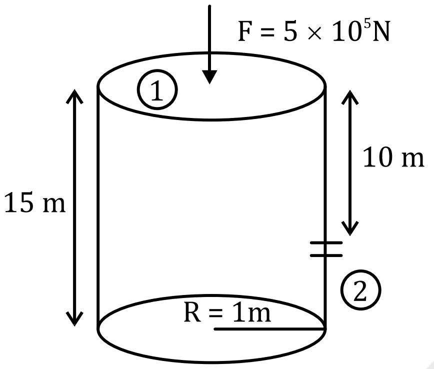
FLUID MECHANICS

DPP 2

SOLUTION

1. Say there is no atmospheric pressure on piston, $P_1 = \frac{F}{\pi R^2}$ and $P_2 = P_0$.



Bernoulli's equation at point 1 and 2,

$$p_1 + \rho g h_1 + \frac{1}{2}\rho v_1^2 = p_2 + \rho g h_2 + \frac{1}{2}\rho v_2^2$$

$$\Rightarrow \frac{F}{\pi R^2} + \rho g h_1 - h_2 = P_0 + \frac{1}{2}\rho v_2^2$$

$$\Rightarrow v_2 = \sqrt{\frac{2F}{\pi R^2 \rho} - \frac{2P_0}{\rho} + 2gh_1 - h_2}$$

$$v_2 = \sqrt{\frac{2 \times 5 \times 10^5}{3.14 \times 1^2 \times 1000} - \frac{2 \times 1.01 \times 10^5}{1000} + 2 \times 10 \times 10}$$

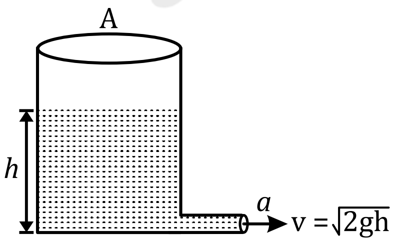
$\approx 17.8 \text{ m s}^{-1}$

2. Say the coefficient of friction is μ.

Mass of fluid $= m = (Ah)\rho$

The velocity of efflux, $v = \sqrt{2gh}$

where h is the height of liquid in vessel.



Force due to emerging liquid

$f = \rho a v^2$

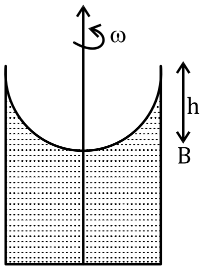
So, for preventing from sliding $\mu mg \geq \rho a^2$

$\therefore \mu \times Ah\rho g \geq \rho a v^2$

or $\mu Ahg \geq a \times 2gh$

$\therefore \mu \geq \frac{2a}{A}$

3. By Bernoulli's theorem



$P_A + \frac{1}{2}\rho v_A^2 + \rho g h_A = P_B + \frac{1}{2}\rho v_B^2 + \rho g h_B$

As $h_A = h_B \therefore\ P_A + \frac{1}{2}\rho v_A^2 = P_B + \frac{1}{2}\rho v_B^2$

$\Rightarrow\ P_A - P_B = \frac{1}{2}\rho(v_B^2 - v_A^2)$

Now, $v_A = 0, v_B = r\omega$ and $P_A - P_B = h\rho g$

$\therefore h\rho g = \frac{1}{2}\rho(r^2\omega^2 - 0)$

or $h = \frac{25\omega^2}{2g}$ cm [Given, $\omega r = 5$ cm]

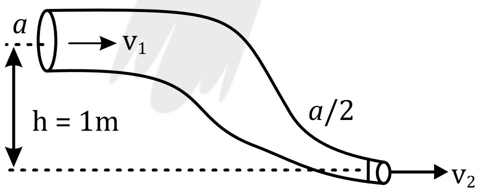
4. The momentum per second carried by liquid per second is $\rho v^2 A$. A is area of cross section of pipe

The force exerted due to reflected back molecules is $2\left(\frac{1}{4}\rho a v^2\right)$.

So, the resultant pressure is

$\frac{\left(\frac{1}{2}\rho A v^2 + \frac{1}{4}\rho A v^2\right)}{A} = \frac{3}{4}\rho v^2$

5. Density, $\rho = 500\ \text{kg/m}^3$



$A_1 = a, A_2 = a/2; P_1 - P_2 = 4100$ Pa

$v_1 = \frac{\sqrt{x}}{6}$

By Bernoulli's theorem,

$P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$

$P_1 - P_2 = \frac{1}{2}\rho v_2^2 - \frac{1}{2}\rho v_1^2 - \rho g h_1$

By equation of continuity

$A_1v_1 = A_2v_2$

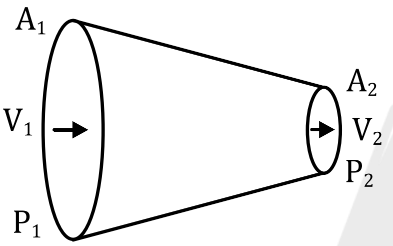
$a \cdot \frac{\sqrt{x}}{6} = \frac{a}{2} \times v_2; v_2 = \frac{\sqrt{x}}{3}$

$4100 = \frac{1}{2} \times 800 \left(\frac{x}{9} - \frac{x}{36}\right) - 800 \times 10 \times 1$

$4100 = 400 \left(\frac{3x}{36}\right) - 8000$

$\Rightarrow \frac{(4100 + 8000) \times 36}{400 \times 3} = x; x = 363$

6. $A_2 = \frac{A_1}{2}$



$P_1 - P_2 = 4500$ Pa

$P_1 + \frac{1}{2}\rho V_1^2 + \rho gh = P_2 + \frac{1}{2}\rho V_2^2 + \rho gh$

$P_1 - P_2 = \frac{1}{2}\rho(V_2^2 - V_1^2)$

And $A_1 V_1 = A_2 V_2$

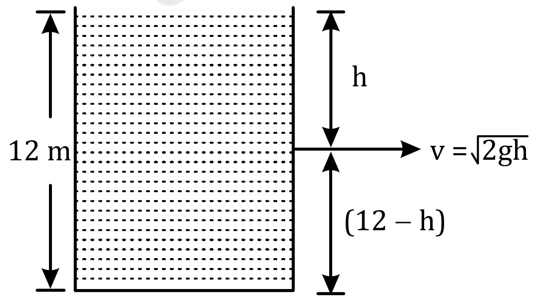
$\Rightarrow V_2 = 2 V_1$

$4500 = \frac{1}{2} \times 750 \times 3 V_1^2$

$V_1 = 2$ m/s

Volume flow rate $= A_1V_1 = 24 \times 10^{-3}$ m^3 s^{-1}

7. The velocity of efflux is, $v = \sqrt{2gh}$



Let it takes time t sec in covering a horizontal range R.

Then, $12 - h = \frac{1}{2}gt^2 \Rightarrow t = \sqrt{\frac{2(12-h)}{g}}$

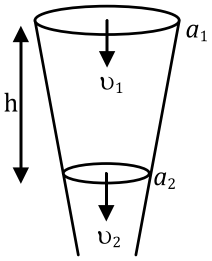
Range, $R = vt = \sqrt{2gh}\sqrt{\frac{2(12-h)}{g}}$

$\Rightarrow R = \sqrt{4h(12-h)}$

For maximum range $\frac{dR}{dh} = 0$

$\Rightarrow\ h = 6$ m.

8. Here,



$d_1 = 8 \times 10^{-3}$ m

$v_1 = 0.4\ \text{m s}^{-1}, h = 0.2$ m

$sv_2 = \sqrt{v_1^2 + 2gh} = \sqrt{(0.4)^2 + 2 \times 10 \times 0.2}$

$\approx 2\ \text{m s}^{-1}$

$\therefore$ As per equation of continuity

$a_1 v_1 = a_2 v_2$

$$\pi \times \left(\frac{8 \times 10^{-3}}{2}\right)^2 \times 0.4 = \pi \times \left(\frac{d_2}{2}\right)^2 \times 2$$

$d_2 = 3.6 \times 10^{-3}$ m

9. Let v = velocity of efflux

then $v^2 = 2gh\ 1 - \left(\frac{a}{2}\right)^2$

$h = 3.0 - 0.525 = 2.475$ m

$$v^2 = \frac{2 \times 10 \times 2.475}{1-(0.1)^2} = \frac{49.50}{0.99}$$

$v^2 = 50\ \text{m}^2\ \text{s}^{-2}$

10. $v = \sqrt{2gh} = \sqrt{2 \times 10 \times 20} = 20$ m/s