Nishant Jindal

Q $\displaystyle\int_0^{2a} \frac{dx}{\sqrt{\boxed{2a}x-x^2}}$ $\to a$ half.

$\displaystyle\int_0^{2a} \frac{dx}{\sqrt{(a)^2-(x-a)^2}}$

$\displaystyle \sin^{-1}\frac{x-a}{a}\Big|_0^{2a}$

$\sin^{-1}(1)-\sin^{-1}(-1)$

$\frac{\pi}{2}+\frac{\pi}{2}=\pi$

7h Chapter बनाया H.W 3500 Book ✓

Q $\displaystyle\int_0^1 \frac{dx}{\sqrt{x-x^2}}$ $\to 1/2$

$\Rightarrow \displaystyle\int_0^1 \frac{dx}{\sqrt{\left(\frac{1}{2}\right)^2-\left(x-\frac{1}{2}\right)^2}}$

$\Rightarrow \displaystyle \sin^{-1}\frac{x-1/2}{1/2}\Big|_0^1$

$\sin^{-1}\left(\frac{1/2}{1/2}\right)-\sin^{-1}\left(\frac{-1/2}{1/2}\right)$

$\sin^{-1}(1)-\sin^{-1}(-1)$

π

Q1 $\Rightarrow \displaystyle\int_0^{2a} \frac{dx}{\sqrt{(x-0)(2a-x)}} = \pi$

Q2 $\displaystyle\int_0^1 \frac{dx}{\sqrt{(x-0)(1-x)}} = \pi$

$\displaystyle\int_\alpha^\beta \frac{dx}{\sqrt{(x-\alpha)(\beta-x)}} = \pi$

Yad Rakha hai

$\displaystyle\int \frac{dx}{\sqrt{(x-\alpha)(\beta-x)}}$ या $\displaystyle\int \sqrt{\frac{x-\alpha}{\beta-x}}\,dx$

$x=\alpha\sin^2 x+\beta\cos^2 x$ लेना है।

Q $\displaystyle\int_2^5 \frac{dx}{\sqrt{-10+7x-x^2}} = \int_2^5 \frac{dx}{\sqrt{(x-2)(5-x)}} = \pi$

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Q $\displaystyle\int_3^8 \frac{\sin\sqrt{x+1}}{\sqrt{x+1}}\,dx$

$\boxed{x+1=t^2}$

$dx = 2t\,dt$

x	t
3	2
8	3

$\displaystyle\int_2^3 \frac{\sin t}{\cancel{t}} \times 2\cancel{t}\,dt$

$-2\left[\cos t\right]_2^3$

$-2\left[\cos 3 - \cos 2\right]$

Q $\displaystyle\int_0^{1/2} \frac{dx}{(1-2x^2)\sqrt{1-x^2}}$

$0 = \sin\theta$

$\sin\theta = 1/2$

$\boxed{x = \sin\theta}$

$dx = \cos\theta \cdot d\theta$

x	θ
0	0
1/2	$\frac{\pi}{6}$

$\displaystyle\int_0^{\pi/6} \frac{\cancel{\cos\theta}\cdot d\theta}{(1-2\sin^2\theta)\cancel{\sqrt{1-\sin^2\theta}}}$

$\displaystyle\int_0^{\pi/6} \frac{d\theta}{\cos 2\theta} = \int_0^{\pi/2} \sec 2\theta \cdot d\theta$

$= \frac{1}{2}\ln\left|\sec 2\theta + \tan 2\theta\right|_0^{\pi/6}$

$= \frac{1}{2}\left\{\ln(2+\sqrt{3}) - \ln(1+0)\right|$

$\frac{1}{2}\ln(2+\sqrt{3})$

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Q $\int_0^1 \sqrt{\frac{x+\sqrt{x^2+1}}{1+x^2}}\,dx$ $\quad \tan\theta = 0$

$\boxed{x = \tan\theta}$

$dx = \sec^2\theta\,d\theta$

x	θ
0	0
1	$\frac{\pi}{4}$

$$I = \int_0^{\pi/4} \sqrt{\frac{\tan\theta + \sec\theta}{(\cancel{1+\tan^2\theta})}} \times \cancel{\sec^2\theta}\,d\theta$$

$$= \int_0^{\pi/4} \sqrt{\sec\theta + \tan\theta}\ \sec\theta\,d\theta$$

$\boxed{\sec\theta + \tan\theta = t^2}$

$\sec\theta(\tan\theta + \sec\theta)\,d\theta = 2t\,dt$

$\sec\theta\,d\theta = \frac{2t\,dt}{t^2}$

$t^2 = \sec 0 + \tan 0 = 1$

$t^2 = \sec\frac{\pi}{4} + \tan\frac{\pi}{4} = \sqrt{2}+1$

θ	t
0	1
$\frac{\pi}{4}$	$\sqrt{\sqrt{2}+1}$

$$= \int_1^{\sqrt{\sqrt{2}+1}} \sqrt{\cancel{t^2}} \times \frac{2\,dt}{\cancel{t}}$$

$$2\left[t\right]_1^{\sqrt{\sqrt{2}+1}} \Rightarrow 2\left[\sqrt{\sqrt{2}+1} - 1\right]$$

Elementry.

Q1 $\int_0^1 \frac{dx}{\sqrt{16+9x^2}} + \int_0^2 \frac{dx}{\sqrt{9+4x^2}}$

$$\frac{1}{3} \ln\left|3x + \sqrt{9x^2+16}\right|\Big|_0^1 + \frac{1}{2}\ln\left|2x + \sqrt{4x^2+9}\right|\Big|_0^2$$

Q $\int x \cdot e^{-x}\,dx$ $\quad$ U.V

Q $I = \int_1^e \frac{1}{\sqrt{x\ln x}} + \int \sqrt{\frac{\ln x}{x}}\,dx$

$$= \int_1^e \frac{1+\ln x}{\sqrt{x\ln x}}\,dx$$

$x\ln x = t^2$

$\left(\frac{x}{x} + \ln x\right) dx = 2t\,dt$

$$\int_0^{\sqrt{e}} \frac{2t\,dt}{\sqrt{t^2}} = 2\left[t\right]_0^{\sqrt{e}}$$

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Q4 $f'(x) = \frac{\cos x}{x}$, $f\left(\frac{\pi}{2}\right) = a$, $f\left(\frac{3\pi}{2}\right) = b$.

$$\int_{\pi/2}^{3\pi/2} f(x)\cdot dx = ?$$

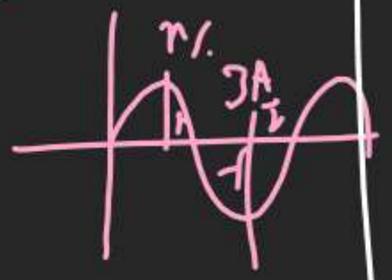
$$\int_{\pi/2}^{3\pi/2} f(x)\cdot 1\cdot dx$$

$$f(x)\cdot\int 1\cdot dx - \int\left(f'(x)\cdot\int 1\cdot dx\right)dx$$

$$x f(x)\Big|_{\pi/2}^{3\pi/2} - \int_{\pi/2}^{3\pi/2} \frac{\cos x}{\cancel{x}} \times \cancel{x}\cdot dx$$

$$\left(\frac{3\pi}{2}\cdot b - \frac{\pi}{2}\cdot a\right) - (\sin x)_{\pi/2}^{3\pi/2}$$

$$-(-1-1)$$



Q $$\int_{-1}^{1} \frac{x+0\,dx}{\sqrt{5-4x}}$$

$$-\frac{1}{4}\int \frac{5-4x}{\sqrt{5-4x}} + \left(0+\frac{5}{4}\right)\int \frac{dx}{\sqrt{5-4x}}$$

$$-\frac{1}{4}\int \sqrt{5-4x} + \frac{5}{4}\int \frac{dx}{\sqrt{5-4x}}$$

$$-\frac{1}{4}\left[\frac{2}{3}\frac{(5-4x)^{3/2}}{-4}\Bigg|_{-1}^{1} + \frac{5}{4}\times\frac{2\sqrt{5-4x}}{-4}\Bigg|_{-1}^{1}\right]$$

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Q $\int_2^e \left(\frac{1}{\ln x} - \frac{1}{\ln^2 x}\right) dx$ Bahut Classic.

$\left(\frac{1}{\ln x}\right)' = -\frac{1}{\ln^2 x} \times \frac{1}{x}$

$\int \frac{1}{\ln x} + \left(\frac{-1}{\ln^2 x} \times \frac{1}{x}\right) \times x \cdot dx$

$\int \left(\underset{u}{f(x)} \cdot \underset{v'}{1} + \underset{u'}{f'(x)} \cdot \underset{v}{x}\right) \cdot dx$

$= x \cdot \frac{1}{\ln x}\Big|_2^e$

$= \frac{e}{\ln e} - \frac{2}{\ln 2}$

$\Rightarrow e - \frac{2}{\ln 2}$

7) $\int \frac{\sin 2x \cdot dx}{\sin^4 x + \cos^4 x}$

$\frac{1}{S^4 + C^4}, \frac{1}{S^6 + C^6}$

$\div C^4 \quad \div C^6$

$\int \frac{2 \tan x \cdot \sec^2 x}{1 + (\tan^2 x)^2}$

$\tan^2 x = t$

Trick

Q8 $\int \frac{\cos x \cdot dx}{(\sin x + 1)(\sin x + 2)}$ diff diff $= \frac{1}{1} \ln\left|\left(\frac{\sin x + 1}{\sin x + 2}\right)\right|$

Q9 $\int_0^{\pi/4} \frac{\sin^2 x \cdot \cos^2 x \cdot dx}{(\sin^3 x + \cos^3 x)^2}$

$\div \cos^6 x$

$\int \frac{\tan^2 x \cdot \sec^2 x \cdot dx}{(1 + \tan^3 x)^2}$ $1 + \tan^3 x = t$

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Q.10 $x = \tan\theta$ & U.V.

Q11 ~~Trick~~ $\int_2^3 \frac{dx}{\sqrt{(x-1)(5-x)}}$

$\int_2^3 \frac{dx}{\sqrt{-5+6x-x^2}}$

$\int_2^3 \frac{dx}{\sqrt{(3)^2-(x-3)^2-5}}$

$\int \frac{dx}{\sqrt{(2)^2-(x-3)^2}} = \sin^{-1}\frac{x-3}{2}\Big|_2^3$

Q12 Repeat $\int_{3/2}^{2} \sqrt{\frac{x-1}{3-x}}\,dx$

$x = 1\cdot\sin^2\theta + 3\cdot\cos^2\theta = 3-2\sin^2\theta$ | $dx = 0-2\sin 2\theta$

$x-1 = 2-2\sin^2\theta = 2\cos^2\theta$ | $3-x = 3-3+2\sin^2\theta = 2\sin^2\theta$

x	θ
$\frac{3}{2}$	$\frac{\pi}{3}$
2	$\frac{\pi}{4}$

$\int_{\pi/3}^{\pi/4} \sqrt{\frac{\cancel{2}\cos^2\theta}{\cancel{2}\cancel{\sin^2\theta}}} \times -2\times 2\cancel{\sin\theta}\cdot\cos\theta\cdot d\theta$

$-4\int_{\pi/3}^{\pi/4} \cos^2\theta\, d\theta = -4\int \frac{1}{2}+\frac{\cos 2\theta}{2}\,d\theta$

$= -4\left[\frac{\theta}{2}+\frac{\sin 2\theta}{4}\right]_{\pi/3}^{\pi/4}$

$= -4\left[\left(\frac{\pi}{8}+\frac{1}{4}\right)-\left(\frac{\pi}{6}+\frac{\sqrt{3}}{8}\right)\right]$

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Class.

Q $\frac{1}{2}\int_0^{\pi/4} x\,(2\cos x\cdot\cos 3x)\cdot dx$

$\frac{1}{2}\int_0^{\pi/4} x\left[\cos 4x+\cos 2x\right]dx$

$\frac{1}{2}\int \underset{U\cdot V}{x\cos 4x\cdot dx} + \frac{1}{2}\int \underset{U\cdot V}{x\cdot\cos 2x\cdot dx}$

Q14 half angle

Q15 $\int \frac{dx}{L\sqrt{Q}}$ $\quad L=\frac{1}{t}$

Q $I=\int_4^5 \underbrace{\sqrt{x+2\sqrt{2x-4}}+\sqrt{x-2\sqrt{2x-4}}}\cdot dx$

$f(x)=\sqrt{x+2\sqrt{2x-4}}+\sqrt{x-2\sqrt{2x-4}}$

$f^2(x)=2x+2\sqrt{x^2-4(2x-4)}$

$=2x+2\sqrt{x^2-8x+16}$

$=2x+2\sqrt{(x-4)^2}$

$f^2(x)=2x+2|x-4| = 4x-8$

$f^2(x)=4(x-2) \Rightarrow f(x)=2\sqrt{x-2}$

$x\in(4,5)$

$x-4\in(0,1)$

$x-4=+ve$

$I=2\int_4^5\sqrt{x-2}\,dx = 2\times\frac{2}{3}(x-2)^{3/2}\Big|_4^5$

$=\frac{4}{3}\left[3\sqrt{3}-2\sqrt{2}\right]$

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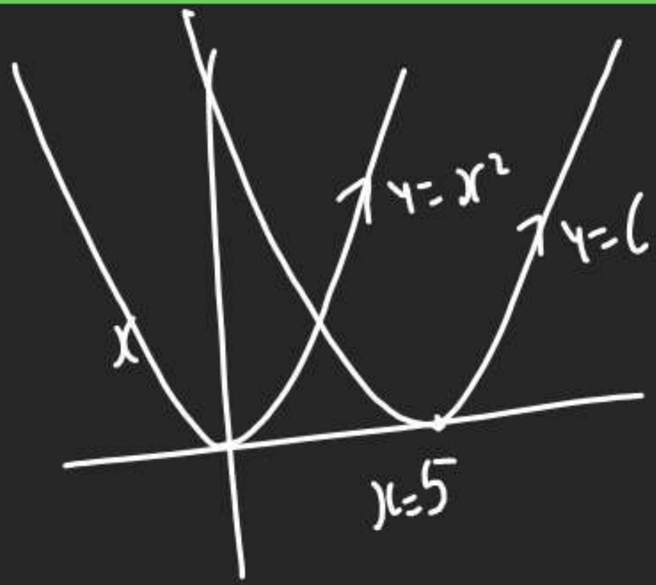
Shifting Property.

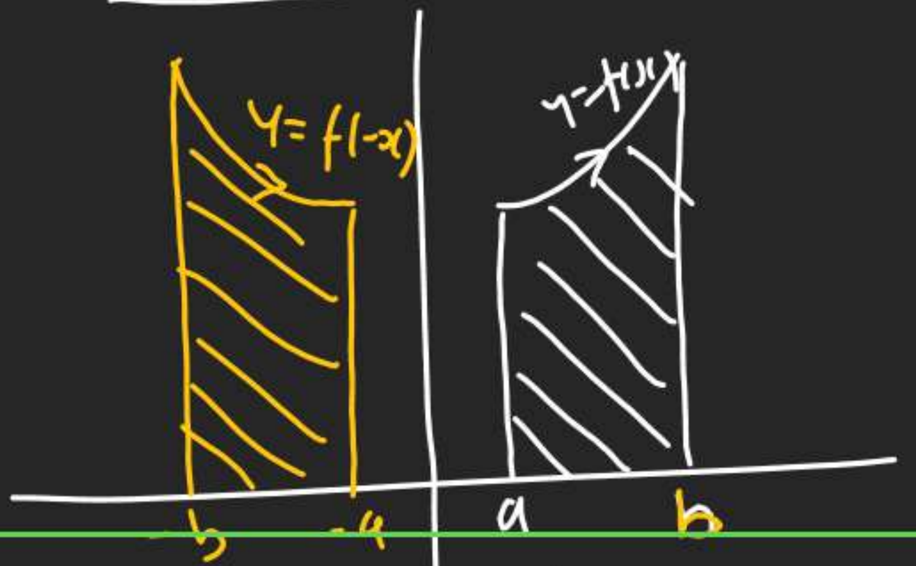
$$\int_a^b f(x)\cdot dx = \int_{a+k}^{b+k} f(x-k)\cdot dx$$



M2

$x-k=t$

Reflecting Property.



$$\int_{-b}^{-a} f(-x)\cdot dx = \int_a^b f(x)\cdot dx$$

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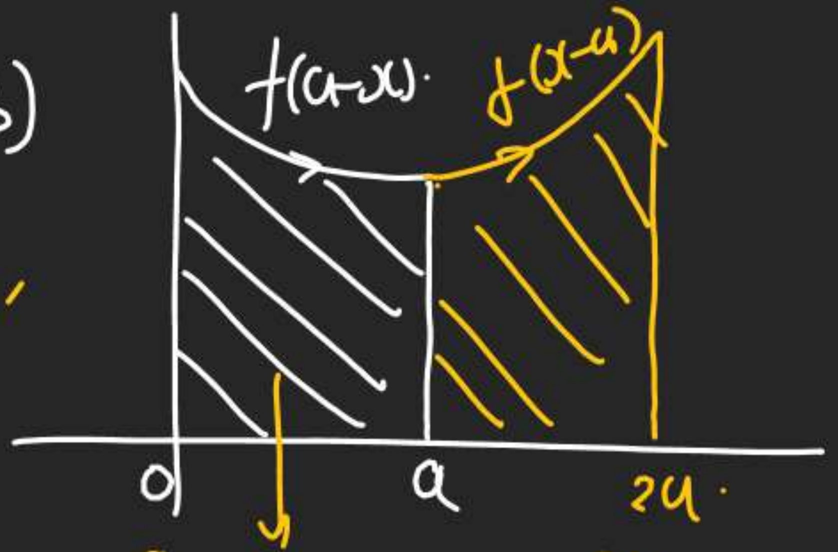
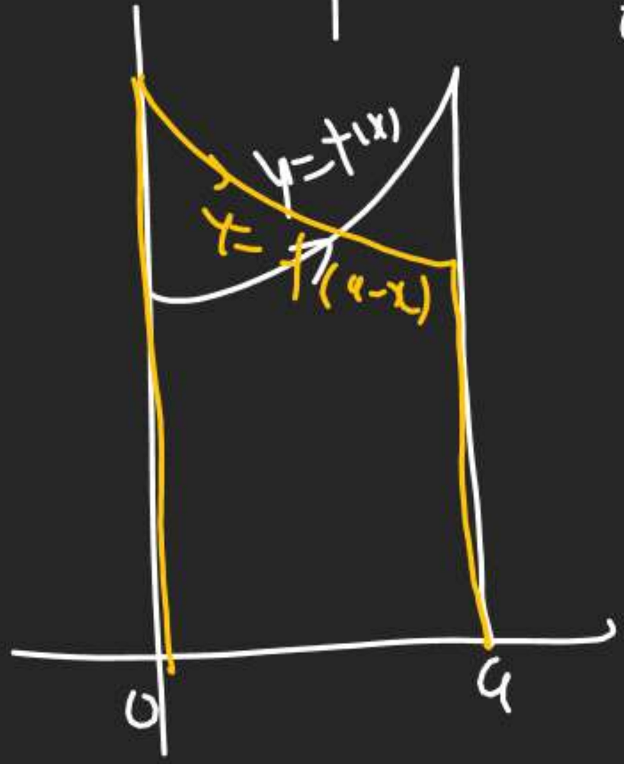


$$\int_0^a f(a-x)\cdot dx = \int_a^{2a} f(x-a)\cdot dx$$



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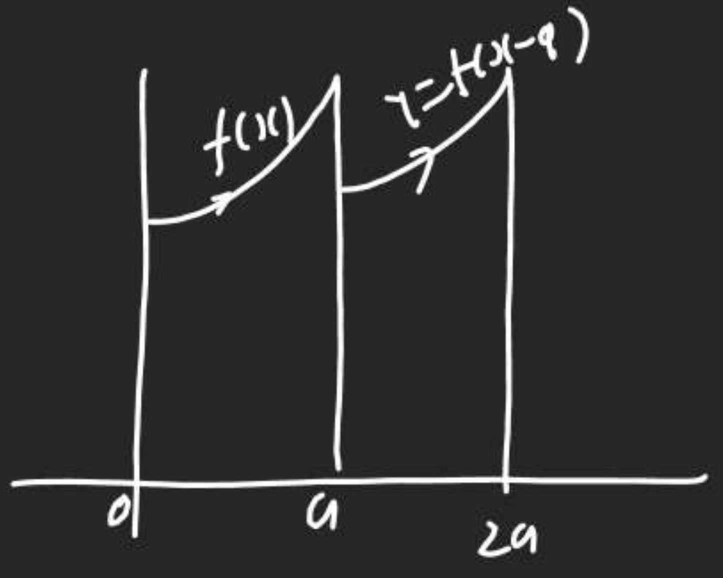

$$\int_0^a f(x)\cdot dx = \int_a^{2a} f(x-a)\, dx$$

H.W.

E.E 15 - 30