Q A is a pt. on $y^2=4ax$

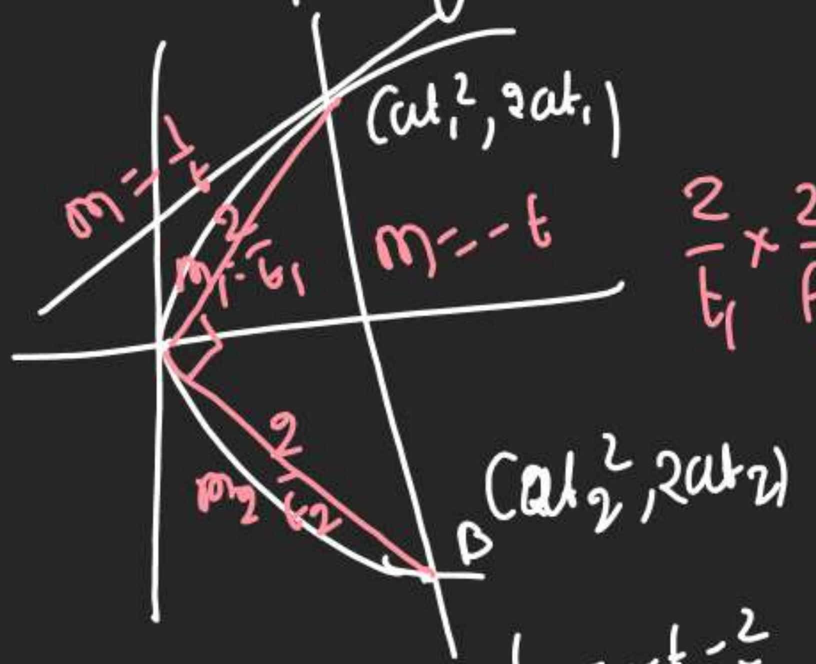
The normal at A cuts

Parabola again at B

If AB subtends Rt. angle

at vertex of Parabola, then

find Slope of Normal at B.



$\frac{2}{t_1}\times\frac{2}{t_2}=-1$

$t_1t_2=-4$

$t_2=-\frac{4}{t_1}$

$t_2=-t_1-\frac{2}{t_1}$

$-\frac{4}{t_1}=-t_1-\frac{2}{t_1}$

$+\frac{2}{t_1}=+t_1$

$t_1^2=2$

$t_1=\sqrt{2},-\sqrt{2}$

$t_2=-\sqrt{2}-\frac{2}{\sqrt{2}}$

$t_2=-2\sqrt{2},\ 2\sqrt{2}$

$m=2\sqrt{2},-2\sqrt{2}$

Conormal Pts:

Normal $\Rightarrow y=mx-2am-am^3$

$\Rightarrow am^3+m(2a-x)+y=0$

m's Cubic Eqn

It has 3 Roots $\Rightarrow m_1,m_2,m_3$

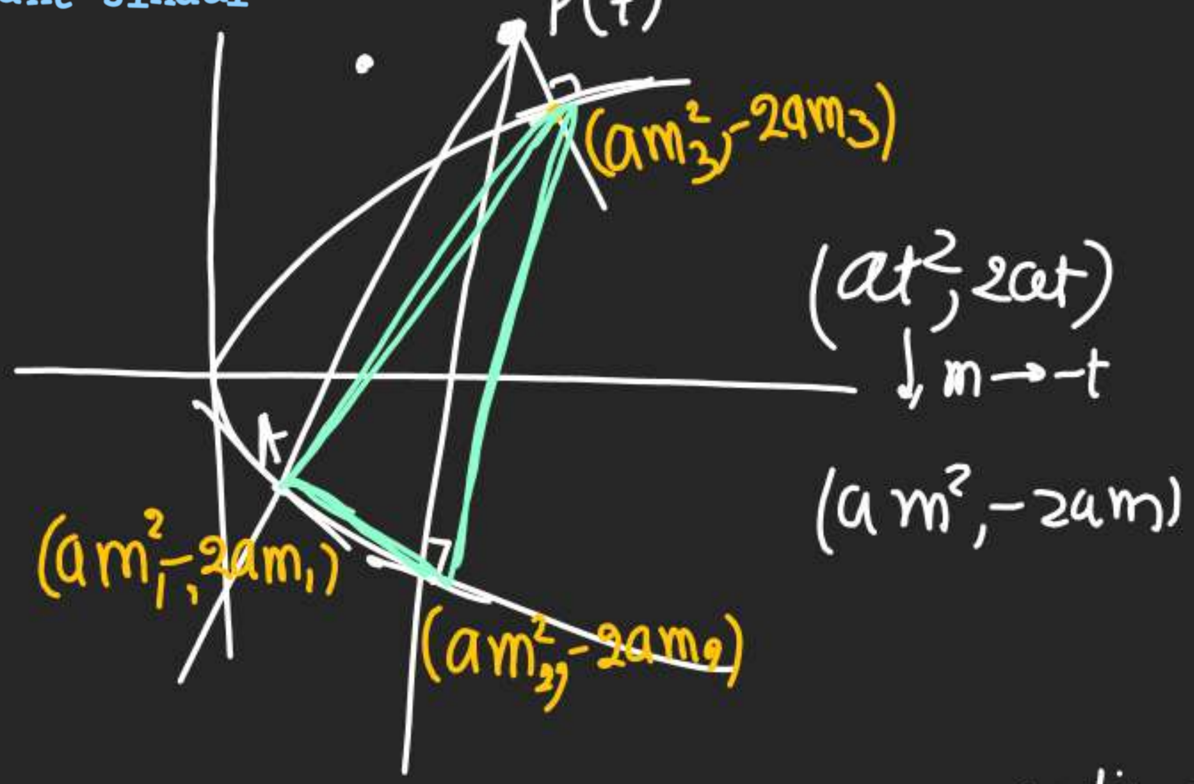
$m_1+m_2+m_2=-\frac{\text{Coff of } m^2}{\text{Const}}=0$

$\Sigma m_1m_2=\frac{c}{a}=\frac{2a-x}{a}$

$m_1m_2m_3=-\frac{d}{a}=\frac{y}{a}$

It shows that from any Pt in Parbola there exist ~~at~~ atmost 3 Normals Possible.

Nishant Jindal



Q Find Algebric Sum of feet ordinate of 3 Normal drawn to the Parabola from any Pt.

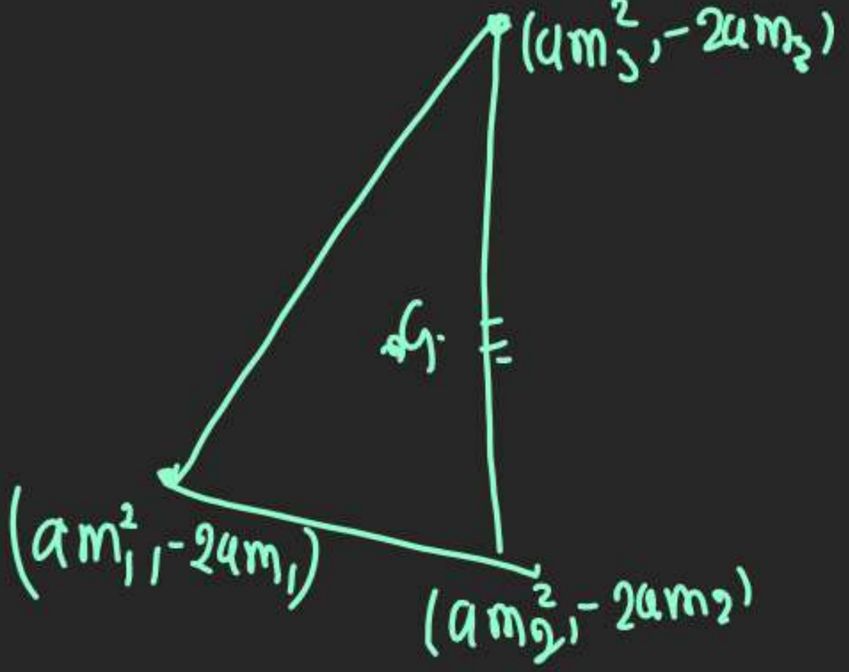
Sum of ord $= -2am_1 + -2am_2 + -2am_3$

$= -2a(m_1+m_2+m_3)$

$= -2a \times 0$

$= 0$

Q P.T. Centroid of Δ formed by Co-normal Pts as vertices lies on axis of Parabola



Y Coord of Centroid (G)

$= \dfrac{-2am_1 - 2am_2 - 2am_3}{3}$

$= \dfrac{0}{3} = 0$

$\Rightarrow$ G will be on Axis

Q If 2 of 3 feet of Normal drawn from a pt to the Parabola $y^2 = 4x$ be the $(1,2)$ & $(1,-2)$ find 3rd foot?

Sum of y coord $= 0$

$2 + -2 + y = 0$

$y = 0$

It has to be on $y^2 = 4x$

$0^2 = 4x \Rightarrow x = 0$

$\Rightarrow (0,0)$ is 3rd foot

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Q If 3 Normals drawn to any Parabola $y^2=4ax$ from a given pt. (h,K) be Real then P.T. $h>2a$

as from (h,K) 3 normals can be drawn. so they are satisfying

$$am^3+(2a-x)m+y=0$$

$$\Rightarrow am^3+(2a-h)m+K=0$$

as all 3 pts are Real in θs

$\Rightarrow m_1\ m_2\ m_3$ all 3 Real.

$$m_1^2+m_2^2+m_3^2>0$$

$$2(m_1^2+m_2^2+m_3^2)>0$$

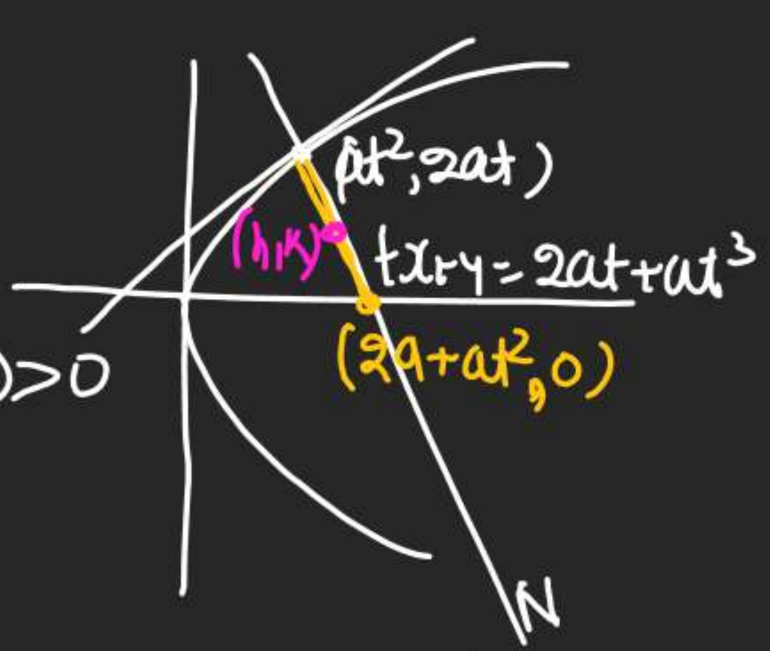
$$\Rightarrow (m_1+m_2+m_3)^2-2(m_1m_2+m_2m_3+m_3m_1)>0$$

$$\Rightarrow 0-2\left(\frac{(2a-h)}{a}\right)>0$$

$$\frac{2a-h}{a}<0$$

$$h>2a.$$

Q P.T. Locus of Middle Pt. of Portion of Normal to $y^2=4ax$ Intercepted betn Curve & Axis is $y^2=a(x-a)$



$$h=\frac{at^2+(2a+at^2)}{2}$$

$$K=\frac{2at+0}{2}$$

$$at=K$$

$$t=\frac{K}{a}.$$

$$h=a+at^2$$

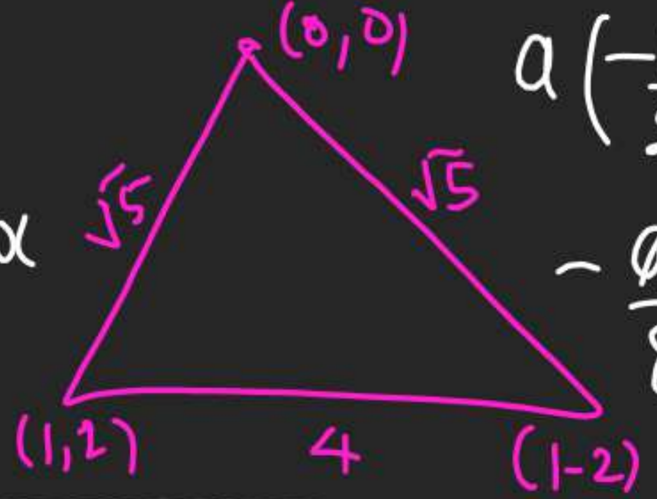
$$h-a=at^2$$

$$(h-a)=a\frac{K^2}{a^2}$$

$$\Rightarrow y^2=a(x-a)$$

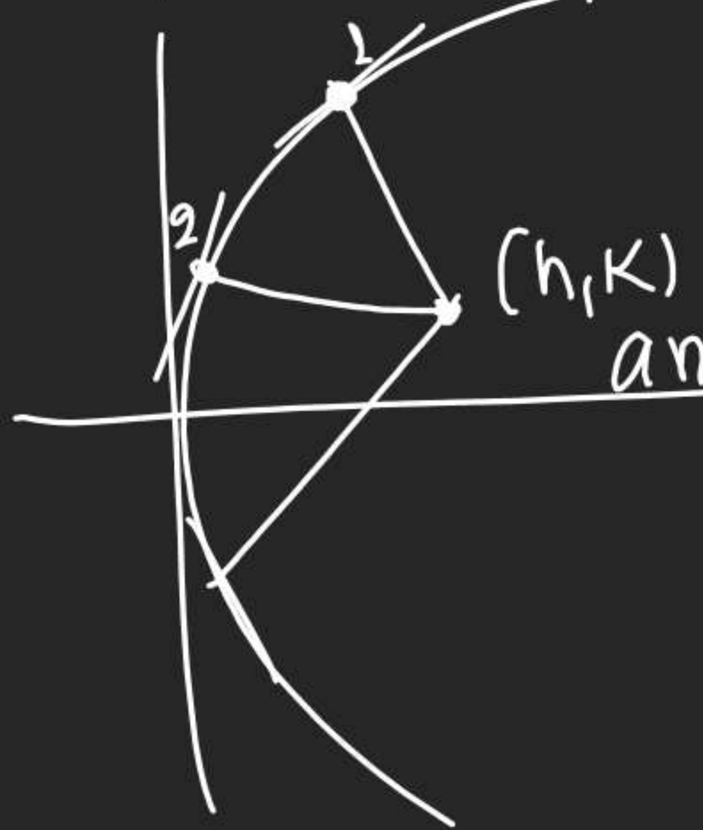
H.P.

Q If 2 Normals are drawn from any pt. on $y^2=4ax$ making angle α & β with axis such that $\boxed{\tan\alpha\cdot\tan\beta=2}$ then find Locus of that Pt.



$m_1=\tan\alpha$

$m_2=\tan\beta$



$am^3+(2a-h)m+K=0 \;\to\; m_1, m_2, m_3$

$\boxed{m_1m_2}m_3=-\frac{K}{a}$

$2m_3=-\frac{K}{a}$

$m_3=-\frac{K}{2a}$

$a\left(-\frac{K}{2a}\right)^3+(2a-h)\left(-\frac{K}{2a}\right)+K=0$

$-\frac{ay^3}{8a^3}-\frac{(2a-x)y}{2a}+y=0$

$-y^3-4ay(2a-x)+8a^2y=0$

$-y^2-4a(2a-x)+8a^2=0$

$-y^2-\cancel{8a^2}+4ax+\cancel{8a^2}=0$

$y^2=4ax$

Q (3,0) is a pt. from which 3 Normals are drawn to $y^2=4x$ which meet the Parabola at P, Q, R find (1) Area of ΔPQR (2) Circumradius (R) (3) Centroid

$am^3+(2a-x)m+y=0$ P.T. (3,0)

$m^3+(2-3)m+0=0$

$m^3-m=0$

$m=0,1,-1$

$(am^2,-2am) \to (0,0),\ (1,-2),\ (1,2)$

$\Delta=\frac{1}{2}\begin{vmatrix}0&0\\1&-2\\1&2\\0&0\end{vmatrix}=\frac{1}{2}\{4\}=2$

(2) Circumrad $=R=\frac{a\cdot b\cdot c}{4\Delta}$

$R=\frac{\sqrt5\times\sqrt5\times4}{4\times2}=\frac{5}{2}$

(3) Centroid $=\left(\frac{2}{3},0\right)$

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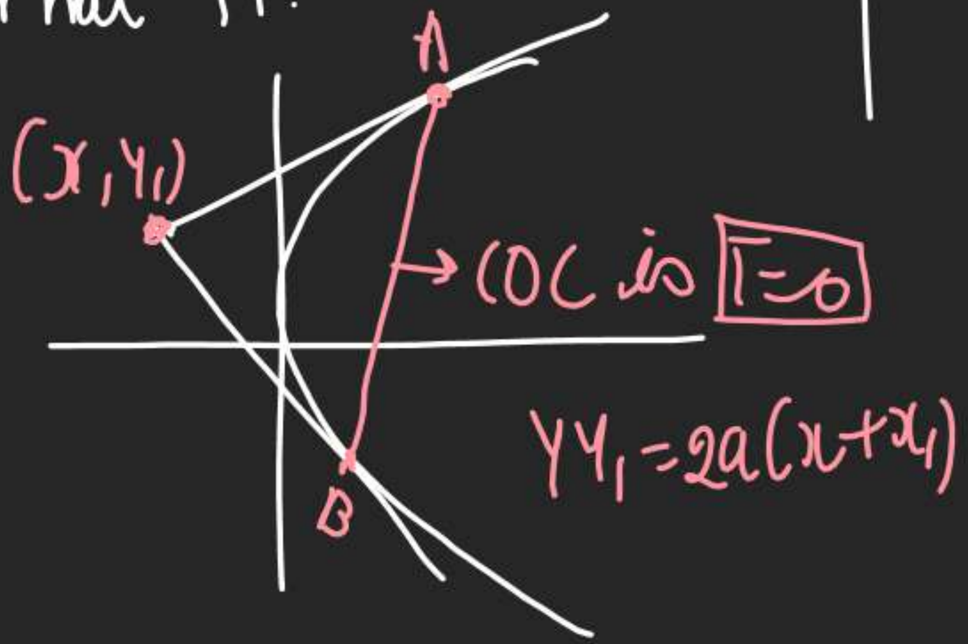
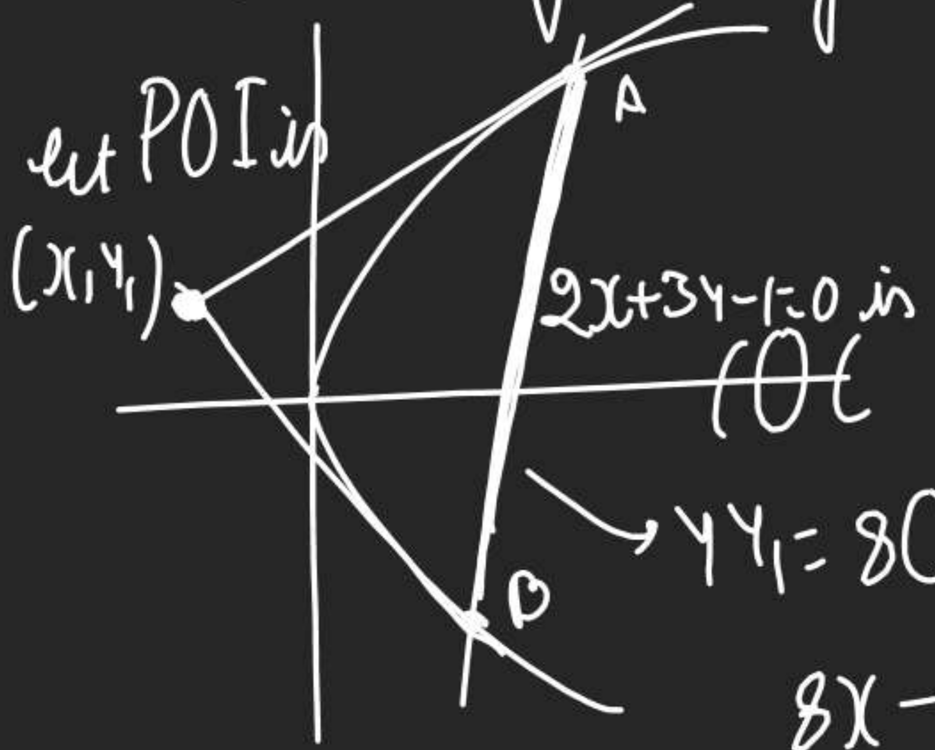
Chord of Contact

A chord joining 2 Pts of Contact of a pair of tangents drawn from an external Pt.



Q Tangents are drawn to $y^2 = 16x$ at Pts where Line $2x+3y-1=0$ meets Parabola. Find Point of Intersection of these tangents.



$\rightarrow yy_1 = 8(x+x_1)$

$$\left.\begin{aligned} 8x - yy_1 + 8x_1 = 0 \\ 2x + 3y - 1 = 0 \end{aligned}\right\} \frac{\cancel{8}\,4}{\cancel{2}} = \frac{-y_1}{3} = \frac{8x_1}{-1}$$

$x_1 = -\frac{1}{2}$

$y_1 = -12$

$(x_1, y_1) \equiv \left(-\frac{1}{2}, -12\right)$

Chord having Mid Pt.

If Mid Pt of chord is (x_1, y_1) given then Eqn of chord will be $T = S_1$

$$\boxed{yy_1 - 2a(x+x_1) = y_1^2 - 4ax_1}$$

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Q If a chord which is not a tangent to $y^2 = 16x$ has the eq^n $2x+y=p$ & mid pt (h, K) then which of the following is/are possible values of p, h & K.

A) $p=-2$, $h=2$, $K=-4$ ✓

B) $p=-1$, $h=1$, $K=-3$ ✗

C) $p=2$, $h=3$, $K=-4$ ✓

D) $p=5$, $h=4$, $K=-3$ ✗

If chord $2x+y=p$ has mid pt (h, K) it must satisfy Eq^n

(2) But if Mid Pt (h, K) then by $T = S_1$

$yK - 8(x+h) = K^2 - 16h$.

$8x - Ky + 8h + K^2 - 16h = 0$

$8x - Ky + K^2 - 8h = 0 \rightarrow$ (B)

$2x + y - p = 0$

$4 \; \frac{\cancel{8}}{\cancel{2}} = \frac{-K}{1} = \frac{K^2-8h}{-p}$

1-15 Ex 1

1-10 Ex 2(1)

$K = -4$ | $4 = \frac{K^2-8h}{-p}$

$4 = \frac{16-8h}{p} = \cancel{\frac{16-8\times 2}{-2}}$

$= \frac{16-8\times 3}{-2} = \frac{-8}{-2} = 4$