Nishant Jindal

S.H.M (Simple harmonic Motion)

Motion

Periodic Motion

Whic occur in regular interval of time.

Non-periodic Motion

(which doesn't occur in fixed interval of time)

Nishant Jindal

Periodic Motion

Oscillatory Motion (To & fro Motion)

⇒ Body oscillate about fixed point called mean position under the influence of a restoring force always acts towards the mean-position

Non-Oscillatory

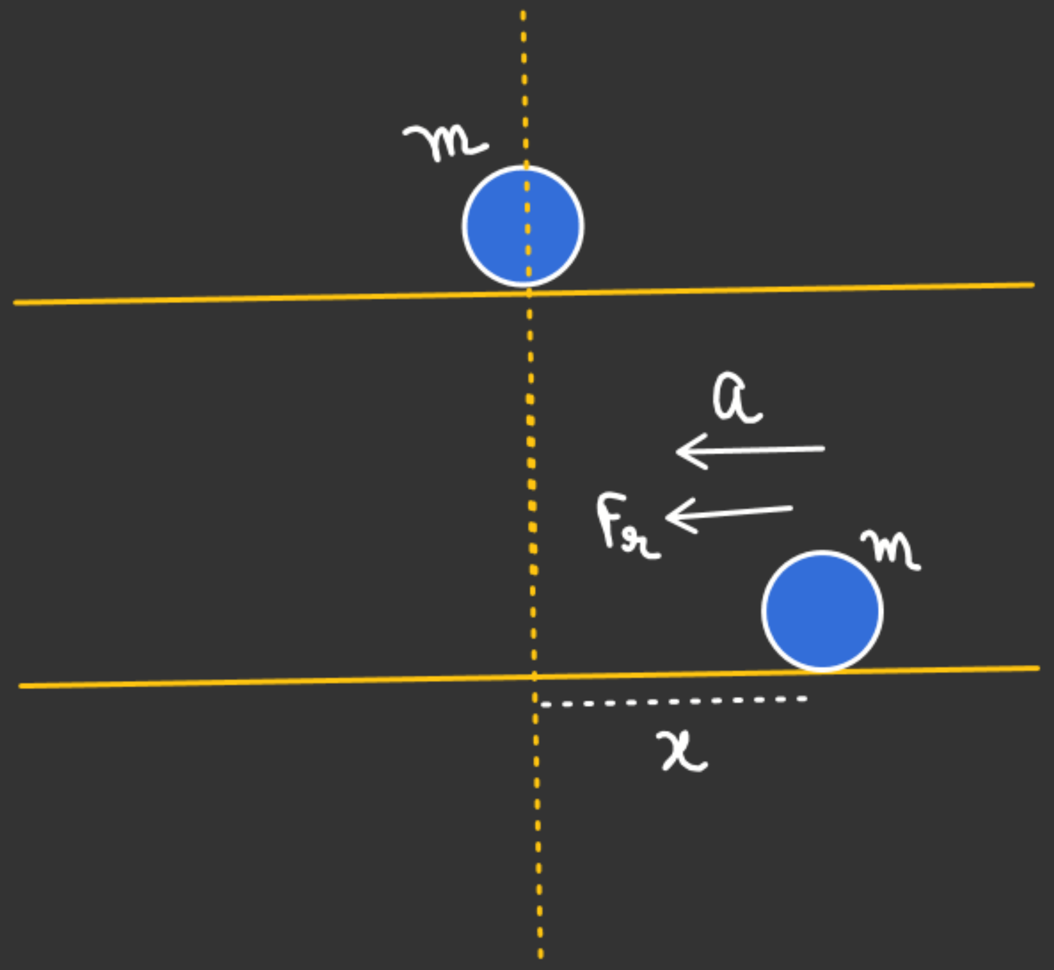
Ex:-
- Uniform Circular Motion
- All planetary Motion

Nishant Jindal

S.H.M

Part of oscillatory Motion.

If restoring force is directly proportional to displacement of particle from the mean position then oscillatory Motion become S.H.M.



$F_r \propto x$

$F_r = -Kx$

$a = -\frac{K}{m}x$

$\omega = \sqrt{\frac{K}{m}}$ → Angular frequency

$$\boxed{a = -\omega^2 x}$$

$$\boxed{\frac{d^2x}{dt^2} = -\omega^2 x}$$

⇓ Equation of S.H.M

$\omega = \frac{2\pi}{T} = 2\pi f$

$\frac{1}{f} = T$

T = Time period

f = frequency.

Nishant Jindal

GENERAL EQUATION OF S.H.M

$a = -\omega^2 x$.

$\Downarrow$

$$v\frac{dv}{dx} = -\omega^2 x$$

$$\int_{v_0}^{v} v\,dv = -\omega^2 \int_0^x x\,dx$$

$$\frac{v^2 - v_0^2}{\cancel{2}} = \frac{-\omega^2 x^2}{\cancel{2}}$$

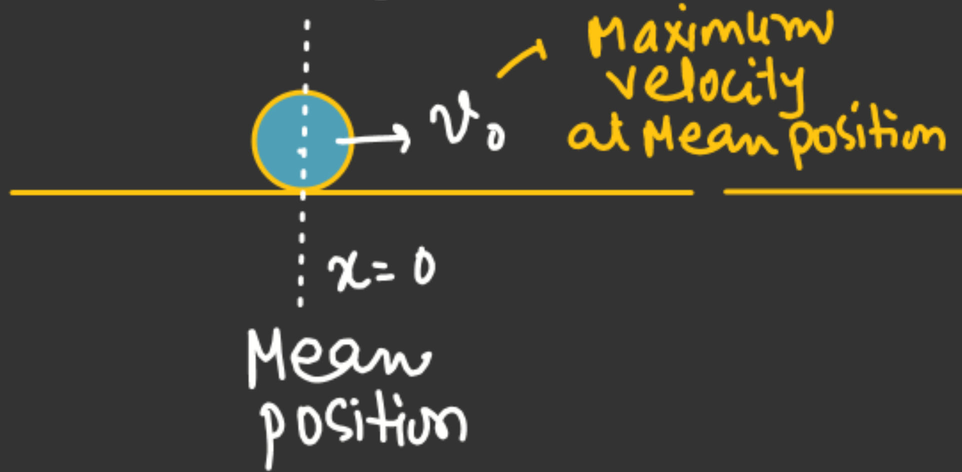
**

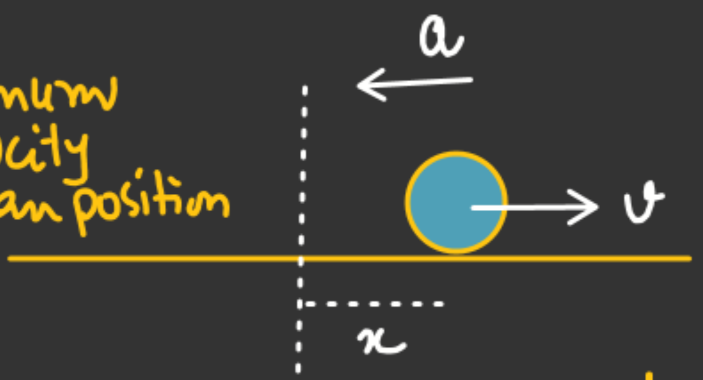
$$\boxed{v = \sqrt{v_0^2 - \omega^2 x^2}}$$

$\downarrow$

$$\frac{dx}{dt} = \sqrt{v_0^2 - \omega^2 x^2}$$

At $t = 0$, $x = 0$, $v = v_0$





$$\boxed{\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\left(\frac{x}{a}\right)}$$

$$\frac{dx}{dt} = \omega\sqrt{\frac{v_0^2}{\omega^2} - x^2}$$

$$\int_0^x \frac{dx}{\sqrt{\underbrace{\left(\frac{v_0}{\omega}\right)^2}_{a} - x^2}} = \omega\int_0^t dt$$

$$\sin^{-1}\left(\frac{\omega x}{v_0}\right) = \omega t$$

$$\frac{\omega x}{v_0} = \sin\omega t$$

$$x = \frac{v_0}{\omega}\sin\omega t$$

Amplitude

$\downarrow$

**

$$\boxed{A = \frac{v_0}{\omega}}$$

$\Downarrow$

**

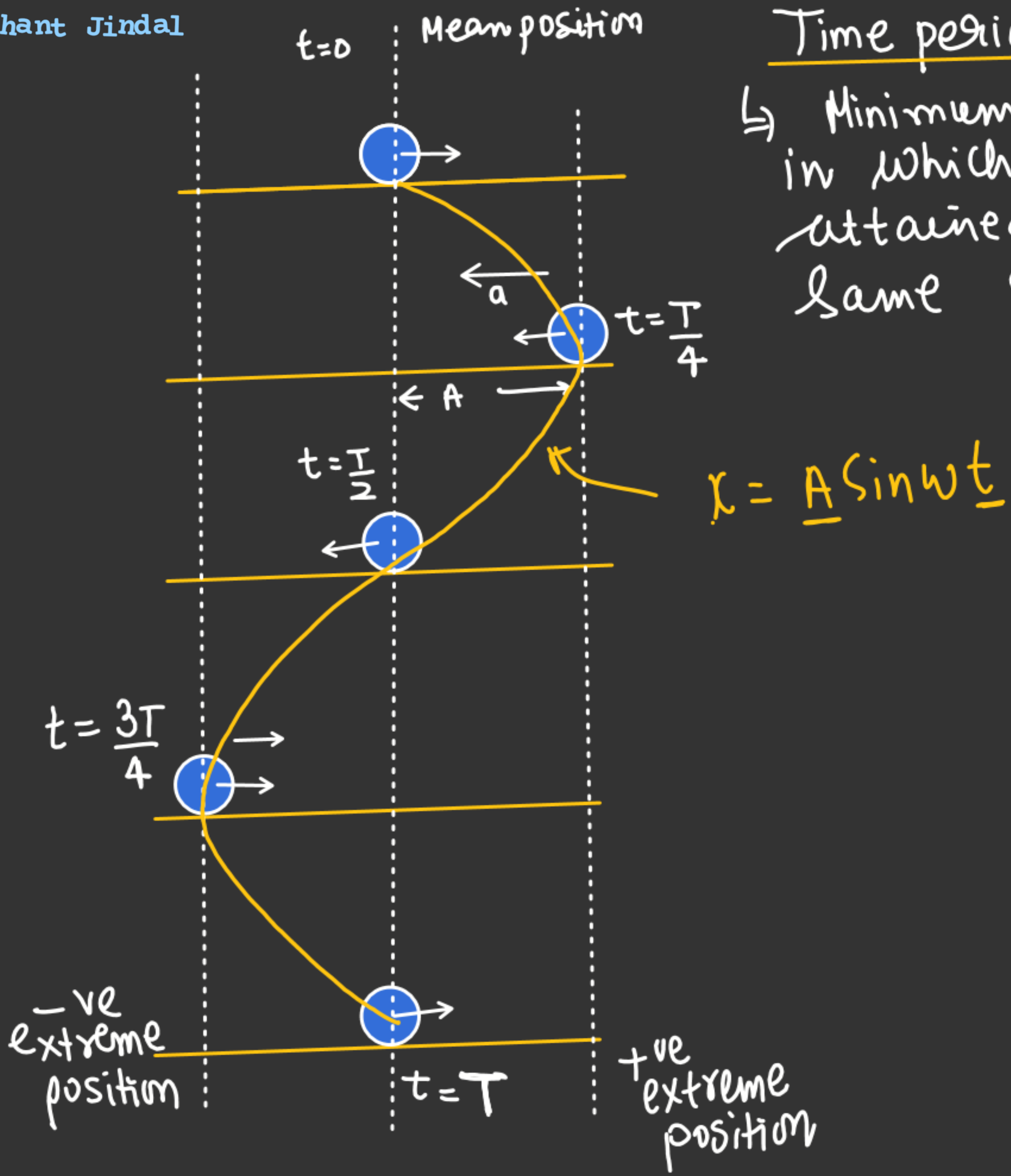
$$\boxed{x = A\sin\omega t}$$

$v_0 = A\omega$

$\downarrow$

Mean Position

Nishant Jindal



Time period

↳ Minimum time in which particle attained its same state.

Amplitude

↳ Maximum displacemet of particle from the mean position is called Amplitude.

Nishant Jindal

$$\boxed{\frac{d^2x}{dt^2} = -\omega^2 x}$$

$$\boxed{a = -\omega^2 x}$$

$$\boxed{x = A\sin(\omega t + \phi)}$$

x = Always from the mean position

(Phase) ↳ give complete information about particle

$$\phi = (\text{Initial phase Constant})$$

$$x = A\sin(\omega t + \phi)$$

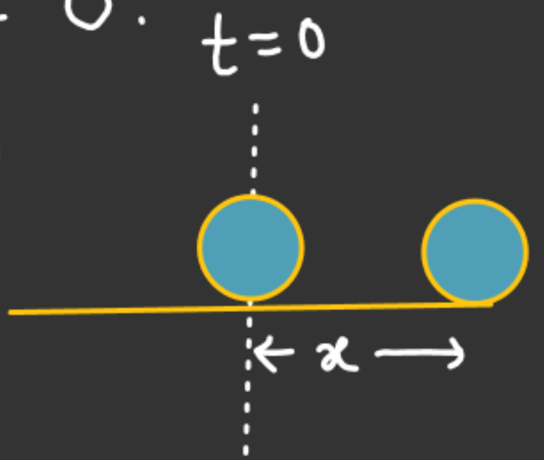
At $t = 0$, $x = 0$.

$$0 = A\sin\phi$$

$$\sin\phi = 0$$

$$\phi = 0$$

$$\boxed{x = A\sin\omega t}$$



At $t = 0$, $x = +A$,

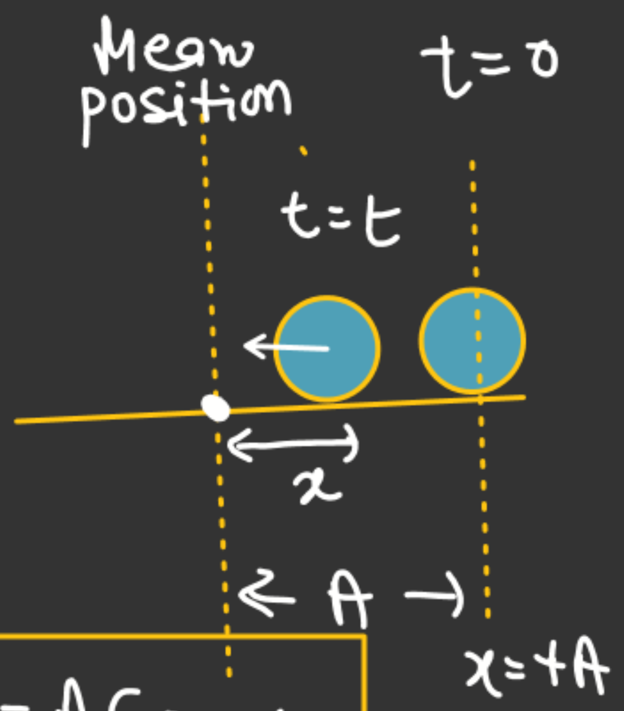
$$+A = A\sin(\omega t + \phi)$$

$$1 = \sin\phi$$

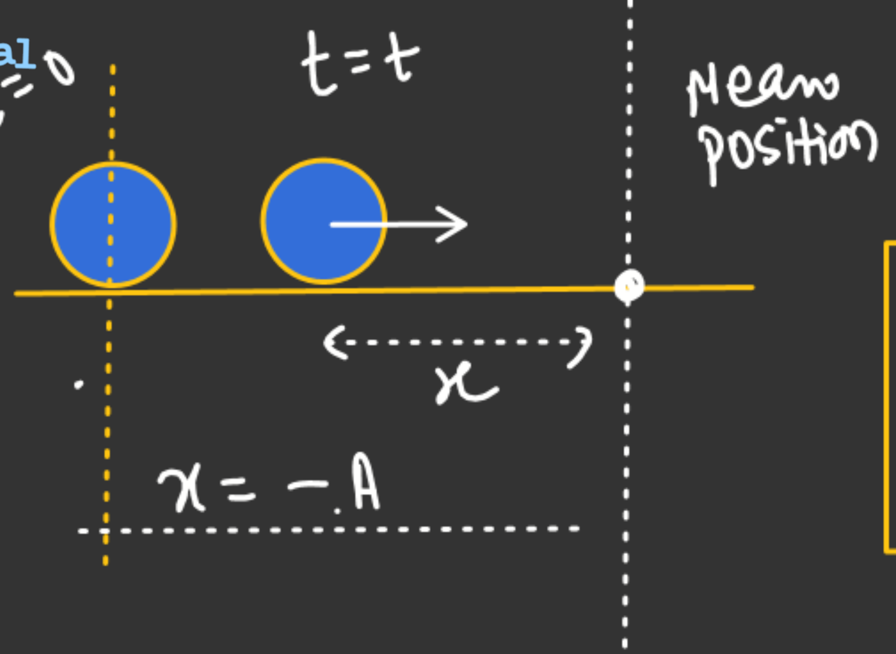
$$\phi = \frac{\pi}{2}$$

$$x = A\sin\left(\omega t + \frac{\pi}{2}\right)$$

$$\boxed{x = A\cos\omega t}$$



Nishant Jindal



$$\boxed{x = A\sin\left(\omega t + \frac{3\pi}{2}\right)}$$

ϕ

At $t=0$, $x = -A$

$$-A = A\sin\phi$$

$$\sin\phi = -1$$

$$\phi = \frac{3\pi}{2}$$

$$x = A\sin\left(\frac{\pi}{2} + (\omega t + \pi)\right)$$

$$x = A\cos(\omega t + \pi)$$

$$\boxed{x = -A\cos\omega t}$$

Nishant Jindal

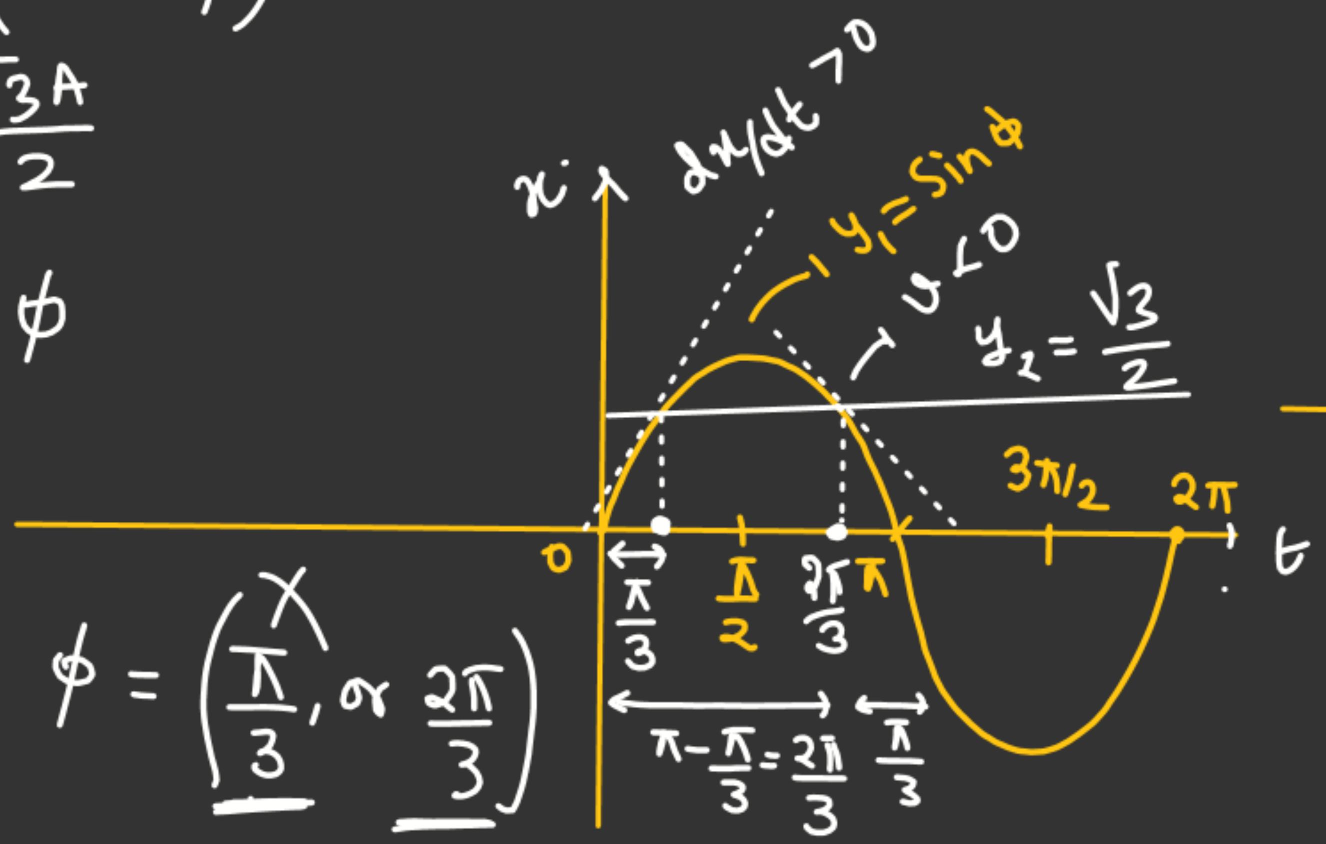
$$x = A\sin(\omega t + \phi)$$

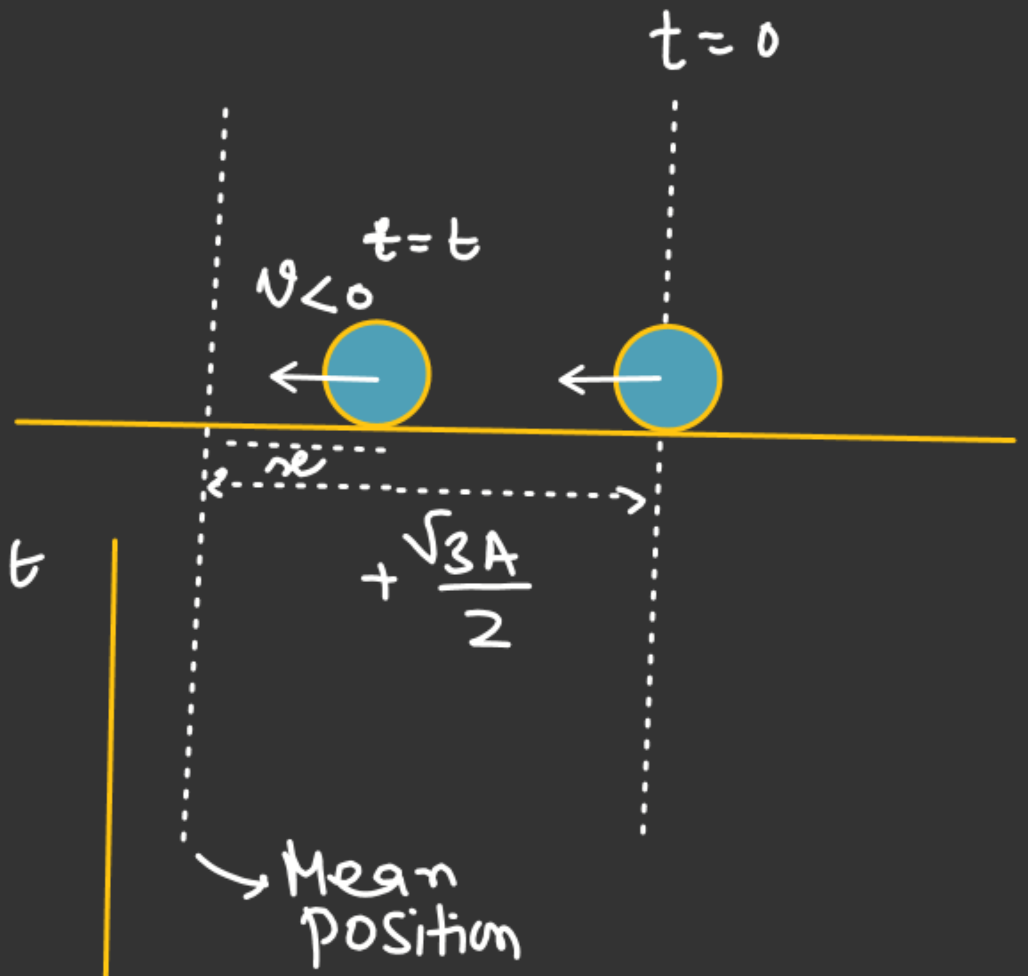
At $t = 0$, $x = \frac{\sqrt{3}A}{2}$

$$\frac{\sqrt{3}A}{2} = A\sin\phi$$

$$\sin\phi = \frac{\sqrt{3}}{2} \rightarrow \phi = \left(\frac{\pi}{3}, \text{ or } \frac{2\pi}{3}\right)$$

y_1 $\qquad$ y_2





✓ $y_1 = \sin\phi$

$y_2 = \frac{\sqrt{3}}{2}$

$$v = \frac{dx}{dt} = A\omega\cos(\omega t + \phi)$$

At $t = 0$,

$$v = A\omega\cos\phi$$

for $\phi = \left(\frac{2\pi}{3}\right)$ $v < 0$

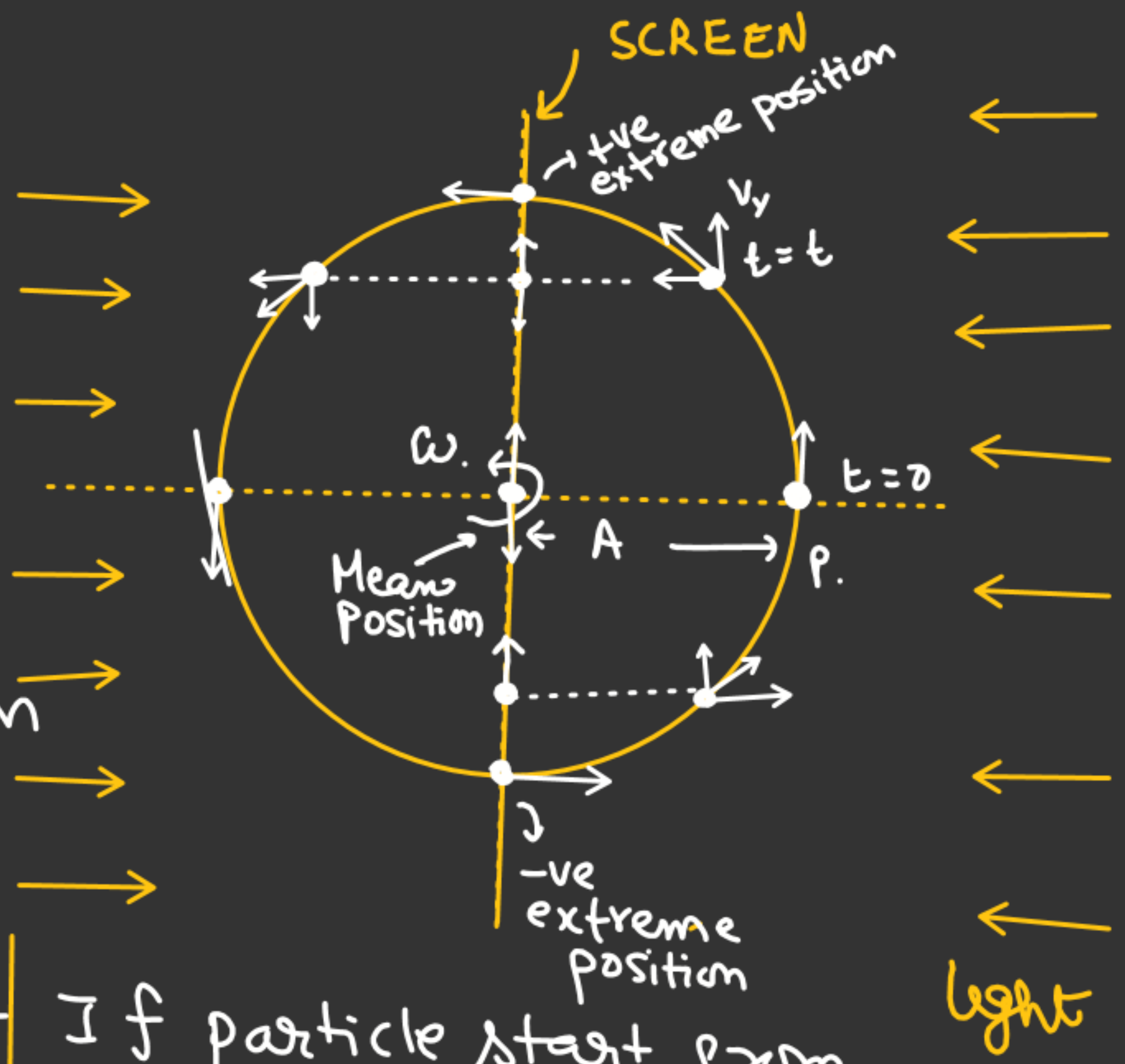
$$\boxed{x = A\sin\left(\omega t + \frac{2\pi}{3}\right)}$$

Nishant Jindal

S.H.M AS a projection of uniform Circular Motion

Assume a uniform circular motion whose radius equals to amplitude of particle & whose angular velocity equals to angular frequency of the particle.

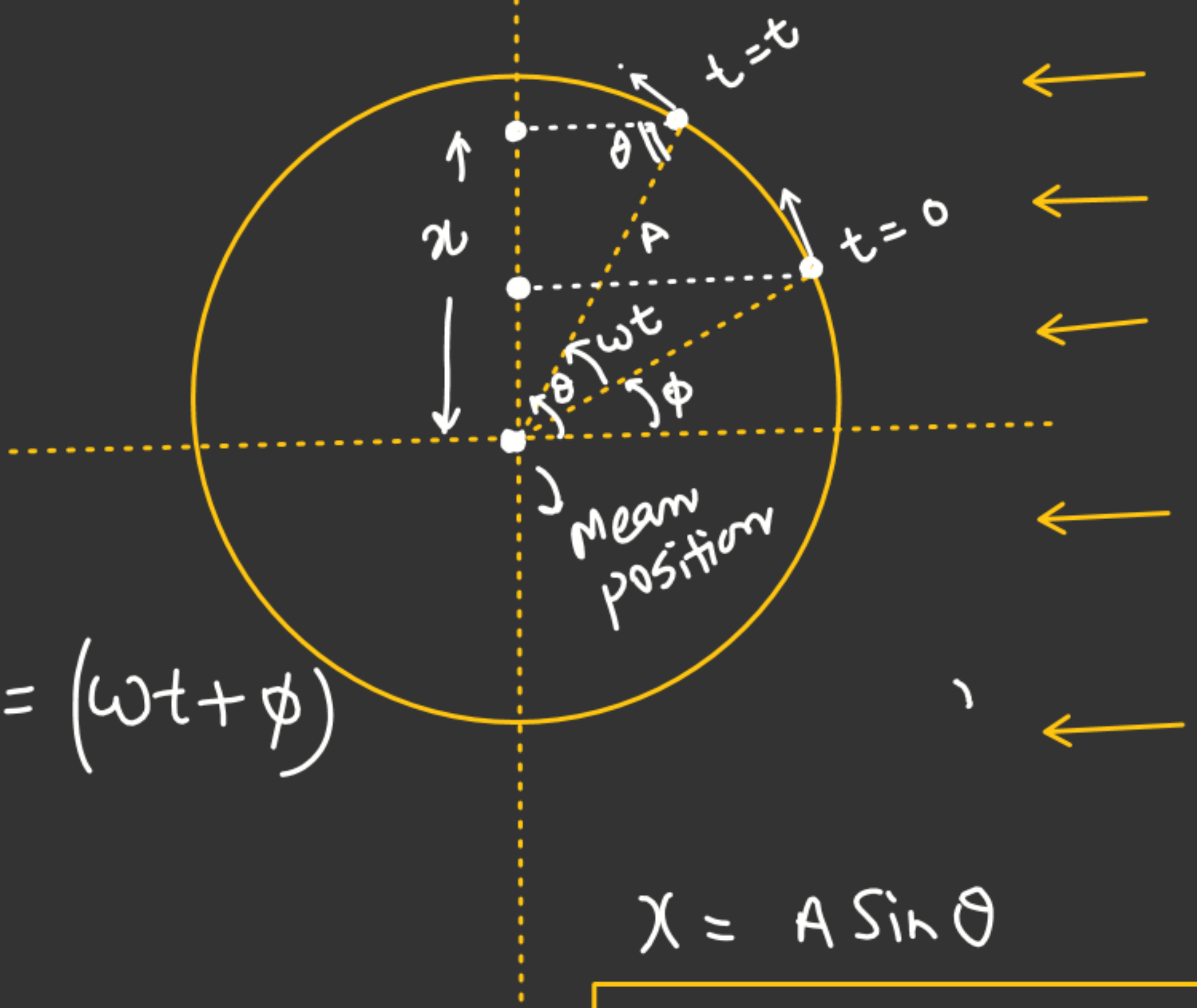
Take projection of uniform circular motion along y or x axis we will get S.H.M



Note:- If particle start from Mean position take projection on y-axis

If at $t=0$, Particle at extreme position take projection of X-axis

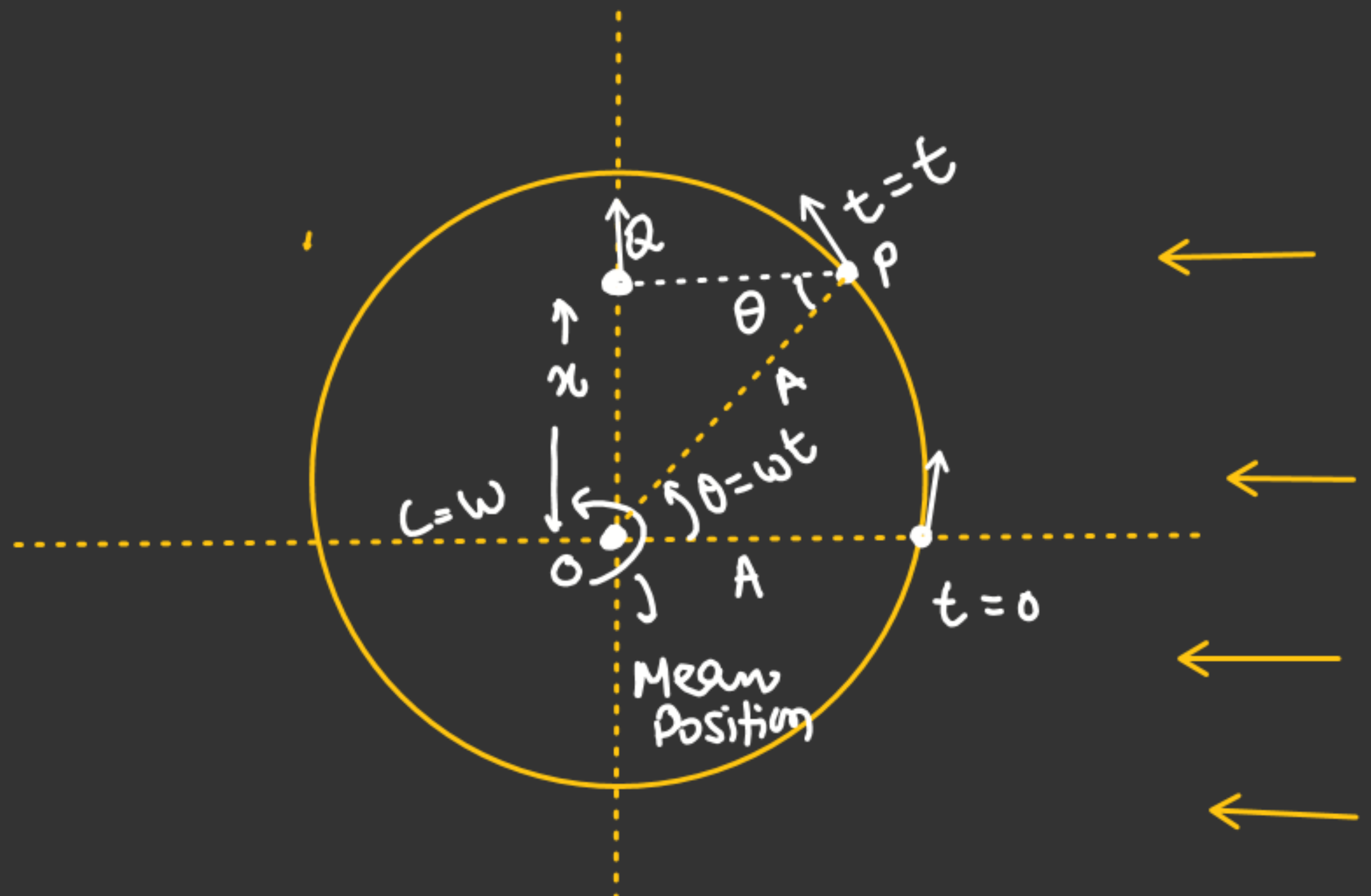
Nishant Jindal



$$\theta = (\omega t + \phi)$$

$$x = A \sin\theta$$

$$\boxed{x = A\sin(\omega t + \phi)}$$



In ΔOPQ

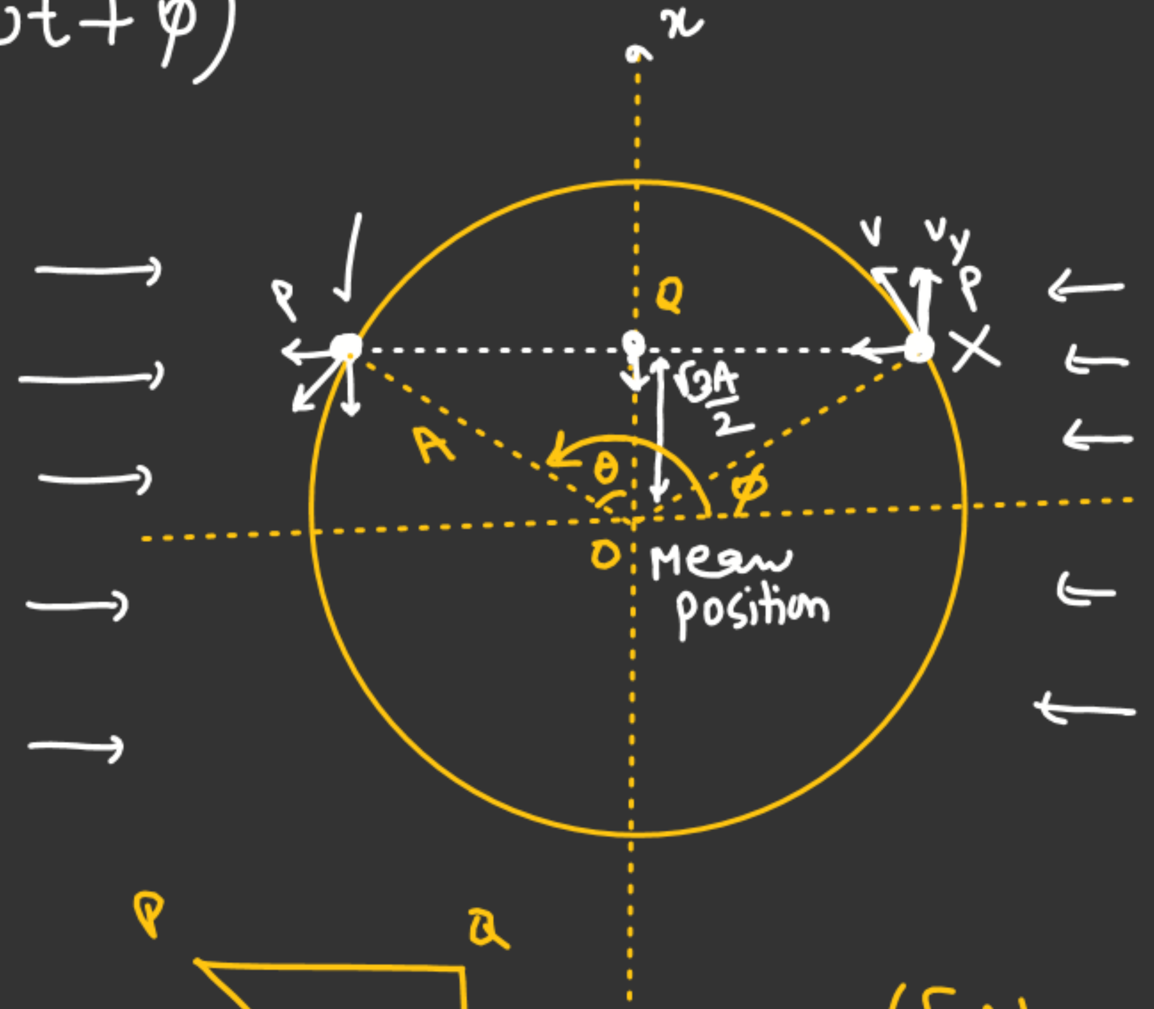
$$\sin\theta = \frac{x}{A}$$

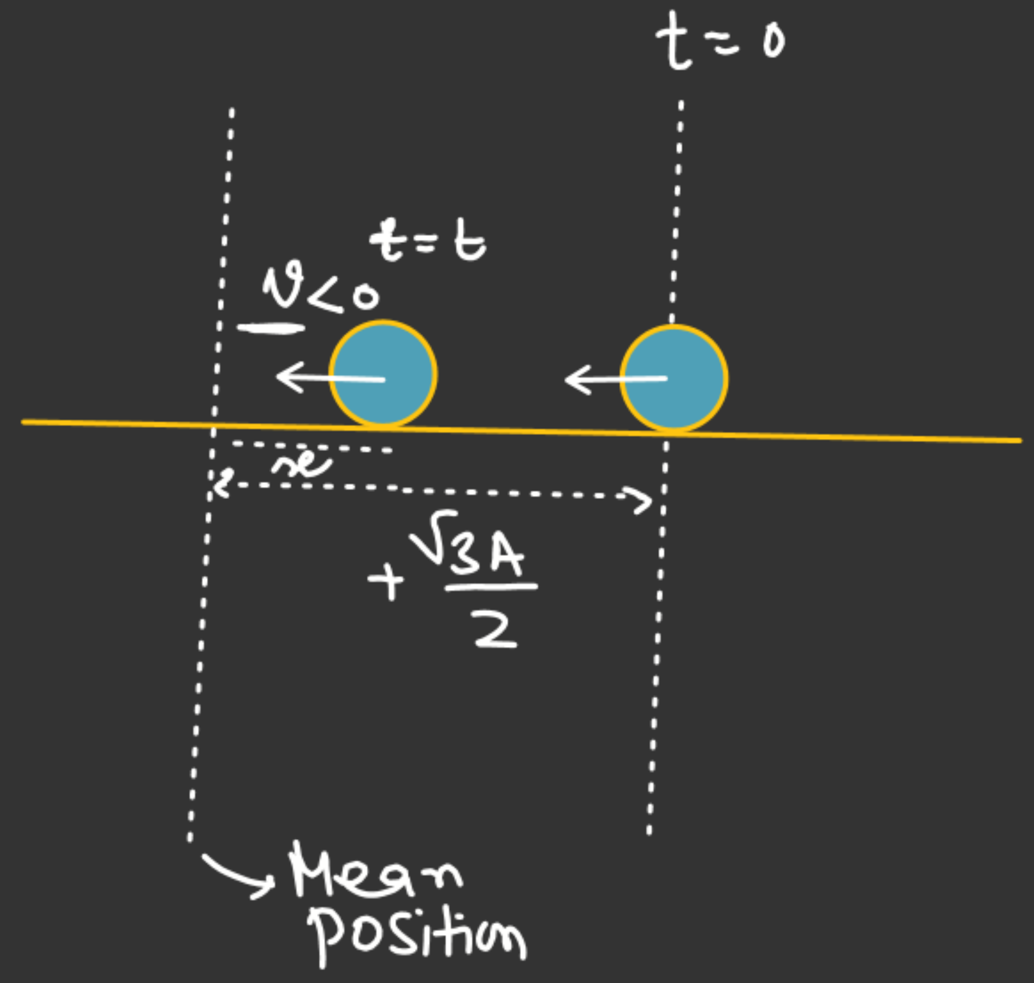
$$x = A\sin\theta$$

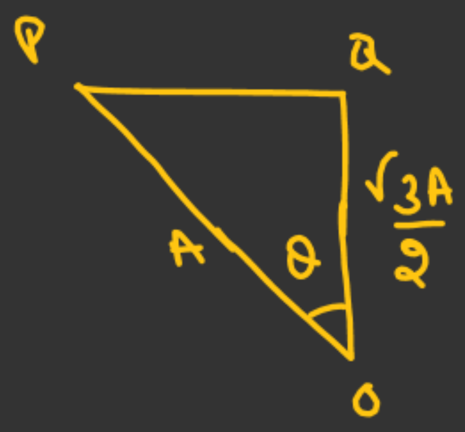
$$\boxed{x = A\sin\omega t}$$

Nishant Jindal

$$x = A\sin(\omega t + \phi)$$







$$\cos\theta = \left(\frac{\sqrt{3}A}{2}\right)/A$$

$$\cos\theta = \frac{\sqrt{3}}{2}$$

$$\theta = \frac{\pi}{6}$$

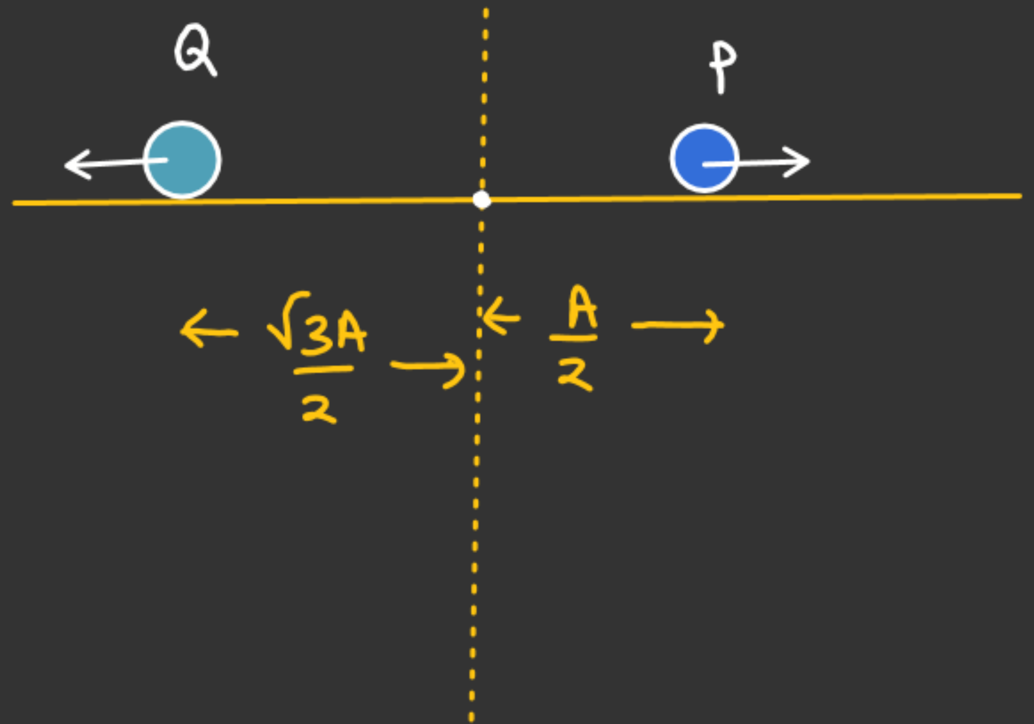
$$\phi = \frac{\pi}{2} + \theta$$

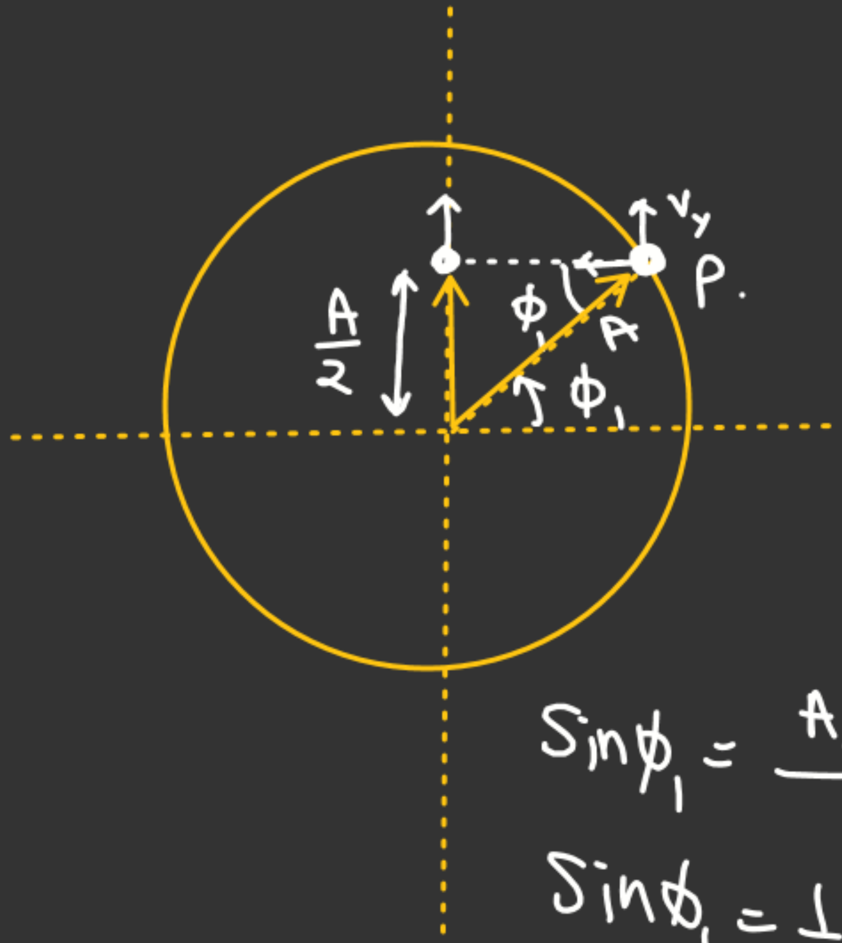
$$= \frac{\pi}{2} + \frac{\pi}{6} = \frac{4\pi}{6} = \frac{2\pi}{3} \checkmark$$

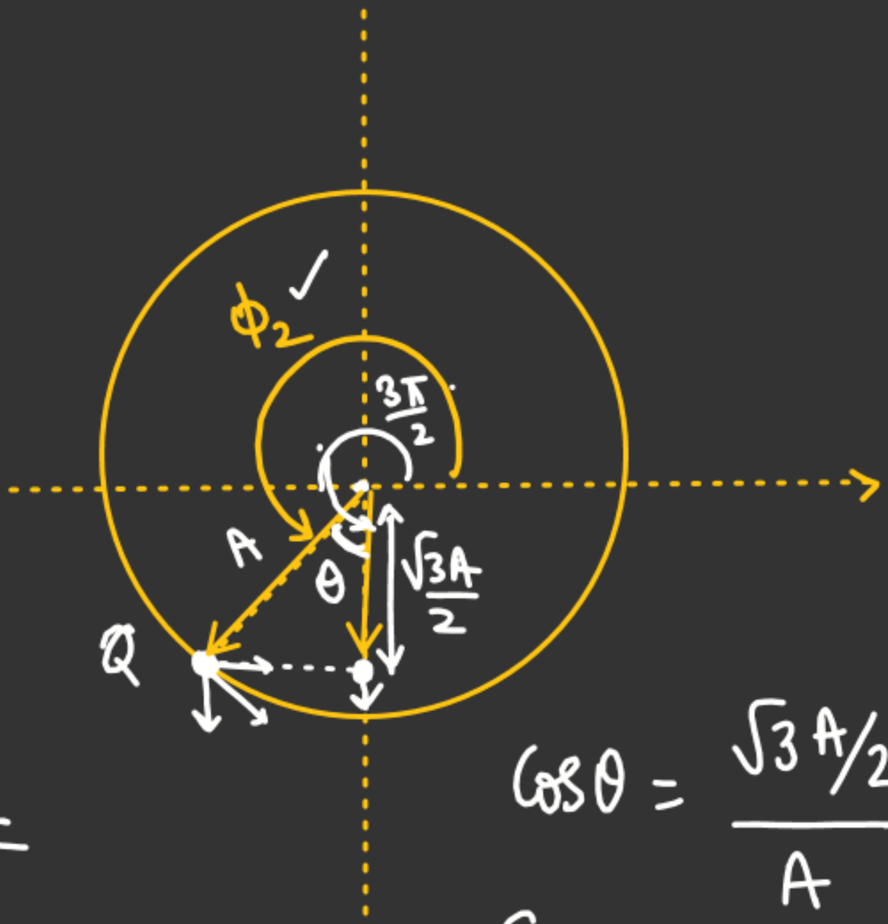
Nishant Jindal

Both the particle P & Q have Same amplitude A. & Same mean position.

At $t = t$, the situation of P & Q as shown in fig. Find $\Delta\phi = ??$







$$\sin\phi_1 = \frac{A/2}{A}$$

$$\sin\phi_1 = \frac{1}{2}$$

$$\phi_1 = \frac{\pi}{6}$$

$$\cos\theta = \frac{\sqrt{3}A/2}{A}$$

$$\cos\theta = \frac{\sqrt{3}}{2}$$

$$\theta = \pi/6$$

$$\phi_2 = \left(\frac{3\pi}{2} - \frac{\pi}{6}\right) \checkmark$$

$$\phi_2 = \frac{8\pi}{6} = \frac{4\pi}{3}$$

$$\Delta\phi = \left(\frac{4\pi}{3} - \pi/6\right)$$

$$= \frac{8\pi - \pi}{6}$$

$$= \left(\frac{7\pi}{6}\right)$$

Nishant Jindal

To Find Time period in Case of S.H.M

Steps :-

1. Locate Mean position (Equilibrium position)
   [$a = 0$, $F_{net} = 0$,] linear   [$\alpha = 0$, $\tau = 0$] Angular S.H.M

2. Displaced the particle from mean position and find restoring force. or restoring torque

3. Find $a = \left(\frac{F_{restoring}}{m}\right)$ if S.H.M is linear.

   or find $\alpha = \left(\frac{\tau_r}{I}\right)$ if S.H.M is angular.

4. Compair the above result with

$$\left[\begin{array}{l} a = -\omega^2 x \text{ for linear} \\ \& \quad \alpha = -\omega^2 \theta \text{ for angular} \end{array}\right] \rightarrow \& \text{ find } \omega = ?$$

$$\omega = \frac{2\pi}{T} \Rightarrow T = \frac{2\pi}{\omega}.$$

Nishant Jindal

S.H.M Spring block system.

$$F_r = -Kx$$

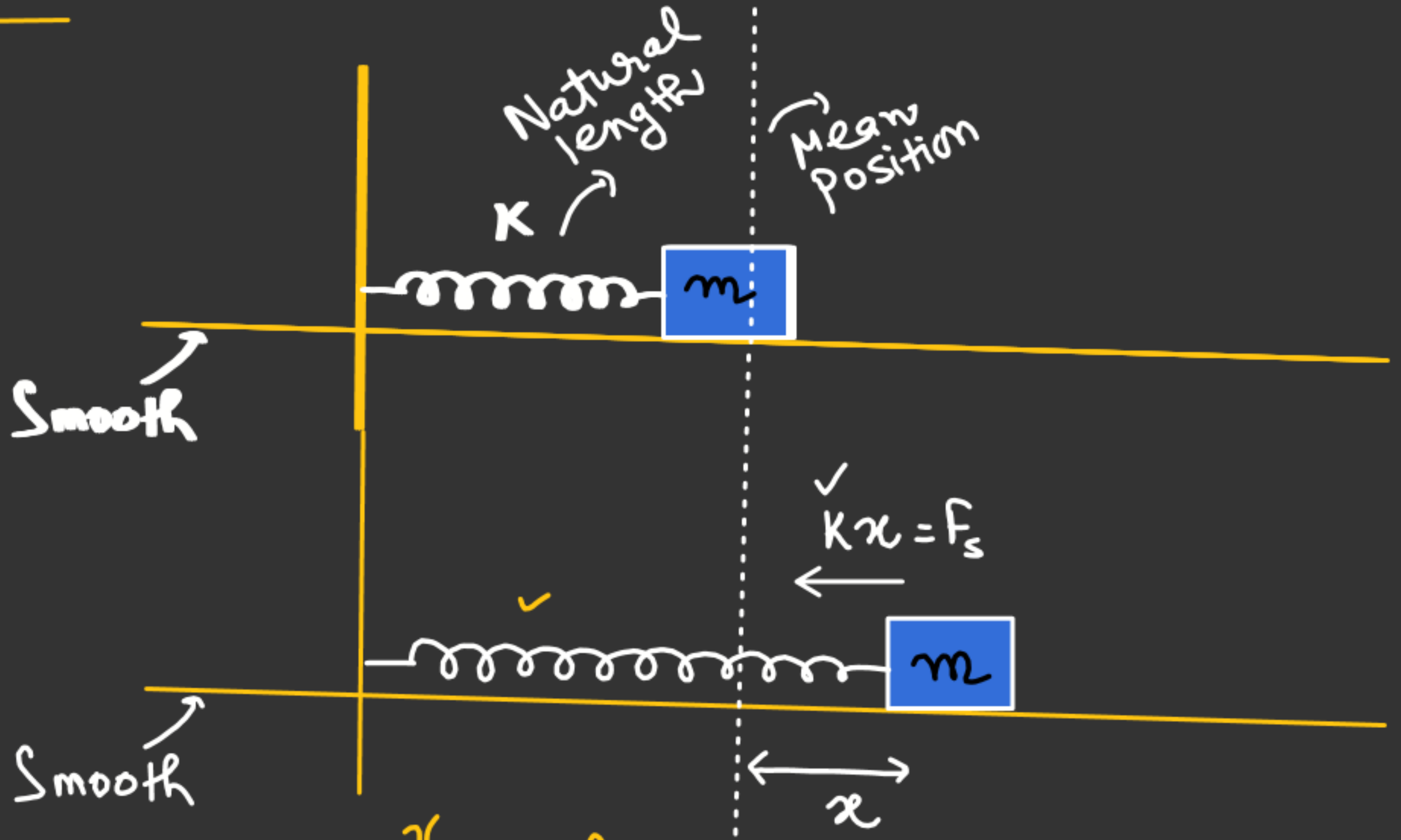
$$a = -\frac{K}{m}x$$

Compare with

$$a = -\omega^2 x.$$

$$\omega = \sqrt{\frac{K}{m}}$$

K = Spring Constant



$$\frac{2\pi}{T} = \sqrt{\frac{K}{m}}$$

$$\boxed{T = 2\pi\sqrt{\frac{m}{K}}}$$

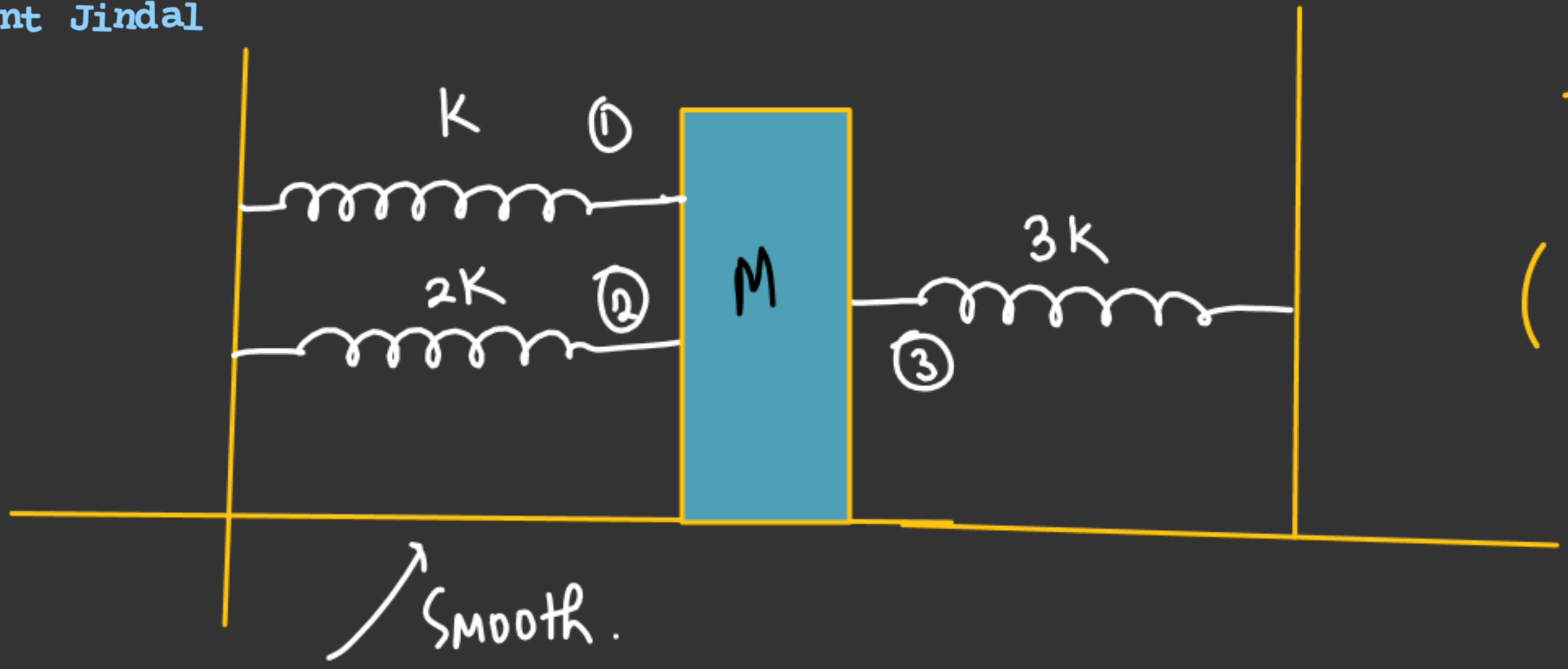
For Amplitude. $x_{max} = A$

$$\frac{1}{2}Mv_0^2 = \frac{1}{2}KA^2$$

$\Downarrow$ Mean Position $\quad\quad$ $\Downarrow$ Extreme position

$$A = \left(\sqrt{\frac{M}{K}}\right)v_0$$

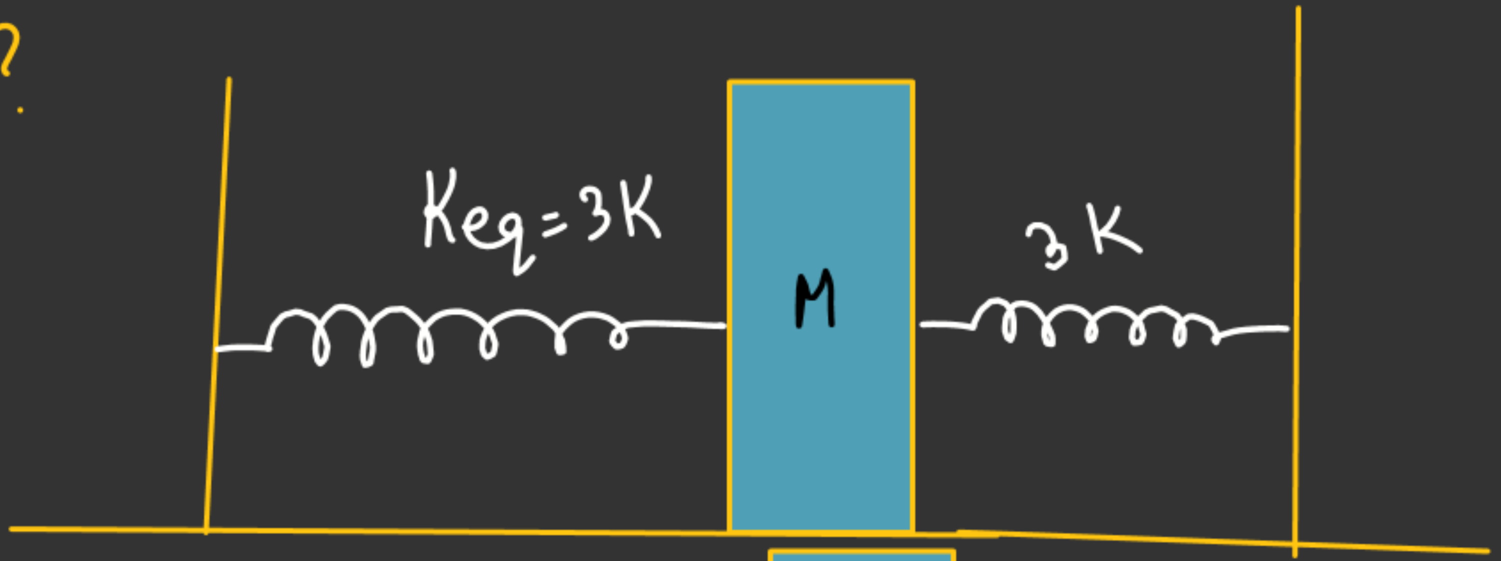
$$v_{max} = v_0 = A\omega.$$



$$\frac{1}{K_{eq}} = \frac{1}{K_1} + \frac{1}{K_2} \cdots \Rightarrow$$ In Series. (Spring force same)

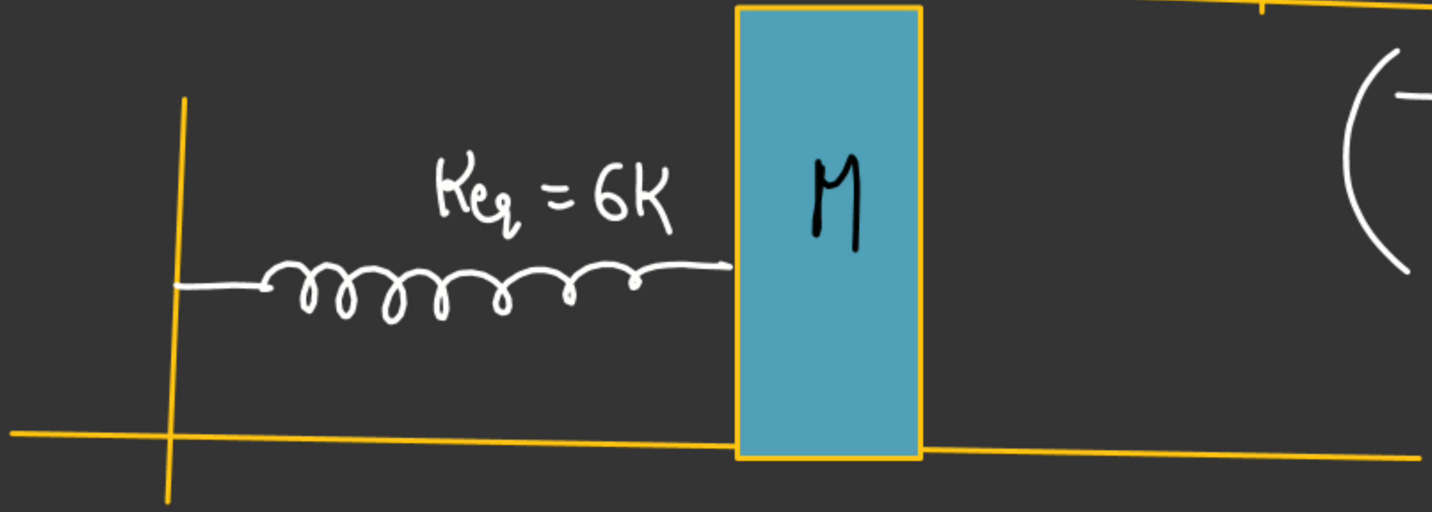
$$\left(K_{eq} = K_1 + K_2 \cdots \right) \Rightarrow$$ In parallel Elongation or Compression in each Spring is same

$T = ??$



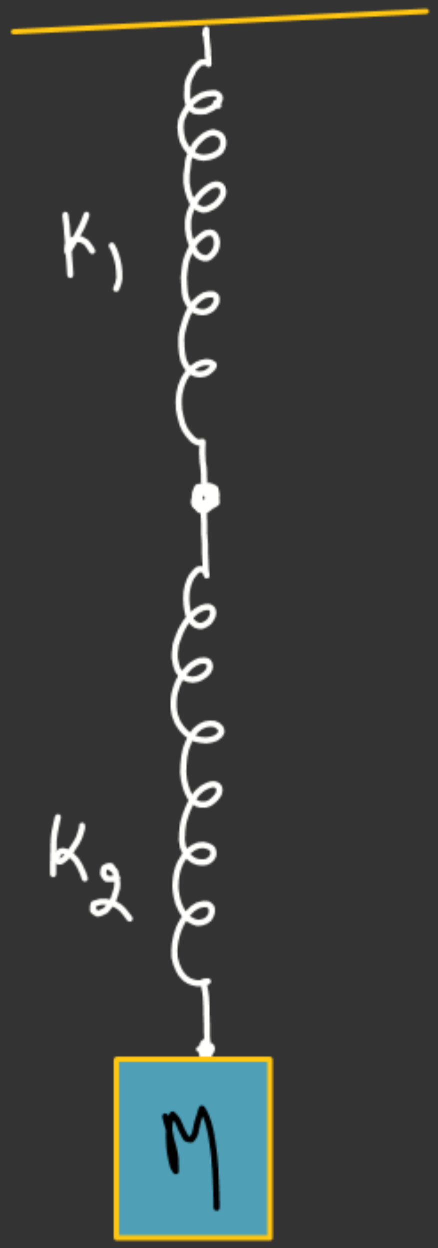
All are in parallel

$$K_{eq} = K + 2K + 3K$$
$$= 6K$$



$$\left(T = 2\pi\sqrt{\frac{m}{6k}} \right) \checkmark$$

Nishant Jindal



Series Combination

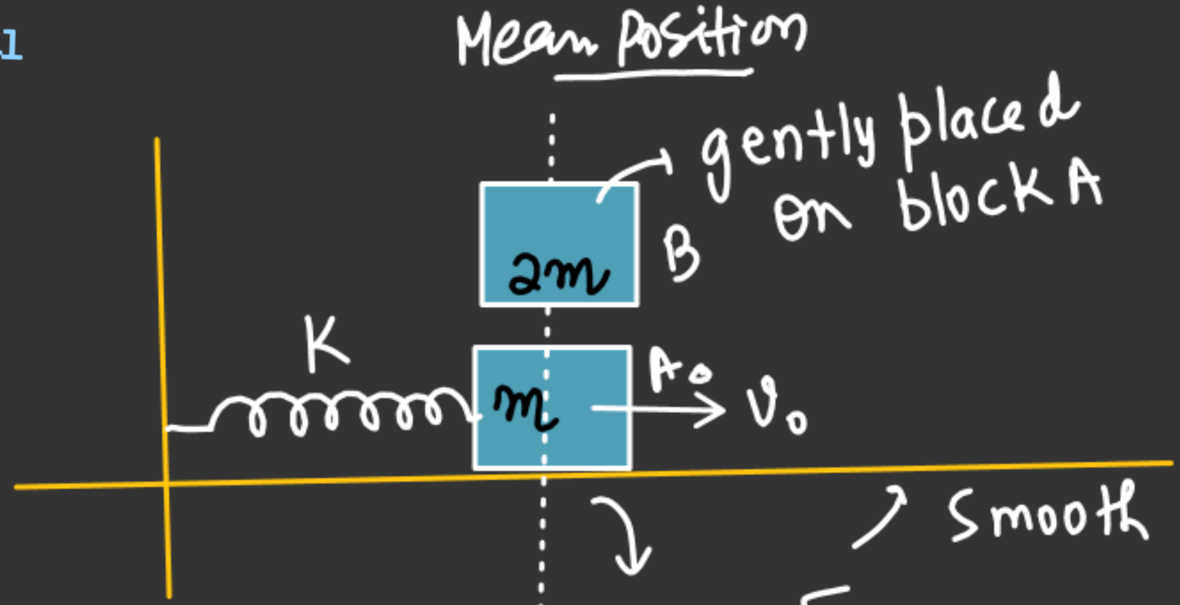
$$\frac{1}{K_{eq}} = \left(\frac{1}{K_1} + \frac{1}{K_2}\right)$$

$$K_{eq} = \left(\frac{K_1K_2}{K_1+K_2}\right)$$

$$T = 2\pi\sqrt{\frac{m}{K_{eq}}}$$

$$T = 2\pi\sqrt{\frac{m(K_1+K_2)}{K_1K_2}}$$

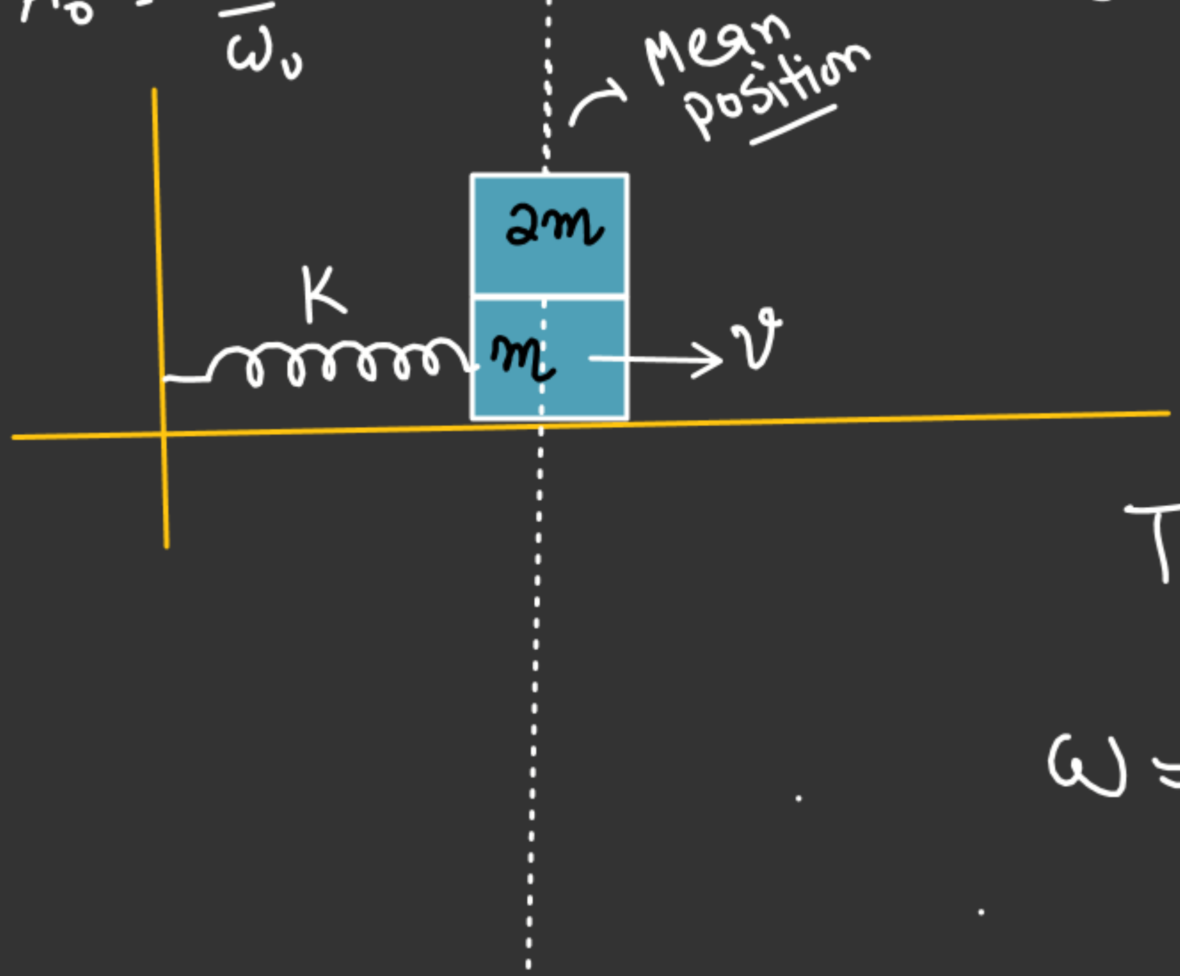
Nishant Jindal



$$T_0 = 2\pi\sqrt{\frac{m}{K}}$$

$$v_0 = A_0\omega_0$$

$$A_0 = \frac{v_0}{\omega_0}$$



No Relative Slipping b/w A & B when B is placed on A.
Find the New time period and New Amplitude.

L.M.C in x- direction

Just befor Loding = Just after Loading

$$mv_0 = 3mv$$

$$\left(v = \frac{v_0}{3}\right) ✓$$

$$T = 2\pi\sqrt{\frac{3M}{K}}$$

$$\omega = \sqrt{\frac{K}{3m}} = \frac{\omega_0}{\sqrt{3}}$$

New Amplitude = ??

$$v = A \cdot \omega$$

$$A = \frac{v}{\omega} = \frac{v_0}{3 \times \omega_0} \times \sqrt{3} \rightarrow A_0$$

$$A = \left(\frac{A_0}{\sqrt{3}}\right) ✓$$