Nishant Jindal

$\boxed{Q\ Eq^n}$

Q Solve $3x^2-2x-1=0$ by factorisation Method.

$$3x^2-3x+x-1=0$$

$$3x(x-1)+1(x-1)=0$$

$$(x-1)(3x+1)=0$$

$$x=1,-1/3$$

Q Solve $3x^2-2x-1=0$ by Per. Sqr Method

$$3\left(x^2-\boxed{\tfrac{2}{3}}x-\tfrac{1}{3}\right)=0 \qquad \tfrac{1}{3}$$

$$3\left(\left(x-\tfrac{1}{3}\right)^2-\left(\tfrac{1}{3}\right)^2-\tfrac{1}{3}\right)=0$$

$$\left(x-\tfrac{1}{3}\right)^2-\tfrac{1}{9}-\tfrac{3}{9}=0 \Rightarrow \left(x-\tfrac{1}{3}\right)^2-\left(\tfrac{2}{3}\right)^2=0$$

$$\left(\left(x-\tfrac{1}{3}\right)-\tfrac{2}{3}\right)\left(\left(x-\tfrac{1}{3}\right)+\tfrac{2}{3}\right)=0$$

$$(x-1)\left(x+\tfrac{1}{3}\right)=0$$

$$x=1,-1/3.$$

Q Solve $3x^2-2x-1=0$ by Sh. Dharacharya Method?

$$x=\frac{2\pm\sqrt{(-2)^2-4\times3\times-1}}{2\times3}$$

$$=\frac{2\pm\sqrt{4+12}}{3\times2}=\frac{2\pm4}{3\times2}$$

$$\frac{2+4}{2\times3}=1$$

$$\frac{2-4}{2\times3}=\frac{-2}{2\times3}=-\frac{1}{3}$$

Rk

1) Values of x by all 3 Methods are Roots of Q. Eq^n

2) Roots can be find out
- Factorisation
- Per. Sq^n
- Sh. Dharacharya.

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Q Find Roots of.

$$x^2+2x-12=0$$

$$x=\frac{-2\pm\sqrt{(2)^2-4\times1\times-12}}{2\times1}$$

$$=\frac{-2\pm\sqrt{52}}{2}$$

$$=\frac{-2\pm2\sqrt{13}}{2}\begin{cases}\to -1+\sqrt{13}\\ \to -1-\sqrt{13}\end{cases}\text{Roots}$$

Do you know?

$x^2+2x-12=0$ factorise karo?

$$\Rightarrow (x-(-1+\sqrt{13}))(x-(-1-\sqrt{13}))=0$$

Per sq. Method

$$(x^2+2x+1)-13=0$$

$$(x+1)^2-(\sqrt{13})^2=0$$

$$(x+1-\sqrt{13})(x+1+\sqrt{13})=0$$

$$x^2+2x-12=0$$

$$POR=\frac{c}{a}=\frac{-12}{1}=-12$$

$$SOR=-\frac{b}{a}=-\frac{2}{1}=-2$$

① SOR = Sum of Roots $=(-1+\cancel{\sqrt{13}})+(-1-\cancel{\sqrt{13}})=-2$

② POR = Prod $=(-1+\sqrt{13})(-1-\sqrt{13})$

$$=(-1)^2-(\sqrt{13})^2=1-13=-12$$

$$-(\alpha+\beta)=\frac{b}{a}$$

$$\alpha\cdot\beta=\frac{c}{a}$$

Q How $\alpha+\beta=-\frac{b}{a}$ & $\alpha\cdot\beta=\frac{c}{a}$?

If $ax^2+bx+c=0$ has 2 Root α & β then QE in term of Roots

$$\boxed{ax^2+bx+c=a(x-\alpha)(x-\beta)}$$

$$ax^2+bx+c=a(x^2-(\alpha+\beta)x+\alpha\cdot\beta)$$

$$x^2+\frac{b}{a}x+\frac{c}{a}$$

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Q Sum of all real Roots of x satisfying

eqⁿ $3^{(x-1)(x^2+5x-50)}=1$

$3^{(x-1)(x^2+5x-50)}=3^0$

Compare.

$(x-1)(\underline{x^2+5x-50})=0$

$x-1=0$ or $x^2+5x-50=0$

$\boxed{x=1}$ | $(x+10)(x-5)=0$

$x=\underline{5,-10}$

$x=1,5,-10$ ← Roots

Sum $=1+5-10=-4$

RK:-

$(var)^{var}=1$ given ho.

Base = Kuchh Bhi Degree = 0	Base = 1 Power = Kuchh Bhi	Base = −1 Power = Even

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1) Sum of all Real Values of x satisfying Eqn.

Mains

$$(x^2-5x+5)^{x^2+4x-60} = 1$$

$0^0 \neq 1$

A) Base = Kuchh B

Deg = $x^2+4x-60=0$

$(x+10)(x-6)=0$

$x=6,-10$

Base check

Base 0 नहीं होना चाहिए

x^2-5x+5

$6^2-5\times6+5 = 35-30 \neq 0$

$(-10)^2+50+5 \neq 0.$

(B) Base = 1 ⇒ $x^2-5x+5=1$

$x^2-5x+4=0$

$(x-1)(x-4)=0 \Rightarrow x=1,4$

Power Kuchh B

$x = 6, -10, 1, 4, 2$

Sum $= 6+1+4+2-10 = \boxed{3}$ Ans.

Rk:-

$(\text{Var})^{\text{Var}} = 1$ given ho. **

Base = Kuchh Bhi Degree = 0	Base = 1 Power = Kuchh Bhi	Base = −1 Power = Even

(C) Base = −1 ⇒ $x^2-5x+5=-1 \Rightarrow x^2-5x+6=0$

$\Rightarrow (x-2)(x-3)=0 \Rightarrow x=2,3$

Now Power should be even at $x=2$, $x=3$

Power $x^2+4x-60 \xRightarrow{x=2} 4+8-60$ = Even.

$x^2+4x-60 \xRightarrow{x=3} 9+12-60$ = odd

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Q If Roots of Q Eqn are 8 & -3.

find Q Eqn

$x^2-(\alpha+\beta)x+\alpha\beta=0$

$x^2-(8+-3)x+8\times-3=0$

$\underline{x^2-5x-24=0}$

Q For Q Eqn $x^2-x-2=0$ find Difference of Roots

$\alpha+\beta=-1$

$\alpha\cdot\beta=-2$

Difference of Roots $=|\alpha-\beta|=\sqrt{(\alpha-\beta)^2}$

$=\sqrt{(\alpha+\beta)^2-4\alpha\beta}$

$=\sqrt{(-1)^2-4\times-2}=3$

Rem.

$|\alpha-\beta|=\sqrt{(\alpha+\beta)^2-4\alpha\beta}$

$=\sqrt{\left(\frac{-b}{a}\right)^2-\frac{4c}{a}}=\sqrt{\frac{b^2-4ac}{a^2}}=\frac{\sqrt{D}}{a}$

$\therefore \boxed{DOR=|\alpha-\beta|=\frac{\sqrt{D}}{a}}$

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$P(5) = \frac{1}{4}\sin^2\left(\frac{\pi}{2\times 5}\right) = \frac{1}{4}\cdot\sin^2 18 = \frac{1-\cos 36}{8} = \frac{1}{8}\left[1-\frac{\sqrt5+1}{4}\right] = \frac{1}{8}\left[\frac{3-\sqrt5}{4}\right]$

Ex 2 Discussion TrigoPhL

Q 11 If $\alpha+\beta=\gamma$ $\quad \cos^2\alpha+\cos^2\beta+\cos^2\gamma = 1+2\cos\alpha\cos\beta\cos\gamma$.

$\sin^2\theta = \frac{1-\cos 2\theta}{2}$

$\sin^2 18 = \frac{1-\cos 36}{2}$

$\cos\frac{(2K-1)\pi}{4K}$

$\cos\left(\frac{2K}{4K}-\frac{1}{4K}\right)\pi$

$\cos\left(\frac{\pi}{2}-\frac{\pi}{4K}\right)$

LHS $\to \cos^2\alpha+\cos^2\beta+\cos^2(\alpha+\beta)$

$\frac{1+\cos 2\alpha}{2}+\frac{1+\cos 2\beta}{2}+\cos^2(\alpha+\beta)$

$\frac{1}{2}\left[2+(\cos 2\alpha+\cos 2\beta)\right]+\cos^2(\alpha+\beta)$

$1+\cancel{2}\cos(\alpha+\beta)\cdot\cos(\alpha-\beta)+\cos^2(\alpha+\beta)$

$1+\cos(\alpha+\beta)\{\cos(\alpha-\beta)+\cos(\alpha+\beta)\}$

$1+\cos(\alpha+\beta)\times 2\cos\alpha\cos\beta$

$1+2\cos\alpha\cos\beta\cos\gamma = $ RHS.

12) good $\quad P(5), P(6)?$

$P(K) = \left(1+\cos\frac{\pi}{4K}\right)\left(1+\cos\frac{(2K-1)\pi}{4K}\right)\left(1+\cos\frac{(2K+1)\pi}{4K}\right)\left(1+\cos\frac{(4K-1)\pi}{4K}\right)$

$= \left(1+\cos\frac{\pi}{4K}\right)\left(1+\cos\left(\frac{\pi}{2}-\frac{\pi}{4K}\right)\right)\left(1+\cos\left(\frac{\pi}{2}+\frac{\pi}{4K}\right)\right)\left(1+\cos\left(\pi-\frac{\pi}{4K}\right)\right)$

$= \left(1+\cos\frac{\pi}{4K}\right)\left(1+\sin\frac{\pi}{4K}\right)\left(1-\sin\frac{\pi}{4K}\right)\left(1-\cos\frac{\pi}{4K}\right)$

$= \left(1-\cos^2\frac{\pi}{4K}\right)\left(1-\sin^2\frac{\pi}{4K}\right)$

$= \frac{4}{4}\sin^2\frac{\pi}{4K}\times\cos^2\frac{\pi}{4K} = \frac{1}{4}\left(2\sin\frac{\pi}{4K}\cos\frac{\pi}{4K}\right)^2$

$P(K) = \frac{1}{4}\left(\sin^2\frac{\pi}{2K}\right)$

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QUADRATIC EQUATION

Q 13(a) $4\cos 20^\circ - \boxed{\sqrt{3}}\cot 20^\circ$

$4\cos 20^\circ - \tan 60^\circ \cdot \dfrac{\cos 20^\circ}{\sin 20^\circ}$

$\dfrac{\left(4\cos 20^\circ \cdot \sin 20^\circ - \dfrac{\sin 60^\circ}{\cos 60^\circ} \cdot \cos 20^\circ\right.}{\sin 20^\circ}$

$\dfrac{2\sin 40^\circ - \dfrac{\sin 60^\circ \cdot \cos 20^\circ}{(\cos 60^\circ)}}{\sin 20^\circ} = \dfrac{2\sin 40^\circ - 2\sin 60^\circ \cdot \cos 20^\circ}{\sin 20^\circ} = \dfrac{2\sin 40^\circ - [\sin(80^\circ) + \sin(40^\circ)]}{\sin 20^\circ}$

$\sin 60^\circ \cdot \cos 20^\circ = \frac{1}{2}(2\sin 60^\circ \cdot \cos 20^\circ)$

$= \frac{1}{2}[\sin(40^\circ) + \sin(20^\circ)]$

$= \dfrac{\sin 40^\circ - \sin 80^\circ}{\sin 20^\circ} = \dfrac{\cancel{2}\cos(60^\circ) \cdot \sin(-20^\circ)}{\sin 20^\circ} = -\dfrac{\sin 20^\circ}{\sin 20^\circ} = -1$

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13(b) $\dfrac{2\cos 40^\circ - \cos 20^\circ}{\sin 20^\circ}$ $\quad \sin C + \sin D$

$\dfrac{\cos 40 + \cos 40 - \cos 20^\circ}{\sin 20^\circ}$

$\dfrac{\cos 40^\circ - \cancel{2}\,\cancel{\sin}(30^\circ)\cdot \sin(10^\circ)}{\sin 20^\circ}$

$\dfrac{\cos 40^\circ - \sin 10^\circ}{\sin 20^\circ} = \dfrac{\sin 50^\circ - \sin 10^\circ}{\sin 20^\circ}$

$= \dfrac{2\cos(30^\circ)\cdot \cancel{\sin(20^\circ)}}{\cancel{\sin 20^\circ}} = \sqrt{3}.$

(c) Copy. $\quad a^3+b^3=(a+b)(a^2+b^2-ab)$

$\sin^4 x + \cos^4 x = 1 - 2\sin^2 x\cos^2 x$

$\left(\cos^2\frac{\pi}{16}\right)^3 + \left(\sin^2\frac{\pi}{16}\right)^3 + \left(\cos^2\frac{3\pi}{16}\right)^3 + \left(\sin^2\frac{3\pi}{16}\right)^3$

$\left(\cancel{\cos^2\frac{\pi}{16} + \sin^2\frac{\pi}{16}}\right)_{1}\left(\cos^4\frac{\pi}{16} + \sin^4\frac{\pi}{16} - \sin^2\frac{\pi}{16}\cdot\cos^2\frac{\pi}{16}\right)$

$\left(1 - 2\sin^2\frac{\pi}{16}\cdot\cos^2\frac{\pi}{16} - \sin^2\frac{\pi}{16}\cdot\cos^2\frac{\pi}{16}\right)$

$\left(1 - \frac{3}{4}\times 4\sin^2\frac{\pi}{16}\cdot\cos^2\frac{\pi}{16}\right) + \left(1 - \frac{3}{4}\times 4\sin^2\frac{3\pi}{16}\cdot\cos^2\frac{3\pi}{16}\right)$

$\left(1 - \frac{3}{4}\cdot\left(2\sin\frac{\pi}{16}\cdot\cos\frac{\pi}{16}\right)^2\right) + \left(1 - \frac{3}{4}\left(2\sin\frac{3\pi}{16}\cdot\cos\frac{3\pi}{16}\right)^2\right)$

$\left(1 - \frac{3}{4}\left(\sin\frac{\pi}{8}\right)^2\right) + \left(1 - \frac{3}{4}\cdot\left(\sin\frac{3\pi}{8}\right)^2\right)$

$\left(1 - \frac{3}{4}\left(\frac{\sqrt{2}-1}{2}\right)^2\right) + \left(1 - \frac{3}{4}\left(\frac{\sqrt{2}+1}{2}\right)^2\right)$

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QUADRATIC EQUATION

14) $(1+\tan 1^\circ)(1+\tan 44^\circ)=2$

$(1+\tan 2^\circ)(1+\tan 43^\circ)=2$

$\vdots$

$(1+\tan 22^\circ)(1+\tan 23^\circ)=2$

LHS. $\underbrace{2\times 2\times 2\times \cdots 2}_{22\text{ times}}\times(1+\tan 45^\circ)$

$=2^{23}=2^n$

$\underline{\underline{n=23}}$

$\underline{\underline{\text{Ex 3}}}$ $\boxed{6}$