# PARABOLA

## EXERCISE - 1

## OBJECTIVE PROBLEMS | JEE MAIN

## SECTION - A

## QUESTIONS BASED ON BASIC DEFINITION & PARAMETRIC REPRESENTATION

**1.** Latus rectum of the parabola whose focus is (3,4) and whose tangent at vertex has the equation $x + y = 7 + 5\sqrt{2}$ is

(A) 5 (B) 10 (C) 20 (D) 15

**2.** Directrix of a parabola is $x + y = 2$. If it's focus is origin, then latus rectum of the parabola is equal to

(A) $\sqrt{2}$ units (B) 2 units (C) $2\sqrt{2}$ units (D) 4 units

**3.** Which one of the following equations represents parametrically, parabolic profile?

(A) $x = 3\cos t; y = 4\sin t$ (B) $x^2 - 2 = -\cos t; y = 4\cos^2 \frac{t}{2}$

(C) $\sqrt{x} = \tan t; \sqrt{y} = \sec t$ (D) $x = \sqrt{1 - \sin t}; y = \sin \frac{t}{2} + \cos \frac{1}{2}$

**4.** The point of intersection of the curves whose parametric equations are $x = t^2 + 1, y = 2t$ and $x = 2s, y = 2/s$ is given by

(A) $(1, -3)$ (B) (2,2) (C) $(-2,4)$ (D) (1,2)

**5.** If the line $x - 1 = 0$ is the directrix of the parabola $y^2 - kx + 8 = 0$, then one of the values of ' k ' is

(A) 1/8 (B) 8 (C) 4 (D) ¼

**6.** Let P be the point (1,0) and Q a point on the locus $y^2 = 8x$. The locus of mid point of PQ is

(A) $y^2 - 4x + 2 = 0$ (B) $y^2 + 4x + 2 = 0$

(C) $x^2 + 4x + 2 = 0$ (D) $x^2 - 4y + 2 = 0$

**7.** PN is an ordinate of the parabola $y^2 = 4ax$. A straight line is drawn parallel to the axis to bisect NP and meets the curve in Q. NQ meets the tangent at the vertex in a point T such that $AT = kNP$, then the value of k is (where A is the vertex)

(A) 3/2 (B) 2/3 (C) 1 (D) none

**8.** The locus of the mid-point of the line segment joining the focus to a moving point on the parabola $y^2 = 4ax$ is another parabola with directrix

(A) $x = -a$ (B) $x = -a/2$ (C) $x = 0$ (D) $x = a/2$

## SECTION - B

### POSITION OF POINT & CHORD JOINING TWO POINTS

**9.** If $(t^2, 2t)$ is one end of a focal chord of the parabola $y^2 = 4x$ then the length of the focal chord will be

(A) $\left(t + \frac{1}{t}\right)^2$ (B) $\left(t + \frac{1}{t}\right)\sqrt{\left(t^2 + \frac{1}{t^2}\right)}$

(C) $\left(t - \frac{1}{t}\right)\sqrt{\left(t^2 + \frac{1}{t^2}\right)}$ (D) none

## SECTION - C

### TANGENT TO PARABOLA

**10.** Locus of the point of intersection of the perpendicular tangents of the curve $y^2 + 4y - 6x - 2 = 0$ is

(A) $2x - 1 = 0$ (B) $2x + 3 = 0$

(C) $2y + 3 = 0$ (D) $2x + 5 = 0$

**11.** Two common tangents to the circle $x^2 + y^2 = 2a^2$ and parabola $y^2 = 8ax$ are

(A) $x = \pm(y + 2a)$ (B) $y = \pm(x + 2a)$

(C) $x = \pm(y + a)$ (D) $y = \pm(x + a)$

**12.** The tangents to the parabola $x = y^2 + c$ from origin are perpendicular then c is equal to

(A) 1/2 (B) 1 (C) 2 (D) ¼

**13.** TP&TQ are tangents to the parabola, $y^2 = 4ax$ at P&Q. If the chord PQ passes through the fixed point $(-a, b)$ then the locus of T is

(A) $ay = 2b(x - b)$ (B) $bx = 2a(y - a)$

(C) $by = 2a(x - a)$ (D) $ax = 2\,b(y - b)$

## SECTION - D & E

### NORMALTO PARABOLA/SUBTANGENT & NORMAL

**14.** PQ is a normal chord of the parabola $y^2 = 4ax$ at P, A being the vertex of the parabola. Through P a line is drawn parallel to AQ meeting the x-axis in R. Then the length of AR is

(A) equal to the length of the latus rectum

(B) equal to the focal distance of the point P.

(C) equal to twice the focal distance of the point P.

(D) equal to the distance of the point P from the directrix

**15.** The normal at the point $(bt_1{}^2, 2bt_1)$ on a parabola meets the parabola again in the point $(b_2{}^2, 2bt_2)$, then

(A) $t_2 = -t_1 - \frac{2}{t_1}$ (B) $t_2 = -t_1 + \frac{2}{t_1}$ (C) $t_2 = t_1 - \frac{2}{t_1}$ (D) $t_2 = t_1 + \frac{2}{t_1}$

**16.** Locus of the intersection of the tangents at the ends of the normal chords of the parabola $y^2 = 4ax$ is

(A) $(2a + x)y^2 + 4a^3 = 0$ (B) $(x + 2a)y^2 + 4a^2 = 0$

(C) $(x + 2a)y^2 + 4a^2 = 0$ (D) none

**17.** Length of the normal chord of the parabola, $y^2 = 4x$, which makes an angle of $\frac{\pi}{4}$ with the axis of x is

(A) 8 (B) $8\sqrt{2}$ (C) 4 (D) $4\sqrt{2}$

**18.** If $x + y = k$ is normal to $y^2 = 12x$, then ' k ' is

(A) 3 (B) 9 (C) -9 (D) -3

## SECTION - F

## CHORD OF CONTACT, LENGTH, CHORD WITH A GIVEN MID POINT

**19.** Tangents are drawn from the points on the line $x - y + 3 = 0$ to parabola $y^2 = 8x$. Then the variable chords of contact pass through a fixed point whose coordinates are

(A) (3,2) (B) (2,4) (C) (3,4) (D) (4,1)

**20.** The line $4x - 7y + 10 = 0$ intersects the parabola, $y^2 = 4x$ at the points A & B. The co-ordinates of the point of intersection of the tangents drawn at the points A&B are

(A) $\left(\frac{7}{2}, \frac{5}{2}\right)$ (B) $\left(-\frac{5}{2}, \frac{7}{2}\right)$ (C) $\left(\frac{5}{2}, \frac{7}{2}\right)$ (D) $\left(-\frac{7}{2}, \frac{5}{2}\right)$

## SECTION - G

## HIGHLIGHTS & BASED RESULTS

**21.** From the point (4,6) a pair of tangent lines are drawn to the parabola, $y^2 = 8x$. The area of the triangle formed by these pair of tangent lines & the chord of contact of the point (4,6) is

(A) 2 (B) 4 (C) 8 (D) none

**22.** Let PSQ be the focal chord of the parabola, $y^2 = 8x$. If the length of $SP = 6$ then, l(SQ) is equal to(where S is the focus)

(A) 3 (B) 4 (C) 6 (D) none

## SECTION - H

## MIXED PROBLEMS

**23.** The equation of the circle drawn with the focus of the parabola $(x - 1)^2 - 8y = 0$ as its centre and touching the parabola at its vertex is

(A) $x^2 + y^2 - 4y = 0$ (B) $x^2 + y^2 - 4y + 1 = 0$

(C) $x^2 + y^2 - 2x - 4y = 0$ (D) $x^2 + y^2 - 2x - 4y + 1 = 0$

**24.** The slope of the focal chords of the parabola $y^2 = 16x$ which are tangents to the circle $(x-6)^2 + y^2 = 2$ are

(A) \pm 2 (B) $-1/2, 2$ (C) \pm 1 (D) $-2, 1/2$

**25.** The equation of the common tangent touching the circle $(x-3)^2 + y^2 = 9$ and the parabola $y^2 = 4x$ above the x-axis is

(A) $\sqrt{3}y = 3x + 1$ (B) $\sqrt{3}y = -(x+3)$ (C) $\sqrt{3}y = x + 3$ (D) $\sqrt{3}y = -(3x+1)$

**26.** AB, AC are tangents to a parabola $y^2 = 4ax \cdot p_1 p_2$ and $p_3$ are the lengths of the perpendiculars from A, B and C respectively on any tangent to the curve, then $p_2, p_1, p_3$ are in

(A) A.P. (B) G.P. (C) H.P. (D) none of these

**27.** A point on the parabola $y^2 = 18x$ at which the ordinate increases at twice the rate of the abscissa is

(A) (2,4) (B) $(2, -4)$ (C) $\left(-\frac{9}{8}, \frac{9}{2}\right)$ (D) $\left(\frac{9}{8}, \frac{9}{2}\right)$

**28.** If the tangent at the point $P(x_1, y_1)$ to the parabola $y^2 = 4ax$ meets the parabola $y^2 = 4a(x+b)$ at Q& R, then the mid point of QR is

(A) $(x_1 + b, y_1 + b)$ (B) $(x_1 - b, y_1 - b)$ (C) $(x_1, y_1)$ (D) $(x_1 + b, y_1)$

## EXERCISE - 2 (LEVEL-I)

### OBJECTIVE PROBLEMS | JEE MAIN

### TANGENT TO PARABOLA

**1.** The locus of a point such that two tangents drawn from it to the parabola $y^2 = 4ax$ are such that the slope of one is double the other is

(A) $y^2 = \frac{9}{2}ax$ (B) $y^2 = \frac{9}{4}ax$ (C) $y^2 = 9ax$ (D) $x^2 = 4\,ay$

**2.** The equation of the common tangent to the curves $y^2 = 8x$ and $xy = -1$ is

(A) $3y = 9x + 2$ (B) $y = 2x + 1$ (C) $2y = x + 8$ (D) $y = x + 2$

### NORMAL TO PARABOLA / SUBTANGENT & NORMAL

**3.** If the tangents and normals at the extremities of a focal chord of a parabola intersect at $(x_1, y_1)$ and $(x_2, y_2)$ respectively, then

(A) $x_1 = x_2$ (B) $x_1 = y_2$ (C) $y_1 = y_2$ (D) $x_2 = y_1$

**4.** The equation of a straight line passing through the point $(3,6)$ and cutting the curve $y = \sqrt{x}$ orthogonally is

(A) $4x + y - 18 = 0$ (B) $x + y - 9 = 0$ (C) $4x - y - 6 = 0$ (D) none

### HIGHLIGHTS & BASED RESULTS

**5.** T is a point on the tangent to a parabola $y^2 = 4ax$ at its point P. TL and TN are the perpendiculars on the focal radius SP and the directrix of the parabola respectively. Then

(A) $SL = 2(TN)$ (B) $3(SL) = 2(TN)$ (C) $SL = TN$ (D) $2(SL) = 3(TN)$

### MIXED PROBLEMS

**6.** From the focus of the parabola $y^2 = 8x$ as centre, a circle is described so that a common chord of the curves is equidistant from the vertex and focus of the parabola. The equation of the circle is

(A) $(x-2)^2 + y^2 = 3$ (B) $(x-2)^2 + y^2 = 9$

(C) $(x+2)^2 + y^2 = 9$ (D) none

**7.** The straight line joining any point P on the parabola $y^2 = 4ax$ to the vertex and perpendicular from the focus to the tangent at P, intersect at R, then the equation of the locus of R is

(A) $x^2 + 2y^2 - ax = 0$ (B) $2x^2 + y^2 - 2ax = 0$

(C) $2x^2 + 2y^2 - ay = 0$ (D) $2x^2 + y^2 - 2ay = 0$

**8.** The tangent and normal at P(t), for all real positive t, to the parabola $y^2 = 4ax$ meet the axis of the parabola in T and G respectively, then the angle at which the tangent at P to the parabola is inclined to the tangent at P to the circle through the points P, T and G is

(A) $\cot^{-1} t$ (B) $\cot^{-1} t^2$ (C) $\tan^{-1} t$ (D) $\sin^{-1}\left(\frac{t}{\sqrt{1+t^2}}\right)$

**9.** Through the vertex O of the parabola, $y^2 = 4ax$ two chords OP and OQ are drawn and the circles on OP and OQ as diameter intersect in R. If $q_1, q_2$ and f are the angles made with the axis by the tangent at P and Q on the parabola and by OR then the value of $\cot q_1 + \cot q_2$ equals

(A) $-2\tan f$ (B) $-2\tan(p - f)$ (C) 0 (D) $2\cot f$

**10.** Two parabolas $y^2 = 4a(x - 1_1)$ and $x^2 = 4a(y - 1_2)$ always touch one another, the quantities $1_1$ and $l_2$ are both variable. Locus of their point of contact has the equation

(A) $xy = a^2$ (B) $xy = 2a^2$ (C) $xy = 4a^2$ (D) none

## EXERCISE - 2 (LEVEL-II)

### MULTIPLE CORRECT | JEE ADVANCED

### QUESTIONS BASED ON BASIC DEFINITION & PARAMETRIC REPRESENTATION

**1.** Let A be the vertex and L the length of the latus rectum of parabola, $y^2 - 2y - 4x - 7 = 0$. The equation of the parabola with point A as vertex, 2 L as the length of the latus rectum and the axis at right angles to that of the given curve is

(A) $x^2 + 4x + 8y - 4 = 0$ (B) $x^2 + 4x - 8y + 12 = 0$

(C) $x^2 + 4x + 8y + 12 = 0$ (D) $x^2 + 8x - 4y + 8 = 0$

**2.** The parametric coordinates of any point on the parabola $y^2 = 4ax$ can be

(A) $(at^2, 2at)$ (B) $(at^2, -2at)$

(C) $(a\sin^2 t, 2a\sin t)$ (D) (asint, 2acost)

**3.** The locus of the mid point of the focal radii of a variable point moving on the parabola, $y^2 = 4ax$ is a parabola whose

(A) latus rectum is half the latus rectum of the original parabola

(B) vertex is (a/2,0)

(C) directrix is y-axis

(D) focus has the co-ordinates $(a, 0)$

### POSITION OF POINT & CHORD JOINING TWO POINTS

**4.** Two parabolas have the same focus. If their directrices are the x-axis & the y-axis respectively, then the slope of their common chord is

(A) 1 (B) -1 (C) 4/3 (D) 3/4

### CHORD OF CONTACT, LENGTH, CHORD WITH A GIVEN MID POINT

**5.** The length of the chord of the parabola $y^2 = x$ which is bisected at the point (2,1) is less than

(A) $5\sqrt{2}$ (B) $4\sqrt{5}$ (C) $4\sqrt{50}$ (D) $2\sqrt{5}$

### HIGHLIGHTS & BASED RESULTS

**6.** Tangent to the parabola $y^2 = 4ax$ at point P meets the tangents at vertex A at point B and the axis of parabola at T, Q is any point on this tangent and N as the foot of perpendicular from Q on SP, where S is focus, M is the foot of perpendicular from Q on the directrix then

(A) B bisects PT (B) B trisects PT (C) QM = SN (D) QM = 2SN

### MIXED PROBLEMS

**7.** A variable circle is described to passes through the point (1,0) and tangent to the curve $y = \tan(\tan^{-1} x)$. The locus of the centre of the circle is a parabola whose

(A) length of the latus rectum is $2\sqrt{2}$ (B) axis of symmetry has the equation $x + y = 1$

(C) vertex has the co-ordinates (3/4,1/4) (D) none of these

## EXERCISE - 3

### | SUBJECTIVE | JEE ADVANCED

**1.** ' O ' is the vertex of the parabola $y^2 = 4ax$&L is the upper end of the latus rectum. If LH is drawn perpendicular to OL meeting OX in H, prove that the length of the double ordinate through H is $4a\sqrt{5}$.

**2.** Through the vertex O of a parabola $y^2 = 4x$, chords OP&OQ are drawn at right angles to one another. Show that for all positions of P, PQ cuts the axis of the parabola at a fixed point. Also find the locus of the middle point of PQ.

**3.** Find the equations of the chords of the parabola $y^2 = 4ax$ which pass through the point $(-6a, 0)$ and which subtends an angle of $45°$ at the vertex.

**4.** Find the equations of the tangents to the parabola $y^2 = 16x$, which are parallel & perpendicular respectively to the line $2x - y + 5 = 0$. Find also the coordinates of their points of contact.

**5.** Find the equations of the tangents of the parabola $y^2 = 12x$, which passes through the point $(2,5)$.

**6.** Through the vertex O of the parabola $y^2 = 4ax$, a perpendicular is drawn to any tangent meeting it at P & the parabola at Q. Show that $OP \cdot OQ = \text{constant}$ .

**7.** Let P be a point on the parabola $y^2 - 2y - 4x + 5 = 0$, such that the tangent on the parabola at P intersects the directrix at point Q. Let R be the point that divides the line segment PQ externally in the ratio $\frac{1}{2}: 1$. Find the locus of R.

**8.** Two tangents to the parabola $y^2 = 8x$ meet the tangent at its vertex in the points P&Q. If $PQ = 4$ units, prove that the locus of the point of the intersection of the two tangents is $y^2 = 8(x + 2)$.

**9.** Show that the normals at the points $(4a, 4a)$& at the upper end of the latus rectum of the parabola $y^2 = 4ax$ intersect on the same parabola.

**10.** In the parabola $y^2 = 4ax$, the tangent at the point P, whose abscissa is equal to the latus ractum meets the axis in T& the normal at P cuts the parabola again in Q. Prove that PT: $PQ = 4: 5$.

**11.** Prove that the locus of the middle point of portion of a normal to $y^2 = 4ax$ intercepted between the curve & the axis is another parabola. Find the vertex & the latus rectum of the second parabola.

**12.** Three normals to $y^2 = 4x$ pass through the point $(15,12)$. Show that if one of the normals is given by $y = x - 3$& find the equations of the others.

**13.** Normals are drawn from the point ' P ' with slopes $m_1$, $m_2$, $m_3$ to the parabola $y^2 = 4x$. If locus of P with $m_1 m_2 = a$ is a part of the parabola itself then find a.

**14.** Prove that, the normal to $y^2 = 12x$ at $(3,6)$ meets the parabola again in $(27, -18)$& circle on this normal chord as diameter is $x^2 + y^2 - 30x + 12y - 27 = 0$.

**15.** P & Q are the points of contact of the tangents drawn from the point T to the parabola $y^2 = 4ax$. If PQ be the normal to the parabola at P, prove that TP is bisected by the directrix.

**16.** A variable chord PQ of the parabola $y^2 = 4x$ is drawn parallel to the line $y = x$. If the parameters of the points P&Q on the parabola are p&q respectively, show that $p + q = 2$. Also show that the locus of the point of intersection of the normals at P&Q is $2x - y = 12$.

**17.** Show that the normals at two suitable distinct real points on the parabola $y^2 = 4ax(a > 0)$ intersect at a point on the parabola whose abscissa $> 8a$.

**18.** Let S is the focus of the parabola $y^2 = 4ax$ and X the foot of the directrix, PP ' is a double ordinate of the curve and PX meets the curve again in Q. Prove that P'Q passes through focus.

**19.** Prove that the parabola $y^2 = 16x$ and the circle $x^2 + y^2 - 40x - 16y - 48 = 0$ meet at the point P(36,24) and one other point Q. Prove that PQ is a diameter of the circle. Find Q.

**20.** Find the equation of the circle which passes through the focus of the parabola $x^2 = 4y$ & touches it at the point (6,9).

**21.** A fixed parabola $y^2 = 4ax$ touches a variable parabola. Find the equation to the locus of the vertex of the variable parabola. Assume that the two parabolas are equal and the axis of the variable parabola remains parallel to the x axis.

**22.** Show that an infinite number of triangles can be inscribed in either of the parabolas $y^2 = 4ax$& $x^2 = 4$ by whose sides touch the other.

**COMPREHENSION**

From the point $P(h, k)$ three normals are drawn to the p rabola $x^2 = 8y$ and $m_1, m_2$ and $m_3$ are the slopes of thre normal

**23.** Find the algebraic sum of the slopes of these thre normals.

**24.** If two of the three normals are at right angles the the locus of point P is a conic, find the latus rectu of conic.

**25.** If the two normals from P are such that they mak complementary angles with the axis then the loct of point P is a conic, find a directrix of conic.

**MATRIX MATCH TYPE**

**26.**

| **Column – I** | **Column - II** |
|---|---|
| (A) The x-coordinate of points on the axis of the parabola $y^2 - 4x - 2y + 5 = 0$ from which all the three normals to the parabola are real is | (P) 4 |
| (B) The x-coordinate of points on the axis of the parabola $4y^2 - 32x + 4y = 65 = 0$ from which all the three normals to the parabola are real is | (Q) 5 |
| (C) The x-coordinate of points on the axis of the parabola $4y^2 - 16x - 4y + 41 = 0$ from which all the three normals to the parabola are real is | (R) 6<br>(S) 7<br>(T) 8 |

## EXERCISE - 4

### | LEVEL-I PREVIOUS YEAR | JEE MAIN

**1.** The locus of the vertices of the family of parabolas $y = \frac{a^3x^2}{3} + \frac{a^2x}{2} - 2a$ is **[AIEEE 2006]**

(A) $xy = \frac{105}{64}$ (B) $xy = 3/4$ (C) $xy = \frac{35}{16}$ (D) $xy = \frac{64}{105}$

**2.** The equation of a tangent to the parabola $y^2 = 8x$ is $y = x + 2$. The point on this line from which the other tangent to the parabola is parapendicular to the given tangent is **[AIEEE 2007]**

(A) $(-1,1)$ (B) $(0,2)$ (C) $(2,4)$ (D) $(-2,0)$

**3.** A parabola has the origin as its focus and the line $x = 2$ as the directrix. Then the vertex of the parabola is at **[AIEEE 2008]**

(A) $(0,2)$ (B) $(1,0)$ (C) $(0,1)$ (D) $(2,0)$

**4.** If two tangents drawn from a point P to the parabola $y^2 = 4x$ are at right angles, then the locus of P is **[AIEEE 2010]**

(A) $x = 1$ (B) $2x + 1 = 0$

(C) $x = -1$ (D) $2x - 1 = 0$

**5.** Given: A circle, $2x^2 + 2y^2 = 5$ and a parabola, $y^2 = 4\sqrt{5}x$.

Statement - I: An equation of a common tangent to these curves is $y = x + \sqrt{5}$.

Statement - II: If the line, $y = mx + \frac{\sqrt{5}}{m}(m \neq 0)$ is their common tangent, then m satisfies $m^4 - 3m^2 + 2 = 0$,

(A) If Statement-I is true but Statement - II is false.

(B) If Statement-I is false but Statement-II is true.

(C) If both Statement - I and Statement - II are true, and Statement - II is the correct explanation of Statement- I.

(D) If both Statement-I and Statement - II are true but Statement - II is not the correct explanation of Statement-I. **[AIEEE 2013]**

**6.** The slope of the line touching both the parabolas $y^2 = 4x$ and $x^2 = -32y$ is : **[AIEEE 2014]**

(A) $\frac{1}{2}$ (B) $\frac{3}{2}$

(C) $\frac{1}{8}$ (D) $\frac{2}{3}$

**7.** Let O be the vertex and Q be any point on the parabola, $x^2 = 8y$. If the point P divides the line segment OQ internally in the ratio 1: 3, then the locus of P is **[AIEEE 2015]**

(A) $y^2 = 2x$ (B) $x^2 = 2y$

(C) $x^2 = y$ (D) $y^2 = x$

**8.** Let P be the point on the parabola, $y^2 = 8x$ which is at a minimum distance from the centre C of the circle, $x^2 + (y + 6)^2 = 1$. Then the equation of the circle, passing through C and having its centre at P is: **[JEE MAIN 2016]**

(A) $x^2 + y^2 - x + 4y - 12 = 0$ (B) $x^2 + y^2 - \frac{x}{4} + 2y - 24 = 0$

(C) $x^2 + y^2 - 4x + 9y + 18 = 0$ (D) $x^2 + y^2 - 4x + 8y + 12 = 0$

**9.** The centres of those circle which touch the circle, $x^2 + y^2 - 8x - 8y - 4 = 0$, externally and also touch the x-axis, lie on: **[JEE MAIN 2016]**

(A) an ellipse which is not a circle (B) a hyperbola

(C) a parabola (D) a circle

**10.** If the tangent at (1,7) to the curve $x^2 = y - 6$ touches the circle $x^2 + y^2 + 16x + 12y + c = 0$ then the value of c is : **[JEE MAIN 2018]**

(A) 95 (B) 195 (C) 185 (D) 85

**11.** Tangent and normal are drawn at P(16,16) on the parabola $y^2 = 16x$, which intersect the axis of the parabola at A and B, respectively. If C is the centre of the circle through the points P, A and B and $\angle CPB = \theta$, then a value of $\tan\theta$ is : **[JEE MAIN 2018]**

(A) $\frac{4}{3}$ (B) $\frac{1}{2}$ (C) 2 (D) 3

## EXERCISE - 4

### | LEVEL-II PREVIOUS YEAR | JEE ADVANCED

**1.(a)** The axis of parabola is along the line $y = x$ and the distance of vertex from origin is $\sqrt{2}$ and that of origin from its focus is $2\sqrt{2}$. If vertex and focus both lie in the $1^{st}$ quadrant, then the equation of the parabola is **[JEE 2006,3]**

(A) $(x+y)^2 = (x-y-2)$ (B) $(x-y)^2 = (x+y-2)$

(C) $(x-y)^2 = 4(x+y-2)$ (D) $(x-y)^2 = 8(x+y-2)$

**(b)** The equations of common tangents ot the parabola $y = x^2$ and $y = -(x-2)^2$ is/are

(A) $y = 4(x-1)$ (B) $y = 0$ **[JEE 2006, 5]**

(C) $y = -4(x-1)$ (D) $y = -30x - 50$

**(c)** **Match The Following** **[JEE 2006, 6]**

Normals are drawn at point P, Q and R lying on the parabola $y^2 = 4x$ which intersect at (3,0). Then

| | |
|---|---|
| (i) Area of $\triangle$ PQR | (A) 2 |
| (ii) Radius of circumcircle of $\triangle$ PQR | (B) 5/2 |
| (iii) Centroid of $\triangle$ PQR | (C) (5/2,0) |
| (iv) Circumcentre of $\triangle$ PQR | (D) (2/3,0) |

**2.** Statement-1: The curve $y = \frac{-x^2}{2} + x + 1$ is symmetric with respect to the line $x = 1$.

Statement -2: A parabola is symmetric about its axis. **[JEE 2007, 4]**

(A) Statement- 1 is true, statement- 2 is true ; statement- 2 is correct explanation for statement-1.

(B) Statement- 1 is true, statement- 2 is true; statement- 2 is NOT a correct explanation for statement-1.

(C) Statement-1 is true, statement- 2 is false

(D) Statement-1 is false, statement- 2 is true

**Comprehension**

**3.** Consider the circle $x^2 + y^2 = 9$ and the parabola $y^2 = 8x$. They intersect at P and Q in the first and the fourth quadrants, respectively. Tangents to the circle at P and Q intersect the X-axis at R and tangents to the parabola at P and Q intersect the X-axis at S. **[JEE 2007]**

**(a)** The ratio of the areas of the triangles PQS and PQR is

(A) $1:\sqrt{2}$ (B) 1: 2 (C) 1: 4 (D) 1: 8

**(b)** The radius of the circumcircle of the triangle PRS is

(A) 5 (B) $3\sqrt{3}$ (C) $3\sqrt{2}$ (D) $2\sqrt{3}$

**(c)** The radius of the incircle of the triangle PQR is

(A) 4 (B) 3 (C) 8/3 (D) 2

**4.** The tangent PT and the normal PN to the parabola $y^2 = 4ax$ at a point P on it meet its axis at points T and N, respectively. The locus of the centroid of the triangle PTN is a parabola whose

(A) vertex is $(2a/3, 0)$

(B) directrix is $x = 0$

(C) latusrectum is $2a/3$

(D) focus is $(a, 0)$ **[JEE 2009]**

**5.** Let A and B be two distinct points on the parabola $y^2 = 4x$. If the axis of the parabola touches a circle of radius r having AB as its diameter, then the slope of the line joining A and B can be

(A) $-\frac{1}{r}$ (B) $\frac{1}{r}$ (C) $\frac{2}{r}$ (D) $-\frac{2}{r}$ **[JEE 2010]**

**6.** Consider the parabola $y^2 = 8x$. Let $\Delta_1$ be the area of the triangle formed by the end points of its latus rectum and the point $P\left(\frac{1}{2}, 2\right)$ on the parabola, and $\Delta_2$ be the area of the triangle formed by drawing tangents at P and at the end points of the latus rectum. Then $\frac{\Delta_1}{\Delta_2}$ is **[JEE 2011]**

**7.** Let $(x, y)$ be any point on the parabola $y^2 = 4x$. Let P be the point that divides the line segment from $(0,0)$ to $(x, y)$ in the ratio $1:3$. Then the locus of P is **[JEE 2011]**

(A) $x^2 = y$ (B) $y^2 = 2x$ (C) $y^2 = x$ (D) $x^2 = 2y$

**8.** Let L be a normal to the parabola $y^2 = 4x$. If L passes through the point $(9,6)$, then L is given by

(A) $y - x + 3 = 0$ (B) $y + 3x - 33 = 0$

(C) $y + x - 15 = 0$ (D) $y - 2x + 12 = 0$ **[JEE 2012]**

**Paragraph for Question 9 and 10**

Let PQ be a focal chord of the parabola $y^2 = 4ax$. The tangents to the parabola at P and Q meet at a point lying on the line $y = 2x + a, a > 0$. **[JEE 2013]**

**9.** Length of chord PQ is

(A) 7a (B) 5a (C) 2a (D) 3a

**10.** If chord PQ subtends an angle $\theta$ at the vertex of $y^2 = 4ax$, then $\tan\theta =$

(A) $\frac{2}{3}\sqrt{7}$ (B) $\frac{-2}{3}\sqrt{7}$ (C) $\frac{2}{3}\sqrt{5}$ (D) $\frac{-2}{3}\sqrt{5}$

**11.** A line L: $y = mx + 3$ meets y-axis at E(0,3) and the are of the parabola $y^2 = 16x, 0 \le y \le 6$ at the point $F(x_0, y_0)$. The tangent to the parabola at $F(x_0, y_0)$ intersects the y-axis at $G(0, y_1)$. The slope m of the L is chosen such that the area of the triangle EFG has a local maximum.

Match List I with List II and select the correct answer using the code given below the lists : **[JEE 2013]**

| **List I** | **List II** |
|---|---|
| P. m = | 1. $\frac{1}{2}$ |
| Q. Maximum area of △ EFG is | 2. 4 |
| R. $y_0$ = | 3. 2 |
| S. $y_1$ = | 4. 1 |

**Codes :**

| | P | Q | R | S |
|---|---|---|---|---|
| (A) | 4 | 1 | 2 | 3 |
| (B) | 3 | 4 | 1 | 2 |
| (C) | 1 | 3 | 2 | 4 |
| (D) | 1 | 3 | 4 | 2 |

**12.** The common tangents to the circle $x^2 + y^2 = 2$ and the parabola $y^2 = 8x$ touch the circle at the points P, Q and the parabola at the points R, S. Then the area of the quadrilateral PQRS is

(A) 3 (B) 6 (C) 9 (D) 15 **[JEE 2014]**

**Paragraph**

Let a, r, s, t be nonzero real number. Let $P(at^2, 2at)$, $Q(ar^2, 2ar)$ and $S(as^2, 2as)$ be distinct points on the parabola $y^2 = 4ax$. Suppose that PQ is the focal chord and lines QR and PK are parallel, where K is the point (2a, 0). **[JEE 2014]**

**13.** The value of r is

(A) $-\frac{1}{t}$ (B) $\frac{t^2+1}{t}$ (C) $\frac{1}{t}$ (D) $\frac{t^2-1}{t}$

**14.** If $st = 1$, then the tangent at P and the normal at S to the parabola meet at a point whose ordinate is **[JEE 2014]**

(A) $\frac{(t^2+1)^2}{2t^3}$ (B) $\frac{a(t^2+1)^2}{2t^3}$ (C) $\frac{a(t^2+1)^2}{t^3}$ (D) $\frac{a(t^2+2)^2}{t^3}$

**15.** Let P and Q be distinct points on the parabola $y^2 = 2x$ such that a circle with PQ as diameter passes through the veriex O of the parabola. if P lies in the first quadrant and the area of the triangle $\triangle$ OPQ is $3\sqrt{2}$, then which of the following is (are) the coordiantes of P ? **[JEE 2015]**

(A) $(4,2\sqrt{2})$ (B) $(9,3\sqrt{2})$ (C) $\left(\frac{1}{4},\frac{1}{\sqrt{2}}\right)$ (D) $(1,\sqrt{2})$

**16.** The circle $C_1: x^2 + y^2 = 3$, with centre at O, intersects the parabola $x^2 = 2y$ at the point P in the first quadrant. Let the tangent to the circle $C_1$ at P touches other two circles $C_2$ and $C_3$ at $R_2$ and $R_3$, respectively. Suppose $C_2$ and $C_3$ have equal radii $2\sqrt{3}$ and centres $Q_2$ and $Q_3$ respectively, If $Q_2$ and $Q_3$ lie on the y-axis, then **[JEE 2016]**

(A) $Q_2Q_3 = 12$ (B) $R_2R_3 = 4\sqrt{6}$

(C) area of the triangle $OR_2R_3$ is $6\sqrt{2}$ (D) area of the triangle $PQ_2Q_3$ is $4\sqrt{2}$

**17.** Let P be the point on the parabola $y^2 = 4x$ which is at the shortest distance from the centre S of the circle $x^2 + y^2 - 4x - 16y + 64 = 0$. Let Q be the point on the circle dividing the lie segment SP internally. Then **[JEE 2016]**

(A) $SP = 2\sqrt{5}$

(B) $SQ: QP = (\sqrt{5} + 1): 2$

(C) the x-intercept of the normal to the parabola at P is 6

(D) the slope of the tangnet to the circle at Q is $\frac{1}{2}$

**18.** If a chord, which is not a tangent, of the parabola $y^2 = 16x$ has the equation $2x + y = p$, and midpoint $(h, k)$, then which of the following is (are) possible value (s) of p, h and k ? [JEE 2017]

(A) $p = -1, h = 1, k = -3$ (B) $p = 2, h = 3, k = -4$

(C) $p = -2, h = 2, k = -4$ (D) $p = 5, h = 4, k = -3$

Chek option.

**19.** Answer the following by appropriately matching the list based on the information given in the paragraph.

Let the circles $C_1: x^2 + y^2 = 9$ and $C_2: (x-3)^2 + (y-4)^2 = 16$, intersect at the points X and Y. Suppose that another circle $C_3: (x-h)^2 + (y-k)^2 = r^2$ satisfies the following conditions

(i) centre of $C_3$ is collinear with the centres of $C_1$ and $C_2$.

(ii) $C_1$ and $C_2$ both lie inside $C_3$, and

(iii) $C_3$ touches $C_1$ at M and $C_2$ at N

Let the line through X and Y intersect $C_3$ at Z and W, and let a common tangent of $C_1$ and $C_3$ be a tangent to the parabola $x^2 =$

$8\alpha y$.

There are some expressions given in the List I whose values are given in List II below :

| List I | List II |
|---|---|
| (I) $2h + k$ | (P) 6 |
| (II) $\frac{\text{Length of ZW}}{\text{Length of XY}}$ | (Q) $\sqrt{6}$ |
| (III) $\frac{\text{Area of triangle MZN}}{\text{Area of triangle ZMW}}$ | (R) $\frac{5}{4}$ |
| (IV) $\alpha$ | (S) $\frac{21}{5}$ |
| | (T) $2\sqrt{6}$ |
| | (U) $\frac{10}{3}$ |

Which of the following is the only CORRECT combination ? **[JEE 2019]**

(A) (II), (T) (B) (I), (U) (C) (I), (S) (D) (II), (Q)

**20.** Answer the following by appropriately matching the list based on the information given in the paragraph.

Let the circles $C_1: x^2 + y^2 = 9$ and $C_2: (x-3)^2 + (y-4)^2 = 16$, intersect at the points X and Y. Suppose that another circle $C_3: (x-h)^2 + (y-k)^2 = r^2$ satisfies the following conditions (i) centre of $C_3$ is collinear with the centres of $C_1$ and $C_2$.

(ii) $C_1$ and $C_2$ both lie inside $C_3$, and (iii) $C_3$ touches $C_1$ at M and $C_2$ at N

Let the line through X and Y intersect $C_3$ at Z and W, and let a common tangent of $C_1$ and $C_3$ be a tangent to the parabola $x^2 = 8\alpha y$.

There are some expressions given in the List I whose values are given in List II below :

| List I | List II |
|---|---|
| (I) $2h + k$ | (P) 6 |
| Length of ZW | |
| (II) Length of XY | (Q) $\sqrt{6}$ |
| (III) $\frac{\text{Area of triangle MZN}}{\text{Area of triangle ZMW}}$ | (R) $\frac{5}{4}$ |
| (IV) $\alpha$ | (S) $\frac{21}{5}$ |
| | (T) $2\sqrt{6}$ |
| | (U) $\frac{10}{3}$ |

Which of the following is the only CORRECT combination ? **[JEE 2019]**

(A) (IV), (S) (B) (IV), (U) (C) (I), (P) (D) (III), (R)

## ANSWER KEY

### EXERCISE – I (JEE MAIN)

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **1.** | C | **2.** | C | **3.** | B | **4.** | B | **5.** | C | **6.** | A | **7.** | B |
| **8.** | C | **9.** | A | **10.** | D | **11.** | B | **12.** | D | **13.** | C | **14.** | C |
| **15.** | A | **16.** | A | **17.** | B | **18.** | B | **19.** | C | **20.** | C | **21.** | A |
| **22.** | A | **23.** | D | **24.** | C | **25.** | C | **26.** | B | **27.** | D | **28.** | C |

### EXERCISE - II (JEE ADVANCE)

#### SINGLE CORRECT OPTION - TYPE QUESTIONS

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **1.** | A | **2.** | D | **3.** | C | **4.** | A | **5.** | C | **6.** | B | **7.** | B |
| **8.** | C | **9.** | A | **10.** | C | | | | | | |

#### MULTIPLE CORRECT OPTION - TYPE QUESTIONS

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **1.** | A,B | **2.** | A,B | **3.** | A,B,C,D | **4.** | A,B | **5.** | A,B,C | **6.** | A,C |
| **7.** | B,C | | | | | | | | | | |

### EXERCISE - III

#### SUBJECTIVE - TYPE QUESTIONS

**2.** $(4,0); y^2 = 2(x-4)$ **3.** $7y \pm 2(x+6a) = 0$

**4.** $2x - y + 2 = 0, (1,4); x + 2y + 16 = 0, (16, -16)$

**5.** $3x - 2y + 4 = 0; x - y + 3 = 0$ **7.** $2(y-1)^2(x-2) = (3x-4)^2$

**11.** $(a, 0); a$ **12.** $4x + y - 72 = 0 \& y - 3x + 33 = 0$

**13.** $a = 2$ **19.** $(4, -8)$

**20.** $x^2 + y^2 + 18x - 28y + 27 = 0$ **21.** $y^2 = 8ax$

#### COMPREHENSION - BASED QUESTIONS

**23.** $\frac{k-4}{h}$ **24.** 2 **25.** $2y - 3 = 0$

#### MATRIX MATCH - TYPE QUESTIONS

**26.** **(A)** $-P, Q, R, S, T$; **(B)** $-S, T$; **(C)** $-Q, R, S, T$

### EXERCISE - IV

#### PREVIOUS YEAR'S QUESTION JEE MAIN

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **1.** | A | **2.** | D | **3.** | B | **4.** | C | **5.** | D | **6.** | A | **7.** | B |
| **8.** | D | **9.** | C | **10.** | A | **11.** | C | | | | |

#### JEE ADVANCED

**1.** **(a)** D, **(b)** A, B, **(c)** **(i)** A, **(ii)** B, **(iii)** D, **(iv)** C **2.** A

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **3.** | **(a)** C; **(b)** B; **(c)** D | **4.** | A,D | **5.** | C,D | **6.** | 0002 | **7.** | C | **8.** | A,B,D |
| **9.** | B | **10.** | D | **11.** | A | **12.** | D | **13.** | D | **14.** | B | **15.** | A,D |
| **16.** | A,B,C | **17.** | A,C,D | **18.** | B | **19.** | D | **20.** | | | |