

$$\int \frac{dx}{x^2 + a^2}$$

$$\frac{1}{a} \tan^{-1} \frac{x}{a} \quad \frac{1}{2a}$$

$$\int \sec x dx = \ln |\sec x + \tan x|$$

$$= \ln \left| \tan \frac{x}{2} \right| + C$$

$$\int \frac{dx}{\sqrt{x^2 + a^2}} = \ln(x + \sqrt{x^2 + a^2})$$

$$\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1} \frac{x}{a} + C$$

$$\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \ln \left| \frac{a+x}{a-x} \right| + C$$

$$\int \sqrt{a^2 - x^2} = \left(\frac{x}{2} \right) \sqrt{a^2 - x^2} + \left(\frac{a^2}{2} \right) \sin^{-1} \frac{x}{a} + C$$

$$\int \frac{dx}{x^2 + a^2} = \left[\frac{1}{a} \right] \left[\tan^{-1} \right] \frac{x}{a} + C$$

$$\int \frac{dx}{\sqrt{x^2 + a^2}} = \left[\ln \right] \left(x + \sqrt{x^2 + a^2} \right)$$

$$Q \int \frac{dx}{3+x^2}$$

$$\int \frac{dx}{x^2 + (\sqrt{3})^2} \rightarrow \int \frac{dx}{x^2 + a^2}$$

$$= \frac{1}{\sqrt{3}} \tan^{-1} \frac{x}{\sqrt{3}} + C$$

$$Q \int \frac{dx}{3-x^2} \rightarrow \int \frac{dx}{(\sqrt{3})^2 - x^2} \leftarrow \int \frac{dx}{a^2 - x^2}$$

$$= \frac{1}{2\sqrt{3}} \ln \left| \frac{\sqrt{3}+x}{\sqrt{3}-x} \right| + C$$

$$Q \int \sqrt{4-x^2} dx \rightarrow \int \sqrt{a^2-x^2} dx$$

$$= \frac{x}{2} \sqrt{2^2-x^2} + \frac{2^2}{2} \sin^{-1} \frac{x}{2} + C$$

$$Q \int \sqrt{19-x^2} dx \rightarrow \int \sqrt{a^2-x^2}$$

$$= \frac{x}{2} \sqrt{19-x^2} + \frac{19}{2} \sin^{-1} \frac{x}{\sqrt{19}} + C$$

$$Q \int \sqrt{x^2-19} dx = \int \sqrt{x^2-(\sqrt{19})^2} dx \rightarrow \int \sqrt{x^2-a^2} dx$$

$$= \frac{x}{2} \sqrt{x^2-19} - \frac{19}{2} \ln(x + \sqrt{x^2-19}) + C$$

$$14) \int \frac{1 + \tan^2 x}{1 + \tan^2 x}$$

$$\int \frac{1 + \tan^2 x}{2 \tan^2 x}$$

$$\Rightarrow \frac{1}{2} \int \frac{dx}{\tan^2 x} + \int \frac{\cancel{\tan^2 x}}{2 \cancel{\tan^2 x}} dx$$

$$\Rightarrow \frac{1}{2} \int \sec^2 x + \int \frac{1}{2} dx$$

$$\Rightarrow \frac{1}{2} \tan x + \frac{x}{2} + C$$

$$5 \int a^x \cdot e^x \cdot dx \quad \int A^x \cdot dx = \frac{A^x}{\ln A}$$

$$= \frac{(ae)^x}{\ln ae} + C$$

Trick

$$14) \int \sin x d(\sin x) \quad \sin x = y$$

$$\int y dy = \frac{y^2}{2} + C$$

$$= \frac{\sin^2 x}{2} + C$$

$$20) \int \tan^3 x \cdot d(\tan x)$$

$$\int y^3 dy = \frac{y^4}{4} + C$$

$$= \frac{\tan^4 x}{4} + C$$

$$Q8 \quad 3 \cdot 4 \int x^{-17} dx$$

$$(3 \cdot 4) \frac{x^{-17+1}}{-17+1} + C$$

$$(3 \cdot 4) \frac{x^{-93}}{-93} + C$$

$$Q7 \int \frac{dh}{\sqrt{2gh}} \quad \begin{matrix} \rightarrow h \text{ var} \\ g = \text{const.} \end{matrix}$$

$$\frac{1}{\sqrt{2g}} \int \frac{dh}{\sqrt{h}} \quad \int \frac{dx}{\sqrt{x}} = 2\sqrt{x}$$

$$\frac{2\sqrt{h}}{\sqrt{2g}} + C$$

$$Q9 \int \frac{\sqrt{x}}{x^3} - \frac{x^3 \cdot e^x}{x^3} + \frac{x^2}{x^3} dx$$

$$\int x^{-\frac{5}{2}} dx - \int e^x dx + \int \frac{1}{x} dx$$

G12 Sir Pro Army 12th.

$$Q23 \int (8-3x)^{\frac{6}{5}} dx$$

$$Q26 \int \frac{\sin x dx}{\cos x} = \frac{(8-3x)^{\frac{6}{5}+1}}{\frac{11}{5} \times (-3)} + C \int \frac{(1-x)^2}{x\sqrt{x}} dx$$

$$\int \frac{\sin x}{\cos x} \times \frac{1}{\cos x} dx$$

$$\int \sec x \tan x dx$$

$$\sec x + C$$

$$\int \frac{1}{x\sqrt{x}} - \frac{2x}{x\sqrt{x}} + \frac{x^2}{x\sqrt{x}}$$

$$\int x^{-3/2} - 2 \cdot x^{-1/2} + x^{1/2} dx$$

$$37x \quad 12) \int \frac{dx}{\sqrt{8-3x^2}} = \frac{1}{\sqrt{3}} \int \frac{dx}{\sqrt{1^2-x^2}} \rightarrow \int \frac{dx}{\sqrt{a^2-x^2}}$$

$$= \frac{1}{\sqrt{3}} \sin^{-1} \frac{x}{1} + C$$

Q 19 $\int \sin x \cos x$

(M1) $\frac{d(\sin x)}{dx} = \cos x$

$d(\sin x) = \cos x dx$

$\int \sin x \cdot \cos x \cdot dx$

$\frac{1}{2} \int 2 \sin x \cdot \cos x dx$

$\frac{1}{2} \int \sin 2x dx$

$-\frac{1}{2} \frac{\cos(2x)}{2} + C$ Ans 1

$\Rightarrow \frac{2 \sin^2 x - 1}{4} + C$

$\Rightarrow \frac{\sin^2 x}{2} + C$

(M2) $\sin x = y$

$\int y \cdot dy$

$= \frac{y^2}{2} + C$

$= \frac{\sin^2 x}{2} + C$ Ans 2

\int Trigo fcn can give 2 or more different Answers.

$\int \frac{dx}{x^2 + a^2}$

$\frac{1}{a} \tan^{-1} \frac{x}{a}$

$\int \frac{dx}{\sqrt{x^2 + a^2}}$

$\ln(x + \sqrt{\dots})$

$\int \frac{dx}{x^2 - a^2}$

$\frac{1}{2a} \ln \left| \frac{x-a}{x+a} \right|$

$\int \frac{dx}{\sqrt{x^2 - a^2}}$

$\ln(x + \sqrt{\dots})$

$\int \frac{dx}{a^2 - x^2}$

$\frac{1}{2a} \ln \left| \frac{a+x}{a-x} \right|$

$\int \frac{dx}{\sqrt{a^2 - x^2}}$

\sin^{-1}

$$Q \int \frac{dx}{4x^2+5} \rightarrow \int \frac{dx}{(2x)^2+(\sqrt{5})^2} \rightarrow \int \frac{dx}{x^2+a^2}$$

$$= \frac{1}{2\sqrt{5}} \tan^{-1} \frac{2x}{\sqrt{5}} + C$$

$$Q \int \frac{dx}{7x^2+3} = \int \frac{dx}{(\sqrt{7}x)^2+(\sqrt{3})^2} \rightarrow \int \frac{dx}{x^2+a^2}$$

$$\frac{1}{\sqrt{7}\sqrt{3}} \tan^{-1} \frac{\sqrt{7}x}{\sqrt{3}} + C$$

$$Q \int \frac{dx}{\sqrt{4x^2+3}} = \int \frac{dx}{\sqrt{(2x)^2+(\sqrt{3})^2}} \rightarrow \int \frac{dx}{\sqrt{x^2+a^2}}$$

$$= \frac{1}{2} \ln \left\{ 2x + \sqrt{4x^2+3} \right\} + C$$

$$Q \int \frac{dx}{\sqrt{4-3x^2}} = \int \frac{dx}{\sqrt{(2)^2-(\sqrt{3}x)^2}} \rightarrow \int \frac{dx}{\sqrt{a^2-x^2}}$$

$$= \frac{1}{\sqrt{3}} \sin^{-1} \frac{\sqrt{3}x}{2} + C$$

$$\int \sqrt{1-x^2} dx \rightarrow \int \sqrt{a^2-x^2}$$

$$\frac{x}{2} \sqrt{\quad} + \frac{a^2}{2} \sin^{-1} \frac{x}{a} + C$$

$$\frac{x}{2} \sqrt{1-x^2} + \frac{1^2}{2} \sin^{-1} \frac{x}{1} + C$$

$$Q \int \frac{dx}{\sqrt{2x^2-1}}$$

$$\int \frac{dx}{\sqrt{(\sqrt{2}x)^2-1^2}} \rightarrow \int \frac{dx}{\sqrt{x^2-a^2}}$$

$$\frac{1}{\sqrt{2}} \ln \left\{ \sqrt{2}x + \sqrt{2x^2-1} \right\} + C$$

$$Q \int \sqrt{3x^2+4} dx \rightarrow \int \sqrt{x^2+a^2}$$

$$\int \sqrt{(\sqrt{3}x)^2+2^2} dx$$

$$\frac{1}{\sqrt{3}} \left(\frac{\sqrt{3}x}{2} \sqrt{3x^2+4} + \frac{2^2}{2} \ln \left\{ \sqrt{3}x + \sqrt{3x^2+4} \right\} \right) + C$$

$$Q \int \sqrt{x^2-4x+6} dx$$

$$\int \sqrt{x^2-4x+4+2} dx \rightarrow \int \sqrt{x^2+a^2} dx$$

$$\int \sqrt{(x-2)^2+(\sqrt{2})^2} dx$$

$$\frac{1}{1} \left(\frac{x-2}{2} \sqrt{x^2-4x+6} + \left(\frac{\sqrt{2}}{2}\right)^2 \ln \left\{ (x-2) + \sqrt{x^2-4x+6} \right\} \right) + C$$

$$Q \int x \cdot (3x+5)^7 dx$$

$$\frac{1}{3} \int 3x (3x+5)^7 dx$$

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$$\Rightarrow \frac{1}{3} \int ((3x+5) - 5) (3x+5)^7 dx$$

$$\Rightarrow \frac{1}{3} \int (3x+5)^8 - 5(3x+5)^7 dx$$

$$\Rightarrow \frac{1}{3} \int (3x+5)^8 dx - \frac{5}{3} \int (3x+5)^7 dx$$

$$\Rightarrow \frac{1}{3} \frac{(3x+5)^9}{9 \times 3} - \frac{5}{3} \frac{(3x+5)^8}{8 \times 3} + C$$

Q

$$\int x \cdot (2x+3)^5 \cdot dx$$

$$\frac{1}{2} \int ((2x+3) - 3) (2x+3)^5 dx$$

$$\frac{1}{2} \int (2x+3)^6 - \frac{3}{2} \int (2x+3)^5 dx$$

$$\frac{1}{2} \frac{(2x+3)^7}{7 \times 2} - \frac{3}{2} \times \frac{(2x+3)^6}{6 \times 2} + C$$

$$Q \int \frac{dx}{x^2 + 2x + 2}$$

$$\int \frac{dx}{(x+1)^2 + 1}$$

$$\int \frac{dx}{(x+1)^2 + 1} \rightarrow \int \frac{dx}{x^2 + 1}$$

$$\frac{1}{1} \tan^{-1} \frac{x+1}{1} + C$$

$$4) \boxed{13}x^2 - x^2$$

$$\left(\frac{13}{2}\right)^2 - \left(x - \frac{13}{2}\right)^2$$

$$5) \boxed{9}x - x^2$$

$$\left(\frac{9}{2}\right)^2 - \left(x - \frac{9}{2}\right)^2$$

$$1) x^2 + \boxed{13}x + 14$$

$$= \left(x + \frac{13}{2}\right)^2 - \left(\frac{13}{2}\right)^2 + 14$$

$$2) x^2 - \boxed{9}x + 3$$

$$\left(x - \frac{9}{2}\right)^2 - \left(\frac{9}{2}\right)^2 + 3$$

$$3) 2x^2 + 7x + 5$$

$$2 \left\{ x^2 + \boxed{\frac{7}{2}}x + \frac{5}{2} \right\}$$

$$2 \left\{ \left(x + \frac{7}{4}\right)^2 - \left(\frac{7}{4}\right)^2 + \frac{5}{2} \right\}$$

$$Q \int \frac{dx}{4x^2 - 4x - 3}$$

$$\frac{1}{4} \int \frac{dx}{x^2 - (x - \frac{3}{4})} \quad -\frac{1}{4} - \frac{3}{4} = -1$$

$$\frac{1}{4} \int \frac{dx}{(x - \frac{1}{2})^2 - (\frac{1}{2})^2 - \frac{3}{4}}$$

$$\frac{1}{4} \int \frac{dx}{(x - \frac{1}{2})^2 - (1)^2} \int \frac{dx}{x^2 - a^2}$$

$$\frac{1}{4} \times \frac{1}{2} \ln \left| \frac{(x - \frac{1}{2}) - 1}{(x - \frac{1}{2}) + 1} \right| + C$$

Q

$$\int \frac{dx}{\sqrt{x - x^2}}$$

in Qs of Q Eqn always try to make Per. Eqn

Use Basic
form

$$\int \frac{dx}{\sqrt{(\frac{1}{2})^2 - (x - \frac{1}{2})^2}} \rightarrow \int \frac{dx}{\sqrt{a^2 - x^2}}$$

$$= \frac{1}{1} \sin^{-1} \left(\frac{x - \frac{1}{2}}{\frac{1}{2}} \right) + C$$

$$Q \int \frac{dx}{\sqrt{2ax - x^2}}$$

$$\int \frac{dx}{\sqrt{a^2 - (x - a)^2}}$$

$$\sin^{-1} \frac{x - a}{a} + C$$

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$$Q \int \frac{1+2x^2}{x^2(1+x^2)} dx$$

$$\int \frac{\cancel{1+x^2}}{x^2(\cancel{1+x^2})} + \frac{x^2}{x^2(1+x^2)} dx$$

$$\int \frac{dx}{x^2} + \int \frac{dx}{1+x^2}$$

$$-\frac{1}{x} + \frac{1}{1} \tan^{-1} \frac{x}{1} + C$$

$$\tan^{-1} x - \frac{1}{x} + C$$

$$Q = \int \frac{(1+x)^2}{x(1+x^2)} dx$$

$$\Rightarrow \int \frac{\cancel{1+x^2}}{x(\cancel{1+x^2})} + \frac{2x}{x(1+x^2)} dx$$

$$\int \frac{dx}{x} + 2 \int \frac{dx}{1+x^2}$$

$$\ln|x| + 2 \tan^{-1} x + C$$

$$\underline{\text{Nr's Deg} \geq \text{Dr's Deg}}$$

If Nr's Deg \geq Dr's Deg then divide.

Q $\int \frac{x^4}{x^2+1} dx$ (11)

$$\int \frac{x^4-1+1}{x^2+1}$$

$$\int \frac{\cancel{x^4-1}}{\cancel{x^2+1}} + \frac{1}{x^2+1} dx$$

$$\frac{x^3}{3} - x + \frac{1}{x} + \tan^{-1} \frac{x}{1} + C$$

Q

$$\int \frac{x^4}{x-1} dx$$

$$\int \frac{x^4-1+1}{(x-1)}$$

$$\int \frac{x^4-1}{x-1} + \int \frac{1}{x-1} dx$$

$$\int \frac{\cancel{x^4-1}}{\cancel{x-1}} (x^2+1) + \ln|x-1|$$

$$\int x^3+x^2+x+1 dx + \ln|x-1|$$

$$\Rightarrow \frac{x^4}{4} + \frac{x^3}{3} + \frac{x^2}{2} + x + \ln|x-1| + C$$

$$(x^2-1) = (x-1)(x+1)$$

$$(x^3-1) = (x-1)(x^2+x+1)$$

$$(x^4-1) = (x-1)(x^3+x^2+x+1)$$

$$(x^{17}-1) = (x-1)(x^{16}+x^{15}+x^{14}+\dots+x+1)$$

$$Q \int \frac{(\sqrt{x}+1)(x^2-\sqrt{x})}{x(\sqrt{x}+x+\sqrt{x})} dx$$

$$M_{\text{Jewar}} \int \frac{(\sqrt{x}+1)(\cancel{\sqrt{x}})(x^{3/2}-1)}{(\cancel{\sqrt{x}})(x+\sqrt{x}+1)} dx$$

$$\int \frac{(\sqrt{x}+1)((\sqrt{x})^3-1)}{((\sqrt{x})^2+\sqrt{x}+1)} dx$$

$$x^3-1 = (x-1)(x^2+x+1)$$

$$\int \frac{(\sqrt{x}+1)(\sqrt{x}-1)(\cancel{x^2+x+1})}{(\cancel{x^2+x+1})} dx$$

$$\int (x-1) dx = \frac{x^2}{2} - x + C$$

$$x^6+1 = (x^2)^3+1^3 \\ = (x^2+1)(x^4-x^2+1)$$

$$Q \int \frac{x^4+1}{x^6+1} dx$$

$$\int \frac{x^4+1}{(x^2+1)(x^4-x^2+1)} dx = \tan^{-1}x + \frac{1}{3} \tan^{-1}(x^3) + C$$

$$\int \frac{\cancel{x^4-x^2+1}}{(x^2+1)(\cancel{x^4-x^2+1})} + \frac{x^2}{(x^2+1)(x^4-x^2+1)} dx$$

$$\int \frac{dx}{x^2+1} + \frac{1}{3} \int \frac{3x^2 dx}{x^6+1}$$

$$\tan^{-1}x + \frac{1}{3} \int \frac{d(x^3)}{(x^3)^2+1^2} = \tan^{-1}x + \frac{1}{3} \int \frac{dy}{y^2+1^2} = \tan^{-1}x + \frac{1}{3} \tan^{-1}y$$