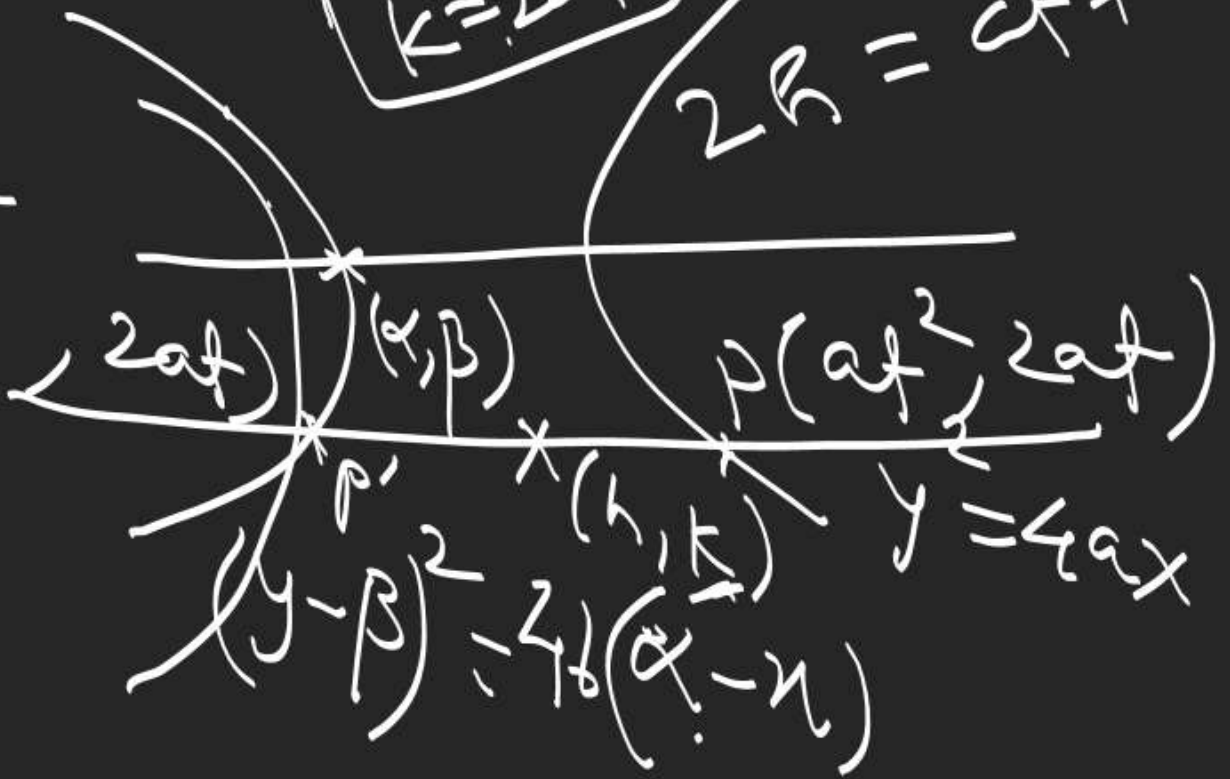


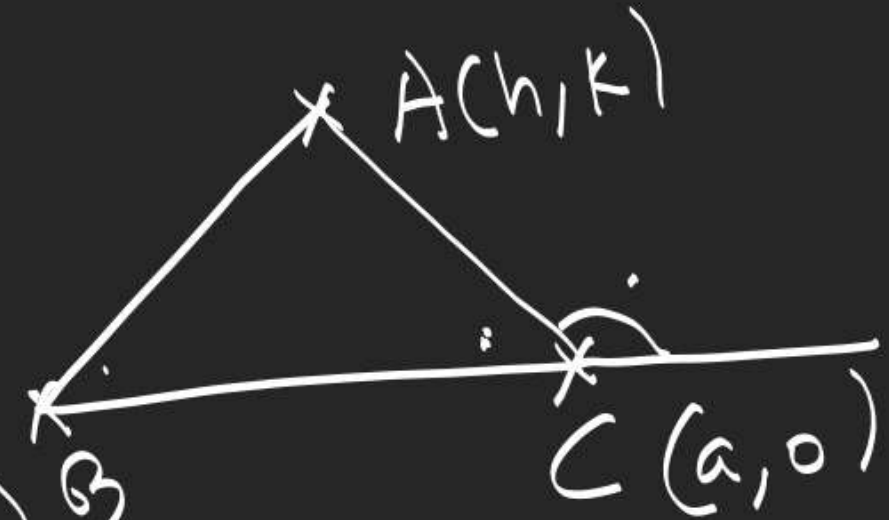
$$k = 2at$$

$$2h = at^2 + 1$$

$$\left(\frac{2at - \beta}{4b} \right)^2$$



$$(-a, 0) B$$



$$\frac{k}{h+a} - \frac{k}{h-a} = \dots$$

$$t_1 t_2 = -1$$

$$Q - y t_1 = x + a + a t_1^2$$

$$(2) - y t_2 = x + a' + a' t_2^2$$

$$(1) \times t_2 - (2) \times t_1$$

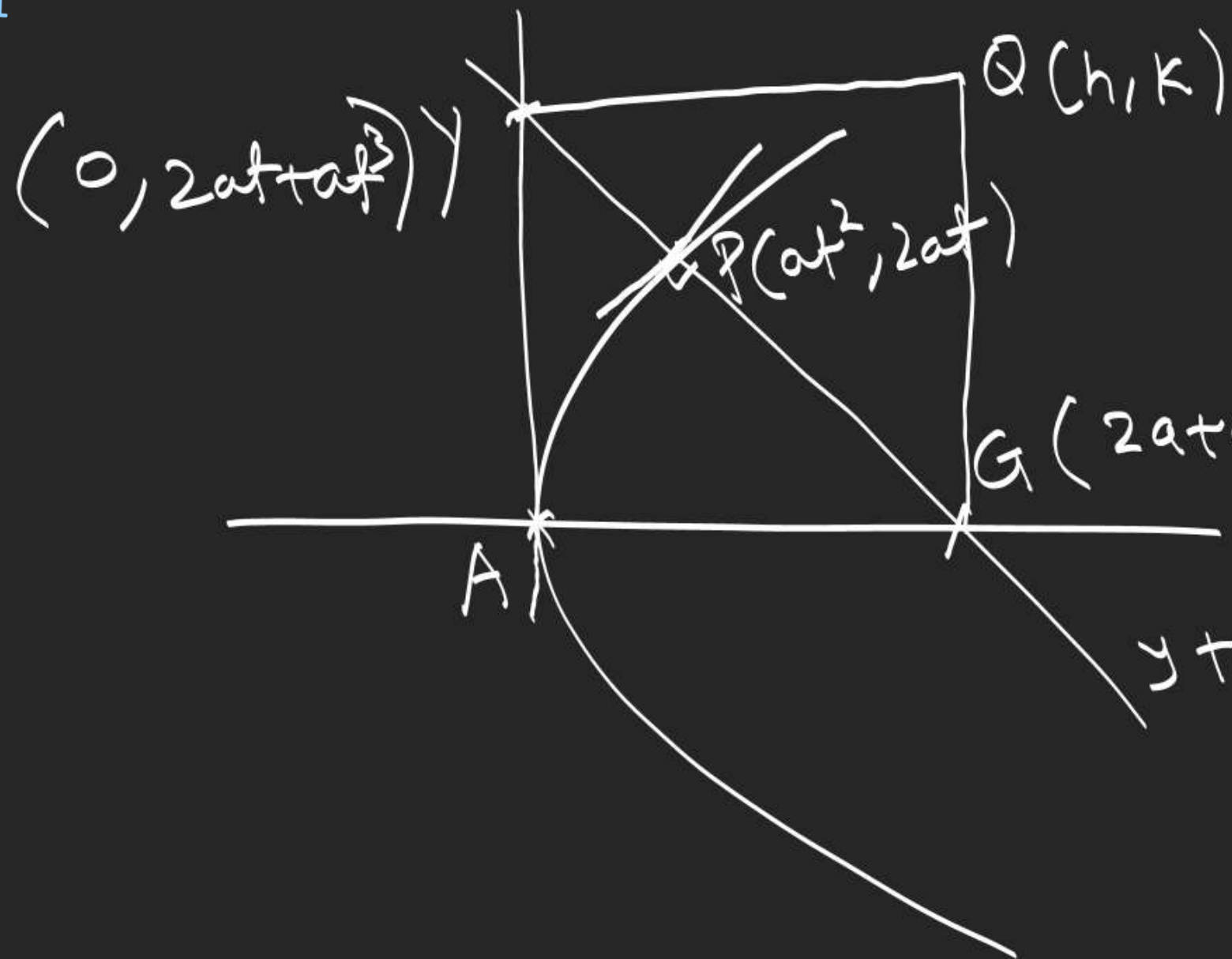
$$0 = x(t_2 - t_1) + (a t_2 - a' t_1) + (-a t_1 + a' t_2)$$

$$(t_2 - t_1)(x + a + a') = 0$$

$$y^2 = 4a(-a')$$

$$y^2 = 4a'(-a)$$

$$x = -a - a'$$



$$h = 2at + at^2$$

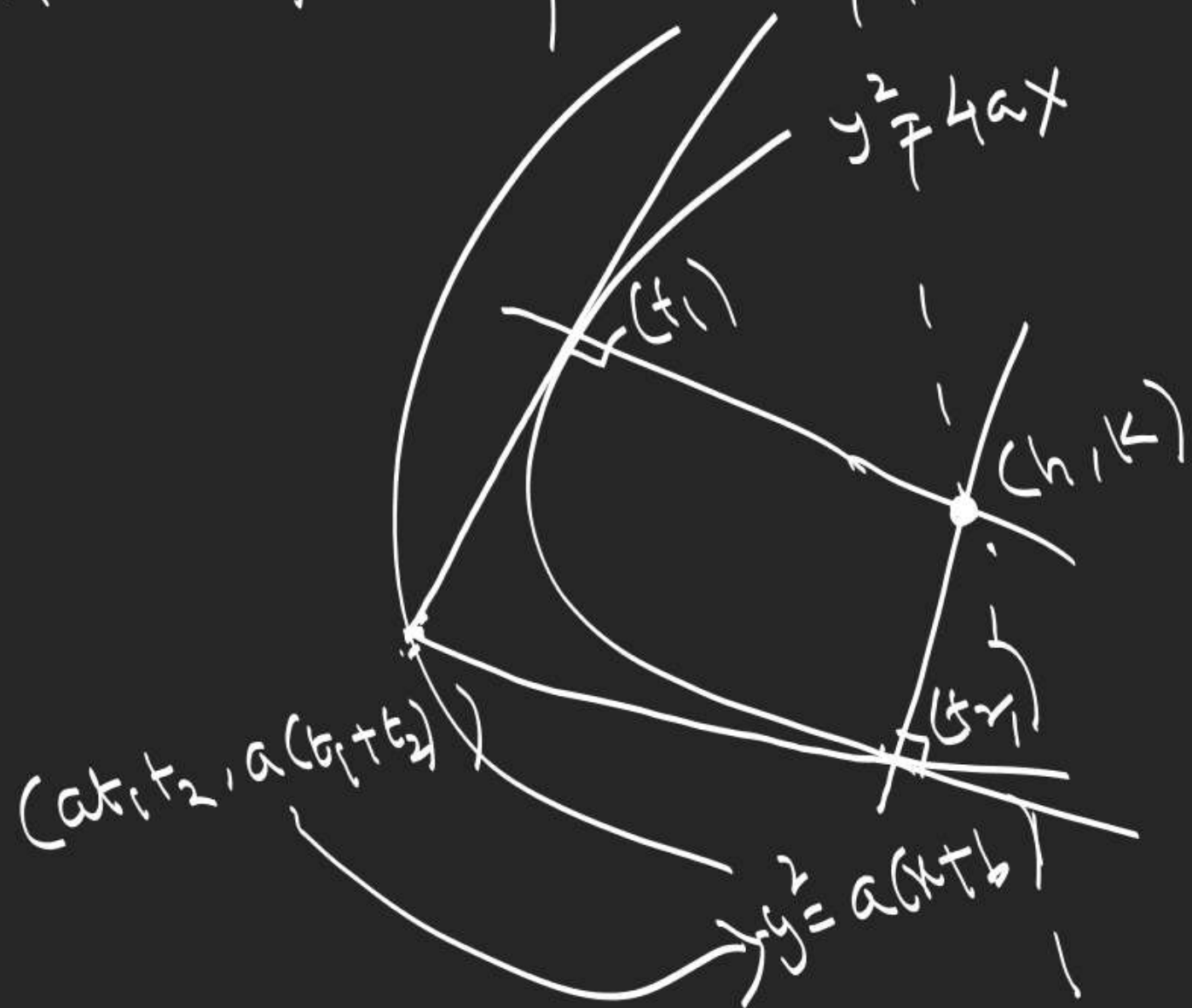
$$k = 2at + at^3$$

$$\boxed{\frac{k}{h} = t}$$

$$y + tx = 2at + at^3$$

$$h = 2a + a\left(\frac{k^2}{h^2}\right)$$

∴ If tangents are drawn to $y^2 = 4ax$ from any point P on parabola $y^2 = a(x+b)$, then show that normals drawn at their point of contact meet on a line.



$$h = a(2 + \underline{t_1^2} + \underline{t_2^2} + \underline{t_1 t_2})$$

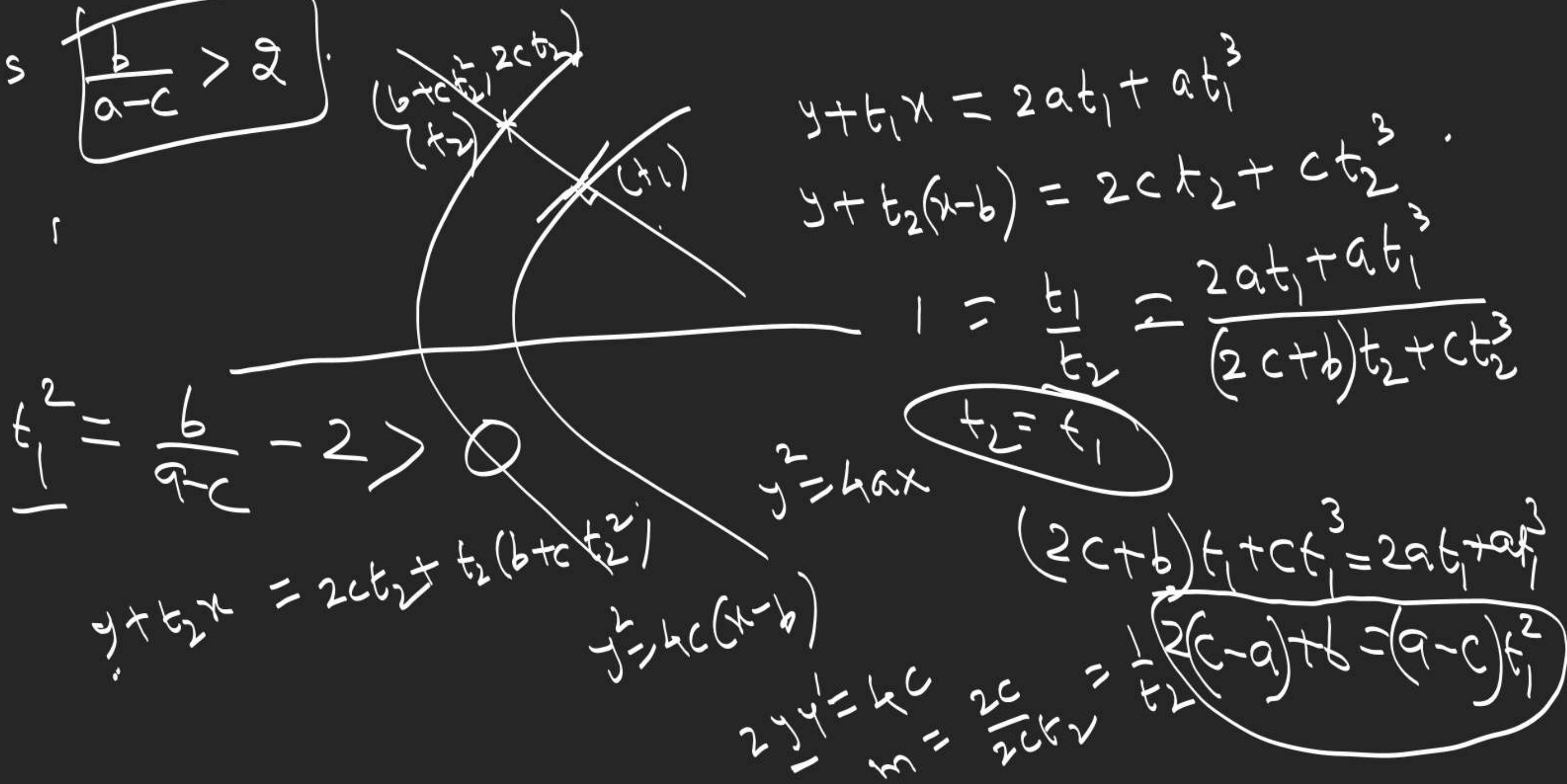
$$k = -a t_1 t_2 (t_1 + t_2)$$

$$\boxed{R = 2a + b}$$

$$\frac{2}{a}(t_1^2 + t_2^2 + 2t_1 t_2) = \cancel{a}(a t_1 t_2 + b)$$

$$a(t_1^2 + t_2^2 + t_1 t_2) = b$$

2. P.T. two parabolas $y^2 = 4ax$ and $y^2 = 4c(x-b)$
 can't have a common normal other than axis
 unless $\boxed{\frac{b}{a-c} > 2}$.



3. TP and TQ are tangents to parabola $y^2 = 4ax$ and normals at P & Q meet at a point R on $y^2 = 4ax$.
 P.T. centre of circle circumscribing $\triangle TPQ$ lies on parabola $2y^2 = a(x-a)$.

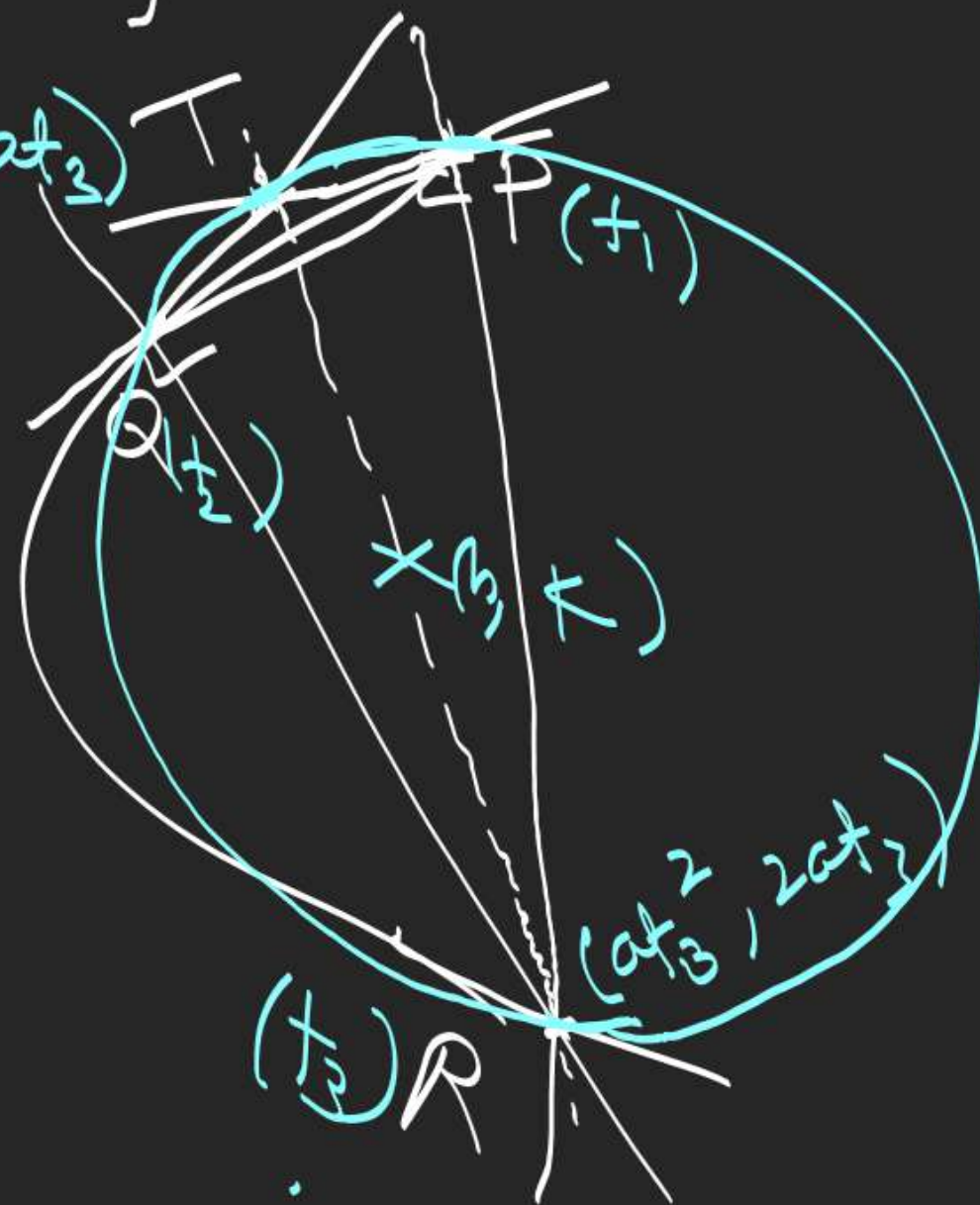
$$2h = 2a + a\left(\frac{2k}{a}\right)^2$$

$$t_1 + t_2 + t_3 = 0$$

$$t_1 + t_2 = 2$$

$$2h = 2a + at_3^2$$

$$2k = at_3$$



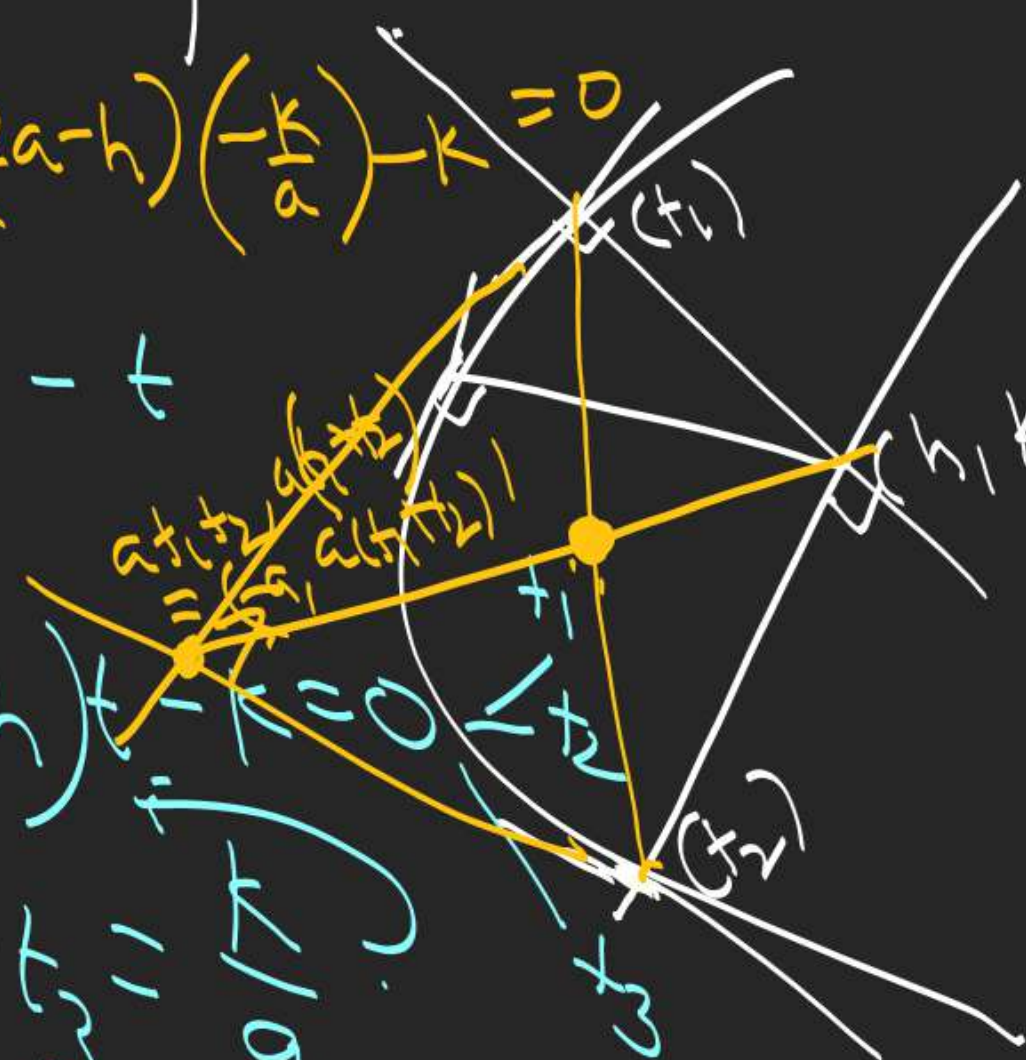
4. Find the locus of point which is such that two of the normals through it to parabola $y^2 = 4ax$ are at right angles.

$$a\left(-\frac{k}{a}\right)^3 + (2a-h)\left(-\frac{k}{a}\right) - k = 0$$

$$\frac{k - 2at}{h - at^2} = -t$$

$$at^3 + (2a-h)t - k = 0$$

$$t_1 t_2 t_3 = -t_3 = \frac{k}{a}$$



$$t_1 t_2 = -1$$

$$(h, k) = (a(t_1^2 + t_2^2 + t_1 t_2 + 2), -at_1 t_2(t_1 + t_2))$$

$$k = (t_1 + t_2)a$$

$$h = a(2 + (t_1 + t_2)^2 - t_1 t_2)$$

$$h = a\left(3 + \left(\frac{k}{a}\right)^2\right)$$

Chord of Contact
of (d, β) w.r.t. $y^2 = 4ax$

P
 (α, β)

A
 (x_1, y_1)

Tangent at A
 $yy_1 = 2a(x + x_1)$

Put (α, β)

$$\beta y_1 = 2a(\alpha + x_1)$$

$$\beta y_2 = 2a(\alpha + x_2)$$

$$\beta y = 2a(\alpha + x) \quad \begin{matrix} (x_1, y_1) \\ (x_2, y_2) \end{matrix}$$

$$S = y^2 - 4ax$$

$$T = y\beta - 2a(x + \alpha)$$

$$\boxed{T = 0}$$

AB

Chord whose midpoint is given

$$y - \beta = \frac{2a}{\beta} (x - \alpha)$$

$$\beta y - 2ax = \beta^2 - 2a\alpha$$

$$\beta y - 2a(x + \alpha) = \beta^2 - 4a\alpha$$

$$\boxed{T = S_1}$$

$$y^2 = 4ax$$

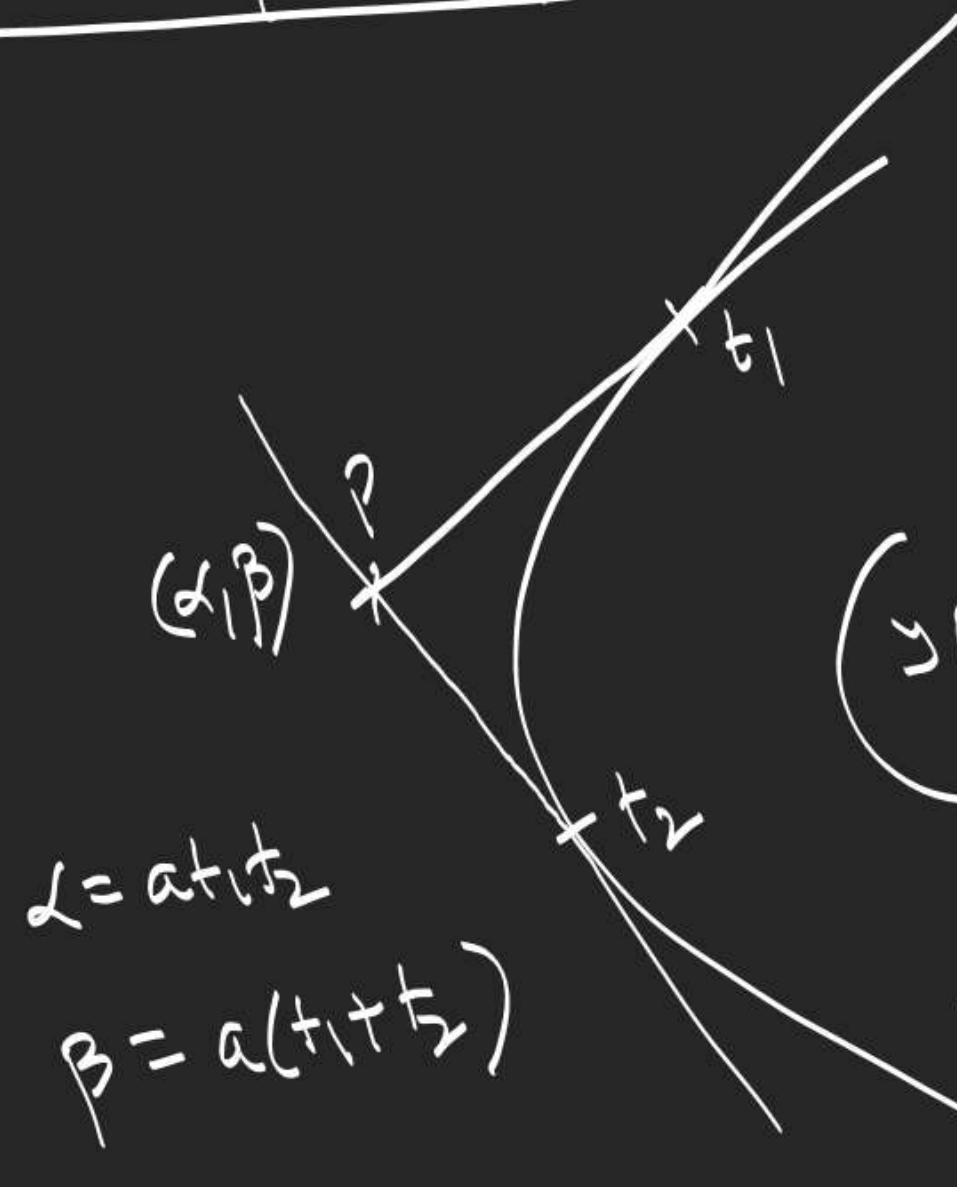
$$m = \frac{2}{t_1 + t_2}$$

$$\beta = \frac{2a(t_1 + t_2)}{2}$$

$$t_1 + t_2 = \frac{\beta}{a}$$

$$m = \frac{2a}{\beta}$$

Pair of Tangents



$$(t_1y - x - at_1^2)(t_2y - x - at_2^2) = 0$$

$$(y\beta - 2a(x + \alpha))^2 = (y^2 - 4ax)(\beta^2 - 4a\alpha)$$

$$y^2 = 4ax$$

$$T^2 = SS_1$$

Diameter
parallel to
axis of
parabola.

$$2at = \frac{2a}{3} \\ m = \frac{1}{3}$$

(ti) $\Sigma x = 26$
25, 26, 27, 28, 33,
34,

$$m = \frac{2}{t_1 + t_2}$$

$$k = \frac{2a(t_1 + t_2)}{2}$$

$$t_1 + t_2 = \frac{k}{a}$$

$$m = \frac{2a}{k}$$

$$y^2 = 4ax$$

$$y = \frac{2a}{3}$$