



DPP 02

SOLUTION

$$1. \quad T_1 = 2\pi \sqrt{\frac{M}{k}} \quad \dots \text{(i)}$$

When a mass m is placed on mass M , Mass of new system $= (M + m)$ attached to the spring. New time period of oscillation

$$T_2 = 2\pi \sqrt{\frac{(m+M)}{k}} \dots \text{(ii)}$$

Say v_1 is the velocity of mass M passing through mean position and v_2 velocity of mass $(m + M)$ passing through mean position.

By conservation of linear momentum

$$Mv_1 = (m+M)v_2$$

$$M(A_1\omega_1) = (m+M)(A_2\omega_2) (\because v_1 = A_1\omega_1 \text{ and } v_2 = A_2\omega_2)$$

$$\text{or } \frac{A_1}{A_2} = \frac{(m+M)}{M} \frac{\omega_2}{\omega_1} = \left(\frac{m+M}{M}\right) \times \frac{T_1}{T_2}$$

$$\left(\because \omega_1 = \frac{2\pi}{T_1} \text{ and } \omega_2 = \frac{2\pi}{T_2} \right)$$

$$\frac{A_1}{A_2} = \sqrt{\frac{m+M}{M}} \quad (\text{Using (i) and (ii)})$$

$$2. \quad v_1 = \frac{d}{dt}(y_1) = (0.1 \times 100\pi) \cos \left(100\pi t + \frac{\pi}{3}\right)$$

$$v_2 = \frac{d}{dt}(y_2) = (-0.1 \times \pi) \sin \pi t \\ = (0.1 \times \pi) \cos \left(\pi t + \frac{\pi}{2}\right) \quad \therefore \Delta\phi = \frac{\pi}{3} - \frac{\pi}{2} = -\frac{\pi}{6}$$

3. Standard of SHM is

$$\frac{d^2x}{dt^2} + \omega^2 x = 0$$

Given equation is $\frac{d^2x}{dt^2} + \alpha x = 0$

$$\therefore \omega^2 = \alpha \text{ or } \omega = \sqrt{\alpha} \quad \therefore T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{\alpha}}$$



4. When springs are in series, $k = \frac{k_1 k_2}{k_1 + k_2}$

$$\text{For first spring, } t_1 = 2\pi \sqrt{\frac{m}{k_1}}$$

$$\text{For second spring } t_2 = 2\pi \sqrt{\frac{m}{k_2}}$$

$$\therefore t_1^2 + t_2^2 = \frac{4\pi^2 m}{k_1} + \frac{4\pi^2 m}{k_2} = 4\pi^2 m \left(\frac{k_1 + k_2}{k_1 k_2} \right)$$

$$\text{or } t_1^2 + t_2^2 = \left[2\pi \sqrt{\frac{m(k_1 + k_2)}{k_1 k_2}} \right]^2 \text{ or } t_1^2 + t_2^2 = T^2$$

5. Maximum velocity under simple harmonic motion

$$v_m = a\omega$$

$$\therefore v_m = \frac{2\pi a}{T} = (2\pi a) \left(\frac{1}{T} \right) = (2\pi a) \left(\frac{1}{2\pi} \sqrt{\frac{k}{m}} \right)$$

$$\text{or } v_m = a \sqrt{\frac{k}{m}}$$

$$\therefore (v_m)_A = (v_m)_B$$

$$\therefore a_1 \sqrt{\frac{k_1}{m}} = a_2 \sqrt{\frac{k_2}{m}} \Rightarrow \frac{a_1}{a_2} = \sqrt{\frac{k_2}{k_1}}$$

6. Initially, $T = 2\pi\sqrt{M/k}$

$$\text{Finally, } \frac{5T}{3} = 2\pi \sqrt{\frac{M+m}{k}}$$

$$\therefore \frac{5}{3} \times 2\pi \sqrt{\frac{M}{k}} = 2\pi \sqrt{\frac{M+m}{k}} \text{ or } \frac{25}{9} \frac{M}{k} = \frac{M+m}{k}$$

$$\text{or } 9m + 9M = 25M \text{ or } \frac{m}{M} = \frac{16}{9}$$

7. $x = 4(\cos \pi t + \sin \pi t)$

$$= 4 \times \sqrt{2} \left[\frac{1}{\sqrt{2}} \cos \pi t + \frac{1}{\sqrt{2}} \sin \pi t \right]$$

$$\text{or } x = 4\sqrt{2} \left[\sin \frac{\pi}{4} \cos \pi t + \cos \frac{\pi}{4} \sin \pi t \right]$$



$$= 4\sqrt{2} \sin \left(\pi t + \frac{\pi}{4} \right)$$

Hence amplitude = $4\sqrt{2}$

- 8.** For simple harmonic motion,

$$\text{Total energy} = \frac{1}{2} m a^2 \omega^2$$

It means total energy is independent of x.

9. $y = \sin^2 \omega t = \frac{1 - \cos 2\omega t}{2} = \frac{1}{2} - \frac{\cos 2\omega t}{2}$

It is a periodic motion but it is not SHM.

∴ Angular speed = 2ω

$$\therefore \text{Period } T = \frac{2\pi}{\text{angular speed}} = \frac{2\pi}{2\omega} = \frac{\pi}{\omega}$$

- 10.** In a simple harmonic oscillator, kinetic energy is maximum and potential energy is minimum at mean position.