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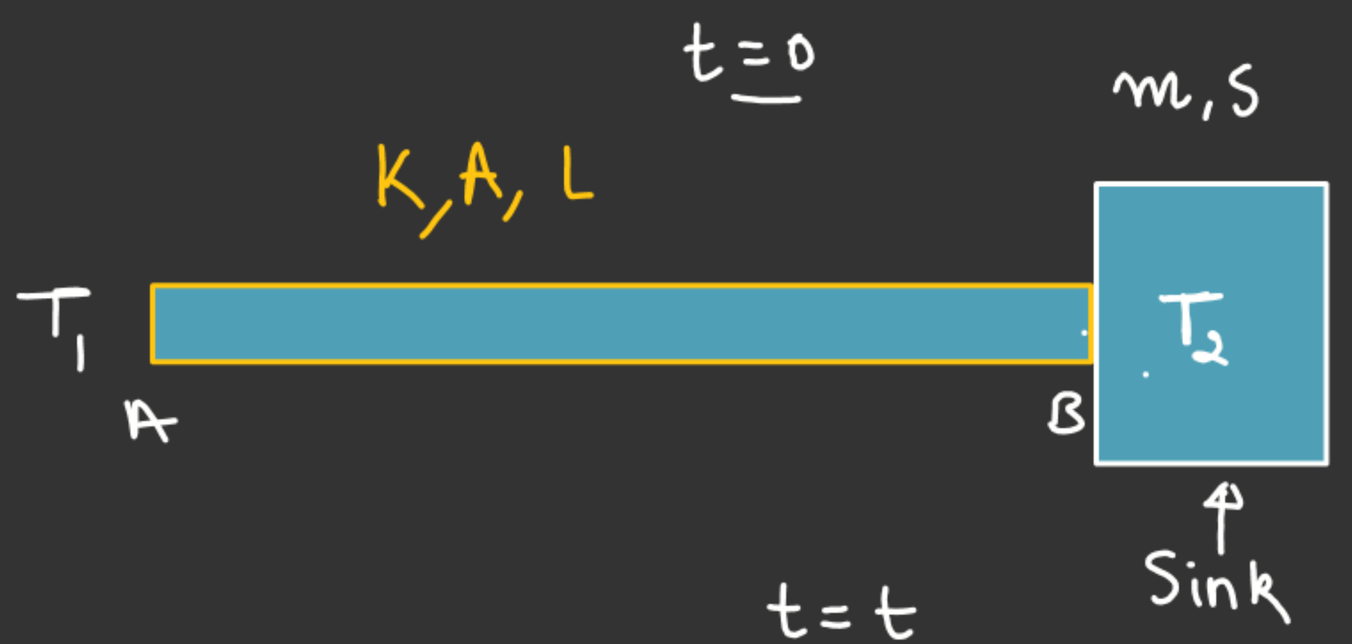
Heat flow in a Sink

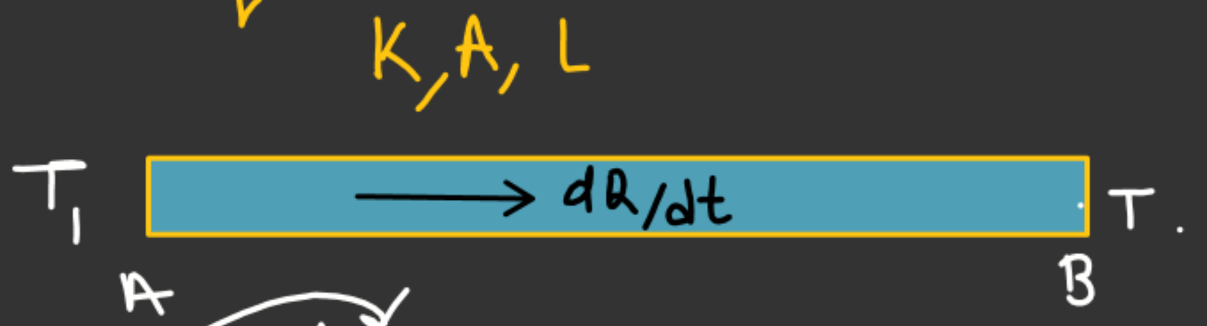
$T_1 > T_2$

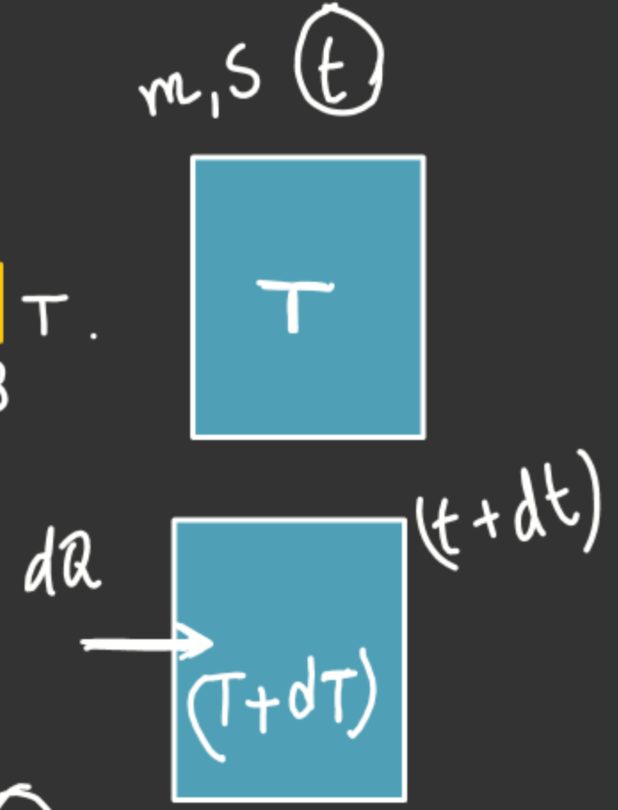


At $t=0$, T_1 & T_2 be the temp of end A & B.

let, at any time 't' temp of end B or Sink be T.

Equation of Rod

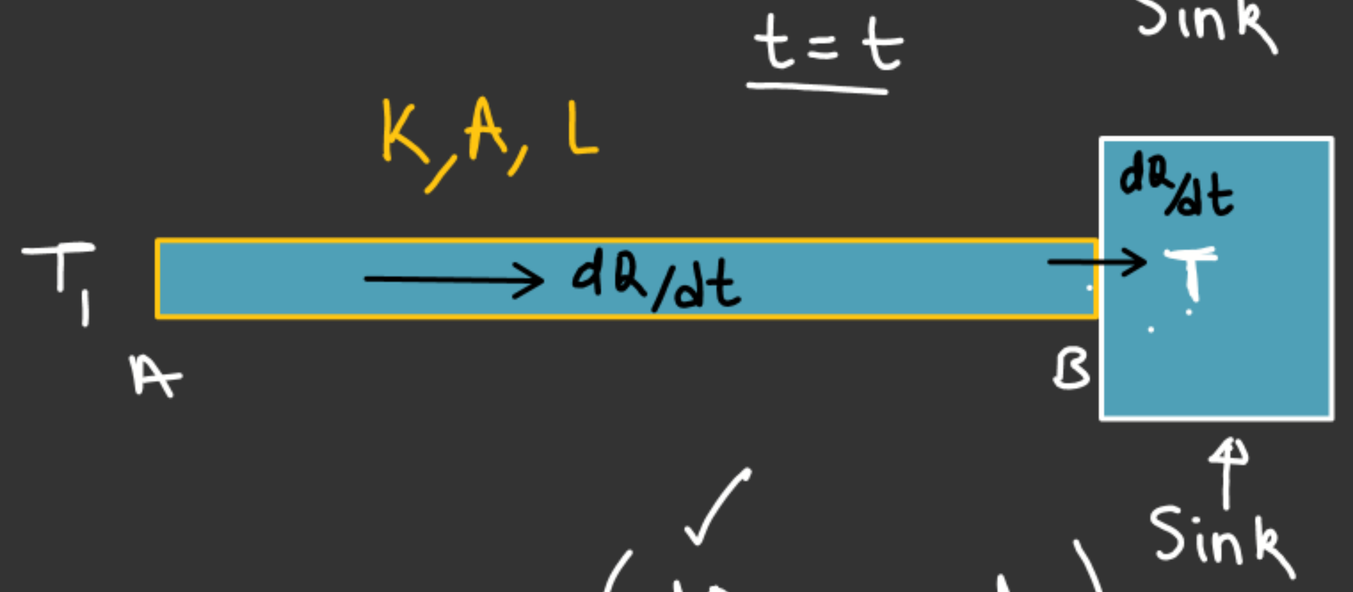




$$\frac{dQ}{dt} = \frac{KA}{L}(T_1 - T) \quad \text{—(1)}$$

Equation of Sink

$$dQ = (msdT) \quad \text{—(2)}$$



$(dQ = msdT)$

s → specific heat

ms → heat capacity

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$$\frac{dQ}{dt} = \frac{KA}{L}(T_1 - T) \quad -(1)$$

Equation of Sink

$$dQ = (ms\,dT) \quad -(2)$$

From (1) & (2)

$$ms\frac{dT}{dt} = \frac{KA}{L}(T_1 - T)$$

$$\int_{T_2}^{T} \frac{dT}{T_1 - T} = \frac{KA}{msL}\int_0^t dt$$

$$\frac{\ln[T_1 - T]_{T_2}^{T}}{(-1)} = \frac{KA}{msL}t$$

$$\ln(T_1 - T) - \ln(T_1 - T_2) = \frac{-KA}{msL}t$$

$$\ln\left(\frac{T_1 - T}{T_1 - T_2}\right) = \frac{-KA}{msL}t$$

$$T_1 - T = (T_1 - T_2)e^{-\frac{KA}{msL}t}$$

$$\boxed{T = T_1 - (T_1 - T_2)e^{-\frac{KA}{msL}t}}$$

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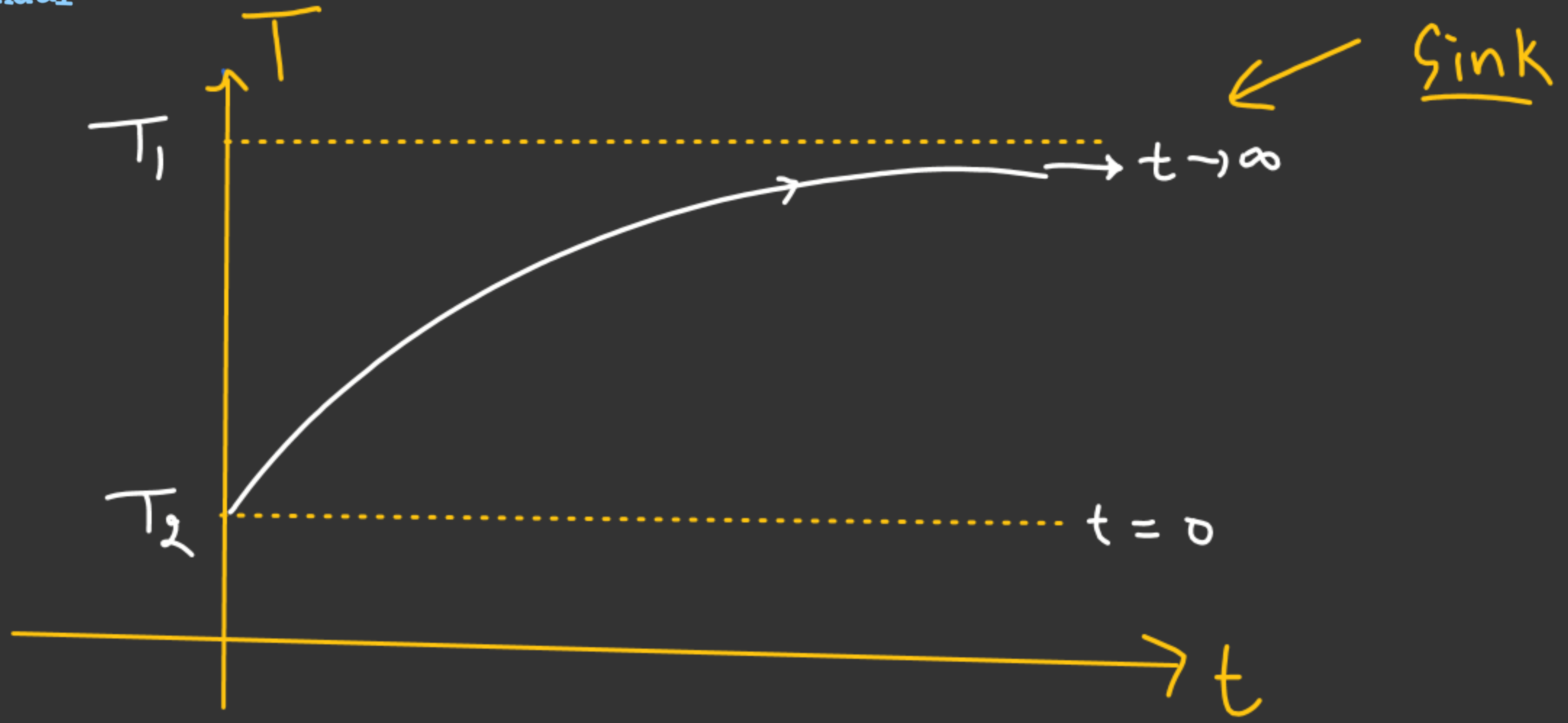

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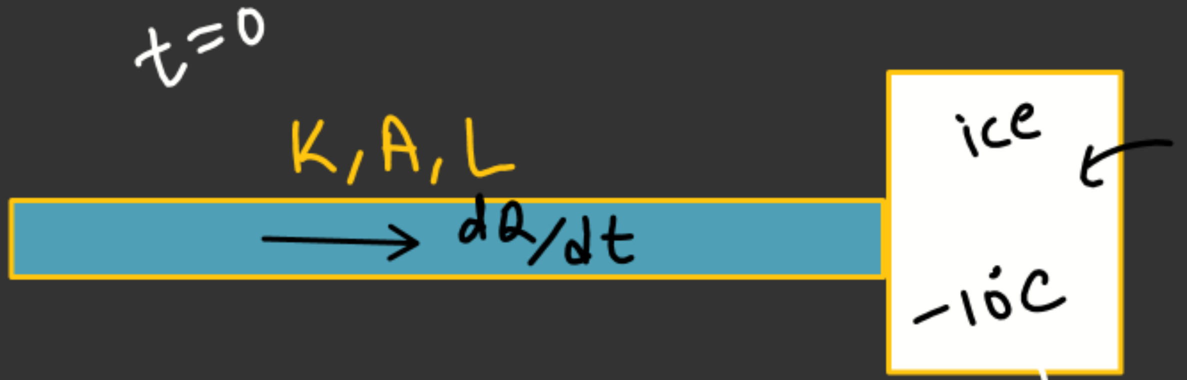
H.W. Find time 't' to melt the ice Cube.



(Maintain Constant)

$m, L_f \rightarrow$ Latent heat of fusion

Temp of Sink as a function of time

Temp of ice as a function of time

$$T = T_1 - (T_1 - T_2)\, e^{-\frac{KA}{msL}t}$$

$$e^a = b$$

$$a = \ln b$$

$T_1 = 100°C$

$T_2 = -10°C$

$T \rightarrow 0°C$

$t \rightarrow t_1$

$$0 = 100 - 110\, e^{-\frac{KA}{msL}t_1}$$

$$110\, e^{-\frac{KA}{msL}t_1} = 100$$

$$-\frac{KA}{msL}t_1 = \ln\left(\frac{100}{110}\right)$$

$$\left(t_1 = \frac{msL}{KA}\ln\left(\frac{11}{10}\right)\right) \checkmark$$

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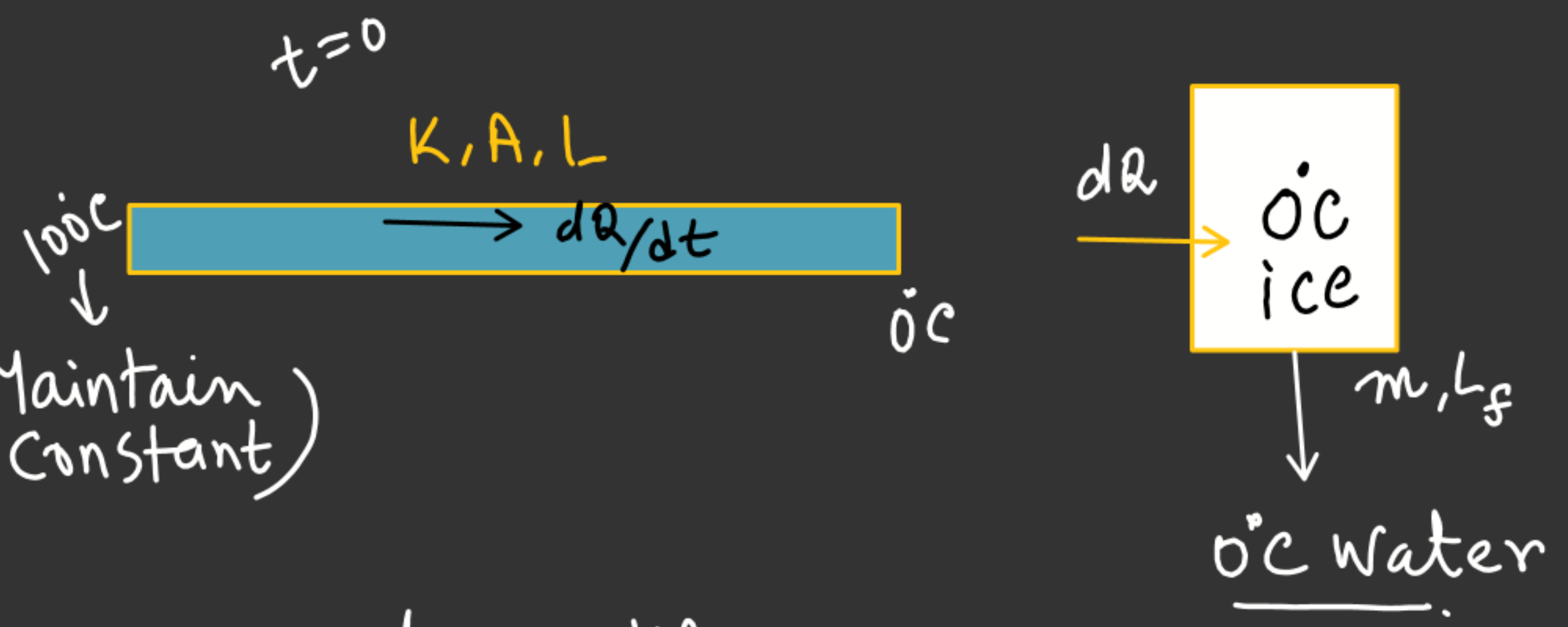
H.W



Equation of Rod

$$\frac{dQ}{dt} = \frac{KA}{L}(100-0) \quad -①$$

For ice cube

$$Q = mL_f$$

$$\frac{dQ}{dt} = L_f\left(\frac{dm}{dt}\right) \quad -②$$

$$L_f \frac{dm}{dt} = \frac{KA}{L} \times 100$$

$$\int_0^m dm = \frac{100KA}{LL_f}\int_0^{t_2} dt$$

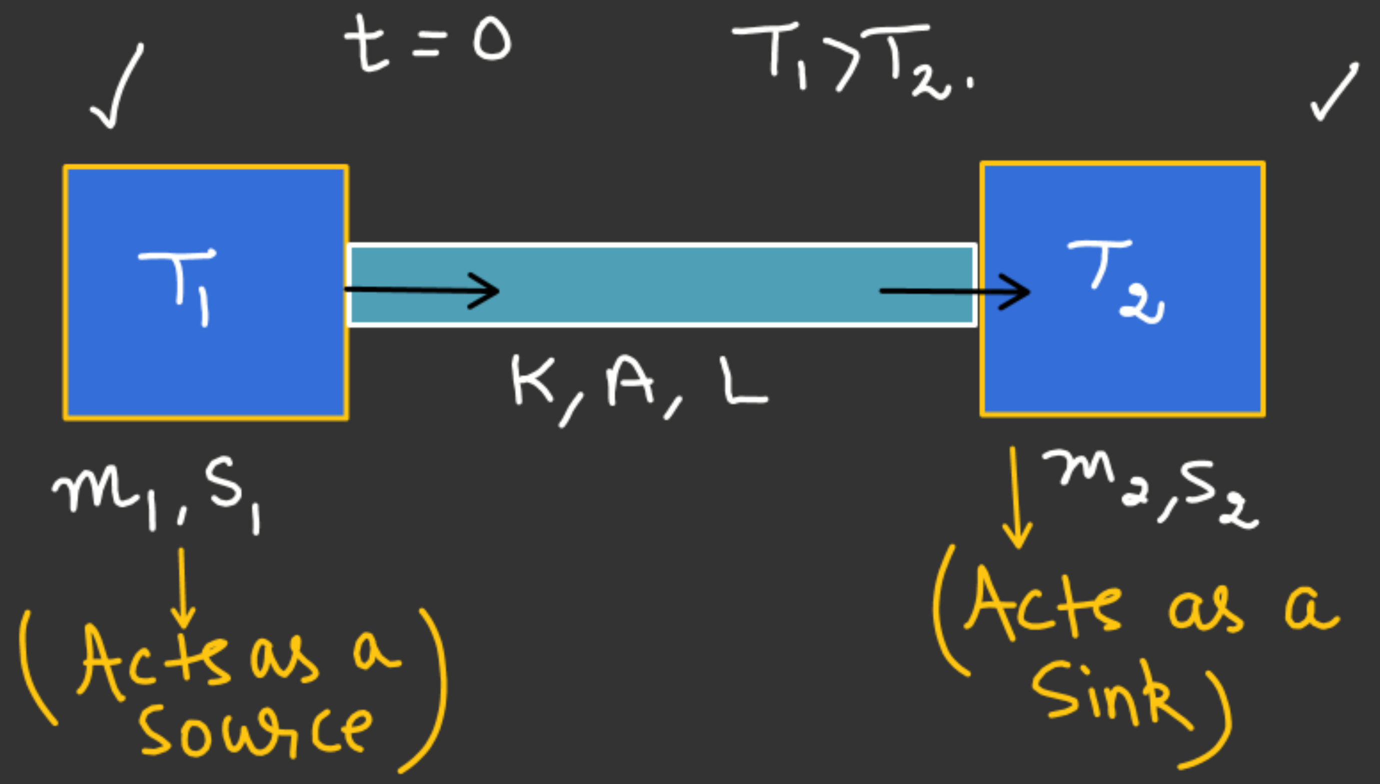
$$m = \frac{100KA}{LL_f} \times t_2$$

$$t_2 = \left(\frac{mLL_f}{100KA}\right) \checkmark$$

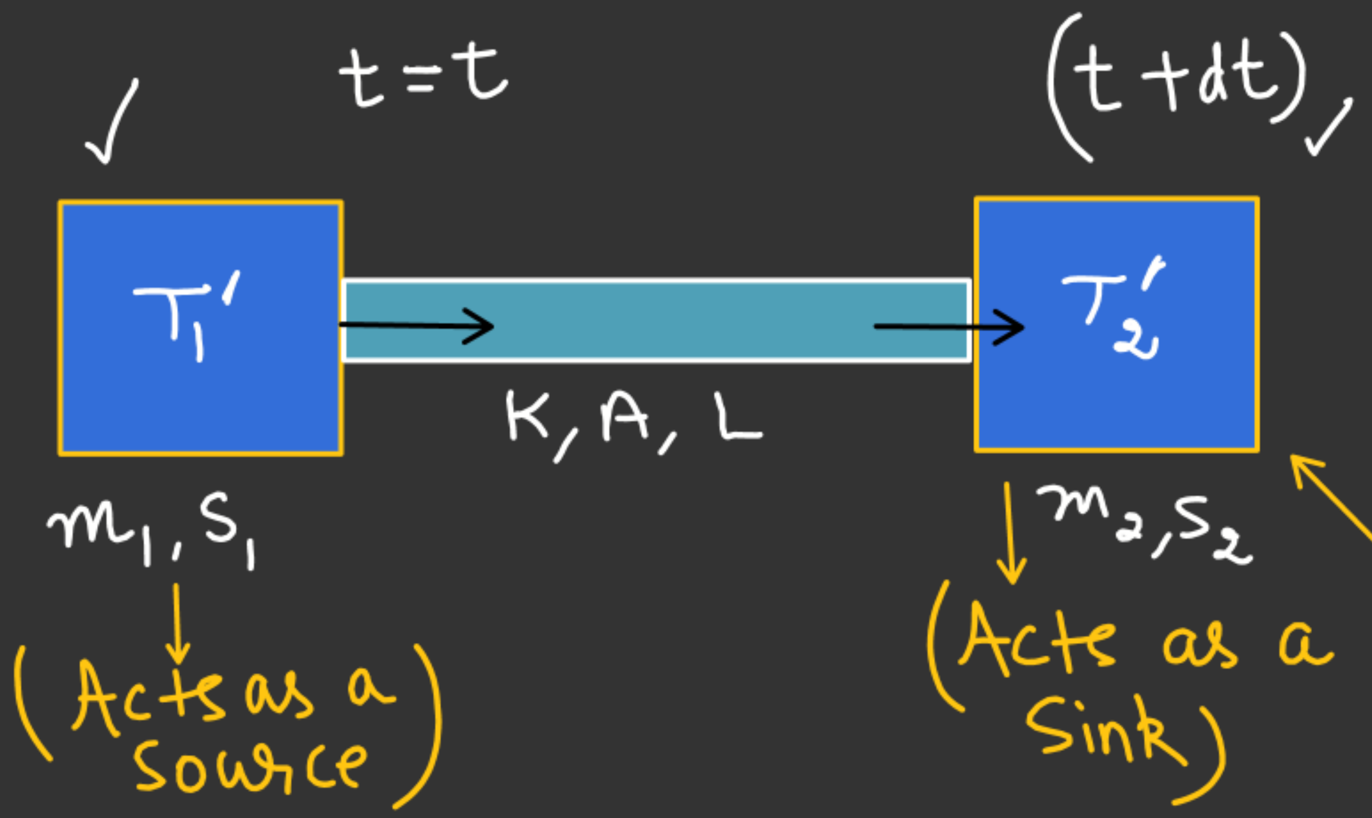
Total time

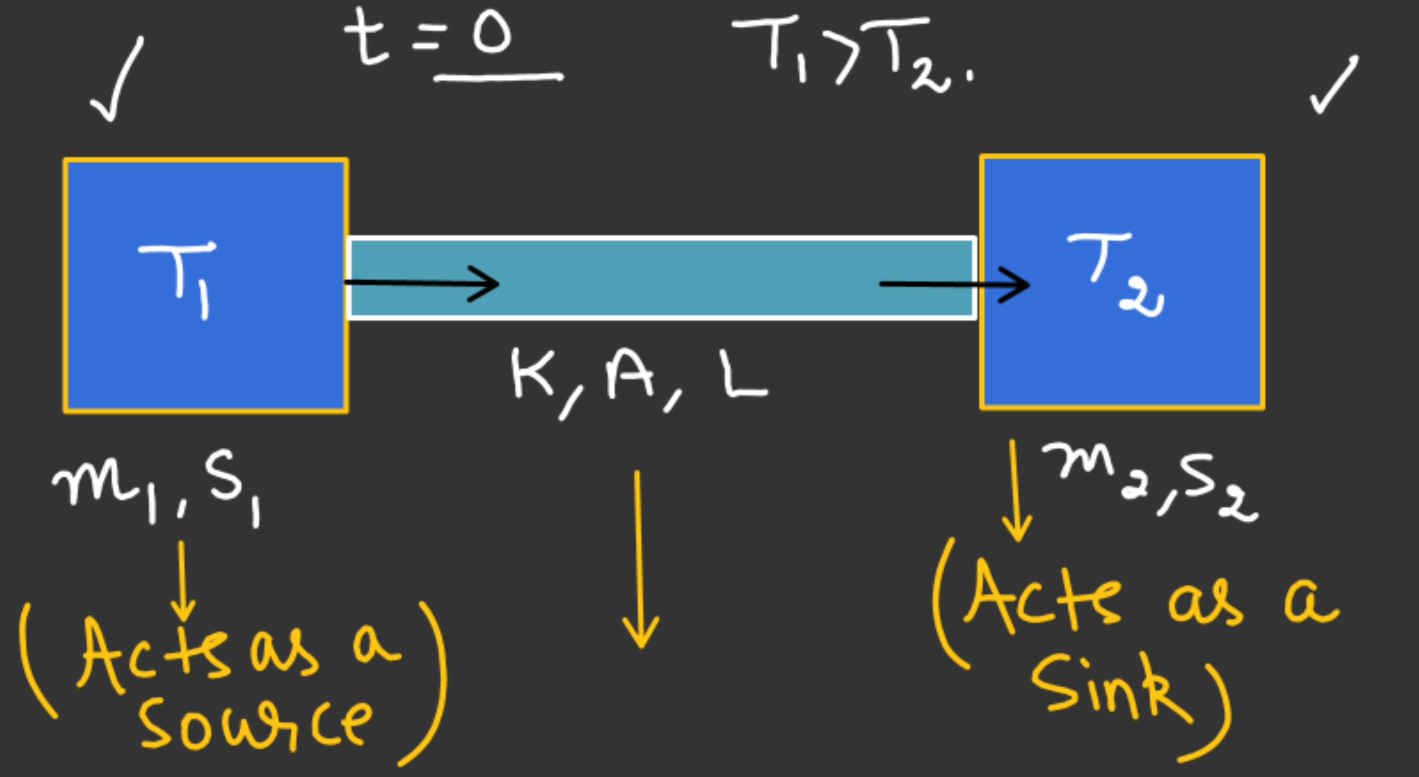
$$t = (t_1 + t_2)$$

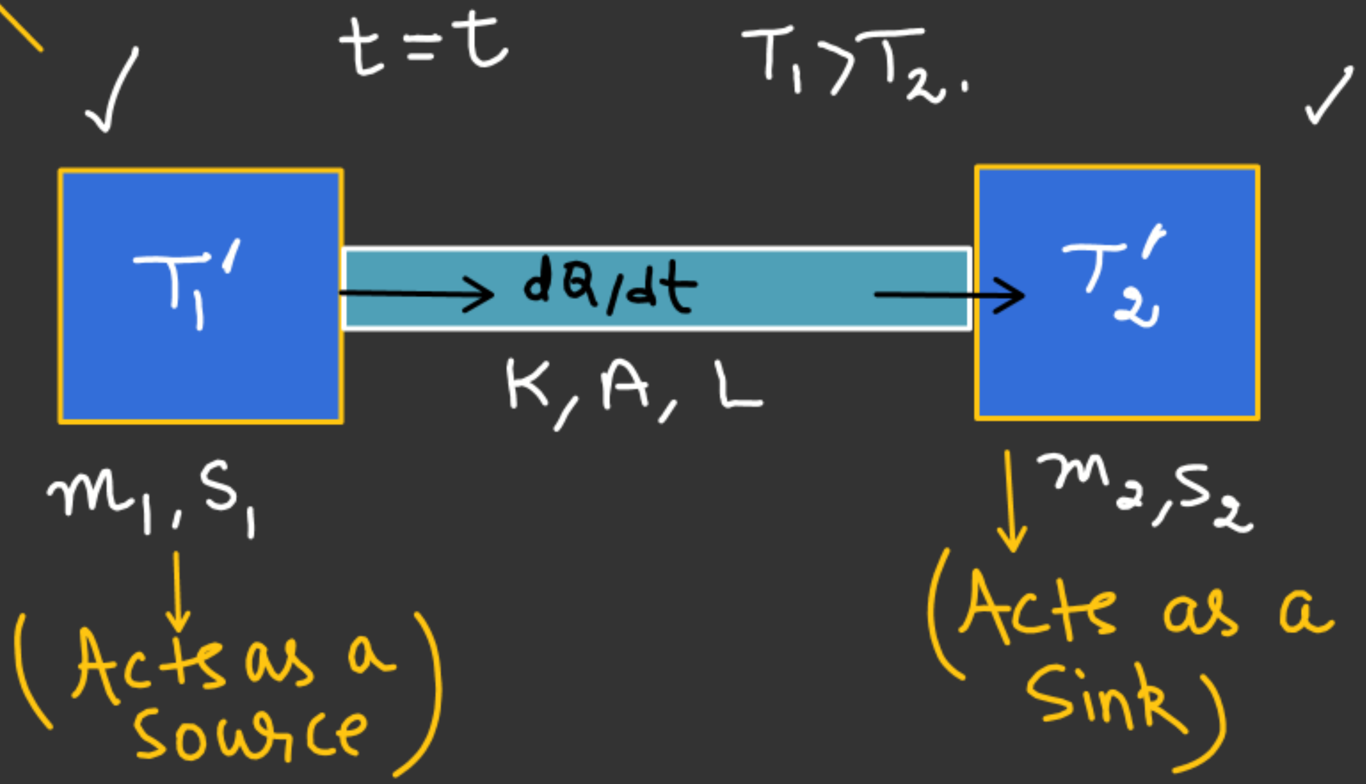
Temp difference of both the reservior as a function of time



Temp difference of both the reservior as a function of time







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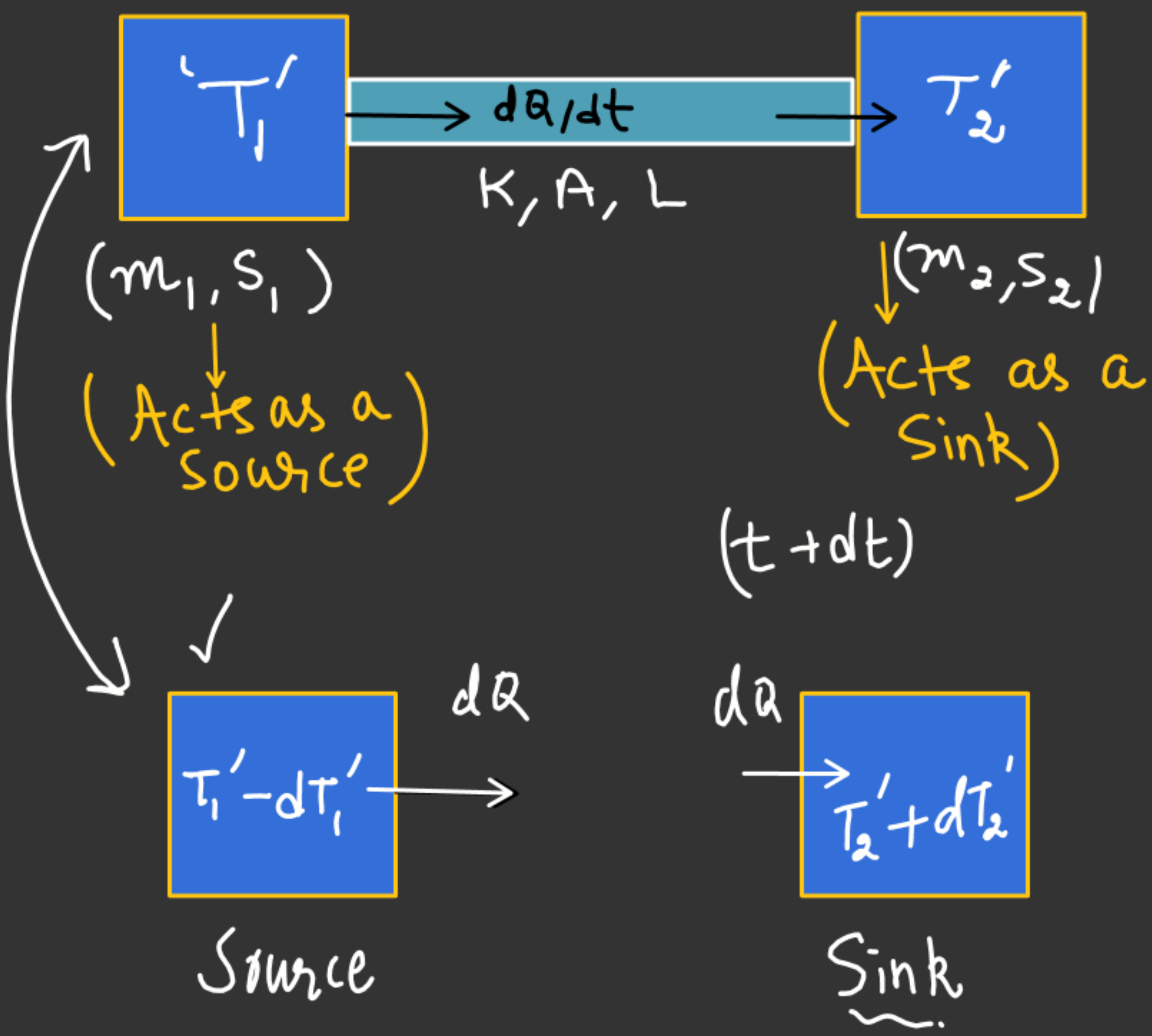
$t = t$ $\qquad T_1 > T_2.$



Equation of rod

$$\frac{dQ}{dt} = \frac{KA}{L}(T_1' - T_2') \quad — (1)$$

For Source

$$dQ = m_1 s_1 [(T_1' - dT_1') - T_1']$$

$$dQ = -m_1 s_1 dT_1' \quad — (2)$$

For Sink

$$dQ = m_2 s_2 (T_2' + dT_2' - T_2')$$

$$dQ = m_2 s_2 dT_2' \quad — (3)$$

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$$\frac{dQ}{dt} = \frac{KA}{L}(T_1' - T_2') \quad -① \longrightarrow dQ = \frac{KA}{L}(T_1' - T_2')\,dt \cdot \text{put in } ④$$

$$\underbrace{d(T_1' - T_2')}_{\Downarrow\ dx} = -\left(\frac{1}{m_1 s_1} + \frac{1}{m_2 s_2}\right)\frac{KA}{L}\underbrace{(T_1' - T_2')}_{x}\,dt$$

$$dQ = -m_1 s_1 \, dT_1' \quad — ②$$

$$(T_1' - T_2')$$

$$dQ = m_2 s_2 \, dT_2' \quad - ③$$

$$\int \frac{d(T_1' - T_2')}{(T_1' - T_2')} = -\frac{KA}{L}\left(\frac{1}{m_1 s_1} + \frac{1}{m_2 s_2}\right)\int_0^t dt$$

$$- \left[\begin{array}{l} dT_1' = -\frac{1}{m_1 s_1}(dQ) \\ dT_2' = \frac{1}{m_2 s_2}(dQ) \end{array}\right.$$

$$(T_1 - T_2)$$

$$d(T_1' - T_2') = -\left(\frac{1}{m_1 s_1} + \frac{1}{m_2 s_2}\right) dQ \quad -④$$

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$$\int \frac{dx}{x} = \ln x$$

$$\int_{(T_1 - T_2)}^{(T_1' - T_2')} \frac{\overbrace{d(T_1' - T_2')}^{dx}}{\underbrace{(T_1' - T_2')}_{x}} = -\frac{KA}{L}\left(\frac{1}{m_1 s_1} + \frac{1}{m_2 s_2}\right)\int_0^t dt$$

$$\downarrow$$

$$\ln\left[T_1' - T_2'\right]_{(T_1 - T_2)}^{(T_1' - T_2')} = -\frac{KA}{L}\left(\frac{1}{m_1 s_1} + \frac{1}{m_2 s_2}\right)t$$

$$\ln\left(\frac{T_1' - T_2'}{T_1 - T_2}\right) = -\frac{KA}{L}\left(\frac{1}{m_1 s_1} + \frac{1}{m_2 s_2}\right)t$$

$$\boxed{(T_1' - T_2') = (T_1 - T_2)\, e^{-\frac{KA}{L}\left(\frac{1}{m_1 s_1} + \frac{1}{m_2 s_2}\right)t}}$$

At $t \to \infty$,

$T_1' - T_2' \to 0$.

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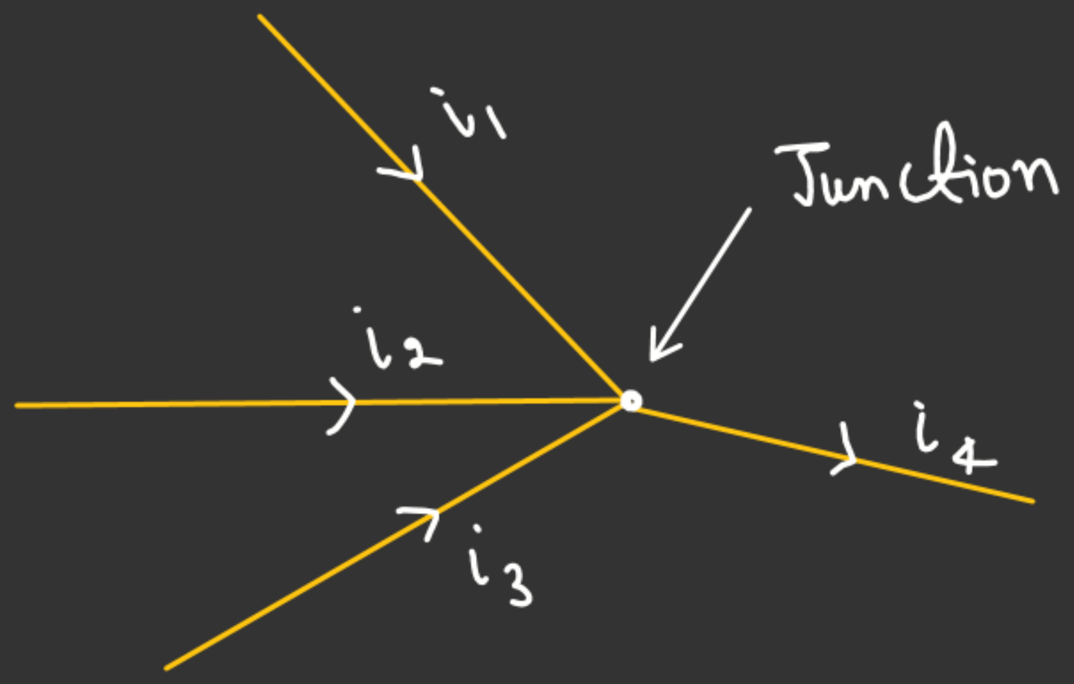
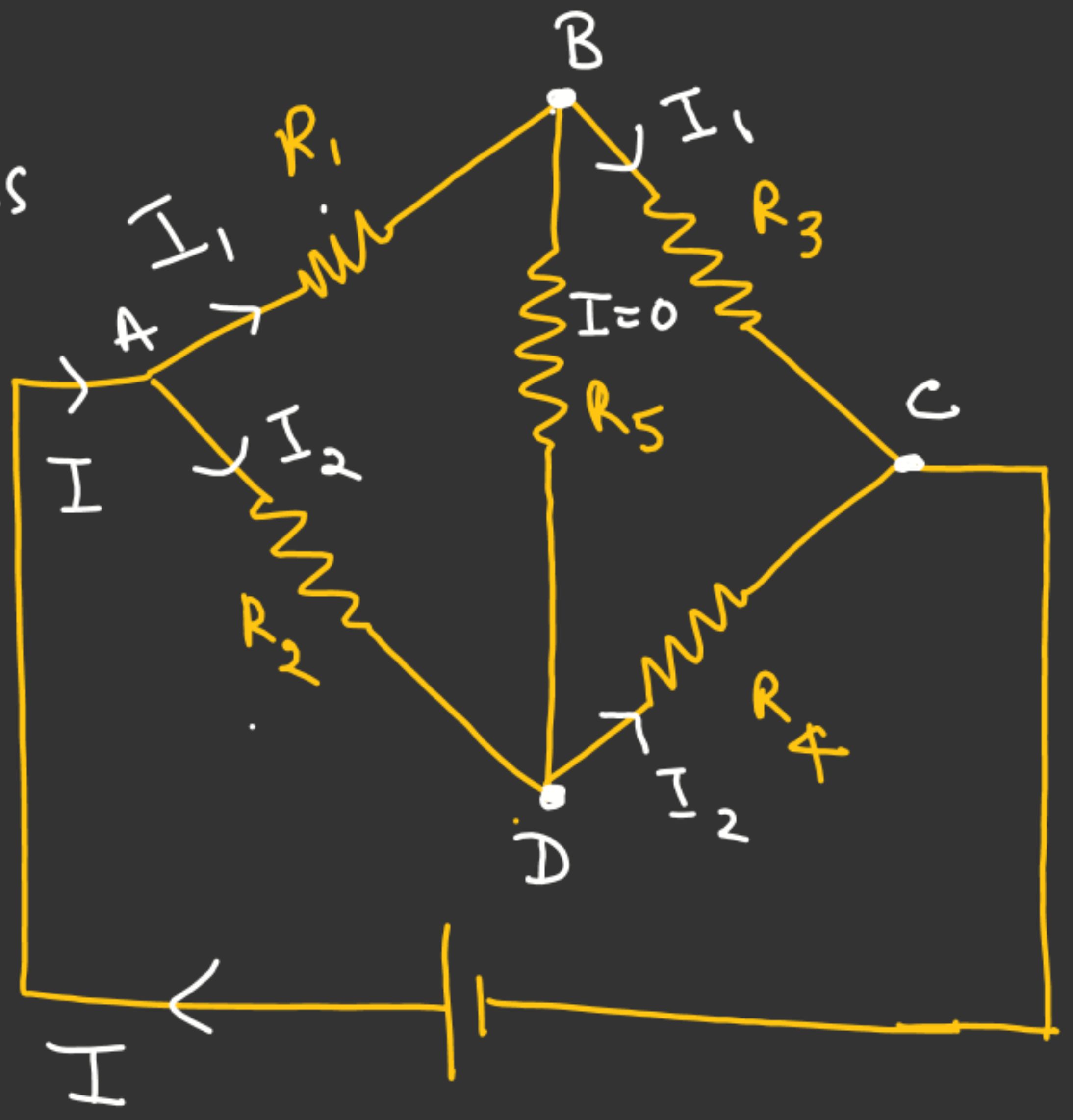
Junction rule



$$\left(i_4 = i_1 + i_2 + i_3 \right)$$

Balance wheat-stone bridge

if $\boxed{V_B = V_D}$, then
No Current across R_5.



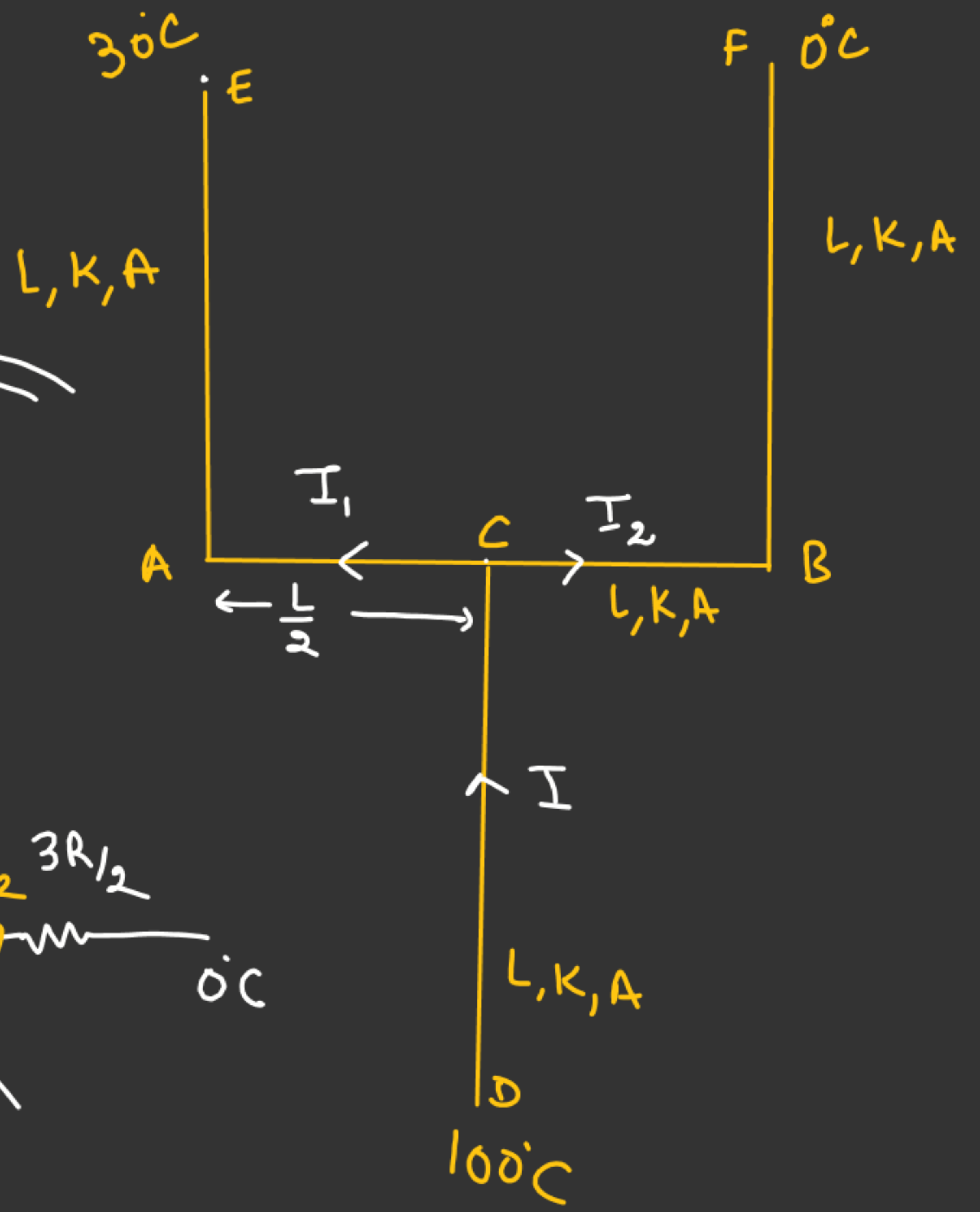
Condition for balance wheat Stone

$$\boxed{\frac{R_1}{R_2} = \frac{R_3}{R_4}}$$

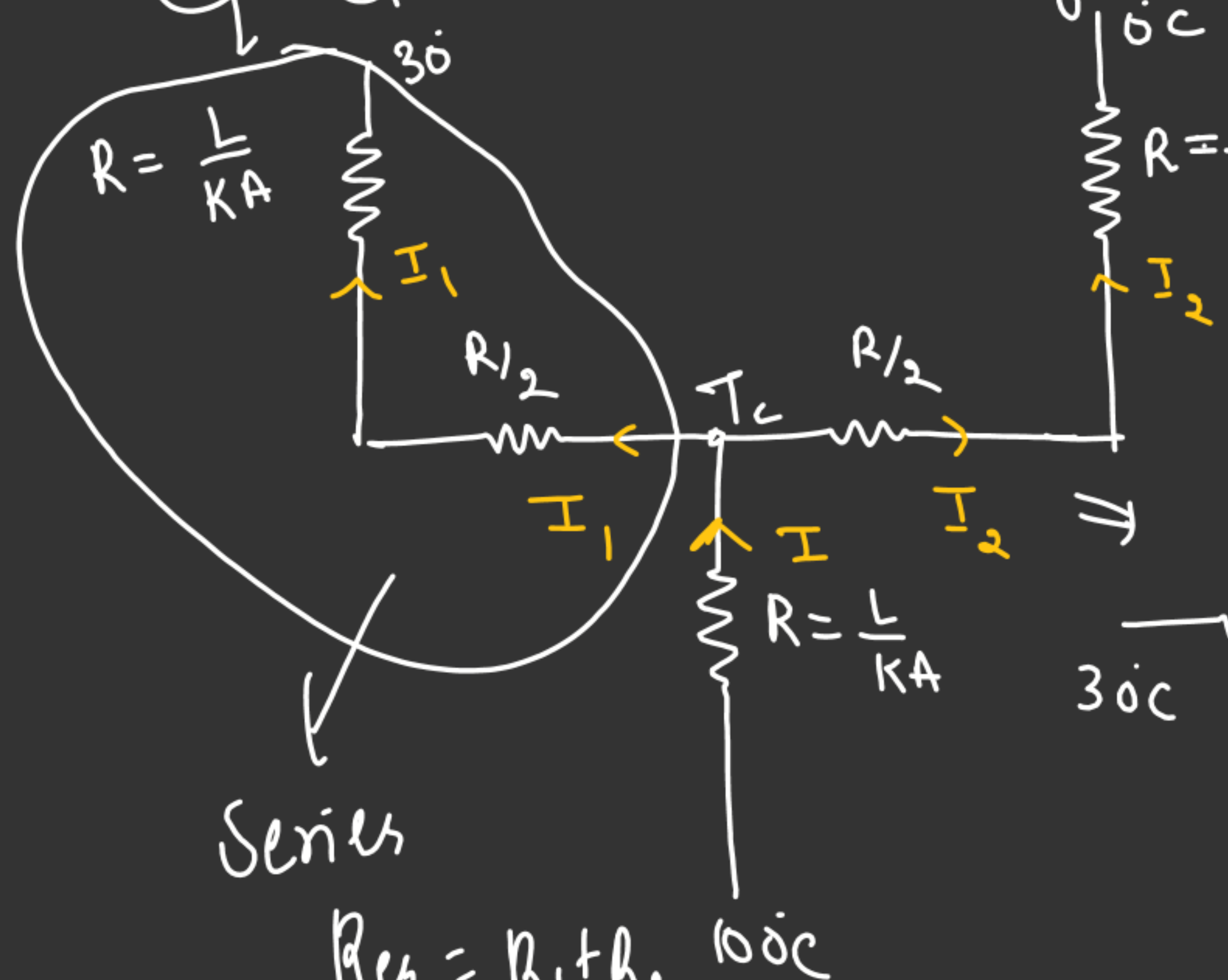
All rods are identical.

C is the mid-point of the rod AB.

Find junction temp i.e. $T_c = ??$



Eq. Electrical Ckt diagram



Series

$R_{eq} = R_1 + R_2$

$= R + R/2 = 3R/2$.

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$$i = i_1 + i_2$$

$$\left(\frac{100 - T_c}{R}\right) = \frac{T_c - 30}{\frac{3R}{2}} + \frac{T_c - 0}{\frac{3R}{2}}$$

$$100 - T_c = \frac{2}{3}(T_c - 30) + \frac{2}{3}T_c$$

$$100 - T_c = \frac{4}{3}T_c - 20$$

$$120 = \frac{7}{3}T_c$$

$$\left(\frac{360}{7} = T_c\right) \checkmark$$

Find heat current in each rod = w

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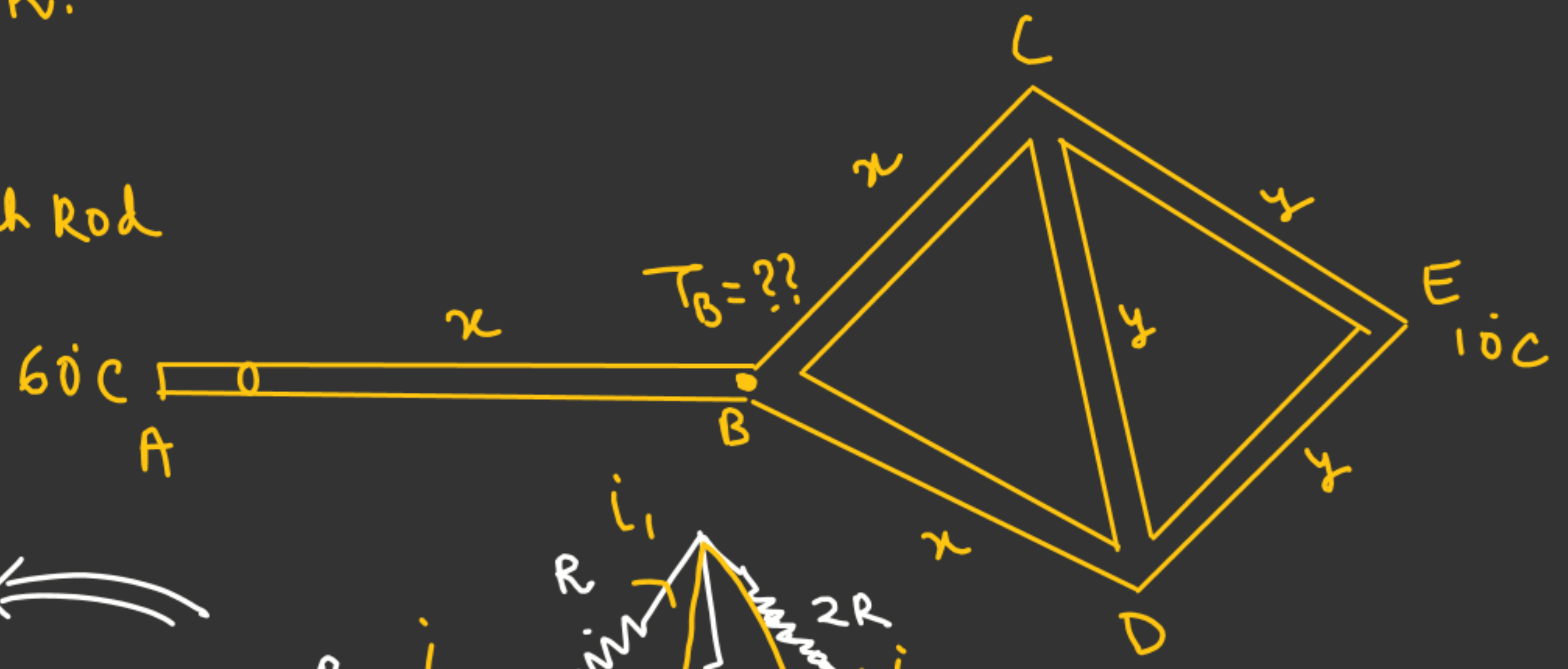
Rod-x, has Conductivity $2K$.

Rod-y has Conductivity K.

L = length of each rod

A = Crossectional area of each Rod

Find $T_B = ??$



$\frac{R}{R} = \frac{2R}{2R}$

$1 = 1$

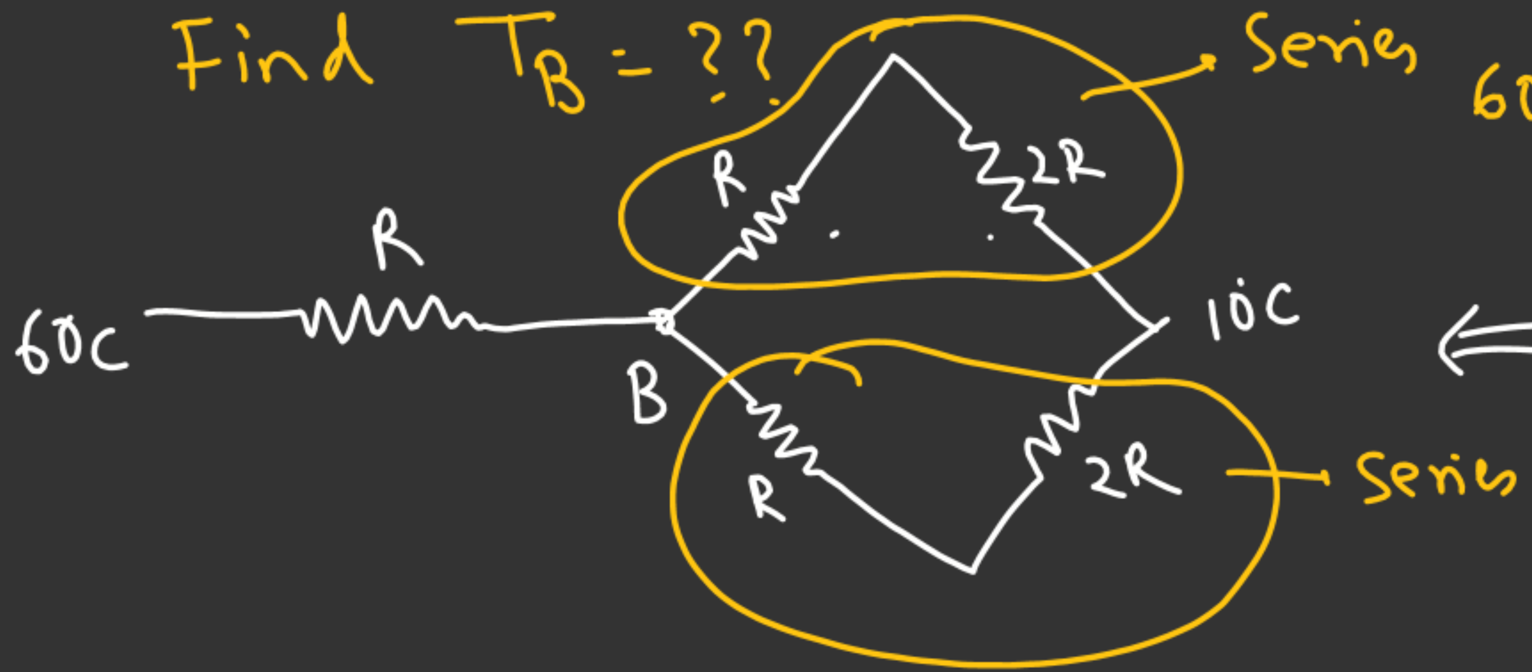
$R_x = \frac{L}{2KA}$

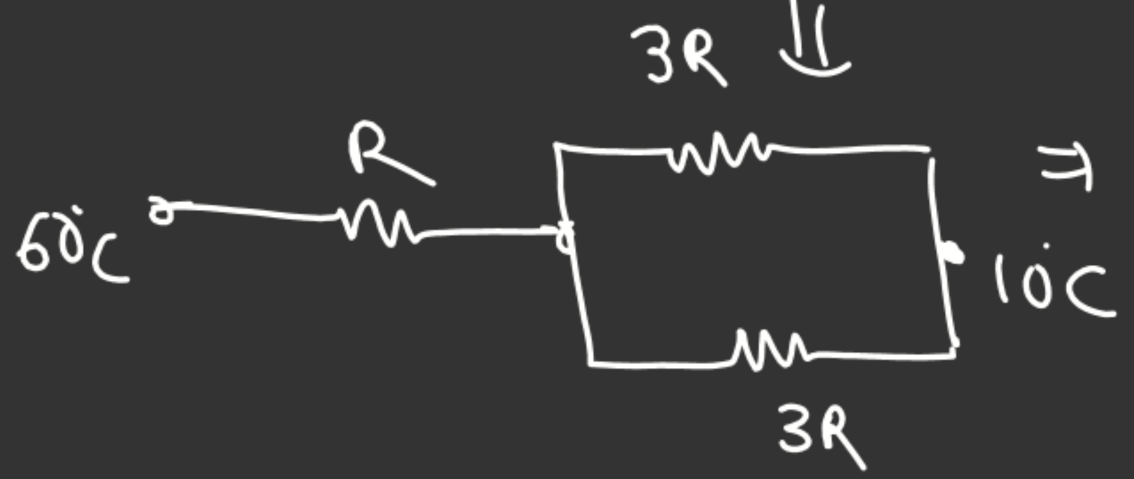
$R_y = \frac{L}{KA}$

$R_y = 2R_x$

$\downarrow$ $\qquad$ $\downarrow$

$2R$ $\qquad$ R





$\Rightarrow$

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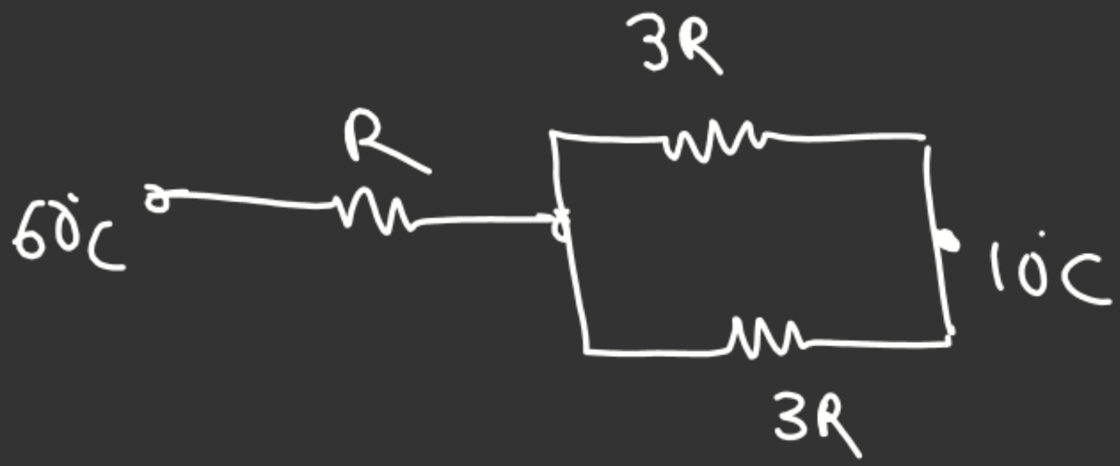

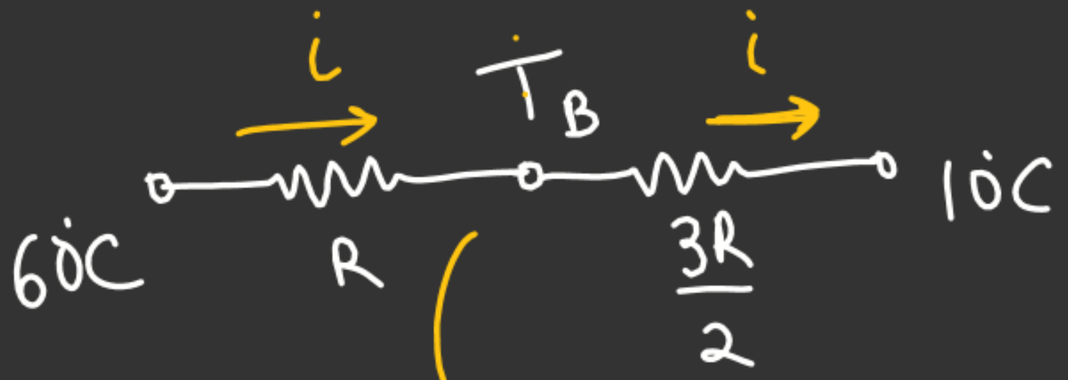

$$\frac{60 - T_B}{R} = \frac{T_B - 10}{\frac{3R}{2}}$$

$$(60 - T_B) = \frac{2}{3}(T_B - 10)$$

$$60 + \frac{20}{3} = \left(\frac{2}{3} + 1\right) T_B$$

$$\frac{200}{\cancel{3}} = \frac{5}{\cancel{3}} \times T_B$$

$$\frac{200}{5} = T_B$$

$$T_B = \underline{40^\circ C}$$

✓