Nishant Jindal

$$\int \frac{dx}{\text{Linear}\sqrt{\text{Linear}}}$$

$$\int \frac{dx}{\text{Quad}\sqrt{\text{Linear}}}$$

$\text{Linear} = t^2$

$$\int \frac{dx}{\text{Linear}\sqrt{\text{Quad}}} \longrightarrow \text{Linear} = \frac{1}{t}$$

$$\int \frac{dx}{\text{Quad}\sqrt{\text{Quad}}} \longrightarrow x = \frac{1}{t}$$

Q $$\int \frac{dx}{(x-2)\sqrt{x+1}}$$

$x+1 = t^2$

$dx = 2t\,dt$

$$\int \frac{2\cancel{t}\,dt}{(t^2-3)\sqrt{\cancel{t^2}}} = 2\int \frac{dt}{t^2-\sqrt{3}^2}$$

$$= \frac{2}{2\sqrt{3}} \ln\left|\frac{t-\sqrt{3}}{t+\sqrt{3}}\right| + C$$

Q $$\int \frac{dx}{x^2\sqrt{x^2+4}} \rightarrow \int \frac{dx}{Q\sqrt{Q}}$$

$x = \frac{1}{t}$

$dx = -\frac{1}{t^2}\,dt$

$$\int \frac{-\frac{1}{\cancel{t^2}}\,dt}{\frac{1}{\cancel{t^2}}\sqrt{\frac{1}{t^2}+4}}$$

$$-\int \frac{t\,dt}{\sqrt{1+(2t)^2}} = -\int \frac{t\cdot dt}{\sqrt{1+4t^2}}$$

$1+4t^2 = z$

$8t\,dt = dz$

$t\,dt = \frac{dz}{8}$

$$-\int \frac{dz}{\sqrt{\cancel{z}}}$$

$-2\sqrt{z} + C$

$-2\sqrt{1+4t^2} + C$

$-2\sqrt{1+\frac{4}{x^2}} + C$

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Q $\int \frac{dx}{(x+1)\sqrt{1-x^2}} \to \int \frac{dx}{L\sqrt{Q}}$

Linear $= x+1 = \frac{1}{t} \Rightarrow x = \frac{1}{t} - 1 =$

$dx = -\frac{1}{t^2}dt$

$$\int \frac{-\frac{1}{t^2}dt}{\frac{1}{t}\sqrt{1-\frac{(1-t)^2}{t^2}}} = -\int \frac{dt}{\sqrt{t^2-(1-t)^2}}$$

$$-\int \frac{dt}{\sqrt{\cancel{t^2}-(1+\cancel{t^2}-2t)}}$$

$$-\int \frac{dt}{\sqrt{2t-1}} = -\frac{\cancel{2}\sqrt{2t-1}}{\cancel{2}} + C$$

$$\therefore = -\sqrt{\frac{2}{x+1}-1} + C$$

Q $\int \frac{(x+2)\,dx}{(x^2+3x+3)\sqrt{x+1}} \to \int \frac{dx}{Q\sqrt{L}}$

$x+1 = t^2$

$dx = 2t\,dt$

$$\int \frac{(t^2+1)\cdot 2\cancel{t}\,dt}{((t^2-1)^2+3(t^2-1)+3)\sqrt{\cancel{t^2}}}$$

$$2\int \frac{(t^2+1)\,dt}{t^4+t^2+1} \quad \{\text{Future}\}$$

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Q $\int \frac{x^2\,dx}{(x-1)\sqrt{x+2}} \to \int \frac{dx}{L\sqrt{L}}$

$x+2=t^2$

$dx=2t\,dt$

$\int \frac{(t^2-2)^2 \cdot 2\cancel{t}\,dt}{(t^2-3)\sqrt{\cancel{t^2}}}$

$2\int \frac{t^4-4t^2+4}{t^2-3}\,dt$

$$\begin{array}{r} t^2-3\,\big)\,t^4-4t^2+4\,\big(\,t^2-1 \\ t^4-3t^2 \\ \hline -t^2+4 \\ -t^2+3 \\ \hline 1 \end{array}$$

$2\int t^2-1+\frac{1}{t^2-3}$

$\frac{2t^3}{3}-2t+\frac{1}{2\sqrt{3}}\ln\left|\frac{t-\sqrt{3}}{t+\sqrt{3}}\right|+C$

Q $\int \frac{dx}{(x^3+3x^2+3x+1)\sqrt{x^2+2x-3}} \to \int \frac{dx}{\text{Cubic}\sqrt{\text{Quad}}}$

$\int \frac{dx}{(x+1)^3\sqrt{(x+1)^2-4}}$ $\quad x+1=\frac{1}{t}$

$\int \frac{-\frac{1}{\cancel{t^2}}dt}{\frac{1}{\cancel{t^3}}\sqrt{\frac{1}{t^2}-4}}$ $\quad dx=-\frac{1}{t^2}dt$

$-\int \frac{t^2\,dt}{\sqrt{1-4t^2}} = \frac{1}{4}\int \frac{(1-4t^2)-1}{\sqrt{1-4t^2}}$

$=\frac{1}{4}\int\sqrt{1-4t^2}-\frac{1}{4}\int\frac{dt}{\sqrt{1-4t^2}}$

$=\frac{1}{2\times 4}\left[\frac{2t}{2}\sqrt{1-4t^2}+\frac{1}{2}\sin^{-1}\frac{2t}{1}\right]-\frac{1}{4}\times\sin^{-1}\frac{2t}{1}+C$

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V. Imp $\int \frac{x^2 \cdot dx}{x^4 \pm Kx^2+1}$

1. Id:- Qs must be having Even deg of x in Nr & Dr.

2. Method $\div$ by x^2

Q $\int \frac{x^2+1 \cdot dx}{x^4+1}$ $\div x^2$

$I = \int \frac{\left(1+\frac{1}{x^2}\right)dx}{x^2+\frac{1}{x^2}}$

$\int \frac{1+\frac{1}{x^2}\,dx}{\left(x^2+\frac{1}{x^2}-2\right)+2}$

$\Rightarrow \int \frac{1+1/x^2\,dx}{\left(x-1/x\right)^2+(\sqrt{2})^2}$

$x-\frac{1}{x}=t$

$\left(1+\frac{1}{x^2}\right)dx=dt$

$I = \int \frac{dt}{t^2+\sqrt{2}^2}$

$= \frac{1}{\sqrt{2}} \tan^{-1}\frac{t}{\sqrt{2}}+C$

$\int \frac{(x^2+1)dx}{x^4+1} = \int \frac{dt}{t^2+2}$ $t = x-\frac{1}{x}$

Q $I = \int \frac{(x^2+1)\,dx}{x^4+3x^2+1}$

$= \int \frac{dt}{t^2+2+3} = \int \frac{dt}{t^2+\sqrt{5}^2} = \frac{1}{\sqrt{5}} \tan^{-1}\frac{t}{\sqrt{5}}+C$

$t = x-1/x$

Q $I = \int \frac{x^2+1 \cdot dx}{x^4-9x^2+1}$

$= \int \frac{dt}{t^2+2-9} = \int \frac{dt}{t^2-\sqrt{7}^2} = \frac{1}{2\sqrt{7}} \ln\left|\frac{t-\sqrt{7}}{t+\sqrt{7}}\right|+C$

$t = x-1/x$

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2nd

Q $I=\int \frac{x^2-1}{x^4+1}\,dx$ $\quad \div x^2$

$$=\int \frac{1-\frac{1}{x^2}\,dx}{\left(x^2+\frac{1}{x^2}+2\right)-2}$$

$$=\int \frac{1-\frac{1}{x^2}\,dx}{\left(x+\frac{1}{x}\right)^2-\sqrt{2}^2}$$

$x+\frac{1}{x}=t$

$1-\frac{1}{x^2}dx=dt$

$$=\int \frac{dt}{t^2-\sqrt{2}^2}=\frac{1}{2\sqrt{2}}\ln\left|\frac{t-\sqrt{2}}{t+\sqrt{2}}\right|+c$$

$t=x+\frac{1}{x}$

Q Step by Step

$I=\int \frac{x^2}{x^4+1}\cdot dx$

$$=\frac{1}{2}\int \frac{2x^2\cdot dx}{x^4+1}$$

$$=\frac{1}{2}\int \frac{x^2+1}{x^4+1}+\frac{x^2-1}{x^4+1}\cdot dx$$

Q 1, Q 4

Q $I=\int \frac{1}{x^4+1}\,dx$

$$=\frac{1}{2}\int \frac{2\cdot dx}{x^4+1}$$

$$=\frac{1}{2}\int \frac{(x^2+1)}{x^4+1}-\frac{(x^2-1)}{x^4-1}$$

Q_1 Q 4

Q $I=\int \sqrt{\tan x}\cdot dx$

$\tan x=t^2$

$\sec^2 x\cdot dx=2t\,dt$

$dx=\frac{2t\,dt}{1+\tan^2 x}$

$dx=\frac{2t\,dt}{1+t^4}$

$$I=2\int \frac{\sqrt{t^2}\cdot t\,dt}{1+t^4}$$

$$=\int \frac{t^2+1}{t^4+1}+\frac{t^2-1}{t^4+1}\cdot dt$$

Q 1 Q 4

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Q $I = \int \frac{dx}{\cos^6 x + \sin^6 x}$

$= \int \frac{dx}{1 - 3\sin^2 x \cdot \cos^2 x} \div \cos^4 x$

$= \int \frac{(1+\tan^2 x)\cdot \sec^2 x \, dx}{(1+\tan^2 x)^2 - 3\tan^2 x}$ $\quad \tan x = t$

$\sec^2 x\, dx = dt$

$\int \frac{(t^2+1)\,dt}{(1+t^2)^2 - 3t^2}$

$\int \frac{x^2+1}{x^4+1}$ $\quad \int \frac{x^2-1}{x^4+1}$

$\int \frac{dt}{t^2-2}$

$t = x + \frac{1}{x}$

Q1 $\int \frac{t^2+1\, dt}{t^4 - t^2 + 1} = \int \frac{dz}{z^2+2-1}$ $\quad z = t - \frac{1}{t}$

$\int \frac{dz}{1+z^2} = \tan^{-1} z = \tan^{-1}\left(t - \frac{1}{t}\right) + C$

$= \tan^{-1}(\tan x - \cot x) + C$

Q $I = \int \frac{e^x \cdot dx}{e^{4x} + e^{2x} + 1}$, $J = \int \frac{e^{-x}\, dx}{e^{-4x} + e^{-2x} + 1}$ then $J - I = ?$

$, J = \int \frac{e^{3x}\, dx}{e^{4x} + e^{2x} + 1}$

$J - I = \int \frac{e^{3x} - e^x \cdot dx}{e^{4x} + e^{2x} + 1}$

$= \int \frac{e^x (e^{2x} - 1)\cdot dx}{e^{4x} + e^{2x} + 1}$ $\quad e^x = t$

$e^x \cdot dx = dt$

$\int \frac{t^2 - 1}{t^4 + t^2 + 1} dt$ $\quad z = t + \frac{1}{t}$

$= \int \frac{dz}{z^2 - 2 + 1} = \int \frac{dz}{z^2 - 1}$

$= \frac{1}{2\times 1} \ln\left|\frac{z-1}{z+1}\right| + C$

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Partial Fraction Based Qs.

$\int \frac{dx}{x(x^n+1)}$ type

Q $\int \frac{dx}{x(x^4+1)}$

$\int \frac{x^3\,dx}{x^4(x^4+1)}$ $\quad x^4=t$, $\quad x^3\,dx=\frac{dt}{4}$

$\frac{1}{4}\int \frac{dt}{(t)(t+1)}$

$\frac{1}{4}\times\frac{1}{1}\int \frac{1}{t}-\frac{1}{t+1}\,dt$

$\frac{1}{4}\ln\frac{t}{t+1}$

$\frac{1}{4}\ln\frac{x^4}{x^4+1}+C$

When Sum & difference of Dr = Nr.

Q $\int \frac{2x^2+3}{(x^2+1)(x^2+2)}\,dx$

$\int \frac{(x^2+1)+(x^2+2)}{(x^2+1)(x^2+2)}\,dx$

$\int \frac{dx}{x^2+\sqrt{2}^2}+\int \frac{dx}{1+x^2}$

$\frac{1}{\sqrt{2}}\tan^{-1}\frac{x}{\sqrt{2}}+\tan^{-1}x+C$

Q $\int \frac{dx}{(x^2+1)(x^2+3)}$ Hota to...

Qs me only x^2 is Present then
① let $x^2=y$ ② take fxn out & solve.
③ Put fxn Back & Integrate

let $x^2=y$

$\frac{1}{(y+1)(y+3)}=\frac{1}{2}\left\{\frac{1}{y+1}-\frac{1}{y+3}\right\}$ $\quad$ diff = 2

$I=\frac{1}{2}\int \frac{1\,dx}{x^2+1}-\frac{1}{2}\int \frac{1\,dx}{x^2+3}$

$=\frac{1}{2}\tan^{-1}x-\frac{1}{2\sqrt{3}}\tan^{-1}\frac{x}{\sqrt{3}}+C$

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When to Use Partial Fraction?

Whenever Deg of Nr < Dr. of Dr.

It has 3 Basic Types.

1) $\int \frac{px+q}{(x+a)(x+b)} \equiv \frac{A}{(x+a)} + \frac{B}{(x+b)}$

2) $\int \frac{px+q\,dx}{(x+a)(x+b)^3} = \frac{A}{(x+a)} + \frac{B}{(x+b)} + \frac{C}{(x+b)^2} + \frac{D}{(x+b)^3}$

3) $\int \frac{(px+q)\,dx}{(x+A)(Bx^2+Cx+D)} = \frac{A}{(x+a)} + \frac{Mx+N}{Bx^2+Cx+D}$

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Q $\int \frac{3x+7}{(x-1)(x-2)(x-3)}\,dx$

$$\frac{3x+7}{(x-1)(x-2)(x-3)} = \frac{\left(\frac{3\times1+7}{-1\times-2}\right)}{(x-1)} + \frac{\left(\frac{3\times2+7}{1\times-1}\right)}{(x-2)} + \frac{\left(\frac{3\times3+7}{2\times1}\right)}{(x-3)}$$

$5\ln|x-1| - 13\ln|x-2| + 8\ln|x-3|$

Q $I = \int \frac{(x^2+1)(x^2+2)}{(x^2+3)(x^2+4)}\,dx$

outside.

$$\frac{(y+1)(y+2)}{(y+3)(y+4)} = 1 + \frac{\left(\frac{-2\times-1}{1}\right)}{(y+3)} + \frac{\left(\frac{-3\times-2}{-1}\right)}{(y+4)}$$

$$= \int 1\cdot dx + 2\int \frac{dx}{x^2+\sqrt{3}^2} - 6\int \frac{dx}{x^2+2^2}$$

$$= x + \frac{2}{\sqrt{3}}\tan^{-1}\frac{x}{\sqrt{3}} - 6\times\frac{1}{2}\tan^{-1}\frac{x}{2}$$

Main + Adv PYQ

+

Upto 45

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Q $I=\int \frac{x^3+2}{(x-1)(x-2)^3}\,dx$

$$\Rightarrow \frac{x^3+2}{(x-1)(x-2)^3} = \frac{\frac{1+2}{(-1)^3}}{(x-1)} + \frac{B}{(x-2)} + \frac{C}{(x-2)^2} + \frac{\frac{8+2}{(1)}}{(x-2)^3} \Bigg\} \times(x-2)$$

B लाने के लिए

B Akela Karo

$$\lim_{x\to\infty} \frac{x^3+2}{(x-1)(x-2)^2} = \lim_{x\to\infty} \frac{-3(x-2)}{x-1} + \lim_{x\to\infty} B + \lim_{x\to\infty} \frac{C}{(x-2)} + \lim_{x\to\infty} \frac{10}{(x-2)^2}$$

$$\frac{1}{1\times1^2} = \frac{-3}{1} + B + 0 + 0 = 1 \quad \boxed{B=4}$$

$x=0$

$$\frac{x^3+2}{(x-1)(x-2)^3} = \frac{-2}{(x-1)} + \frac{4}{(x-2)} + \frac{C}{(x-2)^2} + \frac{10}{(x-2)^3}$$

$$\frac{2}{-1\times-8} = \frac{\cancel{-2}}{\cancel{-1}} + \frac{4}{-2} + \frac{C}{4} + \frac{10}{-8} \Rightarrow \frac{1}{4} = \frac{C}{4} - \frac{5}{4}$$

$$\frac{6}{4} = \frac{C}{4} \Rightarrow C=6$$