DPP - 02

SOLUTION

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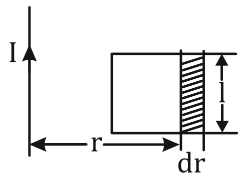
1. Magnetic field at a distance r from the wire

$B = \frac{\mu_0 I}{2\pi r}.$

Magnetic flux for small displacement dr.



$d\phi = B \cdot A = Bldr \qquad [\because A = ldr = \text{ Area }].$

$d\phi = \frac{\mu_0 I}{2\pi r} l \cdot dr = \frac{\mu_0 Il}{2\pi r} dr.$

$l = \frac{d\phi}{dt} = \frac{\mu_0 Il}{2\pi r} \cdot \frac{dr}{dt} = \frac{\mu_0 Ilv}{2\pi r}$

$i = \frac{e}{R} = \frac{\mu_0 Ilv}{2\pi Rr}$

2. $V = 1 \text{ cm/s} = 10^{-2} \text{ m/s}$

$R_{loop} = 1.7\Omega$

$A = 5 \text{ cm} = 5 \times 10^{-2} \text{ m}$

$R_{total} = R_{loop} + R_{wheastone}$

$R_{wheastone} = \frac{(4)(2)}{4+2} = \frac{8}{6} = \frac{4}{3} = 1.3\Omega$

$R_{Total} = 1.7 + 1.3$

$\Rightarrow R_{Total} = 3\Omega$

Induced emf $= VB\ell$

$\Rightarrow$ current $= I = \frac{(VB\ell)}{R_{Total}}$

$= \frac{(10^{-2})(1)(5 \times 10^{-2})}{3}$

$= \frac{5}{3} \times 10^{-4}$

$= 1.67 \times 10^{-4}$

$= 167 \times 10^{-6}$ $=170 \times 10^{-6}$ A (approximately)

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3. Copper rod will acquire terminal velocity when magnetic force = gravitational force

or $IlB = mg\sin\theta$ ...(i)

Also, $I = \frac{\text{induced emf}}{R} = \frac{Blv}{R}$ ...(ii)

From equations (i) and (ii), we get

$$\frac{B^2l^2v}{R} = mg\sin\,\theta;\ \therefore\ v = \frac{mgR\sin\,\theta}{B^2l^2}$$

4. Radius, $r = 8$ cm, number of turns, $N = 20$

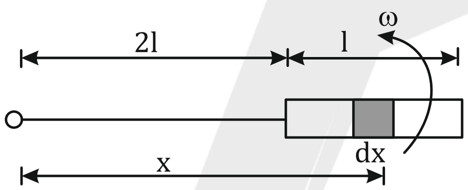
Angular speed, $\omega = 50\text{rad/s}$

Magnetic field, $B = 3 \times 10^{-2}$ T

Let the maximum emf is e

$e\ = NBA\omega = 20 \times 3 \times 10^{-2} \times \pi \times (0.08)^2 \times 50$

$= 0.60288 \simeq 60 \times 10^{-2}$ V

5. 

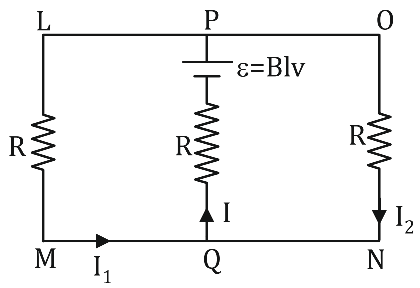
Consider an element of length dx at a distance x from the fixed end of the string. E.m.f. induced is $d\varepsilon = B(\omega x)dx$

Hence, the e.m.f. induced across the ends of the rod is

$$\varepsilon = \int_{2l}^{3l} B\omega x dx\ = B\omega\left[\frac{x^2}{2}\right]_{2l}^{3l}$$

$$= \frac{B\omega}{2}[(3l)^2 - (2l)^2] = \frac{5B\omega l^2}{2}$$

6. The equivalent circuit diagram is as shown in the figure.



Applying Kirchhoff's current law at junction Q, we get

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$I = I_1 + I_2$ ...(i)

Applying Kirchhoff's voltage law for the closed loop PLMQP, we get

$-I_1R - IR + \varepsilon = 0$

$I_1R + IR = Blv$ ...(ii)

Again, applying Kirchhoff's voltage law for the closed loop PONQP, we get

$-I_2R - IR + \varepsilon = 0$

$I_2R + IR = Blv$ ...(iii)

Adding equations (ii) and (iii), we get

$2IR + I_1R + I_2R = 2Blv$

$2IR + R(I_1 + I_2) = 2Blv$

$2IR + IR = 2Blv$ (Using (i))

$3IR = 2Blv$

$I = \frac{2Blv}{3R}$

Substituting this value of I in equation (ii), we get

$I_1 = \frac{Blv}{3R}$

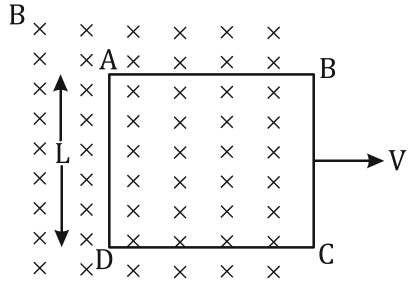
Substituting the value of I in equation (iii), we get

$I_2 = \frac{Blv}{3R}$

Hence, $I_1 = I_2 = (Blu)/R, I = (2Blu)/3R$

7. The emf induced in the circuit is zero because the two emf induced are equal and opposite when one U tube slides inside another tube.

8. As the side BC is outside the field, no emf is induced across BC. Since AB and CD are not cutting any flux, the emf induced across these two sides will also be zero.



The side AD is cutting the flux and emf induced across this side is BvL with corner A at higher potential.

Induced emf = vBL

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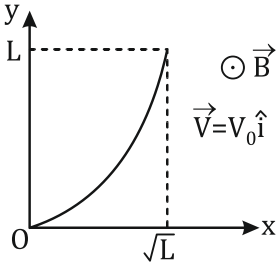
9. Length along y-axis of the wire will have an effective induced emf. Induced emf in small part of the wire,

$d\phi = BV_0 dy = B_0\left[1+\left(\frac{y}{L}\right)^{\beta}\right]V_0 dy$

So, $\Delta\phi = \int \; d\phi = \int_0^L B_0\left[1+\left(\frac{y}{L}\right)^{\beta}\right]V_0 dy$

$= B_0V_0\left[\int_0^L dy + \int_0^L \frac{y^{\beta}}{l^{\beta}}dy\right] = B_0V_0L\left[1+\frac{1}{1+\beta}\right]$

For given value of β, potential difference in the wire is proportional to L.



For $\beta = 0$;

$|\Delta\phi| = 2B_0V_0L$

For $\beta = 2$;

$\therefore |\Delta\phi| = B_0V_0L\left(1+\frac{1}{3}\right) = \frac{4}{3}B_0V_0L$

If parabolic wire is replaced by a straight wire $y = x$, of length $\sqrt{2}L$, then $|\Delta\phi|$ will remain the same.