DPP - 01

SECTION - A

KINDS OF VECTORS

1. Consider points A, B, C and D with position vectors $7\hat{\imath} + 4\hat{\jmath} + 7\hat{k}, \hat{\imath} - 6\hat{\jmath} + 10\hat{k}, -\hat{\imath} - 3\hat{\jmath} + 4\hat{k}$ and $5\hat{\imath} - \hat{\jmath} + 5\hat{k}$ respectively. The $ABCD$ is a

(A) square

(B) rhombus

(C) rectangle

(D) none of these

Ans. (D)

Sol.

$\overline{AB} = -6\hat{i} - 10\hat{j} + 3\hat{k}$

$\overline{AD} = -2\hat{i} - 5\hat{j} - 2\hat{k}$

$\overline{AB} \cdot \overline{AD} \neq 0$

so not a square or rectangle $|\overline{AB}| \neq |\overline{AD}|$ so not a rhombus.

SECTION – B

ADDITION & SUBSTRACTION OF VECTORS

2. The vertices of a triangle are $A(1,1,2), B(4,3,1)$ and $C(2,3,5)$. A vector representing the internal bisector of the angle A is

(A) $\hat{\imath} + \hat{\jmath} + 2\hat{k}$

(B) $2\hat{\imath} - 2\hat{\jmath} + \hat{k}$

(C) $2\hat{\imath} + 2\hat{\jmath} - \hat{k}$

(D) $2\hat{\imath} + 2\hat{\jmath} + \hat{k}$

Ans. (D)

Sol.

$\vec{p} = (3, 2, -1)$

$\hat{p} = \dfrac{(3,2,-1)}{\sqrt{14}}$

$\vec{q} = (1, 2, 3)$

$\hat{q} = \dfrac{1}{\sqrt{14}}(1, 2, 3)$

Angle Bisector = $\hat{p}+\hat{q} = \dfrac{1}{\sqrt{14}}(4, 4, 2)$

3. The vectors $\overline{AB} = 3\hat{\imath} + 4\hat{k}$ and $\overline{AC} = 5\hat{\imath} - 2\hat{\jmath} + 4\hat{k}$ are the sides of a triangle ABC. The length of the median through A is

(A) $\sqrt{18}$

(B) $\sqrt{72}$

(C) $\sqrt{33}$

(D) $\sqrt{288}$

Ans. **(C)**

Sol.

$\overrightarrow{AB} = (3, 0, 4)$

$\overrightarrow{AC} = (5, -2, 4)$

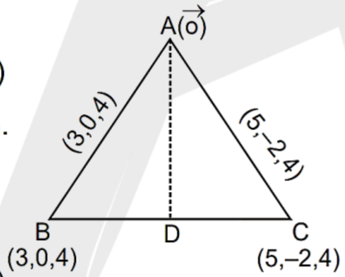
Let $\vec{A}$ be origin.

D is the mid point of BC

D(4, –1, 4)

$\overrightarrow{AD} = (4, -1, 4)$

$|\overrightarrow{AD}| = \sqrt{16+1+16} = \sqrt{33}$



SECTION - C

COLLINEARITY OF THREE POINTS

4. If the vector $\vec{b}$ is collinear with the vector $\vec{a} = (2\sqrt{2}, -1, 4)$ and $|\vec{b}| = 10$, then

(A) $\vec{a} \pm \vec{b} = 0$

(B) $\vec{a} \pm 2\vec{b} = 0$

(C) $2\vec{a} \pm \vec{b} = 0$

(D) none of these

Ans. **(C)**

Sol.

$\vec{a} = (2\sqrt{2}, -1, 4)$ $\qquad |\vec{b}| = 10$

$\vec{b} = \lambda\vec{a}$

$|\vec{b}|^2 = \lambda^2 |\vec{a}|^2$

$100 = \lambda^2 (8 + 1 + 16)$

$\lambda^2 = 4 \quad \Rightarrow \lambda = \pm 2$

$2\vec{a} \pm \vec{b} = 0$

SECTION - D

RELATION BETWEEN TWO PARALLEL VECTORS

5. If $A(-\hat{\imath} + 3\hat{\jmath} + 2\hat{k}), B(-4\hat{\imath} + 2\hat{\jmath} - 2\hat{k})$ and $C(5\hat{\imath} + \lambda\hat{\jmath} + \mu\hat{k})$ are collinear then

(A) $\lambda = 5, \mu = 10$ (B) $\lambda = 10, \mu = 5$

(C) $\lambda = -5, \mu = 10$ (D) $\lambda = 5, \mu = -10$

Ans. (A)

Sol. $\overrightarrow{AB} = -3\hat{i} - \hat{j} - 4\hat{k}$

$\overrightarrow{BC} = 9\hat{i} + (\lambda - 2)\hat{j} + (\mu + 2)\hat{k}$

$\overrightarrow{AB}$ collinear with BC

$\therefore \dfrac{-3}{9} = \dfrac{-1}{\lambda - 2} = \dfrac{-4}{\mu + 2}$

$\lambda = 5, \mu = 10$

6. The vectors $2\hat{\imath} + 3\hat{\jmath}, 5\hat{\imath} + 6\hat{\jmath}$ and $8\hat{\jmath} + \lambda\hat{\jmath}$ have their initial points at $(1,1)$. Find the value of λ so that the vectors terminate on one straight line

(A) 9 (B) 8 (C) 7 (D) 6

Ans. (A)

Sol. Since the vectors $2\hat{i} + 3\hat{j}$ and $5\hat{i} + 6\hat{j}$ have $(1,1)$ as the initial point, therefore their terminal points are $(3,4)$ and $(6,7)$, respectively. The equation of the line joining these two points is $x - y + 1 = 0$. The terminal point of $8\hat{i} + \lambda\hat{j}$ is $(9, \lambda + 1)$. Since the vectors terminate on the same straight line, $(9, \lambda + 1)$ lies on $x - y + 1 = 0$. Therefore,

$9 - \lambda + 1 = 0$

$\Rightarrow \lambda = 9$

7. If $\vec{a}, \vec{b}$ and $\vec{c}$ are three non - zero vectors, no two of which are collinear, $\vec{a} + 2\vec{b}$ is collinear with $\vec{c}$ and $\vec{b} + 3\vec{c}$ is collinear with $\vec{a}$, then find the value of $|\vec{a} + 2\vec{b} + 6\vec{c}|$,

(A) 0 (B) 1 (C) 2 (D) 3

Ans. (A)

Sol. Given $\vec{a}+2\vec{b}=\lambda\vec{c}$

and $\vec{b}+3\vec{c}=\mu\vec{a}$

where no two of $\vec{a},\vec{b}$ and $\vec{c}$ are collinear vectors.

Eliminating $\vec{b}$ from the above relations, we have

$\vec{a}-6\vec{c} = \lambda\vec{c}-2\mu\vec{a}$

$\vec{a}(1+2\mu) = (\lambda+6)\vec{c}$

$\Rightarrow \mu=-\frac{1}{2}$ and $\lambda = -6$ as $\vec{a}$ and $\vec{c}$ are non - collinear

putting $\mu=-\frac{1}{2}$ in (ii) or $\lambda=-6$ in (i), we get

$\vec{a}+2\vec{b}+6\vec{c}=\vec{0}$

$\Rightarrow \left|\vec{a}+2\vec{b}+6\vec{c}\right|=0$

SECTION – L

MIXED PROBLEMS

8. If $\vec{a},\vec{b},\vec{c}$ are linearly independent vectors, then which one of the following set of vectors is linearly dependent?

(A) $\vec{a}+\vec{b},\vec{b}+\vec{c},\vec{c}+\vec{a}$

(B) $\vec{a}-\vec{b},\vec{b}-\vec{c},\vec{c}-\vec{a}$

(C) $\vec{a}\times\vec{b},\vec{b}\times\vec{c},\vec{c}\times\vec{a}$

(D) none of these

Ans. **(B)**

Sol.

$x\vec{a}+y\vec{b}+z\vec{c}=0$

(For linearly independent vector)

$x(\vec{a}-\vec{b})+y(\vec{b}-\vec{c})+z(\vec{c}-\vec{a})$

$x\vec{a}+y\vec{b}+z\vec{c} - (x\vec{b}+y\vec{c}+z\vec{a})$

linear combination $\Rightarrow 0-0=0$

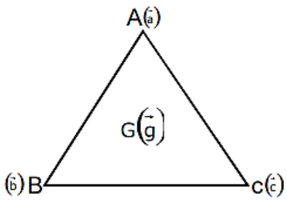
So $\vec{a}-\vec{b},\ \vec{b}-\vec{c},\ \vec{c}-\vec{a}$ are linearly dependent vector.

9. If G is the centroid of a trinagle ABC, then $\overrightarrow{GA}+\overrightarrow{GB}+\overrightarrow{GC}$ is equal to

(A) $\vec{0}$ (B) $3\overrightarrow{GA}$ (C) $3\overrightarrow{GB}$ (D) $3\overrightarrow{GC}$

Ans. **(A)**

Sol.



$$G(\vec{g}) = \left(\frac{\vec{a}+\vec{b}+\vec{c}}{3}\right)$$

$$\overrightarrow{GA} + \overrightarrow{GB} + \overrightarrow{GC} = (\vec{A}-\vec{G}) + (\vec{B}-\vec{G}) + (\vec{C}-\vec{G})$$

$$= (\vec{A}+\vec{B}+\vec{C}) - 3\vec{G}$$

$$= \vec{a}+\vec{b}+\vec{c} - 3\left[\frac{\vec{a}+\vec{b}+\vec{c}}{3}\right]$$

$$\boxed{G\vec{A} + G\vec{B} + G\vec{C} = \vec{0}}$$

10. If vector $\overrightarrow{AB} = -3\hat{\imath} + 4\hat{k}$ and $\overrightarrow{AC} = 5\hat{\imath} - 2\hat{\jmath} + 4\hat{k}$ are the sides of a $\triangle ABC$, then the length of the median through A is

(A) $\sqrt{14}$

(B) $\sqrt{18}$

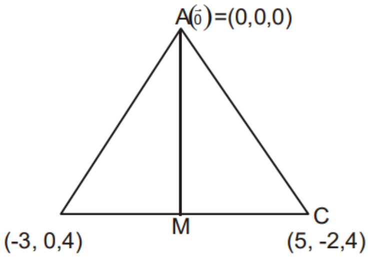
(C) $\sqrt{29}$

(D) 5

Ans. **(B)**

Sol. $\overrightarrow{AB} = -3\hat{i} + 4\hat{k}$

$\Rightarrow \vec{B} - \vec{A} = -3\hat{i} + 4\hat{k}$



$\Rightarrow \boxed{\vec{B} = -3\hat{i} + 4\hat{k}}$

$\overrightarrow{AC} = \vec{C} - \vec{A} = 5\hat{i} - 2\hat{j} + 4\hat{k}$

$\boxed{\vec{C} = 5\hat{i} - 2\hat{j} + 4\hat{k}}$

$M = (1,-1,4)$

$\overrightarrow{AM} = \vec{M} - \vec{A}$

$\overrightarrow{AM} = \hat{i} - \hat{j} + 4\hat{k}$

$\left|\overrightarrow{AM}\right| = \sqrt{1^2 + 1^2 + (4)^2} = \sqrt{18}$

11. If the vectors $\vec{a}$ and $\vec{b}$ are linearly independent satisfying $(\sqrt{3}\tan\ \theta + 1)\vec{a} + (\sqrt{3}\sec\ \theta - 2)\vec{b} = 0$, then the most general values of θ are

(A) $n\pi - \frac{\pi}{6}, n \in Z$

(B) $2n\pi \pm \frac{11\pi}{6}, n \in Z$

(C) $n\pi \pm \frac{\pi}{6}, n \in Z$

(D) $2n\pi + \frac{11\pi}{6}, n \in Z$

Ans. (D)

Sol.

$$\left(\sqrt{3}\tan\theta + 1\right)\vec{a} + \left(\sqrt{3}\sec\theta - 2\right)\vec{b} = 0$$

$\vec{a}$ & $\vec{b}$ are linear independant

So, $\sqrt{3}\tan\theta + 1 = 0$

$$\tan\theta = -\frac{1}{\sqrt{3}}$$

$$\sqrt{3}\sec\theta - 2 \Longrightarrow \sec\theta = \frac{2}{\sqrt{3}}$$

Both are possible in fourth quardrant &

$$\theta = \frac{-\pi}{6} \text{ or } \frac{11\pi}{6}$$

so, $\theta = 2n\pi - \frac{\pi}{6}$ or $\boxed{\theta = 2n\pi + \frac{11\pi}{6}}$

MIXED PROBLEMS

12. The vectors $\vec{a} = -4\hat{\imath} + 3\hat{k}, \vec{b} = 14\hat{\imath} + 2\hat{\jmath} - 5\hat{k}$ are coinitial. The vector $\vec{d}$ which is bisecting the angle between the vectors $\vec{a}$ and $\vec{b}$ and is having the magnitude $\sqrt{6}$, is

(A) $\hat{\imath} + \hat{\jmath} + 2\hat{k}$

(B) $\hat{\imath} - \hat{\jmath} + 2\hat{k}$

(C) $\hat{\imath} + \hat{\jmath} - 2\hat{k}$

(D) none of these

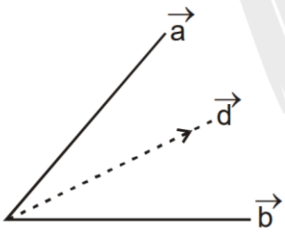
Ans. **(A)**

Sol. $\vec{d} = \hat{a} + \hat{b}$

$$= \frac{-4\hat{i} + 3\hat{k}}{5} + \frac{|14\hat{i} + 2\hat{j} - 5\hat{k}|}{15}$$

$$= \frac{-12\hat{i} + 9\hat{k} + 14\hat{i} + 2\hat{j} - 5\hat{k}}{15}$$

$$= \frac{2\hat{i} + 2\hat{j} + 4\hat{k}}{15} = \frac{2}{15}(\hat{i} + \hat{j} + \hat{k})$$



13. The vector $\hat{\imath} + x\hat{\jmath} + 3\hat{k}$ is rotated through an angle of $\cos^{-1}\frac{11}{14}$ and doubled in magnitude, then it becomes $4\hat{\imath} + (4x - 2)\hat{\jmath} + 2\hat{k}$. The value of ' x ' is

(A) $-\frac{2}{3}$

(B) $\frac{2}{3}$

(C) $\frac{1}{3}$

(D) 2

Ans. **(D)**

Sol.

Let new vector is $\vec{a}$

So, $|\vec{a}| = 2\left|\hat{i} + \hat{j} + 3\hat{k}\right|$

$=2\sqrt{1 + x^2 + 9}$

$|\vec{a}| = 2\sqrt{10 + x^2}$

$|\vec{a}|\left(\hat{i} + x\hat{j} + 3\hat{k}\right)\cos\theta = \vec{a}.\left(\hat{i} + x\hat{j} + 3\hat{k}\right)$

$2\sqrt{10 + x^2}\ \ \sqrt{10 + x^2}.\dfrac{11}{14}$

$= \left(4\hat{i} + (4x - 2)\hat{j} + 2\hat{k}\right).\left(\hat{j} + x\hat{j} + 3k\right)$

$\dfrac{11}{7}\left(10 + x^2\right) = 4+x(4x-2) +6$

$11(10+x^2) = 7\,(10+4x^2 - 2x)$

$110 + 11\,x^2 = 70\,(10+ 4x^2-2x)$

$110 + 11x^2 = 70 + 28x^2 = 14x$

$17x^2 - 14x - 40 = 0$

only "2" is satifies this equation

So, option = D

COLLINEARITY OF THREE POINTS

14. If a, b, c are different real numbers and $a\hat{i} + b\hat{j} + c\hat{k}, b\hat{i} + c\hat{j} + a\hat{k}$ and $c\hat{i} + a\hat{j} + b\hat{k}$ are position vectors of three non-collinear points A, B, and C, then

(A) centroid of triangle ABC is $\frac{a+b+c}{3}(\hat{\imath} + \hat{\jmath} + \hat{k})$

(B) $\hat{\imath} + \hat{\jmath} + \hat{k}$ is equally inclined to the three vectors

(C) perpendicular from the origin to the plane of triangle ABC meet at centroid

(D) triangle ABC is an equilateral triangle.

Ans. **(ABCD)**

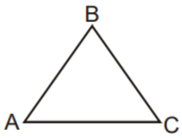
Sol. $G = \dfrac{\vec{A} + \vec{B} + \vec{C}}{3} = \dfrac{(a+b+c)}{3}(\hat{i} + \hat{j} + \hat{k})$

then [A] and [B] are correct.

$|\overrightarrow{AB}| - |\overrightarrow{BC}| = |\overrightarrow{CA}|$

then [D] is also correct.

[C] $\Rightarrow$ (By observation)

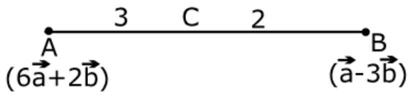


SUBJECTIVE

15. The position vector of two points A and B are $6\vec{a}+2\vec{b}$ and $\vec{a}-3\vec{b}$. If a point C divides AB in the ratio 3: 2 then show that the position vector of C is $3\vec{a}-\vec{b}$

Ans. ()

Sol. $\vec{c}=\dfrac{3(\vec{a}-3\vec{b})+2(6\vec{a}+2\vec{b})}{5}$



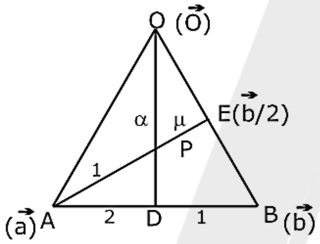
$\vec{c}=3\vec{a}-\vec{b}$

16. In a $\triangle OAB, E$ is the mid-point of OB and D is a point on AB such that AD: DB $=2:1$. if OD and AE intersect at P, then determine the ratio OP: PD using vector methods.

Ans. (3 : 2)

Sol.

$$P=\frac{\alpha\left(\frac{2\vec{b}+\vec{a}}{3}\right)}{\alpha+1}=\frac{\frac{\vec{b}}{2}+\mu\vec{a}}{\mu+1}$$



$$\frac{2\alpha}{3(\alpha+1)}=\frac{1}{2(\mu+1)} \quad \ldots (1)$$

$$\frac{\alpha}{3(\alpha+1)}=\frac{\mu}{(\mu+1)} \quad \ldots (2)$$

(1) − (2)

$$2=\frac{1}{2\mu} \Rightarrow \mu=4 \Rightarrow \alpha=\frac{3}{2} \Rightarrow OP : PD = 3 : 2$$

17. Show that the points

$\vec{a}-2\vec{b}+3\vec{c}; 2\vec{a}+3\vec{b}-4\vec{c}\& -7\vec{b}+10\vec{c}$

are collinear.

Ans. ()

Sol.

(a) Let $\vec{P}, \vec{Q}, \& \vec{R}$ be three vectors

$\overrightarrow{PQ} = \vec{a} + 5\vec{b} - 7\vec{c}$

$\overrightarrow{QR} = -2\vec{a} - 10\vec{b} + 14\vec{c}$

$(\overrightarrow{QR}) = -2(\overrightarrow{PQ})$

Here collinear.

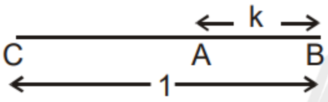
(b) $\overrightarrow{AB} = (2, 2, 4)$

$\overrightarrow{AC} = (-6, -6, -12) \Rightarrow \overrightarrow{AC} = -3\overrightarrow{AB}$

Here $\vec{A}, \vec{B}, \vec{C}$ are collinear

Let the ratio be k : 1

So,



$\frac{-3k-1}{k-1} = 3 \Rightarrow k = \frac{1}{3}$ (externally)

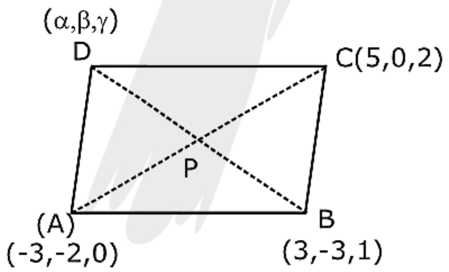
18. If the three successive vertices of a parallelogram have the position vectors as, $A(-3,-2,0)$; $B(3,-3,1)$ and $C(5,0,2)$. Then find

(i) Position vector of the fourth vertex D

Ans. $(D(-1,1,1))$

Sol.

(i) Diagonal bisect each other



$\frac{3+\alpha}{2} = \frac{5-3}{2}$

$\alpha = -1$

$-3 + \beta = -2 \Rightarrow \beta = 1$

$\gamma + 1 = 2 \Rightarrow \gamma = 1$

$D(-1, 1, 1)$

(ii) A vector having the same direction as that of $\overrightarrow{AB}$ but magnitude equal to $\overrightarrow{AC}$

Ans. $\left(\frac{6}{\sqrt{19}}(6,-1,1)\right)$

Sol. $\overrightarrow{AB} = (6, -1, 1)$ $\overrightarrow{AC} = (8, 2, 2)$

$|\overrightarrow{AB}| = \sqrt{38}$ $|\overrightarrow{AC}| = 6\sqrt{2}$

$$\text{Reqd. vector} = \frac{(6,-1,1)}{\sqrt{38}} \cdot 6\sqrt{2}$$

$$= \frac{6}{\sqrt{19}}(6, -1, 1)$$

(iii) The angle between $\overrightarrow{AC}$ and $\overrightarrow{BD}$

Ans. $\left(\frac{2\pi}{3}\right)$

Sol. $\overrightarrow{AC} = (8, 2, 2)$ $\overrightarrow{BD} = (-4,4,0)$

$$\cos\theta = \frac{\overrightarrow{AC}.\overrightarrow{BD}}{|\overrightarrow{AC}||\overrightarrow{BD}|} = -\frac{1}{2} \Rightarrow \theta = \frac{2\pi}{3}$$

19. Find out whether the following pairs of lines are parallel, non parallel; & intersecting, or non-parallel & non-intersecting.

(i) $\vec{r}_1 = \hat{\imath} + \hat{\jmath} + 2\hat{k} + \lambda(3\hat{\imath} - 2\hat{\jmath} + 4\hat{k})$

$\vec{r}_2 = 2\hat{\imath} + \hat{\jmath} + 3\hat{k} + \mu(-6\hat{\imath} + 4\hat{\jmath} - 8\hat{k})$

Ans. (parallel)

Sol. **(i)** $\vec{r}_2 = (2, 1, 3) - 2\mu\,(3, -2, 4)$

Here both lines are parallel.

(ii) $\vec{r}_1 = \hat{\imath} - \hat{\jmath} + 3\hat{k} + \lambda(\hat{\imath} - \hat{\jmath} + \hat{k})$

$\vec{r}_2 = 2\hat{\imath} + 4\hat{\jmath} + 6\hat{k} + \mu(2\hat{\imath} + \hat{\jmath} + 3\hat{k})$

Ans. (intersecting)

Sol. **(ii)** $1 + \lambda = 2 + 2\mu$ ....(1)

$-1 - \lambda = 4 + \mu$ ....(2)

$3 + \lambda = 6 + 3\mu$ ....(3)

from (1) and (2)

$3\mu = -6 \Rightarrow \mu = -2$

then $\lambda = -3$

Since $\lambda = -3$ and $\mu = -2$

satisfies (3), hence both lines are intersecting.

(iii) $\vec{r}_1 = \hat{\imath} + \hat{k} + \lambda(\hat{\imath} + 3\hat{\jmath} + 4\hat{k})$

$\vec{r}_2 = 2\hat{\imath} + 3\hat{\jmath} + \mu(4\hat{\imath} - \hat{\jmath} + \hat{k})$

Ans. (non-intersecting)

Sol. **(iii)** $1 + \lambda = 2 + 4\mu$ ...(1)
$3\lambda = 3 - \mu$ ...(2)
$1 + 4\lambda = \mu$ ...(3)

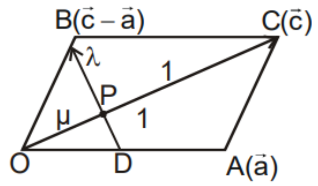
from 2 and 3 $7\lambda + 1 = 3 \Rightarrow \lambda = \frac{2}{7}, \mu = \frac{15}{7}$

since these values of λ and μ not satisfy 1 hence non-intersecting.

20. Let OACB be parallelogram with O at the origin & OC a diagonal. Let D be the mid point of OA. Using vector method prove that BD&CO intersect in the same ratio. Determine this ratio.

Ans. (2:1)

Sol.



P.V. of P are $\dfrac{\left(\frac{\lambda}{2}-1\right)\vec{a}+\vec{c}}{\lambda+1} = \dfrac{\mu\vec{c}}{\mu+1}$

$\lambda = 2, \quad \mu = \frac{1}{2}$

Here BD and CO intersect in the same ratio.

PREVIOUS YEAR

21. If the vectors $\overrightarrow{AB} = 3\vec{\imath} + 4\hat{k}$ and $\overrightarrow{AC} = 5\hat{\imath} - 2\hat{\jmath} + 4\hat{k}$ are the sides of a triangle ABC, then the length of the median through A is: **[JEE-MAIN 2013]**

(A) $\sqrt{33}$ (B) $\sqrt{45}$ (C) $\sqrt{18}$ (D) $\sqrt{72}$

Ans. **(A)**

Sol. If $\overline{AD}$ is given mediam, then

$\overrightarrow{AD} = \frac{1}{2}(\overrightarrow{AB} + \overrightarrow{AC}) = 4\hat{i} - \hat{j} + 4\hat{k}$

$\therefore \ AD = \sqrt{16+1+16}$

$= \sqrt{33}$.