Nishant Jindal

$$16(a-b)^2 - 4\left((a-b)^2 - c^2\right)$$

$$= 12(a-b)^2 + 4c^2 \geq 0.$$

$$= c^2\left(3a^2 - b^2 + 4ab\right)$$

16. $\alpha+\beta = 0 \longrightarrow \boxed{b(m+1) + a(m-1) = 0}$

$$(x^2 - bx)(m+1) = (m-1)(ax - c)$$

$$(m+1)x^2 - \left(b(m+1) + a(m-1)\right)x + c(m-1) = 0$$

$$\frac{\boxed{(\alpha-\beta)^2}(\alpha+\beta)^2}{\alpha^2\beta^2}$$

$$= \frac{\left((\alpha+\beta)^2 - 4\alpha\beta\right)(\alpha+\beta)^2}{(\alpha\beta)^2}$$

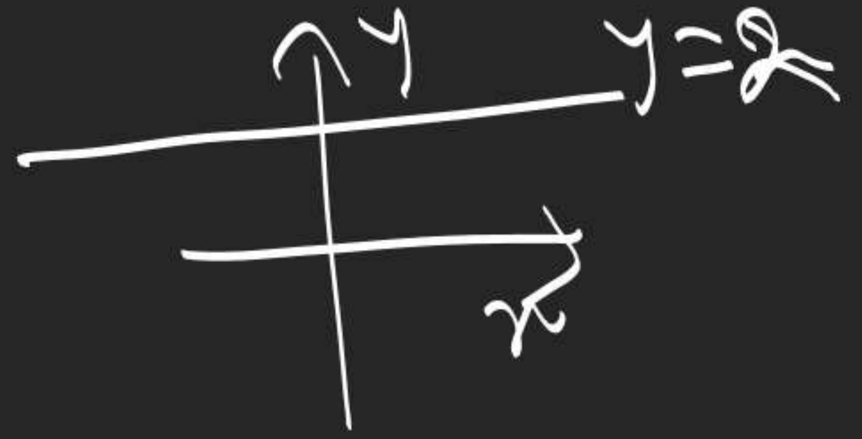
$$= \left(3a^2c + b^2c\right)^2 + 4abc^2\left(6a^2 + ab - 2b^2\right)$$

$$= c^2\left[9a^4 + b^4 + 10a^2b^2 + 24a^3b - 8ab^3\right]$$

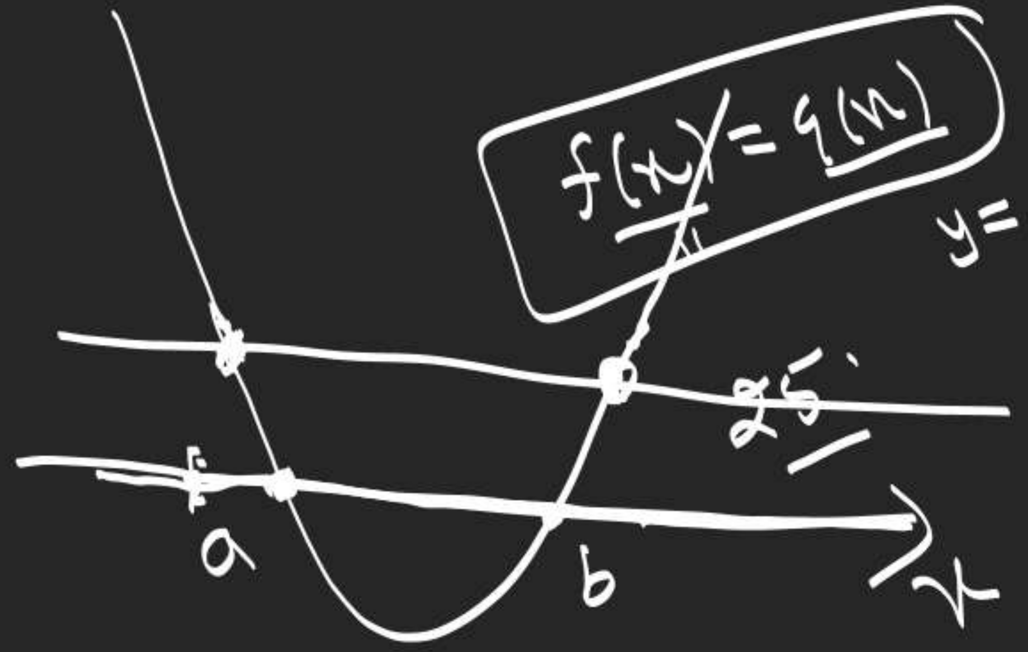
Nishant Jindal

$x = a(1 - i\sqrt{3})$ $\quad$ $a(1 + i\sqrt{3})$

$x^2 - 2ax + 4a^2 = 0$ $\quad$ $a^2(1+3)$



$$x^3 - ax^2 + 2a^2x + 4a^3 = (\underbrace{x^2 - 2ax + 4a^2}_{=0})(x + a) = 0$$



$\boxed{f(x) = g(x)}$

$y = (x - a)(x - b)$

$y = h^2 \geq 0$

$$x^2 - (a+b)x + ab - h^2 = 0$$

$$D = (a+b)^2 - 4(ab - h^2) = (a-b)^2 + 4h^2 \geq 0$$

Nishant Jindal

$$(ax_1+b)^{-3}+(ax_2+b)^{-3} = -\frac{1}{c^3}\left(x_1^3+x_2^3\right)$$

$$ax^2+bx+c=0$$

$$ax_1+b = -\frac{c}{x_1}$$

$$ax^2+bx+c=0 \quad \begin{cases} \alpha \\ n\alpha \end{cases}$$

$$-\frac{b}{a} = (n+1)\alpha \quad —(1)$$

$$\frac{c}{a} = n\alpha^2 \quad —(2)$$

$$\frac{(1)^2}{(2)} \Rightarrow \frac{b^2}{ac} = \frac{(n+1)^2}{n}.$$

Nishant Jindal

$$(\alpha+\beta)^2 \qquad , \quad (\alpha-\beta)^2 = (\alpha+\beta)^2 - 4\alpha\beta$$

$$= (-(m+n))^2 \qquad\qquad = (m+n)^2 - 2(m^2+n^2)$$

$$= (m+n)^2 \qquad\qquad = -(m-n)^2 .$$

$$x^2 - (4mn)x - (m^2-n^2)^2 = 0 .$$

Nishant Jindal

Condition for two quadratic equations $a_1x^2+b_1x+c_1=0$ & $a_2x^2+b_2x+c_2=0$ to have a common root

α

① — $a_1\alpha^2+b_1\alpha+c_1=0$

② — $a_2\alpha^2+b_2\alpha+c_2=0$

$$\frac{\alpha^2}{b_1c_2-b_2c_1}=\frac{\alpha}{c_1a_2-c_2a_1}=\frac{1}{a_1b_2-a_2b_1}$$

① $\times b_2$ − ② $\times b_1$

$(a_1b_2-a_2b_1)\alpha^2+b_2c_1-b_1c_2=0$

① $\times a_2$ − ② $\times a_1$ $\quad \alpha=?$

$$\frac{b_1c_2-b_2c_1}{a_2c_1-c_2a_1}=\frac{c_1a_2-c_2a_1}{a_1b_2-a_2b_1}$$

Nishant Jindal

Condition for two quadratic eqns $a_1x^2+b_1x+c_1=0$ & $a_2x^2+b_2x+c_2=0$ to have both roots common.

$$a_1x^2+b_1x+c_1 = a_1(x-\alpha)(x-\beta)$$

$$a_2x^2+b_2x+c_2 = a_2(x-\alpha)(x-\beta)$$

$$\boxed{\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}}$$

Nishant Jindal

1. Find k for which equations $3x^2+4kx+2=0$ and $2x^2+3x-2=0$ have a common root.

$4x-x$

$(2x-1)(x+2)=0$

$x=\frac{1}{2},-2$

$x=\frac{1}{2}$ satisfy

$\frac{3}{4}+2k+2=0$

$\Rightarrow \boxed{k=-\frac{11}{8}}$

$x=-2$

$12-8k+2=0$

$\boxed{k=\frac{7}{4}}$

Nishant Jindal

Q. If the equations $x^2-4x+5=0$ and $x^2+ax+b=0$, $a, b \in R$ have a common root, find a, b.

imaginary

both roots common

$$\frac{1}{1} = -\frac{4}{a} = \frac{5}{b}$$

$$\boxed{a=-4, \quad b=5}$$

Nishant Jindal

3: If one root of eqn. $x^2-x+3a=0$ is double the root of equation $x^2-x+a=0$, find a, $(a\neq 0)$

$x=\alpha$ $\qquad$ $x=2\alpha$

$$\alpha^2-\alpha+a=0 \quad \text{—} \ (1)$$

$$4\alpha^2-2\alpha+3a=0 \quad \text{—} \ (2)$$

$$(1)\times 4-(2)$$

$$-2\alpha+a=0 \Rightarrow \alpha=\frac{a}{2}$$

$$\frac{a^2}{4}-\frac{a}{2}+a=0$$

$$\frac{a^2}{4}+\frac{a}{2}=0$$

$$a(a+2)=0$$

$$\boxed{a=-2}$$

Nishant Jindal

4. If the quadratic equation ① — $x^2+bx+c=0$ and $x^2+cx+b=0$ — ② $(b \neq c)$ have a common root, then P.T. their uncommon roots are the roots of equation

$x^2+x+bc=0$

$x = \alpha$ $x = \alpha$

1   c   c   b

$\alpha^2+b\alpha+c=0$

$\alpha^2+c\alpha+b=0$

$(b-c)\alpha+c-b=0$

$\boxed{\alpha=1}$

$\boxed{1+b+c=0}$

$x^2-(b+c)x+bc=0$

$\Rightarrow \boxed{x^2+x+bc=0}$

5. If $Q_1(x) = x^2 + (k-29)x - k = 0$ — ① and $Q_2(x) = 2x^2 + (2k-43)x + k = 0$ — ② both are factors of a ~~cubic polynomial~~, find k.

common root.

① × 2 − ②

$-15x - 3k = 0$

$$\boxed{x = -\frac{k}{5}}$$

$$\frac{k^2}{25} - \frac{k}{5}(k-29) - k = 0$$

$$\Rightarrow \boxed{k = 0,\ 30}$$

$(x-1)^2$

$(x-2)(x-3)$

$(x-\alpha)(x-\beta)$

$2(x-\gamma)(x-\delta)$

$(x-\alpha)(x-\beta)(x-\gamma)(x-\delta)$

$(x-1)(x-2)$

$(x-2)(x-3)$

$(x-1)(x-2)(x-3)$

Nishant Jindal

6. Find the cubic each of whose roots is greater by unity than a root of the equation

$x^3 - 5x^2 + 6x - 3 = 0$. α, β, γ

$\alpha+1, \beta+1, \gamma+1$

y

$x \; \alpha, \beta, \gamma$

$y = x + 1$

Put $x = y - 1$

$(y-1)^3 - 5(y-1)^2 + 6(y-1) - 3 = 0$

$y^3 - 8y^2 + 19y - 15 = 0$

$$\boxed{x^3 - 8x^2 + 19x - 15 = 0}$$

Nishant Jindal

$x^3 - x^2 + 1 = 0 \quad \begin{matrix} a \\ b \\ c \end{matrix}$

$\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} = ?$

$\Rightarrow \frac{1}{a^2}, \frac{1}{b^2}, \frac{1}{c^2}$

$x \begin{matrix} a \\ b \\ c \end{matrix}$

$$\boxed{y^3 - 2y^2 + y - 1 = 0} \quad \begin{matrix} \frac{1}{a^2} \\ \frac{1}{b^2} \\ \frac{1}{c^2} \end{matrix}$$

$y = \frac{1}{x^2} \Rightarrow x = \frac{1}{\sqrt{y}}$ ✓

$$\boxed{\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} = 2}$$

$$\boxed{\begin{array}{c} \underline{\text{Sec } 1.4} \\ 46\text{–}60 \end{array}}$$

$$\frac{1}{y\sqrt{y}} - \frac{1}{y} + 1 = 0$$

$$\frac{1}{y\sqrt{y}} = \left(\frac{1}{y} - 1\right) \Rightarrow \frac{1}{\sqrt{y}} = 1 - y$$

$$\Rightarrow \frac{1}{y} = 1 - 2y + y^2$$