Nishant Jindal

FLUID DYNAMICS

** : Time to empty the tank

At $t = t$. $v = \sqrt{2gy}$.

$$AV = av$$

$$V = \frac{a}{A} v$$

$$V = \frac{a}{A}\sqrt{2gy}$$

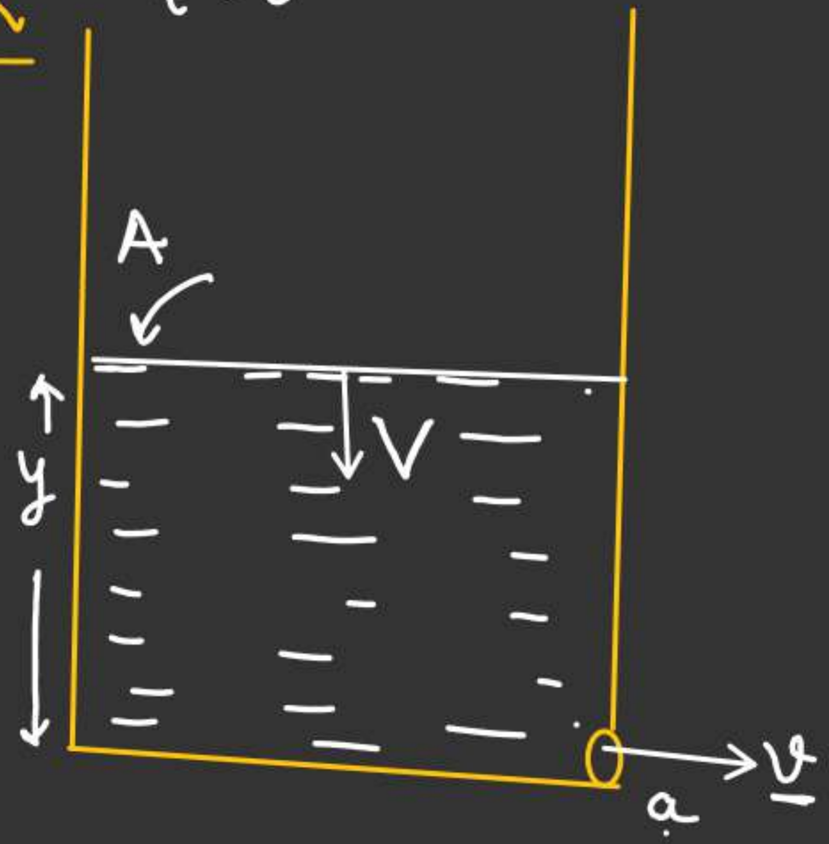
$$-\frac{dy}{dt} = \frac{a}{A}\sqrt{2gy}$$

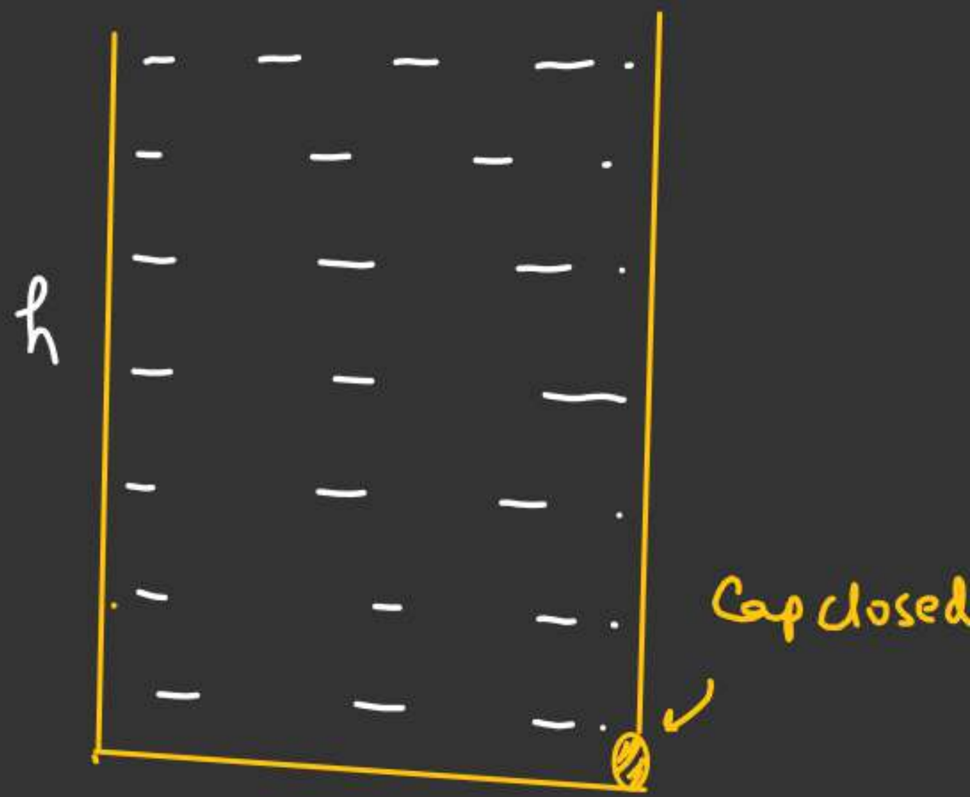
$$\int_h^y \frac{dy}{\sqrt{y}} = -\frac{a}{A}\sqrt{2g}\int_0^t dt$$

$$2\left[\sqrt{y}\right]_h^y = -\frac{a}{A}\sqrt{2g}\, t$$

$$\boxed{\frac{A}{a}\sqrt{\frac{2}{g}}\left[\sqrt{h}-\sqrt{y}\right] = t}$$

$t=t$





** Total time to empty the tank, $y=0,\ t=T$

$$\boxed{T = \frac{A}{a}\sqrt{\frac{2h}{g}}}$$

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FLUID DYNAMICS

Ratio of time taken to empty half of the tank to the time taken to empty the tank.

For half of tank

$$y = \frac{h}{2}$$

$$t_1 = \sqrt{\frac{2}{g}}\left[\sqrt{h} - \sqrt{\frac{h}{2}}\right]$$

$$t_1 = \sqrt{\frac{2h}{g}}\,\frac{[\sqrt{2}-1]}{\sqrt{2}}$$

$$t_1 = \sqrt{\frac{h}{g}}(\sqrt{2}-1)$$

time taken to empty the tank completely

$$t_2 = \sqrt{2h/g}$$

$$\frac{t_1}{t_2} = \left(\frac{\sqrt{2}-1}{\sqrt{2}}\right)$$

By Law of Continuity

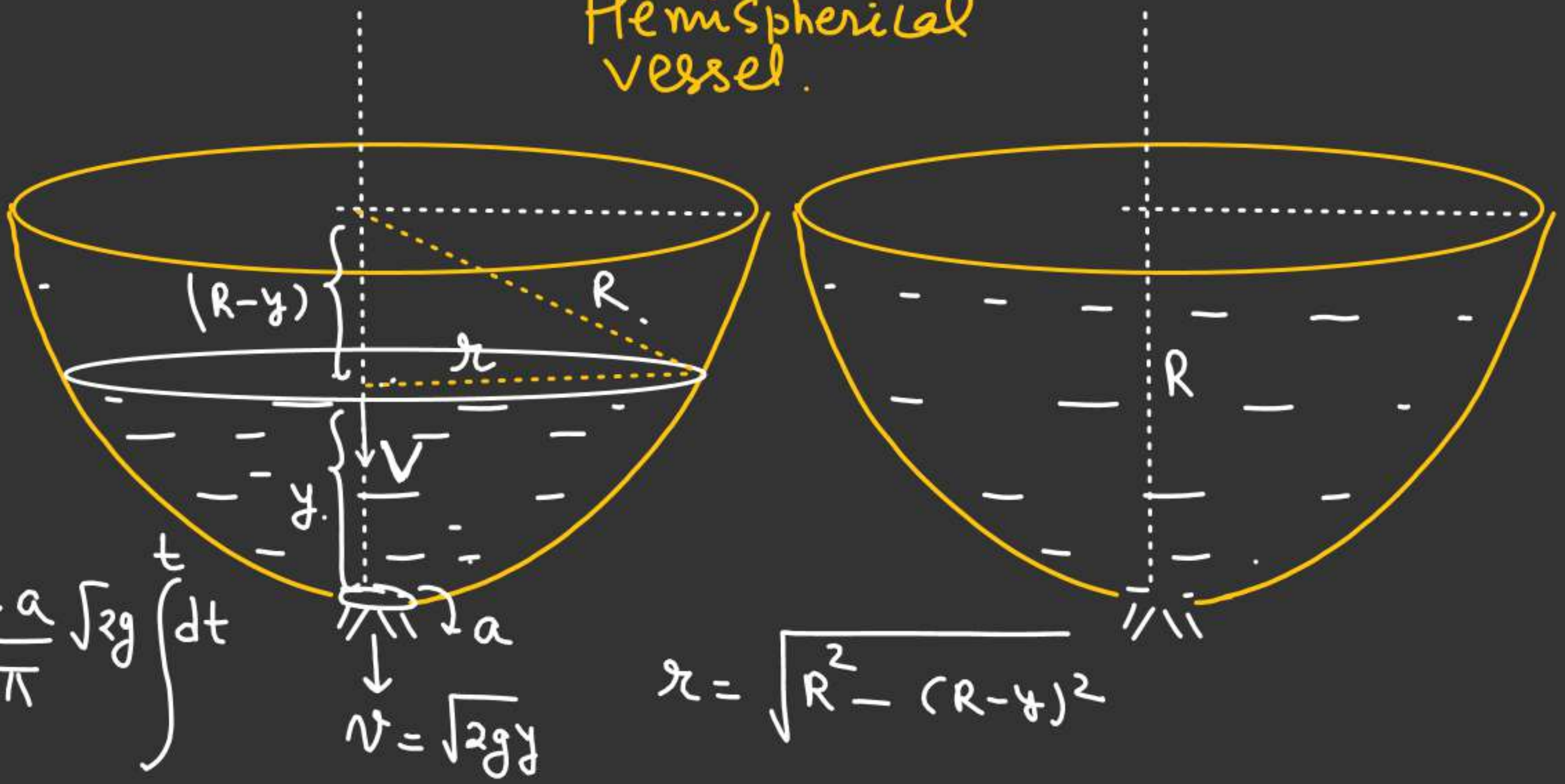
$$(\pi r^2)\underline{V} = a\,\underline{v}$$

$$\pi\left(R^2-(R-y)^2\right)\left(-\frac{dy}{dt}\right) = a\sqrt{2gy}$$

$$\int_R^0 \frac{\left(R^2-(R-y)^2\right)}{\sqrt{y}}\cdot dy = -\frac{a}{\pi}\sqrt{2g}\int_0^t dt$$

$$t = ??$$

Hemispherical vessel.



$$r = \sqrt{R^2-(R-y)^2}$$

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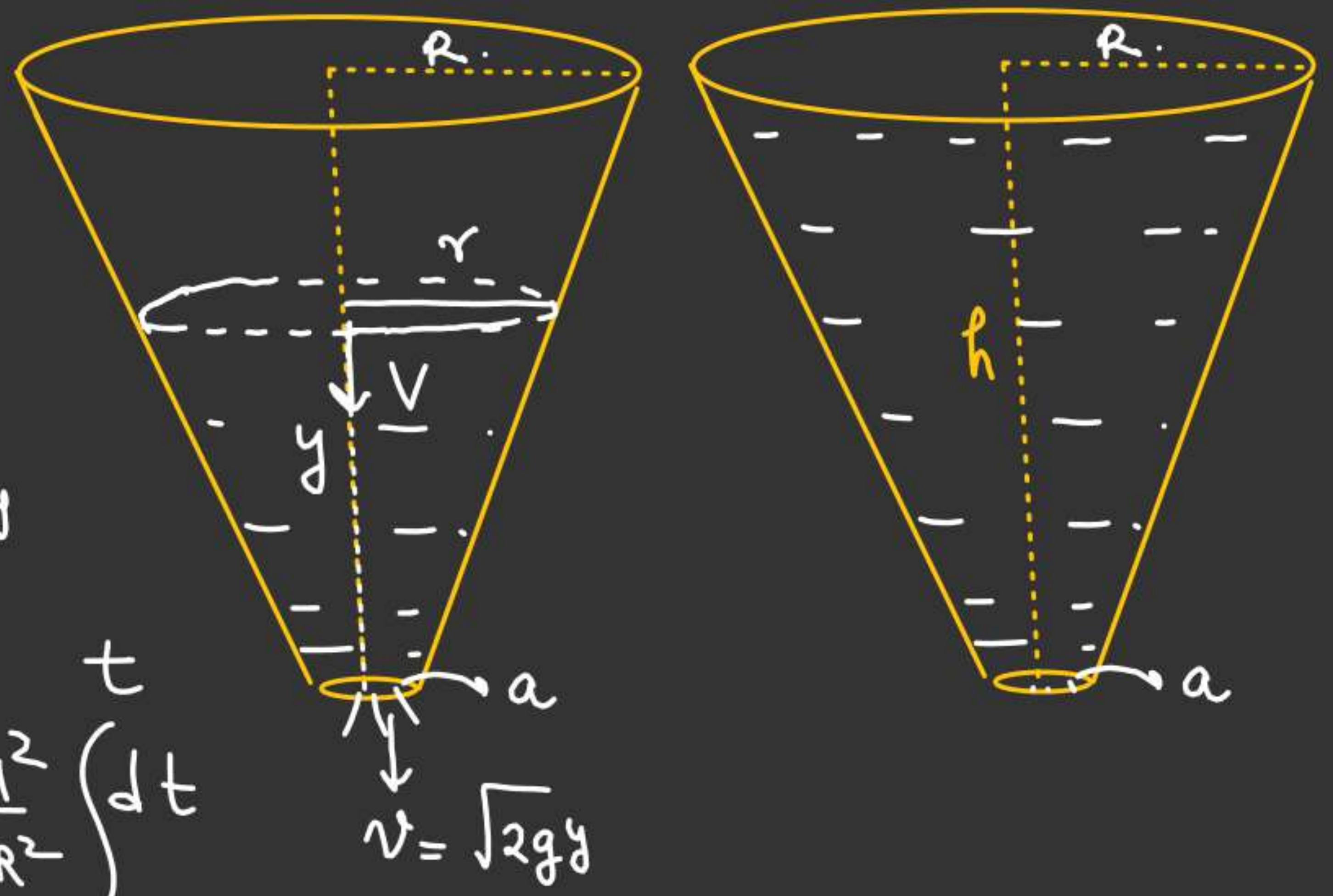
$$\frac{r}{y} = \frac{R}{H}$$

$$r = \left(\frac{R}{H}y\right)$$

$$(\pi r^2)\,\underline{V} = a\sqrt{2gy}$$

$$\pi\left(\frac{R}{H}y\right)^2\left(-\frac{dy}{dt}\right) = a\sqrt{2gy}$$

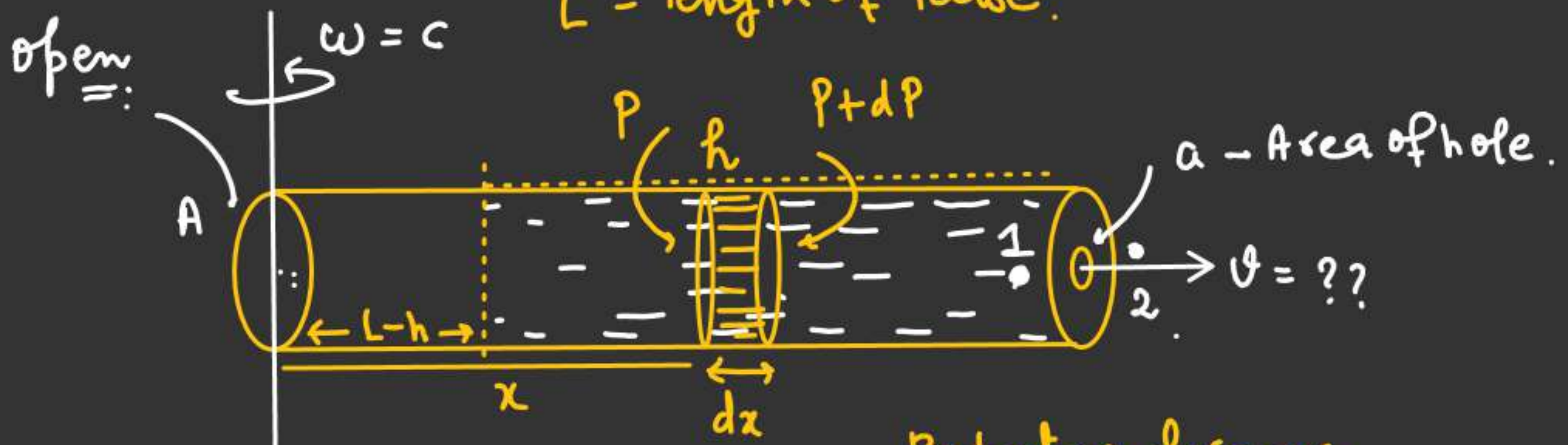
$$\int_h^0 \frac{y^2}{\sqrt{y}}\,dy = -\frac{a}{\pi}\sqrt{2g}\,\frac{H^2}{R^2}\int_0^t dt$$

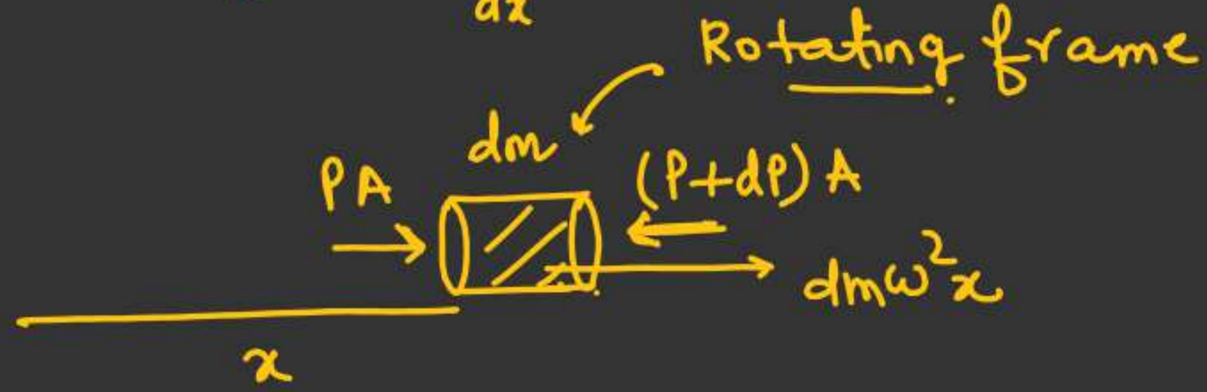


Velocity of efflux in Rotating frame

1 & 2 are points just inside and outside the hole.

L = length of tube.





$$(P+dP)A = PA + \underline{dm}\,\omega^2 x$$

$$dP.\cancel{A} = \rho.\cancel{A}\,dx\,\omega^2 x$$

$$\int dP = \rho\omega^2 \int x\,dx$$

$$\frac{dP}{dx} = \rho\omega^2 x$$

$$\int_{P_{atm}}^{P_1} dP = \rho\omega^2 \int_{(L-h)}^{L} x\,dx$$

$$P_1 - P_{atm} = \frac{\rho\omega^2\left[L^2 - (L-h)^2\right]}{2}$$

$$P_1 = P_{atm} + \frac{\rho\omega^2}{2}\left[L^2 - (L^2 + h^2 - 2Lh)\right]$$

$$P_1 = P_{atm} + \frac{\rho\omega^2}{2}\left[2Lh - h^2\right]$$

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$$P_1 = P_{atm} + \frac{\rho\omega^2}{2}\left[2Lh - h^2\right]$$

Bernoulli's b/w 1 & 2.

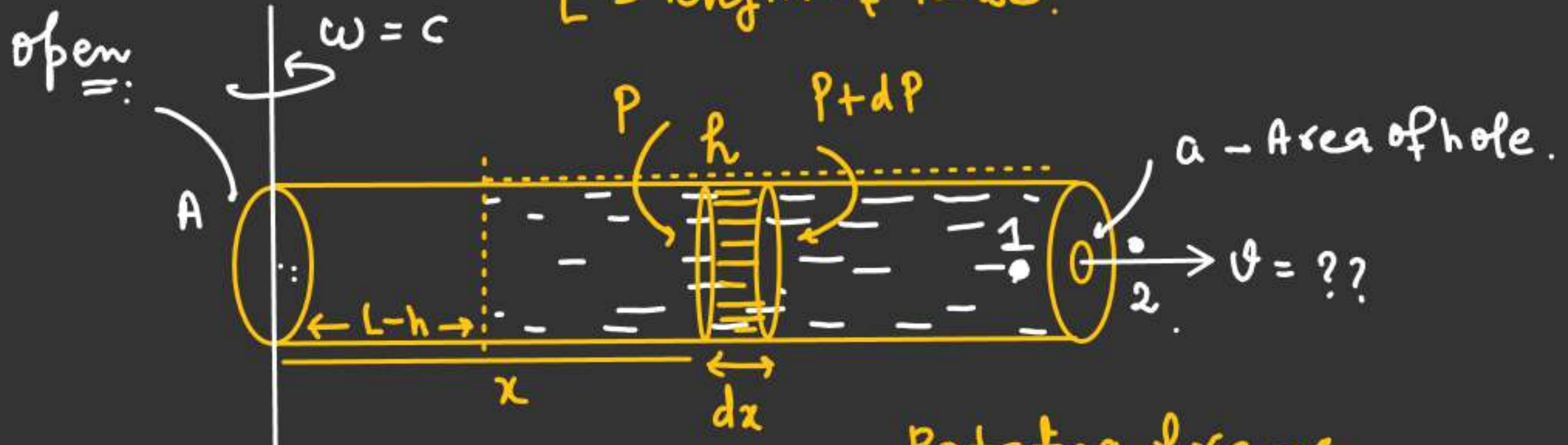
$$P_1 = P_{atm} + \frac{1}{2}\rho v^2$$

$$\cancel{P_{atm}} + \frac{\rho\omega^2}{2}(2Lh - h^2) = \cancel{P_{atm}} + \frac{1}{2}\rho v^2$$

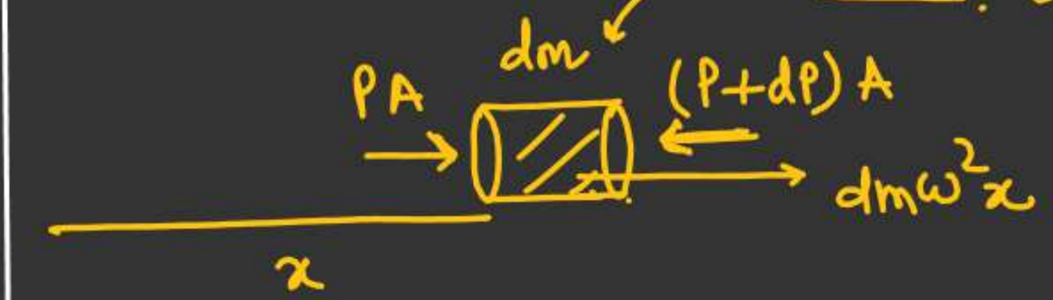
$$\sqrt{\omega^2(2Lh - h^2)} = v$$

$$\left[\, v = \omega h\sqrt{\frac{2L}{h} - 1}\,\right]$$

L = length of tube.



Rotating frame



$$(P+dP)A = PA + dm\,\omega^2 x$$

$$dP\cdot\cancel{A} = \rho\cdot\cancel{A}\,dx\,\omega^2 x.$$

$$\int dP = \rho\omega^2\int x\,dx$$

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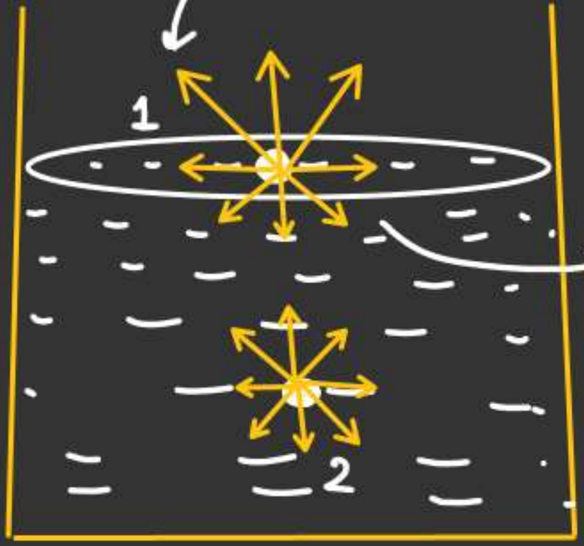
SURFACE TENSION

Two type of forces.

1) Cohesive :- force of attraction b/w molecules of same nature.

2) Adhesive :- force of attraction b/w Molecules of different nature.

Interaction b/w air & liquid Molecule



Interaction b/w liquid-liquid molecule

For Molecule 2 it is surrounded by liquid molecule so force of cohension. No net force on the liquid layer containing molecule-2.

For Molecule 1 it is interacted with air Molecule aswell as liquid-Molecule So there is net force on surface layer. & it acts like a stretch membrane

$$T = \left(\frac{F_T}{L}\right)$$

Surface tension

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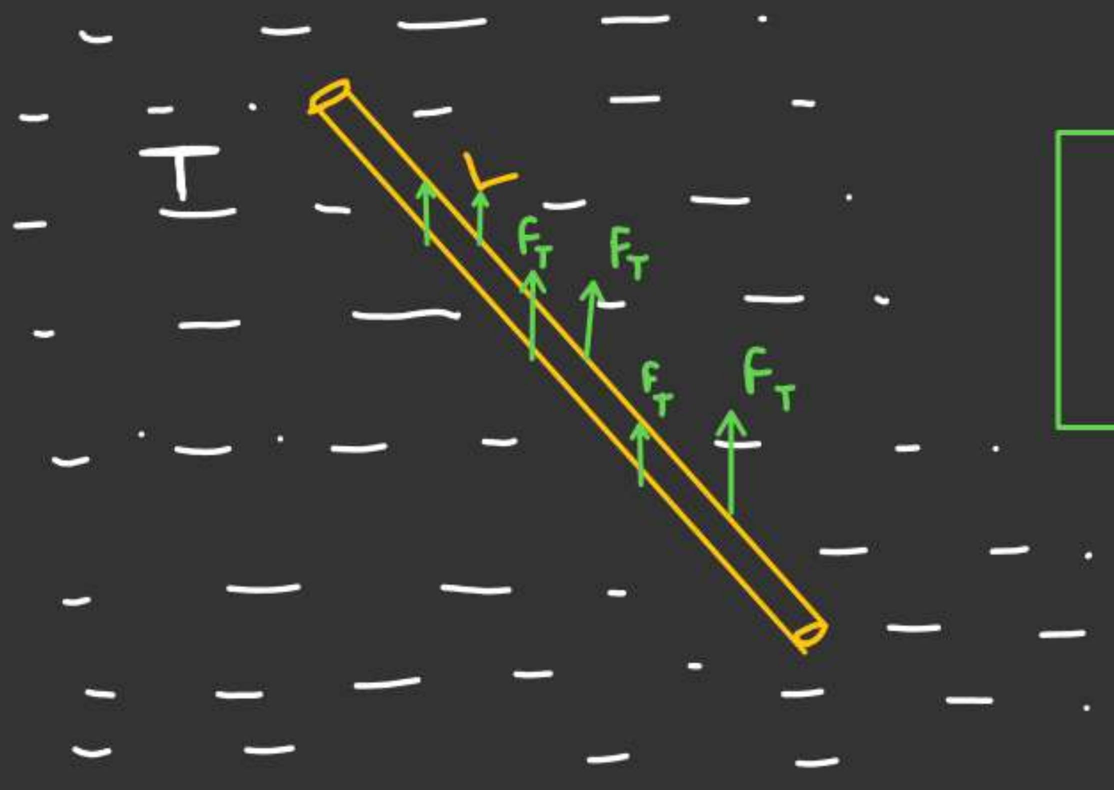
$$\boxed{F = T \cdot L_{eff}}$$

$L_{eff} = (n \times L)$

n = No of contact b/w body and liquid.

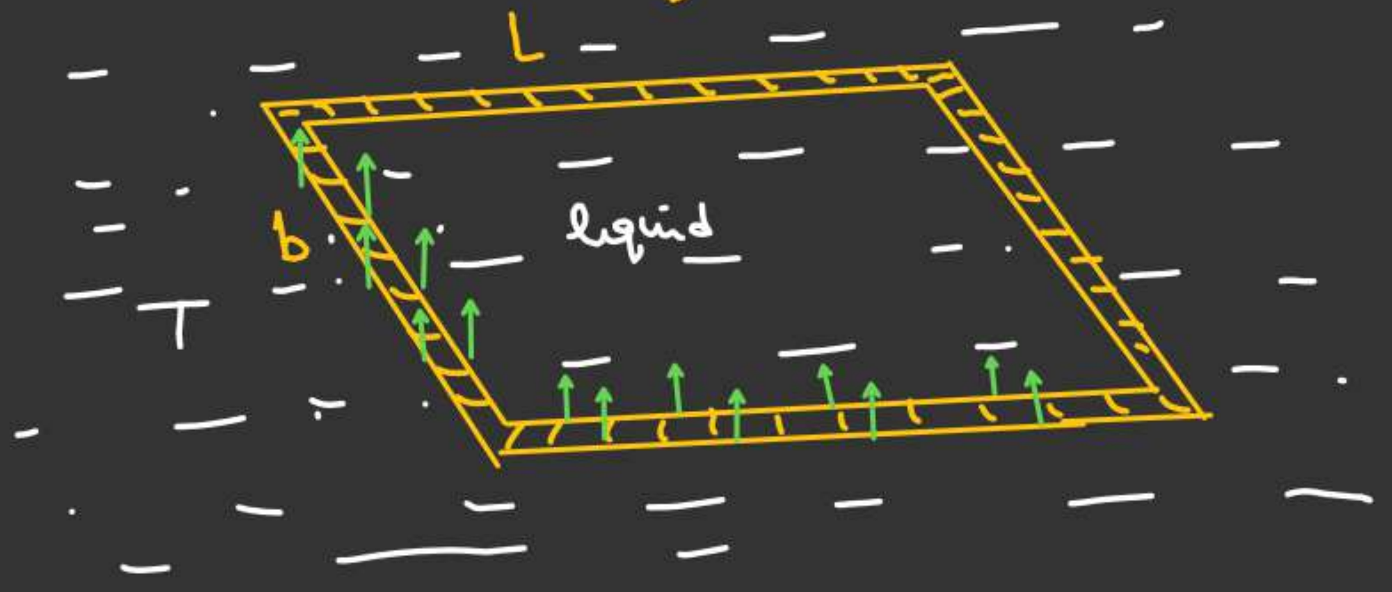
L = length of body in contact with liquid

L_{eff} = Effective length



$$\boxed{F_T = (2TL)}$$

Rectangular wire frame.



$F = T[2(L+b)] \times \underline{2}$

$F = 4T(L+b)$

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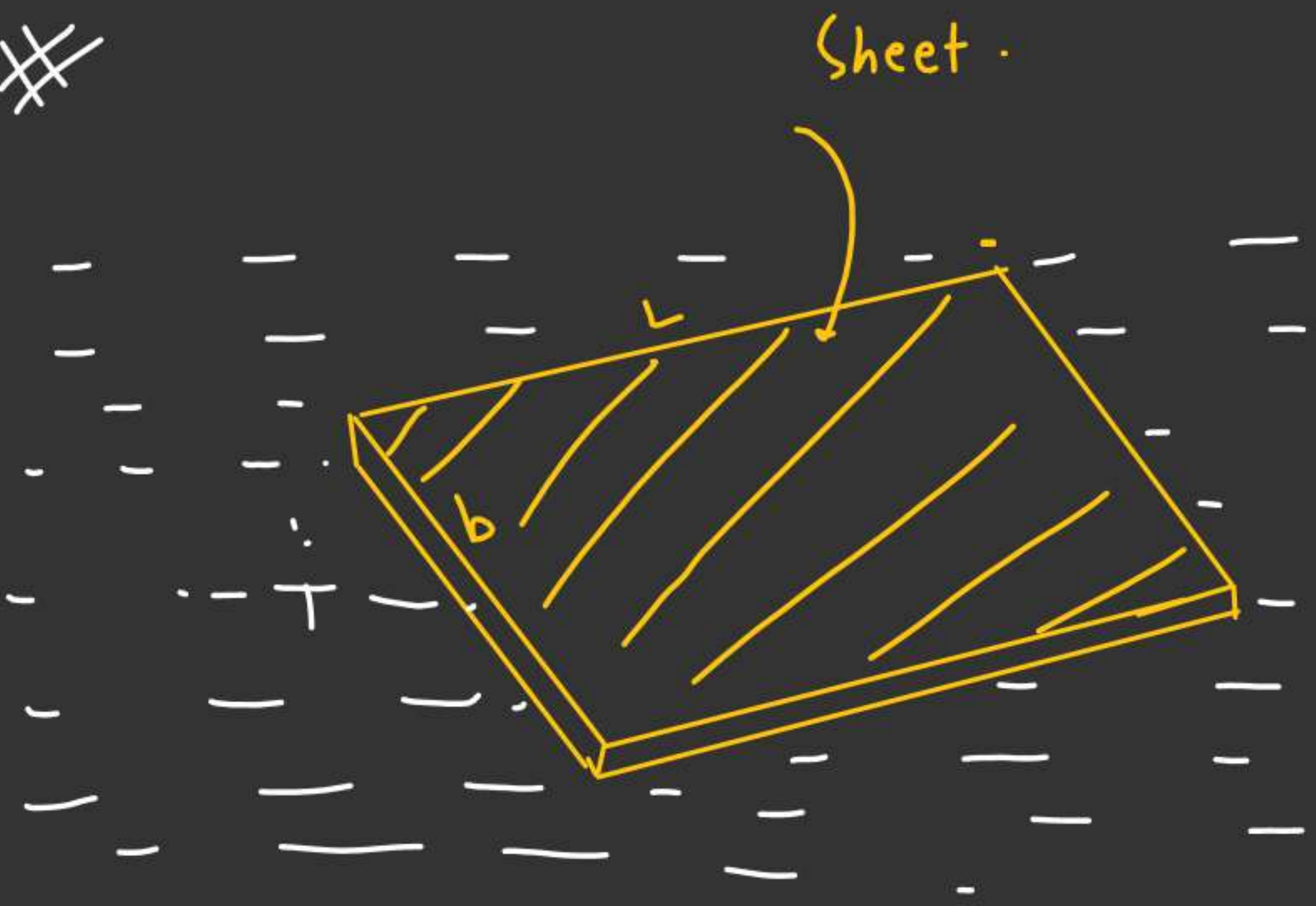
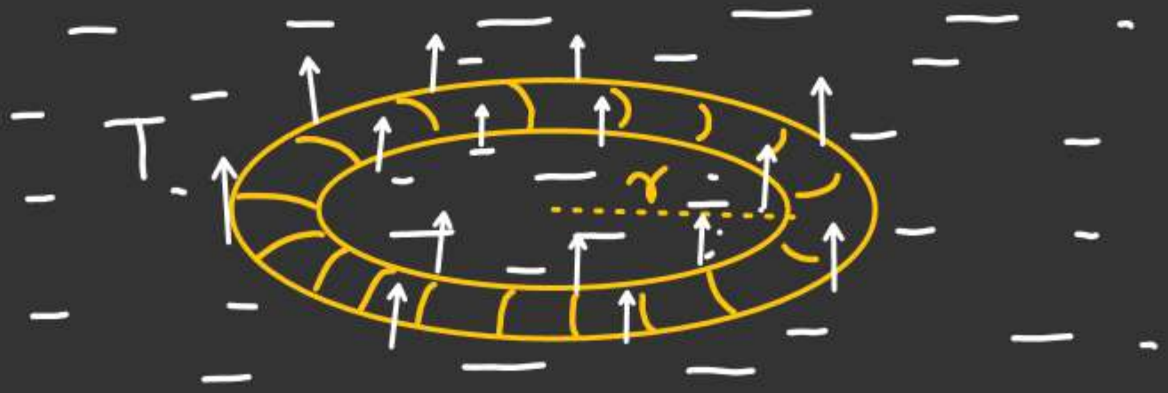
#

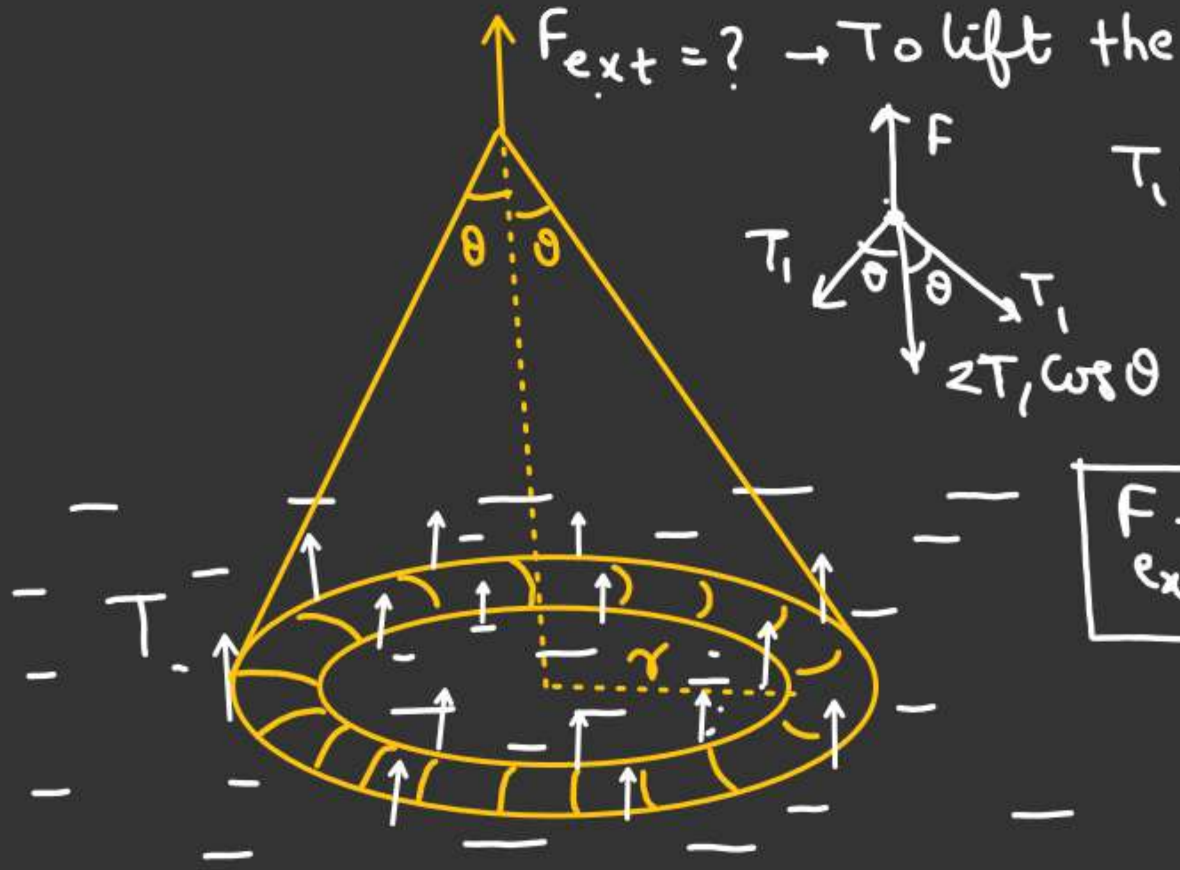


$$\underline{F = 2T(L+b)}$$



$$F_T = [T \times 2\pi r \times 2]$$

$F_{ext} = ?$ → To lift the ring.



T_1 = Tension in string.

$$\boxed{F_{ext} + F_T = mg}$$

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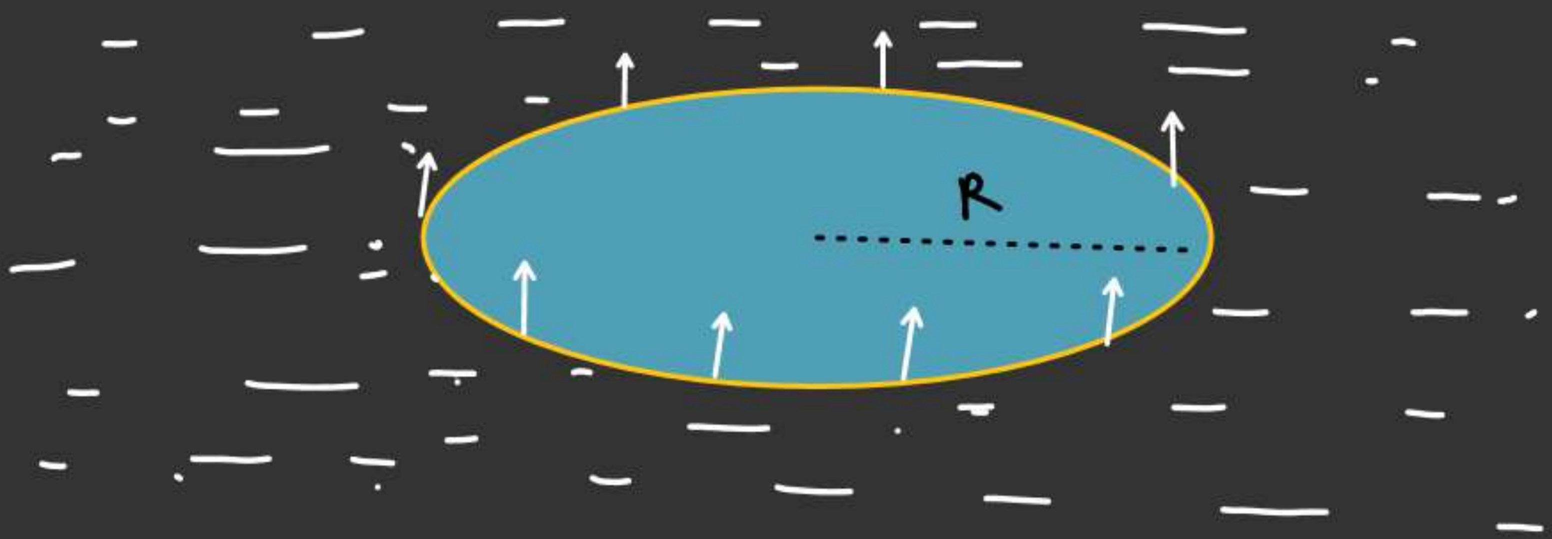
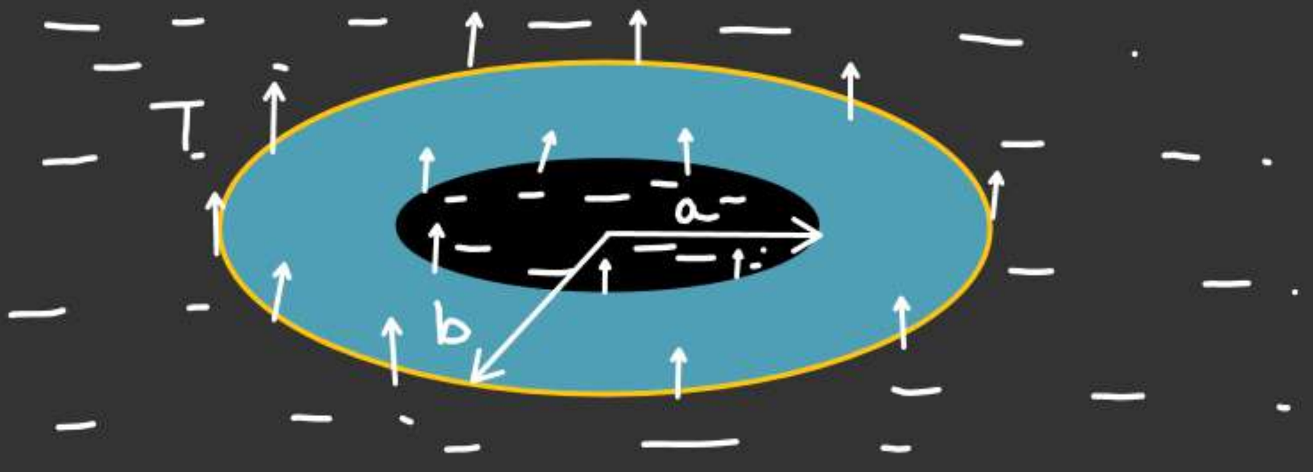


$$\left(F_T = T \times 2\pi R\right)$$



$$F_T = T\,2\pi b + T \times 2\pi a$$

$$F_T = \underline{T\,2\pi\cdot(b+a)}$$