DPP-8

SOLUTION

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1. Given $\vec{E} = 2y\hat{i} + 2x\hat{j}$ & $dV = -\vec{E}\cdot d\vec{r}$

$\therefore dV = -(2y\hat{i} + 2x\hat{j})\cdot(dx\hat{i} + dy\hat{j} + dz\hat{k})$ or $dV = -(2ydx + 2xdy)$

$\therefore \int dV = -2\int (ydx + xdy) = -2\int d(xy)$

$V = -2xy + C$

2. $\therefore \vec{E}(x,y,z) = -\frac{\partial}{\partial x}(V)\,\hat{i} - \frac{\partial}{\partial y}(V)\hat{j} - \frac{\partial}{\partial z}(V)\hat{k}$

$\therefore \vec{E}(x,y,z) = -2xy\hat{i} - (x^2 + 2yz)\hat{j} - y^2\hat{k}$

3. $\because \vec{E}(r) = -\vec{\nabla}V$ or $\vec{E}(r) = -\frac{\partial}{\partial r}(V)\hat{r}$

$\therefore \vec{E}(r) = -\frac{\partial}{\partial r}(2r^2)\hat{r} \Rightarrow \vec{E}(r) = -4\hat{r} = -4\vec{r}$

(i) Given $\vec{r} = \hat{i} - 2\hat{k}$ So, $\vec{E}(r) = -4(\hat{i} - 2\hat{k})$

(ii) $\vec{E}(r = 2) = -4.2\cdot\hat{r}$ or $\vec{E}(r = 2) = -8\hat{r}$

4. $V_b - V_a = -\int_a^b \vec{E}\cdot d\vec{r}$

$\therefore V_{(3,3)} - V_{(0,0)} = -\int_{(0,0)}^{(33)}(10\hat{i} + 20\hat{j})\cdot(dx\hat{i} + dy\hat{j} + dz\hat{k})$

or $V_{(3,3)} = -\int_0^3 10dx - \int_0^3 20dy = -30 - 60$

$\therefore V_{(3,3)} = -90$ volt

5. $V_b - V_a = -\int_a^D \vec{E}\cdot\overrightarrow{dr}$

$V_{(0.0)} - V_{(2,4)} = \int_{(2,4)}^{(0.0)}(20x\hat{i})\cdot(dx\hat{i} + dy\hat{j} + dz\hat{k})$

$\therefore V_{(0,0)} = -\int_2^0 20xdx = [-10x^2]_2^0$

$V_{(0,0)} = 40$ Volt

6. $V(r) = -\int \vec{E}\cdot\overrightarrow{dr}$ or $V(r) = -\int 2r^2dr$

$\therefore V(r) = -\frac{2r^2}{3} + C$

7. $V(x,y,z) = -\int \vec{E}\cdot d\overrightarrow{dr}$

$V(x,y,z) = -\int (2x^2\hat{i} - 3y^2\hat{j})\cdot(dx\hat{i} + dy\hat{j} + dz\hat{k})$ or $V(x,y,z) = -\int 2x^2dx + \int 3y^2dy$

$\therefore V(x,y,z) = -\frac{2x^3}{3} + y^3 + C$

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8. $\vec{E} = -\frac{dv}{dr} \cdot \hat{r}$

$E = -$ [slope of V-or graph]

at $x = 5\text{cm} \Rightarrow E = -\left[-\frac{5V}{2cm}\right] = 2.5V/cm.$

9. E at $x = 0$

$E = -$[slope of V-r graph]

$= -\left[\frac{5\ V}{2\ cm}\right] = -250\ V/m$

$\text{Force} = qE = 2 \times -250$

$= -500\ N$

10. $V = 5x^2 + 10x - 9$

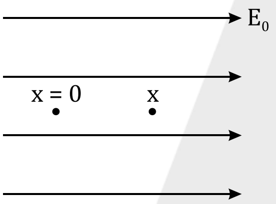
$\vec{E} = -\frac{\partial V}{\partial x}\hat{\imath}$

$\vec{E} = (-10x - 10)\hat{\imath}$

at $x = 1$ m

$\vec{E} = -20\,\hat{\imath}\ V/m$

11.

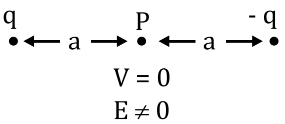


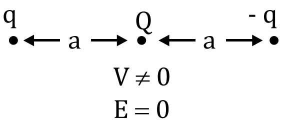
$V_x - V_0 = -\vec{E}_0 \cdot (\vec{x} - 0)$

$V_x - V_0 = -E_0 x$

$V_x = -E_0 x$

12.





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13. $E \propto r$

$E = kr$

$V = -\int \bar{E} \cdot d\bar{r}$

$= -k \int r dr$

$V = -\frac{kr^2}{2}$

$V \propto r^2$

14. $-\int_{\ell \to \infty}^{\ell=0} \vec{E} \cdot d\vec{\ell}$ will be equal to potential at $\ell = 0$ i.e. (at centre) and potential at the centre of the ring is

$$V_{centre} = \frac{Kq_{total}}{R} = \frac{(9 \times 10^9) \times (1.11 \times 10^{-10})}{(0.5)} = +2 \text{ Volt. (Approx)}$$