

$$\text{Case I} \quad x \leq -8$$

$$7-x+3x-6-4x-32-x=21$$

$$x = -\frac{52}{3}$$

$$\text{Case II} \quad x \in [0, 2]$$

$$7-x+3x-6+4x+32+x=21$$

$$7x = -12 \quad \times$$

$$\text{Case III} \quad -8 \leq x \leq 0$$

$$7-x+3x-6+4x+32-x=21$$

$$5x = -12$$

$$x = -\frac{12}{5}$$

$$\text{Case IV} \quad 2 \leq x \leq 7$$

$$7-x-3x+6+4x+32+x=21$$

$$x = -24 \quad \times$$

$$\text{Case V} \quad x \geq 7$$

$$x-7-3x+6+4x+32+x=21$$

$$x = -\frac{10}{3} \quad \times$$

$$\text{2: } \quad x \geq -2$$

$$\frac{x+2-x}{x} < 2$$

OR

$$x < -2$$

$$\frac{-x-2-x}{x} < 2$$

$$\begin{cases} x \in (-\infty, 0) \\ \cup (1, \infty) \end{cases}$$

$$\frac{2}{x} - 2 < 0$$

$$\frac{2(1-x)}{x} < 0$$

$$\frac{x-1}{x} > 0$$

$$x \in (-\infty, 0) \cup (1, \infty)$$

$$x \in [-2, 0) \cup (1, \infty)$$

$$\frac{2+2x}{x} + 2 > 0$$

$$\frac{2(1+2x)}{x} > 0$$

$$x \in (-\infty, -\frac{1}{2}) \cup (0, \infty)$$

$$x \in (-\infty, -2)$$

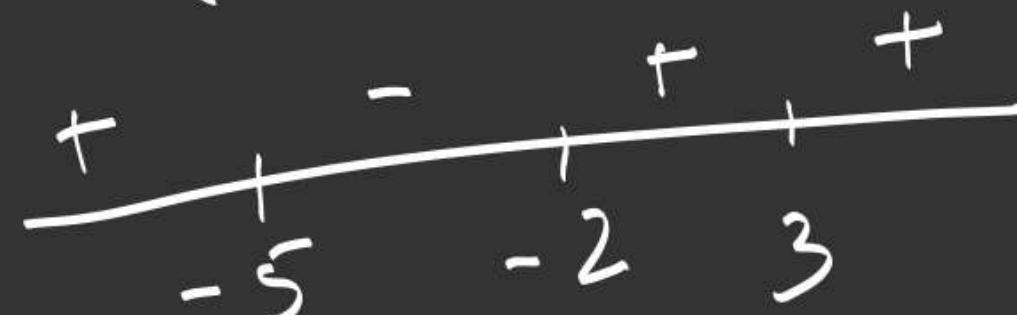
$$\text{3: } x \geq 0$$

$$\frac{x^2 - 7x + 10}{x^2 - 6x + 9} < 0$$

$$\frac{(x-5)(x-2)}{(x-3)^2} < 0$$

OR

$$\frac{(x+2)(x+5)}{(x-3)^2} = \frac{x^2 + 7x + 10}{(x-3)^2} < 0$$



$$x \in (-5, -2) \cup (3, \infty)$$

$$x \in (-5, -2)$$

4:

$$(-\infty, -5) \cup (-4, 1)$$

$$x \in (-5, -2) \cup (2, 3) \cup (3, \infty)$$

Q:  $x \geq 3$

OR

$x < 3$  ✓

$$\frac{x-3}{(x-3)(x-2)} \geq 2$$

$$\frac{1}{x-2} - 2 \geq 0$$

$$\frac{1-2x+4}{x-2} \geq 0$$

$$\frac{2x-5}{x-2} \leq 0$$

$(2, 5/2]$

$x \neq 3$

$x \in \left[\frac{3}{2}, 2\right)$

$$\frac{3-x}{(x-3)(x-2)} \geq 2$$

$$2 + \frac{1}{x-2} \leq 0 \quad x \neq 3$$

$$\frac{2x-3}{x-2} \leq 0$$

$$\left[-\frac{3}{2}, 2\right)$$

$$\underline{6} \quad x > 1$$

$$\frac{x-1}{x+2} - 1 < 0$$

$$\frac{3}{x+2} > 0$$

$$\begin{aligned} (-2, \infty) \\ x \in [1, \infty) \end{aligned}$$

$$x < 1 \quad \checkmark$$

$$1 + \frac{x-1}{x+2} > 0$$

$$\frac{2x+1}{x+2} > 0$$

$$(-\infty, -2) \cup \left(-\frac{1}{2}, \infty\right)$$

$$x \in (-\infty, -2) \cup \left(-\frac{1}{2}, 1\right)$$

$$x \in (-\infty, -2) \cup \left(-\frac{1}{2}, \infty\right)$$

5.

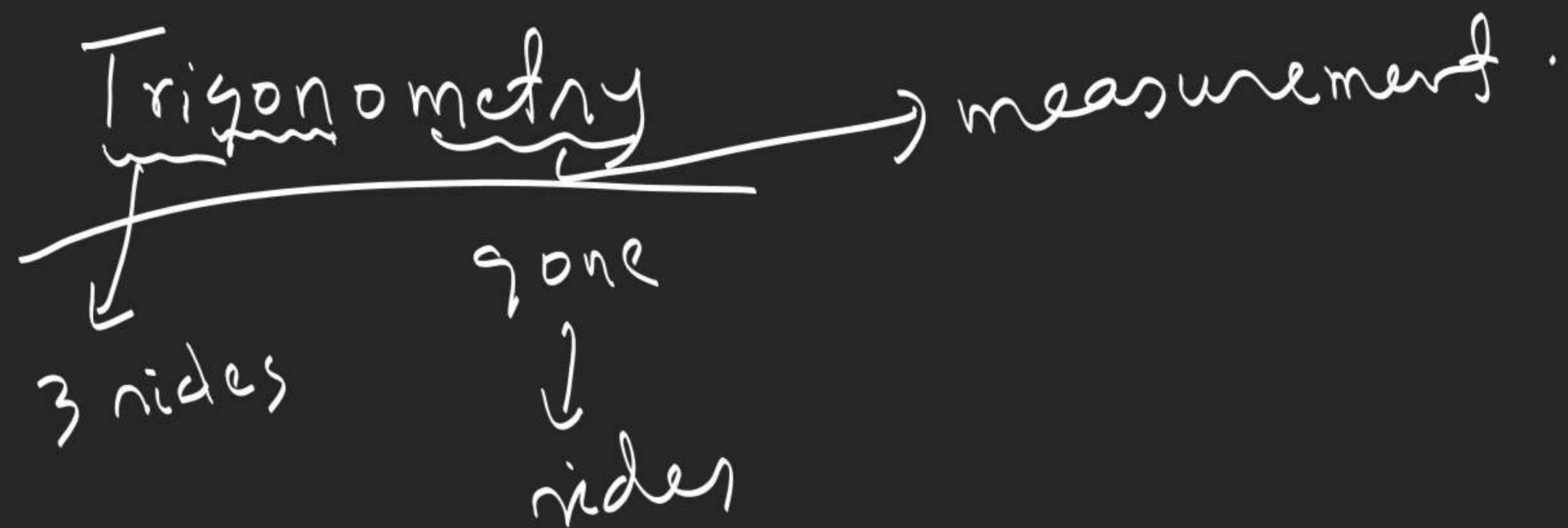
$$\frac{x^2 - 5x + 6}{|x| + 7} < 0$$

$$(x-2)(x-3) < 0$$

S. L. Loney  
Trigonometry

$$x \in (2, 3)$$

Ans.



$$\csc \theta = \frac{h}{P}$$

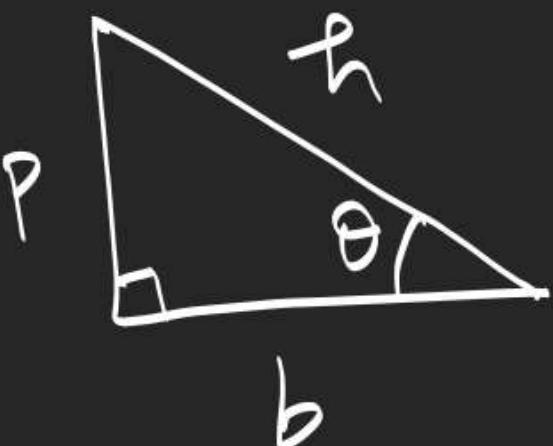
$$\sec \theta = \frac{h}{b}$$

$$\cot \theta = \frac{b}{P}$$

$$\sin \theta = \frac{P}{h}$$

$$\cos \theta = \frac{b}{h}$$

$$\tan \theta = \frac{P}{b}$$



Identity

Equation which holds true for all values  
of parameters whenever defined.

$$x = x \rightarrow \text{Identity}$$

$a^2 + b^2 = c^2$  *not identity*

$$x^2 - 3x + 2 = (x-1)(x-2) \rightarrow \text{Identity}$$

$$(a+b)^2 = a^2 + b^2 + 2ab \rightarrow \text{Identity}$$

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\sec^2 \theta - \tan^2 \theta = 1$$

$$\csc^2 \theta - \cot^2 \theta = 1$$

P.T.

$$\underline{1.} \quad \text{P.T. } (\sec\theta + \csc\theta)(\sin\theta + \cos\theta) = \sec\theta \csc\theta + 2$$

$$\left(\frac{1}{\cos\theta} + \frac{1}{\sin\theta}\right)(\sin\theta + \cos\theta) = \frac{(\sin\theta + \cos\theta)^2}{\sin\theta \cos\theta}$$

$$\text{LHS} = \frac{1 + 2\sin\theta\cos\theta}{\sin\theta\cos\theta} = \frac{(\sin^2\theta + \cos^2\theta) + 2\sin\theta\cos\theta}{\sin\theta\cos\theta} = \frac{\sin\theta\cos\theta + 2\sin\theta\cos\theta}{\sin\theta\cos\theta} = \frac{3\sin\theta\cos\theta}{\sin\theta\cos\theta} = 3$$

$$\underline{2.} \quad (\sec^2\theta + \tan^2\theta)(\csc^2\theta + \cot^2\theta) = 1 + 2\sec^2\theta \csc^2\theta$$

$$\frac{(1 + \sin^2\theta)}{\cos^2\theta} \cdot \frac{(1 + \cos^2\theta)}{\sin^2\theta} = \frac{(1 + \sin^2\theta + \cos^2\theta) + \sin^2\theta \cos^2\theta}{\sin^2\theta \cos^2\theta} = \frac{2 + \sin^2\theta \cos^2\theta}{\sin^2\theta \cos^2\theta} = 2 + \frac{\sin^2\theta \cos^2\theta}{\sin^2\theta \cos^2\theta} = 2 + 1 = 3$$

$$\underline{3.} \quad \tan^2\theta \sin^2\theta = \tan^2\theta - \sin^2\theta$$

$$2\csc^2\theta \sec^2\theta + 1 = \frac{2 + \sin^2\theta \cos^2\theta}{\sin^2\theta \cos^2\theta} = \frac{2 + \sin^2\theta \cos^2\theta}{\sin^2\theta \cos^2\theta} = 2 + \frac{\sin^2\theta \cos^2\theta}{\sin^2\theta \cos^2\theta} = 2 + 1 = 3$$

$$\tan^2 \theta \sin^2 \theta = \tan^2 \theta - \sin^2 \theta$$

$$\cot^2 \theta \cos^2 \theta = \cot^2 \theta - \cos^2 \theta$$

$$\begin{aligned}\tan^2 \theta \sin^2 \theta &= \tan^2 \theta (1 - \cos^2 \theta) \\ &= \tan^2 \theta - \sin^2 \theta\end{aligned}$$

$$\begin{aligned}\cot^2 \theta - \cos^2 \theta &= \cot^2 \theta (1 - \sin^2 \theta) \\ &= \cot^2 \theta \cos^2 \theta\end{aligned}$$

4:

$$\frac{\cot\theta + \cosec\theta - 1}{\cot\theta - \cosec\theta + 1} = \cot\theta + \cosec\theta$$

$$\frac{\cot\theta + \cosec\theta - (\cosec^2\theta - \cot^2\theta)}{\cot\theta - \cosec\theta + 1} = \frac{(\cosec\theta + \cot\theta)(\cosec\theta - \cot\theta)}{(\cosec\theta + \cot\theta)(1 - (\cosec\theta - \cot\theta))}$$

$$= \cosec\theta + \cot\theta$$

5:

$$\left( \frac{1 + \sin\alpha}{1 + \cos\alpha} \right) \left( \frac{1 + \sec\alpha}{1 + \cosec\alpha} \right) = \tan\alpha$$

$$\left( \frac{1 + \sin\alpha}{1 + \cos\alpha} \right) \left( \frac{1 + \frac{1}{\cos\alpha}}{1 + \frac{1}{\sin\alpha}} \right) = \left( \frac{1 + \sin\alpha}{1 + \cos\alpha} \right) \left( \frac{1 + \cos\alpha}{1 + \sin\alpha} \right) \frac{\sin\alpha}{\cos\alpha} = \tan\alpha$$

$$\text{L.H.S.} \frac{\sin x + \cos x}{\cos^3 x} = \tan^3 x + \tan^2 x + \tan x + 1$$

$$\begin{aligned}\frac{\sin x}{\cos x} \frac{1}{\cos^2 x} + \frac{1}{\cos^2 x} &= \tan x \sec^2 x + \sec^2 x \\ &= \tan x (1 + \tan^2 x) + 1 + \tan^2 x \\ &= \tan^3 x + \tan x + \tan^2 x + 1\end{aligned}$$

2. Simplify

$$\csc^2 A \cot^2 A - \sec^2 A \tan^2 A - (\cot^2 A - \tan^2 A)$$

$$(\sec^2 A \csc^2 A - 1)$$

P.T.

$$\underline{3:} \quad \frac{\sin A}{1 + \cos A} + \frac{1 + \cos A}{\sin A} = 2 \csc A$$

$$\underline{4:} \quad \cos^6 A + \sin^6 A = 1 - 3 \sin^2 A \cos^2 A$$

$$\underline{5:} \quad \sin^8 A - \cos^8 A = (\sin^2 A - \cos^2 A)(1 - 2 \sin^2 A \cos^2 A)$$

$$2. \csc^2 A \cot^2 A - \sec^2 A \tan^2 A - (\cot^2 A - \tan^2 A)(\sec^2 A \cosec^2 A - 1)$$

$$= \frac{\cos^2 A}{\sin^4 A} - \frac{\sin^2 A}{\cos^4 A} - \frac{(\cos^4 A - \sin^4 A)}{\sin^2 A \cos^2 A} \frac{(1 - \sin^2 A \cos^2 A)}{\sin^2 A \cos^2 A}$$

$$= \frac{\cos^6 A - \sin^6 A - (\cos^4 A - \sin^4 A)(1 - \sin^2 A \cos^2 A)}{\sin^4 A \cos^4 A}$$

$$\frac{\cos^6 A - \sin^6 A - (\cos^4 A - \sin^4 A)(1 - \sin^2 A \cos^2 A)}{\sin^4 A \cos^4 A} = \frac{(\sin^2 A + \cos^2 A)^2 \cancel{(\cos^4 A - \sin^4 A)}}{\sin^4 A \cos^4 A} = \cancel{(\cos^4 A - \sin^4 A)}$$

$$\frac{(\cos^6 A - \sin^6 A) - (\cos^4 A - \sin^4 A)(\sin^4 A + \cos^4 A + \sin^2 A \cos^2 A)}{\sin^6 A \cos^6 A}$$

Inq:

sec 1.5

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