Q Find EOC P.T. Origin & making Intercept of unit length on Lines $y^2 = x^2$.

$C_1\ \left(x-\frac{1}{\sqrt2}\right)\left(x+\frac{1}{\sqrt2}\right)+\left(y-\frac{1}{\sqrt2}\right)\left(y-\frac{1}{\sqrt2}\right)=0$

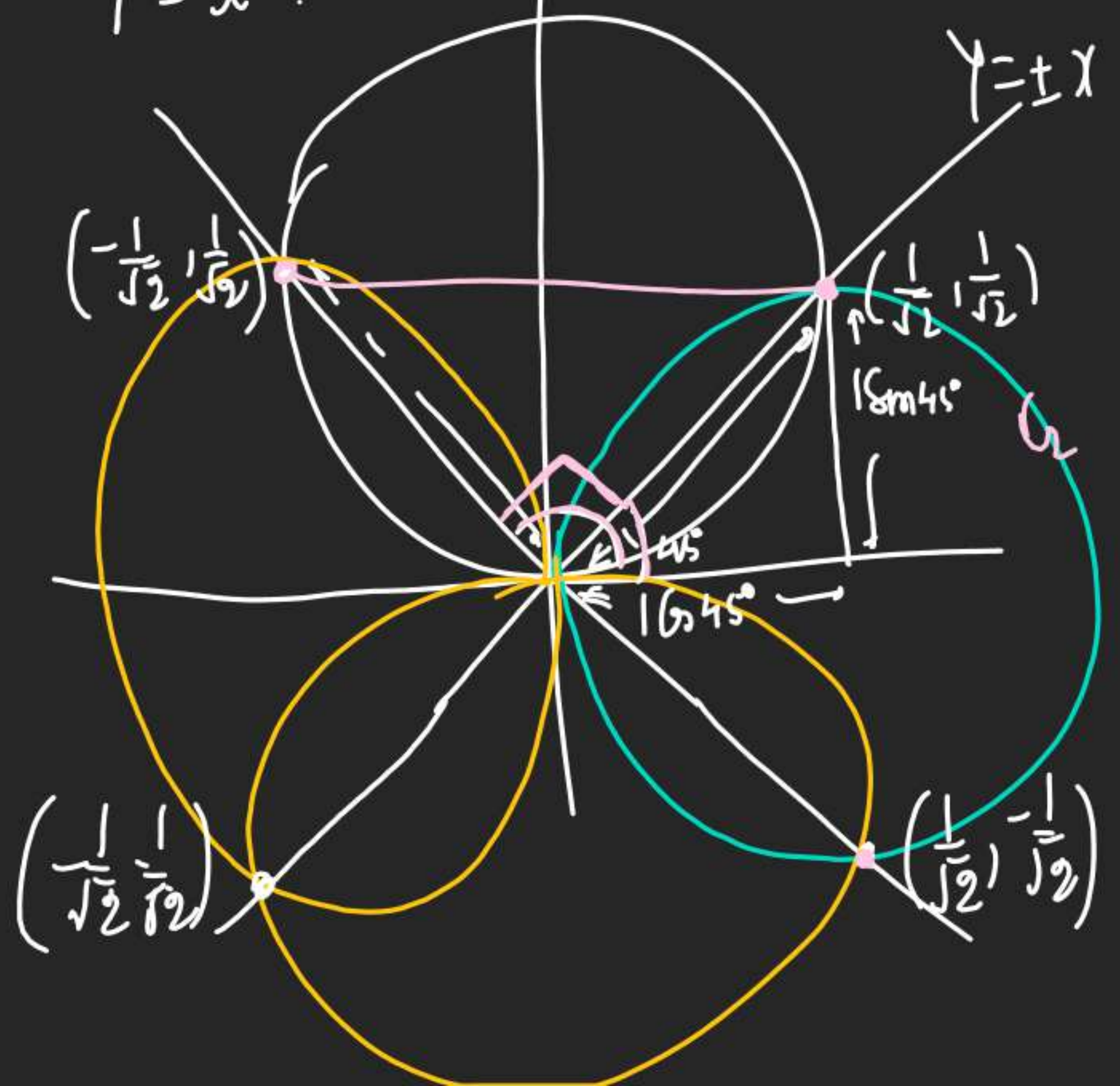
$C_2\ \left(x-\frac{1}{\sqrt2}\right)\left(x-\frac{1}{\sqrt2}\right)+\left(y-\frac{1}{\sqrt2}\right)\left(y+\frac{1}{\sqrt2}\right)=0$

$C_3\ \left(x+\frac{1}{\sqrt2}\right)\left(x-\frac{1}{\sqrt2}\right)+\left(y+\frac{1}{\sqrt2}\right)\left(y+\frac{1}{\sqrt2}\right)=0$

C_4 DY

$y^2=x^2$

$y=\pm x$



Q Find EOC [2] Passing thru Origin & Cutting Intercepts of length a & b on X & Y Axis.

length of Int. on X Axis $= 2\sqrt{g^2-c}$

As Circle is P.T. Origin $\Rightarrow \boxed{c=0}$

∴ length of Intercept on X Axis

$2\sqrt{g^2} = a$ (given)

$|g| = \frac{a}{2} \Rightarrow g=\pm\frac{a}{2}$

Similarly $2\sqrt{f^2-0}=b \Rightarrow |f|=\frac{b}{2} \Rightarrow f=\pm\frac{b}{2}$

$$\boxed{x^2+y^2\pm ax\pm by=0}$$

Q [3] A variable Circle Passing thru a fixed Pt $A(p,q)$ & touches X Axis, then the locus of Other End of Diameter? (Main)

$$\frac{k+q}{2} = \frac{\sqrt{(h-p)^2+(k-q)^2}}{2}$$

$$\cancel{k^2}+\cancel{q^2}+2kq = h^2+\cancel{k^2}-2ph-2kq+p^2+\cancel{q^2}$$

$$x^2-2px-4qy+p^2=0$$

$$(x-p)^2=4qy$$

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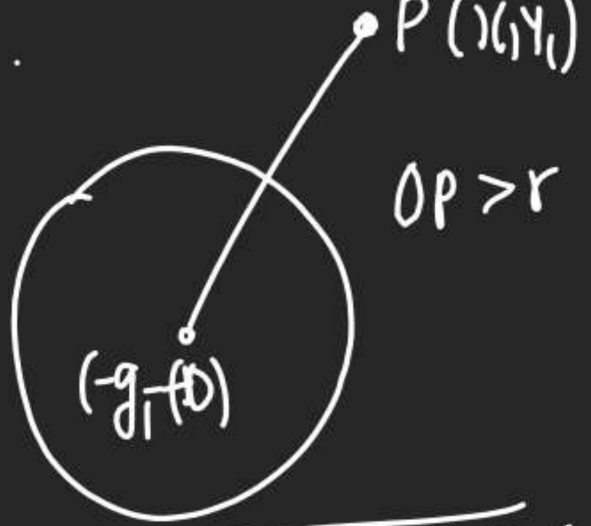
Position of Pt. (x_1, y_1) WRT a Circle.

Here we will study about a given Pt & a given Circle. that Pt. is inside, outside or on Circle.



$OP > r$

$$\sqrt{(x_1+g)^2+(y_1+f)^2} > \sqrt{g^2+f^2-c}$$

$$x_1^2+y_1^2+2gx_1+2fy_1+c>0$$

$S(x_1,y_1)>0$	Pt. is outside Circle
$S(x_1,y_1)<0$	Inside Circle
$S(x_1,y_1)=0$	On Circle

Q4 Find Position of $(0,0)$ WRT. $x^2+y^2-6x-6y=0$

$S(0,0) \Rightarrow 0^2+0^2-6\times0-6\times0=0$

$0=0$

$(0,0)$ lying on Circle.

Q5 Find Position of $(3,2)$ WRT. to $x^2+y^2-6x-6y=0$

$S(3,2)$ $9+4-18-12<0$

Inside Circle.

Q6 If Pt (λ,λ) lying inside $x^2+y^2-50=0$ then No. of Integral value of λ?

$S(\lambda,\lambda)<0$

$\lambda^2+\lambda^2-50<0$

$\lambda^2-25<0$

$(\lambda-5)(\lambda+5)<0$

$-5<\lambda<5$

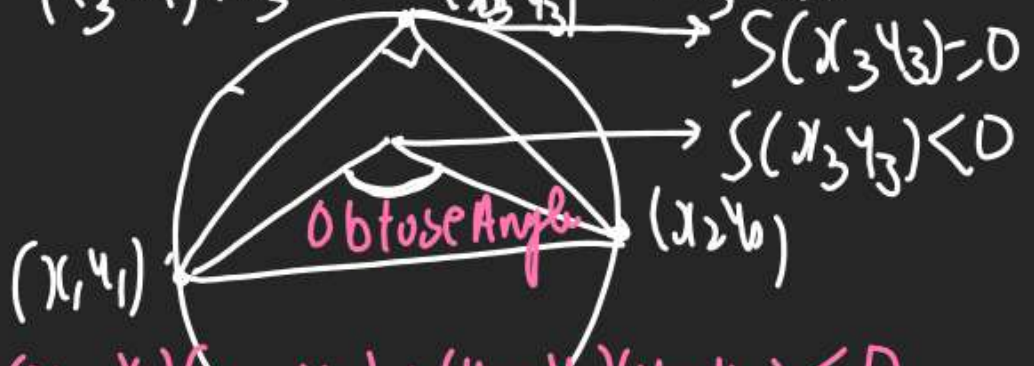
$\lambda = -4,-3,-2,-1,0,1,2,3,4 \Rightarrow 9$ values

Q7 If Line joining (x_3,y_3) to (x_1,y_1) & (x_2,y_2) makes obtuse Angle then P.T.

$$(x_3-x_1)(x_3-x_2)+(y_3-y_1)(y_3-y_2)<0$$

If Recalls Diametric Eqn of Circle.

$$(x_3-x_1)(x_3-x_2)+(y_3-y_1)(y_3-y_2)=0$$



$$(x_3-x_1)(x_3-x_2)+(y_3-y_1)(y_3-y_2)<0$$

When Pt Inside & Angle Obtuse

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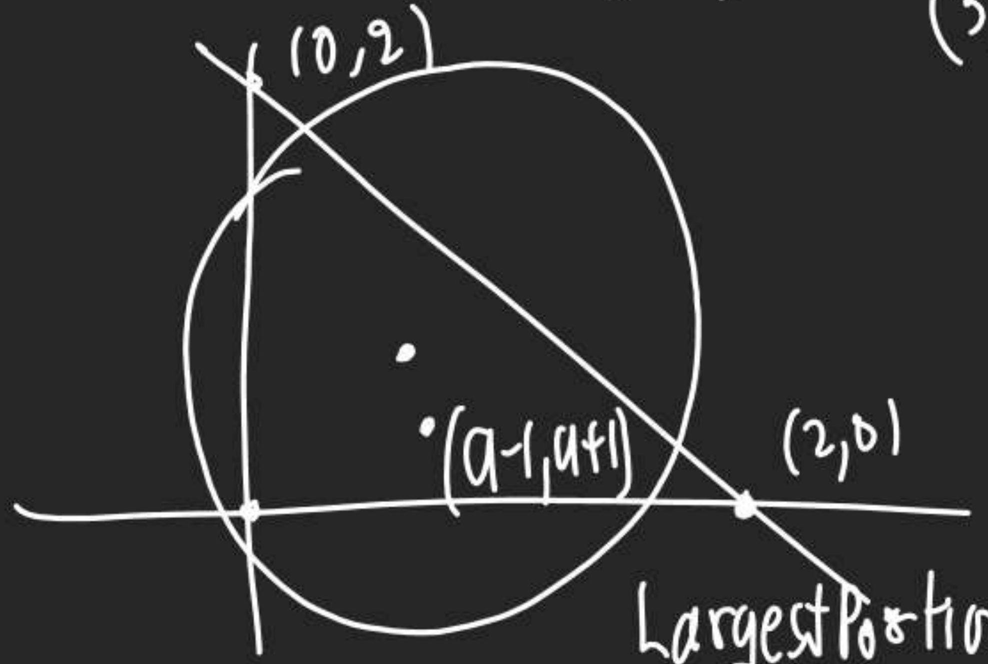
Q8 Find value of a for which the pt. $(a-1, a+1)$ lies in largest section of circle $x^2+y^2-x-y-6=0$ made by the chord $x+y-2=0$

Centre $= \left(\frac{1}{2}, \frac{1}{2}\right)$ Rad $= \sqrt{\frac{1}{4}+\frac{1}{4}+6} = \frac{\sqrt{13}}{\sqrt{2}} < 2$

Chord $\to x+y=2 \Rightarrow \frac{x}{2}+\frac{y}{2}=1$



Largest Portion Below Line

(1) Position of Centre $\left(\frac{1}{2}, \frac{1}{2}\right)$ WRT. $x+y-2=0$

$\frac{1}{2}+\frac{1}{2}-2 = -ve$

$\left(\frac{1}{2}, \frac{1}{2}\right)$ Below Line

(2) $(a-1, a+1)$ Below Line

$(a-1)+(a+1)-2<0$

$a<1 \to$ (1)

(3) $(a-1, a+1)$ Inside Circle

$S(a-1, a+1)<0$

$(a-1)^2+(a+1)^2-(a-1)-(a+1)-6<0$

$2a^2+2-2a-6<0$

$a^2-a-2<0$

$(a-2)(a+1)<0$

$-1<a<2$

$a \in (-1, 1)$ Ans

Max / Min distance of any pt. from Circle.

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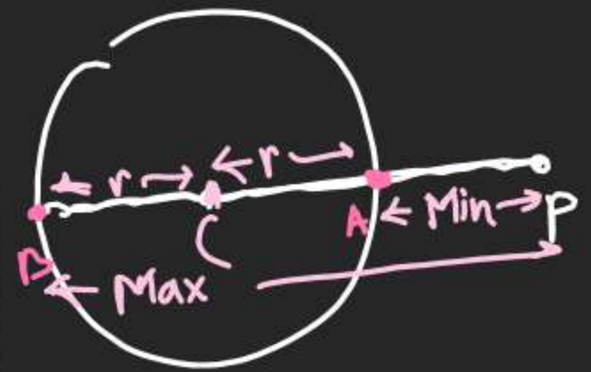
Max / Min distance of any Pt from Circle

Max / Min distance can be find out only when Pt is attached to its diametric line

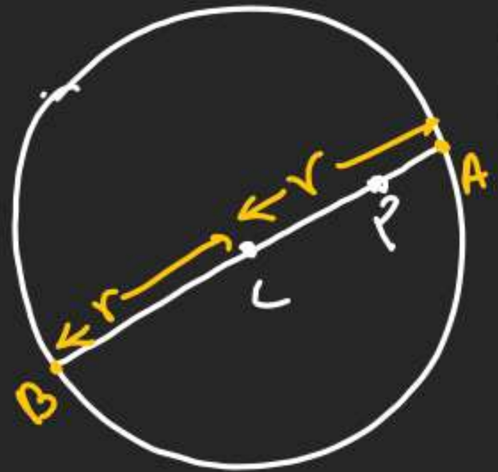
When Pt is outside



AP = Min = CP − r

BP = Max = CP + r

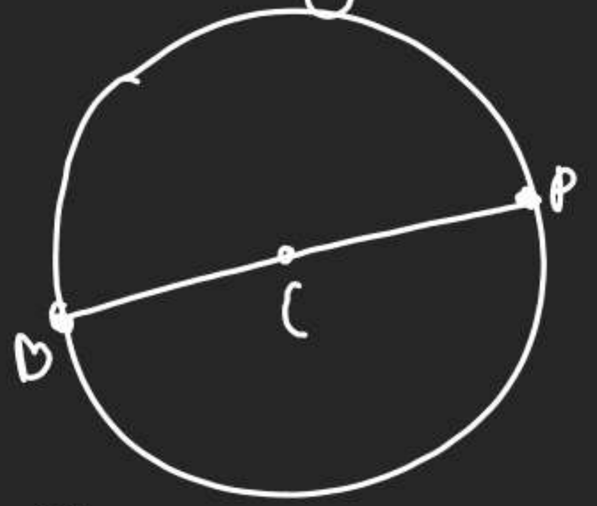
When Pt is Inside



AP = r − CP

BP = r + CP

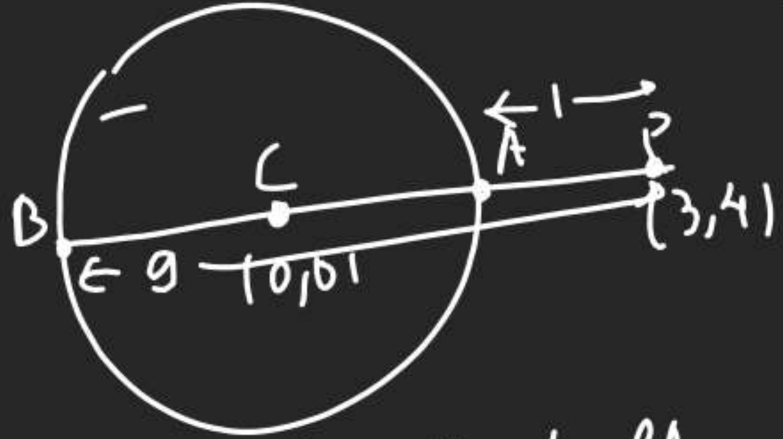
When Pt. lying on Circle.



Min dist = 0

Max Dist = 2r

Q. Find Max & Min distance of (3,4) from Circle $x^2+y^2-16=0$ & also find Coordinate of these Pts.



(1) Position of Pt (3,4) WRT Circle

$9+16-16>0 \Rightarrow$ Pt. outside

(2) $CP = \sqrt{(3-0)^2+(4-0)^2} = 5$ (3) Rad $= r = 4$

(4) Max $BP = CP + r = 5+4 = 9$

Min $AP = CP - r = 5-4 = 1$

(5) Coordinate of A

(1) $(SL)_{CP}$: $\tan\theta = \frac{4-0}{3-0} = \frac{4}{3} \Rightarrow \cos\theta = \frac{3}{5}, \sin\theta = \frac{4}{5}$

(2) $A = \left(0+4\times\frac{3}{5},\ 0+4\times\frac{4}{5}\right) = \left(\frac{12}{5}, \frac{16}{5}\right)$

(6) Coordinate of B

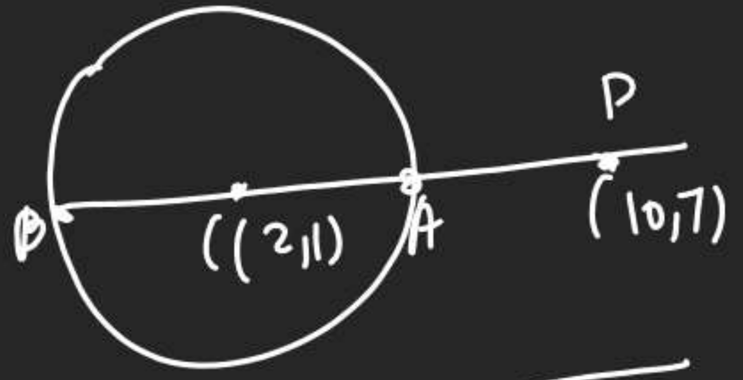
$B = \left(0-4\times\frac{3}{5},\ 0-4\times\frac{4}{5}\right) = \left(-\frac{12}{5}, -\frac{16}{5}\right)$

Q 10 Find Gr. & least dist. of $(10,7)$ from $x^2+y^2-4x-2y-20=0$.

(1) Centre $C=(2,1)$, $r=\sqrt{4+1+20}$
$=5$

(2) Position of $(10,7)$

$100+49-40-14-20>0$



$CP=\sqrt{8^2+6^2}=10$

$AP=CP-r=10-5=5$

$BP=CP+r=10+5=15$

Q 11 St. Line $2x-3y=1$ divides Circular Region $x^2+y^2\le 6$ in 2 Parts. If

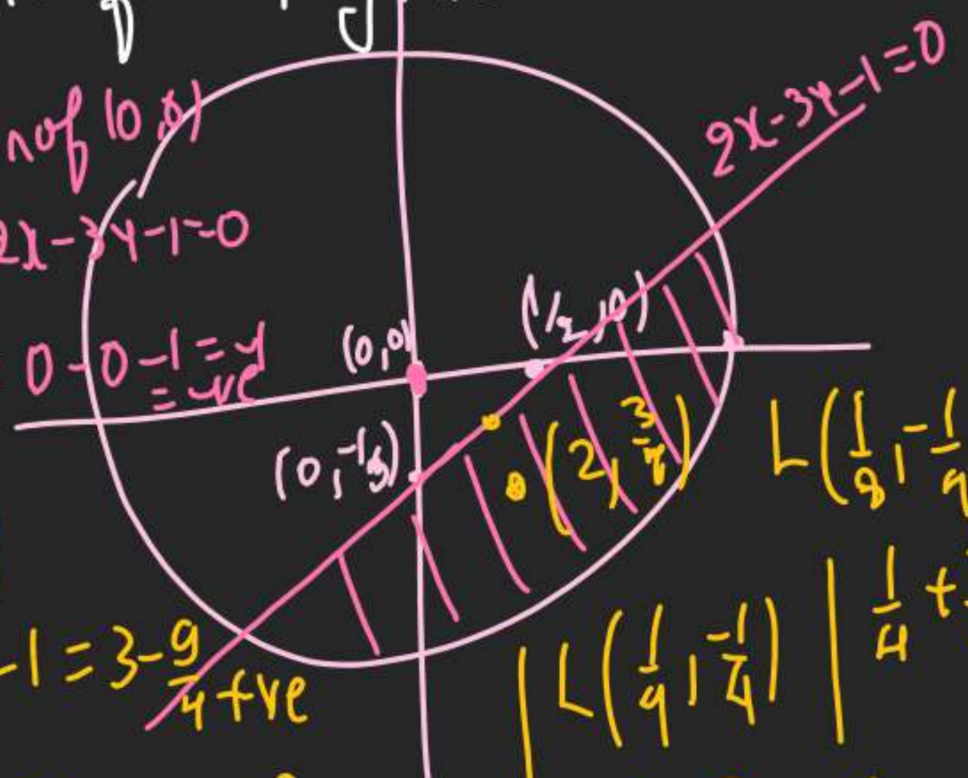
$2x-3y=1 \rightarrow \frac{x}{\frac{1}{2}}+\frac{y}{-\frac{1}{3}}=1$

Inside Circle $S=\left\{\left(2,\frac{3}{4}\right),\left(\frac{5}{2},\frac{3}{4}\right),\left(\frac{1}{4},\frac{-1}{4}\right)\left(\frac{1}{8},\frac{-1}{4}\right)\right\}$

then No of Pts Lying in Smaller Part is

(1) Position of $(0,0)$ WRT $2x-3y-1=0$

$L(0,0)=0-0-1=-1=-ve$



$L\left(2,\frac{3}{4}\right)$

$4-\frac{9}{4}-1=3-\frac{9}{4}$ +ve

$L\left(\frac{5}{2},\frac{3}{4}\right)=5-\frac{9}{4}-1\ (+)$

$L\left(\frac{1}{8},-\frac{1}{4}\right)$ | $\frac{1}{4}+\frac{3}{4}-1=0$

$L\left(\frac{1}{4},\frac{-1}{4}\right)$ | $\frac{1}{2}+\frac{3}{4}-1\ (+)$

3 Pts in Smaller Part.

Q 12 $\left(a,\frac{2}{a}\right),\left(b,\frac{2}{b}\right),\left(c,\frac{2}{c}\right),\left(d,\frac{2}{d}\right)$

4 distinct Pts on a Circle of Radius 4 units then $a\cdot b\cdot c\cdot d=?$

Circle $\rightarrow x^2+y^2+2gx+2fy+c=0$

Pt $\rightarrow \left(m,\frac{2}{m}\right)$ $m\rightarrow a,b,c,d$

$m^2+\frac{4}{m^2}+2gm+\frac{4f}{m}+c=0$

$m^4+2gm^3+cm^2+4fm+4=0$

a b c d

Prod of Root $=abcd$
$=4$