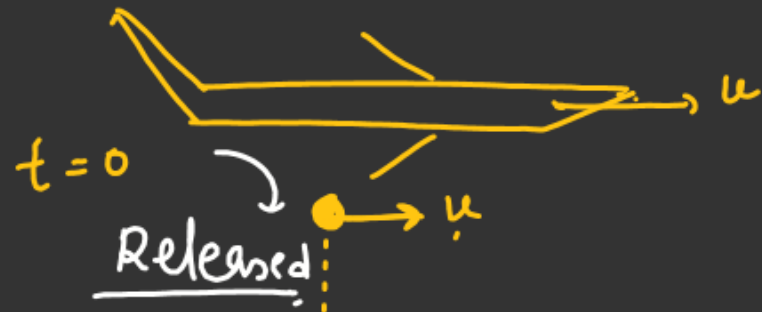




$$\begin{aligned}\vec{v}_{P/Person} &= \vec{v}_{P/E} - \vec{v}_{Person/E} \\ &= u\hat{i} - (-v)\hat{i} \\ &= (u+v)\hat{i}\end{aligned}$$

$$-H = \cancel{u_y T} - \frac{1}{2} g T^2$$

$$T = \sqrt{\frac{2H}{g}}$$



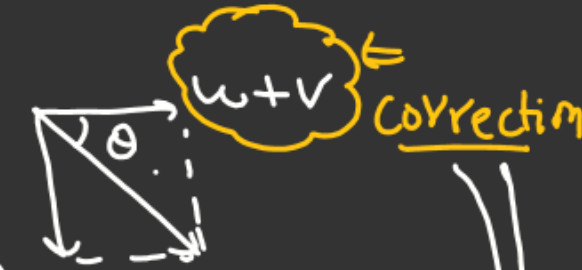
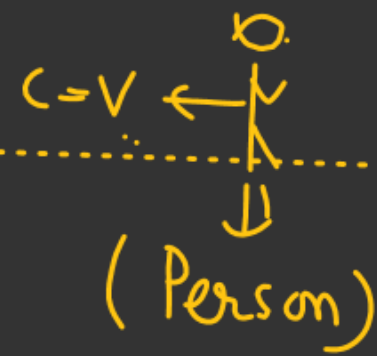
$$x = (u+v) \sqrt{\frac{2H}{g}}$$

$a_y = g$

$$V = \sqrt{u^2 + v_y^2}$$

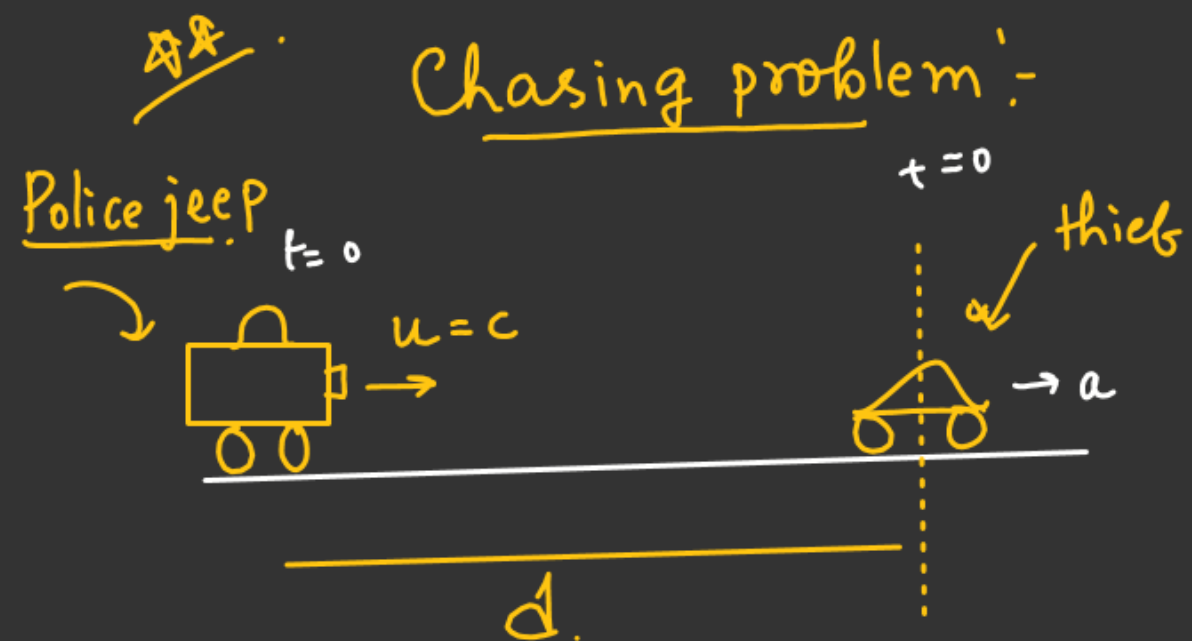
$$v_y = g \sqrt{\frac{2H}{g}}$$

$$v_y = \sqrt{2gH}$$



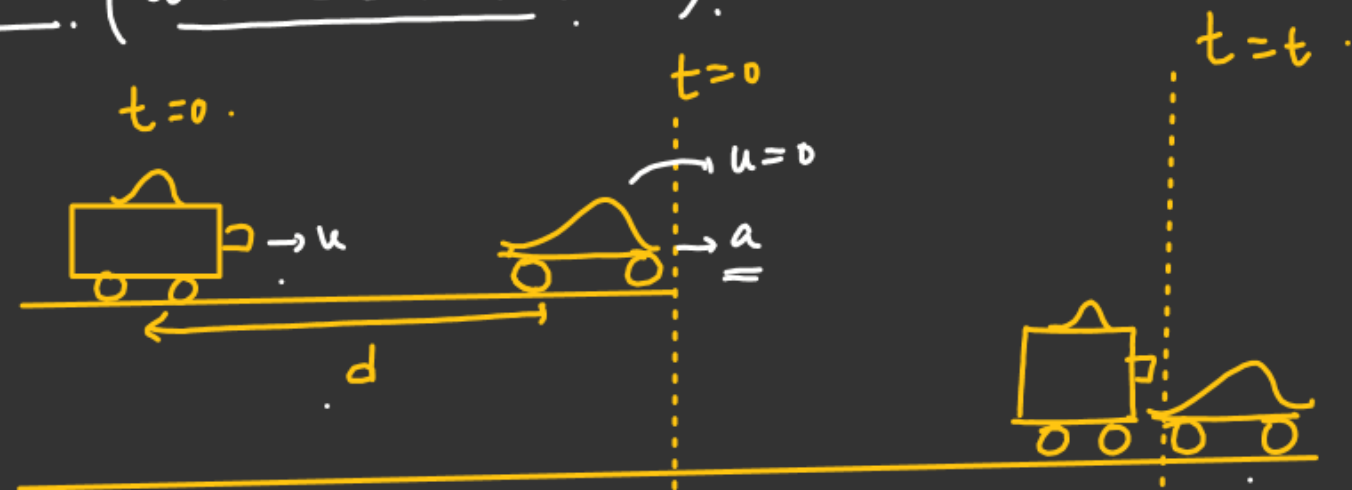
$$\tan \theta = \frac{v_y}{u+v} = \frac{\sqrt{2gH}}{u+v}$$

$$\theta = \tan^{-1} \left( \frac{\sqrt{2gH}}{u+v} \right)$$



When police jeep at a distance 'd' apart from the thief. it starts its bike with constant acceleration 'a'. What should be the min speed of police jeep to catch the thief.

M-1 (w.r.t earth frame.)



For police jeep

$$(d+x) = ut \quad \text{--- (1)} \quad x = (ut-d)$$

For thief

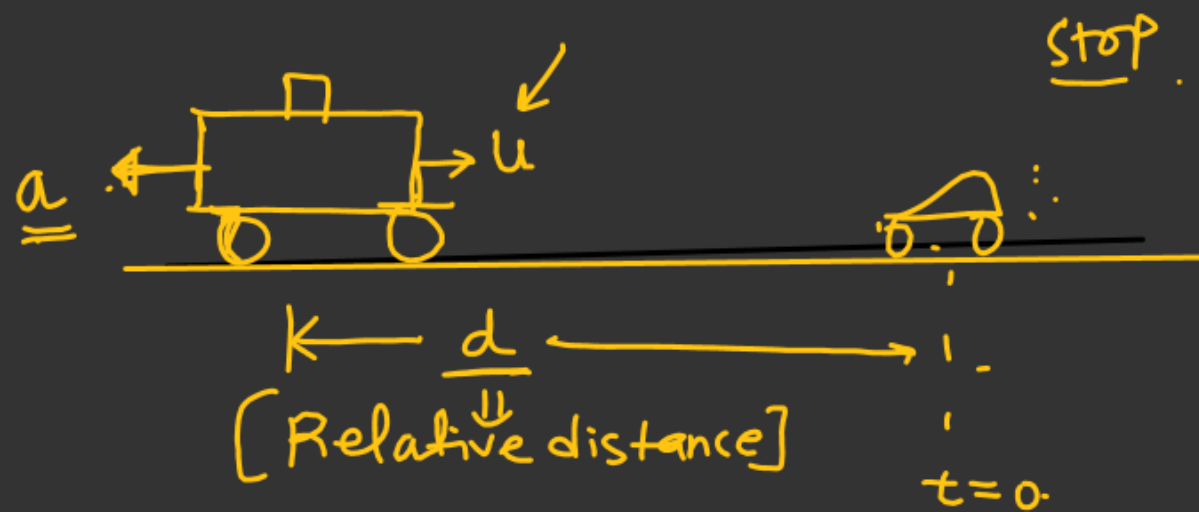
$$x = \frac{1}{2}at^2 \quad \text{--- (2)}$$

Two +ve root ??

$$(ut-d) = \frac{1}{2}at^2 \Rightarrow \left(\frac{a}{2}\right)t^2 - ut + d = 0$$

$$u \geq \sqrt{2ad} \quad \text{where } D \geq 0 \quad u^2 - 4\left(\frac{a}{2}\right)d \geq 0$$

M-2. By relative velocity



$$V^2 = u^2 - 2ad.$$

$$V = \sqrt{u^2 - 2ad}.$$

$$V \geq 0.$$

$$u^2 \geq 2ad$$

$$\boxed{u \geq \sqrt{2ad}}$$

At  $t=0$ .

$$\begin{aligned} \vec{V}_{\text{police jeep/thief}} &= \vec{V}_{\text{police jeep/E}} - \vec{V}_{\text{thief/E}} \\ &= \underline{u \hat{i}}. \end{aligned}$$

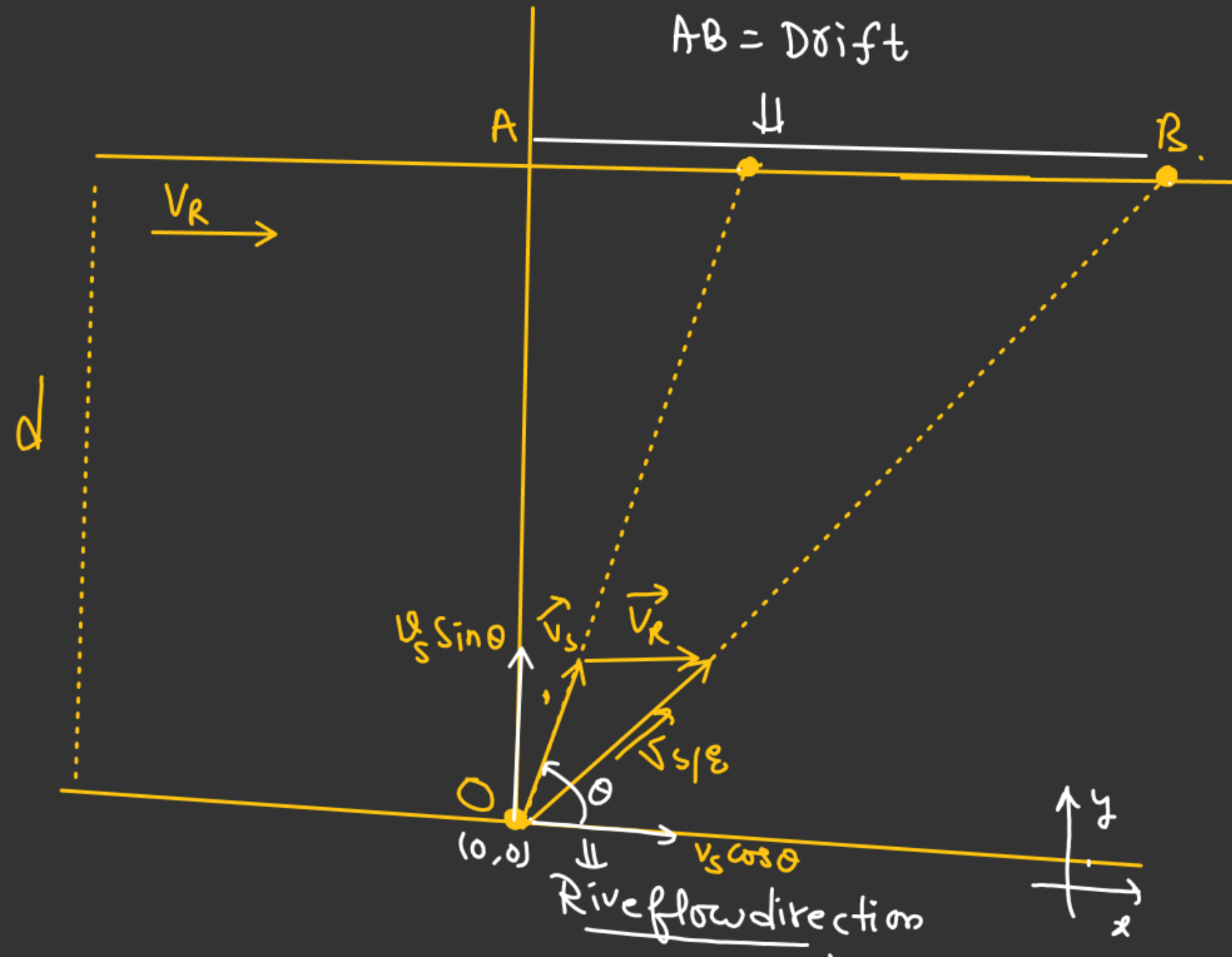
$$\begin{aligned} \vec{a}_{\text{police jeep/thief}} &= \vec{a}_{\text{police jeep/E}} - \vec{a}_{\text{thief/E}} \\ &= \underline{\underline{-a \hat{i}}} \end{aligned}$$

River Swimmer problem $V_R$  = velocity of river $V_s$  = [Velocity of Swimmer w.r.t river  
or  
Velocity of Swimmer in still water] $d$  = width of the river.[ $V_R$  and  $V_s$  uniform velocity]

$$\vec{V}_{s/g} = (\vec{V}_s + \vec{V}_R)$$

$$= V_s \cos \theta \hat{i} + V_s \sin \theta \hat{j} + V_R \hat{i}$$

$$= (\underline{V_R + V_s \cos \theta}) \hat{i} + \underline{V_s \sin \theta} \hat{j}$$



Time of crossing:-

$$T = \frac{d}{|\vec{v}_{s/e}|_y}$$

$$T = \left( \frac{d}{v_s \sin \theta} \right)$$

Drift:- [Net horizontal distance covered by the swimmer from its starting point]

$$\begin{aligned} \text{Drift} &= (v_{s/e})_x \times T \\ &= \left[ (v_R + v_s \cos \theta) \times \frac{d}{v_s \sin \theta} \right] \end{aligned}$$

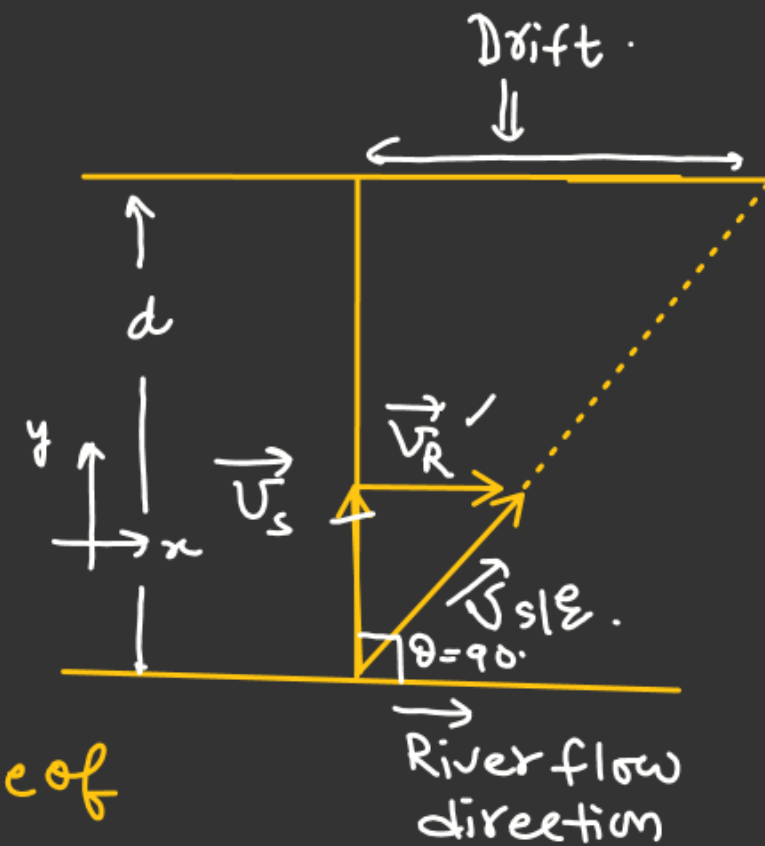
Case of Minimum time Crossing

$$T = \frac{d}{v_s \sin \theta}$$

$$T_{\min}, \sin \theta = +1$$

$$\theta = 90^\circ$$

$$\boxed{T_{\min} = \frac{d}{v_s}}$$



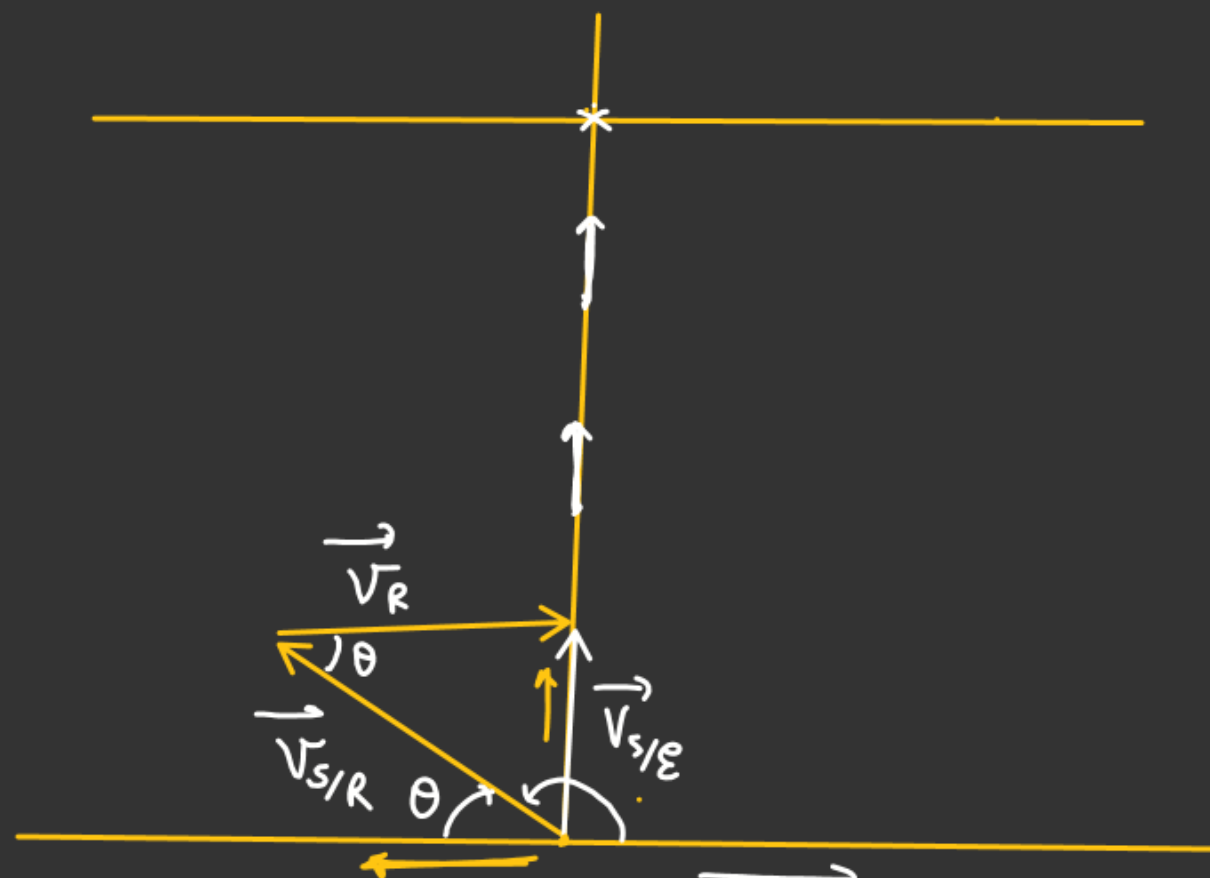
Drift in Case of minimum time of Crossing:-

$$D = (v_{s/e})_x \times T$$

$$D = v_R \times T_{\min} = \left( v_R \times \frac{d}{v_s} \right)$$

$$\begin{aligned} \vec{v}_{s/e} &= \vec{v}_{s/R} + \vec{v}_{R/e} \\ &= v_s \hat{j} + v_R \hat{i} \end{aligned}$$

★: Case when Swimmer reaches directly opposite to bank' -



[Angle w.r.t river flow =  $(\pi - \theta)$ ]

River flow.

$$\text{Drift} = 0$$

$$\vec{V}_{S/R} = -V_S \cos \theta \hat{i} + V_S \sin \theta \hat{j}$$

$$\vec{V}_{R/E} = V_R \hat{i}$$

$$\vec{V}_{S/E} = \vec{V}_{S/R} + \vec{V}_{R/E}$$

$$\vec{V}_{S/E} = (V_R - V_S \cos \theta) \hat{i} + V_S \sin \theta \hat{j}$$

For directly reaching opposite to the bank  $(\vec{V}_{S/E})_x = 0$

$$V_R - V_S \cos \theta = 0$$

$$\cos \theta = \frac{V_R}{V_S}$$

possible only when  $V_R < V_S$

Time of Crossing

$$T = \frac{d}{V_S \sin \theta}$$



$$\sin \theta = \frac{V_R}{\sqrt{V_S^2 - V_R^2}}$$

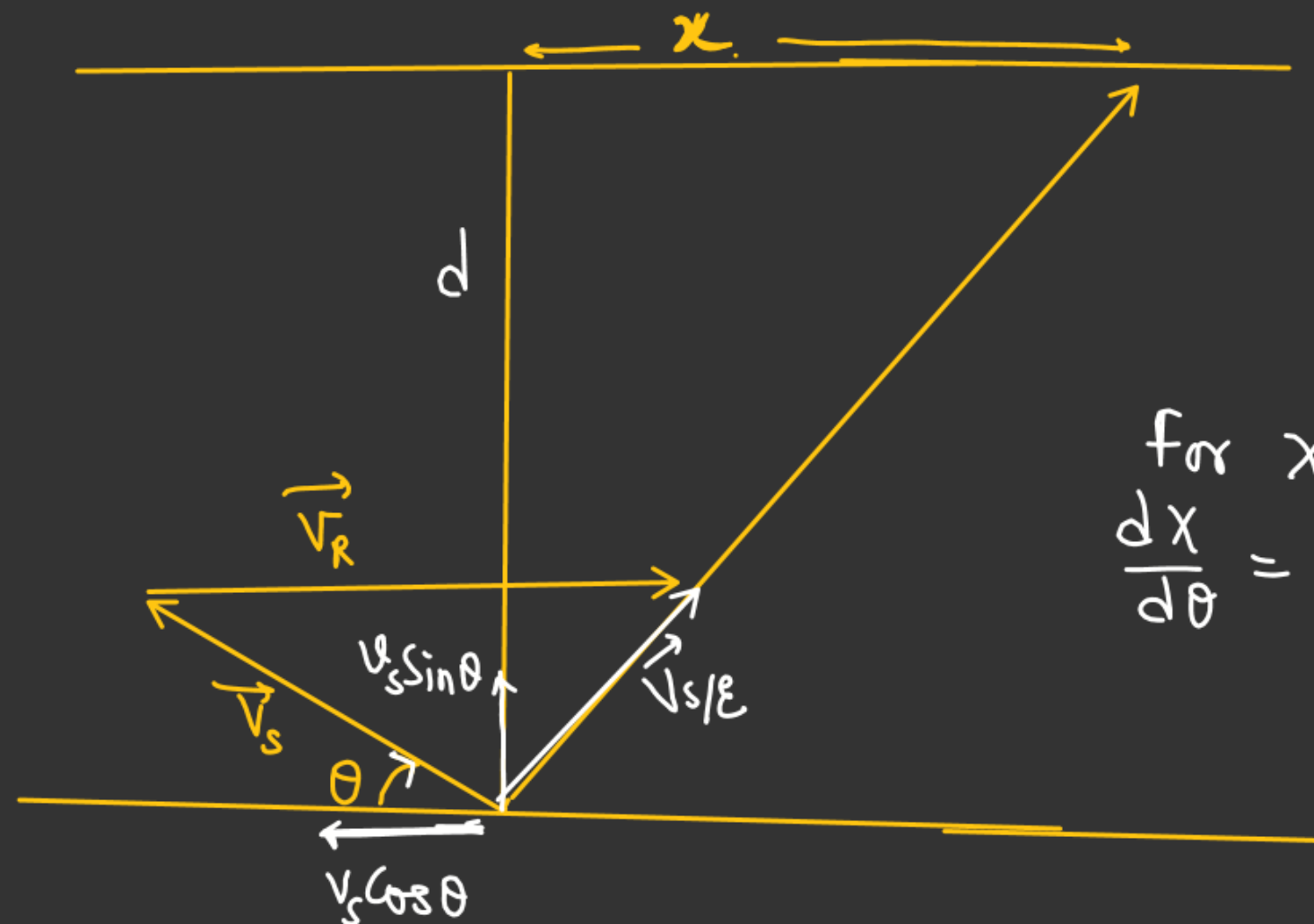
$$T = \frac{d \times V_S}{V_S \sqrt{V_S^2 - V_R^2}}$$

$$T = \frac{d}{\sqrt{V_S^2 - V_R^2}}$$

Condition for minimum drift

$$\textcircled{1} V_s > V_R \Rightarrow \text{Drift} = 0.$$

if  $V_s < V_R$



$$\vec{V}_{s/e} = \vec{V}_{s/R} + \vec{V}_{R/e}$$

$$= -V_s \cos \theta \hat{i} + V_s \sin \theta \hat{j} + V_R \hat{i}$$

$$\vec{V}_{s/e} = (V_R - V_s \cos \theta) \hat{i} + V_s \sin \theta \hat{j}$$

$$\left[ \begin{aligned} \frac{d}{d\theta} (\cot \theta) &= -\operatorname{cosec}^2 \theta \\ \frac{d}{d\theta} (\operatorname{cosec} \theta) &= -\operatorname{cosec} \theta \cdot \cot \theta \end{aligned} \right]$$

$$\text{Drift} = (V_{s/e})_x \times T$$

$$= (V_R - V_s \cos \theta) \times \left( \frac{d}{V_s \sin \theta} \right)$$

$$X = \left[ \frac{V_R d}{V_s} \operatorname{cosec} \theta - d \cot \theta \right]$$

for  $X$  to be maximum or minimum

$$\frac{dX}{d\theta} = 0.$$

$$\frac{V_R d}{V_s} \frac{d}{d\theta} (\operatorname{cosec} \theta) - d \frac{d}{d\theta} (\cot \theta) = 0.$$

$$\frac{V_R d}{V_s} (-\operatorname{cosec} \theta \cdot \cot \theta) + d \operatorname{cosec}^2 \theta = 0$$

$$\cancel{d \operatorname{cosec}^2 \theta} = \cancel{\frac{d v_R}{v_s} \operatorname{cosec} \theta} \cdot \cot \theta$$

$$\frac{v_s}{v_R} = \frac{\cot \theta}{\operatorname{cosec} \theta}$$

$$\frac{v_s}{v_R} = \frac{\cos \theta}{(\sin \theta)} \times (\sin \theta)$$

\*\*

⇓

$$\cos \theta = \left( \frac{v_s}{v_R} \right)$$

⇓  
[Condition for minimum drift]

Time of Crossing in Case of minimum drift:-

$$T = \left( \frac{d}{v_s \sin \theta} \right)$$

$$\sin \theta = \frac{\sqrt{v_R^2 - v_s^2}}{v_R}$$

$$T = \frac{d}{v_s} \times \frac{v_R}{\sqrt{v_R^2 - v_s^2}}$$

$$\left( T = \frac{d v_R}{v_s \sqrt{v_R^2 - v_s^2}} \right) \checkmark$$

