DPP 03

SOLUTION

1. Y_1 & Y_2 are young modulus of rod α_1 & α_2 are linear expansion coefficient of rod.

As per question, there is no shift in Junction when rod was cooled. Which means,

Force exerted on both rod due to contraction will be equal Force = Stress x Area = young modulus x strain x Area.

$= Y_x \dfrac{\Delta L}{L} \times A$

$= Y_x \dfrac{Ka\Delta T}{K} \times A \qquad \therefore \Delta L = La\Delta T$ Force $= Y_x \dfrac{a\Delta T}{1} \times A$

<u>Force on rod</u>

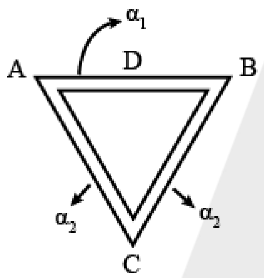
$Y_1\alpha_1\Delta TA = (\text{ Force on rod })_2$

ΔT and A are same for both

$Y, \alpha_1 = Y_2\alpha_2$

2. Let I be the side length of the triangle.

$AD = BD = \frac{1}{2}$ [D is midpoint of AB] Before heating,



$DC^2 = AC^2 - AD^2$

$= l^2 - \dfrac{l^2}{4} = \dfrac{3l^2}{4}$

After heating,

$DC^2 = [l(1 + a_2\Delta t)]^2 - \left[\dfrac{l(1 + a_1\Delta t)}{2}\right]^2$

We can ignore the higher powers of a_1, a_2 as the coefficient of thermal expansion is generally a very small number.

As DC is constant,

$\dfrac{3l^2}{4} = l^2(1 + 2a_2\Delta t) - \dfrac{l^2}{4}(1 + 2a_1\Delta t)$

$\Rightarrow \dfrac{3}{4} = (1 + 2a_2\Delta t) - \dfrac{1}{4}(1 + 2a_1\Delta t)$

$$\Rightarrow 0 = 2a_2\Delta t - \frac{a_1\Delta t}{2}$$

$$\Rightarrow a_2 = \frac{a_1}{4}$$

3. $I = MR^2$

Now $R_1 = R(1 + \alpha\Delta T)$

$R_1^2 = R^2(1 + \alpha\Delta T)^2 = R^2(1 + 2\alpha\Delta T)$, we can neglect α^2 as it is very small.

New moment of Inertia $I_1 = MR^2(1 + 2\alpha\Delta T)$

so $I_1 - I = 2MR^2\alpha\Delta T = 2\alpha I\Delta T$

4. The coefficient of volume expansion (γ) is twice as that of area expansion (β) and thrice as that of linear expansion (α).

i.e., $\gamma = 2\beta$ and $\gamma = 3\alpha$

(a) Increase in diameter is linear expansion.

(b) Increase in surface area is area expansion.

(c) Density decreases with increase in temperature.

5. As change in length,

$$\Delta L = \alpha L\Delta T = \frac{FL}{AY}$$

$$\Rightarrow \text{ Stress } = \frac{F}{A} = Y\alpha\Delta T$$

6. Given $d = 20$ cm

$V = V_0(1 + \gamma t) = V_0(1 + 3\alpha t)(\sin\ ce\gamma = 3\alpha)$

Change in volume $= V - V_0 = 3V_0\alpha t$

$$= 3 \times \frac{4}{3}\pi\left(\frac{d}{2}\right)^3 \times 23 \times 10^{-6} \times 100$$

$$= 3 \times \frac{4}{3}\pi\left(\frac{0.2}{2}\right)^3 \times 23 \times 10^{-6} \times 100$$

$= 28.9\text{cc}(1\text{cc} = 10^{-6}\ \text{m}^3)$

7. The time period of a pendulum is given by,

$$t = 2\pi\sqrt{\frac{l}{g}}$$

$$\Delta t = \frac{2\pi\Delta l}{\sqrt{g}2\sqrt{l}} = \Delta l = \alpha\Delta\theta$$

$$\therefore \Delta t = \frac{\pi}{\sqrt{gl}}\alpha\Delta T$$

$\therefore \frac{\Delta t_1}{\Delta t_2} = \frac{\Delta T_1}{\Delta T_2}$

$\therefore \frac{12}{4} = \frac{40 - T}{T - 20}$

$\therefore 3\,T - 60 = 40 - T$

$\therefore T = 25^\circ$

8. Length is inversely proportional to coefficient of linear expansion, thus $\frac{l_2}{l_1} = \frac{\alpha_A}{\alpha_S}$

or $1 + \frac{l_2}{l_1} = 1 + \frac{\alpha_A}{a_S}$

or $\frac{l_1 + l_2}{l_1} = \frac{a_A + a_S}{a_S}$

or $\frac{l_1}{l_1 + l_2} = \frac{a_S}{a_A + a_S}$

9. Before heating

$Mg = F_b, Mg = V_1\rho_1 g = Ax\rho_1 g$

After heating $Mg = F_b'$, $\quad Mg = V'\rho_1' g$

Equating (1) & (2) $Ax\rho_1^1 g$

$A\rho_1 g = A(l + \beta\Delta t)\frac{\rho_1}{1 + \gamma\,\Delta t}g$

$1 + \gamma\Delta t = 1 + \beta\Delta t, \gamma = \beta \therefore \gamma = 2\alpha$