Nishant Jindal

# Simple pendulum

$$\tau_r = -(mg\sin\theta)\,l \qquad \sin\theta \simeq \theta$$

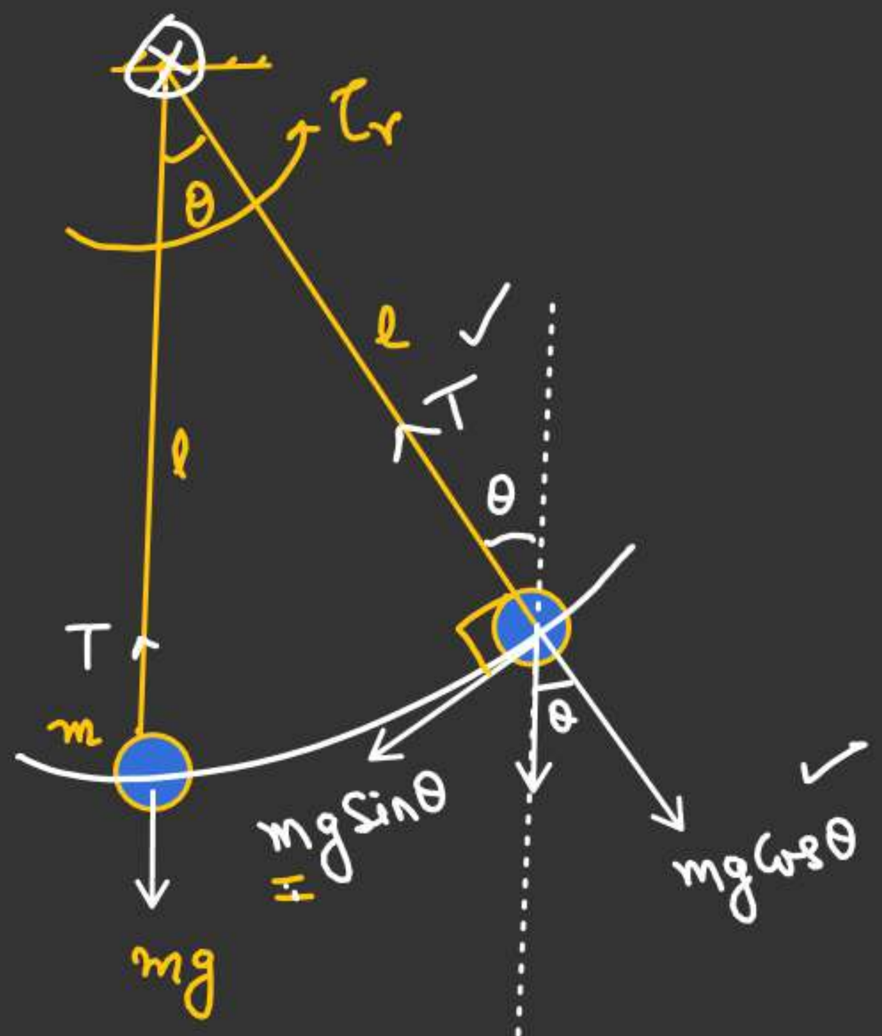
$$\tau_r = -mgl\theta$$

$$\alpha = -\frac{mgl}{ml^2}\theta$$

$$\alpha = -\frac{g}{l}\theta$$

$$\alpha = -\omega^2\theta$$

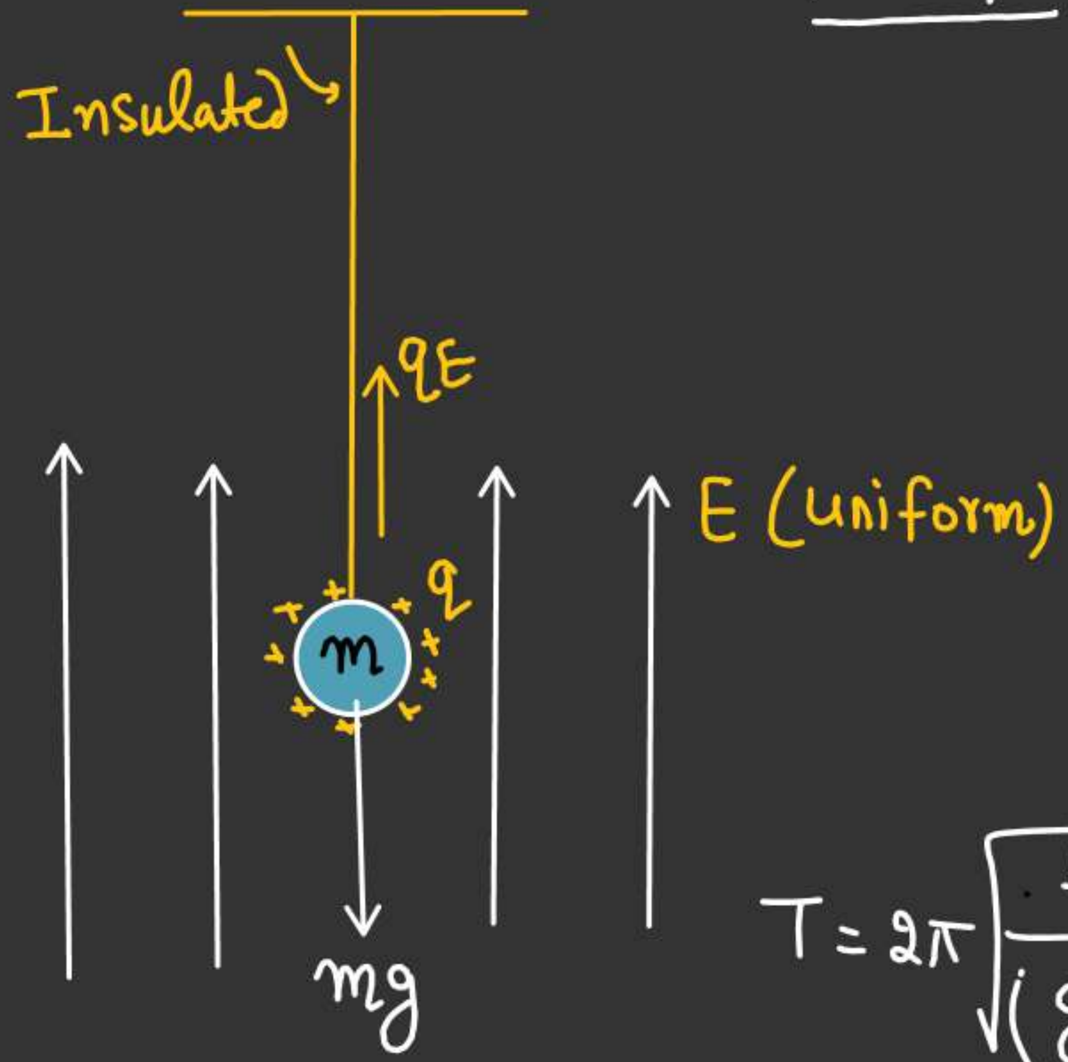
$$\omega^2 = g/l \qquad \boxed{T = 2\pi\sqrt{\frac{l}{g}}}$$



Nishant Jindal

#

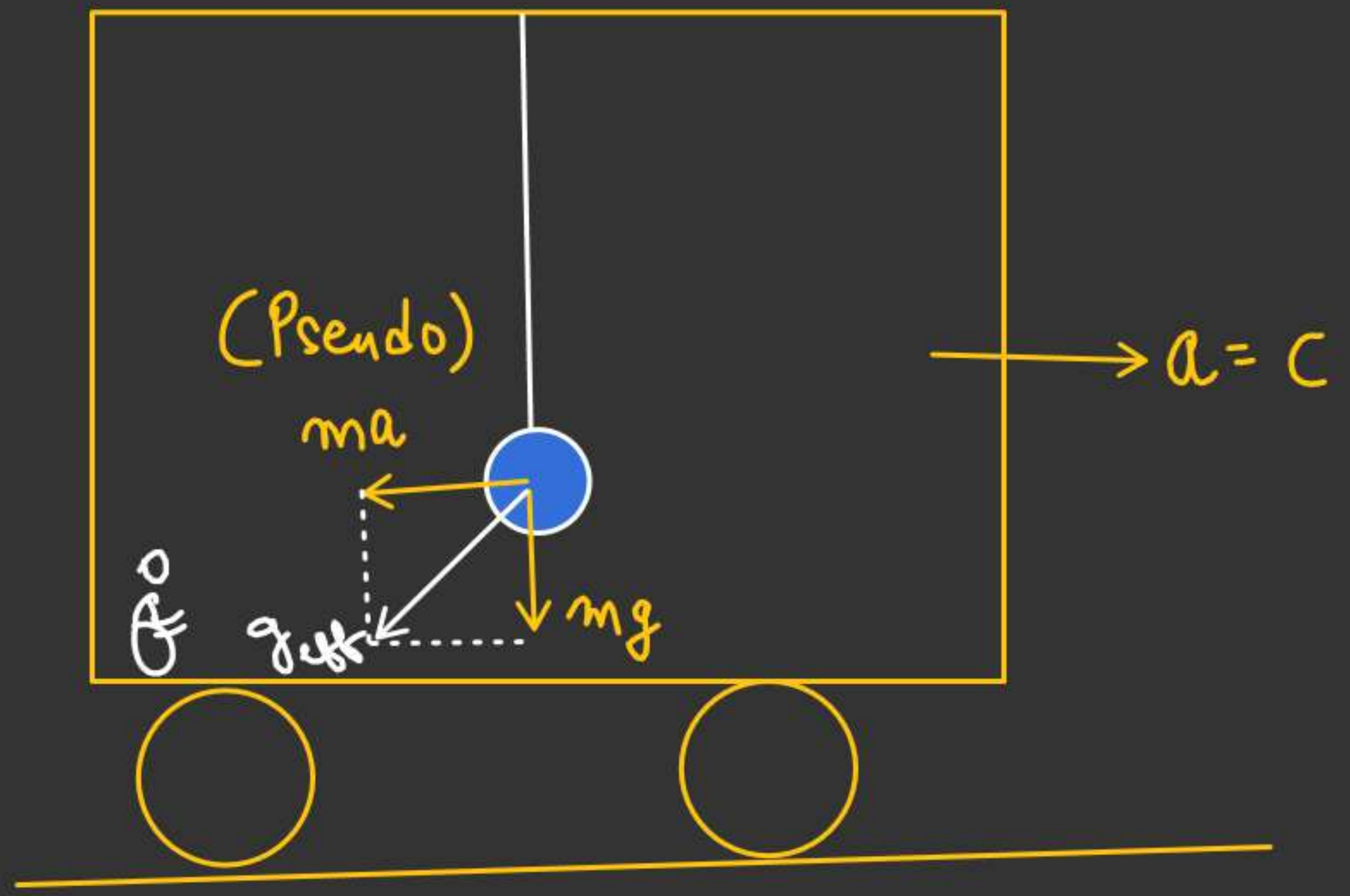
$$\vec{F} = q\vec{E}$$




$$T = 2\pi\sqrt{\frac{l}{\left(g - \frac{qE}{m}\right)}}$$

$$F_{net} = mg - qE$$

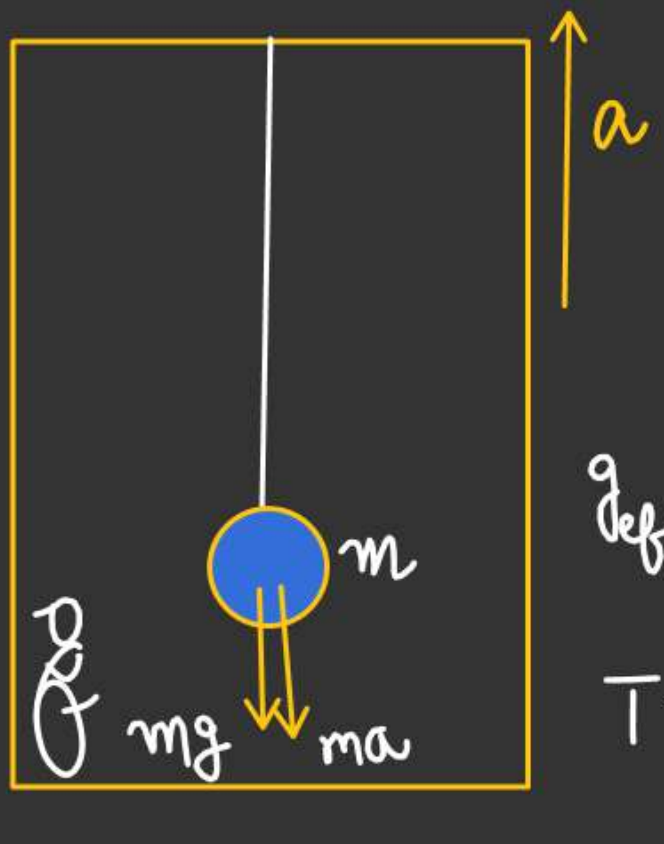
$$a_{net} = g_{eff} = \left(g - \frac{qE}{m}\right) \checkmark$$




$$g_{eff} = \sqrt{g^2 + a^2}$$

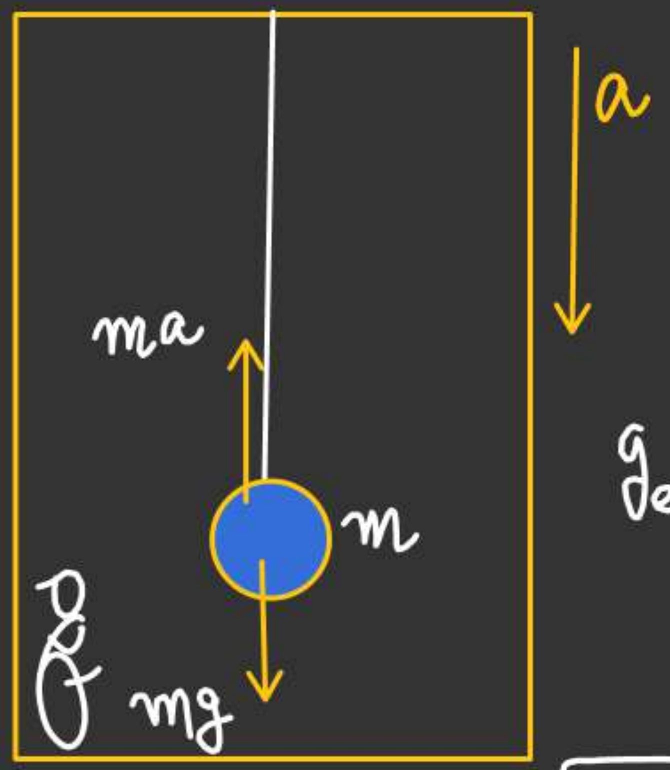
$$T = 2\pi\sqrt{\frac{l}{\sqrt{g^2 + a^2}}}$$

Nishant Jindal



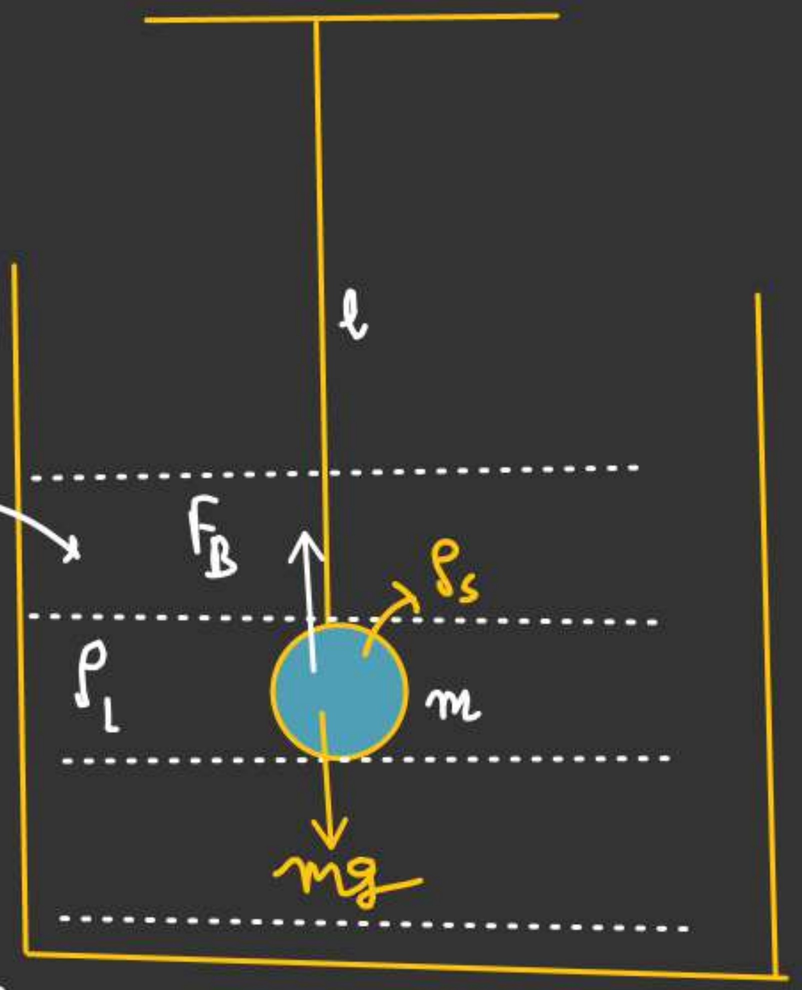

$$g_{eff} = (g+a)$$

$$T = 2\pi\sqrt{\frac{l}{(g+a)}}$$

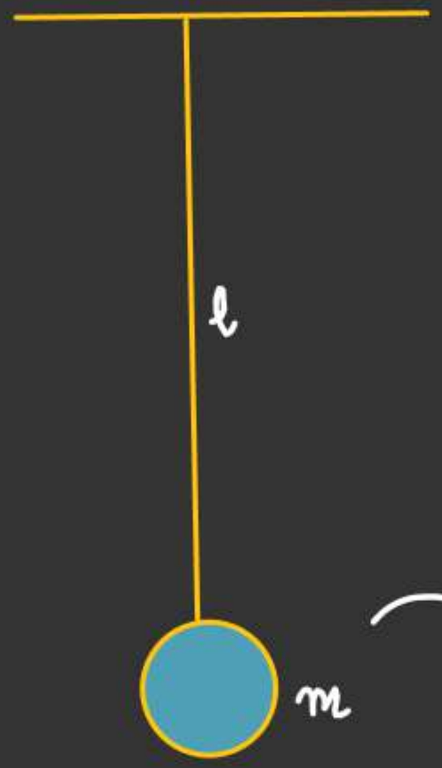



$$g_{eff} = (g-a)$$

$$T = 2\pi\sqrt{\frac{l}{(g-a)}}$$




Non Viscous liquid




$T\uparrow \Rightarrow$ clock slowdown

$T\downarrow \Rightarrow$ clock become fast

$$W_{app} = W\left(1 - \frac{\rho_L}{\rho_s}\right)$$

$$\Downarrow$$

$$\cancel{m} g_{eff} = \cancel{m} g\left(1 - \frac{\rho_L}{\rho_s}\right)$$

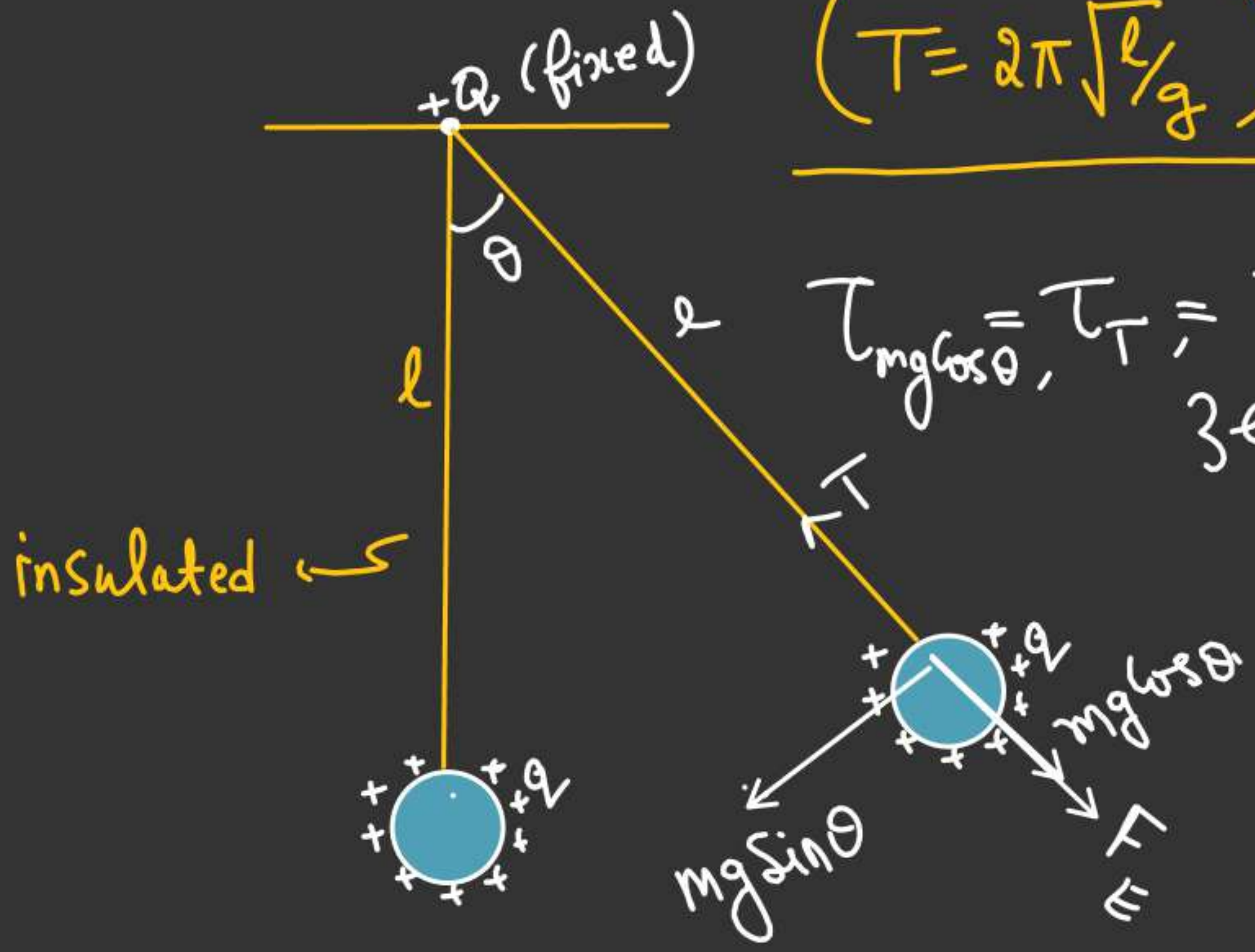
$$g_{eff} = g\left(1 - \frac{\rho_L}{\rho_s}\right)$$

$$T = 2\pi\sqrt{\frac{\ell}{g\left(1 - \frac{\rho_L}{\rho_s}\right)}}$$

$$T = \frac{\underbrace{2\pi\sqrt{\ell/g}}_{T_0}}{\sqrt{1 - \rho_L/\rho_s}}$$

$$T = \frac{T_0}{\sqrt{1 - \frac{\rho_L}{\rho_s}}} \checkmark$$

Nishant Jindal




$$\left(T = 2\pi\sqrt{l/g}\right)$$ ✓ ✓

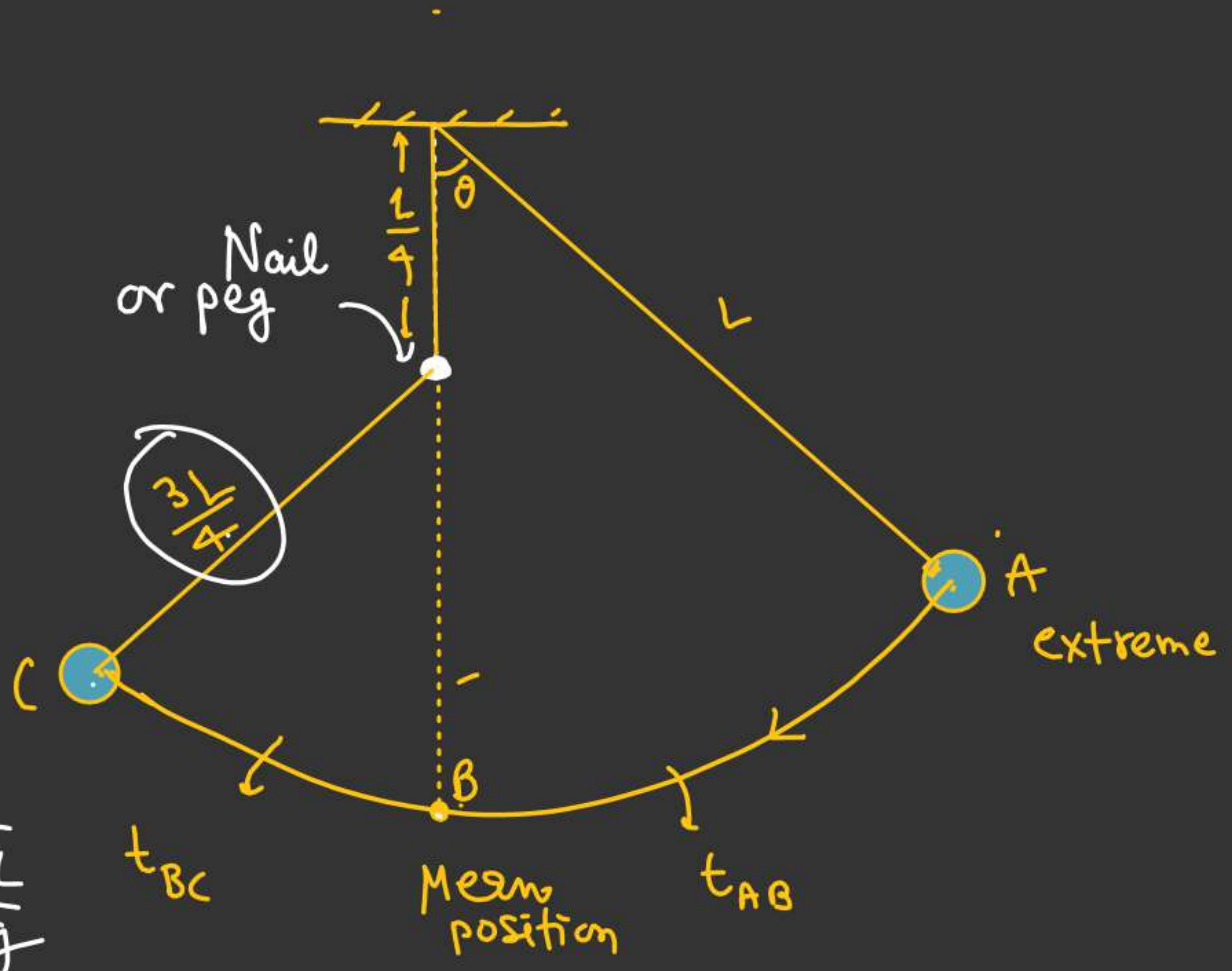
$\tau_{mg\cos\theta}, \tau_T, \tau_{F_E}$ at suspension is zero.

$$\tau_r = -(mg\sin\theta\, L)$$

$$\tau_r = -mgL\theta.$$

Nishant Jindal

Pendulum is released from it's extreme position. Find the time period of the String-bob System. peg or Nail is at a distance $\frac{L}{4}$ from point of suspension.




$$T = 2(t_{BC} + t_{AB})$$

$$t_{AB} = \frac{T_{AB}}{4} = \frac{2\pi}{4}\sqrt{\frac{L}{g}} = \frac{\pi}{2}\sqrt{\frac{L}{g}}$$

$$t_{BC} = \frac{T_{BC}}{4} = \frac{1}{4}\times 2\pi\sqrt{\frac{3L}{4g}} = \frac{\pi}{4}\sqrt{\frac{3L}{g}}$$

$$T = 2\left[\frac{\pi}{2}\sqrt{\frac{L}{g}} + \frac{\pi}{4}\sqrt{\frac{3L}{g}}\right] = \pi\sqrt{\frac{L}{g}}\left(1+\frac{\sqrt{3}}{2}\right) = \frac{\pi}{2}\sqrt{\frac{L}{g}}\left(\sqrt{3}+2\right)$$

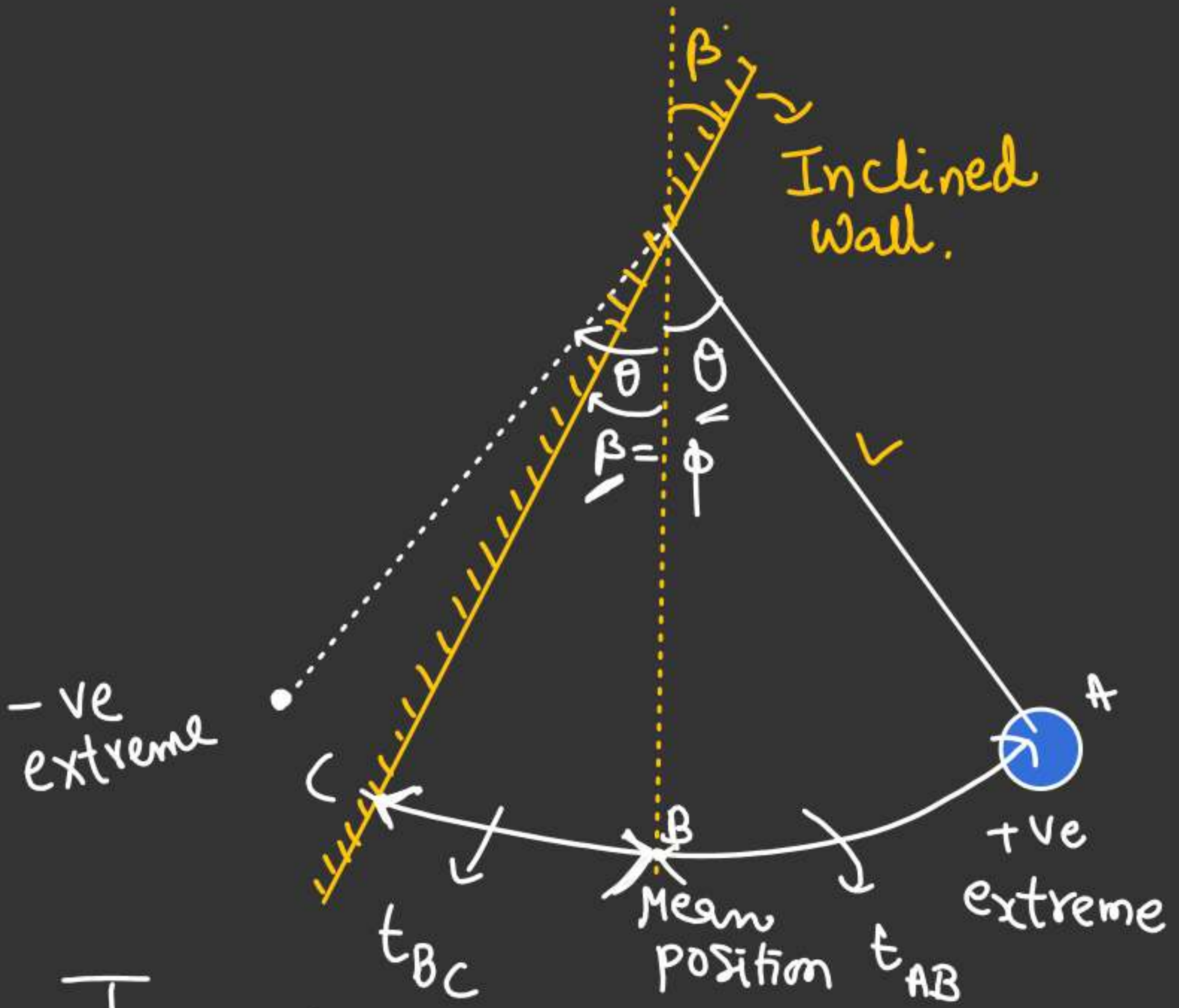
$\beta \rightarrow$ Inclination of wall from vertical.

Find time period of the pendulum if

1) $\theta < \beta$ ✓ $\longrightarrow$ $T = 2\pi\sqrt{l/g}$.

2) $\theta > \beta$.

Collision of bob with wall is perfectly elastic ✓




$$t_{AB} = T/4 = \frac{1}{4} \times 2\pi\sqrt{\frac{L}{g}} = \frac{\pi}{2}\sqrt{\frac{L}{g}}$$

$$t_{BC} = ?? \quad [\phi = \theta_{max} \sin\omega t]$$

$$\beta = \theta \cdot \sin\omega t_{BC}$$

$$\sin\omega t_{BC} = (\beta/\theta)$$

$$\omega t_{BC} = \sin^{-1}(\beta/\theta)$$

$$t_{BC} = \frac{1}{\omega}\sin^{-1}(\beta/\theta)$$

$$T = 2(t_{AB} + t_{BC}) = \pi\sqrt{l/g} + \frac{2}{\omega}\sin^{-1}(\beta/\theta)$$

Nishant Jindal

## Case of Simple pendulum when length of the String is Comparable w.r.t radius of earth.

$$F_r = -[mg\sin\phi + T'\sin\theta]$$

For Vertical Equilibrium

$T'\cos\theta = mg\cos\phi$

$\theta$ & $\phi$ are very small

$\sin\phi \simeq \phi$, $\cos\phi \to 1$, $\phi \to 0$

$\sin\theta \simeq \theta$, $\cos\theta \to 1$, $\theta \to 0$

$(T' = mg)$

$$F_r = -mg[\phi + \theta]$$

In $\Delta DAB$

$\sin\theta \simeq \theta = \frac{x}{l}$

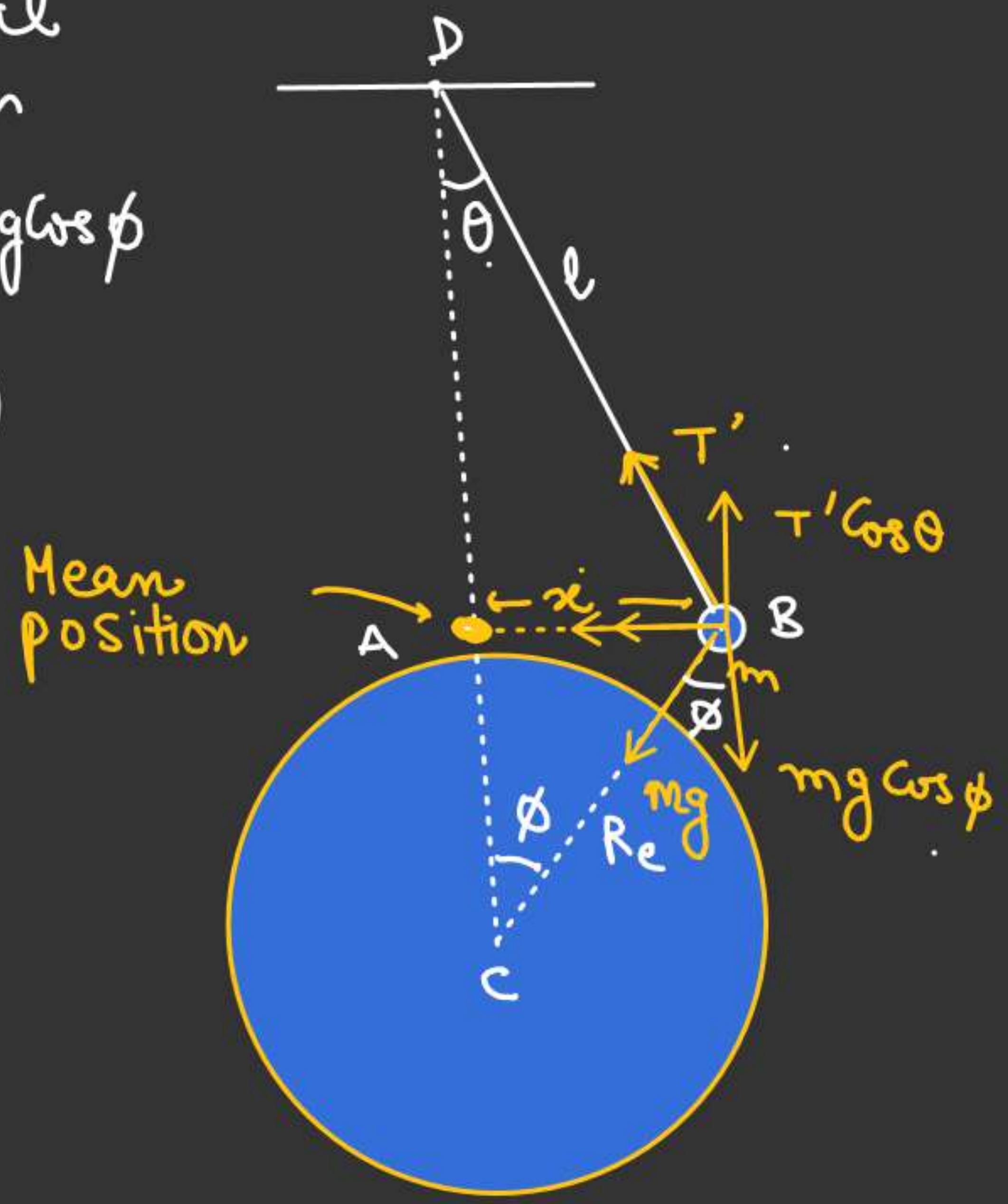
In $\Delta ABC$

$\sin\phi \simeq \phi = \frac{x}{R_e}$

$$a = -g[\theta + \phi] \quad a \propto x$$

$$a = -g\left[\frac{1}{l} + \frac{1}{R_e}\right]x$$

$$a = -\omega^2 x$$



Nishant Jindal

$$a = -g\left[\frac{1}{l}+\frac{1}{R_e}\right]x$$

$$a = -\omega^2 x$$

Compair

$$\omega = \sqrt{g\left(\frac{1}{l}+\frac{1}{R_e}\right)}$$

$$T = \frac{2\pi}{\sqrt{g\left(\frac{1}{l}+\frac{1}{R_e}\right)}}$$

$$\boxed{T = \frac{2\pi}{\sqrt{\frac{g}{l}\left(1+l/R_e\right)}}} \Rightarrow T = \frac{2\pi\sqrt{l/g}}{\sqrt{1+l/R_e}}$$

$$T = \left(\frac{T_0}{\sqrt{1+l/R_e}}\right)$$

if $l << R_e$

$$\frac{l}{R_e} \to 0$$

$$T = T_0$$

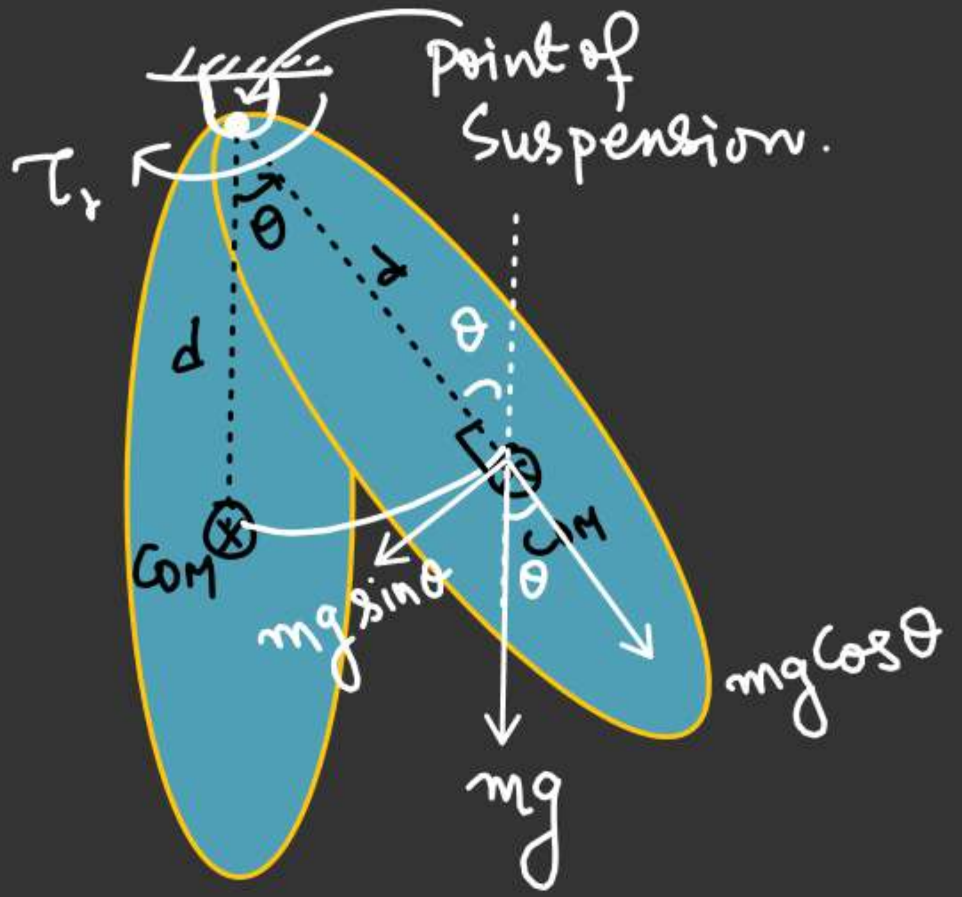
Nishant Jindal

48

Nishant Jindal

# Physical pendulum

Any Rigid body oscillating about any point of suspension.




$$\tau_r = -(mg\sin\theta)d.$$

$\theta$ = very small

$\sin\theta \simeq \theta$

$$\alpha = \frac{\tau_r}{I} = -\left(\frac{mgd}{I}\right)\theta$$

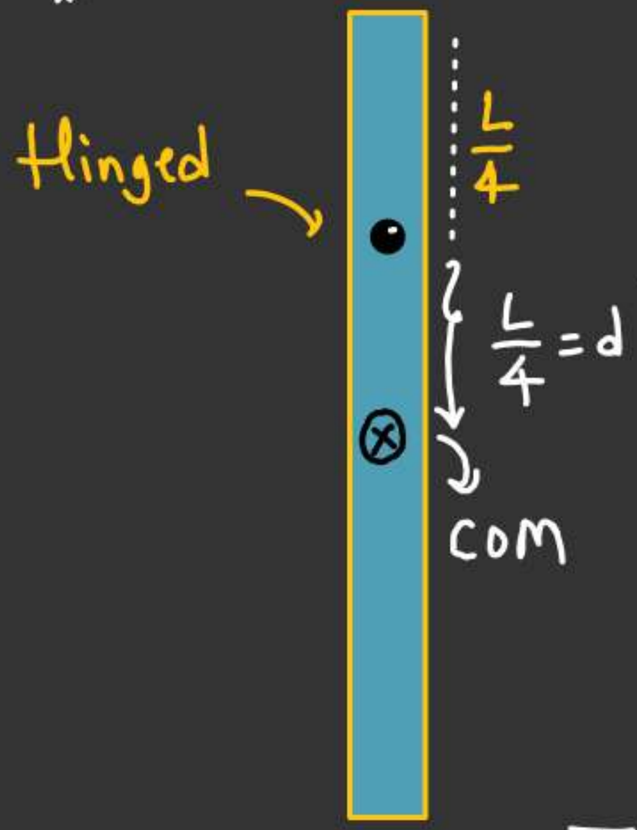
$$\alpha = -\omega^2\theta$$

$$\omega = \sqrt{\frac{mgd}{I}} \Rightarrow \boxed{T = 2\pi\sqrt{\frac{I}{mgd}}}$$

I = M.I of body about axis passing through point of suspension

d = distance b/w COM & point of suspension.

# $T = ?$ M, L




$$T = 2\pi\sqrt{\frac{I}{mgd}}$$

$$I = I_{com} + M\left(\frac{L}{4}\right)^2$$

$$I = \left(\frac{ML^2}{12} + \frac{ML^2}{16}\right)$$

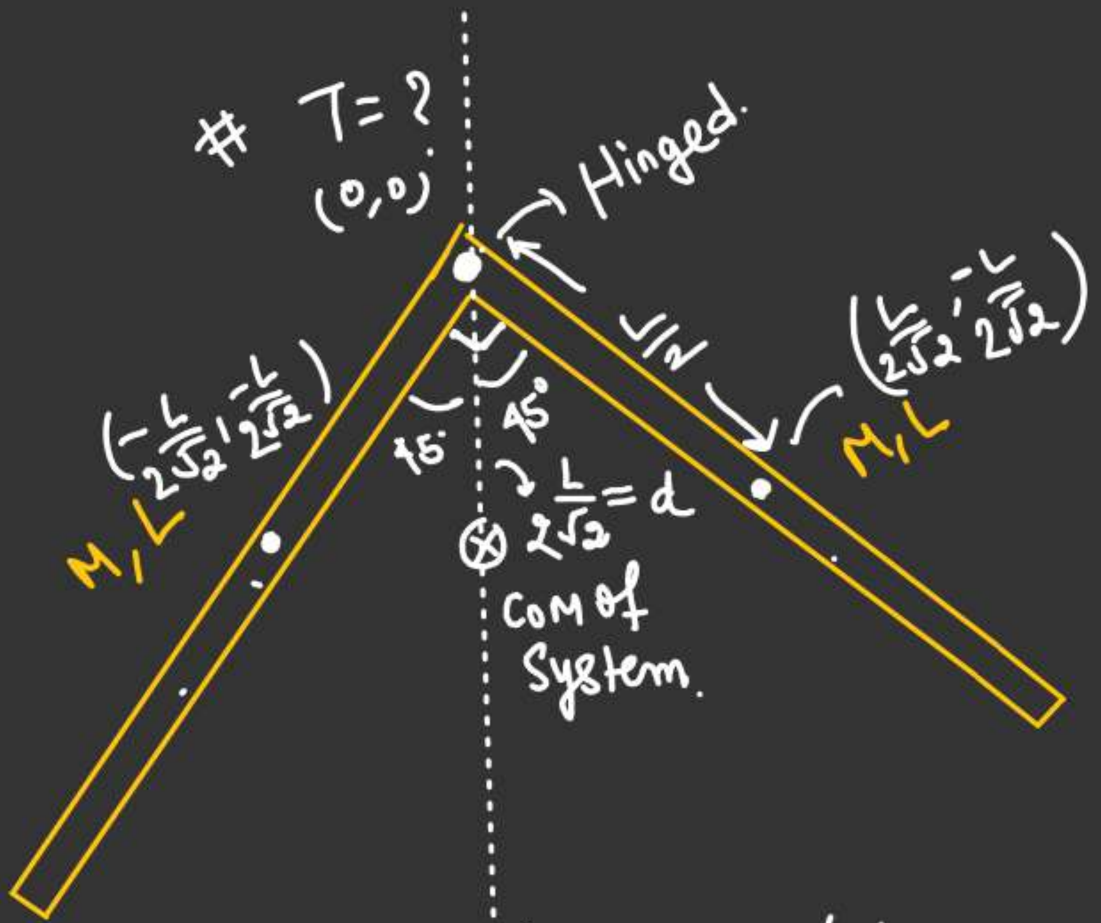
$$I = \left(\frac{7ML^2}{48}\right)$$

$$T = 2\pi\sqrt{\frac{7M\cancel{L^2}}{48 \times \cancel{M} \times g \times \frac{\cancel{L}}{4}}}$$

$$T = 2\pi\sqrt{\frac{7L}{12g}}$$

$4 \,|\, 12, 16$
$\quad\;\; 3, 4$

# $T = ?$




$$d = \frac{L}{2\sqrt{2}}$$

$$I = \frac{ML^2}{3} \times 2$$

$$I = \left(\frac{2ML^2}{3}\right)$$

$$T = 2\pi\sqrt{\frac{\cancel{2M}L^2}{3 \times (\cancel{2M}) g \times \frac{L}{2\sqrt{2}}}}$$

$$T = \left(2\pi\sqrt{\frac{2\sqrt{2}L}{3g}}\right) \checkmark$$

$$y_{com} = \frac{m\left(\frac{-L}{2\sqrt{2}}\right) - m\frac{L}{2\sqrt{2}}}{2m}$$

$$y_{com} = -\frac{\cancel{2m}L}{2\sqrt{2} \times \cancel{2m}}$$

$$= \frac{-L}{2\sqrt{2}}$$

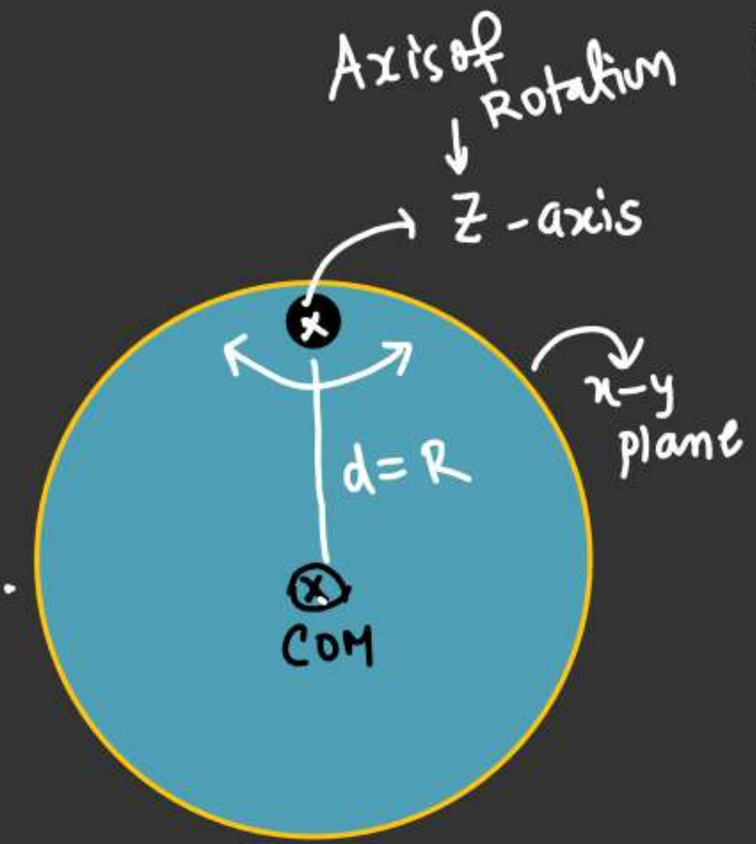
Nishant Jindal

Find the ratio of time period of the uniform disc.

$T_1$ be the time period when disc oscillate in the plane of disc.

$T_2$ be the time period when disc oscillate perpendicular to the plane of disc

$\frac{T_1}{T_2} = ?$




$$I_1 = \frac{MR^2}{2} + MR^2 = \frac{3}{2}MR^2$$

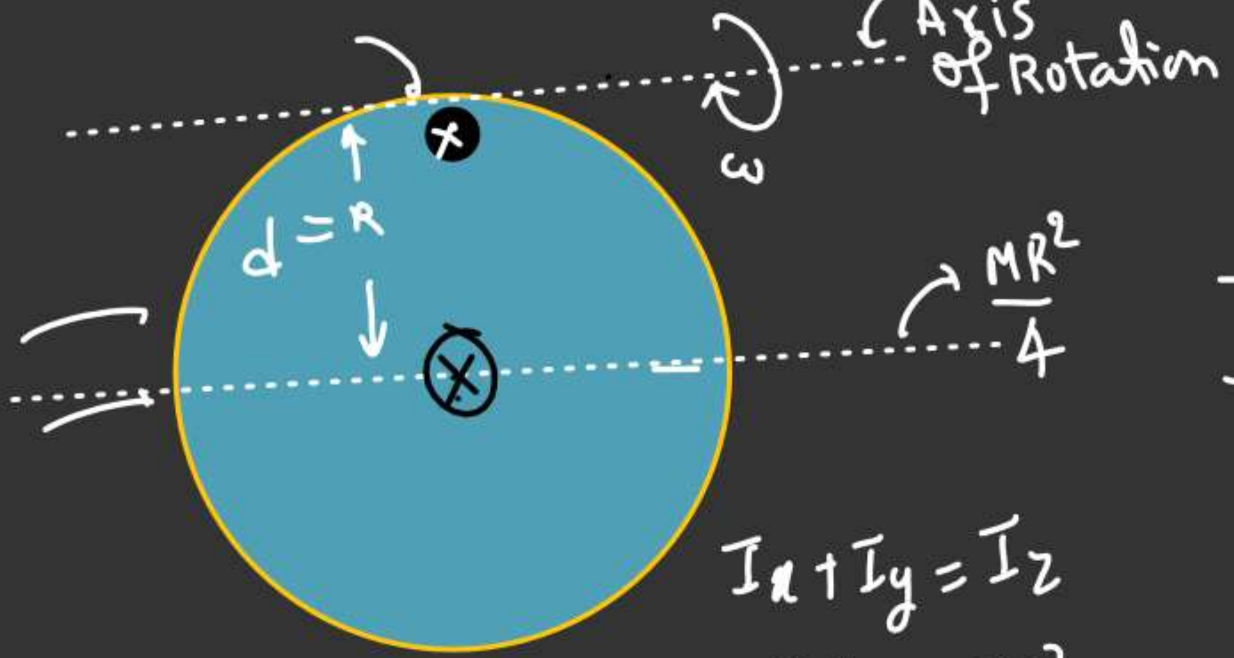
$$I_2 = \frac{MR^2}{4} + MR^2 = \frac{5}{4}MR^2$$

$$T_1 = 2\pi\sqrt{\frac{I_1}{mgR}}$$

$$T_2 = 2\pi\sqrt{\frac{I_2}{mgR}}$$

$$\frac{T_1}{T_2} = \sqrt{\frac{I_1}{I_2}}$$

$$\frac{T_1}{T_2} = \sqrt{\frac{3}{2} \times \frac{4}{5}} = \sqrt{\frac{6}{5}}$$ ✓



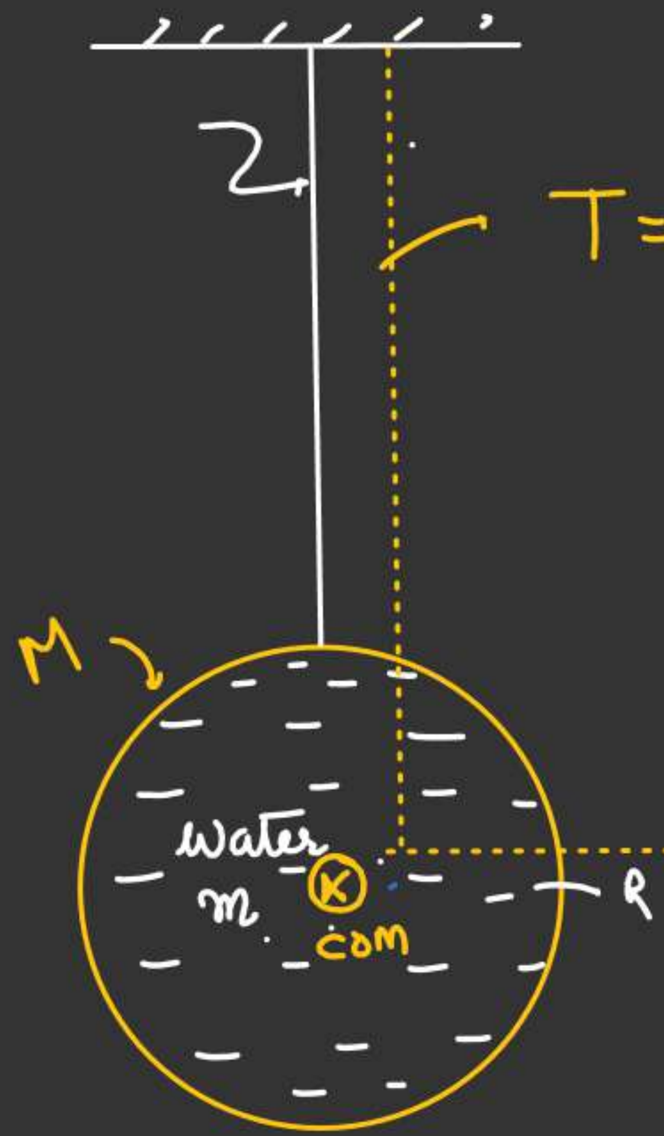

$I_x = I_y$

$$I_x + I_y = I_z$$

$$2I_x = \frac{MR^2}{2}$$

$$I_x = \frac{MR^2}{4}$$

Nishant Jindal



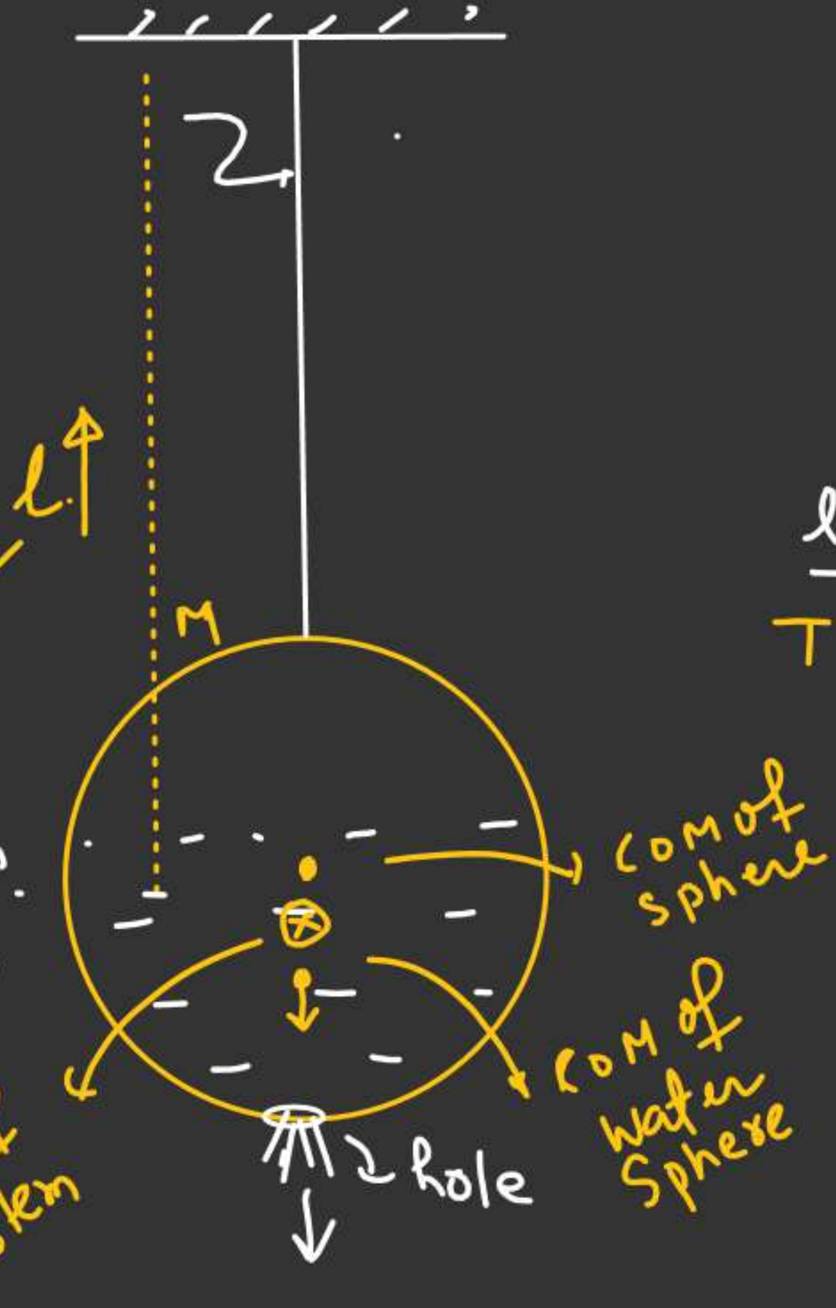

T = Time period of oscillation.

$$T=2\pi\sqrt{l/g}$$

When water drain out

$l \rightarrow$ increases

$T \rightarrow$ increases




Frist increases & then decreases

$l' < l$

$T' < T$