Nishant Jindal

Fundamentals of Mathematics

Remainder $= 0 \cdot x^2 + 2x + 1$

Q Find Remainder if $x^{135} + x^{125} - x^{115} + x^5 + 1$ is divided by $x^3 - x$?

Ans $= 2x + 1$

$P(0) = 0 + 0 - 0 + 0 + 1 \Rightarrow P(0) = 1$

$P(x) = x^{135} + x^{125} - x^{115} + x^5 + 1$

$P(-1) = (-1) + (-1) - (-1) + (-1) + 1$

$P(1) = 1 + 1 - 1 + 1 + 1 = 3$

$P(-1) = -1$

$P(x) = x(x-1)(x+1)Q(x) + ax^2 + bx + c$

Cubic — Q Eqn

$x^3 - x = (x)(x-1)(x+1)$

$a = 0, 1, -1$

$P(0) = 0 + c = 1 \Rightarrow \boxed{c = 1}$

$P(1) = 0 + a + b + c \Rightarrow a + b + c = 3 \Rightarrow \boxed{a + b = 2}$

$P(-1) = 0 + a - b + c \Rightarrow a - b + c = -1 \Rightarrow a - b = -2$

add $\Rightarrow 2a = 0$

$a = 0, b = 2, c = 1$

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Fundamentals of Mathematics

Ratio & Proportion.

A) If a & b are Quantities of same type then Ratio is denoted by $a:b$ or $\frac{a}{b}$.

B) If A & B are same Quantities then Ratio = A : B

Ratio A : B

then $A^2 : B^2$ => Duplicate Ratio

$A^3 : B^3$ => Triplicate Ratio.

$A^{1/2} : B^{1/2}$ => Sub Duplicate Ratio

$A^{1/3} : B^{1/3}$ => Sub Triplicate Ratio

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(C) Ratios can also be Compounded.

If $x:y$, $a:b$, $m:n$ are 3 Ratio

then Compound Ratio $= \frac{x}{y} \times \frac{a}{b} \times \frac{m}{n}$.

(D) $\frac{a}{b} = \frac{2a}{2b} = \frac{17a}{17b} \Rightarrow \boxed{\frac{a}{b} = \frac{ma}{mb} = \frac{na}{nb}}$

(E) If a, b & c, d are in Proportion then it means $a:b::c:d \Rightarrow \frac{a}{b} = \frac{c}{d}$

$1:2::2:4$

$\frac{1}{2} = \frac{2}{4}$

$2:4::3:6$

$\frac{2}{4} = \frac{3}{6}$

$2, 4$ & $3, 6$ are in Proportion.

Q $1, 4$ & $13, 52$ are in Proport?

$\frac{1}{4} = \frac{13}{52}$

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Fundamentals of Mathematics

(F) (1) If a, b, c are in Continued Proportion

$$a : b :: b : c$$

$$\Rightarrow \frac{a}{b} = \frac{b}{c} \Rightarrow \boxed{a \cdot c = b^2}$$

(2) If a, b, c, d are in Continued Proportion

$$\frac{a}{b} = \frac{b}{c} = \frac{c}{d}$$

(G) Componendo

$$\frac{a}{b} = \frac{c}{d}$$

then $\frac{a}{b} + 1 = \frac{c}{d} + 1$

$$\Rightarrow \frac{a+b}{b} = \frac{c+d}{d}$$

$$\frac{a}{b} = \frac{c}{d} \xrightarrow{\text{(om)}} \frac{a+b}{b} = \frac{c+d}{d}$$

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H Dividendo

$$\frac{a}{b} = \frac{c}{d} \Rightarrow \frac{a}{b} - 1 = \frac{c}{d} - 1$$

If $\frac{a}{b} = \frac{c}{d}$ then $\frac{a-b}{b} = \frac{c-d}{d}$.

Niche Wale Ko Jodna.

Niche Wale Ko Ghatana.

Componendo & Dividendo

$$\Rightarrow \boxed{\frac{a}{b} = \frac{c}{d} \Rightarrow \frac{a+b}{a-b} = \frac{c+d}{c-d}}$$

$$\frac{x}{4} = \frac{3}{4}$$

C&D

$$\boxed{\frac{x+4}{x-4} = \frac{3+4}{3-4}}$$

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Fundamentals of Mathematics

(J) If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f}$

Teeno ka Har tareekey Ka linear combination BIntRe equal hoga.

Ex $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \frac{a+c+e}{b+d+f} = \frac{bK+dK+fK}{b+d+f}$ ⇒ $K\frac{(b+d+f)}{(b+d+f)}$

Ex: $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \frac{a-c+e}{b-d+f}$

Ex: $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \frac{2a-3c+5e}{2b-3d+5f}$

Q $\frac{x}{y} = \frac{m}{n}$ then $\frac{x+3m}{y+3n}$ is equal to their Ratio [T/F]

True

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(J) If $\frac{a}{b}=\frac{c}{d}=\frac{e}{f}$

Teeno ka Har tareekey Ka linear combination BIlkul equal hoga.

Ex $\frac{a}{b}=\frac{c}{d}=\frac{e}{f}=\frac{a+c+e}{b+d+f}=\frac{bK+dK+fK}{b+d+f}$

$\Rightarrow K\frac{(b+d+f)}{(b+d+f)}$

Ex: $\frac{a}{b}=\frac{c}{d}=\frac{e}{f}=\frac{a-c+e}{b-d+f}$

Ex: $\frac{a}{b}=\frac{c}{d}=\frac{e}{f}=\frac{2a-3c+5e}{2b-3d+5f}$

Q $\frac{x}{y}=\frac{m}{n}$ then $\frac{x+3m}{y+3n}$ equal to their Ratio [T/F]

True

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(1) $a : b = \frac{a}{b}$

(2) $a, b, c \rightarrow$ Continued Proportion

$$\frac{a}{b} = \frac{b}{c}$$

(3) a, b & c, d Proportion

$$\frac{a}{b} = \frac{c}{d}$$

Componendo

(4) $\frac{a}{b} = \frac{c}{d} \Rightarrow \frac{a+b}{b} = \frac{c+d}{d}$

(5) $\frac{a-b}{b} = \frac{c-d}{d}$ dividendo

(6) $\frac{a+b}{a-b} = \frac{c+d}{c-d} \rightarrow$ C&D

(7) $\frac{a}{b} = \frac{c}{d} = \frac{2a-3c}{2b-3d}$

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(1) $\frac{a}{b}=\frac{c}{d}=\frac{e}{f}=\frac{3a+4c-5e}{3b+4d-5f}$ [T/F]

(2) If $\frac{a}{b}=\frac{c}{d}=\frac{e}{f}=\frac{ax+cy+ez}{bx+dy+fz}$ [T/F]

Is bar 3, 4, -5 ki jgh x, y, z daldia

(3) If $\frac{a}{b}=\frac{c}{d}=\frac{e}{f}=\frac{(xa^3+yc^3+ze^3)^{\frac{1}{3}}}{(xb^3+yd^3+zf^3)^{\frac{1}{3}}}$

Let $\frac{a}{b}=\frac{c}{d}=\frac{e}{f}=K \Rightarrow a=bK, c=dK, e=fK$

$$\frac{(xb^3K^3+yd^3K^3+zf^3K^3)^{\frac{1}{3}}}{(xb^3+yd^3+zf^3)^{\frac{1}{3}}}$$

$$K^{3\times\frac{1}{3}}\frac{(xb^3+yd^3+zf^3)^{\frac{1}{3}}}{(xb^3+yd^3+zf^3)^{\frac{1}{3}}}$$

$$=K$$

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Rem

(4) $\frac{a}{b}=\frac{c}{d}=\frac{e}{f}=\frac{(xa^n+yc^n+ze^n)^{\frac{1}{n}}}{(xb^n+yd^n+zf^n)^{1/n}}$.

Q If $\frac{l}{a}=\frac{m}{b}=\frac{n}{c}$ & $l^2+m^2+n^2=1$ find l,m,n in terms of a,b,c?

$$\frac{l}{a}=\frac{m}{b}=\frac{n}{c}=\frac{(l^2+m^2+n^2)^{1/2}}{(a^2+b^2+c^2)^{1/2}}=\frac{1}{\sqrt{a^2+b^2+c^2}}$$

$$l=\frac{a}{\sqrt{a^2+b^2+c^2}},\ m=\frac{b}{\sqrt{a^2+b^2+c^2}},\ n=\frac{c}{\sqrt{a^2+b^2+c^2}}$$

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Fundamentals of Mathematics

$3bx^2+3b=4ax=0$

Q If $x=\dfrac{\sqrt{2a+3b}+\sqrt{2a-3b}}{\sqrt{2a+3b}-\sqrt{2a-3b}}$ then $3bx^2-4ax+3b=?$ (0)

(C&D) $\dfrac{x+1}{x-1}=\dfrac{(\sqrt{2a+3b}+\sqrt{2a-3b})+(\sqrt{2a+3b}-\sqrt{2a-3b})}{(\sqrt{2a+3b}+\sqrt{2a-3b})-(\sqrt{2a+3b}-\sqrt{2a-3b})}$

$\dfrac{a+b}{a-b}$ Jaisa shKl mile to Samjo Ki (&D लगाना है

$\dfrac{x+1}{x-1}=\dfrac{2\sqrt{2a+3b}}{2\sqrt{2a-3b}}$ Sq $\Rightarrow \dfrac{x^2+1+2x}{x^2+1-2x}=\dfrac{2a+3b}{2a-3b}$ shKl (&D Jaisi

$\dfrac{(x^2+1+2x)+(x^2+1-2x)}{(x^2+1+2x)-(x^2+1-2x)}=\dfrac{2a+3b+2a-3b}{2a+3b-(2a-3b)}$

$\dfrac{2(x^2+1)}{4x}=\dfrac{4a}{6b}$

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Fundamentals of Mathematics

Q $\frac{x}{1} = \frac{\sqrt{2a+3b}+\sqrt{2a-3b}}{\sqrt{2a+3b}-\sqrt{2a-3b}}$ then $3bx^2-4ax+3b=?$

(componendo dividendo)

$\frac{(x+1)}{(x-1)} = \frac{\sqrt{2a+3b}}{\sqrt{2a-3b}}$ sq $\Rightarrow \frac{x^2+1+2x}{x^2+1-2x} = \frac{2a+3b}{2a-3b}$ (componendo dividendo)

$\frac{2(x^2+1)}{4x} = \frac{2a}{3b} \Rightarrow 3bx^2+3b = 4ax$

$\Rightarrow 3bx^2-4ax+3b=0$

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Q If $\frac{x+y}{2} = \frac{y+z}{3} = \frac{z+x}{4}$ then find $x:y:z = ?$

$\frac{x+y}{2} = \frac{y+z}{3} = \frac{z+x}{4} = k$

$x+y = 2k$

$y+z = 3k$

$z+x = 4k$

$2(x+y+z) = 9k$

$x+y+z = \frac{9k}{2}$

$x+y+z = \frac{9k}{2}$

$x+y = 2k$

$z = \frac{5k}{2}$

$x+y+z = \frac{9k}{2}$

$y+z = 3k$

$x = \frac{3k}{2}$

$x+y+z = \frac{9k}{2}$

$z+x = 4k$

$y = \frac{k}{2}$

$x:y:z = \frac{3k}{2} : \frac{k}{2} : \frac{5k}{2}$

$= 3:1:5$

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Q

Q If $\frac{a}{b}=\frac{c}{d}$ then $\frac{a^2+b^2}{c^2+a^2}=?$

700 $\frac{a}{b}=\frac{c}{d}$

$\frac{a}{c}=\frac{b}{d}$

A) $\frac{1}{2}$ B) $\frac{a+b}{c+d}$ (C) $\frac{a-b}{c-d}$ (D) $\frac{ab}{cd}$

① $\frac{a}{b}=\frac{c}{a}=K$ $\left.\begin{matrix} a=bK \\ c=dK \end{matrix}\right\}$ Demand $\frac{a^2+b^2}{c^2+d^2}=\frac{b^2K^2+b^2}{d^2K^2+d^2}$

$=\frac{b^2(K^2+1)}{d^2(K^2+1)}=\frac{b^2}{d^2}$

② $\frac{b^2}{d^2}=\frac{b\times b}{d\times d}=\frac{ba}{dc}$

$\frac{a}{b}=\frac{c}{d}$

$\Rightarrow \frac{a}{c}=\frac{b}{d}=\frac{a+b}{c+d}$

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Inequality

Rule (Wavy Curve Method)

(1) Factorise the given eqn & find values of x by them

(2) Arrange all values of x on No Line in Ascending order.

(3) Put +ve Sign to Right most Side of No Line & change Sign only for odd Power brackets

(4) Check value where Inequality is undefined

(5) for $>$ Sign opt +ve Interval & for $<$ Sign opt $\ominus$ Interval.

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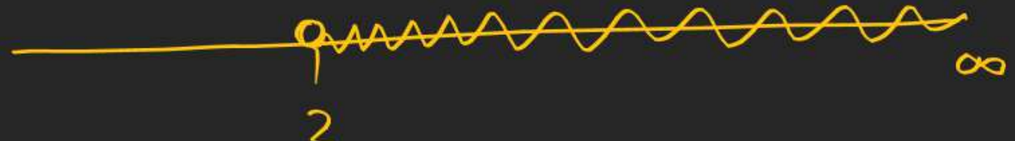
RK

$x \geq 2 \rightarrow$ 

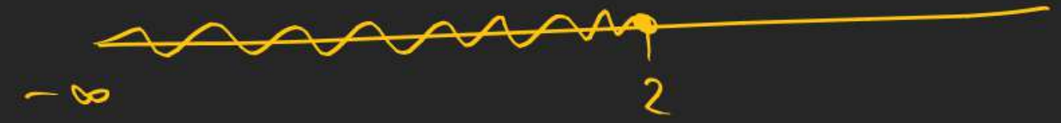
$\Rightarrow x \in [2, \infty)$

( ) open.

[ ] close

$x > 2 \rightarrow$ 

$\Rightarrow x \in (2, \infty)$

$x \leq 2$ 

$\Rightarrow x \in (-\infty, 2]$

$x < -5$ 

$\Rightarrow x \in (-\infty, -5)$

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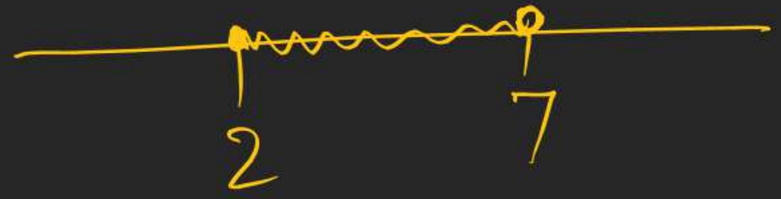
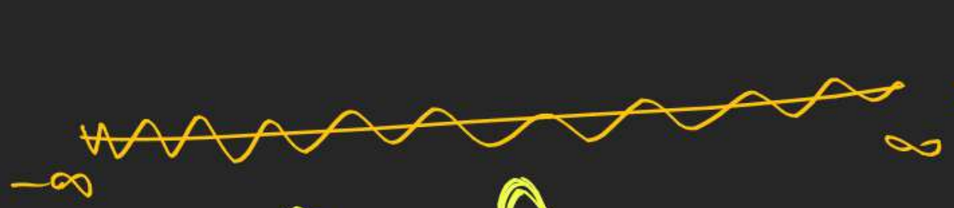
Fundamentals of Mathematics



$\Rightarrow x \in [2, 7)$

$\Rightarrow x \in (-3, 4)$



$\Rightarrow x \in (-\infty, \infty) \Rightarrow \boxed{x \in R}$

$\Rightarrow x \in (-\infty, \infty) - \{-13, 4\}$

$x \in R - \{-13, 4\}$

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Fundamentals of Mathematics

Q. Simplify : $(a^m)^n = a^{m \times n}$

1. $\left(a^{\frac{-3}{4}}\right)^8 = a^{-\frac{3}{4} \times 8^2} = a^{-6} = \frac{1}{a^6}$

2. $\left(a^{\frac{-2}{3}} b^{\frac{5}{6}}\right)^{\frac{3}{4}} = \left(a^{-\frac{2}{3}} \cdot b^{\frac{5}{6}}\right)^{\frac{3}{4}}$

$= a^{-\frac{2}{3} \times \frac{3}{4}^2} \cdot b^{\frac{5}{6}_2 \times \frac{3}{4}} = a^{-\frac{1}{2}} \cdot b^{\frac{5}{8}}$

3. $\left(a^{\frac{-1}{2}} b^{-3}\right)^{-2} = a^{+\frac{1}{2} \times +2} \cdot b^{-3 \times -2}$

$= a^1 b^6$

4. $\left(a^6 b^{\frac{5}{4}}\right)^{\frac{-4}{3}}$

$a^{6^2 \times -\frac{4}{3}} \cdot b^{\frac{5}{4} \times -\frac{4}{3}}$

$c \cdot b^{-\frac{5}{3}}$

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Q. Simplify :

5. $\left(\sqrt[3]{a^4b^3}\right)^6$

$= \left((a^4b^3)^{\frac{1}{3}}\right)^6$

$= a^{\frac{4}{3}\times 6^2 \times \frac{1}{3}\times 6} \cdot b$

$= a^8 \cdot b^6$

6. $\left(\sqrt[6]{x^9y^{-8}}\right)^{-3}$

$= \left((x^9 \cdot y^{-8})^{\frac{1}{6}}\right)^{-3}$

$= x^{9\times\frac{1}{6_2}\times -3} \cdot y^{-\frac{4}{8}\times\frac{1}{6_2}\times +3}$

$= x^{-9/2} \cdot y^4$

7. $\sqrt[8]{x^2 \cdot \sqrt[4]{x^{-3}}}$

$\left(x^2 \cdot (x^{-3})^{\frac{1}{4}}\right)^{\frac{1}{8}} = x^{2\times\frac{1}{8_4}} \cdot x^{-3\times\frac{1}{4}\times\frac{1}{8}}$

$= x^{\frac{1}{4}} \cdot x^{-\frac{3}{32}}$

8. $\sqrt{a^{-3}\,b^4} \times \sqrt[4]{a^2\,b^{-8}}$

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Q. Simplify :

9. $\sqrt[4]{x^{-2}\sqrt{y^5}} \times \sqrt{x^4\sqrt{y^3}}$

$$\left(x^{-2}\cdot(y^5)^{\frac{1}{2}}\right)^{\frac{1}{4}} \times \left(x^4\cdot(y^3)^{\frac{1}{2}}\right)^{\frac{1}{2}} = x^{-\frac{2}{4}} y^{5\times\frac{1}{2}\times\frac{1}{4}} \times x^{4\times\frac{1}{2}} y^{3\times\frac{1}{2}\times\frac{1}{2}}$$

$$= x^{-\frac{1}{2}}\cdot y^{\frac{5}{8}} \times x^2 \cdot y^{\frac{3}{4}} = x^{2-\frac{1}{2}} \cdot y^{\frac{5}{8}+\frac{3}{4}} = x^{\frac{3}{2}} y^{\frac{11}{8}}$$

10. $\left(8x^3 \div 27a^{-3}\right)^{\frac{2}{3}}$

11. $\left(64x^3 \div 27a^{-3}\right)^{\frac{2}{3}}$

12. $\sqrt[3]{a^6b^{-2}c^{-8}} \times \sqrt[4]{a^{-6}b^4c^{-1}}$

$$\left(a^6\cdot b^{-2}(-8)\right)^{\frac{1}{3}} \cdot \left(a^{-6}\cdot b^4\cdot(-1)\right)^{\frac{1}{4}} = a^{\frac{6}{3}} b^{-\frac{2}{3}} \cdot (-8)^{\frac{1}{3}} \times a^{-\frac{6}{4}} b^{\frac{4}{4}} (-1)^{\frac{1}{4}}$$

$$= a^{2-\frac{3}{2}} \cdot b^{-\frac{2}{3}+1} (\quad = a^{\frac{1}{2}} b^{\frac{1}{3}} (^{-\frac{35}{12}}$$

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Q. Simplify :

13. $\sqrt{a^{\frac{-2}{3}} b^4 c^{\frac{-1}{3}}} \div \sqrt{a^2 b^4 c^{-1}}$

$$\left(a^{-\frac{2}{3}} b^4 (-\tfrac{1}{3})\right)^{\frac{1}{2}} \times \frac{1}{(a^2 b^4 c^{-1})^{\frac{1}{2}}} =$$

14. $\sqrt{ab^{-2}c^3} \div \left(\sqrt[3]{a^3 b^2 c^{-3}}\right)^{-1}$

$$a^{-\frac{1}{3}} b^2 (-\tfrac{1}{6} \times \frac{1}{a^1 b^2 (-\frac{1}{2}}$$

$$= a^{-\frac{1}{3}-1} b^{2-2} (-\tfrac{1}{6}+\tfrac{1}{2}$$

$$= a^{-\frac{4}{3}} b^0 c^{\frac{1}{3}}$$

15. $\left(\frac{a^{-1}b^2}{a^2 b^{-4}}\right)^7 \div \left(\frac{a^3 b^{-5}}{a^{-2} b^3}\right)^{-5}$