## HOMEWORK–2

### 1. DIFFERENTIATION OF IMPLICIT FUNCTION

**1.** If $y = \sqrt{\sin x + y}$, then $\frac{dy}{dx} =$

(A) $\frac{\sin x}{2y-1}$ (B) $\frac{\sin x}{1-2y}$ (C) $\frac{\cos x}{1-2y}$ (D) $\frac{\cos x}{2y-1}$

**2.** If $ax^2 + 2hxy + by^2 = 0$, then $\frac{dy}{dx}$ equals-

(A) $\frac{ax+hy}{hx+by}$ (B) $-\frac{ax+hy}{hx+by}$ (C) $\frac{hx+by}{ax+hy}$ (D) $-\frac{hx+by}{ax+hy}$

**3.** If $x\sqrt{y} + y\sqrt{x} = 1$, then $\frac{dy}{dx}$ equals-

(A) $-\frac{y+2\sqrt{xy}}{x+2\sqrt{xy}}$ (B) $-\sqrt{\frac{x}{y}}\left(\frac{y+2\sqrt{xy}}{x+2\sqrt{xy}}\right)$ (C) $-\sqrt{\frac{y}{x}}\left(\frac{y+2\sqrt{xy}}{x+2\sqrt{xy}}\right)$ (D) $\frac{x}{y}$

**4.** If $e^x \sin y - e^y \cos x = 1$, then $\frac{dy}{dx}$ equals-

(A) $\frac{e^x \sin y + e^y \sin x}{e^y \cos x - e^x \cos y}$ (B) $\frac{e^x \sin y + e^y \sin x}{e^y \cos x + e^x \cos y}$

(C) $\frac{e^x \sin y - e^y \sin x}{e^y \cos x - e^x \cos y}$ (D) $\frac{e^x}{e^y}$

**5.** If $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$, then the value of $dy/dx$ is -

(A) $\frac{\sqrt{1-x^2}}{\sqrt{1-y^2}}$ (B) $\frac{\sqrt{1-y^2}}{\sqrt{1-x^2}}$ (C) $-\frac{\sqrt{1-x^2}}{\sqrt{1-y^2}}$ (D) $-\frac{\sqrt{1-y^2}}{\sqrt{1-x^2}}$

**6.** If $x^3 \cos(xy) + y^3 \sin(xy) + 1 = 0$, then $\frac{dy}{dx}$ equals

(A) $\frac{x^3 y \tan(xy) - (3x^2+y^4)}{xy^3 + (3y^2-x^4)\tan xy}$ (B) $\frac{x^3 y \tan(xy) + (3x^2+y^4)}{xy^3 - (3y^2-x^4)\tan xy}$

(C) $\frac{x^3 y - (3x^2+y^4)\tan(xy)}{xy^3 \tan(xy) + (3y^2-x^4)}$ (D) $x^2 y \tan(xy) + 3x^2$

**7.** If $y = 2^{\log_2 x^{2x}} + \left(\tan \frac{\pi x}{4}\right)^{\frac{4}{\pi x}}$ then $\left.\frac{dy}{dx}\right]_{x=1}$ is

(A) 4 (B) 1 (C) 0 (D) Not defined

### 2. DIFFERENTIATION OF LOGARITHMIC FUNCTION

**8.** If $x^m \cdot y^n = (x+y)^{m+n}$, then $\frac{dy}{dx}$ is

(A) $\frac{x+y}{xy}$ (B) $xy$ (C) $\frac{x}{y}$ (D) $\frac{y}{x}$

**9.** If $f(x) = |x|^{|\sin x|}$ then $f'(\pi/4)$ equals

(A) $\left(\frac{\pi}{4}\right)^{1/\sqrt{2}} \left(\frac{\sqrt{2}}{2} \ln \frac{4}{\pi} - \frac{2\sqrt{2}}{\pi}\right)$ (B) $\left(\frac{\pi}{4}\right)^{1/\sqrt{2}} \left(\frac{\sqrt{2}}{2} \ln \frac{4}{\pi} + \frac{2\sqrt{2}}{\pi}\right)$

(C) $\left(\frac{\pi}{4}\right)^{1/\sqrt{2}} \left(\frac{\sqrt{2}}{2} \ln \frac{\pi}{4} - \frac{2\sqrt{2}}{\pi}\right)$ (D) $\left(\frac{\pi}{4}\right)^{1/\sqrt{2}} \left(\frac{\sqrt{2}}{2} \ln \frac{\pi}{4} + \frac{2\sqrt{2}}{\pi}\right)$

**10.** If $y = (1+x)(1+x^2)(1+x^4)\ldots.(1+x^{2n})$, then $\frac{dy}{dx}$ at $x = 0$ is

(A) -1 (B) 1 (C) 0 (D) $2^n$

### 3. DIFFERENTIATION OF INFINITE SERIES

**11.** If $x = e^{y+e^{y+\cdots \text{upto } \infty}}, x > 0$, then $\frac{dy}{dx}$

(A) $\frac{x}{1+x}$ (B) $\frac{1}{x}$ (C) $\frac{1-x}{x}$ (D) $\frac{1+x}{x}$

**12.** If $y = \sqrt{x}^{\sqrt{x}^{\sqrt{x}\ldots\ldots\infty}}$, then the value of $\frac{dy}{dx}$ is

(A) $\frac{xy^2}{2-y\log x}$ (B) $\frac{x^2}{y(2-y\log x)}$ (C) $\frac{y^2}{x(2-y\log x)}$ (D) $\frac{y^2}{x(2+y\log x)}$

**13.** If $y = \sqrt{\log x + \sqrt{\log x + \sqrt{\log x + \ldots\ldots}}}$, then $\frac{dy}{dx}$ equals-

(A) $x/(2y+1)$ (B) $1/x(2y-1)$ (C) $(2y-1)/x$ (D) $x(2y-1)$

**14.** Let $f(x) = x + \frac{1}{2x+\frac{1}{2x+\frac{1}{2x+\ldots\ldots\infty}}}$. Compute the value of $f(100).f'(100)$

**15.** If $y = x + \frac{1}{x+\frac{1}{x+\frac{1}{x+\ldots\ldots}}}$, prove that $\frac{dy}{dx} = \frac{1}{2-\frac{x}{x+\frac{1}{x+\frac{1}{x+\ldots\ldots}}}}$

**16.** If $y = \tan^{-1}\frac{1}{x^2+x+1} + \tan^{-1}\frac{1}{x^2+3x+3} + \tan^{-1}\frac{1}{x^2+5x+7} + \tan^{-1}\frac{1}{x^2+7x+13} + \ldots\ldots$ to n terms.

Find dy/dx, expressing your answer in 2 terms,

### 4. DIFFERENTIATION OF DETERMINANT

**17.** If $f(x), g(x), h(x)$ are polynomials in x of degree 2

and $F(x) = \begin{vmatrix} f & g & h \\ f' & g' & h' \\ f'' & g'' & h'' \end{vmatrix}$, then $F'(x)$ is equal to

(A) 1 (B) 0 (C) -1 (D) $f(x)\cdot g(x)\cdot h(x)$

**18.** If $y = \sin mx$ then the value of $\begin{vmatrix} y & y_1 & y_2 \\ y_3 & y_4 & y_5 \\ y_6 & y_7 & y_8 \end{vmatrix}$

(where subscripts of y shows the order of derivative) is

(A) independent of x but dependent on m (B) dependent of x but independent of m

(C) dependent on both m & x (D) independent of m & x

**19.** If $f(x) = \begin{vmatrix} \cos(x+x^2) & \sin(x+x^2) & -\cos(x+x^2) \\ \sin(x-x^2) & \cos(x-x^2) & \sin(x-x^2) \\ \sin 2x & 0 & \sin 2x^2 \end{vmatrix}$ then

(A) $f(-2) = 0$ (B) $f'(-1/2) = 0$

(C) $f'(-1) = -2$ (D) $f''(0) = 4$

### 5. HIGHER ORDER DERIVATIVE

**20.** If $y = a\cos(\ln x) + b\sin(\ln x)$, then $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx}$

(A) 0 (B) y (C) $-y$ (D) xy

**21.** $\frac{d^2x}{dy^2}$ equals : **[AIEEE 2011]**

(A) $\left(\frac{d^2y}{dx}\right)^{-1}$ (B) $-\left(\frac{d^2y}{dx^2}\right)^{-1}\left(\frac{dy}{dx}\right)^{-3}$

(C) $\left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right)^{-2}$ (D) $-\left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right)^{-3}$

### 6.TRIGONOMETRIC SUBSTITUTIONS

**22.** The derivative of $\sec^{-1}\left(\frac{1}{2x^2-1}\right)$ w.r.t. $\sqrt{1-x^2}$ at $x = \frac{1}{2}$ is

(A) 4 (B) 1/4 (C) 1 (D) 7

**23.** If $y = \sin^{-1}\frac{2x}{1+x^2}$ then $\frac{dy}{dx}\Big]_{x=-2}$ is

(A) $\frac{2}{5}$ (B) $\frac{2}{\sqrt{5}}$ (C) $-\frac{2}{5}$ (D) $\frac{\sqrt{5}}{2}$

**24.** $d/dx\left[\tan^{-1}\left(\frac{\sqrt{x^2+a^2}+x}{\sqrt{x^2+a^2}-x}\right)^{1/2}\right]$, is equal to -

(A) $\frac{a}{2(x^2+a^2)}$ (B) $\frac{a}{x^2+a^2}$

(C) ½ (D) None of these

**25.** $d/dx\left[\sin^2\cot^{-1}\frac{1}{\sqrt{\frac{1+x}{1-x}}}\right]$, is equal to -

(A) 0 (B) 1/2 (C) $-1/2$ (D) -1

**26.** If $y = \tan^{-1}\frac{u}{\sqrt{1-u^2}}$ & $x = \sec^{-1}\frac{1}{2u^2-1}$

$u \in \left(0, \frac{1}{\sqrt{2}}\right) \cup \left(\frac{1}{\sqrt{2}}, 1\right)$ prove that $2\frac{dy}{dx} + 1 = 0$

### 7. MIXED PROBLEMS

**27.** If $y = \sec(\tan^{-1} x)$, then $\frac{dy}{dx}$ at $x = 1$ is equal to: **[JEE Main 2013]**

(A) 1 (B) $\sqrt{2}$ (C) $\frac{1}{\sqrt{2}}$ (D) $\frac{1}{2}$

**28.** For $x \in R$, $f(x) = |\log 2 - \sin x|$ and $g(x) = f(f(x))$, then: **[JEE MAIN 2016]**

(A) $g'(0) = \cos(\log 2)$

(B) $g'(0) = -\cos(\log 2)$

(C) g is differentiable at $x = 0$ and $g'(0) = -\sin(\log 2)$

(D) g is not differentiable at $x = 0$

**29.** If for $x \in \left(0, \frac{1}{4}\right)$, then derivative of $\tan^{-1}\left(\frac{6x\sqrt{x}}{1-9x^3}\right)$ is $\sqrt{x} \cdot g(x)$, then $g(x)$ equals

(A) $\frac{9}{1+9x^3}$ (B) $\frac{3x\sqrt{x}}{1-9x^3}$ **[JEE MAIN 2017]**

(C) $\frac{3x}{1-9x^3}$ (D) $\frac{3x}{1+9x^3}$

**30.** Let $f(\theta) = \sin\left(\tan^{-1}\left(\frac{\sin\theta}{\sqrt{\cos 2\theta}}\right)\right)$ where $-\frac{\pi}{4} < \theta < \frac{\pi}{4}$, Then the value of $\frac{d}{d(\tan\theta)}(f(\theta))$ is

**[JEE 2011]**

**Ans.** 1

## ANSWER KEY

### 1. DIFFERENTIATION OF IMPLICIT FUNCTION

**1.** (D) **2.** (B) **3.** (C) **4.** (A) **5.** (B) **6.** (A) **7.** (A)

### 2. DIFFERENTIATION OF LOGARITHMIC FUNCTION

**8.** (D) **9.** (D) **10.** (B)

### 3. DIFFERENTIATION OF INFINITE SERIES

**11.** (C) **12.** (C) **13.** (B) **14.** 100 **16.** $\frac{1}{1+(x+n)^2}-\frac{1}{1+x^2}$

### 4. DIFFERENTIATION OF DETERMINANT

**17.** (B) **18.** (D) **19.** (B,C,D)

### 5. HIGHER ORDER DERIVATIVE

**20.** (C) **21.** (D)

### 6.TRIGONOMETRIC SUBSTITUTIONS

**22.** (A) **23.** (C) **24.** (A) **25.** (B)

### 7. MIXED PROBLEMS

**27.** (C) **28.** (A) **29.** (A) **30.** 1