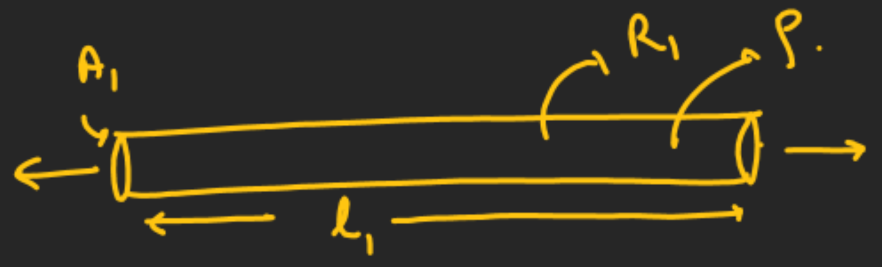
Nishant Jindal

CURRENT ELECTRICITY

$$\boxed{R = \frac{\rho l}{A}}$$



Volume of wire always constant.

$$A_1 l_1 = A_2 l_2.$$

$$\hookrightarrow \frac{A_2}{A_1} = \left(\frac{l_1}{l_2}\right).$$

$$R_1 = \frac{\rho l_1}{A_1}$$

$$R_2 = \frac{\rho l_2}{A_2}$$

$$\frac{R_1}{R_2} = \frac{l_1}{l_2} \times \frac{A_2}{A_1} \rightarrow \boxed{\frac{R_1}{R_2} = \left(\frac{A_2}{A_1}\right)^2}$$

$$\boxed{\frac{R_1}{R_2} = \left(\frac{l_1}{l_2}\right)^2}$$

$$\left(R \propto l^2\right)$$

$$\left(R \propto \frac{1}{A^2}\right) \checkmark$$

percentage Change in resistance

$$= \left(\frac{\Delta R}{R} \times 100\right)$$

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(*)

Resistance of Some Standard Conductors :-

Spherical Conductor :-

[ρ = Constant]

$V = IR$

$I = \left(\frac{\Delta V}{R}\right)$

$I = \left(\frac{\Delta V}{R}\right)$

$\Rightarrow \Delta V = IR$

$\Rightarrow V_1 - V_{(r)} = I\frac{\rho}{4\pi}\left[\frac{1}{r_1} - \frac{1}{r}\right]$

$\Rightarrow \boxed{V_{(r)} = V_1 - \frac{\rho I}{4\pi}\left[\frac{1}{r_1} - \frac{1}{r}\right]}$

V → [Potential difference b/w inner and outer Sphere.]

$$\int_0^R dR = \int_{r_1}^{r_2} \frac{\rho\, dr}{4\pi r^2}$$

dR ⇓ Resistance of 'dr' thickness of the Spherical Shell

A → Area of Spherical Shell having radius r.

$$\Rightarrow R = \frac{\rho}{4\pi}\left[-\frac{1}{r}\right]_{r_1}^{r_2}$$

$$\boxed{R = \frac{\rho}{4\pi}\left[\frac{1}{r_1} - \frac{1}{r_2}\right]}$$

$$\boxed{R = \frac{\rho}{4\pi}\left[\frac{1}{r_1} - \frac{1}{r}\right]}$$

$R \rightarrow f(r)$

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CURRENT ELECTRICITY

$$\vec{J}(r) = \left(\frac{I}{4\pi r^2}\right)(\hat{r})$$

$$\vec{J} = \sigma \vec{E} \qquad \left(\sigma = \frac{1}{\rho}\right)$$

$$\vec{J} = \frac{1}{\rho}\vec{E}$$

$$\vec{E} = \rho \vec{J}$$

$$\vec{E} = \left(\frac{\rho I}{4\pi r^2}\right)\hat{r}$$

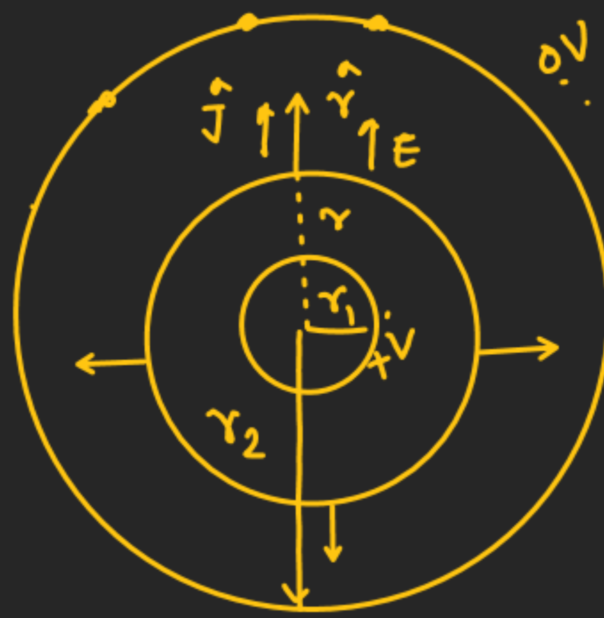
$$\vec{E}_r \parallel d\vec{r}$$

$$\int_V^{V(r)} dV = -\int_{r_1}^{r} \vec{E}_r \cdot d\vec{r}$$

$$V(r) - V = -\int_{r_1}^{r} \frac{\rho I}{4\pi r^2}\, dr$$

$$V(r) - V = -\frac{\rho I}{4\pi}\left[\frac{-1}{r}\right]_{r_1}^{r} \Rightarrow V(r) - V = \frac{\rho I}{4\pi}\left[\frac{1}{r} - \frac{1}{r_1}\right]$$

$$\Rightarrow V(r) - V = -\frac{\rho I}{4\pi}\left[\frac{1}{r_1} - \frac{1}{r}\right]$$



$$\underline{\Delta V = V}$$

$$I = \left(\frac{V}{R_{total}}\right)$$

$$\boxed{I = \frac{V}{\frac{\rho}{4\pi}\left[\frac{1}{R_1} - \frac{1}{R_2}\right]}}$$

$$\boxed{V(r) = V - \frac{\rho I}{4\pi}\left[\frac{1}{r_1} - \frac{1}{r}\right]}$$

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CURRENT ELECTRICITY

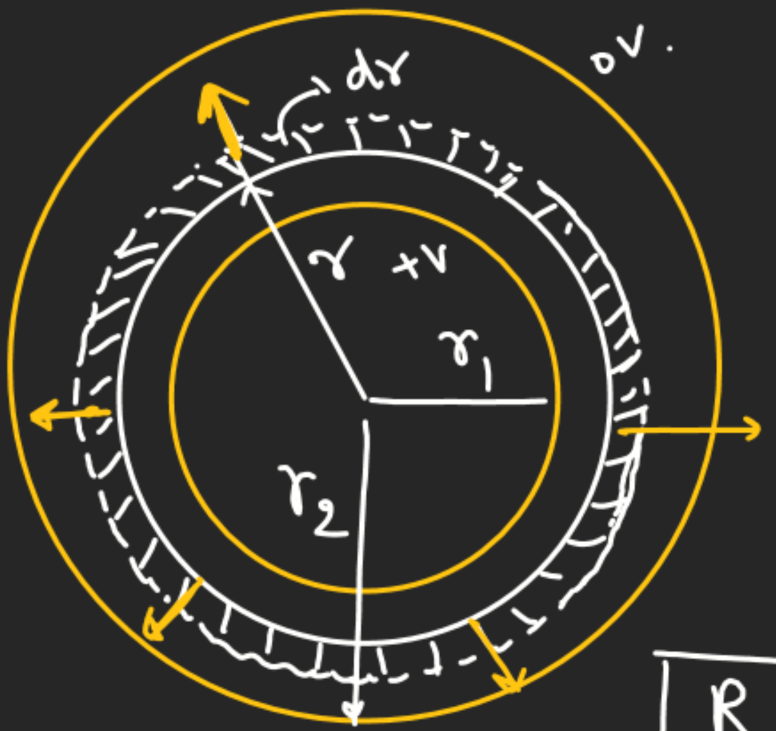
Case-2.

Variable resistivity

$\rho = \left(\frac{\rho_0 r^2}{a^2}\right) \quad [r_1 < r < r_2]$

[ρ_0 and a are constants]

[$\rho_r = \rho_{(r+dr)}$ as dr is very small.
i.e for dr thickness ρ assumed to be Constant.



$$\underline{dR} = \frac{\rho_r \, dr}{4\pi r^2}$$

$$dR = \frac{\rho_0 \cancel{r^2}}{a^2} \frac{dr}{(4\pi \cancel{r^2})}$$

$$\int_0^R dR = \frac{\rho_0}{4\pi a^2} \int_{r_1}^{r_2} dr$$

$$\boxed{R = \frac{\rho_0}{4\pi a^2}(r_2 - r_1)}$$

Total resistance

$$I = \frac{V}{\frac{\rho_0 (r_2 - r_1)}{4\pi a^2}} = \frac{V \, 4\pi a^2}{\rho_0 (r_2 - r_1)} \checkmark$$

$$\boxed{\vec{J}_{(r)} = \left(\frac{I}{4\pi r^2}\right)\hat{r}}$$

Electric field.

$$\vec{J} = \sigma \vec{E}$$

$$\vec{E} = \frac{1}{\sigma}\vec{J}$$

$$\vec{E} = \rho \vec{J} = \rho$$

$$\left(\vec{E}_r = (\rho_r J)\,\hat{r}\right.$$

$$E_r = \left(\frac{\rho_0 r^2}{a^2}\right) \times \frac{I}{4\pi r^2}$$

$$\boxed{E_r = \left(\frac{\rho_0 I}{4\pi a}\right)}$$

if $\left(\rho = \frac{\rho_0 r}{a}\right)$ ✓ (ρ_0 and a constant)

$$dR = \rho_r \frac{dr}{4\pi r^2}$$

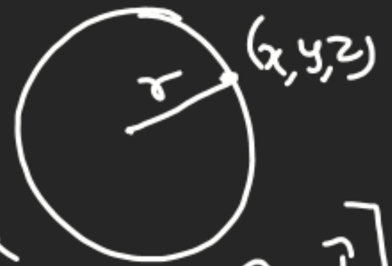
$$dR = \left(\frac{\rho_0 r}{a}\right)\frac{dr}{4\pi r^2}$$

$$\int_0^R dR = \frac{\rho_0}{4\pi a}\int_{r_1}^{r_2}\frac{dr}{r}$$

$$R = \frac{\rho_0}{4\pi}\ln\left(\frac{r_2}{r_1}\right)$$

$$\left(I = \frac{V}{R_T}\right)$$

$$\boxed{J = \left(\frac{I}{4\pi r^2}\right)}$$



$$\left[\hat{r} = \frac{x\hat{i} + y\hat{j} + z\hat{k}}{\sqrt{x^2+y^2+z^2}}\right]$$

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CURRENT ELECTRICITY

$$J = \frac{I}{4\pi r^2}$$

$$E = \rho J$$

$$E_r = \left(\frac{\rho_0 r}{a}\right)\left(\frac{I}{4\pi r^2}\right)$$

$$\boxed{E_r = \frac{\rho_0 I}{4\pi a} \times \left(\frac{1}{r}\right)}$$

$$J = \sigma E$$

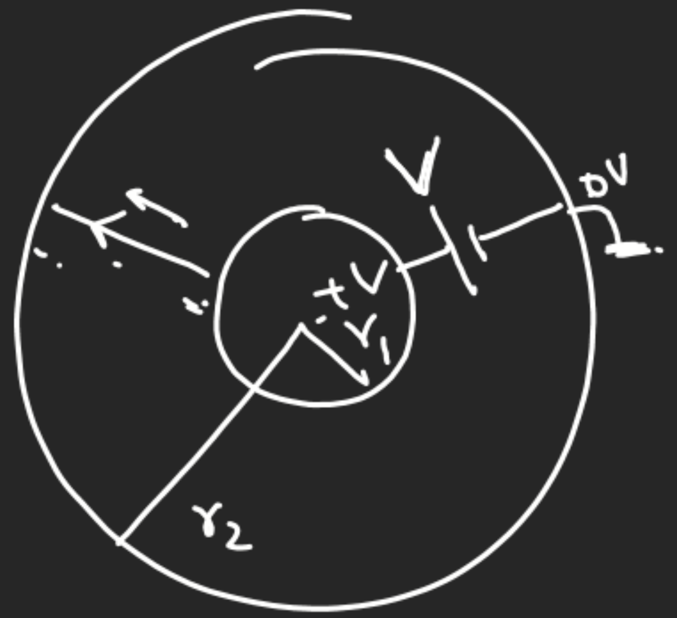
$$E = \frac{1}{\sigma} J$$

$$E = \rho J$$

$$\int_{V}^{V(r)} dV = -\int_{r_1}^{r} E_r \, dr$$

$$V(r) - V = -\frac{\rho_0 I}{4\pi a} \int_{r_1}^{r} \frac{dr}{r}$$

$$V(r) = V - \frac{\rho_0 I}{4\pi a} \ln\left(\frac{r}{r_1}\right)$$



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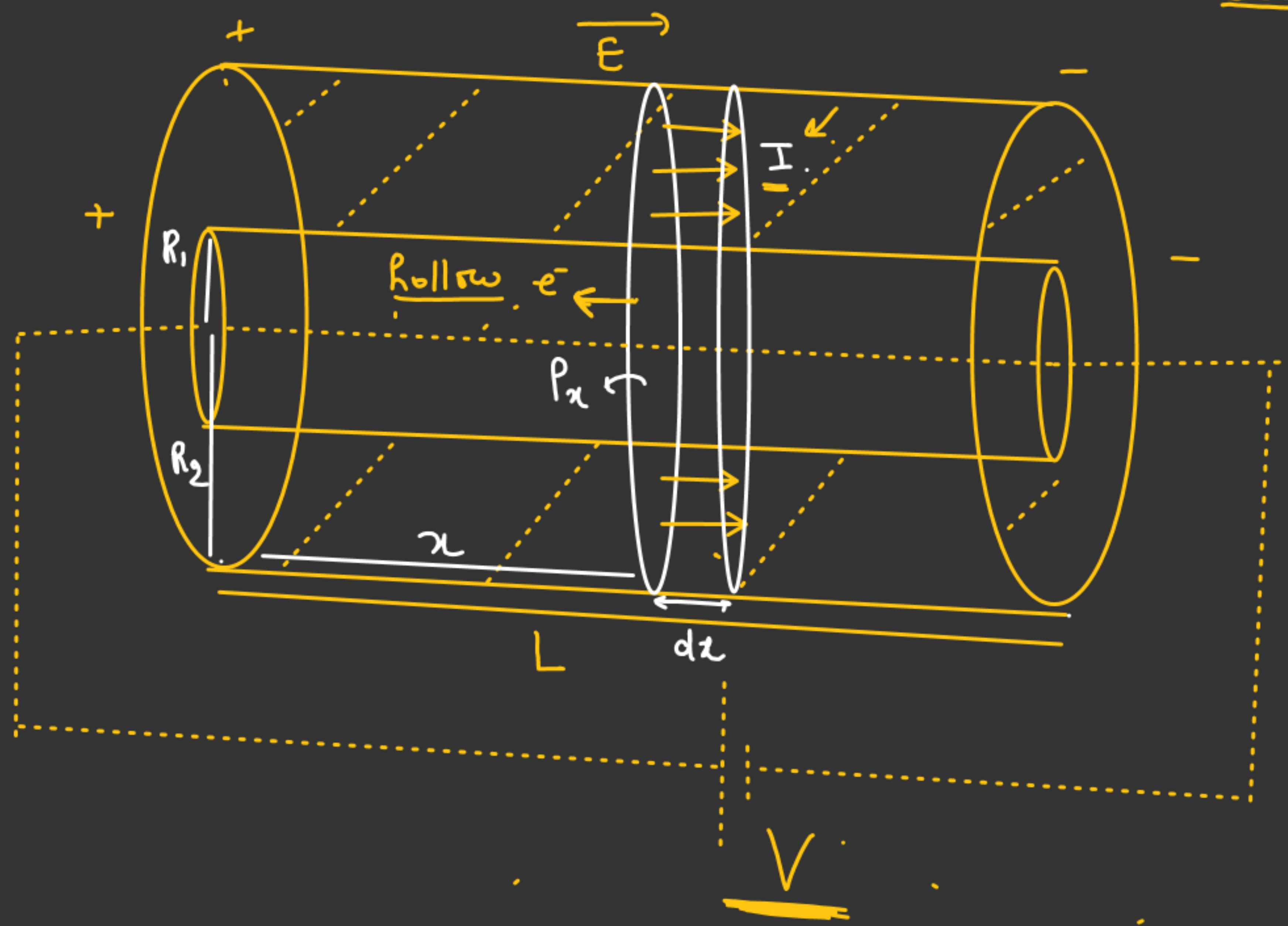
Case of Cylindrical Conductor:-

Case-1 (ρ = constant) $\rightarrow$ $\left[R = \dfrac{\rho L}{\pi(R_2^2 - R_1^2)}\right]$



Case-2 if $\boxed{\rho = \left(\dfrac{\rho_0 x}{L}\right)}$

$(\rho_x = \rho_{x+dx})$ as dx is very small.

$$dR = \frac{\rho_x \, dx}{\pi(R_2^2 - R_1^2)}$$

$\Downarrow$ Resistance of dx length of the cylinder.

$$\int_0^R dR = \frac{\rho_0}{\pi L(R_2^2 - R_1^2)} \int_0^L x\,dx$$

$$R = \frac{\rho_0}{\pi L(R_2^2 - R_1^2)} \times \frac{L^2}{2}$$

$$R = \frac{\rho_0 L}{2\pi(R_2^2 - R_1^2)}$$

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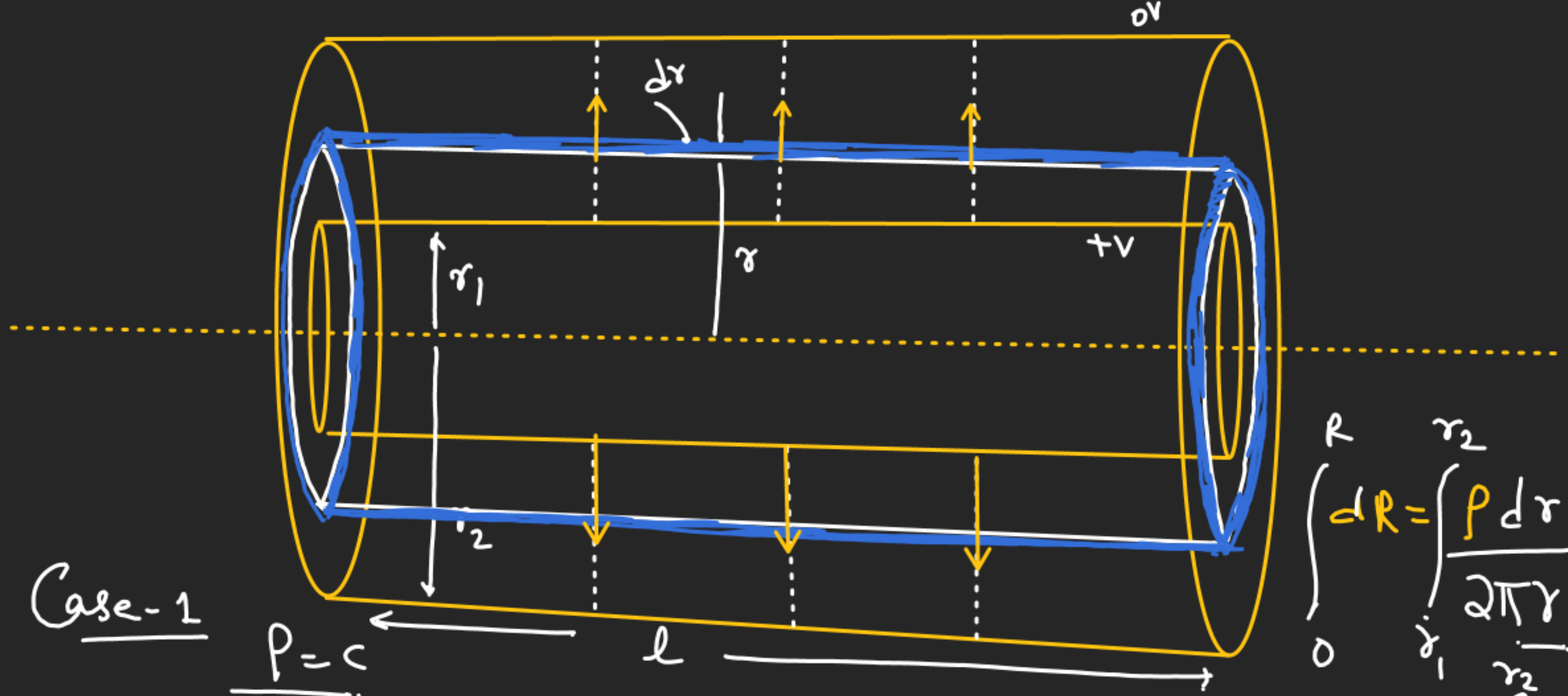
CURRENT ELECTRICITY

Case-1 (i) ρ = Constant

(ii) $\rho = \frac{\rho_0 r}{a}$ → HW



Case-1

$\rho = c$

$$\int_0^R dR = \int_{r_1}^{r_2} \frac{\rho\, dr}{2\pi r l}$$

$$R = \frac{\rho_0}{2\pi l}\int_{r_1}^{r_2}\frac{dr}{r}$$

$$\Rightarrow \boxed{R = \frac{\rho_0}{2\pi l}\ln\left(\frac{r_2}{r_1}\right)}$$