REFRACTION

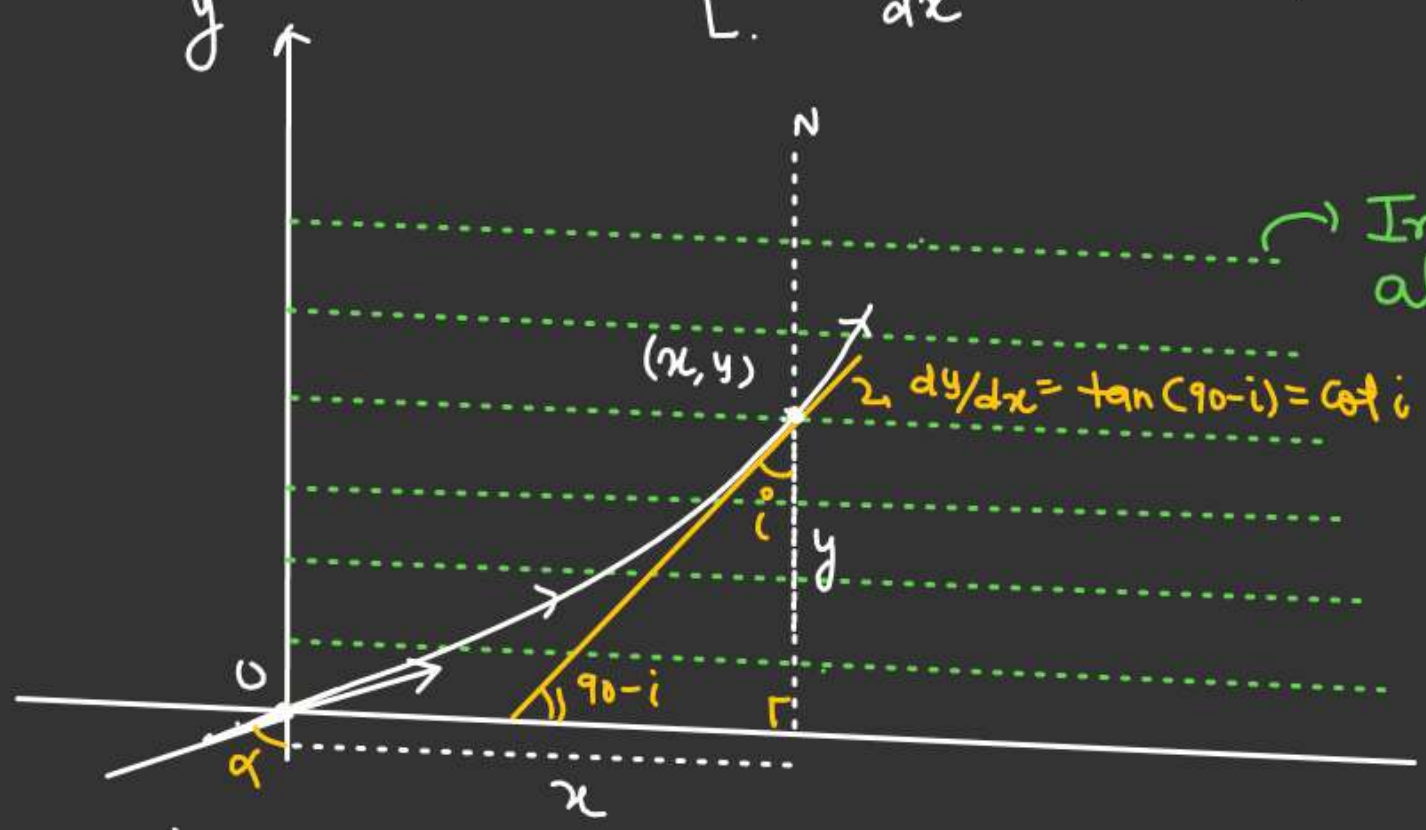
Case when μ vary either along x-axis or y-axis

Approach.

$\mu = f(y)$ ✓

① $\mu_0 \sin\alpha = \mu_y \sin i = \text{Constant}$

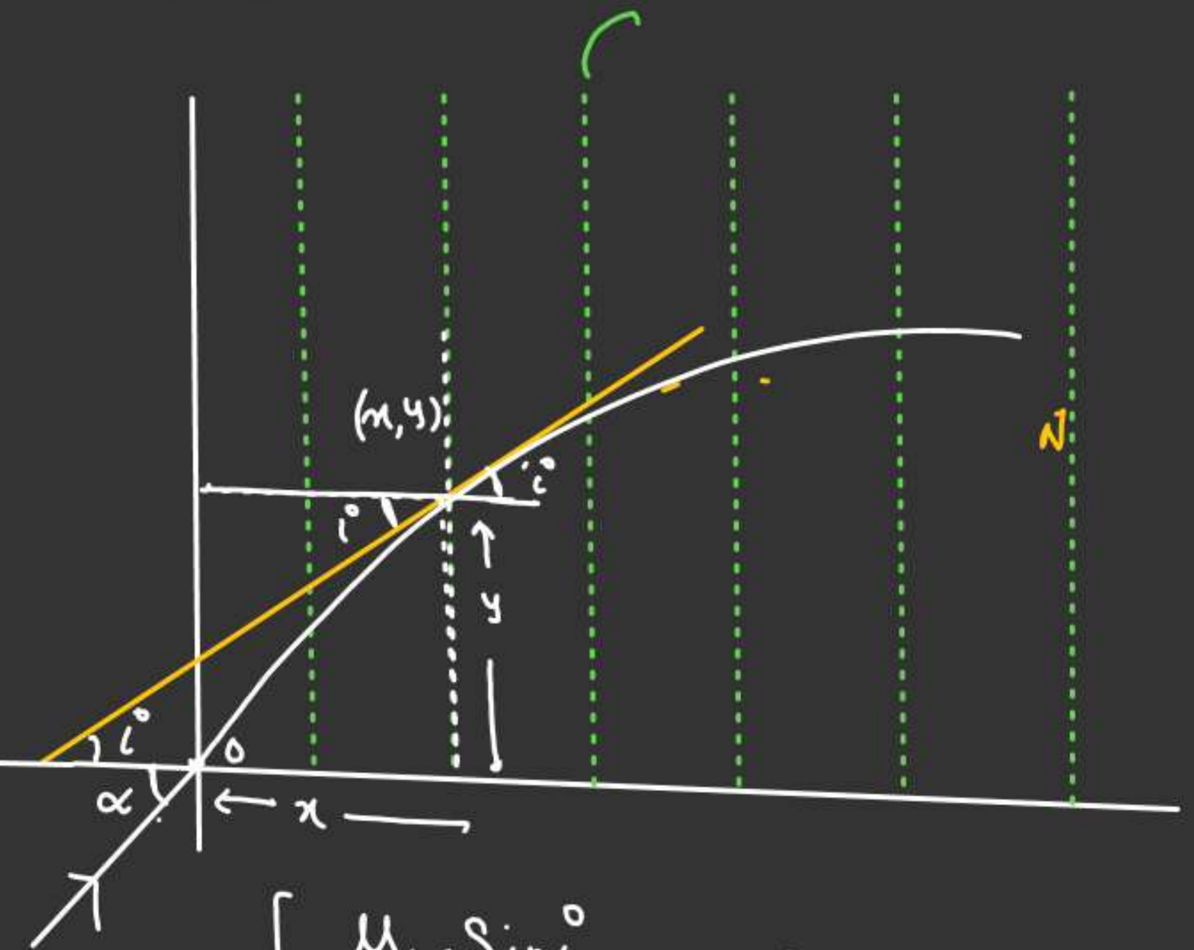
② $\dfrac{dy}{dx} = \tan(90-i) = \cot i$



Interface along x axis assumed to be parallel Slabs

$\mu = f(x)$

Interface along y-axis.



$\mu_y \cdot \sin i = \mu_0 \sin\alpha$ — ①

$\tan i = \left(\dfrac{dy}{dx}\right)$ — ②

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REFRACTION

★★ If refractive index of atmosphere vary as $\mu = \mu_0\left(\sqrt{1+by}\right)$ where y is height from ground.

find $y = f(x)$ ✓.

At $y=0$, $\mu = \mu_0$

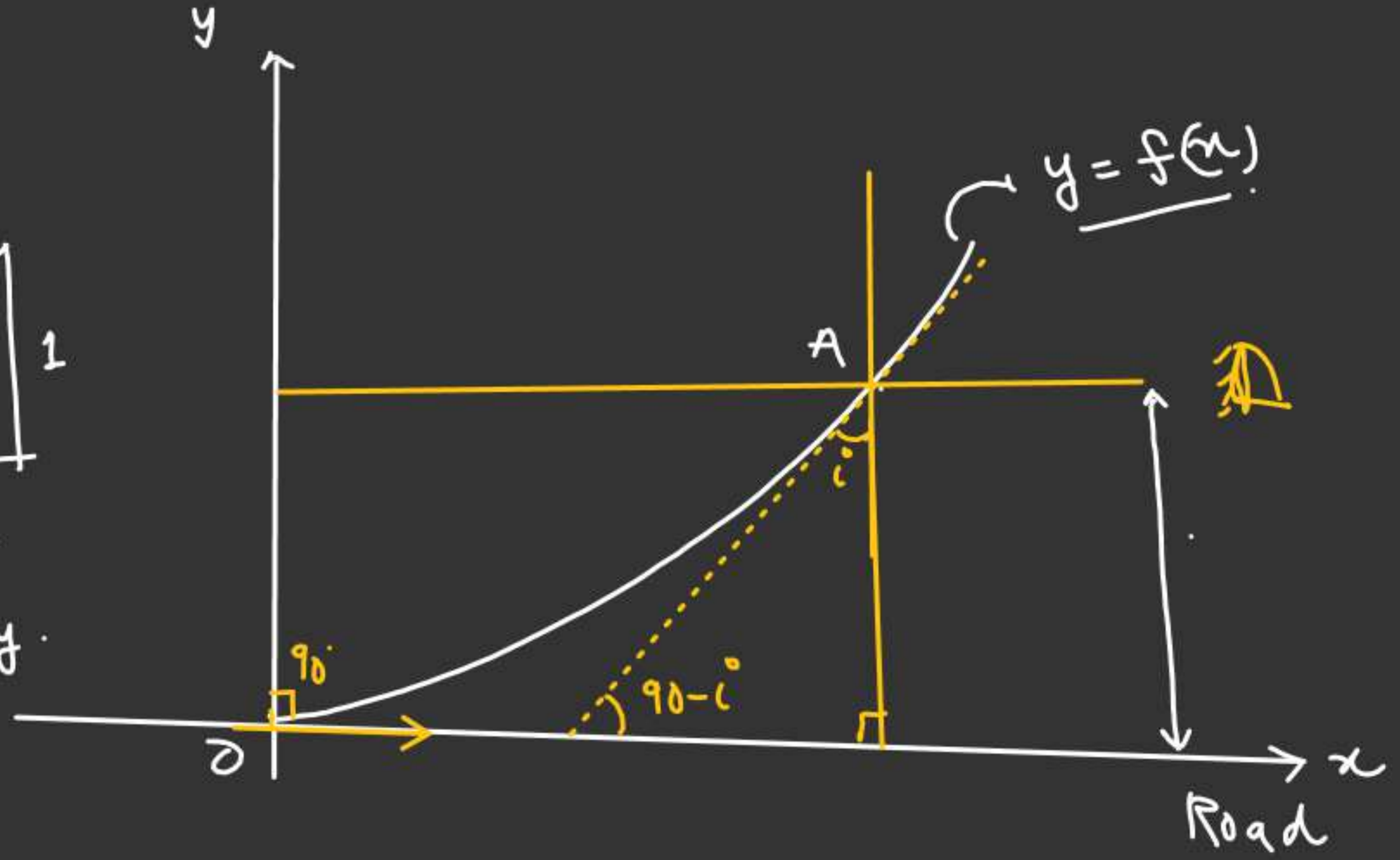
By Snell's Law

$$\mu_0 \sin 90^\circ = \mu_y \cdot \sin i$$

$$\mu_0 = \mu_y \cdot \sin i \quad \text{—(1)}$$

$$\frac{dy}{dx} = \tan(90-i) = \cot i \quad \text{(2)} \qquad \cot i = \sqrt{by}.$$

$$\sin i = \frac{\mu_0}{\mu_y} = \frac{\mu_0}{\mu_0\sqrt{1+by}} = \frac{1}{\sqrt{1+by}}.$$



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REFRACTION

From ②

$$\frac{dy}{dx} = \cot i = \sqrt{by}$$

$$\int_0^y \frac{dy}{\sqrt{by}} = \int_0^x dx$$

$$\frac{1}{\sqrt{b}} \cdot \int_0^y y^{-1/2} \cdot dy = x$$

$$\frac{1}{\sqrt{b}} \left(2\sqrt{y}\right) = x$$

$$\boxed{2\sqrt{\frac{y}{b}} = x}$$

$x = ??$

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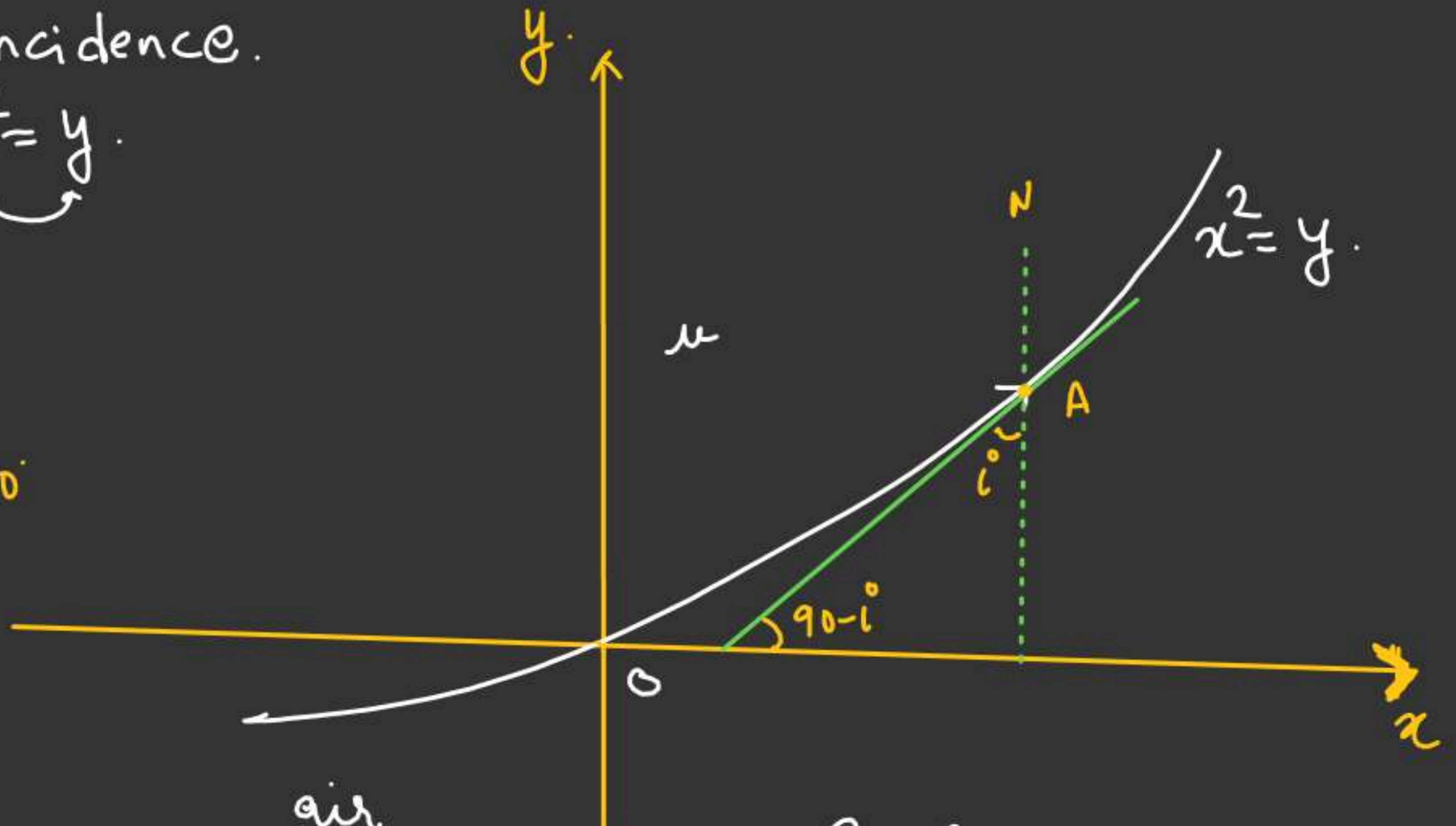
REFRACTION

At origin, Ray have grazing incidence.
Ray follow a parabolic path $x^2 = y$.
Find $\mu = f(y)$



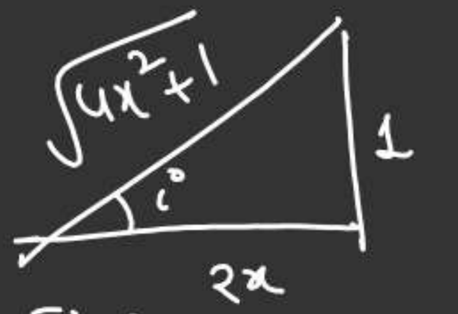
Snell's law from o to A.
At 'o' grazing incidence so, $i = 90°$

$$1 \cdot \sin 90° = \mu \sin i°$$

$$1 = \left(\frac{1}{\sqrt{4x^2+1}}\right)\mu$$

$$\frac{dy}{dx} = \tan(90-i) = \cot i° \quad \text{—}\ (1)$$

$\cot i° = 2x$



$$\sin i° = \frac{1}{\sqrt{4x^2+1}}$$

$$\mu = \left(\sqrt{4x^2+1}\right) \qquad \mu = f(x)$$

From the equation of curve

$$x^2 = y$$

$$\frac{dy}{dx} = 2x \rightarrow \text{Put in (1)}$$

$$\mu = \sqrt{4y+1} \qquad [\mu = f(y)]$$

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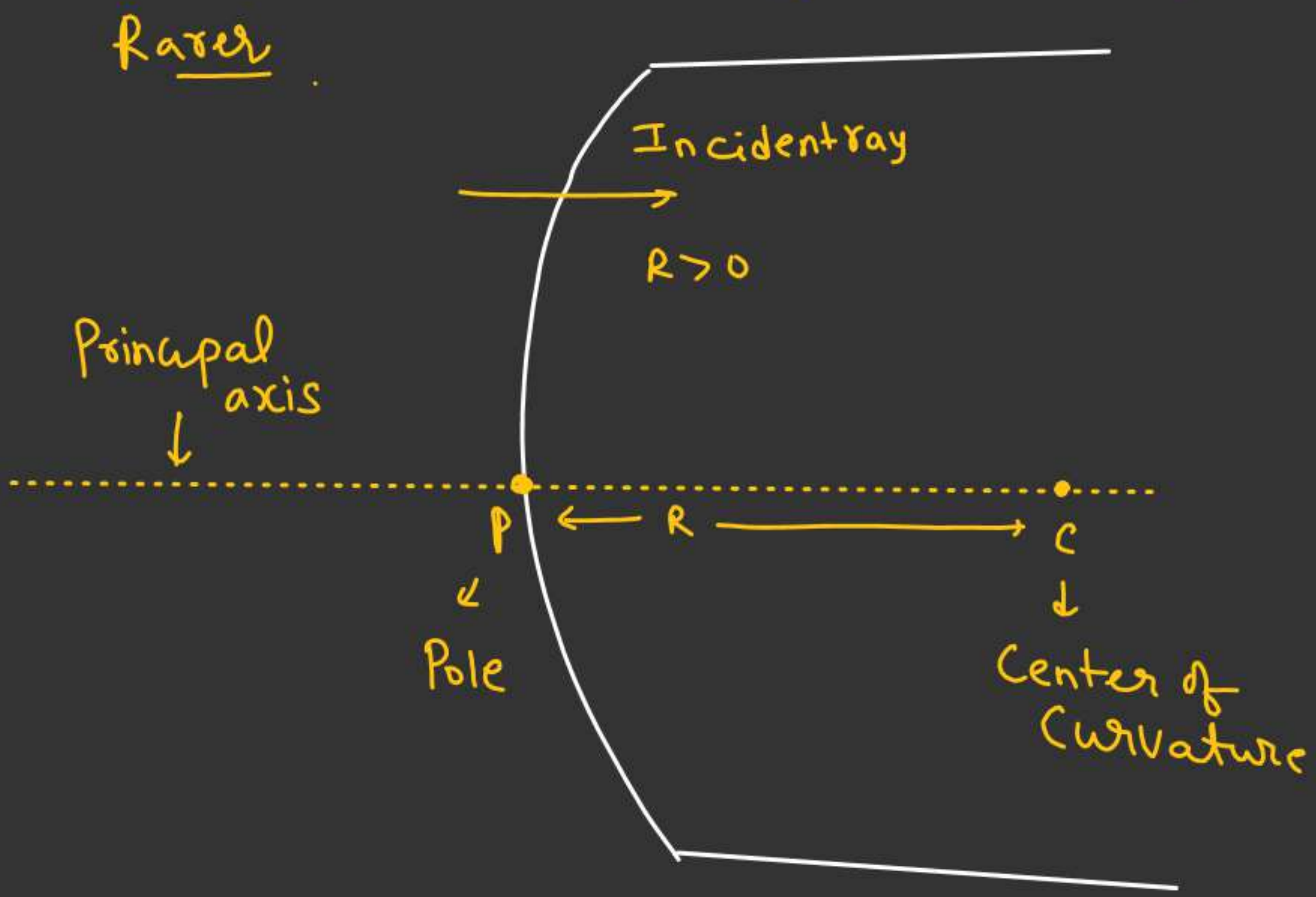
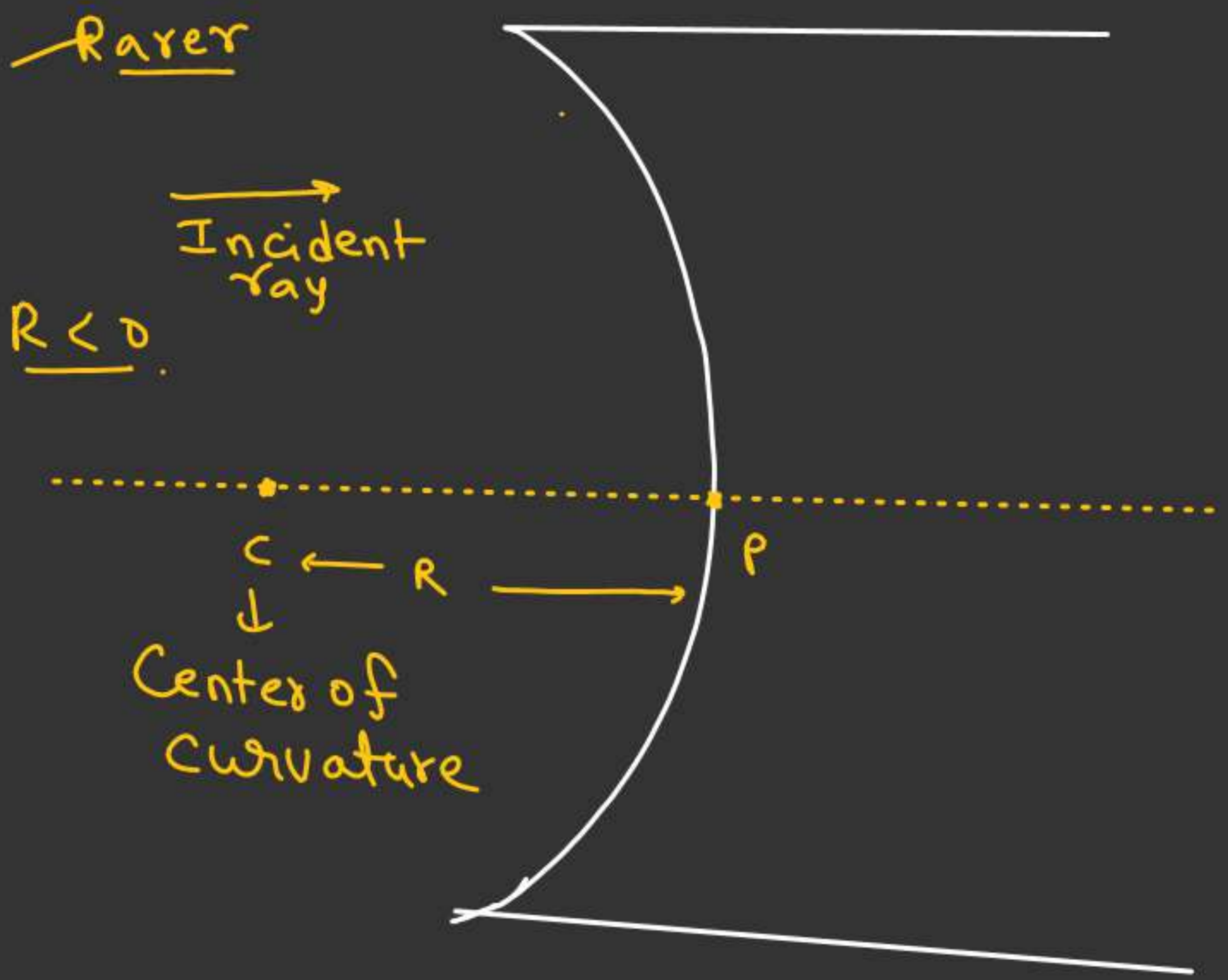
REFRACTION

REFRACTION FROM CURVED SURFACE

If Rarer at Convex Side then convex refractive Surface



If Rarer at Concave Side then Concave refractive surface.



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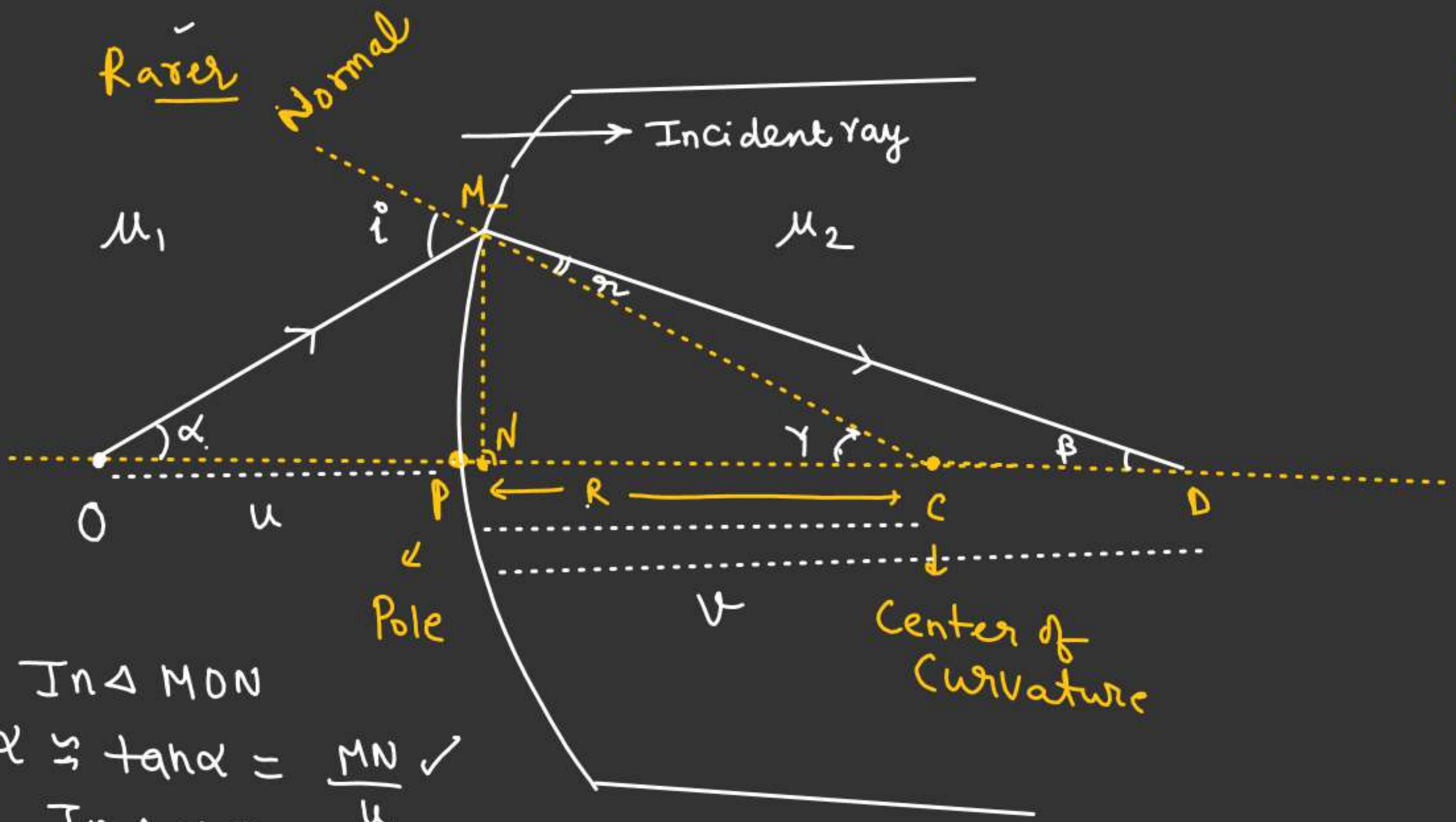
REFRACTION

REFRACTION FROM CURVED SURFACE



In Δ MON

$\alpha \simeq \tan\alpha = \frac{MN}{u}$ ✓

In Δ MND

$\beta \simeq \tan\beta = \frac{MN}{v}$

In Δ MNC

$\gamma \simeq \tan\gamma = \frac{MN}{R}$

Assumptions

1. Point object.
2. Rays very close to principal axis.
3. Angle of incidence, Angle of refraction very small.

$$\left[\begin{array}{l} \tan i \simeq \sin i \simeq i \\ \tan r \simeq \sin r \simeq r \end{array}\right.$$

In Δ OMC

$\underline{i = \alpha + \gamma}$

In Δ MND

$\gamma = r + \beta$

$\underline{r = (\gamma - \beta)}$

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REFRACTION

REFRACTION FROM CURVED SURFACE

By Snell's Law.

$\mu_1 \sin i = \mu_2 \sin r$

$\sin i \simeq i$

$\sin r \simeq r$

$\mu_1 i = \mu_2 r$

$\mu_1(\alpha+\gamma) = \mu_2(\gamma-\beta)$

$\mu_1 \alpha + \mu_2 \beta = (\mu_2-\mu_1)\gamma$

$\mu_1\left(\frac{MN}{u}\right) + \mu_2\left(\frac{MN}{V}\right) = (\mu_2-\mu_1)\frac{MN}{R}$

$\frac{\mu_2}{V} + \frac{\mu_1}{u} = \frac{\mu_2-\mu_1}{R}$ → (for our diagram).

By Sign-Convention

$u \to (-u)$, $v \to (+v)$

$R \to (+R)$

$$\boxed{\frac{\mu_2}{V} - \frac{\mu_1}{u} = \frac{\mu_2-\mu_1}{R}}$$

Put Known value with Sign.

(Take 1st Medium wher light ray incident)

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REFRACTION

Q. REFRACTION FROM CURVED SURFACE

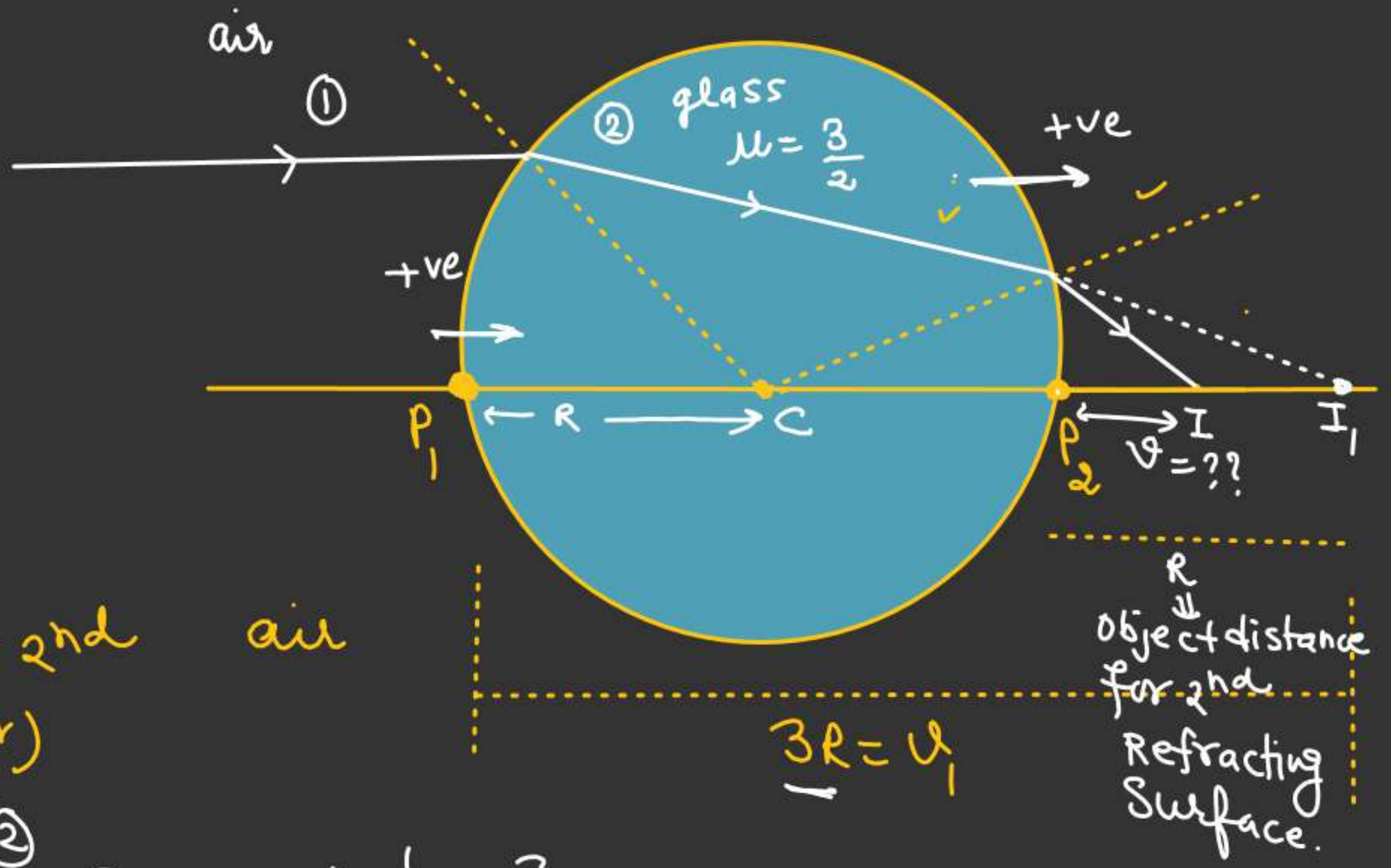
Find location of final image from C

Refraction from air (1) to glass (2)

$u \to \infty, \quad v = v_1$

$$\frac{3/2}{v_1} - \frac{1}{\infty} = \frac{3/2 - 1}{+R}$$

$$\frac{3}{2v_1} = \frac{1}{2R} \Rightarrow v_1 = 3R$$



I_1 acts as a Virtual object for 2nd Refractive Surface (glass (1) → air (2))

$$\frac{1}{v} - \frac{3/2}{+R} = \frac{1 - 3/2}{-R} \Rightarrow \frac{1}{v} - \frac{3}{2R} = \frac{1}{2R} \Rightarrow \frac{1}{v} = \frac{3}{2R} + \frac{1}{2R} = \frac{4}{2R} = \frac{2}{R}$$

$v = R/2$ from P_2

From C $I = 3R/2$ Ans