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2.

$x^2-2cx-5d=0$ (roots a, b)

$\boxed{2c=a+b}$ — ①

$-5d=ab$ — ②

$x^2-2ax-5b=0$ (roots c, d)

$\boxed{2a=c+d}$ — ③

$-5b=cd$ — ④

$a+b+c+d=?$

$=2(a+c)=4\alpha$

① + ③

$a+c=b+d$

② × ④

$25bd=abcd$

$bd=0$ or $\boxed{ac=25}$

$b\ c\ a\ d \rightarrow A.P.$

$\alpha-3\beta\quad \alpha-\beta\quad \alpha+\beta\quad \alpha+3\beta$

$a^2-2ca-5d=0$

$c^2-2ac-5b=0$

$a^2+c^2-\cancel{4ac}^{100}-5(a+c)=0$

$(a+c)^2-5(a+c)-150=0$

$(a+c-15)(a+c+10)=0$

$\alpha^2-2\alpha\beta-3\beta^2=(\alpha+\beta)(\alpha-3\beta)=-5(\alpha+3\beta)$

$\alpha^2+2\alpha\beta-3\beta^2=(\alpha-\beta)(\alpha+3\beta)=-5(\alpha-3\beta)$

$a+c=-10$ ✗

$(a-c)^2=100-100=0$

$-4\alpha\beta=-30\beta$

$\boxed{a+c=15}$

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4.

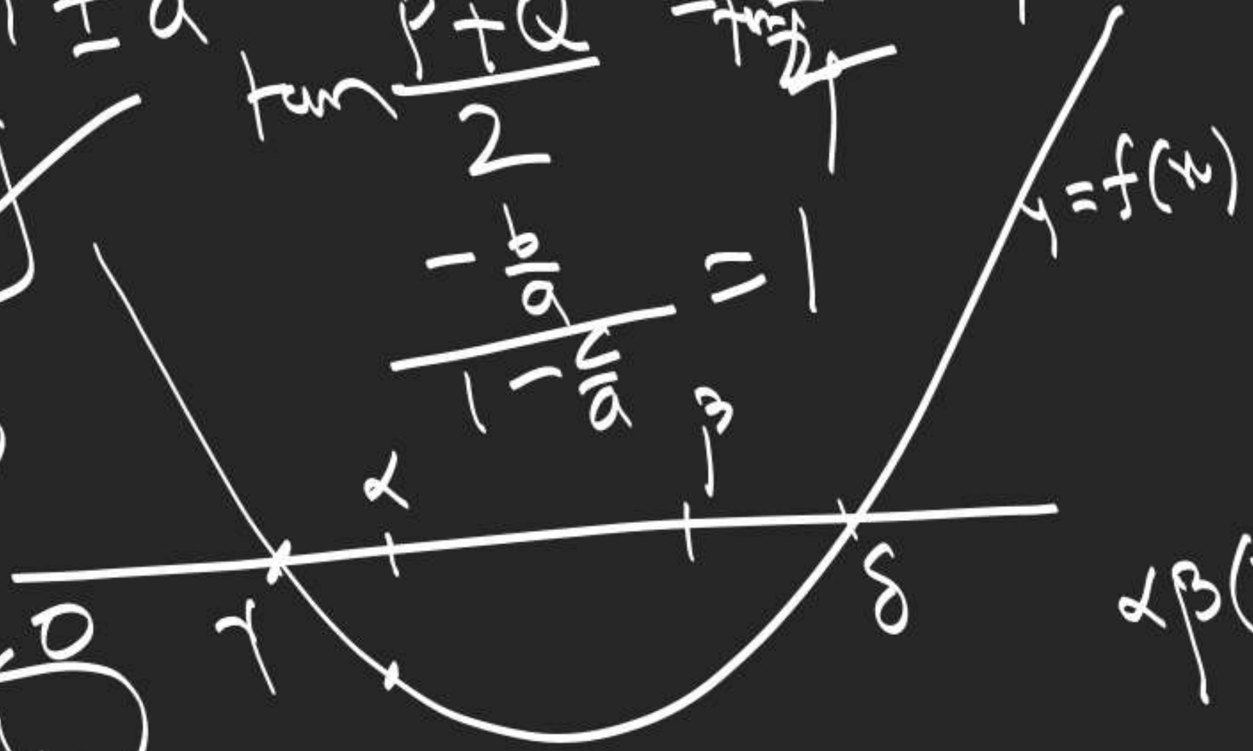
$\alpha, \beta = 1 \pm a$

$\tan\frac{P+Q}{2} = \tan\frac{\pi}{4} = 1$

$\boxed{\sum a^2 \geq \sum ab}$

$\frac{-\frac{b}{a}}{1-\frac{c}{a}} = 1$

$(a+b+c)^2 \geq 0$



$\sum a^2 + 2\sum ab \geq 0$

$\boxed{1 \geq \sum ab \geq -\frac{1}{2}}$

$a+b = -a$

$a = 1 \Leftarrow a_b = b$ ✓

$b = -2$

$\boxed{x > 0}$

$x + \frac{1}{x}$

$f(1-a) < 0$

&

$f(1+a) < 0$

$\alpha\beta = -32, \ \gamma\delta = 62$

$(\alpha+\beta) + (\gamma+\delta) = 18$

$\alpha\beta(\gamma+\delta) + \gamma\delta(\alpha+\beta) = -200$

$-32(\gamma+\delta) + 62(\alpha+\beta) = -200$

$\alpha+\beta = ?, \ \gamma+\delta = ?$

$k = (\alpha+\beta)(\gamma+\delta) + \alpha\beta + \gamma\delta.$

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$4^x - a(2^x) - a + 3 \le 0$ for atleast one real x

$a = ?$

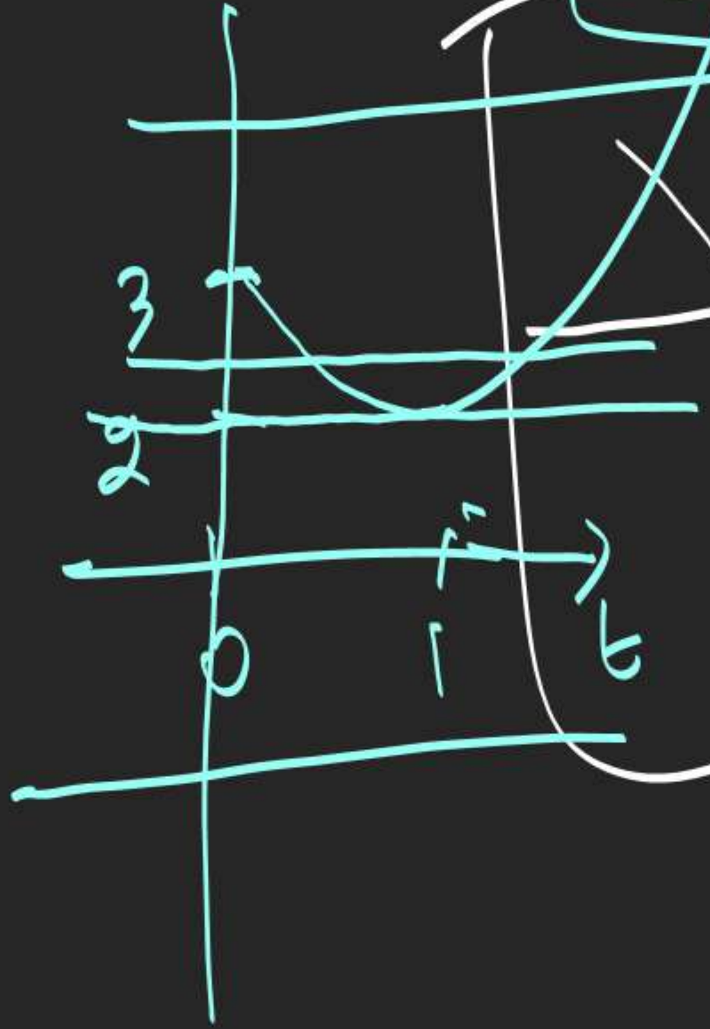
$\boxed{a \in [2, \infty)}$

$f(t) = t^2 - at - a + 3 \le 0$ for atleast one $t > 0$

$\boxed{\frac{t^2+3}{t+1}} \le a$

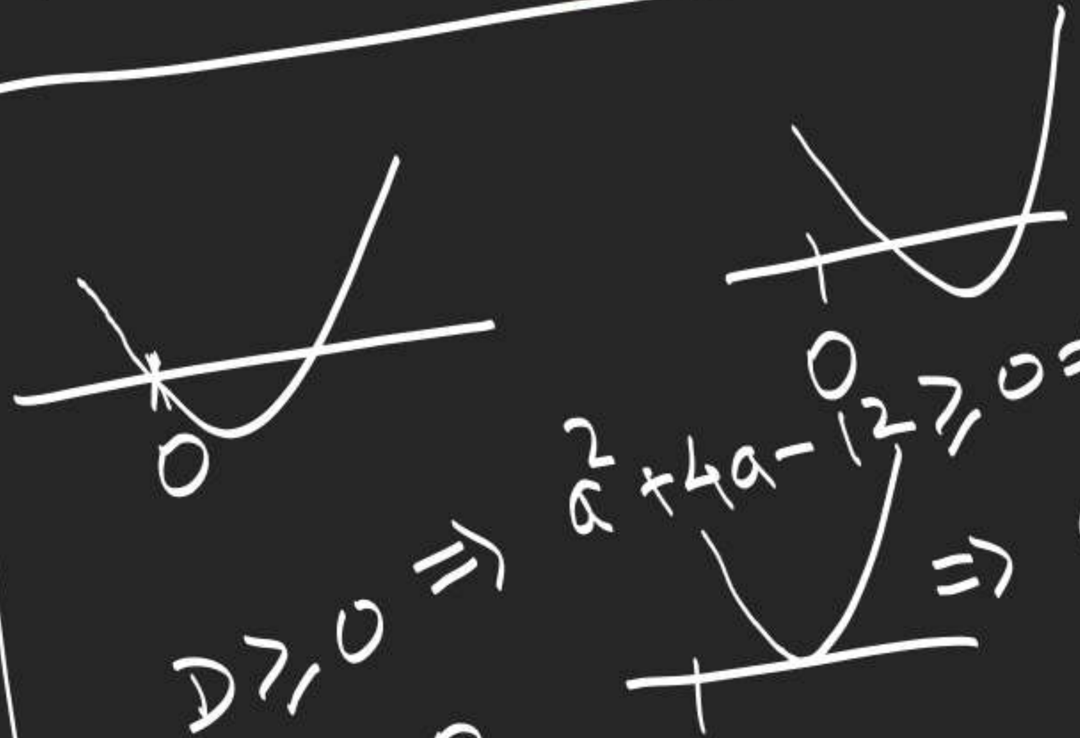
$t - 1 + \frac{4}{t+1}$

$1 - \frac{4}{(t+1)^2} = \frac{(t-1)(t+3)}{(t+1)^2}$



$f(0) < 0 \Rightarrow a > 3$

OR



$D \ge 0 \Rightarrow a^2 + 4a - 12 \ge 0 \Rightarrow (a+6)(a-2) \ge 0$

$(-\infty, -6] \cup [2, \infty)$

$-\frac{b}{2a} > 0 \Rightarrow \frac{a}{2} > 0 \Rightarrow a > 0$

$f(0) \ge 0 \Rightarrow a \le 3$

$\Downarrow$

$a \in [2, 3]$

$\boxed{a \in [2, \infty)}$ ⟹ Ans.

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Geometric Progression (GP)

consecutive terms have same ratio. called common ratio.

$\lim_{n\to\infty}(-1)^n$ not exist $\quad \lim_{n\to\infty} 1^n = 1$

$\{a, ar, ar^2, ar^3, \dots, ar^{n-1}\}$

$\lim_{n\to\infty} 2^n \Rightarrow \infty \quad \lim_{n\to\infty}\left(-\frac{1}{2}\right)^n = 0$

$\lim_{n\to\infty}\left(\frac{1}{2}\right)^n = 0$

Sum of first 'n' terms of G.P.

$$① - S = a + ar + ar^2 + ar^3 + \dots + ar^{n-1}$$

$$② - rS = \quad ar + ar^2 + ar^3 + \dots + ar^{n-1} + ar^n$$

$① - ②$

$$(1-r)S = a - ar^n$$

$$\boxed{S = \frac{a(1-r^n)}{(1-r)}}$$

Sum of infinite terms

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$\lim_{n\to\infty} S_n = \lim_{n\to\infty} \frac{a(1-\cancel{r^n})}{1-r}$$

$$\boxed{S_\infty = \frac{a}{1-r}, \quad -1<r<1}$$

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$$\boxed{\begin{aligned} S_\infty &= \frac{a}{1-r}, \quad |r|<1 \\ &\text{not defined.} \quad |r| \geq 1 \end{aligned}}$$

$$S_n = \frac{a(1-r^n)}{1-r}, \quad r \neq 1$$

$$a+a+a+\dots+a = na$$

$n \in \mathbb{N}$,

$$\lim_{n\to\infty} \alpha^n = \begin{cases} 0 & |\alpha|<1 \\ \infty & |\alpha|>1 \\ 1 & \alpha=1 \\ \text{not exist} & \alpha=-1 \end{cases}$$

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Note :- ① product of equidistant terms from the beginning & the end is the same.

$$T_1 T_n = T_2 T_{n-1} = T_3 T_{n-2} = \cdots$$

$a, ar, ar^2, \ldots, \frac{b}{r^2}, \frac{b}{r}, b$

③ $a, ar, ar^2, ar^3, \ldots, ar^{n-1}$ → G.P.

$\Rightarrow \log a, \log(ar), \log(ar^2), \log(ar^3), \ldots \log(ar^{n-1}) \rightarrow$ A.P.

② 3 terms in G.P. → $\frac{a}{r}, a, ar$

4 —"— → $\frac{a}{r^3}, \frac{a}{r}, ar, ar^3$

5 —"— → $\frac{a}{r^2}, \frac{a}{r}, a, ar, ar^2$

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Geometric Mean of 2 positive numbers, a, b.

G.M. of a, b = G

$\Rightarrow$ $\boxed{a,\ G,\ b \text{ are in G.P.}}$

$ab = GG$

$ab = G^2$

$$\boxed{G = \sqrt{ab}}$$

G.M of 'n' positive numbers

$x_1, x_2, x_3, \ldots, x_n$

$$\boxed{GM = \left(x_1 x_2 x_3 \ldots x_n\right)^{\frac{1}{n}}}$$

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Inserting 'n' G.Ms between 2 positive numbers a, b

$\boxed{\text{Ex-II (11-18)} \quad \text{T1-(Ex-III) remaining}}$

$\boxed{a, \underbrace{G_1, G_2, G_3, \ldots, G_n}_{\text{'n' G.M.s}}, b \quad \text{form G.P.}}$

$$ar^{n+1} = b \Rightarrow r = \left(\frac{b}{a}\right)^{\frac{1}{n+1}}$$ ✓

$$G_1 = ar = a\left(\frac{b}{a}\right)^{\frac{1}{n+1}}$$

$$G_2 = ar^2 = a\left(\frac{b}{a}\right)^{\frac{2}{n+1}}$$

$$\vdots$$