DPP – 02

1. A $(1,-1,-3)$, B$(2,1,-2)$&C$(-5,2,-6)$ are the position vectors of the vertices of a triangle ABC. The length of the bisector of its internal angle at A is:
(A) $\sqrt{10}/4$ (B) $3\sqrt{10}/4$ (C) $\sqrt{10}$ (D) none

Ans. (B)

Sol. We have, $AB = \hat{i} + 2\hat{j} + \hat{k}$, $AC = -6\hat{i} + 3\hat{j} - 3\hat{k}$

$\Rightarrow |AB| = \sqrt{6}$ and $|AC| = 3\sqrt{6}$

Clearly, point D divides BC in the ratio AB:AC, i.e., 1:3

$$\therefore \text{Position vector of } D = \frac{\left(-5\hat{i} + 2\hat{j} - 6\hat{k}\right) + 3\left(2\hat{i} + \hat{j} - 2\hat{k}\right)}{1+3}$$

$$= \frac{1}{4}\left(\hat{i} + 5\hat{9} - 12\hat{k}\right)$$

$$\therefore AD = \frac{1}{4}\left(\hat{i} + 5\hat{j} - 12\hat{k}\right) - \left(\hat{i} - \hat{j} - 3\hat{k}\right) = \frac{3}{4}\left(-2\hat{i} + 3\hat{j}\right)$$

$$|AD| = AD = \frac{3}{4}\sqrt{10}$$

2. Let $\vec{p}$ is the p.v. of the orthocenter & $\vec{g}$ is the p.v. of the centroid of the triangle ABC where circumcentre is the origin. If $\vec{p} = K\vec{g}$, then $K =$
$(A)3$ (B) 2 (C) 1/3 (D) 2/3

Ans. (A)

Sol. Let $\bar{p}$ and $\bar{g}$ be the position vectors of P and G w.r.t. the circumcentre Q.

I.e. $\overline{QP} = p$ and $\overline{QG} = g$.

We know that Q, G, P are collinear and G divides segment QP internally in the ratio 1 : 2

$\therefore$ by section formula for internal division,

$$\bar{g} = \frac{1 \cdot \bar{p} + 2\bar{q}}{1+2} = \frac{\bar{p}}{3}$$

$\ldots \; [\because \bar{q} = 0]$

$\therefore \; \bar{p} = 3\bar{g}$

$\therefore \; \overline{QP} = 3\overline{QG}.$

3. A vector $\vec{a}$ has components 2p & 1 with respect to a rectangular cartesian system. The system is rotated through a certain angle about the origin in the counterclockwise sense. If with respect to the new system, $\vec{a}$ has components p + 1 & 1 then,

(A) p = 0
(B) p = 1 or p = −1/3
(C) $p = -1$ or $p = 1/3$
(D) p = 1 or p = −1

Ans. (B)

Sol. Equate the magnitude i.e. $4p^2 + 1 = (p + 1)^2 + 1 = p^2 + 2p + 2$

$\Rightarrow 3p^2 - 2p - 1 = 0 \Rightarrow p = 1 \text{ or } -1/3.$

4. The number of vectors of unit length perpendicular to vectors $\vec{a} = (1,1,0)$ & $\vec{b}(0,1,1)$ is:

(A) 1
(B) 2
(C) 3
(D) ∞

Ans. (B)

Sol. let the vector be $a\hat{i} + b\hat{j} + c\hat{k}$

$a(1) + b(1) + c(0) = 0$

$\Rightarrow a + b = 0 \ldots..(1)$

$a(0) + b(1) + c(1) = 0$

$\Rightarrow b + c = 0 \ldots..(2)$

$\sqrt{a^2 + b^2 + c^2} = 1$

$a^2 + (-a)^2 + (a)^2 = 1^2$

$3a^2 = 1$

$a = \pm\frac{1}{\sqrt{3}}$

there are two vectors

$\vec{v}_1 = \frac{\hat{i} - \hat{j} + \hat{k}}{\sqrt{3}}; \vec{v}_2 = \frac{-\hat{i} - \hat{j} - \hat{k}}{\sqrt{3}}$

5. Four points A(+1, −1,1); B(1,3,1); C(4,3,1) and D(4, −1,1) taken in order are the vertices of

(A) a parallelogram which is neither a rectangle nor a rhombus
(B) rhombus
(C) an isosceles trapezium
(D) a cyclic quadrilateral.

Ans. (D)

Sol. It is a rectangle

Let, the sides of ABCD be:

$AB = \vec{a}$; $BC = \vec{b}$; $CD = \vec{c}$; $DA = \vec{d}$

$\vec{a} = 4\hat{j}$

$\vec{b} = 3\hat{i}$

$\vec{c} = -4\hat{j}$

$\vec{d} = -3\hat{i}$

Clearly, all angles in this quadrilateral are 90 degree.
Hence, this is cyclic as sum of opposite angles is 180 degree.

6. Let α, β & γ be distinct real numbers. The points whose position vectors are $\alpha\hat{\imath} + \beta\hat{\jmath} + \gamma\hat{k}$; $\beta\hat{\imath} + \gamma\hat{\jmath} + \alpha\hat{k}$ and $\gamma\hat{\imath} + \alpha\hat{\jmath} + \beta\hat{k}$

(A) are collinear
(B) form an equilateral triangle
(C) form a scalene triangle
(D) form a right-angled triangle

Ans. (B)

Sol. Let $\mathbf{OP} = \alpha\mathbf{i} + \beta\mathbf{j} + \gamma\mathbf{k}$,

$\mathbf{OQ} = \beta\mathbf{i} + \gamma\mathbf{j} + \alpha\mathbf{k}$

and $\mathbf{OR} = \gamma\mathbf{i} + \alpha\mathbf{j} + \beta\mathbf{k}$

Now, $\mathbf{PQ} = \mathbf{OQ} - \mathbf{OP}$

$= (\beta - \alpha)\mathbf{i} + (\varphi - \beta)\mathbf{j} + (\alpha - \gamma)\mathbf{k}$

Also, $\mathbf{PR} = \mathbf{OR} - \mathbf{OP}$

$= (\gamma - \alpha)\mathbf{i} + (\alpha - \beta)\mathbf{j} + (\beta - \gamma)\mathbf{k}$

Again, $\mathbf{QR} = \mathbf{OR} - \mathbf{OQ}$

$= (\gamma - \beta)\mathbf{i} + (\alpha - \gamma)\mathbf{j} + (\beta - \alpha)\mathbf{k}$

Thus, $|\mathbf{PQ}| = |\mathbf{QR}| = |\mathbf{PR}|$

$= \sqrt{(\beta - \alpha)^2 + (\gamma - \beta)^2 + (\alpha - \gamma)^2}$

Therefore, ΔPQR is an equilateral triangle.

7. If the vectors $\vec{a} = 3\hat{\imath} + \hat{\jmath} - 2\hat{k}, \vec{b} = -\hat{\imath} + 3\hat{\jmath} + 4\hat{k} \& \vec{c} = 4\hat{\imath} - 2\hat{\jmath} - 6\hat{k}$ constitute the sides of a ΔABC, then the length of the median bisecting the vector $\vec{c}$ is

(A) $\sqrt{2}$ (B) $\sqrt{14}$ (C) $\sqrt{74}$ $(D)\sqrt{6}$

Ans. (D)

Sol. $\vec{m} = \vec{b} + \frac{\vec{c}}{2} = \hat{\imath} + 2\hat{\jmath} + \hat{k}$, hence $|\vec{m}| = 16$

8. Let $A(0,-1,1), B(0,0,1), C(1,0,1)$ are the vertices of a $\triangle$ ABC. If R and r denotes the circumradius and inradius of $\triangle$ ABC, then $\frac{r}{R}$ has value equal to

(A) $\tan\frac{3\pi}{8}$ (B*) $\cot\frac{3\pi}{8}$ (C) $\tan\frac{\pi}{12}$ (D) $\cot\frac{\pi}{12}$

Ans. (B)

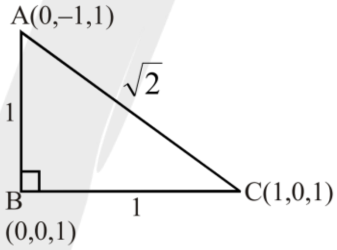
Sol. $A = C = 45^\circ$

$B = 90^\circ$

$\frac{r}{R} = 4\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$

$= 4 \cdot \frac{1}{\sqrt{2}} \cdot \sin^2 22.5 = \frac{(2\sqrt{2})(1-\cos 45^\circ)}{2} = \sqrt{2} - 1 = \cot\frac{3\pi}{8}$ Ans.

Alternatively:



Clearly $\triangle$ ABC is isosceles right angled at B.

By sine rule in $\triangle$ ABC, we get $\frac{\sqrt{2}}{\sin 90^\circ} = 2R \Rightarrow R = \frac{1}{\sqrt{2}}$

Now area ($\triangle$ ABC) $= \frac{1}{2}(1)(1) = \frac{1}{2}$ sq. unit

So, $r = \frac{\Delta}{s} = \frac{\frac{1}{2}}{\frac{2+\sqrt{2}}{2}} = \frac{1}{2+\sqrt{2}} \Rightarrow \frac{r}{R} = \frac{\frac{1}{2+\sqrt{2}}}{\frac{1}{\sqrt{2}}} = \frac{\sqrt{2}}{2+\sqrt{2}} = \frac{1}{\sqrt{2}+1} = \sqrt{2} - 1 = \cot\frac{3\pi}{8}\Big]$

9. If $\vec{a} = x\hat{\imath} - 2\hat{\jmath} + 5\hat{k}$ and $\vec{b} = \hat{\imath} + y\hat{\jmath} - z\hat{k}$ are linearly dependent, then the value of $\frac{xy^2}{z}$ equals

(A) $\frac{4}{5}$ (B) $\frac{-3}{5}$ (C) $\frac{3}{5}$ (D) $\frac{-4}{5}$

Ans. (D)

Sol. As $\vec{a}$ and $\vec{b}$ are collinear, so $\frac{x}{1} = \frac{-2}{y} = \frac{5}{-z} \Rightarrow xy = -2$ and $xz = -5$

Now, $\frac{xy^2}{z} = \frac{(xy)^2}{xz} = \frac{(-2)^2}{-5} = \frac{-4}{5}$. Ans.

10. A vector of magnitude 10 along the normal to the curve $3x^2 + 8xy + 2y^2 - 3 = 0$ at its point $P(1,0)$ can be

(A*) $6\hat{i} + 8\hat{j}$ (B) $-6\hat{i} + 8\hat{j}$ (C) $6\hat{i} - 8\hat{j}$ (D*) $-6\hat{i} - 8\hat{j}$

Ans. (A)

Sol. differentiate the curve

$6x + 8(xy_1 + y) + 4yy_1 = 0$

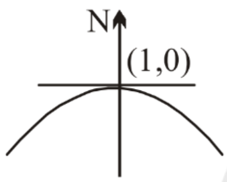
m_T at (1,0) is $6 + 8(y_1(1)) = 0$

$y_1(1) = -\frac{3}{4}$

$m_N = \frac{4}{3} \Rightarrow$ vector along normal $3\hat{i} + 4\hat{j}$

unit vector $= \pm\frac{(3\hat{i}+4\hat{j})}{5}$ along the normal

again normal vector of magnitude $10 = \pm(6\hat{i} + 8\hat{j})$ Ans.]

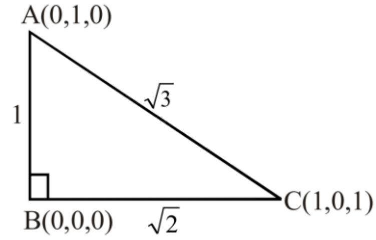


11. If (0,1,0), $B(0,0,0)$, $C(1,0,1)$ are the vertices of a △ ABC. Match the entries of column-I with column-II.

Column-I	**Column-II**
(A) Orthocenter of △ ABC.	(P) $\frac{\sqrt{2}}{2}$
(B) Circumcenter of △ ABC.	(Q) $\frac{\sqrt{3}}{2}$
(C) Area (△ ABC).	(R) $\frac{\sqrt{3}}{3}$
(D) Distance between orthocenter and centroid.	(S) $\frac{\sqrt{3}}{6}$
(E) Distance between orthocenter and circumcenter.	(T) (0,0,0)
(F) Distance between circumcenter and centroid.	(U) $\left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right)$
(G) Incentre of △ ABC.	(V) $\left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$
(H) Centroid of △ ABC	(W) $\left(\frac{1}{\sqrt{1}+\sqrt{2}+\sqrt{3}}, \frac{\sqrt{2}}{\sqrt{1}+\sqrt{2}+\sqrt{3}}, \frac{1}{\sqrt{1}+\sqrt{2}+\sqrt{3}}\right)$

Ans. (A) T; (B) U ; (C) P ; (D) R ; (E) Q; (F) S; (G) W; (H) V

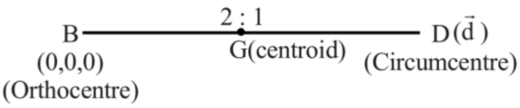
Sol.



Clearly $\triangle$ ABC is right angled at B.

(A) Orthocenter = (0,0,0)

(B)



We have $\frac{2\vec{d}}{3} = \frac{1}{3}\hat{i} + \frac{1}{3}\hat{j} + \frac{1}{3}\hat{k}$

$\Rightarrow$ Circumcentre $= \left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right)$

(C) Area ($\triangle$ ABC) $= \frac{1}{2}(AB)(BC) = \frac{1}{2}(1)(\sqrt{2}) = \frac{\sqrt{2}}{2}$ sq. unit

(D) Distance between orthocenter and centroid

$$= \sqrt{\left(\frac{1}{3} - 0\right)^2 + \left(\frac{1}{3} - 0\right)^2 + \left(\frac{1}{3} - 0\right)^2} = \sqrt{\frac{1}{9} + \frac{1}{9} + \frac{1}{9}} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3} \text{ unit.}$$

(E) Distance between orthocenter and circumcenter

$$= \sqrt{\left(\frac{1}{2} - 0\right)^2 + \left(\frac{1}{2} - 0\right)^2 + \left(\frac{1}{2} - 0\right)^2} = \sqrt{\frac{1}{4} + \frac{1}{4} + \frac{1}{4}} = \frac{\sqrt{3}}{2} \text{ unit.}$$

(F) Distance between centroid and circumcenter

$$= \sqrt{\left(\frac{1}{2} - \frac{1}{3}\right)^2 + \left(\frac{1}{2} - \frac{1}{3}\right)^2 + \left(\frac{1}{2} - \frac{1}{3}\right)^2} = \sqrt{\frac{1}{36} + \frac{1}{36} + \frac{1}{36}} = \frac{1}{\sqrt{12}} = \frac{1}{2\sqrt{3}} = \frac{\sqrt{3}}{6} \text{ unit.}$$

(G) We know that position vector of incentre of $\triangle ABC = \frac{a\vec{a} + b\vec{b} + c\vec{c}}{a+b+c}$

$$\therefore \text{ Position vector of incentre } = \frac{\sqrt{2}(\hat{j}) + \sqrt{3}(\vec{0}) + 1(\hat{i} + \hat{k})}{\sqrt{1} + \sqrt{2} + \sqrt{3}} = \frac{\hat{i} + \sqrt{2}\hat{j} + \hat{k}}{\sqrt{1} + \sqrt{2} + \sqrt{3}}$$

$$\therefore \text{ Incentre } = \left(\frac{1}{\sqrt{1}+\sqrt{2}+\sqrt{3}}, \frac{\sqrt{2}}{\sqrt{1}+\sqrt{2}+\sqrt{3}}, \frac{1}{\sqrt{1}+\sqrt{2}+\sqrt{3}}\right)$$

(H) centriod $= \left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$]