## DPP 02

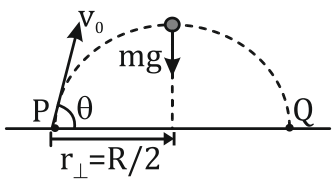
## SOLUTION

**1.** When the particle reaches the maximum height as shown in Figure, then

$\tau = Fr_{\perp} = mg\left(\frac{R}{2}\right) = mg\left(\frac{v_0^2 \sin 2\theta}{2g}\right)$

$\tau = \dfrac{mv_0^2 \sin k\theta}{K}$

k = 2




**2.** Here we can balance torque about wedge point as follows

$16l_1 = l_2\, m \Rightarrow \dfrac{l_1}{l_2} = \dfrac{m}{16}$ ... (i)

$ml_1 = 4l_2 \Rightarrow \dfrac{l_1}{l_2} = \dfrac{4}{m}$ ... (ii)

From (1) and (2)

$\dfrac{m}{16} = \dfrac{4}{m} \Rightarrow m^2 = 64 \Rightarrow m = 8$

**3.** Torque created due to weight of street light remains same in all the three cases. It is balanced by torque created by tension in the string. So if r be the torque created by weight of lamp and T 'be tension in the string and d be perpendicular distance of cable from the axis then, τ = T. d Tension will be least for largest d. This is in pattern A. So Pattern A is more sturdy.

**4.** $\vec{\imath} = \vec{r} \times \vec{F} = \begin{vmatrix} i & j & k \\ 2 & 2 & 1 \\ 5 & 3 & -7 \end{vmatrix} = i(-14-3) - j(-14-5) + \hat{k}(6-10) = -17\hat{\imath} + 19\hat{\jmath} - 4\hat{k}$

**5.** $\vec{F} = 4\hat{\imath} - 3\hat{\jmath}$

$\vec{r}_1 = 5\hat{\imath} + 5\sqrt{3}\hat{\jmath}$

& $\vec{r}_2 = -5\hat{\imath} + 5\sqrt{3}\hat{\jmath}$

Torque about 'O'

$\vec{\tau}_o = \vec{r}_1 \times \vec{F} = (-15 - 20\sqrt{3})\hat{k} = (15 + 20\sqrt{3})(-\hat{k})$

Torque about 'Q'

$\vec{\tau}_Q = \vec{r}_2 \times \vec{F} = (-15 + 20\sqrt{3})\hat{k} = (15 - 20\sqrt{3})(-\hat{k})$

**6.** Find the velocity of the particle.

Formula Used: $\vec{v} = \frac{d\vec{r}}{dt}$

As the position vector is given by

$\vec{r} = x\hat{i} + y\hat{j}$ ... (i)

Given, $x = x_0 + a\cos\ \omega_1 t$

& $y = y_0 + b\sin\ \omega_2 t$

Substituting the values in equation (i)

$\vec{r} = \left(x_0 + a\cos\ (\omega_1 t)\right)\hat{i} + \left(y_0 + b\sin\ (\omega_2 t)\right)\hat{j}$

$\vec{v} = -a\omega_1 \sin\ (\omega_1 t)\hat{i} + b\omega_2 \cos\ (\omega_2 t)\hat{j}$

Find the acceletation of the particle.

Formula Used: $\vec{a} = \frac{d\vec{v}}{dt}$

$\vec{a} = -a\omega_1^2 \cos\ (\omega_1 t)\hat{i} - b\omega_2^2 \sin\ (\omega_2 t)\hat{j}$

At $t = 0$,

$\vec{r} = (x_0 + a)\hat{i} + y_0\hat{j}$

$\vec{F} = -ma\omega_1^2\hat{i}$

$\vec{\tau} = \vec{r} \times \vec{F} = (ma\omega_1^2 y_0)\hat{k}$

**7.** From the given figure,

$\overrightarrow{F_1} = F\hat{k}$

$\overrightarrow{F_2} = F\sin\ 30^\circ(-\hat{i}) + F\cos\ 30^\circ(-\hat{j}) = \frac{F}{2}(-\hat{i}) + \frac{F\sqrt{3}}{2}(-\hat{j})$

$\overrightarrow{r_1} = 2\hat{i} + 3\hat{j}$

$\overrightarrow{r_2} = 0\hat{i} + 6\hat{j}$

Torque due to force $F_1$,

$\overrightarrow{\tau_{F_1}} = \overrightarrow{r_1} \times \overrightarrow{F_1} \Rightarrow \overrightarrow{\tau_{F_1}} = (2\hat{i} + 3\hat{j}) \times F\hat{k} \Rightarrow \overrightarrow{\tau_{F_1}} = 3F\hat{i} + 2F(-\hat{j})$

Torque due to force $F_2$,

$\overrightarrow{\tau_{F_2}} = \overrightarrow{r_2} \times \overrightarrow{F_2} \Rightarrow \vec{\tau}_2 = 6\hat{j} \times \left(\frac{F}{2}(-\hat{i}) + \frac{F\sqrt{3}}{2}(-\hat{j})\right) \Rightarrow \overrightarrow{\tau_{F_2}} = 3F(\hat{k})$

$\overrightarrow{\tau_{net}} = \overrightarrow{\tau_{F_1}} + \overrightarrow{\tau_{F_2}} \Rightarrow \overrightarrow{\tau_{net}} = 3F\hat{i} + 2F(-\hat{j}) + 3F(\hat{k}) \Rightarrow \overrightarrow{\tau_{net}} = (3\hat{i} - 2\hat{j} + 3\hat{k})F$

**8.** Let, 'l' and 'M' be the length and mass of rod.

for equilibrium, taking moment about X, we get

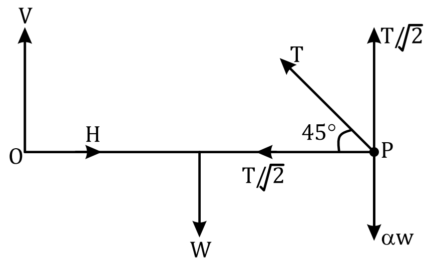
$m \times x = M \times \left(\frac{1}{2} - x\right)$

this can be written as,

$m = \left(\frac{M1}{2}\right)\frac{1}{x} - M$

comparing with straight line equation, $y = mx + c$ we can say that, $m \propto \frac{1}{x}$

9.




In equilibrium $cond^n$

$$V + \frac{T}{\sqrt{2}} - W - \alpha W = 0$$

$$V + \frac{T}{\sqrt{2}} = W(1 + \alpha)$$

$$V = W(1 + \alpha) - \frac{T}{\sqrt{2}} \qquad \text{... (i)}$$

$$\frac{T}{\sqrt{2}}L - \alpha WL - \frac{WL}{2} = 0$$

$$\frac{T}{\sqrt{2}} - \alpha W - \frac{W}{2} = 0$$

$$\frac{T}{\sqrt{2}} = \alpha W + \frac{W}{2} \qquad \text{... (ii)}$$

From (1) and (2)

$$V = W(1 + \alpha) - W\left(\alpha + \frac{1}{2}\right)$$

$$V = W\left[1 + \alpha - \alpha - \frac{1}{2}\right]$$

$$V = W\left[1 - \frac{1}{2}\right]$$

$V = \frac{W}{2}$ option A correct

$$H = \frac{T}{\sqrt{2}}$$

$$H = \alpha W + \frac{W}{2}$$

$$H = W\left(\alpha + \frac{1}{2}\right)$$

for $\alpha = 0.5$

$H = W$ option B is correct

$$\because \frac{T}{\sqrt{2}} = W\left(\alpha + \frac{1}{2}\right)$$

$T = \sqrt{2}W\left(0.5 + \frac{1}{2}\right)$

$T = \sqrt{2}\,W$ ie C option incorrect

from eqn (2)

$\frac{2\sqrt{2}\omega}{\sqrt{2}} = \omega\left(\alpha + \frac{1}{2}\right)$

$2 = \alpha + 1/2 \Rightarrow \alpha = 3/2 = 1.5$

option D correct