DPP-03

(STANDARD INTEGRATION)

1. $\int\left[\sin^2\left(\frac{9\pi}{8}+\frac{x}{4}\right)-\sin^2\left(\frac{7\pi}{8}+\frac{x}{4}\right)\right]dx$

Ans. $-\sqrt{2}\cos\frac{x}{2}+C$

Sol. $I=\int\left[\sin^2\left(\frac{9x}{8}+\frac{x}{4}\right)-\sin^2\left(\frac{7x}{8}+\frac{x}{4}\right)\right]dx$

$$I=\int \sin\left(\frac{9x}{8}+\frac{x}{4}+\frac{7x}{8}+\frac{x}{4}\right)\left(\frac{9x}{8}+\frac{x}{4}-\frac{7x}{8}-\frac{x}{4}\right)dx$$

$$I=\int\left(\sin\left(2x+\frac{x}{2}\right)\sin\frac{x}{4}dx\right.$$

$$I=\frac{1}{\sqrt{2}}\int \sin\frac{x}{2}dx=\frac{-2}{\sqrt{2}}\cos\frac{x}{2}+C$$

$$I=-\sqrt{2}\cos\frac{x}{2}+C$$

2. $\int\frac{\cos 4x+1}{\cot x-\tan x}dx$

Ans. $-\frac{\cos 4x}{8}+C$

Sol. $\int\frac{\cos 4x+1}{\cot x-\tan x}=A\cos 4x+B$

$$I=\int\frac{(\cos 4x+1)}{(\cos^2 x-\sin^2 x)}\cos x\sin x\,dx$$

$$=\int\left(\frac{2\cos^2 2x}{\cos 2x}\right)(\cos x\sin x)dx$$

$$=\int\cos 2x\sin 2x\,dx$$

$$=\frac{1}{2}\int\sin 4x\,dx=-\frac{\cos 4x}{8}+B$$

3. A function g defined for all positive real numbers satisfies $g'(x^2)=x^3$ for all $x>0$ ànd $g(1)=1$. Compute $g(4)$.

Ans. $\frac{67}{5}$

Sol. $g^1(x^2)=x^3$ $\Rightarrow$ put $x^2=t$

$g^1(t)=t^{\frac{3}{2}}$ $\Rightarrow$ Integrating both sides

$g(t)=\frac{t^{\frac{5}{2}}}{\frac{5}{2}}+C$ $\Rightarrow g(1)=\frac{2}{5}\cdot 1+C$

$c=1-\frac{2}{5}=\frac{3}{5}$ $\Rightarrow g(t)=\frac{2}{5}t^{\frac{5}{2}}+\frac{3}{5}$

$g(4)=\frac{2}{5}((2)^2)^{\frac{5}{2}}+\frac{3}{5}$ $\Rightarrow g(4)=\frac{64}{5}+\frac{3}{5}=\frac{67}{5}$

4. $\int \left[\sin\alpha \sin(x-\alpha) + \sin^2\left(\frac{x}{2}-\alpha\right)\right] dx$

Ans. $\frac{1}{2}(x - \sin x) + C$

Sol. $I = \int \left[\sin x \sin(x-2) + \sin^2\left(\frac{x}{2}-\alpha\right)\right] dx$

$I = \int \left[\frac{2\sin\alpha\sin(x-\alpha)}{2} + \frac{1-\cos(x-2\alpha)}{2}\right] dx$

$I = \int \left[\frac{\cos(x-2\alpha)-\cos x}{2} + \frac{1-\cos(x-2\alpha)}{2}\right] dx$

$I = \int \frac{1}{2}(1-\cos x)dx$

$I = \frac{1}{2}(x - \sin x) + C$

5. $\int \frac{\sin 2x + \sin 5x - \sin 3x}{\cos x + 1 - 2\sin^2 2x} dx$

Ans. $-2\cos x + C$

Sol. $I = \int \frac{\sin 2x + \sin 5x - \sin 3x}{\cos x + 1 - 2\sin^2 2x} dx$

$I = \int \frac{2\sin x\cos x + 2\cos 4x\sin x}{\cos x + \cos 4x} dx$

$I = \int \frac{2\sin x(\cos x + \cos 4x)}{(\cos x + \cos 4x)} dx$

$I = \int 2\sin x dx = -2\cos x + C$

6. $\int \left[\frac{\cot^2 2x - 1}{2\cot 2x} - \cos 8x \cot 4x\right] dx$

Ans. $-\frac{\cos 8x}{8} + C$

Sol. $I = \int \left[\frac{\cot^2 2x - 1}{2\cot 2x} - \cos 8x\cot 4x\right] dx$

$I = \int [\cot 4x - \cos 8x\cot 4x]dx$

$I = \int \cot 4x(1 - \cos 8x)dx$

$I = \int \cot 4x(2\sin^2 4x)dx$

$I = \int \sin 8x dx$

$I = \frac{-\cos 8x}{8} + C$

7. $\int \frac{\cos^4 x - \sin^4 x}{\sqrt{1+\cos 4x}} dx \ (\cos 2x > 0)$

Ans. $\frac{x}{\sqrt{2}} + C$

Sol. $I = \int \frac{\cos^4 x - \sin^4 x}{\sqrt{1+\cos 4x}} dx$

$I = \int \frac{(\cos^2 x - \sin^2 x)x(\sin^2 x + \cos^2 x)}{\sqrt{2}\cos 2x} dx$

$I = \int \frac{\cos 2x}{\sqrt{2}\cos 2x} dx = \frac{1}{\sqrt{2}}x + c$

8. $\int \frac{2x^3+3x^2+4x+5}{2x+1} dx$

Ans. $\frac{x^3}{3}+\frac{x^2}{2}+\frac{3x}{2}+\frac{7}{4}\ln(2x+1)$

Sol. $I=\int \frac{2x^3+3x^2+4x+5}{2x+1} dx$

$I=\int \left(\frac{(2x^3+x^2)+(2x^2+x+(3x+5)}{2x+1}\right) dx$

$I=\int \left(x^2+x+\frac{1}{2}\left(\frac{6x+10}{2x+1}\right)\right) dx$

$I=\int \left[x^2+x+\frac{1}{2}\left(\frac{6x+3}{2x+1}+\frac{7}{2x+1}\right)\right] dx$

$I=\frac{x^3}{3}+\frac{x^2}{2}+\frac{3}{2}x+\frac{7}{4}\ln(2x+1)+C$

9. $\int \frac{(x^2+\sin^2 x)\sec^2 x}{1+x^2} dx$

Ans. $\tan x-\tan^{-1}x+C$

Sol. $I=\int \frac{(x^2+\sin^2 x)\sec^2 x}{1+x^2} dx$

$I=\int \frac{(x^2+1-\cos^2 x)\sec^2 x}{1+x^2} dx$

$I=\int \left[\frac{(x^2+1)\sec^2 x}{1+x^2}-\frac{\cos^2 x\cdot\sec^2 x}{1+x}\right] dx$

$I=\int \left(\sec^2 x-\frac{1}{1+x}\right) dx$

$I=\tan x-\tan^{-1}x+C.$

10. $\int \frac{dx}{\sqrt{9-16x^2}}$

Ans. $\frac{1}{4}\sin^{-1}\frac{4}{3}x+C$

Sol. $I=\int \frac{dx}{\sqrt{9-16x^2}}$

$I=\int \frac{dx}{4\sqrt{\frac{9}{16}-x^2}}$

$I=\frac{1}{4}\sin^{-1}\frac{x}{\frac{3}{4}}+C$

$I=\frac{1}{4}\sin^{-1}\frac{4x}{3}+C$

11. $\int \frac{dx}{25+4x^2}$

Ans. $\frac{1}{10}\tan^{-1}\frac{2x}{5}+C$

Sol. $I=\int \frac{dx}{25+4x^2}$

$I=\frac{1}{4}\int \frac{dx}{\frac{25}{4}+x^2}$

$I=\frac{1}{4}\cdot\frac{1}{\frac{5}{2}}\tan^{-1}\frac{x}{\frac{5}{2}}+C$

$I=\frac{1}{10}\tan^{-1}\frac{2x}{5}+C$

12. $\int \frac{2x+3}{3x+2} dx$

Ans. $\frac{2}{3}x + \frac{5}{9}\ln(3x+2) + C$

Sol. $I = \int \frac{2x+3}{3x+2} dx$

$I = \frac{1}{3}\int \frac{6x+9}{3x+2} dx$

$I = \frac{1}{3}\int \left(\frac{6x+4}{3x+2} + \frac{5}{3x+2}\right) dx$

$I = \frac{2}{3}x + \frac{5}{9}\ln(3x+2) + C$

13. $\int \frac{\cos 8x - \cos 7x}{1+2\cos 5x} dx$

Ans. $\frac{\sin 3x}{3} - \frac{\sin 2x}{2} + C$

Sol. $I = \int \frac{\cos 8x - \cos 7x}{1+2\cos 5x} dx$

$I = \int \frac{-2\sin\left(\frac{15x}{2}\right)\sin\frac{x}{2}}{1+2\left(1-2\sin^2\frac{5x}{2}\right)} dx$

$I = \int \frac{-2\sin\left(\frac{15x}{2}\right)\sin\left(\frac{x}{2}\right)}{3-4\sin\frac{5x}{2}} dx$

$I = \int \frac{-2\sin\frac{15x}{2}\sin\frac{x}{2}\sin\frac{5x}{2}}{\left(3-4\sin^2\left(\frac{5x}{2}\right)\right)\cdot\sin\left(\frac{5x}{2}\right)} dx$

$I = \int \frac{-2\sin\frac{15x}{2}\sin\frac{x}{2}\sin\frac{5x}{2}}{3\sin\frac{5x}{2}} dx$

14. $\int \frac{2+3x^2}{x^2(1+x^2)} dx$

Ans. $-\frac{2}{x} + \tan^{-1} x + C$

Sol. $I = \int \frac{2+3x^2}{x^2(1+x^2)} dx$

$I = \int \left(\frac{2}{x^2} + \frac{1}{1+x^2}\right) dx$

$I = \frac{-2}{x} + \tan^{-1} x + C$

15. $\int \frac{(\sin 2x)-(\sin 2k)}{\sin x - \sin k + \cos x - \cos k} dx$

Ans. $(\sin x - \cos x) + (\sin k + \cos k)x + C$

Sol. $I = \int \frac{(\sin 2x)-(\sin 2K)}{\sin x - \sin k + \cos - \cos k} dx$

$$I = \int \frac{2\cos(x+k)\sin(x-k)}{2\cos\left(\frac{x+k}{2}\right)\sin\left(\frac{x-k}{2}\right) + 2\sin\left(\frac{x+k}{2}\right)\sin\left(\frac{k-x}{2}\right)} dx$$

$$I = \int \frac{2\cos(x+k)\sin(x-k)}{2\sin\left(\frac{x+k}{2}\right)\sin\left(\frac{x-k}{2}\right) + 2\sin\left(\frac{x+k}{2}\right)} dx$$

$I = \int \frac{2\cos(x+k)\cos\frac{(x-k)}{2}}{\cos\left(\frac{x+k}{2}\right) - \sin\left(\frac{x+k}{2}\right)} dx$

16. $\int \frac{x^2+3}{x^6(x^2+1)} dx$

Ans. $C - \frac{2}{x} + \frac{2}{3}\frac{1}{x^3} - \frac{3}{5}\frac{1}{x^5} - 2\tan^{-1}x$

Sol. $I = \frac{2\cos(x+k)\cos\left(\frac{x-k}{2}\left[\cos\frac{x+k}{2}\right]\right)}{\cos\left(\frac{x+k}{2}\right)-\sin\left(\frac{x+k}{2}\right)} \times \frac{+\cos\left(\frac{x+k}{2}\right)+\sin\left(\frac{x+k}{2}\right)}{\cos\left(\frac{x+k}{2}\right)+\sin\left(\frac{x+k}{2}\right)}$

$I = \int \frac{2\cos(x+k)\cos\frac{x-k}{2}\left[\cos\left(\frac{x+k}{2}\right)\sin\left[\frac{x+k}{2}\right]\right]}{\cos(x+k)}$

$I = \int 2\cos\left(\frac{x-k}{2}\right)\cos\left(\frac{x+k}{2}\right) dx$

$\int 2\cos\left(\frac{x-k}{2}\right)\sin\left(\frac{x+k}{2}\right) dx$

$I = \int(\cos x + \cos k)dx + \int(\sin x + \sin k)dx$

$I = \sin x + x\cos k - \cos x + x\sin k + c$

$I = \sin x - \cos x + x(\cos k + \sin k) + c$

17. $\int \sin x \cos x \cos 2x \cos 4x dx$

Ans. $-\frac{1}{64}\cos 8x + C$

Sol. $I = \int \sin x\cos x\cos 2x\cos 4x dx$

$I = \frac{1}{2}\int 2\sin x\cos x\cos 2x\cos 4x dx$

$I = \frac{1}{4}\int 2\sin 2x\cos 2x\cos 4x dx$

$I = \frac{1}{8}\int 2\sin 4x\cos 4x dx$

$I = \frac{1}{8}\int \sin 8x dx$

$I = \frac{-1}{64}\cos 8x + C$

18. $\int x^x \ln(ex) dx$

Ans. $x^x + C$

Sol. $I = \int x^x \ln(ex) dx$

$I = \int_{X^x}(1+\ln x)dx$

put $x^x = t$

$xx(1+\ln x)dx = dt$

$I = \int dt = t + c$

$I = x^x + C$

19. $\int \frac{dx}{x^2+x+1}$ is equal to

(A) $\frac{\sqrt{3}}{2}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)+c$

(B) $\frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)+c$

(C) $\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)+c$

(D) $\sqrt{3}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)+c$

Ans. (B)

Sol. $y=\int \frac{dx}{x^2+x+1}$

$=\int \frac{dx}{x^2+x+\frac{1}{4}+1-\frac{1}{4}}=\int \frac{dx}{\left(x+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}$

$=\frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)+c$

20. If $\int \frac{1}{1+\sin x}dx=\tan\left(\frac{x}{2}+a\right)+b$, then

(A) $a=-\frac{\pi}{4}, b\in R$

(B) $a=\frac{\pi}{4}, b\in R$

(C) $a=\frac{5\pi}{4}, b\in R$

(D) $a=\frac{\pi}{2}, b\in R$

Ans. (A)

Sol. $\int \frac{dx}{1+\sin x}\times\frac{1-\sin x}{1-\sin x}$

$=\int \frac{1-\sin x}{\cos^2 x}dx$

$=\int \sec^2 x dx-\int \tan x\sec x dx$

$=\tan x-\sec x+c$

$=\frac{\sin x-1}{\cos x}+c$

$=\frac{2\sin\frac{x}{2}\cos\frac{x}{2}-\left(\cos^2\frac{x}{2}+\sin^2\frac{x}{2}\right)}{\cos^2\frac{x}{2}-\sin^2\frac{x}{2}}+c$

$=-\frac{\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)^2}{\left(\cos^2\frac{x}{2}-\sin^2\frac{x}{2}\right)}+c$

$=-\frac{\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)}{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)}+c=-\left(\frac{1-\tan\frac{x}{2}}{1+\tan\frac{x}{2}}\right)+c$

$=-\tan\left(\frac{\pi}{4}-\frac{x}{2}\right)+c=\tan\left(\frac{x}{2}-\frac{\pi}{4}\right)+c$

$a=-\frac{\pi}{4}, b\in R$

21. $\int\{1+2\tan x(\tan x+\sec x)\}^{1/2}dx$ is equal to

(A) $\ln \sec x(\sec x-\tan x)+c$

(B) $\ln \operatorname{cosec} x(\sec x+\tan x)+c$

(C) $\ln \sec x(\sec x+\tan x)+c$

(D) $\ln(\sec x+\tan x)+c$

Ans. (C)

Sol. $\int\{1+2\tan x(\tan x+\sec x)\}^{1/2}dx$

$\int\{1+2\tan^2 x+2\tan x\sec x\}^{1/2}dx$

$\int\{\sec^2 x-\tan^2 x+2\tan^2 x+2\tan x\sec x\}^{1/2}dx$

$=\int(\sec x+\tan x)dx$

$=\ell n\sec x+\ell n(\sec x+\tan x)+c$

$=\ell n\sec x(\sec x+\tan x)+c$

22. $\int \frac{\cos 2x-1}{\cos 2x+1} dx =$

(A) $\tan x - x + c$

(B) $x + \tan x + c$

(C) $x - \tan x + c$

(D) $-x - \cot x + c$

Ans. **(C)**

Sol. $\int \frac{\cos 2x-1}{\cos 2x+1} dx = \int -\tan^2 x dx = \int (1-\sec^2 x) dx = x - \tan x + c$

23. If $\int \frac{\cos 4x+1}{\cot x-\tan x} dx = A\cos 4x + B$ where A & B are constants, then

(A) $A = -1/4$ & B may have any value

(B) $A = -\frac{1}{8}$ & B may have any value

(C) $A = -\frac{1}{2}$ & $B = -1/4$

(D) $A = \frac{1}{2}$ & $B \in R$

Ans. **(B)**

Sol. $\int \frac{\cos 4x+1}{\cot x-\tan x} = A\cos 4x + B$

$I = \int \frac{(\cos 4x+1)}{(\cos^2 x-\sin^2 x)} \cos x \sin x dx = \int \left(\frac{2\cos^2 2x}{\cos 2x}\right)(\cos x \sin x) dx$

$= \int \cos 2x \sin 2x dx \quad = \frac{1}{2}\int \sin 4x dx = -\frac{\cos 4x}{8} + B$

24. $\int 4\sin x \cos\frac{x}{2}\cos\frac{3x}{2} dx$ is equal to

(A) $\cos x - \frac{1}{2}\cos 2x + \frac{1}{3}\cos 3x + c$

(B) $\cos x - \frac{1}{2}\cos 2x - \frac{1}{3}\cos 3x + c$

(C) $\cos x + \frac{1}{2}\cos 2x + \frac{1}{3}\cos 3x + c$

(D) $\cos x + \frac{1}{2}\cos 2x - \frac{1}{3}\cos 3x + c$

Ans. **(B)**

Sol. $\int 4\sin x \cos\frac{x}{2}\cos\frac{3x}{2} dx$

$2\int \left(2\sin x \cos\frac{x}{2}\right)\cos\frac{3x}{2} dx$

$2\int \left(\sin\frac{3x}{2} + \sin\frac{x}{2}\right)\cos\frac{3x}{2} dx$

$= \int 2\sin\frac{3x}{2}\cos\frac{3x}{2} dx + \int 2\sin\frac{x}{2}\cos\frac{3x}{2} dx$

$= \int \sin 3x dx + \int ((\sin 2x) - \sin x) dx$

$= -\frac{\cos 3x}{2} - \frac{\cos 2x}{2} + \cos x + c$

25. $\int \sin x \cdot \cos x \cdot \cos 2x \cdot \cos 4x \cdot \cos 8x \cdot \cos 16x dx$ is

3 equal to

(A) $\frac{\sin 16x}{1024} + c$

(B) $-\frac{\cos 32x}{1024} + c$

(C) $\frac{\cos 32x}{1096} + c$

(D) $-\frac{\cos 32x}{1096} + c$

Ans. **(B)**

Sol. $\int \sin x \cdot \cos x \cdot \cos 2x \cos 4x \cdot \cos 8x \cdot \cos 16x dx$

$= \frac{1}{2}\int (\sin 2x \cdot \cos 2x)\cos 4x \cdot \cos 8x \cdot \cos 16x dx$

$= \frac{1}{4}\int (\sin 4x \cdot \cos 4x)\cos 8x \cdot \cos 16x dx$

$= \frac{1}{8}\int (\sin 8x \cdot \cos 8x)\cos 16x dx$

$= \frac{1}{16}\int \sin 16x \cos 16x dx$

$= \frac{1}{32}\int \sin 32x dx = -\frac{1}{32} \times \frac{\cos 32x}{32} = -\frac{1}{1024}\cos 32x + c$

26. $\int \frac{\sin^8 x - \cos^8 x}{1-2\sin^2 x \cos^2 x} dx$ is equal to

(A) $\frac{1}{2}\sin 2x + c$
(B) $-\frac{1}{2}\sin 2x + c$
(C) $-\frac{1}{2}\sin x + c$
(D) $-\sin^2 x + c$

Ans. **(B)**

Sol. $\int \frac{\sin^8 x - \cos^8 x}{1-2\sin^2 x\cos^2 x} dx$

$\int \frac{(\sin^4 + \cos^4 x)(\sin^2 x + \cos^2 x)(\sin^2 x - \cos^2 x)}{(\sin^4 x + \cos^4 x)} dx$

$= -\int \cos 2x dx = -\frac{\sin 2x}{2} + c$

27. If $\int \frac{\sin x}{\sin(x-\alpha)} dx = Ax + B\log \sin(x-\alpha) + C$, then value of (A, B) is-

(A) $(\sin\alpha, \cos\alpha)$
(B) $(\cos\alpha, \sin\alpha)$
(C) $(-\sin\alpha, \cos\alpha)$
(D) $(-\cos\alpha, \sin\alpha)$

Ans. **(B)**

Sol. Put $x - \alpha = t \Rightarrow dx = dt$

$I = \int \frac{\sin (t+\alpha)dt}{\sin t} = \int (\cos \alpha + \cot t \sin \alpha)dt$

$= t\cos \alpha + \sin \alpha \ell n|\sin t| + c$

$= x\cos \alpha - \alpha\cos \alpha + \sin \alpha \ln (\sin |x-\alpha|) + c$

$= x\cos \alpha + \sin \alpha \ln (\sin (x-\alpha)) + c$
$\therefore A = \cos \alpha; B = \sin \alpha$

28. $\int (\sin 2x - \cos 2x)dx = \frac{1}{\sqrt{2}}\sin (2x - a) + b$, then

(A) $a = \frac{5\pi}{4}, b \in R$
(B) $a = -\frac{5\pi}{4}, b \in R$
(C) $a = \frac{\pi}{4}, b \in R$
(D) $a = -\frac{\pi}{4}, b \in R$

Ans. **(B)**

Sol. $\int (\sin 2x - \cos 2x)dx = \frac{1}{\sqrt{2}}\sin(2x - a) + b$

$= -\frac{\cos 2x}{2} - \frac{\sin 2x}{2} + b$

$= \frac{-1}{\sqrt{2}}\left[\frac{1}{\sqrt{2}}\cos 2x + \frac{1}{\sqrt{2}}\sin 2x\right] + b$

$= -\frac{1}{\sqrt{2}}\left[\sin\frac{\pi}{4}\cos 2x + \cos\frac{\pi}{4}\sin 2x\right] + b$

$= \frac{1}{\sqrt{2}}\left[\sin\left(2x + \frac{5\pi}{4}\right)\right] + b$

29. $\int [1 + \tan x \cdot \tan(x+\alpha)]dx$ is equal to

(A) $\cos\alpha \cdot \ln\left|\frac{\sin x}{\sin(x+\alpha)}\right| +$

(B) $\tan\alpha \cdot \ln\left|\frac{\sin x}{\sin(x+\alpha)}\right| + C$

(C) $\cot\alpha \cdot \ln\left|\frac{\sec(x+\alpha)}{\sec x}\right| + C$

(D) $\cot\alpha \cdot \ln\left|\frac{\cos(x+\alpha)}{\cos x}\right| + C$

Ans. **(C)**

Sol. $\int [1 + \tan x \tan(x+\alpha)]dx$

$\int \frac{\tan(x+\alpha) - \tan x}{\tan(x+\alpha-x)}dx$

$= \frac{1}{\tan\alpha}\int \tan(x+\alpha)dx - \int \tan x dx$

$= \frac{1}{\tan\alpha}\ell n\left|\frac{\sec(x+\alpha)}{\sec x}\right| + c$

30. $\int \left(\sqrt{\frac{a+x}{a-x}} - \sqrt{\frac{a-x}{a+x}}\right)dx$ is equal to

(A) $-2\sqrt{a^2 - x^2} + C$

(B) $\sqrt{a^2 - x^2} + C$

(C) $-\sqrt{x^2 - a^2} + C$

(D) None of these

Ans. **(A)**

Sol. $\int \sqrt{\frac{a+x}{a-x}} - \sqrt{\frac{a-x}{a+x}}dx = \int \frac{(a+x)-(a-x)}{\sqrt{a^2-x^2}}dx$

$= \int \frac{2x}{\sqrt{a^2 - x^2}}dx$

$a^2 - x^2 = t^2 \Rightarrow xdx = tdt$

$= -2\int \frac{tdt}{t} = -2t + c = -2\sqrt{a^2 - x^2} + c$

31. $\int \frac{x^2+\cos^2 x}{1+x^2}\operatorname{cosec}^2 x dx$ is equal to :

(A) $\cot x - \cot^{-1} x + c$

(B) $c - \cot x + \cot^{-1} x$

(C) $-\tan^{-1} x - \frac{\operatorname{cosec} x}{\sec x} + c$

(D) $-e^{\ell n \tan^{-1} x} - \cot x + c$

Ans. **(B,C,D)**

Sol. $\int \frac{x^2+\cos^2 x}{1+x^2}\operatorname{cosec}^2 x dx$

$$= \int \frac{x^2+1-1+\cos^2 x}{1+x^2}\operatorname{cosec}^2 x dx$$

$$= \int \operatorname{cosec}^2 x dx - \int \frac{dx}{1+x^2}$$

$$= -\cot x - \tan^{-1} x + c$$

$$= -\cot x - \left(\frac{\pi}{2} - \cot^{-1} x\right) + c$$

$$= -\cot x + \cot^{-1} x + c$$
$$= -\cot x - e^{\ln \tan^{-1} x} + c$$

32. $\int \frac{4x^5-7x^4+8x^3-2x^2+4x-7}{x^2(x^2+1)^2}dx$

Ans. $\mathbf{4\ln x + \frac{7}{x} + 6\tan^{-1}(x) + \frac{6x}{1+x^2} + C}$

Sol. $I = \int \frac{4x^5-7x^4+8x^3-2x^2+4x-7}{x^2(x^2+1)^2}dx$

We can split it into partial fractions as $\frac{A}{x^2} + \frac{B}{x} + \frac{Cx+D}{x^2+1} + \frac{Ex+F}{(x^2+1)^2}$ ....(i)
Multiplying both sides by x^2 and putting $x = 0$ we get $A = -7$
Similarly multiplying by $(x^2+1)^2$ and substituting $x = i$ we get

$Ei + F = 12 \therefore E = 0, F = 12$

Since there are repeating factors B, C and D which can not be found by this method.
to solve it we use the substitution.
This is done by substituting different and convenient values for x.

$x = 1$ on either side gave $2B + C + D = 8 \dots\dots(2)$

$x = -1$ gives $-2B - C + D = -8$ .....(3)

From these 2 relations we get $D = 0$
Next $x = 2$ gives $5B + 4C = 20 \dots\dots(4)$
From (2) and (4) we get $B = 4$ and $C = 0$

$$\therefore I = \int \frac{-7}{x^2} + \frac{4}{x} + \frac{0\times x+0}{x^2+1} + \frac{0\times x+12}{(x^2+1)^2}dx$$

$$= \int \frac{-7}{x^2} + \frac{4}{x} + \frac{12}{(x^2+1)^2} dx$$

First two are direct
For third put $x = \tan\theta \Rightarrow dx = \sec^2\theta d\theta$

$$I = \frac{14}{x} + 4\log|x| + 12\int \frac{\sec^2\theta}{\sec^4\theta} d\theta$$

$$I = \frac{14}{x} + 4\log|x| + 12\int \cos^2\theta d\theta$$

$$\int \cos^2\theta d\theta = \int \frac{1+\cos 2\theta}{2} d\theta = \frac{\theta}{2} + \frac{\sin 2\theta}{4} = \frac{\tan^{-1} x}{2} + \frac{2x}{1+x^2} \times \frac{1}{4}$$

$$\therefore I = \frac{14}{x} + 4\log|x| + 6\tan^{-1} x + \frac{6x}{1+x^2}$$

33. $\int \frac{xdx}{x^4+1}$

Ans. $\frac{1}{2}$**arc** $\tan x^2 + C.$

Sol. $\int \frac{x}{1+x^4} dx = \frac{1}{2}\int \frac{2xdx}{1+(x^2)^2} = \frac{1}{2}\int \frac{d(x^2)}{1+(x^2)^2} = I$

let $x^2 = u$

$$\Rightarrow I = \frac{1}{2}\int \frac{du}{1+u^2} = \frac{1}{2}\arctan u = \frac{1}{2}\arctan(x^2)$$

34. $\int \frac{xdx}{\sqrt{a^2-x^4}}$.

Ans. $\frac{1}{2}\arcsin\frac{x^2}{a} + C.$

35. $\int \frac{x^2dx}{x^6+4}$

Ans. $\frac{1}{6}\arctan\frac{x^3}{2} + C$

36. $\int \frac{x^3dx}{\sqrt{1-x^8}}$

Ans. $\frac{1}{4}$**arc** $\sin x^4 + C.$

Sol. $I = \int \frac{x^3}{\sqrt{1-x^8}} dx \quad \Rightarrow \quad I = \int \frac{x^3}{\sqrt{1-(x^4)^2}} dx$

Let $t = x^4$

$\frac{dt}{dx} = 4x^3 \quad \Rightarrow \quad \frac{dt}{4} = x^3 dx$

Therefore, $I = \frac{1}{4}\int \frac{dt}{\sqrt{1-t^2}}$

$I = \frac{1}{4}\sin^{-1} t + C$

On putting the value of t, we get

$I = \frac{1}{4}\sin^{-1} x^4 + C$

37. $\int \frac{e^x dx}{e^{2x}+4}$.

Ans. $\frac{1}{2}\arctan\frac{e^x}{2} + C.$

Sol. $I_1 = \int \frac{e^x}{e^{2x}+4}dx$

Put $t = e^x \Rightarrow dt = e^x dx$

$I_1 = \int \frac{1}{t^2+4}dt = \frac{1}{2}\tan^{-1}\frac{t}{2} = \frac{1}{2}\tan^{-1}\frac{e^x}{2}$

38. $\int \frac{\cos\alpha d\alpha}{a^2+\sin^2\alpha}$.

Ans. $\frac{1}{a}\arc\tan\frac{\sin\alpha}{a} + C.$

39. $\int (e^x+1)^3 dx.$

Ans. $\frac{1}{3}e^{3x} + \frac{3}{2}e^{2x} + 3e^x + x + C$

40. $\int \frac{1+x}{\sqrt{1-x^2}}dx.$

Ans. $\arcsin x - \sqrt{1-x^2} + C$

41. $\int \frac{3x-1}{x^2+9}dx.$

Ans. $\frac{3}{2}\ln(x^2+9) - \frac{1}{3}\arctan\frac{x}{3} + C.$

Sol. Let $I = \int \frac{3x-1}{\sqrt{x^2+9}}dx$
$= \int \frac{3x}{\sqrt{x^2+9}}dx - \int \frac{1}{\sqrt{x^2+9}}dx$

Now $I_1 = \int \frac{3x}{\sqrt{x^2+9}}dx$

Put $x^2+9 = t^2 \Rightarrow 2xdx = 2tdt \Rightarrow xdx = tdt$

$\therefore\ I_1 = 3\int \frac{tdt}{t} = 3\int dt$

$= 3t + C_1 = 3\int \sqrt{x^2+9} + C_1$

And $I_2 = \int \frac{1}{\sqrt{x^2+9}}dx = \int \frac{1}{\sqrt{x^2+3^2}}dx$

$= \log\left|x+\sqrt{x^2+9}\right|+C_2$

$\therefore I = 3\sqrt{x^2+9}+C_1-\log\left|x+\sqrt{x^2+9}\right|-C_2$

$= 3\sqrt{x^2+9}-\log\left|x+\sqrt{x^2+9}\right|+C$

42. $\int\sqrt{\frac{1-x}{1+x}}dx.$

Ans. $\mathbf{\arcsin x+\sqrt{1-x^2}+C.}$

43. $\int\frac{dx}{1+\sin x}$

Ans. $\mathbf{\tan\left(\frac{x}{2}-\frac{\pi}{4}\right)+C}$

Sol. $\int\frac{dx}{1+\sin x}=\int\frac{dx}{1+\sin x}\times\frac{1-\sin x}{1-\sin x}=\int\frac{(1-\sin x)}{1-\sin^2 x}dx$

$= \int\frac{1-\sin x}{\cos^2 x}dx=\int\left(\frac{1}{\cos^2 x}-\frac{\sin x}{\cos x\cos x}\right)dx$

$= \int(\sec^2 x-\tan x\sec x)dx$

$= \int\sec^2 x dx-\int\tan x\sec x dx$

$= \tan x-\sec x+c$

44. $\int\frac{1-\cos x}{1+\cos x}dx.$

Ans. $\mathbf{2\tan\frac{x}{2}-x+C}$

Sol. $\frac{1-\cos x}{1+\cos x}=\frac{2\sin^2\frac{x}{2}}{2\cos^2\frac{x}{2}}$

$= \tan^2\frac{x}{2}=\left(\sec^2\frac{x}{2}-1\right)$

$\therefore\int\frac{1-\cos x}{1+\cos x}dx=\int\left(\sec^2\frac{x}{2}-1\right)dx$

$= \left[\frac{\tan\frac{x}{2}}{\frac{1}{2}}-x\right]+C$

$= 2\tan\frac{x}{2}-x+C$

45. $\int\frac{1+\sin x}{1-\sin x}dx.$

Ans. $\mathbf{2\tan\left(\frac{x}{2}+\frac{\pi}{4}\right)-x+C}$

Sol. $I=\int\left\{\frac{(1+\sin x)}{(1-\sin x)}\times\frac{(1-\sin x)}{(1+\sin x)}\right\}dx=$

$\int \frac{(1+\sin\ x)^2}{(1-\sin\ x^2 x)} dx$

$= \int \frac{(1+\sin^2\ x + 2\sin\ x)}{\cos^2\ x} dx =$

$\int (\sec^2\ x + \tan^2\ x + 2\sec\ x\tan\ x) dx$

$= \int (2\sec^2 - 1 + 2\sec\ x\tan\ x) dx.$

46. $\int\ \cos^3\ xdx$

Ans. $\mathbf{\sin x - \frac{\sin^3 x}{3} + C}$

Sol. $I = \int \frac{\cos\ 3x + 3\cos\ x}{4} dx\ [\because \cos\ 3A = (4\cos^3\ A - 3\cos\ A)]$

$I = \frac{1}{4} \int \cos\ 3x + 3\cos\ xdx$

$I = \frac{1}{4} \int \cos\ 3xdx + \frac{1}{4} \int 3\cos\ xdx$

$I = \frac{1}{4}\left[\frac{\sin\ 3x}{3} + 3\sin\ x\right] + C$

47. $\int \tan^4\ xdx$

Ans. $\mathbf{\frac{1}{3}\tan^3 x - \tan x + x + C}$

Sol. $\int \tan^4\ xdx$

$= \int \tan^2\ \tan^2\ xdx \quad \Rightarrow \quad = \int (\sec^2\ x - 1)\tan^2\ xdx$

$= \int (\sec^2\ x\tan^2\ x - \tan^2\ x) dx \quad \Rightarrow \quad = \int [\sec^2\ x\tan^2\ x - (\sec^2\ x - 1)] dx$

$= \int (\sec^2\ x\tan^2\ x - \sec^2\ x + 1) dx$

$= \int \sec^2\ x\tan^2\ xdx - \int \sec^2\ xdx + \int 1dx$

Putting $\tan x = t$

$\sec^2\ xdx = dt$

$= \int t^2 dt - \tan\ x + x$

$= \frac{t^3}{3} - \tan\ x + x + c$

$= \frac{1}{3}\tan^3\ x - \tan\ x + x + c$

48. $\int \sin^4\ xdx$

Ans. $\mathbf{\frac{3}{8}x - \frac{1}{4}\sin 2x + \frac{1}{32}\sin 4x + C}$

Sol. Since, $\sin^2 x = \frac{1-\cos 2x}{2}$.

Take square on both sides :

$$\sin^4 x = \left(\frac{1-\cos 2x}{2}\right)^2$$

To find the integration of $\sin^4 x$, put integration on both sides:

$$\int \sin^4 x dx = \int \left(\frac{1-\cos 2x}{2}\right)^2 dx$$

$$\Rightarrow \int \sin^4 x dx = \frac{1}{4}\int 1 + \cos^2 2x - 2\cos 2x dx$$

$$\Rightarrow \int \sin^4 x dx = \frac{1}{4}\int \left(1 + \frac{1+\cos 4x}{2} - 2\cos 2x\right) dx$$

$$\left[\begin{array}{l} \because \cos^2 \mathbf{x} = \frac{1+\cos 2x}{2} \\ \therefore \cos^2 \mathbf{2x} = \frac{1+\cos 4x}{2} \end{array}\right]$$

$$\Rightarrow \int \sin^4 x dx = \frac{1}{4}\left(x + \frac{x}{2} + \frac{\sin 4x}{8} - \frac{2\sin 2x}{2}\right) + C$$

49. $\int \tan^3 x dx.$

Ans. $\mathbf{\frac{1}{2}\tan^2 x + \ln|\cos x| + C}$

Sol. $\int \tan^3 x dx = \int \tan x \times \tan^2 x dx$

$$= \int \tan x(\sec^2 x - 1)dx$$

$$\int (\tan x\sec^2 x - \tan x)dx$$

$$= \int \tan x\sec^2 x dx - \int \tan x dx$$

Let $\tan x = t$

$\sec^2 x dx = dt$

$\int t dt - \log |\sec x| + c$

$$= \frac{t^2}{2} - \log |\sec x| + c$$

$= (\tan^2 x/2) - \log |\sec x| + c$

ANSWER KEY

1. $-\sqrt{2}\cos\frac{x}{2}+C$
2. $-\frac{\cos 4x}{8}+C$
3. $\frac{67}{5}$
4. $\frac{1}{2}(x-\sin x)+C$
5. $-2\cos x+C$
6. $-\frac{\cos 8x}{8}+C$
7. $\frac{x}{\sqrt{2}}+C$
8. $\frac{x^3}{3}+\frac{x^2}{2}+\frac{3x}{2}+\frac{7}{4}\ln(2x+1)$
9. $\tan x-\tan^{-1}x+C$
10. $\frac{1}{4}\sin^{-1}\frac{4}{3}x+C$
11. $\frac{1}{10}\tan^{-1}\frac{2x}{5}+C$
12. $\frac{2}{3}x+\frac{5}{9}\ln(3x+2)+C$
13. $\frac{\sin 3x}{3}-\frac{\sin 2x}{2}+C$
14. $-\frac{2}{x}+\tan^{-1}x+C$
15. $(\sin x-\cos x)+(\sin k+\cos k)x+C$
16. $C-\frac{2}{x}+\frac{2}{3}\frac{1}{x^3}-\frac{3}{5}\frac{1}{x^5}-2\tan^{-1}x$
17. $-\frac{1}{64}\cos 8x+C$
18. x^x+C
19. (B) 20. (A) 21. (C) 22. (C) 23. (B) 24. (B) 25. (B)
26. (B) 27. (B) 28. (B) 29. (C) 30. (A) 31. (B,C,D)
32. $4\ln x+\frac{7}{x}+6\tan^{-1}(x)+\frac{6x}{1+x^2}+C$
33. $\frac{1}{2}\arctan x^2+C.$
34. $\frac{1}{2}\arcsin\frac{x^2}{a}+C.$
35. $\frac{1}{6}\arctan\frac{x^3}{2}+C.$
36. $\frac{1}{4}\arcsin x^4+C.$
37. $\frac{1}{2}\arctan\frac{e^x}{2}+C.$
38. $\frac{1}{a}\arctan\frac{\sin\alpha}{a}+C.$
39. $\frac{1}{3}e^{3x}+\frac{3}{2}e^{2x}+3e^x+x+C$
40. $\arcsin x-\sqrt{1-x^2}+C.$
41. $\frac{3}{2}\ln(x^2+9)-\frac{1}{3}\arctan\frac{x}{3}+C.$
42. $\arcsin x+\sqrt{1-x^2+C.}$
43. $\tan\left(\frac{x}{2}-\frac{\pi}{4}\right)+C.$
44. $2\tan\frac{x}{2}-x+C$
45. $2\tan\left(\frac{x}{2}+\frac{\pi}{4}\right)-x+C$
46. $\sin x-\frac{\sin^3 x}{3}+C$
47. $\frac{1}{3}\tan^3 x-\tan x+x+C$
48. $\frac{3}{8}x-\frac{1}{4}\sin 2x+\frac{1}{32}\sin 4x+C$
49. $\frac{1}{2}\tan^2 x+\ln|\cos x|+C$