

$$\Delta^2 = \begin{vmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{vmatrix} \begin{vmatrix} ax_1 & ay_1 & az_1 \\ ay_1 & by_1 & bz_1 \\ az_1 & bz_1 & cz_1 \end{vmatrix} \frac{1}{abc} = \frac{1}{abc} \begin{vmatrix} d & f & f \\ f & d & f \\ f & f & d \end{vmatrix}$$

$$\downarrow R_1 \rightarrow R_1 + R_2 + R_3$$

$$\begin{vmatrix} 1+1+1 & \alpha'+\beta'+\gamma' & \alpha'^2+\beta'^2+\gamma'^2 \\ \alpha'+\beta'+\gamma' & \alpha'^2+\beta'^2+\gamma'^2 & \alpha'^3+\beta'^3+\gamma'^3 \\ \alpha'+\beta'+\gamma' & \alpha'^2+\beta'^2+\gamma'^2 & \alpha'^3+\beta'^3+\gamma'^3 \end{vmatrix} \frac{d+2f}{abc}$$

$$= \begin{vmatrix} 1 & 1 & 1 \\ \alpha' & \beta' & \gamma' \\ \alpha'^2 & \beta'^2 & \gamma'^2 \end{vmatrix} \begin{vmatrix} 1 & \alpha' & \alpha'^2 \\ 1 & \beta' & \beta'^2 \\ 1 & \gamma' & \gamma'^2 \end{vmatrix}$$

$$\begin{vmatrix} y^2 & -xy & x^2 \\ a & b & c \\ a' & b' & c' \end{vmatrix} \xrightarrow{\substack{C_2 \rightarrow yC_2 + xC_1 \\ C_3 \rightarrow yC_3 + xC_2}}$$

$$= \frac{1}{y^2} \begin{vmatrix} y^2 & 0 & 0 \\ a & by+ax & cy+bx \\ a' & b'y+a'x & c'y+b'x \end{vmatrix} = \begin{vmatrix} by+ax & cy+bx \\ b'y+a'x & c'y+b'x \end{vmatrix}$$

$\downarrow C_2 \Rightarrow yC_2 + xC_1$

$$\frac{1}{y} \begin{vmatrix} ax+by & y \\ a'x+b'y & y' \end{vmatrix}$$

$$\textcircled{3} -\cos q \textcircled{1} + \sin q \textcircled{2}$$

$$z(1 - \cos q + \sin q) = \underline{2 - \cos^2 q - \cos q + \sin q - \sin^2 q}$$

$$\boxed{z = 1}$$

$$\boxed{c=2}$$

$$\Delta =$$

$$-3z = 0$$

$$\boxed{z = 0}$$

$$(x, y, z) = \left(1, \frac{1}{2}, 1, 0\right) \cdot \frac{2x+2y-2z}{4x+4y-z} = 1$$

$$6x+6y+2z=3$$

$$\boxed{x+y = \frac{1}{2}}$$

$$\text{Circumcentre} = \left(\frac{(\sin 2A)x_1 + (\sin 2B)x_2 + (\sin 2C)x_3}{\sin 2A + \sin 2B + \sin 2C}, - \right)$$

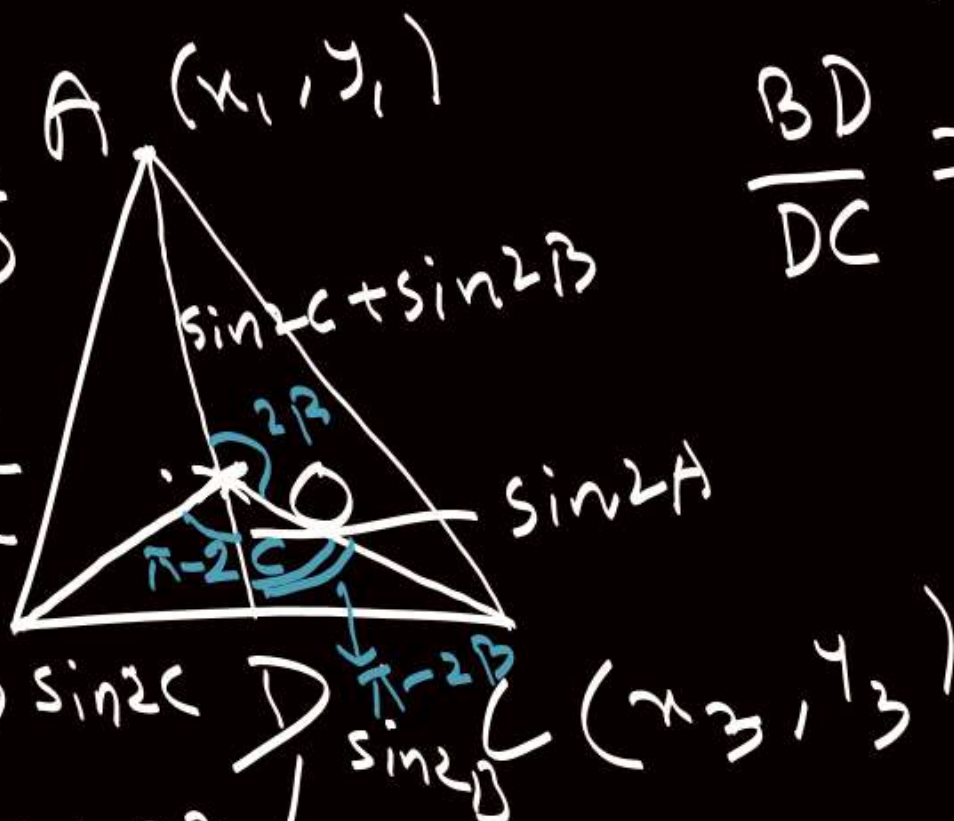
$$\frac{OA}{OD} = \frac{\Delta AOB}{\Delta BOD} = \frac{\Delta AOC}{\Delta COD}$$

$$= \frac{\Delta AOB + \Delta AOC}{\Delta BOD + \Delta COD}$$

$$= \frac{\Delta AOB + \Delta AOC}{\Delta BOC}$$

$$= \frac{\frac{1}{2}(OA)(OB)\sin 2C + \frac{1}{2}R \times R \sin 2B}{\frac{1}{2}R \times R \sin 2A}$$

$$= \frac{\sin 2C + \sin 2B}{\sin 2A}$$

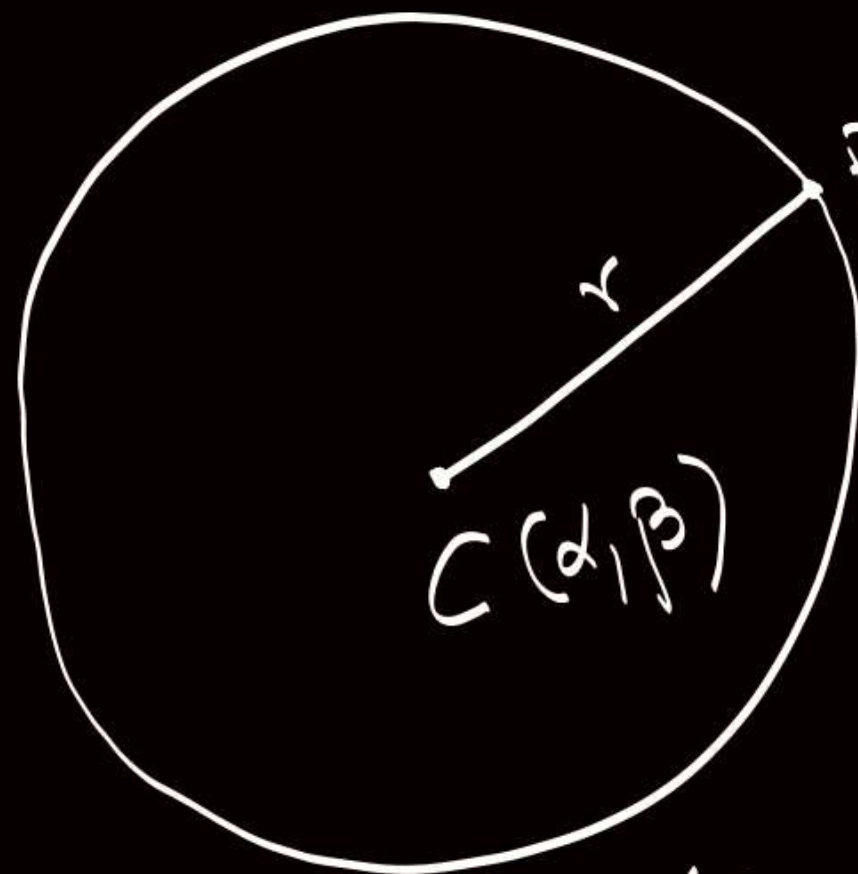


$$\frac{BD}{DC} = \frac{\Delta BOD}{\Delta COD} = \frac{\frac{1}{2}(OB)(OD)\sin(\pi - 2C)}{\frac{1}{2}(OC)(OD)\sin(\pi - 2B)}$$

$$= \frac{\sin 2C}{\sin 2B}$$

$$\left(\frac{(\sin 2B)x_2 + (\sin 2C)x_3}{\sin 2B + \sin 2C}, - \right)$$

Locus



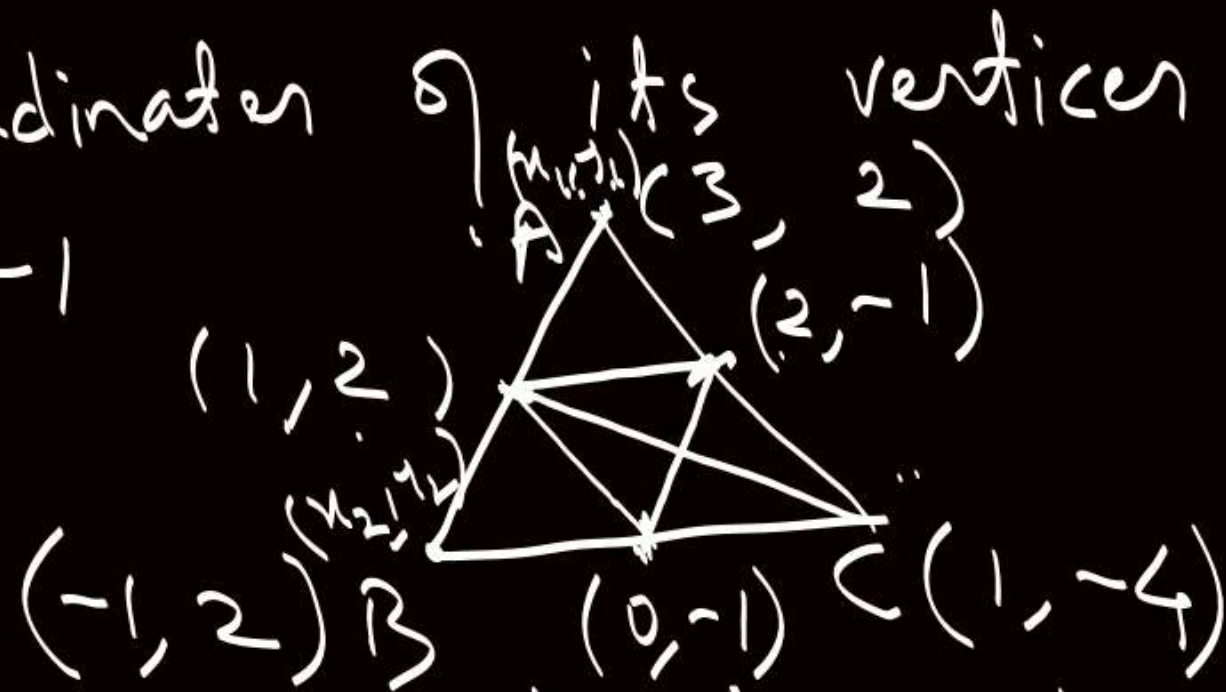
$P(h, k)$ find locus of P moving in plane
so that its distance from
[is constant $= r$.

$$(h - \alpha)^2 + (k - \beta)^2 = r^2$$

$$(x - \alpha)^2 + (y - \beta)^2 = r^2$$

1. The midpoints of the sides of a triangle are $(1, 2)$, $(0, -1)$ and $(2, -1)$. Find the coordinates of its vertices.

$$2 - 1 = y - 1$$



$$x_1 + x_2 = 2$$

$$x_2 + x_3 = 0$$

$$x_3 + x_1 = 4$$

$$x + 1 = 2 + 0$$

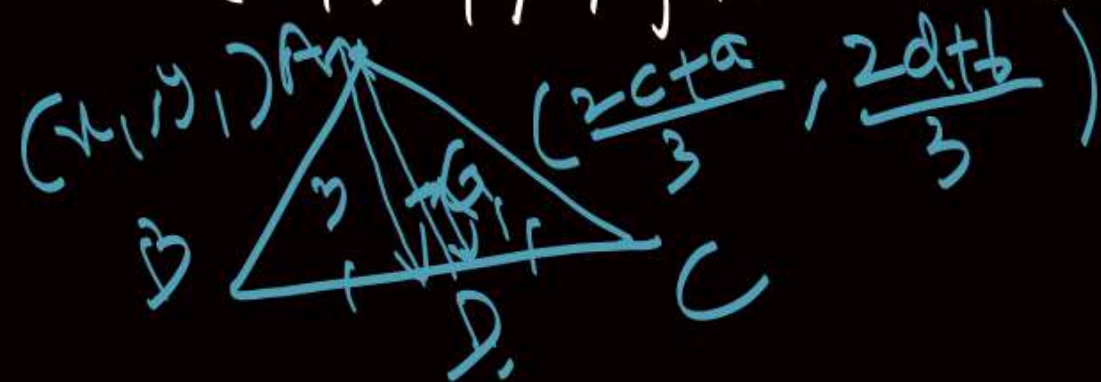
$$D = \frac{3G - 1A}{3 - 1}$$

$$D = \left(\frac{2c + a - x_1}{2}, \frac{2d + b - y_1}{2} \right)$$

$$x_1 + x_2 + x_3 = 3$$

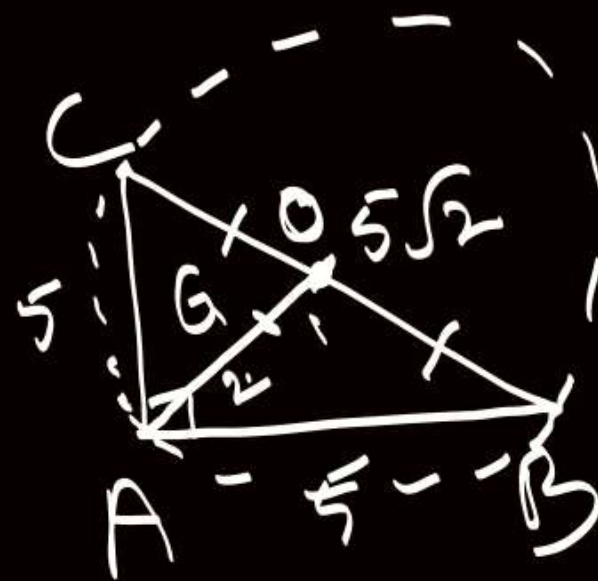
2. Orthocentre and circumcentre of $\triangle ABC$ are (a, b) & (c, d) respectively. If coordinates of A is (x_1, y_1) , find the coordinate of middle point of BC.

$$\left(\frac{2c + a}{3}, \frac{2d + b}{3} \right)$$



3. If the vertices of triangle are $A(2, -2)$, $B(-2, 1)$ and $C(5, 2)$. Find the distance b/w circumcentre and centroid.

$$\begin{aligned} AB &= 5 \\ BC &= 5\sqrt{2} \\ CA &= 5 \end{aligned}$$



$$\begin{aligned} OG &= \frac{1}{3} \left(\frac{5\sqrt{2}}{2} \right) \\ &= \frac{5\sqrt{2}}{6} \end{aligned}$$

4. If the area of triangle formed by points $(1, 2)$, $(2, 3)$, $(x, 4)$ is 40, find x .

$$\frac{1}{2} \begin{vmatrix} 1 & 2 & 1 \\ 2 & 3 & 1 \\ x & 4 & 1 \end{vmatrix} = \pm 40 \Rightarrow \boxed{x = 83, -77}$$

5. If points $(a, 0)$, $(0, b)$ and $(1, 1)$ are collinear, find

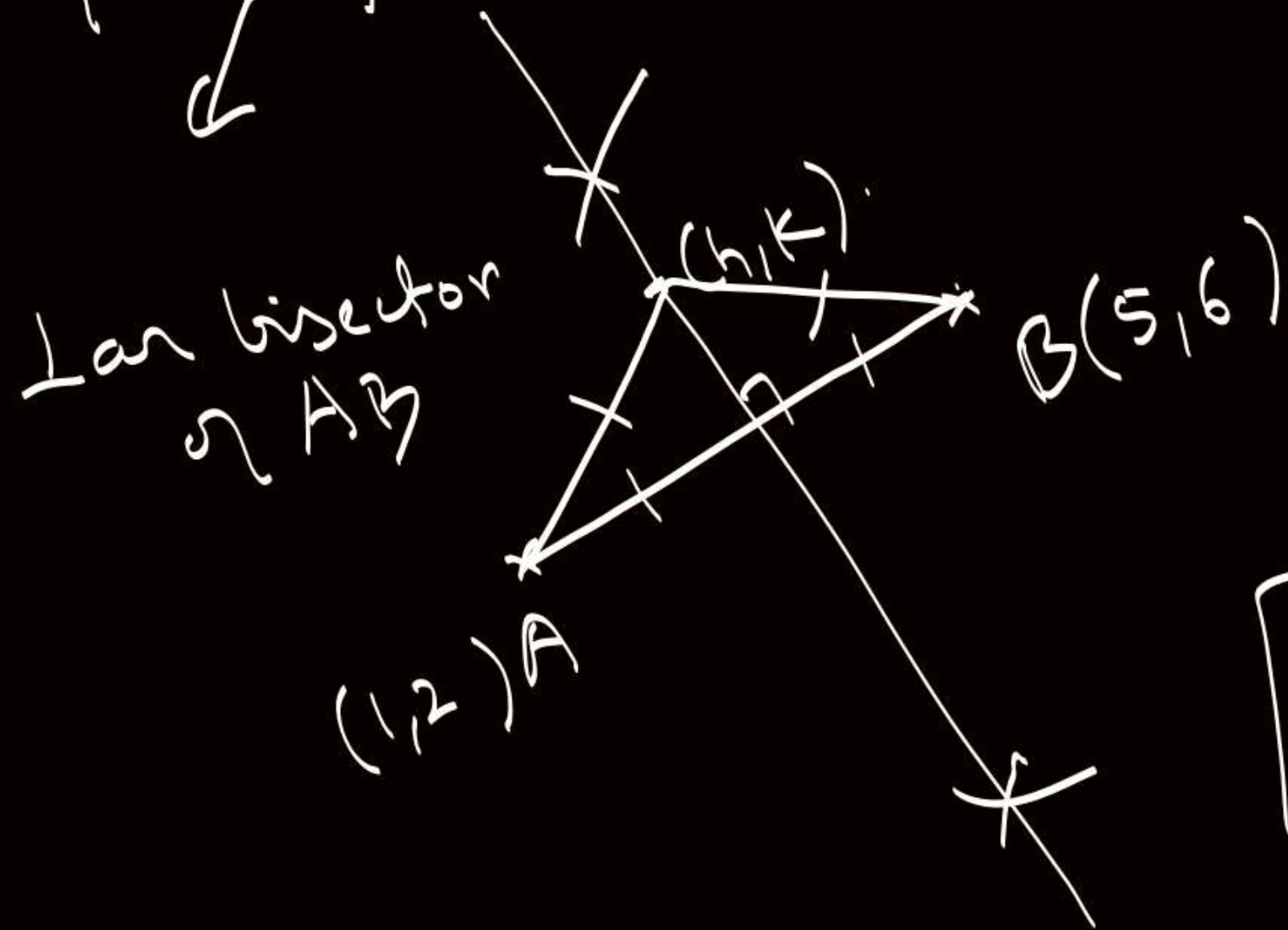
$$\frac{1}{a} + \frac{1}{b}$$

$$\begin{vmatrix} a & 0 & 1 \\ 0 & b & 1 \\ 1 & 1 & 1 \end{vmatrix} = 0 = a(b-1) - b = 0$$

$$ab = a + b$$

$$\frac{1}{a} + \frac{1}{b} = 1$$

6. $A = (1, 2)$, $B = (5, 6)$, find locus of point P such that $PA = PB$.



$$(h-1)^2 + (k-2)^2 = (h-5)^2 + (k-6)^2$$

$$8h + 8k = 56$$

$$x + y = 7$$

7. $A(1,2)$ is a fixed point. A variable point B lies on curve whose equation is $x^2 + y^2 = 4$.

Find the locus of midpoint of AB .

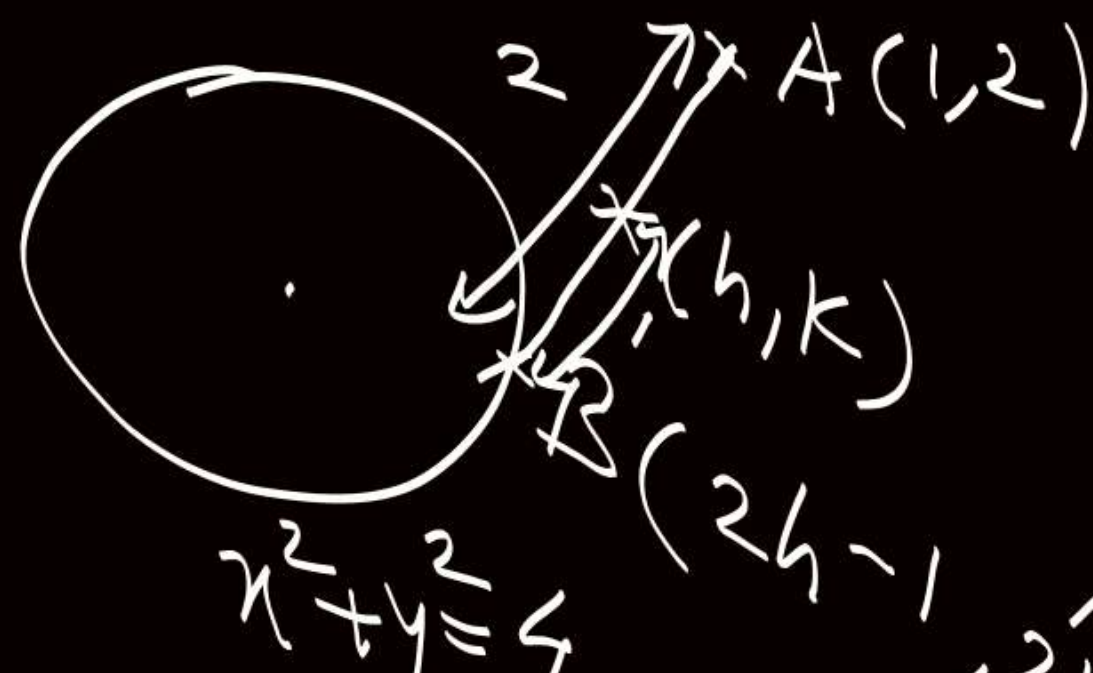


Diagram illustrating the locus of the midpoint of AB . The circle is labeled $x^2 + y^2 = 4$. The fixed point is $A(1,2)$. The variable point $B(h,k)$ lies on the circle. The midpoint of AB is labeled $(2h-1, 2k-2)$.

The locus equation is derived as follows:

$$(2h-1)^2 + (2k-2)^2 = 4$$

$$4x^2 + 4y^2 - 4x - 8y + 1 = 0$$

The final locus equation is boxed and labeled "Determinants":

$$4x^2 + 4y^2 - 4x - 8y + 1 = 0$$