

## FUNCTIONS

3.  $f: \mathbb{R} \rightarrow \mathbb{R}, f(x) = x^3 + x^2 + 3x + \sin x \rightarrow \text{continuous}$

$$f(x) = x^3 \left( 1 + \frac{1}{x} + \frac{3}{x^2} + \frac{\sin x}{x^3} \right)$$

$$x \rightarrow -\infty, y \rightarrow -\infty$$

$$x \rightarrow \infty, y \rightarrow \infty$$

$$R_f = \mathbb{R}$$

$$f'(x) = 3x^2 + 2x + 3 + \cos x = \underbrace{3x^2 + 2x + 2}_{>0} + \underbrace{(1 + \cos x)}_{\geq 0} > 0$$

Bijjective

4.  $f: \mathbb{R} \rightarrow \mathbb{R}$ ,  $f(x) = \frac{2x^2 - x + 5}{7x^2 + 2x + 10}$  Intro & M-1

continuous

$$y = \frac{1}{2}$$

$$\frac{1}{2} = \frac{2x^2 - x + 5}{7x^2 + 2x + 10} \Rightarrow 3x^2 + 4x = 0$$

$$x = 0, -\frac{4}{3}$$

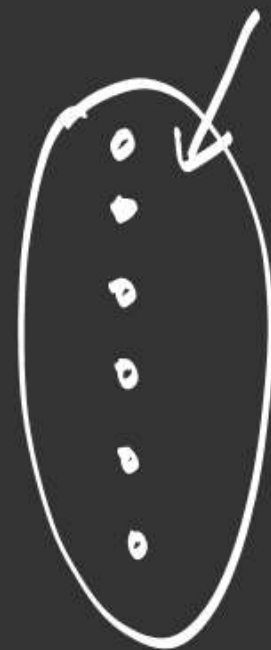
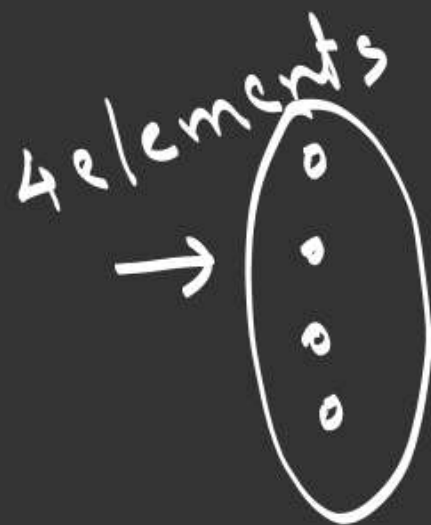
M-1

$f$  is cont  
 $x \rightarrow -\infty, y \rightarrow \frac{2}{7}$   
 $f(x) =$

$x \rightarrow \infty, y \rightarrow \frac{2}{7}$

$$\frac{2 - \frac{1}{x} + \frac{5}{x^2}}{7 + \frac{2}{x} + \frac{10}{x^2}}$$

$$y = \frac{2}{7}$$

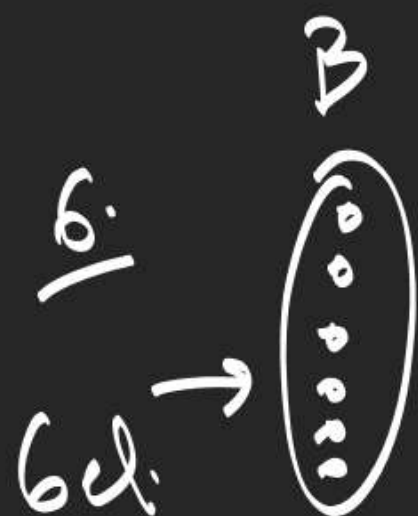
5. $f: A \rightarrow B$ ①  $6^4 \rightarrow$ ②  $(1-1) \text{ fns}$  ${}^6C_4 4! \text{ or } {}^6P_4$ ③  $M-1 = 6^4 - (1-1) = 6^4 - {}^6P_4$ 

④ Onto fns = 0

⑤ Into fns =  $6^4$ 

⑥ Bijective = 0

# FUNCTIONS



①  $4^6$

②  $r-1 \text{ fn} = 0$

$B_1, B_2, B_3, B_4$

Distribute 6 diff books over 4 boys  $\rightarrow$  each boy get atleast one

④ Onto

1 1 1 3

1 1 2 2

$\frac{6!}{(1!)^3 3! 3!} \times 4!$

$\frac{6!}{(1!)^2 (2!)^2 2! 2!} \times 4!$

Onto =

$4^6 - \left( {}^4C_1 3^6 - {}^4C_2 2^6 + {}^4C_3 1^6 \right)$

Into

③  $M-1 = 4^6$

$E_i = B_i \text{ get none}$   
 $i=1,2,3,4$

3. (iv) ~~(vi)~~ <sup>leave</sup>  $y = f(x) = \frac{x}{1+|x|}$

$$R_f = (-1, 1)$$

⇓  
Ans.

I)  $x \geq 0$  ✓

$$y = \frac{x}{1+x}$$

$$y + yx = x$$

$$x = \frac{y}{1-y} \geq 0$$

$$y \in [0, 1)$$

OR

II)  $x < 0$

$$y = \frac{x}{1-x}$$

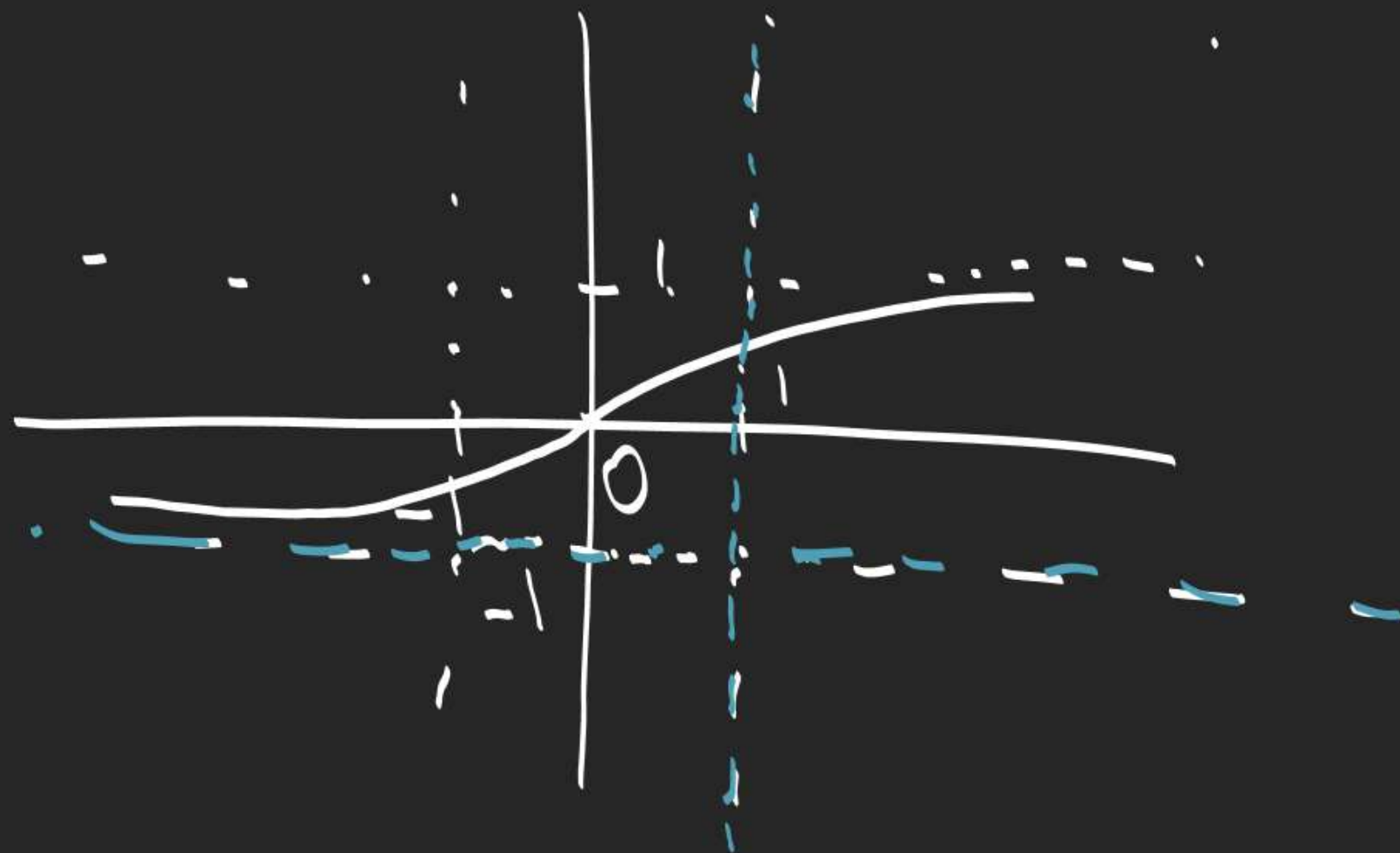
$$y - xy = x$$

$$x = \frac{y}{1+y} < 0$$

$$y \in (-1, 0)$$

# FUNCTIONS

$$y = \frac{x}{1+|x|} = \begin{cases} \frac{x}{1+x} = 1 - \frac{1}{x+1} & x \geq 0 \Rightarrow y = -\frac{1}{x} \\ \frac{x}{1-x} = -1 - \frac{1}{x-1} & x < 0 \Rightarrow y = -\frac{1}{x} \end{cases}$$



5.

$$f(x) = 2|\sin x| - 3|\cos x|$$

$$y_{\max} = 2 - 0$$

$$y_{\min} = 0 - 3 = -3$$

$$R_f = [-3, 2]$$

6. (ii)

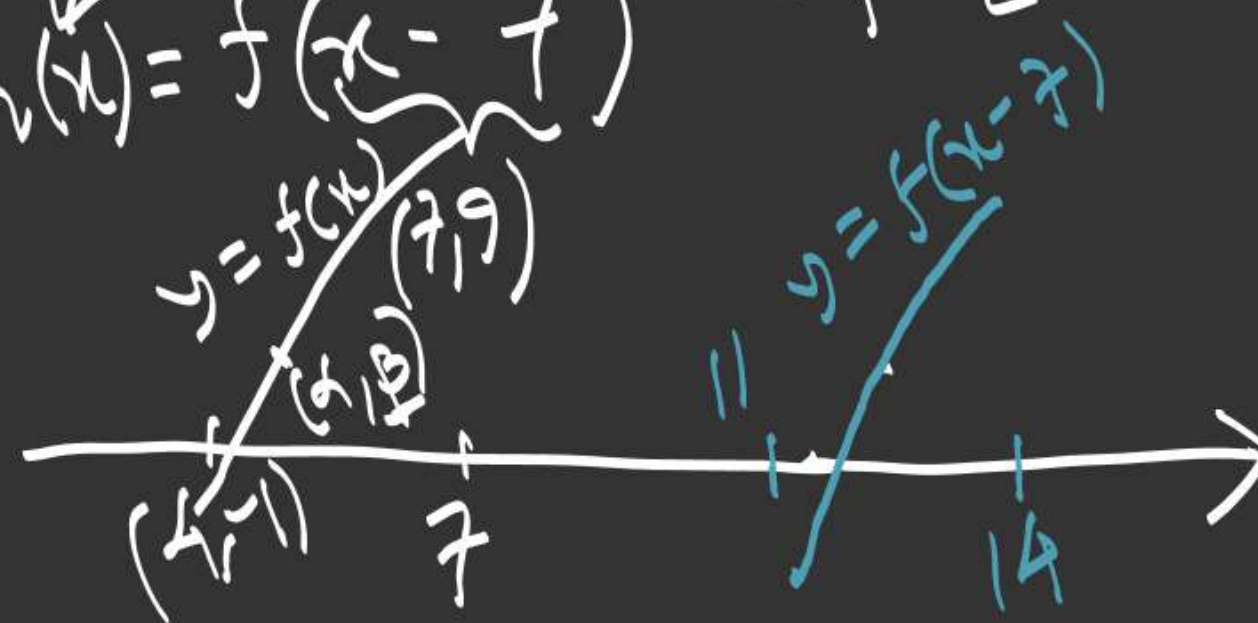
$$D_f = [4, 7]$$

$$R_f = [-1, 9]$$

$$4 \leq x-7 \leq 7$$

$$h(x) = f(x-7) \quad x \in [11, 14]$$

$$R_f = [-1, 9]$$





Find ① No. of bijective function defined from  $A \rightarrow B$

Ans  $\rightarrow 24 = 4!$

②

~ ||

$= 9$

such that  $f(x_i) \neq i$   
 $\forall x_i, i = 1, 2, 3, 4$ .  
Derangement

**FUNCTIONS**Composite Function

$$f: A \rightarrow B, g: C \rightarrow D$$

$$(f \circ g)(x) = f(g(x))$$

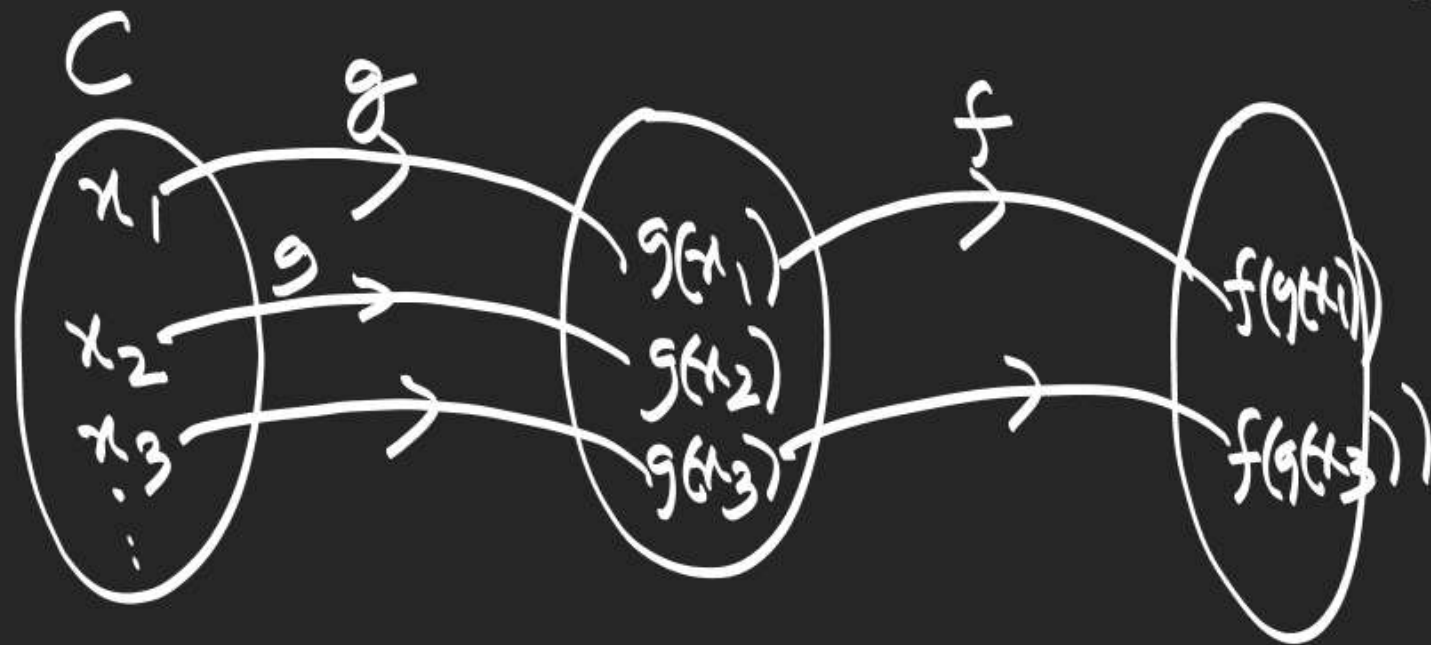
$$(f \circ f)(x) = f(f(x))$$

## FUNCTIONS

$$f: A \rightarrow B, g: C \rightarrow D$$

$$f \circ g(x) = f(\underline{g(x)})$$

$$D_{f \circ g} = \left\{ x \mid x \in D_g \text{ \& } g(x) \in D_f \right\}$$



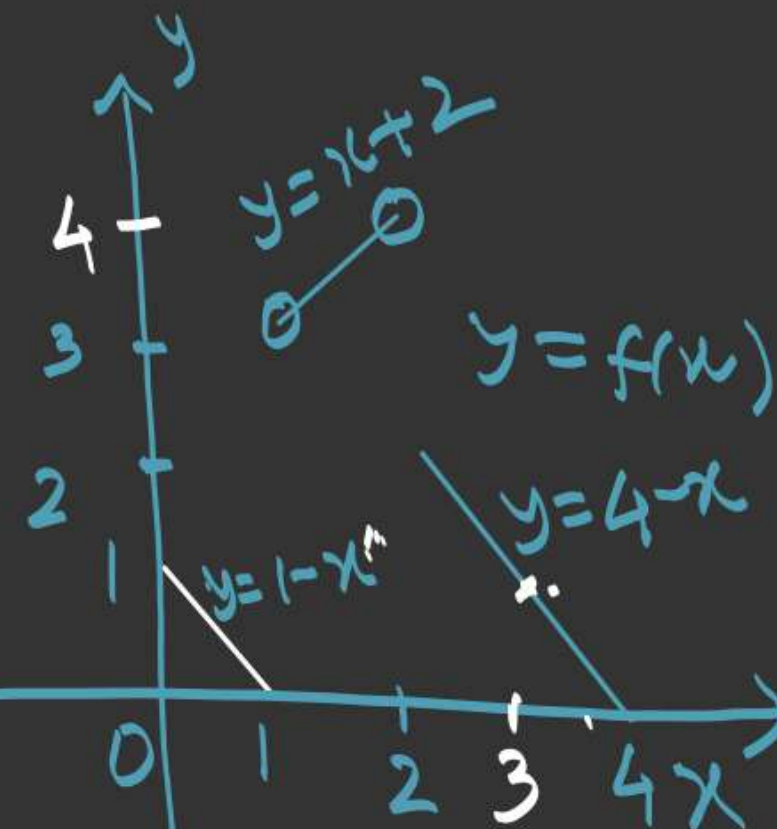
1.  $f(x) = \begin{cases} 1-x & \text{if } 0 \leq x \leq 1 \\ x+2 & \text{if } 1 < x < 2 \\ 4-x & \text{if } 2 \leq x \leq 4 \end{cases}$

Find  $f \circ f(x)$ 

$$f(f(x)) = \begin{cases} 1-f(x) & 0 \leq f(x) \leq 1 \\ f(x)+2 & 1 < f(x) < 2 \\ 4-f(x) & 2 \leq f(x) \leq 4 \end{cases}$$

$$f(f(x)) = \begin{cases} 4-(x+2) & ; x \in (1, 2) \\ 1-(1-x) & ; x \in [0, 1] \\ 1-(4-x) & , x \in [3, 4] \\ (4-x)+2 & ; x \in (2, 3) \\ 4-2 & ; x=2 \end{cases}$$

$f \circ f = [0, 4]$



$$\{ 4-(x+2) \quad x \in (1, 2) \}$$

$$f(x) = \begin{cases} x^2 - 1 & -2 \leq x \leq 1 \\ x^2 + 3 & 1 < x \leq 3 \end{cases}$$

$$f(g(x)) = \begin{cases} g^2(x) - 1 & -2 \leq g(x) \leq 1 \\ g^2(x) + 3 & 1 < g(x) \leq 3 \end{cases}$$

$$g(x) = \begin{cases} \frac{x}{|x|} - x^2 & 0 < |x| \leq 1 \\ x - 2 & |x| > 1 \end{cases}$$

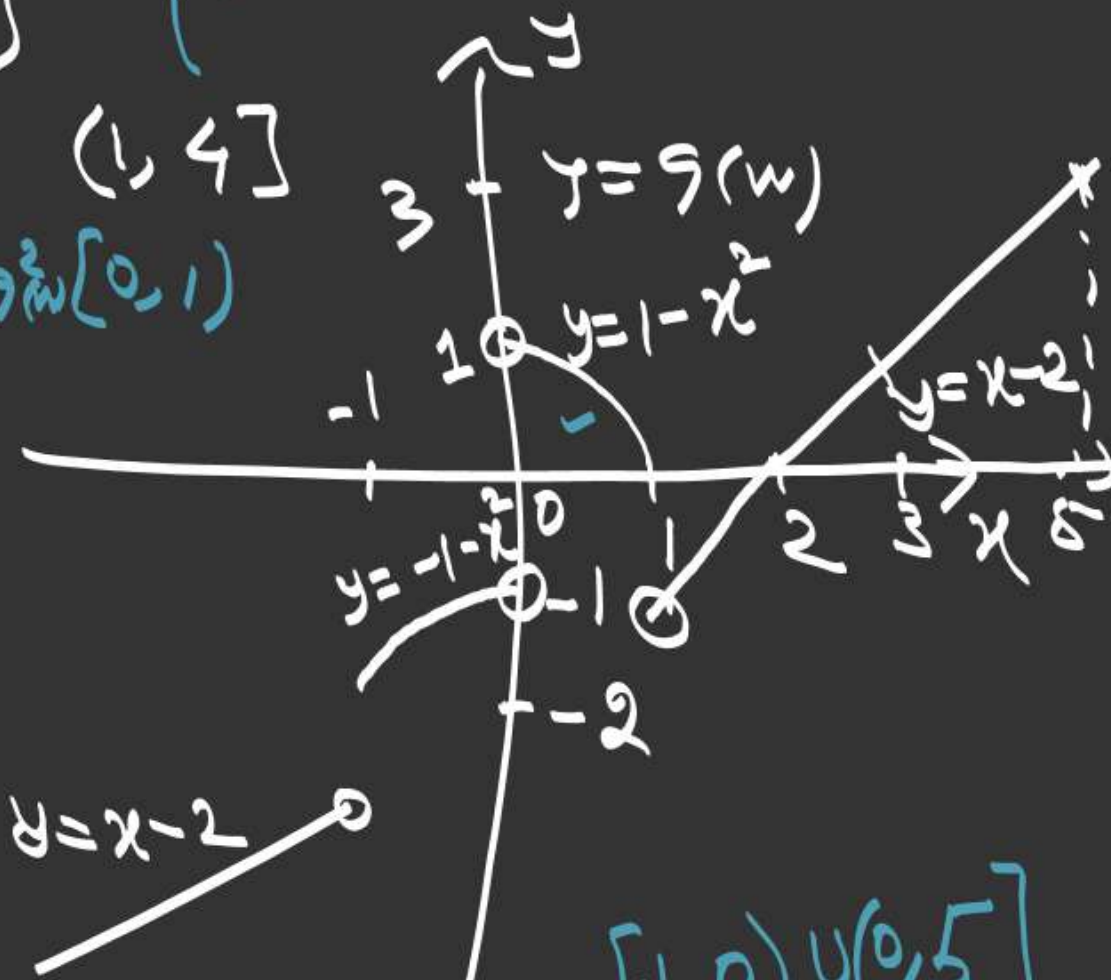
$$[-1, 0) \cup (0, 1]$$

$$0 < |x| \leq 1$$

$$|x| > 1$$

$$(1, 4]$$

$$g^2(x) [0, 1]$$



HW  
PT-2

$$f(g(x)) = \begin{cases} (-1 - x^2)^2 - 1 & x \in [-1, 0) \\ (1 - x^2)^2 - 1 & x \in (0, 1] \end{cases}$$

leaving

Q3, 4, 10

$$(1 - x^2)^2 - 1 \quad x \in (0, 1]$$

$$(x + 2)^2 - 1 \quad x \in (1, 3]$$

$$(x + 2)^2 + 3 \quad x \in (3, 5]$$

$$D_{f \circ g} = [-1, 0) \cup (0, 5]$$

$$R_{f \circ g} = [-1, 3] \cup (4, 12]$$

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