EXAMPLES(XVII)

SL LONEY

Find the equation to the circle:

1. Whose radius is 3 and whose centre is $(-1,2)$.

2. Whose radius is 10 and whose centre is $(-5,-6)$.

3. Whose radius is $a+b$ and whose centre is $(a,-b)$.

4. Whose radius is $\sqrt{a^2-b^2}$ and whose centre is $(-a,-b)$.

Find the coordinates of the centres and the radii of the circles whose equations are:

5. $x^2+y^2-4x-8y=41$

6. $3x^2+3y^2-5x-6y+4=0$

7. $x^2+y^2=k(x+k)$

8. $x^2+y^2=2gx-2fy$

9. $\sqrt{1+m^2}(x^2+y^2)-2cx-2mcy=0$

Draw the circles whose equations are:

10. $x^2+y^2=2ay$

11. $3x^2+3y^2=4x$

12. $5x^2+5y^2=2x+3y$

13. Find the equation to the circle which passes through the points $(1,-2)$ and $(4,-3)$ and which has its centre on the straight line $3x+4y=7$.

14. Find the equation to the circle passing through the points $(0,a)$ and (b,h), and having its centre on the axis of x.

Find the equations to the circles which pass through the points;

15. $(0,0)$, $(a,0)$ and $(0,b)$

16. $(1,2)$, $(3,-4)$ and $(5,-6)$

17. $(1,1)$, $(2,-1)$ and $(3,2)$

18. $(5,7)$, $(8,1)$ and $(1,3)$

19. (a,b), $(a,-b)$ and $(a+b,a-b)$

20. ABCD is a square whose side is a; taking AB and AD as axes, prove that the equation to the circle circumscribing the square is, $x^2+y^2=a(x+y)$.

21. Find the equation to the circle which passes through the origin and cuts off intercepts equal to 3 and 4 from the axes.

22. Find the equation to the circle passing through the origin and the points (a,b) and (b,a). Find the lengths of the chords that it cuts off from the axes.

23. Find the equation to the circle which goes through the origin and cuts off intercepts equal to h and k from the positive parts of the axes.

24. Find the equation to the circle, of radius a, which passes through the two points on the axis of x which are at a distance b from the origin.

Find the equation to the circle which:

25. touches each axis at a distance 5 from the origin.

26. touches each axis and is of radius a.

27. touches both axes and passes through the point $(-2, -3)$.

28. touches the axis of x and passes through the two points $(1, -2)$ and $(3, -4)$

29. touches the axis of y at the origin and passes through the point (b, c).

30. touches the axis of x at a distance 3 from the origin and intercepts a distance 6 on the axis of y.

31. points (1,0) and (2,0) are taken on the axis of x, the axes being rectangular. On the line joining these points an equilateral triangle is described, its vertex being in the positive quadrant. Find the equations to the circles described on its sides as diameters.

32. If $y = mx$ be the equation of a chord of a circle whose radius is ^{4} the origin of coordinates being one extremity of the chord and the axis of ? being a diameter of the circle, prove that the equation of a circle of which this chord is the diameter is,

$(1 + m^2)(x^2 + y^2) - 2a(x + my) = 0.$

33. Find the equation to the circle passing through the points (12,43), (18,39), and (42,3) and prove that it also passes through the points $(-54, -69)$ and $(-81, -38)$.

34. Find the equation to the circle circumscribing the quadrilateral formed by the straight lines, $2x + 3y = 2, 3x - 2y = 4, x + 2y = 3$ and $2x - y = 3$

35. Prove that the equation to the circle of which the points (x_1, y_1) and (x_2, y_2) are the ends of a chord of a segment containing an angle θ is,

$(x - x_1)(x - x_2) + (y - y_1)(y - y_2) \pm \cot\ \theta[(x - x_1)(y - y_2) - (x - x_2)(y - y_1)] = 0.$

36. Find the equations to the circles in which the line joining the points (a, b) and $(b, -a)$ is a chord subtending an angle of 45° at any point on its circumference.

ANSWER KEY

1. $x^2 + y^2 + 2x - 4y = 4$

2. $x^2 + y^2 + 10x + 12y = 39$

3. $x^2 + y^2 - 2ax + 2by = 2ab$

4. $x^2 + y^2 + 2ax + 2by + 2b^2 = 0$

5. $(2,4); \sqrt{61}$

6. $\left(\frac{5}{6}, 1\right) : \frac{1}{6}\sqrt{13}$

7. $\left(\frac{k}{2}, 0\right) : \frac{\sqrt{5}}{2}k$

8. $(g, -f); \sqrt{f^2 + g^2}$

9. $\left(\frac{c}{\sqrt{1+m^2}}, \frac{mc}{\sqrt{1+m^2}}\right); c$

13. $15x^2 + 15y^2 - 94x + 18y + 55 = 0$

14. $b(x^2 + y^2 - a^2) = x(b^2 + h^2 - a^2)$

15. $x^2 + y^2 - ax - by = 0$

16. $x^2 + y^2 - 22x - 4y + 25 = 0$

17. $x^2 + y^2 - 5x - y + 4 = 0$

18. $3x^2 + 3y^2 - 29x - 19y + 56 = 0$

19. $b(x^2 + y^2) - (a^2 + b^2)x + (a - b)(a^2 + b^2) = 0$

21. $x^2 + y^2 - 3x - 4y = 0$

22. $x^2 + y^2 - \frac{a^2+b^2}{a+b}(x + y) = 0; \frac{a^2+b^2}{a+b}$

23. $x^2 + y^2 - hx - ky = 0$

24. $x^2 + y^2 \pm 2y\sqrt{a^2 - b^2} = b^2$

25. $x^2 + y^2 - 10x - 10y + 25 = 0$

26. $x^2 + y^2 \pm 2ax \pm 2ay + a^2 = 0$

27. $x^2 + y^2 + 2(5 \pm \sqrt{12})(x + y) + 37 \pm 10\sqrt{12} = 0$

28. $x^2 + y^2 - 6x + 4y + 9 = 0$, or $x^2 + y^2 + 10x + 20y + 25 = 0$

29. $b(x^2 + y^2) = x(b^2 + c^2)$

30. $x^2 + y^2 \pm 6\sqrt{2}y - 6x + 9 = 0$

31. $x^2 + y^2 - 3x + 2 = 0; 2x^2 + 2y^2 - 5x - \sqrt{3}y + 3 = 0$

$2x^2 + 2y^2 - 7x - \sqrt{3}y + 6 = 0$

33. $(x + 21)^2 + (y + 13)^2 = 65^2$

34. $8x^2 + 8y^2 - 25x - 3y + 18 = 0$

36. $x^2 + y^2 = a^2 + b^2; x^2 + y^2 - 2(a + b)x + 2(a - b)y + a^2 + b^2 = 0$