Centre of mass

- The center of mass (CM) is a point within or outside the body where the total mass of the body can be considered to be concentrated. It always lies within the boundary of the body. The center of mass does not necessarily coincide with the physical location of mass; it can exist even if there is no mass at that specific point.
- Centre of gravity: The center of gravity is the point at which the weight of an object is considered to act.
- **Coordinates of CM**

Coordinates of CM of a system of 'n' different particles

$$X_{cm} = \frac{m_1x_1 + m_2x_2 + \ldots\ldots m_nx_n}{m_1 + m_2 + \ldots\ldots m_n}$$

$$Y_{cm} = \frac{m_1y_1 + m_2y_2 + \ldots\ldots m_ny_n}{m_1 + m_2 + \ldots\ldots m_n}$$

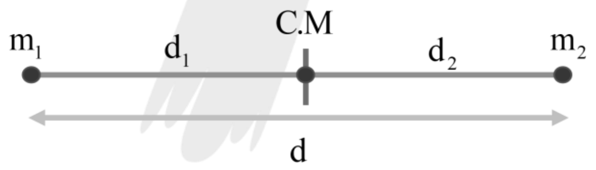
$$Z_{cm} = \frac{m_1z_1 + m_2z_2 + \ldots\ldots m_nz_n}{m_1 + m_2 + \ldots\ldots m_n}$$

Case – 1:

Position of CM of two particle system:

In the case of two bodies, the ratio of the distances of their center of mass from the bodies is inversely proportional to their masses. If the masses of the two bodies are represented by m_1 and m_2, and they are separated by a distance 'd', then the sum of the moments of their weights about the center of mass is zero.

$m_1gd_1 = m_2gd_2$ (or) $\frac{d_1}{d_2} = \frac{m_2}{m_1}$



In figure, $d = d_1 + d_2$

On solving, $m_1d_1 = m_2d_2$

$m_1d_1 = m_2 \times (d - d_1)$

$\Rightarrow d_1 = \frac{m_2d}{m_1+m_2}$ and $d_2 = \frac{m_1d}{m_1+m_2}$

Here, d_1, d_2 are the distances of CM from m_1, m_2.

Thus, CM locates nearer to heavier body.

Note: If m_1, m_2 are located at x_1, x_2 from origin then

$x_{CM} = \frac{m_1x_1+m_2x_2}{m_1+m_2}$

Case – 2:

Center of mass of a system of n particles in one dimension:

Consider n-particles having masses $m_1, m_2, \ldots\ldots, m_n$ along X-axis. The centre of mass of this system is given by

$X_{cm} = \frac{m_1x_1+m_2x_2+\ldots..m_nx_n}{m_1+m_2+\ldots\ldots m_n}$

Case – 3:

Centre of mass of a system of particles in (two dimensional) plane:

Consider n-particles in x-y plane having masses $m_1, m_2, \ldots.., m_n$ with coordinates $(x_1, y_1), (x_2, y_2), \ldots\ldots, (x_n, y_n)$ respectively. The distance of centre of mass from origin in a plane is $d = \sqrt{x_{CM}^2 + y_{CM}^2}$

Where $x_{CM} = \frac{\sum_{i=1}^{n} m_i x_i}{M}$ and $y_{CM} = \frac{\sum_{i=1}^{n} m_i y_i}{M}$

Note – 1: If masses are arranged at vertices of a rectangle then $\theta = 90°$

$\therefore [x_{CM}, y_{CM}] = \left[\frac{a+b\cos 90°}{2}, \frac{b\sin 90°}{2}\right] = \left[\frac{a}{2}, \frac{b}{2}\right]$

Note – 2: If masses are arranged at vertices of a square, then $\theta = 90°$ and lengths of sides are equal $(a = b)$.

$\therefore [x_{CM}, y_{CM}] = \left[\frac{a}{2}, \frac{a}{2}\right]$

Case – 4:

Centre of mass of a system of 'n' particles in (Three dimensional) Space:

Consider n-particles in space having masses $m_1, m_2, \ldots\ldots, m_n$ with coordinates.

$(x_1, y_1, z_1), (x_2, y_2, z_2)\ldots\ldots (x_n, y_n, z_n)$ respectively, then distance of centre of mass from origin in space is $d = \sqrt{x_{cm}^2 + y_{cm}^2 + z_{cm}^2}$

Where, $x_{CM} = \frac{\sum_{i=1}^{n} m_i x_i}{M}$, $y_{CM} = \frac{\sum_{i=1}^{n} m_i y_i}{M}$,

$z_{CM} = \frac{\sum_{i=1}^{n} m_i z_i}{M}$

Case – 5:

Position vector of centre of mass:

Let $\vec{r}_1, \vec{r}_2, \vec{r}_3, \ldots\ldots, \vec{r}_n$ be the position of vectors of n-particles having masses $m_1, m_2, \ldots., m_n$ respectively. If $\vec{r}_{CM}$ is position vector of their CM then

$\vec{r}_{CM} = \frac{m_1\vec{r}_1+m_2\vec{r}_2+\ldots\ldots m_n\vec{r}_n}{m_1+m_2+\ldots..m_n} = \frac{\sum_{i=1}^{n} m_i\vec{r}_i}{M}$

Where $\vec{r}_1 = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$, $\vec{r}_2 = x_2\hat{i} + y_2\hat{j} + z_2\hat{k}$, ………….,

$\vec{r}_n = x_n\hat{i} + y_n\hat{j} + z_n\hat{k}$

Motion of centre of mass

(a) Velocity of centre of mass ($\vec{v}_{cm}$):

If $\vec{v}_1, \vec{v}_2, \ldots\ldots, \vec{v}_n$ are velocities of particles of masses $m_1, m_2, \ldots, m_n$ respectively and $\vec{v}_{cm}$ is velocity of their centre of mass then

- $\overline{v_{CM}} = \dfrac{\overline{dr_{CM}}}{dt} = \dfrac{1}{M}\sum_{i=1}^{n} m_i \left(\dfrac{d\vec{r}_i}{dt}\right)$
- $\overline{v_{CM}} = \dfrac{m_1\vec{v}_1 + m_2\vec{v}_2 + \ldots\ldots + m_n\vec{v}_n}{m_1 + m_2 + \ldots m_n} = \dfrac{1}{M}\sum_{i=1}^{n} m_i\vec{v}_i$

Where $M = m_1 + m_2 + \ldots\ldots m_n$ = Total mass of the system

- When two particles of masses m_1 and m_2 are moving from a point with velocities v_1 and v_2 at an angle 'θ' with each other, then the velocity of their centre of mass is given by

$$v_{CM} = \frac{\sqrt{m_1^2 v_1^2 + m_2^2 v_2^2 + 2(m_1 v_1)(m_2 v_2)\cos\theta}}{(m_1 + m_2)}$$

- If they move in the same direction, then $\theta = 90°$ and $v_{CM} = \dfrac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$
- If they move at right angles to each other, then $\theta = 90°$ and $v_{CM} = \dfrac{\sqrt{m_1^2 v_1^2 + m_2^2 v_2^2}}{m_1 + m_2}$
- If they move in opposite directions, then $\theta = 180°$ and $v_{CM} = \dfrac{m_1 v_1 - m_2 v_2}{m_1 + m_2}$

(b) Linear momentum of centre of mass ($\vec{p}_{cm}$):

If $\vec{p}_1, \vec{p}_2, \ldots\ldots, \vec{p}_n$ are linear momenta of particles of masses $m_1, m_2, \ldots\ldots, m_n$ respectively and $\vec{p}_{cm}$ is linear momentum of their centre of mass then

- $\vec{p}_{cm} = \vec{p}_1 + \vec{p}_2 + \vec{p}_n = \sum_{i=1}^{n} \vec{p}_i,$
- $\vec{p}_{cm} = M\vec{v}_{cm} = \sum_{i=1}^{n} m_i\vec{v}_i = \vec{p}_{system}$

$M = m_1 + m_2 + \ldots + m_n$ = Total mass of the system

(c) Acceleration of centre of mass ($\vec{a}_{cm}$):

If $\vec{a}_1, \vec{a}_2, \ldots\ldots, \vec{a}_n$ are accelerations of particles of masses $m_1, m_2 \ldots m_n$ respectively and $\vec{a}_{cm}$ is the acceleration of their centre of mass then $\vec{a}_{cm} = \dfrac{d\vec{v}_{cm}}{dt} = \dfrac{1}{M}\sum_{i=1}^{n} m_i \dfrac{d\vec{v}_i}{dt}$

- $\vec{a}_{cm} = \dfrac{m_1\vec{a}_1 + m_2\vec{a}_2 + \ldots + m_n\vec{a}_n}{m_1 + m_2 + \ldots + m_n} = \dfrac{1}{M}\sum_{i=1}^{n} m_i\vec{a}_i$ Where $M = m_1 + m_2 + \ldots\ldots + m_n$ = Total mass of the system.
- When two particles of masses m_1 and m_2 are moving from a point with accelerations a_1 and a_2 at an angle θ with each other, then the acceleration of their centre of mass is given by

$$|\vec{a}_{CM}| = \frac{\sqrt{m_1^2 a_1^2 + m_2^2 a_2^2 + 2(m_1 a_1)(m_2 a_2)\cos\theta}}{m_1 + m_2}$$

- If they move in the same direction, then $\theta = 0°$ and $a_{CM} = \dfrac{m_1 a_1 + m_2 a_2}{m_1 + m_2}$

- If they move at right angles to each other, then $\theta = 90°$ and $a_{CM} = \frac{\sqrt{m_1^2 a_1^2 + m_2^2 a_2^2}}{m_1 + m_2}$
- If they move in opposite directions, then $\theta = 180°$ and $a_{CM} = \frac{m_1 a_1 - m_2 a_2}{m_1 + m_2}$

Note: The magnitude of displacement of centre of mass in time 't' is $S_{CM} = \frac{1}{2} a_{CM} t^2$

Effect of external forces on CM

- We know $\vec{a}_{cm} = \frac{1}{M} \sum_i m_i \vec{a}_i$ Therefore $M\vec{a}_{cm} = \sum^{\Sigma} \vec{F}_{ext} + \sum^{\Sigma} \vec{F}_{int}$ But the internal forces are in the form of action-reaction pairs. Hence they cancel each other. Thus $\sum^{\Sigma \vec{0}} \vec{F}_{int}$; $M\vec{a}_{cm} = \sum \vec{F}_{ext}$
- The center of mass is only affected by external forces acting on the system. Internal forces have no effect on the motion of the center of mass. When no external force acts on the system
  (a) acceleration of centre of mass is zero i.e., $\vec{F}_{ext} = M\vec{a}_{CM} \Rightarrow M\vec{a}_{CM} = \vec{0} \Rightarrow \vec{a}_{CM} = \vec{0}$
  (b) Velocity of centre of mass is constant $\vec{v}_{cm}$ = constant
  (c) Linear momentum of the system is constant $\vec{p}_{cm}$ = constant. It is called the law of conservation of linear momentum.

Characteristics of centre of mass

- The center of mass of a system of particles is determined by the masses of the particles and their relative positions.
- For continuous distribution of mass, centre of mass depends on mass distribution and shape of the body.
- Sum of moments of masses about centre of mass is zero i.e., $\sum_i m_i \vec{r}_i = \vec{0}$
- The center of mass is independent of the chosen frame of reference.
- Mass may or may not be present at the center of mass.
- The motion of the center of mass is purely translational.
- The motion of the center of mass follows Newton's second law.

Examples for the motion of centre of mass

(a) When a bomb, initially at rest at the origin of an x, y, z coordinate system, explodes due to internal forces, its fragments scatter randomly in different directions with varying velocities. However, the center of mass (CM) is unaffected by this explosion and remains at rest at the origin.∴ $\sum_i m_i \vec{r}_i = \vec{0}$, where $\vec{r}_i$ is position vector of i^{th} particle about origin

(b) When a bomb is projected on the ground, following a parabolic path, and explodes during its motion due to internal forces, the resulting fragments disperse randomly in various directions. However, the center of mass of the system continues to follow the same parabolic path as the unexploded bomb. Therefore, at any given moment, the vector sum of the moments of mass of all the fragments about the center of mass remains zero.

(c) When a wheel rolls on a road, the paths followed by different particles within the wheel can be complex due to the combined motion of translation and rotation. However, the motion of the center of mass of the wheel remains purely translational, following a straight line path.

Note: Internal forces within a system, such as the gravitational force between two masses or the electric force between two charges, are unable to generate acceleration in the center of mass of the system.

Mutual forces between two bodies:

- When two particles approach each other due to their mutual interaction, then they always meet at their centre of mass.
- To a system of particles $m_1(x_1y_1)$, $m_2(x_2y_2)$ another particle of mass m_3 is added so that centre of mass shifts to the origin then coordinates of third particle are

$$x_3 = -\frac{(m_1x_1 + m_2x_2)}{m_3}; y_3 = \frac{-(m_1y_1 + m_2y_2)}{m_3}$$

- In a system of two particles of masses m_1 and m_2, when m_1 is pushed towards m_2 through a distance d then shift in m_2 towards m_1 without altering CM position is $\frac{-m_1}{m_2}d$.
- A boy of mass m is at one end of a flat boat of mass M and length ℓ which floats stationary on water. If boy moves to the other end,

(i) The boat moves in opposite direction through a distance $d = \frac{m\ell}{(M+m)}$

(ii) The displacement of boy with respect to ground is $d' = \frac{-M\ell}{(M+m)}$

- A boy of mass m is standing on a flat boat is floating stationary on the surface of water. If the boy starts moving on the boat with velocity V_r with respect to boat, then

(i) Velocity of the boat w.r.t. grounds is $V = \frac{-mV_r}{M+m}$, '–' indicates boat moves in opposite direction to the velocity of the boy.

(ii) velocity of boy w.r.t. ground is $V' = \frac{MV_r}{M+m}$

Shifts in centre of mass in different cases:

Shift is the distance of final location of centre of mass of the system from its initial location. Shifts in the centre of mass generally occurs due to

(a) Addition of matter

(b) Removal of matter

(c) Change in shape

(d) Change in mass distribution

(a) Addition of mass:

Due to addition of mass, the C.M. of a system generally shifts towards or into the region where mass is added. If C_1 is the CM before addition and C_2 is the CM of added mass and $C_1C_2 = d$, then $\Delta X_{shift} = \left[\frac{m_{added} \times d}{m_{initial} + m_{added}}\right]$ CM shifts towards the side of added mass

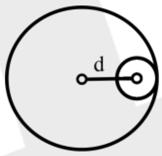
(b) Removal of mass:

Due to removal of mass the C.M of a system shifts away from the region where mass is removed. If C_1 is the CM of the body before removal and C_2 is CM of the removed part and $C_1C_2 = d$ then

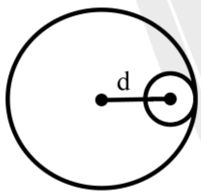
$\Delta X_{shift} = \left[\frac{-m_{removed}\times d}{m_{initial}-m_{removed}}\right]$

'$-$' indicates CM shifts opposite to the side of removed mass.

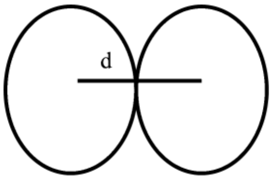
➢ From a uniform circular disc of radius R, if a circular sheet of radius 'r' is removed, then the centre of mass of remaining part shifts by a distance $\frac{-r^2 d}{R^2-r^2}$ where 'd' is the distance of the C.M. of the removed part from the centre of the original disc. In this case the circular sheet is removed from the edge of disc, then the shift in centre of mass is maximum. Here $d = R - r$. Maximum shift $\Delta x = \frac{-r^2}{R+r}$



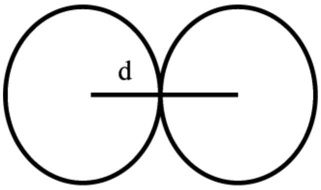
➢ From a uniform solid sphere of radius R, if a sphere of radius 'r', is removed, then the centre of mass of the remaining part shifts by $\Delta x = \frac{-r^3 d}{(R^3+r^3)}$, where 'd' is the distance of the CM of removed part from the centre of the original sphere. In this case spherical cavity is made at the edge of large sphere, then shift in C.M. is maximum. It is given by $\Delta x = \frac{-r^3(R-r)}{(R^3+r^3)}$



➢ To a circular disc of radius R_1 another disc of radius R_2 and of the same material is added then shift in the CM is $\Delta x = \frac{R_2^2(R_1+R_2)}{R_1^2+R_2^2}$



➢ If two spheres of same material and radii r_1 and r_2 are kept in contact, distance of centre of mass from the centre of the first sphere is equal to $\Delta x = \frac{r_2^3}{r_1^3+r_2^3}(r_1 + r_2)$. Similarly, distance of centre of mass from the centre of the second sphere is $\Delta x = \frac{r_1^3}{r_1^3+r_2^3}(r_1 + r_2)$.



- The position of the center of mass of a system is determined by the distribution of mass within the system. Consequently, the center of mass changes as the shape of the system and the relative positions of particles change.

Methods of locate CM:

Locating the Centre of Mass can be done in four different ways. They are,

(1) Method of symmetry

(2) Method of Decomposition

(3) Method of using theorems of Pappus's

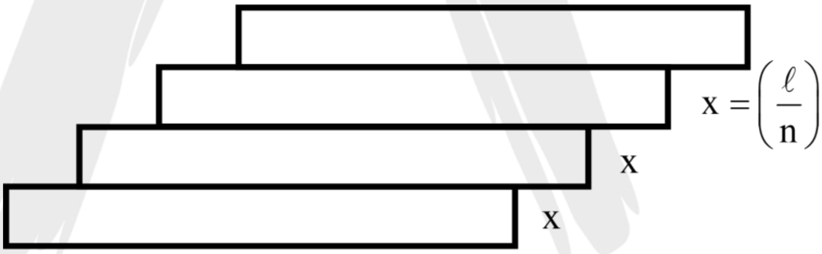
(4) Method of integration

- For continuous distribution of mass, the coordinates of centre of mass are given by

$$x_{cm} = \frac{\int x\,dm}{\int dm}; y_{cm} = \frac{\int y\,dm}{\int dm}; z_{cm} = \frac{\int z\,dm}{\int dm}$$

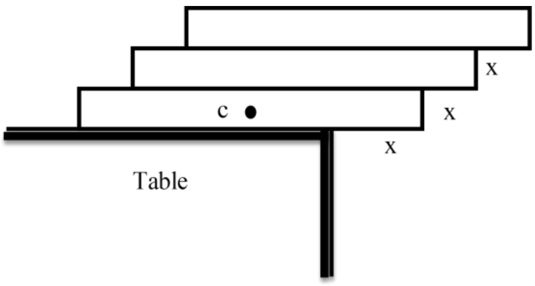
Note – 1:

When the above blocks are arranged in such a manner, that each block projects out by same distance, so that the blocks will not fall then the distance of projection of each block from the edge of its bottom block is $\left(\frac{\ell}{n}\right)$.

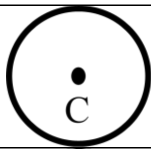
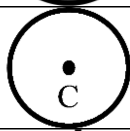
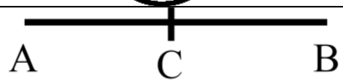
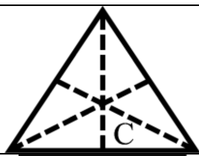
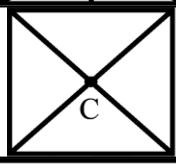
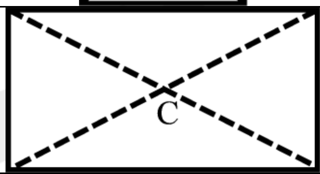
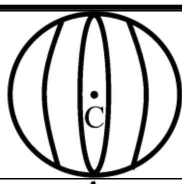
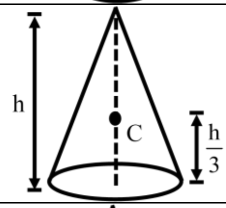
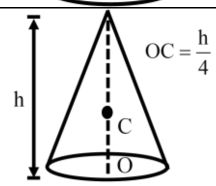
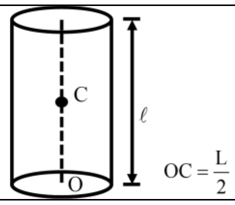
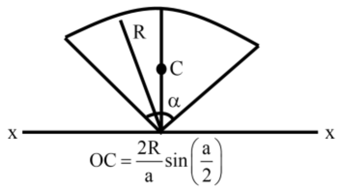


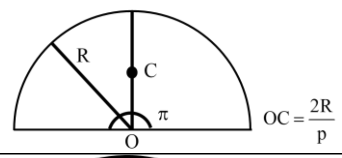
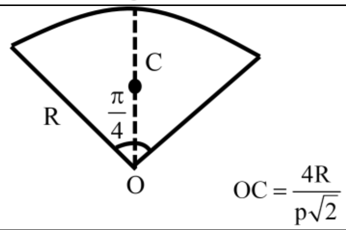
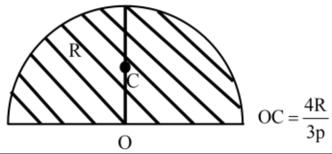
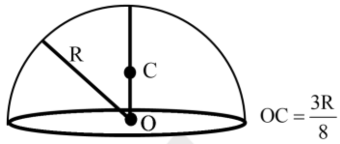
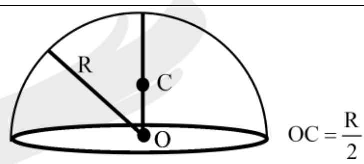
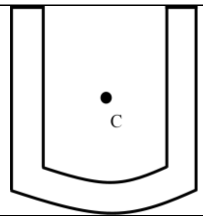
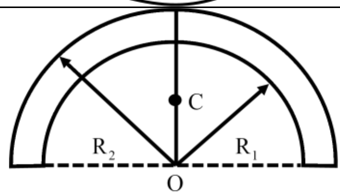
Note – 2:

If the entire system is placed at the edge of a table, so that the blocks will not fall then the equal distance of projection of each block from the edge of its bottom block is $\left(\frac{\ell}{n+1}\right)$



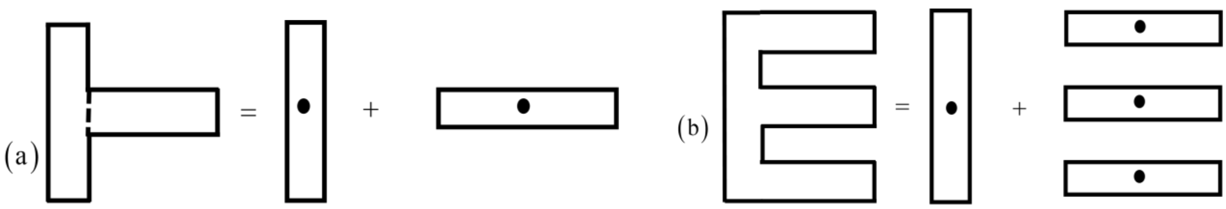
SYSTEM OF PARTICLE

S.No	Shape of the body	Position of centre of mass	Figure
1.	Circular ring	At the centre of the ring	
2.	Circular disc	At the centre of the disc	
3.	Thin uniform straight rod	At the geometric centre	
4.	Triangular plate	At the centroid	
5.	Square plate	At the point of intersection of the diagonals	
6.	Rectangular plate	At the point of intersection of the diagonals	
7.	Hollow solid sphere	At the centre of the sphere	
8.	Hollow cone	At a height of h/3, from the base	
9.	Solid cone or Pyramid	At a height of h/4 from the base	
10.	Solid hollow cylinder	At the mid-point of its own	
11.	An arc of radius R subtending an angle α at its centre of curvature	At a distance of $\frac{2R}{a}\sin\left(\frac{a}{2}\right)$ from its centre of curvature on the axis of symmetry	

	(i) A semi-circle of radius 'R'	At a distance of $\frac{2R}{p}$ from its centre on the axis of symmetry	
	(ii) A quadrant of a circle of radius 'R'	At a distance of $\frac{4R}{p\sqrt{2}}$ from its centre 'O' on the axis of symmetry	
12.	Semi-circular disc	At a distance of $\frac{4R}{3p}$ from its centre 'O' on the axis of symmetry	
13.	Solid hemi-sphere	At a distance of $\frac{3R}{8}$ from its centre 'O' on the axis of symmetry	
14.	Hollow hemi-sphere (or) Hemi-sphere shell	At a distance of $\frac{R}{2}$ from its centre 'O' on the axis of symmetry	
15.	Horse – shoe magnet	At its centre within the boundary limits	
16.	Semi-circular annular plate	At a distance of $OC = \frac{4(R_1^2+R_1R_2+R_2^2)}{3p(R_1+R_2)}$ from its centre of symmetry	

➢ **Method of decomposition:**

The center of mass of a body or system can be determined using a method that involves decomposing it into smaller, geometrically symmetrical parts. Here are a few examples



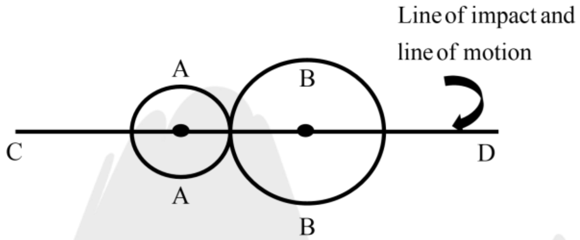
In this method each part is considered as a point object, hence the system will be converted into the system of n particles. Now the coordinates of CM of system w.r.t. some reference point are $x_{CM} = \frac{1}{M}\sum_i m_i x_i$; $y_{CM} = \frac{1}{M}\sum_i m_i y_i$ and $z_{CM} = \frac{1}{M}\sum_i m_i z_i$

Collisions

- A collision refers to a strong interaction between bodies where there is an exchange of momentum occurring within a short interval of time.
- Collisions can occur between bodies with or without physical contact. For example, in the collision of a particle with a nucleus, the coulombic repulsive forces cause the particle to be scattered away without any physical contact.
- Collisions are classified based on the direction of motion of the colliding bodies:

  (i) Head-on or one-dimensional collision (ii) Oblique collision

  Head on (or) one dimensional collision



- A one-dimensional collision is a type of collision where the velocities of the colliding bodies are confined to the same straight line before and after the collision.

Oblique Collision :

- An oblique collision is a type of collision where the velocities of the colliding bodies are not confined to the same straight line before and after the collision.
- Oblique collision may be two dimensional or three dimensional.
- When a particle elastically and obliquely collides with another stationary particle of the same mass, they move perpendicular to each other after the collision.

Types of Collision : Based on conservation kinetic energy collisions are classified into

(i) Elastic Collision (ii) Inelastic collision

(i) Elastic Collision:

- Kinetic energy and momentum are conserved during the collision.
- Forces involved in the collision are conservative in nature.

Examples:

1. Collision between atomic particles.

2. Collision between two smooth billiard balls.

3. Collision of α particle with a nucleus.

(ii) Inelastic Collision:

- Momentum is conserved, but kinetic energy is not conserved.
- Some or all of the forces involved in the collision are non-conservative.
- Collision between two vehicles.

Perfectly inelastic collision :

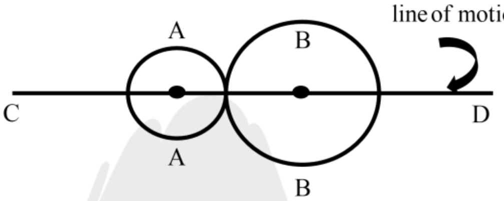
- It is the collision in which the colliding bodies stick together and move as a single body after collision.

- In perfectly inelastic collision the momentum remains conserved but the loss of kinetic energy is maximum.

Ex : A bullet is fired into a wooden block and remains embedded in it.

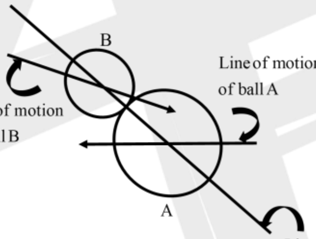
Line of impact : The line that passes through the common normal to the surfaces in contact during an impact is referred to as the line of impact. The force exerted during the collision acts along this line on both bodies.

Ex 1 : Two balls A and B are approaching each other such that their centres are moving along line CD.



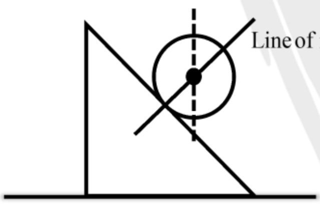
Head on collision

Ex 2: Balls A and B are moving towards each other, with their centers following the paths indicated in the figure.



Oblique collision

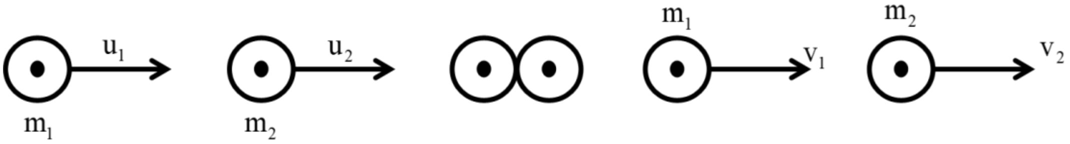
Ex 3: Ball is falling on a stationary wedge



Oblique Collision

Elastic collision in one dimension :

When two particle of masses m_1 and m_2 are moving along the line joining their centers with velocities u_1 and $u_2 (u_1 > u_2)$ before collision. Then v_1 and v_2 are their velocities after collision



Before Collision During Collision After Collision

From the conservation of linear momentum

$m_1(\vec{u}_1 - \vec{v}_1) = m_2(\vec{v}_2 - \vec{u}_2)$

From Law of conservation of K.E

$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$

$\therefore \vec{u}_1 - \vec{u}_2 = \vec{v}_2 - \vec{v}_1$

i.e Relative velocity of approach before collision = Relative velocity of separation after collision

➢ **Velocities after collision are**

$\vec{v}_1 = \left(\frac{m_1-m_2}{m_1+m_2}\right)\vec{u}_1 + \left(\frac{2m_2}{m_1+m_2}\right)\vec{u}_2$

$\vec{v}_2 = \left(\frac{2m_1}{m_1+m_2}\right)\vec{u}_1 + \left(\frac{m_2-m_1}{m_1+m_2}\right)\vec{u}_2$

Special cases :

(1) If colliding particles have equal masses

i.e $m_1 = m_2 = m; \vec{v}_1 = \vec{u}_1, \vec{v}_2 = \vec{u}_1$

(2) If two bodies are of equal masses and the second body is at rest ie., $m_1 = m_2 = m$ and $\vec{u}_2 = \vec{0}$

then $\vec{v}_1 = \vec{0}; \vec{v}_2 = \vec{u}_1$

(3) A lighter particle collides with heavier particle which is at rest $m_1 <<< m_2, \vec{u}_2 = \vec{0}$

$\vec{v}_1 = -\vec{u}_1, \vec{v}_2 = \vec{0}$

(4) A heavier body collides with lighter body at rest

$m_1 >>> m_2, \vec{u}_2 = \vec{0};$

$\vec{v}_1 = \vec{u}_1, \vec{v}_2 = 2\vec{u}_1$

Applications :

➢ A body of mass m_1 moving with a velocity v_1 collides elastically with a stationary mass m_2

(1) Velocity of first body after collision $\vec{v}_1 = \left(\frac{m_1-m_2}{m_1+m_2}\right)\vec{u}_1$

(2) Velocity of second body after collision $\vec{v}_2 = \left(\frac{2m_1}{m_1+m_2}\right)\vec{u}_1$

(3) KE of first body after collision (or) Ke retained by first body $K.E_1 = \frac{1}{2}m_1v_1^2 = \frac{1}{2}m_1\left(\frac{m_1-m_2}{m_1+m_2}\right)^2 u_1^2$

$K.E_{ret} = \frac{1}{2}m_1u_1^2\left[\frac{m_1-m_2}{m_1+m_2}\right]^2 = KE_i\left[\frac{m_1-m_2}{m_1+m_2}\right]^2$

(4) Fraction of KE retained by 1st body $\frac{K.E_{ret}}{K.E_i} = \left[\frac{m_1-m_2}{m_1+m_2}\right]^2$

(5) KE of second body after collision (or) KE transferred to the second body

$KE_2 = \frac{1}{2}m_2v_2^2 = \frac{1}{2}m_2\left(\frac{2m_1}{m_1+m_2}\right)^2 u_1^2$

$KE_2 = \left(\frac{4m_1m_2}{(m_1+m_2)^2}\right)\left(\frac{1}{2}m_1u_1^2\right)$

$KE_{tra} = \left(\frac{4m_1m_2}{(m_1+m_2)^2}\right) KE_i$

(6) Fraction of KE transferred from 1st body to second body (or) Fraction of KE lost by 1st body is

$$\frac{KE_{tra}}{KE_i} = \frac{4m_1m_2}{(m_1+m_2)^2}$$

(7) Fraction of momentum retained by $m_1 \Rightarrow \frac{P_1}{P_i} = \frac{m_1v_1}{m_1u_1} = \frac{m_1-m_2}{m_1+m_2}$

(8) Fraction of momentum transferred from 1st body to second body

$$\frac{P_2}{P_i} = \frac{P_i - P_1}{P_i} = 1 - \frac{P_1}{P_i} = 1 - \left(\frac{m_1 - m_2}{m_1 + m_2}\right) = \frac{2m_2}{m_1 + m_2}$$

Coefficient of restitution

- The coefficient of restitution (e), introduced by Newton, is a dimensionless parameter used to quantify the elasticity of a collision. It is defined as the ratio of the relative velocity of separation to the relative velocity of approach between the two colliding bodies. $e = \frac{\text{Relative velocity of separation}}{\text{Relative velocity of approach}} = \frac{|\vec{v}_2-\vec{v}_1|}{|\vec{u}_1-\vec{u}_2|}$
- This formula is applied along the line of impact. Here the velocities mentioned in the expression should be taken along the line of impact.

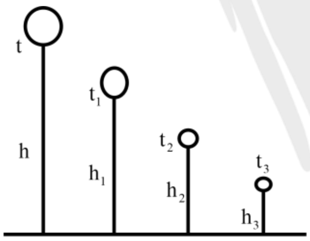
For a perfectly elastic collision $e = 1$

For an inelastic collision $0 < e < 1$

For completely inelastic collision e = 0

- A body dropped freely from a height 'h' strikes the floor and rebounds to a height h_1

  $e = \sqrt{\frac{h_1}{h}}$ and after nth rebound $h_n = e^{2n}h$



- When a freely falling ball strikes the ground with a velocity 'v' and rebounds with a velocity v_1 then $e = \frac{v_1}{v}$ and after n^{th} rebound $v_n = e^n v$
- Total distance travelled by the ball before it stops bouncing

  $d = h + 2h_1 + 2h_2 + 2h_3 + \ldots\ldots$

  $= h + 2e^2h + 2e^4h + 2e^6h + \ldots\ldots$

  $= h + 2e^2h[1 + e^2 + e^4 + \ldots\ldots]$

  $d = h\left[\frac{1+e^2}{1-e^2}\right]$

➢ Total time taken by the ball to stop bouncing

$T = t + 2t_1 + 2t_2 + 2t_3 + \ldots\ldots$

$= \sqrt{\frac{2h}{g}} + 2\sqrt{\frac{2h_1}{g}} + 2\sqrt{\frac{2h_2}{g}} + 2\sqrt{\frac{2h_3}{g}} + \ldots\ldots$

$= \sqrt{\frac{2h}{g}} + 2e\sqrt{\frac{2h}{g}}[1 + e + e^2 + \ldots\ldots]$

$= \sqrt{\frac{2h}{g}}\left[\frac{1+e}{1-e}\right]$

➢ Average speed of the ball during its entire jouney is given by

$\text{Averagespeed} = \frac{\text{Totaldistancetravelled}}{\text{Totaltimetaken}}$

$= \frac{h\left[\frac{1+e^2}{1-e^2}\right]}{\sqrt{\frac{2h}{g}}\left[\frac{1+e}{1-e}\right]} = \sqrt{\frac{gh}{2}}\frac{(1+e^2)}{(1+e)^2}$

➢ Average velocity of the ball during its entire journey is given by

$\text{Averagevelocity} = \frac{\text{Netdisplacement}}{\text{Totaltimetaken}}$

$= \frac{h}{\sqrt{\frac{2h}{g}}\left[\frac{1+e}{1-e}\right]} = \sqrt{\frac{gh}{2}}\frac{(1-e)}{(1+e)}$

➢ Change in momentum in 1st collision

$\Rightarrow mv_1 - (-mu) = (mv_1 + mu)$

$= meu + mu = mu(1 + e)$

Change in momentum in 2nd collision

$\Rightarrow m(v_2 + v_1) = m(e^2u + eu) = meu(1 + e)$

Total change in momentum before it stops is

$\Delta p = mu(1 + 2e + 2e^2 + \ldots\ldots)[u = \sqrt{2gh}]$

$= mu\left[\frac{1+e}{1-e}\right] = m\sqrt{2gh}\left[\frac{1+e}{1-e}\right]$

➢ Distance travelled before second impact is $d_2 = h + 2h_1 = h(1 + 2e^2)$

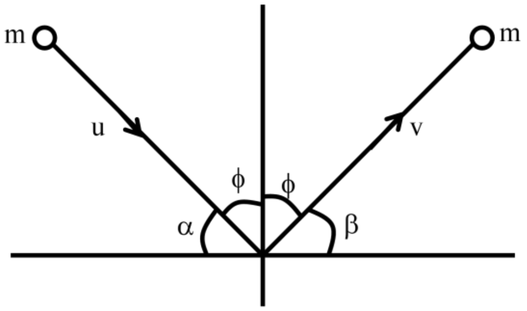
Distance travelled before third impact is $d_3 = h + 2h_1 + 2h_2 = h(1 + 2e^2 + 2e^4)$

Time taken for second impact is $t_2 = t + 2t_1 = \sqrt{\frac{2h}{g}}(1 + 2e)$

Time taken for third impact is $t_3 = t + 2t_1 + 2t_2 = \sqrt{\frac{2h}{g}}(1 + 2e + 2e^2)$

Application

➢ A particle of mass m moving with a speed u strikes a smooth horizontal surface at an angle α. The particle rebounds at an angle β with a speed v. The coefficient of restitution is ‘e’.



Since no external impulse acts in the horizontal direction, momentum of the ball is conserved in the horizontal direction.

$mu\cos\alpha = mv\cos\beta$

$u\cos\alpha = v\cos\beta$ ....(1)

By defination of coefficient of restitution we get

$eu\sin\alpha = v\sin\beta$ (2)

from (1) and (2), $\tan\beta = e\tan\alpha$

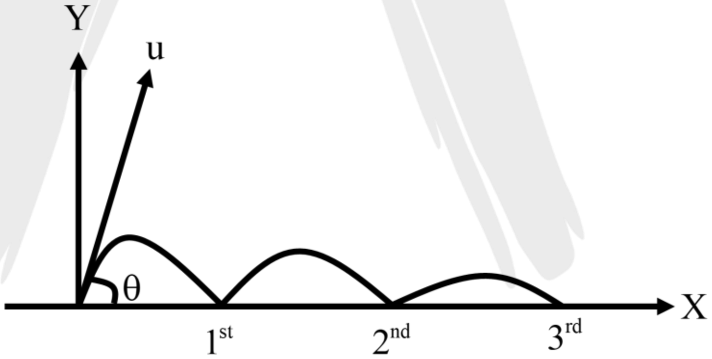
$\tan\alpha = \frac{\tan\beta}{e}$

On squaring equations (1) and (2) and adding we get

$v^2 = u^2(\cos^2\alpha + e^2\sin^2\alpha)$

$v = u\sqrt{\cos^2\alpha + e^2\sin^2\alpha}$

➢ A ball is projected with an initial velocity u at an angle θ to the horizontal surface. If 'e' is the coefficient of restitution between the ball and the surface then



(1) Time taken for 1st collision, $T = \frac{2u\sin\theta}{g}$

(2) Time interval between 1st and 2nd collisions, $T_1 = \frac{2v_1\sin\theta}{g}$ $(\because v_1 = eu)$

$T_1 = \frac{2(eu)\sin\theta}{g} = eT$

(3) Time interval between 2nd and 3rd collisions,

$T_2 = \frac{2v_2\sin\theta}{g} = \frac{2(e^2u)\sin\theta}{g} = e^2T$ $(\because v_2 = e^2u)$

(4) The total time of flight is $T^1 = T + T_1 + T_2 + \ldots\ldots\ldots$

$= T + eT + e^2T + e^3T + \cdots\ldots\ldots$

$= T[1 + e + e^2 + e^3 + \ldots\ldots\ldots]$

$T^1 = \frac{T}{1-e}$

If collision is elastic, $e = 1$ then $T^1 = \infty$

(5) The horizontal distance covered by the ball before 1st collision is

$R = \frac{u^2 \sin 2\theta}{g} = u\cos\theta \times T$

(6) The horizontal distance covered by it between 1st and 2nd collisions, $R_1 = u\cos\theta \times eT = eR$

(7) Horizontal distance covered between 2nd and 3rd collisions, $R_2 = u\cos\theta \times e^2T = e^2R$

(8) Total horizontal distance covered by the ball is

$R^1 = R_0 + R_1 + R_2 + R_3 + \cdots\ldots\ldots$

$= R + eR + e^2R + \cdots\ldots\ldots.$

$= R[1 + e + e^2 + \ldots\ldots\ldots]\ R^1 = \frac{R}{1-e}$

For perfectly elastic collision e = 1 and $R^1 = \infty$

(9) The maximum height reached by the ball before 1st collision $H = \frac{u^2\sin^2\theta}{2g} = \frac{(u\sin\theta)^2}{2g}$

(10) Maximum height it reaches between 1st and 2nd collisions is $H^1 = \frac{(eu\sin\theta)^2}{2g} = e^2H$

(11) The sum of maximum heights reached by the ball is

$H^1 = H + H_1 + H_2 + \ldots\ldots.$

$= H + e^2H + e^4H + \ldots\ldots\ldots$

$= H[1 + e^2 + e^4 + \ldots\ldots.], H^1 = \frac{H}{1-e^2}$

If the collision is elastic $e = 1$ and $H' = \infty$

Head on inelastic collision

Two bodies of masses m_1 and m_2 moving with initial velocities $\vec{u}_1$ and $\vec{u}_2 (u_1 > u_2)$ collide. After collision two bodies will move with velocities $\vec{v}_1$ and $\vec{v}_2$.

From Law of conservation of linear momentum

$m_1(\vec{u}_1 - \vec{v}_1) = m_2(\vec{v}_2 - \vec{u}_2)$

By the definition of coefficient of restitution

$\vec{v}_2 - \vec{v}_1 = e(\vec{u}_1 - \vec{u}_2)$

$\vec{v}_1 = \left(\frac{m_1 - em_2}{m_1 + m_2}\right)\vec{u}_1 + \left(\frac{(1+e)m_2}{m_1 + m_2}\right)\vec{u}_2$

$\vec{v}_2 = \left(\frac{(1+e)m_1}{m_1 + m_2}\right)\vec{u}_1 + \left(\frac{m_2 - em_1}{m_1 + m_2}\right)\vec{u}_2$

- If $m_1 = m_2 = m, u_2 = 0$ then

$v_1 = (1-e)\frac{u_1}{2}; v_2 = (1+e)\frac{u_1}{2}$

$\frac{v_1}{v_2} = \frac{1-e}{1+e}$

Loss of kinetic energy of the system :

$\Delta KE = KE_I - KE_F$

$\Delta KE = \frac{1}{2}\left(\frac{m_1 m_2}{m_1+m_2}\right)(\vec{u}_1 - \vec{u}_2)^2(1-e^2)$

In case of perfectly in-elastic collision, e = 0

∴ loss in KE of system is

$\Delta KE = \frac{1}{2}\left(\frac{m_1 m_2}{m_1+m_2}\right)(\vec{u}_1 - \vec{u}_2)^2$

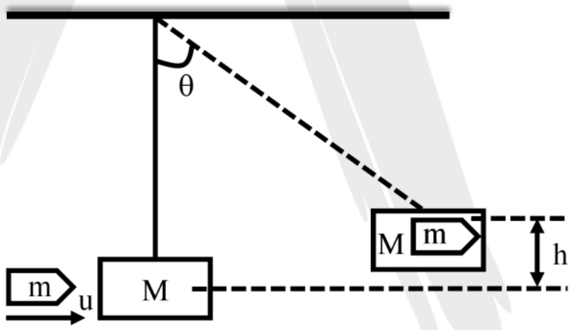
- If two bodies are approaching each other then loss in KE of the system is maximum

$\Delta KE_{max} \frac{1}{2}\left(\frac{m_1 m_2}{m_1+m_2}\right)(u_1 + u_2)^2$

Ballistic pendulum :

It is an experimental setup designed to measure the velocities of bullets. The setup involves suspending a wooden log of mass 'M' using a string of length 'ℓ' as depicted in the diagram. A bullet of mass 'm' is then horizontally fired into the wooden block with an initial velocity 'u'.

Case 1 : Let the bullet gets embedded in the block and system rises to a height 'h' as shown in the figure.



From the law of conservation of linear momentum, $m_1u_1 + m_2u_2 = (m_1 + m_2)v$

$mu + 0 = (m+M)v \Rightarrow v = \frac{mu}{m+M}$ ......(1)

KE of the system after collision is given by $KE = \frac{1}{2}(m+M)v^2$

PE at highest point $= (m+M)gh$

From LCE, $\frac{1}{2}(m+M)v^2 = (m+M)gh$

$v^2 = 2gh (or) v = \sqrt{2gh}$ ....(2)

From (1) and (2) velocity of the bullet

$u = \frac{M+m}{m}\sqrt{2gh} = \frac{M+m}{m}\sqrt{2g\ell(1-\cos\theta)}$

Loss in KE of the system $= KE_1 - KE_2$

$\Delta KE = \frac{1}{2}mu^2 - \frac{1}{2}(m+M)v^2$

$$\Delta KE = \frac{1}{2}\left[mu^2 - (m+M)\frac{m^2u^2}{(m+M)^2}\right]$$

$$\Delta KE = \frac{1}{2}\left[\frac{mM}{m+M}\right]u^2$$

Case II :

If the bullet emerges out of the block with velocity 'v' then

$mu = mv + MV$ where $V = \sqrt{2gh}$

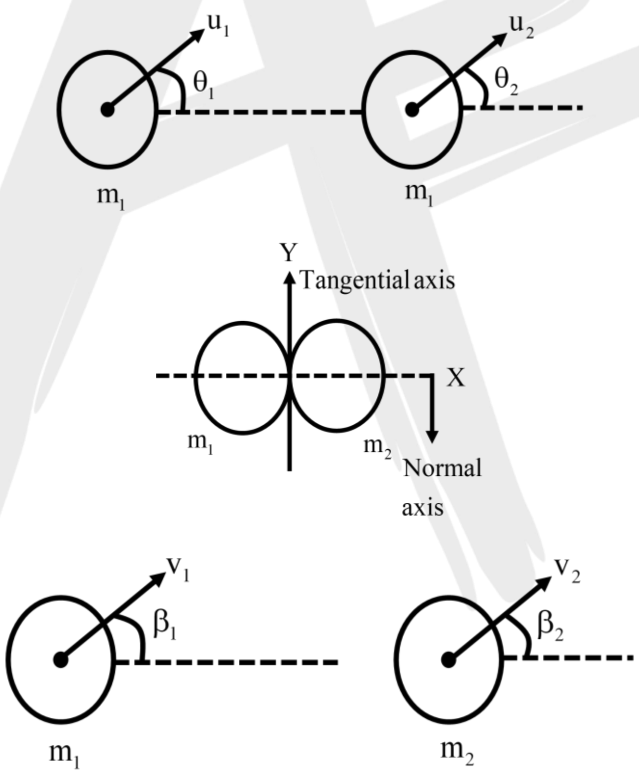
Collisions in two dimensions (oblique collisions)

1. A pair of equal and opposite impulses act along common normal direction. Hence, linear momentum of individual particles changes along common normal direction.

2. No component of impulse acts along common tangent direction. Hence, linear momentum (or) linear velocity of individual particles remains unchanged along this direction.

3. Net impulse on both the particles is zero during collision. Hence, net momentum of both the particles remain conserved before and after collision in any direction.

4. Definition of coefficient of restitution can be applied along common normal direction.



From law of conservation of linear momentum along x-axis :

$$m_1u_1\cos\theta_1 + m_2u_2\cos\theta_2 = m_1v_1\cos\beta_1 + m_2v_2\cos\beta_2$$

Along y-axis :

$$m_1u_1\sin\theta_1 + m_2u_2\sin\theta_2 = m_1v_1\sin\beta_1 + m_2v_2\sin\beta_2$$

Coefficient of restitution

$$e = -\frac{v_1\cos\beta_1 - v_2\cos\beta_2}{u_1\cos\theta_1 - u_2\cos\theta_2}$$

EXCERCISE-I

1. Four particles are in x-y plane at :

[P] 1 kg at (0, 0) [Q] 2 kg at (1, 0) [R] 3 kg at (1, 2) [S] 4 kg at (2, 0)

The centre of mass is located at :

(A) (0.3,1.2) (B) (1.3,0.6) (C) (0.5,1.4) (D) (1.2,0.3)

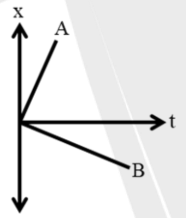
2. If a particle of mass m_1 is located at $(x, y, z) = (0, a, 0)$ and a second particle of mass m_2 is located at $(x, y, z) = (b, c, 0)$, what is the location of their center of mass?

(A) $\left(\frac{b}{2}, \frac{a+c}{2}, 0\right)$ (B) $\left(\frac{m_2 b}{m_1+m_2}, \frac{m_1 a+m_2 c}{m_1+m_2}, 0\right)$

(C) $\left(\frac{m_1 b}{m_1+m_2}, \frac{m_2 a+m_1 c}{m_1+m_2}, 0\right)$ (D) None of the above

3. One end of a thin uniform rod of length L and mass M is riveted to the centre of a uniform circudlar disc of radius r and mass 2M so that the rod is normal to the disc. The centre of mass of the combination from the centre of the disc is at distance of $\frac{L}{n}$. Find n?

4. An initially stationary box on a frictionless floor explodes into two pieces : piece A with mass m_A and piece B with mass m_B. There pieces then move across the floor along x-axis. Graphs of position versus time for the two pieces is given in figure.



(A) The graph is not possible (B) $m_A > m_B$

(C) $m_A < m_B$ (D) $m_A = m_B$

5. Two particles A and B initially at rest, move towards each other by mutual force of attraction. At the instant when the speed of A is v and the speed of B is 2v, the speed of the centre of mass of the system is :

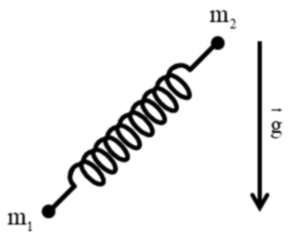
(A) 3v (B) mass should be known

(C) 1.5v (D) zero

6. On a frictionless surface, a ball of mass M moving at speed v collides elastically with another ball of the same mass that is initially at rest. After the collision, the first ball moves at an angle θ to its initial direcction and has a speed $\frac{v}{2}$. The second ball's speed after the collision is :

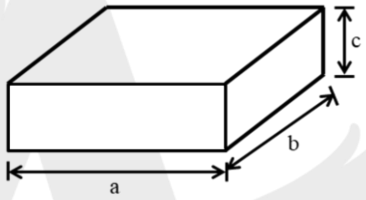
(A) $\frac{\sqrt{3}}{4}v$ (B) $\frac{v}{2}$ (C) $\frac{\sqrt{3}}{2}v$ (D) $v + \frac{v}{2}\cos\theta$

7. Two particles are inter connected by an ideal spring (see figure). The spring is compressed and system is projectd in air under gravity. If the acceleration of m_1 is $\vec{a}$ find acceleration of m_2.



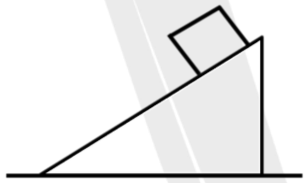
(A) $\vec{g}$ (B) $\vec{g} + \vec{a}$ (C) $\frac{m_1\vec{g} - m_1\vec{a}}{m_2}$ (D) $\frac{m_1\vec{g} - m_2\vec{g} - m_1\vec{a}}{m_2}$

8. A brick has 3 sides a, b, c $\{a > b > c\}$. The brick can be kept on ground on any face. The work required to be done to move from minimum potential energy to maximum potential energy is :



(A) $mg\left(\frac{a}{2} - \frac{b}{2}\right)$ (B) $mg\left(\frac{a}{2} - \frac{c}{2}\right)$ (C) $mg\left(\frac{b}{2} - \frac{c}{2}\right)$ (D) $mg\left(\frac{a+c}{2} - \frac{b}{2}\right)$

9. A large wedge rests on a horizontal frictionless surface, as shown. A block starts from rest and slides down the inclined surface of the wedge, which is rough. During the motion of the block, the centre of mass of the block and wedge system :



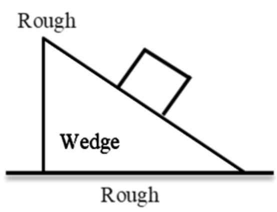
(A) does not move

(B) moves vertically with increasing speed

(C) moves horizontally with constant speed

(D) moves both horizontally and vertically

10. When a block is placed on a wedge as shown in figure, the block starts sliding down and the wedge also start sliding on ground. All surfaces are rough. The centre of mass of (wedge + block) system will move :



(A) leftward and downward (B) rightward and downward

(C) leftward and upward (D) only downward

11. If a body moves in such a way that its linear momentum is constant, then :

(A) its kinetic energy is zero

(B) the net force acting on it must be zero

(C) its acceleration must be non-zero and constant

(D) its centre of mass must remain at rest

12. **Statemene-1:** In centre of mass frame the linear momentum of system is conserved only if no external forces are present.

Statement-2: In presence of external force, the centre of mass has acceleration in inertial frame equal to $\frac{\Sigma\vec{F}_{ext}}{m_{total}}$.

(A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1

(B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1

(C) Statement-1 is true, statement-2 is false

(D) Statement-1 is false, statement-2 is true

13. A body A is at rest and body B is moving at uniform velocity of $40ms^{-1}$. Force F_1 acts on A and F_2 acts on B for 10 seconds. The velocity of A and B now is $40ms^{-1}$ and $80ms^{-1}$ respectively. If A and B are of equal mass, then $F_1 = nF_2$. Find n?

14. In a particular fission process of a uranium atom at rest, fragments with mass numbers A_1 and A_2 are produced. A good estimate of the ratio of kinetic energies of the fragments, $\frac{K_1}{K_2}$ is :

(A) $\frac{A_2}{A_1}$ (B) $\left(\frac{A_2}{A_1}\right)^2$ (C) $\frac{A_1}{A_2}$ (D) $\left(\frac{A_1}{A_2}\right)^2$

15. **Statement-1 :** If we want to throw a bottle from a moving train so that danger of being broken on hitting ground is least. We should throw bottle in forward direction.

Statement-2 : While jumping out from side of a moving carriage it is safer to jump in forwards direction.

(A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1

(B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1

(C) Statement-1 is true, statement-2 is false

(D) Statement-1 is false, statement-2 is true

16. Two identical carts constrained to move on a straight line, on which sit two twins of same mass, are moving with same velocity. At some time snow begins to drop uniformly vertically downward. Ram, sitting on one of the trolleys, throws off the falling snow sideways and in the second car Shyam is asleep. (Assume that friction is absent)

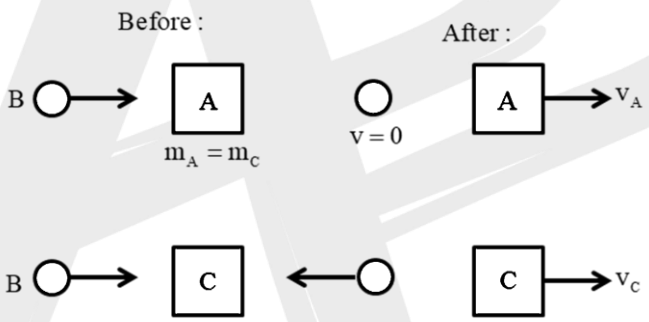
(A) Cart carrying Ram will speed up while cart carrying Shyam will slow down

(B) Cart carrying Ram will remain at the sme speed while cart carrying Shyam will slow down

(C) Cart carrying Ram will speed up while cart carrying Shyam will remain at the same speed

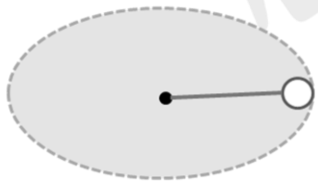
(D) Cart carrying Ram as well as Shyam will slow down

17. Objects A and C are made of different materials, with different "springiness", but they have the same mass and are initially at rest. When ball B collides with object A, the ball ends up at rest. When ball B is thrown with the same speed and collides with object C, the ball rebounds to the left. Compare the velocities of A and C after the collisions. Is v_A greater than, equal to, or less than v_C?

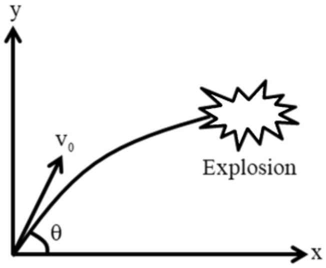


(A) $V_A > V_C$ (B) $V_A = V_C$ (C) $V_A < V_C$ (D) Can't be predicted

18. A ball of mass 2 kg is connected to a string of length 1m and made to move in a horizontal circle on a smooth friction less table with a constant speed of 2 m/s. The other end of the string is connected to a fixed peg on the table. Find the magnitude of the impulse (in N-sec) exerted by the string on the ball in half a revolution.



19. A projectile is projected in x-y plane with velocity v_0. At top most point of its trajectory projectile explodes into two identical fragments. Both the fragments land simultaneously on ground and stick there. Taking point of projection as origin and R as range of projectile if explosion has not taken place. Which of the following can not be position vectors of two pieces, when they land on ground?



(A) $\frac{R}{2}\hat{i}, \frac{3R}{2}\hat{i}$ (B) $0\hat{i}, 2R\hat{i}$

(C) $R\hat{i} - R\hat{k}, R\hat{i} + R\hat{k}$ (D) $2R\hat{i} + \frac{R}{2}\hat{k}, R\hat{i} - \frac{R}{2}\hat{k}$

20. A balloon whose mass is M is attached to a rope ladder on which a man of mass $m(< M)$ is standing. The balloon is stationary. The person begins to rise at a constant speed v on the stairs.

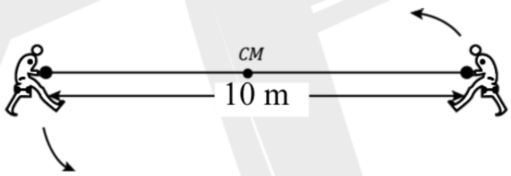
(A) The balloon rises with the same speed v

(B) The balloon descends with the same speed v

(C) The balloon descends with a speed greater than v

(D) The balloon descends with a speed lesser than v

21. Two astronauts each have a mass of 75.0 kg are connected by a 10.0 m long rope of negligible mass. They are isolated in space and orbit around common centre of mass with a speed of 10.0m/s relative to each other, as shown in figure.



(A) Tension in the string will be 375 N

(B) Tension in the string will be 1500 N

(C) Acceleration of astronaut will be $5 m/s^2$

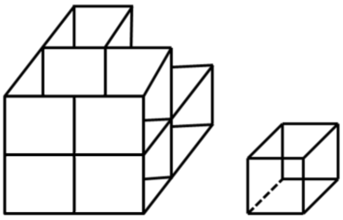
(D) Acceleration of astronaut will be $20 m/s^2$

22. A two particle system masses 2kg and 3kg present at (15,0) and (0,20) in a x,y plane then its centre of mass lies on lines:

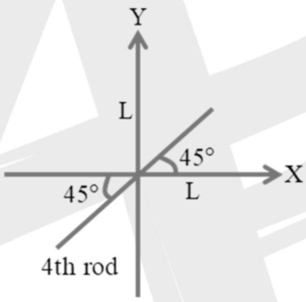
(A) $y = 2x$ (B) $\frac{x}{12} + \frac{y}{24} = 1$ (C) $3x - y = 6$ (D) $y = x$

EXCERCISE-II

1. Eight solid unifform cubes of edge ℓ are stacked together to form a signel cube with centre O. One cube is removed from this system. Distance of the centre of mass of remaining 7 cubes from O is $\frac{\sqrt{n}\ell}{7+x}$. Find n + x?

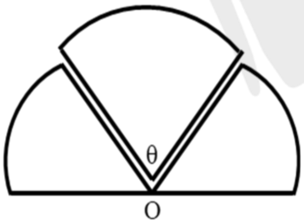


2. Three identical uniform rods of the same mass M and length L are arranged in xy plane as shown in the figure. A fourth uniform rod of mass 3M has been placed as shown in the xy plane. What should be the value of the length of the fourth rod such that the centre of mass of all the four rods lie at the origin?



(A) $L\frac{(2\sqrt{2}+1)}{3}$ (B) 2L (C) $L\frac{(\sqrt{2}+1)}{3}$ (D) 3L

3. A segment of angle θ is cut from a half disc, symmetrically as shown. If the centre of mass of the remaining part is at a distance 'a' from O and the centre of mass of the original half disc was at distance d then it can be definitely said that :



(A) $a = d$ (B) $a > d$

(C) $a < d$ (D) All the above are possible depending on θ

4. A 4.0 kg particle-like object is located at x = 0, y = 2.0 m; a 3.0 kg particle-like object is located at x = 3.0 m, y = 1.0 m. At what (a) x and (b) y-coordinates must a 2.0 kg particle-like object be placed for the center of mass of the three-particle system to be located at the origin?

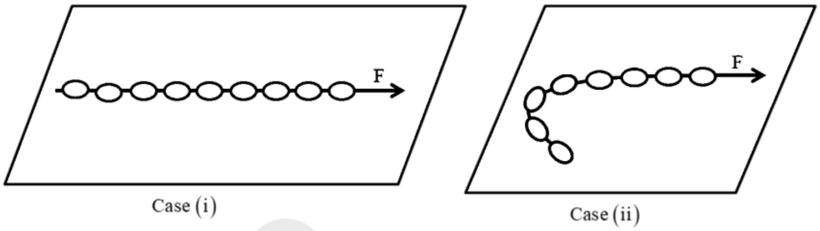
(A) (−5.5m, −4.5m) (B) (−3.5m, −4.5m)

(C) (−4.5m, −5.5m) (D) (−4.5m, −3.5m)

5. Sachin (55 kg) and Kapil (65 kg) are sitting at the two ends of a boat at rest in still water. The boat weighs 100 kg and is 3.0 m long. Sachin walks down to Kapil and shakes hand. The boat gets displaced by :

(A) zero m (B) 0.75 m (C) 3.0 m (D) 2.3 m

6. Consider a loose chain placed on a smooth table. An external force constant in magnitude acts on same chain in its two different configurations as shown in figure.



Statement-1 : Acceleration of cente of mass in both cases is same.

Statement-2 : Acceleration of centre of mass depends on resultant external force only.

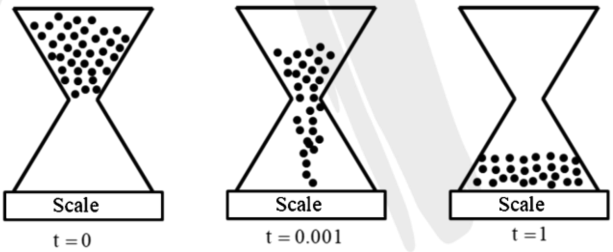
(A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1

(B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1

(C) Statement-1 is true, statement-2 is false

(D) Statement-1 is fasel, statement-2 is true

7. Consider an hourglass on a scale pictured below at times t = 0,0.001, and 1 hour. What happens to the scale's measure of weight of the hourglass plus sand combination s the sand falls?



(A) the weight is constant

(B) the weight decreases and then increases

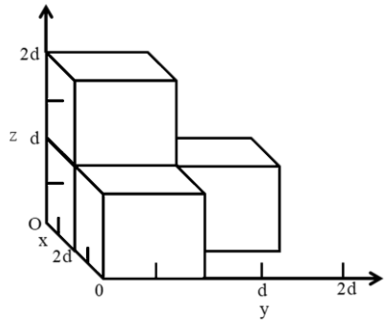
(C) the weight increases

(D) the weight increases and then decreases

8. A catapult on a level field tosses a 3 kg stone a horizontal distance of 100 m. A second 3 kg stone tossed in an identical fashion breaks apart in the air into 2 pieces, one with a mass of 1 kg and one with a mass of 2 kg. Both of the pieces hit the ground at the same time. If the 1 kg piece lands a distance of 180 m away from the catapult, how far away from the catapult does the 2 kg piece land? Ignore air resistance.

(A) 20 m (B) 100 m (C) 180 m (D) 60 m

9. Find the centre of mass of the arrangement of uniform identical cubes shown in the figure. The length of the sides of the each cube is d.



(A) (d, d, d) (B) $\left(\frac{3d}{2}, \frac{3d}{2}, \frac{3d}{2}\right)$ (C) $\left(\frac{3d}{4}, \frac{3d}{4}, \frac{3d}{4}\right)$ (D) $\left(\frac{3d}{8}, \frac{3d}{8}, \frac{3d}{8}\right)$

10. Two particles P and Q are falling freely under gravity. Then false statement is :

(A) their relative acceleration is constant

(B) their relative velocity is constant

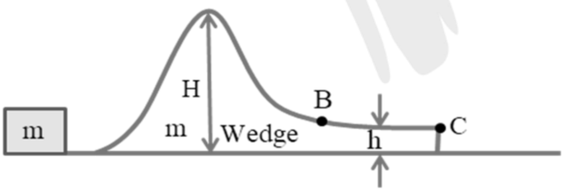
(C) their centre of mass has constant velocity

(D) their centre of mass has constant acceleration

11. A boy jumps from rest straight upward from a flat, stationary concrete surface. The boy, of mass M, leaves the concrete surface with speed v and his centre of mass rises a distance d to the highest point of the motion. How much work did the normal force of contact (N) between the boy's feet and the concrete do on the boy ?

(A) Nd (B) $\frac{Mv^2}{2}$ (C) $\frac{Nd}{2}$ (D) 0

12. Figure shows an irregular wedge of mass m placed on a smooth horizontal surface. Part BC is rough. What minimum velocity should be imparted to a small block of same mass m so that its may reach point B :



(A) $2\sqrt{gH}$ (B) $\sqrt{2gH}$ (C) $2\sqrt{g(H-h)}$ (D) $\sqrt{gh}$

13. Two smooth spheres A and B of equal radii but of masses 1 kg and 2 kg move with speeds 21 m/s and 4 m/s respectively in opposite directions and collide. The velocity of A is reduced to 1 m/s in the same direction. Then, which of the following statements is incorrect?

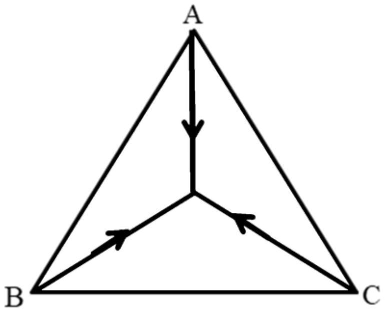
(A) The velocity of B becomes 6 m/s and its direction is reversed

(B) The coefficient of restitution is 0.2

(C) The loss of kinetic energy of the system due to the collision is 200 J

(D) The magnitude of impulse applied by the two spheres on each other is 10 Ns

14. Three particles with equal mass move with equal speed v along the medians of an equilateral triangle as shown. They all collide at the centroid simultaneously. After the collision, A comes to rest, B retraces it's path with same speed. What is speed of C after the collision?



(A) v (B) $\frac{v}{2}$ (C) 2v (D) Information insufficient

15. A bomb travelling horizontally at 90 km/hr suddenly explodes into two parts of mass ratio 1: 3. Immediately after explosion, the lighter mass is found to be travelling vertically upwards with speed $360\sqrt{3}km/hr$. The heavier one must be travelling at :

(A) 240 km/hr vertically downwards

(B) $240\sqrt{3}km/hr$ downwards 30° to horizontal (initial direction)

(C) 240 km/hr downwards 60° to horizontal (initial direction)

(D) None of the above

16. Choose the correct statement(s).

(A) The locations of centre of mass and centre of gravity may be different for an object

(B) Internal forces can change the momentum of a non-rigid body

(C) If the resultant force on a system of particles is non-zero, then the distance of the centre of mass of system may remain constant from a fixed point

(D) If net external force on a two-body system is always zero, then direction of velocity of the centre of mass of given system may change.

17. Two skaters, initially at rest, are 5 m apart. They each have one end of a single rope and each pull on the rope with a force of 50 N for a period of 1 s. One skater weighs 80 kg and the other weighs 45 kg. (Assume no friction between the skates and the ice.)

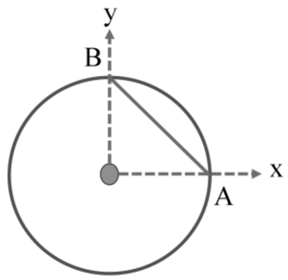
(A) The two skaters meet at a distance of 1.8 m from the initial position of the heavy skater

(B) The two skaters meet at a distance of 3.2 m from the initial position of the heavy skater

(C) The relative velocity of the skaters when they meet is $\frac{125}{72}$ m/s

(D) The relative velocity of the skaters when they meet is $\frac{25}{72}$ m/s

18. An object comprises of a uniform ring of radius R and its uniform chord AB (not necessarily made of the same material) as shown. Which of the following can be the centre of mass of the object?



(A) $\left(\frac{R}{3}, \frac{R}{3}\right)$ (B) $\left(\frac{R}{3}, \frac{R}{2}\right)$ (C) $\left(\frac{R}{4}, \frac{R}{4}\right)$ (D) $\left(\frac{R}{\sqrt{2}}, \frac{R}{\sqrt{2}}\right)$

19. Which of the following are not correct about centre of mass?

(A) Centre of mass of a system of four particles in a plane must lie within the quadrilateral formed by the four particles

(B) In centre of mass frame momentum of a system is always zero

(C) Internal force may affect the motion of centre of mass

(D) Centre of mass and centre of gravity are synonymous in all situations

20. Two toy cars with different masses originally at rest are pushed apart by a spring. Which of the following statements would be true?

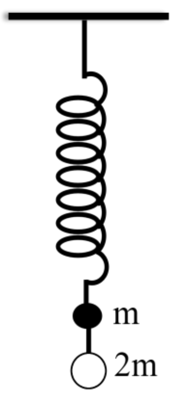
(A) Both toy cars will acquire equal but opposite momenta

(B) Both toy cars will acquire equal kinetic energies

(C) The more massive toy car will acquire less speed

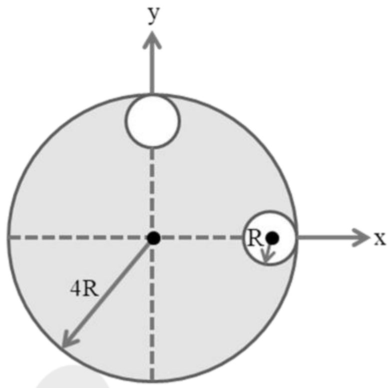
(D) The smaller toy car will experience an acceleration of greatest magnitude

21. A ball of mas m = 1 kg is hung from a spring of constant 400 N/m and another ball of mass 2m is hanging from it as shown with the help of a thread. The system is in equilibrium. Now the thread is burned. As a result, ball of mass m starts moving upwards. Eventually it will stop under the influence of gravity and spring. In between, there is a point at which its speed is maximum. what is that maximum speed (in m/s)?



EXCERCISE-III

1. From the circular disc of radius 4R two small disc of radius R are cut off. The centre of mass of the new structure will be :

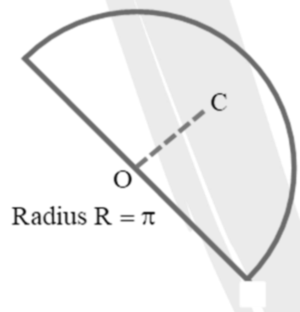


(A) $i\frac{R}{5}+j\frac{R}{5}$ (B) $-i\frac{R}{5}+j\frac{R}{5}$ (C) $\frac{-3R}{14}(\hat{i}+\hat{j})$ (D) None of these

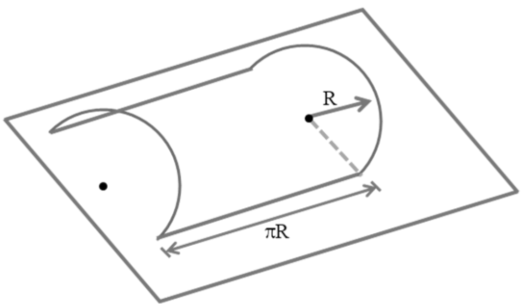
2. A square plate has a diagonal of length 21 cm. One of it's corner is cut along the line joining the midpoint of neighbouring two sides. How far is new centre of mass from original centre of mass ?

(A) 0.25 cm (B) 1.75 cm (C) 1.25 cm (D) −1cm

3. A uniform semicircular disc is suspended from a point C on it such that it remains in vertical plane. It is found that point C is such that disc remains in equilibrium is every orientation. What is distance OC is $\frac{133}{n}$ m. Find n?



4. A wire is bent into the structure as shown in the figure, and placed on a table. It consists of two half rings of radius R and two straight parts of length πR. The height of COM from the table is :



(A) $\frac{2R}{\pi}$ (B) $\frac{R}{\pi}$ (C) $\frac{R}{2}$ (D) zero

5. Two identical cars start at the same point, but travel in opposite directions on a circular path of radius R, each at speed v. While each car travels a distance less than $\left(\frac{\pi}{2}\right)$ R, (one quarter circle) the centre of mass of the two cars :

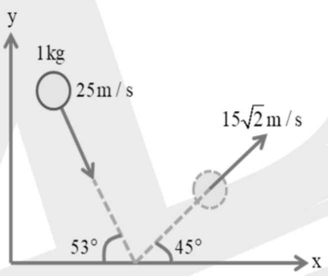
(A) remains at the initial point

(B) travels along a diameter of the circle at speed < v

(C) travels along a diameter of the circle at speed = v

(D) travels along a diameter of the circle at speed > v

6. A ball of mass 1 kg bounces against the ground as shown in the figure. the approaching velocity is 25 m/s and the velocity after hitting the ground is $15\sqrt{2}$m/s, the impulse (in Ns) exerted on the ball is :



7. Two blocks A and B of the same mass are connected to a light spring and placed on a smooth horizontal surface. B is given velocity v_0 (as shown in the figure) when the spring is in natural length. In the subsequent motion.

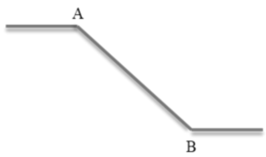
(A) the maximum velocity of B will be v_0

(B) as seen from ground, A can move towards right only

(C) the spring will have maximum extension when A and B both stop

(D) the spring will be at natural length again when B is at rest

8. Two blocks of mass 2m and m slide down the slope AB having same friction coefficient. Then :



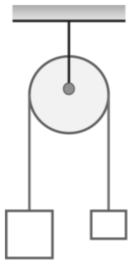
(A) thr heavier mass reaches the bottom in a shorter time

(B) both the masses have the same kinetic energy on arriving at B

(C) the heavier mass has twice the momentum than that of the other when reaching B

(D) the lighter mass reaches the bottom in a short time

9. A light rope passes over a light frictionless pulley attached to the ceiling. An object with a large mass is tied to one end and an object with a smaller mass is tied to the other end. Both masses are released from rest. Which of the following statement(s) is/are false for the system consisting of the two moving masses while string remains taut?



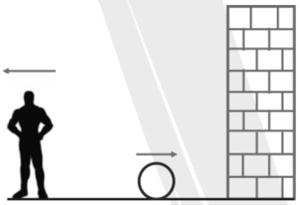
(A) The centre of mass remains at rest

(B) The net external force is zero

(C) The velocity of the centre of mass is a constant

(D) The acceleration of the centre of mass is g downward

10. A man of mass M is carrying a ball of the mass $\frac{M}{2}$. The man is initially in the state of rest at a distance D from fixed vertical wall. He throws the ball towards the wall with a velocity V with respect to earth at t = 0. As a result of throwing, the man also starts moving backwards. The ball rebounds elastically from the wall. The man finally collects the ball. Assuming friction to be absent.



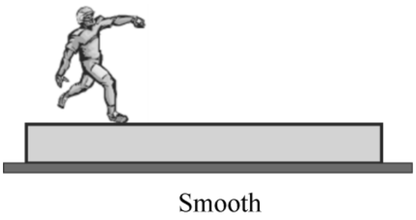
(A) The velocity of the man + ball system after the man has collected the ball is $\frac{2V}{3}$

(B) Impulse by ball on man is $\frac{MV}{3}$

(C) Impulse by ball on man is $\frac{MV}{6}$

(D) He catches the ball again at $t = \frac{4D}{V}$

11. A man of mass m_0 is standing on a long plank of mass M. Man starts to run on plank of mass M. Man starts to run on plank with velocity u_0 with respect to plank.



(A) During running, the velocity of centre of mass of (plank + man) is non-zero.

(B) During running, net work done by static friction between plank and man is non-zero

(C) During running, work done by friction on the plank is positive

(D) Total mechanical energy of plank and man increases

12. A particle of mass m makes a head-on elastic collision with a particle of mass 2m initially at rest. The velocity of the first particle before and after the collision is u_1 and v_1. Assume no external force is acting. Then which of the following statement(s) is/are true for this collision?

(A) For all values of u_1, v_1 will always be less than u_1 in magnitude and $|v_1| = \frac{|u_1|}{3}$

(B) The fractional loss in KE of the first particle is 8/9

(C) During collision KE is not constant

(D) During collision momentum is not constant

13. Consider a particle at rest which may decay into two (daughter) particles or into three (daughter) particles. Which of the following is true in the two-body case but false in the three-body case? (There are no external forces and the masses of daughter particles are known.)

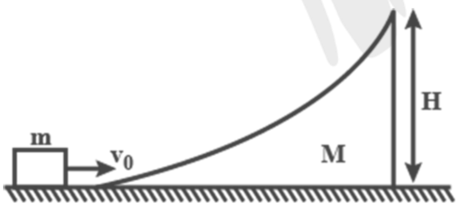
(A) Velocity vectors of the daughter particles must lie in a single plane.

(B) Given the total kinetic energy of the system it is possible to determine the speed of each daughter particle.

(C) Given the speed(s) of all but one daughter particle, it is possible to determine the speed of the remaining particle.

(D) The total momentum of the daughter particles is zero.

14. Figure shows a block of mass m projected with velocity v_0 towards a wedge. Consider all the surfaces to be smooth. Block does not have sufficient energy to negotiate (overcome) wedge. Mark the correct option(s).



(A) When block is at the maximum height on wedge, block and wedge have velocity equal to velocity of centre of mass of block wedge system.

(B) Wedge acquires maximum speed with respect to ground when block returns to lowest point on wedge.

(C) Momentum of wedge and block is conserved at all times.

(D) Centre of mass of wedge and block remains stationary.

15. Two men, of masses 60 kg and 80 kg are sitting at the ends of a boat of mass 60 kg and length 4 m. The boat is stationary. If the men now exchange their positions, then:

(A) the centre of mass of the two men shifts by $\frac{4}{7}$m

(B) the boat moves by 0.4 m

(C) the centre of mass of the two men shift by $\frac{6}{35}$m

(D) the boat moves by 0.6 m

16. An object of mass 3m, initially at rest on a frictionless horizontal surface, explodes breaking into two fragments of mass m and 2m, respectively. Which one of the following statements after the explosion is true?

(A) Kinetic energy of centre of mass increases.

(B) The smaller fragment will have twice the speed of the larger fragment.

(C) Fragments have equal magnitude of momentum in ground frame but different magnitude of momentum in centre of mass frame.

(D) Kinetic energy of system increases.

17. A blast breaks a body initially at rest of mass 0.5 kg into three pieces, two smaller pieces of equal mass and the third double the mass of either of small piece. After the blast the two smaller masses move at right angles to one another with equal speed. Find the statements that is/are true for this case assuming that the energy of blast is totally transferred to masses.

(A) All the three pieces share the energy of blast equally

(B) The speed of bigger mass is $\sqrt{2}$ times the speed of either of the smaller mass

(C) The direction of motion of bigger mass makes an angle of 135^{0} with the direction of smaller pieces

(D) The bigger piece carries double the energy of either piece

18. Block A of mass 0.50 kg initially travelling towards right at 4.00 m/s on a frictionless horizontal track collides elastically with another block B of mass 1.50 kg initially at rest. If the collision is head-on, find magnitude of momentum of the block B after the collision.

(A) 1 kg m/s (B) 2 kg m/s (C) 3 kg m/s (D) 4 kg m/s

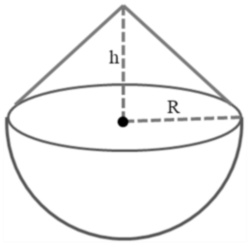
19. A ball is dropped freely from a height of 20 m on to a hard flat surface. The coefficient of resistution between the ball and the surface is 0.5. Find the time it takes to strike the surface for the second time.

(A) 1 sec (B) 2 sec (C) 3 sec (D) 4 sec

20. A ball is dropped on a horizontal floor from a height of 64 m. If $e = \left(\frac{1}{2}\right)^{1/6}$ then find height (in meters) attained by ball after rebounding thrice.

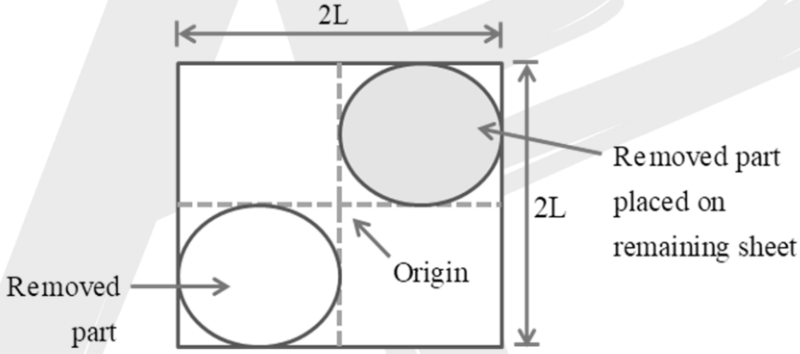
EXCERCISE-IV

1. A uniform solid right circular cone of base radius R is joined to a uniform solid hemisphere of radius R and of the same density, as shown. The centre of mass of the composite solid lies at the centre of base of the cone. The height of the cone is :



(A) 1.5 R (B) $\sqrt{3}$R (C) 3R (D) $2\sqrt{3}$R

2. Figure shows a square lamina with a disc of radius $\frac{L}{2}$ removed from it which is now placed symmetrically over upper right quarterr. What is location of centre of mass of system relative to origin shown in figure.



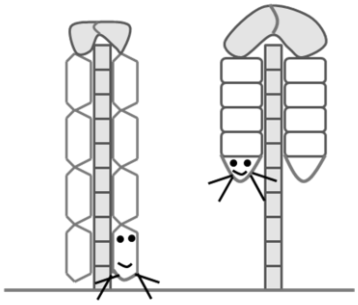
(A) $\frac{\pi L}{12}(\hat{i}+\hat{j})$ (B) $\frac{\pi L}{8}(\hat{i}+\hat{j})$ (C) $\frac{\pi L}{4}(\hat{i}+\hat{j})$ (D) $\frac{\pi L}{16}(\hat{i}+\hat{j})$

3. Pull down in the middle a heavy chain, fixed at both ends, in such a way that it forms a triangle. Does the centre of mass takes up a higher or lower position?

(A) Centre of mass comes down (B) Centre of mass goes up

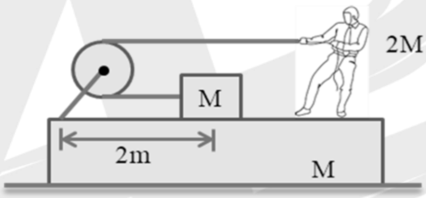
(C) Centre of mass remains stationary (D) Cannot be predicted

4. Two 20 g worms climb over a 10 cm high, very thin wall. One worm is thein and 20 cm long the other is fat and only 10 cm long. What is the ratio of the potntial energy (w.r.t. the base of wall) of the thin worm as compared to that of the fat worm when each is half way over the top of the wall as shown?



(A) 1 : 1 (B) 2 : 1 (C) 2 : 3 (D) 1 : 2

5. A block of mass M is tied to one end of a massless rope. The other end of the rope is in the hands of a man of mass 2M as shown in the figure. The block and the man are resting on a rough plnak of mass M as shown in the figure. The whole system is resting on a smooth horizontal surface. The man pulls the rope. Pulley is massless and frictionless. What is the displacement of the plank when the block meets the pulley? (Man does not leave his position on plank during the pull)



(A) 0.5 m (B) 1 m (C) Zero (D) $\frac{2}{3}$m

6. A projectile is launched vertically upward. It explodes into two pieces at the top point of its trajectory. One piece has twice the mass of the other. Immediately after the explosion, the more massive piece has kinetic energy E. What is the total kinetic energy of both pieces immedeiately after the explosion?

(A) 1.5 E

(B) 2 E

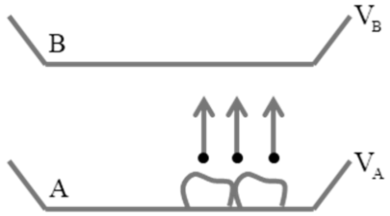
(C) 3 E

(D) Not enough information to answer

7. A 5000 kg rocket is set of vertical firing. The relative speed of burnt gas is $800 ms^{-1}$. To give an initial upwards acceleration of $20 ms^{-2}$, the amount of gas ejected per second to supply the needed thrust will be :

(A) $127.5 kgs^{-1}$ (B) $187.5 kgs^{-1}$ (C) $185.5 kgs^{-1}$ (D) $137.5 kgs^{-1}$

8. Two long motor boats are moving in the same direction in still water, the boat A with speed of 10 km/h and other with a speed of 20 km/h. While they are passing each other coal is shoveled from the slower boat to the faster one at a rate 1000 kg/min. Assume that the shoveling is always normal relative to the boat A and resistance offered by water is negligible.



How much approximate additional force must be provided by driving engine of faster boat B if its velocity is to be maintained as constant?

(A) 46 N (B) 32 N (C) 20 N (D) 167 N

9. Which of the following is/are correct?

(A) If centre of mass of three particles is at rest and it is known that two of them are moving along different lines then the third particle must also be moving

(B) If centre of mass remains at rest, then net work done by the forces acting on the system must be zero

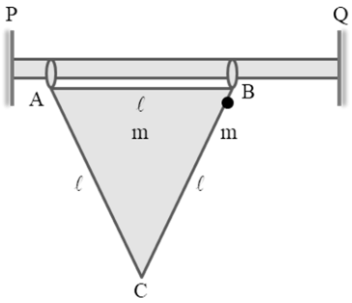
(C) If centre of mass remains at rest then the net external force must be zero

(D) If speed of centre of mass is changing then there must be some work being done by internal forces on the system

10. Inside a hollow sphere of mass M, a rod of length $R\sqrt{2}$ is released from the state of rest. The mass of the rod is same as that of the sphere. If the inner radius of the hollow sphere is R then find out its horizontal displacement with respect to earth in the time in which the rod becomes horizontal.

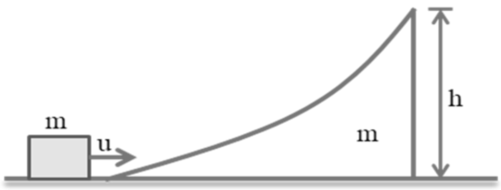
(A) $\frac{R}{2}$ (B) $\frac{R}{4}$ (C) $\frac{R}{2\sqrt{2}}$ (D) None of these

11. A rigid triangular frame ABC of mass m is hanging from a rigid horizontal rod PQ. The frame is constrained to move along horizontal without friction. A bead of mass m is released from B that moves along BC. Displacement of frame when bead reaches C is :



(A) $\frac{\ell}{2}$ (B) $\frac{\ell}{4}$ (C) $\frac{3\ell}{\sqrt{2}}$ (D) None of these

12. A small block of mass m is pushed towards a movable wedge of mass m and height h with initial velocity u. All surfaces are smooth. The minimum value of u for which the block will reach the top of the wedge :

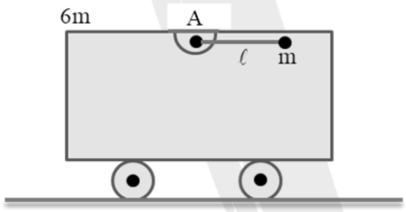


(A) $\sqrt{gh}$ (B) $\sqrt{2gh}$ (C) $2\sqrt{gh}$ (D) None of these

13. A steam of water droplets, each of mass m = 0.001 kg, are fired horizontally at a velocity of 10 m/s towards a steel plate where they collide. The droplets are spaced equidistantly with a spacing of 1 cm. What is the approximate average force exerted on the plate by the water droplets assuming that they do not rebound after their collision?

(A) 10 N (B) 100 N (C) 1 N (D) 0.1 N

14. In the figure shown the cart of mass 6m is initially at rest. A particle of mass m is attached to the end of the light rod which can rotate freely about A. If the rod is released from rest in a horizontal position shown, determine the velocity v_{rel} of the particle with respect to the cart when the rod is vertical.



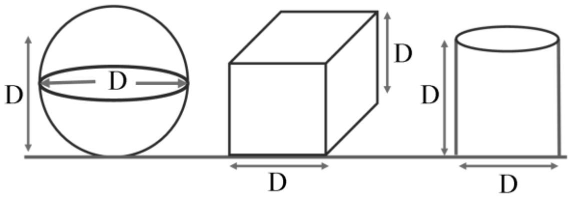
(A) $\sqrt{\frac{7}{3}g\ell}$ (B) $\sqrt{\frac{7}{6}g\ell}$ (C) $\sqrt{\frac{14}{3}g\ell}$ (D) $\sqrt{\frac{8}{3}g\ell}$

15. Assume gravitational potential energy 'U' at ground level to be zero. All objects are made up of same material.

U_P = Gravitational potential energy of solid sphere

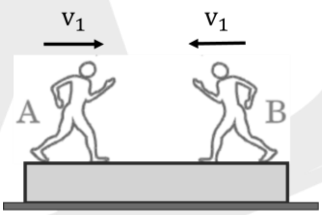
U_Q = Gravitational potential energy of solid cube

U_S= Gravitational potential energy of solid cylinder



(A) $U_S > U_P$ (B) $U_Q > U_S$ (C) $U_P > U_Q$ (D) $U_P > U_S$

16. In the figure shown the system is at rest initially. Two persons 'A' and 'B' of masses 40 kg each move with speeds v_1 and v_2 respectively towards each other on a plank lying on a smooth horizontal surface as shown in figure. Plank travels a distance of 20 m towards right direction in 5 sec.



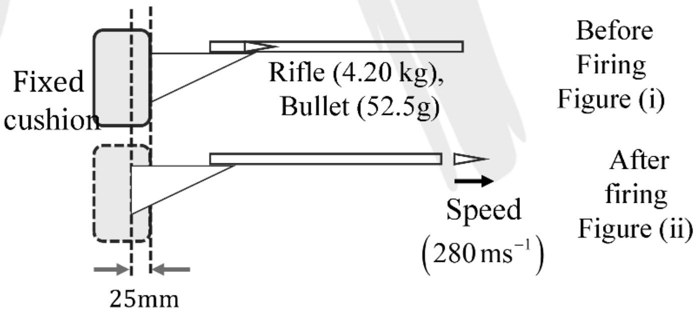
(Here v_1 and v_2 are given with respect to the plank).

Then the possible condition(s) can be:

(A) $v_1 = 0 m/s, v_2 = 10 m/s$ (B) $v_1 = 5 m/s, v_2 = 15 m/s$

(C) $v_1 = 10 m/s, v_2 = 20 m/s$ (D) $v_1 = 2 m/s, v_2 = 12 m/s$

17. A rifle resting against a cushion fires a bullet as shown. Figure (i) below shows the event before firing the bullet. After firing the bullet, the rifle compresses the cushion by 25mm and comes to rest.



(A) The change in momentum of the bullet is 14.7Kg m/s.

(B) The recoil speed of the rifle immediately after firing 3.5 m/s.

(C) The average force exerted by the cushion in stopping the rifle 1.03 KN.

(D) Work done by the rifle on the bullet is 25.73 J.

18. Two balls A and B having masses 1 kg and 2 kg, moving with speeds 21 m/s and 4 m/s respectively in opposite direction, collide head on. After collision A moves with a speed of 1 m/s in the same direction, then:

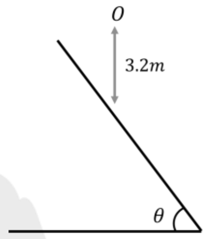
(A) The velocity of B after collision is 6 m/s opposite to the direction before collision.

(B) The coefficient of restitution is 0.2

(C) The loss of kinetic energy due to collision is 200 J.

(D) The impulse of the force between the two balls is 40 Ns.

19. A ball of mass 1 kg is dropped from a height of 3.2 m on smooth inclined plane. The coefficient of restitution for the collision is $e = \frac{1}{2}$ The ball's velocity becomes horizontal after the collision.



(A) The angle $\theta = \tan^{-1}\left(\frac{1}{\sqrt{2}}\right)$

(B) The speed of the ball after the collision $= 4\sqrt{2}$m/s

(C) The total loss in kinetic energy during the collision is 8 J

(D) The ball hits the inclined plane again while travelling vertically downward

20. A tennis racket (held firmly in the hand) moving at a speed u_1 hit's a small ball moving towards it at a speed u_0

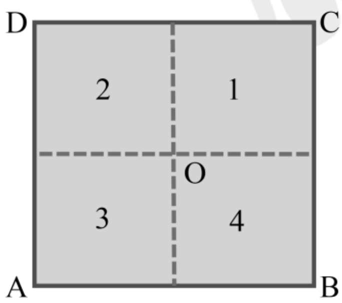
(A) The maximum possible speed of the ball after it is hit is $u_0 + 2u_1$

(B) Magnitude of impulse on ball due to tennis racket is $2mu_1$

(C) Work done by tennis racket on ball is $2mu_1(u_0 + u_1)$

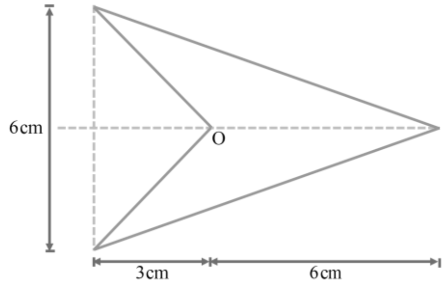
(D) Work done by tennis racket on ball is zero.

21. In the given figure four rods AB, BC, CD and DA have mass m, 2m, 3m and 4m respectively. In which of the regions (numbered 1, 2, 3, 4) the centre of mass of system lies?



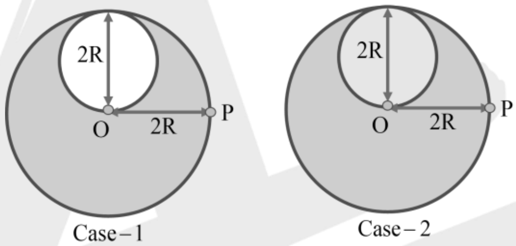
(A) 1 (B) 2 (C) 3 (D) 4

22. A uniformly thick plate in the shape of an arrow head has dimensions as shown. Find the distance of centre of mass from point O.

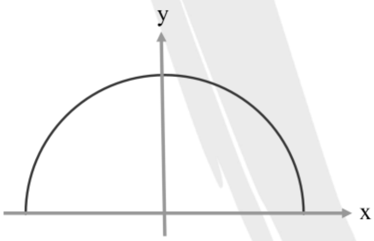


(A) 1cm (B) 2cm (C) 0.5cm (D) 1.5cm

23. A lamina is made by removing a small disc of diameter 2R from a bigger disc of uniform mass density and radius 2R, as shown in the figure. A second similar disc is made but instead of hole a disc of double the density as of first is filled in the hole. Centre of mass is calculate in both the cases and was found at a distance r_1 and r_2 from centre O respectively. Find the ratio $\left|\frac{5r_2}{r_1}\right|$

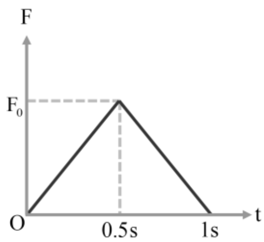


24. In the figure below, a uniform piece of wire is bent in the form of a semicircular arc as shown. Find the distance of centre of mass of the wire from the origin if radius of the semicircular ring is $R = 3\pi$cm.



(A) 2cm (B) 4cm (C) 6cm (D) 8cm

25. A body of mass 1kg moving with velocity 1 m/s makes an elastic one dimensional collision with an identical stationary body. They are in contact for brief time 1s. Their force of interaction increases from zero to F_0 linearly in time 0.5s and decreases linearly to zero in further time 0.5 s as shown in figure. Find the magnitude of force F_0

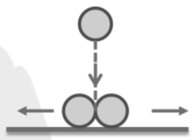


(A) 4N (B) 3N (C) 2N (D) 1N

26. A bomb moving in certain direction suddenly explodes into three identical pieces which fly off with equal speed. Two of the three pieces go in one direction while the third one goes perpendicular to the other two. If K_f and K_i are final and initial kinetic energies of the system. Find the ratio of $\frac{K_f}{K_i}$ is

(A) $\frac{9}{5}$ (B) $\frac{5}{9}$ (C) $\frac{6}{5}$ (D) $\frac{5}{6}$

27. Two identical balls of mass M and radius R are placed in contact with each other on a frictionless horizontal surface as shown. The third ball of mass M and radius R/2 moves vertically downward and hits the two balls symmetrically with speed $\sqrt{5}$m/s and comes to rest. Find the speed of any one of the bigger balls after collision if bigger balls are moving horizontally after collision.



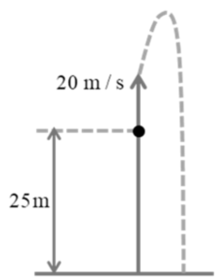
(A) 1 m/s (B) 2 m/s (C) 3 m/s (D) 4 m/s

28. For shown situtation, if collision between block A and B is perfectly elastic, then find the maximum energy stored in spring in joules.

29. A ball of mass m is released from height $\frac{9}{2}$R collides with identical ball kept at the bottom of the loop (as shown in figure). After collision second ball just reaches at top of loop. If height reached by first ball after collision is h and 6h = nR. Find the value of n

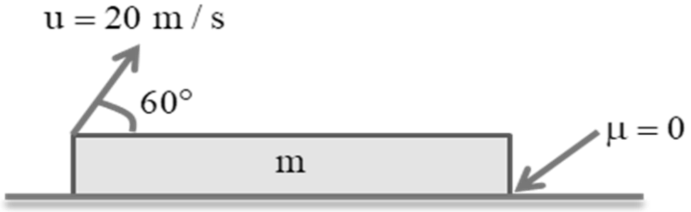
30. An object A of mass 1kg is projected vertically upward with a speed of 20 m/s. At the same moment another object B of mass 3kg, which is initially above the object A, is dropped from a height h = 20 m. The two points like objects (A and B) collide and stick to each other. The kinetic energy is K (in J) of the combined mass just after collision, find the value of $\frac{K}{25}$.

31. A boy throws a ball at time t = 0 with upward initial velocity = 20 m/s, from the corner of the roof of a building, whose height above the ground is 25 m. If the velocity of the ball is simply reversed after collision from the ground (magnitude remaining same), and the 3rd collision of the ball with the ground occurs at $t = t_1$sec. What is the value of t_1?



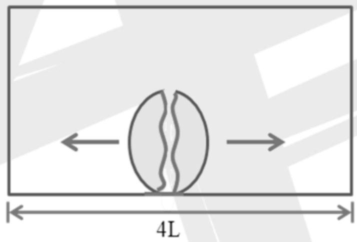
EXCERCISE-V

1. A particle of mass m is projected at an angle of 60° with a velocity of 20 m/s relative to the ground from a plank of same mass m which is placed on smooth surface. Initially plank was at rest. The minimum length of the plank for which the ball will fall on the plank itself is $(g = 10 m/s^2)$



(A) $40\sqrt{3}$m (B) $20\sqrt{3}$m (C) $10\sqrt{3}$m (D) $60\sqrt{3}$m

2. A bomb of mass 3m is kept inside a closed block of mass 3m and length 4L at it's centre. It explodes in two parts of mass m and 2m. The two parts move in opposite direction and stick to the opposite side of the walls of box. Box is kept on a smooth horizontal surface. What is the distance moved by the box during this time interval?



(A) 0 (B) $\frac{L}{6}$ (C) $\frac{L}{12}$ (D) $\frac{L}{3}$

3. A man of mass m walks from end A to the other end B of a boat of mass M and length ℓ. The coefficient of friction between man and boat is μ and neglect any resistive force between boat and water.

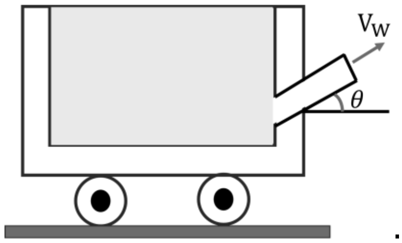
(A) If man runs at his maximum acceleration the acceleration of boat is $\frac{m}{M}\mu g$

(B) Minimum time take by man to reach other end of the boat is $\sqrt{\frac{2M\ell}{(M+m)\mu g}}$

(C) Magnitude of displacement of centre of mass of boat is $\frac{M\ell}{m+M}$

(D) Velocity of CM of man and boat system is zero.

4. Consider a cart of mass M on a fricitonless surface that can hold a full tank of water with mass M. A fire-hose sprays water with a constant ejection speed V_W at a constant mass rate $r = \frac{dm}{dt}$ and at an angle θ relative to the horizontal.



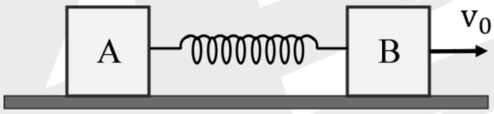
(A) The acceleration at any time t of the cart while it is spraying water is given by $\frac{V_W r \cos\theta}{M-rt}$

(B) The speed of the cart as a function of time $\left(t < \frac{M}{r}\right)$ is $\frac{V_W rt \cos\theta}{M-rt}$

(C) The speed of the cart as a function of time $\left(t < \frac{M}{r}\right)$ is $V_W \cos\theta \ln(M - rt)$

(D) The external horizontal force that must be applied to keep the cart stationary while spraying water is $rV_W \cos\theta$

5. Two blocks A and B of the same mass are connected to a light spring and placed on a smooth horizontal surface. B is given velocity v_0 (as shown in the figure) when the spring is in natural length. In the subsequent motion.



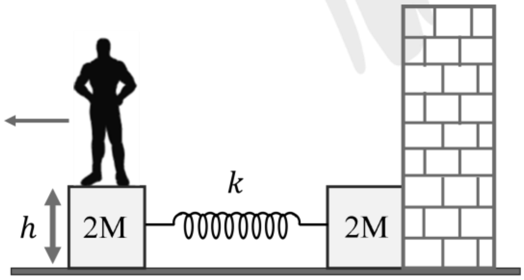
(A) The maximum velocity of B will be v_0

(B) As seen from ground, A can move towards right only

(C) The spring will have maximum extension when A and B both stop

(D) The spring will be at natural length again when B is at rest

6. Two identical blocks of mass 2M are joined by means of a light spring of spring constant k. A man of mass M is standing on one of the blocks as shown in the diagram. If man jumps horizontally with a velocity V relative to block and horizontal surface is smooth, then:



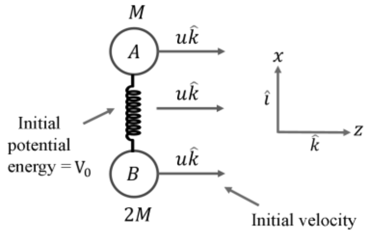
(A) the maximum compression in the spring is $\sqrt{\frac{2M}{k}}\left(\frac{V}{3}\right)$

(B) man lands at horizontal distance $V\sqrt{\frac{2h}{g}}$ from initial position of the block

(C) right block loses contact with wall when the elongation in spring is maximum

(D) velocity of centre of mass of two blocks after 2M loses contact with wall is $\frac{V}{6}$

7. Two masses A and B are connected by a spring. A has mass M, B a mass 2M, and the spring has a negligible mass. The spring is compressed so that the potential energy stored in it is V_0. The system is placed on a horizontal frictionless table and is given a velocity $u\hat{k}$ in the z-direction as shown, and the spring is released. In the subsequent motion the line from B to A always points along the $\hat{i}$ unit vector. The initial total mechanical energy is E.



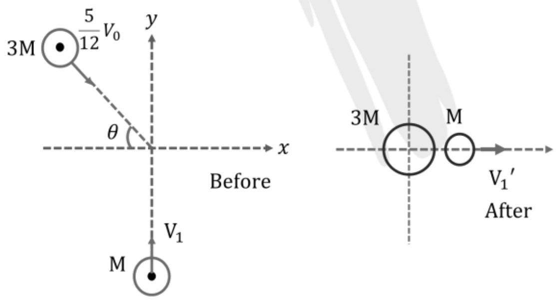
(A) In terms of the given quantities $E = V_0 + \frac{3}{2}Mu^2$

(B) Potential energy is stored in the spring when A and B have kinetic energy K_A and K_B is
$P.E. = E - K_A - K_B$

(C) At some instant of time B is at point P and has a component of velocity in the x-direction of $v_x\hat{i}$. At that instant of time, in terms of the given quantities, the velocity vector $\vec{v}_A$ of A is $\vec{v}_A = -2v_x\hat{i} + u\hat{k}$

(D) In subsequent motion A and B undergo circular motion about centre of mass.

8. Two masses M and 3M collide on a horizontal frictionless surface as shown. Before the collision the mass M has a velocity V_1 in the y-direction. The mass 3M has a velocity $\left(\frac{5}{12}\right)V_0$ making an angle θ to the x-axis as shown. After the collision the mass 3M comes to rest and the mass M moves along the x-axis with the velocity V'_1.



$\left(\text{Given } \sin\theta = \frac{3}{5}\right)$ Neglect gravity.

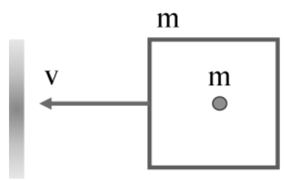
(A) The speed V_1 of mass M before collision is $\frac{3}{4}V_0$

(B) The speed V_1 of mass M before collision is $\frac{V_0}{4}$

(C) The speed V'_1 of mass M after collision is $\frac{V_0}{2}$

(D) The speed V'_1 of mass M after collision is V_0

9. In the middle of a box of mass m is a particle of the same mass m. The whole structure is moving at a speed v in the horizontal plane towards the wall (see fig). Friction is absent everywhere, all the collisions are absolutely elastic. Choose the correct option(s).



(A) There will be a total of three collisions.

(B) Finally, the box will be moving towards right with speed v.

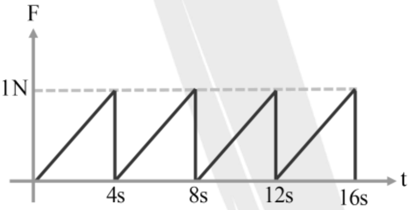
(C) Finally, the particle will be at the edge of the box.

(D) The particle will oscillate back and forth relative to the box after the collision with the wall is over.

10. A cylindrical pot is slowly filled with water. The centre of mass of the empty pot is at a height of 10 cm, the mass of the pot is 1kg, and its inner area is $0.4m^2$. What is the height of the water in it, if the centre of mass of the system is at the lowest position? (Take density of water $1000 kg/m^3$)

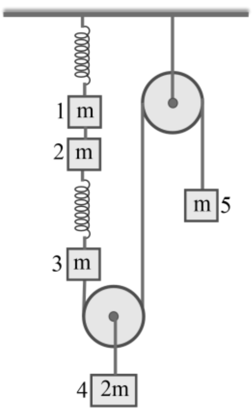
(A) 1cm (B) 2cm (C) 3cm (D) 4cm

11. A particle of mass 500g is projected along x-axis with a velocity 6 m/s. It is acted upon by variable force acting along y-axis as shown in figure. Find the modulus of velocity at t = 8s is



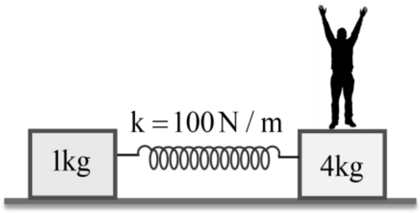
(A) 2.5 m/s (B) 5 m/s (C) 7.5 m/s (D) 10 m/s

12. All the strings, springs and pulleys shown in figure are ideal. Initially the system is in equilibrium and blocks are at rest. Now, the upper spring is cut. Find the acceleration of block 1 just after cutting the upper spring is



(A) gm/s^2 (B) $2gm/s^2$ (C) $3gm/s^2$ (D) $4gm/s^2$

13. A boy of mass 10 kg is standing on a 4 kg block as shown. The whole system is on smooth level ground. The boy jumps to the right with a velocity of 7 m/s relative to the 4kg block. Find the maximum velocity of 1 kg block in the subsequent motion.

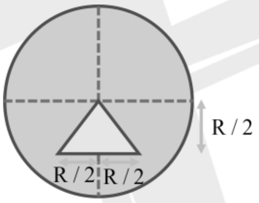


(A) 2 m/s (B) 4 m/s (C) 6 m/s (D) 8 m/s

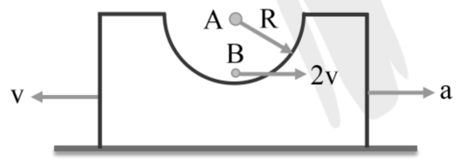
14. A cannon of mass 10×10^3kg is rigidly bolted to the earth so it can recoil only by a negligible amount. The cannon fires a 2.1×10^3kg shell horizontally with an initial velocity of 550 m/s. Suppose the cannon is then unbolted from the earth, and no external force hinders its recoil. What would be the velocity of a shell fired horizontally by this loose cannon? (Hint: In both cases assume that the burning gunpowder imparts the same kinetic energy to the system).

(A) 400 m/s (B) 425 m/s (C) 450 m/s (D) 500 m/s

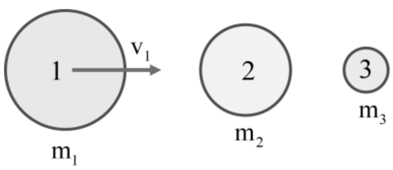
15. In the figure as shown, a triangular portion is cut from a circular disc of radius R. The distance of centre of mass of the remaining portion from the centre of disc is $\frac{R}{\alpha(\beta\pi-1)}$. Calculate $\left(\frac{\alpha\beta}{2}\right)$.



16. A bead kept at the bottom of a wedge moves towards right with a velocity 2v. If the wedge moves towards left with a velocity v while accelerating towards right with an acceleration a as shown then the magnitude of acceleration of the bead is x units. Find x if v = 3 and R = 9 units.

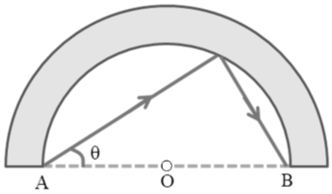


17. The centres of the spheres 1, 2 and 3 lies on a single straight line. sphere 1 is moving with an (initial) velocity v_1 directed along this line and hits sphere 2. Sphere 2, acquiring after collision a velocity v_2, hits sphere 3. Both collisions are absolutely elastic. What must be the mass of sphere 2 for the sphere 3 to acquire maximum velocity (The masses m_1 and m_3 of spheres 1 and 3 are 9 kg and 1 kg respectively)?



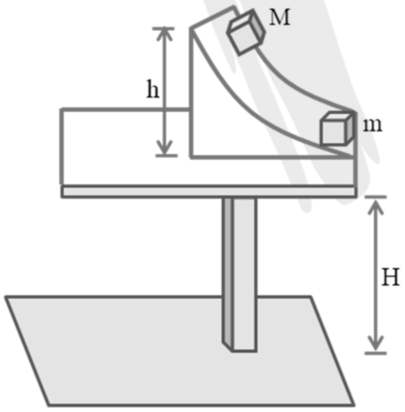
(A) 1 kg (B) 2 kg (C) 3 kg (D) 4 kg

18. Line AB is a diameter of the semi-circular wall shown. The ground is frictionless. A ball thrown along the ground collides the ball and reaches point B. If $\theta = 30°$ and coefficient of restitution is e, find the value of 9e.

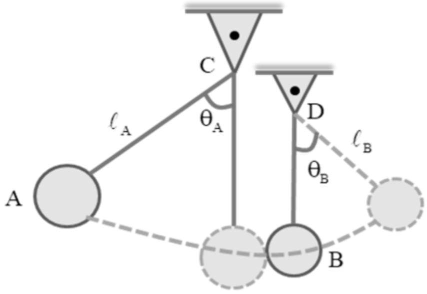


19. A thin metal plate is being bombarded by a perpendicular beam of gas particles from both sides. The mass of these particles is m, their velocity before striking is v_0 and there are n particles/vol in each beam. Each beam has an area A and the collisions are elastic. What force (in N) is needed to move the plate with a constant velocity u? Take $m = 10^{-27}$kg, $n = 10^{26}$perm^3, $A = 1m^2$, $v_0 = 5m/s$, $u = 1m/s$.

20. In a physics lab, a small cube slides down a frictionless incline as shown in figure, it strikes elastically and horizontally a cube that is only one-half of its mass. If the incline is 20 cm high and the table is 90 cm off the floor, big and small cubes strikes the ground at a distance x and y meter respectively from the table. Then $\frac{y}{x}$ is equal to

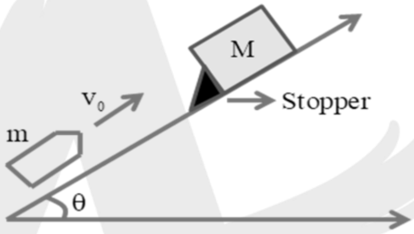


21. In figure, ther are two pendulums with bobs having identical mass. The pendulum A is released from rest in the position shown. If the maximum angle θ_B formed by string BD with the vertical in the subsequent motion of sphere B is to be equal to the angle θ_A, determine the value of the ratio $\frac{\ell_B}{\ell_A}$ of the lengths of the two strings. Coefficient of restitution e between the two spheres is $\frac{1}{2}$. If your answer is $\frac{3N}{32}$. Fill value of N.



22. A block of mass M is kept at rest on a long smooth inclined plane by a stopper. A bullet of mass m travelling at velocity v_0 parallel to inclined plane collides with block at t = 0 and gets embedded. The time elapsed since t = 0 when block again hits the stopper is $\frac{2}{n}$ sec. Find n.

(Given: $m = 5\text{kg}, M = 10\text{kg}; v_0 = 5\text{ms}^{-1}; g = 10\text{ms}^{-2}; \theta = 30°$)



Exercise (ALP)

1. If no external force acts on a system

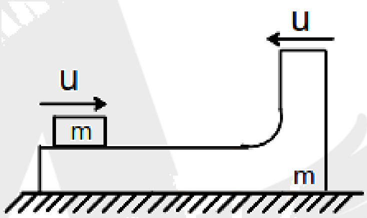
(A) Velocity of centre of mass remains constant

(B) Velocity of centre of mass is not constant

(C) Both velocity and acceleration of centre of mass remains same but not zero

(D) Acceleration of centre of mass is zero

2. A small block of mass m is placed on a smooth wedge of mass 'm'. The combination is placed on a smooth horizontal surface and both blocks are given velocities u towards each other as shown in the diagram. The maximum height to which the small block of mass m rises after breaking off the vertical section of the wedge is H, relative to its initial level



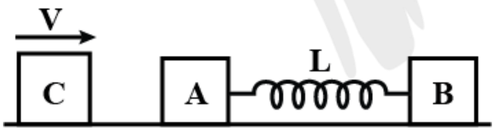
(A) $u = \sqrt{gH}$

(B) Initial momentum of system is zero

(C) The final velocity of the small block at its highest point is zero

(D) The horizontal speed of the system when the smaller block is at it at the highest point of wedge $\frac{2L}{v_{cm}}$

3. Two blocks A and B each of mass m, are connected by a massless spring of natural length L and spring constant k. The blocks are initially resting on a smooth horizontal floor with the spring at its natural length, as shown in figure. A third identical block C, also of mass m, moves on the floor with a speed v along the line joining A and B and collides elastically with A. Then



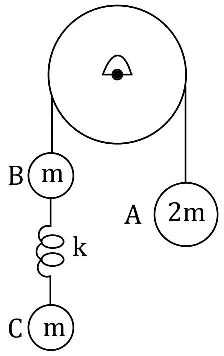
(A) The kinetic energy of the A-B system at maximum compression of the spring is zero

(B) The kinetic energy of a A-B system at maximum compression of the spring is $mv^2 / 4$

(C) The maximum compression of the spring is $v\sqrt{m/k}$

(D) The maximum compression of the spring is $v\sqrt{\frac{m}{2k}}$

4. Particles A and B are connected by a massless, inextensible cord which goes around the massless pulley. If the system is released from rest with the spring initially unstretched. Then



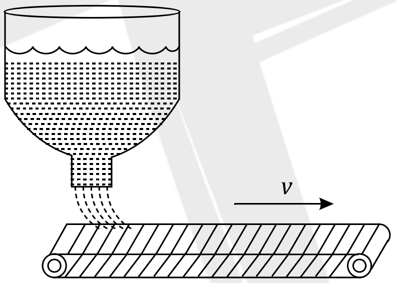
(A) The maximum stretched in the spring as $\frac{2mg}{K}$

(B) The maximum velocity of particle C is $\sqrt{\frac{3m}{4K}}g$

(C) The maximum stretched in the spring is $\frac{mg}{K}$

(D) The maximum velocity of particle C is $\sqrt{\frac{3m}{2K}}g$

5. A hopper drops gravel at a rate of 75 kg/s onto a conveyor belt moving at a constant speed v = 2.20 m/s. Select the correct alternative(s).



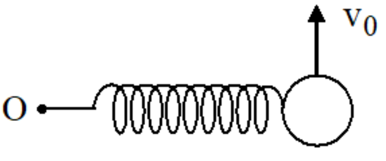
(A) The force needed to keep the conveyor belt moving uniformly is 165 N

(B) The force needed to keep the conveyor belt moving uniformly is 330 N

(C) Output power must motor have in order to drive the conveyor belt is 363 W

(D) The rate at which energy is lost is 181.5 W

6. One end of an ideal spring is fixed at point O and other end is attached to a small disc of mass m which is given an initial velocity v_0 perpendicular to its length on a smooth horizontal surface. If the maximum elongation of the spring is $\frac{l_0}{4}$ then (l_0 = natural length, k = stiffiness of the spring)



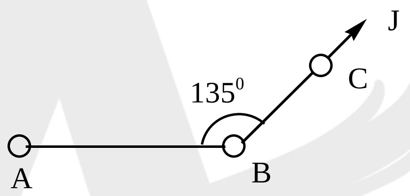
(A) Velocity at maximum elongation is $\frac{4v_0}{5}$

(B) Velocity at maximum elongation is $\frac{3v_0}{5}$

(C) $v_0 = \frac{5l_0}{12}\sqrt{\frac{k}{m}}$

(D) $v_0 = \frac{l_0}{12}\sqrt{\frac{k}{m}}$

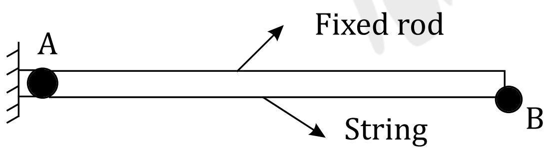
7. Three identical particles A, B and C lie on a smooth horizontal table. Light inextensible strings which are just taut connect AB and BC and is 135^0. An impulse J is applied to the particle C in the direction BC. and the mass of each particle is 'm' then



(A) the initial speed of $A = \frac{\sqrt{2}J}{7m}$ (B) the initial speed $C = \frac{3J}{7m}$

(C) the initial speed of $B = \frac{\sqrt{10}J}{7m}$ (D) the initial speed of B $= \frac{\sqrt{11}J}{7m}$

8. A ball A of mass m = 2kg can slide without friction on a fixed horizontal rod which is led through a diametric hole across the ball. There is another ball B of the same mass 'm' attached to the first ball by a thin thread of length l = 1.6 m. Initially the balls are at rest. The thread is taut and is initially oriented in horizontal direction as shown in figure. Then, the ball B is released with zero initial velocity. At the instant when the string becomes vertical.



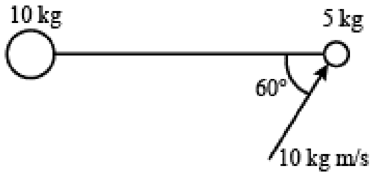
(A) velocity of A is $\sqrt{gl}$ towards right

(B) Acceleration of A is zero

(C) Acceleration of B is g in vertically upward direction

(D) Tension in the string is 2mg

9. Two-point masses are connected by a light inextensible string are lying on a frictionless surface as shown in figure. An impulse of magnitude 10kg/m/s is given to 5 kg block. (as shown)



(A) Speed of 10 kg block immediately after impulse is given $\frac{1}{3}$ m/s

(B) Speed of 10 kg block immediately after impulse is given 2 m/s

(C) Speed of 5 kg block immediately after impulse is given $\sqrt{\frac{28}{9}}$ m/s

(D) Speed of 5 kg block immediately after impulse is given $\frac{2}{3}$ m/s

10. A uniform chain of mass m and length l hangs from a hook in ceiling as shown in figure I. The bottom of link is now raised vertically with the help of upward external force F(y) very slowly as shown in figure II. Choose correct option(s)

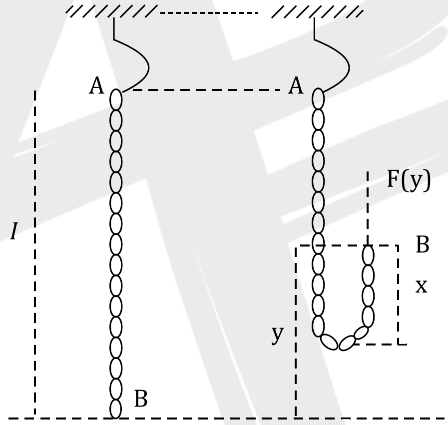


Figure I Figure II

(A) $F(y)=\frac{mgy}{4l}$

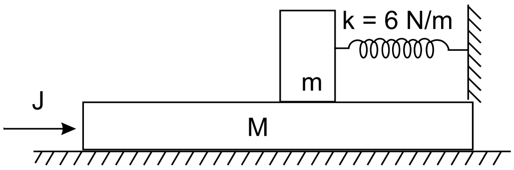
(B) $F(y)=\frac{mgy}{2l}$

(C) Work done by F(y) in raising and B to the position of end A is $\frac{mgl}{4}$

(D) Work done by F(y) in raising end B to the position of end A is $\frac{mgl}{8}$

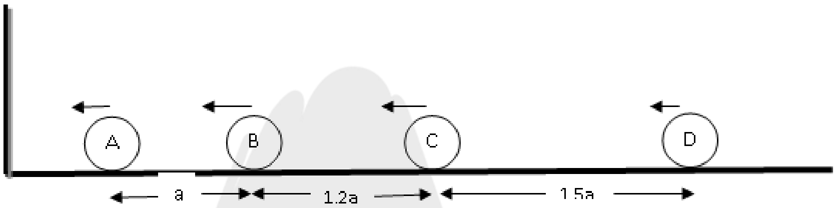
11. A block of mass m=1 kg is placed on a plank of mass M=2 kg, which is placed on a horizontal fritionless surface. There is not friction between the block and the plank. Block and plank are connected by a spring of spring constant 6 N/m as shown in figure. An impulse J=5 newton-sec is applied on the plank.

Possible magnitude of acceleration of the block during the motion is/are

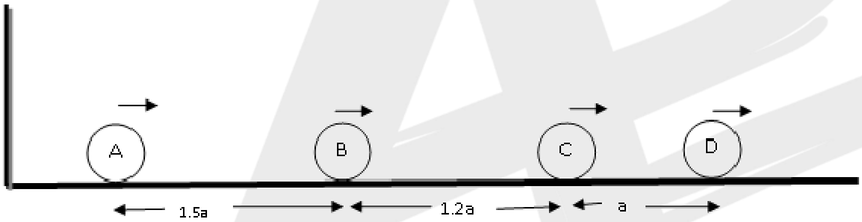


(A) $1\ m/s^2$ (B) $3\ m/s^2$ (C) $5\ m/s^2$ (D) $7\ m/s^2$

12. Four identical spheres with seperation shown in figure are moving on a frictionless floor in aline with a velocity v towards the wall. All collisions are perfectly elastic and head on. From this identify the correct option(s) from the following.(initially A and B are seperated by a, B and C are seperated by 1.2a,C and D are seperated by 1.5a.



(A) After collision at some inistant the position of spheres may be as shown each moving with speed v

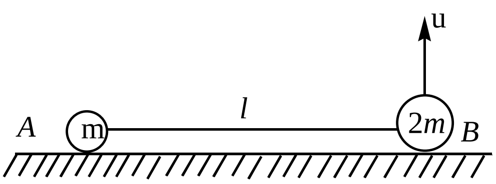


(B) D will make only one collision and C will make two collisions(all possible)

(C) B will make only 5 collisions(all possible)

(D) A will make only 4 collisions(all possible)

13. Two small balls A, B of masses m and 2m respectively are connected by an ideal thread of length l are kept on a smooth horizontal surface as shown. The ball B is given sudden vertical velocity 'u' as shown. Ball B acquires a speed 'v' when thread becomes vertical for the first time. Acceleration due to gravity is g. The value of 'u' is such that ball 'A' is on the verge of lifting off floor when thread is vertical. Mark the **INCORRECT** option(s)



(A) The acceleration of B at the instant when its speed is v is $\frac{3g}{2}$

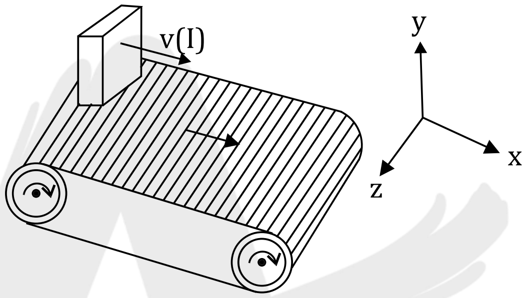
(B) The value of u is $\sqrt{3gl}$

(C) The value of v is $\sqrt{gl}$

(D) The radius of curvature of path of B when its speed is v is $\frac{l}{9}$

PASSAGE

A suitcase of mass M is placed on a level conveyor belt at an airport. The coefficient of static friction between the suitcase and the conveyor belt is μ_1, and the coefficient of kinetic friction is μ_k, with $\mu_k < \mu_1$. The conveyor belt moves with constant speed u. and at time t = 0 the suitcase is placed on the conveyor with speed v = 0. At a time t_f. After moving a distance l. the suitcase catches up with the conveyor belt, and starts to move at speed u. Gravity acts downward with acceleration g > 0. Work can depend on one's frame of reference, so be sure to answer the following two parts in the frame of reference of the airport



14. How much work does friction do on the suitcase during this period?

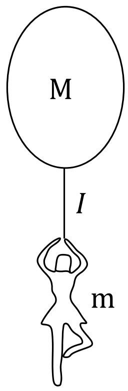
(A) $\frac{1}{2}Mu^2$ (B) $-\frac{1}{2}Mu^2$ (C) Mgl (D) $\mu_k Mgl$

15. How much work does the force of friction from the suitcase do on the belt, during this time period?

(A) $\mu_k Mgl$ (B) $-\mu_k Mgl$ (C) $\mu_k Mgut_f$ (D) $-\mu_k Mgut_f$

PASSAGE

A rope ladder with a length l carrying a woman with a mass m at its end is attached to a balloon with a mass M. The entire system is at rest in equilibrium in the air. As the woman climbs up the ladder to reach balloon, the balloon descends by a height h.



16. The potential energy of the women

(A) decrease by mg (l - h) (B) increase by mg (l- h)

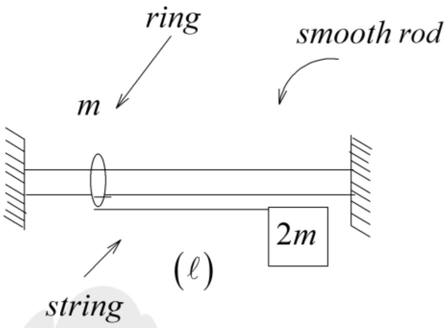
(C) increase by Mg l (D) increase by mg l

17. The ratio of masses of woman and balloon (m/M) is:

(A) $\frac{\ell - h}{h}$ (B) $\frac{h}{\ell - h}$ (C) $\frac{\ell - h}{\ell}$ (D) None of these

PASSAGE

In given figure, the small block of mass 2m is released from rest when the string is in horizontal position.



18. Displacement of the ring when string makes an angle $\theta = 37^0$ with the vertical will be:

(A) $\frac{4l}{15}$ (B) $\frac{l}{15}$ (C) $\frac{2l}{15}$ (D) $\frac{3l}{15}$

19. Maximum possible velocity of ring of mass 'm' is (Assuming zero friction):

(A) $\sqrt{2gl}$ (B) $\sqrt{\frac{4gl}{3}}$ (C) $\sqrt{\frac{8gl}{3}}$ (D) $\sqrt{\frac{2gl}{3}}$

20. Find the tension in the string when the block has maximum velocity.

(A) 12mg (B) 14 mg (C) 8 mg (D) 20mg

PROFICIENCY TEST-I

1. **Statement-1 :** Two particles of mass 1 kg and 3 kg move towards each other under their mutual force of attraction. No other force acts on them. When the relative velocity of approach of the two particles is 2 m/s, their centre of mass has a velocity of 0.5 m/s. When the relative velocity of approach becomes 3 m/s, the velocity of the centre of mass is 0.75 m/s.

Statement-2 : The total kinetic energy as seen from ground is $\frac{1}{2}mv_{rel}^2 + \frac{1}{2}mv_c^2$ and in absence of external force, total energy remains conserved.

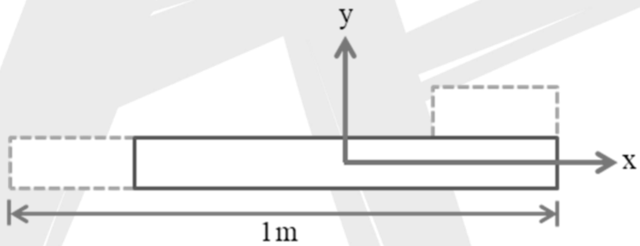
(A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1

(B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1

(C) Statement-1 is true, statement-2 is false

(D) Statement-1 is false, statement-2 is true

2. First 10 cm piece of a uniform meter stick is cut and kept on the other end as shown in figure. Find the shift (in cm) in the x-coordinate of the centre of mass.



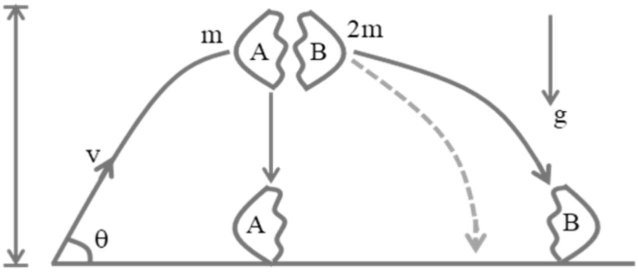
3. Two students were given a physics problem for finding maximum extension of spring if blocks are imparted velocities v_1 and v_2 when spring is unstretched.

By Student A : $\frac{1}{2}m(v_1 + v_2)^2 = \frac{1}{2}kx^2$

By Student B : $\frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2 = \frac{1}{2}kx^2$

(A) Student A is correct, student B is wrong
(B) Student B is correct, student A is wrong
(C) Both are correct
(D) Both are wrong

4. A projectile is launched from the origin with speed v at an angle θ from the horizontal. At the heighest point in the trajectory, the projectile breaks into two pieces, A and B, of mass m and 2m, respectively. Immediately after the breakup piece A is at rest relative to the ground. Neglect air resistance. Which of the following sentences most accurately describes what happens next?



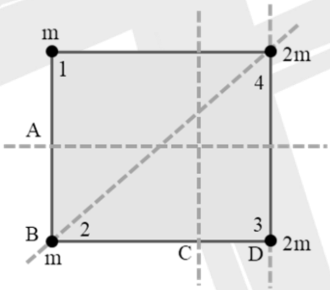
(A) Piece B will hit the ground first, since it is more massive

(B) Both pieces have zero vertical velocity immediately after the breakup, and therefore they hit the ground at the same time

(C) Piece A will hit the ground first, because it will have a downward velocity immediately after the breakup

(D) There is no way of knowing which piece will het he ground first, because not enough information is given about the breakup

5. In the adjacent diagram, objects 1 and 2 each have mass m while objects 3 and 4 each have mass 2m. Note four lines A, B, C and D. The centre of mass of the system is most likely to be at the intersection of lines :

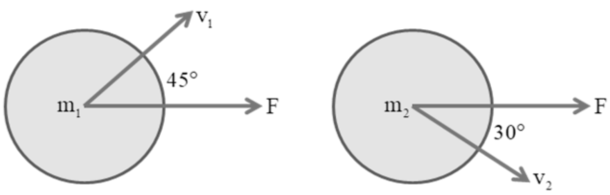


(A) A and B (B) B and C (C) A and D (D) A and C

6. Two astronauts, each of mass 75 kg, are floating next to each other in space, outside the space shuttle. One of them pushes the other through a distance of 1 m (an arm's length) with a force of 300 N. What is the final relative velocity of the two?

(A) $4\sqrt{2}m/s$ (B) $8.0m/s$ (C) $4.0m/s$ (D) $16.0m/s$

7. Two pucks are initially moving along a frictionless surface as shown in the diagram. The pucks have mass $m_1 < m_2$ and begin with equal magnitude of momentum. A constant force F is applied to each puck directly to the right for the same amount of non-zero time. After the pushes are complete, what is the relationship for the size of the momenta of pucks ($p_1 and p_2$)?



(A) $p_1 < p_2$

(B) $p_1 = p_2$

(C) $p_1 > p_2$

(D) More information about the masses, speeds, force and time are required to answer the questions

8. Your physical education teacher throws a ping-pong ball to you at a certain speed and you catch it. The teacher is next going to throw you a basket ball whose mass is ten times the mass of the ping-pong ball. You are given the following choices. You can have the basket ball thrown with :

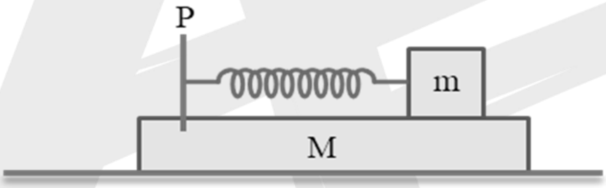
[P] The same speed as the ping-pong ball,

[Q] The same momentum, or

[R] The same kinetic energy. Rank these choices from easiest to hardest to catch i.e. in order of impulse needed to apply.

(A) P, Q, R (B) R, Q, P (C) Q, R, P (D) All 3 are equally easy

9. Which of the following may effect the motion of centre of mass of the two block + spring system during their motion :



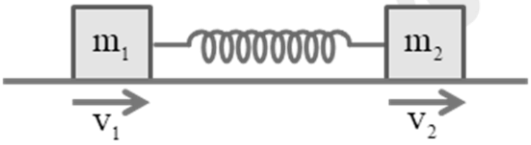
(A) Extension in spring

(B) Friction coefficient between the two blocks

(C) Friction coefficient between M and ground

(D) Force applied by the spring on the peg P

10. Two blocks are connected by a spring and given velocity v_1 and v_2 as shown in figure when spring is unstretched.



Statement-1 : In centre of mass frame, both the blocks come to rest simultaneously.

Statement-2 : Momentum of a system in centre of mass frame is always zero.

(A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1

(B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1

(C) Statement-1 is true, statement-2 is false

(D) Statement-1 is false, statement-2 is true

11. A particle of mass m makes a head-on elastic collision with a particle of mass 2m initially at rest. The velocity of the first particle before and after collision is given to be u_1 and v_1. Then which of the following statements is true in respect of this collision?

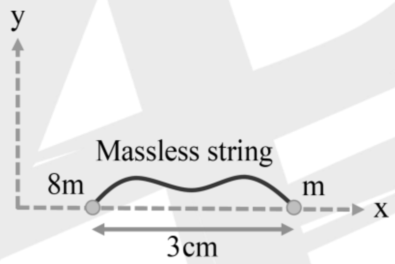
(A) For all values of u_1, v_1 will always be less than u_1 in magnitude and $|v_1| = \frac{u_1}{3}$

(B) The fractional loss in kinetic energy of the first particle is $\frac{8}{9}$

(C) The fractional loss in kinetic energy of the first particle is $\frac{5}{9}$

(D) The sum of kinetic energy of the particles remains constant at every instant during the collision.

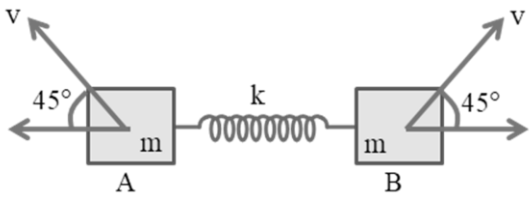
12. In gravity free space two particles of masses 8m and m separated by 3cm are at rest. The particles are connected by a massless string of length 5 cm. If the particle of mass m is imparted velocity v along y-axis, find the velocity of the particle of mass 8m, just after the string becomes taut, is



(A) $\frac{v}{45}$ (B) $\frac{3v}{45}$ (C) $\frac{2v}{45}$ (D) $\frac{4v}{45}$

PROFICIENCY TEST-II

1. Blocks A and B of mass m each are connected with spring of constant k. Both blocks lie on frictionless ground and are imparted horizontal velocity v as shown when spring is unstretched. Find the maximum stretch of spring.



(A) $v\sqrt{\frac{m}{k}}$ (B) $v\sqrt{\frac{m}{2k}}$ (C) $v\sqrt{\frac{2m}{k}}$ (D) None of these

2. A stationary rocket of total initial mass m_0 is expelling exhaust gas at a steady speed v_e with respect to itself. There is no external force on rocket. When its mass is reduced to $\frac{m_0}{2}$, the velocity attained by the rocket will be :

(A) $2v_e$ (B) v_e (C) $0.69v_e$ (D) $0.5v_e$

3. Seven identical birds are flying south together at constant velocity. A hunter shoots one of them, which immediately dies and falls to the ground. The other six continue flying south at the original velocity. After the one bird has hit the ground, the centre of mass of all seven birds :

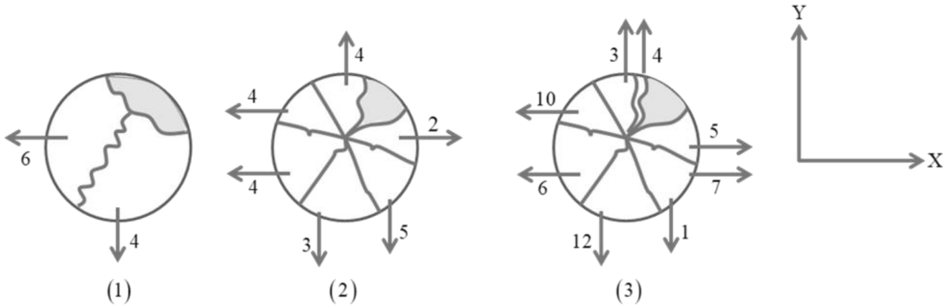
(A) continues south at the original speed, but is now located some distance behind the flying birds

(B) continues south, but a $\frac{6}{7}$ the original velocity

(C) continues south, but at $\frac{1}{7}$ the original velocity

(D) stops with the dead bird

4. Figure shows top views of three, two-dimensional explosions in which a stationary firecracker is blown into three pieces, seven pieces, and nine pieces. The pieces then slide over a frictionless floor. For each situation, figure also shows the directions and magnitudes of (linear) momentum vectors of all except one piece which is darkened in the figures; that piece has momentum $\vec{P}'$. The directions of momenta shown are either parallel to x or y axis. The numberrs next to the vectors are the magnitudes of the momentum (in kilogram-meters per second). Take coordinate axes x and y as shown. The magnitude of momentum $\vec{P}'$ is :



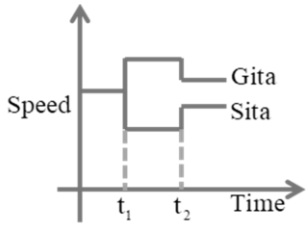
(A) Maximum for situation (1) (B) Maximum for situation (2)

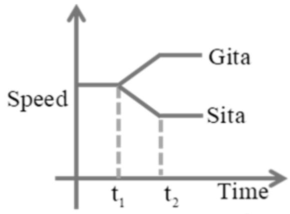
(C) Maximum for situation (3) (D) Same for all situations

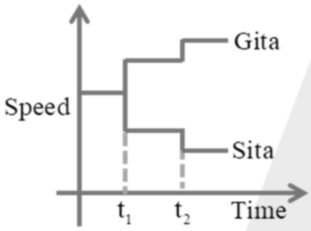
5. At t = 0, a ball is dropped from rest, it bounces from the floor repeatedly. The coefficient of restitution is 0.5 and the speed just before hitting the ground for the first time is 5 m/s. The value of t at which the ball stops bouncing is :

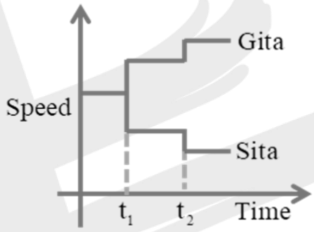
(A) 2 s (B) 1 s (C) 0.5 s (D) None of these

6. Sita and Gita are skating on an ice ring so that they are moving with the same velocity in the same straight line. Sita is skating backwards with her face towards Gita. Sita throws a ball to Gita at time t_1 and receives it back at t_2. Consider negligible time of flight and throw of the ball and equal masses of the skaters, the correct speed-time graph for the girls is :

(A) 

(B) 

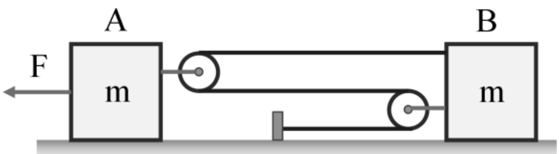
(C) 

(D) 

7. In the situation shown below, block A, having mass m, and block B, having mass 3m, are held at rest on a horizontal, frictionless surface with a massless spring compressed between them. After the blocks are released from rest, what is the ratio of the magnitude of B's momentum to the magnitude of A's momentum?

(A) $\frac{9}{1}$ (B) $\frac{3}{1}$ (C) $\frac{1}{1}$ (D) $\frac{1}{3}$

8. Two blocks A and B of equal mass are connected by a light inextensible taut string passing over two light smooth pulleys fixed to the blocks. The parts of the string not in contact with the pulleys are horizontal. A horizontal force F is applied to the block A as shown. There is no friction, then:



(A) the acceleration of A will be more than that of B

(B) the acceleration of A will be less than that of B

(C) the sum of rate of changes of momentum of A and B is greater than the magnitude of F

(D) the sum of rate of changes of momentum of A and B is equal to the magnitude of F

9. Object A strikes the stationary object B with a certain given speed u head-on in an elastic collision. The mass of A is fixed, you may only choose the mass of B appropriately for following cases. Then after the collision:

(A) for B to have the greatest speed, choose $m_B = m_A$

(B) for B to have the greatest momentum, choose $m_B < m_A$

(C) for B to have the greatest speed, choose $m_B < m_A$

(D) for B to have the greatest kinetic energy, choose $m_B = m_A$

10. A particle of mass m_1 = 4 kg moving at $6\hat{i}$ m/s collides perfectly elastically with a particle of mass $m_2 = 2$kg moving at $3\hat{i}$m/s.

(A) Velocity of centre of mass (CM) is $5\hat{i}$m/s

(B) The velocities of the particles relative to the centre of mass have same magnitude.

(C) Speed of individual particle before and after collision remains same.

(D) The velocity of particles relative to CM after collision are $\vec{v}_{1f} = -\hat{i}\text{m/s}, \vec{v}_{2f} = 2\hat{i}\text{m/s}$

11. A ball A moving with a velocity 5 m/s collides elastically with another identical ball B at rest such that the velocity of A makes an angle of 30° with the line joining the centres of the balls just before collision. Then:

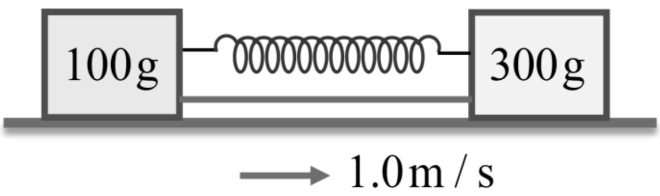
(A) speed of A after collision is $\left(\frac{5}{2}\right)$ m/s

(B) speed of B after collision is $\frac{5\sqrt{3}}{2}\,m/s$

(C) balls A and B move at right angles after collision

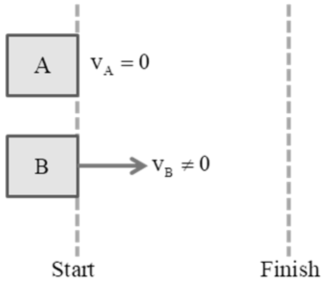
(D) kinetic energy is not conserved as the collision is not head-on

12. The carts in figure are sliding to the right at 1.0 m/s on a smooth level ground. The spring between them has a spring constant of 120 N/m and is compressed 40 cm. The carts slide part a flame that burns through the string holding them together. The carts are not attached to the spring. Afterward, what is the speed (in m/s) of 300 g cart after it loses contact with the spring.



PROFICIENCY TEST-III

1. Identical constant forces push two identical objects A and B continuously from a starting line to a finish line. If A is initially at rest and B is initially moving to the right.



(A) Object A has the larger change in momentum

(B) Object B has the larger change in momentum

(C) Both objects have the same change in momentum

(D) Not enough information is given to decide

2. A cart initially moving on a frictionless track filled with sand has a hole at its bottom from where sand leaks out.

Student-A says : The velocity of cart remains constant although mass of sand in cart decrease.

Student-B says : Since mass is decreasing, the velocity of cart must increase.

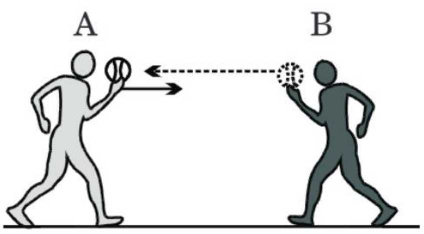
(A) Student-A is correct Student-B is wrong

(B) Student-A is wrong Student-B is correct

(C) Both are correct

(D) Both are wrong

3. Consider two skaters A and B initially at rest on ice (friction is negligible) with A holding a ball. A has greater mass than B and the ball has some significant mass. A throws the ball to B. B catches it and throws it back to A who catches it again. The magnitudes of the skater's (excluding ball) final velocities, momentum and kinetic energies (denoted below as v, p and K respectively) are related as:



(A) $v_A = v_B$ (B) $v_A < v_B, p_A < p_B$

(C) $p_A = p_B, K_A < K_B$ (D) $p_A < p_B, K_A < K_B$

4. Two colliding particles of masses m_1 and m_2 moving with v_1 and v_2 collide head on resulting into (where $\mu^{-1} = m_1^{-1} + m_2^{-1}$ and e is the coefficient of restitution)

(A) loss of kinetic energy of $\frac{1}{2}\mu(v_1 - v_2)^2(1-e)^2$

(B) loss of kinetic energy $\frac{1}{2}\mu(v_1 - v_2)^2(1-e^2)$

(C) momentum transfer from the first particle to the second of $\mu(1+e)(v_1 - v_2)$

(D) momentum transfer from the first particle to the second of $\mu(1+e)(v_1 + v_2)$

5. A particle A suffers an oblique elastic collision with a particle B that is at rest initially. If their masses are the same, then, after the collision:

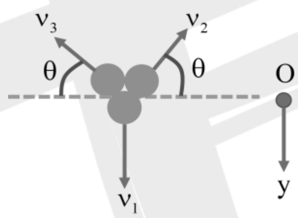
(A) B moves along line of impact

(B) velocities of A and B will be interchanged

(C) they will move in mutually perpendicular directions

(D) they must move in symmetric directions with respect to the initial line of motion of A

6. A rocket is projected straight up and explodes into three equally massive fragments just as it reaches the top of its flight (refer figure). One of the fragments is observed to come straight down in 2 sec, while the other two take 4sec to come to ground, after the burst. Find the height h at which the fragmentation occurred.

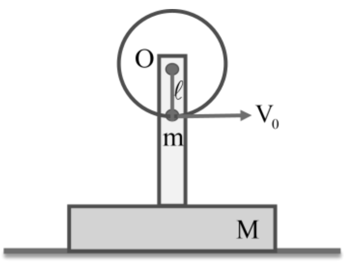


(A) 12.5 m (B) 25 m (C) 37.5 m (D) 50 m

7. Two small boats both having a mass of 150 kg including passengers in it are at rest. A sack of mass 50 kg makes 1st boat having total mass of 200 kg. It is thrown to the second boat with a velocity whose horizontal component is $2m/s$, relative to water. Calculate the distance between the boat 4.5 sec. After the throw if the sack spent 0.5 sec. in air. Neglect resistance of air and water.

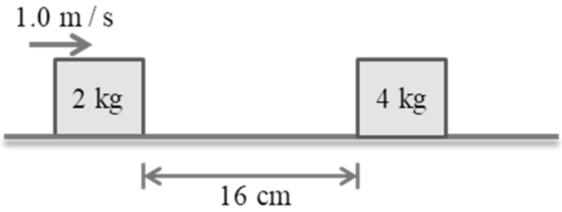
(A) 2m (B) 4m (C) 6m (D) 8m

8. One end of a pendulum is attached to point O on the wedge of mass M = 4m. Mass of the pendulum is m and its length is ℓ. Initially system is at rest on a smooth floor, now velocity $v_0 = 12m/s$ is given to the pendulum in horizontal direction. v_0 is sufficient for the bob to move in circle with respect to O. If at highest point of the trajectory speed of bob is equal to speed of wedge then find this speed.

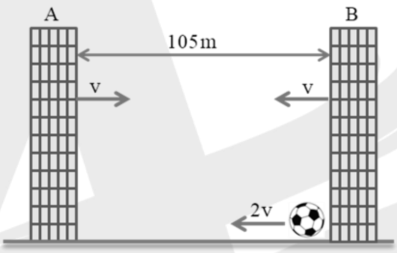


(A) 2 m/s (B) 4 m/s (C) 6 m/s (D) 8 m/s

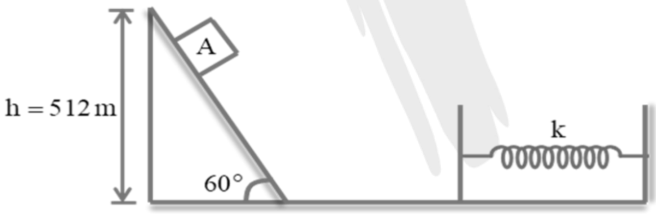
9. The friction coefficient between the horizontal surface and each of the block shown in the figure is 0.2. The collision between the blocks is perfectly elastic. Find the separation (in cm) between them when they come to rest.



10. Initially there is a small ball moving with speed $2v$ towards wall A and both walls are moving with constant velocity towards each other as shown in figure. Then find speed v (in m/s) if time taken by the ball in first three collision is 5 sec. (Assume all collision are perfectly elastic and friction is absent).



11. A block A is released from the top of a fixed wedge at height h = 512 m. All surfaces are frictionless. A collides with spring elastically while moving from inclined to plane surface or vice-versa and A never looses contact with surface. Collision between block and horizontal surface/incline plane are perfectly inelastic. Find the height (in meter) attained by block when it climbs the wedge second time.



ANSWER KEY

EXCERCISE-I_KEY

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
B	B	6	C	D	C	D	B	B	B
11	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
B	D	1	A	D	D	C	8	D	D
21	**22**								
AC	ABC								

EXCERCISE-II_KEY

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
10	C	C	C	B	A	D	D	C	C
11	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
D	A	D	A	C	AC	AC	AC	CD	ACD
21									
1									

EXCERCISE-III_KEY

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
C	D	100	B	B	35	A	C	ABCD	ACD
11	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
D	ABC	BC	AB	BC	BD	AC	C	D	32

EXCERCISE-IV_KEY

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
B	D	B	C	A	C	B	A	C	B
11	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
B	C	A	A	AB	ABCD	ABCD	ABC	AB	A
21	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
B	A	3	C	C	A	A	4	3	2
31									
17									

EXCERCISE-V_KEY

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
A	D	ABD	AD	ABD	AD	ABC	AD	AB	B
11	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
D	B	D	D	6	9	C	3	4	4
21	**22**								
6	3								

Exercise (ALP)

1	2	3	4	5	6	7	8	9	10
A,D	A,B,C	B,D	A,B	A,C,D	A,C	A,B,C	A,B	A,C	B,C
11	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
A,B,C	A,C	B,C	A	D	B	B	A	C	B

PROFICIENCY TEST-I_KEY

1	2	3	4	5	6	7	8	9	10
D	9	D	B	D	C	A	C	C	A
11	**12**								
AB	D								

PROFICIENCY TEST-II_KEY

1	2	3	4	5	6	7	8	9	10
A	C	B	D	D	D	C	AC	CD	ACD
11	**12**								
ABC	5								

PROFICIENCY TEST-III_KEY

1	2	3	4	5	6	7	8	9	10
A	A	BD	BC	AC	D	C	B	5	9
11									
2									