PROBABILITY

Q. Let X and Y be two events such that $P(X) = 1/3, P(X \mid Y) = 1/2$ and $P(Y \mid X) = 2/5$. Then

Adv 18

(A) $P(Y) = \frac{4}{15}$ ✓✓

(B) $P(X' \mid Y) = \frac{1}{2}$ ✓✓

(C) $P(X \cap Y) = \frac{1}{5}$ ✗

(D) $P(X \cup Y) = \frac{2}{5}$ ✗

① $P\left(\frac{X}{Y}\right) = \frac{1}{2} \Rightarrow \frac{P(X \cap Y)}{P(Y)} = \frac{1}{2} \Rightarrow \frac{2}{15} \times \frac{2}{1} = P(Y) = \frac{4}{15}$

② $P\left(\frac{Y}{X}\right) = \frac{P(X \cap Y)}{P(X)} = \frac{2}{5} \Rightarrow P(X \cap Y) = \frac{2}{5} \times \frac{1}{3} = \frac{2}{15}$

(3) $P(X \cup Y) = P(X) + P(Y) - P(X \cap Y)$

$= \frac{1}{3} + \frac{4}{15} - \frac{2}{15} = \frac{7}{15}$

(4) $P\left(\frac{X'}{Y}\right) = \frac{P(X' \cap Y)}{P(Y)} = \frac{\overbrace{P(Y) - P(X \cap Y)}^{\text{only } Y}}{P(Y)} = \frac{\frac{4}{15} - \frac{2}{15}}{\frac{4}{15}} = \frac{1}{2}$

Nishant Jindal

PROBABILITY

Q. The minimum number of times a fair coin needs to be tossed, so that the probability of getting at least two heads is at least 0.96, is

IT

8 Coins

At least two heads कम से कम 2 head तो आना चाहिए

$1-(\text{No head})-(\text{One head}) \geq .96$

$1-\left(\frac{1}{2}\right)^n - {}^nC_1\left(\frac{1}{2}\right)\times\left(\frac{1}{2}\right)^{n-1} \geq \frac{96}{100}$

n में से 1 coin selected — head come — n-1 coins tail

$1-\left(\frac{1}{2}\right)^n(1+n) \geq \frac{96}{100}$

$1-\frac{96}{100} \geq \frac{1+n}{2^n} \Rightarrow \frac{4}{100} \geq \frac{1+n}{2^n}$

$n=1$

$\frac{4}{100} \geq \frac{2}{2}$ ✗

$n=2$

$\frac{4}{100} \geq \frac{3}{4}$

$n=7$ $\frac{4}{100} \geq \frac{8}{128}$

$n=8$

$\frac{4}{100} \geq \frac{9}{256} \to \frac{4}{128}$

If it were like

At least one head

$= 1-(\text{No head on } n \text{ coins})$

(fail on n coins)

$= 1-\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\cdots\left(\frac{1}{2}\right)$

$= 1-\left(\frac{1}{2}\right)^n$

Q. Of the three independent events E_1, E_2, and E_3, the probability that only E_1 occurs is α, only E_2 occurs is β and only E_3 occurs is γ. Let the probability P that none of events E_1, E_2 or E_3 occurs satisfy the equations $(\alpha - 2\beta)P = \alpha\beta$ and $(\beta - 3\gamma)P = 2\beta\gamma$. All the given probabilities are assumed to lie in the interval $(0, 1)$.

Adv 16

Then $\dfrac{\text{Probability of occurrence of } E_1}{\text{Probability of occurrence of } E_3} = \dfrac{P(E_1)}{P(E_3)} = \dfrac{x}{z} = \dfrac{\frac{\alpha}{P+\alpha}}{\frac{\gamma}{P+\gamma}}$

$= \dfrac{\alpha}{P+\alpha} \times \dfrac{P+\gamma}{\gamma} = \dfrac{\left(1+\frac{P}{\gamma}\right)}{\left(1+\frac{P}{\alpha}\right)}$

$= 6$

① $P(E_1 \bar{E}_2 \bar{E}_3) = \alpha \Rightarrow x(1-y)(1-z) = \alpha$

2) $P(\bar{E}_1 E_2 \bar{E}_3) = \beta \Rightarrow (1-x)(y)(1-z) = \beta$

3) $P(\bar{E}_1 \bar{E}_2 E_3) = \gamma \Rightarrow (1-x)(1-y)z = \gamma$

4) $P(\bar{E}_1 \bar{E}_2 \bar{E}_3) = P \Rightarrow (1-x)(1-y)(1-z) = P$

$\dfrac{x}{1-x} = \dfrac{\alpha}{P}$ | $\dfrac{z}{1-z} = \dfrac{\gamma}{P}$

$xP = \alpha - \alpha x \Rightarrow x(P+\alpha) = \alpha$

$x = \dfrac{\alpha}{P+\alpha}$ $\quad$ S/y $z = \dfrac{\gamma}{P+\gamma}$

(5) $\alpha P - 2\beta P = \alpha\beta$

$\beta P - 3\gamma P = 2\beta\gamma$

$\Rightarrow \alpha P = \beta(\alpha + 2P)$ | $3\gamma P = \beta(P - 2\gamma)$

$\Rightarrow \dfrac{\alpha \cancel{P}}{3\gamma \cancel{P}} = \dfrac{\cancel{\beta}(\alpha + 2P)}{\cancel{\beta}(P - 2\gamma)}$

$\dfrac{P - 2\gamma}{3\gamma} = \dfrac{\alpha + 2P}{\alpha} \Rightarrow \dfrac{P}{3\gamma} - \dfrac{2}{3} = 1 + \dfrac{2P}{\alpha}$

$\dfrac{P}{3\gamma} = \dfrac{5}{3} + \dfrac{2P}{\alpha} \Rightarrow \dfrac{P}{\gamma} = 5 + \dfrac{6P}{\alpha}$

$1 + \dfrac{P}{\gamma} = 6\left(1 + \dfrac{P}{\alpha}\right)$

Q. A Boy has 3 white, 4 red, 5 black balls. If any two balls are drawn, then find

15

Probability that

(A) Both are white

(B) Both same colour

(C) 1 R & 1 black

(D) At least one black.

$\boxed{3W, 4R, 5B}$

	एकसाथ 2 Balls.	One after another without Replacement	one after another with Replacement
A)	$P(W) = \dfrac{{}^3C_2}{{}^{12}C_2}$	$= \dfrac{3}{12} \times \dfrac{2}{11}$	$= \dfrac{3}{12} \times \dfrac{3}{12}$
(B)	$\dfrac{{}^3C_2 + {}^4C_2 + {}^5C_2}{{}^{12}C_2}$	$\dfrac{3}{12}\times\dfrac{2}{11} + \dfrac{4}{12}\times\dfrac{3}{11} + \dfrac{5}{12}\times\dfrac{4}{11}$	$\dfrac{3}{12}\times\dfrac{3}{12} + \dfrac{4}{12}\times\dfrac{4}{12} + \dfrac{5}{12}\times\dfrac{5}{12}$
(C)	$\dfrac{{}^5C_1 \times {}^4C_1}{{}^{12}C_2}$	$\dfrac{4}{12}\times\dfrac{5}{11} + \dfrac{5}{12}\times\dfrac{4}{11}$	$\dfrac{4}{12}\times\dfrac{5}{12} + \dfrac{5}{12}\times\dfrac{4}{12}$
(D)	$1 -$ No Black $= 1 - \dfrac{{}^7C_2}{{}^{12}C_2}$	$1 - 1^{st}$ Black × 2^{nd} B. (NW NW) $: 1 - \left(\dfrac{7}{12}\times\dfrac{6}{11}\right)$	$1 - \left(\dfrac{7}{12}\times\dfrac{7}{12}\right)$

Nishant Jindal

PROBABILITY

Q. **A Bag has 3 white, 3red balls. One after another balls are drawn tell bag is not**

14

empty. Find Probability that balls are coming in alternate colour.

$$\frac{\cancel{3}}{\cancel{6}_{2}} \times \frac{3}{5} \times \frac{\cancel{2}}{\cancel{4}_{\cancel{2}}} \times \frac{2}{3} \times \frac{1}{\cancel{2}} \times 1 \times \cancel{2} = \frac{1}{10}$$

PROBABILITY

Q. Cards are drawn one by one from 52 cards till 2 Queen are not coming. Find Probability that for this 15 cards are required

13

2 Queens need 15 Cards

1st Queen in Any of 14 Cards & 2nd on 15th Card.

$$\frac{{}^{4}C_{1} \times {}^{48}C_{13}}{{}^{52}C_{14}} \times \boxed{\frac{{}^{3}C_{1}}{{}^{38}C_{1}}}$$

15th Card is Queen.

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PROBABILITY

Ind. Event

Q. Board 12 **Probability of solving one question from 3 Student is $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$. Find Probability that if question is given to all 3 students then Probability of question being solved, is**

1 − (NoBody Solves)

$1 - \frac{1}{2} \times \frac{2}{3} \times \frac{3}{4}$

$= \frac{3}{4}$

Nishant Jindal

PROBABILITY

Q. 55 boys & 25 girls among 80 students in class, 20 are rich & rest poor, 10 are fair & rest whitish. Find probability of selecting a fair, rich girl?

11

$$P(G \cap F \cap R) = P(G) \times P(F) \times P(R)$$

$$= \frac{25}{80} \times \frac{10}{80} \times \frac{20}{80}$$

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PROBABILITY

Q. Probability of living a person is $\frac{7}{15}$ & Probability of living his wife is $\frac{9}{16}$. for next 10 yrs. Find Probability that one of them living for next 10 year

10

$$1 - \underset{\text{dead}}{\text{Men}} \times \underset{\text{dead}}{\text{Wife}}$$

$$1 - \frac{8}{15} \times \frac{7}{16}$$

Nishant Jindal

Q. A speak 75% truth, B 60% truth. If both speaks on same matter, then

9

Probability of contradiction?

happens when any of them is speaking lie.

$$A\cdot\bar{B} + B\cdot\bar{A}$$

$$P(\text{Cnt.}) = \frac{3}{4}\times\frac{4}{10} + \frac{6}{16}\times\frac{1}{4}$$

Nishant Jindal

PROBABILITY

(1, 6) (6, 1) (2, 5) (5, 2) (3, 4) (4, 3)

Q. 8 A, B throws a pair of dice. If sum 6 comes for on A before 7 comes on B then A is Winner. If 7 Comes early on B before 6 comes to A then B is in inner. If As tarts process then Probability of both winning.

$= \frac{6-1}{36}$ $\quad = \frac{21}{36} = \left(\frac{6}{36} = \frac{1}{6}\right)$

A Wins $\to$ if $S = 6$ Comes Before $S = 7$ Coming.

$$P(A \text{ Wins}) = A + \bar{A}\cdot\bar{B}A + \bar{A}\bar{B}\bar{A}\bar{B}A + \bar{A}\bar{B}\bar{A}\bar{B}\bar{A}\bar{B}A + \cdots$$

$$= \frac{5}{36} + \frac{31}{36}\times\frac{5}{6}\times\frac{5}{36} + \left(\frac{31}{36}\times\frac{5}{6}\right)^2\cdot\frac{5}{36} + \left(\frac{31}{36}\times\frac{5}{6}\right)^3\cdot\frac{5}{36} + \cdots$$

$$= \frac{5}{36}\times\left\{\frac{1}{1-\left(\frac{31\times 5}{36\times 6}\right)}\right\} = \frac{5}{\cancel{36}}\left\{\frac{\cancel{36}\times 6}{216-155}\right\} = \frac{30}{61}$$

$$P(B \text{ Wins}) = \bar{A}B + \bar{A}\bar{B}\bar{A}B + \bar{A}\bar{B}\bar{A}\bar{B}\cdot\bar{A}B + \cdots = \bar{A}B\left\{1 + (\bar{B}\bar{A}) + (\bar{B}\bar{A})^2 + \cdots\right\}$$

$$= \bar{A}B \times \frac{1}{1-\bar{A}\bar{B}} = \frac{31}{36}\times\frac{1}{6}\times\left\{\frac{1}{1-\frac{31}{36}\times\frac{5}{6}}\right\}$$

$$= \frac{31}{61}$$

PROBABILITY

6 Adv.

Q. Football teams T_1 and T_2 have to play two games against each other. It is assumed that the outcomes of the two games are independent. The probabilities of T_1 winning, drawing and losing a game against T_2 are $\frac{1}{2}, \frac{1}{6}$ and $\frac{1}{3}$, respectively. Each team gets 3 points for a win, 1 point for a draw and 0 point for a loss in a game. Let X and Y denote the total points scored by teams T_1 and T_2 respectively, after two games then

(i) $P(X > Y)$ is

(ii) $P(X = Y)$ is

T_1 $P(W) = \frac{1}{2}$, $P(D) = \frac{1}{6}$, $P(L) = \frac{1}{3}$

T_2 $P(T_2) = \frac{1}{3}$ $P(D) = \frac{1}{6}$

$P(T_1 \text{ की Score} > T_2 \text{ की Score}) = T_1 \cdot T_1 + T_1 D + D \cdot T_1$

$= \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{6} + \frac{1}{6} \times \frac{1}{2}$

$P(T_1 \text{ का Score} = T_2 \text{ का Score}) = T_1 T_2 + T_2 T_1 + D D$

$= \frac{1}{2} \times \frac{1}{3} + \frac{1}{3} \times \frac{1}{2} + \frac{1}{6} \times \frac{1}{6}$

Nishant Jindal

PROBABILITY

Q. 3 Boxes B_1 has $1W, 3R$ & $2B$, B_2 has $2W, 3R, 4B$ & B_3 has $3W, 4R, 5B$ balls.

5 If one ball is drawn from B_1, B_2, B_3. Find the Prob. that all 3 balls are of same colour.

$\boxed{1W, 3R, 2B}$ B_1 $\quad \boxed{2W, 3R, 4B}$ B_2 $\quad \boxed{3W, 4R, 5B}$ B_3

$\underline{W} \qquad R \qquad B$

$$\frac{1}{6} \times \frac{2}{9} \times \frac{3}{12} + \frac{3}{6} \times \frac{3}{9} \times \frac{4}{12} + \frac{2}{6} \times \frac{4}{9} \times \frac{5}{12}$$

PROBABILITY

Q. If 2 balls are drawn (without replacement) from a randomly selected box and one of the balls is white and the other ball is red. Find the probability that these 2 balls are drawn from box B_2.

Wait

Nishant Jindal

PROBABILITY

Q. If drawn ball is red from a Selected bag find Prob. of it be bag 2

Wait

3W	4W	2R
4R	5R	7B

Q. 5 persons entered the lift cabin on the ground floor of an 8 floor building. Suppose that each of them independently and with equal probability, can leave the cabin at any other floor, starting from the first. The probability that all 5 persons leave at different floors is

Listen again

(A) $\left(\frac{5}{8}\right)^5$

(B) $\frac{{}^8C_5}{8^5}$

(C) $\frac{5!}{8^5}$

(D) $\frac{{}^8C_5 5!}{8^5}$

→ 8 options

$$P = \frac{{}^8C_5 \times \lfloor 5}{8 \times 8 \times 8 \times 8 \times 8}$$

Nishant Jindal

PROBABILITY

$1, 3 = 9$

Q. From a pack of 52 playing cards, face cards and tens are removed and kept aside then a card is drawn at random from the remaining cards. If

3

A : The event that the card drawn is an ace

H : The event that the card drawn is a heart

S: The event that the card drawn is a spade then which of the following holds ?

(A) $9P(A) = 4P(H)$

(B) $P(S) = 4P(A \cap H)$

(C) $3P(H) = 4P(A \cup S)$

(D) $P(H) = 12P(A \cap S)$

$52 \to \boxed{12} + 4 = 36$ Cards

$P(A) = \frac{4}{36}$ | $P(H) = \frac{9}{36}$

$\frac{P(A)}{P(H)} = \frac{4}{9}$

$9P(A) = 4P(H)$

PROBABILITY

Q. There are ten prizes, five A's, three B's and two C's, placed in identical sealed envelopes for the ten contestants in a mathematics contest. The prizes are awarded by allowing winners to select an envelope at random from those remaining. When the contestant goes to select the prize, the probability that the remaining three prizes are one A, one B one C, is

Listen 2 Again

(A) 1/4

(B) 1/3

(C) 1/12

(D) 1/10

10 Bnde

A - 5 Prizes, B → 3 Prizes, C → 2 Prizes

1 A, 1 B, 1 C Prize Remaining ⇒ 7 Bnde Prize le gye hai |

$$= \frac{{}^5C_4 \times {}^3C_2 \times {}^2C_1}{{}^{10}C_7}$$

PROBABILITY

Q. A determinant is chosen at random from the set of all determinant of order 2 with elements 0 or 1 only. The probability that the determinant chosen has the value non negative is

(A) $3/16$

(B) $6/16$

(C) $10/16$

(D) $13/16$

$$\begin{vmatrix} \underline{a} & \overline{b} \\ \underline{c} & \underline{d} \end{vmatrix} = \begin{vmatrix} 2 & 2 \\ 2 & 2 \end{vmatrix}$$

$$^2C_1 \times {}^2C_1 \times {}^2C_1 \times {}^2C_1 = 16 \text{ option} \rightarrow 16 \ \underline{\underline{\text{det}}}.$$

$$P = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} + \begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix} \quad \begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix}$$

$$= 1 \qquad = 1 \qquad = 1$$

$$\frac{3}{16}$$