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JEE Adv 2021

P_1 & P_2 be pressure at 1 & 2.

$$\beta = \frac{P_1 - P_2}{\rho g d} \text{ (given)}$$

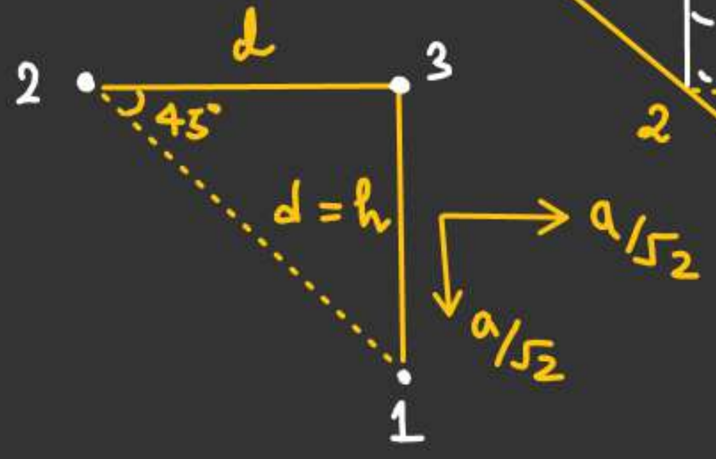
Which of the following is Correct

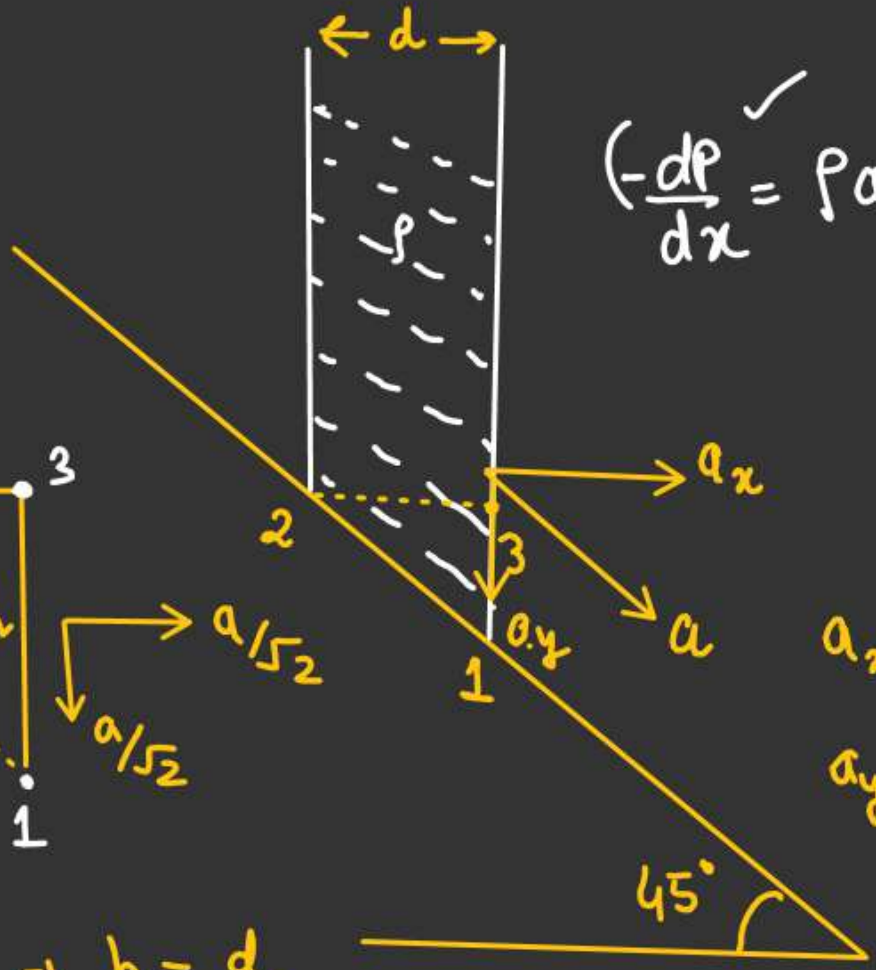
a) $\beta = 0$ when $a = g/\sqrt{2}$ ✓

b) $\beta > 0$ when $a = g/\sqrt{2}$

c) $\beta = \frac{\sqrt{2}-1}{\sqrt{2}}$ when $a = g/2$ ✓

d) $\beta = \frac{1}{\sqrt{2}}$ when $a = g/2$





$$\left(-\frac{dP}{dx} = \rho a_x\right) ✓$$

$a_x = \frac{a}{\sqrt{2}}$

$a_y = \frac{a}{\sqrt{2}}$

$\tan 45° = \frac{h}{d} \Rightarrow \underline{h = d}$.

$$P_1 - P_3 = d\rho\left(g - \frac{a}{\sqrt{2}}\right) \quad -(1)$$

$$P_2 - P_3 = \rho\frac{a}{\sqrt{2}}d \quad -(2)$$

(1) − (2)

$$P_1 - P_2 = \rho d\left(g - \frac{a}{\sqrt{2}}\right) - \frac{\rho a d}{\sqrt{2}}$$

$$P_1 - P_2 = \rho d\left[g - \sqrt{2}a\right]$$

$$\beta \Leftarrow \frac{P_1 - P_2}{\rho d g} = \left[1 - \frac{\sqrt{2}a}{g}\right]$$

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Q: U-tube

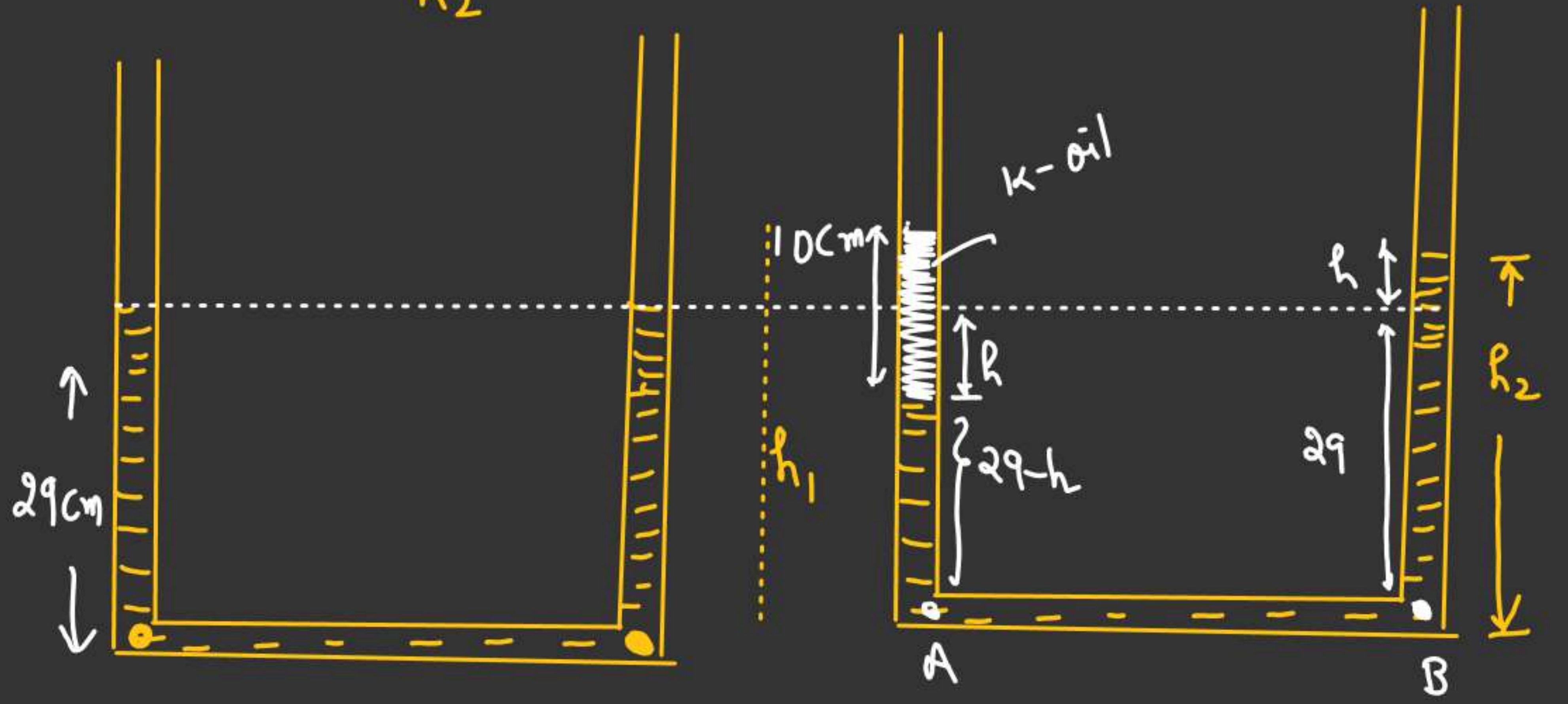
Uniform crossectional area contains water ($\rho_w = 10^3\ kg/m^3$)

Initially water level stands at 0.29m from the bottom in each arm.

Kerosene oil (immiscible) of density $800\ kg/m^3$ is added to the left arm untill its length is 0.1m

the ratio $\frac{h_1}{h_2} = ??$

JEE AdV 2020



$P_A = P_B$

$\cancel{P_{atm}} + 800\times10\times10 + 1000\times10\times(29-h)$

$= \cancel{P_{atm}} + 1000\times10(29+h)$

$8\times10^4 + (29-h)\times10^4 = (29+h)\times10^4$

$8 + 29 - h = 29 + h$

$8 = 2h$

$h = 4Cm$

$h_1 = 39-4 = 35$

$h_2 = 29+4 = 33$

$\frac{h_1}{h_2} = \frac{35}{33}$ ✓

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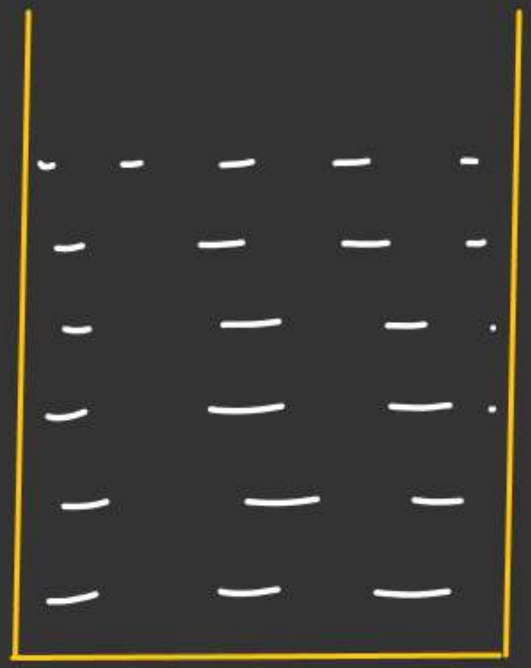
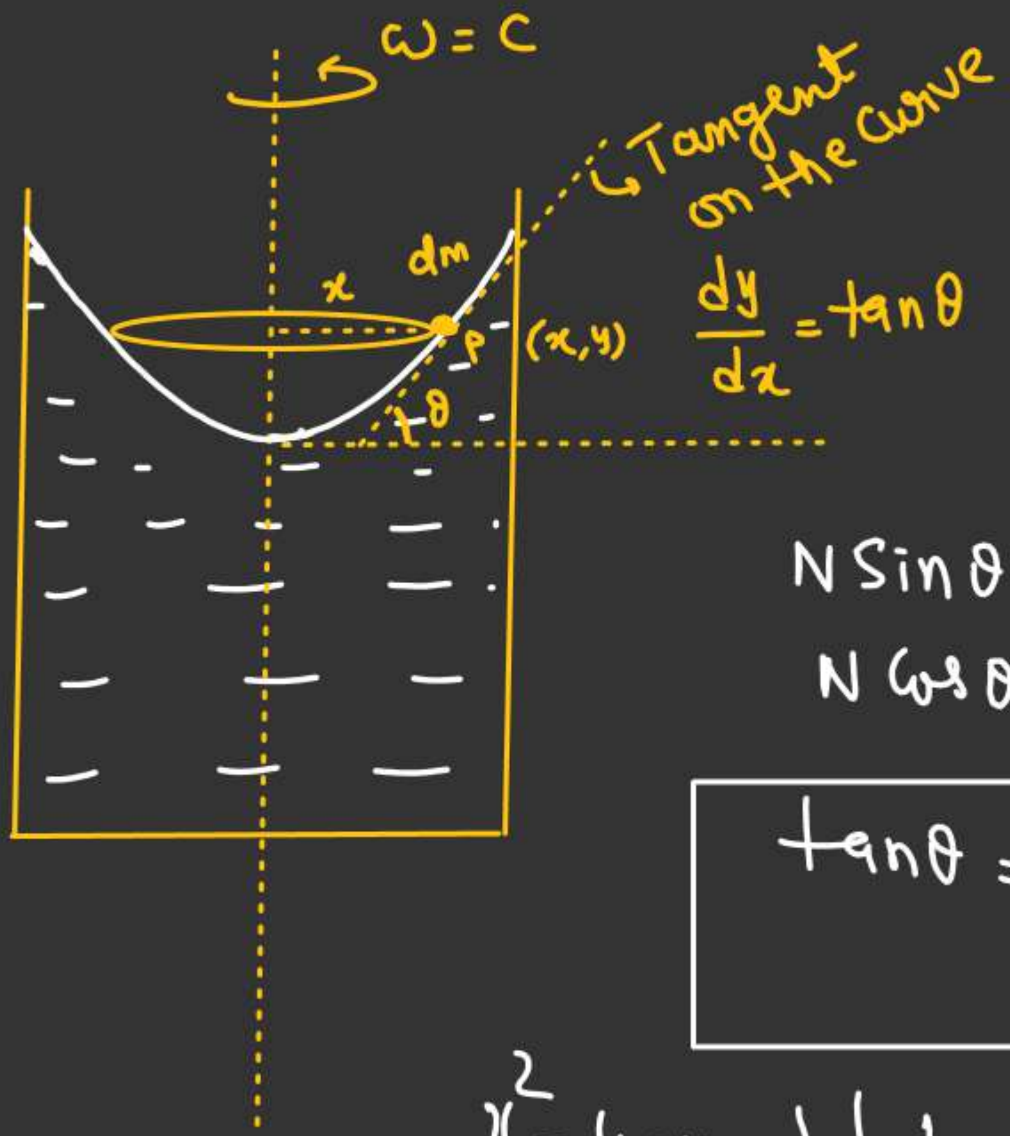
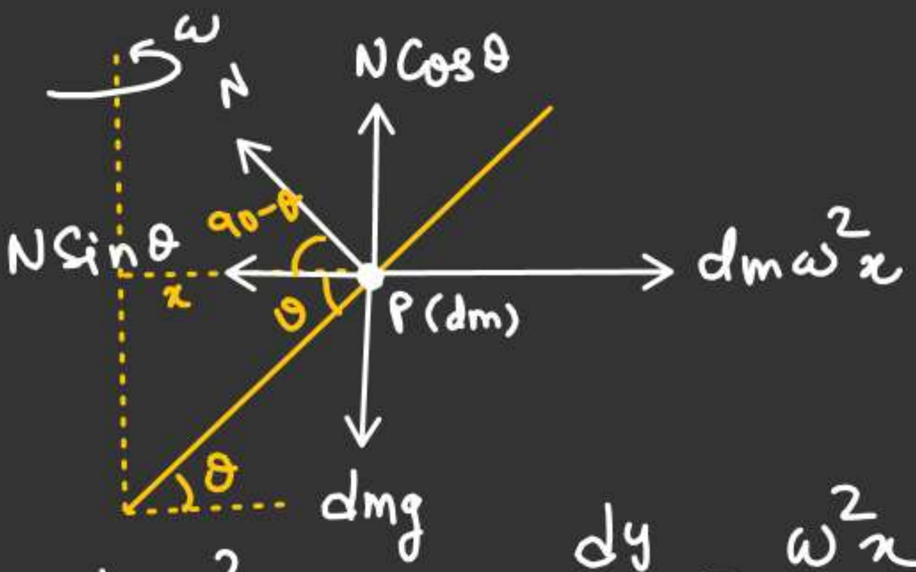
Pressure gradient in Rotating frame





$\frac{dy}{dx} = \tan\theta$

F.B.D of P in rotating frame



$N\sin\theta = dm\,\omega^2 x$

$N\cos\theta = dm\,g$

$$\boxed{\tan\theta = \frac{\omega^2 x}{g}}$$

$$\frac{dy}{dx} = \frac{\omega^2 x}{g}$$

$$\int_0^y dy = \frac{\omega^2}{g}\int_0^x x\,dx$$

$$\boxed{y = \frac{\omega^2 x^2}{2g}}$$

$x^2 = 4ay$

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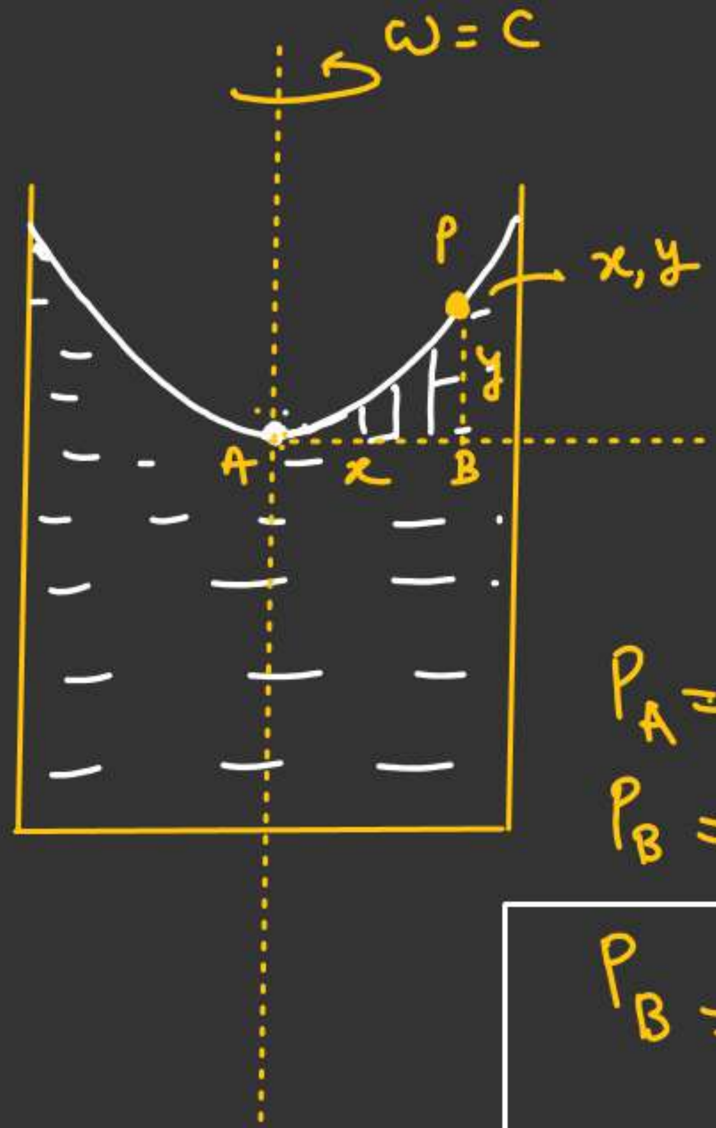
Pressure gradient in Rotating frame



$$\boxed{y = \frac{\omega^2 x^2}{2g}}$$

$P_A = P_{atm}$ ✓

$P_B = P_{atm} + \rho g y$

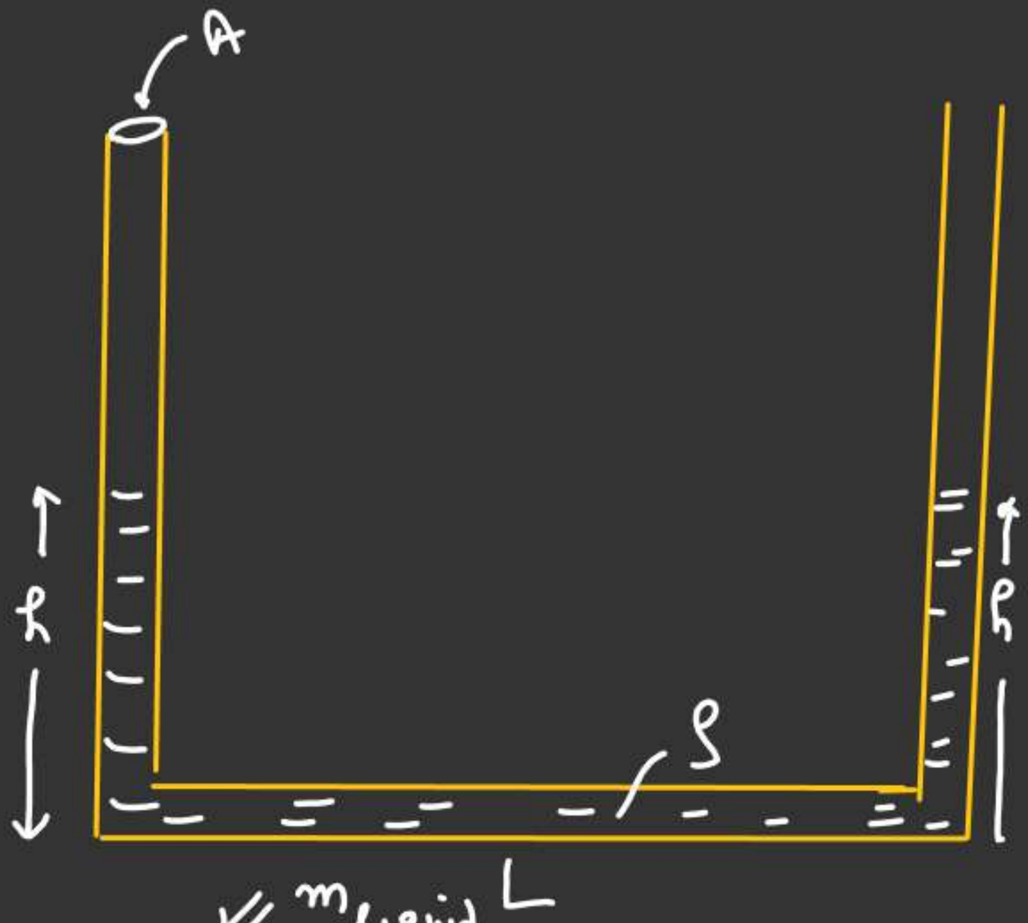
$$\boxed{P_B = P_A + \frac{\rho g \omega^2 x^2}{2g}}$$

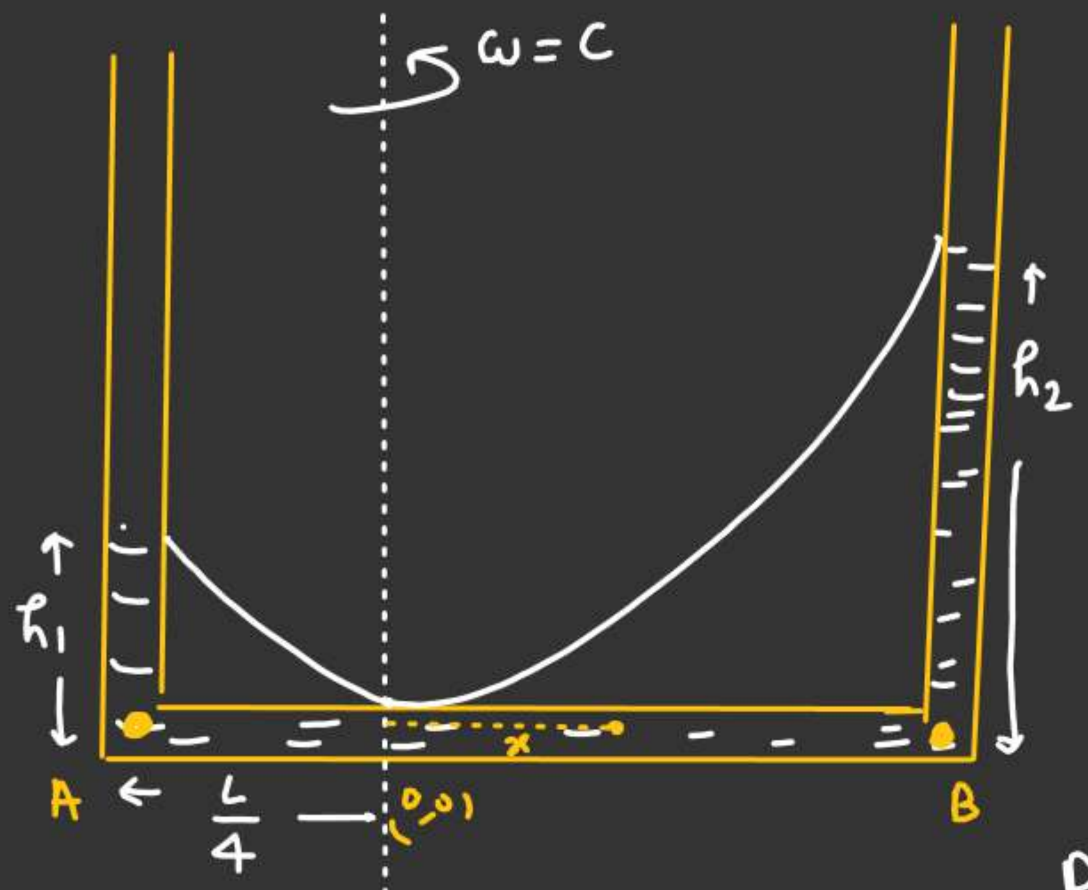
$P_B = P.$

$$\frac{dP}{dx} = \frac{\rho \cancel{g} \omega^2}{2\cancel{g}} \frac{d}{dx}(x^2)$$

$$\frac{dP}{dx} = \frac{\rho \omega^2}{\cancel{2}} \times \cancel{2} x$$

$$\boxed{\frac{dP}{dx} = \rho \omega^2 x}$$





m_{liquid}

$(\rho A h)2 + \cancel{AL}\rho$

$= \underbrace{\rho A h_1 + \rho A h_2}_{m_{liquid}} + \cancel{AL}\rho$

$\frac{h_1+h_2}{2} = h$

$h_1 + h_2 = 2h$ — (2)

$P_A = P_{atm} + \rho g h_1$

$P_B = P_{atm} + \rho g h_2$

$P_B - P_A = \rho g (h_2 - h_1)$

$\frac{\cancel{\rho}\omega^2 l^2}{4} = \cancel{\rho} g (h_2 - h_1)$

$\frac{dP}{dx} = \rho\omega^2 x$

$\int_{P_A}^{P_B} dP = \rho\omega^2 \int_{-\frac{l}{4}}^{\frac{3l}{4}} x\,dx$

$P_B - P_A = \rho\omega^2 \left[\frac{x^2}{2}\right]_{-\frac{l}{4}}^{\frac{3l}{4}}$

$P_B - P_A = \frac{\rho\omega^2}{2}\left[\frac{9l^2}{16} - \frac{l^2}{16}\right]$

$P_B - P_A = \frac{\rho\omega^2 l^2}{4}$

$h_2 - h_1 = \frac{\omega^2 l^2}{4g}$ — (1)

From (1) & (2)

$h_1 =$

$h_2 =$

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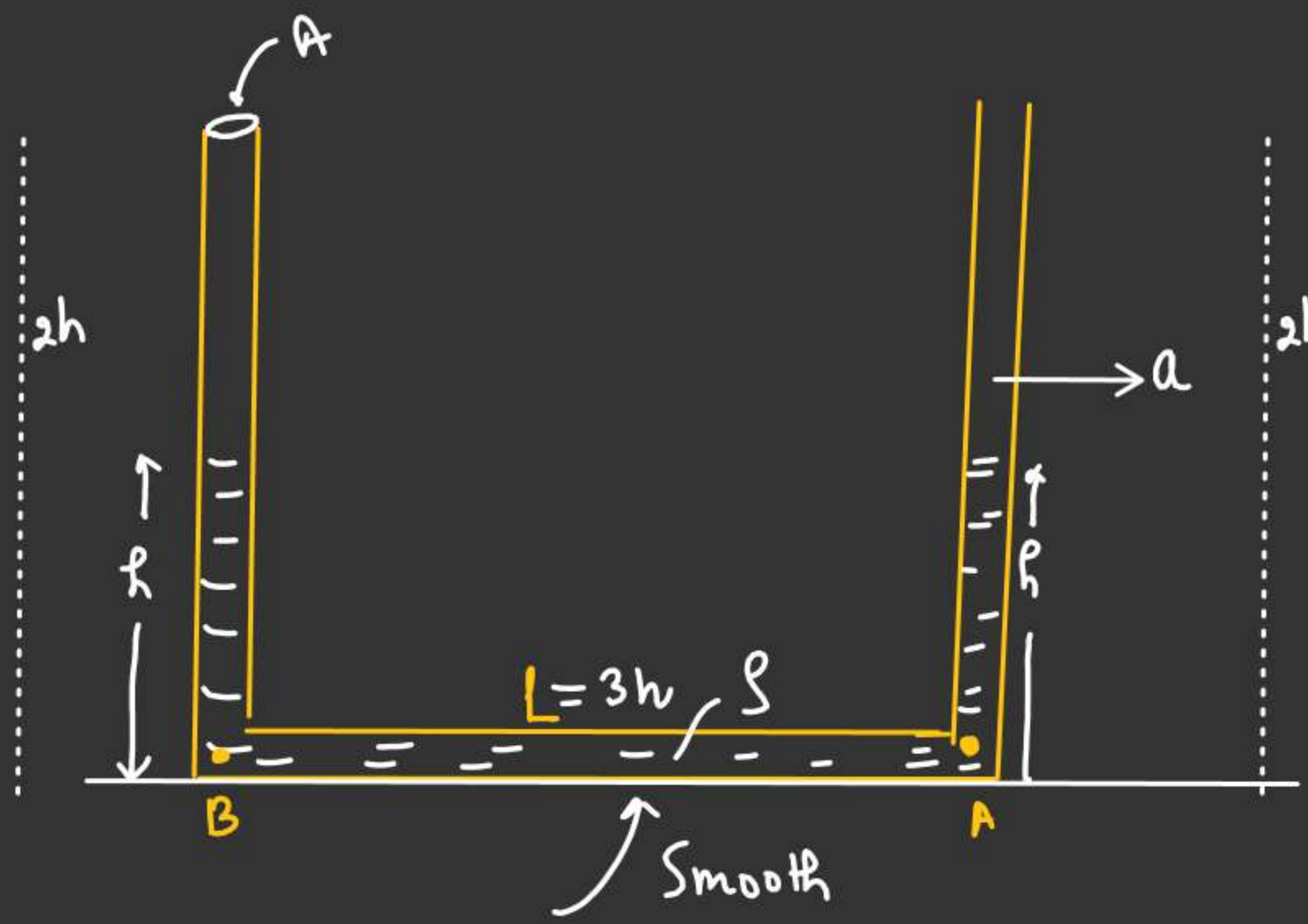
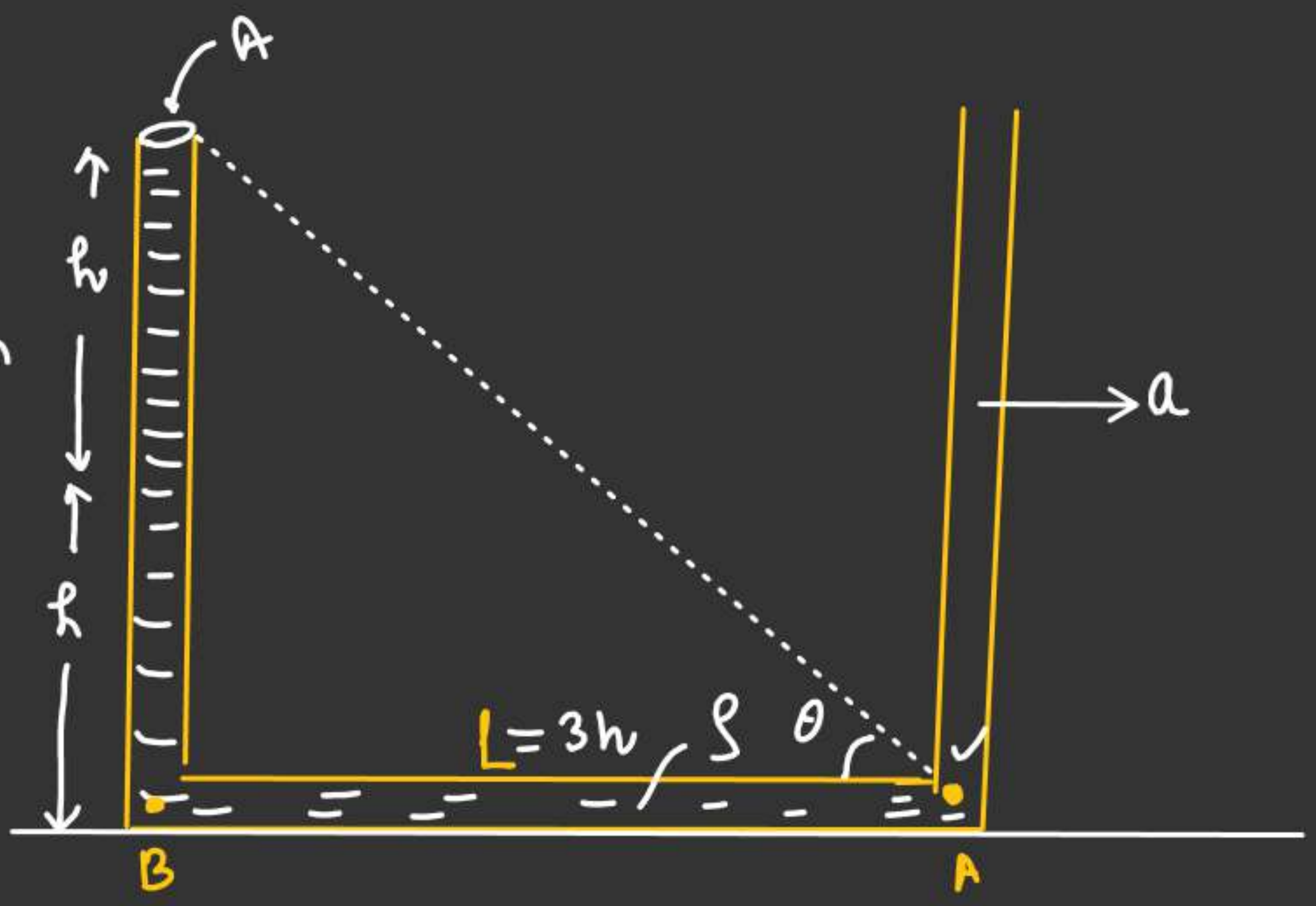
Fin a_{min} so that liquid doesnt over flow from the vertical limb.





$$\tan\theta = \frac{a}{g}$$

$$\frac{2h}{3h} = \frac{a}{g} \rightarrow a_{min} = \left(\frac{2g}{3}\right) ✓$$

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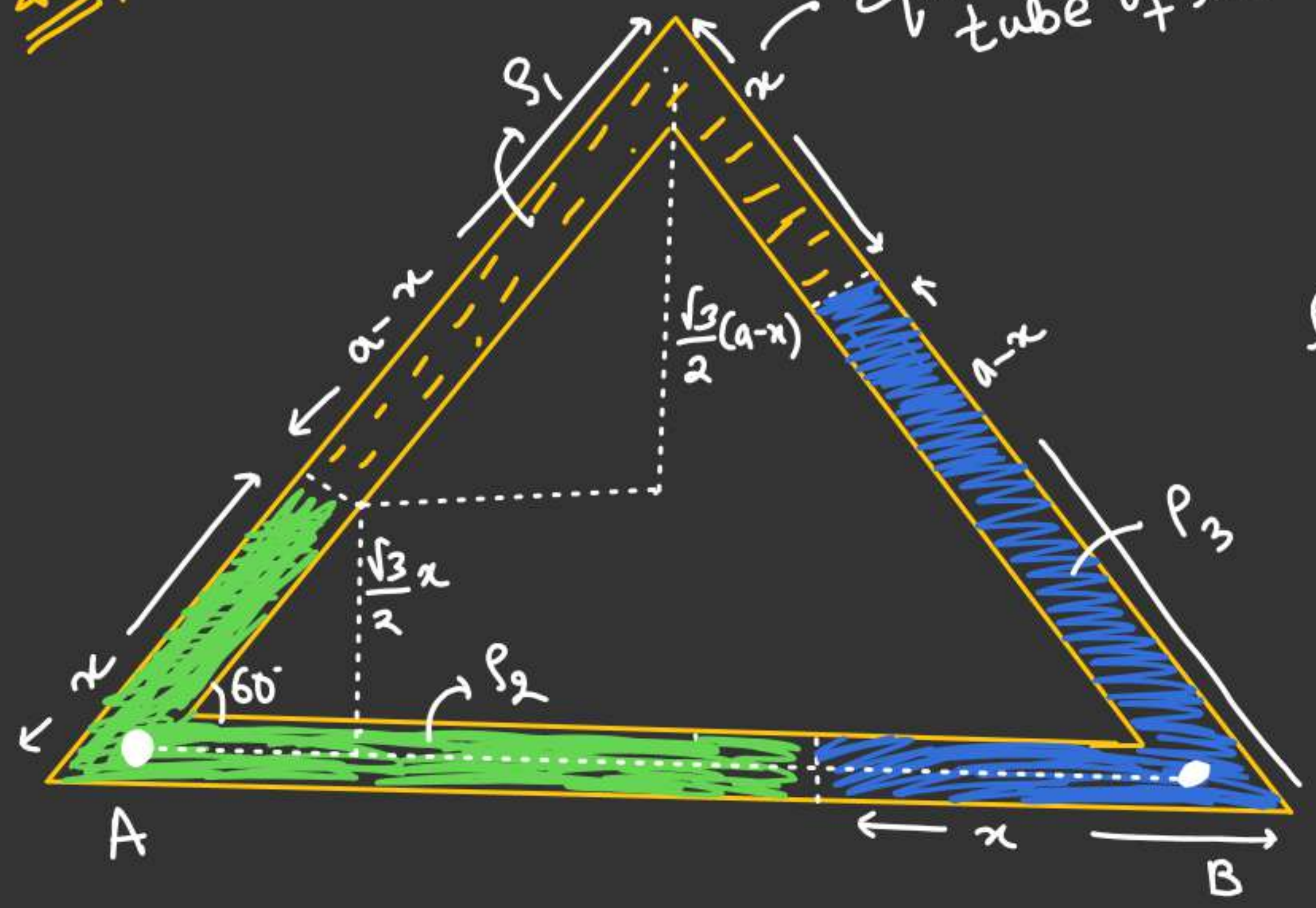

Find $x = ??$

$$P_A = P_B.$$

$$\rho_1 g \frac{\sqrt{3}}{2}(a-x) + \rho_2 g \frac{\sqrt{3}}{2} x = x\frac{\sqrt{3}}{2}\rho_1 g + \rho_3 g(a-x)\frac{\sqrt{3}}{2}$$

$$\rho_1(a-x) + \rho_2 x = \rho_1 x + \rho_3(a-x)$$

$$(\rho_1 - \rho_3)a = (2\rho_1 - \rho_2 - \rho_3)x$$

$$x = \left(\frac{\rho_1 - \rho_3}{2\rho_1 - \rho_2 - \rho_3}\right)a$$