

DPP - 04

Spring system & Impulse

SOLUTION

1. When the spring regains its natural length, then this energy gets converted into kinetic energy, so we have

$$\frac{1}{2}kx^2 = \frac{1}{2}\mu(v_r^2 - 0^2) \dots (1)$$

μ is reduced mass of system

$$\mu = \frac{(6m)(3m)}{6m + 3m} = 2m$$

and v_r is the final relative velocity of the blocks when the spring comes to its natural length.

$$kx^2 = \mu v_r^2$$

$$\Rightarrow kx^2 = (2m)v_r^2$$

$$\Rightarrow v_r = x \sqrt{\frac{k}{2m}}$$

$$\Rightarrow \beta = 2$$

2. At maximum extension in the spring, both the rings move with same common velocity.

Work Energy Theorem from the centre of mass reference frame

$$W = W_{\text{int}} + W_{\text{ext}} = (\Delta K)_{\text{cm}} = \frac{1}{2}\mu(v_{\text{rel}}^2 - u_{\text{rel}}^2)$$

μ is reduced mass of system

$$\mu = \frac{(6m)(12m)}{6m + 12m} = 4m$$

Since no external forces are there, so $W_{\text{ext}} = 0$ and due to extension of the spring, we have

$$\text{Since, } W_{\text{int}} = W_{\text{spring}} = -\Delta U = -\frac{1}{2}kx_{\text{max}}^2$$

$$\Rightarrow -\frac{1}{2}kx_{\text{max}}^2 = -\frac{1}{2}(4m)v_0^2 - 0$$

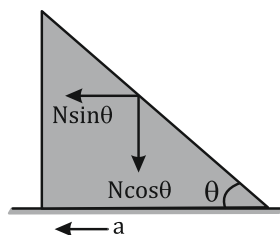
$$\Rightarrow x_{\text{max}} = 2v_0\sqrt{\frac{m}{k}}$$

3. Let a be the acceleration of wedge leftwards and a , the relative acceleration of block down the plane.

Then absolute acceleration of block in horizontal direction will be $(a_r \cos \theta - a)$ towards right.

Net force on the system in horizontal direction is zero. Therefore, acceleration of COM in

horizontal direction will be zero or acceleration of wedge towards left is equal to the acceleration of block towards right.



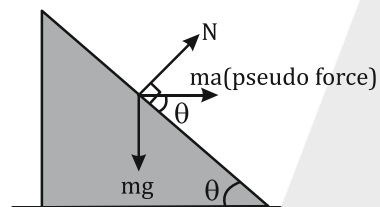
So, $a_r \cos \theta - a = a$

$$\Rightarrow 2a = a_r \cos \theta \dots (1)$$

N be the normal reaction between the block and the wedge. Then free body diagram of wedge gives

$$N \sin \theta = ma \dots (2)$$

Free body diagram of block with respect to wedge is

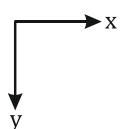


Net force on block perpendicular to plane is zero.

$$\text{Hence, } N + ma \sin \theta = mg \cos \theta \dots (3)$$

Solving equations (1), (2) and (3)

$$a_r = \frac{2g \sin \theta}{1 + \sin^2 \theta}$$



Acceleration of block vertically downwards is

$$a_y = a_r \sin \theta$$

$$\Rightarrow a_y = \frac{2g \sin^2 \theta}{1 + \sin^2 \theta}$$

So, acceleration of COM is

$$a_{cm} = \frac{a_y}{2} = \frac{g \sin^2 \theta}{(1 + \sin^2 \theta)}$$

4. At maximum extension, velocity of both the blocks will be same

Law of Conservation of Linear Momentum

$$6(2) + 3(-1) = (3 + 6)v$$

$$\Rightarrow v = 1 \text{ ms}^{-1}$$

If x be the maximum extension in the spring, then by Law of Conservation of Mechanical Energy

$$\frac{1}{2}(3)(1)^2 + \frac{1}{2}(6)(2)^2 = \frac{1}{2}(200)x^2 + \frac{1}{2}(9)(1)^2$$

$$\Rightarrow 3 + 24 = 200x^2 + 9$$

$$\Rightarrow x = \sqrt{\frac{18}{200}}$$

$$\Rightarrow x = 0.3 \text{ m}$$

$$\Rightarrow x = 30 \text{ cm}$$

5. Let the displacement of wedge be x (leftwards).

Horizontal displacement of A and B with respect to wedge is $10\cos 45^\circ$ or $5\sqrt{2}$ cm (rightwards)

or the horizontal displacement of A and B with respect to ground is $(5\sqrt{2} - x)$ cm rightwards.

The centre of mass of the whole system will not move in horizontal direction.

$$(2m)x = (5\sqrt{2} - x)(m + 2m)$$

$$\text{or } 5mx = 15\sqrt{2} \text{ m}$$

$$\text{or } x = 3\sqrt{2} \text{ cm}$$

6. Initially, $v_{\text{c.m.}} = 0$

Since there is no external force, $v_{\text{c.m.}}$ remains same.

Therefore $v_{\text{c.m.}}$ is always zero.

7. Acceleration of system

$$a = \frac{mg\sin 60^\circ - mg\sin 30^\circ}{2m}$$

Here m = mass of each block

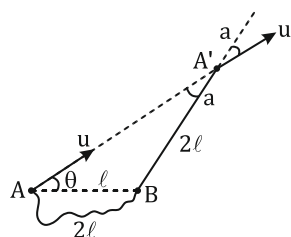
$$\text{or } a = \left(\frac{\sqrt{3}-1}{4}\right)g$$

$$\text{Now } \vec{a}_{\text{com}} = \frac{m\vec{a}_1 + m\vec{a}_2}{2m}$$

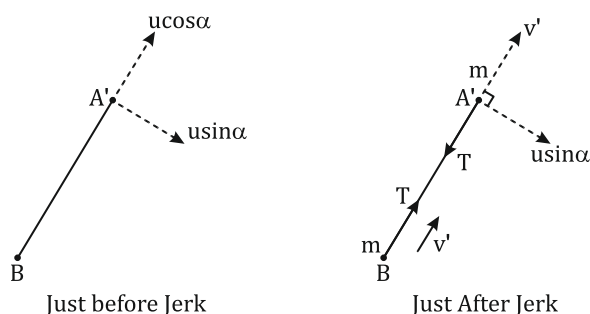
Here \vec{a}_1 and \vec{a}_2 are $\left(\frac{\sqrt{3}-1}{4}\right)g$ at the right angles.

$$\text{Hence, } |\vec{a}_{\text{com}}| = \frac{\sqrt{2}}{2}a = \left(\frac{\sqrt{3}-1}{4\sqrt{2}}\right)g$$

8. General case



At position A', the string gets taut and the components of velocities along and perpendicular to the string just before and after the jerk are shown in Figure.



Applying sine rule to $\triangle AA'B$, we get

$$\frac{\sin \theta}{2l} = \frac{\sin \alpha}{l}$$

$$\Rightarrow \sin \alpha = \frac{1}{2} \sin \theta = \frac{1}{2} \sqrt{1 - \cos^2 \theta} \dots (1)$$

Since, velocity of A perpendicular to the string remains constant during jerk, so applying the Impulse Momentum theorem, we get

$$\text{For A, } \int -T dt = mv' - m u \cos \alpha \dots (2)$$

$$\text{For B, } \int T dt = mv' - 0 \dots (3)$$

Adding equations (2) and (3), we get

$$0 = mv' + mv' - m u \cos \alpha$$

$$\Rightarrow v' = \frac{u \cos \alpha}{2} \dots (4)$$

For (a), u is along BA, i.e., $\theta = \pi$ and $\alpha = 0^\circ$

$$\Rightarrow v' = \frac{u}{2}$$

For (b), u makes an angle of 90° with AB,

$$\text{i.e., } \theta = \frac{\pi}{2} \text{ and}$$

$$\alpha = \frac{\pi}{6}$$

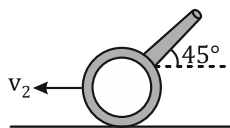
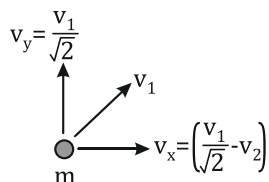
$$\Rightarrow v' = \frac{u\sqrt{3}}{4}$$

9. If shell is fired at muzzle at speed v_1

then by conservation of linear momentum

$$m \left(\frac{v_1}{\sqrt{2}} - v_2 \right) = kmv_2$$

$$\Rightarrow v_2 = \frac{v_1}{\sqrt{2}(k+1)}$$



The ratio of kinetic energies is given by

$$\frac{K_{\text{shell}}}{K_{\text{gun}}} = \frac{\frac{1}{2} m \left[\frac{v_1^2}{2} + \left(\frac{v_1}{\sqrt{2}} - v_2 \right)^2 \right]}{\frac{1}{2} k m v_2^2}$$

$$\Rightarrow \frac{K_{\text{shell}}}{K_{\text{gun}}} = \frac{\frac{v_1^2}{2} + \frac{k^2 v_1^2}{2(k+1)^2}}{\frac{k v_1^2}{2(k+1)^2}}$$

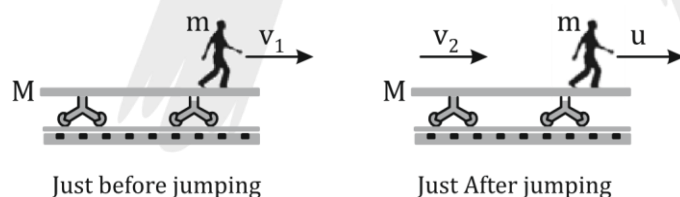
$$\Rightarrow \frac{K_{\text{shell}}}{K_{\text{gun}}} = \frac{2k^2 + 2k + 1}{k}$$

10. Let the velocity of the car just after jumping of man be v .

The net velocity of the man with respect to the ground will be $u - v$, as u is its velocity with respect to the car.

If we take man and car as system.

Linear momentum of the system should be conserved in horizontal direction (No external force is present).



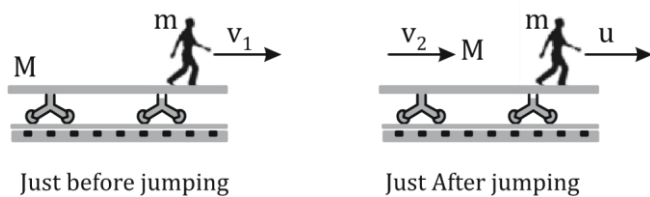
$$m(u - v) = Mv$$

$$\Rightarrow v = \frac{mu}{m + M}$$

11. There is no external force is present, so linear momentum will be conserved

After the man jumps, the car attains a velocity v_2 in the same direction, which is less than v_1 , due to backward push of the man for jumping.

After the jump, the man attains a velocity $(u + v_2)$ in the direction of motion of car with respect to ground.



Law of Conservation of Linear Momentumssss

$$(M + m)v_1 = Mv_2 + m(u + v_2)$$

So, velocity of car after jump is

$$v_2 = \frac{(M + m)v_1 - mu}{M + m}$$

and velocity of man after jump is

$$u + v_2 = \frac{(M + m)v_1 + Mu}{M + m}$$