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5.

$$f(x+T) = \frac{f(x)-5}{f(x)-3}$$

$$f(x+2T) = \frac{\frac{f(x)-5}{f(x)-3} - 5}{\frac{f(x)-5}{f(x)-3} - 3} = \frac{2f(x)-5}{f(x)-2}$$

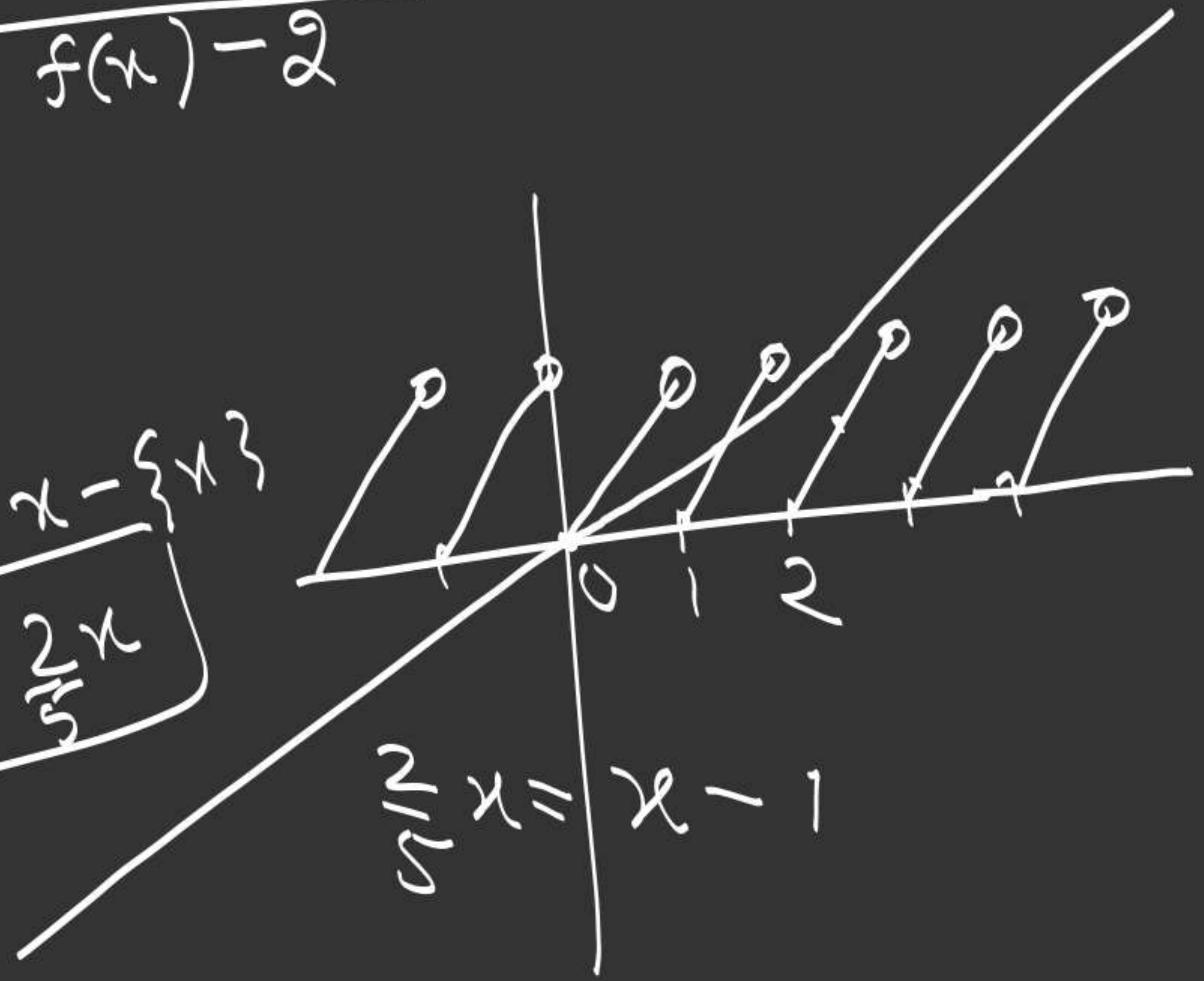
$\vdots$

$$f(x+4T) = f(x)$$

7.

$$4\{x\} = x + x - \{x\}$$

$$\boxed{\{x\} = \frac{2}{5}x}$$



$$\frac{2}{5}x = x - 1$$

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8. $S = f\left(\frac{1}{2006}\right) + f\left(\frac{2}{2006}\right) + \cdots + f\left(1-\frac{2}{2006}\right) + f\left(1-\frac{1}{2006}\right)$

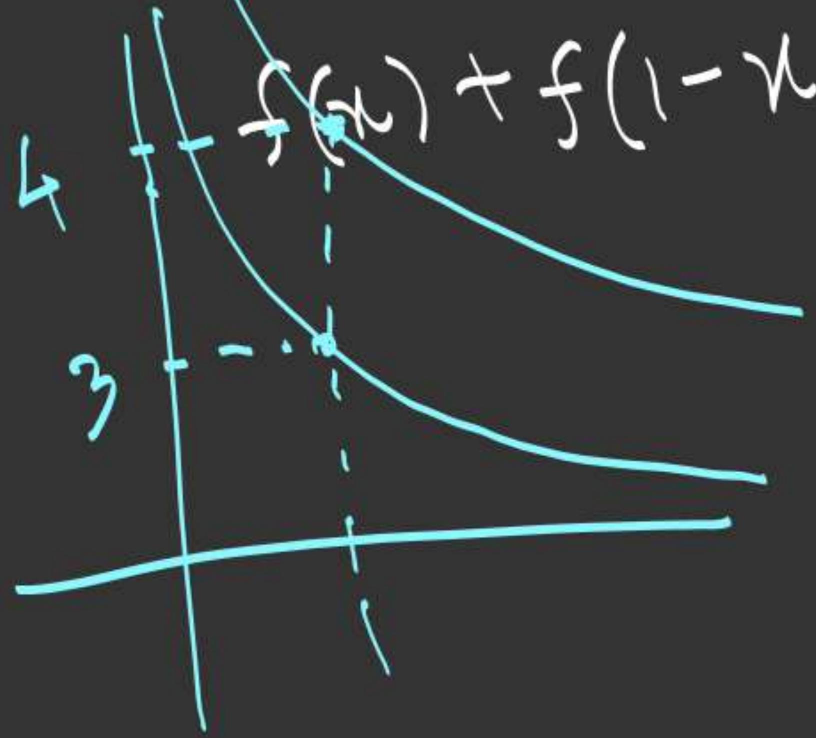
$S = f\left(1-\frac{1}{2006}\right) + f\left(1-\frac{2}{2006}\right) + \cdots + f\left(\frac{1}{2006}\right)$

$$\boxed{2S = 2005}$$

$$f(x) + f(1-x) = \frac{9^x}{9^x+3} + \frac{9^{1-x}}{9^{1-x}+3} = 1$$

Ans $\boxed{x \in \left(1, \frac{4}{3}\right]}$

9.



0 5 ✗
1 4 ✗
√2 3

$\frac{3}{x} \in [2, 3) \Rightarrow x \in \left(1, \frac{3}{2}\right]$

$\frac{4}{x} \in [3, 4) \Rightarrow x \in \left(1, \frac{4}{3}\right]$

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11.

$u=0,\ v=x$

$f(x)+f(-x)=2f(0)\cos x=2a\cos x$

(ii) $u=\frac{\pi}{2}-x,\ v=\frac{\pi}{2}$

$f(\pi-x)+f(-x)=2f\left(\frac{\pi}{2}-x\right)\cos\frac{\pi}{2}=0$

$\boxed{f(x)=(0,4),(17,5)}$

$10\to$ leave

10.

$f(0)=4,\ g(5)=17$

$f(2006)=?$

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$\underline{12.}$ $n \in I$ $\cos(nx + \underline{3\pi n})\sin\left(\frac{5}{n}x + \frac{15\pi}{n}\right) = \cos nx \sin\frac{5}{n}x$

$\underline{\text{If } n \text{ is even}}$

$$\sin\left(\frac{5}{n}x + \frac{15\pi}{n}\right) = \sin\frac{5}{n}x$$

$$\frac{15\pi}{n} = 2k\pi \Rightarrow \boxed{\frac{15}{n} = 2k} \quad \phi$$

$\underline{\text{If } n \text{ is odd}}$

$$\sin\left(\frac{5}{n}x + \frac{15\pi}{n}\right) = -\sin\frac{5}{n}x$$

$$\frac{15\pi}{n} = (2k+1)\pi \Rightarrow \boxed{\frac{15}{n} = 2k+1}$$

$$n = \pm 1, \pm 3, \pm 5, \pm 15$$

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15.

$f(x) \geq 0$

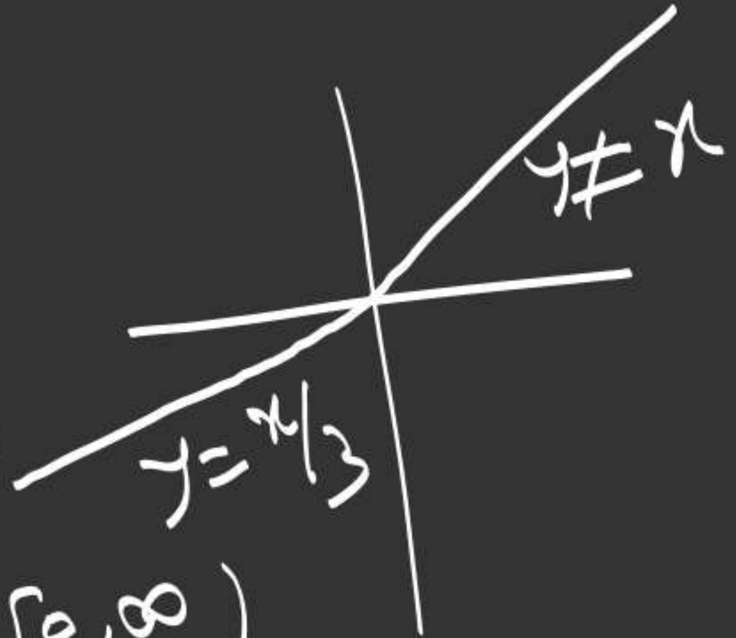
$f(x) = x$

$\Rightarrow x \geq 0$

$f(x) < 0$

$f(x) = \frac{x}{3}$

$\Rightarrow x < 0$



$$f(x) = \begin{cases} x & x \geq 0 \\ \frac{x}{3} & x < 0 \end{cases}$$

$$f^{-1}(x) = \begin{cases} x & x \in [0, \infty) \\ 3x & x \in (-\infty, 0) \end{cases}$$

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14.

$$f(x) = -\frac{x|x|}{1+x^2} = \begin{cases} \dfrac{-x^2}{1+x^2} & x \geq 0 \\[2ex] \dfrac{x^2}{1+x^2} & x < 0 \checkmark \end{cases}$$

$$\frac{-\left(f^{-1}(x)\right)^2}{1+\left(f^{-1}(x)\right)^2} = x$$

$$(1+x)\left(f^{-1}(x)\right)^2 = -x$$

$$x + x\left(f^{-1}(x)\right)^2 = \left(f^{-1}(x)\right)^2$$

$$f^{-1}(x) = \begin{cases} \sqrt{\dfrac{-x}{1+x}} & x<0 \\[2ex] -\sqrt{\dfrac{x}{1-x}} & x>0 \end{cases}$$

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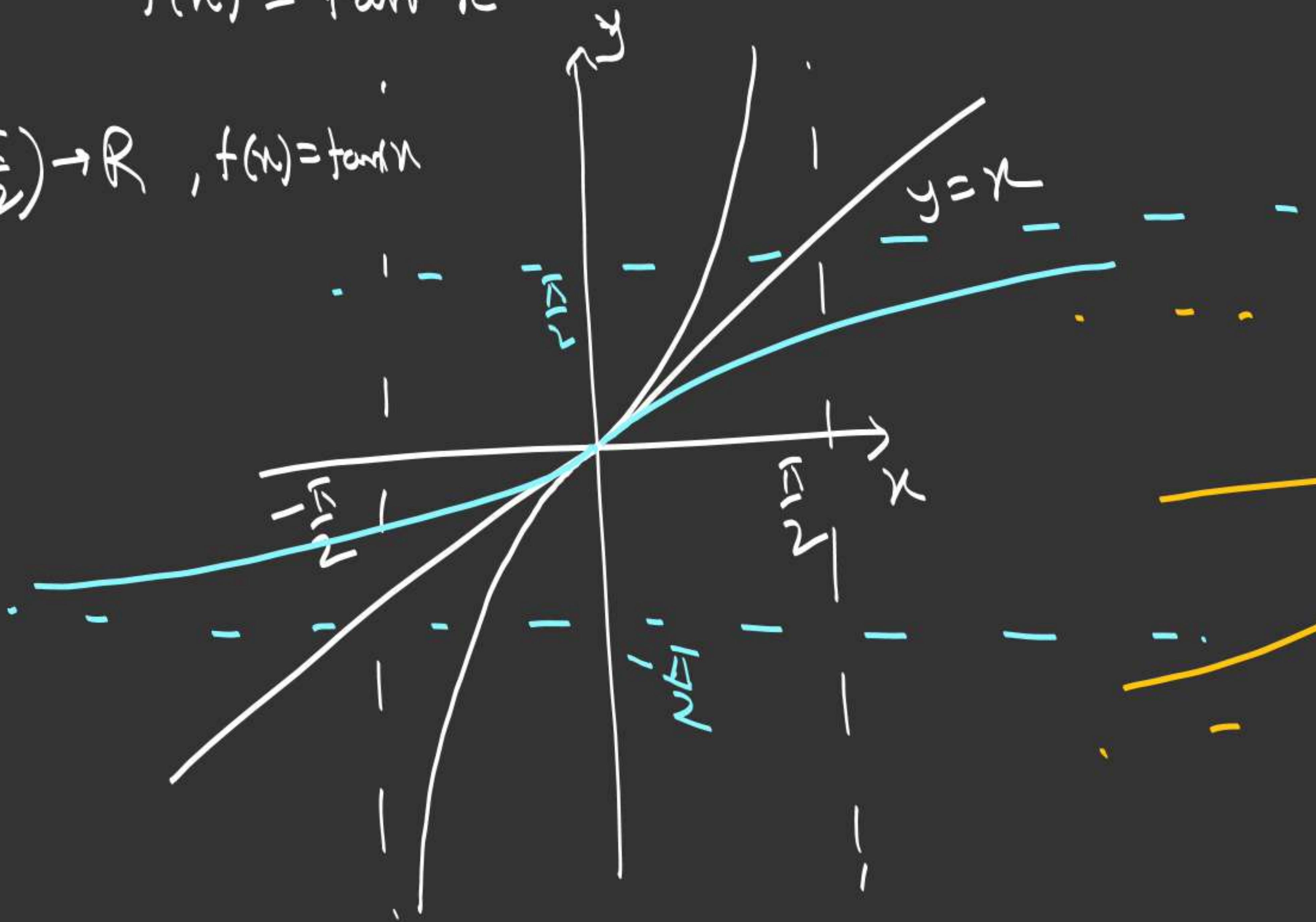
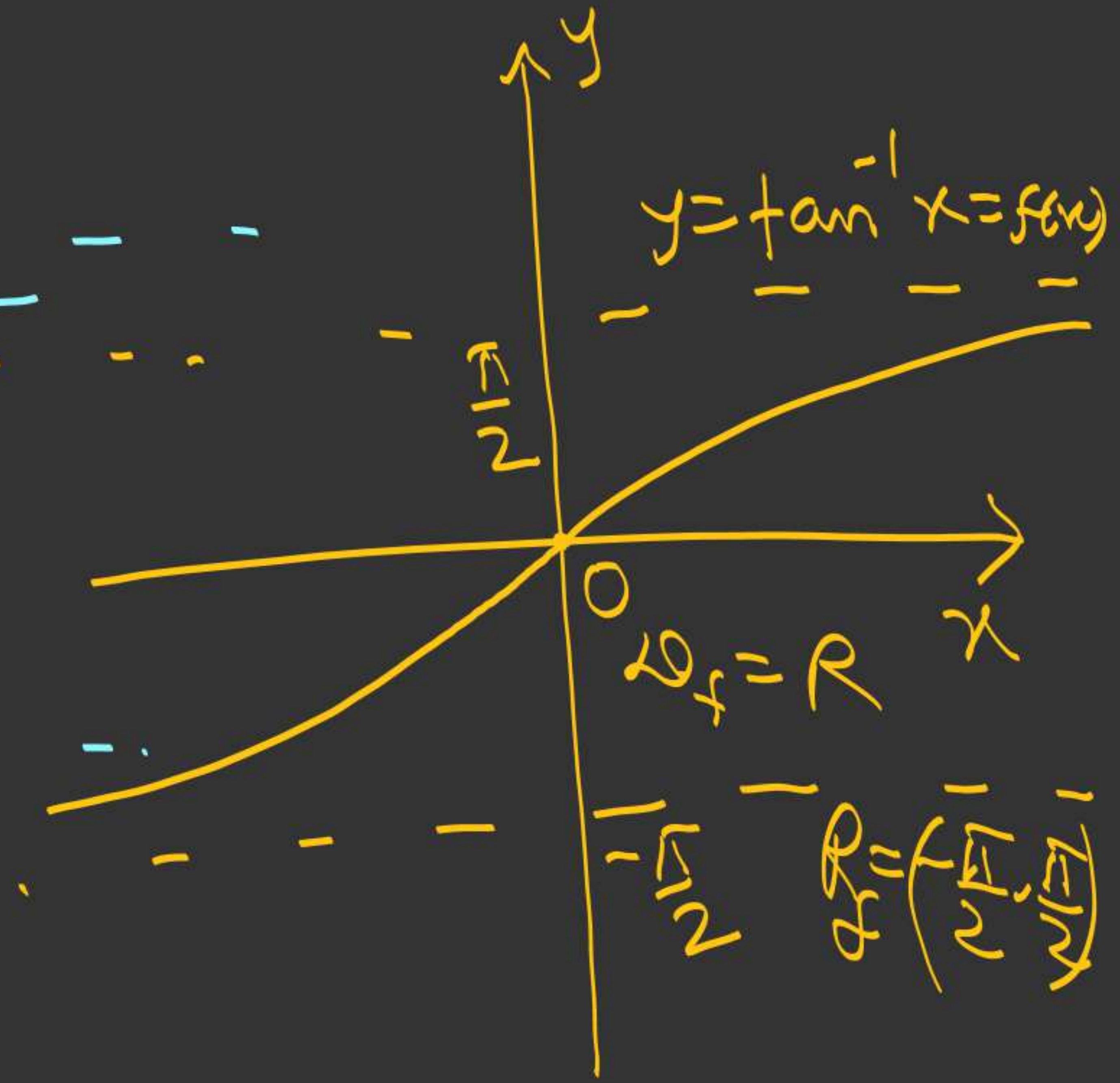
$f(x) = \tan^{-1}x$

$f : \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \to R$, $f(x) = \tan x$





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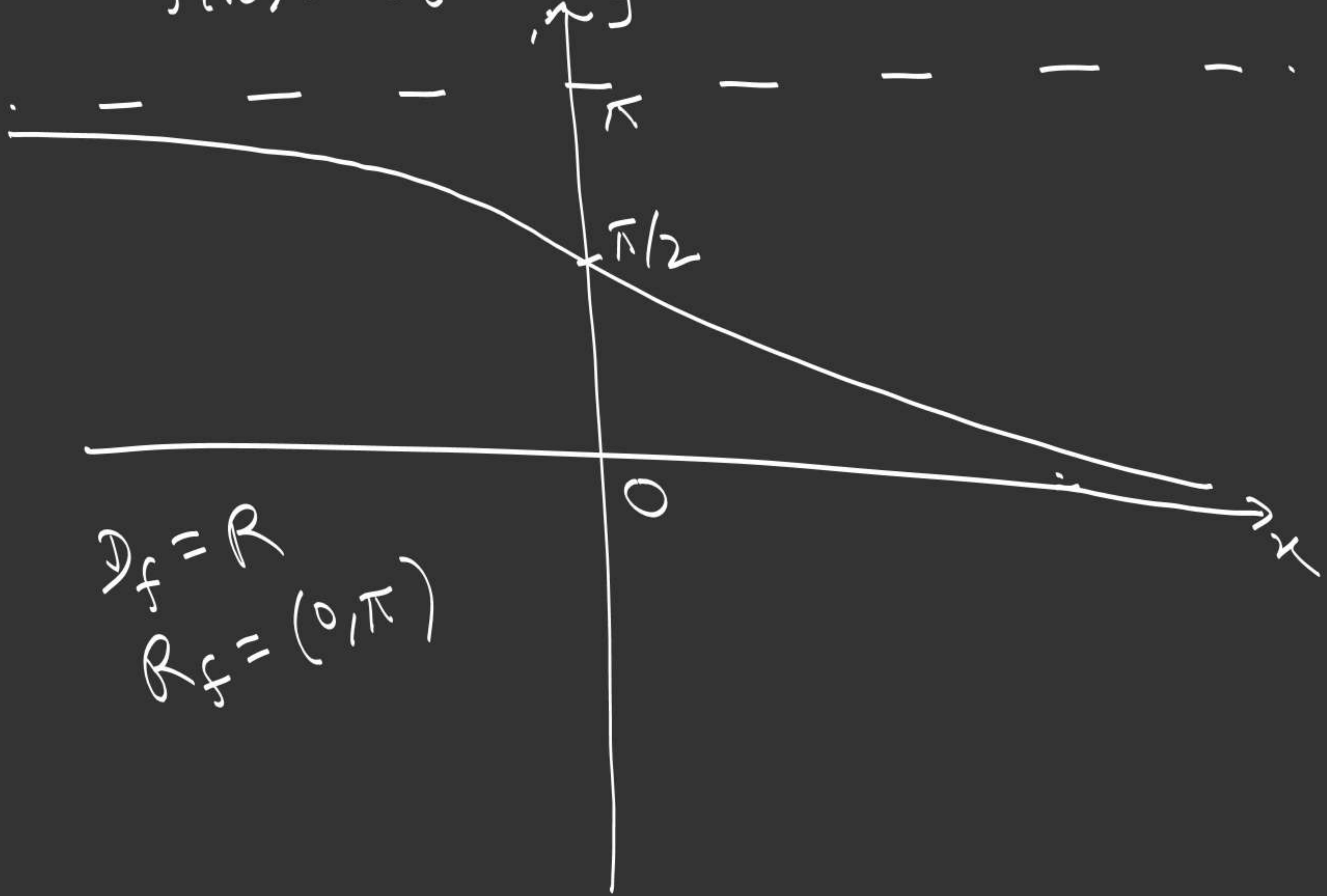
$$f(x) = \cot^{-1}x$$



$D_f = R$

$R_f = (0, \pi)$

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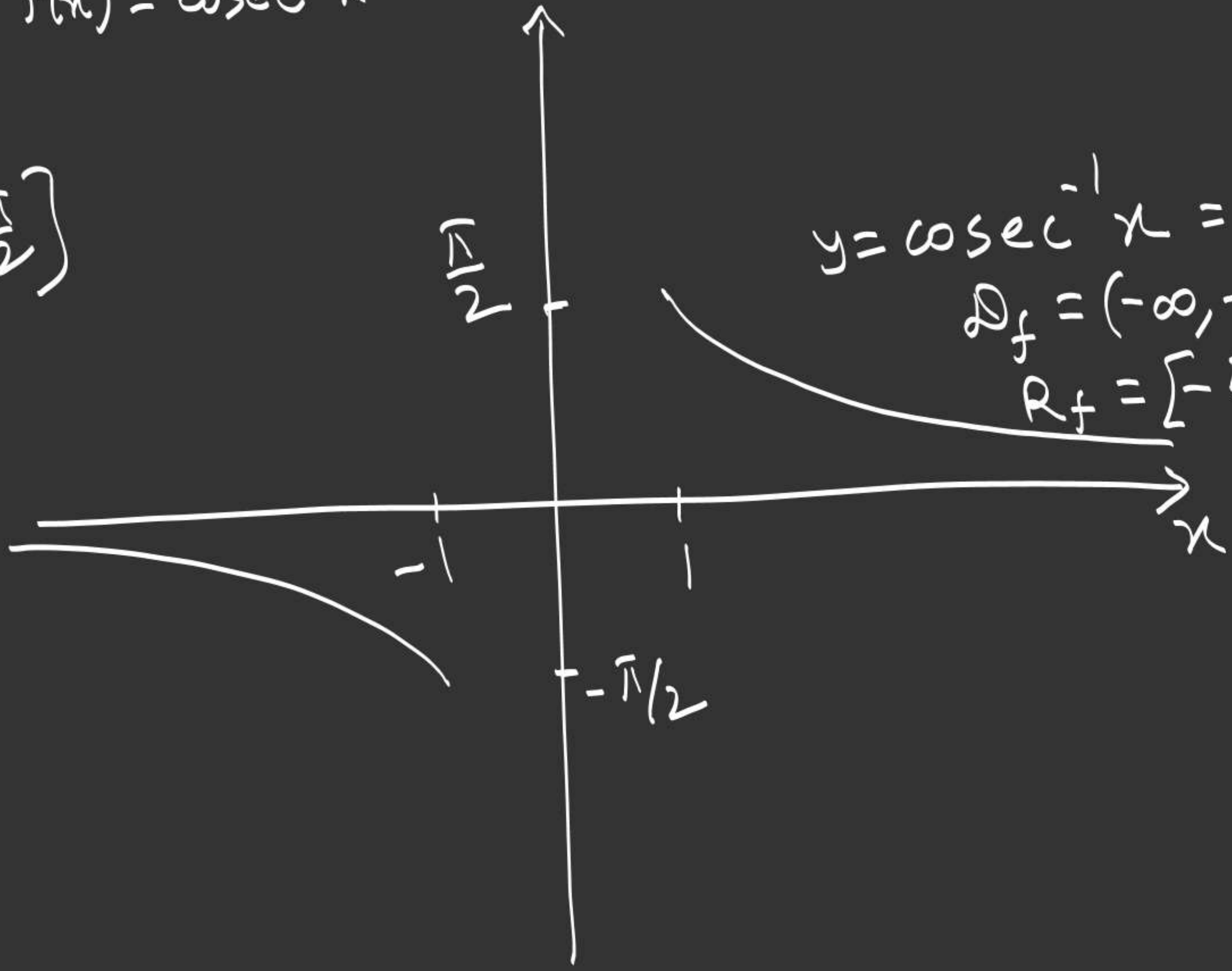
$$f(x) = \operatorname{cosec}^{-1} x$$

$$\left[-\frac{\pi}{2}, 0\right) \cup \left(0, \frac{\pi}{2}\right]$$

$$\operatorname{cosec}^{-1} x = \theta$$

$$\boxed{\operatorname{cosec}\theta = x}$$



$$y = \operatorname{cosec}^{-1} x = f(x)$$

$$D_f = (-\infty, -1] \cup [1, \infty)$$

$$R_f = \left[-\frac{\pi}{2}, 0\right) \cup \left(0, \frac{\pi}{2}\right]$$

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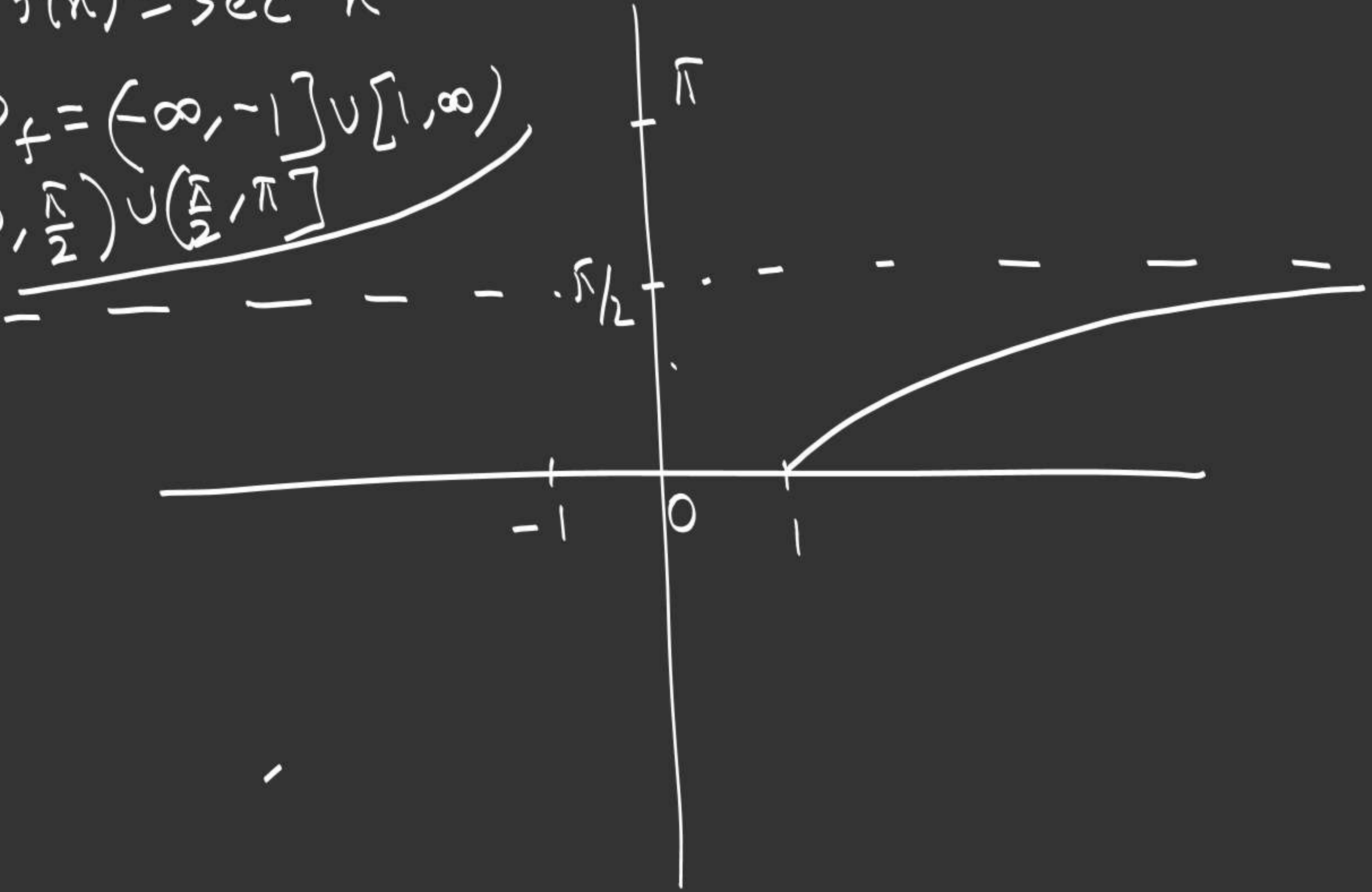
$f(x) = \sec^{-1} x$

$D_f = (-\infty, -1] \cup [1, \infty)$

$R_f = [0, \frac{\pi}{2}) \cup (\frac{\pi}{2}, \pi]$



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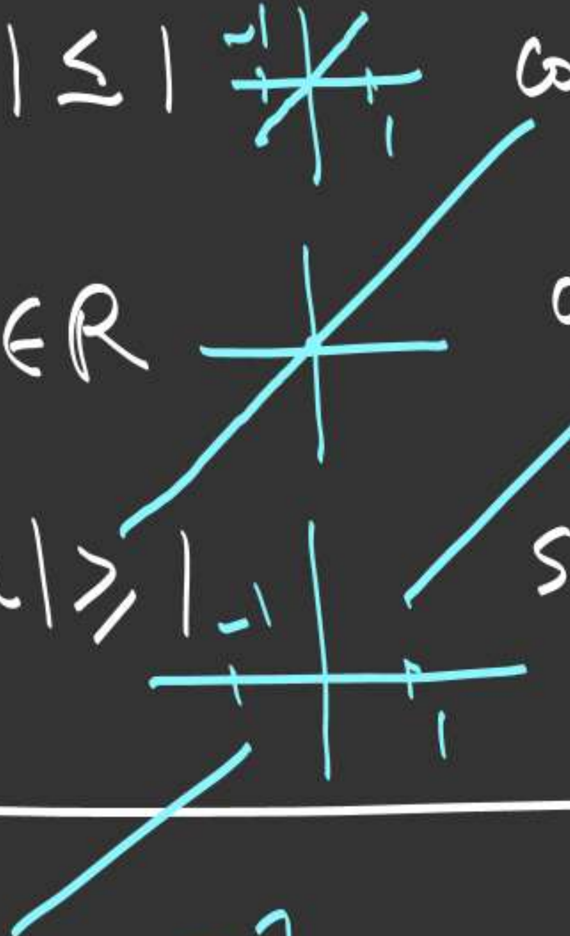
$\sin(\sin^{-1}x) = x \qquad |x| \le 1 \qquad \cos(\cos^{-1}x) = x$

$\tan(\tan^{-1}x) = x \qquad x \in R \qquad \cot(\cot^{-1}x) = x$

$\operatorname{cosec}(\operatorname{cosec}^{-1}x) = x \qquad |x| \geq 1 \qquad \sec(\sec^{-1}x) = x$



$f : \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \to [-1, 1], \quad f(x) = \sin x$

$f^{-1} : [-1, 1] \to \left[-\frac{\pi}{2}, \frac{\pi}{2}\right],$

$f^{-1}(x) = \sin^{-1}x$

$f(f^{-1}(x)) : [-1, 1] \to [-1, 1]$

$f(f^{-1}(x)) = x$

$\sin(\sin^{-1}x) = x$

$\sin^{-1}x = \theta, \quad \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$\sin\theta = x$

$\sin(\sin^{-1}x) = x$

$f(x) = \cot(\cot^{-1}x)$

$g(x) = \tan(\tan^{-1}x)$

Identical

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$$f(x) = \sin^{-1}(\sin x)$$

$$D_f = R$$

$$T = 2\pi$$

$$\sin^{-1}(\sin x) = \begin{cases} x & x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \\ \pi - x & x \in \left[\frac{\pi}{2}, \frac{3\pi}{2}\right] \end{cases}$$

$$\theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

$$\sin^{-1}(\sin x) = \theta$$

$$\sin\theta = \sin x$$

$$\theta = n\pi + (-1)^n x \quad , n \in I$$

$$\boxed{\theta = 2k\pi + x \quad \text{or} \quad (2k+1)\pi - x \quad , k \in I}$$

I) $x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right], \theta = x$

II) $x \in \left[\frac{\pi}{2}, \frac{3\pi}{2}\right], \theta = \pi - x$

$$\pi - x \in \left[\pi - \frac{3\pi}{2}, \pi - \frac{\pi}{2}\right]$$

$f : \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \to [-1, 1]$, $f(x) = \sin x$

$f^{-1} : [-1, 1] \to \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, $f^{-1}(x) = \sin^{-1} x$
$= g(x)$

$g\circ f : \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \to \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, $g\circ f(x) = x$

$$\sin^{-1}(\sin x) = x \quad , x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

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$$y = \sin^{-1}(\sin x) = f(x)$$

$\boxed{\begin{array}{l} Ex-II\ (16-19) \\ Ex-III\ (\text{Complete}) \end{array}}$

$D_f = R$

$R_f = \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, $T = 2\pi$