$\int e^x dx = e^x$

$\int e^{kx} = \frac{e^x}{k}$

ELECTRIC FIELD

$dV = (4\pi r^2)dr$

Q. Charge is distributed within a sphere of radius R with volume charge density $\boxed{\rho(r) = \frac{A}{r^2} e^{\frac{-2r}{a}}}$, where A and a are constants. If Q is the total charge of this charge distribution, the radius R is

$dq = \rho_r\, dV$

[JEE Mains - 2019]

(A) $\frac{a}{2}\log\left(\frac{1}{1-\frac{Q}{2\pi aA}}\right)$

(B) $\frac{a}{2}\log\left(1-\frac{Q}{2\pi aA}\right)$

(C) $a\log\left(1-\frac{Q}{2\pi aA}\right)$

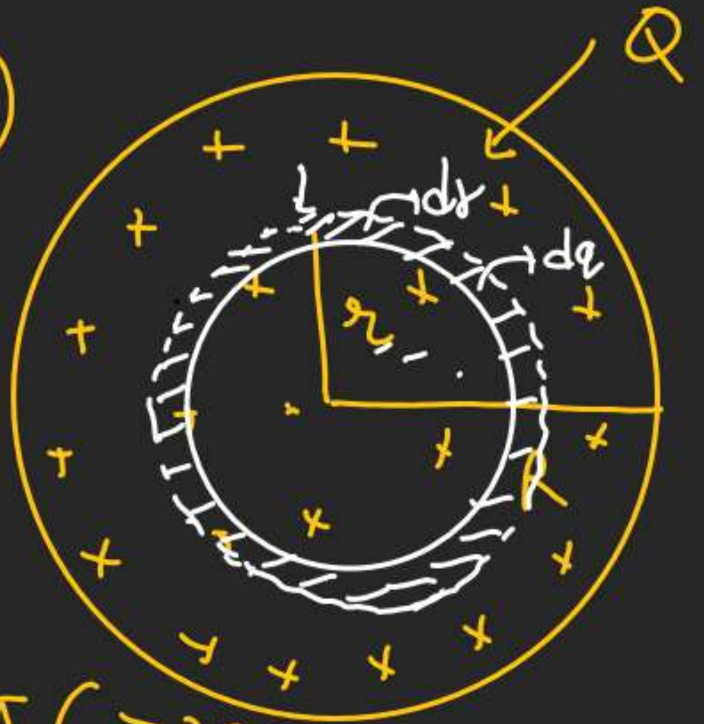
(D) $a\log\left(\frac{1}{1-\frac{Q}{2\pi aA}}\right)$

$dq = \left(\frac{A}{\cancel{r^2}} e^{-\frac{2r}{a}}\right)(4\pi \cancel{r^2}\, dr)$

$\int_0^Q dq = A4\pi \int_0^R e^{-\frac{2r}{a}}\, dr$

$Q = \frac{A4\pi}{\left(-\frac{2}{a}\right)}\left[e^{-\frac{2r}{a}}\right]_0^R \Rightarrow Q = -Aa2\pi\left(e^{-\frac{2R}{a}} - 1\right)$

$\Rightarrow Q = Aa2\pi\left(1 - e^{-2R/a}\right)$



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$$Q = Aa2\pi\left(1 - e^{-\frac{2R}{a}}\right)$$

$e^{x} = y$

$x = \ln y$

$$\boxed{-\ln x = \ln\left(\frac{1}{x}\right)}$$

$$\frac{Q}{Aa2\pi} = \left(1 - e^{-\frac{2R}{a}}\right)$$

$$e^{-\frac{2R}{a}} = \left(1 - \frac{Q}{Aa2\pi}\right)$$

$$-\frac{2R}{a} = \ln\left(1 - \frac{Q}{Aa2\pi}\right)$$

$$R = -\frac{a}{2}\ln\left(1 - \frac{Q}{Aa2\pi}\right)$$

$$\boxed{R = \frac{a}{2}\ln\frac{1}{\left(1 - \frac{Q}{Aa2\pi}\right)}}$$

ELECTRIC FIELD

$\vec{E} = \frac{kq}{r^2}\hat{r} = \left(\frac{kq}{r^3}\vec{r}\right)$

Q. Two point charges $q_1(\sqrt{10}\mu C)$ and $q_2(-25\mu C)$ are placed on the x-axis at $x = 1$ m and $x = 4$ m respectively. The electric field (in V/m) at a point $y = 3$ m on y-axis is, $\left[\text{take } \frac{1}{4\pi\varepsilon_0} = 9 \times 10^9 \text{Nm}^2\text{C}^{-2}\right]$

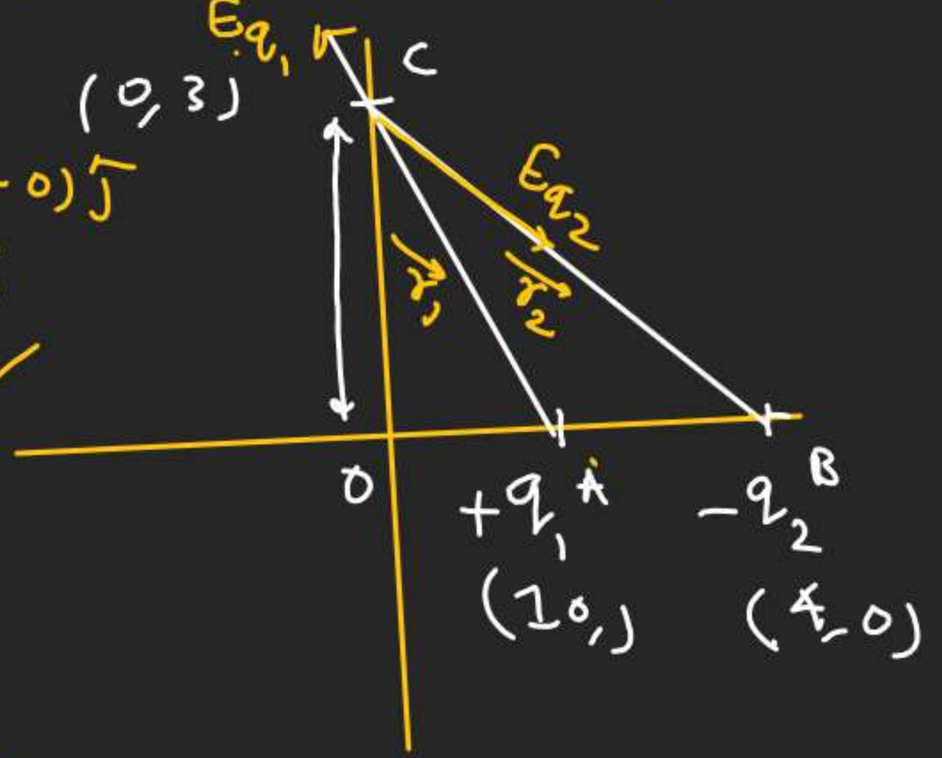
[JEE Mains - 2019]

(A) $(63\hat{i} - 27\hat{j}) \times 10^2$

(B) $(81\hat{i} - 81\hat{j}) \times 10^2$

(C) $(-81\hat{i} + 81\hat{j}) \times 10^2$

(D) $(-63\hat{i} + 27\hat{j}) \times 10^2$



$\vec{r}_1 = (0-1)\hat{i} + (3-0)\hat{j}$

$\vec{r}_1 = -\hat{i} + 3\hat{j}$

$\vec{r}_2 = -4\hat{i} + 3\hat{j}$

$|\vec{r}_1| = \sqrt{10}$

$|\vec{r}_2| = 5$

$\vec{E}_1 = \frac{K(\sqrt{10} \times 10^{-6})}{(10\sqrt{10})}(-\hat{i} + 3\hat{j})$

$= \left(\frac{K}{10} \times 10^{-6}\right)(-\hat{i} + 3\hat{j})$ — (1)

$\vec{E}_2 = \frac{K(\cancel{25} \times 10^{-6})}{(\cancel{25} \times 5)}\left[-(-4\hat{i} + 3\hat{j})\right] \Rightarrow \vec{E}_2 = \frac{K}{5} \times 10^{-6}(4\hat{i} - 3\hat{j})$ — (2)

$\vec{E} = \vec{E}_1 + \vec{E}_2$

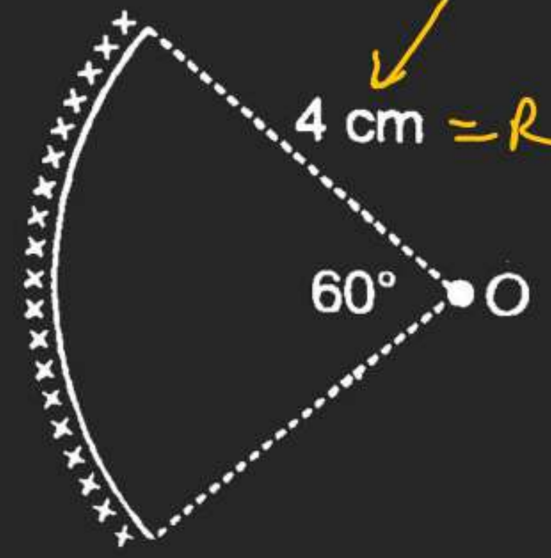
$= \frac{K}{10} \times 10^{-6}\left[(-\hat{i} + 3\hat{j}) + 8\hat{i} - 6\hat{j}\right]$

$= \frac{K}{10} \times 10^{-6}\left[7\hat{i} - 3\hat{j}\right]$

$K = 9 \times 10^9$

ELECTRIC FIELD

Q. The electric field at point O, due to the segment of a ring, whose linear charge density is $8 C/cm$ is $n \times 10^{13}$ V/m. The value of n is ___18___.



$\lambda \Downarrow$

$\lambda = \frac{8}{10^{-2}}$

$\lambda = 8 \times 10^{+2}$

$E = \frac{2K\lambda}{R} \sin\left(\frac{\theta}{2}\right)$

$E = \frac{2 \times 9 \times 10^{9} \times 8 \times 10^{2}}{(4 \times 10^{-2})} \sin\left(\frac{60}{2}\right)$

$E = \frac{\cancel{2} \times 9 \times 10^{13} \times \cancel{8}^{2}}{\cancel{4}} \times \frac{1}{\cancel{2}}$

$E = \underline{18 \times 10^{13}}$ ✓

ELECTRIC FIELD

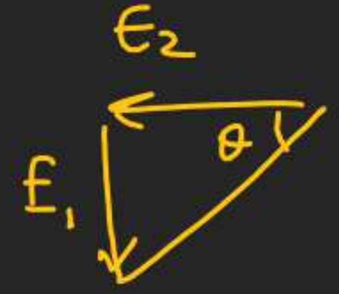
Q. Charges Q_1 and Q_2 are at points A and B of a right angle triangle OAB (see figure). The resultant electric field at point O is perpendicular to the hypotenuse, then Q_1/Q_2 is proportional to **[JEE Mains - 2020]**

(A) $\frac{x_1^3}{x_2^3}$

(B) $\frac{x_2^2}{x_1^2}$

(C) $\frac{x_1}{x_2}$

(D) $\frac{x_2}{x_1}$



$\tan\theta = \frac{E_1}{E_2}$

$\tan\theta = \frac{\frac{KQ_1}{x_1^2}}{\frac{KQ_2}{x_2^2}}$

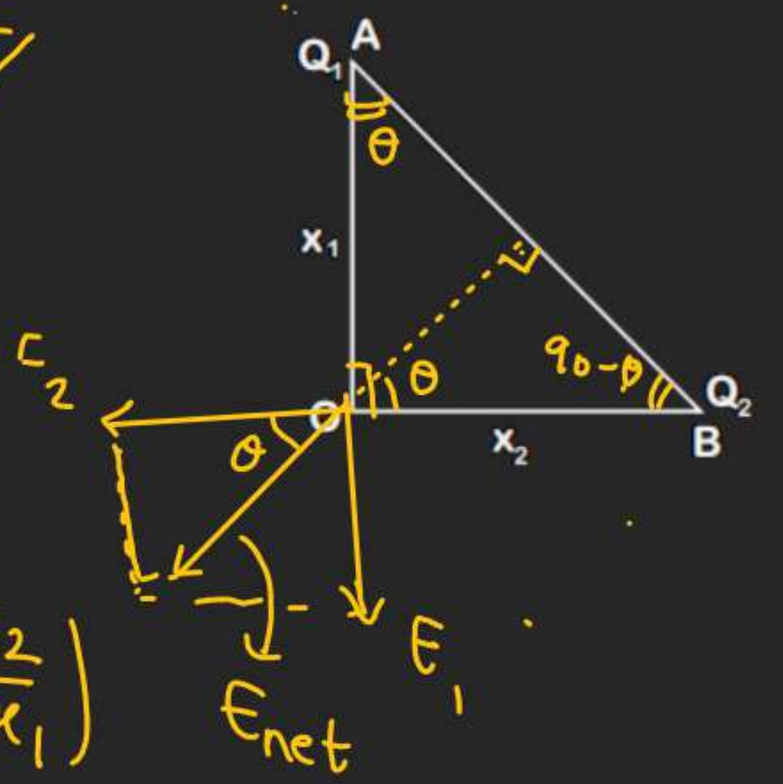
$\tan\theta = \frac{Q_1}{Q_2} \times \frac{x_2^2}{x_1^2}$

In ΔOAB

$\tan\theta = \left(\frac{x_2}{x_1}\right)$

$\frac{x_2}{x_1} \times \frac{x_1^2}{x_2^2} = \frac{Q_1}{Q_2}$

$\boxed{\frac{Q_1}{Q_2} = \frac{x_1}{x_2}}$



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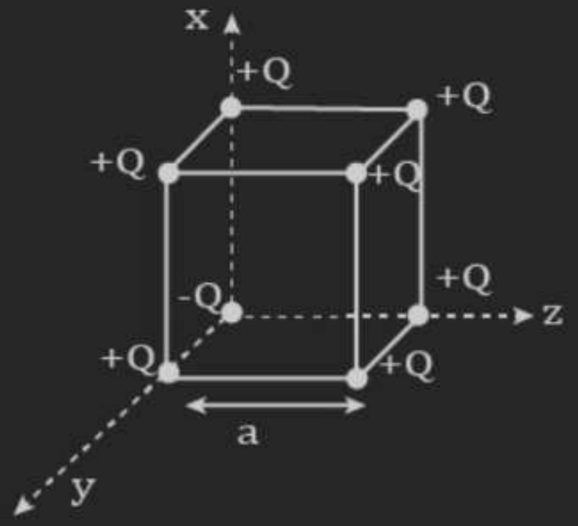
H.W ✓.

ELECTRIC FIELD

Q. A cube of side 'a' has point charges $+Q$ located at each of its vertices except at the origin where the charge is $-Q$. The electric field at the centre of cube is :

[JEE Mains - 2021]



(A) $\frac{-Q}{3\sqrt{3}\pi\varepsilon_0 a^2}(\hat{x}+\hat{y}+\hat{z})$

(B) $\frac{Q}{3\sqrt{3}\pi\varepsilon_0 a^2}(\hat{x}+\hat{y}+\hat{z})$

(C) $\frac{2Q}{3\sqrt{3}\pi\varepsilon_0 a^2}(\hat{x}+\hat{y}+\hat{z})$

(D) $\frac{-2Q}{3\sqrt{3}\pi\varepsilon_0 a^2}(\hat{x}+\hat{y}+\hat{z})$

ELECTRIC FIELD

Q. Find the electric field at point P (as shown in figure) on the perpendicular bisector of a uniformly charged thin wire of length L carrying a charge Q. The distance of the point P from the centre of the rod is $\overset{d}{\Downarrow}{a} = \frac{\sqrt{3}}{2}L$ **[JEE Mains - 2021]**

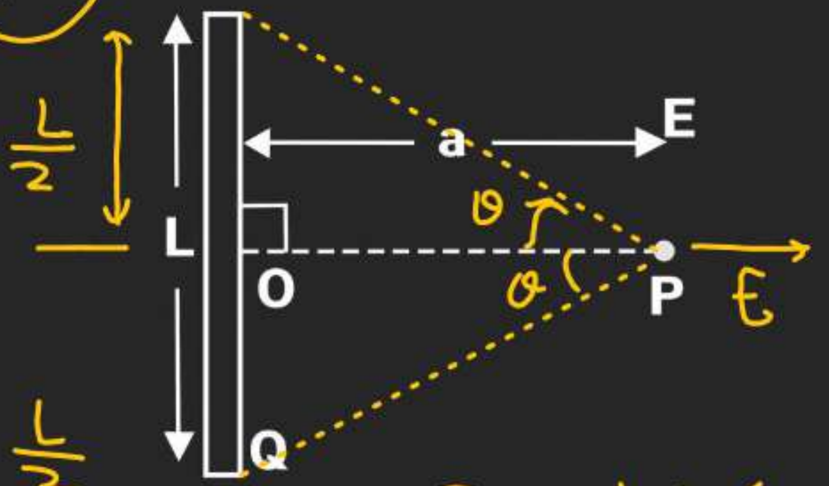
$\left(\lambda = \frac{Q}{L}\right)$

(A) $\frac{Q}{3\pi\varepsilon_0 L^2}$

(B) $\frac{Q}{4\pi\varepsilon_0 L^2}$

(C) $\frac{\sqrt{3}Q}{4\pi\varepsilon_0 L^2}$

(D) $\frac{Q}{2\sqrt{3}\pi\varepsilon_0 L^2}$ ✓



$$E = \frac{K\lambda}{a}(2\sin\theta)$$

$$E = \frac{2K\lambda}{a}\sin\theta$$

$$E = \cancel{2}\times\cancel{2}\times\frac{\lambda}{\sqrt{3}L}\times\frac{1}{\cancel{4}\pi\varepsilon_0}\times\frac{1}{\cancel{2}}$$

$$E = \frac{\lambda}{2\sqrt{3}L\pi\varepsilon_0} = \frac{Q}{2\sqrt{3}L^2\pi\varepsilon_0}$$

$$\tan\theta = \frac{L/2}{a}$$

$$\tan\theta = \frac{L\times 2}{2\times\sqrt{3}L}$$

$$\tan\theta = \frac{1}{\sqrt{3}}$$

$$\theta = 30^\circ$$ ✓

$$E_\perp = \frac{K\lambda}{d}(\sin\alpha + \sin\beta)$$

$$E_{\parallel} = \frac{K\lambda}{d}(\cos\beta - \cos\alpha)$$

$$\alpha = \beta = \theta$$

ELECTRIC FIELD

Q. A uniformly charged disc of radius R having surface charge density σ is placed in the xy plane with its center at the origin. Find the electric field intensity along the z-axis at a distance Z from origin: [JEE Mains - 2021]

(A) $E = \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{Z}{(Z^2+R^2)^{1/2}}\right)$

(B) $E = \frac{2\varepsilon_0}{\sigma}\left(\frac{1}{(Z^2+R^2)^{1/2}} + Z\right)$

(C) $E = \frac{\sigma}{2\varepsilon_0}\left(\frac{1}{(Z^2+R^2)} + \frac{1}{Z^2}\right)$

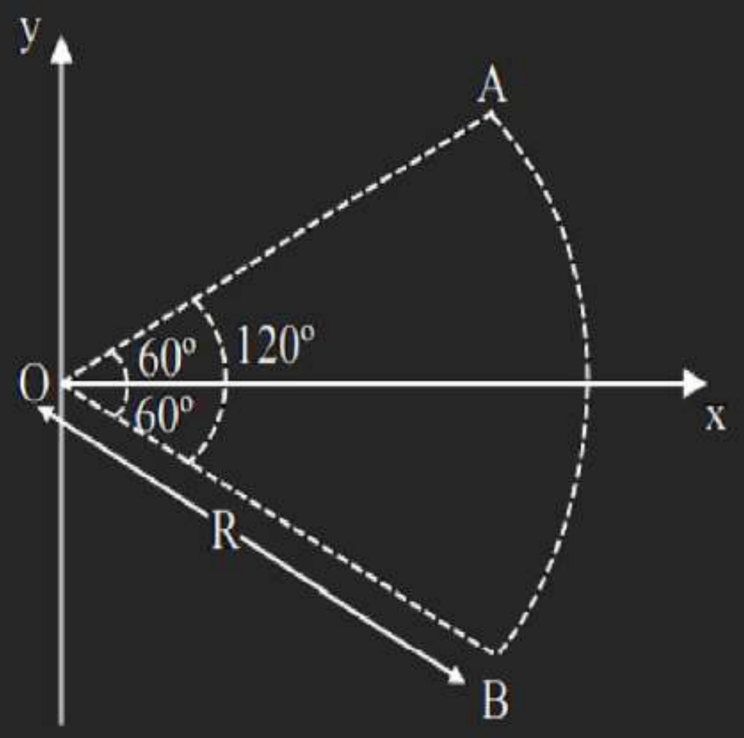
(D) $E = \frac{\sigma}{2\varepsilon_0}\left(1 + \frac{Z}{(Z^2+R^2)^{1/2}}\right)$

ELECTRIC FIELD

H-W.

Q. Figure shows a rod AB, which is bent in a 120° circular arc of radius R. A charge $(-Q)$ is uniformly distributed over rod AB. What is the electric fieid $\vec{E}$ at the centre of curvature O ?

[JEE Mains - 2021]



(A) $\frac{3\sqrt{3}Q}{8\pi^2\varepsilon_0 R^2}(\hat{i})$

(B) $\frac{3\sqrt{3}Q}{8\pi^2\varepsilon_0 R^2}(-\hat{i})$

(C) $\frac{3\sqrt{3}Q}{8\pi\varepsilon_0 R^2}(\hat{i})$

(D) $\frac{3\sqrt{3}Q}{16\pi^2\varepsilon_0 R^2}(\hat{i})$

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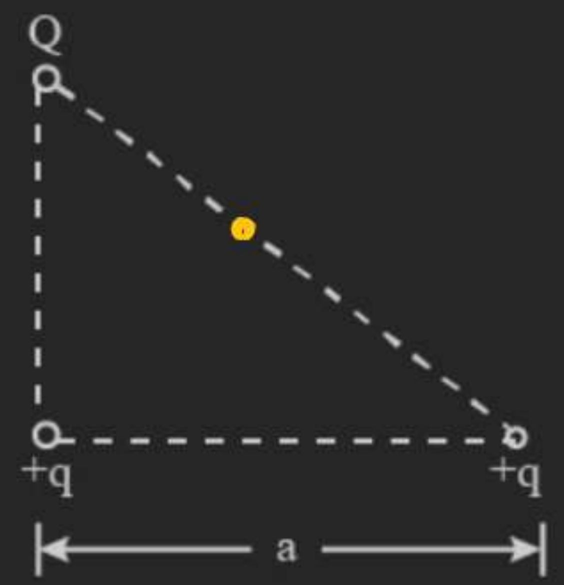
H.W ✓

ELECTRIC FIELD

Q. Three point charges $+Q, -Q$ and $+2Q$ are placed at the vertices of an isosceles right angled triangle as shown in figure. The magnitude of electric field intensity produced by these charges at the middle point of hypotenuse is



(A) $\dfrac{(2+\sqrt{2})Q}{4\pi\varepsilon_0 a^2}$

(B) $\dfrac{2Q}{\pi\varepsilon_0 a^2}$

(C) $\dfrac{\sqrt{2}Q}{\pi\varepsilon_0 a^2}$

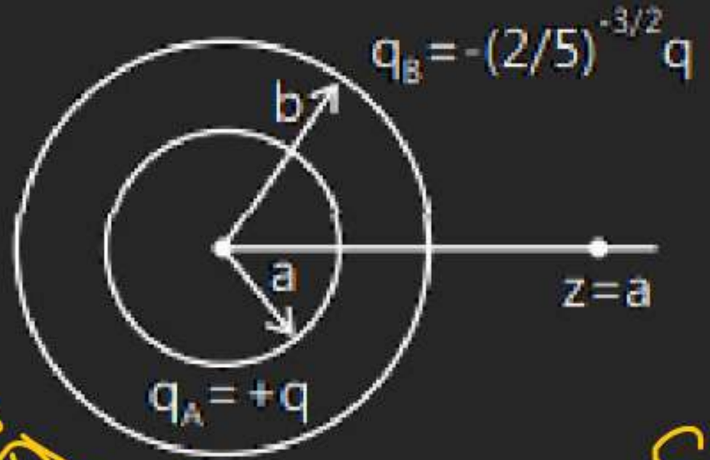
(D) $\dfrac{Q}{8\pi\varepsilon_0 a^2}$

ELECTRIC FIELD

$$\boxed{E = \frac{KQx}{(x^2+R^2)^{3/2}}}$$

Q. Two concentric rings, one of radius a and the other of radius b, have the charges $+q$, and $-(2/5)^{-3/2}q$, respectively, as shown in Fig. Find the ratio b/a if a charge particle placed on the axis at $z = a$ is in equilibrium.

$$F = q E$$
$$\downarrow \qquad \downarrow$$
$$0 \qquad 0$$

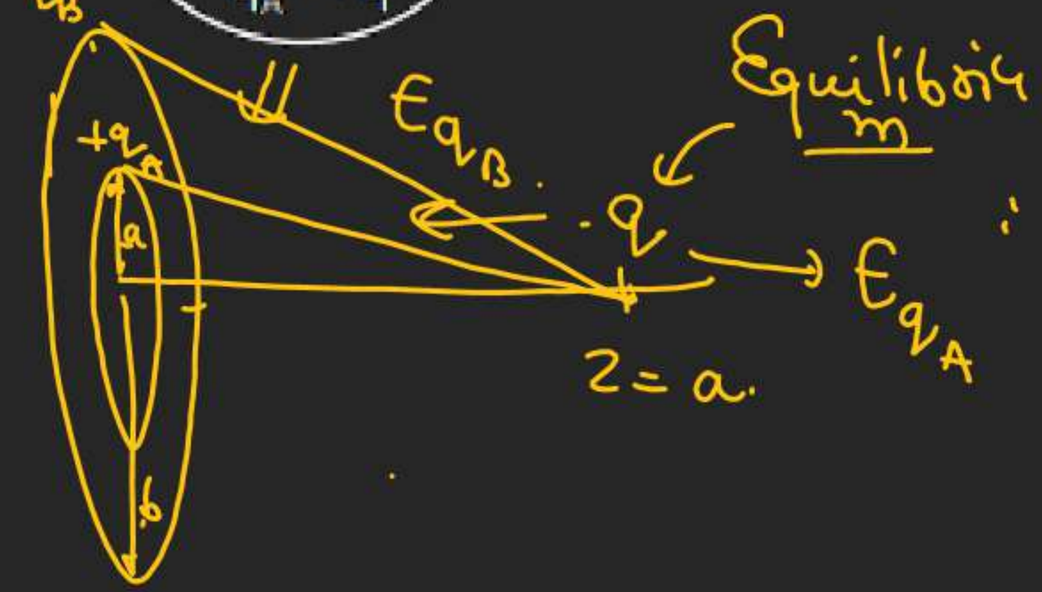


For Equilibrium of the Charge

$$E_{q_A} = E_{q_B}$$

$$\frac{Kqa}{(2a)^{3/2}} = \frac{K\left(\frac{2}{5}\right)^{-3/2} q\, a}{(b^2+a^2)^{3/2}}$$

$\downarrow$ Hw.



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H.W.

ELECTRIC FIELD

Q. A ring of radius R has charge $-\mathrm{Q}$ distributed uniformly over it. Calculate the charge (q) that should be placed at the center of the ring such that the electric field becomes zero at a point on the axis of the ring distant R from the center of the ring. The value of q is $(\mathrm{Q}/4)\sqrt{\square}$.

ELECTRIC FIELD

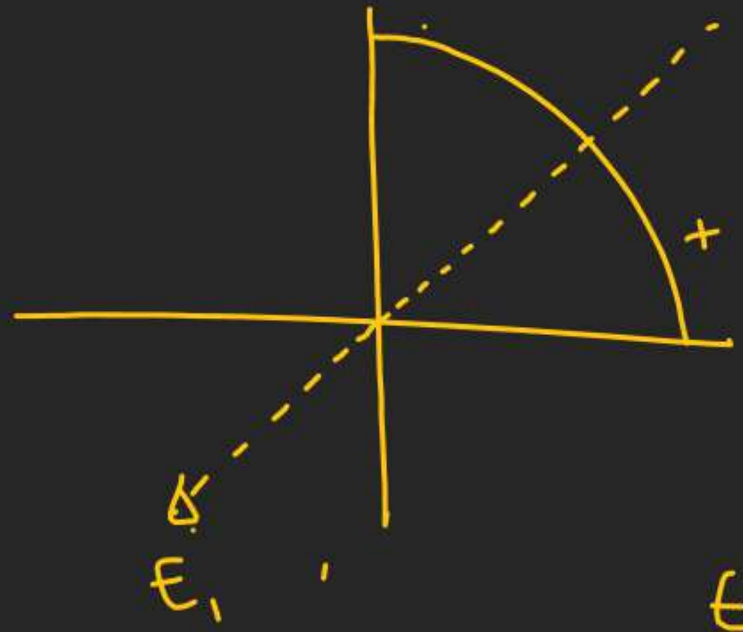
Q. The charge per unit length of the four quadrant of the ring is $2\lambda, -2\lambda, \lambda$ and λ respectively. The electric field at the centre is:

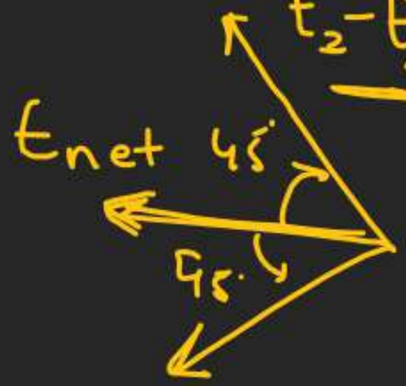
(A) $\frac{\lambda}{2\pi\varepsilon_0 R}\hat{i}$

(B) $\frac{\lambda}{2\pi\varepsilon_0 R}\hat{j}$

(C) $\frac{\sqrt{2}\lambda}{4\pi\varepsilon_0 R}\hat{j}$

(D) None of these





$$E_1 - E_2 = \frac{2K}{R}\sin\frac{\theta}{2}\left[2\lambda - \lambda\right]$$

$$= \frac{2K\lambda}{R}\sin\frac{\theta}{2}$$

$$= \frac{\sqrt{2}K\lambda}{R}$$

$$E_2 - E_4 = \frac{\sqrt{2}K\lambda}{R}$$

$$E = \frac{2K\lambda}{R}\sin\frac{\theta}{2}$$

$$E_1 - E_2 = \frac{\sqrt{2}K\lambda}{R}$$

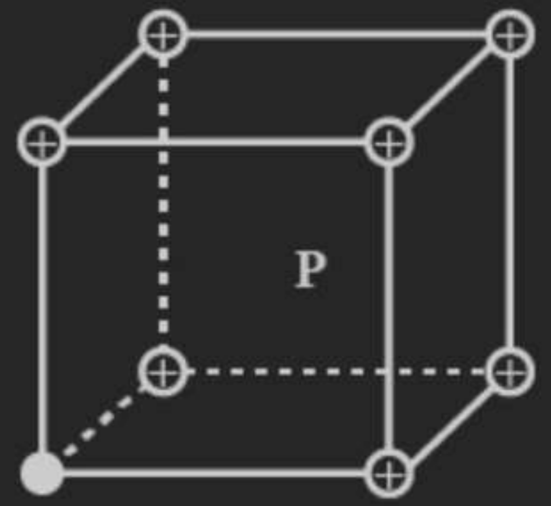
$$E_{net} = \frac{\sqrt{2}K\lambda}{R}\cos 45 \times 2$$

$$= \left(\frac{2K\lambda}{R}\right) = \frac{2}{R}\times\frac{1}{4\pi\varepsilon_0}\times\lambda$$

ELECTRIC FIELD

H.W

Q. Consider a regular cube with positive point charge $+Q$ in all corners except for one which has a negative point charge $-Q$. Let the distance from any corner to the center of the cube be r. What is the magnitude of electric field at point P, the center of the cube ?



(A) $E = 7k_eQ/r^2$

(B) $E = 1k_eQ/r^2$

(C) $E = 2k_eQ/r^2$

(D) $E = 6k_eQ/r^2$

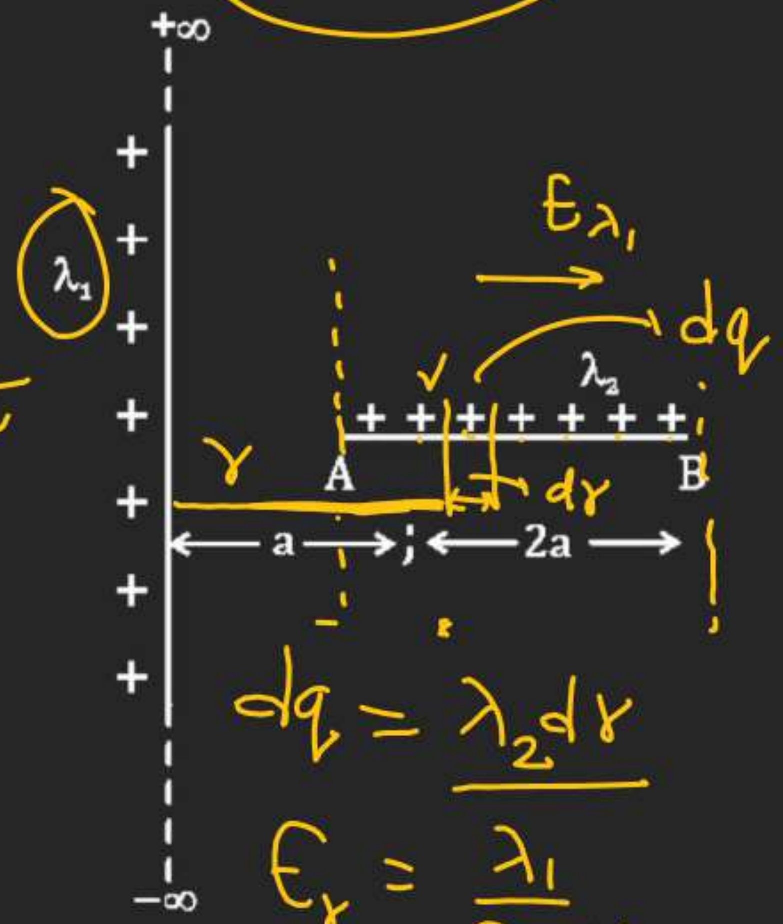
ELECTRIC FIELD

$$\int \frac{dx}{x} = \ln x$$

❖ **Force of Interaction between finite and Infinite Line Charges**

$$E = \frac{2K\lambda}{d} = 2 \times \frac{1}{4\pi\varepsilon_0} \times \frac{\lambda}{d} = \left(\frac{\lambda}{2\pi\varepsilon_0 d}\right)$$

Let, a differential element at a distance 'r' having length 'dr' is cut. Electric field at r distance is same as at a distance (r+dr) as dr is very small



$$dq = \lambda_2 dr$$

$$E_r = \frac{\lambda_1}{2\pi\varepsilon_0 r}$$

$$dF = dq \cdot E_r$$

$$dF = (\lambda_2 dr)\frac{\lambda_1}{2\pi\varepsilon_0 r}$$

$$\int_0^F dF = \frac{\lambda_1\lambda_2}{2\pi\varepsilon_0}\int_a^{3a}\frac{dr}{r}$$

$$F = \frac{\lambda_1\lambda_2}{2\pi\varepsilon_0}\ln[r]_a^{3a}$$

$$F = \frac{\lambda_1\lambda_2}{2\pi\varepsilon_0}[\ln(3a) - \ln(a)]$$

$$\boxed{F = \frac{\lambda_1\lambda_2}{2\pi\varepsilon_0}\ln 3}$$

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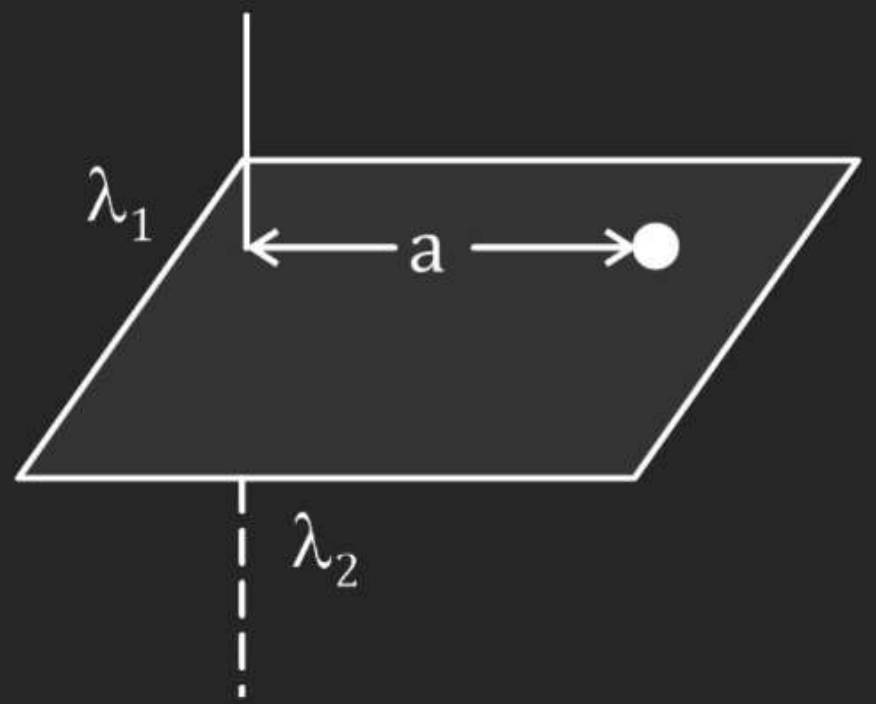
ELECTRIC FIELD

H.W.

❖ Two mutually perpendicular infinitely long lines of charge having charge per unit length as λ_1 and λ_2 are located in air as shown in Fig. Show that the force of interaction between them is $\frac{\lambda_1 \lambda_2}{2\varepsilon_0}$



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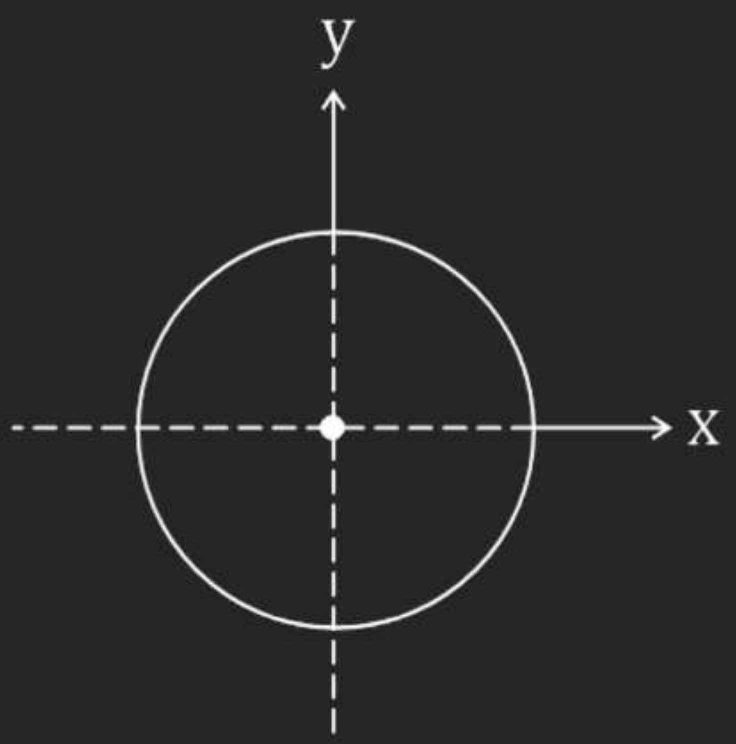
ELECTRIC FIELD

H.W.

❖ **Electric field due to a non-uniformly Charge ring whose linear charge density is $\lambda = \lambda_0 \cos\theta$. Where angle from $+ve$ x-axis**



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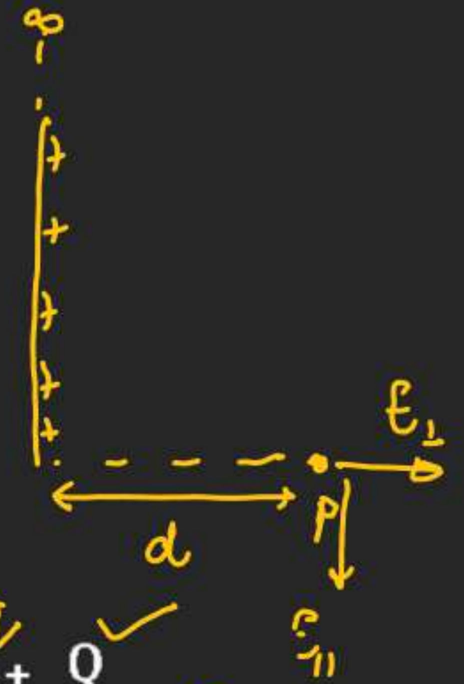
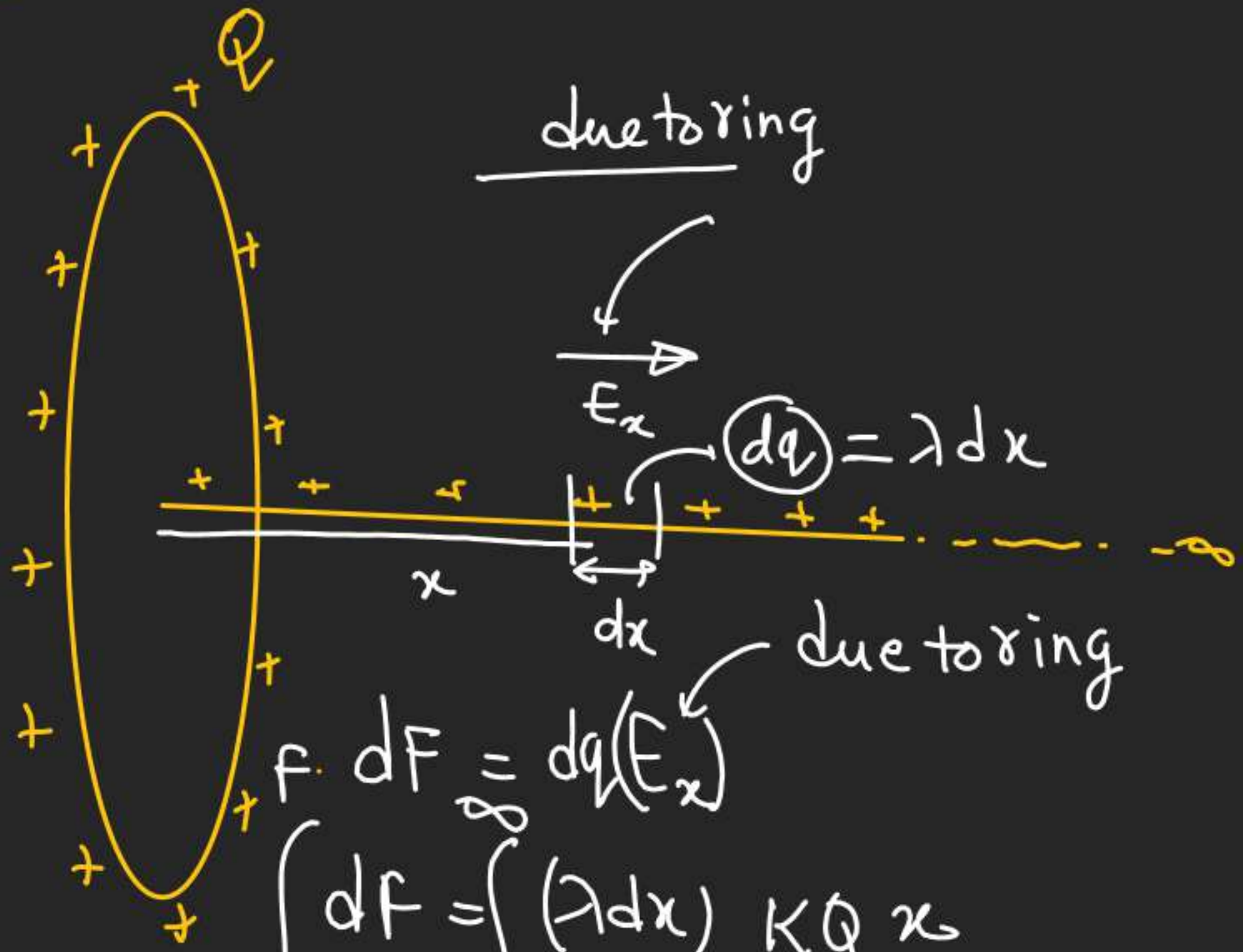
ELECTRIC FIELD

$E_\perp = E_{\parallel} = \left(\frac{K\lambda}{d}\right)$ ✓



❖ **Force of Interaction between ring and Infinitly long line charge:-**

Frist Method



due to ring

E_x

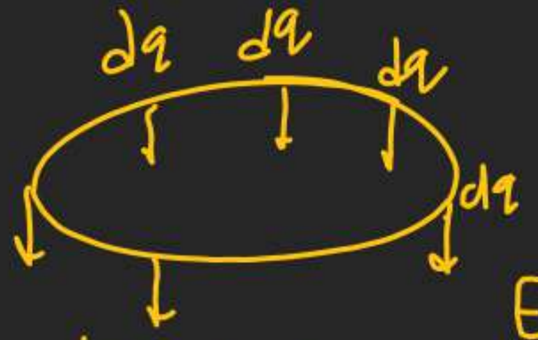
$dq = \lambda dx$

due to ring

$F \cdot dF = dq(E_x)$

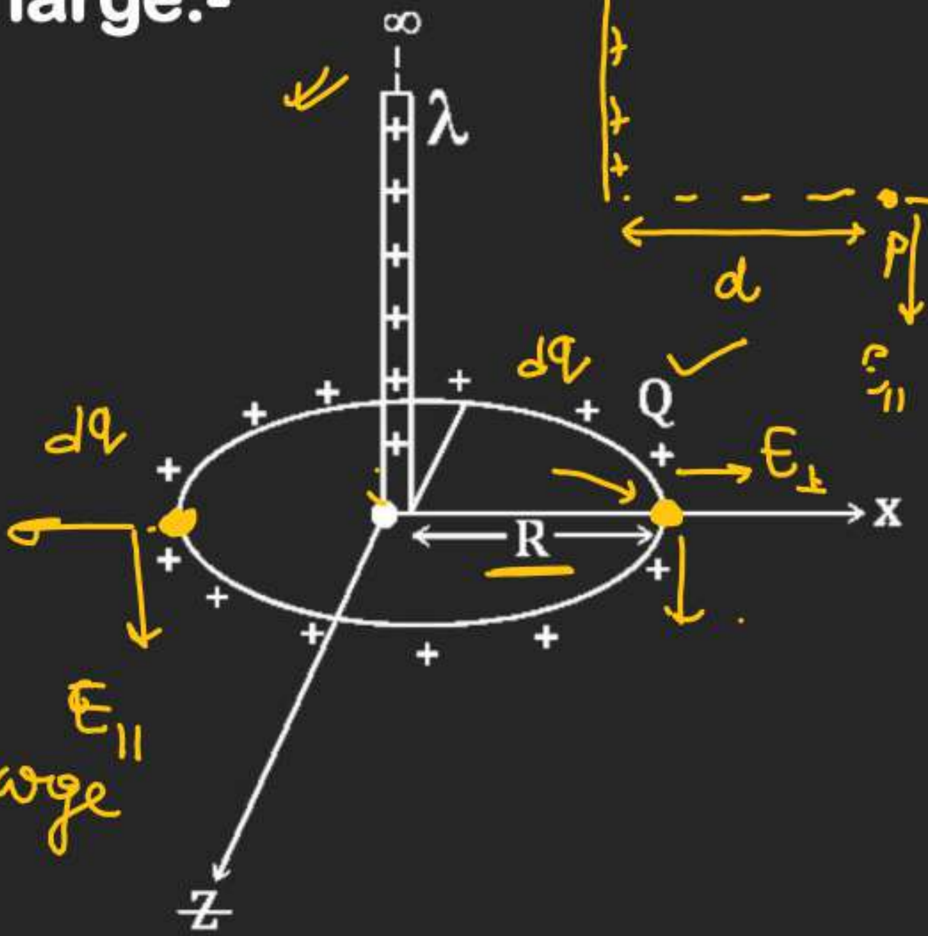
$$\int_0^F dF = \int_0^\infty (\lambda dx)\frac{KQx}{(x^2+R^2)^{3/2}}$$

2nd Method



$dF = dq\,E_{\parallel}$

due to line charge



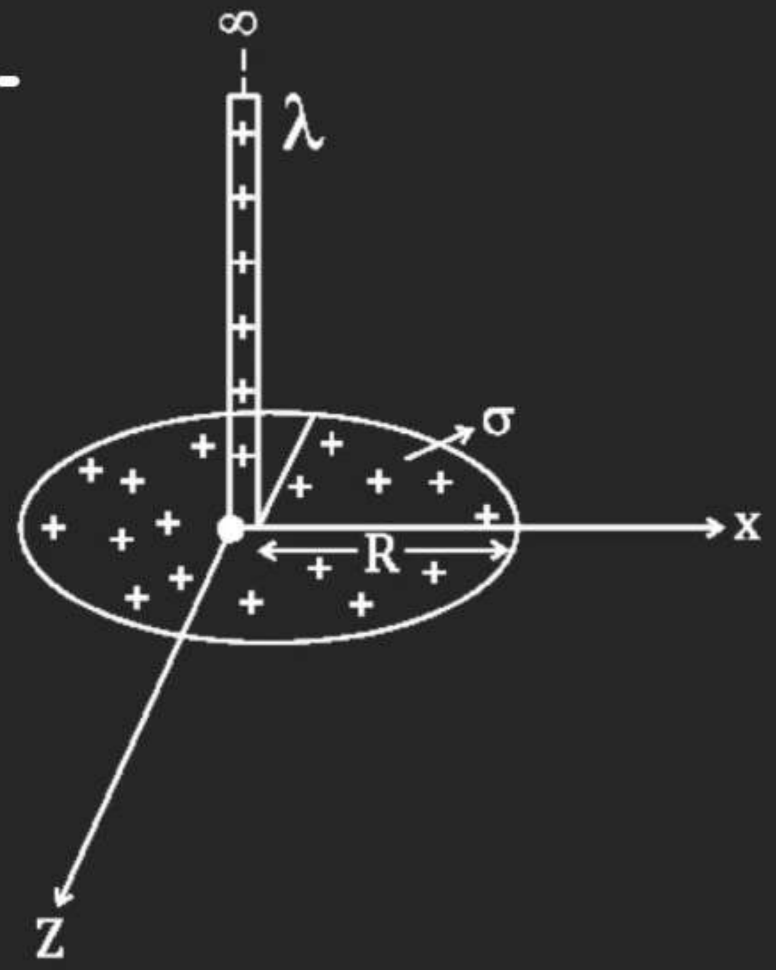
$$F_{net} = \int dq \frac{K\lambda}{R}$$

$$= \frac{K\lambda}{R}\int dq = \left(\frac{K\lambda Q}{R}\right)$$

ELECTRIC FIELD

H.W

❖ **Force of interaction between disc and infinitly long line charge:-**



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ELECTRIC FIELD

H.W.

❖ **Tension in a ring due to +q.**