EXERCISE - I

SECTION - A

SAMPLE SPACE DEFINITION, ODDS IN FAVOUR, A PERMUTATION & COMBINATION BASED PROBLEMS

1. From a pack of 52 playing cards, face cards and tens are removed and kept aside then a card is drawn at random from the remaining cards. If

A: The event that the card drawn is an ace

H: The event that the card drawn is a heart

S: The event that the card drawn is a spade then which of the following holds?

(A) $9P(A) = 4P(H)$ (B) $P(S) = 4P(A \cap H)$

(C) $3P(H) = 4P(A \cup S)$ (D) $P(H) = 12P(A \cap S)$

2. 5 persons entered the lift cabin on the ground floor of an 8 floor building. Suppose that each of them independently and with equal probability, can leave the cabin at any other floor, starting from the first. The probability that all 5 persons leave at different floors is

(A) $\left(\frac{5}{8}\right)^5$ (B) $\frac{{}^8C_5}{8^5}$ (C) $\frac{5!}{8^5}$ (D) $\frac{{}^8C_5 5!}{8^5}$

3. There are ten prizes, five A's, three B's and two C 's, placed in identical sealed envelopes for the top ten contestants in a mathematics contest. The prizes are awarded by allowing winners to select an envelope at random from those remaining. When the 8th contestant goes to select the prize, the probability that the remaining three prizes are one A, one B and one C, is

(A) ¼ (B) 1/3 (C) 1/12 (D) 1/10

4. A license plate is 3 letters (of English alphabets) followed by 3 digits. If all possible license plates are equally likely, the probability that a plate has either a letter palindrome or a digit palindrome (or both), is

(A) $\frac{7}{52}$ (B) $\frac{9}{65}$ (C) $\frac{8}{65}$ (D) None of these

5. A determinant is chosen at random from the set of all determinant of order 2 with elements 0 or 1 only. The probability that the determinant chosen has the value non negative is

(A) 3/16 (B) 6/16 (C) 10/16 (D) 13/16

6. Two cubes have their faces painted either red or blue. The first cube has five red faces and one blue face. When the two cubes are rolled simultaneously, the probability that the two top faces show the same colour is 1/2. Number of red faces on the second cube, is

(A) 1 (B) 2 (C) 3 (D) 4

7. A is one of the 6 horses entered for a race and is to be ridden by one of two jockeys B or C. It is 2 to 1 that B rides A, in which case all the horses are equally likely to win, if C rides A, his chance is trebled. Then the odds against his winning are

(A) 5/13 (B) 18/5 (C) 13/18 (D) 13/5

8. Lot A consists of 3G and 2D articles. Lot B consists of 4G and 1D article. A new lot C is formed by taking 3 articles from A and 2 from B. The probability that an article chosen at random from C is defective, is

(A) 1/3 (B) 2/5 (C) 8/25 (D) None of these

9. An Urn contains ' m ' white and ' n ' black balls. All the balls except for one ball, are drawn from it. The probability that the last ball remaining in the Urn is white, is

(A) $\frac{m}{m+n}$ (B) $\frac{n}{m+n}$ (C) $\frac{1}{(m+n)!}$ (D) $\frac{mn}{(m+n)!}$

10. If two of the 64 squares are chosen at random on a chess board, the probability that they have a side in common is

(A) 1/9 (B) 1/18 (C) 2/7 (D) none

11. A child throws 2 fair dice. If the numbers showing are unequal, he adds them together to get his final score. On the other hand, if the numbers showing are equal, he throws 2 more dice & adds all 4 numbers showing to get his final score. The probability that his final score is 6 is

(A) $\frac{145}{1296}$ (B) $\frac{146}{1296}$ (C) $\frac{147}{1296}$ (D) $\frac{148}{1296}$

12. If a, b and c are three numbers (not necessarily different) chosen randomly and with replacement from the set {1,2,3,4,5}, the probability that (ab+ c) is even, is

(A) $\frac{50}{125}$ (B) $\frac{59}{125}$ (C) $\frac{64}{125}$ (D) $\frac{75}{125}$

SECTION - B

VENN DIAGRAM BASED PROBLEMS

13. Three numbers are chosen at random without replacement from {1,2,3 ….,10}. The probability that the minimum of the chosen numbers is 3 or their maximum is 7 is

(A) 1/2 (B) 1/3 (C) 1/4 (D) 11/40

14. Let A& B be two events. Suppose $P(A) = 0.4, P(B) = p \& P(A \cup B) = 0.7$. The value of p for which A& B are independent is

(A) 1/3 (B) 1/4 (C) 1/2 (D) 1/5

SECTION - C

CONDITIONAL PROBABILITY

15. Before a race the chance of three runners, A, B, C were estimated to be proportional to 5,3,2, but during the race A meets with an accident which reduces his chance to 1/3. If the respective chance of B and C are P(B) and P(C) then

(A) $P(B) = \frac{4}{15}$ (B) $P(C) = \frac{4}{15}$ (C) $P(C) = \frac{2}{5}$ (D) $P(A) = \frac{4}{15}$

16. A bowl has 6 red marbles and 3 green marbles. The probability that a blind folded person will draw a red marble on the second draw from the bowl without replacing the marble from the first draw, is

(A) 2/3 (B) 1/4 (C) 5/12 (D) 5/8

17. Two cards are drawn from a well shuffled pack of 52 playing cards one by one. If

A: the event that the second card drawn is an ace and

B: the event that the first card drawn in an ace card.

then which of the following is true?

(A) $P(A) = \frac{4}{17}; P(B) = \frac{1}{13}$ (B) $P(A) = \frac{1}{13}; P(B) = \frac{1}{13}$

(C) $P(A) = \frac{1}{13}; P(B) = \frac{1}{17}$ (D) $P(A) = \frac{16}{221}; P(B) = \frac{4}{51}$

SECTION - D

MUTUALLY EXCLUSIVE, EXHAUSTIVE, INDEPENDENT EVENTS

18. A & B are two independent events such that $P(\overline{A}) = 0.7, P(\overline{B}) = a \& P(A \cup B) = 0.8$, then a is

(A) 5/7 (B) 2/7 (C) 1 (D) None of these

19. If atleast one child in a family with 3 children is a boy then the probability that 2 of the children are boys, is

(A) 3/7 (B) 4/7 (C) 1/3 (D) 3/8

SECTION - E

TOTAL PROBABILITY THEOREM

20. An urn contains 3 red balls and n white balls. Mr. A draws two balls together from the urn. The probability that they have the same colour is 1/2. Mr. B draws one ball from the urn, notes its colour and replaces it. He then draws a second ball from the urn and finds that both balls have the same colour is, 5/8. The possible value of n is

(A) 9 (B) 6 (C) 5 (D) 1

21. A purse contains 2 six sided dice. One is a normal fair die, while the other has 2 ones, 2 threes and 2 fives. A die is picked up and rolled. Because of some secret magnetic attraction of the unfair die, there is 75% chance of picking the unfair die and a 25% chance of picking a fair die. The die is rolled and shows up the face 3. The probability that a fair die was picked up, is

(A) $\frac{1}{7}$ (B) $\frac{1}{4}$ (C) $\frac{1}{6}$ (D) $\frac{1}{24}$

SECTION - F

BAYE'S THEOREM

22. Two buses A and B are scheduled to arrive at a town central bus station at noon. The probability that bus A will be late is 1/5. The probability that bus B will be late is 7/25. The probability that the bus B is late given that bus A is late is 9/10. Then the probabilities

(i) neither bus will be late on a particular day and

(ii) bus A is late given that bus B is late, are respectively

(A) 2/25 and 12/28 (B) 18/25 and 22/28

(C) 7/10 and 18/28 (D) 12/25 and 2/28

23. In a certain factory, machines A, B and C produce bolts. Of their production, machines A, B, and C produce 2%, 1% and 3% defective bolts respectively. Machine A produces 35% of the total output of bolts, machine B produces 25% and machine C produces 40%. A bolts is chosen at random from the factory's production and is found to be defective. The probability it was produced on machine C, is

(A) $\frac{6}{11}$ (B) $\frac{23}{45}$ (C) $\frac{24}{43}$ (D) $\frac{3}{11}$

SECTION - G

PROBABILITY DISTRIBUTION

24. The probability that a radar will detect an object in one cycle is p. The probability that the object will be detected in n cycles is

(A) $1 - p^n$ (B) $1 - (1 - p)^n$

(C) p^n (D) $p(1 - p)^{n-1}$

25. A biased coin with probability P, $0 < P < 1$, of heads is tossed until a head appears for the first time. If the probability that the number of tosses required is even is 2/5, then the value of P is

(A) 1/4 (B) 1/6 (C) 1/3 (D) ½

26. A examination consists of 8 questions in each of which one of the 5 alternatives is the correct one. On the assumption that a candidate who has done no preparatory work chooses for each question any one of the five alternatives with equal probability, the probability that he gets more than one correct answer is equal to

(A) $(0.8)^8$ (B) $3(0.8)^8$ (C) $1 - (0.8)^8$ (D) $1 - 3(0.8)^8$

SECTION - H

MISCELLANEOUS PROBLEMS

27. A quadratic equation is chosen from the set of all quadratic equations which are unchanged by squaring their roots. The chance that the chosen equation has equal roots is

(A) 1/2 (B) 1/3 (C) 1/4 (D) 2/3

28. Two numbers a and b are selected from the set of natural number then the probability that $a^2 + b^2$ is divisible by 5 is

(A) 9/25 (B) 7/18 (C) 11/36 (D) 17/81

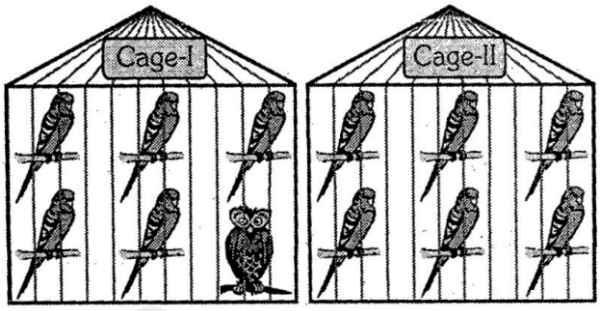
EXERCISE – II (LEVEL-I)

SAMPLE SPACE DEFINITION, ODDS IN FAVOR, A PERMUTATION & COMBINATION BASED PROBLEMS

1. The number ' a ' is randomly selected from the set {0,1,2,3, . 98,99}. The number ' b ' is selected from the same set. Probability that the number $3^a + 7^b$ has a digit equal to 8 at the units place, is.

(A) 1/16 (B) 2/16 (C) 4/16 (D) 3/16

2. Shalu brought two cages of birds : Cage - I contains 5 parrots and 1 owl and Cage - II contains 6 parrots, as shown. One day Shalu forgot to lock both cages and two birds flew from Case - I to Cage - II. Then two birds flew back from Cage - II to Cage - I. Assume that all birds have equal chance of flying,



Birds like to fly

the probability that the Owl is still in Cage - I, is

(A) $\frac{1}{6}$ (B) $\frac{1}{3}$ (C) $\frac{2}{3}$ (D) $\frac{3}{4}$

3. Sixteen players s_1, s_2, ... , s_{16} play in a tournament. They are divided into eight pairs at random. From each pair a winner is decided on the basis of a game played between the two players of the pair. Assume that all the players are of equal strength. The probability that "Exactly one of the two players s_1&s_2 is among the eight winners" is

(A) 4/15 (B) 7/15 (C) 8/15 (D) 9/15

CONDITIONAL PROBABILITY

4. A traffic light runs repeatedly through the following cycle: green for 30 seconds, then yellow for 3 seconds and then red for 30 seconds. Leah picks a random three-second time interval to watch the light. What is the probability that the colour changes while she is watching?

(A) 1/63 (B) 1/21 (C) 1/10 (D) 1/7

5. ' A ' and 'B' each have a bag that contains one ball of each of the colours blue, green, orange, red and violet. 'A' randomly selects one ball from his bag and puts it into B's bag. 'B' then randomly selects one ball from his bag and puts it into A's bag. The probability that after this process the contents of the two bags are the same, is

(A) 1/2 (B) 1/3 (C) 1/5 (D) 1/6

MUTUALLY EXCLUSIVE, EXHAUSITVE, INDEPENDENT EVENTS

6. In a horse race there are 18 horse numbered from 1 to 18 . The probability that horse 1 would win is $\frac{1}{6}$, horse 2 is $\frac{1}{10}$ and 3 is $\frac{1}{8}$. Assuming a tie is impossible, the chance that one of the three horses wins the race, is

(A) $\frac{143}{420}$ (B) $\frac{119}{120}$ (C) $\frac{47}{120}$ (D) $\frac{1}{5}$

7. From an urn containing six balls, 3 white and 3 black ones, a person selects at random an even number of balls (all the different ways of drawing an even number of balls are considered equally probable, irrespective of their number). Then the probability that there will be the same number of black and white balls among them

(A) 4/5 (B) 11/15 (C) 11/30 (D) 2/5

BAYE'S THEOREM

8. An urn contains ' m ' white and 'n' black balls. Balls are drawn one by one till all the balls are drawn. Probability that the second drawn ball is white, is

(A) $\frac{m}{m+n}$ (B) $\frac{m(m+n-1)}{(m+n)(m+n+1)}$ (C) $\frac{m(m-1)}{(m+n)(m+n-1)}$ (D) $\frac{mn}{(m+n)(m+n-1)}$

9. Mr. Dupont is a professional wine taster. When given a French wine, he will identify it with probability 0.9 correctly as French and will mistake it for a Californian wine with probability 0.1. When given a Californian wine, he will identify it with probability 0.8 correctly as Californian and will mistake it for a French wine with probability 0.2 . Suppose that Mr. Dupont is given ten unlabeled glasses of wine, three with French and seven with Californian wines. He randomly picks a glass, tries the wine and solemnly says : "French". The probability that the wine he tasted was Californian, is nearly equal to

(A) 0.14 (B) 0.24 (C) 0.34 (D) 0.44

10. A can hit a target 4 times out of 5 shots, B thrice in 4 shots and C twice in 3 shots. They fire a volley. Two shots hit the target. Find the probability that it is C who has missed –

(A) $\frac{1}{5}$ (B) $\frac{13}{30}$ (C) $\frac{6}{13}$ (D) None

MIXED PROBLEMS

11. 7 persons are stopped on the road at random and asked about their birthdays. If the probability that 3 of them are born on Wednesday, 2 on Thursday and the remaining 2 on Sunday is $\frac{K}{7^6}$, then K is equal to

(A) 15 (B) 30 (C) 105 (D) 210

EXERCISE – II (LEVEL-II)

VENN DIAGRAM BASED PROBLEMS

1. Let $0 < P(A) < 1, 0 < P(B) < 1$ and $P(A \cup B) = P(A) + P(B) - P(A)P(B)$. Then

(A) $P(B \mid A) = P(B) - P(A)$ (B) $P(\bar{A} \cup \bar{B}) = P(\bar{A}) + P(\bar{B})$

(C) $P(\overline{A \cup B}) = P(\bar{A})P(\bar{B})$ (D) $P(A \mid B) = P(A)$

CONDITIONAL PROBABILITY

2. If E_1 and E_2 are two events such that $P(E_1) = 1/4$, $P(E_2 \mid E_1) = 1/2$ and $P(E_1 \mid E_2) = 1/4$ then

(A) E_1 and E_2 are independent

(B) E_1 and E_2 are exhaustive

(C) E_2 is twice as likely to occur as E_1

(D) probabilities of the events $E_1 \cap E_2$, E_1 and E_2 are in G.P.

3. In a maths paper there are 3 sections A, B&C. Section A is compulsory. Out of sections B & C a student has to attempt any one. Passing in the paper means passing in A & passing in B or C. The probability of the student passing in A, B&C are p, q & 1/2 respectively. If the probability that the student is successful is 1/2 then, which of the following false

(A) $p = q = 1$ (B) $p = q = 1/2$

(C) $p = 1, q = 0$ (D) $p = 1, q = 1/2$

MUTUALLY EXCLUSIVE, EXHAUSTIVE, INDEPENDENT EVENTS

4. For any two events A & B in a sample space

(A) $P(A \mid B) \geq \frac{P(A)+P(B)-1}{P(B)}$, $P(B) \neq 0$ is always true

(B) $P(A \cap \bar{B}) = P(A) - P(A \cap B)$ does not hold

(C) $P(A \cup B) = 1 - P(\bar{A})P(\bar{B})$, if A& B are independent

(D) $P(A \cup B) = 1 - P(\bar{A})P(\bar{B})$, if A& B are disjoint

5. Two real numbers, x&y are selected at random. Given that $0 \leq x \leq 1; 0 \leq y \leq 1$. Let A be the event that $y^2 \leq x$; B be the event that $x^2 \leq y$, then

(A) $P(A \cap B) = \frac{1}{3}$

(B) A & B are exhaustive events

(C) A & B are mutually exclusive

(D) A & B are independent events.

6. If A& B are two events such that $P(B) \neq 1$. $\bar{B}$ denotes the event complementary to B, then

(A) $P(A \mid \bar{B}) = \frac{P(A)-P(A \cap B)}{1-P(B)}$

(B) $P(A \cap B) \geq P(A) + P(B) - 1$

(C) $P(A) >< P(A \mid B)$ according as $P(A \mid \bar{B}) >< P(A)$

(D) $P(A \mid \bar{B}) + P(\bar{A} \mid \bar{B}) = 1$

7. A pair of fair dice having six faces numbered from 1 to 6 are thrown once, suppose two events E and F are defined as

E: Product of the two numbers appearing is divisible by 5.

F: At least one of the dice shows up the face one. Then the events E and F are

(A) mutually exclusive

(B) independent

(C) neither independent nor mutually exclusive

(D) are equiprobable

EXERCISE – III - SUBJECTIVE (JEE-ADVANCED)

1. Numbers are selected at random, one at a time, from the two digit numbers 00,01,02, ... ,99 with replacement. An event E occurs if &only if the product of the two digits of a select number is 18. If four numbers are selected, find the probability that the event E occurs at least 3 times.

2. The chance of one event happening is the square of the chance of a 2 nd event, but odds against the first are the cubes of the odds against the 2^{nd} . Find the chances of each. (assume that both events are neither sure nor impossible).

3. Two cards are drawn from a well shuffled pack of 52 cards. Find the probability that one of them is a red card & the other is a queen.

4. In a box, there are 8 alphabets cards with the letters: S, S, A, A, A, H, H, H. Find the probability that the word 'ASH' will form if
(i) the three cards are drawn one by one & placed on the table in the same order that they are drawn.
(ii) the three cards are drawn simultaneously.

5. In a building programme the event that all the materials will be delivered at the correct time is M, and the event that the building programme will be completed on time is F. Given that $P(M) = 0.8$ and $P(M \cap F) = 0.65$, find $P(F \mid M)$. If $P(F) = 0.7$, find the probability that the building programme will be completed on time if all the materials are not delivered at the correct time.

6. The probability that an archer hits the target when it is windy is 0.4 ; when it is not windy, her probability of hitting the target is 0.7 . On any shot, the probability of a gust of wind is 0.3 . Find the probability that
(a) She hit the target on first shot
(b) Hits the target exactly once in two shots

7. A box contains 5 radio tubes of which 2 are defective. The tubes are tested one after the other until the 2 defective tubes are discovered. Find the probability that the process stopped on the. **(i)** Second test; **(ii)** Third test. If the process stopped on the third test, find the probability that the first tube is non defective.

8. A cube with all six faces coloured is cut into 64 cubical blocks of the same size which are thoroughly mixed. Find the probability that the 2 randomly chosen blocks have 2 coloured faces each.

9. Let A & B be two events defined on a sample space. Given $P(A) = 0.4; P(B) = 0.80$ and $P(\bar{A}/\bar{B}) = 0.10$. Then find (i) $P(\bar{A} \cup B)$ & $P[(\bar{A} \cap B) \cup (A \cap \bar{B})]$

10. A certain drug, manufactured by a Company is tested chemically for its toxic nature. Let the event "THE DRUG IS TOXIC" be denoted by H & the event "THE CHEMICAL TEST REVEALS THAT THE DRUG IS TOXIC" be denoted by S. Let $P(H) = a, P(S \mid H) = p(\bar{S} \mid \bar{H}) = 1 - a$. Then show that the probability that the drug is not toxic given that the chemical test reveals that it is toxic, is free from ' a '.

11. Consider the following events for a family with children
$A = \{$ of both the genders ; $B = \{$ at most one boy $\}$ In which of the following (are/is) the events A and B are independent.
(a) if a family has 3 children
(b) if a family has 2 children
Assume that the birth of a boy or a girl is equally likely mutually exclusive and exhaustive.

12. The probabilities that three men hit a target are, respectively, 0.3, 0.5 and 0.4. Each fires once at the target. (As usual, assume that the three events that each hits the target are independent)
(a) Find the probability that they all
(i) hit the target; (ii) miss the target
(b) Find the probability that the target is hit
(i) at least once; (ii) exactly once.
(c) If only one hits the target, what is the probability that it was the first man?

13. During a power blackout, 100 persons are arrested on suspect of looting. Each is given a polygraph test. From past experience it is know that the polygraph is 90% reliable when administered to a guilty person and -98% reliable when given to some one who is innocent. Suppose that of the 100 persons taken into custody, only 12 were actually involved in any wrong doing. If the probability that a given suspect is innocent given that the photograph says he is guilty is a/b where a and b are relatively prime, find the value of (a + b).

14. A covered basket of flowers has some lilies and roses. In search of rose, Sweety and Shweta alternately pick up a flower from the basket but puts it back if it is not a rose. Sweety is 3 times more likely to be the first one to pick a rose. If sweety begin this 'rose hunt' and if there are 60 lilies in the basket, find the number of roses in the basket.

15. A bomber wants to destroy a bridge. Two bombs are sufficient to destroy it. If four bombs are dropped, what is the probability that it is destroyed, if the chance of a bomb hitting the target is 0.4.

16. A game is played with a special fair cubic die which has one red side, two blue sides, and three green sides. The result is the colour of the top side after the die has been rolled. If the die is rolled repeatedly, the probability that the second blue result occurs on or before the tenth roll, can be expressed in the form $\frac{3^p-2^q}{3^r}$ where p, q, r are positive integers, find the value of $p^2+q^2+r^2$.

17. A uniform unbiased die is constructed in the shape of a regular tetrahedron with faces numbered 2,2, 3 and 4 and the score is taken from the face on which the die lands. If two such dice are thrown together, find the probability of scoring.
(i) exactly 6 on each of 3 successive throws.
(ii) more than 4 on at least one of the three successive throws.

18. An examination consists of 8 questions in each of which the candidate must say which one of the 5 alternative is correct one. Assuming that the student has not prepared earlier chooses for each of the question any one of 5 answers with equal probability.
(i) prove that the probability that he gets more than one correct answer is $(5^8-3\times4^8)/5^8$
(ii) find the probability that he gets correct answers to six or more questions.
(iii) find the standard deviation of this distribution.

19. An aircraft gun can take a maximum of four shots at an enemy's plane moving away from it. The probability of hitting the plane at first, second, third & fourth shots are 0.4,0.3,0.2&0.1 respectively. What is the probability that the gun hits the plane.

20. In a batch of 10 articles, 4 articles are defective. 6 articles are taken from the batch for inspection. If more than 2 articles in this batch are defective, the whole batch is rejected Find the probability that the batch will be rejected.

21. One hundred management students who read at least one of the three business magazines are surveyed to study the readership pattern. It is found that 80 read Business India, 50 read Business world and 30 read Business Today. Five students read all the three magazines. A student was selected randomly. Find the probability that he reads exactly two magazines.

22. A player tosses an unbiased coin and is to score two points for every head turned up and one point for every tail turned up. If P_n denotes the probability that his score is exactly n points, prove that $P_n - P_{n-1} = \frac{1}{2}(P_{n-2} - P_{n-1}); n \geq 3$. Also compute P_1 and P_2 and hence deduce the pr that he scores exactly 4.

23. A train consists of n carriages, each of which may have a defect with probability p. All the carriages are inspected, independently of one another, by two inspectors; the first detects defects (if any) with probability p_1& the second with probability p_2 . If none of the carriages is found to have a defect, the train departs. Find the probability of the event; "THE TRAIN DEPARTS WITH ATLEAST ONE DEFECTIVE CARRIAGE".

24. A box contains three coins two of them are fair and one two - headed. A coin is selected at random and tossed. If the head appears the coin is tossed again, if a tail appears, then another coin is selected from the remaining coins and tossed.
(i) Find the probability that head appears twice.
(ii) If the same coin is tossed twice, find the probability that it is two headed coin.
(iii) Find the probability that tail appears twice.

25. In a given race, the odds in favour of four horses A, B, C & D are 1: 3,1: 4,1: 5 and 1: 6 respectively. Assuming that a dead heat is impossible, find the chance that one of them wins the race.

26. Integers a, b, c and d not necessarily distinct, are chosen independently and at random from the set $S = \{0,1,2,3, \ldots \ldots .2006,2007\}$. If the probability that $|ad - bc|$ is even, is $\frac{p}{q}$ where p and q are relatively prime the find the value of $(p + q)$

COMPREHENSION

If n positive integers taken at random and multiplied together, then the chance that the last digit of the product would be

27. 1,3,5,7, or 9 is

(A) $\left(\frac{2}{5}\right)^n$ (B) $\left(\frac{1}{2}\right)^n$ (C) $\frac{2^n-1}{5^n}$ (D) $\frac{5^n-4^n}{10^n}$

28. 1,3,7 or 9 is

(A) $\frac{2^n-1}{5^n}$ (B) $\frac{5^n-4^n}{10^n}$ (C) $\left(\frac{2}{5}\right)^n$ (D) $\left(\frac{1}{2}\right)^n$

29. 5 is

(A) $\frac{5^n-4^n}{10^n}$ (B) $\frac{2^n-1}{5^n}$ (C) $\frac{10^n-8^n-5^n+4^n}{10^n}$ (D) $\left(\frac{2}{5}\right)^n$

30. There are n urns each containing $n+1$ balls such that the i^{th} urn contains i white balls and $(n+1-i)$ red balls. Let u_i be the event of selecting i^{th} urn, $i = 1,2,3.\ n$ and W denotes the event of getting a white ball.

(a) If $P(u_i) \propto i$, where $i = 1,2,3, \ldots\ldots, n$ then $\lim_{n\to\infty} P(W)$ is equal to

(A) 1 (B) 2/3 (C) 3/4 (D) ¼

(b) If $P(u_i) = c$ where c is a constant then $P(u_n/W)$ is equal to

(A) $\frac{2}{n+1}$ (B) $\frac{1}{n+1}$ (C) $\frac{n}{n+1}$ (D) $\frac{1}{2}$

(c) If n is even and E denotes the event of choosing even numbered urn $\left(P(u_i) = \frac{1}{n}\right)$, then the value of P(W/E), is

(A) $\frac{n+2}{2n+1}$ (B) $\frac{n+2}{2(n+1)}$ (C) $\frac{n}{n+1}$ (D) $\frac{1}{n+1}$

MATRIX MATCH TYPE

31.

Column – I	**Column - II**
(A) A, B and C in order toss a coin. The first one to throw a head wins. If p, q and r are their respective chances of winning, assume that the game may continue identifitely, then	(P) $q - r = \frac{1}{7}$
	(Q) $p - r = \frac{3}{7}$
(B) Three persons A, B and C throw a dice in succession till one gets a 'six' and wins the game. If p, q and r are them respective chances of winning, then	(R) $p: q = 6: 5$
(C) A, B and C play a game and chances of their winning it in an attempt are $\frac{2}{3}, \frac{1}{2}$ and $\frac{1}{4}$ respectively. A has the first chance, followed by B and then by C. This cycle is repeated till one of them wins the game. If p, q, r are their respective chances of winning the game, then	(S) $p - r = \frac{5}{7}$
	(T) $p: q = 2: 1$

EXERCISE – IV – LEVEL - I (JEE-MAIN)

1. In a binomial distribution $B\left(n, p = \frac{1}{4}\right)$, if the probability of at least one success is greater than or equal to $\frac{9}{10}$, then n is greater than: **[AIEEE 2009]**

(A) $\frac{1}{\log_{10} 4+\log_{10} 3}$ (B) $\frac{9}{\log_{10} 4-\log_{10} 3}$ (C) $\frac{4}{\log_{10} 4-\log_{10} 3}$ (D) $\frac{1}{\log_{10} 4-\log_{10} 3}$

2. One ticket is selected at random from 50 tickets numbered 00,01,02, … … . . ,49. Then the probability that the sum of the digits on the selected ticket is 8, given that the product of these digits is zero, equals **[AIEEE 2009]**

(A) $\frac{1}{7}$ (B) $\frac{5}{14}$ (C) $\frac{1}{50}$ (D) $\frac{1}{14}$

3. An urn contains nine balls of which three are red, four are blue and two are green. Three balls are drawn at random without replacement from the urn. The probability that the three balls have different colours is- **[AIEEE 2010]**

(A) $\frac{1}{3}$ (B) $\frac{2}{7}$ (C) $\frac{1}{21}$ (D) $\frac{2}{23}$

4. Four numbers are chosen at random (without replacement) from the set {1,2,3, … . ,20}.

Statement -1: The probability that the chosen numbers when arranged in some order will form an AP is $\frac{1}{85}$.

Statement-2: If the four chosen numbers form an AP, then the set of all possible values of common diference is $\{\pm 1, \pm 2, \pm 3, \pm 4, \pm 5\}$ **[AIEEE 2010]**

(A) Statement (1) is true and statement (2) is true and statement (2) is correct explanation for Statement (1)

(B) Statement (1) is true and statement (2) is true and statement (2) is NOT a correct explanation for Statement (1)

(C) Statement (1) is true but (2) is false

(D) Statement (1) is false but (2) is true

5. Consider 5 independent Bernoulli's trials each with probability of success p. If the probability of at least one failure is greater than or equal to $\frac{31}{32}$, then p lies in the interval: **[AIEEE 2011]**

(A) $\left(\frac{1}{2}, \frac{3}{4}\right]$ (B) $\left(\frac{3}{4}, \frac{11}{12}\right]$ (C) $\left[0, \frac{1}{2}\right]$ (D) $\left(\frac{11}{12}, 1\right]$

6. If C and D are two events such that $C \subset D$ and $P(D) \neq 0$, then the correct statement among the following is: **[AIEEE 2011]**

(A) $P(C \mid D) = P(C)$ (B) $P(C \mid D) \geq P(C)$ (C) $P(C \mid D) < P(C)$ (D) $P(C \mid D) = \frac{P(D)}{P(C)}$

7. Let A, B and C be pairwise independent events with $P(C) > 0$ and $P(A \cap B \cap C) = 0$. Then, $P(A^C \cap B^C / C)$ is equal to **[AIEEE 2011]**

(A) $P(A^C) - P(B)$ (B) $P(A) - P(B^C)$ (C) $P(A^C) + P(B^C)$ (D) $P(A^C) - P(B^C)$

8. Three numbers are chosen at random without replacement from {1,2,3, ….8}. The probability that their minimum is 3, given that their maximum is 6, is: **[AIEEE 2012]**

(A) $\frac{1}{4}$ (B) $\frac{2}{5}$ (C) $\frac{3}{8}$ (D) $\frac{1}{5}$

9. A multiple choice examination has 5 questions. Each question has three alternative answers of which exactly one is correct. The probability that a student will get 4 or more correct answers just by guessing is **[JEE-MAIN 2013]**

(A) $\frac{11}{3^5}$ (B) $\frac{10}{3^5}$ (C) $\frac{17}{3^5}$ (D) $\frac{13}{3^5}$

10. Let A and B be two events such that $P(\overline{A \cup B}) = \frac{1}{6}$, $P(A \cap B) = \frac{1}{4}$ and $P(\bar{A}) = \frac{1}{4}$, where $\bar{A}$ stands for the complement of the event A. Then the events A and B are: **[JEE-MAIN 2014]**

(A) mutually exclusive and independent. (B) equally likely but not independent

(C) independent but not equally likely (D) independent and equally likely

11. For there events A,B and C,P(Exactly one of A or B occurs) = P (Exctly one of B or C occures) = P (Exactly one of C or A occurs) = $\frac{1}{4}$ and P(All the three events occur simultaneously) = $\frac{1}{16}$. Then the probability that at least one of the events occurs, is: **[JEE-MAIN 2017]**

(A) $\frac{7}{32}$ (B) $\frac{7}{16}$ (C) $\frac{7}{64}$ (D) $\frac{3}{16}$

12. A bag contains 4 red and 6 black balls. A ball is drawn at random from the bag, its colour is observed and this ball along with two additional balls of the same colour are returned to the bag. If now a ball is drawn at random from the bag, then the probability that this drawn ball is red, is: **[JEE-MAIN 2018]**

(A) $\frac{3}{4}$ (B) $\frac{3}{10}$ (C) $\frac{2}{5}$ (D) $\frac{1}{5}$

EXERCISE – IV – LEVEL - II (JEE-ADVANCED)

1. A signal which can be green or red with probability $\frac{4}{5}$ and $\frac{1}{5}$ respectively, is received by station A and then transmitted to station B. The probability of each station receiving the signal correctly is $\frac{3}{4}$, if the signal received at station B is green, then the probability that the original signal was green is. **[JEE 2010]**

(A) 3/4 (B) 6/7 (C) 20/23 (D) 9/20

Paragraph for Question Nos. 2 to 3

Let U_1 and U_2 be two urns such that U_1 contains 3 white and 2 red balls, and U_2 contains only 1 white ball. A fair coin is tossed. If head appears then 1 ball is drawn at random from U_1 and put into U_2. However, if tail appears then 2 balls are drawn at random from U_1 and put into U_2. Now 1 ball is drawn at random from U_2. **[JEE 2011]**

2. The probability of the drawn ball from U_2 being white is

(A) 13/30 (B) 23/30 (C) 19/30 (D) 11/30

3. Given that the drawn ball from U_2 is white, the probability that head appeared on the coin is

(A) 17/23 (B) 11/23 (C) 15/23 (D) 12/23

4. Let E and F be two independent events. The probability that exactly one of them occurs is $\frac{11}{25}$ and the probability of none of them occurring is $\frac{2}{25}$. If P(T) denotes the probability of occurrence of the event T, then **[JEE 2011]**

(A) $P(E) = \frac{4}{5}, P(F) = \frac{3}{5}$ (B) $P(E) = \frac{1}{5}, P(F) = \frac{2}{5}$

(C) $P(E) = \frac{2}{5}, P(F) = \frac{1}{5}$ (D) $P(E) = \frac{3}{5}, P(F) = \frac{4}{5}$

5. A ship is fitted with three engines E_1, E_2 and E_3. The engines function independently of each other with respective probabilities $\frac{1}{2}, \frac{1}{4}$ and $\frac{1}{4}$. For the ship to be operational at least two of its engines must function. Let X denote the event that the ship is operational and let X_1, X_2 and X_3 denote respectively the events that the engines E_1, E_2 and E_3 are functioning. Which of the following is (are) true? **[JEE 2012]**

(A) $P[X_1^c \mid X] = \frac{3}{16}$

(B) P [Exactly two engines of the ship are functioning | X] $= \frac{7}{8}$

(C) P[X | X_2] $= \frac{5}{16}$

(D) P[X | X_1] $= \frac{7}{16}$

6. Four fair dice D_1, D_2, D_3 and D_4, each having six faces numbered 1,2,3,4,5 and 6, are rolled simultaneously. The probability that D_4 shows a number appearing on one of D_1, D_2 and D_3 is **[JEE 2012]**

(A) $\frac{91}{216}$ (B) $\frac{108}{216}$ (C) $\frac{125}{216}$ (D) $\frac{127}{216}$

7. Let X and Y be two events such that P(X | Y) $= \frac{1}{2}$. P(Y | X) $= \frac{1}{3}$ and P(X ∩ Y) $= \frac{1}{6}$. Which of the following is (are) correct? **[JEE 2012]**

(A) P(X ∪ Y) $= \frac{2}{3}$ (B) X and Y are independent

(C) X and Y are not independent (D) P(X^C ∩ Y) $= \frac{1}{3}$

8. Four persons independently solve a certain problem correctly with probabilities $\frac{1}{2}, \frac{3}{4}, \frac{1}{4}, \frac{1}{8}$. Then the probability that the problem is solved correctly by at least one of them is **[JEE 2013]**

(A) $\frac{235}{256}$ (B) $\frac{21}{256}$ (C) $\frac{3}{256}$ (D) $\frac{253}{256}$

9. Of the three independent events E_1, E_2 and E_3 the probability that only E_1 occurs is α, only E_2 occurs is β and only E_3 occurs is γ. Let the probability p that none of events E_1, E_2 or E_3 occurs satisfy the equations $(\alpha - 2\beta)p = \alpha\beta$ and $(\beta - 3\gamma)p = 2\beta\gamma$. All the given probabilities are assumed to lie in the interval (0,1). **[JEE 2013]**

Then $\frac{\text{Probability of occurrence of } E_1}{\text{Probability of occurrence of } E_3} =$

Paragraph for Question 10 and 11

A box B_1 contains 1 white ball, 3 red balls and 2 black ballls, Another box B_2 contains 2 whilte balls, 3 red balls and 4 black balls. A third box B_3 contains 3 while balls, 4 red balls and 5 black balls. **[JEE 2013]**

10. If 1 ball is drawn fron each of the boxes B_1, B_2 are B_3, the probability that all 3 drawn balls are of the same colour is

(A) $\frac{82}{648}$ (B) $\frac{90}{648}$ (C) $\frac{558}{648}$ (D) $\frac{566}{648}$

11. If 2 balls are drawn (without replacement) from a randomly selected box and one of the balls is white and the other ball is red, the probability that these 2 balls are drawn from box B_2 is

(A) $\frac{116}{181}$ (B) $\frac{126}{181}$ (C) $\frac{65}{181}$ (D) $\frac{55}{181}$

12. Three boys and two girls stand in a queue. The probability, that the number of boys ahead of every girl is at least one more than the number of girls ahead of her, is **[JEE 2014]**

(A) $\frac{1}{2}$ (B) $\frac{1}{3}$ (C) $\frac{2}{3}$ (D) $\frac{3}{4}$

Paragraph for Question 13 and 14

Box 1 contains three cards bearing numbers 1,2,3; box 2 contains five cards bearing number 1,2,3,4,5; and box 3 contains seven cards bearing numbers 1,2,3,4,5,6,7. A card is drawn from each of the boxes. Let x_i be the number on the card drawn from the i^{th} box, $\mathbf{i} = 1,2,3$.

13. The probability that $x_1 + x_2 + x_3$ is odd **[JEE 2014]**

(A) $\frac{29}{105}$ (B) $\frac{53}{105}$ (C) $\frac{57}{105}$ (D) $\frac{1}{2}$

14. The probability that x_1, x_2, x_3 are in an arithmetic progression, is **[JEE 2014]**

(A) $\frac{9}{105}$ (B) $\frac{10}{105}$ (C) $\frac{11}{105}$ (D) $\frac{7}{105}$

15. The minimum number of times a fair coin needs to be tossed, so that the probability of getting at least two heads is at least 0.96 is. **[JEE 2015]**

PARAGRAPH Question No. 16 to 17

Let n_1 and n_2 be the number of red and black balls, respectively, in box I. Let n_3 and n_4 be the number of red and black balls, respectively, in box II. **[JEE 2016]**

16. One of the two boxes, box I and box II, was selected at random and a ball was drawn randomly out of this box. The ball was found to be red. If the probability that this red ball was drawn from box II is $\frac{1}{3}$, then the correct option(s) with the possible values of n_1, n_2, n_3 and n_4 is(are)

(A) $n_1 = 3, n_2 = 3, n_3 = 5, n_4 = 15$ (B) $n_1 = 3, n_2 = 6, n_3 = 10, n_4 = 50$

(C) $n_1 = 8, n_2 = 6, n_3 = 5, n_4 = 20$ (D) $n_1 = 6, n_2 = 12, n_3 = 5, n_4 = 20$

17. A ball is drawn at random from box I and transferred to box II. If the probability of drawing a red ball from box I, after this transfer, is $\frac{1}{3}$, then the correct option(s) with the possible values of n_1 and n_2 is(are)

(A) $n_1 = 4$ and $n_2 = 6$ (B) $n_1 = 2$ and $n_2 = 3$

(C) $n_1 = 10$ and $n_2 = 20$ (D) $n_1 = 3$ and $n_2 = 6$

18. Let X and Y be two events such that $P(X) = \frac{1}{3}$, $P(X \mid Y) = \frac{1}{2}$ and $P(Y \mid X) = \frac{2}{5}$.

Then **[JEE 2017]**

(A) $P(Y) = \frac{4}{15}$ (B) $P(X' \mid Y) = \frac{1}{2}$ (C) $P(X \cup Y) = \frac{2}{5}$ (D) $P(X \cap Y) = \frac{1}{5}$

19. Three randomly chosen nonnegative integers x, y and z are found to satisfy the equation $x + y + z = 10$. Then the probability that z is even, is **[JEE Adv. 2017]**

(A) $\frac{5}{11}$ (B) $\frac{6}{11}$ (C) $\frac{1}{2}$ (D) $\frac{36}{55}$

Paragraph for 20 to 21

There are five students S_1, S_2, S_3, S_4 and S_5 in a music class and for them there are five seats R_1, R_2, R_3, R_4 and R_5 arranged in a row, where intially the seat R_i is allotted to the student $S_i, i = 1,2,3,4,5$. But, on the examination day, the five students are randomly allotted the five seats. **[JEE Adv. 2018]**

20. The probability that, on the examination day, the student S_1 gets the previously allotted seat R_1 and NONE of the remaining students gets the seat previously allotted to him/ her is

(There are two questions based on Paragraph " A ". the question given below is one of them)

(A) $\frac{3}{40}$ (B) $\frac{1}{8}$ (C) $\frac{7}{40}$ (D) $\frac{1}{5}$

21. For $i = 1,2,3,4$ let T_i denote the event that the students S_i and S_{i+1} do NOT sit adjacent to each other on the day of the examination. Then, the probability of the event $T_1 \cap T_2 \cap T_3 \cap T_4$ is

(A) $\frac{1}{15}$ (B) $\frac{1}{10}$ (C) $\frac{7}{60}$ (D) $\frac{1}{5}$

22. There are three bags B_1, B_2 and B_3. The bag B_1 contains 5 red and 5 green balls, B_2 contains 3 red and 5 green balls, and B_3 contains 5 red and 3 green balls. Bags B_1, B_2 and B_3 have probabilities $\frac{3}{10}, \frac{3}{10}$ and $\frac{4}{10}$ respectively of being chosen. A bag is selected at random and a ball is chosen at random from the bag. Then which of the following options is/are correct?

[JEE Adv. 2019]

(A) Probability that the chosen ball is green, given that the selected bag is B_3, equals $\frac{3}{8}$

(B) Probability that the selected bag is B_3 and the chosen ball is green equals $\frac{3}{10}$

(C) Probability that the selected bag is B_3, given that chosen ball is green, equals $\frac{5}{13}$

(D) Probability that the chosen ball is green equals $\frac{39}{80}$

23. Let $|X|$ denote the number of elements in a set X, Let $S = \{1,2,3,4,5,6\}$ be a sample space, where each element is equally likely to occur. If A and B are independent events associated with S, then the number of ordered pairs (A, B) such that $1 \leq |B| < |A|$, equals **[JEE Adv. 2019]**

24. Let S be the sample space of all 3×3 matrices with entries from the set $\{0,1\}$, Let the events E_1 and E_2 be given by $E_1 = \{A \in S: det\ A = 0\}$ and $E_2 = \{A \in S:$ sum of entries of A is $7\}$ If a matrix is chosen at random from S, then the conditional probability $P(E_1 \mid E_2)$ equals

[JEE Adv. 2019]

ANSWER KEY

EXERCISE - I

1.	(A)	**2.**	(D)	**3.**	(A)	**4.**	(A)	**5.**	(D)
6.	(C)	**7.**	(D)	**8.**	(C)	**9.**	(A)	**10.**	(B)
11.	(D)	**12.**	(B)	**13.**	(D)	**14.**	(C)	**15.**	(B)
16	(A)	**17.**	(B)	**18.**	(B)	**19.**	(A)	**20.**	(D)
21.	(A)	**22.**	(C)	**23.**	(C)	**24.**	(B)	**25.**	(C)
26.	(D)	**27.**	(A)	**28.**	(A)				

EXERCISE – II (LEVEL-I)

1.	(D)	**2.**	(B)	**3.**	(C)	**4.**	(D)	**5.**	(B)
6.	(C)	**7.**	(B)	**8.**	(A)	**9.**	(C)	**10.**	(C)
11.	(B)								

EXERCISE – II (LEVEL-II)

1.	(C,D)	**2.**	(A,C,D)	**3.**	(A,B,C)	**4.**	(A,C)	**5.**	(A,B)
6.	(A,B,C,D)	**7.**	(C, D)						

EXERCISE – III - SUBJECTIVE (JEE-ADVANCED)

1. $\frac{97}{(25)^4}$ **2.** $\frac{1}{9}, \frac{1}{3}$ **3.** $\frac{101}{1326}$ **4.** (i) $\frac{3}{56}$ (ii) $\frac{9}{28}$

5. $P(F \mid M) = \frac{13}{16}; P(F \mid \bar{M}) = \frac{1}{4}$ **6.** (a) 0.61; (b) 0.4758

7. (i) $\frac{1}{10}$, (ii) $\frac{3}{10}$, (iii) $\frac{2}{3}$ **8.** $\frac{^{24}C_2}{^{64}C_2}$ or $\frac{23}{168}$ **9.** (i) 0.82, (ii) 0.76

10. $P(\bar{H} \mid S) = \frac{1}{2}$

11. Independent in (a) and not independent in (b)

12. (a), (b) (i) 0.72, (ii) 0.41, (c) $\frac{9}{41}$ **13.** 179 14. 120

15. $\frac{328}{625}$ **16.** 283 **17.** (i) $\frac{125}{16^3}$; (ii) $\frac{63}{64}$

19. 0.6976 **20.** $\frac{19}{42}$ **21.** $\frac{1}{2}$

23. $1 - [1 - p(1 - p_1)(1 - p_2)]^n$ **24.** $\frac{1}{2}, \frac{1}{2}, \frac{1}{12}$ 25. $\frac{319}{420}$

26. 13 **27.** $\left(\frac{1}{2}\right)^n$ **28.** $\left(\frac{2}{5}\right)^n$ 29. $\frac{5^n - 4^n}{10^n}$

30. (a) B; (b) A; (c) B **31.** A $\rightarrow \frac{4}{7}$; B $\rightarrow \frac{1}{7}$; C $\rightarrow \frac{1}{7}$

EXERCISE – IV – LEVEL - I (JEE-MAIN)

1.	(D)	**2.**	(D)	**3.**	(B)	**4.**	(C)	**5.**	(C)
6.	(B)	**7.**	(A)	**8.**	(D)	**9.**	(A)	**10.**	(C)
11.	(B)	**12.**	(C)						

EXERCISE – IV – LEVEL - II (JEE-ADVANCED)

1.	(C)	**2.**	(B)	**3.**	(D)	**4.**	(A)	**5.**	(B,D)
6.	(A)	**7.**	(A,B)	**8.**	(A)	**9.**	(6)	**10.**	(A)
11.	(D)	**12.**	(A)	**13.**	(B)	**14.**	(C)	**15.**	(8)
16.	(A,B)	**17.**	(C,D)	**18.**	(A,B)	**19.**	(B)	**20.**	(A)
21.	(C)	**22.**	(A,D)	**23.**	(422)	**24.**	(1/2)		