Nishant Jindal

Q Let d be $\perp^r$ distance from centre to E: $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ to tangent at a pt. P on ellipse. If F_1 & F_2 are Focii of Ellipse then S.T. $(PF_1-PF_2)^2=\boxed{4a^2\left(1-\frac{d^2}{a^2}\right)} \rightarrow 4a^2\left(1-\frac{b^2}{d^2}\right)$

RHS: $4a^2\times\frac{x_1^2}{a^2}\times e^2=4e^2x_1^2=$ LHS.

$(a-ex_1-a-ex_1)^2=4e^2x_1^2$ ✓

EOT: $\frac{xx_1}{a^2}+\frac{yy_1}{b^2}-1=0$

$d=\frac{|-1|}{\sqrt{\frac{x_1^2}{a^4}+\frac{y_1^2}{b^4}}} \Rightarrow \frac{1}{d^2}=\frac{x_1^2}{a^4}+\frac{y_1^2}{b^4}$

$\frac{x_1^2}{a^2}+\frac{y_1^2}{b^2}=1$

$\frac{x_1^2}{a^2}=1-\frac{y_1^2}{b^2}$

$1-\frac{b^2}{d^2}=1-\frac{x_1^2\cdot b^2}{a^4}-\frac{y_1^2}{b^2}$

$=\frac{x_1^2}{a^2}-\frac{x_1^2b^2}{a^4}=\frac{x_1^2}{a^2}\left(1-\frac{b^2}{a^2}\right)=\frac{x_1^2}{a^2}\times e^2$

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Q. Suppose that the foci of the ellipse $\frac{x^2}{9}+\frac{y^2}{5}=1$ are $(f_1,0)$ and $(f_2,0)$ where $f_1>0$ and $f_2<0$. Let P_1 and P_2 be two parabolas with a common vertex at $(0,0)$ and with foci at $(f_1,0)$ and $(2f_2,0)$, respectively. Let T_1 be a tangent to P_1 which passes through $(2f_2,0)$ and T_2 be a tangent to P_2 which passes through $(f_1,0)$. If m_1 is the slope of T_1 and m_2 is the slope of T_2, then the value of $\left(\frac{1}{m_1^2}+m_2^2\right)$ is

$2+2=4$

$E_1: \frac{x^2}{9}+\frac{y^2}{5}=1$

1) $a=3$

$b=\sqrt{5}$

$e^2=1-\frac{b^2}{a^2}=1-\frac{5}{9}=\frac{4}{9}$

2) $e=\frac{2}{3}$

(3) Focii $=(\pm ae,0)$

$=(\pm 2,0)$

$F_1=(2,0),\ F_2=(-2,0)$

(4) Parabola as Focus $=(a,0)$

$P_1\equiv(2,0),\quad P_2:\to(-4,0)$

$a=2 \qquad a=-4$

$y^2=8x \qquad y^2=-16x$

(5) $T_1\to P_1 \Rightarrow y^2=8x$

$y=m_1x+\frac{2}{m_1}$ P.T. $(-4,0)$

$0=-4m_1+\frac{2}{m_1} \Rightarrow \frac{2}{m_1}=4m_1 \Rightarrow m_1^2=\frac{1}{2}$

$T_2\to P_2: y^2=-16x$

$y=m_2x-\frac{4}{m_2}$ P.T. $(2,0)$

$0=2m_2-\frac{4}{m_2} \quad \frac{4}{m_2}=2m_2$

$m_2^2=2$

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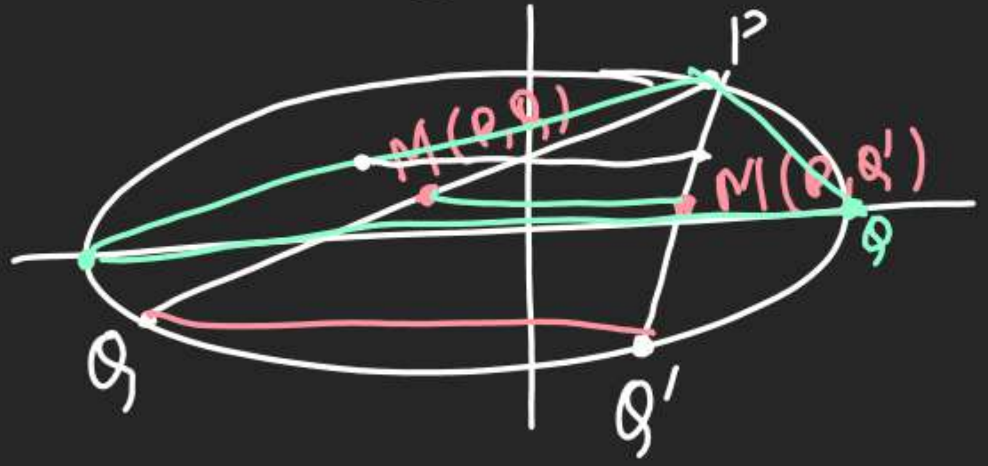
Q. Let E be the ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$. For any three distinct points P, Q and Q' on E, let $M(P, Q)$ be the mid-point of the line segment joining P and Q, and $M(P, Q')$ be the midpoint of the line segment joining P and Q'. Then the maximum possible value of the distance between $M(P, Q)$ and $M(P, Q')$, as P, Q and Q' vary on E, is

1) $E : \frac{x^2}{16}+\frac{y^2}{9}=1$



$a =$

(2) Qs : is PQQ' a Δ or not ?

(3) Qs $M-M = \frac{1}{2} QQ'$ (10^{th} Class)
(MidPt. Thm)

(4) Qs: Q, Q' variable or not ?

$(M-M)_{Max}$ when QQ' Max

& QQ' can be max^m on Ellipse when Q & Q' are on Vertex $\Rightarrow QQ' = 2a$

(5) $MM' = \frac{QQ'}{2} = \frac{2a}{2}$

$MM' = 4$
Max.

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Q. Consider two straight lines, each of which is tangent to both the circles $x^2 + y^2 = \frac{1}{2}$ and the parabola $y^2 = 4x$. Let these lines intersect at the point Q. consider the ellipse whose center is at the origin $O(0,0)$ and whose semi - major axis is OQ. If the length of the minor axis of this ellipse is $\sqrt{2}$, Then which of the following statment(s) is (are) TRUE?

(A) For the ellipse, the eccentricity is $\frac{1}{\sqrt{2}}$ and the length of the latus rectum is 1

(B) For the ellipse, the eccentricity is $\frac{1}{2}$ and the length of the latus rectum is $\frac{1}{2}$

(C) The area of the region bouded by the ellipse between the lines $x = \frac{1}{\sqrt{2}}$ and $x = 1$ is $\frac{1}{4\sqrt{2}}(\pi - 2)$

(D) The area of the region bounded by the ellipse between the lines $x = \frac{1}{\sqrt{2}}$ and $x = 1$ is $\frac{1}{16}(\pi - 2)$

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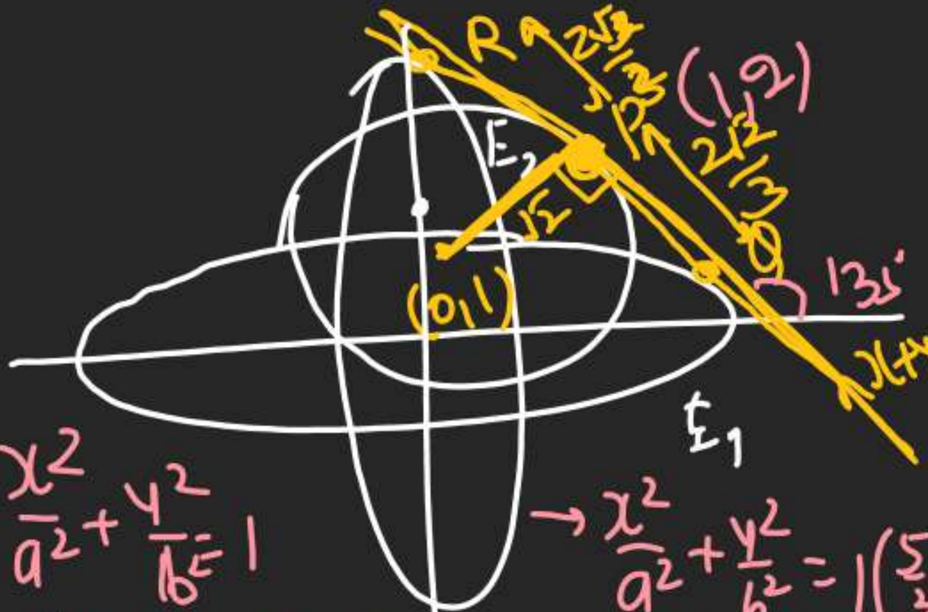
Q let E_1, E_2 be 2 Ellipse whose centres are at origin. Major Axis of E_1, E_2 lie along x Axis & y Axis. Let S be the circle $x^2+(y-1)^2=2$. St. line $x+y=3$ touches S, E_1, E_2 at P, Q, R. Supp. that $PQ=PR=\frac{2\sqrt{2}}{3}$. If e_1, e_2 are ecc. of E_1 & E_2 then

(A) $e_1^2+e_2^2=\frac{43}{40}$ (B) $e_1e_2=\frac{\sqrt{7}}{2\sqrt{10}}$

(C) $|e_1^2-e_2^2|=\frac{5}{8}$ (D) $e_1e_2=\frac{\sqrt{3}}{4}$.

$\frac{7}{8}+\frac{1}{5}=\frac{35+8}{40}=\frac{43}{40}$

$\left|\frac{7}{8}-\frac{1}{5}\right|=\left|\frac{35-7}{40}\right|=\left|\frac{28}{40}\right|$ $e_1\times e_2=\frac{\sqrt{7}}{2\sqrt{2}}\times\frac{1}{\sqrt{5}}$



$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ $\left(\frac{1}{3},\frac{8}{3}\right)$ EOT. $\frac{x}{3a^2}+\frac{8y}{3b^2}=1$

$\rightarrow \frac{x^2}{a^2}+\frac{y^2}{b^2}=1\left(\frac{5}{3},\frac{4}{3}\right)$ T=0 EOT. $\frac{5x}{3a^2}+\frac{4y}{3b^2}=1$

$\frac{x}{3}+\frac{y}{3}=1$

(2) $S: x^2+(y-1)^2=2$ $\frac{x}{3}+\frac{y}{3}=1$

$C:(0,1), r=\sqrt{2}$

(3) $x+y=3 \Rightarrow \frac{x}{3}+\frac{y}{3}=1$

$y=-x+3$

$m=-1 \Rightarrow \theta=135°$

$a^2=1$, $b^2=8$ $\Rightarrow 1-e^2=\frac{1}{8}$ $\Rightarrow e_1^2=\frac{7}{8}$

$a^2=5$, $b^2=4$ $\Rightarrow 1-e^2=\frac{4}{5}$ $e_2^2=\frac{1}{5}$

(4) $x+y=3$ is tangent to $x^2+(y-1)^2=2$ at P

$\Rightarrow$ P is Foot of $\perp^r$ from (0,1)

$x-y=K$ (0,1)

$0-1=K \Rightarrow K=-1$

$\Rightarrow L: \left.\begin{array}{l}x-y=-1\\x+y=3\end{array}\right\}$ P = POI

$x=1, y=2 \Rightarrow P\equiv(1,2)$

$R\equiv\left(1+\frac{2\sqrt{2}}{3}\cos 135°, 2+\frac{2\sqrt{2}}{3}\sin 135°\right)$

$=\left(\frac{1}{3},\frac{8}{3}\right)\rightarrow E_1$

$Q\equiv\left(1-\frac{2\sqrt{2}}{3}\cos 135°, 2-\frac{2\sqrt{2}}{3}\sin 135°\right)$

$=\left(\frac{5}{3},\frac{4}{3}\right)\rightarrow E_2$

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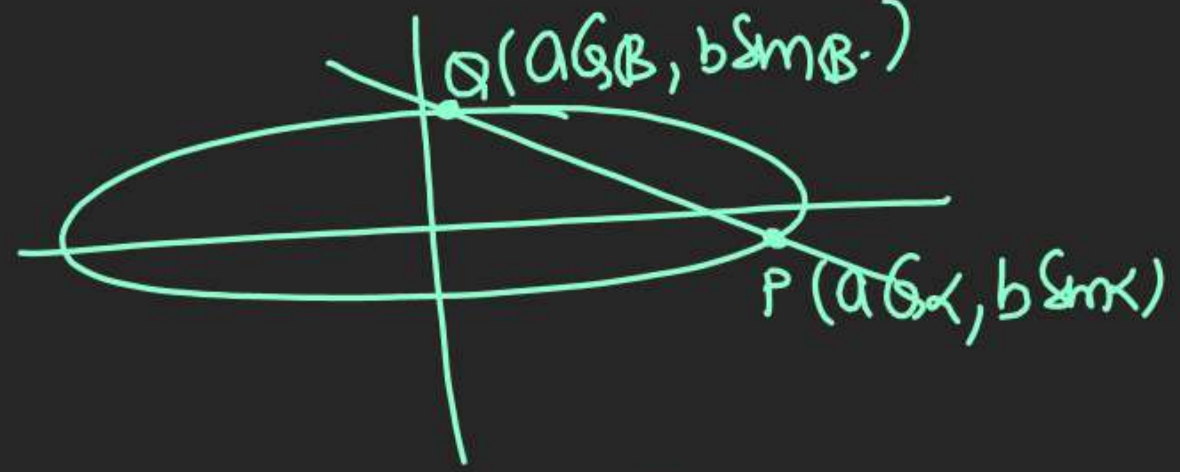
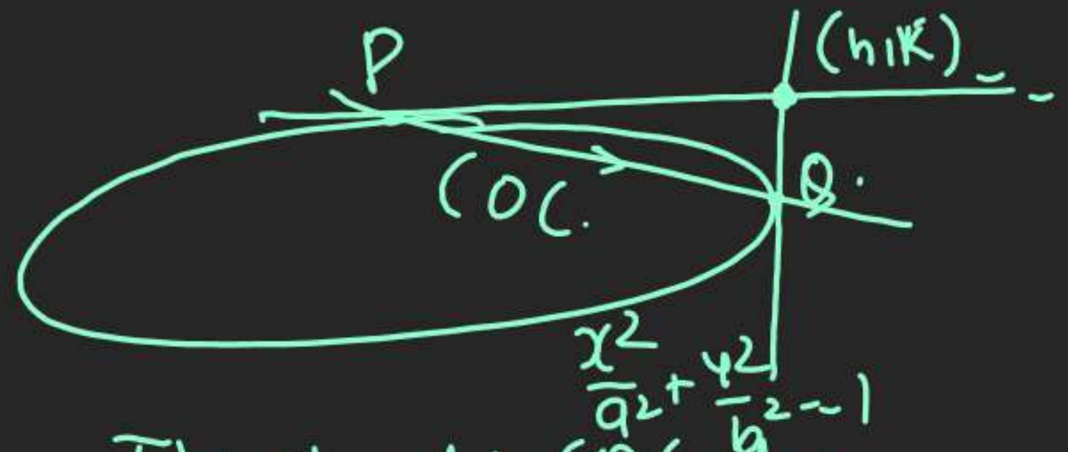
Chord of Ellipse

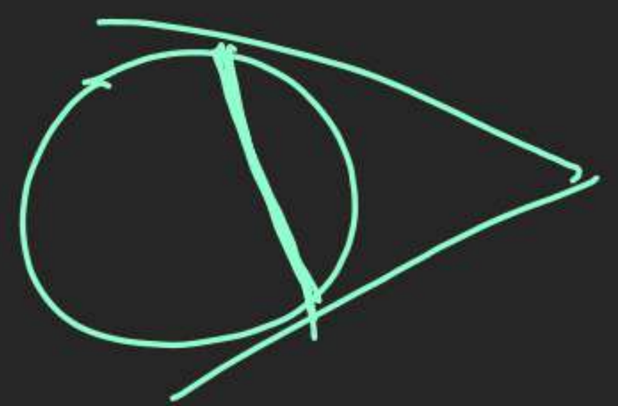


$$PQ: (y - b\sin\alpha) = \frac{b\sin\alpha - b\sin\beta}{a\cos\alpha - a\cos\beta}(x - a\cos\alpha)$$

$$\Rightarrow \boxed{\frac{x}{a}\cos\left(\frac{\alpha+\beta}{2}\right) + \frac{y}{b}\sin\left(\frac{\alpha+\beta}{2}\right) = \cos\left(\frac{\alpha-\beta}{2}\right)}$$

Q If 2 tangents at the ends of chord are intersecting each other at (h,k). find (h,k)?





This chord is COC also

& COC = $\boxed{T=0}$

$$(PQ): COC: \frac{hx}{a^2} + \frac{ky}{b^2} = 1$$

$$PQ: \frac{x}{a}\cdot\cos\left(\frac{\alpha+\beta}{2}\right) + \frac{y}{b}\cdot\sin\left(\frac{\alpha+\beta}{2}\right) = \cos\left(\frac{\alpha-\beta}{2}\right)$$

$$\frac{\frac{h}{a^2}}{\frac{\cos\left(\frac{\alpha+\beta}{2}\right)}{a}} = \frac{\frac{k}{b^2}}{\frac{\sin\left(\frac{\alpha+\beta}{2}\right)}{b}} = \frac{1}{\cos\left(\frac{\alpha-\beta}{2}\right)}$$

$$(h,k) \equiv \left\{\frac{a\cos\left(\frac{\alpha+\beta}{2}\right)}{\cos\left(\frac{\alpha-\beta}{2}\right)}, \frac{b\sin\left(\frac{\alpha+\beta}{2}\right)}{\cos\left(\frac{\alpha-\beta}{2}\right)}\right\}$$

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Q If a chord made at $P(\alpha), Q(\beta)$

P.T. a fix Pt $(d,0)$ find $\tan\frac{\alpha}{2}\cdot\tan\frac{\beta}{2} = ?$

Chord $\Rightarrow \frac{x}{a}\cos\left(\frac{\alpha+\beta}{2}\right) + \frac{y}{b}\sin\left(\frac{\alpha+\beta}{2}\right) = \cos\left(\frac{\alpha-\beta}{2}\right)$

P.T. $(d,0)$

$$\frac{d}{a}\cos\left(\frac{\alpha+\beta}{2}\right) + 0 = \cos\left(\frac{\alpha-\beta}{2}\right)$$

$$\frac{d}{a} = \frac{\cos\left(\frac{\alpha-\beta}{2}\right)}{\cos\left(\frac{\alpha+\beta}{2}\right)}$$

S + S
S − S
C + C
C − C

C & D:

$$\frac{d+a}{d-a} = \frac{\cos\left(\frac{\alpha}{2}-\frac{\beta}{2}\right) + \cos\left(\frac{\alpha}{2}+\frac{\beta}{2}\right)}{\cos\left(\frac{\alpha}{2}-\frac{\beta}{2}\right) - \cos\left(\frac{\alpha}{2}+\frac{\beta}{2}\right)}$$

$$\frac{d+a}{d-a} = \frac{\cancel{2}\cos\left(\frac{\alpha}{2}\right)\cos\left(+\frac{\beta}{2}\right)}{+\cancel{2}\sin\left(\frac{\alpha}{2}\right)\sin\left(+\frac{\beta}{2}\right)}$$

$$\Rightarrow \boxed{\frac{d-a}{d+a}} = \tan\frac{\alpha}{2}\cdot\tan\frac{\beta}{2}$$

Ans

Q. Find $\tan\frac{\alpha}{2}\cdot\tan\frac{\beta}{2}$ for Focal chord of ellipse?

Focus $= (ae, 0) \equiv (d, 0)$

$$\therefore \tan\frac{\alpha}{2}\cdot\tan\frac{\beta}{2} = \frac{ae-a}{ae+a} = \frac{e-1}{e+1}$$

Q. find $\tan\frac{\alpha}{2}\cdot\tan\frac{\beta}{2}$

for E: $\frac{x^2}{16} + \frac{y^2}{4} = 1$

$a^2 = 16$, $b^2 = 4$

$1 - e^2 = \frac{4}{16} \left(\frac{1}{4}\right)$, $e = \frac{\sqrt{3}}{2}$

$$\tan\frac{\alpha}{2}\cdot\tan\frac{\beta}{2} = \frac{\frac{\sqrt{3}}{2}-1}{\frac{\sqrt{3}}{2}+1}$$

$$\frac{\sqrt{3}-2}{\sqrt{3}+2}$$

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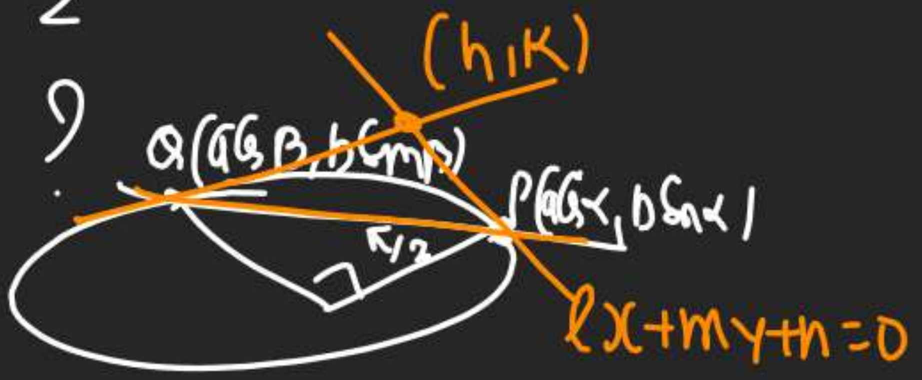
Q Line $lx+my+n=0$ cuts ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ at those pts whose difference of ecc. angle $=\frac{\pi}{2}$ then $l^2a^2+m^2b^2=?$



(1) diff. of ecc. angle $= |\alpha-\beta| = \frac{\pi}{2}$

(2) $PQ: \frac{x}{a}\cos\left(\frac{\alpha+\beta}{2}\right)+\frac{y}{b}\sin\left(\frac{\alpha+\beta}{2}\right)=\cos\left(\frac{\pi}{4}\right)$

$lx + my = -n$

$$\frac{l}{\frac{\cos\left(\frac{\alpha+\beta}{2}\right)}{a}}=\frac{m}{\frac{\sin\left(\frac{\alpha+\beta}{2}\right)}{b}}=\frac{-n}{\frac{1}{\sqrt{2}}}$$

$$\frac{al}{\cos\left(\frac{\alpha+\beta}{2}\right)}=-\sqrt{2}n \quad \Bigg\| \quad \frac{bm}{\sin\left(\frac{\alpha+\beta}{2}\right)}=-\sqrt{2}n$$

$$\cos\left(\frac{\alpha+\beta}{2}\right)=\frac{al}{-\sqrt{2}n} \quad \Bigg| \quad \sin\left(\frac{\alpha+\beta}{2}\right)=\frac{bm}{-\sqrt{2}n}$$

$\sin^2(\ )+\cos^2(\ )=1$

$$\frac{a^2l^2}{2n^2}+\frac{b^2m^2}{2n^2}=1$$

$$\Rightarrow a^2l^2+b^2m^2=2n^2$$