Nishant Jindal

$$\int \frac{dx}{\sqrt[4]{1+x^4}} = \int \frac{x^4 \cdot dx}{x^5 \cdot \sqrt[4]{1+\frac{1}{x^4}}}$$

$$= -\int \frac{t^{\cancel{3}\,2}\,dt}{(t^4-1)\cancel{t}}$$

$$1+\frac{1}{x^4} = t^4$$

$$-\frac{4}{x^5}dx = 4t^3\,dt$$

$$-\frac{1}{2}\int\left(\frac{1}{t^2-1} + \frac{1}{t^2+1}\right)dt$$

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$$\int \frac{dx}{x^{11}\sqrt{1+x^4}} = \int \frac{dx}{x^8 x^5\sqrt{1+\frac{1}{x^4}}}$$

$$1+\frac{1}{x^4} = t^2$$

$$-\frac{4}{x^5}dx = 2t\,dt$$

$$-\frac{1}{2}\int \frac{(t^2-1)^2 \cancel{t}\,dt}{\cancel{t}}$$

$$\int \frac{x^4}{x^3}\sqrt[3]{\frac{1}{x^2}-1}\,dx = -\frac{3}{2}\int \frac{t\cdot t^2\,dt}{(t^3+1)^2}$$

$$\frac{1}{x^2}-1 = t^3$$

$$-\frac{2}{x^3}dx = 3t^2\,dt$$

Nishant Jindal

$$dx = \left(\frac{1}{2} + \frac{3}{2(2t-1)^2}\right)dt \qquad x = \frac{t}{2} + \frac{1}{4} - \frac{\frac{3}{4}}{2t-1}$$

$$\int \frac{2(x+1)}{\left((x+1)^2+2\right)\sqrt{(x+1)^2+3}}\,dx + \int \frac{dx}{\left((x+1)^2+2\right)\sqrt{(x+1)^2+3}}$$

$$(2t-1)\left(\frac{t}{2}+\frac{1}{4}\right) - \frac{3}{4}$$

$$\boxed{x = \frac{t^2-1}{2t-1}}$$

$$x+1 = \frac{1}{t}\ ; \quad \sqrt{3}\tan\theta$$

$$\frac{1}{2}\int \frac{\left(1+\frac{3}{(2t-1)^2}\right)dt}{t}$$

$$x-1$$

$$(x+1)^2+3 = t^2$$

$$2t-(2t-1)$$

$$\int \frac{dt}{t(2t-1)^2}$$

$$x - \sqrt{x^2-x+1} = t.$$

$$x + \sqrt{x^2-x+1} = \frac{x-1}{t}.$$

$$2x = t - \frac{1}{t} + \frac{x}{t}.$$

$$\frac{1}{\sqrt{3}}\int \frac{\cos\theta\, d\theta}{2+\sin^2\theta}$$

$$3t^2+2.$$

$$3s^2+2c^2$$

$$x\left(2-\frac{1}{t}\right) = \frac{t^2-1}{t}$$

Nishant Jindal

1. Compute the intervals of monotonocity of functions and draw the graph.

(i) $f(x) = x^2 e^{-x}$

$f'(x) = e^{-x}(2x - x^2)$

$- \quad + \quad -$; $0 \quad 2$

$\uparrow$ $(0, 2)$

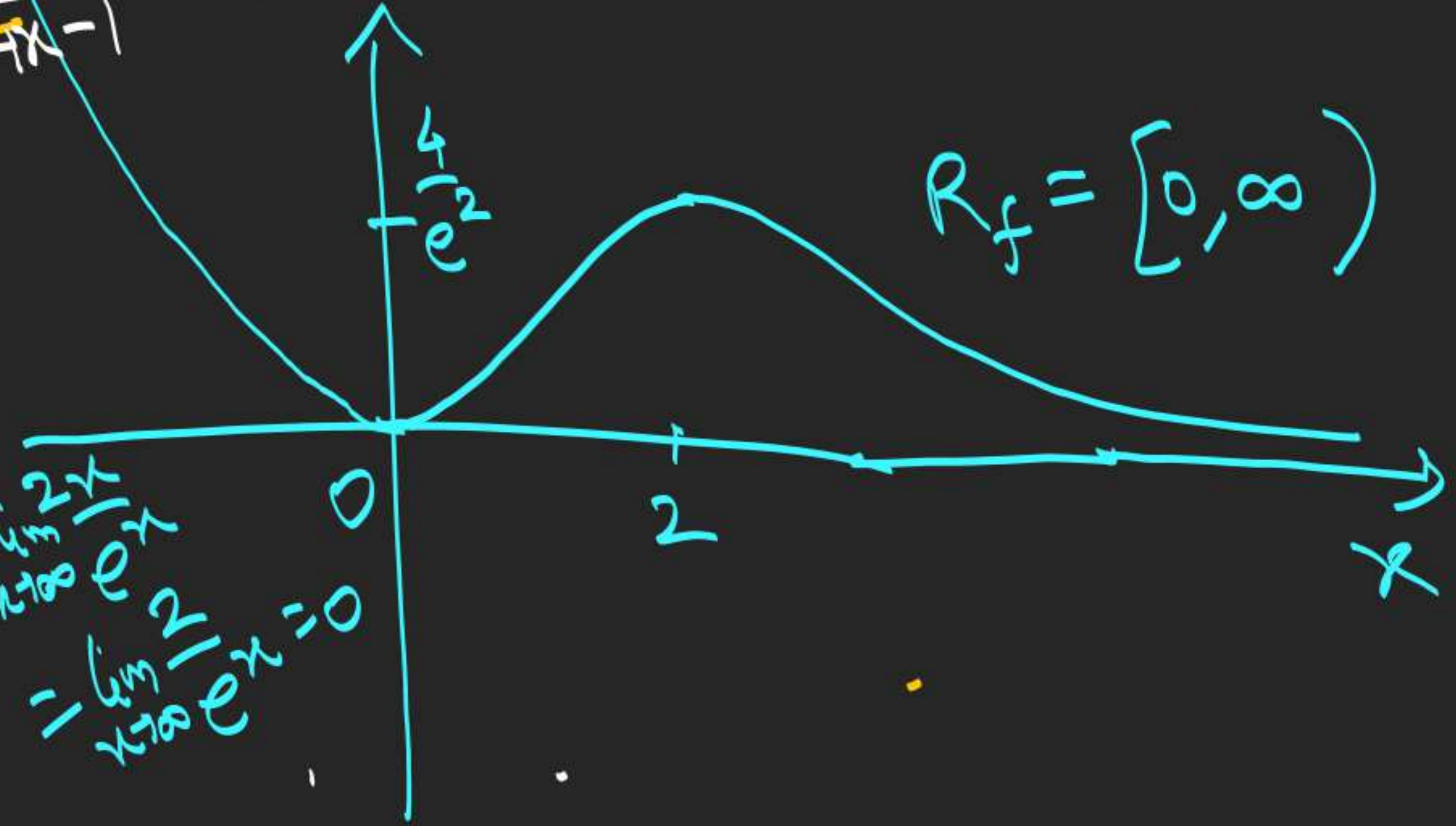
$\downarrow$ $(-\infty, 0) \cup (2, \infty)$

$x \to -\infty$, $f(x) \to \infty$

$x \to \infty$, $f(x)$

$\lim_{x\to\infty} \frac{x^2}{e^x} = \lim_{x\to\infty} \frac{2x}{e^x} = \lim_{x\to\infty} \frac{2}{e^x} = 0$

$R_f = [0, \infty)$



(ii) $f(x) = x + \ln(1-4x)$

$f'(x) = 1 - \frac{4}{1-4x} = \frac{4x+3}{4x-1}$

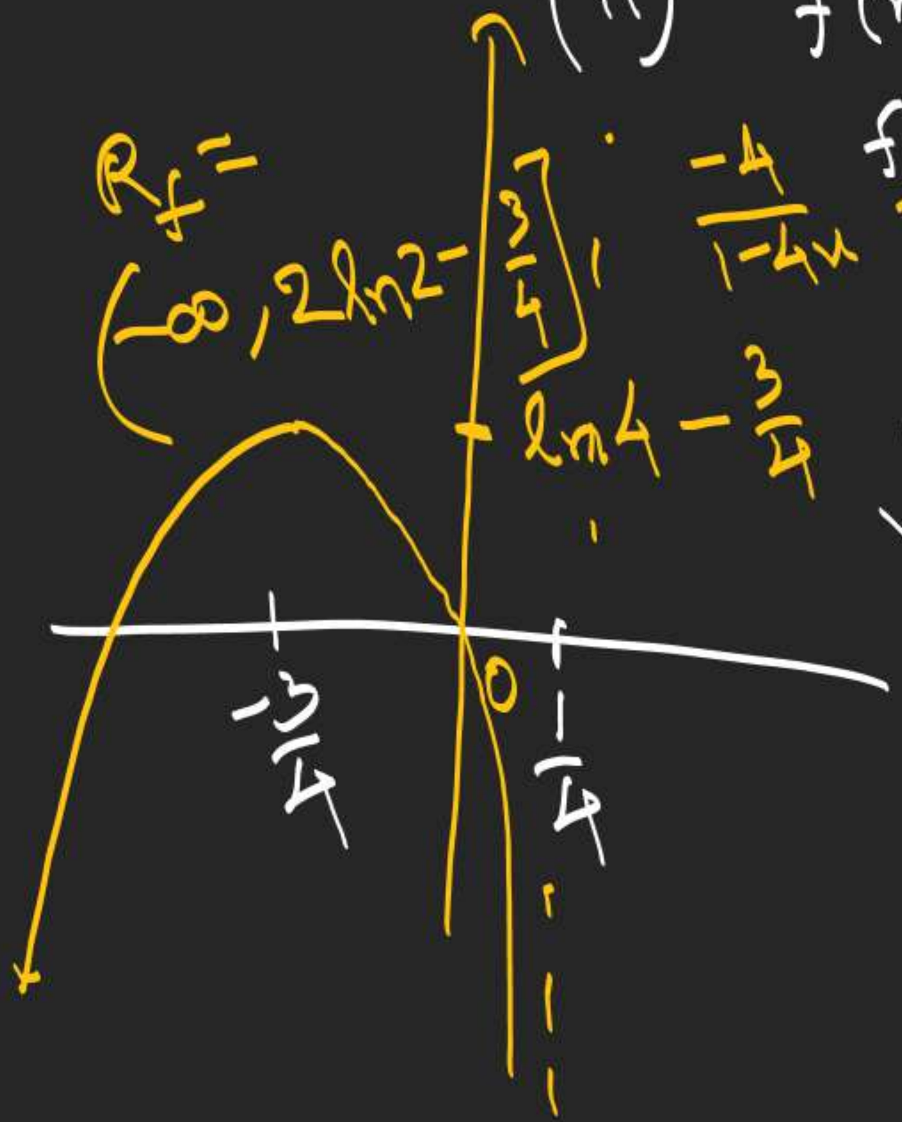
$\uparrow$ $\left(-\infty, -\frac{3}{4}\right)$

$\downarrow$ $\left(-\frac{3}{4}, \frac{1}{4}\right)$

$x \to -\infty$, $y \to -\infty$, $y = x\left(1 + \boxed{\frac{\ln(1-4x)}{x}}\right)$

$\frac{-4}{1-4x} \to 0$

$R_f = \left(-\infty, 2\ln 2 - \frac{3}{4}\right]$



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(iii) $f(x) = \dfrac{x}{\ln x}$ $\quad f'(x) = \dfrac{\ln x - 1}{\ln^2 x}$

$\uparrow (e, \infty)$

$\downarrow (0,1) \cup (1, e)$

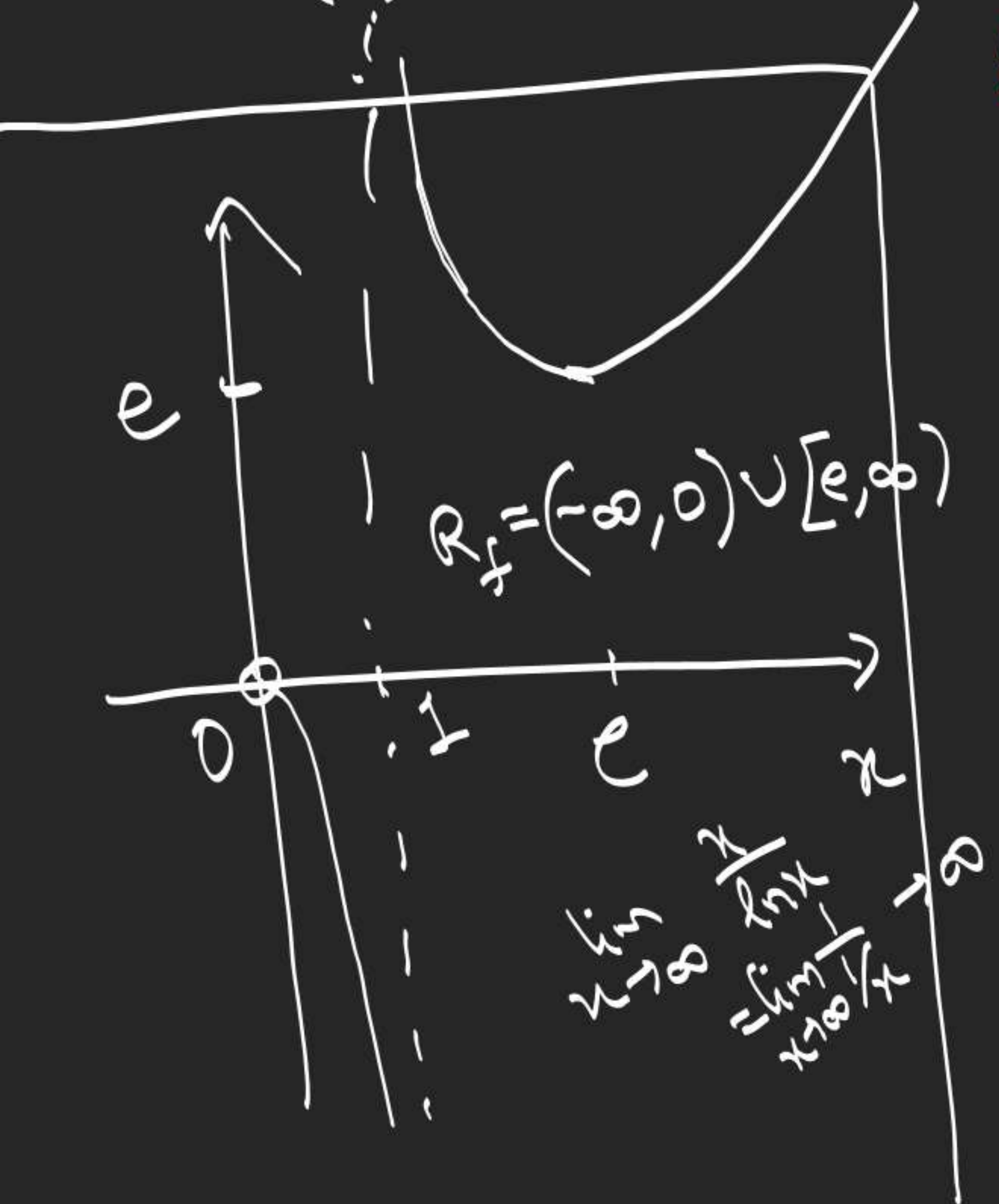
(iv) $f(x) = 2x^2 - \ln|x|$

$f'(x) = 4x - \dfrac{1}{x} = \dfrac{(2x-1)(2x+1)}{x}$

$-\quad +\quad -\quad +$ at $-\frac{1}{2}$, 0, $\frac{1}{2}$

$\uparrow \left(-\frac{1}{2}, 0\right) \cup \left(\frac{1}{2}, \infty\right)$

$\downarrow \left(-\infty, -\frac{1}{2}\right) \cup \left(0, \frac{1}{2}\right)$



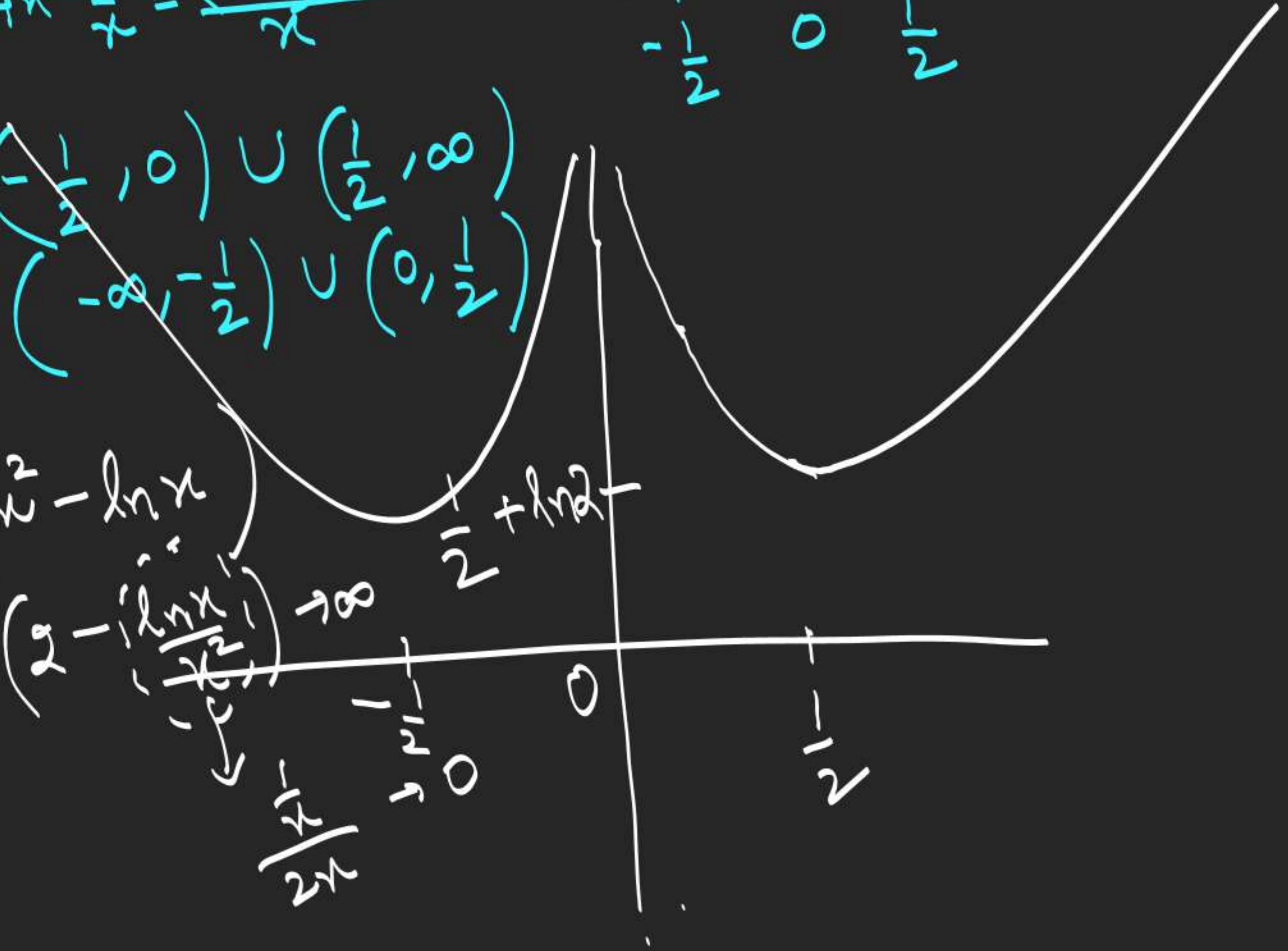
$R_f = (-\infty, 0) \cup [e, \infty)$

$\lim\limits_{x \to \infty} \dfrac{x}{\ln x} = \lim\limits_{x \to \infty} \dfrac{1}{1/x} \to \infty$

$\lim\limits_{x \to \infty} \left(2x^2 - \ln x\right)$

$\lim\limits_{x \to \infty} x^2 \left(2 - \dfrac{\ln x}{x^2}\right) \to \infty$

$\dfrac{\ln x}{x^2} \to \dfrac{\frac{1}{x}}{2x} \to 0$



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2. If the function $f(x)=\sin x - a\sin 2x - \frac{1}{3}\sin 3x + 2ax$ is strictly increasing $\forall x \in R$, find 'a'.

$$\boxed{f'(x) \geq 0 \ \forall x \in R}$$

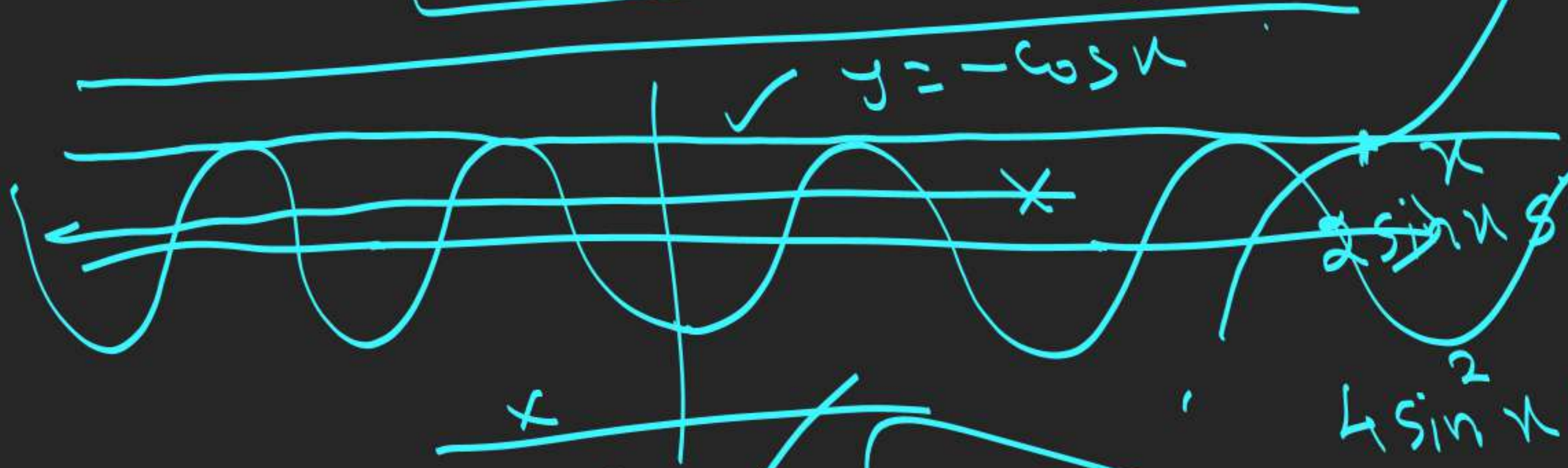
$$f'(x) = \cos x - 2a\cos 2x - \cos 3x + 2a \geq 0 \ \forall x \in R.$$

$$2\sin x \sin 2x + 4a\sin^2 x \geq 0$$

$$4\sin^2 x\,(\cos x + a) \geq 0 \ \forall x \in R.$$

$$\cos x + a \geq 0 \ \forall x \in R.$$

$$\boxed{a \geq -\cos x \ \forall x \in R}$$



$a \in [1, \infty)$

3. If function $f(x)=(a+2)x^3-3ax^2+9ax-1$ is strictly decreasing $\forall x \in R$, find 'a'.

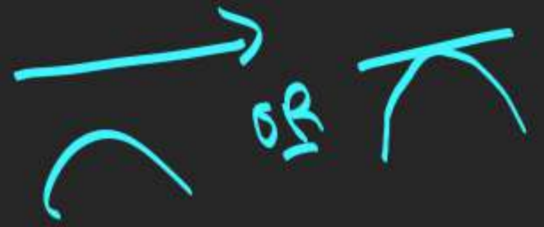
$$(a+2)x^2-2ax+3a \le 0 \quad \forall x \in R$$

$a+2=0$ ✗

$4x-6 \le 0$ ✗

$a+2<0$ ✓ $\quad a<-2$

$$D \le 0 \Rightarrow a^2-3a(a+2) \le 0$$



$$2a^2+6a \ge 0$$

$$(-\infty,-3] \cup [0,\infty)$$

$$\boxed{a \in (-\infty,-3]}$$

Nishant Jindal

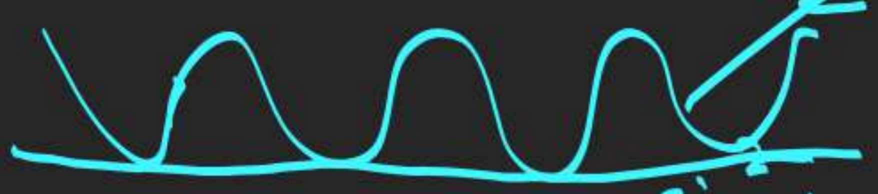
4. Find 'b' for which

$f(x) = \sin 2x - 8(b+2)\cos x - (4b^2+16b+6)x$ is monotonically decreasing $\forall x \in R$ and has no critical point.

$\boxed{b \in (-\infty, -3-\sqrt{3}) \cup (\sqrt{3}-1, \infty)}$

$f'(x) < 0 \ \forall x \in R \Rightarrow 2\cos 2x + 8(b+2)\sin x - (4b^2+16b+6) < 0$

$\cancel{2} - 4\sin^2 x + 8(b+2)\sin x - (4b^2+16b+4) < 0 \ \forall x \in R.$



$\sin^2 x - 2(b+2)\sin x + (b^2+4b+1) > 0 \ \forall x \in R.$

$+3 \quad +3$

$(\sin x - b - 2)^2 > 3 \ \forall x \in R$

$\sin x - b - 2 > \sqrt{3} \ \forall x \in R$ OR $\sin x - b - 2 < -\sqrt{3} \ \forall x \in R.$

$b+2+\sqrt{3} < \sin x \ \forall x \in R.$

$\boxed{b+2+\sqrt{3} < -1}$

$\boxed{b < -3-\sqrt{3}}.$

$b+2-\sqrt{3} > \sin x \ \forall x \in R$

$b+2-\sqrt{3} > 1$

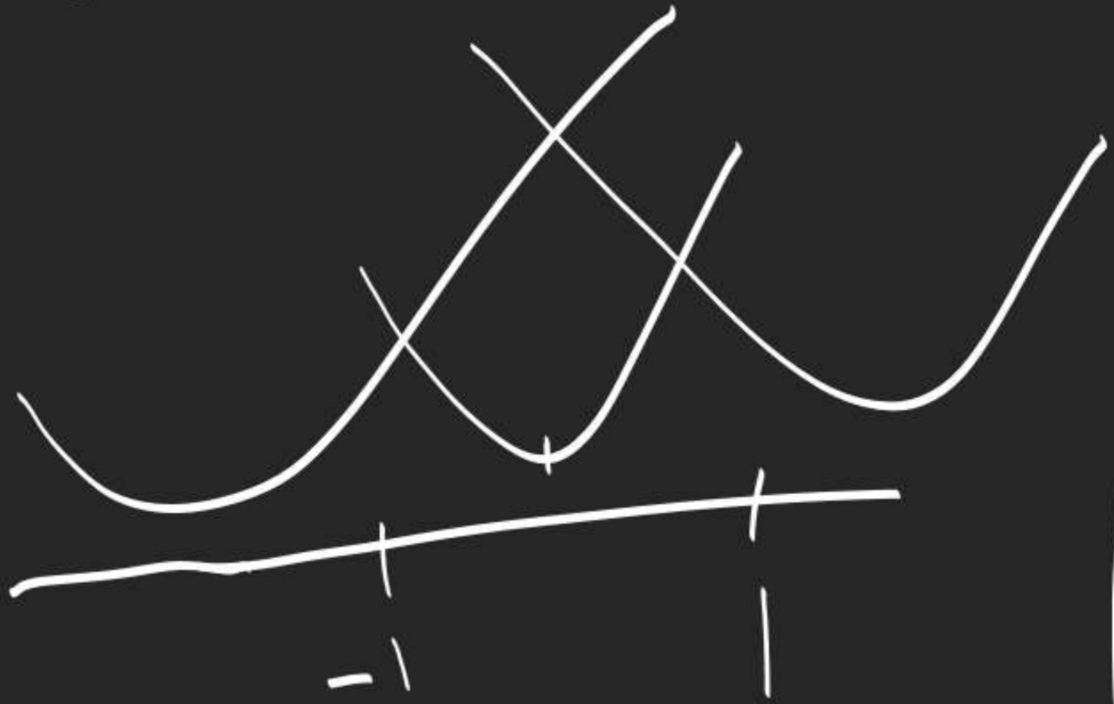
$\boxed{b > \sqrt{3}-1}$

Nishant Jindal

$$g(t) = t^2 - 2(b+2)t + (b^2+4b+1) > 0 \quad \forall\ t \in [-1, 1]$$

$\boxed{g_{min}}$

$b+2$



$b+2 \le -1 \Rightarrow b \le -3$ &

$g(-1) > 0$

$\Rightarrow b \in (-\infty, -3-\sqrt{3}) \cup (-3+\sqrt{3}, \infty)$

$\boxed{b \in (-\infty, -3-\sqrt{3})}$

$b+2 \ge 1 \Rightarrow \boxed{b \ge -1}$

$g(1) > 0 \Rightarrow (-\infty, -1-\sqrt{3}) \cup (-1+\sqrt{3}, \infty)$

$\boxed{b \in (\sqrt{3}-1, \infty)}$

$-1 < b+2 < 1 \Rightarrow b \in (-3, -1)$

$D < 0$

$\Rightarrow b \in$

$b \in \phi$