Rigid body:

A body is referred to as a rigid body when there is no relative motion between any two particles of the body along the line connecting them, even when external forces are applied. Real bodies are not completely rigid, as they undergo deformation when subjected to external forces.

Types of motion of a rigid body

- **Translational motion**: When all particles of a body move along parallel paths, and their displacements are identical to that of the body, we describe the motion as translational.
- **Rotational motion:** Pure rotation is observed in a body when each particle within it moves in a circular path with the same angular velocity, and the centers of all particles align along a straight line referred to as the axis of rotation.
- **Rolling motion:** The motion that involves a combination of rotational and translational motion, subject to specific constraints, is referred to as rolling motion.
- **Kinematics of rotational motion about a fixed axis:**

  The kinematic equations that describe rotational motion with uniform angular acceleration are as follows:

  (1) $\omega = \omega_0 + \alpha t$ (2) $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$

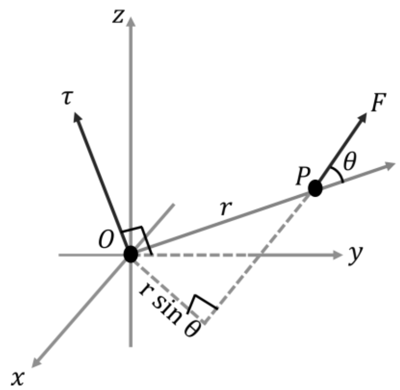
  (3) $\omega^2 = \omega_0^2 + 2\alpha\theta$ (4) $\theta_n = \omega_0 + \alpha\left(n - \frac{1}{2}\right)$

Moment of force (Torque)

Torque is defined as the rotational effect or turning force produced by a force around a fixed point. The magnitude of torque can be calculated by multiplying the magnitude of the force by the perpendicular distance between the line of action of the force and the fixed point.

$\tau = F(r\sin\theta) \Rightarrow \vec{\tau} = \vec{r} \times \vec{F}$

S.I. Unit: Nm Dimensional formula: $\left[ML^2T^{-2}\right]$.



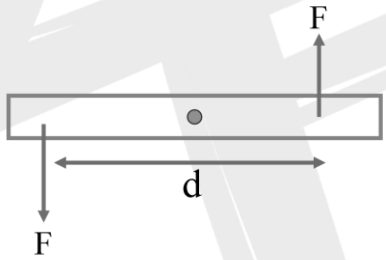
Application: The maximum effectiveness in producing rotation is achieved when a force of a given magnitude is applied perpendicular to the door at its outer edge.

- The moment of a force becomes zero if the magnitude of the force is zero or if the line of action of the force passes through the fixed point.
- Reversing the direction of force (F) results in reversing the direction of the moment of force as well.
- Reversing both the directions of the displacement vector (r) and the force vector (F) does not change the direction of the moment of force.

Sign convention: Positive torque is defined as the torque that generates counterclockwise rotation, while negative torque is associated with clockwise rotation.

Moment of couple:

A couple refers to a pair of forces that are equal in magnitude, opposite in direction, and have different lines of action. A couple generates rotational motion without any translation. In the case of an unconstrained object that is not on a pivot, a couple causes the object to rotate about its center of mass.



This couple has the ability to exert a turning effect or torque on the body. The moment of the couple serves as a measure of the magnitude of this turning effect.

$\therefore \tau$ = moment of couple=magnitude of either force × perpendicular distance between the forces

$\therefore \tau = Fd$

Mechanical Equilibrium of a rigid body:

A rigid body is considered to be in a state of mechanical equilibrium when both its linear momentum and angular momentum remain constant over time. This means that the body experiences neither linear acceleration nor angular acceleration.

Condition for translational equilibrium

- The vector sum of the forces, on the rigid body is zero;

$$\vec{F}_1 + \vec{F}_2 + \ldots + \vec{F}_n = \sum_{i=1}^{n} \vec{F}_i = 0$$

If the total force on the body is zero, then the total linear momentum of the body does not change with time. P = constant

Condition for rotational equilibrium:

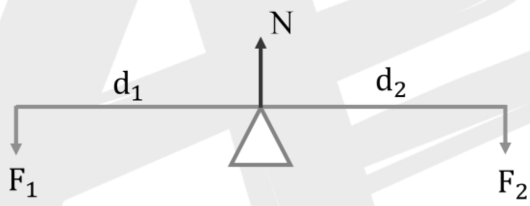
- The vector sum of the torques on the rigid body is zero, $\vec{\tau}_1 + \vec{\tau}_2 + ...\vec{\tau}_n = \sum_{i=1}^{n} \vec{\tau}_i = 0$

  When the total torque acting on a rigid body is zero, the total angular momentum of the body remains constant over time.
  This rotational equilibrium condition is independent of the choice of the origin or the reference point from which the torques are measured.

Principle of moments:

- An ideal lever is essentially a light rod pivoted at a point along its length. This point is called the fulcrum. Two forces F_1 and F_2 parallel to each other and usually perpendicular to the lever act on the lever at distances d_1 and d_2 respectively from the fulcrum. N is directed opposite to the forces F_1 and F_2 (N= Reaction at fulcrum) For translational equilibrium.

  $N - F_1 - F_2 = 0$



- For rotational equilibrium take the moments about the fulcrum; the sum of moments must be zero, $d_1F_1 - d_2F_2 = 0$

  N acts at the fulcrum itself and has zero moment about the fulcrum.

- In the case of the lever force F_1 is usually some weight to be lifted. It is called the load and its distance from the fulcrum d_1 is called the load arm. Force F_2 is the effort applied to lift the load; distance d_2 of the effort from the fulcrum is the effort arm.

Principle of moments for a lever

Load arm x load = effort arm x effort

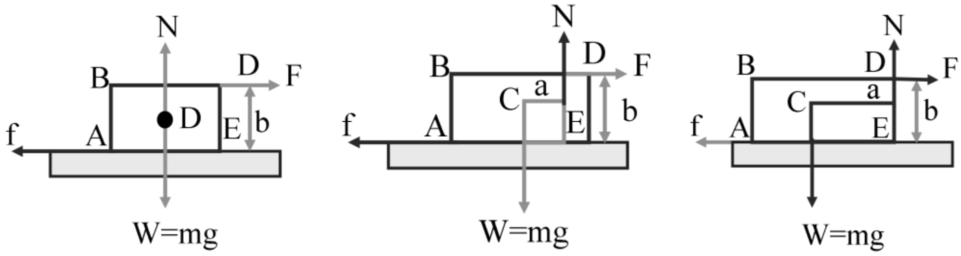
Mechanical advantage:

- The ratio F_1 / F_2 is called the Mechanical Advantage

  $$M.A = \frac{F_1}{F_2} = \frac{d_2}{d_1}$$

  If the effort arm d_2 is larger than the load arm, the mechanical advantage is greater than one. It means

Toppling:



Consider a scenario where a force F is applied at a height b above the base AE of a block. Let's assume that there is enough friction (f) to prevent sliding. In this situation, if the normal reaction N also passes through point C, an unbalanced torque is present despite the block being in translational equilibrium (F = f and N = mg). This torque, caused by the couple of forces F and f, has the tendency to topple the block around point E. To counteract the effect of this unbalanced torque, the normal reaction N is shifted to the right by a distance 'a' so that the net counterclockwise torque is equal to the net clockwise torque. $Fb = mg(a) \Rightarrow a = \frac{Fb}{mg}$

Now, as F (or) b (or) both are increased distance a also increases. But it cannot go beyond the right edge of the block. So in extreme case the normal reaction passes through E. Now if F or b are further increased, the block will topple down. This is why the block having the broader base has less chances of toppling in comparison to a block of smaller base.

Moment of inertia [Rotational Inertia]:

- A stationary body cannot initiate its own rotation, and a rotating body cannot cease rotating spontaneously. Therefore, a body possesses rotational inertia, also known as moment of inertia.
- The moment of inertia is the property that quantifies the rotational inertia of a body.
- The moment of inertia of a particle with mass m is given by

  $I = mr^2$

  Where r = perpendicular distance of particle from axis of rotation.

  S.I. unit: kgm^2; Its D.F - ML^2

Dimensional formula: ML Moment of inertia of a group or system of particles is

$$I = m_1r_1^2 + m_2r_2^2 + m_2r_2^2 + m_nr_n^2 \quad I = \sum mr^2$$

Where $m_1, m_2.......m_n$ are masses of particles and $r_1, r_2..........r_n$ are their perpendicular distances from axis of rotation.

The moment of inertia in rotational motion is analogous to mass in translational motion.

The moment of inertia of a rigid body is influenced by the following three factors:

(a) mass of the body

(b) position of axis of rotation

(c) Nature of distribution of mass.

Note-1: Moment of inertia of a rotating rigid body is independent of its angular velocity.

Note-2: Moment of inertia of a metallic body depends on its temperature.

Radius of Gyration(K): The radius of gyration of a rigid body about an axis of rotation is defined as the distance between the axis of rotation and a hypothetical point at which the entire mass of the body can be considered concentrated. This hypothetical point is chosen in such a way that the moment of inertia of the body, calculated with this concentration of mass, matches the actual distribution of mass in the body.

Moment of inertia of a rigid body of mass M is $I = MK^2$

Where K = radius of gyration

$$K = \sqrt{\frac{r_1^2 + r_2^2 + \ldots\ldots + r_n^n}{n}}$$

Where n is total number of particles in the body and $r_1, r_2 \ldots\ldots\ r_n$ are their perpendicular distances from axis of rotation.

S.I unit: metre CGS unit: cm

Dimensional formula: $\left[M^0LT^0\right]$

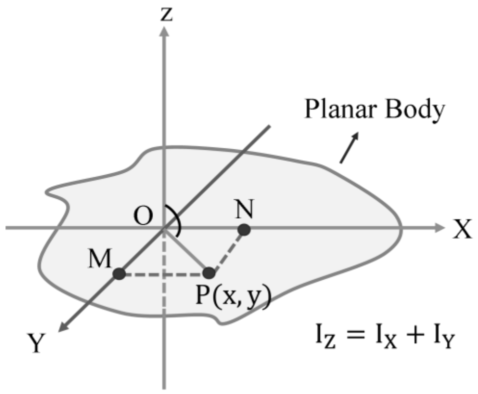
Note: K is not the distance of centre of mass of body from the axis considered.

Radius of gyration of a rigid body depends on the following two factors

(a)Position of axis of rotation. (b)Nature of distribution of mass.

Perpendicular axes theorem

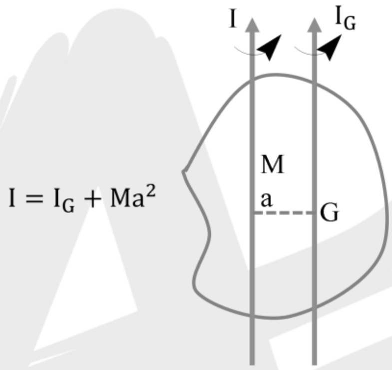
Statement: The parallel axis theorem states that the moment of inertia of a planar lamina about an axis perpendicular to its plane is equal to the sum of its moments of inertia about two mutually perpendicular axes. These perpendicular axes intersect at the axis perpendicular to the lamina's plane and lie within the plane of the body.



- This theorem is applicable to bodies which are planar.
- This theorem applies to flat bodies whose thickness is very small compared to their other dimensions.
- $K_z = \sqrt{K_x^2 + K_y^2}$

Parallel axes theorem:

Statement: The moment of inertia of a body around an axis can be determined by adding the moment of inertia of the body around a parallel axis passing through its center of gravity to the product of its mass and the square of the distance between the two parallel axes.



- This theorem is applicable to a body of any shape.
- $K = \sqrt{K_G^2 + a^2}$

Angular momentum of a particle

Definition: The moment of linear momentum of a body w.r.t. an axis of rotation is known as angular momentum.

- The angular momentum $\bar{L}$ of the particle with respect to fixed point O is represented as $\vec{L} = \vec{r} \times \vec{p} = m(\vec{r} \times \vec{v})$
- The magnitude of angular momentum vector is $L = rp\sin\theta$ where p is the magnitude of $\vec{p}$ and θ is the angle between and p.
- It is always directed perpendicular to the plane of rotation and along the axis of rotation.

Angular momentum of rigid body:

The angular momentum of a rigid body, while it is rotating, is defined as the vector sum of the angular momenta of all its constituent particles about the axis of rotation. It can be calculated by multiplying the moment of inertia of the body by its angular velocity.

$\therefore \bar{L} = \sum_i(\bar{r}_i \times m_i\bar{v}_i) = I\bar{\omega}$

S.I. Unit: kgm^2/sec

Dimensional formula : ML^2T^{-1}

When a body is rolling, its total angular momentum is the combined vector sum of two components: the angular momentum about its center of mass and the angular momentum about a fixed point on the ground.

Law of conservation of angular momentum:

If there is no external torque acting on the rotating body (or system of particles), then its angular

momentum is conserved.

If $\tau_{ext} = \bar{0}$ and $\frac{d\bar{L}}{dt} = \bar{0}$ $\left[\because \frac{d\bar{L}}{dt} = \vec{\tau}_{ext}\right]$

$\Rightarrow \bar{L} = I\omega = \text{constant} \therefore I_1\omega_1 = I_2\omega_2$

Rotational dynamics

Relation between Torque and angular momentum of a rigid body:

The vector sum of torques acting on various particles of a rigid body gives the net torque acting on the body.

$\bar{\tau} = \sum \bar{\tau}_i$ and $\tau = \frac{d\bar{L}}{dt}$, $\bar{L}$ is total angular momentum of the body. The time rate of change of the angular momentum of a particle is equal to the torque acting on it.

Relation between torque and angular acceleration:

$\tau = \frac{d\bar{L}}{dt}$ But $\bar{L} = I\bar{\omega}$

$\therefore \tau = I\frac{d\bar{\omega}}{dt} \Rightarrow \bar{\tau} = I\bar{\alpha}$

This equation is called equation of rotatory motion and analogous to Newton's 2^{nd} law in dynamics.

Rotational kinetic energy:

The sum of the kinetic energies of various particles of rotating body is called rotational kinetic energy.

$$KE_{rot} = \frac{L^2}{2I} = \frac{1}{2}I\omega^2 = \frac{1}{2}\omega L$$

Work, Power & Angular Impulse

Work: Work done by external **torque on rotating body is** $W = \int \tau d\theta$

If τ is constant, then, $\boxed{W = \tau\theta}$

Work energy theorem: The work done by an external torque on a rotating body is equal to the change in its rotational kinetic energy.

$$\boxed{\tau\theta = \frac{1}{2}I\omega^2 - \frac{1}{2}I\omega_0^2}$$

- Work done by retarding torque to stop the rotating body is equal to initial rotational kinetic energy of body.

  $\tau\theta = \frac{1}{2}I\omega^2$ and $\theta = 2\pi N$

  where N = no. of rotations made by the body before coming to rest.

Power: The rate of work done by torque is called power. Instantaneous power is given by

$$P = \frac{dW}{dt} = \frac{d}{dt}(\tau\theta) = \tau\frac{d\theta}{dt} = \tau\omega$$

As the power is a scalar, it is written as $P = \vec{\tau}.\vec{\omega}$

Average power is

$$P_{ave} = \frac{\text{Totalworkdone}}{\text{Totaltime}} = \frac{\frac{1}{2}I\omega^2 - \frac{1}{2}I\omega_0^2}{t}$$

Angular Impulse: The large torque acts on a body for relatively very short interval of time is called impulsive torque.

- The product of impulsive torque and its time of action is called angular impulse J. It is a vector. It is always equal to change in angular momentum.

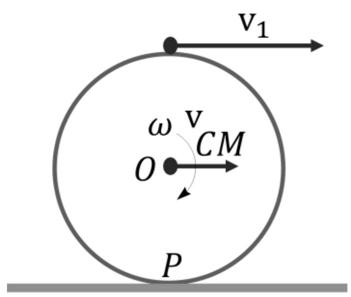
$$\therefore \ \vec{J} = \int \vec{\tau}dt; \text{ As } \vec{\tau} = \frac{d\vec{L}}{dt}; \int \vec{\tau}dt = \Delta\vec{L}$$

$$\therefore \vec{J} = I(\vec{\omega} - \vec{\omega}_0)$$

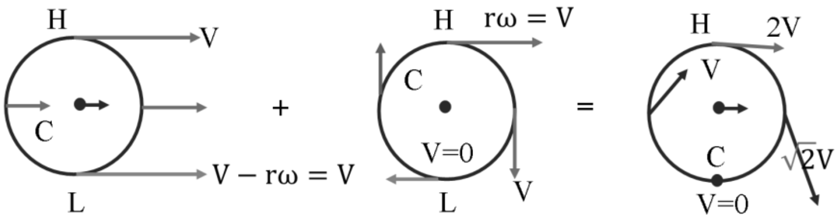
Rolling Motion

Pure rolling motion: The motion of a rolling body on a horizontal surface is a combination of translational motion and rotational motion. At the point of contact between the body and the surface, there is no relative motion. All points of the rolling rigid body have the same angular speed but different linear velocities. A body rolls on a surface when there is frictional force present, which generates torque on the rolling body due to the frictional force.

The condition that point of contact is instantaneously at rest requires $\upsilon_{cm} = R\omega$



- It means that the velocity of point at the top of the disc (v_1) has a magnitude $v_{cm} + R\omega$ or $2\upsilon_{cm}$ and is directed parallel to the level surface



(A) Translation (B) Rotation (C) Rolling

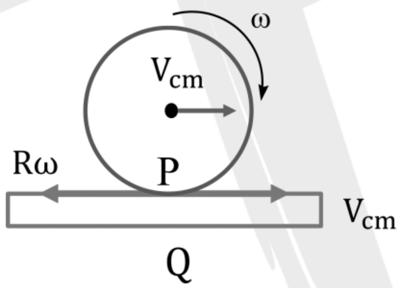
(i) Linear speed at H=2V (max)

(ii) Linear speed at L=0 (min)

(iii) Linear speed at $M = \sqrt{2}V$

Uniform pure rolling:

Pure rolling, also known as uniform pure rolling, refers to the condition where there is no relative motion at the point of contact between a body and the surface it rolls on. Consider a disc of radius R rolling on a stationary horizontal surface. In order for the disc to roll without slipping, the following conditions must be satisfied:



(i) $(V_{cm})_P = V_{surface} \Rightarrow V_{cm} - R\omega = 0 \quad \Rightarrow \quad v_{cm} = R\omega$

(ii) If $V_P > V_Q \Rightarrow v_{cm} - R? > 0 \qquad \Rightarrow v_{cm} > R?$

(Forward Slipping)

(iii) If $v_p < v_Q \Rightarrow v_{cm} - R? < 0 \Rightarrow v_{cm} < R?$

(Backward slipping)

so, $v_{cm} = R?$ is the condition for a body to be in pure rolling on a stationary horizontal surface/ ground. It is sometimes simply called as Rolling.

Non uniform pure rolling:

- If $a_{CM} = R\alpha$, then no friction arises and the body is in pure rolling.

- If $a_{CM} > R\alpha$, then static friction arises and acts opposite to motion of the body to support rotatory motion such that $R\alpha$ is made equal to a_{CM} to keep the body still in pure rolling motion.

  (Instantaneously $V_{CM} = R\omega$)

  If the above two conditions fail then the body slips and the friction present is kinetic.

Total K.E. of a rolling body:

- A rolling body has both translational and rotational kinetic energies

  (i) $KE_{trans} = \frac{1}{2}mV_c^2$ (ii) $KE_{rot} = \frac{1}{2}I\omega^2$

  (iii) $K.E_{total} = K.E_{translatory} + K.E_{rotational}$

  $= \frac{1}{2}mV_c^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}mV_c^2\left(1+\frac{K^2}{R^2}\right)$

  Where V_c = velocity of C.M

  K = radius of gyration, R = radius

  $KE_{trans} ; KE_{rot} ; KE_{tot} = 1 : \frac{K^2}{R^2} : \left(1+\frac{K^2}{R^2}\right)$

Note – 1: Fraction of KE associated with translatory motion

$$\frac{KE_{tra}}{KE_{total}} = \frac{1}{1+\frac{K^2}{R^2}}$$

Note – 2: Fraction of KE associated with rotatory motion

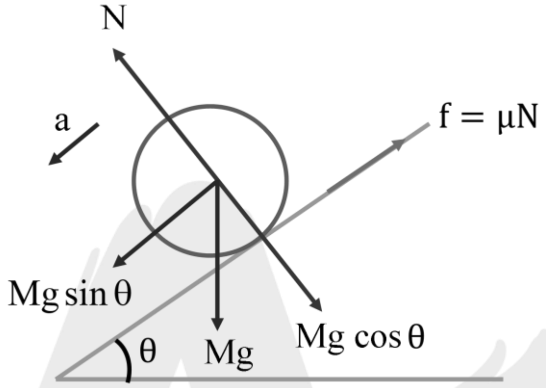
$$\frac{KE_{rot}}{KE_{total}} = \frac{1}{1+\frac{R^2}{K^2}}$$

Note – 3: The K.E. of some rolling bodies:

- The K.E of rolling solid cylinder is $E = \frac{3}{4}mv^2$
- The K.E of rolling solid sphere is $E = \frac{7}{10}mv^2$
- The K.E. of a rolling hollow sphere is $E = \frac{5}{6}mv^2$
- The K.E. of a hollow cylinder or thin ring is $E = mv^2$

Rolling of a body on an inclined plane without slipping:

Let's consider a body with mass M, radius R, and moment of inertia I rolling without slipping on an inclined plane, which forms an angle θ with the horizontal. In order for the body to roll without slipping, it is necessary for there to be a frictional force (f) present. From the given figure, we can observe that all forces (except for friction) are acting along the radius of the body. As a result, these forces do not generate any torque in the body.



At the point of contact velocity is zero, so the net torque due to the various forces about the IAOR is $\tau = (Mg\sin\theta)R = I\alpha$ .... (1)

$$(Mg\sin\theta)R = (MR^2 + Mk^2)\frac{a}{R}$$

$$a = \frac{g\sin\theta R^2}{R^2+k^2} \Rightarrow a = \frac{g\sin\theta}{1+\frac{k^2}{R^2}} \quad ....(2)$$

Torque due to friction $fR = I\left(\frac{a}{R}\right)$ .... (3)

$$\Rightarrow f = \frac{Ia}{R^2} = \frac{(Mk^2)}{R^2}\frac{g\sin\theta}{\left(1+\frac{k^2}{R^2}\right)} \Rightarrow f = \frac{Mg\sin\theta}{\left(1+\frac{R^2}{k^2}\right)}$$

If μ be the coefficient of friction, then

$f \le \mu N$., where $N = Mg\cos\theta$

Using (3), we get $\mu_{min} = \frac{\tan\theta}{\left(1+\frac{R^2}{k^2}\right)}$ .... (4)

Condition for a body to roll without slipping:

For a body to roll without slipping, the force of friction 'f calculated above must be less than or equal to the maximum value of friction i.e. $\mu Mg\cos\theta$

$$\Rightarrow \frac{Mg\sin\theta}{\left(1+\frac{R^2}{k^2}\right)} \le \mu Mg\cos\theta \Rightarrow \mu \ge \frac{\tan\theta}{\left(1+\frac{R^2}{k^2}\right)}$$

(i) Velocity of the body when it reaches the bottom is given by

$$\boxed{v = \sqrt{\frac{2gh}{1+\frac{k^2}{R^2}}} = \sqrt{\frac{2g\ell\sin\theta}{1+\frac{k^2}{R^2}}} (\text{since } h = \ell\sin\theta)}$$

(ii) Acceleration of the body is given by $\boxed{a = \frac{g\sin\theta}{1+\frac{k^2}{R^2}}}$

(iii) Time taken by the body to reach the bottom is given by $t = \sqrt{\frac{2\ell\left(1+k^2/R^2\right)}{g\sin\theta}}$

If all these are allowed to roll down from the top of an inclined plane, they will reach the bottom in the following order

(1) Solid sphere (2) Disc (or) Solid cylinder

(3) Hollow sphere (4) Ring (or) Hollow cylinder

Note -1 : If a_1, a_2, a_3 and a_4 are the accelerations of centre of masses of rolling solid sphere, solid cylinder, hollow sphere and hollow cylinder respectively when they roll down the same inclined plane then $\boxed{a_1 > a_2 > a_3 > a_4}$

Note – 2: If t_1, t_2, t_3 and t_4 are the times of travel of rolling solid sphere, solid cylinder, hollow sphere and hollow cylinder respectively to reach the bottom from top of an inclined plane then

$\boxed{t_1 < t_2 < t_3 < t_4}$

(a) For solid sphere $(\mu) = \frac{2}{7}\tan\theta$

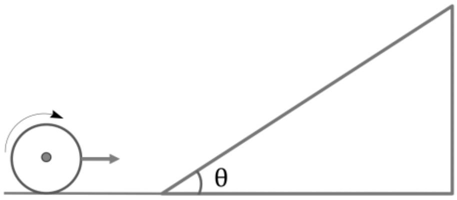
(b) For Hollow sphere $(\mu) = \frac{2}{5}\tan\theta$

(c) For solid cylinder (or) Disc $(\mu) = \frac{1}{3}\tan\theta$

(d) For Ring (or) Hollow Cylinder $(\mu) = \frac{1}{2}\tan\theta$

Note: When a body rolls down an incline without slipping, no work is done against friction because the point of contact between the body and the surface is momentarily at rest.

A body rolls on a smooth horizontal surface with speed v and then rolls up a rough inclined plane of inclination θ.



(i) The height reached by the body before coming to rest is given by $h = \frac{v^2}{2g}\left(1 + \frac{k^2}{r^2}\right)$

(a) For solid sphere, $h = \frac{7v^2}{10g}$

(b) For Hollow sphere, $h = \frac{5v^2}{6g}$

(c) For Disc (or) Solid cylinder, $h = \frac{3v^2}{4g}$

(d) For Ring (or) Hollow cylinder, $h = \frac{v^2}{g}$

Note : If all these bodies travel with same velocity on horizontal surface then

(i) Solid sphere reaches the minimum height.

(ii) Ring reaches maximum height.

Angular momentum in case of rotation about a fixed axis:

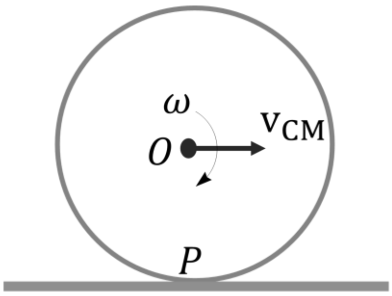
When the total external torque is zero, the total angular momentum of the system is conserved.

The general expression for the total angular momentum ofthe system is $L = \sum_{i=1}^{N} r_i \times p_i$

For rotation about a fixed axis, the component of angular momentum perpendicular to the fixed axis is constant.

Angular Momentum of Rolling Wheel in combined Rotation & Translation:

Angular momentum of a rolling wheel about an axis passing through the point of constant P and perpendicular to the plane of wheel:



$\vec{L} = \vec{L}_{translational} + \vec{L}_{rotation} = m\left(\vec{R} \times \vec{v}_{CM}\right) + I_{cm}\vec{\omega}$ or

$\vec{L} = m\vec{\omega}R^2 + I_{CM}\vec{\omega}$ or $\vec{L} = \left(I_{CM} + mR^2\right)\vec{\omega} = I_p\vec{\omega}$

Instantaneous axis of rotation:

Pure rolling motion is characterized by purely rotational motion around the instantaneous point of contact with the ground. The axis passing through this instantaneous point of contact and perpendicular to the plane of rotation is referred to as the instantaneous axis of rotation.

$\therefore$ The total kinetic energy of rolling body is written

$$KE_{total} = \frac{1}{2} I_P \omega^2$$

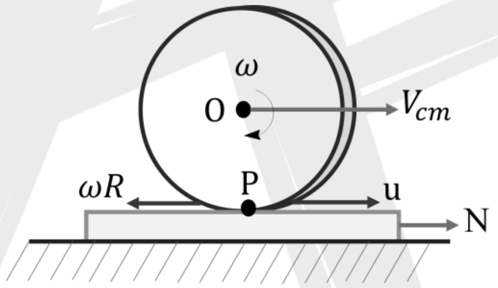
where I_p is moment of inertia about instantaneous axis of rotation.

Rolling bodies over moving platform:

Rolling bodies do not experience sliding on the surface they are moving on. When a rolling body is placed on a moving platform, the point of contact between the body and the platform must have the same velocity as the platform itself.

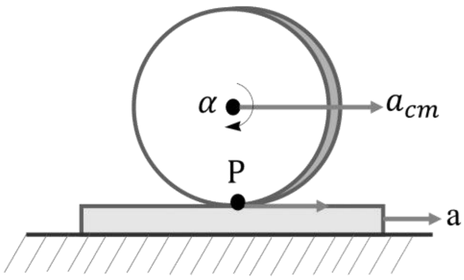
Case 1: If point of contact of surface is moving with velocity u with respect to ground, then

$$V_{cm} - \omega R = u$$



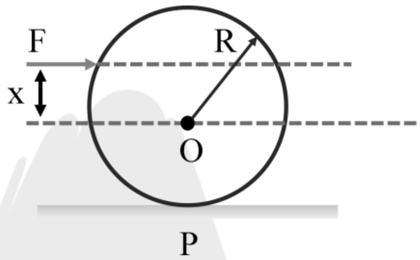
Case – 2: For no sliding on the moving platform, $u = \omega R - v_{cm}$

Case – 3: For accelerated surface, $a_{cm} - \alpha r = a$



Direction of friction in case of translation & rotation combined:

In rotational motion, the direction of friction cannot be determined through direct observation alone, as the body is undergoing both translation and rotation simultaneously. The direction of the frictional force is determined by considering the motion tendency of the point of contact between the body and the ground. For instance, in the case of a rolling object with mass M and radius R placed on a rough horizontal surface and subjected to an applied force F, the direction of the frictional force can be determined based on the motion tendency of the point of contact.



- Acceleration of point P due to translation $a_t = \frac{F}{M}$ (towards right)
- Acceleration of point P due to rotation only, $a_r = \alpha R = \frac{\tau}{I} R = \frac{FR}{I} x$ (towards left)

net acceleration of point P is $\vec{a}_P = \vec{a}_t + \vec{a}_r$

$a_P = \frac{F}{M} - \frac{FRx}{I}$ (towards right) ...(i)

From the above equation it is clear that motion tendency at point P depends upon both x and I,

Eq (i) can be written as,

$$\boxed{a_P = \frac{F}{M}\left(1 - \frac{Rx}{K^2}\right)} \quad \text{... (ii)}$$

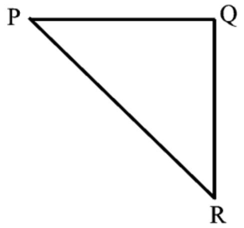
If $K^2 > Rx$: friction will act in backward direction.

If $K^2 = Rx$: no friction will act.

If $K^2 < Rx$: friction will act in forward direction.

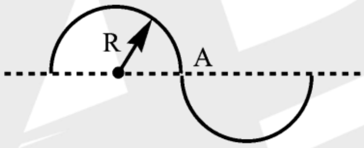
EXCERCISE-I

1. In the given triangular sheet, $PQ = QR = \ell$ and $\angle PQR = 90°$. If M is the mass of the sheet, then its moment of inertia about PR is:



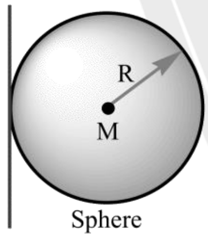
(A) $\frac{M\ell^2}{24}$ (B) $\frac{M\ell^2}{12}$ (C) $\frac{M\ell^2}{6}$ (D) $\frac{M\ell^2}{3}$

2. The S-shaped uniform wire shown in figure has a mass M, and the radius of curvature of each half is R. The moment of inertia about an axis through A and perpendicular to the plane of the paper is:



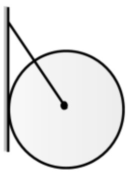
(A) $\frac{3}{4}MR^2$ (B) MR^2 (C) $\frac{3}{2}MR^2$ (D) $2MR^2$

3. What is the moment of inertia I of a uniform solid sphere of mass M and radius R, pivoted about an axis that is tangent to the surface of the sphere?



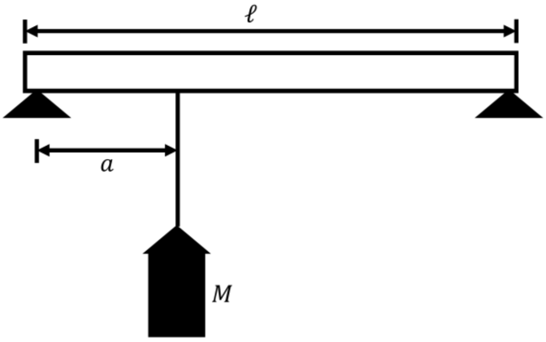
(A) $\frac{2}{5}MR^2$ (B) $\frac{3}{5}MR^2$ (C) $\frac{6}{5}MR^2$ (D) $\frac{7}{5}MR^2$

4. A uniform disk of radius R and mass m is connected to a wall by string of length 2R. The normal reaction of wall is:



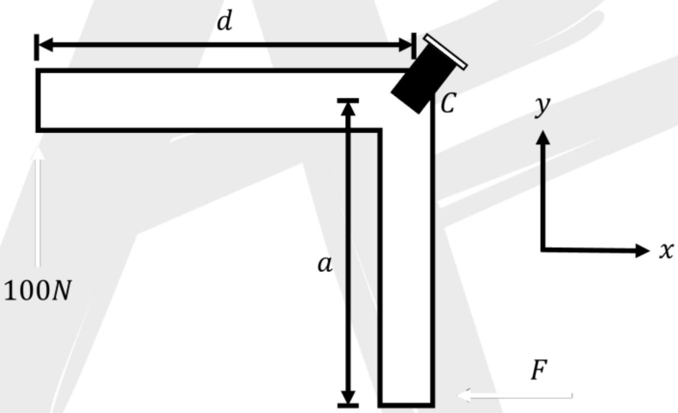
(A) mg (B) $\frac{mg}{2}$ (C) $\frac{mg}{\sqrt{3}}$ (D) 2 mg

5. A horizontal bar of length ℓ and negligible mass is supported at its two ends. A mass M is hung from the bar at a distance 'a' from the left end, as shown. What is the magnitude of the force that the support of the right side applies on the bar?



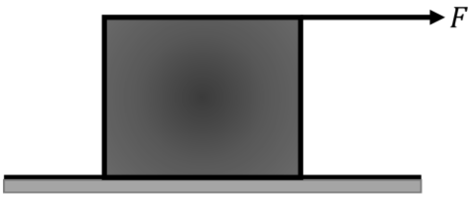
(A) $Mg\frac{a}{\ell}$ (B) $Mg\frac{\ell}{a}$ (C) $Mg\frac{a}{\ell+a}$ (D) $Mg\frac{\ell}{\ell+a}$

6. Find force F required to keep the system in equilibrium. The dimensions of the system are $d = 0.3\text{m}$ and $a = 0.2\text{m}$. Assume the rods to be massless.



(A) $150(\hat{i})$
(B) $150(-\hat{k})$
(C) $150(-\hat{i})$
(D) It cannot be in equilibrium

7. A uniform block of mass m is pulled at the top by a horizontal force but the block not move. Then about the centre of mass, the torque due to:



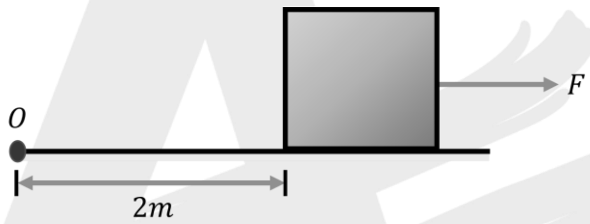
(A) Friction as well as normal reaction is clockwise
(B) Friction as well as normal reaction is anticlockwise
(C) Friction is clockwise but normal reaction is anticlockwise
(D) Friction is anticlockwise but normal reaction is clockwise

8. One end of meter stick is pinned to a table so that stick can rotate freely in a plane parallel to the table surface. Two horizontal forces are applied to the stick such that the rod does not rotate at all. One force has a magnitude of 2N and is applied perpendicular to the stick at the free end. The other force has a magnitude of 8N and acts at 30° with respect to the length of the stick. At what distance (in cm) from the pinned end is the 8N force applied?

9. A generator's flywheel, which is a homogeneous cylinder of radius R and mass M, rotates about its longitudinal axis. The linear velocity of a point on the rim (side) of the flywheel is v. What is the kinetic energy of the flywheel?

(A) $K=\frac{1}{2}Mv^2$ (B) $K=\frac{1}{4}Mv^2$ (C) $K=\frac{1}{2}Mv^2/R$ (D) $K=\frac{1}{2}Mv^2R$

10. A cubical block of side 1 m and mass 1 kg is pulled by a force F applied to central line so that it slides with an acceleration of $1\,m/s^2$. What is the net torque on it about O?



(A) 1 Nm (B) $\frac{1}{2}$ Nm (C) $\frac{1}{9}$ Nm (D) Zero

11. A wheel rotating freely about its axis is brought to rest in time t_0 by the application of two brake blocks to its surface. Each block produces a constant force F acting tangentially to the rim of the wheel. The disc radius is a. The initial angular momentum of the wheel about its axle is:

(A) $2Fat_0$ (B) $\frac{Ft_0}{2a}$ (C) Fat_0 (D) $\frac{2at_0}{F}$

12. A particle performs uniform circular motion with an angular momentum L. If the frequency of particle's motion is doubled and its kinetic energy is halved, the angular momentum is $\frac{\alpha L}{\beta}$. Find $\alpha+\beta$?

13. If $\vec{r}\times\vec{L}=0$ for a rigid body, where $\vec{\tau}$ = resultant torque and $\vec{L}$ = angular momentum about a point and both are non-zero. Then:

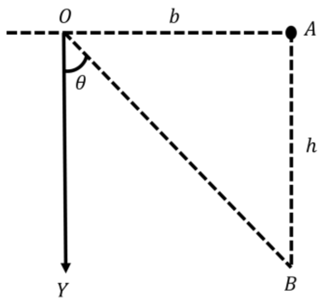
(A) $\vec{L}$ = constant

(B) $|\vec{L}|$ = constant

(C) $|\vec{L}|$ will increase

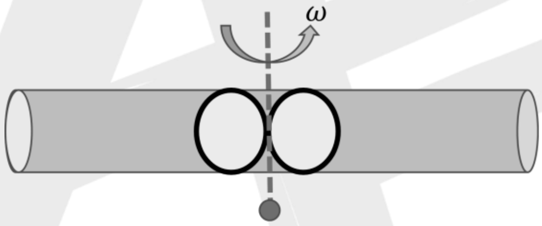
(D) $|\vec{L}|$ may increase

14. A particle of mass m is released from rest at point A in the figure falling freely under gravity parallel to the vertical Y axis. The magnitude of angular momentum of particle about point O when it reaches B is: (Where OA = b and AB = h)



(A) $\frac{mh}{(bg)}$ (B) $mb\sqrt{2gh}$ (C) $mh\sqrt{2gh}$ (D) None of these

15. A smooth tube of certain mass containing two identical balls is rotated as shown in gravity free space and released. The two balls move towards ends of the tube. Which of the following quantity is not conserved:



(A) Angular momentum of the whole system

(B) Net linear momentum of the two balls

(C) Kinetic energy of the whole system

(D) Angular speed of the tube

16. A uniform disc is rolling without sliding on a horizontal plane with centre of mass moving with constant velocity v. Number of points on the disc which has speed v at any instant are :

(A) 1 (B) 2 (C) 3 (D) None of these

17. Starting from rest at the same time, a coin and a ring roll down an incline without slipping. Which reaches the bottom of the incline first?

(A) The ring reaches the bottom first

(B) The coin reaches the bottom first

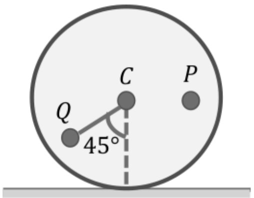
(C) They arrive at the bottom simultaneously

(D) The winner depends on the masses of the two

18. Your snapshot of a street also contains a car that was passing by while you were taking the photograph. Which part of a wheel of the car is least blurry?

(A) the center (B) the bottom (C) the top (D) all sides-equally

19. A disc is rolling without slipping with angular velocity ω. P and Q are two points equidistant from the centre C. The order of magnitude of velocity is:



(A) $v_Q > v_C > v_P$ (B) $v_P > v_C > v_Q$ (C) $v_P = v_C = \frac{v_P}{2}$ (D) $v_P > v_C > v_Q$

20. A wheel of radius 0.1m (wheel A) is attached by a non-stretching belt to a wheel of radius 0.2m (wheel B). The belt does not slip. By the time wheel B turns through 1 revolution, wheel A will rotate through:

(A) $\frac{1}{2}$ revolution (B) 1 revolution (C) 2 revolution (D) 4 revolution

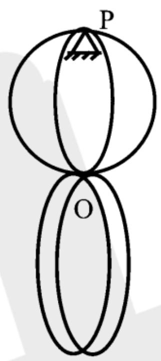
21. Consider the same bowling ball of above problem in pure rolling motion and suppose that it is rotating with an angular velocity of magnitude ω. In applying the principles of classical mechanics to a rigid body, it is useful to regard the rigid body as being composed of an infinite number of point masses. The point masses that make up the sphere will have linear speeds (relative to the ground):

(A) that are all exactly $R\omega$
(B) that range from - $R\omega$ to $R\omega$
(C) that range from 0 to $R\omega$
(D) that range from 0 to $2R\omega$

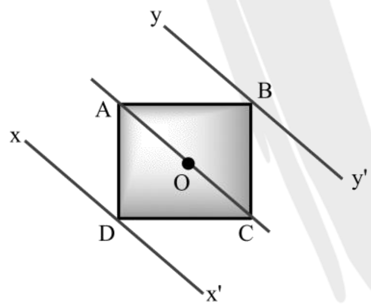
EXCERCISE-II

1. The moment of inertia of hollow sphere of inner radius $\frac{R}{2}$ and outer radius R, having material of uniform density, about a diametral axis is $I = \frac{\alpha m R^2}{\beta}$. Find $\alpha + \beta$?

2. Two identical rings of mass m with their planes mutually perpendicular, radius R are welded at their point of contact O. If the system is free to rotate about an axis passing through the point P perpendicular to the plane of the paper the moment of inertia of the system about this axis is equal to $\frac{115}{n} mR^2$. Find n?



3. Let I_1, I_2 and I_3 be the moment of inertia of a uniform square plate about axes AOC, xDx' and yBy' respectively as shown in the figure. The moments of inertia of the plate $I_1 : I_2 : I_3$ are in the ratio:



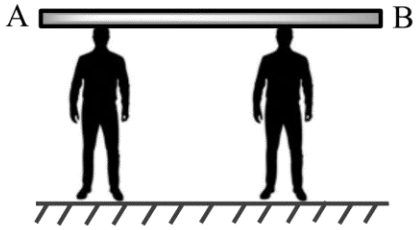
(A) $1 : \frac{1}{7} : \frac{1}{7}$ (B) $1 : \frac{12}{7} : \frac{12}{7}$ (C) $1 : \frac{7}{12} : \frac{7}{12}$ (D) $1 : 7 : 7$

4. An equilateral triangular wire frame is made from 3 rods of equal mass and length ℓ each. The frame is rotated about an axis perpendicular to the plane of the frame and passing through its end. What is the radius of gyration of the frame?

(A) $\frac{\ell}{2}$ (B) ℓ

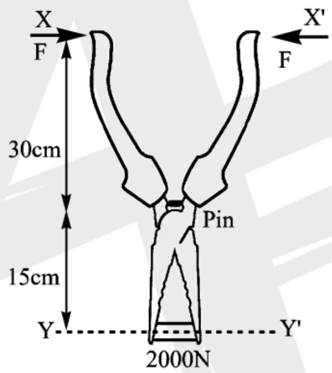
(C) $\frac{\ell}{\sqrt{2}}$ (D) $\frac{\ell}{2\sqrt{3}}$

5. Two persons of equal height are carrying a long uniform wooden beam of length ℓ. They are at distance $\frac{\ell}{4}$ and $\frac{\ell}{6}$ from nearest end of the rod. The ratio of normal reaction at their heads is:

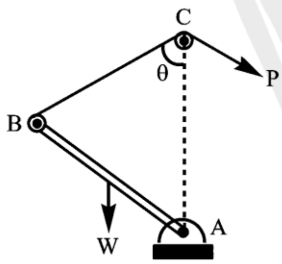


(A) 2 : 3 (B) 1 : 3 (C) 4 : 3 (D) 1 : 2

6. Figure shows a pair of pin jointed gripper tongs holding an object weighing 2000 N. The coeffiecient of friction μ at the gripping surface is 0.1. $X - X'$ is the line of action of the input force and $Y - Y'$ is the line of application of normal gripping force. If the pin-joint is assumed as frictionless, the magnitude of force F required to hold the weight is $n \times 100\,N$. Find n?



7. A uniform rod AB of weight W is movable in a vertical plane about a smooth hinge at A, and is sustained in equilibrium by a force P acting along a string BCP passing over a smooth peg C as shown. AC being vertical. If AC is equal to AB, then the force P is:

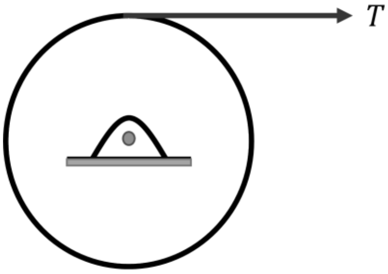


(A) $\frac{W}{\cos\theta}$ (B) $W\cos\theta$ (C) $\frac{W}{\sin\theta}$ (D) $W\sin\theta$

8. At t = 0 a flywheel is rotating with angular velocity ω_0. It then undergoes uniform angular acceleration for a time t_1, at the end of which the angular velocity is ω_1. How many revolutions did the flywheel make during this time interval?

(A) $\frac{1}{2}(\omega_0 + \omega_1)t$ (B) $\frac{\omega_0 t}{2\pi}$ (C) $\frac{\omega_1 t}{2\pi}$ (D) $\frac{(\omega_0 + \omega_1)t}{4\pi}$

9. A wheel 4m is diameter rotates about a fixed frictionless horizontal axis, about which its moment of inertia is $10 kgm^2$. A constant tension of 40 N is maintained on a rope wrapped around the rim of the wheel. If the wheel starts from rest at $t = 0s$, find the length (in m) of rope unwounded till $t = 3s$:



10. A mass of 10 kg connected at the end of a rod of negligible mass is rotating in a circle of radius 30 cm with an angular velocity of 10 rad/s. If this mass is brought to rest in 10 s by a brake, the magnitude of the torque applied is $\frac{9}{n}$ Nm. Find n?

11. Two identical rings A and B are acted upon by torques τ_A and τ_B respectively. A is rotating about an axis passing through the centre of mass and perpendicular to the plane of the ring. B is rotating about a chord at a distance $\frac{1}{\sqrt{2}}$ times the radius from the centre of the ring. If the angular acceleration of the rings is the same, then

(A) $\tau_A = \tau_B$

(B) $\tau_A > \tau_B$

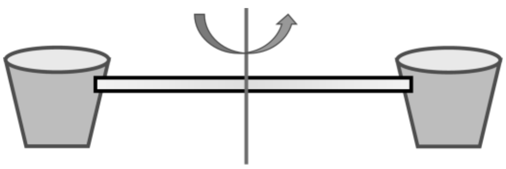
(C) $\tau_A < \tau_B$

(D) Nothing can be said about τ_A and τ_B as data are insufficient

12. A uniform rod of mass m and length ℓ is hinged at it's end so that it can rotate freely in a vertical plane. If it is released from horizontal position, what is it's initial angular acceleration?

(A) $\frac{3g}{\ell}$ (B) $\frac{3g}{2\ell}$ (C) $\sqrt{\frac{3g}{\ell}}$ (D) $\sqrt{3g\ell}$

13. Two empty bucket spin around in a horizontal circle on frictionless bearings. Suddenly, it starts to rain. As a result.



(A) The buckets continue to rotate at constant angular velocity because the rain is falling vertically while the buckets move in a horizontal plane.

(B) The buckets continue to rotate at constant angular velocity because the total mechanical energy of the bucket + rain system is conserved

(C) The buckets speed up because the angular momentum of the bucket + rain system is conserved

(D) The buckets slow down because the angular momentum of the bucket + rain system is conserved

14. Two pendulum bobs of unequal mass are suspended from the same fixed point by strings of equal length. The lighter bob is drawn aside and then released so that it collides with the other bob on reaching the vertical position. The collision is elastic. What quantities are conserved in the collision?

(A) Both kinetic energy and angular momentum of the system

(B) Only kinetic energy

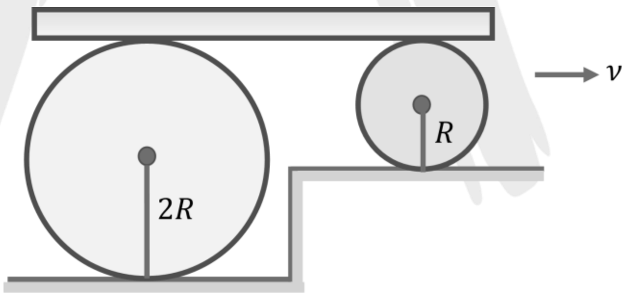
(C) Only angular momentum

(D) Angular speed of lighter bob

15. A solid sphere and a solid cylinder having the same mass and radius roll down the same incline. The ratio of their acceleration is:

(A) 14: 15 (B) 1: 2 (C) 15: 14 (D) 2: 1

16. Velocity of the centre of smaller cylinder is v. There is no slipping anywhere. The velocity of the centre of larger cylinder is nv? Find n?



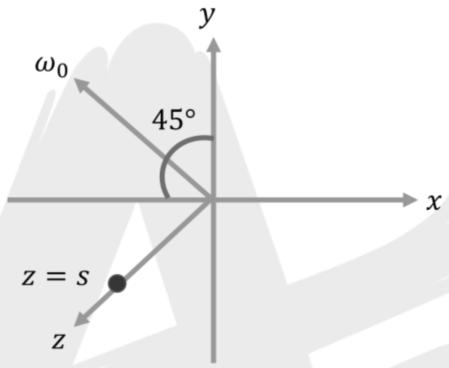
17. Two particles have angular momenta $\left|\vec{L}_1\right| = 30\,\text{kg}\cdot\text{m}^2/\text{s}$ and $\left|\vec{L}_2\right| = 40\,\text{kg}\cdot\text{m}^2/\text{s}$ as measured about the origin. Originally particle 1 moves in the xy plane and particle 2 moves in the yz plane. If there are no external torques, then the total angular momentum is a constant of magnitude:

(A) $\left|\vec{L}\right| = 10\,\text{kg}\cdot\text{m}^2/\text{s}$ (B) $\left|\vec{L}\right| = 50\,\text{kg}\cdot\text{m}^2/\text{s}$

(C) $\left|\vec{L}\right| = 70\,\text{kg}\cdot\text{m}^2/\text{s}$ (D) $10\,\text{kg}\cdot\text{m}^2/\text{s} \le \left|\vec{L}\right| \le 70\,\text{kg}\cdot\text{m}^2/\text{s}$

18. On a train moving with acceleration $10\,m/s^2$ a ball starts rolling on floor of train along the width of the train with angular acceleration $2\ rad/s^2$, radius of ball is 1 m. The acceleration of the top point of ball at the time t = 3 sec. as seen from ground is $n\sqrt{353}\ m/s^2$? Find n?

19. An object which is pivoted at the origin rotates with an angular velocity of magnitude ω_0 directed in the x-y plane at $45°$ from both the y-axis and the negative x-axis, as shown. What is the velocity $\vec{V}$ of a point on the rotating object that is located along the positive z-axis at a distance s from the origin?



(A) $\omega_0 s(\hat{i}+\hat{j})$

(B) $\omega_0 s(-\hat{i}-\hat{j})$

(C) $\frac{\omega_0 s}{\sqrt{2}}(\hat{i}+\hat{j})$

(D) $\frac{\omega_0 s}{\sqrt{2}}(-\hat{i}+\hat{j})$

20. A body is rolling without slipping on a horizontal plane. If the rotational energy of the body is 40% of the total kinetic energy, then the body might be:

(A) Cylinder

(B) Hollow sphere

(C) Solid cylinder

(D) Ring

21. A body of radius R and mass m is rolling on a horizontal surface without slipping with a speed v. It rolls up a rough incline to a maximum height h. If $h=\frac{v^2}{g}$, what is the moment of inertia of the body?

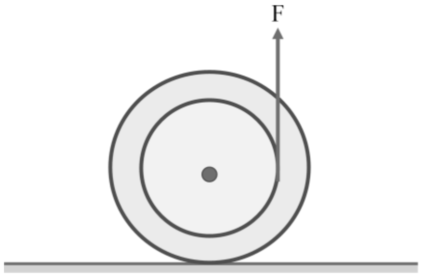
(A) $\frac{2mR^2}{5}$

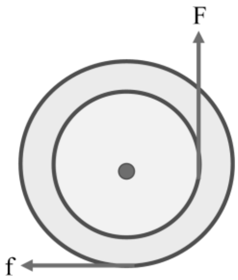
(B) $\frac{mR^2}{2}$

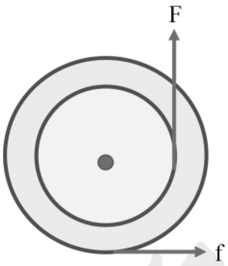
(C) mR^2

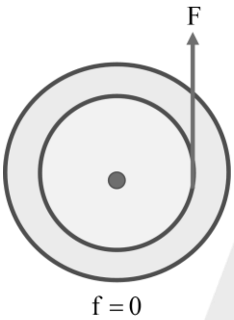
(D) $\frac{mR^2}{4}$

22. A spool is pulled vertically by a constant force $F(<Mg)$ as shown in figure. The friction force can be given by which of the following diagrams?



(A) 

(B) 

(C) 

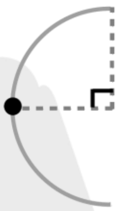
(D) Cannot be interpreted

EXCERCISE-III

1. A thin ring of mass m and radius r rolls without slipping across the floor with a speed v with respect to ground. Which of the following would be the best estimate of the ring's total kinetic energy as it rolls across the floor as seen from a frame moving in opposite direction with the same speed v with respect to ground?

(A) mv^2 (B) $2mv^2$ (C) $\frac{5}{2}mv^2$ (D) None of these

2. Moment of inertia of a thin uniform semicircular wire having mass M, radius R and centre C about an axis passing through point O and perpendicular to its plane is:

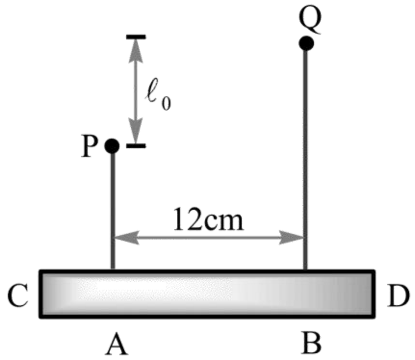


(A) $2MR^2$ (B) $2MR^2\left(1-\frac{1}{\pi}\right)$ (C) $MR^2\left(1-\frac{2}{\pi}\right)$ (D) $2MR^2\left(1-\frac{2}{\pi}\right)$

3. A ring of radius 1m is in xy-plane with its centre at the origin. In each of options below, the moment of inertia of the ring about the line is calculated. In which case does the ring have same moment of inertia as that about z-axis?

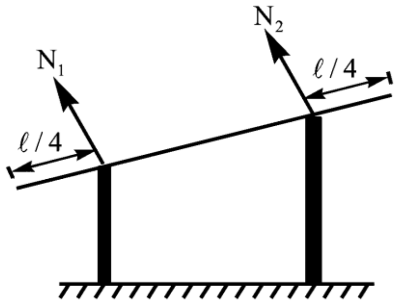
(A) $y=1$ (B) $x+y=\frac{1}{\sqrt{2}}$ (C) $-x-y=\frac{1}{\sqrt{2}}$ (D) $x=-\frac{1}{\sqrt{2}}$

4. An elastic string of natural length $3\ell_0$ is cut into two parts so that their natural lengths are $2\ell_0$ and ℓ_0 respectively. They are attached to points P and Q the vertical distance between P and Q being ℓ_0 and the horizontal distance being 12cm. A uniform rod $CD = 40\,\text{cm}$ is in equilibrium supported by the string at A and B. if the rod CD is horizontal then the lengths of the parts CA and BD in cms are:



(A) 12, 16 (B) 14, 14 (C) 16, 12 (D) 10, 18

5. A uniform rod of length ℓ is placed symmetrically on two walls as shown in figure. The rod is in equilibrium. If N_1 and N_2 are the normal forces exerted by the walls on the rod then:



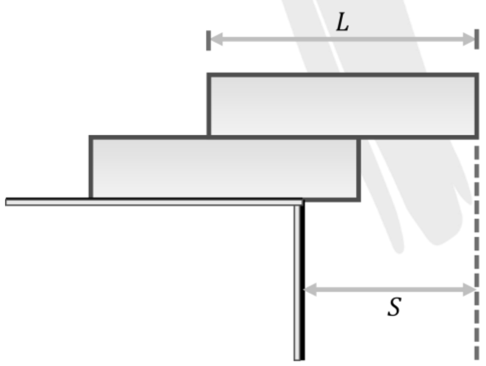
(A) $N_1 > N_2$ (B) $N_1 < N_2$

(C) $N_1 = N_2$ (D) N_1 and N_2 would be in the vertical directions

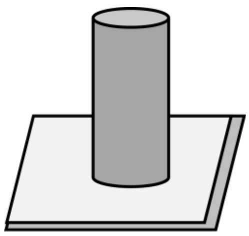
6. A ladder of length 10 m and mass 20 kg (and with uniform mass distribution) leans against a slippery vertical wall. The ladder makes an angle of 30° with the vertical. Friction between the ladder and the ground prevents it from sliding downwards. The magnitude of the force exerted on the ladder by the wall is $\frac{58}{n}$ N. Find n?

[Take $\sqrt{3} = 1.732; g = 10 m/s^2$]

7. Two identical bricks of length L are piled one on top of the other on a table as shown in the figure. The maximum distance S the top brick can overhang the table with the system still balanced is $\frac{\alpha}{\beta}$L. Find $\alpha + \beta$?

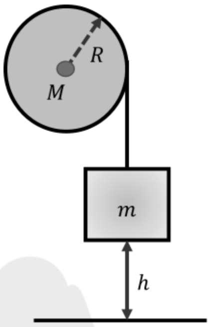


8. A cylindrical column has a uniform density. It has a radius of 20 cm and a height of 2.5 m. It stands vertically on solid ground. The maximum angle of lean before it topples over is closest to:



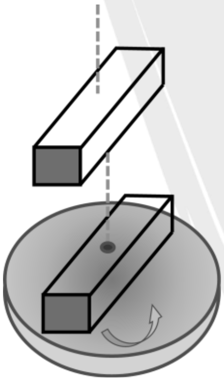
(A) 2° (B) 5° (C) 10° (D) 15°

9. A solid uniform cylinder of mass M and radius R is pivoted at its centre free to rotate about horizontal axis. A massless inextensible string is wrapped around it, and attached to a block of mass m which is initially at a height h above the floor. The acceleration due to gravity is g, directed downward. The block is released from rest. By what total angle $\Delta\theta$ (in radians) has the cylinder turned by the time the block hits the floor if string does not slip over cylinder?

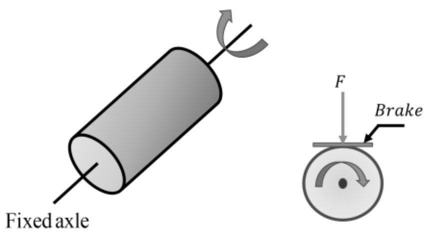


(A) $\sqrt{1+\frac{2M}{m}}$ (B) $\frac{h}{R}$ (C) $\sqrt{1+\frac{h}{2R}}$ (D) $\frac{MR}{mh}$

10. A uniform disk turns at 2.4 rev/s around a frictionless axis. A non-rotating rod, of the same mass as the disk and length equal to the disk's diameter is dropped onto the freely spinning disk figure. They then both turn around the axis with their centres superposed. The angular frequency in rev/s of the combination is $\frac{144}{n}$. Find n?



11. The figure shows a 15 kg solid cylinder mounted on a fixed axle, with a radius 25 cm rotating at an angular speed of 500 rotations per minute. If a 100 N braking force is applied normal to the curved surface of the cylinder bringing it to rest in 15 sec., what is the coefficient of kinetic friction between the brake shoe and the cylinder (approx..)?

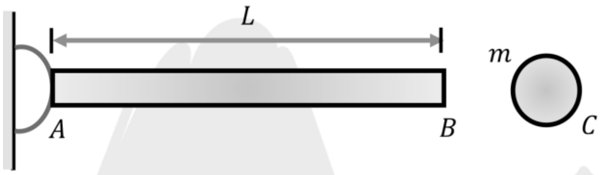


(A) 0.027 (B) 0.042 (C) 0.066 (D) 0.140

12. A constant force acting at the centre of a uniform disc of radius r is always perpendicular to the plane of the disc. The disc can rotate about a chord at a distance 'x' from the centre of the disc. For what value of 'x' will the angular acceleration of the disc be maximum?

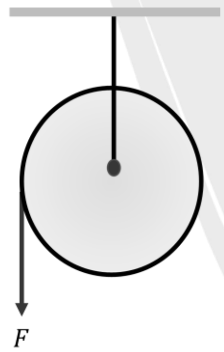
(A) $\frac{r}{\sqrt{3}}$ (B) $\frac{r}{3}$ (C) $\frac{r}{2}$ (D) $\frac{r}{\sqrt{2}}$

13. A uniform bar AB of mass m and a ball of the same mass are released from rest from the same horizontal position. The bar is hinged at end A. there is gravity downwards. What is the distance of the point from point B that has the same acceleration as the ball, immediately after release?



(A) $\frac{2L}{3}$ (B) $\frac{L}{3}$ (C) $\frac{L}{2}$ (D) $\frac{3L}{4}$

14. Suppose the force F_T in the cord hanging from the disc is given by the relation $F_T = 3t - 2t^2$ (newtons) where t is in seconds. If the wheel starts from rest, the linear speed of a point on its rim 3s later is $\frac{225}{n}$ m/s. Find n? (Mass of disc = 1 kg, Radius of disc = 1m.)

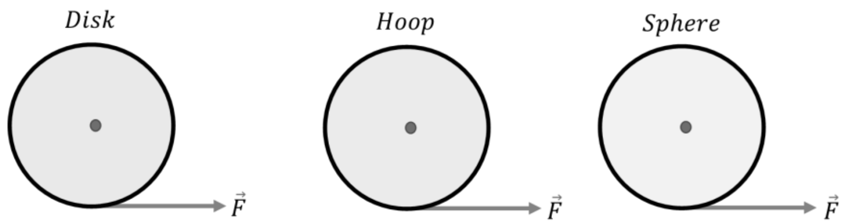


15. A student is standing on the edge of a stationary turntable, made from a uniform disc of mass M and radius R on a frictionless bearing at its centre. She starts to run with speed v around the perimeter of the table. (This is her speed relative to the ground – not the turntable). If her mass is m, what is the magnitude of the angular velocity of the turntable (relative to the ground)?

(A) $\frac{2mv}{MR}$ (B) $\frac{Mv}{(mR)}$

(C) $\frac{mv}{(MR)}$ (D) $\frac{2Mv}{(mR)}$

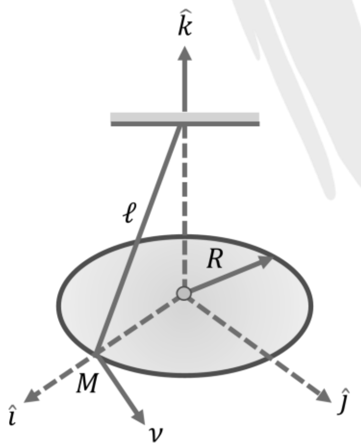
16. A uniform disk, a thin hoop (ring), and a uniform sphere, all with the same mass and same outer radius, are each free to rotate about a fixed axis through its centre. Assume the hoop is connected to the rotation axis by light spokes. With the objects starting from rest, identical forces are simultaneously applied to the rims, as shown. Rank the objects according to their angular momentum after a given time t, least to greatest.



(A) all tie
(B) disk, hoop, sphere
(C) hoop, disk, sphere
(D) hoop, sphere, disk

17. A disk of clay is rotating with angular velocity ω. A blob of clay is now stuck to the outer rim of the disk, and it has a mass $\frac{1}{10}$ of that of the disk. If the blob detaches and flies off tangentially to the outer rim of the disk, the angular velocity of the disk after the blob separates is $\frac{n}{5}\omega$. Find n?

18. A conical pendulum consists of a mass M suspended from a string of length ℓ. The mass executes a circle of radius R in a horizontal plane with speed v. At time t, the mass is at position $R\hat{i}$ and has velocity $v\hat{j}$. At time t, the angular momentum vector of the mass M about the point from which the string suspended is:



(A) $Mv R\hat{k}$
(B) $Mv\ell\hat{k}$
(C) $Mv\ell\left[\frac{\sqrt{\ell^2-R^2}}{\ell}\hat{i}+\frac{R}{\ell}\hat{k}\right]$
(D) $-Mv\ell\left[\frac{\sqrt{\ell^2-R^2}}{\ell}\hat{i}+\frac{R}{\ell}\hat{k}\right]$

19. Three cylinders, all of mass M, roll without slipping down an inclined plane of height H. The cylinders are described as follows:

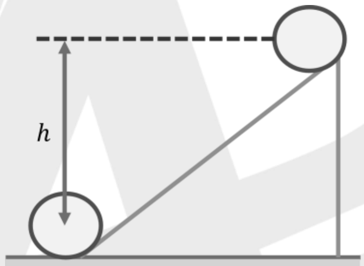
[P] Hollow of radius R [Q] Solid of radius $\frac{R}{\sqrt{2}}$

[R] Solid of radius R

If all cylinders are released simultaneously from the same height, the cylinder (or cylinders) reaching the bottom first is (are):

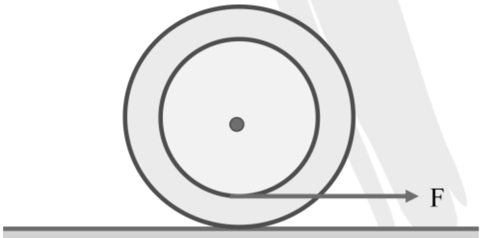
(A) Q (B) R (C) P and Q (D) Q and R

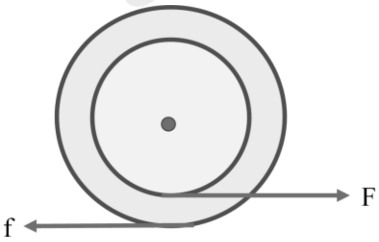
20. A ring of mass M and radius R is at rest at the top of an incline as shown. The ring rolls down the plane without slipping. When the ring reaches bottom, its angular momentum about its center of mass is :

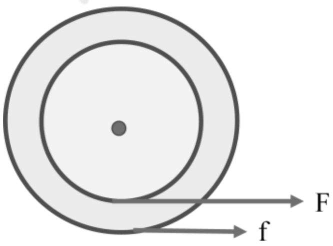


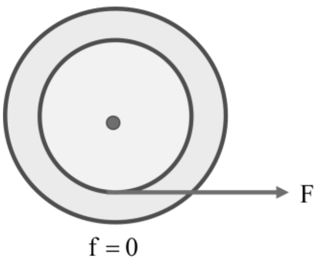
(A) $MR\sqrt{gh}$ (B) $MR\sqrt{\frac{gh}{2}}$ (C) $MR\sqrt{2gh}$ (D) None of these

21. A spool is pulled horizontally by a constant force F below the centre of mass. The friction force can be given by which of the following diagrams?



(A)


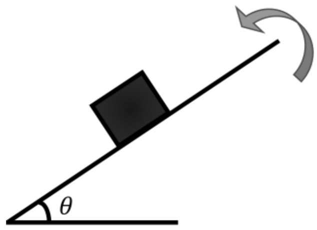
(B)


(C)


(D) Cannot be interpreted

EXCERCISE-IV

1. Consider the arrangement shown in figure. The block is initially at rest. Now θ is slowly increased (consider $0 < \theta < 90°$)



Statement – 1: If sliding starts before toppling and θ is kept on increasing even after that then block won't topple thereafter.

Statement – 2: Line of action of resultant normal force shifts to keep the block from toppling.

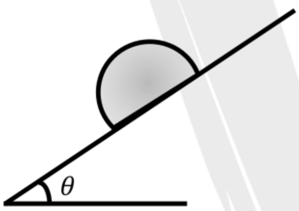
(A) Statement – 1 is true, statement – 2 is true and statement – 2 is correct explanation for statement-1

(B) Statement – 1 is true, statement – 2 is true and statement – 2 is not the correct explanation for statement – 1

(C) Statement – 1 is true, statement – 2 is false

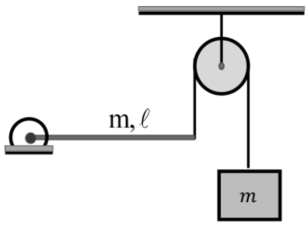
(D) Statement – 1 is false, statement – 2 is true.

2. An uniform hemi-solid sphere is placed with flat surface on rough inclined plane as shown in figure. If friction is large for no sliding, then the minimum angle θ at which toppling occur is:



(A) $\tan^{-1}\left(\frac{1}{2}\right)$ (B) $45°$ (C) $\tan^{-1}\left(\frac{8}{3}\right)$ (D) $\tan^{-1}\left(\frac{4}{3}\right)$

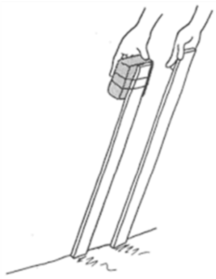
3. Uniform rod AB is hinged at end A in horizontal position as shown in the figure. The other end is connected to a block through a massless string m as shown. The pulley is smooth and massless. Masses of block and rod is same and is equal to m. Then acceleration of block just after release from thin position is:



(A) $\frac{6g}{13}$ (B) $\frac{g}{4}$ (C) $\frac{3g}{8}$ (D) None of these

4. **Statement – 1:** A pair of upright meter sticks, with their lower ends against a wall, are allowed to fall to the floor. One is bare, and the other has a heavy weight attached to its upper end. The stick to hit the floor first is the weighted stick.

Statement – 2: The torque acting on weighted stick is more than the bare stick.



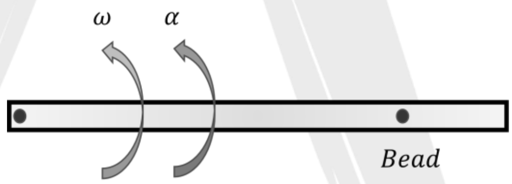
(A) Statement – 1 is true, statement – 2 is true and statement – 2 is the correct explanation for statement – 1.

(B) Statement – 1 is true, statement – 2 is true and statement – 2 is not the correct explanation for statement – 1.

(C) Statement – 1 is true, statement – 2 is false

(D) Statement – 1 is false, statement – 2 is true

5. A bead is constrained to move on rod in gravity free space as shown in figure. The rod is rotating with angular velocity ω and angular acceleration α about its end. If μ is coefficient of friction. Mark the correct option. Rod rotates in the plane of paper:



(A) If $\mu = \frac{\omega^2}{\alpha}$ friction on bead is static in nature

(B) If $\mu > \frac{\omega^2}{\alpha}$ friction on bead is kinetic in nature

(C) If $\mu < \frac{\omega^2}{\alpha}$ friction is static

(D) If bead does not slide relative to rod. Friction will not exist between bead and rod.

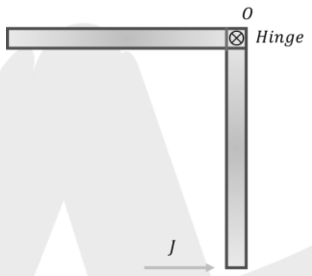
6. A ring of radius R is rolling purely on the outer surface of a pipe of radius 4R. At some instant, the centre of the ring has a constant speed $= v$. Then, the acceleration of the point on the ring which is in contact with the surface of the pipe is:

(A) $\frac{4v^2}{5R}$ (B) $\frac{3v^2}{5R}$ (C) $\frac{v^2}{4R}$ (D) Zero

7. A ring of radius R is rolling purely on the outer surface of a pipe of radius 4R. At some instant, the centre of the ring has a constant speed $= v$. Then, the acceleration of the point on the ring which is farthest from the centre of the pipe, at the given moment, is:

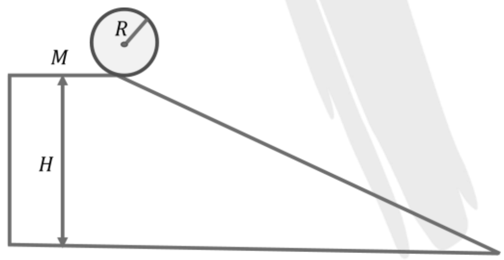
(A) $\frac{4v^2}{5R}$ (B) $\frac{3v^2}{5R}$ (C) $\frac{3v^2}{4R}$ (D) $\frac{6v^2}{5R}$

8. Two rods of mass M and length L each are rigidly joined at point O to form a L-shaped composite rod. Assembly can turn about a frictionless hinge at O. An impulse acts at lower end of rod. What is angular velocity immediately after impulse acts? Entire assembly lies in horizontal plane:



(A) $\frac{2J}{3ML}$ (B) $\frac{3J}{2ML}$ (C) $\frac{3J}{4ML}$ (D) $\frac{3J}{ML}$

9. The body shown, with mass M and radius R, starts from rest and rolls without slipping down an inclined plane of height H. At the bottom of the plane its translational speed is $\left(\frac{8gH}{7}\right)^{1/2}$. Which of the following is the moment of inertia of the body?



(A) $\frac{1}{2}MR^2$ (B) $\frac{3}{4}MR^2$ (C) $\frac{7}{8}MR^2$ (D) MR^2

10. A uniform solid disc is imparted an angular velocity and placed on a rough horizontal surface such that its plane is vertical.

(A) It will continue to rotate at the same place till its rotation ceases

(B) If the angular velocity is clockwise it will move to the left

(C) If the angular velocity is anti-clockwise it will move to the left

(D) Friction will act opposite to the direction of motion of the disc

11. Suppose you are standing on the edge of a spinning plant form and step off at right angles to the edge (radially outward). Now consider it the other way. You are standing on the ground next to a spinning carousel and you step onto the platform at right angles to the edge (radially inward).

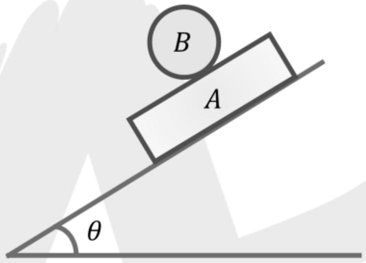
(A) There is no change in rotational speed of the carousel in either situation

(B) There is a change in rotational speed in the first situation but not the second

(C) There is a change in rotational speed in the second situation but not the first

(D) There is a change in rotational speed in both instances

12. On a rough inclined plane, a long plank A is placed. A uniform disc B having mass same as plank is rolling without slipping on the plank. The minimum value of coefficient of friction between the plank and the inclined plane such that the plank can remain stationary is:



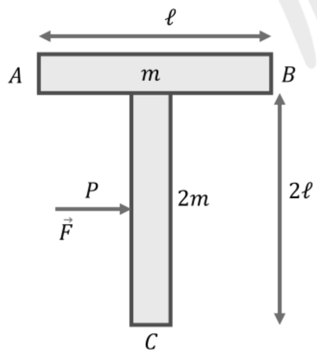
(A) $\dfrac{2\tan\theta}{3}$

(B) $\dfrac{3\tan\theta}{2}$

(C) $\dfrac{4\tan\theta}{3}$

(D) None of these

13. A T shaped object with dimensions shown in the figure is lying on a smooth floor. A force $\vec{F}$ is applied at the point P parallel to AB such that the object has only the translational motion without rotation. Find the location of P with respect to C:



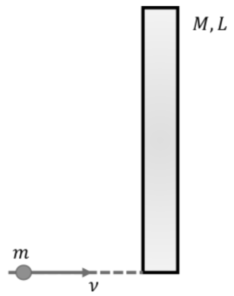
(A) $\left(\dfrac{3}{4}\right)\ell$

(B) ℓ

(C) $\left(\dfrac{4}{3}\right)\ell$

(D) $\left(\dfrac{3}{2}\right)\ell$

14. A uniform stick of length L and mass M lies on a frictionless horizontal surface. A point particle of mass m approaches the stick with speed v on a straight line passing through one end and perpendicular to the stick, as shown in figure. After the collision, which is elastic, the particle comes to rest. The speed V of the center of mass of the stick after the collision is:



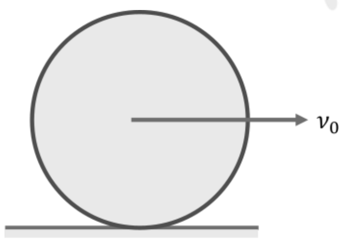
(A) $\frac{m}{M}v$ (B) $\frac{m}{M+m}v$ (C) $\sqrt{\frac{m}{M}}v$ (D) $\sqrt{\frac{m}{M+m}}v$

15. A hollow thin walled pipe is projected on a rough horizontal surface with speed v. As soon as the pipe begins to roll, what will be its speed ?

(A) $\left(\frac{1}{4}\right)\times v$ (B) $\left(\frac{1}{3}\right)\times v$ (C) $\left(\frac{1}{2}\right)\times v$ (D) v

16. A thin hollow sphere of mass m is completely filled with non-viscous liquid of mass m. when the sphere rolls without sliding on horizontal ground such that centre moves with veloicity v, kinetic energy of the system is $\frac{\alpha mv^2}{\beta}$. Find $\alpha+\beta$?

17. A sphere of mass m is projected on a rough ground with a velocity of v_0 without any spin. This is observed from ground and by an observer moving with constant velocity v_0. For both the frames, origin is on ground and direction of motion is positive x-axis. Choose the incorrect statement.



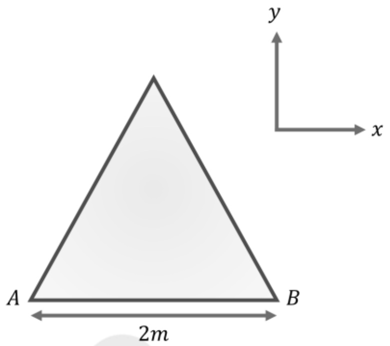
(A) Change in angular momentum of the ball about origin in any time interval is same from both frames

(B) Work done by friction on the ball in any time interval is same from both frames

(C) Total heat dissipated in any time interval is same from both frames

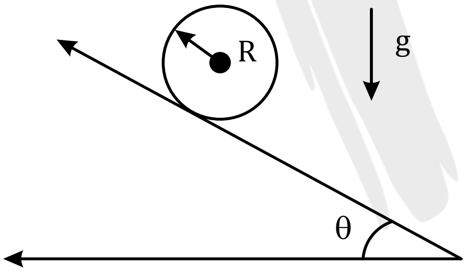
(D) Change in momentum of ball in any time interval is same from the both frames

18. A rigid equilateral triangular plate ABC of side 2m, is in motion in the x-y plane. At the instant shown in the figure, the point B has velocity $\vec{v}_B = (3\hat{i} + 8\hat{j})\,m/s$ and the plate has angular velocity $\vec{\omega} = 2\hat{k}\,rad/sec$. Find the speed (in m/s) of point A:

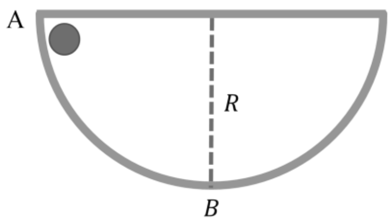


19. A uniform circular disc placed on a rough horizontal surface has initially a velocity v_0 and an angular velocity ω_0. The disc comes to rest after moving some distance in the direction of motion. Then $\frac{v_0}{(r\omega_0)}$ is $\frac{\alpha}{\beta}$. Find $\alpha + \beta$?

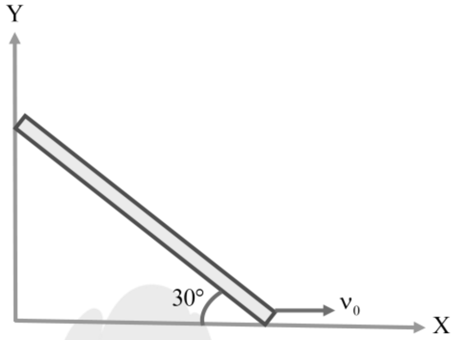
20. The same bowling ball of above problem now rolls without slipping down an inclined plane inclined at an angle θ to the horizontal, as shown. Again the coefficient of static friction between the ball and the surface is μ_s, and the coefficient of kinetic friction is μ_k. The magnitude of the force of friction acting on the ball is $\frac{\alpha}{\beta} mg\sin\theta$. Find $\alpha + \beta$?



21. A small solid sphere A rolls without slipping inside a large fixed hemispherical bowl of radius R as shown in figure. If the sphere starts from rest at the top point of the hemisphere, the normal force exerted by the small sphere on the hemisphere when it is at the bottom B of the hemisphere is $\frac{\alpha}{\beta}$mg. Find $\alpha + \beta$?

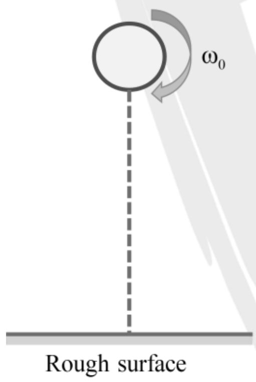


22. A rod of mass m and length ℓ is sliding against a smooth vertical wall as shown. The floor is assumed to be frictionless. The speed of bottom end of the rod at the instant shown is v_0. The magnitude of angular momentum of the rod about ICR (instantaneous axis of rotation) at the instant when angle $\theta = 30°$ is:

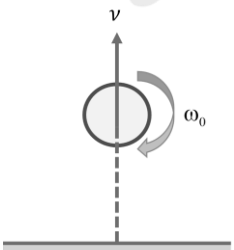


(A) $\frac{2}{3}mv_0\ell$ (B) $\frac{1}{6}mv_0\ell$ (C) $\frac{1}{12}mv_0\ell$ (D) None of these

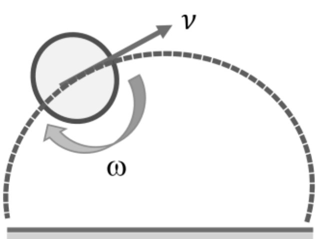
23. A disc is given an angular speed ω_0 and released from a certain height (as shown in figure). Motion of disc is observed after collision with the rough surface. Velocity of centre of mass of ball and direction of ω is shown in figure after the collision. Mark possible path(s), disc cannot follow after the collision:



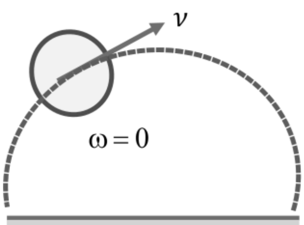
(A)



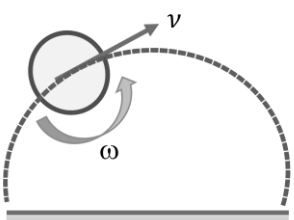
(B)



(C)

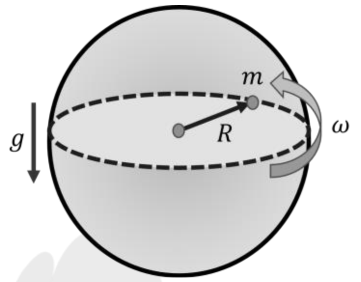


(D)



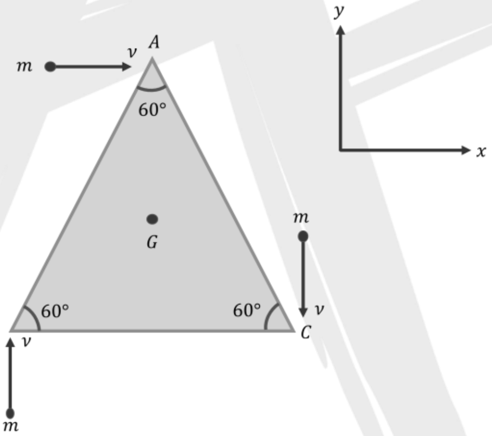
EXCERCISE-V

1. A particle constrained to move inside a smooth fixed spherical surface of radius R is projected horizontally (and tangent to the spherical surface at that point) from a point at the level of the centre so that its angular velocity relative to the vertical axis is ω. Find approximately the maximum depth z below the level of the centre that the ball goes. Take $\omega^2 R >> g$.



(A) $\frac{g}{2\omega^2}$ (B) $\frac{g}{\omega^2}$ (C) $\frac{2g}{\omega^2}$ (D) $\frac{4g}{\omega^2}$

2. A triangular sheet ABC of mass m and sides 2a lies on a smooth horizontal plane as shown. three point masses of mass m each strikes the sheet at A, B and C with speed v as shown. After the collision the particle come to rest. Select the correct alternatives:



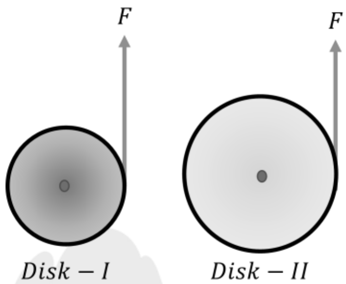
(A) The centre of mass of ABC remains stationary after collision

(B) The centre of mass of ABC moves with a velocity v along x-axis after collision

(C) The triangular sheet rotates with an angular velocity $\omega = \frac{2\sqrt{3}mva}{I}$ abouts its centre of mass (here I is the moment of inertia of triangular sheet about its centroid axis perpendicular to its plane)

(D) A point lying at a distance of $\left(\frac{I}{2\sqrt{3}ma}\right)$ from G on perpendicular bisector of BC (below G) is at rest just after collision.

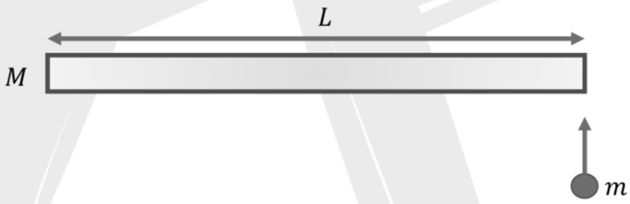
3. Two solid uniform disks of equal mass each are mounted to rotate about an axis fixed through the centre of the disk. Each disk is initially at rest. The radii of the disks are $r_1 < r_2$. A force F is applied to each disk at its edge for the same amount of time. Assume that friction at pulley axis is negligible. What statement is true about the kinetic energy (K) and magnitude of angular momentum (L) of the disks?



(A) $L_1 = L_2$ and $K_1 < K_2$ (B) $L_1 < L_2$ and $K_1 = K_2$

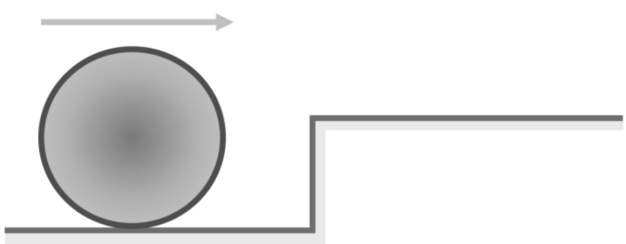
(C) $L_1 < L_2$ and $K_1 > K_2$ (D) $L_1 < L_2$ and $K_1 < K_2$

4. A stick of length L and mass M lies on a frictionless horizontal surface on which it is free to move in any way. A ball of mass m moving with speed v collides elastically with the stick as shown in figure. If after the collision ball comes to rest, then what should be the mass of the ball?

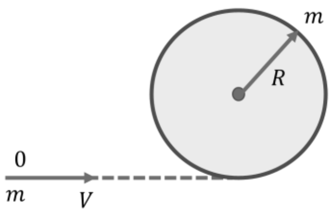


(A) $m = 2M$ (B) $m = M$ (C) $m = \frac{M}{2}$ (D) $m = \frac{M}{4}$

5. A disc of mass M and radius R is rolling purely with centre's velocity v_0 on a flat horizontal floor when it hits a step in the floor of height $\frac{R}{4}$. The corner of the step is sufficiently rough to prevent any slipping of the disc against itself. The velocity of the centre of the disc just after impact is $\frac{Pv_0}{Q}$. Find $P+Q$?

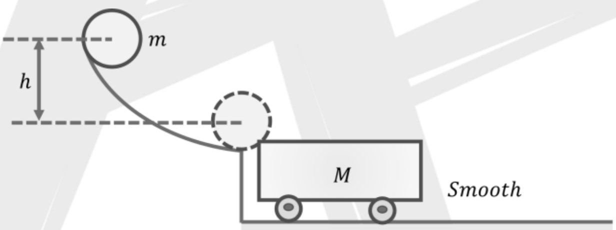


6. A circular hoop of mass m and radius R rests flat on a horizontal frictionless surface. A bullet, also of mass m and moving with a velocity v, strikes the hoop and gets embedded in it. The thickness of the hoop is much smaller than R. The angular velocity with which the system rotates after the bullet strikes the hoop is :



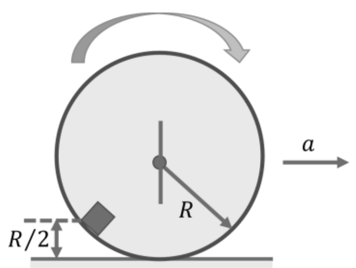
(A) $\frac{V}{(4R)}$ (B) $\frac{V}{(3R)}$ (C) $\frac{2V}{(3R)}$ (D) $\frac{3V}{(4R)}$

7. A uniform disc of mass $m = 12$kg slides down along smooth, frictionless hill, which ends in a horizontal plane without break. The disc is released from rest at a height of $h = 1.25$m (it has no initial speed and it does not rotate), and lands on the top of a cart of mass $M = 6$kg, which can move on a frictionless surface. The coefficient of kinetic friction between the cart and the disc is $\mu = 0.4$. Find minimum length of the cart (in m) so that the disc begins to roll without slipping before losing contact with the cart.



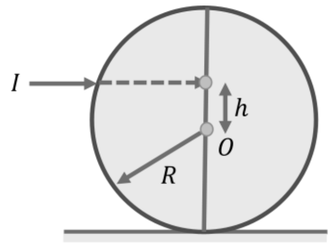
(A) $\frac{7}{8}$ (B) $\frac{7}{4}$ (C) 5.25 (D) None of these

8. Inside a uniformly accelerating thin-walled spherical shell of radius R, which is rolling on horizontal surface, there is a small body slipping around. Angle of friction between body and inner surface of sphere is $23°$. Which of the following can be the acceleration a of the centre of sphere to ensure that the small body stays at $\frac{R}{2}$ distance from the surface?



(A) $\frac{g}{\sqrt{3}}$ (B) $\frac{3g}{4}$ (C) $g \tan 23°$ (D) $\frac{g\sqrt{3}}{2}$

9. An impulse I is applied horizontally at a height h above the centre O of uniform disc at rest on a rough horizontal plane. The velocity of O immediately after the impulse is applied is v_0 but it begins to increase after motion started. Then:



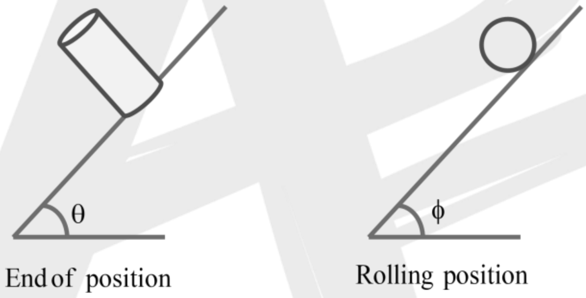
(A) $h < \frac{R}{2}$

(B) $h = \frac{R}{2}$

(C) $h > \frac{R}{2}$

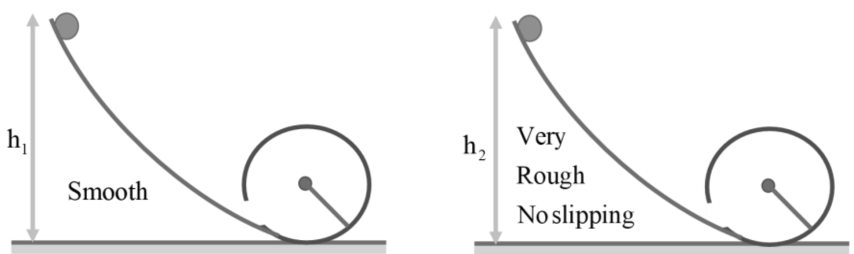
(D) Impulse I must not be less than a fixed minimum value

10. A solid cylinder is placed in ends on position on an inclined plane. It is found that at an angle θ the cylinder starts to slide. When the cylinder turns on its sides and allowed to roll, it is found that the steepest angle at which cylinder performs pure rolling is ϕ. The ratio $\frac{\tan\phi}{\tan\theta}$, is:



(A) $\frac{1}{3}$ (B) 1 (C) 3 (D) $\frac{1}{2}$

11. The following figure shows two situations in which a uniform round rigid body is released from rest from the positions shown, such that it is just able to loop the loop without leaving contact with the track. Assuming that radius of the track is large in comparision to the radius of round body, the ratio $\frac{h_1}{h_2}$:



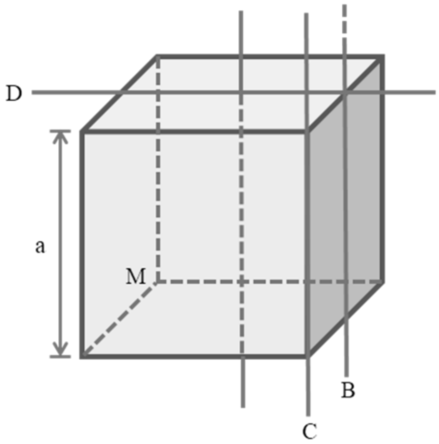
(A) must be greater than 1

(B) must be less than 1

(C) must be equal to 1

(D) can be greater than or less than 1, depending on the moment of inertia of the round body.

12. Shown below is a uniform cubical block of mass M and side a. Mark the correct statement(s).



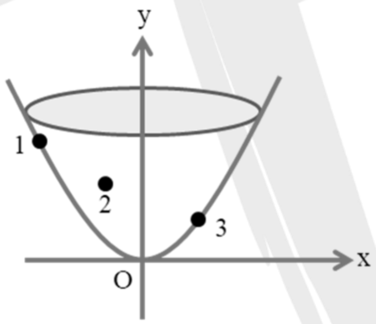
(A) The moment of inertia about axis A, passing through the centre of mass is $I_A = \frac{1}{6}Ma^2$

(B) The moment of inertia about axis B, which bisects one of the cube faces is $I_B = \frac{5}{12}Ma^2$

(C) The moment of inertia about axis C, along one of the cube edges is $I_C = \frac{2}{3}Ma^2$

(D) The moment of inertia about axis D, which bisects one of the horizontal cube faces is $\frac{7}{12}Ma^2$

13. Three particles, each of mass m are placed at the points (x_1, y_1, z_1), (x_2, y_2, z_2) and (x_3, y_3, z_3) on the inner surface of a paraboloid of revolution obtained by rotating the parabola, $x^2 = 4ay$ about the y-axis. Neglect the mass of the paraboloid. (y-axis is along the vertical)



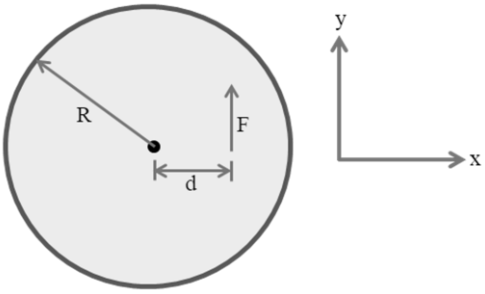
(A) The moment of inertia of the system about the axis of the paraboloid is
$I = 4ma(y_1 + y_2 + y_3)$.

(B) If potential energy at O is taken to be zero, the potential energy of the system is
$mg(y_1 + y_2 + y_3)$

(C) If the particle at (x_1, y_1, z_1) slides down the smooth surface, its speed at O is $\sqrt{2gy_1}$

(D) If the paraboloid spins about OY with an angular speed ω, the kinetic energy of the system will be $2ma(y_1 + y_2 + y_3)\omega^2$

14. A uniform thin flat isolated disc is floating in space. It has radius R and mass m. A force F is applied to it at distance $d = \frac{R}{2}$ from the center in the y-direction. Treat this problem as two-dimensional. At the instant shown the :



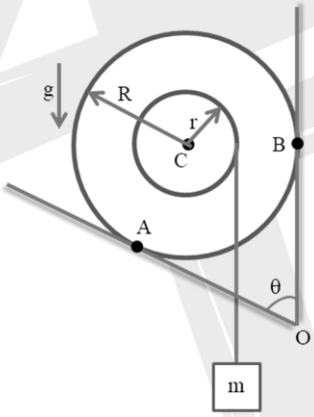
(A) acceleration of the center of the disc is $\frac{F}{m}$

(B) angular acceleration of the disk is $\frac{F}{mR}$

(C) acceleration of leftmost point on the disc is zero

(D) point which is instantaneously unaccelerated is the rightmost point

15. A massless spool of inner radius r, outer radius R is placed against vertical wall and tilted split floor as shown. A light inxtensible thread is tightly wound around the spool through which a mass m is hanging. There exists no friction at point A, while the coefficient of friction between spool and point B is μ. The angle between two surface is θ.



(A) the magnitude of force on the spool at B in order to maintain equilibrium is

$$mg\sqrt{\left(\frac{r}{R}\right)^2 + \left(1-\frac{r}{R}\right)^2 \frac{1}{\tan^2\theta}}$$

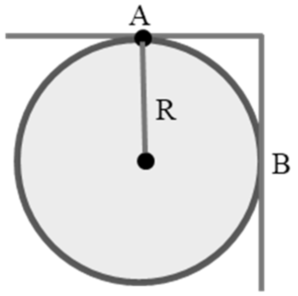
(B) the magnitude of force on the spool at B in order to maintain equilibrium is

$$mg\left(1-\frac{r}{R}\right)\frac{1}{\tan\theta}$$

(C) the minimum value of μ for the system to remain in equilibrium is $\frac{\cot\theta}{\left(\frac{R}{r}\right)-1}$

(D) the minimum value of μ for the system to remain in equilibrium is $\frac{\tan\theta}{\left(\frac{R}{r}\right)-1}$

16. A rod bent at right angle along its centre line, is placed on a rough horizontal fixed cylinder of radius R as shown in figure. Mass of rod is 2 kg and rod is in equilibrium. Assume that friction force on rod at A and B are equal in magnitude.



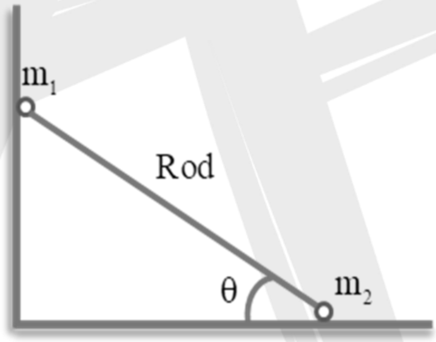
(A) Normal force applied by cylinder on rod at A is $\frac{3mg}{2}$

(B) Normal force applied by cylinder on rod at B must be zero

(C) Friction force acting on rod at B is upward

(D) Normal force applied by cylinder on rod at A is mg

17. Two beads of mass m_1 and m_2 are connected by a light rigid rod. System is placed between a smooth wall and a rough floor having coefficient of friction μ. Which of the following are correct? (T is the force exerted by the rod. N_1 is the normal force exerted by the wall and N_2 is the normal force exerted by the ground. f is the friction force exerted by the ground.)



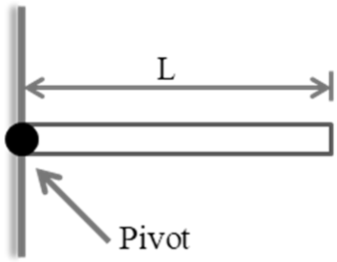
(A) Minimum value of θ so that system does not slip is $\cot^{-1}\left[\mu\left(1+\frac{m^2}{m_1}\right)\right]$

(B) $N_1 = T\cos\theta$

(C) $N_2 = (m_1 + m_2)g$

(D) $f = T\cos\theta$

18. A uniform rod of length L and mass M is free to rotate about a frictionless pivot at one end as shown in the figure. The rod is released form rest in the horizontal position :



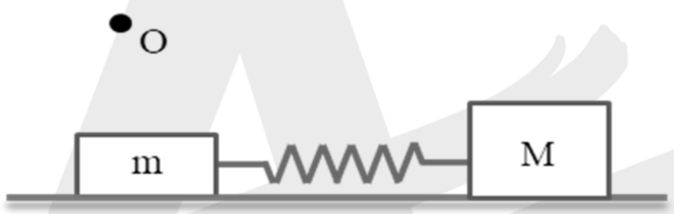
(A) initial angular acceleration of right end of the rod is $\frac{3g}{2L}$ and is common to all points on the rod

(B) initial linear acceleration of the right end of the rod is $\frac{3g}{2}$ and is common to all points on the rod

(C) initial linear acceleration of the right end of the rod is $\frac{3g}{2}$ and its value is different at different points

(D) no points on the rod has a linear acceleration greater than g

19. A light spring is permanently connected between two trolleys of masses M and m which can move over a smooth horizontal table, along straight line parallel to the length of the spring as shown in figure. The trolleys are brought nearer to compress the spring and then released. In the subsequent motion.



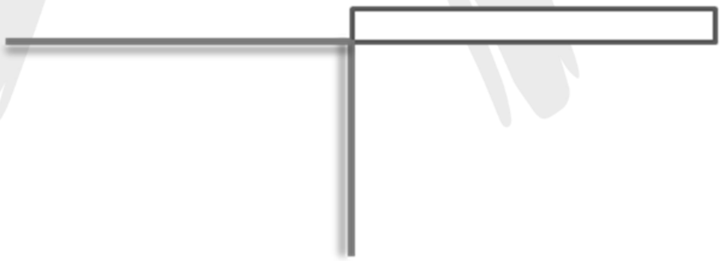
(A) Initially they move in opposite directions with speeds inversely proportional to their masses

(B) Angular momentum of system about O is increasing initially

(C) linear momentum and energy of the system must be conserved

(D) their centre of mass remains stationary

20. A uniform rod of length ℓ is falling down with a velocity V_0 when one of it's end hits a fixed edge as shown.



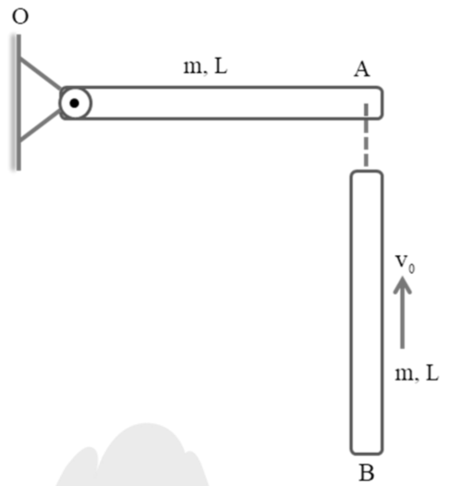
(A) If the collision is elastic, the centre of mass may have upwards velocity just after the collision

(B) If the collision is partially elastic, the centre of mass may come to rest just after the collision

(C) If the collision is perfectly inelastic, the centre of mass has a downward velocity just after the collision

(D) If the collision is elastic, the centre of mass has a downward velocity just after the collision

21. Rod B sticks to rod A on collision. Collision takes place on horizontal plane. Rod-A is hinged at O. Friction is absent everywhere.



(A) Angular velocity of system just after collision is $\frac{2v_0}{5L}$

(B) Velocity of centre of mass of system just after collision is $\sqrt{\frac{9}{40}}v_0$

(C) Centre of mass of system is at a distance of $\frac{\sqrt{10}\,L}{4}$ from O

(D) Kinetic energy of system just after collision is $\frac{9}{40}mv^2$

22. A hollow spherical ball is given an initial push up an incline of inclination angle α. The ball rolls purely. Coefficient of static friction between ball and incline = μ. During its upward journey :

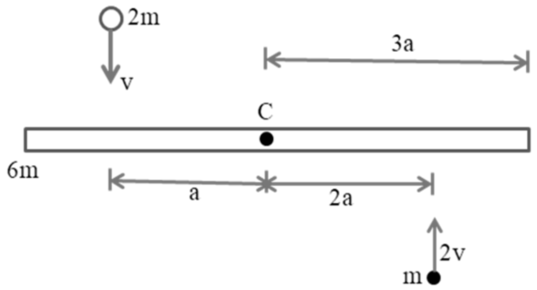
(A) friction acts up along the incline

(B) $\mu \geq \frac{2}{5}\tan\alpha$

(C) friction acts down along the incline

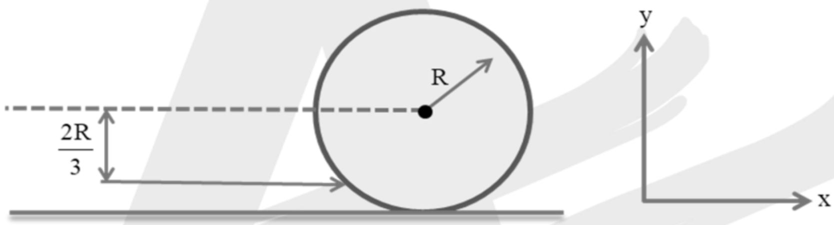
(D) $\mu \geq \frac{2}{7}\tan\alpha$

23. A uniform bar of length 6a and mass 8m lies on a smooth horizontal table. Two point masses m and 2m moving in the same horizontal plane with speeds 2v and v, respectively, strike the bar (as shown in figure) and stick to the bar after collision. Denoting angular velocity, total energy and velocity of centre of mass by ω, E and V_c respectively, we have after collision :



(A) $V_c = 0$ (B) $\omega = \frac{3v}{5a}$ (C) $\omega = \frac{v}{5a}$ (D) $E = \frac{3mv^2}{5}$

24. A billiard ball initially at rest is given a sharp blow by a cue stick. The force is horizontal and is applied at a distance $\frac{2R}{3}$ below the centreline, as shown in figure. The initial speed of the ball is V_0, and the coefficient of kinetic friction is μ_k.



(A) Initiallly kinetic friction acts in $-\hat{i}$ direction

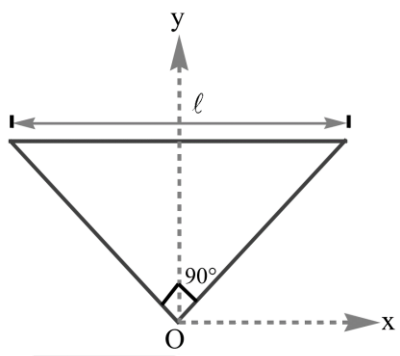
(B) Initially kinetic friction in $\hat{i}$ direction

(C) Ball instantaneously starts pure rolling

(D) Initial anggular velocity of ball is $\frac{5v_0}{3R}$

PROFICIENCY TEST-I

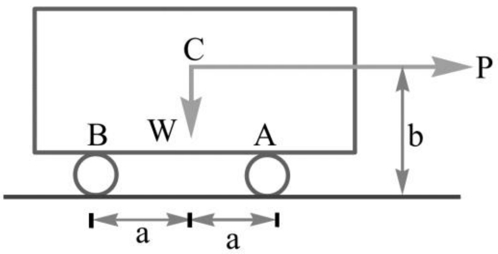
1. Figure shows an isosceles triangular plate of mass M and base length ℓ. The apex lies at the origin and the angle at the apex is $90°$. The base is parallel to x-axis. The moment of inertia of the plate about the x-axis is:



(A) $\frac{M\ell^2}{6}$ (B) $\frac{M\ell^2}{8}$ (C) $\frac{M\ell^2}{12}$ (D) $\frac{M\ell^2}{4}$

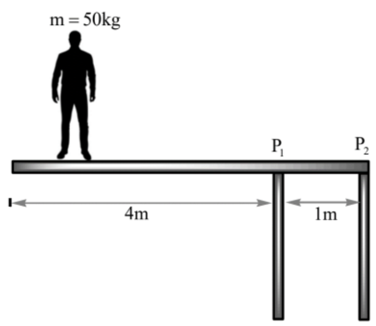
2. A uniform disc of radius R lies in the x-y plane with its centre at origin. Its moment of inertia about z-axis is equal to its moment of inertia about line $y = x + c$. The value of c is $\frac{R}{\sqrt{\alpha}}$. Find α?

3. An automobile of weight W is shown. A pull P is applied. The reaction at the front wheel (location A) is: [Floor is smooth and C represents centre of gravity]



(A) $\left(\frac{W}{2}\right)-\left(\frac{Pb}{2a}\right)$ (B) $\left(\frac{W}{2}\right)+\left(\frac{Pb}{2a}\right)$ (C) $\left(\frac{W}{2}\right)-\left(\frac{Pa}{2b}\right)$ (D) None of these

4. A physics professor, mass = 50 kg, stands at the end of a diving board, as shown. The board is uniform, massive and solid. There is no vibration or motion of any kind. The board is firmly attached to two supports at points P_1 and P_2



The professor now begins to walk slowly in (right) from the end of the board towards P_1. While he was walking how would you describe the force on the board by the support at point P_2 (i.e., over at the far right hand end)?

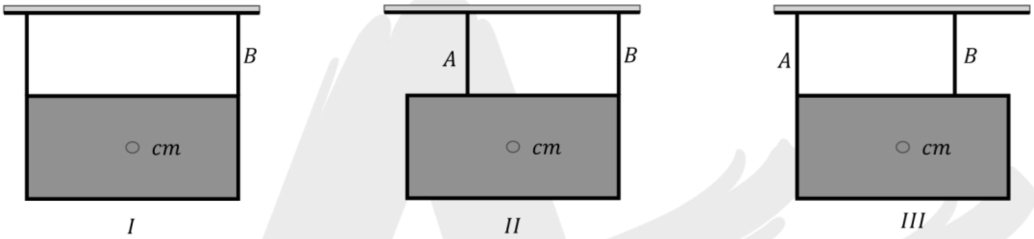
(A) upwards and increasing in magnitude

(B) upwards and decreasing in magnitude

(C) downwards and increasing in magnitude

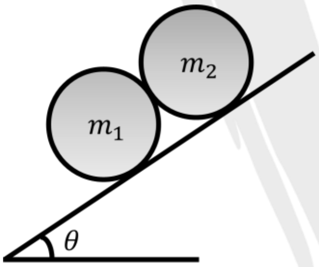
(D) downwards and decreasing in magnitude

5. A picture is to be hung from the ceiling by means of two wires. Order the following arrangements of the wires according to the tension force of wire B, from least to greatest:



(A) I, II, III (B) III, I, II (C) I and II ties, then III (D) II, I, III

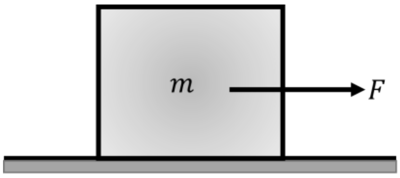
6. Two spheres having masses m_1 and m_2 are kept on an inclined plane as shown. Both of them are in equilibrium. What is a necessary condition for this? (μ_1 is coefficient of friction between m_1 and inclined plane and μ_2 is coefficient of friction between m_2 and inclined plane) :

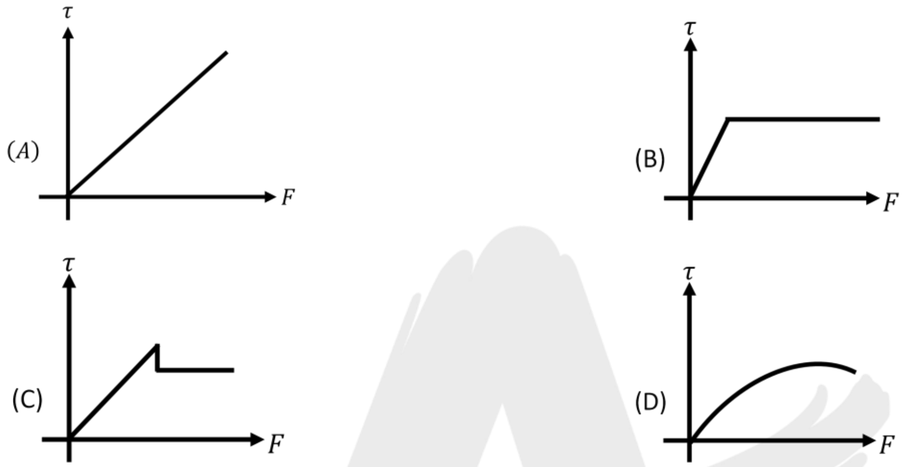


(A) $m_1 > m_2$ (B) $m_1 < m_2$ (C) $\mu_1 > \mu_2$ (D) $\mu_1 < \mu_2$

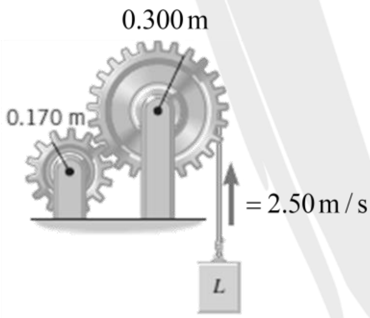
7. 198 cm tall person lies on a light (massless) board which is supported by two scales one under the top of his head and one beneath the bottom of his feet (figure). The two scales read respectively 36 and 30 kg. What distance (in cm) is the centre of gravity of this person from the bottom of his feet?

8. A force is applied horizontally at the centre of mass of a cube. The cube lies on a rough table. Which of the following graphs shown the variation of torque of normal reaction with F.

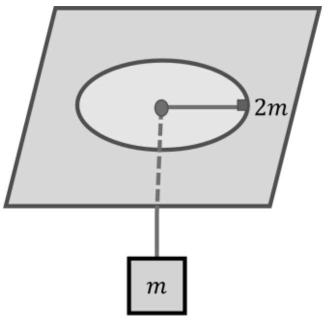




9. The two-gear combination shown in the drawing is being used to lift the load L with a constant upward speed of 2.50 m/s. The rope that is attached to the load is being wound onto a cylinder behind the big gear. The depth of the teeth of gears is negligible compared to the radii. The angular velocity (magnitude) of the smaller gear is $\frac{10n}{17}$ rad / s. Find n?



10. A mass 2m rotating freely in a horizontal circle of radius 1m on a frictionless smooth table supports a stationary mass m, attached to the other end of the string passing through smooth hole O in table, hanging vertically. The angular velocity of rotation is $\sqrt{n}$ rad / s. Find n?

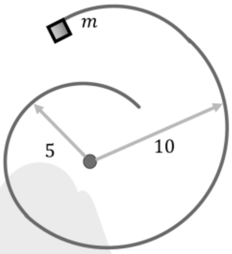


11. If the laws were changed so that traffic in India travelled on the right hand side of the road (instead of on the left), what would happen to the length of the day?

(A) Slightly increase (B) Slightly decrease

(C) Will remain unaltered (D) Can't be said

12. A block enters a horizontal smooth spiral track in which the radius of the track decreases from 10m to 5m. If the block enters the spiral at a speed of 10m/s, what is it's speed (in m/s) at the end of the spiral?

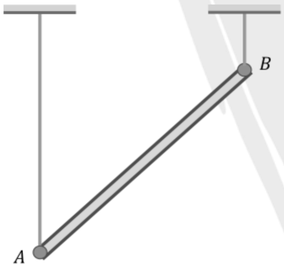


13. A spherical ball of mass m and radius r is released from top of a fixed rough circular track of radius R so that it always rolls purely. Its velocity at the bottom does not depend on:

(A) r (B) R

(C) m (D) acceleration due to gravity

14. Two strings support a uniform rod as shown. String at end B is cut. Which of the following is true just after cut?



[P] initial acceleration of A is vertical

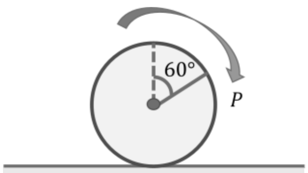
[Q] initial acceleration of A is horizontal

[R] initial acceleration of centre of mass of rod is vertical

[S] initial acceleration of centre of mass of rod is horizontal

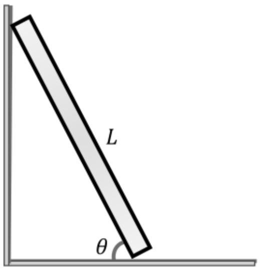
(A) P and Q (B) Q and R (C) R and S (D) P and S

15. A wheel of radius R = 0.1 m is rolling without slipping on a horizontal surface as shown in the figure. Centre of the wheel moves with a constant speed $\sqrt{3}\,m/s$. The speed (in m/s) of the point P with respect to ground is :



PROFICIENCY TEST-II

1. A uniform ladder of length L rests against a smooth frictionless wall. The floor is rough and the coefficient of static friction between the floor and ladder is μ. When the ladder is positioned at angle θ, as shown in the accompanying diagram, it is just about to slip. What is θ?

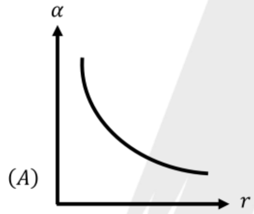


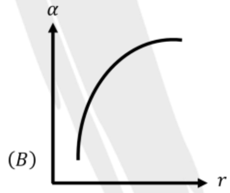
(A) $\cos\theta = \mu$ (B) $\tan\theta = 2\mu$ (C) $\tan\theta = \frac{1}{2\mu}$ (D) $\sin\theta = \frac{1}{\mu}$

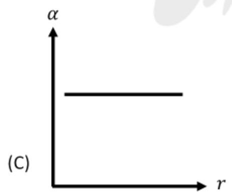
2. A solid homogeneous cylinder of height 'h' and base radius 'r' is kept vertically on conveyer belt moving horizontally with an increasing velocity $v = a + bt^2$. If the cylinder in not allowed to slip then time when the cylinder is about to topple, will be:

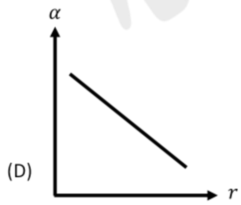
(A) $\frac{2rg}{bh}$ (B) $\frac{rg}{bh}$ (C) $\frac{2bg}{rh}$ (D) $\frac{rg}{2bh}$

3. A magnetic tape is being played on a cassette deck. The tension in the tape applies a torque to the supply reel. Assuming the tension remains constant during play, plot this angular acceleration with reel radius r as the reel becomes empty. Neglect the moment of inetia of empty reel.





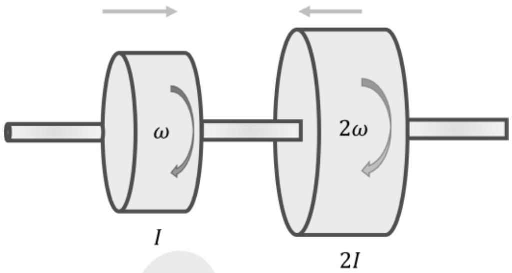




4. A recording disc rotates steadily at n_1 rps on a table. When a small mass m is dropped gently on the disc at a distance x from its axis, it sticks to the disc, the rate of revolution falls to n_2 rps. The original moment of inertia of the disc about a perpendicular axis through its centre is:

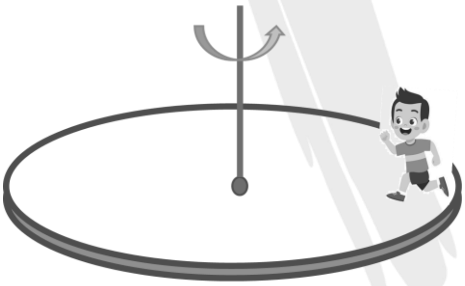
(A) $I = \frac{mx^2}{n_1 - n_2}$ (B) $I = \frac{n_1 mx^2}{n_1 - n_2}$ (C) $I = \frac{n_2 mx^2}{n_1 - n_2}$ (D) $I = \frac{n_2 mx^2}{n_1}$

5. Two disks are mounted on low-friction bearings on a common shaft. The first disc has rotational inertia I and is spinning with angular velocity ω. The second disc has rotational inertia 2I and is spinning in the same direction as the first disc with angular velocity 2ω as shown. The two disks are slowly forced toward each other along the shaft until they couple and have a final common angular velocity of:

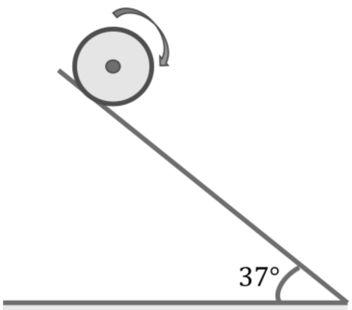


(A) $\frac{5\omega}{3}$ (B) $\frac{\omega}{\sqrt{3}}$ (C) $\omega\sqrt{\frac{7}{3}}$ (D) 3ω

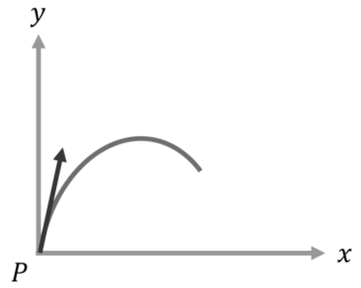
6. A child is standing on the edge of a merry-go-round that has the shape of a disk, as shown in the figure. The mass of the child is 40 kilograms. The merry-go-round has a mass of 200 kilograms and a radius of 2.5 meters, and it is rotating with an angular velocity of $\omega = 2.0$ radians per second. The child then walks slowly towards the centre of the merry-go-round. the final angular velocity of the merry-go-round when the child reaches the centre is $\frac{28}{n}$ rad / s. Find n? (The size of the child can be neglected):



7. A uniform cylinder having radius 0.4 m, initially rotating (at t = 0) with $\omega_0 = 54\,\text{rad/sec}$ is placed on a rough inclined plane with $\theta = 37°$ having friction coefficient $\mu = 0.5$. The time taken by the cylinder to start pure rolling is $\frac{12}{n}$ sec. Find n?



8. At time t = 0, a ball of mass 'm' is thrown vertically from a point P (x = 0, y = 0) with initial velocity $\left(v_x\hat{i}+v_y\hat{j}\right)$. The acceleration due to gravity is $\vec{g}=-g\hat{j}$. The angular momentum vector $\vec{L}(t)$ of the ball about the point P at time t is :



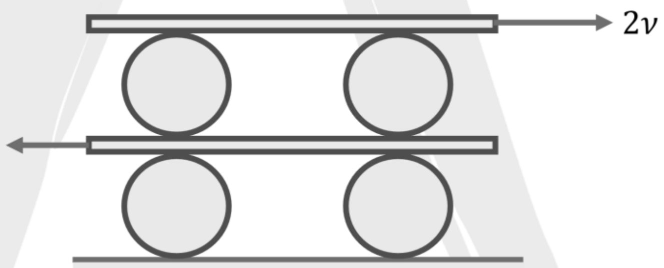
(A) $-\dfrac{mv_xgt^2}{2}\hat{k}$

(B) $2mv_xv_yt\hat{k}$

(C) $\left(-2mv_xv_yt+\dfrac{v_xgt^2}{2}\right)\hat{k}$

(D) $-2mv_xv_yt\hat{i}+\dfrac{mv_xgt^2}{2}\hat{j}$

9. A system of uniform cylinders and plates is shown. All the cylinders are identical and there is no slipping at any contact. Velocity of lower and upper plate is V and 2V respectively as shown. Then the ratio of angular speed of the upper cylinders to angular speed of lower cylinders is :



10. Let r be the distance of a particle from a fixed point to which it is attracted by an inverse square law force given by $f=\dfrac{k}{r^2}$ (k = constant). Let m be the mass of the particle and L be its angular momentum with respect to the fixed point. Which of the following formulae is correct about the total energy of the system?

(A) $\dfrac{1}{2}m\left(\dfrac{dr}{dt}\right)^2-\dfrac{k}{r}+\dfrac{L^2}{2mr^2}=\text{constant}$

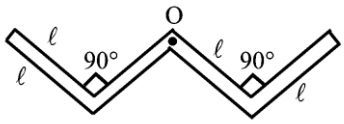
(B) $\dfrac{1}{2}m\left(\dfrac{dr}{dt}\right)^2-\dfrac{k}{r}=\text{constant}$

(C) $\dfrac{1}{2}m\left(\dfrac{dr}{dt}\right)^2+\dfrac{k}{r}+\dfrac{L^2}{2mr^2}=\text{constant}$

(D) None of the above

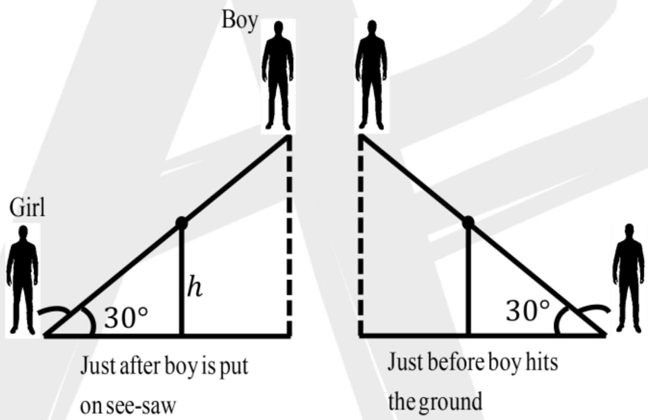
PROFICIENCY TEST-III

1. A thin rod of length 4ℓ, mass 4m is bent at the points as shown in the figure. What is the moment of inertia of the rod about the axis passing point O and perpendicular to the plane of the paper?



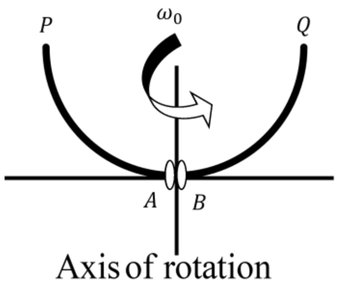
(A) $\frac{M\ell^2}{3}$ (B) $\frac{10M\ell^2}{3}$ (C) $\frac{M\ell^2}{12}$ (D) $\frac{M\ell^2}{24}$

2. A uniform see-saw consists of a uniform thin plank of mass 'M' pivoted at its centre at a height 'h' above the ground. Initially a girl of mass 2 M is sitting at one end of the see-saw. If a boy of mass 3 M now sits onto the other end, with what speed will this end hit the ground. (Angle $\theta = 30°$)

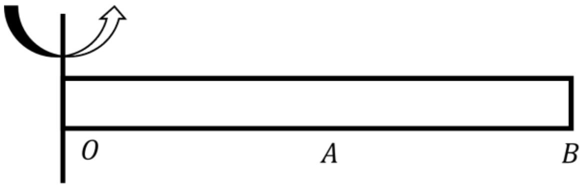


(A) $2\sqrt{\frac{3gh}{19}}$ (B) $\sqrt{\frac{3gh}{4}}$ (C) $\sqrt{3gh}$ (D) None of these

3. Two insects P and q are firmly sitting at the ends of a massless semi-circular wire of radius R and two more insects A and B are firmly sitting at the bottom of the wire. The wire is given an angular velocity ω_0 about a vertical axis through its centre as shown in the figure. Mass of each insect is M. Now A and B crawl to the opposite ends to meet P and Q. Final angular velocity attained by the rod is equal to $\frac{\omega_0}{n}$. Find n?

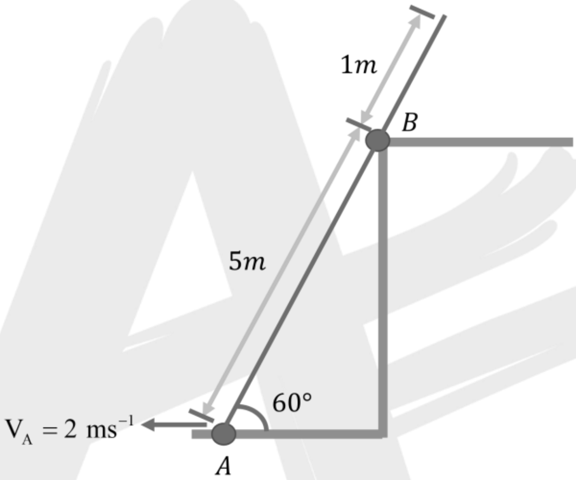


4. A rod OB of length ℓ, hinged about its end O is rotating with an angular velocity ω. The rod breaks at its mid point A and as a result the part OA starts rotating with angular velocity 2ω. Then the angular velocity of AB about its centre of mass is :

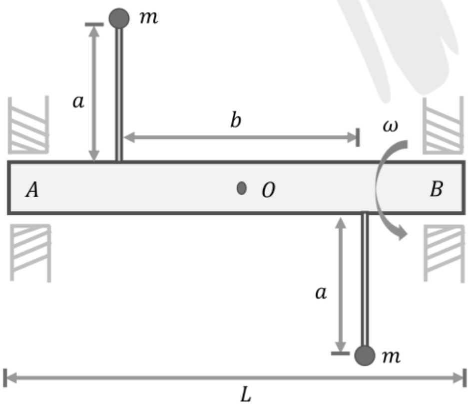


(A) ω (B) 3ω (C) 4ω (D) None of these

5. Velocity of point A on the rod is 2 m/s at the instant shown in the figure. The velocity (in m/s) of the point B on the rod at this instant is :



6. A horizontal rod AB which is held by frictionless bearings at its ends, can rotate freely around its horizontal axis. Two equal masses held in position, as shown, by rigid rods of negligible mass are symmetrically located relative to the centre of rod. If the system is rotating with angular velocity ω in absence of gravity, then choose the correct alternative(s):



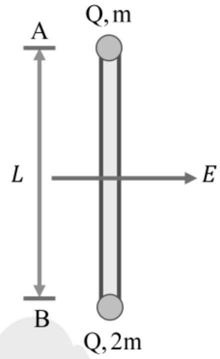
(A) The angular momentum of system about point O is along axis of rotation

(B) The reaction force at bearings on rod is upwards at A and downwards at B

(C) The reaction force at bearings on rod is downwards at A and upwards at B

(D) There would be no reaction force at bearings

7. Two small balls A and B of positive charge Q each and masses m and 2m respectively are connected by a non conducting light rod of length L. This system is released in a uniform electirc field of strenght E as shown. Just after the release (assume no other force acts on the system): [Force on Q due to E is $F = QE$]



(A) Rod has zero angular acceleration

(B) Rod has angular acceleration $\frac{QE}{2mL}$ in anticlockwise direction

(C) Acceleration of point A is $\frac{2QE}{3m}$ towards right

(D) Acceleration of point A is $\frac{Qe}{m}$ towards right

8. A rod of length L is held vertically on a smooth horizontal surface. The top end of the rod is given a gentle push. At a certain instant of time, when the rod makes an angle θ with horizontal the velocity of COM of the rod is v_0. The velocity of the end of the rod in contact with the surface at that instant is:

(A) $v_0 \cot\theta$ (B) $v_0 \cos\theta$ (C) $v_0 \sin\theta$ (D) $v_0 \tan\theta$

9. A ball is given a velocity v and angular velocity ω such that the ball rolls purely on a plank whose upper surface is rough enough to prevent slipping but lower surface in contact with the ground is smooth. No other force is acting on system:

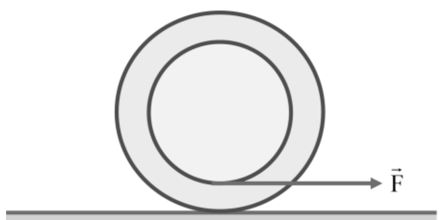
(A) The plank will recoil back

(B) The plank will also move forward but with a lesser velocity than that of the ball

(C) The plank will also move forward but with a greater velocity than that of the ball

(D) The plank will remain at rest

10. A yo-yo, arranged a shown, rests on a frictionless surface. When a force $\vec{F}$ is applied to the string as shown, the yo-yo:



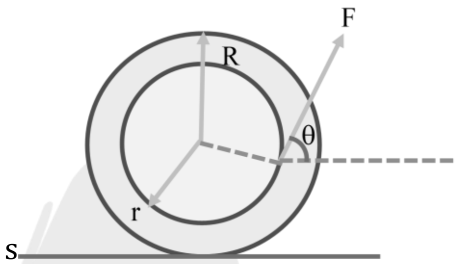
(A) moves to the left and rotates counterclockwise

(B) moves to the right and rotates counterclockwise

(C) moves to the left and rotates clockwise

(D) moves to the right and rotates clockwise

11. The spool shown in figure is placed on rough horizontal surface and has inner radius r and outer radius R. The angle θ between the applied force and the horizontal can be varied. The critical angle (θ) for which the spool does not roll and remains stationary is given by:



(A) $\theta=\cos^{-1}\left(\frac{r}{R}\right)$ (B) $\theta=\cos^{-1}\left(\frac{2r}{R}\right)$ (C) $\theta=\cos^{-1}\sqrt{\frac{r}{R}}$ (D) $\theta=\sin^{-1}\left(\frac{r}{R}\right)$

ANSWER KEY

EXCERCISE-I_KEY

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
B	D	D	C	A	A	C	50	B	B
11	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
A	5	D	B	D	D	B	B	B	C
21									
D									

EXCERCISE-II_KEY

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
101	10	D	C	C	50	B	D	72	10
11	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
A	B	D	A	C	1	B	2	C	B
21	**22**								
C	A								

EXCERCISE-III_KEY

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
C	D	D	C	C	10	7	C	B	100
11	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
C	C	B	100	A	A	6	C	D	A
21									
A									

EXCERCISE-IV_KEY

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
A	C	C	D	A	A	D	B	B	C
11	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
C	A	C	A	C	7	B	5	3	9
21	**22**	**23**							
24	A	A							

EXCERCISE-V_KEY

1	2	3	4	5	6	7	8	9	10
C	B	B	D	11	B	A	B	C	C
11	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
B	ABC	ABCD	ABC	AD	AC	AB	AC	ACD	CD
21	**22**	**23**	**24**						
BC	AB	ACD	AD						

PROFICIENCY TEST-I_KEY

1	2	3	4	5	6	7	8	9	10
B	2	D	D	D	B	108	C	25	5
11	**12**	**13**	**14**	**15**					
A	10	C	B	3					

PROFICIENCY TEST-II_KEY

1	2	3	4	5	6	7	8	9	10
C	B	A	C	A	10	10	A	3	A

PROFICIENCY TEST-III_KEY

1	2	3	4	5	6	7	8	9	10
B	B	2	B	1	C	D	D	D	B
11									
A									