Nishant Jindal

Method of differentiation (L3)

Find diff of $a^{\sin^{-1}x}$ (y) WRT $\sin^{-1}x$ (z) $\Rightarrow \frac{dy}{dz}$

Q3 $y = x - x^2$ derivative of y^2 (z) WRT x^2 (α)? $\rightarrow \frac{dz}{d\alpha}$.

$z = y^2 = x^2 + x^4 - 2x^3$

$\alpha = x^2$

$\frac{dz}{dx} = 4x^3 - 6x^2 + 2x$

$\frac{d\alpha}{dx} = 2x$.

$\frac{dz}{d\alpha} = \frac{dz}{dx} \times \frac{dx}{d\alpha} = \frac{\cancel{2x}(2x^2 - 3x + 1)}{\cancel{2x}}$

(4) **M₁**

$y = a^{\sin^{-1}x}$

$\frac{dy}{dx} = \frac{a^{\sin^{-1}x}}{\sqrt{1-x^2}} \times \ln a$

$z = \sin^{-1}x$

$\frac{dz}{dx} = \frac{1}{\sqrt{1-x^2}}$

$\frac{dy}{dz} = \frac{dy/dx}{dz/dx} = \frac{\frac{a^{\sin^{-1}x}\ln a}{\cancel{\sqrt{1-x^2}}}}{\frac{1}{\cancel{\sqrt{1-x^2}}}} = a^{\sin^{-1}x}\ln a$

M₂

$y = a^{\sin^{-1}x}$

$z = \sin^{-1}x$

$y = a^z$

$\frac{dy}{dz} = a^z \ln a$

$= a^{\sin^{-1}x}\ln a$

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Q5 hold

Q7 $8f(x)+6f\left(\frac{1}{x}\right)=x+5$ $\qquad y=x^2 f(x)$ $\qquad \left.\frac{dy}{dx}\right|_{x=-1}$

$x \longrightarrow \frac{1}{x}$

$8f\left(\frac{1}{x}\right)+6f(x)=\frac{1}{x}+5 \quad \times 6$

$6f\left(\frac{1}{x}\right)+8f(x)=x+5 \quad \times 8$

$\cancel{48f\left(\frac{1}{x}\right)}+36f(x)=\frac{6}{x}+30$

$\cancel{48f\left(\frac{1}{x}\right)}+64f(x)=8x+40$

$28f(x)=8x-\frac{6}{x}+10$

$f(x)=\frac{1}{28}\left(8x-\frac{6}{x}+10\right)$

$y=x^2 \cdot f(x)$

$y=\frac{1}{28}\left(8x^3-6x+10x^2\right)$

$\frac{dy}{dx}=\boxed{DY}$

Q1 $y=ax^{-5/4}+bx^{1/4}$

$\left.\frac{dy}{dx}\right|_{x=5}=-\frac{5}{4}ax^{-9/4}+\frac{1}{4}bx^{-3/4}=0$

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$f(x)=x^n$ then $f(1)-\frac{f'(1)}{1!}+\frac{f''(1)}{2!}-\frac{f'''(1)}{3!}\cdots$ n.

Q8

$f(x)=x^n \longrightarrow f(1)=1 \longrightarrow 1={}^nC_0$

$f'(x)=nx^{n-1} \longrightarrow f'(1)=n \longrightarrow \frac{f'(1)}{1}=\frac{n}{1}={}^nC_1$

$f''(x)=(n)(n-1)x^{n-2} \longrightarrow f''(1)=(n)(n-1) \longrightarrow \frac{f''(1)}{2!}=\frac{(n)(n-1)}{1\cdot 2}={}^nC_2$

$f'''(x)=(n)(n-1)(n-2)x^{n-3} \longrightarrow f'''(1)=(n)(n-1)(n-2) \longrightarrow \frac{f'''(1)}{3!}=\frac{(n)(n-1)(n-2)}{1\cdot 2\cdot 3}={}^nC_3$

${}^nC_0-{}^nC_1+{}^nC_2-{}^nC_3+\cdots=0$

(B.T.)

$\ln f(x)=\frac{x}{2}+c$

$f(x)=e^{\frac{x}{2}+c}$

$f(0)=e^c=e^0$

$\boxed{c=0}$

$\therefore f(x)=e^{x/2}$

Q10 $n=2$ (Check)

Q11 $Y=f(e^x)$ Second der.

$Y'=f'(e^x)\times e^x$

$Y''=f''(e^x)\cdot e^{2x}+f'(e^x)e^x$

Q15 $f(x)=f'(x)+f''(x)+f'''(x)-\cdots\infty$

$f(0)=1$ $f(x)=?$

diff $f'(x)=f''(x)+f'''(x)+f^4(x)-\cdots\infty$

$\therefore f(x)=f'(x)+f'(x)\Rightarrow f(x)=2f'(x)$

$\Rightarrow \int\frac{f'(x)dx}{\underbrace{f(x)}_{t}}=\int\frac{1}{2}dx \Rightarrow \int\frac{dt}{t}=\frac{1}{2}\int dx \Rightarrow \ln t=\frac{1}{2}x+c$

Q 13 hold

19, 20 Sub.

21 → take log.

22 Copy, 23 (Mains)

Q 24, 25, 26 (hold)

Q27

$$f(0) = -1,\ f'(0) = 1.$$

$$g(x) = \left[f(2f(x)+2)\right]^2 \text{ then } g'(0)$$

$$g'(x) = 2f(2f(x)+2) \times f'(2f(x)+2) \times 2f'(x)$$

$$g'(0) = 2f(2f(0)+2) \times f'(2f(0)+2) \times 2f'(0)$$

$$= 2f(-\cancel{2}\overset{0}{+}\cancel{2}) \times f'(-2+2) \times 2$$

$$= 2 \times -1 \times 1 \times 2 = -4$$

$$a\cos^2(x+y) = b.$$

$$\cos^2(x+y) = \frac{b}{a}.$$

$$\cos(x+y) = \sqrt{\frac{b}{a}}.$$

$$x+y = \cos^{-1}\sqrt{\frac{b}{a}}.$$

$$y = -x + \cos^{-1}\sqrt{\frac{b}{a}}$$

$$\frac{dy}{dx} = -1 + 0$$

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Q29 $y = \sqrt{\dfrac{1-\cos x}{1+\cos x}}$ $\dfrac{dy}{dx}$ $x \in (0, \pi)$

$= \sqrt{\dfrac{\cancel{2}\sin^2 x/_2}{\cancel{2}\cos^2 x/_2}}$

$= \left|\tan\dfrac{x}{2}\right|$ $\dfrac{x}{2} \in \left(0, \dfrac{\pi}{2}\right)$

$\oplus$ 1^{st}

$y = \tan\dfrac{x}{2}$.

$\dfrac{dy}{dx} = \sec^2\dfrac{x}{2} \times \dfrac{1}{2}$

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Q. $y=\sqrt{x-1}+\sqrt{x+24-10\sqrt{x-1}}$; $\left.\frac{dy}{dx}\right|=?$ $\quad 1<x<26$

$10\sqrt{x-1} \to 2\times5\sqrt{x-1}$

$y=\sqrt{x-1}+\sqrt{x-1+25-2\times5\times\sqrt{x-1}}$

$+\sqrt{(\sqrt{x-1})^2+5^2-2\times5\times\sqrt{x-1}}$

$+\sqrt{(\sqrt{x-1}-5)^2}$

$y=\sqrt{x-1}+|\sqrt{x-1}-5|$ $\quad 1<x<26$

$\sqrt{24}-5$ $\quad x=25$

$-ve$

$y=\cancel{\sqrt{x-1}}-(\cancel{\sqrt{x-1}}-5)$

$y=5$ $\boxed{y'=0}$

log Based Qs

1) When degree of fxn is fractional

2) When More than 2 fxn are multiplied or divided.

3) $h(x)=(f(x))^{g(x)}$ type

Q $y=\frac{x^{1/4}(2x-5)^{2/7}}{(4-3x)^{5/4}(-x)^{1/19}}$; $y'=?$

$\log y=\frac{1}{4}\log x+\frac{2}{7}\log(2x-5)-\frac{5}{4}\log(4-3x)-\frac{1}{19}\log(-x)$

$\frac{1}{y}\frac{dy}{dx}=$ — — — —

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Q If $F(x) = f(x) \cdot g(x) \cdot h(x)$ diff$^{b/e}$ at $x = x_0$

$\boxed{F'(x_0) = 2F(x_0)}$, $f'(x_0) = 4f(x_0) \rightarrow \dfrac{f'(x_0)}{f(x_0)} = 4.$

$\dfrac{F'(x_0)}{F(x_0)} = 2$

$g'(x_0) = -7g(x_0)$, $h'(x_0) = K\,h(x_0)$.

find $K = ?$

$$\log F(x) = \log f(x) + \log g(x) + \log h(x)$$

at $x = x_0$
$$\frac{F'(x_0)}{F(x_0)} = \frac{f'(x_0)}{f(x_0)} + \frac{g'(x_0)}{g(x_0)} + \frac{h'(x_0)}{h(x_0)}$$

$$2 = 4 + -7 + K.$$

$$\boxed{K = 5}$$

Q If $x^m \cdot y^n = (x+y)^{m+n}$ then $\dfrac{dy}{dx} = ?$

TLBTS

$$m\log x + n\log y = (m+n)\log(x+y)$$

$$\frac{m}{x} + \frac{n}{y}\cdot\frac{dy}{dx} = \frac{(m+n)}{(x+y)} \times \left(1 + \frac{dy}{dx}\right)$$

$$\frac{m}{x} + \frac{n}{y}\cdot\frac{dy}{dx} = \frac{m+n}{x+y} + \frac{m+n}{x+y}\cdot\frac{dy}{dx}$$

$$\frac{dy}{dx}\left(\frac{n}{y} - \frac{m+n}{x+y}\right) = \frac{m+n}{x+y} - \frac{m}{x}$$

$$\frac{dy}{dx}\left(\frac{\overset{n}{\cancel{m}}x + \cancel{ny} - my - \cancel{ny}}{\cancel{(x+y)}\,(y)}\right) = \frac{\cancel{mx} + nx - \cancel{mx} - my}{(x)\,(\cancel{x+y})}$$

$$\frac{dy}{dx}\left(\frac{\cancel{nx - my}}{y}\right) = \frac{\cancel{nx - my}}{x} \Rightarrow \boxed{\frac{dy}{dx} = \frac{y}{x}}$$

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$(f(x))^{g(x)}$
- ① $\to$ TLBTS & diff (Board)
- ② $\to$ Use Formula.
- ③ $\to$ Using e^x funda

Formula.

$$\left((f(x))^{g(x)}\right)' = (f(x))^{g(x)}\left\{\frac{d}{dx}\, g(x)\cdot \ln f(x)\right\}$$

Q $Y = (\sin x)^x$ $\quad Y' = ?$

$$Y' = (\sin x)^x\left\{\frac{d}{dx}\, x\cdot \ln \sin x\right\} = (\sin x)^x\{x\cdot \cot x + \ln \sin x\}$$

(M3) Q. $Y = x^x$ then $y' = ?$

$$x^a = e^{a\ln x}$$

$$Y = e^{x\ln x}$$

$$Y' = e^{x\ln x}\cdot\left(x\cdot\frac{1}{x} + \ln x\right)$$

$$= x^x(1+\ln x)$$

Q. $Y = x^{2x}$ then y'

$$Y' = x^{2x}\left\{\frac{d}{dx}\, 2x\cdot \ln x\right\}$$

$$= x^{2x}\left(\frac{2x}{x} + 2\ln x\right)$$

$$= 2x^{2x}(1+\ln x)$$

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$$y = 2^{\log_2 (x^{2x})} + \left(\tan \frac{\pi x}{4}\right)^{\frac{4}{\pi x}} \text{ then } y' = ?$$

$$y = x^{2x} + \left(\tan \frac{\pi x}{4}\right)^{\frac{4}{\pi x}} \quad y'$$

$$y' = 2x^{2x}(1 + \ln x) + \left(\tan \frac{\pi x}{4}\right)^{\frac{4}{\pi x}} \left(\frac{d}{dx} \frac{4}{\pi x} \ln\left(\tan \frac{\pi x}{4}\right)\right)$$

$$y' = 2x^{2x}(1 + \ln x) + \left(\tan \frac{\pi x}{4}\right)^{\frac{4}{\pi x}} \left\{ \frac{4}{\pi x} \times \frac{\sec^2 \frac{\pi x}{4}}{\tan\left(\frac{\pi x}{4}\right)} \times \frac{\pi}{4} - \frac{4}{\pi} \times \frac{1}{x^2} \ln\left(\tan \frac{\pi x}{4}\right) \right\}$$

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Q. $y = \tan x \cdot \tan 2x \cdot \tan 3x$ then $\frac{dy}{dx} =$?

MCQ all answer different.

Using Trigo

(1) $y = \tan(3x) - \tan(2x) - \tan x$

then y'

(2) Using U.V.W.

(3) TLBTS.

Q. $\int (\sin x)^{\cos x} \{\cos x \cdot \cot x - \sin x \cdot \ln \sin x\}\, dx$

(1) $(\sin x)^{\cos x} = t$

$(\sin x)^{\cos x} \left\{ \frac{d}{dx} \cos x \ln \sin x \right\}$

$(\sin x)^{\cos x} \{\cos x \cdot \cot x - \sin x \ln \sin x\}\, dx = dt$

$\int dt = t + C$

$= (\sin x)^{\cos x} + C$

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$$\boxed{\begin{aligned} &(1)\ (x^x)' = x^x(1+\ln x) \\ &(2)\ x^{-x} = -x^{-x}(1+\ln x) \\ &(3)\ x^{2x} = 2x^{2x}(1+\ln x) \end{aligned}}$$

Differentiation of Implicit fxn.

Differentiation of all fxn having x, y both in given Eqn.

2 Method → 1) Simple diffn

→ 2) Using Partial diffn $\frac{dy}{dx} = -\frac{\left(\frac{\partial f}{\partial x}\right)}{\left(\frac{\partial f}{\partial y}\right)} = -\frac{(\text{Keeping } y \text{ const})}{(\text{Keepin } x \text{ Const})}$

Q $ax^2+by^2+2hxy=0$ then $\frac{dy}{dx}=?$

Simple

$$2ax+2by\cdot\frac{dy}{dx}+2h\cdot x\cdot\frac{dy}{dx}+2hy\cdot 1=0$$

$$\frac{dy}{dx}(by+hx) = -(hy+ax)$$

$$\frac{dy}{dx} = -\frac{(hy+ax)}{(by+hx)}$$

M2

$$\frac{dy}{dx} = -\frac{2ax+0+2hy\cdot 1}{0+2by+2hx\cdot 1} \quad \begin{matrix}(y\text{ const})\\(x\text{ const})\end{matrix}$$

$$y' = -\frac{(ax+hy)}{(by+hx)} = +\left(+\frac{y}{x}\right) - \left(\frac{y}{x}\right)$$ Ans 1, Ans 2

$$(ax^2+hxy)+\underbrace{hxy+by^2}=0$$

$$x(ax+hy) = -y(hx+by) \Rightarrow \frac{(ax+hy)}{(hx+by)} = -\frac{y}{x}$$

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Q If $x^y = e^{x-y}$ then $(1+\ln x)^2 \cdot \frac{dy}{dx} = ?$

$\ln x$

$$y \ln x = x - y$$

$$y(1+\ln x) = x$$

$$y = \frac{x}{(1+\ln x)}$$

$$y' = \frac{(\cancel{1}+\ln x)\cdot 1 - \cancel{x}\cdot\frac{1}{\cancel{x}}}{(1+\ln x)^2}$$

$$(1+\ln x)^2 \cdot \frac{dy}{dx} = \underline{\underline{\ln x}}$$

Q $x^y + y^x = a^b$ then $\frac{dy}{dx} = ?$

$$\boxed{\frac{dy}{dx} = -\frac{\left(y \cdot x^{y-1} + y^x \ln y\right)}{\left(x^y \ln x + x \cdot y^{x-1}\right)}}$$

$$\frac{(y\ \text{const})}{(x\ \text{const})}$$

$x^y \rightarrow x^n$ जैसा

$y^x \rightarrow a^x$ जैसा

$x^y = a^x$ जैसा

$y^x = x^n$ जैसा