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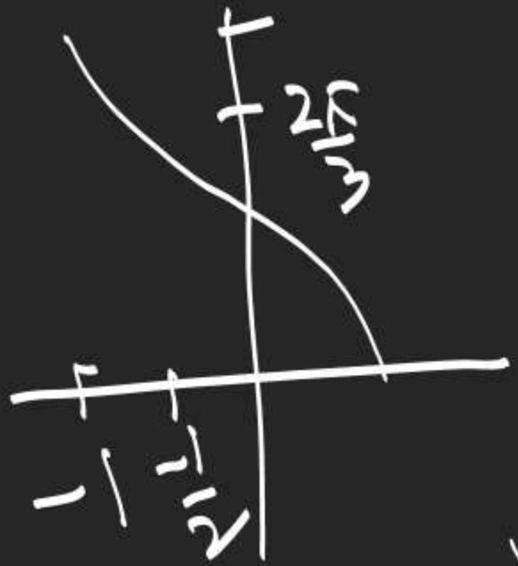
2.

$$\frac{\pi}{2} - \cos^{-1}(3x-4x^3) + \cos^{-1}(4x^3-3x)$$

$$= \frac{\pi}{2} - \left(\pi - \cos^{-1}(4x^3-3x)\right) + \cos^{-1}(4x^3-3x)$$



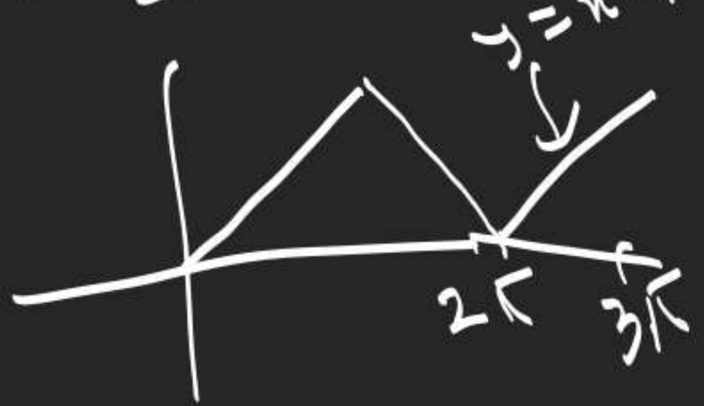
$$= 2\cos^{-1}(4x^3-3x) - \frac{\pi}{2}$$

$\cos^{-1}x = \theta \in \left[\frac{2\pi}{3}, \pi\right]$

$3\theta \in [2\pi, 3\pi]$

$$= 2\cos^{-1}(\cos 3\theta) - \frac{\pi}{2} = 2(3\theta - 2\pi) - \frac{\pi}{2}$$

$$= 6\cos^{-1}x - \frac{9\pi}{2}$$



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3. (d) $\sum_{r=1}^{n} \sin^{-1}\left(\frac{\sqrt{r}-\sqrt{r-1}}{\sqrt{r(r+1)}}\right)$

$$= \sum_{r=1}^{n} \tan^{-1}\left(\frac{\sqrt{r}-\sqrt{r-1}}{1+\sqrt{r}\sqrt{r-1}}\right)$$

$$\sqrt{(r^2-r)+1+2\sqrt{r}\sqrt{r-1}}$$

$$= \sqrt{\left(\sqrt{r^2-r}+1\right)^2}$$

$$= \sum_{r=1}^{n}\left(\tan^{-1}\sqrt{r} - \tan^{-1}\sqrt{r-1}\right) = 1+\sqrt{r^2-r}$$

$$= \tan^{-1}\sqrt{n}$$

$$\sin^{-1}\left(\frac{1}{\sqrt{r}}\sqrt{1-\frac{1}{r+1}} - \sqrt{1-\frac{1}{r}}\,\frac{1}{\sqrt{r+1}}\right) = \sin^{-1}\sin(\underbrace{\theta_1-\theta_2}_{-\frac{\pi}{2},\,\frac{\pi}{2}}) = \sin^{-1}\frac{1}{\sqrt{r}} - \sin^{-1}\frac{1}{\sqrt{r+1}}$$

$$\sin^{-1}\frac{1}{\sqrt{r}} = \theta_1$$

$$\sin^{-1}\frac{1}{\sqrt{r+1}} = \theta_2$$

d, g, h

$$3\cos^{-1}x = \sin^{-1}\left(\sqrt{1-x^2}\,(4x^2-1)\right)$$

$$\cos^{-1}x = \theta \in [0, \pi]$$

$$3\theta = \sin^{-1}\left(\sin\theta\,(\underbrace{4\cos^2\theta-1}_{3-4\sin^2\theta})\right) = \sin^{-1}\sin 3\theta$$

$$3\theta \in [0, 3\pi] \checkmark$$

$$\tan^{-1}a = \theta, \in \left(0, \frac{\pi}{2}\right)$$

$$\cancel{2}\tan^{-1}y = \cancel{2}\tan^{-1}a - \cancel{2}\tan^{-1}b$$

$$\boxed{x = \frac{a-b}{1+ab}}$$

$$3\cos^{-1}x \in \left[0, \frac{\pi}{2}\right]$$

$$\cos^{-1}x \in \left[0, \frac{\pi}{6}\right]$$

$$x \in \left[\frac{\sqrt{3}}{2}, 1\right]$$

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(k) $\pi - \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) - \tan^{-1}\frac{2x}{1-x^2} = \frac{2\pi}{3}$.

$\tan^{-1}x = \theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ $\quad 2\theta \in (-\pi, \pi)$

$\pi - \cos^{-1}\cos 2\theta - \tan^{-1}\tan 2\theta = \frac{2\pi}{3}$

$2\theta \in \left(-\pi, -\frac{\pi}{2}\right)$ OR $2\theta \in \left(-\frac{\pi}{2}, 0\right)$ OR $2\theta \in \left(0, \frac{\pi}{2}\right)$ OR $2\theta \in \left(\frac{\pi}{2}, \pi\right)$

$\pi + 2\theta - (\pi + 2\theta) = \frac{2\pi}{3}$

ϕ

$\pi + 2\theta - 2\theta = \frac{2\pi}{3}$

ϕ

$\pi - 2\theta - 2\theta = \frac{2\pi}{3}$

$2\theta = \frac{\pi}{6}$

$\boxed{x = \tan\frac{\pi}{12} = 2 - \sqrt{3}}$

$\pi - 2\theta - (2\theta - \pi) = \frac{2\pi}{3}$

$2\theta = \frac{2\pi}{3}$

$\boxed{x = \sqrt{3}}$

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5.

$$x^2 - \frac{k\pi^2}{4}x + \frac{\pi^4}{16} = 0$$

$\cos^{-1}x$

$(\sin^{-1}y)^2$

$$D \geq 0 \Rightarrow \frac{k^2}{16}\pi^4 \geq \frac{\pi^4}{4} \Rightarrow k^2 \geq 4$$

$$x^2 - \frac{\pi^2}{2}x + \frac{\pi^4}{16} = 0$$

$$\left(x - \frac{\pi^2}{4}\right)^2 = 0$$

$$\underbrace{\cos^{-1}x}_{[0,\pi]} + \underbrace{(\sin^{-1}y)^2}_{[0,\frac{\pi^2}{4}]} = \frac{k\pi^2}{4} \in \left[0, \pi + \frac{\pi^2}{4}\right]$$

$$0 \leq 2.5k \leq 5.6$$

$$\cos^{-1}x = \frac{\pi^2}{4} = (\sin^{-1}y)^2$$

$\boxed{k = 2}$

$\boxed{k \leq 2}$

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$$\frac{K\pi^2}{8} \geq \frac{\pi^2}{4} \qquad \boxed{K \geq 2}$$

6. $\tan^{-1} x + \tan^{-1} \frac{1}{y} = \tan^{-1} 3$

$$\frac{xy+1}{y-x} = \frac{x+\frac{1}{y}}{1-\frac{x}{y}} = 3$$

$10 \quad -1$

-2

$xy - 3y + 3x + 1 = 0$

$(y+3)(x-3) = -10$

-10

-5

1

2

5

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Fundamental Theorems

If $\lim_{x \to a} f(x)$ exists $= l$ & $\lim_{x \to a} g(x)$ exists $= m$,

then

* $\lim_{x \to a} (f+g)(x) = \lim_{x \to a} f(x) + \lim_{x \to a} g(x)$
* $\lim_{x \to a} (f-g)(x) = \lim_{x \to a} f(x) - \lim_{x \to a} g(x)$
* $\lim_{x \to a} (fg)(x) = \left(\lim_{x \to a} f(x)\right)\left(\lim_{x \to a} g(x)\right)$
* $\lim_{x \to a} \left(\frac{f}{g}\right)(x) = \frac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)}$, $m \neq 0$

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$x=3 \begin{cases} LHL = 2 \\ RHL = 1 \end{cases}$

$\boxed{x = 0, 1, 2, 3, 6}$

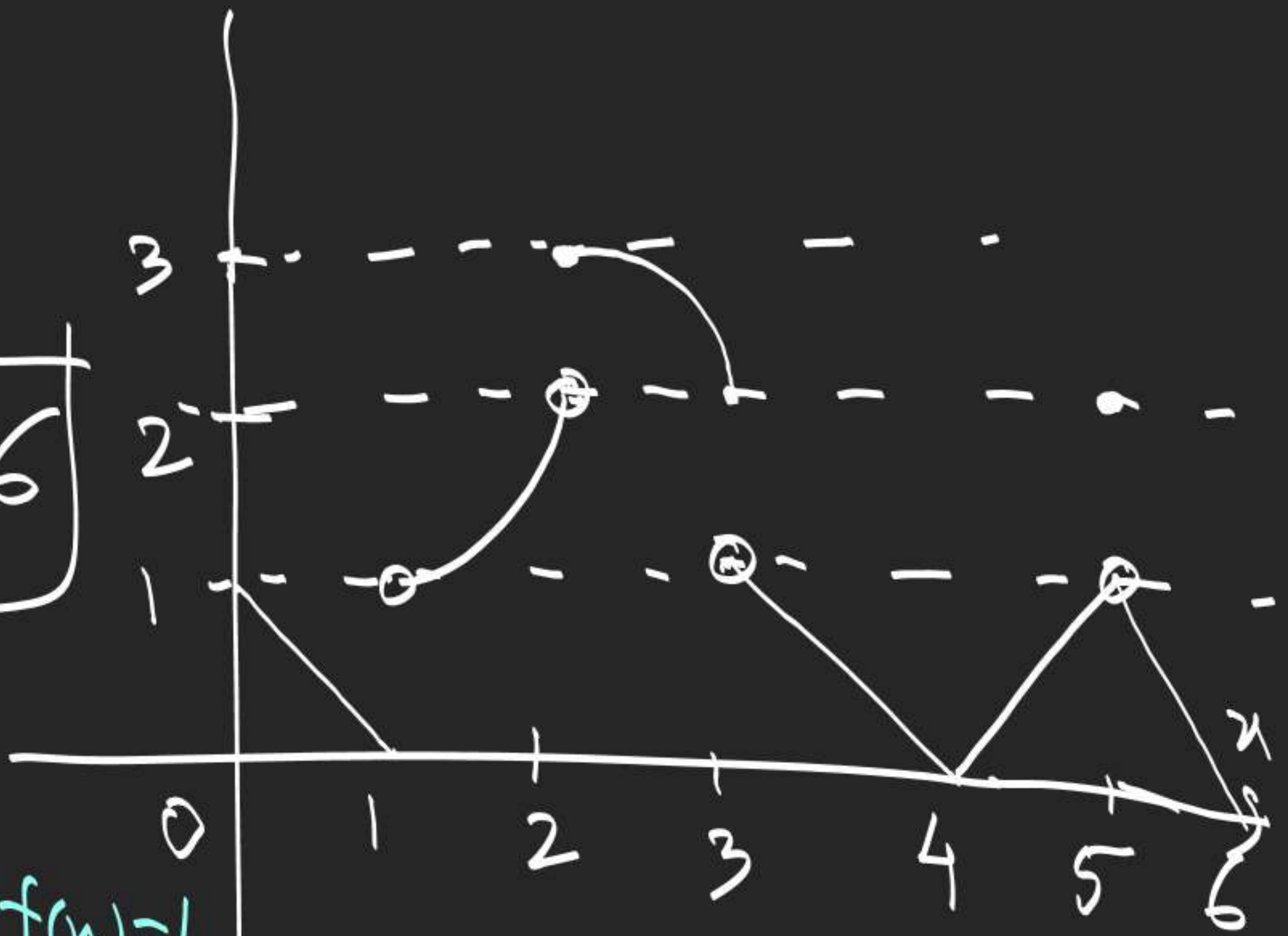
$x=4 \rightarrow$

$\lim_{x\to 4} f(x) = 0.$

$x=1 \begin{cases} LHL = 0 \\ RHL = 1 \end{cases}$

$\lim_{x\to 5} f(x) = 1$

$x=2 \begin{cases} LHL = 2 \\ RHL = 3. \end{cases}$



$\lim_{x\to 0^+} f(x) = 1$

$D_f = [0, 6]$

$\lim_{x\to 6^-} f(x) = 0.$

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Indeterminate form

$$\lim_{x \to a} f(x) \to \frac{0}{0}, \frac{\infty}{\infty}, \infty \times 0, 0^0, \infty^0, \infty - \infty, 1^\infty$$

$$\lim_{x\to 0} \frac{\cancel{x}}{3\cancel{x}} = \frac{1}{3} \qquad \frac{0}{0}$$

$x \neq 0$

$$\lim_{x\to 1} \frac{x+2}{2x-3} = \frac{3}{-1}$$

$\frac{2}{3}$

$$\lim_{x\to 1} \frac{x^2-5x+4}{x^2-3x+2} = \frac{0}{0}$$

$$3 = \lim_{x\to 1} \frac{\cancel{(x-1)}(x-4)}{\cancel{(x-1)}(x-2)} =$$

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Power Series

$\rightarrow$ Infinite series in which every term has exponent of x a whole number.

$$a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \dots \infty$$

Taylor's Series

$$\boxed{f(x) = f(0) + \frac{f'(0)}{1!}x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \dots \infty}$$

derivative any no. of times exist.

Convergent

$|x| < 1$, $1 + x + x^2 + x^3 + \dots \infty = \frac{1}{1-x}$

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Polynomial

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \cdots + a_2 x^2 + a_1 x + a_0$$

$$\boxed{a_r = \frac{f^{(r)}(0)}{r!}}$$

$$f(0) = a_0$$

$$f'(x) = n a_n x^{n-1} + \cdots + 3a_3 x^2 + 2a_2 x + a_1$$

$$\frac{f'(0)}{1} = a_1$$

$$f''(x) = n(n-1) a_n x^{n-2} + \cdots + 3\cdot 2\cdot a_3 x + 2\cdot 1\cdot a_2$$

$$\frac{f''(0)}{1\cdot 2} = a_2$$

$$f'''(x) = n(n-1)(n-2) a_n x^{n-3} + \cdots + 3\cdot 2\cdot 1\, a_3$$

$$\frac{f'''(0)}{1\cdot 2\cdot 3} = a_3$$

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Convergent Series. $\to$ sum exist

Divergent $\to$ not exist.

$1+x+x^2+\cdots\infty$

$\lim_{n\to\infty}\left|\frac{T_{n+1}}{T_n}\right| = \lim_{n\to\infty}|x| = |x|<1$

Ratio Test

$\lim_{n\to\infty}\left|\frac{T_{n+1}}{T_n}\right|$ ✓ $<1 \Rightarrow$ Series is convergent.

$>1 \Rightarrow$ —"— divergent.

$=1 \Rightarrow$ Method fails.

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$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots \infty, \ x \in \mathbb{R}.$$

$$\lim_{n \to \infty} \left| \frac{T_{n+1}}{T_n} \right| = \lim_{n \to \infty} \left| \frac{\frac{x^n}{n!}}{\frac{x^{n-1}}{(n-1)!}} \right| = \lim_{n \to \infty} \left| \frac{x}{n} \right| = 0 < 1$$

HW $\rightarrow$ $\boxed{\text{Ex-II (remaining)}}$