Nishant Jindal

Types of Matrix

Matrix + Det — 3Qs

C.S

(5) Trace of Matrix

A) Tr(A)

B) Sum of values of Pr. diag

C) $Tr(A) = \sum_{i=1}^{n} a_{ii} = a_{11} + a_{22} + a_{33} \cdots a_{nn}$.

D) Prop.

(1) $Tr(KA) = K\,Tr(A)$

(2) $Tr(A+B) = Tr(A) + Tr(B)$

(3) $Tr(AB) = Tr(BA)$

(4) $Tr(A) = Tr(A^T)$

Q

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 5 & 0 & 9 \\ -1 & 2 & -3 \end{bmatrix} \quad Tr(2A) = ?$$

$Tr(2A) = 2\,Tr(A)$

$= 2(1 + 0 + -3) = -4$

Nishant Jindal

Matrix

Q Find Matrix (A) if $3A + \underline{4B^T} = \begin{bmatrix} 1 & 3 & 7 \\ -5 & 4 & 6 \end{bmatrix}$

$(A^T)^T = A$

$\&\ 2B - 3A^T = \begin{bmatrix} -1 & 5 \\ 3 & 6 \\ 8 & 2 \end{bmatrix}$

① $(KA)^T = KA^T$

③ $(A \pm B)^T = (A^T) \pm B^T$

$(2B - 3A^T)^T = \begin{bmatrix} -1 & 5 \\ 3 & 6 \\ 8 & 2 \end{bmatrix}^T$

$(2B)^T - (3A^T)^T = \begin{bmatrix} -1 & 3 & 8 \\ 5 & 6 & 2 \end{bmatrix}$

$2B^T - 3A = \begin{bmatrix} -1 & 3 & 8 \\ 5 & 6 & 2 \end{bmatrix}$

$\times 2$

$\cancel{4B^T} - 6A = \begin{bmatrix} -2 & 6 & 16 \\ 10 & 12 & 4 \end{bmatrix}$

$\cancel{4B^T} + 3A = \begin{bmatrix} 1 & 3 & 7 \\ -5 & 4 & 6 \end{bmatrix}$

$- \quad - \quad -$

$-9A = \begin{bmatrix} -3 & 3 & 9 \\ 15 & 8 & -2 \end{bmatrix}$

$A = \begin{bmatrix} 1/3 & -1/3 & -1 \\ -5/3 & -8/9 & +2/9 \end{bmatrix}$ ✓

Nishant Jindal

Prop. of Add/Sub of Matrices

A & B are 2 same order Matrices then

(1) $A+B = B+A$ (Commutative)

(2) $A-B \neq B-A$

(3) $A+(B+C) = (A+B)+C$

(4) $A+O = O+A = A$

(5) $A+(-A) = O$ → $-A$ is additive Inverse of A

(6) $A+C = B+C \Rightarrow A = B$ (Cancellation law)

(C) If $(A \cdot B)$ Exist that doesnt imply that $(B \cdot A)$ will exist.

(2) Matrix Multiplication

A) Matrix A & B Can be multiplied only when if No of Columns in A = No of Rows in B then $(A \cdot B)$ Exists.

B) $A_{m \times n}$ & $B_{n \times s}$ then if $(A \cdot B)$ is asked

No of Col. No of Row

So $A \cdot B$ exists

$B \cdot A$ Exists or not?

$B_{n \times \boxed{s}}$ $A_{\boxed{m} \times n}$ $\Rightarrow (B \cdot A)$ D.NE

$s \neq m$

Q Let $A_{m\times(n+5)}$ & $B_{2\times3}$ & AB, BA Both exists find $(m,n) = ?$ $(3,-3)$

(AB) Exists $\Rightarrow$ $A_{m\times\boxed{(n+5)}}, B_{\boxed{2}\times3}$.

$n+5=2$

$n=-3$

(BA) Exist $\Rightarrow$ $B_{2\times\boxed{3}}, A_{\boxed{m}\times(n+5)}$

$\boxed{m=3}$

(4) Mostly we multiply using Row×Column in Matrix Multiplication

$$A = \begin{bmatrix} 2 & 3 \\ -1 & 4 \end{bmatrix}, \quad B = \begin{bmatrix} 4 & -5 \\ 0 & 2 \end{bmatrix}$$

$A \cdot B$ Exist? yes

$2\times2 \quad 2\times2$

$$A\cdot B = \begin{bmatrix} 2 & 3 \\ -1 & 4 \end{bmatrix} \cdot \begin{bmatrix} 4 & -5 \\ 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 2\times4+3\times0 & 2\times-5+3\times2 \\ -1\times4+4\times0 & -1\times-5+4\times2 \end{bmatrix} = \begin{bmatrix} 8 & -4 \\ -4 & 13 \end{bmatrix}$$

Nishant Jindal

Q $A = \begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & -2 \end{bmatrix}_{2\times 3}$, $B = \begin{bmatrix} 1 & -5 \\ 2 & 4 \\ 0 & 3 \end{bmatrix}_{3\times 2}$

(1) A.B Exist

$A_{2\times\boxed{3}}$, $B_{\boxed{3}\times 2}$

Yes

(2) B.A Exist

$B_{3\times\boxed{2}}$, $A_{\boxed{2}\times 3}$ ✓

Yes.

(3) $A.B = \begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & -2 \end{bmatrix} . \begin{bmatrix} 1 & -5 \\ 2 & 4 \\ 0 & 3 \end{bmatrix}$

$= \begin{bmatrix} R_1 C_1 & R_1 C_2 \\ R_2 C_1 & R_2 C_2 \end{bmatrix}$

$= \begin{bmatrix} 1\times1 + 3\times2 + 5\times0 & 1\times-5 + 3\times4 + 5\times3 \\ 2\times1 + 4\times2 + -2\times0 & 2\times-5 + 4\times4 + -2\times3 \end{bmatrix}$

$= \begin{bmatrix} 7 & 22 \\ 10 & 0 \end{bmatrix}$

(4) $B.A = \begin{array}{c} R_1 \\ R_2 \\ R_3 \end{array}\begin{bmatrix} 1 & -5 \\ 2 & 4 \\ 0 & 3 \end{bmatrix} . \begin{array}{c} \begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & -2 \end{bmatrix} \\ \begin{array}{ccc} \downarrow & \downarrow & \downarrow \\ C_1 & C_2 & C_3 \end{array} \end{array} = \begin{bmatrix} R_1C_1 & R_1C_2 & R_1C_3 \\ R_2C_1 & R_2C_2 & R_2C_3 \\ R_3C_1 & R_3C_2 & R_3C_3 \end{bmatrix}$

$= \begin{bmatrix} 1\times1 + -5\times2 & 1\times3 + -5\times4 & 1\times5 + -5\times-2 \\ 2\times1 + 4\times2 & 2\times3 + 4\times4 & 2\times5 + 4\times-2 \\ 0\times1 + 3\times2 & 0\times3 + 3\times4 & 0\times5 + 3\times-2 \end{bmatrix} = \begin{bmatrix} -9 & -17 & 15 \\ 10 & 22 & 2 \\ 6 & 12 & -6 \end{bmatrix}$

Nishant Jindal

Q Find A^2 if $A = \begin{bmatrix} -5 & -8 & 0 \\ 3 & 5 & 0 \\ 1 & 2 & -1 \end{bmatrix}$

$$A^2 = A \cdot A = \begin{bmatrix} -5 & -8 & 0 \\ 3 & 5 & 0 \\ 1 & 2 & -1 \end{bmatrix} \times \begin{bmatrix} -5 & -8 & 0 \\ 3 & 5 & 0 \\ 1 & 2 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 25 + -24 + 0 & 40 + -40 + 0 & 0 + 0 + 0 \\ -15 + 15 + 0 & -24 + 25 + 0 & 0 + 0 + 0 \\ -5 + 6 + -1 & -8 + 10 - 2 & 0 + 0 + 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$$

* $A^2 = I \Rightarrow$ A is Involutory Matrix.

Q $A = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$ find $A^2 = ?$

Sol $\rightarrow$ here $A^2 = A$

$\Rightarrow$ A is Idempotent Matrix.

Nishant Jindal

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}, \quad B = \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix}$$

$$A \cdot B = \begin{bmatrix} a_{11} \cdot b_{11} + a_{12} b_{21} + a_{13} b_{31} & a_{11} \cdot b_{12} + a_{12} b_{22} + a_{13} \cdot b_{32} & a_{11} b_{13} + a_{12} \cdot b_{23} + a_{13} \cdot b_{33} \\ a_{21} b_{11} + a_{22} b_{21} + a_{23} b_{31} & a_{21} b_{12} + a_{22} b_{22} + a_{23} b_{32} & \boxed{a_{21} b_{13} + a_{22} b_{23} + a_{23} b_{33}} \\ a_{31} b_{11} + a_{32} b_{21} + a_{33} b_{31} & a_{31} b_{12} + a_{32} b_{22} + a_{33} b_{32} & a_{31} b_{13} + a_{32} b_{23} + a_{33} b_{33} \end{bmatrix} \rightarrow (AB)_{23}$$

$$\boxed{\text{Note} \rightarrow (AB)_{ij} = \sum_{r=1}^{3} a_{ir} \, b_{rj}}$$

$A \rightarrow R_3 \quad B \rightarrow C_2 \leftarrow (AB)_{32} = a_{31} b_{12} + a_{32} b_{22} + a_{33} b_{32}$

$R_3 C_2$

$$(AB)_{12} = \sum_{r=1}^{3} a_{1r} \, b_{r2} = a_{11} b_{12} + a_{12} b_{22} + a_{13} \cdot b_{32}$$

Nishant Jindal

RK

★ 1) $\boxed{A \cdot I = A = I \cdot A}$

2) Easiest Multiplication of Matrices is diag Matrix's Multiplication.

Q $A = diag(1, -1, 8)$, $B = diag(3, 4, 0)$

① $A \cdot B =$

② $A^2 B =$

$A \cdot B = diag(1 \times 3, -1 \times 4, 8 \times 0)$

$= diag(3, -4, 0)$

(3) diag × diag = diag

(4) Scalar × Scalar = Scalar.

(5) $\Delta^r \times \Delta^r = \Delta^r$.

(6) If A^8 is Asked (PYQ) then Always check Pattern.

$A^2, A^3, - \longrightarrow$ direct

② $A^2 B = diag(1, -1, 8) \times diag(1, -1, 8) \times diag(3, 4, 0)$

$= dig(1 \times 1 \times 3, -1 \times -1 \times 4, 8 \times 8 \times 0)$

$= diag(3, 4, 0)$

Nishant Jindal

Q If $A_\alpha = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$ then $\lim\limits_{K\to\infty} \dfrac{A_\alpha^{K+1}}{K+1} = ?$

(A) $A_\alpha^2 = \begin{bmatrix} C & S \\ -S & C \end{bmatrix} \times \begin{bmatrix} C & S \\ -S & C \end{bmatrix} = \begin{bmatrix} C^2 - S^2 & SC + SC \\ -SC - SC & -S^2 + C^2 \end{bmatrix} = \begin{bmatrix} \cos 2\alpha & \sin 2\alpha \\ -\sin 2\alpha & \cos 2\alpha \end{bmatrix}$

$-2\sin\alpha\cos\alpha$

(B) $A_\alpha^3 = \begin{bmatrix} \cos 2\alpha & \sin 2\alpha \\ -\sin 2\alpha & \cos 2\alpha \end{bmatrix} \times \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix} = \begin{bmatrix} \cos 2\alpha\cos\alpha - \sin 2\alpha \sin\alpha & \cos 2\alpha \sin\alpha + \sin 2\alpha \cos\alpha \\ -\sin 2\alpha \cos\alpha - \cos 2\alpha \sin\alpha & -\sin 2\alpha \sin\alpha + \cos 2\alpha \cos\alpha \end{bmatrix} = \begin{bmatrix} \cos 3\alpha & \sin 3\alpha \\ -\sin 3\alpha & \cos 3\alpha \end{bmatrix}$

$$\therefore \lim_{K\to\infty} \frac{A_\alpha^{K+1}}{K+1} = \begin{bmatrix} \lim\limits_{K\to\infty} \dfrac{\cos(K+1)\alpha}{K+1} & \lim\limits_{K\to\infty} \dfrac{\sin(K+1)\alpha}{K+1} \\ \lim\limits_{K\to\infty} \dfrac{-\sin(K+1)\alpha}{K+1} & \lim\limits_{K\to\infty} \dfrac{\cos(K+1)\alpha}{K+1} \end{bmatrix}$$

$\to \dfrac{(-1 \text{ से } +1 \text{ के Bich})}{\infty} = 0$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = \text{Null Matrix}$$

Q (IIT 2003) $A = \begin{bmatrix} \alpha & 0 \\ 1 & 1 \end{bmatrix}$ & $B = \begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix}$

then a value of α for which $A^2 = B$ is ?

$$A^2 = \begin{bmatrix} \alpha & 0 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} \alpha & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} \alpha^2 & 0 \\ \alpha+1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix}$$

$\alpha^2 = 1$ & $\alpha + 1 = 5$

$\alpha = 1, -1$ $\quad$ $\alpha = 4$

No com. value

$\Rightarrow \alpha = \phi$

Q (Main 2019) $A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$ $\alpha \in R$, Such that $A^{32} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

then value of $\alpha = ?$ $\quad \frac{\pi}{64} \quad 0 \quad \frac{\pi}{32} \quad \frac{\pi}{16}$.

$$A^2 = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} \times \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} = \begin{bmatrix} C^2 - S^2 & -SC - SC \\ SC + SC & -S^2 + C^2 \end{bmatrix}$$

$$= \begin{bmatrix} \cos 2\alpha & -\sin 2\alpha \\ \sin 2\alpha & \cos 2\alpha \end{bmatrix}$$

$$A^{32} = \begin{bmatrix} \cos 32\alpha & -\sin 32\alpha \\ \sin 32\alpha & \cos 32\alpha \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

$\cos 32\alpha = 0$ & $+\sin 32\alpha = +1$

$\cos \cancel{32}\frac{\pi}{\cancel{64}2} = \cos\frac{\pi}{2} = 0$ $\quad\Big|\quad$ $\sin \cancel{32} \times \frac{\pi}{\cancel{64}2} = \sin\frac{\pi}{2} = 1$

Nishant Jindal

Rk

1) $\omega^3 = 1$, $1+\omega+\omega^2 = 0$

2) $\omega^4 = \omega^3 \omega = 1 \cdot \omega = \omega$

3) $\omega^5 = \omega^3 \omega^2 = 1 \cdot \omega^2 = \omega^2$

4) $\omega^6 = \omega^3 \cdot \omega^3 = 1 \times 1 = 1$

5) $\omega^{91} = (\omega^3)^{30} \cdot \omega^1 = (1)^{30} \cdot \omega = \omega$

Q $A = \begin{bmatrix} \omega & 0 \\ 0 & \omega \end{bmatrix}$ then $A^{70} = ?$ ω = Cube Root of Unity

KHH

$A^2 = \begin{bmatrix} \omega & 0 \\ 0 & \omega \end{bmatrix} \times \begin{bmatrix} \omega & 0 \\ 0 & \omega \end{bmatrix} = \begin{bmatrix} \omega^2 & 0 \\ 0 & \omega^2 \end{bmatrix}$

$A^3 = \begin{bmatrix} \omega^2 & 0 \\ 0 & \omega^2 \end{bmatrix} \begin{bmatrix} \omega & 0 \\ 0 & \omega \end{bmatrix} = \begin{bmatrix} \omega^3 & 0 \\ 0 & \omega^3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$

Demand $A^{70} = (A^3)^{23} \cdot A^1 = (I)^{23} \cdot A = I \cdot A = A$

Ans

Nishant Jindal

Q $A = \begin{bmatrix} 1 & 2 & 1 \\ -1 & 0 & 1 \end{bmatrix}$ & $B = \begin{bmatrix} 1 & 1 & 2 \\ -1 & 1 & 0 \\ 2 & 3 & 1 \end{bmatrix}$ find $\sum_{r=1}^{3} b_{r3} \cdot a_{2r} = ?$

$$\sum_{r=1}^{3} \underbrace{b_{r3} \cdot a_{2r}}_{\text{Wrong Way}} = \sum a_{2r} \cdot b_{r3} = (AB)_{23} \to \text{A का 2 Row} \times \text{B का 3 Colu}$$

$$= -1 \times 2 + 0 \times 0 + 1 \times 1$$

$$= -1$$

Q Total No. of Matrices $A = \begin{pmatrix} 0 & 2y & 1 \\ 2x & y & -1 \\ 2x & -y & 1 \end{pmatrix}$ $(x, y \in R,\ x \neq y)$

Main 2020

for which $A^T \cdot A = 3I$ is?

x+	y+	x-	y+
x+	y-		
x-	y-		

4 dif. Matrix Possible

3 1

$$A^T \cdot A = \begin{pmatrix} 0 & 2x & 2x \\ 2y & y & -y \\ 1 & -1 & 1 \end{pmatrix} \times \begin{pmatrix} 0 & 2y & 1 \\ 2x & y & -1 \\ 2x & -y & 1 \end{pmatrix} = \begin{pmatrix} 8x^2 & 0 & 0 \\ 0 & 6y^2 & 0 \\ 0 & 0 & 3 \end{pmatrix} = \begin{pmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{pmatrix}$$

$8x^2 = 3$ | $6y^2 = 3 \to y = \sqrt{\frac{1}{2}}, -\sqrt{\frac{1}{2}}$

$x = \sqrt{\frac{3}{8}}, -\sqrt{\frac{3}{8}}$

Nishant Jindal

Q let α be root of $x^2+x+1=0$ & $\longrightarrow$ $x = \frac{-1 \pm \sqrt{-3}}{2} = \frac{-1 \pm \sqrt{3}i}{2}$

$\nearrow -\frac{1}{2} + \frac{\sqrt{3}i}{2} = \omega = \alpha$

$\searrow -\frac{1}{2} - \frac{\sqrt{3}i}{2} = \omega^2 = \alpha$

Mani 2021

matrix $A = \frac{1}{\sqrt{3}}\begin{pmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha^4 \end{pmatrix}$ then $A^{31} = ?$

A $\quad$ A3 $\quad$ A2 $\quad$ I

$$A^2 = \frac{1}{\sqrt{3}}\begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega^4 \end{pmatrix} \times \frac{1}{\sqrt{3}}\begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega^4 \end{pmatrix}$$

Exr

1, 8, 9

Ex 1 (3, 4, 8)

7, 8, 9

11, 18

$$= \frac{1}{3}\begin{pmatrix} 3 & 1+\omega+\omega^2 & \underset{1+\omega^2+\omega}{1+\omega^2+\omega^4} \\ 1+\omega+\omega^2 & 1+\omega^2+\omega^4 & \underset{1+1+1}{1+\omega^3+\omega^3} \\ \underset{1+\omega^2+\omega}{1+\omega^2+\omega^4} & 1+\omega^3+\omega^6 & \underset{1+\omega+\omega^2}{\underbrace{1+\omega^4+\omega^8}} \end{pmatrix} = \frac{1}{3}\begin{pmatrix} 3 & 0 & 0 \\ 0 & 0 & 3 \\ 0 & 3 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$\omega^4 = \omega$

$\omega^6 = 1$

$\omega^8 = \omega^3 \cdot \omega^3 \cdot \omega^2 = \omega^2$

$$A^4 = A^2 \cdot A^2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = I$$

Dem and 4

$$= A^{31} = (A^4)^7 \cdot A^3 = (I)^7 \cdot A^3 = I \cdot A^3 = A^3.$$

JM 19, 15, 16, 14, 17