

$$\frac{1}{5} + i\frac{2}{5} - 4 - i\frac{5}{2}$$

$$\left(\frac{1}{5} - 4\right) + i\left(\frac{2}{5} - \frac{5}{2}\right) \quad (20) \quad \left(\frac{1+i}{1-i}\right)^m = (i)^m = 1$$

$m=4$

$$-\frac{19}{5} + i\left(-\frac{21}{10}\right)$$

$$9(a+b)^3 = \dots$$

$$(1) \sim$$

$$(2) \left(1^2 + (-i)^{25}\right)$$

$$\frac{(-1 - i)}{(-1 - i)^3}$$

$$16 \quad x=0 \quad z = \underbrace{(iy)}_{y \neq 0} \rightarrow \text{Purely Imag. N.}$$

Qs
44 HW.

$$Q \text{ If } |z_1 + z_2|^2 = |z_1|^2 + |z_2|^2 \text{ then } \frac{z_1}{z_2} \text{ is}$$

Purely Imag. = ?

$$|z_1 + z_2|^2 = |z_1|^2 + |z_2|^2$$

$$(z_1 + z_2) \cdot (\overline{z_1 + z_2}) = |z_1|^2 + |z_2|^2$$

$$(z_1 + z_2)(\overline{z_1} + \overline{z_2}) =$$

$$z_1 \overline{z_1} + z_1 \overline{z_2} + z_2 \overline{z_1} + z_2 \overline{z_2} = |z_1|^2 + |z_2|^2$$

$$|z_1|^2 + z_1 \overline{z_2} + z_2 \overline{z_1} + |z_2|^2 = |z_1|^2 + |z_2|^2$$

$$z_1 \overline{z_2} = -z_2 \overline{z_1}$$

$$\frac{z_1}{z_2} = -\left(\frac{\overline{z_1}}{\overline{z_2}}\right) \rightarrow \frac{z_1}{z_2} \text{ is Purely Imag.}$$

Q If z is a (N.Such that

$\frac{z-1}{z+1}$ is purely Imag. then $|z| = ?$

$$\downarrow$$

$$z = -\bar{z}$$

$$\frac{z-1}{z+1} = -\left(\frac{\bar{z}-1}{\bar{z}+1}\right)$$

$$\frac{z-1}{z+1} = -\left(\frac{\bar{z}-1}{\bar{z}+1}\right)$$

$$z \cdot \bar{z} - \bar{z} + z - 1 = -(z\bar{z} - z + \bar{z} - 1)$$

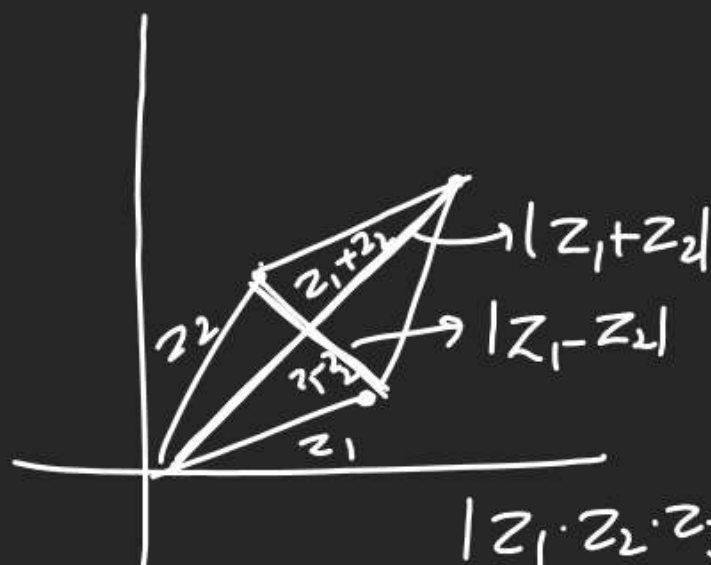
$$|z|^2 - \bar{z} + z - 1 = -|z|^2 + z - \bar{z} + 1$$

$$|z|^2 = 1$$

$$|z| = 1$$

Q What is $|z_1 + z_2|$

& $|z_1 - z_2|$?



Q $z = (1+i)(1+2i)(1+3i)$

then $|z| = ?$

$$|z| = |(1+i)(1+2i)(1+3i)|$$

$$= |1+i| |1+2i| |1+3i|$$

$$= \sqrt{2} \sqrt{5} \sqrt{10} = 10$$

Q $z = \frac{(1+i)(1+2i)}{(1+3i)}$ then $|z| = ?$

$$|z| = \left| \frac{(1+i)(1+2i)}{(1+3i)} \right|$$

$$= \frac{|1+i| |1+2i|}{|1+3i|}$$

$$= \frac{\sqrt{2} \times \sqrt{5}}{\sqrt{10}} = 1$$

Q If $z = \frac{(1-i)^2}{(1+2i)}$ then $|z| = ?$

$$|z| = \frac{(1-i)^2}{(1+2i)} = \frac{(-2i)}{1+2i}$$

$$= \frac{-2i}{1+2i} = \frac{2}{\sqrt{5}}$$

Q $z = 1 + 6\cos 2\theta + i 8\sin 2\theta$
then $|z| = ?$; $\theta \in (\pi, \frac{3\pi}{2})$

$$\begin{aligned} |z| &= \sqrt{(1+6\cos 2\theta)^2 + 8^2 \sin^2 2\theta} \\ &= \sqrt{1 + 2 \cdot 6 \cos 2\theta + 6^2 \cos^2 2\theta + 8^2 \sin^2 2\theta} \\ &= \sqrt{2 + 2 \cdot 6 \cos 2\theta} = \sqrt{2} \sqrt{1 + 6 \cos 2\theta} \end{aligned}$$

$$\begin{aligned} &= \sqrt{2} \sqrt{2 \cos^2 \theta} \quad \theta \in (180^\circ, 270^\circ) \\ &= 2 \cos \theta \\ &= -2 \cos \theta \end{aligned}$$

Q If $\left(\frac{3}{2} + i\frac{\sqrt{3}}{2}\right)^{50} = 3^{24}(x+iy)$

then $x^2 + y^2 = ?$
then $|z|$ Demand

$$\left|\left(\frac{3}{2} + i\frac{\sqrt{3}}{2}\right)^{50}\right| = |3^{24}(x+iy)|$$

$$\left|\frac{3}{2} + i\frac{\sqrt{3}}{2}\right|^{50} = 3^{24} |x+iy|$$

$$\left(\sqrt{\frac{9}{4} + \frac{3}{4}}\right)^{50} = 3^{24} \sqrt{x^2 + y^2}$$

$$3 \cdot 3^{25} = 3^{24} \sqrt{x^2 + y^2}$$

$$9 = x^2 + y^2$$

Q Prove that

$$|z_1 + z_2|^2 = |z_1|^2 + |z_2|^2 + 2\operatorname{Re}(z_1 \bar{z}_2)$$

LHS $|z_1 + z_2|^2$ $\left[z + \bar{z} = 2\operatorname{Re}(z) \right]$

$$= (z_1 + z_2)(\bar{z}_1 + \bar{z}_2)$$

$$= (z_1 + z_2)(\bar{z}_1 + \bar{z}_2)$$

$$= z_1 \bar{z}_1 + z_1 \bar{z}_2 + z_2 \bar{z}_1 + z_2 \bar{z}_2$$

$$= |z_1|^2 + |z_2|^2 + (z_1 \bar{z}_2 + z_2 \bar{z}_1)$$

$$= |z_1|^2 + |z_2|^2 + (z_1 \bar{z}_2 + \overline{z_1 \bar{z}_2})$$

$$= |z_1|^2 + |z_2|^2 + 2\operatorname{Re}(z_1 \bar{z}_2)$$

$$= \text{RHS}$$

Q Find $|z_1 - z_2|^2$?

$$|z_1 - z_2|^2 = (z_1 - z_2)(\overline{z_1 - z_2})$$

$$= (z_1 - z_2)(\overline{z_1} - \overline{z_2})$$

$$= z_1 \overline{z_1} - z_1 \overline{z_2} - z_2 \overline{z_1} + z_2 \overline{z_2}$$

$$= |z_1|^2 + |z_2|^2 - (z_1 \overline{z_2} + z_2 \overline{z_1})$$

$$= |z_1|^2 + |z_2|^2 - (z_1 \overline{z_2} + \overline{z_1} z_2)$$

$$= |z_1|^2 + |z_2|^2 - 2R(z_1 \overline{z_2})$$

Q $|z_1 + z_2|^2 + |z_1 - z_2|^2 = ?$

$$|z_1 + z_2|^2 = |z_1|^2 + |z_2|^2 + 2R(z_1 \overline{z_2})$$

$$|z_1 - z_2|^2 = |z_1|^2 + |z_2|^2 - 2R(z_1 \overline{z_2})$$

$$|z_1 + z_2|^2 + |z_1 - z_2|^2 = 2(|z_1|^2 + |z_2|^2)$$

Q Find $|az_1 - bz_2|^2 + |bz_1 + az_2|^2 = ?$

HW

$$= (a^2 + b^2)(|z_1|^2 + |z_2|^2)$$

Q If $|z - 2 + 3i| = |z - 1 + 2i|$ then locus of z ?

$$z = x + iy \text{ Put}$$

$$|x + iy - 2 + 3i| = |x + iy - 1 + 2i|$$

$$|(x-2) + i(y+3)| = |(x-1) + i(y+2)|$$

$$\sqrt{(x-2)^2 + (y+3)^2} = \sqrt{(x-1)^2 + (y+2)^2}$$

$$(x-2)^2 + (y+3)^2 = (x-1)^2 + (y+2)^2$$

$$x^2 + y^2 - 4x + 6y + 13$$

$$= x^2 + y^2 - 2x + 4y + 5$$

$$2x - 2y = 8$$

$$x - y = 4 \rightarrow \text{St. line}$$

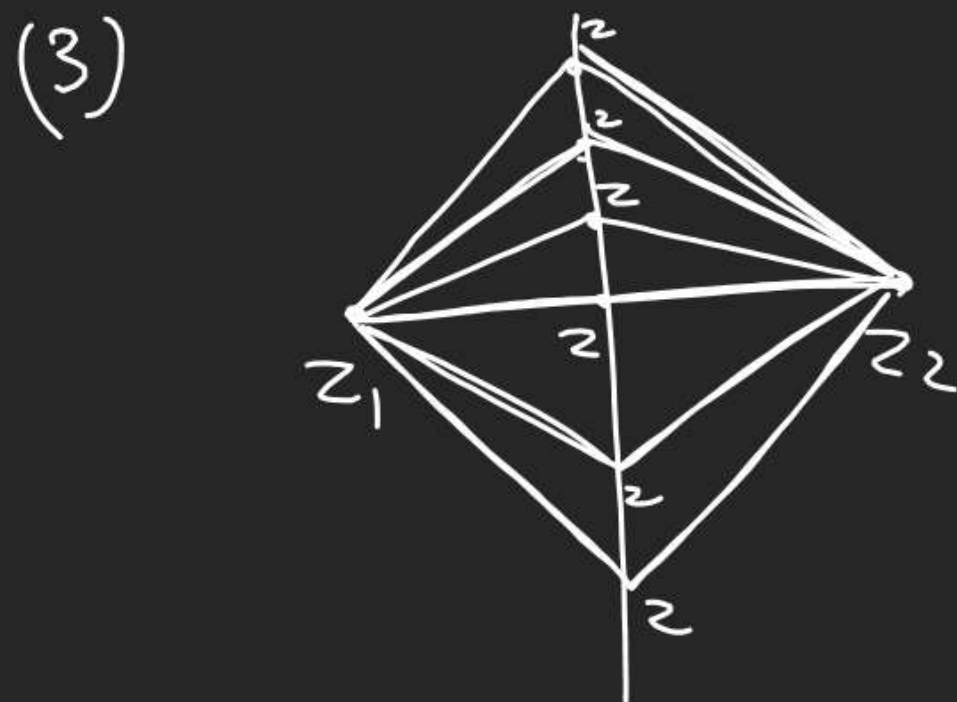
Locus of given Ex is
a St. Line

Imp Note:-

$$Q 3 |z - (2-3i)| = |z - (1-2i)|$$



(2) $|z - z_1| = |z - z_2|$ given then.
 It Rep. Locus of \perp ^{Bisector} to
 Line joining z_1 & z_2



(4) $|z - z_1|$ Rep. Dist. of z to z_1
 Q $|z - 2 + 3i| = 5$ then locus of z .

(1) Meaning of $|z - 2 + 3i|$
 ,, $|z - (2 - 3i)|$
 = Dist. of z to Pt $(2, -3)$

2) Meaning of this Qs.

$$|z - (2 - 3i)| = 5$$

3) dist of $(2, -3)$ to $z = 5$
 ↑ ↑
 fix pt var. pt

$$(4) |x + i4 - 2 + 3i| = 5$$

$$|(x-2) + i(4+3)| = 5$$

$$\sqrt{(x-2)^2 + (4+3)^2} = 5$$

$$(x-2)^2 + (4+3)^2 = 5^2$$

Circle $(2, -3)$ Rad = 5

Q $|z - 2 + 3i| = -5$ find
 locus of z ?

dist of $(2, -3)$ from $z = -5i$

which is purely meaningless.

No Locus.

Q find z if

$$|z+1| = |z+2(1+i)|$$

$$|x+iy+1| = |x+iy+2+2i|$$

$$|(x+1)+iy| = |(x+2)+i(y+2)|$$

$$\sqrt{\underbrace{(x+1)^2}_{\text{Real}} + \underbrace{y^2}_{\text{Real}}} = \sqrt{\underbrace{(x+2)^2}_{\text{Real}} + \underbrace{(y+2)^2}_{\text{Imag.}}}$$

$$\Rightarrow \sqrt{(x+1)^2 + y^2} = (x+2) \quad \left| \begin{array}{l} y+2=0 \\ y=-2 \end{array} \right.$$

$$\sqrt{(x+1)^2 + (-2)^2} = (x+2)$$

$$x^2 + 2x + 1 + 4 = x^2 + 4x + 4$$

$$2x = 1$$

$$x = \frac{1}{2}$$

$$\therefore z = x+iy = \frac{1}{2} - 2i$$

Q $\left| \left(\frac{\sqrt{3}+1}{2\sqrt{2}} \right) - i \left(\frac{\sqrt{3}-1}{2\sqrt{2}} \right) \right| = ?$

$$\downarrow$$

$$|\cos 15^\circ - i \sin 15^\circ|$$

$$\sqrt{\cos^2 15^\circ + \sin^2 15^\circ}$$

$$= 1$$

Q If $|z_1|=1, |z_2|=2, |z_3|=3$

$$|z_1 + z_2 + z_3| = 1$$

then $|z_2 z_3 + 4 z_1 z_3 + 9 z_1 z_2| = ?$

Demand: $|z_1 z_2 z_3 \left(\frac{1}{z_1} + \frac{4}{z_2} + \frac{9}{z_3} \right)|$

$$= |z_1| |z_2| |z_3| \left| \frac{1}{z_1} + \frac{4}{z_2} + \frac{9}{z_3} \right|$$

$$= 1 \times 2 \times 3 \times \left| \frac{\bar{z}_1}{|z_1|^2} + \frac{4\bar{z}_2}{|z_2|^2} + \frac{9\bar{z}_3}{|z_3|^2} \right|$$

$$\left(\frac{1}{z} = \frac{\bar{z}}{|z|^2} \right), |z| = |z|$$

$$6 \left| \bar{z}_1 + \frac{4\bar{z}_2}{2} + \frac{9\bar{z}_3}{3} \right| = 6 \left| \bar{z}_1 + \bar{z}_2 + \bar{z}_3 \right|$$

$$= 6 \left| \overline{z_1 + z_2 + z_3} \right| = 6 |z_1 + z_2 + z_3|$$

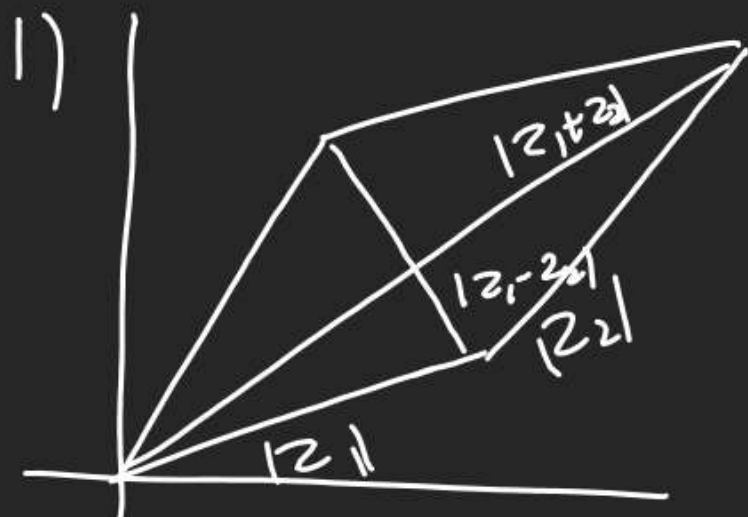
$$= 6 \times 1 = 6$$

Q for $z=a+ib$ Check $\frac{1}{z} = \frac{\bar{z}}{|z|^2}$

$$\text{LHS} \Rightarrow \frac{1}{z} = \frac{1}{a+ib} \times \frac{a-ib}{a-ib} = \frac{a-ib}{a^2+b^2}$$

$$= \frac{\bar{z}}{|z|^2} = \text{RHS}$$

Triangular Inequality



$$|z_1| + |z_2| \geq |z_1 + z_2| \geq |z_1 - z_2| \geq ||z_1| - |z_2||$$

$$2) \quad -\sqrt{x^2 + y^2} \leq x \leq \sqrt{x^2 + y^2}$$

$$3) \quad \operatorname{Re}(z) \leq |z|$$

$$(4) \quad |z_1 + z_2|^2 = |z_1|^2 + |z_2|^2 + 2 \operatorname{Re}(z_1 \bar{z}_2) \leq |z_1|^2 + |z_2|^2 + 2|z_1||z_2|$$

$$|z_1 + z_2|^2 \leq |z_1|^2 + |z_2|^2 + 2|z_1||z_2| \quad |z_1| = |z_1| = |z_1|$$

$$\leq |z_1|^2 + |z_2|^2 + 2|z_1||z_2|$$

$$|z_1 + z_2|^2 \leq (|z_1| + |z_2|)^2$$

$$|z_1 + z_2| \leq |z_1| + |z_2|$$

$$\leq |z_1| + |z_2|$$

$$|z_1 - z_2| \geq ||z_1| - |z_2||$$

$$||z_1| - |z_2|| \leq |z_1 - z_2| \leq |z_1 + z_2| \leq |z_1| + |z_2|$$

$$(5) \quad \begin{aligned} \text{Min. of } |z_1 + z_2| &= ||z_1| - |z_2|| \\ \text{Max. of } |z_1 + z_2| &= |z_1| + |z_2| \end{aligned}$$

Q If z is C.N. S.T. $|x| \leq a \Rightarrow -a \leq x \leq a$

$|z - 2 + i| \leq 2$ then gr. & Least value of $|z|$

$$|z - (2 - i)| \leq 2$$

$$||z| - |2 - i|| \leq |z - (2 - i)| \leq 2$$

$$||z| - \sqrt{5}| \leq 2$$

$$-2 \leq |z| - \sqrt{5} \leq 2$$

$$\frac{-2 + \sqrt{5}}{\text{Least}} \leq |z| \leq \frac{2 + \sqrt{5}}{\text{gr.}}$$

Q z is a C.N. S.T.

$|z + 4| \leq 3$ find gr. value of $|z + 1|$

$$|z + 4| \leq 3$$

$$||z + 1| - 3| \leq |(z + 1) + 3| \leq 3$$

$$|z + 1| - 3 \leq 3$$

$$-3 \leq |z + 1| - 3 \leq 3$$

$$0 \leq |z + 1| \leq 6$$

Max.

Q If $|z - \frac{6}{z}| = 4$ find

gr. value of $|z|$?

$$|z - \frac{6}{z}| = 4 \text{ (given)}$$

$$|z - \frac{6}{z}| = \left| \left(z - \frac{6}{z} \right) + \left(\frac{6}{z} \right) \right|$$

$$4 - \left| \frac{6}{z} \right| \leq |z|$$

$$-|z| \leq 4 - \frac{6}{|z|} \leq |z|$$

$$\textcircled{+} \quad 4 - \frac{6}{|z|} \leq |z|$$

$$|z| + \frac{6}{|z|} \geq 4$$

$$|z|^2 - 4|z| + 6 \geq 0$$

Solve for $|z|$