

$$\underline{3.} \quad f'\left(\frac{\pi}{2}\right)^+ = \lim_{x \rightarrow \frac{\pi}{2}^+} \frac{\cancel{2} + \left(x - \frac{\pi}{2}\right)^2 - \cancel{2}}{\left(x - \frac{\pi}{2}\right)} = 0$$

$$f'\left(\frac{\pi}{2}\right)^- = \lim_{x \rightarrow \frac{\pi}{2}^-} \frac{1 + |\sin x| - 2}{x - \frac{\pi}{2}} = \lim_{x \rightarrow \frac{\pi}{2}^-} \frac{\sin x - \sin \frac{\pi}{2}}{x - \frac{\pi}{2}}$$

$$= \cos \frac{\pi}{2} = 0$$

$$\lim_{h \rightarrow 0} \frac{f(3+h^2) - f(3-h^2)}{(3+h^2) - (3-h^2)} = \lim_{h \rightarrow 0} \left[ \frac{f(3+h^2) - f(3)}{h^2} + \frac{f(3-h^2) - f(3)}{-h^2} \right]$$

$h^2 > 0$   
 $h^2 < 0$

$f(x) = \begin{cases} x^2 & x \in (-\infty, -\sqrt{2}] \cup [\sqrt{2}, \infty) \\ 2 & x \in (-\sqrt{2}, \sqrt{2}) \end{cases}$

$x \in (-\infty, -\sqrt{2}] \cup [\sqrt{2}, \infty)$   
 $x \in (-\sqrt{2}, \sqrt{2})$

$f'(3)$

$$\max(2, 3, 5) = 5$$

$$-f'(-x) = f'(x)$$

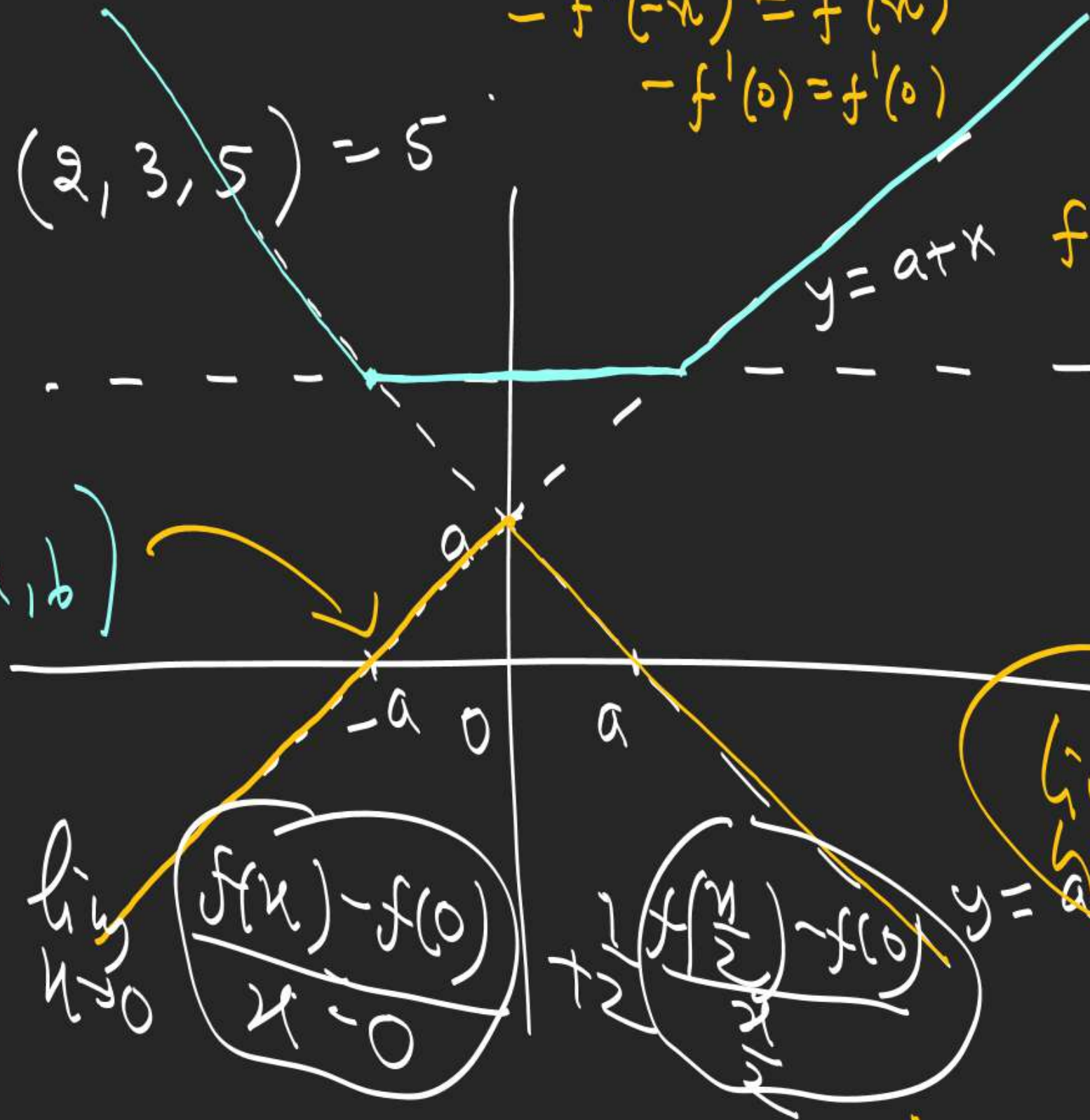
$$-f'(0) = f'(0)$$

$$f'(0) = \lim_{h \rightarrow 0} \frac{f(h) - f(0)}{h}$$

$$f'(0^-) = \lim_{h \rightarrow 0} \frac{f(-h) - f(0)}{-h}$$

$$= - \lim_{h \rightarrow 0} \frac{f(h) - f(0)}{h}$$

$$f(x) = \min(a-x, a+x, b)$$



$$\lim_{h \rightarrow 0}$$

$$\frac{f(h) - f(0)}{h - 0}$$

$$\lim_{h \rightarrow 0}$$

$$\frac{f(h) - f(0)}{h - 0}$$

$$y = a - x$$

$$\lim_{h \rightarrow 0} \frac{f(h) - f(0)}{h} = 0$$

1.  
 $h > 0 \checkmark$

$$\frac{f(x+h) - f(x)}{h} \leq 6h$$

$$\lim_{h \rightarrow 0^+} \frac{f(x+h) - f(x)}{h} \leq \lim_{h \rightarrow 0^+} 6h$$

$$\Rightarrow \lim_{h \rightarrow 0^+} 6h = 0$$

$$\Rightarrow f'(x^+) \leq 0$$

$$f'(x) \leq 0, f'(x) \geq 0$$

$$f(x) > g(x)$$

$$f'(x) = 0$$

$$y = f(x)$$

$$y = g(x)$$

$$\lim_{x \rightarrow a} f(x) > \lim_{x \rightarrow a} g(x)$$

$$x = a$$

$h < 0$

$$\frac{f(x+h) - f(x)}{h} \geq 6h$$

$$\lim_{h \rightarrow 0^-} \frac{f(x+h) - f(x)}{h} \geq \lim_{h \rightarrow 0^-} (6h)$$

$$f'(x^-) \geq 0$$

$$\lim_{x \rightarrow 0} \frac{x^2 (|x|^2 + 2|x|) - 0}{x - 0} = \lim_{x \rightarrow 0} \left( x + \frac{2|x|}{x} \right) \begin{cases} \rightarrow LHD = -2 \\ \rightarrow RHD = 2 \end{cases}$$

$$y = \begin{cases} \sin^{-1} x & x \in [0, 1] \\ \frac{1}{3} \sin^{-1} x & x \in [-1, 0) \end{cases}$$

$$f(x) = \begin{cases} 1 + 0 \\ \end{cases}$$

$$\left(0, \frac{\pi}{2}\right] f'(0) = \lim_{x \rightarrow 0} \frac{(\sin^{-1} x) \cos \frac{1}{x} - 0}{x}$$

$$\lim_{x \rightarrow 0} x \left( \frac{\sin^{-1} x}{x} \right)^2 \cos \frac{1}{x} = 0$$

1. Let  $F(x) = f(x)g(x)h(x)$ . If for some  $x = x_0$ ,  
 $F'(x_0) = 21 F(x_0)$ ,  $f'(x_0) = 4 f(x_0)$ ,  $g'(x_0) = -7 g(x_0)$   
 and  $h'(x_0) = K h(x_0)$ , find  $K$ .

$$\frac{F'}{F} = \sum \frac{f'}{f} \Rightarrow 21 = 4 - 7 + K \Rightarrow \boxed{K = 24}$$

2. If  $f(x) = (1+x)(3+x^2)^{1/2} (9+x^3)^{1/3}$ , find  $f'(-1)$

$$f'(-1) = (3+(-1)^2)^{1/2} (9+(-1)^3)^{1/3} = 4.$$

3. Let  $f(0)=1, g(0)=2, h(0)=3$  and  $(fg)'(0)=6$

$(gh)'(0)=4$  and  $(hf)'(0)=5$ , find  $(fgh)'(0) = \boxed{16}$

$$(fg' + f'g)(0) = 6 \Rightarrow \underline{h(0)}(\underline{fg'} + f'g)(0) = 6 \times 3$$

$$(gh' + g'h)(0) = 4 \Rightarrow \underline{f(0)}(\underline{gh'} + g'h)(0) = 4 \times 1$$

$$(hf' + h'f)(0) = 5 \Rightarrow g(0)(hf' + h'f)(0) = 5 \times 2$$

$$(fgh)^2 = \frac{(fg)(gh)(hf)}{2 \frac{(fg)(gh)(hf)}{(fgh)^2}}(0) = \left( \frac{6}{1 \times 2} + \frac{4}{2 \times 3} + \frac{5}{1 \times 3} \right) (fgh)(0) = 3^2$$

$$(fgh)'(0) = \frac{1 \times 2 \times 3}{2} \left( \frac{6}{2} + \frac{4}{6} + \frac{5}{3} \right)$$

4. 2]  $y = \frac{\sec x + \tan x - 1}{\tan x - \sec x + 1}$ , find  $\frac{dy}{dx}$  at  $x = \frac{\pi}{4}$ .

$$y = \frac{(\sec x + \tan x) - (\sec^2 x - \tan^2 x)}{\tan x - \sec x + 1} = \sec x + \tan x$$

$$\frac{dy}{dx} = \sec x \tan x + \sec^2 x$$

5. 1]  $y = \frac{x^3 + x^2 + x}{1 + x^2}$ , find  $\frac{dy}{dx}$ .  $\left. \frac{dy}{dx} \right|_{x = \frac{\pi}{4}} = \sqrt{2} + 2$ .

$$y = \frac{(x^2 + 1)(x + 1) - 1}{1 + x^2} = x + 1 - \frac{1}{x^2 + 1}$$

$$\frac{dy}{dx} = 1 - \frac{(x^2 + 1)(0) - 1(2x)}{(x^2 + 1)^2} = 1 + \frac{2x}{(x^2 + 1)^2}$$

$$\frac{d}{dx} \left( e^{\underbrace{x^2 \cos x}} \right) = \frac{d}{dt} (e^t) \frac{dt}{dx} = e^{x^2 \cos x} \frac{d}{dx} (x^2 \cos x)$$

$$= e^{x^2 \cos x} (2x \cos x - x^2 \sin x)$$

$$\begin{aligned} y &= f(t) \\ t &= g(x) \\ \frac{dy}{dx} &= \frac{dy}{dt} \frac{dt}{dx} \end{aligned}$$

$$\frac{d}{dx} \left( \ln^3 (\tan^2(x^4)) \right) = \frac{d}{dt} t^3 \underbrace{\frac{d}{dx} (\ln(\tan^2(x^4)))}_{\frac{d}{dx} \tan^2(x^4)}$$

$$t = \ln(\tan^2(x^4))$$

$$= 3 \ln^2(\tan^2(x^4)) \frac{d}{dt} \ln t \frac{d}{dx} \tan^2(x^4)$$

$$\frac{d}{dx} \left( \sec^2(f^3(x)) \right) = 3 \ln^2(\tan^2(x^4)) \frac{1}{\tan^2(x^4)} \frac{d}{dt} (t^2) \underbrace{\frac{d}{dx} \tan(x^4)}$$

$$\frac{3 \ln^2(\tan^2(x^4)) \tan(x^4) \sec^2(x^4) (4x^3)}{\tan^2(x^4)} = 3 \ln^2(\tan^2(x^4)) \frac{1}{\tan^2(x^4)} 2 \tan(x^4) \frac{d}{dx} (\tan x)$$

$$D \left( e^{\frac{\sin(\ln(x^2+7)^5)}{5\ln(x^2+7)}} \right)$$

$$= e^{\frac{\sin(\ln(x^2+7)^5)}{5\ln(x^2+7)}} \times \frac{1}{2\sqrt{\sin(\ln(x^2+7)^5)}} \times \cos(\ln(x^2+7)^5) \times \frac{5x^{2x}}{(x^2+7)}$$

# Logarithmic Differentiation

$$\frac{y'}{y} = \frac{f_1'}{f_1} + \frac{f_2'}{f_2} + \dots$$

$$y = (f_1 f_2 f_3 \dots f_n)(x)$$

$$\ln y = \ln f_1 + \ln f_2 + \dots$$

$$y = f(x)^{g(x)} = e^{g(x) \ln f(x)}$$

$$\ln y = g(x) \ln f(x)$$

$$\frac{1}{y} \frac{dy}{dx} = g'(x) \ln f(x) + \frac{g(x)}{f(x)} f'(x)$$

$$\therefore \quad \text{I} \quad f(x) = \prod_{n=1}^{100} (x-n)^{n(101-n)}$$

$$\text{, find } \frac{f(101)}{f'(101)} = \frac{1}{5050}.$$

Ex-2 (Differentiability)

$$\frac{f'(x)}{f(x)} = \sum_{r=1}^{100} \frac{r(101-r)}{x-r} \Rightarrow \frac{f'(101)}{f(101)} = \sum_{r=1}^{100} \frac{r(101-r)}{(101-r)}$$

$$\sum_{r=1}^{100} \frac{r}{1} = \frac{r(101-r)(x-r)}{(x-r)^r(101-r)} = 5050$$

$$\ln f(x) =$$

$$\sum_{n=1}^{100} n(101-n) \ln(x-n)$$

$$\frac{f'(x)}{f(x)} = \sum_{n=1}^{100} \frac{n(101-n)}{x-n}$$

2. 2)  $y = x^{\tan x} + (\sin x)^{\cos x}$ , find  $\frac{dy}{dx}$ .

$$y = e^{\tan x \ln x} + e^{\cos x \ln \sin x}$$

$$y' = x^{\tan x} \left( \sec^2 x \ln x + \frac{\tan x}{x} \right) + (\sin x)^{\cos x} \left( -\sin x \ln(\sin x) \right)$$

$$y_1 = x^{\tan x}$$

$$y_2 = (\sin x)^{\cos x}$$

$$\ln y_1 = \tan x \ln x$$

$$\frac{1}{y_1} y_1' = \sec^2 x \ln x + \frac{\tan x}{x} \Rightarrow \frac{d(y_1)}{dx} = x^{\tan x} \left( \sec^2 x \ln x + \frac{\tan x}{x} \right)$$

$$\frac{dy}{dx} = \frac{d}{dx} y_1 + \frac{d}{dx} y_2$$