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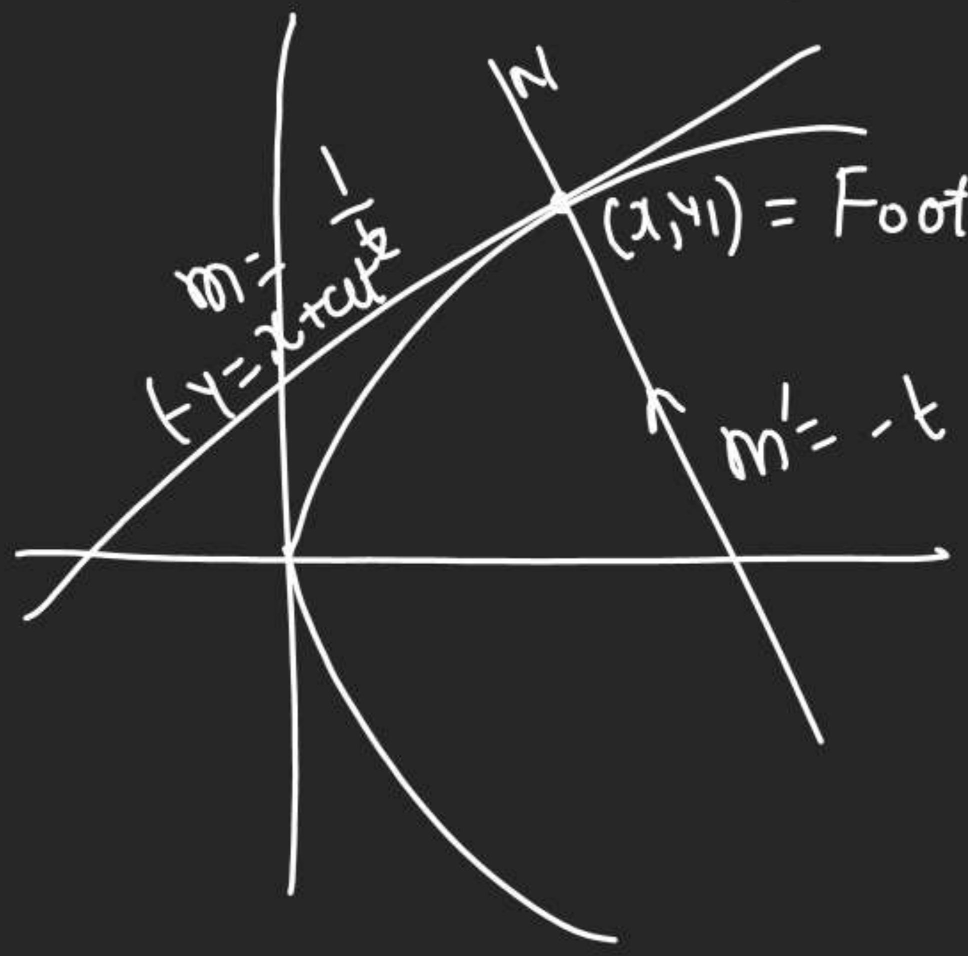
Normal

Line $\perp^r$ to tangent



(x_1, y_1) = Foot of Normal — EOT

$y^2 = 4ax$

(1) Pt. form / Cartesian form

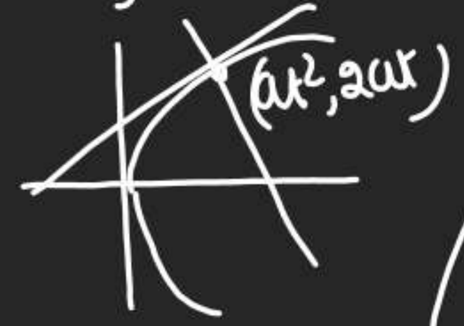
$$yy_1 = 2a(x + x_1)$$

$$(Sl)_{Tangent} = \frac{2a}{y_1}$$

$$(Sl)_{Normal} = -\frac{y_1}{2a}$$

$$\text{EON} = \boxed{(y - y_1) = -\frac{y_1}{2a}(x - x_1)}$$

(2) Parametric form



$$(y - 2at) = -\frac{2at}{2a}(x - at^2)$$

$$y - 2at = -tx + at^3$$

$$\boxed{y + tx = 2at + at^3}$$

(3) Slope form.

$t \to -m$

$$y - mx = -2am - am^3$$

$$\boxed{y = mx - 2am - am^3}$$

$$m = (Sl)_{Normal}$$

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	Slope form.	Par. form
$y^2=4ax \Rightarrow (y-y_1)=\frac{-y_1}{2a}(x-x_1)$	$y^2=4ax \Rightarrow y=mx-2am-am^3$ $(am^2,-2am)$	$y^2=4ax \Rightarrow (y-2at)=-t(x-at^2)$ $(at^2,2at)$ $t=-m$
$x^2=4ay \Rightarrow (y-y_1)=-\frac{2a}{x}(x-x_1)$	$x^2=4ay \Rightarrow y=mx+2a+\frac{a}{m^2}$ $\left(-\frac{2a}{m},\frac{a}{m^2}\right)$	$x^2=4ay \Rightarrow x+ty=2at+at^3$ $(2at,at^2)$

$2x=4a\frac{dy}{dx}$

T $m=\frac{dy}{dx}=\frac{x}{2a}$

N $\Rightarrow m'=-\frac{2a}{x}$

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Q Find EON to Parabola.

$y^2 = 8x$ at lower end of LR? $\to a = 2$

Lower end of LR $\equiv (a, -2a)$

$= (2, -4)$

$\frac{dy}{dx} \Rightarrow 2y\frac{dy}{dx} = 8$

$(SL)_T = \left.\frac{dy}{dx}\right|_{(2,-4)} = \frac{4}{y} = \frac{4}{-4} = -1$

$(SL)_N = \frac{-1}{-1} = 1$ //

$(y-(-4)) = 1(x-2) \Rightarrow x - y = 6$

Q Find EON to Par. $y^2 = 8x$ which is $\parallel^{r}$ to $y = 2x+3$?

as Line $\parallel^{r}$ Normal.

$m = (SL)_L = (SL)_N = 2$

$y = mx - 2am - am^3$

$y = 2x - 8 - 2 \times 2^3$

$y = 2x - 24$

Q What is Foot of Normal to Par. $x^2 + 8y = 0$ which is $\perp^{r}$ to $\boxed{x - 3y = 1}$ $\Rightarrow m = \frac{1}{3}$

$x^2 = -8y \Rightarrow x^2 = 4ay$ $\to a = -2$

$m = (SL)_{Nor} = -3$

$\left(-\frac{2a}{m}, \frac{a}{m^2}\right)$

$\left(\frac{-2 \times -2}{-3}, \frac{-2}{(-3)^2}\right)$

$\left(-\frac{4}{3}, -\frac{2}{9}\right)$

If I forgot:.... then???

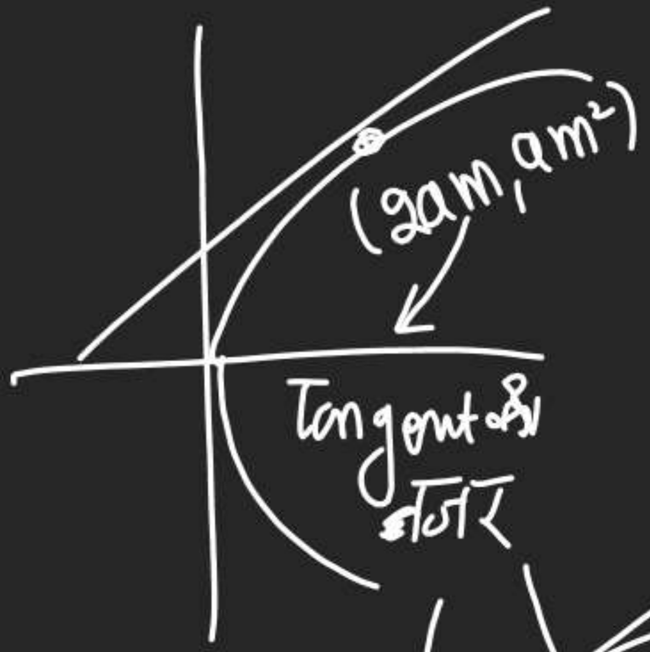
$x^2 = 4ay$

(1) first think about Slope of Tangat

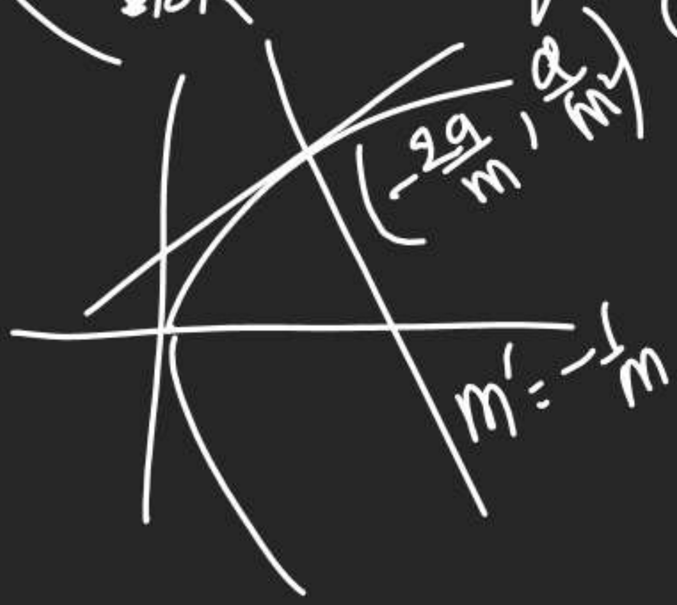
$2x = 4a\frac{dy}{dx} \Rightarrow \cancel{2}x = \cancel{4}^{2}am$

$x = 2am$

$\cancel{4}a^{\cancel{2}}m^2 = \cancel{4}ay \Rightarrow y = am^2$



Pt of tangency.



$x^2 = -4ay \Rightarrow \cancel{2}x = -\cancel{4}^{2}a\frac{dy}{dx}$

$x = -2am \Rightarrow$

$\cancel{4}a^2m^2 = -\cancel{4}ay \Rightarrow y = -am^2$

$m \to -\frac{1}{m}$ Pt of Tange $(-2am, -am^2)$

$\Downarrow$

Foot of Normal $\left(\frac{2a}{m}, -\frac{a}{m^2}\right)$

Q $x^2 = -8y$

$a = 2$

$m = -3$

$\left(\frac{2\times 2}{-3}, \frac{-2}{(-3)^2}\right)$

$\left(-\frac{4}{3}, -\frac{2}{9}\right)$

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Q What is cond^n of Normality for $y^2 = 4ax$.

EON $\Rightarrow$ $y = mx - 2am - am^3$

$y = mx + c$

$c = -2am - am^3$

Q What is cond^n of tangency for $y^2 = 4ax$

$y = mx + \boxed{\frac{a}{m}}$

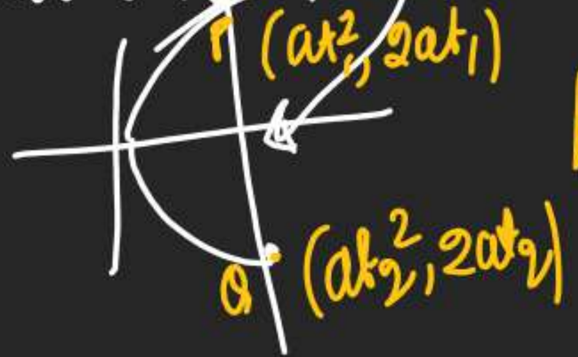
$c = \frac{a}{m}$ is cond^n of T.

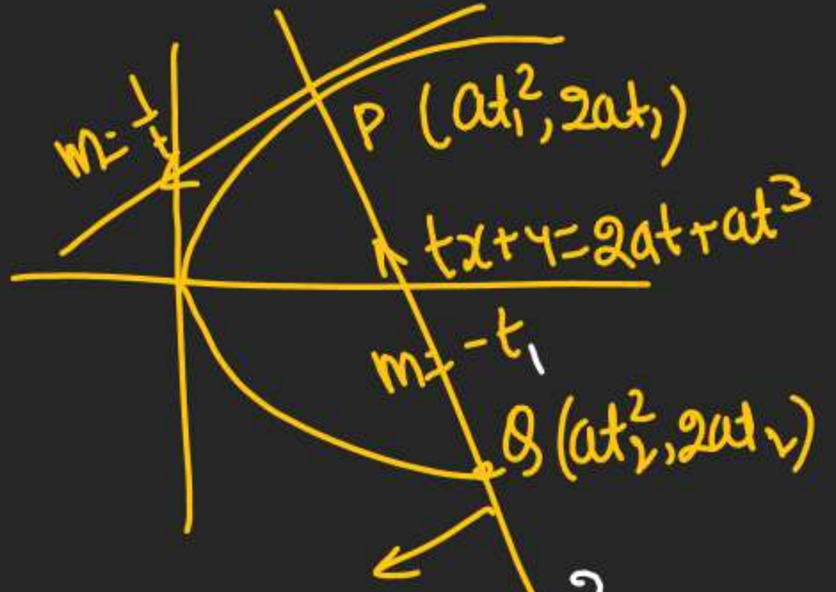
Prop of Normal

(1) Axis is a Normal.

(2) Axis is only Normal which P.T. Focus

(3) If Normal at any Pt. cuts Parabola again then it is Normal Chord.





$(SL)_{chord} = \frac{2}{t_1 + t_2} = -t_1$

$\frac{t_1 + t_2}{2} = -\frac{1}{t_1}$

$t_1 + t_2 = -\frac{2}{t_1}$

$t_2 = -t_1 - \frac{2}{t_1}$

here $\boxed{t_2 = -t_1 - \frac{2}{t_1}}$

This Relation is also true for $x^2 = 4ay$

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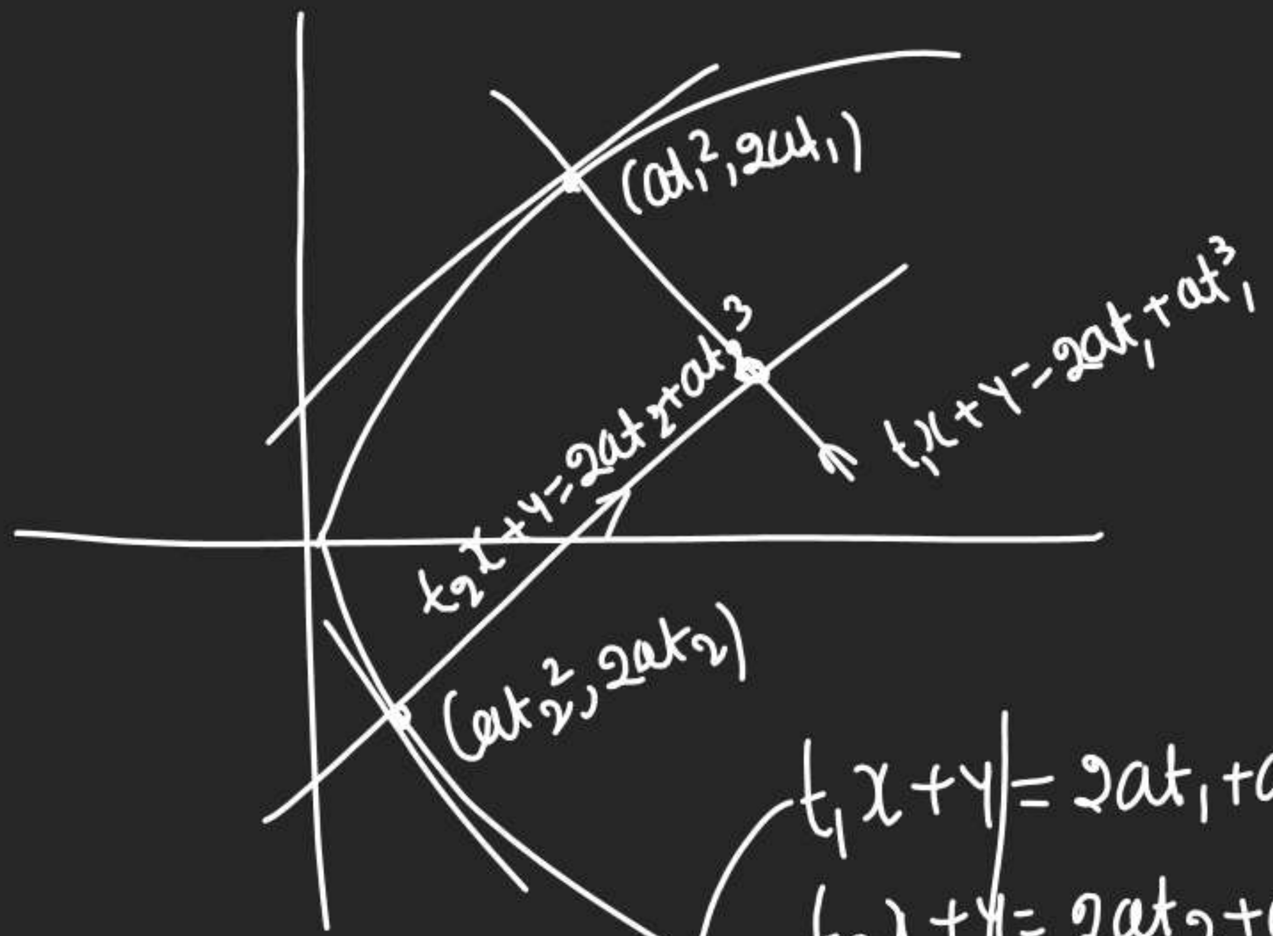
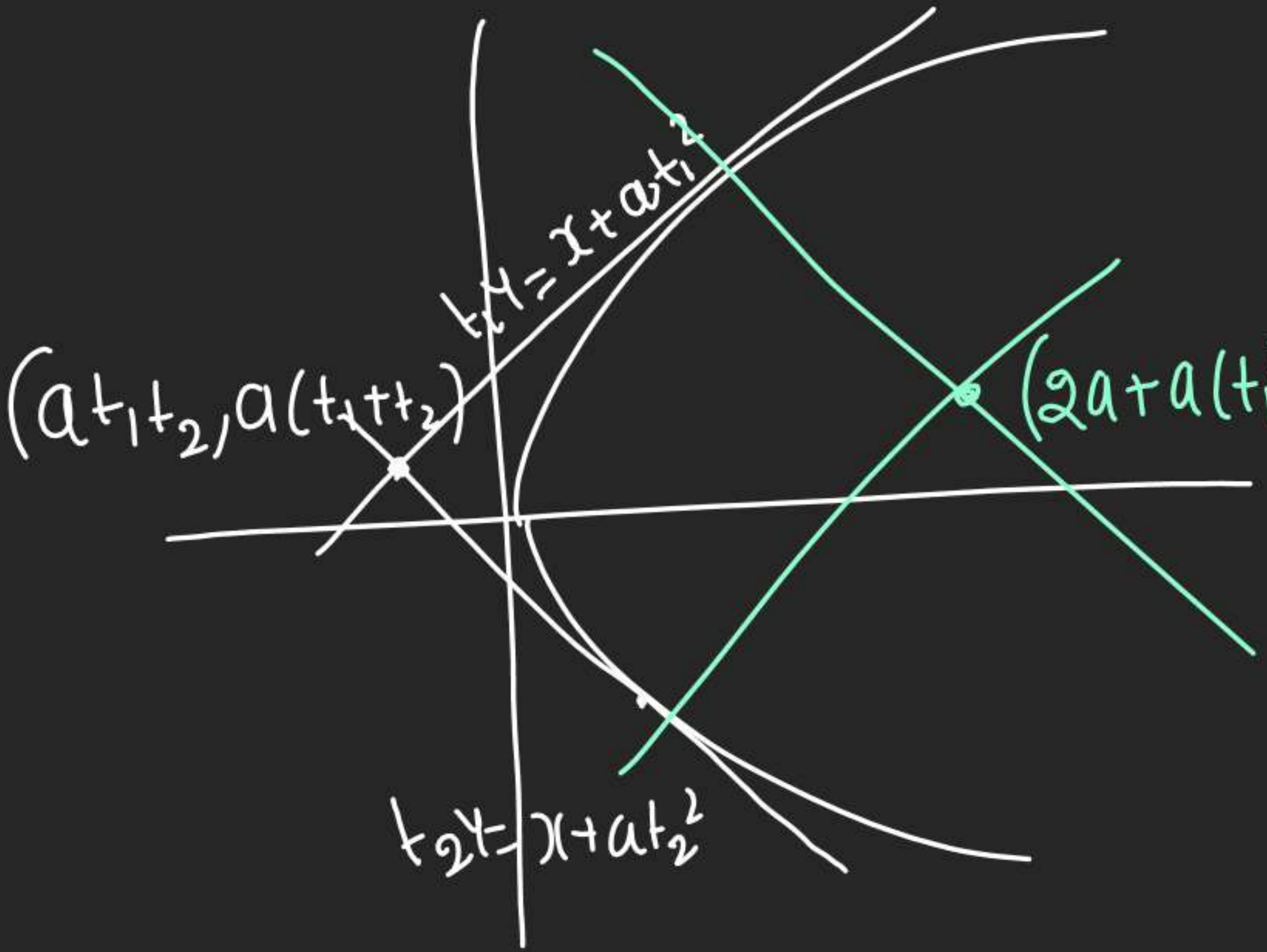
(4) Point of Int. of 2 Normal.

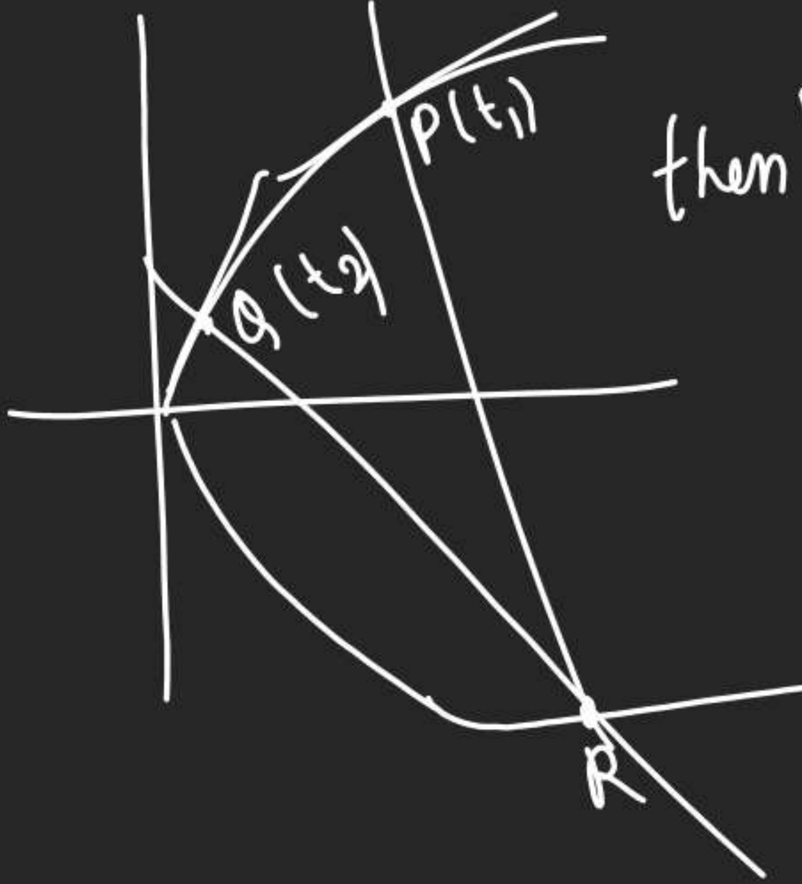


$$t_1x + y = 2at_1 + at_1^3$$
$$t_2x + y = 2at_2 + at_2^3$$
$$x(t_1 - t_2) = 2a(t_1 - t_2) + a(t_1^3 - t_2^3)$$
$$x = 2a + a(t_1^2 + t_2^2 + t_1t_2)$$
$$y =$$

$$\cancel{2at_1} + \cancel{at_1^3} + at_1t_2^2 + at_1^2t_2 + y = \cancel{2at_1} + \cancel{at_1^3}$$
$$y = -at_1t_2(t_1 + t_2)$$



(5) If 2 Normals at $P(t_1), Q(t_2)$ meet at Parabola at $R(t_3)$ then $\boxed{t_1t_2 = 2}$



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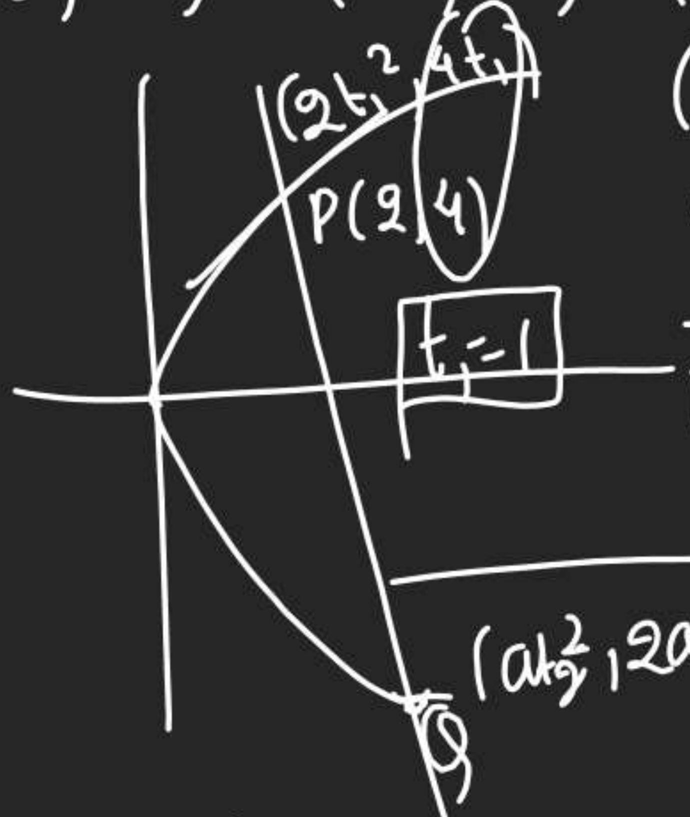
Q If Normal to Parabola $y^2=8x$ at Pt $(2,4)$ meets Parabola again, the Point is

$(-18,-12)$ $(-18,12)$ $(18,12)$ $(18,-12)$



$y^2=8x$

① $a=2$

Normal Chord

$t_2=-t_1-\frac{2}{t_1}$

$=-1-\frac{2}{1}$

$t_2=-3$

$Q=(2t_2^2, 4t_2)$

$=(18,-12)$

Q Let L be a Normal to $y^2=4x$. If L P.T. Pt. $(9,6)$

Eqn of L are.

$y-x-3=0$, $y+3x-33=0$

$y+x-15=0$ $y-2x+12=0$

$a=1$

Normal $\Rightarrow y=mx-2am-am^3$

$m=1$ $\leftarrow y=mx-2m-m^3$ P.T. $(9,6)$

$y=x-2-1$

$y=x-3$

$m=-3$

$y=-3x+6+27$

$m=2$

$y=2x-4-8$

$6=9m-2m-m^3$

$m^3-7m+6=0 \Rightarrow (m-1)(m^2+m-6)=0$

$(m-1)(m+3)(m-2)=0$

$m=1, -3, 2$