

(MATHEMATICS)

DIWALI ASSIGNMENT

ROUND-01

FUNCTION

1. If $f(x) = \cos[\pi^2]x + \cos[-\pi^2]x$, where $[x] = \text{greatest integer } \leq x$, then
 (A) $f(\pi/2) = -1$ (B) $f(\pi) = 1$ (C) $f(-\pi) = -1$ (D) $f(\pi/4) = 2$
2. $f: (-\pi/2, \pi/2) \rightarrow (-\infty, \infty)$, $f(x) = \tan x$ is
 (A) onto but not one-one (B) one-one but not onto
 (C) one-one onto (D) neither one-one nor onto
3. The domain of the function $f(x) = \frac{\sin^{-1}(x-3)}{\sqrt{9-x^2}}$ is
 (A) $[1, 2]$ (B) $[2, 3]$ (C) $(1, 2)$ (D) $[2, 3]$
4. If x be real then the range of the function $f(x) = \frac{x}{1+x^2}$ is
 (A) $[-1/2, 1/2]$ (B) $(-2, 2)$ (C) $(-1, 1)$ (D) $(-1/2, 1/2)$
5. If $g(x) = x^2 + x - 2$ and $\frac{1}{2}(g \circ f)(x) = 2x^2 - 5x + 2$, then $f(x)$ is equal to
 (A) $2x - 3$ (B) $2x + 3$ (C) $2x^2 + 3x + 1$ (D) $2x^2 - 3x - 1$

LIMITS

6. $\lim_{x \rightarrow \pi/4} \frac{\int_2^{\sec^2 x} f(t) dt}{x^2 - \pi^2/16}$ is equal to
 (A) $\frac{8}{\pi} f(2)$ (B) $\frac{2}{\pi} f(2)$ (C) $\frac{2}{\pi} f\left(\frac{1}{2}\right)$ (D) $4f(2)$
7. For $a \in \mathbb{R}$, $a \neq -1$,

$$\lim_{n \rightarrow \infty} \frac{1^a + 2^a + \dots + n^a}{(n+1)^{a-1}[(na+1) + (na+2) + \dots + (na+n)]} = \frac{1}{60}.$$
 Then a is equal to
 (A) 5 (B) 7 (C) $-15/2$ (D) $-17/2$
8. $\lim_{x \rightarrow 0} \frac{\tan x - \sin x}{x^3}$ is equal to
 (A) $1/2$ (B) -1 (C) 2 (D) -2
9. $\lim_{x \rightarrow 0} \frac{[\sin(x+a) + \sin(a-x) - 2\sin a]}{x \sin x}$ is equal to
 (A) $\sin a$ (B) $-\sin a$ (C) 1 (D) 0
10. If $f(x)$ is differentiable and strictly increasing function, then $\lim_{x \rightarrow 0} \frac{f(x^2) - f(x)}{f(x) - f(0)}$ is equal to
 (A) 1 (B) 0 (C) 2 (D) -1
11. $\lim_{x \rightarrow 0} \left(\frac{\tan x}{x}\right)^{1/x^2}$ is equal to
 (A) e^3 (B) $e^{1/3}$ (C) 1 (D) e

CONTINUITY

12. If $f(x) = \begin{cases} \frac{\log(1+ax) - \log(1-bx)}{x} & , x \neq 0 \\ k & , x = 0 \end{cases}$ is continuous at $x = 0$, then the value of k is
 (A) 0 (B) $a + b$ (C) $a - b$ (D)
13. If $f(x) = \begin{cases} x \sin\left(\frac{1}{x}\right) & , x \neq 0 \\ k & , x = 0 \end{cases}$ is continuous at $x = 0$, then the value of k will be
 (A) 1 (B) -1 (C) 0 (D) none of these
14. If $f(x) = \begin{cases} x^\alpha \cos\left(\frac{1}{x}\right) & , x \neq 0 \\ 0 & , x = 0 \end{cases}$ is continuous at $x = 0$, then
 (A) $\alpha < 0$ (B) $\alpha > 0$ (C) $\alpha = 0$ (D) $\alpha \geq 0$
15. If $f(x) = \begin{cases} -2\sin x & , x \leq -\pi/2 \\ a\sin x + b & , -\pi/2 < x < \pi/2 \\ \cos x & , x \geq \pi/2 \end{cases}$ is a continuous function, then
 (A) $a = 1, b = 1$ (B) $a = -1, b = 1$ (C) $a = 1, b = -1$ (D) $a = b = -1$
16. Let $f(x) = [x] \sin\left(\frac{\pi}{[x+1]}\right)$ where $[x]$ denotes the greatest integer function. The set of points of discontinuity of f in its domain is
 (A) \mathbb{Z} (B) \mathbb{Z}_0 (C) \mathbb{N} (D) none of these
17. Function $f: \mathbb{R}/\{0\} \rightarrow \mathbb{R}, f(x) = \frac{1}{x} - \frac{2}{e^{2x}-1}$ can be made continuous at $x = 0$ by defining $f(0)$ as
 (A) 1 (B) 2 (C) -1 (D) 0

DIFFERENTIATION

18. Let $f: (-1, 1) \rightarrow \mathbb{R}$ be a differentiable function with $f(0) = -1$ and $f'(0) = 1$.
 Let $g(x) = \{f(2f(x) + 2)\}^2$. Then $g'(0)$ is equal to
 (A) -2 (B) 4 (C) -4 (D) 0
19. If $f(x) = \begin{cases} xe^{-\left(\frac{1}{|x|} + \frac{1}{x}\right)} & , x \neq 0 \\ 0 & , x = 0 \end{cases}$, then $f(x)$ is
 (A) discontinuous everywhere
 (B) continuous as well as differentiable for all x
 (C) continuous for all x but not differentiable at $x = 0$
 (D) neither differentiable nor continuous at $x = 0$

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20. $\frac{d}{dx} \left[\tan^{-1} \left(\frac{\cos x}{1 + \sin x} \right) \right]$ equals
 (A) $\frac{1}{2}$ (B) $-1/2$ (C) 1 (D) -1
21. If $x^p y^q = (x + y)^{p+q}$, then $\frac{dy}{dx}$ equals
 (A) y/x (B) x/y (C) $(x + y)/x$ (D) $(x + y)/y$
22. If $x^2 + y^2 = t - \frac{1}{t}$, $x^4 + y^4 = t^2 + \frac{1}{t^2}$, then $\frac{dy}{dx}$ equals
 (A) $\frac{1}{x^2 y}$ (B) $\frac{1}{xy^3}$ (C) $-\frac{1}{x^3 y}$ (D) $-\frac{1}{xy^3}$

TANGENTS AND NORMALS

23. The equation of the tangent to the curve $y = be^{-xa}$ at the point where it meets y-axis is
 (A) $\frac{x}{b} + \frac{y}{a} = 1$ (B) $\frac{x}{a} + \frac{y}{b} = 1$ (C) $\frac{x}{b} + \frac{y}{a} = 2$ (D) $\frac{x}{a} + \frac{y}{b} = 2$
24. The sum of the squares of intercepts on axes made by a tangent at any point on the curve $x^{2n} + y^{2/3} = a^{2/3}$ is
 (A) a (B) $2a$ (C) a^2 (D) $2a^2$
25. The normal to the curve $x = a(\cos \theta + \theta \sin \theta)$, $y = a(\sin \theta - \theta \cos \theta)$ at any point ' θ '
 (A) makes angle $(\pi/2 + \theta)$ with x-axis.
 (B) passes through the origin.
 (C) is at a constant distance from origin.
 (D) passes through the point $(a\pi/2, -a)$.
26. The angle of intersection of the curves $y = 4 - x^2$ and $y = x^2$ is
 (A) $\pi/2$ (B) $\tan^{-1}(4/3)$ (C) $\tan^{-1}(4\sqrt{2}/7)$ (D) none of these
27. If the line $\frac{x}{a} + \frac{y}{b} = 2$ touches the curve $\left(\frac{x}{a}\right)^n + \left(\frac{y}{b}\right)^n = 2$ at point (a, b) , then n is equal to
 (A) 1 (B) 2 (C) 3 (D) all non-zero values
28. The normal to the curve at $P(x, y)$ meets the x-axis at G. If the distance of G from the origin is twice the abscissa of P, then the curve is a
 (A) circle (B) parabola (C) ellipse (D) hyperbola

MONOTONICITY

29. Function $f(x) = 2x^3 - 9x^2 + 12x + 29$ is monotonic decreasing when
 (A) $x < 2$ (B) $x > 2$ (C) $x > 3$ (D) $1 < x < 2$

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30. $f(x) = 2x^2 - \log|x|$ ($x \neq 0$) is monotonic increasing in the interval
 (A) $(1/2, \infty)$ (B) $(-\infty, -1/2) \cup (1/2, \infty)$
 (C) $(-\infty, -1/2) \cup (0, 1/2)$ (D) $(-1/2, 0) \cup (1/2, \infty)$
31. Function $f(x) = x^2 e^{-x}$ is monotonic increasing when
 (A) $x \in \mathbb{R} - [0, 2]$ (B) $0 < x < 2$ (C) $2 < x < \infty$ (D) $x < 0$
32. If $f(x) = x e^{x(1-x)}$, then $f(x)$ is
 (A) increasing in $[-1/2, 1]$ (B) decreasing in \mathbb{R}
 (C) increasing in \mathbb{R} (F) decreasing in $[-1/2, 1]$
33. Function $f(x) = \int_{x^2}^{x^2+1} e^{-t^2} dt$ is increasing in
 (A) $(2, 2)$ (B) $(0, \infty)$ (C) $(-\infty, 0)$ (D) no where

MAXIMA AND MINIMA

34. $f(x) = 2x^3 - 21x^2 + 36x + 7$ has
 (A) a local maxima at $x = 1$ and minima at $x = 6$.
 (B) a local maxima at $x = 6$ and minima at $x = 1$.
 (C) a local maxima at $x = 1$ and no local minima.
 (D) a local minima at $x = 6$ and no local maxima.
35. The maximum value of $\sin x + \cos x$ is
 (A) 1 (B) 2 (C) $\sqrt{2}$ (D) none of these
36. The maximum and minimum values of $\sin x - x$ are
 (A) 1, -1 (B) $\frac{3\sqrt{3}-\pi}{6}, \frac{\pi-3\sqrt{3}}{6}$ (C) $\frac{\pi-3\sqrt{3}}{6}, \frac{3\sqrt{3}-\pi}{6}$ (D) do not exist
37. The height of a cylinder of maximum volume inscribed in a sphere of radius a is
 (A) $a/\sqrt{3}$ (B) $2a/\sqrt{3}$ (C) $\sqrt{3}a$ (D) $2\sqrt{3}a$
38. The height of a right circular cone of maximum volume inscribed in a sphere of diameter a is
 (A) $(2/3)a$ (B) $(3/4)a$ (C) $(1/3)a$ (D) $(1/4)a$
39. The semi-vertical angle of a right circular cone of given slant height and maximum volume is
 (A) $\tan^{-1} 2$ (B) $\tan^{-1} \sqrt{2}$ (C) $\tan^{-1} 1/2$ (D) $\tan^{-1} 1/\sqrt{2}$
40. A rectangular sheet of fixed perimeter with side having their lengths in the ratio 8: 15 is converted into an open rectangular box by folding after removing squares of equal area from all four corners. If the total area of removed squares is 100, the resulting box has maximum volume, then the lengths of sides of the rectangular sheet are
 (A) 24 (B) 32 (C) 45 (D) 60

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INTEGRATION

41. $\int \frac{\sqrt{\tan x}}{\sin x \cos x} dx$ equals
 (A) $2\sqrt{\sec x} + c$ (B) $2\sqrt{\tan x} + c$ (C) $\frac{2}{\sqrt{\tan x}} + c$ (D) $\frac{2}{\sqrt{\sec x}} + c$
42. $\int \frac{e^x(1+x)}{\cos^2(xe^x)} dx$ equals
 (A) $\tan(xe^x) + c$ (B) $\cot(xe^x) + c$ (C) $\tan(e^x) + c$ (D) $\cot(e^x) + c$
43. $\int \frac{x}{1+\cos x} dx$ is equal to
 (A) $x \tan \frac{x}{2} + \log \cos \frac{x}{2} + c$ (B) $x \tan \frac{x}{2} + 2 \log \cos \frac{x}{2} + c$
 (C) $x \tan \frac{x}{2} - 2 \log \cos \frac{x}{2} + c$ (D) none of these
44. $\int e^x \left(\frac{1+\sin x}{1+\cos x} \right) dx$ is equal to
 (A) $e^x \tan \frac{x}{2} + c$ (B) $e^x \cot \frac{x}{2} + c$ (C) $e^x \tan x + c$ (D) $e^x \cot x + c$
45. $\int \frac{dx}{x(x^n+1)}$ is equal to
 (A) $\log \left(\frac{x^n}{x^n+1} \right) + c$ (B) $\log \left(\frac{x^n+1}{x^n} \right) + c$
 (C) $\frac{1}{n} \log \left(\frac{x^n}{x^n+1} \right) + c$ (D) $\frac{1}{n} \log \left(\frac{x^n+1}{x^n} \right) + c$
46. $\int \left\{ \frac{\log x - 1}{1+(\log x)^2} \right\}^2 dx$ is equal to
 (A) $\frac{x}{(\log x)^2+1} + c$ (B) $\frac{xe^x}{1+x^2} + c$ (C) $\frac{x}{x^2+1} + c$ (D) $\frac{\log x}{(\log x)^2+1} + c$
47. $\int \sec^{2/3} x \operatorname{cosec}^{4/3} x dx$ is equal to
 (A) $-3(\cot x)^{1/3} + c$ (B) $-3(\tan x)^{-2/3} + c$
 (C) $3(\operatorname{cosec} x)^{1/3} + c$ (D) $3(\cos 2x)^{1/3} + c$

DEFINITE INTEGRAL

48. The value of the integral $\int_{-4}^4 (ax^3 + bx + c) dx$ depends on
 (A) b and c (B) a, b and c (C) only c (D) a and c
49. $\int_0^\pi \frac{x \sin x}{1+\cos^2 x} dx$ equals
 (A) 0 (B) $\pi/4$ (C) $\pi^2/4$ (D) $\pi^2/2$
50. $\int_2^3 \frac{\sqrt{x}}{\sqrt{5-x}+\sqrt{x}} dx$ equals
 (A) $3/2$ (B) $1/2$ (C) $1/4$ (D) 1
51. $\int_0^{2\pi} \frac{x \sin^{2n} x}{\sin^{2n} x + \cos^{2n} x} dx$ ($n \in \mathbb{N}$) is equal to
 (A) π^2 (B) $2\pi^2$ (C) π (D) 2π

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52. If $f(x) = \begin{cases} e^{\cos x} \sin x & , |x| \leq 2 \\ 2, & \text{otherwise} \end{cases}$, then $\int_{-2}^3 f(x) dx$ is equal to
 (A) 0 (B) 1 (C) 2 (D) 3
53. Let $\frac{d}{dx} F(x) = \frac{e^{\sin x}}{x}$, $x > 0$. If $\int_1^4 \frac{3}{x} e^{\sin x^3} dx = F(k) - F(1)$, then one of the possible value of k, is
 (A) 15 (B) 16 (C) 63 (D) 64
54. $\lim_{n \rightarrow \infty} \left[\frac{1}{n^2} \sec^2 \frac{1}{n^2} + \frac{2}{n^2} \sec^2 \frac{4}{n^2} + \dots + \frac{1}{n} \sec^2 1 \right]$ equals
 (A) $\frac{1}{2} \tan 1$ (B) $\tan 1$ (C) $\frac{1}{2} \operatorname{cosec} 1$ (D) $\frac{1}{2} \sec 1$

AREA UNDER CURVE

55. The area between the parabola $y^2 = 4ax$ and its latus rectum is
 (A) $(8/3)a$ (B) $(8/3)a^2$ (C) $(4/3)a$ (D) $(4/3)a^2$
56. The area bounded by the circle $x^2 + y^2 = 4$, the line $x = \sqrt{3}y$ and x-axis lying in the first quadrant is
 (A) π (B) $\pi/2$ (C) $\pi/3$ (D) $2\pi/3$
57. The area between the curves $y = x$ and $y = x^3$ is
 (A) $1/4$ (B) $1/2$ (C) $1/3$ (D) 1
58. The area bounded by the curves $y = e^x$, $y = e^{-x}$ and $y = 2$ is
 (A) $\log(16/e)$ (B) $\log(4/e)$ (C) $2\log(4/e)$ (D) none of these
59. The area of the smaller portion between curves $x^2 + y^2 = 8$ and $y^2 = 2x$ is
 (A) $\pi + 2/3$ (B) $2\pi + 2/3$ (C) $2\pi + 4/3$ (D) $\pi + 4/3$

DIFFERENTIAL EQUATIONS

60. The order and degree of differential equation
 $\left[1 + \left(\frac{dy}{dx} \right)^2 \right]^{3/2} = k \frac{d^2y}{dx^2}$ are
 (A) 2,2 (B) 2,3 (C) 2,1 (D) 1,6
61. Solution of the differential equation
 $\sec^2 x \tan y dx + \sec^2 y \tan x dy = 0$ is
 (A) $\tan x \sec y = c$ (B) $\tan x \tan y = c$
 (C) $\tan x = c \tan(x + y)$ (D) $\tan x = c \tan(x - y)$

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62. Solution of differential equation $\frac{dy}{dx} = e^{x-y} + x^2 e^{-y}$ is
 (A) $e^y = e^x + \frac{1}{3}x^3 + c$ (B) $y = e^x + \frac{1}{3}x^3 + c$
 (C) $e^{-y} = e^x + \frac{1}{3}x^3 + c$ (D) none of these
63. Solution of differential equation $\frac{dy}{dx} + y \sec^2 x = \tan x \sec^2 x$ is
 (A) $y = \tan x - 1 + ce^{-\tan x}$ (B) $y^2 = \tan x - 1 + ce^{\tan x}$
 (C) $ye^{\tan x} = \tan x - 1 + c$ (D) $ye^{-\tan x} = \tan x - 1 + c$
64. Solution of differential equation $(1 + y^2)dx + (x - e^{\tan^{-1}y})dy = 0$ is
 (A) $ye^{\tan^{-1}x} = \tan^{-1}x + c$ (B) $xe^{\tan^{-1}y} = \frac{1}{2}e^{2\tan^{-1}y} + c$
 (C) $2x = e^{\tan^{-1}y} + c$ (D) $y = xe^{-\tan^{-1}x} + c$
65. The order and degree of the differential equation of all parabolas with their axes as x-axis will be
 (A) 1,2 (B) 2,2 (C) 3,2 (D) 2,1
66. Solution of differential equation $x \frac{dy}{dx} = y(\log y - \log x + 1)$ is
 (A) $y + e^x = 0$ (B) $y = xe^{cx}$ (C) $y + xe^{cx} = 0$ (D) none of these

VECTORS

67. If $2\mathbf{i} - \mathbf{j} + \mathbf{k}$, $\mathbf{i} - 3\mathbf{j} - 5\mathbf{k}$ and $3\mathbf{i} - 4\mathbf{j} - 4\mathbf{k}$ are position vectors of the vertices of a triangle then this triangle is
 (A) equilateral (B) isosceles
 (C) right angled isosceles (D) right angled
68. If two adjacent sides of a parallelogram are represented by vectors $\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$ and $-3\mathbf{i} + 2\mathbf{j} + \mathbf{k}$, then its area is
 (A) $5\sqrt{6}$ (B) $6\sqrt{2}$ (C) $6\sqrt{5}$ (D) 180
69. $\mathbf{a} \cdot [(\mathbf{b} + \mathbf{c}) \times (\mathbf{a} + \mathbf{b} + \mathbf{c})]$ equals
 (A) 0 (B) $[\mathbf{a} \ \mathbf{b} \ \mathbf{c}] + [\mathbf{b} \mathbf{c} \ \mathbf{a}]$
 (C) $[\mathbf{a} \ \mathbf{b} \ \mathbf{c}]$ (D) none of these
70. If $\mathbf{a}, \mathbf{b}, \mathbf{c}$ are coplanar unit vectors, then $[2\mathbf{a} - \mathbf{b} \ 2\mathbf{b} - \mathbf{c} \ 2\mathbf{c} - \mathbf{a}]$ is equal to
 (A) 0 (B) 1 (C) $-\sqrt{3}$ (D) $\sqrt{3}$
71. Let $\mathbf{a} = 2\mathbf{i} + \mathbf{j} - 2\mathbf{k}$ and $\mathbf{b} = \mathbf{i} + \mathbf{j}$. If \mathbf{c} is a vector such that $\mathbf{a} \cdot \mathbf{c} = |\mathbf{c}|$, $|\mathbf{c} - \mathbf{a}| = 2\sqrt{2}$ and the angle between $\mathbf{a} \times \mathbf{b}$ and \mathbf{c} is 30° , then $|(\mathbf{a} \times \mathbf{b}) \times \mathbf{c}|$ is equal to
 (A) $2/3$ (B) $3/2$ (C) 2 (D) 3

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72. Let $u = i + j$, $v = i - j$ and $w = i + 2j + 3k$. If \hat{n} is a unit vector such that $u \cdot \hat{n} = 0$ and $v \cdot \hat{n} = 0$, then $|w \cdot \hat{n}|$ is equal to
 (A) 3 (B) 0 (C) 1 (D) 2
73. If C is the middle point of AB and P is any point outside AB, then
 (A) $\overrightarrow{PA} + \overrightarrow{PB} = \overrightarrow{PC}$ (B) $\overrightarrow{PA} + \overrightarrow{PB} = 2\overrightarrow{PC}$
 (C) $\overrightarrow{PA} + \overrightarrow{PB} + \overrightarrow{PC} = \vec{0}$ (D) $\overrightarrow{PA} + \overrightarrow{PB} + 2\overrightarrow{PC} = \vec{0}$

3D-GEOMETRY

75. If a line makes α, β, γ angles with coordinate axes, then $\cos^2\alpha + \cos^2\beta + \cos^2\gamma$ is equal to
 (A) -2 (B) -1 (C) 1 (D) 2
76. If dc's of two lines satisfy $l + m + n = 0$ and $l^2 + m^2 - n^2 = 0$, then angle between these lines is
 (A) $2\pi/3$ (B) $\pi/6$ (C) $\pi/2$ (D) none of these
77. If a line makes $\alpha, \beta, \gamma, \delta$ angles with four diagonals of a cube, then $\cos^2\alpha + \cos^2\beta + \cos^2\gamma + \cos^2\delta$ is equal to
 (A) 1 (B) $2/3$ (C) $4/3$ (D) none of these

PLANE STRAIGHT LINE & SPHERE

78. Equation of the plane through point (4,2,4) and perpendicular to planes $2x + 5y + 4z + 1 = 0$ and $4x + 7y + 6z + 2 = 0$ is
 (A) $x + 2y - 3z + 4 = 0$ (B) $x + 2y - 3z - 4 = 0$
 (C) $x - 2y + 3z + 4 = 0$ (D) none of these
79. Equation of the plane passing through the point (4,3,7) and through the line $\frac{x-1}{5} = \frac{y+2}{6} = \frac{z-3}{4}$ will be
 (A) $4x + 8y + 7z = 41$ (B) $4x - 8y + 7z = 41$
 (C) $4x - 8y - 7z = 41$ (D) $4x - 8y + 7z = 39$
80. Equation of the plane containing lines $\frac{x-5}{4} = \frac{y-7}{4} = \frac{z+3}{-5}$ and $\frac{x-8}{7} = \frac{y-4}{1} = \frac{z-5}{3}$ is
 (A) $17x - 47y - 24z + 172 = 0$ (B) $17x + 47y - 24z + 172 = 0$
 (C) $17x + 47y + 24z + 172 = 0$ (D) $17x - 47y + 24z + 172 = 0$
81. The distance between point $(-1, -5, -10)$ and the point of intersection of line $\frac{x-2}{3} = \frac{y+1}{4} = \frac{z-2}{12}$ and the plane $x - y + z = 5$ is
 (A) 10 (B) 8 (C) 21 (D) 13

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82. The vector equation of the plane $r = (1 + s - t)i + (2 - s)j + (3 - 2s + 2t)k$ in scalar product form is

(A) $r \cdot (i + 2j) = 5$ (C) $r \cdot (2i + k) = 5$
(B) $r \cdot (i + 2k) = 5$ (D) none of these

83. Two lines

$\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4}$ and $\frac{x-3}{1} = \frac{y-k}{2} = \frac{z}{1}$ intersect at a point, then k is equal to
(A) $3/2$ (B) 2 (C) $9/2$ (D) $2/9$

PROBABILITY

84. A coin is tossed 4 times. The probability of showing tail at least once will be
(A) $15/16$ (B) $1/16$ (C) $1/4$ (D) $3/4$
85. If A, B, C can hit a target 4 times in 5 shots, 3 times in 4 shots and 2 times in 3 shots respectively, then the probability that exactly two of them will hit the target is
(A) $13/30$ (B) $5/6$ (C) $17/30$ (D) none of these
86. A speaks truth in 75% cases and B in 80% cases. What is the probability that they contradict each other in stating the same fact?
(A) $7/20$ (B) $13/20$ (C) $3/20$ (D) $1/5$
87. The probability that a leap year has 53 Sundays is
(A) $2/7$ (B) $3/5$ (C) $2/3$ (D) $1/7$
88. For three events A, B and C, if
P (Happening of exactly A or B) = p
P (Happening of exactly B or C) = p
P (Happening of exactly C or A) = p
P (happening A, B, C together) = p^2 ,
where $0 < p < 1/2$; then probability of happening of atleast one of A, B, C is
(A) $\frac{3p+2p^2}{2}$ (B) $\frac{p+3p^2}{4}$
(C) $\frac{p+3p^2}{2}$ (D) $\frac{3p+2p^2}{4}$
89. Two dice are thrown together 4 times. The probability that both dice will show same numbers twice is
(A) $1/3$ (B) $25/36$
(C) $25/216$ (D) none of these

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90. The probability distribution of a variate X is as follows:

$X:$	1	2	3	4	5	6	7	8
$P(X):$	0.15	0.23	0.12	0.10	0.20	0.08	0.07	0.05

If $E = \{X \text{ is a prime number}\}$ and $F = \{X < 4\}$, then $P(E \cup F)$ is equal to

- (A) 0.35 (B) 0.77 (C) 0.87 (D) 0.50

MATHEMATICAL REASONING

91. Which one of the following statements is a tautology?

- (A) $p \wedge q \equiv p \vee q$ (B) $(p \wedge q) \vee r \Leftrightarrow (p \vee q) \wedge r$
 (C) $p \Rightarrow q \Leftrightarrow (\sim q) \Rightarrow (\sim p)$ (D) none of these

92. The contrapositive statement of the statement $(\sim p \wedge q) \Rightarrow (q \wedge \sim r)$ is

- (A) $(\sim p \vee r) \Rightarrow (\sim p \wedge \sim r)$ (B) $(\sim q \vee r) \Rightarrow (\sim p \vee q)$
 (C) $(p \vee \sim q) \Rightarrow (\sim q \vee p)$ (D) $(\sim q \vee r) \Rightarrow (p \vee \sim q)$

93. The statement equivalent to $(\sim p \wedge q) \vee (\sim q)$ is

- (A) $\sim (p \vee q)$ (B) $\sim (p \wedge q)$ (C) $p \vee q$ (D) $p \wedge q$

94. The negation of $p \rightarrow (\sim p \vee q)$ is

- (A) $p \vee (p \vee \sim q)$ (B) $p \rightarrow q$ (C) $p \wedge \sim q$ (D) $p \rightarrow \sim (p \vee q)$

SETS & RELATIONS

95. If $A = \{0, \phi, \{\phi\}\}$, then

- (A) $\phi \in P(A)$ (B) $\{\phi\} \in P(A)$ (C) $\{\{\phi\}\} \in P(A)$ (D) $0 \in P(A)$

96. Two finite sets have m and n elements. If total number of subsets of first set is 56 more than that of second set, then (m, n) is equal to

- (A) (7,6) (B) (6,3) (C) (5,1) (D) (8,7)

97. Let R be a relation defined as $R = \{(a, b) \mid a \leq b\}$ where a, b are real numbers. Then relation R is

- (A) reflexive, symmetric and transitive.
 (B) reflexive and transitive but not symmetric.
 (C) symmetric and transitive but not reflexive.
 (D) symmetric but neither reflexive nor transitive.

(MATHEMATICS)

DIWALI ASSIGNMENT

98. Let R be a relation defined on N as follows:

$$'xRy \Leftrightarrow x + 2y = 10'$$

Then range of R is

- (A) N (B) {1,2,3,4} (C) {2,3,4,6} (D) {10,12,14, ...}

99. Let R be the real line. Consider the following subsets of the plane $R \times R$:

$$S = \{(x, y) : y = x + 1 \text{ and } 0 < x < 2\}$$

$$T = \{(x, y) : x - y \text{ is an integer}\}$$

Which one of the following is true?

- (A) Both S and T are equivalence relations on R.
 (B) S is an equivalence relation on R but T is not.
 (C) T is an equivalence relation on R but S is not.
 (D) Neither S nor T is an equivalence relation on R

DETERMINANTS

100. $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$ is negative when

- (A) a, b, c are positive (B) a, b, c are negative
 (C) a, b, c are positive and unequal (D) never

101. If x, y, z are positive numbers, then value of the determinant $\begin{vmatrix} 1 & \log_x y & \log_x z \\ \log_y x & 1 & \log_y z \\ \log_z x & \log_z y & 1 \end{vmatrix}$ is equal to

- (A) 0 (B) 3 (C) \log_{xyz} (D) none of these

102. If for a fixed integer n, $\Delta = \begin{vmatrix} n! & (n+1)! & (n+2)! \\ (n+1)! & (n+2)! & (n+3)! \\ (n+2)! & (n+3)! & (n+4)! \end{vmatrix}$, then $\left(\frac{\Delta}{(n!)^3} - 4\right)$ is divisible by

- (A) (n+1) (B) n (C) (n+2) (D) none of these

103. If $\Delta_1 = \begin{vmatrix} x & b & b \\ a & x & b \\ a & a & x \end{vmatrix}$ and $\Delta_2 = \begin{vmatrix} x & b \\ a & x \end{vmatrix}$, then

- (A) $\Delta_1 = 3\Delta_2^2$ (B) $\frac{d}{dx}(\Delta_1) = 3\Delta_2^2$
 (C) $\frac{d}{dx}(\Delta_1) = 3\Delta_2$ (D) none of these

104. If system of equations $x + 4ay + az = 0$, $x + 3by + bz = 0$, $x + 2cy + cz = 0$ has a non-zero solution, then a, b, c are in

- (A) AP (B) GP (C) HP (D) none of these

(MATHEMATICS)

DIWALI ASSIGNMENT

105. Consider the system of equations

$$x - 2y + 3z = -1 \quad -x + y - 2z = k \quad x - 3y + 4z = 1$$

Statement I: The system of equations has no solution for $k \neq 3$, and

Statement II: The determinant $\begin{vmatrix} 1 & 3 & -1 \\ -1 & -2 & k \\ 1 & 4 & 1 \end{vmatrix} \neq 0$ for $k \neq 3$.

In the following correct answer is

- (A) Statement I is true, Statement II is true, Statement II is a correct explanation for Statement I.
- (B) Statement I is true, Statement II is true; Statement II is not a correct explanation for Statement I.
- (C) Statement I is true, Statement II is false.
- (D) Statement I is false, Statement II is true.

106. Let a, b, c be any real numbers. Suppose that there are real numbers x, y, z not all zero such that and

$$x = cy + bz$$

$$y = az + cx$$

$$z = bx + ay$$

Then $a^2 + b^2 + c^2 + 2abc$ is equal to

- (A) -1
- (B) 0
- (C) 1
- (D) 2

MATRICES

107. If $A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$ and $A^2 - 4A - nI = 0$, then n is equal to

- (A) 3
- (B) -3
- (C) $1/3$
- (D) $-1/3$

108. If $A = \begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix}$, then $|A||\text{adj}A|$ is equal to

- (A) a^3
- (B) a^6
- (C) a^9
- (D) a^{27}

109. If A is a square matrix and B is a nonsingular matrix of the same order, then $\det(B^{-1}AB)$ equals

- (A) $\det(A)$
- (B) $\det(B)$
- (C) $\det(A^{-1})$
- (D) $\det(B^{-1})$

110. If $A = \begin{bmatrix} \alpha & 2 \\ 2 & \alpha \end{bmatrix}$ and $|A^3| = 125$, then α is equal to

- (A) 0
- (B) ± 5
- (C) ± 2
- (D) ± 3

(MATHEMATICS)

DIWALI ASSIGNMENT

111. If $P = \begin{pmatrix} \sqrt{3}/2 & 1/2 \\ -1/2 & \sqrt{3}/2 \end{pmatrix}$, $A = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$ and $Q = PAP^T$, $X = P^T Q^{2005} P$, then X is equal to

(A) $\begin{pmatrix} 1 & 2005 \\ 0 & 1 \end{pmatrix}$

(B) $\frac{1}{4} \begin{pmatrix} 4 + 2005\sqrt{3} & 6015 \\ 2005 & 4 - 2005\sqrt{3} \end{pmatrix}$

(C) $\frac{1}{4} \begin{pmatrix} 2005 & 2 - \sqrt{3} \\ 2 + \sqrt{3} & 2005 \end{pmatrix}$

(D) none of these

112. If $P = \begin{bmatrix} 1 & \alpha & 3 \\ 1 & 3 & 3 \\ 2 & 4 & 4 \end{bmatrix}$ is the adjoint of a 3×3 matrix A and $|A| = 4$, then α is equal to

(A) 5

(B) 0

(C) 4

(D) 11

113. How many 3×3 matrices M with entries from $\{0,1,2\}$ are there, for which the sum of the diagonal entries of $M^T M$ is 5 ?

(A) 198

(B) 126

(C) 135

(D) 162

MEASURES OF CENTRAL TENDENCY AND DISPERSION

114. If the mean of the series x_1, x_2, \dots, x_n is \bar{x} , then the mean of the series $x_i + 2i, i = 1, 2, \dots, n$ will be

(A) $\bar{x} + n$

(B) $\bar{x} + n + 1$

(C) $\bar{x} + 2$

(D) $\bar{x} + 2n$

115. The variance of a distribution is σ^2 . If each value of the distribution is increased by λ , then the variance of the new distribution is

(A) $\lambda^2 \sigma^2$

(B) $\lambda^2 + \sigma^2$

(C) $\lambda + \sigma^2$

(D) σ^2

116. If frequencies of the values $0, 1, 2, \dots, n$ of a variate are proportional to ${}^n C_0, {}^n C_1, {}^n C_2, \dots, {}^n C_n$, then $\text{Var}(X)$ is equal to

(A) $\frac{(n^2-1)}{12}$

(B) $\frac{n}{2}$

(C) $n/4$

(D) none of these

117. The mean and variance of 5 observations of an experiment are 4 and 5.2 respectively. If from these observations three are 1, 2 and 6, then the remaining will be

(A) 2, 9

(B) 5, 6

(C) 4, 7

(D) 3, 8

118. If $a > 0$, then the minimum sum of the real numbers $a^{-5}, a^{-4}, 3a^{-3}, 1, a^8$ and a^{10} will be

(A) 7

(B) 8

(C) 9

(D) 10

(MATHEMATICS)

DIWALI ASSIGNMENT

ANSWER KEY

1.	(A)	2.	(C)	3.	(B)	4.	(A)	5.	(A)	6.	(A)	7.	(B)
8.	(A)	9.	(B)	10.	(D)	11.	(D)	12.	(B)	13.	(C)	14.	(B)
15.	(B)	16.	(B)	17.	(A)	18.	(C)	19.	(C)	20.	(B)	21.	(A)
22.	(A)	23.	(B)	24.	(C)	25.	(AC)	26.	(C)	27.	(D)	28.	(D)
29.	(D)	30.	(D)	31.	(B)	32.	(A)	33.	(C)	34.	(A)	35.	(C)
36.	(B)	37.	(B)	38.	(A)	39.	(B)	40.	(AC)	41.	(B)	42.	(A)
43.	(B)	44.	(A)	45.	(C)	46.	(A)	47.	(A)	48.	(C)	49.	(C)
50.	(B)	51.	(A)	52.	(C)	53.	(D)	54.	(A)	55.	(B)	56.	(C)
57.	(B)	58.	(C)	59.	(C)	60.	(A)	61.	(B)	62.	(A)	63.	(A)
64.	(B)	65.	(D)	66.	(B)	67.	(D)	68.	(C)	69.	(A)	70.	(A)
71.	(B)	72.	(A)	73.	(B)	75.	(B)	76.	(A)	77.	(C)	78.	(A)
79.	(B)	80.	(A)	81.	(D)	82.	(C)	83.	(C)	84.	(A)	85.	(A)
86.	(A)	87.	(A)	88.	(A)	89.	(C)	90.	(B)	91.	(C)	92.	(D)
93.	(B)	94.	(C)	95.	(ABC)	96.	(B)	97.	(B)	98.	(B)	99.	(C)
100.	(C)	101.	(A)	102.	(B)	103.	(C)	104.	(C)	105.	(A)	106.	(C)
107.	(B)	108.	(C)	109.	(A)	110.	(D)	111.	(A)	112.	(D)	113.	(A)
114.	(B)	115.	(D)	116.	(C)	117.	(C)	118.	(B)				