Nishant Jindal

RELATION FUNCTION

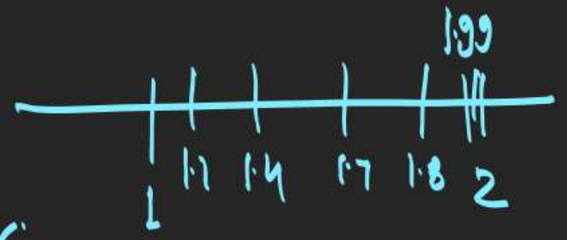
Greatest Integer fxn → ① $f(x) = [x]$

② left side's Integer

(3) $[x] = 1$ When

$x \in [1, 2)$

$[x] = 3 \Rightarrow x \in [3, 4)$



$[1.7] = 1$

$[1.4] = 1$

$[1.1] = 1$

$[1.99] = 1$

$[2] = 2$

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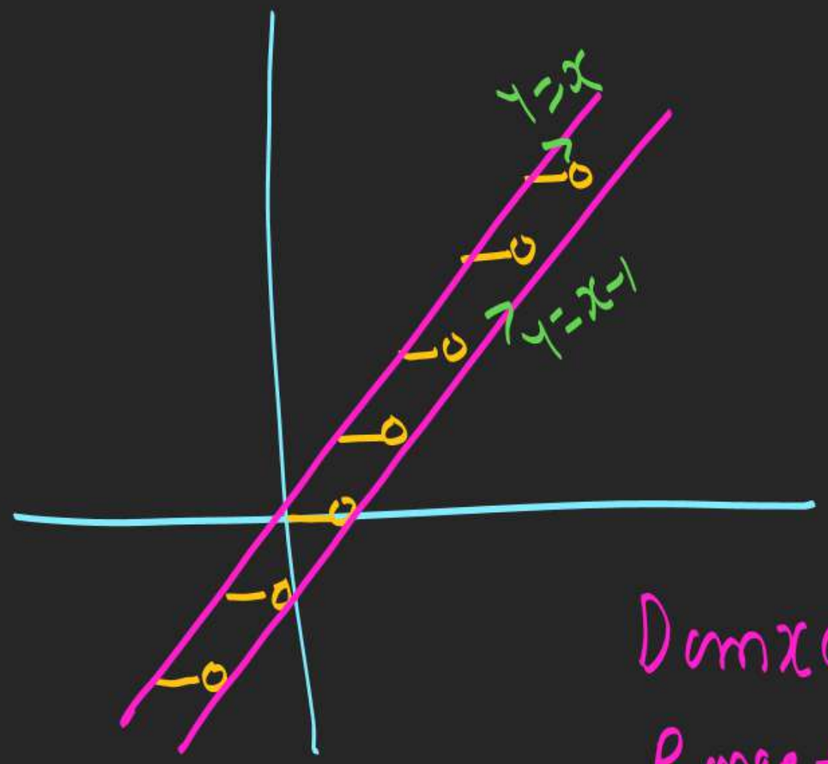
RELATION FUNCTION



Dom x ∈ R

Range ⇒ y ∈ {I}

$x \geq [x] > x-1$

$[x] \leq x$

$x - 1 < [x]$

$x < [x] + 1$

$[x] \leq x < [x] + 1$

Integer

Pichhla Int + 1

$0 \leq x < 1 \Rightarrow [x] = 0$

$2 \leq x < 3 \Rightarrow [x] = 2$

ULTA

$[x] = 2 \Rightarrow x \in [3, 3)$

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RELATION FUNCTION

$x \geq [x] > x-1$

① $x-[x] \geq 0$ (I-Hold!!)

② $[x] > x-1$

$1 > x-[x]$

$x-[x] < 1$

By ① & ②

$$1 > x-[x] \geq 0$$

Q Why $[x+n] = [x]+n$?

$[x-7] = [x]-7$ $\quad$ $[x+3] = [x]+3$

Kyu Ka Jubab

$[x] \leq x < [x]+1$

adding n $\quad [x]+n \leq x+n < [x]+n+1$

$$[x+n] = [x]+n$$

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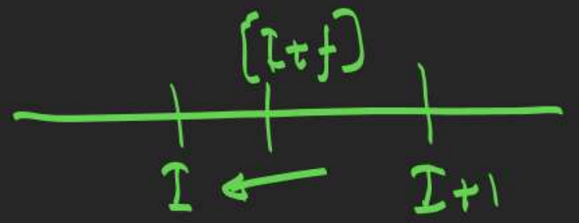
RELATION FUNCTION

$[4]+[-4]=0$

$[x]+[-x]=\begin{cases} 0 & x=I \\ -1 & x\neq I \end{cases}$

Why?

$[16]=16$, $[-16]=-16$



$.4=\frac{4}{10}$

Q.

Any No

$1.4=1+4$

$I+f$

$3.29=3+.29$

$I+f$

$16=16+0$

$I+f$

B.T. $=4+(-5)=-1$

$x=I+f$

$x=I$ or $f=0$

$LHS=[x]+[-x]$

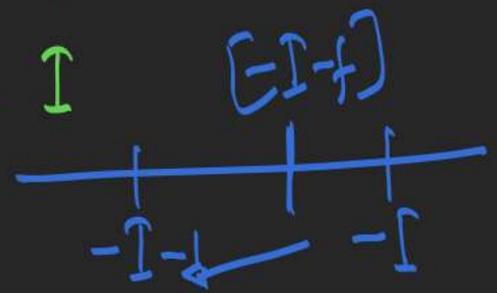
$=[I]+[-I]$

$=I+-I=0$

$x\neq I$ then $x=I+f$

$[x]=[I+f]=I$

$[-x]=[-I-f]$



$=-I-1$

Add

$[x]+[-x]=I+(-I-1)$

$=-1$

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RELATION FUNCTION

P.T.

Q $[x+y] = [x]+[y]$ OR $[x]+[y]+1$

Adv

Repeat

$I_1 \le x < I_1+1 \Rightarrow [x] = I_1$

$I_2 \le y < I_2+1 \Rightarrow [y] = I_2$

$I_1+I_2 \le x+y < I_1+I_2+2$

Integer $[x+y] = I_1+I_2$

$[x+y] = [x]+[y]$

Agr Q(+y) yahan hai to ??

$[x+y] = I_1+I_2+1$

$[x+y] = [x]+[y]+1$

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RELATION FUNCTION

Properties

① $\lceil I \rceil = I$

② $\lceil x+n \rceil = \lceil x \rceil + n$

③ $\lceil x \rceil + \lceil -x \rceil = \begin{cases} 0 & x = I \\ -1 & x \neq I \end{cases}$

④ If $\lceil x \rceil = n$ then $x \in [n, n+1)$

$\lceil x \rceil = 5 \rightarrow x \in [5, 6)$

$\lceil x \rceil = 0 \rightarrow x \in [0, 1)$

$\lceil x \rceil = -1$ then $x \in [-1, -1+1)$

$\in [-1, 0)$

Q If $\lceil x + \lceil x + \lceil x + \lceil x \rceil \rceil \rceil = 8$, $\text{Int} \geq n$ then $x \in ?$

$\lceil x + \lceil x + \lceil x \rceil + \lceil x \rceil \rceil = 8$

$\lceil x + \lceil x \rceil + \lceil x \rceil + \lceil x \rceil = 8$

$\lceil x \rceil + \lceil x \rceil + \lceil x \rceil + \lceil x \rceil = 8$

$4\lceil x \rceil = 8 \Rightarrow \lceil x \rceil = 2$

$x \in [2, 3)$

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RELATION FUNCTION

Q Eqn $x^2-12x+35=[x]+[-x]$ has

how many Sol. ?

IgR to 2 Answer 0, -1 hote

$x=I$

$x^2-12x+35=0$

$(x-7)(x-5)=0$

$x=5,7$

2 Sol.

$x \neq I$

$x^2-12x+35=-1$

$x^2-12x+36=0$

$(x-6)^2=0$

$x-6=0$

No Sol. $x=I$ ⊗ x to Int.

Agya

hi BD

Ans = 2 Sol.

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RELATION FUNCTION

Sum = 21+36+55+78

$[I]=I$ $[7]=7$

Q Sum of all PSBL values of n when $n \in N$ & $10 < n \leq 100$ such that

Adv & Mains

Eqn $[2x^2]+x-n=0$ has a sol., equal to ?

$n = x(2x+1)$

$x=2$

$n = 2x(5) = 10$ ✗

$x=3$

$n = 3(7) = 21$ ✓

$x=4$

$n = 4(9) = 36$ ✓

$x=5$

$n = 5(11) = 55$ ✓

$x=6$

$n = 6(13) = 78$ ✓

Common Sense

① $\underbrace{[2x^2]}_{\text{Int हो}} + x = \underbrace{n}_{\text{Int हो}}$ → Int + Something = Int.

⇒ Something has to be Int

x Ko Int. होना पड़ेगा!

② If $x = \text{Int} \Rightarrow x^2 = \text{Int} \Rightarrow 2x^2 = \text{Int}$ & $[2x^2] = 2x^2$

③ $2x^2 + x = n$ ⇒ $n = x(2x+1)$

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RELATION FUNCTION

Q. $[x]^2 - 3[x] + 2 = 0$ find x?

$\Rightarrow ([x]-1)([x]-2) = 0$

$\Rightarrow [x]-1=0$ OR $[x]-2=0$

$[x]=1$ OR $[x]=2$

$x \in [1,2) \cup x \in [2,3)$

$x \in [1,3)$

Next Prop → Prop 5

here I will make Qs to Explain Prop.

Q $1 \le [x] \le 5$ find x ∈ ?

Jab b [x] Ko Le Kar Inequality Aaye then Pronounce [x] = ALSA INTER.

$1 \le [x] \le 5$ → Aisa Int. Jo 1 Se 5 tak ho.

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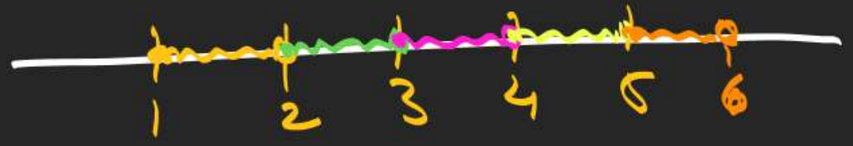
RELATION FUNCTION

$1 \le [x] \le 5$



$1 \le x < 6$

Q If $1 \le [x] < 5$ then $x \in ?$

$1 \le x < 5$

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RELATION FUNCTION

Q $1 < [x] < 5$

$\Rightarrow 2 \leq x < 5$

Q If $-2 \leq [x] < 7$ then $x \in ?$

$-2 \leq x < 7$

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RELATION FUNCTION

Q If $[x] \geq 2$ then x ∈ ?

Aisa Int Jo 2 Aur 2 se bada ho.

$\Rightarrow x \geq 2$

Q If $[x] > -4$ then x ∈ ?

Aisa Int Jo -4 se Bada ho.

$x \geq -3$.

$x \in [-3, \infty)$

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RELATION FUNCTION

Q If $[x] \le -5$ then $x \in$?

$x < -4$

$x \in (-\infty, -4)$

Q find Dom of $f(x) = \frac{1}{\sqrt{[x]^2 - 3[x] + 2}}$

$[x]^2 - 3[x] + 2 > 0$

$([x] - 1)([x] - 2) > 0$

2 Br. BHALA

$[x] < 1 \cup [x] > 2$

Aisa Int. $\nexists$ R

$\cup$

$x \in (-\infty, 1) \cup [3, \infty)$

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RELATION FUNCTION

$[x] = n \Rightarrow x \in [n, n+1)$

Prop 6

$$[x] + \left[x + \frac{1}{n}\right] + \left[x + \frac{2}{n}\right] + \left[x + \frac{3}{n}\right] + \cdots + \left[x + \frac{n-1}{n}\right] = [nx]$$

Q $\left[\frac{1}{4}\right] + \left[\frac{1}{4} + \frac{1}{200}\right] + \left[\frac{1}{4} + \frac{2}{200}\right] + \left[\frac{1}{4} + \frac{3}{200}\right] + \cdots + \left[\frac{1}{4} + \frac{199}{200}\right] = ?$ $n \in \mathbb{N}$

$$\text{Ans} = [nx] = \left[200 \times \frac{1}{4}\right] = [50] = 50$$

Q If $[x] + \left[x + \frac{1}{100}\right] + \left[x + \frac{2}{100}\right] + \cdots + \left[x + \frac{99}{100}\right] = 7$ find $x \in ?$

$$\text{Ans} = [nx] = [100x] = 7$$

$$100x \in [7, 8)$$

$$x \in \left[\frac{7}{100}, \frac{8}{100}\right)$$

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RELATION FUNCTION

Q If A, B, C are 3 Decimal No (fraction No) & $\alpha = [A+B+C]_{Max}$ & $\beta = [A]+[B]+[C]$

find Q Eqn having Roots α & β?

$0 \le A < 1 \Rightarrow [A] = 0$

$0 \le B < 1 \Rightarrow [B] = 0$

$0 \le C < 1 \Rightarrow [C] = 0$

$[A]+[B]+[C] = 0 = \beta$

$0 \le A+B+C < 3$

$A+B+C_{Max} = 2.99999$

$\alpha = [A+B+C]_{Max} = 2$

If α, β are Roots of Q Eqn.

Q Eqn $\Rightarrow x^2 - (\alpha+\beta)x + \alpha\beta = 0$

$x^2 - (0+2)x + 0 \times 2 = 0$

$x^2 - 2x = 0$

Ans