Nishant Jindal

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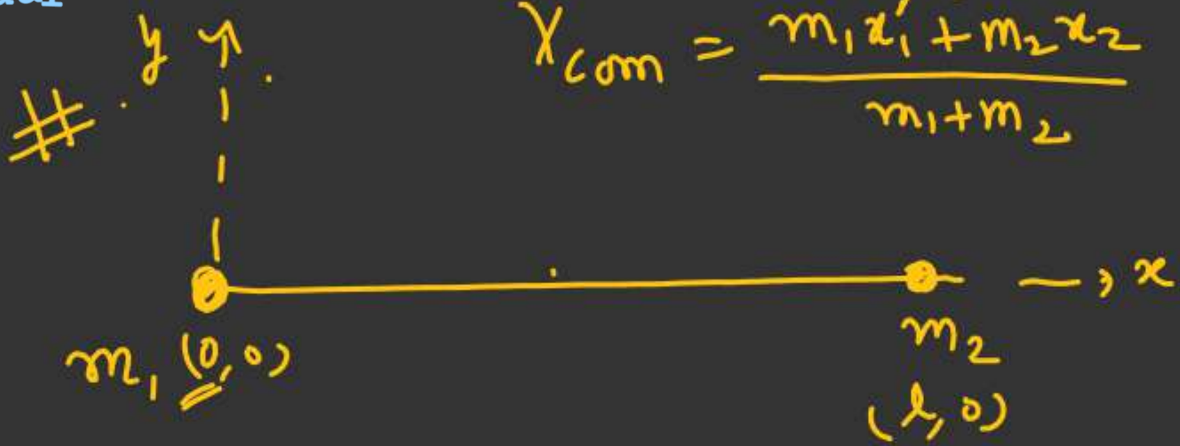


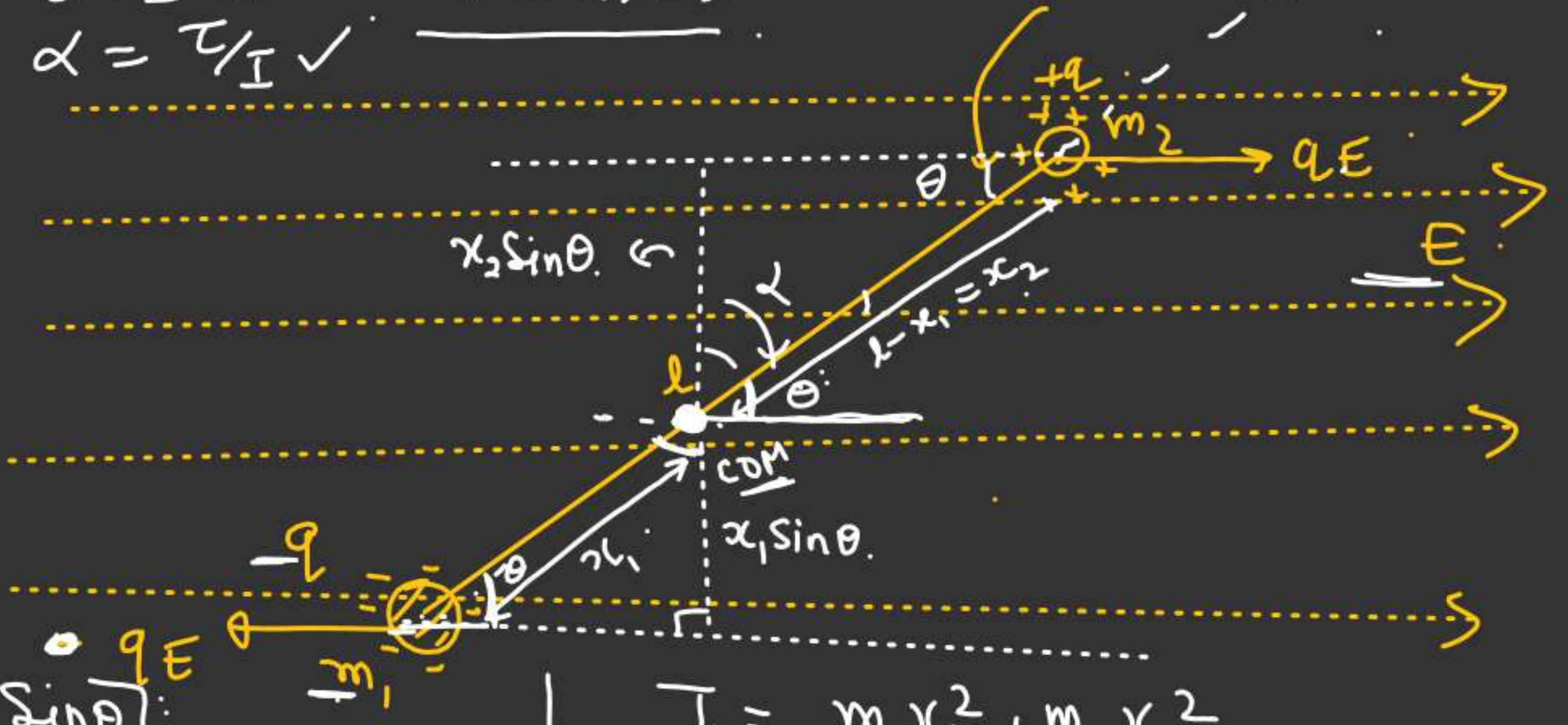
$$X_{com} = \frac{m_1 \overset{\to 0}{x_1} + m_2 x_2}{m_1+m_2}$$

$\tau = I\alpha$

$\alpha = \tau / I$ ✓

$p = (ql)$

massless rod



$$x_1 = X_{com} = \frac{m_2 l}{m_1+m_2}$$

$$x_2 = l - \left(\frac{m_2 l}{m_1+m_2}\right) = \left(\frac{m_1 l}{m_1+m_2}\right)$$

$$\tau_r = -\left[qE\, x_2 \sin\theta + qE\, x_1 \sin\theta\right]$$

$$\tau_r = -\left[qE\,(x_1+x_2)\sin\theta\right]$$

$$\sin\theta \simeq \theta$$

$$\alpha = -\frac{qEl\theta}{I}$$

$$\tau_r = -(qEl)\,\theta$$

$$\alpha = \frac{-qEl}{\left(\frac{m_1 m_2}{m_1+m_2}\right) l^2}\,\theta$$

$$\omega = \sqrt{\frac{qE}{\left(\frac{m_1 m_2 l}{m_1+m_2}\right)}} = \sqrt{\frac{pE}{\mu l^2}}$$

$$\left(\frac{m_1 m_2}{m_1+m_2}\right) \Downarrow \mu$$

$$\mu = \frac{m_1 m_2}{m_1+m_2}$$

$$\alpha = -\omega^2\,\theta$$

$$I = m_1 x_1^2 + m_2 x_2^2$$

$$= m_1\left(\frac{m_2 l}{m_1+m_2}\right)^2 + m_2\left(\frac{m_1 l}{m_1+m_2}\right)^2$$

$$I = \frac{m_1 m_2^2 l^2}{(m_1+m_2)^2} + \frac{m_2 m_1^2 l^2}{(m_1+m_2)^2}$$

$$I = \frac{m_1 m_2 l^2}{\cancel{(m_1+m_2)^2}}\left[\cancel{m_1+m_2}\right] = \left[\frac{m_1 m_2 l^2}{m_1+m_2}\right]$$

DIPOLE

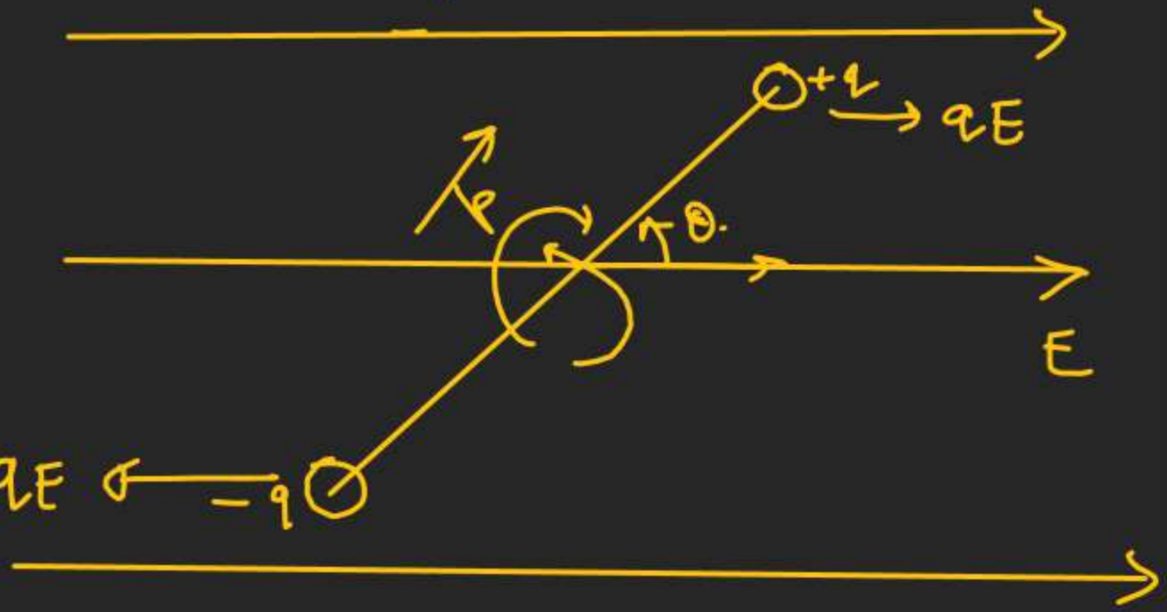
Stored

❖ **Potential Energy stared in a dipole placed in an uniform electric field.**

$$W = \int F \cdot dx, \quad \left[W = \int_{\theta_1}^{\theta_2} \tau \, d\theta \right]$$

$$\tau = PE \sin\theta$$

$$dW_{extagent} = \int_{U(\theta_1)}^{U(\theta_2)} dU = PE \int_{\theta_1}^{\theta_2} \sin\theta \, d\theta$$



$$U(\theta_2) - U(\theta_1) = PE \left[-\cos\theta \right]_{\theta_1}^{\theta_2}$$

$$= PE \left[-\cos\theta_2 + \cos\theta_1 \right]$$

$$\boxed{U(\theta_2) - U(\theta_1) = PE \left[\cos\theta_1 - \cos\theta_2 \right]}$$

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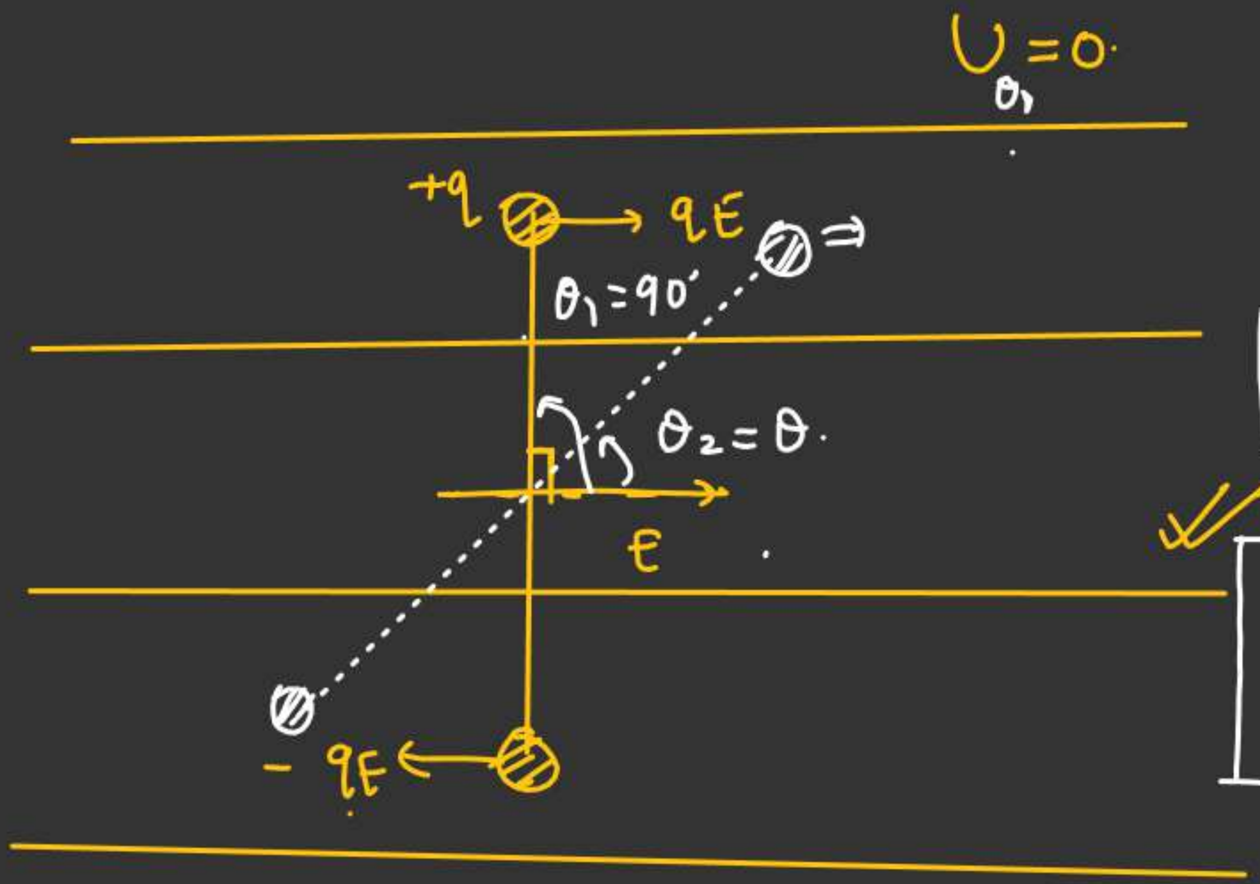
Absolute P.E of a dipole placed in an uniform Electric field.

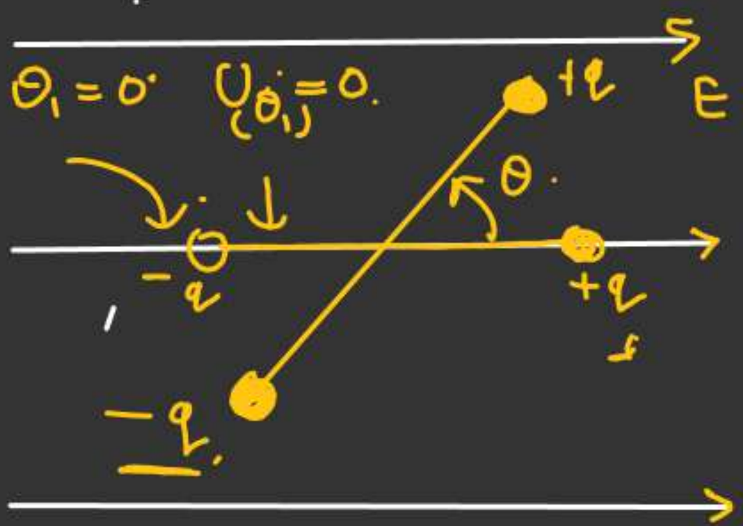


$\theta_1 = 90°$, $\theta_2 = \theta$

$$U_{(\theta_2)} - \cancel{U_{(\theta_1)}}^{0} = PE\left[\cos 90° - \cos\theta\right]$$

$$\boxed{U_{(\theta)} = -PE\cos\theta}$$

$$\boxed{U_{(\theta)} = -\vec{P}\cdot\vec{E}}$$ **



$$U_{(\theta_2)} = U_{(\theta)} = PE\left[\cos 0° - \cos\theta\right]$$

$$\boxed{U_{(\theta)} = PE\left[1 - \cos\theta\right]}$$ **

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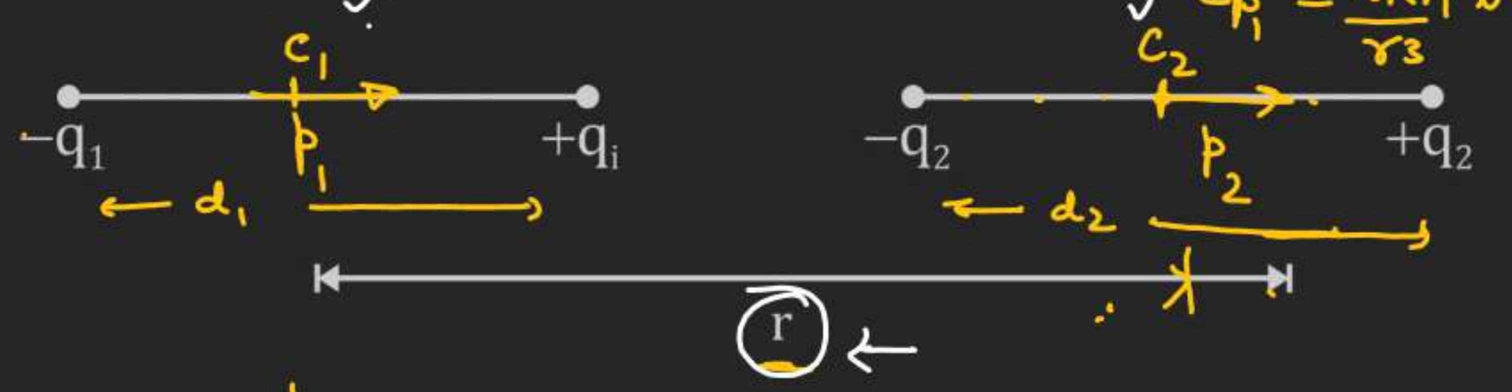
DIPOLE

$r >> d_1$ or d_2

①



$\vec{E}_{p_1} = \frac{2Kp_1}{r^3}\hat{i}$

❖ Force of interaction between two dipoles

$$E = -\frac{dV}{dr} \qquad \frac{U}{q} = V$$

$$qE = -\frac{d}{dr}(qV)$$

$$\boxed{F_c = -\frac{dU}{dr}}$$

(Conservative force)

$$U = -\vec{E}_{p_1} \cdot \vec{p}_2$$

$$U = -\left(\frac{2Kp_1}{r^3}(\hat{i})\right) \cdot p_2(\hat{i})$$

$$\boxed{U = -\frac{2Kp_1p_2}{r^3}}$$

$$F = -\frac{dU}{dr}$$

$$F = -\frac{d}{dr}\left(-\frac{2Kp_1p_2}{r^3}\right)$$

$$F = +2Kp_1p_2\frac{d}{dr}(r^{-3})$$

$$F = 2Kp_1p_2(-3)\,r^{-4}$$

$$\boxed{F = -\frac{6Kp_1p_2}{r^4}}$$

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DIPOLE

❖ Find the work done in rotating the dipole by 180°. as shown in fig [r ≫ d]

$U_i = -\vec{p}\cdot\vec{E}$

$U_i = -(q2d)\dfrac{\lambda}{2\pi\varepsilon_0 r}$

$U_i = -\dfrac{\lambda q d}{\pi\varepsilon_0 r}$

$U_f = -\vec{p}\cdot\vec{E}$

$= -pE\cos\pi$

$= (pE) = q(2d)\dfrac{\lambda}{2\pi\varepsilon_0 r} = \dfrac{qd\lambda}{\pi\varepsilon_0 r}$

$\Delta U = U_f - U_i$

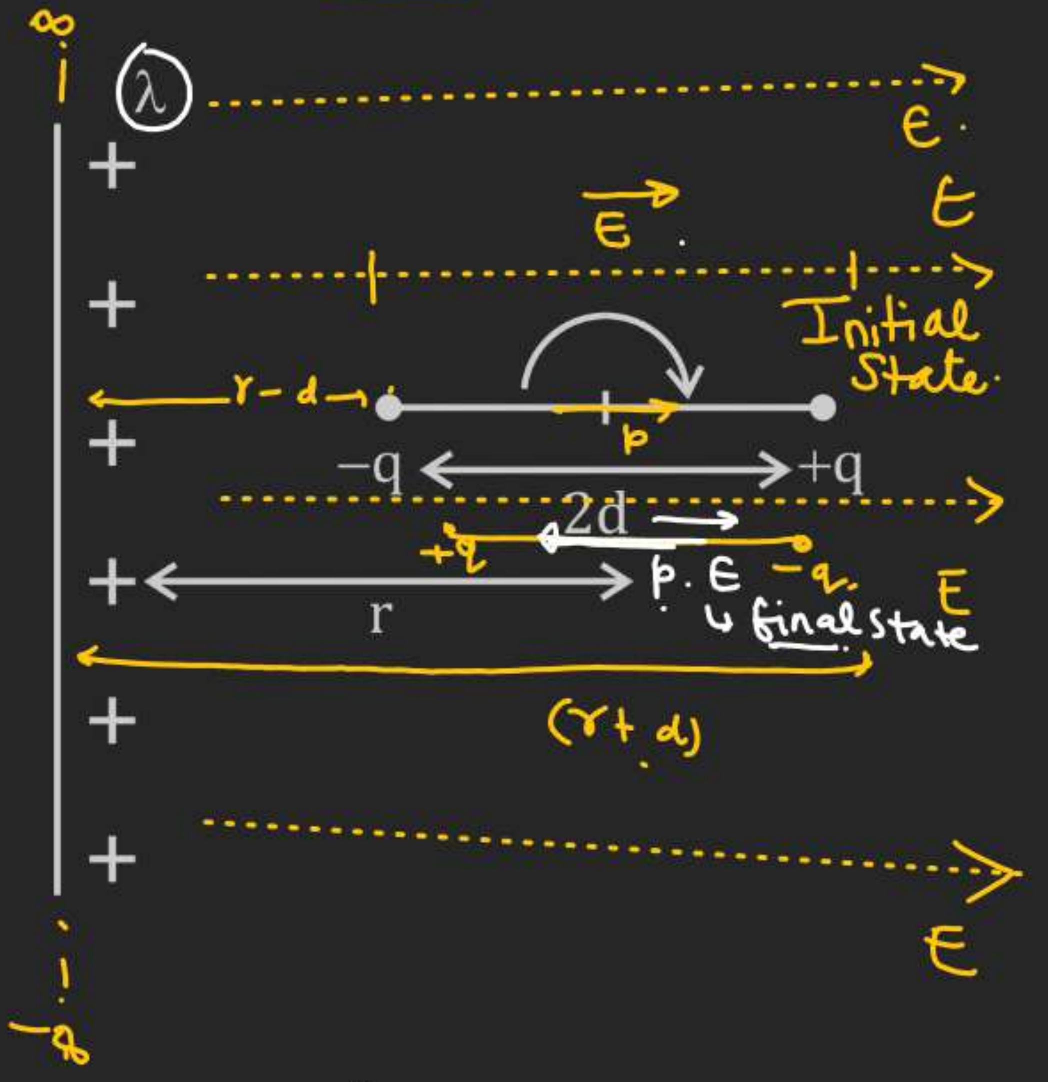
$= \dfrac{qd\lambda}{\pi\varepsilon_0 r} - \left(\dfrac{-qd\lambda}{\pi\varepsilon_0 r}\right) = \left(\dfrac{2qd\lambda}{\pi\varepsilon_0 r}\right)$ ✓

$E = \dfrac{\lambda}{2\pi\varepsilon_0 r}$

$\begin{cases} r-d \simeq r \\ r+d \simeq r \end{cases}$

$W_{ext\,agent} = \Delta U = \left[\dfrac{2qd\lambda}{\pi\varepsilon_0 r}\right]$

$W_{system} = -\Delta U = \dfrac{-2qd\lambda}{\pi\varepsilon_0 r}$ ✓



Sepecial : Concept of Monopole : → **DIPOLE**

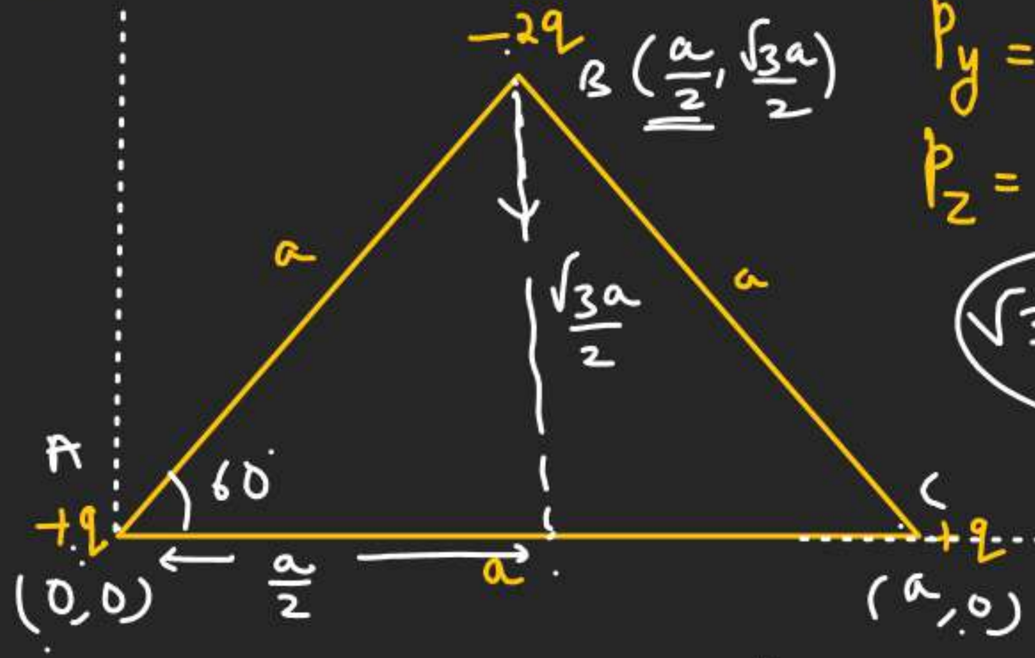
❖ Dipole Moment due to Continious charge distribution :-

$$\vec{p} = \left[p_x \hat{i} + p_y \hat{j} + p_z \hat{k} \right]$$

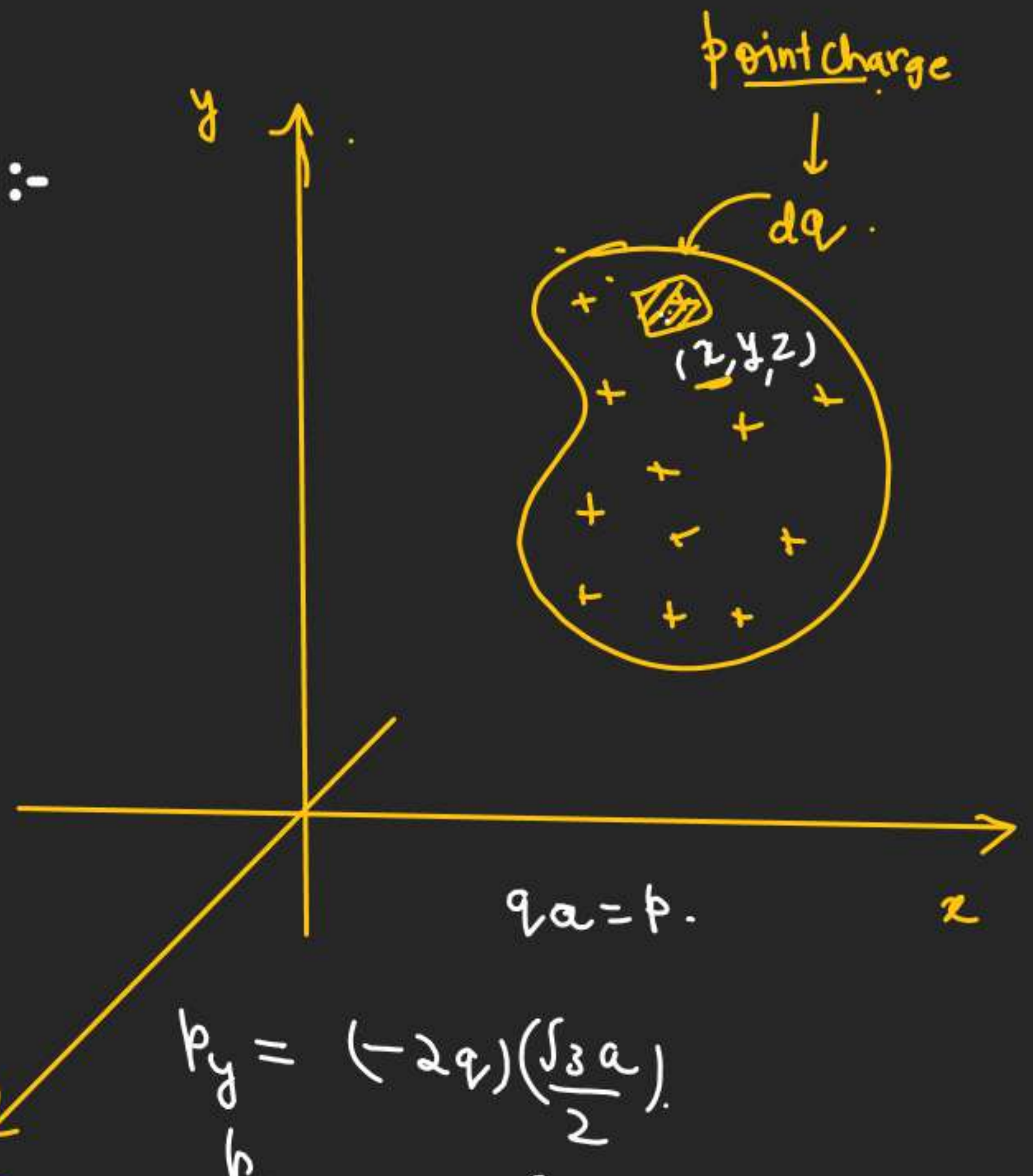
$$p_x = \int dq \cdot x$$

$$p_y = \int dq \cdot y$$

$$p_z = \int dq \cdot z$$



$$\sqrt{3}p = p_{net}$$



$$\tan 60^\circ = \frac{BC}{\frac{a}{2}}$$

$$\frac{\sqrt{3}a}{2} = BC$$

$$p_x = q(0) + q(a) + (-2q)\left(\frac{a}{2}\right)$$

$$p_x = 0$$

$$qa = p$$

$$p_y = (-2q)\left(\frac{\sqrt{3}a}{2}\right)$$

$$p_y = -q\sqrt{3}a$$

$$= -\sqrt{3}p$$

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DIPOLE

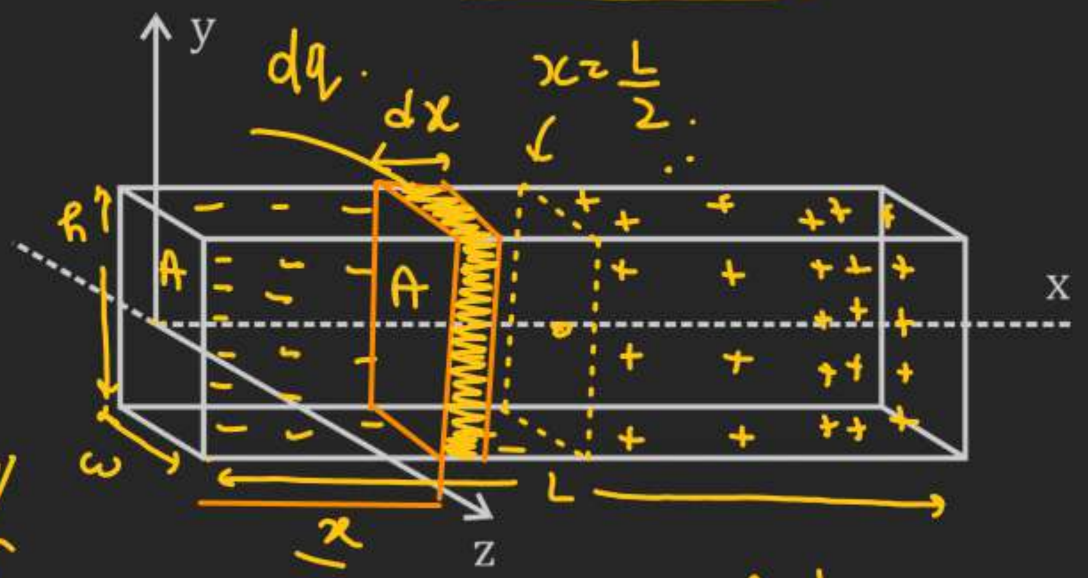
❖ **Calculate the dipole moment of a rectangular rod of charge density.**

$$\boxed{\rho = \rho_0(x - \ell/2)} \text{ for } 0 \leqslant x \leqslant \ell$$

$L >> (w/h)$

when $0 \le x \le \frac{\ell}{2}$ | $\frac{\ell}{2} < x \le \ell$

$\rho < 0$ | $\rho > 0$



$$dp_x = dq\, x$$

$$dq = \rho_x \underbrace{dV}_{\text{differential Volume of 'dx' thickness}}$$

$dV = A\,dx$

$$p_x = \int_0^{\ell} dp_x = \rho_0 A \int_0^{\ell} \left(x - \frac{\ell}{2}\right) x \cdot dx$$

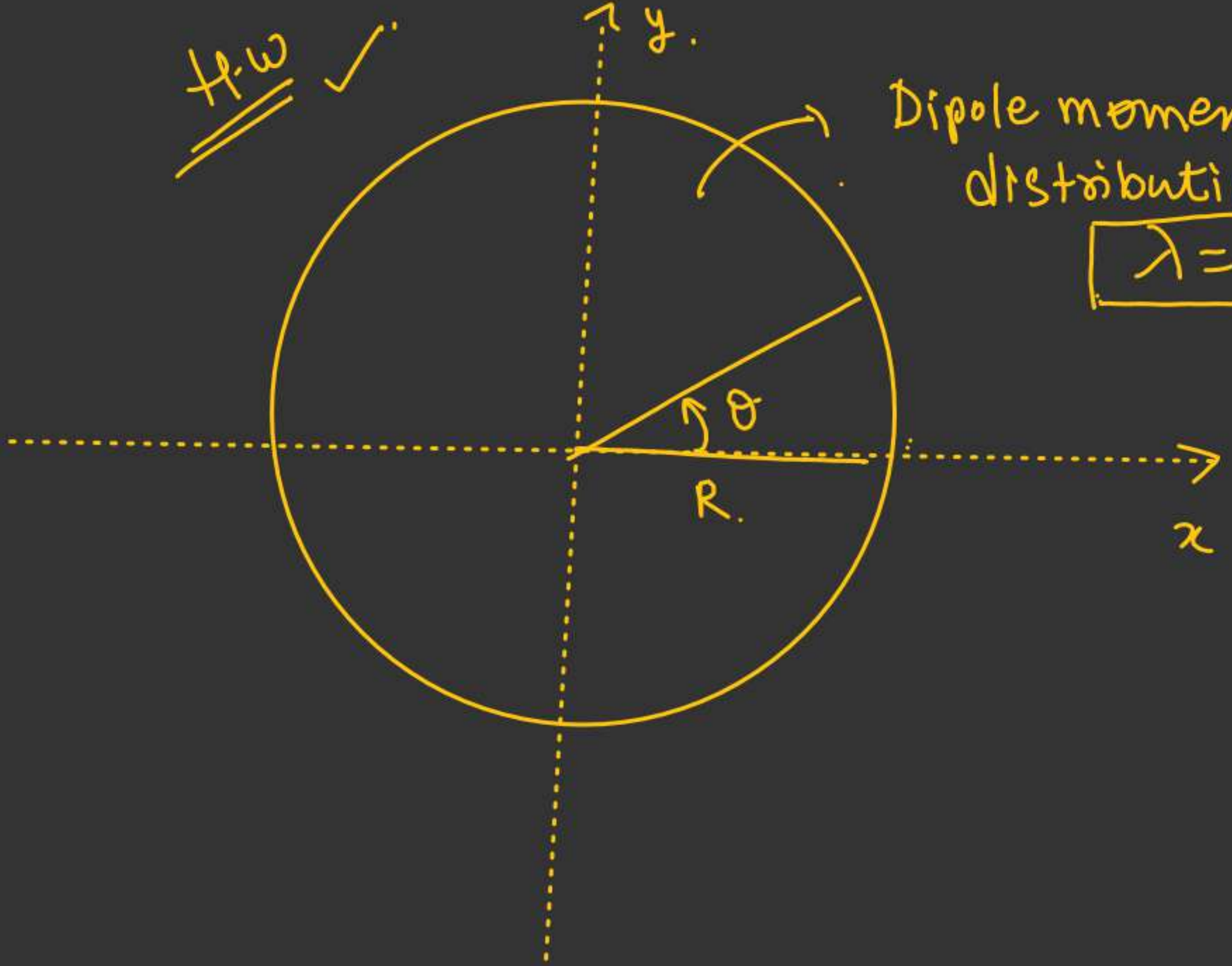
$dq = \rho_0 \left(x - \frac{\ell}{2}\right) A\,dx$

$dq = \rho_0 A \left(x - \frac{\ell}{2}\right) dx$

$$p_x = \rho_0 A \left[\int_0^{\ell} x^2 dx - \frac{\ell}{2}\int_0^{\ell} x\,dx\right]$$

$$p_x = \rho_0 A \left[\frac{\ell^3}{3} - \frac{\ell^3}{4}\right] = \left(\frac{\rho_0 A \ell^3}{12}\right)$$

H.W ✓



Dipole moment of ring if Charge distribution

$$\boxed{\lambda = \lambda_0 \cos\theta}$$