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# Projectile on an Inclined plane :→

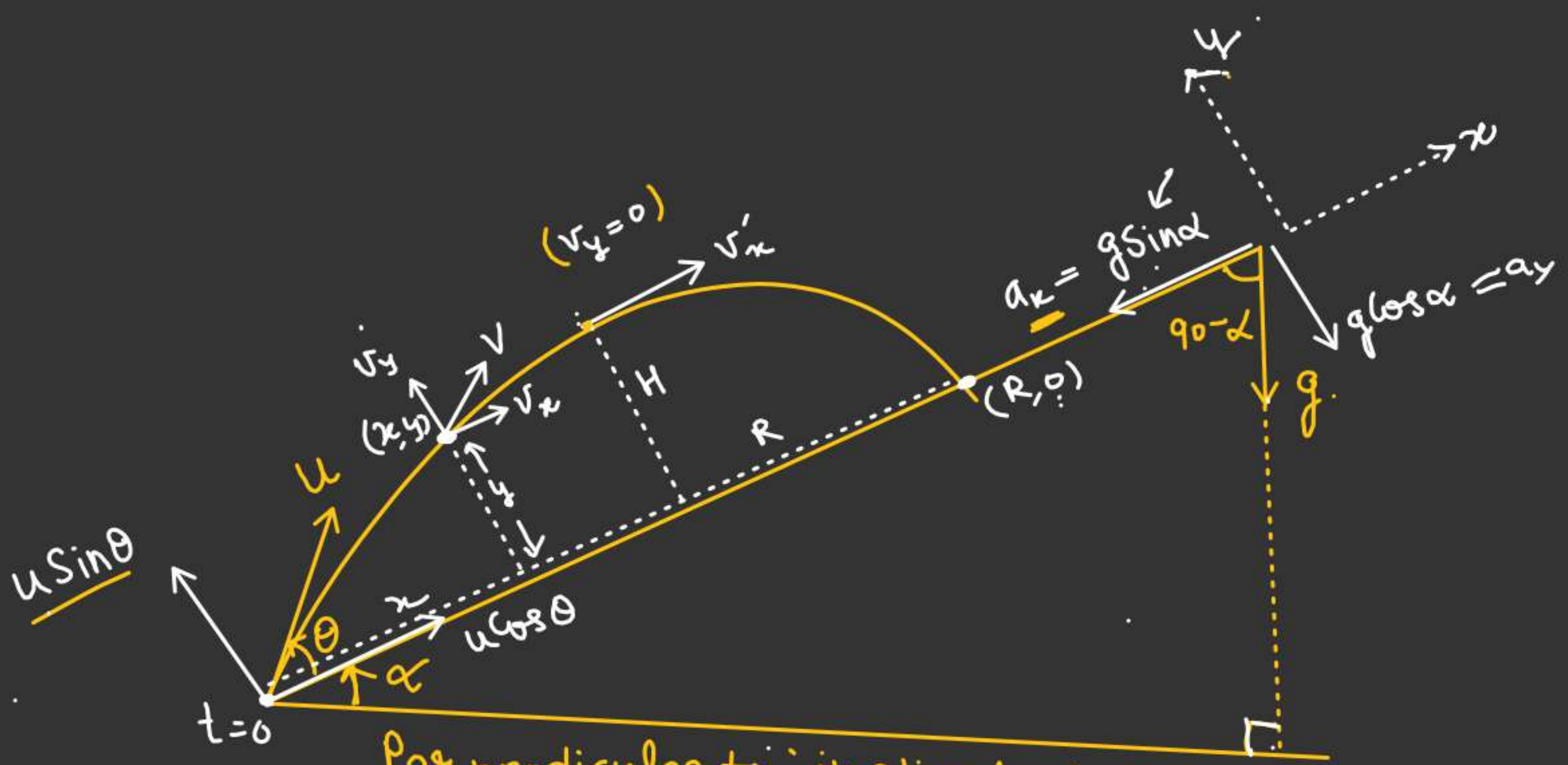
## Along the inclined plane

↳ (Motion is Constant retardation with $g\sin\alpha$)

① $v_x = (u\cos\theta) - (g\sin\alpha)t$

② $x = (u\cos\theta)t - \frac{1}{2}(g\sin\alpha)t^2$

③ $v_x^2 = (u\cos\theta)^2 - 2(g\sin\alpha)x$




## Perpendicular to inclined plane :-

$v_y = u\sin\theta - (g\cos\alpha)t$

$y = (u\sin\theta)t - \frac{1}{2}(g\cos\alpha)t^2$

$v_y^2 = (u\sin\theta)^2 - 2(g\cos\alpha)y$

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# Projectile Motion

## Time of flight:

At $t = T$, $y = 0$.

$$0 = (u\sin\theta)t - \frac{1}{2}g\cos\alpha\, t^2$$

$$t\left[u\sin\theta - \frac{1}{2}(g\cos\alpha)t\right] = 0$$

$$t = 0, \quad t = \left[\frac{2u\sin\theta}{g\cos\alpha}\right]$$

$$\Downarrow$$

$$\boxed{T = \frac{2u\sin\theta}{g\cos\alpha}}$$

$$\downarrow$$

$$\left(T = \frac{2u_y}{g_{eff}}\right)^{**}$$

$g_{eff}$ = [effective component of 'g' perpendicular to inclined plane]

## Maximum height:

For maximum height

$V_y = 0$, $y = H_{max}$

$$0 = u^2\sin^2\theta - 2g\cos\alpha \cdot H.$$

$$\boxed{H = \left(\frac{u^2\sin^2\theta}{2g\cos\alpha}\right).}$$

$$\Downarrow$$

$$\boxed{H = \frac{u_y^2}{2g_{eff}}}$$

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# Projectile Motion

Range: →

$$x = (u\cos\theta)t - \frac{1}{2}(g\sin\alpha)t^2$$

When $t = T$, $x = R$

$$R = (u\cos\theta)\left(\frac{2u\sin\theta}{g\cos\alpha}\right) - \frac{1}{2}(g\sin\alpha)\left(\frac{2u\sin\theta}{g\cos\alpha}\right)^2$$

$$R = \frac{2u\sin\theta}{g\cos\alpha}\left[u\cos\theta - \frac{1}{\cancel{2}} \times \frac{\cancel{2}u\sin\theta}{\cancel{g}\cos\alpha} \times g\sin\alpha\right]$$

$$R = \frac{2u^2\sin\theta}{g\cos\alpha}\left[\cos\theta - \frac{\sin\theta \cdot \sin\alpha}{\cos\alpha}\right]$$

$$R = \frac{2u^2\sin\theta}{g\cos^2\alpha}\left[\cos\theta \cdot \cos\alpha - \sin\theta \cdot \sin\alpha\right]$$

$$\boxed{R = \frac{2u^2\sin\theta}{g\cos^2\alpha}\cos(\theta+\alpha)}$$

$R = f(\theta)$

$R_{max} = ??$

For R to be maximum or minimum $\frac{dR}{d\theta} = 0$.

$$R = \underbrace{\left(\frac{2u^2}{g\cos^2\alpha}\right)}_{C}\left(\underbrace{\sin\theta}_{\mathrm{I}} \cdot \underbrace{\cos(\theta+\alpha)}_{\mathrm{II}}\right)$$

$$\frac{dR}{d\theta} = \frac{2u^2}{g\cos^2\alpha}\frac{d}{d\theta}\left[\underbrace{\sin\theta}_{\mathrm{I}} \cdot \underbrace{\cos(\theta+\alpha)}_{\mathrm{II}}\right]$$

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# Projectile Motion

$$\frac{dR}{d\theta} = \frac{2u^2}{g\cos^2\alpha}\left[\sin\theta \cdot \frac{d}{d\theta}\cos(\theta+\alpha) + \cos(\theta+\alpha)\frac{d}{d\theta}(\sin\theta)\right]$$

$$\frac{d}{d\theta}\cos(\theta+\alpha) =$$

$$\theta+\alpha = t$$

$$\frac{d}{dt}\cos(t) \times \frac{dt}{d\theta} = -\sin(\theta+\alpha)$$

$$\frac{dR}{d\theta} = \frac{2u^2}{g\cos^2\alpha}\left[\sin\theta\,[-\sin(\theta+\alpha)] + \cos(\theta+\alpha)\cos\theta\right]$$

$$\frac{dR}{d\theta} = \frac{2u^2}{g\cos^2\alpha}\left[-\underset{A}{\sin\theta}\cdot\underset{B}{\sin(\theta+\alpha)} + \cos\alpha\cdot\cos(\theta+\alpha)\right]$$

$$\frac{dR}{d\theta} = \frac{2u^2}{g\cos^2\alpha}\left[\cos[\theta+\theta+\alpha]\right]$$

$$\frac{dR}{d\theta} = \left(\frac{2u^2}{g\cos^2\alpha}\right)\left[\cos(2\theta+\alpha)\right]$$

$$\left(\frac{dR}{d\theta} = 0\right)$$

$$\cos(2\theta+\alpha) = 0$$

$$2\theta+\alpha = \frac{\pi}{2}$$

$$2\theta = \frac{\pi}{2}-\alpha$$

$$\boxed{\theta = \frac{\pi}{4}-\frac{\alpha}{2}}\ **$$

$\theta \rightarrow$ Angle from inclined plane

$\alpha \rightarrow$ Angle of Inclination

[Critical '$\theta$' for Range to be maximum]

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# Projectile Motion

$R_{max} = ??$

$$R = \frac{2u^2 \sin\theta}{g\cos^2\alpha}\left(\cos(\theta+\alpha)\right)$$

For $R_{max}$, $\theta = \left(\frac{\pi}{4} - \frac{\alpha}{2}\right)$

$$R = \frac{u^2}{g\cos^2\alpha}\left[2\sin\underset{A}{\theta}\cdot\cos\underset{B}{(\theta+\alpha)}\right]$$

$$R = \frac{u^2}{g\cos^2\alpha}\left[\sin(2\theta+\alpha) + \sin(\theta-(\theta+\alpha))\right]$$

$$R = \frac{u^2}{g\cos^2\alpha}\left[\sin(2\theta+\alpha) - \sin\alpha\right]$$

$2\sin A\cdot\cos B = \sin(a+b) + \sin(a-b)$

$$R_{max} = \frac{u^2}{g\cos^2\alpha}\left[\sin\left(2\times\left(\frac{\pi}{4}-\frac{\alpha}{2}\right)+\alpha\right) - \sin\alpha\right]$$

$$R_{max} = \frac{u^2}{g\cos^2\alpha}\left[\sin\left(\frac{\pi}{2}-\alpha+\alpha\right) - \sin\alpha\right]$$

$$R_{max} = \frac{u^2}{g\cos^2\alpha}\left[\sin\left(\frac{\pi}{2}\right) - \sin\alpha\right]$$

$$R_{max} = \frac{u^2}{g(1-\sin^2\alpha)}(1-\sin\alpha)$$

$$R_{max} = \frac{u^2}{g\cancel{(1-\sin\alpha)}(1+\sin\alpha)}\cancel{(1-\sin\alpha)}$$

**

$$\boxed{R_{max} = \frac{u^2}{g(1+\sin\alpha)}}$$

$\alpha$ = Angle of Inclination

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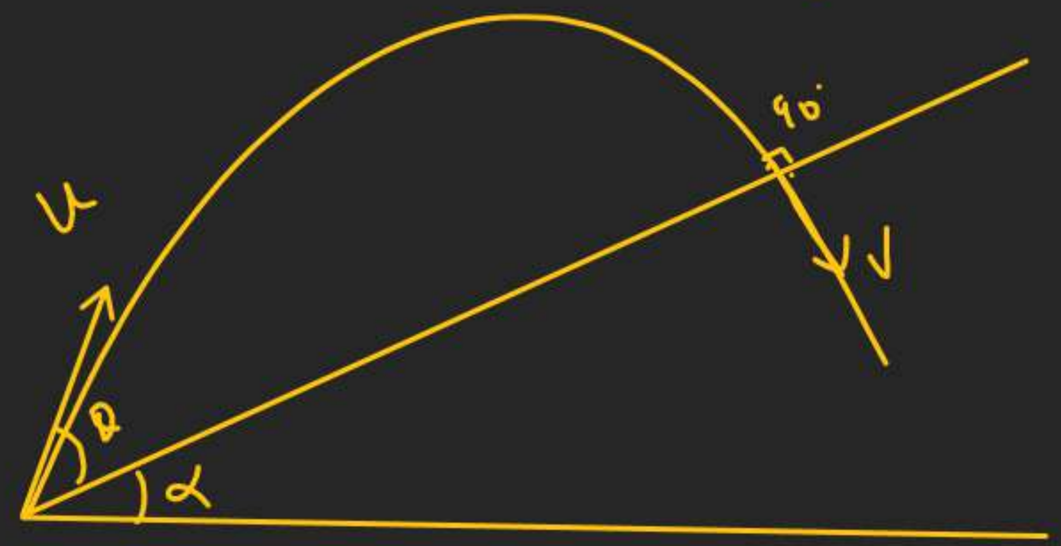
# Projectile Motion

H.W

Condition for projectile to hit the inclined plane perpendicularly :- ?? (Relation b/w $\theta$ & $\alpha$)




* Condition to hit the inclined plane horizontally.