EXERCISE–2

MIXED QUESTIONS

1. If $a_1, a_2, a_3, \ldots, a_n, \ldots$ are in G.P, then the value of the determinant

$$\begin{vmatrix} \log a_n & \log a_{n+1} & \log a_{n+2} \\ \log a_{n+3} & \log a_{n+4} & \log a_{n+5} \\ \log a_{n+6} & \log a_{n+7} & \log a_{n+8} \end{vmatrix} \text{ is}$$

(A) 0 (B) 1 (C) 2 (D) -2

2. Let $a + b + c = s$ and $\begin{vmatrix} s+c & a & b \\ c & s+a & b \\ c & a & s+b \end{vmatrix}$ is equal to 54 , then the value of s is

(A) 2 (B) 3 (C) 4 (D) 5

3. There are two numbers x making the value of the determinant $\begin{vmatrix} 1 & -2 & 5 \\ 2 & x & -1 \\ 0 & 4 & 2x \end{vmatrix}$ equal to 86.

The sum of these two numbers , is

(A) –4 (B) 5 (C) –3 (D) 9

4. Let $A = \begin{bmatrix} a & b & c \\ p & q & r \\ x & y & z \end{bmatrix}$ and suppose that det $(A) = 2$ then the det (B) equals, where

$$B = \begin{bmatrix} 4x & 2a & -p \\ 4y & 2b & -q \\ 4z & 2c & -r \end{bmatrix}$$

(A) det $(B) = -2$ (B) det $(B) = -8$ (C) det $(B) = -16$ (D) det $(B) = 8$

5. Let $D_1 = \begin{vmatrix} a & b & a+b \\ c & d & c+d \\ a & b & a-b \end{vmatrix}$ and $D_2 = \begin{vmatrix} a & c & a+c \\ b & d & b+d \\ a & c & a+b+c \end{vmatrix}$ then the value of $\frac{D_1}{D_2}$ where $b \neq 0$ and $ad \neq bc$, is

(A) –2 (B) 0 (C) $-2b$ (D) $2b$

6. If a, b, c are all different from zero and $\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = 0$ then the value of

$a^{-1} + b^{-1} + c^{-1}$ is

(A) abc (B) $a^{-1}b^{-1}c^{-1}$ (C) $-a-b-c$ (D) -1

7. If $\begin{vmatrix} p & q-b & r-c \\ p-a & q & r-c \\ p-a & q-b & r \end{vmatrix} = 0$, then the value of $\frac{p}{a} + \frac{q}{b} + \frac{r}{c}$ is

(A) 0 (B) 2 (C) 4 (D) 5

8. If $A + B + C = \pi$, then the value of the determinant $D = \begin{vmatrix} \sin^2 A & \cot A & 1 \\ \sin^2 B & \cot B & 1 \\ \sin^2 C & \cot C & 1 \end{vmatrix}$

(A) 1 (B) -1 (C) 0 (D) None of these

9. Let $f(x) = \begin{vmatrix} 1+\sin^2 x & \cos^2 x & 4\sin 2x \\ \sin^2 x & 1+\cos^2 x & 4\sin 2x \\ \sin^2 x & \cos^2 x & 1+4\sin 2x \end{vmatrix}$, then the maximum value of $f(x) =$

(A) 2 (B) 4 (C) 6 (D) 8

10. If $\alpha + \beta + \gamma = \pi$, then the value of $\begin{vmatrix} \sin(\alpha+\beta+\gamma) & \sin\beta & \cos\gamma \\ -\sin\beta & 0 & \tan\alpha \\ \cos(\alpha+\beta) & -\tan\alpha & 0 \end{vmatrix}$ is

(A) 0 (B) 1 (C) 2 (D) $2 \cdot \sin\beta \cdot \cos\gamma \cdot \tan\alpha$

11. If α, β and γ are the roots of the equation $x^3 + px + q = 0$, then the value of the determinant

$\begin{vmatrix} \alpha & \beta & \gamma \\ \beta & \gamma & \alpha \\ \gamma & \alpha & \beta \end{vmatrix} =$

(A) p (B) q (C) $p^2 - 2q$ (D) None of these

12. Given $(1 + x + x^2)^8 = \sum_{k=0}^{16} a_k x^k$, then the value of the determinant $\begin{vmatrix} a_3 & a_7 & a_{13} \\ a_4 & a_6 & a_{12} \\ a_5 & a_4 & a_{11} \end{vmatrix}$ is

(A) 1 (B) 2 (C) 0 (D) 3

13. Consider the system of equations $x + ky = 0, y + kz = 0$ and $z + kx = 0$. The set of all real values of k for which the system has a unique solution, is

(A) $R - \{-1\}$ (B) $R - \{1\}$ (C) $\{-1\}$ (D) $\{-1,1\}$

14. If the lines

$x + ay + a = 0$

$bx + y + b = 0$

$cx + cy + 1 = 0,$

where a, b, c are non-zero and non-unity, pass through the same point then the value of

$\frac{a}{1-a} + \frac{b}{1-b} + \frac{c}{1-c}$ is equal to

(A) -1 (B) 2 (C) 1 (D) 3

15. If a, b, c, d, e and f are in G.P. then the value of $\begin{vmatrix} a^2 & d^2 & x \\ b^2 & e^2 & y \\ c^2 & f^2 & z \end{vmatrix}$

(A) x and y (B) x and z (C) y and z (D) Independent of x, y and z

16. If $\begin{vmatrix} a+1 & a+2 & a+p \\ a+2 & a+3 & a+q \\ a+3 & a+4 & a+r \end{vmatrix} = 0$, then p, q, r are in

(A) A.P. (B) G.P. (C) H.P. (D) None of these

17. The value of $\begin{vmatrix} 2 & 3 \\ 3 & 7 \end{vmatrix} + \begin{vmatrix} 1 & 2 \\ 3 & 7 \end{vmatrix} + \begin{vmatrix} \frac{1}{2} & \frac{4}{3} \\ 3 & 7 \end{vmatrix} + \begin{vmatrix} \frac{1}{2^2} & \frac{8}{3^2} \\ 3 & 7 \end{vmatrix} + \cdots \infty$ is equal to

(A) 13 (B) 5 (C) $\frac{-13}{2}$ (D) 1

18. If $D_r = \begin{vmatrix} \frac{2}{2r-1} & 3 \\ 0 & \frac{1}{2r+1} \end{vmatrix}$ then $\sum_{r=1}^{n} D_r$ equals

(A) $\frac{n}{2n+1}$ (B) $\frac{1}{2n+1}$ (C) $\frac{2n}{2n+1}$ (D) $\frac{2n-1}{2n+1}$

19. If $\Delta_r = \begin{vmatrix} r-1 & n & 6 \\ (r-1)^2 & 2n^2 & 4n-2 \\ (r-1)^3 & 3n^2 & 3n^2-3n \end{vmatrix}$, then $\sum_{r=1}^{n} \Delta_r$ equals

(A) 1 (B) -1 (C) 0 (D) n

NUMERICAL TYPE QUESTIONS

20. If $\Delta_k = \begin{vmatrix} 1 & n & n \\ 2k & n^2+n+1 & n^2+n \\ 2k-1 & n^2 & n^2+n+1 \end{vmatrix}$ and $\sum_{k=1}^{n} \Delta_k = 56$, then what is the value of n ?

21. If $D = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$ and corresponding cofactors of elements be written as c_{11}, c_{12}, c_{21} and c_{22}, then find the value of $a_{11}c_{21} + a_{12}c_{22} + a_{21}c_{11} + a_{22}c_{12}$

22. If the system of equations $3x - 2y + z = 0, \lambda x - 14y + 15z = 0, x + 2y - 3z = 0$ has a non-zero solution, then $\lambda =$

23. If $\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} = \ell(1+a^2+b^2)^m$ then find the value of $\ell + m$.

24. The system of equation $2x + 3y - z = 0, 3x + 2y + kz = 0, 4x + y + z = 0$ have a set of nonzero integral solution then find the least positive value of z.

25. If $\alpha, \beta \neq 0$ and $f(n) = \alpha^n + \beta^n$ and

$$\begin{vmatrix} 3 & 1+f(1) & 1+f(2) \\ 1+f(1) & 1+f(2) & 1+f(3) \\ 1+f(2) & 1+f(3) & 1+f(4) \end{vmatrix} = K(1-\alpha)^2(1-\beta)^2(\alpha-\beta)^2$$, then what is the value of K ?

26. Let a third order determinant $\Delta_1 = \{a_{ij}\}; i, j \in \{1,2,3\}$ and the determinant Δ_2 is constructed by multiplying all the element of Δ_1 by 2^{i-j}, i.e., $\Delta_2 = \{2^{i-j}a_{ij}\}$ and $\Delta_2 = \lambda\Delta_1$ then find the value of λ.

27. If $\begin{vmatrix} a^2+1 & ab & ac \\ ba & b^2+1 & bc \\ ca & cb & c^2+1 \end{vmatrix} = k\sqrt{(abc)}$, where a, b, c are positive reals then find the minimum possible value of k

28. If $\begin{vmatrix} 1 & 1 & 1 \\ m & m+3 & m+6 \\ m(m-1) & (m+3)(m+2) & (m+6)(m+5) \end{vmatrix} = 2^\alpha \cdot 3^\beta \cdot 5^\gamma$ then find the value of $(\alpha + \beta + \gamma)$.

ANSWER KEY

1.	(A)	**2.**	(B)	**3.**	(A)	**4.**	(C)	**5.**	(A)	**6.**	(D)	**7.**	(B)
8.	(C)	**9.**	(C)	**10.**	(A)	**11.**	(D)	**12.**	(C)	**13.**	(A)	**14.**	(A)
15.	(D)	**16.**	(A)	**17.**	(D)	**18.**	(C)	**19.**	(C)	**20.**	7	**21.**	0
22.	5	**23.**	4	**24.**	5	**25.**	1	**26.**	1	**27.**	4	**28.**	4