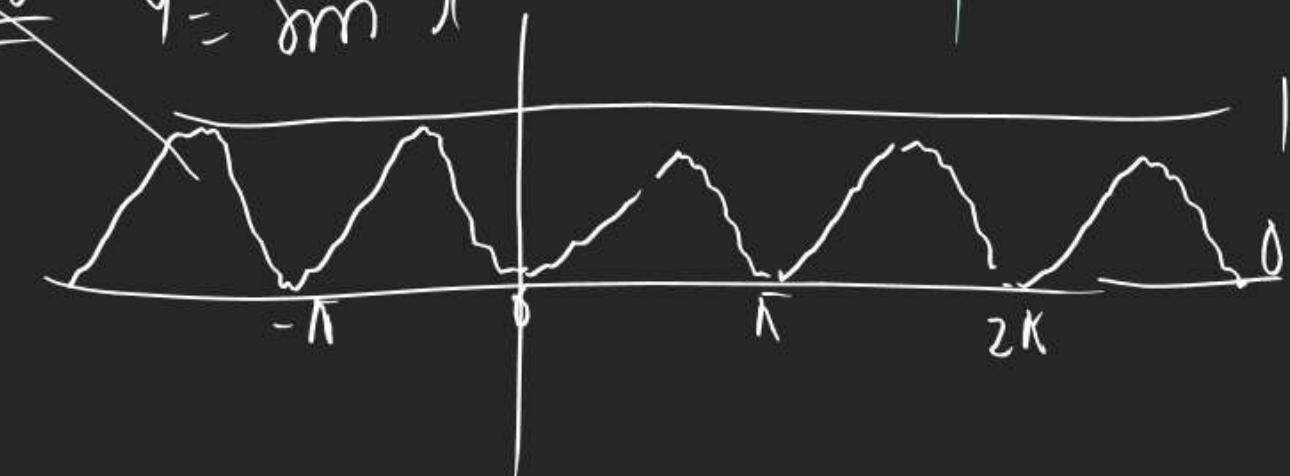
$$Q \propto \rho m^2 x' \propto R^2$$

1

$$\infty > t m^2 x \geq 0$$



$$\text{Ans} \quad y = \sin^2(x)$$

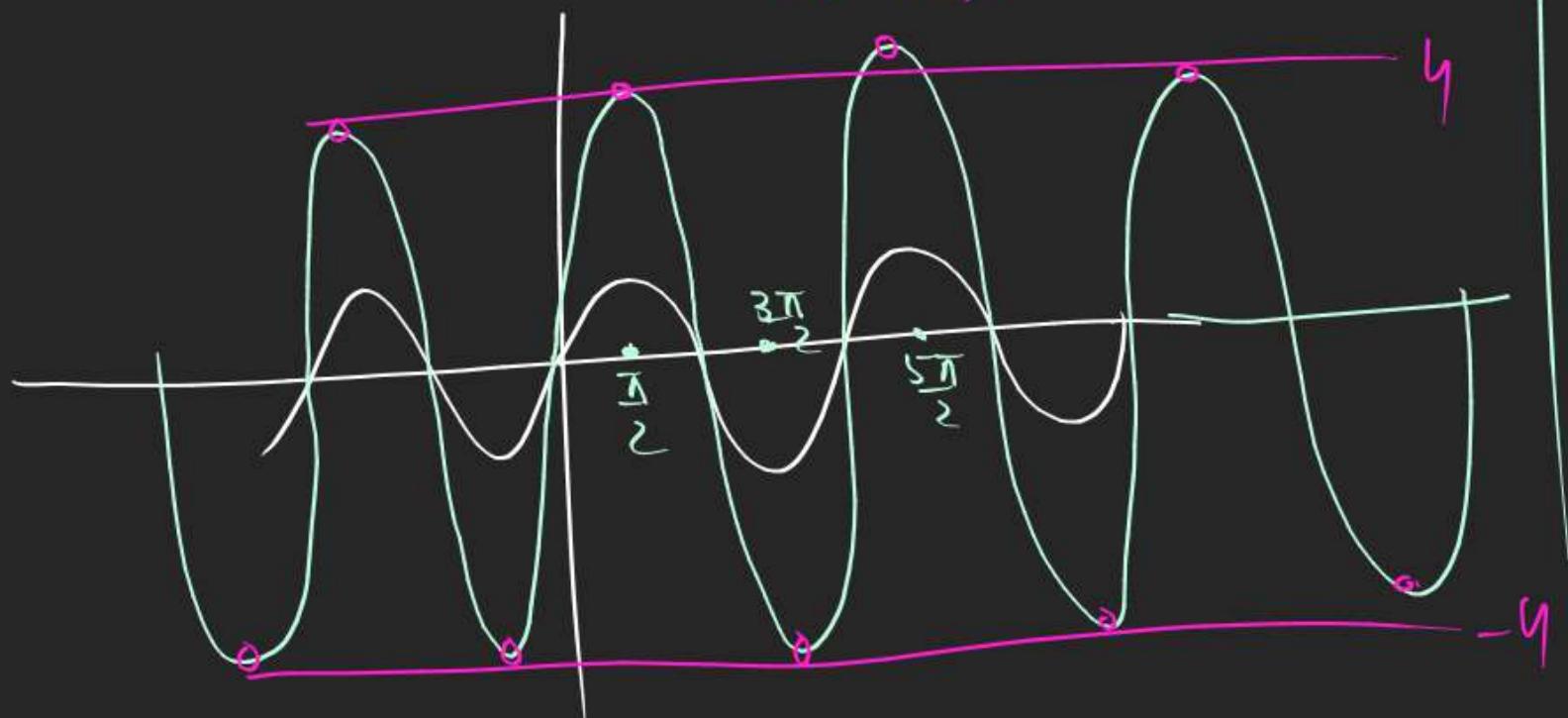


Trigonometry

$$\text{Q. } \boxed{y = 4 + \sin x} \text{ (Ans R+)}$$

$$y = 4 + \frac{\sin x}{\sin x} \times \cancel{\sin x} \quad (\text{as } x \neq 0)$$

$$y = 4 + \sin x \in (-4, 4) \quad \boxed{x + (2n+1)\frac{\pi}{2}}$$



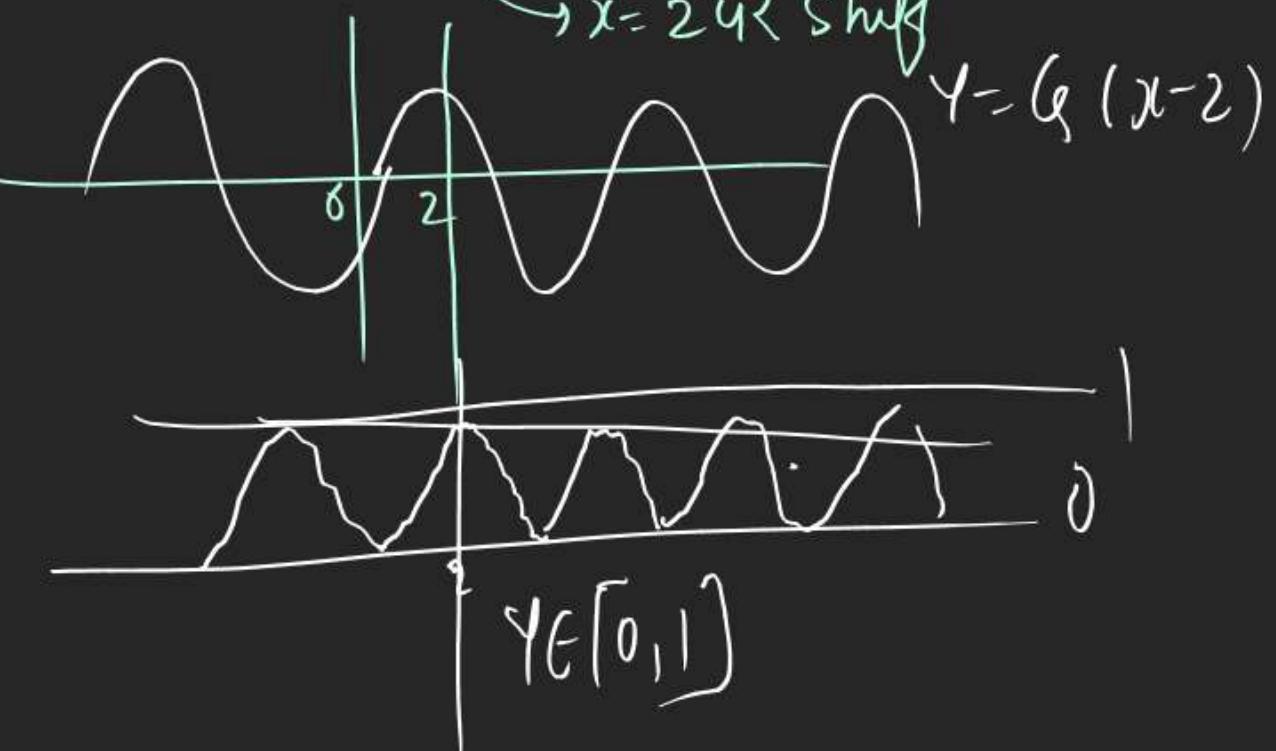
$$\text{Q. Rf of } y = \left(2^2 \left(\frac{x}{4}\right) - \sin^2 \left(\frac{x}{4}\right)\right)$$

$$\text{Ans Aanya} \rightarrow 6^2 \theta - \sin^2 \theta = 6 \sin^2 \theta$$

$$y = \sin^2 \left(\frac{x}{4}\right) - \sin^2 \left(\frac{\pi}{4}\right) = \sin^2 \frac{x}{4}$$

$$R+ \in [-1, 1]$$

$$\text{Q. } y = 6^2(x-2) \text{ R+ } \xrightarrow{x=2\sqrt{2} \text{ shift}}$$



Trigonometry

Ques: $y = \sqrt{\sin x}$ is Rf?

Intuition: Concept: $\sqrt{\quad}$ gives +ve Quantities only

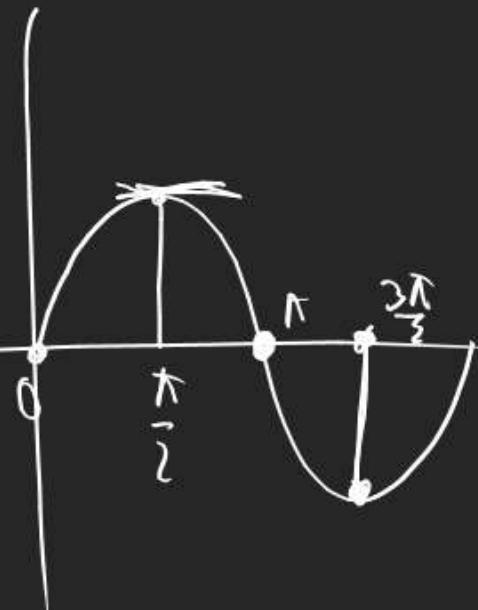
$$\sqrt{\sin x} \in [0, 1]$$

Ques: $y = \sin \sqrt{x}$ is Rf?

$y \in [-1, 1]$

$$\left\{ \begin{array}{l} x = \frac{\pi^2}{4} \rightarrow y = \sin \sqrt{\frac{\pi^2}{4}} = \sin \frac{\pi}{2} = 1 \\ x = \frac{9\pi^2}{4} \rightarrow y = \sin \sqrt{\frac{9\pi^2}{4}} = \sin \frac{3\pi}{2} = -1 \\ x = \pi^2 \rightarrow y = \sin \sqrt{\pi^2} = \sin \pi = 0 \end{array} \right.$$

-2 Ques: $y = 2 \sin x$ is Rf?
 $-1 \leq \sin x \leq 1$
 $-2 \leq 2 \sin x \leq 2$
 $\therefore y \in [-2, 2]$



Trigonometry

$$\text{Q } y = \frac{1}{\sqrt{2}} Gx \text{'s R}_+$$

$$-1 \leq Gx \leq 1$$

$$-\frac{1}{\sqrt{2}} \leq \frac{1}{\sqrt{2}} Gx \leq \frac{1}{\sqrt{2}}$$

$$\therefore y \in \left[-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right]$$

~~graph~~

$$\text{Q } y = \sin^2\left(\frac{15\pi}{8} - 4x\right) - \sin^2\left(\frac{17\pi}{8} - 4x\right) \text{'s R}_+$$

This matches to $\sin^2 A - \sin^2 B$

$$= \sin(A+B) \cdot \sin(A-B)$$

$$\sin\left\{\left(\frac{15\pi}{8} - 4x\right) + \left(\frac{17\pi}{8} - 4x\right)\right\}, \sin\left\{\left(\frac{15\pi}{8} - 4x\right) - \left(\frac{17\pi}{8} - 4x\right)\right\}$$

$$+ \sin\left(\frac{32\pi}{8} - 8x\right) \cdot \sin\left(-\frac{2\pi}{8}\right)$$

$$y = + \sin \frac{1}{4} \sin(8x) = \frac{1}{\sqrt{2}} \boxed{\sin 8x} \text{ Range}$$

$$\begin{aligned} -1 &\leq \sin 8x \leq 1 \\ -\frac{1}{\sqrt{2}} &\leq \sin 8x \leq \frac{1}{\sqrt{2}} \quad \therefore y \in \left[-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right] \end{aligned}$$