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$$\left(b\left(\frac{c}{b}\right)^{\frac{1}{3}}\right)^3 + \left(b\left(\frac{c}{b}\right)^{\frac{2}{3}}\right)^3 = b^2c + bc^2$$

$$a^x = b^y = c^z = d^u = k$$

$$x\,\underline{\log a} = y\,\underline{\log b} = z\log c \geqslant 2\sqrt{a}\quad 2\sqrt{b}\quad 2\sqrt{c}\quad 2\sqrt{d}$$

$$= u\log d = k$$

$$\frac{1}{x} = \frac{\log a}{k}$$

$$\frac{\frac{1}{a^2} - \left(\frac{1}{a} + 3d\right)^2}{\left(\frac{1}{a}+d\right)^2 - \left(\frac{1}{a}+2d\right)^2}$$

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$$S_{p+q} - \cancel{S_p} \to 0 = \frac{p+q}{2}\left(2a + (p+q-1)d\right)$$

$$\frac{p}{2}\left(2a + (p-1)d\right) = 0 \qquad = \frac{(p+q)}{2}\, q\, d$$

$$d = \frac{-2a}{p-1} \qquad = -\frac{a(p+q)\, q}{p-1}$$

$a+b+c = 25 \Rightarrow c = 27 - 3a$

$2a = 2 + b \Rightarrow 2a - 2 = b$

$c^2 = 18b \Rightarrow (27 - 3a)^2 = 18(2a-2)$

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$$\sum_{r=1}^{\infty} r^2 x^r = \frac{-x}{(x-1)^2} - \frac{2x}{(x-1)^3}$$

$$\sum_{r=1}^{1999} \sqrt{1+\frac{1}{r^2}+\frac{1}{(r+1)^2}} = \sum \frac{\sqrt{r^4+2r^3+3r^2+2r+1}}{r(r+1)} = \sum \frac{r^2+r+1}{r(r+1)}$$

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1. $\sin x + \sin 5x = \sin 2x + \sin 4x$.

$2\sin 3x \cos 2x = 2\sin 3x \cos x$

$x = \frac{n\pi}{3}$ or $\cos 2x = \cos x$

$2x = 2n\pi \pm x$

$x = 2n\pi, \frac{2n\pi}{3}$

$\boxed{x = \frac{n\pi}{3}, n \in I}$

2. Find the number of solutions in $[0, \pi]$ of the eqn.

$(3 - 4\sin^2\theta)\sin\theta = \sin 3\theta = 4\sin\theta \sin 2\theta \sin 4\theta$.

$\boxed{8}$

$\theta = 0, \pi$ or $3 - 4\sin^2\theta = 4\sin 2\theta \sin 4\theta$

$1 + 2\cos 2\theta = 3 - 2(1 - \cos 2\theta) = 2(\cos 2\theta - \cos 6\theta)$

$\cos 6\theta = -\frac{1}{2}$. $\rightarrow$ 6.

$6\theta = \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{8\pi}{3}, \frac{10\pi}{3}, \frac{14\pi}{3}, \frac{16\pi}{3}$

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3. Find general soln. of eqn.

$$\cos^2 x + \cos^2 2x + \cos^2 3x + \cos^2 4x = 2.$$

$$(\cos^2 2x - \sin^2 x) + (\cos^2 4x - \sin^2 3x) = 0$$

$$\cos x \cos 3x + \cos x \cos 7x = 0$$

$$\cos x \cos 5x \cos 2x = 0$$

$$(2n+1)\frac{\pi}{2}, \quad \boxed{(2n+1)}\frac{\pi}{10}, \quad (2n+1)\frac{\pi}{4}$$

$$5(2m+1)$$

$$\boxed{x = (2n+1)\frac{\pi}{4}, (2n+1)\frac{\pi}{10} \quad n \in I}$$

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4. Find no. of solution in $\boxed{[0, 2\pi]}$ of the eqn.

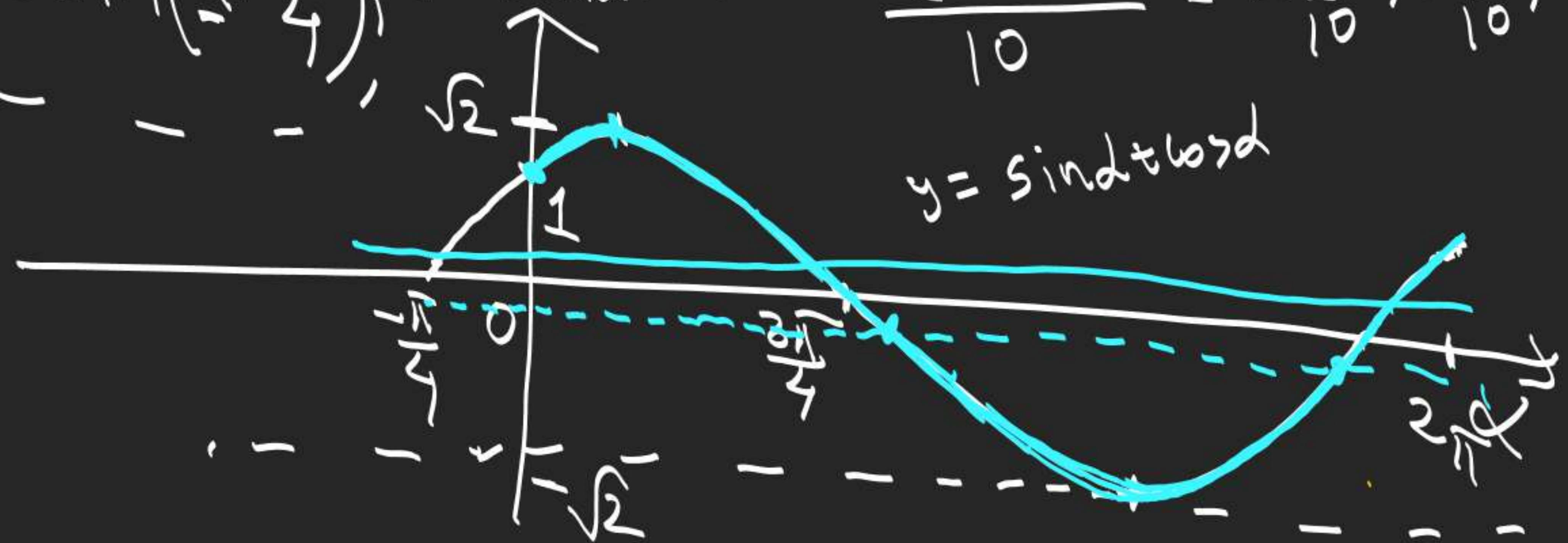
$$\tan(5\pi\cos\alpha) = \cot(5\pi\sin\alpha)$$

$$\boxed{14 \times 2}$$

$$\tan(5\pi\cos\alpha) = \tan\left(\frac{\pi}{2} - 5\pi\sin\alpha\right)$$

$$5\pi\cos\alpha = n\pi + \frac{\pi}{2} - 5\pi\sin\alpha \quad , n \in I.$$

$$\sqrt{2}\sin\left(\alpha + \frac{\pi}{4}\right) = \sin\alpha + \cos\alpha = \frac{(2n+1)}{10} = \pm\frac{1}{10}, \pm\frac{3}{10}, \pm\frac{5}{10}, \pm\frac{7}{10}, \pm\frac{9}{10}, \pm\frac{11}{10}, \pm\frac{13}{10}$$



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5. $\operatorname{cosec} x - \operatorname{cosec} 2x = \operatorname{cosec} 4x$.

$$\frac{1}{\sin x} - \frac{1}{\sin 2x} = \frac{1}{\sin 4x} \Rightarrow \frac{\sin 2x - \sin x}{\sin x \cancel{\sin 2x}} = \frac{1}{2\cancel{\sin 2x}\cos 2x}$$

$$\sin 4x - (\sin 3x - \sin x) = \sin x$$

$$\sin 4x = \sin 3x \Rightarrow 2\sin\frac{x}{2}\cos\frac{7x}{2} = 0$$

$$\frac{x}{2} = n\pi, \quad \frac{7x}{2} = (2n+1)\frac{\pi}{2}$$

$$4x = n\pi + (-1)^n 3x$$

$$x = 2m\pi, \quad \frac{(2m+1)\pi}{7}$$

$$x = \frac{(2m+1)\pi}{7}, \quad m \in I$$

$$2m+1 = 7(2k+1), \quad k \in I$$

$$m = 7k+3 \checkmark$$

$$\boxed{x = \frac{(2m+1)\pi}{7}, \quad m \in I - \{7k+3\}, \quad k \in I}$$

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6.
$$\sin^4 2x + \cos^4 2x = \sin 2x \cos 2x$$

$$1 - \frac{1}{2}\sin^2 4x = \frac{1}{2}\sin 4x$$

$$\sin^2 4x + \sin 4x - 2 = 0$$

$$\underbrace{(\sin 4x + 2)}_{\times}(\sin 4x - 1) = 0$$

$$\sin 4x = 1$$

$$4x = 2n\pi + \frac{\pi}{2}$$

$$\boxed{x = \frac{n\pi}{2} + \frac{\pi}{8}, \quad n \in I}$$

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7. $\sin^3 x - \cos^3 x = 1 + \sin x \cos x$

$(\sin x - \cos x)(1 + \sin x \cos x) = (1 + \sin x \cos x)$

$1 + \sin x \cos x = 0 \Rightarrow \sin 2x = -2$ ✗

or

$\sin x - \cos x = 1$

$\sin\left(x - \frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$

$x - \frac{\pi}{4} = n\pi + (-1)^n \frac{\pi}{4}$

$x - \frac{\pi}{4} = 2m\pi + \frac{\pi}{4},\ (2m+1)\pi - \frac{\pi}{4}$

$\boxed{x = 2m\pi + \frac{\pi}{2},\ (2m+1)\pi \quad m \in I.}$

Ex - I (11 - 20) -

Ex - II (1 - 5).

Ex - III (1 - 10)

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8. $\cos x + \cos 2x + \cos 3x = 3$