DPP-3

1. The value of $\text{Tan}\left[\text{Sin}^{-1}\left(\frac{3}{5}\right)+\text{Tan}^{-1}\left(\frac{2}{3}\right)\right]$ is

(A) $\frac{6}{17}$ (B) $\frac{7}{16}$ (c) $\frac{5}{7}$ (D) $\frac{17}{6}$

2. $\text{Tan}\left(\frac{\pi}{4}+\frac{1}{2}\cos^{-1}\text{x}\right)+\text{Tan}\left(\frac{\pi}{4}-\frac{1}{2}\cos^{-1}\text{x}\right), \text{x}\neq 0$, is equal to

(A) x (B) 2x (C) $\frac{2}{\text{x}}$ (D) $\frac{\text{x}}{2}$

3. The value of $\sin^{-1}[\cos\{\cos^{-1}(\cos\text{x})+\sin^{-1}(\sin\text{x})\}]$, where $\text{x}\in\left(\frac{\pi}{2},\pi\right)$ is

(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{4}$ (C) $\frac{-\pi}{4}$ (D) $\frac{-\pi}{2}$

4. If $\text{x}<0$ then value of $\tan^{-1}\text{x}+\tan^{-1}\left(\frac{1}{\text{x}}\right)$ is equal to

(A) $\frac{\pi}{2}$ (B) $\frac{-\pi}{2}$ (C) 0 (D) None of these

5. $\text{Tan}^{-1}\text{a}+\text{Tan}^{-1}\text{b}$, where $\text{a}>0,\text{b}>0,\text{ab}>1$ is equal to

(A) $\text{Tan}^{-1}\left(\frac{\text{a}+\text{b}}{1-\text{ab}}\right)$ (B) $\text{Tan}^{-1}\left(\frac{\text{a}+\text{b}}{1-\text{ab}}\right)-\pi$

(C) $\pi+\text{Tan}^{-1}\left(\frac{\text{a}+\text{b}}{1-\text{ab}}\right)$ (D) $\pi-\text{Tan}^{-1}\left(\frac{\text{a}+\text{b}}{1-\text{ab}}\right)$

6. The number of solution of the equation $\text{Tan}^{-1}(1+\text{x})+\text{Tan}^{-1}(1-\text{x})=\frac{\pi}{2}$ is

(A) 3 (B) 2 (C) 1 (D) 4

7. The number of real solution of $\text{Tan}^{-1}\sqrt{\text{x}(\text{x}+1)}+\sin^{-1}\sqrt{\text{x}^2+\text{x}+1}=\frac{\pi}{2}$ is

(A) Zero (B) one (c) Two (D) Infinite

8. If $\cot^{-1}\frac{\text{n}}{\pi}>\frac{\pi}{6}, \text{n}\in\text{N}$, then the maximum Value of ' n ' is

(A) 1 (B) 5 (c) 9 (D) None of these

9. Which of the following is correct?

(A) $\text{Tan}\ 1>\text{Tan}^{-1}\ 1$ (B) $\text{Tan}\ 1<\text{Tan}^{-1}\ 1$

(c) $\text{Tan}\ 1=\text{Tan}^{-1}\ 1$ (D) None of these

10. If $\sum_{\text{i}=1}^{\text{n}}\cos^{-1}\alpha_{\text{i}}=0$ then $\sum_{\text{i}=1}^{\text{n}}\alpha_{\text{i}}=$

(A) n (B) $-\text{n}$ (c) 0 (D) None of these

11. If $\sum_{i=1}^{2n} \sin^{-1} x_i = n\pi$ then $\sum_{i=1}^{2n} x_i$ is equal to

(A) n (B) 2n (c) $\frac{n(n+1)}{2}$ (D) None of these

12. The number of solution (s) of the equation $\sin^{-1} x + \cos^{-1}(1-x) = \sin^{-1}(-x)$ is/are

(A) 0 (B) 1 (c) 2 (D) More than 2

13. The value of $\text{Tan}^{-1}\left(\frac{a}{b+c}\right) + \text{Tan}^{-1}\left(\frac{b}{c+a}\right)$, if $\angle C = 90°$ in triangle ABC, is

(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{3}$ (c) $\frac{\pi}{2}$ (D) π

14. If $x > 0, \cos^{-1}\left(\frac{12}{x}\right) = \frac{\pi}{2} - \cos^{-1}\left(\frac{16}{x}\right)$ then x equals

(A) 12 (B) 16 (c) 20 (D) None of these

15. Number of integral ordered pairs (a, b) for which

$\sin^{-1}(1 + b + b^2 + \cdots \infty) + \cos^{-1}\left(a - \frac{a^2}{3} + \frac{a^2}{9} - \cdots \infty\right) = \frac{\pi}{2}$is

(A) 0 (B) 4 (c) 9 (D) infinitely many

16. If $\cos^{-1}(2x^2 - 1) = 2\pi - 2\cos^{-1} x$, then

(A) $x \in [-1,0]$ (B) $x \in [0,1]$

(c) $x \in \left[0, \frac{1}{\sqrt{2}}\right]$ (D) $x \in \left[\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right]$

17. Evaluate each of the following

(A) $\sin^{-1}\left(\sin \frac{7\pi}{6}\right)$ (ii) $\tan^{-1}\left(\tan \frac{2\pi}{3}\right)$

(iii) $\cos^{-1}\left(\cos \frac{5\pi}{4}\right)$ (iv) $\sec^{-1}\left(\sec \frac{7\pi}{4}\right)$

18. Find the value of the following

(i) $\sin^{-1}(\sin 5)$ (ii) $\cos^{-1}(\cos 10)$

(iii) $\tan^{-1}(\tan(-6))$ (iv) $\cot^{-1}(\cot(-10))$

(v) $\cos^{-1}\left(\frac{1}{\sqrt{2}}\left(\cos \frac{9\pi}{10} - \sin \frac{9\pi}{10}\right)\right)$

19. Find $\sin^{-1}(\sin \theta), \cos^{-1}(\cos \theta), \tan^{-1}(\tan \theta)$ and $\cot^{-1}(\cot \theta)$ for $\theta \in \left[\frac{3\pi}{2}, 3\pi\right]$

20. Prove each of the following:

(i) $\tan^{-1} x = -\pi + \cot^{-1}\frac{1}{x} = \sin^{-1}\frac{x}{\sqrt{1+x^2}} = -\cos^{-1}\frac{1}{\sqrt{1+x^2}}$ when $x < 0$.

21. Prove that $\sin^{-1}\cos(\sin^{-1}x)+\cos^{-1}\sin(\cos^{-1}x)=\frac{\pi}{2}, |\pi|\leq 1$

22. $\tan(\tan^{-1}x+\tan^{-1}y+\tan^{-1}z)=\cot(\cot^{-1}x+(\int\cot^{-1}y+\cot^{-1}z)$

23. Prove that $\cos^{-1}\left(\sqrt{\frac{1}{3}}\right)-\cos^{-1}\left(\sqrt{\frac{1}{6}}\right)+\cos^{-1}\left(\frac{\sqrt{10}-1}{3\sqrt{2}}\right)=\cos^{-1}\left(\frac{2}{3}\right)$

24. Prove that $2\tan^{-1}(\text{cosec}\ \tan^{-1}x-\tan\ \cot^{-1}x)=\tan^{-1}x$

25. Prove that $\cos^{-1}\left(\frac{63}{65}\right)+2\tan^{-1}\left(\frac{1}{5}\right)=\sin^{-1}\left(\frac{3}{5}\right)$

26. Prove that

(i) $2\cos^{-1}\frac{1}{\sqrt{13}}+\cot-\frac{16}{63}+\frac{1}{2}\text{con}-\frac{7}{25}=\pi$

(ii) $\cos^{1}\left(\frac{5}{13}\right)+\cos^{-1}\left(-\frac{7}{25}\right)=\sin-\frac{36}{325}=\pi$

(iii) are $\cos\sqrt{\frac{2}{3}}-$ are $\cos\frac{\sqrt{5}+1}{2\sqrt{3}}-\frac{k}{6}$

27. Show that:

$\sin^{-1}(\sin\frac{33\pi}{7})+\cos\left(\text{ax}\ \frac{4\pi\pi}{7}\right)+\tan^{-1}(-\tan^{13\pi})+\sin\left(\cos\left(\frac{19\pi}{8}\right)\right)=\frac{45\pi}{28}$

28. Solve for x: $\sin^{-1}\left(\sin\left(\frac{2x^2+4}{1+x^3}\right)\right)<x-3$

29. If the sum $\sum_{n=1}^{m}\sum_{m=1}^{10}\tan^{-1}\left(\frac{m}{n}\right)=k\pi$, find the walue of k.

30. Let $y=\sin^{-1}(\sin\ 8)-\text{tant}^{-1}(\tan\ 10)+\cos(\text{con}\ 12)$

$\text{sec}^{-}(\sec\ 9)+\text{cot}^{-}(\cot\ 6)-\text{cosec}(\text{conec}\ 7)$. If y simplifies to a s t $^{+}$b then find $(a-b)$.

31. Prove that $\tan\left[\frac{\pi}{4}+\frac{1}{2}\cos^4\frac{a}{b}\right]+\tan\left[\frac{\pi}{4}\cdot\frac{1}{2}\cos^{-4}\frac{a}{b}\right]=\frac{b}{a}$

32. Solve the following inequalities

(i) $\sin^{-1}x>-1$ (ii) $\cos^{-1}x<2$

(iii) $\cot^{-1}x<-\sqrt{3}$ (iv) Solve the inequality:

$(\text{arcsec}\ x)^2-6(\text{arc sec}\ x)+8>0$