Nishant Jindal

23.

$7r^{n-1} \times r = 448 \times r$

$$\frac{7(1-r^n)}{1-r} = 889$$

$$\frac{7-448r}{1-r} = 889$$

$\alpha^3 = 216 \Rightarrow \alpha = 6$

$S_p + s_p = \frac{2}{1-r^{2p}} = 2S_{2p}$

$S_p = \frac{1}{1-r^p}$

$s_p = \frac{1}{1-(-r^p)} = \frac{1}{1+r^p}$

$\frac{\alpha}{r} \quad \alpha \quad \alpha r$

$\alpha^2\left(\frac{1}{r}+r+1\right) = 156$

$36\left(\frac{1}{r}+r+1\right) = 156$

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$$\sum_{r=1}^{n} r = 1+2+3+4+\cdots+(n-1)+n = \frac{n}{2}(1+n)$$

$$\boxed{\sum_{r=1}^{n} r = \frac{n(n+1)}{2}}$$

$$\boxed{\sum_{r=1}^{n} r^2 = \frac{n(n+1)(2n+1)}{6}}$$

$$\sum_{r=1}^{n} r^2 = 1^2+2^2+3^2+\cdots+n^2 \quad \checkmark$$

$$(k+1)^3 - k^3 = 3k^2+3k+1$$

$k=1$, $2^3 - 1^3 = 3(1^2)+3(1)+1$

$k=2$, $3^3 - 2^3 = 3(2^2)+3(2)+1$

$k=3$, $4^3 - 3^3 = 3(3^2)+3(3)+1$

$\vdots$

$k=n$, $(n+1)^3 - n^3 = 3(n^2)+3(n)+1$

$$(n+1)^3 - 1^3 = 3\sum_{r=1}^{n} r^2 + 3\sum_{r=1}^{n} r + n$$

$$n^3+3n^2+2n - \frac{3}{2}n(n+1) = 3\sum_{r=1}^{n} r^2$$

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$$\boxed{1^3+2^3+3^3+\cdots+n^3=\sum_{r=1}^{n} r^3=\left(\frac{n(n+1)}{2}\right)^2}$$

$$(k+1)^4-k^4 = 4k^3+6k^2+4k+1$$

$$(k^2+2k+1)^2$$

$k=1,\quad 2^4-1^4 = 4(1^3)+6(1^2)+4(1)+1$

$k=2,\quad 3^4-2^4 = 4(2^3)+6(2^2)+4(2)+1$

$k=3,\quad 4^4-3^4 = 4(3^3)+6(3^2)+4(3)+1$

$\vdots$

$k=n\quad (n+1)^4-n^4 = 4(n^3)+6(n^2)+4(n)+1$

$$(n+1)^4-1^4 = 4\sum_{r=1}^{n} r^3+6\sum_{r=1}^{n} r^2+4\sum_{r=1}^{n} r+n$$

$$(n+1)^4-1^4 = 4\sum_{r=1}^{n} r^3+n(n+1)(2n+1)+2n(n+1)+n$$

Nishant Jindal

1. Find $(31)^2+(32)^2+(33)^2+\cdots+(50)^2$

$$\sum_{r=1}^{50} r^2 - \sum_{r=1}^{30} r^2 = \frac{50(51)(101)}{6} - \frac{30(31)(61)}{6}$$

$$\sum_{r=1}^{20}(30+r)^2 = \sum_{r=1}^{20}\left(900+60r+r^2\right) = 900\sum_{r=1}^{20}1 + 60\sum_{r=1}^{20} r + \sum_{r=1}^{20} r^2$$

$$= 900\times 20 + \frac{60\times 20\times 21}{2} + \frac{20\times 21\times 41}{6}$$

2. $3^2+7^2+11^2+15^2+\cdots$ + upto n terms

$$\sum_{r=1}^{n}\left(f(r)+g(r)+h(r)\right) = \sum_{r=1}^{n} f(r) + \sum_{r=1}^{n} g(r) + \sum_{r=1}^{n} h(r)$$

$$\left(f(1)+g(1)+h(1)\right)+\left(f(2)+g(2)+h(2)\right)+\left(f(3)+g(3)+h(3)\right)+\cdots+\left(f(n)+g(n)+h(n)\right)$$

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2. $3^2+7^2+11^2+15^2+\cdots +$ upto n terms.

$$= \sum_{r=1}^{n}(4r-1)^2 = 16\sum_{r=1}^{n} r^2 - 8\sum_{r=1}^{n} r + \sum_{r=1}^{n} 1$$

$$= 16\left(\frac{n(n+1)(2n+1)}{6}\right) - \frac{8n(n+1)}{2} + n$$

3. $$\sum_{r=1}^{n}\left(r(r+1)(3r-1)\right) = 3\sum_{r=1}^{n} r^3 + 2\sum_{r=1}^{n} r^2 - \sum_{r=1}^{n} r$$

$$= 3\frac{n^2(n+1)^2}{4} + 2\frac{n(n+1)(2n+1)}{6} - \frac{n(n+1)}{2}$$

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4. Find the value of $n$ for which the quadratic

eqn. $\sum_{K=1}^{n}\left((x+K-1)(x+K)\right) = 10n$ has solutions $\alpha$, $\alpha+1$ for some $\alpha$.

$x^2+(2K-1)x+K^2-K$

$x^2\sum_{K=1}^{n} 1 + x\sum_{K=1}^{n}(2K-1) + \sum_{K=1}^{n} K(K-1) = 10n$

$nx^2 + n^2x + \frac{n(n+1)(2n+1)}{6} - \frac{n(n+1)}{2} = 10n$

$\frac{(n+1)(2n+1)}{6} - \frac{n+1}{2} - 10$

$\frac{(n+1)(2n+1-3)}{6} - 10$

$\frac{n^2-1}{3} - 10 = $

$x^2 + nx + \frac{n^2-31}{3} = 0 \quad \begin{matrix} \alpha \\ \alpha+1 = \beta \end{matrix}$

$1^2 = (\alpha+\beta)^2 - 4\alpha\beta = (-n)^2 - \frac{4}{3}(n^2-31) = \frac{124-n^2}{3}$

$n^2 = 121$

$\boxed{n = 11}$ ✓

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5.

$$\sum_{i=1}^{n}\sum_{j=1}^{i}\left(\sum_{k=1}^{j} 1\right) = \sum_{i=1}^{n}\left(\sum_{j=1}^{i} j\right)$$

$i = i,\ j = 1, 2, 3, \dots i$

$i = i,\ j = j,\ k = 1, 2, 3, \dots, j$

$1 + 1 + 1 + \dots + 1$

$i = 1,\ j = 1,\ k = 1$

1

$i = 3$

$j = 1 \rightarrow k = 1$ : 1

$j = 2 \rightarrow k = 1,\ k = 2$ : 2

$j = 3 \rightarrow k = 1,\ k = 2,\ k = 3$ : 3

$1 + 2 + 3 + \dots i$

$$\sum_{i=1}^{n} \frac{i(i+1)}{2} = \frac{1}{2}\left(\sum_{i=1}^{n} i^2 + \sum_{i=1}^{n} i\right)$$

$$= \frac{1}{2}\left(\frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2}\right)$$

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$$(1+2+3+\cdots+n)^2 = (1^2+2^2+\cdots n^2) + 2S$$

$$\frac{n^2(n+1)^2}{4} = \frac{n(n+1)(2n+1)}{6} + 2\sum\sum ij$$

$$(a_1+a_2+\cdots+a_n)^2 = (a_1+a_2+\cdots a_n)(a_1+a_2+\cdots+a_n) = (a_1^2+a_2^2+\cdots a_n^2) + 2(a_1a_2+a_1a_3+\cdots+a_{n-1}a_n)$$

5(b).

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7, 8, 9, 10

$$S = \boxed{\sum_{i=1}^{n}\sum_{j=i+1}^{n} ij} = (1\cdot 2)+(1\cdot 3)+(1\cdot 4)+\cdots(1\cdot n) + (2\cdot 3)+(2\cdot 4)+\cdots(2\cdot n) + (3\cdot 4)+(3\cdot 5)+\cdots(3\cdot n) + \cdots (n-1)n$$

$i=1,\ j=2,3,\ldots,n$

$i=2,\ j=3,4,5,\ldots,n$

$i=3,\ j=4,5,6,\ldots,n$

$\vdots$

$i=n-1,\ j=n$