Nishant Jindal

Q If $f(x)$ is Integrable over $[1,2)$ then $\int_1^2 f(x)\,dx =$

A) $\lim_{n\to\infty} \frac{1}{n}\sum_{r=1}^{n} f\left(\frac{r}{n}\right) = \int_0^1 f(x)\,dx$ ×

B) $\lim_{n\to\infty} \frac{1}{n}\sum_{r=n+1}^{2n} f\left(\frac{r}{n}\right) = \int_{1+0}^{2} f(x)\,dx$

C) $\lim_{n\to\infty} \frac{1}{n}\sum_{r=1}^{n} f\left(\frac{r+n}{n}\right) = \int_0^1 f(x+1)\,dx$

$x+1=t$, $dx=dt$

x	t
0	1
1	2

$= \int_1^2 f(t)\,dt$

D) NOT

Prop 10 Newton Liebnitz Theorem.

(NL)

$f(x)$ Int $= F \Rightarrow F(x)$ diff $= f$

Proof $\int_U^V f(t)\,dt = F(x)\Big|_U^V = F(V) - F(U)$

Composite fxn

$$\boxed{\frac{d\left(\int_{\phi(x)}^{\psi(x)} f(t)\,dt\right)}{dx} = f(UL)\cdot(UL)' - f(LL)\cdot(LL)'}$$

$\frac{d}{dx}$

$= F'(V)\cdot V' - F'(U)\cdot U'$

$= f(V)\cdot V' - f(U)\cdot U'$

Subjective

Q If $g(x) = \int_1^x \sqrt{t^4+1}\,dt$ then $g'(x)$?

$g'(x) = \sqrt{x^4+1}\cdot(x)' - \sqrt{1^4+1}\times(1)'$

$= 1\times\sqrt{x^4+1} - 0$

$g'(x) = \sqrt{x^4+1}$

Q 32, 33, 34
35, 45, 46
47, 48
52, 53, 54, 55
56, 57, 58, 59, 60
61

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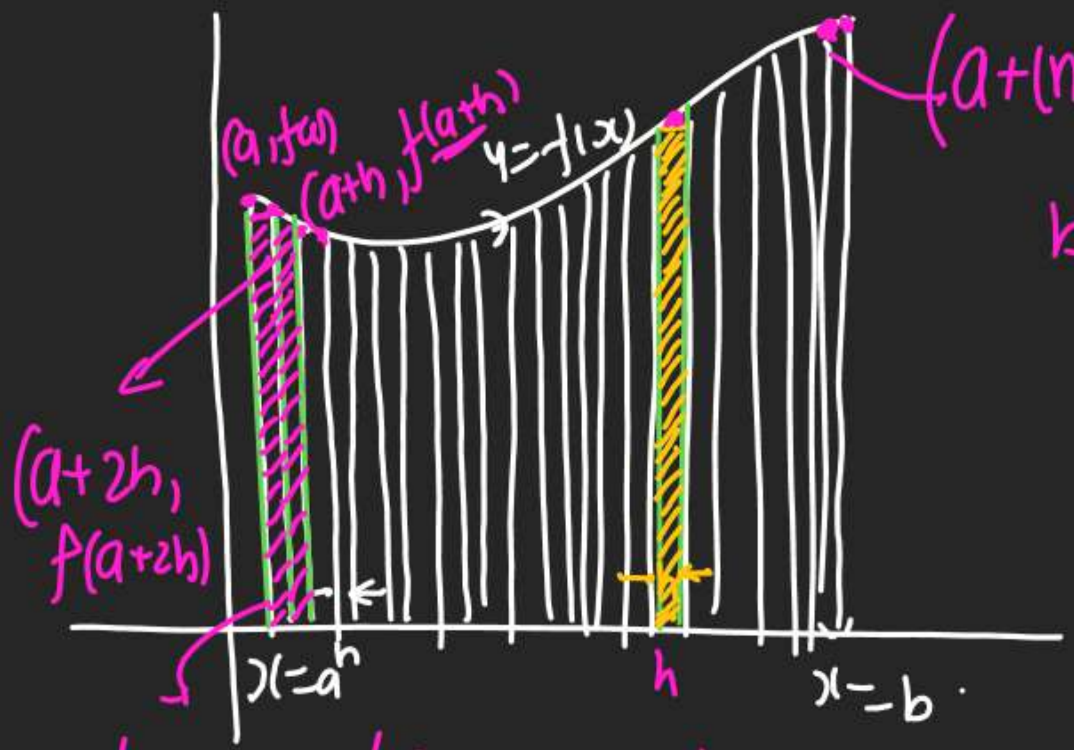

$b = a + nh.$

$h = \frac{b-a}{n}$

$f(a)\times h + f(a+h)\times h + f(a+2h)\times h \ldots\ldots h\left[f(a) + f(a+h) + f(a+2h) \ldots - f(a+(n-1)h)\right]$

$\int_a^b f(x)\cdot dx =$

$h\sum_{r=0}^{n-1} f(a+rh)$

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Q If $f(x) = \int_1^x \sqrt{2-t^2}\,dt$ then No of real roots in $x^2 - f'(x) = 0$

Adv

NL

$x^2 - \{\sqrt{2-x^2} \times (x)' + \cancel{\sqrt{2-1^2} \times (1)'}\} = 0$

$x^2 - \sqrt{2-x^2} \times 1 = 0$

$x^2 - \sqrt{2-x^2} = 0$

$x^2 = \sqrt{2-x^2}$

$x^4 = 2 - x^2$

$x^4 + x^2 - 2 = 0$

$t^2 + t - 2 = 0 \Rightarrow (t+2)(t-1) = 0$

$(\underbrace{x^2+2}_{\text{Imgx}})(x^2-1) = 0$

$x = \pm 1$ 2 Real Root

Q If $f(x) = \int_0^x (a-1)(t^2+t+1)^2 - (a+1)(t^4+t^2+1)\cdot dt$ ($x \to$ var. limit)

Adv

then if $f'(x) = 0$. Find value of a for real & distinct roots.

NL.

$(a-1)(x^2+x+1)^2 - (a+1)(x^4+x^2+1) \times 1 - 0 = 0$

$(\underbrace{x^2+x+1}_{\neq 0})\{(a-1)(x^2+x+1) - (a+1)(x^2-x+1)\} = 0$

$\{a\cancel{x^2} + ax + \cancel{a} - x^2 - \cancel{x} - 1 - \cancel{ax^2} + ax - \cancel{a} - x^2 + \cancel{x} - 1\} = 0$

$-2x^2 + 2ax - 2 = 0$

$x^2 - ax + 1 = 0$ roots Real & Distinct

$D > 0$

$a^2 - 4 > 0$

$(a-2)(a+2) > 0$

$a < -2 \vee a > 2$

$a \in (-\infty, -2) \cup (2, \infty)$

Nishant Jindal

Q Adv If $\int_0^{t^2} x\cdot f(x)\cdot dx = \frac{2}{5}t^5$ then $f\left(\frac{4}{25}\right) = ?$ $\longrightarrow$ ①

Q Adv If $\int_{\sin x}^{1} t^2\cdot f(t)\cdot dt = 1-\sin x$ then $f\left(\frac{1}{\sqrt{3}}\right) = ?$

Q. Adv $\int_0^x f(t)\,dt = x + \int_x^1 t\cdot f(t)\,dt$ then $f(1) = ?$

Q Adv If $F(x^2) = \int_0^{x^2} F(t)\,dt$ & $F(x^2) = x^2(1+x)$ then $F(4) = ?$

Q Adv If $\int_0^x \sqrt{1-(F'(t))^2}\,dt = \int_0^x F(t)\,dt$ & $F(0)=0$ then P.T

$F\left(\frac{1}{2}\right) < \frac{1}{2}$ & $F\left(\frac{1}{3}\right) < \frac{1}{3}$

NL

① $\cancel{t^2}\cdot f(t^2)\times 2\cancel{t} - 0 = \frac{2}{\cancel{5}}\times \cancel{5}\,\cancel{t^4}\,t$

$f(t^2) = t \Rightarrow f\left(\frac{4}{25}\right) = \frac{2}{5}$ Ans.

$\int \frac{dt}{\sqrt{1-t^2}}$

$\sin^{-1}t$

② Diff $0 + \sin^2 x\cdot f(\sin x)\times \cancel{\cos x} = 0 + \cancel{\cos x}$

$f(\sin x) = \frac{1}{\sin^2 x} \Rightarrow f(t) = \frac{1}{t^2} \Rightarrow f\left(\frac{1}{\sqrt{3}}\right) = 3$ A.

③ diff NL $f(x)\times 1 - 0 = 1 + \{0 - x\cdot f(x)\}$

$x f(x) + f(x) = 1 \Rightarrow f(x)(x+1) = 1$

$f(x) = \frac{1}{x+1} \Rightarrow f(1) = \frac{1}{2}$

(5) diff NL $\sqrt{1-(F'(x))^2}\times 1 = F(x) \Rightarrow 1-(F'(x))^2 = F^2(x)$

$\Rightarrow 1 - F^2(x) = (F'(x))^2 \Rightarrow F'(x) = \sqrt{1-F^2(x)}$

$\Rightarrow \frac{dy}{dx} = \sqrt{1-y^2} \Rightarrow \int \frac{dy}{\sqrt{1-y^2}} = \int dx \Rightarrow \boxed{\sin^{-1}y = x + C}$

$f(x) = y = \sin(x+C) \overset{x=0}{\Rightarrow} 0 = \sin(C) \Rightarrow C = 0$

$\sin^{-1}y = x \Rightarrow y = \sin x$

$F(x) = \sin x < x$

$F(x) < x$

$F\left(\frac{1}{2}\right) < \frac{1}{2}$

$F\left(\frac{1}{3}\right) < \frac{1}{3}$

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Q let $f: R \to R$ be a cont^s fxn which satisfies $f(x) = \int_0^x f(t)\,dt$ then value of $f(\ln 5)$?

Adv

$e^{\ln 5} = 5$

NL

diff^n

$$\boxed{f'(x) = f(x)\cdot 1}$$

$$\frac{dy}{dx} = y \Rightarrow \int \frac{dy}{y} = \int dx$$

$$\Rightarrow \ln y = x + c$$

$$y = e^{x+c} = e^x \cdot e^c$$

$$f(x) = y = Ke^x$$

$x=0$: $0 = K e^0 \Rightarrow \boxed{K=0}$

$$f(x) = 0\cdot e^x$$

$$f(x) = 0$$

$$f(\ln 5) = 0$$

$$f(x) = \int_0^x f(t)\,dt$$

$x=0$: $f(0) = \int_0^0 \sim$

$$f(0) = 0$$

Q If f be a real valued fxn in $(-1,1)$ such that

$$\boxed{e^{-x}\cdot f(x) = 2 + \int_0^x \sqrt{t^4+1}\cdot dt}$$ for $x \in (-1,1)$

Adv

& let f^{-1} be its inverse fxn then $(f^{-1})'(2) = ?$

demand f^{-1} का

x की जगह 2

① $e^{-x} f(x) = 2 + \int_0^x \sqrt{t^4+1}\cdot dt$

x की किस value पर $f(x) = 2$ होगा Think??

$$e^0\cdot f(0) = 2 + \int_0^0 \sim \Rightarrow f(0) = 2 \Rightarrow f^{-1}(2) = 0$$

② $f(f^{-1}(x)) = x \Rightarrow f'(f^{-1}(x)) \times f^{-1\prime}(x) = 1$

$$f^{-1\prime}(x) = \frac{1}{f'(f^{-1}(x))} \xRightarrow{x=2} f^{-1\prime}(2) = \frac{1}{f'(f^{-1}(2))} = \frac{1}{f'(0)}$$

③ NL

$$e^{-x} f'(x) - f(x) e^{-x} = 0 + \sqrt{x^4+1}\times 1 \xRightarrow{x=0} e^0 f'(0) - 2e^0 = \sqrt{1}$$

$$f'(0) = 3 \Rightarrow f^{-1\prime}(2) = \frac{1}{3}$$

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Q let $f(x) = \begin{cases} 1-x & \text{if } 0 \le x \le 1 \\ 0 & \text{if } 1 < x \le 2 \\ (2-x)^2 & \text{if } 2 < x \le 3. \end{cases}$

define $F(x) = \int_0^x f(t)\,dt$

& Show that F is cont^s in $[0,3]$ & diff^{ble} in $(0,3)$?

$$F(x) = \begin{cases} \int_0^x (1-t)\,dt & 0 \le x \le 1 \\ \int_0^1 (1-t)\,dt + \int_1^x 0\cdot dt & 1 < x \le 2 \\ \int_0^1 (1-t)\,dx + \int_1^2 0\cdot dt + \int_2^x (2-t)^2\,dt & 2 < x \le 3 \end{cases}$$

$$F(x) = \begin{cases} x - \frac{x^2}{2} & 0 \le x \le 1 \\ \frac{1}{2} & 1 < x \le 2 \\ \frac{1}{2} + \frac{(x-2)^3}{3} & 2 < x \le 3 \end{cases}$$

Now check Cont^y/diff^y

Q. $\int_0^{\sin^2 x} \sin^{-1}\sqrt{t}\,dt + \int_0^{\cos^2 x} \cos^{-1}\sqrt{t}\,dt = ?$ ← LL Same

Q $\int_{1/e}^{\tan x} \frac{t\,dt}{1+t^2} + \int_{1/e}^{\cot x} \frac{dt}{(t)(1+t^2)} = ?$ $x = \frac{\pi}{4}$ $\Rightarrow \int_{1/e}^{1} \frac{t}{(1+t^2)} + \frac{1}{(t)(1+t^2)}$

let

① $F(x) = \int_0^{\sin^2 x} \sin^{-1}\sqrt{t}\,dt + \int_0^{\cos^2 x} \cos^{-1}\sqrt{t}\,dt = \frac{\pi}{4}$

$F'(x) = \sin^{-1}\sqrt{\sin^2 x}\,.\sin 2x - \cos^{-1}\sqrt{\cos^2 x}\,.\sin 2x$

$= \sin^{-1}(\sin x)\cdot \sin 2x - \cos^{-1}\cos x \cdot \sin 2x$

$= x\cdot \cancel{\sin 2x} - x\cdot\cancel{\sin 2x}$

$F'(x) = 0 \Rightarrow F(x) = \text{Constant}$

② $x = \frac{\pi}{4} \Rightarrow F\left(\frac{\pi}{4}\right) = \int_0^{1/2} \sin^{-1}\sqrt{t}\,dt + \int_0^{1/2} \cos^{-1}\sqrt{t}\,dt$

$= \int_0^{1/2} \sin^{-1}\sqrt{t} + \cos^{-1}\sqrt{t}\,dt = \frac{\pi}{2}(t)_0^{1/2} = \frac{\pi}{4}$

Nishant Jindal

Q If x Satisfied Eqn $\left(\int_0^1 \frac{dt}{t^2+2t\cos\alpha+1}\right)\cdot x^2-\left(\int_{-3}^{3}\frac{t^2\cdot\sin 2t\,dt}{t^2+1}\right)x-2=0$

$+x\sim$ $E\times O=O$

$(0<\alpha<\pi)$ then value of x?

$$\left(\int_0^1 \frac{dt}{\{t^2+2t\cos\alpha+\underbrace{1-\sin^2\alpha}\}+\sin^2\alpha}\right)x=2$$

$\int \frac{dx}{x^2+a^2}$

$$\left(\int_0^1 \frac{dt}{(t+\cos\alpha)^2+(\sin\alpha)^2}\right)x^2=2$$

$$\left(\frac{1}{\sin\alpha}\cdot\tan^{-1}\frac{t+\cos\alpha}{\sin\alpha}\Big|_0^1\right)\times x^2=2$$

$$\left(\frac{\alpha}{2\sin\alpha}\right)x^2=2$$

$$\left\{\frac{1}{\sin\alpha}\cdot\tan^{-1}\left(\frac{1+\cos\alpha}{\sin\alpha}\right)-\frac{1}{\sin\alpha}\cdot\tan^{-1}\left(\frac{\cos\alpha}{\sin\alpha}\right)\right\}x^2=2$$

$\left(\cot\frac{\alpha}{2}\right)$ $\left(\cot\alpha\right)$

Q $\lim_{x\to 0}\frac{\int_0^x \frac{t^2}{\sqrt{a+t}}dt}{x-\sin x}=1$ $(a>0)$ then a?

$\frac{0}{0}$ DL

NL

$$\lim_{x\to 0}\frac{\frac{x^2}{\sqrt{a+x}}-0}{1-\cos x}=\frac{0}{0}=1$$

$$\lim_{x\to 0}\frac{x^2}{(1-\cos x)\sqrt{a+x}}=1$$

$$\frac{2}{\sqrt{a+0}}=1\Rightarrow\sqrt{a}=2$$

$$a=4$$

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Q $y = \lim_{n\to\infty}\left[\dfrac{n^n (x+n)\left(x+\frac{n}{2}\right)\cdots\left(x+\frac{n}{n}\right)}{\lfloor n \left(x^2+n^2\right)\left(x^2+\frac{n^2}{4}\right)\cdots\left(x^2+\frac{n^2}{n^2}\right)}\right]^{\frac{x}{n}}$

for all $x>0$ then

A) $f'\left(\frac{1}{2}\right) > \frac{1}{2}$ B) $f\left(\frac{1}{3}\right) \le f\left(\frac{2}{3}\right)$ (C) $f'(2) \le 0$

(D) $\dfrac{f'(3)}{f(3)} > \dfrac{f'(2)}{f(2)}$

$$\ln y = \lim_{n\to\infty} \frac{x}{n}\ln\left[\frac{\left(1+\frac{x}{n}\right)\left(\frac{1}{2}+\frac{x}{n}\right)\left(\frac{1}{3}+\frac{x}{n}\right)\cdots\left(\frac{1}{n}+\frac{x}{n}\right)}{\lfloor n \left(1+\frac{x^2}{n^2}\right)\left(\frac{1}{2^2}+\frac{x^2}{n^2}\right)\cdot\left(\frac{1}{3^2}+\frac{x^2}{n^2}\right)\cdots\left(\frac{1}{n^2}+\frac{x^2}{n^2}\right)}\right]$$

$$= \lim_{n\to\infty} \frac{x}{n}\left[\sum \ln\left(\frac{x}{n}+\frac{1}{r}\right) - \sum \ln\left(\frac{x^2}{n^2}+\frac{1}{r^2}\right)\right]$$

$$\frac{x}{n}\left[\sum_{r=1}^{n} \ln\left(1+\frac{rx}{n}\right)\frac{1}{r} - \sum_{r=1}^{n} \ln\left(1+\frac{r^2x^2}{n^2}\right)\frac{1}{r}\right]$$

Nishant Jindal

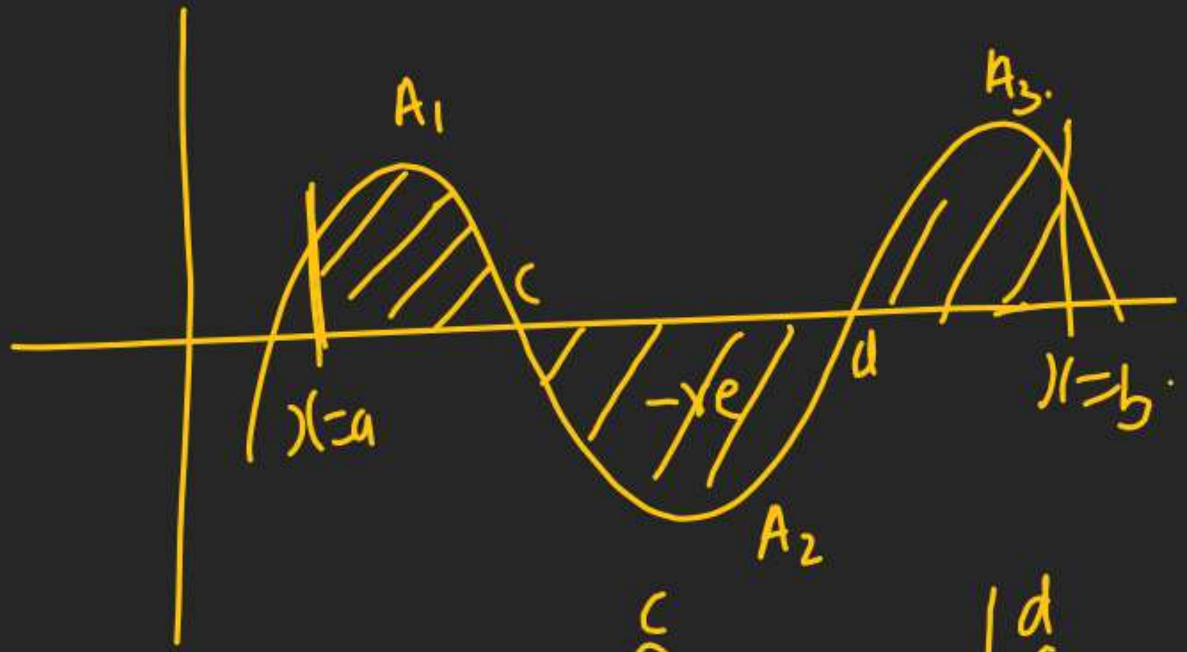
Area under Curve. [2days]

1Qs Sure.

① $\int_a^b f(x)\cdot dx$ = Algebric Sum of Area Under $y=f(x)$ in $x=a$, $x=b$ & X Axis

$A=\int_a^b f(x)dx$

Now in this Chapter. [Basic Graph]



$$\text{Area}=\int_a^c f(x)\,dx+\left|\int_c^d f(x)\cdot dx\right|+\int_d^b f(x)\cdot dx$$