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IRODOV

A particle whose velocity as a function of x is $v = a\hat{i} + bx\hat{j}$, where a & b are constant.
Particle is moving in x-y plane starting from origin.
Find radius of curvature of the particle as a function of x.

Sol^n :-

$v_x = a$, $v_y = bx$

$\frac{dx}{dt} = a$ — ① $\frac{dy}{dt} = bx$ — ②

② ÷ ①

$$\frac{dy}{dx} = \frac{b}{a}x$$

$$\int_0^y dy = \frac{b}{a}\int_0^x x\,dx$$

$$y = \frac{b}{a}\left(\frac{x^2}{2}\right)$$

$$r = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\left[\frac{d^2y}{dx^2}\right]}$$

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$$y = \frac{b}{2a}x^2$$

$$\frac{dy}{dx} = \frac{b}{2a} \times 2x = \left(\frac{b}{a}x\right)$$

$$\frac{d^2y}{dx^2} = \left(\frac{b}{a}\right)$$

$$r = \frac{\left[1 + \left(\frac{b}{a}x\right)^2\right]^{\frac{3}{2}}}{\left(b/a\right)}$$

Ans.

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Another Formula for radius of Curvature

$$F_r = F\sin\theta$$

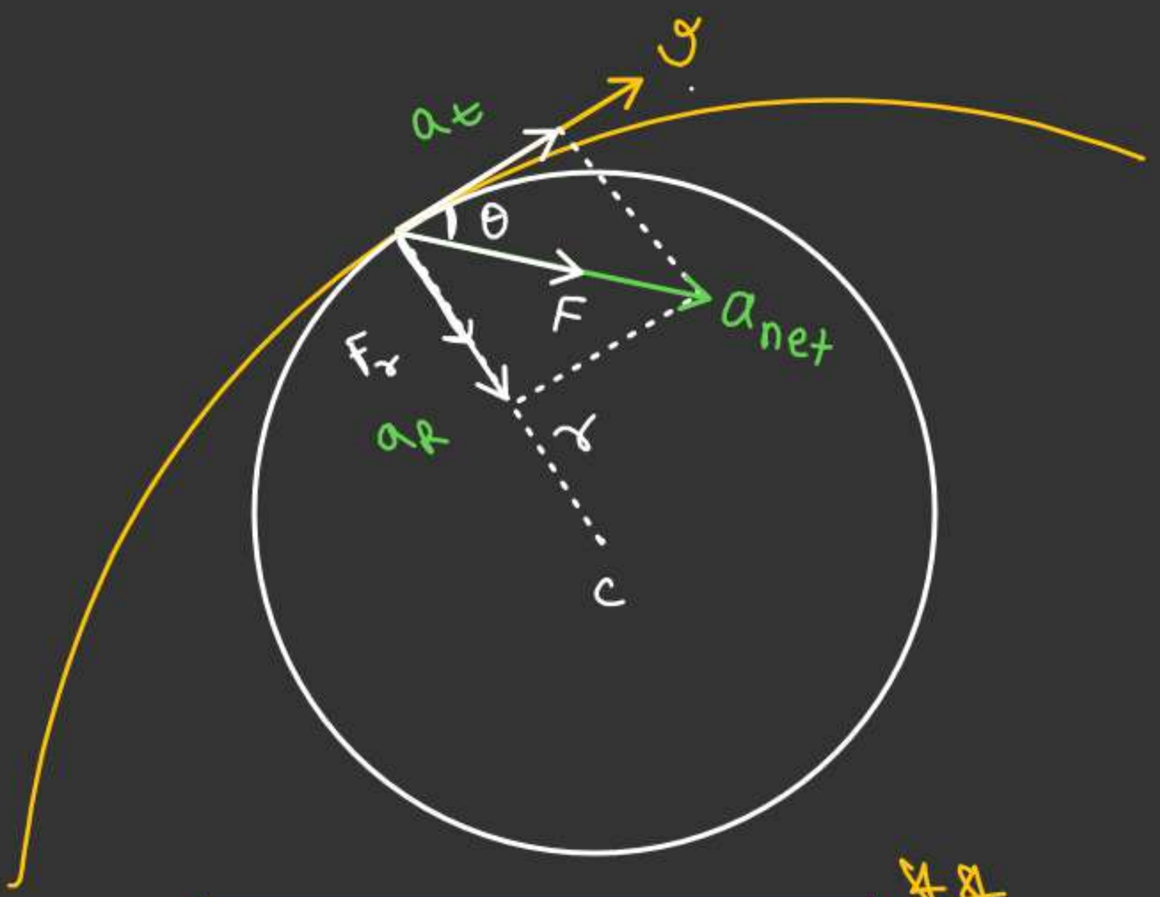
$$ma_r = F\sin\theta$$

$$\frac{mv^2}{r} = F\sin\theta$$

$$\frac{mv^2}{r} \times \frac{v}{v} = F\sin\theta$$

$$\frac{mv^3}{rv} = F\sin\theta$$

$$r = \left(\frac{mv^3}{v(F\sin\theta)}\right) = \left(\frac{mv^3}{|\vec{v}\times\vec{F}|}\right)$$



$$\boxed{r = \frac{mv^3}{|\vec{v}\times\vec{F}|}}$$

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Radius of Curvature in Case of projectile motion

Q: $\dfrac{R_{max}}{R_{min}} = ??$

$R_{min} = ??$ (At top most point)

$a_R = \dfrac{v^2}{r}$

$g\cos\theta = \dfrac{u^2}{R_{max}}$

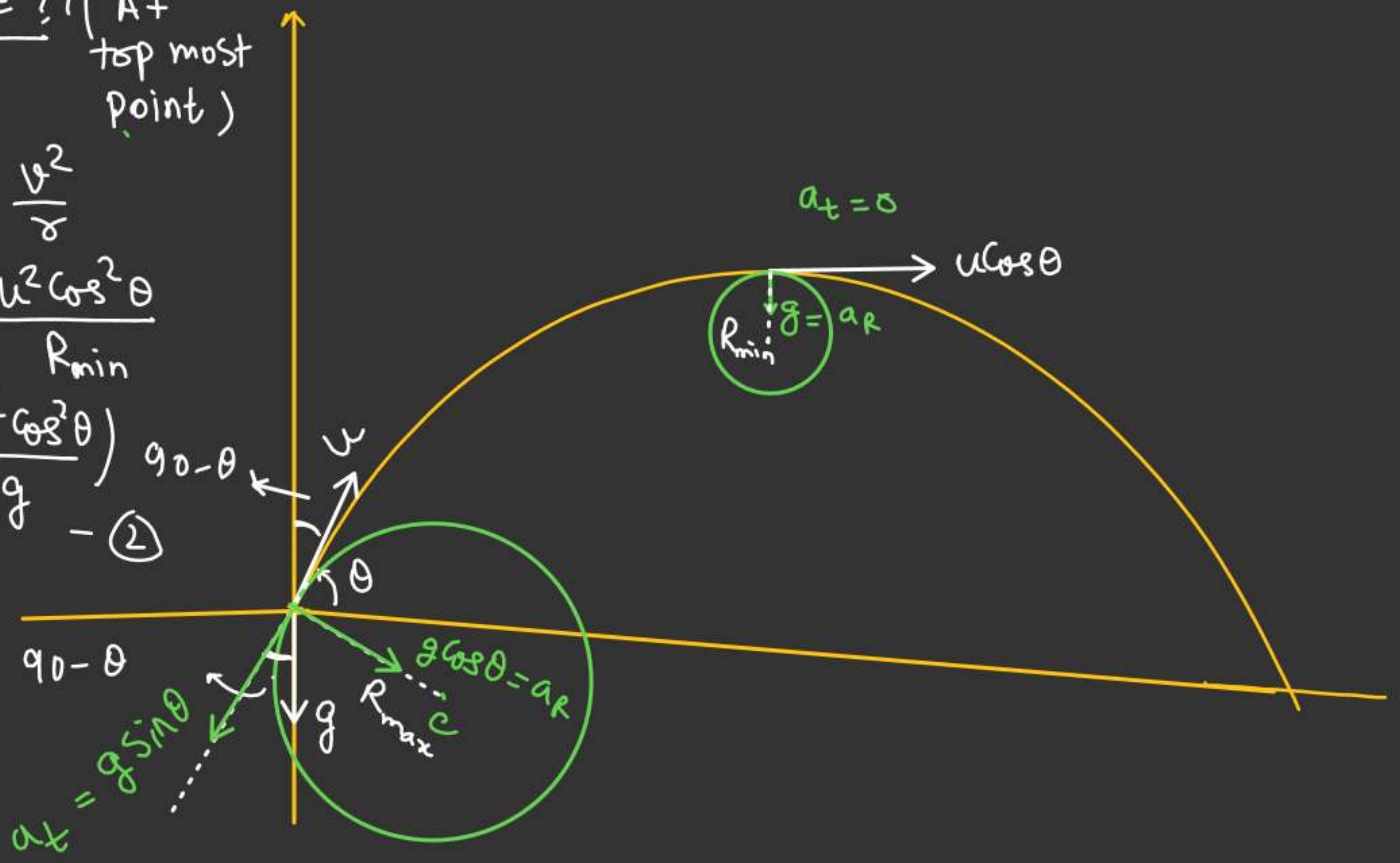
$R_{max} = \left(\dfrac{u^2}{g\cos\theta}\right)$ — ①

$a_R = \dfrac{v^2}{r}$

$\Downarrow$

$g = \dfrac{u^2\cos^2\theta}{R_{min}}$

$\left(R_{min} = \dfrac{u^2\cos^2\theta}{g}\right)$ — ②



$\dfrac{R_{min}}{R_{max}} = \dfrac{u^2\cos^2\theta}{g} \times \dfrac{g\cos\theta}{u^2}$

$= (\cos^3\theta)$

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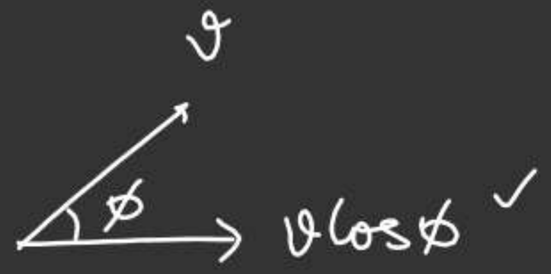
$$a_R = \frac{v^2}{r}$$

$$g\cos\phi = \frac{v^2}{r}$$

$$r = \left(\frac{v^2}{g\cos\phi}\right)$$

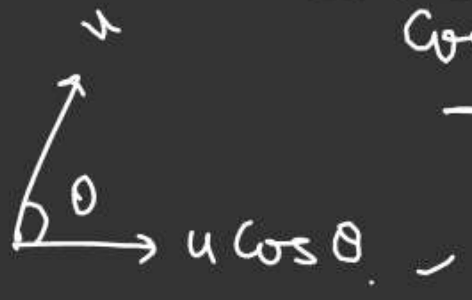
$$r = \frac{v^3}{g(v\cos\phi)}$$

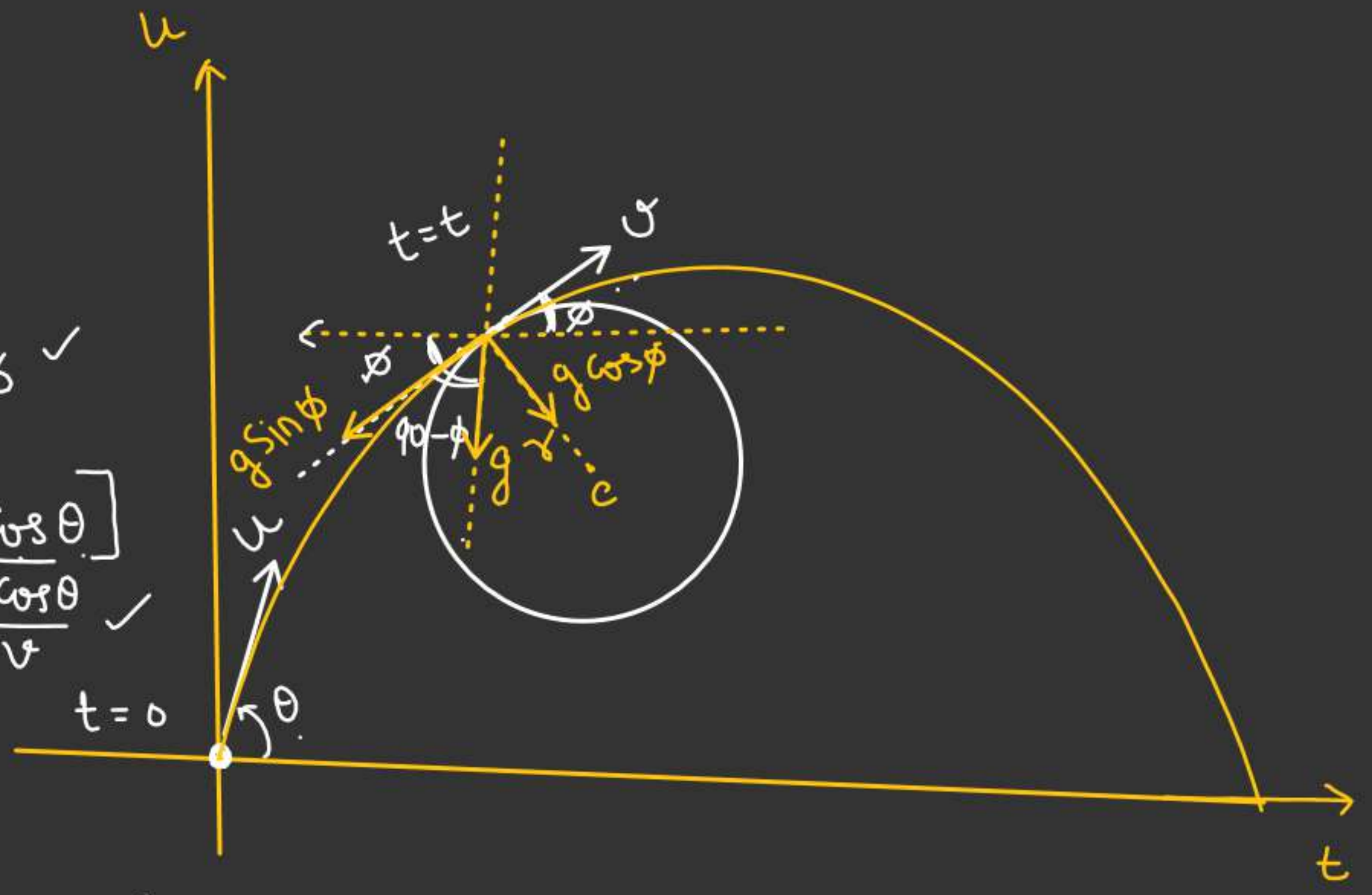
$$r = \left(\frac{v^3}{g\,u\cos\theta}\right) \checkmark$$



$$\boxed{v\cos\phi = u\cos\theta}$$

$$\cos\phi = \frac{u\cos\theta}{v} \checkmark$$





$$\vec{v} = v_x\hat{i} + v_y\hat{j}$$

$$v = u\cos\theta\,\hat{i} + (u\sin\theta - gt)\hat{j}$$

$$|v| = \sqrt{u^2\cos^2\theta + (u\sin\theta - gt)^2}$$

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2nd Approach

$$\boxed{r = \frac{mv^3}{|\vec{v} \times \vec{F}|}}$$

At Origin

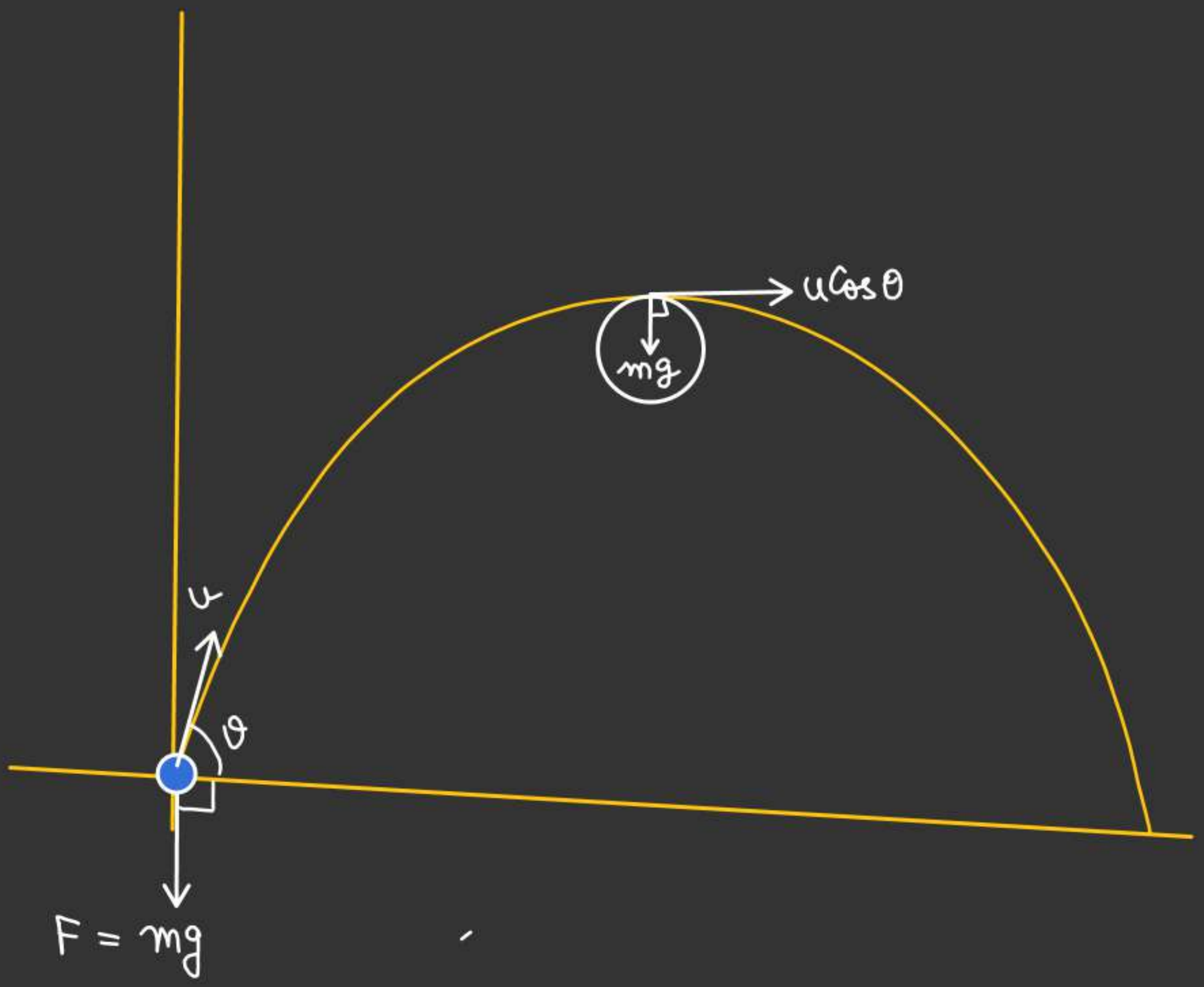
$$R_{max} = \frac{mu^3}{uF\sin(90+\theta)} = \frac{mu^3}{uF\cos\theta}$$

$$R_{max} = \frac{mu^3}{u \times mg \times \cos\theta} = \left(\frac{u^2}{g\cos\theta}\right) \checkmark$$

At highest point

$$R_{min} = \frac{\cancel{m}(u\cos\theta)^3}{u\cos\theta \cdot \cancel{m}g \cdot \sin 90}$$

$$R_{min} = \left(\frac{u^2\cos^2\theta}{g}\right) \checkmark$$



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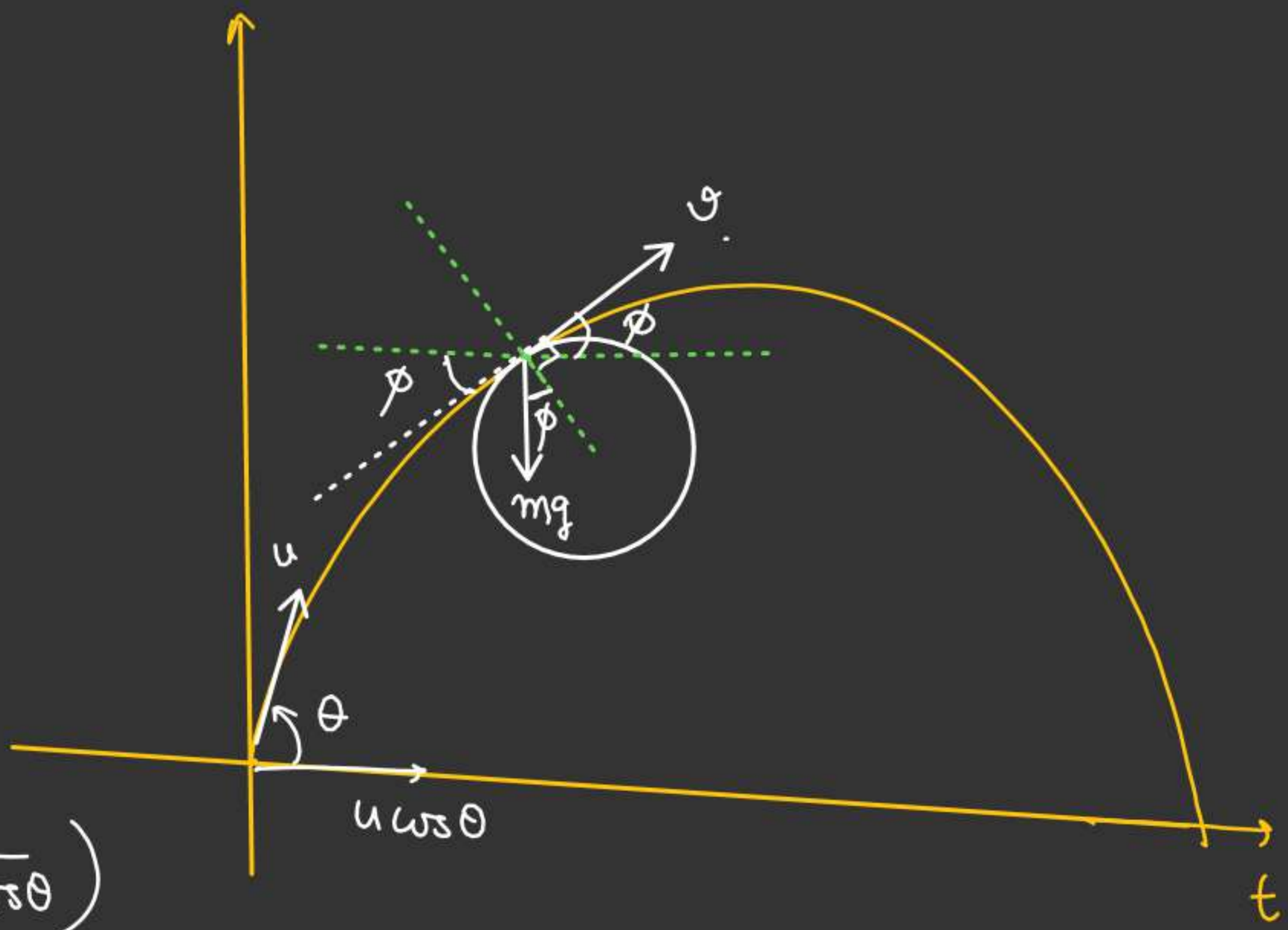
2nd Approach.

$$r = \frac{mv^3}{|\vec{v} \times \vec{F}|}$$

$$r = \frac{mv^3}{vF\sin(90+\phi)}$$

$$r = \frac{mv^3}{v \times mg \times \cos\phi} = \left(\frac{v^2}{g\cos\phi}\right)$$

$$r = \frac{v^2}{g\cos\phi} \times \frac{v}{v} = \frac{v^3}{g(v\cos\phi)} = \left(\frac{v^3}{gu\cos\theta}\right)$$



$(u\cos\theta = v\cos\phi)$