DPP 03

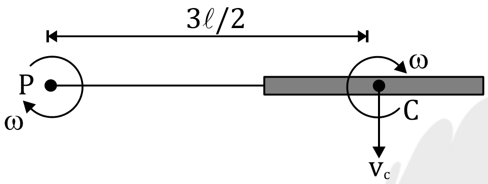
SOLUTION

1. Translational kinetic energy of rod,

$$K_{translational} = \frac{1}{2}mv_{cm}^2$$

Velocity of center of mass is

$$v_{cm} = \omega\left(\frac{3}{2}l\right)$$



The translational kinetic energy of rod is

$$K_{translational} = K_T = \frac{1}{2}m\left(\frac{3\omega l}{2}\right)^2 = \frac{9}{8}m\omega^2 l^2$$

The rotational kinetic energy of rod is

$$K_{rotational} = \frac{1}{2}I_{cm}\omega^2 = \frac{1}{2}\left(\frac{ml^2}{12}\right)\omega^2 = \frac{1}{24}m\omega^2 l^2$$

Total kinetic energy of the rod is

$$K_{total} = K_{translational} + K_{rotational}$$

$$\Rightarrow K_{total} = \frac{9}{8}m\omega^2 l^2 + \frac{1}{24}m\omega^2 l^2 = \frac{7}{6}m\omega^2 l^2$$

$$K_{total} = \frac{1}{2}I_P\omega^2$$

where I_P is the moment of inertial of the rod about the axis passing through the point P. According to parallel axis theorem, we have

$$I_P = I_{cm} + md^2 = I_{cm} + m\left(\frac{3}{2}l\right)^2$$

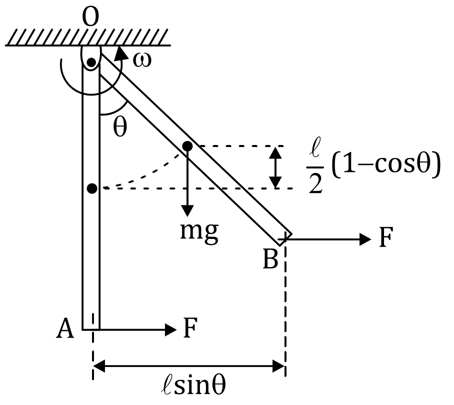
$$\Rightarrow I_P = \frac{ml^2}{12} + \frac{9}{4}ml^2 = \frac{7}{3}ml^2$$

Hence total kinetic energy of the rod is

$$K_{total} = \frac{1}{2}\left(\frac{7}{3}ml^2\right)\omega^2 = \frac{7}{6}m\omega^2 l^2$$

2. According to work energy theorem, we have $W_{ext} = \Delta U + \Delta K$,

where, $F_{ext} = F\Delta r = Fl\sin\,\theta$



ΔU is the rise in potential energy of the centre of mass of the rod, so

$$\Delta U = mg\left(\frac{l}{2}\right)(1-\cos\theta)$$

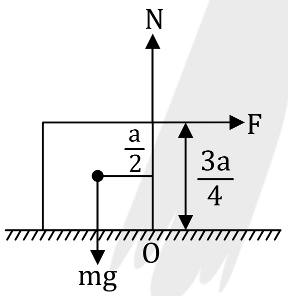
ΔK is the rotational kinetic energy of the rod about rotational axes passing through hinge and can also be called as the total kinetic energy of the rod, so

$$\Delta K = \frac{1}{2}I_O\omega^2$$

$$\Rightarrow Fl\sin\theta = \frac{mgl}{2}(1-\cos\theta) + \frac{1}{2}\left(\frac{ml^2}{3}\right)\omega^2$$

$$\Rightarrow \omega = \sqrt{\frac{6F\sin\theta}{ml} - \frac{3g}{l}(1-\cos\theta)}$$

3. In the limiting case normal reaction will pass through O. The cube will topple about O if torque of F exceeds the torque of mg.



$$\Rightarrow F\left(\frac{3a}{4}\right) > mg\left(\frac{a}{2}\right)$$

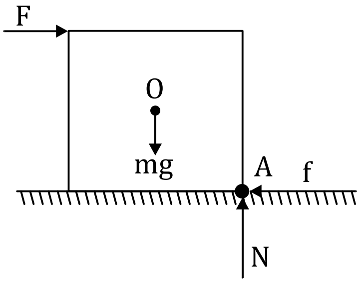
$$\Rightarrow F > \frac{2}{3}mg$$

So, the minimum value of F is $\frac{2}{3}mg$

4. For translational equilibrium of the system, we have

$N = mg$

$f = F$



For rotational equilibrium of the system, we have

$\tau_0 = 0 = F\frac{a}{2} + f\frac{a}{2} - N\frac{a}{2}$

$\Rightarrow \ Fa = mg\frac{a}{2}$

$\Rightarrow F = \frac{mg}{2}$

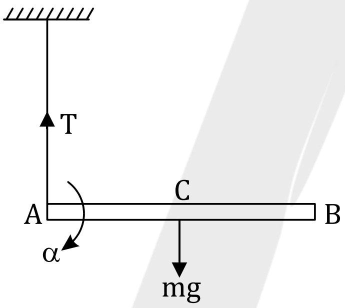
(OR)

We can also think that when the body is in complete equilibrium, then net torque will be zero about any point taken. So net torque about the point A is zero.

$\Rightarrow \ Fa = mg\frac{a}{2}$

$\Rightarrow \ F = \frac{mg}{2}$

5. $\alpha = \frac{\tau}{I} = \frac{mg\left(\frac{\ell}{2}\right)}{\frac{m\ell^2}{3}} = \frac{3}{2}\frac{g}{\ell}$



(a) $a_B = \ell\alpha = \frac{3g}{2}$

(b) $a_C = \frac{\ell}{2}\alpha = \frac{3g}{4}$

(c) Since $mg - T = ma_C$

$\Rightarrow \ mg - T = m\left(\frac{3g}{4}\right)$

$\Rightarrow \ T = \frac{mg}{4}$

6. The rod will rotate about point A with angular acceleration given by

$\alpha = \frac{\tau}{I} = \frac{Fx}{\left(\frac{m\ell^2}{3}\right)} = \frac{3Fx}{m\ell^2}$

$\Rightarrow\ a = \frac{\ell}{2}\alpha = \frac{3}{2}\frac{Fx}{m\ell}$

$\Rightarrow\ a \propto x$

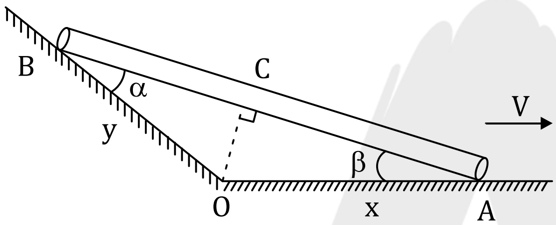
So, $a - x$ graph is a straight line passing through origin with slope $\frac{3F}{2m\ell}$.

7. Net torque about O due to weights acting at midpoints of the respective rods should be zero. Hence,

$mg\left(\frac{\ell}{2}\sin\ 60^\circ\right) = Mg\left(\frac{\ell}{2}\sin\ 30^\circ\right)$

$\Rightarrow \frac{M}{m} = \frac{\sin\ 60^\circ}{\sin\ 30^\circ} = \sqrt{3}$

8. Let $OA = x$ and $OB = y$ as shown in figure



Then $BC + CA = \ell$

$\Rightarrow y\cos\ \alpha + x\cos\ \beta = \ell$

Differentiating both sides

$\left(\frac{dy}{dt}\right)\cos\ \alpha + \left(\frac{dx}{dt}\right)\cos\ \beta = 0$

Since, $\frac{dx}{dt} = v_A = v$

So, $\left(\frac{dy}{dt}\right) = v_B = -\frac{v\cos\ \beta}{\cos\ \alpha}$