L-C-R Series

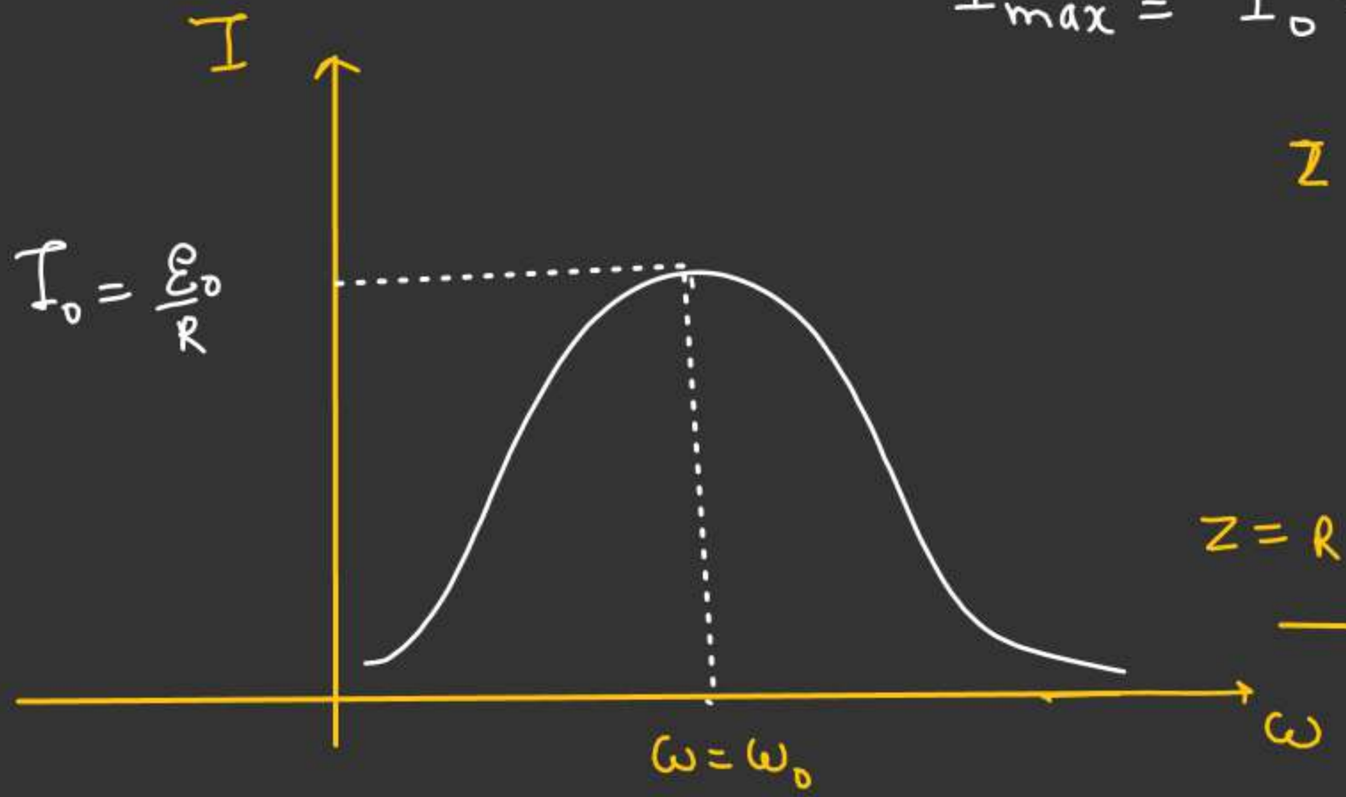
graph:

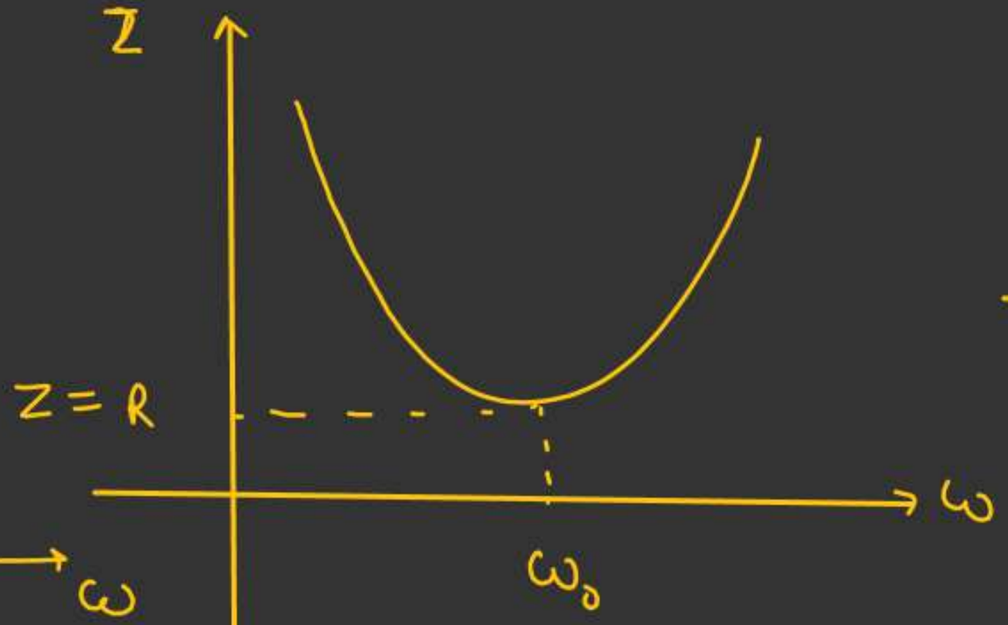
At the time of Resonance ($Z_{min} = R$)

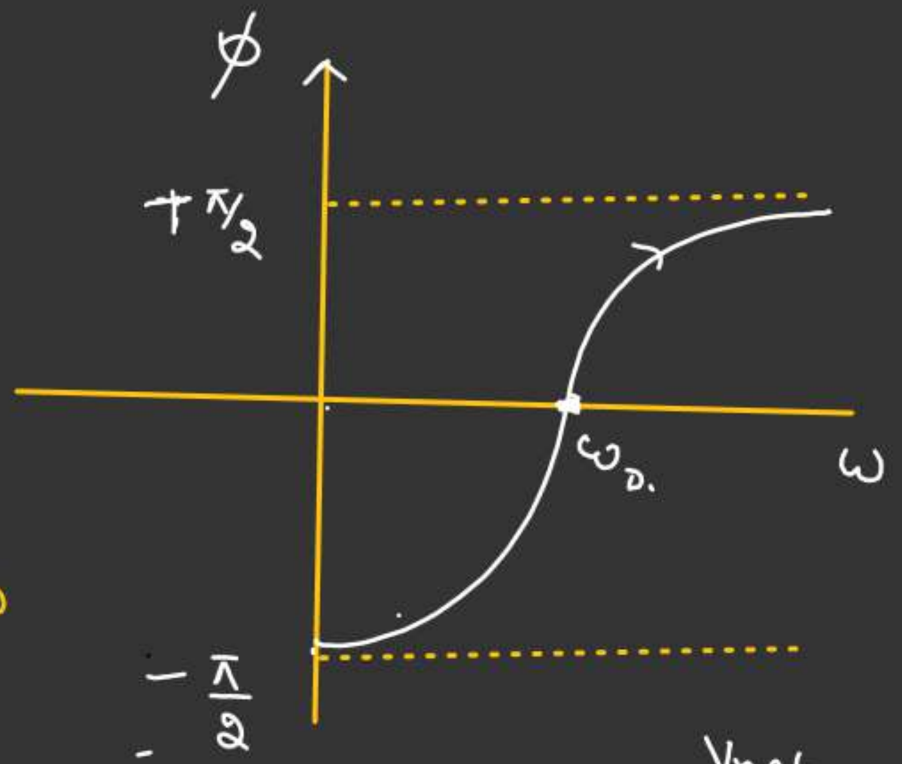
$$I_{max} = I_0 = \left(\frac{\mathcal{E}_0}{R}\right)$$

$$I = \frac{\mathcal{E}}{|Z|}\sin(\omega t + \phi)$$

$$|Z| = \sqrt{R^2 + \left|\omega L - \frac{1}{\omega C}\right|^2}$$







$$\tan\phi = \left|\frac{\omega L - \frac{1}{\omega C}}{R}\right|$$

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Quality Factor — [JEE MAINS]

↳ gives the Idea about Sharpness of I Vs ω Curve at the time of resonance.

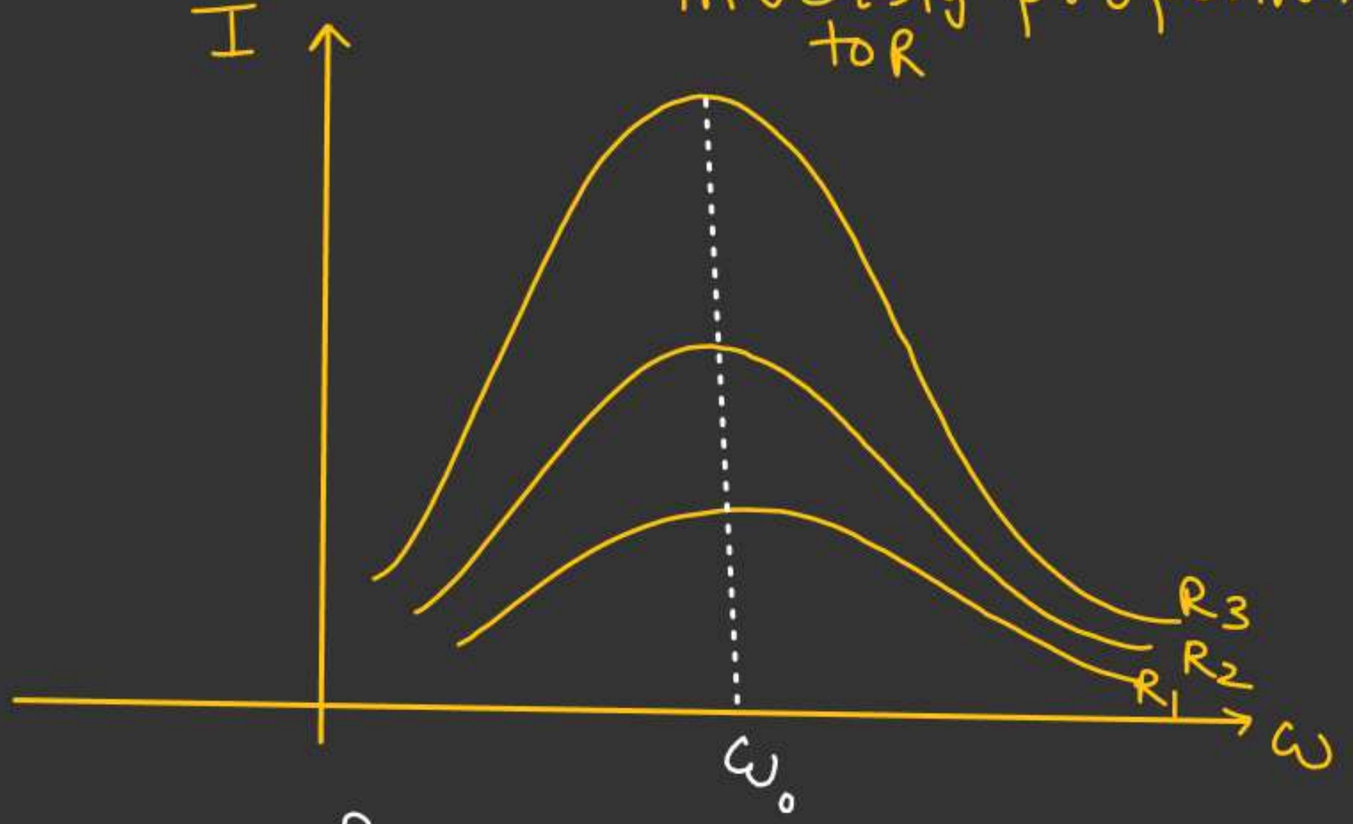
↳ $$Q = \left(\frac{\text{Voltage across L or C}}{\text{Maximum applied voltage}}\right)$$

$$Q = \frac{I X_L}{I R} \qquad \omega_0 = \frac{1}{\sqrt{LC}}$$

$$Q = \frac{X_L}{R} = \frac{\omega_0 L}{R}$$

$$\boxed{Q = \frac{1}{R}\sqrt{\frac{L}{C}}}$$

$Q \propto \frac{1}{R} \Rightarrow$ Sharpness of the Curve is inversly proportional to R



$\Rightarrow$ $\underline{R_3 < R_2 < R_1}$ ✓

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Another way to defined Quality factor

$$Q = 2\pi \frac{\left[\begin{array}{l}\text{Maximum Energy}\\ \text{Stored in a inductor or Capacitor}\end{array}\right]}{\text{Energy loss per Cycle.}}$$

$$U_{max} = \frac{1}{2} L I_0^2$$

$$P_{avg} = i_{rms}^2 R = \left(\frac{i_0}{\sqrt{2}}\right)^2 R$$

$$E = \left(\frac{i_0^2 R}{2}\right) T$$

$$Q = \frac{2\pi}{T} \left(\frac{\frac{1}{2} L I_0^2}{\frac{i_0^2 R}{2}} \right)$$

$$Q = \frac{\omega_0 L}{R}$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$\boxed{Q = \frac{1}{R}\sqrt{L/C}}$$

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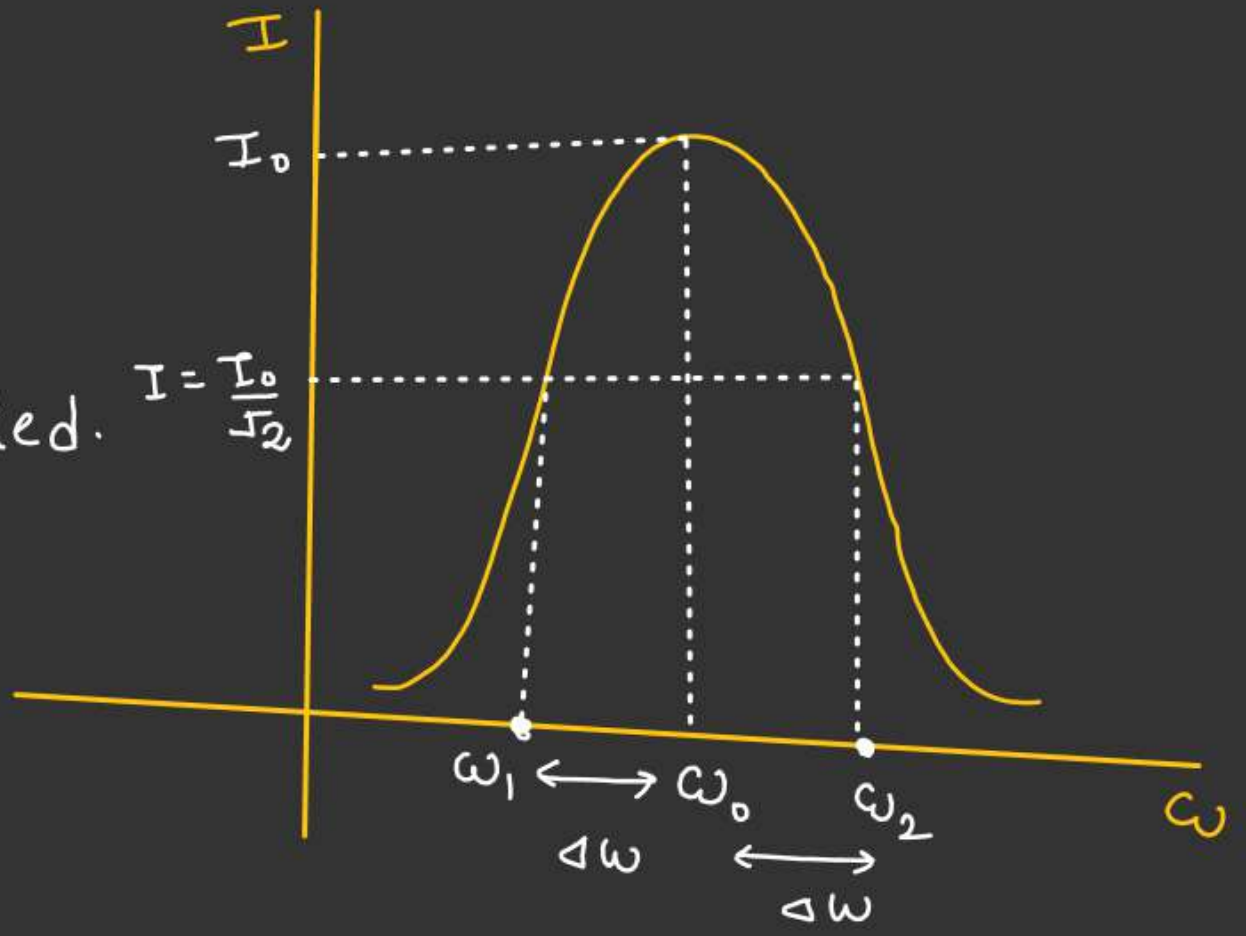
☆☆: Band Width of L-C-R Series Ckt

let, I be the Current in L-C-R Series Ckt for which power dissipiation is half of the maximum power dissipiated.



$$I = \frac{\mathcal{E}_0}{\sqrt{R^2 + \left(\frac{1}{\omega C} - \omega L\right)^2}}$$

$$P_{max} = I_0^2 R$$

$$I^2 R = \frac{I_0^2 R}{2}$$

$$I = \left(\frac{I_0}{\sqrt{2}}\right) = \left(\frac{\mathcal{E}_0}{R\sqrt{2}}\right)$$

$$\frac{\mathcal{E}_0}{R\sqrt{2}} = \frac{\mathcal{E}_0}{\sqrt{R^2 + \left(\frac{1}{\omega C} - \omega L\right)^2}}$$

$$R^2 + \left(\frac{1}{\omega C} - \omega L\right)^2 = 2R^2$$

$$\left(\frac{1}{\omega C} - \omega L\right)^2 = R^2$$

$$\omega_2 - \omega_1 = (2\Delta\omega)$$

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$$\left(\frac{1}{\omega C}-\omega L\right)^2 = R^2$$

$$\frac{1}{\omega C}-\omega L = \pm R$$

$$1-\omega^2(LC) = \pm RC\omega$$

$$(LC)\omega^2 \pm (RC)\omega - 1 = 0.$$

$$\omega$$

$$\omega = \frac{\mp RC + \sqrt{R^2C^2+4LC}}{2LC}$$

$$\checkmark\ \omega_1 = \frac{-RC+\sqrt{R^2C^2+4LC}}{2LC}, \quad \omega_2 = \frac{RC+\sqrt{R^2C^2+4LC}}{2LC}$$

$$\omega_2-\omega_1 = \Delta\omega = \frac{2RC}{2LC}$$

$$\boxed{\omega_2-\omega_1 = 2\Delta\omega = \frac{R}{L}}$$

(Bandwidth directly proportional to R)

$$\boxed{\omega_1\omega_2 = \frac{1}{LC}}$$

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Relation b/w Quality factor and bandwidth

$$Q = \left(\frac{\omega_0}{2\Delta\omega}\right)$$

$$\omega_2 - \omega_1 = R/L$$

$$\left(Q_0 = \frac{\omega_0 R}{L}\right)$$

$$\boxed{Q = \left(\frac{\omega_0}{\omega_2 - \omega_1}\right)}$$

[Quality factor is inversly proportional to bandwidth]

Q.1 A series $R - C$ combination is connected to an AC voltage of angular frequency $\omega = 500 \text{radian/s}$. If the impedance of the $R - C$ circuit is $R\sqrt{1.25}$, the time constant (in millisecond) of the circuit is **(2011)**

Sol^n :-

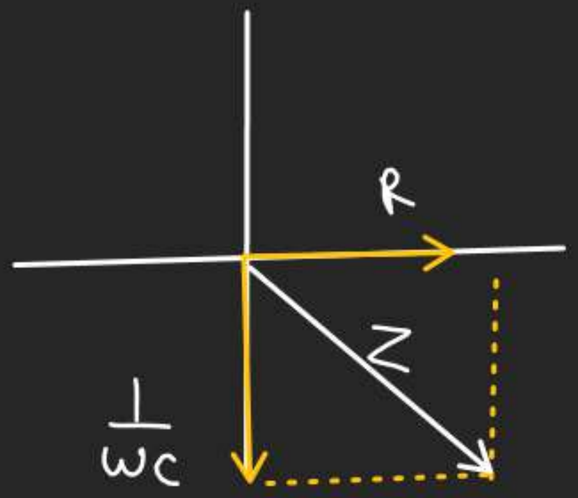
$$R^2 + \frac{1}{\omega^2 C^2} = Z^2$$

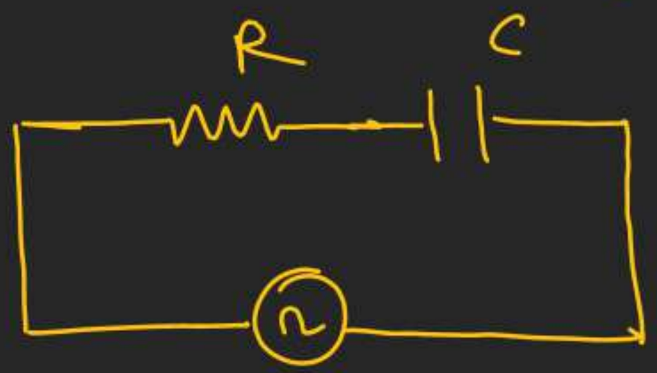
$$R^2 + \frac{1}{\omega^2 C^2} = R^2(1.25)$$

$$\frac{1}{\omega^2 C^2} = 0.25R^2$$

$$C^2 = \frac{1}{\omega^2 \times 0.25R^2}$$

$$C = \frac{1}{\omega \times 0.5 \times R} = \frac{1}{500 \times 0.5 \times R}$$





$$\Rightarrow \tau = RC = \frac{1}{25} \times 10^{-1}$$

$$= 0.04 \times 10^{-1}$$

$$= 4 \times 10^{-3}$$

(4) Ans

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ALTERNATING CURRENT

Q.2 In a circuit, a metal filament lamp is connected in series with a capacitor of capacitance $C\mu F$ across a $200\ V$, $50\ Hz$ supply. The power consumed by the lamp is $500\ W$ while the voltage drop across it is $100\ V$. Assume that there is no inductive load in the circuit. Take rms values of the voltages. The magnitude of the phase-angle (in degrees) between the current and the supply voltage is ϕ. Assume $\pi\sqrt{3} \approx 5$.

f

1. The value of C is ________ μF

2. The value of ϕ is ________ (2021)

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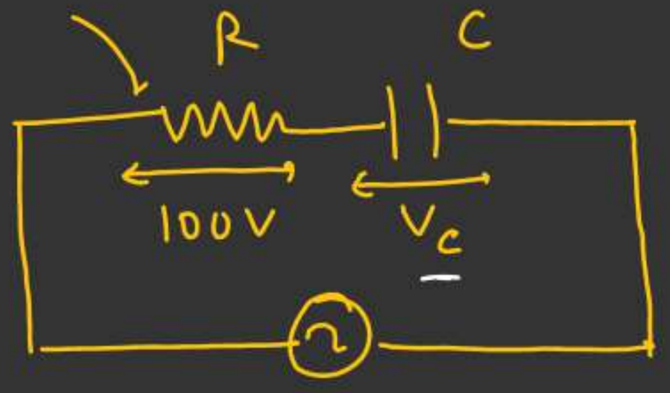
$P_R = 500\,\text{Watt}$



200V, 50Hz = f

$\omega = 2\pi f$
$= 2\pi \times 50$
$= 100\pi$

$\tan\phi = \frac{V_c}{V_R}$

$\tan\phi = \frac{\sqrt{3}\times 10^2}{10^2}$

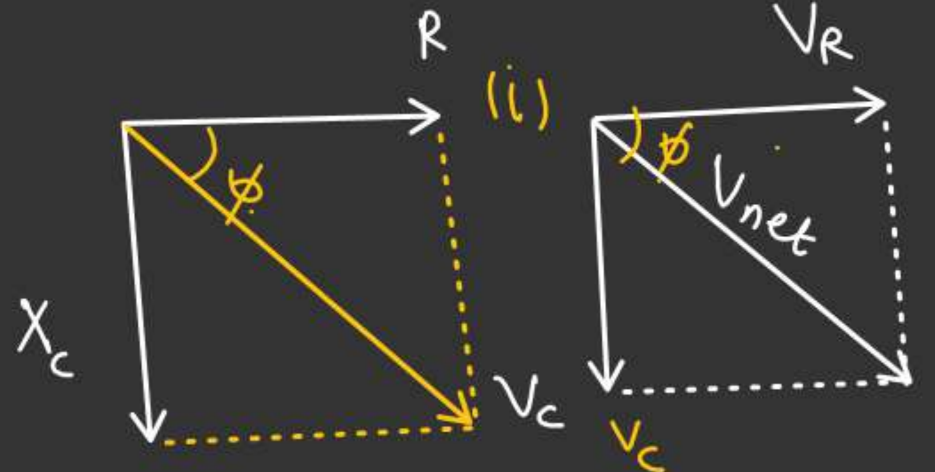
$\boxed{\phi = 60^\circ}$

$P_R = \frac{V^2}{R}$

$R = \frac{V^2}{P_R} = \frac{10^4}{5\times 10^2}$

$R = 0.2\times 10^2$

$R = 20\,\Omega$ ✓



$\tan\phi = \left(\frac{X_c}{R}\right)$

$V_{net}^2 = V_R^2 + V_c^2$

$(200)^2 = (100)^2 + V_c^2$

$V_c^2 = (4\times 10^4 - 1\times 10^4)$

$V_c^2 = 3\times 10^4$

$V_c = \sqrt{3}\times 10^2$

$\tan 60^\circ = \frac{X_c}{R}$

$20\sqrt{3} = X_c = \frac{1}{\omega C}$

$C = \frac{1}{\omega\times 20\times\sqrt{3}} = \frac{1}{100\pi\times 20\times\sqrt{3}} = \frac{1}{5\times 2\times 10^3}$

$= 10^{-4} = 100\times 10^{-6} = 100\,\mu F$

ALTERNATING CURRENT

H.W

Q.3 Consider an LC circuit, with inductance $L = 0.1 H$ and capacitance $C = 10^{-3}$ F, kept on a plane. The area of the circuit is $1 \, m^2$. It is place in a constant magnetic field of strength B_0 which is perpendicular to the plane of the circuit. At time $t = 0$, the magnetic field strength starts increasing linearly as $B = B_0 + \beta t$ with $\beta = 0.04 \, T \, s^{-1}$. The maximum magnitude of the current in the circuit is ______ mA. **(2022)**

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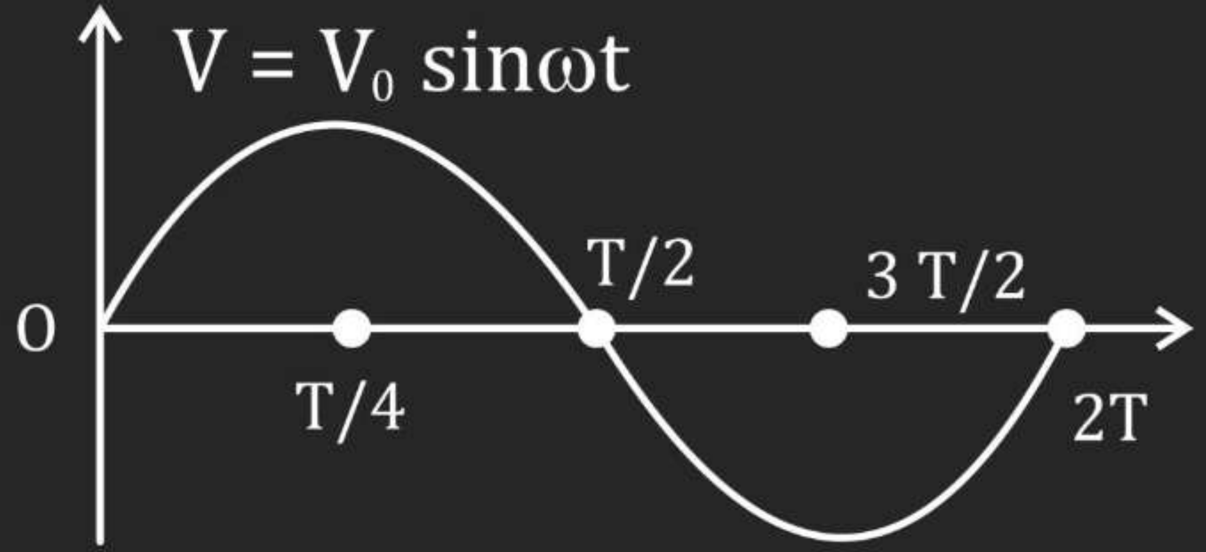
ALTERNATING CURRENT

H-W

Q.4 In a series $L - R$ circuit $(L = 35\,\text{mH}$ and $R = 11\Omega$), a variable emf source $(V = V_0 \sin \omega t)$ of $V_{rms} = 220\text{ V}$ and frequency 50 Hz is applied. Find the current amplitude in the circuit and phase of current with respect to voltage. Draw current-time graph on given graph $(\pi = 22/7)$. **(2004)**



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ALTERNATING CURRENT

H.W

Q.5 The instantaneous voltages at three terminals marked X, Y and Z are given by

$V_X = V_0 \sin \omega t,$

$V_Y = V_0 \sin\left(\omega t + \frac{2\pi}{3}\right)$ and $V_Z = V_0 \sin\left(\omega t + \frac{4\pi}{3}\right)$

An ideal voltmeter is configured to read rms value of the potential difference between its terminals. It is connected between points X and Y and then between Y and Z. The reading(s) of the voltmeter will be

(A) independent of the choice of the two terminals

(B) $V_{XY}^{rms} = V_0$

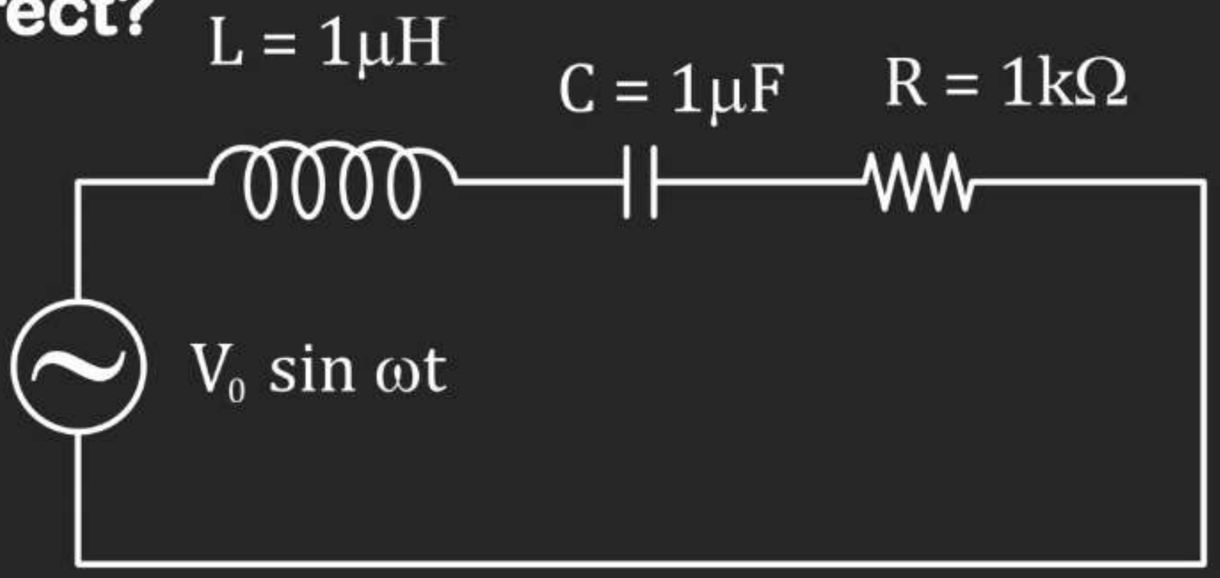
(C) $V_{YZ}^{rms} = V_0\sqrt{\frac{1}{2}}$

(D) $V_{XY}^{rms} = V_0\sqrt{\frac{3}{2}}$

ALTERNATING CURRENT

H-W

Q.6 In the circuit shown, $L = 1\mu H, C = 1\mu F$ and $R = 1k\Omega$. They are connected in series with an a.c. source $V = V_0 \sin \omega t$ as shown. Which of the following options is/are correct? **(2017)**



(A) At $\omega \sim 0$ the current flowing through the circuit becomes nearly zero

(B) The frequency at which the current will be in phase with the voltage is independent of R

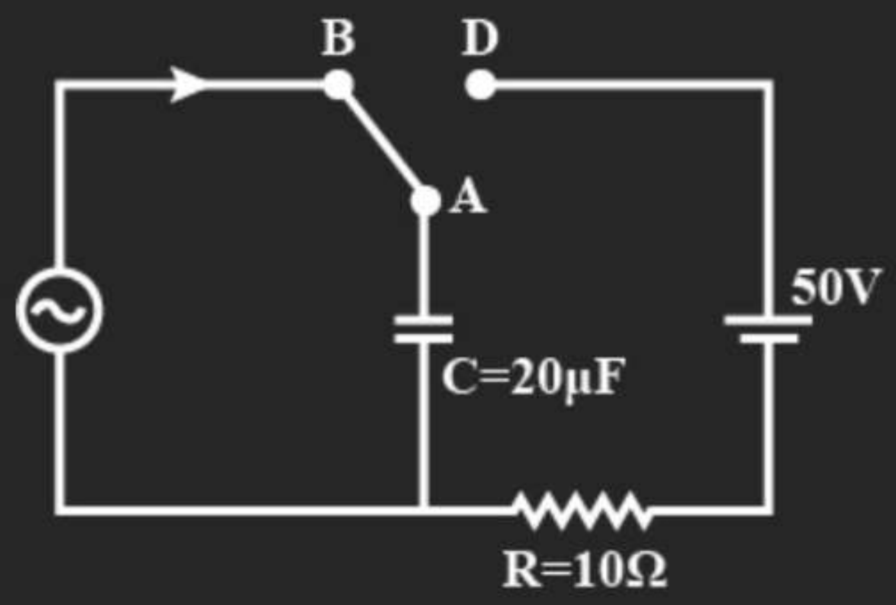
(C) The current will be in phase with the voltage if $\omega = 10^4 \text{rads}^{-1}$.

(D) At $\omega \gg 10^6 \text{rads}^{-1}$, the circuit behaves like a capacitor.

ALTERNATING CURRENT

H-W

Q.7 At time $t = 0$, terminal A in the circuit shown in the figure is connected to B by a key and an alternating current $I(t) = I_0\cos(\omega t)$, with $I_0 = 1\ A$ and $\omega = 500\text{rads}^{-1}$ starts flowing in it with the initial direction shown in the figure. At $t = \frac{7\pi}{6\omega}$, the key is switched from B to D. Now onwards only A and D are connected. A total charge Q flows from the battery to charge the capacitor fully. If $C = 20\mu F, R = 10\Omega$ and the battery is ideal with emf of $50\ V$, identify the correct statement(s). **(2014)**



(A) Magnitude of the maximum charge on the capacitor before $t = \frac{7\pi}{6\omega}$ is $1 \times 10^{-3}C$.

(B) The current in the left part of the circuit just before $t = \frac{7\pi}{6\omega}$ is clockwise.

(C) Immediately after A is connected to D, the current in R is $10\ A$.

(D) $Q = 2 \times 10^{-3}C$.

ALTERNATING CURRENT

HW

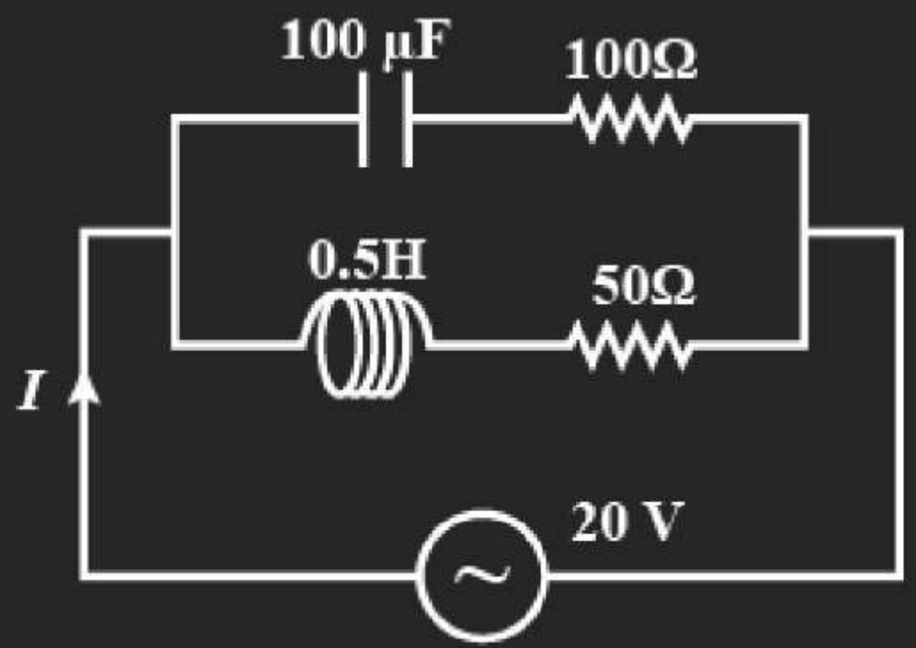
Q.8 In the given circuit, the AC source has $\omega = 100 \text{rad/s}$. Considering the inductor and capacitor to be ideal, the correct choice(s) is/are **(2012)**

(A) the current through the circuit, I is 0.3 A.

(B) the current through the circuit, I is $0.3\sqrt{2}$ A.

(C) the voltage across 100Ω resistor $= 10\sqrt{2}$ V.

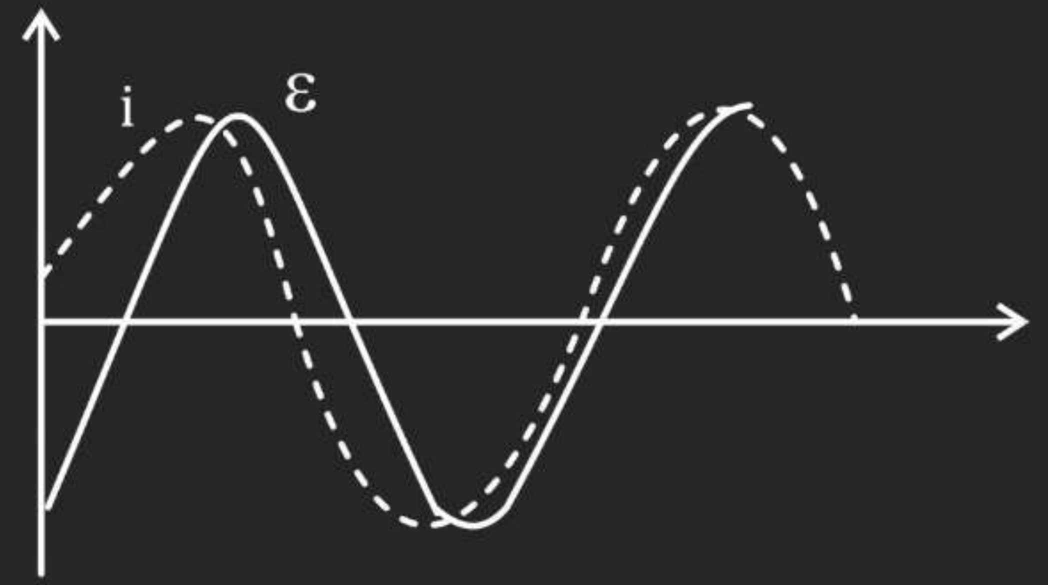
(D) the voltage across 50Ω resistor $= 10$ V.



H-W

ALTERNATING CURRENT

Q.9 When an AC source of emf $\varepsilon = E_0 \sin(100t)$ is connected across a circuit, the phase difference between the emf e and the current i in the circuit is observed to be $\pi/4$, as shown in the diagram. If the circuit consists possibly only of $R - C$ or $R - L$ or $L - C$ in series, find the relationship between the two elements.



(A) $R = 1k\Omega, C = 10\mu F$

(B) $R = 1k\Omega, C = 1\mu F$

(C) $R = 1k\Omega, L = 10H$

(D) $R = 1k\Omega, L = 1H.$