

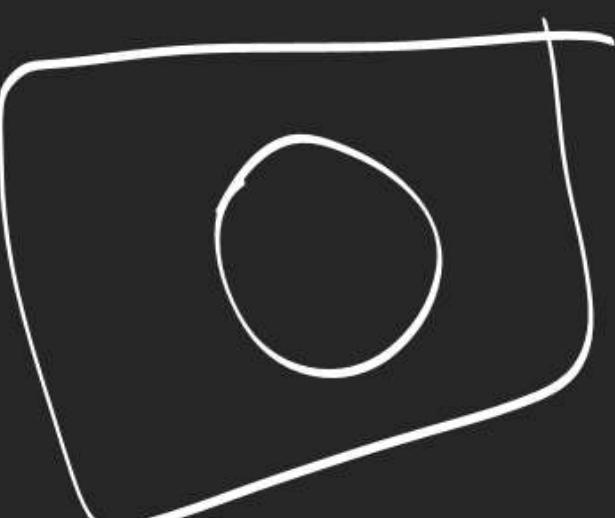
3. (iv) ~~$\cos 24^\circ + \cos 55^\circ + \cos 125^\circ + \cos 204^\circ + \cos 300^\circ$~~

~~$\sin 20^\circ + \cos 50^\circ + \tan 80^\circ$~~

$= 0$

$= \cos 60^\circ$

$360 - 60$



(vi) $\sin\left(1560 - 1440\right)$

+ $\cos(-3030 + 2880)$

+ $\tan(-1260 + 1080)$

$= \frac{1}{2}$

4.

$$2\sin^2 x + 2\sin x + \sin x + 1 = 0$$

$$\alpha \in (0, \pi/2) \\ \sin \alpha = \frac{1}{3}$$

$$(2\sin x + 1)(\sin x + 1) = 0$$

$$\sin x = -\frac{1}{2}, -1$$



$$\begin{aligned} & 180^\circ + 30^\circ, \\ & 360^\circ - 30^\circ \\ & \pi + \alpha, 2\pi - \alpha \\ & \pi + \beta, 2\pi - \beta \end{aligned}$$

$$\beta \in (0, \pi/2) \\ \sin \beta = \frac{1}{4}$$

$$\frac{\pi}{6}, \frac{11\pi}{6}, \frac{3\pi}{2}$$

$$\gamma \in (0, \pi) \\ \sin \gamma = -\frac{1}{3}$$

$$(3\sin x + 1)(4\sin x + 1) = 0$$

$$\frac{(1+t^2)}{(1-t^2)} > 1 \\ 6\pi \quad \left(\frac{1+t^2}{1-t^2} \right) > 1$$

Find sum of all values satisfying

$$[0, 2\pi] \\ Q10 \rightarrow \text{case}$$

$$\therefore \frac{\sin A(\cos(B+C)) + \cos A \sin(B+C)}{\cos A \cos B \cos C}$$

$$= \frac{\sin A \cos B \cos C - \sin A \sin B \sin C + \cos A \sin B \cos C + \cos A \sin C \cos B}{\cos A \cos B \cos C}$$

$$\sin(A+B) = \frac{3}{5} \times \left(-\frac{12}{13}\right) + \cdot \frac{5}{13} \left(-\frac{4}{5}\right)$$

$\therefore \cot(A-B) = \frac{1 + \tan A \tan B}{\tan A - \tan B} = \frac{1 + \left(-\frac{3}{4}\right)}{\frac{5}{13}}$

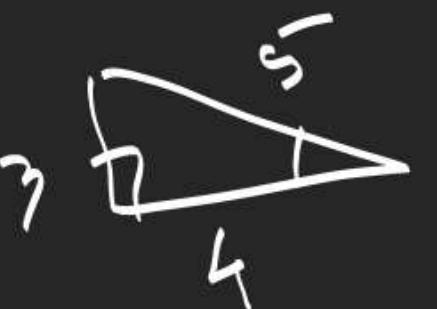
$$\frac{\tan A - \tan B}{\tan A \tan B - 1} = \gamma$$



$$\begin{aligned}
 & \frac{\left| \sec \frac{\pi}{12} \right|}{\tan \frac{\pi}{12}} - \frac{\left| \csc \frac{\pi}{12} \right|}{\cot \frac{\pi}{12}} \\
 & = \frac{1}{\cos \frac{\pi}{12}} - \frac{-2 \left(\cos \frac{\pi}{12} - \sin \frac{\pi}{12} \right)}{2 \sin \frac{\pi}{12} \cos \frac{\pi}{12}} \\
 & = \frac{1}{\cos 22^\circ} - \frac{2 \left(\frac{\sqrt{3}+1}{2\sqrt{2}} - \frac{\sqrt{3}-1}{2\sqrt{2}} \right)}{2 \sin 68^\circ} \\
 & = \frac{\cos 22^\circ}{\sin 34^\circ \cos 22^\circ} = \frac{2 \cos 22^\circ}{\sin 68^\circ} = 2
 \end{aligned}$$

$$\underline{12.} \quad \tan \frac{x}{2} = \frac{1 - \cos x}{\sin x} = \frac{1 - \left(-\frac{4}{5}\right)}{-\frac{3}{5}} = -3.$$

$\tan x =$



$$a \sin x + b \cos x$$

$$f(x) = \sin x + \sqrt{3} \cos x = 2 \left(\frac{1}{2} \sin x + \frac{\sqrt{3}}{2} \cos x \right) = 2 \sin \left(x + \frac{\pi}{3} \right)$$

↓ ↓

$\cos \frac{\pi}{3}$ $\sin \frac{\pi}{3}$

find range

$$R_f = \begin{bmatrix} -2 & 2 \end{bmatrix}$$

$$\Omega_f = R$$

Q. Find the domain and range of

$$(i) f(x) = \frac{3\sin x - 4\cos x + 15}{10}$$

$$\boxed{R_f = [1, 2] \\ D_f = \mathbb{R}}$$

$$10 = \sqrt{5^2 + 3^2} \leq \sqrt{3\sin x - 4\cos x + 15} \leq \sqrt{15+5^2} = 20$$

$$(ii) f(x) = \sin^2\left(\frac{15\pi}{8} - 4x\right) - \sin^2\left(\frac{17\pi}{8} - 4x\right)$$

$$= \frac{\sin\left(\frac{\pi}{8}\right)}{\sqrt{2}} \sin\left(\frac{3\pi}{8}\right) - \frac{\sin\left(-\frac{3\pi}{8}\right)}{\sqrt{2}} \sin\left(\frac{5\pi}{8}\right) - 8x$$

$$\boxed{D_f = \mathbb{R} \\ R_f = \left[-\frac{1}{2}, \frac{1}{2}\right]}$$

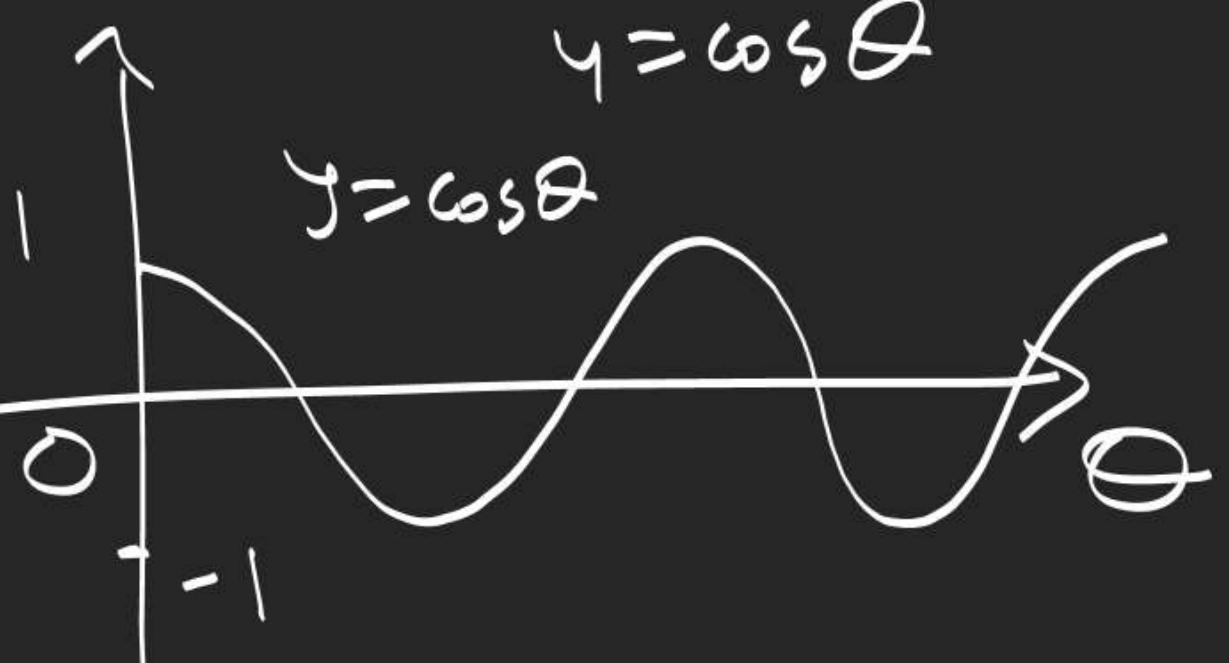
Q. $f(x) = \cos(x^2)$, find Domain & Range

$$D_f = \mathbb{R}$$

$$R_f = [-1, 1]$$

$$x^2 \in [0, \infty)$$

$$\theta = x^2$$



$$f(x) = \tan(x^2)$$

$$\theta = x^2 > 0$$

$$y = \tan \theta$$

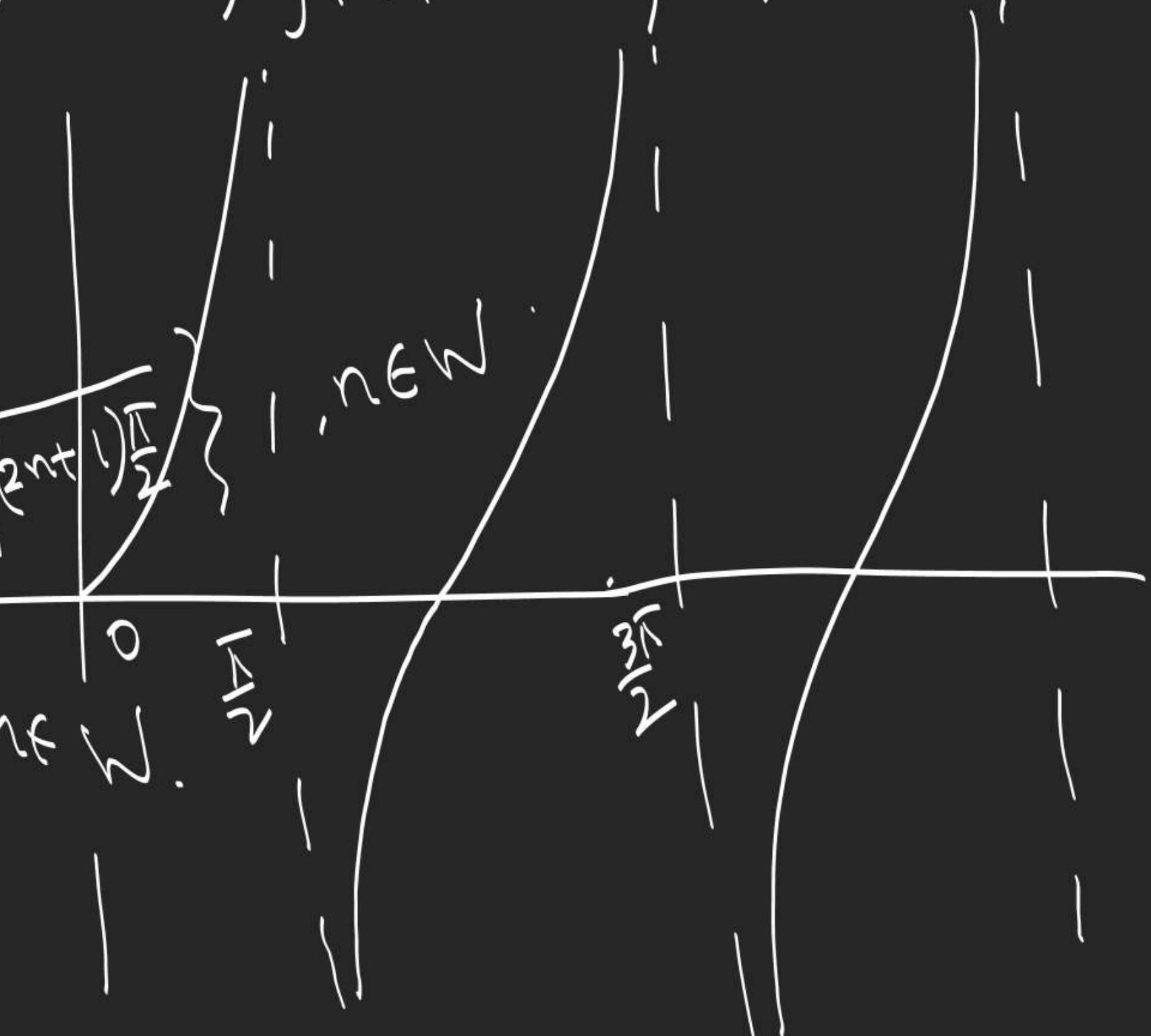
$$R_f = R$$

$$D_x = R - \left\{ \pm \sqrt{(2n+1)\frac{\pi}{2}} \right\}, n \in \mathbb{Z}$$

$$x^2 = (2n+1)\frac{\pi}{2}$$

$$\frac{x^2}{k} = \frac{\pi}{2}$$

, find range & domain



2: Find domain & range of

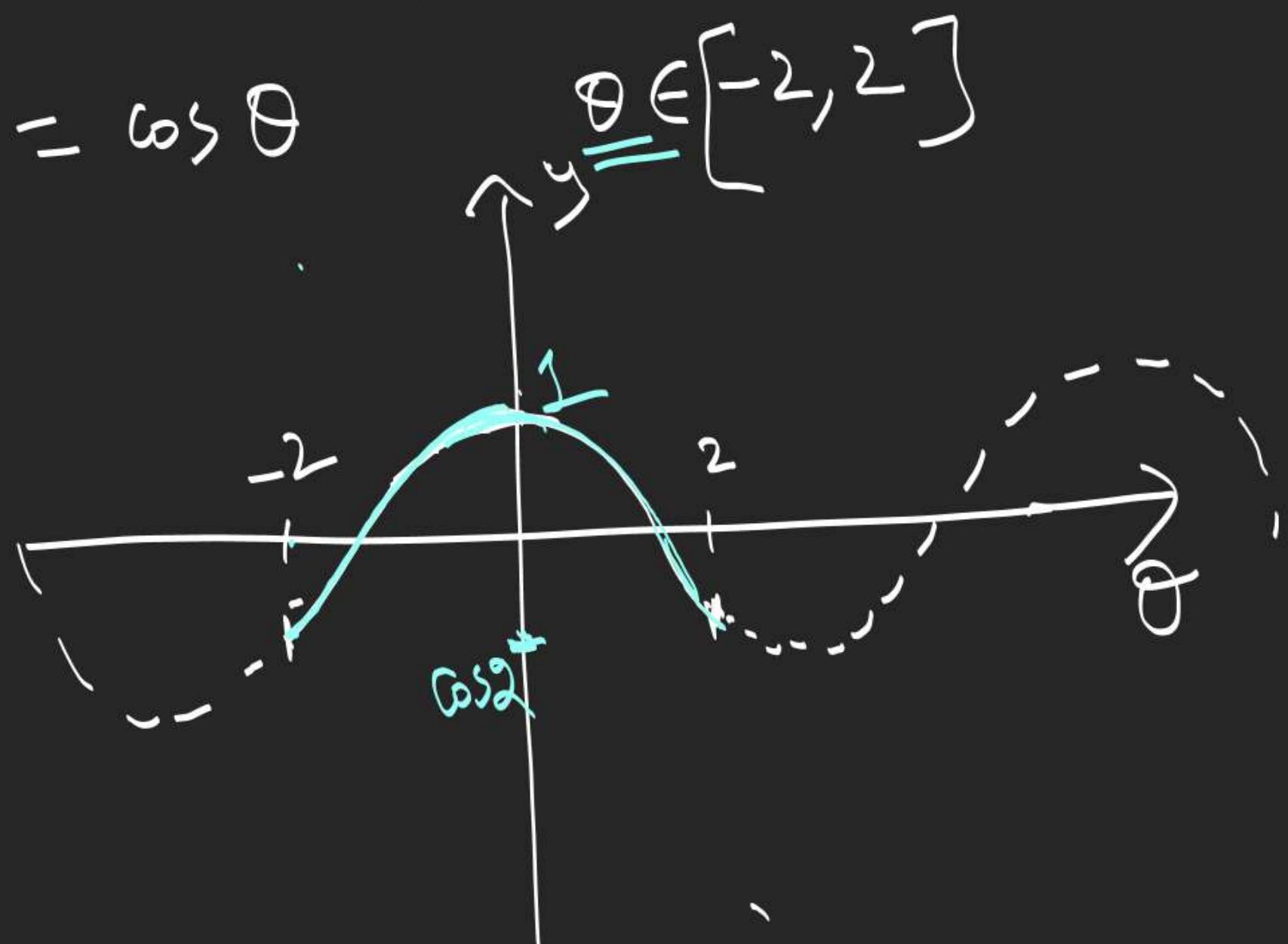
$$f(x) = \cos(2\sin x)$$

$$D_f = \mathbb{R}$$

$$f(\theta) = \cos \theta$$

$$\theta \in [-2, 2]$$

$$Q_f = [\cos 2, 1] \checkmark$$



Find domain & range of

$$1. \quad f(x) = 3 \cos\left(x + \frac{\pi}{3}\right) + 5 \cos x + 3$$

$$D_x = \mathbb{R} \quad [-4, 10] = 3\left(\frac{1}{2} \cos x - \frac{\sqrt{3}}{2} \sin x\right) + 5 \cos x + 3$$

$$R_f = \left[3 - 7, 3 + 7 \right] = \frac{13}{2} \cos x - \frac{3\sqrt{3}}{2} \sin x + 3$$

$$-7 = -\sqrt{\frac{169}{4} + \frac{27}{4}} \leq \frac{13}{2} \cos x - \frac{3\sqrt{2}}{2} \sin x \leq \sqrt{\frac{169}{4} + \frac{27}{4}} = 7$$

$$\therefore f(x) = \sin\left(x + \frac{\pi}{3}\right) + 3 \cos\left(x - \frac{\pi}{3}\right)$$

$$= \frac{1}{2} \sin x + \frac{\sqrt{3}}{2} \cos x + 3 \left(\frac{1}{2} \cos x + \frac{\sqrt{3}}{2} \sin x \right)$$

$$= \left(\frac{3+\sqrt{3}}{2} \right) \cos x + \left(\frac{3\sqrt{3}+1}{2} \right) \sin x$$

$$\frac{9+3+6\sqrt{3}+27+1}{+6\sqrt{3}} \in \left[-\sqrt{\frac{(3+\sqrt{3})^2}{4} + \frac{(3\sqrt{3}+1)^2}{4}}, \sqrt{\frac{(3+\sqrt{3})^2}{4} + \frac{(3\sqrt{3}+1)^2}{4}} \right]$$

$$R_f = \left[-\sqrt{10+3\sqrt{3}}, \sqrt{10+3\sqrt{3}} \right]$$

$$\therefore f(x) = \frac{\cos^2 x - 4 \cos x + 13}{\cos^2 x - 2(2) \cos x + 4} = \frac{(\cos x - 2)^2 + 9}{R_f = [10, 18]} \quad -1 \leq \cos x \leq 1$$

$$-3 \leq \cos x - 2 \leq -1$$

$$\therefore f(x) = \cos 2x + 3 \sin x \Rightarrow 1 \leq (\cos x - 2)^2 \leq 9$$

$$-1 < x < 2 \Rightarrow 0 \leq x^2 \leq 4 \leq (\cos x - 2)^2 + 9 \leq 9 + 9$$

$$-3 < x < -1$$

$$-3 < x < 2$$

$$x^2 \in [0, 9]$$

$$-1 < x < 3$$

$$\Rightarrow -1 < x < 9$$

$$-1 < x^2 < 9$$

$$f(x) = \cos 2x + 3 \sin x$$

$$= 1 - 2 \sin^2 x + 3 \sin x$$

$$= 1 - 2 \left(\sin x - \frac{3}{2} \sin x \right)$$

$$= 1 - 2 \left(\left(\sin x - \frac{3}{4} \right)^2 - \frac{9}{16} \right) = \frac{17}{8} - 2 \left(\sin x - \frac{3}{4} \right)^2$$

$$-\frac{7}{4} = -1 - \frac{3}{4} \leq \sin x - \frac{3}{4} \leq 1 - \frac{3}{4} = \frac{1}{4}$$

$$0 \leq \left(\sin x - \frac{3}{4} \right)^2 \leq \frac{49}{16}$$

$$-\frac{49}{16} \leq -2 \left(\sin x - \frac{3}{4} \right)^2 \leq 0$$

$$D_f = R$$

$$R_f = \left[\frac{17}{8} - \frac{49}{8}, \frac{17}{8} + 0 \right] = \left[-\frac{49}{8}, \frac{17}{8} \right]$$

Ex-I (Complete)