Nishant Jindal

ITF

Q. Find the value of $\sin\left(2\sin^{-1}\left(\frac{1}{4}\right)\right)$

$$\frac{\sqrt{15}}{8}$$

Q. Find the value of $\cos\left(2\cos^{-1}\left(\frac{1}{3}\right)\right)$

$-\frac{7}{9}$

Q. Find the value of $\cos\left(2\tan^{-1}\left(\frac{1}{3}\right)\right)$

$\frac{4}{5}$

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ITF

Q. Find the value of $\sin\left(\frac{1}{2}\cot^{-1}\left(\frac{3}{4}\right)\right)$ $\Rightarrow$ Demand: $\sin\left(\frac{\theta}{2}\right) = \sqrt{\frac{1-\cos\theta}{2}}$

$\cot^{-1}\frac{3}{4} = \theta \Rightarrow \cot\theta = \frac{3}{4}$

$1-\cos 2\theta = 2\sin^2\theta$

$\frac{1-\cos 2\theta}{2} = \sin^2\theta$

$\frac{1-\cos\theta}{2} = \sin^2\theta/2$

$\sin\theta/2 = \sqrt{\frac{1-\cos\theta}{2}}$

$= \sqrt{\frac{1-\frac{\cos\theta}{\sin\theta\cdot\operatorname{cosec}\theta}}{2}} = \sqrt{\frac{1-\frac{\cot\theta}{\sqrt{1+\cot^2\theta}}}{2}}$

$= \sqrt{\frac{1-\frac{3/4}{\sqrt{1+9/16}}}{2}} = \sqrt{\frac{1-3/5}{2}}$

$= \sqrt{\frac{1}{5}} = \frac{1}{\sqrt{5}}$

ITF

Q. **Find the value of** $\tan\left(\frac{3\pi}{4} - 2\boxed{\tan^{-1}\left(\frac{3}{4}\right)}\right)$

5

$\frac{41}{17}$

$\tan^{-1}\frac{3}{4} = \theta$

$\tan\theta = \frac{3}{4}$

$\tan\left(\frac{3\pi}{4} - 2\theta\right) = \dfrac{\tan\frac{3\pi}{4} - \tan 2\theta}{1 + \tan\frac{3\pi}{4}\cdot\tan 2\theta} = \dfrac{-1 - \frac{2\tan\theta}{1-\tan^2\theta}}{1 - \left(\frac{2\tan\theta}{1-\tan^2\theta}\right)}$

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Q. Prove that $\sin\left(2\sin^{-1}\left(\frac{1}{2}\right)\right) = \frac{\sqrt{3}}{2}$

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ITF

Q. Prove the $\sin\left(3\sin^{-1}\left(\frac{1}{3}\right)\right) = \frac{23}{27}$

Q. Prove that $\cos\left(\frac{1}{2}\cos^{-1}\left(\frac{1}{8}\right)\right) = \frac{3}{4}$

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Q. Prove that $\cos\left(\frac{1}{2}\cos^{-1}\left(\frac{1}{10}\right)\right) = \frac{3\sqrt{5}}{10}$

Q. Prove that $\sin\left(\frac{1}{2}\cos^{-1}\left(\frac{1}{9}\right)\right) = \frac{2}{3}$

10

Q. Prove that $\sin\left(\frac{1}{4}\tan^{-1}\sqrt{63}\right) = \frac{1}{2\sqrt{2}}$

Demand = $\sin\theta$

$$\frac{1}{4}\tan^{-1}\sqrt{63} = \theta$$

$$\tan^{-1}\sqrt{63} = 4\theta$$

$$\tan 4\theta = \sqrt{63}$$

$$\frac{\sin 4\theta}{\cos 4\theta} = \frac{\sqrt{63}}{1} = \frac{P}{B}$$

$$\cos 4\theta = \frac{B}{H} = \frac{1}{8}$$

$$2\cos^2 2\theta - 1 = \frac{1}{8} \Rightarrow 2\cos^2 2\theta = \frac{9}{8}$$

$$\cos^2 2\theta = \frac{9}{16} \Rightarrow \cos 2\theta = \frac{3}{4}$$

$$2\cos^2\theta - 1 = \frac{3}{4}$$

$$2\cos^2\theta = \frac{7}{4}$$

$$\cos^2\theta = \frac{7}{8} \Rightarrow \cos\theta = \frac{\sqrt{7}}{2\sqrt{2}}$$

$$\sin\theta = \sqrt{1 - \frac{7}{8}} = \frac{1}{2\sqrt{2}}$$

Nishant Jindal

ITF

Q. **Prove that** $\cos\left(\boxed{\frac{1}{4}\left(\tan^{-1}\left(\frac{24}{7}\right)\right)}\right) = \frac{3}{\sqrt{10}}$

Demand = $\cos\theta$

$\frac{1}{4}\tan^{-1}\frac{24}{7} = \theta$

$\tan 4\theta = \frac{24}{7}$

$\frac{\sin 4\theta}{\cos 4\theta} = \frac{24}{7}$

$\cos 4\theta = \frac{7}{25}$

$2\cos^2 2\theta - 1 = \frac{7}{25}$

ITF

Q. The domain of the function $f(x) = \sin^{-1}\left(\frac{x^2 - 3x + 2}{x^2 + 2x + 7}\right)$ is. **[Main, 2022]**

(A) $[1, \infty)$
(B) $(-1, 2]$
(C) $[-1, \infty)$
(D) $(-\infty, 2]$

$$-1 \le \frac{x^2-3x+2}{x^2+2x+7} \le 1$$

$$\frac{x^2-3x+2}{(x^2+2x+7)} \geq -1 \quad \text{And} \quad \frac{x^2-3x+2}{(x^2+2x+7)} \le 1$$

$D = 4 - 28$

$x^2 - 3x + 2 \geq -x^2 - 2x - 7$

$\cancel{x^2} - 3x + 2 \le \cancel{x^2} + 2x + 7$

$2x^2 - x + 9 \geq 0$

$-5 \le 5x$

$x \geq -1$

Q. The domain of the function $cos^{-1}\left(\frac{2sin^{-1}\left(\frac{1}{4x^2-1}\right)}{\pi}\right)$ is: **[Main, 2022]**

(A) $R-\left\{-\frac{1}{2},\frac{1}{2}\right\}$

(B) $(-\infty,-1]\cup[1,\infty)\cup\{0\}$

(C) $\left(-\infty,\frac{-1}{2}\right)\cup\left(\frac{1}{2},\infty\right)\cup\{0\}$

(D) $\left(-\infty,\frac{-1}{\sqrt{2}}\right]\cup\left[\frac{1}{\sqrt{2}},\infty\right)\cup\{0\}$

$-1\le\frac{2\sin^{-1}\left(\frac{1}{4x^2-1}\right)}{\pi}\le 1$

$\underbrace{-\frac{\pi}{2}\le\sin^{-1}\left(\frac{1}{4x^2-1}\right)\le\frac{\pi}{2}}$

It is True Always

$-1\le\frac{1}{4x^2-1}\le 1$

$\frac{1}{4x^2-1}\ge -1$ And $\frac{1}{4x^2-1}\le 1$

$\frac{1}{4x^2-1}+1\ge 0 \;\cap\; \frac{1}{4x^2-1}-1\le 0$

Yhi Ans

ITF

Q. Considering only the principal values of the inverse trigonometric functions, the domain of the function $f(x)=\cos^{-1}\left(\frac{x^2-4x+2}{x^2+3}\right)$ is: **[Main, 2022]**

(A) $\left(-\infty, \frac{1}{4}\right]$

(B) $\left[-\frac{1}{4}, \infty\right)$

(C) $\left(-\frac{1}{3}, \infty\right)$

(D) $\left(-\infty, \frac{1}{3}\right]$

$$-1 \le \frac{x^2-4x+2}{(x^2+3)} \le 1$$

DY

$$\frac{x^2-4x+2}{x^2+3} \ge -1 \quad \text{And} \quad \frac{x^2-4x+2}{x^2+3} \le 1$$

$$x^2-4x+2 \ge -x^2-3$$

$$\cancel{x^2}-4x+2 \le \cancel{x^2}+3$$

$$4x \ge -1$$

$$x \ge -\frac{1}{4}$$

How to Convert One ITF into Another.

When x<0 given

Basic

$x = -ve$

$x = -1/2$

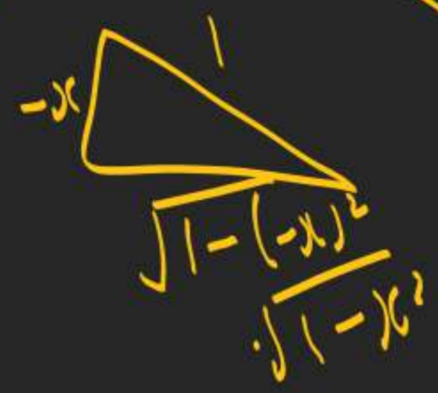
$\sin^{-1}\left(-\frac{1}{2}\right) = -\sin^{-1}\frac{1}{2}$

$\cos^{-1}\left(-\frac{1}{2}\right) = \pi - \cos^{-1}\frac{1}{2}$

Q Convert $\sin^{-1}x$ into $\cos^{-1}$__ When $x<0$

$$\sin^{-1}(x) = -\sin^{-1}(-x) = -\cos^{-1}\sqrt{1-x^2}$$

$\sin^{-1}(-x) = \theta$



$\sin\theta = -\frac{x}{1}$ $\frac{P}{H}$

$\cos\theta = \frac{B}{H} = \sqrt{1-x^2}$

$\theta = \cos^{-1}\sqrt{1-x^2}$

$$\sin^{-1}(-x) = \cos^{-1}\sqrt{1-x^2}$$

Nishant Jindal

$x<0$

Q Convert $\cos^{-1}x$ into $\tan^{-1}$ ... $(x=-ve)$

$$\cos^{-1}(x) = \pi - \underbrace{\cos^{-1}(-x)} = \pi - \left(-\tan^{-1}\frac{\sqrt{1-x^2}}{x}\right) = \pi + \tan^{-1}\frac{\sqrt{1-x^2}}{x}$$

$\cos^{-1}\left(-\frac{1}{2}\right)$

$= \pi - \cos^{-1}\left(\frac{1}{2}\right)$

$\cos^{-1}(-x) = \theta$

$\cos\theta = -\frac{x}{1}\ \frac{B}{H}$

$$\tan\theta = \frac{P}{B} = \frac{\sqrt{1-x^2}}{-x}$$

$$\theta = \tan^{-1}\left(\frac{\sqrt{1-x^2}}{-x}\right) = -\tan^{-1}\frac{\sqrt{1-x^2}}{x}$$

$$\cos^{-1}(-x) = -\tan^{-1}\frac{\sqrt{1-x^2}}{x}$$

Q Convert $\cot^{-1}(x)$ in $\tan^{-1}$ ___ $(x<0)$

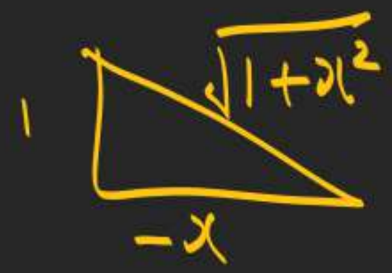
$$\cot^{-1}(x) = \pi - \underbrace{\cot^{-1}(-x)} = \pi - \left(-\tan^{-1}\frac{1}{x}\right) = \pi + \tan^{-1}\frac{1}{x}$$

$\cot^{-1}(-1)$

$= \pi - \cot^{-1}(1)$

$\cot^{-1}(-x) = \theta$

$\cot\theta = -\frac{x}{1} = \frac{B}{P}$



$\tan\theta = \frac{P}{B} = \frac{1}{-x}$

$\theta = \tan^{-1}\left(\frac{1}{-x}\right)$

$\cot^{-1}(-x) = \theta = -\tan^{-1}\left(\frac{1}{x}\right)$

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Q $\sec^{-1}(x)$ into $\sin^{-1}$ ... $(x<0)$

$\sec^{-1}(-2)$

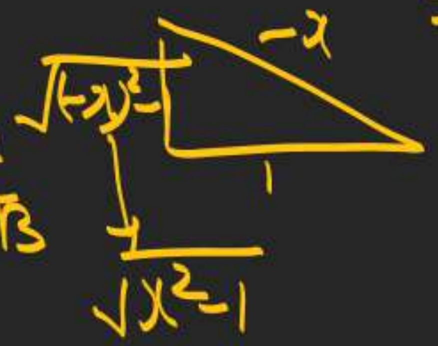
$\pi - \sec^{-1}(2)$

$$\sec^{-1}(x) = \pi - \sec^{-1}(-x) = \pi + \left(+\sin^{-1}\frac{\sqrt{x^2+1}}{x}\right)$$

$$= \pi + \sin^{-1}\frac{\sqrt{x^2+1}}{x}$$

$\sec^{-1}(-x) = \theta$

$\sec\theta = -\frac{x}{1} = \frac{H}{B}$



$$\sin\theta = \frac{P}{H} = \frac{\sqrt{x^2-1}}{-x}$$

$$\sec^{-1}(-x) = \theta = \sin^{-1}\frac{\sqrt{x^2-1}}{-x} = -\sin^{-1}\left(\frac{\sqrt{x^2-1}}{x}\right)$$

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Prop 5:- Sum & difference of 2 or more ITF

A) $\sin^{-1}x + \sin^{-1}y = \sin^{-1}\left(x\sqrt{1-y^2} + y\sqrt{1-x^2}\right) \rightarrow x \geq 0, y \geq 0 \quad x^2+y^2 \leq 1$

$\sin^{-1}x - \sin^{-1}y = \sin^{-1}\left(x\sqrt{1-y^2} - y\sqrt{1-x^2}\right)$

B) $\cos^{-1}x \pm \cos^{-1}y = \cos^{-1}\left(xy \mp \sqrt{1-x^2}\sqrt{1-y^2}\right) \rightarrow x \geq 0, y \geq 0 \quad x \leq y$

C) $\tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right) \rightarrow \boxed{xy < 1}$

$\tan^{-1}x + \tan^{-1}y = \pi + \tan^{-1}\left(\frac{x+y}{1-xy}\right) \rightarrow xy > 1$

Nishant Jindal

Q P.T.

$$2\boxed{\cos^{-1}\frac{3}{\sqrt{13}}} + \cot^{-1}\frac{16}{63} + \frac{1}{2}\cos^{-1}\frac{7}{25} = \pi$$

$$\frac{12}{5}\times\frac{3}{4} = \frac{36}{20} > 1$$

$$\cos^{-1}\frac{3}{\sqrt{13}} = \theta$$

$$\cos\theta = \frac{3}{\sqrt{13}}$$

$$\tan\theta = \frac{P}{B} = \frac{2}{3}$$

$$\theta = \tan^{-1}\frac{2}{3}$$

$$\frac{1}{2}\cos^{-1}\frac{7}{25} = \theta$$

$$2\theta = \cos^{-1}\frac{7}{25}$$

$$\cos 2\theta = \frac{7}{25}$$

$$\frac{1-\tan^2\theta}{1+\tan^2\theta} = \frac{7}{25}$$

$$25 - 25\tan^2\theta = 7 + 7\tan^2\theta$$

$$18 = 32\tan^2\theta \Rightarrow \tan^2\theta = \frac{18}{32} = \frac{9}{16} \Rightarrow \tan\theta = \frac{3}{4}$$

Qs

$$\frac{2}{3}\times\frac{2}{3} = \frac{4}{9} < 1$$

$$2\tan^{-1}\frac{2}{3} + \cot^{-1}\frac{16}{63} + \tan^{-1}\frac{3}{4}$$

$$\rightarrow \tan^{-1}\left(\frac{2}{3}\right) + \tan^{-1}\left(\frac{2}{3}\right) + \cot^{-1}\frac{16}{63} + \tan^{-1}\frac{3}{4}$$

$$\tan^{-1}\left(\frac{\frac{2}{3}+\frac{2}{3}}{1-\frac{2}{3}\times\frac{2}{3}}\right) + \;,, + \tan^{-1}\frac{3}{4}$$

$$\tan^{-1}\left(\frac{4}{\cancel{3}}\times\frac{\cancel{9}^{\,3}}{5}\right) + \tan^{-1}\frac{3}{4} + \cot^{-1}\frac{16}{63}$$

$$\rightarrow \tan^{-1}\left(\frac{12}{5}\right) + \tan^{-1}\frac{3}{4} + \cot^{-1}\frac{16}{63}$$

$$\pi + \tan^{-1}\left(\frac{\frac{12}{5}+\frac{3}{4}}{1-\frac{12}{5}\times\frac{3}{4}}\right) + \cot^{-1}\frac{16}{63}$$

$$= \pi + \cancel{\tan^{-1}\left(\frac{63}{-16}\right)} + \cancel{\tan^{-1}\frac{63}{16}}$$

$$= \pi$$

Nishant Jindal

Q $\cos^{-1}\sqrt{\frac{2}{3}} - \cos^{-1}\frac{\sqrt{6}+1}{2\sqrt{3}} = \frac{\pi}{6}$ (P.T.)

$\cos\theta = \frac{\sqrt{2}}{\sqrt{3}} = \frac{B}{H}$

$\tan\theta = \frac{1}{\sqrt{2}}$

$\cos\theta = \frac{\sqrt{6}+1}{2\sqrt{3}}$

$x = \sqrt{12-(\sqrt{6}+1)^2}$

$= \sqrt{12-(6+1+2\sqrt{6})}$

$= \sqrt{5-2\sqrt{6}}$ TP1

$= \sqrt{(\sqrt{3})^2+(\sqrt{2})^2-2\sqrt{3}\sqrt{2}}$

$x = (\sqrt{3}-\sqrt{2})$

$\tan\theta = \frac{\sqrt{3}-\sqrt{2}}{\sqrt{6}+1}$

$\Rightarrow \tan^{-1}\left(\frac{1}{\sqrt{2}}\right) - \tan^{-1}\left(\frac{\sqrt{3}-\sqrt{2}}{\sqrt{6}+1}\right)$ *T.P.2

$\Rightarrow \tan^{-1}\left(\frac{1}{\sqrt{2}}\right) - \tan^{-1}\left(\frac{\sqrt{3}-\sqrt{2}}{1+\sqrt{3}\cdot\sqrt{2}}\right)$

$= \cot^{-1}\sqrt{2} - (\tan^{-1}\sqrt{3} - \tan^{-1}\sqrt{2}) = \frac{\pi}{2} - \frac{\pi}{3} = \frac{\pi}{6}$

$\tan^{-1}(x) - \tan^{-1}(y) = \tan^{-1}\left(\frac{x-y}{1+xy}\right)$

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Q $\tan^{-1}\frac{1}{7}+\tan^{-1}\frac{1}{13}=?$ $\quad$ $Pr=\frac{1}{7}\times\frac{1}{13}=\frac{1}{91}<1$.

$$\tan^{-1}\left(\frac{\frac{1}{7}+\frac{1}{13}}{1-\frac{1}{7}\times\frac{1}{13}}\right)$$

$$\tan^{-1}\left(\frac{\frac{20}{\cancel{13\times7}}}{\frac{91-1}{\cancel{13\times7}}}\right)=\tan^{-1}\left(\frac{20}{90}\right)$$

$$=\tan^{-1}\left(\frac{2}{9}\right)$$

Q $\tan^{-1}2+\tan^{-1}3$. $\quad$ $Pr=2\times3=6>1$

$$\pi+\tan^{-1}\left(\frac{2+3}{1-2\times3}\right)$$

$$\pi+\tan^{-1}\left(\frac{5}{-5}\right)$$

$$\pi+\tan^{-1}(-1)$$

$$\pi-\tan^{-1}(1)=\pi-\frac{\pi}{4}=\frac{3\pi}{4}$$

Nishant Jindal

Q $\tan^{-1}\frac{1}{2}+\tan^{-1}\frac{1}{3}=?$ $\quad Pr=\frac{1}{2}\times\frac{1}{3}=\frac{1}{6}<1$

$$\tan^{-1}\left(\frac{\frac{1}{2}+\frac{1}{3}}{1-\frac{1}{2}\times\frac{1}{3}}\right)=\tan^{-1}\left(\frac{\frac{5}{6}}{1-\frac{1}{6}}\right)$$

$$=\tan^{-1}(1)=\frac{\pi}{4}$$

Q $\underline{\tan^{-1}\frac{1}{2}+\tan^{-1}\frac{1}{3}}+\underbrace{\tan^{-1}\frac{1}{4}+\tan^{-1}\frac{1}{5}}=?$

$\frac{\pi}{4}+\tan^{-1}\frac{9}{19}$

$\tan^{-1}\frac{1}{4}+\tan^{-1}\frac{1}{5}$ $\quad Pr=\frac{1}{20}<1$

$$\tan^{-1}\left(\frac{\frac{1}{4}+\frac{1}{5}}{1-\frac{1}{4}\times\frac{1}{5}}\right)=\tan^{-1}\left(\frac{\frac{9}{\cancel{20}}}{\frac{19}{\cancel{20}}}\right)=\tan^{-1}\frac{9}{19}$$

Q If $\tan\left(\underline{\tan^{-1}\frac{1}{2}+\tan^{-1}\frac{1}{3}}+\tan^{-1}\frac{1}{4}+\tan^{-1}\frac{1}{5}\right)$ is expressed in lowest form of $\frac{a}{b}$ then $(\underline{a+b})=?$

$$\tan\left(\frac{\pi}{4}+\tan^{-1}\frac{9}{19}\right)=\frac{1+\tan\left(\tan^{-1}\frac{9}{19}\right)}{1-\tan\left(\tan^{-1}\frac{9}{19}\right)}$$

$$\tan\left(\frac{\pi}{4}+\theta\right)=\frac{1+\tan\theta}{1-\tan\theta}$$

$$=\frac{1+\frac{9}{19}}{1-\frac{9}{19}}=\frac{19+9}{19-9}=\frac{28}{10}$$

$$=\frac{14}{5}=\frac{a}{b}$$

$$a+b=14+5$$
$$=19$$

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Q $\tan^{-1}\frac{a}{b} + \tan^{-1}\left(\frac{b-a}{b+a}\right)$

$\tan^{-1}\frac{a}{b} + \tan^{-1}\left(\frac{\frac{b}{a}-1}{\frac{b}{a}+1}\right)$

$\tan^{-1}\frac{a}{b} + \tan^{-1}\left(\frac{\frac{b}{a}-1}{1+\frac{b}{a}\times 1}\right)$

$\tan^{-1}\frac{a}{b} + \left(\tan^{-1}\frac{b}{a} - \tan^{-1}1\right)$

$\left(\tan^{-1}\frac{a}{b} + \cot^{-1}\frac{a}{b}\right) - \frac{\pi}{4}$

$\frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$

Q P.T. HP

$\tan^{-1}\left(\frac{yz}{xr}\right) + \tan^{-1}\left(\frac{zx}{yr}\right) + \tan^{-1}\left(\frac{xy}{zr}\right) = \frac{\pi}{2}$

if $x^2+y^2+z^2 = r^2$

$Pr = \frac{\cancel{yx}z^2}{\cancel{xy}r^2} = \frac{z^2}{x^2+y^2+z^2} < 1$

LHS

$\tan^{-1}\left(\frac{\frac{yz}{xr}+\frac{zx}{yr}}{1-\frac{\cancel{xy}z^2}{\cancel{xy}r^2}}\right) + \tan^{-1}\left(\frac{xy}{zr}\right)$

$\tan^{-1}\left(\frac{\frac{y^2zr+zx^2r}{xy\cancel{r^2}}}{\frac{x^2+y^2+\cancel{z^2}-\cancel{z^2}}{\cancel{r^2}}}\right) + '' \Rightarrow \tan^{-1}\left(\frac{zr\cancel{(x^2+y^2)}}{xy\cancel{(x^2+y^2)}}\right) + ''$

$\tan^{-1}\left(\frac{zr}{xy}\right) + \tan^{-1}\left(\frac{xy}{zr}\right) = \frac{\pi}{2}$

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Q If $a, b, c \geq 0$ then S.T.

$$\tan^{-1}\sqrt{\frac{a(a+b+c)}{bc}} + \tan^{-1}\sqrt{\frac{b(a+b+c)}{ac}} + \tan^{-1}\sqrt{\frac{c(a+b+c)}{ab}} = \pi$$

$$Pr = \sqrt{\frac{a(a+b+c)}{bc}} \times \sqrt{\frac{b(a+b+c)}{ac}} = \sqrt{\frac{(a+b+c)^2}{c^2}} = \frac{a+b+c}{c} > 1$$

$$\pi + \tan^{-1}\left\{\frac{\sqrt{\frac{a(a+b+c)}{bc}} + \sqrt{\frac{b(a+b+c)}{ac}}}{1 - \sqrt{\frac{a(a+b+c)}{bc}}\sqrt{\frac{b(a+b+c)}{ac}}}\right\} + \text{,,}$$

$$\pi + \tan^{-1}\left\{\frac{\sqrt{\frac{a+b+c}{c}}\left(\sqrt{\frac{a}{b}} + \sqrt{\frac{b}{a}}\right)}{1 - \frac{a+b+c}{c}}\right\} + \text{,,}$$

$$\pi + \tan^{-1}\left(\frac{\sqrt{\frac{a+b+c}{c}}\left(\frac{\cancel{a+b}}{\sqrt{ab}}\right)}{-\frac{\cancel{(a+b)}}{c}}\right) + \tan^{-1}\sqrt{}$$

$$\pi \cancel{+} \tan^{-1}\left(\cancel{\sqrt{\frac{c(a+b+c)}{ab}}}\right) + \tan^{-1}\left(\cancel{\sqrt{\frac{c(a+b+c)}{ab}}}\right)$$

$$\pi$$

Nishant Jindal

$\cos^{-1}x+\cos^{-1}y=\cos^{-1}\left(xy-\sqrt{1-x^2}\sqrt{1-y^2}\right)$

Q If $\cos^{-1}\left(\frac{x}{a}\right)+\cos^{-1}\left(\frac{y}{b}\right)=\alpha$.

then S.T. $\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{2xy\cos\alpha}{ab}=\sin^2\alpha$

$$\cos^{-1}\left(\frac{xy}{ab}-\sqrt{1-\frac{x^2}{a^2}}\sqrt{1-\frac{y^2}{b^2}}\right)=\alpha$$

$$\frac{xy}{ab}-\sqrt{1-\frac{x^2}{a}}\sqrt{1-\frac{y^2}{b^2}}=\cos\alpha$$

$$\left(\frac{xy}{ab}-\cos\alpha\right)^2=\left(\sqrt{\left(1-\frac{x^2}{a}\right)}\sqrt{\left(1-\frac{y^2}{b^2}\right)}\right)^2$$

$$\cancel{\frac{x^2y^2}{a^2b^2}}+\cos^2\alpha-\frac{2xy\cos\alpha}{ab}=1-\frac{y^2}{b^2}-\frac{x^2}{a^2}+\cancel{\frac{x^2y^2}{a^2b^2}}$$

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{2xy\cos\alpha}{ab}=1-\cos^2\alpha$$

$$=\sin^2\alpha$$

$\cos^{-1}(y)+\cos^{-1}(bxy)=\frac{\pi}{2}-\sin^{-1}(ax)$

Q $\sin^{-1}(ax)+\cos^{-1}(y)+\cos^{-1}(bxy)=\frac{\pi}{2}$ then (x,y)

Adv

Col 1	Col 2
A) $a=1$ $b=0$	P) $x^2+y^2=1$
B) $a=1$ $b=1$	Q) $(x^2-1)(y^2-1)=0$
C) $a=1$ $b=2$	R) $y=x$
D) $a=2$ $b=2$	S) $(4x^2-1)(y^2-1)=0$

$$\cos^{-1}\left(bxy^2-\sqrt{1-y^2}\sqrt{1-b^2x^2y^2}\right)=\cos^{-1}(ax)$$

$$\left(bxy^2-ax\right)^2=\left(\sqrt{1-y^2}\sqrt{1-b^2xy^2}\right)^2$$

$$\cancel{b^2x^2y^4}+a^2x^2-2abx^2y^2=1-y^2-b^2xy^2+\cancel{b^2x^2y^4}$$

$a=1,\ b=0$ $\quad x^2=1-y^2\Rightarrow x^2+y^2=1$

$a=1,\ b=1$ $\quad x^2-2x^2y^2=1-y^2-xy^2$