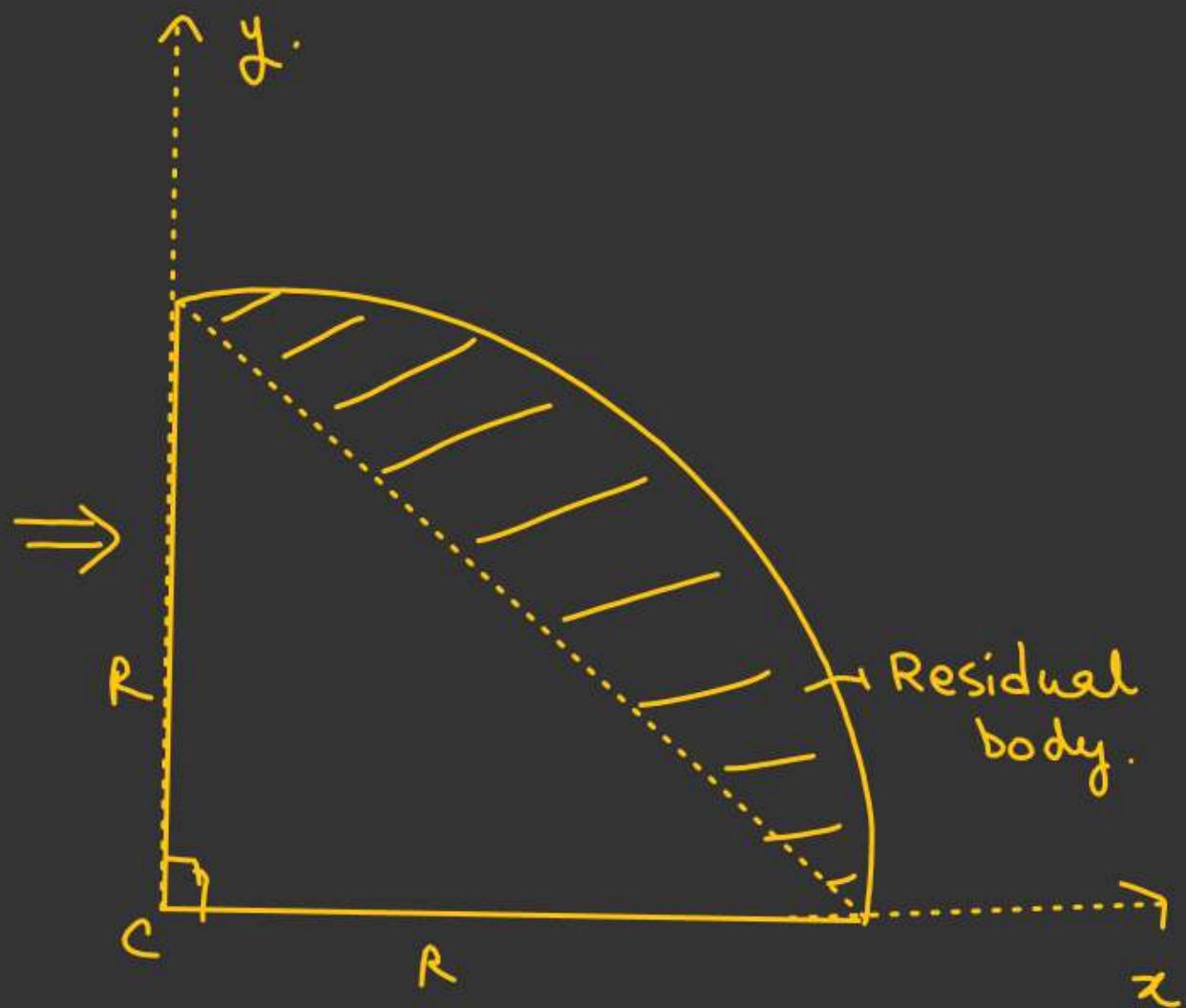
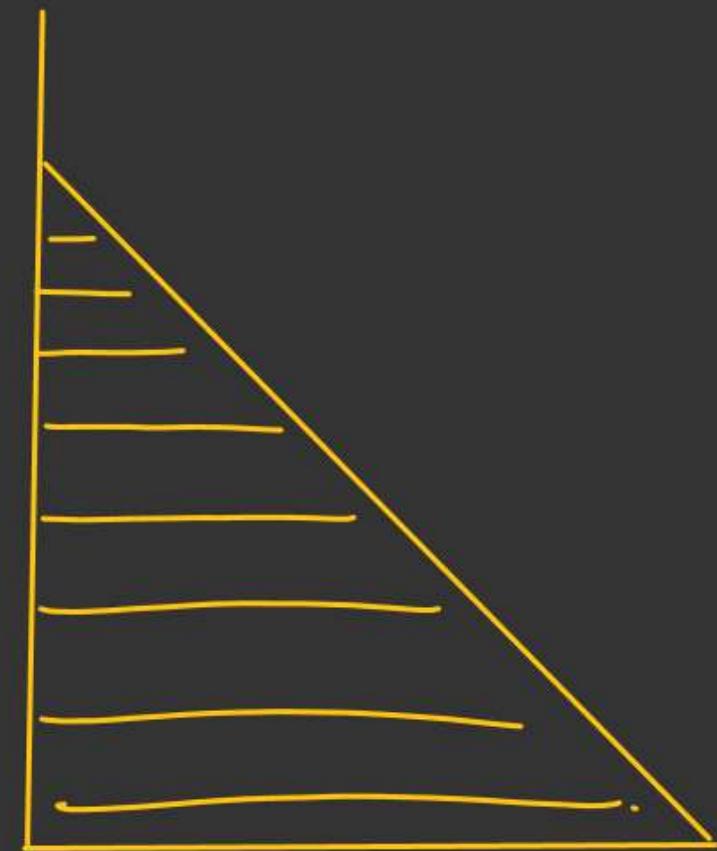
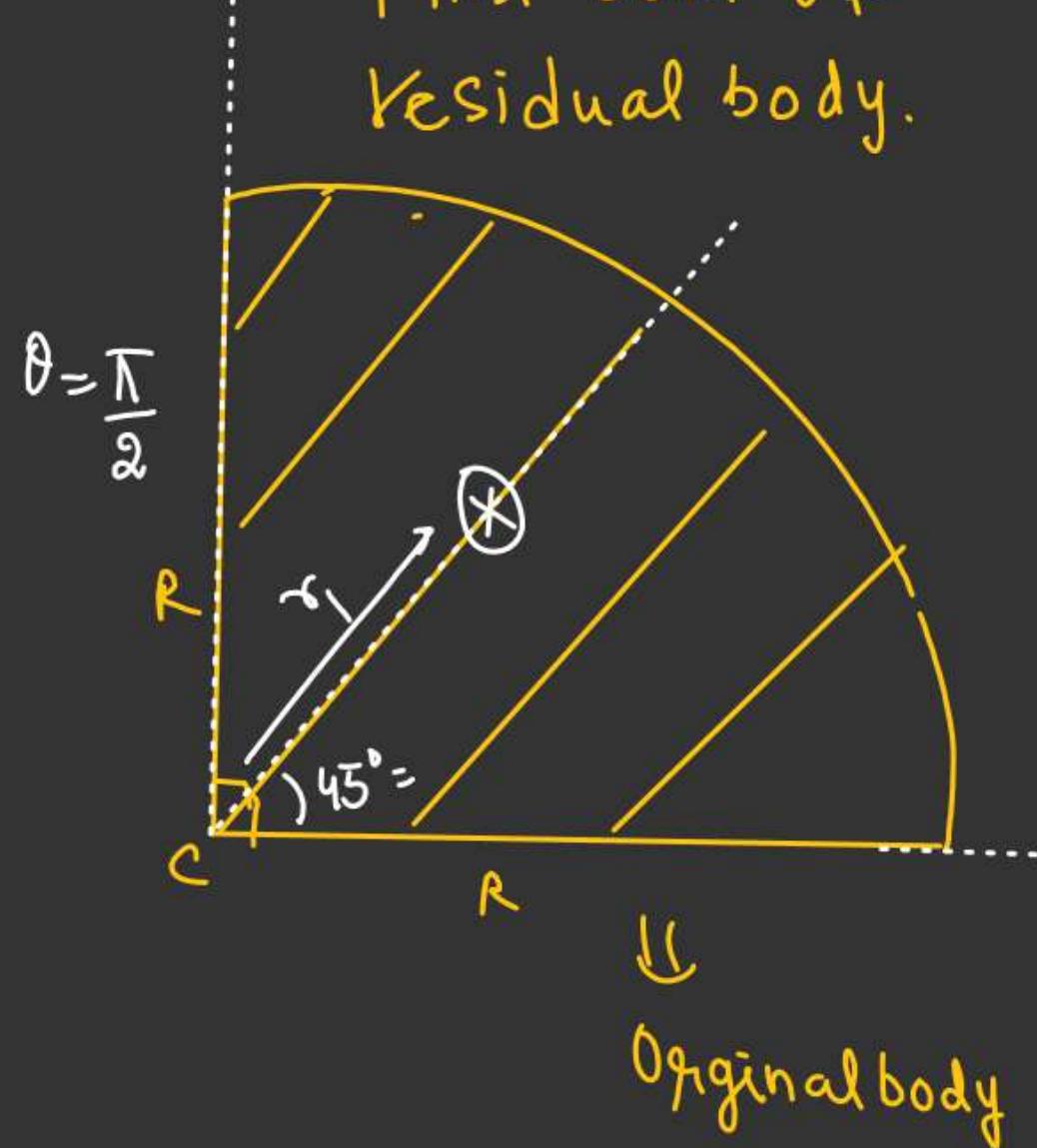
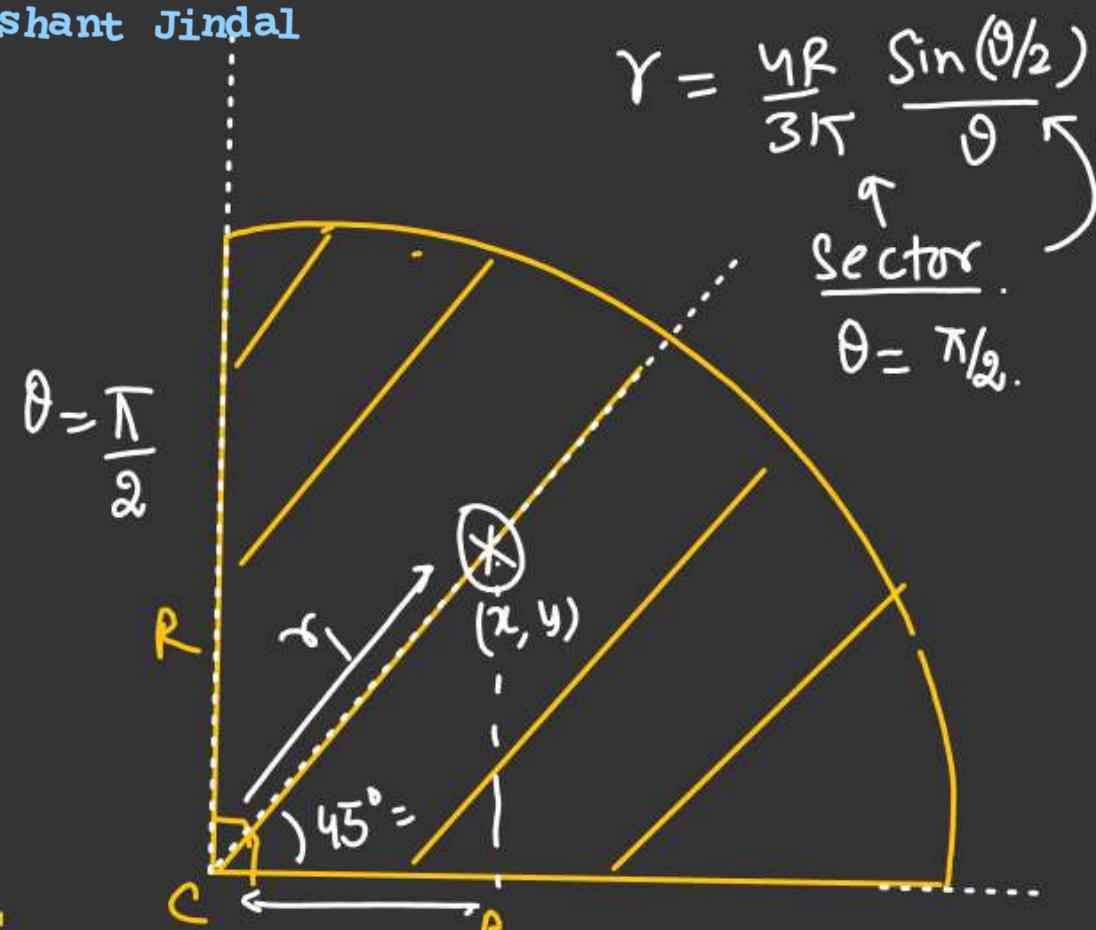


#H.W.: A triangle is cut from a Sector of radius R.

Find COM of

Residual body.

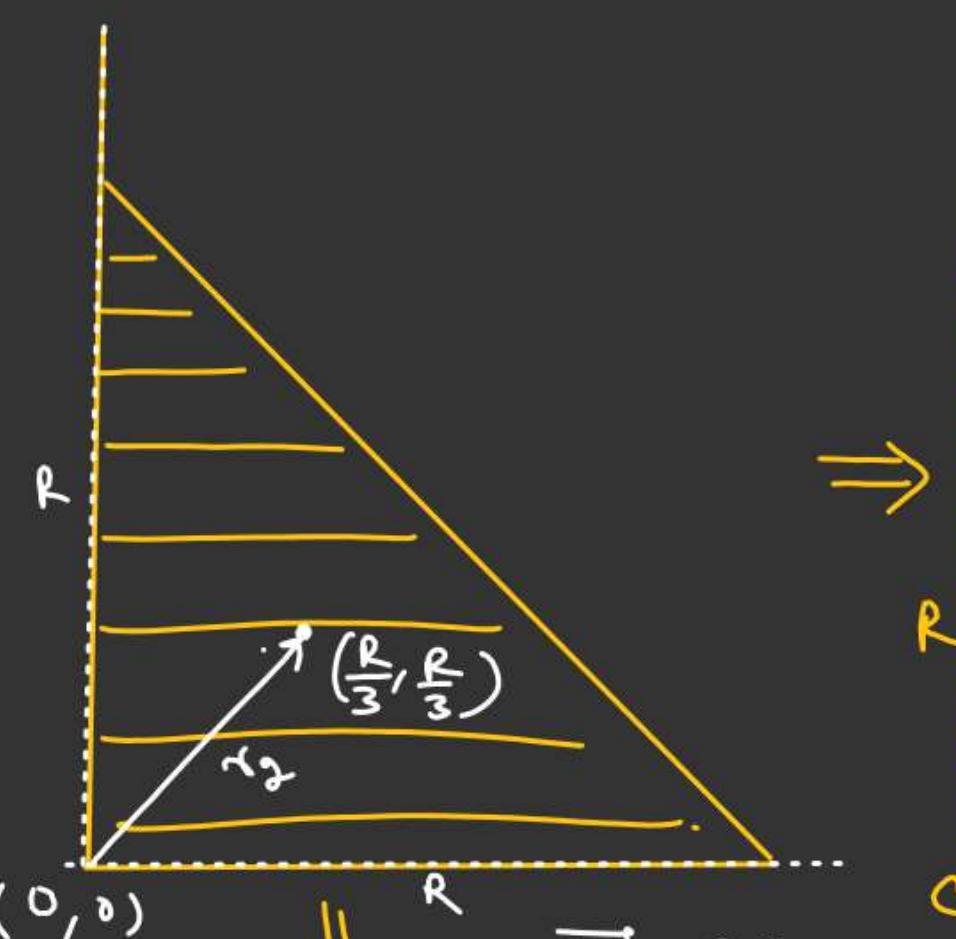




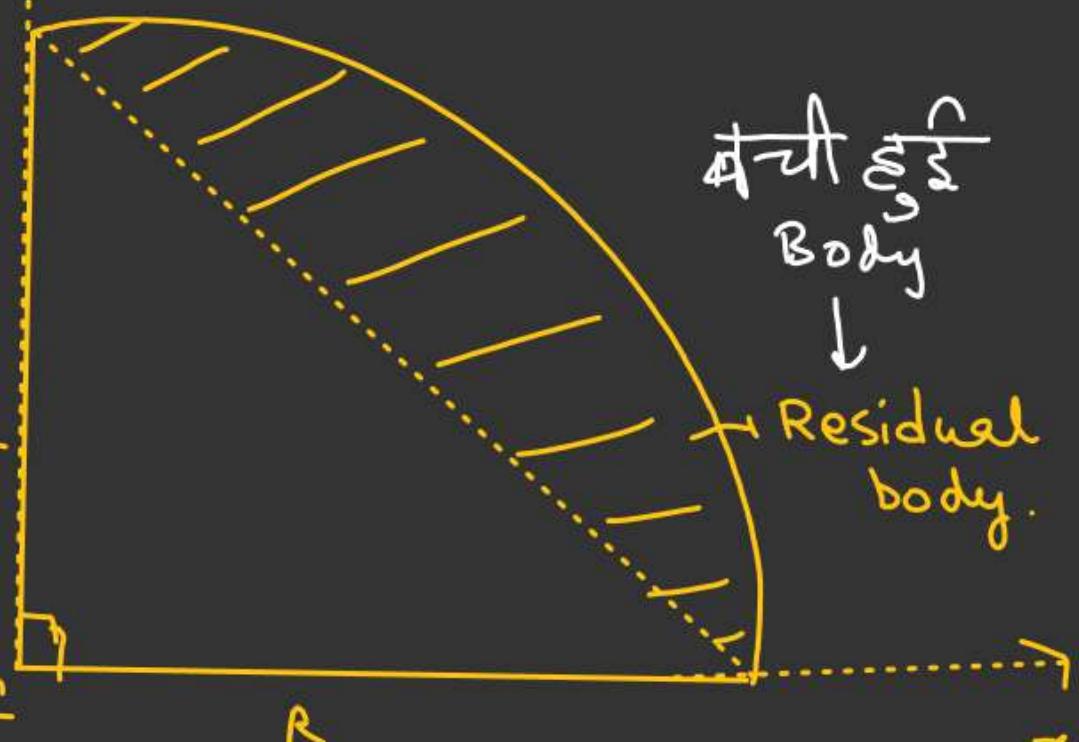
$$\gamma_1 = \frac{4R}{3} \frac{\sin(\pi/4)}{\pi/2} \quad \vec{\gamma}_1 = \frac{8R}{3\pi\sqrt{2}} (\cos 45^\circ \hat{i} + \sin 45^\circ \hat{j})$$

$$\gamma_1 = \left(\frac{8R}{3\pi} \times \frac{1}{\sqrt{2}} \right) \quad \vec{\gamma}_1 = \frac{4R}{3\pi} (\hat{i} + \hat{j})$$

$$A_1 = \frac{R^2}{2} \pi/2 = \frac{R^2}{4}\pi$$



$$\begin{aligned} \vec{\gamma}_2 &= \frac{R}{3} \hat{i} + \frac{R}{3} \hat{j} \\ A_2 &= \frac{1}{2} \times R \times R \\ &= \frac{R^2}{2} \end{aligned}$$



$$\begin{aligned} \bar{\gamma}_{\text{residual}} &= \frac{(A_1 \vec{\gamma}_1 - A_2 \vec{\gamma}_2)}{A_1 - A_2} \\ &= \frac{\left[\frac{\pi R^2}{4} \times \frac{4R}{3\pi} (\hat{i} + \hat{j}) \right] - \left[\frac{R^2}{2} \times \frac{R}{3} (\hat{i} + \hat{j}) \right]}{\left(\frac{\pi R^2}{4} - \frac{R^2}{2} \right)} \\ &\quad ?! \end{aligned}$$

H.W

$$\# M = \frac{\text{Mass of Residual body}}{\text{Volume of Residual body}}$$

A Solid Sphere of radius R_1 is cut from a Solid Sphere of radius R .

Find COM of Residual body.

$$\vec{X}_{\text{COM}} = \left(\frac{V_1 \vec{R}_1 - V_2 \vec{R}_2}{V_1 - V_2} \right)$$

$$\vec{R}_1 = \vec{0}$$

$$\vec{R}_2 = +\frac{R}{2} \hat{i}$$

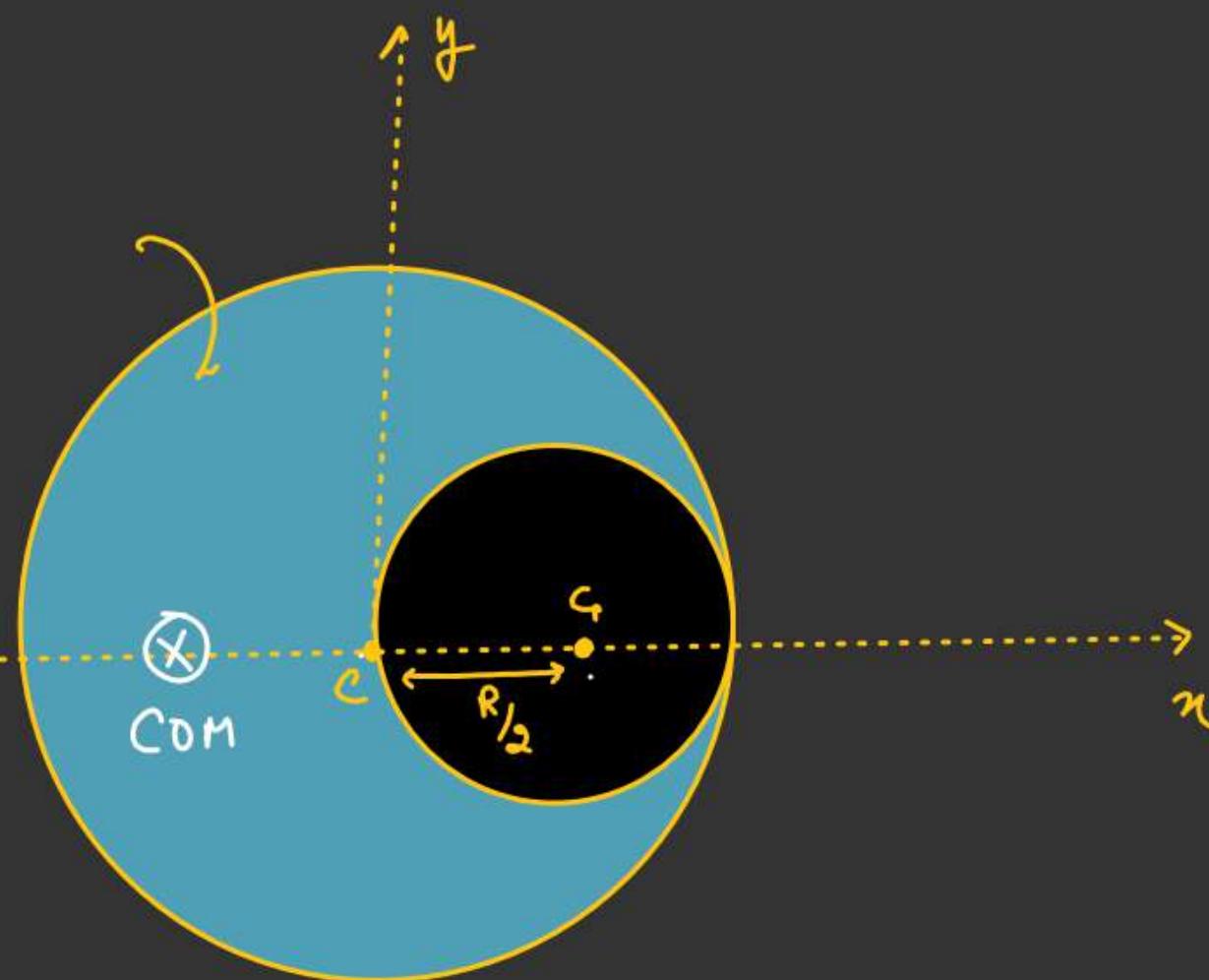
$$V_1 = \left(\frac{4}{3}\pi R^3\right)$$

$$V_2 = \frac{4}{3}\pi \left(\frac{R}{2}\right)^3$$

$$V_2 = \left(\frac{4}{3}\pi R^3\right) \frac{1}{8}$$

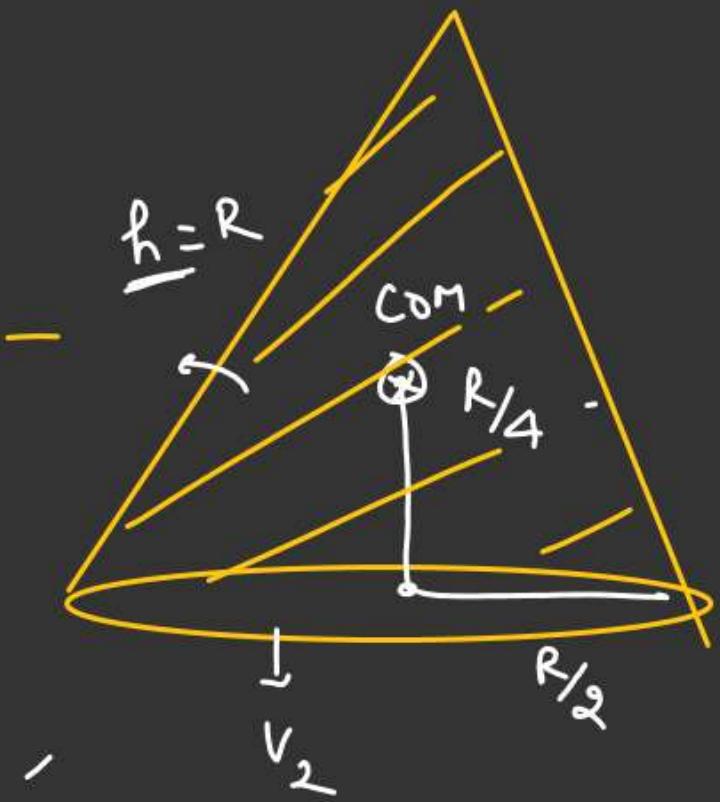
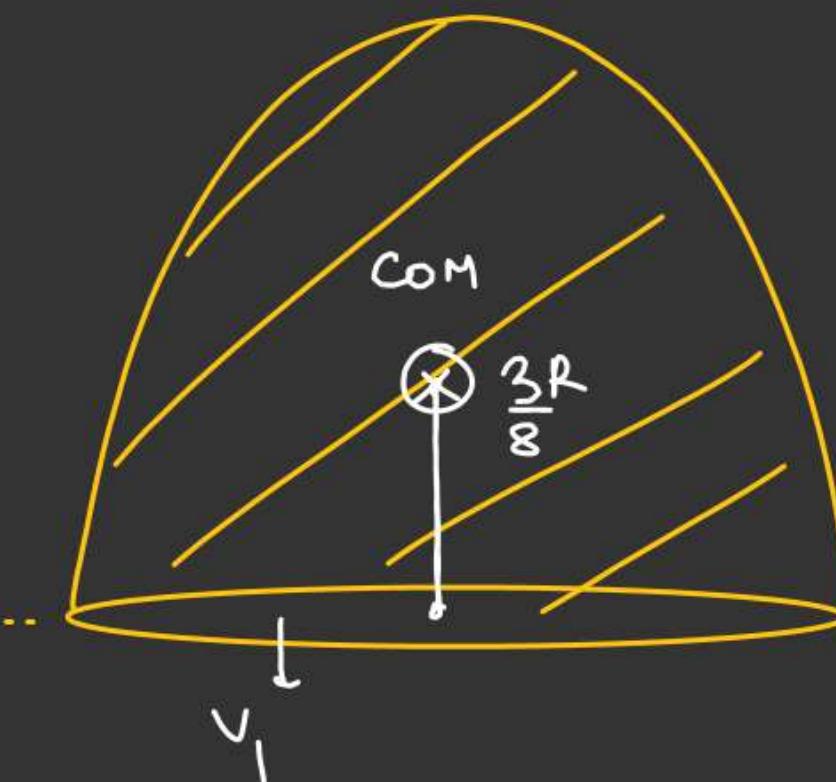
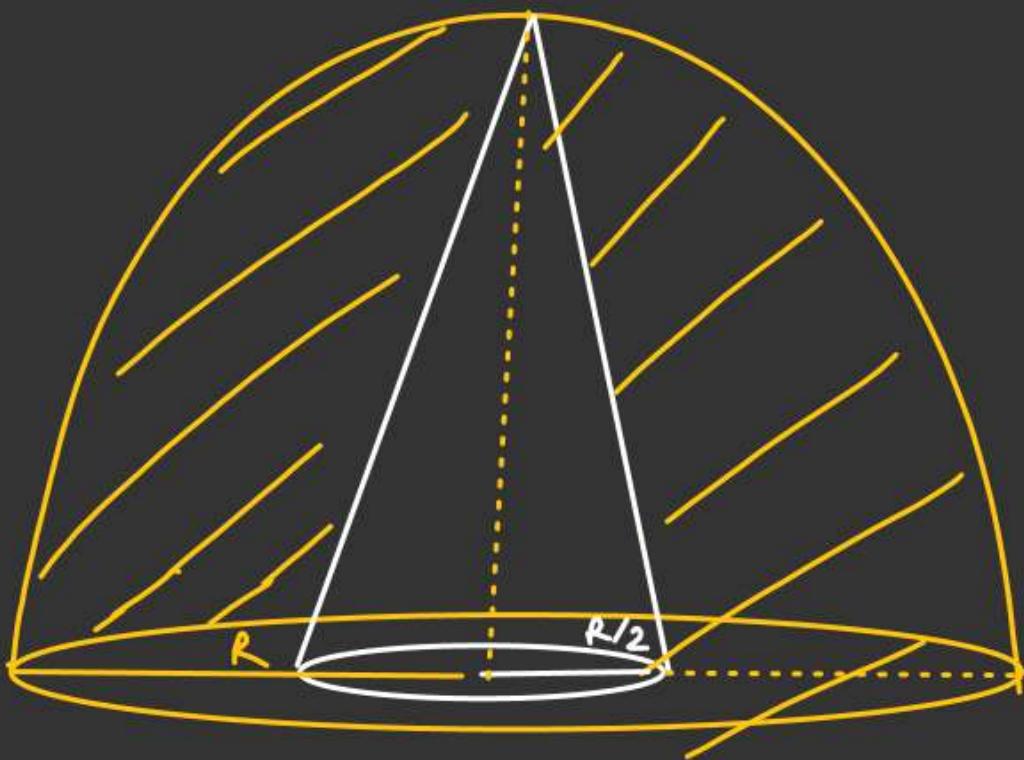
$$\vec{X}_{\text{COM}} = \frac{-\left(\frac{4}{3}\pi R^3\right) \frac{1}{8} \times \frac{R}{2} \hat{i}}{\frac{4}{3}\pi R^3 \left(1 - \frac{1}{8}\right)} = -\frac{R/16}{7/8} \hat{i}$$

$$\vec{X}_{\text{COM}} = -\frac{R/16}{7/8} \hat{i}$$



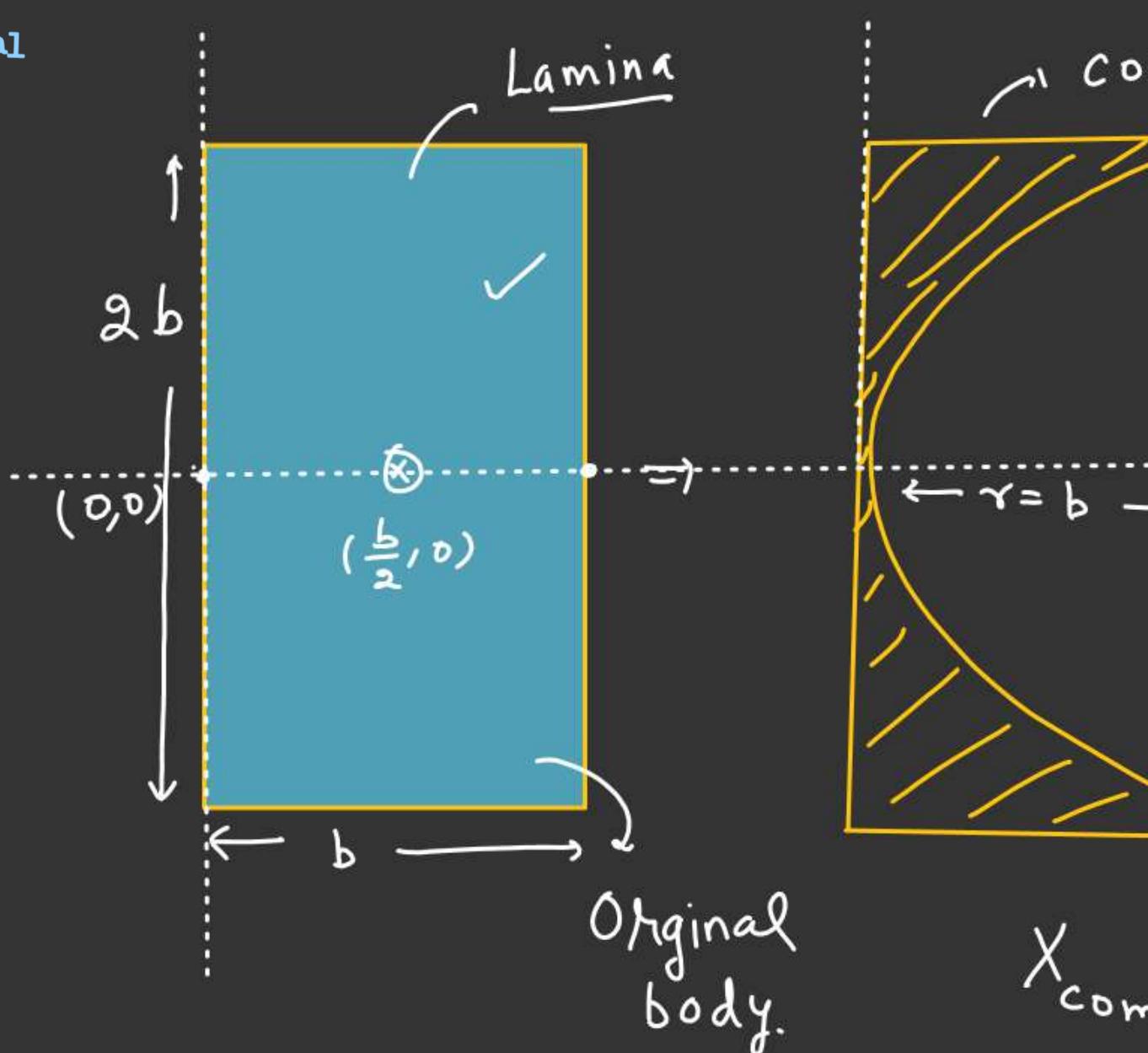
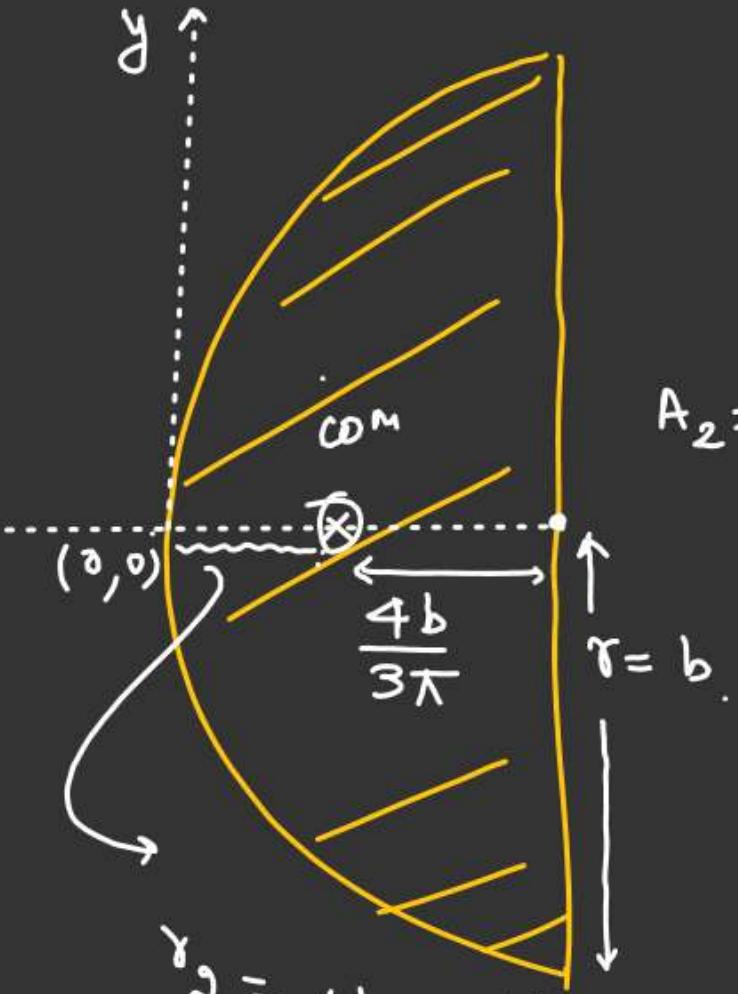
~~A&E~~

Find COM of remaining body if a cone of radius $R/2$ is cut from a Solid hemisphere.



$$\gamma = \frac{\left(\frac{2}{3}\pi R^3\right)\left(\frac{3R}{8}\right)}{\left[\frac{1}{3}\pi \left(\frac{R}{2}\right)^2 \times R\right] \times \left(\frac{R}{4}\right)} = \frac{\left[\frac{2}{3}\pi R^3 - \frac{1}{3}\left(\frac{R}{2}\right)^2 \times R\right]}{\left[\frac{33R}{56}\right]}$$

$\frac{33R}{56}$ → Check ??

 $COM = ??$ x_{com} 

$$x_{com} = \frac{\underline{A_1} \underline{r_1} + \underline{A_2} \underline{r_2}}{\underline{A_1} + \underline{A_2}}$$

$$= \frac{(2b)(b)\left(\frac{b}{2}\right) - \left(\frac{\pi b^2}{2}\right)\left(b - \frac{4b}{3\pi}\right)}{\left(2b^2 - \frac{\pi b^2}{2}\right)}$$



Motion of COM :

$$\vec{r}_{\text{com}} = \frac{(m_1 \vec{r}_1 + m_2 \vec{r}_2 + \dots + m_n \vec{r}_n)}{m_1 + m_2 + \dots + m_n}$$

Differentiating both side w.r.t time.

$$\frac{d(\vec{r}_{\text{com}})}{dt} = \frac{m_1 \frac{d(\vec{r}_1)}{dt} + m_2 \frac{d(\vec{r}_2)}{dt} + \dots + m_n \frac{d(\vec{r}_n)}{dt}}{(m_1 + m_2 + \dots + m_n)}$$

↓

$$\vec{v}_{\text{com}} = \frac{m_1 \vec{v}_1 + m_2 \vec{v}_2 + \dots + m_n \vec{v}_n}{(m_1 + m_2 + \dots + m_n)}$$

↓

$$\frac{d(\vec{v}_{\text{com}})}{dt} = \frac{m_1 \frac{d(\vec{v}_1)}{dt} + m_2 \frac{d(\vec{v}_2)}{dt} + \dots + m_n \frac{d(\vec{v}_n)}{dt}}{m_1 + m_2 + \dots + m_n}$$

$$\vec{a}_{\text{com}} = \frac{m_1 \vec{a}_1 + m_2 \vec{a}_2 + \dots + m_n \vec{a}_n}{m_1 + m_2 + \dots + m_n}$$

$$\vec{A}_{com} = \frac{m_1 \vec{a}_1 + m_2 \vec{a}_2 + \dots + m_n \vec{a}_n}{(m_1 + m_2 + \dots + m_n)}$$

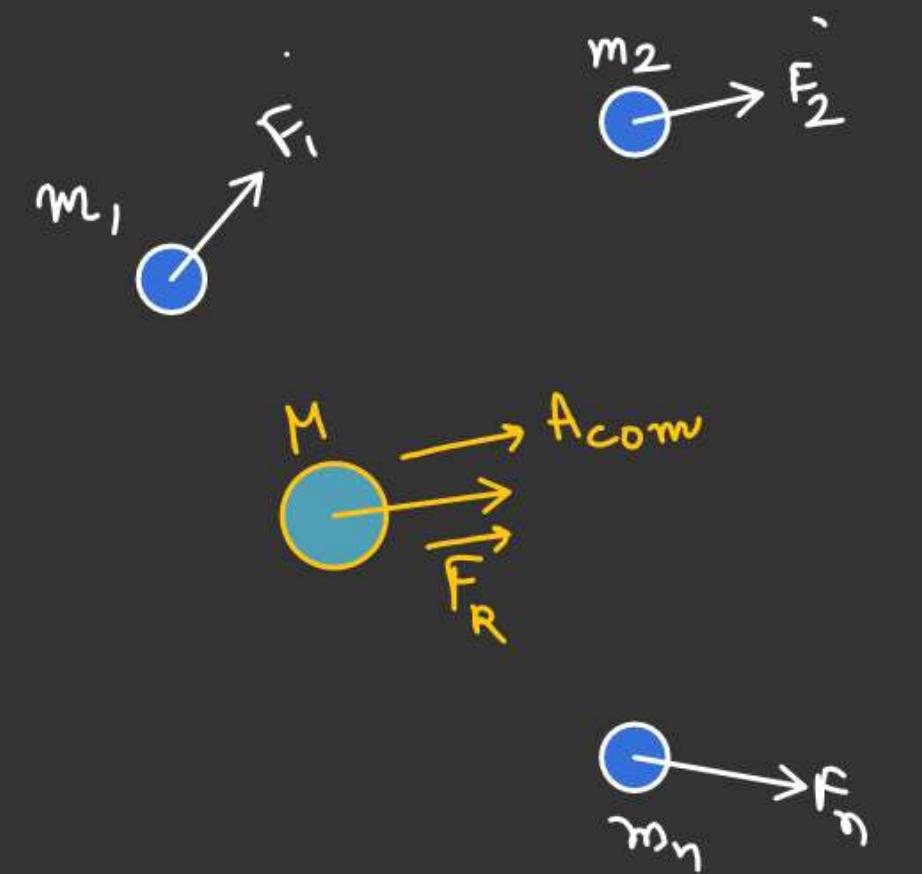
$$(m_1 + m_2 + \dots + m_n) \vec{A}_{com} = (m_1 \vec{a}_1 + m_2 \vec{a}_2 + \dots + m_n \vec{a}_n)$$

 \Downarrow \Downarrow

$$M \vec{A}_{com} = \left(\underbrace{\vec{F}_1 + \vec{F}_2 + \dots + \vec{F}_n}_{\text{ }} \right)$$

 \Downarrow

$$M \vec{A}_{com} = \vec{F}_R$$

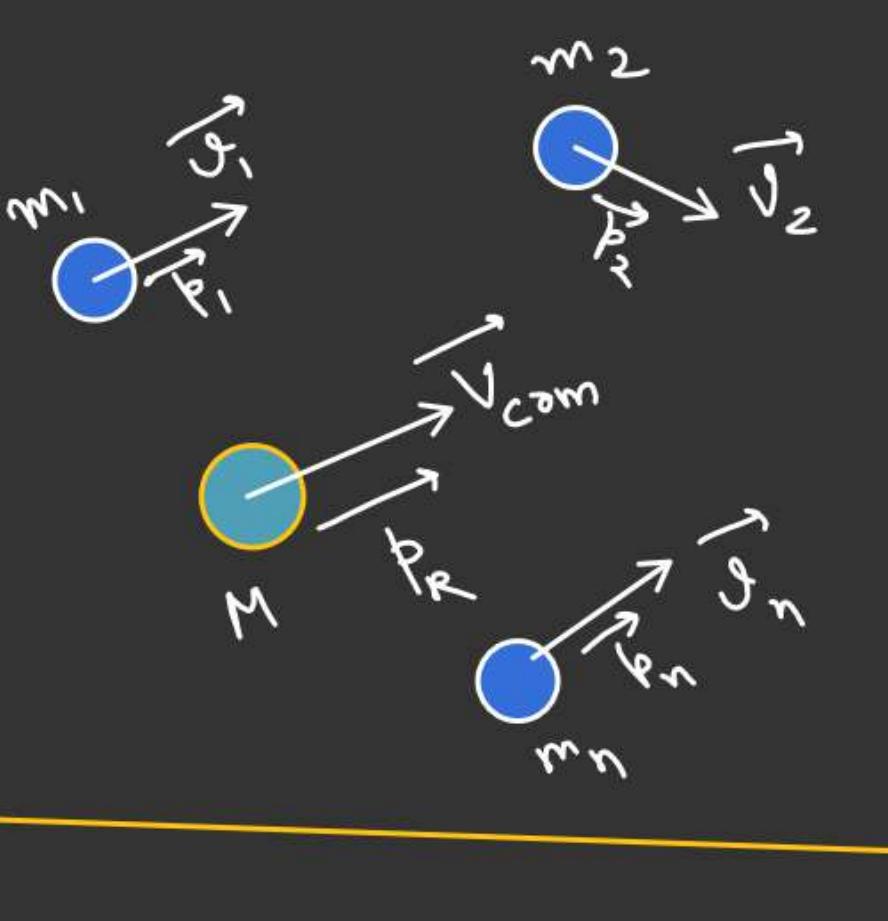


$$\vec{V}_{\text{com}} = \frac{m_1 \vec{v}_1 + m_2 \vec{v}_2 + \dots + m_n \vec{v}_n}{(m_1 + m_2 + \dots + m_n)}$$

$$(m_1 + m_2 + \dots + m_n) \vec{V}_{\text{com}} = m_1 \vec{v}_1 + m_2 \vec{v}_2 + \dots + m_n \vec{v}_n$$

$$= \left(\vec{p}_1 + \vec{p}_2 + \dots + \vec{p}_n \right)$$

$$\Downarrow \vec{p}_R \text{ or } \vec{p}_{\text{net}}$$



~~AA:~~

$$\text{If } \vec{F}_R = 0 \cdot \checkmark \quad F_R = \text{Resultant force.}$$

Net Force

$$\rightarrow M \vec{A}_{com} = 0.$$

$$\rightarrow \vec{A}_{com} = 0 \checkmark$$

$$\frac{d \vec{V}_{com}}{dt} = 0$$

$$(\vec{V}_{com})_i = (\vec{V}_{com})_f$$

$$M(\vec{V}_{com})_i = M(\vec{V}_{com})_f$$

$M = \text{Total Mass of the System}$

$$\vec{F}_R = 0, \vec{A}_{com} = 0.$$

$$(\vec{V}_{com})_i = (\vec{V}_{com})_f$$

$$\text{If } (\vec{V}_{com})_i = 0 \Rightarrow (\vec{V}_{com})_f = 0.$$

\Downarrow

$$\frac{d(\vec{R}_{com})}{dt} = 0$$

$$(\vec{R}_{com})_i = (\vec{R}_{com})_f$$

$$\Rightarrow \vec{R}_{com} = 0.$$

$$(\vec{p}_i)_{\text{system}} = (\vec{p}_f)_{\text{system}}$$

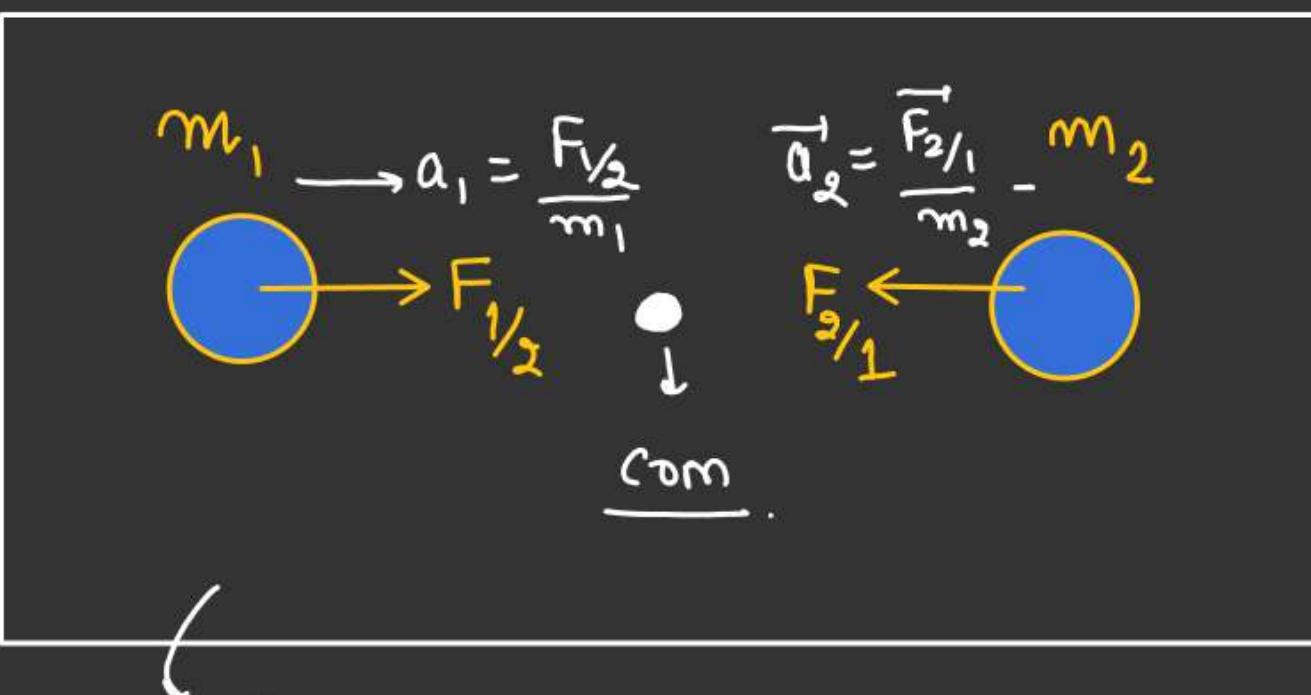
Momentum of the System Conserved.

~~Ans~~:

F = Gravitational force
of interaction
b/w two particle.

$$\vec{F}_{1/2} = -\vec{F}_{2/1}$$

$$|\vec{F}_{1/2}| = |\vec{F}_{2/1}| = \frac{Gm_1m_2}{r^2}$$



System boundary.

$$\vec{A}_{com} = \frac{\underline{m}_1 \vec{a}_1 + \underline{m}_2 \vec{a}_2}{\underline{m}_1 + \underline{m}_2}$$

$$\vec{A}_{com} = \frac{\underline{m}_1 \left(\frac{\vec{F}_{1/2}}{\underline{m}_1} \right) + \underline{m}_2 \left(\frac{\vec{F}_{2/1}}{\underline{m}_2} \right)}{\underline{m}_1 + \underline{m}_2} = \frac{\vec{F}_{1/2} + \vec{F}_{2/1}}{\underline{m}_1 + \underline{m}_2} = 0$$

$$F_{net} = 0 \rightarrow A_{com} = 0$$

$$\vec{v}_{com} = \text{constant}$$

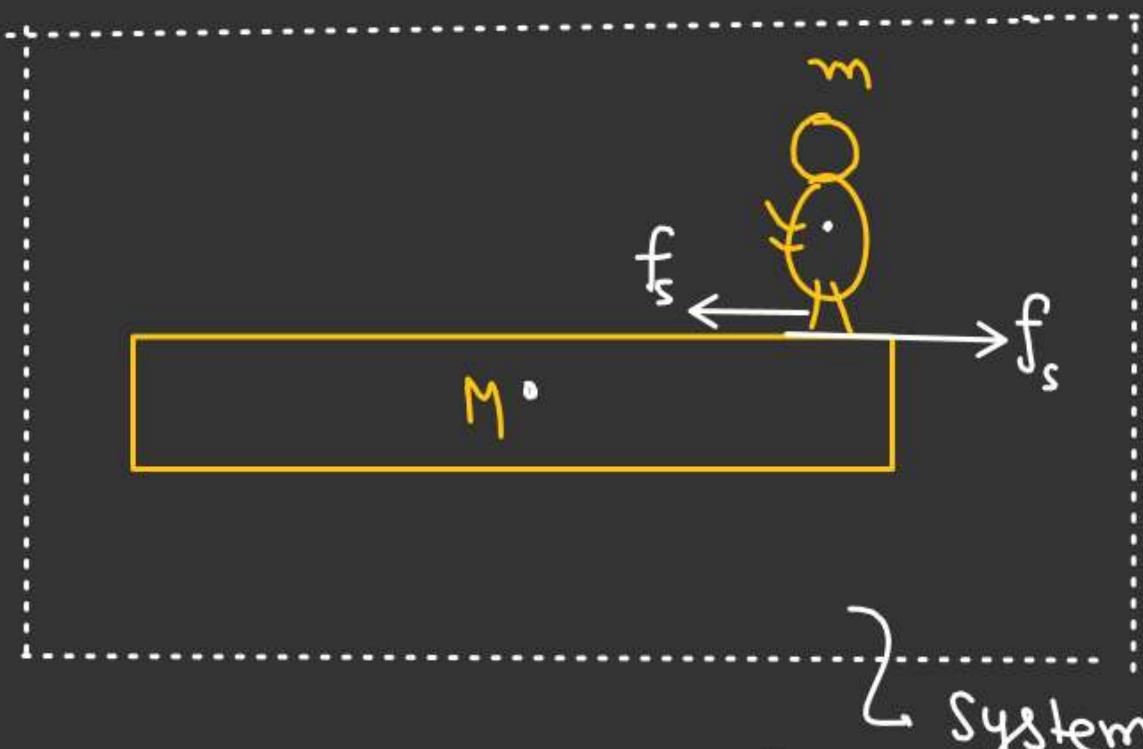
$$\vec{P}_{com} = \text{constant}$$

$$\text{As } (\vec{F}_{1/2} = -\vec{F}_{2/1})$$

MAN - PLANK SYSTEM

Man starts walking on the plank.

Find displacement of plank when reaches the other end of the plank.



Smooth

(0, 0)

(0, 0)

In x-direction

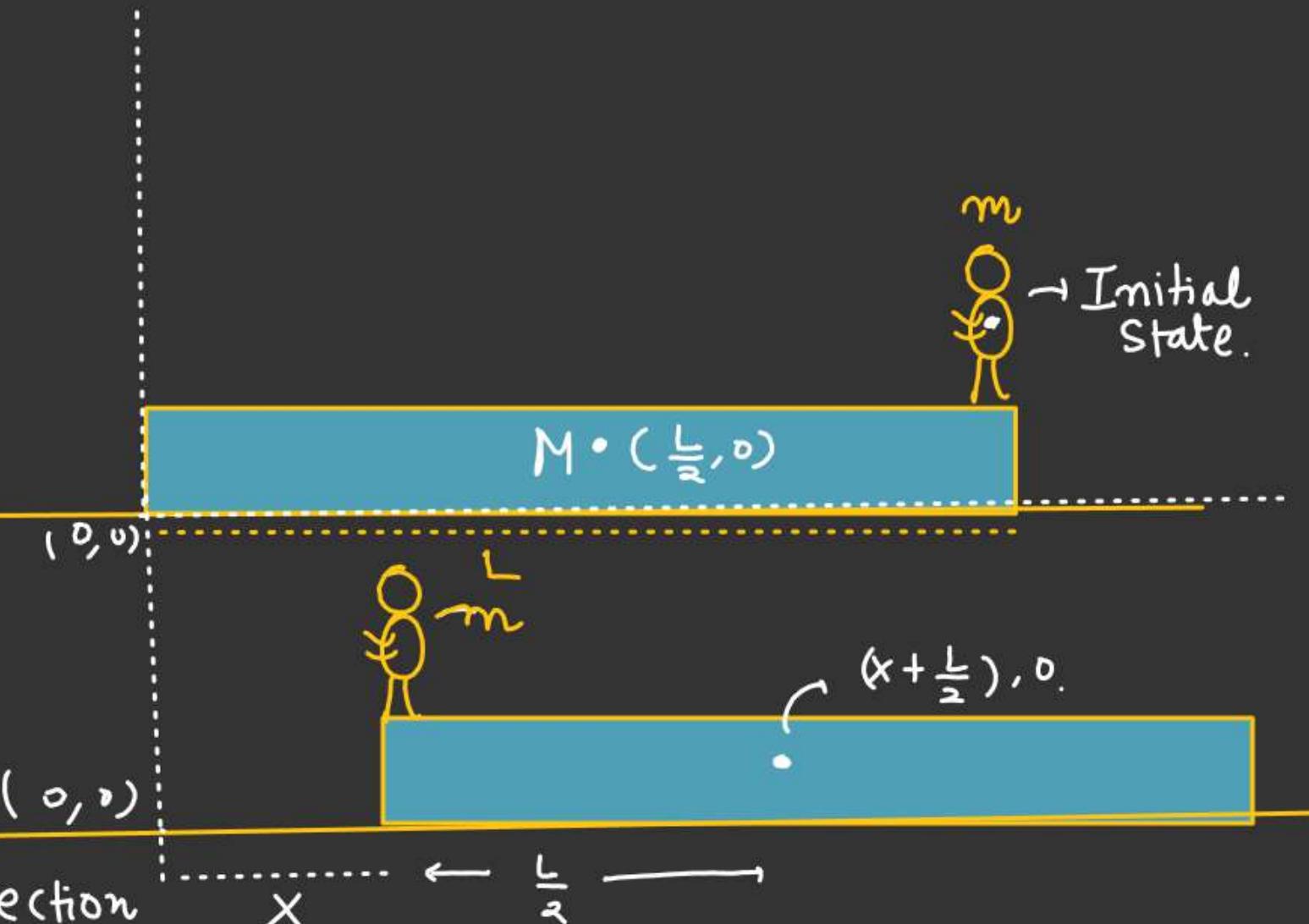
$$F_{\text{net}} = 0.$$

$$A_{\text{com}} = 0.$$

$$V_{\text{com}} = 0.$$

$$\Rightarrow (R_{\text{com}})_i = (R_{\text{com}})_f$$

$$\Rightarrow (X_{\text{com}})_i = (X_{\text{com}})_f$$



m → Initial State.

In x-direction

$$\left. \begin{array}{l} F_{\text{net}} = 0 \\ A_{\text{com}} = 0 \\ V_{\text{com}} = 0 \end{array} \right\} \Rightarrow$$

$$\begin{aligned} (R_{\text{com}})_i &= (R_{\text{com}})_f \\ (X_{\text{com}})_i &= (X_{\text{com}})_f \end{aligned}$$

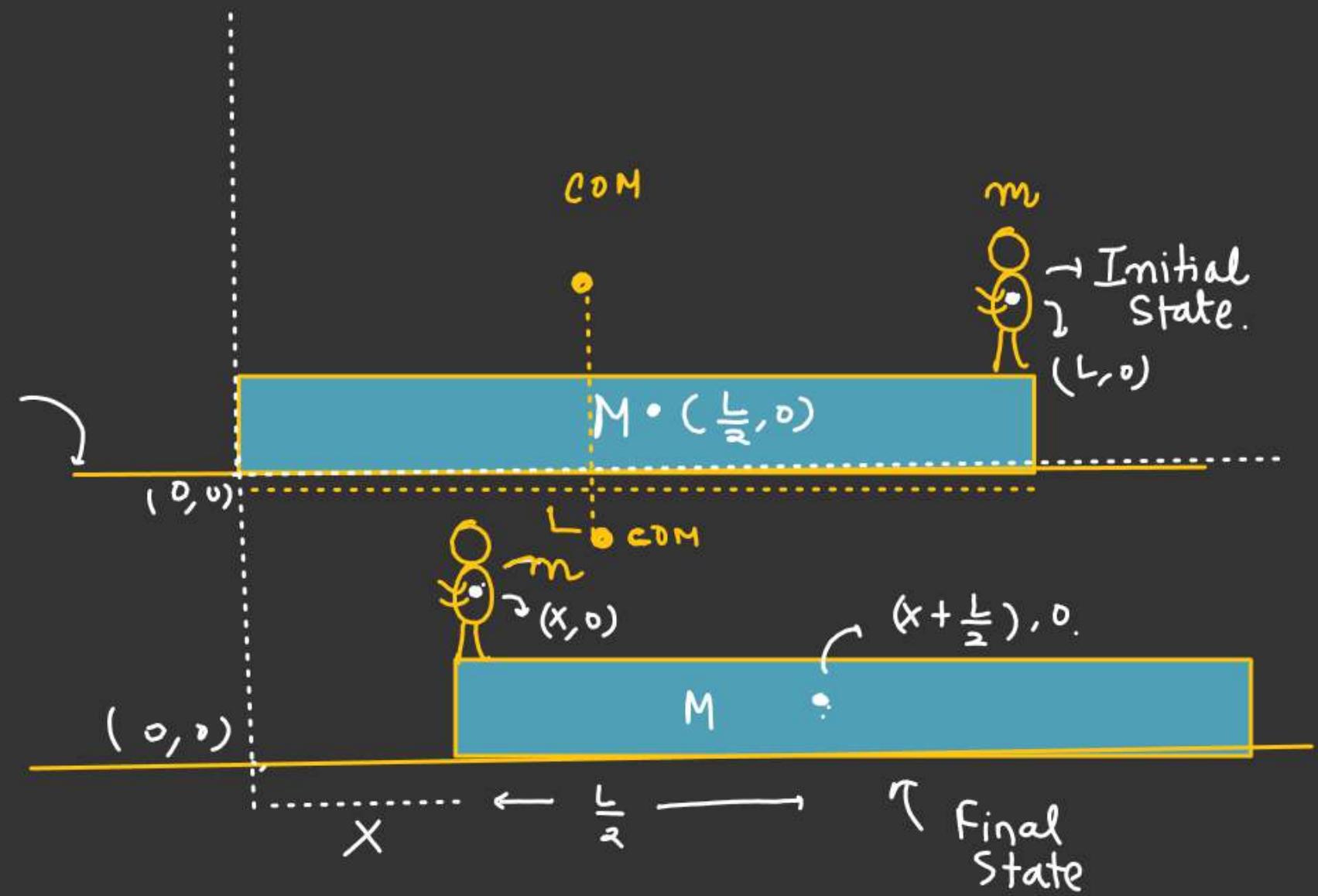
$$(X_{\text{com}})_i = \frac{M\left(\frac{L}{2}\right) + (mL)}{M+m}$$

$$(X_{\text{com}})_f = \frac{mx + M\left(\frac{L}{2} + x\right)}{M+m}$$

$$(X_{\text{com}})_i = (X_{\text{com}})_f$$

$$\cancel{M\left(\frac{L}{2}\right) + mL} = mx + \cancel{M\frac{L}{2}} + Nx$$

$$\frac{mL}{M+m} = x \quad \checkmark$$



H-W

D.P.P \rightarrow (1)

L Module questions. Mention in D.P.P - 1.

H-C-V Page-No (159 \rightarrow 160)

Q.No \rightarrow 1 to 11 ✓ .