24) $\frac{d}{dx}\left[\tan^{-1}\left(\frac{\sqrt{x^2+a^2}+x}{\sqrt{x^2+a^2}-x}\right)^{1/2}\right]$

$x = a\tan\theta \Rightarrow \tan\theta = \frac{x}{a} \Rightarrow \theta = \tan^{-1}\frac{x}{a} \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$\frac{d}{dx}\left[\tan^{-1}\left(\frac{\sqrt{a^2+a^2\tan^2\theta}+a\tan\theta}{\sqrt{a^2+a^2\tan^2\theta}-a\tan\theta}\right)^{1/2}\right]$

$\frac{d}{dx}\left[\tan^{-1}\left(\frac{\sec\theta+\tan\theta}{\sec\theta-\tan\theta}\right)^{1/2}\right]$

$\frac{d}{dx}\left[\tan^{-1}\left(\frac{\left(\cos\frac{\theta}{2}+\sin\frac{\theta}{2}\right)^2}{\left(\cos\frac{\theta}{2}-\sin\frac{\theta}{2}\right)^2}\right)^{1/2}\right] = \frac{d}{dx}\left[\tan^{-1}\left(\frac{\cos\frac{\theta}{2}+\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}-\sin\frac{\theta}{2}}\right)\right] = \frac{d}{dx}\left[\tan^{-1}\left(\frac{1+\tan\frac{\theta}{2}}{1-\tan\frac{\theta}{2}}\right)\right] = \frac{d}{dx}\left[\tan^{-1}\left(\tan\left(\frac{\pi}{4}+\frac{\theta}{2}\right)\right)\right]$

भाग

$\frac{d}{dx}\left[\frac{1}{2}\times\tan^{-1}\frac{x}{a}\right]$

$= \frac{1}{2}\times\frac{1}{1+\left(\frac{x}{a}\right)^2}\times\frac{1}{a}$

$\frac{d}{dx}\left(\frac{\theta}{2}\right)$

Nishant Jindal

$f(x) = \left|\log 2 - \sin x\right|,\ g(x) = f(f(x))$

$g = \left|\log 2 - \sin\left(\left|\log 2 - \sin x\right|\right)\right|$

$g(0) = \left|\log 2 - \sin(\log 2)\right|$

$\sin x < x$

$\sin(\log 2) < \log 2$

Q 29 $\quad f(x) = \tan^{-1}\left(\dfrac{6x\sqrt{x}}{1-9x^3}\right)$

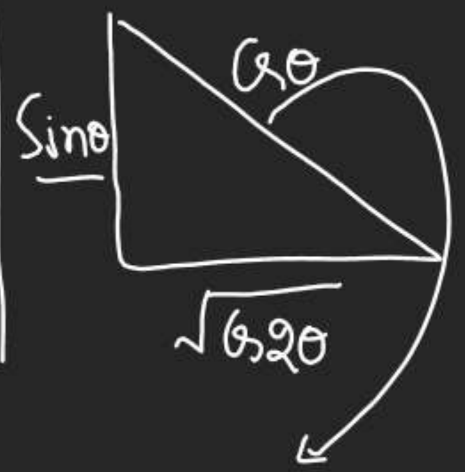
$= \tan^{-1}\left(\dfrac{3x\sqrt{x} + 3x\sqrt{x}}{1-(3x\sqrt{x})(3x\sqrt{x})}\right) = \tan^{-1}(3x\sqrt{x}) + \tan^{-1}(3x\sqrt{x})$

$= 2\tan^{-1}(3x\sqrt{x})$

Q 30 $\quad f(\theta) = \sin\left(\tan^{-1}\left(\dfrac{\sin\theta}{\sqrt{\cos 2\theta}}\right)\right)$

$\dfrac{d(f(\theta))}{d(\tan\theta)}$

$\sin(\sin^{-1}x) = x$



$f(\theta) = \sin\left(\sin^{-1}\left(\dfrac{\sin\theta}{\cos\theta}\right)\right)$

$= \sin(\sin^{-1}(\tan\theta))$

$f(\theta) = \tan\theta$

$\dfrac{d(f(\theta))}{d(\tan\theta)}$

$\dfrac{d(\tan\theta)}{d(\tan\theta)} = 1$

$\sqrt{\sin^2\theta + \cos 2\theta}$

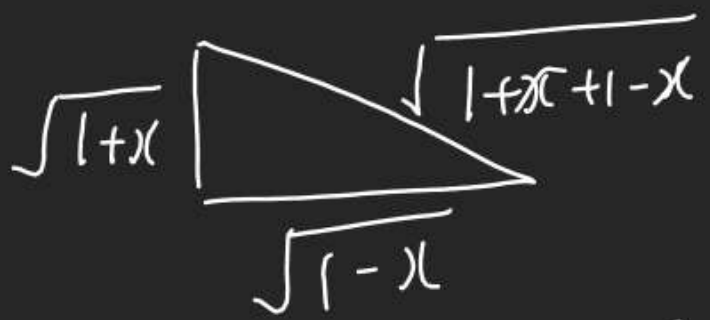
$\sqrt{\sin^2\theta + 1 - 2\sin^2\theta}$

$= \sqrt{1-\sin^2\theta}$

$= \cos\theta$

25) $\frac{d}{dx}\left[\left(\sin\cdot\cot^{-1}\frac{1}{\sqrt{\frac{1+x}{1-x}}}\right)^2\right]$

$\sin\left(\cot^{-1}\sqrt{\frac{1-x}{1+x}}\right) \to \frac{B}{P}$



$\frac{d}{dx}\left(\sin\left(\sin^{-1}\sqrt{\frac{1+x}{2}}\right)\right)^2$

$\frac{d}{dx}\left(\left(\sqrt{\frac{1+x}{2}}\right)\right)^2 = \frac{d}{dx}\left(\frac{1+x}{2}\right) = \frac{1}{2}$

23) $x = -2$

$y = \sin^{-1}\left(\frac{2x}{1+x^2}\right) = 2\tan^{-1}x = \pi - 2\tan^{-1}x, -\pi - 2\tan^{-1}x$

$\left(-\frac{\pi}{2},\frac{\pi}{2}\right) = -\pi$

$\left(-\frac{\pi}{4},\frac{\pi}{4}\right)$

$x \in (-1,1)$ x

$\left(-\frac{2}{1+x^2}\right)_{x=-2} = -\frac{2}{5}$

Nishant Jindal

diffⁿ of Inverse fxn.

(1) $f(x) \rightarrow$ Inverse fxn $= f^{-1}(x)$

(2) $f: A \rightarrow B \rightarrow y = f(x)$

then $\exists$ a fxn $g: B \rightarrow A$, $f(x) = y \Rightarrow g(y) = x$

$f(3) = 5 \Rightarrow g(5) = 3$

(3) $g(f(x)) = x$, $f(g(x)) = x$

$\therefore$ f & g are inverse to each other.

$f(g(x)) = g(f(x)) = x$

(4) $f(x)$ is Inverse of $g(x)$

$\Rightarrow g(x)$ is inverse of $f(x)$

$$\boxed{f(x) = g^{-1}(x)}$$

$$g(f(x)) = x$$

$$g'(f(x)) \times f'(x) = 1$$

$$g'(f(x)) = \frac{1}{f'(x)}$$

$$\boxed{f'(x) = \frac{1}{g'(f(x))}}$$

$$\boxed{g(x) = f^{-1}(x)}$$

$$f(g(x)) = x$$

$$f'(g(x)) \times g'(x) = 1$$

$$f'(g(x)) = \frac{1}{g'(x)}$$

$$\boxed{g'(x) = \frac{1}{f'(g(x))}}$$

Nishant Jindal

Q If $\boxed{f(x) = e^{x^3+x^2+x}}$ & $g(x) = f^{-1}(x)$

then $g'(e^3) = ?$

f(x) me Kya Rakha Jaye Ki e^3 Ban Jaye?

$f(1) = e^{1^3+1^2+1}$

$= e^3$

$f(x) = e^{x^3+x^2+x}$

$f'(x) = e^{x^3+x^2+x} \times (3x^2+2x+1)$

$f'(1) = e^3 \times (6)$

① $g'(x) = \dfrac{1}{f'(g(x))}$

② $f'(x) = \dfrac{1}{g'(f(x))}$ ← $x = 1$

$f'(1) = \dfrac{1}{g'(f(1))} = \dfrac{1}{g'(e^3)}$

$g'(e^3) = \dfrac{1}{f'(1)} = \dfrac{1}{6e^3}$ //

Q If $f(x) = e^{\frac{x}{2}+x^3}$, $g(x) = f^{-1}(x)$

then $g'(1) = ?$

f(x) me Kya Rakhe Ki 1 आ Jaye.

$f(0) = e^{\frac{0}{2}+0^2} = e^0 = 1$

$g'(f(x)) = \dfrac{1}{f'(x)}$

$g'(f(0)) = \dfrac{1}{f'(0)}$

$f(x) = e^{\frac{x}{2}+x^3}$

$f'(x) = e^{\frac{x}{2}+x^3} \times \left(\frac{1}{2}+3x^2\right)$

$f'(0) = e^0 \times \left(\frac{1}{2}+3\times 0\right) = \frac{1}{2}$

$= \dfrac{1}{1/2}$

$= 2$

Nishant Jindal

Q $f(x) = \sin^{-1}\{[3x+2] - \{3x + (x - \{2x\})\}\}$; $x \in (0, \frac{\pi}{12})$ & $g \circ f(x) = x \ \forall x \in (0, \frac{\pi}{12})$ find $g'(\frac{\pi}{6}) = ?$

$\{3x + (x - 2x)\}$

1) $2x \in (0, \frac{\pi}{6}) \simeq (0, \frac{1}{2})$

$\{2x\} = 2x \qquad \{\frac{1}{3}\} = \frac{1}{3}$

2) $\sin^{-1}\{[3x+2] - \{3x - x\}\}$

3) $x \in (0, \frac{\pi}{12})$

$3x \in (0, \frac{\pi}{4})$

$3x \in (0, \frac{3.14}{4})$

$3x \in (0, .75)$

$3x + 2 \in (2, 2.75)$

$[3x+2] = 2$

1) $\{x+n\} = \{x\}$

2) $\{-h\} = 1 - h$

4) $= \sin^{-1}\{\cancel{2} - 2x\}$

5) $f(x) = \sin^{-1}\{-2x\} = \sin^{-1}(1-2x)$

6) $g(f(x)) = x$

$f(x) = g^{-1}(x)$

$g'(f(x)) = \frac{1}{f'(x)}$

7) $f(x) = \frac{\pi}{6}$ Kb hoga

$1 - 2x = \frac{1}{2} \Rightarrow \frac{1}{2} = 2x$

$x = \frac{1}{4}$

8) $f'(x) = \frac{1}{\sqrt{1-(1-2x)^2}} \times -2$

$f'(\frac{1}{4}) = \frac{-2}{\sqrt{1-(1-\frac{1}{2})^2}} = -\frac{4}{\sqrt{3}}$

9) $g'(\frac{\pi}{6}) = \frac{1}{f'(\frac{1}{4})}$

$= \frac{1}{-\frac{4}{\sqrt{3}}} = -\frac{\sqrt{3}}{4}$