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45(a) $= \int_1^5 f(x)\,dx + \int_2^{10} g(y)\,dy$

$= 50 - 2 = 48.$

(b) $f(0)=0,\ f(1)=1$

$\int_0^1 f(x) = \frac{1}{3}$ & $\int_0^1 f^{-1}(y)\,dy = 2$

$\Rightarrow \int_0^1 f(x)\,dx + \int_0^1 f^{-1}(x)\,dx = |x| - 0$

$\frac{1}{3} + \int_0^1 f^{-1}(x)\,dx = 1$

$\int_0^1 f^{-1}(x)\,dx = \frac{2}{3}.$

44) $\int x \cdot f''(x)\,dx$

$x \cdot f'(x)\Big| - \int 1 \cdot f'(x)\,dx$

43) $I = \int \frac{(\sec x - \tan x)\cdot \csc x}{\sqrt{1+2\csc x}}$ Sin / Cos.

$\sec x + \tan x = \frac{1}{\sec x - \tan x}$

42) $f(x) = e^{-x} + 2e^{-2x} + 3e^{-3x} + \cdots \infty$ (AGP)

$f(x)\cdot e^{-x} = \quad + e^{-2x} + 2e^{-3x} + \cdots \infty$

$f(x)(1-e^{-x}) = e^{-x} + e^{-2x} + e^{-3x} + \cdots \infty = \frac{e^{-x}}{1-e^{-x}}$

∞ GP

$\int f(x) = \int \frac{e^{-x}}{(1-e^{-x})^2}\,dx \qquad 1 - e^{-x} = t$

$e^{-x}\,dx = dt$

$\int \frac{dt}{t^2} = -\frac{1}{1-e^{-x}} + C$

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41) $\int_a^b \frac{|x|}{x}\cdot dx$ $\frac{d(|x|)}{dx} = \frac{|x|}{x}$ ($x\neq 0$)

$= |x|\Big|_a^b = |b| - |a|$

40)

39) $\cos^2 A - \cos^2 B$

$= \sin(A+B)\cdot\sin(A-B)$

37) $\left(\frac{1}{1+e^{-1/x}}\right)_{-1}^{0^-} + \left(\frac{1}{1+e^{-1/x}}\right)_{0^+}^{1}$

Copy में $\cot^{-1}\frac{1}{x}$ के Qs जाँचो

38) $\int_1^e \frac{dx}{x(1+\ln x)}$ $1+\ln x = t$

35) $\int \frac{x^2+x}{(x+1)(x^2+1)} + \frac{x^2+1}{(x+1)(x^2+1)}\,dx$

33) $\int \frac{1+2\cos x}{(2+\cos x)^2}$ $\rightarrow$ $\div$ by $\sin^2 x$

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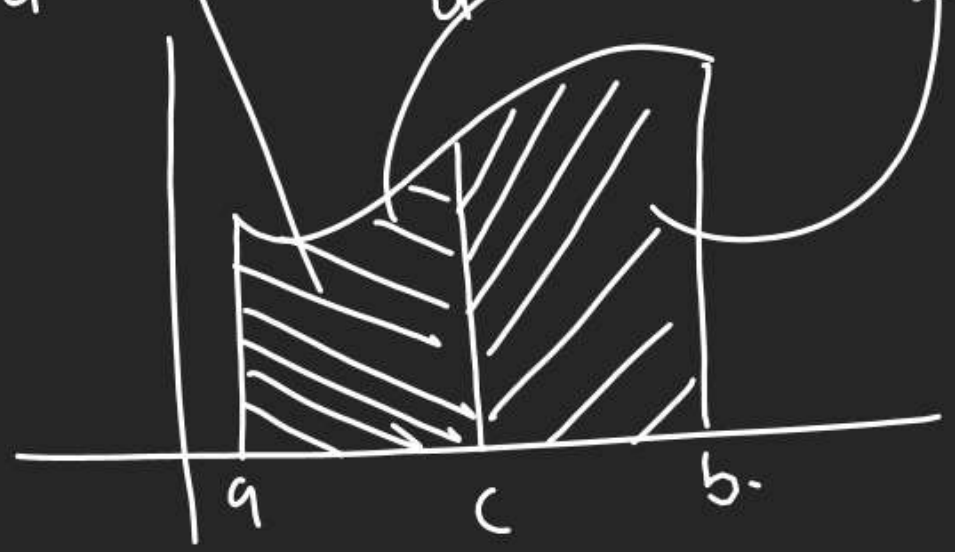
Prop3 → Splitting Limit Based Prop

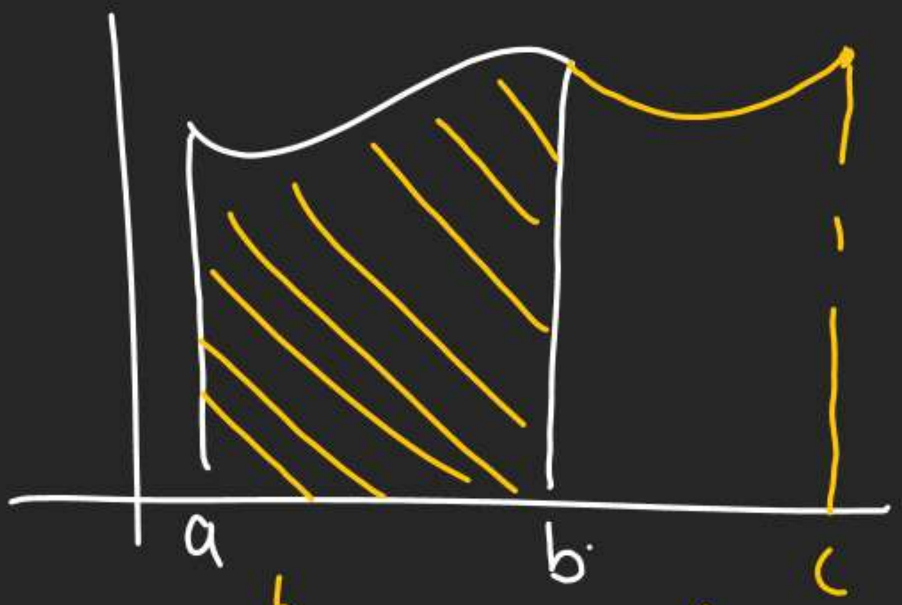
1) $\int_a^b f(x)\cdot dx = \int_a^c f(x)\cdot dx + \int_c^b f(x)\cdot dx$ $c \in (a,b)$



2) If $c_1, c_2, c_3 \ldots c_n \in (a,b)$ then

$$\int_a^b f(x)\cdot dx = \int_a^{c_1} f(x)\cdot dx + \int_{c_1}^{c_2} f(x)\cdot dx + \int_{c_2}^{c_3} f(x)\cdot dx + \ldots + \int_{c_n}^{b} f(x)\cdot dx$$

(3) c can be outside to (a,b)



$$\int_a^b f(x)\cdot dx = \int_a^c f(x)\cdot dx - \int_b^c f(x)\cdot dx$$

Pr 2

$$\boxed{\int_a^b f(x)\cdot dx = \int_a^c f(x)\cdot dx + \int_c^b f(x)\cdot dx}$$

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Q $\int_{-1}^{4} f(x)\cdot dx = 4$ & $\int_{2}^{4} (3-f(x))\cdot dx = 7$

then $\int_{-1}^{2} f(x)\cdot dx = ?$

$-1 \quad 2 \quad 4$

$\int_{2}^{4} 3\cdot dx - \int_{2}^{4} f(x)dx = 7$

$3x\Big|_2^4 - \int_{2}^{4} f(x)\cdot dx = 7$

$\Rightarrow \boxed{\int_{2}^{4} f(x)\cdot dx = -1}$

Demand $= \int_{-1}^{2} f(x)\, dx$

$= \int_{2}^{4} f(x)\cdot dx + \int_{4}^{-1} f(x)\cdot dx$

$= -1 + (-4) = -5$

Q2 If $\int_{0}^{100} f(x)\cdot dx = a$ then $\sum_{r=1}^{100} \int_{0}^{1} f(r-1+x)\cdot dx = ?$

$\int_{0}^{1} f(x)\cdot dx + \int_{0}^{1} f(1+x)\cdot dx + \int_{0}^{1} f(2+x)\cdot dx + \int_{0}^{1} f(3+x)\cdot dx + \cdots + \int_{0}^{1} f(99+x)$

$1+x = t \; \begin{array}{c|c} x & t \\ 0 & 1 \\ 1 & 2 \end{array}$ $\quad 2+x = t \; \begin{array}{c|c} x & t \\ 0 & 2 \\ 1 & 3 \end{array}$

$\Rightarrow \int_{0}^{1} f(x)\cdot dx + \int_{1}^{2} f(t)dt + \int_{2}^{3} f(t)dt + \int_{3}^{4} f(t)dt + \cdots \int_{99}^{100} f(t)dt$

$= \int_{0}^{100} f(x)\cdot dx = a$ Ans

Q3 If $f(x) = \begin{cases} \sqrt{x} & 0 \le x \le 1 \\ x^2 & 1 < x \le 2 \end{cases}$ then $\int_{0}^{2} f(x)\cdot dx$

$= \int_{0}^{1} \sqrt{x}\cdot dx + \int_{1}^{2} x^2\cdot dx = \frac{2}{3}(x)^{3/2}\Big|_0^1 + \frac{x^3}{3}\Big|_1^2$

$\frac{2}{3} + \frac{7}{3} = 3$

Props Mostly We Use in Qs of Mod x, GIF, { }, Sgn fxn & Defined fxn

Q4 $\int_{-1}^{1} e^{|x|}\,dx$

T.P $\Rightarrow x = 0$

Break Limit at Turning Pt of Mod

$= \int_{-1}^{0} e^{-x}\,dx + \int_{0}^{1} e^{x}\,dx$

$= -e^{-x}\Big|_{-1}^{0} + e^{x}\Big|_{0}^{1}$

$= -(e^{0} - e^{1}) + (e^{1} - e^{0})$

$= -1 + e + e - 1$

$= 2(e-1)$

$x \in (0,1)$
$x = +ve$
$|x| = x$

$x \in (-1,0)$
$x = -ve$
$|x| = -x$

Q $\int_{0}^{5} |x-3|\,dx$

T.P. $\Rightarrow x = 3$.

$x \in (3,5)$
$x-3 \in (0,2)$
$x-3 = +ve$

$x \in (0,3)$
$x-3 \in (-3,0)$
$x-3 = -ve$

$= -\int_{0}^{3} (x-3)\,dx + \int_{3}^{5} (x-3)\,dx$

$= 3x - \frac{x^2}{2}\Big|_{0}^{3} + \frac{x^2}{2} - 3x\Big|_{3}^{5}$

$= \left(9 - \frac{9}{2}\right) - 0 + \left(\frac{25}{2} - 15\right) - \left(\frac{9}{2} - 9\right)$

Board

$= 2\left(\frac{9}{2}\right) - \frac{5}{2} = \frac{13}{2}$

$= \frac{3\times 3}{2} + \frac{2\times 2}{2} = \frac{13}{2}$

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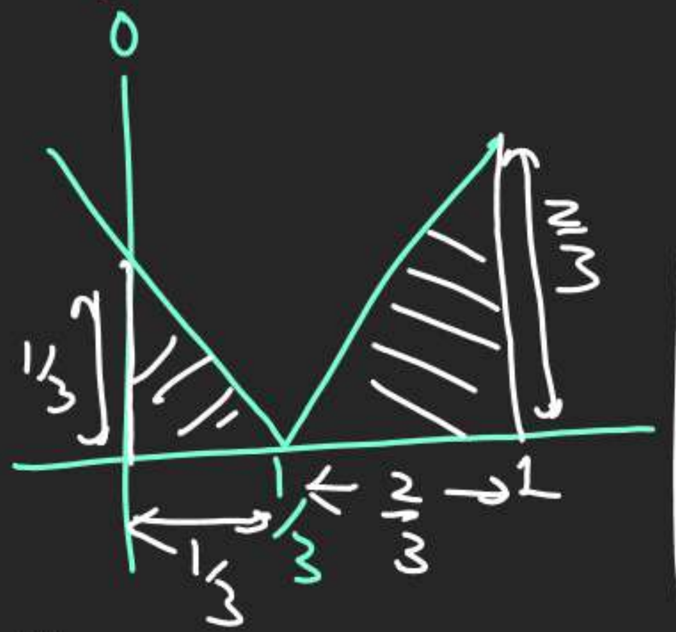
Q6 $\int_0^1 |3x-1| \cdot dx$

$\Rightarrow 3\int_0^1 \left|x-\frac{1}{3}\right| \cdot dx$



$3\left\{\frac{\frac{1}{3}\times\frac{1}{3}}{2}+\frac{\frac{2}{3}\times\frac{2}{3}}{2}\right\}$

$= \frac{1}{18}+\frac{4}{18}=\frac{5}{18}\times 3=\frac{5}{6}$

Q7 $\int_0^1 |x^2+x+1| \cdot dx$

$D=-3=-ve$

$x^2+x+1=+ve$

1) |Q uadl ho then factorise

2) If factorisation not Possible then check $D<0$

$\int_0^1 (x^2+x+1)dx$

$=\left[\frac{x^3}{3}+\frac{x^2}{2}+x\right]_0^1$

$=\frac{1}{3}+\frac{1}{2}+1$

$=\frac{11}{6}$

Q $\int_1^2 |x^2-3x+2| \cdot dx$

$\Rightarrow \int_1^2 |(x-1)(x-2)| \cdot dx$

$+ \quad 1 \quad - \quad 2 \quad +$

in $x\in(1,2)$ $x^2-3x+2=-ve$

$\Rightarrow I=-\int_1^2 (x^2-3x+2) \cdot dx$

$=-\left[\frac{x^3}{3}-\frac{3x^2}{2}+2x\right]_1^2$

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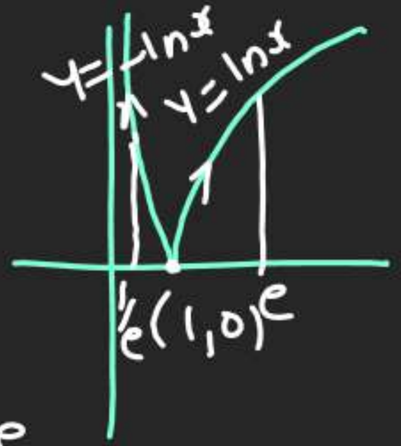
Q9 $I=\int_{1/e}^{e}|\ln x|\cdot dx$



$$=-\int_{1/e}^{1}\ln x\cdot dx+\int_{1}^{e}\ln x\cdot dx$$

$$=-\Big[x\ln x-x\Big]_{1/e}^{1}+\Big[x\ln x-x\Big]_{1}^{e}$$

$$=+\left[(0+1)+\left(-\frac{1}{e}-\frac{1}{e}\right)\right]+\left[(e\times 1-e)-(0-1)\right]$$

$$=1-\frac{2}{e}+1=2-\frac{2}{e}$$

Q10 $I=\int_{1/e}^{e}\left|\frac{\ln x}{x}\right|\cdot dx$

$x\in\left(\frac{1}{e},e\right)$

$x\in\left(\frac{1}{3},3\right)$

$x=+ve$

$$I=\int_{1/e}^{e}\frac{|\ln x|}{x}\cdot dx$$

$$=\int_{1/e}^{1}-\frac{\ln x}{x}\cdot dx+\int_{1}^{e}\frac{\ln x}{x}\cdot dx$$

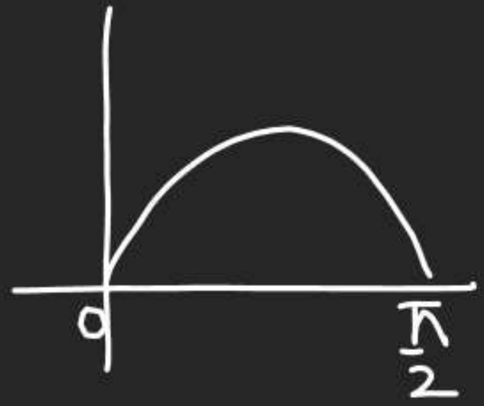
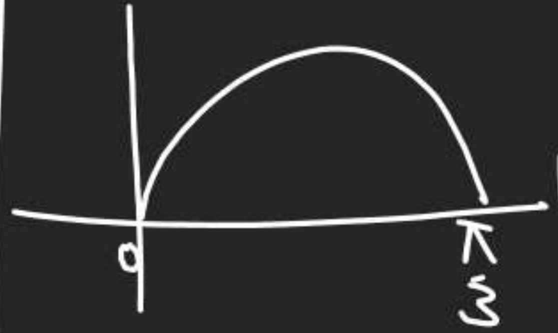
$$=-\frac{(\ln x)^2}{2}\Bigg|_{1/e}^{1}+\frac{(\ln x)^2}{2}\Bigg|_{1}^{e}$$

$$=-\left(0+\frac{(-1)^2}{2}\right)+\left(\frac{1}{2}-0\right)$$

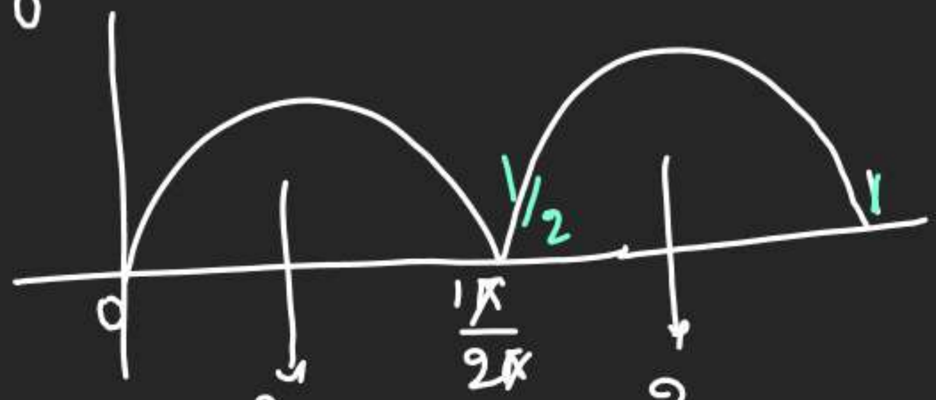
$$=1$$

Q

Base.

		Area
1) $\int_0^{\pi} \sin x \cdot dx$		2
2) $\int_0^{\pi/2} \sin 2x \cdot dx$		$\frac{2}{2}$
3) $\int_0^{\pi/3} \sin 3x\, dx$		$\frac{2}{3}$.
4) $\int_0^{\pi/n} \sin nx$		$\frac{2}{n}$.

Q $\int_0^{1} \lvert \sin 2\pi x \rvert \cdot dx$



$$\frac{2}{2\pi} + \frac{2}{2\pi}$$

$$\frac{1}{\pi} + \frac{1}{\pi} = \frac{2}{\pi}$$

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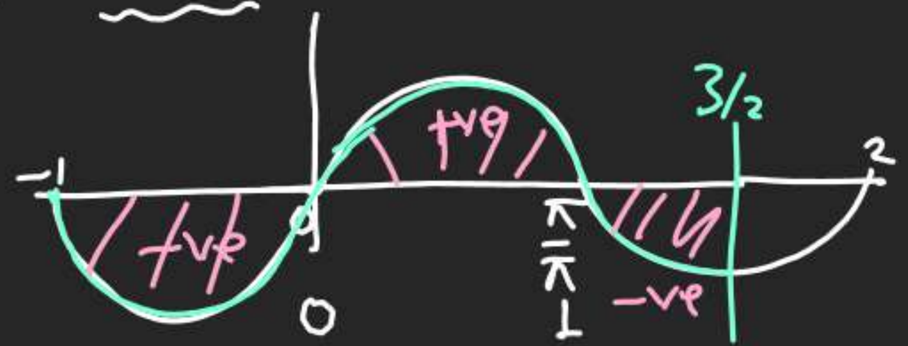
Q $\int_{-1}^{3/2} |x \cdot \sin \pi x| \cdot dx$

Adv



$$\Rightarrow \int_{-1}^{0} x \cdot \sin \pi x \cdot dx + \int_{0}^{1} x \cdot \sin \pi x \cdot dx + \int_{1}^{3/2} x \cdot \sin \pi x \, dx$$

$x \in (-1, 0)$ $x = -ve$ $\sin \pi x = -ve$ $x \cdot \sin \pi x = +ve$	$x \in (0, 1)$ $x = +ve$ $\sin \pi x = +ve$ $x \cdot \sin \pi x = +ve$	$x \in (1, 3/2)$ $x = +ve$ $\sin \pi x = -ve$ $x \cdot \sin \pi x = -ve$

$$\Rightarrow \int_{-1}^{1} x \cdot \sin \pi x \cdot dx - \int_{1}^{3/2} x \cdot \sin \pi x \cdot dx \Rightarrow \left[x \cdot \left(-\frac{\cos \pi x}{\pi}\right) - 1 \cdot \left(-\frac{\sin \pi x}{\pi^2}\right)\right]_{-1}^{1} - \left[x\left(-\frac{\cos \pi x}{\pi}\right) - 1 \cdot \left(-\frac{\sin \pi x}{\pi^2}\right)\right]_{1}^{3/2}$$

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[ ] & { } Based Qs

Q12 $I = \int_0^5 [x]\,dx$

$= \int_0^1 0\,dx + \int_1^2 1\,dx + \int_2^3 2\,dx + \int_3^4 3\,dx + \int_4^5 4\,dx$

$x \in (0,1)$, $[x] = 0$ $\qquad$ $x \in (1,2)$, $[x] = 1$

$= 0 + 1\cdot(x)_1^2 + 2(x)_2^3 + 3(x)_3^4 + 4(x)_4^5$

$= 0 + 1\cdot(2-1) + 2(3-2) + 3(4-3) + 4(5-4)$

$= 0 + 1 + 2 + 3 + 4 = 10$

Q13 $I = \int_0^n [x]\,dx$, $n \in N$

$= 0+1+2+3+\dots+(n-1) = \frac{(n-1)(n-1+1)}{2} = \frac{(n)(n-1)}{2}$

Result

$$\int_0^n [x]\,dx = \frac{n(n-1)}{2}$$

Result

$$\int_0^n \{x\}\,dx = \frac{n}{2}$$

Q $I = \int_0^5 \{x\}\,dx$

M1 $= \int_0^5 x - [x]\,dx = \left.\frac{x^2}{2}\right|_0^5 - \int_0^5 [x]\,dx = \frac{25}{2} - 10 = \frac{5}{2}$

M2

$\int_0^5 \{x\}\,dx =$ Area of 5 chhote Δ

$= 5 \times \frac{1\times1}{2} = \frac{5}{2}$

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Q15 $\dfrac{\int_0^n [x]\,dx}{\int_0^n \{x\}\,dx}$

$$= \frac{\frac{n(n-1)}{2}}{\frac{n}{2}} = (n-1) \text{ Ans}$$

Q16 $\int_0^{[x]} \{x\}\,dx = \dfrac{[x]}{2}$ Ans.

Q17 $\int_0^{[11.7]} \{x\}\,dx = \int_0^{11} \{x\}\,dx = \dfrac{11}{2}$

Q $\int_0^{11.7} \{x\}\,dx$

← 11 Δ's Area

$$\frac{11}{2} + \frac{.7\times .7}{2} = \frac{11.49}{2}$$

Same Result can be seen from this point of view

$$\int_0^{11.7} \{x\}\,dx = \frac{[11.7]}{2} + \frac{\{11.7\}\times\{11.7\}}{2}$$

Result

$$\boxed{\int_0^{t} \{x\}\,dx = \frac{[t]}{2} + \frac{\{t\}^2}{2}}$$

Q18 $\int_0^1 [2x]\,dx$

$x \to 0 - 1$

$2x \to 0 - 2$

$2x \to 0 - 1 - 2$

$x \to 0 - \frac{1}{2} - 1$

$$= \int_0^{1/2} 0\,dx + \int_{1/2}^{1} 1\,dx$$

$$= 0 + (x)_{1/2}^{1} = 1 - \frac{1}{2} = \frac{1}{2}$$

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Q19 $\int_0^1 [4x]\cdot dx$

$x \to 0-1$

$4x \to 0-4$

$4x \to 0-1-2-3-4$

$x \to 0-\frac{1}{4}-\frac{2}{4}-\frac{3}{4}-\frac{4}{4}$

$= \int_0^{1/4} 0\cdot dx + \int_{1/4}^{2/4} 1\cdot dx + \int_{2/4}^{3/4} 2\cdot dx + \int_{3/4}^{4/4} 3\cdot dx$

$0 + 1(x)_{1/4}^{2/4} + 2(x)_{2/4}^{3/4} + 3(x)_{3/4}^{4/4}$

$= DY$

Prop3 वाले Qs

वर्कशीट

अरगर्म

Q Adv $\int_0^{3/2} [x^2]\, dx$

$x \to 0-3/2$

$x^2 \to 0-\frac{9}{4}$

$x^2 \to 0-1-2-\frac{9}{4}$

$x \to 0-1-\sqrt{2}-\frac{3}{2}$

$= \int_0^1 0\cdot dx + \int_1^{\sqrt{2}} 1\cdot dx + \int_{\sqrt{2}}^{3/2} 2\cdot dx$

$0 + (x)_1^{\sqrt{2}} + 2(x)_{\sqrt{2}}^{3/2}$