Nishant Jindal

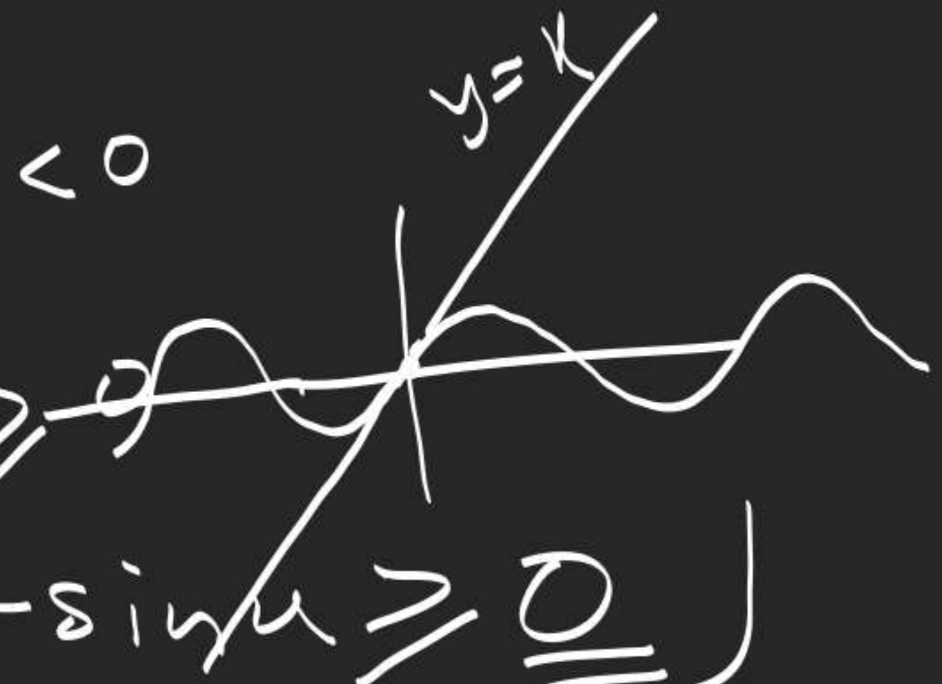
$$\frac{3\ln^2(\sec x+\tan x)}{\sec x+\tan x}\left(\sec x\tan x+\sec^2 x\right)$$

$$= 3\ln^2(\sec x+\tan x)\sec x > 0.$$

S: (c)

Nishant Jindal

4. (c) $f_2 \circ f_1 = f_1^2 = \begin{cases} x^2 & x < 0 \\ e^{2x} & x \geq 0 \end{cases}$



7 (c)

$f'(x) = x(\underbrace{2-\cos x}_{\geq 1}) - \sin x \geq x - \sin x \geq 0$

$x \geq 0$

4 → leave

7. $f(x) = x(x - \sin x)$

$= x^2\left(1 - \frac{\sin x}{x}\right)$

odd

$x \to \infty,\ y \to \infty$

$x = 0,\ y = 0$

$f'(x) = \boxed{2x - x\cos x} - \sin x$

Nishant Jindal

17. $(f+g)(4) = 2+5$

$(\quad)(5) = 1+6$

$(f+g)(3) = 1+4$

$(2) = 1+3$

$\boxed{x=6}$

Nishant Jindal

Find

1. $\sin\left(\underbrace{2\sin^{-1}\frac{3}{5}}_{\theta\in[0,\frac{\pi}{2}]}\right) = 2\sin\theta\cos\theta = 2\times\frac{3}{5}\times\frac{4}{5} = \frac{24}{25}$

2. $\sin\left(\underbrace{\arcsin\frac{3}{5}}_{\theta_1\in(0,\frac{\pi}{2})} - \underbrace{\arccos\frac{3}{5}}_{\theta_2\in(0,\frac{\pi}{2})}\right) = \sin\theta_1\cos\theta_2 - \sin\theta_2\cos\theta_1$

$= \frac{3}{5}\times\frac{3}{5} - \frac{4}{5}\times\frac{4}{5} = \frac{-7}{25}$

3. $\tan\left(\underbrace{2\tan^{-1}\frac{1}{5}}_{\theta\in(0,\pi/2)} - \frac{\pi}{4}\right) = \frac{\tan 2\theta - 1}{1+\tan 2\theta} = \frac{\frac{5}{12}-1}{1+\frac{5}{12}} = \boxed{-\frac{7}{17}}$

$\tan 2\theta = \frac{\frac{2}{5}}{1-\frac{1}{25}} \Leftarrow \tan\theta = \frac{1}{5}$

$= \frac{10}{24} = \frac{5}{12}$

4. $\tan\left(\underbrace{\frac{1}{2}\cos^{-1}\frac{\sqrt{5}}{3}}_{\theta\in(0,\frac{\pi}{2})}\right) = \frac{1-\cos\theta}{\sin\theta}$

$= \frac{1-\frac{\sqrt{5}}{3}}{\frac{2}{3}}$

$\frac{3-\sqrt{5}}{2}$

Nishant Jindal

2. Find domain and range of

$[\cdot] = G.I.F$

$\{\cdot\} = FPF.$

(i) $f(x) = \cos^{-1}[x]$

$D_f = [-1, 2)$

$R_f = \{0, \frac{\pi}{2}, \pi\}$

(ii) $f(x) = \cos^{-1}\underbrace{\{x\}}_{[0,1)}$

$D_f = R$

$R_f = (0, \frac{\pi}{2}]$

(iii) $f(x) = \cot^{-1}(\operatorname{sgn} x)$

$D_f = R$

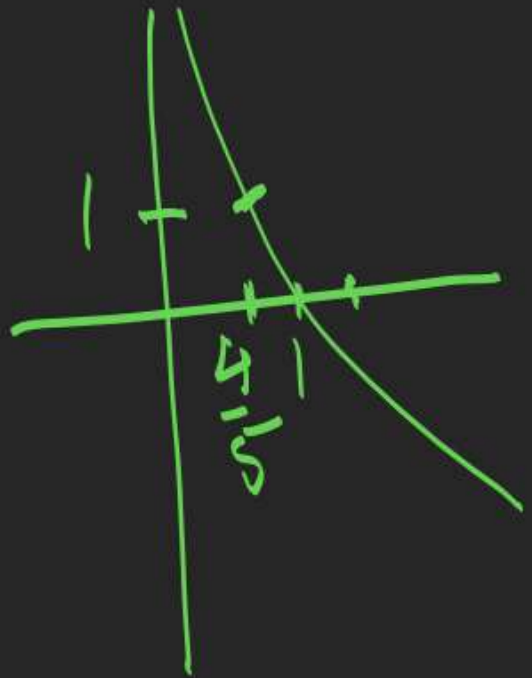
$R_f = \{\frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}\}$

(iv) $f(x) = \cot^{-1} \overbrace{\log_{\frac{4}{5}} \underbrace{(5x^2 - 8x + 4)}_{[\frac{4}{5}, \infty)}}^{(-\infty, 1]}$

$\frac{16}{5} - \frac{32}{5} + 4 = 4 - \frac{16}{5}$

$D_f = R$

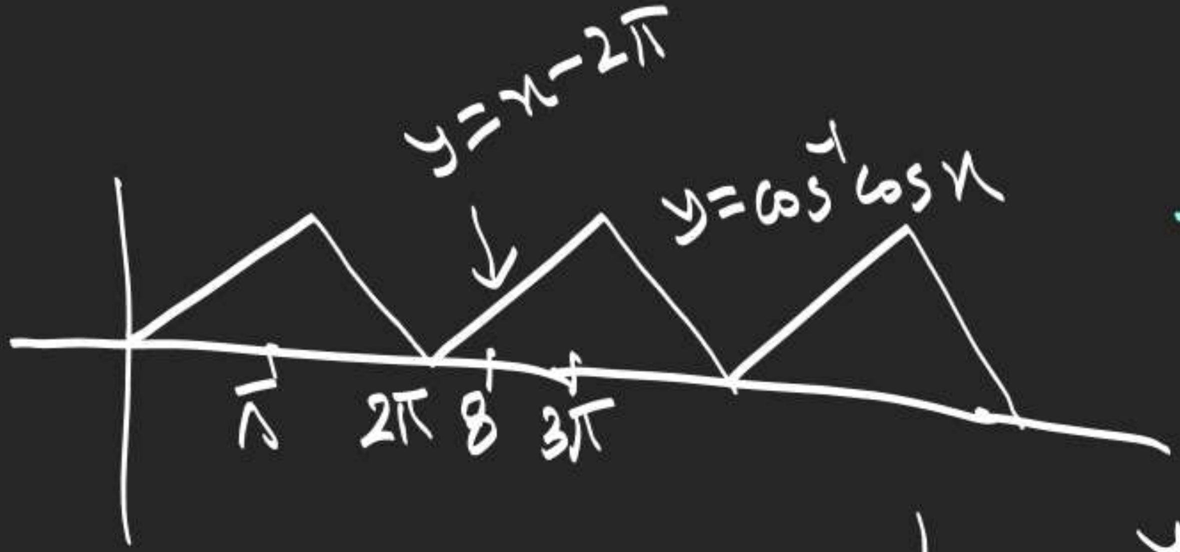
$R_f = [\frac{\pi}{4}, \pi)$

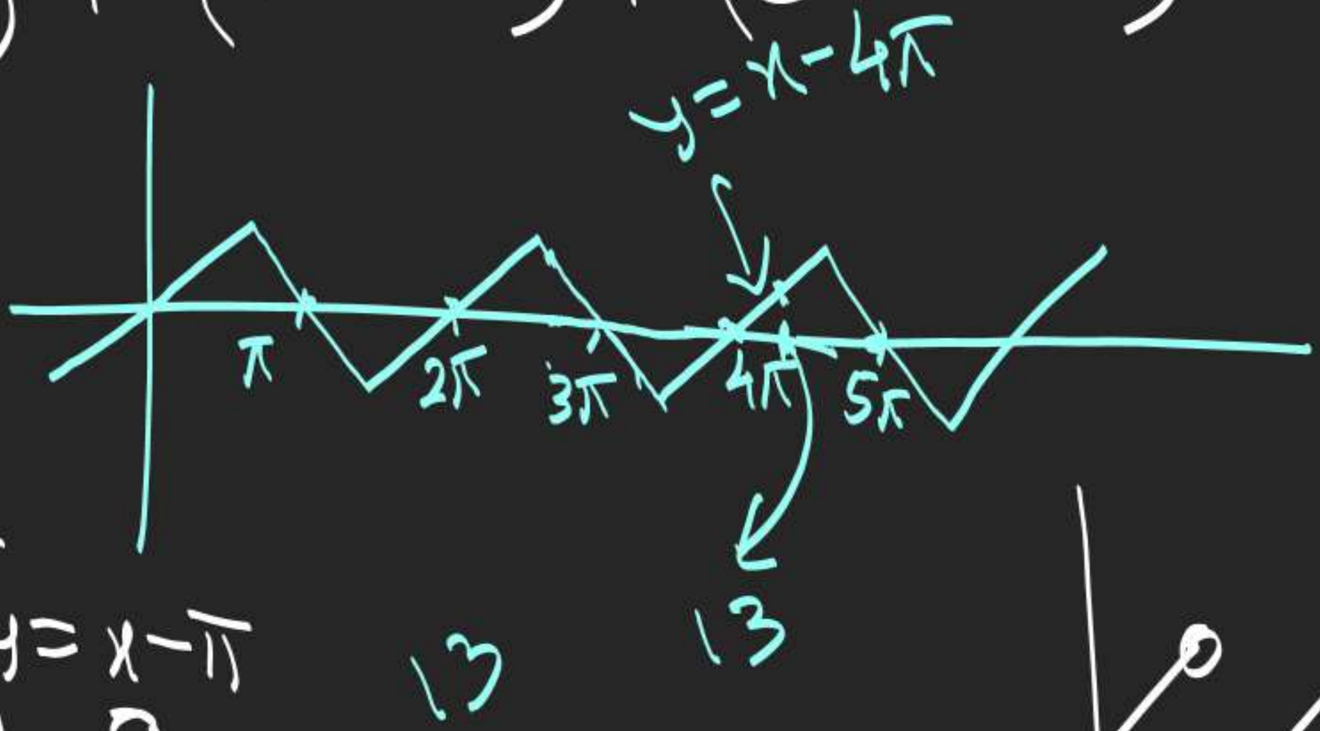


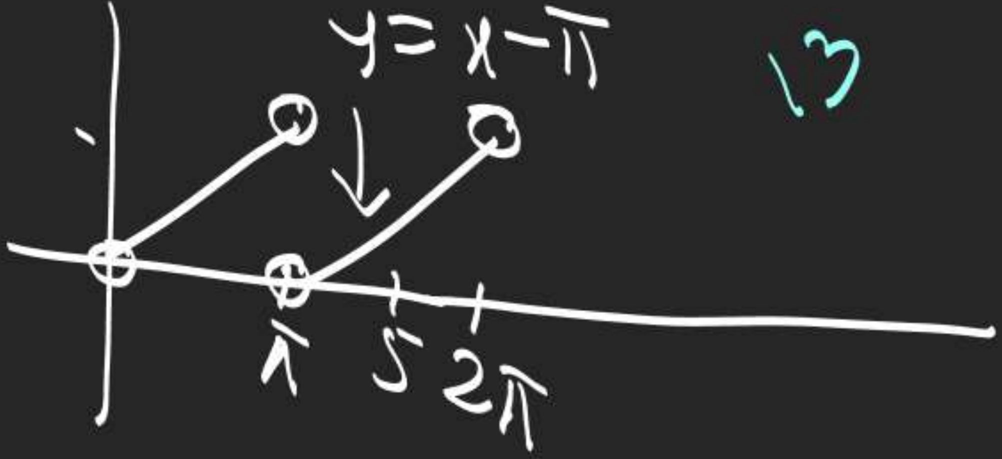
1. Simplify

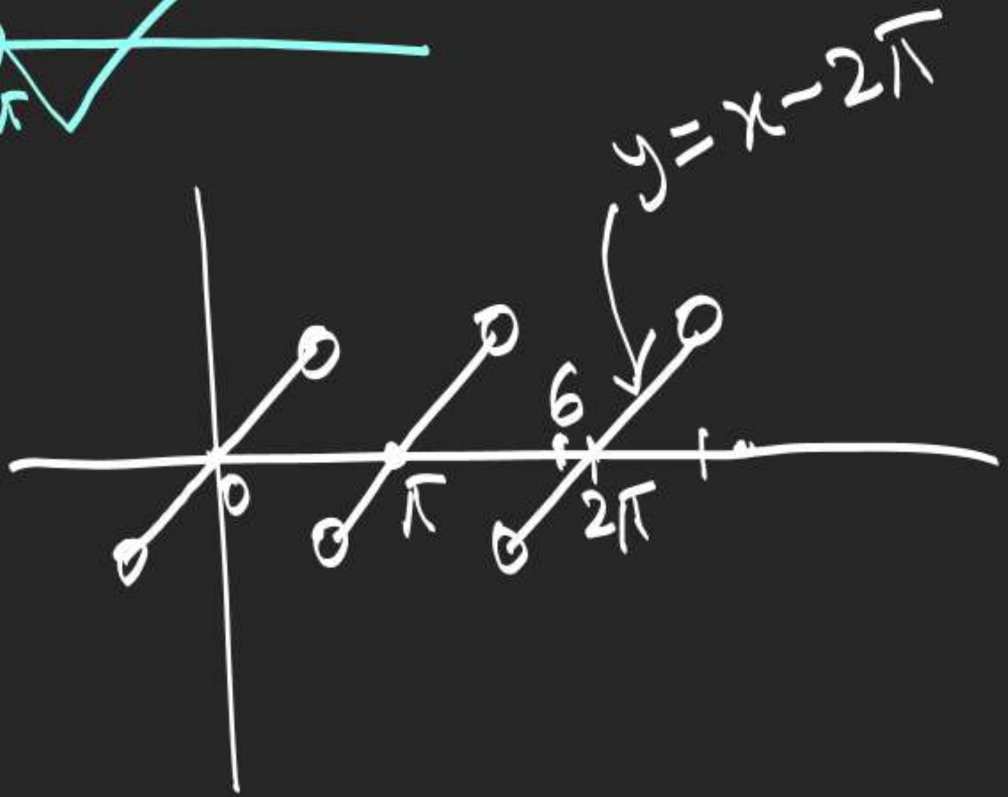
$$\cos^{-1}\cos 8 + \sin^{-1}\sin(13) + \cot^{-1}\cot 5 + \tan^{-1}\tan 6$$

$$= (8-2\pi) + (13-4\pi) + (5-\pi) + (6-2\pi) = 32 - 9\pi$$







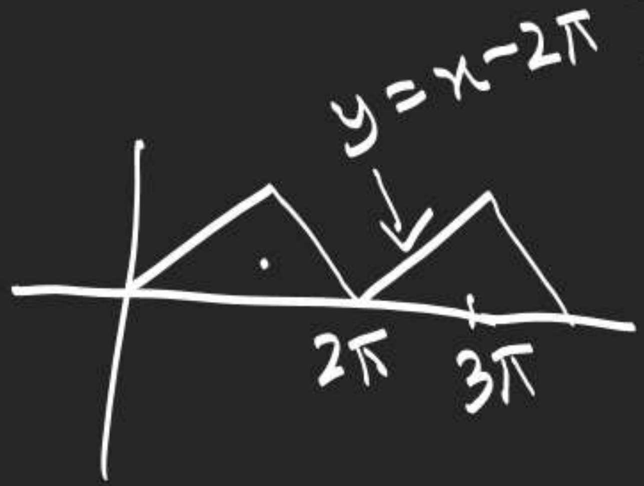


Nishant Jindal

$\sin^{-1}(-x) = -\sin^{-1}x \qquad |x| \le 1 \qquad \cos^{-1}(-x) = \pi - \cos^{-1}x$

$\tan^{-1}(-x) = -\tan^{-1}x \qquad x \in R \qquad \cot^{-1}(-x) = \pi - \cot^{-1}x$

$\operatorname{cosec}^{-1}(-x) = -\operatorname{cosec}^{-1}x \qquad |x| \ge 1 \qquad \sec^{-1}(-x) = \pi - \sec^{-1}x$



P.T. $\cos^{-1}(-x) = \pi - \cos^{-1}x \quad |x| \le 1$

$\cos^{-1}x = \theta \;, \theta \in [0, \pi]$ ✓

$\cos\theta = x \Rightarrow -\cos\theta = -x$

$(3\pi - \theta) - 2\pi = \cos^{-1}(\cos(\underbrace{3\pi - \theta}_{\in [2\pi, 3\pi]})) = \cos^{-1}(-\cos\theta) = \cos^{-1}(-x)$

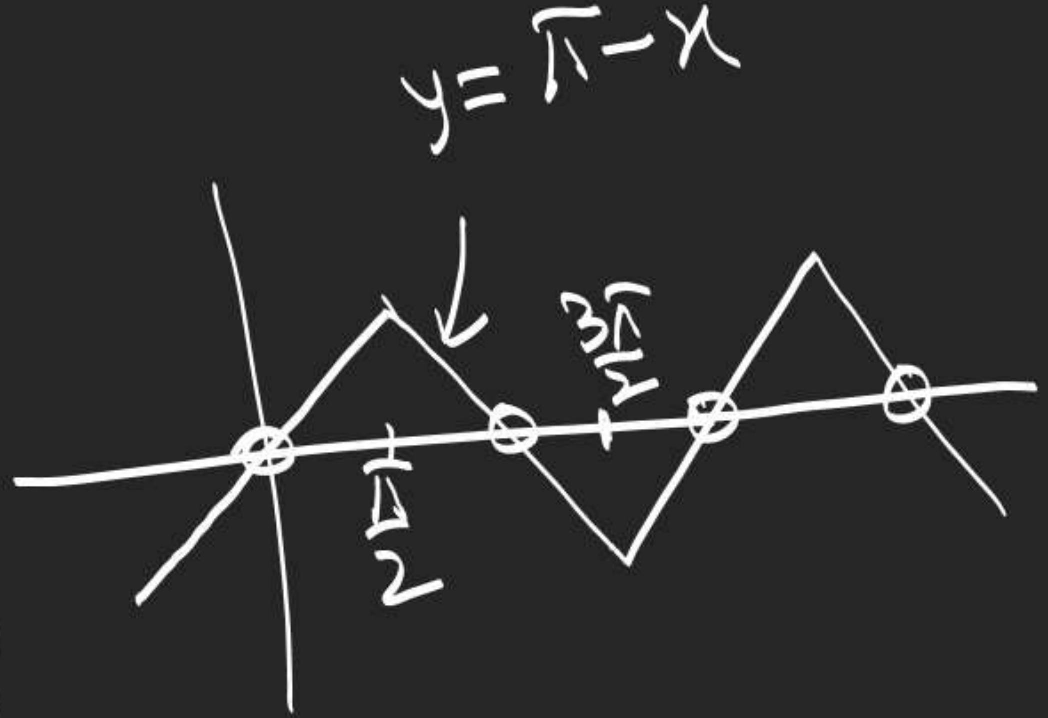
$= \boxed{\pi - \theta}$

$\frac{1}{2} = \frac{1}{2}$

$\cos^{-1}\frac{1}{2} = \cos^{-1}\frac{1}{2}$

Nishant Jindal

$$\boxed{\begin{array}{ll} \sin^{-1}x+\cos^{-1}x=\frac{\pi}{2} & |x|\le 1 \\ \tan^{-1}x+\cot^{-1}x=\frac{\pi}{2} & x\in R \\ \sec^{-1}x+\operatorname{cosec}^{-1}x=\frac{\pi}{2} & |x|\ge 1 \end{array}}$$



$\sec^{-1}x=\theta \quad , \theta\in\left[0,\frac{\pi}{2}\right)\cup\left(\frac{\pi}{2},\pi\right]$

$\sec\theta=x$

$\operatorname{cosec}^{-1}\sec\theta = \operatorname{cosec}^{-1}x = \operatorname{cosec}^{-1}\operatorname{cosec}\left(\underbrace{\frac{\pi}{2}+\theta}_{\left[\frac{\pi}{2},\pi\right)\cup\left(\pi,\frac{3\pi}{2}\right]}\right) = \pi-\left(\frac{\pi}{2}+\theta\right)$

$= \frac{\pi}{2}-\theta$

Nishant Jindal

Solve for x

1. $4\sin^{-1}x+\cos^{-1}x=\frac{3\pi}{4}=3\sin^{-1}x+\frac{\pi}{2}\Rightarrow \boxed{\sin^{-1}x=\frac{\pi}{12}}$

$x=\sin\frac{\pi}{12}$

$=\frac{\sqrt{3}-1}{2\sqrt{2}}$

2. $5\tan^{-1}x+3\cot^{-1}x=\frac{7\pi}{4}$

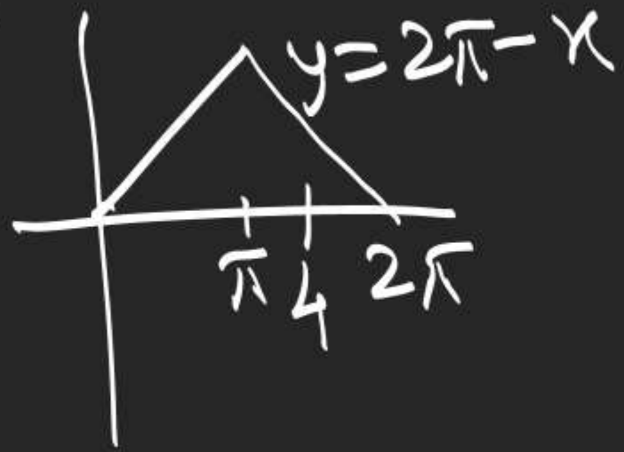
$2\tan^{-1}x+\frac{3\pi}{2}=\frac{7\pi}{4}$

3. $3x^2+8x<2\sin^{-1}\sin(4)-\cos^{-1}\cos 4$

$\tan^{-1}x=\frac{\pi}{8}\Rightarrow \tan\frac{\pi}{8}=x=\sqrt{2}-1$

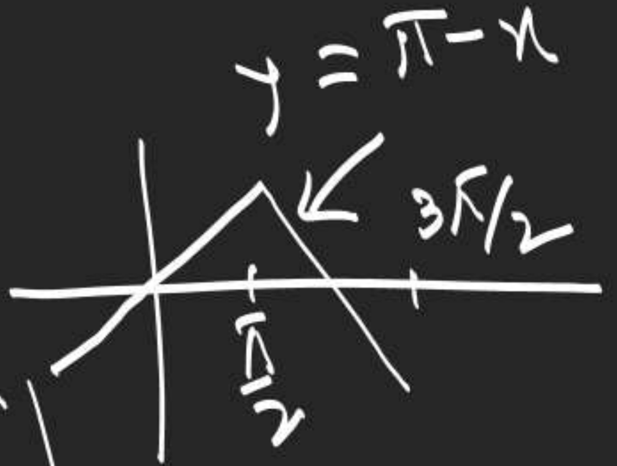
3. $3x^2+8x < 2\sin^{-1}\sin 4 - \cos^{-1}\cos 4$

$2(\pi-4)-(2\pi-4)$



$3x^2+8x+4<0$

$6x+2x$



$(3x+2)(x+2)<0$

$$\boxed{x\in\left(-2,-\frac{2}{3}\right)}$$

Nishant Jindal

$$\operatorname{cosec}^{-1}\left(\frac{1}{x}\right) = \sin^{-1}x \qquad x \in [-1,0) \cup (0,1]$$

$$\sec^{-1}\left(\frac{1}{x}\right) = \cos^{-1}x \qquad x \in [-1,0) \cup (0,1]$$

$$\cot^{-1}\left(\frac{1}{x}\right) = \begin{cases} \tan^{-1}x & , x > 0 \quad \text{PT-}1 \\ \pi + \tan^{-1}x & , x < 0 \quad \text{PT-}2 \end{cases}$$

(ITF)

$\tan^{-1}x = \theta \quad , \theta \in \left(-\frac{\pi}{2}, 0\right) \cup \left(0, \frac{\pi}{2}\right)$

$\tan\theta = x$

$\cot\theta = \frac{1}{x} \Rightarrow \cot^{-1}\cot\theta = \cot^{-1}\frac{1}{x}$

$$= \begin{cases} \theta & , \theta \in \left(0, \frac{\pi}{2}\right) \Rightarrow x > 0 \\ \pi + \theta & , \theta \in \left(-\frac{\pi}{2}, 0\right) \Rightarrow x < 0 \end{cases}$$