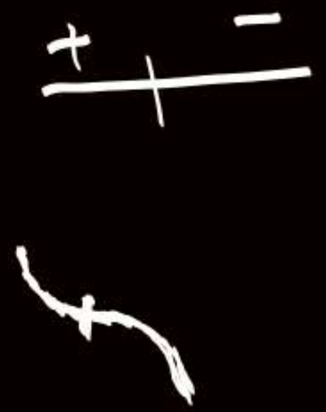
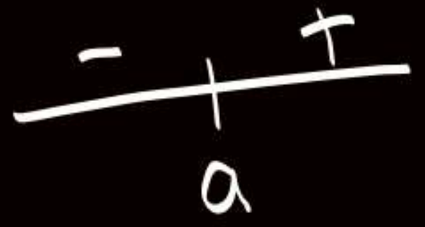
1.

$f'(a)=0$, $f''(a)=0$, $f'''(a)>0$

f'' ↑ $f'''(a)<0$



sign of $f''(x)$



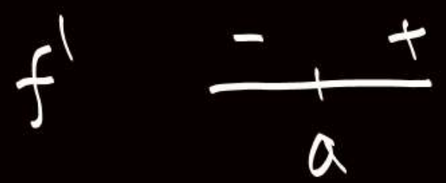
$y=f(x)$

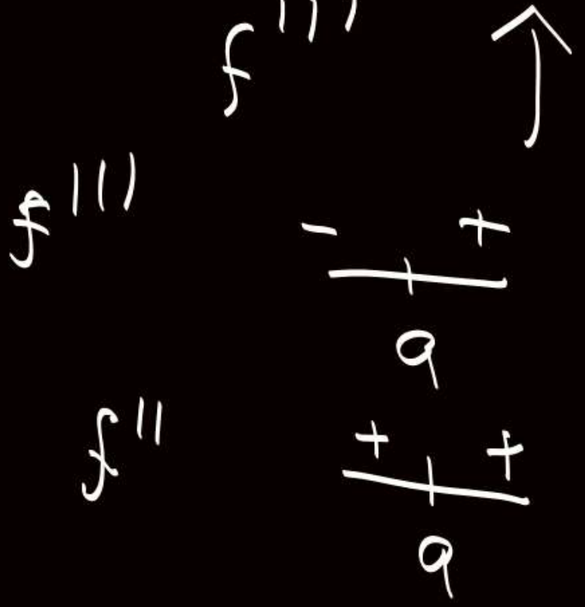
$\underline{f(x)}$

2. $f'(a) = 0 = f''(a) = f'''(a)$

$f''''(a) > 0$.

f ∪

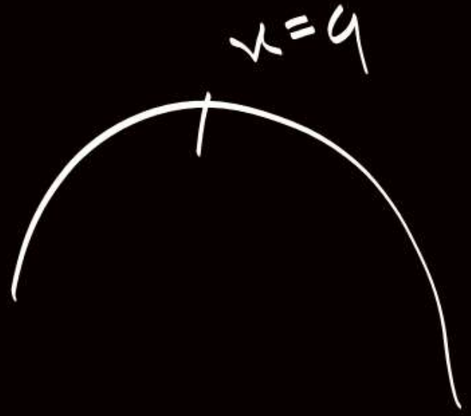




Point of Extremum Value.

↓

local minimum / local maximum



↓

point of extremum

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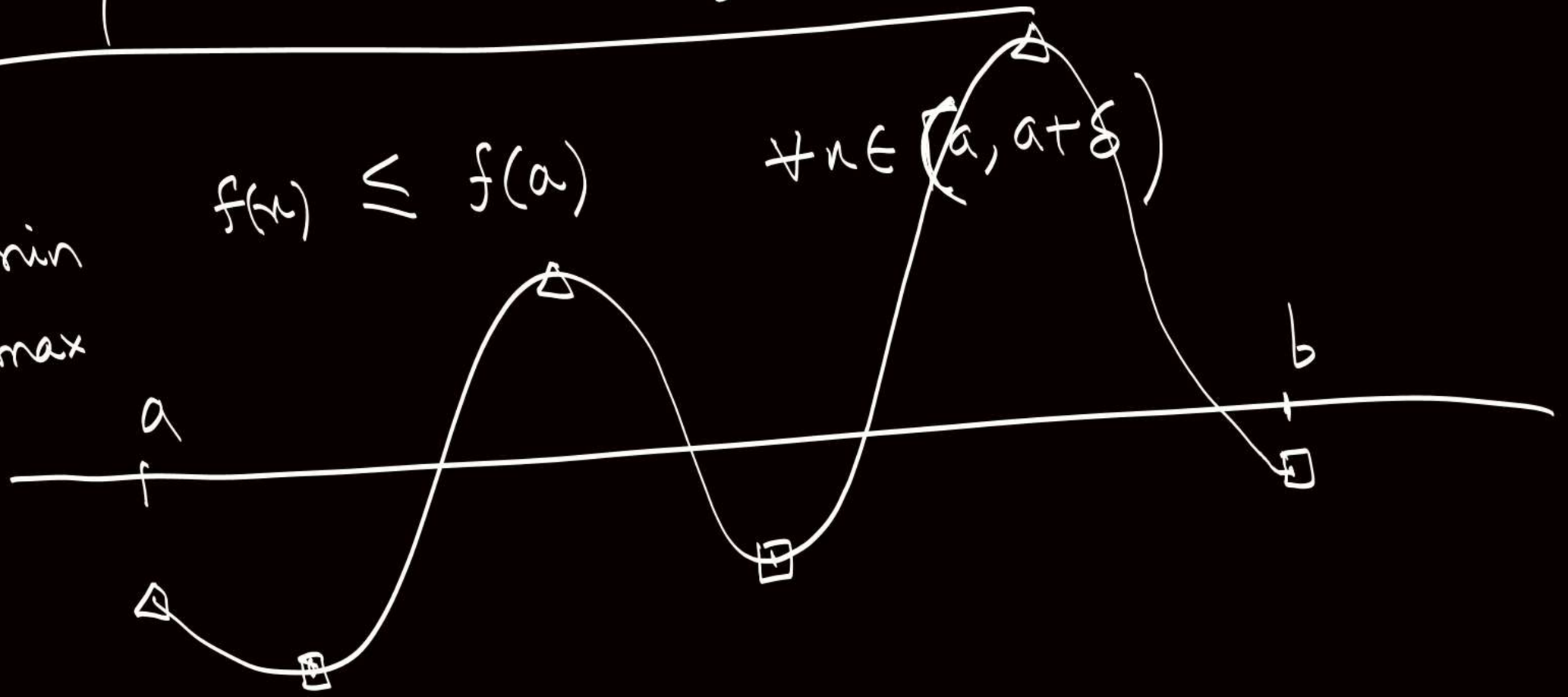
Point of Extremum in $[a, b]$

$f(x) \leq f(a) \quad \forall x \in (a, a+\delta)$

□ → min

△ → max



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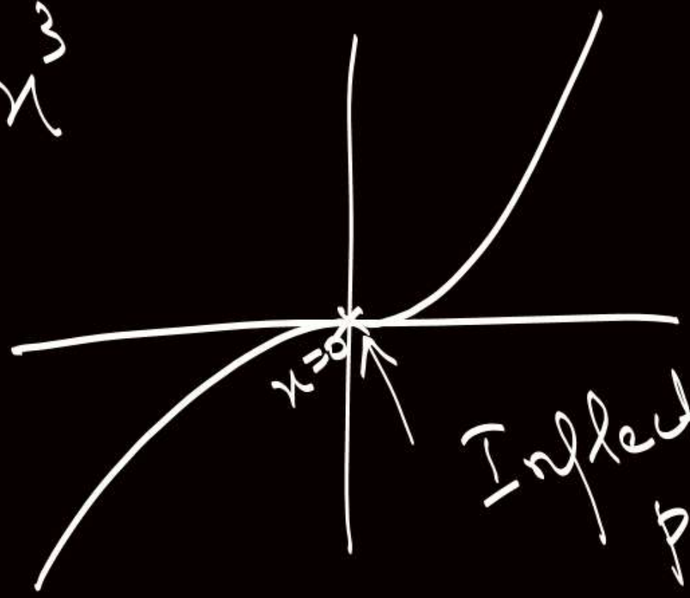
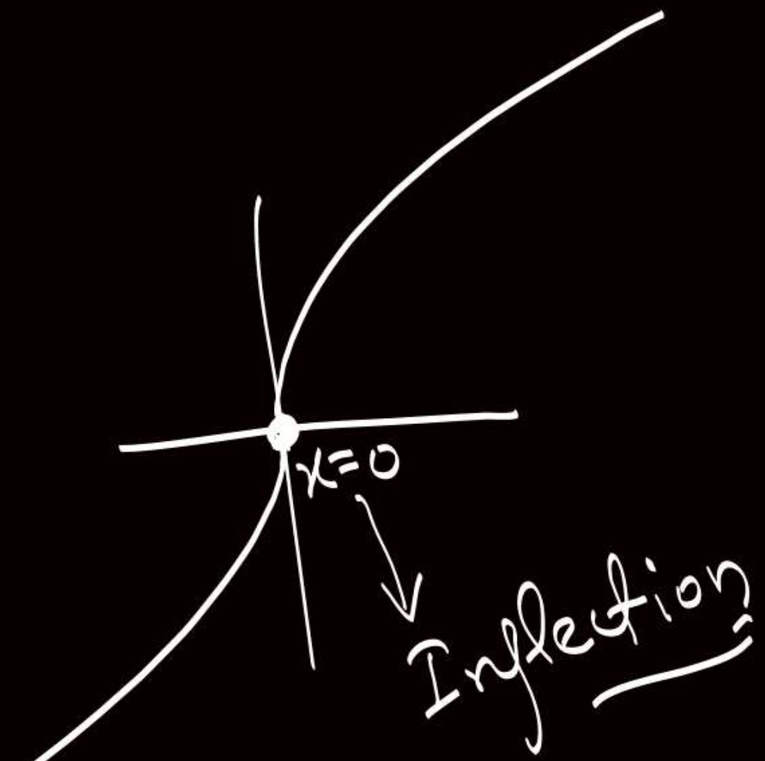
Inflection Point

$f(x) = x^{\frac{1}{7}}$

$f(x) = x^5$ at $x = 0$

$f''(x) = 20x^3$





Vertical tangent for function

$f(x) = x^{2/3}$ at $x=0$

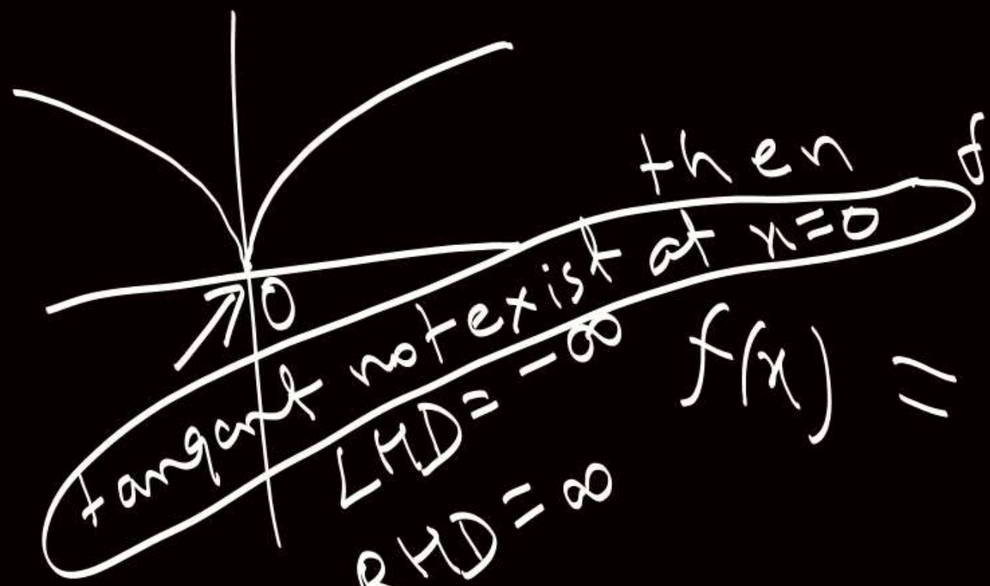
If LHD & RHD both ∞

or

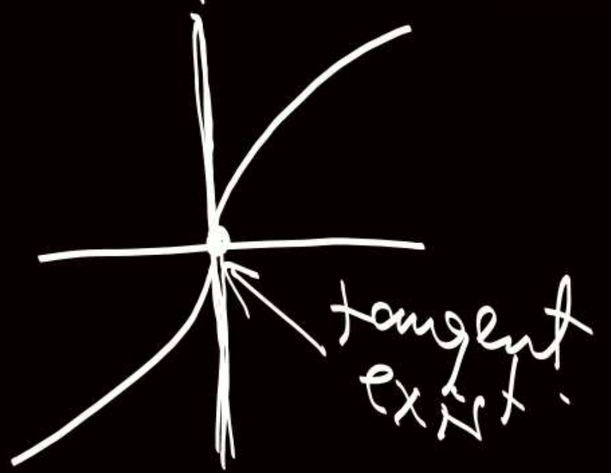
both LHD & RHD $\to -\infty$

$$\lim_{x\to 0} \frac{x^{2/3}-0}{x} = \frac{1}{x^{1/3}}$$

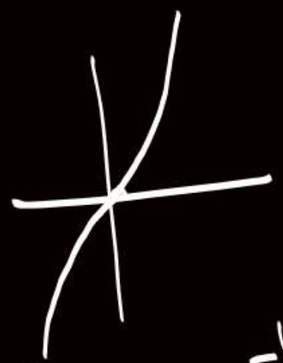
then $f(x)$ has vertical tangent at $x=0$.



$f(x) = x^{1/3}$ at $x=0$



tangent not exist at $x=0$



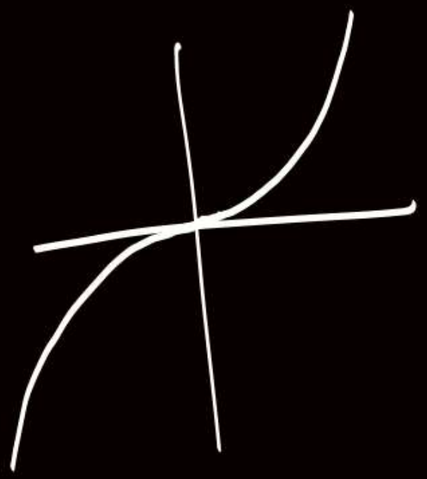
$x=a \rightarrow$ inflection.

$f''(a) = 0$ or not exist.

$f(x) = \sin^{-1} x$ at $x = 0$

$f(x) = x^{1/3}$ at $x = 0$

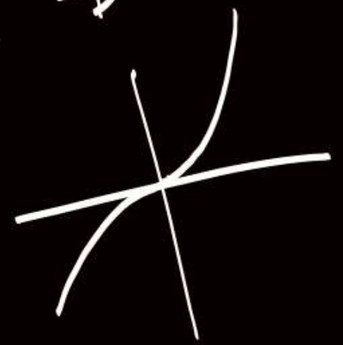
$f(x) = x^{7/5} \rightarrow x = 0$



$f'(x) = \frac{1}{\sqrt{1-x^2}}$

$f''(x) = \frac{x}{(1-x^2)^{3/2}}$

$f(x) = x^3 \Rightarrow$



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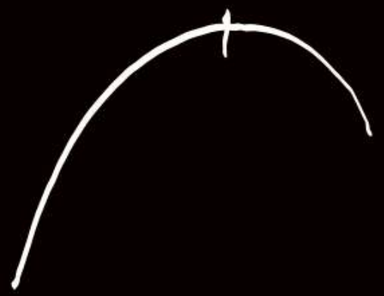
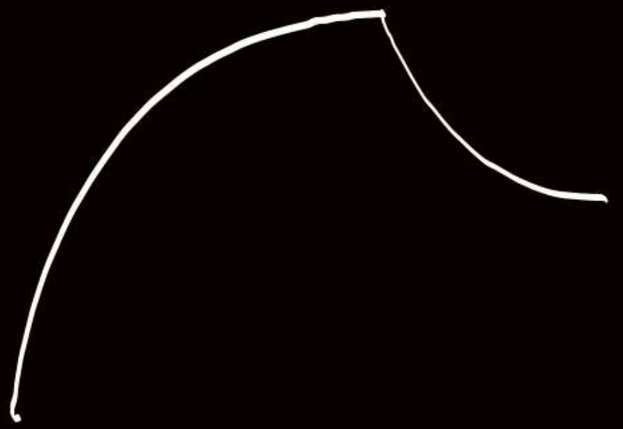
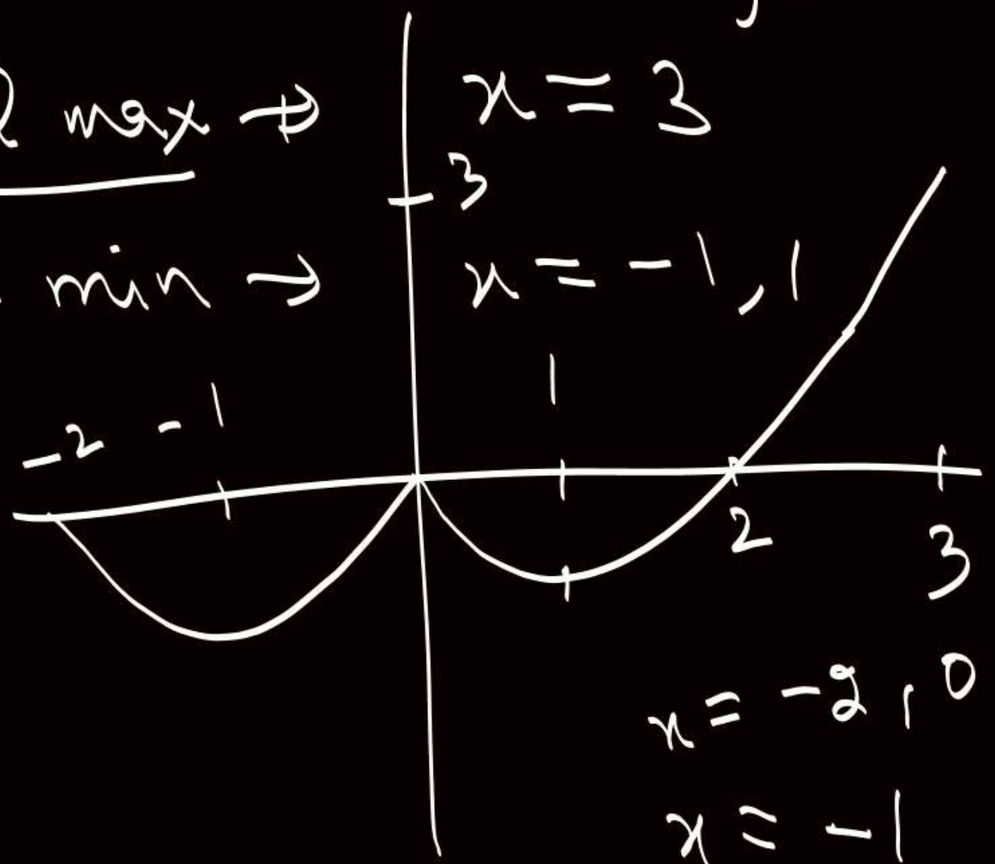
Note





1: Discuss $f(x) = x^2 - 2|x|$ in $[-2, 3]$

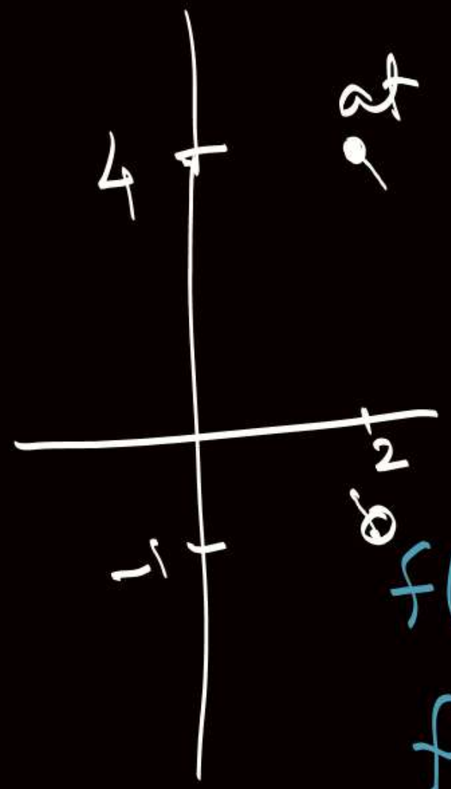
for extremum



$x = -2, 0, 3$ → maximum

$x = -1, 1$ → minimum

Q: Discuss $f(x) = 2x - 5\{x\}$, $\{\cdot\} = FPF$.



at $x = 2$:

$2x - 5(x-1) = 5 - 3x \quad x \in (2-h, 2)$

local maximum

$(a-h, a+h)$

$f(2) \; ? \; f(x)$

$(2, 2+h)$

$f(2+h) = 2(2+h) - 5(2+h - 2) = 4 - 3h < 4$ $\quad (2x - 5(x-2) = 10 - 3x)$

$f(2-h) = 2(2-h) - 5(2-h-1) = -1 + 3h > -1$

$\therefore f(2) = 4$