DPP–1(Continuity at a Point)

1. A function f(x) satisfies the following property : $f(x+y)=f(x)f(y)$ Show that the function is continuous for all values of x if it is continuous at $x=1$.

2. Find the points of discontinuity of the following functions.

(i) $f(x)=\frac{1}{2\sin x-1}$

(ii) $f(x)=[[x]]-[x-1]$, where [.] represent the greatest integer function.

3. Let $f(x)=\begin{cases}\frac{\log_e \cos x}{\sqrt[4]{1+x^2}-1}, & x>0\\ \frac{e^{\sin 4x}-1}{\log_e(1+\tan 2x)}, & x<0\end{cases}$. Find the value of f(0) which makes the function continuous at $x=0$,

4. $f(x)=\begin{cases}\cos^{-1}\{\cot x\} & x<\frac{\pi}{2}\\ \pi[x]-1 & x\geq\frac{\pi}{2}\end{cases}$; find jump of discontinuity , where [ ] denotes greatest integer & { } denotes fractional part function.

5. $f(x)=\begin{cases}|x+1|\,; & x\leq 0\\ x\,; & x>0\end{cases}$ and $g(x)=\begin{cases}|x|+1; & x\leq 1\\ -|x-2|; & x>1\end{cases}$

Draw its graph and discuss the continuity of $f(x)+g(x)$.

6. Draw the graph and discuss continuity of $f(x)=[\sin x+\cos x], x\in[0,2\pi]$, where [.] represents the greatest integer function.

7. Let $f(x)=\lim_{n\to\infty}\frac{1}{1+n\sin^2 x}$, then find $f\left(\frac{\pi}{4}\right)$ and also comment on the continuity at $x=0$

8. Discuss the continuity of $f(x)=\begin{cases}x\{x\}+1, & 0\leq x<1\\ 2-\{x\}, & 1\leq x\leq 2\end{cases}$. where {x} denotes the fractional part function.

9. If $f(x)=\begin{cases}x+2, & \text{when } x<1\\ 4x-1, & \text{when } 1\leq x\leq 3\\ x^2+5, & \text{when } x>3\end{cases}$, then correct statement is -

(A) $\lim_{x\to 1}f(x)=\lim_{x\to 3}f(x)$ (B) f(x) is continuous at $x=3$
(C) f(x) is continuous at $x=1$ (D) f(x) is continuous at $x=1$ and 3

10. If $f(x)=\frac{x-e^x+\cos 2x}{x^2}, x\neq 0$ is continuous at $x=0$, then

(A) $f(0)=\frac{5}{2}$ (B) $[f(0)]=-2$

(C) $\{f(0)\}=-0.5$ (D) $[f(0)]\cdot\{f(0)\}=-1.5$

where [x] and {x} denotes greatest integer and fractional part function

11. A function f(x) is defined as below $f(x)=\frac{\cos(\sin x)-\cos x}{x^2}, x\neq 0$ and $f(0)=a$

f(x) is continuous at $x=0$ if 'a' equals

(A) 0 (B) 4 (C) 5 (D) 6

12. Consider the function $f(x)=\begin{cases}x\{x\}+1 & 0\leq x<1\\ 2-\{x\} & 1\leq x\leq 2\end{cases}$ where {x} denotes the fractional part function.

Which one of the following statements is NOT correct?

(A) $\lim_{x\to 1} f(x)$ exists (B) $f(0)\neq f(2)$

(C) f(x) is continuous in [0,2] (D) Rolle's theorem is not applicable to f(x) in [0,2]

13. Given $f(x)=\frac{e^x-\cos 2x-x}{x^2}$ for $x\in R-\{0\}$

$$g(x)=\begin{cases}f(\{x\}) & \text{for } n<x<n+\frac{1}{2}\\ f(1-\{x\}) & \text{for } n+\frac{1}{2}\leq x<n+1, n\in I\\ \frac{5}{2} & \text{otherwise}\end{cases}\left\{\begin{array}{l}\text{where } \{x\} \text{ denotes}\\ \text{fractional part function}\end{array}\right.$$

then g (x) is

(A) discontinuous at all integral values of x only

(B) continuous everywhere except for $x=0$

(C) discontinuous at $x=n+\frac{1}{2}; n\in I$ and at some $x\in I$

(D) continuous everywhere

14. If $f(x)=\begin{cases}\frac{\log(1+2ax)-\log(1-bx)}{x} & ,x\neq 0\\ k & ,x=0\end{cases}$, is continuous at $x=0$, then k is equal to -

(A) $2a+b$ (B) $2a-b$ (C) $b-2a$ (D) $a+b$

15. If $f(x)=\begin{cases}\frac{1-\cos 4x}{x^2} & ,x<0\\ a & ,x=0\\ \frac{\sqrt{x}}{\sqrt{16+\sqrt{x}}-4}, & x>0\end{cases}$, then correct statement is -

(A) f(x) is discontinuous at $x=0$ for any value of a

(B) f(x) is continuous at $x=0$ when $a=8$

(C) f(x) is continuous at $x=0$ when $a=0$

(D) none of these

16. Let $f(x)=\begin{cases}\frac{2^x+2^{3-x}-6}{\sqrt{2^{-x}}-2^{1-x}} & \text{if } x>2\\ \frac{x^2-4}{x-\sqrt{3x-2}} & \text{if } x<2\end{cases}$ then

(A) $f(2)=8\Rightarrow f$ is continuous at $x=2$

(B) $f(2)=16\Rightarrow f$ is continuous at $x=2$

(C) $f(2^-)\neq f(2^+)\Rightarrow f$ is discontinuous

(D) f has a removable discontinuity at $x=2$

More than one answer type

17. Let $f(x)=\frac{|x+\pi|}{\sin x}$, then

(A) $f(-\pi^+)=-1$ (B) $f(-\pi)=1$

(C) $\lim_{x\to-\pi} f(x)$ does not exist (D) $\lim_{x\to\pi} f(x)$ does not exist

18. On the interval $I=[-2,2]$, the function $f(x)=\begin{cases}(x+1)e^{-\left[\frac{1}{|x|}+\frac{1}{x}\right]} & (x\neq 0)\\ 0 & (x=0)\end{cases}$ then which one of the following hold good?

(A) is continuous for all values of $x\in I$

(B) is continuous for $x\in I-\{0\}$

(C) assumes all intermediate values from $f(-2)\&f(2)$

(D) has a maximum value equal to $3/e$

19. The function, $f(x)=[|x|]-|[x]|$ where $[x]$ denotes greatest integer function

(A) is continuous for all positive integers

(B) is discontinuous for all non positive integers

(C) has finite number of elements in its range

(D) is such that its graph does not lie above the x-axis.

20. f is a continous function in $[a,b]$; g is a continuous function in $[b,c]$

A function h(x) is defined as $h(x)=\begin{cases}f(x) & \text{for } x\in[a,b)\\ g(x) & \text{for } x\in(b,c]\end{cases}$ if $f(B)=g(B)$, then

(A) h(x) has a removable discontinuity at $x=b$.

(B) h(x) may or may not be continuous in $[a,c]$

(C) $h(b^-)=g(b^+)$ and $h(b^+)=f(b^-)$

(D) $h(b^+)=g(b^-)$ and $h(b^-)=f(b^+)$

21. Function whose jump (non-negative difference of LHL & RHL) of discontinuity is greater than or equal to one, is/are -

(A) $f(x)=\begin{cases}\frac{(e^{1/x}+1)}{(e^{1/x}-1)}; & x<0\\ \frac{(1-\cos x)}{x}; & x>0\end{cases}$ (B) $g(x)=\begin{cases}\frac{x^{1/3}-1}{x^{1/2}-1}; & x>1\\ \frac{\ell nx}{(x-1)}; & \frac{1}{2}<x<1\end{cases}$

(C) $u(x)=\begin{cases}\frac{\sin^{-1}2x}{\tan^{-1}3x}; & x\in\left(0,\frac{1}{2}\right]\\ \frac{|\sin x|}{x}; & x<0\end{cases}$ (D) $v(x)=\begin{cases}\log_3(x+2); & x>2\\ \log_{1/2}(x^2+5); & x<2\end{cases}$

Answer Key

2. **(i)** $x = 2n\pi + \frac{\pi}{6}$ or $x = 2n\pi + \frac{5\pi}{6}, n \in Z$ **(ii)** continuous $\forall\, x \in R$.

3. f(0) cannot be defined. **4.** $\frac{\pi}{2} - 1$ **5.** f(x) + g(x) is discontinuous at x = 0,1

6. discontinuous at $x = 0, \frac{\pi}{2}, \frac{3\pi}{4}, \pi, \frac{3\pi}{2}, \frac{7\pi}{4}, 2\pi$ **7.** $f\left(\frac{\pi}{4}\right) = 0$, f(x) is discontinuous at x = 0

8. discontinuous at x = 2

9.	C	**10.**	D	**11.**	A	**12.**	C	**13.**	D	**14.**	A	**15.**	B	**16.**	C
17.	ABCD	**18.**	BCD	**19.**	ABCD	**20.**	AC	**21.**	ACD						