Introduction

- Fluid Statics delves into the examination of fluid behaviour when it is at rest.
- Fluid dynamics on the other hand, concerns itself with the study of fluids in motion

Pressure:

- Static pressure refers to the perpendicular force exerted per unit area of cross section. SP = $\frac{F}{A}$

  Unit area of cross section. $P = \frac{F}{A}$

  SI unit: Nm^{-2} = Pascal, CGS unit: $dyne/cm^2$

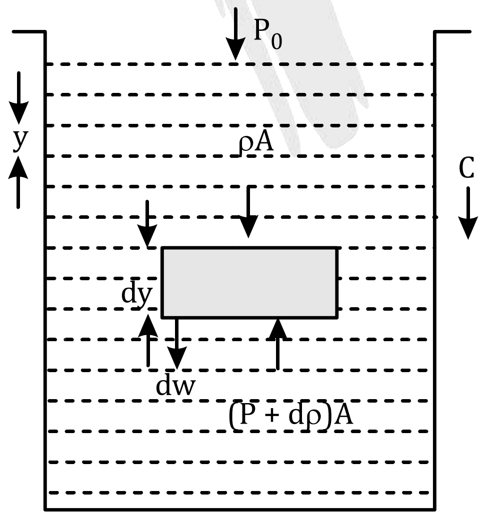
  $1Nm^{-2} = 10\,dyne/cm^2$, D.F: $ML^{-1}T^{-2}$

- The average pressure on the surface area ΔA is the result of the normal force $\Delta F_{\perp}$ applied to it. $P_{avg} = \frac{\Delta F_{\perp}}{\Delta A}$

  Pressure at a specific point is determined by the following equation: $P = \underset{\Delta A \to 0}{Lt} \frac{\Delta F_{\perp}}{\Delta A} = \frac{dF}{dA}$

- Taking into account the atmospheric pressure (P_o), the net pressure at the bottom of the container is given by $P = P_o + h\rho g$

Variation of pressure with height and depth:



$$\Rightarrow P = P_0 = \rho g y \Rightarrow dP = \rho g y$$

Gauge pressure, Absolute pressure:

The Absolute pressure at a depth 'h' within a liquid column is determined by the equation $P = P_0 + h\rho g$, where 'P' represents the absolute pressure, P_0 is the atmospheric pressure, and $h\rho g$ is the hydrostatic pressure (gauge pressure)

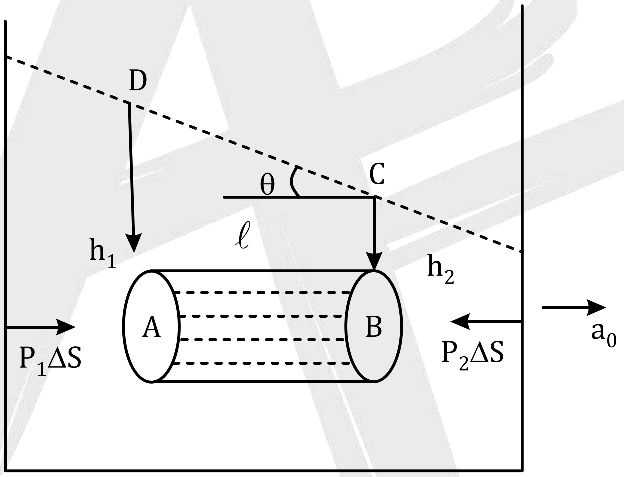
ρ-density of liquid, $P_{absolute} = P_{atm} + P_{gauge}$

Pressure difference when liquid is accelerating in horizontal direction:

- Let the pressure at point A be P_1 and the pressure at point B be P_2. The forces acting along the line AB are:

  (a) $P_1\Delta S$ towards right due to the liquid on the left

  (b) $P_2\Delta S$ towards left due to the liquid on the right



Here, $P_A - P_C = (P_A - P_B) + (P_B - P_C)$

$$= \rho \ell a + \rho g h_2 = \rho(\ell a + g h_2)$$

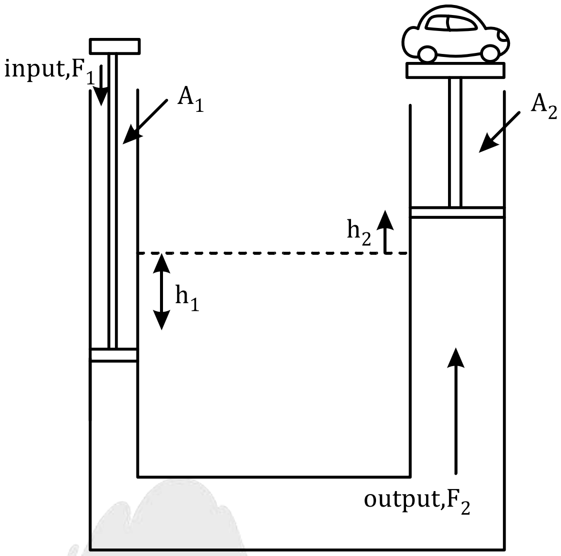
Pascal's Law:

- The pressure exerted on an enclosed imcompressible liquid is transmitted without any reduction to every point on the liquid's container walls
- In a stationary liquid, the pressure is uniform at all points if we disregard the effects of gravity.

  Mechanical Gain: It is the ratio of output force to input force (or)

  Mechanical gain $= \left(\dfrac{F_2}{F_1}\right)$

Hydraulic lift:



$$\Delta P = \frac{F_1}{A_1} = \frac{F_2}{A_2} \Rightarrow F_2 = F_1\frac{A_2}{A_1}\text{; As } A_2 > A_1; F_2 > F_1$$

- As the same volume of fluid is displaced at both pistons $A_1h_1 = A_2h_2 \Rightarrow h_2 < h_1$

Pressure Energy:

- The energy inherent in a fluid due to its pressure is referred to as pressure energy
- Pressure energy per unit volume $= \dfrac{P \times A \times x}{A \times x} = P$ (static pressure)
- Pressure energy per unit mass $= \dfrac{P \times A \times x}{\rho \times A \times x} = \dfrac{P}{\rho} = \dfrac{\text{Pressure}}{\text{density}}$

Density of a mixture:

- When two liquids of masses m_1, m_2 and densities ρ_1, ρ_2 respectively are mixed then the effective density of the mixture is

$$\rho = \frac{M_{total}}{V_{total}} = \frac{m_1 + m_2}{V_1 + V_2} = \frac{m_1 + m_2}{\left(\dfrac{m_1}{\rho_1} + \dfrac{m_2}{\rho_2}\right)} = \frac{(m_1 + m_2)\rho_1\rho_2}{m_1\rho_2 + m_2\rho_1}$$

Variation of density with pressure:

With increase in pressure, volume decreases and density increases

$$\rho_0 = \rho\left[1 - \frac{\Delta P}{K}\right] \Rightarrow \rho = \rho_0\left[1 - \frac{\Delta P}{K}\right]^{-1} \approx \rho_0\left(1 + \frac{\Delta P}{K}\right)$$

Buoyancy:

When a body is partially or completely submerged in a fluid, the upward force exerted by the fluid on the object is known as buoyancy

Laws of floatation:

Let W and W_1 be the weight of a body and the buoyant force on it and

(1) If $W > W_1$ body sinks

(2) $W = W_1$ body just submerged (body floats with its volume completely under the liquid)

(3) $W < W_1$ body floats (a part of the body lies outside the liquid)

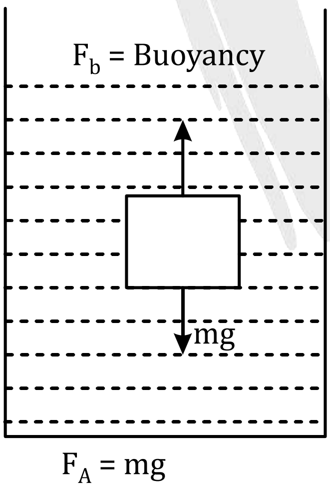
Fraction of Volume of the Body outside the Liquid:

The fraction of volume outside the liquid is

$$f_{out} = \frac{V_{out}}{V_{total}} = \left(1 - \frac{\rho_b}{\rho_1}\right)$$

Archimede's Principle:

- When an object is partially or entirely submerged in a fluid, it seems to lose some of its weight. This loss of weight is preceisely equal to the weight of the fluid that the object displaces (which is also known as the force of buoyancy)



- Apparent loss of weight of a body or weight of fluid displaced $= V_{in}\rho_1 g$

  V_{in} = Volume of body immersed or volume of fluid displaced

- By using Archimede's principle we can determine

(a) Relative density (Specific gravity) of a solid:

$$\text{RD} = \frac{\text{density of the body}}{\text{density of water at } 4°\text{C}} \quad \text{(or)}$$

$$\text{R.D.} = \frac{w_1}{w_1 - w_2};$$

w_1 = weight of the body in air

w_2 = weight of the body in water

Loss of weight of body in water $= w_1 - w_2$.

(b) Relative density of a liquid:

If loss of weight of a body in water is 'a' and that in liquid is 'b' then, $V\rho_w g = a$; $V\rho_L g = b$

$$\text{RD of liquid} = \frac{\rho_L}{\rho_w} = \frac{\text{Loss of weight in liquid}}{\text{Loss of weight in water}} = \frac{b}{a} = \frac{W_{air} - W_{liquid}}{W_{air} - W_{water}}$$

(c) Volume of a cavity in a body:

$$\text{Volume of cavity} = V' - V = \left(\frac{W_1 - W_2}{\rho_w g}\right) - \frac{M}{\rho_m}$$

(d) The amount of impurity in a given metal:

Let w_1 be the weight of an alloy in air and W_2 be the weight in water. Let the alloy consists of two metals having masses m_1 and m_2 such that total mass $m = (m_1 + m_2)$. The buoyant force on the alloy is,

$$F_b = w_1 - w_2 = V\rho_w g$$

$\Rightarrow \dfrac{w_1 - w_2}{\rho_w g} = V = \dfrac{m_1}{\rho_1} + \dfrac{m_2}{\rho_2}$ = Volume of the first metal in the alloy + Volume of the second metal in the alloy = Volume of the alloy (ρ_1, ρ_2 are the densities of the metals)

$$\frac{w_1 - w_2}{\rho_w g} = \frac{m_1}{\rho_1} + \frac{(m - m_1)}{\rho_2}$$

Fluid Dynamics:

Fluid dynamics is the study of fluid behaviour in motion. The movement of fluids is induced by variations in pressure between two distinct points within the fluid.

The rate of flow of a liquid:

The rate of liquid flow refers to the volume of liquid passing through any given cross section per unit of time and is expressed as follows

Volume flow rate:

$$Q = \frac{\text{Volume}}{\text{time}} = \frac{V}{t} = A\left(\frac{\ell}{t}\right) = Av \text{ (v = vel. of the fluid)}$$

SI units $\left[\frac{m^3}{sec}\right]$; D.F: L^3T^{-1}

Mass flow rate:

Mass of the liquid that flows per unit time

$$\frac{M}{t} = \frac{\text{Volume}}{\text{time}} \times \text{density} = A\left(\frac{\ell}{t}\right)\rho = Av\rho$$

Where A is the area of cross section of the tube, v is the velocity of the liquid and ρ is the density of the liquid.

Types of liquid flow:

- There are two types of liquid flow.

 (1) Stream line flow

 (2) Turbulent flow

Characteristics of fluid flow:

- Streamlines can be either straight lines or curves
- At any point on a curved streamline, the tangent drawn to that point indicates the direction of the velocity of the fluid particle at that location.
- Streamlines never intersect with each other.

Equation of Continuity:

$$\Rightarrow A_1v_1 = A_2v_2 = \text{constant}$$

$$\Rightarrow v \propto \frac{1}{A} \Rightarrow v \propto \frac{1}{r^2}$$

Equation of continuity represents the law of Conservation of mass in case of moving fluids.

Critical velocity:

Critical velocity is the velocity threshold at which streamline flow transitions gradually into turbulent flow.

Critical velocity $v_c = \frac{R\eta}{D\rho} \Rightarrow R = \frac{D\rho v_c}{\eta}$

- The Reynolds number relies on the pipe's diameter, the density of the liquid, and the coefficient of viscosity.

  (i) If $0 < R < 1000$, the liquid flow is called stream line

  (ii) If the value of R > 2000 the liquid flow becomes turbulent

  (iii) If $1000 < R < 2000$ the flow is unsteady

Types of Energies in fluid flow:

In motion, a fluid possesses three distinct types of energy: kinetic energy, potential energy and pressure energy.

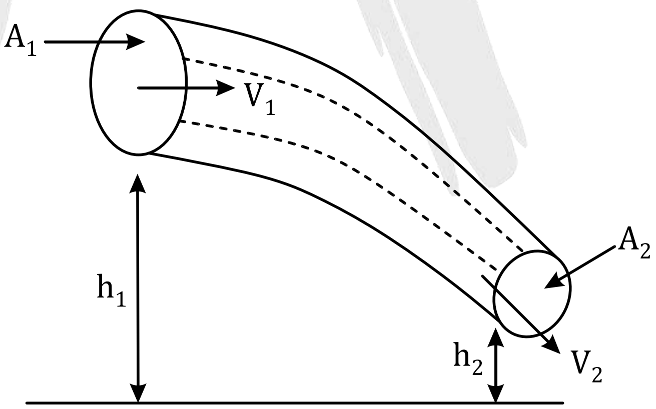
(i) $KE = \frac{1}{2}mv^2$ (ii) PE = mgh (iii) Pressure energy $= P \times V$

Bernoulli's Theorem:

- Bernoulli's Theorem asserts that the total energy, comprising pressure energy, kinetic energy, and potential energy, remains constant at any given point in a steady flow, calculated either per unit mass or per unit volume.

  (or)

- Alternatively, Bernoulli's Theorem can be expressed as follows: "In a streamlined flow of fluid, the total energy, consisting of gravitational head, pressure head, and velocity head at any location along the flow path, remains constant." $P + \frac{1}{2}\rho v^2 + \rho gh$ = constant (per unit volume)

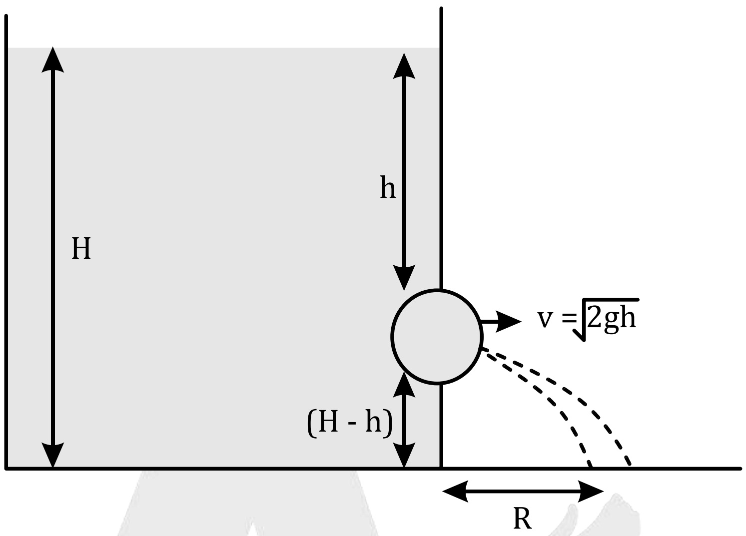


- In the case of horizontal liquid flow,there is an inverse relationship between pressure and velocity: maximum pressure corresponds to minimum velocity, and vice versa

$$\left(P + \frac{1}{2}\rho v^2 = \text{constant}\right)$$

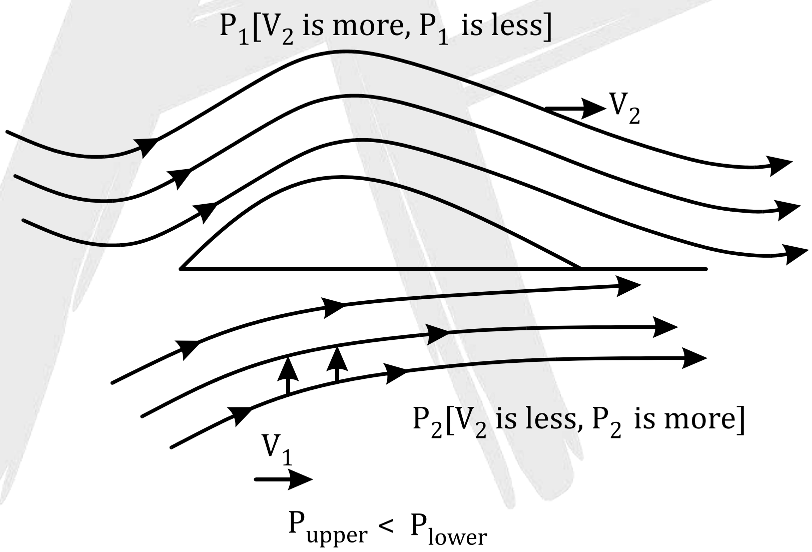
Torricelli's theorem:

➢ The velocity of liquid efflux through an orifice is equal to the velocity that a freely falling body acquires from a height equivalent to the liquid level above the orifice. $v = \sqrt{2gh}$



Applications of Bernoulli's theorem:

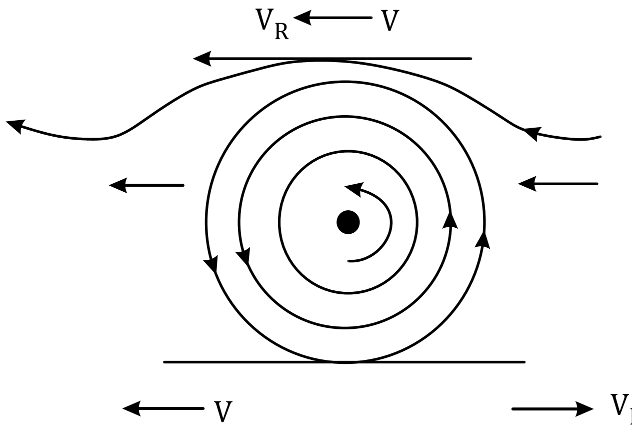
Dynamic lift: The upward lift experienced by a body in motion in a fluid is called dynamic lift.



The wing of an airplane has a more curved upper surface compared to the lower surface. As a result, air flows with greater speed over the upper surface of the wing, causing lower pressure above the wing than below it. This difference in pressures creates the aerodynamic lift, which enables airplanes to fly.

Dynamic lift $= (P_2 - P_1)A = \frac{1}{2}\rho.\left(V_1^2 - V_2^2\right) \times A$

Spinning ball:

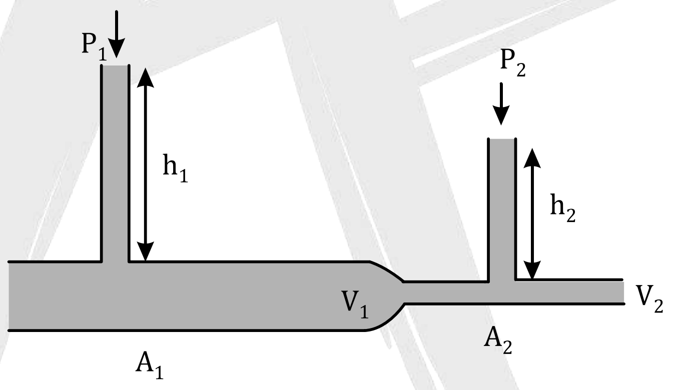


The plane of motion of spinning ball gets changed due to an effect called Magnus effect.

Venturimeter: It is a highly efficient device used to measure the rate of liquid flow through a pipe, known as a Venturi tube or flow meter

$$Q = A_1A_2 = \sqrt{\frac{2(P_1 - P_2)}{(A_1^2 - A_2^2)\rho}} = A_1A_2\sqrt{\frac{2(h_1 - h_2)\rho g}{(A_1^2 - A_2^2)\rho}} = A_1A_2\sqrt{\frac{2hg}{(A_1^2 - A_2^2)}}$$

$(\because h = h_1 - h_2)$



Viscosity:

- Viscous force is the frictional force that acts between two layers of a fluid, resisting their relative motion.

Viscous Force (Newton's Formula)

The viscous force acting between two adjacent layers of a liquid is given by $F = -\eta A\frac{dv}{dx}$

Coefficient of Kinetic Viscosity:

- The ratio between the coefficient of viscosity and density of the liquid is called Coefficient of Kinematic viscosity.

  Coefficient of Kinematic viscosity $= \frac{\eta}{\rho}$

- Its SI unit is m^2s^{-1}

Effect of Temperature:

- For liquids, the coefficient of viscosity decreases with increase in temperature due to the reduction of cohesive forces between the molecules at higher temperatures.
- In contrast, for gases, the coefficient of viscosity increases with the rising temperature as the number of collisions between gas molecules intensities.

Effect of Pressure:

- For liquids the value of η increases with increase of pressure
- Above $33°C$ the viscosity of water increases with pressure, and that below this temperature, initially the pressure effect is negative

Poiseuille's Forumal:

According to Poiseuill'e Formula the rate of flow of a liquid through a horizontal capillary tube is $V = \frac{\pi P r^4}{8\eta\ell}$

Where, V is volume of the liquid flowing out per second,
'P' is the pressure difference across the capillary pipe,
'r' is the radius of the pipe,
'ℓ' is the length of the capillary pipe,
'η' is the coefficient of viscosity of the liquid,
$\pi / 8$ is a proportionality constant

Poiseuille's equation is valid for:

(a) Flow through a horizontal capillary tube
(b) Steady and laminar flow
(c) The liquid in contact with the walls of the capillary tube must be at rest
(d) The pressure at any cross-section must be same

Stoke's law – Terminal velocity:

- When a spherical body is immersed in a fluid and dropped, the fluid in direct contact with the body is dragged along with it. Simultaneously, the remaining layers of the fluid exert a viscous force on the body to counteract its motion

According to stoke,

(i) the viscous force (F) $= 6\pi\eta r v_t$.

Where η = coefficient of viscosity of the fluid

r = radius of the body and v_t = terminal velocity

(ii) Weight of the body (W) = mg = (volume of sphere $\times$ density) $\times g = \frac{4}{3}\pi r^3 \rho g$

(iii) Upward thrust (T) = Weight of the fluid displaced

= (volume of the fluid displaced $\times$ density of the fluid) $\times g = \frac{4}{3}\pi r^3 \sigma g$

Surface Tension

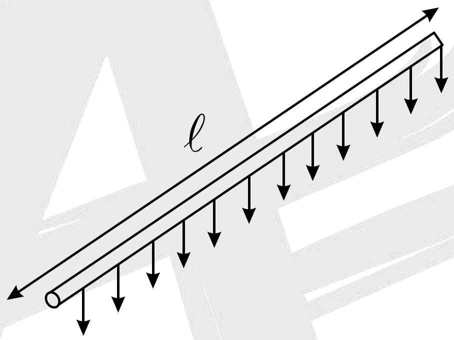
- Cohesive forces refer to the attractive forces between molecules of the same substance.
- Adhesive forces pertain to the attractive forces between molecules of different substances.
- Surface tension is defined as the force per unit length, perpendicular to any imaginary line drawn on the free surface of a liquid.

  $T = \frac{F}{\ell}$; SI Unit : N/m, CGS Unit : dyne/cm.

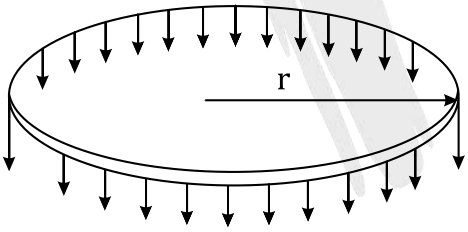
- Dimensional formula of surface tension is ML^0T^{-2}

Applications of force due to surface tension :

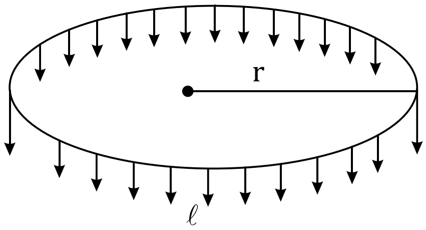
- Force required in addition to weight to pull a wire of length 'ℓ' from the surface of water of surface tension T is $F = 2\ell T$



- Force requied to pull a circular disc of radius R from the surface of water of surface tension T is $F = 2\pi rT$



- Force required to pull a thin circular ring of radius r from the surface of water of surface tension T is $F = 4\pi rT$



Surface Energy :

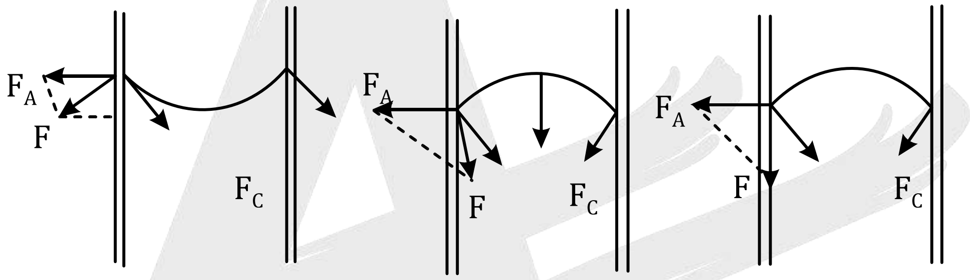
- Work done to increase surface area of a film by one unit is known as surface energy. It is numerically equal to surface tension. (or) The additional potential energy possessed due to increase in surface area by one unit is called surface energy

  $S = \frac{W}{\Delta A}$; SI Unit : J/m^2; D.F = MT^{-2}

Angle of Contact:

- The angle of contact between a liquid and a solid refers to the angle formed between the tangent drawn to the liquid's surface at the point of contact and the solid's surface, all measured within the liquid medium.

Shape of a Liquid Surface in a Tube:



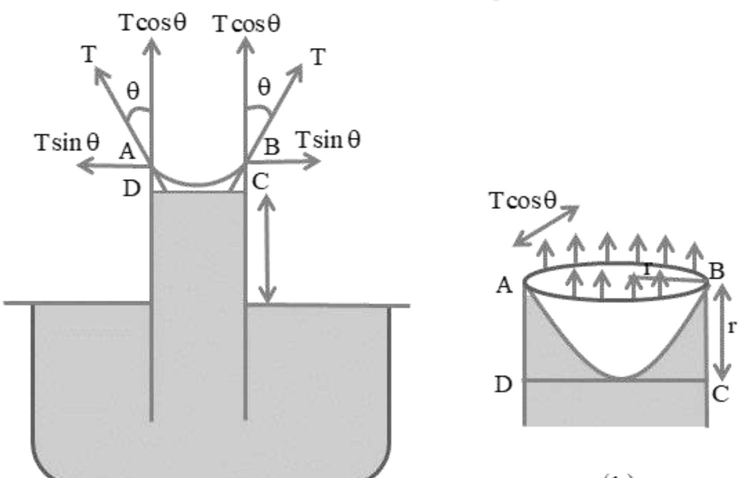
$F_A > F_C$ Case(i) $F_A < F_C$ Case(ii) $F_A = F_C$ Case(iii)

(F_A : Adhesive forces, F_C : Cohesive forces,

P_A : Pressure at A, P_B : Pressure at B)

Capillarity:

- Capillarity refers to the phenomenon of a liquid column rising or falling when a capillary tube is submerged in the liquid.
- Capillarity is determined by the balance between cohesive and adhesive forces in the liquid.



(a) (b)

$$2\pi rT\cos\theta = Mg$$

$$2\pi rT\cos\theta = \left(\pi r^2 h + \frac{1}{3}\pi r^3\right)dg$$

$$T = \frac{r\left(h + \frac{r}{3}\right)dg}{2\cos\theta}$$

If r is very small compared to h, then $T = \frac{hrdg}{2\cos\theta}$

Where r = radius of the capillary tube

h = height of liquid column;

d = density of the liquid;

g = acceleration due to gravity;

θ = angle of contact

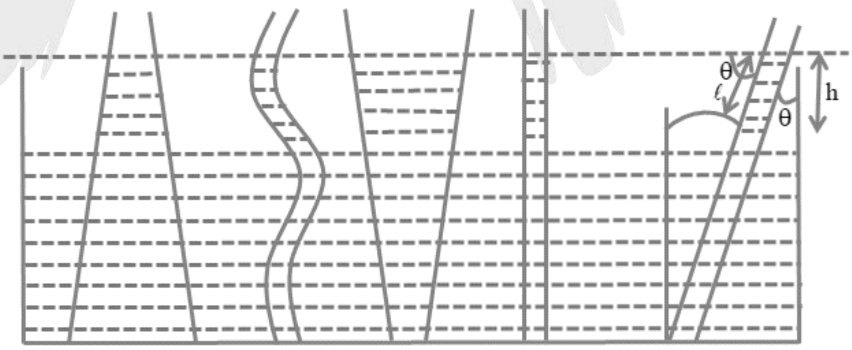
- Gravitational Potential energy of a liquid that rises in a tube is

U = Weight $\times \frac{h}{2} = \frac{mgh}{2} = \frac{2\pi T^2 \cos^2\theta}{dg}$ (When r is very less than h)

- When diameter of capillary tube increases twice, the height of liquid column falls down to half.

$\because r_1 h_1 = r_2 h_2$

- Since $h \propto \frac{1}{r}$, the graph between h and r is s rectangular hyperbola
- For a given radius, the capillary rise in a capillary tube does not depend either on the angle of inclination or on the shape of the tube



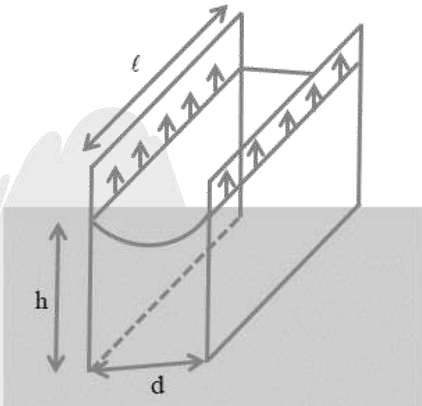
$$\cos\theta = \frac{h}{\ell} \Rightarrow h = \ell\cos\theta;\ \ell_1\cos\theta_1 = \ell_2\cos\theta_2$$

h = height of water in the tube

ℓ = length of water in the tube

- If the radii of the two limbs of a U-tube are r_1 and r_2, the difference between the levels of a liquid in 'u' tube is $\Delta h = \frac{2T}{dg}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$ (if $r_1 < r_2 << h$)
- If two parallel plates with the spacing 't' are placed in water reservoir, then height of rise $2T\ell = mg = V\rho g \approx \ell ht\rho g$ (ρ density of the liquid)

$$h = \frac{2T}{\rho tg}$$

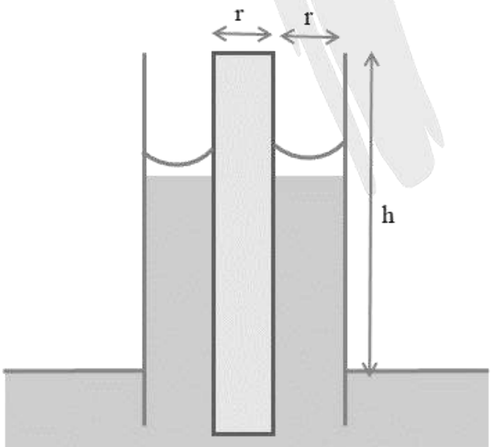


- If two concentric cylinders of radii r_1 & r_2 (inner one is solid are placed in water reservoir.)

$$T\cos\theta(\ell_1 + \ell_2) = mg$$

$$T[2\pi r_1 + 2\pi r_2] = [\pi r_2^2 h - \pi r_1^2 h]\rho g \qquad (\because \theta = 0°)$$

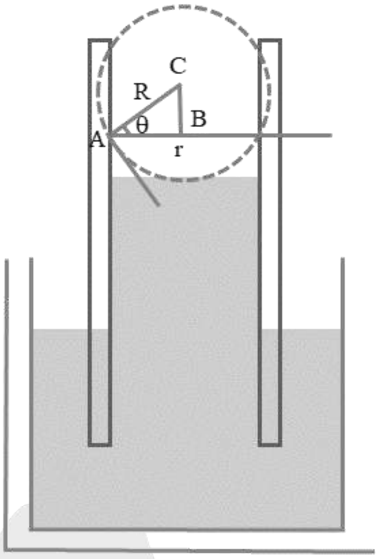
$$h = \frac{2T}{(r_2 - r_1)\rho g} \qquad \text{(if } r_1 < r_2 << h\text{)}$$



- A drop of liquid of density d_1 is floating with half immershed in a liquid of density d_2. If T is the surface tension of the liquid, then the radius of the drop is $(\text{if } \theta = 0°)$;

$$F_{\text{surface tension}} + F_{\text{buoyancy}} = mg \Rightarrow 2\pi r T\cos\theta + \frac{2}{3}\pi r^3 d_2 g = \frac{4}{3}\pi r^3 d_1 g \Rightarrow r = \sqrt{\frac{3T}{(2d_1 - d_2)g}}$$

The relation between radius of tube and radius of meniscus is:



AB = Radius of tube (r)

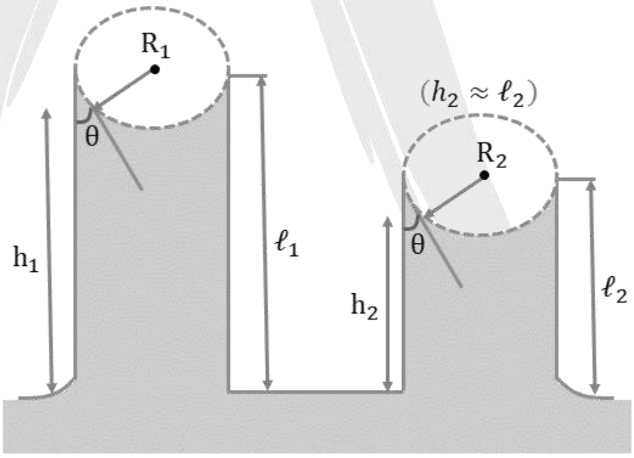
AC = Radius of meniscus (R)

In ΔABC, $\cos\theta = \frac{AB}{AC} = \frac{r}{R} \Rightarrow r = R\cos\theta$

If $\theta = 0°$ then $r = R$; If $\theta = 90°$, $R = \frac{r}{\cos 90°} = \infty$

i.e., The liquid meniscus is plane

capillary tube of insufficient length:



Theoretically the rise of liquid in the tube is

$$h = \frac{2T}{R\rho g}.$$

Therefore, if a capillary tube with insufficient length is dipped in the liquid, the liquid will not overflow. Instead, it will remain within the tube, and the meniscus radius will increase to accommodate the confined space.

Effect of Temperature on Surface Tension :

Over small ranges of temperature, the surface tension of a liquid decreases linearly with the rise of temperature due to increase in inter molecular distances according to the relation

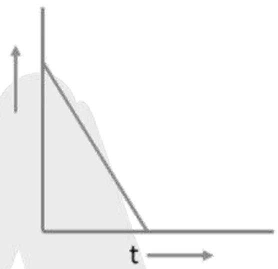
$S_t = S_0(1-\alpha\Delta t)$ where S_t = Surface tension at $t°C$

S_0 = Surface tension at $0°C$

Δt = Change in temperature.

α = Temperature coefficient of surface tension.

Dependence of Surface Tension On temperature :



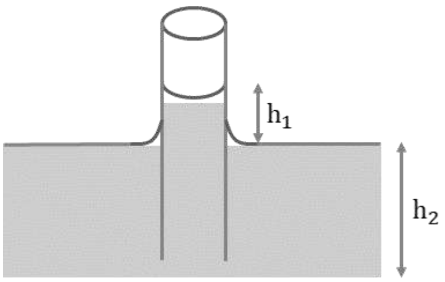
- For molten copper and molten cadmium, the surface tension rises as the temperature increases.
- At the critical temperature of any liquid, the surface tension becomes zero.

Effect of impurities on Surface Tension :

- The introduction of impurities can lead to either an increase or decrease in surface tension, depending on the type of impurity.
- Impurities can be categorized into two types:
  (i) Weakly soluble impurities (ii) Highly soluble impurities

Excess Pressure inside a liquid drop & Soap bubble :

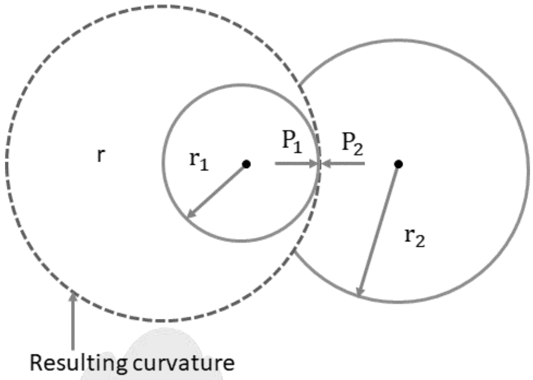
- Excess pressure inside a liquid drop of radius r is given by $P = 2T/r$. (single surface)
- Excess pressure in a soap bubble of radius r is given by $P = 4T/r$. (two surfaces)
- Total pressure inside an air bubble, which is just below a liquid surface, $P_t = P_0 + \frac{2T}{R}$
- Total pressure inside an air bubble, which is at a depth "h" below the liquid surface, $P_t = P_0 + h\rho g + \frac{2T}{R}$



$P_1 = P_0 + (h_1 + h_2)\rho g$ (or) $P_t = P_0 + h_2\rho g + \frac{2T}{R}$

➢ When a soap bubble of radius r_1 and another of radius r_2 are brought together, then the radius of curvature 'r' on the common interface is

$$r = \frac{r_1 r_2}{r_2 - r_1} \quad (\text{If } r_2 > r_1)$$



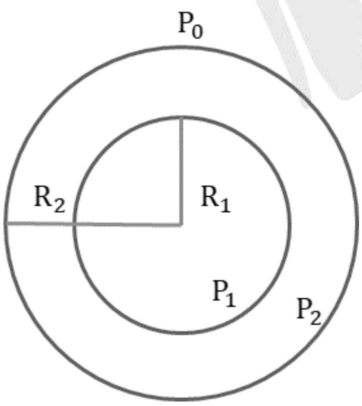
If P_1 and P_2 be the excess pressure in soap bubbles of radii r_1 and r_2 respectively, then the resultant excess pressure is given by $P = P_1 - P_2$

If r be the radius of the interface, then

$$\frac{4T}{r} = \frac{4T}{r_1} - \frac{4T}{r_2} \text{ or } \frac{1}{r} = \frac{1}{r_1} - \frac{1}{r_2}$$

$$\frac{1}{r} = \frac{r_2 - r_1}{r_1 r_2} \text{ or } r = \frac{r_1 r_2}{r_2 - r_1}$$

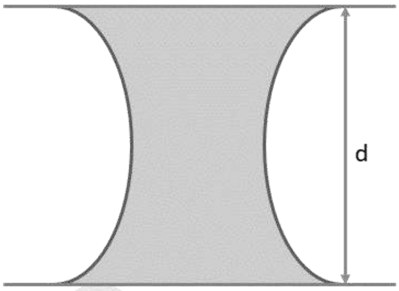
➢ A soap bubble of radius R_1 is formed inside a soap bubble of radius R_2. A soap bubble of radius R whose excess of pressure is equal to the pressure difference between that of the outside of the soap bubble of R_2 and the inside the R_1 is to be formed.



$$\frac{4T}{R} = \frac{4T}{R_1} + \frac{4T}{R_2} \Rightarrow \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$\Rightarrow R = \frac{R_1 R_2}{R_1 + R_2}$$

- **Liquid between two plates :** When a small drop of water is positioned between two glass plates placed face to face, it creates a thin film that exhibits a concave outward shape along its boundary. Let 'R_1' and 'R_2' be the radii of curvature of the enclosed film in two perpendicular directions.



Detergent And Surface Tension :

- To clean dirty clothes with grease and oil stains adhering to cotton or other fabrics, we utilize detergents or soap by adding them to water.
  Adding detergent or soap to water makes the angle of contact less than $90°$ and there by wets the clothes.

Wetting Agents :

A wetting agent is a substance that, when mixed with a liquid, reduces the angle of contact between the liquid and a given solid.

On the other hand, **a water-proofing agent** is a material applied to the surface of a solid to increase the angle of contact between the solid and water.

Eg : Wax

MECHANICAL PROPERTIES OF SOLIDS:

- **Deformation force:** A deformation force is a type of force that alters the size or shape, or both, of a body without causing its overall movement.
- **Restoring force:** The force responsible for returning a body to its original size and shape once the deformation forces are removed is known as the restoring force. It's essential to note that deformation force and restoring force are not an action-reaction pair.
- **Elasticity:** Elasticity is the inherent property of a body that enables it to recover its original size and shape when the deformation forces are no longer applied. This remarkable characteristic is a molecular phenomenon.
- **Elastic body :** A body which shows elastic behaviour is called elastic body.
- **Plastic body :** A body which does not show elastic behaviour is called plastic body.
  Eg : Putty, chewing gum, lead solder, wax.

- **Stress :** The restoring force developed per unit area of the deformated body is called stress

$$\text{Stress} = \frac{\text{Restoring force}}{\text{cross sectional area}} = \frac{F}{A}$$

F = External force applied

$A = \pi r^2$ = area of cross section

r = radius of the wire

Unit : $\frac{N}{m^2}$ or pascal, $\frac{\text{dyne}}{\text{cm}^2}$

Dimensional Formula: $M^1L^1T^{-2}$

Stress is a tensor

- **Stress is of three types :**

(i) **Longitudinal stress :** Longitudinal stress occurs when the restoring forces act perpendicular to the cross-sectional area and along the length of the wire.

(ii) **Tangential stress (or shearing stress) :** Shearing stress occurs when the restoring forces act parallel to the surface of the object.

(iii) **Bulk stress (or volume stress) :** Bulk stress refers to the type of stress that occurs when a body is subjected to equal forces acting normally on all of its faces.

- **Strain :** The deformation produced per unit magnitude is called strain

(a) longitudinal strain $= \frac{\text{change in length}}{\text{original length}} = \frac{\Delta\ell}{\ell}$

(b) Bulk strain $= \frac{\text{change in volume}}{\text{original volume}} = \frac{\Delta v}{v}$

(c) Shearing strain $= \theta = \frac{\text{lateral displacement between two layers}}{\text{perpendicular distance between the two layers}}$

(d) Transverse strain $= \frac{\text{change in radius}}{\text{original radius}} = \frac{\Delta r}{r}$

(e) Shearing strain = 2 × longitudinal strain

(f) Bulk strain = 3 × longitudinal strain

(g) Longitudinal strain : Shearing strain : Bulk strain = 1 : 2 : 3

- **Hooke's law :** Within the elastic limit, stress is directly proportional to strain.

$$\frac{\text{stress}}{\text{strain}} = E = \text{constant}$$

E is modulus of elasticity

Unit of E : $\frac{N}{m^2}$ or pascal, $\frac{\text{dyne}}{\text{cm}^2}$

(i) within the elastic limit, stress – strain graph is a straight line passing through the origin

Slope of the straight line is E

- **Breaking stress :**

  (i) The breaking stress of a wire is the maximum stress the material can withstand.

  (ii) Breaking stress $=\dfrac{\text{Breaking Force}}{\text{initial area of cross section}}$

- **Elastic fatigue:**

  (a) The state of temporary loss of elastic nature due to continuous strain is called elastic fatigue.

- **Elastic after affect :** Elastic After Effect refers to the delay observed in a material's ability to regain its original length after the removal of a deforming force within the elastic limit.

- **Young's modulus :**

$$Y=\frac{\text{longitudinal stress}}{\text{longitudinal strain}}$$

$$Y=\frac{F\ell}{Ae}$$

If load is M, then $F=Mg$ and $y=\dfrac{Mg\ell}{\pi r^2 e}$ where r is radius of the wire.

Thermal force.

Thermal force $F=YA\alpha\theta$

where α is co-efficient of linear expansion of the bar

θ is rise in temperature.

Y is young's modulus,

A is area of cross section.

Thermal force is independent of length of the bar.

Thermal stress : Thermal force per unit area is called thermal stress

Thermal stress $=\dfrac{\text{Thermal force}}{\text{area}}=\dfrac{YA\alpha\theta}{A}=Y\alpha\theta$

Thermal stress is independent of area of cross section and length of the bar.

A vertically suspended long wire can experience elongation caused by its self-weight.

Elongation of a wire due to its own weight

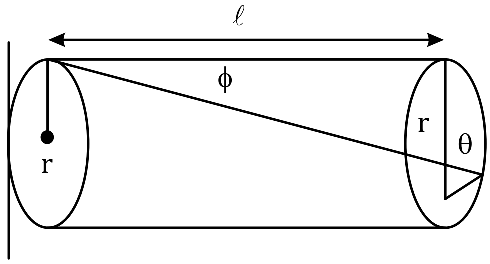
$$e=\frac{\ell^2 dg}{2Y}$$

ℓ = length of the wire, d = density of the wire

Y = young's modulus of the material of the wire

g = acceleration due to gravity

- Rigidity modulus, $G = \dfrac{\text{Tangential stress}}{\text{shearing strain}} = \dfrac{F}{A\theta}$ where θ is shearing strain
- A rod of length 'ℓ' and radius 'r' is fixed at one end. If the other free end is twisted through an angle 'θ' then the angle of shear ϕ is given by $\ell\phi = r\theta$



- One end of the rod is fixed. The other free end is twisted through an angle 'θ' by applying a torque 'τ' then the work done on the rod (or) energy stored in the rod is $W = \dfrac{1}{2}\tau\theta$ where θ is in radians.
- Bulk modulus $B = \dfrac{\text{volume stress}}{\text{volume strain}} = \dfrac{V\Delta P}{\Delta V}$
  - (i) If a block of coefficient of cubical expansion γ is heated through a rise in temperature of θ, the pressure to be applied on it to prevent its expansion, $P = K\gamma\theta$ where K is its bulk modulus,
  - (ii) When a rubber ball of volume V, bulk modulus K is taken to a depth h in water, then decrease in its volume $\Delta V = \dfrac{hdgV}{K}$ (d = density of material)
  - (iii) Solids posses Y, G and B, but liquids and gases posses only B.
  - (iv) Isothermal bulk modulus of a gas = pressure of the gas (P)
    Adiabatic bulk modulus of a gas $= \gamma P$
    where γ is the ratio of two specific heats.
  - (v) **Compressibility :** The reciprocal of bulk modulus is called compressibility, C = 1/B
    For incompressible substances, $C = 0;\ B = \infty$.
  - (vi) A material is easily compressed if it has a small bulk modulus.
- Poisson's ratio $(\sigma) = \dfrac{\text{lateral contraction strain}}{\text{longitudinal elongation strain}} = \left(\dfrac{-\Delta r / r}{\Delta\ell / \ell}\right)$
  - (i) Poisson's ratio has no units and no dimensions
  - (ii) Theoretical limits of $\sigma : -1$ to 0.5
  - (iii) Practical limit of $\sigma : 0$ to 0.5

- Work done in stretching a wire,

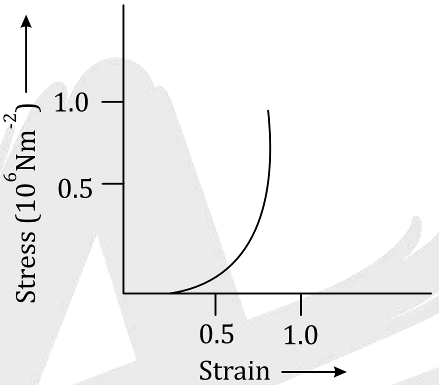
  $w = \frac{1}{2} \times$ stretching force $\times$ extension.

  (i) $w = \frac{1}{2} Fe = \frac{1}{2} \frac{Y\pi r^2 e}{\ell} = \frac{1}{2} \frac{F^2 \ell}{AY} = \frac{1}{2} \frac{F^2 \ell}{2\pi r^2 y}$

  (ii) $w = \frac{1}{2} \times$ stress $\times$ strain $\times$ volume of the wire

- **Elastomers**

  The stress-strain behaviour varies from material to material.



The curve does not exhibit a well-defined plastic region. Substances such as the aortic tissue and rubber, which can undergo significant deformation (large strains) upon stretching, are referred to as elastomers.

- Relation among elastic contents Y, G, B, σ

  (i) $\frac{9}{Y} = \frac{1}{B} + \frac{3}{G}$

  (ii) $Y = 2G(1+\sigma)$

  (iii) $Y = 3B(1-2\sigma)$

  (iv) $\sigma = \frac{3B-2G}{2(G+3B)}$

EXCERCISE-I

1. Three objects A, B and C all individually float on top of water. A and B have identical masses and densities but different shaped while B and C have identical sizes and shapes but C has less mass and density than B. If three identical weights are then tied to the objects and all three are pulled completely beneath the surface of the water, which object will displace the greatest volume of water?

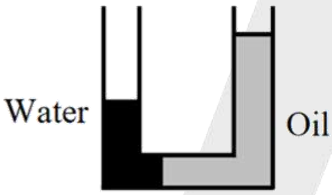
(A) object A only

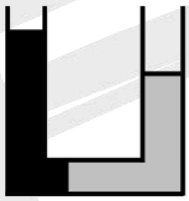
(B) object C only

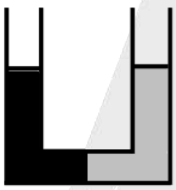
(C) objects A and B only

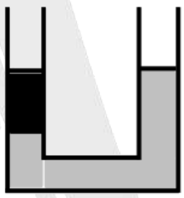
(D) All three displace equal volumes of water

2. A U shaped tube of constant cross-sectional area is filled with equal masses of oil and water. These do not mix and stay in the left and right parts of the tube respectively. The water has twice the density of the oil. The diagram best representing the case is :
(water is the darker shade on the left).

(A) 

(B) 

(C) 

(D) 

3. A table tennis ball is floating in a bucket of water. If the bucket is taken inside an elevator accelerating upwards, choose the correct statement :

(A) The buoyant force on the ball increases and the volume submerged also increases

(B) The buoyant force on the ball decreases and the volume submerged remains same

(C) The buoyant force on the ball increases and the volume submerged remains same

(D) The buoyant force on the ball remain same and the volume submerged remains same

4. When submerged under water, the mass of one cubic centimeter of pure gold is 19.3 g. What would be its mass in air?

(A) 16.3 g (B) 17.3 g

(C) 18.3 g (D) 19.3 g

5. An iceberg is floating in ocean. What fraction of its volume is above the water? (Given: density of ice $= 900 \text{ kg/m}^3$ and density of ocean water $= 1030 \text{ kg/m}^3$)

(A) $\frac{90}{103}$ (B) $\frac{13}{103}$ (C) $\frac{103}{10}$ (D) $\frac{31}{103}$

6. Why do raindrops fall with constant speed during the later stages of their descent?

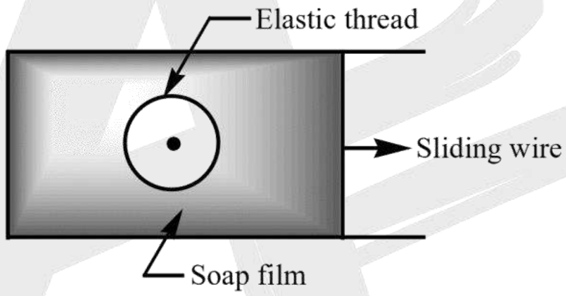
(A) All drops fall from the same height

(B) The gravitational force is negligible for objects as small as raindrops

(C) The gravitational force cannot increase the speed of a falling object to more than 9.8 m/s

(D) Air resistance balances the gravitational force on a drop

7. The figure shows a soap film in which a closed elastic thread is lying. The film inside the thread is pricked. Now the sliding wire is moved out so that the surface area increases. The radius of the circle formed by elastic thread will :

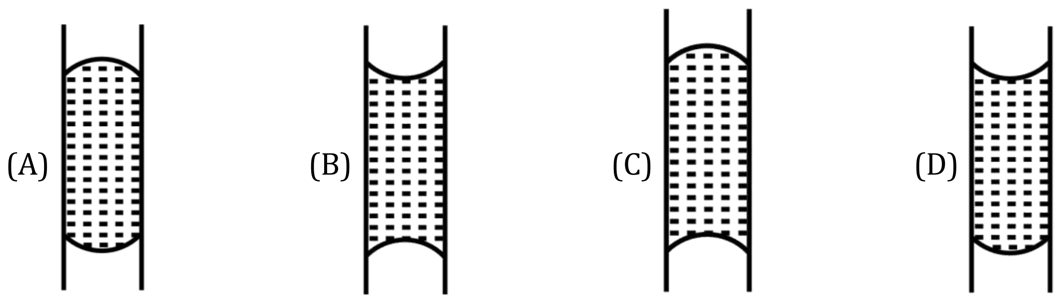


(A) increase (B) decrease (C) remains same (D) data insufficient

8. A mosquito with 8 legs stands on water surface and each leg makes depression of radius 'a'. If the surface tension and angle of contact are 'T' and zero respectively, then the weight of mosquito is:

(A) $12\pi Ta$ (B) $16\pi Ta$ (C) $4\pi Ta$ (D) $8\pi aT$

9. A vertical glass capillary tube, open at both ends contains some water. Which of the following shapes may be taken by the water in the tube? Assuming radius of both curved liquid surfaces are same.



10. A body floats on water and also on an oil of density 1.25. Which of the following is/are true?

(A) The body loses more weight in oil than in water

(B) The volume of water displaced is 1.25 times that of oil displaced.

(C) The body experiences equal upthrust from water and oil

(D) to make the body just sink, one will need 1.25 times load in case of oil than in case of water

11. To determine Young's modulus of a wire, the relation $Y = \frac{\text{Stress}}{\text{Strain}} = \frac{F/A}{\Delta\ell/\ell}$ is used. The conversion factor to change value of Young's modulus from SI to CGS system is :

(A) 1 (B) 10

(C) 0.1 (D) 100

12. Statement-1: When a wire is stretched within the proportionality limit such that its length becomes n times that of its initial value, the resistance of wire may become n^2 times of its initial resistance.

Statement-2: The poisson's ratio of the wire's material can be $\frac{\ell}{2}$.

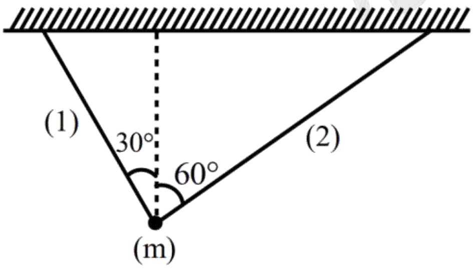
(A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1

(B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1

(C) Statement-1 is true, statement-2 is false

(D) Statement-1 is false, statement-2 is true

13. A point mass m is suspended using two wires of different materials as shown in the figure. If cross-section area of wire-1 and wire-2 are $3\ \text{mm}^2$ and $\sqrt{3}\ \text{mm}^2$ respectively, which of the following is correct?



(A) Stress in wire-1 > stress in wire-2

(B) Stress in wire-1 < stress in wire-2

(C) Stress in wire-1 = stress in wire-2

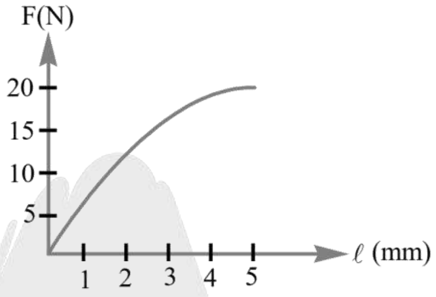
(D) Value of Young's modulus of both the wires is needed

14. The following four wires of length L and radius r are made of the same material. Which of these will have the largest extension, when the same tension is applied?

(A) $L = 100cm, r = 0.2mm$ (B) $L = 200cm, r = 0.4mm$

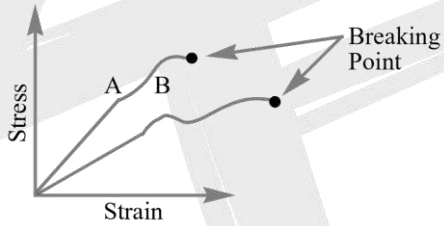
(C) $L = 300cm, r = 0.6mm$ (D) $L = 400cm, r = 0.8mm$

15. The force (F)-extension (ℓ), graph shows that the strain energy stored in the material under test, for an extension of 4 mm, is greater than which of the following values ?



(A) 80 mJ (B) 60 mJ (C) 40 mJ (D) None of these

16. Select the correct stateemnt on the basis of the given graph :



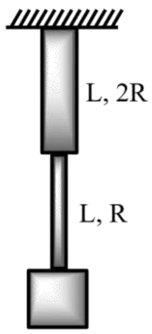
(A) Young's modulus of A is greater but it is less ductile

(B) Young's modulus of A is greater and it is more ductile

(C) Young's modulus of A is less and it is less ductile

(D) Young's modulus of A is less but it is more ductile

17. Two light wires of the same material (Young's modulus Y) and same length L but different radii R and 2R, as shown in the figure, are joined end to end and supported from a fixed support. A weight W is suspended from the combination. The elastic potential energy in the system is :

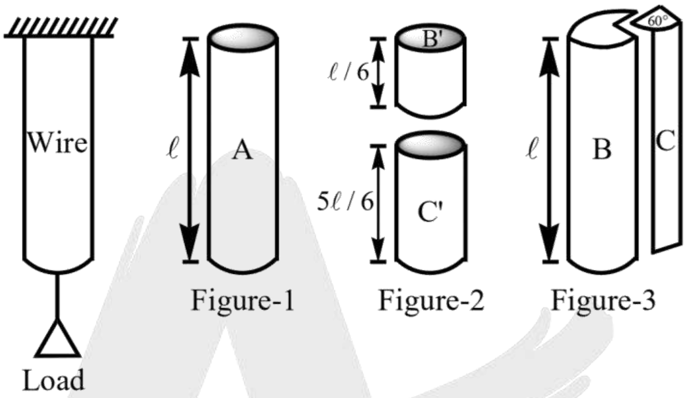


(A) $\frac{3W^2L}{4\pi R^2Y}$ (B) $\frac{3W^2L}{8\pi R^2Y}$ (C) $\frac{5W^2L}{8\pi R^2Y}$ (D) $\frac{W^2L}{\pi R^2Y}$

18. A steel wire, 3.2 m long, has a diameter of 1.2 mm. The wire stretches by 1.6 mm when it bears a load. Young's modulus for steel is 2.0×10^{11} Pa. The mass of the load is closest to :

(A) 24 kg (B) 28 kg (C) 12 kg (D) 20 kg

19. A light wire of length ℓ (figure-1) is cut into two pieces in two different ways as shown in (figure-2 and 3). Different pieces can be arranged in place of wire as shown and a load can be placed on the massless hanger. Choose the correct statement(s) :



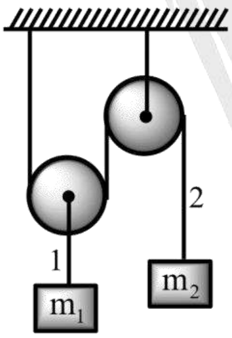
(A) The load required to break the wire B' is 6 times that required to break B

(B) The stress required to break the wire B and C is same but to break B and B' is different

(C) The stress required to break C and C' is same

(D) The load required to break A and B' is same, but different for B and B'

20. In the figure shown the system is in equilibrium. In this system we have used two wires 1 and 2. The Young's modulus and radius of the wire 1 are two times and $2^{1/4}$ times respectively that of wire 2, then :



(A) Elastic energy density in wire 1 is equal to that of in wire 2

(B) Elastic energy density in wire 1 is greater than that of in wire 2

(C) Elastic energy density in wire 1 is less than that of in wire 2

(D) Stress in wire 1 is greater than stress in wire 2

EXCERCISE-II

1. A balloon has a volume of $0.09\ \text{m}^3$ below the surface of the water (density $=10^3\ \text{kg}/\text{m}^3$) at a depth of 60 m. If the temperature remains constant, what is its volume in m^3 when it is at the surface where the pressure is $1\times10^5\ \text{N}/\text{m}^2$?

(A) 0.43 (B) 0.45 (C) 0.36 (D) 0.63

2. A cubical block of wood of specific gravity 0.5 and chunk of concrete of specific gravity 0.25 are fastened together. The ratio of mass of wood to the mass of concrete which makes the combination to float with entire volume of the combination submerged in water is $\frac{1.5\alpha}{2.5\beta}$. Find $\alpha+\beta$?

3. Two objects labeled K and L have equal mass but densities $0.95\ D_0$ and D_0 respectively. Each of these objects float after being thrown into a deep swimming pool. Which is true about the buoyant forces acting on these objects?

(A) The buoyant force is greater on Object K since it has a lower density and displaces more water

(B) The buoyant force is greater on Object K since it has lower density and lower density objects always float "higher" in the fluid

(C) The buoyant force is greater on Object L since it is denser than K and therefore "heavier"

(D) The buoyant forces are equal on the objects since they have equal mass

4. A steel ball with a volume of $2\ \text{cm}^3$ is sinking at a speed of $2\ \text{cm s}^{-1}$ in a closed jar filled with honey. What is the magnitude of momentum of the honey if its density is 2 g-cm^{-3}?

(A) 6 gm-cm/sec

(B) 0.5 g-cm/s

(C) 2 g-cm/s

(D) 1 g-cm/s

5. A completely filled closed aquarium is kept on weighing machine. It can be assumed that the density of the fish is greater than the density of the water. The total mass of the aquarium and its contents put together is M. If now all the fish start accelerating upwards with an acceleration A, then the correct option is :

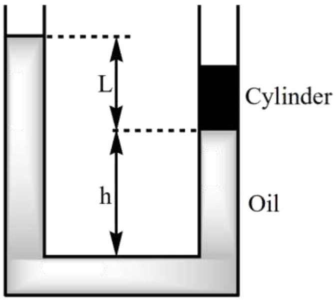
(A) The weight recorded will be equal to Mg

(B) The weight reading will be less than Mg

(C) The weight reading will be more than Mg

(D) No conclusion can be drawn from the given information

6. The diagram shows a U-tube with cross-sectional area A and partially filled with oil of density ρ. A solid cylinder, which fits the tube tightly but can slide without friction, is placed in the right arm. The system is in equilibrium. The weight of the cylinder is :



(A) $AL\rho g$ (B) $L^3\rho g$ (C) $A\rho(L^3+h)g$ (D) $A\rho(L^2-h)g$

7. An aquarium is in the shape of a cube with sides of length 60 cm. The height (in cm) we should pour water in it so that the force on the side is one sixth of that at the bottom is (Atmospheric pressure is not taken into account).

8. A ball is thrown vertically upwards into the air. Taking into account air resistance, the force acting on the ball during upward flight are :

(A) a decreasing upward force and a constant downward force

(B) a decreasing upward force and a decreasing downward force

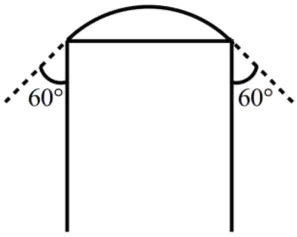
(C) an increasing downward force and a constant downward force

(D) a decreasing downward force and a constant downward force

9. An open capillary tube is lowered in a vessel with mercury. The difference between the levels of the mercury in the vessel and in the capillary tube $\Delta h = 4.6$ mm. What is the radius of curvature of the mercury meniscus in the capillary tube? Surface tension of mercury is 0.46 N/m, density of mercury is 13.6 gm/cc.

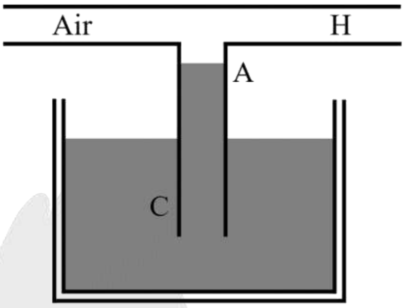
(A) $\frac{1}{340}$ m (B) $\frac{1}{680}$ m (C) $\frac{1}{1020}$ m (D) $\frac{1}{720}$ m

10. A soap bubble is being blown on a tube of radius 1 cm. The surface tension of the soap solution is 0.05 N/m and the bubble makes an angle of $60°$ with the tube as shown. The excess of pressure over the atmospheric pressure in the tube (in Pa) is :



11. A body floats with $\left(\frac{1}{3}\right)$ of its volume outside water when submerged in water and $\left(\frac{5}{9}\right)$ of its volume outside another liquid when submerged in another liquid. The density (in kg/m^3) of the liquid is

12. Figure shows a capillary tube C dipped in a liquid that wets it. The liquid rises to a point A. If we blow air through the horizontal tube H, what will happen to the liquid column in the capillary tube?

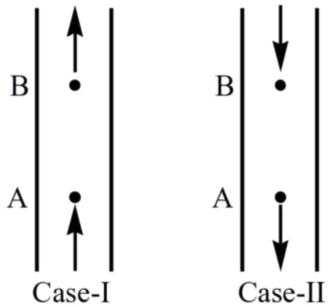


(A) Level will rise above A
(B) Level will fall below A
(C) Level will remain at A
(D) Remain at the same level

13. Each of the following system begins moving upwards with a constant acceleration. Select these cases in which quantity will change due to this upward acceleration :

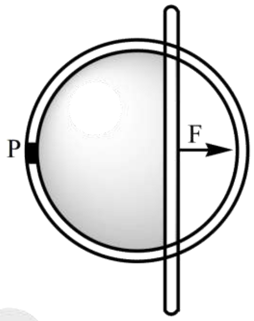
(A) time period of simple pendulum
(B) fraction of floating body submerged in a liquid
(C) time period of a spring block system
(D) pressure on the base of a container containing liquid

14. Two tubes of uniform cross-section are held vertically. v_A and v_B are the velocities of fluid flow at A and B respectively, and p_A and p_B are pressure at A and B respectively. Arrow show the direction of fluid flow.



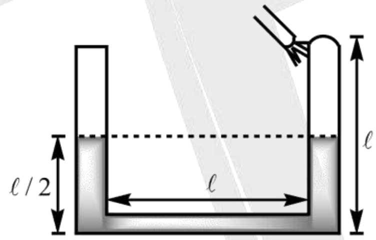
(A) In case I, $v_A < v_B$ and $p_A = p_B$
(B) In case I, $v_A = v_B$ and $p_A > p_B$
(C) In case II, $v_A > v_B$ and $p_A < p_B$
(D) In case II, $v_A = v_B$ and $p_A > p_B$

15. A circular wire, 10 cm in diameter, with a slider wire on it, is in a horizontal plane. A liquid film is formed, bounded by the wires, on the left side of the slider, as shown. The surface tension of the liquid is 0.1 N/m. An applied force 16 mN, perpendicular to the slider, maintains the film in equilibrium. Ignore the sag in the film. What can be the distance between point P and slider?



(A) 8 cm (B) 2 cm (C) 5 cm (D) 6 cm

16. A rectangular narrow U-tube has equal arm lengths and base length, each equal to ℓ. The vertical arms are filled with mercury up to $\frac{\ell}{2}$ and then one end is sealed. By heating the enclosed gas all the mercury is expelled. If atmospheric pressure is P_0, the density of mercury is ρ and cross-sectional area is S, then : [Neglect thermal expansion of glass and mercury]



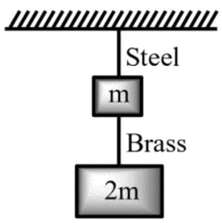
(A) Work done by the gas against the atmospheric pressure is $\frac{5\ell}{2} P_0 S$

(B) Work done by the gas against the gravity is $4S\rho g\ell^2$

(C) Work done by the gas against the atmospheric pressure is $P_0 S\ell$

(D) Work done by the gas against the gravity is $S\rho g\ell^2$

17. If the ratio of lengths, radii and Young's moduli of steel and brass wires in the figure are a, b and c respectively, then the corresponding ratio of increase in their lengths is :



(A) $\frac{2a^2c}{b}$ (B) $\frac{3a}{2b^2c}$ (C) $\frac{2ac}{b^2}$ (D) $\frac{3c}{2ab^2}$

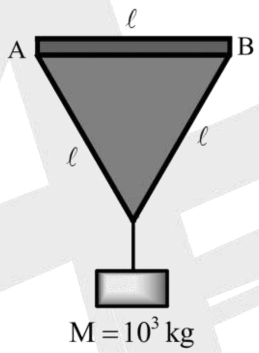
18. A steel ring of radius r is made of wire of cross-sectional area A is fitted on to a wooden disc of radius $R(R > r)$. If Young's modulus be Y, then the force with which the steel ring is expanded is :

(A) $AY\left(\frac{R}{r}\right)$ (B) $AY\left(\frac{R-r}{r}\right)$ (C) $\frac{Y}{A}\left(\frac{R-r}{r}\right)$ (D) $\frac{Yr}{AR}$

19. A copper wire $\left(Y = 1\times10^{11}\ N/m^2\right)$ of length 8 m, and a steel wire $\left(Y = 2\times10^{11}\ N/m^2\right)$ of length 4 m, each of cross-section $0.5\ cm^2$ are fastened end to end and stretched with a tension of 500 N. The elastic potential energy of the system is :

(A) 0.125 J (B) 0.2 J (C) 0.25 J (D) 0.5 J

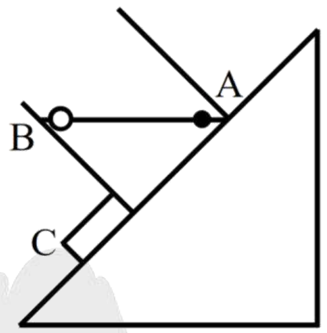
20. The diagram shows a horizontal girder AB of length ℓ, from the ends of which a load of 10^3 kg is suspended by two strings of length = ℓ. The compression in the girder is $\left(g = 10\ ms^{-2}\right)$:



(A) $\frac{5000}{\sqrt{3}}N$ (B) $\frac{10^4}{\sqrt{3}}N$ (C) 5×10^3N (D) $5\sqrt{3}\times10^3N$

EXCERCISE-III

1. An open cubical vessel is standing on an inclined plane, angle 45°, as seen in figure. Its walls are thin and it is kept from sliding down by a small wedge C. The vessel is filled to its half with mercury and an iron sphere is floating on the surface from point A in the direction of point B. When does the vessel tip over?



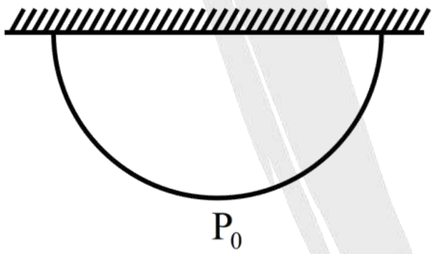
(A) When the sphere is at A

(B) When the sphere is at B

(C) When the sphere is at midpoint of A and B

(D) In all the positions of the sphere, the probability of tipping is equal

2. A suction cup is attached to a smooth metal ceiling. Pressure inside suction cup is assumed to be nearly zero. The maximum weight that can be supported by the suction cup is dependent on:



(A) its area of contact with the ceiling

(B) the air pressure outside the cup

(C) both (a) and (b)

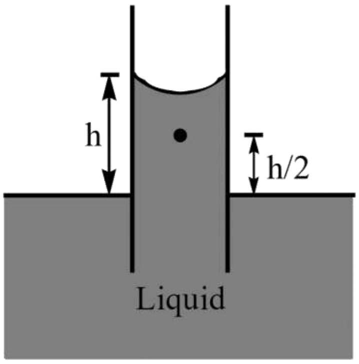
(D) neither (a) nor (b)

3. A metal ball of density $7800 \text{ kg} / \text{m}^3$ is suspected to have a large number of cavities. Its weight 9.8 kg when weighed directly on a balance and 2 kg less when immersed in water. The fraction by volume of the cavities in the metal ball is approximately :

(A) 20% (B) 30%

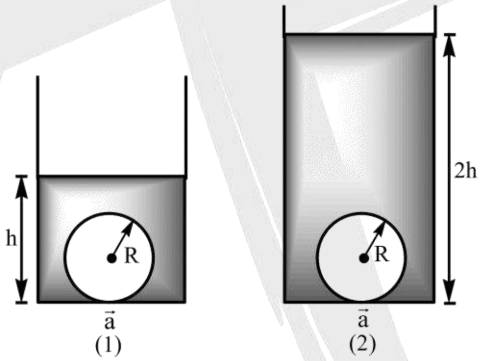
(C) 16% (D) 37%

4. In the diagram shown the liquid has density ρ. Atmospheric pressure is P_0. The pressure at point A is :



(A) $P_0 + \frac{\rho gh}{2}$ (B) $P_0 - \frac{\rho gh}{2}$ (C) $\frac{\rho gh}{2}$ (D) $\frac{P_0}{2} + \rho gh$

5. Two identical cylinders have a hole of radius a $(a < R)$ at its bottom. A ball of radius R is kept on the hole and water is filled in the cylinder such that there is no water leakage from bottom. In case-1 water is filled upto height h and in second case it is filled upto height 2h. If F_1 is net force by liquid on sphere in case-1 and F_2 is net force by liquid on sphere in case-2 then :



(A) $F_1 = F_2 = 0$ (B) $F_1 > F_2$ (C) $F_2 > F_1$ (D) $F_1 = F_2 \neq 0$

6. A solid floats with 2/3 of its volume immersed in a liquid and with 3/4 of its volume immersed in another liquid. What fraction of its volume will be immersed if it floats in a homogeneous mixture formed of equal volumes of the liquids?

(A) $\frac{7}{6}$ (B) $\frac{11}{8}$ (C) $\frac{16}{11}$ (D) $\frac{12}{17}$

7. The ratio of the diameters of certain air bubble at the bottom and at the surface is 1 : 2. What is the depth (in m) of the lake (1 atmosphere = 10 m depth of water) is

8. A barometer tube of length 0.99 m reads 0.76 m. The volume of air measured at atmospheric pressure to be introduced into space to cause the length of mercury column to drop to 0.57 m is $\frac{105}{n}$ *cc*. Find n? (the cross-section of the barometer tube is $0.1\ cm^2$)?

9. You are studying for an exam on the eight floor of your luxurious apartment building. You look out from the window and notice that one of you neighbours is giving a party on the ground floor terrace and has placed a huge punch bowl full of an interesting looking beverage (specific gravity 1) directly below your window. You quickly string together 80 drinking straws to form a giant straw that can reach the punch bowl 80 feet below. You dip the straw into the punch and begin to suck. When you use a single drinking straw to drink something, it takes you 0.1 seconds to raise the liquid to your lips. But when you use this giant drinking straw :

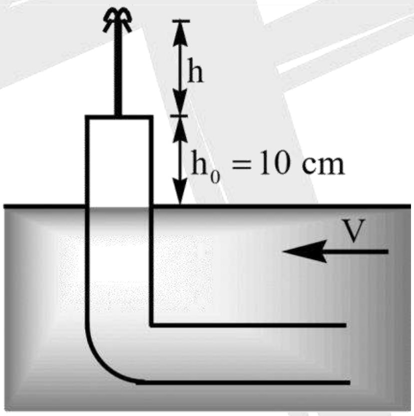
(A) you find that you can't raise the liquid to your lips no matter how hard you try

(B) it takes you 8 seconds (80 times 0.1 second) to raise the liquid to your lips

(C) it takes you 800 seconds (80 divided by 0.1 second) to raise the liquid to your lips

(D) it takes you 640 seconds (80 times 80 times 0.1 second) to raise the liquid to your lips

10. A bent tube is lowered into water stream as shown in the figure. The velocity of the stream relative to the tube is equal to V = 2 m/s. The closed upper end of the tube located at height $h_0 = 10\text{ cm}$ above free surface of water has a small orifice. The height h (in cm) will the water get spurt is



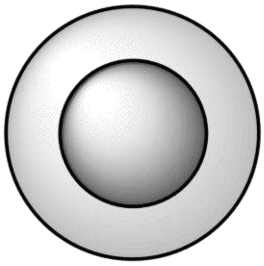
11. A sphere of brass released in a long liquid column attains a terminal speed v_0. If the terminal speed attained by the sphere of marble of the same radius and released in the same liquid is nv_0, then the value of n will be :

Given : The specific gravities of brass, marble and the liquid are 8.5, 2.5 and 0.8 respectively.

(A) $\frac{55}{77}$ (B) $\frac{17}{77}$ (C) $\frac{44}{77}$ (D) $\frac{33}{77}$

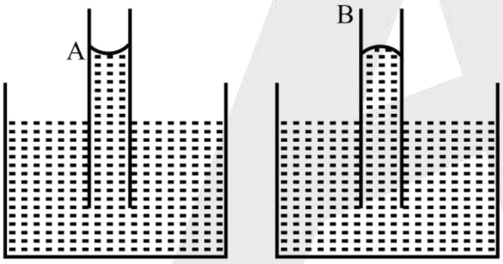
12. A sphere of radius 10 cm and density 500 kg/m^3 is under water of density 1000 kg/m^3. The acceleration of the sphere is 9.8 m/s^2 upward. Viscosity of water is 1.0 centripoise. If $g = 9.81 m/s^2$, the velocity (in m/s) of the sphere approximately is

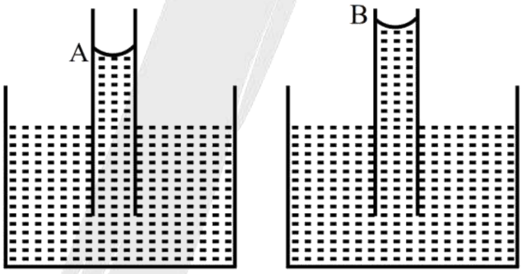
13. A soap bubble of radius R is surrounded by another soap bubble of radius 2R, as shown. Take surface tension = S. Then, the pressure inside the smaller soap bubble, in excess of the atmospheric pressure, will be :

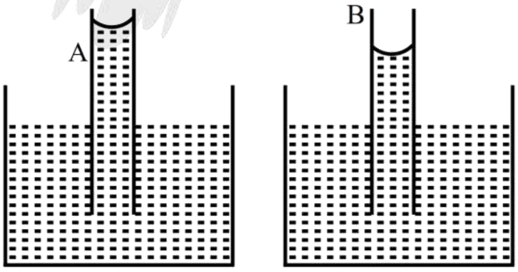


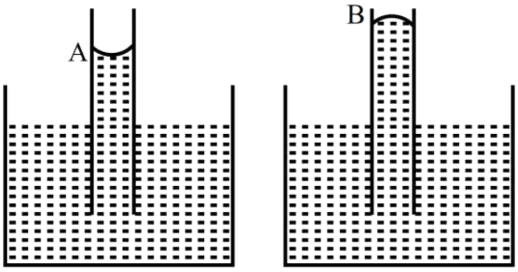
(A) $\frac{9S}{R}$ (B) $\frac{12S}{R}$ (C) $\frac{6S}{R}$ (D) $\frac{3S}{R}$

14. A capillary tube (A) is dipped in water. Another identical tube (B) is dipped in a soap water solution. Which of the following shows the relative nature of the liquid columns in the two tubes?

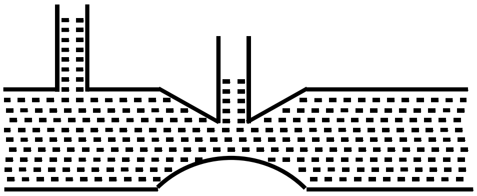
(A)


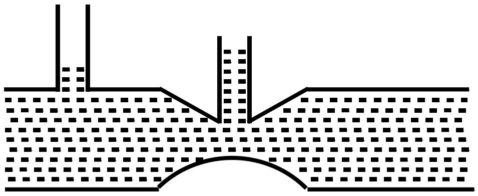
(B)


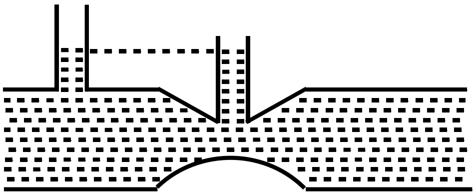
(C)


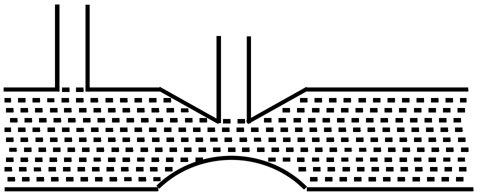
(D)


15. For a fluid which is flowing steadily, the level in the vertical tubes is best represented by :

(A) 

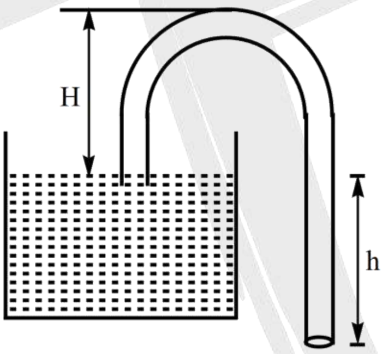
(B) 

(C) 

(D) 

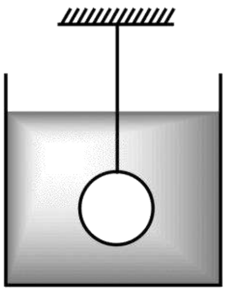
16. An isolated and charged spherical soap bubble has a radius 'r' and the pressure inside is atmospheric. If 'T' is the surface tension of soap solution, then charge on drop is $\alpha\pi r\sqrt{\beta r T\varepsilon_0}$. Find $\alpha+\beta$?

17. A large tank filled with water of density ρ and surface tension γ is being siphoned out using a glass capillary tube of cross-sectional radius r. If the siphon action starts without any external agent then (angle of contact is zero).



(A) $H < \frac{\gamma}{\rho g r}$ (B) $H < \frac{2\gamma}{\rho g r}$ (C) $H - h < \frac{2\gamma}{\rho g r}$ (D) $\sqrt{H^2 - h^2} < \frac{\gamma}{\rho g r}$

18. A solid sphere of mass m, suspended through a string in a liquid as shown. The string has some tension. Magnitudes of net force due to liquid on upper hemisphere and that on lower hemisphere are F_A and F_B respectively. Which of the following is/are true?



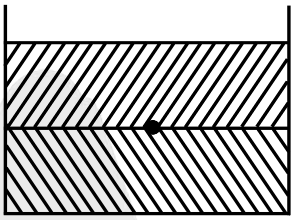
(A) Density of material of the sphere is less than density of liquid

(B) Difference of F_B and F_A is independent of atmospheric pressure

(C) $F_B - F_A = mg$

(D) $F_B - F_A < mg$

19. A solid uniform ball of volume V floats on the interface of two immiscible liquids (see figure). The specific gravity of the upper liquid is ρ_1 and of the lower liquid is ρ_2, while that of ball is ρ. If $(\rho_1 < \rho < \rho_2)$, then :



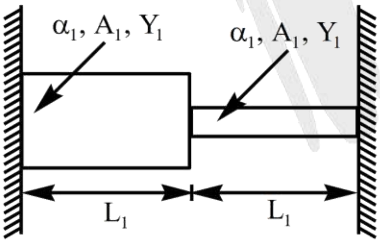
(A) The fraction of volume of the ball in the upper liquid (ρ_1) is $\left(\frac{\rho_2-\rho}{\rho_2-\rho_1}\right)$

(B) The fraction of volume of the ball in the upper liquid (ρ_1) is $\left(\frac{\rho-\rho_1}{\rho_2-\rho_1}\right)$

(C) The fraction of volume of the ball in the lower liquid (ρ_2) is $\left(\frac{\rho-\rho_1}{\rho_2-\rho_1}\right)$

(D) The fraction of volume of the ball in the lower liquid (ρ_2) is $\left(\frac{\rho_2-\rho}{\rho_2-\rho_1}\right)$

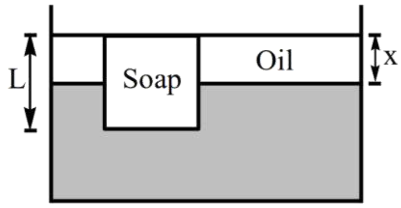
20. Two rods are joined between fixed supports as shown in the figure. Condition for no change in the lengths of individual rods with the increase of temperature will be : (α_1, α_2 = linear expansion coefficient; A_1, A_2 = Area of rods; Y_1, Y_2 = Young modulus)



(A) $\frac{A_1}{A_2} = \frac{\alpha_1 Y_1}{\alpha_2 Y_2}$ (B) $\frac{A_1}{A_2} = \frac{L_1\alpha_1 Y_1}{L_2\alpha_2 Y_2}$ (C) $\frac{A_1}{A_2} = \frac{L_2\alpha_2 Y_2}{L_1\alpha_1 Y_1}$ (D) $\frac{A_1}{A_2} = \frac{\alpha_2 Y_2}{\alpha_1 Y_1}$

EXCERCISE-IV

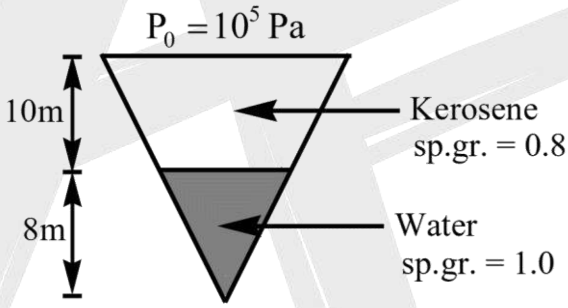
1. A rectangular bar of soap has density $700kg/m^3$ floats in water of density $1000kg/m^3$. Oil of density $300kg/m^3$ is slowly added, forming a layer that does not mix with the water. When the top surface of the oil is at the same level as the top surface of the soap. What is the ratio of the oil layer thickness to the soap's thickness, x/L ?



(A) $\frac{2}{10}$ (B) $\frac{2}{7}$ (C) $\frac{3}{7}$ (D) $\frac{3}{8}$

2. The figure shows a conical container of half-apex angle 37° filled with certain quantities of kerosene and water. The force exerted by the water on the kerosene is $2 \times 10^{\alpha}$. Find α?

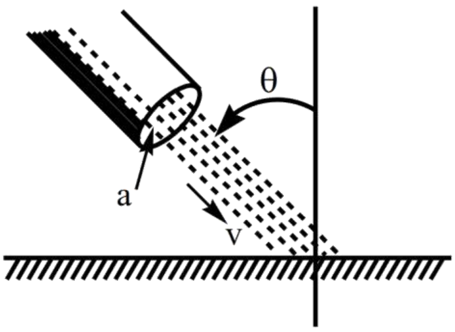
(Take atmospheric pressure $= 10^5 Pa$)



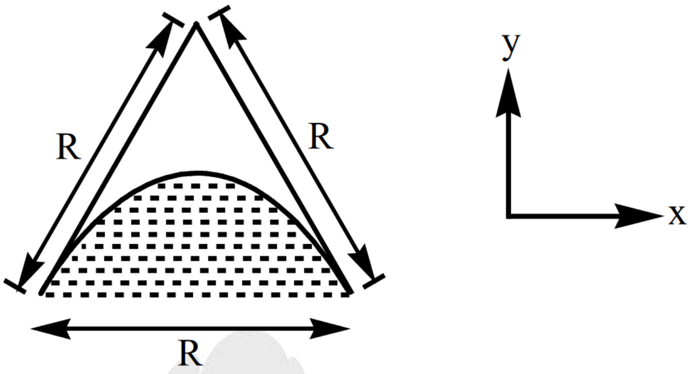
3. There is a small hole of area A at the bottom of a cylindrical vessel. Water is filled upto a height 'h' and water flows out in time 't'. If water is filled to a height '4h, it will flow out in time αt. Find α ?

4. A stream of liquid, set at an angle θ, is directed against a plane surface (figure). The liquid, after hitting the surface, spreads over it. The pressure on the surface is $\rho v^{\alpha} \cos^{\beta} \theta$. Find $\alpha + \beta$?

(The density of the liquid is ρ and its velocity is v.)



5. A light, rigid sheet of triangular shape has a curved portion cut from it as shown in figure. It floats on the surface of water. Some soap solution is dropped over dotted region. Surface tension of water and soap film are T_1 and T_2 respectively. $T_1 = 1.5T_2$. Mark correct option.



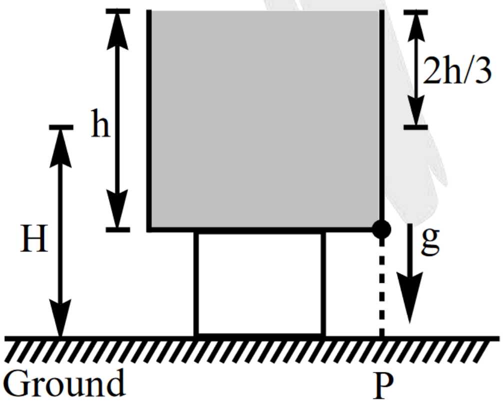
(A) The frame experiences a net force $F = (T_1 - T_2)$ R in y < 0 direction

(B) The frame experiences force $F = (T_1 - T_2)$ R in y > 0 direction

(C) The frame experiences force $(T_1 - T_2)(2 - \pi)$ R in y > 0 direction

(D) Resultant force on wire frame is zero

6. An open vessel full of water is falling freely under gravity. There is a small hole in one face of the vessel, as shown in the figure. The water which comes out from the hole at the instant when hole is at height H above the ground, strikes the ground at a distance of x from P. Which of the following is correct for the situation described?



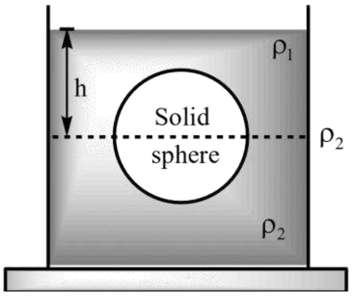
(A) The value of x is $2\sqrt{\frac{2hH}{3}}$

(B) The value of x is $\sqrt{\frac{4hH}{3}}$

(C) The value of x can't be computed from information provided

(D) The question is irrelevant as no water comes out from the hole

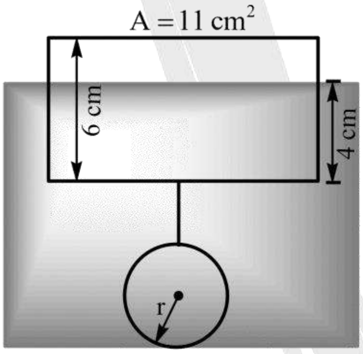
7. A solid sphere of radius r is floating at the interface of two immiscible liquids of densities ρ_1 and $\rho_2(\rho_2 > \rho_1)$, half of its volume lying in each. The height of the upper liquid column from the interface of the two liquids is h. The force exerted on the sphere by the upper liquid is (atmospheric pressure $= p_0$ and acceleration due to gravity is g) :



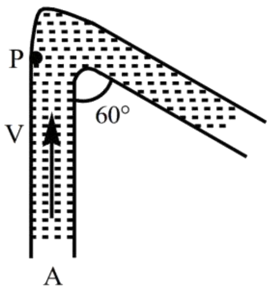
(A) $P_0\pi r^2 + \left(h - \frac{2}{3}r\right)\pi r^2\rho_1 g$ (B) $\left(h + \frac{2}{3}r\right)\pi r^2\rho_1 g + P_0\pi r^2$

(C) $\frac{2}{3}\pi r^3\rho_1 g$ (D) $P_0\pi r^2 - \left(h + \frac{2}{3}r\right)\pi r^2\rho_1 g$

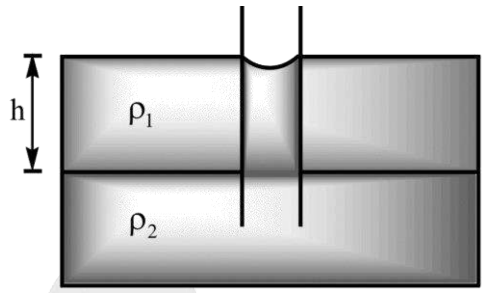
8. Figure shows a metal ball suspended by thread of negligible mass from an upright cylinder that floats partially submerged in water. The cylinder has height 6 cm, face area $11cm^2$ on the top and bottom and density $0.5\ g/cm^3$. 4 cm of cylinder's height is inside the water surface. If density of the metal ball is $8gm/cm^3$ then its radius is $r = \left(\frac{3}{8}\right)^{\alpha/\beta} cm$. Find $\alpha + \beta$?$(\rho_w = 1gm/cm^3)$



9. Water (density ρ) is flowing through the uniform tube of cross-sectional area A with a constant speed v as shown in the figure. The magnitude of force exerted by the water on the curved corner of the tube is $\sqrt{\alpha}\rho A v^2$. Find α ? (neglect viscous forces)



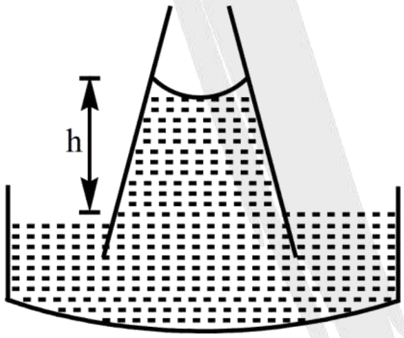
10. A container has two immiscible liquids of densities ρ_1 and ρ_2. A capillary tube of radius r is inserted in the liquid so that its bottom reaches upto the denser liquid. The denser liquid rises in the capillary and attains a height h from the interface of the liquids, which is equal to the column length of the lighter liquid. Assuming angle of contact to be zero, the surface tension of heavier liquid is:



(A) $2r(\rho_1 - \rho_2)gh$ (B) $r(\rho_2 - \rho_1)gh$

(C) $\frac{r}{2}(\rho_2 - \rho_1)gh$ (D) $2\pi r(\rho_2 - \rho_1)gh$

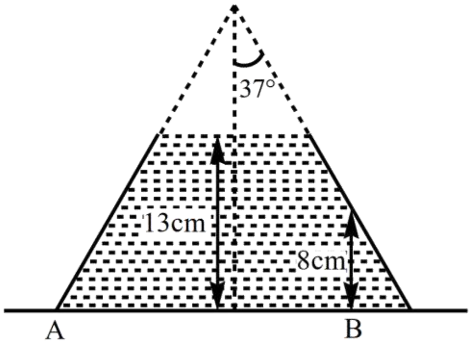
11. A capillary of the shape as shown in dipped in a liquid. Contact angle between the liquid and the capillary is $0°$ and effect of liquid inside the meniscus is to be neglected. T is surface tension of the liquid, r is radius of the meniscus, g is acceleration due to gravity and ρ is density of the liquid then height h in equilibrium is :



(A) greater than $\frac{2T}{r\rho g}$ (B) equal to $\frac{2T}{r\rho g}$

(C) less than $\frac{2T}{r\rho g}$ (D) equal to $\frac{4T}{r\rho g}$

12. Curved surface of a vessel has shape of a truncated cone having semi vertex angle $37°$. Vessel is full of water (density $\rho = 1000\,kg/m^3$) upto a height of 13 cm and is placed on a smooth horizontal plane. Upper surface is opened to atmosphere. A hole of $1.5\ cm^2$ is made on curved wall at a height of 8 cm from bottom as shown in figure. Area of water surface in the vessel is large as compared to the area of hole.



(A) Initial velocity of efflux is 1 m/sec

(B) Initial horizontal range of water jet from point B is 3.2 cm

(C) Horizontal force required to keep the vessel in static equilibrium is 0.15 N

(D) Horizontal force required to keep the vessel in static equilibrium is 0.12 N

13. Water jet coming out of a stationary horizontal tube at speed v strikes horizontally a massive wall moving in opposite direction with same speed. Water comes to rest relative to wall after striking. Treating A as cross-section of jet and density of water as ρ. Select the correct alternative(s).

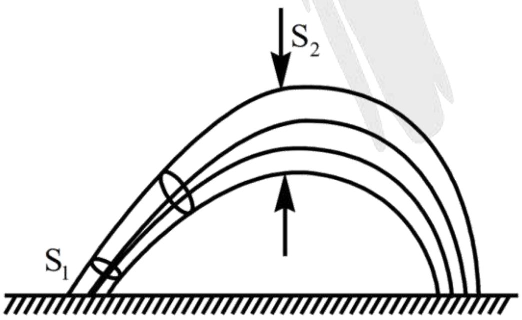
(A) force exerted on the wall is $2\rho Av^2$

(B) force exerted on the wall is $4\rho Av^2$

(C) rate of change of kinetic energy of water jet striking the wall is $8\rho Av^3$

(D) rate of change of kinetic energy of water jet striking the wall is zero

14. Water jet is projected at an angle to the horizontal. At the point of projection, the area of the jet is S_1 and at the highest point, the area of the jet is S_2. The initial velocity of projection is u.



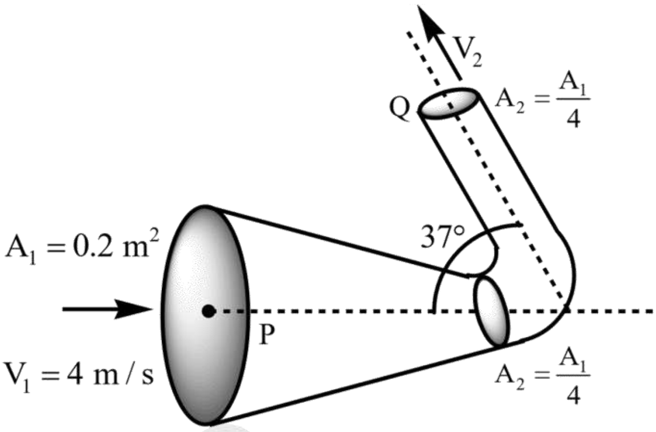
(A) The angle of projection is $cos^{-1}\left(\frac{S_1}{S_2}\right)$

(B) The range on the level ground is $\frac{2u^2}{g}\frac{S_1}{S_2}\sqrt{1-\frac{S_1^2}{S_2^2}}$

(C) The maximum height reached from the ground is $\frac{u^2}{2g}\left(1-\frac{S_1^2}{S_2^2}\right)$

(D) The rate of volume flow is S_2u

15. Oil enters the bend of a pipe in the horizontal plane with velocity $4ms^{-1}$ and pressure $280 \times 10^3 Nm^{-2}$ as shown in figure. (Take specific gravity of oil as 0.9 and $sin\,37° = 0.6$)



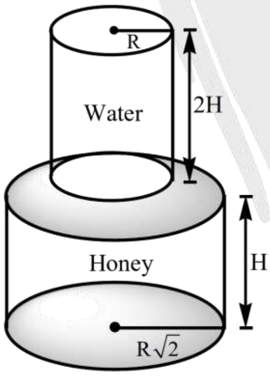
(A) the pressure of oil at the point Q, $(P_2) = 172 \times 10^3 N/m^2$

(B) the pressure of oil at the point Q, $(P_2) = 388 \times 10^3 N/m^2$

(C) the force required to hold the bend in the place is $63 \times 10^3 N$

(D) the force required to hold the bend in the place is $76 \times 10^3 N$

16. A bottle is kept on the ground as shown in the figure. The bottle can be modeled as having two cylindrical zones. The lower zone of the bottle has a cross-sectional radius of $R\sqrt{2}$ and is filled with honey of density 2ρ. The upper zone of the bottle is filled with the water of density ρ and has a cross-sectional radius R. The height of the lower zone is H while that of the upper zone is 2H. If now the honey and the water parts are mixed together to form a homogeneous solution. (Assume that total volume does not change)



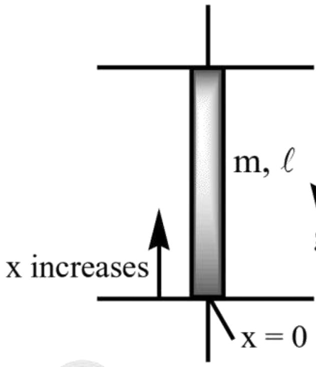
(A) The pressure inside the bottle at the base will remain unaltered

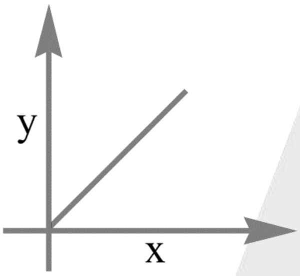
(B) The normal reaction on the bottle from the ground will remain unaltered

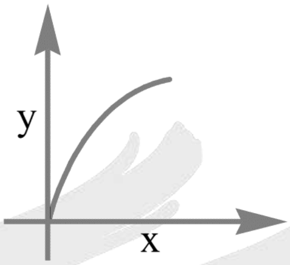
(C) The pressure inside the bottle at the base will increase by an amount $\left(\frac{1}{2}\right)\rho gH$

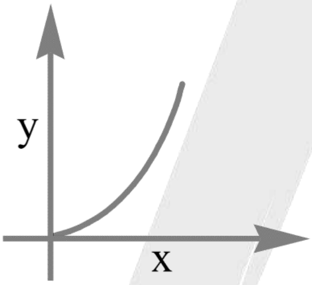
(D) The pressure inside the bottle at the base will decrease by an amount $\left(\frac{1}{4}\right)\rho gH$

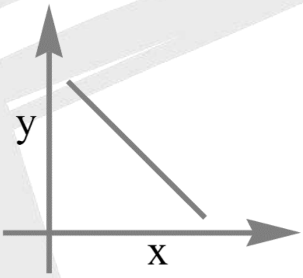
17. A uniform dense rod with non-uniform Young's modulus is hanging from ceiling under gravity. If elastic energy density at every point is same then Young's modulus with x will as which of the shown graph :



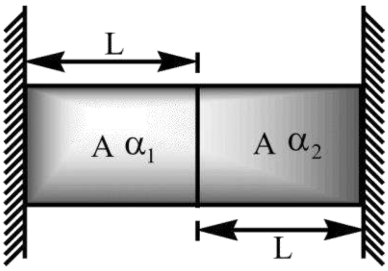
(A) 

(B) 

(C) 

(D) 

18. Figure shows two rods A and B of same length L and same cross sectional area S but of different material having coefficient of linear expansion α_1 and α_2 respectively. They are clamped between two rigid walls, separated by a distance 2L. This all refers to temperature $t°C$. (Take the Young's modulus for the two rods to be Y_1 and Y_2 respectively).



(A) $\frac{S(Y_1\alpha_1+Y_2\alpha_2)t}{Y_1+Y_2}$

(B) $\frac{SY_1Y_2(\alpha_1+\alpha_2)t}{Y_1+Y_2}$

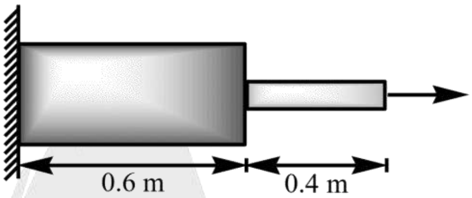
(C) $\frac{St\alpha_1\alpha_2(Y_1+Y_2)t}{Y_1Y_2}$

(D) $\frac{StY_1Y_2(\alpha_1+\alpha_2)}{2(Y_1+Y_2)}$

19. A liquid of volumetric thermal expansion coefficient = γ and bulk modulus B is filled in a spherical tank of negligible heat expansion coefficient. Its radius is R and wall thickness is 't' ($t << R$). When the temperature of the liquid is raised by θ, the tensile stress developed in the walls of the tank is :

(A) $\frac{B\gamma\theta R}{2t}$ (B) $\frac{B\gamma\theta R}{t}$ (C) $\frac{2B\gamma\theta R}{t}$ (D) $\frac{B\gamma\theta R}{4t}$

20. Two bars of steel ($Y = 2 \times 10^{11} N/m^2$) are joined together as shown. The area of cross-section of the left bar is $15cm^2$ and the area of right bar is unknown. The extension in both bars is the same.



(A) The area of right bar is $10\ cm^2$

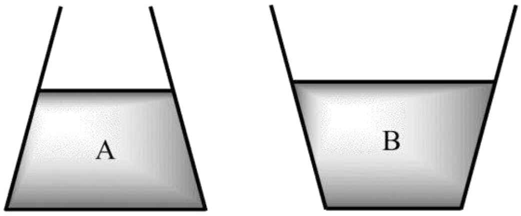
(B) The stresses in left and right bar are in ratio 3 : 2

(C) The decrease in thickness of bar is more for the left

(D) The decrease in thickness of bar is more for right bar

EXCERCISE-V

1. Two vessels A and B of cross-sections as shown contain a liquid up to the same height. As the temperature rises, the liquid pressure at the bottom (neglecting expansion of the vessels) will :



(A) increase in A, decrease in B

(B) increase in B, decrease in A

(C) increase in both A and B

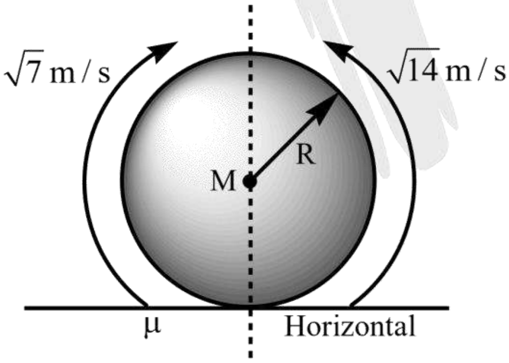
(D) decrease in both A and B

2. A cylindrical vessel of 92 cm height is kept filled up to the brim. It has four holes 1, 2, 3, 4 which are respectively at heights of 20 cm, 30 cm, 46 cm and 80 cm from the horizontal floor. The water falling at the maximum horizontal distance from the vessel comes from :

(A) hole no. 4 (B) hole no. 3 (C) hole no. 2 (D) hole no. 1

3. A solid sphere of mass M and radius R is kept on a rough surface. The velocities of air (density ρ) around the sphere are as shown in figure. Assuming R to be small and $M = \frac{4\pi\rho R^2}{g}$, the minimum value of coefficient of friction so that the sphere starts pure rolling is $\frac{25}{n}$.

Find n? (Assume force due to pressure difference is acting on centre of mass of the sphere).

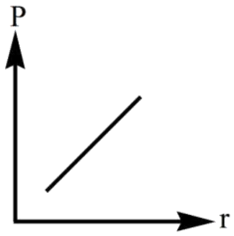


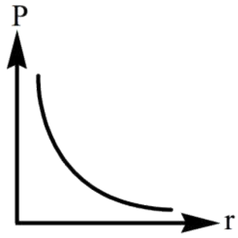
4. A plastic circular disc of radius r is placed on a thin oil film, spread over a flat horizontal surface. The torque required to spin the disc about its central vertical axis with a constant angular velocity is proportional to :

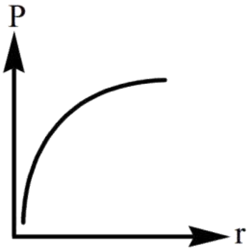
(A) R^2 (B) R^3

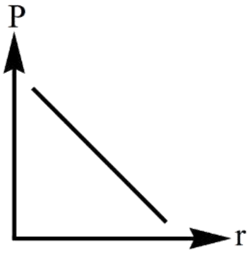
(C) R^4 (D) R^6

5. A spherical soap bubble is blown such that its radius increases at a constant rate. Which of the following curves represents power required to increase surface energy of the bubble versus radius of drop?

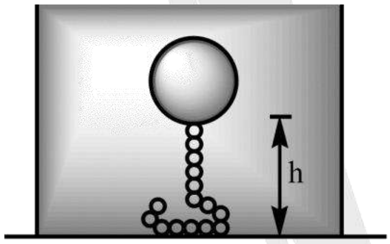
(A)


(B)


(C)


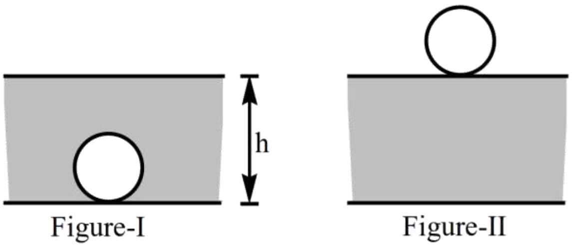
(D)


6. One end of a long iron chain of linear mass density λ is fixed to a sphere of mass m and specific density $\frac{1}{3}$ while the other end is free. The sphere along the chain is immersed in a deep lake. If specific density of iron is 7, the height h above the bed of the lake at which the sphere will float in equilibrium is (Assume that the part of the chain lying on the bottom of the lake exerts negligible force on the upper part of the chain) :



(A) $\frac{16m}{3\lambda}$ (B) $\frac{7m}{3\lambda}$ (C) $\frac{5m}{3\lambda}$ (D) $\frac{8m}{3\lambda}$

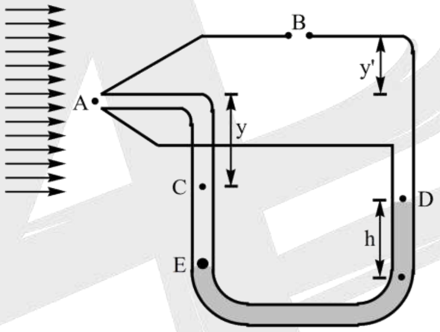
7. In figure-I is shown a sphere of a mass m and radius r resting at the bottom of a large container filled with water. Depth of the container is h. Density of material of the sphere is the same as that of water. Now the whole sphere is slowly pulled water as shown in figure-II. Work done by the agent pulling the sphere equals to :



(A) mgr (B) 0.5 mgr (C) 2 mgr (D) 4 mgr

8. A container of cross-section area 'S' and height 'h' is filled with mercury upto the brim. Then the container is sealed airtight and a hole of small cross section area 'S/n' (where 'n' is a positive constant) is punched in its bottom. The time interval upto which the mercury will come out from the bottom hole is $t = \alpha n\sqrt{\frac{\beta}{g}(h - h_0)}$. Find $\alpha + \beta$? [Take the atmospheric pressure to be equal to h_0 height of mercury column: $h > h_0$]

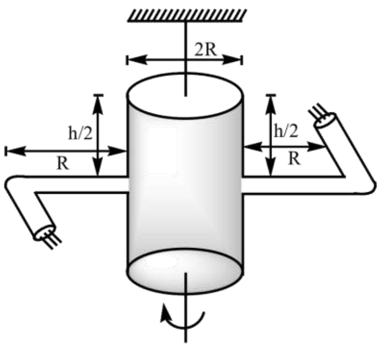
9. A Pitot tube is shown in figure. Wind blows in the directions shown. Air at inlet A is brought to rest, whereas its speed just outside of opening B is unchanged. The U tube contains mercury of density ρ_m. The speed of wind with respect to Pitot tube is $v = \sqrt{\frac{\alpha(\rho_m - \rho_a)gh}{\beta\rho_a}}$. Find $\alpha + \beta$? (Neglect the height difference between A and B and take the density of air as ρ_a.)



10. A cylindrical wooden float whose base area S and the height H drifts on the water surface. Density of wood d and density of water is ρ. What minimum work must be performed to take the float out of the water?

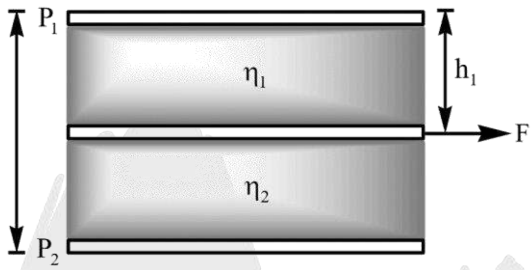
(A) $\frac{Sdg^2H^2}{4\rho}$ (B) $\frac{Sdg^2H^2}{2\rho}$ (C) $\frac{Sdg^2H^2}{\rho}$ (D) $\frac{Sdg^2H^2}{3\rho}$

11. A cylindrical container of radius 'R' and height 'h' is completely filled with a liquid. Two horizontal L shaped pipes of small cross-section area 'a' are connected to the cylinder as shown in the figure. Now the two pipes are opened and fluid starts coming out of the pipes horizontally in opposite directions. Then the torque due to ejected liquid on the system is :

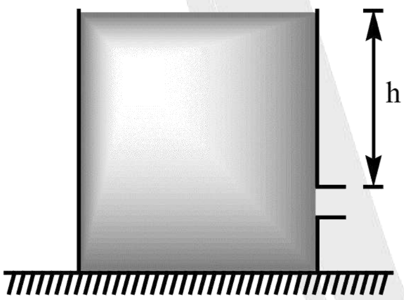


(A) 4aghρR (B) 8aghρR (C) 2aghρR (D) 6ρaghR

12. A thin horizontal movable plate is separated from two fixed horizontal plates P_1 and P_2 by two highly viscous liquids of coefficient of viscosity η_1 and η_2 as shown, where $\eta_2 = 4\eta_1$. Area of contact of movable plate with each fluid is same. If the distance between two fixed plates is h, then the distance h_1 of movable plate from upper fixed plate such that the movable plate can be moved with a constant velocity by applying a minimum constant horizontal force F on movable plate is $\frac{h}{\alpha}$. Find α ? (Assume velocity gradient to be uniform in each liquid).

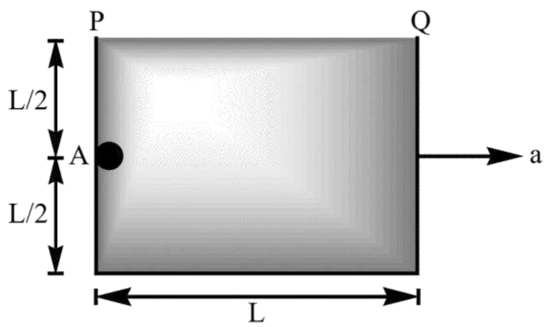


13. In the figure shown, a light container is kept on a horizontal rough surface of coefficient of friction $\mu = \frac{Sh}{V}$. A very small hole of area S is made at depth 'h'. Water of volume 'V' is filled in the container. The friction is not sufficient to keep the container at rest. The acceleration of the container initially is :



(A) $\frac{V}{Sh}g$ (B) $\frac{Sv}{h}g$ (C) $\frac{2Sh}{v}g$ (D) $\frac{Sh}{V}g$

14. A small solid ball of density ρ is held inside at point A of a cubical container of side L, filled with an ideal liquid of density 4ρ as shown. Now, if the container starts moving with constant acceleration a horizontally and the ball is released from point A, simultaneously, then :



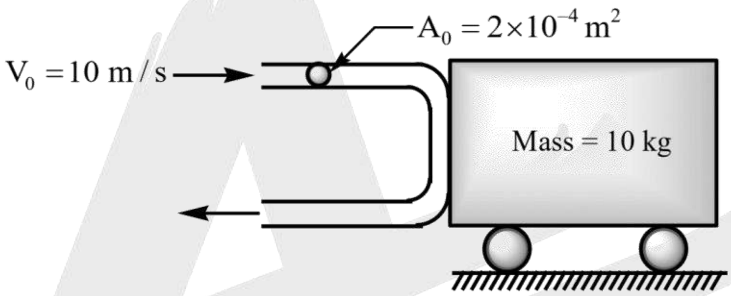
(A) For the ball to hit top of container at end Q, a = 3g

(B) For the ball to hit top of container at end Q, a = 2g

(C) Ball hits the top of container at end Q after a time $t = \sqrt{\frac{L}{3g}}$

(D) Ball hits the top of container at end Q after a time $t = \sqrt{\frac{L}{2g}}$

15. The figure shows a cart of mass 10 kg initially kept at rest on a frictionless surface. A jet of liquid of density $1000\ kg/m^3$ is directed on the cart. Velocity of the jet is 10 m/s and its cross section area is $2\times10^{-4}\ m^2$. If at t = 0, cart is released and allowed to move freely in horizontal direction then choose the correct statement(s) :



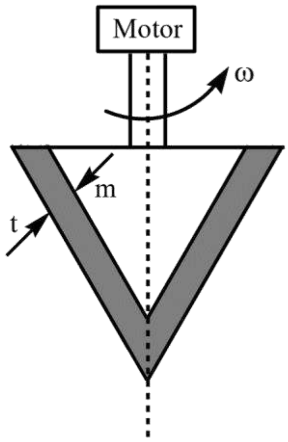
(A) Force on the cart at t = 0 s is 40 N

(B) Velocity of cart at t = 10 s is 8 m/s

(C) Acceleration of cart at t = 10 s is $1\ m/s^2$

(D) Power supplied to the cart when cart is moving with velocity 5 m/s is 50 W

16. In the figure shown, there is a conical shaft rotating on a bearing of very small clearance t. The space between the conical shaft and the bearing, is filled with viscous fluid having coefficient of viscosity η. The shaft is having radius R and height h. If the external torque applied by the motor is τ and the power delivered by the motor is P working in 100% efficiency to rotated the shaft with constant ω. Then :

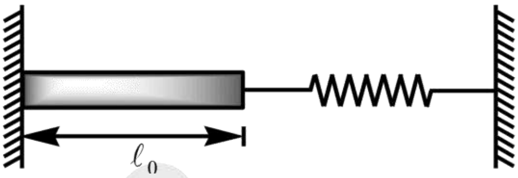


(A) $P = \frac{\pi\omega^2\eta R^3\sqrt{R^2+h^2}}{2t}$ (B) $\tau = \frac{\pi\omega\eta R^3\sqrt{R^2+h^2}}{2t}$

(C) $P = \frac{\pi\omega^2\eta R^3 h}{2t}$ (D) $\tau = \frac{\pi\omega\eta R^3 h^2}{2t\sqrt{R^2+h^2}}$

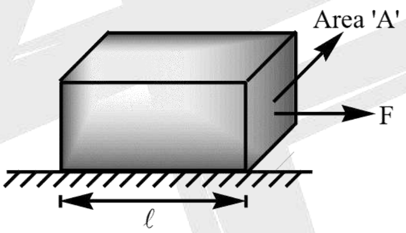
17. A rod of mass M, area of cross section A and length ℓ_0 is connected with a spring as shown in figure. If coefficient of linear expansion of rod is α and initially no extension was there in the spring of spring constant k, then the stress developed in rod when its temperature is increased by ΔT, is :

[Young's modulus of material of the rod is Y]



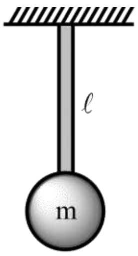
(A) $\frac{k[Y\alpha\Delta T]\ell_0}{[k\ell_0+YA]}$ (B) $Y\alpha\Delta T$ (C) $\left[\frac{k\ell_0+YA}{k\ell_0}\right]Y\alpha\Delta T$ (D) $\left[\frac{k\ell_0+Y}{k\ell_0}\right]Y\alpha\Delta T$

18. A block of mass 'M', area of cross-section 'A' and length 'ℓ' is placed on smooth horizontal floor. A for 'F' is applied on the block as shown. If 'y' is young modulus of material, then total extension in the block will be :



(A) $\frac{F\ell}{Ay}$ (B) $\frac{F\ell}{2Ay}$ (C) $\frac{F\ell}{3Ay}$ (D) cannot extend

19. A ball of mass m is suspended from a uniform elastic wire of negligible mass, having cross-section diameter d, length ℓ, Young's modulus Y and heat capacity C. Now, when the ball is snapped then:



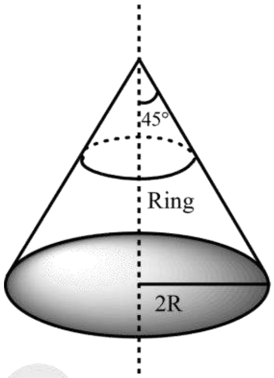
(A) initial elongation of wire is $\Delta\ell = \frac{4mg\ell}{Y\pi d^2}$

(B) initial elongation of wire is $\Delta\ell = \frac{2mg\ell}{\pi Y d^2}$

(C) increase in the temperature of wire is $\Delta\theta = \frac{m^2g^2\ell}{CY\pi d^2}$

(D) increase in the temperature of wire is $\Delta\theta = \frac{2m^2g^2\ell}{CY\pi d^2}$

20. A ring of mass m, radius R, cross-sectional area A and Young's modulus Y is kept on a smooth cone of radius 2R and semi vertical angle 45°, as shown in the figure. Assume that the extension in the ring is small :



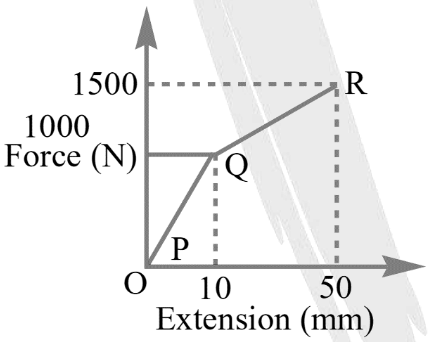
(A) The tension in the ring will be same throughout

(B) The tension in the ring will be independent of the radius of ring

(C) The extension in the ring will be $\frac{mgR}{AY}$

(D) Elastic potential energy stored in the ring will be $\frac{m^2g^2R}{8\pi YA}$

21. The graph shows the force-extension curve for a steel wire that is stretched by a tensile force. In the graph, PQ and QR are two idealized straight line sections. The unextended length of the wire is 2.00 m. Assume that Q is proportionality limit and R is yield point.



(A) The strain in the wire at R is 0.025

(B) The work done in stretching the wire from Q to R is equal to 50 J

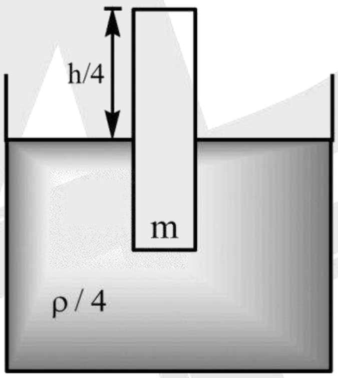
(C) If release the wire at point Q, it won't have any permanent set

(D) The work done in stretching the wire from Q to R is equal to 55 J

PROFICIENCY TEST-I

1. A cone of height 1m and half angle $45°$ is kept inside a tank. Both the surfaces are perfectly smooth so that no air is trapped in between the base of the cone and the bottom of the tank. Then the tank is filled with water upto height of 1m. (Take $P_{atm} = 1\times10^5$ Pa, Density of water = Average density of the cone $= 10^3$ kg / m^3) the normal reaction exerted by the tank on the cone is $1.1\,\pi\times10^{\alpha}$ N. Find α

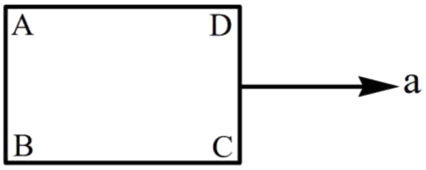
2. A solid cylinder of height h and mass m floats in a liquid of density $\rho/4$ as shown in figure. Now the cylinder is accelerated downward. Determine the magnitude of acceleration A of vessel, for which cylinder sinks with relative acceleration A/3 with respect to vessel. Neglect any dissipated force.



(A) $\frac{2}{3}g$ (B) $\frac{4}{3}g$

(C) $\frac{6}{5}g$ (D) $\frac{2}{3}g$

3. A closed rectangular tank is completely filled with water and is accelerated horizontally with an acceleration towards right. Pressure is (i) maximum at and (ii) minimum at :



(A) (i) B (ii) D

(B) (i) C (ii) D

(C) (i) B (i) C

(D) (i) B (ii) A

4. A tennis ball receives a top spin when struck by a racket and describes a curved trajectory. The top spin implies that the rotator motion of the top surface of the ball is in the direction of the translator motion of the ball. Which one of the following statements is the best description of the trajectory?

(A) Pressure on the top surface is lower; trajectory rises

(B) Pressure on the top surface is lower; trajectory dips

(C) Pressure on the top surface is higher; trajectory rises

(D) Pressure on the top surface is higher; trajectory dips

5. A hole is punched in the side of a bucket so that water flows out and follows a parabolic trajectory. If the container is dropped from a height, falls freely accelerating under gravity and air resistance can be ignored, the water flow :

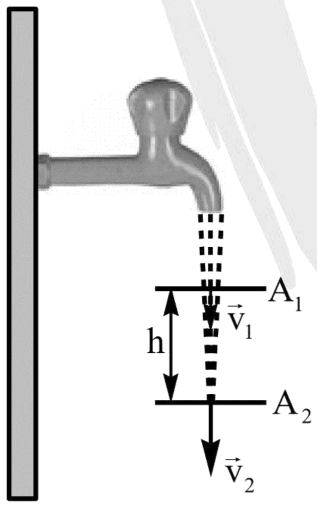
(A) stops

(B) follows a straight line trajectory relative to the falling bucket

(C) follows an upward curving trajectory relative to the bucket

(D) follows a downward curving trajectory relative to the bucket

6. A steady stream of water-falls straight down from a pipe. Assume the flow is incompressible; the flow is similar to figure. How does the pressure in the water vary with height in the falling stream?



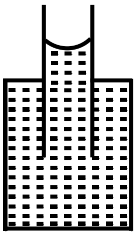
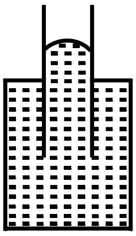
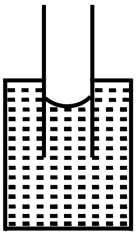
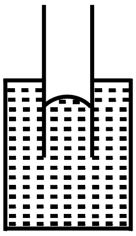
(A) The pressure in the water is higher at lower points in the stream

(B) The pressure in the water is lower at lower points in the stream

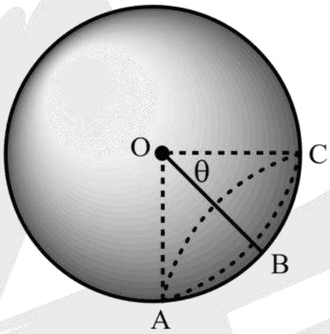
(C) The pressure in the water is the same at all points in the stream

(D) The pressure of water is maximum at the middle of the stream

7. You dip a capillary tube into a beaker of different fluids, and from observation of the shape of the fluid surface deduce that in which figure cohesive forces dominate over adhesive forces?

(A)  (B)  (C)  (D) 

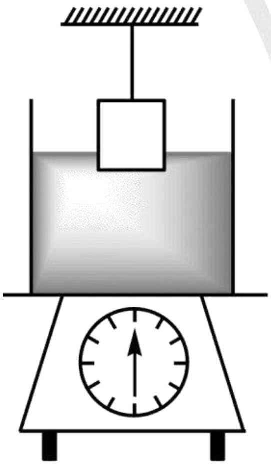
8. Consider a small water drop in air. If T is the surface tension, then what is the force due to surface tension acting on the smaller section ABC ?



(A) $2\pi TR\cos^2\theta$ (B) $2\pi TR\sin\theta$ (C) $2\pi TR\sin^2\theta$ (D) $2\pi TR\cos\theta$

9. A soap bubble of radius R and surface tension T is formed in vacuum. It is slowly charged so that it slowly expands. It is stopped charging when the radius becomes 2R. The amount of charge given to the bubble is $Q = \sqrt{768\pi^{\alpha}R^{\beta}\varepsilon_0 T}$. Find $\alpha + \beta$?

10. The mass of block is m_1 and that of liquid with the vessel is m_2. The block is suspended by a string (tension T) partially in the liquid. The reading of the weighing machine placed below the vessel :



(A) can be $(m_1 + m_2)g$ (B) can be greater than $(m_1 + m_2)g$

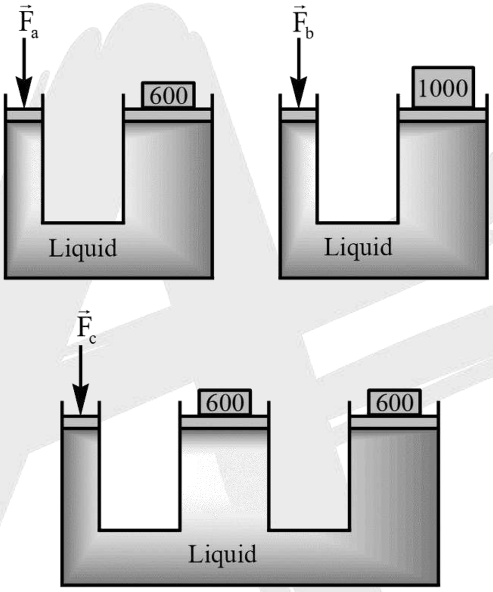
(C) is equal to $(m_1g + m_2g + T)$ (D) can be less than $(m_1 + m_2)g$

PROFICIENCY TEST-II

1. A small body with relative density d_1 falls in air from a height 'h' on to the surface of a liquid of relative density d_2 where $d_2 > d_1$. The time elapsed after entering the liquid to the instant when it comes to instantaneous rest inside liquid :

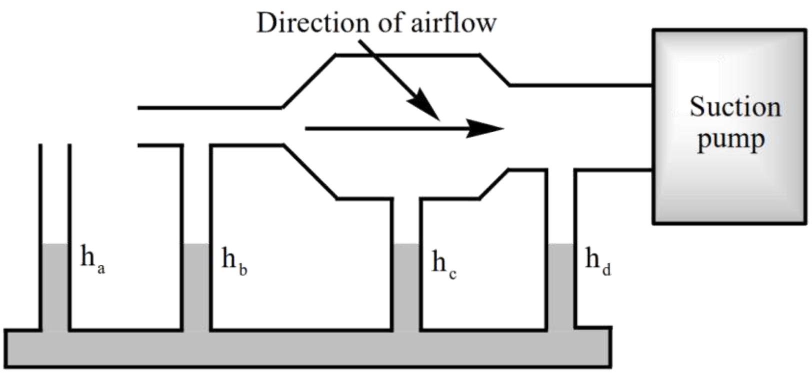
(A) $\sqrt{\frac{2h}{g}}\frac{d_2}{d_1}$ (B) $\sqrt{\frac{2h}{g}}\frac{d_1}{d_2-d_1}$ (C) $\sqrt{\frac{2h}{g}}\frac{d_1}{d_2}$ (D) $\sqrt{\frac{2h}{g}}\frac{d_2-d_1}{d_2}$

2. Rank in order, from largest to smallest, the magnitude of the forces $\vec{F}_a$, $\vec{F}_b$ and $\vec{F}_c$ required to balance the masses. The masses are in kilograms.



(A) $F_a = F_b = F_c$ (B) $F_a > F_b = F_c$ (C) $F_b > F_a = F_c$ (D) $F_c < F_b < F_a$

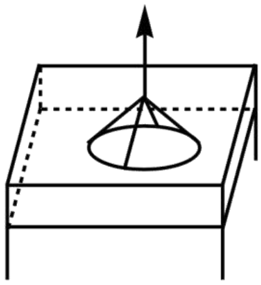
3. Rank in order, from highest to lowest, the liquid heights h_a to h_c. The air flow is from left to right. The liquid columns are not drawn to scale but pipe diameter is drawn to scale.



(A) $h_b = h_d > h_a > h_c$ (B) $h_b = h_d > h_c > h_a$

(C) $h_d < h_c < h_b < h_a$ (D) $h_b > h_d > h_c > h_a$

4. The surface tension of water is 75 dyne/cm. The minimum vertical force required to pull a thin wire ring up (refer figure) if it is initially resting on a horizontal water surface. The circumference of the ring is 20 cm and its weight is 0.1 N is $\frac{13}{n}$N. Find n?

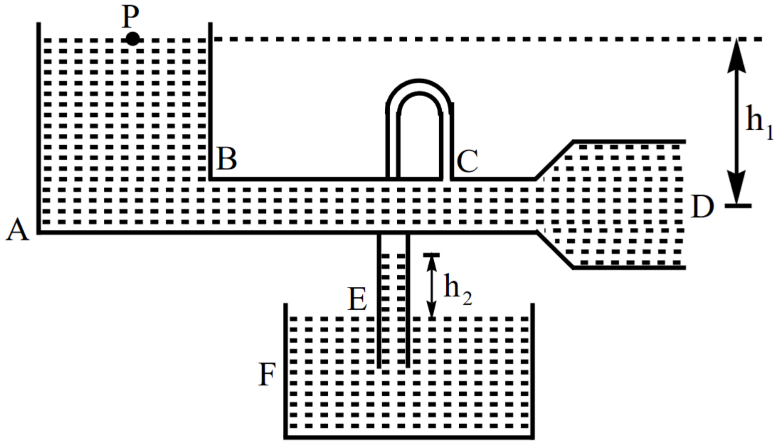


5. A solid ice block (of any shape) is floating remaining in equilibrium in water. Some part of it is outside water because its density is less than the density of water. What is change in level of water when ice melts completely? Neglect the changes in volume due to temperature changes.

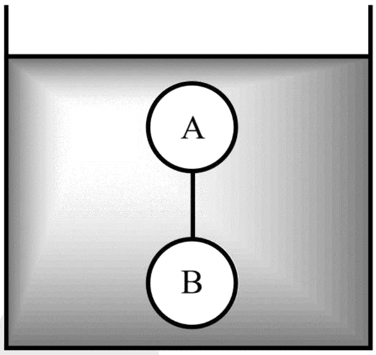
(A) Level goes down
(B) Level goes up
(C) Level remain same
(D) Data insufficient to decide

6. Two very large open tanks A and F both contains the same liquid. A horizontal pipe BCD, having a small constriction at C, leads out of the bottom of tank A, and a vertical pipe E containing air opens into the constriction at C and dips into the liquid in tank F. Assume streamline flow and no viscosity. If the cross section area at C is one-half that at D, and if D is at distance h_1 below the level of the liquid in A, to what height h_2 will liquid rise in pipe E in terms of h_1 is $h_2 = \alpha h_1$. Find α?

[Neglect changes in atmospheric pressure with elevation. In the container there is atmosphere above the water surface and D is also open to atmosphere.]



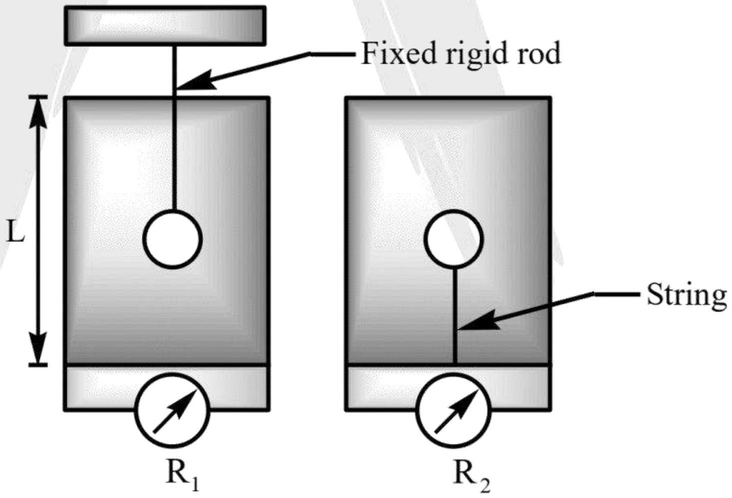
7. Two solid spheres A and B of equal volumes but of different densities d_A and d_B are connected by a string. They are fully immersed in a fluid of density d_F. They get arranged into an equilibrium state as shown in the figure with a tension in the string. The arrangement is possible only if :



(A) $d_A = d_F$ (B) $d_B > d_F$

(C) $d_A > d_F$ (D) $d_A + d_B = 2d_F$

8. A cylindrical container of length L is full to the brim with a liquid which has density ρ. It is placed on a weigh-scale, which reads W. A light ball (which would float on the liquid if allowed to do so) of volume V and mass m is pushed gently down and held beneath the surface of the liquid with a rigid rod of negligible volume, as shown.



(A) The mass M of liquid which overflowed while the ball was being pushed into the container is ρV

(B) The reading R_1 on the scale when the ball is fully immersed is 2W

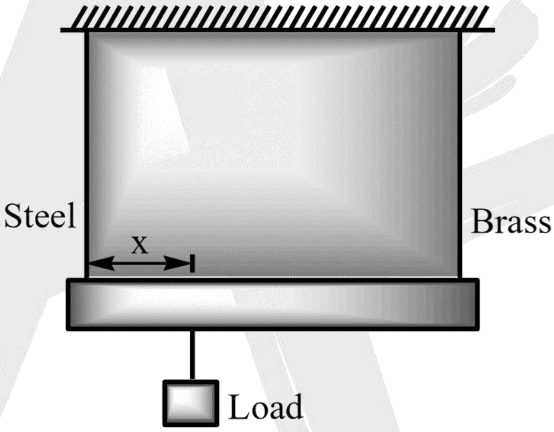
(C) If instead of being pushed down by a rod the ball is held in place by a fine string attached to the bottom of the container, the tension T in the string $\rho V g - mg$

(D) In part (c), the reading R_2 on the scale is $W - \rho Vg + mg$

9. The bar shown in the figure is made of a single piece of material. It is fixed at one end and consists of two segments of equal length $\frac{L}{2}$ but different cross-sectional area A and 2A. What is the change in length of the entire system under the action of an axial force F? [Consider the shape of joint to remain circular, Y is Young's modulus].

(A) $\frac{3FL}{4AY}$ (B) $\frac{3FL}{8AY}$ (C) $\frac{3FL}{2AY}$ (D) $\frac{2FL}{3AY}$

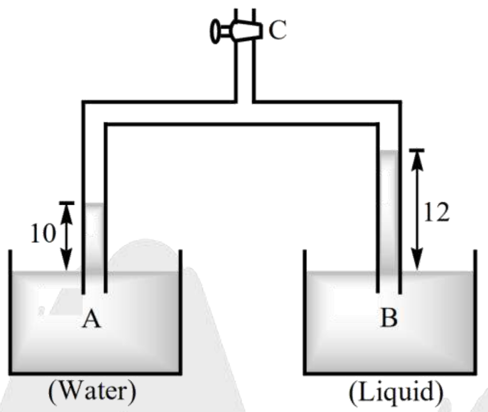
10. A light rod of length 2 m is suspended from the ceiling horizontally by means of two vertical wires of equal length tied to its end. One of the wires is made of steel and is of cross-section $0.1\ cm^2$. The other wire is of brass of cross-section $0.2\ cm^2$. A weight is suspended from a certain point of the rod such that equal stresses are produced in both the wires. The rod remains horizontal in this case also. Find out the position of the load from the steel wire.



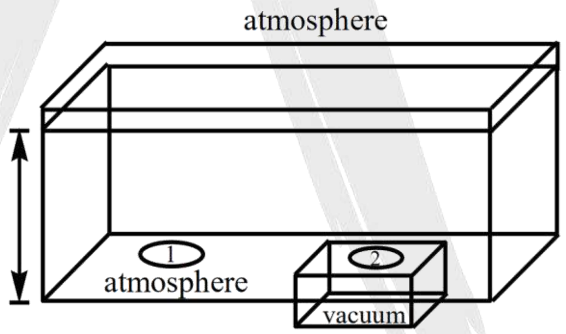
(A) $\frac{4}{3}$ (B) $\frac{3}{2}$ (C) $\frac{2}{3}$ (D) $\frac{1}{3}$

PROFICIENCY TEST-III

1. The limbs of a glass U-tube are lowered into vessel A and B, A containing water. Some air is pumped out through the top of the tube C. The liquids in the left hand limb A and the right hand limb B rise to heights of 10 cm and 12 cm respectively. The density of liquid B is $\frac{8J}{n}\ g/cm^3$. Find n



2. Two identical discs sit at the bottom of a 3 m pool of water whose surface is exposed to atmospheric pressure. The first disc acts as a plug to seal the drain as shown. The second disc covers a container containing nearly a perfect vacuum. If each disc has an area of $1\ m^2$, what is the approximate difference in the force necessary to open the discs? (Note : 1 atm = 101,300 Pa)



(A) 6000 N
(B) 3000 N
(C) 101,300 N
(D) 101,3000 N

3. A large cylindrical tank has a hole of area A at its bottom. Water is poured in the tank by a tube of equal cross-sectional area A, ejecting water at speed v.

(A) The water level in the tank will keep rising

(B) The water can be stored in the tank

(C) The water level will raise to a height $\frac{v^2}{2g}$ and then becomes constant

(D) The water level will oscillate

4. One end of a long iron chain of linear mass density λ is fixed to a sphere of mass m and specific density $\frac{1}{3}$ while the other end is free. The sphere along the chain is immersed in a deep lake. If specific density of iron is 7, the sphere is slightly displaced vertically from its equilibrium position, the time period of the resulting SHM is :

(A) $\frac{3\pi}{7}\sqrt{\frac{34m}{\lambda g}}$ (B) $\frac{2\pi}{3}\sqrt{\frac{35m}{\lambda g}}$ (C) $\frac{2\pi}{3}\sqrt{\frac{35\lambda}{mg}}$ (D) $\frac{3\pi}{7}\sqrt{\frac{64m}{\lambda g}}$

5. A fixed container of height 'H' with large cross-sectional area 'A' is completely filled with water. Two small orifice of cross-sectional area 'a' are made, one at the bottom and the other on the vertical side of the container at a distance $\frac{H}{2}$ from the top of the container. Find the time taken by the water level to reach a height of $\frac{H}{2}$ from the bottom of the container.

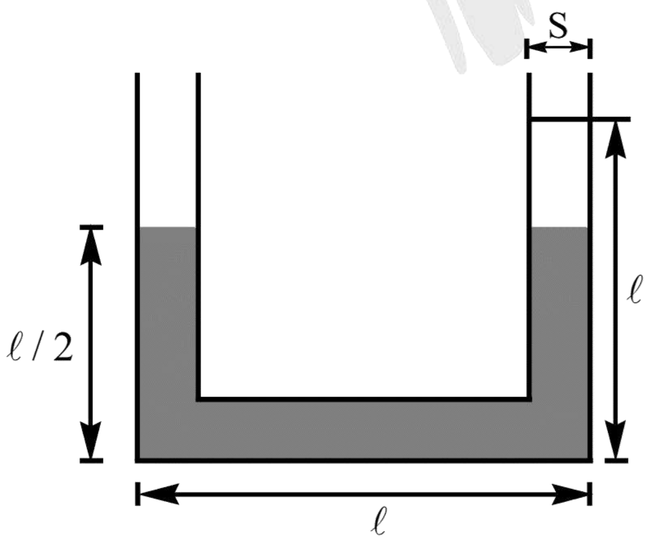
(A) $t = \frac{2A}{a}(\sqrt{2}+1)\sqrt{\frac{H}{g}}$

(B) $t = \frac{4a}{3a}(\sqrt{2}+1)\sqrt{\frac{H}{g}}$

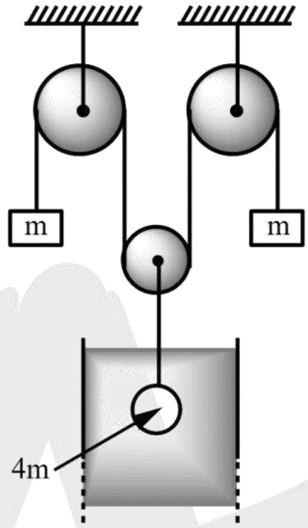
(C) $t = \frac{2A}{3a}(\sqrt{2}-1)\sqrt{\frac{H}{g}}$

(D) $t = \frac{4A}{3a}(\sqrt{2}-1)\sqrt{\frac{H}{g}}$

6. A thin U-tube sealed at one end consists of three bends of length $= 250$ mm each forming right angles. The vertical parts of the tube are filled with mercury to half the height as in figure. All of mercury can be displaced from the tube by heating slowly the gas in the sealed end of the tube which is separated from the atmospheric air by mercury. The work done by the gas thereby if the atmospheric pressure is $p_0 = 10^5$ Pa, density of mercury is $\rho_{mer} = 13.6\times10^3$ kg / m^3 and the cross-sectional area of the tube is $S = 1$ cm^2 is $\frac{619}{8n}$ joules. Find n?



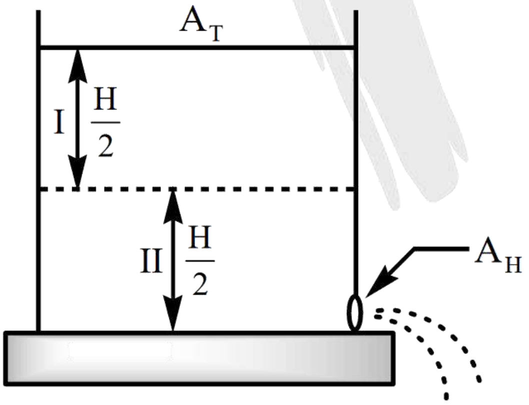
7. A spherical ball of mass 4m, density σ and radius r is attached to a pulley-mass system as shown in figure. The ball is released in a liquid of coefficient of viscosity η and density $\rho\left(<\frac{\sigma}{2}\right)$. If the length of the liquid column is sufficiently long, the terminal velocity attained by the ball is given by (assume all pulleys to be massless and string as massless and inextensible) :



(A) $\frac{2}{9}\frac{r^2(2\sigma+\rho)g}{\eta}$ (B) $\frac{2}{9}\frac{r^2(\sigma-2\rho)g}{\eta}$

(C) $\frac{2}{9}\frac{r^2(\sigma-4\rho)g}{\eta}$ (D) $\frac{2}{9}\frac{r^2(\sigma+3\rho)g}{\eta}$

8. A tank having a hole at bottom, water is filled upto height H, area of hole in A_H and area of top is A_T choose the correct statements.



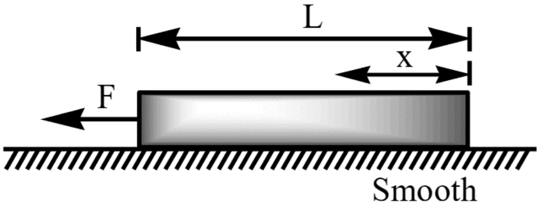
(A) The ratio of time taken to emply I[st] half and II[nd] half is directly proportional to the ratio of $\frac{A_T}{A_H}$

(B) It is independent of the ratio of area of hole and top

(C) It is inversely proportional to the ratio $\frac{A_T}{A_H}$

(D) This ratio is equal to $\frac{(\sqrt{2}-1)}{\sqrt{2}}$

9. A rod of mass m, uniform cross sectional area A and length L is accelerated by applying force F as shown in figure on a smooth surface. If Young's modulus of elasticity of the material of rod is Y. (Consider x as measured from the right end). Then :



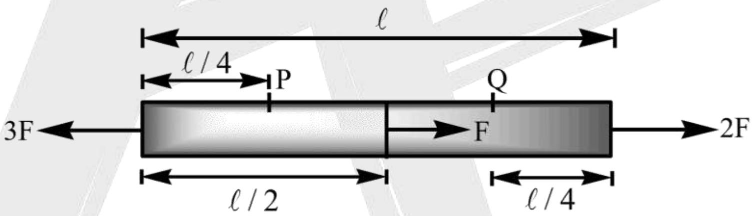
(A) Tension in rod as a function of distance x is $\frac{Fx}{2L}$

(B) Strain in rod is $\frac{F}{2AY}$

(C) Elastic potential energy stored in the rod is $\frac{F^2L}{8AY}$

(D) There is no stress in rod

10. In the figure shown a rod is kept on floor and applied some forces. The area of cross-section is A and Young's modulus is Y (Assume all forces are applied uniformly distributed throughout cross section of rod).



(A) Stress at P and Q are equal

(B) Stress at P is $\frac{3}{2}$ times stress at Q

(C) Elongation of rod under the forces is $\frac{5}{2}\frac{F\ell}{AY}$

(D) Strain at Q is $\frac{2F}{AY}$

ANSWER KEY

EXERCISE-I_KEY

1	2	3	4	5	6	7	8	9	10
D	A	C	D	B	D	C	B	D	BC
11	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
B	A	C	A	C	A	C	C	CD	AD

EXERCISE-II_KEY

1	2	3	4	5	6	7	8	9	10
D	4	D	A	C	A	20	D	B	10
11	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
1500	A	AD	BD	AB	A	B	B	C	A

EXERCISE-III_KEY

1	2	3	4	5	6	7	8	9	10
D	C	D	B	C	D	70	10	A	10
11	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
B	11	C	C	A	10	B	BD	AC	D

EXERCISE-IV_KEY

1	2	3	4	5	6	7	8	9	10
C	7	2	4	B	D	A	4	3	C
11	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
B	ABD	BD	ABC	AD	BC	C	B	A	AD

EXERCISE-V_KEY

1	2	3	4	5	6	7	8	9	10
A	B	100	C	A	B	A	3	3	B
11	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
A	3	D	BC	ABD	AB	A	B	AD	ABC
21									
ABC									

PROFICIENCY TEST-I_KEY

1	2	3	4	5	6	7	8	9	10
5	A	A	D	A	C	A	C	5	AD

SPROFICIENCY TEST-II_KEY

1	2	3	4	5	6	7	8	9	10
B	C	D	100	C	3	BD	ACD	A	A

PROFICIENCY TEST-III_KEY

1	2	3	4	5	6	7	8	9	10
100	C	C	B	C	10	B	BD	B	BCD