Nishant Jindal

EXERCISE - 1

Objective Problems | JEE Main

SECTION - A
QUESTIONS BASED ON BASIC DEFINITION & PARAMETRIC REPRESENTATION

1. If distance between the foci of an ellipse is equal to its minor axis, then eccentricity of the ellipse is-
(A) $e = \frac{1}{\sqrt{2}}$ (B) $e = \frac{1}{\sqrt{3}}$ (C) $e = \frac{1}{\sqrt{4}}$ (D) $e = \frac{1}{\sqrt{6}}$

2. The eccentricity of an ellipse, with its centre at the origin, is $\frac{1}{2}$. If one of the directrices is $x = 4$, then the equation of the ellipse is-
(A) $3x^2 + 4y^2 = 1$ (B) $3x^2 + 4y^2 = 12$ (C) $4x^2 + 3y^2 = 12$ (D) $4x^2 + 3y^2 = 1$

3. If distance between the directrices be thrice the distance between the foci, then eccentricity of ellipse is
(A) $\frac{1}{2}$ (B) $\frac{2}{3}$ (C) $\frac{1}{\sqrt{3}}$ (D) $\frac{4}{5}$

4. The curve represented by $x = 3(\cos t + \sin t)$, $y = 4(\cos t - \sin t)$, is
(A) ellipse (B) parabola (C) hyperbola (D) circle

5. An ellipse having foci at $(3,3)$ and $(-4,4)$ and passing through the origin has eccentricity equal to
(A) $\frac{3}{7}$ (B) $\frac{2}{7}$ (C) $\frac{5}{7}$ (D) $\frac{3}{5}$

SECTION - B
POSITION OF POINT & CHORD JOINING TWO POINTS

6. The position of the point $(4, -3)$ with respect to the ellipse $2x^2 + 5y^2 = 20$ is-
(A) outside the ellipse (B) on the ellipse
(C) on the major axis (D) None of these

SECTION – C

TANGENT TO PARABOLA

7. A tangent having slope of $-\frac{4}{3}$ to the ellipse $\frac{x^2}{18} + \frac{y^2}{32} = 1$ intersects the major & minor axes in points A&B respectively. If C is the centre of the ellipse then the area of the triangle ABC is
(A) 12 sq. units (B) 24 sq. units (C) 36 sq. units (D) 48 sq. units

8. Angle between the tangents drawn from point $(4,5)$ to the ellipse $\frac{x^2}{16} + \frac{y^2}{25} = 1$ is
(A) $\frac{\pi}{3}$ (B) $\frac{5\pi}{6}$ (C) $\frac{\pi}{4}$ (D) $\frac{\pi}{2}$

9. The equation to the locus of the middle point of the portion of the tangent to the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$ included between the co-ordinate axes is the curve
(A) $9x^2 + 16y^2 = 4x^2y^2$ (B) $16x^2 + 9y^2 = 4x^2y^2$
(C) $3x^2 + 4y^2 = 4x^2y^2$ (D) $9x^2 + 16y^2 = x^2y^2$

SECTION - D

NORMAL TO PARABOLA

10. The normal at a variable point P on an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ of eccentricity e meets the axes of the ellipse in Q and R then the locus of the mid-point of QR is a conic with an eccentricity e' such that

(A) e' is independent of e (B) $e'=1$

(C) $e'=e$ (D) $e'=1/e$

11. The equation of the normal to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ at the positive end of latus rectum is

(A) $x+ey+e^2a=0$ (B) $x-ey-e^3a=0$

(C) $x-ey-e^2a=0$ (D) none of these

12. PQ is a double ordinate of the ellipse $x^2+9y^2=9$, the normal at P meets the diameter through Q at R, then the locus of the mid point of PR is

(A) a circle (B) a parabola (C) an ellipse (D) a hyperbola

SECTION - F
CHORD OF CONTACT, LENGTH, CHORD WITH A GIVEN MID POINT

13. The line, $lx+my+n=0$ will cut the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ in points whose eccentric angles differ by $\pi/2$ if

(A) $x^2l^2+b^2n^2=2\,m^2$ (B) $a^2\,m^2+b^2l=2n^2$

(C) $a^2l^2+b^2m^2=2n^2$ (D) $a^2n^2+b^2m^2=2l$

SECTION – G

HIGHLIGHTS & BASED RESULTS

14. Q is a point on the auxiliary circle of an ellipse. P is the corresponding point on ellipse. N is the foot of perpendicular from focus S, to the tangent of auxiliary circle at Q. Then

(A) SP = SN (B) SP = PQ (C) PN = SP (D) NQ = SP

SECTION - H : MIXED PROBLEMS

15. The area of the rectangle formed by the perpendiculars from the centre of the standard ellipse to the tangent and normal at its point whose eccentric angle is $\pi/4$ is

(A) $\frac{(a^2-b^2)ab}{a^2+b^2}$ (B) $\frac{(a^2+b^2)ab}{a^2-b^2}$ (C) $\frac{(a^2-b^2)}{ab(a^2+b^2)}$ (D) $\frac{(a^2+b^2)}{(a^2-b^2)ab}$

Nishant Jindal

EXERCISE - 2 (LEVEL-I)

Objective Problems | JEE Main

QUESTIONS BASED ON BASIC DEFINITION & PARAMETRIC REPRESENTATION

1. The equation of an ellipse, whose major axis $= 8$ and eccentricity $= 1/2$, is

(A) $3x^2 + 4y^2 = 12$ (B) $3x^2 + 4y^2 = 48$
(C) $4x^2 + 3y^2 = 48$ (D) $3x^2 + 9y^2 = 12$

2. If the eccentricity of an ellipse be $5/8$ and the distance between its foci be 10 , then its latus rectum is

(A) $\frac{39}{4}$ (B) 12 (C) 15 (D) $\frac{37}{2}$

3. If the distance of a point on the ellipse $\frac{x^2}{6} + \frac{y^2}{2} = 1$ from the centre is 2 , then the eccentric angle is

(A) $\pi/3$ (B) $\pi/4$ (C) $\pi/6$ (D) $\pi/2$

4. The eccentricity of the ellipse $\frac{x^2}{4} + \frac{y^2}{3} = 1$ is decreasing at the rate of $0.1/$ second due to change in semi minor axis only. The time at which ellipse become auxiliary circle is

(A) 2 seconds (B) 3 seconds (C) 4 seconds (D) 5 seconds

5. An ellipse is such that the length of the latus rectum is equal to the sum of the lengths of its semi principal axes. Then

(A) Ellipse becomes a circle
(B) Ellipse becomes a line segment between the two foci
(C) Ellipse becomes a parabola
(D) none of these

POSITION OF POINT & CHORD JOINING TWO POINTS

6. P is a variable point on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ with AA' as the major axis. Then, the maximum value of the area of the triangle APA' is-

(A) ab (B) 2ab (C) ab/2 (D) None of these

7. If PSQ is a focal chord of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, $a > b$, then the harmonic mean of SP and SQ is

(A) $\frac{b^2}{a}$ (B) $\frac{a^2}{b}$ (C) $\frac{2b^2}{a}$ (D) $\frac{2a^2}{b}$

TANGENT TO PARABOLA

8. Which of the following is the common tangent to the ellipses $\frac{x^2}{a^2+b^2} + \frac{y^2}{b^2} = 1 \& \frac{x^2}{a^2} + \frac{y^2}{a^2+b^2} = 1$?

(A) $ay = bx + \sqrt{a^4 - a^2b^2 + b^4}$ (B) $by = ax - \sqrt{a^4 + a^2b^2 + b^4}$
(C) $ay = bx - \sqrt{a^4 + a^2b^2 + b^4}$ (D) $by = ax - \sqrt{a^4 - a^2b^2 + b^4}$

9. The point of intersection of the tangents at the point P on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, and its corresponding point Q on the auxiliary circle meet on the line

(A) $x = a/e$ (B) $x = 0$ (C) $y = 0$ (D) none

10. Q is a point on the auxiliary circle corresponding to the point P of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$. If T is the foot of the perpendicular dropped from the focus S onto the tangent to the auxiliary circle at Q then the $\triangle$ SPT is

(A) isosceles (B) equilateral (C) right angled (D) right isosceles

11. $x-2y+4=0$ is a common tangent to $y^2=4x\&\frac{x^2}{4}+\frac{y^2}{b^2}=1$. Then the value of b and the other common tangent are given by

(A) $b=\sqrt{3};x+2y+4=0$ (B) $b=3;x+2y+4=0$

(C) $b=\sqrt{3};x+2y-4=0$ (D) $b=\sqrt{3};x-2y-4=0$

NORMAL TO PARABOLA

12. The length of the normal (terminated by the major axis) at a point of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is

(A) $\frac{b}{a}(r+r_1)$ (B) $\frac{b}{a}|r-r_1|$ (C) $\frac{b}{a}\sqrt{rr_1}$ (D) independent of r, r_1

where r and r_1 are the focal distance of the point.

13. The eccentric angle of the point where the line, $5x-3y=8\sqrt{2}$ is a normal to the ellipse $\frac{x^2}{25}+\frac{y^2}{9}=1$ is

(A) $\frac{3\pi}{4}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{6}$ (D) $\tan^{-1}2$

Nishant Jindal

EXERCISE - 2 (LEVEL-II)

Multiple Correct | JEE Advanced

QUESTIONS BASED ON BASIC DEFINITION & PARAMETRIC REPRESENTATION

1. A circle has the same centre as an ellipse & passes through the foci F_1& F_2 of the ellipse, such that the two curves intersect in 4 points. Let ' P ' be any one of their point of intersection. If the major axis of the ellipse is 17& the area of the triangle PF_1F_2 is 30 , then the distance between the foci is less than

(A) 12 (B) 13 (C) 14 (D) 15

POSITION OF POINT & CHORD JOINING TWO POINTS

2. If the chord through the points whose eccentric angles are θ&ϕ on the ellipse, $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ passes through the focus, then the value of $\tan(\theta/2)\tan(\phi/2)$ is

(A) $\frac{e+1}{e-1}$ (B) $\frac{e-1}{e+1}$ (C) $\frac{1+e}{1-e}$ (D) $\frac{1-e}{1+e}$

TANGENT TO PARABOLA

3. The tangent at any point P on a standard ellipse with foci as S&S′ meets the tangents at the vertcies A & A′ in the points V&V′, then

(A) $\ell(AV)\cdot\ell(A'V')=b^2$ (B) $\ell(AV)\cdot\ell(A'V')=a^2$

(C) $\angle V'SV=90°$ (D) V′S′SV is a cyclic quadrilateral

MIXED PROBLEMS

4. Point ' O ' is the centre of the ellipse with major axis AB and minor axis CD. Point F is one focus of the ellipse. If $OF=6$ and the diameter of the inscribed circle of triangle OCF is 2 , then the product (AB)(CD) is less than

(A) 65 (B) 52 (C) 78 (D) 87

5. If P is a point of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, whose focii are S and S′. Let $\angle PSS'=\alpha$ and $\angle PS'S=\beta$, then

(A) $SP+PS'=2a$, if $a>b$ (B) $PS+PS'=2b$, if $a<b$

(C) $\tan\frac{\alpha}{2}\tan\frac{\beta}{2}=\frac{1-e}{1+e}$ (D) $\tan\frac{\alpha}{2}\tan\frac{\beta}{2}=\frac{\sqrt{a^2-b^2}}{b^2}\left[a-\sqrt{a^2-b^2}\right]$ when $a>b$

Nishant Jindal

EXERCISE – 3

| Subjective | JEE Advanced

1. (a) Find the equation of the ellipse with its centre (1,2), focus at (6,2) and passing through the point (4,6).
(b) An ellipse passes through the points $(-3,1)$ & $(2,-2)$& its principal axis are along the coordinate axes in order. Find its equation.

2. If any two chords be drawn through two points on the major axis of an ellipse equidistant from the centre, show that $\tan\frac{\alpha}{2}\cdot\tan\frac{\beta}{2}\cdot\tan\frac{\gamma}{2}\cdot\tan\frac{\delta}{2}=1$, where $\alpha,\beta,\gamma,\delta$ are the eccentric angles of the extremities of the chords.

3. The tangent at any point P of a circle $x^2+y^2=a^2$ meets the tangent at a fixed point $A(a,0)$ in T and T is joined to B, the other end of the diameter through A, prove that the locus of the intersection of AP and BT is an ellipse whose eccentricity is $1/\sqrt{2}$.

4. An ellipse has foci at $F_1(9,20)$ and $F_2(49,55)$ in the xy-plane and is tangent to the x-axis. Find the length of its major axis.

5. The tangent at the point α on a standard ellipse meets the auxiliary circle in two points which subtends a right angle at the centre. Show that the eccentricity of the ellipse is $(1+\sin^2\alpha)^{-1/2}$.

6. Find the equations of the lines with equal intercepts on the axes & which touch the ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$.

7. A tangent having slope $-\frac{4}{3}$ to the ellipse $\frac{x^2}{18}+\frac{y^2}{32}=1$, intersects the axis of x&y in points A&B respectively. If O is the origin, find the area of triangle OAB.

8. Let d be the perpendicular distance from the centre of the ellipse $\frac{x^2}{a^2}+\frac{x^2}{b^2}=1$ to the tangent drawn at a point P on the ellipse. If F_1& F_2 are the two foci of the ellipse, then show that $(PF_1-PF_2)^2=4a^2\left[1-\frac{b^2}{d^2}\right]$.

9. The tangent at a point P on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ intersects the major axis in T& N is the foot of the perpendicular from P to the same axis. Show that the circle on NT as diameter intersects the auxiliary circle orthogonally.

10. If the tangent at any point of an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ makes an angle α with the major axis and an angle β with the focal radius of the point of contact then show that the eccentricity ' e ' of the ellipse is given by the absolute value of $\frac{\cos\beta}{\cos\alpha}$.

11. Common tangents are drawn to the parabola $y^2=4x$ & the ellipse $3x^2+8y^2=48$ touching the parabola at A & B and the ellipse at C & D. Find the area of the quadrilateral.

12. P & Q are the corresponding points on a standard ellipse & its auxiliary circle. The tangent at P to the ellipse meets the major axis in T. Prove that QT touches the auxiliary circle.

13. A tangent to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ touches at the point P on it in the first quadrant & meets the coordinate axes in A & B respectively. If P divides AB in the ratio 3: 1 reckoning from the x-axis find the equation of the tangent.

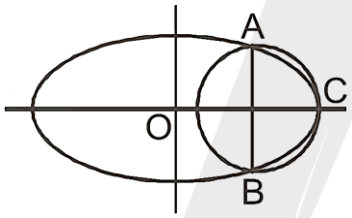
14. If the normal at the point P(θ) to the ellipse $\frac{x^2}{14}+\frac{y^2}{5}=1$, intersects it again at the point Q(2θ), show that $\cos\theta=-(2/3)$.

15. PG is the normal to a standard ellipse at P, G being on the major axis. GP is produced outwards to Q so that PQ = GP. Show that the locus of Q is an ellipse whose eccentricity is $\frac{a^2-b^2}{a^2+b^2}$.

16. Suppose x and y are real numbers and that $x^2+9y^2-4x+6y+4=0$ then find the maximum value of $(4x-9y)$.

17. ABC is an isosceles triangle with its base BC twice its altitude. A point P moves within the triangle such that the square of its distance from BC is half the rectangle contained by its distance from the two sides. Show that the locus of P is an ellipse with eccentricity $\sqrt{2/3}$ passing through B & C.

18. A circle intersects an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ precisely at three points A, B, C as shown in the figure. AB is a diameter of the circle and is perpendicular to the major axis of the ellipse. If the eccentricity of the ellipse is 4/5, find the length of the diameter AB is terms of a.



19. The point P on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is joined to the ends A,A' of the major axis. If the lines through perpendicular to PA, PA' meet the major axis in Q and R then prove that $\ell(QR)$ = length of latus rectum.

20. Given the equation of the ellipse $\frac{(x-3)^2}{16}+\frac{(y+4)^2}{49}=1$, a parabola is such that its vertex is the lowest point of the ellipse and it passes through the ends of the minor axis of the ellipse. The equation of the parabola is in the form $16y=a(x-h)^2-k$. Determine the value of

$(a+h+k)$.

COMPREHENSION

21. Consider the parabola $y^2=4x$ and the ellipse $2x^2+y^2=6$, intersecting at P and Q.
(a) Find the area enclosed by the parabola and the common chord of the ellipse and parabola.
(b) If tangent and normal at the point P on the ellipse intersect the x-axis at T and G respectively then find the area of the triangle PTG.

Nishant Jindal

MATRIX MATCH TYPE

22. Consider an ellipse $\frac{x^2}{25}+\frac{y^2}{9}=1$ with centre C and al point P on it with eccentric angle $\frac{\pi}{4}$. Normal drawn at P intersects the major and minor axes in A and B respectively. N_1 and N_2 are the feet of the perpendiculars from the foci S_1 and S_2 respectively on the tangent at P and N is the foot of the perpendicular from the centre of the ellipse on the normal at P. Tangent at P intersects the axis of x at T. Match the entries of Column-I with the entries of Column-II.

Column-I	**Column-II**
(A) (CA)(CT) is equal to	(P) 9
(B) (PN)(PB) is equal to	(Q) 16
(C) $(S_1\ N_1)(S_2\ N_2)$ is equal to	(R) 17
(D) $(S_1P)(S_2P)$ is equal to	(S) 25

Nishant Jindal

EXERCISE - 4

| Level-I Previous Year | JEE Main

1. In an ellipse, the distance between its foci is 6 and minor axis is 8 . Then its eccentricity is
(A) $\frac{1}{2}$ (B) $\frac{4}{5}$ (C) $\frac{1}{\sqrt{5}}$ (D) $\frac{3}{5}$ **[AIEEE 2006]**

2. A focus of an ellipse is at the origin. The directrix is the line $x = 4$ and the eccentricity is $\frac{1}{2}$. Then the length of the semi-major axis is - **[AIEEE 2008]**
(A) $\frac{2}{3}$ (B) $\frac{4}{3}$ (C) $\frac{5}{3}$ (D) $\frac{8}{3}$

3. **Statement1:** An equation of a common tangent to the parabola $y^2 = 16\sqrt{3}x$ and the ellipse $2x^2 + y^2 = 4$ is $y = 2x + 2\sqrt{3}$
Statement 2: If the line $y = mx + \frac{4\sqrt{3}}{m}$, $(m \neq 0)$ is a common tangent to the parabola $y^2 = 16\sqrt{3}x$ and the ellipse $2x^2 + y^2 = 4$,then m satisfies $m^4 + 2m^2 = 24$ **[AIEEE 2012]**
(A) Statement 1 is true, Statement 2 is true, Statement 2 is not a correct explanation for statement 1 .
(B) Statement 1 is true, Statement 2 is false.
(C) Statement 1 is false, Statement 2 is true.
(D) Statement 1 is true, Statement 2 is true, Statement 2 is a correct explanation for statement 1 .

4. An ellipse is drawn by taking a diameter of the circle $(x - 1)^2 + y^2 = 1$, as its semi-minor axis and a diameter of the circle $x^2 + (y - 2)^2 = 4$ as its semi-major axis. If the centre of the ellipse is at the origin and its axes are the coordinate axes, then the equation of the ellipse is : **[AIEEE 2012]**
(A) $4x^2 + y^2 = 8$ (B) $x^2 + 4y^2 = 16$ (C) $4x^2 + y^2 = 4$ (D) $x^2 + 4y^2 = 8$

5. The equation of the circle passing through the foci of the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$, and having centre at (0,3) is : **[JEE MAIN 2013]**
(A) $x^2 + y^2 - 6y - 5 = 0$ (B) $x^2 + y^2 - 6y + 5 = 0$
(C) $x^2 + y^2 - 6y - 7 = 0$ (D) $x^2 + y^2 - 6y + 7 = 0$

6. The locus of the foot of perpendicular drawn from the centre of the ellipse $x^2 + 3y^2 = 6$ on any tangent to it is : **[JEE MAIN 2014]**
(A) $(x^2 - y^2)^2 = 6x^2 + 2y^2$ (B) $(x^2 - y^2)^2 = 6x^2 - 2y^2$
(C) $(x^2 + y^2)^2 = 6x^2 + 2y^2$ (D) $(x^2 + y^2)^2 = 6x^2 - 2y^2$

7. The area (in sq. units) of the quadrilateral formed by the tangents at the end points of the latera recta to the ellipse $\frac{x^2}{9} + \frac{y^2}{5} = 1$, is : **[JEE MAIN 2015]**
(A) $\frac{27}{2}$ (B) 27 (C) $\frac{27}{4}$ (D) 18

8. If the curves $y^2 = 6x$, $9x^2 + by^2 = 16$ intersect each other at right angles, then the value of b is :
(A) $\frac{9}{2}$ (B) 6 (C) $\frac{7}{2}$ (D) 4 **[JEE MAIN 2018]**

EXERCISE - 4

| Level-II Previous Year | JEE Advanced

1. Let $P(x_1, y_1)$ and $Q(x_2, y_2), y_1 < 0, y_2 < 0$, be the end points of the latus rectum of the ellipse $^2 + 4y^2 = 4$. The equations of parabolas with latus rectum PQ are **[JEE 2008, 4]**

(A) $x^2 + 2\sqrt{3}y = 3 + \sqrt{3}$ (B) $x^2 - 2\sqrt{3}y = 3 + \sqrt{3}$

(C) $x^2 + 2\sqrt{3}y = 3 - \sqrt{3}$ (D) $x^2 - 2\sqrt{3}y = 3 - \sqrt{3}$

2. The line passing through the extremity A of the major axis of extremity B of the minor axis of the ellipse $x^2 + 9y^2 = 9$ meets the auxiliary circle at the point M. Then the area of the triangle with vertices at A, M and the origin O is **[JEE 2009]**

(A) 31/10 (B) 29/10 (C) 21/10 (D) 27/10

3. The normal at a point P on the ellipse $x^2 + 4y^2 = 16$ meets the X-axis at Q. If M is the midpoint of the line segment PQ, then the locus of M intersects the latus rectums of the given ellipse at the points **[JEE 2009]**

(A) $\left(\pm\frac{3\sqrt{5}}{2}, \pm\frac{2}{7}\right)$ (B) $\left(\pm\frac{3\sqrt{5}}{2}, \pm\sqrt{\frac{19}{4}}\right)$ (C) $\left(\pm 2\sqrt{3}, \pm\frac{1}{7}\right)$ (D) $\left(\pm 2\sqrt{3}, \pm\frac{4\sqrt{3}}{7}\right)$

Paragraph for questions 4 to 6

Tangents are drawn from the point P(3,4) to the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ touching the ellipse at points A and B **[JEE 2010]**

4. The coordinates of A and B are

(A) (3,0) and (0,2) (B) $\left(-\frac{8}{5}, \frac{2\sqrt{161}}{15}\right)$ and $\left(-\frac{9}{5}, \frac{8}{5}\right)$

(C) $\left(-\frac{8}{5}, \frac{2\sqrt{161}}{15}\right)$ and (0,2) (D) (3,0) and $\left(-\frac{9}{5}, \frac{8}{5}\right)$

5. The orthocenter of the triangle PAB is

(A) $\left(5, \frac{8}{7}\right)$ (B) $\left(\frac{7}{5}, \frac{25}{8}\right)$ (C) $\left(\frac{11}{5}, \frac{8}{5}\right)$ (D) $\left(\frac{8}{25}, \frac{7}{5}\right)$

6. The equation of the locus of the point whose distances from the point P and the line AB are equal is

(A) $9x^2 + y^2 - 6xy - 54x - 62y + 241 = 0$

(B) $x^2 + 9y^2 + 6xy - 54x + 62y - 241 = 0$

(C) $9x^2 + 9y^2 - 6xy - 54x - 62y - 241 = 0$

(D) $x^2 + y^2 - 2xy + 27x + 31y - 120 = 0$

7. A vertical line passing through the point (h, 0) intersects the ellipse $\frac{x^2}{4} + \frac{y^2}{3} = 1$ at the points P and Q. Let the tangents to the ellipse at P and Q meet at the point R. If $\Delta(h) =$ area of the triangle PQR, $\Delta_1 = \max_{1/2 \le h \le 1} \Delta(h)$ and $\Delta_2 = \min_{1/2 \le h \le 1} \Delta(h)$, then $\frac{8}{\sqrt{5}}\Delta_1 - 8\Delta_2 =$ **[JEE 2013]**

8. Let E_1 and E_2 be two ellipses whose centers are at origin. The major axes of E_1 and E_2 lie along the x axis and the y-axis, respectively. Let S be the circle $x^2 + (y-1)^2 = 2$. The straight line

$x + y = 3$ touches the curves S, E_1 and E_2 at P, Q and R, respectively.

Suppose that $PQ = PR = \frac{2\sqrt{2}}{3}$. If e_1 and e_2 are the eccentricites of E_1 and E_2, respectively, then

Nishant Jindal

the correct expression(s) is (are) **[JEE 2015]**

(A) $e_1^2 + e_2^2 = \frac{43}{40}$ (B) $e_1 e_2 = \frac{\sqrt{7}}{2\sqrt{10}}$ (C) $|e_1^2 + e_2^2| = \frac{5}{8}$ (D) $e_1 e_2 = \frac{\sqrt{3}}{4}$

9. Suppose that the foci of the ellipse $\frac{x^2}{9} + \frac{y^2}{5} = 1$ are $(f_1, 0)$ and $(f_2, 0)$ where $f_1 > 0$ and $f_2 < 0$. Let P_1 and P_2 be two parabolas with a common vertex at (0,0) and with foci at $(f_1, 0)$ and $(2f_2, 0)$, respectively. Let T_1 be a tangent to P_1 which passes through $(2f_2, 0)$ and T_2 be a tangent to P_2 which passes through $(f_1, 0)$. If m_1 is the slope of T_1 and m_2 is the slope of T_2, then the value of $\left(\frac{1}{m_1^2} + m_2^2\right)$ is. **[JEE 2015]**

Paragraph for Question Nos. 10 to 11

Let $F_1(x_1, 0)$ and $F_2(x_2, 0)$, for $x_1 < 0$ and $x_2 > 0$, be the foci of the ellipse $\frac{x^2}{9} + \frac{y^2}{8} = 1$. Suppose a parabola having vertex at the origin and focus at F_2 intersects the ellipse at point M in the first quadrant and at point N in the fourth quadrant. **[JEE 2016]**

10. The orthocentre of the triangle F_1MN is

(A) $\left(-\frac{9}{10}, 0\right)$ (B) $\left(\frac{2}{3}, 0\right)$ (C) $\left(\frac{9}{10}, 0\right)$ (D) $\left(\frac{2}{3}, \sqrt{6}\right)$

11. If the tangents to the ellipse at M and N meet at R and the normal to the parabola at M meets the x axis at Q, then the ratio of area of the triangle MQR to area of the quadrilateral MF_1NF_2 is

(A) 3: 4 (B) 4: 5 (C) 5: 8 (D) 2: 3 **[JEE MAIN 2018]**

12. Consider two straight lines, each of which is tangent to both the circles $x^2 + y^2 = \frac{1}{2}$ and the parabola $y^2 = 4x$. Let these lines intersect at the point Q. consider the ellipse whose center is at the origin O(0,0) and whose semi - major axis is OQ. If the length of the minor axis of this ellipse is $\sqrt{2}$, Then which of the following statment(s) is (are) TRUE ? **[JEE 2018]**

(A) For the ellipse, the eccentricity is $\frac{1}{\sqrt{2}}$ and the length of the latus rectum is 1

(B) For the ellipse, the eccentricity is $\frac{1}{2}$ and the length of the latus rectum is $\frac{1}{2}$

(C) The area of the region bouded by the ellipse between the lines $x = \frac{1}{\sqrt{2}}$ and $x = 1$ is $\frac{1}{4\sqrt{2}}(\pi - 2)$

(D) The area of the region bounded by the ellipse between the lines $x = \frac{1}{\sqrt{2}}$ and $x = 1$ is $\frac{1}{16}(\pi - 2)$

13. Define the collections $\{E_1, E_2, E_3 \ldots \ldots\}$ of ellipse and $\{R_1, R_2, R_3 \ldots \ldots\}$ of rectangles as follows :

$E_1: \frac{x^2}{9} + \frac{y^2}{4} = 1$;

R_1 : rectangle of largest area, with sides parallel to the axes, inscribed in E_1;

E_n : ellipse $\frac{x^2}{a_n^2} + \frac{y^2}{b_n^2} = 1$ of largest area inscribed in $R_{n-1}, n > 1$;

R_n : rectangle of largest area, with sides parallel to the axes, inscribed in E_n , $n > 1$

Then which of the following options is/are correct? **[JEE 2019]**

(A)The eccentricities of E_{18} and E_{19} are NOT equal

(B) The distance of a focus from the centre in E_9 is $\frac{\sqrt{5}}{32}$

(C) $\sum_{n=1}^{N}$ (area of R_n) < 24, for each positive integer N

(D) The length of latus rectum of E_9 is $\frac{1}{6}$

Nishant Jindal

ANSWER KEY

EXERCISE - I JEE Main

1.	A	**2.**	B	**3.**	C	**4.**	A	**5.**	C	**6.**	A	**7.**	B
8.	D	**9.**	A	**10.**	C	**11.**	B	**12.**	C	**13.**	C	**14.**	A
15.	A												

EXERCISE - II
JEE Advance Single correct Option - type Questions

1.	B	**2.**	A	**3.**	B	**4.**	D	**5.**	A	**6.**	A	**7.**	A
8.	B	**9.**	C	**10.**	A	**11.**	A	**12.**	C	**13.**	B		

Multiple correct Option - type Questions

1.	C, D	**2.**	A,B	**3.**	A,C,D	**4.**	C,D	**5.**	A,B,C

EXERCISE - III Subjective - type Questions

1. **(a)** $\frac{(x-1)^2}{45}+\frac{(y-2)^2}{20}=1$ **(b)** $\frac{3x^2}{32}+\frac{3y^2}{32}=1$

4. 85 **6.** $x+y-5=0, x+y+5=0$ **7.** 24 sq. units **11.** $55\sqrt{2}$ sq. units

13. $bx+a\sqrt{3}y=2ab$ **16.** 16 **18.** $\frac{18a}{17}$ **20.** 186

Comprehension - based Questions

21. **(a)** 8/3, **(b)** 4

Matrix Match - type Questions

22. (A) Q; (B) S; (C) P; (D) R

EXERCISE - IV

Previous Year's Question JEE Main

1.	D	**2.**	D	**3.**	D	**4.**	B	**5.**	C	**6.**	C	**7.**	B
8.	A												

JEE Advanced

1.	B, C	**2.**	D	**3.**	C	**4.**	D	**5.**	C	**6.**	A	**7.**	9
8.	AB	**9.**	4	**10.**	A	**11.**	C	**12.**	A,C	**13.**	C,D		