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Relation

Cartesian Product of 2 sets A, B

$$A \times B = \{(a,b) \mid a \in A, b \in B\}.$$

$A = \{a_1, a_2\}$ $\quad n(A) = m_1$

$B = \{b_1, b_2, b_3\}$ $\quad n(B) = m_2$ $\quad n(A \times B) = m_1 m_2$

$A \times B = \{(a_1, b_1), (a_1, b_2), (a_1, b_3), (a_2, b_1), (a_2, b_2), (a_2, b_3)\}.$

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Relation

R $\qquad$ $A \times B$

Domain = $\{a, b, c\}$

Range = $\{$Ali, Bhanu, Binoy, Chandra$\}$.

subset of $A \times B$ is called relation

$R = \{(a, \text{Ali}), (b, \text{Bhanu}), (b, \text{Binoy}), (c, \text{Chandra})\}$

$A = \{a, b, c\}$, $B = \{$Ali, Bhanu, Binoy, Chandra, Divya$\}$.

$R = \{(x, y) \mid x$ is the first letter of the name y, $x \in A, y \in B\}$.

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$n(A) = m_1$ $n(A \times B) = m_1 m_2$.

$n(B) = m_2$

How many no. of relations can be defined from A to B.

$A = \{1, 2\}$

$B = \{3, 4, 5\}$

$R = \{(x, y) \mid |x-y| \geq 0,\ x \in A, y \in B\}$ → universal

$$n(R) = 2^{m_1 m_2}$$

→ Empty relation: $R = \phi$

→ Universal relation: $R = A \times B$

$R_2 = \{(x, y) \mid x - y > 5,\ x \in A,\ y \in B\}$ → Empty

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Domain of relation

A to B

Set of all first elements in ordered pairs of the relation.

$(a,b) \in R \Rightarrow$ a is related to b by relation R.

$\Rightarrow a R_b$

Range of relation

Set of all second elements in ordered pairs of the relation.

b is image of a under the relation R

B = CoDomain of Relation

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Types

Relation defined on set A $\boxed{A \times A}$

Reflexive

$\forall a \in A$, $(a,a) \in R$.

If $(a,b) \in R \Rightarrow (a,a) \in R$, then R is said to be reflexive

aRa

Symmetric

R is symm, If $(a,b) \in R \Rightarrow (b,a) \in R$

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Transitive

R is ~~tran~~ tran-sitive If $(a,b) \in R$, $(b,c) \in R \Rightarrow (a,c) \in R$

Equivalence

if Relation is Reflexive, Symm & Transitive

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1. Let T be the set of all triangles in a plane with R a relation in T given by

$$R = \{(T_1, T_2) : T_1 \text{ is congruent to } T_2\}.$$

$T \times T$

Equivalence

$(T_1, T_1) \in R \Rightarrow$ Reflexive

If $(T_1, T_2) \in R \Rightarrow (T_2, T_1) \in R$ R is symm

$(T_1, T_2) \in R, (T_2, T_3) \in R \Rightarrow (T_1, T_3) \in R$ Transitive

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2. Let P be the relation defined on set of all real numbers such that

$$P = \{(a,b) \mid \sec^2 a - \tan^2 b = 1\}$$

Equivalence

$\tan^2 a - \tan^2 a = 0$

Reflexive

$(a,b) \in R$

$\downarrow$

$(b,a) \in R$

$\tan^2 a = \tan^2 b$

$\Rightarrow$ **Symm**

$(a,b) \in R$

$(b,c) \in R$

$\tan^2 a = \tan^2 b$

$\tan^2 b = \tan^2 c$

$\Rightarrow \tan^2 a = \tan^2 c$

$\Downarrow$

$(a,c) \in R$

$\tan^2 a - \tan^2 a = 0$

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3. Relation R is given by

$\{(x,y) \mid x^2-3xy+2y^2=0 \ \forall x,y \in Z\}$ Z = set of integers

$(x-y)(x-2y)=0$

$_xR_x$

$_2R_1$ is true, $_1R_2$ is not true $\Rightarrow$ not symmetric

$_4R_2$, $_2R_1$ $\not\Rightarrow$ $_4R_1$ $\Rightarrow$ not transitive

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4. Let R be relation on set $\mathbb{R}$ of all real numbers defined by setting

aR_b if $|a-b| \le \frac{1}{2}$.

reflexive & symm. only

$|a-a| = 0 \le \frac{1}{2} \Rightarrow aR_a$ is true

$|a-b| \le \frac{1}{2}$, $|b-a| \le \frac{1}{2}$

$-\frac{1}{2} \le a-b \le \frac{1}{2}$

$-\frac{1}{2} \le b-c \le \frac{1}{2}$

$-1 \le a-c \le 1$

$a = 1$, $b = \frac{1}{2}$, $c = \frac{1}{4}$

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5: If relation R_1 and R_2 from set A to set B are defined as $R_1 = \{(1,2),(3,4),(5,6)\}$ and $R_2 = \{(2,1),(4,3),(6,5)\}$. Then $n(A \times B)$ can be equal to

$A \times B = m \times n$, $m \geq 6$, $n \geq 6$.

(a) 35 (b) 53 (c) 91 ✓ (d) 55

$A = \{1,2,3,4,5,6, \ldots\}$

$B = \{1,2,3,4,5,6, \ldots\}$

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6. Let R be the set of real numbers.

Statement-1 : $A = \{(x, y) \in R \times R : \underline{y - x} \text{ is an integer}\}$ is an equivalence relation on R.

$0 = 0(2)$ $\quad 0 = 0(\frac{1}{2})$ $\quad 0 = 0 (2)$

S-2 : $B = \{(x, y) \in R \times R : x = \alpha y$ for some rational number $\alpha\}$ is an equivalence relation on R.

$\boxed{\text{S-1 is true}}$

$0 = 0(2)$ $\quad 2 = \alpha(0)$ $\quad x = \alpha^2 z$

(a) S-1 is True, S-2 is false ✓

$x = \alpha y$

$y = \alpha z$

(b) —"— False, —"— true

(c) both True, S-2 is correct explanation of S-1.

(d) both True, S-2 is not correct explanation of S-1