Force on a dielectric slab

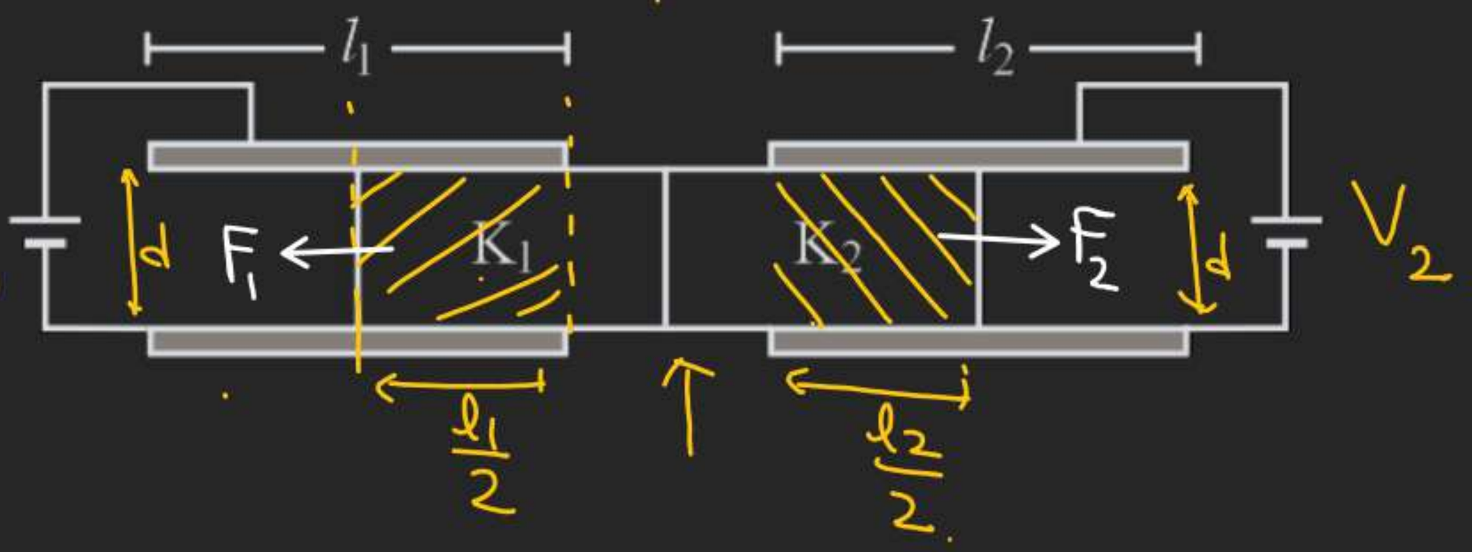
Q.1 Figure shows two parallel plate capacitors with fixed plates and connected to two batteries. The separation between the plates is the same for the two capacitors. The plates are rectangular in shape with width b and length l_1 and l_2. The left half of the dielectric slab has a dielectric constant k_1 and the right half k_2. Neglecting any friction, find the ratio of the voltage of the left battery to that of the right battery for which the dielectric slab may remain in equilibrium.

For Equilibrium

$$F_1 = F_2$$

$$\frac{\varepsilon_0 b V_1^2 (K_1 - 1)}{2d} = \frac{\varepsilon_0 b V_2^2 (K_2 - 1)}{2d}$$

$$\frac{V_1}{V_2} = \sqrt{\frac{(K_2 - 1)}{(K_1 - 1)}} \checkmark$$



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CAPACITOR

during insertion of Slab in the Capacitor:-

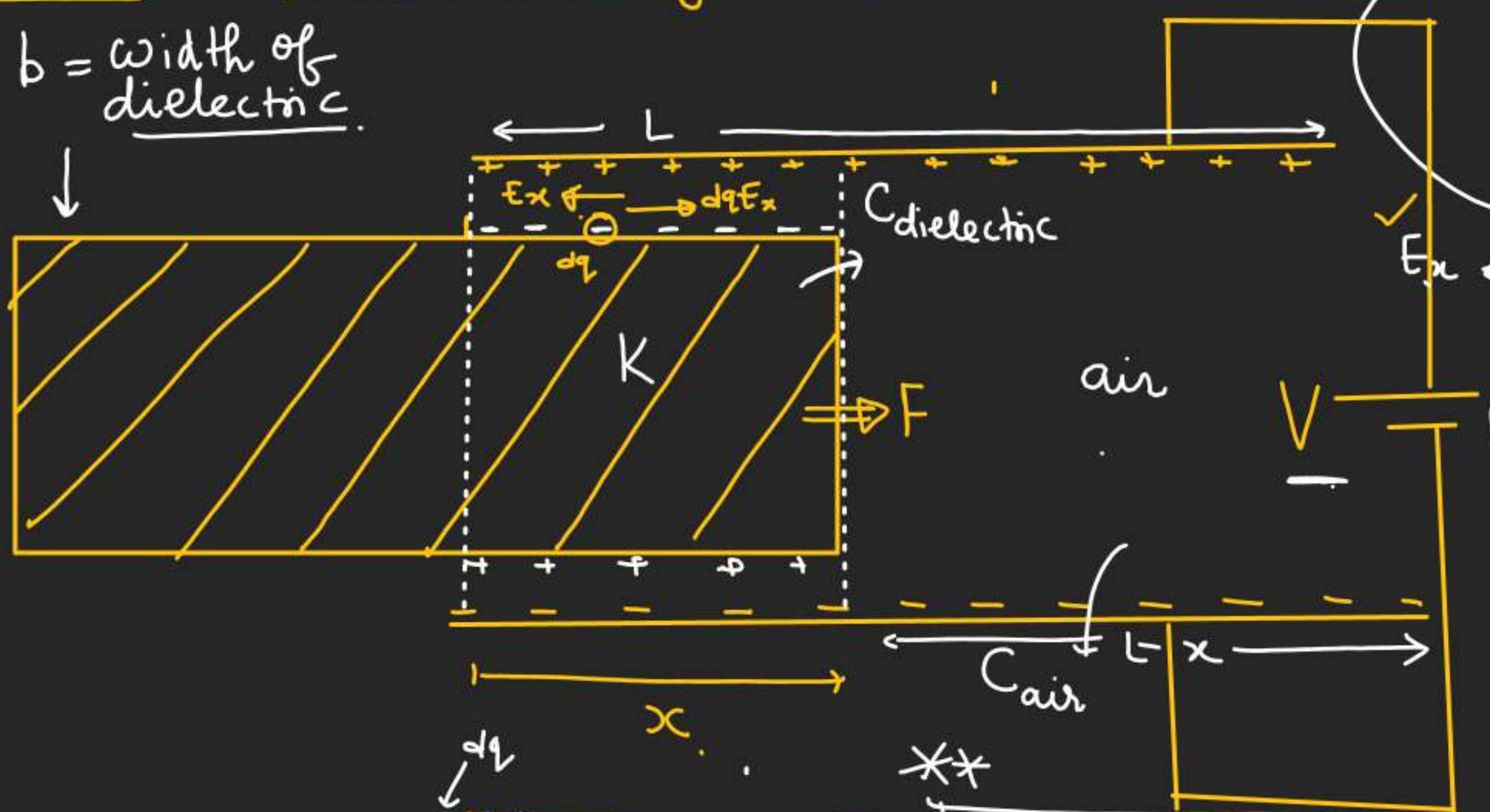
$q = CV$

$V = q/C \uparrow$

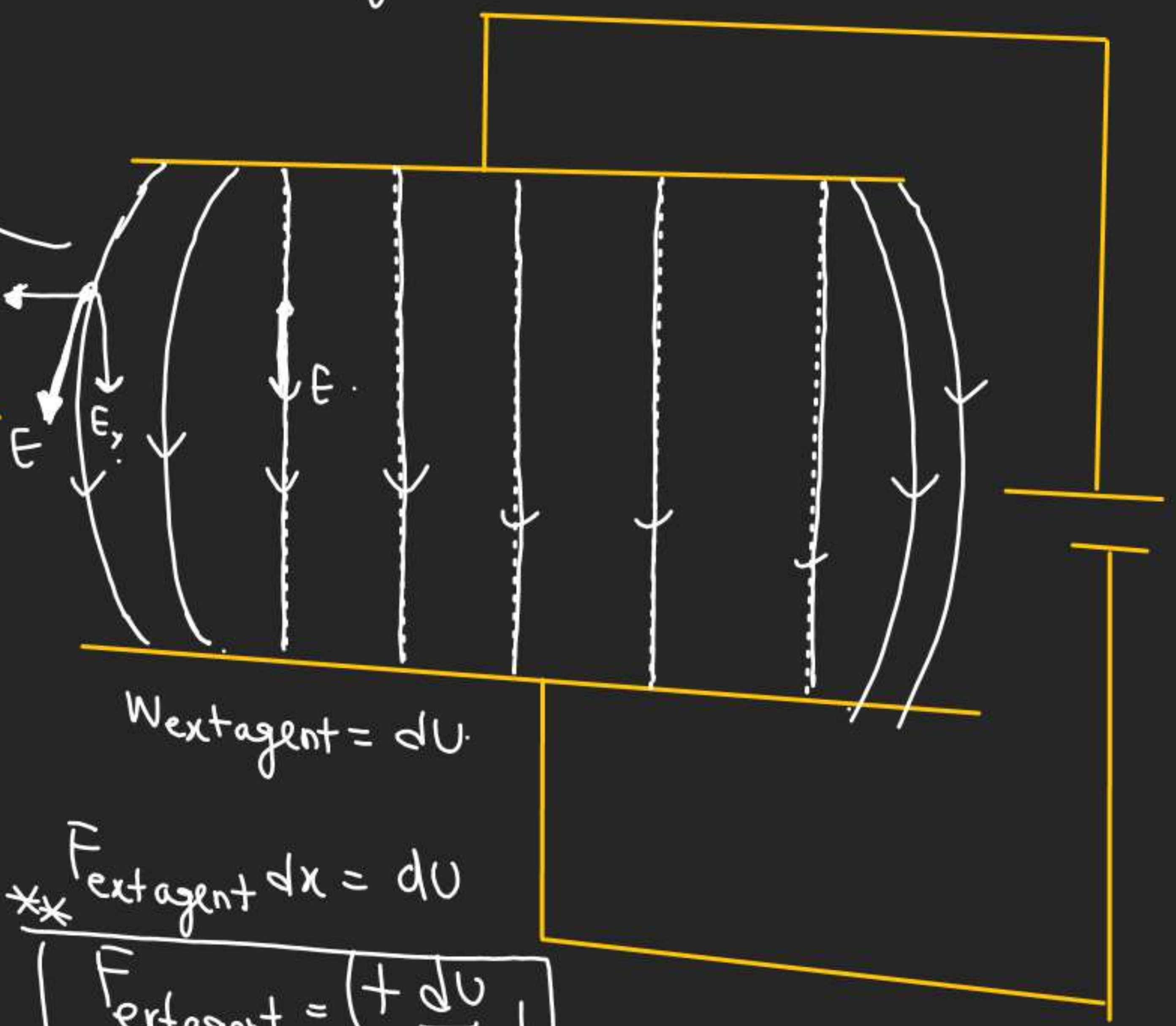
$$\boxed{q_{ind} = q\left[1 - \frac{1}{k}\right]}$$

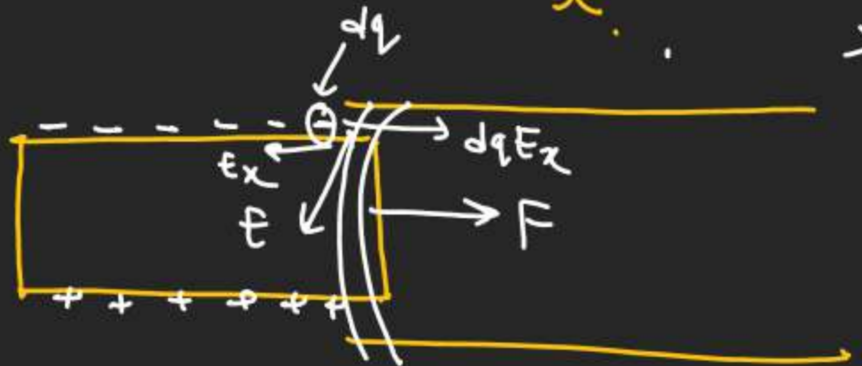
Case-1 :- When battery is Connected →

b = width of dielectric



At the end of





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$$\boxed{F_{system} = \left(-\frac{dU}{dx}\right)}$$

$W_{extagent} = dU$

** $F_{extagent}\, dx = dU$

$$\boxed{F_{extagent} = \left(+\frac{dU}{dx}\right)}$$

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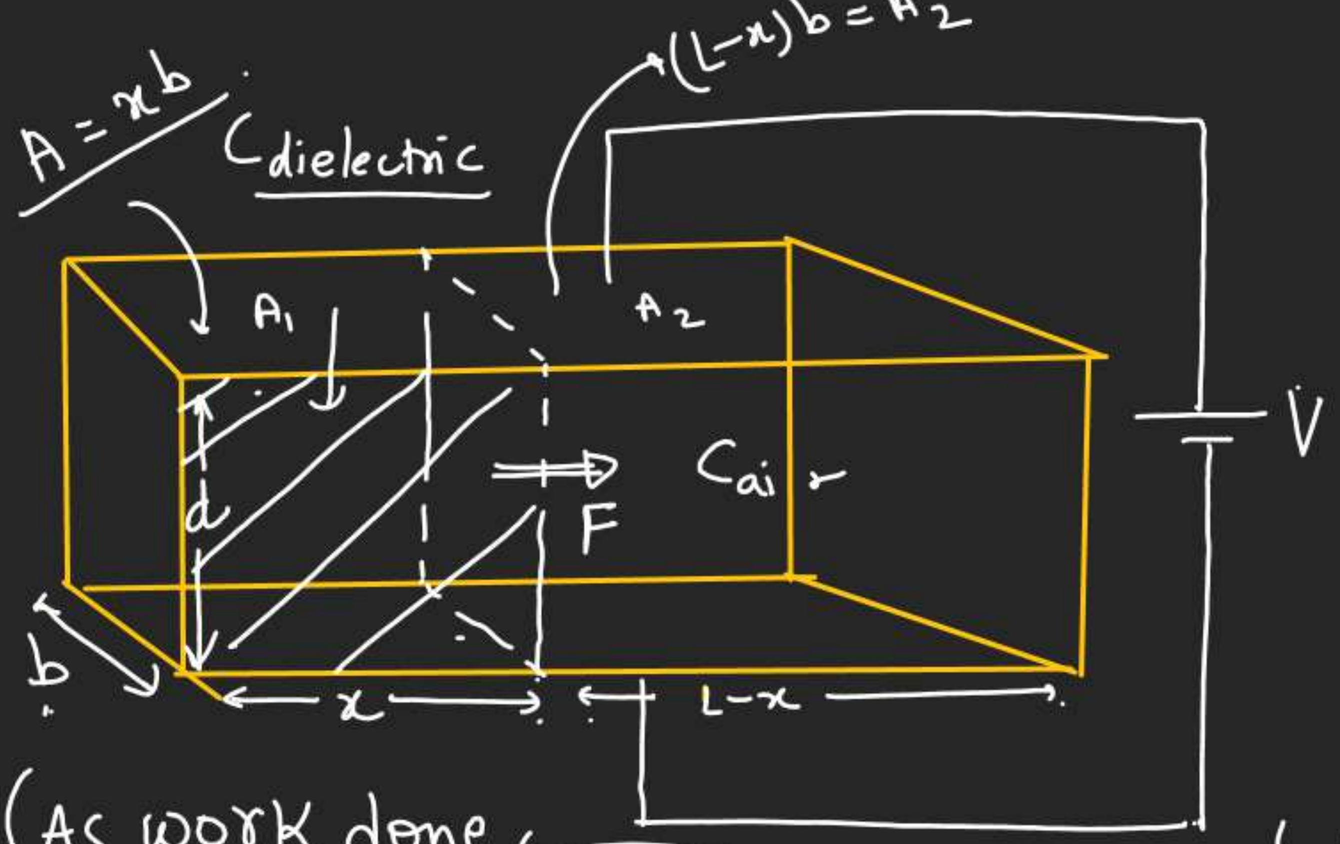


$$C = C_{air} + C_{dielectric}$$

$$C = \frac{\varepsilon_0 (L-x) b}{d} + \frac{K\varepsilon_0 b x}{d}$$

$$C = \frac{\varepsilon_0 L b}{d} - \frac{\varepsilon_0 b}{d}x + \frac{K\varepsilon_0 b x}{d}$$

$$C = \frac{\varepsilon_0 L b}{d} + \frac{\varepsilon_0 b}{d}(K-1)x$$

$$C = \frac{\varepsilon_0 b}{d}\left[L + (K-1)x\right]$$

$$U_{system} = \frac{1}{2}CV^2 = \frac{\varepsilon_0 b V^2}{2d}\left[L + (K-1)x\right]$$

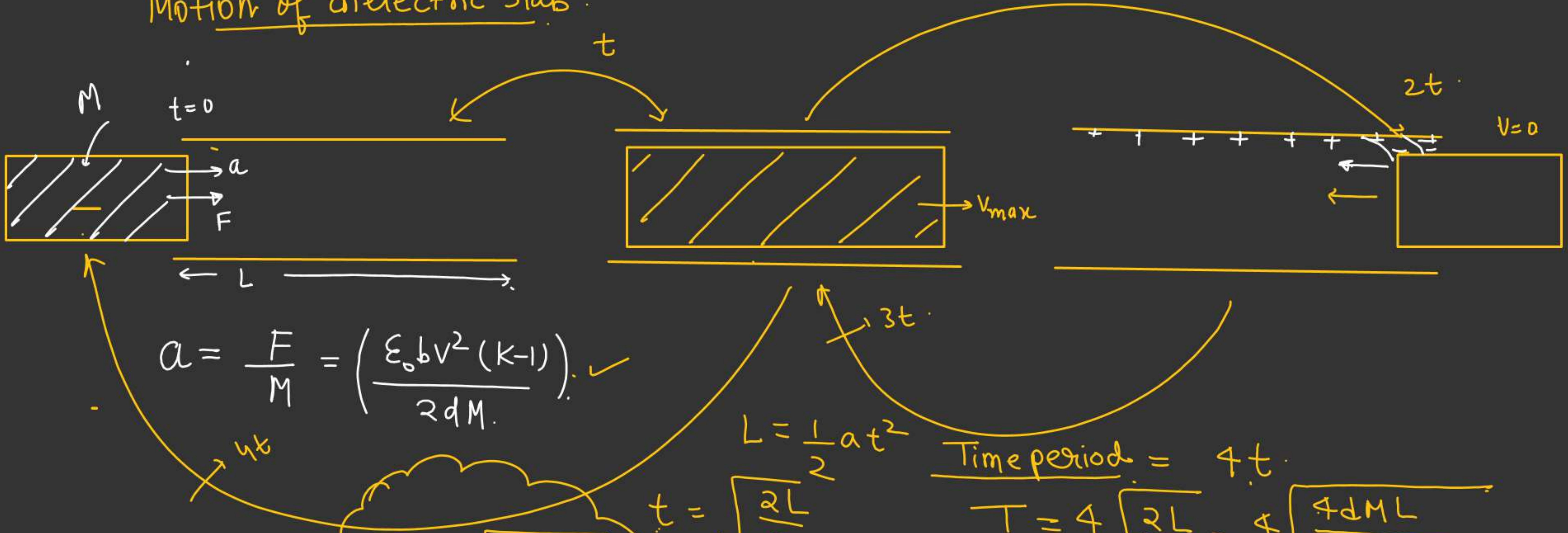
(As work done on the System)

$$F = +\left(\frac{dU}{dx}\right) = \frac{\varepsilon_0 b V^2}{2d}(K-1)$$

$$\boxed{F = \frac{\varepsilon_0 b V^2}{2d}(K-1)}$$

**
- $b \rightarrow$ width of dielectric slab.
- $d \rightarrow$ seperation b/w plate.
- $V \rightarrow$ potential of battery.

Motion of dielectric slab.



$$a = \frac{F}{M} = \left(\frac{\varepsilon_0 b V^2 (K-1)}{2dM}\right)$$

$$L = \frac{1}{2} a t^2$$

$$t = \sqrt{\frac{2L}{a}}$$

Time period $= 4t$

$$T = 4\sqrt{\frac{2L}{a}} = 4\sqrt{\frac{4dML}{\varepsilon_0 b V^2 (K-1)}}$$

$$\boxed{T = 8\sqrt{\frac{dML}{\varepsilon_0 b V^2 (K-1)}}}$$

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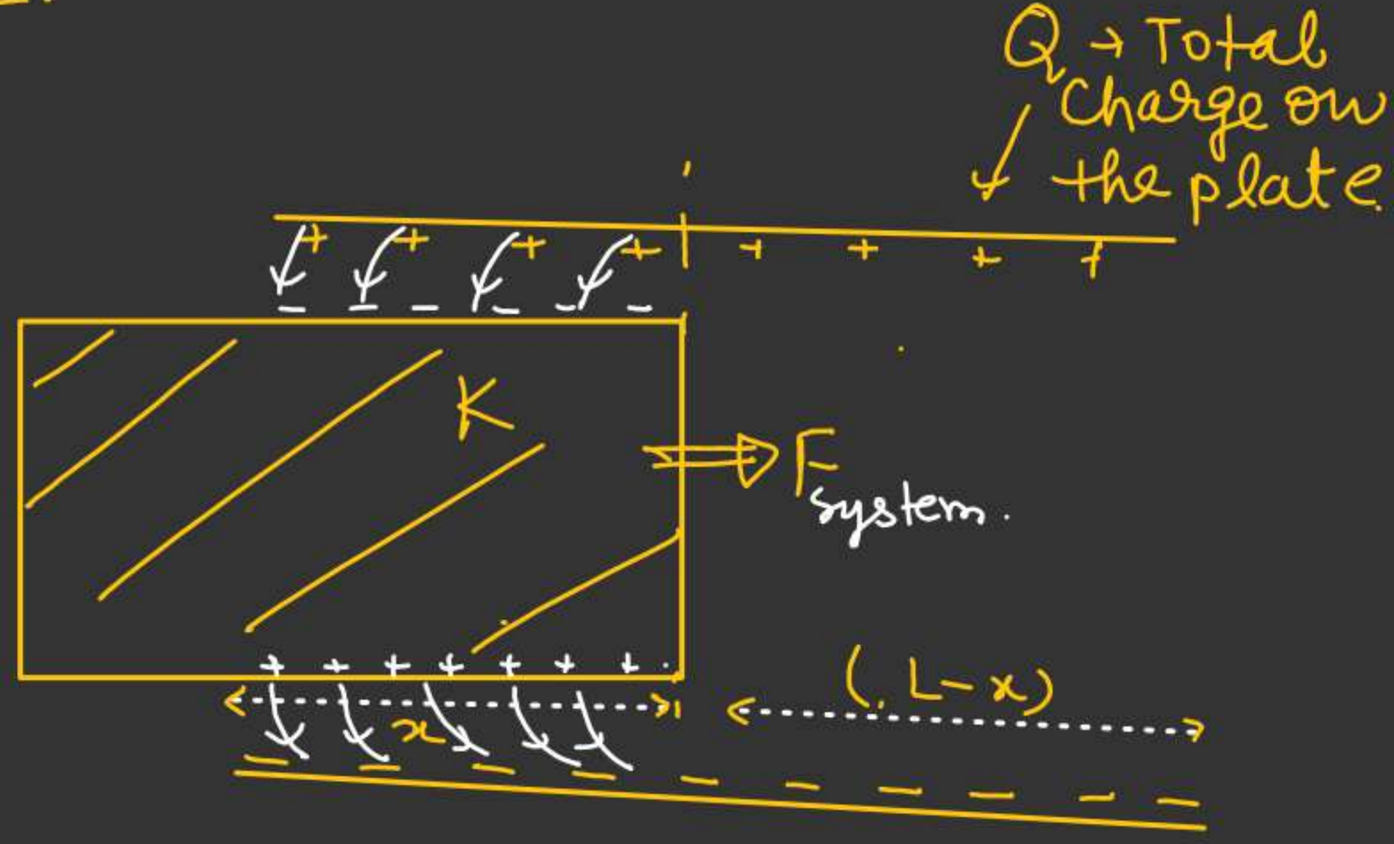
during insertion in the Capacitor:—

Case-2 :- When Capacitor is Isolated:—

↳ [In isolated Capacitor Charge is always Constant]



$$C = \frac{\varepsilon_0 b}{d}\left[L + (K-1)x\right]$$

$$U = \frac{Q^2}{2C}$$

$$\boxed{F_{system} = \left(-\frac{dU}{dx}\right)} \Rightarrow F_{system} = -\frac{Q^2}{2}\frac{d}{dx}\left(\frac{1}{C}\right)$$

$$= -\frac{Q^2}{2}\left[\frac{d}{dC}\left(\frac{1}{C}\right)\times\left(\frac{dC}{dx}\right)\right] = -\frac{Q^2}{2}\left(-\frac{1}{C^2}\right)\frac{dC}{dx} = \frac{Q^2}{2C^2}\left(\frac{dC}{dx}\right)$$

$$\frac{dC}{dx} = \left(\frac{\varepsilon_0 b(K-1)}{d}\right)$$

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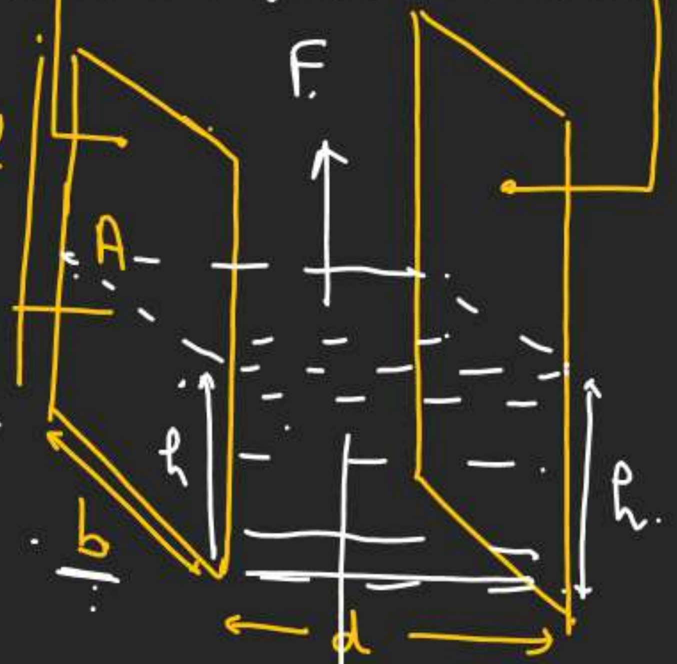
$$F = \frac{Q^2}{2C^2}\left(\frac{dC}{dx}\right)$$

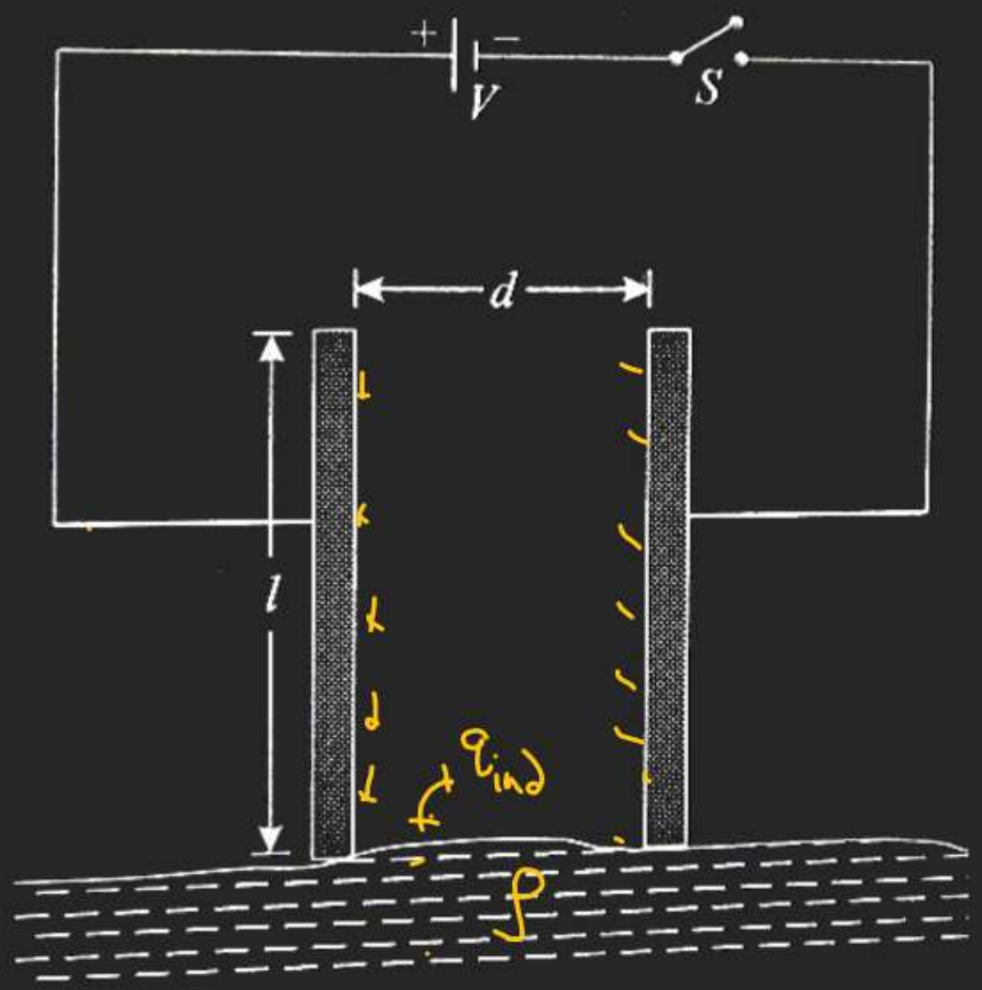
$$F = \frac{Q^2}{2\,\frac{\varepsilon_0^2 b^2}{d^2}\left[L+(K-1)x\right]^2} \times \frac{\varepsilon_0 b}{d}(K-1)$$

$$F = \frac{Q^2 d^2}{2\varepsilon_0^2 b^2\left[L+(K-1)x\right]^2} \times \frac{\varepsilon_0 b}{d}(K-1)$$

$$\boxed{F = \frac{Q^2 d(K-1)}{2\varepsilon_0 b\left[L+(K-1)x\right]^2}}$$

Q.2 Figure-shows a parallel plate capacitor of plate area $A = lb$ with separation d connected to a battery via a switch S. Capacitor plates are kept vertical and touched on the surface of a liquid of density ρ as shown. If S is closed find the height between plates to which the liquid level will rise.





Sol^n

$A = lb$

$$F = \frac{\varepsilon_0 b V^2}{2d}(K-1)$$

$m = \rho$(Volume of liquid b/w the plate)

$= \rho(bdh)$

At maximum height:-

$$F = mg = \rho b d g h$$

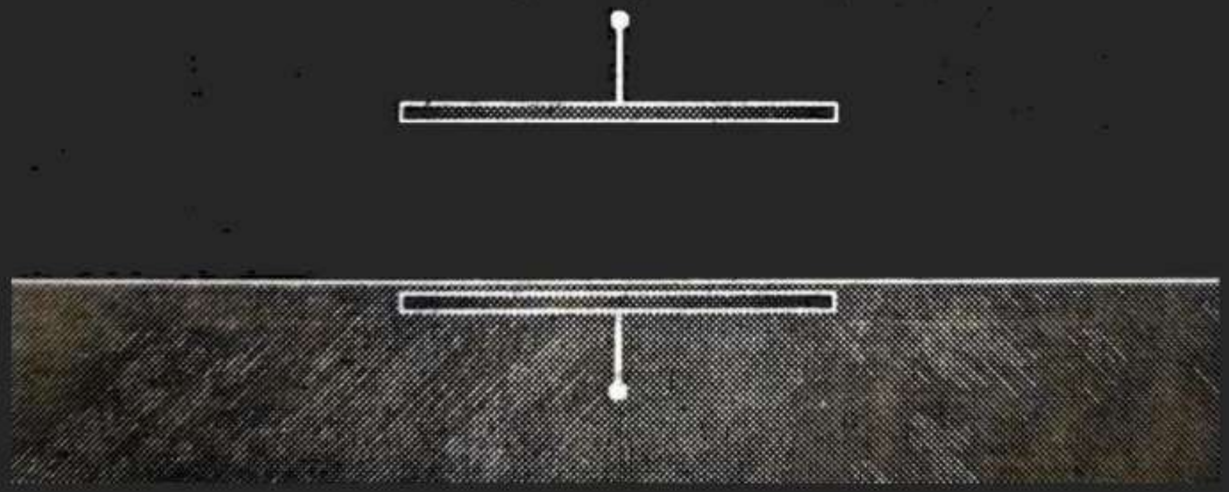
$$h = \frac{F}{\rho b d g} = \frac{\varepsilon_0 V^2}{2\rho g d^2}(K-1)$$

Force on a dielectric slab

H.W

CAPACITOR

Q.3 Figure-shows a horizontal parallel plate capacitor is lowered on a liquid surface in such a way that its lower plate is just submerged in the liquid of dielectric constant k. Find the height to which the liquid level will be raised between the plates if the capacitor plates are given a surface charge density $+\sigma$ and $-\sigma$ on its plates.



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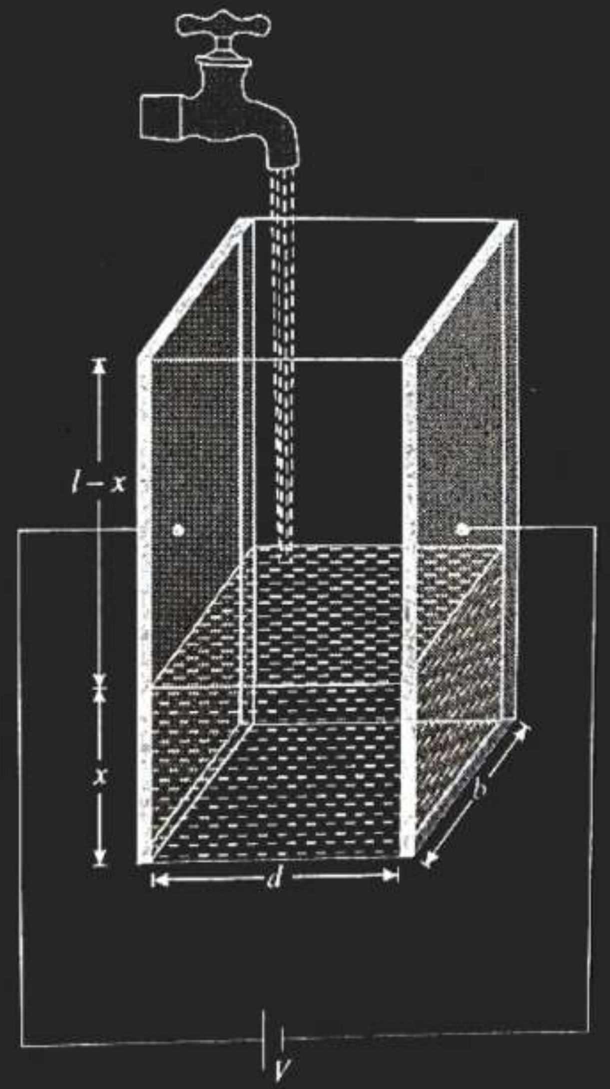
Force on a dielectric slab

CAPACITOR

H-W

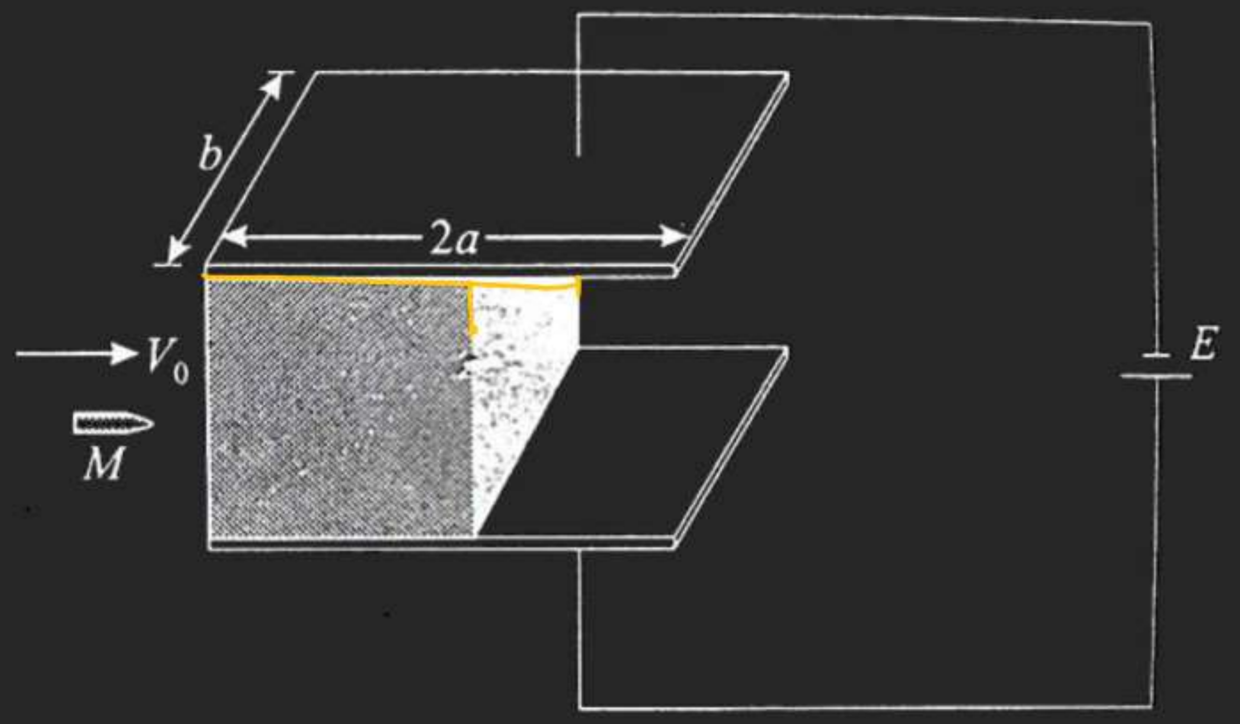
Q.4 A parallel plate capacitor is made by fixing two plates inside of a container as shown in figure. The plates are connected to a battery of voltage V. If at $t = 0$ the tap is opened from which a liquid of dielectric constant k starts filling in the container at a constant rate of $r\ m^3/s$, find the current in connecting wires as a function of time. Neglect any resistance in connecting wires.



Force on a dielectric slab

H.W

Q.5 A parallel plate capacitor is half filled with a dielectric of relative permittivity k and of mass M. Capacitor is attached with a cell of voltage E. Plates are held fixed on smooth insulating horizontal surface. A bullet of equal mass M hits the dielectric elastically and its found that dielectric just leaves out the capacitor. Find speed of bullet.



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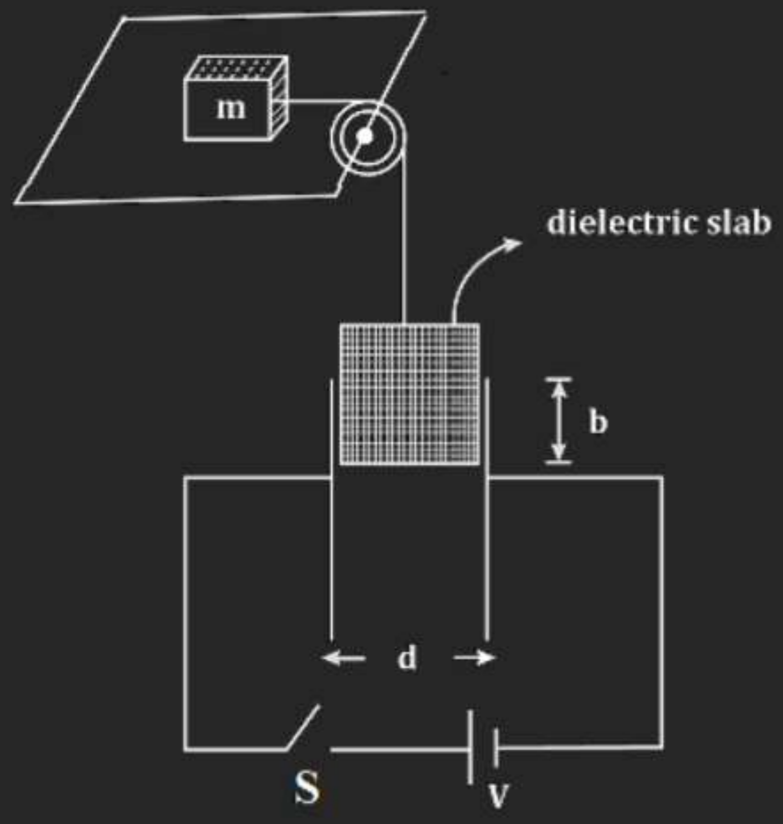
Force on a dielectric slab

CAPACITOR

H/W

Q.6 A block A of mass m kept on a rough horizontal table is connected to a dielectric slab of mass $m/6$ and dielectric constant K by means of a light and inextensible string passing over a fixed pulley as shown in the figure. The dielectric can completely fill the space between the parallel plate capacitor of plate area $l \times l$ and separation between the plates d kept in vertical position. Initially switch S is open and length of the dielectric inside the capacitor is b. The coefficient of friction between the block A and the surface is $\mu = 1/4$.



(A) Find the minimum value of the applied emf E so that block A starts sliding on the table when switch S is closed.

(B) If the applied emf is twice the above value, find the speed of block A when the dielectric completely fills the space between the capacitor. Ignore any other friction.