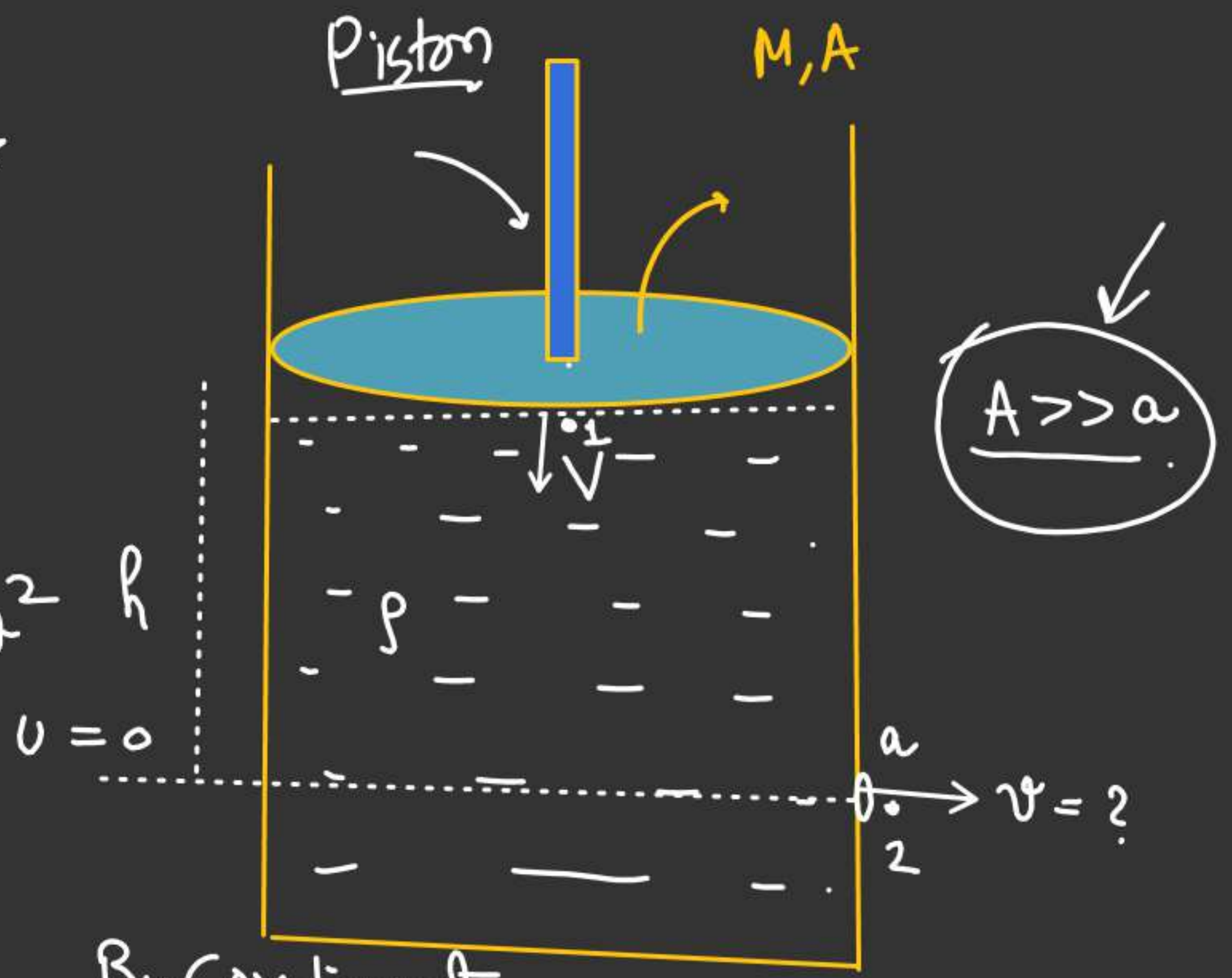
Nishant Jindal

FLUID DYNAMICS

Bernoulli's b/w 1 & 2.

$$\frac{P_{atm}\downarrow \ \downarrow \frac{Mg}{A}}{\bullet\, 1}$$



$$P_1 + \frac{1}{2}\rho V^2 + \rho g h$$

$$= P_{atm} + \frac{1}{2}\rho v^2$$

$$\cancel{P_{atm}} + \frac{Mg}{A} + \left(\frac{1}{2}\rho V^2\right) + \rho g h = \cancel{P_{atm}} + \frac{1}{2}\rho v^2$$

$$\frac{Mg}{A} + \rho g h = \frac{1}{2}\rho\left(v^2 - V^2\right)$$

$$= \frac{1}{2}\rho v^2\left(1 - \frac{a^2}{A^2}\right)$$

$$\frac{a^2}{A^2} \to 0$$

By Continuity

$$AV = av$$

$$V = \left(\frac{av}{A}\right)$$

$$\frac{Mg}{A} + \rho g h = \frac{1}{2}\rho v^2$$

Velocity of efflux ← $v = \sqrt{\frac{2Mg}{\rho A} + 2gh}$

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Time to empty the tank

$$-\frac{dy}{dt} = V$$

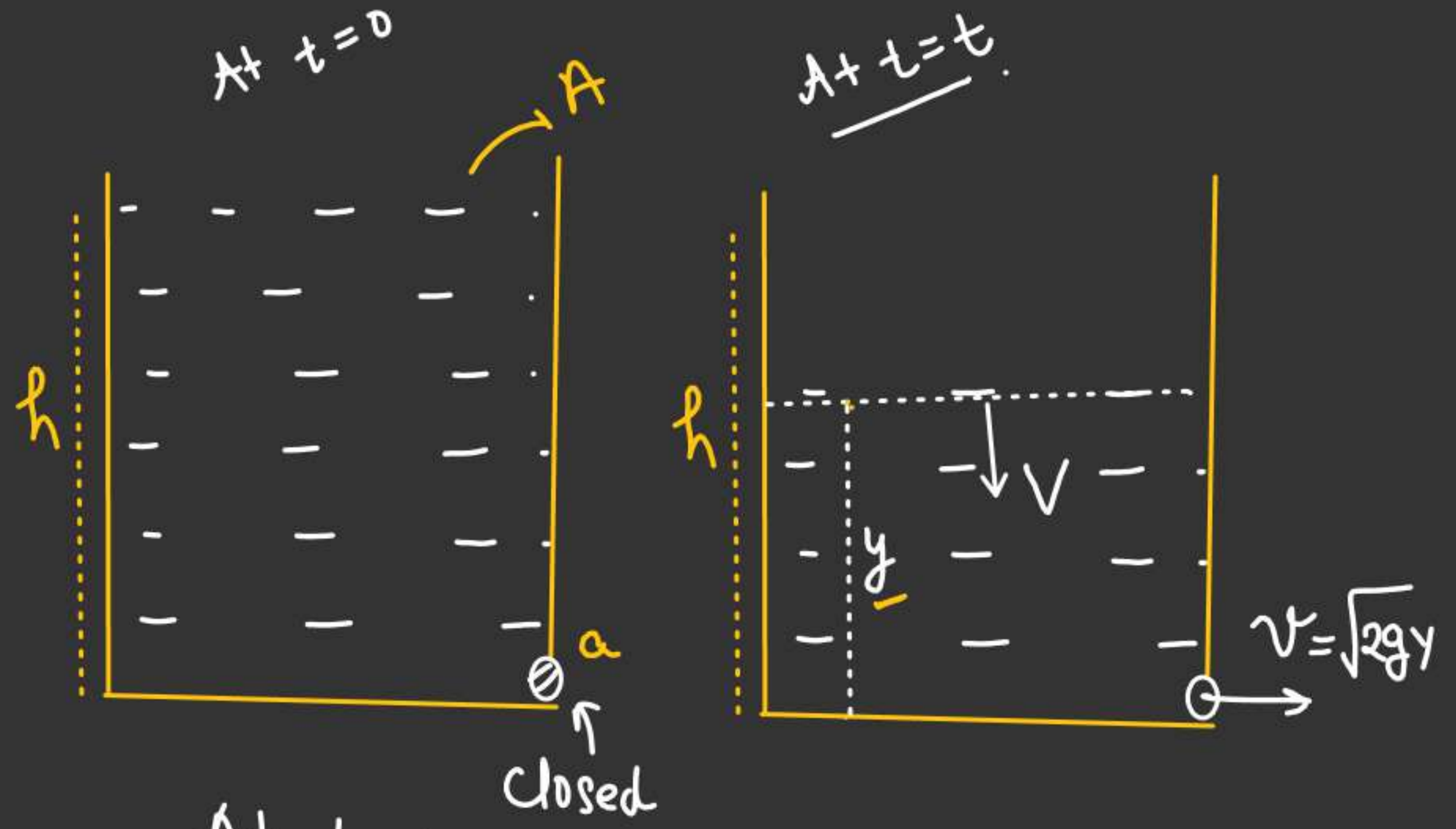
By continuity equation

$$AV = av$$

$$V = \frac{a}{A}v$$

$$-\frac{dy}{dt} = \frac{a}{A}\sqrt{2gy}$$

$$-\int_h^y \frac{dy}{\sqrt{y}} = \frac{a}{A}\sqrt{2g}\int_0^t dt$$



At $t=0$, opened.

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FLUID DYNAMICS

$$-\int_h^y \frac{dy}{\sqrt{y}} = \frac{a}{A}\sqrt{2g}\int_0^t dt$$

$$-2\left[\sqrt{y}\right]_h^y = \frac{a}{A}\sqrt{2g}\, t$$

$$-2\left[\sqrt{y}-\sqrt{h}\right] = \frac{a}{A}\sqrt{2g}\, t$$

$$\boxed{t = \frac{A}{a}\sqrt{\frac{2}{g}}\left(\sqrt{h}-\sqrt{y}\right)}$$

$\Rightarrow$ Time to empty the tank.

$y = 0$

$$\boxed{T = \frac{A}{a}\sqrt{\frac{2h}{g}}}$$

$\Rightarrow$ Find Ratio of time taken to empty half of the tank to time taken to empty the tank completely

$y = \frac{h}{2}$

$$t_1 = \frac{A}{a}\sqrt{\frac{2h}{g}}\left(\frac{\sqrt{2}-1}{\sqrt{2}}\right),$$

$y = 0$

$$t_2 = \frac{A}{a}\sqrt{\frac{2h}{g}}$$

$$\frac{t_1}{t_2} = \left(\frac{\sqrt{2}-1}{\sqrt{2}}\right) ✓$$

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FLUID DYNAMICS

By Continuity

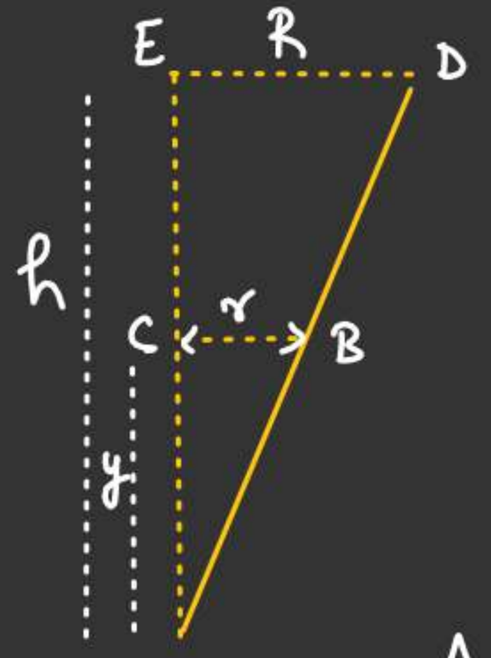
$$(\pi r^2)\underline{V} = a\underline{v}$$

$$(\pi \underline{r^2})\left(-\frac{dy}{dt}\right) = a\sqrt{2gy}$$

$$\left(\pi \frac{R^2}{h^2}\underline{y^2}\right)\left(-\frac{dy}{dt}\right) = a\sqrt{2gy}$$

$$-\left(\frac{\pi R^2}{h^2}\right)\int_h^0 \frac{y^2\,dy}{\sqrt{y}} = a\sqrt{2g}\int_0^T dt$$

$$-\frac{\pi R^2}{h^2}\int_h^0 y^{3/2}\,dy = a\sqrt{2g}\;T$$

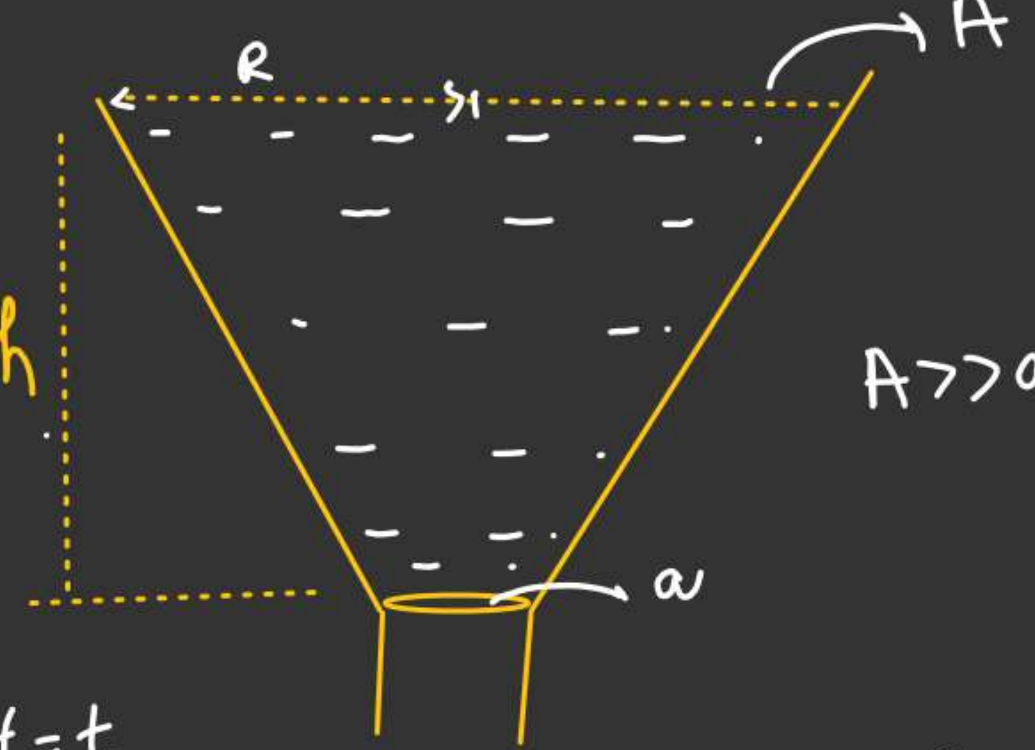


By Similarity

$$\frac{r}{y} = \frac{R}{h}$$

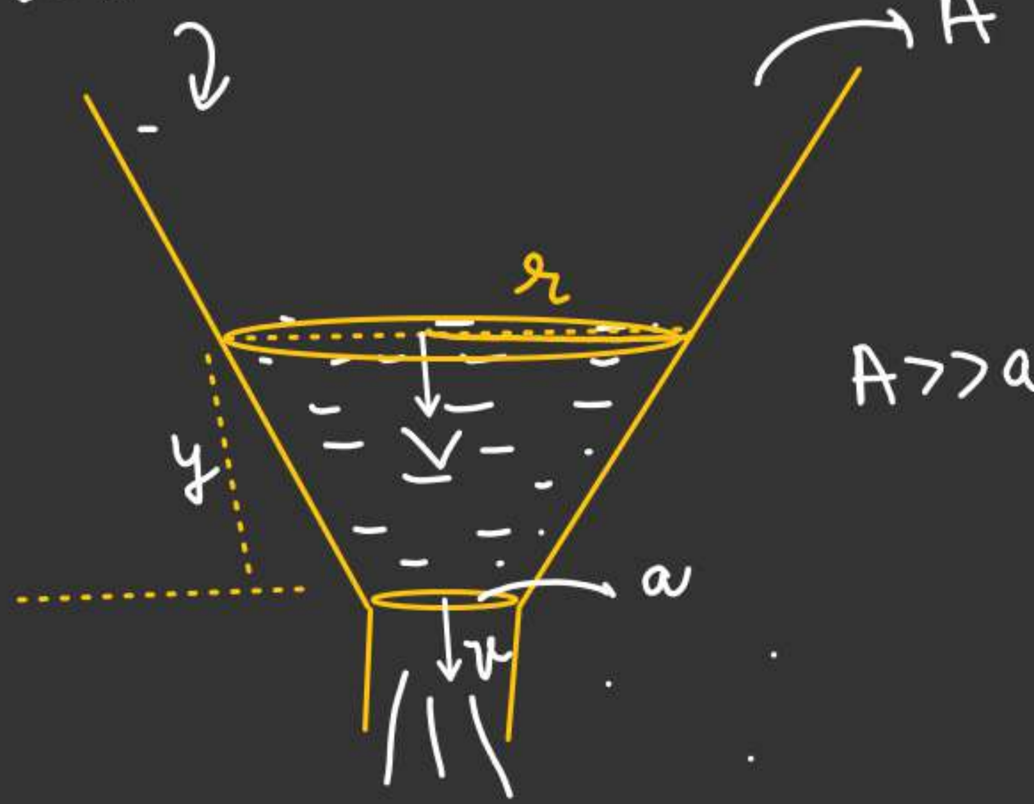
$$r = \left(\frac{R y}{h}\right)$$

At $t=0$, orffice opened.



$A >> a$

At $t=t$



$A >> a$

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$$-\frac{\pi R^2}{h^2}\int_h^0 y^{3/2}\,dy = a\sqrt{2g}\,T$$

$$-\frac{\pi R^2}{h^2}\left[\frac{y^{5/2}}{5/2}\right]_h^0 = a\sqrt{2g}\,T$$

$$\frac{2\pi R^2}{5h^2}\times h^{5/2} = a\sqrt{2g}\,T$$

$$\frac{\pi R^2}{5a}\sqrt{\frac{2h}{g}} = T$$

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FLUID DYNAMICS

By Continuity

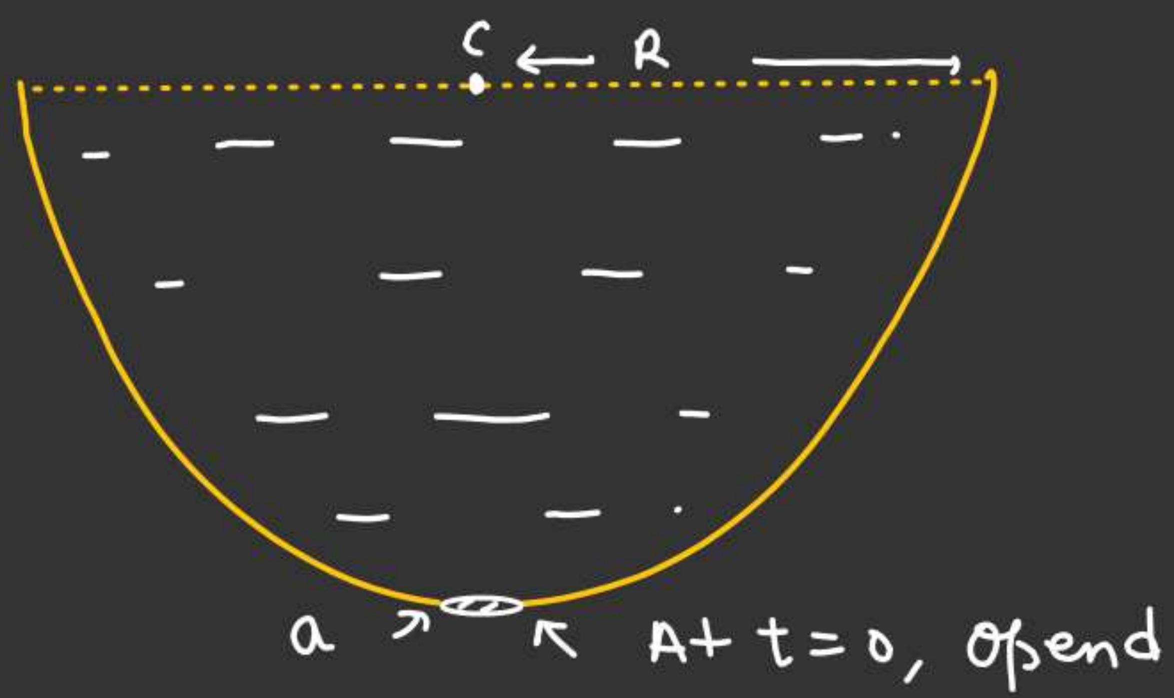
$$(\pi r^2)\left(-\frac{dy}{dt}\right) = a\sqrt{2gy}$$

$$(R-y)^2 + \underline{r^2} = R^2$$

$$r^2 = R^2 - (R-y)^2$$

$$r^2 = R^2 - (R^2 + y^2 - 2Ry)$$

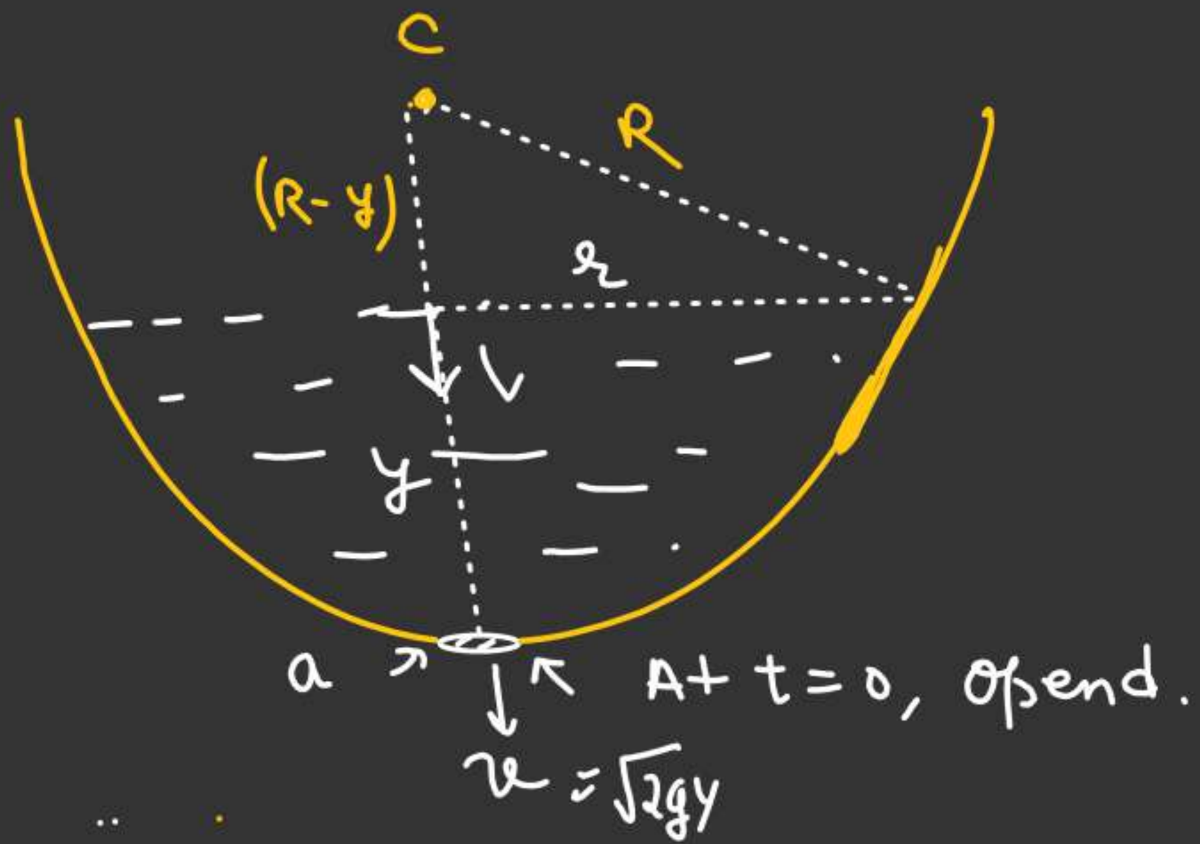
$$r^2 = (2Ry - y^2)$$



$$\pi(2Ry - y^2)\,\frac{-dy}{dt} = a\sqrt{2g}\,\sqrt{y}$$

$$-\int_R^0 \pi\left(\frac{2Ry - y^2}{y^{1/2}}\right)dy = a\sqrt{2g}\int_0^T dt$$

$T = ??$



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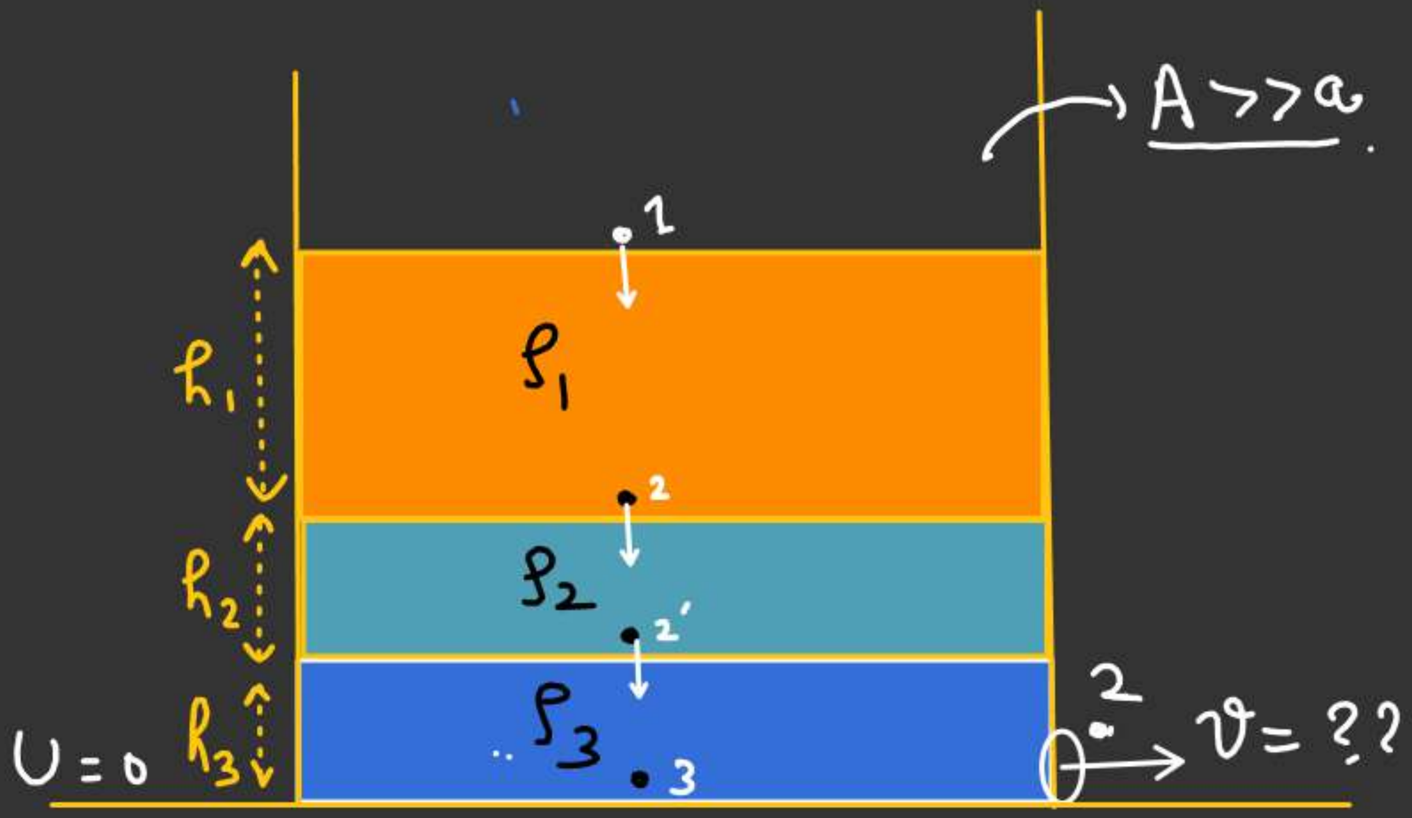
Bernoulli's Equation in Case of two or more than two liquids

$$P_1 = P_2 = P_{atm}.$$

$$\cancel{P_{atm}} + \rho_1 g h_1 + \rho_2 g h_2 + \rho_3 g h_3$$

$$= \cancel{P_{atm}} + \frac{1}{2}\rho_3 v^2$$

$$v = \sqrt{2g\left(\frac{\rho_1 h_1 + \rho_2 h_2 + \rho_3 h_3}{\rho_3}\right)}$$



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Bernoulli's Equation in Case of two or more than two liquids

$P_1 = P_2 = P_{atm}$.

Bernoulli's b/w 1 & 1'

$$P_{atm} + \cancel{\tfrac{1}{2}\rho_1 v_1^2} + \rho_1 g h_1 = P_1' + \cancel{\tfrac{1}{2}\rho v_1^2}$$

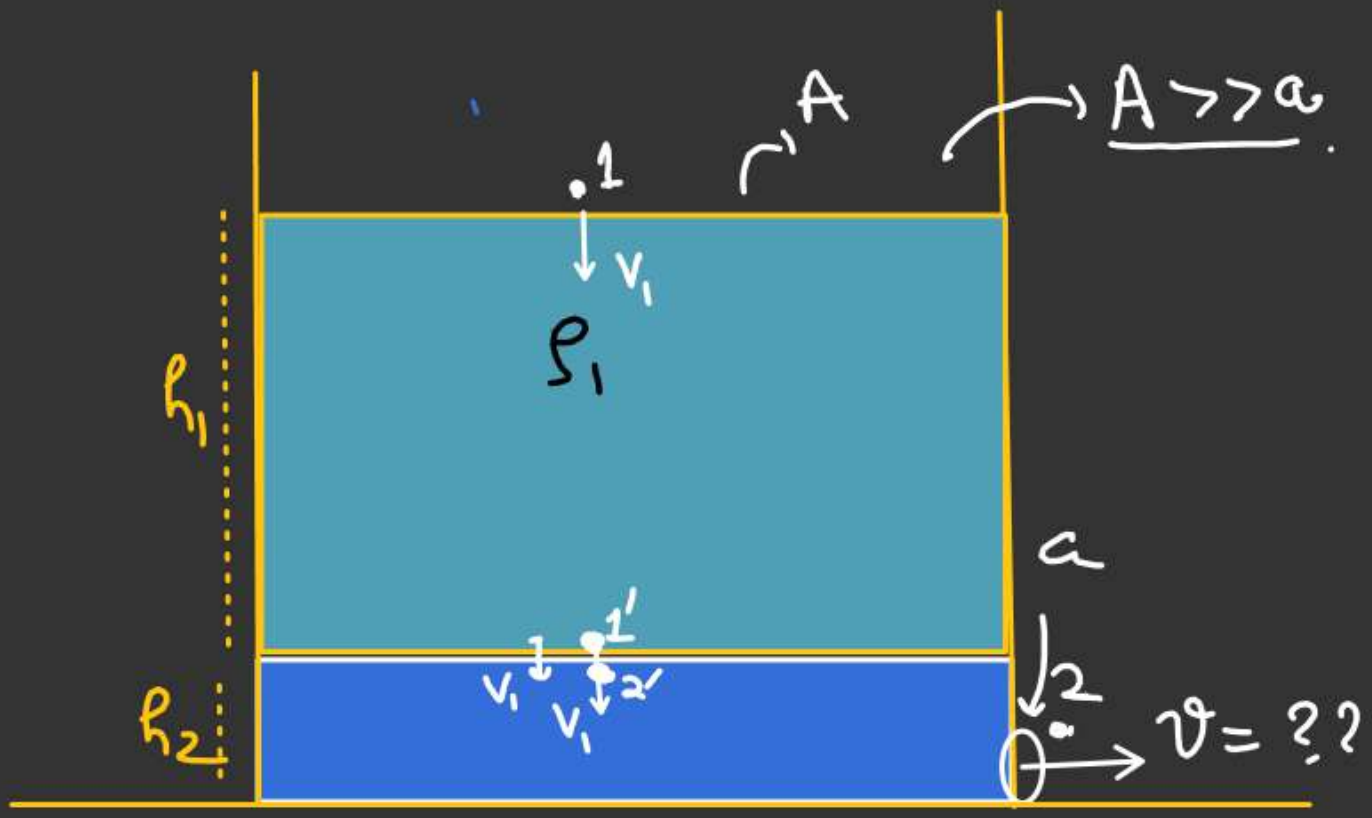
$$P_1' = P_{atm} + \rho_1 g h_1$$

Bernoulli's b/w 2 & 2'

$P_1' = P_2'$

$$P_1' + \tfrac{1}{2}\rho_2 V_1^2 + \rho_2 g h_2 = P_{atm} + \tfrac{1}{2}\rho_2 v^2$$

$$\cancel{P_{atm}} + \rho_1 g h_1 + \rho_2 g h_2 = \cancel{P_{atm}} + \tfrac{1}{2}\rho_2 v^2 - \tfrac{1}{2}\rho_2 V_1^2$$

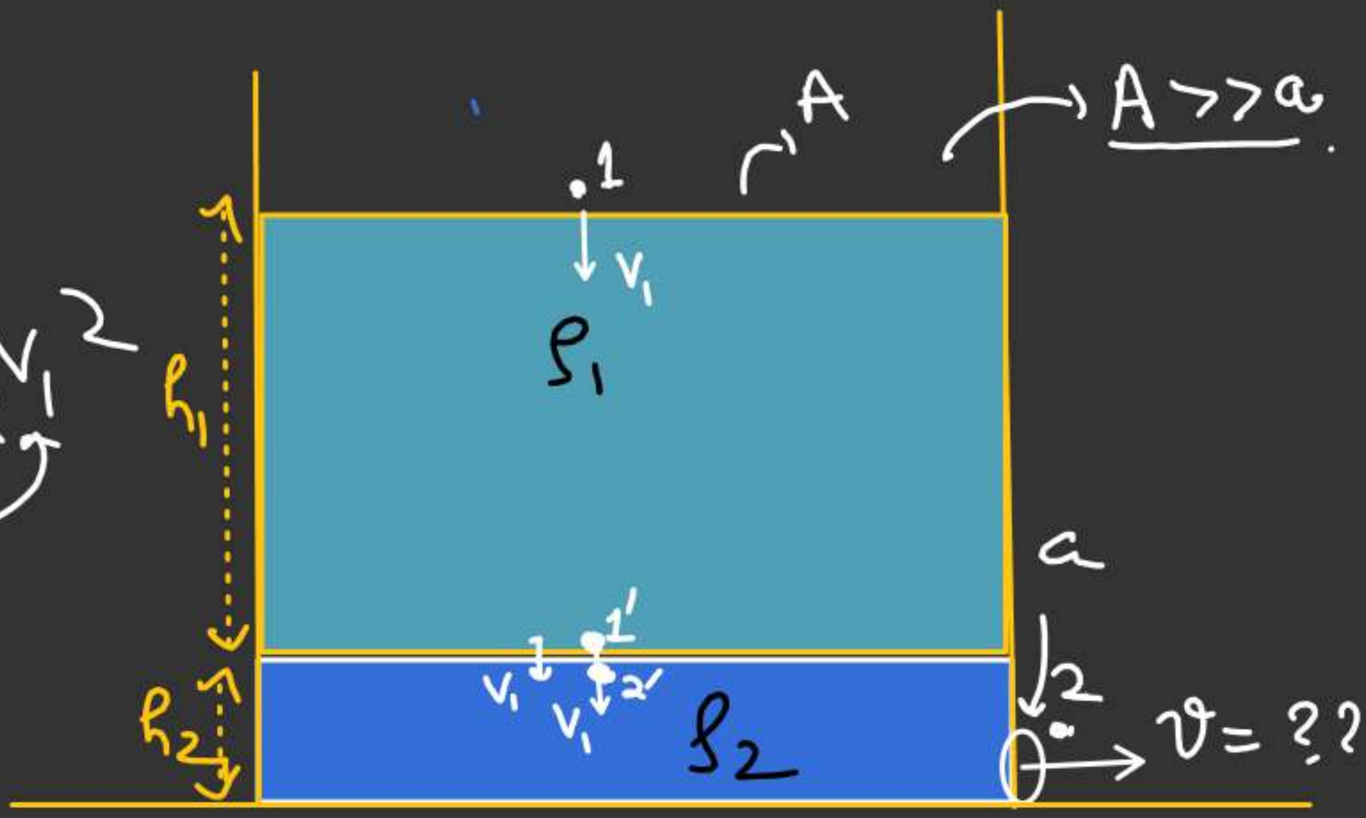


By Continuity

$$AV_1 = av$$

$$V_1 = av/A$$

Bernoulli's Equation in Case of two or more than two liquids



$$\cancel{P_{atm}} + \rho_1 g h_1 + \rho_2 g h_2 = \cancel{P_{atm}} + \frac{1}{2}\rho_2 v^2 - \frac{1}{2}\rho_2 V_1^2$$

By Continuity

$$AV_1 = av$$

$$V_1 = av/A$$

$$\rho_1 g h_1 + \rho_2 g h_2 = \frac{1}{2}\rho_2 v^2\left(1 - \underbrace{a^2/A^2}_{0}\right)$$

$a << A$

$$v = \sqrt{2g\left(\frac{\rho_1 h_1 + \rho_2 h_2}{\rho_2}\right)}$$

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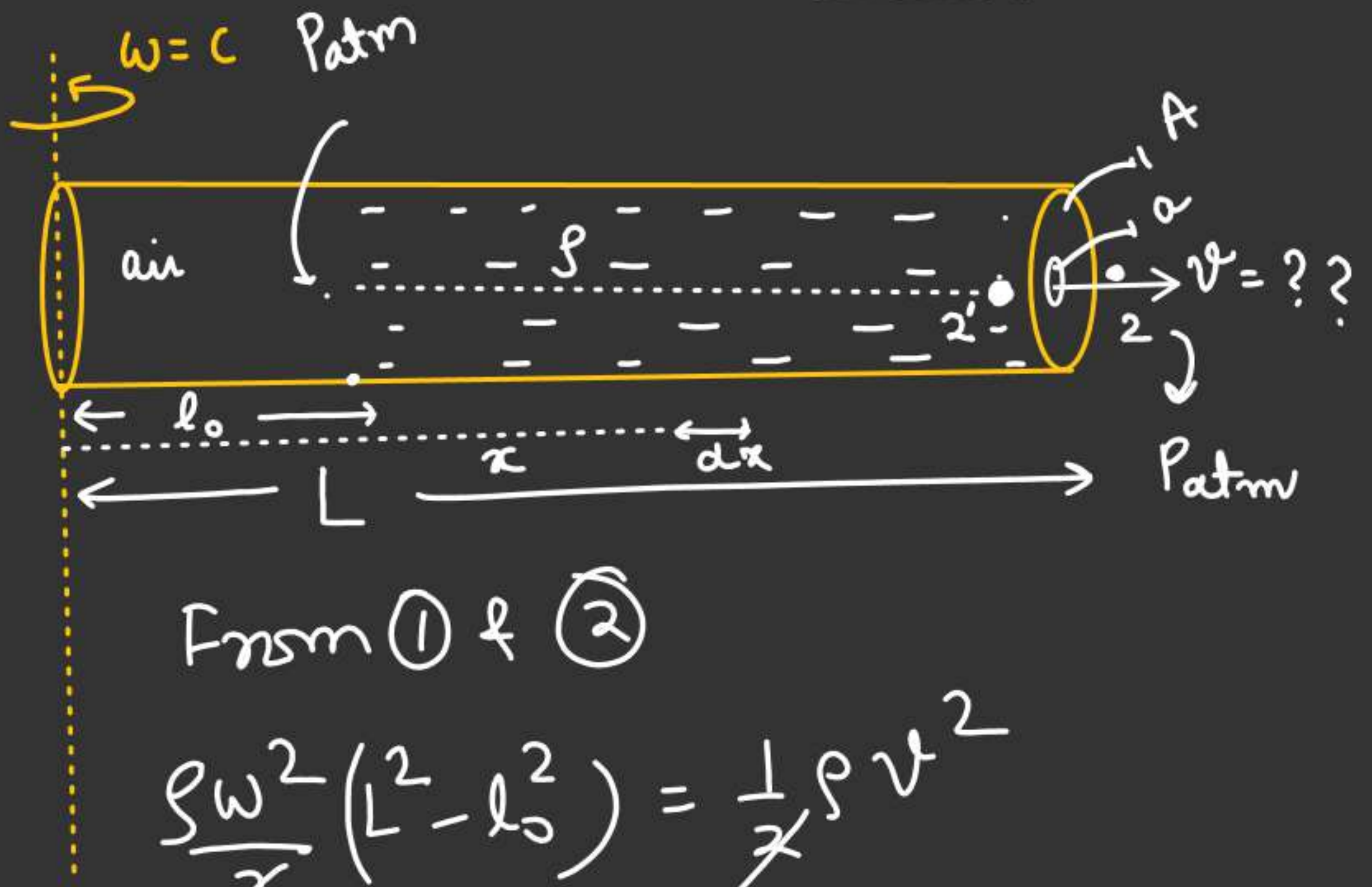
Velocity of efflux in rotating frame

$A >> a$

Bernoulli's b/w $2'$ & 2.

$$P_{2'} + \cancel{\tfrac{1}{2}\rho v^2}^{\,0} = P_{atm} + \frac{1}{2}\rho v^2$$

$$P_{2'} = P_{atm} + \frac{1}{2}\rho v^2 \quad \text{—(1)}$$



$$\frac{dP}{dx} = \rho \omega^2 x$$

$$\int_{P_{atm}}^{P_{2'}} dP = \rho\omega^2 \int_{l_0}^{L} x\,dx$$

$$P_{2'} - P_{atm} = \frac{\rho\omega^2}{2}\left(L^2 - l_0^2\right)$$

$$P_2' = P_{atm} + \frac{\rho\omega^2}{2}\left(L^2 - l_0^2\right) \quad \text{—(2)}$$

From (1) & (2)

$$\frac{\rho\omega^2}{\cancel{2}}\left(L^2 - l_0^2\right) = \frac{1}{\cancel{2}}\rho v^2$$

$$v = \omega\sqrt{L^2 - l_0^2}$$

$$v = \left(\omega L\sqrt{1 - \frac{l_0^2}{L^2}}\right) \checkmark$$