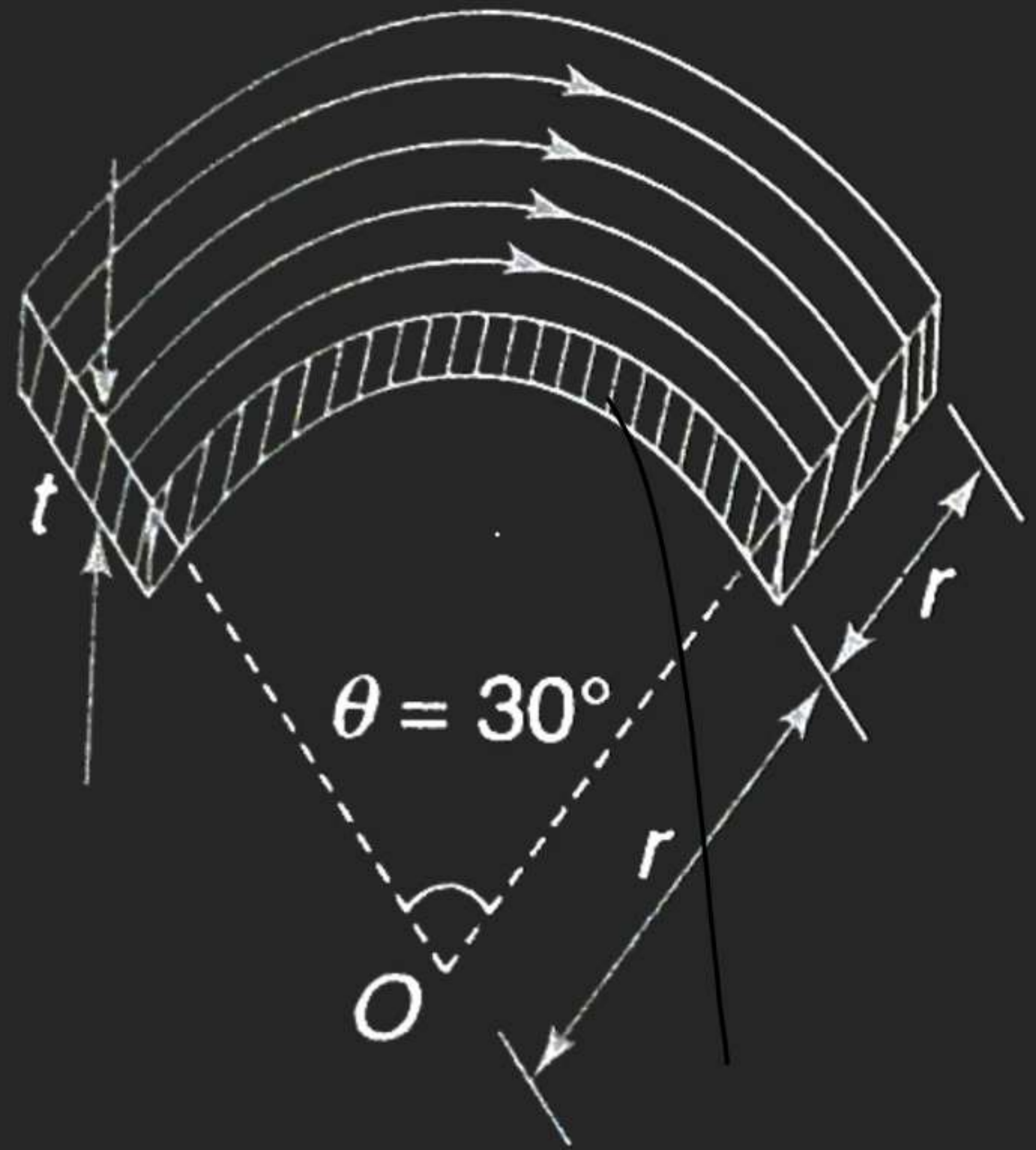
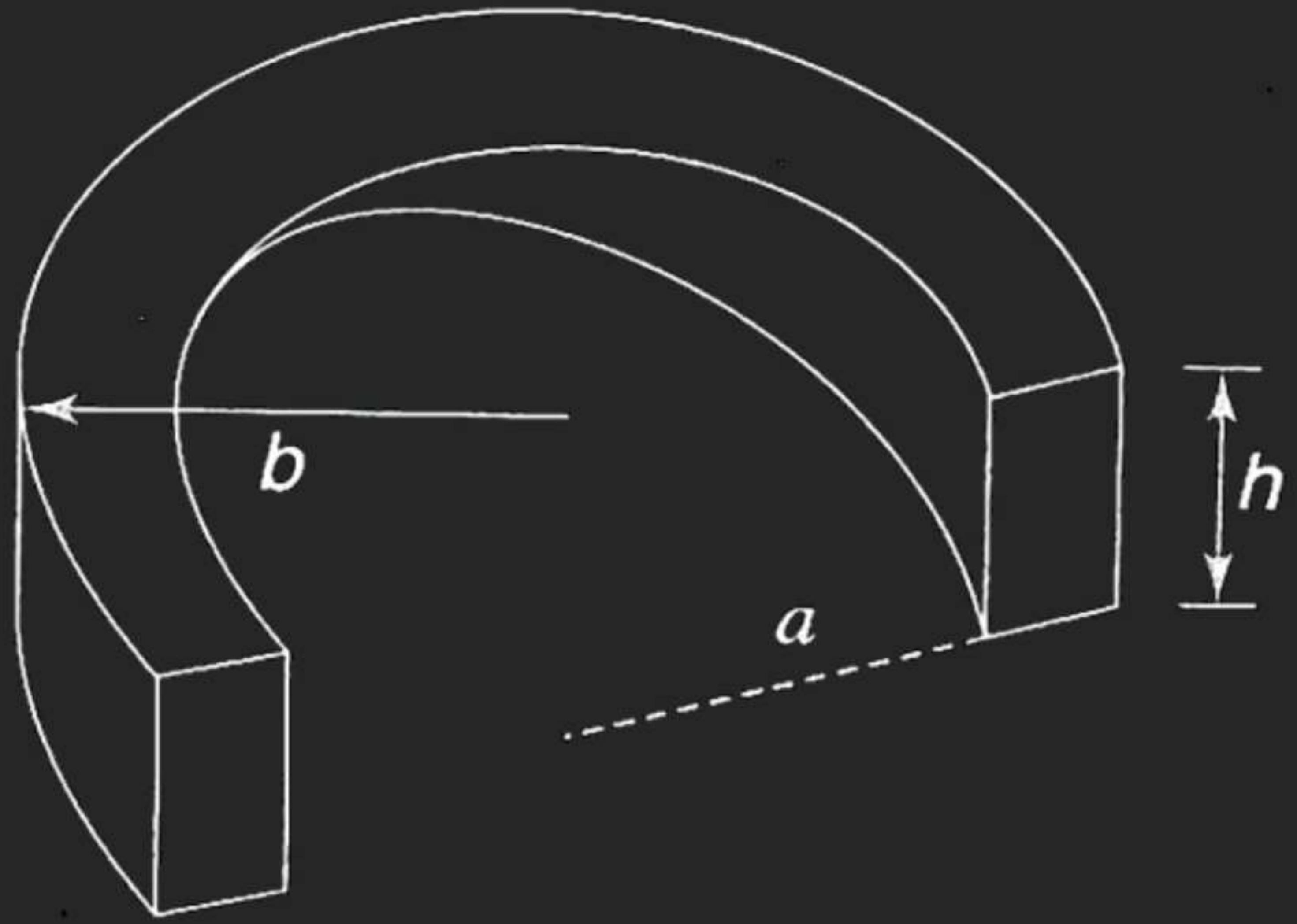
Nishant Jindal

CURRENT ELECTRICITY

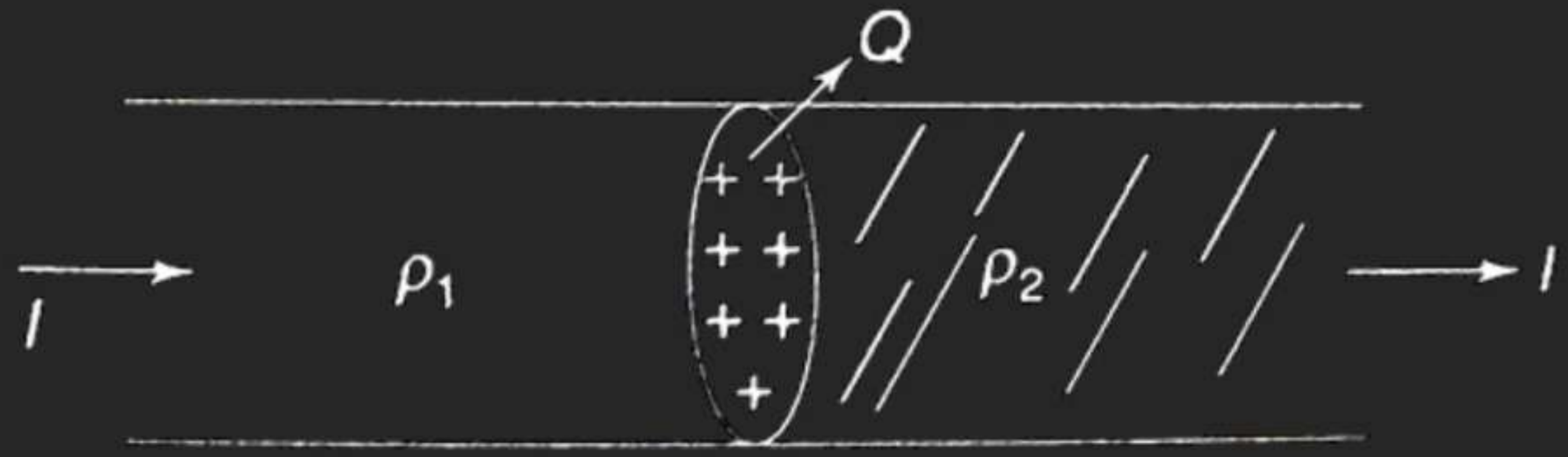


CURRENT ELECTRICITY



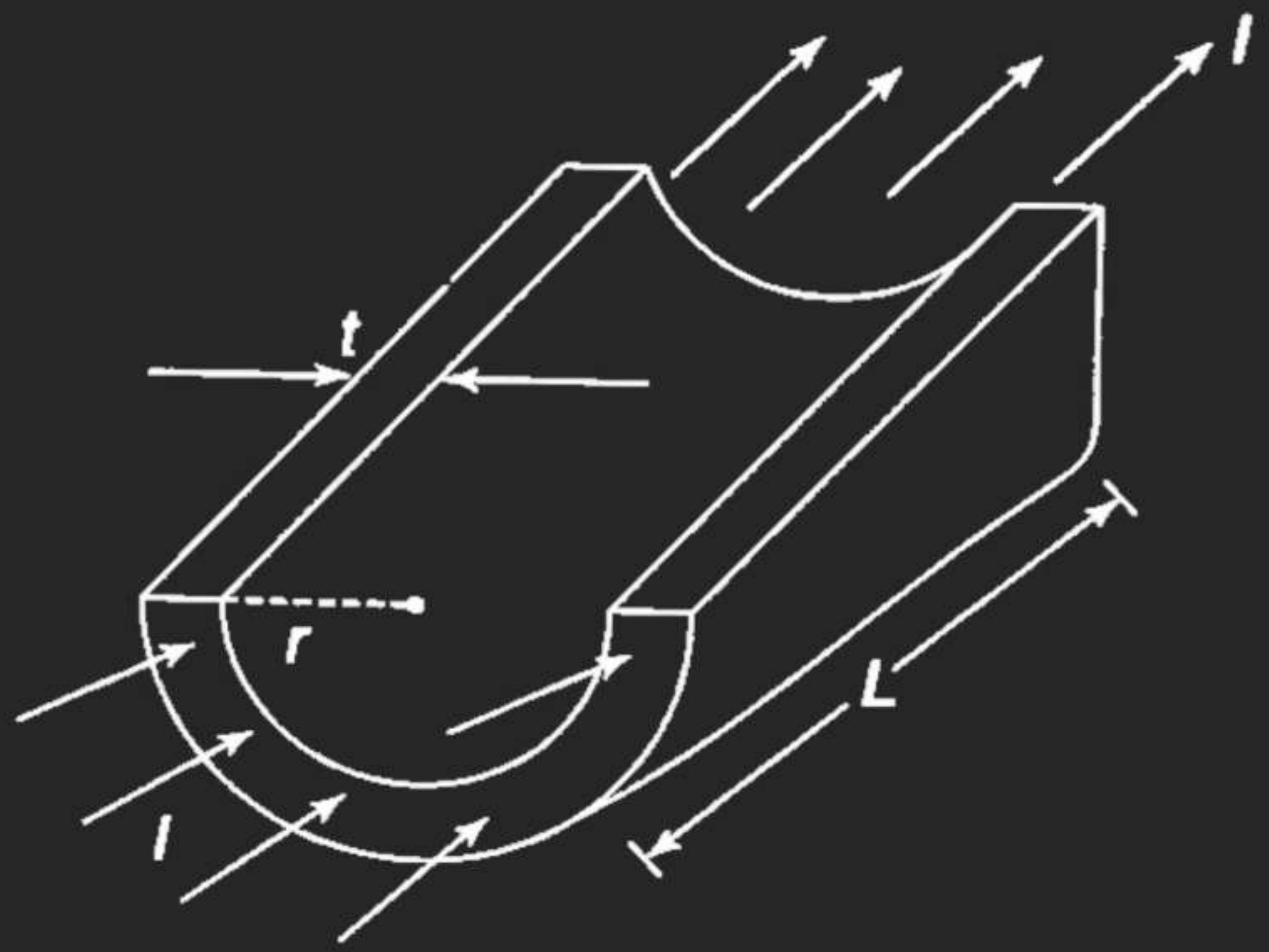
CURRENT ELECTRICITY

Q. Two cylindrical rods, of different material, are joined as shown. The rods have same cross section (A) and their electrical resistivities are ρ_1 and ρ_2. When a current I is passed through the rods, a charge (Q) gets piled up at the junction boundary. Assuming the current density to be uniform throughout the cross section, calculate Q. Under what condition the charge Q is negative?



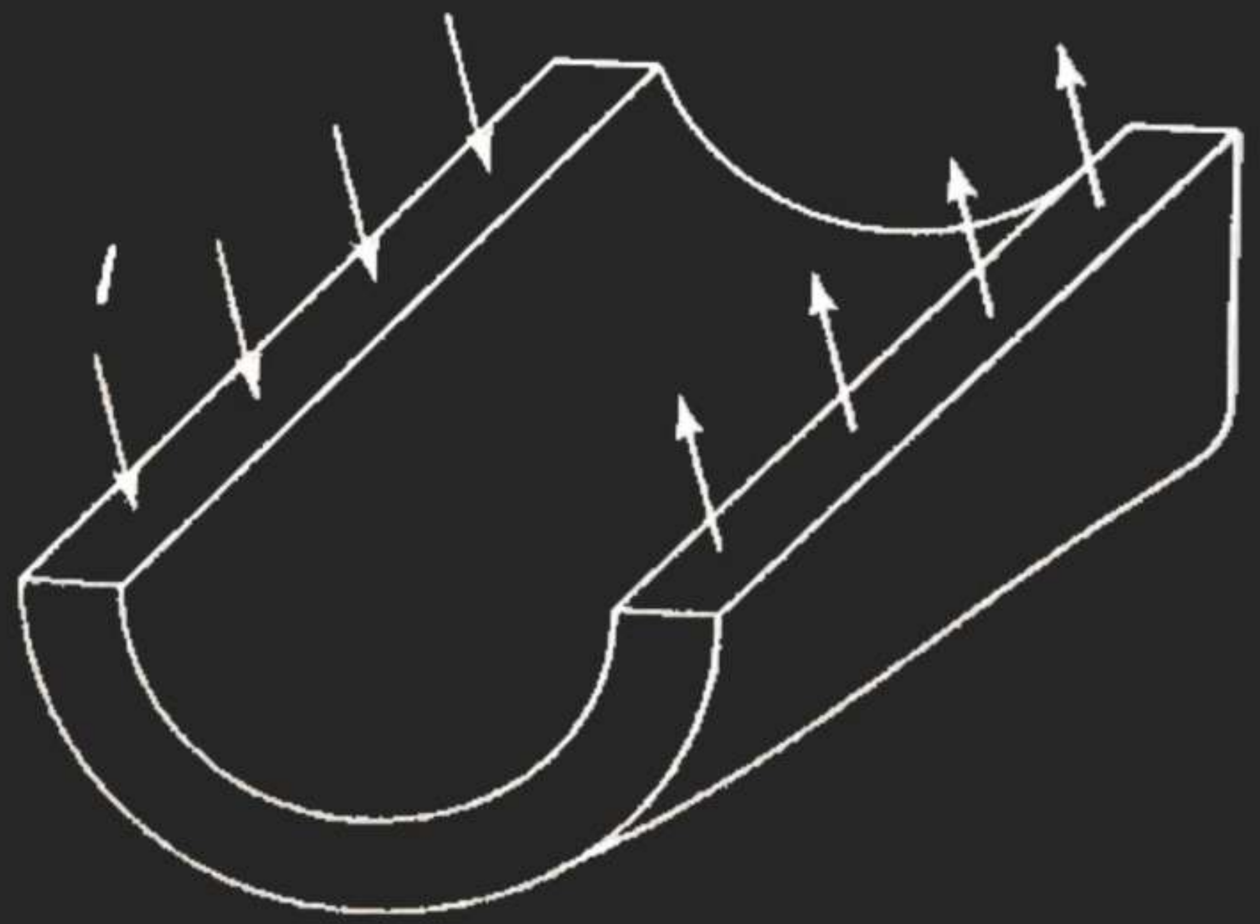
Nishant Jindal

CURRENT ELECTRICITY



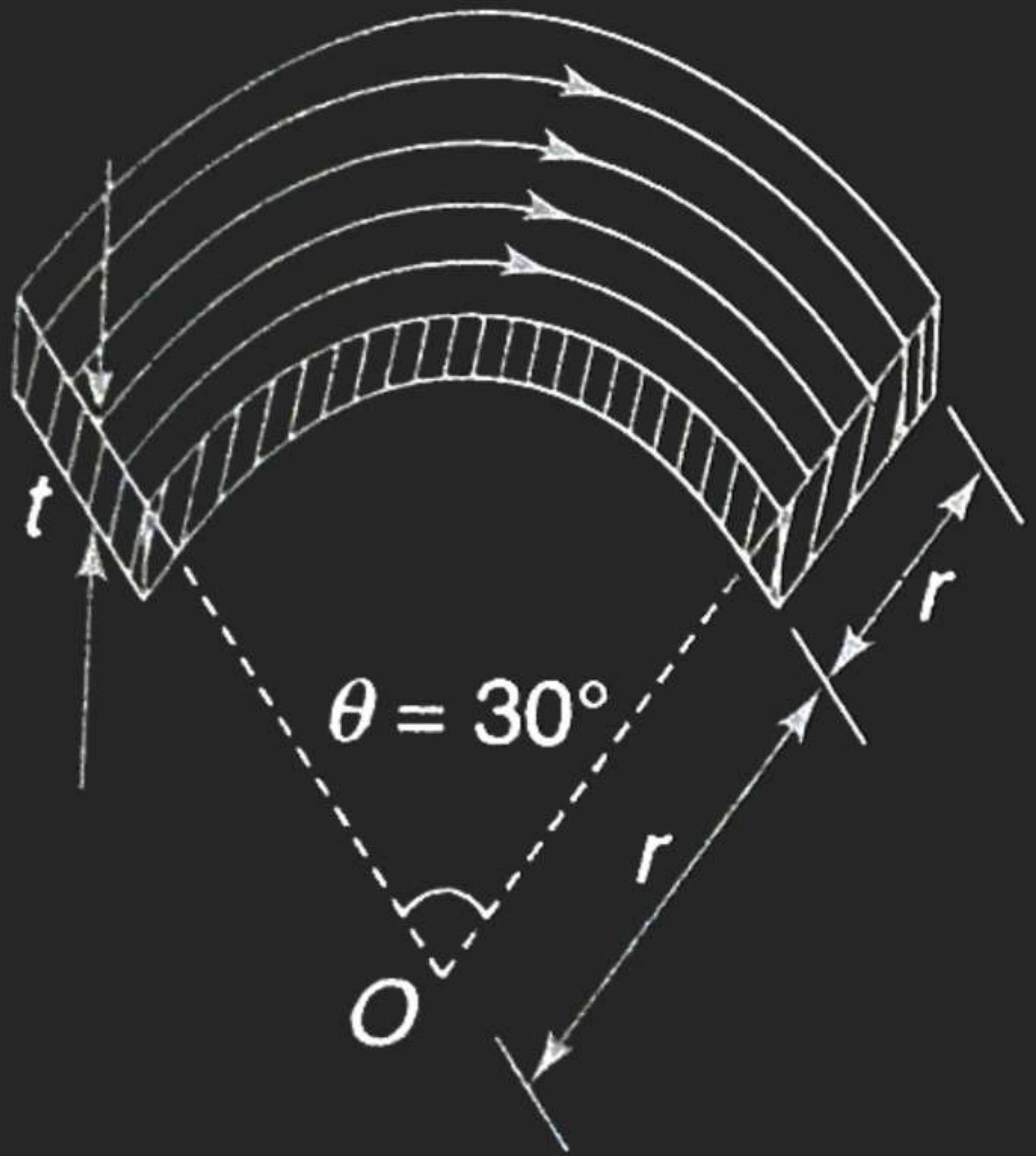
Nishant Jindal

CURRENT ELECTRICITY



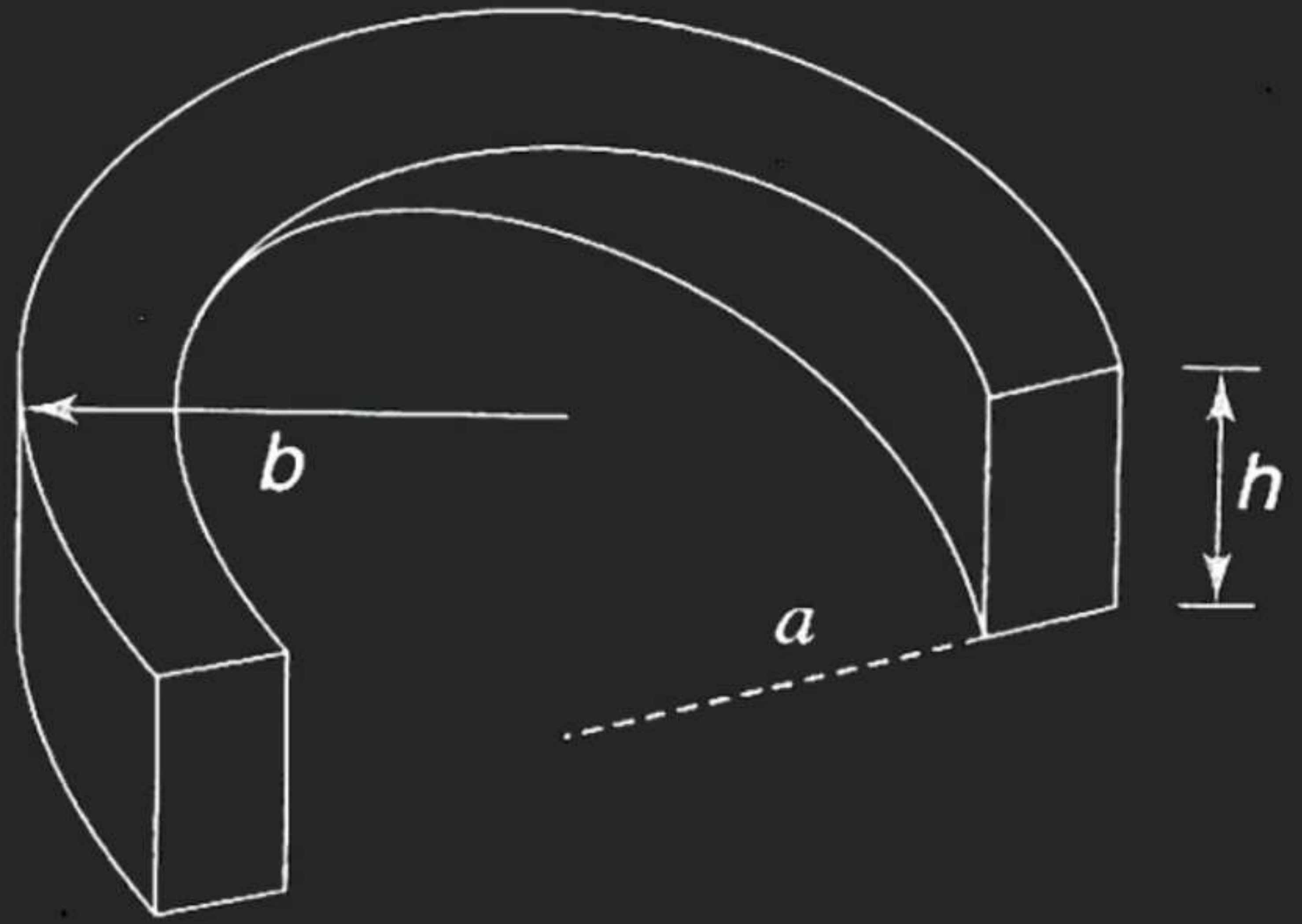
Nishant Jindal

CURRENT ELECTRICITY



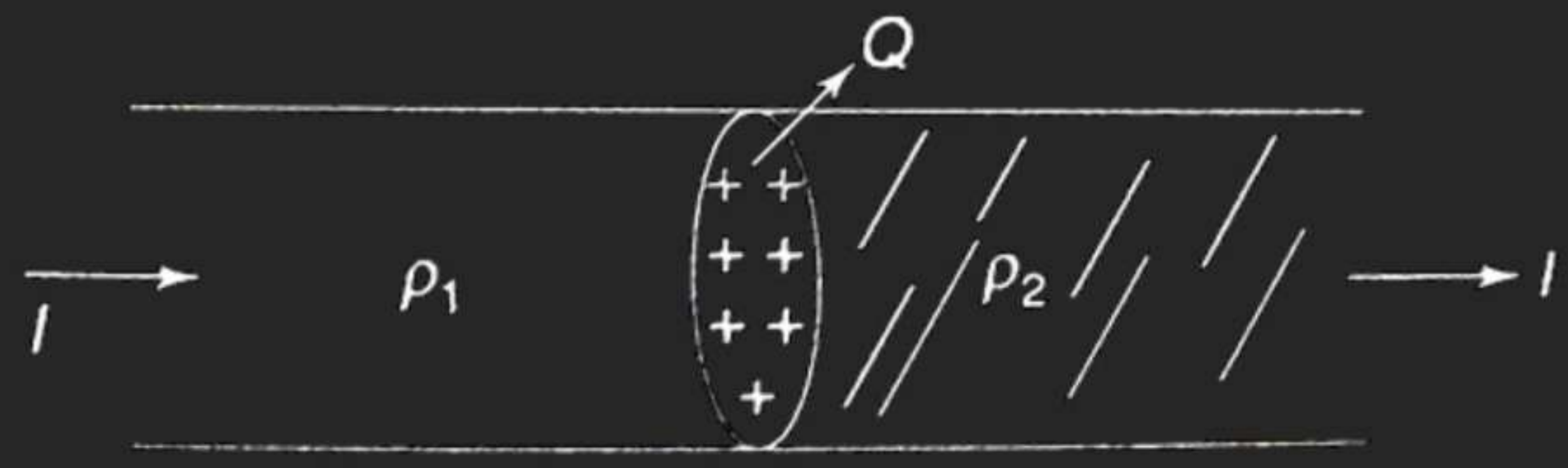
Nishant Jindal

CURRENT ELECTRICITY



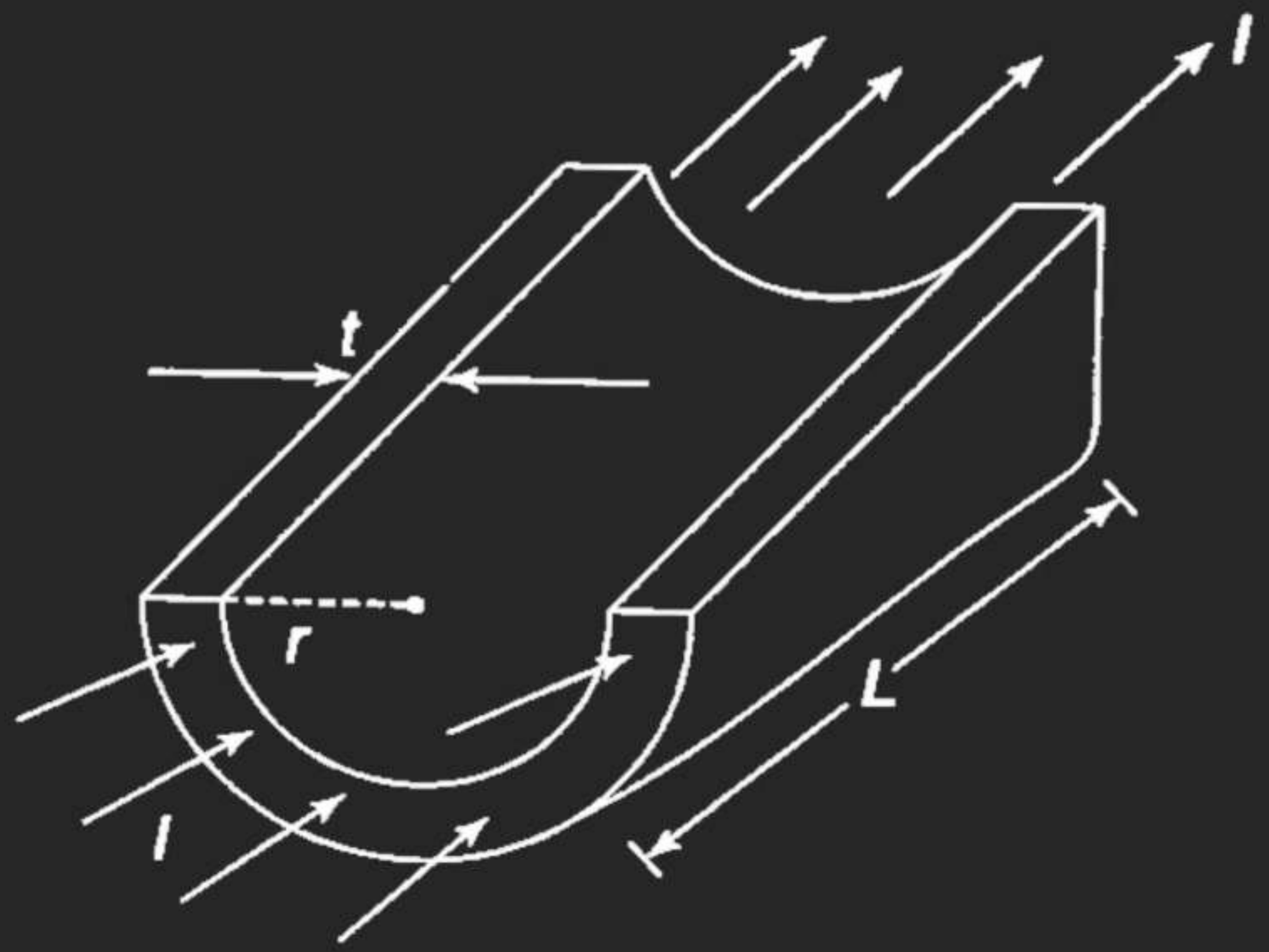
CURRENT ELECTRICITY

Q. Two cylindrical rods, of different material, are joined as shown. The rods have same cross section (A) and their electrical resistivities are ρ_1 and ρ_2. When a current I is passed through the rods, a charge (Q) gets piled up at the junction boundary. Assuming the current density to be uniform throughout the cross section, calculate Q. Under what condition the charge Q is negative?

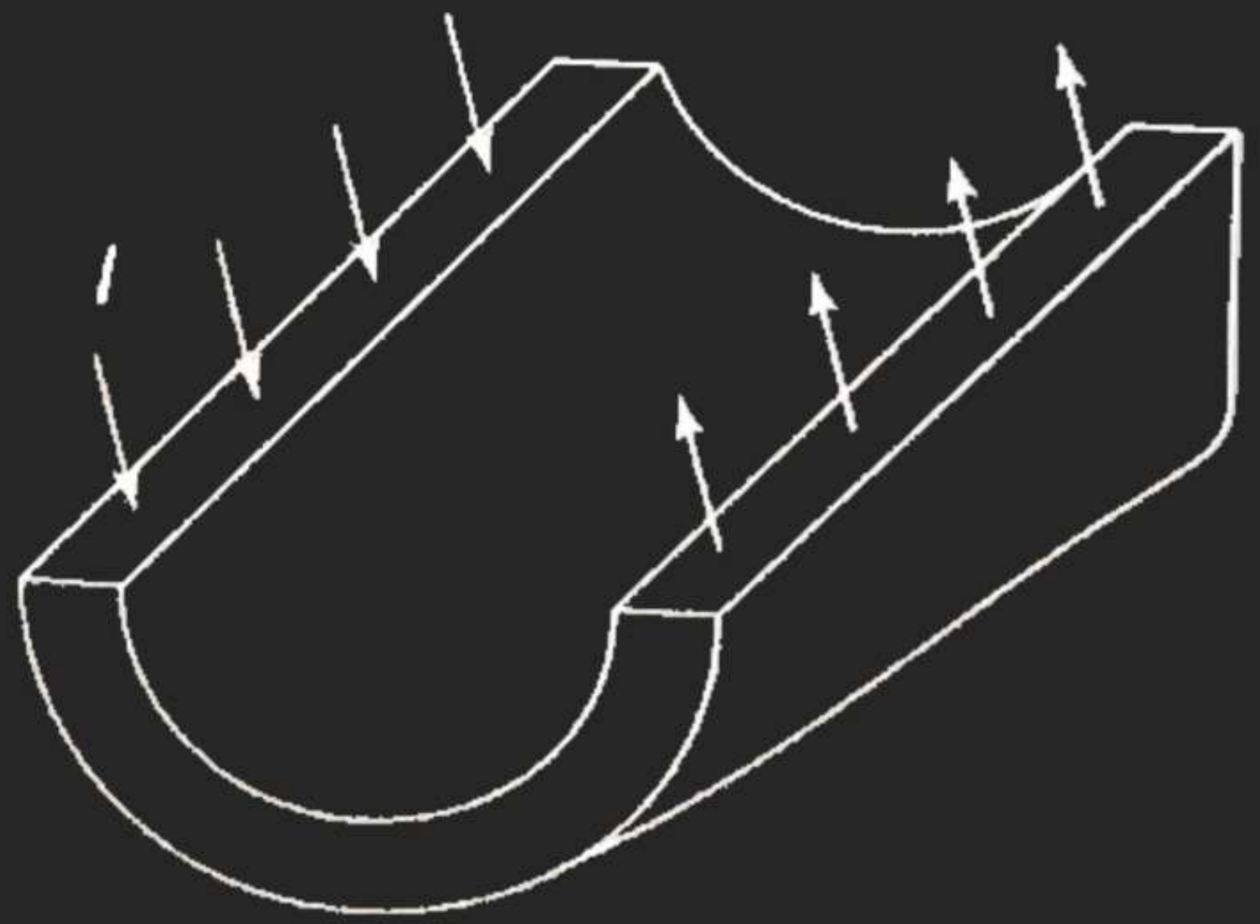


Nishant Jindal

CURRENT ELECTRICITY



CURRENT ELECTRICITY



Nishant Jindal

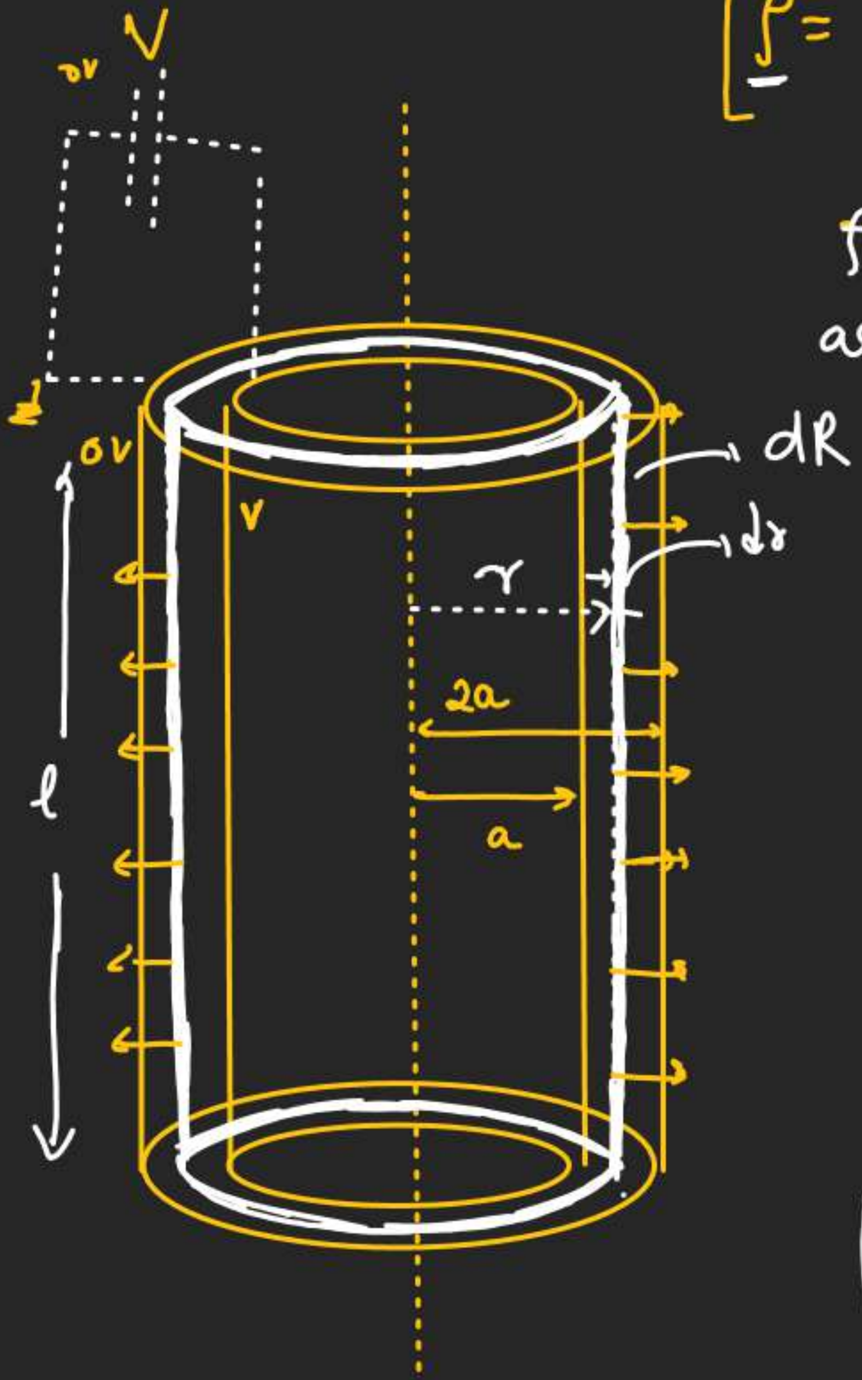
CURRENT ELECTRICITY

Nishant Jindal

CURRENT ELECTRICITY

Case-2 $\rho \rightarrow$ Variable

$$\boxed{\rho = \frac{\rho_0 r^2}{a^2}}$$



$\rho_r = \rho_{r+dr}$

as 'dr' is very small.

$$dR = \frac{(\rho_r)\, dr}{(2\pi r l)}$$

$$dR = \frac{\rho_0}{a^2} \frac{\cancel{r^2}\, r}{2\pi \cancel{r} l} dr$$

$$\int_0^R dR = \frac{\rho_0}{2\pi a^2 l} \int_a^r r\, dr$$

$$\boxed{R = \frac{\rho_0}{4\pi a^2 l}(r^2 - a^2)}$$

$$R_T = \frac{\rho_0}{4\pi a^2 l}\left((2a)^2 - a^2\right)$$

$$R_T = \frac{\rho_0}{4\pi \cancel{a^2} l}(3\cancel{a^2})$$

$$\boxed{R_T = \frac{3\rho_0}{4\pi l}}$$

$$\boxed{I = \left(\frac{V}{R_T}\right)} \qquad I = \left(\frac{V 4\pi l}{3\rho_0}\right)$$

Constant

Nishant Jindal

$\rho_r = \frac{\rho_0 r^2}{a^2}$

CURRENT ELECTRICITY

$$J = \frac{I}{A}$$

$$J = \left(\frac{I}{2\pi r L}\right)$$

$$J = \frac{1}{2\pi r L} \times \left(\frac{V 4\pi L}{3\rho_0}\right)$$

$$J = \left(\frac{2V}{3\rho_0 r}\right)$$

$$J = \sigma E$$

$$J_r = \frac{E_r}{\rho_r}$$

$$E_r = J_r \rho_r$$

$$E_r = \frac{2V}{3\cancel{\rho_0} r} \times \frac{\cancel{\rho_0} r^2}{a^2}$$

$$E_r = \left(\frac{2V}{3a^2}\right) r$$

$$\int_V^{V(r)} dV = -\int_a^r E_r \, dr$$

$$V(r) - V = -\frac{2V}{3a^2}\int_a^r r\,dr$$

$$V(r) - V = -\frac{2V}{3a^2}\left[\frac{r^2}{2}\right]_a^r$$

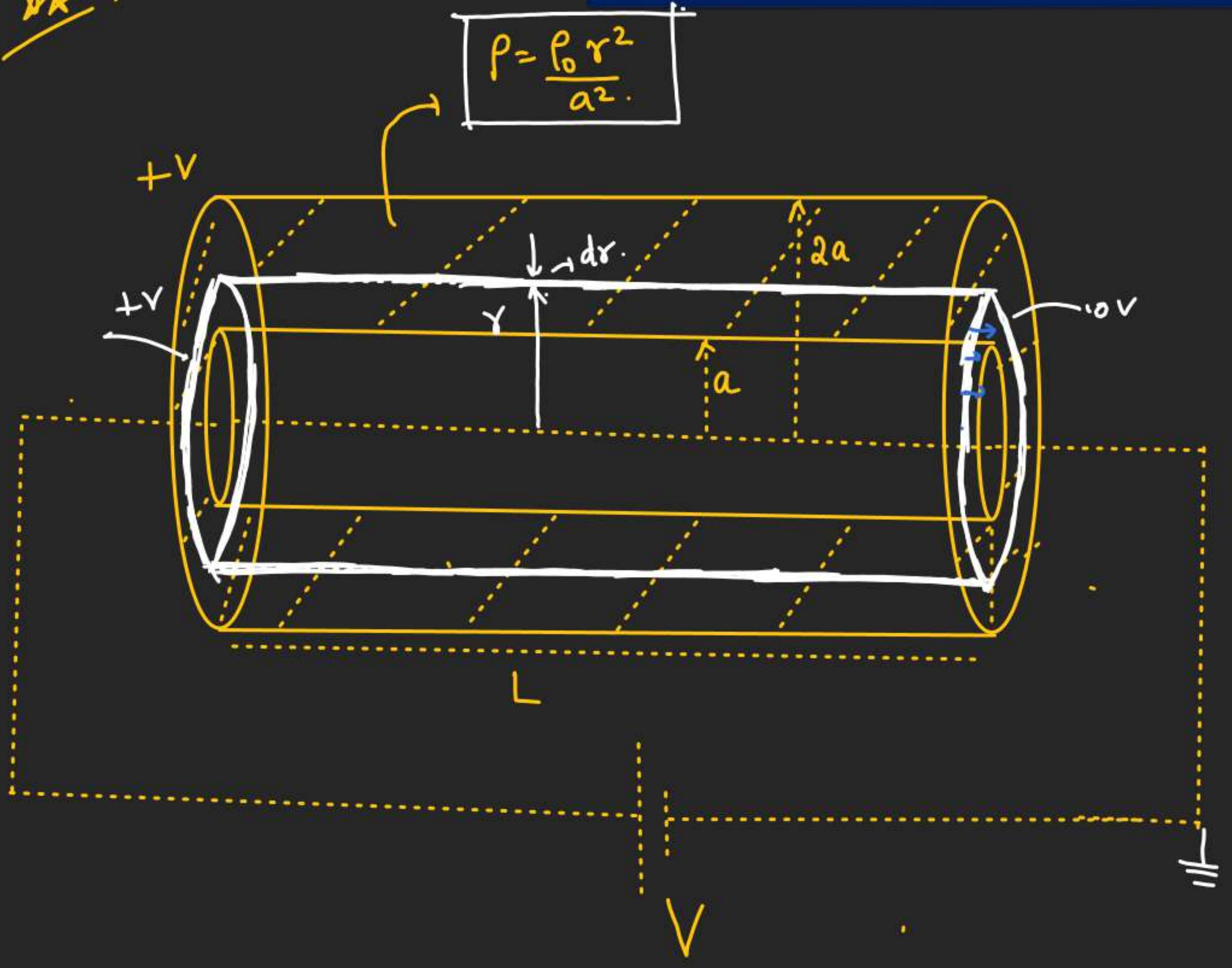
$$V(r) - V = -\frac{2V}{6a^2}(r^2 - a^2)$$

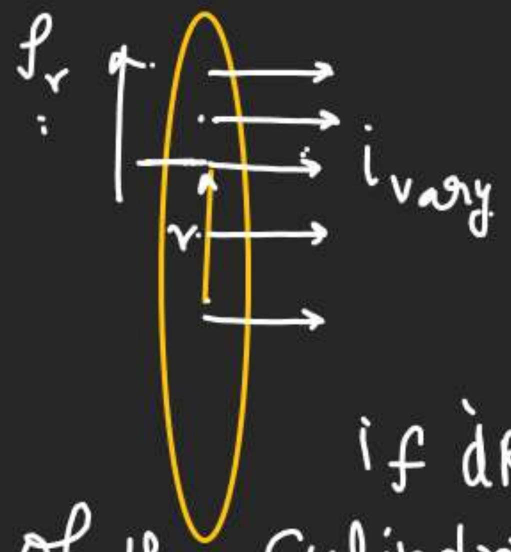
$$V(r) = V - \frac{V}{3a^2}(r^2 - a^2)$$

$$V(r) = V_0 - (I_r \cdot R_r)$$

Nishant Jindal

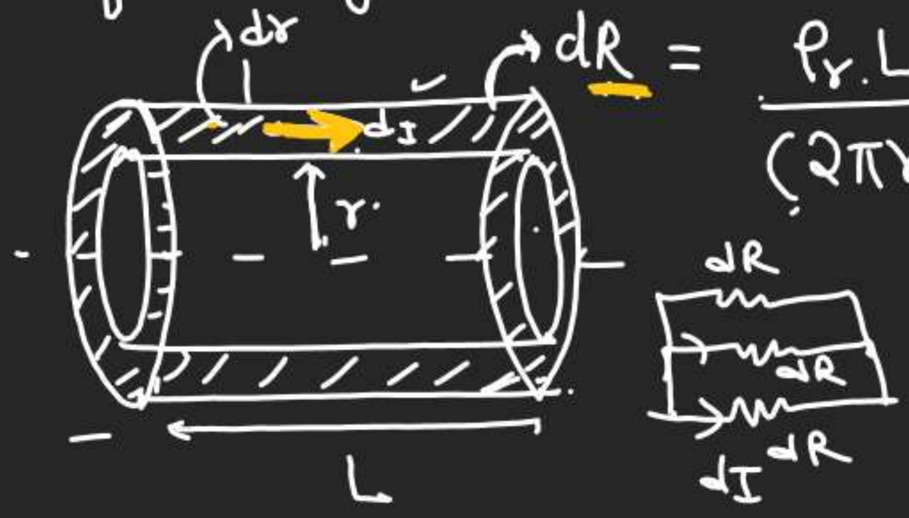
CURRENT ELECTRICITY





For every cylindrical shell potential difference is same and is equal to potential of battery. if 'dR' be the resistance of the cylindrical shell then.

$$dR = \frac{\rho_r . L}{(2\pi r dr)}$$



CURRENT ELECTRICITY

$$dR = \left(\frac{\rho_r L}{2\pi r\,dr}\right)$$

$$dR = \frac{\rho_0 r^2}{a^2} \times \left(\frac{L}{2\pi r\,dr}\right)$$

$$dR = \frac{\rho_0 L}{2\pi a^2}\left(\frac{r}{dr}\right)$$

$\Downarrow$

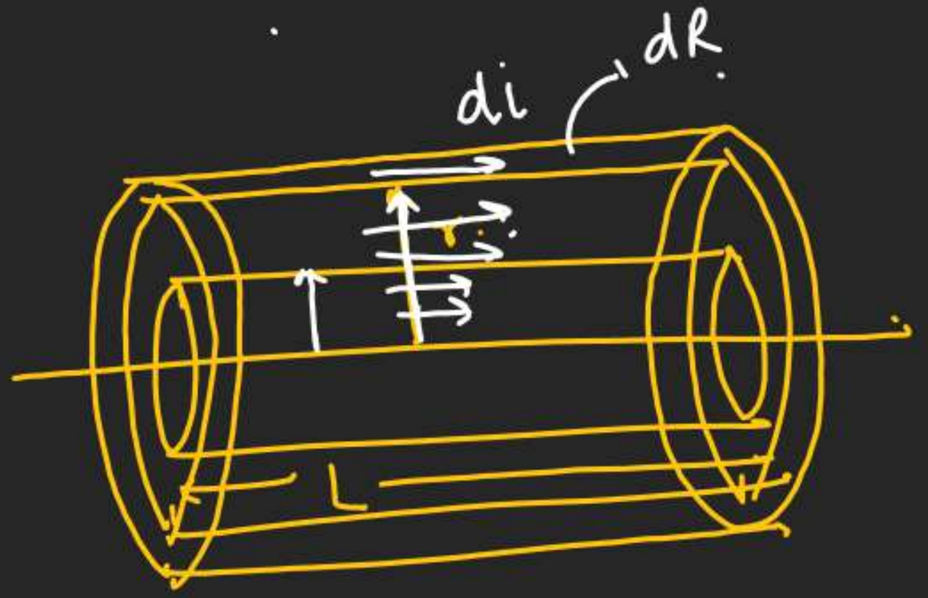
(All 'dR' in parallel)

$$\frac{1}{R_{eq}} = \int_a^{2a} \frac{1}{dR}$$

$$\frac{1}{R_{eq}} = \frac{2\pi a^2}{\rho_0 L}\int_a^{2a} \frac{dr}{r}$$

$$\frac{1}{R_{eq}} = \frac{2\pi a^2}{\rho_0 L}\ln(2)$$

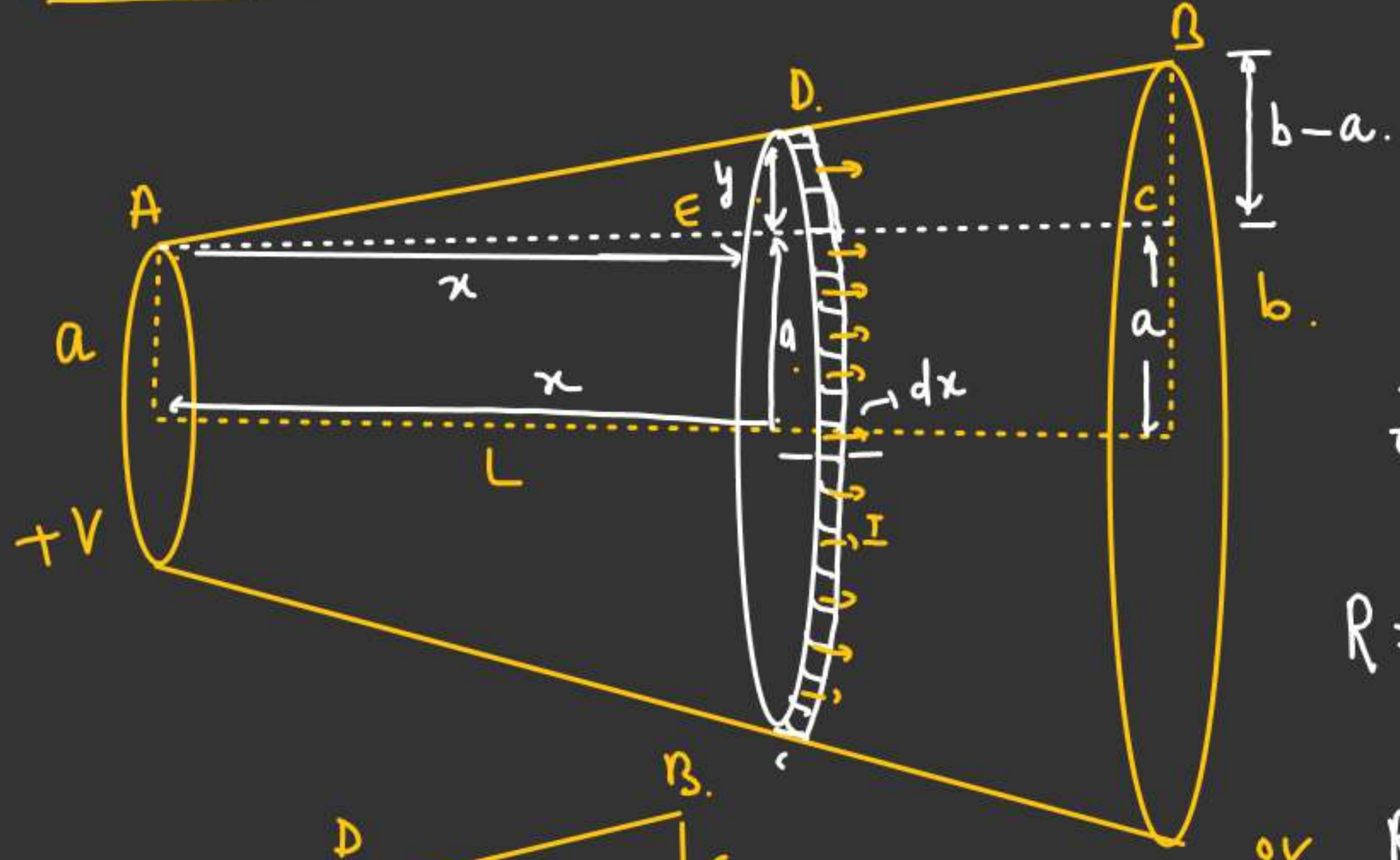
$$\boxed{R_{eq} = \frac{\rho_0 L}{2\pi a^2 \ln(2)}}$$

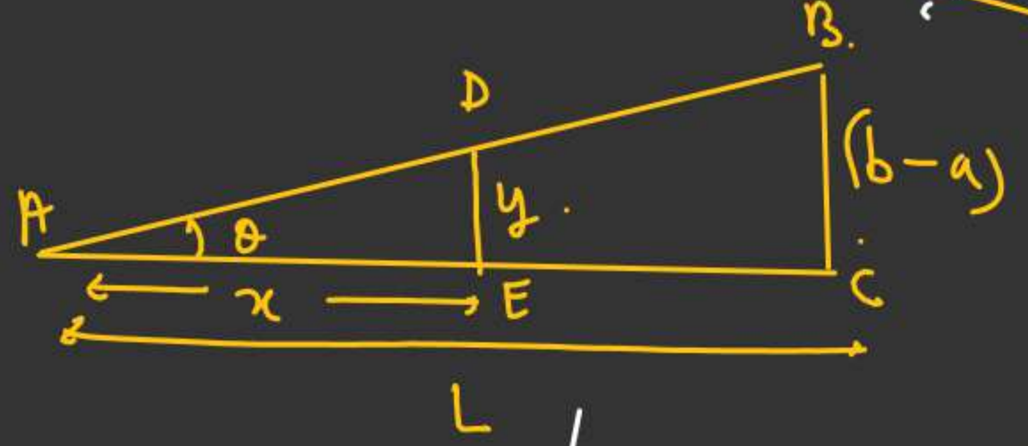


Nishant Jindal

Resistance of a frustum

ρ = Constant





$$\Rightarrow \frac{y}{x} = \frac{b-a}{L}$$

or

$$y = \left(\frac{b-a}{L}x\right)$$

$$dR = \frac{\rho\, dx}{\pi (a+y)^2}$$

$$\int_0^R dR = \int_0^L \frac{\rho\, dx}{\pi\left[a + \left(\frac{b-a}{L}\right)x\right]^2}$$

put

$$t = a + \left(\frac{b-a}{L}\right)x$$

$$\frac{dt}{dx} = \left(\frac{b-a}{L}\right)$$

$$dt = \left(\frac{b-a}{L}\right)dx$$

$$dx = \left(\frac{L}{b-a}\right)dt$$

$$R = \frac{\rho}{\pi} \times \frac{L}{(b-a)} \int_0^L \frac{dt}{t^2}$$

$$R = \frac{\rho L}{\pi (b-a)}\left[-\frac{1}{t}\right]_0^L \Rightarrow R = \frac{-\rho L}{\pi(b-a)}\left[\frac{1}{a + \left(\frac{b-a}{L}\right)x}\right]_0^L$$

$$R = \frac{-\rho L}{\pi(b-a)}\left[\frac{1}{b} - \frac{1}{a}\right]$$

$$R = \frac{\rho L}{\cancel{\pi(a-b)}} \frac{\cancel{(a-b)}}{ab}$$

$$R = \frac{\rho L}{\pi a b}$$