Nishant Jindal

Ex 16 Q 1

$A = B$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \cdot \tan B}$$

$$\tan(A+A) = \frac{\tan A + \tan A}{1 - \tan A \cdot \tan A} = \frac{2\tan A}{1 - \tan^2 A}$$

1) $\boxed{\tan A = \frac{1}{2}}$, $\boxed{\tan B = \frac{1}{3}}$

$\tan(2A+B) = ?$ & $\tan(2A-B) = ?$

Step 1) $$\tan 2A = \frac{2\tan A}{1-\tan^2 A} = \frac{2 \times \frac{1}{2}}{1 - \frac{1}{4}} = \frac{4}{3}$$

Step 2) $$\tan(2A+B) = \frac{\tan 2A + \tan B}{1 - \tan 2A \cdot \tan B} = \frac{\frac{4}{3} + \frac{1}{3}}{1 - \frac{4}{3} \times \frac{1}{3}} = \frac{\frac{5}{3}}{1 - \frac{4}{9}} = \frac{\frac{5}{3}}{\frac{5}{9}} = \frac{\cancel{5}}{\cancel{3}} \times \frac{\cancel{9}^{3}}{\cancel{5}} = 3$$

Step 3) $$\tan(2A-B) = \frac{\tan 2A - \tan B}{1 + \tan 2A \cdot \tan B} = \frac{\frac{4}{3} - \frac{1}{3}}{1 + \frac{4}{3} \times \frac{1}{3}} = \frac{1}{1 + \frac{4}{9}} = \frac{9}{13}$$

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Trigonometry

Q2 $\tan A = \frac{\sqrt{3}}{\sqrt{4}-\sqrt{3}}$ & $\tan B = \frac{\sqrt{3}}{\sqrt{4}+\sqrt{3}}$ then P.T. $\tan(A-B) = \boxed{\cancel{.375}}$

$$\tan(A-B) = \frac{\tan A - \tan B}{1+\tan A \cdot \tan B} = \frac{\frac{\sqrt{3}}{4-\sqrt{3}} - \frac{\sqrt{3}}{4+\sqrt{3}}}{1+\left(\frac{\sqrt{3}}{4-\sqrt{3}}\right)\left(\frac{\sqrt{3}}{4+\sqrt{3}}\right)} = \frac{\frac{(4\cancel{\sqrt{3}}+3)-(\cancel{4\sqrt{3}}-3)}{\cancel{(4-\sqrt{3})(4+\sqrt{3})}}}{\frac{(16-\cancel{3})+\cancel{3}}{\cancel{(4-\sqrt{3})(4+\sqrt{3})}}} = \frac{6}{16}$$

$$= \frac{3}{8}$$

$$= .375$$

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Q $\tan A = \frac{n}{n+1}$, $\tan B = \frac{1}{2n+1}$.

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B} = \frac{\frac{n}{n+1} + \frac{1}{2n+1}}{1 - \left(\frac{n}{n+1}\right)\left(\frac{1}{2n+1}\right)}$$ $\boxed{DY}$ $\boxed{1}$

Q $\tan\alpha = \frac{5}{6}$, $\tan\beta = \frac{1}{11}$ P.T. $\alpha + \beta = \frac{\pi}{4}$.

$$\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha \cdot \tan\beta} = \frac{\frac{5}{6} + \frac{1}{11}}{1 - \frac{5}{6} \times \frac{1}{11}} = \frac{\frac{55+6}{\cancel{66}}}{\frac{66-5}{\cancel{66}}} = \frac{61}{61} = 1 = \tan\frac{\pi}{4}$$

Q5 $\tan\left(\frac{\pi}{4}+\theta\right) \cdot \tan\left(\frac{3\pi}{4}+\theta\right) = -1$

LHS $\left(\frac{1 + \cancel{\tan\theta}}{1 - \cancel{\tan\theta}}\right) \times \left(\frac{-1 + \cancel{\tan\theta}}{1 + \cancel{\tan\theta}}\right) = -1 = RHS$

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$\cot\frac{\pi}{4} = \cot 45^\circ = 1$

Q To Prove $\cot\left(\frac{\pi}{4}+\theta\right)\cdot\cot\left(\frac{\pi}{4}-\theta\right) = 1$.

$$\frac{\cot\frac{\pi}{4}\cdot\cot\theta - 1}{\cot\frac{\pi}{4}+\cot\theta} \times \frac{\cot\frac{\pi}{4}\cdot\cot\theta + 1}{\cot\theta - \cot\frac{\pi}{4}} = \left(\frac{\cancel{1\cdot\cot\theta - 1}}{1+\cot\theta}\right)\times\left(\frac{\cancel{\cot\theta + 1}}{\cancel{-1+\cot\theta}}\right) = +1$$

Q To prove $\boxed{1+\tan A\cdot\tan\frac{A}{2}} = \tan A\cdot\cot\frac{A}{2} - 1 = \boxed{\sec A}$.

LHS $1+\tan A\cdot\tan\frac{A}{2} = 1+\frac{\sin A}{\cos A}\times\frac{\sin\frac{A}{2}}{\cos A/2} = \frac{\cos A\cdot\cos\frac{A}{2}+\sin A\cdot\sin\frac{A}{2}}{\cos A\cdot\cos A/2} = \frac{\cos\left(A-\frac{A}{2}\right)}{\cos A\cdot\cos\frac{A}{2}}$

$$\frac{\cancel{\cos A/2}}{\cos A\cdot\cancel{\cos A/2}}$$

$\frac{1}{\cos A} = \sec A$

RHS

$\cot(A+B) = \frac{\cot A\cot B - 1}{\cot A+\cot B}$, $\cot(A-B) = \frac{\cot A\cdot\cot B+1}{\cot B-\cot A}$

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Ex 15

1) $2\sin 5\theta \cdot \sin 7\theta$

$= \cos(5\theta-7\theta) - \cos(5\theta+7\theta)$

$= \cos(+2\theta) - \cos(12\theta)$

$\boxed{3}$

2) $2\cos 7\theta \cdot \cos 5\theta$

Sin

$\cos(7\theta+5\theta) + \cos(7\theta-5\theta)$

$\cos 12\theta + \cos(2\theta$

$\boxed{3}$

3) $2\cos 11\theta \cdot \cos 3\theta$

$\cos(14\theta) + \cos(8\theta)$

$\boxed{4}$

(4) $2\sin 54^\circ \cdot \sin 66^\circ$

$\cos(54-66) - \cos(54+66)$

$\cos(-12^\circ) - \cos(120^\circ)$

$\cos 12^\circ - (-\frac{1}{2}) = \cos 12^\circ + \frac{1}{2}$

$\boxed{6, 7, 8, 9, 10}$

(5) $\sin\frac{\theta}{2} \cdot \sin\frac{7\theta}{2} + \sin\frac{3\theta}{2} \cdot \sin\frac{11\theta}{2} = \sin 5\theta \cdot \sin 2\theta$

LHS $\frac{1}{2}\left(2\sin\frac{\theta}{2} \cdot \sin\frac{7\theta}{2}\right) + \frac{1}{2}\left(2\sin\frac{3\theta}{2} \cdot \sin\frac{11\theta}{2}\right)$

$\boxed{4}$ $\frac{1}{2}\left(\cos\left(+\frac{\cancel{6\theta}}{\cancel{2}}\right)^{3\theta} - \cos\left(\frac{\cancel{8\theta}}{\cancel{2}}\right)^{4\theta}\right) + \frac{1}{2}\left(\cos\left(+\frac{\cancel{8\theta}}{\cancel{2}}\right)^{4\theta} - \cos\left(\frac{\cancel{14\theta}}{\cancel{2}}\right)^{7\theta}\right)$

$\frac{1}{2}(\cos 3\theta - \cos 7\theta) = \frac{1}{2}(+2\sin(5\theta) \cdot \sin(+2\theta))$

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Trigonometry

Q7 $\sin A \cdot \sin(A+2B) - \sin B \cdot \sin(B+2A) = \sin(A-B) \cdot \sin(A+B)$

LHS

$$= \frac{1}{2}\left(2\sin A \cdot \sin(A+2B)\right) - \frac{1}{2}\left(2\sin B \cdot \sin(B+2A)\right)$$

$$= \frac{1}{2}\left(\cos(-2B) - \cos(2A+2B)\right) - \frac{1}{2}\left(\cos(-2A) - \cos(2B+2A)\right)$$

$$= \frac{1}{2}\left(\cos 2B - \cos 2A\right)$$

$$= \frac{1}{\cancel{2}}\left(-\cancel{2}\sin\left(\frac{2B+2A}{2}\right) \cdot \sin\left(\frac{2B-2A}{2}\right)\right)$$

$$= -\sin(B+A) \cdot \sin(B-A)$$

$$= +\sin(A+B)\sin(A-B)$$

RHS

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$(36^\circ - A) - (36^\circ + A) = -2A$

Q8 $(\sin 3A + \sin A)\sin A + (\cos 3A - \cos A)\cdot\cos A = 0$

$\underline{\sin 18^\circ} = \underline{\cos 72^\circ} \Rightarrow 18^\circ + 72^\circ = 90^\circ$

LHS

Sum and diff

$\frac{1}{2}\left[\underbrace{2\sin 3A\cdot\sin A}_{\text{Prod}} + \underbrace{2\sin A\cdot\sin A}_{\text{Prod}} + \underbrace{2\cos 3A\cdot\cos A}_{\text{Prod}} - \underbrace{2\cos A\cdot\cos A}_{\text{Prod.}}\right]$

$\frac{1}{2}\left[\cancel{\cos(2A)} - \cancel{\cos(4A)} + \cancel{\cos(0)} - \cancel{\cos(2A)} + \cancel{\cos(4A)} + \cancel{\cos(2A)} - \left(\cancel{\cos(2A)} + \cancel{\cos(0)}\right)\right]$

$= 0$

$(54^\circ + A) + (54^\circ - A) = 108^\circ$

Q12 $\frac{1}{2}\left[\underbrace{2\cos(36^\circ - A)\cdot\cos(36^\circ + A)}_{\text{Prod}} + \underbrace{2\cos(54^\circ + A)\cos(54^\circ - A)}_{\text{Prod}}\right] = \cos 2A$

$(36-A)+(36+A)$

$\frac{1}{2}\left[\cos(72^\circ) + \cos(+2A) + \underline{\underline{\cos(108^\circ)}} + \cos(2A)\right]$

$\frac{1}{2}\left[\sin(18^\circ) + \cos\left(\frac{\pi}{2} + 18^\circ\right) + 2\cos 2A\right] = \frac{1}{2}\left[2\cos 2A + \cancel{\sin 18^\circ} - \cancel{\sin 18^\circ}\right]$

$= \cos 2A = \text{RHS}$

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Q15

① Out of syllabus.

Ver Sin (A+B). Ver Sin (A−B)

$(1-\cos(A+B))\ (1-\cos(A-B))$

Ex17 KamKA

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2θ

1) $\sin 2\theta = 2\sin\theta \cdot \cos\theta$

2) $\cos 2\theta = \cos^2\theta - \sin^2\theta$
$= 2\cos^2\theta - 1$
$= 1 - 2\sin^2\theta$

$1 + \cos 2A = 2\cos^2 A$
$1 - \cos 2A = 2\sin^2 A$

3) $\tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta}$

3θ

$$\boxed{\begin{aligned} \sin 3\theta &= 3\sin\theta - 4\sin^3\theta \\ \cos 3\theta &= 4\cos^3\theta - 3\cos\theta \\ \tan 3\theta &= \frac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta} \end{aligned}}$$

(A) $\sin\theta \cdot \sin(60-\theta) \cdot \sin(60+\theta) = \dfrac{\sin 3\theta}{4}$

(B) $\cos\theta \cdot \cos(60-\theta)\cos(60+\theta) = \dfrac{\cos 3\theta}{4}$

(C) $\tan\theta \cdot \tan(60-\theta) \cdot \tan(60+\theta) = \tan 3\theta$

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Q. $A+B=\frac{\pi}{4}$ $\longrightarrow$ $11^\circ+34^\circ=45$

then $(1+\tan A)(1+\tan B)=?$ $\quad (1+\tan 11)(1+\tan 34)$

$= 2$

$\boxed{2}$

Q. If $\underline{A+B=225^\circ}$

Value of $\left(\frac{\cot A}{1+\cot A}\right)\left(\frac{\cot B}{1+\cot B}\right)=?$

$A+B=225^\circ$

$\boxed{3}$

$\cot(A+B)=\cot 225^\circ=\cot(\pi+45^\circ)=\cot 45^\circ=1$

$\cot(A+B)=1$ $\quad$ <u>Experience</u>

$\frac{\cot A\cdot\cot B-1}{\cot A+\cot B}=1$

$\cot A\cdot\cot B-1=\cot A+\cot B$

$2\cot A\cot B\cancel{-1}=\cot A+\underbrace{\cot B+\cot A\cot B}+1$

$2\cot A\cot B=(\cot A+1)+\cot B(1+\cot A)$

$2\cot A\cot B=(1+\cot A)(1+\cot B)$

$\left(\frac{\cot A}{1+\cot A}\right)\cdot\left(\frac{\cot B}{1+\cot B}\right)=\frac{1}{2}$ Ans

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$(a-b)^2 \div (a^2-b^2)$

$(a-b)^2 \div (a-b)(a+b) = (a-b)$

$11^\circ + 34^\circ = 45^\circ$

Q If $f(\theta) = \dfrac{(1-\sin 2\theta) + \cos 2\theta}{2\cos 2\theta}$ then find value of $\boxed{8}\, f(11^\circ) \cdot f(34^\circ)$

$(1+\tan 11)(1+\tan 34) = 2$

$(a-b)^2 = a^2 + b^2 - 2a\cdot b$

$$f(\theta) = \frac{(\sin^2\theta + \cos^2\theta - 2\sin\theta\cos\theta) + (\cos^2\theta - \sin^2\theta)}{2(\cos^2\theta - \sin^2\theta)}$$

$(\cos\theta - \sin\theta)^2$

$= \cos^2\theta + \sin^2\theta - 2\sin\theta\cdot\cos\theta$

$$= \frac{(\cos\theta-\sin\theta)^2 + (\cos^2\theta - \sin^2\theta)}{2(\cos^2\theta-\sin^2\theta)} = \frac{\cancel{(\cos\theta-\sin\theta)}\{(\cos\theta-\sin\theta) + (\cos\theta+\sin\theta)\}}{2\,\underbrace{\cancel{(\cos^2\theta-\sin^2\theta)}}_{(\cos\theta+\sin\theta)}}$$

$$f(\theta) = \frac{\cos\theta}{\cos\theta + \sin\theta} = \frac{1}{(1+\tan\theta)}$$

$$\text{Demand} = 8f(11^\circ)\cdot f(34^\circ) = 8 \times \frac{1}{(1+\tan 11^\circ)} \times \frac{1}{(1+\tan 34^\circ)} = 8 \times \frac{1}{2} = 4$$

$\boxed{1+\cos 2A = 2\cos^2 A}$

Practice

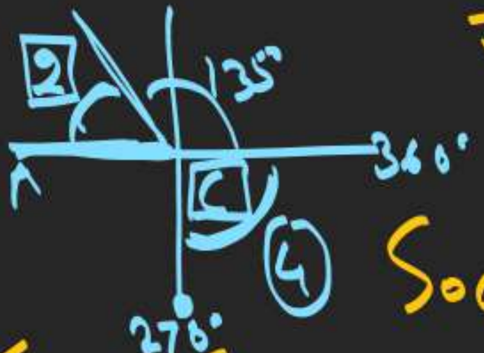
$\sqrt{x^2} = |x|$

Q (Imp) If $(4\cos^2 9^\circ - 3)\times(4\cos^2 27^\circ - 3) = \tan\theta$ then $\theta = ?$

43 No. Ki Bus.

$\Rightarrow \cos 3\theta = 4\cos^3\theta - 3\cos\theta$

LHS. $\dfrac{(4\cos^3 9^\circ - 3\cos 9^\circ)}{\cos 9^\circ}\times\dfrac{(4\cos^3 27^\circ - 3\cos 27^\circ)}{\cos 27^\circ}$



Sochhhhhhhna.

$\dfrac{\cos(3\times 9^\circ)}{\cos 9^\circ}\times\dfrac{(\cos 3\times 27^\circ)}{\cos 27^\circ} = \dfrac{\cancel{\cos 27^\circ}}{\cos 9^\circ}\times\dfrac{\cos 81^\circ}{\cancel{\cos 27^\circ}} = \dfrac{\sin 9^\circ}{\cos 9^\circ} = \tan\boxed{9^\circ} = \tan\boxed{\theta}$

$\theta = 9^\circ$

Q (Imp) $\sqrt{2+\sqrt{2+2\cos 4\theta}} = ?$; $3\pi < 4\theta < 4\pi$

$135^\circ < \theta < \pi$

$\frac{3\pi}{2} < 2\theta < 2\pi$

$\sqrt{2+\sqrt{2(1+\cos 4\theta)}} = \sqrt{2+\sqrt{2\times 2\cos^2 2\theta}} = \sqrt{2+2|\cos 2\theta|}$ $(+)$

$= \sqrt{2+2\cos 2\theta} = \sqrt{2(1+\cos 2\theta)} = \sqrt{2\times 2\cos^2\theta}$

$= 2|\cos\theta| = -2\cos\theta$ $(-ve)$

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$\tan\theta\cdot\tan(60-\theta)\cdot\tan(60+\theta)$
$=\tan 3\theta$

Q $\tan 6^\circ\cdot\tan 42^\circ\cdot\tan 66^\circ\cdot\tan 78^\circ$

Imp.

$(\tan 6^\circ\cdot\tan 66^\circ \quad)\times(\quad \tan 42^\circ\cdot\tan 78^\circ)$

$\left(\tan\boxed{6^\circ}\tan(60+6^\circ)\cdot\tan(60-6^\circ)\right)\times\left(\dfrac{\tan\boxed{18^\circ}\tan(60-18^\circ)\cdot\tan(60+18^\circ)}{\tan 18^\circ}\right)$

$\tan 54^\circ$

Ex17

4085

2085 + 2085

$\dfrac{\cancel{\tan(3\times 6^\circ)}}{\cancel{\tan 54^\circ}}\times\dfrac{\cancel{\tan(3\times 18^\circ)}}{\cancel{\tan 18^\circ}}=1$

$\tan 3A-\tan 2A-\tan A=\tan 3A\cdot\tan 2A\cdot\tan A$

Q $\tan 9^\circ-\tan 27^\circ-\tan 63^\circ+\tan(81^\circ)=?$

Sol $\tan 9^\circ+\cot 9^\circ-(\tan 27^\circ+\cot 27^\circ)$