Nishant Jindal

$\theta_1 \in \left(0,\frac{\pi}{2}\right) \qquad \theta_2 \in \left(0,\frac{\pi}{2}\right)$

1. $\sin^{-1}\frac{2}{3} + \sin^{-1}\frac{6}{7} = \sin^{-1}\sin(\theta_1+\theta_2) = \sin^{-1}\left(\frac{2}{3}\frac{\sqrt{13}}{7} + \frac{6}{7}\frac{\sqrt{5}}{3}\right)$

$= \pi - (\theta_1+\theta_2)$

$\theta_1+\theta_2 \in (0,\pi)$

(a) $\sin^{-1}\left(\frac{2\sqrt{13} + 6\sqrt{5}}{21}\right)$

(b) $\pi - \sin^{-1}\left(\frac{2\sqrt{13}+6\sqrt{5}}{21}\right)$

(c) $\cos^{-1}\left(\frac{\sqrt{65} - 12}{21}\right)$

(d) $2\pi - \cos^{-1}\left(\frac{\sqrt{65}-12}{21}\right)$

$\theta_1+\theta_2 = \cos^{-1}\left(\cos(\theta_1+\theta_2)\right) = \cos^{-1}\left(\frac{\sqrt{5}}{3}\frac{\sqrt{13}}{7} - \frac{2}{3}\frac{6}{7}\right)$

sin

$\theta_1+\theta_2 \in \left(\frac{\pi}{2},\pi\right) \Leftarrow \cos(\theta_1+\theta_2) = \frac{\sqrt{65}-12}{21} < 0$

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$\theta_1 \in (0, \pi/2)$ $\theta_2 \in (0, \frac{\pi}{2})$ $\cos^{-1}\cos(\theta_1 - \theta_2) = \cos^{-1}\left(\frac{3\times 1 + \sqrt{7}\sqrt{8}}{12}\right)$

Q. $\cos^{-1}\frac{3}{4} - \cos^{-1}\frac{1}{3} = -(\theta_1 - \theta_2)$ $\theta_1 - \theta_2 \in (-\frac{\pi}{2}, 0)$

$-\theta_2 \in (-\frac{\pi}{2}, 0)$

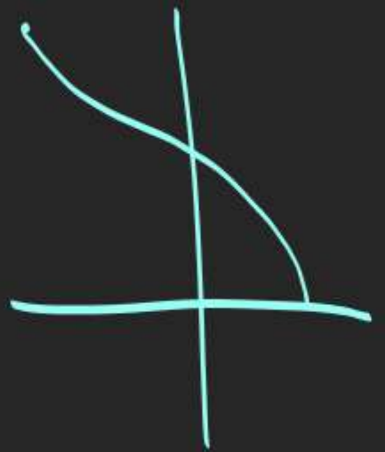
$\theta_1 - \theta_2 \in (-\frac{\pi}{2}, \frac{\pi}{2})$

(a) $\cos^{-1}\left(\frac{3+\sqrt{56}}{12}\right)$

(b) $-\cos^{-1}\left(\frac{3+\sqrt{56}}{12}\right)$

(c) $\sin^{-1}\left(\frac{\sqrt{7} - 3\sqrt{8}}{12}\right)$

(d) $\pi - \sin^{-1}\left(\frac{\sqrt{7} - 3\sqrt{8}}{12}\right)$



$\frac{3}{4} > \frac{1}{3}$

$\theta_1 < \theta_2$

$\sin^{-1}\sin(\theta_1 - \theta_2) = \sin^{-1}\left(\frac{\sqrt{7}}{4}\frac{1}{3} - \frac{\sqrt{8}}{3}\frac{3}{4}\right)$

$= (\theta_1 - \theta_2)$

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$$\sin^{-1}x + \sin^{-1}y = \begin{cases} \sin^{-1}\left(x\sqrt{1-y^2} + y\sqrt{1-x^2}\right) & 0<x<1,\ 0<y<1,\ x^2+y^2<1 \\ \pi - \sin^{-1}\left(x\sqrt{1-y^2} + y\sqrt{1-x^2}\right), & 0<x<1,\ 0<y<1,\ x^2+y^2>1 \end{cases}$$

$\theta_1 \in (0, \frac{\pi}{2})$ $\theta_2 \in (0, \frac{\pi}{2})$

$\theta_1 + \theta_2 \in (0, \pi)$

$\sin^{-1}x - \sin^{-1}y$

$\sin^{-1}\left(x\sqrt{1-y^2} - y\sqrt{1-x^2}\right)$

$0<x<1,\ 0<y<1$

$\cos^{-1}x + \cos^{-1}y$ $0<x<1,\ 0<y<1$

$\cos^{-1}x - \cos^{-1}y$ $0<x<1,\ 0<y<1$

$\sin^{-1}\sin(\theta_1+\theta_2) = \sin^{-1}\left(x\sqrt{1-y^2} + y\sqrt{1-x^2}\right)$

$\pi - (\theta_1+\theta_2)$

$\cos(\theta_1+\theta_2) = \sqrt{1-x^2}\sqrt{1-y^2} - xy > 0$ $\theta_1+\theta_2 \in (0, \frac{\pi}{2})$

$x^2+y^2<1 \Leftarrow (1-x^2)(1-y^2) > x^2y^2$

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$$\cos^{-1}x+\cos^{-1}y = \cos^{-1}\left(xy-\sqrt{1-x^2}\sqrt{1-y^2}\right) \quad 0<x<1,\ 0<y<1$$

$$\cos^{-1}x-\cos^{-1}y = \begin{cases} \cos^{-1}\left(xy+\sqrt{1-x^2}\sqrt{1-y^2}\right) & 0<x<1,\ 0<y<1,\ x\le y \\ -\cos^{-1}\left(xy+\sqrt{1-x^2}\sqrt{1-y^2}\right) & 0<x<1,\ 0<y<1,\ x>y \end{cases}$$

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1. $f: R \to R$, $f(x) = \dfrac{x(x^4+1)(x+1)+x^4+2}{x^2+x+1}$

Continuous

cont. $\forall x>0$

$x \to \pm\infty, y \to \infty$ $\quad \dfrac{x^6\left(1+\dfrac{1}{\ }\right)}{x^2\left(1+\frac{1}{x}+\frac{1}{x^2}\right)}$

$x \to \infty, y \to \infty$

2. Find range of $f(x) = \dfrac{(1+x+x^2)(1+x^4)}{x^3}$ for $\boxed{x>0}$

$y=6$ at $x=1$

$\left(\underbrace{\frac{1}{x}+x}_{}+1\right)\left(\underbrace{\frac{1}{x^2}+x^2}_{}\right) \geq 6$

$\geq 3 \qquad \geq 2$

$\boxed{\text{M-1, Into}}$

$\boxed{R_f = [6, \infty)}$

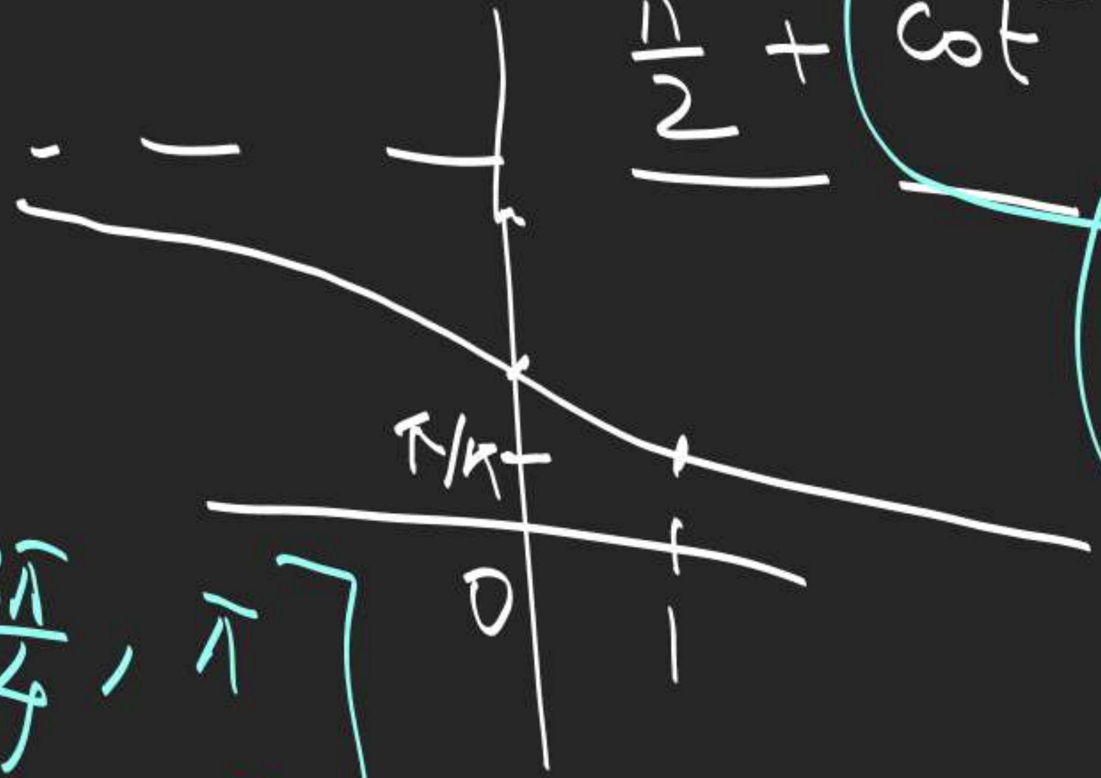
3: Find the range of

$$f(x) = \cot^{-1}\{-x\} + \sin^{-1}\{x\} + \cos^{-1}\{x\}$$

$D_f = R$.

$\{\cdot\} = FPF$

$$\frac{\pi}{2} + \cot^{-1}\underbrace{\{-x\}}_{\boxed{[0,1)}} \rightarrow \left(\frac{\pi}{4}, \frac{\pi}{2}\right]$$



$$R_f = \left(\frac{3\pi}{4}, \pi\right]$$

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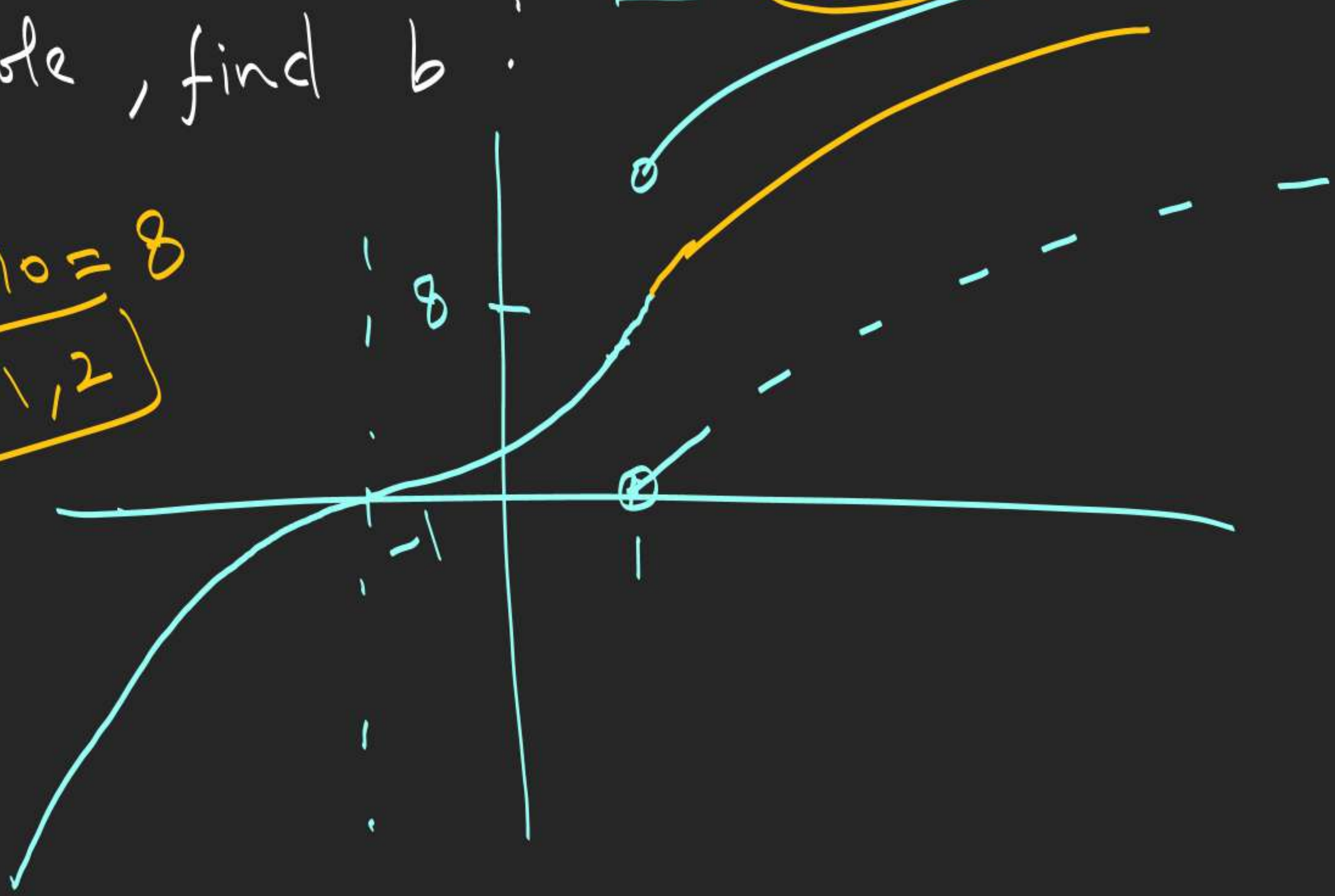
4. Let $f : R \to R$, $f(x) = \begin{cases} (x+1)^3 & , x \le 1 \\ \ln x + (b^2 - 3b + 10) & , x > 1 \end{cases}$

is invertible, find b.

$b^2 - 3b + 10 = 8$

$\boxed{b = 1, 2}$



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5. $f:(-\infty,2]\to(-\infty,4]$, $f(x)=x(4-x)$, find $f^{-1}(x)$

$f^{-1}:(-\infty,4]\to(-\infty,2]$

$f^{-1}(x)\left(4-f^{-1}(x)\right)=x$

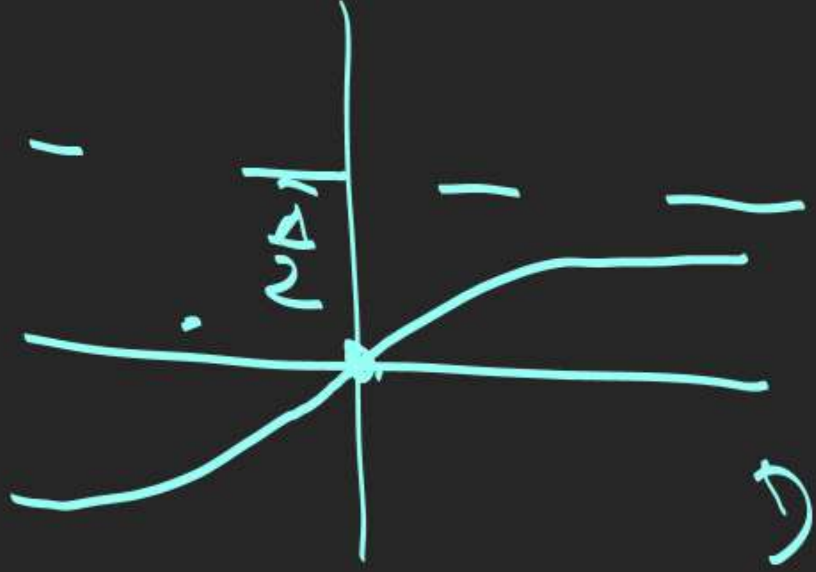
$t^2-4t+x=0 \Rightarrow f^{-1}(x)=2\pm\sqrt{4-x}$

$$\boxed{f^{-1}(x)=2-\sqrt{4-x}}$$

6. $f:R\to\left[0,\frac{\pi}{2}\right)$, $f(x)=\tan^{-1}(3x^2+bx+c)$ is surjective, then

(a) $b^2=3c$ (b) $b^2=4c$ (c) $b^2=8c$ (d) $b^2=12c$.

$3x^2+bx+c \in[0,\infty)\ \forall x\in R$.



$D=0$ $\boxed{b^2-12c=0}$