

$$C_1 \rightarrow C_1 + C_3 - 2 \cos d \times C_2$$

$$R_1 \rightarrow R_1 + R_2 + R_3$$

$$\sum A+P = 0 \quad \left| \begin{array}{ccc} 1 & \sum \tan(A+P) + \tan(B+Q) + \tan(C+R) & (a+b+c) \\ \tan(A+P) & \tan(B+Q) & \tan(C+R) \\ 0 & 0 & 0 \end{array} \right| \quad \left| \begin{array}{ccc} 1 & 1 & 1 \\ & & \end{array} \right|$$

$$\text{pr} \sum a^3 - \frac{(\text{pr}^3)}{6} abc$$

$$C_1 \rightarrow C_1 - 6C_3$$

$$C_2 \rightarrow C_2 + aC_3$$

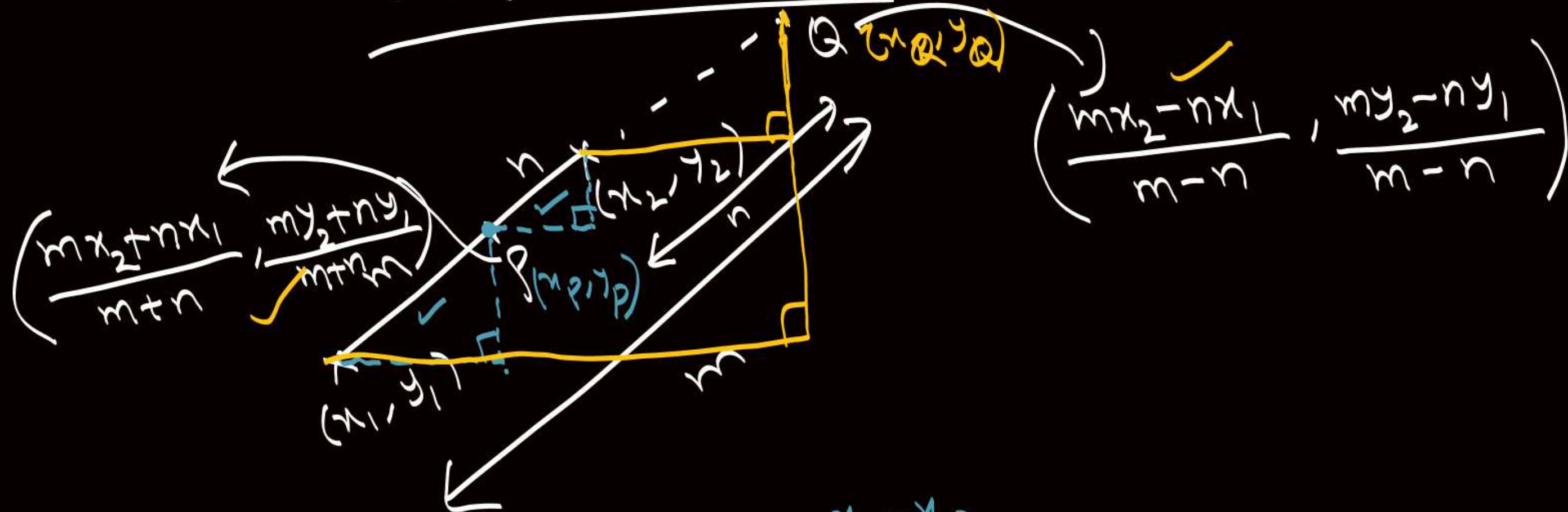
$$C_2 \rightarrow C_2 - C_1$$

$$C_3 \rightarrow C_3 - C_1$$

$$(1+a^2+b^2) \left| \begin{array}{ccc} 1 & 0 & -2b \\ 0 & 1 & -a \\ b & -a & \end{array} \right|$$

$$C_3 \rightarrow C_3 + 2bC_1$$

Section Formula

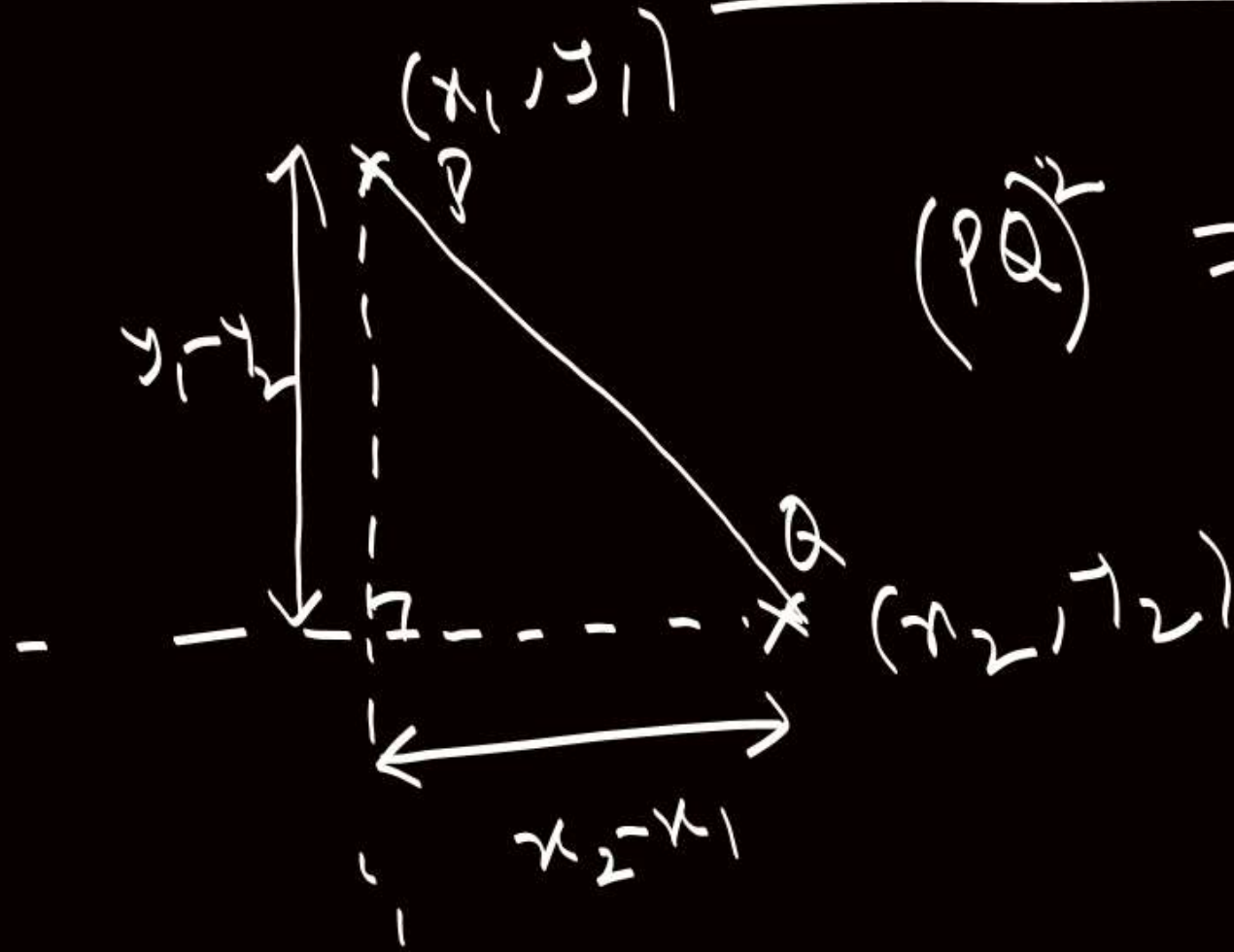


$$\left(\frac{mx_2 - nx_1}{m-n}, \frac{my_2 - ny_1}{m-n} \right)$$

$$\frac{y_2 - y_r}{y_r - y_1} = \frac{b}{a} = \frac{x_2 - x_r}{x_r - x_1}$$

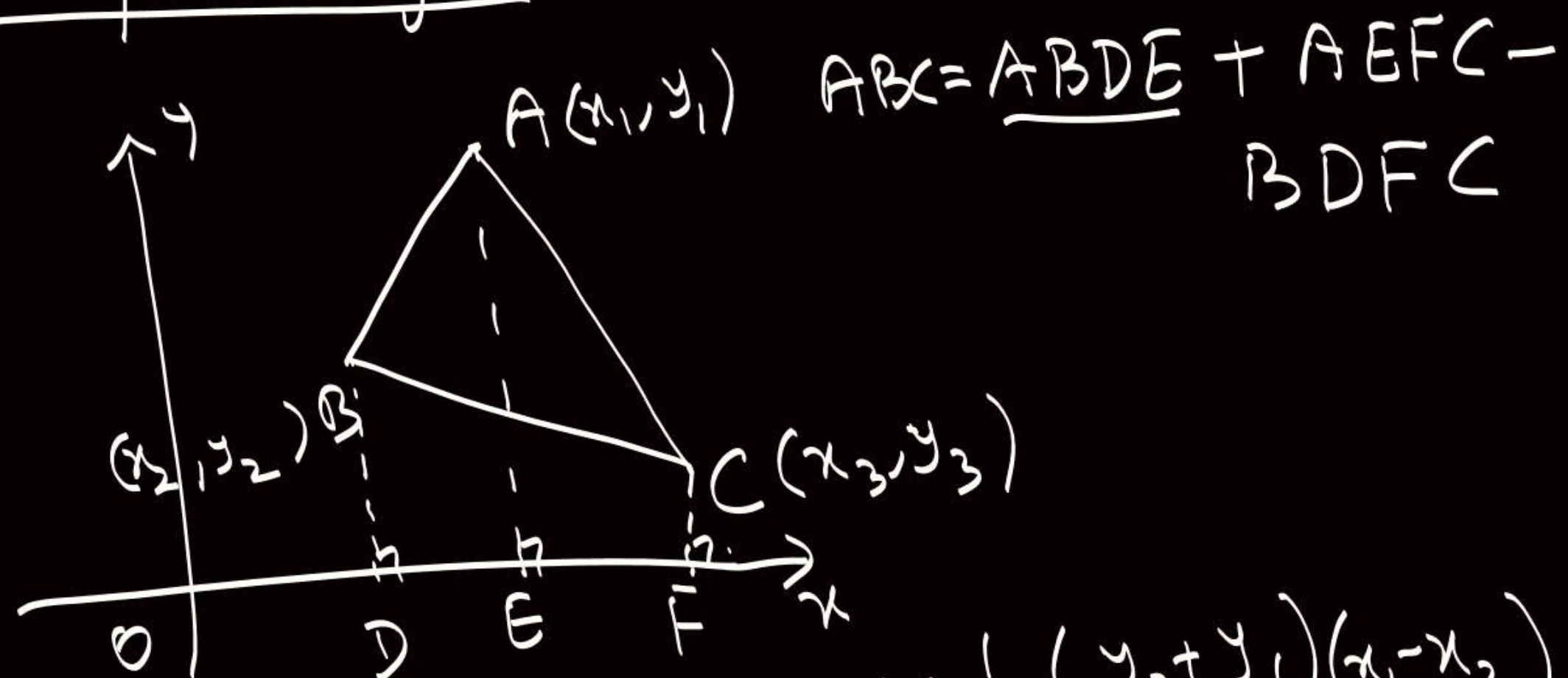
$$\frac{x_Q - x_2}{x_Q - x_1} = \frac{a}{b} = \frac{y_Q - y_2}{y_Q - y_1}$$

Distance b/w 2 points



$$(PQ)^2 = (x_2 - x_1)^2 + (y_1 - y_2)^2$$

Area of triangle



$$ABC = \underline{ABDE} + AEFC - BDFC$$

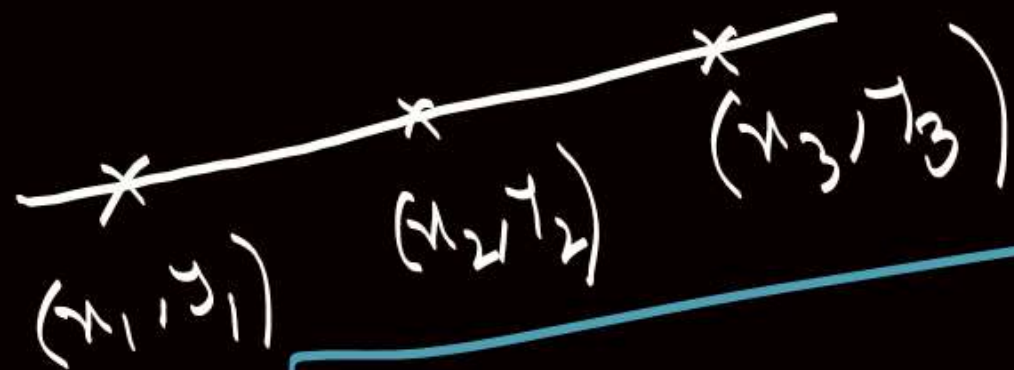
$$\Delta ABC = \frac{1}{2} (y_2 + y_1)(x_1 - x_2)$$

$$+ \frac{1}{2} (y_1 + y_3)(x_3 - x_1) - \frac{1}{2} (y_2 + y_3)(x_3 - x_2)$$

$$\frac{1}{2} |\vec{BA} \times \vec{BC}|$$


$$= \text{modulus of } \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

Condition for Collinear of 3 points



$$\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$$

Harmonic Conjugates



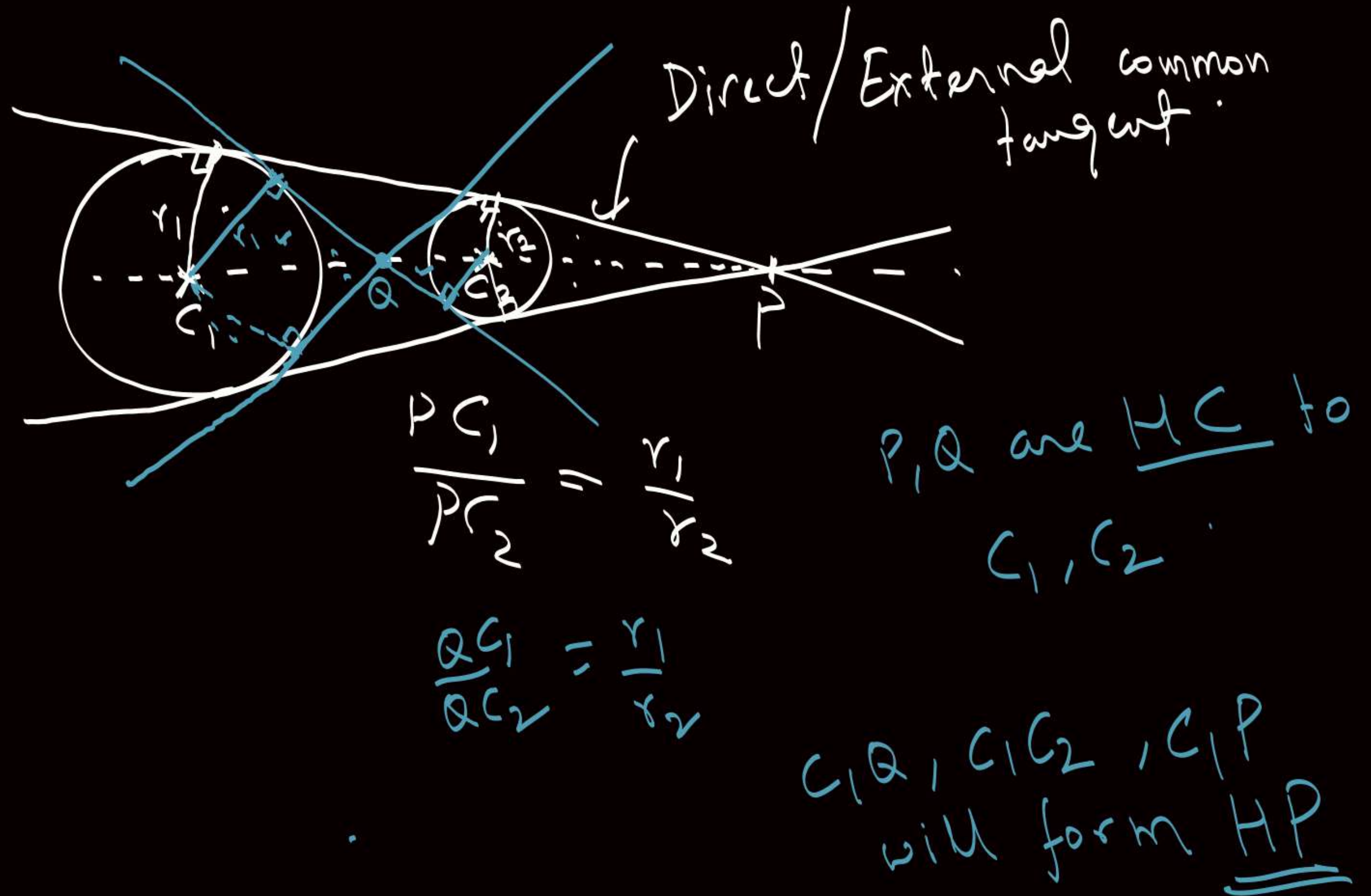
$\Rightarrow \frac{AB}{AP} = \frac{(AQ - AB) + AQ}{AQ} \Rightarrow \frac{AB}{AP} = 2 - \frac{AB}{AQ} \Rightarrow \boxed{\frac{1}{AP} + \frac{1}{AQ} = \frac{2}{AB}}$

$\Rightarrow 1 + \frac{PB}{PA} = \frac{QB}{QA} + 1 \Leftrightarrow \frac{PA}{PB} = \frac{QA}{QB}$

$\Rightarrow \boxed{AP, AB, AQ \text{ will be in HP}}$

HC to B, C

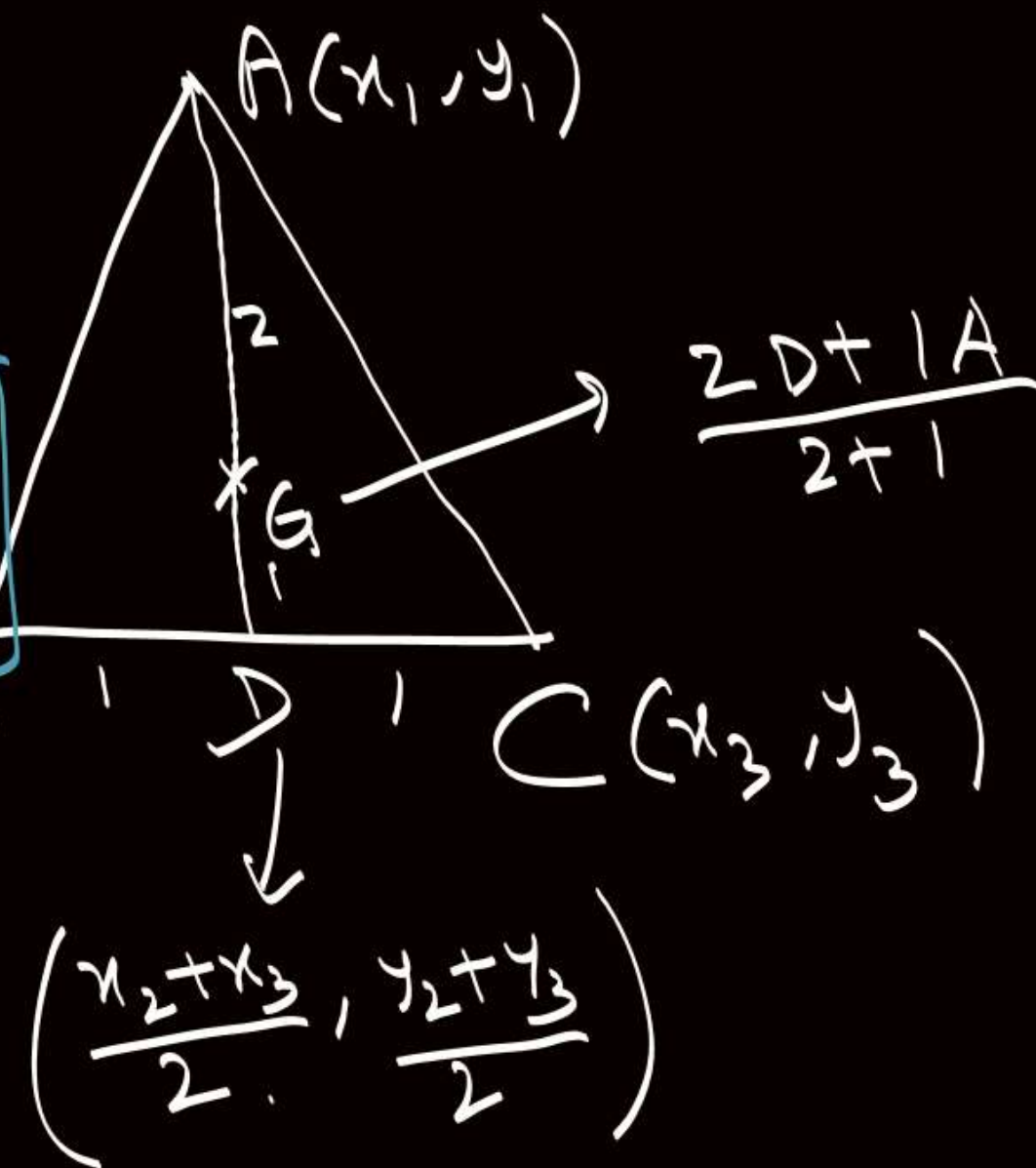
BD, BC, BE will form harmonic progression



Centroid

$$G = \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3} \right)$$

(x_2, y_2) B



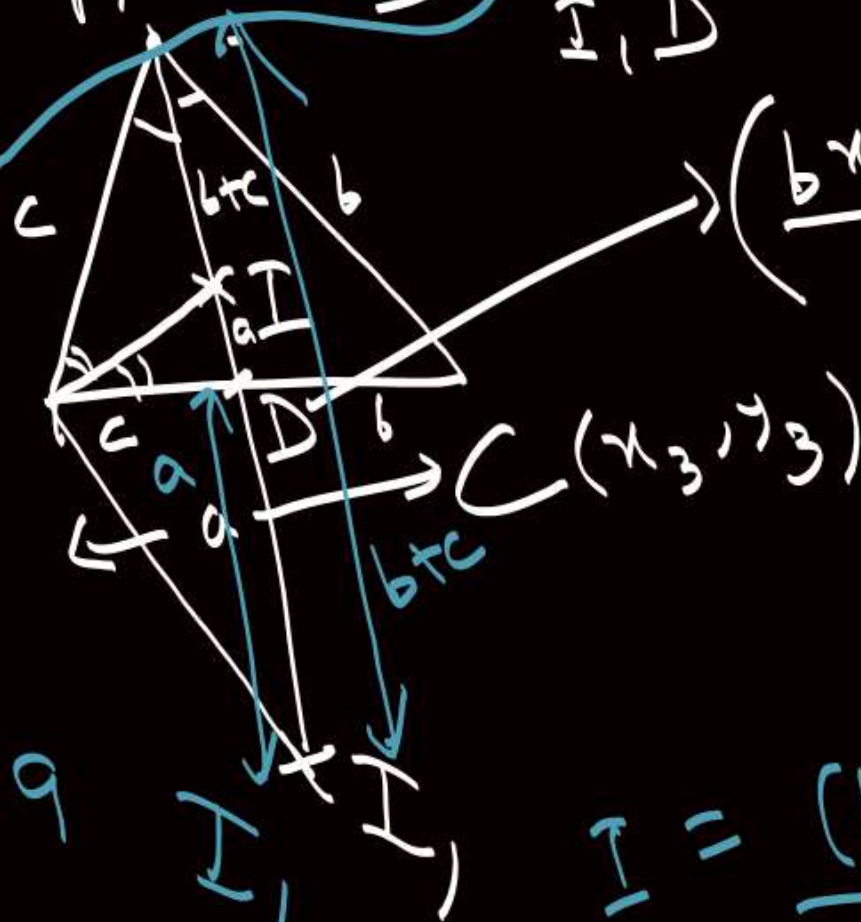
Incentre / Excentre

$$I = \left(\frac{ax_1 + bx_2 + cx_3}{a+b+c}, \frac{ay_1 + by_2 + cy_3}{a+b+c} \right)$$

$$I_1 = \left(\frac{-ax_1 + bx_2 + cx_3}{-a+b+c}, \frac{-ay_1 + by_2 + cy_3}{-a+b+c} \right)$$

$$\frac{AI}{ID} = \frac{\Delta ABD}{\Delta IBD} = \frac{AB}{BD} = \frac{c}{\frac{ca}{b+c}} = \frac{b+c}{a}$$

$$\left(\frac{bx_2 + cx_3}{b+c}, \frac{by_2 + cy_3}{b+c} \right)$$



$I \Rightarrow$

$a \rightarrow -a$

$b \rightarrow -b \Rightarrow I_1$

$c \rightarrow -c \Rightarrow I_2$

$$I = \frac{(b+c)D + aA}{(b+c) + a}$$

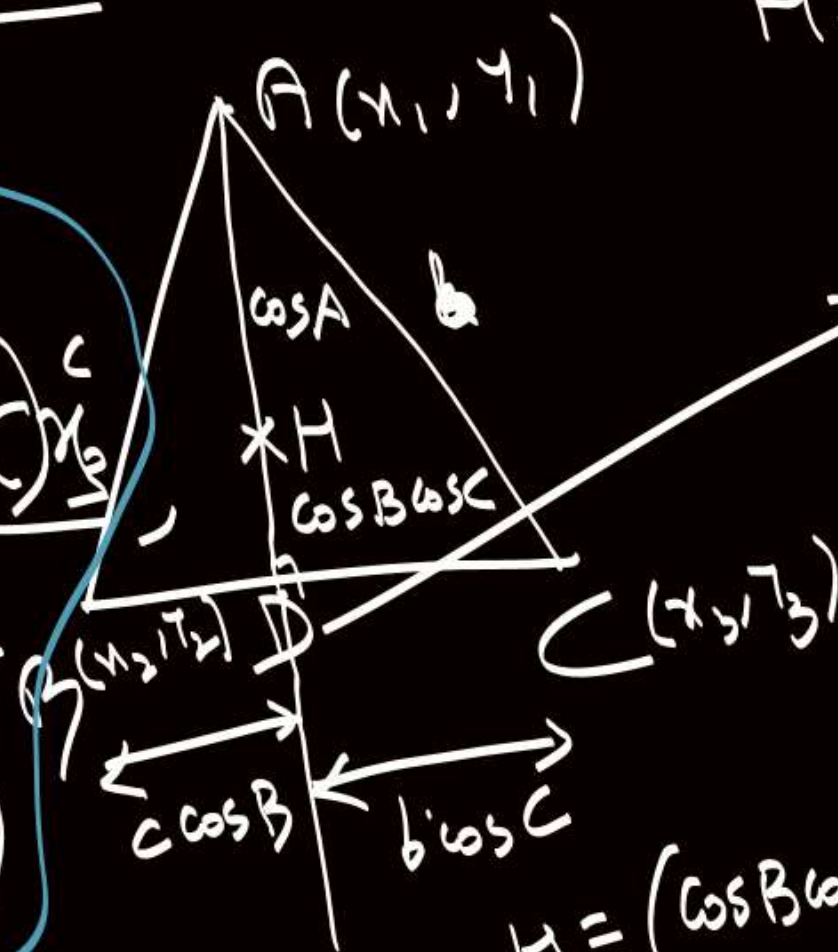
$$I_1 = \frac{(b+c)D - aA}{(b+c) - a}$$

Orthocentre

$$H = \frac{(\tan A)x_1 + (\tan B)x_2 + (\tan C)x_3}{\tan A + \tan B + \tan C}$$

$$\frac{\cos B \cos C x_1 + \cos A c \cos B x_3 + \cos A b \cos C x_2}{a}$$

$$\frac{\cos B \cos C - \cos(B+C)}{\sin A (\sin B \sin C)}$$



$$H = \frac{\sum \tan A x_1}{\sum \tan A} \cdot \frac{\sum \tan A y_1}{\sum \tan A}$$

$$\left(\frac{\cos B x_3 + b \cos C x_2}{a}, - \right)$$

$$H = \frac{(\cos B \cos C)A + (\cos A)D}{\cos B \cos C + \cos A}$$

$$= \frac{\sin A \cos B \cos C x_1 + \sin A \cos A \cos B x_3 + \sin B \cos A \cos C x_2}{\sin A (\sin B \sin C)}$$