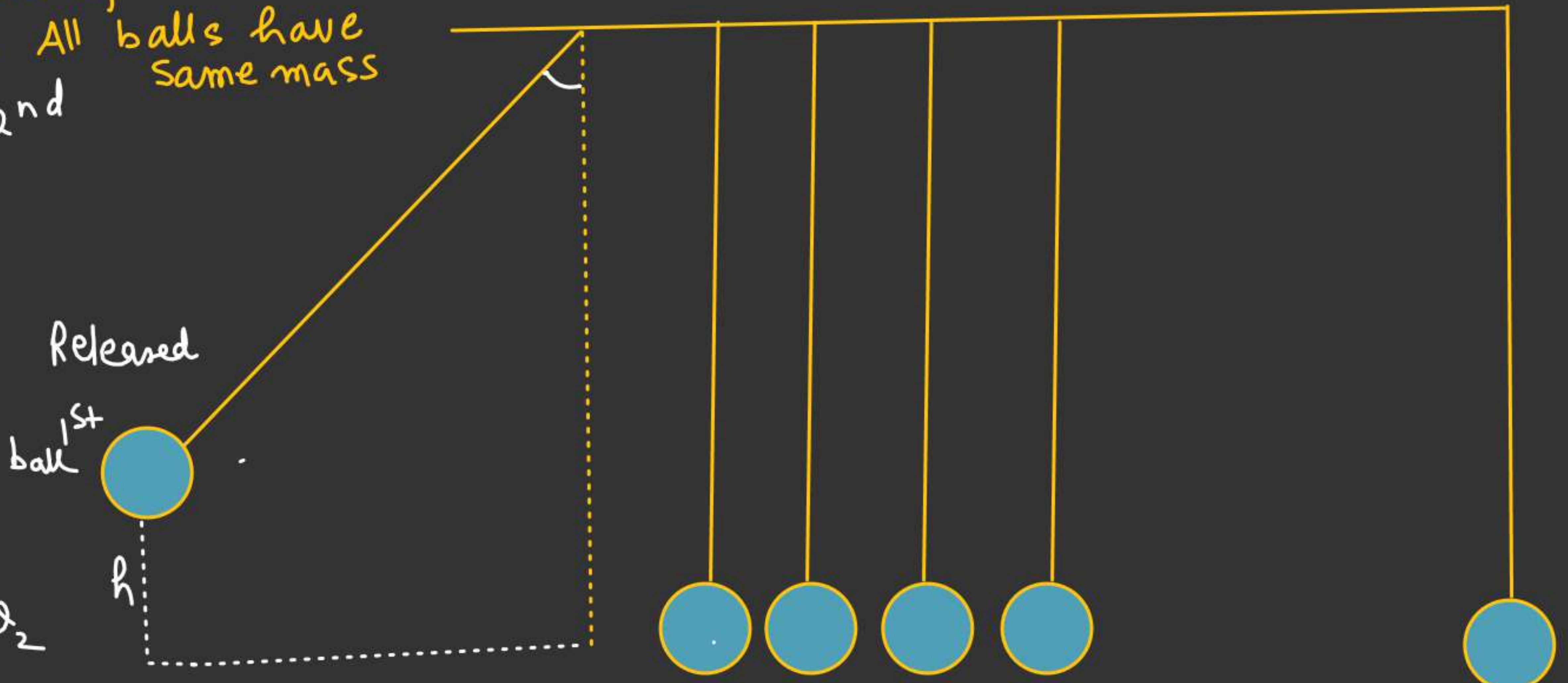
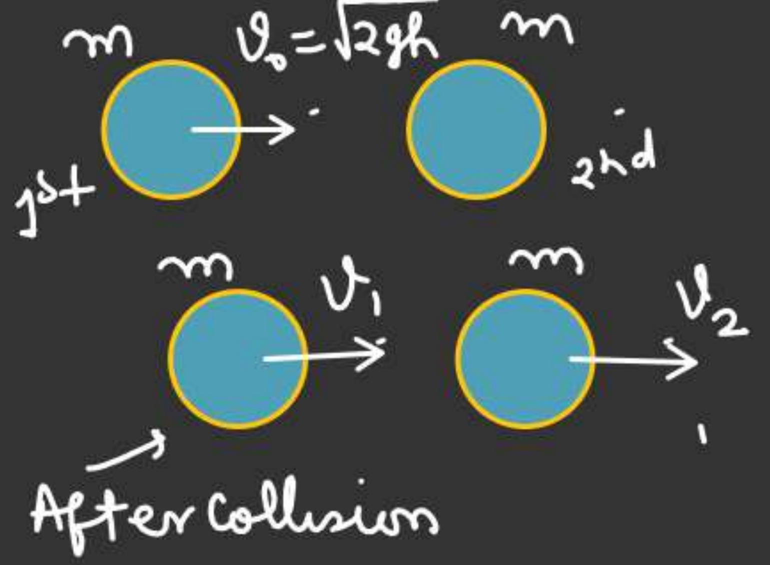
Nishant Jindal

Find velocity of nth ball.

e be the coefficient of restitution b/w each collision. All balls have same mass



Collision b/w 1st and 2nd



$$mv_0 = mv_1 + mv_2$$

$$v_1 + v_2 = v_0 \quad \text{—(1)}$$

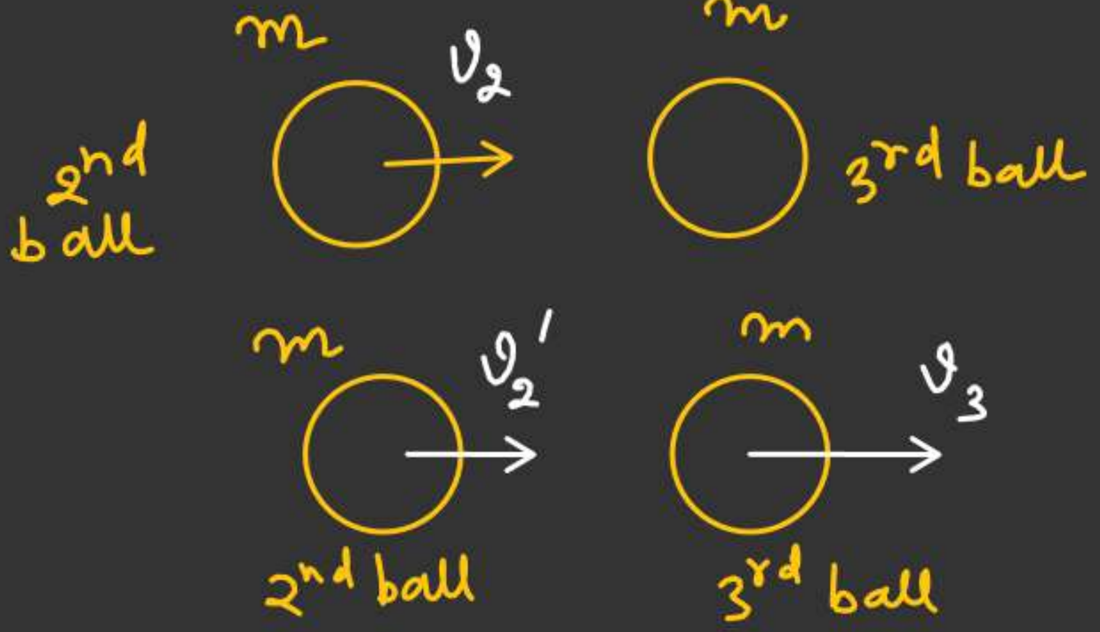
$$e = \frac{v_2 - v_1}{v_0}$$

$$ev_0 = v_2 - v_1 \quad \text{—(2)}$$

$$\underline{\underline{v_2}} = (e+1)\frac{v_0}{2} \checkmark$$

$$v_1 = (1-e)\frac{v_0}{2}$$

Nishant Jindal



$$m v_2 = m v_2' + m v_3$$

$$v_2 = v_2' + v_3$$

$$e = \frac{v_3 - v_2'}{v_2}$$

$$e v_2 = v_3 - v_2'$$

Adding

$$v_3 = \frac{(e+1)}{2} v_2$$

$$v_3 = \frac{(e+1)}{2}\left[\frac{(e+1)}{2} v_0\right]$$

$$v_3 = \left(\frac{e+1}{2}\right)^2 v_0$$

$$v_n = \left(\frac{e+1}{2}\right)^{n-1} v_0$$

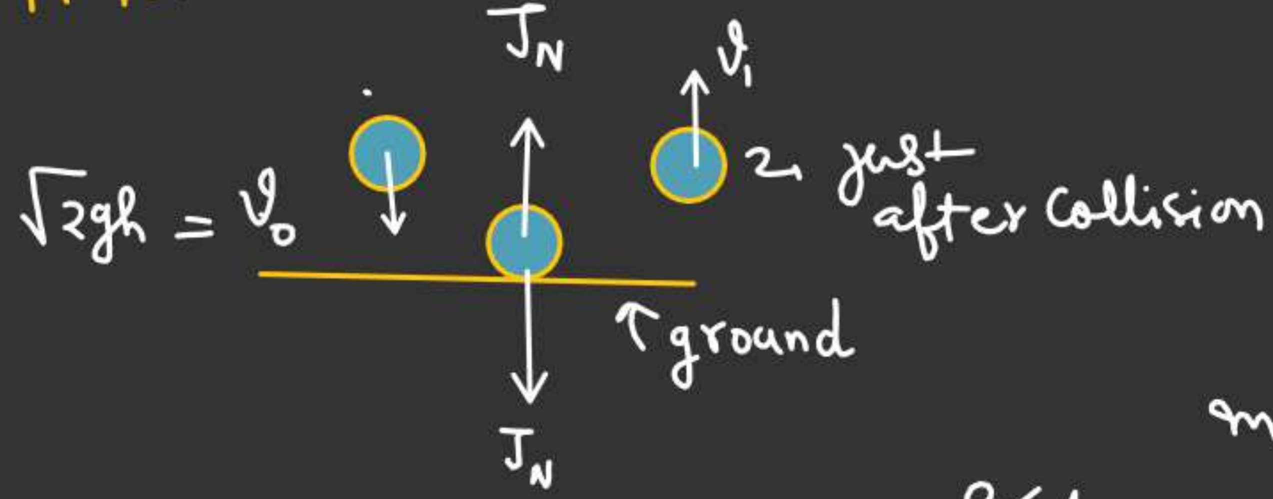
$$v_n = \left(\frac{e+1}{2}\right)^{n-1} \sqrt{2gh}$$ Ans

Nishant Jindal

e = Cofficient of restitution b/w ground and ball.

Ball released from vertical height h

Find Avg force imparted by ground on the ball after a very long time.



$$e = \frac{v_1}{v_0}$$

$$\underline{v_1 = e v_0} = e\sqrt{2gh}$$

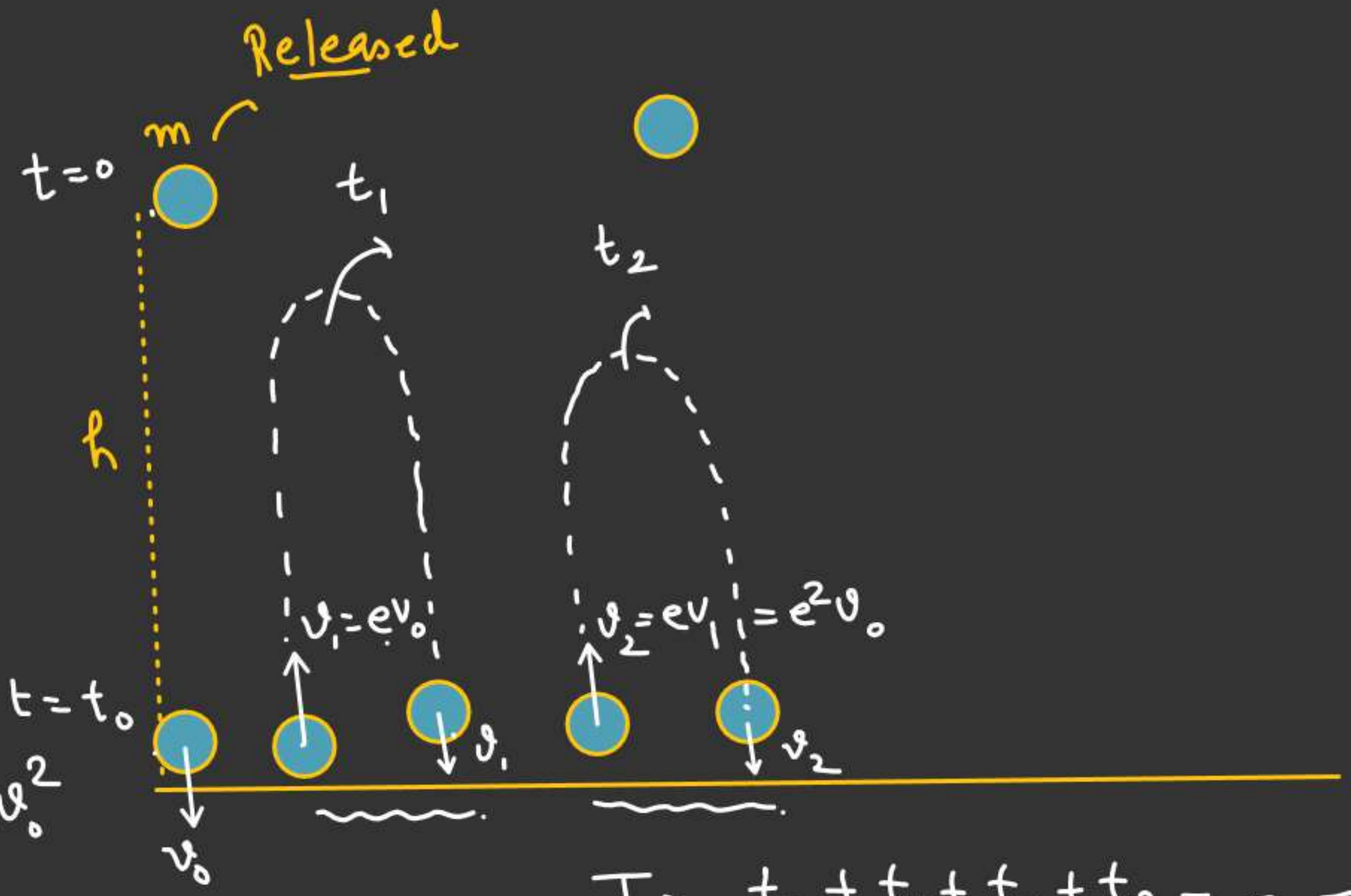
$\underline{e<1}$

$$mgh = \frac{1}{2} m v_0^2$$

$$v_0 = \sqrt{2gh}$$

$$t_0 = \sqrt{\frac{2h}{g}}$$

$$t_1 = \frac{2v_1}{g} = \frac{2ev_0}{g}$$



$$t_2 = \frac{2v_2}{g} = \frac{2e^2 v_0}{g}$$

$$T = t_0 + t_1 + t_2 + t_3 - - -$$

$$T = \sqrt{\frac{2h}{g}} + \frac{2ev_0}{g} + \frac{2e^2 v_0}{g} + \frac{2e^3 v_0}{g} \ldots$$

Nishant Jindal

$$T = t_0 + t_1 + t_2 + t_3 - - -$$

$$v_0 = \sqrt{2gh}$$

$$T = \sqrt{\frac{2h}{g}} + \frac{2ev_0}{g} + \frac{2e^2v_0}{g} + \frac{2e^3v_0}{g} \ldots$$

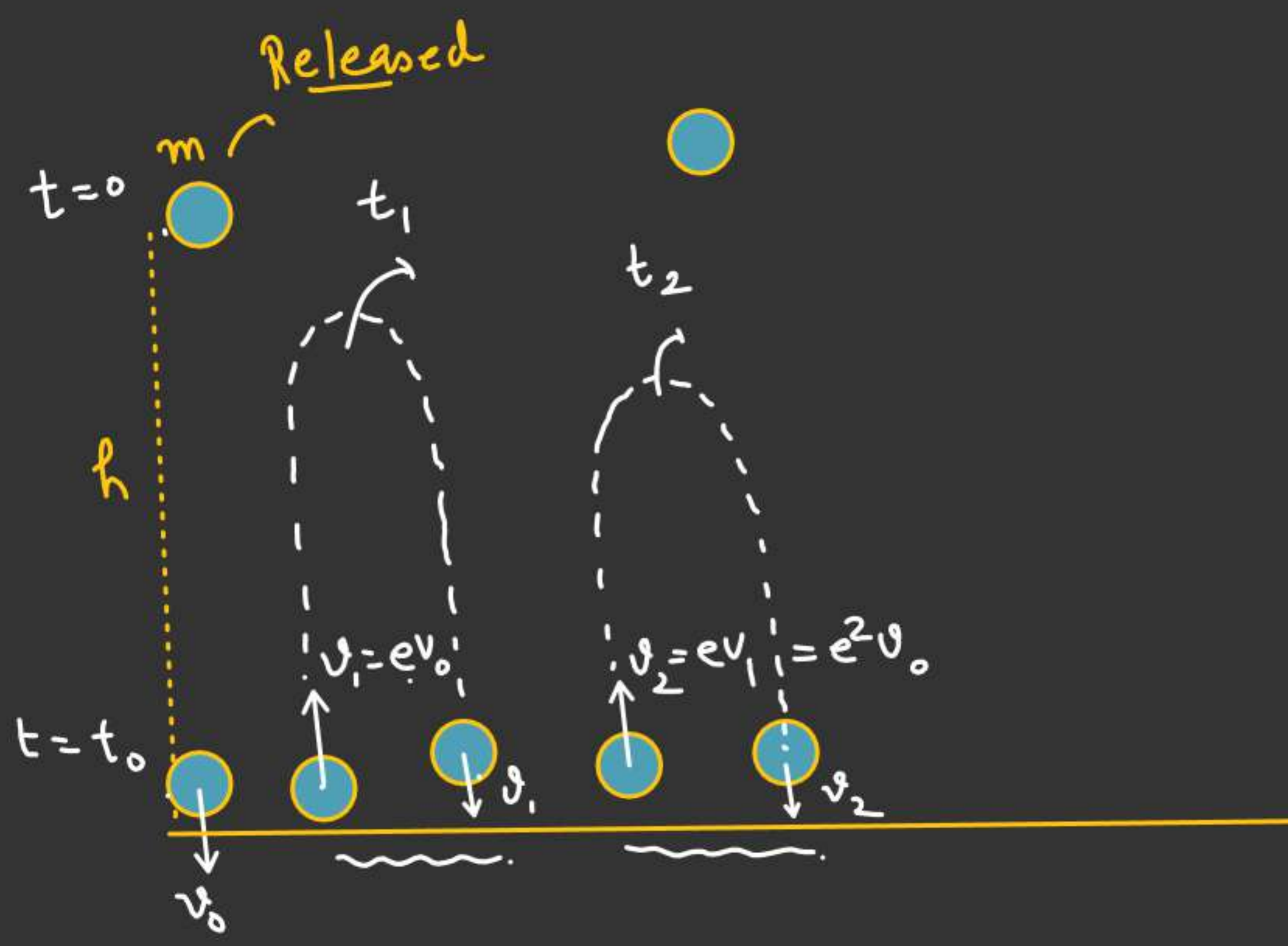
$$T = \sqrt{\frac{2h}{g}} + \frac{2ev_0}{g}\left[1 + e + e^2 - - - \infty\right]$$

$$T = \sqrt{\frac{2h}{g}} + \frac{2e(\sqrt{2gh})}{g}\left[\frac{1}{1-e}\right]$$

$$T = \sqrt{\frac{2h}{g}} + 2e\sqrt{\frac{2h}{g}}\left(\frac{1}{1-e}\right)$$

$$T = \sqrt{\frac{2h}{g}}\left(1 + \frac{2e}{1-e}\right)$$

$$\boxed{T = \sqrt{\frac{2h}{g}}\left(\frac{1+e}{1-e}\right)}$$



Nishant Jindal

Total Change in linear Momentum

Change in linear Momentum during 1st Collision

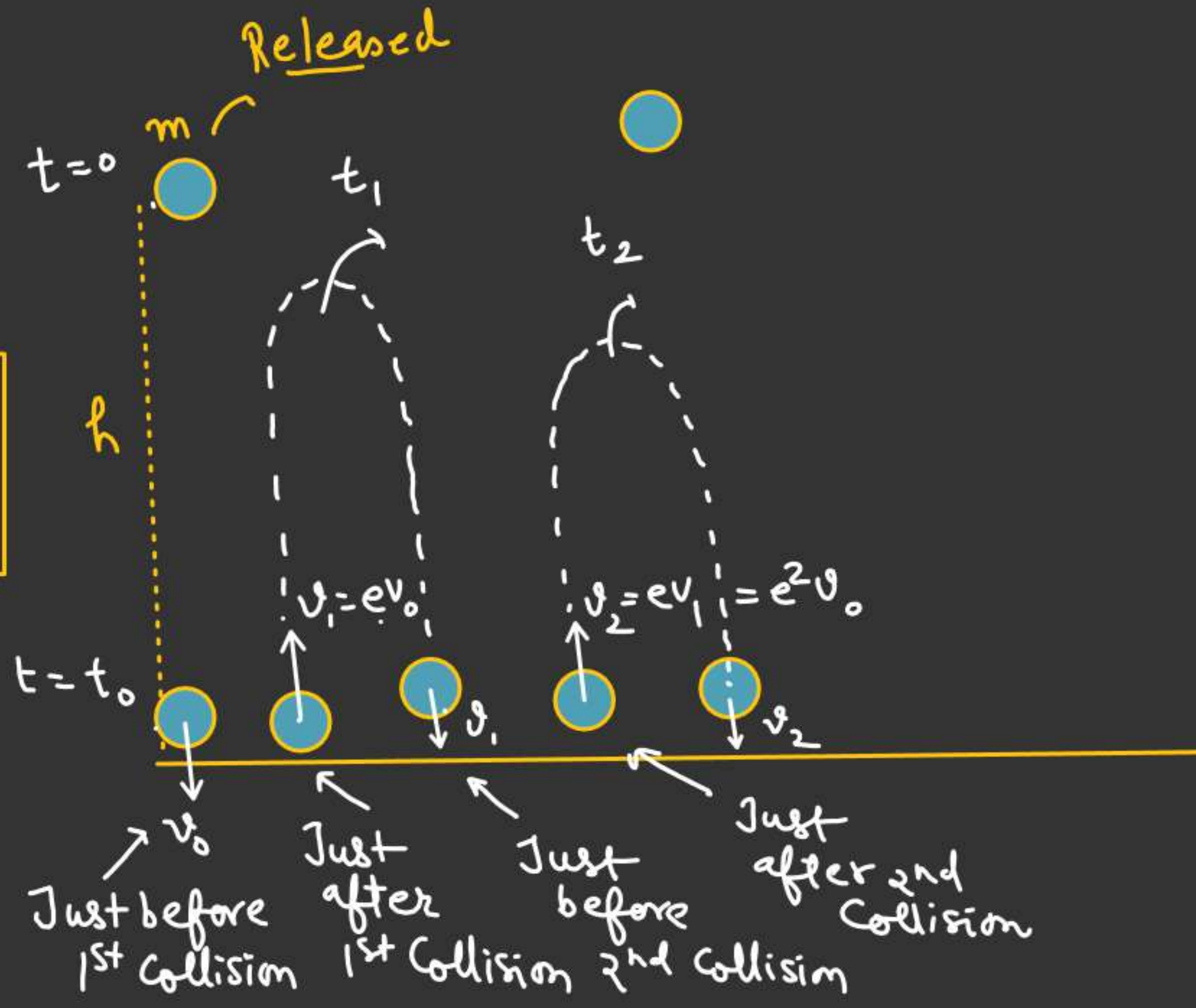
$$\Delta p_1 = mv_1 - (-mv_0)$$
$$= m(v_1 + v_0) = m(ev_0 + v_0)$$
$$= mv_0(e+1)$$

$$\boxed{\Delta p_{net} = mv_0 \frac{(e+1)}{(1-e)}}$$

Change during 2nd Collision

$$\Delta p_2 = mv_2 - (-mv_1)$$
$$= m(v_2 + v_1)$$
$$= m(e^2 v_0 + e v_0)$$
$$= mv_0(e+1)e$$

$$\Delta p_{net} = \Delta p_1 + \Delta p_2 + \Delta p_3 \ldots$$
$$= mv_0(e+1) + mv_0(e+1)e + mv_0(e+1)e^2 \ldots \infty$$
$$= mv_0(e+1)\left[1 + e + e^2 \ldots \infty\right]$$
$$= \frac{mv_0(e+1)}{(1-e)}$$



Nishant Jindal

$$F_{avg} = \left(\frac{\Delta p_{net}}{T}\right) \qquad v_0 = \sqrt{2gh}$$

$$F_{avg} = \frac{mv_0\left(\frac{1+e}{1-e}\right)}{\sqrt{\frac{2h}{g}}\left(\frac{1+e}{1-e}\right)}$$

$$F_{avg} = m\sqrt{2gh} \times \sqrt{\frac{g}{2h}}$$

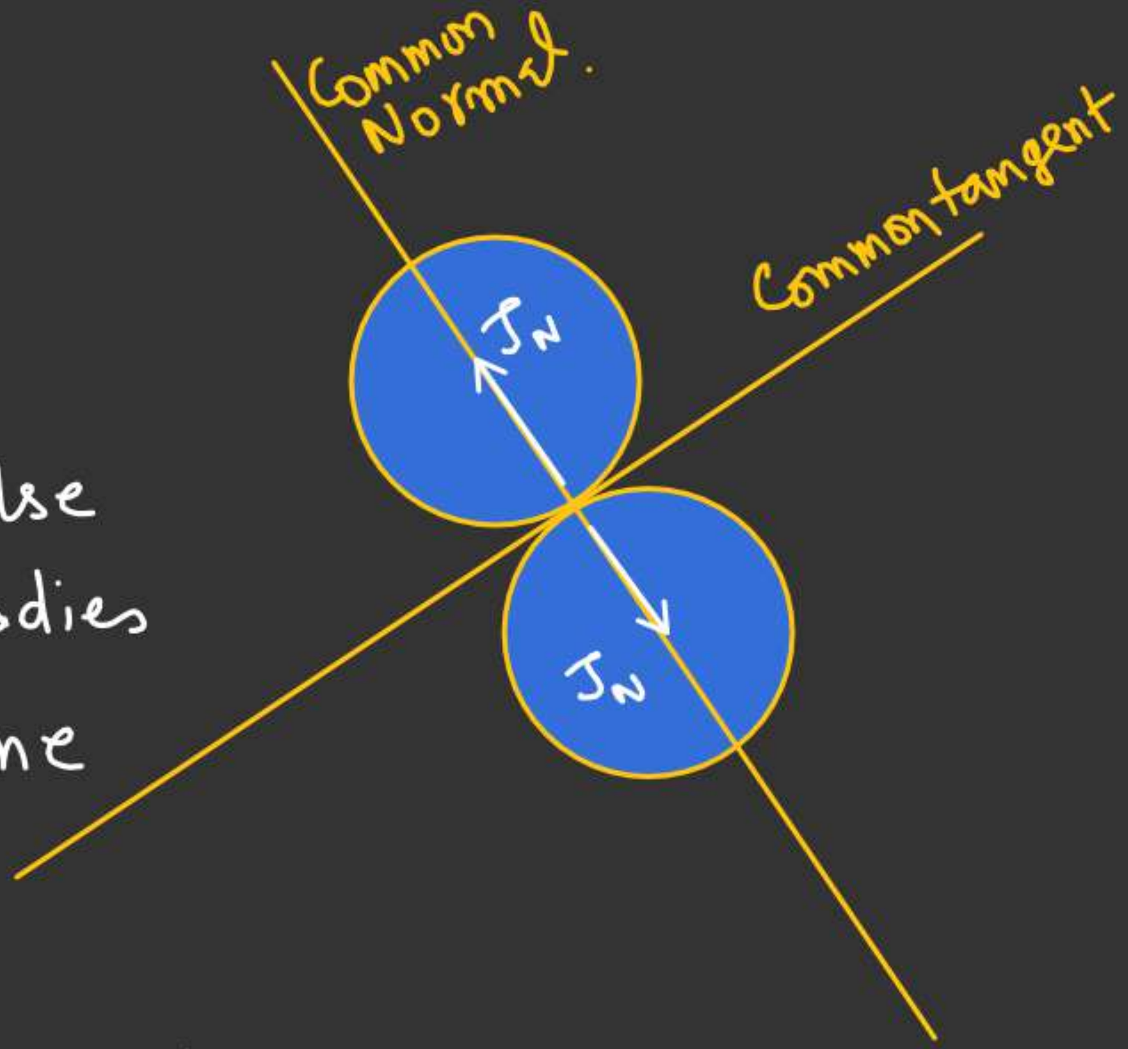
$$\boxed{F_{avg} = mg}$$

Nishant Jindal

OBLIQUE COLLISION

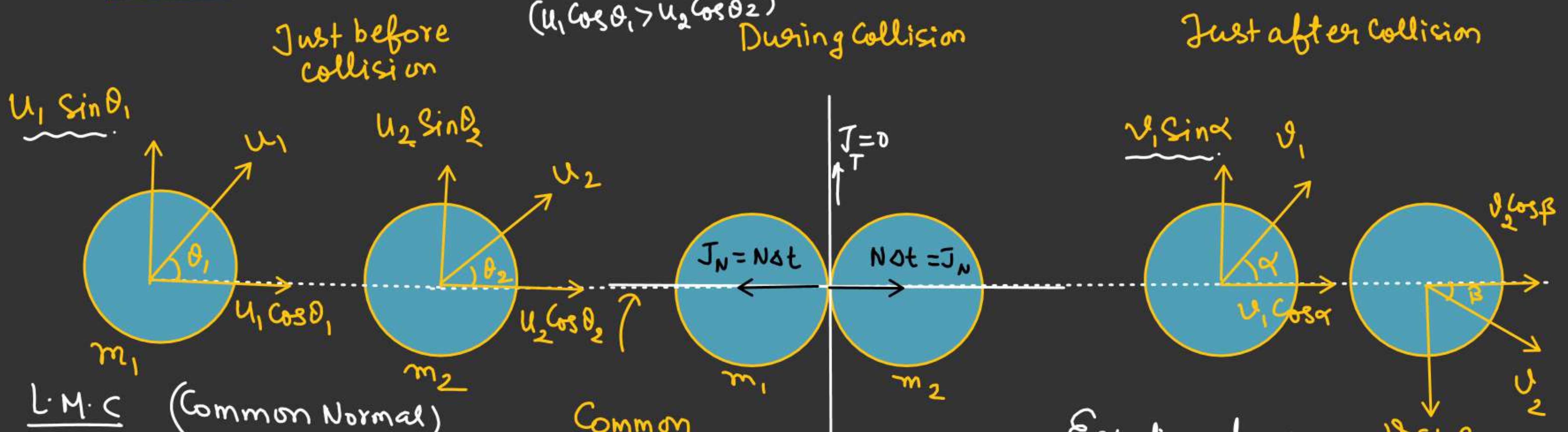
Important Points

- L·M·C alway along Common Normal. ✓
- If along Common tangent net impulse is zero then velocity of Colliding bodies along Common tangent remain same Just before & just after collision
- Always equation of e along Common Normal.



Nishant Jindal

OBLIQUE COLLISION



L.M.C (Common Normal)

$$m_1 u_1 \cos\theta_1 + m_2 u_2 \cos\theta_2 = m_1 v_1 \cos\alpha + m_2 v_2 \cos\beta$$

Equation of e (Common Normal)

$$e = \left(\frac{v_2 \cos\beta - v_1 \cos\alpha}{u_1 \cos\theta_1 - u_2 \cos\theta_2} \right)$$

Equation along Common tangent

If impulse along common tangent is zero

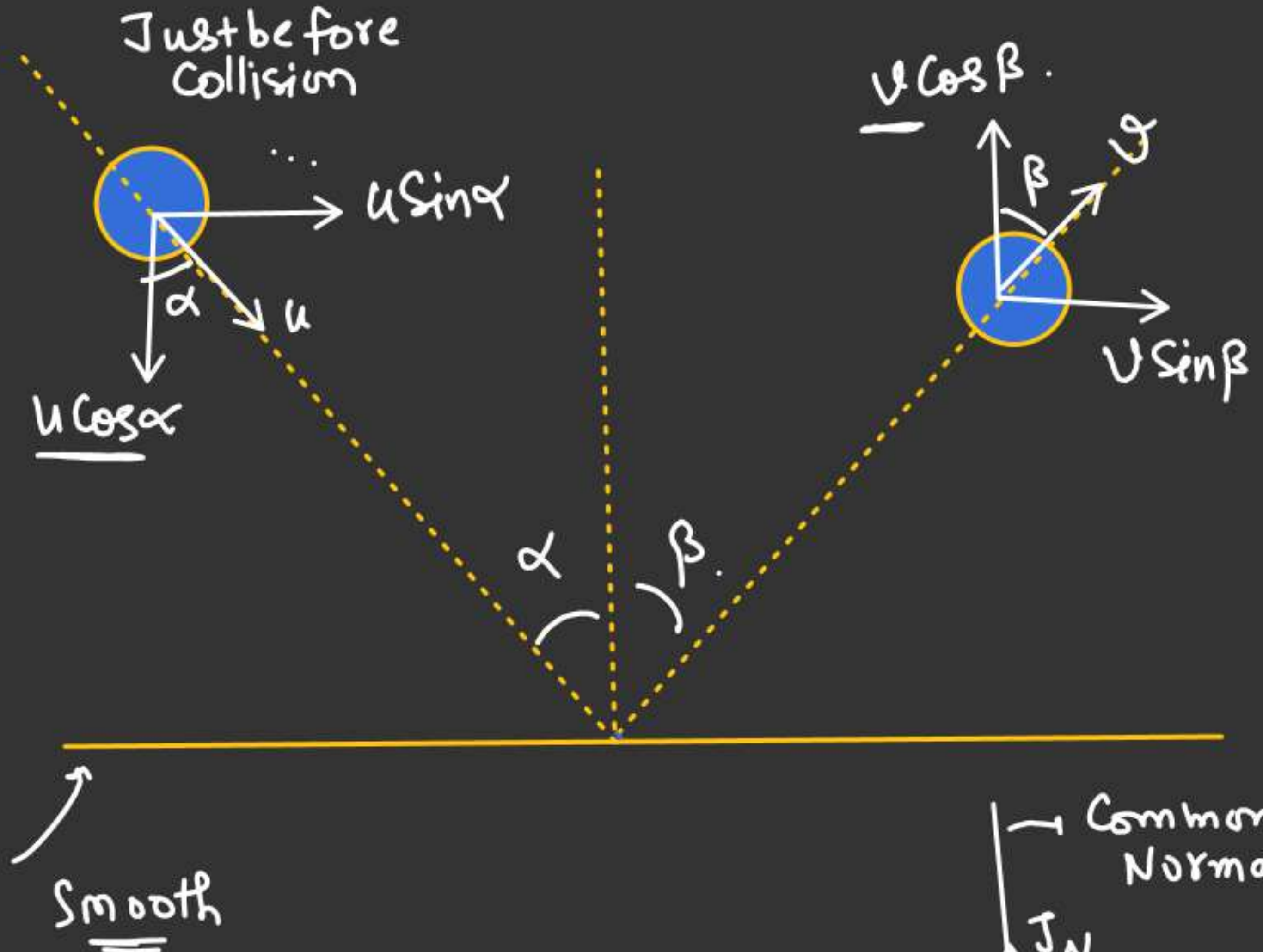
$$\boxed{\begin{aligned} u_1 \sin\theta_1 &= v_1 \sin\alpha \\ u_2 \sin\theta_2 &= v_2 \sin\beta \end{aligned}}$$

Nishant Jindal

e = Coffⁿ of Restitution b/w ground & ball.

a) Find $v = ?$

b) Ralation b/w $\tan\alpha$ & $\tan\beta$.

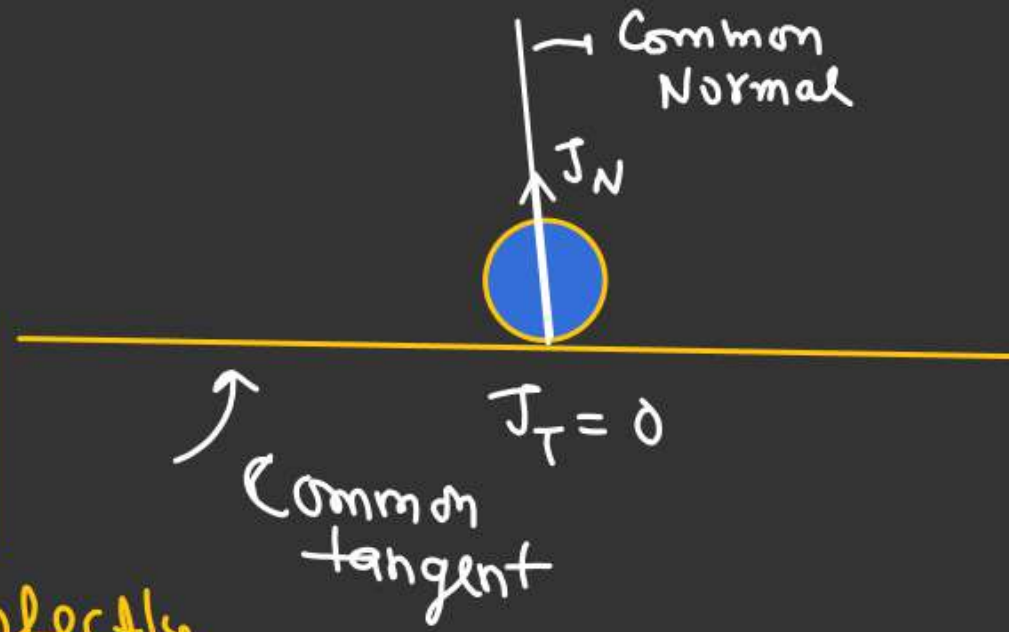


Along Common tangent

$$u\sin\alpha = v\sin\beta \quad -(1)$$

$$e = \frac{v\cos\beta}{u\cos\alpha}$$

$$e\,u\cos\alpha = v\cos\beta \quad -(2)$$



Squaring & adding both side

$$u^2\sin^2\alpha + e^2u^2\cos^2\alpha = v^2(\sin^2\beta + \cos^2\beta)$$

$$v = \left(u\sqrt{\sin^2\alpha + e^2\cos^2\alpha}\right) ✓$$

Dividing Eqⁿ (1) & (2)

$$\boxed{\tan\beta = \frac{\tan\alpha}{e}}$$

If $e = 1$ (Perfectly Elastic collision)

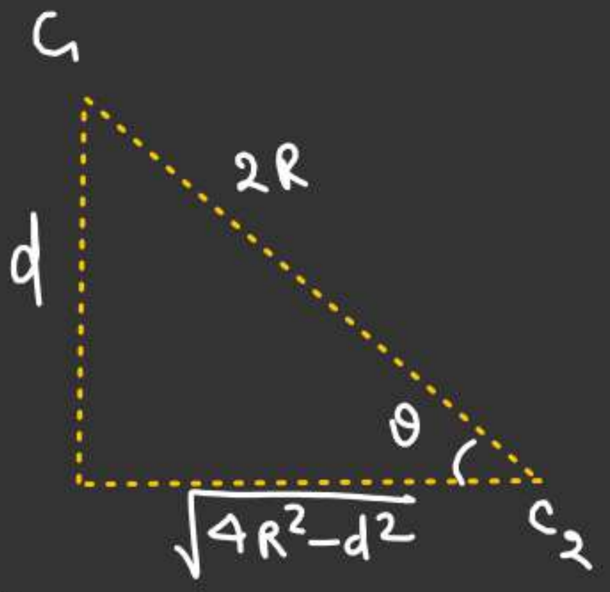
$\tan\beta = \tan\alpha$

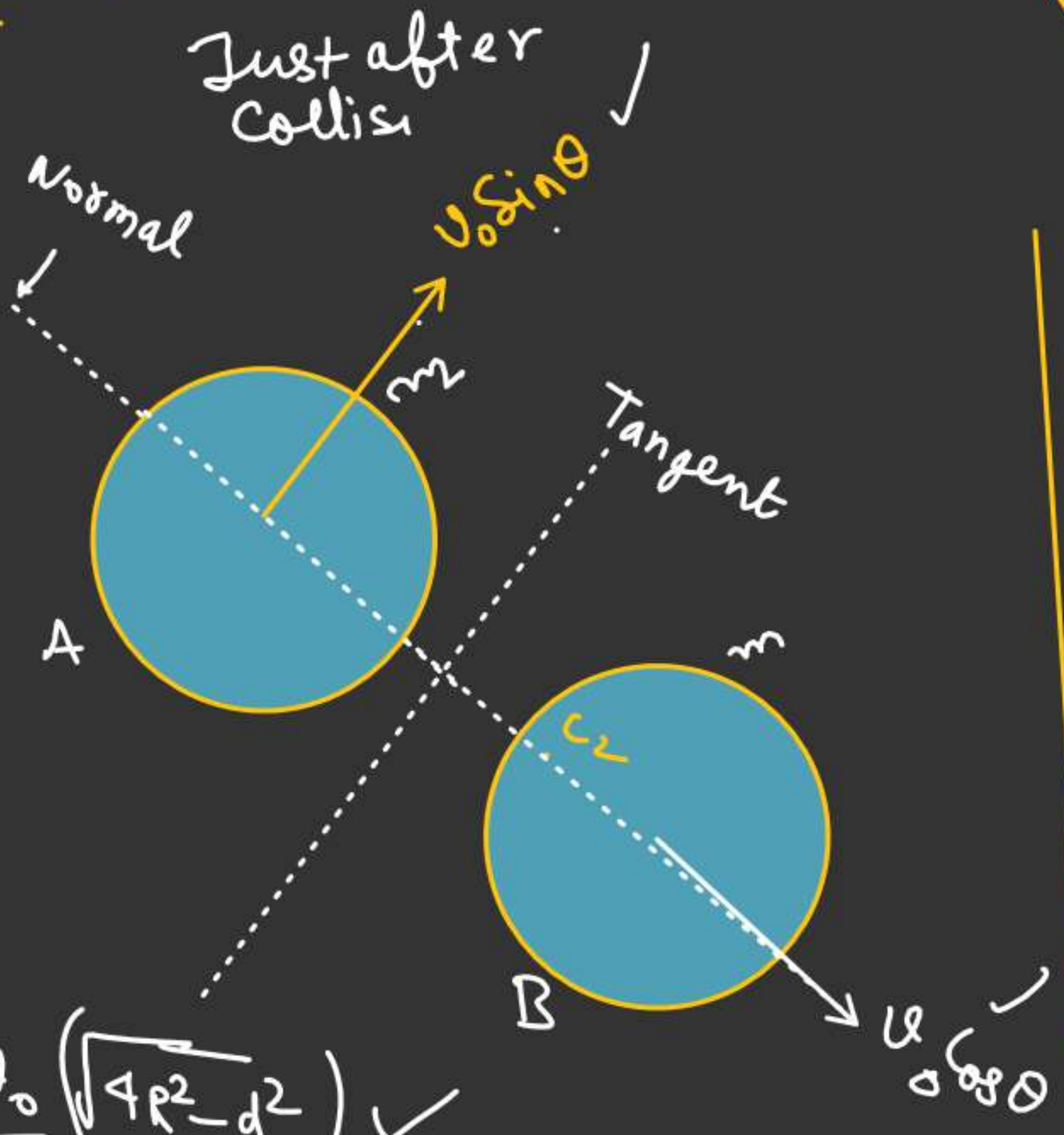
$\Rightarrow \boxed{\beta = \alpha}$

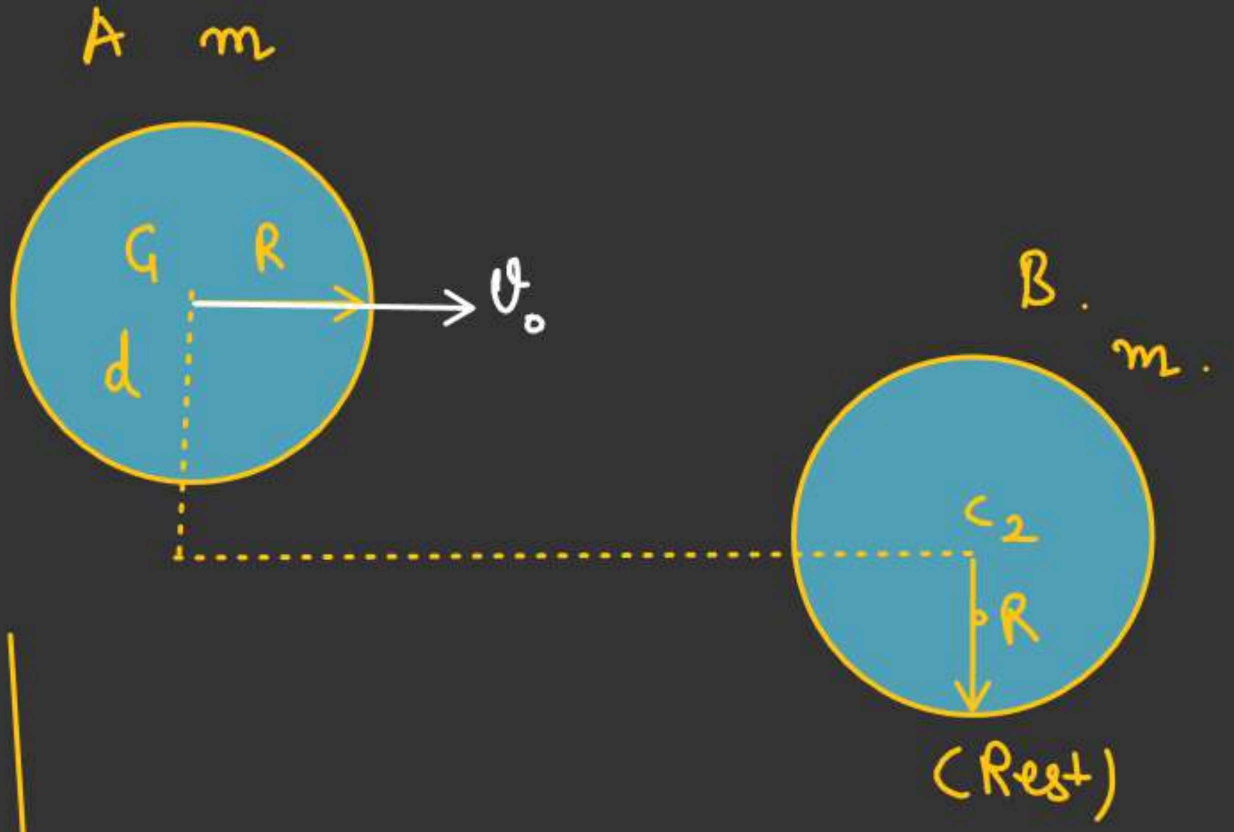
Nishant Jindal

AQ. After collision velocity of A and B..

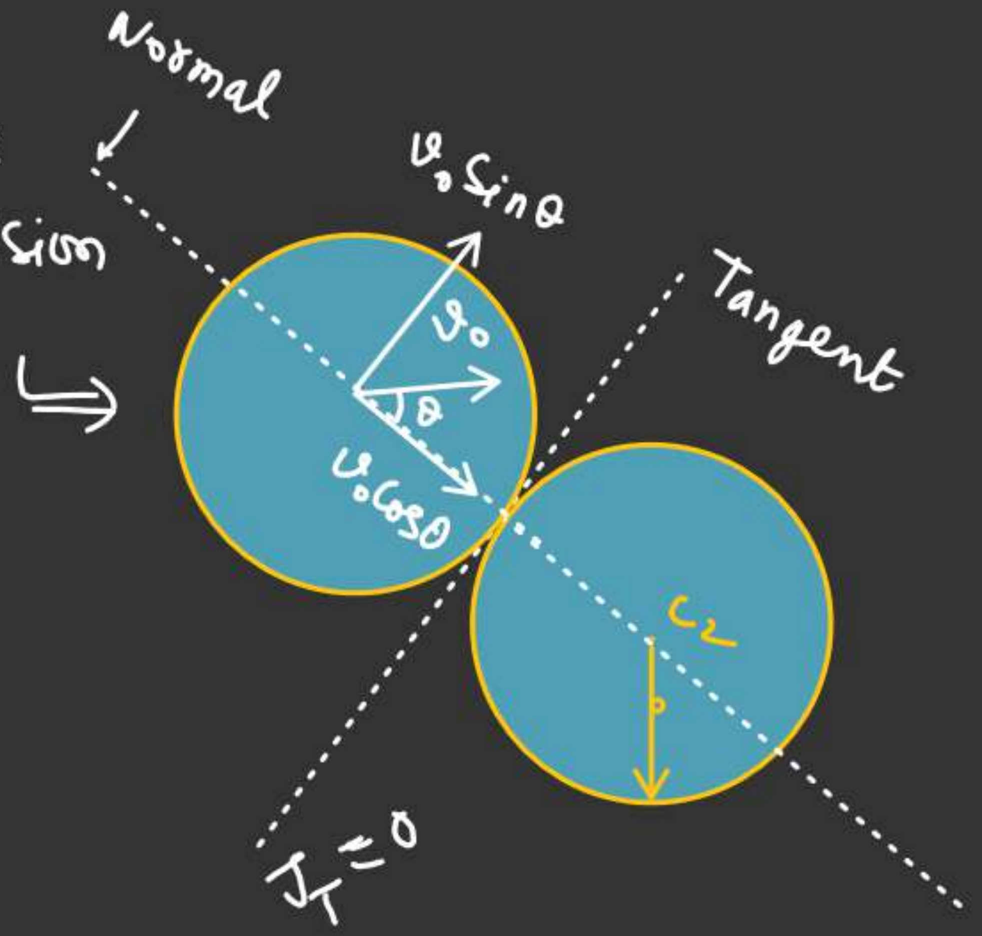
Collision is perfectly elastic.







Just before Collision



$$v_A = v_0 \sin\theta$$

$$= \left(\frac{v_0 d}{2R}\right)$$

$$v_B = v_0 \cos\theta = \frac{v_0}{2R}\left(\sqrt{4R^2-d^2}\right)$$

Nishant Jindal

H.W

If $e = \frac{1}{2}$.

Find Speed of ball A and B.
Just after Collision