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FUNCTIONS

1. (iv), (v)

$\log_{\left[x+\frac{1}{x}\right]} |x^2 - x - 6| +$

$x > 0$

$x + \frac{1}{x} \geq 2$

$x^2 - x - 6 \neq 0$

$x \neq 3, -2$

$x \in (0,3) \cup (3, \infty)$

$^{16-x}C_{2x-1} + {}^{20-3x}P_{2x-5}$

$16 - x \geq 2x - 1$

$x \leq \frac{17}{3}$

$x = 1, 2, 4, 5$

$\mathcal{D}_f = \{4, 5\}$

$x = 4, 5$

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(v) $\log\left(\log_{|\sin x|}(x^2-8x+23)-\log_{|\sin x|}2^3\right)$

$$\log_{|\sin x|}\left(\frac{x^2-8x+23}{8}\right) > 0 = \log_{|\sin x|}1$$

$0<|\sin x|<1$

$$0<\frac{x^2-8x+23}{8}<1$$

$x\in(3,5)$

$D_f=\left(3,\pi\right)\cup\left(\pi,\frac{3\pi}{2}\right)\cup\left(\frac{3\pi}{2},5\right)$

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(ii) $$\frac{1}{[x]} + \log_{(1-\{x\})}(x^2-3x+10) + \frac{1}{\sqrt{2-|x|}} + \frac{1}{\sqrt{\sec(\sin x)}}$$

$x^2-3x+10 > 0$

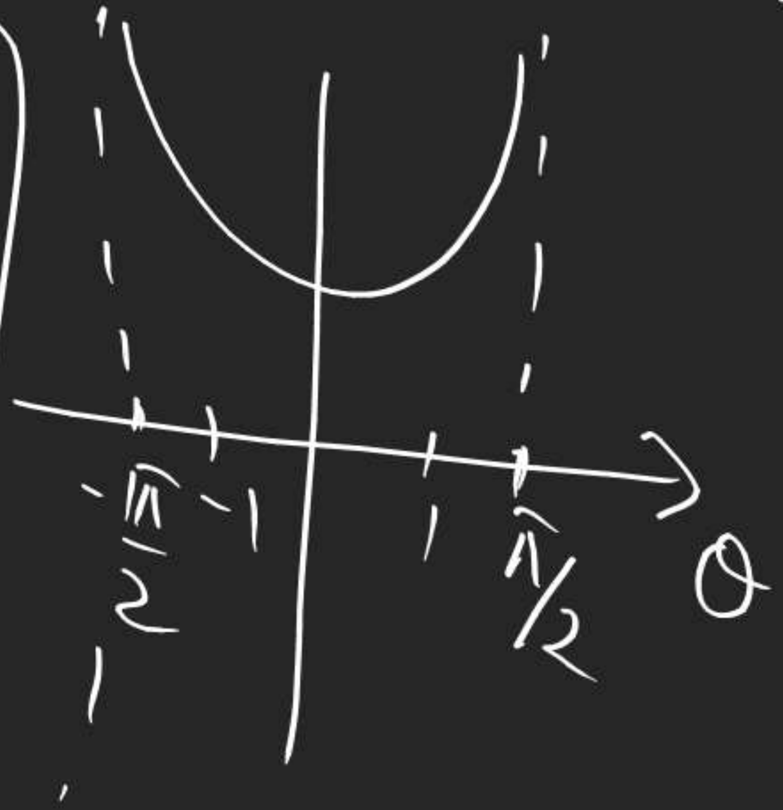
$R-[0,1)$

$x \notin I$

$(-2,2)$

$\sin x \in [-1,1]$, $\sec(\sin x) \geq 0$

$$\boxed{D_f = (-2,-1)\cup(-1,0)\cup(1,2)}$$



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2: (ii) $f(x) = \log_{(\operatorname{cosec} x - 1)}\left(2 - [\sin x] - [\sin x]^2\right)$

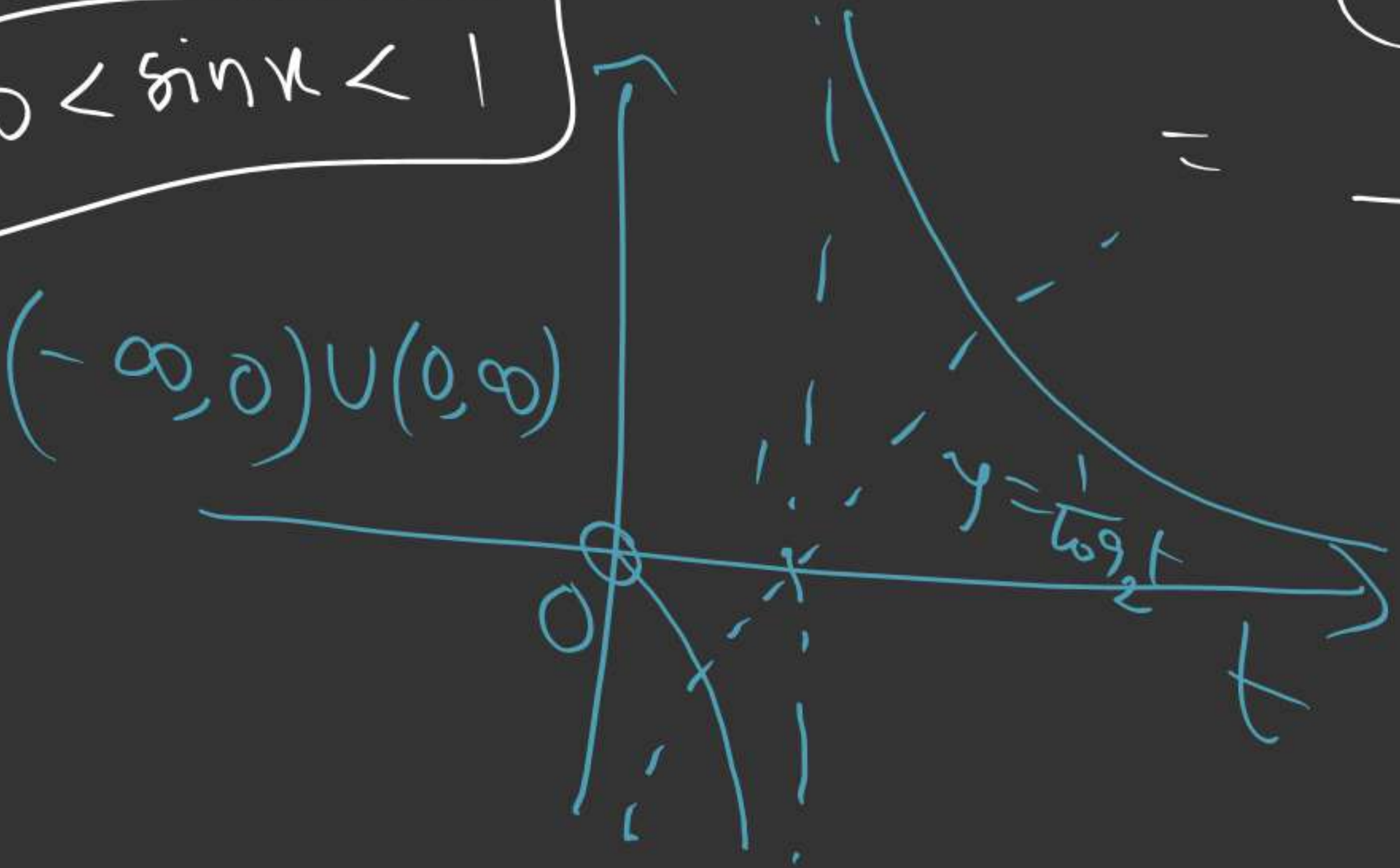
$\operatorname{cosec} x > 1$, $\neq 2$

$\boxed{0 < \sin x < 1}$

$$= \log_{(\operatorname{cosec} x - 1)} 2$$

$$= \frac{1}{\log_2(\operatorname{cosec} x - 1)}$$

$$= \frac{1}{\log_2 t}$$



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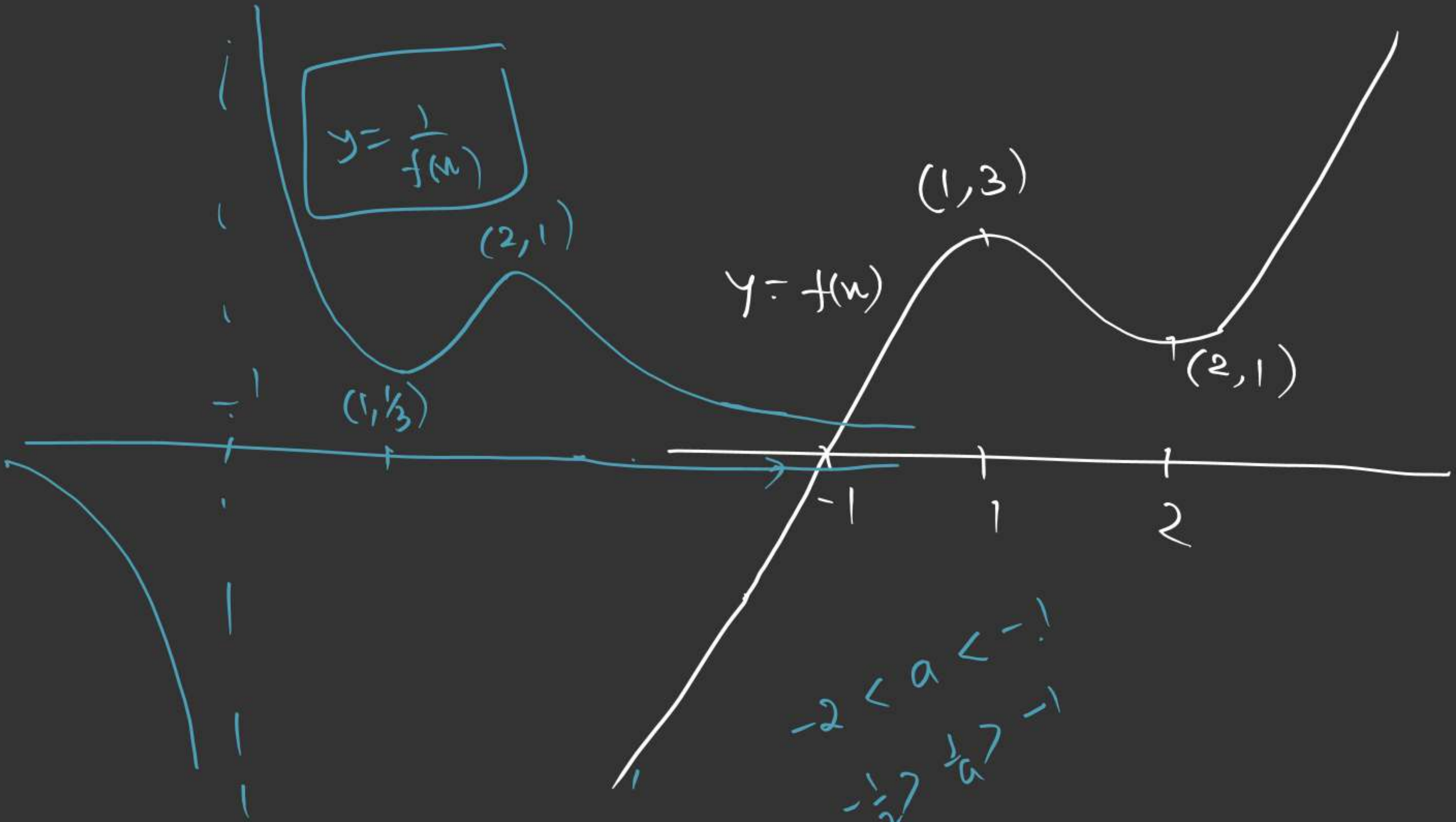

$-2 < a < -1$

$-\frac{1}{2} > \frac{1}{a} > -1$

f

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$f: R \to R$, $f(x) = x^3 + (a+2)x^2 + 3ax + 5$ $\boxed{\text{onto}}$

$$f'(x) = 3x^2 + 2(a+2)x + 3a \geq 0 \quad \forall x \in R$$

$$D \leq 0$$

$$(a+2)^2 - 9a \leq 0$$

$$a^2 - 5a + 4 \leq 0$$

$$\boxed{a \in [1, 4]}$$

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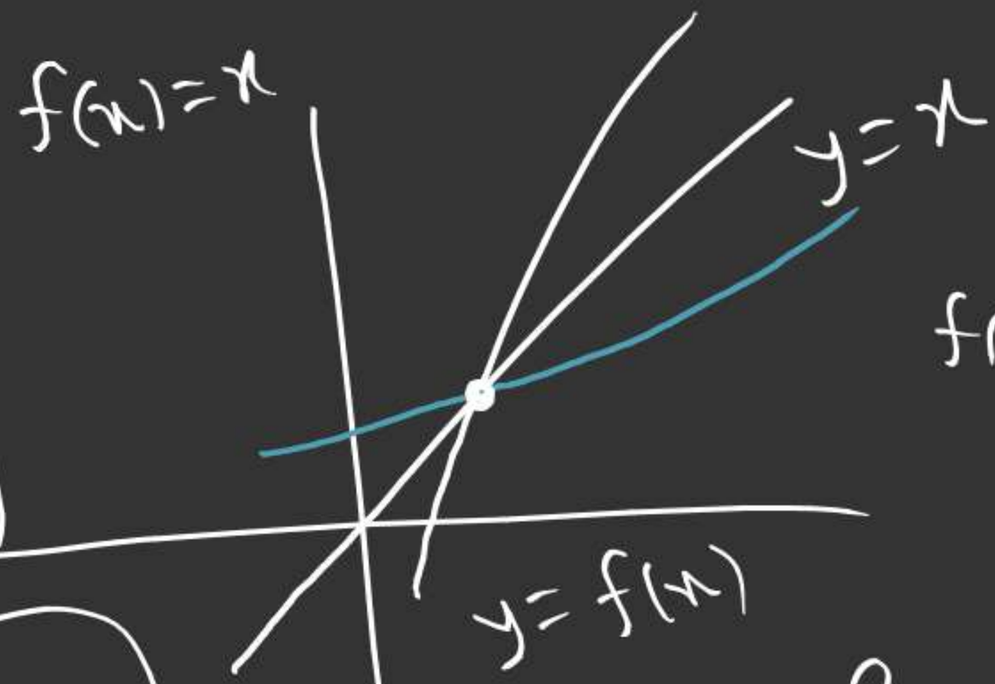
③ $f(x)=f^{-1}(x)$, $x=?$

$f(x)=x$

$\Rightarrow f(x)=f^{-1}(x)$



$f(x)=f^{-1}(x)$

$\not\Rightarrow f(x)=x$

always exist in pair

$f(x)=x \Rightarrow 2-x=x$

$\boxed{x=1}$

$f: R\to R$, $f(x)=2-x$

$\boxed{x\in R}$

$f^{-1}(x)=2-x$.

$\Uparrow$

$2-x=2-x \Leftarrow f(x)=f^{-1}(x)$

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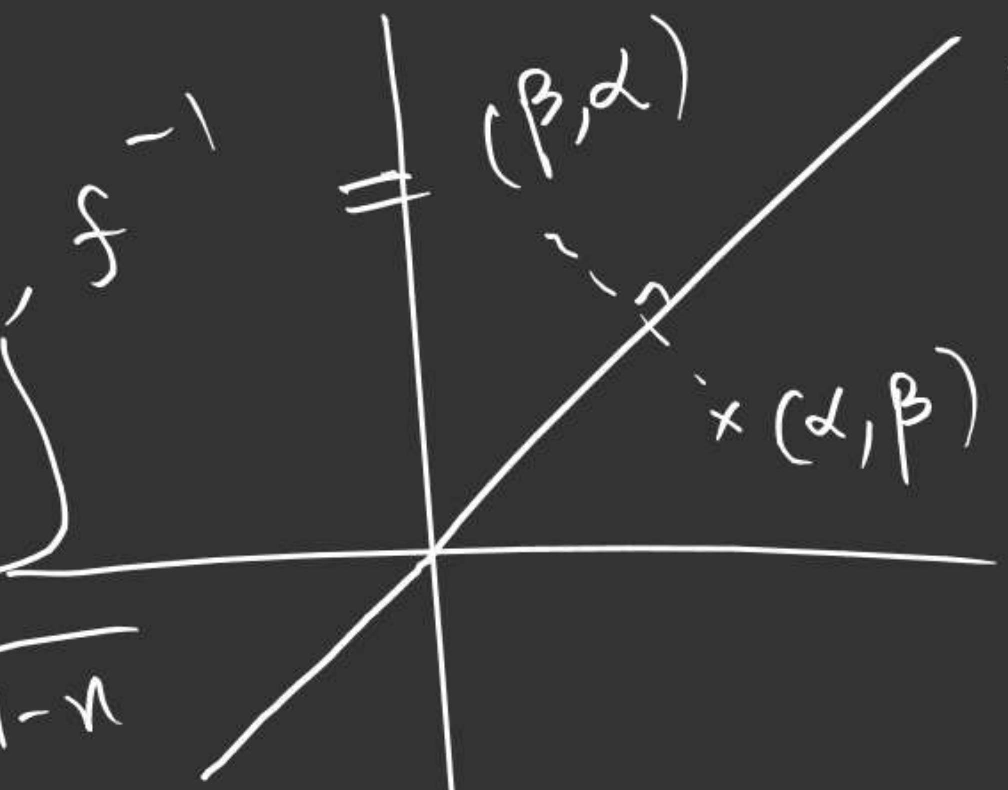
$\boxed{1-x^3 = \sqrt[3]{1-x}}$

$f, f^{-1} = (\beta, \alpha)$



$(\alpha, \beta) = f, \boxed{f^{-1}}$

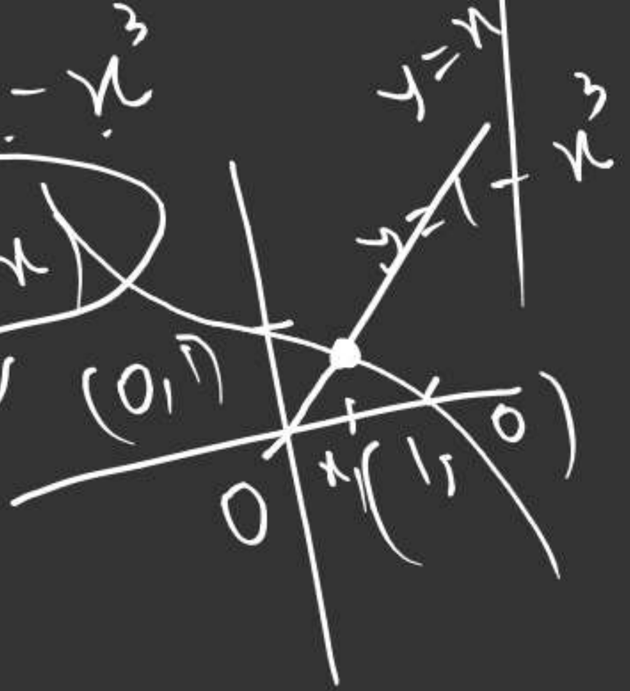
$f(x) = x$

$f^{-1}(x) = \sqrt[3]{1-x}$

$f: R \to R, \ f(x) = 1 - x^3$

$f(x) = f^{-1}(x)$

$x = 0, 1, \ldots$



$f(x) = f^{-1}(x)$

$f(x) \neq x$

$x = \alpha$

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Odd & Even Function

$y=f(x)$

$(\alpha,\beta), (-\alpha,-\beta)$

$$f(-x) = -f(x) \quad \forall\, x \in D_f$$

$\Rightarrow f$ is odd.

$(\alpha,\beta), (-\alpha,\beta)$

$$f(-x) = f(x) \quad \forall\, x \in D_f$$

$\Rightarrow f$ is even

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① Graph of odd fn is symmetric about origin.

② Graph of even function is symmetric about y-axis

$\boxed{f(x) = x \quad x \in [-2, 3]}$

neither odd nor even.

③ Every function which is defined both at $x = -a$ & $x = a$, can always be expressed as sum of an odd and an even function in a unique way.

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Let $f(x) = \underset{\text{Odd}}{g(x)} + \underset{\text{Even}}{h(x)}$ —— ①

$$f(x) = \underbrace{\left(\frac{f(x)+f(-x)}{2}\right)}_{\text{Even}} + \underbrace{\left(\frac{f(x)-f(-x)}{2}\right)}_{\text{Odd}}$$

$f(-x) = g(-x) + h(-x)$

$f(-x) = -g(x) + h(x)$ —— ②

① + ②

$$h(x) = \frac{f(x)+f(-x)}{2}$$

$$g(x) = \frac{f(x)-f(-x)}{2}$$

$$3^x = \frac{3^x+3^{-x}}{2} + \frac{3^x-3^{-x}}{2}$$

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$$f(x) = \log\left(x + \sqrt{x^2+1}\right)$$

$> |x|$

Odd

$$f(-x) = \log\left(-x + \sqrt{x^2+1}\right)$$

$$= \log\left(\frac{1}{\sqrt{x^2+1} + x}\right)$$

$$= -f(x)$$

② $f(x) = \cos x + x^2$

Even

③ $f(x) = \cos x - x$

neither odd nor even.

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$$f(x) = \begin{cases} x^2 & x \in (0,1] \\ 2-x & x \in (1,\infty) \end{cases}$$

Define $f(x)$ for $x < 0$ if $f(x)$ is

(I) odd

(II) even.

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$$f(x)=\begin{cases} x^2 & x\in(0,1] \\ 2-x & (1,\infty) \end{cases}$$

$$f(-x)=\begin{cases} (-x)^2 & -x\in(0,1] \\ 2-(-x) & -x\in(1,\infty) \end{cases}$$

$$f(-x)=\begin{cases} x^2 & x\in[-1,0) \\ 2+x & x\in(-\infty,-1) \end{cases}$$

① $-f(x)=\begin{cases} x^2 & x\in[-1,0) \\ 2+x & x\in(-\infty,-1) \end{cases}$

$$f(x)=\begin{cases} -x^2 & x\in[-1,0) \\ -2-x & x\in(-\infty,-1) \end{cases}$$

② $f(x)=\begin{cases} x^2 & x\in[-1,0) \\ 2+x & x\in(-\infty,-1) \end{cases}$

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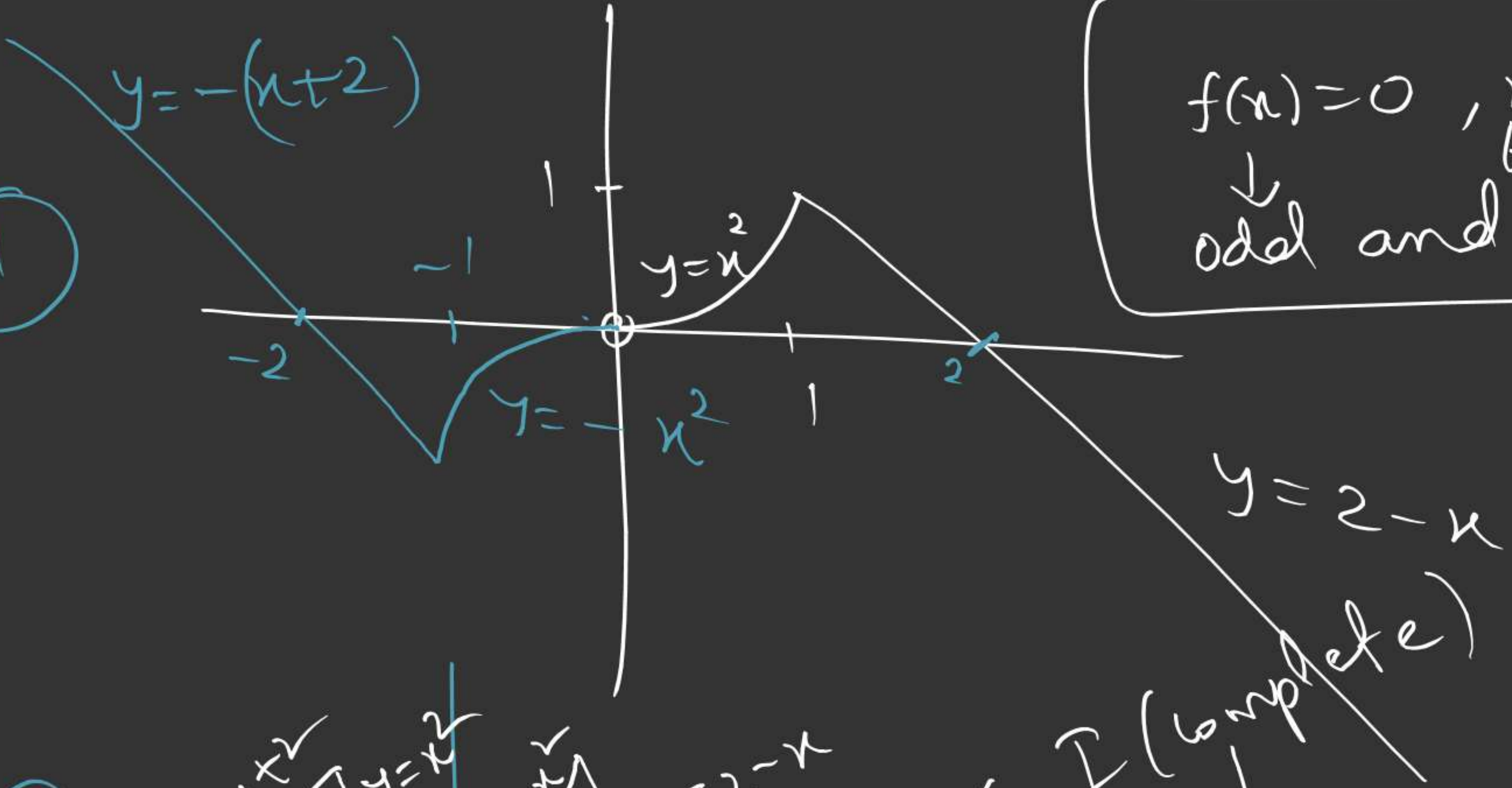
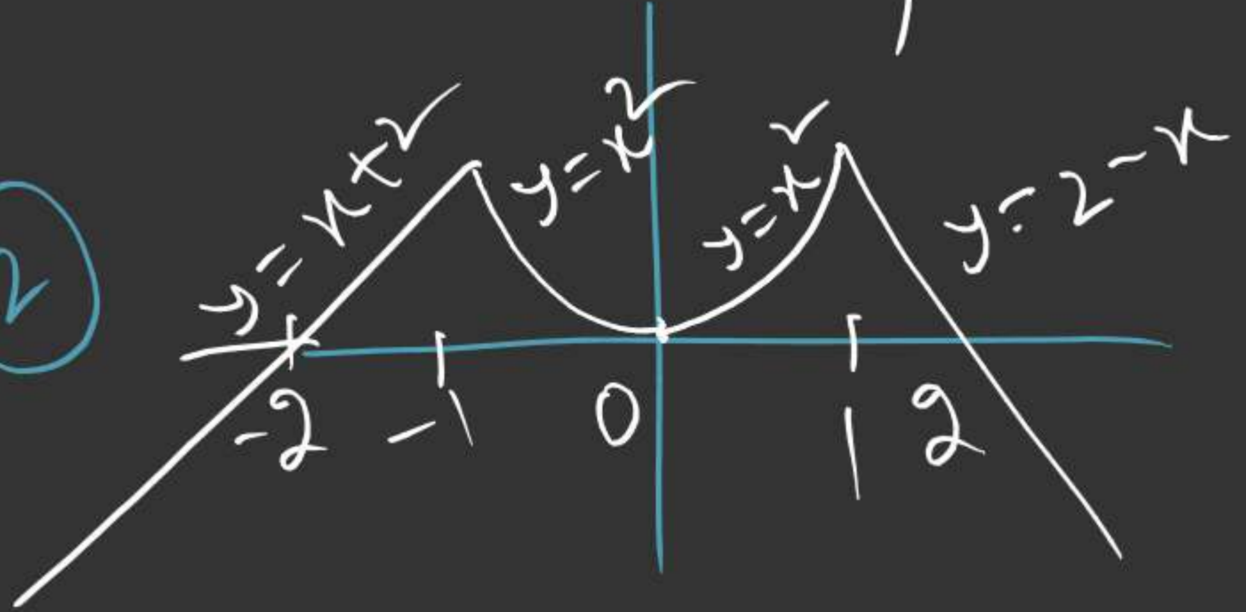
① 

$f(x)=0$, $x=-a$ & $x=a$ both in domain
$\downarrow$
odd and even

② 

Ex I (complete)
$\downarrow$
leaving Q15, 17(a)