Nishant Jindal

Dscsn

16) $I = \int_0^{\pi/2} \frac{dx}{1+\cos\theta\left(\frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}\right)}$

$\int \frac{\sec^2 x/_2 . dx}{1+\tan^2\frac{x}{2}(1-\cos\theta)+\cos\theta}$ $\tan\frac{x}{2}=t$

$\int \frac{dt}{1+t^2 A+B.}$

Q17) $\int_0^{\frac{\ln 3}{2}} \frac{e^x}{e^{2x}+1}.dx + \int_0^{\frac{\ln 3}{2}} \frac{1}{e^{2x}+1} \times \frac{e^{-2x}}{e^{-2x}}$

$e^x = t$

$\int \frac{dt}{t^2+1} + \int \frac{e^{-2x}dx}{e^{-2x}+1}$ | $e^{-2x}=z$, $e^{-2x}dx = -\frac{dz}{2}$

18) $1-\sin x = t^2$, 20 Copy , 19 Copy

21) Ind → P.F. (22) $a^2\sin^2\phi + b^2\cos^2\phi = t$

23) A) $\int_0^{\frac{3\pi}{4}} (\sin x + \cos x) + x(\sin x - \cos x)$

$f \quad + x \cdot f'(x)$

(B) $\int_{\frac{\pi}{2}}^{\pi} x^{\sin x}(1 + x\cos x \cdot \ln x + \sin x)$

$\int$ $\int x^{\sin x+1}\left(\frac{1}{x} + \cos x \cdot \ln x + \frac{\sin x}{x}\right)$

$x^{\sin x+1} = t$

$\Rightarrow \int dt$

$= x^{\sin x+1} + C$

$x^{\sin x+1}\left[\frac{d}{dx}((\sin x+1)\cdot \ln x)\right] = dt$

$x^{\sin x+1}\left[\frac{\sin x+1}{x} + \cos x \cdot \ln x\right].dx = dt$

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24) $\int x\cdot(\tan^{-1}x)^2\cdot dx$

$\tan^{-1}x = t$

$x = \tan t$

$dx = \sec^2 t\, dt$

$\int \tan t \cdot t^2 \cdot \sec^2 t\, dt$

$\int \underset{I}{\tan t}\cdot \underset{II}{\sec^2 t}$

$= \frac{\tan^2 t}{2}$

$\int t^2 (\sec^2 t \cdot \tan t)\, dt$

$t^2 \cdot \frac{\tan^2 t}{2} - \int \underbrace{\cancel{2}\, t \cdot \frac{\tan^2 t}{\cancel{2}}}_{U\cdot V}\cdot dt$

Q25 $f, f', f''(x) \to [0, \ln 2]$

$f(0)=0,\ f'(0)=3,\ f(\ln 2)=6\ \ f'(\ln 2)=4$

PS (smp

Q $\int_0^{\ln 2} e^{-2x}\cdot f(x)\cdot dx = 3.$

then $\int_0^{\ln 2} e^{-2x}\cdot f''(x)\cdot dx = ?$

$I = \int_0^{\ln 2} \underset{V}{e^{-2x}}\cdot \underset{U}{f(x)}\cdot dx = 3.$

$= f(x)\cdot \frac{e^{-2x}}{-2}\Big|_0^{\ln 2} + \frac{1}{2}\int f'(x)\cdot e^{-2x}\cdot dx$

$3 = \quad + \frac{1}{2}\left\{ f'(x)\cdot \frac{e^{-2x}}{2}\Big|_0^{\ln 2} + \frac{1}{2}\left\{\underbrace{\int f''(x)\cdot e^{-2x}\cdot dx}_{\text{dem.}}\right\}\right\}$

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Dscsn

Q $\int_0^1 \frac{dx}{(x+\cos x)^2+(\sin x)^2}$

27) $\ln|x+\sqrt{1+x^2}|\Big|_0^{5}$

28) $\int_{0^+}^{1} \frac{x^x(x^{2x}+1)(\ln x+1)}{x^4+1}\,dx$

$x^x = t$

$x^x(1+\ln x)\,dx$

$\int \frac{t^2+1}{t^4+1}\,dt = \int \frac{dz}{z^2-2}$

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IIT/NIT most Imp chapter (Not for AZAR

Q If $a_n = \int_0^{\pi/4} \tan^n x \cdot dx$ then a_2+a_4, a_3+a_5, a_4+a_6. are in

Exam

$$a_n = \int_0^{\pi/4} \tan^n x \cdot dx$$

$$a_{n+2} = \int_0^{\pi/4} \tan^{n+2} x \cdot dx$$

$$a_n + a_{n+2} = \int_0^{\pi/4} \tan^n x + \tan^{n+2} x \, dx$$

$$= \int \tan^n x (1+\tan^2 x) \cdot dx$$

$$= \int_0^{\pi/4} \underline{\tan^n x} \cdot \sec^2 x$$

$\boxed{\tan x = t}$

$\sec^2 x \, dx = dt$

x	t
0	0
$\frac{\pi}{4}$	1.

$$a_n + a_{n+2} = \int_0^1 t^n \cdot dt = \left.\frac{t^{n+1}}{n+1}\right|_0^1 = \frac{1}{n+1}$$

$$a_n + a_{n+2} = \frac{1}{n+1}$$

$n=2$ $\quad a_2 + a_4 = \frac{1}{3}$

$n=3$ $\quad a_3 + a_5 = \frac{1}{4}$

$n=4$ $\quad a_4 + a_6 = \frac{1}{5}$

$\frac{1}{3}, \frac{1}{4}, \frac{1}{5}$ are in H.P.

Q If $a_n = \int_0^{\pi/4} \tan^n x \cdot d(x - \cancel{[x]})$ then $a_n + a_{n+2}$?

Shed

$\{x\} \in (0, \frac{3}{4})$

$$= \int_0^{\pi/4} \tan^n x \cdot dx$$

fract

$x \in (0, \frac{3}{4})$

$[x] = 0$

$$a_n + a_{n+2} = \frac{1}{n+1}$$

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Q $\displaystyle\int_{\pi/4}^{\cancel{3\pi/4}\ \pi/3} \frac{(\sin^3\theta-\cos^3\theta-\cos^2\theta)(\sin\theta+\cos\theta+\cos^2\theta)^{2007}}{(\sin\theta)^{2009}\cdot(\cos\theta)^{2009}}\,d\theta = \frac{(a+\sqrt{b})^n-(1+\sqrt{c})^n}{d}$; $a,b,c,d\in I^+$ then $a+b+c+d=$?

$$\int_{\pi/4}^{\pi/3}\left(\frac{\sin^3\theta-\cos^3\theta-\cos^2\theta}{\sin^2\theta\cdot\cos^2\theta}\right)\left(\frac{\sin\theta+\cos\theta+\cos^2\theta}{\sin\theta\cdot\cos\theta}\right)^{2007}$$

$$\int(\sec\theta\cdot\tan\theta-\csc\theta\cdot\cot\theta-\csc^2\theta)(\sec\theta+\csc\theta+\cot\theta)^{2007}$$

$\boxed{\sec\theta+\csc\theta+\cot\theta=t}$

$(\sec\theta\cdot\tan\theta-\csc\theta\cdot\cot\theta-\csc^2\theta)\,d\theta=dt$

θ	t
$\frac{\pi}{4}$	$\sqrt{2}+\sqrt{2}+1$
$\frac{\pi}{3}$	$2+\frac{2}{\sqrt3}+\frac{1}{\sqrt3}$
	$2+\sqrt3$

$$\int_{1+2\sqrt2}^{2+\sqrt3} t^{2007}\cdot dt = \left.\frac{(t)^{2008}}{2008}\right|_{1+\sqrt8}^{\dots} = \frac{(2+\sqrt3)^{2008}}{2008}-\frac{(1+\sqrt8)^{2008}}{2008}$$

$a=2$

$b=3$

$c=8$ $\quad d=2008$

$a+b+c=2021$

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Q $I = \int_0^\infty \frac{1 \cdot dx}{a^2 + (x - 1/x)^2} = \frac{\pi}{5050}$ then $a = ?$ $\rightarrow A$

$\boxed{x = \frac{1}{t}} \Rightarrow dx = -\frac{1}{t^2} \cdot dt$

x	t
0	∞
∞	0

$$I = \int_\infty^0 \frac{-\frac{1}{t^2} dt}{(a)^2 + \left(t - \frac{1}{t}\right)^2} \rightarrow$$

$t = x$
$dt = dx$

t	x
0	0
∞	∞

$$I = \int_\infty^0 \frac{-\frac{1}{x^2} dx}{(a)^2 + (x - 1/x)^2} = \int_0^\infty \frac{\frac{1}{x^2} dx}{(a)^2 + (x - 1/x)^2} \rightarrow B$$

A + B

$$2I = \int_0^\infty \frac{\left(1 + \frac{1}{x^2}\right) dx}{a^2 + (x - 1/x)^2}$$

$x - \frac{1}{x} = t$

$\left(1 + \frac{1}{x^2}\right) dx = dt$

x	t
0	$-\infty$
∞	∞

$$= \int_{-\infty}^{\infty} \frac{dt}{a^2 + t^2} = \frac{1}{a} \tan^{-1} \frac{t}{a} \Big|_{-\infty}^{\infty} = \frac{1}{a} \left(\tan^{-1} \infty - \tan^{-1}(-\infty)\right) = \frac{1}{a}(\pi) = \frac{\pi}{a}$$

$I = \frac{\pi}{2a} = \frac{\pi}{5050}$

$a = 2525$

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Q $I = \int_0^1 \frac{(1-x)\cdot dx}{(1+x)\sqrt{x^3+x^2+x}}$

$\boxed{\text{Yad}}$

Self

$= \int \frac{(1-x^2)\cdot dx}{\underbrace{(x^2+2x+1)}_{x\text{ com}}\sqrt{\underbrace{x^3+x^2}_{x^2\text{ com}}+x}}$

$I = \int_0^1 \frac{\left(\frac{1}{x^2}-1\right)dx}{\left(x+\frac{1}{x}+2\right)\sqrt{x+\frac{1}{x}+1}}$

$x+\frac{1}{x}+1 = t^2$

$(1-1/x^2)\,dx = 2t\,dt$

x	t
0	∞
1	$\sqrt{3}$

$= \int_{\infty}^{\sqrt{3}} \frac{-2\cancel{t}\,dt}{t^2+1\ \sqrt{\cancel{t^2}}} = +2\int_{\sqrt{3}}^{\infty} \frac{dt}{t^2+1} = 2\left[\tan^{-1} t\right]_{\sqrt{3}}^{\infty}$

$= 2\left[\frac{\pi}{2}-\frac{\pi}{3}\right] = \frac{\pi}{3}$

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Wallis formula

$$I_n = \int_0^{\pi/2} \sin^n x \cdot dx, \int_0^{\pi/2} \cos^n x \cdot dx, n \geq 2 \quad (I)$$

Acc. to Wallis

$$\boxed{\int_0^{\pi/2} \sin^n x \cdot dx = \frac{(n-1)\cdot(n-3)(n-5)\ldots}{(n)(n-2)(n-4)\ldots \; 1 \text{ or } 2} \times 1 \text{ or } \frac{\pi}{2}}$$

Q $\int_0^{\pi/2} \sin^3 x \cdot dx = \frac{2}{3 \cdot 1} \times 1 = \frac{2}{3}$.

Q $\int_0^{\pi/2} \cos^8 x \cdot dx = \frac{7 \cdot 5 \cdot 3 \cdot 1}{8 \cdot 6 \cdot 4 \cdot 2} \times \frac{\pi}{2}$

$= \frac{35\pi}{256}$

Gamma's fxn.

$$I_n = \int_0^{\pi/2} \sin^n x \cdot \cos^m x \cdot dx \qquad n \geq 2, m \geq 2$$

$$\int_0^{\pi/2} \sin^n x \cdot \cos^m x \cdot dx$$

$$= \frac{(n-1)\cdot(n-3)\ldots \times (m-1)(m-3)\ldots}{(m+n)(m+n-2)(m+n-4)\ldots 1 \text{ or } 2} \times 1 \text{ or } \frac{\pi}{2}$$

Q $I = \int_0^{\pi/2} \sin^3 x \cdot \cos^4 x \cdot dx$ Gamma

$= \frac{2 \times 3 \times 1}{7 \cdot 5 \cdot 3 \cdot 1} \times 1 = \frac{2}{35}$

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Q $\int_0^{\pi/4} \sin^6(2x)\,dx$

$2x=t$ | x | t
$dx=\frac{dt}{2}$ | 0 | 0
| $\frac{\pi}{4}$ | $\frac{\pi}{2}$

$$I=\frac{1}{2}\int_0^{\pi/2}\sin^6(t)\,dt$$

$$=\frac{1}{2}\times\frac{5\cdot\cancel{3}\cdot1}{\cancel{6}_{2}\cdot4\cdot2}\times\frac{\pi}{2}=\frac{5\pi}{64}$$

Q $\int_0^1 \frac{x^3}{\sqrt{1-x^2}}\,dx$

$x=\sin\theta$ | x | θ
$dx=\cos\theta\,d\theta$ | 0 | 0
| 1 | $\frac{\pi}{2}$

$$=\int_0^{\pi/2}\frac{\sin^3\theta\cdot\cancel{\cos\theta}}{\cancel{\sqrt{1-\sin^2\theta}}}\,dx=\frac{2}{3\cdot1}\times1=\frac{2}{3}$$

Q $\int_0^{\infty}\frac{x\,dx}{(1+x^2)^{3/2}}$

$x=\tan\theta$ | x | θ
$dx=\sec^2\theta\,d\theta$ | 0 | 0
| ∞ | $\frac{\pi}{2}$

$$I=\int_0^{\pi/2}\frac{\tan\theta\,\sec^2\theta\,d\theta}{(1+\tan^2\theta)^{3}}$$

$$=\int\frac{\tan\theta\,\cancel{\sec^2\theta}\,d\theta}{\sec^3\theta}=\int_0^{\pi/2}\sin\theta\,d\theta$$

$$=1$$

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Q $$\int_0^3 \sqrt{\frac{x3}{3-x}} \cdot dx$$

$$x = \underline{3\sin^2\theta}$$