EXERCISE-01

SECTION - A

BASIC DEFINITION OF CIRCLE

1. If a be the radius of a circle which touches x-axis at the origin, then its equation is

(A) $x^2 + y^2 + ax = 0$

(B) $x^2 + y^2 \pm 2ya = 0$

(C) $x^2 + y^2 \pm 2xa = 0$

(D) $x^2 + y^2 + ya = 0$

2. The equation of the circle passing through $(3,6)$ and whose centre is $(2,-1)$ is

(A) $x^2 + y^2 - 4x + 2y = 45$

(B) $x^2 + y^2 - 4x - 2y + 45 = 0$

(C) $x^2 + y^2 + 4x - 2y = 45$

(D) $x^2 + y^2 - 4x + 2y + 45 = 0$

3. The equation of a circle which passes through the three points $(3,0)(1,-6),(4,-1)$ is

(A) $2x^2 + 2y^2 + 5x - 11y + 3 = 0$

(B) $x^2 + y^2 - 5x + 11y - 3 = 0$

(C) $x^2 + y^2 + 5x - 11y + 3 = 0$

(D) $2x^2 + 2y^2 - 5x + 11y - 3 = 0$

4. B and C are fixed point having co-ordinates $(3,0)$ and $(-3,0)$ respectively. If the vertical angle BAC is $90°$, then the locus of the centroid of the $\triangle$ ABC has the equation

(A) $x^2 + y^2 = 1$

(B) $x^2 + y^2 = 2$

(C) $9(x^2 + y^2) = 1$

(D) $9(x^2 + y^2) = 4$

5. The area of an equilateral triangle inscribed in the circle $x^2 + y^2 - 2x = 0$ is

(A) $\frac{3\sqrt{3}}{2}$

(B) $\frac{3\sqrt{3}}{4}$

(C) $\frac{3\sqrt{3}}{8}$

(D) none

6. A circle is drawn touching the x-axis and centre at the point which is the reflection of (a, b) in the line $y - x = 0$. The equation of the circle is

(A) $x^2 + y^2 - 2bx - 2ay + a^2 = 0$

(B) $x^2 + y^2 - 2bx - 2ay + b^2 = 0$

(C) $x^2 + y^2 - 2ax - 2by + b^2 = 0$

(D) $x^2 + y^2 - 2ax - 2by + a^2 = 0$

7. $y = \sqrt{3}x + c_1$ & $y = \sqrt{3}x + c_2$ are two parallel tangents of a circle of radius 2 units, then $|c_1 - c_2|$ is equal to

(A) 8

(B) 4

(C) 2

(D) 1

SECTION - B

DIAMETRIC FORM OF CIRCLE

8. The length of intercept on y-axis, by a circle whose diameter is the line joining the points $(-4,3)$ and $(12,-1)$ is

(A) $3\sqrt{2}$

(B) $\sqrt{13}$

(C) $4\sqrt{13}$

(D) none of these

9. If $(6, -3)$ is the one extremity of diameter to the circle $x^2 + y^2 - 3x + 8y - 3 = 0$ then its other extremity is -

(A) $(3/2, -4)$

(B) $(-3, -5)$

(C) $(3, -5)$

(D) $(3,5)$

10. If $y = 2x + K$ is a diameter to the circle $2(x^2 + y^2) + 3x + 4y - 1 = 0$, then K equals

(A) 0

(B) 1

(C) 2

(D) ½

SECTION – C

INTERCEPTS OF A CIRCLE

11. A circle touches x - axis at +3 distance and cuts an intercept of 8 in +ve direction of y-axis. Its equation is -

(A) $x^2+y^2+6x+10y-9=0$

(B) $x^2+y^2-6x-10y-9=0$

(C) $x^2+y^2-6x-10y+9=0$

(D) $x^2+y^2+6x+10y+9=0$

12. The gradient of the tangent line at the point ($a\cos\alpha, a\sin\alpha$) to the circle $x^2+y^2=a^2$, is

(A) $\tan(\pi-\alpha)$

(B) $\tan\alpha$

(C) $\cot\alpha$

(D) $-\cot\alpha$

13. $\ell x+my+n=0$ is a tangent line to the circle $x^2+y^2=r^2$, if

(A) $\ell^2+m^2=n^2r^2$

(B) $\ell^2+m^2=n^2+r^2$

(C) $n^2=r^2(\ell^2+m^2)$

(D) none of these

14. If $y=c$ is a tangent to the circle $x^2+y^2-2x+2y-2=0$ at (1,1), then the value of c is

(A) 1

(B) 2

(C) -1

(D) -2

SECTION-D

POSITIONOFA POINTW.R.T.CIRCLE

15. Line $3x+4y=25$ touches the circle $x^2+y^2=25$ at the point

(A) (4,3)

(B) (3,4)

(C) $(-3,-4)$

(D) none of these

16. The greatest distance of the point $P(10,7)$ from the circle $x^2+y^2-4x-2y-20=0$ is

(A) 5

(B) 15

(C) 10

(D) none of these

SECTION – E

PARAMETRIC FORM OF A CIRCLE

17. The parametric coordinates of any point on the circle $x^2+y^2-4x-4y=0$ are

(A) $(-2+2\cos\alpha, -2+2\sin\alpha)$

(B) $(2+2\cos\alpha, 2+2\sin\alpha)$

(C) $(2+2\sqrt{2}\cos\alpha, 2+2\sqrt{2}\sin\alpha)$

(D) none of these

18. Cartesian equations of a circle whose parametric equation are $x=-7+4\cos q, y=3+4\sin q$ is -

(A) $(x+7)^2+(y-3)^2=16$

(B) $(x-7)^2+(y-3)^2=16$

(C) $(x-7)^2+(y+3)^2=16$

(D) $(x+7)^2+(y+3)^2=16$

SECTION - F : TANGENT AND NORMAL

19. The equations of the tangents drawn from the point $(0,1)$ to the circle $x^2+y^2-2x+4y=0$ are

(A) $2x-y+1=0, x+2y-2=0$

(B) $2x-y-1=0, x+2y-2=0$

(C) $2x-y+1=0, x+2y+2=0$

(D) $2x-y-1=0, x+2y+2=0$

20. The equation of the normal to the circle $x^2+y^2=9$ at the point $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ is

(A) $x-y=\frac{\sqrt{2}}{3}$

(B) $x+y=0$

(C) $x-y=0$

(D) none of these

21. The length of the tangent drawn from the point (2,3) to the circles $2(x^2+y^2)-7x+9y-11=0$.

(A) 18

(B) 14

(C) $\sqrt{14}$

(D) $\sqrt{28}$

22. The angle between the two tangents from the origin to the circle $(x-7)^2+(y+1)^2=25$ equals

(A) $\frac{\pi}{2}$

(B) $\frac{\pi}{3}$

(C) $\frac{\pi}{4}$

(D) none

23. The point from which the tangents to the circles $x^2+y^2-8x+40=0, 5x^2+5y^2-25x+80=0$ $x^2+y^2-8x+16y+160=0$ are equal in length is

(A) $\left(8,\frac{15}{2}\right)$

(B) $\left(-8,\frac{15}{2}\right)$

(C) $\left(8,-\frac{15}{2}\right)$

(D) none of these

24. The tangent from the point of intersection of the lines $2x-3y+1=0$ and $3x-2y-1=0$ to the circle $x^2+y^2+2x-4y=0$ is

(A) $x+2y=0, x-2y+1=0$

(B) $2x-y-1=0$

(C) $y=x, y=3x-2$

(D) $2x+y+1=0$

25. The equation of the circle having the lines $y^2-2y+4x-2xy=0$ as its normals & passing through the point (2,1) is

(A) $x^2+y^2-2x-4y+3=0$

(B) $x^2+y^2-2x+4y-5=0$

(C) $x^2+y^2+2x+4y-13=0$

(D) none

SECTION - G : DIRECTOR CIRCLE

26. The equation of director circle to the circle $x^2 + y^2 = 8$ is-

(A) $x^2 + y^2 = 8$

(B) $x^2 + y^2 = 16$

(C) $x^2 + y^2 = 4$

(D) $x^2 + y^2 = 12$

27. Two perpendicular tangents to the circle $x^2 + y^2 = a^2$ meet at P. Then the locus of P has the equation-

(A) $x^2 + y^2 = 2a^2$

(B) $x^2 + y^2 = 3a^2$

(C) $x^2 = y^2 = 4a^2$

(D) None of these

SECTION – H

CHORD WITH A GIVEN MIDDLE POINT

28. The locus of the mid-points of the chords of the circle $x^2 + y^2 - 2x - 4y - 11 = 0$ which subtend 60° at the centre is

(A) $x^2 + y^2 - 4x - 2y - 7 = 0$

(B) $x^2 + y^2 + 4x + 2y - 7 = 0$

(C) $x^2 + y^2 - 2x - 4y - 7 = 0$

(D) $x^2 + y^2 + 2x + 4y + 7 = 0$

29. Find the locus of mid point of chords of circle $x^2 + y^2 = 25$ which subtends right angle at origin-

(A) $x^2 + y^2 = 25/4$

(B) $x^2 + y^2 = 5$

(C) $x^2 + y^2 = 25/2$

(D) $x^2 + y^2 = 5/2$

30. The equation to the chord of the circle $x^2 + y^2 = 16$ which is bisected at $(2, -1)$ is-

(A) $2x + y = 16$

(B) $2x - y = 16$

(C) $x + 2y = 5$

(D) $2x - y = 5$

31. The locus of the centres of the circles such that the point (2,3) is the mid point of the chord $5x + 2y = 16$ is

(A) $2x - 5y + 11 = 0$

(B) $2x + 5y - 11 = 0$

(C) $2x + 5y + 11 = 0$

(D) none

SECTION - I : CHORD OF CONTACT

32. Tangents are drawn from (4,4) to the circle $x^2 + y^2 - 2x - 2y - 7 = 0$ to meet the circle at A and B. The length of the chord AB is

(A) $2\sqrt{3}$

(B) $3\sqrt{2}$

(C) $2\sqrt{6}$

(D) $6\sqrt{2}$

33. Pair of tangents are drawn from every point on the line $3x + 4y = 12$ on the circle $x^2 + y^2 = 4$. Their variable chord of contact always passes through a fixed point whose co-ordinates are

(A) $\left(\frac{4}{3}, \frac{3}{4}\right)$

(B) $\left(\frac{3}{4}, \frac{3}{4}\right)$

(C) (1,1)

(D) $\left(1, \frac{4}{3}\right)$

SECTION - J : PAIR OF TANGENTS

34. The equation of pair of tangents drawn from the point (0,1) to the circle $x^2 + y^2 - 2x + 4y = 0$ is-

(A) $4x^2 - 4y^2 + 6xy + 6x + 8y - 4 = 0$

(B) $4x^2 - 4y^2 + 6xy - 6x + 8y - 4 = 0$

(C) $x^2 - y^2 + 3xy - 3x + 2y - 1 = 0$

(D) $x^2 - y^2 + 6xy - 6x + 8y - 4 = 0$

35. From the point P(16,7) tangents PQ and PR are drawn to the circle $x^2 + y^2 - 2x - 4y - 20 = 0$. If C be the centre of the circle then area of the quadriteral PQCR is-

(A) 450 sq. units

(B) 15 sq. units

(C) 50 sq. units

(D) 75 sq. units

SECTION - K : FAMILY OF CIRCLE

36. Equation of the circle touching the circle $x^2 + y^2 - 15x + 5y = 0$ at the point (1,2) and passing through the point (0,2) is

(A) $13x^2 + 13y^2 - 13x - 61y + 70 = 0$

(B) $x^2 + y^2 + 2x = 0$

(C) $13x^2 + 13y^2 - 13x - 61y + 9 = 0$

(D) none of these

SECTION – L

NUMBER OF COMMON TANGENTS AND POSITION OF TWO CIRCLE

37. The number of common tangents of the circles $x^2 + y^2 - 2x - 1 = 0$ and $x^2 + y^2 - 2y - 7 = 0$

(A) 1

(B) 3

(C) 2

(D) 4

38. If the circle $x^2 + y^2 = 9$ touches the circle $x^2 + y^2 + 6y + c = 0$, then c is equal to

(A) -27

(B) 36

(C) -36

(D) 27

SECTION - M : RADICAL AXIS

39. If the circumference of the circle $x^2 + y^2 + 8x + 8y - b = 0$ is bisected by the circle $x^2 + y^2 - 2x + 4y + a = 0$, then $a + b =$

(A) 50

(B) 56

(C) -56

(D) -34

40. The distance of the centre of the circle $x^2 + y^2 = 2x$ from the common chord of the circles $x^2 + y^2 + 5x - 8y + 1 = 0$ and $x^2 + y^2 - 3x + 7y - 25 = 0$ is

(A) 1

(B) 3

(C) 2

(D) $\frac{1}{3}$

SECTION - N

RADICAL CENTRE

41. The equation of three circles are given $x^2 + y^2 = 1$,

$x^2 + y^2 - 8x + 15 = 0, x^2 + y^2 + 10y + 24 = 0$. Determine the coordinates of the point P such that the tangents drawn from it to the circles are equal in length.

(A) $(2, -5/2)$

(B) $(-2, -5/2)$

(C) $(2, 5/2)$

(D) $(3, -5/3)$

SECTION - O

ORTHOGONALITY OF TWO CIRCLES

42. The locus of the centers of the circles which cut the circles $x^2 + y^2 + 4x - 6y + 9 = 0$ and $x^2 + y^2 - 5x + 4y - 2 = 0$ orthogonally is

(A) $9x + 10y - 7 = 0$

(B) $x - y + 2 = 0$

(C) $9x - 10y + 11 = 0$

(D) $9x + 10y + 7 = 0$

43. Two given circles $x^2 + y^2 + ax + by + c = 0$ and $x^2 + y^2 + dx + ey + f = 0$ will intersect each other orthogonally, only when-

(A) $ad + be = c + f$

(B) $a + b + c = d + e + f$

(C) $ad + be = 2c + 2f$

(D) $2ad + 2be = c + f$

44. If the circles of same radius a and centres at (2,3) and (5,6) cut orthogonally, then a is equal to-

(A) 6

(B) 4

(C) 3

(D) 10

SECTION – P

MIXED PROBLEMS

45. If $a^2 + b^2 = 1,\ m^2 + n^2 = 1$, then

(A) $| am + bn | \leq 1$

(B) $| am - bn | \geq 1$

(C) $| am + bn | \geq 1$

(D) none of these

46. Tangents are drawn to the circle $x^2 + y^2 = 1$ at the points where it is met by the circles. $x^2 + y^2 - (\lambda + 6)x + (8 - 2\lambda)y - 3 = 0, \lambda$ being the variable. The locus of the point of intersection of these tangents is

(A) $2x - y + 10 = 0$

(B) $x + 2y - 10 = 0$

(C) $x - 2y + 10 = 0$

(D) $2x + y - 10 = 0$

47. AB is a diameter of a circle. CD is a chord parallel to AB and $2CD = AB$. The tangent at B meets the line AC produced at E then AE is equal to

(A) AB

(B) $\sqrt{2}AB$

(C) $2\sqrt{2}AB$

(D) 2AB

48. (6,0), (0,6) and (7,7) are the vertices of a triangle. The circle inscribed in the triangle has the equation

(A) $x^2 + y^2 - 9x + 9y + 36 = 0$

(B) $x^2 + y^2 - 9x - 9y + 36 = 0$

(C) $x^2 + y^2 + 9x - 9y + 36 = 0$

(D) $x^2 + y^2 - 9x - 9y - 36 = 0$

EXERCISE-02(LEVEL-I)

BASIC DEFINITION OF CIRCLE

1. The equation of the circle which touches the axis of y at the origin and passes through (3,4) is

(A) $4(x^2+y^2)-25x=0$

(B) $3(x^2+y^2)-25x=0$

(C) $2(x^2+y^2)-3x=0$

(D) $4(x^2+y^2)-25x+10=0$

2. The equation to the circle whose radius is 4 and which touches the negative x-axis at a distance 3 units from the origin is

(A) $x^2+y^2-6x+8y-9=0$

(B) $x^2+y^2\pm 6x-8y+9=0$

(C) $x^2+y^2+6x+8y+9=0$

(D) $x^2+y^2\pm 6x-8y-9=0$

3. Number of different circles that can be drawn touching 3 lines, no two of which are parallel and they are neither coincident nor concurrent, are

(A) 1

(B) 2

(C) 3

(D) 4

4. If a circle of constant radius 3k passes through the origin ' O ' and meets co-ordinate axes at A and B then the locus of the centroid of the triangle OAB is

(A) $x^2+y^2=(2k)^2$

(B) $x^2+y^2=(3k)^2$

(C) $x^2+y^2=(4k)^2$

(D) $x^2+y^2=(6k)^2$

5. The circle passing through the distinct points $(1,t),(t,1)\&(t,t)$ for all values of ' t '. passes through the point

(A) $(-1,-1)$

(B) $(-1,1)$

(C) $(1,-1)$

(D) $(1,1)$

6. The lines $2x - 3y = 5$ and $3x - 4y = 7$ are diameters of a circle having area as 154 sq. units. Then the equation of the circle is

(A) $x^2 + y^2 - 2x + 2y = 62$

(B) $x^2 + y^2 + 2x - 2y = 62$

(C) $x^2 + y^2 + 2x - 2y = 47$

(D) $x^2 + y^2 - 2x + 2y = 47$

7. If the lines $2x + 3y + 1 = 0$ and $3x - y - 4 = 0$ lie along diameters of a circle of circumference 10π, then the equation of the circle is -

(A) $x^2 + y^2 - 2x + 2y - 23 = 0$

(B) $x^2 + y^2 - 2x - 2y - 23 = 0$

(C) $x^2 + y^2 + 2x + 2y - 23 = 0$

(D) $x^2 + y^2 + 2x - 2y - 23 = 0$

8. If the pair of line $ax^2 + 2(a + b)xy + by^2 = 0$ lie along diameters of a circle and divide the circle into four sectors such that the area of one of the sectors is thrice the area of another sector then -

(A) $3a^2 - 10ab + 3b^2 = 0$

(B) $3a^2 - 2ab + 3b^2 = 0$

(C) $3a^2 + 10ab + 3b^2 = 0$

(D) $3a^2 + 2ab + 3b^2 = 0$

9. If the lines $3x - 4y - 7 = 0$ and $2x - 3y - 5 = 0$ are two diameters of a circle of area 49p square units, the equation of the circle is -

(A) $x^2 + y^2 + 2x - 2y - 62 = 0$

(B) $x^2 + y^2 - 2x + 2y - 62 = 0$

(C) $x^2 + y^2 - 2x + 2y - 47 = 0$

(D) $x^2 + y^2 + 2x - 2y - 47 = 0$

DIAMETRIC FORM OF CIRCLE

10. If $(x, 3)$ and $(3,5)$ are the extremities of a diameter of a circle with centre at $(2, y)$. Then the value of x and y are-

(A) $x = 1, y = 4$

(B) $x = 4, y = 1$

(C) $x = 8, y = 2$

(D) None of these

11. A variable circle passes through the fixed point A(p, q) and touches the x-axis. The locus of the other end of the diameter through A is

(A) $(x-p)^2 = 4qy$

(B) $(x-q)^2 = 4py$

(C) $(y-p)^2 = 4qx$

(D) $(y-q)^2 = 4px$

INTERCEPTS OF A CIRCLE

12. The centre of the circle touching the y-axis at (0,3) and making an intercept of 2 units on the positive x axis is

(A) $(10, \sqrt{3})$

(B) $(\sqrt{3}, 10)$

(C) $(\sqrt{10}, 3)$

(D) $(3, \sqrt{10})$

13. A circle touches a straight line $lx + my + n = 0$ and cuts the circle $x^2 + y^2 = 9$ orthogonally, The locus of centres of such circles is

(A) $(lx + my + n)^2 = (l^2 + m^2)(x^2 + y^2 - 9)$

(B) $(1x + my - n)^2 = (1^2 + m^2)(x^2 + y^2 - 9)$

(C) $(lx + my + n)^2 = (l^2 + m^2)(x^2 + y^2 + 9)$

(D) none of these

14. A circle touches the x-axis and also touches the circle with centre at (0,3) and radius 2 . The locus of the centre of the circle is -

(A) an ellipse

(B) a circle

(C) a hyperbola

(D) a parabola

POSITION OF A POINT W.R.T. CIRCLE

15. The locus of the centre of a circle which touches externally the circle, $x^2 + y^2 - 6x - 6y + 14 = 0$ and also touches the y-axis is given by the equation

(A) $x^2 - 6x - 10y + 14 = 0$

(B) $x^2 - 10x - 6y + 14 = 0$

(C) $y^2 - 6x - 10y + 14 = 0$

(D) $y^2 - 10x - 6y + 14 = 0$

16. If $\left(a,\frac{1}{a}\right),\left(b,\frac{1}{b}\right),\left(c,\frac{1}{c}\right)$ & $\left(d,\frac{1}{d}\right)$ are four distinct points on a circle of radius 4 units then, abed =

(A) 4

(B) 1/4

(C) 1

(D) 16

17. Number of points (x, y) having integral coordinates satisfying the condition $x^2 + y^2 < 25$ is

(A) 69

(B) 80

(C) 81

(D) 77

TANGENT AND NORMAL

18. The square of the length of tangent from $(3, -4)$ on the circle $x^2 + y^2 - 4x - 6y + 3 = 0$ -

(A) 20

(B) 30

(C) 40

(D) 50

19. Three equal circles each of radius r touch one another. The radius of the circle touching all the three given circle internally is

(A) $(2+\sqrt{3})r$

(B) $\frac{(2+\sqrt{3})}{\sqrt{3}}r$

(C) $\frac{(2-\sqrt{3})}{\sqrt{3}}r$

(D) $(2-\sqrt{3})r$

CHORD WITH A GIVEN MIDDLE POINT

20. The locus of the mid points of the chords of the circle $x^2 + y^2 - ax - by = 0$ which subtend a right angle at $\left(\frac{a}{2},\frac{b}{2}\right)$ is

(A) $ax + by = 0$

(B) $ax + by = a^2 + b^2$

(C) $x^2 + y^2 - ax - by + \frac{a^2+b^2}{8} = 0$

(D) $x^2 + y^2 - ax - by - \frac{a^2+b^2}{8} = 0$

21. Let C be the circle with centre (0,0) and radius 3 units. The equation of the locus of the mid points of the chords of the circle C that subtend an angle of $\frac{2\pi}{3}$ at its centre is

(A) $x^2 + y^2 = \frac{27}{4}$

(B) $x^2 + y^2 = \frac{9}{4}$

(C) $x^2 + y^2 = \frac{3}{2}$

(D) $x^2 + y^2 = 1$

CHORD OF CONTACT

22. The chord of contact of tangents from three points P, Q, R to the circle $x^2 + y^2 = c^2$ are concurrent, then P, Q, R

(A) form a triangle

(B) are concyclic

(C) are collinear

(D) none of these

23. Distance between the chord of contact with respect to point (0,0) and (g, f) of circle $x^2 + y^2 + 2gx + 2fy + c = 0$ is -

(A) $\frac{1}{2}(g^2 + f^2 + c)$

(B) $g^2 + f^2$

(C) $\frac{g^2+f^2+c}{2\sqrt{g^2-f^2}}$

(D) $\frac{g^2+f^2-c}{2\sqrt{g^2+f^2}}$

PAIR OF TANGENTS

24. A pair of tangents are drawn from the origin to the circle $x^2 + y^2 + 20(x + y) + 20 = 0$. The equation of the pair of tangents is

(A) $x^2 + y^2 + 5xy = 0$

(B) $x^2 + y^2 + 10xy = 0$

(C) $2x^2 + 2y^2 + 5xy = 0$

(D) $2x^2 + 2y^2 - 5xy = 0$

25. The equation of the pair of tangents drawn to the circle $x^2 + y^2 - 2x + 4y + 3 = 0$ from $(6, -5)$ is-

(A) $7x^2 + 23y^2 + 30xy + 66x + 50y - 73 = 0$

(B) $7x^2 + 23y^2 - 30xy - 66x - 50y + 73 = 0$

(C) $7x^2 + 23y^2 + 30xy - 66x - 50y - 73 = 0$

(D) None of these

FAMILY OF CIRCLE

NUMBER OF COMMON TANGENTS AND POSITION OF TWO CIRCLE

26. The length of the common chord of circles $x^2 + y^2 - 6x - 16 = 0$ and $x^2 + y^2 - 8y - 9 = 0$ is

(A) $10\sqrt{3}$

(B) $5\sqrt{3}$

(C) $5\sqrt{3}/2$

(D) none of these

27. If the two circles, $x^2 + y^2 + 2g_1x + 2f_1y = 0$ and $x^2 + y^2 + 2g_2x + 2f_2y = 0$ touches each other, then

(A) $f_1g_1 = f_2g_2$

(B) $\frac{f_1}{g_1} = \frac{f_2}{g_2}$

(C) $f_1f_2 = g_1\, g_2$

(D) none

28. If the two circles $(x - 1)^2 + (y - 3)^2 = r^2$ and $x^2 + y^2 - 8x + 2y + 8 = 0$ intersect in two distinct points, then

(A) $r > 2$

(B) $2 < r < 8$

(C) $r < 2$

(D) $r = 2$

29. If the circles $x^2 + y^2 + 2ax + cy + a = 0$ and $x^2 + y^2 - 3ax + dy - 1 = 0$ intersect in two distinct point P and Q then the lines $5x + by - a = 0$ passes through P and Q for -

(A) exactly one value of a

(B) no value of a

(C) infinitely many values of a

(D) exactly two values of a

ORTHOGONALITY OFTWO CIRCLE

30. If a circle passes through the point (a, b) and cuts the circle $x^2 + y^2 = p^2$ orthogonally, then the equation of the locus of its centre is -

(A) $x^2 + y^2 - 3ax - 4by + (a^2 + b^2 - p^2) = 0$

(B) $2ax + 2by - (a^2 - b^2 + p^2) = 0$

(C) $x^2 + y^2 - 2ax - 3by + (a^2 - b^2 - p^2) = 0$

(D) $2ax + 2by - (a^2 + b^2 + p^2) = 0$

RADICAL AXIS

31. If the circle $x^2 + y^2 + 4x + 22y + c = 0$ bisects the circumference of the circle $x^2 + y^2 - 2x + 8y - d = 0$, then $c + d$ is equal to

(A) 60

(B) 50

(C) 40

(D) 56

32. The equation of the circle which passes through $(2a, 0)$ and has the radical axis $2x - a = 0$ with the circle $x^2 + y^2 = a^2$ is

(A) $x^2 + y^2 - 2ax = 0$

(B) $x^2 + y^2 + 2ax = 0$

(C) $x^2 + y^2 + 3ax = 0$

(D) $x^2 + y^2 - 3ax = 0$

RADICAL CENTRE

33. Find the equation of the circle which cuts the three circles

$x^2 + y^2 - 3x - 6y + 14 = 0, x^2 + y^2 - x - 4y + 8 = 0$, and $x^2 + y^2 + 2x - 6y + 9 = 0$

orthogonally.

(A) $x^2 + y^2 - 2x - 4y + 1 = 0$

(B) $x^2 - y^2 - 2x - 4y + 1 = 0$

(C) $x^2 + y^2 + 2x - 4y + 1 = 0$

(D) $x^2 + y^2 - 2x + 4y + 1 = 0$

MIXED PROBLEMS

34. The common chord of two intersecting circles C_1 and C_2 can be seen from their centres at the angles of 90° and 60° respectively. If the distance between their centres is equal to $\sqrt{3} + 1$ then the radius of C_1 and C_2 are

(A) $\sqrt{3}$ and 3

(B) $\sqrt{2}$ and $2\sqrt{2}$

(C) $\sqrt{2}$ and 2

(D) $2\sqrt{2}$ and 4

35. What is the length of shortest path by which one can go from $(-2,0)$ to $(2,0)$ without entering the interior of circle, $x^2 + y^2 = 1$

(A) $2\sqrt{3}$ (B) $\sqrt{3} + \frac{2\pi}{3}$ (C) $2\sqrt{3} + \frac{\pi}{3}$ (D) none of these

36. In a right triangle ABC, right angled at A, on the leg AC as diameter, a semicircle is described. The chord joining A with the point of intersection D of the hypotenuse and the semicircle, then the length AC equals to

(A) $\frac{AB \cdot AD}{\sqrt{AB^2+AD^2}}$

(B) $\frac{AB \cdot AD}{AB+AD}$

(C) $\sqrt{AB \cdot AD}$

(D) $\frac{AB \cdot AD}{\sqrt{AB^2-AD^2}}$

37. A circle is inscribed into a rhombus ABCD with one angle 60°. The distance from the centre of the circle to the nearest vertex is equal to 1 . If P is any point of the circle, then $|PA|^2+|PB|^2+|PC|^2+|PD|^2$ is equal to

(A) 12

(B) 11

(C) 9

(D) none

EXERCISE-02(LEVEL-II)

BASIC DEFINITION OF CIRCLE

1. Circles are drawn touching the co-ordinate axis and having radius 2 , then

(A) Centre of these circles lie on the pair of lines

$y^2 - x^2 = 0$

(B) centre of these circles lie only on the line $y = x$

(C) Area of the quadrilateral whose vertices are centre of these circles is 16 sq. units.

(D) Area of the circle touching these four circles internally is $4\pi(3 + 2\sqrt{2})$

2. $x^2 + y^2 + 6x = 0$ and $x^2 + y^2 - 2x = 0$ are two circles, then

(A) They touch each other externally

(B) They touch each other internally

(C) Area of triangle formed by their common tangents is $3\sqrt{3}$ sq. units.

(D) Their common tangents do not form any triangle.

3. The centre(s) of the circle(s) passing through the points (0,0), (1,0) and touching the circle $x^2 + y^2 = 9$ is/are

(A) $\left(\frac{3}{2}, \frac{1}{2}\right)$

(B) $\left(\frac{1}{2}, \frac{3}{2}\right)$

(C) $\left(\frac{1}{2}, 2^{1/2}\right)$

(D) $\left(\frac{1}{2}, -2^{1/2}\right)$

POSITION OF A POINT W.R.T. CIRCLE

4. Consider the circle $x^2 + y^2 - 2x - 2y - 14 = 0$, which of following is/are true :

(A) Point (3,4) lies insides the circle

(B) Point $(-5, -3)$ lies outside the circle

(C) Line $x + 3y + 4 = 0$ is diameter of circle

(D) Area of circle is 16π square units

5. If A and B are two points on the circle $x^2 + y^2 - 4x + 6y - 3 = 0$ which are farthest and nearest respectively from the point (7,2), then

(A) $A \equiv (2 - 2\sqrt{2}, -3 - 2\sqrt{2})$

(B) $A \equiv (2 + 2\sqrt{2}, -3 + 2\sqrt{2})$

(C) $B \equiv (2 + 2\sqrt{2}, -3 + 2\sqrt{2})$

(D) $B \equiv (2 - 2\sqrt{2}, -3 - 2\sqrt{2})$

PARAMETRIC FORM OF A CIRCLE

6. Let d be the shortest and D be the longest distance between two circles $x = \cos\theta, y = \sin\theta$ and $x = 3 + 2\cos\theta, y = 3 + 2\sin\theta$, then the value of $(D - d)$ cannot be

(A) $6\sqrt{2}$

(B) 6

(C) $3\sqrt{2} + 2$

(D) 5

TANGENT AND NORMAL

7. Slope of tangent to the circle $(x - r)^2 + y^2 = r^2$ at the point (x, y) lying on the circle is

(A) $\frac{x}{y-r}$

(B) $\frac{r-x}{y}$

(C) $\frac{y^2-x^2}{2xy}$

(D) $\frac{y^2+x^2}{2xy}$

8. Point M moved along the circle $(x - 4)^2 + (y - 8)^2 = 20$. Then it broke away from it and moving along a tangent to the circle cuts the x-axis at the point $(-2,0)$. The co-ordinates of the point on the circle at which the moving point broke away can be

(A) $\left(-\frac{3}{5}, \frac{46}{5}\right)$

(B) $\left(-\frac{2}{5}, \frac{44}{5}\right)$

(C) $(6,4)$

(D) $(3,5)$

NUMBER OF COMMON TANGENTS AND POSITION OF TWO CIRCLE

9. Consider the circles $x^2 + y^2 = 1$ & $x^2 + y^2 - 2x - 6y + 6 = 0$. Then equation of a common tangent to the two circles is

(A) $4x - 3y - 5 = 0$

(B) $x + 1 = 0$

(C) $3x + 4y - 5 = 0$

(D) $y - 1 = 0$

10. Let number of points of intersection and number of common tangents of two circles $x^2 + y^2 - 6x - 2y + 1 = 0$ and $x^2 + y^2 + 2x - 6y + 9 = 0$ be m and n respectively. Which of the following is/are CORRECT ?

(A) $m + n = 4$ (B) $n - m = 4$ (C) $mn = 4$ (D) $m^n + n^m = 1$

11. Circles $x^2 + y^2 + 2x = 0$&$x^2 + y^2 - 2x = 0$ Now which of following is/are correct?

(A) Number of common tangents is 3

(B) 2 of common tangents are parallel

(C) Both circles touches the y-axis

(D) None of these

RADICAL AXIS

12. For the circles $S_1 \equiv x^2 + y^2 - 4x - 6y - 12 = 0$ and $S_2 \equiv x^2 + y^2 + 6x + 4y - 12 = 0$ and the line $L \equiv x + y = 0$

(A) L is common tangent of S_1 and S_2

(B) L is common chord of S_1 and S_2

(C) L is radical axis of S_1& S_2

(D) L is Perpendicular to the line joining the centre of S_1& S_2

13. Let $S_2 = 0$ is the mirror image of $S_1: x^2 + y^2 - 4x - 6y + 12 = 0$ w.r.t the line $L_1: 10^4x + (10^4 + 10)y + (10^4 + 20) = 0$. Let $L_2: 2^{11}x + (2^{11} + 2^{12})y + (2^{11} + 2^{13}) = 0$ be a line then the equations of line passing through the point of intersection of the line $L_2 = 0$ with radical axes of $S_1 = 0,\ S_2 = 0$ and making equal intercepts in magnitude with the coordinate axes is/are

(A) $x - y - 3 = 0$

(B) $x + y + 1 = 0$

(C) $2x - 2y + 1 = 0$

(D) $2x + 2y + 3 = 0$

MIXED PROBLEMS

14. 3 circle of radii 1,2 and 3 and centres at A, B and C respectively, touch each other. Another circle whose centre is P touches all these 3 circles externally. and has radius r. Also $\angle PAB = \theta$&$\angle PAC = \alpha$.

(A) $\cos\ \theta = \frac{3-r}{3(1+r)}$

(B) $\cos\ \alpha = \frac{2-r}{2(1+r)}$

(C) $r = \frac{6}{23}$

(D) $r = \frac{6}{\sqrt{23}}$

EXERCISE-03

SUBJECTIVE | JEE ADVANCED

1. Let L_1 be a straight line through the origin and L_2 be the straight line $x+y=1$. If the intercepts made by the circle $x^2+y^2-x+3y=0$ on $L_1\&L_2$ are equal, then find the equation(s) which represent L_1

2. A circle passes through the points $(-1,1)$, $(0,6)$ and $(5,5)$. Find the points on the circle the tangents at which are parallel to the straight line joining origin to the centre.

3. A circle is drawn with its centre on the line $x+y=2$ to touch the line $4x-3y+4=0$ and pass through the point $(0,1)$. Find its equation.

4. Find the locus of the middle points of portions of the tangents to the circle $x^2+y^2=a^2$ terminated by the coordinates axes.

5. Find the equation of the circle passing through the three points $(4,7)$, $(5,6)$ and $(1,8)$. Also find the coordinates of the point of intersection of the tangents to the circle at the points where it is cut by the straight line $5x+y+17=0$.

6. The lines $2x-3y+1=0$ is tangent to a circle $S=0$ at $(1,1)$. If the radius of the circle is $\sqrt{13}$. Find the equation of the circle S.

7. A circle with centre in the first quadrant is tangent to $y=x+10$, $y=x-6$, and the y-axis. Let (h,k) be the centre of the circle. If the value of $(h+k)=a+b\sqrt{a}$ where $\sqrt{a}$ is a surd, find the value of $a+b$.

8. A circle C is tangent to the x and y axis in the first quadrant at the points P and Q respectively. BC and AD are parallel tangents to the circle with slope -1 . If the points A and B are on the y-axis while C and D are on the x-axis and the area of the figure ABCD is $900\sqrt{2}$ sq. units then find the radius of the circle.

9. Tangents are drawn to the concentric circles $x^2+y^2=a^2$ and $x^2+y^2=b^2$ at right angle to one another. Show that the locus of their point of intersection is a 3rd concentric circle. Find its radius.

10. Find the locus of the mid point of the chord of a circle $x^2+y^2=4$ such that the segment intercepted by the chord on the curve $x^2-2x-2y=0$ subtends a right angle at the origin.

11. Find the equation of circle passing through $(1,1)$ belonging to the system of co-axial circles that are tangent at $(2,2)$ to the locus of the point of intersection of mutually perpendicular tangent to the circle $x^2+y^2=4$.

12. A circle $S=0$ is drawn with its centre at $(-1,1)$ so as to touch the circle $x^2+y^2-4x+6y-3=0$ externally. Find the intercept made by the circle $S=0$ on the coordinate axes.

13. Find the equation of the circle which passes through the point (1,1)& which touches the circle $x^2+y^2+4x-6y-3=0$ at the point (2,3) on it.

14. Find the equation of the circle whose radius is 3 and which touches the circle $x^2+y^2-4x-6y-12=0$ internally at the point $(-1,-1)$.

15. Let K denotes the square of the diameter of the circle whose diameter is the common chord of the two circles $x^2+y^2+2x+3y+1=0$ and $x^2+y^2+4x+3y+2=0$ and W denotes the sum of the abscissa and ordinate of a point P where all variable chords of the curve $y^2=8x$ subtending right angles at the origin, are concurrent. and H denotes the square of the length of the tangent from the point (3,0) on the circle $2x^2+2y^2+5y-16=0$. Find the value of KWH.

16. The radical axis of the circles $x^2+y^2+2gx+2fy+c=0$ and $2x^2+2y^2+3x+8y+2c=0$ touches the circle $x^2+y^2+2x-2y+1=0$. Show that either $g=3/4$ or $f=2$.

17. Find the equation of the circle through the points of intersection of circles $x^2+y^2-4x-6y-12=0$ and $x^2+y^2+6x+4y-12=0$& cutting the circle $x^2+y^2-2x-4=0$ orthogonally.

18. The centre of the circle $S=0$ lie on the line $2x-2y+9=0$&$S=0$ cuts orthogonally the circle $x^2+y^2=4$. Show that circle $S=0$ passes through two fixed points & find their coordinates.

19. (a) Find the equation of a circle passing through the origin if the line pair, $xy-3x+2y-6=0$ is orthogonal to it. If this circle is orthogonal to the circle $x^2+y^2-kx+2ky-8=0$ then find the value of k.

(b) Find the equation of the circle which cuts the circle $x^2+y^2-14x-8y+64=0$ and the coordinates axes orthogonally.

20. Show that the locus of the centres of a circle which cuts two given circles orthogonally is a straight line & hence deduce the locus of the centres of the circles which cut the circles $x^2+y^2+4x-6y+9=0$ &$x^2+y^2-5x+4y+2=0$ orthogonally. Interpret the locus.

21. Find the equation of a circle which touches the line $x+y=5$ at the point $(-2,7)$ and cuts the circle $x^2+y^2+4x-6y+9=0$ orthogonally

22. Find the equation of the circle passing through the point $(-6,0)$ if the power of the point (1,1) w.r.t. the circle is 5 and it cuts the circle $x^2+y^2-4x-6y-3=0$ orthogonally.

23. Find the equation of the circle which passes through the origin, meets the x-axis orthogonally & cuts the circle $x^2+y^2=a^2$ at an angle of 45°.

24. Determine the nature of the quadrilateral formed by four lines $3x+4y-5=0$; $4x-3y-5=0$; $3x+4y+5=0$ and $4x-3y+5=0$. Find the equation of the circle inscribed and circumscribing this quadrilateral.

25. The line $lx+my+n=0$ intersects the curve $ax^2+2hxy+by^2=1$ at the point P and Q. The circle on PQ as diameter passes through the origin. Prove that $n^2(a+b)=l^2+m^2$.

26. One of the diameters of the circle circumscribing the rectangle ABCD is $4y = x + 7$. If A&B are the points $(-3,4)$&$(5,4)$ respectively, then find the area of the rectangle.

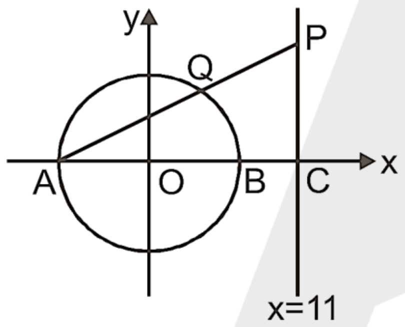
27. Find the equations of straight lines which pass through the intersection of the lines $x - 2y - 5 = 0$, $7x + y = 50$ & divide the circumference of the circle $x^2 + y^2 = 100$ into two arcs whose lengths are in the ratio 2: 1.

28. In the given figure, the circle $x^2 + y^2 = 25$ intersects the x-axis at the point A and B. The line $x = 11$ intersects the x-axis at the point C. Point P moves along the line $x = 11$ above the x-axis and AP intersects the circle at Q. Find

(i) The coordinates of the point P if the triangle AQB has the maximum area.

(ii) The coordinates of the point P if Q is the middle point of AP.

(iii) The coordinates of P if the area of the triangle AQB is $(1/4)^{\text{th}}$ of the area of the triangle APC.



29. A point moving around circle $(x + 4)^2 + (y + 2)^2 = 25$ with centre C broke away from it either at the point A or point B on the circle and moved along a tangent to the circle passing through the point $D(3, -3)$. Find the following.

(i) Equation of the tangents at A and B.

(ii) Coordinates of the points A and B.

(iii) Angle ADB and the maximum and minimum distances of the point D from the circle.

(iv)Area of quadrilateral ADBC and the $\triangle$ DAB.

(v) Equation of the circle circumscribing the $\triangle$ DAB and also the intercepts made by this circle on the coordinate axes.

30. Find the equation of a line with gradient 1 such that the two circles $x^2 + y^2 = 4$ and $x^2 + y^2 - 10x - 14y + 65 = 0$ intercept equal length on it.

31. Consider a circle S with centre at the origin and radius 4 . Four circles A, B, C and D each with radius unity and centres $(-3,0)$, $(-1,0)$, $(1,0)$ and $(3,0)$ respectively are drawn. A chord PQ of the circle S touches the circle B and passes through the centre of the circle C. If the length of this chord can be expressed as $\sqrt{x}$, find x.

32. Obtain the equations of the straight lines passing through the point A(2,0)& making 45° angle with the tangent at A to the circle $(x+2)^2+(y-3)^2=25$. Find the equations of the circles each of radius 3 whose centres are on these straight lines at a distance of $5\sqrt{2}$ from A.

33. A variable circle passes through the point A(a, b) & touches the x-axis; show that the locus of the other end of the diameter through A is $(x-a)^2=4by$.

34. Consider a family of circles passing through two fixed points A(3,7)& B(6,5). The chords in which the circle $x^2+y^2-4x-6y-3=0$ cuts the members of the family are concurrent at a point. Find the coordinates of this point.

35. The circle $C: x^2+y^2+kx+(1+k)y-(k+1)=0$ passes through two fixed points for every real number k. Find.

(i) the coordinates of these two points.

(ii) the minimum value of the radius of a circle C.+

36. Let A, B, C be real numbers such that (i) $(\sin A, \cos B)$ lies on a unit circle centred at origin. (ii) $\tan C$ and $\cot C$ are defined. If the minimum value of $(\tan C-\sin A)^2+(\cot C-\cos B)^2$ is $a+b\sqrt{2}$ where $a, b \in I$, find the value of a^3+b^3.

37. A Rhombus ABCD has sides of length 10. A circle with centre 'A' passes through C (the opposite vertex) likewise, a circle with centre B passes through D. If the two circles are tangent to each other. Find the area of the rhombus.

38. Circles C_1 and C_2 are externally tangent and they are both internally tangent to the circle C_3. The radii of C_1 and C_2 are 4 and 10 , respectively and the centres of the three circles are collinear. A chord of C_3 is also a common internal tangent of C_1 and C_2. Given that the length of the chord is $\frac{m\sqrt{n}}{p}$ where m, n and p are positive integers, m and p are relatively prime and n is not divisible by the square of any prime, find the value of $(m+n+p)$.

39. Determine the range of values of $\theta \in [0,2\pi]$ for which the point $(\cos \theta, \sin \theta)$ lies inside the triangle formed by the lines $x+y=2$; $x-y=1$ and $6x+2y-\sqrt{10}=0$.

COMPREHENSION FOR QUE. 40 TO 42

If $7\ell^2-9m^2+8\ell+1=0$ and we have to find equation of circle having $\ell x+my+1=0$ is a tangent and we can adjust given condition as $16\ell^2+8\ell+1=9(\ell^2+m^2)$ or $(4\ell+1)^2=9(\ell^2+m^2) \Rightarrow \frac{|4\ell+1|}{\sqrt{(\ell^2+m^2)}}=3$ Centre of circle $=(4,0)$ and radius $=3$ when any two non parallel lines touching a circle, then centre of circle lies on angle bisector of lines.

On the basis of above information, answer the following questions :

40. If $16\,m^2 - 8\ell - 1 = 0$, then equation of the circle having $\ell x + my + 1 = 0$ is a tangent is

(A) $x^2 + y^2 + 8x = 0$

(B) $x^2 + y^2 - 8x = 0$

(C) $x^2 + y^2 + 8y = 0$

(D) $x^2 + y^2 - 8y = 0$

41. If $16\ell^2 + 9\,m^2 = 24\ell m + 6\ell + 8\,m + 1$ and if S be the equation of the circle having $\ell x + my + 1 = 0$ is a tangent when the equation of director circle of S is

(A) $x^2 + y^2 + 6x + 8y = 25$

(B) $x^2 + y^2 - 6x + 8y = 25$

(C) $x^2 + y^2 - 6x - 8y = 25$

(D) $x^2 + y^2 + 6x - 8y = 25$

42. If $4\ell^2 - 5\,m^2 + 6\ell + 1 = 0$, then the centre and radius of the circle which have $\ell x + my + 1 = 0$ is a tangent is

(A) $(0,4); \sqrt{5}$

(B) $(4,0); \sqrt{5}$

(C) $(0,3); \sqrt{5}$

(D) $(3,0); \sqrt{5}$

MATRIX MATCH TYPE

43.

Column – I	**Column - II**
(A) If the straight line $y = mx \forall m \in I$ touches or lies outside the circle $x^2 + y^2 - 20y + 90 + 0$, then the value of $\|m\|$ can be	(P) 0
(B) If the straight line $3x - 4y - 5k = 0, \forall k \in I$ touches or lies inside the circle, $x^2 + y^2 - 4x - 8y - 5 = 0$, then the value of $\|k + 2\|$ can be	(Q) 1
(C) Two circle $x^2 + y^2 + px + py - 7 = 0$ and $x^2 + y^2 - 10x + 2py + 1 = 0$ will orthogonally, then the value of p is	(R) 2
	(S) 3
	(T) 4

EXERCISE-04(LEVEL-I)

PREVIOUS YEAR | JEE MAIN

1. Consider a family of circles which are passing through the point $(-1,1)$ and are tangent to x-axis. If (h, k) are the co-ordinates of the centre of the circles, then the set of values of k is given by the interval **[AIEEE-2007]**

(A) $0 < k < 1/2$

(B) $k \geq 1/2$

(C) $-1/2 \leq k \leq 1/2$

(D) $k \leq 1/2$

2. The point diametrically opposite to the point $P(1,0)$ on the circle $x^2 + y^2 + 2x + 4y - 3 = 0$ is **[AIEEE-2008]**

(A) $(-3,4)$

(B) $(-3,-4)$

(C) $(3,4)$

(D) $(3,-4)$

3. The circle $x^2 + y^2 = 4x + 8y + 5$ intersects the line $3x - 4y = m$ at two distinct points if **[AIEEE-2010]**

(A) $-85 < m < -35$

(B) $-35 < m < 15$

(C) $15 < m < 65$

(D) $35 < m < 85$

4. The two circles $x^2 + y^2 = ax$ and $x^2 + y^2 = c^2 (c > 0)$ touch each other if : **[AIEEE-2011]**

(A) $2|a| = c$

(B) $|a| = c$

(C) $a = 2c$

(D) $|a| = 2c$

5. The length of the diameter of the circle which touches the x-axis at the pint $(1,0)$ and passes through the point $(2,3)$ **[AIEEE-2012]**

(A) $6/5$

(B) $5/3$

(C) $10/3$

(D) $3/5$

6. The circle passing through $(1,-2)$ and touching the axis of x at $(3,0)$ also passes through the point : **[JEE-MAIN 2013]**

(A) $(5,-2)$

(B) $(-2,5)$

(C) $(-5,2)$

(D) $(2,-5)$

7. Let C be the circle with centre at $(1,1)$ and radius $= 1$. If T is the circle centred at $(0, y)$, passing through origin and touching the circle C externally, then the radius of T is equal to: **[JEE-MAIN 2014]**

(A) $\frac{\sqrt{3}}{\sqrt{2}}$

(B) $\frac{\sqrt{3}}{2}$

(C) $\frac{1}{2}$

(D) $\frac{1}{4}$

8. The number of common tangents to the circles $x^2 + y^2 - 4x - 6y - 12 = 0$ and $x^2 + y^2 + 6x + 18y + 26 = 0$, is : **[JEE-MAIN 2015]**

(A) 3

(B) 4

(C) 1

(D) 2

9. If one of the diameters of the circle, given by the equation, $x^2 + y^2 - 4x + 6y - 12 = 0$, is a chord of a circle S, whose centre is at $(-3,2)$, then the radius of S is : **[JEE-MAIN 2016]**

(A) $5\sqrt{3}$

(B) 5

(C) 10

(D) $5\sqrt{2}$

10. Let the orthocentre and centroid of a triangle be $A(-3,5)$ and $B(3,3)$ respectively. If C is the circumcentre of this traingle, then the radius of the circle having line segment AC as diameter, is : **[JEE-MAIN 2018]**

(A) $\frac{3\sqrt{5}}{2}$ (B) $\sqrt{10}$ (C) $2\sqrt{10}$ (D) $3\sqrt{\frac{5}{2}}$

EXERCISE-04(LEVEL-II)

PREVIOUS YEAR | JEE ADVANCED

1. **(a)** Let ABCD be a quadrilateral with area 18 , with side AB parallel to the side CD and $AB = 2CD$. Let AD be perpendicular to AB and CD. If a circle is drawn inside the quadrilateral ABCD touching all the sides, then its radius is **[JEE 2007]**

(A) 3

(B) 2

(C) 3/2

(D) 1

(b) Tangents are drawn from the point (17,7) to the circle $x^2 + y^2 = 169$.

Statement-I : The tangents are mutually perpendicular. because

Statement-II : The locus of the points from which mutually perpendicular tangents can be drawn to the given circle is $x^2 + y^2 = 338$.

(A) Statement-I is true, statement-II is true; statement-II is correct explanation for statement-I

(B) Statement-I is true, statement-II is true; statement-II is NOT correct explanation for statement-I

(C) Statement-I is true, Statement-II is False

(D) Statement-I is False, Statement-II is True

2. **(a)** Consider the two curves

$C_1: y^2 = 4x; C_2: x^2 + y^2 - 6x + 1 = 0$. Then, **[JEE 2008]**

(A) C_1 and C_2 touch each other only at one point

(B) C_1 and C_2 touch each other exactly at two points

(C) C_1 and C_2^2 intersect (but do not touch) at exactly two points

(D) C_1 and C_2 neither intersect nor touch each other

(b) Consider,$L_1: 2x + 3y + p - 3 = 0; L_2: 2x + 3y + p + 3 = 0$,

where p is a real number, and $C: x^2 + y^2 + 6x - 10y + 30 = 0$

Statement-I : If line L_1 is a chord of circle C, then line L_2 is not always a diameter of circle C.

Statement-II : If line L_1 is a diameter of circle C, then line L_2 is not a chord of circle C.

(A) Statement-I is true, statement-II is true; statement-II is correct explanation for statement-I

(B) Statement-I is true, statement-II is true; statement-II is NOT comect explanation for statement-I (C) Statement-I is true, Statement-II is False

(D) Statement-I is False, Statement-II is True

(c) Comprehension

A circle C of radius 1 is inscribed in an equilateral triangle PQR. The points of contact of C with the sides PQ, QR, RP and D, E, F respectively. The line PQ is given by the equation $\sqrt{3}x + y - 6 = 0$ and the point D is $\left(\frac{3\sqrt{3}}{2}, \frac{3}{2}\right)$. Further, it is given that the origin and the centre of C are on the same side of the line PQ.

(i) The equation of circle C is

(A) $(x - 2\sqrt{3})^2 + (y - 1)^2 = 1$

(B) $(x - 2\sqrt{3})^2 + \left(y + \frac{1}{2}\right)^2 = 1$

(C) $(x - \sqrt{3})^2 + (y + 1)^2 = 1$

(D) $(x - \sqrt{3})^2 + (y - 1)^2 = 1$

(ii) Points E and F are given by

(A) $\left(\frac{\sqrt{3}}{2}, \frac{3}{2}\right), (\sqrt{3}, 0)$

(B) $\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right), (\sqrt{3}, 0)$

(C) $\left(\frac{\sqrt{3}}{2}, \frac{3}{2}\right), \left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

(D) $\left(\frac{3}{2}, \frac{\sqrt{3}}{2}\right), \left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

(iii) Equations of the sides RP, RQ are

(A) $y = \frac{2}{\sqrt{3}}x + 1, y = -\frac{2}{\sqrt{3}}x - 1$

(B) $y = \frac{1}{\sqrt{3}}x, y = 0$

(C) $y = \frac{\sqrt{3}}{2}x + 1, y = -\frac{\sqrt{3}}{2}x - 1$

(D) $y = \sqrt{3}x, y = 0$

3. **(a)** Tangents drawn from the point P(1,8) to the circle $x^2 + y^2 - 6x - 4y - 11 = 0$ touch the circle at the points A and B. The equation of the circumcircle of the triangle PAB is **[JEE 2009]**

(A) $x^2 + y^2 + 4x - 6y + 19 = 0$

(B) $x^2 + y^2 - 4x - 10y + 19 = 0$

(C) $x^2 + y^2 - 2x + 6y - 29 = 0$

(D) $x^2 + y^2 - 6x - 4y + 19 = 0$

(b) The centres of two circles C_1 and C_2 each of unit radius are at a distance of 6 units from each other. Let P be the mid point of the line segment joining the centres of C_1 and C_2 and C be a circle touching circles C_1 and C_2 externally. If a common tangent to C_1 and C passing through P is also a common tangent to C_2 and C, then the radius of the circle C is

4. Two parallel chords of a circle of radius 2 are at a distance $\sqrt{3}+1$ apart. If the chords subtend at the center, angles of $\frac{\pi}{k}$ and $\frac{2\pi}{k}$, where $k > 0$, then the value of $[k]$ is **[JEE 2010]**

{ Note : [k] denotes the largest integer less than or equal to k}

5. The circle passing through the point $(-1,0)$ and touching the y-axis at $(0,2)$ also passes through the point **[JEE 2011]**

(A) $\left(-\frac{3}{2},0\right)$

(B) $\left(-\frac{5}{2},2\right)$

(C) $\left(-\frac{3}{2},\frac{5}{2}\right)$

(D) $(-4,0)$

6. The straight line $2x-3y=1$ divides the circular region $x^2+y^2 \le 6$ into two parts. If

$S=\left\{\left(2,\frac{3}{4}\right),\left(\frac{5}{2},\frac{3}{4}\right),\left(\frac{1}{4},-\frac{1}{4}\right),\left(\frac{1}{8},\frac{1}{4}\right)\right\}$,

then the number of point(s) in S lying inside the smaller part is **[JEE 2011]**

7. The locus of the mid-point of the chord of contact of tangents drawn from points lying on the straight line $4x-5y=20$ to the circle $x^2+y^2=9$ is **[JEE 2012]**

(A) $20(x^2+y^2)-36x+45y=0$

(B) $20(x^2+y^2)+36x-45y=0$

(C) $36(x^2+y^2)-20x+45y=0$

(D) $36(x^2+y^2)+20x-45y=0$

Paragraph(8 - 9)

A tangent PT is drawn to the circle $x^2+y^2=4$ at the point $P(\sqrt{3},1)$. A straight line L, perpendicular to PT is a tangent to the circle $(x-3)^2+y^2=1$. **[JEE 2012]**

8. A possible equation of L is

(A) $x-\sqrt{3}y=1$

(B) $x+\sqrt{3}y=1$

(C) $x-\sqrt{3}y=-1$

(D) $x+\sqrt{3}y=5$

9. A common tangent of the two circles is

(A) $x=4$

(B) $y=2$

(C) $x+\sqrt{3}y=4$

(D) $x+2\sqrt{2}y=6$

10. Circle(s) touching x − axis at a distance 3 from the origin and having an intercept of length $2\sqrt{7}$ on y-axis is (are) **[JEEAdv. 2013]**

(A) $x^2 + y^2 - 6x + 8y + 9 = 0$

(B) $x^2 + y^2 - 6x + 7y + 9 = 0$

(C) $x^2 + y^2 - 6x - 8y + 9 = 0$

(D) $x^2 + y^2 - 6x + 7y + 9 = 0$

11. A circle S passes through the point (0,1) and is orthogonal to the circles $(x-1)^2 + y^2 = 16$ and $x^2 + y^2 = 1$. Then **[JEE Adv.2014]**

(A) Radius of S is 8

(B) radius of S is 7

(C) centre of S is $(-7,1)$

(D) centre of S is $(-8,1)$

12. Let RS be the diameter of the circle $x^2 + y^2 = 1$, where S is the point (1,0). Let P be a variable point (other than R and S) on the circle and tangents to the circle at S and P meet at the point Q. Then normal to the circle at P intersects a line drawn through Q parallel to RS at point E. Then the locus of E passes through the point(s) **[JEE Adv. 2016]**

(A) $\left(\frac{1}{3}, \frac{1}{\sqrt{3}}\right)$

(B) $\left(\frac{1}{4}, \frac{1}{2}\right)$

(C) $\left(\frac{1}{3}, -\frac{1}{\sqrt{3}}\right)$

(D) $\left(\frac{1}{4}, -\frac{1}{2}\right)$

13. For how many values of p, the circle $x^2 + y^2 + 2x + 4y - p = 0$ and the coordinate axes have exactly three common points **[JEE Adv. 2017]**

14. Let T be the line passing the points $P(-2,7)$ and $Q(2,-5)$. Let F be the set of all pairs of circles (S_1, S_2) such that T is tangent to S_1 at P and tangent to S_2 at Q, and also such that S_1 and S_2 touch each other at a point, say M. Let E_1^2 be the set representing the locus of M as the pair $(S_1,\ S_2)$ varies in F_1. Let the set of all straight line segments joining a pair of distinct points of E_1 and passing through the point R(1,1,)beF_2. Let E_2 be the set of the mid-points of the line segments in the set F_2. Then, which of the following statment(s) is (are) TRUE ?

[JEE Adv.2018]

(A) The point $(-2,7)$ lies in E_1

(B) The point $\left(\frac{4}{5}, \frac{7}{5}\right)$ does NOT lie in E_2

(C) The point $\left(\frac{1}{2}, 1\right)$ lies in E_2

(D) The point $\left(0, \frac{3}{2}\right)$ does NOT lie in E_1

PARAGRAPH 15 to 16

Let S be the circle in the xy - plane defined by the equation $x^2 + y^2 = 4$ **[JEE Adv.2018]**

15. Let E_1E_2 and F_1F_2 be the chords of S passing through the point $P_0(1,1)$ and parallel to the x-axis and the y-axis, respectively. Let G_1G_2 be the chord of S passing through P_0 and having slope -1 . Let the tangents to S at E_1 and E_2 meet at E_3, the tangents to S at F_1 and F_2 meet at F_3 and the tangents to S at G_1 and G_2 meet at G_3. Then, the points E_3, F_3, and G_3 lie on the curve

(A) $x + y = 4$

(B) $(x-4)^2 + (y-4)^2 = 16$

(C) $(x-4)(y-4) = 4$

(D) $xy = 4$

16. Let P be a point on the circle S with both coordinates being positive. Let the tangent to S at P intersect the coordinate axes at the points M and N. Then, the mid-point of the line segment MN must lie on the curve.

(A) $(x+y)^2 = 3xy$

(B) $x^{2/3} + y^{2/3} = 2^{4/3}$

(C) $x^2 + y^2 = 2xy$

(D) $x^2 + y^2 = x^2y^2$

17. A line $y = mx + 1$ intersects the circle $(x-3)^2 + (y+2)^2 = 25$ at the points P and Q. If the midpoint of the line segment PQ has x - coordinate $\frac{-3}{5}$, then which one of the following options is correct ? **[JEE Adv. 2019]**

(A) $-3 \leq m < -1$

(B) $6 \leq m < 8$

(C) $4 \leq m < 6$

(D) $2 \leq m < 4$

18. Let the point B be the reflection of the point A(2,3) with respect to the line $8x - 6y - 23 = 0$. Let Γ_A and Γ_B be circles of radii 2 and 1 with centres A and B resepectively. Let T be a common tangent to the circles Γ_A and Γ_B such that both the circles are on the same side of T. If C is the point of intersection of T and the line passing through A and B, then the length of the line segment AC is **[JEE Adv. 2019]**

19. Let O be the centre of the circle $x^2 + y^2 = r^2$, where $r > \frac{\sqrt{5}}{2}$. Suppose PQ is a chord of this circle and the equation of the line passing through P and Q is $2x + 4y = 5$. If the centre of the circumcircle of the triangle OPQ lies on the line $x + 2y = 4$, then the value of r is

[JEE Adv. 2020]

20. Consider a triangle Δ whose two sides lie on the x axis and the line $x + y + 1 = 0$. If the orthocenter of Δ is $(1,1)$, then the equation of the circle passing through the vertices of the triangle Δ is **[JEE Adv. 2021]**

(A) $x^2 + y^2 - 3x + y = 0$

(B) $x^2 + y^2 + x + 3y = 0$

(C) $x^2 + y^2 + 2y - 1 = 0$

(D) $x^2 + y^2 + x + y = 0$

Paragraph for Q.21 & Q.22

Let $M = \{(x, y) \in R \times R: x^2 + y^2 \le r^2\}$,

Where $r > 0$. Consider the geometric progression

$a_n = \frac{1}{2^{n-1}}$, $n = 1,2,3, ...$ Let $S_0 = 0$ and, for $n \ge 1$, let

S_n denote the sum of the first n terms of this progression. For $n \ge 1$, let C_n denote the circle with center $(S_{n-1}, 0)$ and radius a_n and D_n denote the circle with center (S_{n-1}, S_{n-1}) and radius a_n. **[JEE Adv. 2021]**

21. Consider M with $r = \frac{1025}{513}$. Let k be the number of all those circles C_n that are inside M. Let 1 be the maximum possible number of circles among these k circles such that no two circles intersect. Then

(A) $k + 2l = 22$

(B) $2k + 1 = 26$

(C) $2k + 3l = 34$

(D) $3k + 2l = 40$

22. Consider M with $r = \frac{(2^{199}-1)\sqrt{2}}{2^{198}}$. The number of all those circles D_n that are inside M is

(A) 198 (B) 199 (C) 200 (D) 201

23. Let ABC be the triangle with $AB = 1$, $AC = 3$ and $\angle BAC = \frac{\pi}{2}$. If a circle of radius $r > 0$ touches the sides AB, AC and also touches internally the circumcircle of the triangle ABC, then the value of r is **[JEE Adv. 2022]**

24. Let G be a circle of radius $R > 0$. Let $G_1, G_2, \dots \dots, G_n$ be n circles of equal radius $r > 0$. Suppose each of the n circles $G_1, G_2, \dots, G_n$ touches the circle G externally. Also, for $i = 1,2, \dots, n-1$, the circle G_i touches G_{i+1} externally, and G_n touches G_1 externally. Then, which of the following statements is/are TRUE? **[JEE Adv. 2022]**

(A) If $n = 4$, then $(\sqrt{2} - 1)r < R$

(B) If $n = 5$, then $r < R$

(C) If $n = 8$, then $(\sqrt{2} - 1)r < R$

(D) If $n = 12$, then $\sqrt{2}(\sqrt{3} + 1)r > R$

ANSWER KEY

EXERCISE - I JEE Main

1.	B	**2.**	A	**3.**	D	**4.**	A	**5.**	B	**6.**	B	**7.**	A
8.	C	**9.**	B	**10.**	D	**11.**	C	**12.**	D	**13.**	C	**14.**	A
15.	B	**16.**	B	**17.**	C	**18.**	A	**19.**	A	**20.**	C	**21.**	C
22.	A	**23.**	C	**24.**	B	**25.**	A	**26.**	B	**27.**	A	**28.**	C
29.	C	**30**	D	**31.**	A	**32.**	B	**33.**	D	**34.**	B	**35.**	D
36.	A	**37.**	A	**38.**	A	**39.**	C	**40.**	C	**41.**	A	**42.**	C
43.	C	**44.**	C	**45.**	A	**46.**	A	**47.**	D	**48.**	B		

EXERCISE - II JEE Advance (Level - I)

Single correct Option - type Questions

1.	B	**2.**	C	**3.**	D	**4.**	A	**5.**	D	**6.**	D	**7.**	A
8.	D	**9.**	C	**10.**	A	**11.**	A	**12.**	C	**13.**	A	**14.**	D
15.	D	**16.**	C	**17.**	A	**18.**	C	**19.**	B	**20.**	C	**21.**	B
22.	C	**23.**	D	**24.**	C	**25.**	A	**26.**	B	**27.**	B	**28.**	B
29.	B	**30.**	D	**31.**	B	**32.**	A	**33.**	A	**34.**	C	**35.**	C
36.	D	**37.**	B										

(Level - II)

Multiple correct Option - type Questions

1.	A,C,D	**2.**	A,C	**3.**	C,D	**4.**	A,B,D	**5.**	A,C	**6.**	A,C,D	**7.**	B,C
8.	B,C,	**9.**	A,B,C,D			**10.**	A,B,D	**11.**	A,B,C	**12.**	B,C,D	**13.**	A,B
14.	A,B,C												

EXERCISE – III

Subjective - type Questions

1. $x - y = 0; x + 7y = 0$

2. $(5,1)\&(-1,5)$

3. $x^2 + y^2 - 2x - 2y + 1 = 0$ OR $x^2 + y^2 - 42x + 38y - 39 = 0$

4. $a^2(x^2 + y^2) = 4x^2y^2$

5. $(-4,2), x^2 + y^2 - 2x - 6y - 15 = 0$

6. $x^2 + y^2 - 6x + 4y = 0$ OR $x^2 + y^2 + 2x - 8y + 4 = 0$

7. 10

8. $r = 15$

9. $x^2 + y^2 = a^2 + b^2; r = \sqrt{a^2 + b^2}$

10. $x^2 + y^2 - 2x - 2y = 0$

11. $x^2 + y^2 - 3x - 3y + 4 = 0$

12. zero, zero

13. $x^2 + y^2 + x - 6y + 3 = 0$

14. $5x^2 + 5y^2 - 8x - 14y - 32 = 0$

15. 64

17. $x^2 + y^2 + 16x + 14y - 12 = 0$

18. $(-4,4); (-1/2,1/2)$

19. (a) $x^2 + y^2 + 4x - 6y = 0; k = 1;$(b) $x^2 + y^2 = 64$

20. $9x - 10y + 7 = 0$; radical axis

21. $x^2 + y^2 + 7x - 11y + 38 = 0$

22. $x^2 + y^2 + 6x - 3y = 0$

23. $x^2 + y^2 \pm a\sqrt{2}x = 0$

24. square of side, 2; $x^2 + y^2 = 1; x^2 + y^2 = 2$

26. 32 sq. unit

27. $4x - 3y - 25 = 0$ OR $3x + 4y - 25 = 0$

28. (i) (11,16)(ii) (11,8),(iii) (11,12)

29. (i) $3x - 4y = 21; 4x + 3y = 3$;(ii) A(0,1) and B$(-1,-6)$;(iii) $90^\circ, 5(\sqrt{2} \pm 1)$ units
(iv) 25 sq. units, 12.5 sq. units ; (v) $x^2 + y^2 + x + 5y - 6$, x intercept 5 ; y intercept 7

30. $2x - 2y - 3 = 0$

31. 63

32. $x - 7y = 2, 7x + y = 14; (x-1)^2 + (y-7)^2 = 3^2; (x-3)^2 + (y+7)^2 = 3^2; (x-9)^2 + (y-1)^2 = 3^2; (x+5)^2 + (y+1)^2 = 3^2$

34. $\left(2, \frac{23}{3}\right)$

35. $(1,0)\&(1/2,1/2); r = \frac{1}{2\sqrt{2}}$

36. 19

37. 75 sq. unit

38. 19

39. $0 < \theta < \frac{5\pi}{6} - \tan^{-1} 3$

Comprehension - based Questions

40. B **41.** C **42.** D

Matrix Match - type Questions

43. (A) $-$P, Q, R, S;(B) $-$P, Q, R, S, T;(C) $-$R, S

EXERCISE - IV
Previous Year's Question JEE Main

1. B **2.** B **3.** B **4.** B **5.** C **6.** A **7.** D
8. A **9.** A **10.** D

JEE Advanced

1. (a) B; **(b)** A
2. (a) B; (b) C; (c) (i) D, (ii) A, (iii) D
3. (a) B ; (b) 8
4. 3 **5.** D **6.** 0002 **7.** A **8.** A **9.** D **10.** A,C
11. B,C **12.** A,C **13.** 2 **14.** D **15.** A **16.** D **17.** D
18. 10 **19.** 2 **20.** B **21.** D **22.** B **23.** 0.84 **24.** CD