DPP-01

ORDER/DEGREE

1. The order and degree of the differential equation $\left(1+3\frac{dy}{dx}\right)^{\frac{2}{3}}=4\frac{d^3y}{dx^3}$ are

(A) $1,\frac{2}{3}$ (B) 3, 1 (C) 1,2 (D) 3,3

Ans. (D)

Sol.

D

$$\left(1+3\frac{dy}{dx}\right)^{2/3}=4\frac{d^3y}{dx^3}$$

$$\left(1+\frac{3dy}{dx}\right)^{2}=\left(4\frac{d^3y}{dx^3}\right)^{3}$$

order = 3

Degree = 3

2. The order and degree of the differential equation $\sqrt[3]{\frac{dy}{dx}}-4\frac{d^2y}{dx^2}-7x=0$ are a and b, then $a+b$ is

(A) 3 (B) 4 (C) 5 (D) 6

Ans. (C)

Sol.

C

$$\left(\frac{dy}{dx}\right)^{1/3}=4\frac{d^2y}{dx^2}+7x$$

$$\left(\frac{dy}{dx}\right)=\left(4\frac{d^2y}{dx^2}+7x\right)^{3}$$

order =2 = a

degree = 3 = b

a + b = 5

3. The degree of the differential equation $\left(\frac{d^3y}{dx^3}\right)^{2/3}+4-3\frac{d^2y}{dx^2}+5\frac{dy}{dx}=0$ is

(A) 1 (B) 2 (C) 3 (D) Not defined

Ans. (B)

Sol.

[B]

We have, $\left(\frac{d^3y}{dx^3}\right)^{2/3} + 4 - 3\frac{d^2y}{dx^2} + 5\frac{dy}{dx} = 0$

$\Rightarrow \left(\frac{d^3y}{dx^3}\right)^2 = \left(3\frac{d^2y}{dx^2} - 5\frac{dy}{dx} - 4\right)^3$

Clearly, it is a differential equation of degree 2

4. The degree and order of the differential equation of the family of all parabolas whose axis is x - axis, are respectively-

(A) 2,3 (B) 2,1 (C) 1,2 (D) 3,2

Ans. (C)

Sol.

C

General equation of parabola whose axis is x-axis, is

$y^2 = 4a(x + h)$

On differentiationg w.r.t. x, we get

$2y\frac{dy}{dx} = 4a \Rightarrow y\frac{dy}{dx} = 2a$

Again, differentating w.r.t. x, we get

$y\left(\frac{dy}{dx}\right)^2 + y\frac{d^2y}{dx} = 0$

This is a differential equation whose degree and order are one and two respectively.

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5. The differential equation representing the family of curves $y^2 = 2c(x + \sqrt{c})$, where $c > 0$, is a parameter, is of order and degree as follows -

(A) order 1, degree 2 (B) order 1, degree 1

(C) order 1, degree 3 (D) order 2, degree 2

Ans. (C)

Sol.

C

Given equation of family of curves is

$y^2 - 2c(x + \sqrt{c})$ ...(i)

On differentiating Eq. (i) w.r.t. x, we get

$2yy_1 = 2c$

$\Rightarrow c = yy_1$

On putting the value of c in Eq. (i), we get

$y^2 = 2yy_1(x + \sqrt{yy_1})$

$\Rightarrow (y^2 - 2yy_1x)^2 = 4(yy_1)^3$

Hence, the degree and order of above equation are three and one respectively.

6. The differential equation whose solution is $Ax^2 + By^2 = 1$, where A and B are arbitrary constants is of -

(A) first order and second degree
(B) first order and first degree
(C) second order and first degree
(D) second order and second degree

Ans. (C)

Sol.

C

The given equation is $Ax^2 + By^2 = 1$

On differentiating w.r.t. x, we get

$$2Ax + 2By\frac{dy}{dx} = 0 \quad ...(i)$$

On again differentiating, we get

$$2A + 2B\left\{\left(\frac{dy}{dx}\right)^2 + y\frac{d^2y}{dx^2}\right\} = 0 \quad ...(ii)$$

Eliminating A and B from Eqs. (i) and (ii), we get

$$y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 - \frac{y}{x}\cdot\frac{dy}{dx} = 0$$

This is the required differential equation whose order is two and degree is one.

7. Which of the following differential equations has the same order and degree?

(A) $\frac{d^4y}{dx^4} + 8\left(\frac{dy}{dx}\right)^6 + 5y = e^x$
(B) $5\left(\frac{d^3y}{dx^3}\right)^4 + 8\left(1 + \frac{dy}{dx}\right)^2 + 5y = x^8$
(C) $\left[1 + \left(\frac{dy}{dx}\right)^3\right]^{2/3} = 4\frac{d^3y}{dx^3}$
(D) $y = x^2\frac{dy}{dx} + \sqrt{1 + \left(\frac{dy}{dx}\right)^2}$

Ans. (C)

Sol.

C

Clearly

$$\left[1 + \left(\frac{dy}{dx}\right)^3\right]^2 = \left[4\frac{d^3y}{dx^3}\right]^3$$

Has order = 3 and degree = 3

8. The differential equation $\frac{d^2y}{dx^2} + \frac{dy}{dx} + \sin\ y + x^2 = 0$ is of the following type

(A) linear
(B) homogeneous
(C) order two
(D) degree one

Ans. (CD)

Sol.

C,D

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} + \sin y + x^2 = 0$$

Order = 2, Degree = 1

FORMATION OF DIFFERENTIAL EQUATION

9. Number of values of $m \in N$ for which $y = e^{mx}$ is a solution of the differential equation $D^3y - 3D^2y - 4Dy + 12y = 0$ is

(A) 0 (B) 1 (C) 2 (D) more than 2

Ans. (C)

Sol.

C

$y = e^{mx}$

$D^3y - 3D^2y - 4\,Dy + 12\,y = 0$

$m^3 e^{mx} - 3m^2 e^{mx} - 4me^{mx} + 12\,e^{mx} = 0$

$m^3 - 3m^2 - 4m + 12 = 0$

$m^2(m-3) - 4(m-3) = 0$

$m = 3, 2, -2$

Two Natural numnber of m possible

10. The value of the constant ' m ' and ' c ' for which $y = \mathrm{mx} + \mathrm{c}$ is a solution of the differential equation $D^2y - 3Dy - 4y = -4x$

(A) is $\mathrm{m} = -1, \mathrm{c} = 3/4$ (B) is $m = 1, c = 3/4$

(C) no such real m, c (D) is $m = 1, c = -3/4$

Ans.

Sol.

D

$y = mx + c$

$y' = m$

$D^2y - 3Dy - 4y = -4x$

$0 - 3m - 4(mx + c) = -4x$

$-3m - 4mx - 4C = -4x$

$-4m = -4 \quad \Rightarrow m = 1$

$-3m - 4C = 0 \quad \Rightarrow \quad 4C = -3m \quad \Rightarrow \quad C = -\frac{3}{4}$

11. The differential equation of the family of curves represented by $y = a + bx + ce^{-x}$ (where a, b, c are arbitrary constants) is

(A) $y''' = y'$ (B) $y''' + y'' = 0$

(C) $y''' - y'' + y' = 0$ (D) $y''' + y'' - y' = 0$

Ans. (B)

Sol.

B

$y = a + bx + ce^{-x}$

$y' = b - ce^{-x}$

$y'' = ce^{-x}$

$y''' = -ce^{-x}$

$y''' = -y'' \quad \Rightarrow \quad y''' + y'' = 0$

12. The differential equation whose solution is $(x-h)^2+(y-k)^2=a^2$ is (where a is a constant)

(A) $\left[1+\left(\frac{dy}{dx}\right)^2\right]^3 = a^2\left(\frac{d^2y}{dx^2}\right)^2$ (B) $\left[1+\left(\frac{dy}{dx}\right)^2\right]^3 = a^2\frac{d^2y}{dx^2}$

(C) $\left[1+\left(\frac{dy}{dx}\right)\right]^3 = a^2\left(\frac{d^2y}{dx^2}\right)^2$ (D) $\left[1+\left(\frac{dy}{dx}\right)^3\right]^2 = a^2\left(\frac{d^2y}{dx^2}\right)^2$

Ans. (A)

Sol.

$(x-h)^2+(y-k)^2=a^2$

$(x-h)+(y-k)\,y'=0 \quad\Rightarrow\quad y' = \frac{-(x-h)}{(y-k)}$

$1+(y-k)\,y''+(y')^2=0 \quad\Rightarrow\quad y'' = \frac{-a^2}{4(y-k)^3}$

(A) option satisfy the given conditions

13. The differential equation representing all line at a distance p from the origin is-

(A) $(x^2+y^2)\frac{dy}{dx} = 2y\left\{x-p\left(\frac{dy}{dx}\right)^2\right\}$ (B) $\left(x\frac{dy}{dx}-y\right)^2 - p^2\left\{1+\left(\frac{dy}{dx}\right)^2\right\}=0$

(C) $\left(x\frac{dy}{dx}-y\right)\left(p\frac{dy}{dx}+x\frac{dy}{dx}\right)=0$ (D) $(x-y)\left(\frac{dy}{dx}-\frac{dx}{dy}\right)=0$

Ans. (B)

Sol.

All lines at a constant distance p from the origin are tangent to the circle, $x^2+y^2=p^2$ $\Rightarrow$ Equation to the family of such line are, $y = mx \pm p\sqrt{1+m^2}$

Put $m = \frac{dy}{dx}$ and get the result.

14. If $y = e^{(K+1)x}$ is a solution of differential equation $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 0$, then k equals

(A) -1 (B) 0 (C) 1 (D) 2

Ans. (C)

Sol.

$y = e^{(k+1)x}$

$y' = (k+1)e^{(k+1)x}$

$y'' = (k+1)^2\,e^{(k+1)x}$

$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 0$

$(k+1)^2 - 4(k+1) + 4 = 0$

$k^2 + 2k + 1 - 4k = 0$

$(k-1)^2 = 0$

$k = 1$

15. The differential equation, which represents the family of plane curves $y = e^{cx}$, is-
(A) $y' = \text{cy}$ (B) $xy' - \log y = 0$ (C) $x\log y = yy'$ (D) $y\log y = xy'$

Ans. (D)

Sol.

$y = e^{cx} \Rightarrow \log y = cx$

Diff. w.r.t x,

$$\frac{1}{y} \cdot y' = C \quad \Rightarrow \frac{y'}{y} = \frac{\log y}{x}$$

$\Rightarrow y \log y = xy'$

16. The differential equation $2xydy = (x^2 + y^2 + 1)dx$ determines
(A) A family of circles with centre on x-axis
(B) A family of circles with centre on y-axis
(C) A family of rectangular hyperbola with centre on x-axis
(D) A family of rectangular hyperbola with centre on yaxis

Ans. (C)

Sol.

$$2xy \frac{dy}{dx} = (x^2 + y^2 + 1)$$

Put $x^2 + y^2 + 1 = t$

$$2x + 2y \frac{dy}{dx} = \frac{dt}{dx}$$

$$2x^2 + 2xy \frac{dy}{dx} = x \frac{dt}{dx}$$

$$x \frac{dt}{dx} - 2x^2 = t \quad \Rightarrow \quad x \frac{dt}{dx} = t + 2x^2$$

$$\frac{dt}{dx} - \frac{t}{x} = 2x$$

$$\frac{t}{x} = 2x + C \Rightarrow t = 2x^2 + Cx$$

$x^2 + y^2 + 1 = 2x^2 + Cx$

$x^2 - y^2 + Cx - 1\, x = 0$

17. Let $y = (A + Bx)e^{3x}$ be a solution of the differential equation $\frac{d^2y}{dx^2} + m\frac{dy}{dx} + ny = 0, m, n \in I$, then
(A) $m + n = 3$
(B) $n^2 - m^2 = 64$
(C) $m = -6$
(D) $n = 9$

Ans. (ACD)

Sol.

$y = (A + Bx)\,e^{3x}$

$y' = 3(A + Bx)\,e^{3x} + Be^{3x} = e^{3x}(3A + B + 3Bx)$

$y' = 3y + Be^{3x}$

$y'' = 3y' + 3Be^{3x}$

By adding something and subtracting it will convert into

$y'' - 3y' = 3y + 3(B + 3A + 3Bx)\,e^{3x} - 12(A + Bx)\,e^{3x}$

$y'' - 3y' = 3y + 3y' - 12y$

$y'' - 6y' + 9y = 0$

$m = -6,\ n = 9$

VARIABLE SEPARABLE

18. The solution to the differential equation $y/ny + xy' = 0$, where $y(1) = e$, is

(A) $x(\text{ eny }) = 1$

(B) $xy(\text{ ny }) = 1$

(C) $(\text{ lny })^2 = 2$

(D) $\ln\ y + \left(\frac{x^2}{2}\right) y = 1$

Ans. (A)

Sol.

$y\ell ny + xy' = 0$

$y\ell ny + x\frac{dy}{dx} = 0 \quad \Rightarrow \quad \frac{dx}{x} + \frac{dy}{y\ell ny} = 0$

$\ell nx + \ell n(\ell ny) = \ell nC$

$x(\ell ny) = C$

$y(1) = e$

$\ell ne = C \Rightarrow C = 1 \qquad x(\ell ny) = 1$

19. The solution of $\frac{xdy}{x^2+y^2} = \left(\frac{y}{x^2+y^2} - 1\right) dx$ is

(A) $y = x\cot(c - x)$

(B) $\cos^{-1}\ y/x = -x + c$

(C) $y = x\tan(c - x)$

(D) $y^2/x^2 = x\tan(c - x)$

Ans. (C)

Sol.

$$\frac{xdy}{x^2+y^2} = \frac{ydx}{x^2+y^2} - dx$$

$$\frac{xdy - ydx}{x^2+y^2} = -dx$$

$$\frac{\frac{xdy - ydx}{x^2}}{1+\left(\frac{y}{x}\right)^2} = -dx \Rightarrow \frac{d(y/x)}{1+(y/x)^2} = -dx$$

$$d\left(\tan^{-1}\frac{y}{x}\right) = -dx \Rightarrow \tan^{-1}\frac{y}{x} = -x + C$$

$$\frac{y}{x} = \tan(C - x) \Rightarrow \quad y = x\tan(C - x)$$

20. The solution of the differential equation $dy = \sec^2 x\, dx$ is-

(A) $y = \sec x \tan x + c$ (B) $y = 2\sec x + c$

(C) $y = \frac{1}{2}\tan x + c$ (D) None of these

Ans. (D)

Sol.

$dy = \sec^2 x\, dx$

on integrating

$y = \tan x + c$

21. The solution of the differential equation $\frac{dy}{dx} = (1+x)(1+y^2)$ is-

(A) $y = \tan(x^2 + x + c)$ (B) $y = \tan(2x^2 + x + c)$

(C) $y = \tan(x^2 - x + c)$ (D) $y = \tan\left(\frac{x^2}{2} + x + c\right)$

Ans. (D)

Sol.

[D]

$$\frac{dy}{dx} = (1+x)(1+y^2)$$

$$\int \frac{dy}{1+y^2} = \int (1+x)\,dx$$

$$\Rightarrow \tan^{-1} y = x + \frac{x^2}{2} + c$$

$$\Rightarrow y = \tan\left(\frac{x^2}{2} + x + c\right)$$

22. The general solution of the differential equation, $y' + y\phi'(x) - \phi(x)\cdot\phi'(x) = 0$ where $\phi(x)$ is a known function is

(A) $y = ce^{-\phi(x)} + \phi(x) - 1$ (B) $y = ce^{\phi(x)} + \phi(x) + K$

(C) $y = ce^{-\phi(x)} - \phi(x) + 1$ (D) $y = ce^{-\phi(x)} + \phi(x) + K$

Ans. (A)

Sol.

[A]

$y' + y\phi' - \phi\phi' = 0$

$y' + \phi'(y - \phi) = 0$

$dy + \phi'(y - \phi)\,dx = 0$

Let $\phi = t$ $\Rightarrow \phi'\,dx = dt$

$dy + (y - t)\,dt = 0$

$$\frac{dy}{dt} + y = t$$

I.F. $= e^t$

$$ye^t = \int te^t\,dt$$

$ye^t = te^t - e^t + C$

$y = t - 1 + ce^{-t}$

$y = \phi(x) - 1 + ce^{-\phi(x)}$

23. The equation of the curve through the point (1,0), whose slope is $\frac{y-1}{x^2+x}$, is-

(A) $(y-1)(x+1)+2x=0$
(B) $2x(y-1)+x+1=0$
(C) $x(y-1)(x+1)+2=0$
(D) $x(y+1)+y(x+1)=0$

Ans. (A)

Sol.

A

$$\frac{dy}{dx}=\frac{y-1}{x(x+1)} \Rightarrow \frac{dy}{y-1}=\frac{dx}{x(x+1)}$$

$$\Rightarrow \int\frac{dy}{y-1}=\int\left(\frac{(x+1)-x}{x(x+1)}\right)dx$$

$$\Rightarrow \int\frac{dy}{y-1}=\int\left(\frac{1}{x}-\frac{1}{x+1}\right)dx$$

$$\Rightarrow \ln(y-1)=\ln|x|-\ln|x+1|+\ln C$$

$$\frac{(y-1)}{x}(x+1)=C\ (1,0)$$

$$\Rightarrow \frac{0-1}{1}(1+1)=C \Rightarrow C=-2$$

$$\Rightarrow (x+1)(y-1)+2x=0$$

24. The solution of the differential equation $ydx+(x+x^2y)dy=0$ is-

(A) $-\frac{1}{xy}=C$
(B) $-\frac{1}{xy}+\log y=C$
(C) $\frac{1}{xy}+\log y=C$
(D) $\log y=Cx$

Ans. (B)

Sol.

B

Given that, $y\,dx+(x+x^2y)\,dy=0$

$$\therefore \frac{y\,dx+x\,dy}{x^2y^2}=-\frac{1}{y}dy$$

$$\Rightarrow d\left(-\frac{1}{xy}\right)=-\frac{1}{y}dy$$

On integrating both sides, we get

$$-\frac{1}{xy}=-\log y+c$$

$$\Rightarrow -\frac{1}{xy}+\log y=c$$

25. A curve passing through (2,3) and satisfying the differential equation $\int_0^x ty(t)dt=x^2y(x),(x>0)$ is

(A) $x^2+y^2=13$
(B) $y^2=\frac{9}{2}x$
(C) $\frac{x^2}{8}+\frac{y^2}{18}=1$
(D) $xy=6$

Ans. (D)

Sol.

$$\int_0^x t\,y(t)dt = x^2 y(x)$$

Using leibnitz

$xy = 2xy + x^2y'$

$y = 2y + xy'$

$xy' + y = 0$

$$x\frac{dy}{dx} + y = 0 \Rightarrow \frac{dy}{y} + \frac{dx}{x} = 0$$

$\ell n\, xy = C$

$xy = k$; (2, 3) $\Rightarrow k = 6$

$xy = 6$

26. The solution of $x^2dy - y^2dx + xy^2(x-y)dy = 0$ is

(A) $\ln\left|\frac{x-y}{xy}\right| = \frac{y^2}{2} + c$

(B) $\ell\left|\frac{xy}{x-y}\right| = \frac{x^2}{2} + c$

(C) $\ell n\left|\frac{x-y}{xy}\right| = \frac{x^2}{2} + c$

(D) $\ell n\left|\frac{x-y}{xy}\right| = x + c$

Ans. (A)

Sol.

A

$x^2\,dy - y^2dx + x^2y^2\,dy - xy^3\,dy = 0$

$$\frac{1}{y^2}dy - \frac{1}{x^2}dx + dy - \frac{y}{x}dy = 0$$

$$d\left(\frac{1}{x} - \frac{1}{y}\right) = \left(\frac{y-x}{xy}\right)y\,dy$$

$$\frac{d\left(\frac{1}{x} - \frac{1}{y}\right)}{\left(\frac{1}{x} - \frac{1}{y}\right)} = ydy$$

$$\Rightarrow \ell n\left(\frac{1}{x} - \frac{1}{y}\right) = \frac{y^2}{2} + c$$

$$\ell n\left|\frac{y-x}{yx}\right| = \frac{y^2}{2} + c$$

$$\ell n\left|\frac{y-x}{yx}\right| = \frac{y^2}{2} + c$$

27. If $\int_a^x ty(t)dt = x^2 + y(x)$ then y as a function of x is

(A) $y = 2 - (2+a^2)e^{\frac{x^2-a^2}{2}}$

(B) $y = 1 - (2+a^2)e^{\frac{x^2-a^2}{2}}$

(C) $y = 2 - (1+a^2)e^{\frac{x^2-a^2}{2}}$

(D) $y = 2 + (2+a^2)e^{\frac{x^2+a^2}{2}}$

Ans. (A)

Sol.

$$\int_0^x t\,y(t)dt = x^2 + y(x)$$

$xy(x) = 2x + y'(x)$; $y(a) = -a^2$

$x(y-2) = y'$

$$xdx = \frac{dy}{y-2}$$

$$\frac{x^2}{2} = \ell n(y-2) + C$$

$$y - 2 = e^{\left(\frac{x^2}{2}-c\right)}$$

$$\Rightarrow c = \frac{a^2}{2} - \ell n(-a^2-2)$$

$$y = 2 - (a^2+2)\,e^{\frac{x^2-a^2}{2}}$$

28. $y = ae^{-1/x} + b$ is a solution of $\frac{dy}{dx} = \frac{y}{x^2}$ then

(A) $a \in R$

(B) $b = 0$

(C) $b = 1$

(D) a takes finite number of values

Ans. (AB)

Sol.

$$\frac{dy}{dx} = \frac{y}{x^2} \Rightarrow \frac{dy}{y} = \frac{dx}{x^2} \Rightarrow \text{In}y = -\frac{1}{x} + \text{Inc}$$

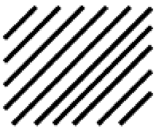
$$\Rightarrow \frac{y}{c} = e^{-\frac{1}{x}}$$

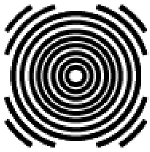
$$\Rightarrow y = ce^{-\frac{1}{x}}$$

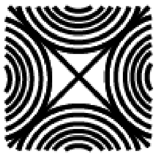
Comparing with $y = a\,e^{-1/x} + b$, $a \in R$, $b = 0$

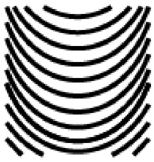
HOMOGENEOUS

29. The general solution of the differential equation $\frac{dy}{dx} = \frac{1-x}{y}$ is a family of curves which looks most like which of the following?

(A) 

(B) 

(C) 

(D) 

Ans. (B)

Sol.

$$ydy = (1 - x)\,dx$$

$$\frac{y^2}{2} = x - \frac{x^2}{2} + C$$

$$x^2 + y^2 - 2x - C = 0$$

30. The solution of the differential equation $(x^2 + y^2)dx = 2xy\,dy$ is-

(A) $x = c(x^2 + y^2)$ (B) $x = c(x^2 - y^2)$

(C) $x + c(x^2 + y^2) = 0$ (D) $y = c(x^2 - y^2)$

Ans. (B)

Sol.

$$(x^2 + y^2)dx = 2xy\,dy$$

$$\Rightarrow \frac{dy}{dx} = \frac{x^2 + y^2}{2xy}$$

$$\text{Let } y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$\Rightarrow x\frac{dv}{dx} = \frac{1+v^2}{2v} - v$$

$$\Rightarrow \int \frac{2v}{1-v^2}dv = \int \frac{dx}{x}$$

$$\Rightarrow -\log(1 - v^2) = \log x + \log c$$

$$\Rightarrow \frac{x^2}{x^2 - y^2} = cx \Rightarrow c(x^2 - y^2) = x$$

31. The solution of the equation $x\frac{dy}{dx} = y - x\tan\left(\frac{y}{x}\right)$ is-

(A) $x\sin\left(\frac{x}{y}\right) + c = 0$ (B) $x\sin\, y + c = 0$

(C) $x\sin\left(\frac{y}{x}\right) = c$ (D) $y\sin\left(\frac{x}{y}\right) = c$

Ans. (C)

Sol.

$$x\frac{dy}{dx} = y - x\tan\frac{y}{x}$$

$$\Rightarrow \frac{dy}{dx} = \frac{y}{x} - \tan\frac{y}{x}$$

$$\text{Let } y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$\Rightarrow x\frac{dv}{dx} = v - \tan v - v \Rightarrow \int \cot v\,dv = -\int \frac{dx}{x}$$

$$\Rightarrow \log\sin v = -\log x + \log c$$

$$\Rightarrow x\sin\frac{y}{x} = c$$

32. The solution of the differential equation $x\frac{dy}{dx} = y(\log\ y - \log\ x + 1)$ is-

(A) $y = xe^{cx}$ (B) $y + xe^{cx} = 0$

(C) $y + e^{x} = 0$ (D) $x = ye^{cy}$

Ans. (A)

Sol.

A

$x\frac{dy}{dx} = y\ (\log y - \log x + 1)$

Let $\log y - \log x + 1 = t$

$\Rightarrow \frac{1}{y}\frac{dy}{dx} - \frac{1}{x} = \frac{dt}{dx} \Rightarrow \frac{x}{y}\frac{dy}{dx} = x\frac{dt}{dx} + 1$

$\Rightarrow x\frac{dt}{dx} + 1 = t \Rightarrow \int\frac{dt}{t-1} = \int\frac{dx}{x}$

$\Rightarrow \log\ (t - 1) = \log x + \log c$

$\Rightarrow t - 1 = xc$

$\Rightarrow \log\frac{y}{x} = xc \Rightarrow y = xe^{cx}$

33. The solution of the differential equation $x^2\frac{dy}{dx} = x^2 + xy + y^2$ is-

(A) $\tan^{-1}\left(\frac{y}{x}\right) = \log\ x + c$ (B) $\tan^{-1}\left(\frac{y}{x}\right) = -\log\ x + c$

(C) $\sin^{-1}\left(\frac{y}{x}\right) = \log\ x + c$ (D) $\tan^{-1}\left(\frac{x}{y}\right) = \log\ x + c$

Ans. (A)

Sol.

A

$\frac{dy}{dx} = \frac{x^2 + xy + y^2}{x^2}$

Let $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dy}{dx}$

$\Rightarrow v + x\frac{dv}{dx} = 1 + v + v^2$

$\Rightarrow x\frac{dv}{dx} = 1 + v^2 \Rightarrow \int\frac{dv}{1+v^2} = \int\frac{dx}{x}$

$\Rightarrow \tan^{-1} v = \log x + c$

$\Rightarrow \tan^{-1}\frac{y}{x} = \log x + c$

34. If $x\frac{dy}{dx} = y(\log\ y - \log\ x + 1)$, then the solution of the equation is -

(A) $y\log\left(\frac{x}{y}\right) = cx$ (B) $x\log\left(\frac{y}{x}\right) = cy$

(C) $\log\left(\frac{y}{x}\right) = cx$ (D) $\log\left(\frac{x}{y}\right) = cy$

Ans. (C)

Sol.

C

Given that,

$$x\frac{dy}{dx} = y(\log y - \log x + 1)$$

$$\therefore \quad \frac{dy}{dx} = \left(\frac{y}{x}\right)\left(\log\left(\frac{y}{x}\right)+1\right)$$

Put $y = t\,x \Rightarrow \frac{dy}{dx} = t + x\frac{dt}{dx}$

$$\therefore \quad t + x\frac{dt}{dx} = t\log t + t$$

$\Rightarrow \quad t\log t\,dx = x\,dt$

$\Rightarrow \quad \frac{dt}{t\log t} = \frac{dx}{x}$

$\Rightarrow \quad \log t = \log x + \log c$

$\Rightarrow \quad \log\left(\frac{y}{x}\right) = cx$

35. The solution of the differential equation $(x^2 - y^2)dx + 2xydy = 0$ is-

(A) $x^2 + y^2 = cx$ (B) $x^2 - y^2 + cx = 0$

(C) $x^2 + 2xy = y^2 + cx$ (D) $x^2 + y^2 = 2xy + cx^2$

Ans. (A)

Sol.

$$\frac{dy}{dx} = \frac{y^2 - x^2}{2xy} = \frac{(y/x)^2 - 1}{2(y/x)}$$

put $y = v\,x \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

$$\therefore \quad v + x\frac{dv}{dx} = \frac{v^2+1}{2v} \Rightarrow x\frac{dv}{dx} = -\frac{(1+v^2)}{2v}$$

$\Rightarrow \quad \frac{2v}{1+v^2}dv = -\frac{dx}{x} \Rightarrow \int\frac{2v}{1+v^2}dv = \int -\frac{dx}{x}$

$\Rightarrow \quad \ell n\,|1 + v^2| + \ell n\,|x| = \ell n\,C$

$\Rightarrow \quad (1 + v^2)\,x = C \Rightarrow \left(1+\frac{y^2}{x^2}\right)x = C$

$\Rightarrow \quad x^2 + y^2 = cx$

36. The equation of the curve passing through origin and satisfying the differential equation $\frac{dy}{dx} = \sin(10x + 6y)$ is

(A) $y = \frac{1}{3}\tan^{-1}\left(\frac{5\tan 4x}{4-3\tan 4x}\right) - \frac{5x}{3}$ (B) $y = \frac{1}{3}\tan^{-1}\left(\frac{5\tan 4x}{4+3\tan 4x}\right) - \frac{5x}{3}$

(C) $y = \frac{1}{3}\tan^{-1}\left(\frac{3+\tan 4x}{4-3\tan 4x}\right) - \frac{5x}{3}$ (D) $y = \frac{1}{3}\tan^{-1}\left(\frac{\tan 4x}{4-3\tan 4x}\right) - \frac{5x}{3}$

Ans. (A)

Sol.

$$\frac{dy}{dx} = \sin(10x + 6y)$$

Put $10x + 6y = t \Rightarrow 10 + 6\frac{dy}{dx} = \frac{dt}{dx}$

$$\frac{dt}{dx} - 10 = 6 \sin t \Rightarrow \frac{dt}{10 + 6\sin t} = dx$$

$$\frac{dt}{10 + 6\left(\frac{2\tan\frac{t}{2}}{1 + \tan^2\frac{t}{2}}\right)} = dx$$

$$\frac{\sec^2\frac{t}{2}dt}{10 + 10\tan^2\frac{t}{2} + 12\tan\frac{t}{2}} = dx$$

put $\tan\frac{t}{2} = z \Rightarrow \sec^2\frac{t}{2}\, dt = 2dz$

$$\frac{2dz}{10(1+z^2)+12z} = dx$$

$$\frac{dz}{5z^2+6z+5} = dx \Rightarrow \frac{dz}{\left(z+\frac{3}{5}\right)^2 + \left(\frac{4}{5}\right)^2} = 5\ dx$$

$$\frac{5}{4}\tan^{-1}\frac{z+\frac{3}{5}}{\frac{4}{5}} = 5x + 5k$$

$$\tan^{-1}\frac{5z+3}{4} = 4x + C$$

$(0, 0) \Rightarrow C = \tan^{-1}\frac{3}{4}$

$$\tan^{-1}\frac{5z+3}{4} - \tan^{-1}\frac{3}{4} = 4x$$

$$\frac{\frac{5z+3}{4} - \frac{3}{4}}{1 + \left(\frac{5z+3}{4}\right)\frac{3}{4}} = \tan 4x \Rightarrow \frac{20z}{25+15z} = \tan 4x$$

$4z = (5 + 3z)\tan 4x$
$z(4 - 3\tan 4x) = 5\tan 4x$

$$z = \frac{5\tan 4x}{4 - 3\tan 4x} \Rightarrow \tan\frac{t}{2} = \left(\frac{5\tan 4x}{4 - 3\tan 4x}\right)$$

$$5x + 3y = \tan^{-1}\left(\frac{5\tan 4x}{4 - 3\tan 4x}\right)$$

$$y = \frac{1}{3}\tan^{-1}\left(\frac{5\tan 4x}{4 - 3\tan 4x}\right) - \frac{5x}{3}$$

37. A curve passes through the point $\left(1,\frac{\pi}{4}\right)$ & its slope at any point is given by $\frac{y}{x}-\cos^2\left(\frac{y}{x}\right)$. Then the curve has the equation

(A) $y = x\tan^{-1}\left(\ln\frac{e}{x}\right)$ (B) $y = x\tan^{-1}(\ell n+2)$

(C) $y = \frac{1}{x}\tan^{-1}\left(\ln\frac{e}{x}\right)$ (D) $y = x\tan^{-1}(\ell o g e)$

Ans. (A)

Sol. $\frac{dy}{dx} = \frac{y}{x} - \cos^2\left(\frac{y}{x}\right)$

Put $y = tx \Rightarrow \frac{dy}{dx} = t + x\frac{dt}{dx}$

$t + x\frac{dt}{dx} = t - \cos^2(t) \Rightarrow x\frac{dt}{dx} = -\cos^2 t$

$\sec^2 t\,dt + \frac{dx}{x} = 0$

$\tan t + \ell n\, x = C \Rightarrow \tan t = C - \ell n\, x$

$\tan\frac{y}{x} = C - \ell n\, x$ $\left(1,\frac{\pi}{4}\right)$

$C = 1$

$\tan\frac{y}{x} = 1 - \ell n\, x \Rightarrow \tan\frac{y}{x} = \ell n\frac{e}{x}$

$y = x\tan^{-1}\left(\ell n\frac{e}{x}\right)$

38. A function $f(x)$ satisfying $\int_0^1 f(tx)dt = nf(x)$, where $x > 0$, is

(A) $f(x) = c.x^{\frac{1-n}{n}}$ (B) $f(x) = c\cdot x^{\frac{n}{n-1}}$ (C) $f(x) = c.x^{\frac{1}{n}}$ (D) $f(x) = c\cdot x^{(1-n)}$

Ans. (A)

Sol. $\int_0^1 f(tx)\,dt = n\,f(x)$

let $tx = z \Rightarrow dt = dz/x$

$\int_0^x f(z).\frac{dz}{x} = n\,f(x)$

$\int_0^x f(z)\,dz = nx\,f(x)$ use leibnitz to differentiate

$f(x) = n\,f(x) + n\,x\,f'(x)$

$nx\,f'(x) = (1-n)\,f(x)$

$\frac{f'(x)}{f(x)} = \left(\frac{1-n}{n}\right)\frac{1}{n}$

$\ell n\,f(x) = \left(\frac{1-n}{n}\right)\ell n\,x + \ell n\,c$

$\Rightarrow f(x) = k\,.\,x^{\left(\frac{1-n}{n}\right)}$

39. The general solution of the differential equation $\frac{dy}{dx}+\frac{1+\cos 2y}{1-\cos 2x}=0$, is given by-

(A) $\tan y+\cot x=c$ (B) $\tan y-\cot x=c$

(C) $\tan x-\cot y=c$ (D) $\tan x+\cot y=c$

Ans. (B)

Sol.

$$\frac{dy}{dx}+\frac{\cos^2 y}{\sin^2 x}=0$$

$$\Rightarrow \int \sec^2 y\,dy+\int \operatorname{cosec}^2 x\,dx$$

$$\Rightarrow \tan y-\cot x=c$$

40. The solution of the differential equation $\frac{dy}{dx}=\frac{xy}{x^2+y^2}$ is-

(A) $ay^2=e^{x^2/y^2}$ (B) $ay=e^{x/y}$

(C) $y=++c$ (D) $y=+y^2+c$

Ans. (A)

Sol.

$$\frac{dy}{dx}=\frac{xy}{x^2+y^2}$$

$$\text{Let } y=vx \Rightarrow \frac{dy}{dx}=v+x\frac{dv}{dx}$$

$$\Rightarrow x\frac{dv}{dx}=\frac{v}{1+v^2}-v$$

$$\Rightarrow \int\frac{1+v^2}{v^3}dv=-\int\frac{dx}{x}$$

$$\Rightarrow -\frac{1}{2v^2}+\log v=-\log x-\log c$$

$$\Rightarrow 2\log\frac{y}{x}+2\log x+\log a=\frac{x^2}{y^2}$$

$$\Rightarrow ay^2=e^{x^2/y^2}$$

41. Which one of the following is homogeneous function?

(A) $f(x,y)=\frac{x-y}{x^2+y^2}$

(B) $f(x,y)=x^{\frac{1}{3}}\cdot y^{-\frac{2}{3}}\tan^{-1}\frac{x}{y}$

(C) $f(x,y)=x\left(\ell n\sqrt{x^2+y^2}-\ell ny\right)+ye^{x/y}$

(D) $f(x,y)=x\left[\ln\frac{2x^2+y^2}{x}-\ln(x+y)\right]+y^2\tan\left(\frac{x+2y}{3x-y}\right)$

Ans. (ABC)

Sol.

Its not asking homogeneous diff. equation

It is asking homogeneous function.

So (A) ✓ (B)✓ (C) ✓(D) ×

EXACT DIFFERENTIAL EQUATION

42. Solution of $\sec^2 x\tan y dx + \sec^2 y\tan x dy = 0$ is
(A) $\tan x\tan y = C$ (B) $\tan x/\tan y = C$
(C) $\sec x\sec y = C$ (D) $\tan y/\tan x = C$

Ans. (A)

Sol.

A

$\tan x \tan y = C$

$\frac{\sec^2 x}{\tan x}dx + \frac{\sec^2 y}{\tan y}dy = 0$

integrating

$\log(\tan x) + \log(\tan y) = C$

$\tan x\tan y = C$

43. Solution of $y\log y\, dx - x dy = 0$ is
(A) $y = e^{cx}$ (B) $y = e^{-cx}$
(C) $y = \log x$ (D) $y = e^{x\log c}$

Ans. (A)

Sol.

$\frac{dx}{x} = \frac{dy}{y\log y}$

$\log x = \log(\log y) + \log C$

$xc = \log y \Rightarrow \boxed{y = e^{cx}}$

44. Solution of $x\cos y\, dy = (xe^x\log x + e^x)dx$ is
(A) $\sin y = \log x + c$ (B) $\sin y = e^{-x}\log x + c$
(C) $\sin y = e^x\log x + c$ (D) $\sin y = e^x\log x + c$

Ans. (C)

Sol.

Consider $x \cos y\, dy = (xe^x \log x + e^x)\, dx$

$\therefore \cos y dy \frac{e^x(x\log x + 1)}{x}dx$

$\therefore \int \cos y dy = \int e^x\left(\log x + \frac{1}{x}\right)dx$

$\therefore \sin y = e^x \log x + C$ is the required solution.

45. Solve the differential equation $(2xy - 3x^2)dx + (x^2 - 2y)dy = 0$
(A) $x^2y - x^3 - y^2 = c$
(B) $x^2y + x^3 + y^2 = c$
(C) $x^2y + x^3 - y^2 = c$
(D) None of these

Ans. (A)

Sol.

A

$2xydx - x^2dx + x^2dy - 2ydy = 0$

$(2xydx + x^2dy) - x^2dx - 2ydy = 0$

$dx^2y - x^2dx - 2ydy = 0$

Integrate the equation, we get

$x^2y - x^3 - y^2 = c$

46. Find the particular solution of $(\cos\ x - x\sin\ x + y^2)dx + 2xydy = 0$ that satisfies the initial condition $y = 1$ when $x = \pi$

(A) $xy^2 + x\cos\ x = 0$
(B) $xy^2 - x\cos\ x = 0$
(C) $xy^2 + x\sin\ x = 0$
(D) $x^2y + x\sin\ x = 0$

Ans. (A)

Sol.

A

$\cos x dx - x\sin x\ dx + y^2dx + 2xy\ dy = 0$

$d(x\cos x) + dxy^2 = 0$

Integrate the equation, we get

$x\ \cos x + xy^2 = c$

Given $y = 1$ when $x = \pi$

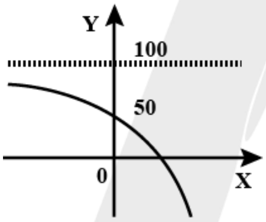
$\pi(-1) + \pi = c \Rightarrow c = 0$

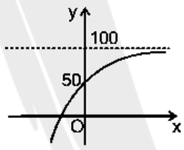
$x\ \cos x + xy^2 = 0$

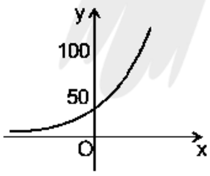
$xy^2 + x\cos\ x = 0$

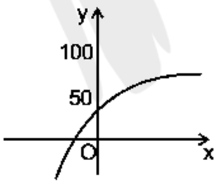
MIXED PROBLEMS

47. Which one of the following curves represents the solution of the initial value problem $Dy = 100 - y$ where $y(0) = 50$

(A) 

(B) 

(C) 

(D) 

Ans. (B)

Sol.

$\frac{dy}{dx} = 100 - y$

$-\ell n\ (100 - y) = x + C$; $y\ (0) = 50$

$-\ell n\ (100 - y) = x - \ell n\ 50 \Rightarrow C = -\ell n\ 50$

$\ell n \left(\frac{100 - y}{50}\right) = -x$

$100 - y = 50\ e^{-x}$

$y = 100 - 50\ e^{-x}$

48. Which of the following transformation reduce the differential equation $\frac{dz}{dx} + \frac{z}{x}\log\, z = \frac{z}{x^2}(\log\, z)^2$ into the form $\frac{du}{dx} + \mathrm{P(x)u} = \mathrm{Q(x)}$?

(A) $\mathrm{u} = \log\, \mathrm{z}$ (B) $\mathrm{u} = \mathrm{e^z}$

(C) $\mathrm{u} = (\log\, \mathrm{z})^{-1}$ (D) $\mathrm{u} = (\log\, \mathrm{z})^2$

Ans. (C)

Sol.

C

Dividing the given equation by z (log z)², we get

$$\frac{1}{z(\log z)^2}\frac{dz}{dx} + \frac{1}{\log z}\frac{1}{x} = \frac{1}{x^2} \qquad \ldots (1)$$

Putting $\frac{1}{\log z} = u$,

we have $\frac{du}{dx} = -(\log z)^{-2}\frac{1}{z}\frac{dz}{dx}$

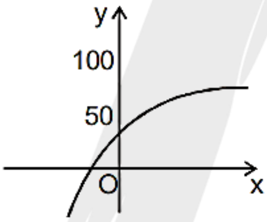
So (1) can be written as

$$-\frac{du}{dx} + \frac{u}{x} = \frac{1}{x^2} \Rightarrow \frac{du}{dx} - \frac{u}{x} = \frac{1}{x^2}$$

which is the required form with

$P(x) = \frac{-1}{x}$ and $Q(x) = \frac{-1}{x^2}$.

Hence (C) is the correct answer.

(D) 

49. If $y = \frac{x}{\ell n|cx|}$ (where c is an arbitrary constant) is the general solution of the differential equation $\frac{dy}{dx} = \frac{y}{x} + \phi\left(\frac{x}{y}\right)$ then the function $\phi\left(\frac{x}{y}\right)$ is

(A) $\frac{x^2}{y^2}$ (B) $-\frac{x^2}{y^2}$

(C) $\frac{y^2}{x^2}$ (D) $-\frac{y^2}{x^2}$

Ans. (D)

Sol.

D

$$y = \frac{x}{\ell n(cx)}$$

$$y' = \frac{\ell n(x).1 - \frac{x}{cx}}{(\ell ncx)^2} = \frac{(\ell ncx) - \frac{1}{c}}{(\ell ncx)^2}$$

$$\frac{dy}{dx} = \frac{y}{x} + \phi\left(\frac{x}{y}\right)$$

$$\frac{\ell ncx - \frac{1}{c}}{(\ell ncx)^2} = \frac{y}{x} + \phi\left(\frac{x}{y}\right)$$

$$\frac{\frac{x}{y} - \frac{1}{c}}{\left(\frac{x}{y}\right)^2} = \frac{y}{x} + \phi\left(\frac{x}{y}\right)$$

$$\frac{y}{x} - \frac{1}{c\left(\frac{x}{y}\right)^2} = \frac{y}{x} + \phi\left(\frac{x}{y}\right)$$

$$\phi\left(\frac{x}{y}\right) = -\frac{1}{c}\left(\frac{y^2}{x^2}\right)$$

50. If $f''(x) + f'(x) + f^2(x) = x^2$ be the differential equation of a curve and let P be the point of maxima then number of tangents which can be drawn from point P to $x^2 - y^2 = a^2$ is

(A) 2 (B) 1 (C) 0 (D) either 1 or 2

Ans. (D)

Sol.

D

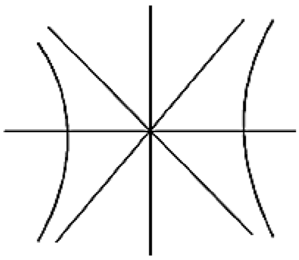
$f''(x) + f'(x) + f^2(x) = x^2$

put $f'(x) = 0$

$$\frac{d^2y}{dx} = x^2 - y^2 \le 0$$

$y^2 \ge x^2$

$|y| \ge |x|$



51. If $y = e^{(K+1)x}$ is a solution of differential equation $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 0$, then k equals

(A) -1 (B) 0 (C) 1 (D) 2

Ans. (C)

Sol.

C

$y = e^{(k+1)x}$

$y' = (k+1)e^{(k+1)x}$

$y'' = (k+1)^2 e^{(k+1)x}$

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 0$$

$(k+1)^2 - 4(k+1) + 4 = 0$

$k^2 + 2k + 1 - 4k = 0$

$(k-1)^2 = 0$

$k = 1$

SUBJECTIVE (JEE ADVANCED)

52. State the order & degree of the following differential equations:

(i) $\left[\frac{d^2x}{dt^2}\right]^3 + \left[\frac{dx}{dt}\right]^4 - xt = 0$ (ii) $\frac{d^2y}{dx^2} = \left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}$

Ans. (i) order 2 & degree 3 (ii) order 2 & degree 2

Sol.

(i) $\left[\frac{d^2x}{dt^2}\right]^3 + \left[\frac{dx}{dt}\right]^4 - xt = 0$

Order = 2

Degree = 3

(ii) $\frac{d^2y}{dx^2} = \left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}$

$\Rightarrow \left(\frac{d^2y}{dx^2}\right)^2 = \left[1 + \left(\frac{dy}{dx}\right)^2\right]^3$

Order = 2

Degree = 2

53. $\frac{\ln(\sec x + \tan x)}{\cos x} dx = \frac{\ell n(\sec y + \tan y)}{\cos y} dy$

Ans. $\ell n^2(\sec x + \tan x) - \ell n^2(\sec y + \tan y) = c$

Sol.

Sol.2 $\frac{\ell n(\sec x + \tan x)}{\cos x} dx = \frac{\ell n(\sec y + \tan y)}{\cos y} dy$

$\int \sec x\, \ell n(\sec x + \tan x) dx = \int \sec y\, \ell n(\sec y + \tan y) dy$

$[\ln(\sec x + \tan x)]^2 = [\ell n(\sec y + \tan y)]^2 + k$

54. $\frac{dy}{dx}+\frac{\sqrt{(x^2-1)(y^2-1)}}{xy}=0$

Ans. $\sqrt{x^2-1}-\sec^{-1}x+\sqrt{y^2-1}=c$

Sol.

$$\frac{dy}{dx}+\frac{\sqrt{(x^2-1)(y^2-1)}}{xy}=0$$

$$\frac{y\,dy}{\sqrt{y^2-1}}+\frac{\sqrt{x^2-1}}{x}dx=0$$

$$\frac{y\,dy}{\sqrt{y^2-1}}+\frac{x^2-1}{x\sqrt{x^2-1}}dx=0$$

$$\frac{ydy}{\sqrt{y^2-1}}+\frac{x}{\sqrt{x^2-1}}dx-\frac{1}{x\sqrt{x^2-1}}dx=0$$

Integrate

$$\sqrt{y^2-1}+\sqrt{x^2-1}-\sec^{-1}x=C$$

55. $\frac{dy}{dx}=\sin(x+y)+\cos(x+y)$

Ans. $\ell n\left[1+\tan\frac{x+y}{2}\right]=x+c$

Sol.

$$\frac{dy}{dx}=\sin(x+y)+\cos(x+y)$$

Put $x+y=t$

$$1+\frac{dy}{dx}=\frac{dt}{dx}$$

$$\frac{dt}{dx}-1=\sin t+\cos t$$

$$\frac{dt}{dx}=\sin t+\cos t+1$$

$$\frac{dt}{\sin t+\cos t+1}=dx$$

$$\frac{\sec^2\frac{t}{2}dt}{1+\tan\frac{t}{2}}=2dx$$

$$\ell n\left(1+\tan\frac{t}{2}\right)=x+C$$

$$\ell n\left(1+\tan\frac{x+y}{2}\right)=x+C$$

56. $e^{(dy/dx)} = x + 1$ given that when $x = 0, y = 3$

Ans. $y = (x+1) \cdot \ln(x+1) - x + 3$

Sol.

$e^{dy/dx} = x + 1$

$\frac{dy}{dx} = \ell n\,(x+1)$

$\int dy = \int \ell n(x+1)\,dx$

$y = (x+1)\ \int \ell n(x+1)\,dx$

$y = (x+1)\,[\ln(x+1)] + C$

$(0, 3) \Rightarrow C = 4$

$y = (x+1)\,\ell n\,(x+1) - x - 1 + 4$

$y = (x+1)\,\ell n\,(x+1) - x + 3$ Particular soluton

57. $(x - y^2)dx + 2xydy = 0$

Ans. $y^2 + x\ell nax = 0$

Sol.

$(x - y^2)\,dx + 2xy\,dy = 0$

$\frac{dy}{dx} = \frac{y}{2x} - \frac{1}{2y}$; put $y^2 = t$

$2y\frac{dy}{dx} = \frac{y^2}{x} - 1$

$2y\frac{dy}{dx} = \frac{dt}{dx}$

$\frac{dt}{dx} = \frac{t}{x} - 1$

$\frac{dt}{dx} - \frac{t}{x} = -1$

$\text{I.F.} = e^{-\int \frac{1}{x}dx} = -\frac{1}{x}$

$\frac{t}{x} = -\int \frac{1}{x}dx$

$\frac{t}{x} = -\ell nx - \ell nC$

$\frac{y^2}{x} + \ell n\,(xC) = 0$

$y^2 + x\,\ell n\,(xc) = 0$

58. $\frac{dy}{dx} = e^{x-y}(e^x - e^y)$

Ans. $e^y = c \cdot \exp(-e^x) + e^x - 1$

Sol.

$$\frac{dy}{dx} = e^{x-y}(e^x - e^y)$$

$$\frac{dy}{dx} = \frac{e^{2x}}{e^y} - e^x$$

$$e^y \frac{dy}{dx} + e^x e^y = e^{2x}$$

$$e^y = t \quad \Rightarrow e^y \frac{dy}{dx} = \frac{dt}{dx}$$

$$\frac{dt}{dx} + e^x t = e^{2x}$$

$$\text{I.F.} = e^{\int e^x dx} = e^{e^x}$$

$$t\, e^{e^x} = \int e^{e^x} . \underset{\substack{e^x = z \\ e^x dx = dz}}{e^{2x} dx}$$

$$= \int z e^z \, dz$$

$$t e^{e^x} = z e^z - e^z + C$$

$$t e^{e^x} = e^x e^{e^x} - e^{e^x} + C$$

$$e^y = (e^x - 1) + C \exp.(-e^x)$$

59. $xdy + ydx + \frac{xdy - ydx}{x^2 + y^2} = 0$

Ans. $xy + \tan^{-1}\frac{y}{x} = c$

Sol. $xdy + ydx = \frac{ydx - xdy}{x^2 + y^2}$

$\Rightarrow d(xy) = -d(\tan^{-1}(y/x))$

$\Rightarrow xy + \tan^{-1}(y/x) = c$

60. (a) $\frac{dy}{dx}=\frac{x^2+xy}{x^2+y^2}$ (b) $(x^3-3xy^2)dx=(y^3-3x^2y)dy$

Ans. (a) $c(x-y)^{2/3}(x^2+xy+y^2)^{1/6}=\exp\left[\frac{1}{\sqrt{3}}\tan^{-1}\frac{x+2y}{x\sqrt{3}}\right]$ where $\exp x\equiv e^x$,

(b) $y^2-x^2=c(y^2+x^2)^2$

Sol.

(a) $\frac{dy}{dx}=\frac{x^2+xy}{x^2+y^2} \Rightarrow \frac{dy}{dx}=\frac{1+y/x}{1+(y/x)^2}$

Put y = tx $\Rightarrow \frac{dy}{dx}=t+x\frac{dt}{dx}$

$t+x\frac{dt}{dx}=\frac{1+t}{1+t^2} \Rightarrow x\frac{dt}{dx}=\frac{1+t}{1+t^2}-t$

$x\frac{dt}{dx}=\frac{1+t-t-t^3}{1+t^2} \Rightarrow \frac{(1+t^2)dt}{(1-t^3)}=\frac{dx}{x}$

$\frac{t^2}{1-t^3}dt+\frac{1}{1-t^3}dt=\frac{dx}{x}$

$\frac{t^2}{1-t^3}dt+\left(\frac{1-t^2+t^2}{1-t^3}\right)dt=\frac{dx}{x}$

$\left(\frac{2t^2}{1-t^3}\right)dt+\frac{(1-t)(1+t)dt}{(1-t)(1-t^2+t)}=\frac{dx}{x}$

$\frac{2}{3}\left[\frac{3t^2}{1-t^3}\right]dt+\frac{1}{2}\left[\frac{2t+1}{t^2+t+1}\right]dt+\frac{1}{2}\frac{dt}{t^2+t+1}=\frac{dx}{x}$

Integrate both the side

$-\frac{2}{3}\ell n(1-t^3)+\frac{1}{2}\ell n(t^2+t+1)$

$+\frac{1}{2.\frac{\sqrt{3}}{2}}\tan^{-1}\left(\frac{2t+1}{\sqrt{3}}\right)=\ell n\, x+C$

After putting value of 't'

$C(x-y)^{2/3}(x^2+xy+y^2)^{1/6}$

$=\exp\left[\frac{1}{\sqrt{3}}\tan^{-1}\frac{x+2y}{x\sqrt{3}}\right]$

(b) $(x^3-3xy^2)\,dx=(y^3-3x^2y)\,dy$

$\frac{dy}{dx}=\frac{x^3-3xy^2}{y^3-3x^2y}$

$\frac{dy}{dx}=\frac{1-3\left(\frac{y}{x}\right)^2}{\left(\frac{y}{x}\right)^3-3\left(\frac{y}{x}\right)}$

Put y = tx $\Rightarrow \frac{dy}{dx}=t+x\frac{dt}{dx}$

$$t + x\frac{dt}{dx} = \frac{1-3t^2}{t^3-3t}$$

$$x\frac{dt}{dx} = \frac{1-3t^2-t^4+3t^2}{t(t^2-3)}$$

$$\frac{t(t^2-3)}{1-t^4}dt = \frac{dx}{x}$$

After integration

$y^2 - x^2 = c\,(y^2 + x^2)^2$

61. $\left[x\cos\frac{y}{x} + y\sin\frac{y}{x}\right]y = \left[y\sin\frac{y}{x} - x\cos\frac{y}{x}\right]x\frac{dy}{dx}$

Ans. $xy\cos\frac{y}{x} = c$

Sol.

$$\left[x\cos\frac{y}{x} + y\sin\frac{y}{x}\right]y = \left[y\sin\frac{y}{x} - x\cos\frac{y}{x}\right]x\,\frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{\left[x\cos\frac{y}{x} + y\sin\frac{y}{x}\right]y}{\left[y\sin\frac{y}{x} - x\cos\frac{y}{x}\right]x}$$

$$\frac{dy}{dx} = \frac{\left[\cos\frac{y}{x} + \left(\frac{y}{x}\right)\sin\frac{y}{x}\right]\frac{y}{x}}{\left[\frac{y}{x}\sin\frac{y}{x} - \cos\frac{y}{x}\right]}$$

$y = tx$

$$x\frac{dt}{dx} = \frac{t[\cos t + t\sin t]}{[t\sin t - \cos t]} - t$$

$$x\frac{dt}{dx} = \frac{t\cos t + t^2\sin t - t^2\sin t + t\cos t}{t\sin t - \cos t}$$

$$\frac{t\sin t - \cos t}{2t\cos t}\,dt = \frac{dx}{x}$$

$-\ell n\,(t\cos t) = 2\ell nx + \ell nC$

$\ell n\,(t\cos t)\,x^2 = \ell nk$

$(t\cos t)\,x^2 = k'$

$$\left(\frac{y}{x}\right)\cos\left(\frac{y}{x}\right)x^2 = k' \quad \Rightarrow \quad xy\cos\left(\frac{y}{x}\right) = k'$$

62. $(x-y)dy = (x+y+1)dx$

Ans. $\arctan\frac{2y+1}{2x+1} = \ln c\sqrt{x^2+y^2+x+y+\frac{1}{2}}$

Sol.

$$\frac{dy}{dx}=\frac{(x+y+1)}{x-y}$$

Let $x = X + h \Rightarrow dx = dX$

$y = Y + k \Rightarrow dy = dY$

$$\frac{dY}{dX}=\frac{(X+h)+(Y+k)+1}{(X+h)-(Y+k)}$$

$$\frac{dY}{dX}=\frac{(X+Y)+(h+k+1)}{(X-Y)+(h-k)}$$

$h + k + 1 = 0$ & $h - k = 0$

$\Rightarrow h = -\frac{1}{2},\ k = -\frac{1}{2}$

$\frac{dY}{dX}=\frac{X+Y}{X-Y}$ Put $Y = tX$

$X\frac{dt}{dX}=\frac{1+t}{1-t}-t \Rightarrow \frac{1-t}{1+t^2}dt=\frac{dX}{X}$

$\tan^{-1} t-\frac{1}{2}\ \ell n\,(1+t^2)=\ell nx+\ell nC$

$\tan^{-1}t-\frac{1}{2}\ \ell n\,(y^2+x^2)=\ell nC$

$\tan^{-1}\frac{y}{x}=\ell nC\sqrt{y^2+x^2}$

$$\tan^{-1}\left(\frac{y+\frac{1}{2}}{x+\frac{1}{2}}\right)=\ell n\, C\sqrt{\left(x+\frac{1}{2}\right)^2+\left(y+\frac{1}{2}\right)^2}$$

$$\tan^{-1}\left(\frac{2y+1}{2x+1}\right)=\ell nC\sqrt{x^2+y^2+x+y+\frac{1}{2}}$$

63. $\frac{dy}{dx}=\frac{x+y+1}{2x+2y+3}$

Ans. $x+y+\frac{4}{3}=ce^{3(x-2y)}$

Sol.

$$\frac{dy}{dx} = \frac{x+y+1}{2x+2y+3}$$

$$2\frac{dy}{dx} = \frac{x+y+1}{x+y+1+\frac{1}{2}} \quad \text{Put } x + y + 1 = t$$

$$2\left[\frac{dt}{dx}-1\right] = \frac{t}{t+\frac{1}{2}} \quad ; \quad 1 + \frac{dy}{dx} = \frac{dt}{dx}$$

$$\frac{dt}{dx} - 1 = \frac{t}{2t+1}$$

$$\frac{dt}{dx} = \frac{3t+1}{2t+1}$$

$$\left(\frac{2t+1}{3t+1}\right) dt = dx$$

$$\frac{2}{3}\left[\frac{3t+\frac{3}{2}}{3t+1}\right] dt = dx$$

$$\frac{2}{3}\left[\frac{3t+1+\frac{1}{2}}{3t+1}\right] dt = dx$$

$$\frac{2}{3}\left[1+\frac{1}{2(3t+1)}\right] dt = dx$$

$$\frac{2}{3}t + \frac{1}{9}\ell n\,(3t + 1) = x + C$$

$$(3t + 1) = ke^{3(x-2y)}$$

$$3(x + y + 1) + 1 = ke^{3(x-2y)}$$

$$x + y + \frac{4}{3} = k'e^{3(x-2y)}$$

64. Show that the curve such that the distance between the origin and the tangent at an arbitrary point is equal to the distance between the origin and the normal at the same point.

$\sqrt{x^2 + y^2} = ce^{\pm\tan^{-1}\frac{y}{x}}$

Sol.

tangent $Y - y = m(X - x)$

Normal $Y - y = \frac{-1}{m}(X - x)$

$\perp^r$ from origin to tangent $= \perp^r$ to Normal

$$\left|\frac{-y + mx}{\sqrt{1+m^2}}\right| = \left|\frac{-y - x/m}{\sqrt{1+1/m^2}}\right|$$

$(y - mx)^2 = (my + x)^2$

$\Rightarrow m^2(x^2 - y^2) - 4m\,xy + y^2 - x^2 = 0$

$$m = \frac{dy}{dx} = \frac{2xy \pm (x^2 + y^2)}{x^2 - y^2}$$

+ve $\frac{dy}{dx} = \frac{(x+y)^2}{x^2 - y^2} = \frac{x+y}{x-y}$;

–ve $\frac{dy}{dx} = -\frac{(x-y)}{x+y}$

Homogenous $y = tx$

$$x\frac{dt}{dx} = \frac{1+t^2}{1-t}$$

$$\frac{1}{1+t^2} - \frac{1}{2}\left(\frac{2t}{1+t^2}\right)dt = \frac{dx}{x}$$

$$\tan^{-1}\frac{1}{1+t^2} - \frac{1}{2}\ln(1 + t^2) = \ln x + \ln k$$

$$\sqrt{x^2 + y^2} = c\, e^{\tan^{-1} y/x}$$

Similarly if we take –ve sign

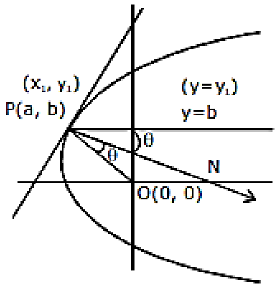
$$\sqrt{x^2 + y^2} = c\, e^{-\tan^{-1} y/x}$$

65. The light rays emanating from a point source situated at origin when reflected from the mirror of a search light are reflected as beam parallel to the x-axis. Show that the surface is parabolic, by first forming the differential equation and then solving it.

Sol.

Equation of PN (angle bisector of emanating

and reflected ray) $\frac{y-b}{1} = \pm \frac{bx-ay}{\sqrt{a^2+b^2}}$



$$\frac{b}{\sqrt{a^2+b^2}}x - \left(\frac{a+\sqrt{a^2+b^2}}{\sqrt{a^2+b^2}}\right)y + b = 0$$

$$\frac{y}{x+\sqrt{a^2+b^2}} = \frac{dy}{dx}$$

$$\frac{ydx - xdy}{y^2} = \frac{\sqrt{x^2+y^2}}{y^2}dx$$

$$\int \frac{d(x/y)}{\sqrt{\left(\frac{x}{y}\right)^2+1}} = \int \frac{dy}{y}$$

$$\ln\left(\frac{x}{y}+\sqrt{\frac{x^2}{y^2}+1}\right) = \ln y + \ln c$$

$$x + \sqrt{x^2+y^2} = ky^2$$

$$1 = k^2y^2 - 2kx \text{ (Parabola)}$$

66. The perpendicular from the origin to the tangent at any point on a curve is equal to the abscissa of the point of contact. Find the equation of the curve satisfying the above condition and which passes through (1,1).

Ans. $x^2 + y^2 - 2x = 0$

Sol.

$Y - y = m(X - x)$
$Y - y - m(X - x) = 0$

$$\left|\frac{-y + mx}{\sqrt{1+m^2}}\right| = x$$

$(mx - y)^2 = x^2(1 + m^2)$
$m^2x^2 - 2mxy + y^2 = x^2 + x^2m^2$
$2mxy = y^2 - x^2$

$$m = \frac{y^2 - x^2}{2xy}$$

$$\frac{dy}{dx} = \frac{\frac{y^2}{x^2} - 1}{\frac{2y}{x}} \qquad \text{Put } y = tx$$

$$t + \frac{xdt}{dx} = \frac{t^2 - 1}{2t}$$

$$x\frac{dt}{dx} = \frac{t^2 - 1 - 2t^2}{2t}$$

$$\frac{2t}{-t^2 - 1}dt = \frac{dx}{x}$$

$\ell n(t^2 + 1) = -\ell nx + \ell nC$
$x(t^2 + 1) = C$

$$\left(\frac{y^2 + x^2}{x}\right) = C$$

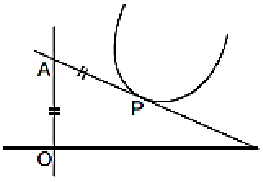
$C = 2$ (1, 1) Passes
$y^2 + x^2 = 2x$
$x^2 + y^2 - 2x = 0$

67. Find the curve for which any tangent intersects the y-axis at the point equidistant from the point of tangency and the origin.

Ans. $x^2 + y^2 = cx$

Sol.

$Y - y = m(X - x)$
A $(0, y - mx)$
O $(0, 0)$
P (x, y)
OA = PA



$|y - mx| = \sqrt{x^2 + m^2x^2}$

$y^2 + m^2x^2 - 2mxy = x^2 + m^2x^2$

$\frac{y^2 - x^2}{2xy} = m$

$\frac{dy}{dx} = \frac{y^2 - x^2}{2xy}$

Put $y = tx$

$\frac{dy}{dx} = t + x\frac{dt}{dx}$

$t + \frac{xdt}{dx} = \frac{t^2 - 1}{2t}$

$x\frac{dt}{dx} = \frac{t^2 - 1}{2t}$

$\frac{2t}{1 + t^2}dt = \frac{-dx}{x}$

$\ell n(1 + t^2) + \ell nx = \ell nC$

$(1 + t^2)x = C \quad x^2 + y^2 = Cx$

68. The population P of a town decrease at a rate proportional to the number by which the population exceeds 1000, proportionality constant being $k > 0$. Find
(a) Population at any time t, given initial population of the town being 2500.
(b) If 10 years later the population has fallen to 1900, find the time when the population will be 1500.
(c) Predict about the population of the town in the long run.

Ans. (a) $P = 1000 + 1500e^{-kt}$ where $k = \frac{1}{10}\ln\left(\frac{5}{3}\right)$;
(b) $T = 10\log_{5/3}(3)$; (c) $P = 1000$ as $t \to \infty$

Sol.

(a) $\frac{-dP}{dt} = k\,(P - 1000)$

$\ln(P - 1000) = -kt + c$

$P = 1000 + c_1\, e^{-kt}$

$t = 0 \Rightarrow P = 2500$

$\Rightarrow c_1 = 1500$

$P = 1000 + 1500\, e^{-kt}$

(b) $t = 10 \Rightarrow P = 1900$

$900 = 1500\, e^{-k(10)}$

$\Rightarrow k = \frac{1}{10} \ln\left(\frac{5}{3}\right)$

$\Rightarrow P = 1000 + 1500\, e^{-kt}$

$1500 = 1000 + 1500\, e^{-kt}$

$\Rightarrow e^{kt} = 3 \Rightarrow kt = \ln 3$

$t = \frac{\ln 3}{k} = 10\,\frac{\ln 3}{\ln\left(\frac{5}{3}\right)}$

$= 10.\ \ln_{5/3} 3$

(c) $P = 1000 + 1500\, e^{-kt}$

at $t \to \infty$

$P = 1000$

PREVIOUS YEAR | JEE MAIN

69. The differential equation of all circles passing through the origin and having their centres on the x-axis is- [AIEEE 2007]

(A) $x^2 = y^2 + xy\frac{dy}{dx}$ (B) $x^2 = y^2 + 3xy\frac{dy}{dx}$

(C) $y^2 = x^2 + 2xy\frac{dy}{dx}$ (D) $y^2 = x^2 - 2xy\frac{dy}{dx}$

Ans. (C)

Sol.

C

General equation of all such circles which pass through the origin and whose centre lie on x-axis is

$x^2 + y^2 + 2gx = 0$ ...(i)

On differentiating w.r.t. x, we get

$2x + 2y\frac{dy}{dx} + 2g = 0$

$\Rightarrow 2g = -\left(2x + 2y\frac{dy}{dx}\right)$

On putting the value of 2g in Eq. (i), we get

$x^2 + y^2 + \left(-2x^2 - 2y\frac{dy}{dx}\right) x = 0$

$\Rightarrow x^2 + y^2 - 2x^2 - 2xy\frac{dy}{dx} = 0$

$\Rightarrow y^2 = x^2 + 2xy\frac{dy}{dx}$

70. The differential equation which represents the family of curves $y = c_1 e^{c_2 x}$ where c_1 and c_2 are arbitrary constants, is - [AIEEE 2009]

(A) $y' = y^2$ (B) $y'' = y'y$

(C) $yy'' = y'$ (D) $yy'' = (y')^2$

Ans. (D)

Sol.

D

Given, $y = c_1 e^{c_2 x}$

$\Rightarrow \quad y' = c_2 c_1 e^{c_2 x}$

$\Rightarrow \quad y' = c_2 y \quad \quad ...(i)$

$\Rightarrow \quad y' = c_2 y' \quad \quad ...(ii)$

$\Rightarrow \quad y'' = \dfrac{(y')^2}{y}$

$\left[\text{From Eq. (i)}, c_2 = \dfrac{y'}{y}\right]$

$\Rightarrow \quad yy'' = (y')^2$

71. If $\frac{dy}{dx} = y + 3 > 0$ and $(0) = 2$, then $y(\ln\ 2)$ is equal to : [AIEEE 2011]

(A) 7 (B) 5 (C) 13 (D) -2

Ans. (A)

Sol.

A

Here, $\dfrac{dy}{dx} = y + 3 > 0$ and $y(0) = 2$

$\Rightarrow \quad \int \dfrac{dy}{y+3} \int dx$

$\Rightarrow \quad \log|y+3| = x + C$, since,

$y(0) = 2$

$\Rightarrow \quad \log|2+3| = 0 + C$

$\therefore \quad C = \log_e 5$

$\Rightarrow \quad \log|y+3| = x + \log_e 5$

$\therefore$ when $\quad x = \log_e = 2$

$\Rightarrow \quad \log|y+3| = \log_e 2 + \log_e 5 = \log_e 10$

$\Rightarrow \quad y + 3 = 10 \Rightarrow y = 7$

72. The population p(t) at time t of a certain mouse species satisfies the differential equation $\frac{dp(t)}{dt} = 0.5\,p(t) - 450$. If $p(0) = 850$, then the time at which the population becomes zero is: [AIEEE 2012]

(A) $\frac{1}{2}\ln\ 18$ (B) $\ln\ 18$ (C) $2\ln\ 18$ (D) $\ln\ 9$

Ans. (C)

Sol.

C

Let $y = P(t)$

$$\int_{850}^{0} \frac{2dy}{\frac{1}{2}y - 900} = \int_{0}^{t} dt$$

$$\left[2\ln|y-900|\right]_{850}^{0} = (t)_0^t$$

$2\ln 900 - 2\ln 50 = t \qquad t = 2\ln 18$

73. Let $y = y(x)$ be the solution of the differential equation

$\sin x\frac{dy}{dx} + y\cos x = 4x, x \in (0, \pi)$.

If $y\left(\frac{\pi}{2}\right) = 0$, then $y\left(\frac{\pi}{6}\right)$ is equal to : [JEE-MAIN 2018]

(A) $-\frac{4}{9}\pi^2$ (B) $\frac{4}{9\sqrt{3}}\pi^2$ (C) $\frac{-8}{9\sqrt{3}}\pi^2$ (D) $-\frac{8}{9}\pi^2$

Sol.

$$\frac{dy}{dx} + y\cot x = \frac{4x}{\sin x}$$

$$\text{If} = e^{\int \cot x dx} = \sin x$$

$$y\sin x = \int \sin x \cdot \frac{4x}{\sin x} = 2x^2 + C$$

$$\text{at } y\left(\frac{\pi}{2}\right) = 0$$

$$y\frac{1}{2} = \frac{\pi^2}{18} - \frac{\pi^2}{2}$$

$$y = \frac{-8}{9}\pi^2$$

PREVIOUS YEAR | JEE ADVANCED

74. A curve passes through the point $\left(1, \frac{\pi}{6}\right)$. Let the solpe of the curve at each point (x, y) be $\frac{y}{x} + \sec\left(\frac{y}{x}\right), x > 0$. Then the equation of the curve is [JEE 2013]

(A) $\sin\left(\frac{y}{x}\right) = \log x + \frac{1}{2}$ (B) $\text{cosec}\left(\frac{y}{x}\right) = \log x + 2$

(C) $\sec\left(\frac{2y}{x}\right) = \log x + 2$ (D) $\cos\left(\frac{2y}{x}\right) = \log x + \frac{1}{2}$

Ans. (D)

Sol.

D

$\left(1, \frac{\pi}{6}\right)$

$\frac{dy}{dx} = \frac{y}{x} + \sec\frac{y}{x}$

$\frac{y}{x} = v \Rightarrow y = vx$

$\frac{dy}{dx} = v + x\frac{dv}{dx}$

$v + \frac{xdv}{dx} = v + \sec V$

$\frac{dv}{\sec(v)} = \frac{dx}{x}$

$\Rightarrow \sin v = \ln x + C$

$\sin\frac{y}{x} = \ln x + C$

$C = \frac{1}{2}\left(1, \frac{\pi}{6}\right)$

$\sin\frac{y}{x} = \log x + \frac{1}{2}$

75. consider the family of all circles whose centers lie on the straight line y = x. If this family of circles is represented by the differential equation Py" + Qy' +1 = 0, where P, Q are fucntions of x, y and y' (here $y' = \frac{dy}{dx}, y'' = \frac{d^2y}{dx^2}$), then which of the following statement is (are) true? [JEE 2015]

(A) $P = y + x$ (B) $P = y - x$

(C) $P + Q = 1 - x + y + y' + (y')^2$ (D) $P - Q = x + y - y' - (y')^2$

Ans. (BC)

Sol.

centre lie on

y = x

$x^2 + y^2 + 2ax + 2ay + c = 0$

$2x + 2yy' + 2a + 2ay' = 0$

$2a = \frac{-(2\times + 2yy')}{1+y'}$

$2 + 2(y')^2 + 2yy'' + 2ay'' = 0$

76. If $y = y(x)$ satisfies the differential equation $8\sqrt{x}(\sqrt{9+\sqrt{x}})dy = (\sqrt{4+\sqrt{9+\sqrt{x}}})^{-1}dx, x > 0$ and $y(0) = \sqrt{7}$, then $y = (256) =$ [JEE 2017]

(A) 3 (B) 16 (C) 9 (D) 80

Ans. (D)

Sol.

$$\sqrt{4+\sqrt{9+\sqrt{x}}}=t$$

$$\frac{1}{2\sqrt{4+\sqrt{9+\sqrt{x}}}}\times\frac{1}{2\sqrt{9+\sqrt{x}}}\cdot\frac{1}{2\sqrt{x}}dx=dt$$

$\Rightarrow$ $dy = dt$

$y = t + \lambda$

$$y=\sqrt{4+\sqrt{9+\sqrt{x}}}+\lambda$$

$y(0)=\sqrt{7}+\lambda \Rightarrow \boxed{\lambda=0}$

$\Rightarrow$ $y(256)=\sqrt{4+\sqrt{9+16}}=\sqrt{4+5}$

$= 3$

ANSWER KEY

1.	(D)	**2.**	(C)	**3.**	(B)	**4.**	(C)	**5.**	(C)	**6.**	(C)	**7.**	(C)
8.	(CD)	**9.**	(C)	**10.**	(D)	**11.**	(B)	**12.**	(A)	**13.**	(B)	**14.**	(C)
15.	(D)	**16.**	(C)	**17.**	(ACD)	**18.**	(A)	**19.**	(C)	**20.**	(D)	**21.**	(D)
22.	(A)	**23.**	(A)	**24.**	(B)	**25.**	(D)	**26.**	(A)	**27.**	(A)	**28.**	(AB)
29.	(B)	**30.**	(B)	**31.**	(C)	**32.**	(A)	**33.**	(A)	**34.**	(C)	**35.**	(A)
36.	(A)	**37.**	(A)	**38.**	(A)	**39.**	(B)	**40.**	(A)	**41.**	(ABC)	**42.**	(A)
43.	(A)	**44.**	(C)	**45.**	(A)	**46.**	(A)	**47.**	(B)	**48.**	(C)	**49.**	(D)
50.	(D)	**51.**	(C)	**52.**	(i) order 2 & degree 3 (ii) order 2 & degree 2								

53. $\ell n^2(\sec x+\tan x)-\ell n^2(\sec y+\tan y)=c$

54. $\sqrt{x^2-1}-\sec^{-1}x+\sqrt{y^2-1}=c$

55. $\ell n\left[1+\tan\frac{x+y}{2}\right]=x+c$

56. $y=(x+1)\cdot\ln(x+1)-x+3$

57. $y^2+x\ell nax=0$

58. $e^y=c\cdot\exp(-e^x)+e^x-1$

59. $xy+\tan^{-1}\frac{y}{x}=c$

60. (a) $c(x-y)^{2/3}(x^2+xy+y^2)^{1/6}=\exp\left[\frac{1}{\sqrt{3}}\tan^{-1}\frac{x+2y}{x\sqrt{3}}\right]$ where $\exp x\equiv e^x$,

(b) $y^2-x^2=c(y^2+x^2)^2$

61. $xy\cos\frac{y}{x}=c$

62. $\arctan\frac{2y+1}{2x+1}=\ln c\sqrt{x^2+y^2+x+y+\frac{1}{2}}$

63. $x+y+\frac{4}{3}=ce^{3(x-2y)}$

66. $x^2+y^2-2x=0$

67. $x^2+y^2=cx$

68. (a) $P=1000+1500e^{-kt}$ where $k=\frac{1}{10}\ln\left(\frac{5}{3}\right)$;(b) $T=10\log_{5/3}(3)$; (c) $P=1000$ as $t\to\infty$

69.	(C)	**70.**	(D)	**71.**	(A)	**72.**	(C)	**73.**	**(D)**	**74.**	(D)	**75.**	(BC)
76.	(D)												