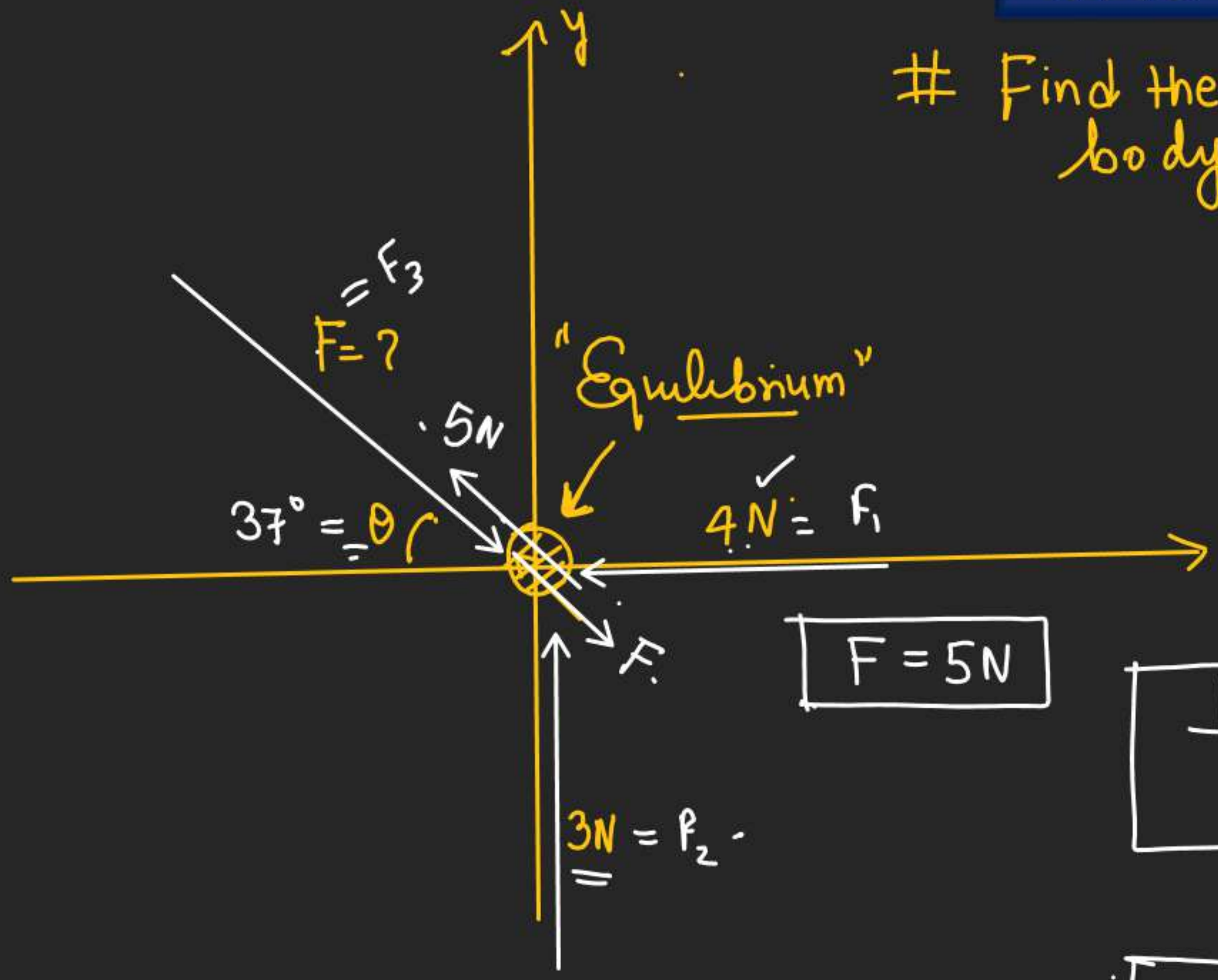
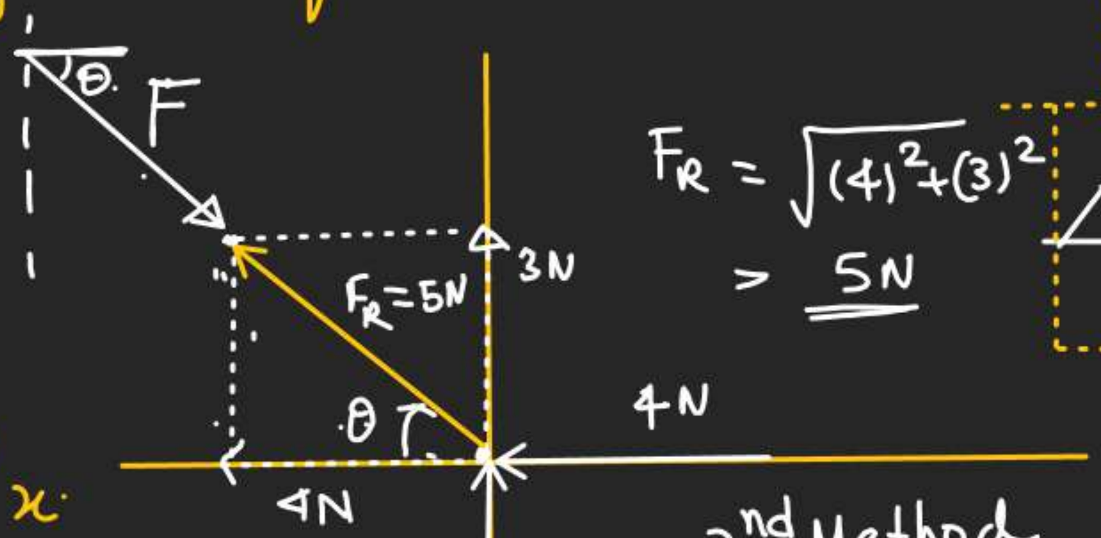
Nishant Jindal

VECTOR

\# Find the value of F and 'θ' so that body is in equilibrium.



$$\boxed{F = 5N}$$



$$F_R = \sqrt{(4)^2 + (3)^2}$$
$$= \underline{5N}$$

$$\boxed{\tan\theta = \frac{3}{4}}$$

$$\boxed{\theta = 37^\circ}$$

$$\boxed{|\vec{F_3}| = 5}$$

2^{nd} Method

$$\vec{F_R} = 0$$
$$\vec{F_1} + \vec{F_2} + \vec{F_3} = 0$$
$$\vec{F_3} = -(\vec{F_1} + \vec{F_2})$$
$$= -(-4\hat{i} + 3\hat{j})$$
$$\vec{F_3} = (4\hat{i} - 3\hat{j})$$

Cross-product :-
or
Vector product :

$\Rightarrow$ If $\vec{A}$ and $\vec{B}$ are two vectors then

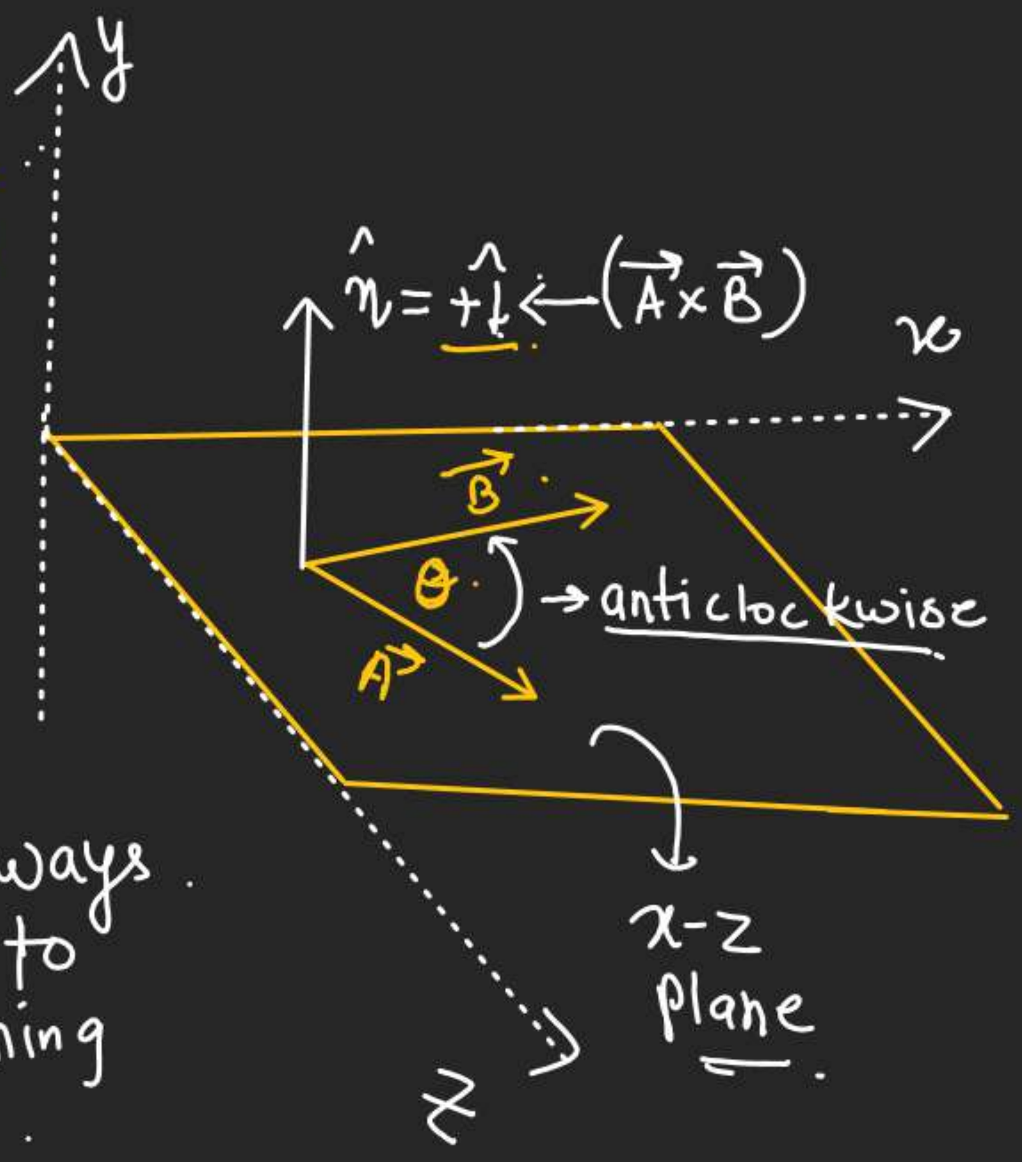
$$\boxed{\vec{A} \times \vec{B} = \left[|\vec{A}||\vec{B}| \sin\theta\right] \cdot \hat{n}}$$

$\Downarrow$

Magnitude of $(\vec{A} \times \vec{B}) = |\vec{A} \times \vec{B}|$

$\hat{n}$ = It is a Unit vector always perpendicular to plane containing $\vec{A}$ and $\vec{B}$.

Ex:-



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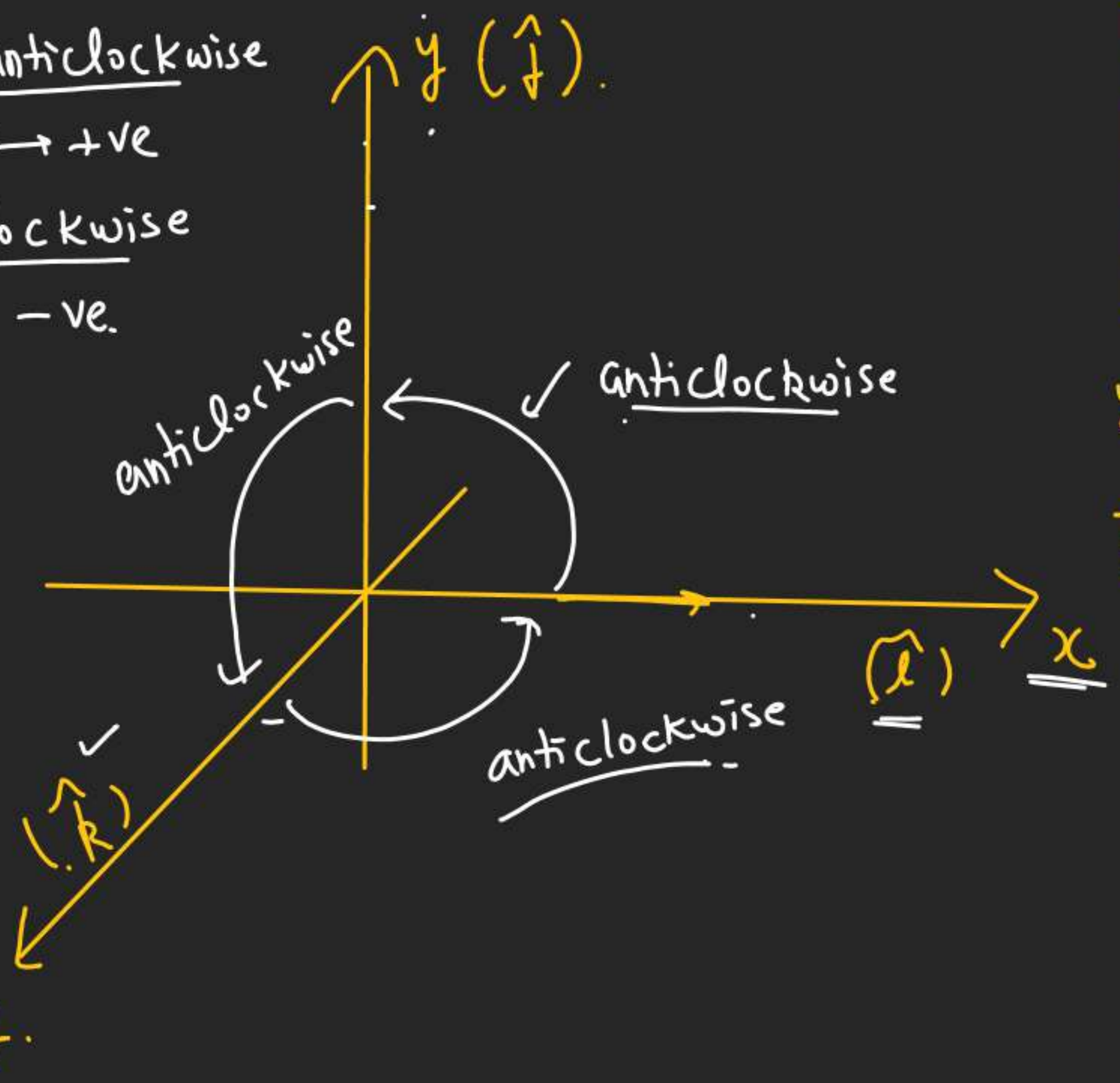
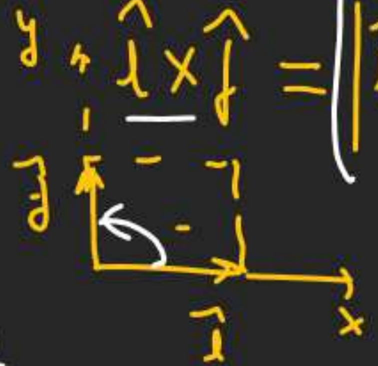
Vector product of Standard unit vector :→

anticlockwise ↳ +ve

clockwise → −ve.



$$\left[\begin{aligned} \hat{i}\times\hat{i} &= |\hat{i}||\hat{i}|\sin 0 = 0 \\ \hat{j}\times\hat{j} &= 0. \\ \hat{k}\times\hat{k} &= 0. \end{aligned}\right]$$



$$\hat{i}\times\hat{j} = \left[\underbrace{|\hat{i}||\hat{j}|\sin 90}_{1.}\right]\hat{k} \Rightarrow \left[\hat{i}\times\hat{j} = (1)\hat{k}\right]$$

$$\Rightarrow \hat{j}\times\hat{k} = +\hat{i}.$$

$$\Rightarrow \hat{k}\times\hat{i} = +\hat{j}.$$

$$\begin{aligned} \hat{j}\times\hat{i} &= -\hat{k}. \\ \hat{k}\times\hat{j} &= -\hat{i}. \\ \hat{i}\times\hat{k} &= -\hat{j}. \end{aligned}$$

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Vector product of two position vectors :-

$$\vec{r}_1 = a_1\hat{i} + b_1\hat{j} + c_1\hat{k}$$
$$\vec{r}_2 = a_2\hat{i} + b_2\hat{j} + c_2\hat{k}$$

$$(\vec{r}_1 \times \vec{r}_2) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = +\hat{i}\begin{vmatrix} b_1 & c_1 \\ b_2 & c_2 \end{vmatrix} - \hat{j}\begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix} + \hat{k}\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$$

$(+)$ $(-)$ $(+)$

$$\vec{r}_1 \times \vec{r}_2 = \left[+\hat{i}(b_1c_2 - b_2c_1) - \hat{j}(a_1c_2 - a_2c_1) + \hat{k}(a_1b_2 - a_2b_1)\right]$$

[If $\vec{r}_1$ and $\vec{r}_2$ are parallel. $\theta = 0$.]

$\vec{r}_1$ $\longrightarrow$

$\vec{r}_2$ $\longrightarrow$

$$\boxed{\vec{r}_1 \times \vec{r}_2 = 0}$$

$$0 = +\hat{i}(\underbrace{b_1c_2 - b_2c_1}_{0}) - \hat{j}(\underbrace{a_1c_2 - a_2c_1}_{0}) + \hat{k}(\underbrace{a_1b_2 - a_2b_1}_{0})$$

$b_1c_2 - b_2c_1 = 0$ $\qquad$ $a_1c_2 = a_2c_1$

$$\boxed{\frac{b_1}{b_2} = \frac{c_1}{c_2}} \longleftrightarrow \boxed{\frac{a_1}{a_2} = \frac{c_1}{c_2}} \Rightarrow \boxed{\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}} \Rightarrow$$

Condition for two vector to be parallel

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a) Find $|\vec{r}_1 \times \vec{r}_2| = ??$

if $\left[\begin{array}{l} \vec{r}_1 = 3\hat{i} - \hat{j} + 2\hat{k} \\ \vec{r}_2 = 2\hat{i} + 4\hat{j} + \hat{k} \end{array}\right]$

$$\vec{r}_1 \times \vec{r}_2 = \begin{array}{ccc} (+) & (-) & (+) \\ \end{array}$$
$$\vec{r}_1 \times \vec{r}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & (-1) & 2 \\ 2 & 4 & 1 \end{vmatrix} = +\hat{i}\begin{vmatrix} -1 & 2 \\ 4 & 1 \end{vmatrix} - \hat{j}\begin{vmatrix} 3 & 2 \\ 2 & 1 \end{vmatrix} + \hat{k}\begin{vmatrix} 3 & -1 \\ 2 & 4 \end{vmatrix}$$

$$= \hat{i}[(-1) - 8] - \hat{j}[3 - 4] + \hat{k}[12 - (-2)]$$

$$= \boxed{-9\hat{i} + \hat{j} + 14\hat{k}}$$

← This vector is always perpendicular to the plane containing $\vec{r}_1$ and $\vec{r}_2$.

$$|\vec{r}_1 \times \vec{r}_2| = \sqrt{(-9)^2 + (1)^2 + (14)^2}$$
$$= \sqrt{81 + 1 + 196}$$
$$= \sqrt{278}$$

b) Find a unit vector which is perpendicular to $\vec{r}_1$ and $\vec{r}_2$.

$\hat{n}$ = unit vector of $(-9\hat{i} + \hat{j} + 14\hat{k})$

$$\hat{n} = \left[\frac{-9\hat{i} + \hat{j} + 14\hat{k}}{\sqrt{278}}\right] ✓$$

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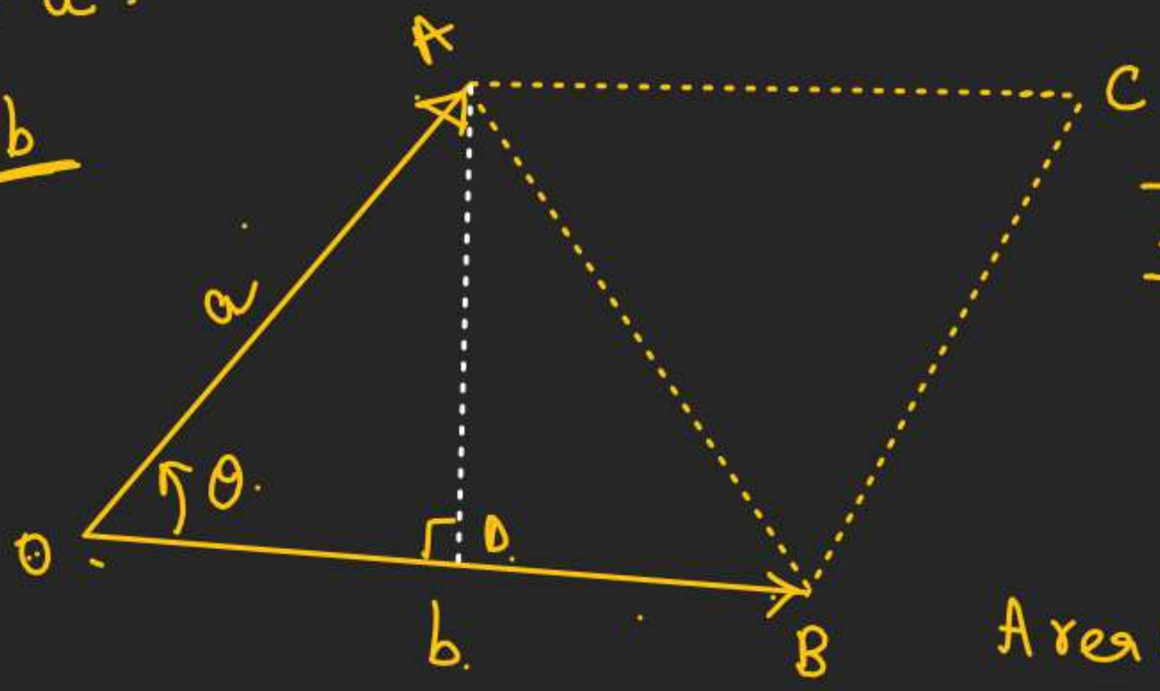
Geometrical meaning of VECTOR Cross product :→

$|\overrightarrow{OA}| = a$.

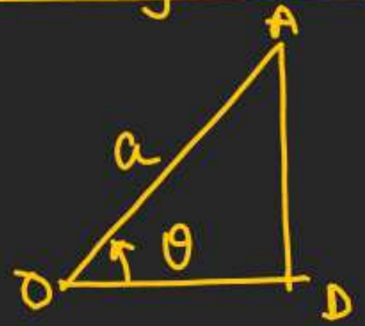
$|\overrightarrow{OB}| = b$



OAB and ABC are Congruant.

Area of Δ OAB = Area of Δ ABC

In right angle Δ OAD



$\sin\theta = \frac{AD}{a}$

$AD = a\sin\theta$.

Area of $\Delta OAB = \frac{1}{2} \times OB \times AD$.

$= \frac{1}{2} \times b \times a\sin\theta$.

$= \frac{1}{2} \times |\vec{a} \times \vec{b}|$

Area of parallelogram

$= 2 \times$ Area of ΔOAB.

$= ab\sin\theta$

$\Downarrow$

$= |\vec{a} \times \vec{b}|$

Find the Area of a parallelogram whose adjacent sides are $\vec{a} = \hat{i} - \hat{j} + 2\hat{k}$ and $\vec{b} = 3\hat{i} - \hat{j} + \hat{k}$.

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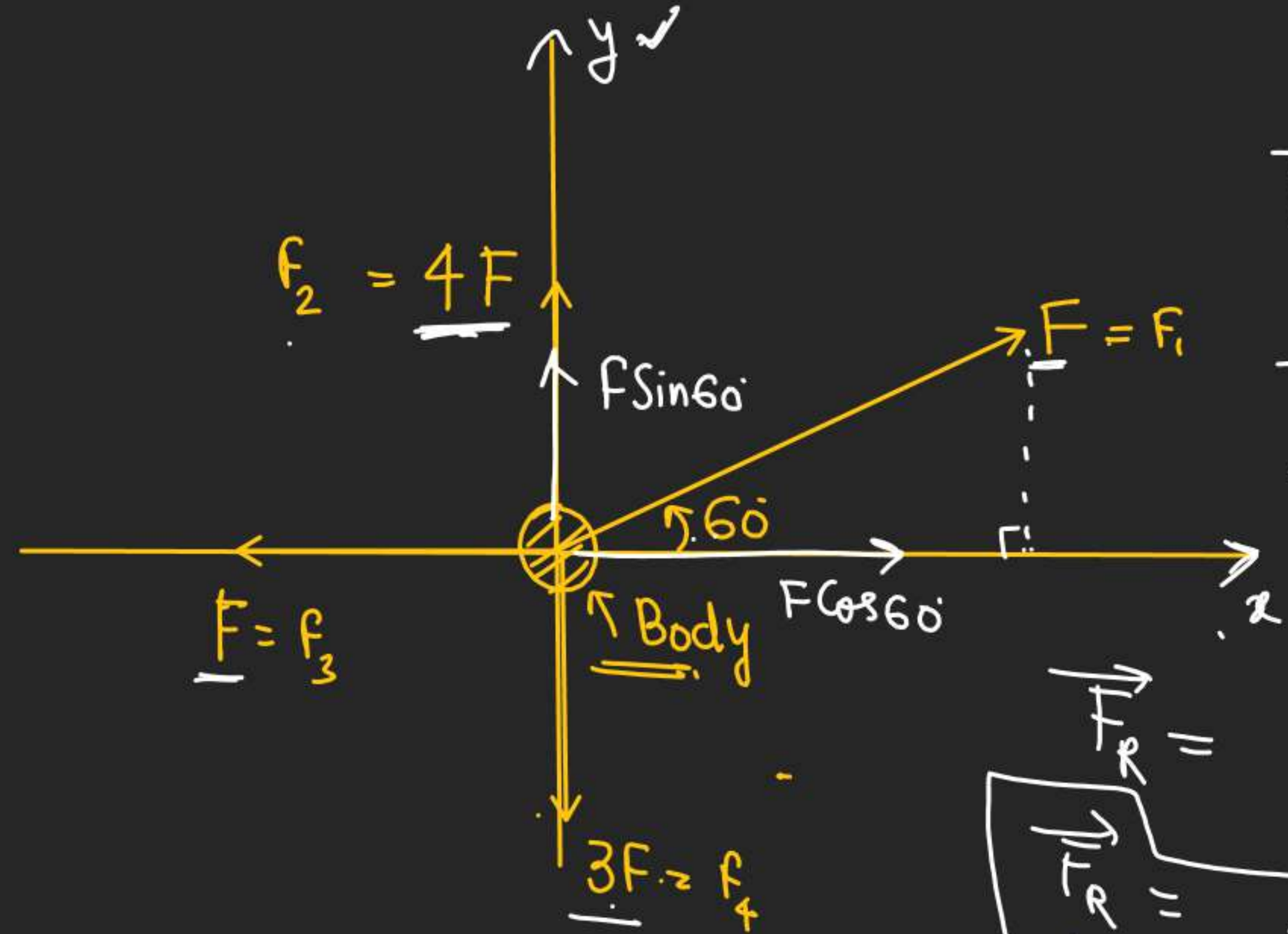
(*) To find resultant force →



Find resultant force.

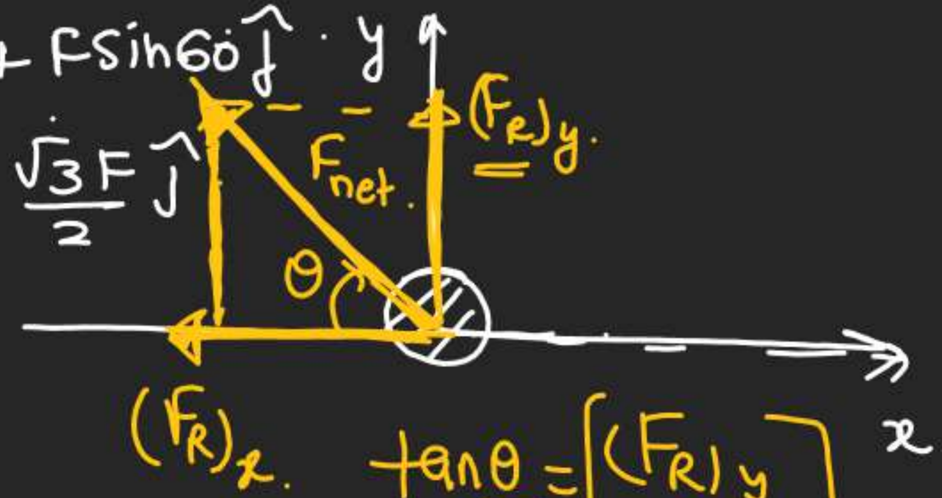
$$\vec{F_R} = \vec{F_1} + \vec{F_2} + \vec{F_3} + \vec{F_4}$$

$$\vec{F_1} = F\cos 60\,\hat{i} + F\sin 60\,\hat{j}$$
$$= \frac{F}{2}\hat{i} + \frac{\sqrt{3}F}{2}\hat{j}$$

$$\vec{F_2} = (4F)\hat{j}$$

$$\vec{F_3} = (-F)\hat{i}$$

$$\vec{F_4} = (-3F)\hat{j}$$



$$\tan\theta = \boxed{\frac{(F_R)_y}{(F_R)_x}}$$

$$\vec{F_R} = \frac{F}{2}\hat{i} + \frac{\sqrt{3}F}{2}\hat{j} + 4F\hat{j} - F\hat{i} - 3F\hat{j}$$

$$\boxed{\vec{F_R} = \underbrace{-\frac{F}{2}\hat{i}}_{(F_R)_x} + \underbrace{\left(\frac{\sqrt{3}F}{2} + F\right)\hat{j}}_{(F_R)_y}}$$