

$$\begin{aligned}
 & \left(2R \sin \frac{\pi}{n}\right)^2 + \left(2R \sin \frac{2\pi}{n}\right)^2 + \left(2R \sin \frac{3\pi}{n}\right)^2 + \dots + \left(2R \sin \frac{(n-1)\pi}{n}\right)^2 \\
 &= 2R^2 \left(\left(1 - \cos \frac{2\pi}{n}\right) + \left(1 - \cos \frac{4\pi}{n}\right) + \left(1 - \cos \frac{6\pi}{n}\right) + \dots + \left(1 - \cos \frac{2(n-1)\pi}{n}\right) \right) \\
 &= 2R^2 \left((n-1) - \frac{\sin \frac{(n-1)\pi}{n}}{\sin \frac{\pi}{n}} \omega_s \pi \right) \\
 &= 2nR^2
 \end{aligned}$$

$$\underline{13} \cdot \frac{\cos^2 x(1-\sin^2 y)}{\cos^2 y} + \frac{\sin^2 x(1-\cos^2 y)}{\sin^2 y} = 1$$

$$\frac{\cos^2 x}{\cos^2 y} + \frac{\sin^2 x}{\sin^2 y} - \sin x \cos^2 x \left(\frac{\sin^2 y + \cos^2 y}{\cos^2 y \sin^2 y} \right) = 1$$

$$\underbrace{\cos^2 x \sin^2 y + \sin^2 x \cos^2 y}_{=} - \underbrace{\sin x \cos^2 x}_{=} - \underbrace{\sin^2 y \cos^2 y}_{=} = 0.$$

$$(\cos^2 x - \cos^2 y)(\sin^2 y - \sin^2 x) = 0$$

$$\cos^2 y - \cos^2 x \in \boxed{\sin^2 y = \sin^2 x}$$

$$\frac{\cos^2 y}{\cos^2 x} + \frac{\sin^2 y}{\sin^2 x} = \frac{\cos^2 x + \sin^2 x}{\cos^2 x \sin^2 x} = \frac{1}{\sin^2 x}$$

$$\frac{b}{a} \sin^4 \alpha + \sin^4 \alpha + \frac{a}{b} \cos^4 \alpha + \cos^4 \alpha = 1$$

$$0 = \left(\sqrt{\frac{b}{a}} \sin^2 \alpha - \sqrt{\frac{a}{b}} \cos^2 \alpha \right)^2 + \frac{\tan^4 \alpha}{a} + \frac{1}{b} = \frac{\frac{b}{a} \sin^4 \alpha + \frac{a}{b} \cos^4 \alpha - 2 \sin^2 \alpha \cos^2 \alpha}{ab} = 0.$$

$$\cancel{\tan^4 \alpha + \frac{b}{a} \tan^2 \alpha + \frac{1}{b}} + \cancel{y + \frac{9}{b}} = \cancel{x + \tan^4 \alpha + 2 \tan^2 \alpha}$$

$$\frac{b}{a} \tan^4 \alpha - 2 \tan^2 \alpha + \frac{9}{b} = 0.$$

$$\boxed{\tan^2 \alpha = \frac{9}{b}}$$

$$\Leftrightarrow \left(\sqrt{\frac{b}{a}} \tan^2 \alpha - \sqrt{\frac{9}{b}} \right)^2 = 0.$$

15.

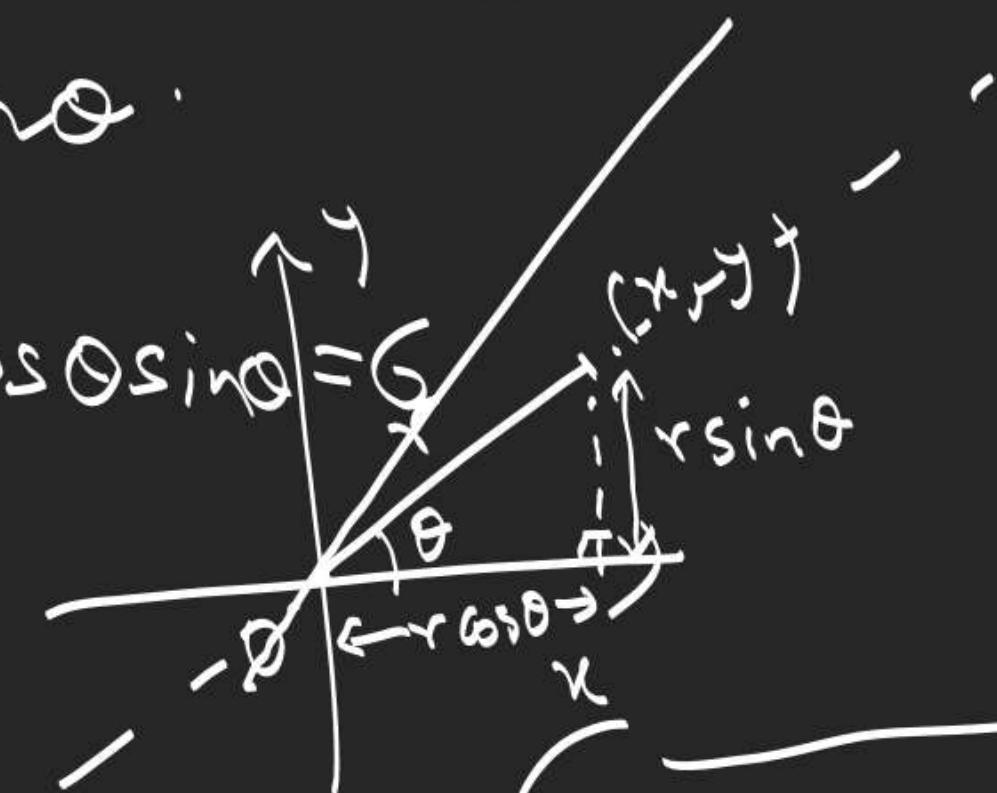
$$\underline{x^2 + 2xy - y^2 = 6}$$

$$x = r \cos \theta, \quad y = r \sin \theta$$

$$(x^2 + y^2)^2 \min = r^4 \min.$$

$$r^2(\cos^2 \theta - \sin^2 \theta) + 2r^2 \cos \theta \sin \theta = 6$$

$$r^2 = \frac{6}{\cos 2\theta + \sin 2\theta} \geq \frac{6}{\sqrt{2}}$$



$$(x^2 + y^2)^2 = (x^2 - y^2)^2 + 4x^2 y^2$$

$$= (6 - 2xy)^2 + 4x^2 y^2 = 8x^2 y^2 - 24xy + 36$$

$$\therefore 8 \left(\underbrace{(xy - \frac{3}{2})^2}_{\geq 0} - \frac{9}{4} \right) + 36 = 8 \underbrace{(xy - \frac{3}{2})^2}_{\geq 0} + 18 \geq [18]$$

x, y varies

$$\therefore 5^{1+\log_4 x} + 5^{\frac{(\log x)-1}{4}} = \frac{26}{5}$$

$$5^{1+\log_4 x} + 5^{\frac{-\log x - 1}{4}} = \frac{26}{5}$$

$$t + \frac{1}{t} = \frac{26}{5}$$

$$5t^2 + 5 - 26t = 0$$

$$5t^2 - 25t - t + 5 = 0$$

$$(5t-1)(t-5) = 0$$

$$5^{1+\log_4 x} = 5^{-1}, 5^1 \Rightarrow 1+\log_4 x = -1 \text{ or } 1 \\ \log_4 x = -2 \text{ or } 0$$

~~$t > 0$~~

$x = \frac{1}{16}, 1$

$$\therefore \log_5(5^{\frac{1}{x}} + 125) = \log_5(6) + 1 + \frac{1}{2x}$$

$$\log_5(5^{\frac{1}{x}} + 125) = \log_5 6 + \log_5 5 + \log_5(5^{\frac{1}{2x}}) \quad 5^{\frac{1}{2x}} = 5^1 \text{ or } 5^2$$

$$\log_5(5^{\frac{1}{x}} + 125) = \log_5(30(5^{\frac{1}{2x}})) \quad \frac{1}{2x} = 1 \text{ or } 2$$

$$5^{\frac{1}{2x}} = t \Rightarrow \boxed{x = \frac{1}{2}, \frac{1}{4}}$$

$$\log_a x_1 = \log_a x_2$$

$$\Rightarrow x_1 = x_2$$

$$\Rightarrow 5^{\frac{1}{x}} + 125 = 5^N$$

$$t^2 - 30t + 125 = 0$$

$$(t-5)(t-25) = 0$$

$$= 5^{\log_5 6 + 1 + \frac{1}{2x}}, 5^{\log_5 6}, 5^1, 5^{\frac{1}{2x}}$$

$$= 6 \cdot 5 \cdot 5^{\frac{1}{2x}}$$

$$= 30 \cdot (5^{\frac{1}{2x}})$$

$$\stackrel{?}{=} \log_4 \left(2 \log_3 \left(1 + \log_2 \left(1 + 3 \log_2 x \right) \right) \right) = \frac{1}{2}$$

$$2 \log_3 \left(1 + \log_2 \left(1 + 3 \log_2 x \right) \right) = 4^{\frac{1}{2}} = 2$$

$$\log_3 \left(1 + \log_2 \left(1 + 3 \log_2 x \right) \right) = 1$$

$$1 + \log_2 \left(1 + 3 \log_2 x \right) = 3^1$$

$$\log_2 \left(1 + 3 \log_2 x \right) = 2$$

$$1 + 3 \log_2 x = 2^2 = \boxed{\log_2 x = 1}$$

$$\Rightarrow x = 2$$

$$\log_a b = x$$

$$b = a^x$$

$$\log_2 N = \frac{1}{2} \Rightarrow N = 2^{\frac{1}{2}}$$

$$\text{L.H.S.} \quad (x+1)^{\log_{10}(x+1)} = 100(x+1)$$

$$(\log_a b)^2 \Rightarrow \log_{10}\left((x+1)^{\log_{10}(x+1)}\right) = \log_{10}(100(x+1))$$

$$n_1 = n_2 \Rightarrow (\log_{10}(x+1))^2 = \log_{10}100 + \log_{10}(x+1)$$

$$\log_a n_1 = \log_a n_2 \Rightarrow (\log_{10}(x+1))^2 = 2 + \log_{10}(x+1)$$

$$x+1=100 \text{ or } \frac{x+1}{100}=1 \Rightarrow x+1=100 \text{ or } x+1=1$$

$$t^2 - t - 2 = (t-2)(t+1) = 0$$

$$5. \quad 3^{\log_3^2 x} + x^{\log_3 x} = 162$$

$$\log_3^2 x = (\log_3 x)^2 = (\log_3 x)(\log_3 x)$$

Ans

$$(3^{\log_3 x})^{\log_3 x} + x^{\log_3 x} = 162$$

$$x^{\log_3 x} + x^{\log_3 x} = 162$$

$$x^{\log_3 x} = 81 \Rightarrow \log_3(x^{\log_3 x}) = \log_3 81$$

$$x = 9, \frac{1}{9}$$

$$\log_3 x = 2, -2 \Leftrightarrow (\log_3 x)^2 = 4 \Leftrightarrow (\log_3 x)(\log_3 x) = 4$$

$$\underline{6:} \quad x^{\log_{\sqrt{x}}(x-2)} = 9$$

$$\frac{2 \log_x(x-2)}{x} = 9 \quad \boxed{x=5}$$

$$(x^{\log_x(x-2)})^2 = 9$$

$$(x-2)^2 = 9 \Rightarrow x-2 = 3 \text{ or } -3$$

$$x = 5, -1$$

$x = -1$ rejected

Sec 1.9

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NCERT

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Natural logarithm

$\log x = \log e^{\ln x} = \ln x$

$e = \text{irrational no. } \approx 2.71828\ldots$

$\log x = \log_{10} x + \log_e 10 = \log_{10} x + 2 \log_e 10 = \log_{10} x + 2 \log(2.7)$

$\therefore \log_{10} x = \log_e x - 2 \log_e 10$

Ex-II (Q. 16, Q. 17)

10² log(x-2)

2 log(x-2)

10

Sec 1.9

125-140

Ex-II (Q. 16, Q. 17)