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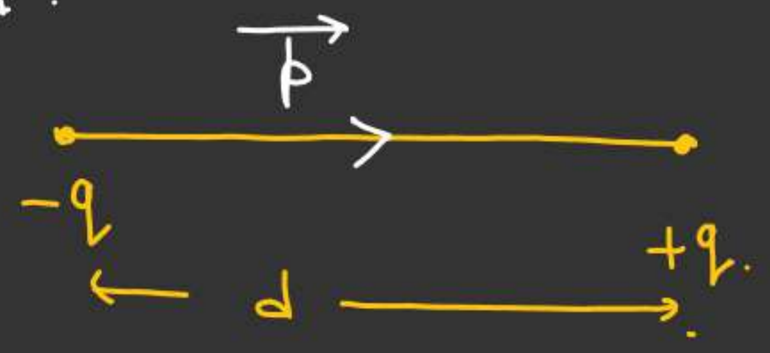
Dipole & Dipole moment

Dipole :- "Two equal and opposite Charges seperated by a very Small distance form a dipole."

Dipole moment :→ A vector always directed from −ve Charge to +ve charge.



$$|\vec{p}| = |q|d$$

$= \underline{C\text{-}m}$

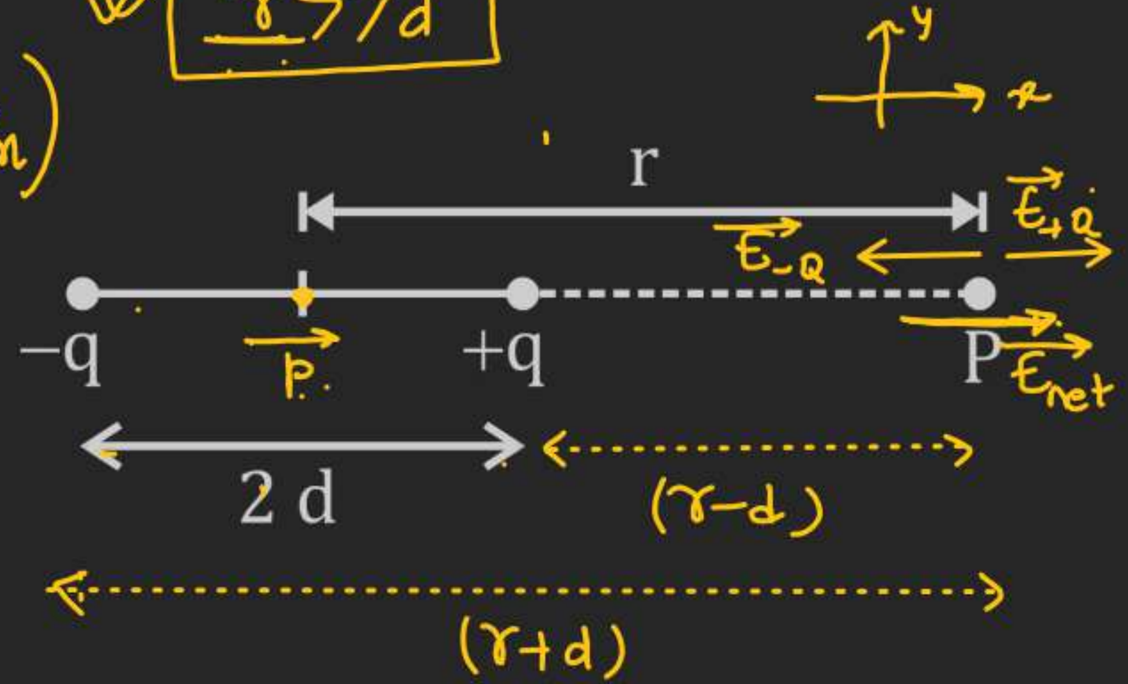
DIPOLE

❖ **Electric field and potential at the axis of Dipole.** (Axial position)

$\boxed{r >> d}$

(*) Electric field →

$|\vec{p}| = q(2d)$



↳ $\vec{E}_{net} = \vec{E}_{-q} + \vec{E}_{+q}$

$$= \frac{kq}{(r+d)^2}(-\hat{i}) + \frac{kq}{(r-d)^2}(\hat{i})$$

$$= kq\left[\frac{1}{(r-d)^2} - \frac{1}{(r+d)^2}\right]\hat{i}$$

$$\vec{E}_{net} = kq\left[\frac{(r+d)^2-(r-d)^2}{(r^2-d^2)^2}\right]\hat{i}$$

$$\vec{E}_{net} = \frac{kq}{(r^2-d^2)^2}[4rd]\,\hat{i}$$

$$\vec{E}_{net} = \left\{2k(\underbrace{2qd}_{p})\frac{\cancel{r}}{\cancel{r^4}\left[1-\underbrace{\frac{d^2}{r^2}}_{0}\right]^2}\right\}\hat{i}$$

$$\boxed{\vec{E}_{net} = \left[\frac{2kp}{r^3}\right]\hat{i}}\ **$$

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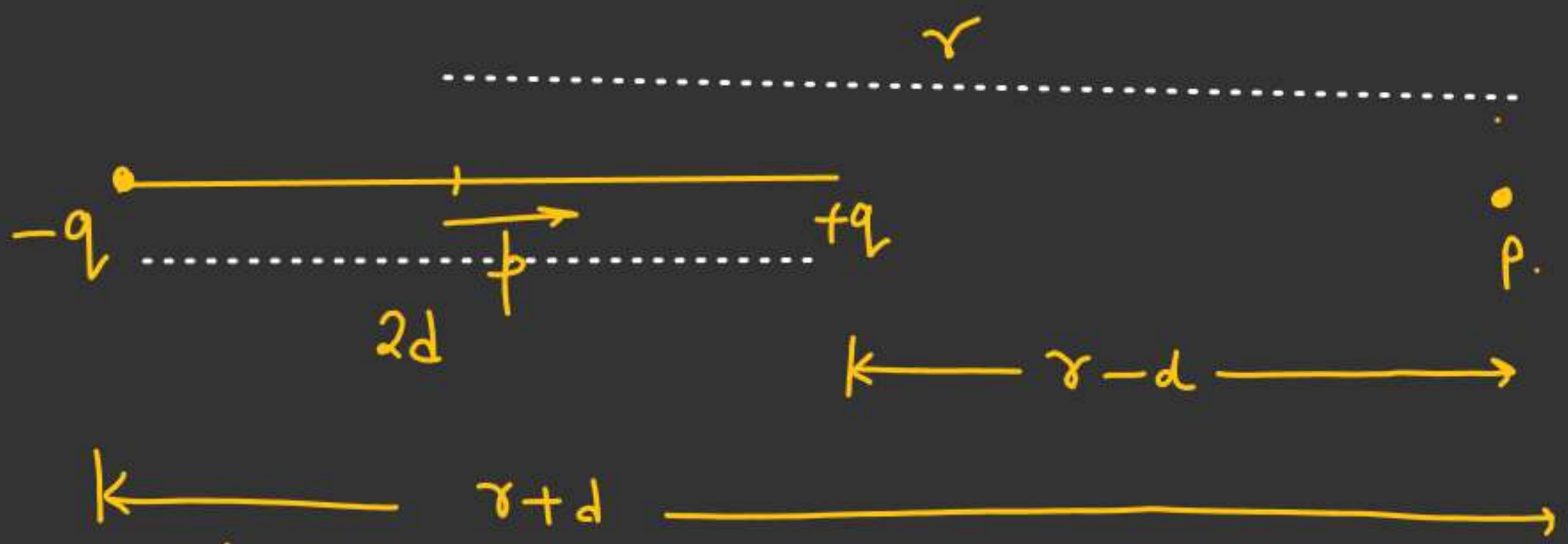
(*) Potential on the axis of the dipole! →

$|\vec{p}| = (q \cdot 2d)$



$$V_P = (V_{-q})_P + (V_{+q})_P.$$

$$= \frac{-kq}{(r+d)} + \frac{kq}{(r-d)}$$

$$= kq\left[\frac{1}{(r-d)} - \frac{1}{r+d}\right]$$

$$= kq\left[\frac{(r+d)-(r-d)}{r^2 - d^2}\right]$$

$$= \frac{kq\,2d}{(r^2 - d^2)} = \frac{kp}{r^2\left(1 - \frac{d^2}{r^2}\right)}$$

$\frac{d^2}{r^2} \approx 0$

$r >> d$

$$\boxed{V_{\text{at axis of dipole}} = \frac{kp}{r^2}}$$

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DIPOLE

$\boxed{r >> d}$

Perpendicular to the axis

❖ Electric field and potential at the equatorial position of dipole.

$$E_{net} = 2E\sin\theta$$

$$= 2\frac{Kq}{(\sqrt{d^2+r^2})^2} \times \frac{d}{\sqrt{d^2+r^2}}$$

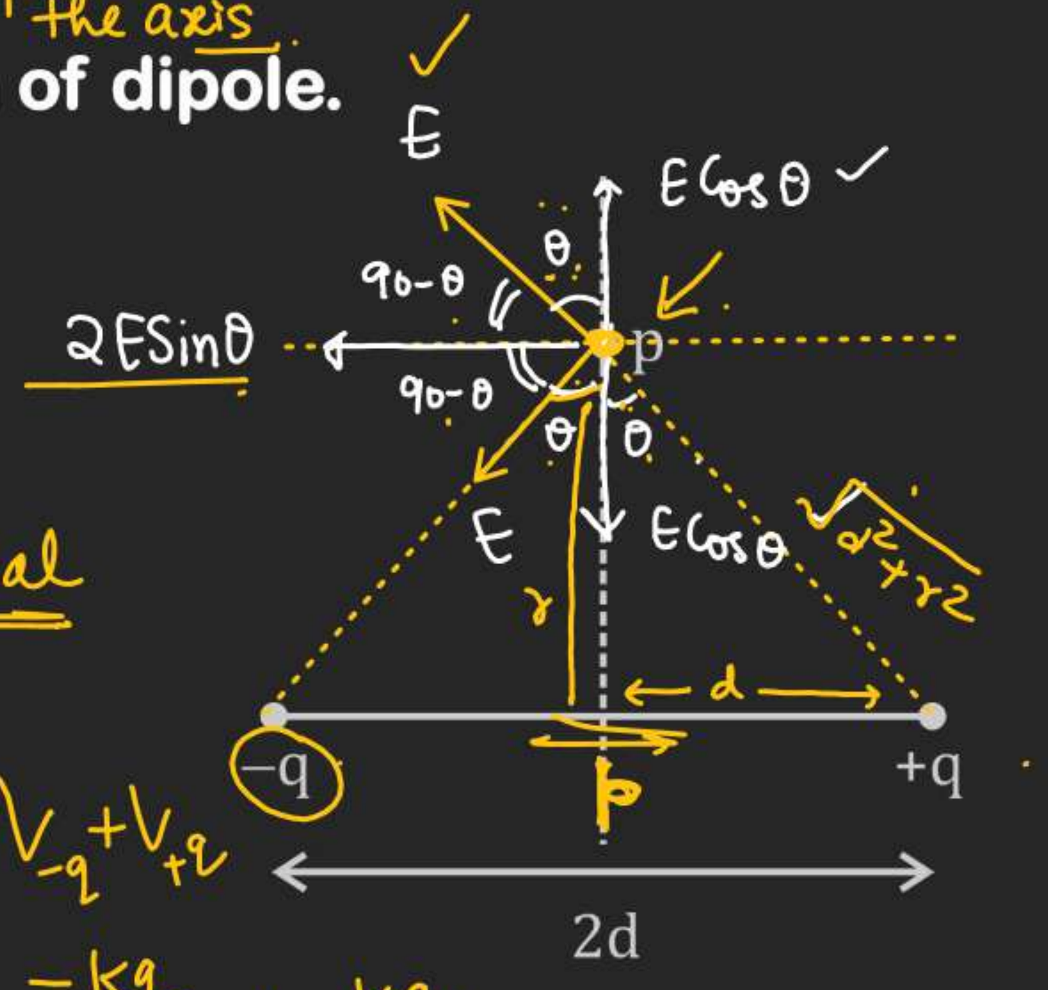
$$|\vec{p}| = (q2d)$$

$$\vec{E}_{net} = \frac{K(q2d)}{(d^2+r^2)^{3/2}}(-\hat{i})$$

$$\vec{E}_{net} = \frac{Kp}{r^3\left[1+\frac{d^2}{r^2}\right]^{3/2}}(-\hat{i}) = \left(\frac{Kp}{r^3}\right)(-\hat{i})$$

$\frac{d^2}{r^2} \Downarrow 0$

$$\boxed{\vec{E} = -\frac{K\vec{p}}{r^3}} \;**$$



Potential

$$V_{net} = V_{-q} + V_{+q}$$

$$= \frac{-Kq}{\sqrt{d^2+r^2}} + \frac{Kq}{\sqrt{d^2+r^2}} = 0$$

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DIPOLE

❖ **Electric field and potential at any general point due to dipole:-**

$$E_{p\cos\theta} = \frac{2Kp\cos\theta}{r^3}.$$

$$E_{(p\sin\theta)} = \left(\frac{Kp\sin\theta}{r^3}\right)$$

$$|\vec{E}_{net}| = \sqrt{\left(\frac{2Kp\cos\theta}{r^3}\right)^2 + \left(\frac{Kp\sin\theta}{r^3}\right)^2}$$

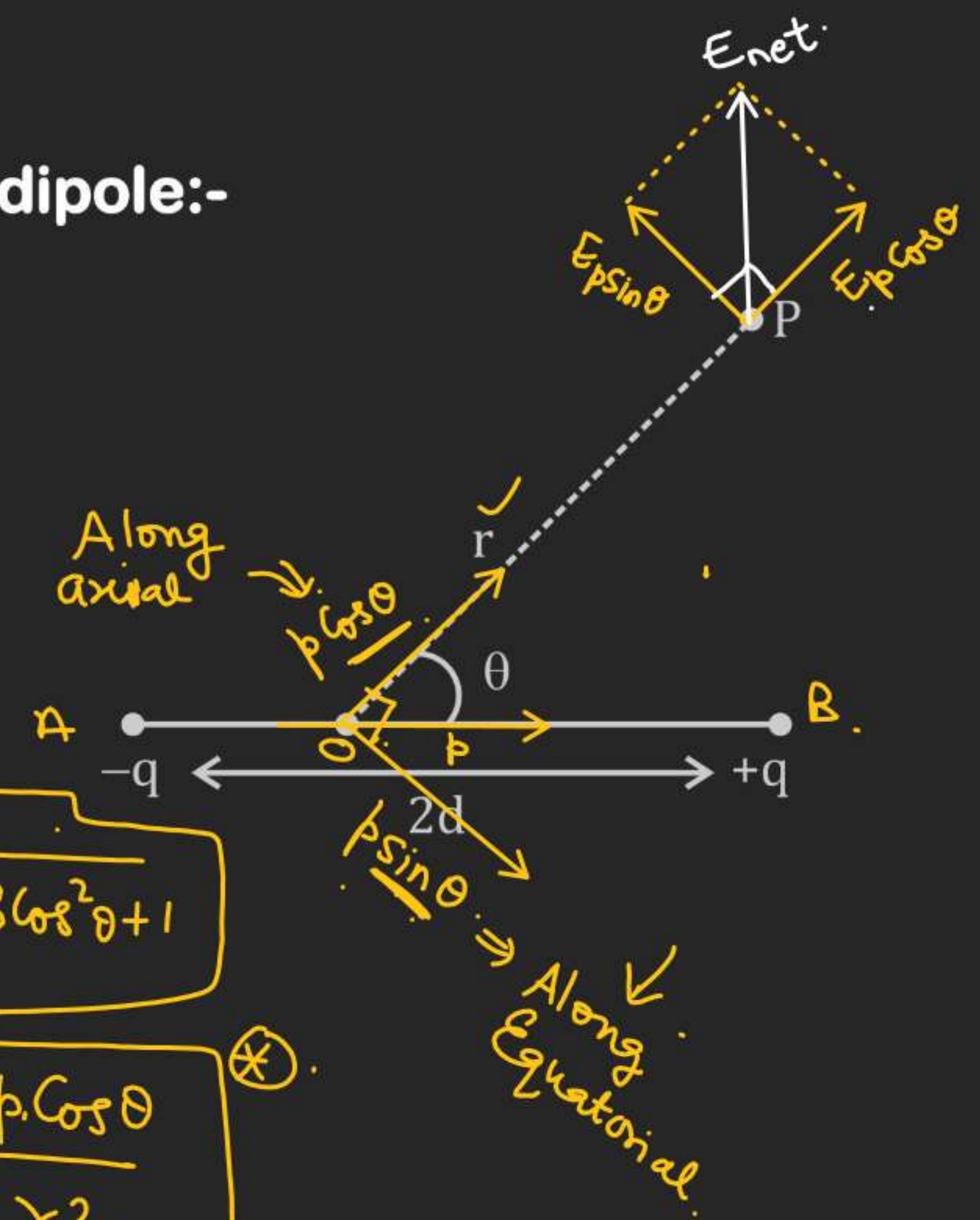
$$|\vec{E}_{net}| = \frac{Kp}{r^3}\sqrt{4\cos^2\theta + \sin^2\theta}.$$

$$|\vec{E}_{net}| = \frac{Kp}{r^3}\sqrt{3\cos^2\theta + \underbrace{\cos^2\theta + \sin^2\theta}_{1}}$$

**

$$\boxed{(E_{net})_p = \frac{Kp}{r^3}\sqrt{3\cos^2\theta + 1}}$$

$$\boxed{(V_p)_{net} = \frac{Kp\cos\theta}{r^2}.}$$



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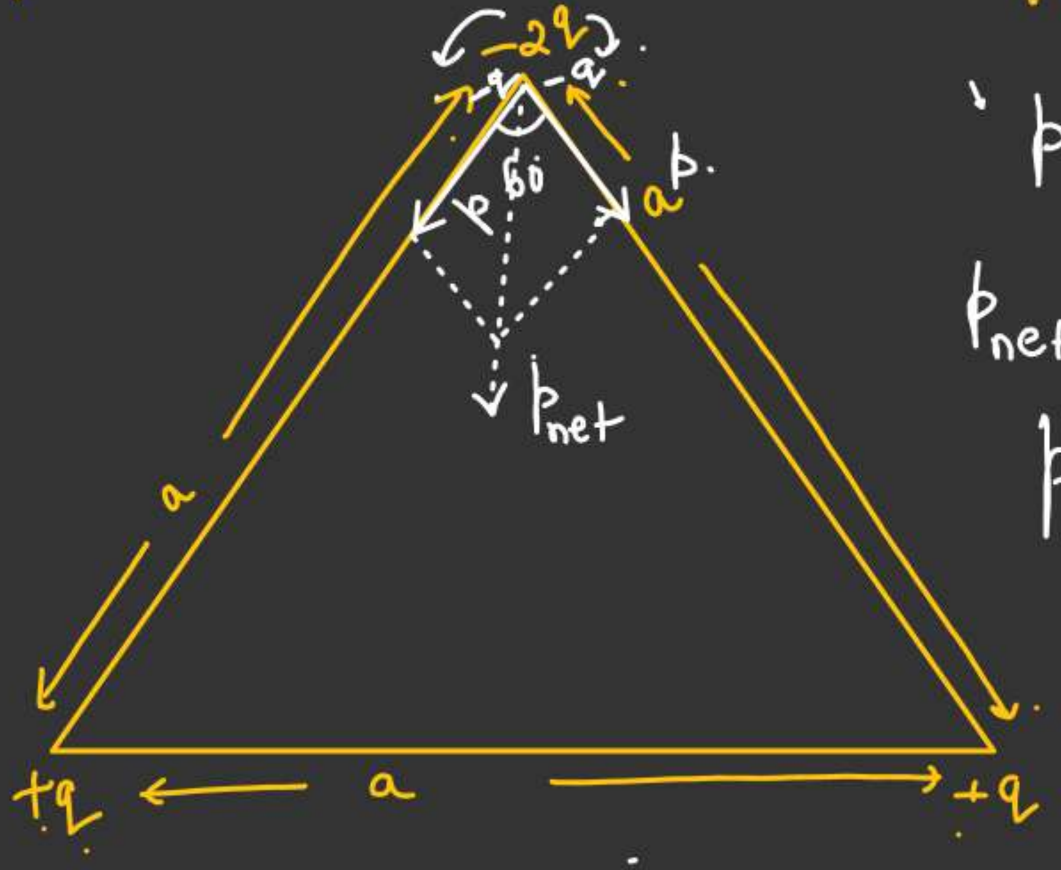
Find net dipole moment of the System:-



$p = (qa)$

$p_{net} = \sqrt{p^2 + p^2 + 2 \cdot p \cdot p \cdot \cos 60}$

$p_{net} = \sqrt{2p^2 + p^2}$

$= (\sqrt{3}\, p)$

$= \sqrt{3}\, qa$ Ans

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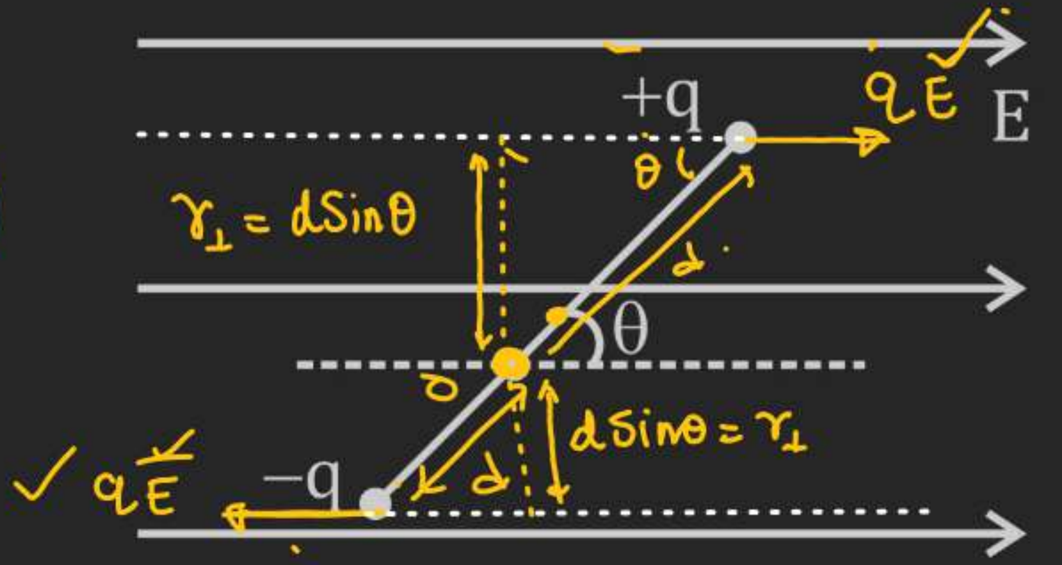
DIPOLE

❖ Dipole placed in an uniform Electric field

[Net force on the dipole placed in a uniform electric field is always zero]

$$|\vec{p}| = (q\,2d)$$

$$\tau_{net} = 2(qE\,d\sin\theta)$$

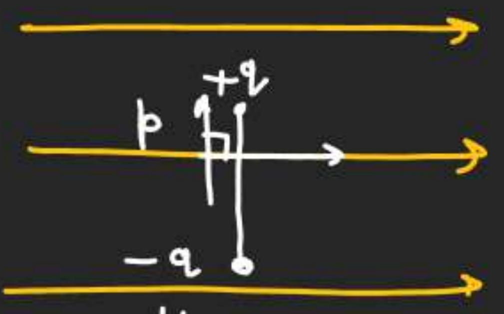


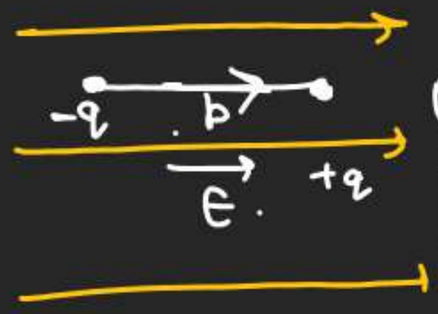
$$\tau_{net} = pE\sin\theta$$

$$\boxed{\tau_{max} = pE}$$

$\theta = 90$

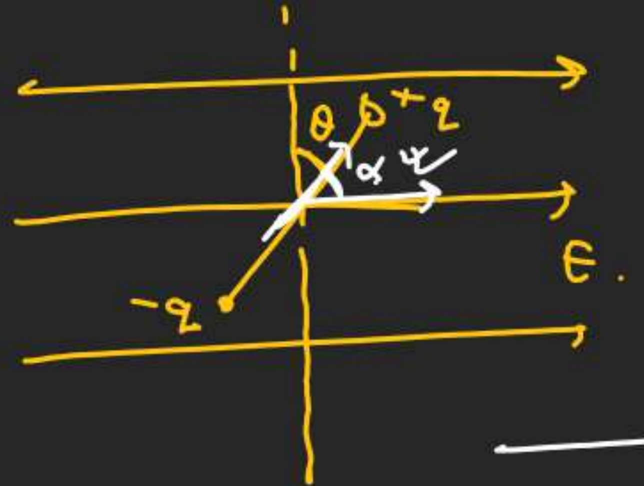
$$\boxed{\vec{\tau}_{net} = \vec{p} \times \vec{E}}$$





$\theta = 0$

$$\boxed{\tau = 0}$$



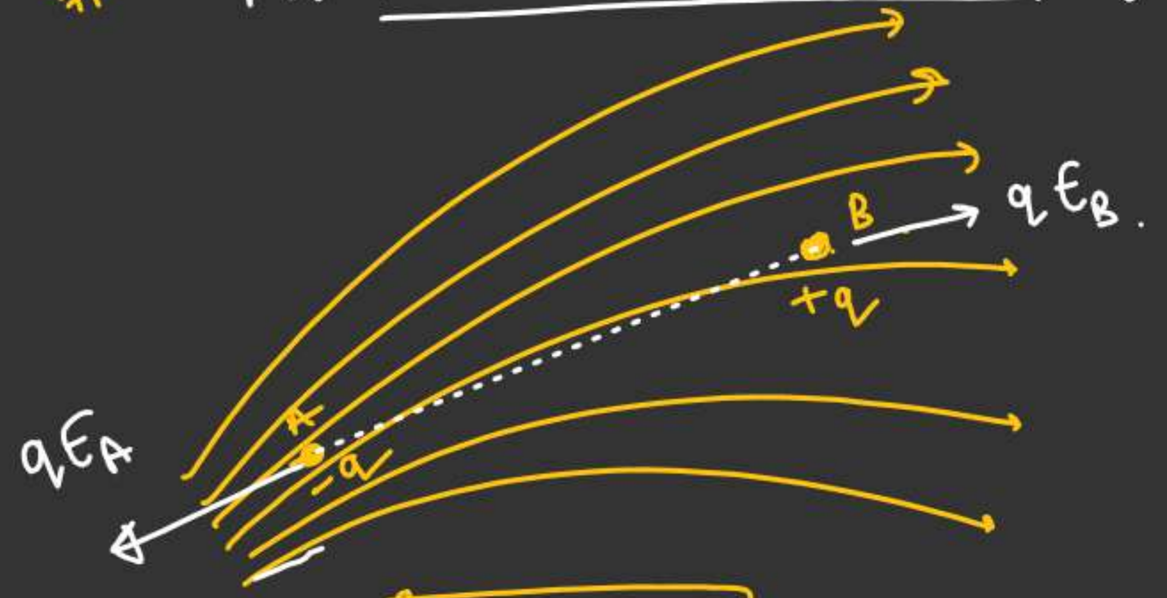
$\alpha \rightarrow$ Angle b/w $\vec{p}$ and $\vec{E}$.

$$\tau = pE\sin\alpha$$

$\alpha = (90 - \theta)$.

$$\tau = pE\sin(90-\theta) = pE\cos\theta$$

For Non-uniform Electric field net force is not zero on the dipole.



$$\boxed{E_A > E_B}$$

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#. Time period of a dipole placed in uniform electric field :→

Angular S.H.M

$\tau_{rest} = -K\theta$ (I = Moment of Inertia)

$\alpha = \frac{\tau_{rest}}{I} = -\frac{K}{I}\theta$

$\alpha = -\omega^2\theta$

$\omega = \sqrt{\frac{K}{I}}$

$\left(T = 2\pi\sqrt{\frac{I}{k}}\right)$

$\sin\theta \simeq \theta$

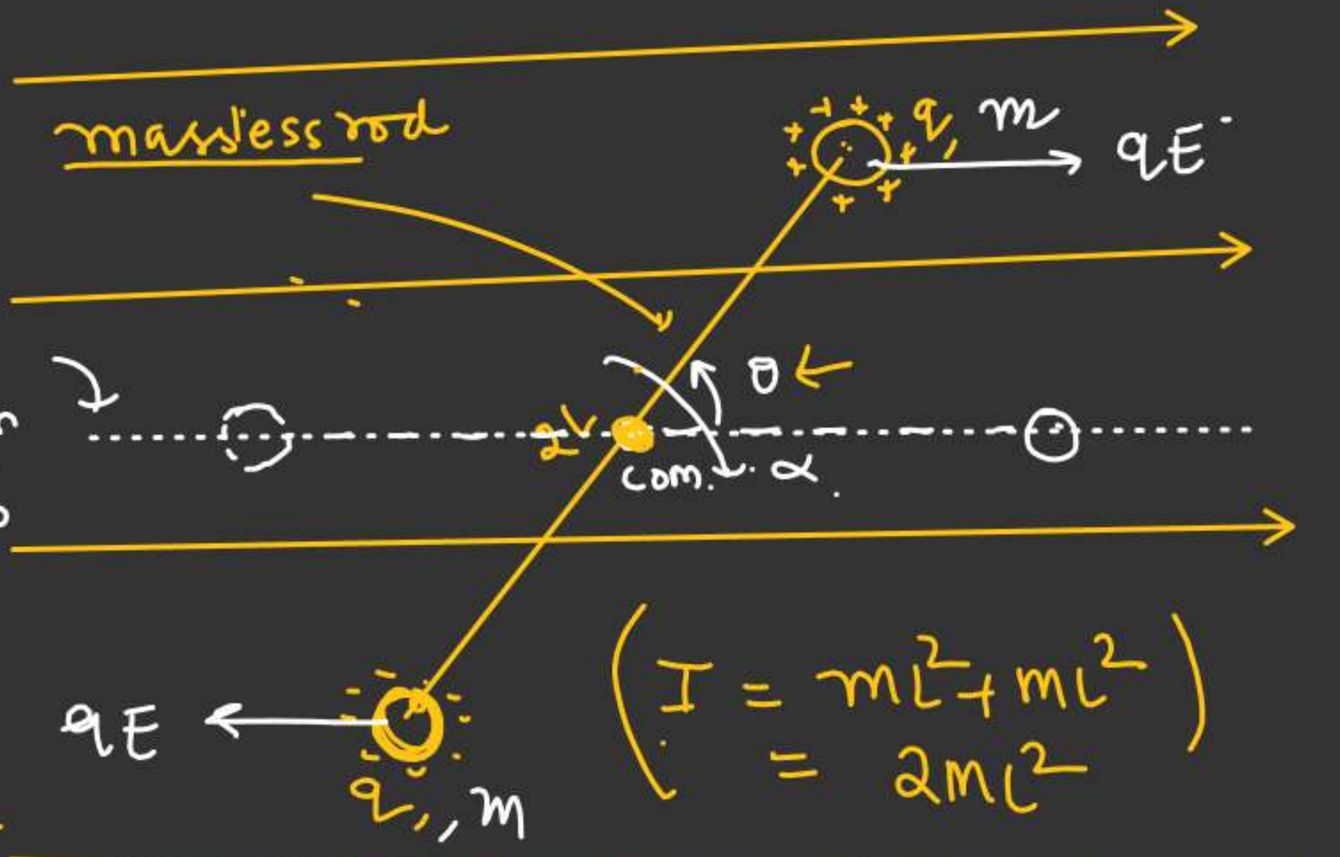
$\tau_{restoring} = -pE\sin\theta$

$\theta \rightarrow$ is very small

$\tau_{restoring} = -pE\,\theta$

$\alpha = -\frac{pE}{I}\theta$

$\alpha = -\omega^2\theta$



$\left(I = ml^2 + ml^2 = 2ml^2\right)$

$\omega = \sqrt{\frac{pE}{I}} \Rightarrow T = 2\pi\sqrt{\frac{I}{pE}}$