HOME WORK–2

(PROBLEMS BASED ON FUNDAMENTALS)

A trigonometric equation is of the form $a\cos\theta \pm b\sin\theta = c$

Q. Solve for θ :

1. $\sin(\theta) + \cos(\theta) = 1$

2. $\sqrt{3}\sin(\theta) + \cos(\theta) = 2$

3. $\sin(2\theta) + \cos(2\theta) = \sin(\theta) + \cos(\theta) + 1 = 0$

4. $\sin^3\theta + \sin\theta\cos\theta + \cos^3\theta = 1$

5. $\sin\theta + \sqrt{3}\cos\theta = \sqrt{2}$

6. $\sqrt{2}\sec\theta + \tan\theta = 1$

7. $\cot\theta + \operatorname{cosec}\theta = \sqrt{3}$

8. $\sin\theta + \cos\theta = \sqrt{2}$

9. $\sqrt{3}\cos\theta + \sin\theta = 1$

10. $\sin\theta + \cos\theta = 1$

11. $\operatorname{cosec}\theta = 1 + \cot\theta$

12. $\tan\theta + \sec\theta = \sqrt{3}$

13. $\cos\theta + \sqrt{3}\sin\theta = 2\cos2\theta$

14. $\sqrt{3}(\cos\theta - \sqrt{3}\sin\theta) = 4\sin2\theta \cdot \cos3\theta$

Q. Solve for x:

15. $\sin x + \cos x - 2\sqrt{2}\sin x\cos x = 0$

16. $\sin^3 x + \sin x\cos x + \cos^3 x = 1$

17. $\sin x + \cos x = 1 - \sin x\cos x$

18. $1 + \sin^3 x + \cos^3 x = \frac{3}{2}\sin 2x$

19. $\sin 2x - 12(\sin x - \cos x) + 12 = 0$, where $0 \le x \le 2\pi$

SOLUTIONS IN CASE IF TWO EQUATIONS ARE GIVEN:

20. If $(1 + \tan A)(1 + \tan B) = 2$, then find all the values of $A + B$

21. If $\tan(A - B) = 1$ and $\sec(A + B) = \frac{2}{\sqrt{3}}$, then find the smallest +ve values of A and B and their most general values.

22. If $\sin(\pi\cos\theta) = \cos(\pi\sin\theta)$, then prove that, $\cos\left(\theta \pm \frac{\pi}{4}\right) = \frac{1}{2\sqrt{2}}$.

23. If $\tan(\pi\cos\theta) = \cot(\pi\sin\theta)$, then prove that $\cos\left(\theta - \frac{\pi}{4}\right) = \frac{1}{2\sqrt{2}}$

24. If $\sin A = \sin B$ and $\cos A = \cos B$, then find the values of A in terms of B.

25. If A and B are acute +ve angles satisfying the equations $3\sin^2 A + 2\sin^2 B = 1$ and $3\sin A - 2\sin B = 0$, then find $A + 2B$.

DIFFERENT TYPES OF TRIGNOMETRIC EQUATION

TYPE – 4 (Product to Sum)

Q. Solve:

26. $4\sin x \cdot \sin 2x \cdot \sin 4x = \sin 3x$

27. $\cos x \cdot \cos 2x \cdot \cos 3x = \frac{1}{4,0} \leq x \leq 2\pi$

28. $\sin 3\alpha = 4\sin\alpha \cdot \sin(x+\alpha) \cdot \sin(x-\alpha)$

29. $\sin 2x \cdot \sin\ 4x + \cos\ 2x = \cos 6x$

30. $\sec x \cdot \cos 5x + 1 = 0, 0 \leq x \leq 2\pi$

31. $\cos x \cdot \cos 6x = -1$

TYPE – 5 (Make one Variable)

Q. Solve:

32. $2\sin^2 x - 5\sin x \cos x - 8\cos^2 x = -2$

33. $5\sin^2 x - 7\sin x \cos x + 10\cos^2 x = 4$

34. $2\sin^2 x - 5\sin x \cos x - 8\cos^2 x = -3$

35. $\sin^3 x \cos x + \sin^2 x \cos^2 x + \sin x \cos^3 x = 1$

TYPE – 6 (Boundness)

Q. Solve for x:

36. $\sin 6x + \cos 4x + 2 = 0$

37. $\sin^6 x = 1 + \cos^4 3x$

38. $\sin^4\ x = 1 + \tan^8 x$

39. $\sin^2 x + \cos^2 y = 2\sec^2 z$

40. $\sin\ 3x + \cos\ 2x + 2 = 0$

41. $\cos\ 4x + \sin\ 5x = 2$

Type – 7 ($a^{f(x)} = b$ Type)

Q. Solve for x:

42. Find the values of x in $(-\pi, \pi)$ which satisfy the equation

$8^{1+|\cos x| + \cos^2 x + |\cos x|^3 + \cos^4 x + |\cos x|^5 + \cdots \text{ to } \infty} = 64$

43. $2^{1+|\cos x| + \cos^2 x + |\cos x|^3 + \cos^4 x + |\cos x|^5 + \cdots \text{ to } \infty} = 4$

44. $1 + \sin\theta + \sin^2\theta + \sin^3\theta + \cdots \text{ to } \infty = 4 + 2\sqrt{3}$

45. $|\cos x|^{\sin^2 x - \frac{3}{2}\sin x + \frac{1}{2}} = 1$

46. $e^{\sin\ x} - e^{-\sin\ x} - 4 = 0$

47. If $e^{[\sin^2 x + \sin^4 x + \sin^6 x + \cdots + \text{ to } \infty]\log_e 2}$ satisfies the equations $x^2 - 9x + 8 = 0$, then find the value of $\frac{\cos x}{\cos x + \sin x}, 0 < x < \frac{\pi}{2}$

TYPE – 8

Q. Solve for x:

48. $2\cos^2\left(\frac{x}{2}\right)\sin^2 x = x^2 + \frac{1}{x^2}, 0 < x < \frac{\pi}{2}$

49. $2\cos^2\left(\frac{x^2+x}{6}\right) = 2^x + 2^{-x}$

TYPE – 9 (Miscellaneous)

Q. Solve:

50. $1 + 2\text{cosec } x = -\frac{\sec^2\left(\frac{x}{2}\right)}{2}$.

51. $\cot\left(\frac{x}{2}\right) - \text{cosec}\left(\frac{x}{2}\right) = \cot x$

52. If $\theta_1, \theta_2, \theta_3, \theta_4$ be the four roots of the equation $\sin(\theta + \alpha) = k \sin 2\theta$, no two of which differ by a multiple of 2π, then prove that $\theta_1 + \theta_2 + \theta_3 + \theta_4 = (2n+1)\pi, (n \in Z)$

TYPE- 10 (Putting Value of y in Function)

Q. Solve for x & y:

53. Solve $x + y = \frac{\pi}{4}$ and $\tan x + \tan y = 1$.

54. Solve $x + y = \frac{2\pi}{3}$ and $\sin x = 2 \sin y$.

55. Solve $x + y = \frac{2\pi}{3}$ and $\cos x + \cos y = \frac{3}{2}$

Answer Key

1. $\left(n\pi + (-1)^n\left(\frac{\pi}{4}\right) - \frac{\pi}{4}\right)$

2. $n\pi + (-1)^n\left(\frac{\pi}{2}\right) - \frac{\pi}{6}$

3. $n\pi - \frac{\pi}{4}$ and $\theta = 2n\pi \pm \left(\frac{2\pi}{3}\right)$

4. $\frac{1}{2}\left(n\pi + (-1)^n\left(\frac{\pi}{6}\right)\right)$

5. $n\pi + (-1)^n\left(\frac{\pi}{4}\right) - \frac{\pi}{3}$

13. $\frac{2n\pi}{3} + \frac{\pi}{9}$

14. $(2k+1)\frac{\pi}{4} - \frac{\pi}{12}$

15. $x = n\pi - (-1)^n\frac{\pi}{6} - \frac{\pi}{4}, n \in Z$

17. $x = n\pi + (-1)^n\left(\frac{\pi}{4}\right) - \frac{\pi}{4}, n \in I$

18. $x = n\pi + (-1)^n\left(-\frac{\pi}{4}\right) - \frac{\pi}{4}, n \in I$

19. $x = 0, \frac{\pi}{2}, 2\pi$ 20. $(A+B) = n\pi + \left(\frac{\pi}{4}\right)$

21. $\begin{cases} A = (2n+m)\frac{\pi}{2} + \frac{\pi}{24} \\ B = (2n-m)\frac{\pi}{2} - \frac{5\pi}{24} \end{cases}$ 24. $A = n\pi + B$

25. $(A+2B) = \frac{\pi}{2}$

26. $x = n\pi, x = (3n \pm 1)\frac{\pi}{9}, n \in Z$ 29. $x = \frac{n\pi}{4}, x = \frac{n\pi}{2}, n \in I$

30. $x = \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \frac{5\pi}{6}$

31. $x = (2n+1)\frac{\pi}{7}, x = (2n+1)\frac{\pi}{5}, n \in I$

32. $\alpha = \tan^{-1}(2), \beta = \tan^{-1}\left(-\frac{3}{4}\right), n \in Z.$

33. $x = n\pi + \frac{\pi}{4}, x = n\pi + \alpha, \alpha = \tan^{-1}(5)$

34. $x = n\pi + \alpha, \alpha = \tan^{-1}\left(\frac{1 \pm \sqrt{5}}{2}\right)$

35. no solution

36. $x = m\pi + \frac{\pi}{4}, m \in Z$

38. $x = n\pi \pm \frac{\pi}{2}, x = n\pi, n \in I$

39. $x = (2n+1)\frac{\pi}{2}, y = m\pi, z = k\pi$

40. $x = 2n\pi + \frac{\pi}{2}$

41. $x = 2n\pi + \frac{\pi}{2} = (4n+1)\frac{\pi}{2}$

42. $x = \pm\frac{\pi}{3}, \pm\frac{2\pi}{3}$

43. $x = \pm\frac{\pi}{3}, \pm\frac{2\pi}{3}$

44. $n\pi + (-1)^n\left(\frac{\pi}{3}\right)$

45. $n\pi + (-1)^n\left(\frac{\pi}{6}\right), 2n\pi, (2n+1)\pi$

46. no solution

47. $\frac{(\sqrt{3}-1)}{2}$

48. no solution

50. $x = 2n\pi - \frac{\pi}{2}, n \in I$

51. $x = 2n\pi$

53. no solutions

54. $\begin{cases} x = (2n+1)\frac{\pi}{2} \\ y = n\pi - \frac{\pi}{6} \end{cases}, n \in I$

55. No Solutions.