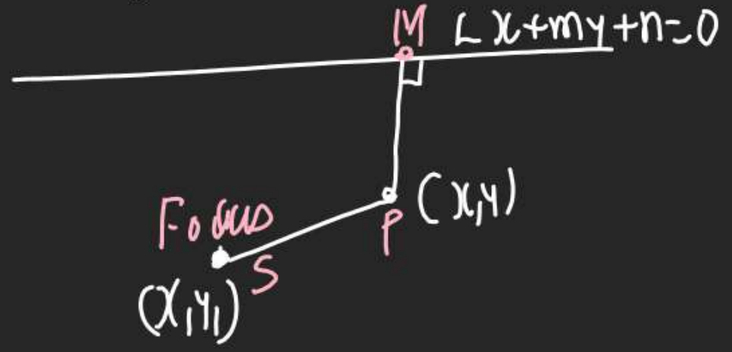
Nishant Jindal

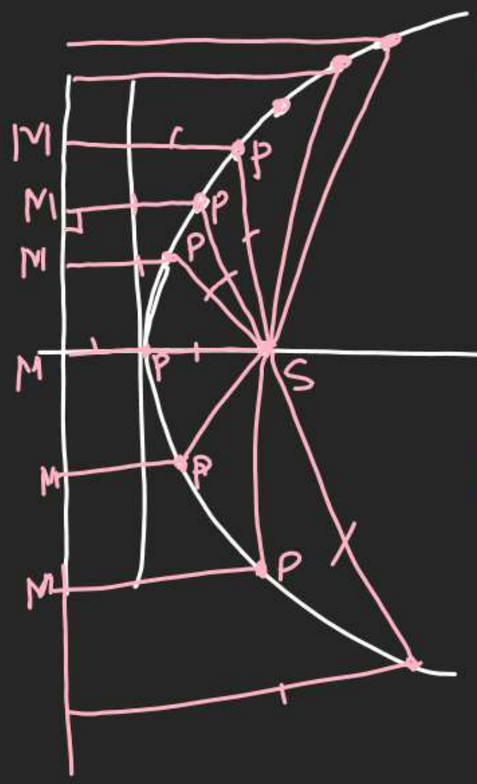
Conic Section

Parabola

(1) Locus of a Pt. whose distance from a fix Pt. & a fix Line Remains Constant.

(2) Here fix pt = Focus
fix Line = Directix

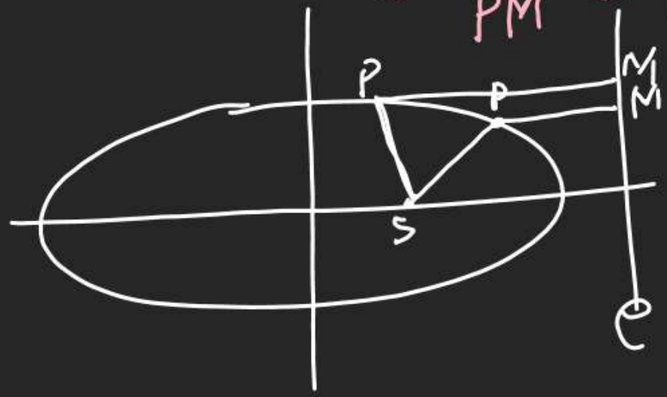




(3) SP = PM

(4) Ratio of SP & PM = Eccentricity e = 0 Circle

$e = \frac{SP}{PM} = 1$



SP < PM

$e = \frac{SP}{PM} < 1$

(5) Eccentricity (e)

e = 1 Parabola
e > 1 hyperbola
e < 1 Ellipse

(6) 2nd Non Hom Eq

$f(x,y) = ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$

(A) $h^2 = ab$ Par.
(B) $h^2 > ab$ hyp.
(C) $h^2 < ab$ Ellipse

(7) $f(x,y) = ax^2 + 2hxy + by^2 + 2gx + 2fy + c$

$\Delta = abc + 2fgh - af^2 - bg^2 - ch^2$

- $\Delta = 0$ → Degenerated Conic
  - Pair of st. line
    - $h^2 = ab$ → Coincident
    - $h^2 > ab$ Pair of ST L
    - $h^2 < ab$ No Real Lines.
- $\Delta \neq 0$ → Non Degenerated Conics.
  - Par. $h^2 = ab$
  - Ellipse $h^2 < ab$
  - hyperbola $h^2 > ab$
  - Circle → $h = 0$ & $a = b$

Q Find Nature of

$x^2 - 2xy + y^2 + 3x + 2 = 0$

$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$

$a = 1,\ h = -1,\ b = 1,\ g = \frac{3}{2},\ f = 0,\ c = 2$

1) $\Delta = abc + 2fgh - af^2 - bg^2 - ch^2$

$= 1 \times 1 \times 2 + 2 \times 0 - 1 \times 0^2 - 1 \times \frac{9}{4} - 2 \times 1$

$= 2 - \frac{9}{4} - 2 \neq 0$ → It is N.S.

(2) $h^2 = (-1)^2 = 1$

$ab = 1 \times 1 = 1$

$h^2 = ab$ ⇒ Parabola.

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Q Find value of K

for which $6x^2+11xy-10y^2+x+31y+K=0$

Rep. Pair of St L?

$\rightarrow$ $\boxed{K=-15}$

Q In what Conic $\sqrt{ax}+\sqrt{by}=1$ Represents?

Sol $ax+by+2\sqrt{ab}\sqrt{xy}=1$

$ax+by-1=2\sqrt{ab}\sqrt{xy}$

Sq $(ax+by-1)^2=(2\sqrt{ab}\sqrt{xy})^2$

$a^2x^2+b^2y^2+1-2ax-2by+2abxy$

$=4abxy$

$\Rightarrow a^2x^2+b^2y^2-2ahxy-2ax-2by+1=0$

$Ax^2+By^2+2Hxy+2hx+2Fy+C=0$

$H=-ab \mid h=-a \mid F=-b, C=1$

1) $\Delta = a^2b^2+2ab(-ab)-a^2b^2-b^2a^2+a^2b^2$

$=-2a^2b^2 \neq 0$ (N.S.)

2) $h^2=ab$ (heck

$a^2b^2=a^2b^2 \rightarrow$ Parabola

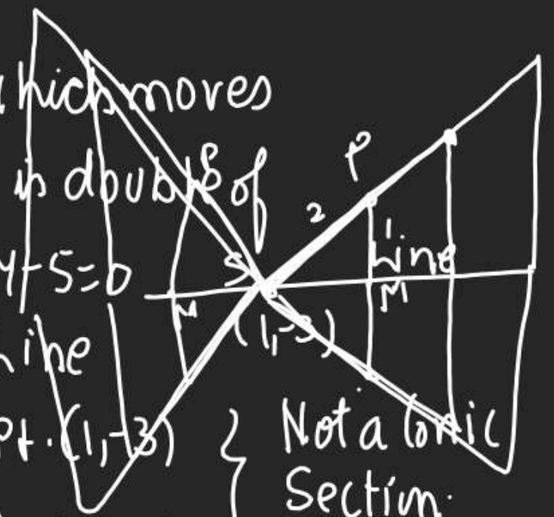
Q Find Natire of Locus of Pt. which moves S.T. its distance from $(1,-3)$ is double of its distance from Line $2x-y+5=0$

(A) Check Positim of Pt. and line

Line $\rightarrow 2x-y-5=0$ & Pt. $(1,-3)$

$2\times1-(-3)-5=0$

$5-5=0=1$ Satisfy } Not a Conic Sectim: Pair of St. Line



(B) $SP=2PM \Rightarrow \frac{SP}{PM}=e=2>1$ (distinct Pair of St Line)

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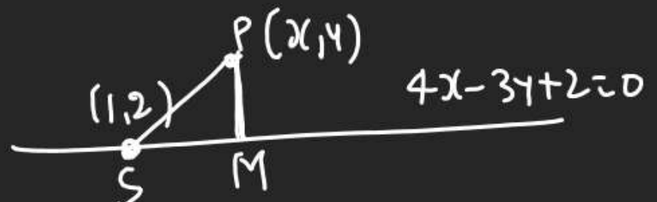
Q Find Locus of a pt. which moves S.T. the Ratio of its distance from (1,2) & Line $4x-3y+2=0$ is $\sqrt{3}$.

① (1,2) Satisfies Lines Pair of STL

②



$$\frac{SP}{PM}=\sqrt{3} \Rightarrow SP=\sqrt{3}PM$$

$$\sqrt{(x-1)^2+(y-2)^2}=\sqrt{3}\,\frac{|4x-3y+2|}{\sqrt{4^2+(-3)^2}}$$

Sq

$$25\{(x-1)^2+(y-2)^2\}=3(4x-3y+2)^2$$

$$23x^2+2y^2+\qquad\qquad =0$$

Q Locus of $x^2+(y-1)^2=2\,|3x+4y-1|^2$ Rep?

↓ Actual Scene

$$\sqrt{(x)^2+(y-1)^2}=5\sqrt{2}\,\frac{|3x+4y-1|}{\sqrt{3^2+4^2}} \Rightarrow \frac{SP}{PM}=5\sqrt{2}$$

↓ Back to Scene

$$x^2+(y-1)^2=50\,\frac{|3x+4y-1|^2}{25}$$

dist. Pair of STL — hyperbola

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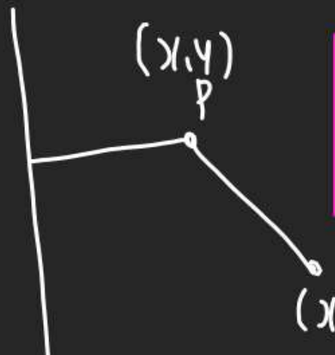
(8) General Eqn of Parabola.

SP = PM



$$\sqrt{(x-x_1)^2+(y-y_1)^2} = \frac{|Lx+my+n|}{\sqrt{L^2+M^2}}$$

$Lx+my+n=0$

$$(L^2+M^2)\{(x-x_1)^2+(y-y_1)^2\} = (Lx+My+N)^2$$

$$(L^2+M^2)x^2+(L^2+M^2)y^2-L^2x^2-M^2y^2 = 2LMxy \quad \text{---}$$

$$(M^2x^2+L^2y^2-2LMxy)+ \text{---} = \text{---}$$

$$(Ly-Mx)^2+ \text{---} = \text{---}$$

General EOP Consists ① It is always a 2nd Deg Eqn.

② always have a Perfect sqr

③ It always keep Linear term & Constant term.

Q In/OTF Represents Parabola?

A) $(x^2-y^2+2xy)-5=0$ → It is not a Perfect Sqr. Not a Par.

(B) $(4x^2+y^2-4xy)=8$ → Per. Sqr But not Containing Linear term. Not a Parabola.

(C) $(4x^2+9y^2-12xy)+x+1=0$

$= (2x-3y)^2+x+1=0$

Per Sqr, Linear, Constant. Yes it is a Parabola

(D) $4x^2+3x-5y+1=0$

Per Sqr, L.T., Const. Yes it is a Parabola.

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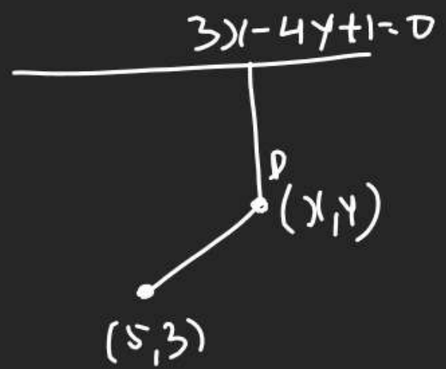
Q Find Eqn of Parabola.

whose focus is (5,3)

& directrix is $3x-4y+1=0$

As

$SP = PM$



$$16x^2+9y^2+24xy-256x-142y+849=0$$

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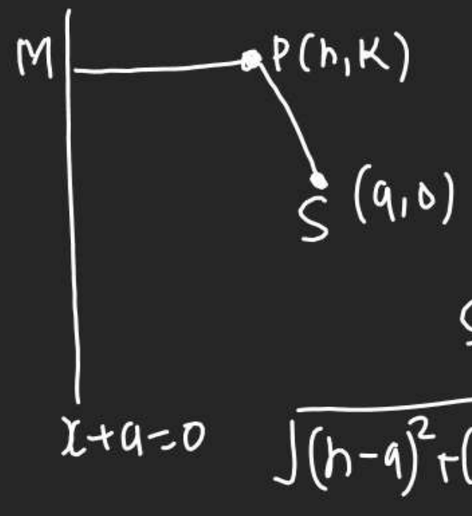
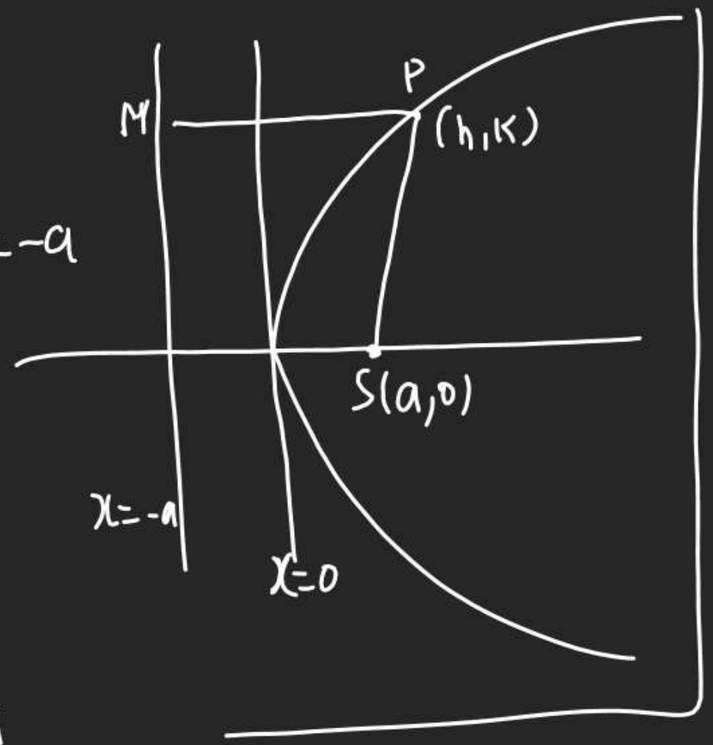
(9) Standard Eqⁿ of Parabola.

1) Fix Line : directrix → $x = -a$

2) Fix pt = Focus = $(a, 0)$





$SP = PM$

$$\sqrt{(h-a)^2+(K-0)^2} = \frac{|h+a|}{\sqrt{1^2+0^2}}$$

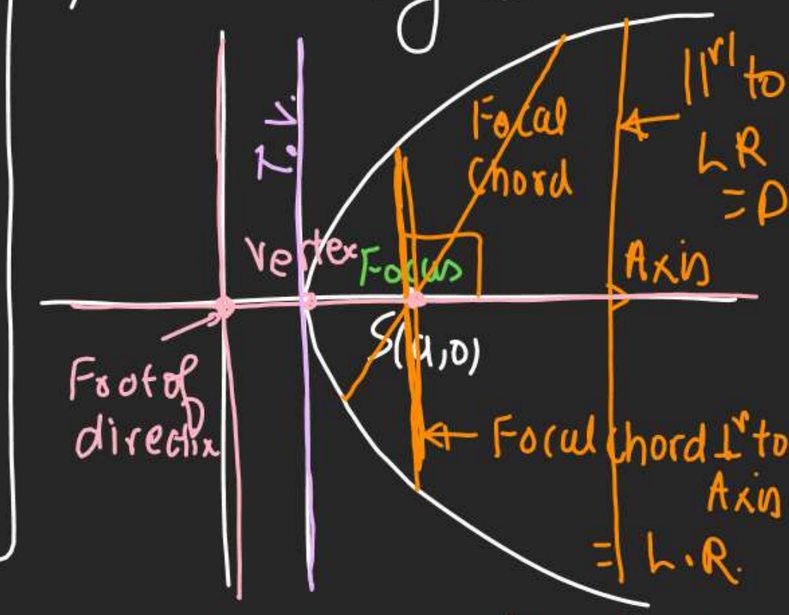
Sqr

$$(h-a)^2+(K)^2 = (h+a)^2$$

$$h^2+a^2-2ah+K^2 = h^2+a^2+2ah$$

$$K^2 = 4ah \Rightarrow \boxed{y^2 = 4ax}$$

(10) Terminologies.



(1) Axis = STL P.T. Focus & ⊥ʳ to directrix.

(2) Vertex = Mid Pt. of FD & Focus.

(3) Directrix = Fix Line ⊥ʳ to Axis

(4) T.V. = Tangent at vertex.

(5) Focal Chord = Chord P.T. Focus.

(6) L.R. = Latus Rectum = Focal chord ⊥ʳ to Axis

(7) D.O. = Double Ordinate Line ∥ᵍ to L.R.

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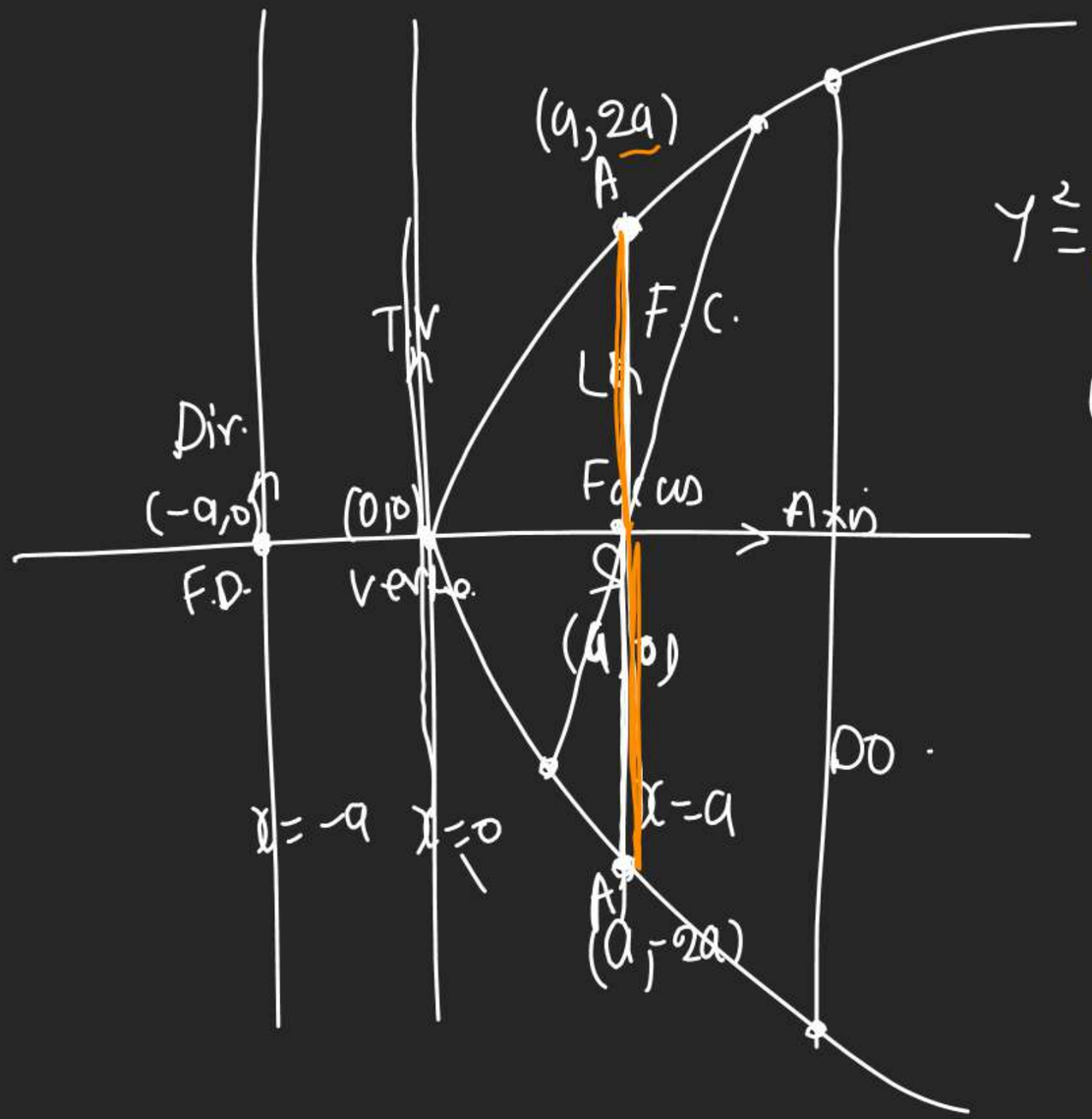

$y^2 = 4ax$

Q) What is L.L.R for $y^2 = \frac{x}{2}$?

1) L.L.R $= \frac{1}{2} = 4a$

2) Focus $= (a,0)$

$4a = \frac{1}{2}$

$a = \frac{1}{8}$

$S \equiv (\frac{1}{8}, 0)$

① Eqn of L.R → $x = K$ P.T. Focus $= (a,0)$

$a = K \Rightarrow K = a$

L.R $\Rightarrow x = a$

(2) End Pts of LR?

Demand A, A'??

A = Pt of Parabola & LR

$y^2 = 4ax$ & $x = a$

$y^2 = 4a \times a \Rightarrow y = 2a, -2a$

$\therefore A = (a, 2a)$ & $A' = (a, -2a)$

(3) L.L.R = Length of LR?

$= 4a$ = (coeff of x)

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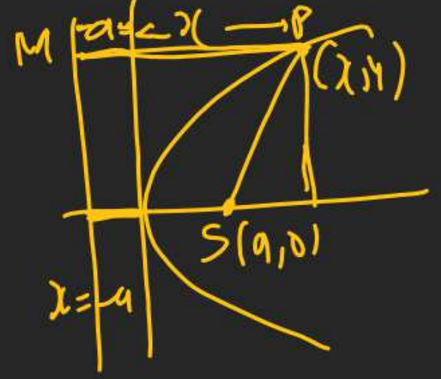
(11) Focal distance.

Distance bet^n any point on Parabola & Focus = SP



We don't Use dist. formula here

Focal dist :- $SP = PM = x + a$

(12) 4 Kinds of Stndard Parabola