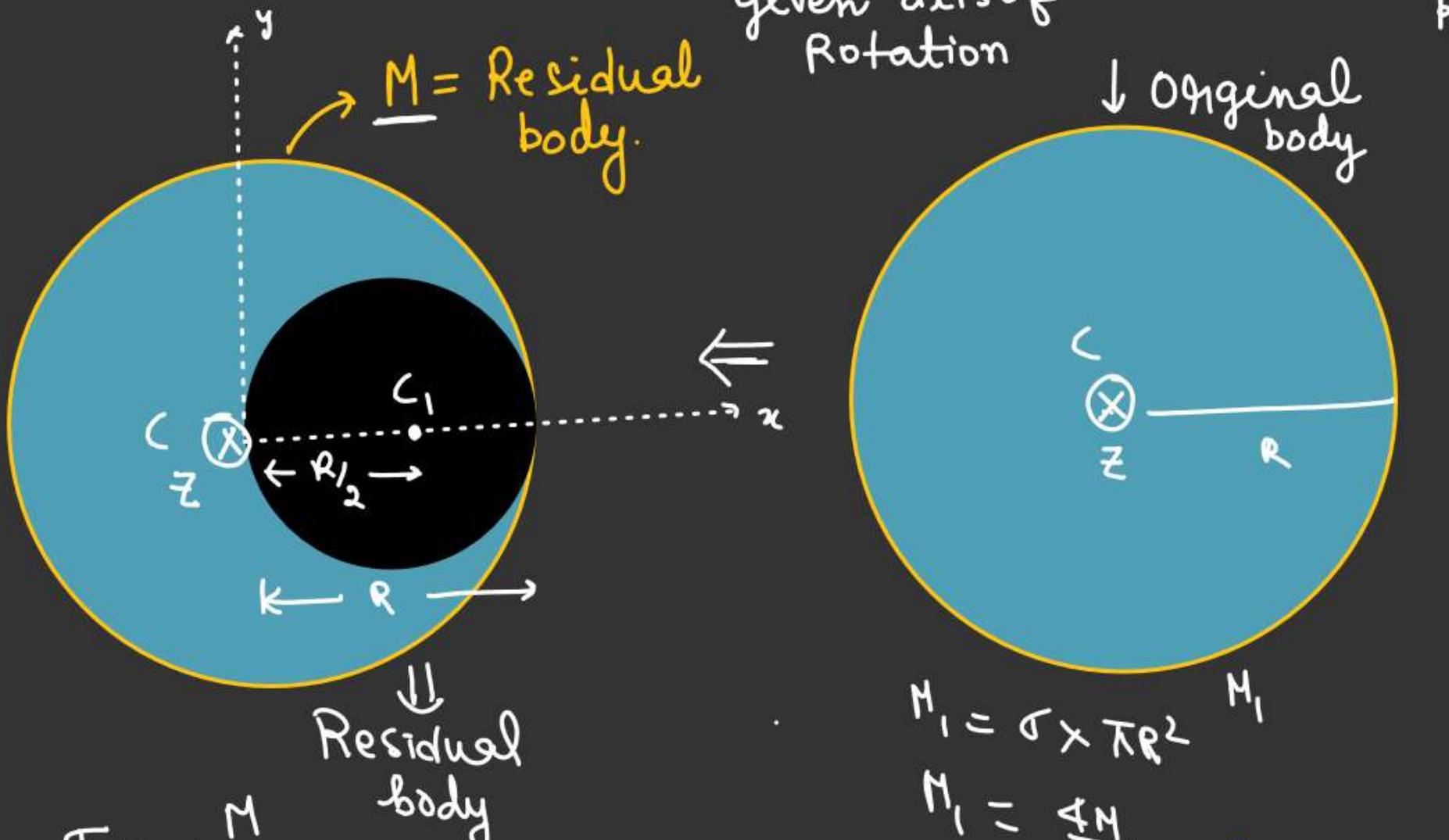




## M-I of Residual body

$$I_{\text{Residual}} = I_{\text{Original body about given axis of Rotation}} - I_{\text{Cut body about given axis of Rotation}}$$



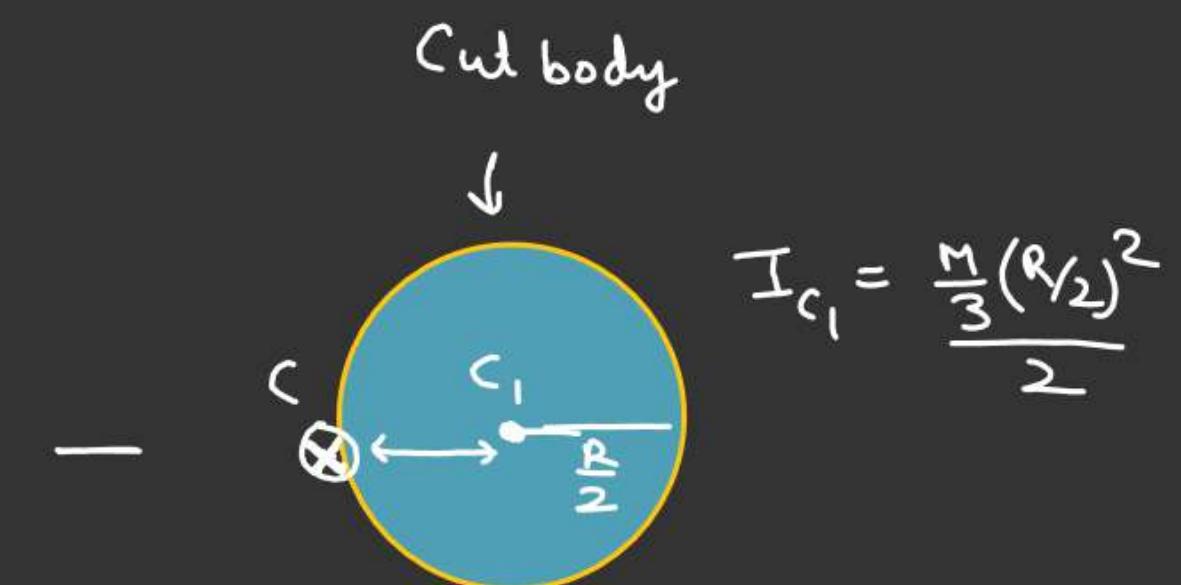
$$\sigma = \frac{M}{\pi R^2 - \pi (\frac{R}{2})^2}$$

$$\sigma = \frac{4M}{3\pi R^2}$$

$$M_1 = \sigma \times \pi R^2$$

$$M_1 = \frac{4M}{3}$$

$$I_1 = \frac{M_1 R^2}{2} = \left(\frac{2MR^2}{3}\right)$$



$$I_{C1} = \frac{\frac{M}{3}(\frac{R}{2})^2}{2}$$

$$M_2 = \sigma \left(\frac{\pi R^2}{4}\right)$$

$$(I_2) = \left(\frac{M}{3}(\frac{R}{2})^2 \cdot \frac{1}{2}\right) = \frac{4M}{3\pi R^2} \times \frac{\pi R^2}{4}$$

$$\text{Cut about } + \frac{M}{3}(\frac{R}{2})^2 = \frac{M}{3}$$

$$= \frac{MR^2}{24} + \frac{MR^2}{12}$$

$$= \frac{MR^2}{8}$$

$$\begin{aligned} I_{\text{residual}} &= I_1 - I_2 \\ &= \frac{2}{3}MR^2 - \frac{MR^2}{8} \\ &= \frac{16MR^2 - 3MR^2}{24} \\ &= \left( \frac{13MR^2}{24} \right) \end{aligned}$$

From a hollow Sphere a vertical Cylinder of radius  $R/2$  is cut.

Find M-I of the remaining body about vertical axis passing through center of sphere.

$$dm = \text{mass of ring}$$

$$= \sigma(2\pi R \sin\phi) R d\phi$$

$$dm = \sigma 2\pi R^2 \sin\phi d\phi$$

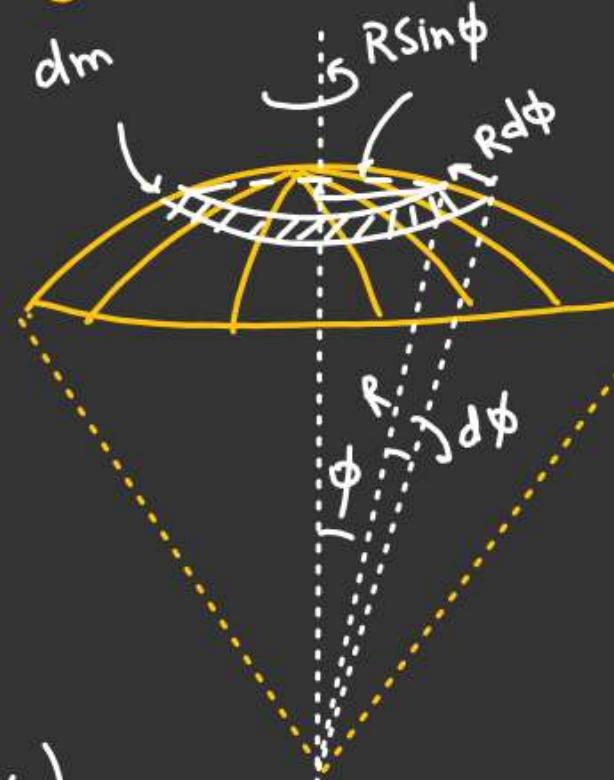
$dI \rightarrow$  M.I of  $dm$

$$dI = dm (R \sin\phi)^2$$

$$I_{\text{cut}} = \sigma (2\pi R^2 \sin\phi d\phi) (R^2 \sin^2\phi)$$

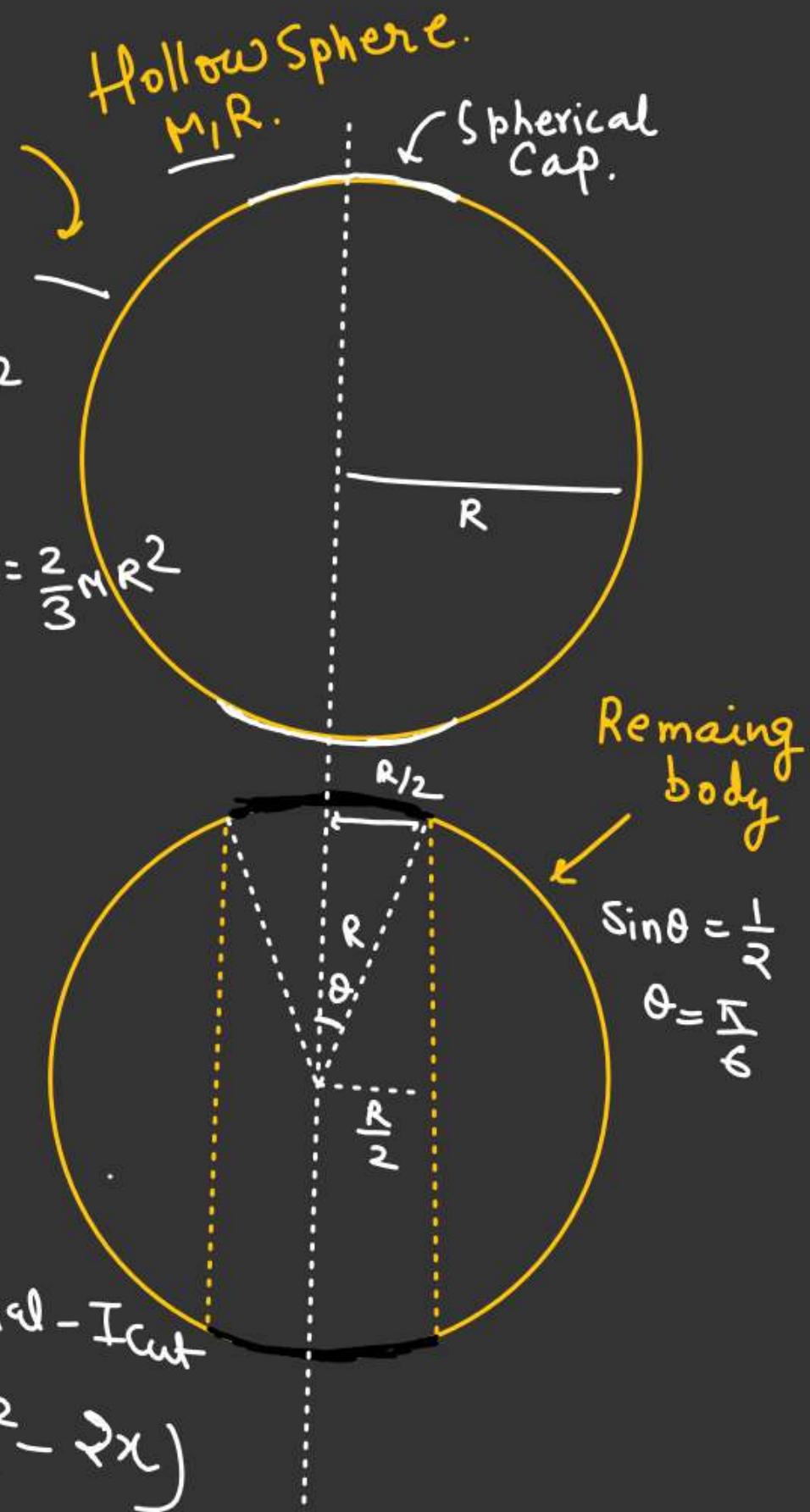
$$\int_0^{\pi/2} dI = \sigma 2\pi R^4 \int_0^{\pi/2} \sin^3\phi d\phi \quad \sin 3\phi = 3 \sin\phi - 4 \sin^3\phi$$

$$= \frac{M}{4\pi R^2} \times 2\pi R^4 \int_0^{\pi/2} (3 \sin\phi - \sin 3\phi) d\phi = x$$



$$\sigma = \frac{M}{4\pi R^2}$$

$$I_{\text{original}} = \frac{2}{3} MR^2$$



$$I_{\text{residual}} = I_{\text{original}} - I_{\text{cut}}$$

$$= \left( \frac{2}{3} MR^2 - 2x \right)$$



## TORQUE

( Moment of force)

Torque about a point

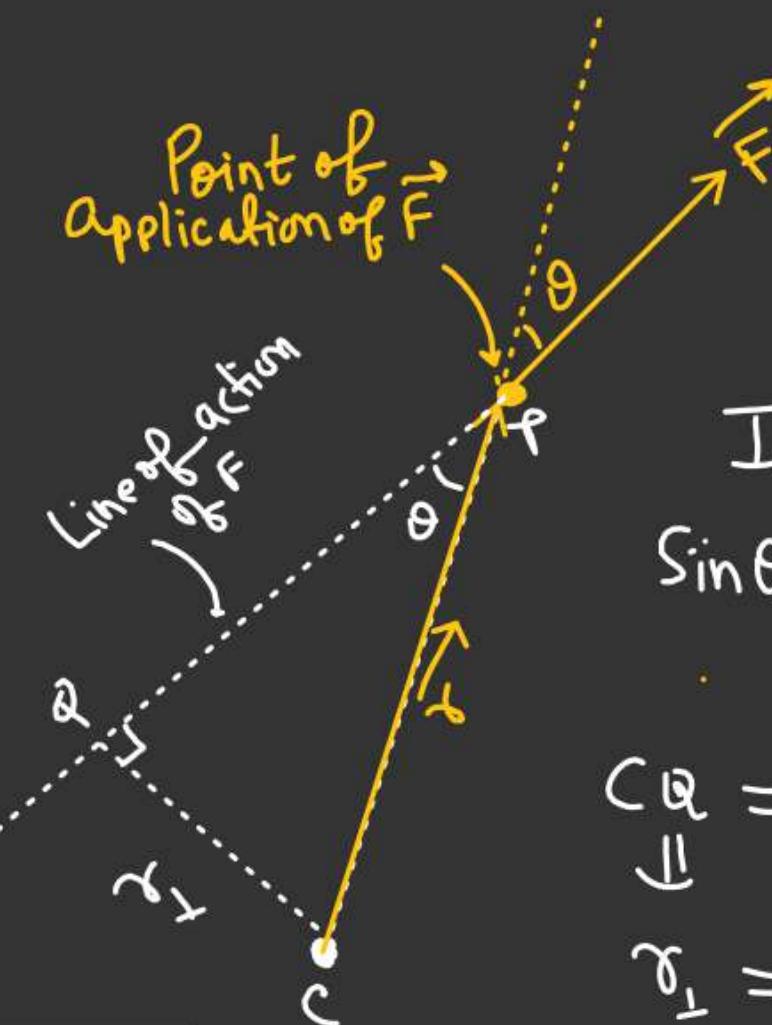
$$\vec{\tau}_{F/C} = (\vec{r} \times \vec{F}) \rightarrow$$

$$\begin{aligned} |\vec{\tau}_{F/C}| &= r F \sin\theta \\ &= (r \sin\theta) F \\ &= r_{\perp} F \end{aligned}$$

$\vec{r}$  = Position vector joining the point from where we have to calculate the torque to the point of application of force  $F$

$\Rightarrow \vec{\tau}$  is perpendicular to the plane containing  $\vec{r}$  &  $\vec{F}$

$$|\vec{\tau}_{F/C}| = r_{\perp} F$$



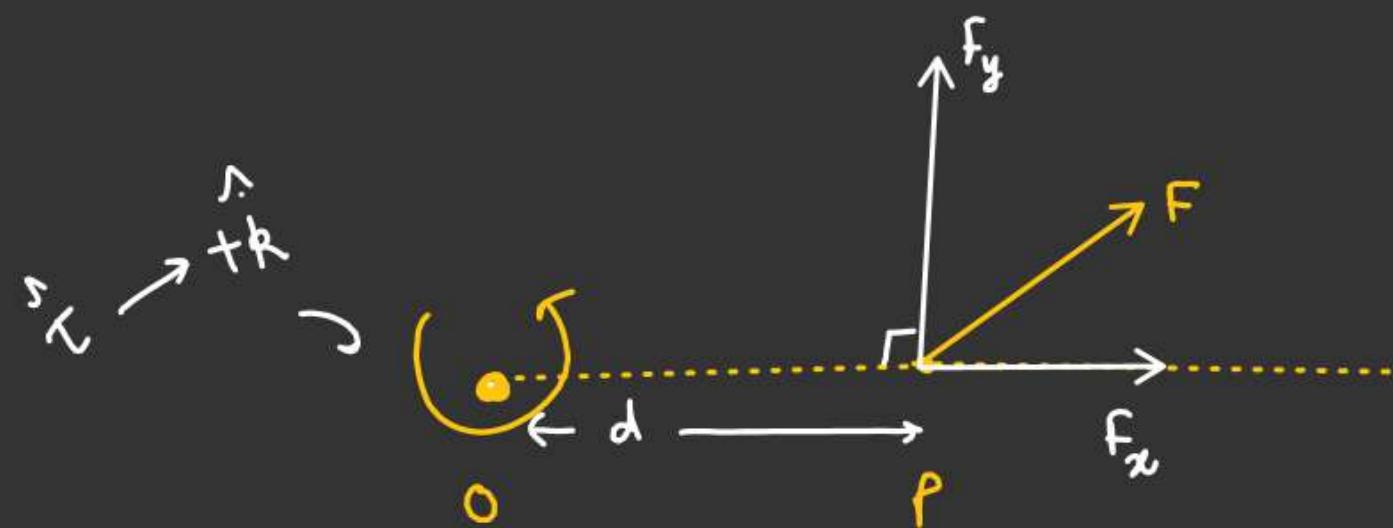
In  $\triangle PCQ$

$$\sin\theta = \frac{CQ}{PC}$$

$$CQ = PC \sin\theta$$

$$r_{\perp} = |r| \sin\theta$$

$r_{\perp} \rightarrow$  Perpendicular distance from the point where we have to calculate the torque to the line of action of  $F$



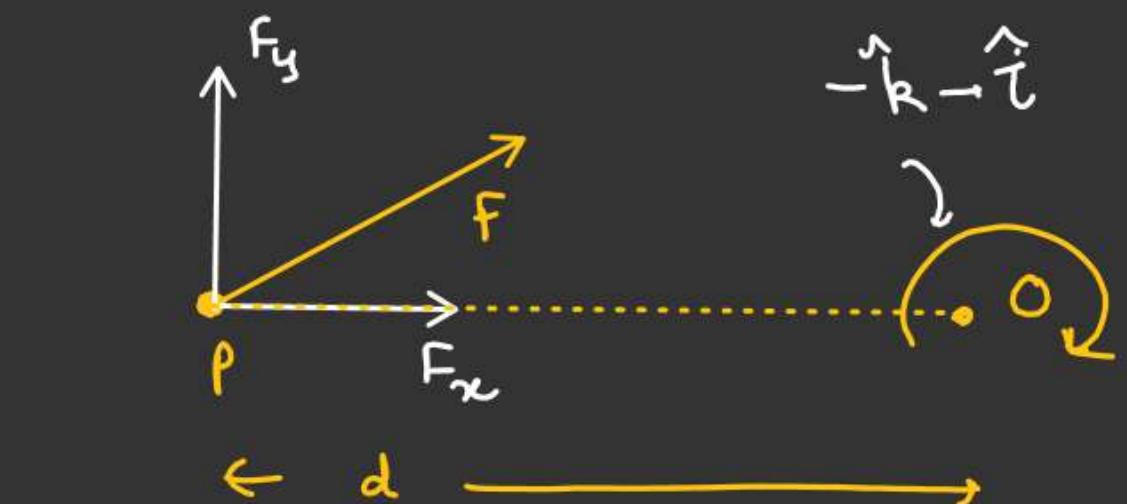
$$|\vec{\tau}_{F/O}| = |\vec{\tau}_{F_y/O}| = (F_y \cdot d)$$

$$\vec{\tau}_{F_x/O} = 0$$



line of action of  $F_x$   
passing through O

$$\text{So } \gamma_{\perp} = 0$$



$$|\vec{\tau}_{F/O}| = |\vec{\tau}_{F_y/O}| = F_y \cdot d$$

$$\vec{\tau} = (F_y d) (-\hat{k})$$

Find

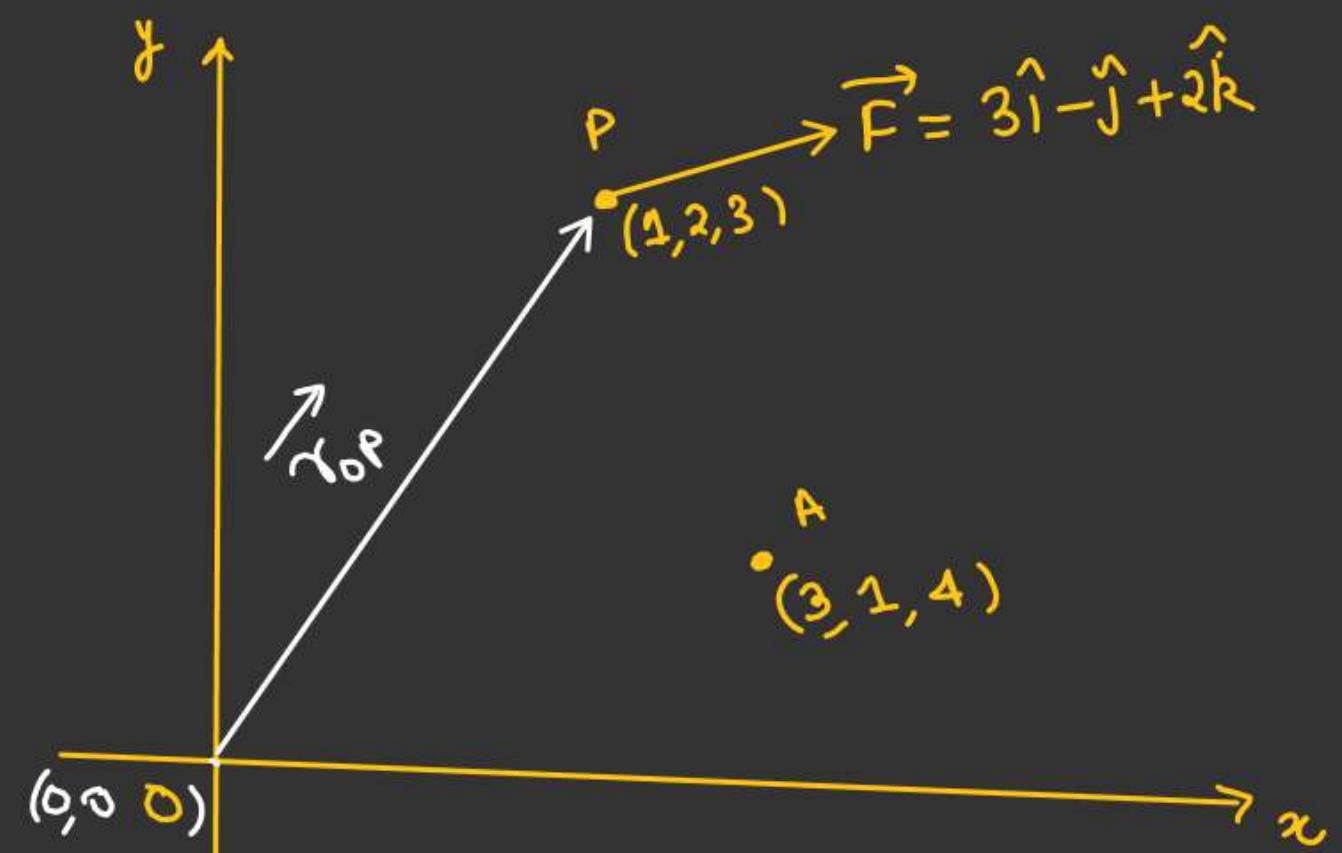
$$\overline{\tau}_{F/O} = ?$$

$$\overline{\tau}_{F/A} = ?$$

$$\begin{aligned}\overline{\tau}_{F/O} &= \overrightarrow{\gamma_{OP}} \times \vec{F} \\ &= (\hat{i} + 2\hat{j} + 3\hat{k}) \times (3\hat{i} - \hat{j} + 2\hat{k})\end{aligned}$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 3 & -1 & 2 \end{vmatrix} = \hat{i}(4 - (-3)) - \hat{j}(2 - 9) + \hat{k}(-1 - 6)$$

$$\underline{\underline{\tau}_{F/O}} = 7\hat{i} + 7\hat{j} - 7\hat{k} \quad |\overline{\tau}_{F/O}| = \sqrt{7^2 + 7^2 + 7^2} = 7\sqrt{3} \text{ N-mm}$$



Find  $\vec{\tau}_{F/A} = ?$

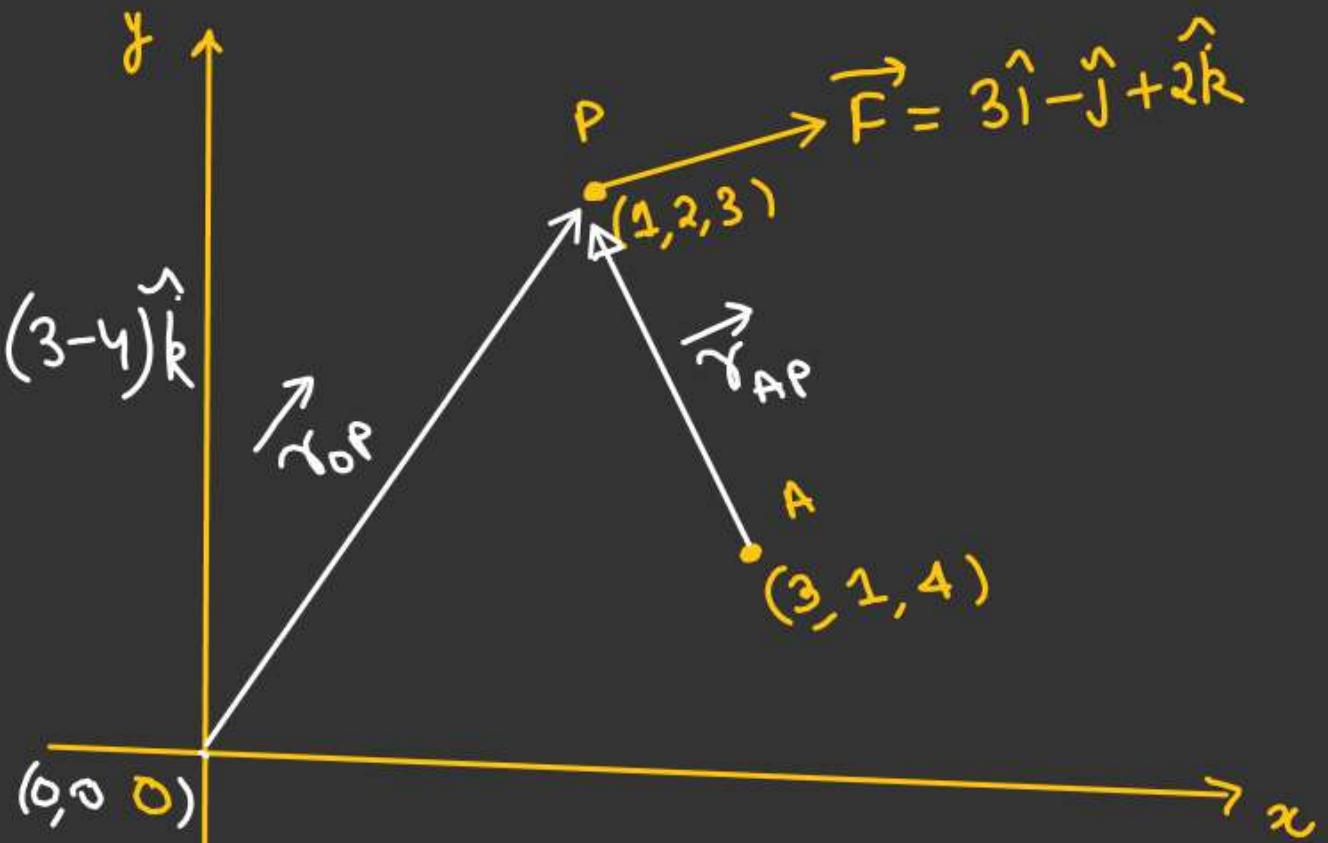
$$\vec{\tau}_{F/A} = (\vec{r}_{AP} \times \vec{F})$$

$$\Rightarrow \vec{r}_{AP} = (1-3)\hat{i} + (2-1)\hat{j} + (3-4)\hat{k}$$

$$\vec{\tau}_{F/A} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2 & 1 & -1 \\ 3 & -1 & 2 \end{vmatrix} = (-2\hat{i} + \hat{j} - \hat{k})$$

$$= \hat{i}(2-1) - \hat{j}(-4+3) + \hat{k}(2-3)$$

$$= \hat{i} + \hat{j} - \hat{k}$$



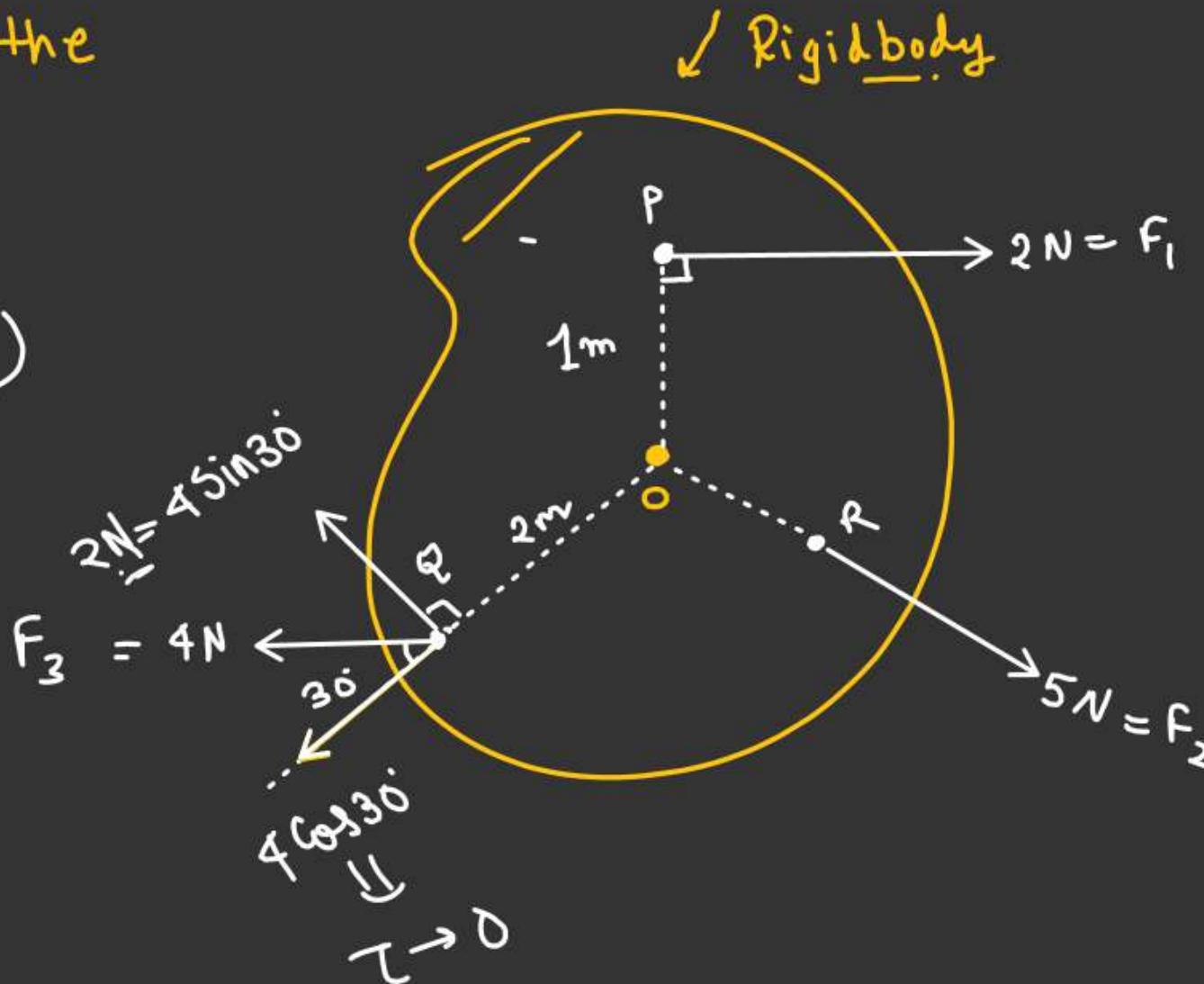
\* Find net torque acting on the body about O

$$\vec{\tau}_{F_1/O} = F_1 \tau_L = (2 \times 1)(-\hat{k})$$

$$\vec{\tau}_{F_2/O} = 0$$

$$\begin{aligned}\vec{\tau}_{F_3/O} &= (2 \times 2) \hat{k} \\ &= -4\hat{k}\end{aligned}$$

$$\vec{\tau}_{net/O} = \underline{-6\hat{k}} \text{ N-m}$$



✓ Rigid body