Formula

(i) $\sin(A+B) = \sin A \cos B + \sin B \cos A$

$\sin(A-B) = \sin A \cos B - \sin B \cos A$

$\cos(A+B) = \cos A \cos B - \sin A \sin B$

$\cos(A-B) = \cos A \cos B + \sin A \sin B$

$\sin(A+B)\sin(A-B) = \sin^2 A - \sin^2 B$

$\cos(A+B)\cos(A-B) = \cos^2 A - \sin^2 B$

(ii) $\sin\frac{\pi}{12} = \sin 15° = \frac{\sqrt{3}-1}{2\sqrt{2}} = \cos 75° = \cos\frac{5\pi}{12}$

$\cos\frac{\pi}{12} = \cos 15° = \frac{\sqrt{3}+1}{2\sqrt{2}} = \sin 75° = \sin\frac{5\pi}{12}$

$\tan\frac{\pi}{12} = \tan 15° = 2-\sqrt{3} = \cot 75° = \cot\frac{5\pi}{12}$

$\tan 75° = 2+\sqrt{3} = \cot 15°$

(iii) $2\sin A \cos B = \sin(A+B) + \sin(A-B)$

$2\cos A \cos B = \cos(A+B) + \cos(A-B)$

$2\sin A \sin B = \cos(A-B) - \cos(A+B)$

$\sin C + \sin D = 2\sin\frac{C+D}{2}\cos\frac{C-D}{2}$

$\sin C - \sin D = 2\sin\frac{C-D}{2}\cos\frac{C+D}{2}$

$\cos C + \cos D = 2\cos\frac{C+D}{2}\cos\frac{C-D}{2}$

$\cos C - \cos D = 2\sin\frac{D-C}{2}\sin\frac{C+D}{2}$

$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$

$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$

$\cot(A+B) = \frac{\cot A \cot B - 1}{\cot A + \cot B}$

$\cot(A-B) = \frac{\cot A \cot B + 1}{\cot B - \cot A}$

$\tan(A+B) - \tan A - \tan B = \tan(A+B)\tan A \tan B$

(iv) $\tan A + \tan B = \frac{\sin(A+B)}{\cos A \cos B}$

$\tan A - \tan B = \frac{\sin(A-B)}{\cos A \cos B}$

$\cot A + \cot B = \frac{\sin(A+B)}{\sin A \sin B}$

$\cot A - \cot B = \frac{\sin(B-A)}{\sin A \sin B}$

(v) $\tan(\theta_1 + \theta_2 + \theta_3 + \cdots + \theta_n) = \frac{s_1 - s_3 + s_5 - s_7 + \cdots}{1 - s_2 + s_4 - s_6 + \cdots}$

$s_1 = \sum \tan \theta_1$

$s_2 = \sum \tan \theta_1 \tan \theta_2$

$s_3 = \sum \tan \theta_1 \tan \theta_2 \tan \theta_3$

(vi) $\sin 2\theta = 2 \sin \theta \cos \theta = \frac{2 \tan \theta}{1 + \tan^2 \theta}$

(vii) $\cos 2\theta = \cos^2 \theta - \sin^2 \theta$

$= 2\cos^2 \theta - 1 = 1 - 2\sin^2 \theta = \frac{1 - \tan^2 \theta}{1 + \tan^2 \theta}$

(viii) $1 + \cos 2\theta = 2 \cos^2 \theta$

$1 - \cos 2\theta = 2 \sin^2 \theta$

$1 + \sin 2\theta = (\cos \theta + \sin \theta)^2$

$1 - \sin 2\theta = (\cos \theta - \sin \theta)^2$

(ix) $\tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta}$

(x) $\sin 3\theta = 3 \sin \theta - 4 \sin^3 \theta$

$\cos 3\theta = 4 \cos^3 \theta - 3 \cos \theta$

$\tan 3\theta = \frac{3 \tan \theta - \tan^3 \theta}{1 - 3 \tan^2 \theta}$

(xi) $\sin^4 \theta + \cos^4 \theta = 1 - \frac{1}{2} \sin^2 2\theta$

$\sin^6 \theta + \cos^6 \theta = 1 - \frac{3}{4} \sin^2 2\theta$

$\cos \theta \cos 2\theta \cos 2^2\theta \cos 2^3\theta \cdots\cdots \cos 2^{n-1}\theta$

$= \frac{\sin 2^n\theta}{2^n \sin \theta}$

(xii) $\sin\theta \sin\left(\frac{\pi}{3}-\theta\right)\sin\left(\frac{\pi}{3}+\theta\right) = \frac{1}{4}\sin 3\theta$

$\cos\theta \cos\left(\frac{\pi}{3}-\theta\right)\cos\left(\frac{\pi}{3}+\theta\right) = \frac{1}{4}\cos 3\theta$

$\tan\theta \tan\left(\frac{\pi}{3}-\theta\right)\tan\left(\frac{\pi}{3}+\theta\right) = \tan 3\theta$

Conditional Identities

if $\tan A + \tan B + \tan C = \tan A \tan B \tan C$

$\Rightarrow\ A + B + C = n\pi, n \in I.$

(i) If $A + B + C = \pi$, then

$\tan A + \tan B + \tan C = \tan A \tan B \tan C$

$\cot A \cot B + \cot B \cot C + \cot C \cot A = 1$

$\tan\frac{A}{2}\tan\frac{B}{2} + \tan\frac{B}{2}\tan\frac{C}{2} + \tan\frac{C}{2}\tan\frac{A}{2} = 1$

$\cot\frac{A}{2} + \cot\frac{B}{2} + \cot\frac{C}{2} = \cot\frac{A}{2}\cot\frac{B}{2}\cot\frac{C}{2}$

(ii) $\sin 18^\circ = \sin\frac{\pi}{10} = \frac{\sqrt{5}-1}{4} = \cos 72^\circ = \cos\frac{2\pi}{5}$

$\cos 36^\circ = \cos\frac{\pi}{5} = \frac{\sqrt{5}+1}{4} = \sin 54^\circ = \sin\frac{3\pi}{10}$

$\tan 22.5^\circ = \tan\frac{\pi}{8} = \sqrt{2}-1 = \cot\frac{3\pi}{8} = \cot(67\cdot 5^\circ)$

$\tan\ 67.5^\circ = \tan\ \frac{3\pi}{8} = \sqrt{2}+1 = \cot\ \frac{\pi}{8} = \cot\ 22.5^\circ$

(iii) $\sin\alpha + \sin(\alpha+\beta) + \sin(\alpha+2\beta) + \sin(\alpha+3\beta) + \cdots \sin(\alpha+(n-1)\beta)$

$= \frac{\sin\left(\frac{n\beta}{2}\right)}{\sin\left(\frac{\beta}{2}\right)}\sin\left(\frac{2\alpha+(n-1)\beta}{2}\right)$

$\cos\alpha + \cos(\alpha+\beta) + \cos(\alpha+2\beta) + \cos(\alpha+3\beta) + \cdots + \cos(\alpha+(n-1)\beta)$

$= \frac{\sin\left(\frac{n\beta}{2}\right)}{\sin\left(\frac{\beta}{2}\right)}\cos\left(\frac{2\alpha+(n-1)\beta}{2}\right)$