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Props — Even odd fxn Based Prop.

Id → $\int_{-a}^{a} f(x)\,dx \rightarrow \int_{-1}^{1} f(x)$

$\int_{-\pi/2}^{\pi/2} f(x)$

Prop → $\int_{-a}^{a} f(x)\cdot dx = \begin{cases} 0 & f(x) = \text{odd.} \\ 2\int_0^a f(x)\cdot dx & f(x) = \text{Even.} \\ \int_0^a (f(x)+f(-x))dx & f(x) = \text{NENO} \end{cases}$

Q If $f(x)$ is Even fxn.

then evaluate $\int_{-2}^{2} \left(x^3\cdot f(x) + x\cdot f''(x) + 2\right) dx$

$= \int_{-2}^{2} \boxed{x^3} \cancel{f(x)}\,dx + \int_{-2}^{2} \cancel{x' f''(x)}\,dx + \int_{-2}^{2} 2\,dx$

Odd x Even   Odd x Even

$\ominus \times \oplus = \ominus$ $\ominus \times \oplus = \ominus$

$= 0 + 0 + 2(x)_{-2}^{2}$

$= 2(2+2)$

$= 8$

$f(x) = \text{Even fxn}$

$f'(x) = \text{odd fxn}$

$f''(x) = \text{Even fxn}$

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Q $\displaystyle\int_{-a}^{a}\left(\frac{f(x)+f(-x)}{g(x)-g(-x)}\right)^{2n}(h(x)-h(-x))^{2n-1}dx = ?$

Rem:- $\boxed{\begin{aligned} f(x)+f(-x) &= \text{Even} \\ f(x)-f(-x) &= \text{odd.}\end{aligned}}$

$\Rightarrow \displaystyle\int_{-a}^{a}\left(\frac{E(+)}{O(-)}\right)^{2n}(\text{odd})^{2n-1}$

$\displaystyle\int_{-a}^{a}(\text{odd})^{2n}(\text{odd})^{2n-1}$

$= \displaystyle\int \text{Even} \times \text{odd} = 0$

$(+) \times (-)$

Q $I = \displaystyle\int_{-1}^{3}\tan^{-1}\left(\frac{x}{x^2+1}\right)+\tan^{-1}\left(\frac{x^2+1}{x}\right).dx \longrightarrow x\in(-1,3)$

(x) Step wrong.

$= \cancel{\displaystyle\int_{-1}^{3}\tan^{-1}\left(\frac{x}{x^2+1}\right)+\cot^{-1}\left(\frac{x}{x^2+1}\right)dx}$

$-1 \quad 0 \quad 3$; $-ve$; $+ve$

$\tan^{-1}\left(\frac{1}{x}\right)=\cot^{-1}(x)$

if $x>0$

Odd+odd

$I = \displaystyle\int_{-1}^{1}\tan^{-1}\left(\frac{x}{x^2+1}\right)+\tan^{-1}\left(\frac{x^2+1}{x}\right)dx + \int_{1}^{3}\underbrace{\tan^{-1}\left(\frac{x}{x^2+1}\right)+\cot^{-1}\left(\frac{x}{x^2+1}\right)}_{\frac{\pi}{2}}dx$

$x = +ve$

$f(x) = \tan^{-1}\left(\frac{x}{x^2+1}\right)$

$f(-x) = \tan^{-1}\left(\frac{-x}{x^2+1}\right)$

$= -f(x)$

$= 0 + \displaystyle\int_{1}^{3}\frac{\pi}{2}.dx = \frac{\pi}{2}(x)_1^3 = \frac{\pi}{2}(3-1)$

$= \pi$

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Q If $F(x) = \begin{vmatrix} \cos x & x\sin x & x^3\cos x \\ x^2 & \sec x & x+\sin x \\ 1 & 2 & \tan x \end{vmatrix}$

odd × Even = odd

then $I = \int_{-1}^{1} (F(x)+F''(x))(x^2+1)\,dx = ?$

(odd + odd) × Even

$F(-x) = \begin{vmatrix} \cos(-x) & (-x)\sin(-x) & (-x)^3\cos(-x) \\ (-x)^2 & \sec(-x) & -x-\sin x \\ 1 & 2 & -\tan x \end{vmatrix}$

$= \begin{vmatrix} \cos x & x\sin x & -x^3\cos x \\ x^2 & \sec x & -(x+\sin x) \\ 1 & 2 & -\tan x \end{vmatrix}$

$F(-x) = -F(x) \Rightarrow F(x) = \text{odd} \rightarrow F'(x) = \text{Even}$

$F''(x) = \text{odd}$

Q If $f(x) = \begin{vmatrix} \boxed{1 \quad 2x \quad 3x^2} \\ 2 & a & 27 \\ 1 & 3 & 9 \end{vmatrix}$ & $\int_0^3 f(x)\cdot dx = 0$ then $a = ?$

Int. of det.

$\int_0^3 f(x) = \begin{vmatrix} x & x^2 & x^3 \\ 2 & a & 27 \\ 1 & 3 & 9 \end{vmatrix}_0^3$

$= \begin{vmatrix} 3 & 9 & 27 \\ 2 & a & 27 \\ 1 & 3 & 9 \end{vmatrix} = 0$

Identical Rows.

$\Rightarrow$ Ans $= 0$

for $a \in R$

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Q $\int_{-1/2}^{1/2} [x] + \ln\left(\frac{1-x}{1+x}\right) dx$ → Prop 3.

$\int_{-1/2}^{1/2} [x]\, dx + \cancel{\int_{-1/2}^{1/2} \ln\left(\frac{1-x}{1+x}\right) dx}$

$\int_{-1/2}^{0} -1 \cdot dx + \int_{0}^{1/2} 0 \cdot dx + 0$

$= -(x)_{-1/2}^{0} + 0$

$-(0 + \frac{1}{2}) = -\frac{1}{2}$

Q $\int_{-1}^{1} [|x|] \frac{d}{dx}\left(\frac{1}{1+e^{-1/x}}\right)$

Integer Pr Brkt!!!

$\Rightarrow \int_{-1}^{0} 0 \cdot \frac{d}{dx}\left(\frac{1}{1+e^{-1/x}}\right) + \int_{0}^{1} 0 \cdot \frac{d}{dx}\left(\frac{1}{1+e^{-1/x}}\right)$

$x \in (-1, 0)$ | $x \in (0, 1)$

$|x| \in (0, 1)$ | $|x| \in (0, 1)$

$[|x|] = 0$ | $[|x|] = 0$

$0 + 0 = 0$

$$\boxed{\begin{aligned} 1 + \sin x &= NENO \\ 1 + a^x &= NENO \end{aligned}}$$

Q $\int_{-\pi/2}^{\pi/2} \cos^3\theta (1 + \underbrace{\sin\theta}_{NENO}) d\theta$

$\int_{-\pi/2}^{\pi/2} \underbrace{\cos^3\theta \cdot d\theta}_{\text{Even}} + \cancel{\int_{-\pi/2}^{\pi/2} \cos^3\theta \cdot \sin\theta\, d\theta}$

E × O

⊖ × ⊖ = ⊖

$= 2 \int_{0}^{\pi/2} \cos^3\theta \cdot d\theta$

Walli

$= 2 \times \frac{2}{3 \times 1} \times 1$

$= \frac{4}{3}$

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$$\int_0^a f(x)\,dx = \int_0^a f(x)+f(-x)\,dx$$

Q Adv. $I = \int_{-\pi}^{\pi} \frac{\cos^2 x}{1+a^x}\,dx \to$ NENO $\quad \int \frac{\sin 9x}{\sin x}$

$$= \int_0^{\pi} \frac{\cos^2 x}{1+a^x} + \frac{\cos^2 x}{1+a^{-x}}\,dx$$

$$= \int_0^{\pi} \frac{\cos^2 x}{1+a^x} + \frac{a^x \cdot \cos^2 x}{1+a^x}\,dx$$

$$= \int_0^{\pi} \frac{\cos^2 x(\cancel{1+a^x})}{\cancel{1+a^x}}\,dx$$

$$= \int_0^{\pi} \frac{1}{2}+\frac{\cos 2x}{2}\,dx$$

$$= \left[\frac{x}{2}+\frac{\sin 2x}{4}\right]_0^{\pi} = \left(\frac{\pi}{2}\right)$$

Q Adv. If $I_n = \int_{-\pi}^{\pi} \frac{\sin nx}{\underbrace{(1+\pi^x)}_{\text{NENO}}\cdot \sin x}\,dx$; $n = 0,1,2,3 \ldots$ then.

~~(A)~~ $I_n = I_{n+2}$ (B) $\sum_{m=1}^{10} I_{2m+1} = 10\pi$

~~(C)~~ $\sum_{m=1}^{10} I_{2m} = 0$ $\quad$ ~~(D)~~ $I_n = I_{n+1}$

$$10 I_2 = \int \frac{\sin 2x}{\sin x} = \int_0^{\pi} \frac{2\cancel{\sin x}\cos x}{\cancel{\sin x}} = 2\sin x\Big|_0^{\pi} = 0$$

$$I_n = \int_0^{\pi} \frac{\sin nx}{\sin x}\,dx$$

$$I_{n+2} = \int_0^{\pi} \frac{\sin(n+2)x}{\sin x}\,dx$$

$$I_{n+2} - I_n = \int_0^{\pi} \frac{\sin(n+2)x - \sin nx}{\sin x}$$

$$I_{n+2} - I_n = \int_0^{\pi} \frac{2\cos(n+1)x \cdot \cancel{\sin x}}{\cancel{\sin x}}\,dx$$

$$= 2\left.\frac{\sin(n+1)x}{(n+1)}\right|_0^{\pi}$$

$$= 2\left[\cancel{\frac{\sin(n+1)\pi}{n+1}} - \cancel{\frac{\sin(n+1)\times 0}{n+1}}\right]$$

$$I_{n+2} - I_n = 0 \Rightarrow I_{n+2} = I_n$$

(B) $\sum_{m=1}^{10} I_{2m+1} = I_3 + I_5 + I_7 + \ldots I_{21}$

$$= 10 \cdot I_3 = 10\pi$$

$$I_3 = \int_0^{\pi} \frac{\sin 3x}{\sin x}\,dx = \int_0^{\pi} \frac{\sin 3x - \sin x + \sin x}{\sin x}$$

$$= \int_0^{\pi} \frac{2\cos 2x \cdot \cancel{\sin x}}{\cancel{\sin x}} + 1 = \cancel{\frac{\sin 2x}{2}}\Big|_0^{\pi} + x\Big|_0^{\pi} = \pi$$

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Q Adv $I = \int_{-1}^{1} \frac{dx}{1+e^x}$

$\int_{-a}^{a} f(x)\,dx = \int_{0}^{a} f(x)+f(-x)\,dx$

$= \int_0^1 \frac{1}{1+e^x} + \frac{e^x}{1+e^x} = \int \frac{1+e^x}{1+e^x} = \int 1\,dx$

$= \int_0^1 \frac{1}{1+e^x} + \frac{1}{1+e^{-x}}\,dx$

$= \int_0^1 1\,dx = x\Big|_0^1$

$= 1$

Q Jee mains $f(x) = \frac{2-x\cos x}{2+x\cos x}$ & $g(x) = \ln x\ (x>0)$

then value of $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} g(f(x))\,dx = ?$

$= \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \ln\left(\cancel{\frac{2-x\cos x}{2+x\cos x}}\right)dx = 0$

Q Jee mains $\int_{-2}^{2} \frac{|x^3+x|\,dx}{e^{x|x|}+1}$

NENO

$x^3+x = +ve$

$x^3 +ve$

$x = +ve$

$x \in (0,2)$

$= \int_0^2 |x^3+x|\,dx$

$= \int_0^2 x^3+x\,dx = \frac{x^4}{4}+\frac{x^2}{2}\Big|_0^2$

$= \frac{16}{4}+\frac{4}{2} = 4+2 = 6$

Q Jee mains $I = \int_{-\frac{\pi}{2}}^{\pi/2} \frac{dx}{(1+e^x)(\sin^6 x+\cos^6 x)}$

$I = \int_0^{\pi/2} \frac{dx}{\sin^6 x + \cos^6 x}$

$I = \int_0^{\pi/2} \frac{dx}{1-3\sin^2 x\cdot\cos^2 x}$ $\div \cos^4 x$

$= \int_0^{\pi/2} \frac{\sec^2 x\cdot(1+\tan^2 x)\cdot dx}{(1+\tan^2 x)^2 - 3\tan^2 x}$

$= \int_0^{\pi/2} \frac{(1+\tan^2 x)\sec^2 x\cdot dx}{1+\tan^4 x - \tan^2 x}$

$= \int_0^{\infty} \frac{t^2+1}{t^4-t^2+1}\cdot dt = \int \frac{1+\frac{1}{t^2}}{\left(t-\frac{1}{t}\right)^2+1}$

$\infty \quad t^2+\frac{1}{t^2}-1 \quad -2+2 \qquad z = t-\frac{1}{t}$

$= \int_{-\infty}^{\infty} \frac{dz}{z^2+1}$ ✓

Aap hi Ko Karna hai!!

Nishant Jindal

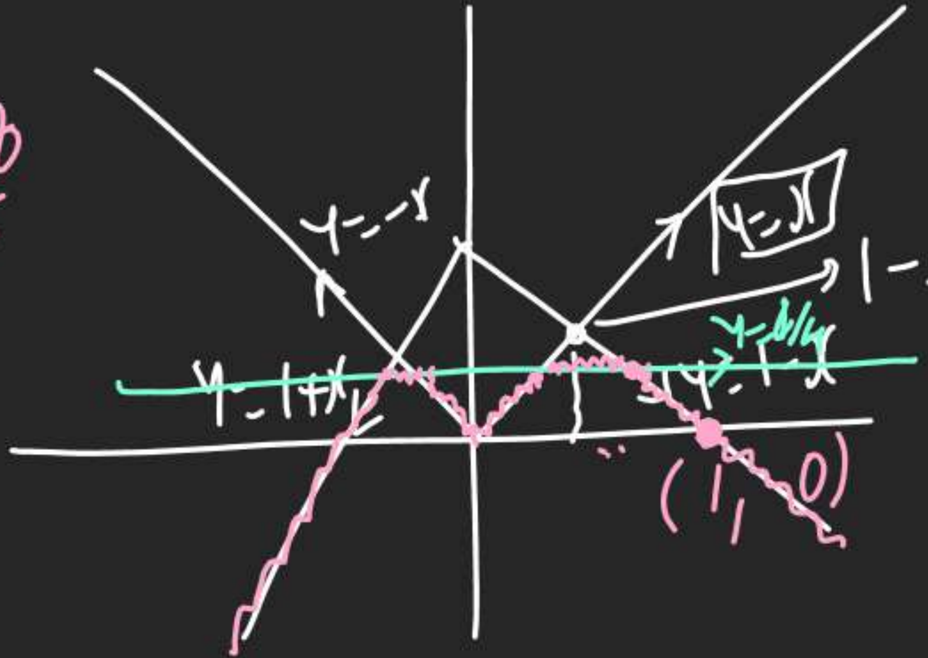
Q41) $\int_{-1/2}^{1/2} \left|\frac{x+1}{x-1}-\frac{x-1}{x+1}\right| dx$

$= \int_{-1/2}^{1/2} \left|\frac{4x}{x^2-1}\right| dx$ 44 Copy

even ⊕ ⊖ $f(-x) = \left|\frac{-4x}{x^2-1}\right|$

$= -2\times 2\int_0^{1/2} \frac{2x}{x^2-1}\,dx = -4\ln|x^2-1|\Big|_0^{1/2}$

Q38



$1-x = x$

$2x=1$

$x=\frac{1}{2}$

Q36 $\left[\frac{|\sin x|}{2}\right]=0$

$|\sin x| \in [0,1]$

$\frac{|\sin x|}{2} \in [0,\frac{1}{2}]$

$\left[\frac{|\sin x|}{2}\right]=0$

49) $I = \int_{-\pi/2}^{\pi/2} \frac{|x|}{8\cos^2 2x+1}$ even

$= 2\int_0^{\pi/2} \frac{x\,dx}{1+8\cos^2(2x)}$ Removal of x

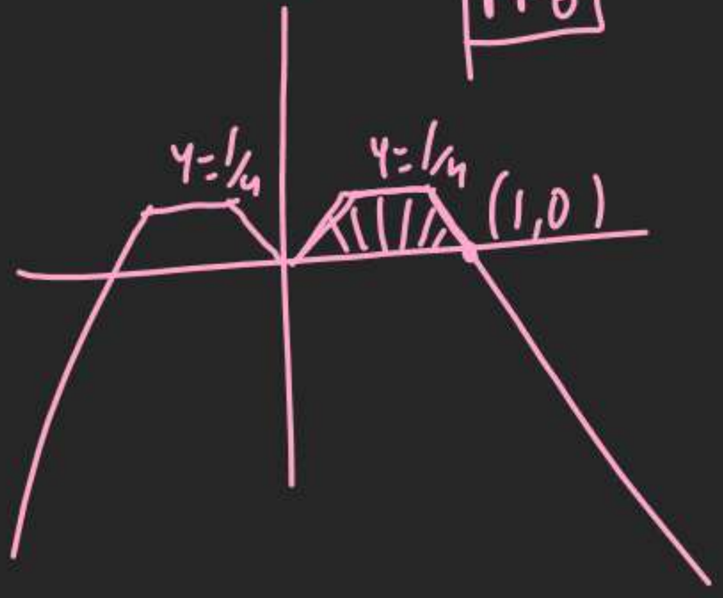
$\div \cos^2(2x)$

$= 2\times\frac{\pi}{4}\int_0^{\pi/2} \frac{dx}{1+8\cos^2(2x)}$

$= \frac{\pi}{2}\int_0^{\pi/2} \frac{\sec^2(2x)}{1+\tan^2(2x)+8}\,dx$

$\tan 2x = t$

46 hold $\boxed{\text{Pr6}}$



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NENO

Q52 $I = \int_{-1}^{1} \frac{\sin x + x^2}{3-|x|} \, dx$

$= \int_{-1}^{1} \cancel{\frac{\sin x}{3-|x|}} \, dx + \int_{-1}^{1} \frac{x^2 \cdot dx}{3-|x|} \quad \frac{O}{E} \quad \frac{E}{E} = E.$

$0 + 2\int_{0}^{1} \frac{x^2 \cdot dx}{3-x} \cdot dx$

Divide

NENO

Q54 $\int_{-\pi}^{\pi} \frac{2x(1+\sin x)}{1+\cos^2 x} = \int_{-\pi}^{\pi} \cancel{\frac{2x\,dx}{1+\cos^2 x}} + 2\int_{-\pi}^{\pi} \frac{x \cdot \sin x \cdot dx}{1+\cos^2 x} \quad \frac{E}{E}$

$O \times O = E$

$0 + 2\times 2 \boxed{\int_{0}^{\pi} \frac{x \sin x}{1+\cos^2 x} \cdot dx} = 4\times\frac{\pi^2}{16} = \pi^2/4$

Pr 4
Removal of x

ans of Q6

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Prop. 6 Limit half Karne Wali Prop.

$$\boxed{\int_0^{2a} f(x)\cdot dx = \begin{cases} 2\int_0^{a} f(x)\cdot dx & \text{if } f(2a-x) = f(x) \\ 0 & f(2a-x) = -f(x) \end{cases}}$$

① $I = \int_0^{2\pi} \cos^4 x \cdot dx = ?$

Pr6 $\cos^4(2\pi - x) = \cos^4 x$

If Limit wld have been $\int_0^{\pi/2}$ I were using Walli

$= 2\int_0^{\pi} \cos^4 x \cdot dx$ $\quad \{(\cos(\pi - x))^4 = (-\cos x)^4 = \cos^4 x\}$

Pr 6

$= 2\times 2\int_0^{\pi/2} \cos^4 x\, dx = 4\times \frac{3\times 1}{4\times 2}\times \frac{\pi}{2} = \frac{3\pi}{4}.$