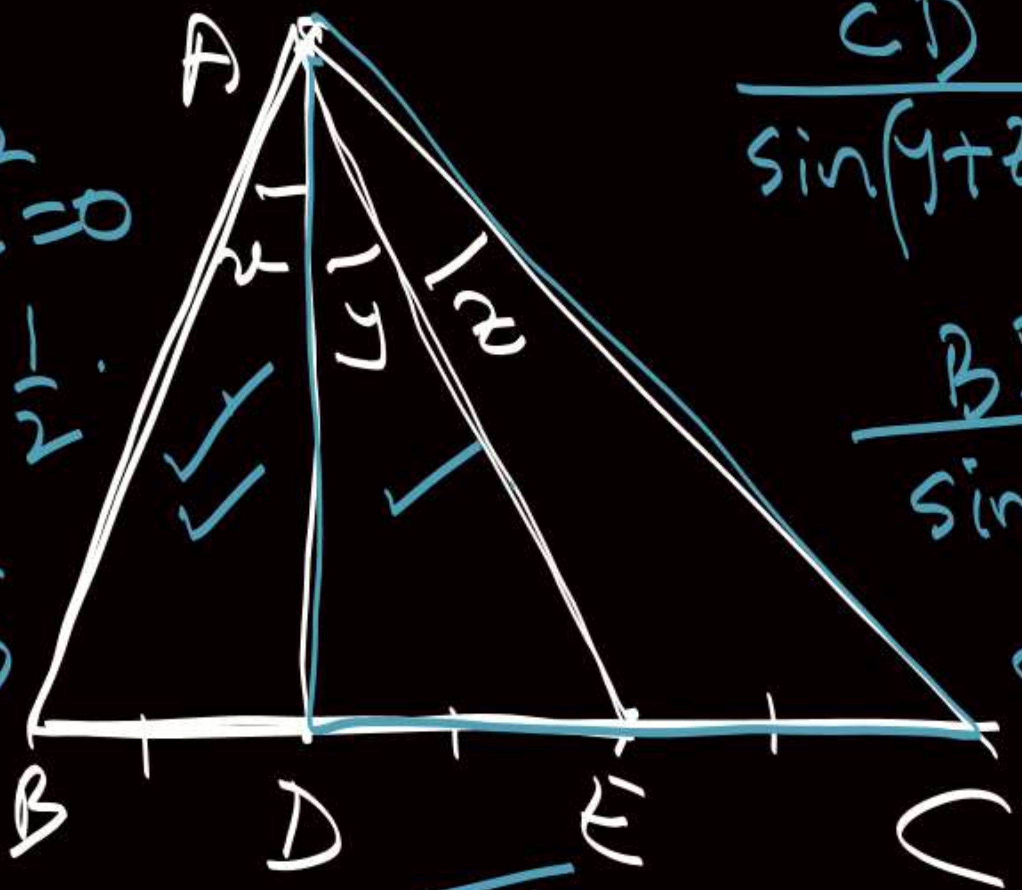
Nishant Jindal



$$\left(a^2-b^2+c^2\right)^2-2a^2c^2=0$$

$$\left(\frac{a^2+c^2-b^2}{2ac}\right)^2=\frac{1}{2}$$

$$\frac{BE}{\sin(x+y)}=\frac{AE}{\sin B}$$

$$\frac{CE}{\sin z}=\frac{AE}{\sin C}$$

$$\frac{CD}{\sin(y+z)}=\frac{AD}{\sin C}$$

$$\frac{BD}{\sin x}=\frac{AD}{\sin B}$$

$$\frac{2\sin x}{\sin(y+z)}=\frac{\sin B}{\sin C}$$

$$\sqrt{s(s-a)(s-b)(s-c)}=(2s-2b)(2s-2c)$$

$$\frac{3}{\sqrt{7}}=\frac{9}{\sqrt{63}}=\cot\frac{C}{2}$$

$$\sqrt{\frac{1-\frac{31}{32}}{1+\frac{31}{32}}}$$

$$\sqrt{\frac{s(s-a)}{(s-b)(s-c)}}=4C=\cot\frac{A}{2}$$

$$\frac{s-4}{s+4}\cot\frac{C}{2}$$

$$\cos C=?$$

$$=\frac{1-\frac{7}{9}}{1+\frac{7}{9}}$$

1. Simplify

$$\begin{vmatrix} \cancel{\dfrac{1-a_1^3b_1^3}{1-a_1b_1}}\ 1+a_1b_1+a_1^2b_1^2 & \dfrac{1-a_1^3b_2^3}{1-a_1b_2} & \dfrac{1-a_1^3b_3^3}{1-a_1b_3} \\ \dfrac{1-a_2^3b_1^3}{1-a_2b_1} & \dfrac{1-a_2^3b_2^3}{1-a_2b_2} & \dfrac{1-a_2^3b_3^3}{1-a_2b_3} \\ \dfrac{1-a_3^3b_1^3}{1-a_3b_1} & \dfrac{1-a_3^3b_2^3}{1-a_3b_2} & \dfrac{1-a_3^3b_3^3}{1-a_3b_3} \end{vmatrix}$$

$$\begin{vmatrix} 1 & a_1 & a_1^2 \\ 1 & a_2 & a_2^2 \\ 1 & a_3 & a_3^2 \end{vmatrix} \begin{vmatrix} 1 & 1 & 1 \\ b_1 & b_2 & b_3 \\ b_1^2 & b_2^2 & b_3^2 \end{vmatrix}$$

$$= (a_1-a_2)(a_2-a_3)(a_3-a_1)$$
$$(b_1-b_2)(b_2-b_3)(b_3-b_1)$$

$$\begin{vmatrix} u+a^2x & l+abx & m+acx \\ l+abx & v+b^2x & n+bcx \\ m+acx & n+bcx & w+c^2x \end{vmatrix} = 0 \xrightarrow[C_3 \to aC_3 - cC_1]{C_2 \to aC_2 - bC_1}$$

$$0 = \begin{vmatrix} u & al-bu & am-cu \\ l & av-bl & an-cl \\ m & an-bm & aw-cm \end{vmatrix} + x\begin{vmatrix} a^2 & al-bu & am-cu \\ ab & av-bl & an-cl \\ ac & an-bm & aw-cm \end{vmatrix}$$

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System of Equations (Cramer's Rule)

$$a_1x+b_1y+c_1z=d_1$$
$$a_2x+b_2y+c_2z=d_2$$
$$a_3x+b_3y+c_3z=d_3$$

a_i, b_i, c_i, d_i given

$x, y, z = ?$

$$\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}, \quad \Delta_1 = \begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix}, \quad \Delta_2 = \begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix}$$

$$\Delta_3 = \begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix}$$

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$$\Delta_1 = \begin{vmatrix} a_1x+b_1y+c_1z & b_1 & c_1 \\ a_2x+b_2y+c_2z & b_2 & c_2 \\ a_3x+b_3y+c_3z & b_3 & c_3 \end{vmatrix} \xrightarrow{C_1 \to C_1 - yC_2 - zC_3}$$

$$= x\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

$$\boxed{\begin{aligned} \Delta_1 &= x\Delta \\ \Delta_2 &= y\Delta \\ \Delta_3 &= z\Delta \end{aligned}}$$

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System of Equations

$\Delta_1 = x\Delta$

$\Delta_2 = y\Delta$

$\Delta_3 = z\Delta$

$\Delta \neq 0$, Unique solution

$$(x, y, z) = \left(\frac{\Delta_1}{\Delta}, \frac{\Delta_2}{\Delta}, \frac{\Delta_3}{\Delta}\right)$$

$\Delta = 0 = \Delta_1 = \Delta_2 = \Delta_3$

Infinite solution

$\Delta = 0$ & atleast one of $\Delta_1, \Delta_2, \Delta_3 \neq 0$, no solution

Let $z = K$

$$a_1x + b_1y = d_1 - c_1K$$

$$a_2x + b_2y = d_2 - c_2K$$

$z = K$, $x = f(K)$, $y = g(K)$

$$(x, y, z) = (2K, K, 5)$$

Exception

$$x + y + z = 1$$
$$x + y + z = 2$$
$$x + y + z = 3$$

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Consistent → If the system has atleast one solution.

Inconsistent → If the system has no solution

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Homogeneous System

$$a_1x+b_1y+c_1z=0$$
$$a_2x+b_2y+c_2z=0$$
$$a_3x+b_3y+c_3z=0$$

$(x,y,z)=(0,0,0) \Rightarrow$ Trivial solution

Non trivial solution

atleast one of x, y, z is non zero.

Condition for homogeneous system to have non trivial solutions

$$\Delta = 0$$

$$\boxed{\Rightarrow \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0}$$

1. Find p, q so that the system of

$$2x + py + 6z = 8$$
$$x + 2y + qz = 5$$
$$x + y + 3z = 4$$

has

(i) unique soln $\Rightarrow p \in R - \{2\}$, $q \in R - \{3\}$

(ii) infinite —"—

(iii) no solution.

② $p = 2$, $q \in R$.

③ $p \in R - \{2\}$, $q = 3$.

$= 6 - 2q - 3p + pq$

$= 12 - 2q - 3p + pq - 6$

$$\Delta = \begin{vmatrix} 2 & p & 6 \\ 1 & 2 & q \\ 1 & 1 & 3 \end{vmatrix}$$

$$\Delta_2 = \begin{vmatrix} 2 & 8 & 6 \\ 1 & 5 & q \\ 1 & 4 & 3 \end{vmatrix} = 2(15 - 4q) - 8(3 - q) + 6(-1)$$

$\Delta_2 = 0$

$\Delta_1 = (p-2)(4q - 15)$

$\Delta = (p-2)(q-3)$

$\Delta_3 = (p-2)$

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2: Find 'θ' for which eqns

$$(\sin 3\theta)x - y + z = 0$$
$$(\cos 2\theta)x + 4y + 3z = 0$$
$$2x + 7y + 7z = 0$$

has non trivial solutions.

$\boxed{\text{SOT Ex-5}}$

$\boxed{\sin\theta = 0, \frac{1}{2}}$

$\boxed{\theta = n\pi,\ n\pi + (-1)^n \frac{\pi}{6} \quad n \in I}$

$$\begin{vmatrix} \sin 3\theta & -1 & 1 \\ \cos 2\theta & 4 & 3 \\ 2 & 7 & 7 \end{vmatrix} = 0 = (\sin 3\theta)7 + 14\cos 2\theta - 14$$

$$3\sin\theta - 4\sin^3\theta - 4\sin^2\theta = 0$$

$\sin\theta = 0,$

$$4\sin^2\theta + 4\sin\theta - 3 = 0 = (2\sin\theta + 3)(2\sin\theta - 1)$$

$-2 + 6$