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Partial Fraction ✓

$$\int \left(\frac{\overbrace{a_m x^m + a_{m-1} x^{m-1} + \cdots + a_0}^{N(x)}}{\underbrace{b_n x^n + b_{n-1} x^{n-1} + \cdots + b_0}_{D(x)}}\right) dx$$

$$= \begin{cases} \text{Use Partial Fraction}, & m < n \\ \displaystyle\int \left(\frac{D(x)Q(x) + R(x)}{D(x)}\right) dx, & m \geq n \end{cases}$$

$$= \int Q(x)\, dx + \int \frac{R(x)}{D(x)}\, dx$$

Using P.F

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$$\boxed{\frac{f(x)}{(x-1)(x-2)(x-3)} = \frac{A}{x-1} + \frac{B}{x-2} + \frac{C}{x-3}}$$

$$f(x) = A(x-2)(x-3) + B(x-1)(x-3) + C(x-1)(x-2)$$

Equate coeff. of x^2, x, const.

$A, B, C = ?$

$$\frac{x}{(x-1)(x-3)} = \frac{3(x-1)-(x-3)}{2(x-1)(x-3)}$$ wt

$x=1$,

$x=2$

$x=3$.

$A, B, C = ?$

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$$\frac{f(x)}{(x-1)^2(x-2)^3(x+7)} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x-2} + \frac{D}{(x-2)^2} + \frac{E}{(x-2)^3} + \frac{F}{x+7}$$

$$\frac{f(x)}{(x-1)(x^2+9)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+9}$$

$$\frac{f(x)}{(x-2)^2(x^2+9)} = \frac{A}{x-2} + \frac{B}{(x-2)^2} + \frac{Cx+D}{x^2+9}$$

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1. $$\int \frac{x\,dx}{(x-1)(x^2+4)} = \int \left(\frac{A}{x-1} + \frac{Bx+C}{x^2+4}\right) dx$$

$$= \frac{1}{5}\ln|x-1| - \frac{1}{10}\ln|x^2+4| + \frac{4}{5}\frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right) + C$$

$$x = A(x^2+4) + (Bx+C)(x-1)$$

$x=1,\quad 1 = 5A \Rightarrow \boxed{A = \frac{1}{5}}$

const $\quad 0 = 4A - C \Rightarrow C = \frac{4}{5}$

coeff of x^2 $\quad 0 = A + B \Rightarrow B = -\frac{1}{5}$

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2.
$$\int \frac{x^3\,dx}{(x^4+3x^2+2)} = \int \frac{x^2 \quad x\,dx}{(x^2+2)(x^2+1)}$$

$$= \int \frac{\left(2(x^2+1)-(x^2+2)\right)}{(x^2+2)(x^2+1)}\,x\,dx = \int \left(\frac{2x}{x^2+2} - \frac{x}{x^2+1}\right)dx$$

$$= \ln|x^2+2| - \frac{1}{2}\ln|x^2+1| + C$$

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3.

$$\boxed{\int \frac{dx}{x^3+1} = \int \frac{\left(x^2-(x^2-1)\right)}{(x^3+1)}\,dx} = \int\left(\frac{x^2}{x^3+1} - \frac{x-1}{x^2-x+1}\right)dx$$

$$\int \frac{dx}{x^3-1} = \int \frac{x^2-(x^2-1)}{(x^3-1)}\,dx$$

$$\frac{1}{(x+1)(x^2-x+1)} = \frac{A}{x+1} + \frac{Bx+C}{x^2-x+1}$$

$$\int\left(\frac{x^2}{x^3+1} - \frac{\frac{1}{2}(2x-1)-\frac{1}{2}}{x^2-x+1}\right)dx$$

$$\left(x-\frac{1}{2}\right)^2+\frac{3}{4}$$

$$\frac{1}{3}\ln|x^3+1| - \frac{1}{2}\ln|x^2-x+1| = $$

$$+\frac{1}{2}\times\frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{x-\frac{1}{2}}{\frac{\sqrt{3}}{2}}\right)+C.$$

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4. $\int \frac{dx}{(\sin 2x - 2\sin x)} = \int \frac{dx}{2\sin x(\cos x - 1)}$.

$\int \frac{1+\cos x}{2\sin x(\cos^2 x - 1)}$ $= -\int \frac{dx}{8\sin^3\frac{x}{2}\cos\frac{x}{2}} = -\frac{1}{4}\int \frac{(1+\tan^2\frac{x}{2})\sec^2\frac{x}{2}dx}{\tan^3\frac{x}{2}}$

$\int \frac{\operatorname{cosec}^2 x\, dx}{\sqrt{1+\cot^2 x}}$

$= \frac{1}{8}\frac{1}{\tan^2\frac{x}{2}} - \frac{1}{4}\ln\left|\tan\frac{x}{2}\right| + C.$

$\int \frac{\sin x \, dx}{2\sin^2 x(\cos x - 1)} = \frac{1}{2}\int \frac{-\sin x\, dx}{(\underbrace{\cos x - 1}_{=t})^2(1+\cos x)} = -\frac{1}{2}\int \frac{dt}{t^2(2+t)} = \frac{1}{8}\int \frac{(t^2 - (t^2-4))\,dt}{t^2(t+2)}$

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5.

$$\int \frac{\sin x \, dx}{\sin 4x}$$