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Permutation of alike objects taken some at a time

Find no. of ways to (i) select 5 letters

(ii) form 5 letter words using the letters of the word 'INDEPENDENCE'

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'INDEPENDENCE'

I, NNN, DD, EEEE, P, C

Cases	Selections	Arrangements
4 Alike + 1 Diff.	$1 \times {}^5C_1$	$1 \times {}^5C_1 \times \frac{5!}{4!}$
3 A + 2 others Alike	${}^2C_1\,{}^2C_1$	${}^2C_1\,{}^2C_1\,\frac{5!}{3!\,2!}$
3 A + 2 D	${}^2C_1\,{}^5C_2$	${}^2C_1\,{}^5C_2\,\frac{5!}{3!}$
2 A + 2 O A + 1 D	${}^3C_2 \times {}^4C_1$	${}^3C_2\,{}^4C_1\,\frac{5!}{2!\,2!}$
2 A + 3 D	${}^3C_1\,{}^5C_3$	${}^3C_1\,{}^5C_3\,\frac{5!}{2!}$
5 D	${}^6C_5$	${}^6C_5\,5!$

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Q. Find no. of 4 digit numbers that can be formed using the digits 1, 1, 2, 3, 4, 0.

$$2A + 2D \rightarrow 1 \times {}^4C_2\left(\frac{4!}{2!}\right) - 1 \times {}^3C_1\left(\frac{3!}{2!}\right)$$

$$4D \rightarrow 4 \times 4 \times 3 \times 2 \quad \text{or} \quad {}^5C_4\,4! - {}^4C_3\,3!$$

$\boxed{159}$ _ _ _ _ 0 _ _ _

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3: How many 6 letter words can be formed using the letters from the word 'INTEGRATION' if each word has 3 vowels and 3 consonants.

I I, E, A, O ; N N, T T, G, R

$^4C_3 \; ^2C_1 \; ^3C_1 \quad \frac{6!}{2!}$

$^6C_3 (33)(42)$

V V _ V _ _

Vowels → 2A + 1D $\quad 1 \times {}^3C_1 \times \frac{3!}{2!}$

3D $\quad {}^4C_3 \times 3!$

Consonants → 2A + 1D → ${}^2C_1 \; {}^3C_1 \; \frac{3!}{2!} \quad 33$

3D → ${}^4C_3 3!$ $\quad 42$

(2A + 1D, 2A + 1D), (2A + 1D, 3D), (3D, 2A + 1D), (3D, 3D)

V C, V C, V C, V C

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4. Find the coefficient of $x^5 y^4 z^3$ in the expansion of $(x+y+z)^{12} = (x+y+z)(x+y+z)\cdot\cdot\cdot\cdot(x+y+z)$

_ _ _ _   _ _

5x, 4y, 3z in 12 places

$$= \frac{12!}{5!\,4!\,3!}$$

$$(a+b)^2 = a^2 + b^2 + 2ab$$

$$= (a+b)(a+b) = a^2$$

$$(a+b)^3 = a^3 + 3ab^2 + 3a^2b + b^3$$

$$(a+b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$$

_ _ _ _

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Total number of Combinations (selection of atleast one object)

$= (p+1)(q+1)(r+1) - 1$

from 'n' distinct objects

$2 \times 2 \times 2 \times \cdots 2 - 1 = 2^n - 1$

${}^nC_1 + {}^nC_2 + {}^nC_3 + \cdots + {}^nC_n$

$T_1, T_2, \ldots, T_n$

from p alike T-I objects →
q alike T-II objects
r alike T-III objects

T-I → 0 objects → 1 way
→ 1 objects → 1
→ 2 objects → 1
→ 3
→ p → 1

ways

(p+1) ways

${}^nC_1 + {}^nC_2 + \cdots + {}^nC_n = 2^n - 1$

$\boxed{{}^nC_0 + {}^nC_1 + {}^nC_2 + \cdots + {}^nC_n = 2^n}$

p alike T-I
q alike T-II
r alike T-III
s distinct.

$= (p+1)(q+1)(r+1)2^s - 1$

$(p+1)(q+1)(r+1)2^s - 1$

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2 orange, 3 Mango, 4 Apple

(i) atleast one fruit

(ii) atleast one fruit of every species

if Case I : fruits of same species are alike

(i) $3\times4\times5-1$

(ii) $\cancel{2}\times3\times4$

Case II : fruits of same species are different.

(i) 2^9-1

(ii) $(2^2-1)(2^3-1)(2^4-1)$

DPP-4, DPP-5 (1-6)