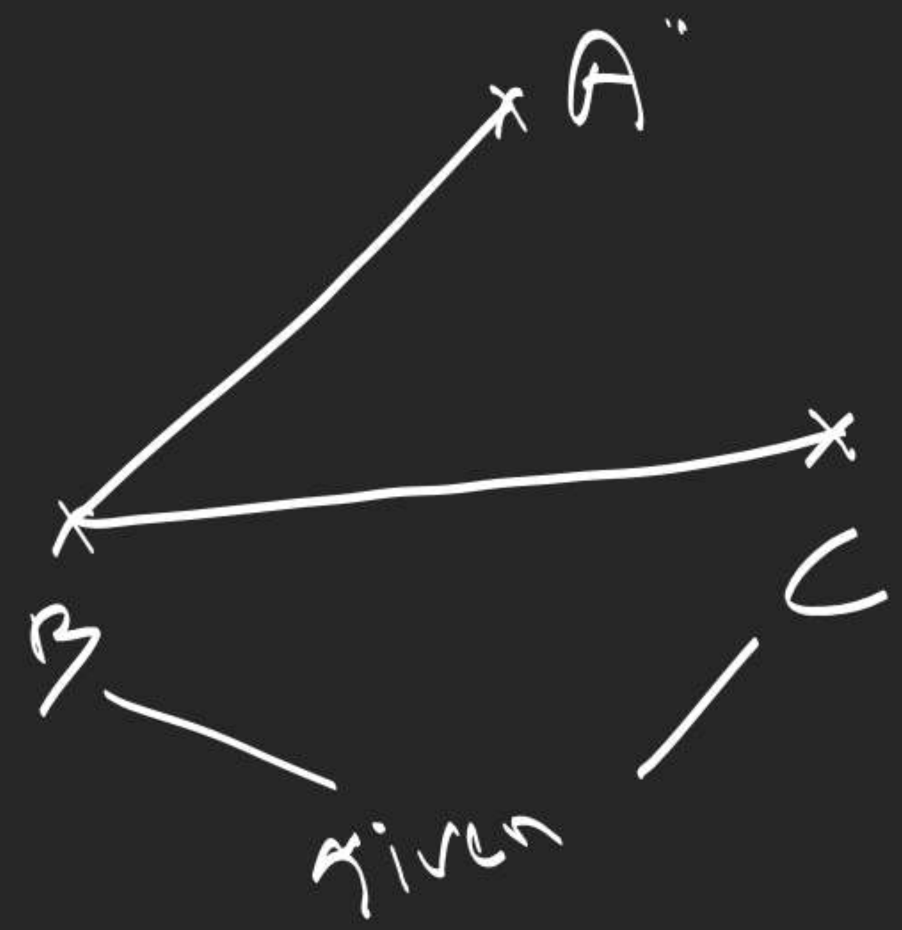
Nishant Jindal



$$\frac{\tan\frac{B}{2}}{\tan\frac{C}{2}} = \frac{\lambda}{1} = \frac{s-c}{s-b}$$

$$\frac{\lambda-1}{\lambda+1} = \frac{b-c}{a} = \frac{AC-AB}{a}$$

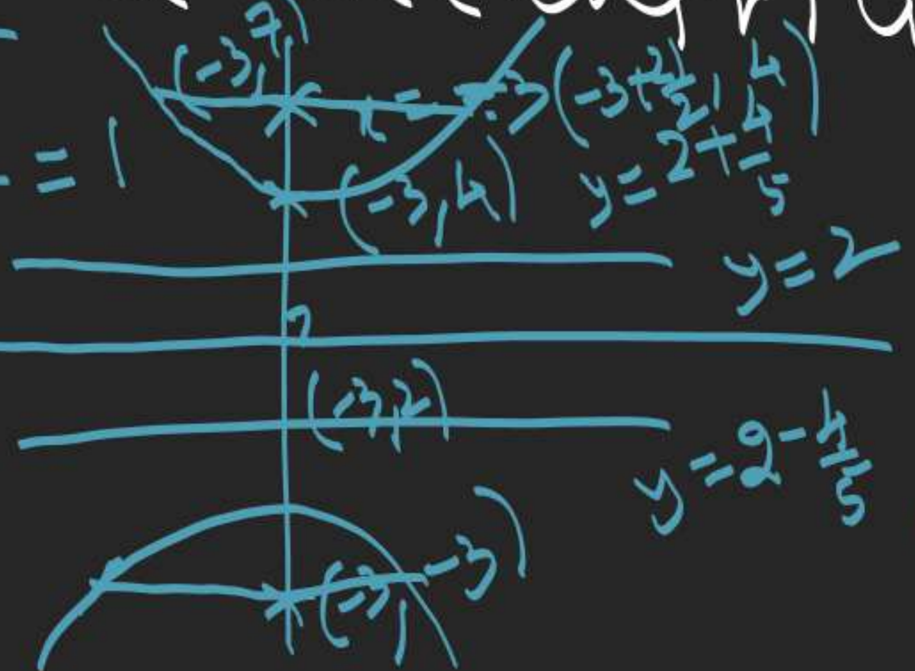
$$\underbrace{AC-AB}_{\text{fixed.}} = \underbrace{a\left(\frac{\lambda-1}{\lambda+1}\right)}_{TA.}$$

1. Find the eqn. of hyperbola

(i) whose centre is $(-3, 2)$, one vertex is $(-3, 4)$ and eccentricity is $\frac{5}{2}$

(ii) whose foci are $(4, 2)$ and $(8, 2)$ and eccentricity is 2.

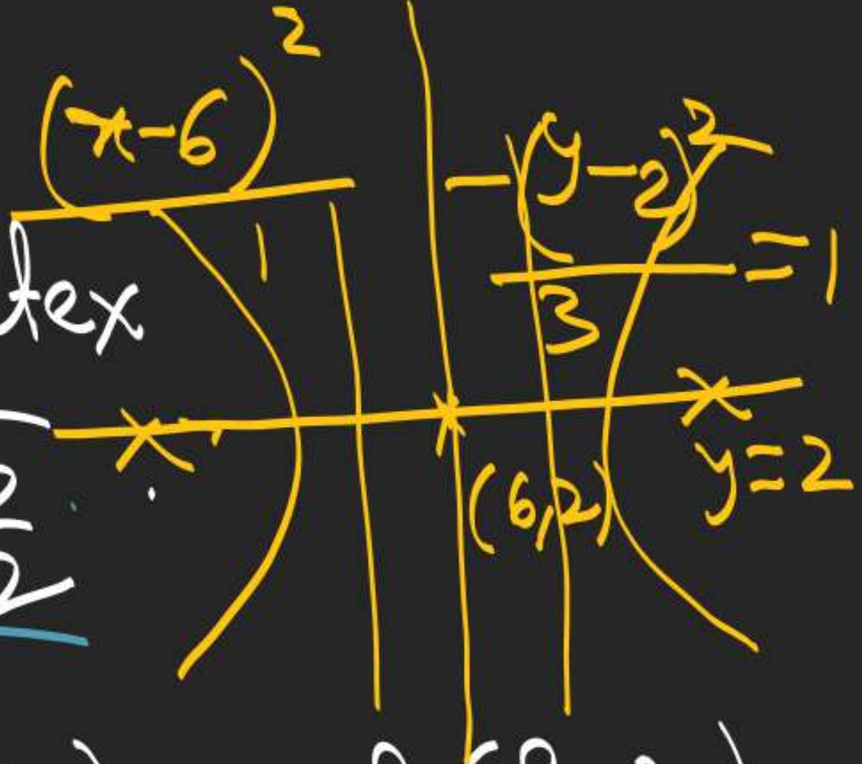
$$\frac{(y-2)^2}{4} - \frac{(x+3)^2}{21} = 1$$



$$b^2 = 4\left(\frac{25}{4} - 1\right) = 21$$

$$\frac{a}{e} = \frac{4}{5} \qquad \frac{b^2}{a} = \frac{21}{2}$$

$$\frac{(x-6)^2}{1} - \frac{(y-2)^2}{3} = 1$$



$2ae = 4$

$\boxed{a = 1}$

$b^2 = 1(4-1)$

$= 3$

Q. An ellipse and a hyperbola are confocal (have the same focus) and the conjugate axis of hyperbola is equal to minor axis of the ellipse. If e_1, e_2 are the eccentricities of ellipse and hyperbola.

Find $\frac{1}{e_1^2} + \frac{1}{e_2^2}$.

$$a^2 e_1^2 = A^2 e_2^2$$

$$b = B$$

$$a^2(1-e_1^2) = A^2(e_2^2-1)$$

$$\frac{1-e_1^2}{e_1^2} = \frac{e_2^2-1}{e_2^2}$$

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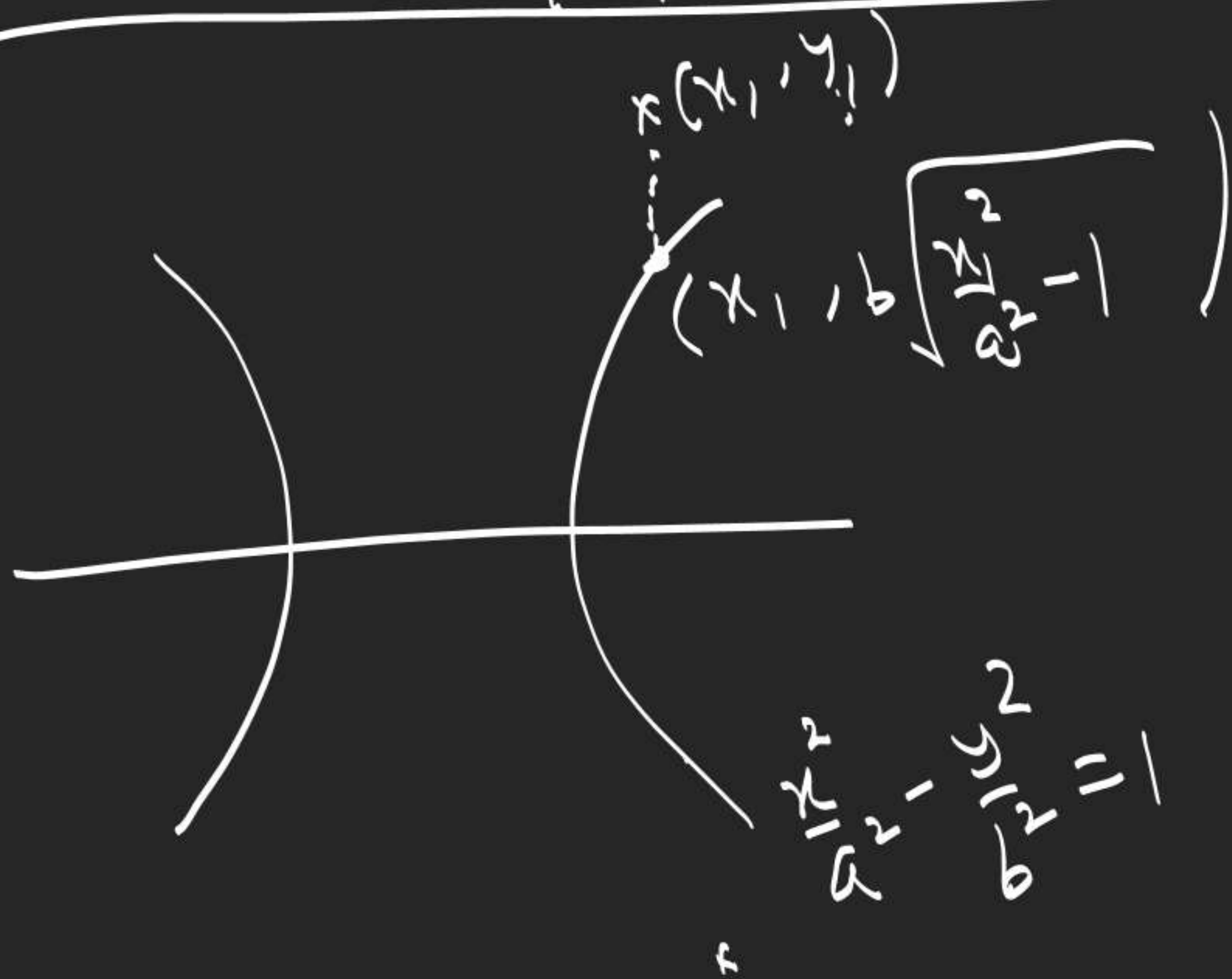
Position of point w.r.t. Hyperbola



$$|y_1| > b\sqrt{\frac{x_1^2}{a^2}-1}$$

$$\frac{x_1^2}{a^2}-\frac{y_1^2}{b^2}-1 < 0$$

$S_1 < 0 \Rightarrow P$ lies outside.

$> 0 \Rightarrow$ inside.

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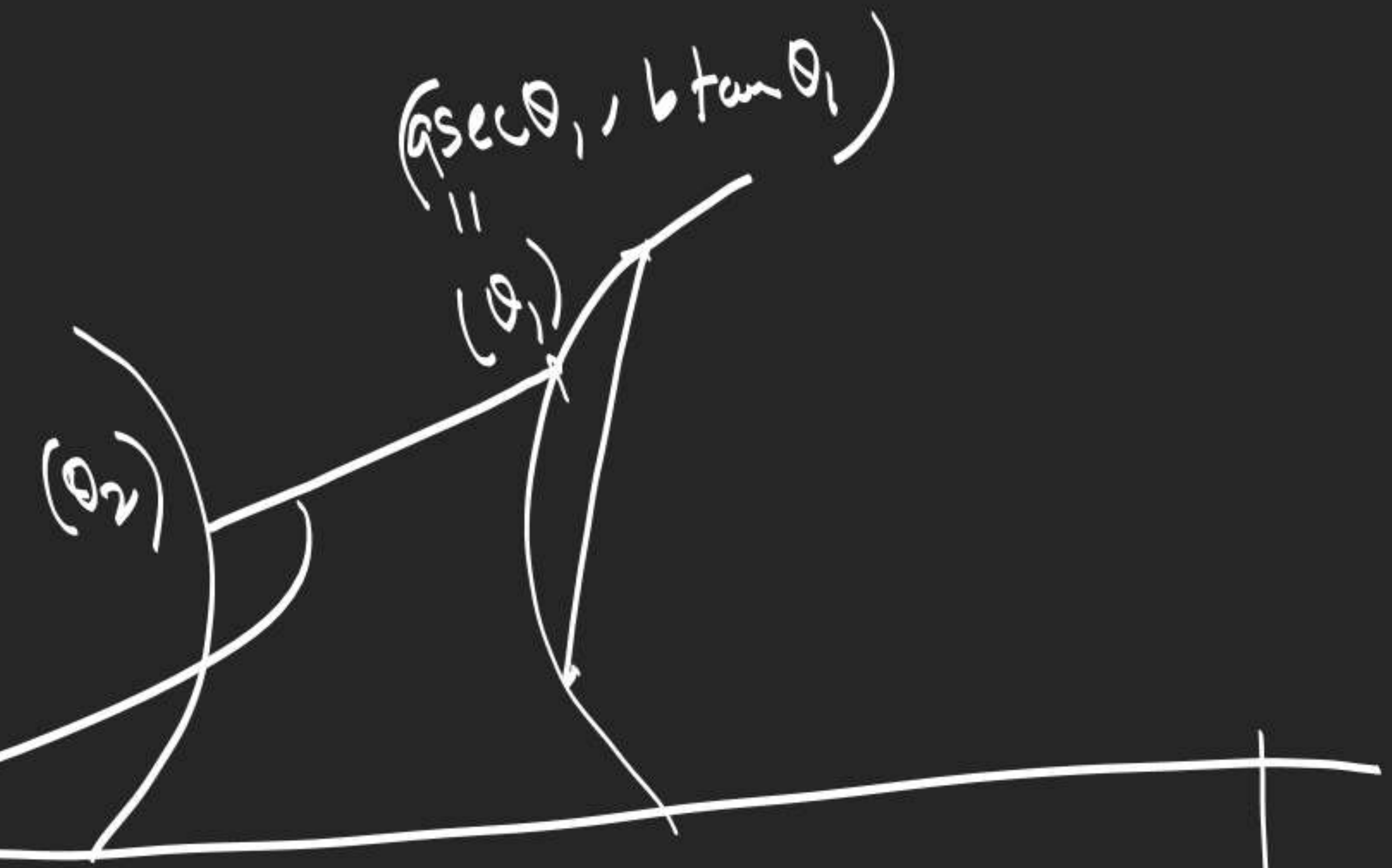
Chord.



$$\boxed{\frac{x}{a}\cos\left(\frac{\theta_1-\theta_2}{2}\right)-\frac{y}{b}\sin\left(\frac{\theta_1+\theta_2}{2}\right)=\cos\left(\frac{\theta_1+\theta_2}{2}\right)}$$

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Tangent

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

$$\frac{x^2}{a^2} - \frac{(mx+c)^2}{b^2} = 1$$

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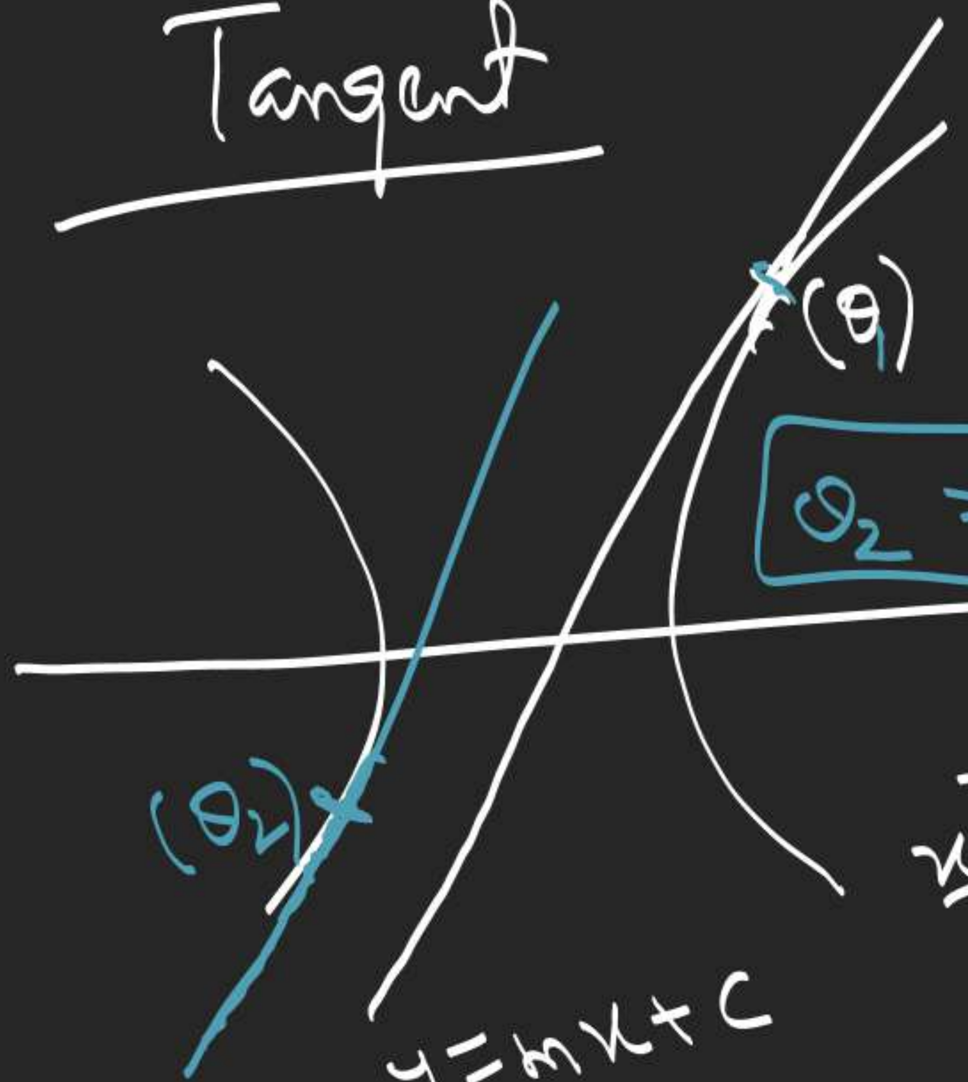
Tangent



$$\boxed{\theta_2 = \pi - \theta_1}$$

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

$$\frac{x}{a}\sec\theta - \frac{y}{b}\tan\theta = 1$$

$$y - mx = c$$

$$\frac{\sec\theta}{-am} = \frac{\tan\theta}{-b} = \frac{1}{c}.$$

$$\left(-\frac{am}{c}\right)^2 - \left(-\frac{b}{c}\right)^2 = 1$$

$$\boxed{c^2 = a^2m^2 - b^2}$$

$$\boxed{y = mx \pm \sqrt{a^2m^2 - b^2} \quad \forall\, m^2 > \frac{b^2}{a^2}}$$

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Director Circle

$$y = mx \pm \sqrt{a^2m^2 - b^2}$$

Put (h,k)

$$(k - mh)^2 = a^2m^2 - b^2$$

$$m^2(h^2 - a^2) - 2hkm + k^2 + b^2 = 0 \quad \begin{cases} m_1 \\ m_2 \end{cases}$$

$$m_1m_2 = \frac{k^2 + b^2}{h^2 - a^2} = -1$$

$$\boxed{x^2 + y^2 = a^2 - b^2 \quad a > b}$$

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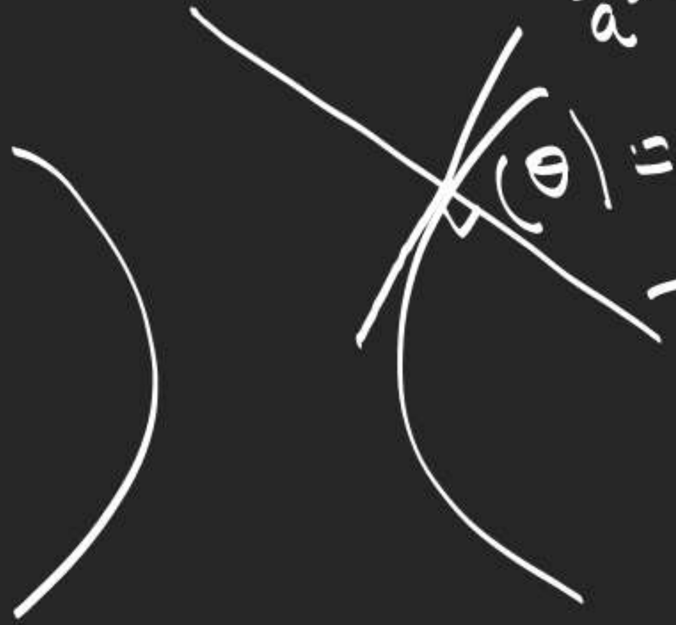
Normal



$$\frac{x}{a}\sec\theta - \frac{y}{b}\tan\theta = 1$$

$$(\theta) = (x_1, y_1)$$

$$\frac{x}{b}\tan\theta + \frac{y}{a}\sec\theta = \sec\theta\tan\theta\left(\frac{a}{b}+\frac{b}{a}\right)$$

$$\frac{ax}{\sec\theta} + \frac{by}{\tan\theta} = a^2+b^2 = a^2e^2.$$

$$\frac{a^2x}{x_1} + \frac{b^2y}{y_1} = a^2+b^2 = a^2e^2$$

Ex-II.