

$$Q \quad \sqrt{\sec^2 \theta + \sec^2 \theta} = \tan \theta + \cot \theta$$

$$\sqrt{1 + \tan^2 \theta + 1 + \cot^2 \theta}$$

$$\sqrt{\tan^2 \theta + \cot^2 \theta + 2 \cdot \tan \theta \cdot \cot \theta}$$

$$\sqrt{(\tan \theta + \cot \theta)^2}$$

$$= \tan \theta + \cot \theta = \text{RHS}$$

$$Q \quad (\sec \theta - \sin \theta)(\sec \theta - \cos \theta)(\tan \theta + \cot \theta) = 1$$

$$\left(\frac{1}{s} - s\right) \left(\frac{1}{c} - c\right) \left(\frac{s}{c} + \frac{c}{s}\right)$$

$$\frac{1-s^2}{s} \times \frac{1-c^2}{c} \times \frac{s^2+c^2}{s \cdot c}$$

$$\frac{\cancel{c^2}}{s} \times \frac{\cancel{s^2}}{\cancel{c}} \times \frac{1}{s \cdot c} = 1 \text{ RHS}$$

Q If  $\tan \theta + \sec \theta = \frac{3}{2}$  then  $\sin \theta = ?$

$$\sec \theta + \tan \theta = \frac{3}{2}$$

$$\sec \theta - \tan \theta = \frac{2}{3}$$

$$\frac{2 \sec \theta = \frac{3}{2} + \frac{2}{3} = \frac{9+4}{6}}$$

$$\sec \theta = \frac{13}{12} \Rightarrow \cos \theta = \frac{12}{13}$$

$$\begin{aligned} \sin \theta &= \sqrt{1 - \cos^2 \theta} \\ &= \sqrt{1 - \frac{144}{169}} = \sqrt{\frac{25}{169}} \\ &= \frac{5}{13} \end{aligned}$$

Q If  $2 \sin \theta = 2 - \cos \theta$  then  $\sin \theta = ?$

$$\cos \theta = 2 - 2 \sin \theta$$

$$\cos^2 \theta = 4 + 4 \sin^2 \theta - 8 \sin \theta$$

$$1 - \sin^2 \theta = 4 + 4 \sin^2 \theta - 8 \sin \theta$$

$$5 \sin^2 \theta - 8 \sin \theta + 3 = 0$$

$$5 \sin^2 \theta - 5 \sin \theta - 3 \sin \theta + 3 = 0$$

$$5 \sin \theta (\sin \theta - 1) - 3 (\sin \theta - 1) = 0$$

$$(\sin \theta - 1)(5 \sin \theta - 3) = 0$$

$$\sin \theta = 1, \frac{3}{5}$$

$\sin \theta \in [0, 1]$   
&  
Solve



Q 5  $3 \sec^4 \theta + 8 = 10 \sec^2 \theta$  then  $\tan \theta = ?$

$$\boxed{\sec^2 \theta = T}$$

$$3T^2 + 8 = 10T$$

$$3T^2 - 10T + 8 = 0$$

$$3T^2 - 6T - 4T + 8 = 0$$

$$3T(T-2) - 4(T-2) = 0$$

$$T = \frac{4}{3}, 2$$

$$\sec^2 \theta = \frac{4}{3}$$

$$1 + \tan^2 \theta = \frac{4}{3} \Rightarrow \tan^2 \theta = \frac{1}{3}$$

$$\tan \theta = \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$$

$$\sec^2 \theta = 2$$

$$1 + \tan^2 \theta = 2$$

$$\boxed{\tan^2 \theta = 1} \Rightarrow \tan \theta = 1, -1$$

Q 6

$$\frac{\sin x + \cos x}{\cos^3 x} = \tan^3 x + \tan^2 x + \tan x + 1$$

$$\text{LHS} \Rightarrow \frac{\sin x + \cos x}{\cos^3 x}$$

$$\Rightarrow \left( \frac{1}{\cos^2 x} \right) \times \left( \frac{\sin x + \cos x}{\cos x} \right)$$

$$= \sec^2 x \times (\tan x + 1)$$

$$(1 + \tan^2 x)(\tan x + 1)$$

$$\tan^3 x + \tan^2 x + \tan x + 1 = \text{RHS}$$

# Fundamentals of Mathematics

Q.  $\sin \theta + \sin^3 \theta + \sin^3 \theta = 1$

then  $\sin^6 \theta - 4\sin^4 \theta + 8\sin^2 \theta = ?$

Hint

$$\sin \theta + \sin^3 \theta = 1 - \sin^2 \theta$$

$$\sin \theta + \sin^3 \theta = \cos^2 \theta$$

$$\sin \theta (1 + \sin^2 \theta) = \cos^2 \theta$$

Sol

$$\sin^2 \theta (1 + \sin^2 \theta)^2 = \cos^4 \theta$$

$$(1 - \cos^2 \theta) (1 + 1 - \cos^2 \theta)^2 = \cos^4 \theta$$

$$(1 - \cos^2 \theta) (2 - \cos^2 \theta)^2 = \cos^4 \theta$$

$$(1 - \cos^2 \theta) (4 + \cos^4 \theta - 4\cos^2 \theta) = \cos^4 \theta$$

$$4 + \cancel{\cos^4 \theta} - \underline{4\cos^2 \theta} - \underline{4\cos^2 \theta} - \underline{\cos^6 \theta} + \underline{4\cos^4 \theta} - \cancel{\cos^4 \theta}$$

$$4\cos^4 \theta - \cos^6 \theta - 8\cos^2 \theta = -4$$

$$\underline{\cos^6 \theta} + \underline{8\cos^2 \theta} - 4\cos^4 \theta = \underline{4}$$



Q If  $\frac{\sin^4 x}{2} + \frac{\cos^4 x}{3} = \frac{1}{5}$  then.

A)  $\frac{\sin^8 x}{8} + \frac{\cos^8 x}{27} = \frac{1}{125}$  //

B)  $\tan^2 x = \frac{1}{3}$

C)  $\tan^2 x = \frac{2}{3}$

D)  $\frac{\sin^8 x}{8} + \frac{\cos^8 x}{27} = \frac{2}{125}$

A)  $\frac{\sin^8 x}{8} + \frac{\cos^8 x}{27} = \left(\frac{2}{5}\right)^4 + \left(\frac{3}{5}\right)^4$   
 $\frac{16^2}{625 \times 8} + \frac{81^2}{625 \times 27} = \frac{16}{625} + \frac{81}{625} = \frac{97}{625} \neq \frac{1}{125}$

$$\frac{\sin^4 x}{2} + \frac{(1 - \sin^2 x)^2}{3} = \frac{1}{5}$$

$$\frac{\sin^4 x}{2} + \frac{\sin^4 x - 2\sin^2 x + 1}{3} = \frac{1}{5}$$

$$3\sin^4 x + 2\sin^4 x - 4\sin^2 x + 2 = \frac{6}{5}$$

$$5\sin^4 x - 4\sin^2 x + 2 = \frac{6}{5}$$

$$25\sin^4 x - 20\sin^2 x + 4 = 0$$

$$(5\sin^2 x - 2)^2 = 0$$

$$5\sin^2 x - 2 = 0$$

$$\sin^2 x = \frac{2}{5} \Rightarrow \cos^2 x = 1 - \sin^2 x = 1 - \frac{2}{5} = \frac{3}{5}$$

$$\tan^2 x = \frac{\sin^2 x}{\cos^2 x} = \frac{\frac{2}{5}}{\frac{3}{5}} = \frac{2}{3}$$

$$Q \quad 81^{\sin^2 \theta} + 81^{\frac{62\theta}{9}} = 30$$

find  $\theta = ?$

$$81^{\sin^2 \theta} + 81^{1 - \sin^2 \theta} = 30$$

$$81^{\sin^2 \theta} + \frac{81}{81^{\sin^2 \theta}} = 30$$

$$t + \frac{81}{t} = 30$$

$$t^2 - 30t + 81 = 0$$

$$t^2 - 27t - 3t + 81 = 0$$

$$t(t - 27) - 3(t - 27) = 0$$

$$t = 3, 27$$

$$\rightarrow \textcircled{81}^{\sin^2 \theta} = 3 \quad \& \quad 81^{\sin^2 \theta} = \underline{27}$$

3 या 4 को

$$3^{4 \sin^2 \theta} = 3^1$$

$$4 \sin^2 \theta = 1$$

$$\sin^2 \theta = \frac{1}{4}$$

$$\sin \theta = \frac{1}{2}, -\frac{1}{2}$$

$$\theta = 30^\circ, -30^\circ$$

$$\frac{\pi}{6}, -\frac{\pi}{6}$$

$$3^{4 \sin^2 \theta} = 3^3$$

$$4 \sin^2 \theta = 3$$

$$\sin^2 \theta = \frac{3}{4}$$

$$\sin \theta = \frac{\sqrt{3}}{2}, -\frac{\sqrt{3}}{2}$$

$$\theta = 60^\circ, -60^\circ$$

$$\theta = \frac{\pi}{3}, -\frac{\pi}{3}$$



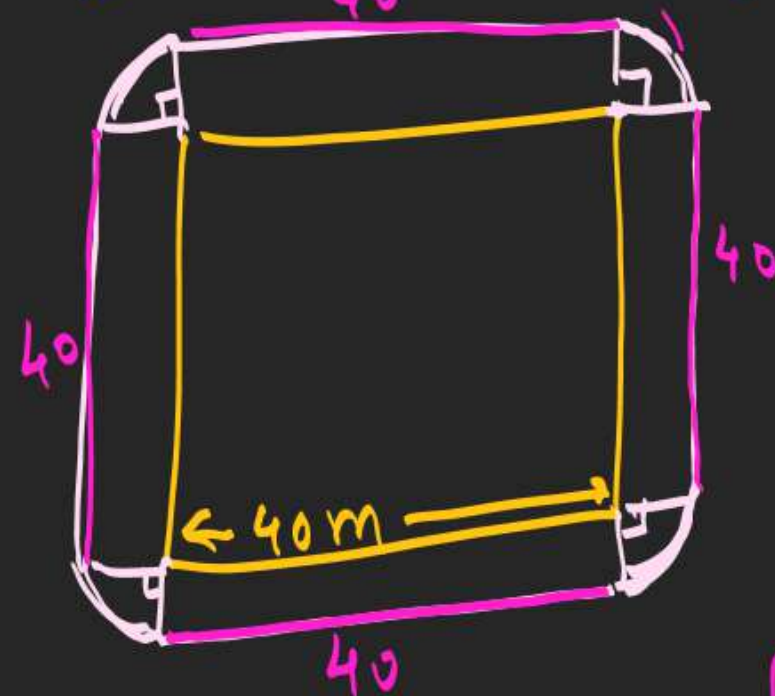
Q Let  $f_k(\theta) = 6m^k\theta + 6^k\theta$

then find value of  $\frac{1}{6}f_6(\theta) - \frac{1}{4}f_4(\theta)$

Q  $\ln m + \ln n = m$ ,  $\ln m - \ln n = n$

then P.T.  $m^2 - n^2 = 4\sqrt{mn}$

Q A garden is in shape of a sq<sup>r</sup> of side length 40m. Now if a man runs around the garden in such a way that his distance from side of sq<sup>r</sup> is 1 meter. How much distance will he travel after 1 Round.



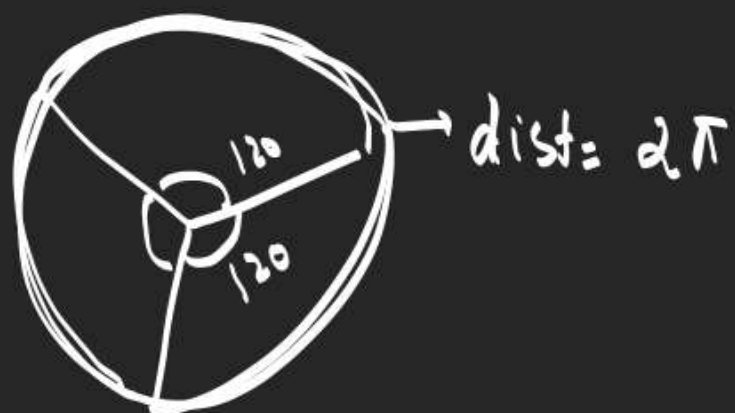
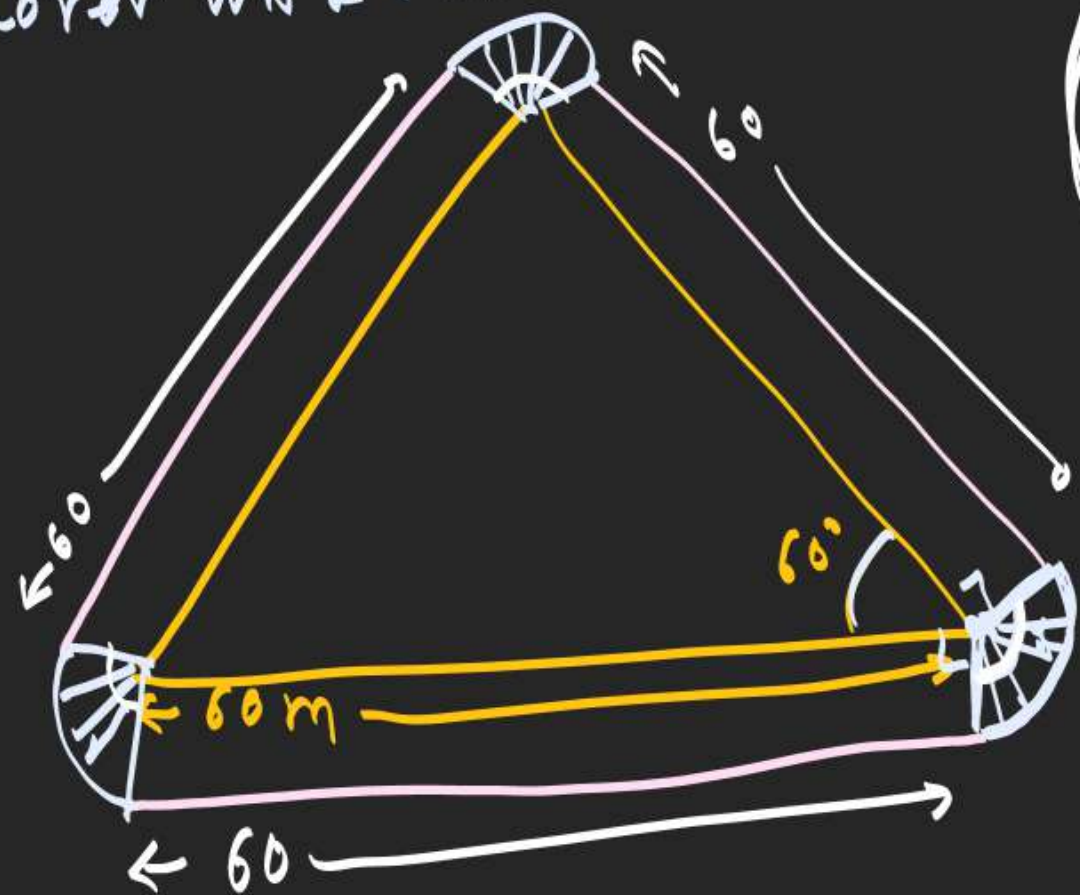
dist =  $2\pi$

$40 + 40 + 40 + 40 + 2\pi$   
 $160 + 2\pi$

Q An eq<sup>t</sup>  $\Delta$  of sides 60m is in the shape of garden. Now if a man runs in such a way that his distance from the side of  $\Delta$  is always  $\perp$  to it. How much distance he will cover in 1 round.

$$\text{Total} = 180 + 2\pi$$

cover in 1 round



dist =  $2\pi$

$$\begin{aligned} x + 90 + 90 + 60 &= 360 \\ x &= 120^\circ \end{aligned}$$



Q If  $\theta \in (0, \frac{\pi}{4})$  then.

$$t_1 = (t \sin \theta)^{\tan \theta}$$

$$t_2 = (t \sin \theta)^{\cot \theta}$$

$$t_3 = (t \cos \theta)^{\tan \theta}$$

$$t_4 = (t \cos \theta)^{\cot \theta}$$

$$t_1 > t_2 > t_3 > t_4$$

$$t_4 > t_3 > t_2 > t_1$$

$$t_3 > t_1 > t_2 > t_4$$

$$t_2 > t_3 > t_1 > t_4$$

$$\theta = (0, 45^\circ)$$

$\theta = 30^\circ$  Assume

$$t_1 = (t \sin 30^\circ)^{\tan 30^\circ} = \left(\frac{1}{\sqrt{3}}\right)^{\frac{1}{\sqrt{3}}}$$

$$t_2 = (t \sin 30^\circ)^{\cot 30^\circ} = \left(\frac{1}{\sqrt{3}}\right)^{\sqrt{3}} = \left(\frac{1}{1.7}\right)^{1.7} = (1.41 \text{ or } 1.414)^{1.7} \text{ Aur chhotu}$$

$$t_3 = (t \cos 30^\circ)^{\tan 30^\circ} = (\sqrt{3})^{\frac{1}{\sqrt{3}}}$$

$$t_4 = (t \cos 30^\circ)^{\cot 30^\circ} = (\sqrt{3})^{\sqrt{3}} = \text{Sabse Bda}$$

# Conversion from Radian to degree

$$\frac{5\pi}{12} \rightarrow \frac{5\pi}{12} \times \frac{180^\circ}{\pi} = 75^\circ$$

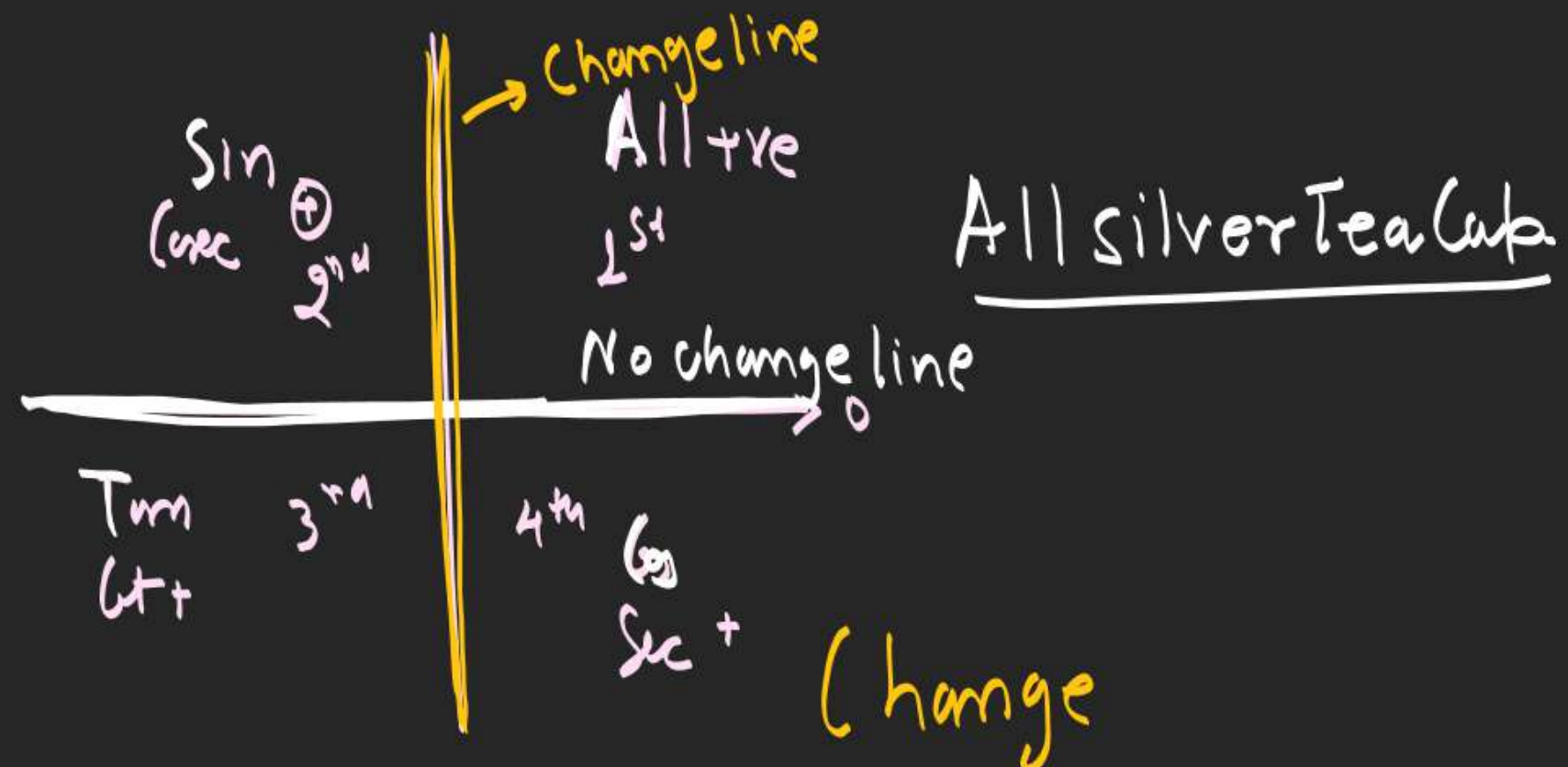
$$\frac{\pi}{10} \rightarrow \frac{\pi}{10} \times \frac{180^\circ}{\pi} = 18^\circ$$

$$\frac{2\pi}{3} \rightarrow \frac{2\pi}{3} \times \frac{180^\circ}{\pi} = 120^\circ$$

$$\frac{4\pi}{5} \rightarrow \frac{4\pi}{5} \times \frac{180^\circ}{\pi} = 144^\circ$$

$$\frac{\pi}{5} \rightarrow \frac{\pi}{5} \times \frac{180^\circ}{\pi} = 36^\circ$$

## Quadrant system.



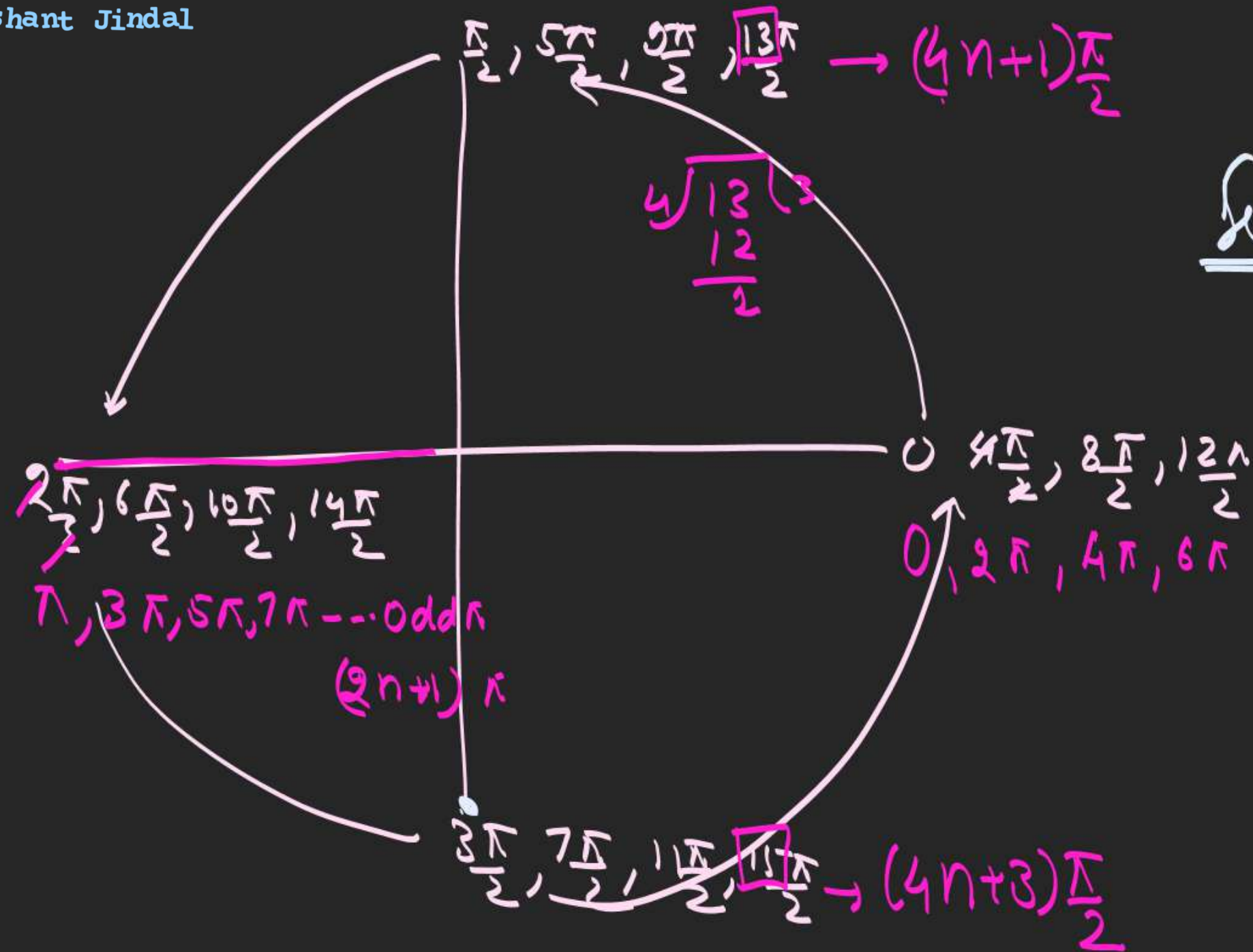
Change

$$\sin \Rightarrow \cos$$

$$\tan \Rightarrow \cot$$

$$\sec \Rightarrow \csc$$





$$\frac{4\sqrt{317}}{28}$$

$$\sin\left(\frac{31\pi}{2} - \theta\right) = -\cos\theta$$

$$\cos\left(\frac{17\pi}{2} + \theta\right)$$

$$\sin(82\pi - \theta)$$

$$\cos(8\pi + \theta)$$

$$\tan\left(\frac{7\pi}{2} - \theta\right)$$

$$\sec\left(\frac{3\pi}{2} + \theta\right)$$