Coordinate system:

A mathematical system to locate the position of particle uniquely.

1. Cartesian co-ordinate system
2. Polar co-ordinate system
3. Spherical co-ordinate system etc.

1. Cartesian co-ordinate system:

Cartesian coordinate system was invented by Rene Descartes and Pierre de Fermat.

Both inventors used only one axis in their writtings. Later during translating writings of Descrartes the concept of pair of axes was introduced by Frans Van Schooten.

Two dimensional co-ordinate system is called rectangular co-ordinate system.

x-coordinate is called abscissa

y-coordinate is called ordinate

- Motion in a straight-line deals with the motion of an object which changes its position with time along a straight line.
- The study of the motion of objects without considering the cause of motion is called kinematics.

Rest and Motion:

If the position of a body does not change with time with respect to the surroundings, then it is said to be at rest, if not it is said to be in motion.

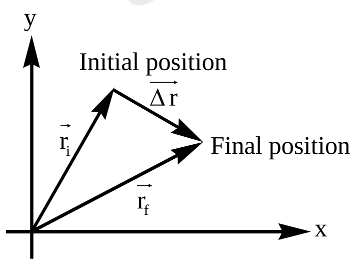
Displacement:

Change in position vector is called displacement vector.

It depends on observer.

$\vec{r_i} + \overrightarrow{\Delta r} = \vec{r_f}$

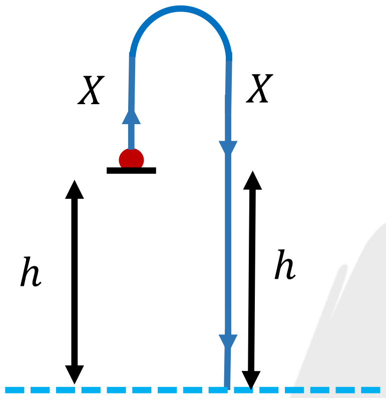
$\overrightarrow{\Delta r} = \vec{r_f} - \vec{r_i}$



- The minimum length between initial and final point is the magnitude of displacement and its direction is from initial point to final point.
- The magnitude of displacement is equal to the shortest distance between two positions; so $\text{Distance} \geq |\text{Displacement}|$

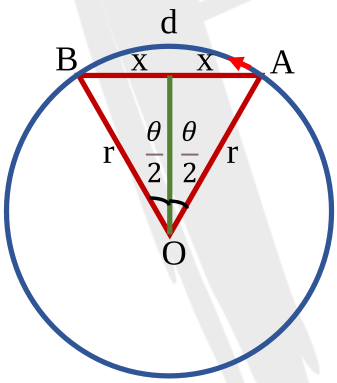
- Displament doesn't depend on path, depend only on initial and final position
- For motion between two points, displacement is single valued while distance depends on actual path and so can have many values.
- For a moving particle, distance can never be negative or zero while displacement can be negative.

1)



$\rightarrow$ distance = h + 2x $\rightarrow$ displacement = h

2) In the figure, displacement $2x = 2r\sin(\theta/2)\left[\because \sin\frac{\theta}{2} = \frac{x}{r}\right]$



Distance:

Actual travelled length by particle is called distance.

It depends on observer.

It depends on path.

It is a scalar quantity.

This cannot be negative.

Distance travelled by any particle never decreases with time, either remains constant or increases.

Velocity:

This is a vector quantity and depends on observer.

'Instantaneous velocity' also called 'velocity'

Instantaneous velocity = Rate of change of position vector

$$\vec{V} = \lim_{\Delta t \to \infty} \frac{\Delta \vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}$$

- The magnitude of velocity is called speed, i.e $\text{Speed} = |\text{velocity}|$ i.e, $V = |\vec{V}|$
- Velocity is a vector while speed is a scalar, both having same units (m/s) and dimensions (LT^{-1}).

$$\vec{V} = \frac{d\vec{r}}{dt}$$

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

$$\vec{V} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} + \frac{dz}{dt}\hat{k}$$

$$\vec{V} = V_x\hat{i} + V_y\hat{j} + V_z\hat{k}$$

$$V_x = \frac{dx}{dt},\ V_y = \frac{dy}{dt},\ V_z = \frac{dz}{dt}$$

V_x = x – component of velocity

$$\text{Average velocity} = \frac{\text{Displacement}}{\text{time interval}}$$

$$\vec{V}_{avg} = \frac{\overrightarrow{\Delta r}}{\Delta t} = \frac{(x_f - x_i)\hat{i} + (y_f - y_i)\hat{j} + (z_f - z_i)\hat{k}}{\Delta t} = \vec{r}_f - \vec{r}_i$$

Direction of average velocity will be along direction of displacement.

Speed:

Rate of change of length traversed is called speed.

In other words, magnitude of instantaneous velocity is called speed. Also called instantaneous speed.

This is scalar quantity and depends on observer

$\text{Speed} = \frac{d\ell}{dt}$ $\qquad$ ℓ = length of travelled path

$$V = \frac{d\ell}{dt}$$

$$d\ell = Vdt$$

$d\ell$ = travelled distance in time interval "dt"

$$\text{Average speed} = \frac{\text{Travelled distance}}{\text{time interval}}$$

$$V_{avg} = \frac{\ell}{\Delta t}$$

For the journey between two points:

Distance $\geq |\text{displacement}|$

$$\frac{\text{distance}}{\Delta t} \geq \left|\frac{\text{displacement}}{\Delta t}\right|$$

$$\text{avg speed} \geq |\text{avg velocity}|$$

Let (i) $\vec{v}$ = constant

$$\frac{d\vec{r}}{dt} = \vec{v}$$

$d\vec{r} = \vec{V}dt$ { $d\vec{r}$ is displacement vector in time dt}

$$\int_{\vec{r}_n}^{\vec{r}} d\vec{r} = \int_{t=0}^{t=t} \vec{v}dt$$

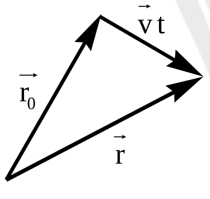
$$\vec{r} - \vec{r}_0 = \vec{v}(t-0)$$

$$\vec{r} = \vec{r}_0 + \vec{v}t$$

$$\frac{\vec{r} - \vec{r}_0}{t-0} = \vec{v}$$

It means

$$\vec{v}_{avg} = \vec{v}$$



Let (ii) $|\vec{v}|$ = constant (means speed is constant)

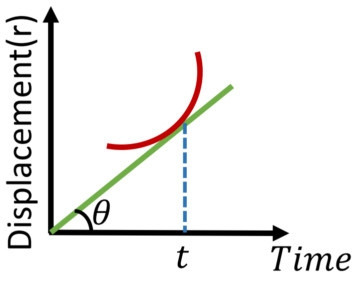
$$V = \frac{d\ell}{dt}$$

$$\int d\ell = \int_{t=t_1}^{t=t_2} v\,dt \Rightarrow \ell = v(t_2 - t_1)$$

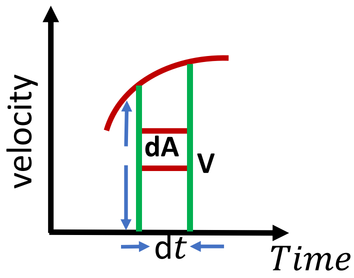
Travelled length = speed × time interval

- As by definition $V = \frac{dr}{dt}$, the slope of displacement – versus time gives velocity , i.e,

$$V = \frac{dr}{dt} = \tan\theta = \text{slope of } r - t \text{ curve}$$



(a)



(b)

- As by definition $V = \frac{dr}{dt}$; i.e., $dr = Vdt$ and from fig. $Vdt = dA$

So, $dA = dr$ i.e., $r = \int dA = \int Vdt$

i.e, area under velocity-time graph gives displacement while without sign gives distance.

- Average speed is the total distance divided by total time

$$v_{avg} = \frac{\text{Total distance travelled}}{\text{Total time taken}}$$

- If a body travels a distance s_1 in time t_1, s_2 time t_2 and s_3 in time t_3 then the average speed is

$$v_{avg} = \frac{s_1 + s_2 + s_3}{t_1 + t_2 + t_3}$$

- If an object travels distances s_1, s_2, s_3 etc. with speeds v_1, v_2, v_3 respectively in the same direction. Then

$$\text{Average speed} = \frac{s_1 + s_2 + s_3}{\frac{s_1}{v_1} + \frac{s_2}{v_2} + \frac{s_3}{v_3}}$$

- If an object travels first half of the total journey with a speed v_1 and next half with a speed v_2 then its average speed is

$$v_{avg} = \frac{s + s}{\frac{s}{v_1} + \frac{s}{v_2}} = \frac{2s}{\frac{s}{v_1} + \frac{s}{v_2}} = \frac{2}{\frac{1}{v_1} + \frac{1}{v_2}} = \frac{2v_1v_2}{v_1 + v_2}$$

- If a body travels first 1/3 rd of the distance with a speed v_1 and second 1/3rd of the distance with a speed v_2 and last 1/3rd of the distance with a speed v_3 ,then the average speed

$$v_{avg} = \frac{\frac{s}{3}+\frac{s}{3}+\frac{s}{3}}{\frac{s}{3v_1}+\frac{s}{3v_2}+\frac{s}{3v_3}}$$

$$v_{avg} = \frac{3v_1v_2v_3}{v_1v_2+v_2v_3+v_3v_1}$$

- If an object travels with speeds v_1, v_2, v_3 etc., during time intervals t_1, t_2, t_3 etc.,

then its average speed $= \dfrac{v_1t_1+v_2t_2+v_3t_3+....}{t_1+t_2+t_3+....}$

If $t_1 = t_2 = t_3 ... = t$, then

$$v_{avg} = \frac{v_1t+v_2t+v_3t+...}{nt} = \frac{v_1+v_2+...}{n}$$

i.e., The average speed is equal to the arithmetic mean of individual speeds.

- The actual path length traversed by a body is called distance.

$$\vec{s} = \vec{r}_f - \vec{r}_i = \int_{t_1}^{t_2} v_x\hat{i}dt + \int_{t_1}^{t_2} v_y\hat{j}dt + \int_{t_1}^{t_2} v_z\hat{k}dt$$

Acceleration:

Rate of change of velocity ($\vec{V}$) is called acceleration. This is a vector quantity and depends on observer.

$$\vec{a} = \lim_{\Delta t\to 0}\frac{\Delta\vec{V}}{\Delta t} = \frac{d\vec{V}}{dt} \qquad .... (2)$$

Regarding acceleration it is worth noting that:

- It is a vector with dimensions $\left\lfloor LT^{-2} \right\rfloor$ and SI units $\left\lfloor m/s^2 \right\rfloor$

$$\vec{a} = \frac{d\vec{v}}{dt}$$

$$d\vec{v} = \vec{a}\,dt$$

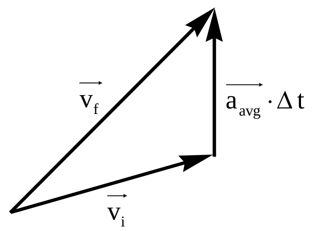
$d\vec{v}$ = change in velocity vector in time interval "dt"

Average acceleration

Average acceleration = $\dfrac{\vec{v}_f - \vec{v}_i}{\Delta t}$

$$\vec{a}_{avg} = \frac{\vec{v}_f - \vec{v}_i}{\Delta t}$$

$$\vec{v}_f = \vec{v}_i + \vec{a}_{avg} \cdot \Delta t$$



Uniform acceleration:

$\vec{a}$ = constant

$$\vec{a} = \frac{d\vec{v}}{dt}$$

$$\int_{\vec{v}=\vec{U}}^{\vec{v}=\vec{v}} d\vec{v} = \int_{t=0}^{t=t} \vec{a}\,dt$$

For constant acceleration

$$\vec{v} - \vec{u} = \vec{a}.(t-0)$$

$$\vec{v} = \vec{u} + \vec{a}.t$$

$\vec{v}$ = final velocity

$\vec{u}$ = Initial velocity

t = time interval

$$v_x\hat{i} + v_y\hat{j} + v_z\hat{k} = u_x\hat{i} + u_y\hat{j} + u_z\hat{k} + \left(a_x\hat{i} + a_y\hat{j} + a_z\hat{k}\right)t$$

$$v_x = u_x + a_x t$$

$$v_y = u_y + a_y t$$

$$v_z = u_z + a_z t$$

$$v = u + at$$

$$\frac{d\vec{r}}{dt} = \vec{u} + \vec{a}t$$

$$\int_{\vec{r}_i}^{\vec{r}_f} d\vec{r} = \int_{t=0}^{t} \vec{u}.dt + \int_{t=0}^{t=t} \vec{a}.tdt$$

$$\vec{r}_i - \vec{r}_f = \vec{u}.t + \frac{1}{2}\vec{a}.t^2$$

$$\vec{d} = \vec{u}t + \frac{1}{2}\vec{a}t^2$$

$$\Delta x = x_f - x_i = u_x t + \frac{1}{2}a_x t^2$$

$$\Delta y = y_f - y_i = u_y t + \frac{1}{2}a_y t^2$$

$$\Delta z = z_f - z_i = u_z t + \frac{1}{2}a_z t^2$$

$$S = u\,t + \frac{1}{2}at^2$$

If velocity and acceleration have same direction, speed will increase. (1)

$$v = u + at \Rightarrow u = v - at$$

$$S = (v - at)t + \frac{1}{2}at^2$$

$$S = vt - at^2 + \frac{1}{2}at^2$$

$$S = vt - \frac{1}{2}at^2$$

If velocity and acceleration have opposite direction, speed will decrease. (2)

$$S = \frac{2vt + at^2}{2} = \frac{vt + vt + at^2}{2}$$

$$S = \frac{vt + t(v + at)}{2} = \frac{u + V}{2}t$$

When speed decreases motion is called retarded motion (3)

$$S = \frac{u + V}{2}.t$$

$$\frac{S}{t} = \frac{u + V}{2}$$

$$\left\{\frac{S}{t} = V_{avg}\,(\text{By definition})\right\}$$

$V_{avg} = \dfrac{u + V}{2}$ (only valid when acceleration is constant)

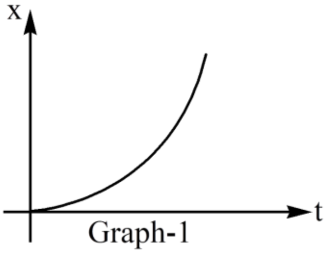
Variable acceleration:

Graph: (For motion in 1-D)

$$V_x = \frac{dx}{dt}$$

Slope of $x - t$ graph represent velocity of particle.

If $x - t$ graph is a straight line it means velocity is constant



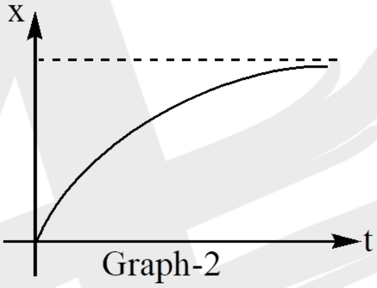
About graph – 1:

1. x increases as t increases

2. Initial velocity of particle is zero

3. At any instant velocity of particle is +ve, it means that the is particle moving in positive x direction always

4. Velocity of particle continuously increasing it means acceleration is also positive



About graph – 2:

1. x increases as t increases and very long time later asymptotically reaches $x = x_0$

2. Initial velocity of particle is positive

3. At any instant velocity of particle positive, it means particle moving in positive x direction always

4. Velocity of particle continuously decreasing and very long time later become zero, it means acceleration is in negative x direction

$$V_x = \frac{dx}{dt} \Rightarrow dx = V_x dt \Rightarrow \int_{x=x_1}^{x=x_2} dx = \int_{t=t_1}^{t=t_2} V_x\, dt$$

$$x_2 - x_1 = \int_{t=t_1}^{t=t_2} V_x dt$$

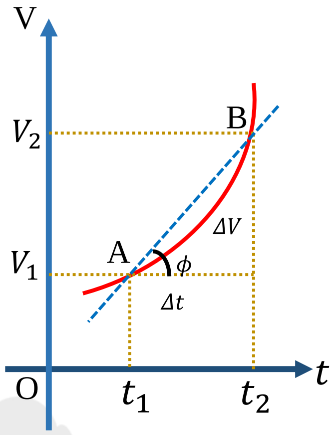
→ Area under v – t graph represent displacement (change in x – coordinate)

$$a_x = \frac{dV_x}{dt} \Rightarrow \int_{V=V_1}^{V=V_2} dV_x = \int_{t=t_1}^{t=t_2} a_x\, dt$$

$$V_2 - V_1 = \int_{t=t_1}^{t=t_2} a_x dt$$

$\rightarrow$ Area under $a_x - t$ represent change in velocity

Velocity - Time Graph



$$\vec{a}_{avg} = \frac{\Delta \vec{V}}{\Delta t} = \tan\phi$$

$\vec{a}_{avg} =$ slope of the line joining two points in v-t graph.

- The slope of a versus t curve, i.e, $\frac{d\vec{a}}{dt}$ is a measure of rate of non-uniformity of acceleration (usually it is known as JERK).
- Acceleration can be positive or negative. Positive acceleration means velocity is increasing with time while negative acceleration called retardation means velocity is decreasing with time.

Equations of motion:

- If a particle starts with an initial velocity u, acceleration a and it attains final velocity v in time t

then $a = \frac{dv}{dt}$ or $dv = adt$ or $\int_u^v dv = a\int_0^t dt$ or $[V]_u^V = a[t]_0^t$ or v = u + at ....(1)

In vector form $\vec{v} = \vec{u} + \vec{a}t$ ....(2)

- By definition of velocity, Eqn. (1) becomes to $\frac{ds}{dt} = u + at$

$\int_0^s ds = \int_0^t (u+at)dt$ or $s = ut + \frac{1}{2}at^2$ ....(3)

In vector form $\vec{s} = \vec{ut} + \frac{1}{2}\vec{a}t^2$ .... (4)

From eqns. (1) and (3), we get

$$s = u\frac{(v-u)}{a} + \frac{1}{2}a\left[\frac{(v-u)}{a}\right]^2$$

or $\quad 2as = 2uv - 2u^2 + v^2 + u^2 - 2uv$

i.e., $\quad v^2 = u^2 + 2as \qquad ....(5)$

- In a scalar form

$\vec{v}.\vec{v} = \vec{u}.\vec{u} + 2\vec{a}.\vec{s}$ or $v^2 = u^2 + 2\vec{a}.\vec{s}$

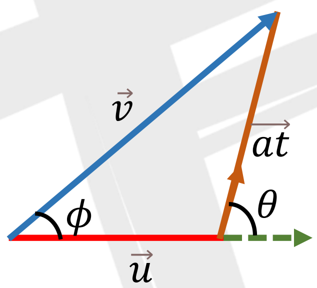
- Distance travelled by the object = average speed x time

$$s = \left(\frac{u+v}{2}\right)t \qquad ....(6)$$

- Distance travelled by the object in n^{th} second $s_n = u + a\left(n - \frac{1}{2}\right)$ .... (7)

- If acceleration and velocity are not collinear, v can be calculated by using

$$v = \left[u^2 + (at)^2 + 2uat\cos\theta\right]^{1/2} \text{ with } \tan\phi = \frac{at\sin\theta}{u + at\cos\theta} .....(8)$$

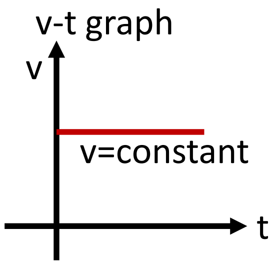
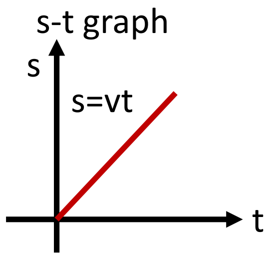
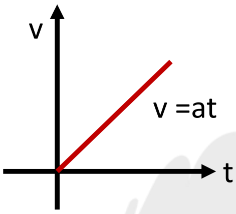
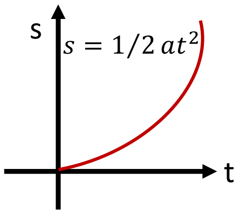
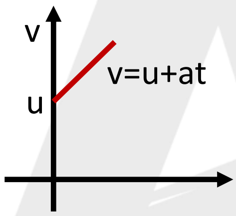
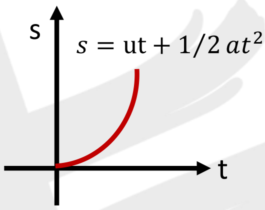
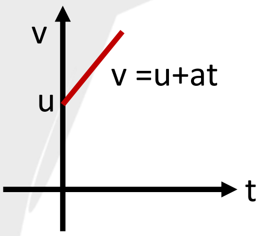
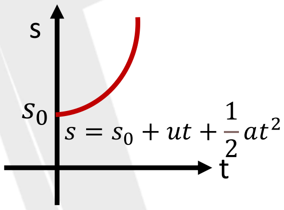
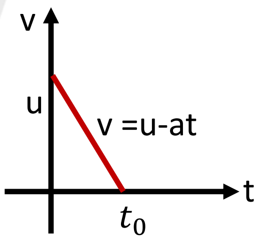
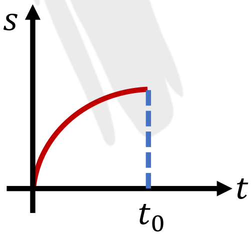
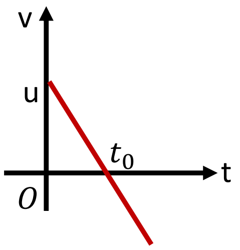
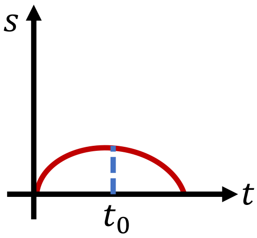


GRAPHS

Characteristics of s-t and v-t graphs

- Slope of displacement time graph gives velocity.
- Slope of velocity-time graph gives acceleration.
- Area under velocity-time graph gives displacement

The plots of v-t and s-t graphs are shown in the table when the particle has got either a one-dimensional motion with uniform velocity or with constant acceleration.

<table>
<tr><th>S.No</th><th>Situation</th><th>v-t graph</th><th>s-t graph</th><th>Interpretation</th></tr>
<tr><td>1.</td><td>Uniform motion</td><td></td><td></td><td>i)Slope of s-t graph = v = constant.
ii) In s-t- graph s = 0 at t = 0</td></tr>
<tr><td>2.</td><td>Uniformly accelerated motion with u=0 and s=0 at t = 0</td><td></td><td></td><td>I) u=0, i.e.,v=0 and t=0
ii) u=0, i.e., slope of s-t graph at t=0, should be zero
iii) a or slope of v-t graph is constant</td></tr>
<tr><td>3)</td><td>Uniformly accelerated motion with $u \neq 0$ and $s = s_0$ at $t = 0$</td><td></td><td></td><td>I) $u \neq 0$, i.e., v or slope of s-t graph at t=0 is not zero
ii) v or slope of s-t graph gradually goes on increasing</td></tr>
<tr><td>4)</td><td>Uniformly accelerated motion with $u \neq 0$ and $s = s_0$ at $t = 0$</td><td></td><td></td><td>I) $s = s_0$ at t=0</td></tr>
<tr><td>5)</td><td>Uniformly retarded motion till velocity be comes zero</td><td></td><td></td><td>I)Slope of s-t graph at t=0 gives u.
ii) Slope of s-t graph at $t = t_0$ becomes zero
iii) In this case u can't be zero.</td></tr>
<tr><td>6)</td><td>Uniformly retarded then accelerated in opposite direction</td><td></td><td></td><td>I)At time $t = t_0$, v=0 or slope of s-t graph is zero.
ii) In s-t graph slope or velocity first decreases then increases with opposite sign.</td></tr>
</table>

Equations of Motion for Variable Acceleration:

- When acceleration 'a' of the particle is a function of time i.e., a = f(t)

  $$\Rightarrow \frac{dv}{dt} = f(t) \Rightarrow dv = f(t)dt$$

  Integrating both sides within suitable limits, we have

  $$\int_u^v dv = \int_0^t f(t)dt \Rightarrow v = u + \int_0^t f(t)\,dt$$

  When acceleration 'a' of the particle is a function of distance $a = f(x)$

  $$\Rightarrow \frac{dv}{dt} = f(x) \Rightarrow \frac{dv}{dx}\frac{dx}{dt} = f(x)$$

  $$\int_u^v v\,dv = \int_0^x f(x)dx \Rightarrow v^2 = u^2 + 2\int_0^x f(x)dx$$

MOTION UNDER GRAVITY

Equation of motion for a body projected vertically downwards :

- When a body is projected vertically downwards with an initial velocity u from a height h then a = g, s = h

  a) $v = u + gt$ b) $h = ut + \frac{1}{2}gt^2$ c) $v^2 - u^2 = 2gh$ d) $S_n = u + \frac{g}{2}(2n-1)$

- In case of freely falling body u = 0, a = +g

  a) v = gt b) $S = \frac{1}{2}gt^2$ c) $v^2 = 2gS$ d) $S_n = g\left(n - \frac{1}{2}\right)$

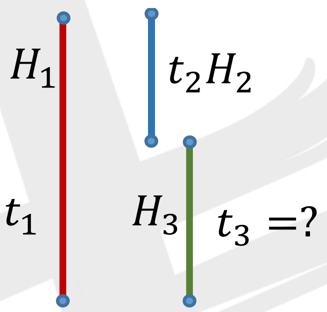
- For a freely falling body, the ratio of distances travelled in 1 second, 2seconds, 3 seconds, ... = 1 : 4 : 9 : 16....
- For a freely falling body, the ratio of distances travelled in successive seconds = 1 : 3 : 5 : 9 ....
- A freely falling body passes through two points A and B in time intervals of t_1 and t_2 from the start, then the distance between the two points A and B is = $\frac{g}{2}\left(t_2^2 - t_1^2\right)$
- A freely falling body passes through two points A and B at distances h_1 and h_2 from the start, then the time taken by it to move from A to B is

  $$t = \sqrt{\frac{2h_2}{g}} - \sqrt{\frac{2h_1}{g}} = \sqrt{\frac{2}{g}}\left(\sqrt{h_2} - \sqrt{h_1}\right)$$

- Two bodies are dropped from heights h_1 and h_2 simultaneously. Then after any time the distance between them is equal to $(h_2 \sim h_1)$.
- A stone is dropped into a river from the bridge and after 'x' seconds another stone is projected down into the river from the same point with a velocity of 'u'. If both the stones reach the water simultaneously, then $S_{1(t)} = S_{2(t-x)}$

$$\frac{1}{2}gt^2 = u(t-x) + \frac{1}{2}g(t-x)^2$$

- A body dropped freely from a multi storeyed building can reach the ground in t_1 sec. It is stopped in its path after t_2 sec and again dropped freely from the point. The further time taken by it to reach the ground is $t_3 = \sqrt{t_1^2 - t_2^2}$



We know that $H_1 = H_2 + H_3$

$$\Rightarrow \frac{1}{2}gt_1^2 = \frac{1}{2}gt_2^2 + \frac{1}{2}gt_3^2$$

$$\Rightarrow t_1^2 = t_2^2 + t_3^2 \qquad \therefore t_3 = \sqrt{t_1^2 - t_2^2}$$

Equations of motion of a body Projected Vertically up :

- Acceleration (a) = -g

a) $v = u - gt$ b) $s = ut - \frac{1}{2}gt^2$ c) $v^2 - u^2 = 2gh$ d) $s_n = u - \frac{g}{2}(2n-1)$

- Angle between velocity vector and acceleration vector is 180° until the body reaches the highest point.
- At maximum height, v = 0 and a = g
- $H_{max} = \frac{u^2}{2g} \Rightarrow H_{max} \propto u^2$ (Independent of mass of the body)
- A body is projected vertically up with a velocity 'u' from ground in the absence of air resistance 'R'. then $(t_a = t_d)$

i) $t_a = t_d = \frac{u}{g}$

ii) Time of flight $T = t_a + t_d = \frac{2u}{g}$

- A body is projected vertically up with a velocity 'u' from ground in the presence of constant air resistance 'R'. If it reaches the ground with a velocity 'v', then

  a) Height of ascent = Height of descent

  b) Time of ascent $t_a = \frac{mu}{mg+R}$

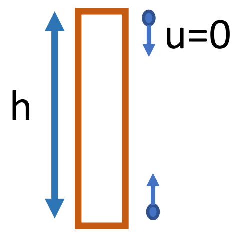
  c) Time of descent $t_d = \frac{mv}{mg-R}$

  d) $t_a < t_d$

  e) $\frac{v}{u} = \sqrt{\frac{mg-R}{mg+R}}\left(v<u\right)$

  f) For a body projected vertically up under air resistance, retardation during its motion is > g

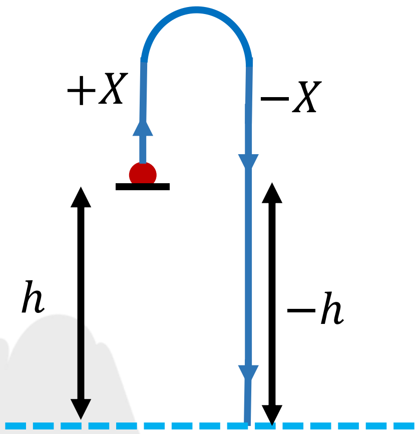
- At any point of the journey, a body possess the same speed while moving up and while moving down.
- Irrespective of velocity of projection, all the bodies pass through a height $\frac{g}{2}$ in the last second of ascent. Distance travelled in the last second of its journey $u-\frac{g}{2}$
- The change in velocity over the complete journey is '2u' (downwards)
- If a vertically projected body rises through a height 'h' in n^{th} second, then in $(n-1)^{th}$ second it will rise through a height (h+g) and in $(n+1)^{th}$ second it will rise through height (h-g).
- If velocity of body in n^{th} second is 'v' then in $(n-1)^{th}$ second it is (v + g) and that in $(n+1)^{th}$ second is (v - g) while ascending
- A body is dropped from the top edge of a tower of height 'h' and at the same time another body is projected vertically up from the foot of the tower with a velocity 'u'.



a) The separation between them after 't' seconds is = $(h-ut)$

Body Projected Vertically up from a Tower

- A body projected vertically up from a tower of height ‘h’ with a velocity ‘u’ (or) a body dropped from a rising balloon (or) a body dropped from an helicopter rising up vertically with constant velocity ‘u’ reaches the ground exactly below the point of projection after a time ‘t’. Then



(a) Height of the tower is $h = -ut + \frac{1}{2}gt^2$

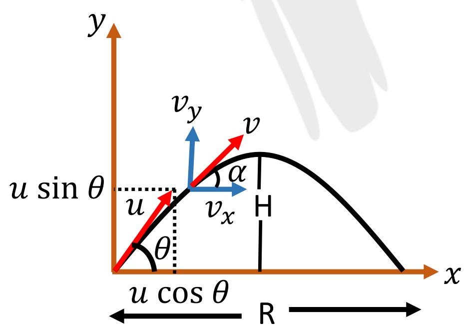
(b) Time taken by the body to reach the ground $t = \frac{u + \sqrt{u^2 + 2gh}}{g}$

(c) The velocity of the body at the foot of the tower $v = \sqrt{u^2 + 2gh}$

Projectiles:

Oblique Projectile:

- A body projected into air at an angle $'\theta'\left[\theta \neq 90^\circ \text{ and } 0^\circ\right]$ with the horizontal is called an oblique projectile.



- **The velocity vector $(\vec{v})$ at any instant ‘t’ is**

$\vec{v} = v_x\hat{i} + v_y\hat{j}$

where $v_x = u\cos\theta$ and $v_y = u_y + a_yt = u\sin\theta - gt$

Hence $\vec{v} = u\cos\theta\hat{i} + (u\sin\theta - gt)\hat{j}$

- The magnitude of velocity is given by

$$v=\sqrt{v_x^2+v_y^2}=\sqrt{(u\cos\theta)^2+(u\sin\theta-gt)^2}$$

- The angle α (direction of velocity) is given by

$$\alpha=\tan^{-1}\left(\frac{v_y}{v_x}\right)=\tan^{-1}\left(\frac{u\sin\theta-gt}{u\cos\theta}\right)$$

Displacement vector $(\vec{s})$

Displacement $\vec{s}=x\hat{i}+y\hat{j}$ here

- horizontal displacement during a time t

$$x=u_x t=(u\cos\theta)t$$

- vertical displacement during a time t

$$y=u_y t-\frac{1}{2}gt^2=(u\sin\theta)t-\frac{1}{2}gt^2$$

- **Equation of projectile**

$$y=(\tan\theta)x-\left(\frac{g}{2u^2\cos^2\theta}\right)x^2=Ax-Bx^2$$

Where A and B are constants

$$A=(\tan\theta),B=\frac{g}{2u^2\cos^2\theta}$$

- Time of ascent $(t_a)=\text{Time of descent}(t_d)$

$$=\frac{u_y}{g}=\frac{u\sin\theta}{g}$$

- Time of flight (T) = Time of ascent (t_a) + Time of descent (t_d)

$$T=t_a+t_d=\frac{2u_y}{g}=\frac{2u\sin\theta}{g}$$

- $H=\frac{u^2{}_y}{2g}=\frac{u^2\sin^2\theta}{2g}$

Range (R)

- $R=u_x T=\frac{2u_x u_y}{g}$ (or)

$$R=(u\cos\theta)T=\cos\theta\times\frac{2u\sin\theta}{g}=\frac{u^2\sin2\theta}{g}$$

1) Range is maximum when $\theta=45^{\circ}$

2) Maximum range, $R_{max} = \frac{u^2}{g}$

3) When 'R' is maximum, $H_{max} = \frac{R_{max}}{4} = \frac{u^2}{4g}$

4) The range, for a given velocity of projection, is same for complimentary angles of projection i.e $\left(\theta_1 + \theta_2 = 90^0\right)$

- Relation between H, T and R

1) $\frac{H}{T^2} = \frac{g}{8}$ (b) $\frac{H}{R} = \frac{\tan\theta}{4}$ (c) $\frac{R}{T^2} = \frac{g}{2\tan\theta}$

2) $R = \frac{gT^2}{2\tan\theta}$ and if $\theta = 45^o$ then

$R = \frac{gT^2}{2} \Rightarrow T = \sqrt{\frac{2R}{g}}$

- The equation representing the projectile is $y = Ax - Bx^2$

1) Angle of projection $\theta = \tan^{-1}(A)$

2) Initial velocity $|\vec{u}| = \sqrt{\frac{g\left(1 + A^2\right)}{2B}}$

3) Range of the projectile $R = \frac{A}{B}$

4) Maximum height $H = \frac{A^2}{4B}$

5) Time of flight $(T) = \sqrt{\frac{A^2}{2Bg}}$

- Let the horizontal and vertical displacement of projectile be $x = at$ and $y = bt - ct^2$ respectively. Then

1) angle of projection $\theta = \tan^{-1}\left(\frac{b}{a}\right)$

2) velocity of projection $u = \sqrt{a^2 + b^2}$

3) acceleration of projectile = 2c

4) maximum height reached $= \frac{b^2}{4c}$

5) horizontal range $= \frac{ab}{c}$

- In case of complimentary angles $(\theta_1+\theta_2=90^\circ)$ of projection

  1) If T_1 and T_2 are the times of flight then

  i) $\frac{T_1}{T_2}=\tan\theta$ ii) $T_1T_2=\frac{2R}{g}\Rightarrow T_1T_2\alpha R$

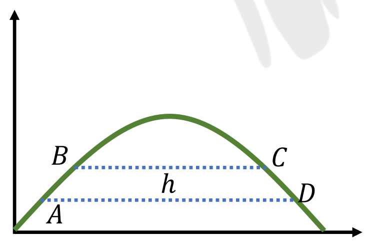
  2) If H_1 and H_2 are maximum heights then

  i) $\frac{H_1}{H_2}=\tan^2\theta$ ii) $H_1+H_2=\frac{u^2}{2g}$

  iii) $R=4\sqrt{H_1H_2}$ iv) $R_{max}=2(H_1+H_2)$

- If a body is thrown to a maximum distance 'R' then it can be projected to a maximum vertical height $R/2$
- If a body is thrown to a maximum distance 'R' then maximum height attained by it in its path is R/4.
- At the point of striking the ground
  1) Horizontal component of velocity $=u\cos\theta$
  2) Vertical component of velocity $=-u\sin\theta$
  3) Speed of projection is equal to striking speed of projectile
  4) Angle of projection is equal to the striking angle of projectile
  5) If the angle of projection with the horizontal is θ then angle of deviation is 2θ
- The projectile crosses the points A, D in time interval t_1 seconds and B, C in time interval t_2 seconds then ${t^2}_1-{t^2}_2=\frac{8h}{g}$

  (h is the distance between BC and AD)



- **The path of projectile as seen from another projectile**

  Suppose two bodies A and B are projected simultaneously from the same point with initial Velocities u_1 and u_2 at angles θ_1 and θ_2 to the horizontal.

  The instantaneous positions of the two bodies are given by

  Body $A: x_1=u_1\cos\theta_1 t, y_1=u_1\sin\theta_1 t-\frac{1}{2}gt^2$

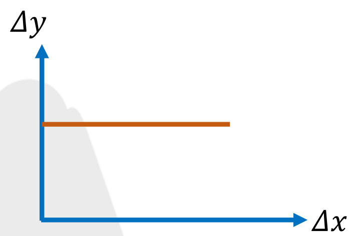
Body B: $x_2 = u_2\cos\theta_2 t,\ y_2 = u_2\sin\theta_2 t - \frac{1}{2}gt^2$

$\Delta x = (u_1\cos\theta_1 - u_2\cos\theta_2)t$

$\Delta y = (u_1\sin\theta_1 - u_2\sin\theta_2)t$

$$\text{slope} = \frac{\Delta y}{\Delta x} = \frac{u_1\sin\theta_1 - u_2\sin\theta_2}{u_1\cos\theta_1 - u_2\cos\theta_2}$$

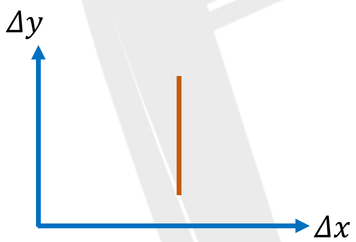
i) If $u_1\sin\theta_1 = u_2\sin\theta_2$ (initial vertical components) then slope $\frac{\Delta y}{\Delta x} = 0$



The path is a horizontal straight line

ii) If $u_1\cos\theta_1 = u_2\cos\theta_2$ (initial horizontal components)

Then slope $\frac{\Delta y}{\Delta x} = \infty$



The path is a vertical straight line

- For a projectile, 'y' component of velocity at $\frac{1}{n^{th}}$ of the maximum height is $\frac{u\sin\theta}{\sqrt{n}}$
- Resultant velocity at a height of $\frac{1}{n^{th}}$ of maximum height

$$v = \sqrt{v^2{}_x + v^2{}_y} = \sqrt{(u\cos\theta)^2 + \left(\frac{u\sin\theta}{\sqrt{n}}\right)^2}$$

$$= u\sqrt{\frac{(n-1)\cos^2\theta + 1}{n}}$$

If n = 2, velocity of a projectile at half of maximum

height $= u\sqrt{\frac{1+\cos^2\theta}{2}}$

- For a projectile, with respect to stationary frame trajectory is a parabola
- Path of projectile with respect to frame of another projectile is a straight line
- Acceleration of a projectile relative to another projectile is zero
- If a gun is aimed towards a target and the bullet is fired, the moment when the target falls, the bullet will always hit the target irrespective of the velocity of the bullet if it is within the range
- A particle is projected with a velocity $\vec{u} = a\hat{i} + b\hat{j}$ then the radius of curvature of the trajectory of the particle at the

  (i) point of projection is $r = \dfrac{(a^2+b^2)^{3/2}}{ga}$

  (ii) Highest point is $r = \dfrac{a^2}{g}$

- Expression for radius of curvature is

  $$r = \frac{(\text{velocity})^2}{\text{normal acceleration}}$$

  $$r = \frac{u^2\cos^2\theta}{g\cos^3\alpha}$$

  α is angle made by $\vec{v}$ with horizontal

- **If a body is projected with a velocity**

  $\vec{u} = \vec{ai} + \vec{bj} + \vec{ck}$

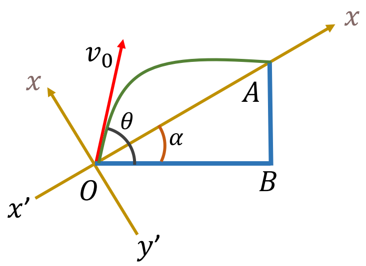
  $(i - \text{east } \hat{j} - \text{north } k - \text{vertical})$ then

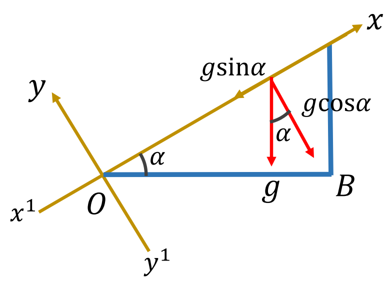
  $u_x = \sqrt{a^2+b^2} : u_y = c$

  $$T = \frac{2c}{g}; H = \frac{c^2}{2g}, R = \frac{2\left(\sqrt{a^2+b^2}\right)c}{g}$$

- **Motion of a Projected Body on an inclined plane:**

  A body is projected up the inclined plane from the point O with an initial velocity v_0 at an angle θ with horizontal.





a) Acceleration along $x-$axis, $a_x = -g\sin\alpha$

b) Acceleration along y axis, $a_y = -g\cos\alpha$

c) Component of velocity along x axis, $u_x = v_0\cos(\theta-\alpha)$

d) Component of velocity along y axis $u_y = v_0\sin(\theta-\alpha)$

e) Time of flight $T = \dfrac{2v_0\sin(\theta-\alpha)}{g\cos\alpha}$

f) Range of projectile (OA)

$$R = \frac{v^2_0}{g\cos^2\alpha}\left[\sin(2\theta-\alpha)-\sin\alpha\right] \quad \text{(or)}$$

$$R = \frac{2v^2_0\sin(\theta-\alpha)\cos\theta}{g\cos^2\alpha}$$

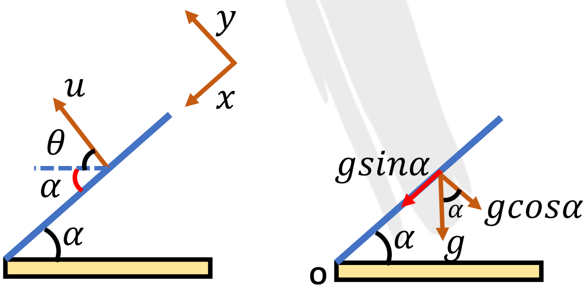
For maximum range $(2\theta-\alpha) = \dfrac{\pi}{2}$

$$\therefore R_{max} = \frac{v_0^2(1-\sin\alpha)}{g\cos^2\alpha}$$

g) $T^2 g = 2R_{max}$

horizontal range (OB) $x = R\cos\alpha$

Down the plane: Here, x and y-directions are down the plane and perpendicular to plane respectively



$u_x = u\cos(\alpha+\theta), a_x = g\sin\alpha$

$u_y = u\sin(\alpha+\theta), a_y = -g\cos\alpha$

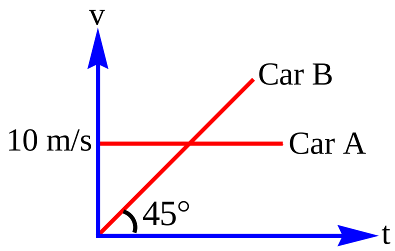
Proceeding in the similar manner, we get the following results

$$T = \frac{2u\sin(\alpha+\theta)}{g\cos\alpha}$$

$$R = \frac{u^2}{g\cos^2\alpha}\left[\sin(2\theta+\alpha)+\sin\alpha\right]$$

EXERCISE 1

1. Initially car A is 10.5 m ahead of car B. Both cars start moving at time t = 0 in the same direction along a straight path. The velocity time graph of two cars is shown in figure. The time when the car B will catch the car A, will be ________ (in secs)



2. A stone is projected vertically upward with initial velocity 25 m/s. For its motion during third second, which of the following statement is correct?

(A) Displacement of the stone is 30 m

(B) Distance covered by the stone is 30 m

(C) Distance covered by the stone is 2.5 m

(D) None of the above

3. A bus, moving with a speed of 50 km h^{-1}, can be stopped by brakes in minimum 6m distance. If the same bus is moving at a speed of 100 km h^{-1}, the minimum stopping distance is (in meters)

4. A ball is thrown vertically upwards with a velocity u from the balloon descending with velocity V. The ball will pass by the balloon after time.

(A) $\frac{u-V}{2g}$ (B) $\frac{u+V}{2g}$

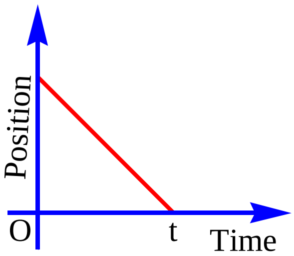
(C) $\frac{2(u-V)}{g}$ (D) $\frac{2(u+V)}{g}$

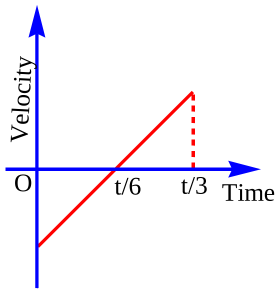
5. During a trip Shyam walks a litter faster than Ram, who started off earlier. Shyam observes that he reaches a certain point time t_1 later than Ram. Then he catches up with Ram after another lapse of time t_2. The ratio of v_{Shyam} to v_{Ram} is :

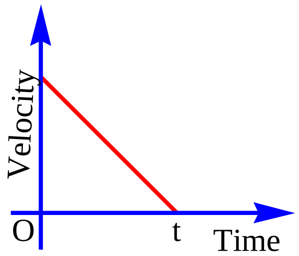
(A) $\frac{t_2}{t_1+t_2}$ (B) $\frac{t_1+t_2}{t_2}$

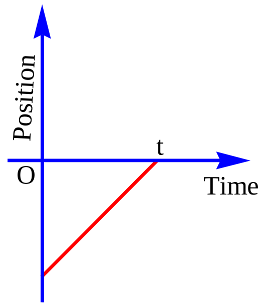
(C) $\frac{t_2}{t_1}$ (D) $\frac{t_1}{t_2}$

6. For which of the following graphs the average velocity of a body moving along a straight line for time interval (0, t) must be negative.

(A) 

(B) 

(C) 

(D) 

7. The equation of the path of the particle is $y = 0.5x - 0.04x^2$. The initial speed of the projectile is :

(A) 10 m/s (B) 15 m/s (C) 12.5 m/s (D) None of these

8. A particle is projected with velocity 20 m/s at an angle of $60°$ with the horizontal. Find the time interval between the two locations of particle where velocity of particle makes an angle of $30°$ with horizontal.

(A) 1.15 sec (B) 0.95 sec (C) 1 sec (D) 1.5 sec

9. Suppose a player hits several tennis balls. Which tennis ball will be in the air for the longest time?

(A) The one with the farthest range

(B) The one which reaches maximum height

(C) The one with the greatest initial velocity

(D) The one leaving the bat at $45°$ with respect to the ground

10. Suppose that a particle moving in three-dimensions has a position vector $\vec{R} = (4+2t)\hat{i} + (3+4t)\hat{j} + (2+2t+3t^2)\hat{k}$ where distance is measured in metres and time in seconds. Mark the incorrect statement :

(A) Instantaneous velocity vector is $2\hat{i} + 4\hat{j} + (2+6t)\hat{k}$

(B) Instantaneous acceleration is $+6\hat{k}$

(C) At t = 0 velocity vector is $2\hat{i} + 4\hat{j} + 2\hat{k}$

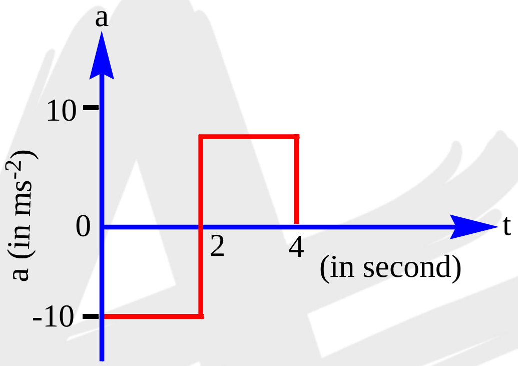
(D) At t = 0 acceleration is zero

11. From the top of a tower, a stone is thrown up. It reaches the ground in time t_1. A second stone thrown down with the same speed reaches the ground in time t_2. A third stone released from rest reaches the ground in time t_3. Then :

(A) $t_3 = \frac{t_1 + t_2}{2}$ (B) $t_3 = \sqrt{t_1 t_2}$

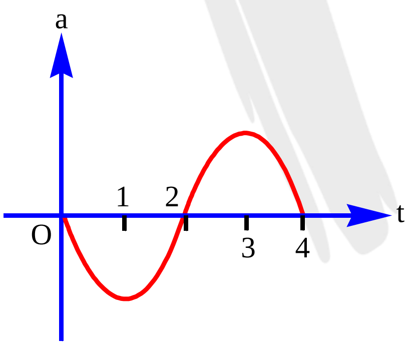
(C) $\frac{1}{t_3} = \frac{1}{t_1} - \frac{1}{t_2}$ (D) $t_3^2 = t_1^2 - t_2^2$

12. A particle starts from rest at time t = 0 and moves on a straight line with acceleration as plotted in figure. The speed of the particle will be maximum at time :



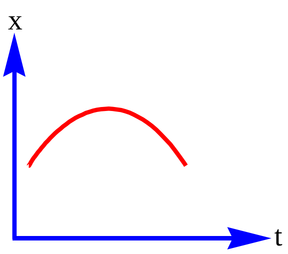
(A) 1 s (B) 2 s (C) 3 s (D) 4 s

13. Acceleration (a)-time(t) graph for a particle starting from rest at t = 0 is as given aside. The particle has maximum speed at :



(A) 1 s (B) 2 s (C) 3 s (D) 4 s

14. Displacement time curve for a particle moving along a straight line is shown. It can be concluded that :



(A) Particle has positive acceleration in initial and negative acceleration in later part of motion

(B) Particle has negative acceleration in initial and positive acceleration in later part of motion

(C) Particle has negative acceleration during whole course of motion

(D) Particle has positive acceleration during whole course of motion

15. A particle moves along a straight line in such a way that its acceleration is increasing at the rate of $2\,m/s^3$. Its initial acceleration and velocity were zero. Then, the distance which it will cover in the 3rd second is $\frac{\alpha}{3}$. Find α

(A) 19 (B)17 (C) 21 (D) 23

16. A bird flies for 4 sec from t = 0 with a velocity of $|t-2|\ m/s$ in a straight line, where t = time in seconds. It covers a distance α meters. Find α?

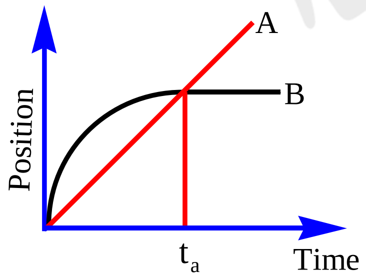
17. Rectilinear motion of a particle is defined by the equation $V^3 = x^2$ (where V is velocity). The acceleration of the particle when x = 8 m is $\frac{\alpha}{\beta} m/s^2$. Find $(\alpha+\beta)$ and (α, β are integers)

18. A point moves in x-y plane according to the law $x = 3\cos 4t$ and $y = 3(1-\sin 4t)$. The distance (in meters) travelled by the particle in 2 sec is (where x and y are in meters) is

19. Some people hire a boat for 4 hours. The speed of the river is 5 km/hr and the speed of the boat is 15 km/hr relative to the water. How far along the flow direction, they can go if they have to return in 4 hrs?

(A) 5 km (B) 7.5 km (C) $\frac{80}{3}$ km (D) 15 km

20. The graph shows position as a function of time for two trains running on parallel tracks. Which one of the following statement is true?



(A) At time t_B both trains have the same velocity

(B) Both trains have the same velocity at some time

(C) Both trains have the same velocity at some time before t_B

(D) Somewhere on the graph, both trains have the same acceleration

EXERCISE 2

1. Two bodies 1 and 2 are moving with velocities $\vec{v}_1 = 4\hat{i} - 3\hat{j}\ m/s$ and $\vec{v}_2 = b\hat{i} - \hat{j}\ m/s$ respectively. The position vectors of the bodies at time t = 0 are $\vec{r}_1 = 5\hat{i} + 2\hat{j}\ m$ and $\vec{r}_2 = -4\hat{i} - 4\hat{j}\ m$. If they collide at t = 3s, the value of b is _______

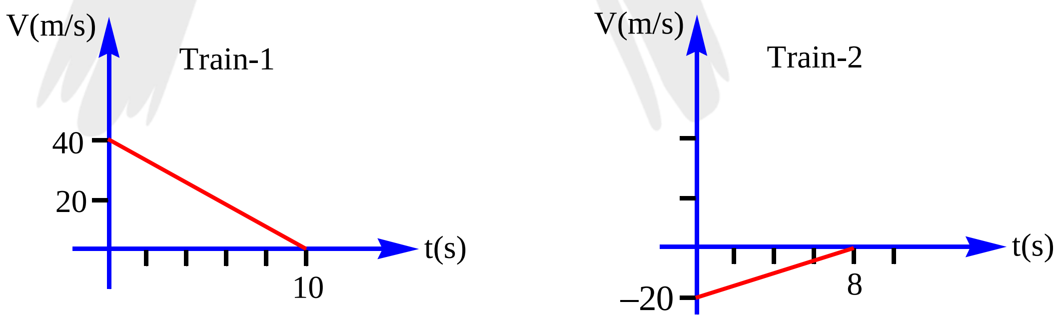
2. An inclined plane of height 10m and angle of inclination $30°$ with the horizontal is having a groove at an angle of $45°$ with the line of greatest slope. A particle is released from rest from the top of incline plane in the groove, the time taken by the particle to reach the bottom of the groove is $\left(g = 10 m/s^2\right)$

(A) 1 sec (B) 2 sec (C) 3 sec (D) 4 sec

3. A car starts from rest with uniform acceleration. Its velocity after 2n second in v_0. The displacement of the body in last n seconds is $\frac{\alpha v_0 n}{\beta}$. Find $\alpha + \beta$?

4. A car is moving on a straight road with initial velocity u and uniform acceleration f. If the sum of the distances travelled in t^{th} and $(t+1)^{th}$ seconds is 100 cm, then its velocity after t seconds, in cm/s, is :

5. Two trains, which are moving along different tracks in opposite directions towards each other, are put on the same track by mistake. Their drivers, on noticing the mistake, start slowing down the trains when the trains are 300 m apart. Graphs given below show their velocities as function of time as the trains slow down. The separation between the trains after both the have stopped, is (in meters)



6. Is it possible for the motion of an object along a straight line so that its velocity increases while its acceleration decreases?

(A) No, because if acceleration is decreasing the object will be slowing down

(B) No, because velocity and acceleration must always be in the same direction

(C) Yes, an example would be a falling object near the surface of the moon

(D) Yes, an example would be a falling object in the presence of air resistance

7. A body starts from rest with constant acceleration a and it is then decelerated with a constant value b till it is brought to rest. The total time taken between these two rest positions is t. The maximum speed acquired by the body is $\frac{\alpha a b}{\beta a+\gamma b}$. Find $\alpha+\beta+\gamma=?$

8. A body starts from the origin at t = 0 and moves in the xy plane with constant acceleration a in the y-direction. Its equation of motion is $y=bx^2$. The x-component of its velocity is $\sqrt{\frac{\alpha a}{\beta b}}$.

Find $\alpha+\beta$

9. Two bodies are simultaneously projected in opposite directions horizontally from a given point in space where gravity g is uniform. If u_1 and u_2 be their initial speeds then the time t after which their velocities are mutually perpendicular to each other, is given by

(A) $\frac{\sqrt{u_1u_2}}{g}$ (B) $\frac{\sqrt{u_1^2+u_2^2}}{g}$ (C) $\frac{\sqrt{u_1(u_1+u_2)}}{g}$ (D) $\frac{\sqrt{u_2(u_1+u_2)}}{g}$

10. The velocity at the maximum height of a projectile is half its initial velocity of projection. If magnitude of initial velocity is u, its range on the horizontal plane is $\frac{u^2\sqrt{\alpha}}{\beta g}$. Find $\alpha+\beta$

11. Velocity of an object depends on displacement as $v^{3/2}=k8(y)^{3/4}$, where v is velocity (in m/s), y is displacement (in meter) and k is constant, then acceleration (in m/s^2) when y = 16 is $\alpha k^{\beta/\gamma}$. Find $\alpha+\beta+\gamma$?

(A) 13 (B) 14 (C) 15 (D) 16

12. Acceleration of a particle is defined as $a=(75v^2-30v+3)(m/s^2)$ find constant speed achieved by the particle :

(A) 3 m/s (B) $\frac{1}{5}m/s$

(C) 5 m/s (D) It will never achieve constant speed

13. There are two values of time for which a projectile is at the same height (say h) from ground. If T is the total time of flight for this projectile on level ground, then the sum of these two time is $T=\frac{\alpha u_y}{g}$. Find α?

(A) 4 (B) 3 (C) 1 (D) 2

14. A body starts from origin such that its velocity varies as $\vec{v} = \left(\frac{x^2}{2}\hat{i} + \frac{y^2}{2}\hat{j}\right)$, the path of body will be :

(A) straight line

(B) in the plane of $x - y$

(B) in 3D space (X, Y, Z)

(D) particle will not move

15. A jogger Ravi runs with constant velocity v through a forest of coconut trees. A coconut starts to fall from a height h when the jogger Ravi is directly below it. The coconut will fall behind the jogger Ravi is $\sqrt{\frac{\alpha h v^2}{\beta g}}$. Find $\alpha + \beta$?

(A) 3 (B) 2 (C) 1 (D) 4

16. A body is projected up so that it can reach maximum height H. The height from ground when its velocity is half the maximum velocity is $\frac{\alpha H}{\beta}$. Find $\alpha\beta$?

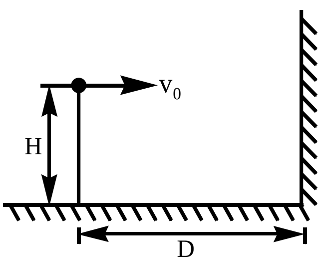
(A) 4 (B) 8 (C) 12 (D) 16

17. A person throws vertical up n balls per second with the same velocity. He throws a ball whenever the previous one is at its highest point. The height to the balls rise is $\frac{g}{\alpha n^{\beta}}$. Find $\alpha + \beta$?

18. On an inclined plane inclined at an angle of $30°$ to the horizontal, a ball is thrown upwards with a velocity of 10 m/s, at an angle of $60°$ to the inclined plane. Its range on the inclined plane is :

(A) 10 m (B) $5\sqrt{3}$ m (C) 0 m (D) $10\sqrt{3}$ m

19. A ball is projected horizontally from a table such that it collides with wall and then with ground. If after collision component of velocity perpendicular to the surface is reversed in direction without change in magnitude and component of velocity parallel to the surface remains unchanged. At what distance from the wall does the ball collide with the ground. (H = 500 m, $v_0 = 20\,m/s$, $D = 100\,m$)



(A) 100 m (B) 200 m (C) 50 m (D) 150 m

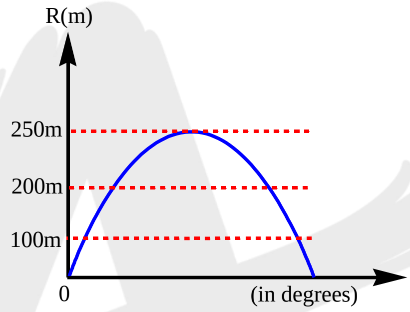
20. A body is dropped from the top of a tall cliff and n second later another body is thrown vertically downwards with a velocity u. Then the second body overtakes the first, below the top of the cliff at a distance given by :

(A) $\frac{g}{2}\left[\frac{n\left(\frac{gn}{2-u}\right)}{(gn-u)}\right]^2$ (B) $\frac{g}{3}\left[\frac{n\left(gn-\frac{u}{2}\right)}{(gn-u)}\right]^2$ (C) $g\left[\frac{(gn-u)}{\left(gn-\frac{u}{3}\right)}\right]^2$ (D) None of these

EXERCISE -3

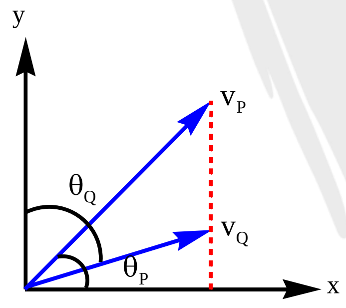
1. The horizontal range of a projectiles is R and the maximum height attained by it is H. A strong wind now begins to blow in the direction of motion of the projectile, giving it a constant horizontal acceleration $= g/2$. Under the same conditions of projection, the horizontal range of the projectile will now be $\alpha R + \beta H$. Find $\alpha + \beta$

2. From the ground level, a body is to be shot with a certain speed. Graph shows the range R it will have versus the launch angle θ. The least speed the body will have during its flight if θ is chosen such that the flight time is half of its maximum possible value, is equal to (Take $g = 10\ m/s^2$) :



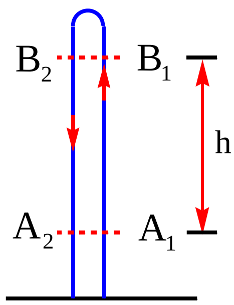
(A) 250 m/s (B) $50\sqrt{3}\,m/s$ (C) 50 m/s (D) $25\sqrt{3}\,m/s$

3. Two projectiles are projected with velocity v_P, v_Q at angles θ_P (from horizontal) and θ_Q (from vertical) as shown in the given figure, such that $v_P > v_Q$ but having same horizontal component of velocity. Which of the following cannot be correct ?



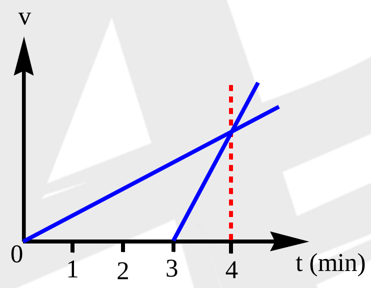
(A) $T_P > T_Q$ (B) $H_P > H_Q$ (C) $R_P > R_Q$ (D) $R_Q > R_P$

4. The acceleration of gravity can be measured by projecting a ball upward and measuring the time that it takes to pass two given lines in both directions (upward motion and downward motion). If the time the ball takes to pass a horizontal line A in both direction (from A_1 to A_2) is T_A, and the time to go by a second line B in both directions is (from B_1 to B_2) T_B, then assuming that the acceleration due to gravity to be constant :



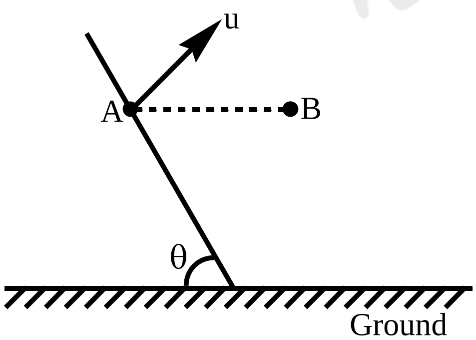
(A) $\frac{8h}{T_A^2 - T_B^2}$ (B) $\frac{8h}{T_A^2 + T_B^2}$ (C) $\frac{8h}{T_A^2 T_B^2}$ (D) $\frac{8hT_A T_B}{T_A^2 T_B^2}$

5. The drawing shows velocity (v) versus time (t) graphs for two cyclists moving along the same straight segment of a highway from the same point. The second cyclist starts moving at t = 2 min. At what time do the two cyclists meet?



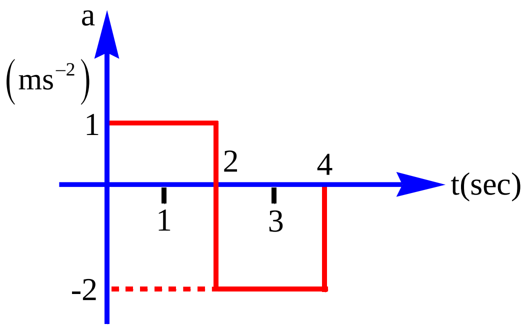
(A) 4 min (B) 6 min (C) 8 min (D) 12 min

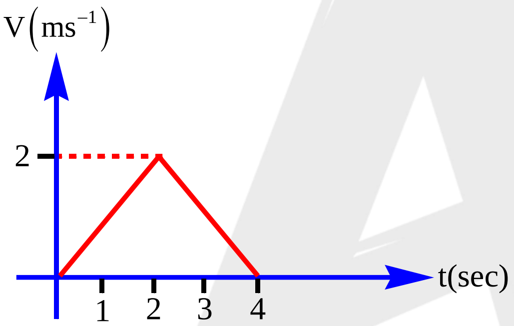
6. A body is projected at point A from an inclined plane with inclination angle θ as shown in figure. The magnitude of projection velocity is u and its direction is perpendicular to the plane. After some time it passes from point B which is in the same horizontal level of A, with velocity $\vec{v}$. Then the angle between $\vec{u}$ and $\vec{v}$ will be :



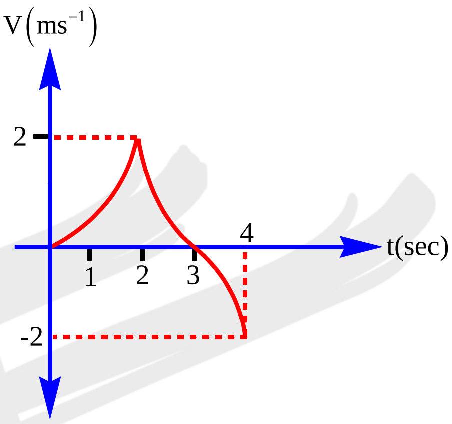
(A) 2π (B) θ (C) $\pi - 2\theta$ (D) $90 - \theta$

7. A particle moves in one dimension. Its velocity is given by V and its acceleration by a. The figure shows its measured acceleration versus time graph. Which of the graph (V vs t) below are consistent with measured acceleration?

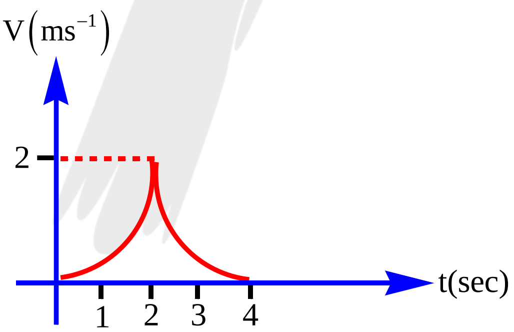




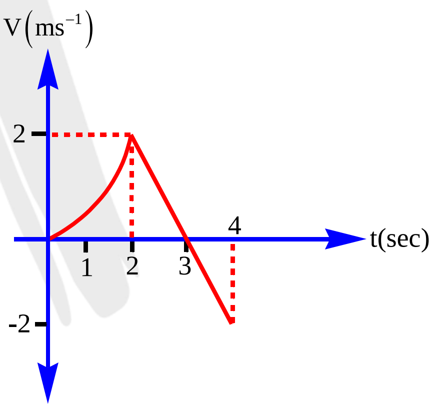
(A)



(B)

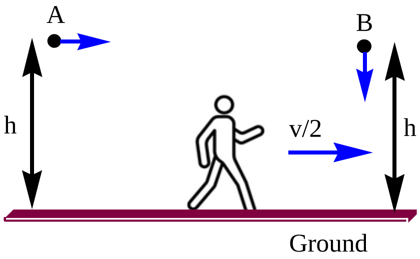


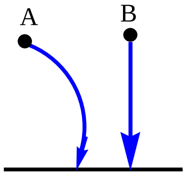
(C)



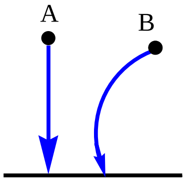
(D)

8. Two identical balls are kept into motion simultaneously from equal heights h. While the ball A is projected horizontally with velocity v, the ball B is just released to fall by itself. Choose the alternative that best represents the motion of A and B with respect to an observer who moves with velocity $\frac{v}{2}$ with respect to the ground as shown in the figure.

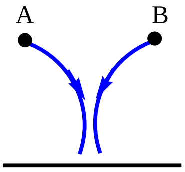




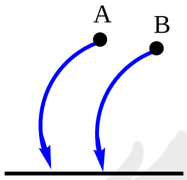
(A)



(B)



(C)



(D)

9. Two swimmers start swimming perpendicular to river flow with different constant speeds at the same time. After reaching the opposite bank they have to walk back to reach the point exactly opposite to the starting point at the same time. Assume constant walking speed for both the swimmers.

(A) Swimmer with greater speed in water will have to walk slowly

(B) Swimmer with smaller speed in water have to walk back slowly

(C) Both the swimmers will have to walk back with the same speed

(D) It is not possible for both swimmer to reach the starting point simultaneously

10. Two boats were going down stream with different velocities. When one overtook the other a plastic ball was dropped from one of the boats. Sometime later both boats turned back simultaneously and went at the same speeds as before (relative to the water) towards the spot where the ball had been dropped. Which boat will reach the ball first?

(A) The boat which has greater velocity (relative to water)

(B) The boat which has lesser velocity (relative to water)

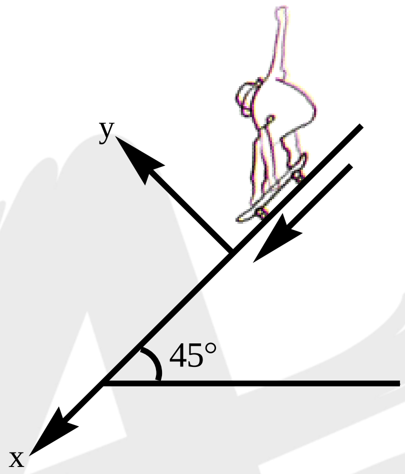
(C) Both will reach the ball simultaneously

(D) Cannot be decided unless we know the actual values of the velocities and the time after which they turned around

11. A river is flows with velocity 2 m/s. A boat is moving downstream. Velocity of boat in still water is 3 m/s. A man standing on boat throws a ball vertically upwards w.r.t. himself with a velocity of 10 m/s. At the top most point, the velocity of ball w.r.t. man standing on boat w.r.t. river and w.r.t. ground respectively is :

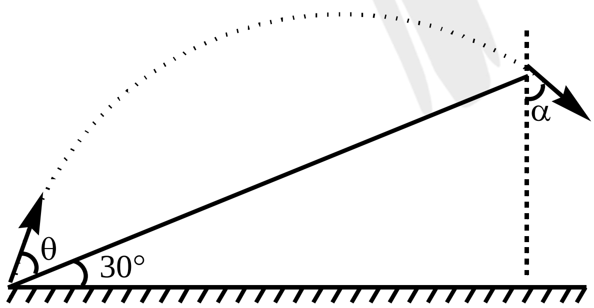
(A) 5, 3, 0 m/s (B) 0, 3, 5 m/s (C) 0, 5, 3 m/s (D) None of these

12. A skater on skateboard is coming down on a smooth incline. He throws a ball such that he catches it back. What should be unit vector of the ball's velocity relative to him.

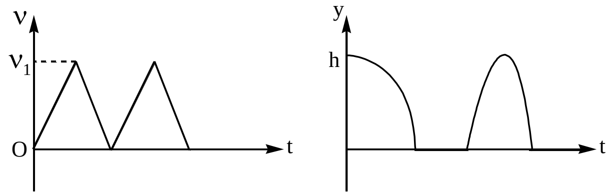


(A) $\hat{j}$ (B) $\frac{\hat{i}}{\sqrt{2}}+\frac{\hat{j}}{\sqrt{2}}$ (C) $-\frac{\hat{i}}{\sqrt{2}}+\frac{\hat{j}}{\sqrt{2}}$ (D) None of these

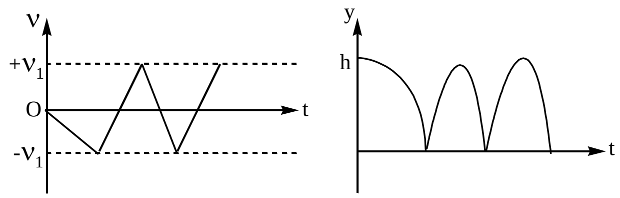
13. A particle is projected at an angle $\theta = \tan^{-1}\left(\frac{\sqrt{3}}{2}\right)$ with the inclined plane of angle $30°$ with horizontal. If α is the angle that its velocity vector makes with vertical at the point of collision (with the inclined plane) if $\alpha = 5K°$. Find the value of 'K'.



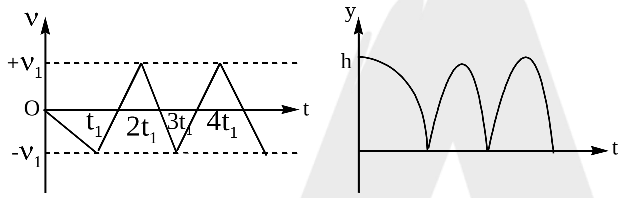
14. Consider a rubber ball freely falling from a height h = 4.9 m onto a horizontal elastic plate. Assume that the duration of collision in negligible and the collision with the plate is totally elastic. Then the velocity as a function of time the heights as function of time will be



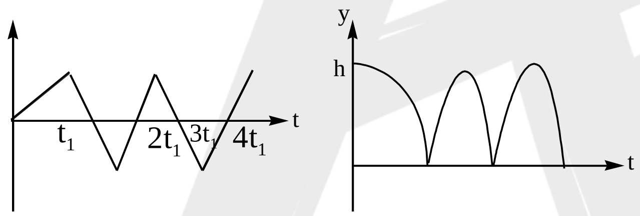
(A)



(B)

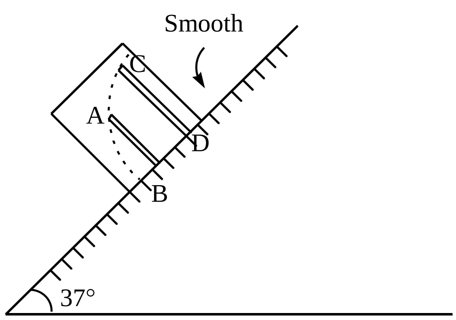


(C)



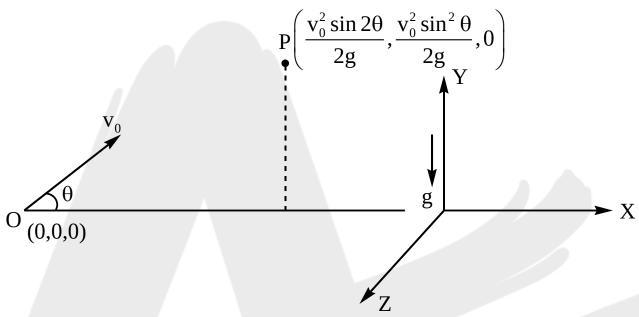
(D)

15. Two plates AB and CD are rigidly attached perpendicular to base of a massive box. Box is released on smooth inclined plane having angle of inclination $37°$.with what minimum speed with respect to box a particle is projected so as to graze top point A and C of plate AB = 2m, CD = 3m. Given separation between top of plate is 3m. Take $g = 10 m/s^2$.



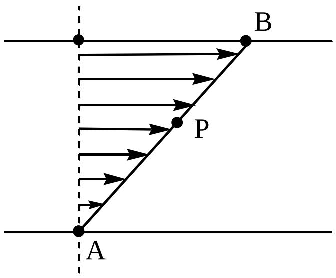
16. Two particles A and B are located at points $(0,-10\sqrt{3})$ and $(0,0)$ in xy plane. They start moving simultaneously at time t = 0 with constant velocities $\vec{V}_A = 5\hat{i}\,m/s$ and $\vec{V}_B = -5\sqrt{3}\hat{j}\,m/s$, respectively. Time when they are closest to each other is found to be K/2 second. Find K. All distances are given in meter.

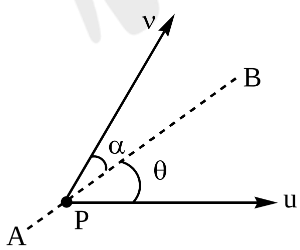
17. A particle of mass m is projected from O with velocity $v_0\cos\theta\hat{i} + v_0\sin\theta\hat{j}$. At the same instant, another particle of mass 2m is projected from point P with $v_0\hat{k}$. The velocity of 2nd particle w.r.t first particle after time $t = \frac{v_0\sin\theta}{g}$ is



(A) $v_0\hat{k} + gt\hat{j} + v_0\cos\theta\hat{i}$
(B) $v_0\hat{k} - gt\hat{j} - v_0\cos\theta\hat{i}$
(C) $v_0\hat{k} - v_0\cos\theta\hat{i}$
(D) None of the above

18. A river is flowing horizontally with a constant velocity gradient along its width. Its velocity from one bank to another varies from zero to u. A swimmer swims with a constant speed v w.r.t water such that he is always heading along the line AB w.r.t ground as shown. If the angle made by velocity of swimmer in still water varies from $45°$ to $75°$ with the river flow while going from A to B, if this angle is $\sin^{-1}\left(\frac{1}{N}\right)$ with line AB at the mid-point of the river, find 'N'.

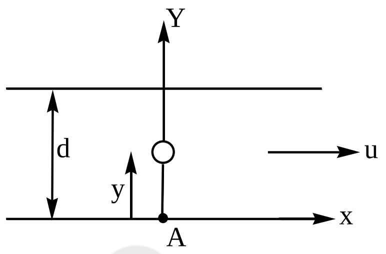




19. A point mass performs straight line motion along positive axis. At t = 0 point mass is at point $A(x_1, 0)$. It moves such that its velocity is given by $v = \frac{a}{x}$, where a is positive constant and x is the x-coordinate of position vector of point mass at a certain time t. Find the time required to move from A to B $(x_2, 0)$

(A) $\frac{x_2^2 - x_1^2}{2a}$ (B) $\frac{x_2^2 - x_1^2}{a}$ (C) $\frac{2x_2^2 - x_1^2}{2a}$ (D) $\frac{2x_2^2 - x_1^2}{a}$

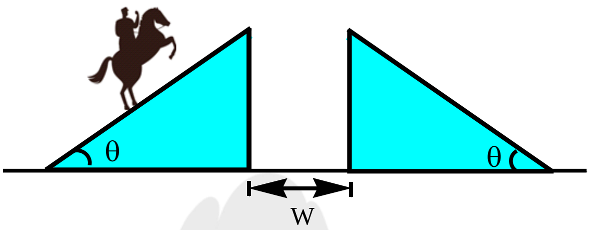
20. A river of width d is flowing with a velocity u. A person starts from point A. He always try to keep himself along y-axis. Speed of man with respect to river at any position is given by $v = k\sqrt{y}$ (k→ +ve constant). Time taken by man to cross the river is (Assume that at t = 0, y = 0)



(A) $\sqrt{\frac{d}{k}}$ (B) $2\sqrt{\frac{d}{k}}$ (C) $\frac{2\sqrt{d}}{k}$ (D) $\frac{2d}{\sqrt{k}}$

EXERCISE 4

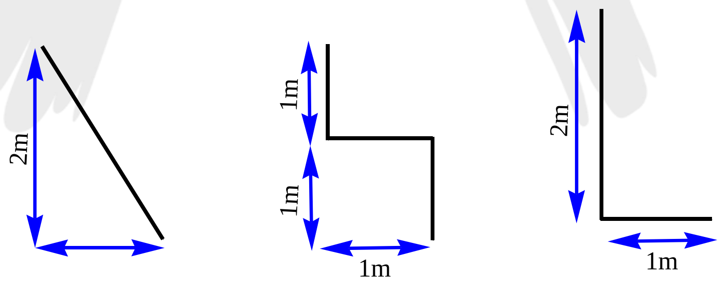
1. A man is riding on a horse. He is trying to jump the gap between two symmetrical ramps of snow separated by a distance W as shown in the figure. He launches off the first ramp with a speed V_L. The man and the horse have a total mass m and their size is small as compared to W. The value of initial launch speed V_L which will put the horse exactly at the peak of the second ramp is :



(A) $\sqrt{\dfrac{Wg}{\sin\theta\times\cos\theta}}$ (B) $\sqrt{\dfrac{Wg}{\sin\left(\frac{\theta}{2}\right)\times\cos\left(\frac{\theta}{2}\right)}}$

(C) $\sqrt{\dfrac{Wg}{2\sin\theta\cos\theta}}$ (D) $\sqrt{\dfrac{2Wg}{\sin\theta\cos\theta}}$

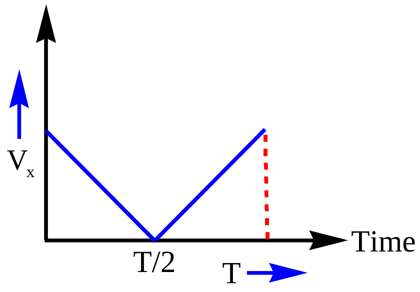
2. A body is pulled downwards by gravity, starting from rest the position labeled A, 2 meters above the ground. The body is constrained to exactly follows each of the frictionless paths shown. Rank these three paths from shortest to longest in terms of the amount of time (T) necessary for the object to travel to point B. Assume that in changing direction that there is no loss of energy and spends negligible time.



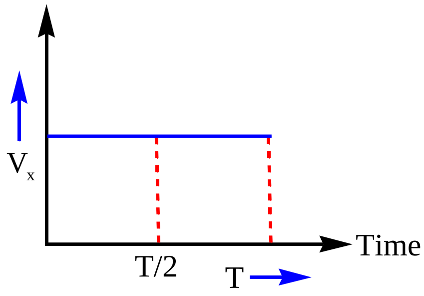
(A) $T_1 < T_3 < T_2$ (B) $T_3 = T_2 < T_1$ (C) $T_1 < T_2 = T_3$ (D) $T_3 < T_2 < T_1$

3. A helicopter is flying horizontally at 8 m/s at an altitude 180 m when a package of emergency medical supplies is ejected horizontally backward with a speed of 12 m/s relative to the helicopter. Ignoring air resistance the horizontal distance between the package and the helicopter when the package hits the ground ________ (in meters)

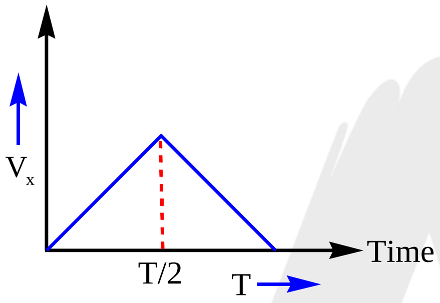
4. A particle is projected with speed u at an angle θ from horizontal at t = 0. Its horizontal component of velocity (v_x) varies with time as following graph :



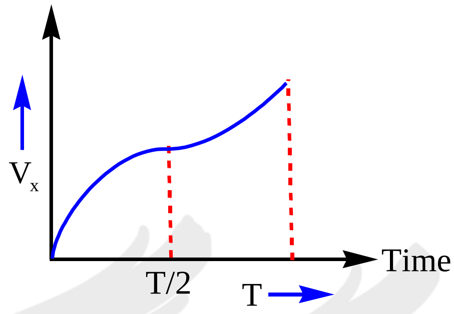
(A)



(B)

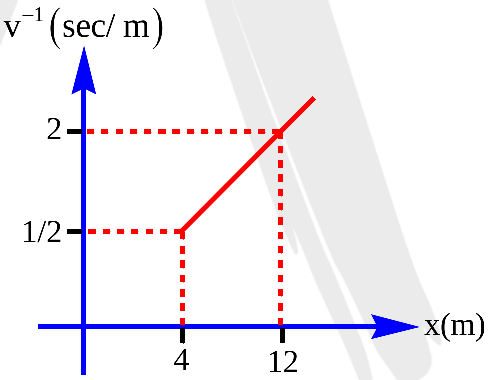


(C)

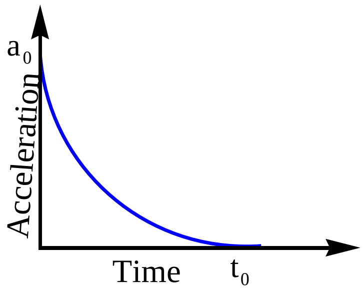


(D)

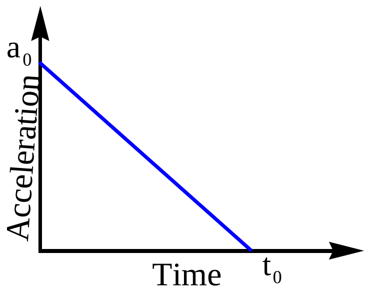
5. Graph of (1/v) vs x for a particle under motion is as shown, where v is velocity and x is position. The time taken by particle to move from x = 4 m to x = 12 m is ________ (in sec)



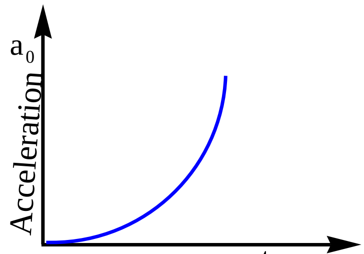
6. Acceleration versus time graphs for four objects are shown below. All axes have the same scale. Which object has the greatest change in velocity during the interval ?



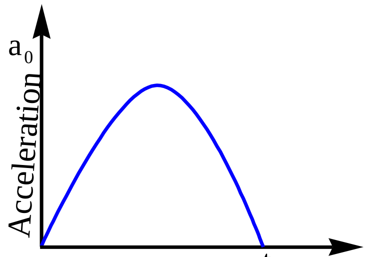
(A)



(B)

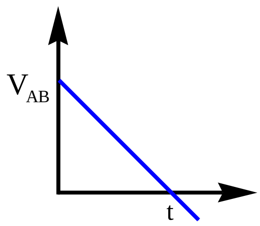


(C)

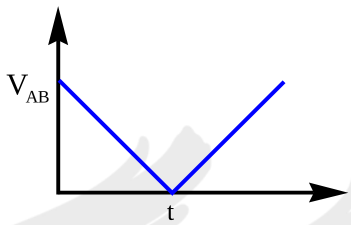


(D)

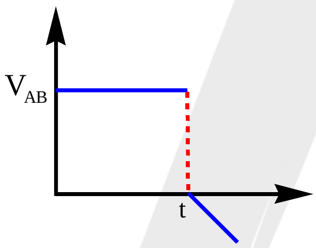
7. A body A is thrown vertically upwards with such a velocity that it reaches a maximum height of h. simultaneously another body B is dropped from height h. It strikes the ground and does not rebound. The velocity of A relative to B v/s time graph is best represented by : (upward direction is positive)



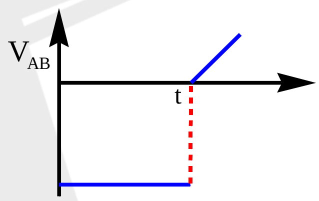
(A)



(B)



(C)



(D)

8. Two transparent elevator cars A and B are moving in front of each other. Car A is moving up and retarding at a_1, while car B is moving down and retarding at a_2. Person in car A drops a coin inside the car. What is the acceleration observed by person in car B?

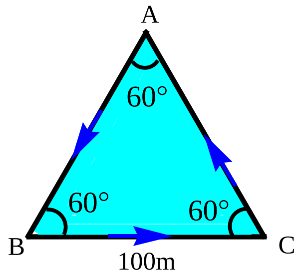
(A) $g + a_2$ downward

(B) $g - a_1 - a_2$ downward

(C) $g - a_1 + a_2$ downward

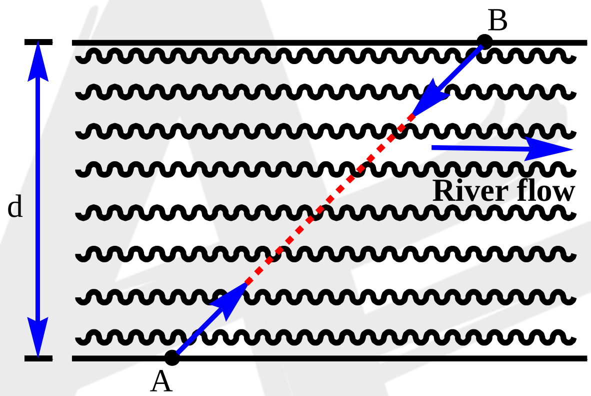
(D) None of the above

9. Three boys are running on a triangular track with same speed 5 m/s. At start, they were at the three corners with velocity along indicated directions. The velocity of approach of any one of them towards another, at t = 10 s equals :



(A) 7.5 m/s (B) 10 m/s (C) 5 m/s (D) 0 m/s

10 . Figure shows two swimmers starting from point A and B on opposite banks. They started at same instant with a constant velocity. Both of them are swimming in a direction parallel to line AB always. The river flows towards east.



(A) Swimmers A and B cannot collide

(B) Swimmers A and B will definitely collide somewhere on line AB

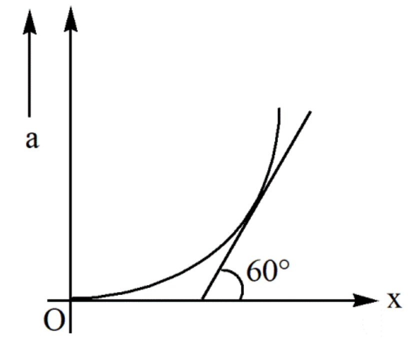
(C) Swimmers A and B will definitely collide somewhere to the right of line AB

(D) Swimmers A and B will definitely collide somewhere to the left of line AB

11. A particle is moving in x-y plane. At certain instant the components of its velocity and acceleration are as follows $V_x = 3m/s, V_y = 4m/s, a_x = 2m/s^2$ and $a_y = 1m/s^2$. The rate of change of speed at this moment is.

(A) $4m/s^2$ (B) $2m/s^2$ (C) $\sqrt{3}m/s^2$ (D) $\sqrt{5}m/s^2$

12. A particle starts moving with initial velocity 3 m/s along x-axis from origin. Its acceleration is varying with x in parabolic nature as shown in figure. At $x = \sqrt{3}m$ tangent to the graph makes an angle $60°$ with positive x-axis as shown in diagram. Then at $x = \sqrt{3}$



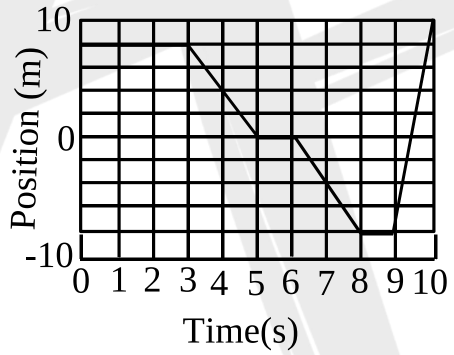
(A) $v=\sqrt{(\sqrt{3}+9)}\,m/s$ (B) $a=1.5m/s^2$

(C) $v=\sqrt{12}m/s$ (D) $a=3m/s^2$

13. From the point on the ground at a distance 30m from the foot of a pole a ball is thrown at an angle of $45°$, which just touches the top of the pole and lands further 60m from the foot of the pole on other side, height of the pole is

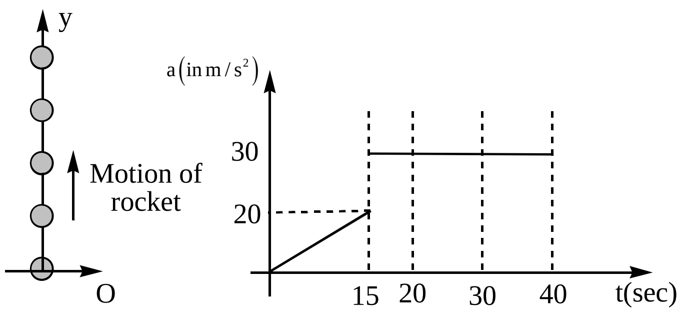
(A) 30m (B) 45m (C) 22.5m (D) 20m

14. The position versus time graph of a particle moving along a straight line is shown. What is the total distance travelled by the particle from t = 0 s to t = 10s?



(A) 2m (B) 18m (C) 26m (D) 34m

15. A two stage rocket is fired vertically upward from rest with acceleration as shown in a-t graphs. After 15 sec, the first stage burns out and second stage ignites. Choose the INCORRECT statement regarding motion of rocket in the time interval $0 \le t \le 40s$.



(A) At t = 15 sec, the speed of rocket is 150 m/s

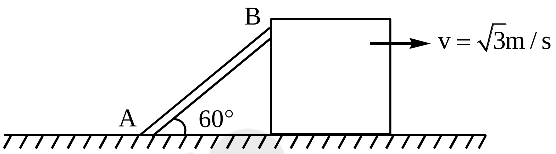
(B) At t = 20 sec, the speed of rocket is 300 m/s

(C) In time interval t = 0 sec to t = 15 sec, the distance travelled by rocket is 750m

(D) In time interval t = 0 sec to t = 20 sec, the distance travelled by rocket is 1500m

16. A rod AB is shown in figure. End A of the rod is fixed on the ground. Block is moving with velocity $\sqrt{3}m/s$ towards right. The speed of end B of the rod when rod makes an angle of $60°$ with the ground is

(A) 3 m/s (B) 2 m/s (C) 5 m/s (D) 2.5 m/s



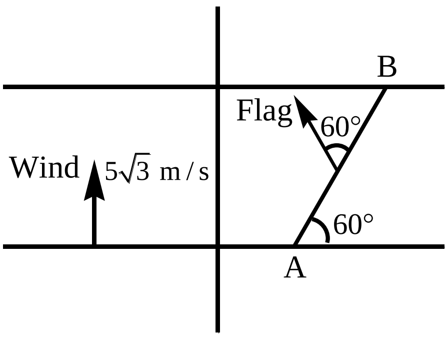
17. Two small boats both having a mass of 150 kg including passengers in it are at rest. A sack of mass 50 kg makes 1st boat having total mass of 200 kg. It is thrown to the second boat with a velocity whose horizontal component is 2 m/s, relative to water. Calculate the distance between the boat 4.5 sec. after the throw if the sack spent 0.5 sec. in air. Neglect resistance of air and water.

(A) 2 m (B) 4 m (C) 6 m (D) 8 m

18. A boat takes 3 hours to go downstream from point A to point B and 6 hours to come back. How long does in take for this boat to cover the distance AB downstream with its engine off ?

(A) 4 hour (B) 8 hour (C) 12 hour (D) 16 hour

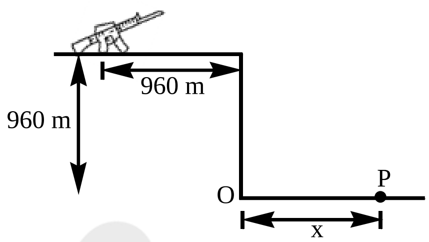
19. A boat travels across a river from a point A to a point B of opposite bank along the line AB forming angle $60°$ with the bank. A flag on the pole (mounted on the boat) flutters at an angle $60°$ with the line AB as shown in figure. If velocity of wind $\left(5\sqrt{3}m/s\right)$ is perpendicular to the flow of river, find the speed (in m/s) of boat with respect to the bank.



20. A ship steaming north at the rate of 12 km/h observes a ship due east to itself and distant 10 km, which steaming due west at the rate of 16 km/h. If they are at least distance after 12α min. Find the value of α.

EXERCISE 5

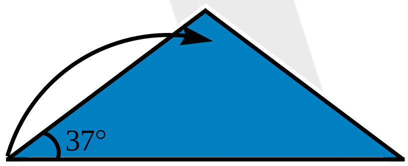
1. A gun is mounted on a plateau 960 m away from its edge as shown. Height of plateau is 960 m. The gun can fires shells with a velocity of 100 m/s at any angle of the following choices, the minimum distance (OP) x from the edge of plateau where the shell of gun can reach is ________ (in meters)



2. A particle at a height h from the ground is projected with an angle 30° from the horizontal, it strikes the ground making an angle 45° with horizontal. It is again projected from the same point with the same speed but with an angle of 60° with horizontal. Angle it makes with the horizontal when it strikes the ground is $\tan^{-1}\sqrt{\alpha}$

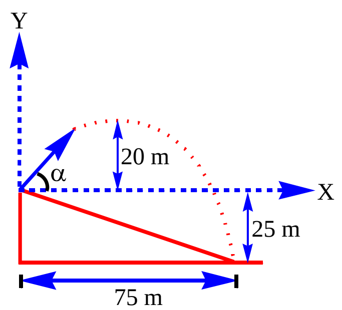
(A) 3 (B) 4 (C) 5 (D) 5

3. A shell is fired at an angle θ to the horizontal such that it strikes the hill while moving horizontally. Find initial angle of projection θ.



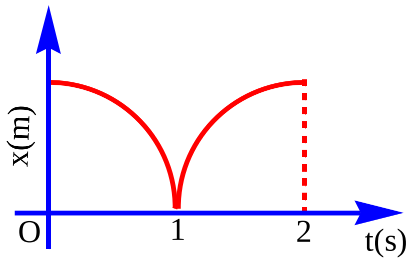
(A) $\tan^{-1}\left(\frac{2}{5}\right)$ (B) $\tan^{-1}\left(\frac{1}{7}\right)$ (C) $\tan^{-1}\left(\frac{3}{2}\right)$ (D) $\tan^{-1}\left(\frac{5}{8}\right)$

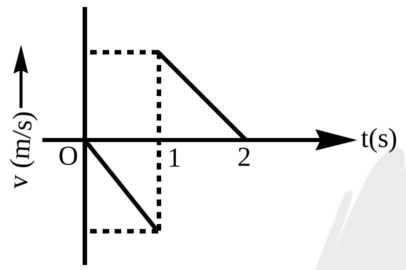
4. A body thrown down the incline strikes at a point on the incline 25 m below the horizontal as shown in the figure. If the body rises to a maximum height of 20 m above the point of projection, the angle of projection α (with horizontal x axis) is $\tan^{-1}\left(\frac{4}{\alpha}\right)$. Find α



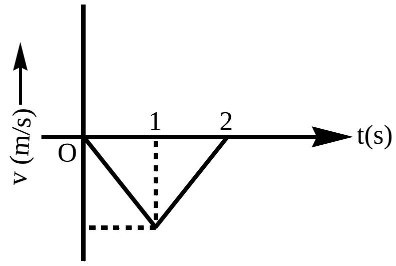
(A) 3 (B) 4 (C) 5 (D) 6

5. The displacement time graph of a moving particle with constant acceleration is shown in the figure. The velocity time graph is best given by :

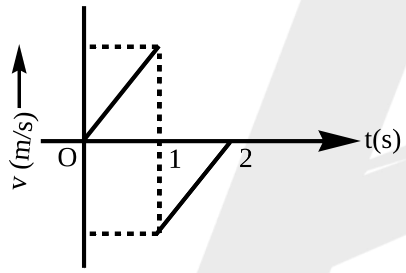




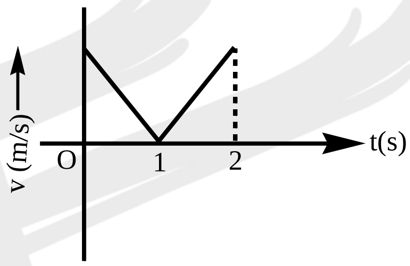
(A)



(B)

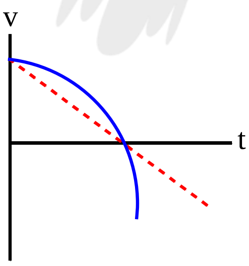


(C)

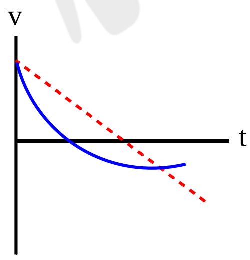


(D)

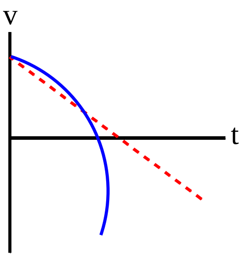
6. You calculate that to throw an object vertically to a height h it needs to be launched with an initial upward velocity v_0, assuming no air resistance. The dashed lines in figure show the motion according to this calculation. Which of the velocity-time graphs shows the motion of an object tossed with initial upward velocity v_1 that will also rise to height h, but this time with air resistance?



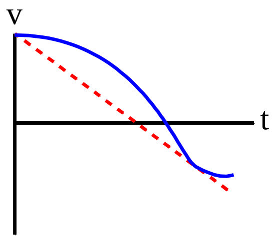
(A)



(B)



(C)



(D)

7. A particle is moving along the locus : $k\sqrt{x}\,(k>0)$ with a constant speed v. At t = 0, it is at the origin and about to enter the first quadrant of x-y axes. At some later time t > 0, $v_x = v_y$. At this moment, $[a_y - a_x]$ is $\frac{-\alpha v^2}{\beta k^2}$. Then $\alpha+\beta$

(A) 1 (B) 2 (C) 3 (D) 4

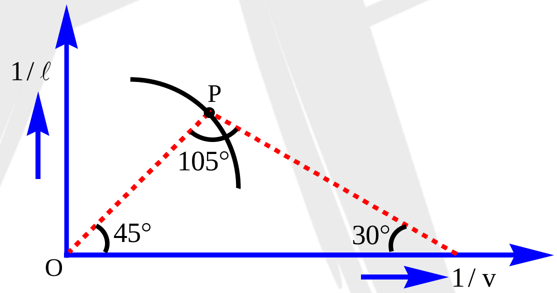
8. An object is moving in the xy plane with the position as a function of time given by $\vec{r} = x(t)\hat{i} + y(t)\hat{j}$. Point O is at $\vec{r} = 0$. The distance of object from O is definitely decreasing when :

(A) $v_x > 0, v_y > 0$ (B) $v_x < 0, v_y < 0$ (C) $xv_x + yv_y < 0$ (D) $xv_x + yv_y > 0$

9. At a given time, an stationary observer on the ground sees a package falling with speed 60m/s at certain angle with the vertical. A pilot in a aeroplane flying at a constant horizontal velocity relative to the ground sees the package falling vertically with a speed 40m/s at the same instant. What is the speed of the aeroplane relative to the ground?

(A)100m/s (B)20m/s (C)$10\sqrt{20}$ m/s (D) $10\sqrt{52}$ m/s

10. For shown graph magnitude of acceleration of particle at point P is :



(A) $\sqrt{3}$ units (B) $\frac{1}{\sqrt{3}}$ units (C) $3\sqrt{31}$ units (D) can't determine

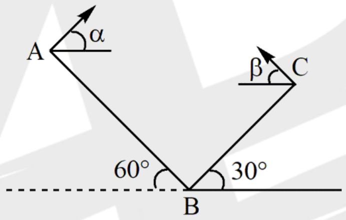
11. A toy train moves due north at a constant speed 2 m/s along a straight track which is parallel to the wall of a room. The wall is to the east of the track at a distance 4 m. There is a toy dart gun on the train with its barrel fixed in a plane perpendicular to the motion of the train. The gun points at an angle $60°$ to the horizontal. There is a vertical line drawn on the wall, stretching from floor to ceiling, and the dart gun is fired at the instant when the line is due east of the gun. If the dart leaves the gun at speed 8 m/s relative to the gun, find the distance by which the dart misses the vertical line. That is, find how far north or south of the vertical line is the point at which the dart hits the wall.

(A) 2 m (B) 3 m (C) 1 m (D) 5 m

12. A particle is placed in a cart. Initial location of the particle is taken as origin in ground frame. At t=0, the cart starts moving along x-axis with uniform velocity 2m/s and at the same instant the particle starts moving with $2x\hat{j}$ relative to cart where x is x-coordinate of the cart. Which of the following is the radius of curvature of the trajectory of the particle in ground frame when it's tangential acceleration is equal to normal acceleration in magnitude?

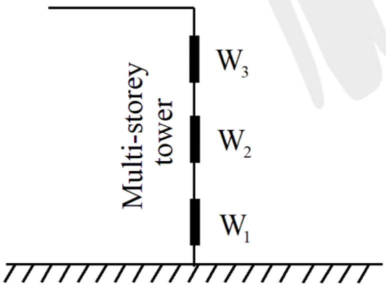
(A) $\frac{1}{\sqrt{2}}$m (B) $\frac{2}{\sqrt{2}}$m (C) $\frac{3}{2\sqrt{2}}$m (D) $\frac{4}{\sqrt{2}}$m

13. Two inclined planes AB and BC are at inclination of $60°$ and $30°$ as shown in the figure. The two projectiles of same mass are thrown simultaneously from A and C with speed 2m/s and v_0 m/s respectively, they strike at B with same speed. If length of AB is $\frac{1}{\sqrt{3}}$m and BC is 1m, then find the value of v_0



(A) (1/2) m/s (B) 1 m/s (C) 2 m/s (D) none of these

14. W_1, W_2 and W_3 are the different sizes of windows 1, 2 and 3 respectively placed in a vertical plane. A particle is thrown up in the vertical plane. Let t_1, t_2 and t_3 are the time taken to cross the window W_1, W_2 and W_3 respectively and $\Delta V_1, \Delta V_2$ and ΔV_3 are the change in speed after respectively window cross.



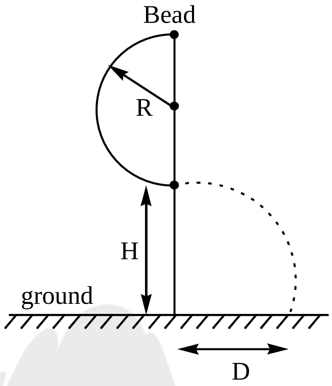
(A) Average speed of the particle passing the windows may be equal if $W_1 < W_2 < W_3$

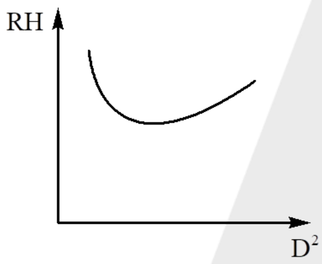
(B) Average speed of the particle passing the windows may be equal if $W_1 > W_2 > W_3$

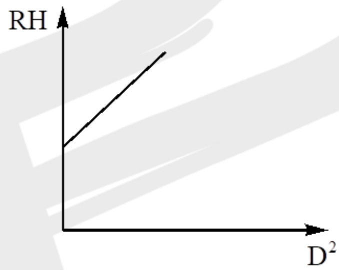
(C) If $W_1 = W_2 = W_3$, the change in speed of the particle while crossing the windows will satisfy $\Delta V_1 < \Delta V_2 < \Delta V_3$

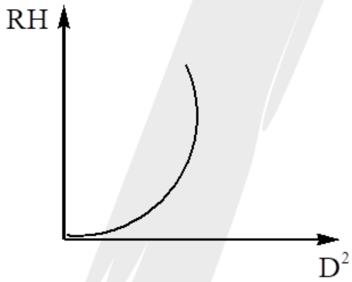
(D) If $W_1 = W_2 = W_3$, the time taken by particle to cross the windows will satisfy $t_1 < t_2 < t_3$

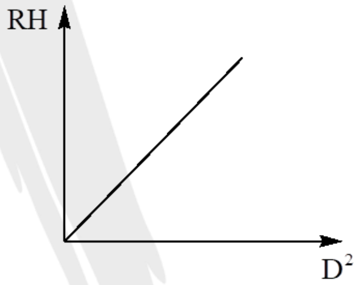
15. A semicircular wire of radius R is oriented vertically. A small bead is released from rest from the top of the wire. It slides without friction under the influence of gravity to the bottom, where it then leaves the wire horizontally and falls distance D away from where it was launched. Which of the following is correct graph of RH versus D^2 ?



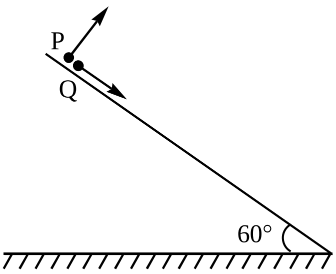
(A)


(B)


(C)


(D)


16. A particle P is projected at t = 0 from a point on the surface of a smooth inclined plane as shown in the figure simultaneously another particle Q is released on the smooth inclined plane from the same position. P and Q collide after t = 4 seconds. Then choose the correct option(s).



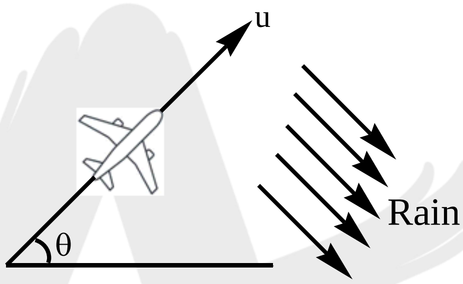
(A) Trajectory of particle P in the frame of Q is parabola during the flight of particle P

(B) Speed of projection of P is 20m/s

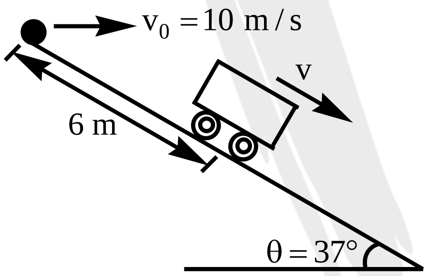
(C) Relative velocity of particle P in the frame of Q changes linearly with time during the flight of P

(D) Acceleration of particle P in the frame of Q is zero during the flight of P

17. Rain is falling with speed $12\sqrt{2}\,m/s$ at an angle of $45°$ with vertical line. A man in a glider going at a speed of v at angle of $37°$ with respect to ground. Find the speed (in m/s) of glider so that rain appears to him falling vertically. Consider motion of glider and rain drop in same vertical plane.

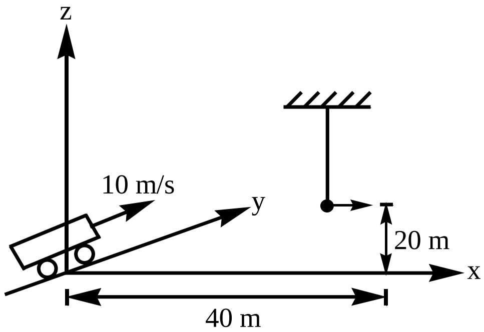


18. A ball is projected horizontally from an incline so as to strike a cart sliding frictionlessly on the incline 6m away (as shown). At the instance the ball is thrown, the speed of cart is 'v' (in m/s). Find 'v' so that the ball strikes the cart. (Neglect height of cart and point of projection of ball above incline.)

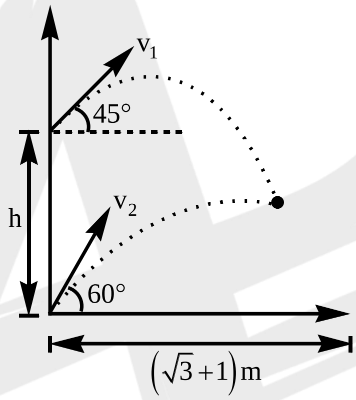


(A) 1 (B) 2 (C) 3 (D) 4

19. A pendulum bob is in x-z plane. A cart is moving along y-axis with velocity 10 m/s from which a ball is to be projected such that it strikes pendulum bob with minimum speed while travelling horizontally in x-z plane. Ball is projected from origin of coordinates. Pendulum bob is located at horizontal distance 40 m from point of projection above x-axis. If ball is projected with velocity $v_x\hat{i} + v_y\hat{j} + v_z\hat{k}$ relative to cart express velocity of ball (in m/s) as sum $\frac{|v_x| + |v_y| + |v_z|}{10}$.

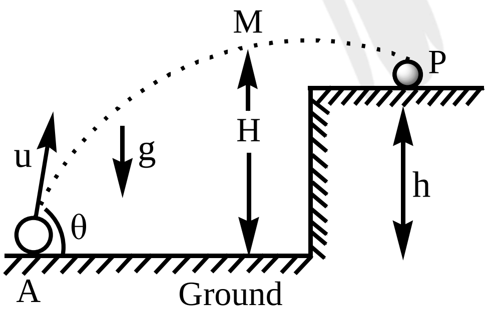


20. Shots are fired from the top of a tower and from bottom simultaneously at angles $45°$ and $60°$ respectively as shown. If horizontal distance of the point of collision is at a distance of $\left(\sqrt{3}+1\right)$m from the tower, then find the height h of tower.



(A) 1 m (B) 2 m (C) 3 m (D) 4 m

21. A ball is projected from ground with initial velocity u at an angle θ (with horizontal) from point A on ground .It strikes a point 'P' at height 'h' from ground level .Provided that the maximum height attained by the balls is H, the angle made by the velocity vector of the ball with horizontal at point P at the moment of striking is:



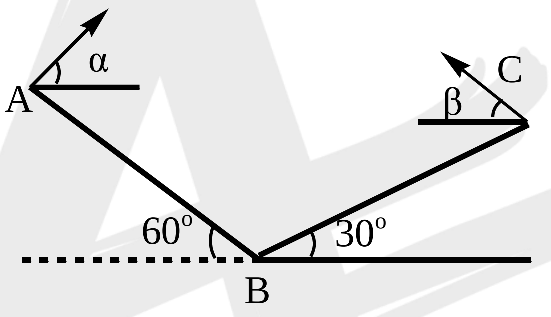
(A) $\tan^{-1}\left(\dfrac{\sqrt{2gh}}{u\cos\theta}\right)$ (B) $\tan^{-1}\left(\dfrac{\sqrt{2gh}}{u\sin\theta}\right)$

(C) $\tan^{-1}\left(\dfrac{\sqrt{2g(H-h)}}{u}\right)$ (D) $\tan^{-1}\left(\dfrac{\sqrt{2g(H-h)}}{u\cos\theta}\right)$

22. Three particles start from the origin at same time. One with a velocity v_1 along x-axis, the second along y-axis with a velocity v_2 and third along the line x = y. The velocity of the third, so that three may always lie on the same line is:

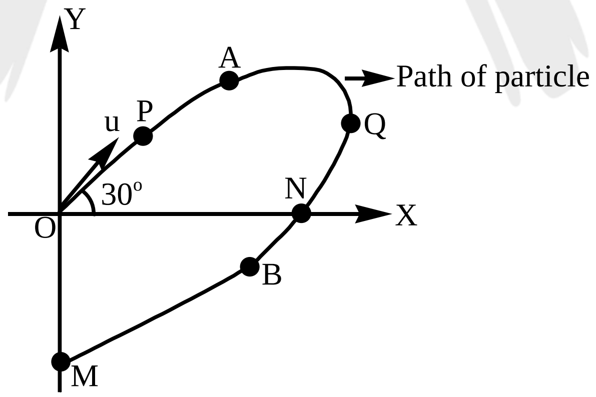
(A) $\frac{v_1 + v_2}{2}$ (B) $\sqrt{v_1 v_2}$ C) $\frac{v_1 v_2}{v_1 + v_2}$ D) $\frac{\sqrt{2} v_1 v_2}{v_1 + v_2}$

23. Two inclined planes AB and BC are at inclination of 60° and 30° as shown in the figure. The two projectiles of same mass are thrown simultaneously from A and C with speed 2 m/s and $v_0\,m/s$ respectively, the strike at B with same speed. If length of AB is $\frac{1}{\sqrt{3}}m$ and BC is 1 m, then find the value of v_0



(A) (1/2) m/s (B) 1 m/s (C) $\frac{2}{\sqrt{3}}$ m/s (D) none of these

24. A particle is projected from origin with speed $u = 50 m/s$ at an angle 30^0 with X-axis. Due to wind particle acquires an extra acceleration $a = 6 m/s^2$ in negative x-direction. Path of particle is shown in figure.



X-coordinates of point A and B are same

Y-coordinates of P and Q are same.

Point N on the X-axis, point M on the Y-axis.

Time taken by particle from O to P,O to Q, O to A and O to B are t_1, t_2, t_3 and t_4 respectively (Assuming Y along vertical and there is no surface in the space) (g=10 m/s^2 and constant).

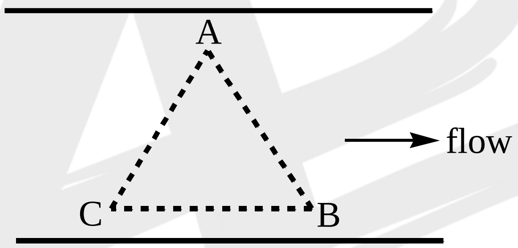
(A) The value of $t_1 + t_2$ is 5sec

(B) The value of $t_3 + t_4$ is 25 sec

(C) The value of ON is $125\sqrt{3}\left(5-\sqrt{3}\right)m$

(D) The length of OM is $\frac{625}{\sqrt{3}}\left(\frac{5}{\sqrt{3}}-1\right)m$

25. A, B and C are three rafts floating in a flowing river such that they always form an equilateral triangle. A swimmer whose swimming speed is constant (w.r.t river) swims from A to B, then B to C and finally C to A along straight lines. If the time taken to go from B to C is t_0, which of the following is/are correct?



(A) Time taken to go from A to B is $\sqrt{3}t_0$

(B) Time taken to go from A to B is t_0

(C) Time taken to go from C to A is $2\sqrt{3}t_0$

(D) Time taken to go from C to A is t_0

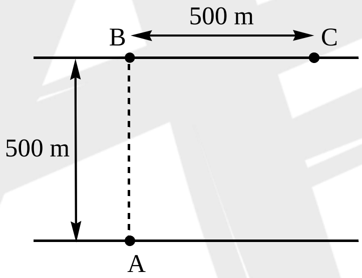
PROFICIENCY TEST 1

1. A body is projected vertically upwards with a velocity u = 5m/s. After time t another body is projected vertically upward from the same point with a velocity v = 3m/s. If they meet in minimum time duration measured from the projection of first body, then $t = \frac{k}{g}\sec$, find k. (where g is gravitational acceleration)

2. A bead moves along straight horizontal wire of length L, starting from the left end with a velocity v_0. Its retardation is proportional to the distance from the right end of the wire. Find the initial retardation (in m/s^2)(at left end of the wire) if the bead reaches the right end of the wire with a velocity $\frac{v_0}{2}$. (given $v_0 = 5m/s$ and $L = 1m$)

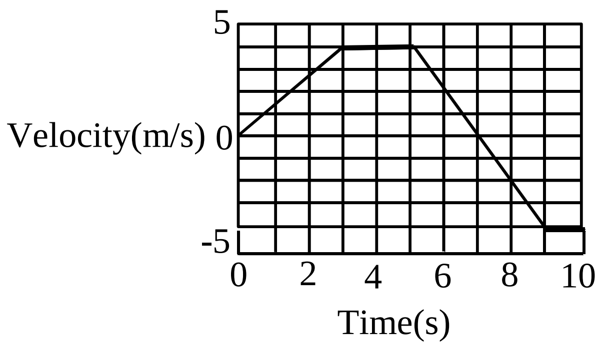
(A) 18.75 (B) 17.75 (C) 16.75 (D) 19.75

3. A river is flowing with a speed of 1m/s. A swimmer wants to go to point C directly from point A. If he swims with a speed of 5m/s at an angle θ with respect to water flow find θ.



(A) 37° (B) 60° (C) 53° (D) 45°

4. An object starts at the origin in a straight line. Its velocity versus time graph is shown in the figure. Which one of the following choices best gives the proper interval(s) of time for which the object is moving away from the origin?



(A) Only for times 0 s < t < 3s (B) Only for times 0 s < t < 5s

(C) Only for times 3s < t < 5s (D) Only for times 0 s < t < 7s

5. A particle is moving along straight line such that dot product of its acceleration $\vec{a}$ and velocity $\vec{v}$ is negative. How many times particle can be found at a distance d from the origin?

(A) One (B) Three

(C) Two (D) Can't be determined with given data

6. Two projectile are thrown at the same time from two different points. The projectile thrown from the origin has initial velocity $3\hat{i}+3\hat{j}$ with respect to earth. The projectile has initial velocity $a\hat{i}+b\hat{j}$ with respect to earth thrown from the point $(10,5)$. ($\hat{i}$ is a unit vector along horizontal, $\hat{j}$ along vertical). If the projectile collides after two second, then the

(A) value of a is -2 (B) value of a is $\frac{1}{2}$

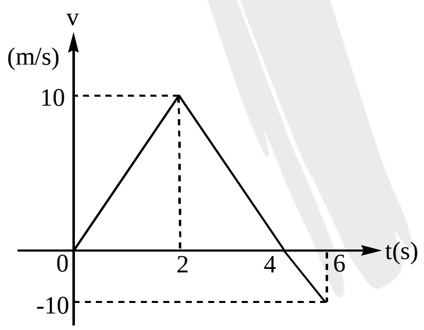
(C) value of b is $\frac{1}{2}$ (D) value of b is -2

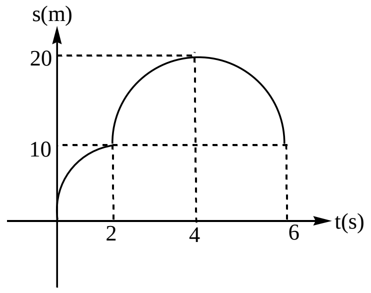
7. A particle is moving with velocity $\vec{v}=K\left(y\hat{i}+x\hat{j}\right)$, where K is a constant. The general equation for its path is

(A) $y=x^2+\text{constant}$ (B) $y^2=x+\text{constant}$

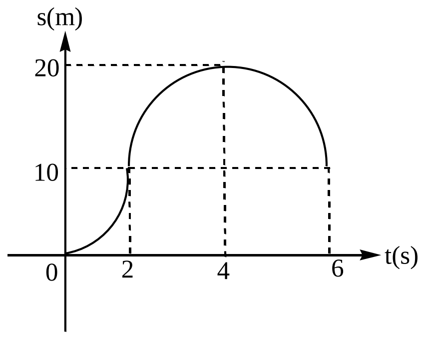
(C) xy = constant (D) $y^2=x^2+\text{constant}$

8. For the given velocity time curve, displacement is zero at t = 0 the corresponding displacement time curve is

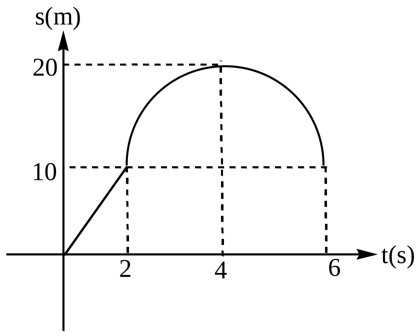




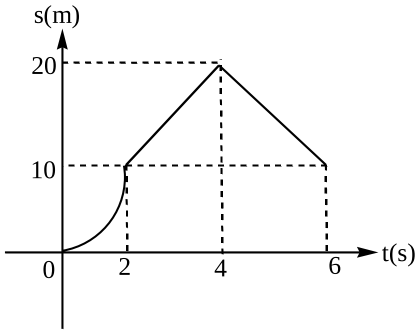
(A)



(B)



(C)



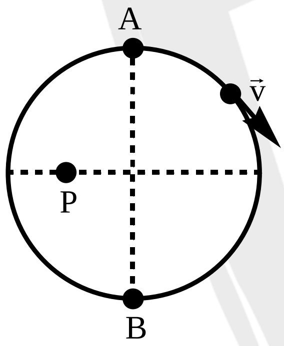
(D)

9. A particle is projected with speed u at an angle θ above horizontal. There are two times for which a projectile is at the same height. If the sum of these two times is equal to $n\dfrac{2u\sin\theta}{g}$, then find the value of 'n'.

10. A particle moves in x-y plane such that its position vector varies with time as $\vec{r}=(2\sin 3t)\hat{i}+2(1-\cos 3t)\hat{j}$. The equation of trajectory of the particle will be

(A) $x^2+y^2=4y$ (B) $x^2+(2-y)^2=4$ (C) $(2-x)^2+y^2=4$ (D) $(1-x)^2+(1-y)^2=4$

11. A particle is moving non-uniformly on a circle in clockwise sense. At all positions on the circle, the acceleration of the particle is always directed towards a fixed point P on the diameter as shown in the figure. Choose the correct statement.



(A) Speed of the particle is increasing at positions A and B

(B) Speed of the particle is increasing at position A and decreasing at position B

(C) Speed of the particle is decreasing at positions A and B

(D) Speed of the particle is decreasing at position A and increasing at position B

12. A swimmer who can swim in a river with speed αv (with respect to still water) where v is the velocity of river current. Jumps into the river from one bank to cross the river.

(A) If $\alpha<1$ he cannot cross the river

(B) If $\alpha\le 1$ he cannot reach a point on other bank directly opposite to his starting point

(C) If $\alpha>1$ he can reach a point on other bank directly opposite to his starting point

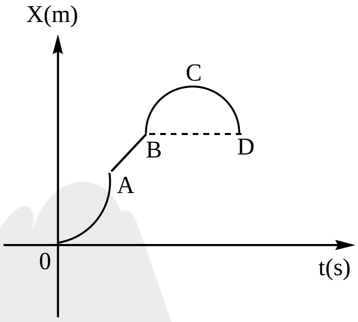
(D) He can reach the other bank at some point for anynon zero value of α

PROFICIENCY TEST 2

1. A particle moves with a speed $v = |t-3|\,m/s$ along a straight line, where t is time in seconds. Distance travelled by the particle during first 5 seconds is equal to:

(A) 1.5 m (B) 2 m (C) 6.5 m (D) None of these

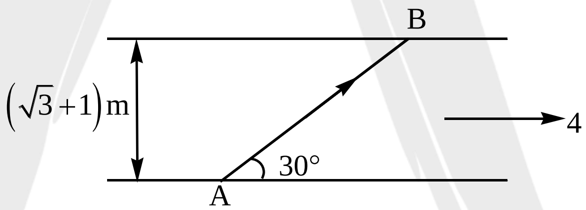
2. The displacement time graph for a particle in motion is shown in figure. For which portion of the graph the force acting on the particle is opposite to the direction of velocity



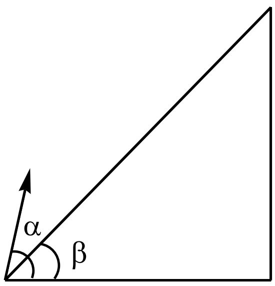
(A) AB (B) OA (C) BC (D) CD

3. A person has to cross the river along the path shown, velocity of river flow is 4 m/s. Speed of person in still water is $2\sqrt{2}m/s$. Time taken by the person to cross the river may be

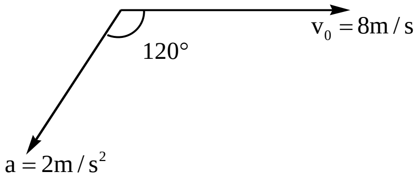
(A) 2 s (B) $\left(2-\sqrt{3}\right)s$ (C) $\left(2+\sqrt{3}\right)s$ (D) $\left(1+\frac{\sqrt{3}}{2}\right)s$



4. A particle is projected up an inclined plane of inclination β to be horizontal as shown. The angle of projection of particle with horizontal is α. If the particle strikes the plane horizontally, then $\tan\beta = \frac{\tan\alpha}{x}$ find the value of x.



5. The following figure shows the velocity and acceleration of a point like body at the initial moment of its motion. The acceleration vector of the body remains constant. The time (in sec) after which its speed becomes 8 m/s again is



6. A ball is dropped from rest at height 4h. After it has fallen a distance d, a second ball is dropped from rest at height h. What should d be (in terms of h) so that the balls hit the ground at the same time?

(A) $d = \frac{3H}{2}$ (B) $d = \frac{H}{3}$ (C) $d = H$ (D) $d = 3H$

7. A particle is moving in space. Let $\vec{r}, \vec{v}$ and $\vec{a}$ are the position vector, velocity vector and acceleration vector of a particle at a given instant. Then

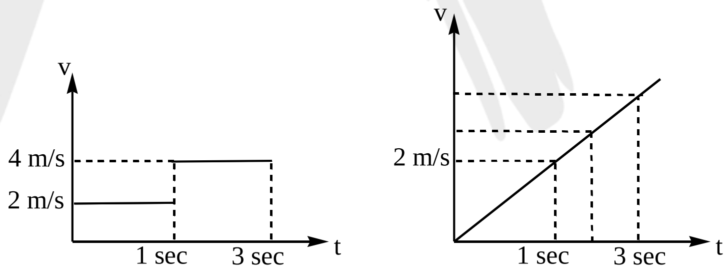
(A) If $\vec{v}.\vec{a} > 0$ then magnitude of velocity must be increasing

(B) If $\vec{r}.\vec{v} > 0$ then magnitude of position vector must be increasing

(C) If $\vec{r}.\vec{a} > 0$ then magnitude of velocity must be increasing

(D) If $\vec{r}.\vec{a} > 0$ then magnitude of velocity must be decreasing

8. Two particles P and Q starts from the origin along x-axis. Velocity time graph of both particles are shown in the figure. During the given time interval, the maximum separation between the particles is



(A) 4m (B) 1m (C) 2m (D) 3m

9. A lamp post is fixed at point (0,0,5m). A man is moving with constant velocity 6 m/s along a line $y = \sqrt{3}x$. Height of man is 2m and initially the man is at origin. The velocity of edge of his shadow along the line of motion of man is

(A) $8\hat{i} + 6\hat{j}$ (B) $6\hat{i} + 8\hat{j}$ (C) $5\sqrt{3}\hat{i} + 5\hat{j}$ (D) $5\hat{i} + 5\sqrt{3}\hat{j}$

10. The displacement of a particle after time t is given by $x = (k/b^2)(1-e^{-bt})$, where b is a constant. What is the acceleration of the particle?

(A) ke^{-bt} (B) $-ke^{-bt}$ (C) $\frac{k}{b^2}e^{-bt}$ (D) $\frac{-k}{b^2}e^{-bt}$

11. A Projectile is fired with speed v_0 at $t=0$ on a planet named 'Increasing Gravity'. This planet is strange one, in the sense that the acceleration due to gravity increase linearly with time t as g(t) = bt, where b is a positive constant.

A) If angle of projection with horizontal is θ, then the time of flight is $\sqrt{\frac{6v_0 \sin\theta}{b}}$

B) If angle of projection with horizontal is θ, then the maximum height attained is $\frac{1}{3}\frac{(2v_0 \sin\theta)^{3/2}}{\sqrt{b}}$

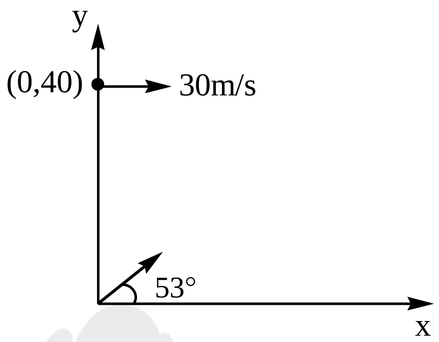
C) The angle with the horizontal at which the projectile should be projected so that it travels the maximum horizontal distance is $\theta = tan^{-1}\frac{1}{\sqrt{2}}$

D) None of the above is correct

12. In an announcement on a railway station, a passenger hears that the last train has passed the station $\Delta t_1 = 30$ min earlier than his train. On the next station that is s = 20 km away from the previous station, in another announcement he hears that the first train arrived $\Delta t_2 = 20\,\text{min}$ earlier than his train. Reading time from his watch, he calculates average speed of his train to be $v_p = 60\,\text{km/h}$. Relying on the announcements and the passenger's calculations, determine average speed of the first train. If the answer is $n \times 10\,\text{km/hr}$ then write the value of n.

PROFICIENCY TEST 3

1. As soon as a monkey jumps horizontally with speed 30m/s from height 40m of a tree, an arrow is projected from the bottom of tree at an angle of $53°$ with the horizontal in the same plane. If the arrow hits the monkey. Then



(A) The angle at which arrow is approaching monkey with the horizontal with respect to monkey is $90°$

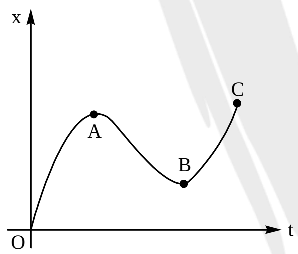
(B) The angle at which arrow is approaching monkey with the horizontal with respect to monkey is $60°$

(C) The time after which arrow hits the monkey is 1 second

(D) The time after which arrow hits the monkey is 2 second

Answer the following by appropriately matching the list based on the information given in the paragraph.

The displacement time (x –t) graph of a body acted by some forces is shown in the figure.



	List – I		List - II
(A)	From 'O' to just before 'A', the physical quantity of the body is	(P)	Zero
(B)	In between the points 'A' and 'B' the physical quantity of the body is	(Q)	Positive
(C)	In between the points 'B' and 'C' the physical quantity of the body is	(R)	Negative

(D)	At the point 'A' the physical quantity of the body is	(S)	First negative then positive
		(T)	First positive then negative
		(U)	Cannot be explained

2. If the physical quantity of the body is velocity then the correct match List – I and List – II.

(A) $A \to Q, B \to R, C \to Q, D \to P$ (B) $A \to R, B \to Q, C \to P, D \to Q$

(C) $A \to U, B \to R, C \to Q, D \to P$ (D) $A \to T, B \to U, C \to S, D \to P$

3. If the physical quantity of the body is acceleration then the correct match List – I and List – II.

(A) $A \to S, B \to T, C \to Q, D \to R$ (B) $A \to R, B \to S, C \to Q, D \to R$

(C) $A \to T, B \to S, C \to Q, D \to P$ (D) $A \to S, B \to Q, C \to T, D \to P$

4. At t = 0, two particles B & C are located at the origin of the coordinate system. Then they start moving simultaneously. B moves under a constant acceleration of $2\hat{k}\,m/s^2$ with an initial velocity of $8\hat{j}\,m/s$. Particle C moves with constant velocity $\vec{V}_0$ in such a way that B & C collides at t = 4sec. Then mark the INCORRECT statement(s).

(A) $\vec{V}_0 = 8\hat{j} + 4\hat{k}\,m/s$

(B) Position vector of location where two particles collides is $16\hat{i} + 32\hat{k}\,m$

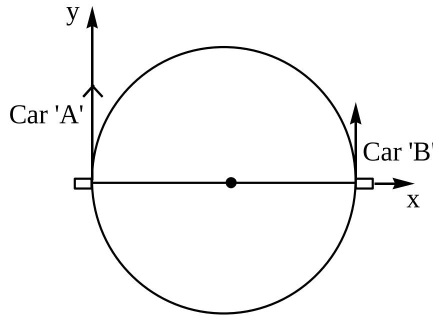
(C) Both (A) & (B) are correct

(D) It is not possible that B & C collide with each other for any value of $\vec{V}_0$

5. During a rainy day, rain is falling vertically down with a velocity 2 m/s. A boy at rest starts his motion with a constant acceleration of $2\,m/s^2$ along a straight road. Find the rate at which the angle of the axis of the umbrella with vertical should be changed so that the rain always appears to fall parallel to the axis of the umbrella.

(A) $\dfrac{1}{1+t^2}$ (B) $\dfrac{2}{1+t^2}$ (C) $\dfrac{1}{2+t^2}$ (D) $\dfrac{1}{1+2t^2}$

6. Two cars 'A' and 'B' are moving along y-axis and in a circle of radius 30 m respectively. Initial coordinate of car 'A' and 'B' are (0,0) and (60,0) respectively. Car 'A' starts from rest and moves with constant acceleration $\dfrac{20}{3}\,m/s^2$ and car 'B' moves with constant speed. Time taken by car 'B' to complete the circle is 12 sec. choose the correct option(s).



(A) The magnitude of average velocity of car 'A' with respect to car 'B' is 10 m/s in time interval 0 to 3 sec.

(B) The magnitude of average velocity of car 'A' with respect to car 'B' is 10 m/s in time interval 0 to 6 sec.

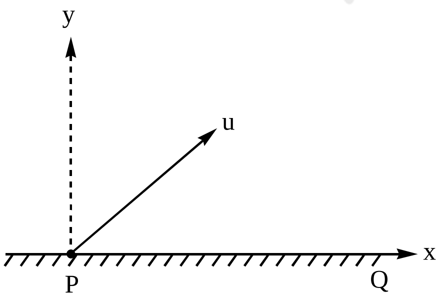
(C) The magnitude of average acceleration of car 'A' with respect to car 'B' is $\frac{5\pi+20}{3} m/s^2$ in time interval 0 to 6 sec.

(D) The magnitude of average acceleration of car 'A' with respect to car 'B' is $\frac{5\pi}{3} m/s^2$ in time interval 0 to 6 sec.

7. A particle is moving in x-y plane with $v = v_x\hat{i} + v_y\hat{j}$ and acceleration $a = a_x\hat{i} + a_y\hat{j}$. If the magnitude of velocity of the particle is constant then choose the correct option.

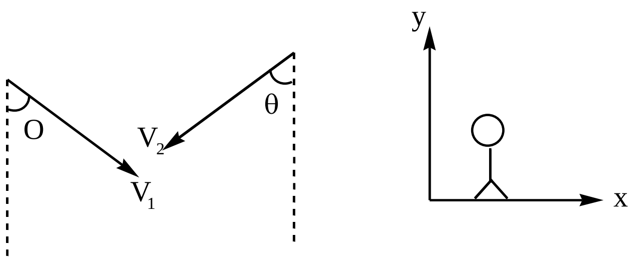
(A) $a_x v_x + a_y v_y = 0$ (B) $a_x v_x - a_y v_y = 0$ (C) $\frac{a_x}{v_x} + \frac{a_y}{v_y} = 0$ (D) $\frac{a_x}{v_x} - \frac{a_y}{v_y} = 0$

8. A small particle is projected from point 'P' in a vertical plane with an initial speed u. It hits the ground at Q. Assume that its collision with the ground which occurs at Q is perfectly inelastic. If area covered between the trajectory and x-axis by particle from P to Q is maximum, then the angle of projection of the particle is



(A) 60° (B) 45° (C) 30° (D) 75°

9. V_1 and V_2 are velocity vectors and θ is angle with vertical



(A) A man moving in positive x direction observes rain in direction of V_1 vector, if it moves with increasing speed it will observe that rain velocity vector is rotating clockwise

(B) A man moving in positive x direction observes rain in direction of V_2 vector, if it moves with decreasing speed it will observe that rain velocity vector is rotating anti-clockwise

(C) A man moving in negative x direction observes rain in direction of V_1 vector, if it moves with decreasing speed it will observe that rain velocity vector is rotating anti-clockwise

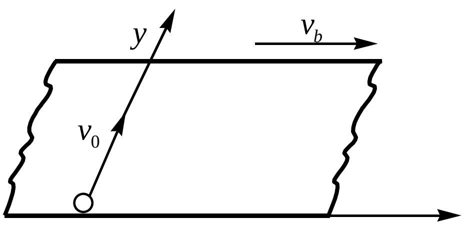
(D) A man moving in negative x direction observes rain in direction of V_2 vector, if it moves with increasing speed it will observe that rain velocity vector is rotating anti-clockwise

10. The deceleration experienced by a moving motor boat, after its engine is cut-off is given by: $\frac{dv}{dt} = -kv^3$, where k is constant. If v_0 is the magnitude of the velocity at cut-off, the magnitude of the velocity at a time t after the cut-off is:

(A) $v_0/2$ (B) v_0 (C) $v_0 e^{-kt}$ (D) $\frac{v_0}{\sqrt{(2v_0^2 kt + 1)}}$

Paragraph for Questions 11 to 12

A horizontal conveyor belt is running at a constant speed $\upsilon_b = 3.0 m/s$. A small disc enters the belt moving horizontally with a velocity $\upsilon_0 = 4.0 m/s$ that is perpendicular to the velocity of the belt. Coefficient friction between the disc and the belt is 0.50



11. What should the minimum width of the belt be so that the disc always remains on the belt?

(A) 0.9m (B) 1.6m (C) 2.0m (D) 2.5m

12. What is the minimum speed of the disc relative to the ground?

(A) 0.0m/s (B) 1.8m/s (C) 2.4m/s (D) 3.0m/s

ANSWER KEY

EXERCISE – 1

1	2	3	4	5	6	7	8	9	10
21	C	24	D	B	A	C	A	B	A
11	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
B	B	B	C	A	4	7	24	C	C

EXERCISE – 2

1	2	3	4	5	6	7	8	9	10
7	D	7	50	20	D	3	3	A	5
11	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
C	B	D	D	A	C	4	C	A	A

EXERCISE – 3

1	2	3	4	5	6	7	8	9	10
3	D	D	A	A	C	B	C	A	C
11	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
B	A	6	C	8	3	B	4	A	C

EXERCISE – 4

1	2	3	4	5	6	7	8	9	10
C	A	72	B	10	D	C	A	D	C
11	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
B	AB	D	D	D	B	C	C	5	2

EXERCISE – 5

1	2	3	4	5	6	7	8	9	10
480	C	C	A	A	B	B	C	C	A
11	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
A	D	C	ACD	D	C	15	D	5	B
21	**22**	**23**	**24**	**25**					
D	D	C	AD	BD					

PROFICIENCY TEST – 1

1	2	3	4	5	6	7	8	9	10
6	A	C	D	D	AC	D	B	1	A
11	**12**								
D	BCD								

PROFICIENCY TEST – 2

1	2	3	4	5	6	7	8	9	10
C	C	C	2	4	C	AD	C	D	B
11	**12**								
ABC	4								

PROFICIENCY TEST – 3

1	2	3	4	5	6	7	8	9	10
AC	A	B	BCD	A	AC	A	A	ABD	D
11	**12**								
C	C								