Nishant Jindal

Q Find LLR, TV, axis

for $(3x+4y-4)^2 = 4(4x-3y+1)$

$(\text{Axis})^2 = \text{LLR}\,(\text{TV})$

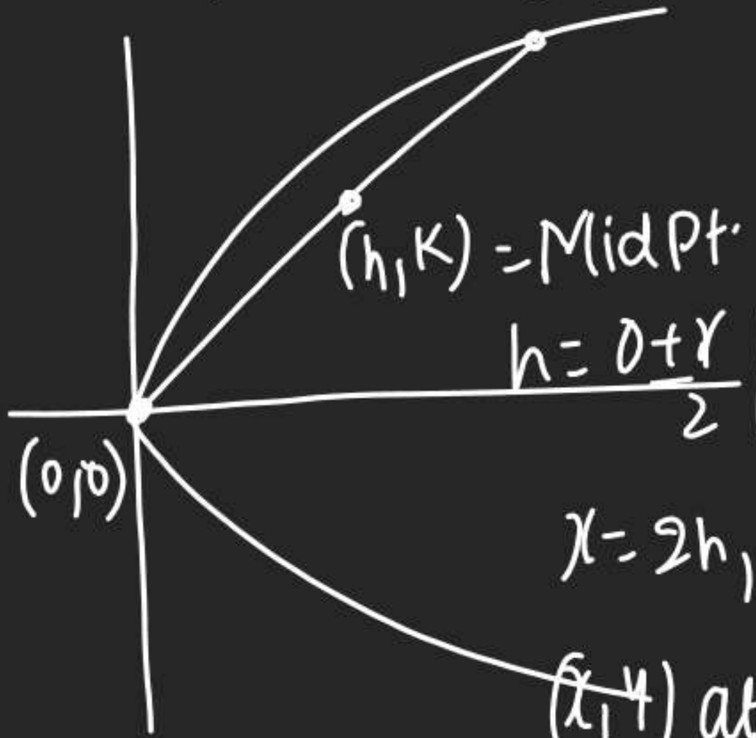
$$\left(\frac{3x+4y-4}{\sqrt{3^2+4^2}}\right)^2 = \frac{4}{5}\left(\frac{4x-3y+1}{\sqrt{4^2+(-3)^2}}\right)$$

Axis $\Rightarrow 3x+4y-4=0$

Tv $\Rightarrow 4x-3y+1=0$

$\text{LLR} = \frac{4}{5}$

Q If one vertex of chord of Parabola $y^2=4ax$ is at $(0,0)$ find Locus of Mid Pt of chord.



(h,k) = Mid Pt.

$h = \frac{0+x}{2}$ | $k = \frac{0+y}{2}$

$x = 2h,\ y = 2k$

(x,y) at Parabola $y^2=4ax$

$(2k)^2 = 4a(2h)$

$4k^2 = 8ah$

$k^2 = 2ah$

$y^2 = 2ax$ ← It is also Para

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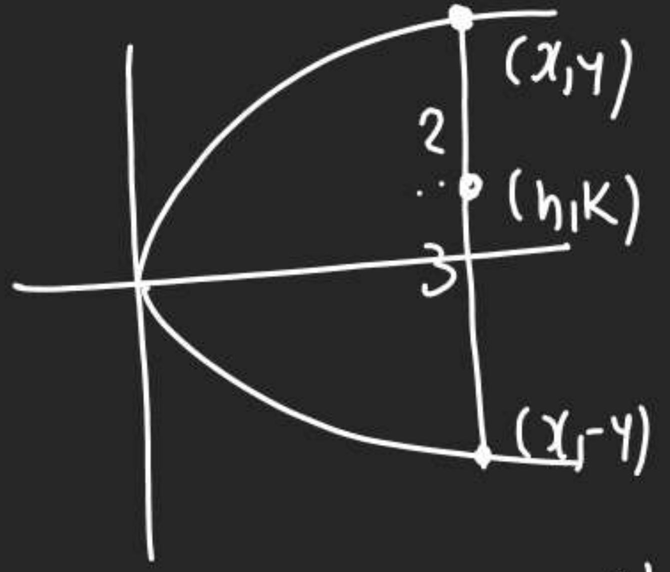
Q Find Locus of Pt. which divide Double Ordinate in 2:3 for $y^2 = 4ax$



$h = \frac{2x+3x}{2+3}$ | $k = \frac{-2y+3y}{2+3}$

$5x = 5h$ | $5k = y$

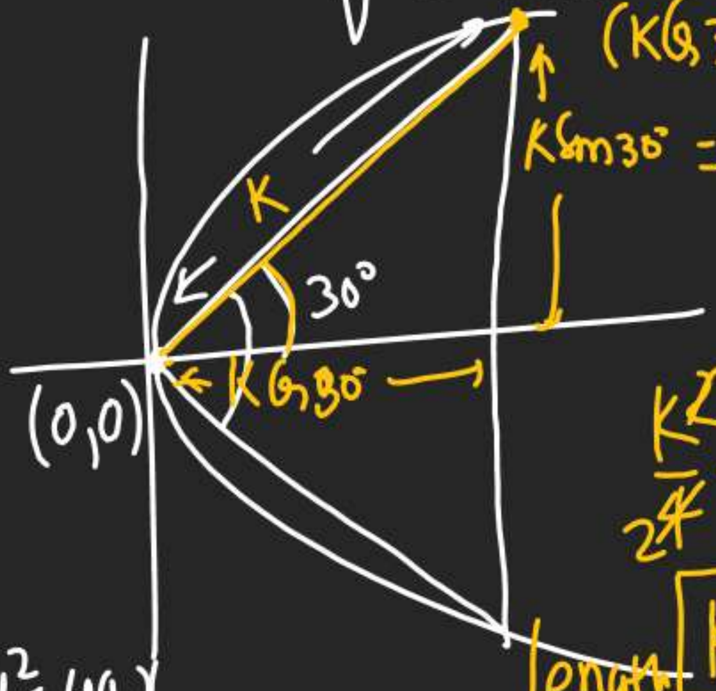
$h = x$

(x,y) lying on Par. $y^2 = 4ax$

$(5k)^2 = 4ah$

$k^2 = \frac{4}{25}ah$ | $\boxed{y^2 = \frac{4}{25}ax}$

Q Find Sides of Eq. Δ inscribed in $y^2 = 4ax$, if one of its vertex coincides with vertex of Parabola.



$= \left(\frac{k\sqrt{3}}{2}, \frac{k}{2}\right)$

on $y^2 = 4ax$

$\frac{k^2}{4} = 4a \times \frac{k\sqrt{3}}{2}$

$\boxed{k = 8a\sqrt{3}}$

length

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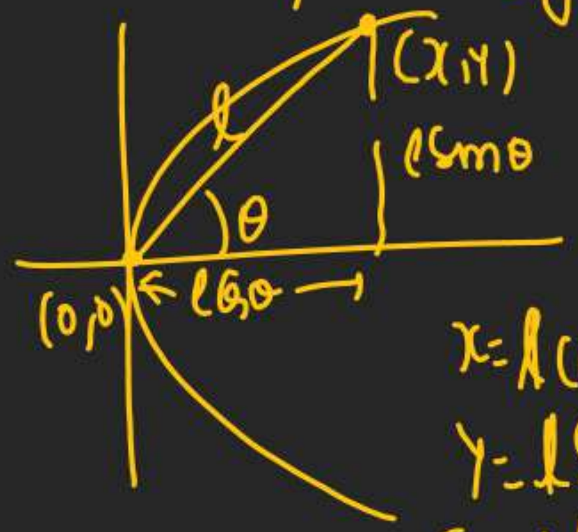
Q If one vertex to the chord to the Parabola $y^2 = 4ax$ is $(0,0)$. If chord makes angle θ with +ve x Axis find length of chord. $\rightarrow l$



$x = l\cos\theta$

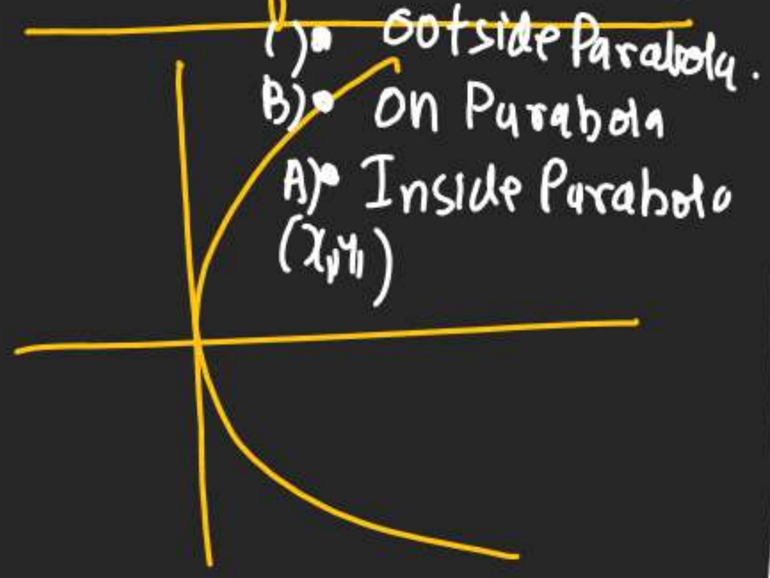
$y = l\sin\theta$

(x,y) lying on $y^2 = 4ax$

$l^2\sin^2\theta = 4a\,l\cos\theta$

$l = 4a\dfrac{\cos\theta}{\sin^2\theta} \Rightarrow l = 4a\cot\theta\cdot\operatorname{cosec}\theta$

Position of Pt. WRT. Parabola



3 Positions Psbl.

$Par(x_1,y_1) > 0$ outside

$= 0$ on Par.

< 0 Inside Par.

Q Find Position of $(1,2)$ WRT $y^2 = 16x$?

Par: $y^2 - 16x = 0$

$(1,2) \rightarrow 4 - 16\times 1 = -ve$

Inside Parabola

Q Find Position $(7,2)$ WRT. $y^2 = 16x$?

Par $y^2 - 16x = 0$

$(7,2) \Rightarrow 4 - 16\times 7 < 0$

Inside.

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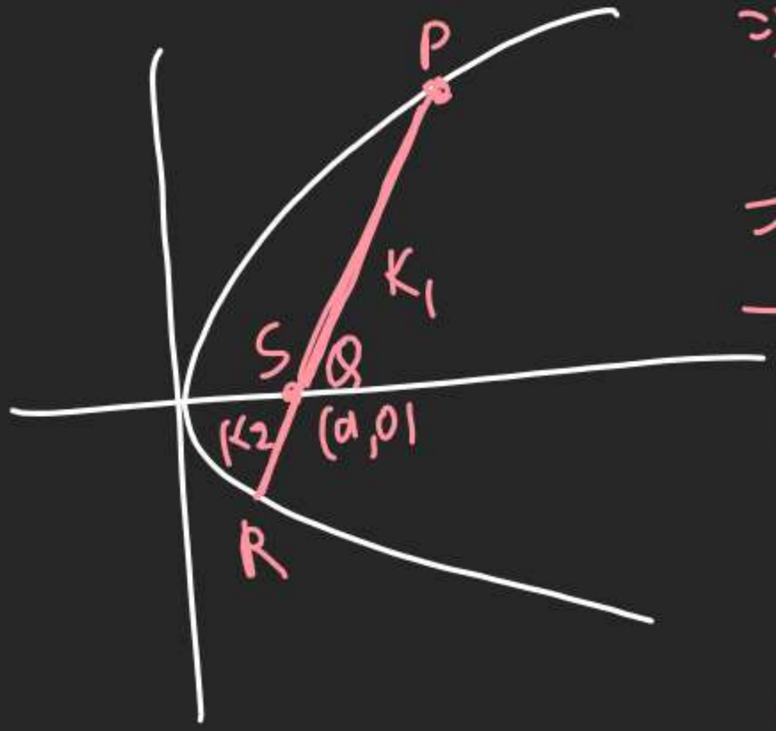
RK.

Semi $\boxed{L.R.}$ ($=2a$) of $y^2=4ax$ is HM betⁿ

Segments of any Focal chord of Parabola.



$a, b, c \to HP$

$\Rightarrow \frac{1}{a}+\frac{1}{c}=\frac{2}{b}$

$\Rightarrow$ b in HM of a, c

$2a$ is HM of K_1, K_2

$\frac{1}{K_1}+\frac{1}{K_2}=\frac{2}{2a}$

$\Rightarrow \boxed{\frac{1}{K_1}+\frac{1}{K_2}=\frac{1}{a}}$

Q If PQ is Focal chord of $y^2=8x$ & SP $=6$ units find SQ?

$4a=8$

$K_1=6$

$K_2=?$

Semi LR $\boxed{2a=4}$

$\frac{1}{K_1}+\frac{1}{K_2}=\frac{1}{a}$

$\frac{1}{6}+\frac{1}{K_2}=\frac{1}{2}$

$\frac{1}{K_2}=\frac{1}{2}-\frac{1}{6}=\frac{3-1}{6}=\frac{2}{6}=\frac{1}{3}$

$K_2=3$

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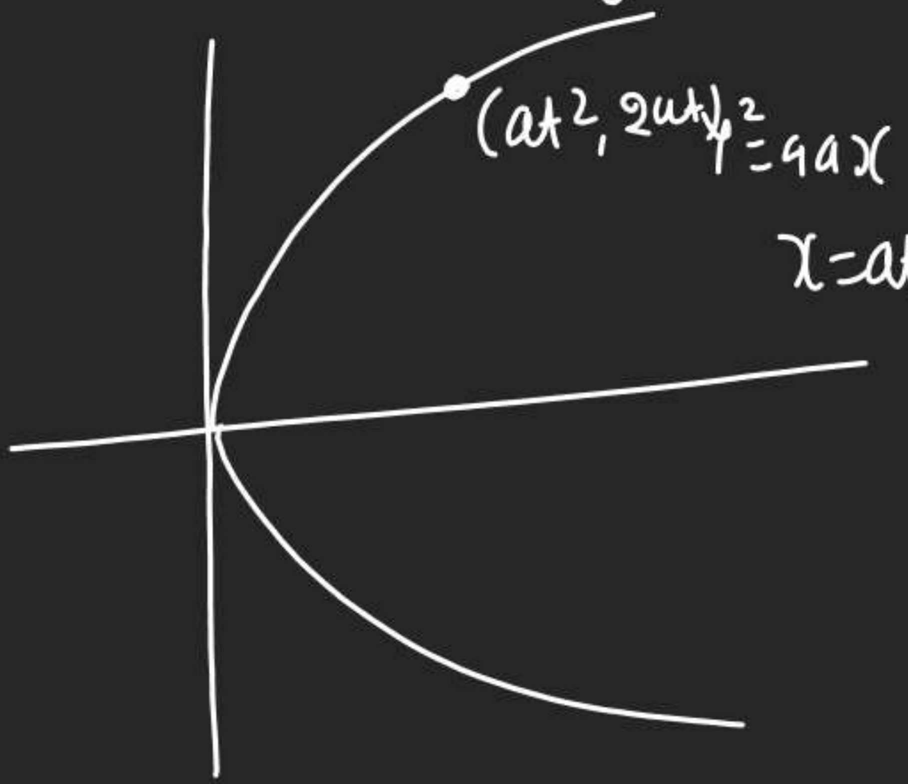
Parameteric Cordinates (Battery)

→ Purpose → Achanak se Koi Point Curve Par Lena Pde we think 1st about Par. Coord.



$x = at^2$ in Par.

$y^2 = 4a^2t^2$

$y = 2at$

$\boxed{x = at^2, y = 2at \text{ is Par. Coord of } y^2 = 4ax.}$

Q Find Par. Coord of

A) $y^2 = 4x$

$a = 1$

$\therefore \left.\begin{array}{l} x = t^2 \\ y = 2t \end{array}\right\} (t^2, 2t)$

(B) $y^2 = 16x$ Par. Coord?

$a = 4$

Par. Coord $(4t^2, 8t)$

Q Find Par. coord of $x^2 = 4y$

$y = t^2$

$x^2 = 4t^2$

$x = 2t$

$(x, y) = (2t, t^2)$

Q Par. Coord $x^2 = 32y$

$y = 2t^2 \Rightarrow x^2 = 32 \times 2t^2$

$x^2 = 64t^2$

$x = 8t$

$(8t, 2t^2)$

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Q $x^2 = 32y$

$y = t^2$

$x^2 = 32t^2$

$x = 4\sqrt{2}t$

$(x,y) \equiv (4\sqrt{2}t, t^2)$

$(8t, 2t^2)$

More favourable

Q Par. Coord $x^2 = -8y$

$y = -t^2$

$x^2 = -8 \times -t^2$

$x^2 = 8t^2$

$x = 2\sqrt{2}t$

$(2\sqrt{2}t, -t^2)$

2nd

$y = -2t^2$

$x^2 = -8 \times -2t^2$

$x^2 = 16t^2$

$x = 4t$

$(x,y) \equiv (4t, -2t^2)$

Q Par. Coord $y^2 = x^3$

$x = t^2$

$y^2 = (t^2)^3 = t^6$

$y = t^3$

$\therefore (x,y) = (t^2, t^3)$

Q. $y^2 = -2x$ Par. Coord

$x = -2t^2$

$y^2 = -2 \times -2t^2$

$y^2 = 4t^2 \Rightarrow y = 2t$

$(-2t^2, 2t)$

$y^2 = 4ax$ $(at^2, 2at)$

$y^2 = -4ax$ $(-at^2, 2at)$

$x^2 = 4ay$ $(2at, at^2)$

$x^2 = -4ay$ $(2at, -at^2)$

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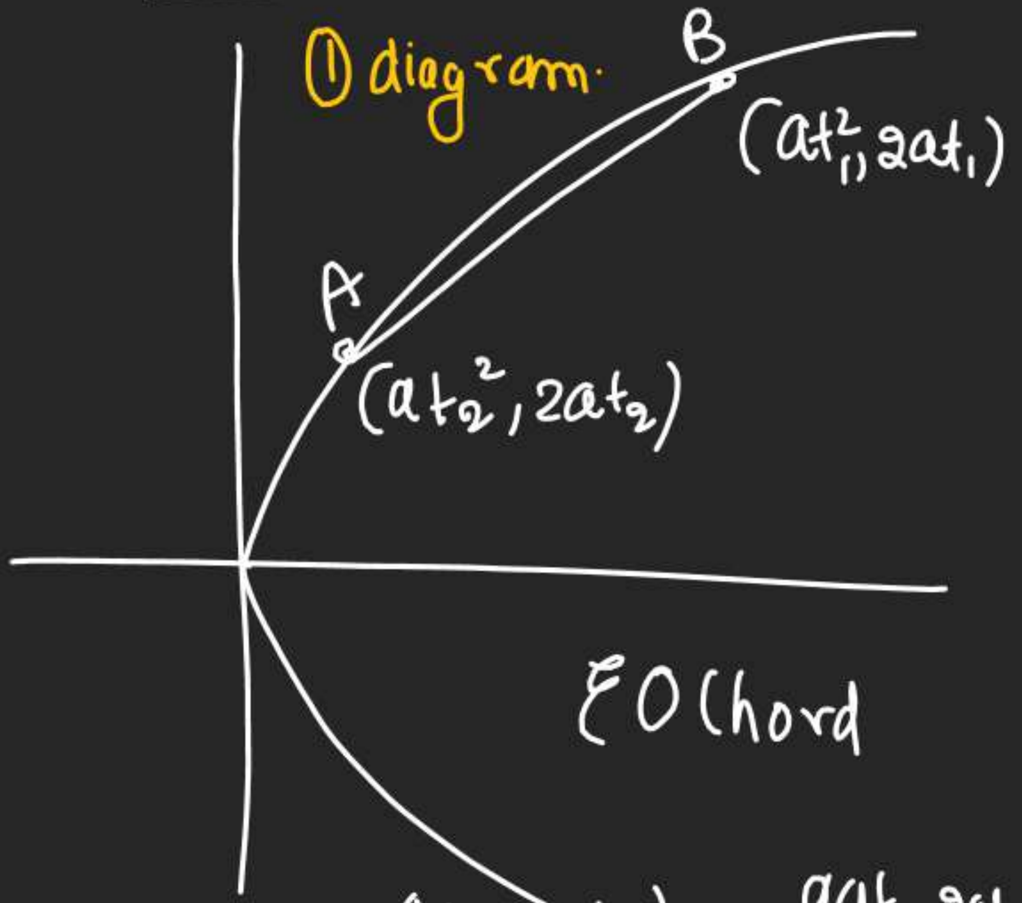
Eq^n of chord

① diagram.



EO Chord

$$(y-2at_1) = \frac{2at_1-2at_2}{at_1^2-at_2^2}(x-at_1^2)$$

$$(y-2at_1) = \frac{2}{(t_1+t_2)}(x-at_1^2)$$

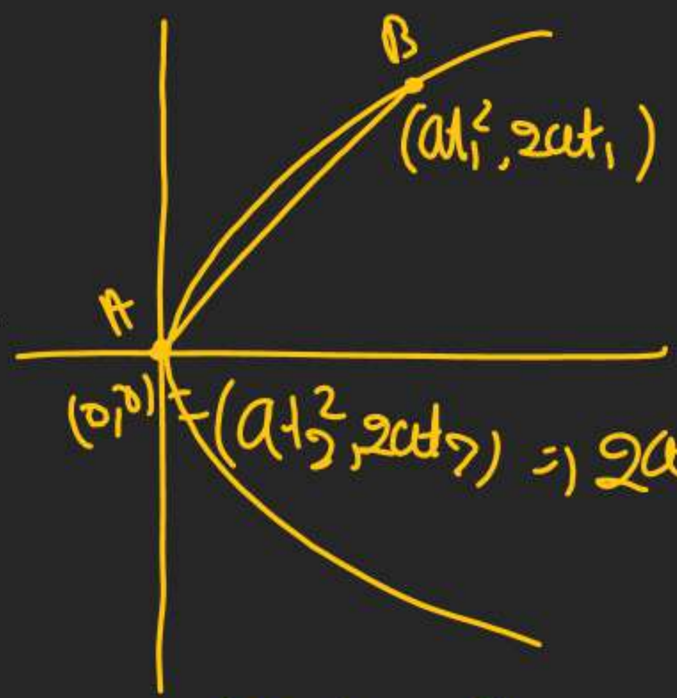
$$(t_1+t_2)(y-2at_1) = 2(x-at_1^2)$$

$$y(t_1+t_2) - \cancel{2at_1^2} - 2at_1t_2 = 2x - \cancel{2at_1^2}$$

$\Rightarrow$ (2) $\boxed{2x - y(t_1+t_2) + 2at_1t_2 = 0}$

(3) $(Sl)_{chord} = \dfrac{2}{t_1+t_2}$

Q Find slope of chord having one of its vertex with vertex of Parabola.



$\Rightarrow 2at_2 = 0$

$t_2 = 0$

$(Sl) = \dfrac{2}{t_1+t_2}$

(4) $\boxed{Sl = \dfrac{2}{t_1}}$

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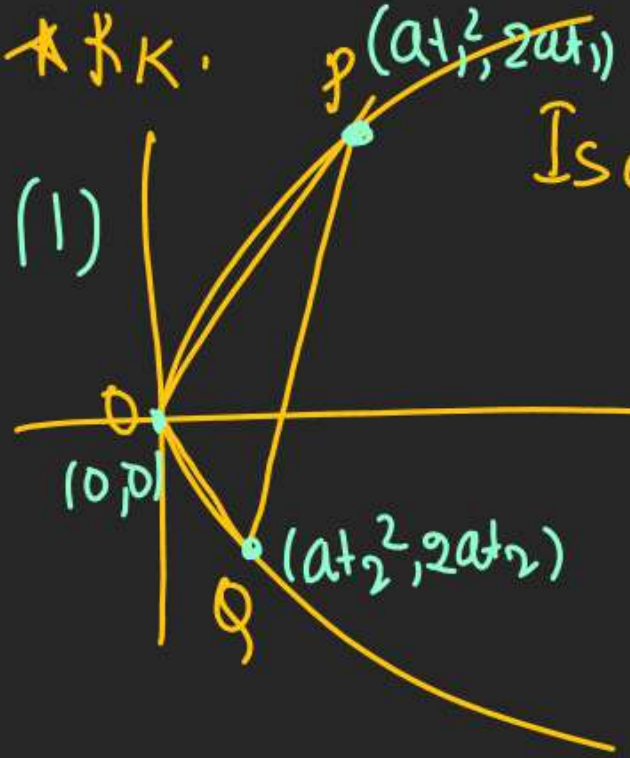
**K.

(1)



Is diagramme vertex Kya hongei

$P \equiv (at_1^2, 2at_1)$

$Q \equiv (at_2^2, 2at_2)$

$O \equiv (0,0)$

$(Sl)_{OP} = \frac{2}{t_1}$

$(Sl)_{OQ} = \frac{2}{t_2}$

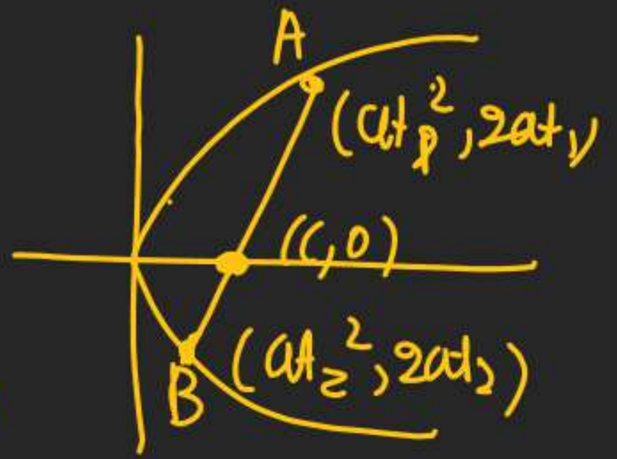
(2) If OPQ is a Rt. angle Δ.

$\frac{2}{t_1} \times \frac{2}{t_2} = -1$

$\boxed{t_1 t_2 = -4}$

Eqn of chord

$2x - y(t_1+t_2) + 2at_1t_2 = 0$



Chord P.T. fixed Pt (c,0)

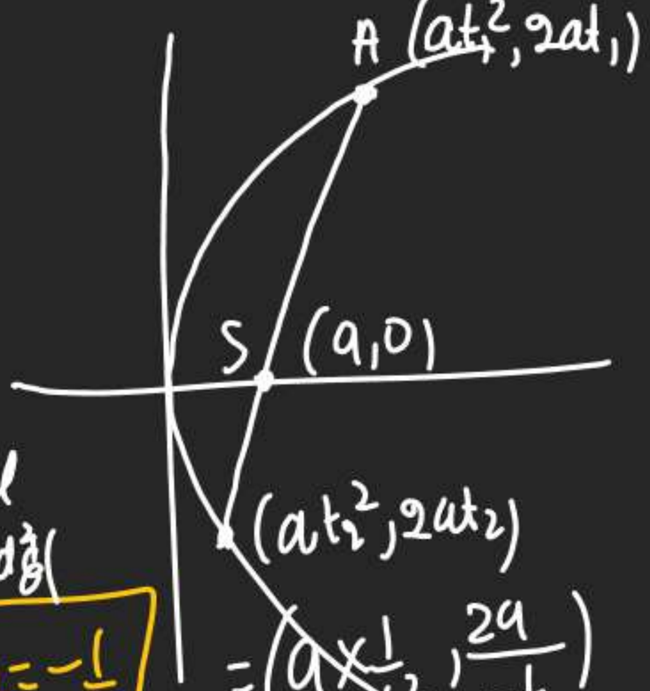
$2c - 0 + 2at_1t_2 = 0$

$\boxed{t_1 t_2 = -\frac{c}{a}}$

Chord P.T. Focus (a,0) ~ Focal Chord

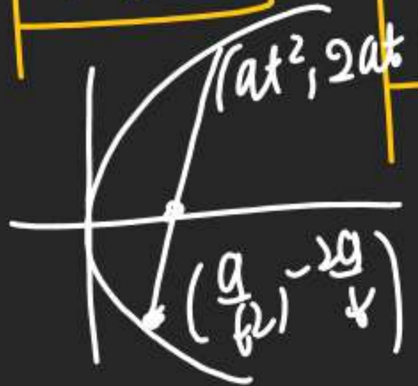
$2a - 0 + 2at_1t_2 = 0$

$\boxed{t_1 t_2 = -1} \Rightarrow \boxed{t_2 = -\frac{1}{t_1}}$



$= \left(a \times \frac{1}{t_1^2}, \frac{2a}{-t_1}\right)$

$= \left(\frac{a}{t_1^2}, \frac{-2a}{t_1}\right)$



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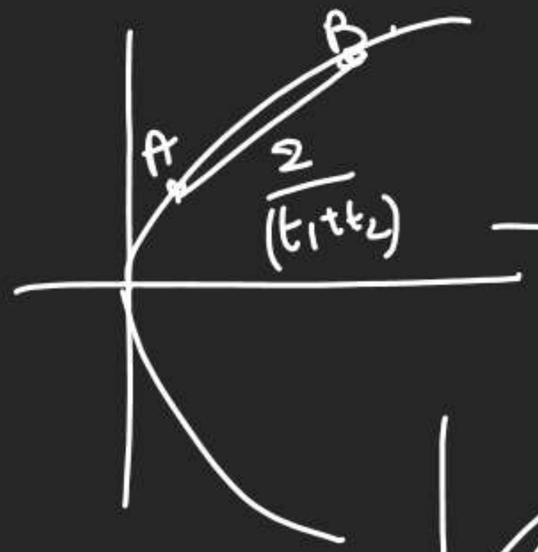
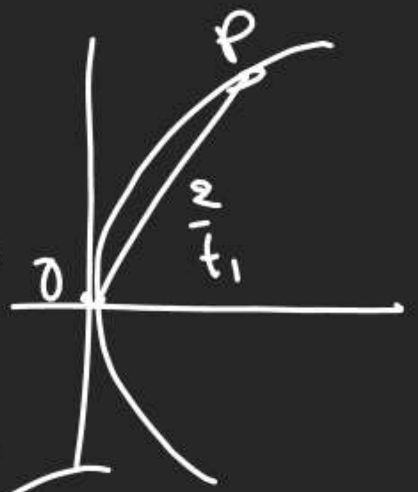
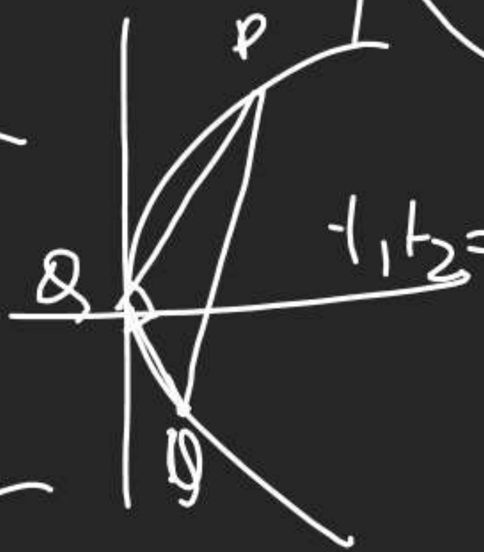
① Par. Coord.

② Chord.

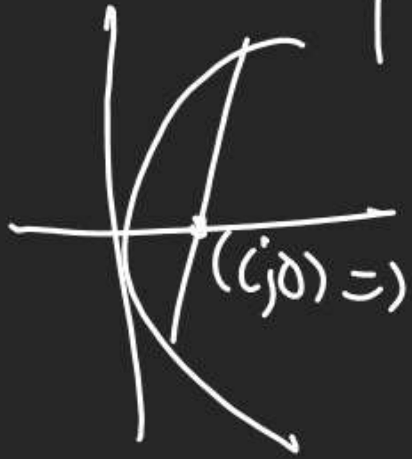


$\longrightarrow$





$t_1t_2 = -4.$



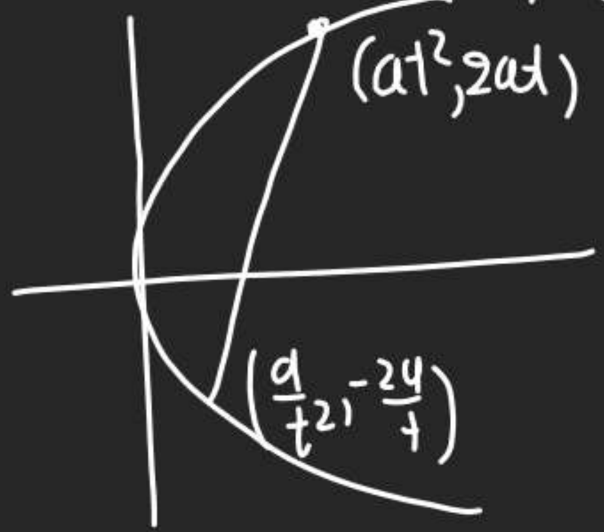
$(c,0) \Rightarrow t_1t_2 = -\frac{c}{a}.$

Focal Chord.

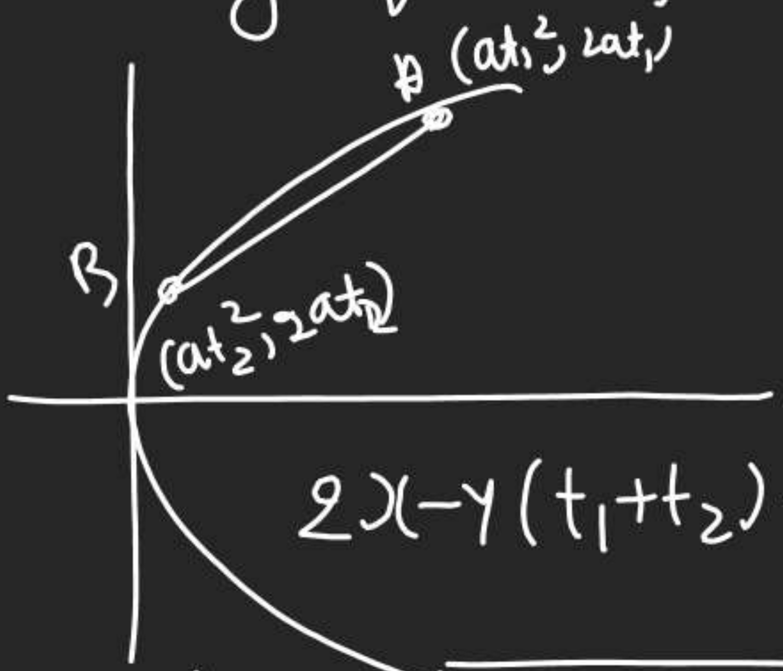
$t_1t_2 = -1$

$t_2 = -\frac{1}{t_1}$

Focal Chord ⇒ [illegible]



Q Find Length of chord for $y^2 = 4ax$



$$2x - y(t_1+t_2) + 2at_1t_2 = 0$$

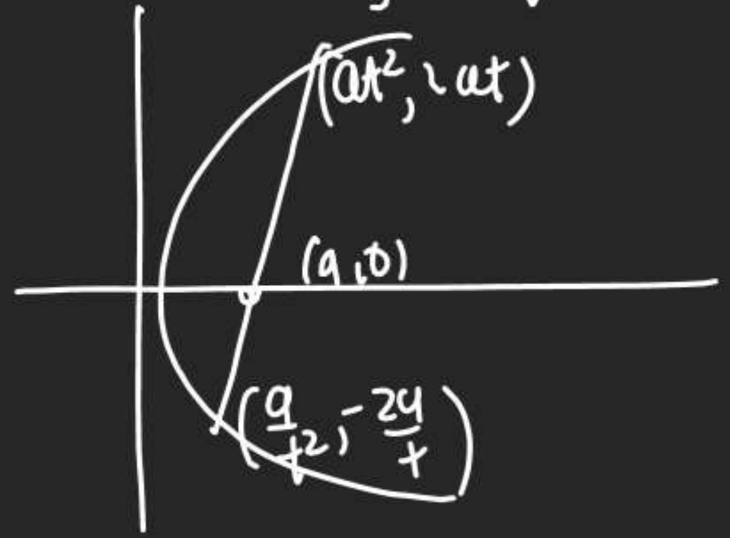
$$AB = \sqrt{(2at_2 - 2at_1)^2 + (at_2^2 - at_1^2)^2}$$

$$= a\sqrt{4(t_2-t_1)^2 + (t_2-t_1)^2(t_2+t_1)^2}$$

$$= a(t_2-t_1)\sqrt{4+(t_1+t_2)^2}$$

$$t_1 = t$$
$$t_2 = -\frac{1}{t}$$

Q Find length of Focal Chord



$$L = a\left(-\frac{1}{t} - t\right)\sqrt{4 + \left(t - \frac{1}{t}\right)^2}$$

$$= -a\left(t+\frac{1}{t}\right)\sqrt{t^2 + \frac{1}{t^2} - 2 + 4}$$

$$= -a\left(t+\frac{1}{t}\right)\sqrt{\left(t+\frac{1}{t}\right)^2}$$

$$= -a\left(t+\frac{1}{t}\right)^2$$