Light Questions

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Q.1 Compute the following without log tables:

(A) $\log_{\sqrt{2}} 16$

Ans. **(8)**

Sol. $\log_{2^{\frac{1}{2}}} 16$ $\left(\log_{b^c} a = \frac{1}{c}\log_b a\right)$

$=2\log_2 16$

$=2\log_2 2^4$ $\left(\log a_c^b = b \log a_c\right)$

$=2\text{x}4(\log_2 2)$

$= 8$

(B) $\log_{\sqrt{2}}\left(32\sqrt[5]{4}\right)$

Ans. $\frac{54}{5}$

Sol. $\log_{2^{1/2}} 2^5(2^2)^{1/5} = \log_{2^{1/2}} 2^5 \cdot 2^{2/5}$

$= \log_{2^{1/2}} 2^{5+2/5} = \left(5+\frac{2}{5}\right)\cdot 2(\log_{2^2})$

$= \left(5+\frac{2}{5}\right)2 = \frac{27}{5}\cdot 2 = \frac{54}{5}$

(C) $\log_{125} 625$

Ans. $\frac{4}{3}$

Sol. $= \log_{5^3} 5^4 = 4\times\frac{1}{3}(\log_5 5) = \frac{4}{3}$

(D) $\log_{\sqrt{3}+\sqrt{2}} \sqrt{3}-\sqrt{2}$

Ans. -1

Sol. $\log_{\sqrt{3}+\sqrt{2}}(1/\sqrt{3}+\sqrt{2}) = -1$

$$\because (\sqrt{3}+\sqrt{2})\cdot(\sqrt{3}-\sqrt{2}) = 1$$
$$\therefore (\sqrt{3}-\sqrt{2}) = \frac{1}{\sqrt{3}+\sqrt{2}}$$
$$\& \log_{\frac{1}{a}} a = -1 = \log_a \frac{1}{a}$$

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(E) $\log_2\left(\frac{1}{512}\right)$

Ans. -9

Sol. $\log_2\left(\frac{1}{2^9}\right) \Rightarrow \log_2\ 2^{-9} = (-9)(\log_2 2) = -9$

(F) $\log_{(\cot\ 45^\circ)}(\operatorname{cosec}\ 45^\circ)$

Ans. **Not possible**

Sol. $\log_{(1)}(1/\sqrt{2})$
But Base Can not be equal to "1"
$\therefore$ Not possible

Q.2 Evaluate:

(A) $\log_{3^{-9}} 27 - 2\log_{3^{-4}} 9$

Ans. $\frac{2}{3}$

Sol. $\log_{3^{-9}}\ 3^3 - 2\log_{3^{-4}}\ 3^2$

$\frac{(3)}{(-9)} - 2\frac{(2)}{(-4)} - \frac{1}{3} + 1 = \frac{2}{3}$

(B) $\log_3 \log_5 125$

Ans. **1**

Sol. $\log_3(\log_5\ 5^3) = \log_3(3\log_5 5) = \log_3\ 3 = 1$

Q.3 Evaluate:

(A) $4^{\log_2 6}$

Ans. **36**

Sol. $6^{\log_2\ 4} = 6^{\log_2\ 2} = 6^{2(\log_2\ 2)} = 6^{2(1)} = 6^2 = 36$

(B) $4^{\log_{10} 100} + 100^{\log_{10} 4}$

Ans. **32**

Sol.

$\because$ (i) $a^{\log_a\ b} = b$

(ii) a $\log_c^b = b^{\log_c\ a}$

$= 4^{\log_{10}\ 100} + 4^{\log_{10}\ 100} = 2\cdot\left(4^{\log_{10}\ 100}\right)$

$= 2\left(4^{\log_{10}\ 10^2}\right) = 2\cdot\left(4^{2\log_{10}\ 10}\right) = 2(4^2) = 32$

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Q.4 The value of $\log 99(0.\overline{9})$ is =__________

Ans. **0**

Sol. Let

$x = 0.\overline{9}$

$x = 0.999999 \ldots(1)$

$10\,(x) = 10 \cdot (0.99999\ldots)$

$10x = 9.999999 \ldots(2)$

(2) – (1)

$10x = 9.999999 \ldots\ldots$

$\underline{\quad x = 0.99999 \ldots \ldots\ldots}$

$9x = 9 \Rightarrow x = 1$

$\therefore \log_{99}(0 \cdot \overline{9}) = \log_{99} 1 \quad = 0$

Q.5 **(i)** $4^{\log_{\frac{1}{2}} 3}$ =________

Ans. $\mathbf{\frac{1}{9}}$

Sol. $= 4^{\log_2^3 -1} = 4^{-\log_2 3} = (2^2)^{\log_2 3^{-1}} = 2^{2\log_2 3^{-1}} = 2^{\log_2 3^{-2}} = 3^{-2} = \frac{1}{9}$

(ii) $8^{\frac{1}{\log_3 2}}$ =________

Ans. **27**

Sol. $= 8^{\log_2 3} = (2^3)^{\log_2 3} = 2^{3\log_2 3} = 2^{\log_2 3^3} = 3^3 = 27$

(iii) $2^{\log_3 7} + 5^{\log_8 11} - 7^{\log_3 2} - 11^{\log_8 5}$

Ans. **0**

Sol. $(\because a^{\log_c b} = b^{\log_c a})$

$2^{\log_3 7} + 5^{\log_8 11} - 2^{\log_3 7} - 5^{\log_8 11}$ =0

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(iv) $\log_4 (\sqrt{4\sqrt{4}\sqrt{4}})$

Ans. **1**

Sol. $\log_4 \left(4^{1/2} \cdot 4^{1/4} 4^{1/4}\right) = \log_4 4^{\left(\frac{1}{2}+\frac{1}{4}+\frac{1}{4}\right)} = \log_4 4^1 = 1$

(v) $\log_5 \left(\log_5 (\sqrt{5\sqrt{5}\sqrt{5}}\right)$

Ans. **0**

Sol. $\log_5 \left\{\log_5 \left(5^{1/2} 5^{\frac{1}{4}} 5^{1/4}\right)\right\} = \log_5 \left\{\log_5 \left(5^{\frac{1}{2}+\frac{1}{4}+\frac{1}{4}}\right)\right\} = \log_5 (\log_5 5^1) = \log_5 1 = 0$

(vi) $\log_{\frac{1}{3}} \left(\sqrt[4]{729 \cdot \sqrt[3]{9^{-1} \cdot 27^{\frac{-4}{3}}}}\right)$

Ans. **–1**

Sol. $= \log_{1/3} \left\{\sqrt[4]{3^6 \sqrt[3]{(3^2)^{-1}(3^3)^{-4/3}}}\right\} = \log_{1/3} \{3^6(3^{-2}3^{-4})^{1/3}\}^{1/4} = \log_{1/3} [3^6(3^{-6})^{1/3}]^{\frac{1}{4}}$

$= \log_{1/3} (3^6 \cdot 3^{-2})^{\frac{1}{4}} = \log_{1/3} (3^4)^{1/4} = \log_{1/3} 3 = -1$

Q.6 Prove that:

(A) $7 \log \frac{16}{15} + 5 \log \frac{25}{24} + 3 \log \frac{81}{80} = \log 2$

Ans.

Sol. $\begin{bmatrix} \log \left(\frac{a}{b}\right) = \log a - \log b \\ \log_a b = b\log a \\ \log a \cdot b = \log a + \log b \end{bmatrix}$

$7(\log 16 - \log 15) + 5(\log 25 - \log 24) + 3(\log 81 - \log 80)$

$7[\log 2^4 - \log (3 \cdot 5)] + 5[\log 5^2 - \log (2^3 \cdot 3)] + 3[\log 3^4 - \log (2^4 \cdot 5)]$

$7[(4\log 2) - (\log 3 + \log 5)] + 5[2\log 5 - (\log 2^3 + \log 3)] + 3[(4\log 3) - (\log 2^4 + \log 5)]$

$7(4\log 2 - \log 3 - \log 5) + 5[2\log 5 - 3\log 2 - \log 3] + 3[4\log 3 - 4\log 2 - \log 5]$

$(28 \log 2 - 15 \log 2 - 12 \log 2) + (-7 \log 3 - 5 \log 3 + 12 \log 3) + (-7 \log 5 + 10 \log 5 - 3 \log 5)$

$= \log 2$

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(B) $\log(1+2+3)=\log 1+\log 2+\log 3$

Ans.

Sol. LHS $= \log(1+2+3) = \log 6 = \log(1\cdot 2\cdot 3) = \log 1 + \log 2 + \log 3 =$ R.H.S

Q.7 Evaluate:

(A) $\log_{10}\sin 1°\log_{10}\sin 2°\log_{10}\sin 3°.......\log_{10}\sin 179°$

Ans. **0**

Sol. $\log_{10}(\sin 1°)\cdot\log_{10}(\sin 2°)\cdots\log_{10}(\sin 89°)\cdot\log_{10}(\sin 90°).\log_{10}(\sin 91°)\ldots\ldots\log_{10}(\sin 179°)$

$\log_{10}(\sin 1°)\ldots.\log_{10}(\sin 89°)\cdot\log_{10}(1)\ldots.\log_{10}(\sin 179°) = 0$

(B) $\log_2(\tan 1°)\log_2(\tan 2°)\log_2(\tan 3°)\ldots\ldots..\log_2(\tan 89°)$

Ans. **0**

Sol. $\log_2(\tan 1°)\cdot\log_2(\tan 2°)\cdots\log_2(\text{tog } 45°)\cdot\log_2(\tan 46°)\ldots.\log_2(\tan 89°)$

$\log_2(\tan 1°)\cdot\log_2(\tan 2°)\cdots(\log_2 1)^0\cdots-\log_2(\tan 89°) = 0$

(C) $\log_{10}(\log_2 3)+\log_{10}(\log_3 4)+\cdots\ldots..+\log_{10}(\log_{1023} 1024) =$________.

Ans. **1**

Sol. $\log_{10}(\log_2 3)+\log_{10}(\log_3 4)+\log_{10}(\log\ 5)+\cdots\log_{10}(\log_{1023} 1024)$

$$\left[\therefore\ \log_b a = \frac{\log a}{\log b}\right] \& \left[\begin{matrix}\log a+\log b+\log c = \\ \log(a\cdot b\cdot c)\end{matrix}\right]$$

$\log_{10}(\log_2 3\cdot\log_3 4\cdot\log_4 5\cdots\log_{1023} 1024)$

$$\log_{10}\left(\frac{\log 3}{\log 2}\cdot\frac{\log 4}{\log 3}\cdot\frac{\log 5}{\log 4}\ldots\ldots\frac{\log 1023}{\log 1022}\cdot\frac{\log 1024}{\log 1023}\right)$$

$$\log_{10}\left(\frac{\log^{1024}}{\log 2}\right) = \log_{10}(\log_2 1024)$$

$= \log_{10}(\log_2 2^{10}) = \log_{10}(10\log_2 2)$

$= \log_{10}(10(1)) = \log_{10} 10 = 1$