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$\underline{1.}$

$$\lim_{h\to 0}\frac{p\left(1-(1+h)^{q}\right)-q\left(1-(1+h)^{p}\right)}{\left(1-(1+h)^{p}\right)\left(1-(1+h)^{q}\right)}$$

$x=1+h$

$$=\lim_{h\to 0}\frac{p\left(\cancel{-{}^{q}C_1h}-{}^{q}C_2h^2-\dots\right)-q\left(\cancel{-{}^{p}C_1h}-{}^{p}C_2h^2-\dots\right)}{\left(-{}^{p}C_1h-{}^{p}C_2h^2-\dots\right)\left(-{}^{q}C_1h-{}^{q}C_2h^2-\dots\right)}$$

$$\frac{q\,{}^{p}C_2-p\,{}^{q}C_2}{pq}=\lim_{h\to 0}\frac{\left(q\,{}^{p}C_2-p\,{}^{q}C_2\right)+h(\dots)}{\left(-p-{}^{p}C_2h-\dots\right)\left(-q-{}^{q}C_2h-\dots\right)}$$

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$$l = \lim_{x\to 1}\left(\frac{p}{1-x^p} - \frac{q}{1-x^q}\right)$$

$$x = \frac{1}{t} \qquad l = \lim_{t\to 1}\left(\frac{p}{1-\frac{1}{t^p}} - \frac{q}{1-\frac{1}{t^q}}\right) = \lim_{t\to 1}\left(\frac{p\,t^p}{t^p-1} - \frac{q\,t^q}{t^q-1}\right)$$

$$= \lim_{t\to 1}\left(\frac{p(t^p-1)+p}{t^p-1} - \frac{q(t^q-1+1)}{(t^q-1)}\right)$$

$$\boxed{l = p-q-l} \Leftarrow l = \lim_{t\to 1}\left((p-q) - \left(\frac{p}{1-t^p} - \frac{q}{1-t^q}\right)\right)$$

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$$\frac{\left(1\cdot 3\cdot 5\cdots(2n-1)\right)\left(2\cdot 4\cdot 6\cdots(2n)\right)}{(n!)^2}$$

$$\frac{\left(2\cdot 6\cdot 10\cdots(4n-2)\right)\cancel{\left(1\cdot 2\cdot 3\cdots n\right)}}{\cancel{n!}\; n!}$$

$$\frac{\sin\left(x-\frac{\pi}{2}\right)}{x\left(x-\frac{\pi}{2}\right)}\;\frac{2\sin\left(-\frac{\pi}{2}+x\right)-1}{\sin\left(x-\frac{\pi}{2}\right)^3}$$

$$\boxed{=\frac{2}{\pi}\ln 2}$$

$$\lim_{x\to\infty}\left(x-\ln\left(\frac{e^x+e^{-x}}{2}\right)\right)$$

$x = \ln e^x$

$$=\lim_{x\to\infty}\ln\left(\frac{2e^x}{e^x+e^{-x}}\right)$$

$$=\lim_{x\to\infty}\ln\left(\frac{2}{1+e^{-2x}}\right)$$

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$\boxed{\text{not exist}}$

$\sin^{-1}x = \theta$

$$\lim_{\theta \to \pi/4} \frac{\cos^{-1} \sin 2\theta}{\sin\theta - \frac{1}{\sqrt{2}}} = \lim_{\theta \to \frac{\pi}{4}} \frac{\frac{\pi}{2} - \sin^{-1}\sin(2\theta)}{\sin\theta - \frac{1}{\sqrt{2}}}$$

$\theta \in \left(\frac{\pi}{4} - \delta, \frac{\pi}{4}\right) \cup \left(\frac{\pi}{4}, \frac{\pi}{4} + \delta\right)$

$$LHL = \lim_{\theta \to \frac{\pi}{4}^-} \frac{\frac{\pi}{2} - 2\theta}{\sin\theta - \sin\frac{\pi}{4}} = \lim_{\theta \to \frac{\pi}{4}^-} \frac{-2\left(\theta - \frac{\pi}{4}\right)}{\left(\sin\theta - \sin\frac{\pi}{4}\right)} = -2\sqrt{2}$$

$x \to \frac{1}{\sqrt{2}}^-$

$$RHL = \lim_{\theta \to \frac{\pi}{4}^+} \frac{\frac{\pi}{2} - (\pi - 2\theta)}{\sin\theta - \sin\frac{\pi}{4}} = \lim_{\theta \to \frac{\pi}{4}^+} \frac{2\left(\theta - \frac{\pi}{4}\right)}{\sin\theta - \sin\frac{\pi}{4}} = 2\sqrt{2}$$

$\boxed{\pi - x}$

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7. $\underline{f_1(x)} = \frac{x}{2} + 10$

$$f_n(x) = f_1(f_{n-1}(x))$$

$$l = \lim_{n\to\infty} f_n(x) = \lim_{n\to\infty} f_1\left(\left(f_{n-1}(x)\right)\right)$$

$$\Rightarrow \boxed{l = f_1(l) = \frac{l}{2} + 10}$$

$$\lim_{n\to\infty} f_n(x) = l$$

$$\lim_{n\to\infty} f_{n-1}(x) = l$$

$$f_2(x) = f_1(f_1(x)) = \frac{f_1(x)}{2} + 10$$

$$= \frac{1}{2}\left(\frac{x}{2} + 10\right) + 10$$

$$= \frac{x}{2^2} + 10\left(\frac{1}{2} + 1\right)$$

$$f_3(x) = \frac{f_2}{2} + 10$$

$$f_3(x) = \frac{x}{2^3} + 10\left(1 + \frac{1}{2} + \frac{1}{2^2}\right)$$

8.

$$\lim_{x\to 0^-} g(\underbrace{f(x)}_{\downarrow})$$

$$2-x \to 2^+$$

$$g(2^+) = 2-5 = -3$$

$$g \downarrow \; x-5$$

$$\lim_{x\to 0^+} g(\underbrace{f(x)}_{1^+}) = 1-2-2 = -3$$

$$\frac{x}{\sin x} \to 1^+$$

$$x^2-2x-2$$

$\{\cdot\} = FPF$

$$\lim_{x\to 0^-}\left\{\underbrace{g(\underset{2^+}{f(x)})}_{-3^+}\right\} = 0$$

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$$\sum_{k=1}^{n}\left(\sin\frac{\pi}{2k}-\sin\frac{\pi}{2(k+2)}\right)=\sum_{k=1}^{n}\left(\cos\frac{\pi}{2k}-\cos\frac{\pi}{2(k+2)}\right)$$

$k=1,2$ $\qquad k=n,n-1$

$$=\left(\sin\frac{\pi}{2}+\sin\frac{\pi}{4}-\sin\frac{\pi}{2(n+2)}-\sin\frac{\pi}{2(n+1)}\right)-\left(\cos\frac{\pi}{2}+\cos\frac{\pi}{4}-\cos\frac{\pi}{2(n+1)}-\cos\frac{\pi}{2(n+2)}\right)$$

$$\lim_{x\to 0}\frac{P_n}{x}=\lim_{x\to 0}\frac{a^{P_{n-1}}-1}{P_{n-1}}\frac{P_{n-1}}{x}=\ln a\lim_{x\to 0}\frac{P_{n-1}}{x}$$

$$\boxed{\lim_{x\to 0}\frac{P_1}{x}=\ln a}$$

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12. $$\lim_{n\to\infty} \prod_{r=3}^{n} \frac{(r-2)(r+2)}{r^2} = \lim_{n\to\infty} \prod_{r=3}^{n} \frac{r-2}{r} \prod_{r=3}^{n} \frac{r+2}{r}$$

$r=3,4$

$r=n, n-1$

$$= \lim_{n\to\infty} \frac{1\cdot 2}{n(n-1)} \cdot \frac{(n+2)(n+1)}{3\cdot 4}$$

$$M = \lim_{n\to\infty} \left(\prod_{r=2}^{n} \left(\frac{r-1}{r+1}\right) \prod_{r=2}^{n} \left(\frac{r^2+r+1}{r^2-r+1}\right) \right)$$

$r=n$

$r=2$

$$= \lim_{n\to\infty} \frac{1\cdot 2}{(n+1)n} \cdot \frac{n^2+n+1}{(4-2+1)}$$

$r=n$

$$\lim_{n\to\infty} \prod_{r=1}^{n} \frac{2\tan\frac{\theta}{2^r}}{\tan\left(2\left(\frac{\theta}{2^r}\right)\right)}$$

$r=1$

$$= \lim_{n\to\infty} \frac{2^n \tan\left(\frac{\theta}{2^n}\right)\theta}{\tan\theta \;\; \theta}$$

$$= \frac{\theta}{\tan\theta}$$

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$$0 < \left(\sqrt{2} - 2^{\frac{1}{3}}\right)\left(\sqrt{2} - 2^{\frac{1}{5}}\right)\left(\sqrt{2} - 2^{\frac{1}{7}}\right)\cdots\left(\sqrt{2} - 2^{\frac{1}{2n+1}}\right) < \left(\sqrt{2} - 1\right)^n$$

$$\sqrt{2} - 2^{\frac{1}{3}} < \sqrt{2} - 1$$

$$\sqrt{2} - 2^{\frac{1}{5}} < \sqrt{2} - 1$$

$$2^{\frac{1}{3}} > 1$$

$$2^{\frac{1}{5}} > 1$$

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(d) $2 < \frac{1}{{}^nC_0} + \frac{1}{{}^nC_1} + \frac{1}{{}^nC_2} + \frac{1}{{}^nC_3} + \cdots \frac{1}{{}^nC_{n-1}} + \frac{1}{{}^nC_n} < 2 + \frac{2}{n} + \frac{n-3}{{}^nC_2}$

$$\lim_{n\to\infty} 2 + \frac{2}{n} + \frac{(n-3)2}{n(n-1)}$$

$= 2$

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15.

$$\lim_{x\to 0} \frac{\ln\left(\frac{x+\sqrt{1+x^2}}{1+x}\right)}{\ln(1+x)\,\ln\left(x+\sqrt{1+x^2}\right)}$$

$$= \boxed{\frac{1}{2}}$$

$$\frac{x^2\left(\sqrt{1+x^2}-(x-1)\right)}{x\left(\sqrt{1+x^2}+1\right)\quad 2x}$$

× white circles

$$\frac{\dfrac{\ln\left(1+(\quad)\right)}{(\quad)}\left(\dfrac{\sqrt{1+x^2}-1}{1+x}\right)}{\left(\dfrac{\ln(1+x)}{x}\right)x\;\dfrac{\ln(1+\quad)}{(\quad)}\left(\sqrt{1+x^2}+x-1\right)}$$

1. $$f(x) = \lim_{n\to\infty}\left(\frac{\tan(\pi x^2) + (x+1)^n \sin x}{x^2 + (x+1)^n}\right), \quad \lim_{x\to 0} f(x) = ?$$

$\boxed{\text{not exist}}$

$$\lim_{x\to 0^-}\lim_{n\to\infty}\left(\frac{\tan(\pi x^2) + \overset{1^-}{(x+1)}^n \sin x}{x^2 + (x+1)^n}\right) = \lim_{x\to 0^-}\left(\frac{\tan(\pi x^2) + 0}{x^2 + 0}\right) = \pi$$

$$\lim_{x\to 0^+}\lim_{n\to\infty}\left(\frac{\frac{\tan(\pi x^2)}{(x+1)^n} + \sin x}{\frac{x^2}{(x+1)^n} + 1}\right) = \lim_{x\to 0^+}\frac{0+\sin x}{0+1} = 0$$

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Q. If $f(x) = \lim_{n\to\infty} \dfrac{\cos(\pi x) - x^{2n}\sin(x-1)}{1+x^{2n+1} - x^{2n}}$, find $\lim_{x\to 1} f(x)$

$= -1$

$$\lim_{x\to 1^-} \lim_{n\to\infty} \frac{\cos(\pi x) - x^{2n}\sin(x-1)}{1+x^{2n+1} - x^{2n}} = \lim_{x\to 1^-} \cos\pi x = -1$$

$$\lim_{x\to 1^+} \lim_{n\to\infty} \frac{\frac{\cos\pi x}{x^{2n}} - \sin(x-1)}{\frac{1}{x^{2n}} + x - 1}$$

$$= \lim_{x\to 1^+} \frac{-\sin(x-1)}{(x-1)} = -1$$

✓ Ex-III → $\boxed{\text{Leave Q 15}}$

Ex-IV (1-10)

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De Arrangement

$E_1 \cap E_2$

To place n letters in n addhered envelopes so that no letter is placed to its corresponding envelope

L_1 is placed in correct envelop $\rightarrow E_1$ ✓

$\vdots$ $\rightarrow E_n$

L_n —— " ——

$$= n! - n(E_1 \cup E_2 \cup E_3 \cup \dots \cup E_n)$$

$$= \cancel{n!} - \left(\cancel{{}^nC_1 \times (n-1)!} - {}^nC_2 \times (n-2)! + {}^nC_3 (n-3)! - \dots\right)$$

$$= n!\left(\frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \frac{1}{5!} + \dots + \frac{(-1)^n}{n!}\right)$$

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Col.-I	Col.-II
Q 1	A-1
2	-2
	-3
4	-4
5	-5
6	-6

One to one.

find no. of ways so that atleast 4 are wrongly matched.

$$^6C_2\left(4!\left(\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}\right)\right)2!\left(\frac{1}{2!}\right)$$

$$+\,^6C_1\left(5!\left(\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}-\frac{1}{5!}\right)\right)$$

$$3!\left(\frac{1}{2!}-\frac{1}{3!}\right) = 2$$

$$+\,6!\left(\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}-\frac{1}{5!}+\frac{1}{6!}\right)$$

$$6! - \left(1 + 0 + {}^6C_2\times1\times1 + {}^6C_3\times1\times2\right)$$