Nishant Jindal

Projectile Motion

(*) Another form of trajectory Equation:-

$$y = (x\tan\theta) - \left(\frac{g}{2u^2\cos^2\theta}\right)x^2$$

$$y = x\tan\theta\left[1 - \frac{g x^2}{2u^2\cos^2\theta \times x\tan\theta}\right]$$

$$y = x\tan\theta\left[1 - \frac{g \cancel{x^2}}{2u^2\cos^{\cancel{2}}\theta \times \cancel{x}\left(\frac{\sin\theta}{\cancel{\cos\theta}}\right)}\right]$$

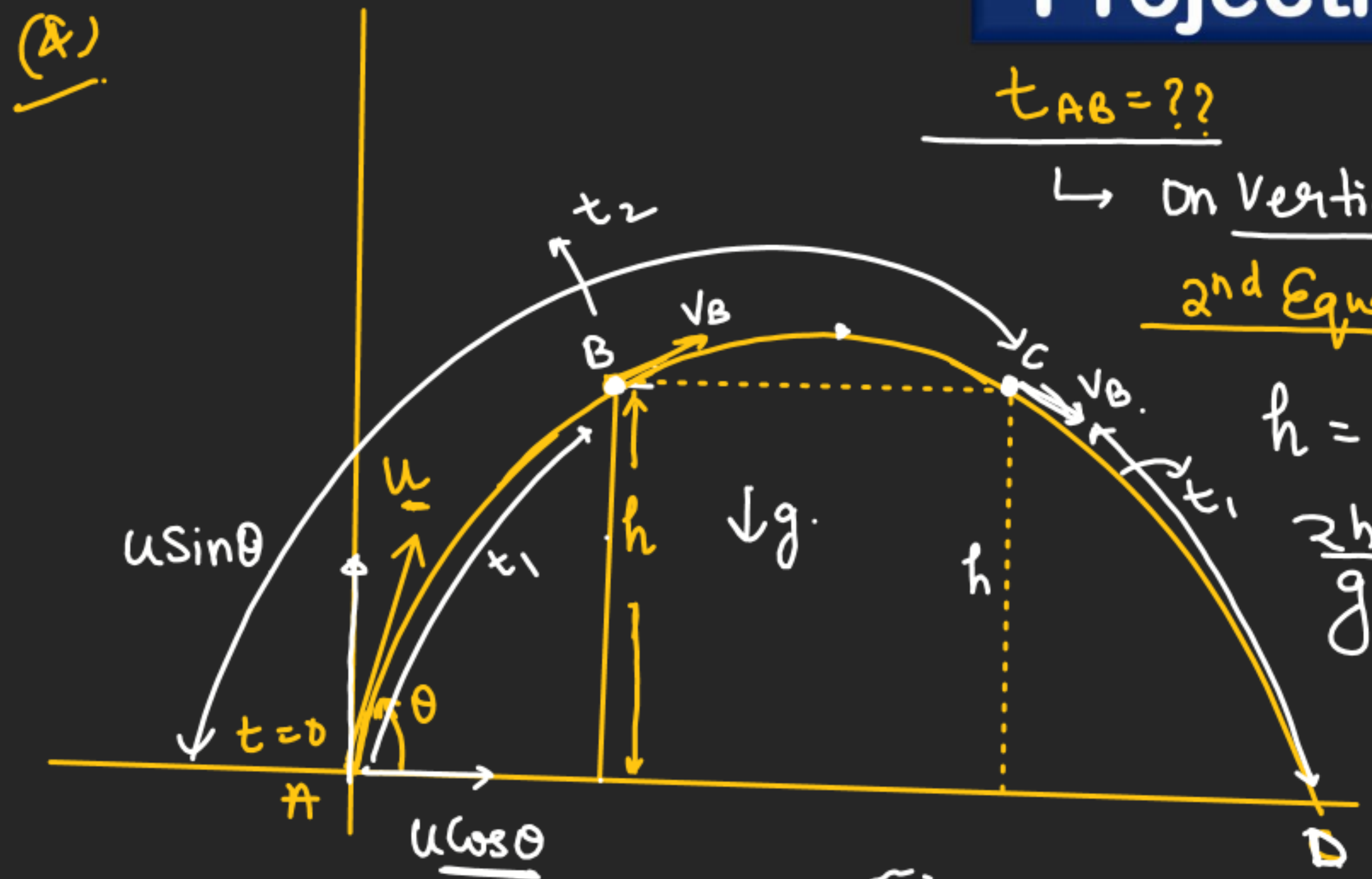
$$y = x\tan\theta\left[1 - \frac{x}{\underbrace{\left(\frac{2u^2\sin\theta\cos\theta}{g}\right)}_{R}}\right]$$

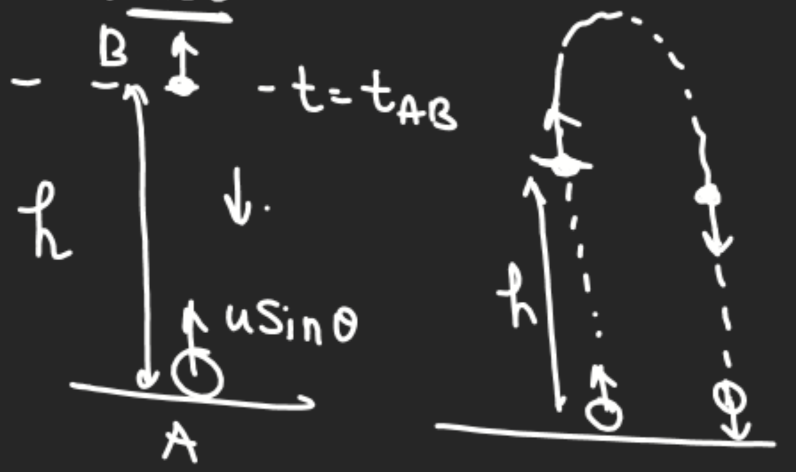
**

$$\boxed{y = x\tan\theta\left[1 - \frac{x}{R}\right]}$$

Projectile Motion

(A)





$t_{AB} = ??$

↳ On Vertical Motion

2nd Equation

$$h = (u\sin\theta)t - \frac{1}{2}gt^2$$

$$\frac{2h}{g} = \left(\frac{2u\sin\theta}{g}\right)t - t^2$$

$$t^2 - \left(\frac{2u\sin\theta}{g}\right)t + \frac{2h}{g} = 0$$

↳ let, t_1 and t_2 be two roots.

$$t_1 + t_2 = \left[\frac{2u\sin\theta}{g}\right] > 0$$

$$t_1 t_2 = \left(\frac{2h}{g}\right) > 0$$

$\Rightarrow$ t_1 & t_2 both +ve.

let, $t_1 < t_2$

$$t_{AB} = t_{CD} = t_1$$

$$t_{ABC} = t_2$$

$$t_{BC} = t_{ABC} - t_{AB} = (t_2 - t_1)$$

$$(t_2 - t_1) = \sqrt{(t_1+t_2)^2 - 4t_1t_2}$$

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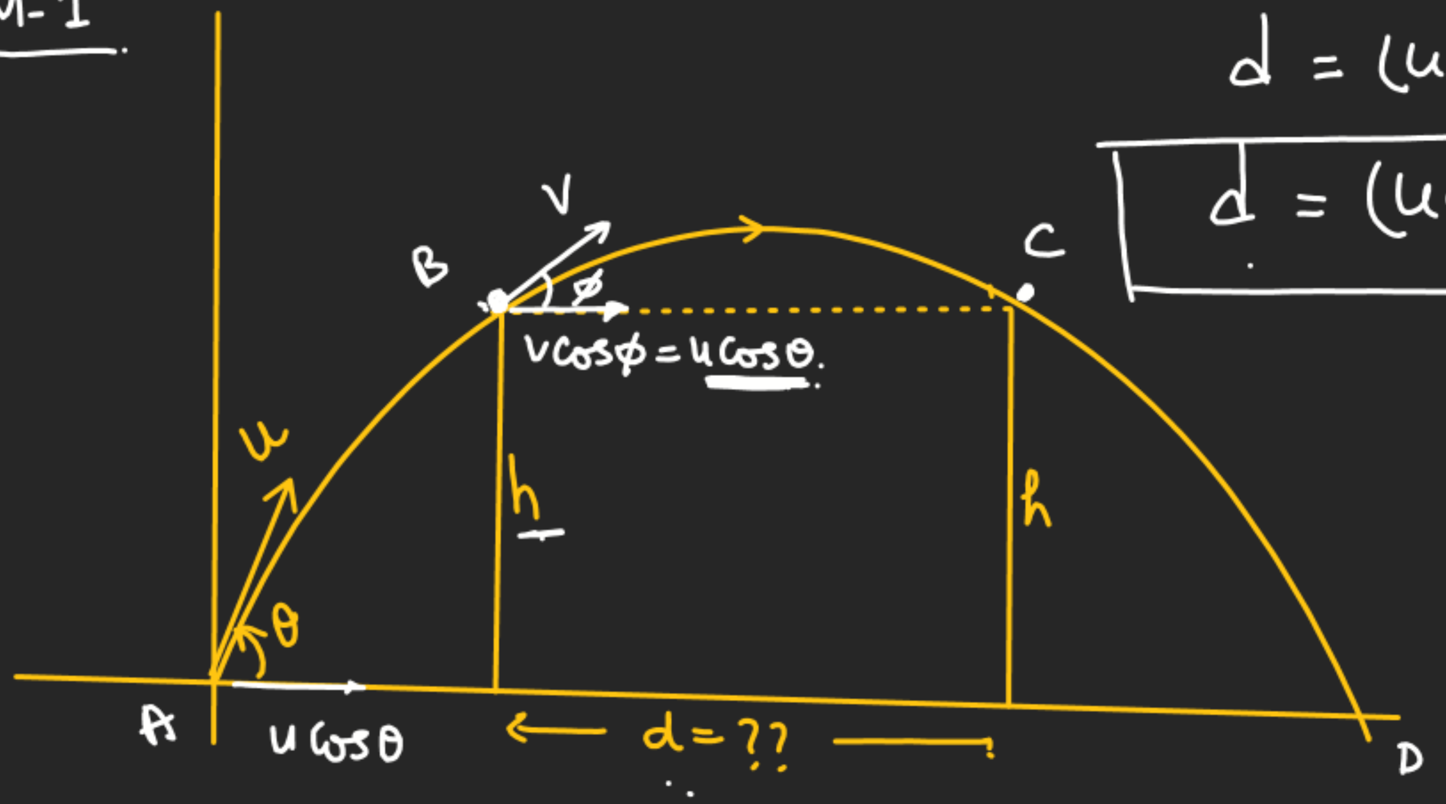
Projectile Motion

\#. Find distance b/w the two towers.

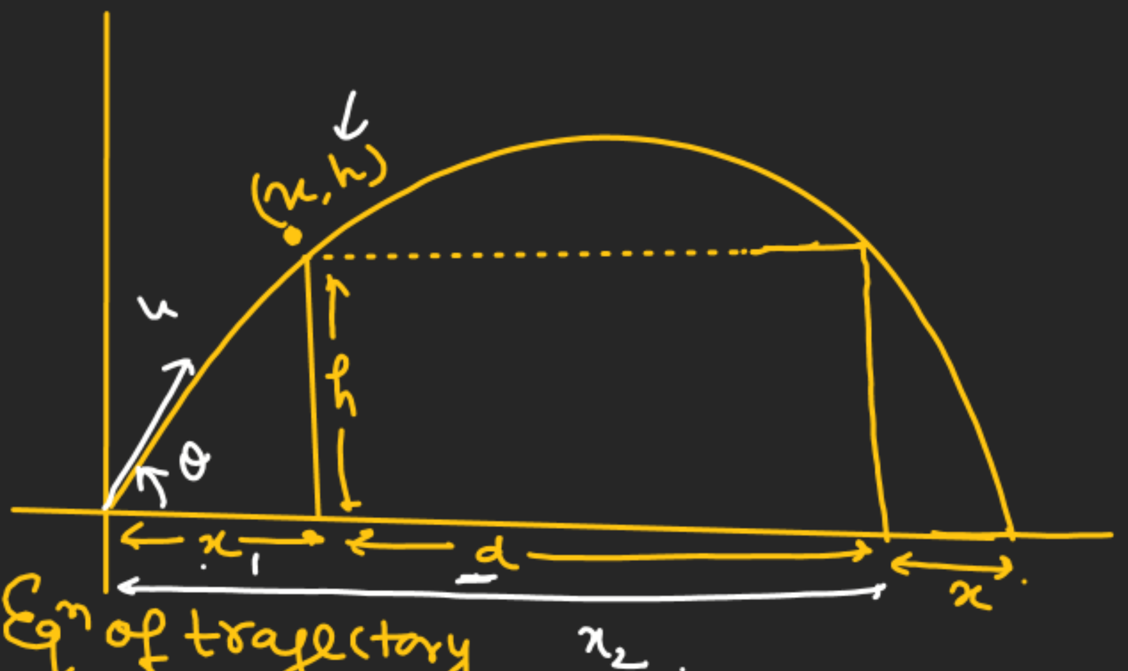
M-1



$$d = (u\cos\theta) \times t_{BC}$$

$$\boxed{d = (u\cos\theta) \times (t_2 - t_1)}$$

M_2 :-



Eq^n of trajectory

$$h = x\tan\theta - \frac{g}{2u^2\cos^2\theta}x^2$$

$$\left(\frac{g}{2u^2\cos^2\theta}\right)x^2 - x\tan\theta + h = 0$$

↳ two roots of x let, x_1 and x_2 be the two roots.

Find x_1 and x_2

$$\boxed{d = (x_2 - x_1)}$$

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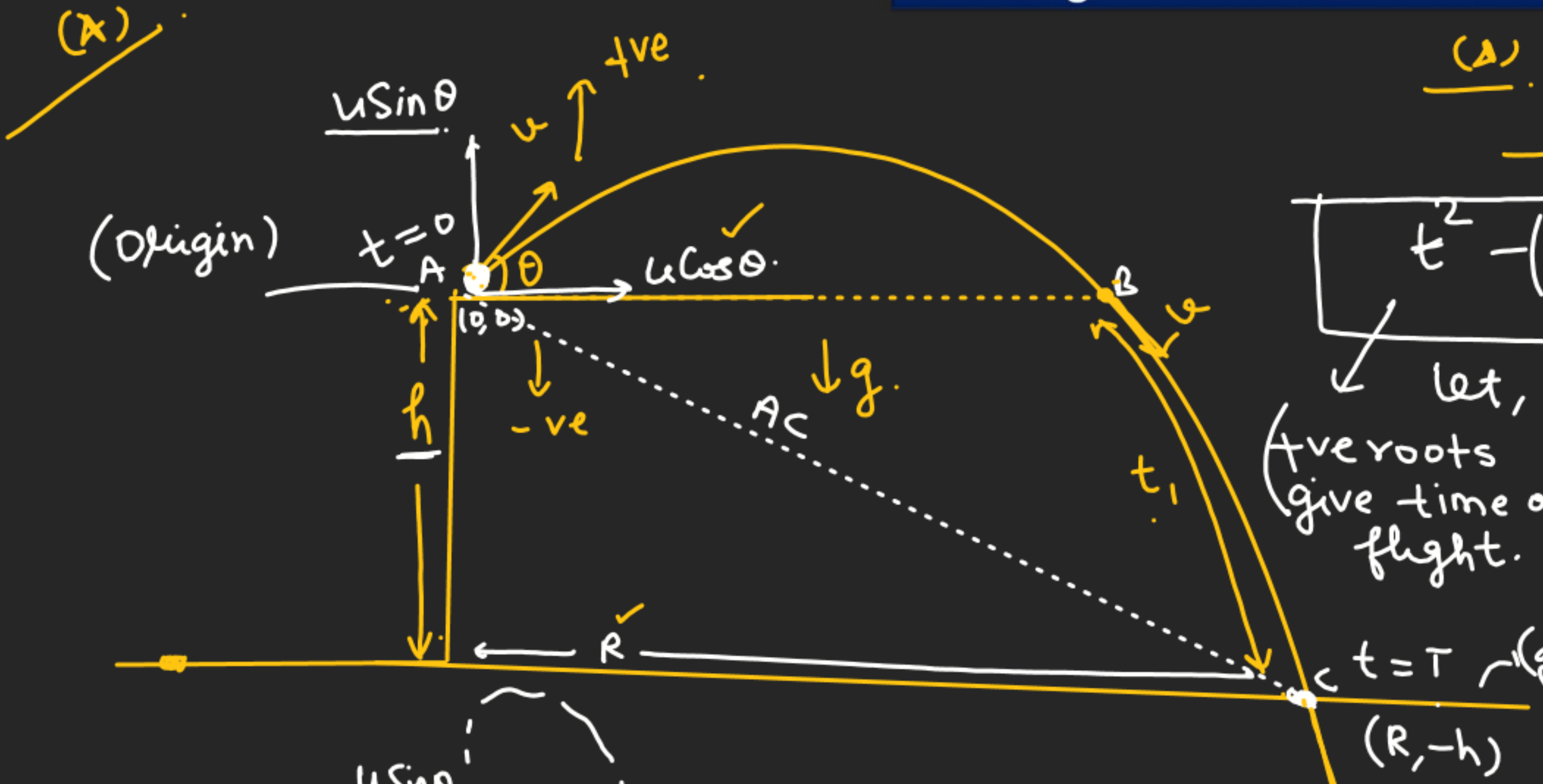
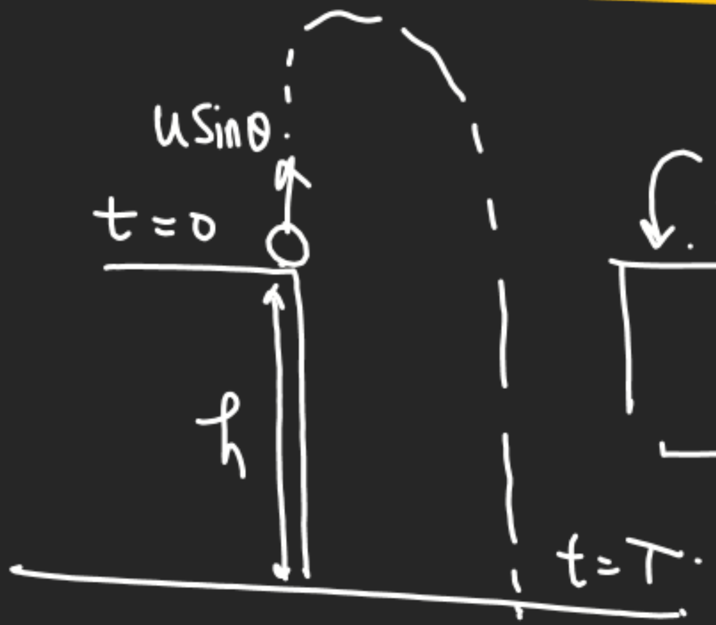
Projectile Motion





AC = (Displacement)

$$\boxed{AC = \sqrt{R^2 + h^2}}$$

(a) In -y direction

$$-h = (u\sin\theta)\,t - \frac{1}{2}gt^2$$

$$\boxed{t^2 - \left(\frac{2u\sin\theta}{g}\right)t - \frac{2h}{g} = 0}$$

let, t_1 and t_2 be two roots.

(+ve roots give time of flight.)

$t_1 t_2 = \left(-\frac{2h}{g}\right) < 0$

↳ t_1 & t_2 be two roots.

let, $t_1 \to -ve$, $t_2 \to +ve$

$t_{AB} = [t_2 - |t_1|]$

Range ⇓ R $= (u\cos\theta) \times t_2$

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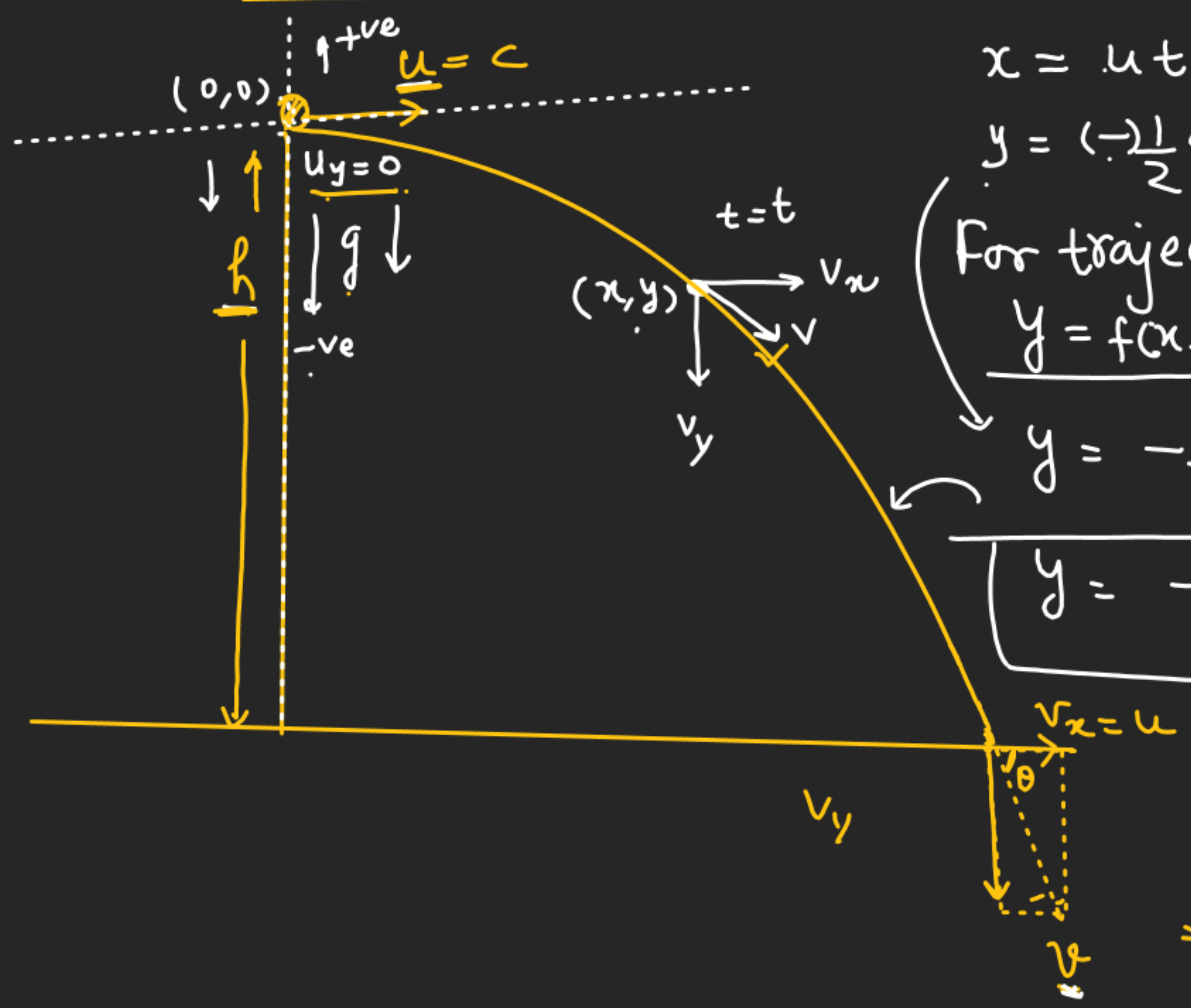
Projectile Motion

(A)

Case of horizontal projectile :-



$x = ut$ — ① $\Rightarrow t = \frac{x}{u}$

$y = (-)\frac{1}{2}gt^2$ — ②

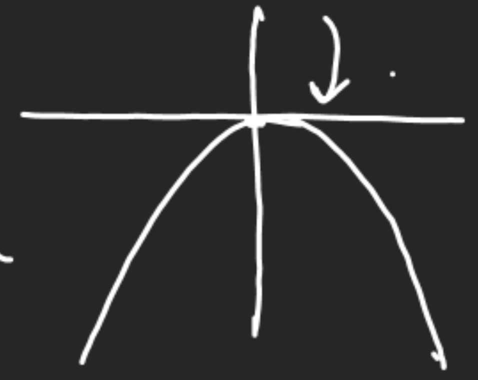
For trajectory

$y = f(x)$

$y = -\frac{1}{2}g\left(\frac{x}{u}\right)^2$

$$\boxed{y = -\frac{g}{2u^2}x^2}$$

$x^2 = -4ay$



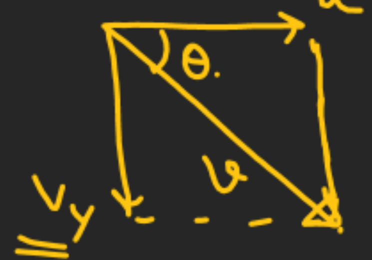
By 3rd Equation

$[v_y^2 = u_y^2 + 2(-g)(-h)]$

$u_y \Rightarrow 0$

$v_y = (\sqrt{2gh})$

$v = \sqrt{v_x^2 + v_y^2} = \sqrt{u^2 + 2gh}$ ✓



$\Rightarrow \tan\theta = \left(\frac{v_y}{u}\right)$

$\theta = \tan^{-1}(v_y/u)$

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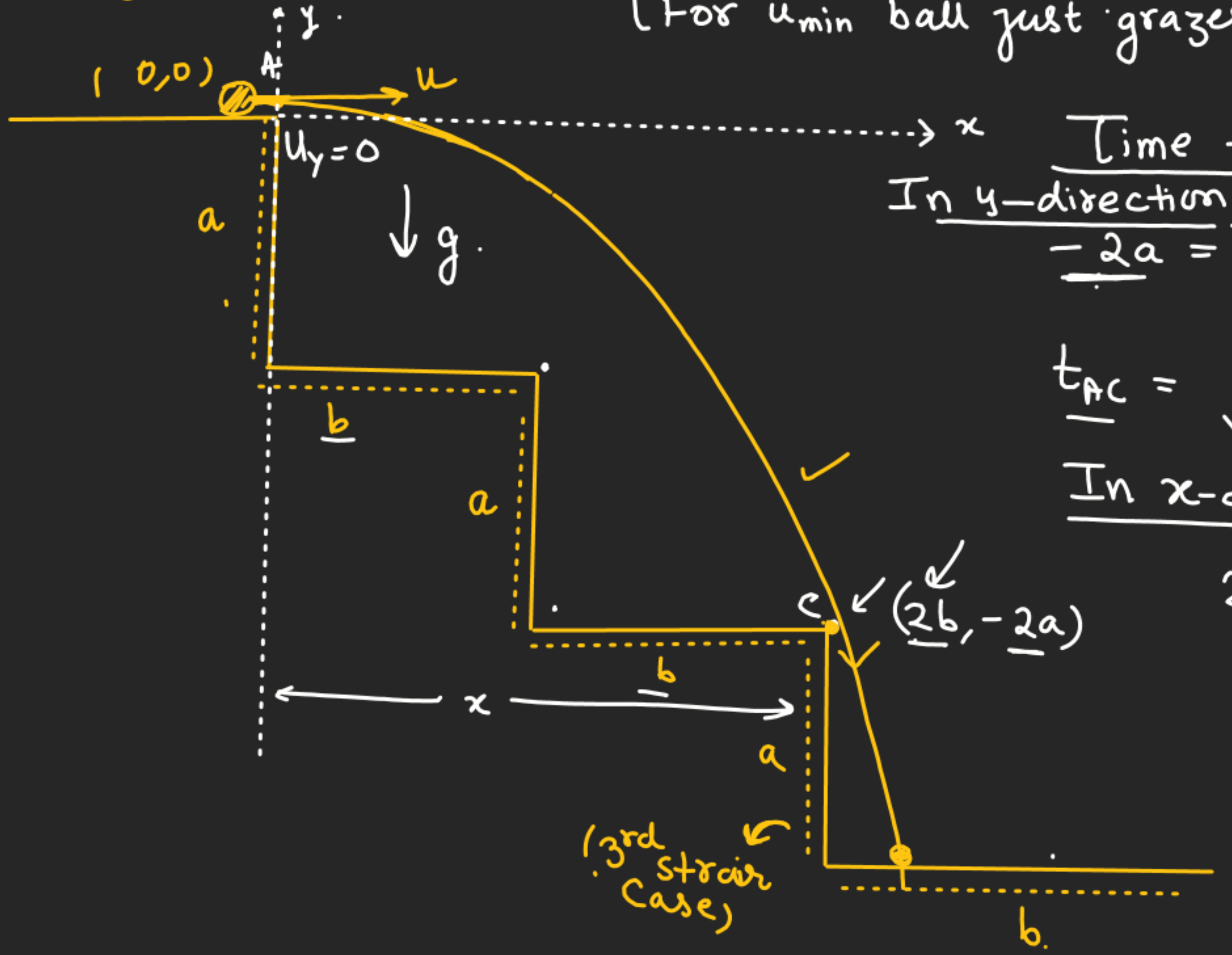
Projectile Motion

Find min 'u' So that ball hit the 3rd Stair-Case.

[For u_{min} ball just grazes point c]



H-W

↳ [generlized this for n-Stair case]

Time for AC motion

In y-direction

$$-2a = \frac{1}{2}(-g)\, t_{AC}^2 \Rightarrow ??$$

$$t_{AC} = \sqrt{\frac{4a}{g}} = 2\sqrt{\frac{a}{g}}$$

In x-direction

$$2b = u \times t_{AC}$$

$$u = \frac{2b}{(t_{AC})} = \frac{2b}{2\sqrt{a/g}}$$

$$u = b\sqrt{g/a}$$ Ans

Min u

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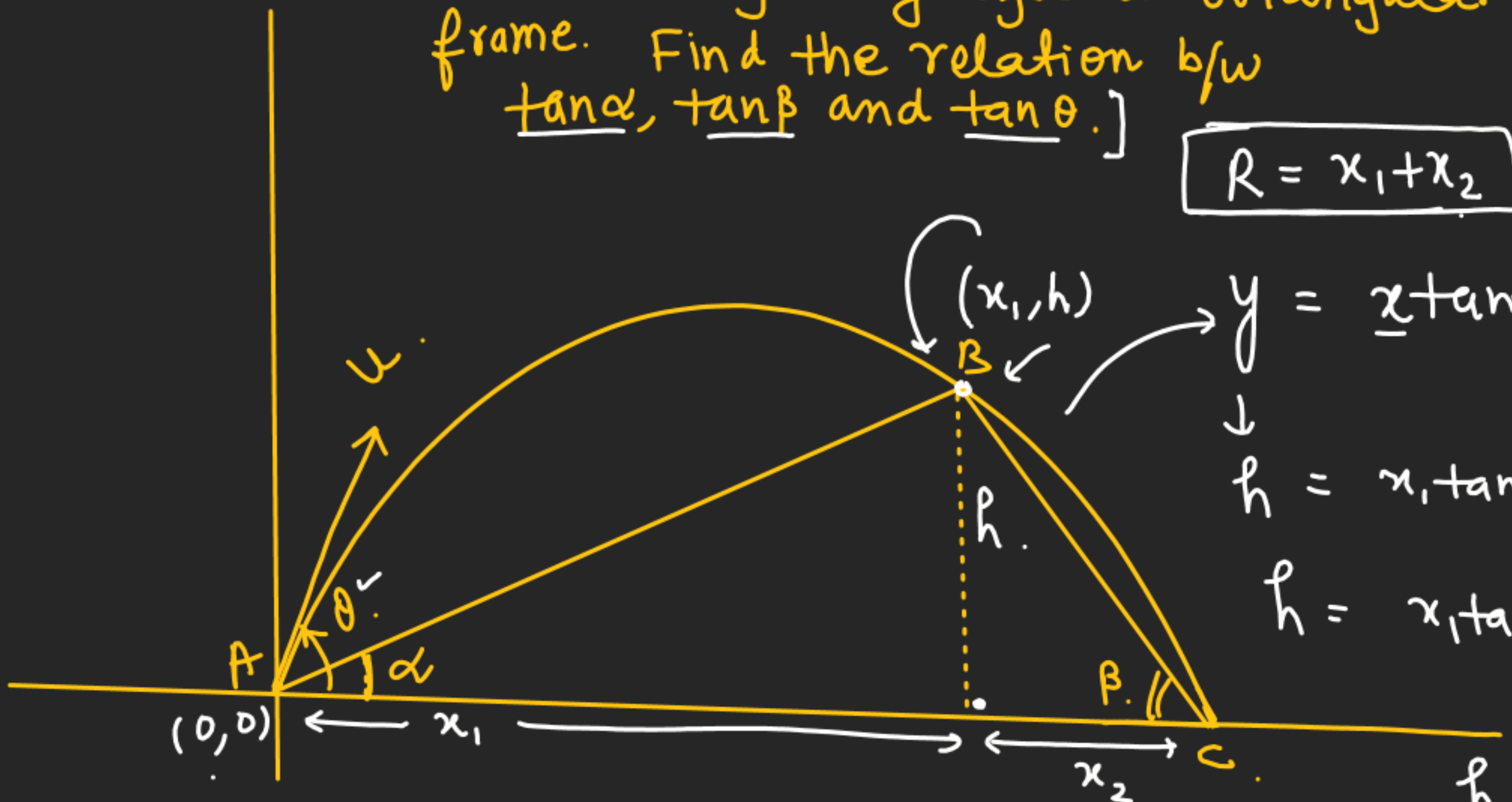
Projectile Motion

(※)

[Projectile is projected in such a way so that it just grazes a triangular frame. Find the relation b/w $\tan\alpha$, $\tan\beta$ and $\tan\theta$.]



$$\boxed{R = x_1 + x_2}$$

$$y = x\tan\theta\left[1 - \frac{x}{R}\right]$$

$$h = x_1\tan\theta\left[1 - \frac{x_1}{x_1+x_2}\right]$$

$$h = x_1\tan\theta\left[\frac{x_2}{x_1+x_2}\right]$$

$$\tan\alpha = \left(\frac{h}{x_1}\right) \Rightarrow x_1 = \left(\frac{h}{\tan\alpha}\right)$$

$$\tan\beta = \left(h/x_2\right) \Rightarrow x_2 = \frac{h}{\tan\beta}$$

$$h = \tan\theta\left[\frac{x_1 x_2}{x_1 + x_2}\right] \Rightarrow \boxed{\tan\alpha + \tan\beta = \tan\theta}$$

Ans

$$h = \tan\theta\left[\frac{\frac{h}{\tan\alpha}\cdot\frac{h}{\tan\beta}}{\frac{h}{\tan\alpha} + \frac{h}{\tan\beta}}\right]$$