

1. Let $\int_0^1 \frac{dx}{\sqrt{16+9x^2}} + \int_0^2 \frac{dx}{\sqrt{9+4x^2}} = \ln a$. Find a.
2. $\int_0^{\ln 2} x e^{-x} dx$
3. $\int_1^e \left(\frac{1}{\sqrt{x \ln x}} + \sqrt{\frac{\ln x}{x}} \right) dx$
4. Given $f'(x) = \frac{\cos x}{x}$, $f\left(\frac{\pi}{2}\right) = a$, $f\left(\frac{3\pi}{2}\right) = b$. Find the value of the definite integral $\int_{\pi/2}^{3\pi/2} f(x) dx$.
5. $\int_{-1}^1 \frac{x dx}{\sqrt{5-4x}}$
6. $\int_2^e \left(\frac{1}{\ln x} - \frac{1}{\ln^2 x} \right) dx$
7. $\int_0^{\pi/4} \frac{\sin 2x}{\sin^4 x + \cos^4 x} dx$
8. $\int_0^{\pi/2} \frac{\cos x dx}{(1+\sin x)(2+\sin x)}$
9. $\int_0^{\pi/4} \frac{\sin^2 x \cdot \cos^2 x}{(\sin^3 x + \cos^3 x)^2} dx$
10. $\int_{1/3}^3 \frac{\sin^{-1} \frac{x}{\sqrt{1+x^2}}}{x} dx$
11. $\int_2^3 \frac{dx}{\sqrt{(x-1)(5-x)}}$
12. $\int_{3/2}^2 \left(\frac{x-1}{3-x} \right)^{1/2} dx$
13. $\int_0^{\pi/4} x \cos x \cos 3x dx$
14. $\int_0^{\pi/2} \frac{dx}{5+4 \sin x}$
15. $\int_2^3 \frac{dx}{(x-1)\sqrt{x^2-2x}}$
16. $\int_0^{\pi/2} \frac{dx}{1+\cos \theta \cos x}$ $\theta \in (0, \pi)$
17. $\int_0^{\ln 3} \frac{e^x + 1}{e^{2x} + 1} dx$
18. $\int_0^{\pi/4} \cos 2x \sqrt{1 - \sin 2x} dx$
19. $\int_0^3 \sqrt{\frac{x}{3-x}} dx$
20. $\int_0^{1/2} \frac{dx}{(1-2x^2)\sqrt{1-x^2}}$
21. $\int_1^2 \frac{dx}{x(x^4+1)}$
22. $\int_0^{\pi/2} \sin \phi \cos \phi \sqrt{(a^2 \sin^2 \phi + b^2 \cos^2 \phi)} d\phi \neq b$ ($a > 0, b > 0$)

(MATHEMATICS)

DEFINITE INTEGRATION

23. (a) $\int_0^{3\pi/4} ((1+x)\sin x + (1-x)\cos x)dx$ (b) $\int_{\pi/2}^{\pi} x^{\sin x} (1 + x \cos x \cdot \ln x + \sin x)dx$
24. $\int_0^1 x(\tan^{-1}x)^2 dx$
25. Suppose that f, f' and f'' are continuous on $[0, \ln 2]$ and that $f(0) = 0, f'(0) = 3, f(\ln 2) = 6, f'(\ln 2) = 4$ and $\int_0^{\ln 2} e^{-2x} \cdot f(x)dx = 3$. Find the value of $\int_0^{\ln 2} e^{-2x} \cdot f''(x)dx$.
26. $\int_0^1 \frac{dx}{x^2 + 2x \cos \alpha + 1}$ where $-\pi < \alpha < \pi$
27. $\int_a^b \frac{dx}{\sqrt{1+x^2}}$ where $a = \frac{e-e^{-1}}{2}$ & $b = \frac{e^2-e^{-2}}{2}$
28. $\int_{0^+}^1 \frac{x^x(x^{2x}+1)(\ln x+1)}{x^{4x}+1} dx$
29. $\int_0^1 x^5 \sqrt{\frac{1+x^2}{1-x^2}} dx$
30. Suppose that the function f, g, f' and g' are continuous over $[0, 1], g(x) \neq 0$ for $x \in [0, 1], f(0) = 0, g(0) = \pi, f(1) = \frac{2009}{2}$ and $g(1) = 1$. Find the value of the definite integral $\int_0^1 \frac{f(x) \cdot g'(x) \{g^2(x) - 1\} + f'(x) \cdot g(x) \{g^2(x) + 1\}}{g^2(x)} dx$
31. $\int_0^{\pi/4} \frac{\sin \theta + \cos \theta}{9 + 16 \sin 2\theta} d\theta$
32. $\int_0^{\pi} \theta \sin^2 \theta \cos \theta d\theta$
33. $\int_0^{\pi/2} \frac{1+2\cos x}{(2+\cos x)^2} dx$
34. $\int_0^{\pi/2} \frac{x+\sin x}{1+\cos x} dx$
35. Let $A = \int_{3/4}^{4/3} \frac{2x^2+x+1}{x^3+x^2+x+1} dx$ then find the value of e^A .
36. $\int_0^1 \frac{2-x^2}{(x+1)\sqrt{1-x^2}} dx$
37. $\int_{-1}^1 \left(\frac{d}{dx} \left(\frac{1}{1+e^{1/x}} \right) \right) dx$
38. $\int_1^e \frac{dx}{\ln(x^x e^x)}$
39. $\int_0^{\pi} \left[\cos^2 \left(\frac{3\pi}{8} - \frac{x}{4} \right) - \cos^2 \left(\frac{11\pi}{8} + \frac{x}{4} \right) \right] dx$
40. If $f(\pi) = 2$ & $\int_0^{\pi} (f(x) + f''(x)) \sin x dx = 5$, then find $f(0)$
41. $\int_a^b \frac{|x|}{x} dx$
42. $\int_{\ln 2}^{\ln 3} f(x)dx$, where $f(x) = e^{-x} + 2e^{-2x} + 3e^{-3x} + \dots, \infty$
43. $\int_0^{\pi/2} \frac{\sqrt{\sec x - \tan x} \operatorname{cosec} x}{\sqrt{\sec x + \tan x} \sqrt{1 + 2 \operatorname{cosec} x}} dx$
44. $\int_0^1 x f''(x) dx$, where $f(x) = \cos(\tan^{-1} x)$

(MATHEMATICS)

DEFINITE INTEGRATION

45. (a) If $g(x)$ is the inverse of $f(x)$ and $f(x)$ has domain $x \in [1, 5]$, where $f(1) = 2$ and $f(5) = 10$ then find the value of $\int_1^5 f(x)dx + \int_2^{10} g(y)dy$.
- (b) Suppose f is continuous, $f(0) = 0$, $f(1) = 1$, $f'(x) > 0$ and $\int_0^1 f(x)dx = \frac{1}{3}$. Find the value of the definite integral $\int_0^1 f^{-1}(y)dy$.



ANSWER KEY

1. $2^{1/3} \cdot 3^{1/2}$
2. $\frac{1}{2} \ln\left(\frac{e}{2}\right)$
3. $2\sqrt{e}$
4. $2 - \frac{\pi}{2}(a - 3b)$
5. $\frac{1}{6}$
6. $e - \frac{2}{\ln 2}$
7. $\frac{\pi}{4}$
8. $\ln \frac{4}{3}$
9. $\frac{1}{6}$
10. $\frac{\pi \ln 3}{2}$
11. $\frac{\pi}{6}$
12. $\frac{\sqrt{3}}{2} - 1 + \frac{\pi}{6}$
13. $\frac{\pi - 3}{16}$
14. $\frac{2}{3} \tan^{-1} \frac{1}{3}$
15. $\frac{\pi}{3}$
16. $\frac{\theta}{\sin \theta}$
17. $\frac{1}{2} \left(\frac{\pi}{6} + \ln 3 - \ln 2 \right)$
18. $\frac{1}{3}$
19. $\frac{3\pi}{2}$
20. $\frac{1}{2} \ln(2 + \sqrt{3})$
21. $\frac{1}{4} \ln \frac{32}{17}$
22. $\frac{1}{3} \frac{a^3 - b^3}{a^2 - b^2}$
23. (a) $2(\sqrt{2} + 1)$; (b) $\left(\pi - \frac{\pi^2}{4} \right)$
24. $\frac{\pi}{4} \left(\frac{\pi}{4} - 1 \right) + \frac{1}{2} \ln 2$
25. 13
26. $\frac{\alpha}{2 \sin \alpha}$ if $\alpha \neq 0$; $\frac{1}{2}$ if $\alpha = 0$
27. 1
28. 0
29. $\frac{3\pi + 8}{24}$
30. 2009
31. $\frac{1}{20} \ln 3$
32. $-\frac{4}{9}$
33. $\frac{1}{2}$
34. $\frac{\pi}{2}$
35. $\frac{16}{9}$
36. $\frac{\pi}{2}$
37. $\frac{2}{1+e}$
38. $\ln 2$
39. $\sqrt{2}$
40. 3
41. $|b| - |a|$
42. $\frac{1}{2}$
43. $\pi/3$
44. $1 - \frac{3}{2\sqrt{2}}$
45. (a) 48, (b) $2/3$