

$$3h = x_1 + x_2 + x_3 = \lambda x_1$$

$$3k = \frac{x_1}{2} - \frac{x_2}{3} + x_3 = \mu x_1$$

$$\frac{x_1 + x_2}{2} = -1 \Rightarrow x_2 = f(x_1)$$

$$\frac{x_3 - \frac{x_1}{2}}{x_1 - x_2} = 3 \Rightarrow x_3 = g(x_1)$$

Family of Circles thru intersection

Given  
a line & a circle

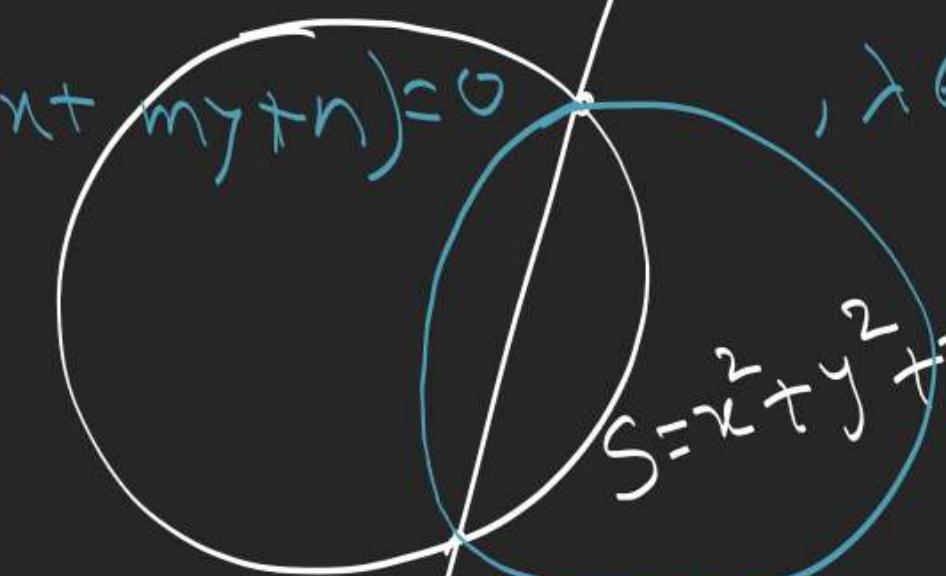
$$x^2 + y^2 + 2gx + 2fy + c + \lambda(lx + my + n) = 0$$

$$L = lx + my + n = 0$$

,  $\lambda \in \mathbb{R}$

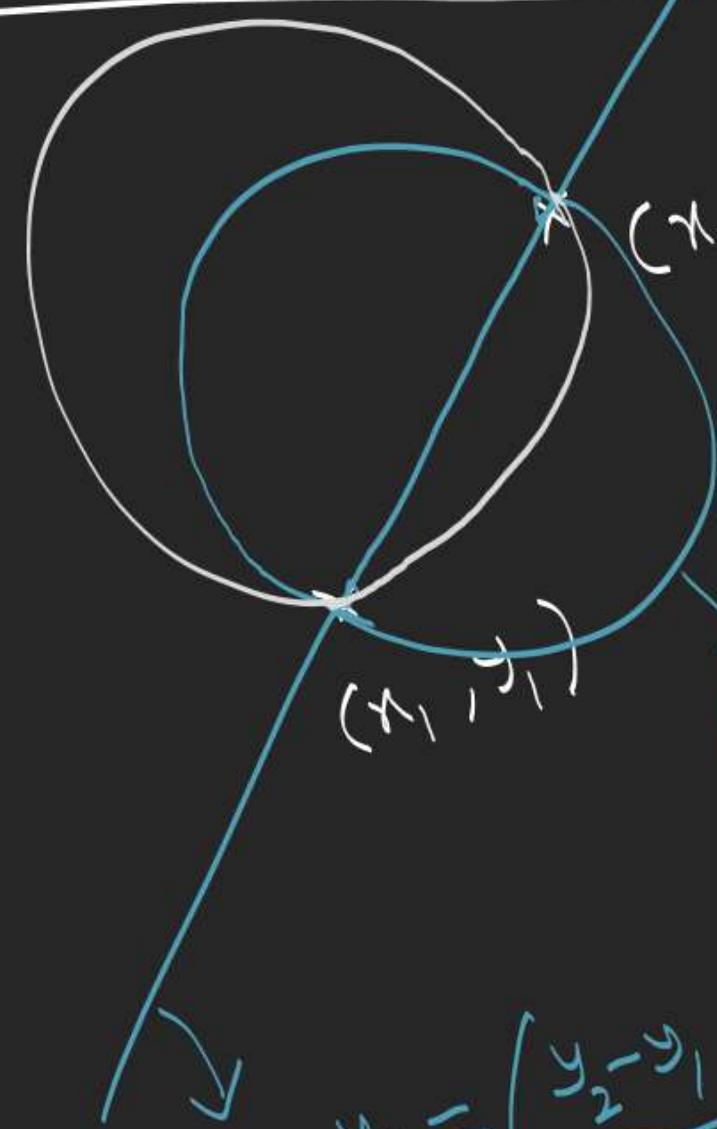
$$S + \lambda L = 0$$

$\lambda \in \mathbb{R}$



$$S = x^2 + y^2 + 2gx + 2fy + c = 0$$

Family of circles thru two given points



$$(x_2, y_2)$$

$$(x_1, y_1)$$

$$(x-x_1)(x-x_2) + (y-y_1)(y-y_2) + \lambda \left( (y-y_1) - \left( \frac{y_2-y_1}{x_2-x_1} \right) (x-x_1) \right) = 0$$

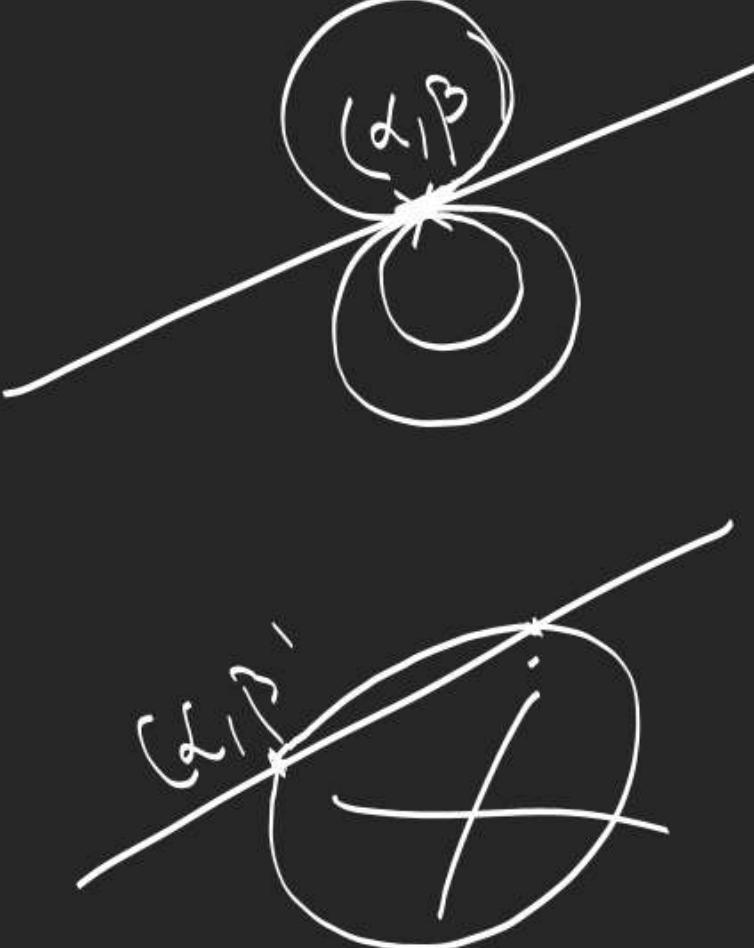
$$= 0$$

$$\lambda \in \mathbb{R}$$

$$y - y_1 = \left( \frac{y_2 - y_1}{x_2 - x_1} \right) (x - x_1)$$

$$x \quad y$$

Family of circles touching a line at a given point on it

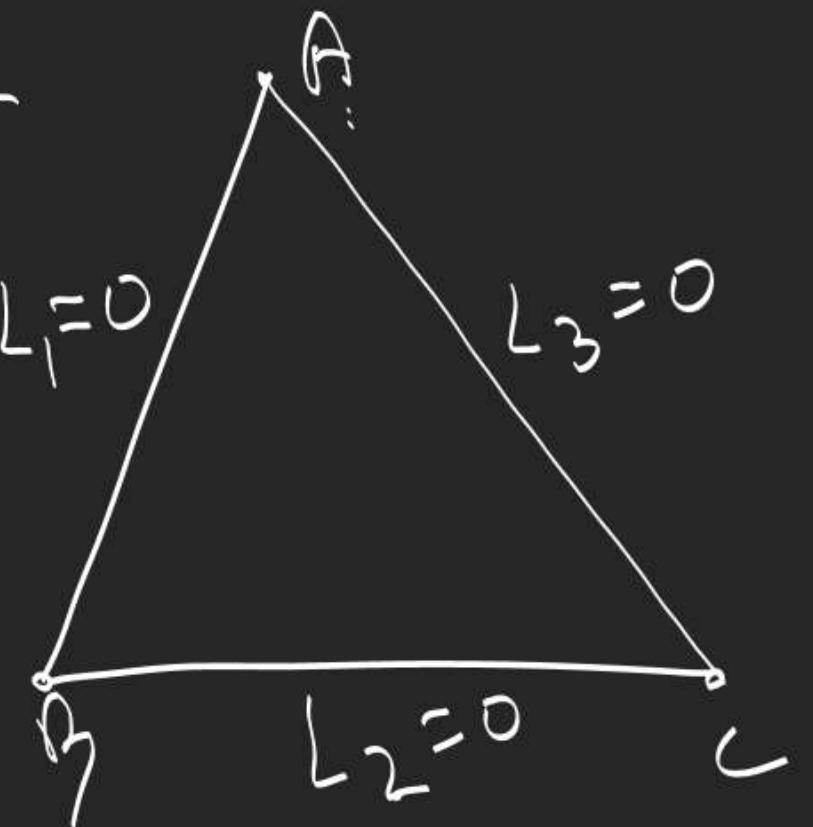


$$lx + my + n = 0$$

$$(x - \alpha)^2 + (y - \beta)^2 + \lambda(lx + my + n) = 0$$

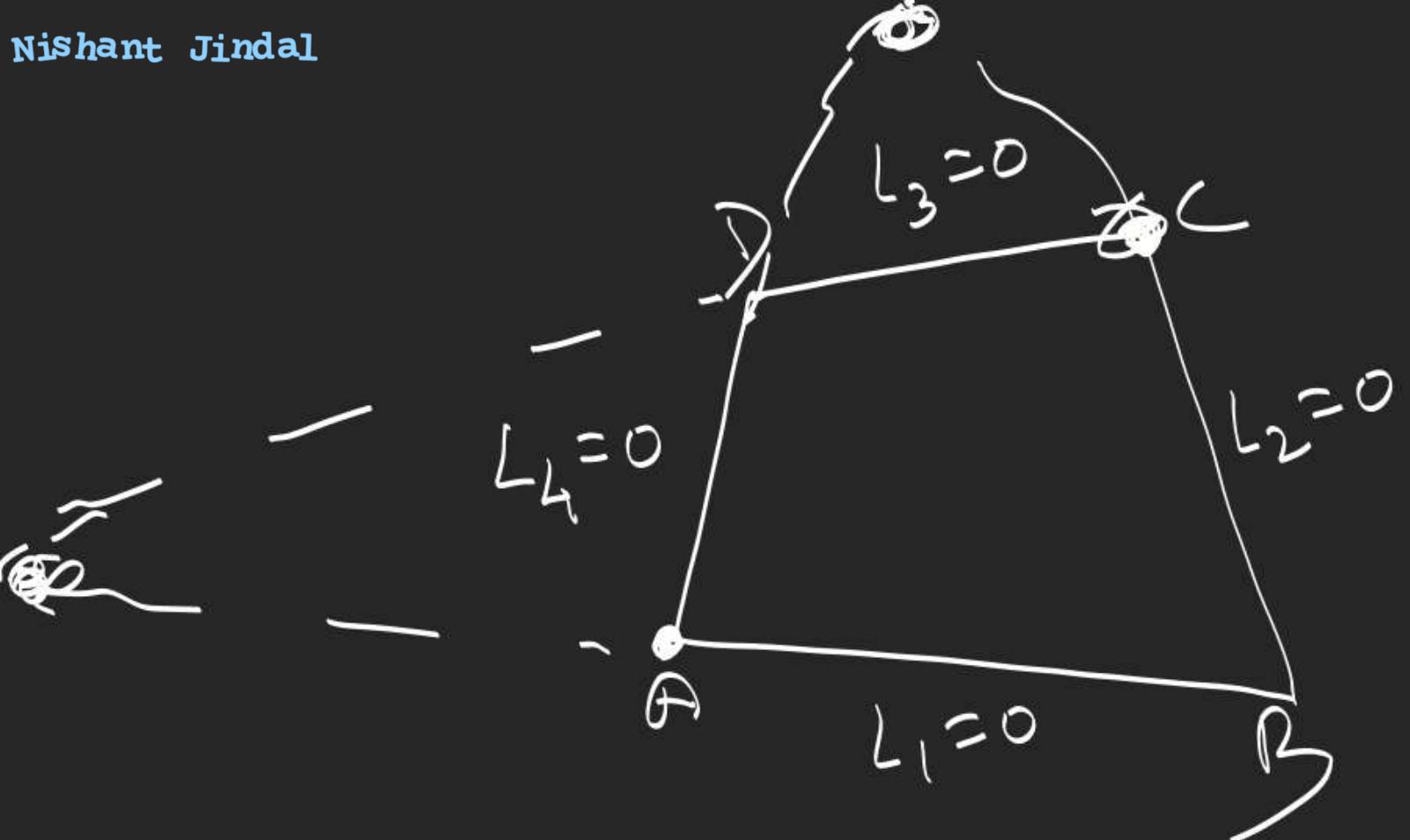
$$\lambda \in \mathbb{R}$$

Circumcircle



$$L_1 L_2 + \lambda L_2 L_3 + \mu L_3 L_1 = 0$$

$$\begin{aligned} \text{coeff. of } x^2 &= \text{coeff. of } y^2 \\ \text{coeff. of } xy &= 0 \end{aligned} \quad \left. \begin{array}{l} \lambda, \mu = ? \end{array} \right\}$$



Circumcircle

$$L_1 L_3 + \lambda L_2 L_4 = 0$$

$$\text{coeff. of } x^2 - \text{coeff. of } y^2$$

$$\text{coeff. of } xy = 0$$

$$L_1 L_2 + \lambda L_3 L_4 = 0$$

~~X~~

$$\lambda = ?$$

L. Find the eqn. of circle which touches the line  $2x-y=4$  at point  $(1, -2)$  and

(i) passes thru  $(3, 4)$

$$(x-1)^2 + (y+2)^2 + \lambda(2x-y-4) = 0$$

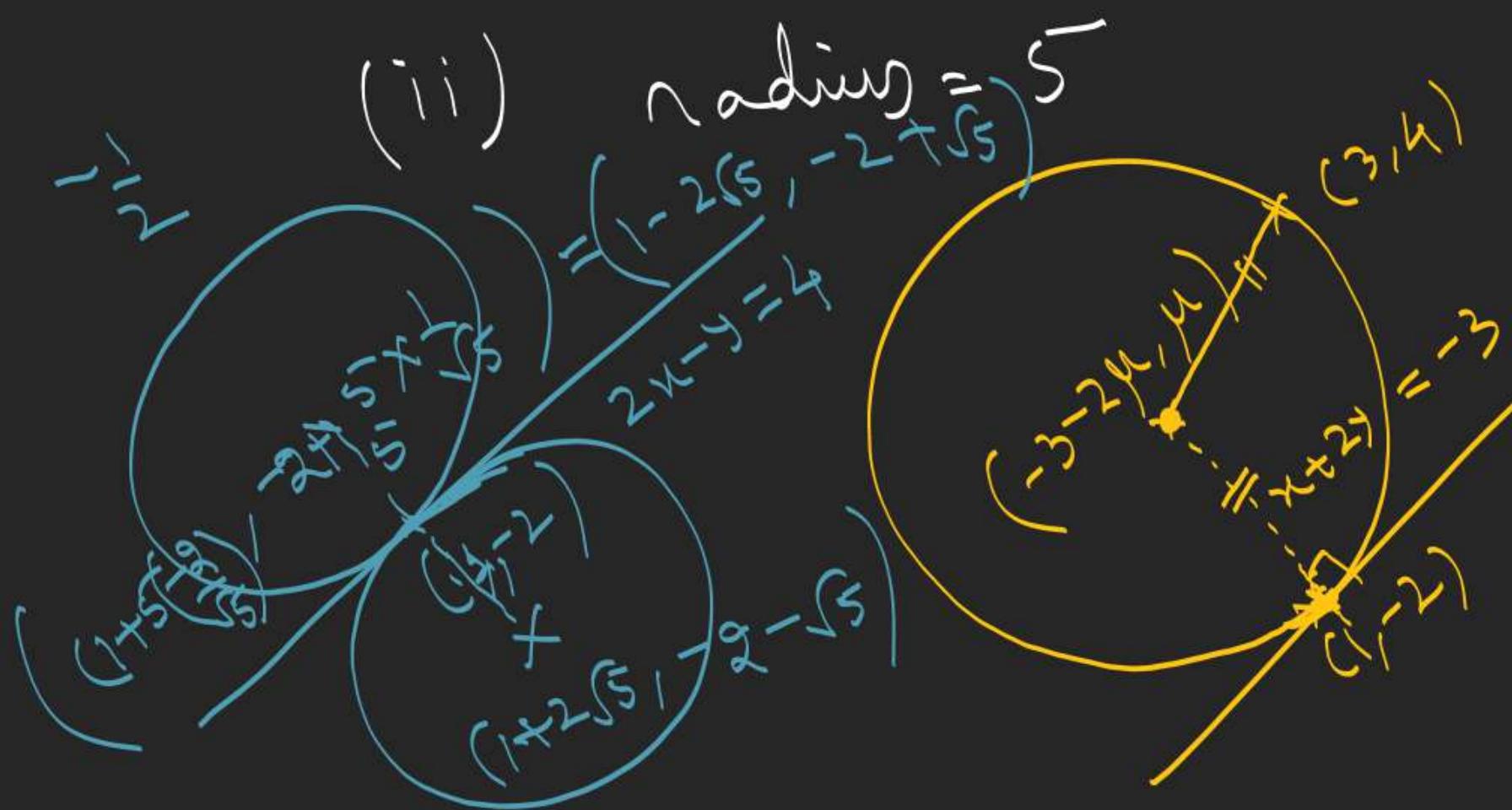
Put  $(3, 4)$

$$4 + 36 + \lambda(-2) = 0$$

$$\lambda = 20$$

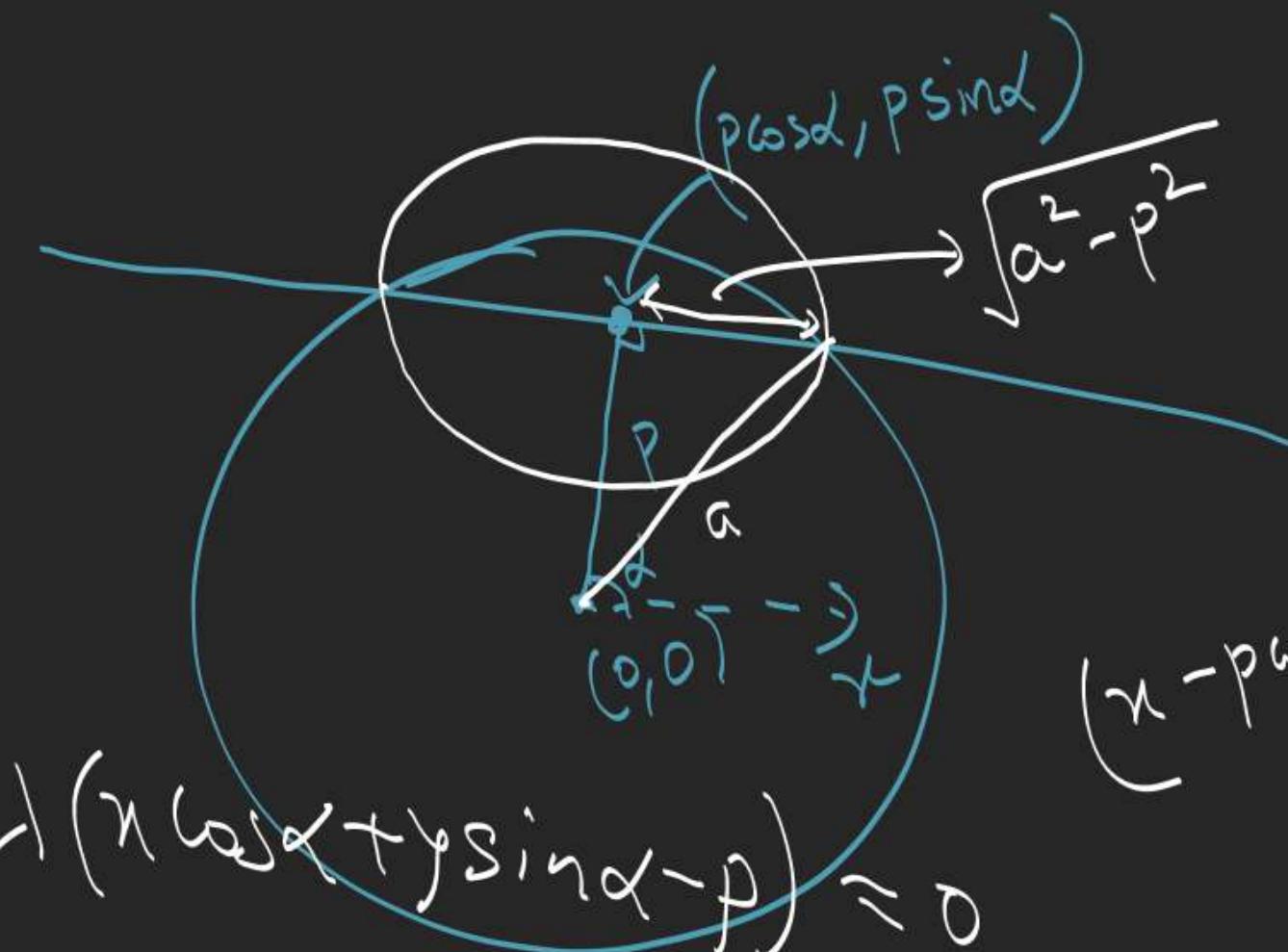
$$(x+2\mu)^2 + (y-\mu)^2 = (4+2\mu)^2 + (\mu+2)^2$$

$$\mu = ?$$



Q2. Find the eqn. of circle drawn on chord

$x\cos\alpha + y\sin\alpha = p$  of the circle  $x^2 + y^2 = a^2$  as its diameter.



$$x^2 + y^2 - a^2 + \lambda(x\cos\alpha + y\sin\alpha - p) = 0$$

$$\text{Put } \left(-\frac{1}{2}\cos\alpha, -\frac{1}{2}\sin\alpha\right) \text{ to } x\cos\alpha + y\sin\alpha = p, \lambda = ?$$

$$(x - p\cos\alpha)^2 + (y - p\sin\alpha)^2 = a^2 - p^2$$

3. Find the equation of circle which passes thru origin and thru the point of contact of tangents drawn from origin to the circle  $x^2 + y^2 - 11x + 13y + 17 = 0$ .

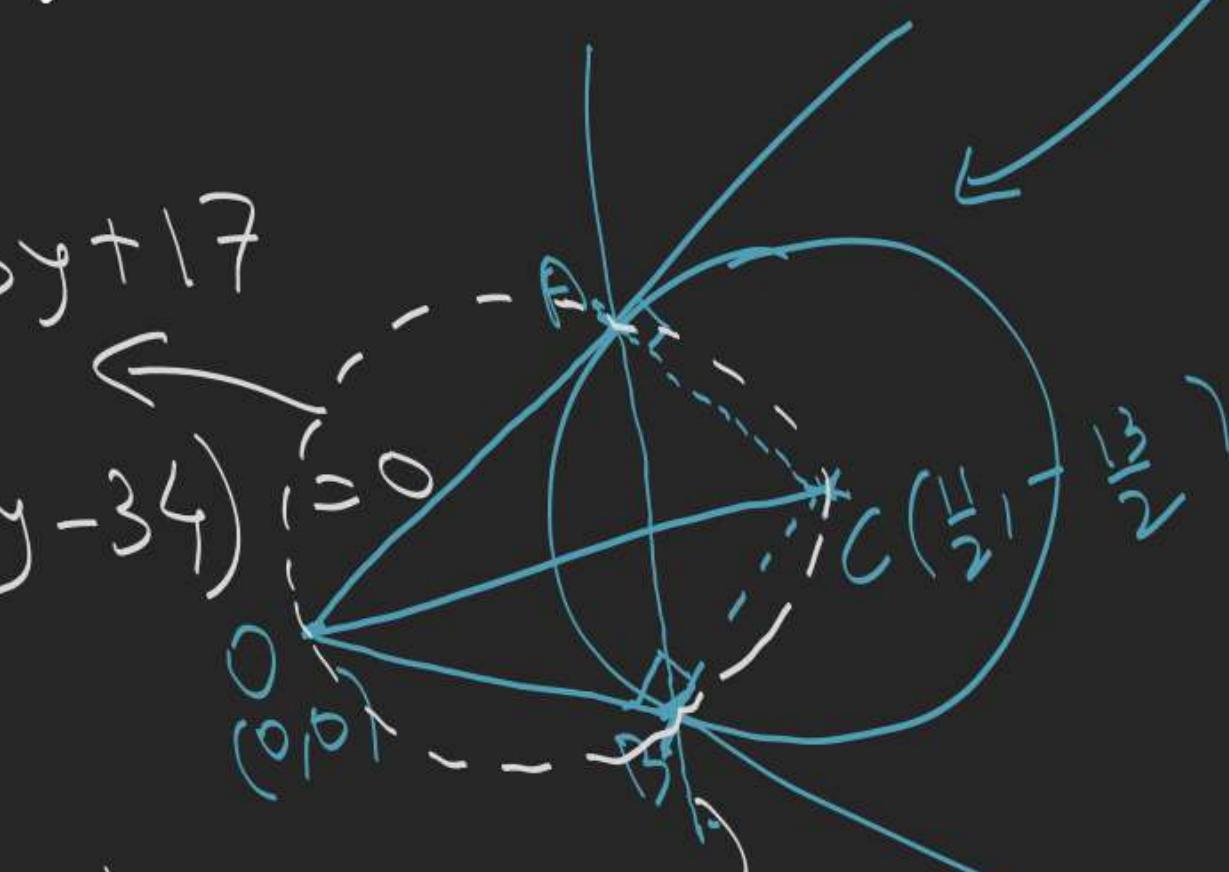
$$x^2 + y^2 - 11x + 13y + 17$$

$$+ \lambda(11x - 13y - 34) = 0$$

Put  $(0,0)$

$$\lambda = ?$$

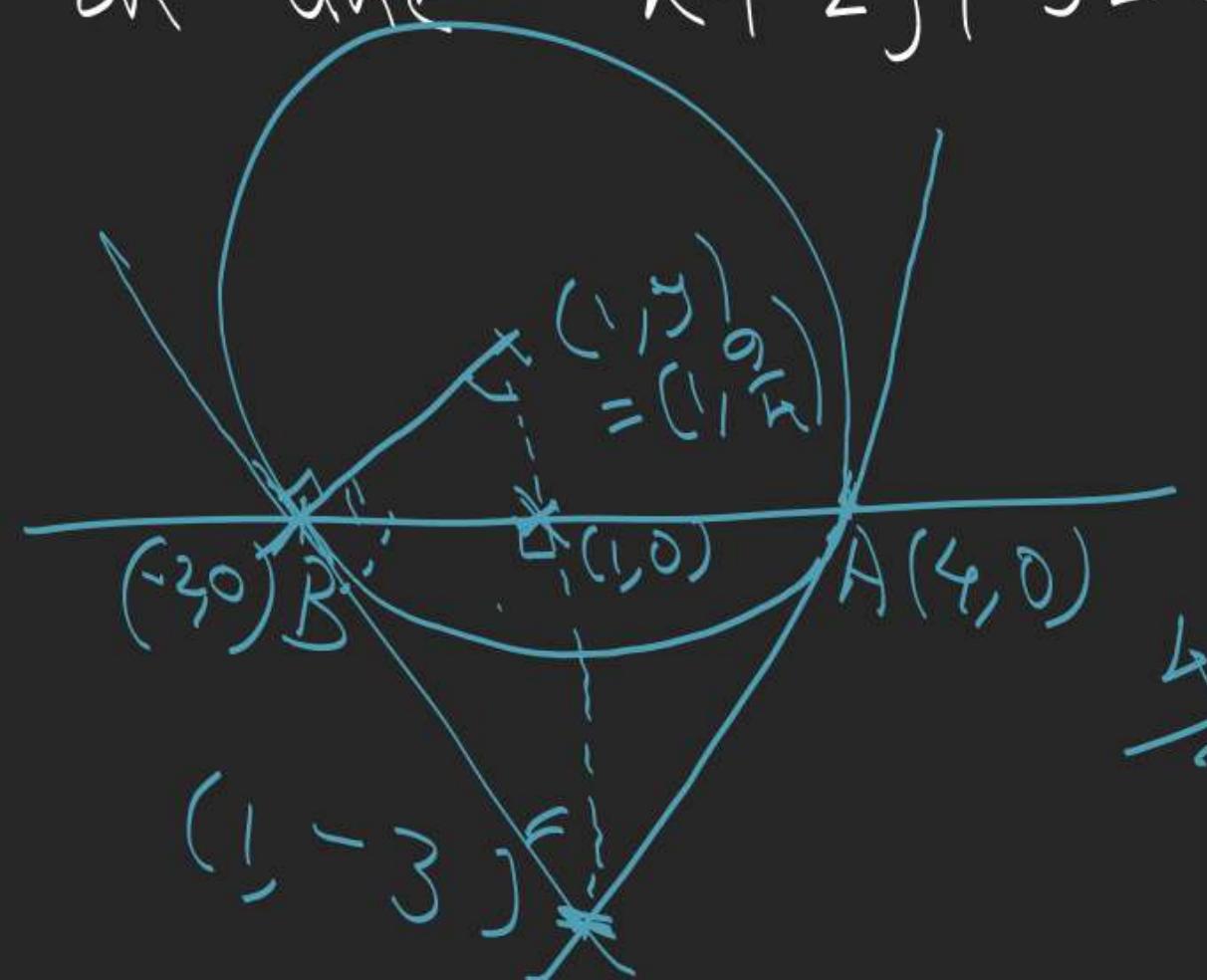
$$-\frac{11}{2}x + \frac{13}{2}y + 17 = 0$$



$$x\left(x - \frac{11}{2}\right) + y\left(y + \frac{13}{2}\right) = 0$$

Q. S.T. eqn  $x^2 + y^2 - 2x - 2\lambda y - 8 = 0$  represents for different values of  $\lambda$ , a system of circles passing through fixed points  $A, B$ . Also find the eqn of that circle of the system, the tangent to which at  $A$  and  $B$

meet on line  $x + 2y + 5 = 0$



$$(x^2 + y^2 - 2x - 8) - 2\lambda(y) = 0$$

$$x^2 - 2x - 8 = 0$$

$$x = 0 \quad x = 4$$

$$\boxed{A(4, 0), B(-2, 0)}$$

$$\frac{4}{3} = \frac{3}{1}$$

$$\alpha_1 = 8$$