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Q. $\int \ln(\sqrt{1+x} - \sqrt{1-x}) \cdot dx$

$\Rightarrow \int \ln(\sqrt{1+x} - \sqrt{1-x}) \cdot 1 \cdot dx$

Q $\int \ln(1+x^2) \cdot dx$

$= \int \ln(1+x^2) \cdot 1 \cdot dx$

Q $\int \ln(x+\sqrt{x^2+a^2}) \cdot dx$

$\int \underbrace{\ln(x+\sqrt{x^2+a^2})}_{u} \cdot \overset{v}{1} \cdot dx$

$= \ln(x+\sqrt{x^2+a^2}) \int 1 \cdot dx - \int \left(\frac{1}{\sqrt{x^2+a^2}} \cdot \int 1 \cdot dx\right) dx$

$= x \cdot \ln(x+\sqrt{x^2+a^2}) - \int \frac{x}{\sqrt{x^2+a^2}} dx$

$x \ln(x+\sqrt{x^2+a^2} - \sqrt{x^2+a^2} + c$

AW Sheet

45, 46, 47, 48, 49
50, 53

11 Qs, 21, 22, 23, 24
26, 29, 30, 31
38, 40, 41, 42, 43, 44

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Q_1 $\frac{1}{\frac{\pi}{2}}\int \tan^{-1}x - \cot^{-1}x \cdot dx$

$\frac{2}{\pi}\int \tan^{-1}x - \left(\frac{\pi}{2} - \tan^{-1}x\right) \cdot dx$

$\frac{2}{\pi}\int 2\tan^{-1}x - \frac{2}{\pi}\int \frac{\pi}{2} \cdot dx$

$\frac{4}{\pi}\int \tan^{-1}x \cdot 1 \cdot dx - \int dx$

Q_2 $\int \frac{\log x}{x^2} \cdot dx$ $\log x = t$

$x = e^t$

$dx = e^t$

$\int \frac{t \cdot e^t \cdot dt}{(e^t)^2}$

$Q\,3.$ $\int x e^{\ln \sin x} \cdot dx$

$\int x \cdot \sin x - \cos x$

Q4 Copy.

Q5 laws of cancellus.

Q6 Copy

Q7 ''

Q8 $\sqrt{x} = t \rightarrow e^x(f + f')$ ''

Q9 laws of cancell.

$Q10 \rightarrow$ $\log x = t$

$x = e^t, dx = e^t\, dt$

$\int e^t \left(\frac{t-1}{t^2+1}\right)^2 dt$

$\int e^t \left(\frac{t^2+1}{(t^2+1)^2} - \frac{2t}{(t^2+1)^2}\right) dt$

$\int e^t \left(\frac{1}{t^2+1} - \frac{2t}{(t^2+1)^2}\right) dt$

f f'

$\ln(1+\sin x) = t$

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21 $f'(x) = \lim_{x\to 0} \frac{2\sin x - 2\sin x\cdot\cos x}{x^3}$

$= \lim_{x\to 0} \cancel{\frac{2\sin x}{x}} \cancel{\frac{(1-\cos x)}{x^2}}$

$2\times 1 \times \frac{1}{2} = 1$

Kiskaden

22 $\int \frac{1}{x^2}\sqrt{\frac{x-1}{x+1}}$

$\int \frac{1}{x^2}\sqrt{\frac{1-1/x}{1+1/x}}$ $\qquad \frac{1}{x} = t$

$\frac{1}{x^2}dx = -dt$

$-\int \sqrt{\frac{1-t}{1+t}}\,dt$

Rat

23) ✓

24 $\int \frac{\boxed{x}\cdot \ln(x+\sqrt{1+x^2})}{\sqrt{1+x^2}}\,dx$

u ✓

v

$\int \frac{x}{\sqrt{1+x^2}}\,dx$

$= \sqrt{1+x^2}$

$\ln(x+\sqrt{1+x^2})\sqrt{1+x^2} - \int \frac{1}{\cancel{\sqrt{1+x^2}}} \times \cancel{(\sqrt{1+x^2})}\,dx$

$\sqrt{1+x^2}\ln(x+\sqrt{1+x^2}) - x + C$

Q 26 $\int \frac{dx}{x^2(x^4+1)^{3/4}} = \int \frac{1\cdot dx}{x^5\left(1+\frac{1}{x^4}\right)^{3/4}}$

$1+\frac{1}{x^4} = t$

$\frac{-4}{x^5}\,dx = dt$

$\frac{1}{x^5}\,dx = -\frac{dt}{4}$

$= -\frac{1}{4}\int \frac{dt}{t^{3/4}}$

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29) $\displaystyle\int \frac{(1-\sin^2 x)\cos x}{\sin x(1+\sin x)}$

30) $\displaystyle\int \frac{3x^4-1}{(x^4+x+1)^2}$

$\displaystyle\int \frac{3x^4-1}{x^2\left(x^3+1+\frac{1}{x}\right)^2}\cdot dx$

$\displaystyle\int \frac{\left(3x^2-\frac{1}{x^2}\right)dx}{\left(\underbrace{x^3+\frac{1}{x}+1}_{t}\right)^2}$ $\to dt$

46) $\displaystyle\int \underbrace{\cos 2x}_{V}\cdot \underbrace{\ln(1+\tan x)}_{U}\cdot dx$

47 $\displaystyle\int \underbrace{\tan^{-1}x}_{U}\cdot \underbrace{\ln(1+x^2)}_{V}\cdot dx$

45 $\displaystyle\int \underbrace{\ln x}_{U}\cdot \underbrace{\frac{x}{(x^2-1)^{3/2}}}_{V}$

44) $\displaystyle\int e^x\left(\frac{x^3-x+2}{(x^2+1)^2}\right)dx$

$\displaystyle\int e^x\left(\frac{x^3+x^2+x+1}{(x^2+1)^2}+\frac{-x^2-2x+1}{(x^2+1)^2}\right)dx$

$\displaystyle\int e^x\left(\frac{x+1}{(x^2+1)^2}+\frac{-(x^2+2x+1)}{(x^2+1)^2}\right)dx$

Q $I = \int \sec^3\theta \cdot d\theta$

Int. पता है

$$= \int \underbrace{\sec\theta}_{U} \cdot \underbrace{\boxed{\sec^2\theta}}_{V} \, d\theta$$

$$= \sec\theta \cdot (\tan\theta) - \int \sec\theta \cdot \tan\theta \cdot \tan\theta \cdot d\theta$$

$$- \int \sec\theta \cdot (\sec^2\theta - 1) \cdot d\theta$$

$$I = \sec\theta \cdot \tan\theta - \underbrace{\int \sec^3\theta \cdot d\theta}_{I} + \int \sec\theta \cdot d\theta$$

$$2I = \sec\theta \cdot \tan\theta + \ln|\sec\theta + \tan\theta| \cdot d\theta$$

Q $\displaystyle\int \frac{x \cdot dx}{1+\sin x}$

$$I = \int \frac{x(1-\sin x)}{\cos^2 x} \cdot dx$$

$$= \int \underbrace{x}_{U} \cdot \underbrace{\sec^2 x}_{V} \cdot dx - \int \underbrace{x}_{U} \cdot \underbrace{\sec x \tan x}_{V} \cdot dx$$

$$= x(\tan x) - \int 1 \cdot \tan x \cdot dx - \left[x \sec x - \int 1 \cdot \sec x \cdot dx\right]$$

$$= x(\tan x - \sec x) - \ln|\sec x| + \ln|\sec x + \tan x| + C$$

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Q $\int \frac{x+\sin x}{1+\cos x}\,dx$ $\quad x+\sin x \neq t$

$\int \frac{x}{1+\cos x}\cdot dx + \int \frac{\sin x}{1+\cos x}\cdot dx$

$\frac{1}{2}\int \underset{U}{x}\cdot \underset{V}{\sec^2\frac{x}{2}}\cdot dx + \int \tan\frac{x}{2}\cdot dx$

Q $\int \frac{\cos^{-1}x}{x^3}\cdot dx$

$\cos^{-1}x = \theta$

$x = \cos\theta$

$dx = -\sin\theta\, d\theta$

$-\int \theta\cdot\sec^3\theta\cdot\sin\theta\cdot d\theta$

$\Rightarrow -\int \underbrace{\theta}_{U}\cdot \underbrace{\sec^2\theta\cdot\tan\theta}_{V}\cdot d\theta$

Bahar.

$\int \sec^2\theta\cdot\tan\theta\cdot d\theta$ $\quad \tan\theta = t$

$\int t\, dt$ $\quad \sec^2\theta\cdot d\theta = dt$

$= \frac{t^2}{2} = \frac{\tan^2\theta}{2} + C$

$\Rightarrow -\left\{\theta\cdot\frac{\tan^2\theta}{2} - \int 1\cdot\frac{\tan^2\theta}{2}\cdot d\theta\right\}$

$\Rightarrow -\frac{\theta\cdot\tan^2\theta}{2} + \frac{1}{2}\int \sec^2\theta - 1\cdot d\theta$

DI

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When $\int (f(x))^n dx$ is asked.

$\int (\log_e x)^2 \cdot dx$, $\int (\sin^{-1} x)^3 \cdot dx$, $\int (x+\sqrt{x^2+a^2})^n \cdot dx$

take fxn Inside $= t$

Yad

Q $\int (x+\sqrt{x^2+a^2})^n \cdot dx$

$x+\sqrt{x^2+a^2}=t$

$\sqrt{x^2+a^2}=t-x$

$\cancel{x^2}+a^2=t^2+\cancel{x^2}-2tx$

$2tx=t^2-a^2$

$x=\frac{t}{2}-\frac{a^2}{2t}$

$dx=\left(\frac{1}{2}+\frac{a^2}{2}\frac{1}{t^2}\right)dt$

$I=\int t^n\left(\frac{1}{2}+\frac{a^2}{2t^2}\right)dt$

$=\int \frac{t^n}{2}\cdot dt+\frac{a^2}{2}\int t^{n-2} dt$

$\Rightarrow \frac{1}{2}\left[\frac{t^{n+1}}{n+1}\right]+\frac{a^2}{2}\times\frac{t^{n-1}}{n-1}+C$

Q $I=\int \frac{dx}{(x+\sqrt{x^2+1})^2}$

$x+\sqrt{x^2+1}=t$

Forced Integration

Q $\int \frac{x^2 \cdot dx}{(x\cos x-\sin x)^2}$

$x\cos x-\sin x=t$

$(-x\sin x+\cancel{\cos x}-\cancel{\cos x})dx=dt$

$-x\cdot\sin x\cdot dx=dt$

$\Rightarrow I=\int \underbrace{\frac{x}{\sin x}}_{U}\times\underbrace{\boxed{\frac{x\sin x\cdot dx}{(x\cos x-\sin x)^2}}}_{V}$ I_n

$=\frac{x}{\sin x}\times\frac{1}{(x\cos x-\sin x)}-\int\frac{(\sin x-x\cdot\cos x)}{(\sin x)^2}\cdot\frac{1}{(\cancel{x\cos x-\sin x})}dx$

$+\cot x+C$

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Q** $\int \frac{x^2 \cdot dx}{(x\cos x - \sin x)(x\sin x + \cos x)}$

↓ try.

$\int \frac{x\cos x}{x\sin x + \cos x} + \frac{x\sin x}{x\cos x - \sin x}\, dx$

Laws of Cancellation.

When $\int f(x)\cdot dx + \int g(x)\cdot dx$ or.
$\int f(x)\cdot dx - \int g(x)\cdot dx$ .. is given

Q $I = \int e^{\cot x}\cdot(\cos x - \operatorname{cosec} x)\cdot dx$

$\int \underset{U}{e^{\cot x}} \cdot \underset{V}{\cos x}\, dx - \int e^{\cot x} \operatorname{cosec} x\, dx$

$= e^{\cot x}(\sin x) + \int \cancel{e^{\cot x}\cdot(+\operatorname{cosec}^2 x)\cdot(\sin x)\cdot dx} - \cancel{\int e^{\cot x}} \operatorname{cosec} x\cdot dx$

$= e^{\cot x}\cdot \sin x + C$

Q $I = \int e^{\tan\theta}(\sec\theta - \sin\theta)\cdot d\theta$
Self.

Q $I = \int ((\operatorname{cosec}^2 x - 2010) \sec^{2010} x\, dx$ → Gndi sideg. ↓ IInd deg. ↓ ILate

$= \int \underset{V}{\operatorname{cosec}^2 x} \cdot \underset{U}{\sec^{2010} x} \cdot dx - \underbrace{2010 \int \sec^{2010} x \cdot dx}$

$= \sec^{2010} x \cdot (-\cot x) + \int \cancel{2010 \cdot \sec^{2009} x \cdot \sec x \cdot \tan x \cdot (+\cot x)}\, dx$

$\cancel{-2010 \int \sec^{2010} x \cdot dx}$

$= -\cot x \cdot \sec^{2010} x + C$

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Integration of Trigo fxn.

Type 1

$$\int \frac{dx}{a\sin x + b\cos x}$$

$a = r\cos\alpha$.

$b = r\sin\alpha$.

See Ex.

Type 2

$$\int \frac{dx}{a + b\sin x}$$

$$\int \frac{dx}{a + b\cos x}$$

Dr me $\sin x, \cos x$ deg = 1 mile.

Method = half Angle

$$\sin x = \frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}$$

$$\cos x = \frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}$$

Type 3

$$\int \frac{dx}{a + b\sin^2 x}$$

$$\int \frac{dx}{a + b\cos^2 x}$$

When $\sin x, \cos x$ in Dr with deg = 2

Method $\div$ by $\cos^2 x$

Type 4

$$\int \frac{a\cos x + b\sin x}{c\cos x + d\sin x}$$

$$\int \frac{ae^x + be^{-x}}{ce^x + de^{-x}}\,dx$$

Method

$Nr = \lambda(Dr) + \mu(Dr)'$

Replace

Type 5

$$\int \frac{dx}{\sin^m x \cdot \cos^n x}$$

try to make $\tan x$ in Dr.

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Q T_2 $\int \frac{dx}{2+3\cos x}$

half A.

$$\int \frac{dx}{2+3\left(\frac{1-\tan^2 x/2}{1+\tan^2 x/2}\right)}$$

$$\int \frac{\sec^2\frac{x}{2}\cdot dx}{2+2\tan^2\frac{x}{2}+3-3\tan^2\frac{x}{2}}$$

$$\int \frac{\sec^2\frac{x}{2}\cdot dx}{5-\left(\tan\frac{x}{2}\right)^2}$$

$\tan\frac{x}{2}=t$

$\sec^2\frac{x}{2}\,dx=2dt$

$$2\int\frac{dt}{(\sqrt{5})^2-(t)^2}=\frac{2}{2\sqrt{5}}\ln\left|\frac{\sqrt{5}+t}{\sqrt{5}-t}\right|+C$$

Q T_3 $\int \frac{dx}{2-3\cos^2 x}$

$\div \cos^2 x$

$$\int\frac{\sec^2 x\cdot dx}{2\sec^2 x-3}$$

$$\int\frac{\sec^2 x\cdot dx}{2+2\tan^2 x-3}$$

$$\int\frac{\sec^2 x\,dx}{(\sqrt{2}\tan x)^2-1^2}$$

$\sqrt{2}\tan x=t$

$\sec^2 x\,dx=\frac{dt}{\sqrt{2}}$

$$\frac{1}{\sqrt{2}}\int\frac{dt}{t^2-1^2}$$

$$\frac{1}{\sqrt{2}}\times\frac{1}{2}\ln\left|\frac{t-1}{t+1}\right|+C$$

Q T_1 $\int\frac{dx}{\sin x-\cos x}$

$a=1,\ b=-1$

$$=\frac{1}{\sqrt{2}}\ln\left|\tan\left(\frac{x}{2}+\frac{1}{2}\tan^{-1}\frac{-1}{1}\right)\right|$$

$$=\frac{1}{\sqrt{2}}\ln\left|\tan\left(\frac{x}{2}-\frac{\pi}{8}\right)\right|+C$$

Q $\int\frac{dx}{\sqrt{1+\sin x}}$

T_5 $\int\frac{dx}{\sin\frac{x}{2}+\cos\frac{x}{2}}$

$a=1,\ b=1$

$$\frac{1}{\sqrt{2}}\ln\left|\tan\left(\frac{x}{4}+\frac{\pi}{8}\right)\right|+C$$

2 & 8 → 1008 → 69

Q $I=\int\frac{dx}{a\sin x+b\cos x}$

$a=r\cos\alpha$, $b=r\sin\alpha$

$a^2=r^2\cos^2\alpha$

$b^2=r^2\sin^2\alpha$

$a^2+b^2=r^2$

$$\int\frac{dx}{r\sin x\cdot\cos\alpha+r\cos x\cdot\sin\alpha}$$

$$\frac{1}{r}\int\frac{dx}{(\sin x\cos\alpha+\cos x\cdot\sin\alpha)}$$

$$\frac{1}{r}\int\frac{dx}{\sin(x+\alpha)}=\frac{1}{r}\int\operatorname{cosec}(x+\alpha)\,dx$$

$$=\frac{1}{\sqrt{a^2+b^2}}\cdot\ln\left|\tan\left(\frac{x}{2}+\frac{\alpha}{2}\right)\right|+C$$

$$=\frac{1}{\sqrt{a^2+b^2}}\ln\left|\tan\left(\frac{x}{2}+\frac{1}{2}\tan^{-1}\frac{b}{a}\right)\right|+C$$

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Q $\int \frac{\sin x}{\sin 3x} \cdot dx$

$\int \frac{dx}{3-4\sin^2 x}$

$\div \cos^2 x$

$\int \frac{\sec^2 x \cdot dx}{3+3\tan^2 x - 4\tan^2 x}$

$\int \frac{\sec^2 x \cdot dx}{3-(\tan x)^2}$

DY

Q
T3 $\int \frac{(a+b\sin x)dx}{(b+a\sin x)^2}$

$\div \cos^2 x$

$\int \frac{a\sec^2 x + \sec x\tan x \, dx}{(b\sec x + a\tan x)^2}$

$b\sec x + a\tan x = t$

$b\sec x\tan x + a\sec^2 x \, dx = dt$

$\int \frac{dt}{t^2} = -\frac{1}{t}$

Q $\int \frac{\sin^2 x \cdot dx}{1+\sin^2 x}$

$\int 1 dx - \int \frac{1}{1+\sin^2 x} dx$

$\div \cos^2 x$

JM | 17, 20, 25, 31, 32.
68, 69 | 33, 35, 36, 37, 38, 39
71, 72 | 52, 54, 55, 56, 57
73, 74 |
76, 75 | 58, 59, 61, 63, 67