Nishant Jindal

GRAVITATION

** Gravitation field inside the Cavity of a uniform Solid Sphere

$$\vec{E} = \frac{\rho}{3\varepsilon_0}\left(\vec{r}_{c_1 c_2}\right)$$

$$\downarrow$$

$$\frac{1}{4\pi\varepsilon_0} = G,$$

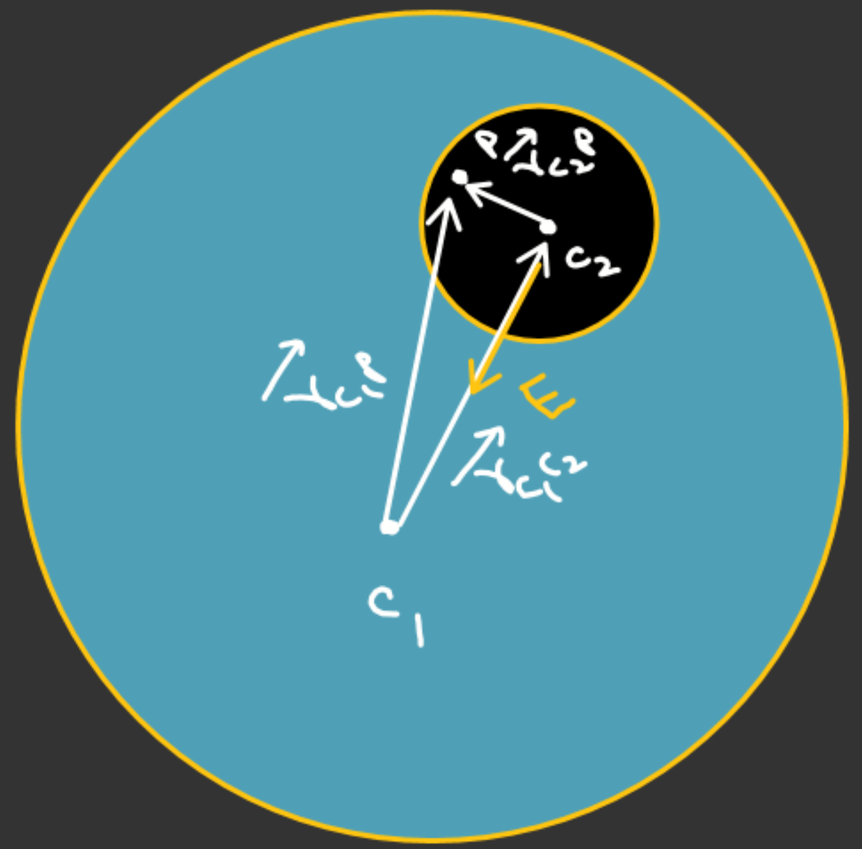
$$\frac{1}{\varepsilon_0} = (4\pi G)$$

**

$$\boxed{\vec{E} = -\frac{\rho 4\pi G}{3}\,\vec{r}_{c_1 c_2}}$$

$$\rho = \left(\frac{M}{\frac{4}{3}\pi R^3}\right)$$

M = Total Mass without Cavity.



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GRAVITATION

Self Energy of a uniform Solid Sphere

$$\boxed{U_{self} = \left(\frac{3}{5}\frac{GM^2}{R}\right)}$$

Electro

$$U = \frac{3}{5}\left(\frac{KQ^2}{R}\right)$$

$K \rightarrow G$

$Q \rightarrow M$

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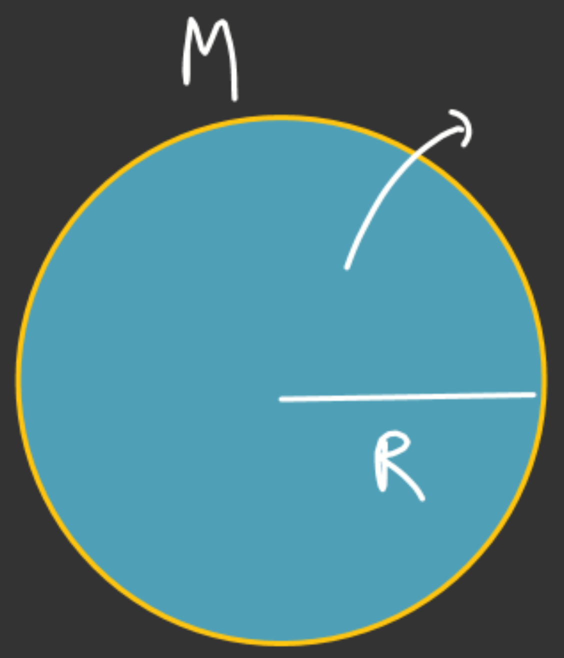
GRAVITATION

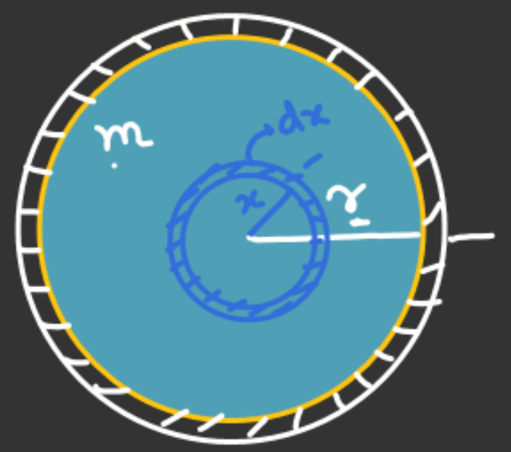
⇒ Find Self Energy of a Solid Sphere whose mass density $\rho = \underline{Kr}$ where K is a constant and r is radial function.



[m = mass of Solid Sphere Of radius r.

dm

∞



$$m = \int_0^r \rho_x \, dV$$

$$= \int_0^r (4\pi x^2 dx)\, Kx$$

$$= 4\pi K \int_0^r x^3 dx$$

$$= \frac{\cancel{4}\pi K r^4}{\cancel{4}} = \underline{\pi K r^4}$$

$$dm = (4\pi r^2 dr)\rho_r$$

$$= (Kr) 4\pi r^2 dr$$

$$= \underline{4\pi K r^3 dr}$$

$$dW = dm\, V_m.$$

$$dW = dm\left(-\frac{Gm}{r}\right)$$

$$dW = (-)\ \frac{G\, m\, dm}{r}$$

$$-dW = \frac{G m\, dm}{r}$$

$$\Downarrow$$

$$dU_{self} = \left(\frac{G m\, dm}{r}\right)$$

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U_{self}

$$dU_{self} = \frac{G\,m\,dm}{r}$$

$$\int_0^{U} dU_{self} = \int_0^R \frac{G\,(\pi K r^4)\,(4\pi K r^3 dr)}{r}$$

$$U_{self} = 4G(\pi K)^2 \int_0^R r^6 \cdot dr$$

$$U_{self} = \frac{4G\pi^2 K^2 R^7}{7}$$

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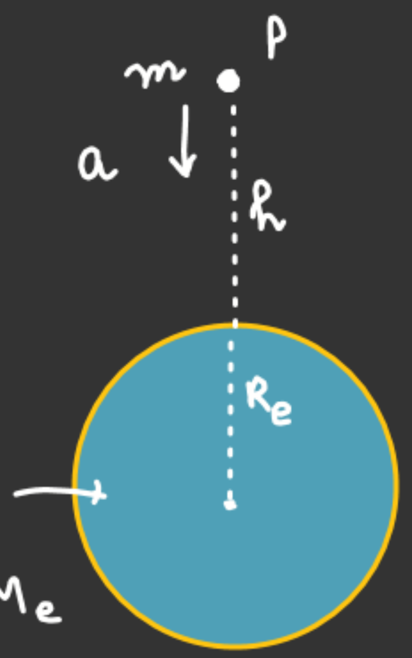
GRAVITATION

Variation of 'g'

Along height



$$F_{m/M_e} = \frac{GM_e m}{(R_e+h)^2}$$

$$\frac{F_{m/M_e}}{m} = g' = \frac{GM_e}{(R_e+h)^2}$$

$$g' = \frac{GM_e}{R_e^2\left(1+\frac{h}{R_e}\right)^2}$$

$$\boxed{g' = \frac{g}{\left(1+\frac{h}{R_e}\right)^2}}$$

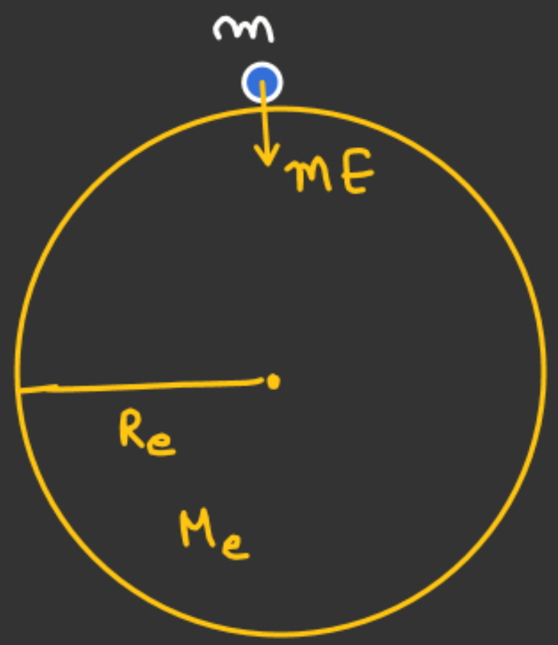
$$E = \frac{GM_e}{R_e^2} \checkmark$$

$$F_{m/M_e} = m\cdot\left(\frac{GM_e}{R_e^2}\right)$$

$$\boxed{g = \left(\frac{F_{m/M_e}}{m}\right) = \frac{GM_e}{R_e^2}}$$

$\Downarrow$

Accleration due to gravity



If $h \ll R_e$

$$g' = g\left(1+\frac{h}{R_e}\right)^{-2}$$

$$\boxed{g' = g\left(1-\frac{2h}{R_e}\right)}$$

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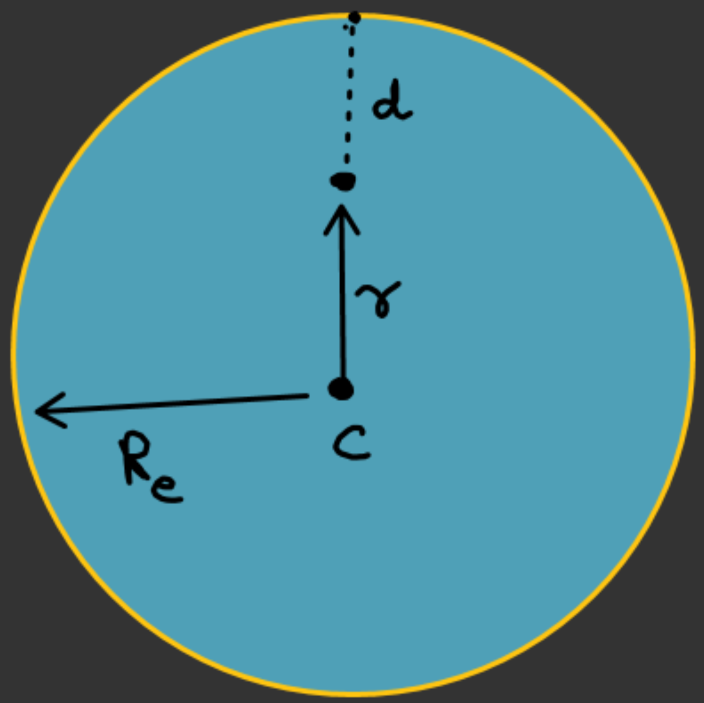
GRAVITATION

$$g' = E = \left(\frac{GM}{R_e^3} r\right)$$

$$g' = \left(\frac{GM}{R_e^2}\right) \times \frac{r}{R_e}$$

$$\boxed{g' = \frac{g}{R_e} r}$$

$$r = (R_e - d)$$



$$g' = \frac{g}{R_e}(R_e - d)$$

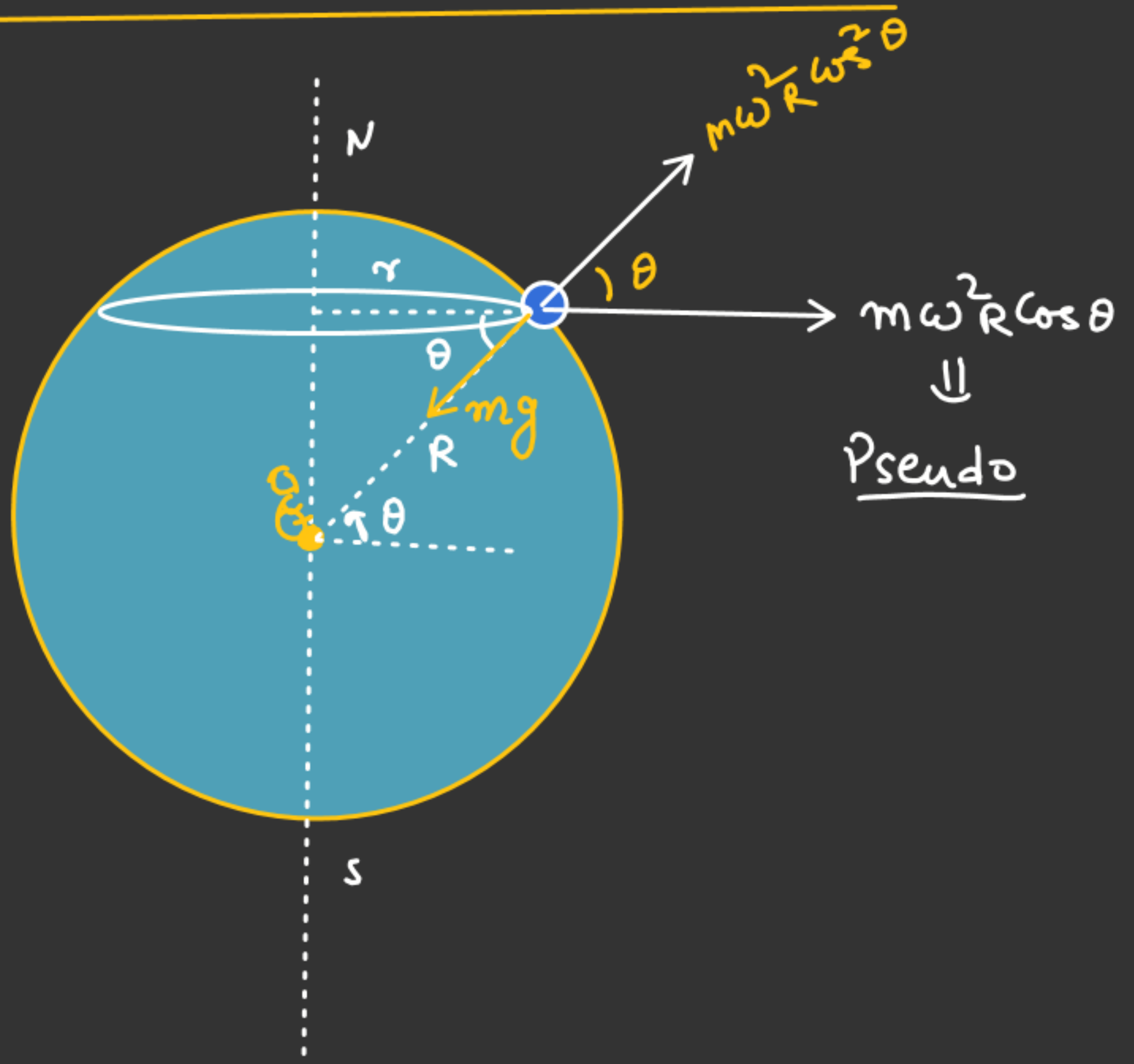
$$\boxed{g' = g\left(1 - \frac{d}{R_e}\right)}$$

GRAVITATION

Variation of 'g' due to rotation of earth about its axis

$$mg' = mg - m\omega^2 R\cos^2\theta$$

$$\boxed{g' = g - \omega^2 R\cos^2\theta}$$



At pole. $\theta = 90°$

$$\boxed{(g' = g)}$$

At Equator. $\theta = 0°$

$$\boxed{g' = g - \omega^2 R}$$

$$T = \frac{2\pi}{\omega}$$

$$\omega = \frac{2\pi}{24\text{ hrs}} = \left(\frac{2\pi}{24 \times 3600}\right)$$

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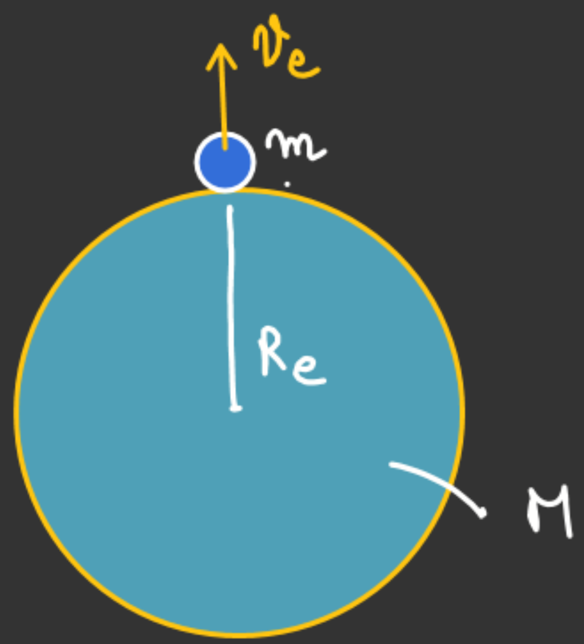
Escape velocity from the surface of earth

$K \cdot E = 0$ ∞

Escape velocity :- Min velocity given to the body so that it can escape from the gravitational field of any body.



$$U_i + K \cdot E_i = U_f + K \cdot E_f$$

$$-\frac{GM_e m}{R_e} + \frac{1}{2} m v_e^2 = 0 + 0$$

$$\boxed{v_e = \sqrt{\frac{2GM_e}{R_e}} \qquad v_e = \sqrt{2gR_e}}$$

$\Leftarrow$ 11.2 Km/s

$$g = \frac{GM_e}{R_e^2}$$

$$g \cdot R_e = \frac{GM_e}{R_e}$$

GRAVITATION

$\cdot (\infty)$ $K.E = 0$.
$P.E = 0$

Find escape velocity of mass m.

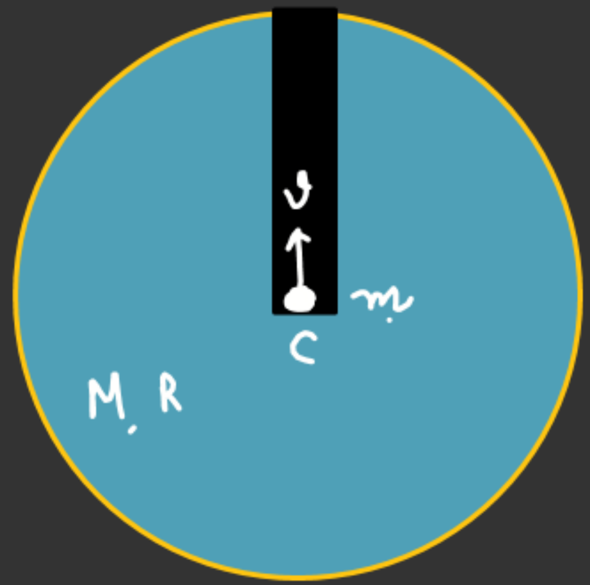
$$U_i + K.E_i = U_f + K.E_f$$

$$-\frac{3}{2}\left(\frac{GMm}{R}\right) + \frac{1}{2}mv_e^2 = 0 + 0$$

$$\frac{\cancel{m}v_e^2}{\cancel{2}} = \frac{3}{\cancel{2}}\left(\frac{GM\cancel{m}}{R}\right)$$

$$v_e^2 = \frac{3GM}{R}$$

$$v_e = \sqrt{\frac{3GM}{R}}.$$



$$V = -\frac{GM}{2R^3}(3R^2 - r^2)$$

For V_c $r = 0$

$$U_c = mV_c$$

$$= -\frac{3}{2}\frac{GMm}{R}$$

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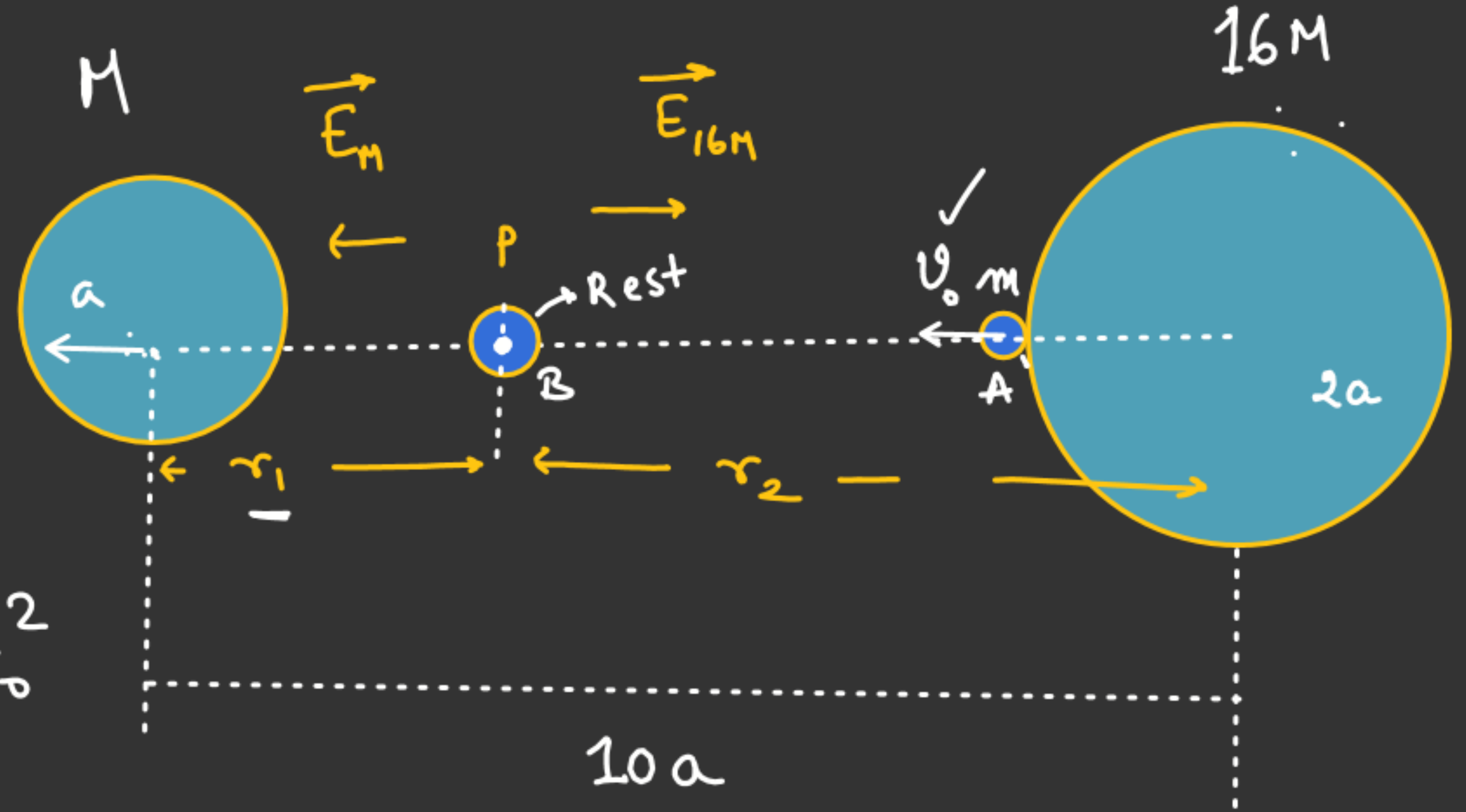
GRAVITATION

\# Find $(v_0)_{min}$ so that particle reaches to surface of smaller planet.



$$r_1 + r_2 = 10a \quad \text{--- (1)}$$

$$\frac{GM}{r_1^2} = \frac{G\,16M}{r_2^2}$$

$$\frac{r_2}{r_1} = 4$$

$$r_2 = 4r_1 \quad \text{--- (2)}$$

$$5r_1 = 10a$$

$$r_1 = 2a$$

$$r_2 = 8a$$

Energy Conservation from A to B.

$$U_A + K.E_A = U_B + K.E_B$$

$$K.E_B \to 0$$

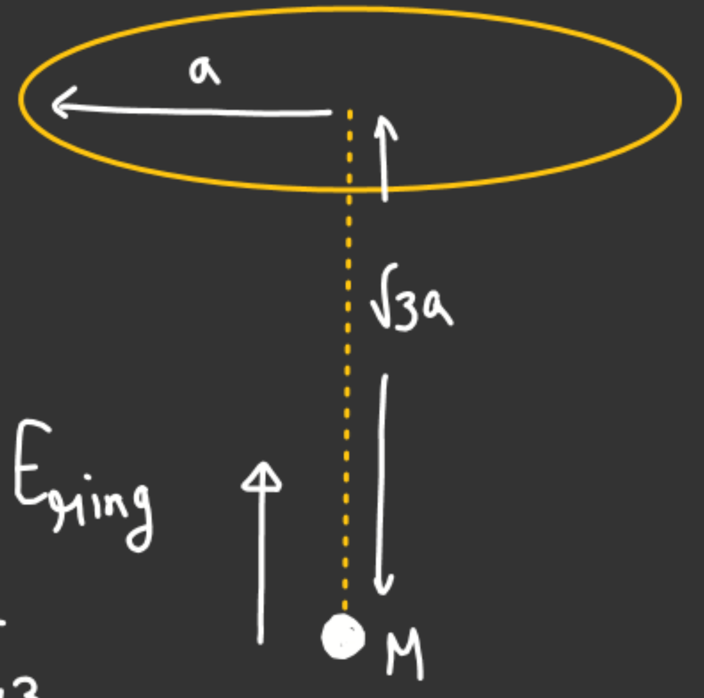
$$-\frac{G\,16Mm}{(2a)} - \left(\frac{GMm}{8a}\right) + \frac{1}{2}mv_0^2$$

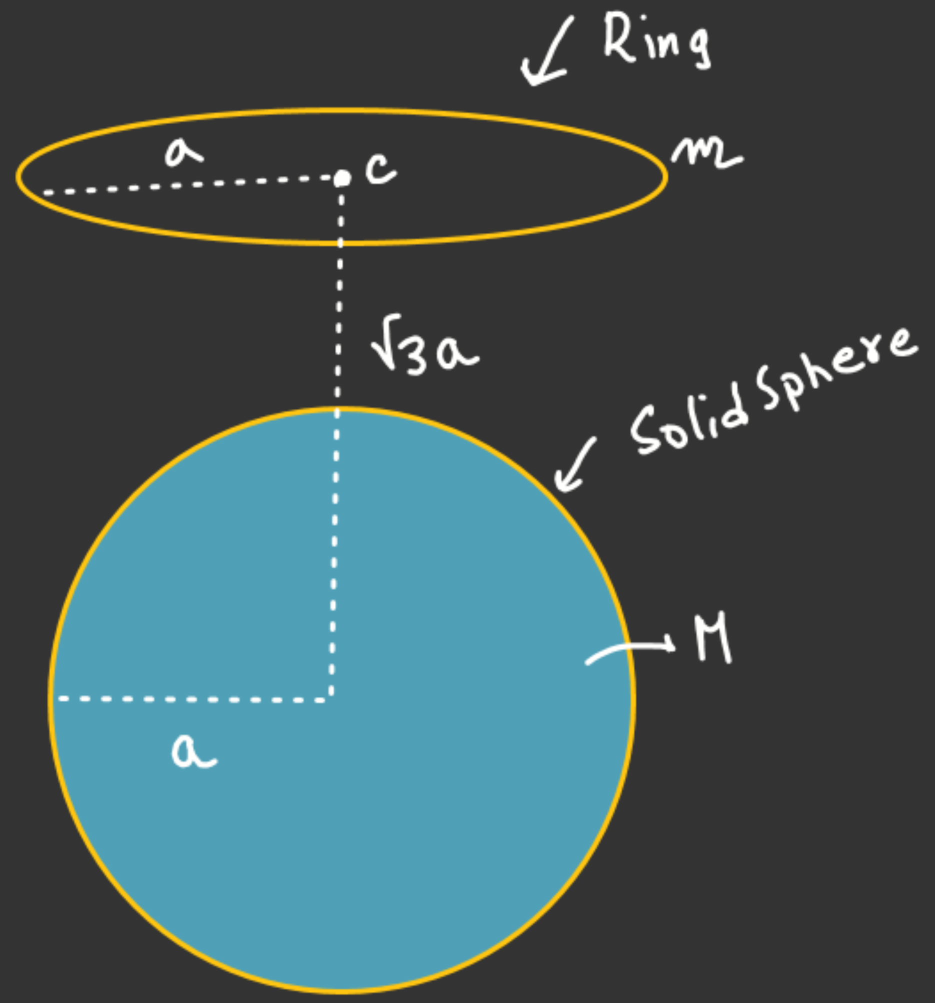
$$= -\frac{GMm}{2a} - \frac{G\,16Mm}{8a} +$$

$$v_0 = \frac{3}{2}\sqrt{\frac{5GM}{a}} \quad \text{Ans}$$

GRAVITATION

Force of interaction b/w ring and Solid sphere





$$F_{M/ring} = M \cdot E_{ring}.$$

$$= M \cdot \frac{Gm(\sqrt{3}a)}{\left[(\sqrt{3}a)^2 + a^2\right]^{3/2}}$$

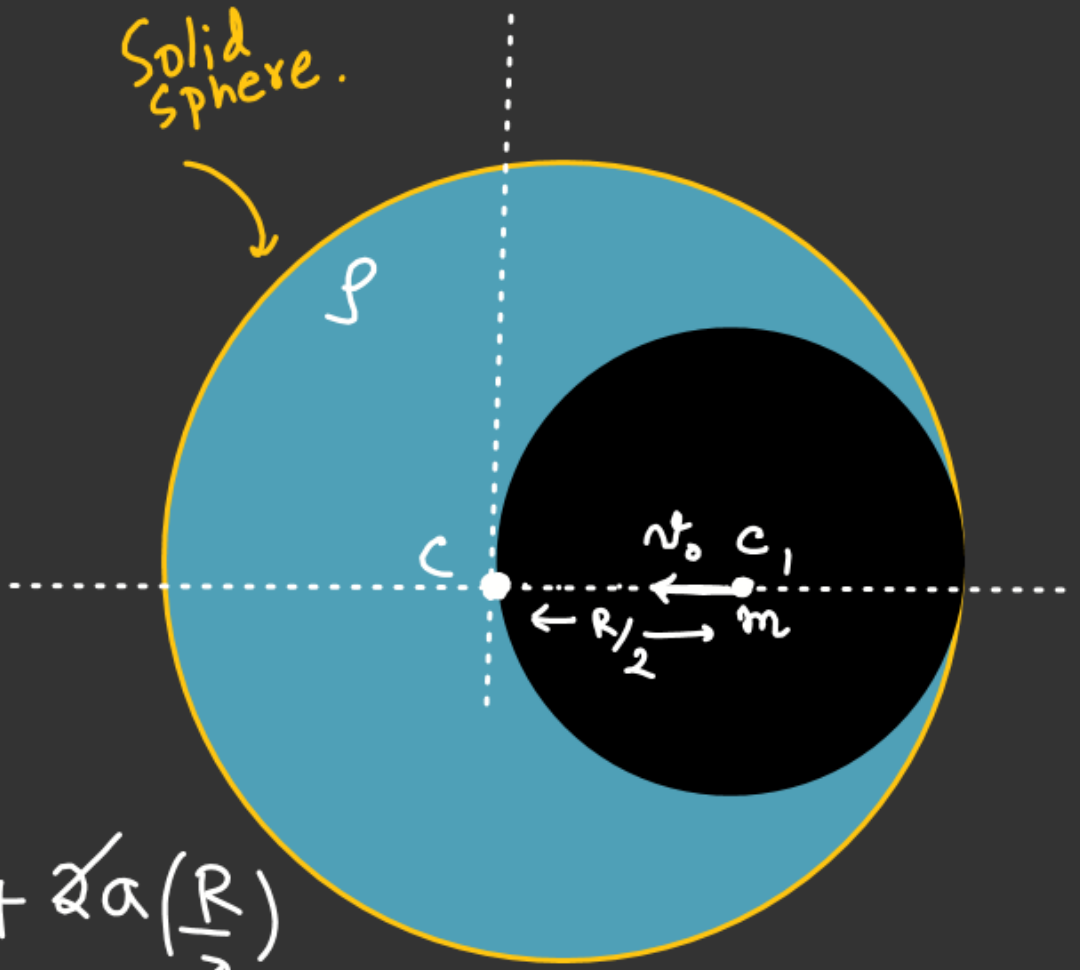
$$= \frac{\sqrt{3}GMm\,a}{(4a^2)^{3/2}} = \left(\frac{\sqrt{3}GMm}{8a^2}\right) \wedge.$$

GRAVITATION

Q:

Find the velocity with which particle of mass m projected with v_0 from C_1 hit at C.

Solid sphere.



Field inside the Cavity uniform.

$$\vec{E} = -\frac{\rho \cdot 4\pi G}{3}\vec{r}_{C_1C_2}$$

$$E = \frac{\rho \cdot 4\pi G}{3}\left(\frac{R}{2}\right)$$

$$F = m \cdot E$$

$$a = \frac{F}{m} = \underline{\underline{E}}$$

$$v^2 = v_0^2 + \cancel{2}a\left(\frac{R}{\cancel{2}}\right)$$

$$v^2 = v_0^2 + aR$$

$$v^2 = \left(v_0^2 + \frac{\rho \cdot 4\pi G R^2}{6}\right)$$

$$v = \sqrt{v_0^2 + \frac{2}{3}\rho\pi G R^2}$$