

THERMAL EXPANSIONQ.4Case of bimetallic strip

$\alpha_1 < \alpha_2$



$\alpha_1 > \alpha_2$



THERMAL EXPANSION

Q.4

Case of bimetallic strip

$$\alpha_1 > \alpha_2$$



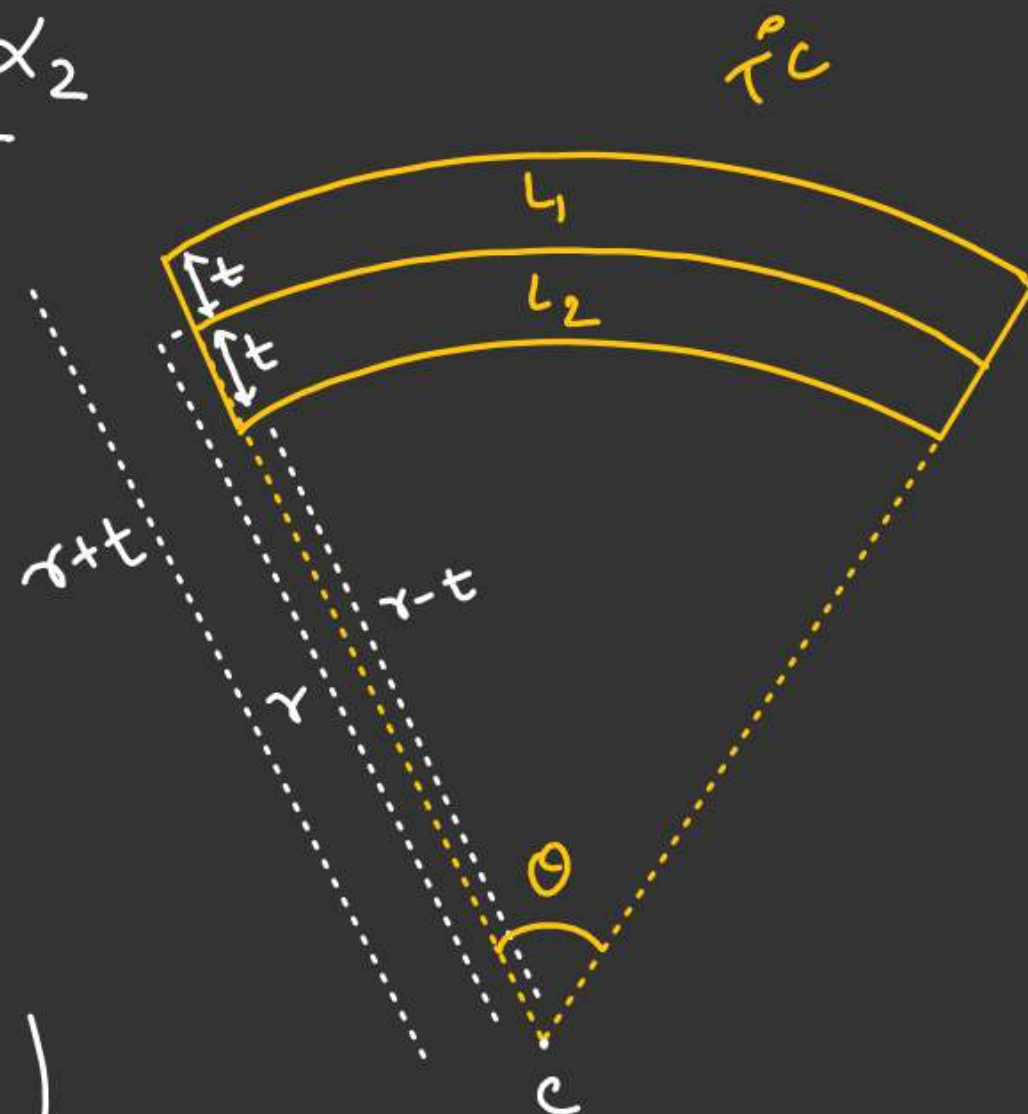
$$\Delta T = (T - T_0)$$

 r = Mean radius

$$L_1 = (r + t)\theta$$

$$L_2 = (r - t)\theta$$

$$\frac{L_1}{L_2} = \left(\frac{r+t}{r-t} \right) \Rightarrow \frac{L(1+\alpha_1\Delta T)}{L(1+\alpha_2\Delta T)} = \left(\frac{r+t}{r-t} \right)$$

 \Rightarrow 

$$\frac{(1 + \alpha_1 \Delta T)}{(1 + \alpha_2 \Delta T)} = \frac{r + t}{r - t}$$

$$\frac{(1 + \alpha_1 \Delta T) + (1 + \alpha_2 \Delta T)}{(1 + \alpha_1 \Delta T) - (1 + \alpha_2 \Delta T)} = \frac{(r + t) + (r - t)}{(r + t) - (r - t)}$$

$$\Rightarrow \frac{2 + (\alpha_1 + \alpha_2) \Delta T}{(\alpha_1 - \alpha_2) \Delta T} = \frac{2r}{2t}$$

$$\Rightarrow r = \frac{t [2 + (\alpha_1 + \alpha_2) \Delta T]}{(\alpha_1 - \alpha_2) \Delta T}$$

$$\rightarrow r = \frac{2t \left[1 + \frac{(\alpha_1 + \alpha_2) \Delta T}{2} \right]}{(\alpha_1 - \alpha_2) \Delta T}$$

$\gg (\alpha_1 + \alpha_2) \Delta T$

$$r = \frac{2t}{(\alpha_1 - \alpha_2) \Delta T}$$

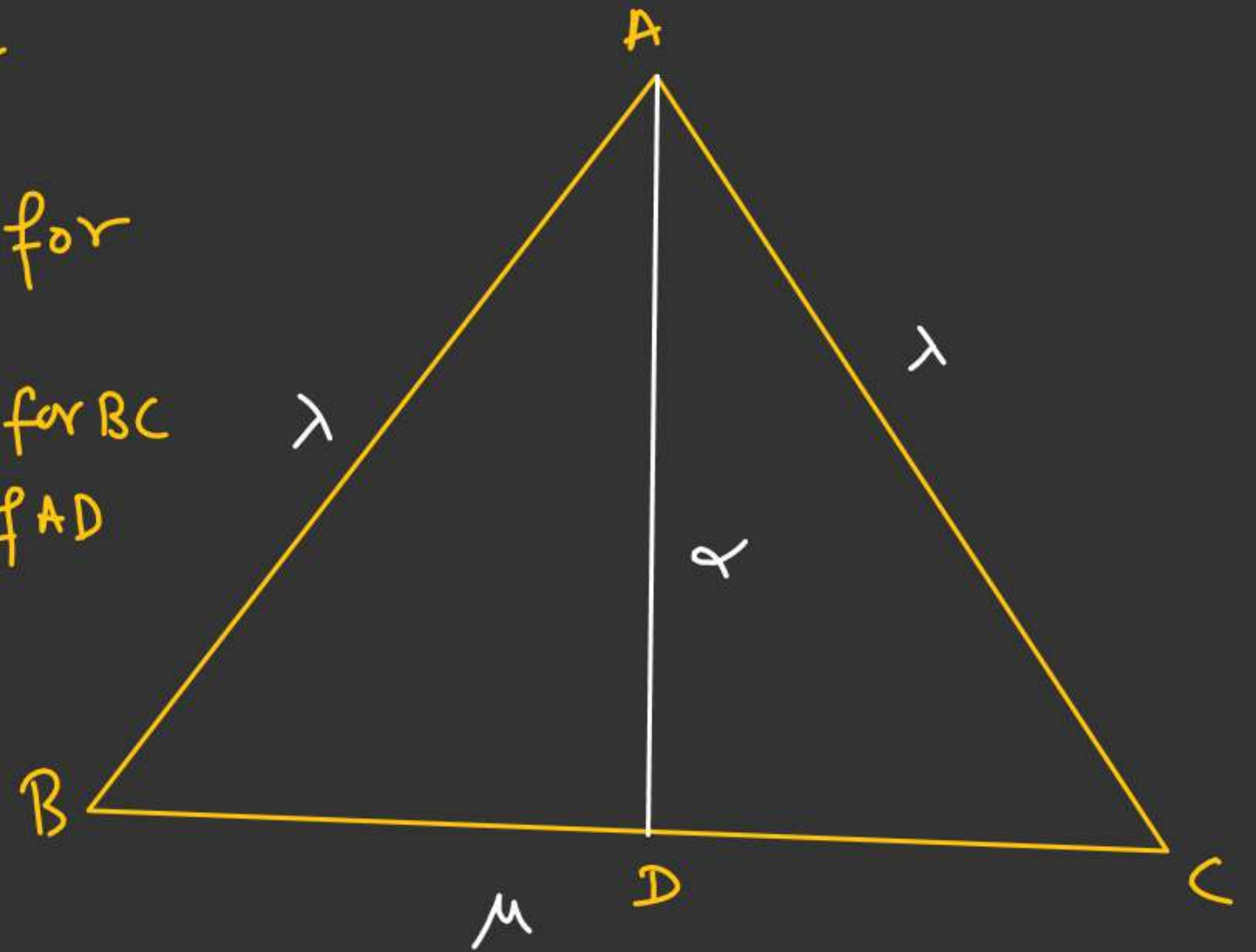
ABC equilateral triangle form by
3-rods AB, BC & CA

λ is the coeffⁿ of linear expansion for
AB and AC

μ be the coeffⁿ of linear expansion for BC

α be the coeffⁿ of linear expansion of AD

Find α so that the frame will
not deformed if heated from
 T_0° to T°



For AB.

$$L' = L(1 + \lambda \Delta T)$$

$$1 \gg \alpha \Delta T \text{ or } \mu \Delta T \text{ or } \lambda \Delta T$$

For BD.

$$\frac{L'}{2} = \frac{L}{2}(1 + \mu \Delta T)$$

$$1 \gg \alpha \Delta T$$

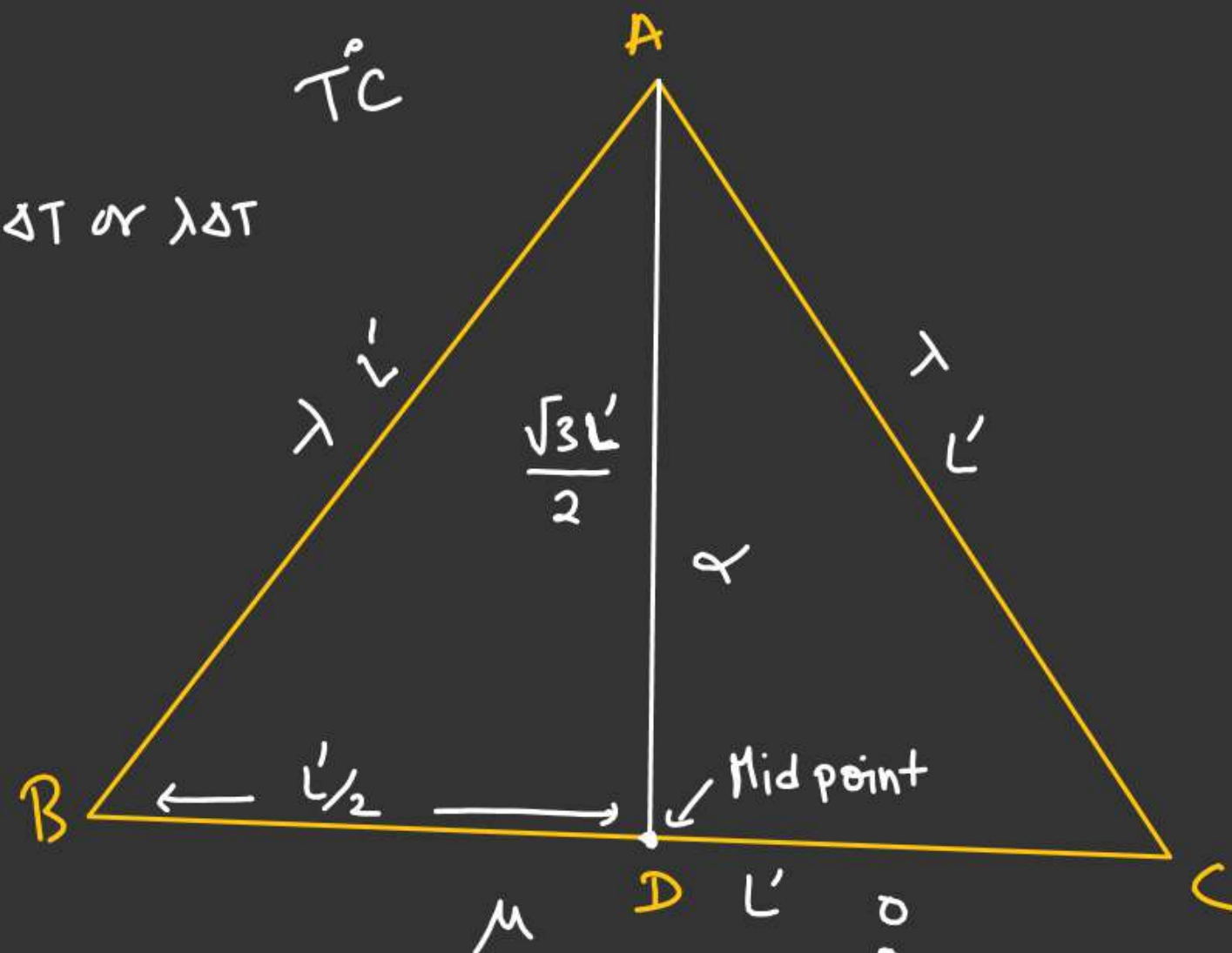
For AD

$$\frac{\sqrt{3} L'}{2} = \frac{\sqrt{3} L}{2}(1 + \alpha \Delta T)$$

$$L'^2 = \left(\frac{L'}{2}\right)^2 + \left(\frac{\sqrt{3} L'}{2}\right)^2$$

$$L'^2(1 + \lambda \Delta T)^2 = \frac{1}{4}(1 + \mu \Delta T)^2 + \frac{3}{4}(1 + \alpha \Delta T)^2$$

$$(1 + \lambda \Delta T)^2 = \frac{1}{4}(1 + \mu \Delta T)^2 + \frac{3}{4}(1 + \alpha \Delta T)^2$$



$$1 + \lambda^2 \Delta T^2 + 2\lambda \Delta T = \frac{1}{4}(1 + \mu^2 \Delta T^2 + 2\mu \Delta T) + \frac{3}{4}(1 + \alpha^2 \Delta T^2 + 2\alpha \Delta T)$$

$$1 + 2\lambda \Delta T = \frac{1}{4} + \frac{\mu}{2} \Delta T + \frac{3}{4} + \frac{3}{2} \alpha \Delta T$$

$$2\lambda - \frac{\mu}{2} = \frac{3}{2} \alpha \Rightarrow \alpha = \frac{4\lambda}{3} - \frac{1}{3} \mu$$

Thermal Expansion in liquid.

Case when only expansion of liquid not vessel

$$\rho = \frac{m}{V}$$

$$(\rho \propto \frac{1}{V})$$

$$\underline{m = c}$$



$$1 \gg \gamma_l \Delta T$$

$$\rho = \rho_0 (1 + \gamma_l \Delta T)^{-1}$$

$$\boxed{\rho = \rho_0 (1 - \gamma_l \Delta T)} \quad \checkmark$$

$\rho_0 \rightarrow$ Density of liquid at T_0^C

$\rho \rightarrow$ Density of liquid at T^C

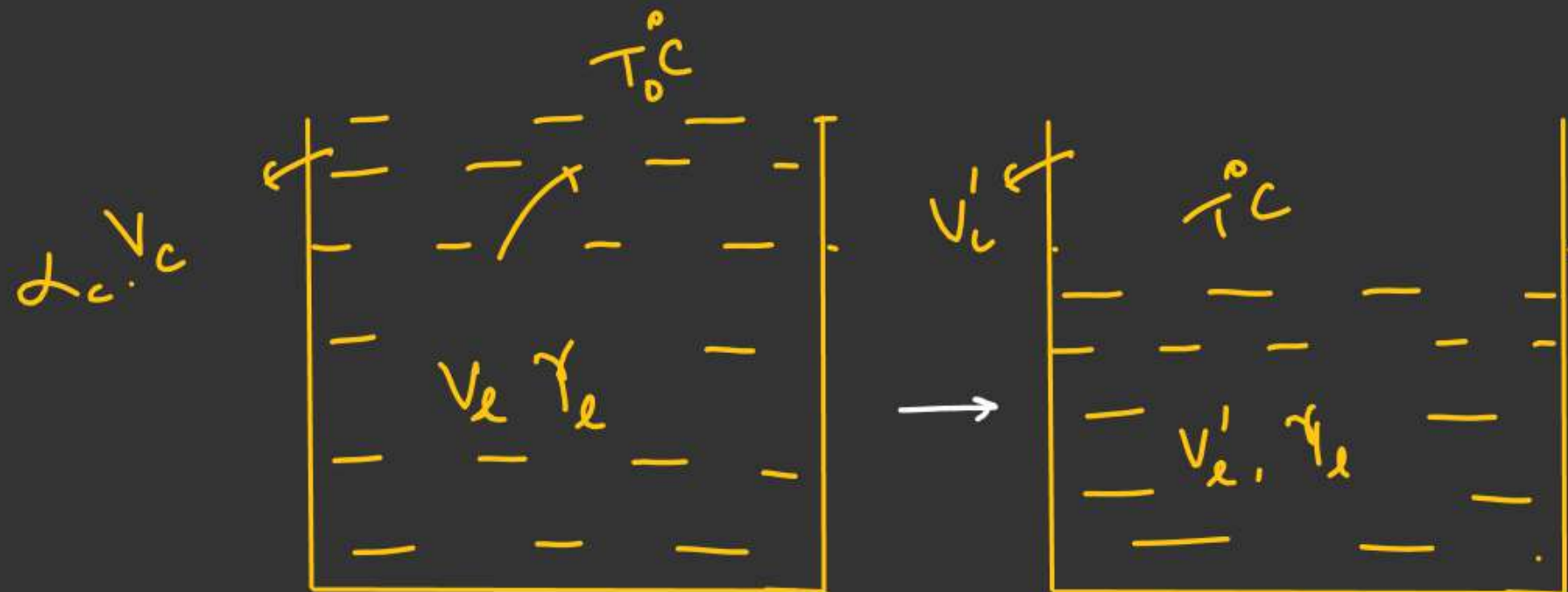
$$V = V_0 (1 + \gamma_l \Delta T)$$

$$\frac{\rho}{\rho_0} = \frac{V_0}{V}$$

$$\frac{\rho}{\rho_0} = \frac{V_0}{V_0 (1 + \gamma_l \Delta T)}$$

$$\boxed{\rho = \frac{\rho_0}{(1 + \gamma_l \Delta T)}}$$

At $T^\circ C$



At $T_0^o C$

- ✓ V_l = Volume of liquid
- ✓ V_c = Volume of container

At $T^o C$

- V'_l = Volume of liquid
- V'_c = Volume of vessel

$\gamma_l > 3\alpha_c \Rightarrow$ level of liquid increase
 $\gamma_l < 3\alpha_c \Rightarrow$ level of liquid decrease.

For liquid

$$V'_l = V_l (1 + \gamma_l \Delta T)$$

$$V'_c = V_c (1 + \gamma_c \Delta T)$$

$$\gamma_c = 3\alpha_c$$

$$V'_c = V_c (1 + 3\alpha_c \Delta T)$$

$$(\gamma = 3\alpha)$$

$$(V_l = V_c = V_0)$$

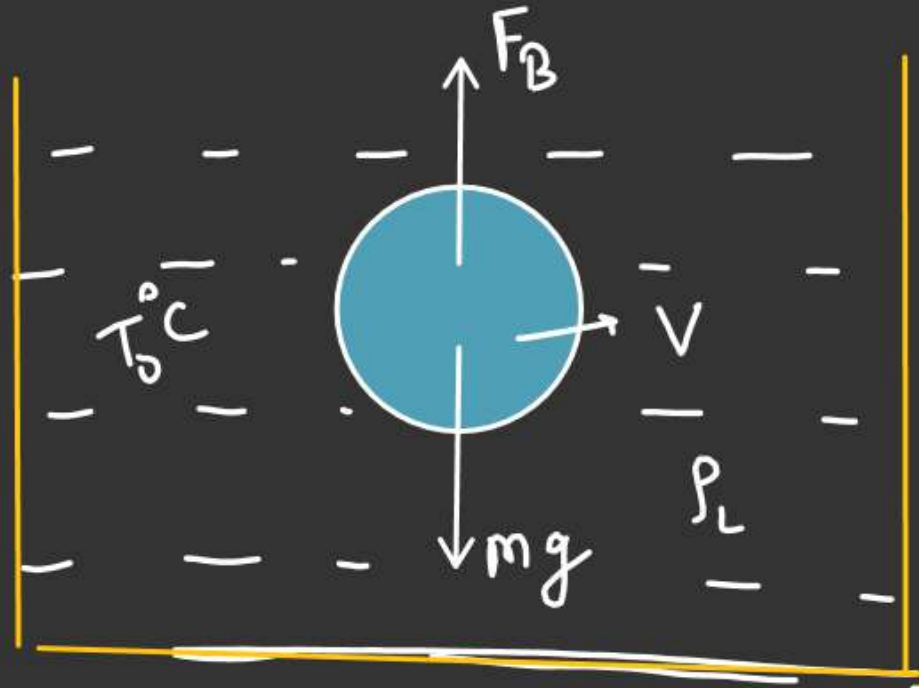
$$\Delta V_l = V'_l - V'_c$$

$$= V_l (1 + \gamma_l \Delta T) - V_c (1 + 3\alpha_c \Delta T)$$

$$\Delta V_l = V_0 (\gamma_l - 3\alpha_c) \Delta T$$

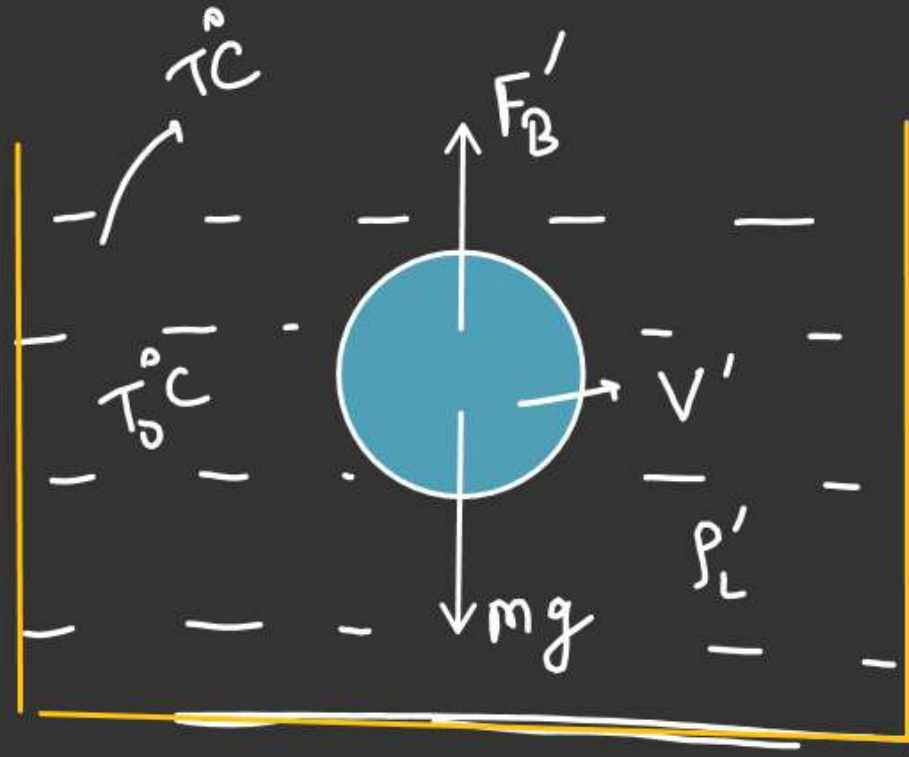
Effect of Temperature on the Apparent weight of the body fully submerged

No expansion of vessel



$$W_{app} = (mg - F_B)$$

$$W_{app} = mg - V \rho_L g$$



$$W'_{app} = mg - F'_B = mg - V' \rho'_L g$$

$$W'_{app} - W_{app} = V \rho_L g - V' \rho'_L g$$

$$F_B = \text{Weight of displaced liquid} \\ = V_L \rho_L g$$

$$V_L = V_s$$

$V_s = \text{Volume of submerged part of body}$

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$$W'_{app} - W_{app} = V\rho_L g - V'\rho'_L g$$

$$V' = V(1 + \gamma_s \Delta T)$$

$$\rho'_L = \frac{\rho_L}{(1 + \gamma_L \Delta T)} = \rho_L (1 - \gamma_L \Delta T)$$

$$\Delta W_{app} = V\rho_L g - [V(1 + \gamma_s \Delta T) \rho_L (1 - \gamma_L \Delta T) g]$$

$$\begin{aligned} \Delta W_{app} &= V\rho_L g - V\rho_L g [(1 + \gamma_s \Delta T)(1 - \gamma_L \Delta T)] \\ &= V\rho_L g - V\rho_L g [1 - \gamma_L \Delta T + \gamma_s \Delta T - \gamma_s \gamma_L \Delta T^2] \\ &= V\rho_L g [1 - (1 + (\gamma_s - \gamma_L) \Delta T)] \end{aligned}$$

$$\Delta W_{app} = \underline{V\rho_L g [\gamma_L - \gamma_s] \Delta T}$$

$$\textcircled{1} \quad \Delta W_{app} > 0 \Rightarrow \gamma_L > \gamma_s$$

$$\textcircled{2} \quad \Delta W_{app} < 0 \Rightarrow (W_{app})_f < (W_{app})_i \Rightarrow \gamma_s > \gamma_L$$

$$\textcircled{3} \quad \text{if } \gamma_s = \gamma_L$$

$$3\alpha_s = \gamma_L$$

No Change in apparent weight.

SCALE ERROR

$$l_a = l_o [1 + \alpha(T - T_o)]$$

l_a = Actual length

l_o = Observed or Measured length

T_o = Temperature at which Scale gives correct reading

T = Temperature at which Observation is made