

① Plane

$$(r - F \cdot P) \cdot \vec{n} = 0$$

② $a\hat{i} + b\hat{j} + (z+d)\hat{k} = 0$

$(a, b, c) = DR$ of Normal.

(3) $\alpha=0 \rightarrow$ Plane $\parallel x \text{ axis}$ (8) 2 Int. Lines

xy Plane \parallel y and z Plane

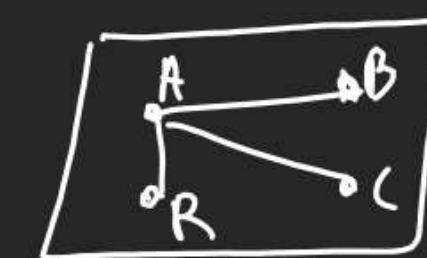
$$\Rightarrow z = d$$

(4) $P_1 \parallel P_2$

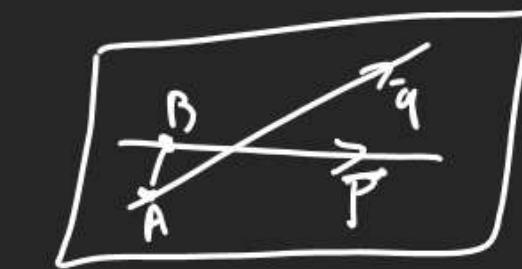
$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} + \frac{d_1}{d_2}$$

(5) $\theta = \frac{\vec{n}_{P_1} \cdot \vec{n}_{P_2}}{|\vec{n}_{P_1}| |\vec{n}_{P_2}|}$ (6) $P_1 \perp P_2$
 $a_1 a_2 + b_1 b_2 + c_1 c_2 = 0$ (11)

(7) $3h\vec{k}$



$$[AR \ AB \ A\vec{l}] = 0$$



$$[\vec{AB} \ \vec{P} \ \vec{Q}] = 0$$

(9) $\vec{r} = \vec{a} + t\vec{p} + s\vec{q}$
 Par. EOP $\vec{n} = \vec{p} \times \vec{q}$



Q Express EOP $\vec{r} = (1-2\lambda) + \lambda(2\hat{i} - \hat{j} + 3\hat{k})$

$$+ \mu(3\hat{i} + 4\hat{j} - \hat{k})$$

① in Scalar Dot Prod. form.

(2) in Cart. form.

$$\vec{r}$$

$$\vec{q}$$

here $\vec{r} = <1-2\lambda> + \lambda<2\hat{i} - \hat{j} + 3\hat{k}> + \mu<3\hat{i} + 4\hat{j} - \hat{k}>$

is Parametric form.

(2) normal vector $= \vec{n} = \begin{vmatrix} 1 & \hat{i} & \hat{k} \\ 2 & -1 & 3 \\ 3 & 4 & -1 \end{vmatrix}$

$$= <-11, 11, 11>$$

(3) $(r - <1, -2, 0>) \cdot <-1, 1, 1> = 0$

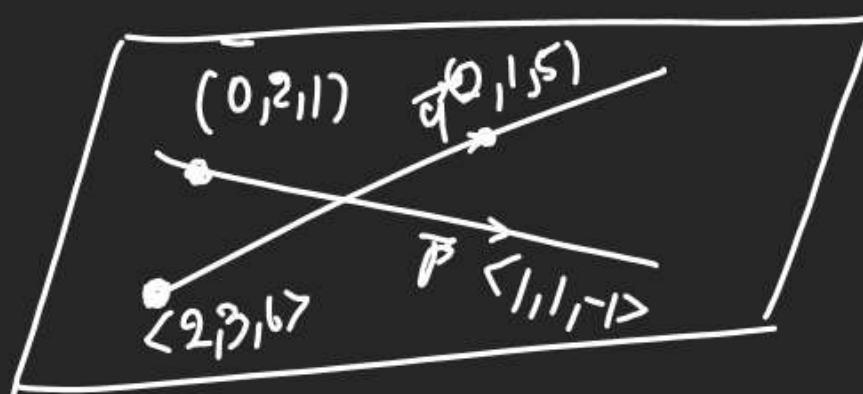
$$\vec{r} \cdot <-1, 1, 1> = -1 - 2 + 0$$

$\vec{r} \cdot <-1, 1, 1> = -3$ SDP form
 (4) (Cart. form)
 $-x + y + z = -3$

Q Find EOP containing Lines

$$L_1: \vec{r} = \langle 0, 2, 1 \rangle + \lambda \langle 1, 1, 5 \rangle$$

$$L_2: \vec{r} = \langle 2, 3, 6 \rangle + \mu \langle 2, 1, 5 \rangle$$



$$(\vec{r} - \langle 2, 3, 6 \rangle) \cdot \langle 6, -7, -1 \rangle = 0$$

$$\vec{r} \cdot \langle 6, -7, -1 \rangle = 12 - 21 - 6$$

$$\begin{aligned} n = \vec{P} \times \vec{q} &= \begin{vmatrix} i & j & k \\ 1 & 1 & -1 \\ 2 & 1 & 5 \end{vmatrix} \quad | \quad 6(-7) - (-15) \\ &= \langle 6, -7, -1 \rangle \end{aligned}$$

Eg of Plane in Normal Form.

$$a\lambda + b\mu + (z - d) \rightarrow \text{EOP}$$

$$\vec{n} = \langle a, b, c \rangle$$

If we convert $\langle a, b, c \rangle$ in $\langle l, m, n \rangle$
DR & DC

then EOP will be in Normal form.

$$P: a\lambda + b\mu + (z - d) \div \sqrt{a^2 + b^2 + c^2}$$

$$\frac{a}{\sqrt{a^2 + b^2 + c^2}} \lambda + \frac{b}{\sqrt{a^2 + b^2 + c^2}} \mu + \frac{c}{\sqrt{a^2 + b^2 + c^2}} z = \frac{d}{\sqrt{a^2 + b^2 + c^2}}$$

$$l\lambda + m\mu + n\lambda = p$$

$$3\lambda + 4\mu - 5 = 0 \text{ at } (0, 0) \text{ dist: } p = \frac{|(0, 0, 5)|}{\sqrt{3^2 + 4^2}} = \frac{5}{\sqrt{3^2 + 4^2}}$$

$$\vec{r} \cdot \vec{n} = d \quad (\text{Vector EOP}) \div |\vec{n}|$$

$$\vec{r} \cdot \hat{n} = \frac{d}{|\vec{n}|}$$

$$\vec{r} \cdot \hat{n} = p$$

This distance of
Plane from
origin

Q Find Dir. Cosines of Normal & find distance of plane $\vec{r} \cdot \langle 6\hat{i} - 3\hat{j} - 2\hat{k} \rangle + 1 = 0$ from $(0,0,0)$

$$\vec{r} \cdot \langle 6\hat{i} - 3\hat{j} - 2\hat{k} \rangle = -1$$

⊕ बताया

$$\vec{r} \cdot \langle -6\hat{i} + 3\hat{j} + 2\hat{k} \rangle = 1$$

$$\div \sqrt{6^2 + 3^2 + 2^2} = 1$$

$$\vec{r} \cdot \langle -\frac{6}{7}\hat{i} + \frac{3}{7}\hat{j} + \frac{2}{7}\hat{k} \rangle = \frac{1}{7}$$

↑
D.C. of Normal are $\langle -\frac{6}{7}, \frac{3}{7}, \frac{2}{7} \rangle$

Distance from origin = $\frac{1}{7}$

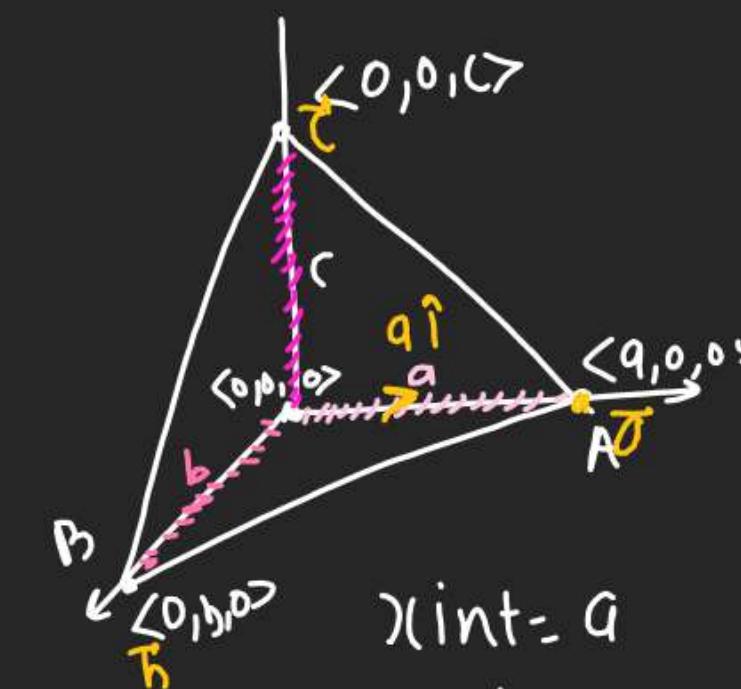
Rem: → distance of $ax+by+cz+d=0$ from:

$$\langle 0,0,1 \rangle = \frac{|d|}{\sqrt{a^2+b^2+c^2}}$$

Q Find \perp distance of $3x - 4y + z - 1 = 0$ from

$$d = \frac{|-1|}{\sqrt{9+16+1}} = \frac{1}{\sqrt{26}} \quad \langle 0,0,1 \rangle$$

Intercept form of Plane



$$x_{\text{int}} = a \\ y_{\text{int}} = b \\ z_{\text{int}} = c$$

1) $EOP \rightarrow \frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$

2) If Plane \parallel to $Z Axn$ \rightarrow Gf of $I = 0$

$$z \rightarrow \infty$$

$$\frac{x}{a} + \frac{y}{b} = 1$$

(4) $\Delta's \text{ Area} = \frac{1}{2} |\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}|$

$$= \frac{1}{2} |a\hat{i} \times b\hat{j} + b\hat{j} \times c\hat{k} + c\hat{k} \times a\hat{i}|$$

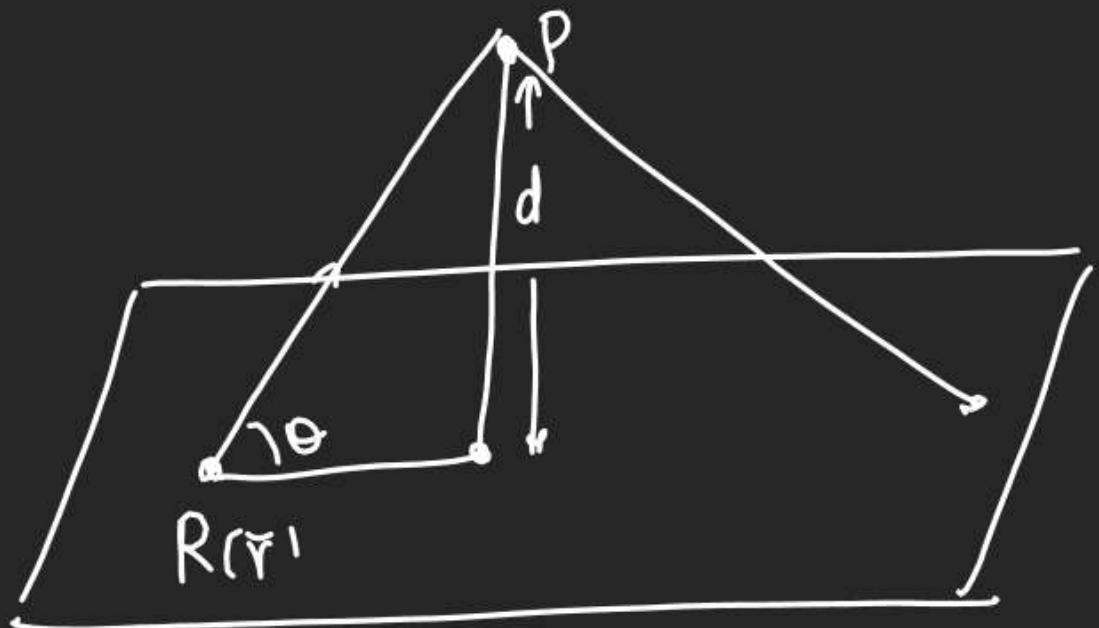
$$= \frac{1}{2} |ab\hat{k} + bc\hat{i} + ca\hat{j}|$$

$$A = \frac{1}{2} \sqrt{(ah)^2 + (bh)^2 + (ch)^2}$$

$$a=0$$

3) $ax+by+cz+d=0$ is \parallel to Axn

1^r distance of a Pt. from a Plane:



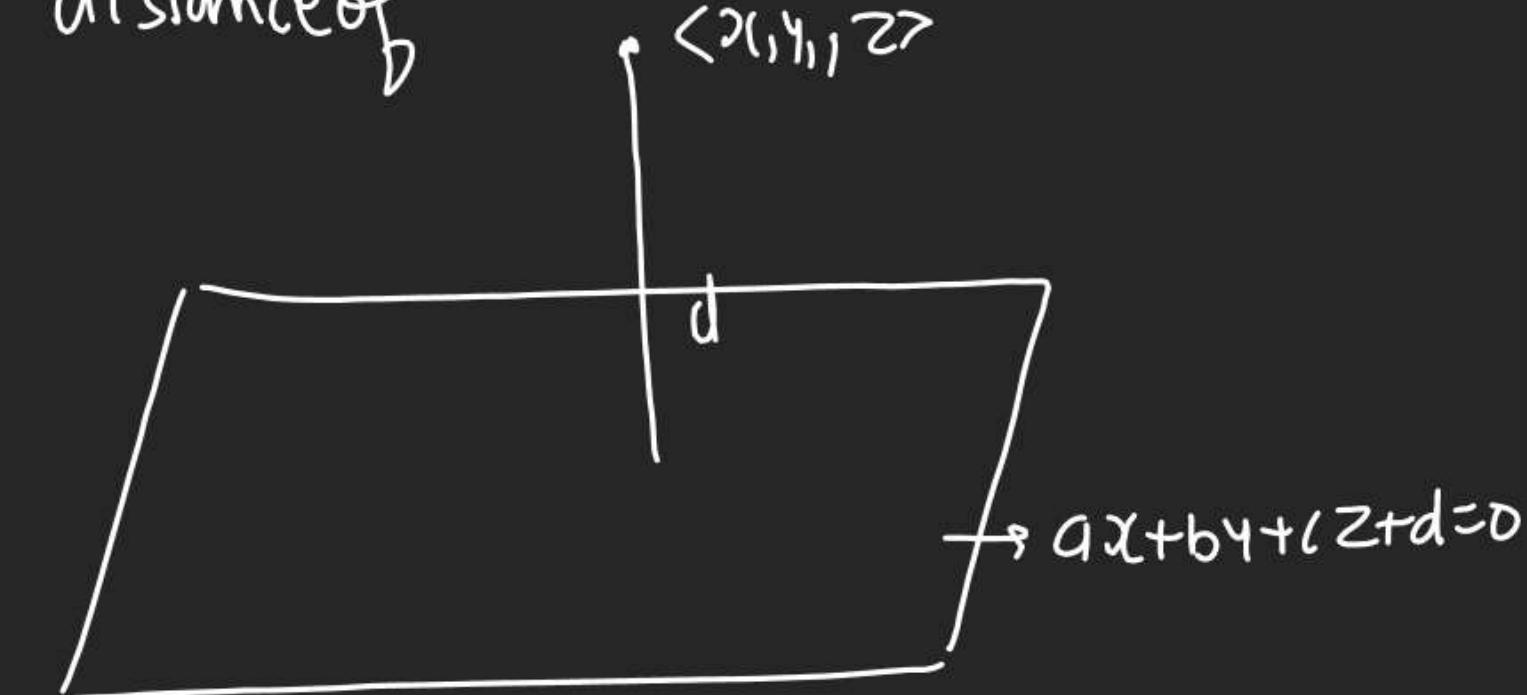
$$(\vec{r} - \vec{a}) \cdot \vec{n}$$

$$d = \text{proj of } \vec{RP} \text{ on } \vec{n}$$

$$\therefore \left| \frac{\vec{RP} \cdot \vec{n}}{|\vec{n}|} \right| = \left| \frac{(\vec{r} - \vec{P}) \cdot \vec{n}}{|\vec{n}|} \right|$$

$$d = \left| \frac{\vec{r} \cdot \vec{n} - \vec{P} \cdot \vec{n}}{|\vec{n}|} \right|$$

distance of
 $\langle x_1, y_1, z_1 \rangle$



$$\Rightarrow d = \frac{|ax_1 + by_1 + cz_1 + d|}{\sqrt{a^2 + b^2 + c^2}}$$

Q. 1^r dist. of $\langle 1, -1, 1 \rangle$ from $(\vec{r} - \langle 6, 7, -1 \rangle) \cdot \langle 2, 0, 1 \rangle = 0$

$$d = \left| \frac{2+0+1-1}{\sqrt{2^2+0^2+1^2}} \right| \quad 2x + 0y + z = 12 + 0 - 1 \\ = \frac{8}{\sqrt{5}} \quad 2x + z = 11$$

Q) Find EOP in which is a distance 4 from -

origin & $\vec{R} = \langle 2, -1, 2 \rangle$

$$\downarrow \quad \vec{n} = \langle a, b, c \rangle = \langle 2, -1, 2 \rangle \rightarrow |n| = 3$$

$$b = 4$$

$$\vec{v} \cdot \vec{n} = b$$

$$\hat{n} = \left\langle \frac{2}{3}, -\frac{1}{3}, \frac{2}{3} \right\rangle$$

$$\langle x, y, z \rangle \cdot \left\langle \frac{2}{3}, -\frac{1}{3}, \frac{2}{3} \right\rangle = 4$$

$$\frac{2x}{3} - \frac{y}{3} + \frac{2z}{3} = 4$$

$$\boxed{2x - y + 2z = 12}$$

Q) Reduce $x - 3y + 5z + 35 = 0$ in

Intercept form?

$$x - 3y + 5z = -35$$

$$\frac{x}{-35} + \frac{y}{35/3} + \frac{z}{-7} = 1 \quad \div (-35)$$

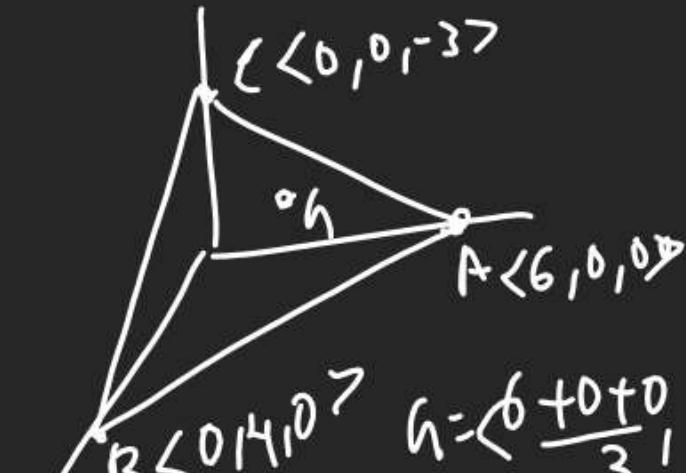
Q) If P(m, n, 2) & $2x + 3y - 4z = 12$

meet Coord Axes at A, B, C
find centroid of $\triangle ABC$

Intercept form

$$2x + 3y - 4z = 12 \quad \div 12$$

$$\frac{x}{6} + \frac{y}{4} + \frac{z}{-3} = 1$$



$$G = \left\langle \frac{6+0+0}{3}, \frac{0+4+0}{3}, \frac{0+0+(-3)}{3} \right\rangle \\ = \left\langle 2, \frac{4}{3}, -1 \right\rangle$$

Q) Find EOP P.T. $\langle 2, -1, 3 \rangle$

& 1st vector

$$\vec{b} = 3\hat{i} - \hat{k}, \vec{c} = -3\hat{i} + 2\hat{j} + 2\hat{k}$$

$$1) \vec{n} = \vec{b} \times \vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 0 & -1 \\ -3 & 2 & 2 \end{vmatrix}$$

$$= \langle 2, -3, 6 \rangle$$

$$2) 2(x-2) - 3(y+1) + 6(z-3) = 0$$

$$2x - 3y + 6z = 4 + 3 + 18$$

$$2x - 3y + 6z = 25$$

Distance between 2 parallel planes.

$$P_1: ax+by+cz+d_1=0$$

$$P_2: ax+by+cz+d_2=0$$

$$d = \frac{|d_1 - d_2|}{\sqrt{a^2 + b^2 + c^2}}$$

Q Find plane P_1' to $P_1: x+5y-4z+5=0$

& sum of whose intercepts on 3 axes
is 19 then find distance between these
2 planes.

① Plane $P_1': x+5y-4z+5=0$

$$P_2: x+5y-4z=\lambda$$

(2) Intercept form

$$\frac{x}{1} + \frac{y}{5} + \frac{z}{-4} = 1$$

$$3) \lambda + \frac{\lambda}{5} + \frac{-\lambda}{4} = 19$$

$$20\lambda + 4\lambda - 5\lambda = 380$$

$$19\lambda = 380$$

$$\lambda = 20$$

$$\therefore \text{parallel plane } P_2: x+5y-4z=20$$

$$x+5y-4z-20=0$$

(4) dist. betw P_1 & P_2

$$d = \frac{|5 - (-20)|}{\sqrt{1+25+16}} = \frac{25}{\sqrt{42}}$$

$$P_2: 2x-6y+3z=-5$$

$$2x-6y+3z=-33$$

Q Find plane P_1' to $P_1: 2x-6y+3z=0$

at a distance of 2 units from

$$\text{pt. m: } \langle 1, 2, -3 \rangle$$

parallel plane $\rightarrow P_2: 2x-6y+3z=\lambda$

dist. of P_2 from m " $\langle 1, 2, -3 \rangle = 2$

$$\frac{|2-12-9-\lambda|}{\sqrt{4+36+9}} = 2$$

$$|\lambda+19|=14$$

$$\lambda+19=14 \quad \lambda+19=-14$$

$$\lambda=-5, \quad \lambda=-33$$

Q) A Plume which remains at a constnt

distance b from origin cuts coord

Axes at A, B, C. Find Locus of

A) (centroid of $\triangle ABC$)

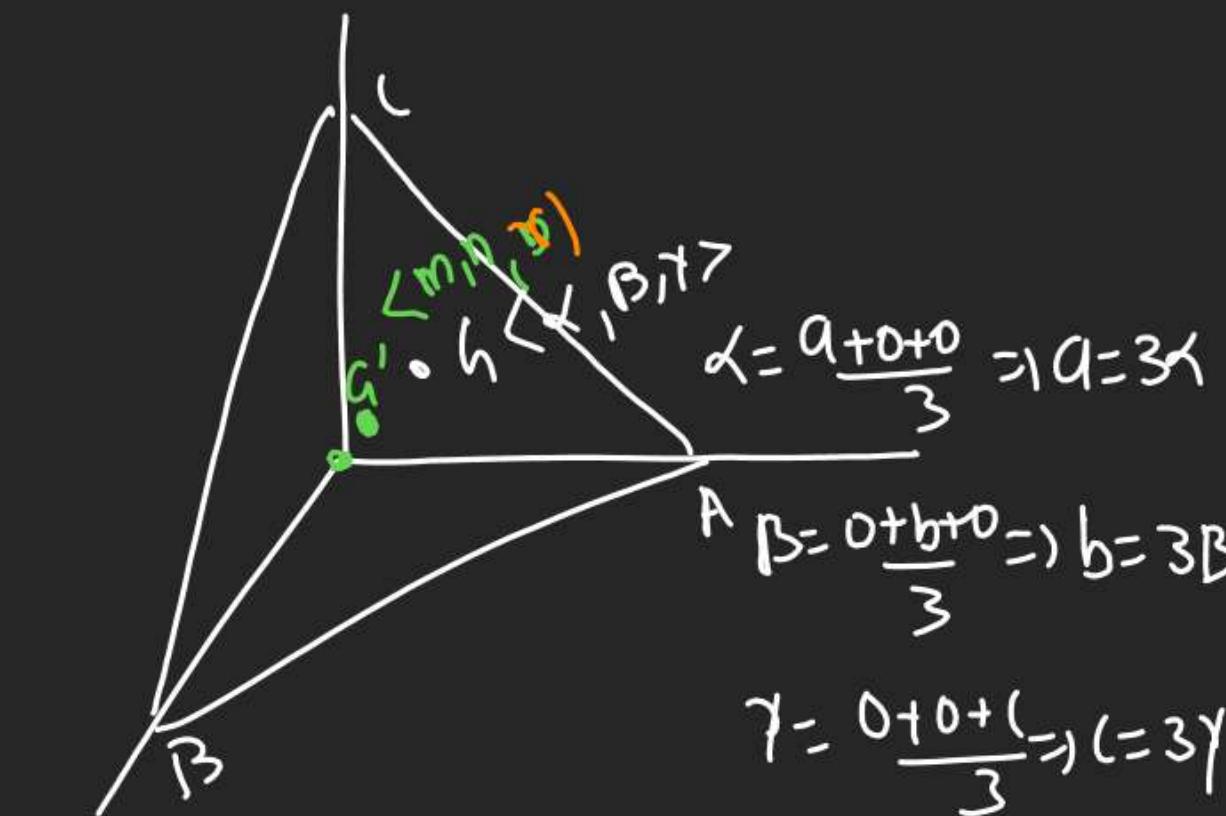
B) (centre of tetrahedron OABC)

$$1) \text{ Let Plane is } \frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$$

b = dist. from $\langle 0, 0, 0 \rangle$

$$b = \sqrt{\frac{|0+0+0-1|}{\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2}}}$$

$$\sqrt{\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2}} = \frac{1}{b} \Rightarrow \boxed{\frac{1}{b^2} = \frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2}}$$



$$a = \frac{a+b+c}{3} \Rightarrow a = 3x$$

$$b = \frac{a+b+c}{3} \Rightarrow b = 3y$$

$$c = \frac{a+b+c}{3} \Rightarrow c = 3z$$

$$\frac{1}{b^2} = \frac{1}{(3x)^2} + \frac{1}{(3y)^2} + \frac{1}{(3z)^2}$$

$$\boxed{y/b^2 = x^2 + y^2 + z^2} \quad \text{Board}$$

(3) $G' = \langle m, n, r \rangle$ - (centroid of tetrahedron).

$$m = \frac{0+a+b+c}{4}, n = \frac{0+a+b+c}{4}, r = \frac{0+a+b+c}{4}$$

$$a = 4m, b = 4n, c = 4r$$

$$\frac{1}{b^2} = \frac{1}{(4m)^2} + \frac{1}{(4n)^2} + \frac{1}{(4r)^2}$$

$$16b^{-2} = m^2 + n^2 + r^2$$

$$\boxed{16b^{-2} = x^2 + y^2 + z^2}$$