

1. a. Prove that $\sin 65^\circ + \cos 65^\circ = \sqrt{2}\cos 20^\circ$.
b. Prove that $\sin 47^\circ + \cos 77^\circ = \cos 17^\circ$.
2. Prove that $\cos 80^\circ + \cos 40^\circ - \cos 20^\circ = 0$.
3. Prove that $\sin 10^\circ + \sin 20^\circ + \sin 40^\circ + \sin 50^\circ = \sin 70^\circ + \sin 80^\circ$.
4. Prove that $\cos \frac{\pi}{5} + \cos \frac{2\pi}{5} + \cos \frac{6\pi}{5} + \cos \frac{7\pi}{5} = 0$.
5. If $\sin \alpha - \sin \beta = \frac{1}{3}$ and $\cos \beta - \cos \alpha = \frac{1}{2}$, show that $\cot \frac{\alpha+\beta}{2} = \frac{2}{3}$.
6. If $\operatorname{cosec} A + \sec A = \operatorname{cosec} B + \sec B$, prove that $\tan A \tan B = \cot \frac{A+B}{2}$.
7. Prove that $\sin 25^\circ \cos 115^\circ = \frac{1}{2}(\sin 40^\circ - 1)$.
8. If $\cos A = \frac{3}{4}$, then find the value of $32\sin\left(\frac{A}{2}\right)\sin\left(\frac{5A}{2}\right)$.
9. If $x \cos \theta = y \cos\left(\theta + \frac{2\pi}{3}\right) = z \cos\left(\theta + \frac{4\pi}{3}\right)$, prove that $xy + yz + zx = 0$.
10. If $y \sin \phi = x \sin(2\theta + \phi)$, show that $(x + y) \cot(\theta + \phi) = (y - x) \cot \theta$.
11. If $\cos(A + B) \sin(C + D) = \cos(A - B) \sin(C - D)$, prove that $\cot A \cot B \cot C = \cot D$.
12. If $\tan(A + B) = 3 \tan A$, prove that
 - a. $\sin(2A + B) = 2 \sin B$
 - b. $\sin 2(A + B) + \sin 2A = 2 \sin 2B$

Answer Key

8. 11