Nishant Jindal

(8) Value of Determinant is unaltered by adding to elements of any Row (Col) with a constant multiple of corresponding elements of any other Row (Col.)

{MOD}

No morning Class

{Discussion.}

Initial 20 min

Discussion

HW 2+3

$$\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

Matrix → JA, JM

$R_1 \to R_1 + P R_3$

$$\Delta' = \begin{vmatrix} a_{11}+Pa_{31} & a_{12}+Pa_{32} & a_{13}+Pa_{33} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} + \begin{vmatrix} Pa_{31} & Pa_{32} & Pa_{33} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

Prop.

$\Delta + 0$

(9) $|KA| = K^n |A|$ (10) $|A \cdot B| = |A| \cdot |B|$ (11) $|A^2| = |A|^2$

$|A^3| = |A|^3$

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Value of det is unchanged by adding or subtracting the multiple of any Row to given Row.

$R_i \to R_i + \alpha R_j + \beta R_k$.

$C_i \to C_i + \alpha C_j + \beta C_k$.

$R_1 \to R_1 + 2R_2$ ✓

1) $R_1 \to R_1 - 2R_2 + R_3$ ✓

2) $R_2 \to R_2 - 3R_1$ ✓

3) $R_1 \to -R_1 + R_2$ ✗

4) $R_2 \to 2R_2 + R_1$ ✗

5) $C_1 \to C_1 + C_2 - 2C_3$ ✓

6) $C_1 \to \frac{1}{2}(2C_1 + C_3) \Rightarrow C_1 \to C_1 + \frac{C_3}{2}$ ✓

(7) $R_1 \leftrightarrow R_3$. ✗ (8) $C_3 \to 2C_1$ ✗

Q Find value of determinant:

$$\Delta = \begin{vmatrix} b^2-ab & b-c & bc-ac \\ ab-a^2 & a-b & b^2-ab \\ bc-ac & c-a & ab-a^2 \end{vmatrix} = ?$$

$$\Delta = \begin{vmatrix} b(b-a) & b-c & c(b-a) \\ a(b-a) & a-b & b(b-a) \\ c(b-a) & c-a & a(b-a) \end{vmatrix}$$

$$= (b-a)^2 \begin{vmatrix} b & b-c & c \\ a & a-b & b \\ c & c-a & a \end{vmatrix} \quad C_2 \to C_2 + C_3$$

$$= (b-a)^2 \begin{vmatrix} b & b & c \\ a & a & b \\ c & c & a \end{vmatrix} = 0$$

Q $\begin{vmatrix} a_1 & l a_1 + m b_1 & b_1 \\ a_2 & l a_2 + m b_2 & b_2 \\ a_3 & l a_3 + m b_3 & b_3 \end{vmatrix} = ?$

$C_2 \to C_2 - l C_1 - m C_3$

$$\begin{vmatrix} a_1 & \overbrace{\cancel{l a_1} + \cancel{m b_1} - \cancel{l a_1} - \cancel{m b_1}}^{0} & b_1 \\ a_2 & \overbrace{\cancel{l a_2} + \cancel{m b_2} - \cancel{l a_2} - \cancel{m b_2}}^{0} & b_2 \\ a_3 & \overbrace{\cancel{l a_3} + \cancel{m b_3} - \cancel{l a_3} - \cancel{m b_3}}^{0} & b_3 \end{vmatrix}$$

$= 0$

Q_3 $\begin{vmatrix} a-b & b-c & c-a \\ x-y & y-z & z-x \\ p-q & q-r & r-p \end{vmatrix}$

$C_1 \to C_1 + C_2 + C_3$

$$\begin{vmatrix} \overbrace{\cancel{a} - \cancel{b} + \cancel{b} - \cancel{c} + \cancel{c} - \cancel{a}}^{0} & b-c & c-a \\ \overbrace{\cancel{x} - \cancel{y} + \cancel{y} - \cancel{z} + \cancel{z} - \cancel{x}}^{0} & y-z & z-x \\ \overbrace{\cancel{p} - \cancel{q} + \cancel{q} - \cancel{r} + \cancel{r} - \cancel{p}}^{0} & q-r & r-p \end{vmatrix}$$

$= 0$

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Q4 $\begin{vmatrix} \log x & \log y & \log z \\ \log 2x & \log 2y & \log 2z \\ \log 3x & \log 3y & \log 3z \end{vmatrix} = ?$

$$\begin{vmatrix} \log x & \log y & \log z \\ \log 2+\log x & \log 2+\log y & \log 2+\log z \\ \log 3+\log x & \log 3+\log y & \log 3+\log z \end{vmatrix}$$

$R_2 \to R_2 - R_1\ ;\ R_3 \to R_3 - R_1$

$$\begin{vmatrix} \log x & \log y & \log z \\ \log 2 & \log 2 & \log 2 \\ \log 3 & \log 3 & \log 3 \end{vmatrix}$$

$$\log 2 \times \log 3 \begin{vmatrix} & & \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix} \leftarrow \text{Identical} = 0$$

Q5 $\begin{vmatrix} (a^x+a^{-x})^2 & (a^x-a^{-x})^2 & 1 \\ (b^y+b^{-y})^2 & (b^y-b^{-y})^2 & 1 \\ (c^z+c^{-z})^2 & (c^z-c^{-z})^2 & 1 \end{vmatrix} = ?$

Concept $(a^x+a^{-x})^2-(a^x-a^{-x})^2 = (\cancel{a^{2x}}+\cancel{a^{-2x}}+2)-(\cancel{a^{2x}}+\cancel{a^{-2x}}-2) = 4$

$C_1 \to C_1 - C_2$

$$\begin{vmatrix} 4 & (\quad)^2 & 1 \\ 4 & (\quad)^2 & 1 \\ 4 & (\quad)^2 & 1 \end{vmatrix} = 0$$

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Q6

$$\begin{vmatrix} \sin^2\left(x+\frac{3\pi}{2}\right) & \sin^2\left(x+\frac{5\pi}{2}\right) & \sin^2\left(x+\frac{7\pi}{2}\right) \\ \sin\left(x+\frac{3\pi}{2}\right) & \sin\left(x+\frac{5\pi}{2}\right) & \sin\left(x+\frac{7\pi}{2}\right) \\ \sin\left(x-\frac{3\pi}{2}\right) & \sin\left(x-\frac{5\pi}{2}\right) & \sin\left(x-\frac{7\pi}{2}\right) \end{vmatrix} = ?$$

$C_3 \to C_3 - C_1$

Concept :- ① $\sin^2 A - \sin^2 B$

$= \sin(A+B)\cdot\sin(A-B)$

② $\sin(A) - \sin B$

$= 2\cos\left(\frac{A+B}{2}\right)\cdot\sin\left(\frac{A-B}{2}\right)$

$$\begin{vmatrix} \sin^2\left(x+\frac{3\pi}{2}\right) & " & \sin^2\left(x+\frac{7\pi}{2}\right)-\sin^2\left(x+\frac{3\pi}{2}\right) \\ \sin\left(x+\frac{3\pi}{2}\right) & " & \sin\left(x+\frac{7\pi}{2}\right)-\sin\left(x+\frac{3\pi}{2}\right) \\ \sin\left(x-\frac{3\pi}{2}\right) & " & \sin\left(x-\frac{7\pi}{2}\right)-\sin\left(x-\frac{3\pi}{2}\right) \end{vmatrix} = \begin{vmatrix} " & " & \sin(5\pi+2x)\cdot\sin(2\pi) \\ " & " & 2\cos\left(x+\frac{5\pi}{2}\right)\cdot\sin(\pi) \\ " & " & 2\cos\left(x-\frac{5\pi}{2}\right)\sin(-\pi) \end{vmatrix} = 0$$

0 0 0

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Q $\begin{vmatrix} 1 & 1+p & 1+p+q \\ 2 & 3+2p & 1+3p+2q \\ 3 & 6+3p & 10+6p+3q \end{vmatrix} = ?$

$C_2 \to C_2 - C_1 \times p,\ C_3 \to C_3 - C_1 \times q$

$\begin{vmatrix} 1 & 1+\cancel{p}-1\times p & 1+p+\cancel{q}-1\times q \\ 2 & 3+2\cancel{p}-2\times p & 1+3p+2\cancel{q}-2\times q \\ 3 & 6+3\cancel{p}-3\times p & 10+6p+\cancel{3}q-\cancel{3}q \end{vmatrix}$

$\begin{vmatrix} 1 & 1 & 1+p \\ 2 & 3 & 1+3p \\ 3 & 6 & 10+6p \end{vmatrix}$ $C_3 \to C_3 - C_2 \times p$

$\begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & 1 \\ 3 & 6 & 10 \end{vmatrix} = (30+3+12) - (9+6+20)$

$= 10$

Q $\begin{vmatrix} a-x & c & b \\ c & b-x & a \\ b & a & c-x \end{vmatrix} = 0$ Solve Eqn.

if $a+b+c \neq 0$

$x = a+b+c$

$C_1 \to C_1 + C_2 + C_3$

$\begin{vmatrix} a+b+c-x & c & b \\ a+b+c-x & b-x & a \\ a+b+c-x & a & c-x \end{vmatrix} = 0$

$(x^2 - a^2 - b^2 - c^2 + ab + bc + ca) = 0$

$x^2 = \frac{1}{2}\{(a-b)^2 + (b-c)^2 + (c-a)^2\}$

Make 2 Zeros.

$(a+b+c-x)\begin{vmatrix} 1 & c & b \\ 1 & b-x & a \\ 1 & a & c-x \end{vmatrix} = 0$ $R_2 \to R_2 - R_1$

$R_3 \to R_3 - R_1$

$\begin{vmatrix} 1 & c & b \\ 0 & (b-c-x) & a-b \\ 0 & a-c & (c-b-x) \end{vmatrix} = 0$

$(a+b+c-x)\left((b-c-x)(c-b-x) - (a^2 - ab - ac + bc)\right) = 0$

$bc - b^2 - b\cancel{x} - c^2 + bc + \cancel{cx} - \cancel{cx} + \cancel{bx} + x^2$

Q $\begin{vmatrix} 3a & -a+b & -a+c \\ -b+a & 3b & -b+c \\ -c+a & -c+b & 3c \end{vmatrix} = ?$

$C_1 \to C_1 + C_2 + C_3$

$$\begin{vmatrix} a+b+c & -a+b & -a+c \\ a+b+c & 3b & -b+c \\ a+b+c & -c+b & 3c \end{vmatrix}$$

$$(a+b+c)\begin{vmatrix} 1 & -a+b & -a+c \\ 1 & 3b & -b+c \\ 1 & -c+b & 3c \end{vmatrix}$$

$R_2 \to R_2 - R_1$

$$(a+b+c)\begin{vmatrix} 1 & -a+b & -a+c \\ 0 & 2b+a & -b+a \\ 0 & -c+a & 2c+a \end{vmatrix} = (a+b+c)\left((2b+a)(2c+a) - (-c+a)(-b+a)\right)$$

$$\left(4bc + 2ab + 2ac + \cancel{a^2} - (bc - ac - ab + \cancel{a^2})\right)$$

$$= 3(a+b+c)(ab+bc+ca)$$

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a, aR, aR^2 $\ln\frac{a_3}{a_1} = \ln\frac{aR^2}{a} = 2\ln R$ | $\frac{a_4}{a_2} = \frac{aR^3}{aR} = R^2$

Q let $a_1, a_2, a_3 \ldots a_{10}$ be in G.P. with $a_i > 0$ for $i = 1, 2, \ldots 10$ & S be the set of pairs such that

Mains 2020

$S = (r, K);\ r, K \in N$ for which

$$\begin{vmatrix} \ln a_1^r a_2^K & \ln a_2^r a_3^K & \ln a_3^r a_4^K \\ \ln a_4^r a_5^K & \ln a_5^r a_6^K & \ln a_6^r a_7^K \\ \ln a_7^r a_8^K & \ln a_8^r a_9^K & \ln a_9^r a_{10}^K \end{vmatrix} = 0$$

Then No of elements in S

10, 2, 4, ∞

$R = \frac{a_2}{a_1} = \frac{a_3}{a_2} = \frac{a_4}{a_3} \ldots$

$\boxed{C_2 \to C_2 - C_1}$, $C_3 \to C_3 - C_1$

$\ln a_2^r a_3^K - \ln a_1^r \cdot a_2^K$

$(r\ln a_2 + K\ln a_3) - (r\ln a_1 + K\ln a_2)$

$r(\ln a_2 - \ln a_1) + K(\ln a_3 - \ln a_2)$

$r\left(\ln\frac{a_2}{a_1}\right) + K\left(\ln\frac{a_3}{a_2}\right)$

$r\ln R + K\ln R$

$\ln R(r+K)$

$C_3 \to C_3 - C_1$

$r(\ln a_3 - \ln a_1) + K(\ln a_4 - \ln a_2)$

$r\ln\frac{a_3}{a_1} + K\ln\frac{a_4}{a_2}$

$r\ln R^2 + K\ln R^2$

$\ln R^2(r+K)$

$2\ln R(r+K)$

$$\begin{vmatrix} " & \ln R(r+K) & 2\ln R(r+K) \\ " & \ln R(r+K) & 2\ln R(r+K) \\ " & \ln R(r+K) & 2\ln R(r+K) \end{vmatrix}$$

$= 0$ for (r, K)

∞ value

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Q Find value of $\theta \in \left(0, \frac{\pi}{3}\right)$

Mains

for which

$\frac{\pi}{18}, \frac{\pi}{9}, \frac{7\pi}{24}, \frac{7\pi}{36}$

$$\begin{vmatrix} 1+\cos^2\theta & \sin^2\theta & 4\cos 6\theta \\ \cos^2\theta & 1+\sin^2\theta & 4\cos 6\theta \\ \cos^2\theta & \sin^2\theta & 1+4\cos 6\theta \end{vmatrix} = 0$$

$R_1 \to R_1 - R_2$; $R_2 \to R_2 - R_3$

$$\begin{vmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ \cos^2\theta & \sin^2\theta & 1+4\cos 6\theta \end{vmatrix} = 0$$

$C_1 \to C_1 + C_2$

$$\begin{vmatrix} 0 & -1 & 0 \\ 1 & 1 & -1 \\ 1 & \sin^2\theta & 1+4\cos 6\theta \end{vmatrix} = 0 \Rightarrow (0+1+0) - (0+0-1-4\cos 6\theta) = 0$$

$2 + 4\cos 6\theta = 0$

$\cos 6\theta = -\frac{1}{2}$

$\cos 6 \times \frac{\pi}{18} = \frac{1}{2}$ ×

$\cos \overset{2}{\cancel{6}} \times \frac{\pi}{\cancel{9}3} = -\frac{1}{2}$ ✓

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$(\ \ )^2 \geq 0$, AM $\geq$ GM

Q let $|M|$ denotes det. of sq^n Matrix M, let $g:[0,\frac{\pi}{2}]\to R$ be the fxn defined by $g(\theta)=\sqrt{f(\theta)-1}+\sqrt{f(\frac{\pi}{2}-\theta)-1}$

Adv. 2022

$$f(\theta)=\frac{1}{2}\begin{vmatrix}1 & \sin\theta & 1\\ -\sin\theta & 1 & \sin\theta\\ -1 & -\sin\theta & 1\end{vmatrix}+\begin{vmatrix}\sin\pi & \cos(\theta+\frac{\pi}{4}) & \tan(\theta-\frac{\pi}{4})\\ \sin(\theta-\frac{\pi}{4}) & -\cos\frac{\pi}{2} & \log\frac{4}{\pi}\\ \cot(\theta+\frac{\pi}{4}) & \log\frac{\pi}{4} & \tan\pi\end{vmatrix}$$

let $P(x)$ be a Quad Poly whose Roots are Max^m & Min^m value of $g(\theta)$ & $P(2)=2-\sqrt{2}$ then W.O.T.F is True

A) $P\left(\frac{3+\sqrt{2}}{4}\right)<0$ ✓

B) $P\left(\frac{1+3\sqrt{2}}{4}\right)>0$

C) $P\left(\frac{5\sqrt{2}-1}{4}\right)>0$

D) $P\left(\frac{5-\sqrt{2}}{4}\right)<0$

$$\begin{vmatrix}0 & \cos(\theta+\frac{\pi}{4}) & \tan(\theta-\frac{\pi}{4})\\ -\sin(\theta-\frac{\pi}{4}) & 0 & \log\frac{4}{\pi}\\ \cot(\theta+\frac{\pi}{4}) & \log\frac{\pi}{4} & 0\end{vmatrix}$$

Skew Symm determinant of order = $\boxed{3}$ odd

1) $f(\theta)=(1-\cancel{\sin^2\theta}+\cancel{\sin^2\theta})-(-1-\sin^2\theta-\sin^2\theta)$

$f(\theta)=\frac{1}{2}(2+2\sin^2\theta)=1+\sin^2\theta$

2) $g(\theta)=\sqrt{f(\theta)-1}+\sqrt{f(\frac{\pi}{2}-\theta)-1}$

$=\sqrt{\cancel{1}+\sin^2\theta-\cancel{1}}+\sqrt{\cancel{1}+\cos^2\theta-\cancel{1}}$

$g(\theta)=|\sin\theta|+|\cos\theta|\in[1,\sqrt{2}]$ ★★

Min^m Max^m

(3) $P(x)=a(x-1)(x-\sqrt{2})$

$x=2$) $\cancel{2-\sqrt{2}}=a(2-1)\cancel{(2-\sqrt{2})}\Rightarrow a=1$

$\therefore P(x)=(x-1)(x-\sqrt{2})$

$P\left(\frac{3+\sqrt{2}}{4}\right)=\left(\frac{3+1.4}{4}-1\right)\left(\frac{3+1.4}{4}-1.4\right)$

$(.1)(-.3)<0$

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Some Special Determinant

Inki value yad Rakhni hai!!

1) $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = (a-b)(b-c)(c-a)$

2) $\begin{vmatrix} 1 & a & a^3 \\ 1 & b & b^3 \\ 1 & c & c^3 \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c)$

3) $\begin{vmatrix} 1 & a^2 & a^3 \\ 1 & b^2 & b^3 \\ 1 & c^2 & c^3 \end{vmatrix} = (a-b)(b-c)(c-a)(ab+bc+ca)$

(4) Cyclic Det.

$$\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = 3abc-a^3-b^3-c^3$$

$$= -(a^3+b^3+c^3-3abc)$$

$$= -(a+b+c)(a^2+b^2+c^2-ab-bc-ca)$$

$$= -\frac{1}{2}(a+b+c)\{(a-b)^2+(b-c)^2+(c-a)^2\}$$

$$= (a+b+c)(a+b\omega+c\omega^2)(a+b\omega^2+c\omega)$$

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Q let $a, b, c \in R$ be all non zero & satisfy $a^3+b^3+c^3=2$

Mains 2020. If Matrix $A=\begin{pmatrix} a & b & c \\ b & c & a \\ c & a & b \end{pmatrix}$ satisfies $A^T A = I$ then value of abc can be.

$\frac{2}{3}, -\frac{1}{3}, 3, \frac{1}{3}$

$$A^T \cdot A = I$$

det

$$|A^T \cdot A| = |I|$$

$$|A^T||A| = 1$$

$$|A||A| = 1$$

$$|A|^2 = 1$$

$$|A| = 1 \quad \text{or} \quad |A| = -1$$

$|A| = 1$

$$3abc - (a^3+b^3+c^3) = 1$$

$$3abc - 2 = 1$$

$$3abc = 3$$

$$abc = 1$$

$|A| = -1$

$$3abc - (a^3+b^3+c^3) = -1$$

$$3abc - (2) = -1$$

$$3abc = 1$$

$$abc = \frac{1}{3}$$

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Q
$$\begin{vmatrix} b+c & \boxed{a-b} & a \\ c+a & \boxed{b-c} & b \\ a+b & \boxed{c-a} & c \end{vmatrix} = ?$$

$C_2 \to C_2 - C_3.$

$$\begin{vmatrix} b+c & -b & a \\ c+a & -c & b \\ a+b & -a & c \end{vmatrix}$$

$C_1 \to C_1 + C_2$

$$\begin{vmatrix} c & -b & a \\ a & -c & b \\ b & -a & c \end{vmatrix}$$

$$-\begin{vmatrix} a & c & b \\ b & a & c \\ c & b & a \end{vmatrix} = +\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = 3abc-(a^3+b^3+c^3)$$

Q
$$\begin{vmatrix} x & y & z \\ x^2 & y^2 & z^2 \\ x^3 & y^3 & z^3 \end{vmatrix}$$

X com Y com Z com.

$$xyz\begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ x^2 & y^2 & z^2 \end{vmatrix}$$

$xyz(x-y)(y-z)(z-x)$

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Q $\Delta = \begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ bc+a^2 & ca+b^2 & ab+c^2 \end{vmatrix} = ?$

$$= \begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ bc & ca & ab \end{vmatrix} + \begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix}$$

$$\begin{matrix} \uparrow & \uparrow & \uparrow \\ a & b & c \end{matrix}$$

$$+(\quad)(\quad)(\quad)$$

$$= \frac{1}{abc} \begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ abc & abc & abc \end{vmatrix}$$

$$= \frac{\cancel{abc}}{\cancel{abc}} \begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ 1 & 1 & 1 \end{vmatrix}$$

$$(\quad)(\quad)(\quad)$$

$$= 2(a-b)(b-c)(c-a)$$

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Q $\begin{vmatrix} x_{C_1} & x_{C_2} & x_{C_3} \\ y_{C_1} & y_{C_2} & y_{C_3} \\ z_{C_1} & z_{C_2} & z_{C_3} \end{vmatrix} = ?$

$$\begin{vmatrix} x & \frac{(x)(x-1)}{1\cdot 2} & \frac{(x)(x-1)(x-2)}{1\cdot 2\cdot 3} \\ y & \frac{(y)(y-1)}{1\cdot 2} & \frac{(y)(y-1)(y-2)}{1\cdot 2\cdot 3} \\ z & \frac{(z)(z-1)}{1\cdot 2} & \frac{(z)(z-1)(z-2)}{1\cdot 2\cdot 3} \end{vmatrix}$$

$$\frac{xyz}{12}\begin{vmatrix} 1 & x-1 & x^2-3x+2 \\ 1 & y-1 & y^2-3y+2 \\ 1 & z-1 & z^2-3z+2 \end{vmatrix} \quad C_2 \to C_2 + C_1$$

$$\frac{xyz}{12}\begin{vmatrix} 1 & x & x^2-3x+2 \\ 1 & y & y^2-3y+2 \\ 1 & z & z^2-3z+2 \end{vmatrix}$$

$C_3 \to C_3 - 2C_1 + 3C_2$

$$\frac{xyz}{12}\begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix}$$

$$\frac{xyz}{12}(\quad)(\quad)(\quad)$$

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$$\begin{vmatrix} 1 & a & a^2-bc \\ 1 & b & b^2-ac \\ 1 & c & c^2-ab \end{vmatrix}$$

$$\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} - \begin{vmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{vmatrix} \begin{array}{l} \leftarrow a \\ \leftarrow b \\ \leftarrow c \end{array}$$

$(\quad)(\quad)(\quad) -$

Q $$\begin{vmatrix} a^2+\lambda & ab & ac \\ ab & b^2+\lambda & bc \\ ac & bc & c^2+\lambda \end{vmatrix} \begin{array}{l} \leftarrow a \\ \leftarrow b \\ \leftarrow c \end{array} = 0 \quad \text{then } \lambda = ?$$

$$\frac{1}{abc}\begin{vmatrix} a(a^2+\lambda) & a^2b & a^2c \\ ab^2 & b(b^2+\lambda) & b^2c \\ ac^2 & bc^2 & c(c^2+\lambda) \end{vmatrix} = 0$$

$$\frac{\cancel{abc}}{\cancel{abc}}\begin{vmatrix} a^2+\lambda & a^2 & a^2 \\ b^2 & b^2+\lambda & b^2 \\ c^2 & c^2 & c^2 \end{vmatrix} = 0$$

$R_1 \to R_1 + R_2 + R_3$

$$\begin{vmatrix} a^2+b^2+c^2+\lambda & a^2+b^2+c^2+\lambda & a^2+b^2+c^2+\lambda \\ b^2 & b^2+\lambda & b^2 \\ c^2 & c^2 & c^2+\lambda \end{vmatrix} = 0$$

$$(a^2+b^2+c^2+\lambda)\begin{vmatrix} 1 & 1 & 1 \\ b^2 & b^2+\lambda & b^2 \\ c^2 & c^2 & c^2+\lambda^2 \end{vmatrix} = 0$$

open & get