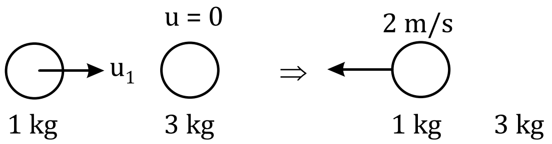
SOLUTION

CENTRE OF MASS

1.



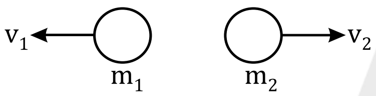
$1 \times u_1 = -2 + 3v \Rightarrow u_1 = -2 + 3v$ .... (i)

$1 = \dfrac{v+2}{u_1} \Rightarrow v + 2 = u_1$ ... (iii)

Solving (1) and (2)

$u_1 = 4\ m/s$

2.



$\dfrac{v_1}{v_2} = \dfrac{3}{2}$

$m_1 v_1 = m_2 v_2 \Rightarrow \dfrac{m_1}{m_2} = \dfrac{2}{3}$

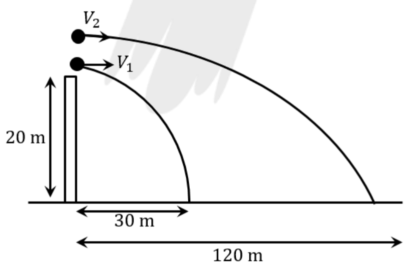
Since, Nuclear mass density is constant

$\dfrac{m_1}{\frac{4}{3}\pi r_1^3} = \dfrac{m_2}{\frac{4}{3}\pi r_2^3}$

$\left(\dfrac{r_1}{r_2}\right)^3 = \dfrac{m_1}{m_2} \Rightarrow \dfrac{r_1}{r_2} = \left(\dfrac{2}{3}\right)^{\frac{1}{3}}$

So, $x = 2$

3.



$v_1 = \dfrac{30}{\sqrt{\frac{2h}{g}}}, v_2 = \dfrac{120}{\sqrt{\frac{2h}{g}}}$

$(0.01)u = (0.2)\dfrac{30\sqrt{g}}{\sqrt{2h}} + (0.01)\dfrac{120\sqrt{g}}{\sqrt{2h}}$

$u = 300 + 60 = 360\ ms^{-1}$

4. The velocities will be interchanged after collision.

Speed of P just before collision $= \sqrt{2gh} = \sqrt{2 \times 10 \times 0.2} = 2\ \text{m/s}$

5. $20 \times 10^{-3} \times \frac{180}{60} \times 100 = 10\ \text{V}$

$\Rightarrow \text{v} = 0.6\ \text{m/s}$

6. $\text{P}_\text{i} = \text{Nmv}\hat{\text{i}}$

$\vec{\text{P}}_\text{f} = -\text{Nmv}\hat{\text{i}}$

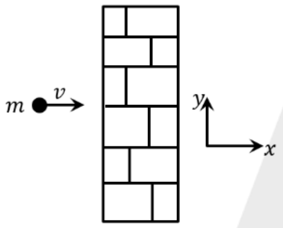
N is Number of balls strikes with wall

$\text{N} = 100$

$\Delta\vec{\text{P}} = \vec{\text{P}}_\text{f} - \vec{\text{P}}_\text{i} = -2\text{Nmv}\hat{\text{i}} = -200\text{Nmu}\hat{\text{i}}$

$$\vec{\text{F}}_{\text{Total}} = \frac{\Delta\vec{\text{P}}}{\Delta\text{t}} = -\frac{200\text{mvt}}{\text{t}}$$

$$|\vec{\text{F}}| = \frac{200\text{mv}}{\text{t}}$$



7. $\frac{1}{2}\text{mv}^2 + \frac{1}{2}\text{I}\omega^2 = 7 \times 10^{-3}$

$$\frac{1}{2}\text{mv}^2 + \frac{1}{2}\left(\frac{2}{5}\text{MR}^2\right)\left(\frac{\text{V}}{\text{R}}\right)^2 = 7 \times 10^{-3}$$

$$\frac{1}{2}\text{MV}^2\left[1 + \frac{2}{5}\right] = 7 \times 10^{-3} \Rightarrow \frac{1}{2}(1)(\text{V}^2)\left(\frac{7}{5}\right) = 7 \times 10^{-3}$$

$\text{V}^2 = 10^{-2} \Rightarrow \text{V} = 10^{-1} = 0.1\ \text{m/s} = 10\ \text{cm/s}$

8. We know $\text{h}' = \text{e}^2\text{h}$

$\text{h}' = (0.5)^2 \times 20\ \text{m} = 5\ \text{m}$

9. Impulse = Area under $F = t$ curve

(a) $\frac{1}{2} \times 1 \times 0.5 = \frac{1}{4}\ N.s$

(b) $0.5 \times 2 = 1\ N.s$ (maximum)

(c) $\frac{1}{2} \times 1 \times 0.75 = \frac{3}{8}$ N.s

(d) $\frac{1}{2} \times 2 \times 0.5 = \frac{1}{2}$ N.s

10. The law of conservation of momentum is given as $m_1u + m_2u' = (m_1 + m_2)v'$

The given data is

$u = v$

$u' = 0\ m\ s^{-1}$

$m_1 = m$

$m_2 = 2m$

Using the conservation of momentum equation,

$$mv + 2m(0) = (m + 2m)v' \Rightarrow v' = \frac{mv}{3m} = \frac{v}{3}$$

11. The formula to calculate the net momentum of the gun and bullet system can be written as

$F = nmv$ ...(i)

Substitute the values of the known parameters into equation (i) and solve to calculate the required number of bullets.

$$125 = n \times \frac{10}{1000} \times (250) \Rightarrow n = 50$$

12. Using the law of Momentum conservation

$m_1v_1 + m_2(-v_2) = 0$

$$\Rightarrow m_1v_1 = m_2v_2 \Rightarrow \left(\frac{m_1}{m_2}\right) = \left(\frac{v_2}{v_1}\right) \Rightarrow \frac{\frac{4\pi r^3}{3}}{\frac{4\pi \rho r^3 2}{3}} = \left(\frac{v_2}{v_1}\right)$$

It is given

$$\frac{r_1}{r_2} = \frac{1}{2^{\frac{1}{3}}}$$

and $\left(\frac{r_1}{r_2}\right)^3 = \left(\frac{v_2}{v_1}\right)$

Therefore,

$$\left(\frac{1}{2}\right) = \left(\frac{v_2}{v_1}\right) \Rightarrow \frac{v_1}{v_2} = \frac{2}{1}$$

13. The kinetic energy and the momentum of a moving object are related by

$K = \frac{p^2}{2m}$ ... (i)

Hence, the ratio of the kinetic energies of two objects having masses m and m ' can be calculated as follows:

$$\frac{K}{K'} = \frac{\frac{p^2}{2m}}{\frac{p^2}{2m'}}$$

$= \frac{m'}{m}$ ... (ii)

Substitute the ratio of the kinetic energies into equation (ii) and simplify to obtain the ratio of their masses.

$\Rightarrow \frac{16}{9} = \frac{m'}{m} \Rightarrow m : m' = 9 : 16$

14. The formula to calculate the impulse (J) on the gun is given by

$J = m(v - u)$ ... (i)

where, m is the mass, u is the initial velocity and v is the final velocity of the bullet.

Substitute the values of the known parameters into equation (i) to calculate the required impulse.

$J = 10\text{ g} \times \frac{1\text{ kg}}{1000\text{ g}} \times (600 - 0)\text{ms}^{-1} = 6\text{ N s}$

MOTION IN ONE DIMENSION

15. Displacement $= \Sigma$ area $= 16 - 8 + 16 - 8 = 16$ m

Distance $= \Sigma\,|\text{area}| = 48$ m

$\frac{\text{displacement}}{\text{Distance}} = \frac{1}{3}$

16. Average velocity $= \frac{\text{Total displacement}}{\text{Total time}}$

$= \frac{x + x}{\underline{x} + \underline{x}} = \frac{2v_1v_2}{v_1 + v_2}$

17. $x = 4t^2$

$v = \frac{dx}{dt} = 8t$

At $t = 5$sec

$v = 8 \times 5 = 40$ m/s

18. $v_i = \sqrt{2gh_i}$

$= \sqrt{2 \times 10 \times 9.8} \downarrow$

$= 14$ m/s $\downarrow$

$v_f = \sqrt{2gh_f}$

$= \sqrt{2 \times 10 \times 5} \uparrow$

$= 10$ m/s $\uparrow$

$|\vec{a}_{avg}| = \left|\frac{\Delta\vec{v}}{\Delta t}\right| = \frac{24}{0.2} = 120\text{ m/s}^2$

19. $t_{AB} = \frac{x}{5\text{ m/s}}$

In motion BC

$x = d_1 + d_2$

where d_1 & d_2 we the distance travelled with 10 m/s and 15 m/s respectively in equal time intervals

$\frac{'t'}{2}$ each

$d_1 = \frac{10t}{2}$, $d_2 = \frac{15t}{2}$

$d_1 + d_2 = x = \frac{t}{2}(10 + 15) = \frac{25t}{2}$

$\langle v \rangle = \frac{2x}{\frac{x}{5} + \frac{2x}{25}} = \frac{2 \times 25}{5 + 2} = \frac{50}{7}$ m/s

20. Loss in PE = Gain in KE

$\left(-\frac{GMm}{2R}\right) - \left(-\frac{GMm}{R}\right) = \frac{1}{2}mv^2$

$\Rightarrow v^2 = \frac{GM}{R} = gR$

$\Rightarrow v = \sqrt{gR}$

21. $\frac{2}{V_{av}} = \frac{1}{3} + \frac{1}{5} = \frac{8}{15}$

$\Rightarrow V_{av} = \frac{15}{4} = 3.75$ km/h

22. Speed, v = constant

Radius, R = 10 m

T = Time period = 4 s

At the end of 3rd second, particle will be at D (Starts from A)

$\therefore$ displacement $S = \sqrt{2}R = \sqrt{2} \times 10$

$= 10\sqrt{2}$ m

23. AB = x

BC = x

2x + CD = 3x

CD = x

$< V >= \frac{3x}{\frac{x}{v_1} + \frac{x}{v_2} + \frac{x}{v_3}} = \frac{3v_1v_2v_3}{v_2v_3 + v_1v_3 + v_1v_2}$

24. $u = 20\ m/s, S_1 = 500\ m, v = 0$

By third equation of motion

$0 = (20)^2 - 2a \cdot 500 \Rightarrow a = \frac{4}{10}\ m/s^2$

$u = 20\ m/s, S_2 = 250\ m, v = ?$

$v^2 = (20)^2 - 2a \cdot 250$

$= v = \sqrt{200}\ m/s$

$x = 200$

25. Acceleration can be written as, $a = v\frac{dv}{dx}$.

Given: $a = -2x$ (-ve sign is due to retardation)

Therefore,

$v\frac{dv}{dx} = -2x$

$\Rightarrow vdv = (-2x)dx$

Integrating both sides, we get

$\frac{1}{2}(v_f^2 - v_i^2) = -2\left(\frac{x^2}{2}\right) = x^2$

$\Rightarrow \left|\frac{1}{2}m(v_f^2 - v_i^2)\right| = mx^2$

$\Rightarrow |\Delta KE| = 0.01x^2 = \left(\frac{10}{x}\right)^{-2}$

Hence, $n = 2$.

26. The given data is

$v = 60\ m\ s^{-1}$

$u = 10\ m\ s^{-1}$

$a = 2\ m\ s^{-2}$

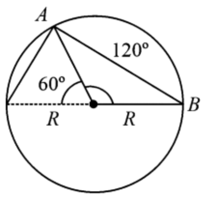
Using the equation $v = u + at$, we get

$60 = 10 + 2t$

$\Rightarrow t = \frac{(60 - 10)}{2}$

$\Rightarrow t = 25\ s$

27. The given data is



$v = \pi ms^{-1}$

By using trigonometric ratios in the triangle shown above the displacement can be found out by

$$\sin\ 60^\circ = \frac{AB}{2R}$$

$$\Rightarrow AB = R\sqrt{3}$$

The formula for distance is

$$d = vt$$

$$\Rightarrow t = \frac{d}{v}$$

$$\Rightarrow t = \frac{120^\circ}{360^\circ} \times \frac{2\pi R}{\pi} = \frac{2R}{3}$$

So, the average velocity is

$$v_{avg} = \frac{\sqrt{3}R}{\frac{2R}{3}} = \frac{3\sqrt{3}}{2}$$

$$= 1.5\sqrt{3}\ m\ s^{-1}$$

28. The definition of displacement is given by the product of velocity and time.

$\int vdt = \Delta x =$ displacement of body in given time.

Distance is a scalar quantity and displacement is a vector quantity defined by the shortest distance between the initial and the final position.

Acceleration is defined by the rate of change of velocity.

$\int adt = \Delta v =$ change in velocity in given time.

Thus, the area under the velocity time graph gives the displacement of the body and the area under the acceleration time graph gives the change in velocity of the body.

29. From the position time graph the velocity is found out by the slope of the line. The slope is given by $\frac{x_2 - x_1}{t_2 - t_1}$.

The slope of B is greater than A. Hence, the velocity of B is greater than A. As can be seen from the graph $X_A < X_B$. So A lives closer to the school and $V_B > V_A$. Hence, this is the correct option.

30. The time (t_1) taken by the person to travel distance x with velocity v_1 is given by $t_1 = \frac{x}{v_1}$ $\cdots (1)$

The time (t_2) taken by the person to travel distance x with velocity v_2 is given by $t_2 = \frac{x}{v_2}$

The formula to calculate the average speed (v) of the person is given by $v = \frac{2x}{t_1 + t_2}$

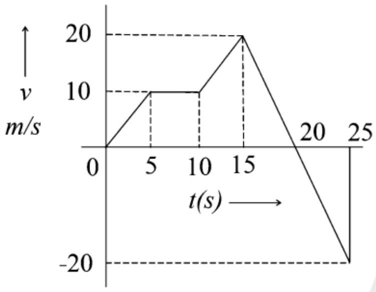
Substitute the expressions from equations (1) and (2) into equation (3) to obtain the required relation.

$$v = \frac{2x}{\left(\frac{x}{v_1} + \frac{x}{v_2}\right)}$$

$$\Rightarrow \frac{x}{v_1} + \frac{x}{v_2} = \frac{2x}{v}$$

$$\Rightarrow \frac{2}{v} = \frac{1}{v_1} + \frac{1}{v_2}$$

31. The area under the velocity time graph gives the displacement. The area for $t = 0$ s to $t = 25$ s is



$$(\text{ Area }) = \left(\frac{1}{2} \times 5 \times 10\right) + (5 \times 10) + \left(\frac{1}{2} \times 30 \times 5\right) + \left(\frac{1}{2} \times 20 \times 5\right) - \left(\frac{1}{2} \times 20 \times 5\right)$$

$$\Rightarrow \text{ Area } = 25 + 50 + 75 + 50 - 50 = 150 \text{ m}$$

Hence, the displacement $= 150$ m

The distance is defined as the path taken by the object to travel from initial to final position. Also note that displacement is a vector quantity, but distance is a scalar quantity.

Distance

$$= \left(\frac{1}{2} \times 5 \times 10\right) + (5 \times 10) + \left(\frac{1}{2} \times 30 \times 5\right) + \left(\frac{1}{2} \times 20 \times 5\right) + \left(\frac{1}{2} \times 20 \times 5\right)$$

$$= (200 + 50) = 250 \text{ m}$$

Therefore, required ratio

$$\frac{\text{distance}}{\text{displacement}} = \frac{250}{150} = \left(\frac{5}{3}\right)$$

32. The formula to calculate the velocity of the ball after 3 s is given by

$v_3 = u - 3g$

The formula to calculate the velocity of the ball after 5 s is given by

$v_5 = u - 5g$...(2)

Divide equation (1) by equation (2) to obtain the required ratio.

$$\frac{v_3}{v_5} = \frac{u - 3g}{u - 5g}$$

Substitute the values of the known parameters into equation (3) to calculate the required ratio.

$$\frac{v_3}{v_5} = \frac{150 - 30}{150 - 50}$$

$$= \frac{120}{100} = \frac{6}{5} = \frac{5+1}{5}$$

Comparing the above equation with the given expression, it can be written that $x = 5$.

33. The work done in stopping the car and the truck can be written as

$W = Fx$

$= \Delta KE$

As both the car and the truck are moving initially with the same kinetic energy, after being stopped, they will have the same change of kinetic energy.

Also, they are stopped with the same applied force.

Hence, from the above equation, it can be written that they have the same displacement,

i.e., $x_1 = x_2$.

Acceleration is a vector quantity. As the direction of motion of the car is different in the two given situations, the acceleration of the car will not be the same.

34. The time taken for A

$$\Delta t_A = \frac{l + L}{V_A}$$

The time taken for B

$$\Delta t_B = \frac{4l + L}{V_B}$$

It is given that B takes more time than A. So,

$$\Rightarrow \frac{4l + L}{20} - \frac{l + L}{30} = 35$$

$$\Rightarrow \frac{64l}{20} - \frac{61l}{30} = 35$$

$$\Rightarrow l = \frac{35 \times 600}{700} = 30 \text{ m}$$

Therefore,

$L = 1800$ m.

35. It is given that

$s = 2.5t^2$

Differentiating,

$$v = \frac{ds}{dt} = \frac{d(2.5t^2)}{dt} = 5t$$

At $t = 5$ s, $v = 25$ m s^{-1}

36. Let the velocity of A be v_A and the velocity of B be v_B. The relative velocity is

$$\vec{v}_{BA} = \vec{v}_B - \vec{v}_A$$

$\Rightarrow v_{BA} = 54 - (-90) = 144 \text{ km h}^{-1}$

$\Rightarrow v_{BA} = 40 \text{ m s}^{-1}$

$\text{Time } = \dfrac{\text{length}}{v_{BA}}$

$\Rightarrow \text{ length } = 40 \text{ m s}^{-1} \times 8 \text{ s} = 320 \text{ m}$

37. It is given that

$x = 5t^2 - 4t + 5$

The velocity is given by

$v = \dfrac{dx}{dt} = \dfrac{d(5t^2 - 4t + 5)}{dt} = 10t - 4$

At $t = 2$ s

$v = 10(2) - 4 = 16 \text{ m s}^{-1}$

MOTION IN TWO DIMENSIONS

38. $H_{max}^{max} = \dfrac{v^2}{2g} = 136 \text{ m}$

$R_{max}^{matho} = \dfrac{v^2}{g} = 2H_{max}$

$= 2(136)$

$= 272 \text{ m}$

39. $\text{Range } = \dfrac{u^2 \sin 2\theta}{g}$

Range for projection angle " α "

$R_1 = \dfrac{u^2 \sin 2\alpha}{g}$

Range for projection angle " β "

$R_2 = \dfrac{u^2 \sin 2\beta}{g}$

$\alpha + \beta = 90^\circ$ (Given)

$\Rightarrow \dfrac{\beta = 90^\circ - \alpha}{u^2 \sin 2(90^\circ - \alpha)}$

$R_2 = \dfrac{u^\circ}{g}$

$R_2 = \dfrac{u^2 \sin(180^\circ - 2\alpha)}{g}$

$R_2 = \dfrac{u^2 \sin 2\alpha}{g}$

$$\Rightarrow \frac{R_1}{R_2} = \frac{\left(\frac{u^2 \sin\ 2\alpha}{g}\right)}{\left(\frac{u^2 \sin\ 2\alpha}{g}\right)} = \frac{1}{1}$$

40. $\frac{KE_{POP}}{KE_{top}} = \frac{\frac{1}{2}M(u)^2}{\frac{1}{2}M(u\cos\ 30°)^2} = \frac{4}{3}$

41. Use $\Delta L = \int_0^t \tau dt$

$$L_0 = \int_0^2 mg\ (v_x t)dt$$

$$= mgv_x \frac{t^2}{2} = (0.1)(10)(10\sqrt{2})\frac{2^2}{2}$$

$$= 20\sqrt{2}$$

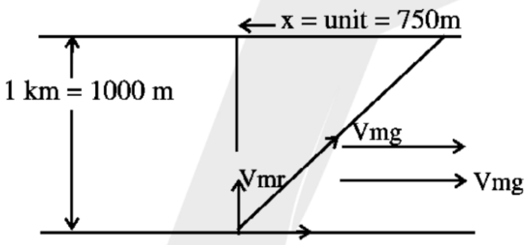
$$= \sqrt{800}\ \text{kg m}^2/\text{s}$$

42. $u\cos\ \theta = \frac{\sqrt{3}u}{2} \Rightarrow \cos\ \theta = \frac{\sqrt{3}}{2}$

$$\Rightarrow \theta = 30°$$

$$T = \frac{2u\sin\ 30°}{g} = \frac{u}{g}$$

43. time to cross the river width $\omega = 1000$ m



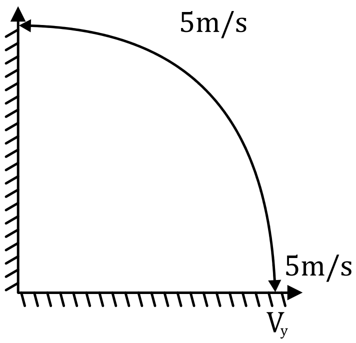
is $= \frac{1\text{ km}}{4\text{ km/h}}$

Drift $x = Vm/g \times t$

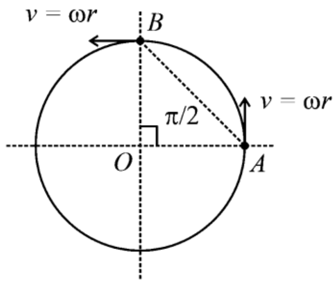
Where Vm/g is velocity of River w.r. to ground.

44. $v_y = \sqrt{2gh} = \sqrt{200}$

$$v_{net} = \sqrt{25 + 200} = 15\ \text{m/s}$$



45. In circular motion with constant speed, magnitude of instantaneous velocity, $v_{ins} = \omega r$.



Time taken to turn by 90° will be $= \frac{\frac{\pi}{2}}{\omega} = \frac{\pi}{2\omega}$.

Displacement will be equal to the length of AB $= \sqrt{r^2 + r^2} = r\sqrt{2}$ Average velocity

$$v_{avg} = \frac{\text{displacement}}{\text{time}}$$

$$= \frac{r\sqrt{2} \times 2\omega}{\pi} = \frac{2\sqrt{2}}{\pi}\omega r$$

Hence, required ratio $\frac{v_{ins}}{v_{avg}} = \frac{\omega r \pi}{2\sqrt{2}\omega r} = \frac{\pi}{2\sqrt{2}}$

Therefore, x = 2.

46. Horizontal range of a projectile from a horizontal plane is given by, $R = \frac{u^2 \sin 2\theta}{g}$. Clearly for maximum range, $\sin 2\theta = 1$.

Therefore, $\theta = 45°$.

47. The angular speed (ω) of the child can be calculated as follows:

$$\omega = \frac{2\pi}{3.14}$$

$= 2\text{rads}^{-1}$

The formula to calculate the centrifugal force on the child is given by

$F_r = m\omega^2 r$

Substitute the values of the known parameters into equation (1) to calculate the required centrifugal force.

$F_r = 5 \text{ kg} \times (2\text{rads}^{-1})^2 \times 2 \text{ m}$

$= 40 \text{ N}$

48. The range (R_1) of the first projectile is given by

$$R_1 = \frac{u_1^2 \sin 2\theta_1}{g} \quad \text{... (1)}$$

The range (R_2) of the second projectile is given by

$$R_2 = \frac{u_2^2 \sin 2\theta_2}{g} \quad \text{... (2)}$$

Divide equation (1) by equation (2) and simplify to obtain the ratio of the ranges.

$$\frac{R_1}{R_2}=\frac{\frac{u_1^2\sin\ 2\theta_1}{g}}{\frac{u_2{}^2\sin\ 2\theta_2}{g}}$$

$$=\frac{u_1{}^2\sin\ 2\theta_1}{u_2{}^2\sin\ 2\theta_2}\ldots$$

Substitute the values of the known parameters into equation (3) to calculate the required ratio.

$$\frac{R_1}{R_2}=\frac{40^2\times\sin\ 60^\circ}{60^2\times\sin\ 120^\circ}$$

$$=\frac{4}{9}$$

49. The trajectory of a projectile is parabolic in nature. At maximum height, it can be written that

$$\frac{dy}{dx}=0$$

Substitute the expression for trajectory into equation (1) and solve to calculate the horizontal distance, for which the projectile attains the maximum height.

$$\frac{d}{dx}\left[x-\frac{x^2}{20}\right]=0$$

$$\Rightarrow 1-\frac{x}{10}=0$$

$$\Rightarrow x=10\ \text{m}$$

Substitute the value of x in the given expression for the trajectory to obtain the maximum height.

$$y=\left(10-\frac{10^2}{20}\right)\text{m}$$

$$=5\ \text{m}$$

50. The data given is

$\theta_1=15^\circ$

$\theta_2=45^\circ$

$R_1=50\ \text{m}$

The formula for range of a projectile is given by

$$R=\frac{u^2\sin\ 2\theta}{g}\ldots(i)$$

Substituting the values in equation (i)

$$R_1=\frac{u^2\sin\ 2\theta_1}{g}$$

$$\Rightarrow 50=\frac{u^2\sin\ 30^\circ}{g}$$

$$\Rightarrow \frac{u^2}{g} = 100$$

Let the new range be R_2. The magnitude of the range is

$$R_2 = \frac{u^2 \sin 2\theta_2}{g}$$

$$\Rightarrow R_2 = \left(\frac{u^2}{g}\right) \sin 90^\circ$$

$$\Rightarrow R_2 = 100 \text{ m}$$

51. The maximum height is given by, $H = \frac{u^2 \sin^2 \theta}{2g}$. The given data is

$\theta_1 = 30^\circ$

$\theta_2 = 60^\circ$

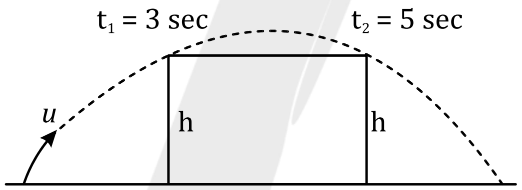
$u_1 = u_2$

Therefore,

$$\frac{(H_1)_{max}}{(H_2)_{max}} = \frac{u_1^2 \sin^2 \theta_1}{u_2^2 \sin^2 \theta_2} = \left(\frac{\sin 30^\circ}{\sin 60^\circ}\right)^2$$

$$= \left(\frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}}\right)^2 = \frac{1}{3}$$

52. The time taken to reach maximum height is



$$T' = \frac{u \sin \theta}{g}.$$

The total time of flight is $T = (3 + 5)\text{s} = 8$ s. Hence,

$$T' = \left(\frac{T}{2}\right) = \left(\frac{3+5}{2}\right) = \left(\frac{u \sin \theta}{g}\right)$$

$$\Rightarrow 4 = \frac{(u) \times \frac{1}{2}}{10}$$

$$\Rightarrow u = 80 \text{ m s}^{-1}$$

53. It is given that $v = 40 \text{ m s}^{-1}$.

The components of the velocity will be

$$v_y = v \sin 30^\circ = \frac{40 \text{ m s}^{-1}}{2} = 20 \text{ m s}^{-1}$$

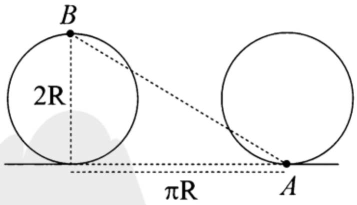
$v_x = v\cos\ 30° = \frac{40\sqrt{3}}{2}\ m\ s^{-1} = 20\sqrt{3}\ m\ s^{-1}$

The time taken to reach maximum height is $T = \frac{u\sin\ 30°}{g} = \frac{20\ m\ s^{-1}}{10\ m\ s^{-2}} = 2\ s$

Hence, at $T = 2$ s, as $v_y = 0\ m\ s^{-1}$ therefore $v_{net} = v_x = 20\sqrt{3}\ m\ s^{-1}$.

54. The displacement is the hypotenuse of the diameter and the distance travelled for half the rotation πR. By using Pythagoras theorem,

Displacement of A $= \sqrt{(2R)^2 + (\pi R)^2} = R\sqrt{\pi^2 + 4}$



55. The centripetal force is given by

$F = \frac{Mv^2}{R} = MR\omega^2$

The given data is

$M = 200$ kg

$R = 70$ m

$\omega = 0.2 rads^{-1}$

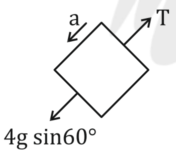
The magnitude of the force is

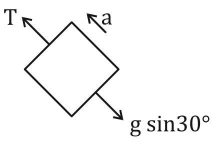
$F = 200 \times 70 \times 0.2^2$ N

$\Rightarrow F = 560$ N

Newtons Laws of Motion

56. $4\ g\sin 60° - T = 4a \ldots .(1)$





$T - g\sin 30° = a \ldots .(2)$

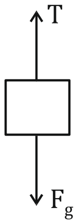
Solving (1) and (2) we get.

$20\sqrt{3} - T = 4\ T - 20$

$T = 4(\sqrt{3} + 1)N$

57. Statement-1

When elevator is moving with uniform speed $T = F_g$



Statement-2

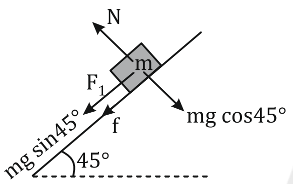
When elevator is going down with increasing speed, its acceleration is downward.

Hence

$$W - N = \frac{W}{g} \times a$$

$N = W\left(1 - \frac{a}{g}\right)$ i.e. less than weight.

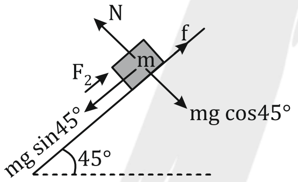
58.



$$F_1 = mg\sin\ 45^\circ + f = mg\sin\ 45^\circ + \mu N$$

$$F_1 = \frac{mg}{\sqrt{2}} + \mu mg\cos\ 45^\circ$$

$$F_1 = \frac{mg}{\sqrt{2}}(1 + \mu)$$



$$F_2 = mg\sin\ 45^\circ - f = mg\sin\ 45^\circ - \mu N$$

$$= \frac{mg}{\sqrt{2}}(1 - \mu)$$

$$F_1 = 2\ F_2$$

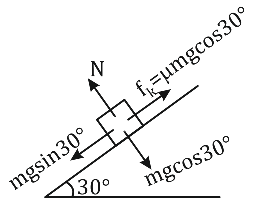
$$\frac{mg}{\sqrt{2}}(1 + \mu) = 2\frac{mg}{\sqrt{2}}(1 - \mu)$$

$$1 + \mu = 2 - 2\mu$$

$$\mu = 1/3 = 0.33$$

59. $Mg\sin\ 30^\circ - \mu mg\cos\ 30^\circ = ma$

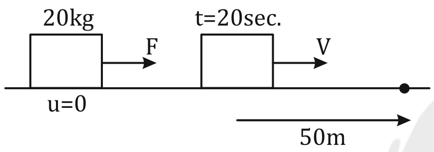
$$\frac{g}{2} - \frac{\sqrt{3}}{2} \cdot \mu g = \frac{g}{4}$$



$$\frac{\sqrt{3}}{2}\mu = \frac{1}{2}$$

$$\mu = \frac{1}{2\sqrt{3}}$$

60. $50 = V \times 10$



$V = 5\ m/s$

$V = 0 + a \times 20$

$5 = a \times 20$

$$a = \frac{1}{4}\ m/s^2$$

$$F = ma = 20 \times \frac{1}{4} = 5\ N$$

61. $a_1 = g\sin\theta = g/\sqrt{2}$

$$a_2 = g\sin\theta - Kg\cos\theta = \frac{g}{\sqrt{2}} - \frac{Kg}{\sqrt{2}}$$

$t_2 = nt_1\ \&\ a_1t_1^2 = a_2t_2^2$

$$\frac{g}{\sqrt{2}}t_1^2 = \left(\frac{g}{\sqrt{2}} - \frac{kg}{\sqrt{2}}\right)n^2t_1^2$$

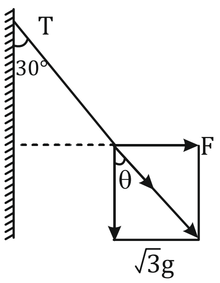
$$K = 1 - \frac{1}{n^2}$$

62. $\left|\frac{d\vec{p}}{dt}\right| = |\vec{F}| \Rightarrow \frac{d\vec{p}}{dt} =$ Slope of curve

max slope (c)

min slope (b)

63. $\theta = 30°$



$\cos\ \theta = \frac{\sqrt{3}\,g}{T}$

$\Rightarrow \frac{\sqrt{3}}{2} = \frac{\sqrt{3}\,g}{T}$

$\Rightarrow T = 20\ N$

64. $a = \omega^2 R = \left(\frac{28\times2\pi}{60}\right)^2 \times 1.8$

$= \left(\frac{56}{60}\times\frac{22}{7}\right)^2 \times 1.8$

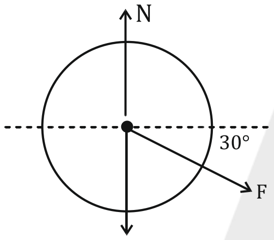
$= \frac{(44)^2}{225}\times 1.8$

$= \frac{1936\times1.8}{225}$

$x = 125$

65. $N = mg + F\sin\ 30^\circ$

$= 700 + 200\times\frac{1}{2} = 800$ newton.



66. $v^2\ = u^2 + 2\ as$

$= 2^2 + 2(2)(6)$

$= 4 + 24 = 28$

$KE = \frac{1}{2}mv^2$

$= \frac{1}{2}(500)28$

$= 7000\ J$

$= 7\ Kj$

67. $S = ut + \frac{1}{2}at^2$

$50 = 0 + \frac{1}{2}\times a\times 100$

$a = 1\ m/s^2$

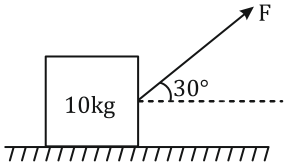
$F - \mu mg = ma$

$30 - \mu\times 50 = 5\times 1$

$50\mu = 25$

$\mu = \frac{1}{2}$

68. $N = Mg - F\text{Sin } 30°$



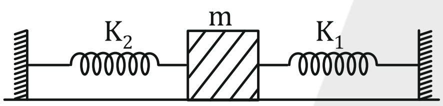
$= mg - \frac{F}{2} = 100 - \frac{F}{2} = \frac{200-F}{2}$

$F\text{Cos } 30° = \mu N$

$\sqrt{3}\frac{F}{2} = 0.25 \times \left(\frac{200-F}{2}\right)$

$4\sqrt{3}\,F = 200 - F$

$F = \frac{200}{4\sqrt{3}+1} = 25.22$

69. 

For the given combination, if the block is displaced towards right, elongation in one spring will be equal to the compression in other and the direction of force due to both spring will be the same and restoring in nature. Hence, both springs are effectively in parallel combination. Therefore,

$\Rightarrow K_{eff} = K_1 + K_2$ and

$\Rightarrow T = 2\pi\sqrt{\frac{m}{K_{eff}}} = 2\pi\sqrt{\frac{m}{K_1+K_2}}$

70. Centripetal force will be provided by the spring force. Let the elongation in the spring be x, then we can write

$\Rightarrow kx = m\omega^2(r+x)$

$\Rightarrow 7.5x = 2.5(0.2+x)$

$\Rightarrow x = \frac{0.5}{5} = 0.1$

Therefore, required tension will be $T = kx = 0.75$ N.

71. The equation for velocity is

$\vec{v} = (2t\hat{i} + 3t^2\hat{j})$

Thus, the acceleration is

$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d(2t\hat{i}+3t^2\hat{j})}{dt} = (2\hat{i} + 6t\hat{j})$

At $t = 1$ s, $\vec{a} = 2\hat{i} + 6\hat{j}$

The force is written as $\vec{F} = m\vec{a} = \frac{2\hat{i}+6\hat{j}}{2} = \hat{i} + 3\hat{j}$

Comparing it with the given value of force, $x = 3$.

72. When a fan is rotating, it acquires some kinetic energy. When the switch is off, because of its motion, it continues rotating for some time until it stops because of the air friction. So, it is due to inertia of motion that fan continues to rotate.

Hence, both A and R are correct and R is the correct explanation of A.

73. The coin will slip when

$\mu mg = m\omega^2 r$

$\Rightarrow \mu g = \omega^2 \times 1\text{ cm} \ldots\ldots (i)$

Now, $\omega' = \frac{\omega}{2}$

$\mu g = \left(\frac{\omega}{2}\right)^2 R \ \ldots (ii)$

Hence we can write from (i) and (ii)

$\omega^2 \times 1\text{ cm} = \frac{\omega^2}{4} R$

$\Rightarrow R = 4\text{ cm}$

74. It is given that

$v = 10\sqrt{x}\text{ m s}^{-1}$

Force is defined by $F = m\frac{dv}{dt} \ \ldots (i)$

Multiplying and dividing equation (i) by dx

$F = m\frac{dv}{dt}\frac{dx}{dx}$

$\Rightarrow F = m\frac{dx}{dt}\frac{dv}{dx} = mv\frac{dv}{dx} \ \ldots (ii)$

Finding the values of the terms in equation (ii)

$\frac{dv}{dx} = \frac{10}{2\sqrt{x}} = \frac{5}{\sqrt{x}}$

$v\frac{dv}{dx} = a = 10\sqrt{x} \times \frac{5}{\sqrt{x}} = 50\text{ m s}^{-2}$

Hence, the force is

$F = \frac{1}{2} \times 50 = 25\text{ N}.$

75. Given, the position vector of the particle is

$\vec{r} = 10t\hat{i} + 15t^2\hat{j} + 7\hat{k}$

The velocity of the particle can be calculated as follows:

$\vec{v} = \frac{d\vec{r}}{dt}$

$= \frac{d}{dt}\left(10t\hat{i} + 15t^2\hat{j} + 7\hat{k}\right)$

$= 10\hat{i} + 30t\hat{j}$

And, the acceleration of the particle is given by

$\vec{a} = \frac{d\vec{v}}{dt}$

$= \frac{d}{dt}(10\hat{i} + 30t\hat{j})$

$= 30\hat{j}$

Hence, the force on the particle can be written as

$\vec{F} = m\vec{a}$

$= 30m\hat{j}$

WORK POWER ENERGY

76. $\frac{1}{2} \times 2 \times v^2 = 10000$

$\Rightarrow v^2 = 10000$

$\Rightarrow v = 100 \text{ m/s}$

$\Rightarrow v = at = a \times 5 = 100$

$\Rightarrow a = 20 \text{ m/s}^2$

$F = ma = 2 \times 20 = 40 \text{ N}$

77. $\vec{F} = t\hat{i} + 3t^2\hat{j}$

$\frac{md\vec{v}}{dt} = t\hat{i} + 3t^2\hat{j}$

$m = 1\text{kg}, \int_0^v dv = \int_0^t t dt\hat{i} + \int_0^t 3t^2 dt\hat{j}$

$\vec{v} = \frac{t^2}{2}\hat{i} + t^3\hat{j}$

Power $= \vec{F} \cdot \vec{V} = \frac{t^3}{2} + 3t^5$

At $t = 2$, power $= \frac{8}{2} + 3 \times 32 = 100$

78. $F\cos\theta = ma$

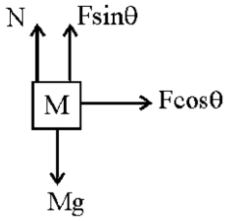
$2\cos(kx) = \frac{mvdv}{dx}$

$\int_0^v vdv = 2\int_0^x \cos(kx)dx$

$$\frac{mv}{2} = \frac{2}{k}\sin\ kx$$

$$K \cdot E. = \frac{2}{k}\sin\ \theta$$

$n = 2$



79. Displacement is 8^{un}sec.

$$S_8 = 0 + \frac{1}{2} \times 10 \times (2 \times 8 - 1)$$

$$S_8 = 5 \times 15$$

$$\Delta U = 0.4 \times 10 \times 5 \times 15$$

$$\Delta U = 20 \times 15 = 300$$

80. $\frac{1}{2}mV^2 = Pt$

$$V = \sqrt{\frac{2Pt}{m}}$$

$$\frac{dx}{dt} = \sqrt{\frac{2Pt}{m}}$$

$$x = \sqrt{\frac{2P}{m}}\frac{2}{3}\left[t^{3/2}\right]_0^4$$

$$x = \frac{16\sqrt{P}}{3} = \frac{1}{3} \times 16\sqrt{P}$$

$\alpha = 4$

81. $W = \vec{F} \cdot (\vec{r}_f - \vec{r}_i)$

$$= (5\hat{i} + 2\hat{j} + 7\hat{k}) \cdot ((5\hat{i} - 2\hat{j} + \hat{k}) - (2\hat{i} + 3\hat{j} - 4\hat{k}))$$

$W = 40$ J

82. $F = 5 + 3y^2$

$$W = \int_2^5 (5 + 3y^2)dy$$

$$= \left[5y + \frac{3y^3}{3}\right]_2^5$$

$= 132$ J

83. The data given is

$$U = \frac{1}{2}m\omega^2 r^2$$

From Bohr's quantization,

$$mvr = \frac{nh}{2\pi}$$

$$\Rightarrow v^2 = \left(\frac{nh}{2\pi rm}\right)^2 \quad \ldots (i)$$

In an orbit the value of the kinetic energy is half of the potential energy,

$$K = \frac{m\omega^2 r^2}{4}$$

Substituting the value of equation (i) in the kinetic energy,

$$K = \frac{1}{2}m\left(\frac{n^2h^2}{4\pi^2 m^2 r^2}\right) = \frac{m\omega^2 r^2}{4}$$

$$\Rightarrow \frac{n^2h^2}{2\pi^2 m^2 \omega^2} = r^4$$

$$\Rightarrow r \propto \sqrt{n}$$

84. The given ratio of the velocities is $\frac{v_1}{v_2} = 4$

The percentage loss in kinetic energy is

$$\frac{\frac{mv^2 1}{2} - \frac{mv^2 2}{2}}{\frac{mv^2 1}{2}} \times 100 = \frac{x}{4}$$

$$\Rightarrow \frac{v^2{}_1 - v^2 2}{v^2{}_1} = \frac{x}{400} \Rightarrow 1 - \frac{v_{2\ 2}}{v^2} = \frac{x}{400}$$

$$\Rightarrow 1 - \frac{1}{16} = \frac{x}{400}$$

$$\Rightarrow x = 375$$

85. The relation of kinetic energy and momentum is given by

$$K = \frac{p^2}{2\,m}$$

The new value of momentum is $p' = 1.5p$

The new value of kinetic energy is

$$\Rightarrow K' = \frac{(1.5p)^2}{2\,m} = 2.25K$$

Hence, the percentage increase in kinetic energy is

$$\frac{K' - K}{K} \times 100 = \frac{2.25K - K}{K} = 125\%$$

86. Let the mass of the bullet be m and mass of the block be M. It is given that $V = 400\ m\ s^{-1}$. By conservation of momentum,

$mV = (m + M)v$

$\Rightarrow 0.1 \times 400 = (0.1 + 3.9)v$

$\Rightarrow v = 10\ m\ s^{-1}$

The force acting on the block

$f = \mu m' g$

$\Rightarrow a = \mu g$

Using $v'^2 - v^2 = -2aS$

$\Rightarrow 0^2 - 10^2 = 2(-\mu g)(S)$

$\Rightarrow \mu = \dfrac{100}{2 \times 10 \times 20} = 0.25$

87. The formula to calculate the change on momentum of the object is given by

$p_f - p_i = \int Fdt$

Substitute the values of the known parameters into equation (1) and solve to calculate the final momentum of the object.

$p_f - 10\ kg\ m\ s = 2\ N \times 5\ s$

$\Rightarrow p_f = (10 + 10) kgms^{-1}$

$= 20\ kg\ m\ s$

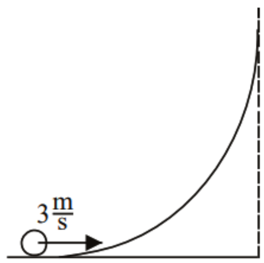
The formula to calculate increase in kinetic energy (ΔK) of the particle is given by

$\Delta K = \dfrac{1}{2m}\left(p_f^2 - p_i^2\right)$

Substitute the values of the known parameters into equation (2) to calculate the required increase in kinetic energy.

$\Delta K = \dfrac{1}{2 \times 5\ kg} \times (20^2 - 10^2) kg^2\ m^2\ s^{-2} = 30\ J$

88. The total kinetic energy of the ball at the bottom of the curve can be calculated as follows:



$K = \dfrac{1}{2}Mv^2 + \dfrac{1}{2} \cdot \dfrac{2}{3}MR^2\omega^2$

$= \frac{1}{2}Mv^2 + \frac{1}{3}M(R\omega)^2$

$= \frac{1}{2}Mv^2 + \frac{1}{3}Mv^2 \; [\because v = \omega R]$

$= \frac{5}{6}Mv^2 \; \dots (1)$

The formula to calculate the potential energy acquired by the ball when it attains the maximum height is given by

$U = Mgh$

Equate equation (1) and (2) and simplify to obtain the required height.

$Mgh = \frac{5}{6}Mv^2$

$\Rightarrow h = \frac{5v^2}{6g}$

Substitute the values of the known parameters into equation (3) to calculate the required height attained by the ball.

$h = \frac{5 \times (3 \text{ m s}^{-1})^2}{6 \times 10 \text{ m s}^{-2}}$

$= 0.75 \text{ m} \times \frac{100 \text{ cm}}{1 \text{ m}}$

$= 75 \text{ cm}$

89. The data given is

$r = 15 \text{ cm}$

$m = 1 \text{ kg}$

$v = 22 \text{ m s}^{-1}$

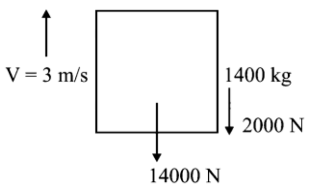
Total work done by the tube on the block is equal to loss in mechanical energy,

$w = \frac{1}{2}mv^2 + mgh$

$\Rightarrow w = \frac{1}{2} \times 1 \times (22)^2 + 1 \times 10 \times 0.3$

$\Rightarrow w = 245 \text{ J}$

90. The maximum force acting is, $F_{max} = (14000 + 2000)N = 16000 \text{ N}$



The formula for power is given by, $P = F_{max}v$.

It is given, $v = 3\ m\ s^{-1}$.

$\Rightarrow P = 16000\ N \times 3\ m\ s^{-1} = 48000\ W = 48\ kW$

$\Rightarrow W = 48\ kW$

91. The formula to calculate the work done by a variable force is given by

$$W = \int_{x_1}^{x_2} F dx$$

Substitute the values of the known parameters into equation (1) to calculate the required work done.

$$W = \int_0^4 (2 + 3x)dx$$

$$= \left[2x + \frac{3}{2}x^2\right]_0^4$$

$= [8 + 24]J$

$= 32\ J$

92. It is given,

$v = 80\ km\ h^{-1}$

$m = 500\ kg$

$S = 4\ km$

$\mu = 0.04$

The power due to friction is

$P_f = Fv$ where $F = \mu mg$

$$= \left((0.04)(500 \times 9.8) \times 80 \times \frac{5}{18}\right) W$$

Hence, the work done by the engine in overcoming the friction is

$$W = P_f t = P_f \frac{d}{v}$$

$$\Rightarrow W = 20 \times 9.8 \times 80 \times \frac{5}{18} \times \frac{4 \times 10^3}{80 \times \frac{5}{18}} = 784\ kJ$$

93. The work done in raising the water is $W = mgh$.

Power is $P = \frac{W}{t}$

Let the powers of both the motors be P_1&P_2 respectively. Hence, the ratio of the powers is,

$$\frac{P_1}{P_2} = \frac{\frac{300 \times g \times 100}{5 \times 100}}{\frac{50 \times g \times 100}{2 \times 60}} = \frac{60}{25}$$

$$\Rightarrow \frac{3\sqrt{x}}{\sqrt{x}+1} = 2.4$$

$$\Rightarrow 3\sqrt{x} = 2.4\sqrt{x} + 2.4$$

$$\Rightarrow x = 16$$

94. The kinetic energy is given by,

$$K = \frac{1}{2}mv^2$$

Differentiating the above equation,

$$dK = \frac{1}{2}(v^2dm + 2mvdv)$$

Dividing by K,

$$\Rightarrow \frac{dK}{K} = \frac{dm}{m} + \frac{2dv}{v}$$

$$\Rightarrow \frac{dK}{1000} = \frac{0.5}{5} + \frac{2 \times 0.4}{20}$$

$$\Rightarrow dK = 100 + 40 = 140$$

$$\Rightarrow KE = (1000 \pm 140)J$$

95. The kinetic energy of a moving particle is given by

$$E = \frac{1}{2}mu^2 \ \dots(1)$$

When the final velocity of the particle becomes twice the initial velocity, the kinetic energy of the particle can be written as

$$nE = \frac{1}{2}m\{(2u)^2 - u^2\}$$

$$= \frac{1}{2}m(3u^2) \dots (2)$$

Divide equation (2) by equation (1) and solve to calculate the value of n.

$$\frac{nE}{E} = \frac{\frac{1}{2}m(3u^2)}{\frac{1}{2}mu^2}$$

$$\Rightarrow n = 3$$

96. Using conservation of mechanical energy,

$$U_i + K_i = U_f + K_f$$

$$\Rightarrow -\frac{GMm}{2R} + 0 = -\frac{GMm}{R} + \frac{1}{2}mv^2$$

$$\Rightarrow \frac{1}{2}mv^2 = \frac{GMm}{2R}$$

$$\Rightarrow v = \sqrt{\frac{GM}{R}} = \sqrt{gR}$$

97. Work done is $W = \int F dx$

It is given that

$F = 5x$ N

So, work done is

$$W = \int_2^4 5x dx = \left.\frac{5x^2}{2}\right|_2^4$$

$$\Rightarrow W = \frac{5(16-4)}{2} = 30 \text{ J}$$