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Q13 $y^2 = x^3$

$2y\frac{dy}{dx} = 3x^2$

$\left.\frac{dy}{dx}\right| = \frac{3x^2}{2y} = \frac{3\cancel{m^4}}{-2\cancel{m^3}} = -\frac{3m}{2}$

$(m^2, -m^3)$

$(y+m^3) = \frac{2}{3m}(x-m^2)$

Compare.
$y = 3mx - 4m^3 \Rightarrow m^2$

(14) $x^3+y^3 = 8xy$

$y^2 = 4x$

$\frac{y^6}{64} + y^3 = \frac{\cancel{8}^{2}y^3}{\cancel{4}}$

$\frac{\cancel{y^6}\,y^3}{64} = \cancel{y^3}$

$y=0$

$y^3 = 64$

$y = 4,\ x = 4$

$(4,4)$

20 $y^2 - 2x^3 - 4y + 8 = 0 \rightarrow y_1^2 - 4y_1 - 2x_1^3 + 8 = 0 \rightarrow (1)$

$(1,2)$

$2y\frac{dy}{dx} - 6x^2 - 4\frac{dy}{dx} = 0$

$\left.\frac{dy}{dx}\right|_{(x_1,y_1)} = \frac{6x^2}{2y-4} = \frac{3x^2}{y-2} = \frac{3x_1^2}{y_1-2} = \frac{y_1-2}{x_1-1}$

$y_1^2 - 4y_1 + 4 = 3x_1^3 - 3x_1^2$

$y_1^2 - 4y_1 - 3x_1^3 + 3x_1^2 + 4 = 0 \rightarrow (2)$

$y_1^2 - 4y_1 - 2x_1^3 + 8 = 0$

$-x_1^3 + 3x_1^2 - 4 = 0$

$x_1^3 - 3x_1^2 + 4 = 0$

Solve $\rightarrow x = \ldots$

(19) find your EQN & compare.

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22) $x = \sec^2 t,\ y = \cot t$

① $\tan t = \frac{1}{y}$

$\tan^2 t = \frac{1}{y^2}$

$x - \frac{1}{y^2} = 1$

$xy^2 - 1 = y^2$

② $t = \frac{\pi}{4}$

$$\left.\frac{dy}{dx}\right|_{t=\frac{\pi}{4}} = \frac{-\operatorname{cosec}^2 t}{2\sec^2 t \cdot \tan t} = -\frac{2}{2\times 2\times 1} = -\frac{1}{2}$$

$x = 2,\ y = 1$

$(y-1) = -\frac{1}{2}(x-2)$ → EOT को Curve के Sath Solve करो। (x, y) Milenge → POT Second Q होगा।

25) $y^2 = x(2-x)^2$

$$2y\frac{dy}{dx} = -x\cdot 2(2-x) + (2-x)^2$$

(1,1) $2\frac{dy}{dx} = -1\times 2 + 1 = -1$

$$\frac{dy}{dx} = -\frac{1}{2}$$

$(y-1) = -\frac{1}{2}(x-1)$

$2y - 2 = -x + 1$

$x + 2y = 3$

$\left(\frac{3-x}{2}\right)^2 = x(2-x)^2$

Solve Cubic Eqⁿ

28) $(y-3) = \frac{-2-3}{5-0}(x-0)$

$y - 3 = -x \Rightarrow x + y - 3 = 0$

is tangent $y = \frac{ax}{1-x}$

If this Tangent then Combine Eqⁿ of Curve & Line will Satisfy Condⁿ of tangency.

$(3-x) = \frac{ax}{1-x} \Rightarrow (3-x)(1-x) = ax$

Quad → $D = 0$

Q30 जैसा Qs already [illegible].

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33) $f(x) = ax^2 + bx + c$

$y = x$ at $x = 1 \Rightarrow$ Pt of Cont $= (1,1)$.

$f(1) = a + b + c = 1 \longrightarrow a + \cancel{b} + c = 2a + b$

$f'(x) = 2ax + b$ $\quad c = a \Rightarrow a$

$f'(1) = 2a + b = 1 \rightarrow$ option A ✓

option (x) $f'(0) = b$ Not sure. $\quad b = 1 - 2a$

$b = 1 - 2c$

$2f(0) = 1 - f'(0)$

$2c = 1 - b$

$f'(0) = 1 - 2f(0)$

Option C ✓

$f(0) = c$

$f'(0) = b$

$f''(0) = 2a$

35) $y = x^2 + ax + b$ $(1,0)$, $y = x(c - x) = -x^2 + cx$

$0 = 1 + a + b$ $\quad 0 = c - 1$

$c = 1$ option (D) ×

$\left.\frac{dy}{dx}\right|_{(1,0)} = 2x + a = a + 2$

$\left.\frac{dy}{dx}\right|_{(1,0)} = -2x + c = c - 2 = -1$

$a + 2 = -1$

$a = -3$. option A ✓

$b = 2$

39) try kro.

45) तक वाले Qs

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Monotonocity

① If $f(x)$ & $g(x)$ are 2 fxns such that
$x_1 > x_2$ & $f(x_1) > f(x_2)$, $g(x_1) < g(x_2)$
then find α such that
$f(g(\alpha^2-2\alpha)) < f(g(3\alpha-4))$

1) $x_1 > x_2 \to f(x_1) > f(x_2) \to f \uparrow$ ing
2) $x_1 > x_2 \to g(x_1) < g(x_2) \to g \downarrow$ ing
3) $f(g(\alpha^2-2\alpha)) < f(g(3\alpha-4))$

f Remove $g(\alpha^2-2\alpha) < g(3\alpha-4)$

g Remove $\alpha^2-2\alpha > 3\alpha-4$

$\alpha^2-5\alpha+4>0$

$(\alpha-1)(\alpha-4)>0$

$\alpha<1 \cup \alpha>4$

Q $f: R \to R$, $g: R \to R$ defined

Main 2022 by $f(x)=\log_e(x^2+1)-e^{-x}+1$

$g(x)=\frac{1-2e^{2x}}{e^x}$ then find α

In which $f\left(g\left(\frac{(\alpha-1)^2}{3}\right)\right) > f\left(g\left(\alpha-\frac{5}{3}\right)\right)$ holds.

$f'(x)=\frac{2x}{1+x^2}+e^{-x} > 0$ $f(x)\uparrow$

$[-1,1]$

$g(x)=e^{-x}-2e^{x}$

$g'(x)=-e^{-x}-2e^{x}<0$ $g \downarrow$ fxn

f Remove $g\left(\frac{(\alpha-1)^2}{3}\right) > g\left(\alpha-\frac{5}{3}\right)$

g Remove $\frac{(\alpha-1)^2}{3} < \frac{3\alpha-5}{3} \Rightarrow \alpha^2-2\alpha+1 < 3\alpha-5$

$\alpha^2-5\alpha+6<0$

$(\alpha-2)(\alpha-3)<0$

$2<\alpha<3$

$f(-\infty) \Rightarrow 0$

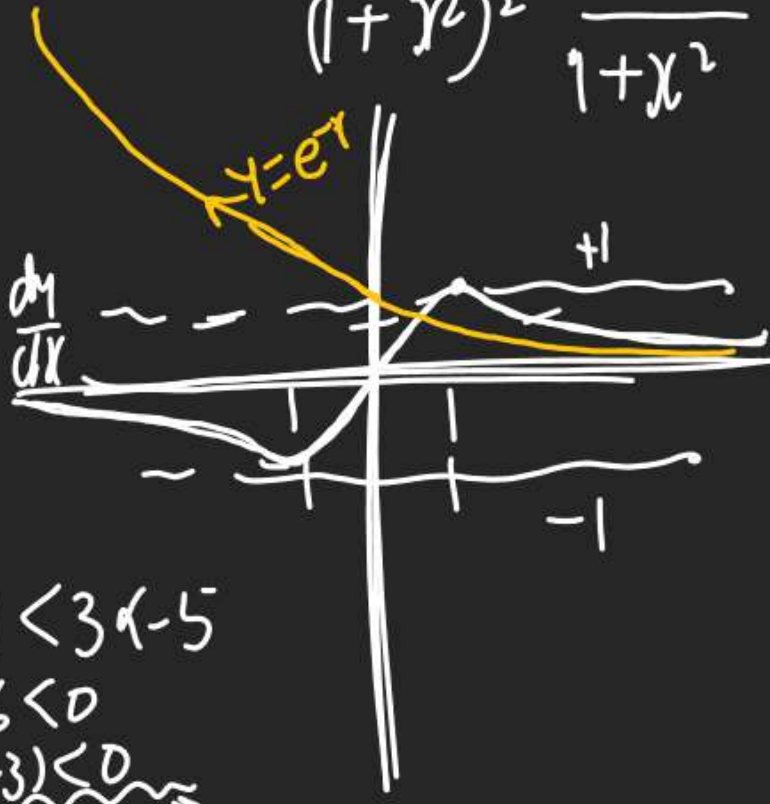
$f(\infty) \Rightarrow 0$

$Y=\frac{2x}{1+x^2}$ $f(0)=0$

$f(-1)=\frac{-2}{2}=-1 \to D=(-\infty,\infty)$

$\frac{dy}{dx}=\frac{2(1+x^2)-2x\times 2x}{(1+x^2)^2}$ $f(1)=1$

$=\frac{2-2x^2}{(1+x^2)^2}=\frac{-2(x-1)(x+1)}{1+x^2}$



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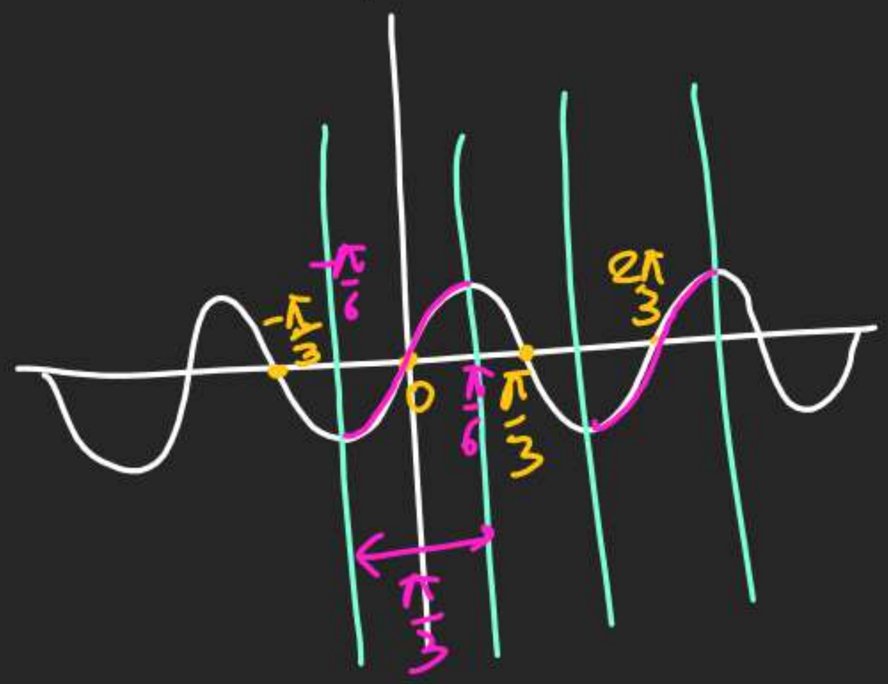
Q Find Max^m length of Interval Where $f(x) = 3\sin x - 4\sin^3 x$ is ↑ing.

$f(x) = \sin 3x$



Max length of gap Where $f(x) = \sin 3x$ is ↑ing is $\frac{\pi}{3}$.

Q $f : [0, \infty) \to R$

$$f(x) = \frac{e^{x^2} - e^{-x^2}}{e^{x^2} + e^{-x^2}}$$

Check fxn is ↑ing or ↓ing.

$$f(x) = \frac{e^{2x^2} - 1 + 1 - 1}{e^{2x^2} + 1} = 1 - \frac{2}{e^{2x^2} + 1}$$

① $x \in [0, \infty)$

x^2 ↑

$2x^2$ ↑

e^{2x^2} ↑

$e^{2x^2} + 1$ ↑

$\frac{1}{e^{2x^2}+1}$ ↓

$\frac{-2}{e^{2x^2}+1}$ ↑

$1 - \frac{2}{e^{2x^2}+1}$ ↑.

$f(x)$ ↑.

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Q Find Interval Where.

$f(x) = x + \frac{4}{x^2}$ is ↓ ?

$f'(x) = 1 - \frac{8}{x^3} < 0$ (↓)

$\frac{8}{x^3} > 1$

$x^3 < 8$ ⊗

$= \frac{x^3 - 8}{x^3} < 0$

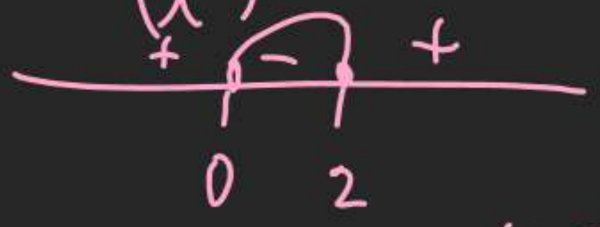
Further Factorise

$= \frac{(x-2)(x^2+2x+4)}{x^3} < 0$

हमेशा ⊕

$D < 0$

$\frac{(x-2)}{(x)^3} < 0$



↓ $x \in (0,2)$

Q $f(x) = (x-1)^3$ is ↑ or ↓ at $x = 1$

$f'(x) = 3(x-1)^2$

$f'(1) = 3(1-1)^2 = 0$

$f(1-h) = (1-h-1)^3 = -h^3$

$f(1) = (1-1)^3 = 0$

$f(1+h) = (1+h-1)^3 = h^3$

$-h^3 < 0 < h^3$

$f(1-h) < f(1) < f(1+h)$

L ⟶ R

Increasing.

Q $f(x) = \ln 5x$ at $x = \frac{1}{5}$

↑ or ↓ ?

$f'(x) = \frac{5}{5x} = \frac{1}{x}$

$f'\left(\frac{1}{5}\right) = \frac{1}{1/5} = 5 > 0$

↑ing.

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Q $f(x) = x \cdot e^{x(1-x)}$ fxn is ↑ in?

$f'(x) = x \cdot e^{x(1-x)} \times (1-2x) + e^{x(1-x)} \cdot 1$

$= e^{x(1-x)} \{ 1 + x - 2x^2 \}$

$= -e^{x(1-x)} \{ 2x^2 - x - 1 \}$

$f'(x) = -e^{x(1-x)} \{ (2x+1)(x-1) \}$

$+$

$- \quad + \quad -$

$-\frac{1}{2} \quad 1$

↑ in $x \in \left[-\frac{1}{2}, 1\right]$

Q Set of values of a & b for which fxn. $f(x) = \sin^2 x + \sin 2x + ax + b$ is always ↑ing.

$f'(x) \geq 0$

$\sin 2x + 2\cos 2x + a \geq 0$

$a \geq -(\sin 2x + 2\cos 2x)_{Max.}$

याद रखना

Same QS: Constant $\geq$ $(\text{Variable})_{Max}$

$-\sqrt{1^2+2^2} \leq (\sin 2x + 2\cos 2x) \leq \sqrt{1^2+2^2}$

$\sqrt{5} \geq -(\sin 2x + 2\cos 2x) \geq -\sqrt{5}$

Max.

$\Rightarrow a \geq \sqrt{5} \Rightarrow a \in [\sqrt{5}, \infty)$

10 QS

Monotonocity