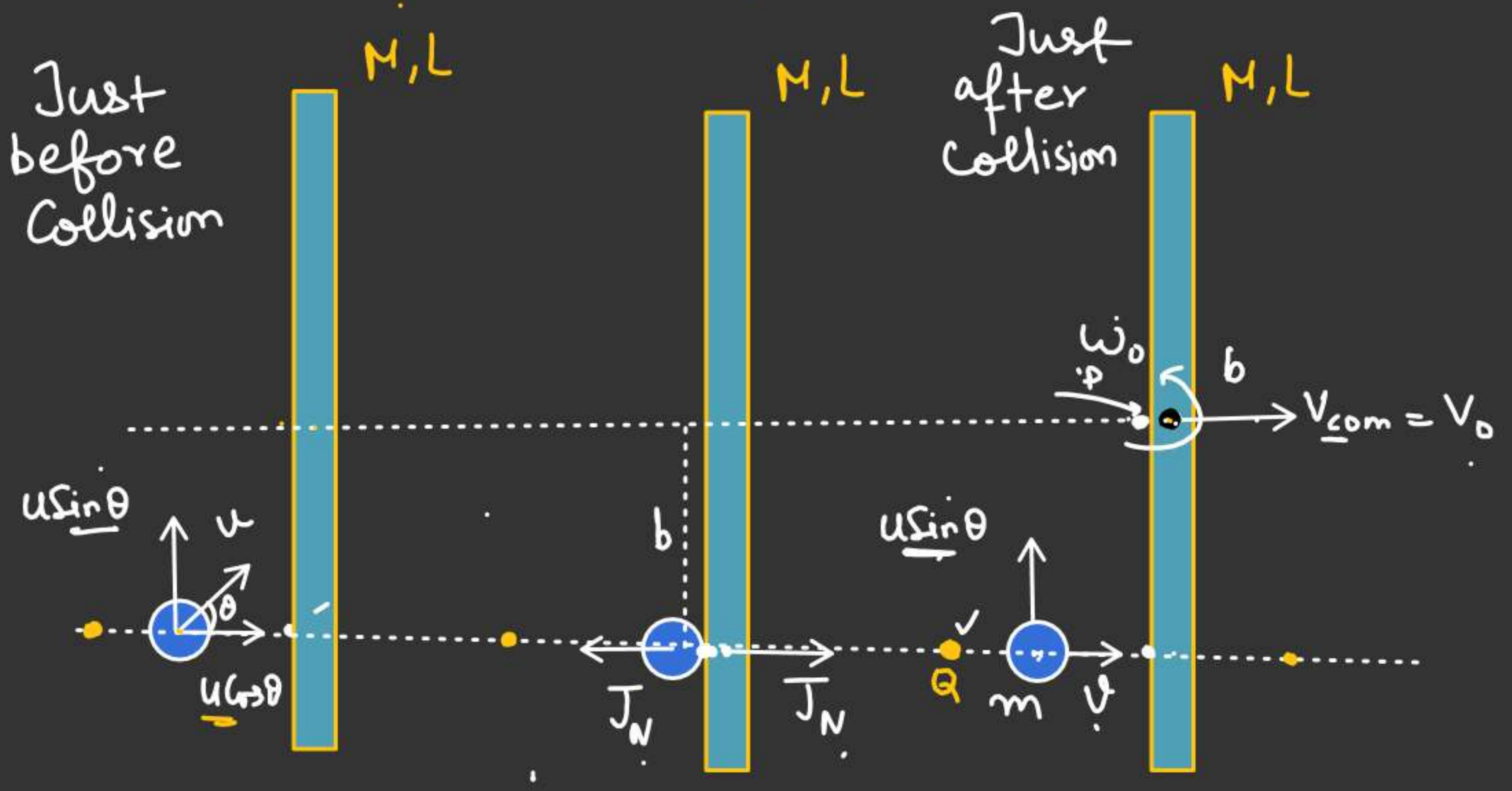
Case of collision of ball with free Rod

The whole System on a Smooth horizontal surface.



Taking (Rod + ball) as System

L.M.C

$$mu\cos\theta = MV_0 + mv \rightarrow (1)$$

A.M.C about P

$$(mu\cos\theta)b = mvb + \frac{ML^2}{12}\omega_0 - (2)$$

$$L = I_{cm}\omega + MVr_\perp$$

About P

$r_\perp = 0$

$V \rightarrow V_{com}$

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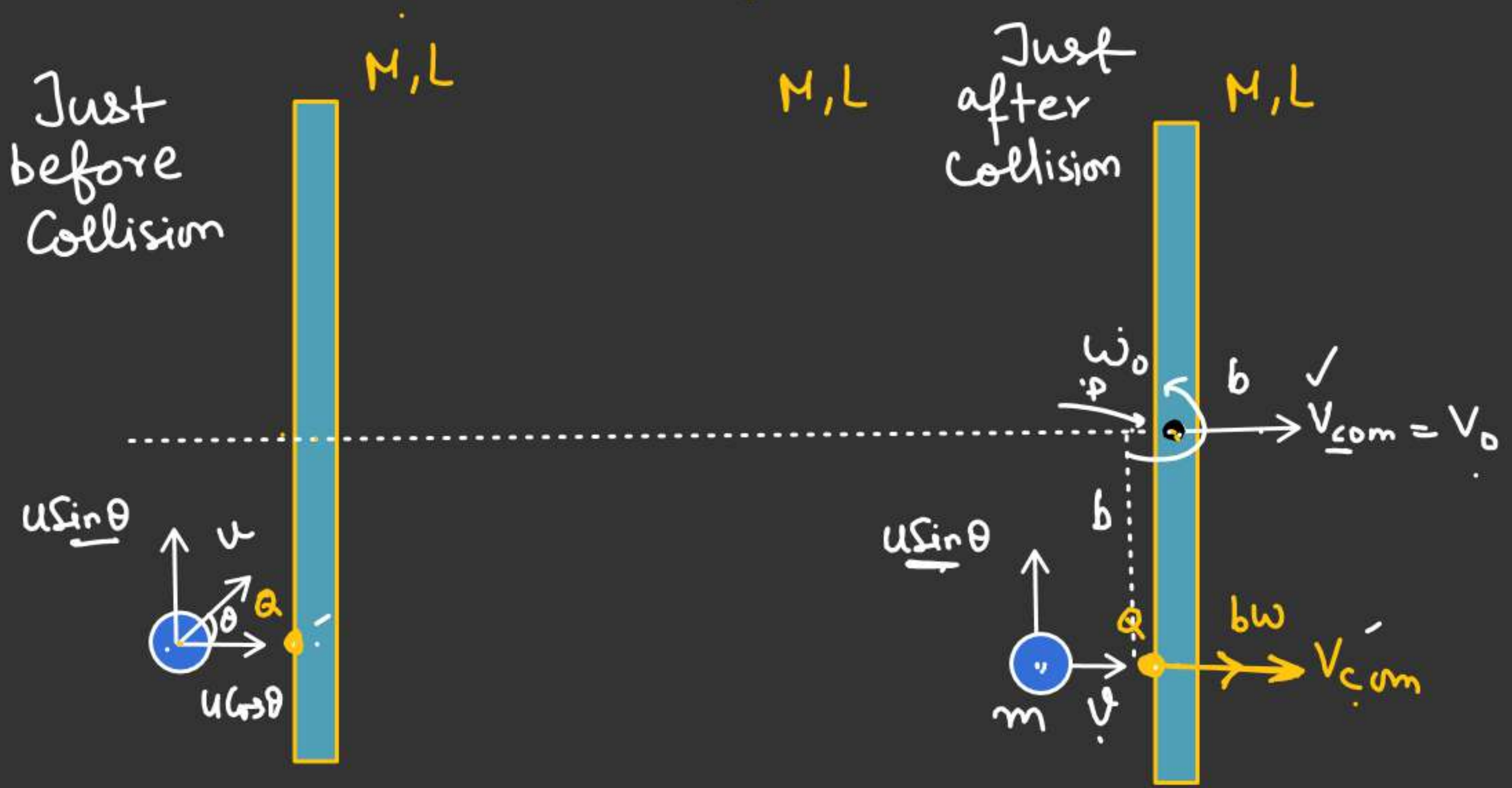
Case of collision of ball with free Rod

The whole System on a Smooth horizontal S



$\vec{V}_Q = \vec{V}_{Q/com\ of\ Rod} + \vec{V}_{com\ of\ Rod/E}$

$= (b\omega\hat{i} + V_{com}\hat{i})$

Equation of e

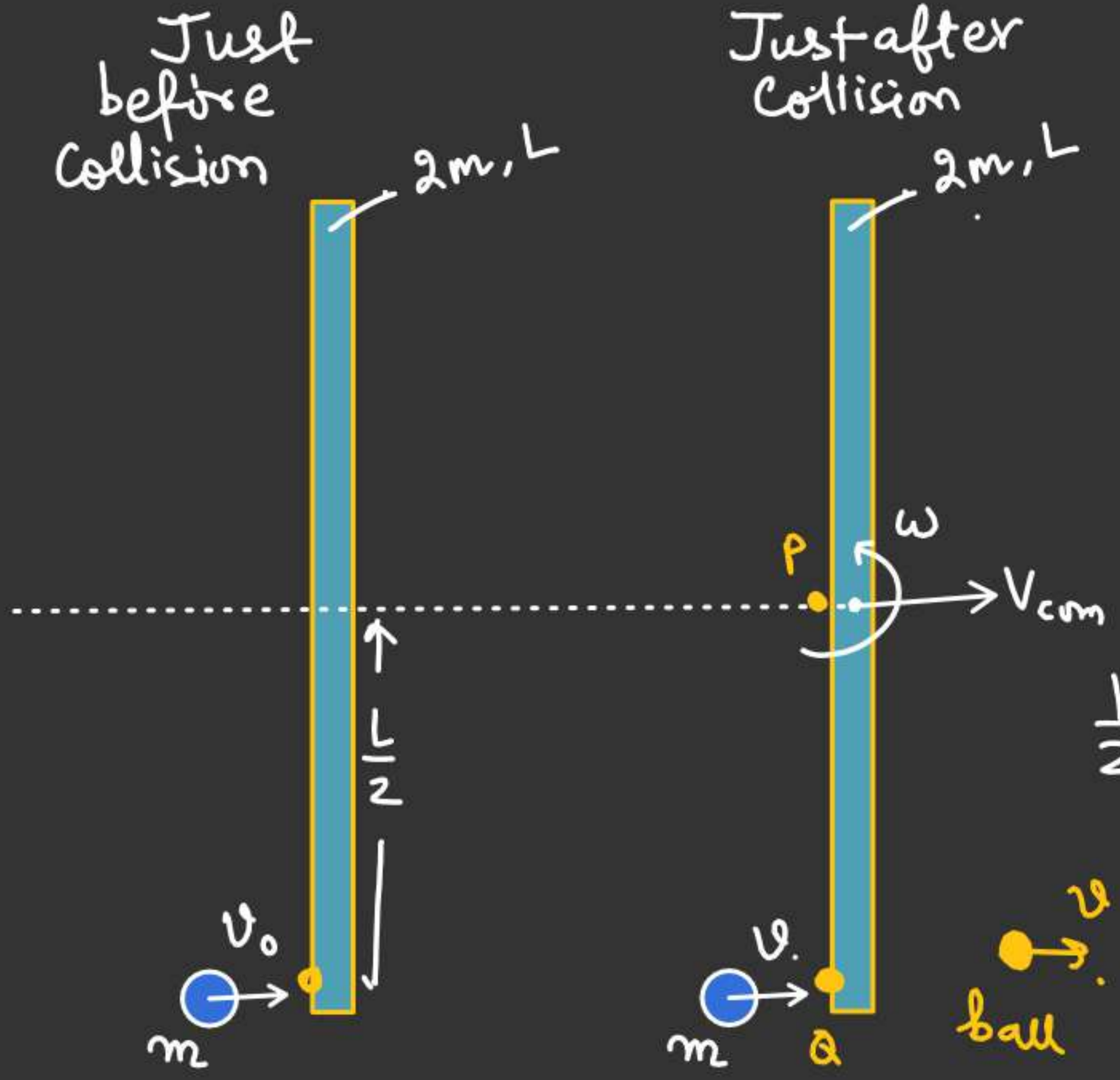
$$e = \frac{(V_{com} + b\omega) - v}{u\cos\theta}.$$

$$[e\,u\cos\theta = (V_{com} + b\omega) - v] \quad (3)$$

Whole System on Smooth horizontal Surface

Find

Angular velocity of Rod & velocity of ball just after Collision if 1) $e = \frac{1}{2}$



L.M.C.

$$m v_0 = 2m V_{com} + m v \quad \text{— (1)}$$

A.M.C about P.

$$m v_0 \frac{L}{2} = \frac{(2m)L^2}{12} \times \omega + m v \frac{L}{2} \quad \text{— (2)}$$

Equation of e.

$$\frac{1}{2} = e = \frac{\left(V_{com} + \frac{L}{2}\omega\right) - v}{v_0}$$

$$\left[\frac{v_0}{2} = V_{com} + \frac{L}{2}\omega - v\right] \rightarrow (3)$$

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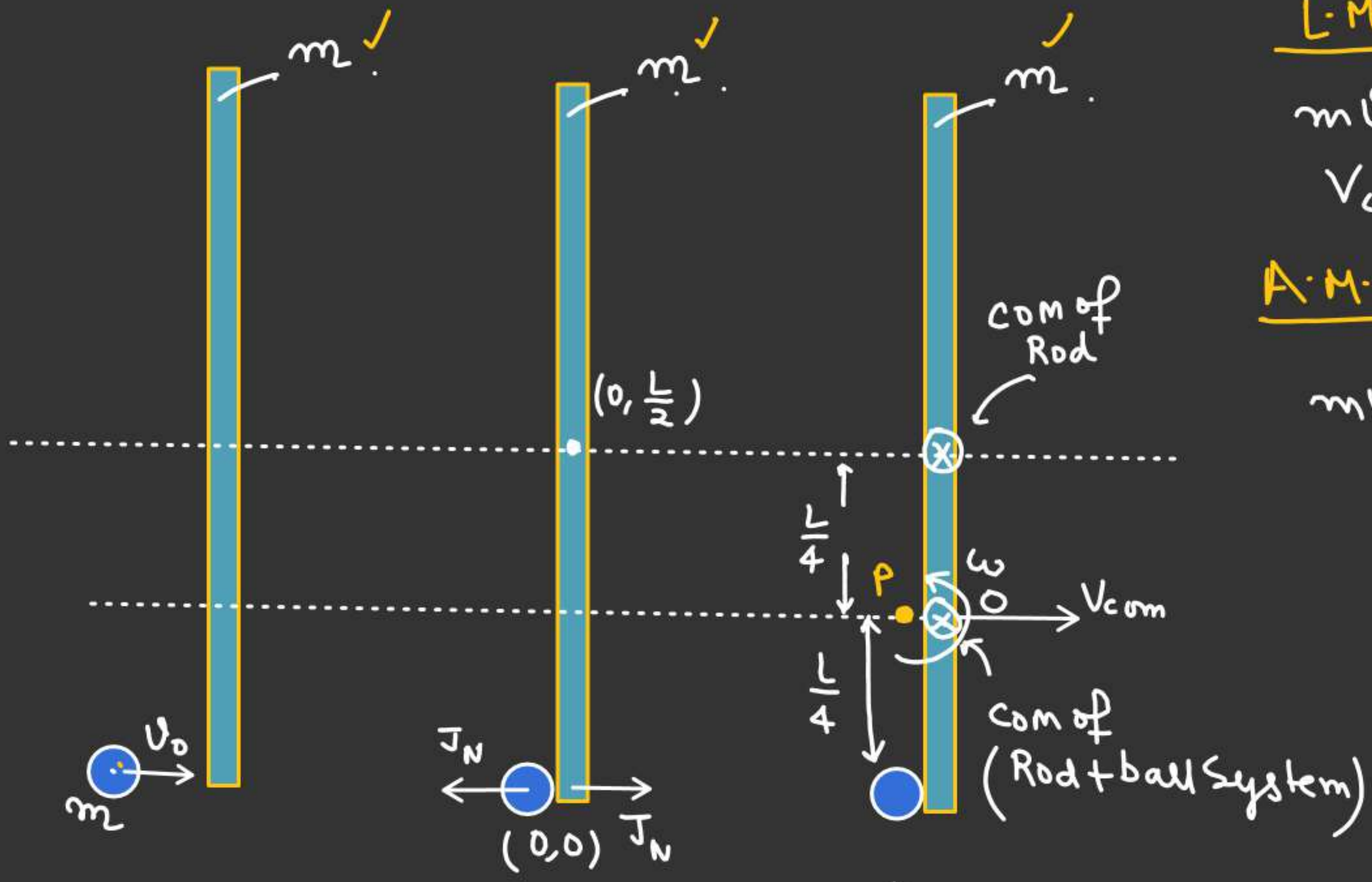
Perfectly Inelastic Collision of a ball with a free rod

$[e=0]$



$$y = \frac{m(L/2)}{m+m} = \frac{L}{4}$$

L·M·C

$$m v_0 = (m+m) V_{com}$$

$$V_{com} = \frac{v_0}{2}$$

A·M·C about P

$$m v_0 \frac{L}{4} = (I_{com})_O \omega$$

$$= (I_{Rod} + I_{ball})_O \omega$$

$$= \left[\frac{mL^2}{12} + m\left(\frac{L}{4}\right)^2 + m\left(\frac{L}{4}\right)^2\right]\omega$$

$$\frac{v_0 L}{4} = \left[\frac{L^2}{12} + \frac{L^2}{8}\right]\omega$$

$$\frac{v_0 L}{4} = \frac{5L^2}{24}\omega$$

$$\omega = \left(\frac{6 v_0}{5L}\right)$$

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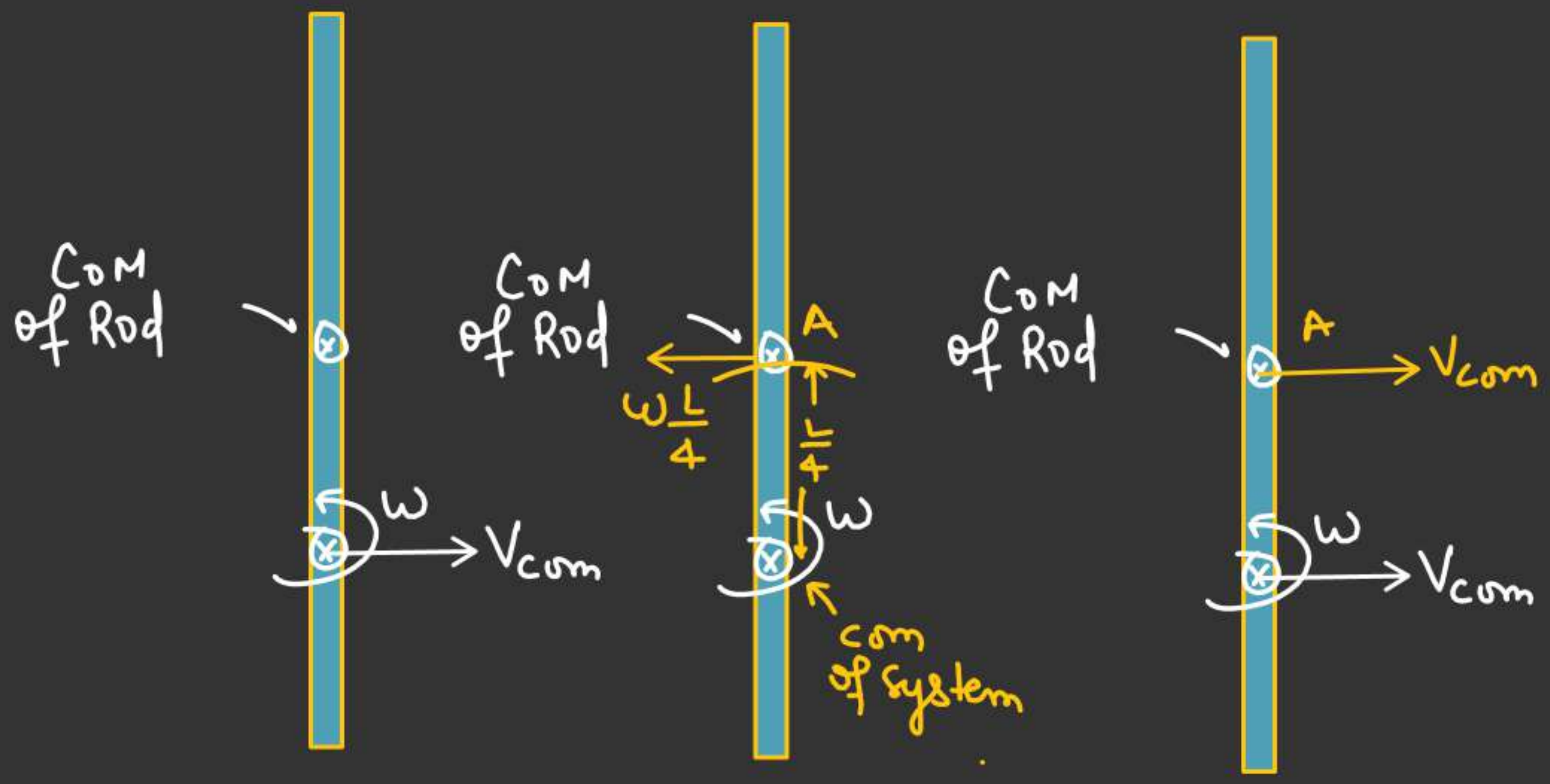
Speed of center of Rod after Collision



$$\vec{V}_A = \vec{V}_{A/com} + \vec{V}_{com/E}$$

$$= -\frac{\omega L}{4}\hat{i} + V_{com}\hat{i}$$

$$= \left(V_{com} - \frac{\omega L}{4}\right)\hat{i}$$

$$= \left(\frac{V_0}{2} - \frac{L}{4} \times \frac{6V_0}{5L}\right)$$

$$= \left(\frac{V_0}{2} - \frac{6V_0}{20}\right)$$

$$= \frac{4V_0}{20} = \frac{V_0}{5}$$

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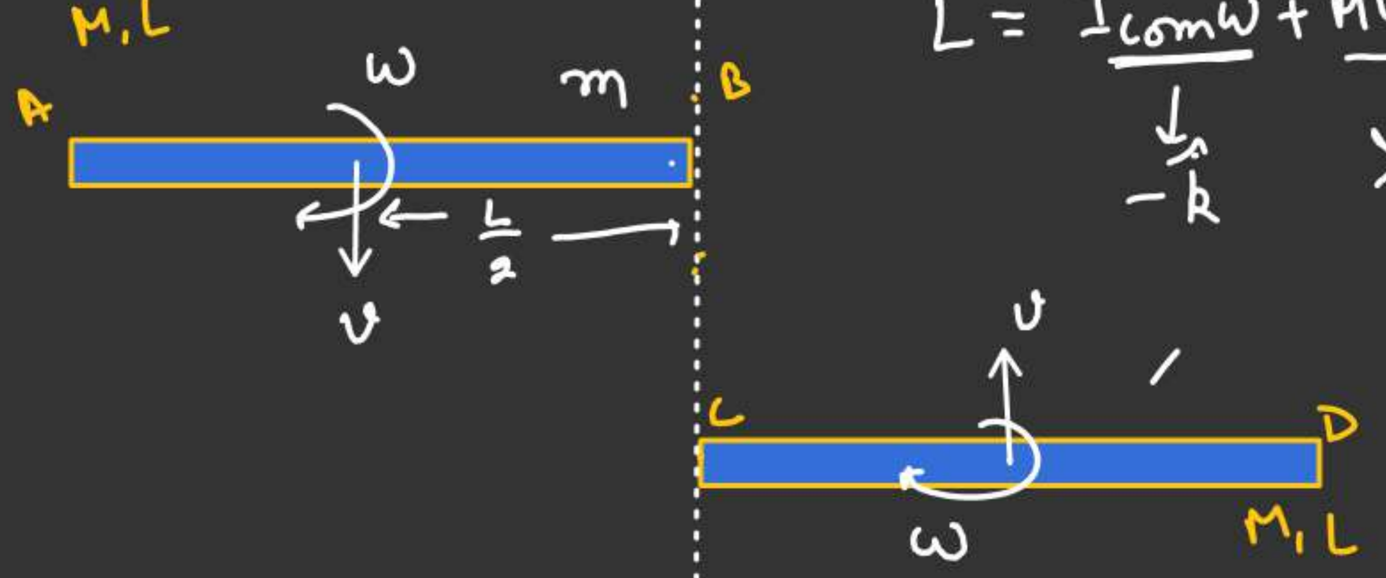


Just before Collision.

$$L = \underline{I_{com}\omega} + \underline{MVr_{\perp}}$$

$$\downarrow \hat{-k} \qquad \downarrow \hat{k}$$

After Collision both the Rod Stick together. Find angular velocity of both the rod just after collision.

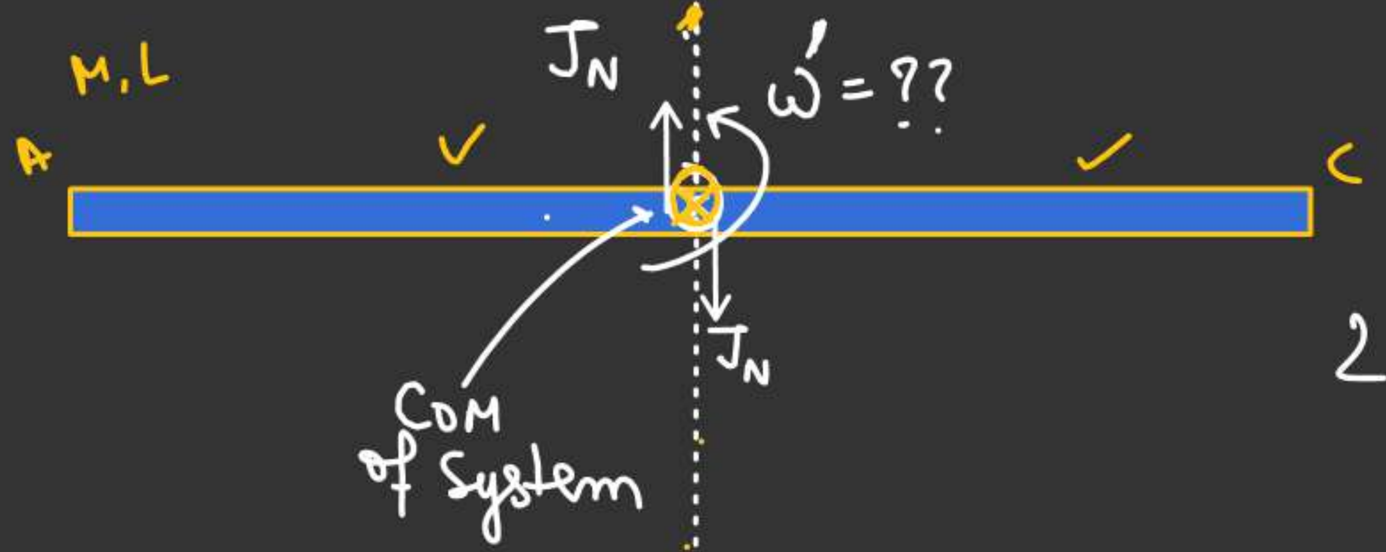
L·M·C

$$-mV + mV = (m+m)V_{com}$$

$$V_{com} = 0$$

A·M·C about COM of System

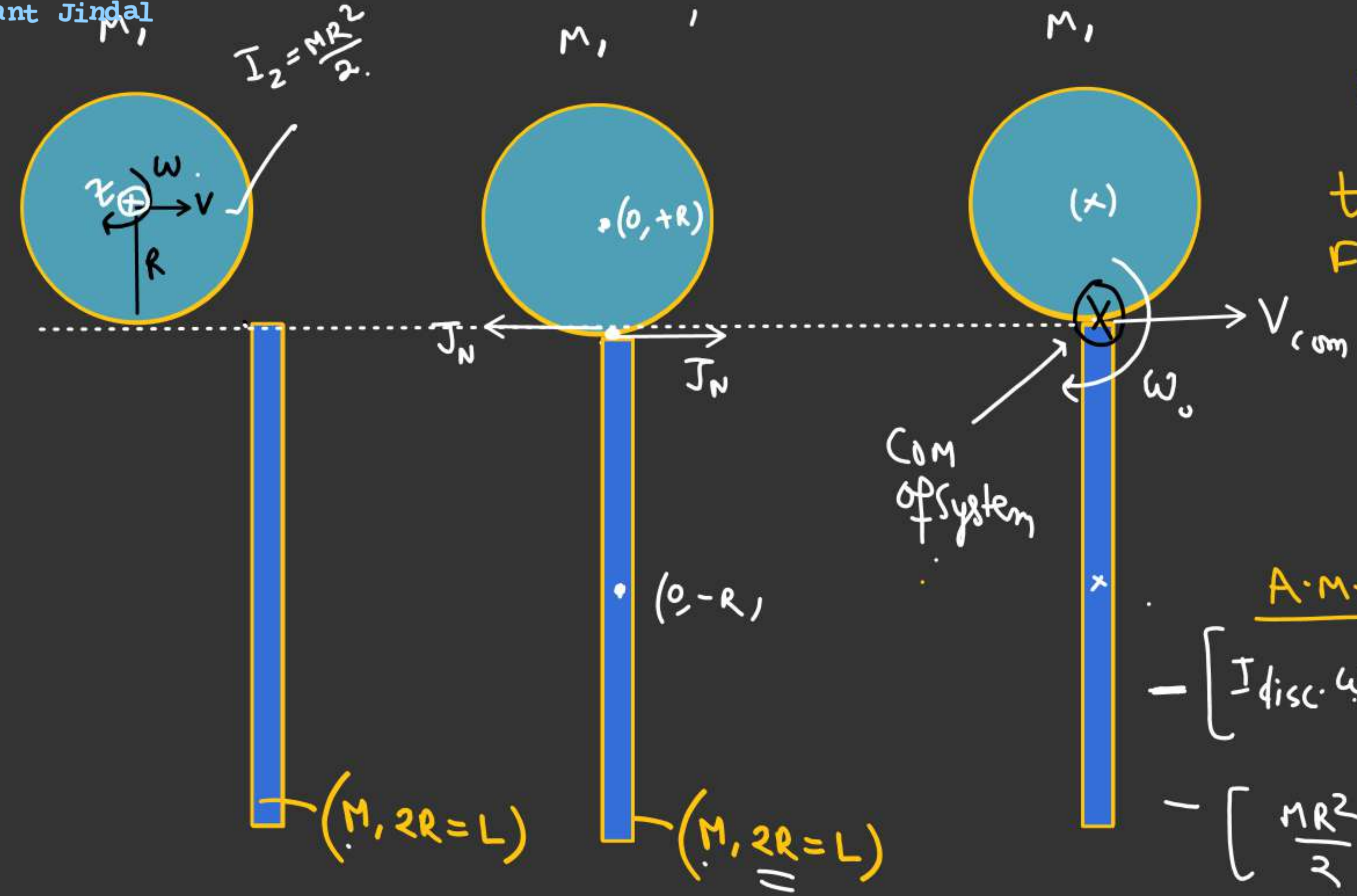
$$L_i = L_f$$



$$2 \times \left[mv\frac{L}{2} - \frac{ML^2}{12}\omega\right] = \left(\frac{ML^2}{3}\omega'\right) \times 2$$

$$\left(mv\frac{L}{2} - \frac{ML^2}{12}\omega\right) = \frac{ML^2}{3} \cdot \omega'$$

$$\omega' = \left(\frac{3v}{2L} - \frac{\omega}{4}\right)$$



Whole System on a smooth horizontal surface. disc stick to the rod after collision. Find ω_{system} just after collision.

($v = R\omega$ given)

$\omega = \frac{v}{R}$

L.M.C

$$MV = 2MV_{com}$$

$$V_{com} = \frac{V}{2}$$

A.M.C

$$-\left[I_{disc}\cdot\omega + MVR\right] = \left[\underbrace{\frac{MR^2}{2}+MR^2}_{\text{Disc of com}} + \frac{M(2R)^2}{3}\right]\omega_0$$

$$-\left[\frac{MR^2}{2}\times\frac{v}{R}+MvR\right] = \left[\frac{3}{2}MR^2+\frac{4MR^2}{3}\right]\omega_0$$

$$-\left[\frac{3}{2}\cancel{M}v\cancel{R}\right] = \frac{17\cancel{M}R^{\cancel{2}}}{6}\omega_0$$

$$-\frac{9v}{17R} = \omega_0 \quad \text{or} \quad \left(\frac{9\omega}{17} = \omega_0\right)$$