DPP-01

ORDER–DEGREE

1. The order and degree of the differential equation $\left(1+3\frac{dy}{dx}\right)^{\frac{2}{3}} = 4\frac{d^3y}{dx^3}$ are

(A) $1, \frac{2}{3}$ (B) 3, 1 (C) 1,2 (D) 3,3

2. The order and degree of the differential equation $\sqrt[3]{\frac{dy}{dx}} - 4\frac{d^2y}{dx^2} - 7x = 0$ are a and b, then $a + b$ is

(A) 3 (B) 4 (C) 5 (D) 6

3. The degree of the differential equation $\left(\frac{d^3y}{dx^3}\right)^{2/3} + 4 - 3\frac{d^2y}{dx^2} + 5\frac{dy}{dx} = 0$ is

(A) 1 (B) 2 (C) 3 (D) Not defined

4. The degree and order of the differential equation of the family of all parabolas whose axis is x - axis, are respectively-

(A) 2,3 (B) 2,1 (C) 1,2 (D) 3,2

5. The differential equation representing the family of curves $y^2 = 2c(x + \sqrt{c})$, where $c > 0$, is a parameter, is of order and degree as follows -

(A) order 1, degree 2
(B) order 1, degree 1
(C) order 1, degree 3
(D) order 2, degree 2

6. The differential equation whose solution is $Ax^2 + By^2 = 1$, where A and B are arbitrary constants is of -

(A) first order and second degree
(B) first order and first degree
(C) second order and first degree
(D) second order and second degree

7. Which of the following differential equations has the same order and degree?

(A) $\frac{d^4y}{dx^4} + 8\left(\frac{dy}{dx}\right)^6 + 5y = e^x$
(B) $5\left(\frac{d^3y}{dx^3}\right)^4 + 8\left(1+\frac{dy}{dx}\right)^2 + 5y = x^8$
(C) $\left[1+\left(\frac{dy}{dx}\right)^3\right]^{2/3} = 4\frac{d^3y}{dx^3}$
(D) $y = x^2\frac{dy}{dx} + \sqrt{1+\left(\frac{dy}{dx}\right)^2}$

8. The differential equation $\frac{d^2y}{dx^2} + \frac{dy}{dx} + \sin y + x^2 = 0$ is of the following type

(A) linear
(B) homogeneous
(C) order two
(D) degree one

FORMATION OF DIFFERENTIAL EQUATION

9. Number of values of $m \in N$ for which $y = e^{mx}$ is a solution of the differential equation $D^3y - 3D^2y - 4Dy + 12y = 0$ is

(A) 0 (B) 1 (C) 2 (D) more than 2

10. The value of the constant ' m ' and ' c ' for which $y = mx + c$ is a solution of the differential equation $D^2y - 3Dy - 4y = -4x$

(A) is $m = -1, c = 3/4$

(B) is $m = 1, c = 3/4$

(C) no such real m, c

(D) is $m = 1, c = -3/4$

11. The differential equation of the family of curves represented by $y = a + bx + ce^{-x}$ (where a, b, c are arbitrary constants) is

(A) $y''' = y'$

(B) $y''' + y'' = 0$

(C) $y''' - y'' + y' = 0$

(D) $y''' + y'' - y' = 0$

12. The differential equation whose solution is $(x-h)^2 + (y-k)^2 = a^2$ is (where a is a constant)

(A) $\left[1 + \left(\frac{dy}{dx}\right)^2\right]^3 = a^2\left(\frac{d^2y}{dx^2}\right)^2$

(B) $\left[1 + \left(\frac{dy}{dx}\right)^2\right]^3 = a^2\frac{d^2y}{dx^2}$

(C) $\left[1 + \left(\frac{dy}{dx}\right)\right]^3 = a^2\left(\frac{d^2y}{dx^2}\right)^2$

(D) $\left[1 + \left(\frac{dy}{dx}\right)^3\right]^2 = a^2\left(\frac{d^2y}{dx^2}\right)^2$

13. The differential equation representing all line at a distance p from the origin is-

(A) $(x^2 + y^2)\frac{dy}{dx} = 2y\left\{x - p\left(\frac{dy}{dx}\right)^2\right\}$

(B) $\left(x\frac{dy}{dx} - y\right)^2 - p^2\left\{1 + \left(\frac{dy}{dx}\right)^2\right\} = 0$

(C) $\left(x\frac{dy}{dx} - y\right)\left(p\frac{dy}{dx} + x\frac{dy}{dx}\right) = 0$

(D) $(x-y)\left(\frac{dy}{dx} - \frac{dx}{dy}\right) = 0$

14. If $y = e^{(K+1)x}$ is a solution of differential equation $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 0$, then k equals

(A) -1

(B) 0

(C) 1

(D) 2

15. The differential equation, which represents the family of plane curves $y = e^{cx}$, is-

(A) $y' = cy$

(B) $xy' - \log y = 0$

(C) $x\log y = yy'$

(D) $y\log y = xy'$

16. The differential equation $2xydy = (x^2 + y^2 + 1)dx$ determines

(A) A family of circles with centre on x-axis

(B) A family of circles with centre on y-axis

(C) A family of rectangular hyperbola with centre on x-axis

(D) A family of rectangular hyperbola with centre on yaxis

17. Let $y = (A + Bx)e^{3x}$ be a solution of the differential equation $\frac{d^2y}{dx^2} + m\frac{dy}{dx} + ny = 0, m, n \in I$, then

(A) $m + n = 3$

(B) $n^2 - m^2 = 64$

(C) $m = -6$

(D) $n = 9$

VARIABLE SEPARABLE

18. The solution to the differential equation $y/ny + xy' = 0$, where $y(1) = e$, is

(A) $x(\ eny\ ) = 1$
(B) $xy(\ ny\ ) = 1$
(C) $(\ \ln y\ )^2 = 2$
(D) $\ln\ y + \left(\frac{x^2}{2}\right) y = 1$

19. The solution of $\frac{xdy}{x^2+y^2} = \left(\frac{y}{x^2+y^2} - 1\right) dx$ is

(A) $y = x\cot\ (c - x)$
(B) $\cos^{-1}\ y/x = -x + c$
(C) $y = x\tan\ (c - x)$
(D) $y^2/x^2 = x\tan\ (c - x)$

20. The solution of the differential equation $dy = \sec^2 x\ dx$ is-

(A) $y = \sec\ x\tan\ x + c$
(B) $y = 2\sec\ x + c$
(C) $y = \frac{1}{2}\tan\ x + c$
(D) None of these

21. The solution of the differential equation $\frac{dy}{dx} = (1 + x)(1 + y^2)$ is-

(A) $y = \tan\ (x^2 + x + c)$
(B) $y = \tan\ (2x^2 + x + c)$
(C) $y = \tan\ (x^2 - x + c)$
(D) $y = \tan\ \left(\frac{x^2}{2} + x + c\right)$

22. The general solution of the differential equation, $y' + y\phi'(x) - \phi(x) \cdot \phi'(x) = 0$ where $\phi(x)$ is a known function is

(A) $y = ce^{-\phi(x)} + \phi(x) - 1$
(B) $y = ce^{\phi(x)} + \phi(x) + K$
(C) $y = ce^{-\phi(x)} - \phi(x) + 1$
(D) $y = ce^{-\phi(x)} + \phi(x) + K$

23. The equation of the curve through the point (1,0), whose slope is $\frac{y-1}{x^2+x}$, is-

(A) $(y - 1)(x + 1) + 2x = 0$
(B) $2x(y - 1) + x + 1 = 0$
(C) $x(y - 1)(x + 1) + 2 = 0$
(D) $x(y + 1) + y(x + 1) = 0$

24. The solution of the differential equation $ydx + (x + x^2y)dy = 0$ is-

(A) $-\frac{1}{xy} = C$
(B) $-\frac{1}{xy} + \log\ y = C$
(C) $\frac{1}{xy} + \log\ y = C$
(D) $\log\ y = Cx$

25. A curve passing through (2,3) and satisfying the differential equation

$\int_0^x ty\ (t)dt = x^2 y(x), (x > 0)$ is

(A) $x^2 + y^2 = 13$
(B) $y^2 = \frac{9}{2}x$
(C) $\frac{x^2}{8} + \frac{y^2}{18} = 1$
(D) $xy = 6$

26. The solution of $x^2dy - y^2dx + xy^2(x-y)dy = 0$ is

(A) $\ln\left|\frac{x-y}{xy}\right| = \frac{y^2}{2} + c$ (B) $\ell\left|\frac{xy}{x-y}\right| = \frac{x^2}{2} + c$

(C) $\ell n\left|\frac{x-y}{xy}\right| = \frac{x^2}{2} + c$ (D) $\ell n\left|\frac{x-y}{xy}\right| = x + c$

27. If $\int_a^x ty(t)dt = x^2 + y(x)$ then y as a function of x is

(A) $y = 2 - (2+a^2)e^{\frac{x^2-a^2}{2}}$ (B) $y = 1 - (2+a^2)e^{\frac{x^2-a^2}{2}}$

(C) $y = 2 - (1+a^2)e^{\frac{x^2-a^2}{2}}$ (D) $y = 2 + (2+a^2)e^{\frac{x^2+a^2}{2}}$

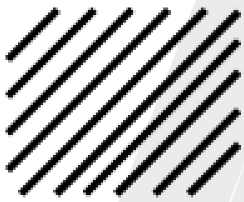
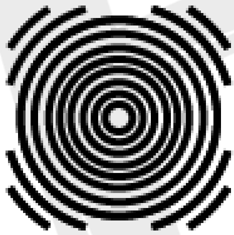
28. $y = ae^{-1/x} + b$ is a solution of $\frac{dy}{dx} = \frac{y}{x^2}$ then

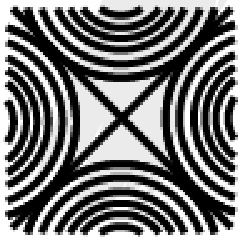
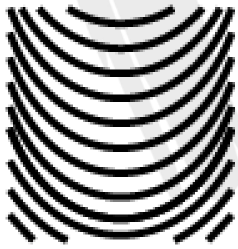
(A) $a \in R$ (B) $b = 0$

(C) $b = 1$ (D) a takes finite number of values

HOMOGENEOUS

29. The general solution of the differential equation $\frac{dy}{dx} = \frac{1-x}{y}$ is a family of curves which looks most like which of the following ?

(A)  (B) 

(C)  (D) 

30. The solution of the differential equation $(x^2+y^2)dx = 2xy\,dy$ is-

(A) $x = c(x^2+y^2)$ (B) $x = c(x^2-y^2)$

(C) $x + c(x^2+y^2) = 0$ (D) $y = c(x^2-y^2)$

31. The solution of the equation $x\frac{dy}{dx} = y - x\tan\left(\frac{y}{x}\right)$ is-

(A) $x\sin\left(\frac{x}{y}\right) + c = 0$ (B) $x\sin\ y + c = 0$

(C) $x\sin\left(\frac{y}{x}\right) = c$ (D) $y\sin\left(\frac{x}{y}\right) = c$

32. The solution of the differential equation $x\frac{dy}{dx} = y(\log y - \log x + 1)$ is-

(A) $y = xe^{cx}$ (B) $y + xe^{cx} = 0$

(C) $y + e^{x} = 0$ (D) $x = ye^{cy}$

33. The solution of the differential equation $x^2\frac{dy}{dx} = x^2 + xy + y^2$ is-

(A) $\tan^{-1}\left(\frac{y}{x}\right) = \log x + c$ (B) $\tan^{-1}\left(\frac{y}{x}\right) = -\log x + c$

(C) $\sin^{-1}\left(\frac{y}{x}\right) = \log x + c$ (D) $\tan^{-1}\left(\frac{x}{y}\right) = \log x + c$

34. If $x\frac{dy}{dx} = y(\log y - \log x + 1)$, then the solution of the equation is -

(A) $y\log\left(\frac{x}{y}\right) = cx$ (B) $x\log\left(\frac{y}{x}\right) = cy$

(C) $\log\left(\frac{y}{x}\right) = cx$ (D) $\log\left(\frac{x}{y}\right) = cy$

35. The solution of the differential equation $(x^2 - y^2)dx + 2xydy = 0$ is-

(A) $x^2 + y^2 = cx$ (B) $x^2 - y^2 + cx = 0$

(C) $x^2 + 2xy = y^2 + cx$ (D) $x^2 + y^2 = 2xy + cx^2$

36. The equation of the curve passing through origin and satisfying the differential equation $\frac{dy}{dx} = \sin(10x + 6y)$ is

(A) $y = \frac{1}{3}\tan^{-1}\left(\frac{5\tan 4x}{4-3\tan 4x}\right) - \frac{5x}{3}$ (B) $y = \frac{1}{3}\tan^{-1}\left(\frac{5\tan 4x}{4+3\tan 4x}\right) - \frac{5x}{3}$

(C) $y = \frac{1}{3}\tan^{-1}\left(\frac{3+\tan 4x}{4-3\tan 4x}\right) - \frac{5x}{3}$ (D) $y = \frac{1}{3}\tan^{-1}\left(\frac{\tan 4x}{4-3\tan 4x}\right) - \frac{5x}{3}$

37. A curve passes through the point $\left(1, \frac{\pi}{4}\right)$ & its slope at any point is given by $\frac{y}{x} - \cos^2\left(\frac{y}{x}\right)$. Then the curve has the equation

(A) $y = x\tan^{-1}\left(\ln\frac{e}{x}\right)$ (B) $y = x\tan^{-1}(\ell n + 2)$

(C) $y = \frac{1}{x}\tan^{-1}\left(\ln\frac{e}{x}\right)$ (D) $y = x\tan^{-1}(\ell oge)$

38. A function f(x) satisfying $\int_0^1 f(tx)dt = nf(x)$, where $x > 0$, is

(A) $f(x) = c.x^{\frac{1-n}{n}}$ (B) $f(x) = c \cdot x^{\frac{n}{n-1}}$

(C) $f(x) = c.x^{\frac{1}{n}}$ (D) $f(x) = c \cdot x^{(1-n)}$

39. The general solution of the differential equation $\frac{dy}{dx} + \frac{1+\cos 2y}{1-\cos 2x} = 0$, is given by-

(A) $\tan y + \cot x = c$ (B) $\tan y - \cot x = c$

(C) $\tan x - \cot y = c$ (D) $\tan x + \cot y = c$

40. The solution of the differential equation $\frac{dy}{dx}=\frac{xy}{x^2+y^2}$ is-

(A) $ay^2=e^{x^2/y^2}$
(B) $ay=e^{x/y}$
(C) $y=++c$
(D) $y=+y^2+c$

41. Which one of the following is homogeneous function?

(A) $f(x,y)=\frac{x-y}{x^2+y^2}$

(B) $f(x,y)=x^{\frac{1}{3}}\cdot y^{-\frac{2}{3}}\tan^{-1}\frac{x}{y}$

(C) $f(x,y)=x\left(\ell n\sqrt{x^2+y^2}-\ell ny\right)+ye^{x/y}$

(D) $f(x,y)=x\left[\ln\frac{2x^2+y^2}{x}-\ln(x+y)\right]+y^2\tan\left(\frac{x+2y}{3x-y}\right)$

EXACT DIFFERENTIAL EQUATION

42. Solution of $\sec^2 x\tan y dx+\sec^2 y\tan x dy=0$ is

(A) $\tan x\tan y=C$
(B) $\tan x/\tan y=C$
(C) $\sec x\sec y=C$
(D) $\tan y/\tan x=C$

43. Solution of $y\log y\, dx-xdy=0$ is

(A) $y=e^{cx}$
(B) $y=e^{-cx}$
(C) $y=\log x$
(D) $y=e^{x\log c}$

44. Solution of $x\cos y\, dy=(xe^x\log x+e^x)dx$ is

(A) $\sin y=\log x+c$
(B) $\sin y=e^{-x}\log x+c$
(C) $\sin y=e^x\log x+c$
(D) $\sin y=e^x\log x+c$

45. Solve the differential equation $(2xy-3x^2)dx+(x^2-2y)dy=0$

(A) $x^2y-x^3-y^2=c$
(B) $x^2y+x^3+y^2=c$
(C) $x^2y+x^3-y^2=c$
(D) None of these

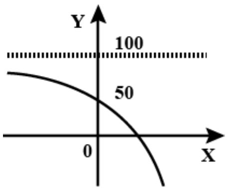
46. Find the particular solution of $(\cos x-x\sin x+y^2)dx+2xydy=0$ that satisfies the initial condition $y=1$ when $x=\pi$

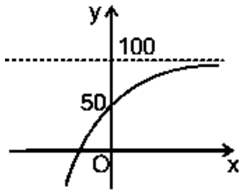
(A) $xy^2+x\cos x=0$
(B) $xy^2-x\cos x=0$
(C) $xy^2+x\sin x=0$
(D) $x^2y+x\sin x=0$

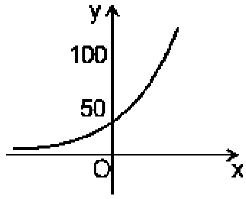
MIXED PROBLEMS

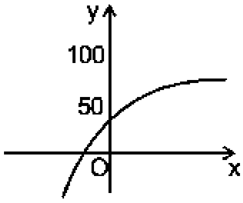
47. Which one of the following curves represents the solution of the initial value problem

$Dy = 100 - y$ where $y(0) = 50$

(A) 

(B) 

(C) 

(D) 

48. Which of the following transformation reduce the differential equation

$\frac{dz}{dx} + \frac{z}{x} log\ z = \frac{z}{x^2}(log\ z)^2$ into the form $\frac{du}{dx} + P(x)u = Q(x)$?

(A) $u = log\ z$ (B) $u = e^z$ (C) $u = (log\ z)^{-1}$ (D) $u = (log\ z)^2$

49. If $y = \frac{x}{\ell n|cx|}$ (where c is an arbitrary constant) is the general solution of the differential equation $\frac{dy}{dx} = \frac{y}{x} + \phi\left(\frac{x}{y}\right)$ then the function $\phi\left(\frac{x}{y}\right)$ is

(A) $\frac{x^2}{y^2}$ (B) $-\frac{x^2}{y^2}$ (C) $\frac{y^2}{x^2}$ (D) $-\frac{y^2}{x^2}$

50. If $f''(x) + f'(x) + f^2(x) = x^2$ be the differential equation of a curve and let P be the point of maxima then number of tangents which can be drawn from point P to $x^2 - y^2 = a^2$ is

(A) 2 (B) 1 (C) 0 (D) either 1 or 2

51. If $y = e^{(K+1)x}$ is a solution of differential equation $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 0$, then k equals

(A) -1 (B) 0 (C) 1 (D) 2

SUBJECTIVE (JEE ADVANCED)

52. State the order & degree of the following differential equations :

(i) $\left[\frac{d^2x}{dt^2}\right]^3 + \left[\frac{dx}{dt}\right]^4 - xt = 0$

(ii) $\frac{d^2y}{dx^2} = \left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}$

53. $\frac{ln\,(sec\ x + tan\ x)}{cos\ x} dx = \frac{\ell n(sec\ y + tan\ y)}{cos\ y} dy$

54. $\frac{dy}{dx} + \frac{\sqrt{(x^2-1)(y^2-1)}}{xy} = 0$

55. $\frac{dy}{dx} = \sin(x+y) + \cos(x+y)$

56. $e^{(dy/dx)} = x + 1$ given that when $x = 0, y = 3$

57. $(x - y^2)dx + 2xydy = 0$

58. $\frac{dy}{dx} = e^{x-y}(e^x - e^y)$

59. $xdy + ydx + \frac{xdy - ydx}{x^2+y^2} = 0$

60. (a) $\frac{dy}{dx} = \frac{x^2+xy}{x^2+y^2}$ (b) $(x^3 - 3xy^2)dx = (y^3 - 3x^2y)dy$

61. $\left[x\cos\frac{y}{x} + y\sin\frac{y}{x}\right]y = \left[y\sin\frac{y}{x} - x\cos\frac{y}{x}\right]x\frac{dy}{dx}$

62. $(x - y)dy = (x + y + 1)dx$

63. $\frac{dy}{dx} = \frac{x+y+1}{2x+2y+3}$

64. Show that the curve such that the distance between the origin and the tangent at an arbitrary point is equal to the distance between the origin and the normal at the same point.

$\sqrt{x^2 + y^2} = ce^{\pm\tan^{-1}\frac{y}{x}}$

65. The light rays emanating from a point source situated at origin when reflected from the mirror of a search light are reflected as beam parallel to the x-axis. Show that the surface is parabolic, by first forming the differential equation and then solving it.

66. The perpendicular from the origin to the tangent at any point on a curve is equal to the abscissa of the point of contact. Find the equation of the curve satisfying the above condition and which passes through (1,1).

67. Find the curve for which any tangent intersects the y-axis at the point equidistant from the point of tangency and the origin.

68. The population P of a town decreases at a rate proportional to the number by which the population exceeds 1000 , proportionality constant being $k > 0$. Find

(a) Population at any time t, given initial population of the town being 2500 .

(b) If 10 years later the population has fallen to 1900 , find the time when the population will be 1500 .

(c) Predict about the population of the town in the long run.

PREVIOUS YEAR (JEE MAIN)

69. The differential equation of all circles passing through the origin and having their centres on the x-axis is- **[AIEEE 2007]**

(A) $x^2 = y^2 + xy\frac{dy}{dx}$ (B) $x^2 = y^2 + 3xy\frac{dy}{dx}$

(C) $y^2 = x^2 + 2xy\frac{dy}{dx}$ (D) $y^2 = x^2 - 2xy\frac{dy}{dx}$

70. The differential equation which represents the family of curves $y = c_1 e^{c_2 x}$ where c_1 and c_2 are arbitrary constants, is - **[AIEEE 2009]**

(A) $y' = y^2$ (B) $y'' = y'y$

(C) $yy'' = y'$ (D) $yy'' = (y')^2$

71. If $\frac{dy}{dx} = y + 3 > 0$ and $(0) = 2$, then $y(ln\ 2)$ is equal to : **[AIEEE 2011]**

(A) 7 (B) 5 (C) 13 (D) -2

72. The population $p(t)$ at time t of a certain mouse species satisfies the differential equation $\frac{dp(t)}{dt} = 0.5\ p(t) - 450$. If $p(0) = 850$, then the time at which the population becomes zero is: [AIEEE 2012]

(A) $\frac{1}{2} ln\ 18$ (B) $ln\ 18$ (C) $2ln\ 18$ (D) $ln\ 9$

73. Let $y = y(x)$ be the solution of the differential equation

$$sin\ x\frac{dy}{dx} + ycos\ x = 4x, x \in (0, \pi).$$

If $y\left(\frac{\pi}{2}\right) = 0$, then $y\left(\frac{\pi}{6}\right)$ is equal to : **[JEE-MAIN 2018]**

(A) $-\frac{4}{9}\pi^2$ (B) $\frac{4}{9\sqrt{3}}\pi^2$ (C) $\frac{-8}{9\sqrt{3}}\pi^2$ (D) $-\frac{8}{9}\pi^2$

PREVIOUS YEAR (JEE ADVANCED)

74. A curve passes through the point $\left(1, \frac{\pi}{6}\right)$. Let the solpe of the curve at each point (x, y) be $\frac{y}{x} + sec\ \left(\frac{y}{x}\right), x > 0$. Then the equation of the curve is **[JEE 2013]**

(A) $sin\ \left(\frac{y}{x}\right) = log\ x + \frac{1}{2}$ (B) $cosec\ \left(\frac{y}{x}\right) = log\ x + 2$

(C) $sec\ \left(\frac{2y}{x}\right) = log\ x + 2$ (D) $cos\ \left(\frac{2y}{x}\right) = log\ x + \frac{1}{2}$

75. consider the family of all circles whose centers lie on the straight line $y = x$. If this family of circles is represented by the differential equation Py" + Qy' +1 = 0, where P, Q are fucntions of x, y and y' (here $y' = \frac{dy}{dx}, y'' = \frac{d^2y}{dx^2}$), then which of the following statement is (are) true?

(A) $P = y + x$ (B) $P = y - x$ **[JEE 2015]**

(C) $P + Q = 1 - x + y + y' + (y')^2$ (D) $P - Q = x + y - y' - (y')^2$

76. If $y = y(x)$ satisfies the differential equation

$8\sqrt{x}(\sqrt{9+\sqrt{x}})dy = (\sqrt{4+\sqrt{9+\sqrt{x}}})^{-1}dx, x > 0$ and $y(0) = \sqrt{7}$, then $y = (256) =$

(A) 3 (B) 16 (C) 9 (D) 80 **[JEE 2017]**

ANSWER KEY

1.	(D)	**2.**	(C)	**3.**	(B)	**4.**	(C)	**5.**	(C)	**6.**	(C)	**7.**	(C)
8.	(CD)	**9.**	(C)	**10.**	(D)	**11.**	(B)	**12.**	(A)	**13.**	(B)	**14.**	(C)
15.	(D)	**16.**	(C)	**17.**	(ACD)	**18.**	(A)	**19.**	(C)	**20.**	(D)	**21.**	(D)
22.	(A)	**23.**	(A)	**24.**	(B)	**25.**	(D)	**26.**	(A)	**27.**	(A)	**28.**	(AB)
29.	(B)	**30.**	(B)	**31.**	(C)	**32.**	(A)	**33.**	(A)	**34.**	(C)	**35.**	(A)
36.	(A)	**37.**	(A)	**38.**	(A)	**39.**	(B)	**40.**	(A)	**41.**	(ABC)	**42.**	(A)
43.	(A)	**44.**	(C)	**45.**	(A)	**46.**	(A)	**47.**	(B)	**48.**	(C)	**49.**	(D)
50.	(D)	**51.**	(C)	**52.**	(i) order 2 & degree 3 (ii) order 2 & degree 2								

53. $\ell n^2(sec\ x + tan\ x) - \ell n^2(sec\ y + tan\ y) = c$

54. $\sqrt{x^2-1} - sec^{-1}\ x + \sqrt{y^2-1} = c$

55. $\ell n\left[1 + tan\ \frac{x+y}{2}\right] = x + c$

56. $y = (x+1)\cdot ln\ (x+1) - x + 3$

57. $y^2 + x\ell nax = 0$

58. $e^y = c\cdot exp\ (-e^x) + e^x - 1$

59. $xy + tan^{-1}\ \frac{y}{x} = c$

60. **(a)** $c(x-y)^{2/3}(x^2+xy+y^2)^{1/6} = exp\ \left[\frac{1}{\sqrt{3}} tan^{-1}\ \frac{x+2y}{x\sqrt{3}}\right]$ where $\exp x \equiv e^x$,

(b) $y^2 - x^2 = c(y^2 + x^2)^2$

61. $\mathrm{xycos}\ \frac{y}{x} = c$

62. $\arctan \frac{2y+1}{2x+1} = \ln\ c\sqrt{x^2+y^2+x+y+\frac{1}{2}}$

63. $x + y + \frac{4}{3} = ce^{3(x-2y)}$

66. $x^2 + y^2 - 2x = 0$ 67. $x^2 + y^2 = cx$

68. **(a)** $P = 1000 + 1500e^{-kt}$ where $k = \frac{1}{10}\ln\left(\frac{5}{3}\right)$;**(b)** $T = 10\log_{5/3}(3)$; **(c)** $P = 1000$ as $t \to \infty$

69.	(C)	**70.**	(D)	**71.**	(A)	**72.**	(C)	**73.**	(D)	**74.**	(D)	**75.**	(BC)
76.	(D)												