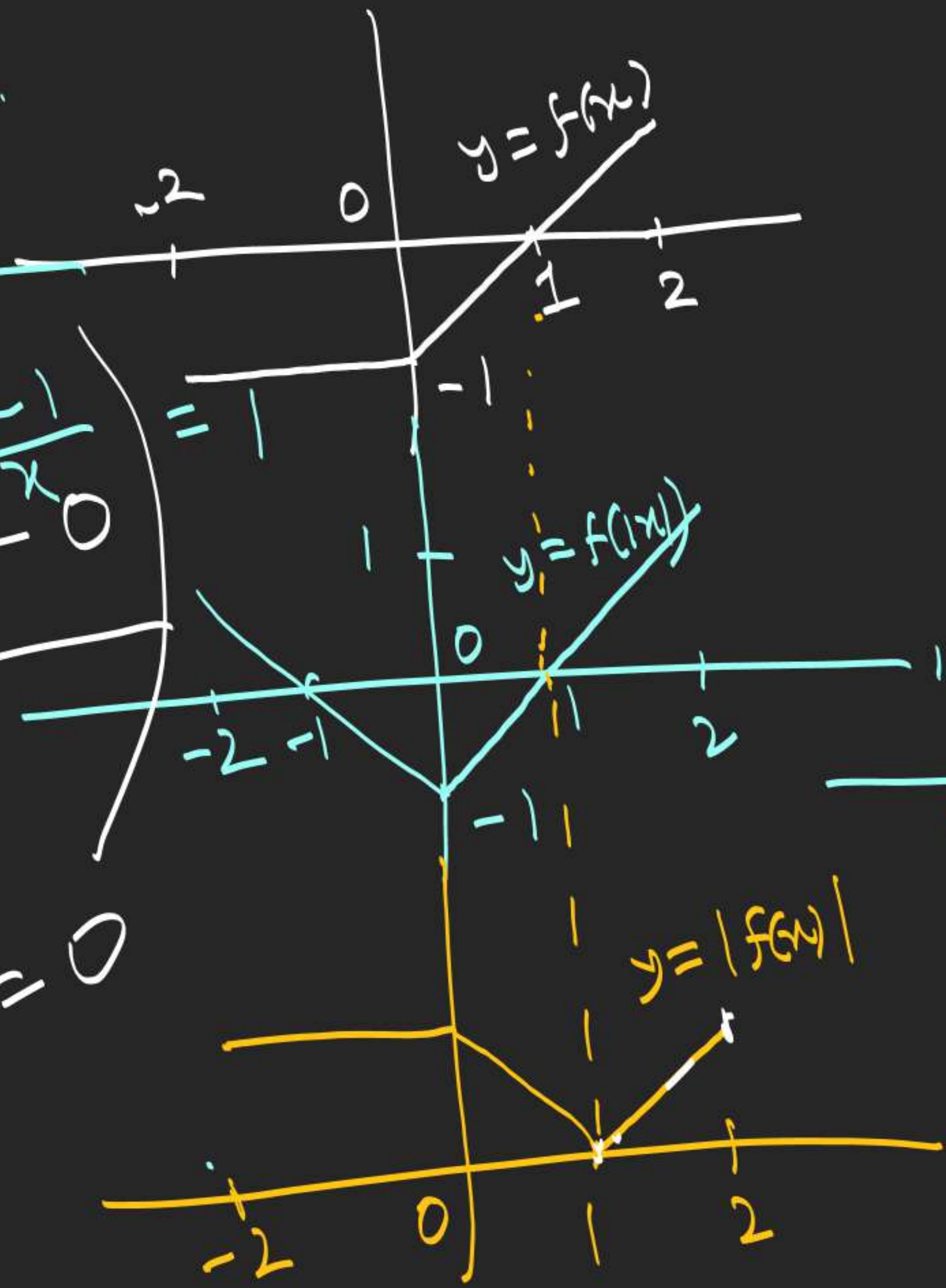
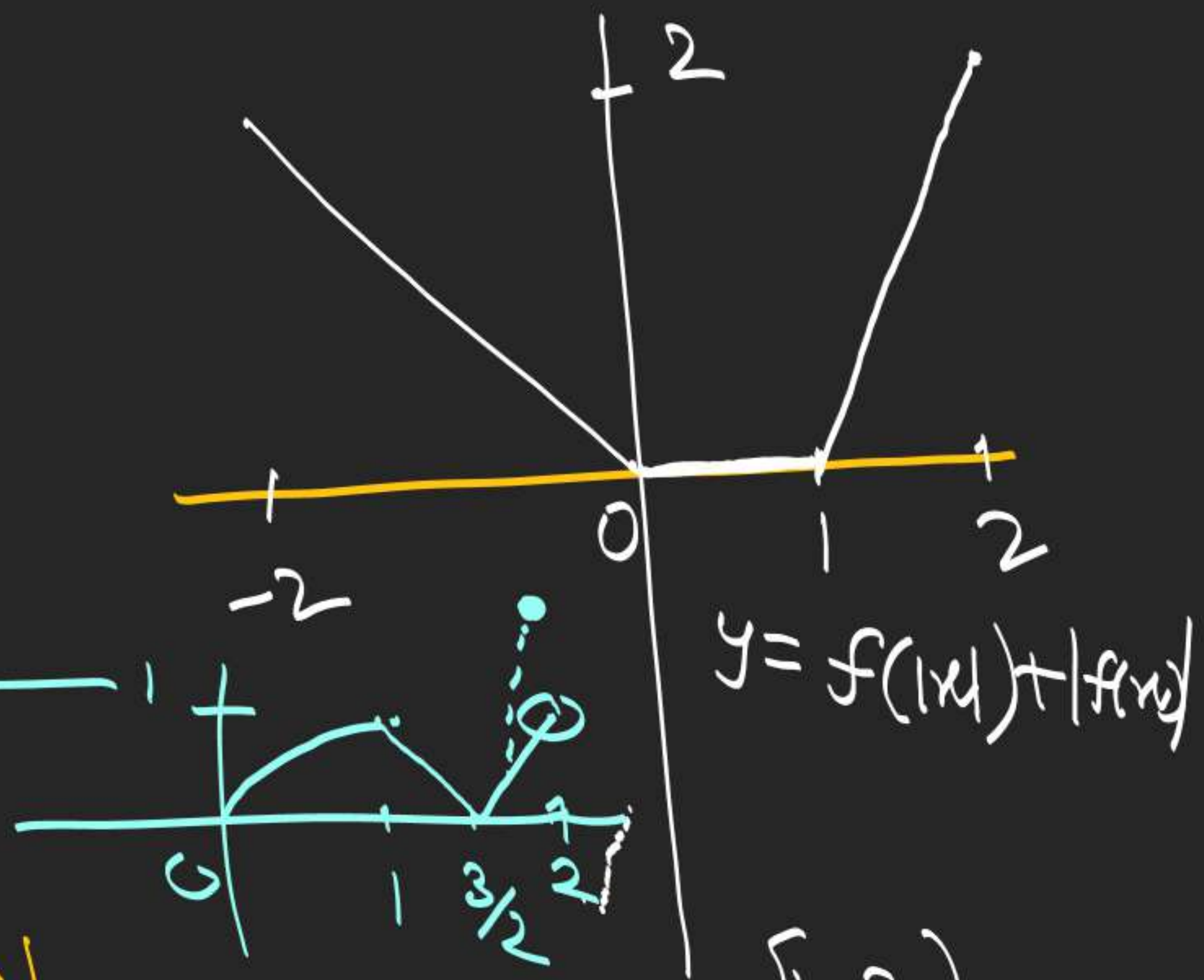
$$\lim_{x\to 0^+} x\left(\frac{e^{\frac{1}{|x|}}-2}{\frac{1}{|x|}}\right)+1 \to 0$$

$$\lim_{x\to -1^-} \frac{\cos^{-1}\operatorname{sgn}\left(\frac{-4}{3x+2}\right)}{x+1}$$

$$\lim_{x\to 0^+}\frac{e^x-1}{x}-0=1$$

$$\lim_{x\to -1^-}\frac{0}{x+1}=0$$





$$\begin{cases} |2x-3| & x\in[1,2) \\ 2|2x-3| & \{2\} \\ \sin\frac{\pi x}{2} & 0\le x<1 \end{cases}$$

Nishant Jindal

$b = 0$

$\cancel{2} = 9p + 3q + \cancel{2}$

$1 = 6p + q$

$3p + q = 0$

$p = ?$

$q = ?$

$$\lim_{x \to 1^-} \frac{ax(x-1)}{x-1} = a$$

$\boxed{a \neq 1}$

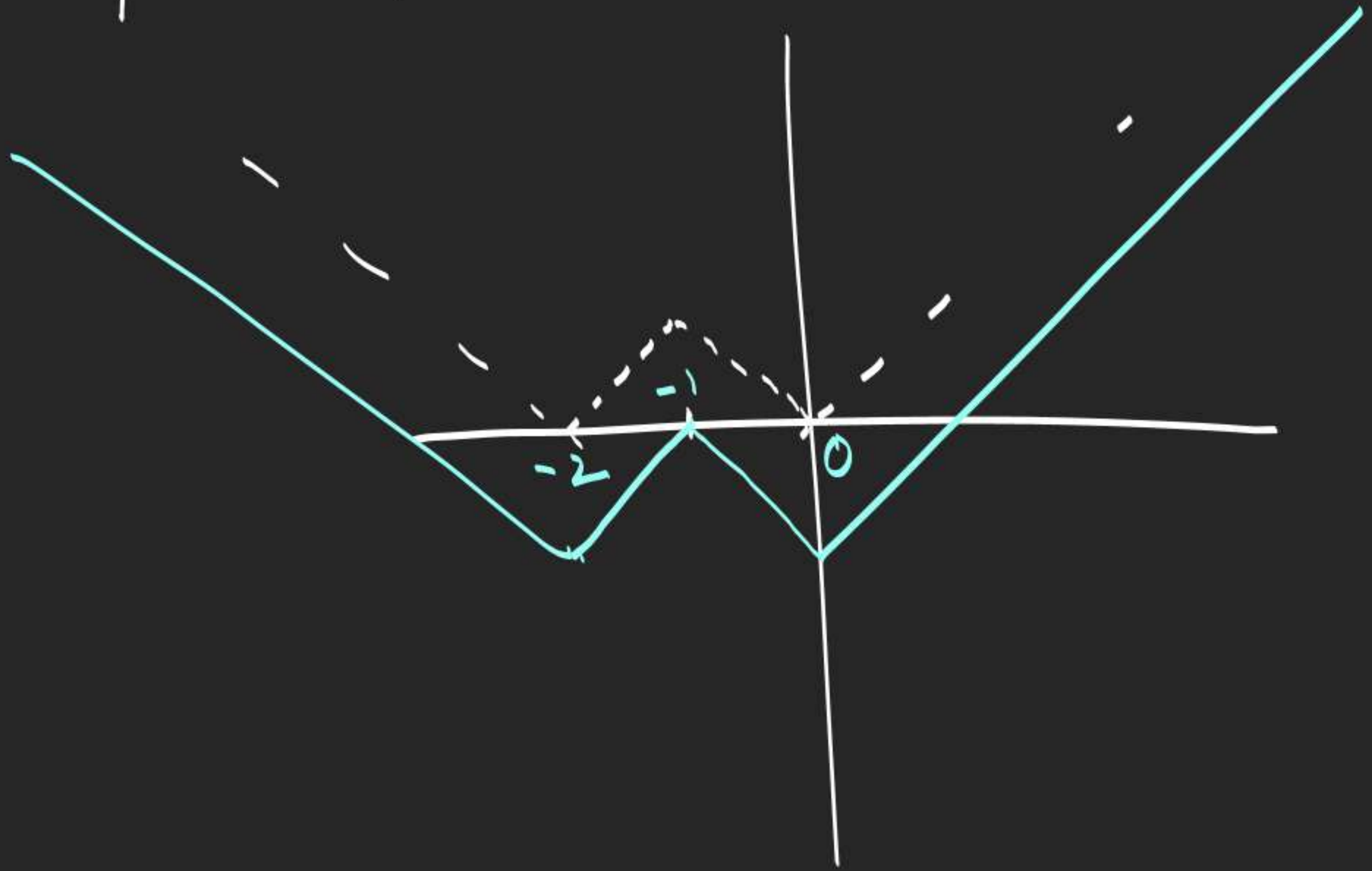
10. $f'(x) = \lim_{h \to 0} \left(\frac{f(h) + 2xh + e^x e^h - e^x - e^h}{h} \right)$

$= \lim_{h \to 0} \left(2x + \left(\frac{f(h)}{h} \right) - \frac{(e^h - 1)}{h} + \frac{e^x(e^h - 1)}{h} \right)$

$\boxed{f'(x) = 2x + f'(0) - 1 + e^x}$

Nishant Jindal

$$f(g(x)) = -1 + \left| \underbrace{g(x) - 1} \right| = -1 + \Big|\, \big| 1 - |x+1| \big| \,\Big|$$



Nishant Jindal

$$x^2\left|\cos\frac{\pi}{2x}\right|$$

$$\frac{\pi}{2x}=(2n+1)\frac{\pi}{2}$$

$$\boxed{x=\frac{1}{2n+1},\ n\in I}$$

$$LHD=\lim_{h\to 0}\frac{\left(\frac{1}{2n+1}-h\right)^2\left|\cos\frac{\pi}{2\left(\frac{1}{2n+1}-h\right)}\right|-0}{-h}$$

$$\frac{(2n+1)\pi}{2(1-(2n+1)h)}$$

$$\lim_{h\to 0}\left(\frac{1}{2n+1}-h\right)^2\left|\frac{\sin\left(\frac{(2n+1)\frac{\pi}{2}h}{1-(2n+1)h}\right)}{\frac{(2n+1)^2\frac{\pi}{2}h}{1-(2n+1)h}}\right|\frac{(2n+1)^2\frac{\pi}{2}(h)}{(1-(2n+1)h)(-h)}$$

$$=-\frac{\pi}{2}$$

Nishant Jindal

$$f'(x) = \lim_{h\to 0} \frac{\left(f\left(h^{1/n}\right)\right)^n}{h} = \lim_{h\to 0}\left(\frac{f(h^{1/n}) - f(0)}{h^{1/n}}\right)^n = \left(f'(0)\right)^n$$

$$\boxed{f(0)=0}$$

$$f'(x) = \left(f'(0)\right)^n$$

$$f'(0) = \left(f'(0)\right)^n \Rightarrow f'(0) = 0,\ \pm 1$$

$$\boxed{f'(0) = 0, 1}$$

$$\boxed{f'(y) = \frac{1}{y} + 1 =}\ g(y)$$

If $f'(0)=0$ $\quad f'(x) = 0 \Rightarrow f(x) = \text{const.}$

$$\boxed{f(x) = 0} \times$$

$$\boxed{f'(0) = 1} \quad f'(x) = 1 \quad f(x) = x + \cancel{c}^{\,0}$$

$$\boxed{f(x) = x}$$

Nishant Jindal

$x \neq y$

$x > y \quad \lim_{x \to y^+} \frac{f(x) - f(y)}{x - y} \geq \lim_{x \to y^+} \frac{\ln x - \ln y}{x - y} + 1$

$\underline{\underline{f'(y^+)}} \geq \frac{1}{y} + 1$

$$\boxed{f'(y) = 1 + \frac{1}{y}}$$

$f(x) \geq g(x)$

$x < y \quad \lim_{x \to y^-} \frac{f(x) - f(y)}{x - y} \leq \lim_{x \to y^-} \frac{\ln x - \ln y}{x - y} + 1$

$\underline{\underline{f'(y^-)}} \leq \frac{1}{y} + 1$

Nishant Jindal

1. $y = (\sin x)\, e^{\sqrt{\sin x}}\, (\ln x)\, (x^x)\, (x^{\cos^{-1} x})$, find $\frac{dy}{dx}$.

$$\frac{y'}{y} = \cot x + \frac{\cos x}{2\sqrt{\sin x}} + \frac{1}{x \ln x} + (1 + \ln x) + \left(\frac{\cos^{-1} x}{x} - \frac{\ln x}{\sqrt{1-x^2}}\right)$$

$$\frac{dy}{dx} = \sin x\, e^{\sqrt{\sin x}} \ln x\, (x^x)\, x^{\cos^{-1} x} \left(\cot x + \frac{\cos x}{2\sqrt{\sin x}} + \frac{1}{x \ln x} + 1 + \ln x + \frac{\cos^{-1} x}{x} - \frac{\ln x}{\sqrt{1-x^2}}\right)$$

$$\ln y = \ln \sin x + \sqrt{\sin x} + \ln(\ln x) + x \ln x + \cos^{-1} x \ln x$$

Nishant Jindal

Q. $y = 2^{\log_2\left(x^{2x}\right)} + \left(\tan\frac{\pi x}{4}\right)^{\frac{4}{\pi x}}$, find $\frac{dy}{dx}$ at $x=1$.

$$y = x^{2x} + \left(\tan\frac{\pi x}{4}\right)^{\frac{4}{\pi x}} = e^{2x\ln x} + e^{\frac{4}{\pi x}\ln\tan\frac{\pi x}{4}}$$

$$y' = x^{2x}\left(2+2\ln x\right) + \left(\tan\frac{\pi x}{4}\right)^{\frac{4}{\pi x}}\left(-\frac{4}{\pi x^2}\ln\tan\frac{\pi x}{4} + \frac{\cancel{4}}{\cancel{\pi} x}\,\frac{\frac{\cancel{\pi}}{\cancel{4}}\sec^2\frac{\pi x}{4}}{\tan\frac{\pi x}{4}}\right)$$

$$y'\Big|_{x=1} = 2 + 1\left(0+2\right)$$

$$\therefore \boxed{4}$$

Nishant Jindal

3:

$$D\left(x^{x^{x^4}}\right) = x^{x^{x^4}}\left(\frac{x^{x^4}}{x} + \ln x \, D\left(x^{x^4}\right)\right)$$

$$= x^{x^{x^4}}\left(\frac{x^{x^4}}{x} + \ln x \left(x^{x^4}\right)\left(\frac{x^4}{x} + 4x^3 \ln x\right)\right)$$

$e^{x^4 \ln x}$

$e^{x^4 \ln x}$

Nishant Jindal

15.

Ex-3/4

(1) $f'(x) = \lim_{h\to 0} \frac{f(x)\left(f(h)-1\right) - g(x)g(h)}{h}$

(2) $g'(x) = \lim_{h\to 0} \frac{g(x)\left(f(h)-1\right) + f(x)g(h)}{h}$

f^2+g^2

$2ff'+2gg'$

$\boxed{f^2(x)+g^2(x)=1}$

$f(x)f'(x)+g(x)g'(x) = \lim_{h\to 0} \frac{\left(f^2(x)+g^2(x)\right)\left(f(h)-1\right)}{h} = 0$

$f(0)$

$\Rightarrow \boxed{f^2(x)+g^2(x) = \text{Const}}$

$f(0) = f^2(0) - g^2(0)$ ✓

$g(0) = 2g(0)f(0)$

$\boxed{g(0)=0}$ ✓ or $f(0)=\frac{1}{2}$ ✗

$f(0) = f^2(0)$

$f(0)=0, 1$

$f(0)=0$ ✓, $y=0$

$f(x) = f(x)f(0) - g(x)g(0)$

$\boxed{f(0)=1}$

$\boxed{f(x)=0}$ ✗