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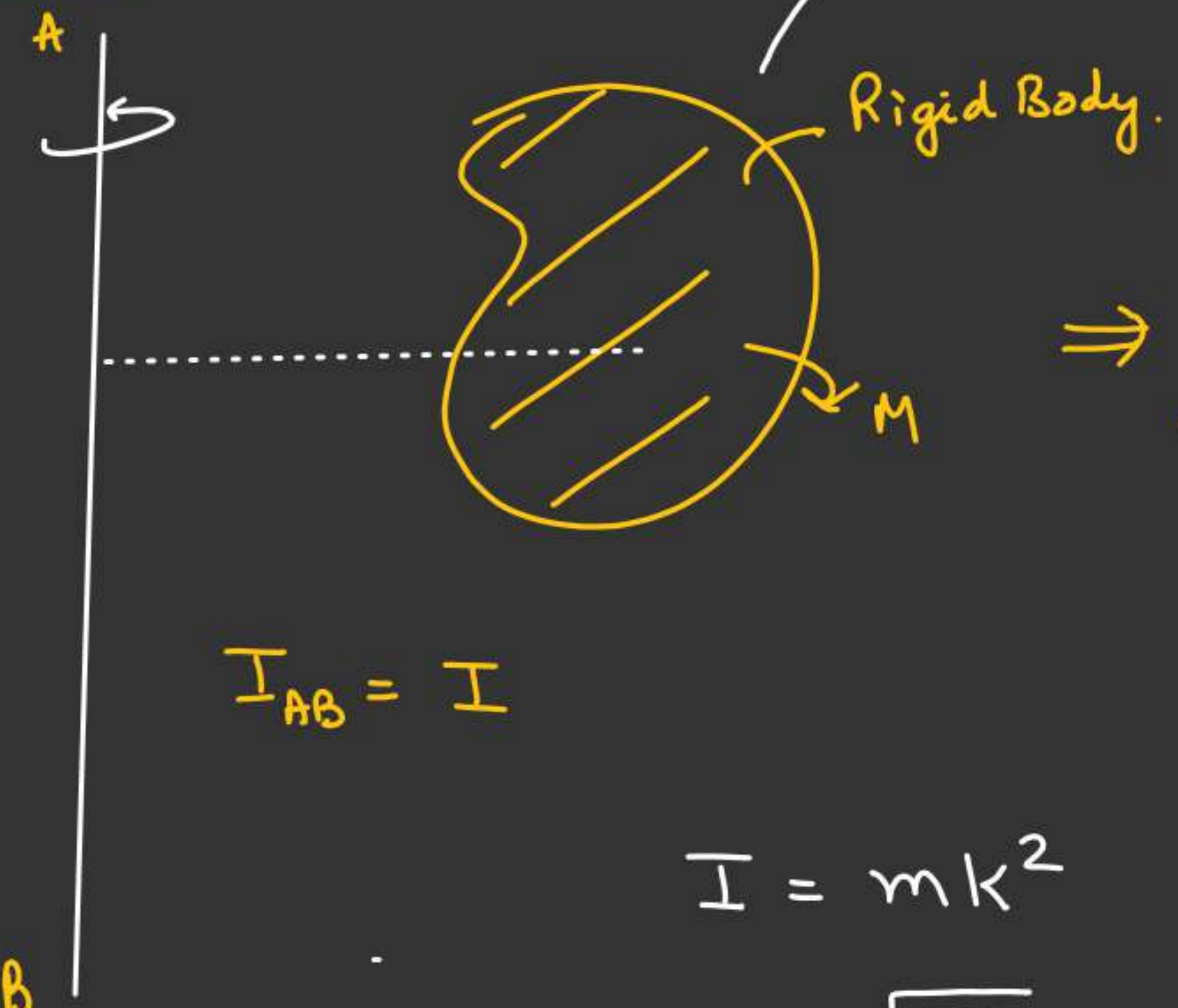
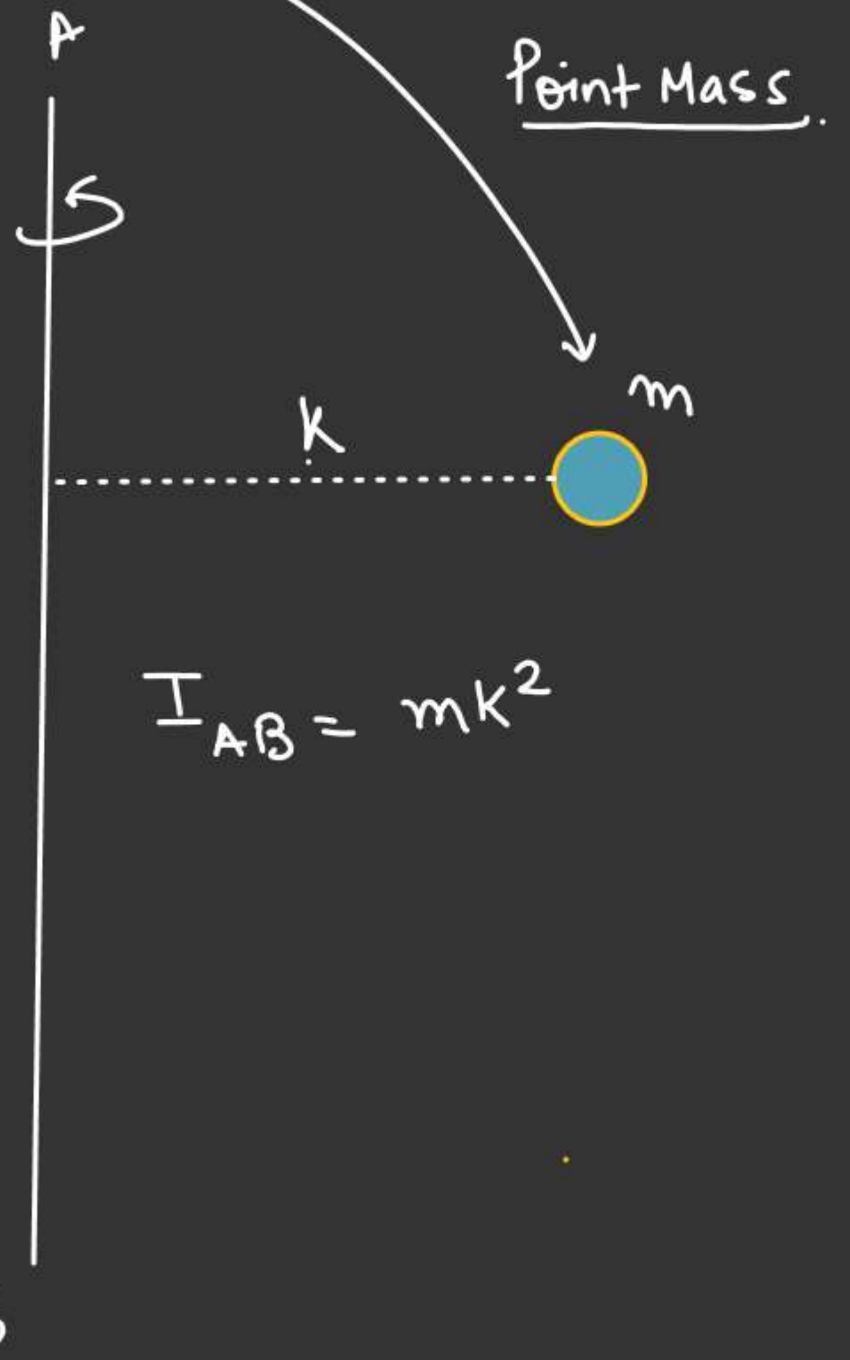
Radius of gyration



$I_{AB} = I$

$\Rightarrow$



$I_{AB} = mK^2$

$I = mK^2$

$K = \sqrt{\frac{I}{m}}$

(Radius of gyration)

Ex:-

Radius of gyration of hollow sphere

$\frac{2}{3}MR^2 = mK^2$

$K = \sqrt{\frac{2}{3}}R$

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Torque due to gravity

$$d\tau_{ith} = \vec{r}_i \times (dm_i \vec{g})$$

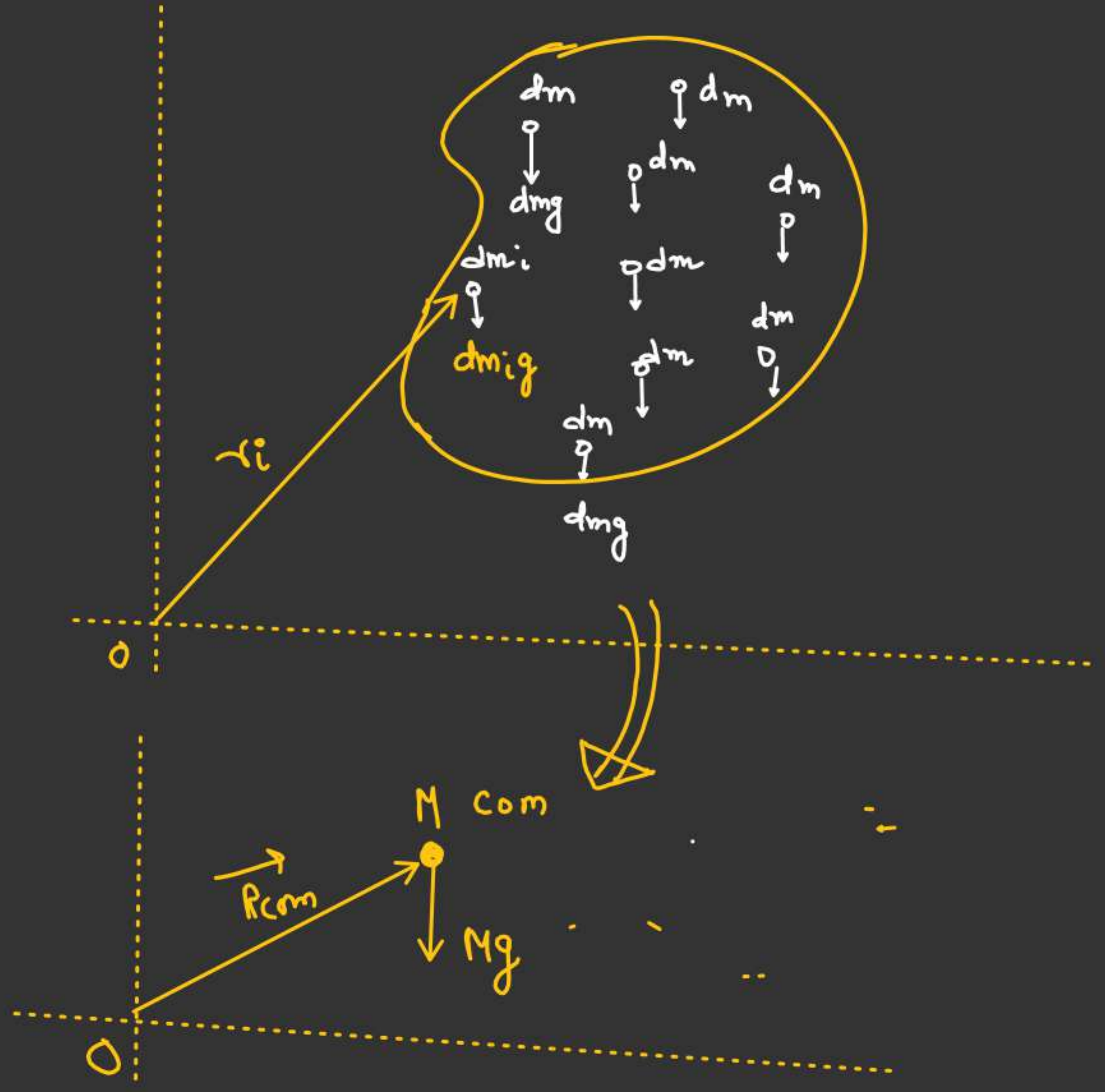
$$d\tau_{ith} = (dm_i \vec{r}_i) \times \vec{g}$$

$$\vec{\tau}_{net} = \sum dm_i \vec{r}_i \times \vec{g}$$

$$\vec{\tau}_{net} = \left(\frac{\sum dm_i \vec{r}_i}{\sum dm_i}\right) \times \underline{\sum dm_i}\, \vec{g}$$

$\Downarrow$

$$\boxed{\vec{\tau}_{net} = \vec{R}_{com} \times M\vec{g}}$$



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TORQUE About an axis

$$\vec{\tau}_{F/O} = (\vec{r}_{OP} \times \vec{F})$$

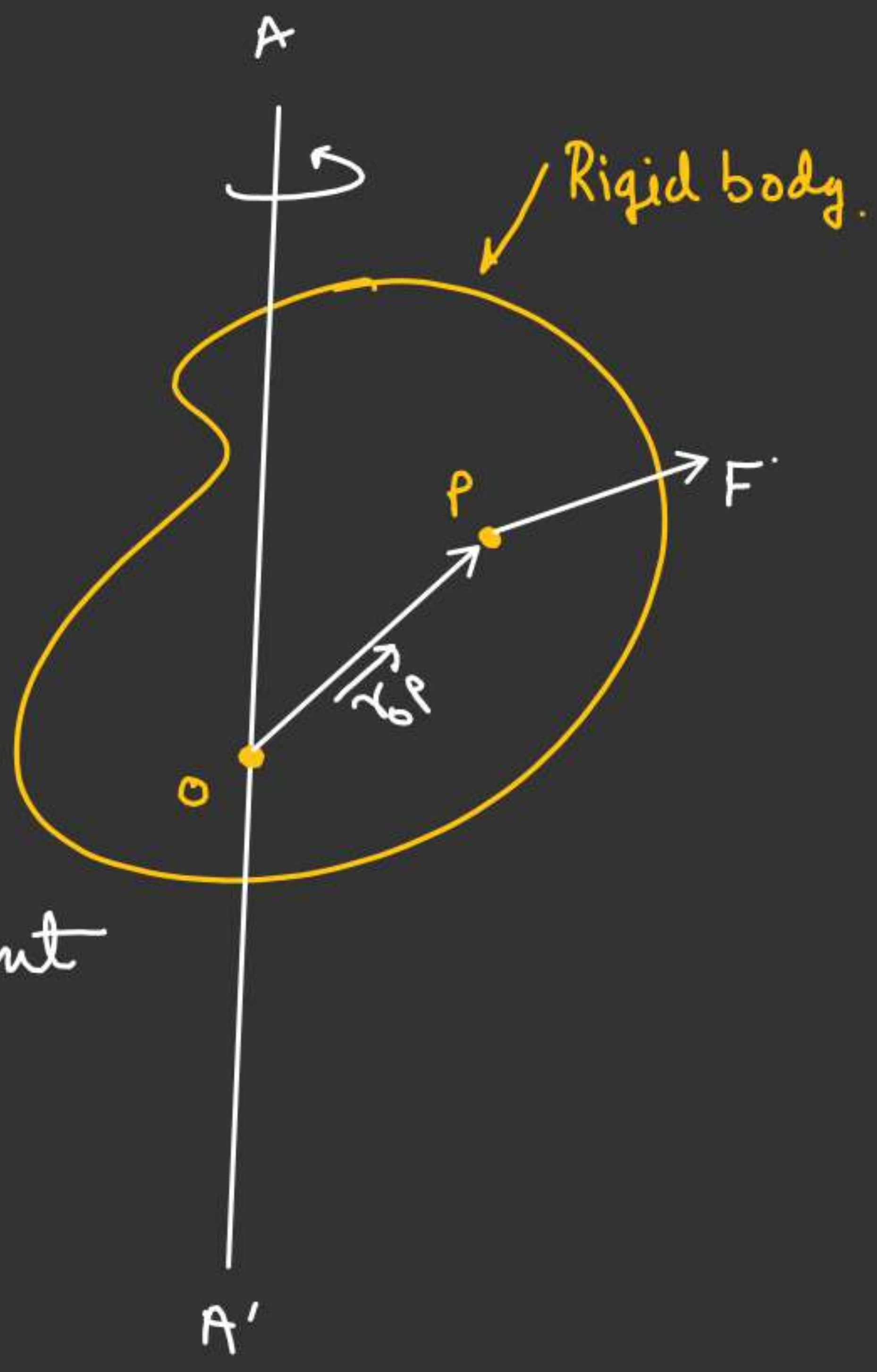
(Torque about a point)

$\vec{\tau}_{F/AA'} \rightarrow$ Torque of F about an axis AA'

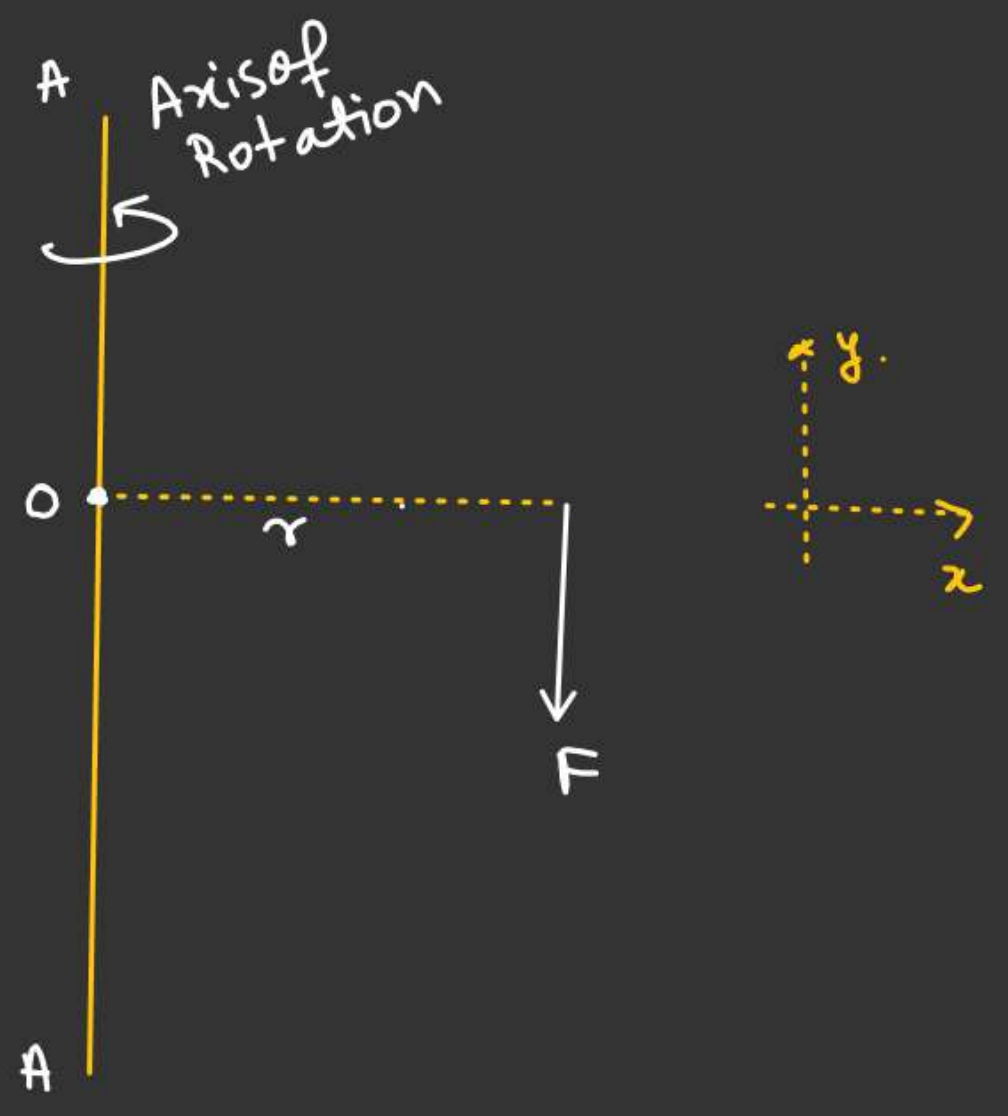
$$\vec{\tau}_{F/AA'} = (\vec{\tau}_{F/O} \cdot \hat{r}_{AA'})$$

↳ Projection or Component of $\vec{\tau}_{F/O}$ along AA'

$$= \left(\vec{\tau}_{F/O} \cdot \frac{\vec{r}_{AA'}}{|\vec{r}_{AA'}|}\right)$$



Torque of force F parallel to axis of Rotation.



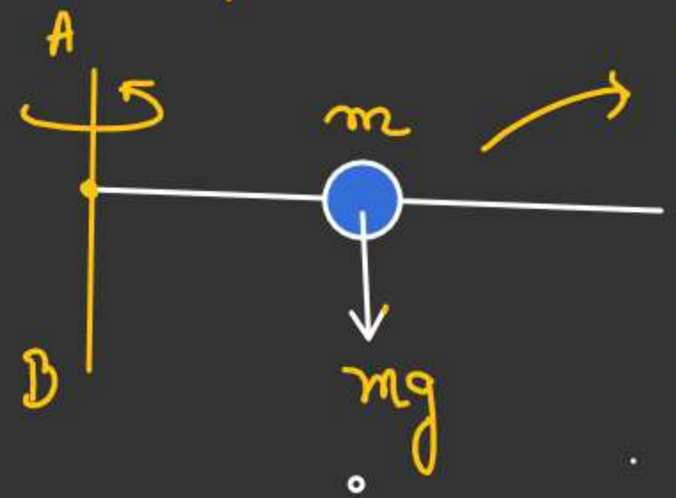
$$\vec{\tau}_{F/O} = (rF\sin 90^\circ)(-\hat{k}) = (rF)(-\hat{k})$$

$$\vec{\tau}_{F/AA'} = \vec{\tau}_{F/O}\cdot(\underbrace{\hat{r}_{AA'}}_{\hat{j}})$$

$$= (rF)(-\hat{k}\cdot\hat{j})$$

$$= 0$$

Note :- Force parallel to axis of rotation have zero torque about axis of Rotation



$$(\tau_{mg})_{AB} = 0$$

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Torque of any force is independent of point taken on the axis of rotation

$\vec{\tau}_{F/O} = \vec{r}_{OP} \times \vec{F}$ By Δ-Law.

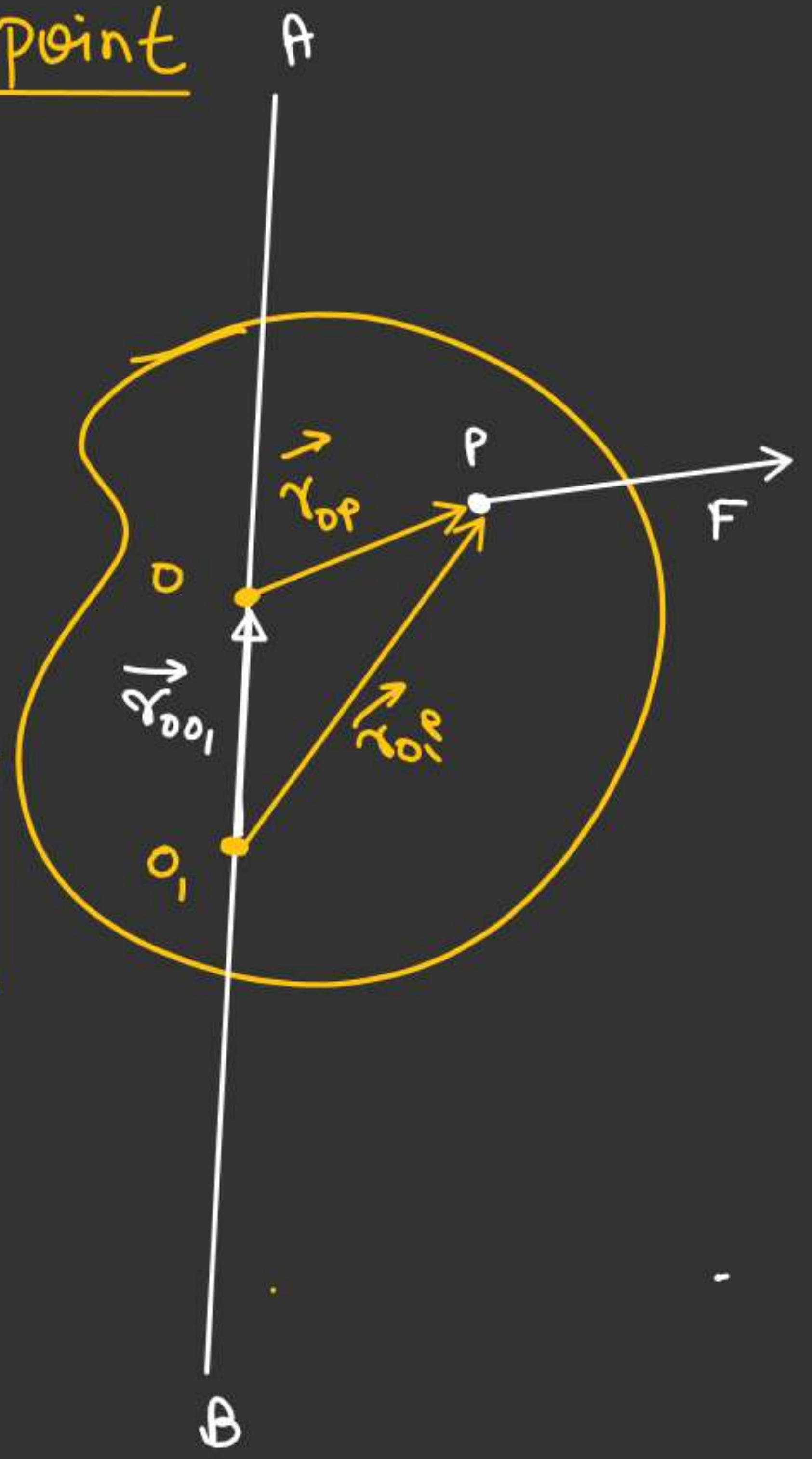
$\vec{\tau}_{F/O_1} = \vec{r}_{O_1P} \times \vec{F}$ $\vec{r}_{OO_1} + \vec{r}_{OP} = \vec{r}_{O_1P}$

$= (\vec{r}_{OO_1} + \vec{r}_{OP}) \times \vec{F}$

$= \underbrace{(\vec{r}_{OO_1} \times \vec{F})}_{\text{Torque of } \vec{F} \text{ about } AB} + \underbrace{(\vec{r}_{OP} \times \vec{F})}_{\text{Torque of } \vec{F} \text{ about } O} \Rightarrow \boxed{\vec{\tau}_{F/O_1} = \vec{\tau}_{F/O}}$

$\vec{r}_{OO_1} \times \vec{F} \Rightarrow$ perpendicular to both $\vec{r}_{OO_1}$ & $\vec{F}$

Component of $\vec{F}$ about AB is zero



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Newton's 1st Law in Rotational dynamics

$\boxed{\vec{\tau}_{net} = 0}$ $\Rightarrow$ Body is in Rotational Equilibrium.

$\boxed{\vec{F}_{net} = 0}$ $\Rightarrow$ Body is in translational Equilibrium.

Newton's 2nd Law in Rotational dynamics

$$\boxed{\vec{\tau}_{ext} = I\vec{\alpha}}$$

I = Moment of inertia of the body about axis of rotation

α = Angular accleration

$$a_t = R\alpha = \left(R\frac{d\omega}{dt}\right)$$

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μ_{min} for ladder to be in equlibrium.

For translational Equilibrium.

$$N_1 = f_s \quad -① $$

$$N_2 = Mg \quad -② $$

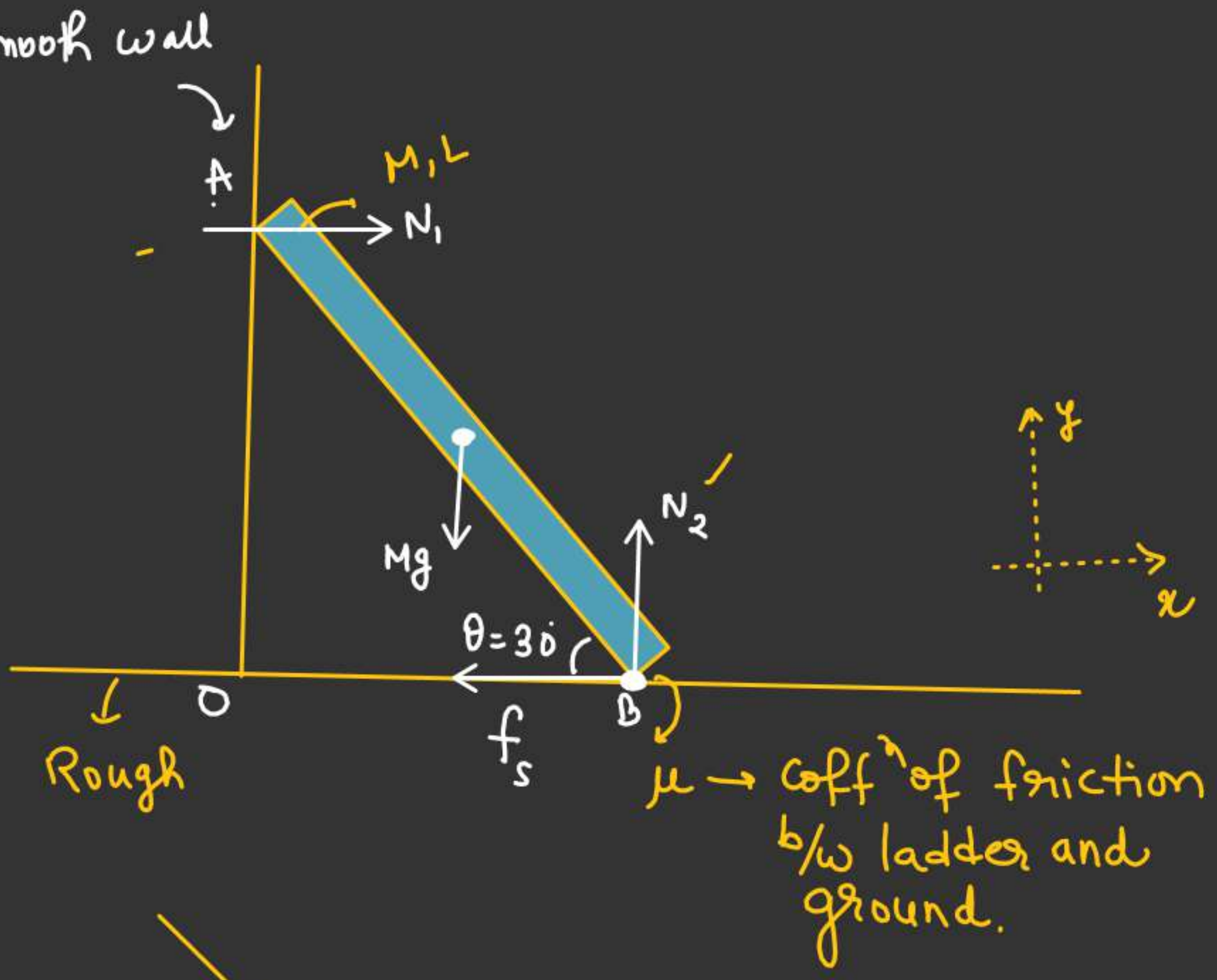
For Rotational Equilibrium

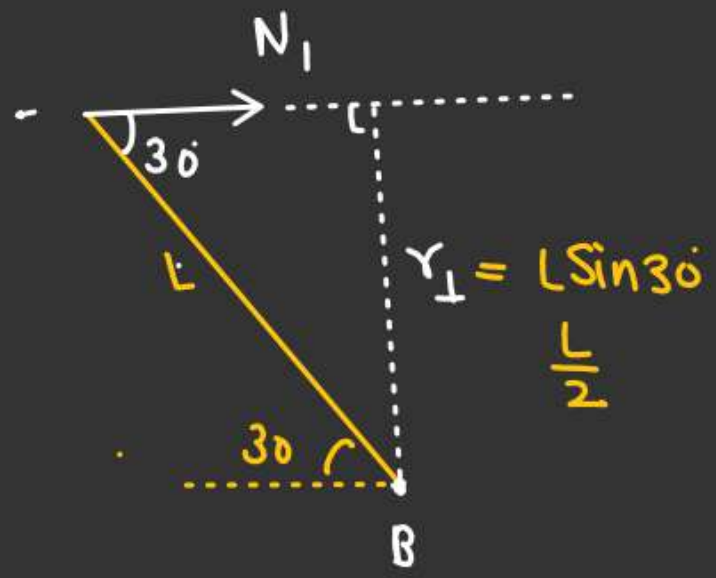
$$(\vec{\tau}_{net})_B = 0$$

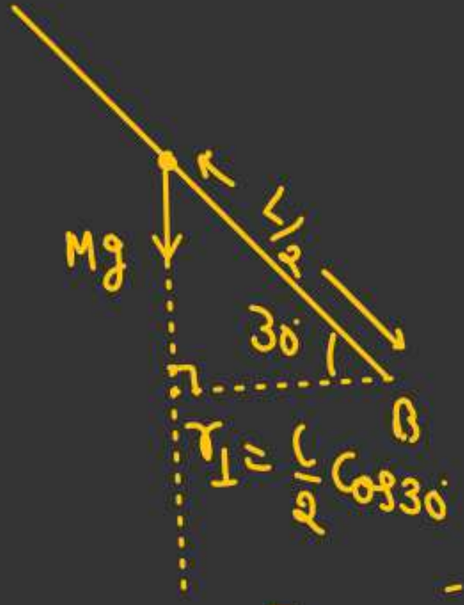
$$\vec{\tau}_{N_1} = \left(N_1 \frac{L}{2}\right)(-\hat{k})$$

$$\vec{\tau}_{Mg} = \left(Mg \frac{L}{2} \cos 30^\circ\right) + \hat{k}$$

$$= \frac{\sqrt{3} MgL}{4} \hat{k}$$







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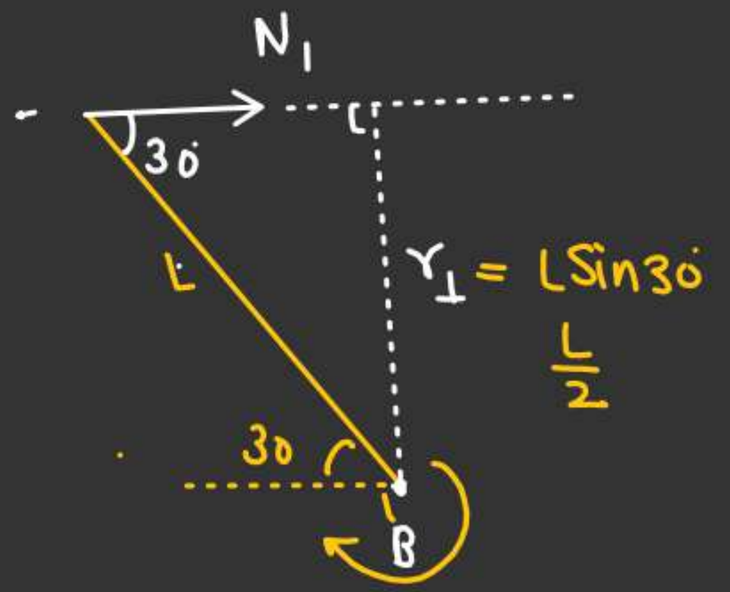


$$N_1 \frac{L}{2} - \frac{\sqrt{3} MgL}{4} = 0$$

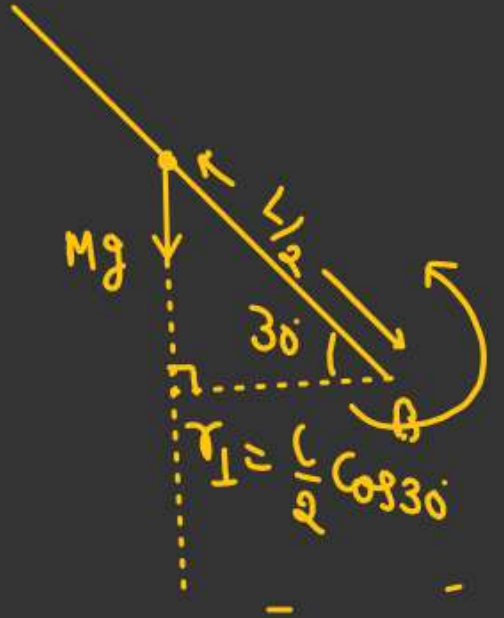
$$N_1 = \frac{\sqrt{3} Mg}{2} \checkmark$$

$$f_s = N_1 = \left(\frac{\sqrt{3} Mg}{2}\right)$$

$$f_s \leq (f_s)_{max}$$

$$\frac{\sqrt{3} Mg}{2} \leq \mu N_2$$

$$\frac{\sqrt{3}}{2} \cancel{Mg} \leq \mu \cancel{Mg} \Rightarrow \mu \geq \frac{\sqrt{3}}{2}$$



$$\mu_{min} = \frac{\sqrt{3}}{2}$$

Ans

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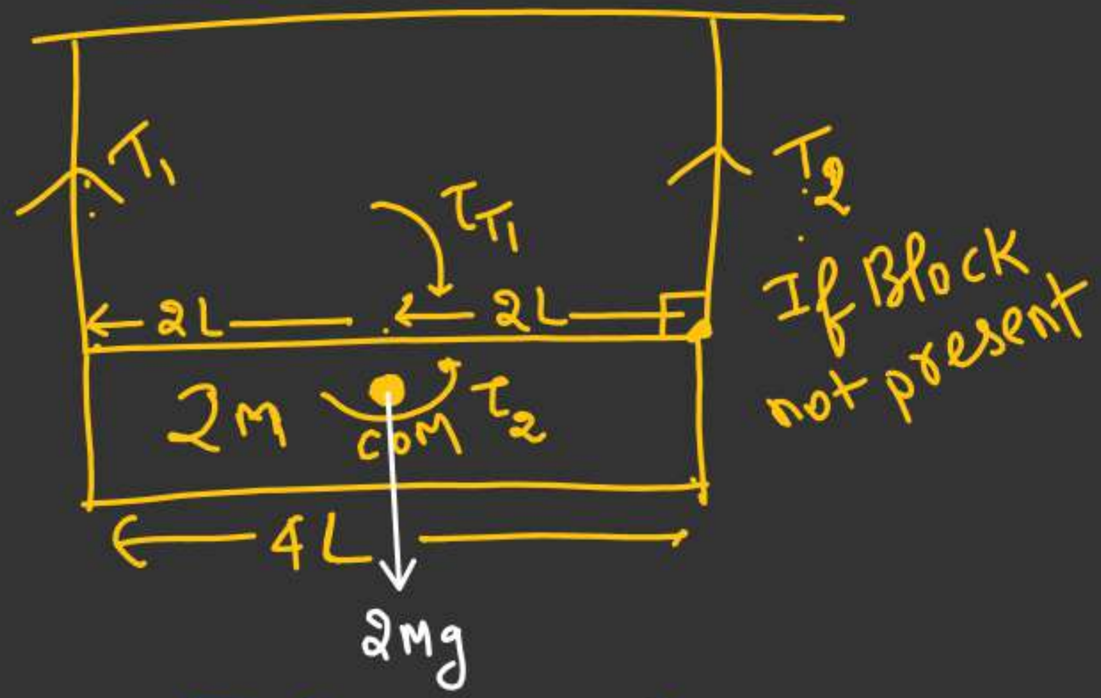
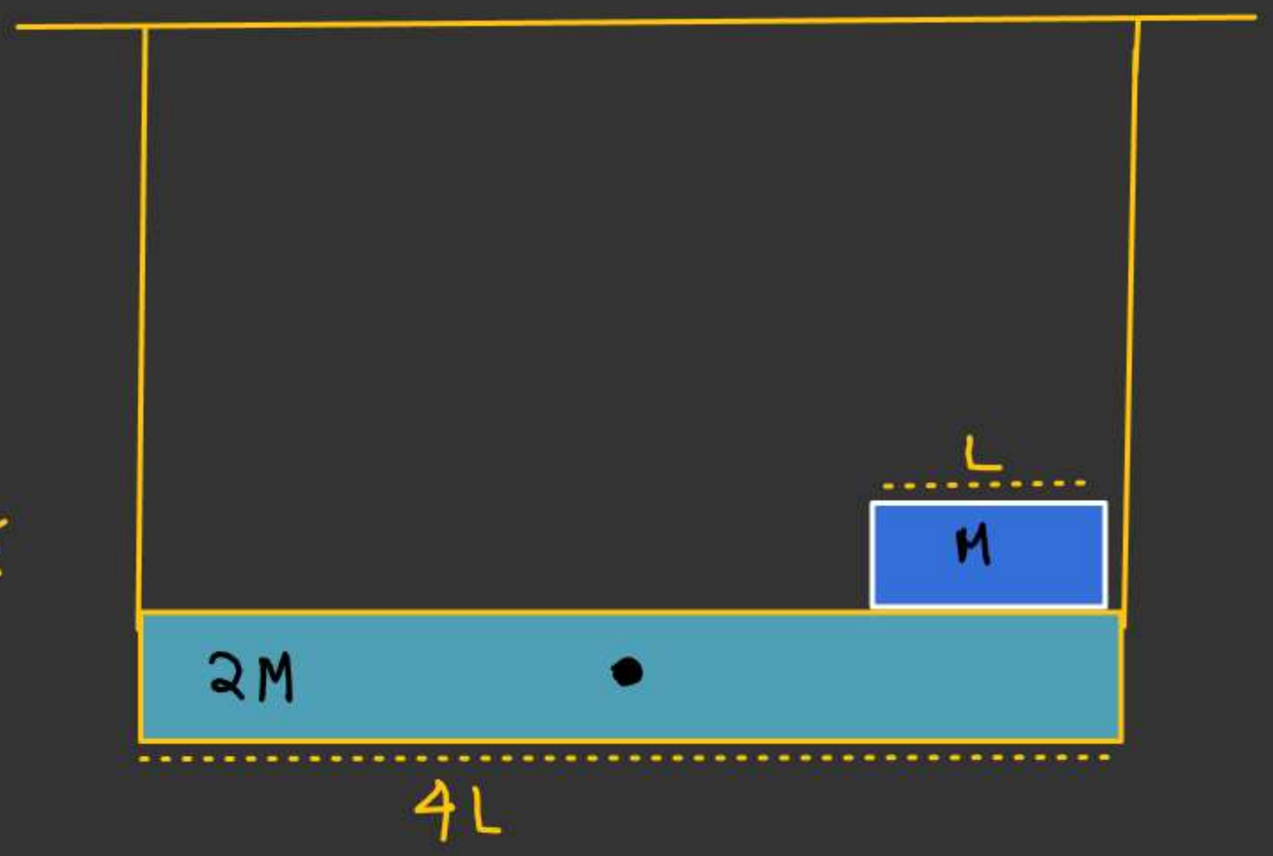
\# The whole System is in equilibrium
Find tension in both the String



$$T_2 \cdot (2L) - T_1(2L) = 0$$

$$T_2 = T_1$$



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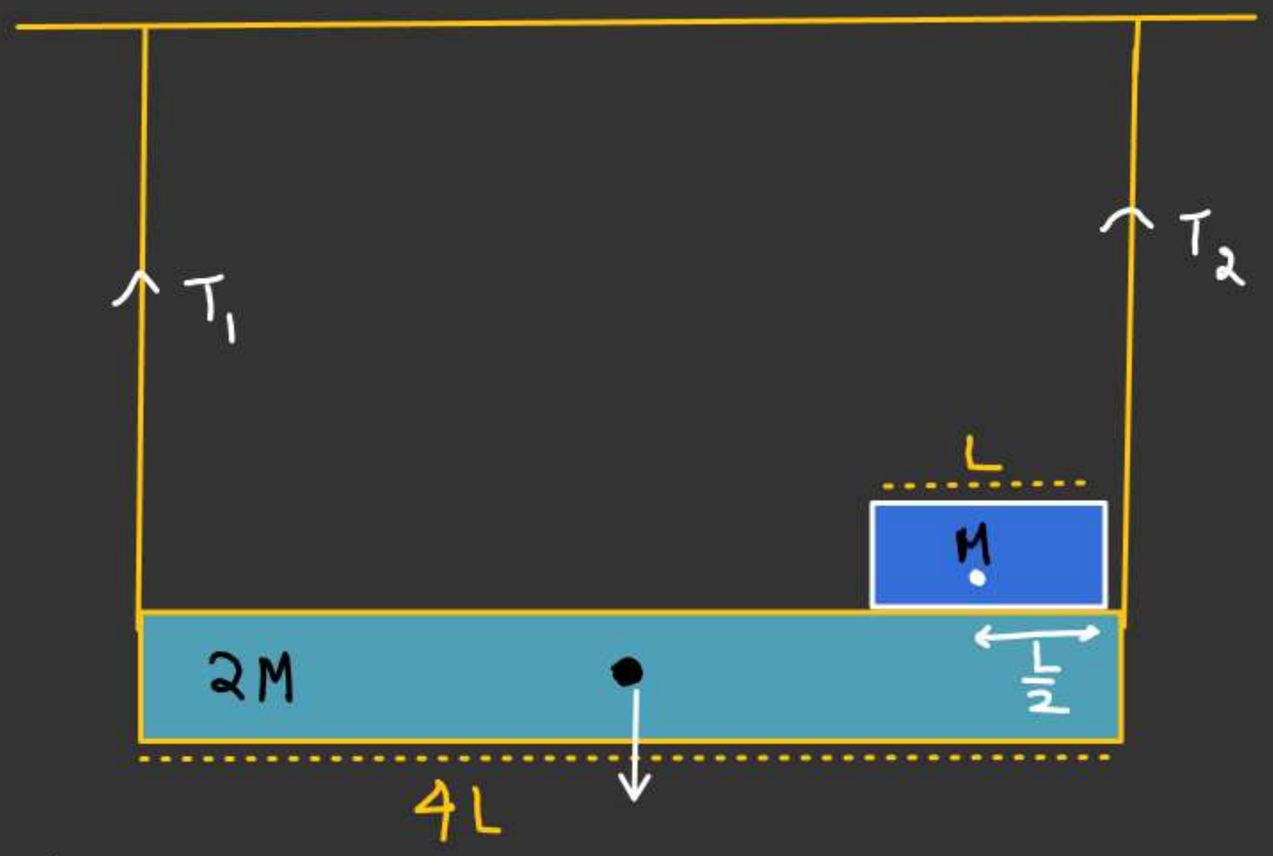
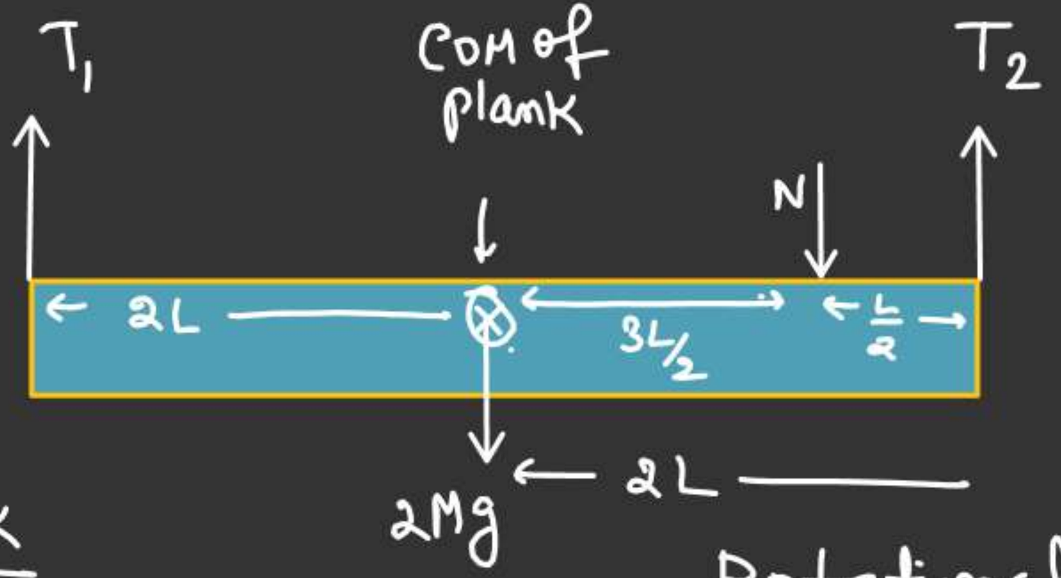




For plank

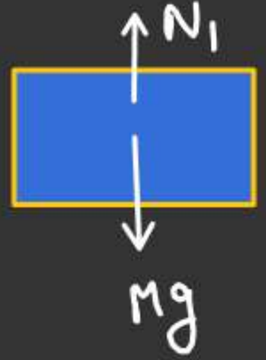
Translational Equilibrium

$$T_1 + T_2 = N_1 + 2Mg$$

$$T_1 + T_2 = 3mg \quad -(1)$$

Rotational Equilibrium

$$-T_1(2L) - N_1\left(\frac{3L}{2}\right) + T_2(2L) = 0$$



$N_1 = Mg$

$$(T_2 - T_1)2L = Mg\left(\frac{3L}{2}\right)$$

$$T_2 - T_1 = \frac{3Mg}{4} \quad -(2)$$

(1) + (2)

$$2T_2 = 3mg + \frac{3mg}{4} = \frac{15mg}{4}$$

$$T_2 = \frac{15mg}{8} \text{ N-m}$$

$$T_1 = T_2 - \frac{3mg}{4}$$

$$= \frac{15mg}{8} - \frac{3mg}{4}$$

$$= \frac{9mg}{8} \text{ N-m}$$

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For Hemisphere to be in equilibrium find $\mu_{min} = ??$