Nishant Jindal

$$\frac{1}{x} + 2bx + a = 0$$

$$\frac{2bx^2 + ax + 1}{x} = \frac{2b(x+1)(x-2)}{x}$$

$-2b + a = 1$

$4b + a = -\frac{1}{2}$

$\boxed{b = -\frac{1}{4}}$

$2b > 0$

$$\frac{\left(x - \frac{1}{x}\right)^2 + 2}{\left(x-\frac{1}{x}\right) +} = \frac{\left(x - \frac{1}{x}\right)}{} + \frac{2}{\left(x - \frac{1}{x}\right)}$$

$-\quad + \quad - \quad +$

$-1 \quad 0 \quad 2$

$(-\infty, -2\sqrt{2}] \cup [2\sqrt{2}, \infty)$

$2b < 0$ ✓

$+$ max $-$ $+$ $-$ max $-$

$-1 \quad 0 \quad 2$

Nishant Jindal

$$\cos\alpha_1 \cos\alpha_2 \cdots \cos\alpha_n = \sin\alpha_1 \sin\alpha_2 \cdots \sin\alpha_n .$$

$$\left(\cos\alpha_1 \cos\alpha_2 \cdots \cos\alpha_n\right)^2 = \frac{1}{2^n} \sin 2\alpha_1 \sin 2\alpha_2 \cdots \sin 2\alpha_n \leq \frac{1}{2^n} .$$

$$\sec^2\alpha_1 = 1 + \tan^2\alpha_1 \geqslant 2\left|\tan\alpha_1\right|$$

$$\sec^2\alpha_2 \geqslant 2\tan\alpha_2 .$$

$$\vdots$$

$$\left(\sec\alpha_1 \sec\alpha_2 \cdots \sec\alpha_n\right)^2 \geqslant 2^n \checkmark$$

Nishant Jindal

$$\lim_{n\to\infty} \sum_{r=0}^{n-1} \frac{(b-a)}{n} f\left(a + \frac{r}{n}(b-a)\right) = \int_a^b f(x)\,dx$$

$$= \int_0^1 (b-a) f(a+(b-a)x)\,dx$$

$$= \int_0^{b-a} f(a+x)\,dx$$

$$= \int_a^b f(x)\,dx$$

$$\boxed{\begin{array}{l} \sum \longrightarrow \int \\ \frac{r}{n} \longrightarrow x \\ \frac{1}{n} \longrightarrow dx \\ \lim_{n\to\infty} \frac{0}{n} = 0, \quad \lim_{n\to\infty} \frac{n-1}{n} = 1 \end{array}}$$

Nishant Jindal

$$\lim_{n\to\infty} \sum_{r=r_1}^{r_2} \frac{1}{n} f\left(\frac{r}{n}\right) = \int_{\lim\limits_{n\to\infty} \frac{r_1}{n}}^{\lim\limits_{n\to\infty} \frac{r_2}{n}} f(x)\,dx$$

Nishant Jindal

1. $$\lim_{n\to\infty}\left(\frac{n^2}{(n^2+1^2)^{3/2}}+\frac{n^2}{(n^2+2^2)^{3/2}}+\frac{n^2}{(n^2+3^2)^{3/2}}+\cdots+\frac{n^2}{(n^2+(n-1)^2)^{3/2}}\right)$$

$$=\lim_{n\to\infty}\sum_{r=1}^{n-1}\frac{n^2}{(n^2+r^2)^{3/2}}=\lim_{n\to\infty}\sum_{r=1}^{n-1}\frac{1}{n\left(1+\left(\frac{r}{n}\right)^2\right)^{3/2}}=\int_0^1\frac{dx}{(1+x^2)^{3/2}}=-\frac{1}{2}\int_0^1\frac{-2\,dx}{x^3\left(\frac{1}{x^2}+1\right)^{3/2}}$$

$$=\left.\frac{1}{\left(\frac{1}{x^2}+1\right)^{1/2}}\right|_0^1=\left.\frac{x}{\sqrt{1+x^2}}\right|_0^1=\frac{1}{\sqrt{2}}$$

2. $$\lim_{n\to\infty}\left(\frac{n+1}{n^2+1^2}+\frac{n+2}{n^2+2^2}+\frac{n+3}{n^2+3^2}+\cdots+\frac{3}{5n}\right)$$

$$\lim_{n\to\infty}\sum_{r=1}^{2n}\frac{n+r}{n^2+r^2}=\lim_{n\to\infty}\sum_{r=1}^{2n}\frac{\left(1+\frac{r}{n}\right)}{1+\left(\frac{r}{n}\right)^2}\frac{1}{n}=\int_0^2\frac{(1+x)\,dx}{1+x^2}=\tan^{-1}x+\frac{1}{2}\ln(1+x^2)\Big|_0^2$$

$$=\boxed{\tan^{-1}2+\frac{1}{2}\ln 5}$$

Nishant Jindal

3: $\lim_{n\to\infty}\left(\left(\tan^{-1}\frac{1}{n}\right)\sum_{k=1}^{n}\left(\frac{1}{1+\tan\frac{k}{n}}\right)\right)$

$\lim_{n\to\infty}\left(\frac{\tan^{-1}\frac{1}{n}}{\frac{1}{n}}\right)\lim_{n\to\infty}\sum_{k=1}^{n}\frac{\frac{1}{n}}{1+\tan\frac{k}{n}} = \int_0^1\frac{dx}{1+\tan x} = \int_0^1\frac{\cos x\,dx}{\cos x+\sin x}$

$= \frac{1}{2}\int_0^1\frac{(\cos x+\sin x)+(\cos x-\sin x)}{(\cos x+\sin x)}dx = \frac{1}{2}\left(1+\ln(\cos 1+\sin 1)\right)$

Nishant Jindal

4. $$\lim_{n\to\infty}\left(\frac{(n+1)(n+2)(n+3)\cdots(n+n)}{n^n}\right)^{\frac{1}{n}}$$

$$l = \lim_{n\to\infty}\left(\prod_{r=1}^{n}\left(\frac{n+r}{n}\right)\right)^{\frac{1}{n}}$$

$$\ln l = \lim_{n\to\infty}\frac{1}{n}\sum_{r=1}^{n}\ln\left(1+\frac{r}{n}\right) = \boxed{\int_0^1 \ln(1+x)\,dx} = \int_1^2 \ln x\,dx$$

$$= x\ln x - x\Big|_1^2 = \underline{2\ln 2 - 1} = \ln\frac{4}{e}$$

$$\boxed{l = \frac{4}{e}}$$

5. $$\lim_{n\to\infty}\left({}^{2n}C_n\right)^{\frac{1}{n}} = \lim_{n\to\infty}\left(\frac{(n+1)(n+2)\cdots(n+n)}{1\quad 2\quad\quad n}\right)^{\frac{1}{n}}$$

$$l = \lim_{n\to\infty}\left(\prod_{r=1}^{n}\frac{n+r}{r}\right)^{\frac{1}{n}} \Rightarrow \ln l = \lim_{n\to\infty}\sum_{r=1}^{n}\frac{1}{n}\ln\left(1+\frac{n}{r}\right)$$

$$= \int_0^1 \ln\left(1+\frac{1}{x}\right)dx = \int_0^1 \ln(1+x)\,dx - \underbrace{\int_{0^+}^1 \ln x\,dx}.$$

$$= 2\ln 2 - 1 - (-1) = \ln 4$$

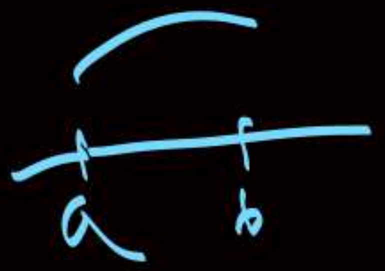
$$\boxed{l = 4}$$

Nishant Jindal

Inequality

• If $f(x) < g(x) < h(x) \quad \forall x \in [a, b]$

$$\Rightarrow \int_a^b f(x)\,dx < \int_a^b g(x)\,dx < \int_a^b h(x)\,dx.$$



$g(x) - f(x) > 0 \qquad \forall x \in [a, b]$

$$\int_a^b \left(g(x) - f(x)\right) dx > 0$$

- $\int_a^b f(x)\,dx \le \left|\int_a^b f(x)\,dx\right| \le \int_a^b |f(x)|\,dx \quad , \quad a<b.$

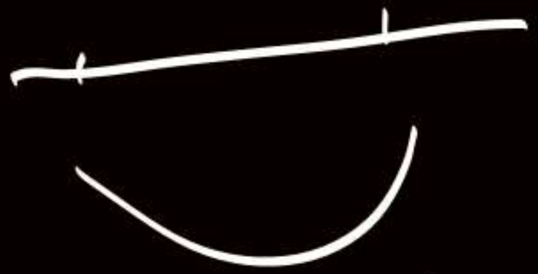
$\{$ Equality holds if $f(x) \geq 0 \;\forall x \in [a,b]$

Equality holds:

$f(x) \geq 0 \quad \forall x \in [a,b]$

or

$f(x) \leq 0 \quad \forall x \in [a,b]$



$|-D|$

Nishant Jindal

Cauchy's Inequality

f, g are continuous in $[a, b]$, then

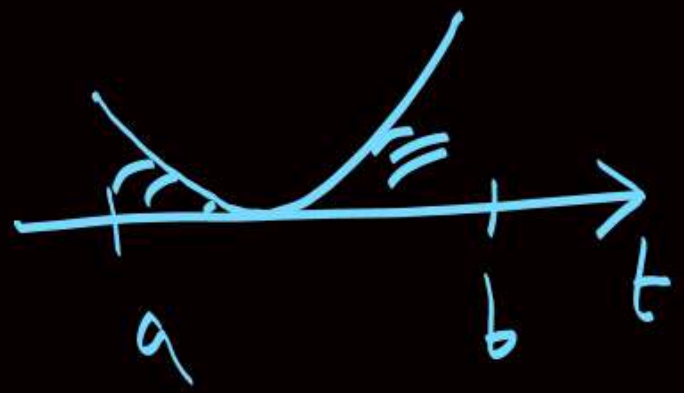
$$\left(\int_a^b f(t)g(t)\,dt\right)^2 \leq \left(\int_a^b f^2(t)\,dt\right)\left(\int_a^b g^2(t)\,dt\right)$$

Equality holds if $\dfrac{f(t)}{g(t)}$ is constant.

Nishant Jindal

$a < b$

$$\int_a^b \left(f(t) + x\, g(t) \right)^2 dt \underset{=}{\geq} 0$$



$$\int_a^b f^2(t)\,dt + 2x \int_a^b f(t)\,g(t)\,dt + x^2 \int_a^b g^2(t)\,dt \underset{=}{\geq} 0 \quad \forall x \in \mathbb{R}.$$

$f(t) + x\,g(t) = 0$

$\dfrac{f(t)}{g(t)} = -x$

$D \leq 0$

$$\left(\int_a^b f(t)\,g(t)\,dt \right)^2 \underset{=}{\leq} \int_a^b f^2(t)\,dt \int_a^b g^2(t)\,dt.$$