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358.

$$\lim_{x\to-\infty}\left(\frac{1+\frac{1}{x}}{2-\frac{1}{x}}\right)^{x} \to \infty$$

$$\left(\frac{1}{2}\right)^{-\infty}$$

55

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379. $n \in N$

$$\lim_{x \to \infty} \frac{\left(a + \frac{1}{x}\right)^n}{1 + \frac{A}{x^n}} = a^n.$$

$a \neq 0$

$a = 0$ $\lim_{x \to \infty} \frac{1}{x^n + A}$

$n \in I^-$ $x^n \to 0$

$n \in I^+$, $x^n \to \infty$.

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380.

$$\lim_{x \to -\infty} x\left(\sqrt{x^2+\sqrt{x^4+1}} - x\sqrt{2}\right) \to -\infty$$

$$-\infty \quad \infty \longrightarrow -\infty$$

$$\lim_{x\to\infty} x\left(\sqrt{x^2+\sqrt{x^4+1}} - x\sqrt{2}\right)$$

$$\lim_{x\to\infty} \frac{x\left(\sqrt{x^4+1} - x^2\right)}{\left(\sqrt{x^2+\sqrt{x^4+1}} + x\sqrt{2}\right)}$$

$$= \lim_{x\to\infty} \frac{x}{\left(\sqrt{x^2+\sqrt{x^4+1}} + x\sqrt{2}\right)\left(\sqrt{x^4+1} + x^2\right)}$$

$$\frac{1}{\left(\sqrt{1+\sqrt{1+\frac{1}{x^4}}} + \sqrt{2}\right)\left(\sqrt{x^4+1}+x^2\right)}$$

$$= 0.$$

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387.

$$\lim_{h\to 0} \frac{2\sin\frac{3h}{2}\cos\left(a+\frac{3h}{2}\right) - 6\sin\frac{h}{2}\cos\left(a+\frac{3h}{2}\right)}{h^3}$$

$$\frac{2\cos\left(a+\frac{3h}{2}\right)\left(-4\sin^3\frac{h}{2}\right)}{8\left(\frac{h}{2}\right)^3}$$

$$= \frac{2\cos a\,(-4)}{8} = -\cos a.$$

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388. $\lim_{x\to\pi/2} \dfrac{\sin^2 x \,\cancel{(\sin x - 1)}^{\,-1}(\sin x - 2)}{\cancel{(1-\sin x)}(1+\sin x)\left(\sqrt{2\sin^2 x + 3\sin x + 4} + \sqrt{\sin^2 x + 6\sin x + 2}\right)}$

$$\frac{1}{12}$$

$s^2 x - 3 s x + 2$

90. $\lim_{n\to\infty} \dfrac{\sin x}{2^n \sin\left(\dfrac{x}{2^n}\right)} = \lim_{n\to\infty} \dfrac{\sin x}{x\left(\dfrac{\sin \frac{x}{2^n}}{\frac{x}{2^n}}\right)} = \dfrac{\sin x}{x}$

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92. $\lim_{x\to\infty}\left(\cos\sqrt{x+1}-\cos\sqrt{x}\right)$

$$=\lim_{x\to\infty}\left(-2\sin\left(\frac{\sqrt{x+1}-\sqrt{x}}{2}\right)\underbrace{\sin\left(\frac{\sqrt{x+1}+\sqrt{x}}{2}\right)}_{[-1,1]}\right)$$

$$\lim_{x\to\infty}\underbrace{-2\sin\left(\frac{1}{2\left(\sqrt{x+1}+\sqrt{x}\right)}\right)}_{\to 0}\underbrace{\sin\left(\frac{\sqrt{x+1}+\sqrt{x}}{2}\right)}_{[-1,1]}$$

$$=0$$

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$$\lim_{x\to 0}\left(\frac{\sin x - x}{x^3}\right) = -\frac{1}{6}$$

$$\lim_{x\to 0}\left(\frac{\tan x - x}{x^3}\right) = \frac{1}{3}$$

$$\lim_{x\to 0}\left(\frac{e^x - 1 - x}{x^2}\right) = \frac{1}{2}$$

$$\lim_{x\to 0}\frac{\ln(1+x) - x}{x^2} = -\frac{1}{2}$$

$$\left(\cancel{1}+\cancel{\frac{x}{1!}}+\cancel{\frac{x^2}{2!}}+\frac{x^3}{3!}+\cdots\right)-\left(\cancel{1}-\cancel{\frac{x}{1!}}+\cancel{\frac{x^2}{2!}}-\frac{x^3}{3!}+\cdots\right)\cancel{-2x}$$

$$\lim_{x\to 0}\left(\frac{e^x - e^{-x} - 2x}{x^3}\right) = \frac{1}{3}$$

$$\lim_{x\to 0}\frac{\sin x - x}{x^3} = \frac{\cancel{\frac{x}{1!}}-\frac{x^3}{3!}+\frac{x^5}{5!}-\cdots\cancel{-x}}{x^3}$$

$$= \lim_{x\to 0}\left(-\frac{1}{3!}+\frac{x^2}{5!}-\cdots\right)$$

$$= -\frac{1}{6}$$

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$$l = \lim_{x \to 0} \left(\frac{e^x - 1 - x}{x^2} \right) \quad \text{—— (1)}$$

$x = -t, \frac{1}{t}, 2t, 3t$

$x = -t$

$$l = \lim_{t \to 0} \frac{e^{-t} - 1 + t}{t^2} = \lim_{x \to 0} \frac{e^{-x} - 1 + x}{x^2} \quad \text{—— (2)}$$

(1) + (2)

$$2l = \lim_{x \to 0} \frac{e^x + e^{-x} - 2}{x^2} = \lim_{x \to 0} \frac{1}{e^x} \left(\frac{e^x - 1}{x} \right)^2$$

$= 1$

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$$\ell = \lim_{x\to 0} \frac{\sin x - x}{x^3}$$

$$\lim_{x\to 0} \frac{\sin x + x}{x^3}$$

$$x = 3t$$

$$\ell = \lim_{t\to 0} \frac{3\sin t - 4\sin^3 t - 3t}{27t^3}$$

$$\ell = \lim_{t\to 0} \left(\frac{1}{9}\left(\frac{\sin t - t}{t^3}\right) - \frac{4}{27}\frac{\sin^3 t}{t^3} \right) = \frac{\ell}{9} - \frac{4}{27}$$

$$\ell = \frac{\ell}{9} - \frac{4}{27}$$

$$\frac{8\ell}{9} = -\frac{4}{27}$$

$$\boxed{\ell = -\frac{1}{6}}$$

1. $$\lim_{x\to\infty}\left(x - x^2\ln\left(1+\frac{1}{x}\right)\right) = \lim_{t\to 0}\left(\frac{1}{t} - \frac{1}{t^2}\ln(1+t)\right)$$

$\frac{1}{x} = t$

$$= \lim_{t\to 0}\frac{t - \ln(1+t)}{t^2} = \frac{1}{2}$$

$\cancel{\frac{1}{x^2}(\sin^{-1}x)^2 - \frac{1}{x^2}}$

$= 0$

$\sin^{-1}x = \theta$

2. $$\lim_{x\to 0}\left(\frac{1}{(\sin^{-1}x)^2} - \frac{1}{x^2}\right)$$

$$\lim_{\theta\to 0}\left(\frac{1}{\theta^2} - \frac{1}{\sin^2\theta}\right) = \lim_{\theta\to 0}\frac{(\sin\theta - \theta)(\sin\theta + \theta)}{\theta^4\left(\frac{\sin^2\theta}{\theta^2}\right)}$$

$$\frac{1}{\left(\frac{\sin^2\theta}{\theta^2}\right)}\left(\frac{\sin\theta - \theta}{\theta^3}\right)\left(\frac{\sin\theta}{\theta} + 1\right) = -\frac{1}{6}\times 2 = \boxed{-\frac{1}{3}}$$

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$\boxed{\begin{array}{l} \underline{PT-1,\ 2} \\ Ex-I\ (1-10) \end{array}}$