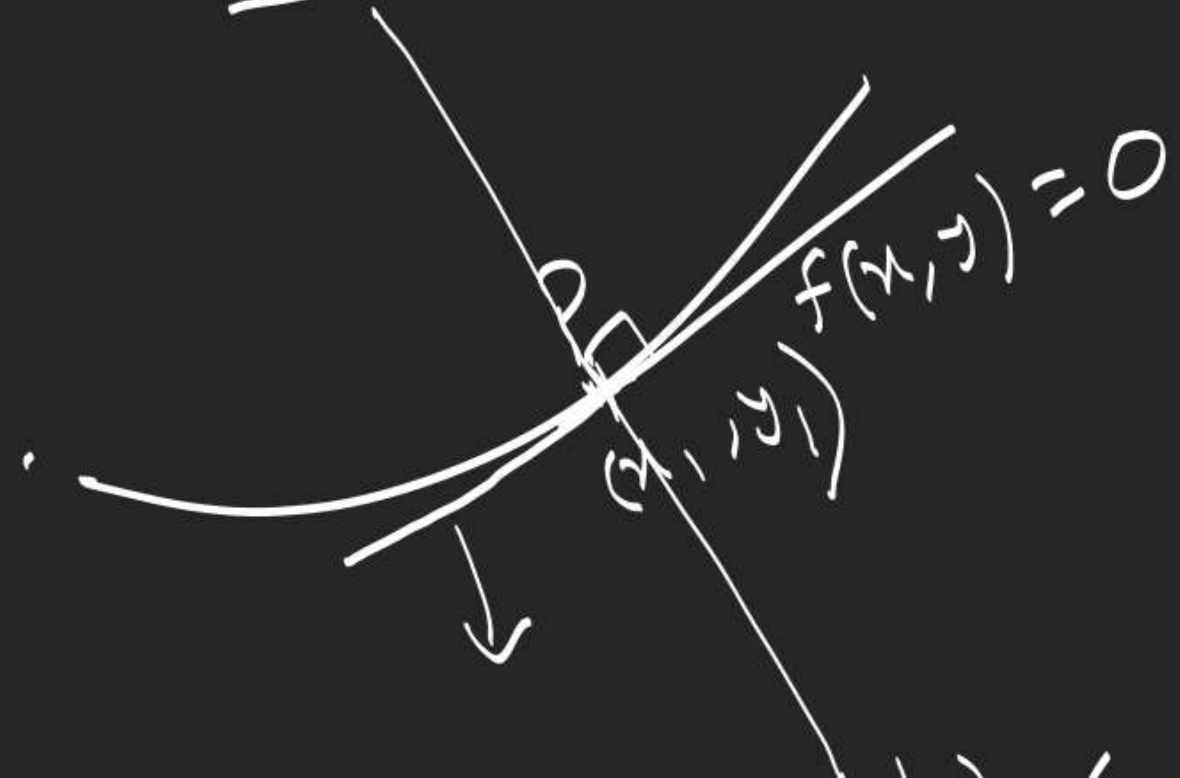
Nishant Jindal

Tangent & Normal



$$y - y_1 = \left(\frac{dy}{dx}\right)_{(x_1, y_1)} (x - x_1) \rightarrow \text{Tangent}$$

$$y - y_1 = \frac{-1}{\left(\frac{dy}{dx}\right)_{(x_1, y_1)}} (x - x_1) \rightarrow \text{Normal.}$$

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Eqn. of tangent to curve $\frac{x^n}{a^n}+\frac{y^n}{b^n}=1$ at (x_1, y_1) on it.

$$\frac{n x_1^{n-1}}{a^n} + \frac{n y_1^{n-1}}{b^n} y'\Big|_{(x_1, y_1)} = 0$$

$$y-y_1 = -\frac{x_1^{n-1}\, b^n}{a^n\, y_1^{n-1}}(x-x_1)$$

$$\frac{y y_1^{n-1}}{b^n} - \frac{y_1^n}{b^n} = -\frac{x x_1^{n-1}}{a^n} + \frac{x_1^n}{a^n}$$

$$\boxed{\frac{x x_1^{n-1}}{a^n} + \frac{y y_1^{n-1}}{b^n} = 1}$$

Tangent to $\alpha x^n + \beta y^n = \gamma$ at (x_1, y_1)

$$\boxed{\alpha x x_1^{n-1} + \beta y y_1^{n-1} = \gamma}$$

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1. Find eqn. of tangent to curve

$y = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$ at $\boxed{x=\sqrt{3}}$

$\tan^{-1}x = \theta$ $\qquad \theta \in \boxed{\frac{\pi}{3}-h, \frac{\pi}{3}+h}$

$\sin^{-1}\sin 2\theta = \pi - 2\theta$ $\qquad 2\theta$ $\qquad \frac{2\pi}{3}-2h, \frac{2\pi}{3}+2h$

$= \pi - 2\tan^{-1}x$

$-\frac{2}{1+x^2}\Big|_{x=\sqrt{3}} = -\frac{1}{2}$

$$\boxed{y - \frac{\pi}{3} = -\frac{1}{2}\left(x-\sqrt{3}\right)}$$

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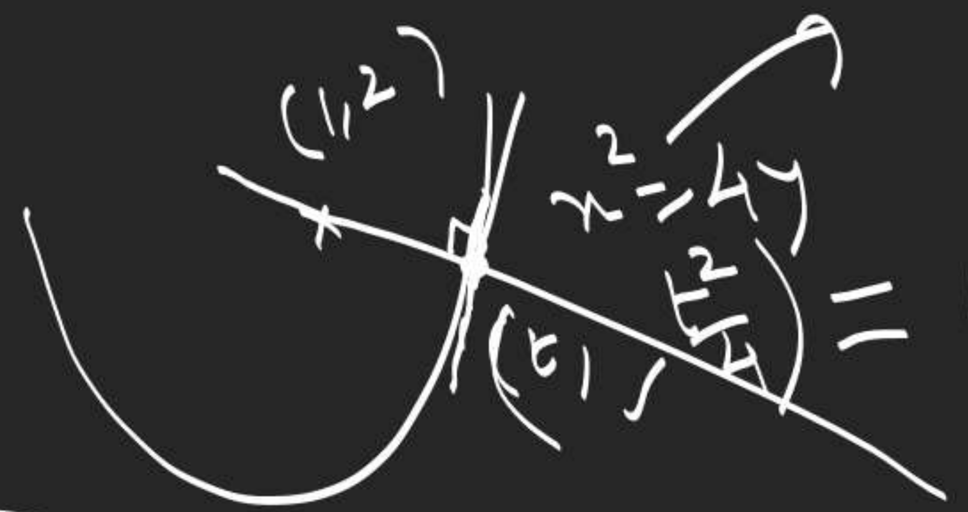
Q. Find eqn. of normal to curve $x^2=4y$ which passes thru $(1,2)$.

$x = 2y'$

$y' = \frac{t}{2}$



$\left(t, \frac{t^2}{4}\right) = (2,1)$

$$\frac{\frac{t^2}{4} - 2}{t-1} = -\frac{2}{t}$$

$$\frac{t^3}{4} - \cancel{2t} = -\cancel{2t} + 2$$

$$t^3 = 8$$

$$\boxed{x+y=3}$$

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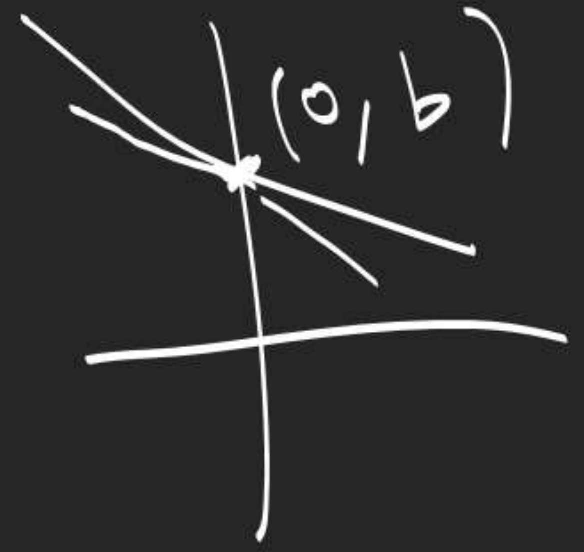
3. Find eqn of tangent to curve $y = be^{-\frac{x}{a}}$ at the point where the curve crosses y-axis.



$$y' = -\frac{b}{a}e^{-\frac{x}{a}}$$

$$y'\Big|_{x=0} = -\frac{b}{a}.$$

$$bx + ay = ab$$

$$\frac{x}{a} + \frac{y}{b} = 1$$

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4. Find the point on curve $y = 2x^3 - 15x^2 + 34x - 20$ where tangents are parallel to line $y + 2x = 0$.

$$\frac{dy}{dx} = -2$$

$$6x^2 - 30x + 34 = -2$$

$$x^2 - 5x + 6 = 0$$

$$x = 2, 3$$

$$\boxed{(2, 4), \ (3, 1)}$$

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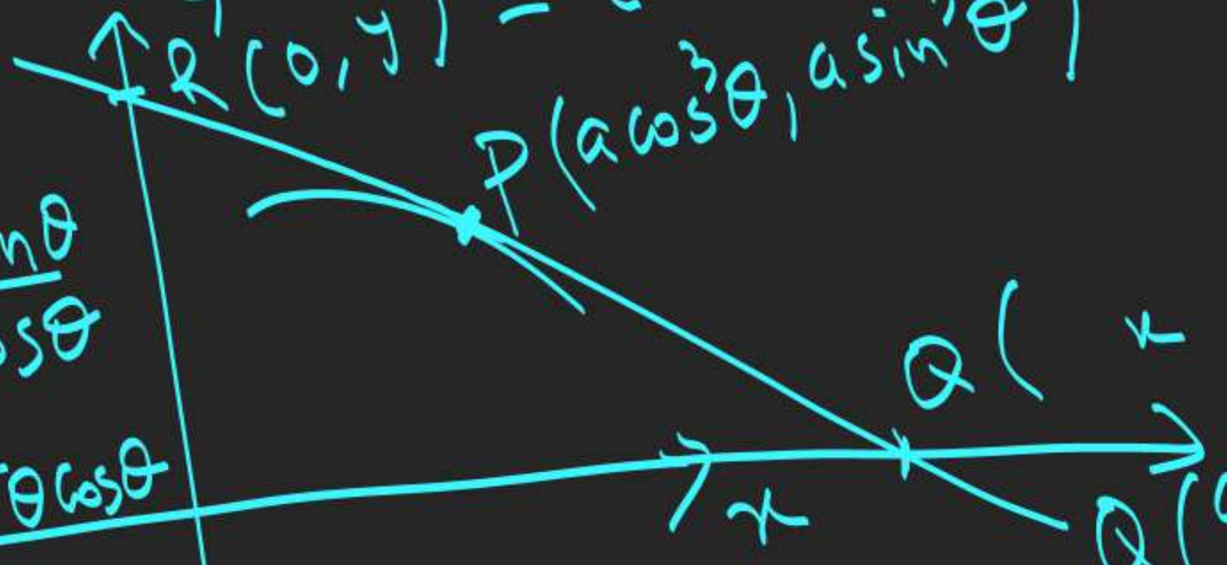
5: Show that portion of tangent to curve $x = a\cos^3\theta$, $y = a\sin^3\theta$ intercepted between the coordinate axes is constant.

$$\frac{dy}{dx} = \frac{3a\sin^2\theta\cos\theta}{3a\cos^2\theta(-\sin\theta)}$$

$$= -\tan\theta$$



$$\frac{0 - a\sin^3\theta}{x - a\cos^3\theta} = -\frac{\sin\theta}{\cos\theta}$$

$$x - a\cos^3\theta = a\sin^2\theta\cos\theta$$

$$x = a\cos\theta$$

$$\frac{y - 0}{0 - a\cos\theta} = -\tan\theta = -\frac{\sin\theta}{\cos\theta}$$

$$y = a\sin\theta$$

$$\boxed{QR = |a|}$$

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6. P.T. the portion of tangent to the curve

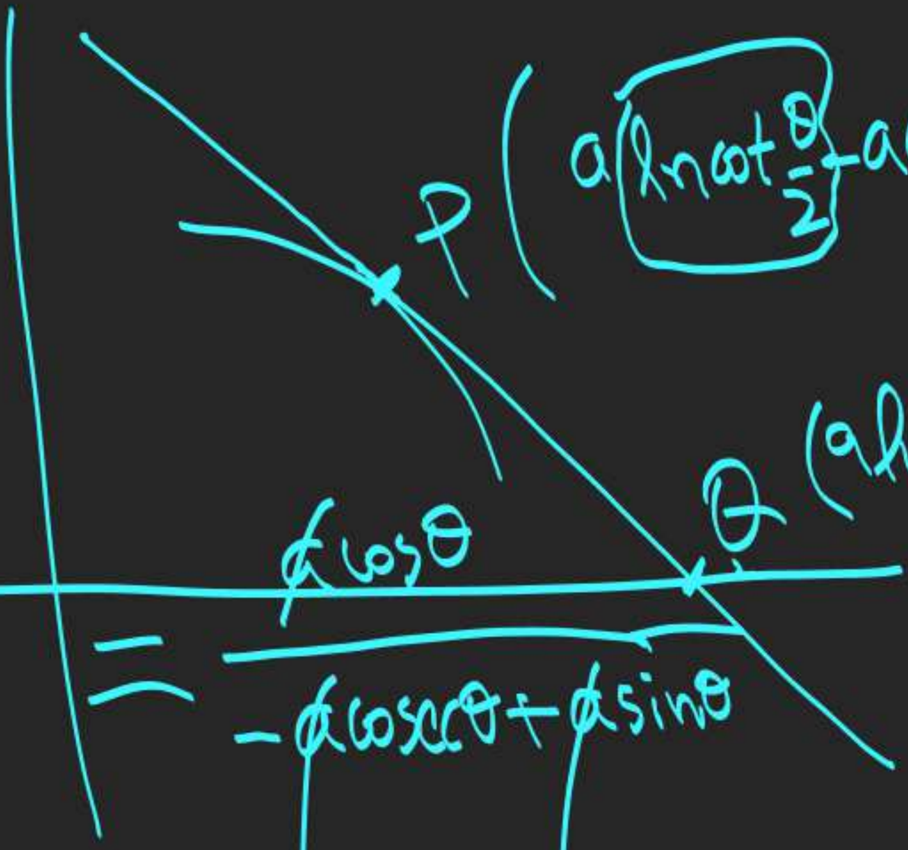
$a\cancel{\cos}\theta = x - a\ln\cot\frac{\theta}{2} + a\cancel{\cos}\theta$

$$\frac{x+\sqrt{a^2-y^2}}{a} = \ln\left(\frac{a+\sqrt{a^2-y^2}}{y}\right)$$ intercepted between

$\ln\tan\frac{\theta}{2}$

the point of contact and the x-axis is constant.

$$\frac{x+a\cos\theta}{a} = \ln\left(\frac{a+a\cos\theta}{a\sin\theta}\right) = \ln\cot\frac{\theta}{2}$$



$P\left(a\boxed{\ln\cot\frac{\theta}{2}} - a\cos\theta,\ a\sin\theta\right)$

$Q\left(a\ln\cot\frac{\theta}{2},\ 0\right)$

$$\boxed{PQ = |a|}$$

$$\frac{-a\sin\theta}{x - a\ln\cot\frac{\theta}{2} + a\cos\theta} = \frac{\cancel{a}\cos\theta}{-\cancel{a}\csc\theta + \cancel{a}\sin\theta} = \frac{\sin\theta\cos\theta}{(\sin^2\theta - 1)} = -\frac{\sin\theta}{\cos\theta}$$

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7. Show that condition for the line $x\cos\theta + y\sin\theta = p$ to touch the curve $\boxed{\dfrac{x^m}{a^m} + \dfrac{y^m}{b^m} = 1}$ is $(a\cos\theta)^{\frac{m}{m-1}} + (b\sin\theta)^{\frac{m}{m-1}} = p^{\frac{m}{m-1}}$

$$\left(\frac{x_1}{a}\right)^m + \left(\frac{y_1}{b}\right)^m = 1$$

$$\frac{x x_1^{m-1}}{a^m} + \frac{y y_1^{m-1}}{b^m} = 1$$

$$x\cos\theta + y\sin\theta = p$$

$$\frac{x_1^{m-1}}{a\, a^{m-1}\cos\theta} = \frac{y_1^{m-1}}{b\, b^{m-1}\sin\theta} = \frac{1}{p}$$

$\Leftarrow$

$$\frac{y_1}{b} = \left(\frac{b\sin\theta}{p}\right)^{\frac{1}{m-1}}$$

$$\frac{x_1}{a} = \left(\frac{a\cos\theta}{p}\right)^{\frac{1}{m-1}}$$

$\Downarrow$

$$\left(\frac{a\cos\theta}{p}\right)^{\frac{m}{m-1}} + \left(\frac{b\sin\theta}{p}\right)^{\frac{m}{m-1}} = 1$$

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12.

$$f(x) = x + \cfrac{1}{2x + \cfrac{1}{2x + \cfrac{1}{2x + \cdots}}}$$

$$f(x) - x = \cfrac{1}{2x + \left(\cfrac{1}{2x + \cfrac{1}{2x + \cfrac{1}{2x + \cdots}}}\right)}$$

$$= \frac{1}{2x + (f(x) - x)}$$

Indefinite Integration

$\boxed{16 - 25}$

$$e^x + e^y y' = e^{x+y}(1 + y')$$

$$e^x + e^y y' = (e^x + e^y)(1 + y')$$

$$e^x y' = -e^y$$

$$y' = -e^{y-x}.$$