Nishant Jindal

Heat Transfer

Heat transfer through Variable crossectional area

$T_1 > T_2$. K = constant.

$$\frac{dQ}{dt} = -K(4\pi r^2)\left(\frac{dT}{dr}\right)$$

$\Downarrow$

$$P\ J/s = -(4\pi r^2)K\frac{dT}{dr}$$

$$P\int_{r_1}^{r_2}\frac{dr}{r^2} = -4\pi K\int_{T_1}^{T_2} dT$$

$$P\left[-\frac{1}{r}\right]_{r_1}^{r_2} = -4\pi K(T_2 - T_1)$$

$$P\left[\frac{1}{r_1} - \frac{1}{r_2}\right] = 4\pi K(T_1 - T_2)$$

$$P = \frac{T_1 - T_2}{\frac{1}{4\pi K}\left[\frac{r_2 - r_1}{r_1 r_2}\right]}$$

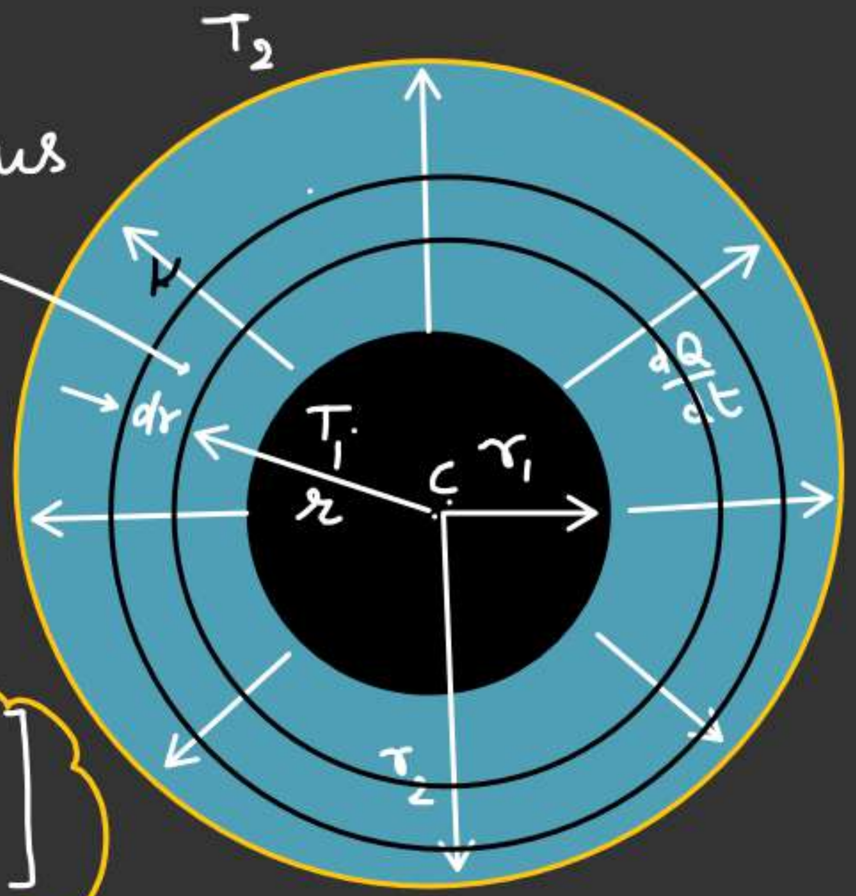
Heat current

$\Downarrow$

Thermal Resistance

$$R_{th} = \frac{1}{4\pi K}\left(\frac{r_2 - r_1}{r_1 r_2}\right)$$

Spherical Shell of radius r and thickness dr



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Heat Transfer

Case when temperature of one end of the rod is a function of time

Total Steady state heat flow from left to right.

$$100 = 20 + 2t$$

$$t = \frac{80}{2} = 40\,\text{sec}$$

At every moment $\left(\frac{dQ}{dt}\right)$ is different

At $t = t$

$$\frac{dQ}{dt} = \frac{KA}{L}\left[100 - (20+2t)\right]$$

$$\frac{dQ}{dt} = \frac{KA}{L}(80 - 2t)$$

$$\int_0^Q dQ = \frac{KA}{L}\int_0^{40}(80-2t)\,dt$$

$$Q = \frac{KA}{L}\left[80\left[t\right]_0^{40} - 2\left[\frac{t^2}{2}\right]_0^{40}\right]$$

$$Q = \frac{KA}{L}\left[80 \times 40 - 1600\right]$$

$$Q = \frac{1600\,KA}{L}\ \text{J}$$

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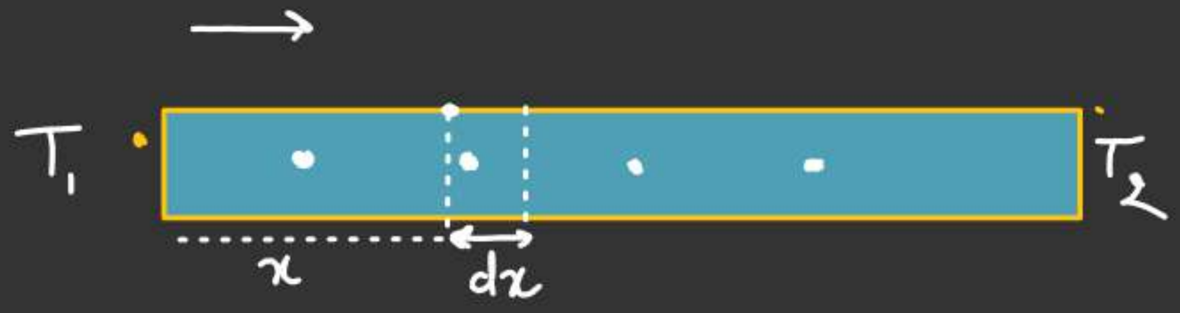
Heat Transfer

$K = \left(\frac{\alpha}{T}\right)$ where α is a constant.

$$\frac{dQ}{dt} = -KA\frac{dT}{dx} = -\left(\frac{A\alpha}{T}\right)\frac{dT}{dx}$$

P J/s

$T_1 > T_2$



$$P\int_0^L dx = -A\alpha\int_{T_1}^{T_2}\frac{dT}{T}$$

$$PL = -A\alpha \ln\left(\frac{T_2}{T_1}\right)$$

$$P = \left(\frac{A\alpha}{L}\right)\ln\left(\frac{T_1}{T_2}\right) \checkmark$$

Temp. as a function of x

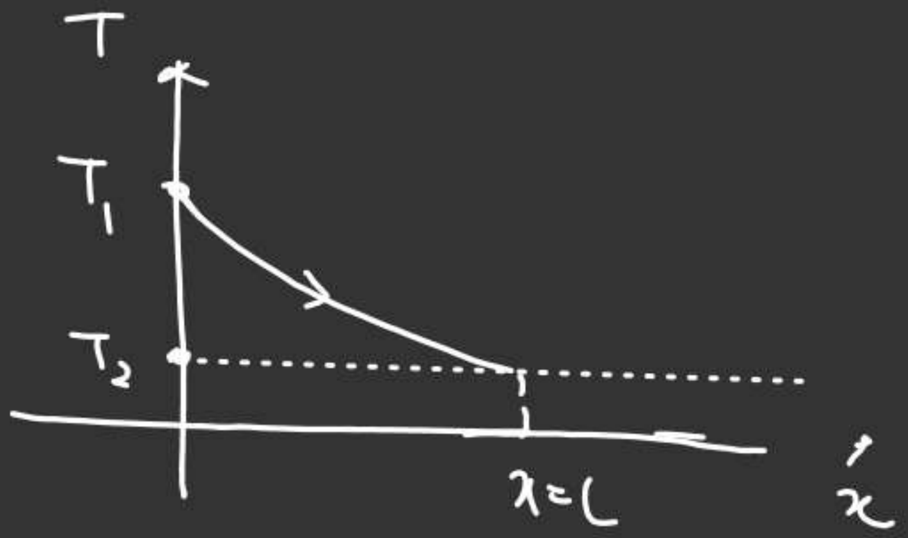
$$P\int_0^x dx = -A\alpha\int_{T_1}^{T}\frac{dT}{T}$$

$$Px = -A\alpha \ln\left(\frac{T}{T_1}\right)$$

$$-\frac{P}{A\alpha}x = \ln\left(T/T_1\right)$$

$$T = T_1 e^{-\frac{P}{A\alpha}x}$$

K, A, L



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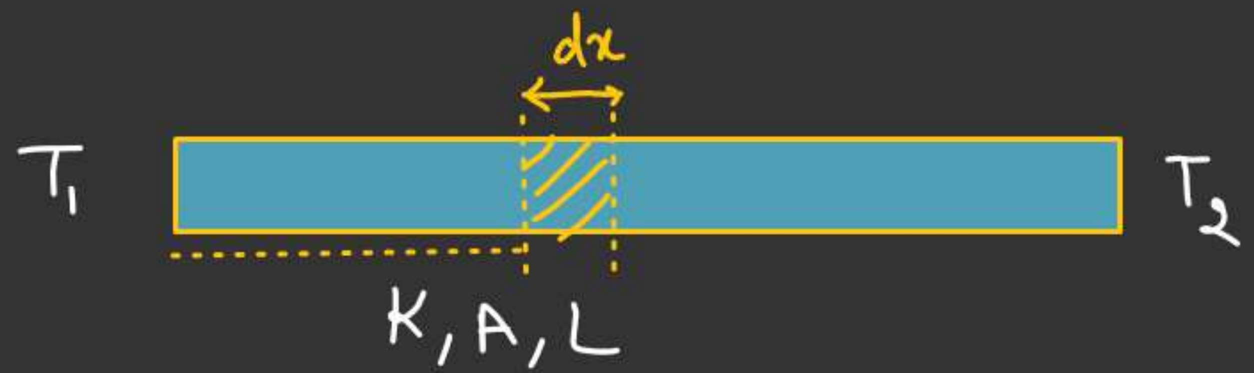
Heat Transfer

$$K = (K_0 + \alpha x)$$ K_0 & α are constant.

$T_1 > T_2$



$$\frac{dQ}{dt} = K_x A \left(-\frac{dT}{dx}\right)$$

$$\Downarrow$$

$$P = A(K_0 + \alpha x) - \left(\frac{dT}{dx}\right)$$

$$P\int_0^L \left(\frac{dx}{K_0 + \alpha x}\right) = -A\int_{T_1}^{T_2} dT$$

$$\frac{P}{\alpha}\ln\left[\frac{K_0 + \alpha L}{K_0}\right] = -A(T_2 - T_1)$$

$$\frac{P}{\alpha}\ln\left(\frac{K_0 + \alpha L}{K_0}\right) = A(T_1 - T_2)$$

$$P = \frac{(T_1 - T_2)}{\frac{1}{\alpha A}\ln\left(\frac{K_0 + \alpha L}{K_0}\right)}$$

Thermal Resistance

$$\int_0^R dR = \int_0^L \frac{dx}{(K_0 + \alpha x)A}$$

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Heat Transfer

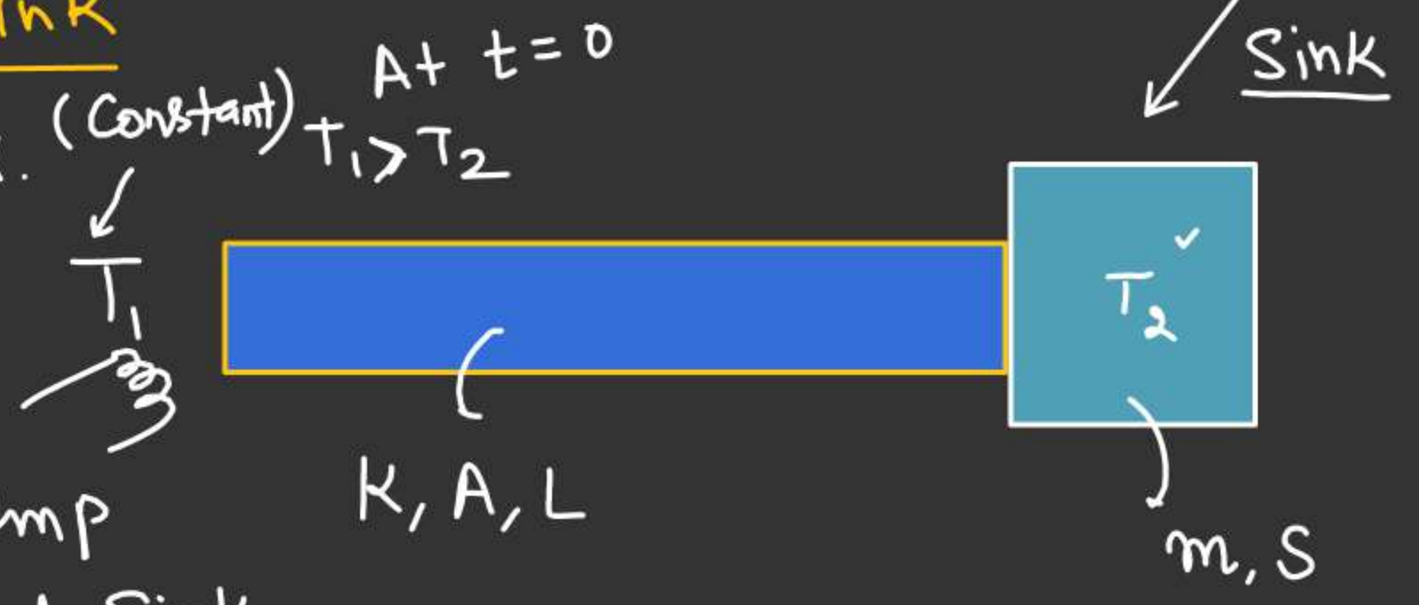
AQ: Imp

Heat Transfer into a Sink

$S \rightarrow$ Specific heat of block.

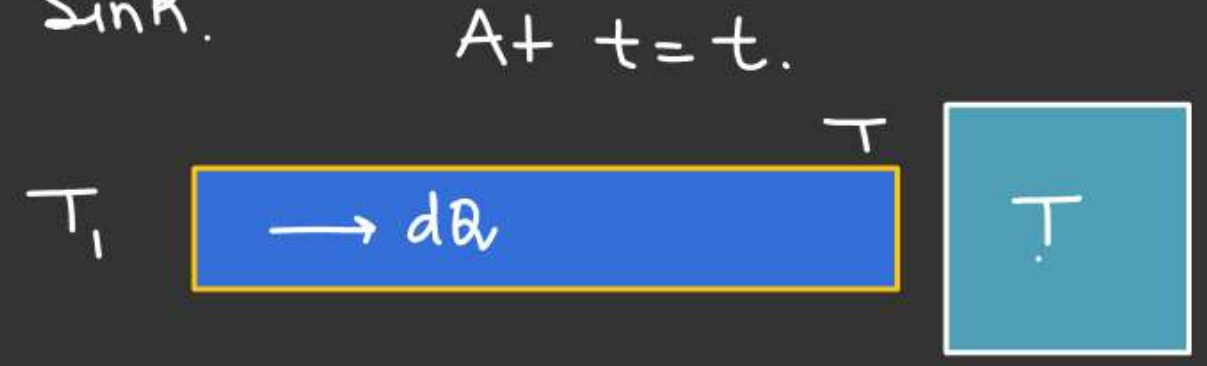
$m \rightarrow$ mass of block.

$$(dQ = ms\,dT)$$



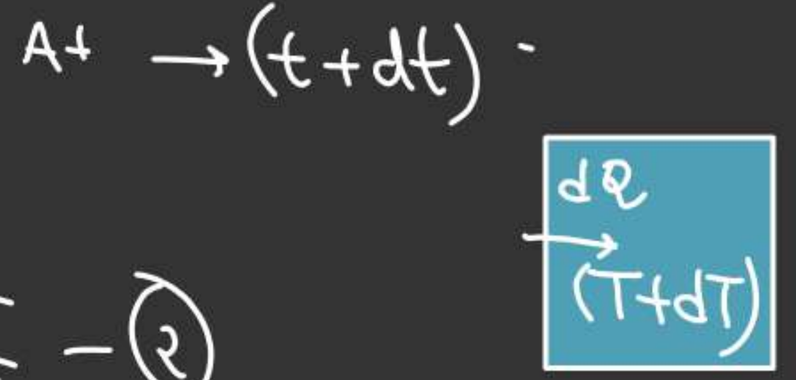
At $t=0$, T_1 & T_2 be the temp of one end of the rod and Sink.

$T_2 \rightarrow f(t)$



Equation of Rod

$$\frac{dQ}{dt} = \frac{KA}{L}(T_1 - T) \quad -(1)$$



Equation of block.

$$dQ = ms\,dT \rightarrow \frac{dQ}{dt} = ms\frac{dT}{dt} \quad -(2)$$

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Heat Transfer

Equation of Rod

$$\frac{dQ}{dt} = \frac{KA}{L}(T_1 - T) \quad \text{—} (1)$$

Equation of block.

$$dQ = ms\,dT \rightarrow \frac{dQ}{dt} = ms\frac{dT}{dt} \quad \text{—} (2)$$

$$ms\frac{dT}{dt} = \frac{KA}{L}(T_1 - T)$$

$$\int_{T_2}^{T}\frac{dT}{(T_1 - T)} = \frac{KA}{msL}\int_0^t dt$$

$$\frac{\ln\left[T_1 - T\right]_{T_2}^{T}}{(-1)} = \frac{KA}{msL}t$$

$$\ln\left(\frac{T_1 - T}{T_1 - T_2}\right) = \frac{-KA}{msL}t$$

$$(T_1 - T) = (T_1 - T_2)\,e^{\frac{-KA}{msL}t}$$

$$\boxed{T = T_1 - (T_1 - T_2)\,e^{\frac{-KA}{msL}t}}$$

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Heat Transfer

$$\boxed{T = T_1 - (T_1 - T_2)\, e^{-\frac{KA}{msL}t}}$$

$ms = C$

$\Downarrow$

heat Capacity

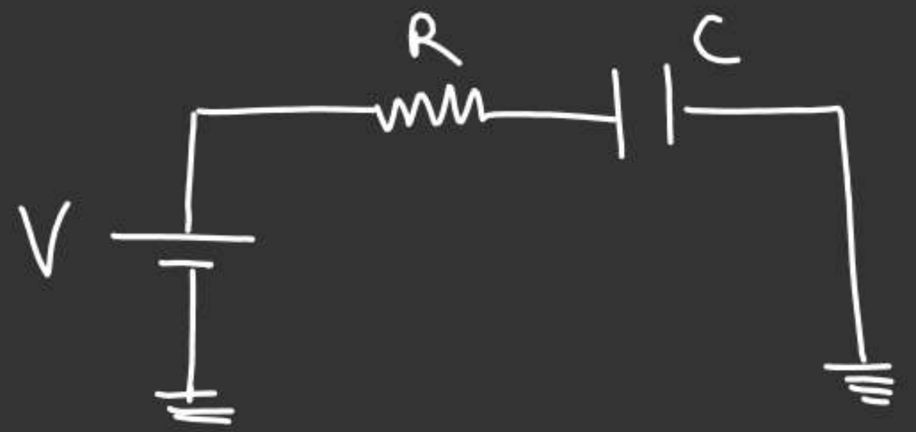
$\frac{L}{KA} = R.$

$$T = T_1 - (T_1 - T_2)\, e^{-\frac{t}{(ms)\left(\frac{L}{KA}\right)}}$$

$$\boxed{T = T_1 - (T_1 - T_2)\, e^{-\frac{t}{RC}}}$$

Block $\rightarrow$ Capacitor

Rod $\rightarrow$ Resistance



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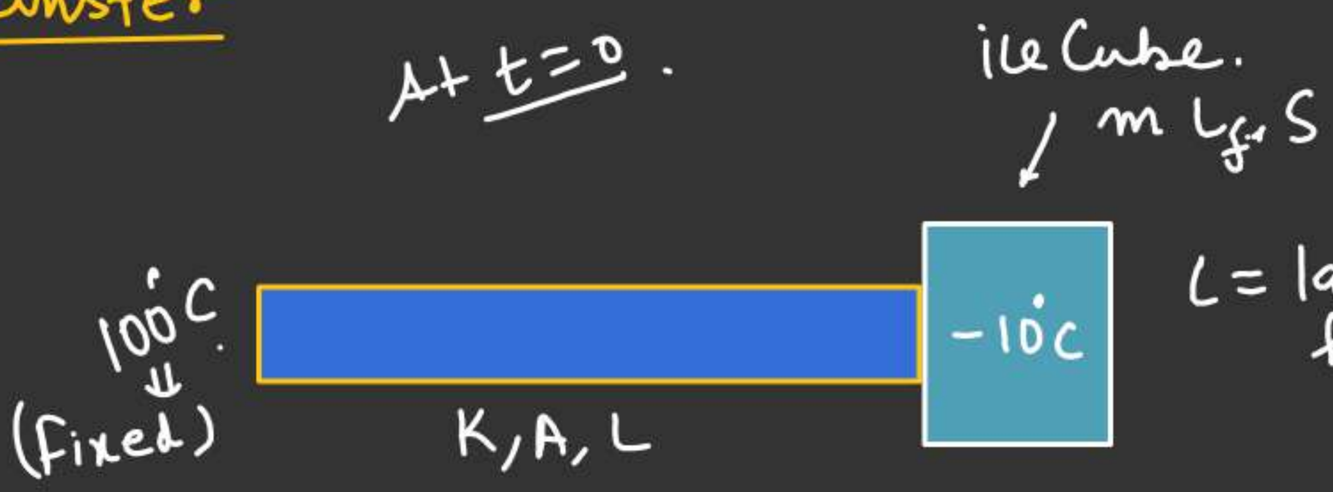
Heat Transfer

$$\boxed{T = T_1 - (T_1 - T_2)\, e^{-\frac{KA}{msL}t}}$$

At $t=0$.



L = latent heat of fusion of ice

S = Specific heat of ice

Find total time taken to convert ice cube to $0^\circ C$ water

Let, t_1 be the time taken to change the temp. of ice cube from $-10^\circ C$ to $0^\circ C$

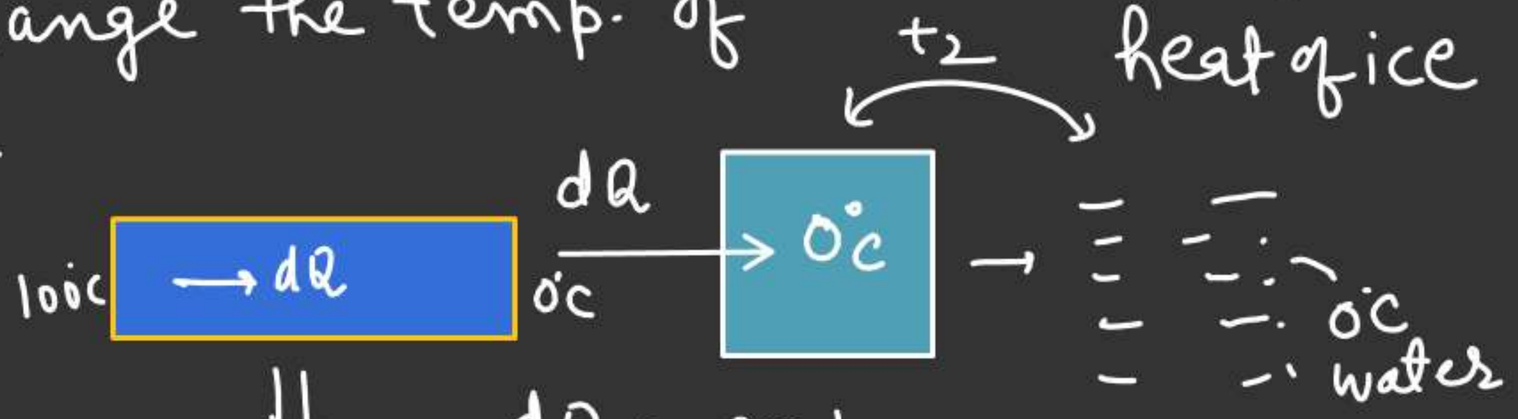
$T_1 = 100^\circ C$, $T_2 = -10^\circ C$, $T = 0^\circ C$

$$0 = 100 - (110)\, e^{-\frac{KA}{msL}t_1}$$

$$e^{-\frac{KA}{msL}t_1} = \frac{10}{11}$$

$$-\frac{KA}{msL}t_1 = \ln\left(\frac{10}{11}\right)$$

$$t_1 = \frac{msL}{KA}\ln\left(\frac{11}{10}\right)$$



$dQ = mL_f$

$$\frac{dQ}{dt} = \frac{100-0}{L/KA}$$

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Heat Transfer

$$L_f\int_0^m dm = \frac{100KA}{L}\int_0^{t_2} dt$$

$$mL_f = \frac{100KA}{L}\,t_2$$

$$t_2 = \left(\frac{mL_f L}{100KA}\right)$$

$$\left(t = t_1 + t_2\right)$$

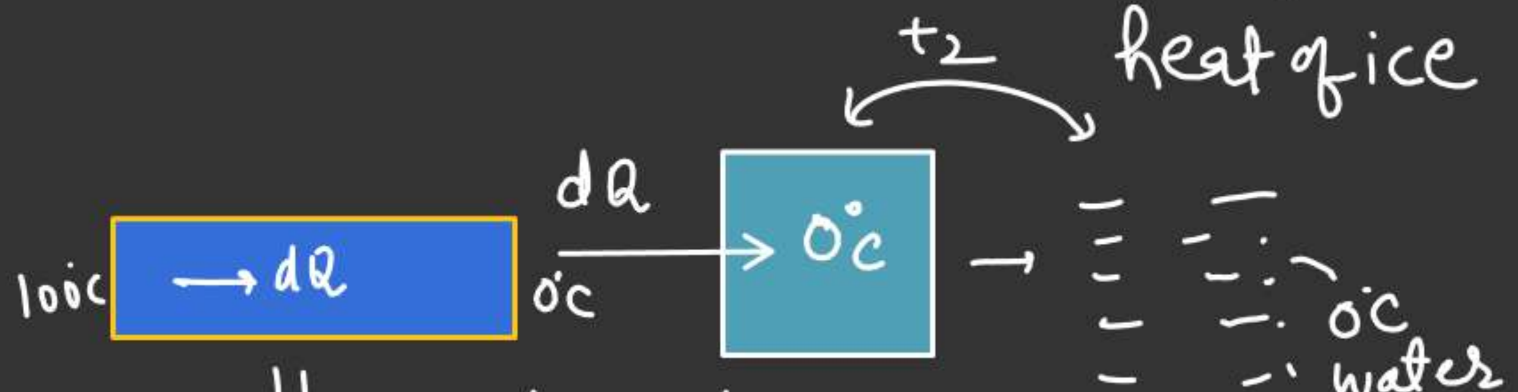
At $\underline{t=0}$.

ice Cube.

m L_f, S

L = latent heat of fusion of ice

S = Specific heat of ice



$$dQ = dm\,L_f$$

$$\frac{dQ}{dt} = \frac{100-0}{L/KA}$$

$$L_f\frac{dm}{dt} = \frac{KA(100)}{L}$$

Heat Transfer

. Find temp defference of vessel as a function of time

At $t=0$. $T_1 > T_2$.