Nishant Jindal

Q DE of all St. line P.T. $(-1,-1)$

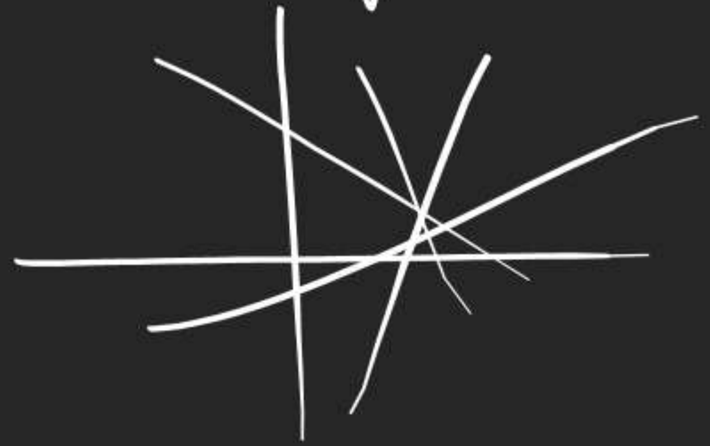
line $\rightarrow (y+1) = m(x+1)$

$\uparrow$ 1 Arb.

$$\frac{dy}{dx} = m$$

$$(y+1) = \frac{dy}{dx}(x+1)$$

Q DE of all lines in xy Plane?



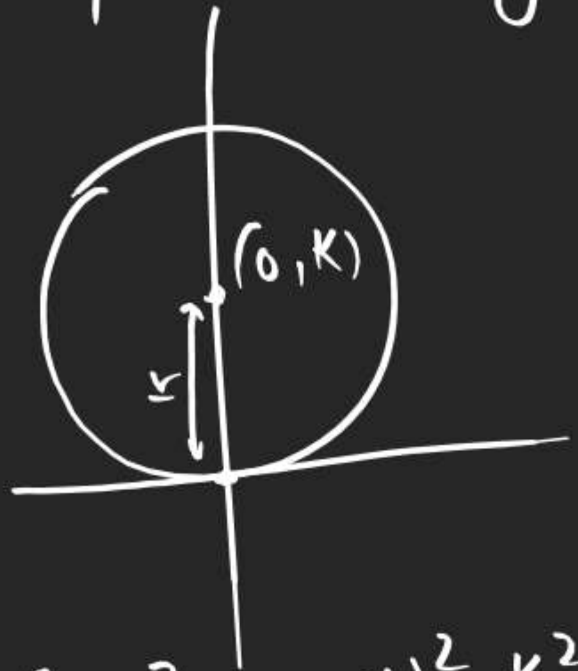
$y = mx + c$

2 Arb.

$$\frac{dy}{dx} = m$$

$$\boxed{\frac{d^2y}{dx^2} = 0}$$ A

Q D.E of all Circles having Centre at y Axis & P.T. origin.



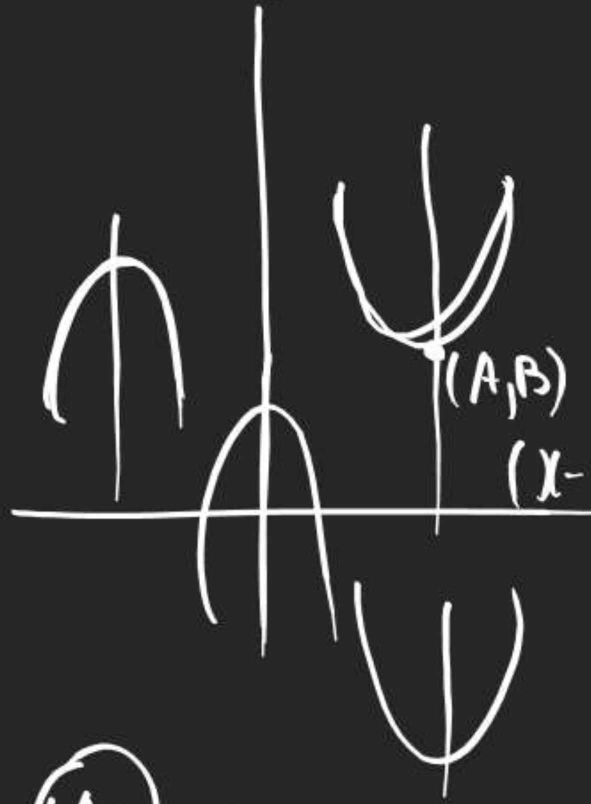
$$(x-0)^2 + (y-K)^2 = K^2$$

$$x^2 + y^2 = 2Ky + \cancel{K^2} = \cancel{K^2}$$

$$x^2 + y^2 - 2Ky = 0$$

$$\frac{dy}{dx}\left(\frac{y^2 - x^2}{2y}\right) = -x$$

Q DE of all Parabolas having their axes $\|^{r}$ to y Axis?



M_1

$$(x-A)^2 = 4a(y-B)$$

3 Arb.

M_2 Parabola $\rightarrow y = ax^2 + bx + c$

$$\frac{dy}{dx} = 2ax + b$$

$$\frac{d^2y}{dx^2} = 2a$$

$$\frac{d^3y}{dx^3} = 0$$

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Variable Seperable.

$\int f_1(x)\cdot dx + \int f_2(y)dy = 0$

$\downarrow$ & Solve.

Solution in the form of C will come, known as general sol. $(y = y(x))$

When some condition for fxn is given then C can be removed & that sol. will be known as Particular Sol. → Ex $y(1)=0$ is given. then $x=1$ & $y=0$ will be used in general sol.

Q $\frac{dy}{dx} = \frac{1}{2y+\cos y}$ find General sol.

$\int 2y+\cos y \cdot dy = \int dx$

$y^2 + \sin y = x + C$ → General sol.

Adv

Q For $\left(\frac{2+\sin x}{1+y}\right)\frac{dy}{dx} = -\cos x$

if $\boxed{y(0)=1}$ then $\boxed{y\left(\frac{\pi}{2}\right)} = ?$

$\int \frac{dy}{1+y} = -\int \frac{\cos x}{2+\sin x} \cdot dx$ | $2+\sin x = t$, $\cos x\, dx = dt$

$\ln(1+y) = -\ln(2+\sin x) + \ln C$ → Gen. Sol. $y = y(x)$

(2) $y(0)=1$ $x=0, y=1$

$\ln(1+1) = -\ln(2+\sin 0) + \ln C$

$\ln 2 = -\ln 2 + \ln C \Rightarrow C = 4.$

(3) $\ln(1+y) = -\ln(2+\sin x) + \ln 4$

$\ln(1+y) = \ln\left(\frac{4}{2+\sin x}\right)$

$1+y = \frac{4}{2+\sin x}$

(4) $y\left(\frac{\pi}{2}\right) = \dots ?$ $x = \frac{\pi}{2}$ Put

$1+y = \frac{4}{2+\sin\frac{\pi}{2}} = \frac{4}{3}$

$y = \frac{1}{3} = y\left(\frac{\pi}{2}\right)$

Nishant Jindal

Q Find the family of Curve P.T. $\left(\frac{\pi}{2}, e\right)$

& Satisfying DE $\sin x\, dy = y \ln y\, dx$?

$$\sin x \cdot dy = y \ln y \cdot dx$$

$$\int \frac{dy}{y \ln y} = \int \frac{dx}{\sin x}$$

$\ln y = t$

$\frac{dy}{y} = dt$

$$\int \frac{dt}{t} = \int \operatorname{cosec} x \cdot dx$$

$$\ln(\ln y) = \ln \tan\frac{x}{2} + \ln c$$

P.T. $\left(\frac{\pi}{2}, e\right)$

$$\ln(\ln e) = \ln \tan\frac{\pi}{4} + \ln c$$

$$0 = 0 + \ln c \Rightarrow \ln c = 0$$

$$\ln(\ln y) = \ln \tan\frac{x}{2}$$

$$y = e^{\tan\frac{x}{2}}\ \text{Ans.}$$

Q Family of Curve P.T. $(1,0)$

& Satisfying. $(1+y^2)\,dx = xy\,dy$

$$\int \frac{dx}{x} = \int \frac{y\,dy}{1+y^2}$$

$1+y^2 = t$

$y\,dy = \frac{dt}{2}$

$$\ln x = \frac{1}{2}\int \frac{dt}{t}$$

$$\ln x = \frac{1}{2}\ln(1+y^2) + \ln c$$

$$\ln x = \ln\left(\sqrt{1+y^2} \cdot c\right)$$

$$x = \sqrt{1+y^2} \cdot c$$

P.T. $(1,0)$.

$$1 = \sqrt{1+0^2} * c \Rightarrow c = 1$$

$$\therefore x = \sqrt{1+y^2}$$

$$\boxed{x^2 - y^2 = 1}$$

Q A Curve P.T. $(2,3)$ & Satisfying

D.Eqn $\int_0^x t \cdot y(t)\,dt = x^2 \cdot y(x)$; $(x>0)$ is?

Rem: $\to y(x) = y \Rightarrow y(t) = y$

$$\int_0^x t \cdot y \cdot dt = x^2 \cdot y$$

NL

$$y \cdot x = x^2 \cdot y' + 2xy$$

$$x^2 y' = -xy \Rightarrow x\frac{dy}{dx} = -y$$

$$\int \frac{dy}{y} = -\int \frac{dx}{x} \Rightarrow \ln y = -\ln x + \ln c$$

$y = \frac{c}{x}$ P.T. $(2,3) \Rightarrow c = 6$

$$\boxed{xy = 6}$$

Q Rep. by graph. Initial value Prob.

$DY = 100 - y$ where $y(0) = 50$

(1) $\dfrac{dy}{dx} = 100 - y$

$\int \dfrac{dy}{100-y} = \int dx$

$-\ln(100-y) = x + C$

(2) $y(0) = 50$

$-\ln 50 = 0 + C$

$-\ln(100-y) = x - \ln 50$

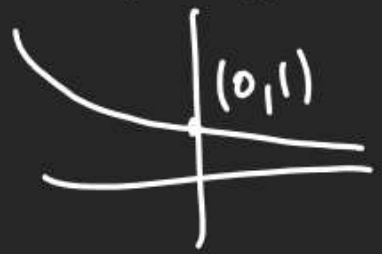
$x = \ln\left(\dfrac{50}{100-y}\right)$

$e^{x}\cdot(100-y) = 50$

$100 - y = \dfrac{50}{e^{x}}$

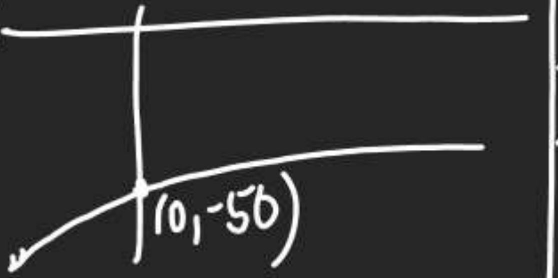
$y = 100 - 50\cdot e^{-x}$

(A) $y = e^{-x}$

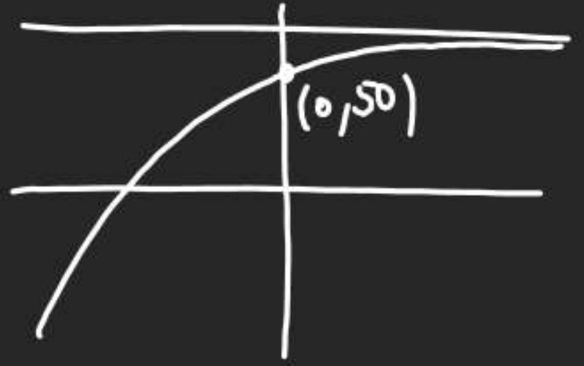


(B) $y = -e^{x}$

(C) $y = -50e^{-x}$



(D) $y = -50e^{-x} + 100$



Mains 3 बार

Q Population of town at time t is given by $\dfrac{dP(t)}{dt} = .5P(t) - 450$ Also $P(0) = 850$. Find the time when Population of town becomes Zero.

$\dfrac{d(P(t))}{dt} = \dfrac{P(t) - 900}{2}$

M Imp

$\int_0^t \dfrac{dP(t)}{P(t)-900} = \int_0^t \dfrac{dt}{2}$

$\ln|P(t)-900| - \ln|P(0)-900| = \dfrac{t}{2}$

$\ln|P(t)-900| - \ln|850-900| = \dfrac{t}{2}$

$\ln|P(t)-900| - \ln 50 = \dfrac{t}{2}$

$\ln\left|\dfrac{P(t)-900}{50}\right| = \dfrac{t}{2}$

$P(t) = 0$ हो जाए तो t चाहिए

$\ln\left|\dfrac{-900}{50}\right| = \dfrac{t}{2}$

$t = 2\ln 18$

Nishant Jindal

Q fxn Satisfying $f^2(x)+4f'(x)\cdot f(x)+(f'(x))^2=0$ is?

$y^2+4y\cdot y'+(y')^2=0$ $\to A=1,\ B=4y,\ C=y^2$

$$y'=-\frac{4y\pm\sqrt{16y^2-4y^2}}{2}$$

$$y'=\frac{-4y\pm\sqrt{12y^2}}{\cancel{2}}$$

$$y'=(-2\pm\sqrt{3})y$$

$$\frac{dy}{dx}=(-2+\sqrt{3})y$$

Solve urself

$$\frac{dy}{dx}=-(2+\sqrt{3})y$$

$$\int\frac{dy}{(2+\sqrt{3})y}=-\int dx$$

$$\frac{1}{2+\sqrt{3}}\cdot\ln y=-x+C$$

Q Eqn of Curve P.T. (3, 4) Satisfying D.E $y\left(\frac{dy}{dx}\right)^2+(x-y)\frac{dy}{dx}-x=0$ can be?

$A=y,\ B=(x-y),\ C=-x$

$$\frac{dy}{dx}=\frac{-(x-y)\pm\sqrt{(x-y)^2+4xy}}{2y}$$

$$\frac{dy}{dx}=\frac{-(x-y)\pm(x+y)}{2y}$$

(+)

$$\frac{dy}{dx}=\frac{\cancel{2y}}{\cancel{2y}}\ 1$$

$y=x+C$

$(3,4)\to C=1$

$\boxed{y=x+1}$

(−)

$$\frac{dy}{dx}=-\frac{2x}{2y}$$

$$\int y\,dy=-\int x\,dx$$

$$\frac{y^2}{2}=-\frac{x^2}{2}+C$$

S Y

Q let $y=y(x)$ be Sol. of DE $\log_e\left(\frac{dy}{dx}\right)=3x+4y$ with $y(0)=0$

If $y\left(-\frac{2}{3}\ln 2\right)=\alpha\ln 2$ find α?

① $\frac{dy}{dx}=e^{3x+4y}=e^{3x}\cdot e^{4y}$

$$\int\frac{dy}{e^{4y}}=\int e^{3x}\,dx$$

$$\Rightarrow\int e^{-4y}dy=\int e^{3x}\,dx$$

$$\Rightarrow\frac{e^{-4y}}{-4}=\frac{e^{3x}}{3}+C \quad \Big|\ y(0)=0$$

$$\Rightarrow-\frac{1}{4}=\frac{1}{3}+C\Rightarrow C=-\frac{1}{4}-\frac{1}{3}=\frac{-7}{12}$$

② $\Rightarrow\frac{e^{3x}}{3}+\frac{e^{-4y}}{4}=\frac{7}{12}$

③ $x=-\frac{2}{3}\ln 2$

$y=\alpha\ln 2$

$$\frac{1}{12}+\frac{1}{4\cdot 2^{4\alpha}}=\frac{7}{12}$$

$$\frac{1}{\cancel{4}\cdot 2^{4\alpha}}=\frac{1}{\cancel{2}}$$

$2^{4\alpha}=2^{-1}$

$\alpha=-\frac{1}{4}$

Nishant Jindal

2^{nd} Method → Reducible to Var. Seperable.

जब x, y Seperate न हो

① Check if we get a linear fxn of x, y.

② If linear fxn of x, y is avaliable take fxn $= t$ & change $\frac{dy}{dx}$ in $\frac{dt}{dx}$

Q $\frac{dy}{dx} = \sin(x+y)$ Solve?

x, y का linear fxn.

let $x + y = t$

$1 + \frac{dy}{dx} = \frac{dt}{dx}$

$\frac{dy}{dx} = \frac{dt}{dx} - 1$

$\frac{dt}{dx} - 1 = \sin t$

$\frac{dt}{dx} = 1 + \sin t$

$\int \frac{dt}{1+\sin t} = \int dx$

$\int \frac{dt}{1 + \frac{2\tan t/2}{1+\tan^2 t/2}} = \int dx$

$\int \frac{\sec^2 \frac{t}{2} \cdot dt}{(1+\tan\frac{t}{2})^2} = \int dx$

$1 + \tan\frac{t}{2} = v$

$\sec^2\frac{t}{2} \times \frac{1}{2}\, dt = dv$

$\sec^2\frac{t}{2}\, dt = 2dv$

$2\int \frac{dv}{v^2} = \int dx$

$\Rightarrow 2 \times -\frac{1}{v} = x + c$

$\Rightarrow \frac{-2}{1 + \tan\frac{t}{2}} = x + c$

$\Rightarrow \frac{-2}{1 + \tan\left(\frac{x+y}{2}\right)} = x + c$

Q $\frac{dy}{dx} = (4x + y + 1)^2$

linear fxn

$4x + y + 1 = t$

$4 + \frac{dy}{dx} = \frac{dt}{dx}$

$\frac{dy}{dx} = \frac{dt}{dx} - 4$

$\frac{dt}{dx} - 4 = t^2$

$\frac{dt}{dx} = t^2 + 4$

$\int \frac{dt}{t^2+4} = \int dx$

$\frac{1}{2}\tan^{-1}\frac{t}{2} = x + c$

Nishant Jindal

Q $\sqrt{1+x+y}\cdot\frac{dy}{dx}=x+y-1$ Solve?

$x+y+1=t^2$

$1+\frac{dy}{dx}=2t\frac{dt}{dx}$

$\frac{dy}{dx}=2t\frac{dt}{dx}-1$

$t\times\left(2t\frac{dt}{dx}-1\right)=t^2-2$

$2t\frac{dt}{dx}-1=\frac{t^2-2}{t}$

$2t\frac{dt}{dx}=\frac{t^2-2}{t}+1$

$=\frac{t^2+t-2}{t}$

$\int\frac{2t^2dt}{t^2+t-2}=\int dx$

$2\int\frac{\cancel{t^2+t-2}}{\cancel{t^2+t-2}}-\frac{(t-2)}{t^2+t-2}$

$2t-2\int\frac{(t-2)dt}{t^2+t-2}=x+C$

$\int\frac{\text{Linear}}{\text{Quad}}$

Solve urself.