Nishant Jindal

$$-\frac{1}{\sin\alpha}\int \frac{-\sin\alpha \operatorname{cosec}^2 x \, dx}{\sqrt{\cos\alpha + \sin\alpha\cot x}}$$

$$\frac{-2}{\sin\alpha}\sqrt{\cos\alpha+\sin\alpha\cot x} + c$$

$$x\cos x\left(x\cos x-\sin x\right)+x\sin x\left(x\sin x+\cos x\right)$$

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Approximation

$$f'(x) \approx \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$\boxed{f(x+\Delta x) \approx f(x) + (\Delta x)\, f'(x)}$$

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Rate Measure

$$\frac{dy}{dx} > 0 \checkmark$$

$$< 0$$

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1. Find approximate value of

$\sqrt[3]{124.9}$ using derivative.

$f(x) = x^{1/3}$

$x = 125$, $\Delta x = -0.1$

$\sqrt[3]{124.9} \approx \sqrt[3]{125} + (-0.1)\frac{1}{3}(125)^{-2/3}$

$\approx 5 - \frac{1}{750}$

$\approx$

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2. In an acute triangle ABC, if sides a, b be constants and the base angles A, B vary, then

P.T. $$\frac{dA}{\sqrt{a^2-b^2\sin^2 A}} = \frac{dB}{\sqrt{b^2-a^2\sin^2 B}}$$

$$\frac{dB}{\sqrt{b^2-a^2\sin^2 B}} = \frac{dA}{\sqrt{a^2-b^2\sin^2 A}}$$

$$a\sin B = b\sin A$$

$$a\cos B\, dB = b\cos A\, dA$$

$$\frac{dB}{b\cos A} = \frac{dA}{a\cos B} \Rightarrow \frac{dB}{b\sqrt{1-\sin^2 A}} = \frac{dA}{a\sqrt{1-\sin^2 B}}$$

$$\Rightarrow \frac{dB}{\sqrt{b^2-b^2\sin^2 A}} = \frac{dA}{\sqrt{a^2-a^2\sin^2 B}}$$

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3. Show that for curve, $y = \frac{c}{2}\left(e^{\frac{x}{c}} + e^{-\frac{x}{c}}\right)$, the length of normal at any point is $\frac{y^2}{|c|}$

$$y' = \frac{1}{\cancel{c}}\frac{\cancel{c}}{2}\left(e^{\frac{x}{c}} - e^{-\frac{x}{c}}\right)$$

$$\sqrt{(yy')^2 + y^2}$$

$$= \sqrt{y^2\left(1 + \frac{1}{4}\left(e^{\frac{x}{c}} - e^{-\frac{x}{c}}\right)^2\right)}$$

$$= \sqrt{\frac{y^2}{4}\left(e^{\frac{x}{c}} + e^{-\frac{x}{c}}\right)^2} = \sqrt{\frac{y^2}{4}\left(\frac{2y}{c}\right)^2} = \sqrt{\frac{y^4}{c^2}}$$

$$= \frac{y^2}{|c|}.$$

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4. The height of right circular cone is 20 cm and is decreasing at the rate of 4 cm/s. At the same time, its radius is 10 cm and is increasing at the rate of 2 cm/s. Find the rate of change of volume in cm^3/s at the same time.

$$V = \frac{\pi r^2 h}{3}$$

$$\frac{dV}{dt} = \frac{\pi}{3}\left(2rh\frac{dr}{dt} + r^2\frac{dh}{dt}\right) = \frac{\pi}{3}\left(2\times10\times20\times2 + (10)^2(-4)\right)$$

$$= \frac{400\pi}{3}\ cm^3/s$$

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Relative/Local Minimum of function at point $x = a$

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$$\int \frac{\sqrt{1+\cot^4 x}}{\sin x \cos x}\,dx.$$

$$\int \frac{\sqrt{1+\cot^4 x}}{\cot^4 x}\cot^3 x \,\mathrm{cosec}^2 x\,dx.$$

$$\boxed{1+\cot^4 x = t^2}$$

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$$\int \frac{e^x\left(1+(1-x^2)\right)}{(1-x)\sqrt{1-x^2}}\,dx \cdot$$

$$= \int e^x\left(\frac{1}{(1-x)\sqrt{1-x^2}} + \underbrace{\sqrt{\frac{1+x}{1-x}}}_{f(x)}\right)$$

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$\boxed{36-50}$ ✓