x^{α}. LDNE type Qs.

Q. $f(x) = \begin{cases} \dfrac{x}{1+e^{1/x}} & x \neq 0 \\ 0 & x = 0 \end{cases}$ then $f(x)$ is diff^{ble} for.

$x \in R^+$ $x \in R$ $x \in R_0$ NOT.

$f(x) = \begin{cases} x^1 \cdot \boxed{\dfrac{1}{1+e^{1/x}}} & \text{LDNE} \\ 0 \end{cases}$

LHL $= \dfrac{1}{1+e^{1/x}} = 1$

Chor $x \neq 0$

RHL $= \dfrac{1}{1+e^{\infty}} = 0$

$x = 0$

$n = 1 \Rightarrow$ Cont^s But Not Diff^{ble} at $x = 0$

$x \in R_0$
$x \in R - \{0\}$

Q. Comment on derivative $f(x)$ at $x = 0$

Where $f(x) = \begin{cases} x \cdot \boxed{tn^{-1} \dfrac{1}{x}} & \text{LDNE} \quad x \neq 0 \\ 0 & x = 0 \end{cases}$

$n = 1 \Rightarrow$ Cont^s But ND at $x = 0$

Q. $f(x) = \begin{cases} \dfrac{x^1 \boxed{(3e^{1/x}+4)}}{2 - e^{1/x}} & \text{LDNE} \quad x \neq 0 \\ 0 & x = 0 \end{cases}$

(Comment on diff^y at $x = 0$)

$n = 1 \Rightarrow$ Cont^s But N.D. at $x = 0$

Q $f(x) = \begin{cases} 3^x & -1 \le x \le 1 \\ 4-x & 1 < x \le 4 \end{cases}$

(Check diff^y at x=1 ?

1) Cont^y at x=1

$3^1 = 4-1 \Rightarrow 3 = 3$ ✓

2) $f'(x) = \begin{cases} 3^x \ln 3 & -1 \le \boxed{x \le 1} \\ \boxed{-1} & 1 < x < 4. \end{cases}$

diff^y at x=1

LHD $= 3^1 \ln 3 = 3 \ln 3$ $\Big\}$ as Both

RHD $= -1$ ✓   ND.

$3^1 \ln 3 \ne -1$

Q $f(x) = \begin{cases} A+Bx^2 & x<1 \\ 3Ax-B+2 & x \ge 1 \end{cases}$ diff$^{b/e}$ at x=1

find A, B?

Cont$s$ x=1

$A+B(1)^2 = 3A-B+2 \to ①$

$f'(x) = \begin{cases} 2Bx & x<1 \\ 3A & x \ge 1 \end{cases}$

LHD = RHD

$2B = 3A \to ②$ ✓

Solve A & B.

Properties of diff^y

1)

$f(x)$	$g(x)$	$f(x) \pm g(x)$	$f(x) \times g(x)$
D	D	D	D
D	ND	ND	M
ND	ND	M	M.

2) Every Poly fxn is diff
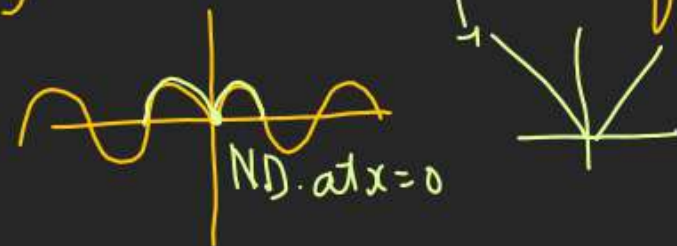
3) Every Const. fxn is diff.

4) Exp. $(a>0)$ are diff.

(5) log fxn are diff in their domain

(6) $\sin x, \cos x, \tan x, \cot x, \sec x, \csc x$ are diff^{ble} in their domain

(7) $|f(x)|$ is doubtfull when $f(x)=0$

$$ND + ND = Maybe$$

Q $f(x) = \sin|x| + |x|$ is diff or Not?

ND. at $x = 0$

$$f(x) = \sin|x| + |x| = \begin{cases} \sin x + x & x \geq 0 \\ -\sin(x) - x & x < 0 \end{cases}$$

1) Cont^y $\sin 0 + 0 = -\sin 0 - 0$

$$0 = 0 \checkmark$$

2) diff^y $f'(x) = \begin{cases} \cos x + 1 & x \geq 0 \\ -\cos x - 1 & x < 0 \end{cases}$

$LHD = -\cos 0 - 1 = \underline{-2}$

$RHD = \cos 0 + 1 = \underline{2}$

ND. at $x = 0$

Q $f(x) = \sin|x| - |x|$ is diff^{ble} or not?

$$f(x) = \sin|x| - |x| = \begin{cases} \sin x - x & x \geq 0 \\ -\sin x + x & x < 0 \end{cases}$$
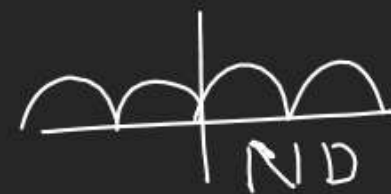
(cont^y)
$x=0$

$\sin 0 - 0 = -\sin 0 + 0$

$0 = 0$

$$f'(x) = \begin{cases} \cos x - 1 & x \geq 0 \\ -\cos x + 1 & x < 0 \end{cases}$$

$LHD = -\cos 0 + 1 = 0$
$RHD = \cos 0 - 1 = 0$ } diff^{ble} at $x = 0$

Q $f(x) = a|\sin x| + b e^{|x|} + c|x^3|$ is diff then.

find a, b, c?

ND

$x=0$ ND.

diff^{ble} at $x=0$

$$f'(0^+) = \lim_{h \to 0} \frac{f(0+h) - f(0)}{h}$$

$$= \lim_{h \to 0} \frac{a|\sin h| + b e^{|h|} + c|h^3| - (0 + b)}{h}$$

$$= \lim_{h \to 0} \frac{ah + (b e^{|h|} - b) + c h^3}{h}$$

$$= \lim_{h \to 0} \frac{h(a + c h^2)}{h} + \frac{b(e^h - 1)}{h}$$

$$= a + b + 0$$

$$f'(0^-) = \lim_{h \to 0} \frac{f(0-h) - f(0)}{-h}$$

$$= \lim_{h \to 0} \frac{a|\sin(-h)| + be^{|-h|} + c(|-h|^3) - b}{-h}$$

$$= \lim_{h \to 0} \boxed{\frac{ah + ch^3}{-h}} + \frac{b(e^h - 1)}{-h} \qquad \frac{h(a + ch^2)}{-h}$$

$$f'(0^-) = -a + 0 - b$$

$$f'(0^+) = a + b$$

$$-a - b = a + b$$

$$2(a+b) = 0$$

$$\boxed{a + b = 0}$$

$$\boxed{\begin{array}{c} a = 0, b = 0 \\ \& \\ a + b = 0 \end{array}}$$

Points of Non diff.
Using diff^n.

(a) then f^n is not a doubtful fxn.

then we can check Non diff^y
Pts Using diff^n.

(Q) $Y = \sqrt{1 - e^{-x^2}}$ is N.D. at.

$$\frac{dy}{dx} = \frac{1}{2\sqrt{1 - e^{-x^2}}} \times (0 + e^{-x^2} x(+2x)) = \frac{x e^{-x^2}}{\sqrt{1 - e^{-x^2}}}$$

$$\frac{dy}{dx} \text{ DNE } 1 - e^{-x^2} = 0$$

$$e^{-x^2} = e^0 \Rightarrow x^2 = 0$$

$$\boxed{x = 0}$$

Q $y = \sin^{-1}(x)$ is N.D. at

$\underline{Dom} \ x \in [-1,1]$

$\dfrac{dy}{dx} = \dfrac{1}{\sqrt{1-x^2}}$

$\dfrac{dy}{dx}$ DNE when $1-x^2 = 0$

$x^2 = 1$

$\boxed{x = 1, -1}$ ND

$y = \sin^{-1} x$

$\dfrac{dy}{dx} = \dfrac{1}{\sqrt{1-x^2}} \ ; \ x \in (-1,1)$

Q $y = \sin^{-1}(\cos x)$ is ND at

M_1 It is N.D. when

$\cos x = \pm 1$

N.D. at $\underline{x = n\pi}$

M_2 $\dfrac{dy}{dx} = \dfrac{1}{\sqrt{1-\cos^2 x}} \times -\sin x$

$\dfrac{dy}{dx} = \dfrac{-\sin x}{|\sin x|}$

$\dfrac{dy}{dx}$ DNE when $|\sin x| = 0$

$\sin x = 0$

$x = n\pi$

Q $f(x) = \sin^{-1}\left(\dfrac{2x}{1+x^2}\right)$ is N.D. at?

$\dfrac{2x}{1+x^2} = \pm 1$

$\left|\dfrac{2x}{1+x^2}\right| = 1$ $\quad \boxed{(x)^2 = |x|^2}$

$\dfrac{2|x|}{1+x^2} = 1$

$1+x^2 = 2|x|$

$1 + |x|^2 = 2|x|$

$|x|^2 - 2|x| + 1 = 0$

$(|x|-1)^2 = 0$

$|x| = 1$

ND $\boxed{x = \pm 1}$

finding fxn from functional Eqn.

Q Determine fxn given by fxnl Eqn.

$$f(x+y) = f(x) + f(y) + 2xy - 1$$

$$f(x+h) = f(x) + f(h) + 2hx - 1$$

① Write down formula of $f'(x)$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

(2) Now Use fxnl eqn in writter formula.

$$f'(x) = \lim_{h \to 0} \frac{f(x) + f(h) + 2hx - 1 - f(x)}{h}$$

(3) Now make Non x value as Constant K

$$f'(x) = \lim_{h \to 0} \left\{ \frac{f(h) - 1}{h} \right\} + \frac{(2hx)}{h} \leftarrow x\text{ वाली term}$$

← Non x

$$f'(x) = K + 2x$$

(4) Now Integrate

$$f(x) = Kx + 2 \cdot \frac{x^2}{2} + C$$

(5)
$$x = 0 = y \text{ in fxnl Eqn}$$

$$f(0+0) = f(0) + f(0) + 2 \times 0 \times 0 - 1$$

$$f(0) = f(0) + f(0) - 1 \implies f(0) = 1$$

$$\xrightarrow{x \to 0} f(0) = 0 + 0^2 + C \implies \boxed{1 = C}$$

$$f(x) = Kx + x^2 + 1 \longrightarrow \text{This is fxn}$$

Satisfied by given fxnl Eqn.

Q If $f\left(\dfrac{x+y}{3}\right) = \dfrac{2+f(x)+f(y)}{3}$; $f'(0)=2$

determine $f(x)$

$f\left(\dfrac{x+y+0}{3}\right) = \dfrac{f(0)+f(x)+f(y)}{3}$

① $f'(x) = \lim\limits_{h \to 0} \dfrac{f(x+h)-f(x)}{h}$

$= \lim\limits_{h \to 0} \dfrac{f\left(\dfrac{3x+3h}{3}\right) - f\left(\dfrac{3x+3\times 0}{3}\right)}{h}$

$= \lim\limits_{h \to 0} \dfrac{\dfrac{2+f(3x)+f(3h)}{3} - \left(\dfrac{2+f(3x)+f(0)}{3}\right)}{h}$

$= \lim\limits_{h \to 0} \dfrac{2+f(3x)+f(3h) - 2 - f(3x) - f(0)}{3h}$ $\underbrace{\left(\lim\limits_{h \to 0} \dfrac{f(3h)-f(0)}{3h}\right)}_{Non$

A Match feel

$f(x) = ax+b \Rightarrow b=2$

$f'(x) = a$

$f'(0) = a = 2$

$\therefore f(x) = 2x+2$

② $f'(x) = K \longrightarrow f'(0) = K \Rightarrow \boxed{K=2}$

③ $f(x) = Kx+c$

④ $x=0=y \Rightarrow f\left(\dfrac{0+0}{3}\right) = \dfrac{2+f(0)+f(0)}{3}$

$\Rightarrow 3f(0) = 2+2f(0) \Rightarrow \boxed{f(0)=2}$

⑤ $\xrightarrow{x=0} f(0) = 0+c \Rightarrow 2 = c \Rightarrow \boxed{c=2}$

$f(x) = Kx+2$

⑥ $f'(x) = K \overset{x=0}{\Rightarrow} f'(0) = K \Rightarrow K=2$

$\therefore f(x) = 2x+2$

Method 2

Practice of diff^n

① $Y = f(x+3)$

$$\frac{dy}{dx} = f'(x+3) \times (1+0)$$

② $y = f\left(\frac{x+3}{2}\right) = f\left(\frac{x}{2} + \frac{3}{2}\right)$

$$\frac{dy}{dx} = f'\left(\frac{x}{2} + \frac{3}{2}\right) \times \left(\frac{1}{2} + 0\right)$$

③ $Y = f\left(\frac{2x+3}{5}\right) = f\left(\frac{2x}{5} + \frac{3}{5}\right)$

$$\frac{dy}{dx} = f'\left(\frac{2x}{5} + \frac{3}{5}\right) \times \left(\frac{2}{5} + 0\right)$$

Q $f\left(\frac{x+y}{3}\right) = \frac{2 + f(x) + f(y)}{3}$; $f'(0) = 2$ find $f(x)$?

$$f\left(\frac{x}{3} + \frac{y}{3}\right) = \frac{2 + f(x) + f(y)}{3} \leftarrow \text{fxn Eqn}$$

① diff fxn Eqn WRT x keeping y constant

$$f'\left(\frac{x}{3} + \frac{y}{3}\right) \times \left(\frac{1}{3} + 0\right) = \frac{0 + f'(x) + 0}{3} \Rightarrow \boxed{\frac{1}{\cancel{3}} f'\left(\frac{x}{3} + \frac{y}{3}\right) = \frac{f'(x)}{\cancel{3}}}$$

(2) Put $x=0$ & put value of y such that it creat $f'(x)$ any how

$x = 0$
$y = 3x$

$$f'\left(0 + \frac{3x}{\cancel{3}}\right) = f'(0) \Rightarrow f'(x) = 2$$

$$f(x) = 2x + C$$

(3) $x = y = 0$

$$f(0) = \frac{2 + f(0) + f(0)}{3} \Rightarrow 3f(0) = 2 + 2f(0)$$

$$\Rightarrow f(0) = 2$$

$x = 0$: $2 = 2 \times 0 + C \Rightarrow C = 2$

$$f(x) = 2x + 2$$

Q let $f: R \to R$ be a Real value diff^ble fxn Satisfying

$$\boxed{f\left(\frac{2x+3y}{5}\right) = \frac{2f(x)+3f(y)}{5}} \quad \forall x, y \in R \ \& \ f(0)=1, f'(0)=2$$

(find $f(3) + f'(4)$)

$f(x) = ax + b$
$= 2x + 1$

$\hookrightarrow f\left(\frac{2x}{5} + \frac{3y}{5}\right) = \frac{2f(x)}{5} + \frac{3}{5} f(y)$

(1) diff WRT x keeping y const

$$f'\left(\frac{2x}{5} + \frac{3y}{5}\right) \times \left(\frac{2}{5} + 0\right) = \frac{2}{5} \cdot f'(x) + 0$$

(2) Put x = 0 & put y = $\frac{5x}{3}$

$f'\left(0 + \frac{3}{5} \times \frac{5x}{3}\right) = f'(0) \Rightarrow f'(x) = f'(0) = 2$

$f'(x) = 2$

$f(x) = 2x + c$

$x=0 \quad f(0) = 0 + c$

$\boxed{1 = c}$

$f(x) = 2x + 1$

$f(3) = 6 + 1 = 7$
$f'(x) = 2$
$f'(4) = 2$

$\Big\rangle f(3) + f'(4)$

$= 7 + 2 = 9$

Q let f be a diff^{ble} fxn Satisfying

$$f(x+y) = f(x) + f(y) + (e^x - 1)(e^y - 1) \cdot \forall x, y \in R.$$

$f'(0) = 2$ Identify correct st.

Now
Check

A) $\lim\limits_{x \to 0} \dfrac{f(f(x))}{f(x) - x} = 4$

B) $\lim\limits_{x \to 0} (f(x) + 6x)^{\frac{1}{e^x - 1}} = e^2$

C) No of sol of Eq^n $f(x) = 0$ is 2

D) Rf of fxn in $(-\infty, 0)$

$$f(x+y) = f(x) + f(y) + (e^x - 1)(e^y - 1)$$

① diff INRTDI keeping y $\underbrace{\text{lim}}$ const!

$$f'(x+y) \cdot (1+0) = f'(x) + 0 + (e^y - 1)(e^x - 0)$$

② Put $x = 0$, $Y = X$

$$f'(0+x) = f'(0) + (e^x - 1) \cdot (e^0)$$

$$f'(x) = 2 + e^x - 1 = e^x + 1$$

③ $f(x) = e^x + x + c$

④ $x = 0 = y =)$ $f(0+0) = f(0) + f(0) + (e^0 - 1)(e^0 - 1)$

$\Rightarrow) f(0) = 0$

⑤ $x = 0)$ $0 = e^0 + 0 + c =) \boxed{c = -1}$ $\therefore f(x) = \boxed{e^x + x - 1}$

Q $\quad f\left(1 \cdot \dfrac{x+2y}{3}\right) = 1 \cdot \dfrac{f(x)+2f(y)}{3}$ $\quad f'(0)=1$

$$f\left(\dfrac{x}{3} + \dfrac{2y}{3}\right) = \dfrac{f(x)}{3} + \dfrac{2}{3}f(y)$$

$$f'\left(\dfrac{x}{3} + \dfrac{2y}{3}\right) \times \left(\dfrac{1}{3} + 0\right) = \dfrac{f'(x)}{3} + 0$$

$x=0 , y = \dfrac{3x}{2}$

$$f'\left(0 + \dfrac{2}{3} \times \dfrac{3x}{2}\right) = f'(0)$$

$$f'(x) = \cdot 1$$

$$f(x) = x + c$$