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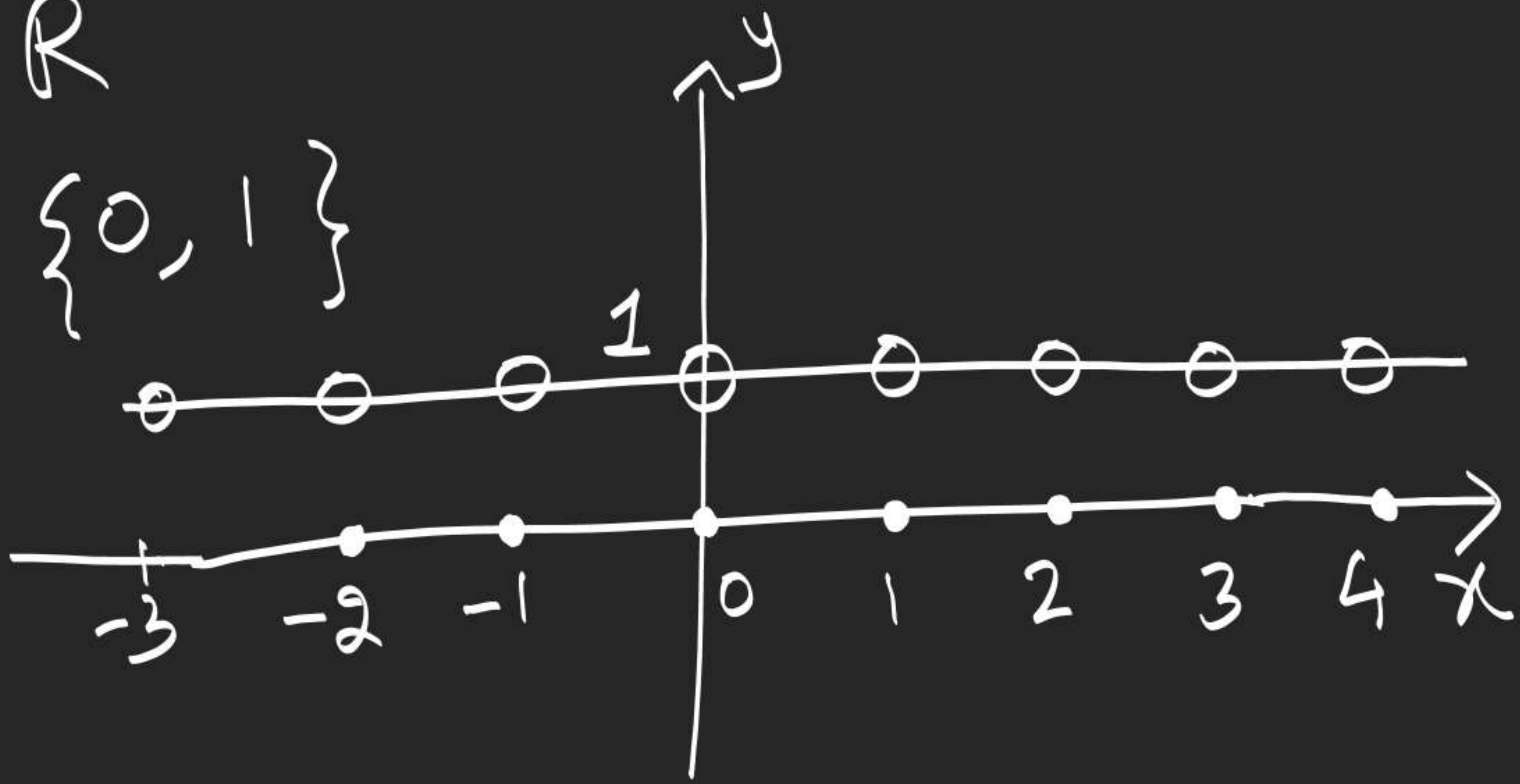
FUNCTIONS

1. $f(x) = \text{sgn}\{x\}$ $\qquad$ $\{\cdot\} = FPF$

$D_f = R$

$R_f = \{0, 1\}$

$T = 1$



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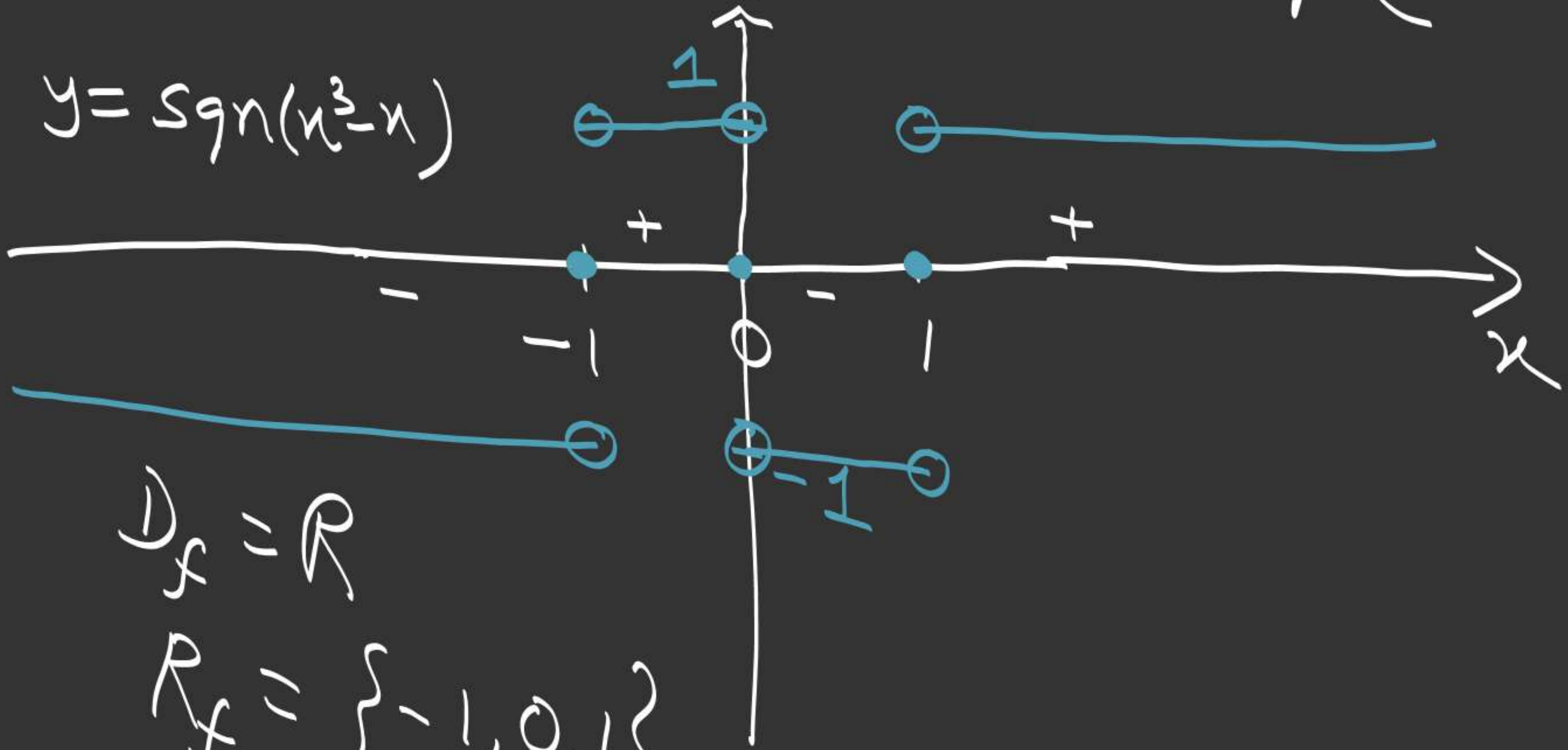
2.

$$f(x) = \operatorname{sgn}(x^3 - x) = \operatorname{sgn}(x(x-1)(x+1))$$

$y = \operatorname{sgn}(x^3 - x)$



$D_f = R$

$R_f = \{-1, 0, 1\}$

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FUNCTIONS

3.

$$f(x) = \sin\{x\}$$

$$\{\cdot\} = FPF$$

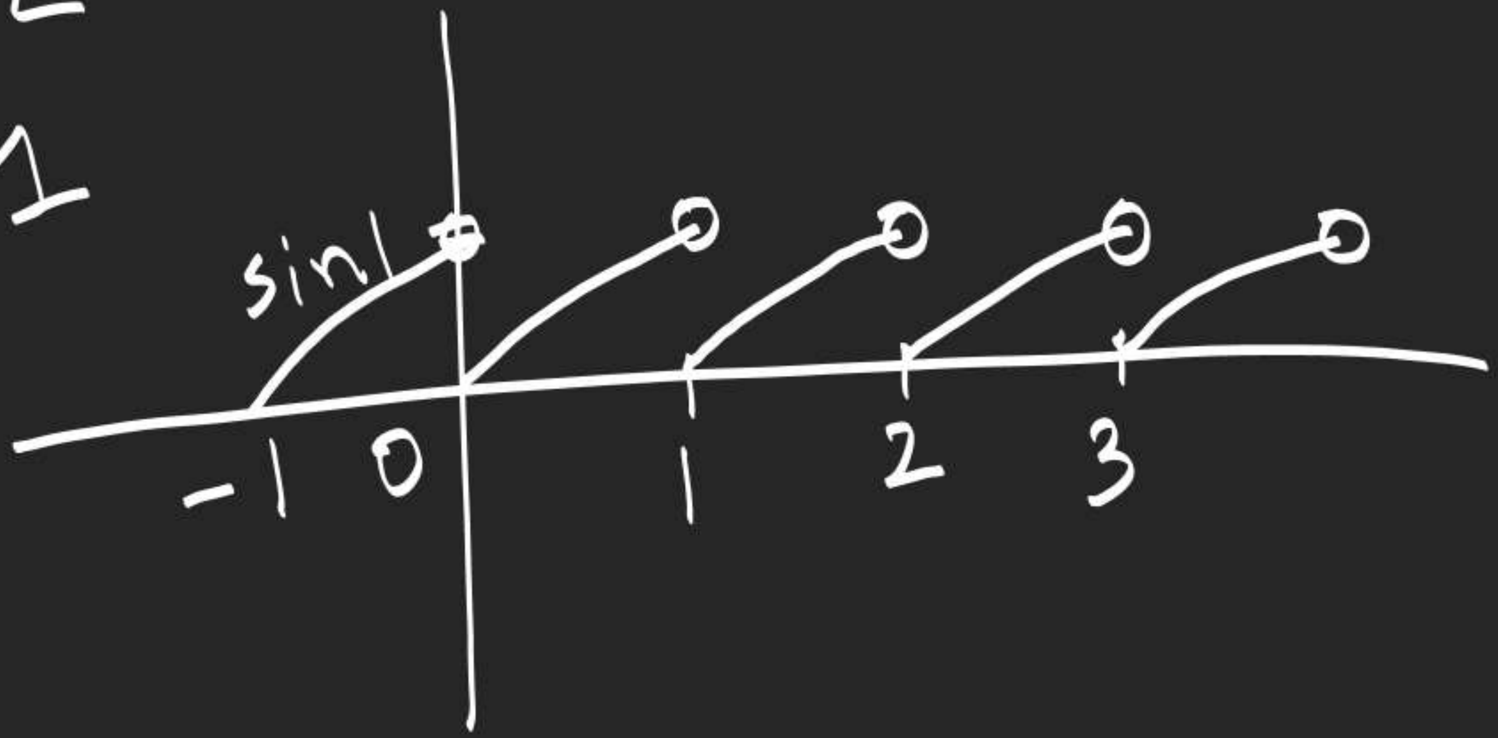
$$D_f = R$$

$$R_f = [\sin 0, \sin 1) = [0, \sin 1)$$

$$T = 1$$

If $x \in [0,1)$

$\{x\} = x$



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Shifting of Origin

$(0,0) \to P(-1, 3)$

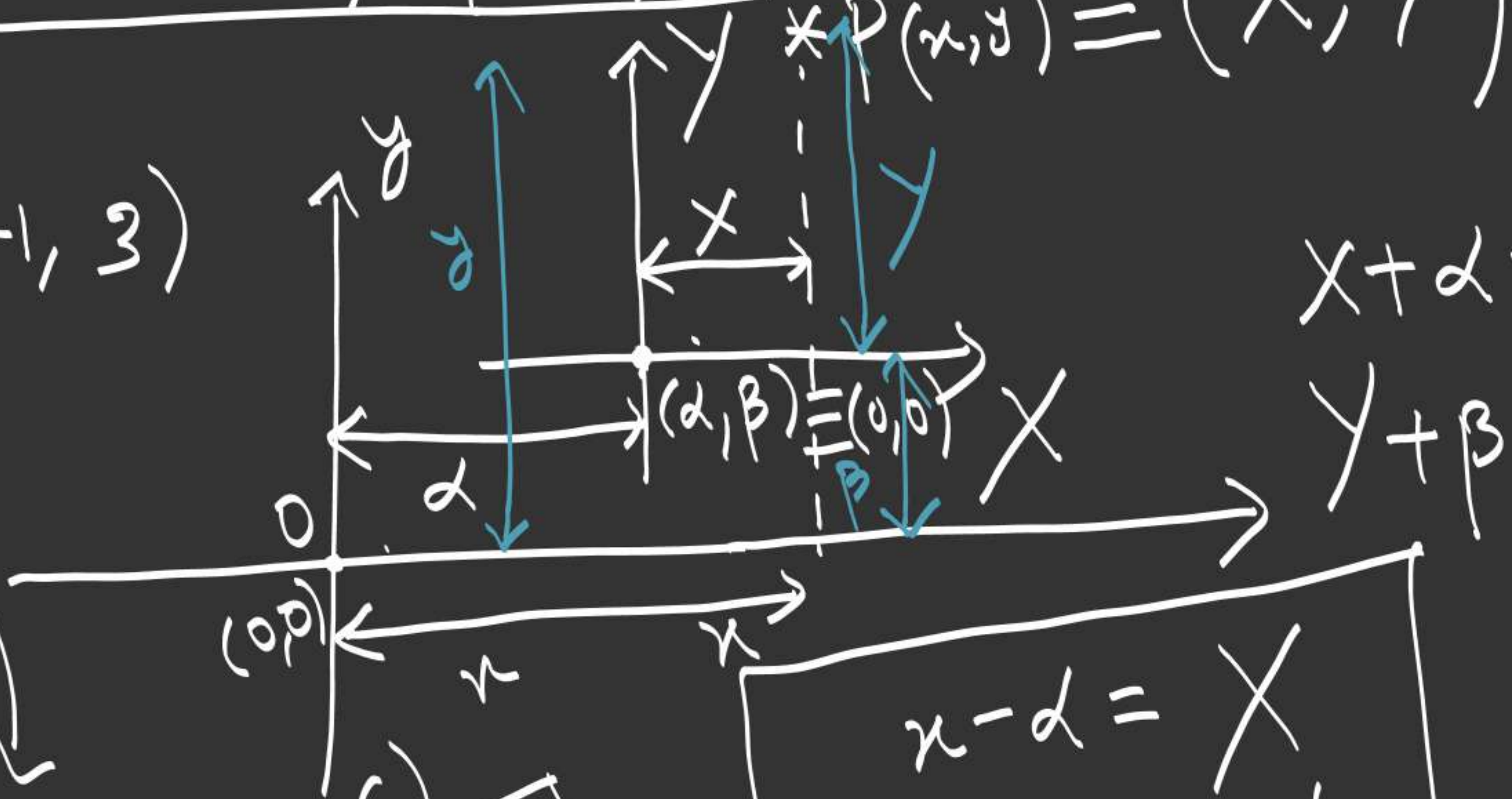
$\downarrow$

?

$(-7, 6)$

$(-1-(-7),\ 3-6)$

$= \boxed{(6, -3)}$



$X + \alpha = x$

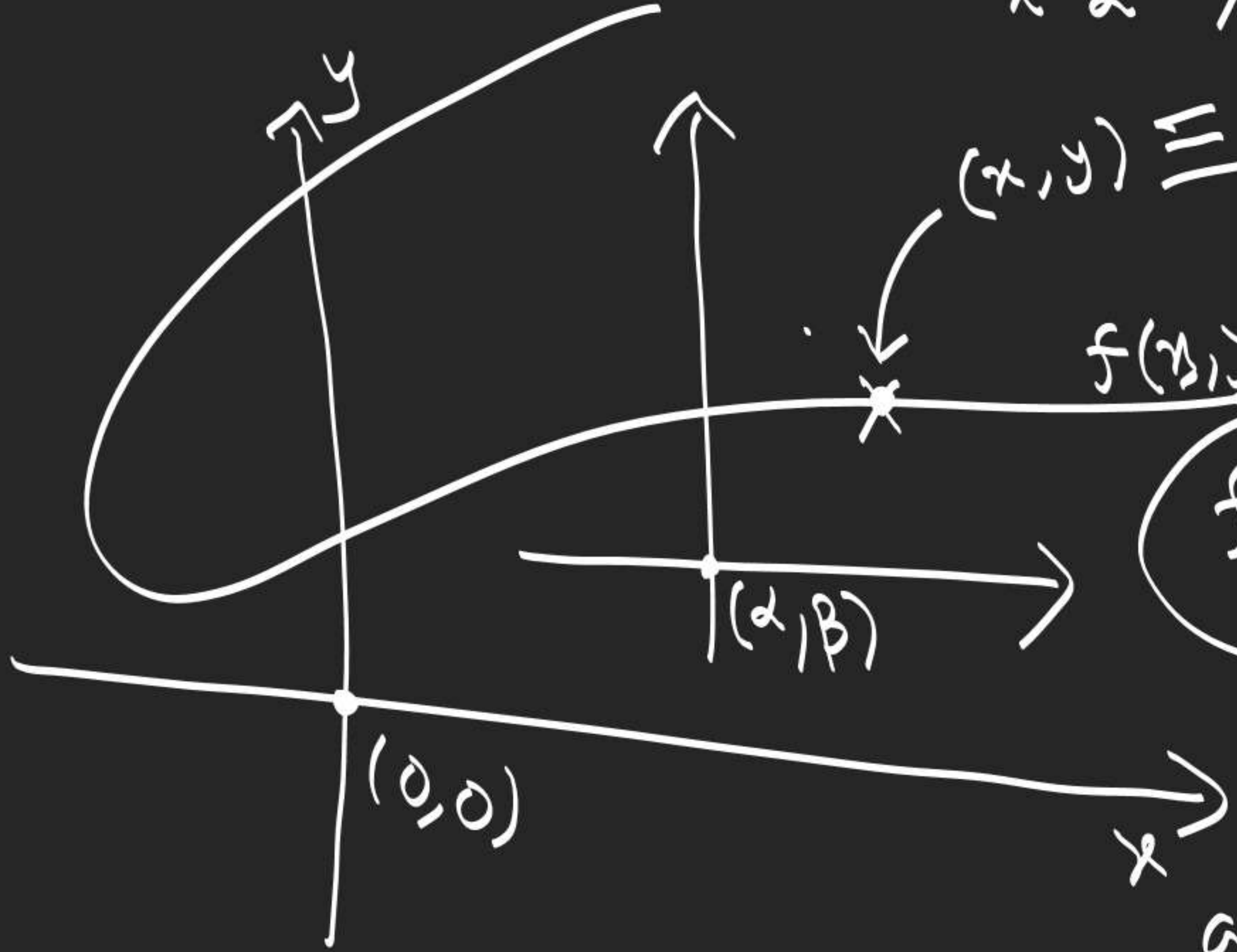
$Y + \beta = y$

$$\boxed{\begin{aligned} x - \alpha &= X \\ y - \beta &= Y \end{aligned}}$$

FUNCTIONS

$x-\alpha = X$

$(x,y) \equiv (X,Y)$



$f(x,y)=0$

$f(X+\alpha,\ Y+\beta)=0$

$g(X,Y)=0$

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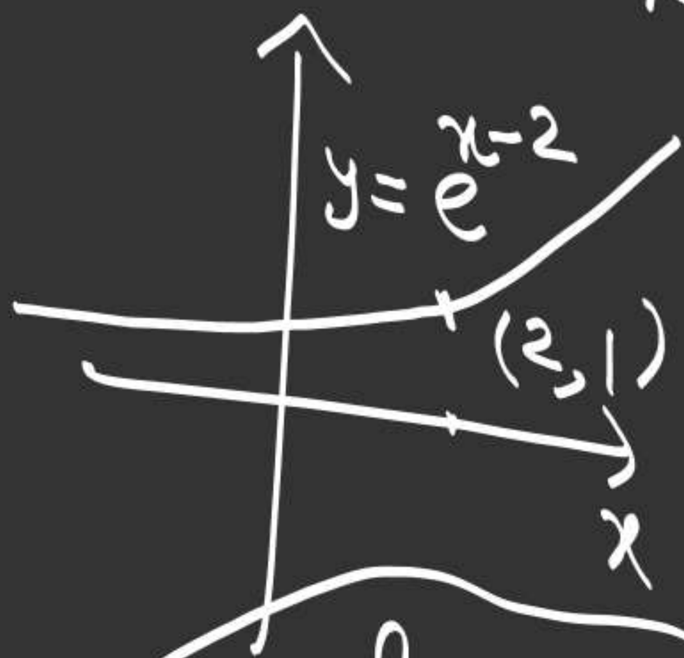
$$y = e^{x-2}$$

$x - 2 = X$

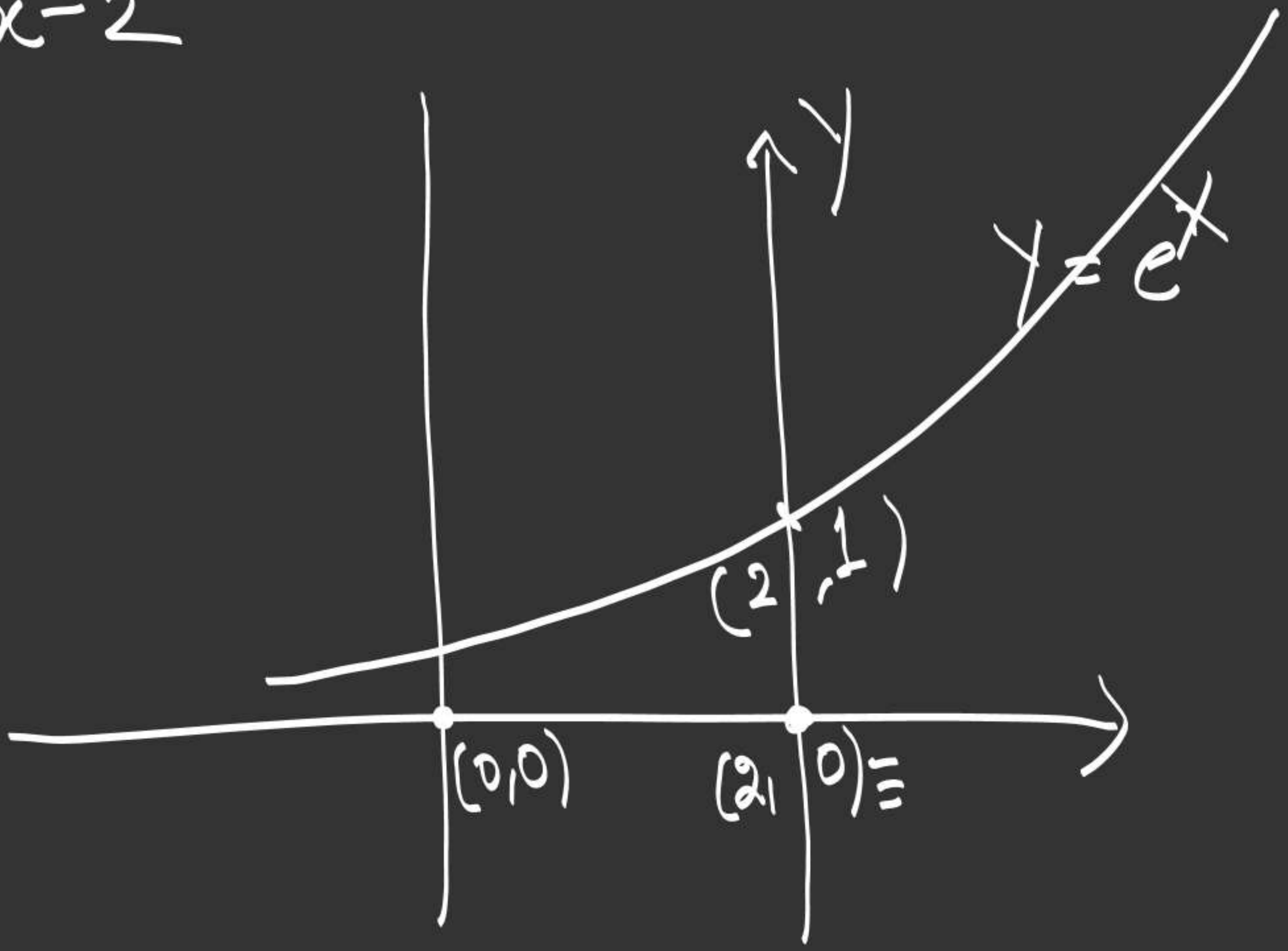
$y = Y$

$$\boxed{Y = e^X}$$



$D_f = R$

$R_f = (0, \infty)$



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FUNCTIONS

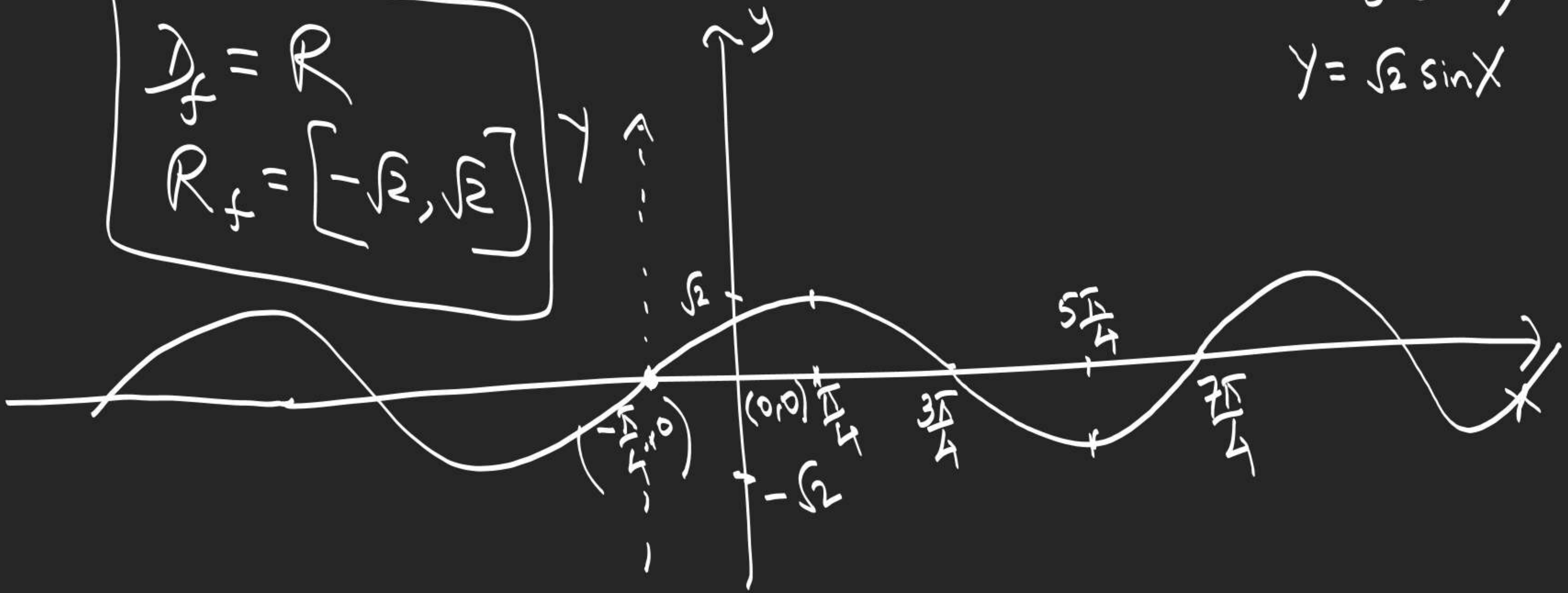
$$y = \sin x + \cos x = \sqrt{2}\sin\left(x + \frac{\pi}{4}\right)$$

$x + \frac{\pi}{4} = X$

$y - 0 = Y$

$Y = \sqrt{2}\sin X$

$$\boxed{\begin{aligned} D_f &= R \\ R_f &= [-\sqrt{2}, \sqrt{2}] \end{aligned}}$$



FUNCTIONS

$y = x^n \quad , x \geq 0$

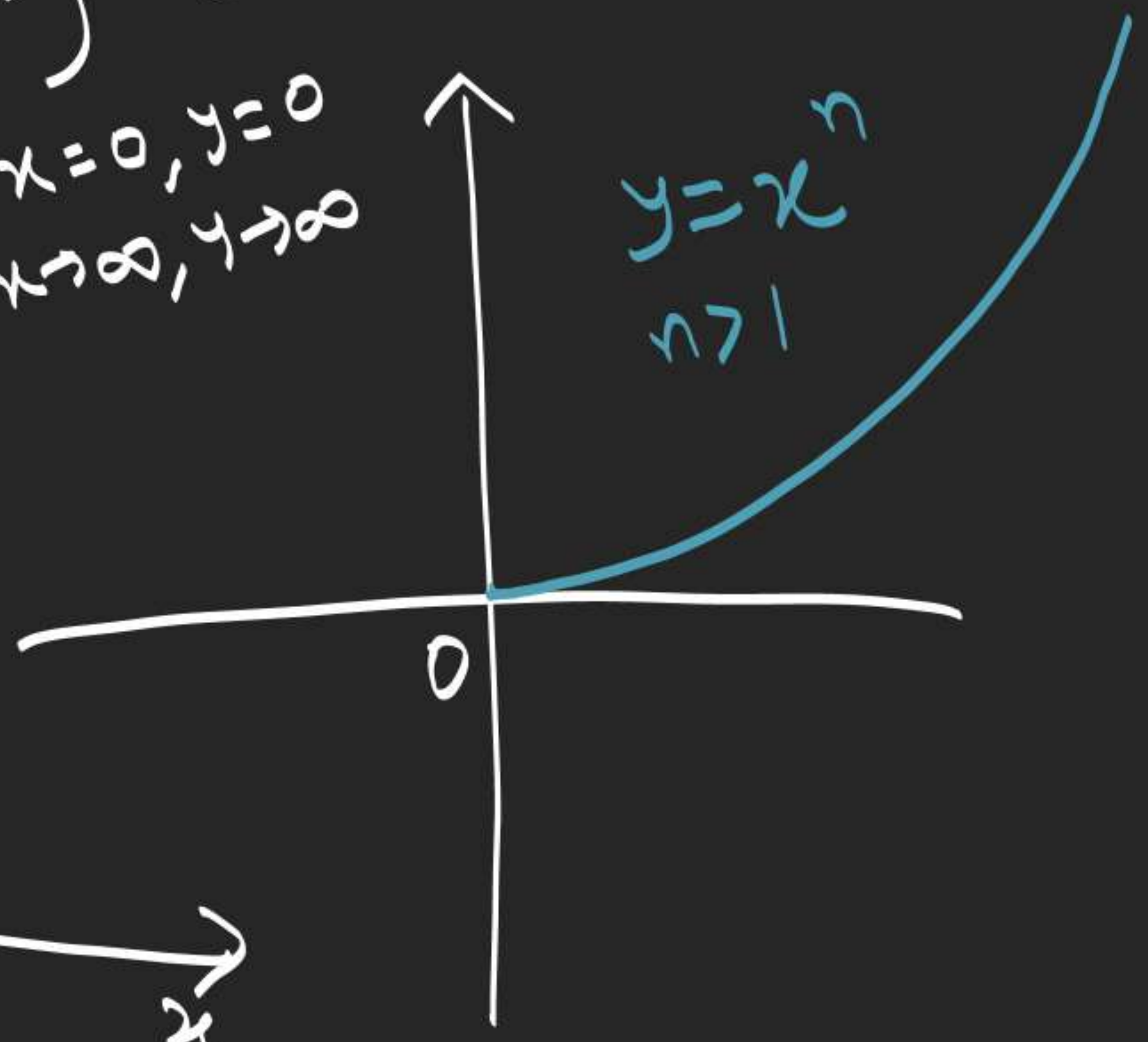
$y' = n x^{n-1}$

$y'' = n(n-1)x^{n-2}$

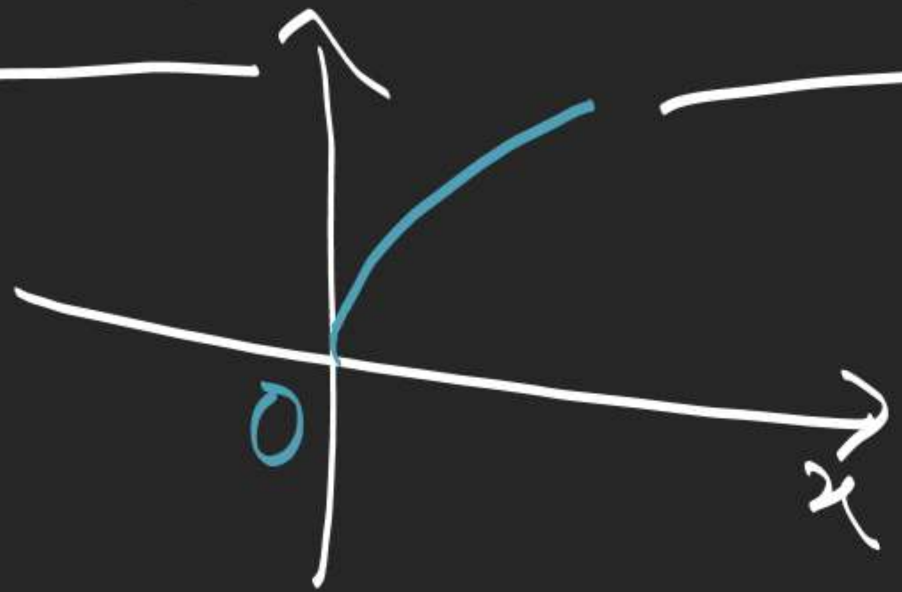
① If $n > 1$

$x = 0, y = 0$

$x \to \infty, y \to \infty$



② If $0 < n < 1$



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(3) $n < 0$ $\underline{x > 0}$

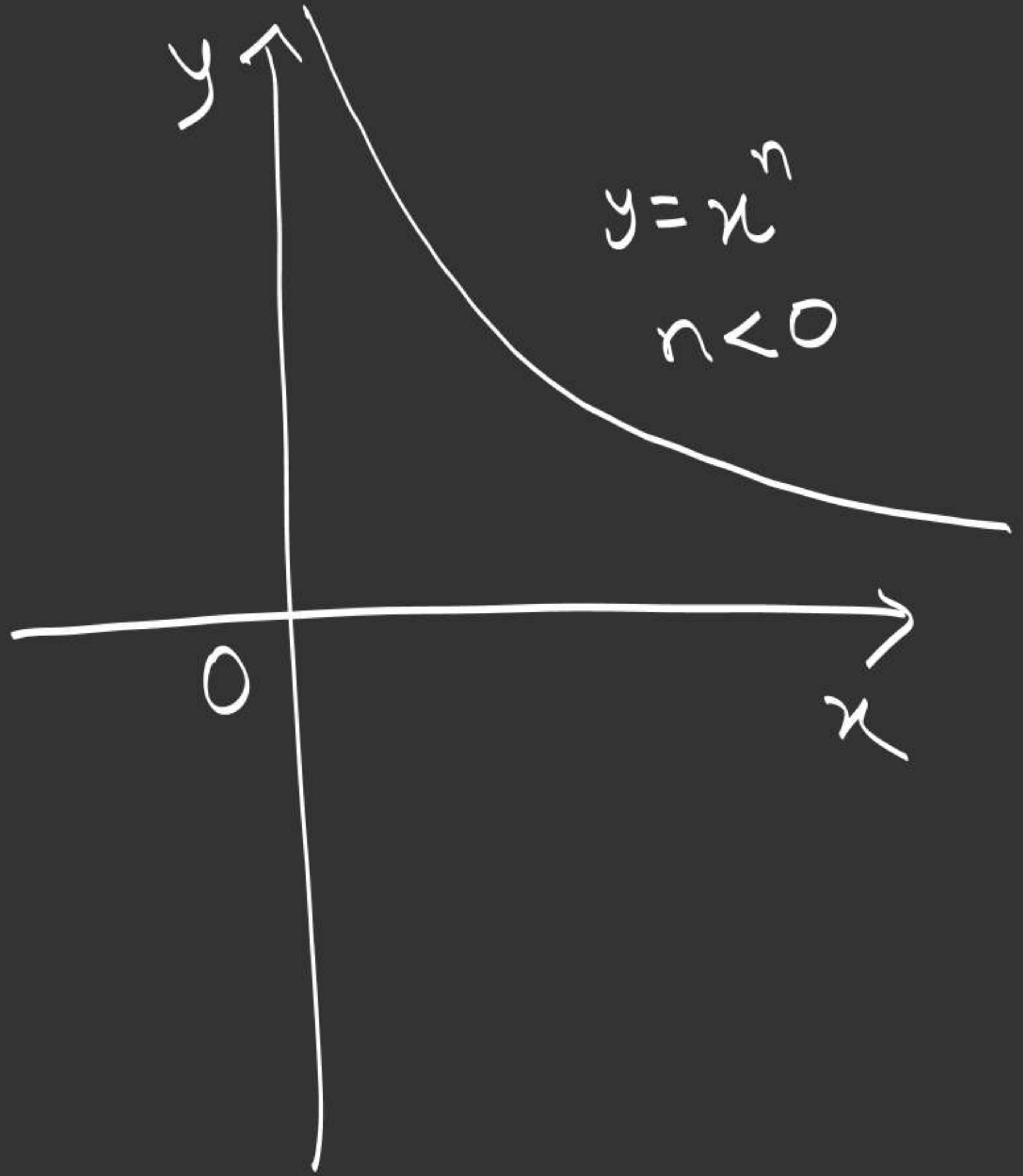
$y = x^n$

$y' = nx^{n-1} < 0$

$y'' = n(n-1)x^{n-2} > 0$

$x \to 0, y \to \infty$

$x \to \infty, y \to 0$



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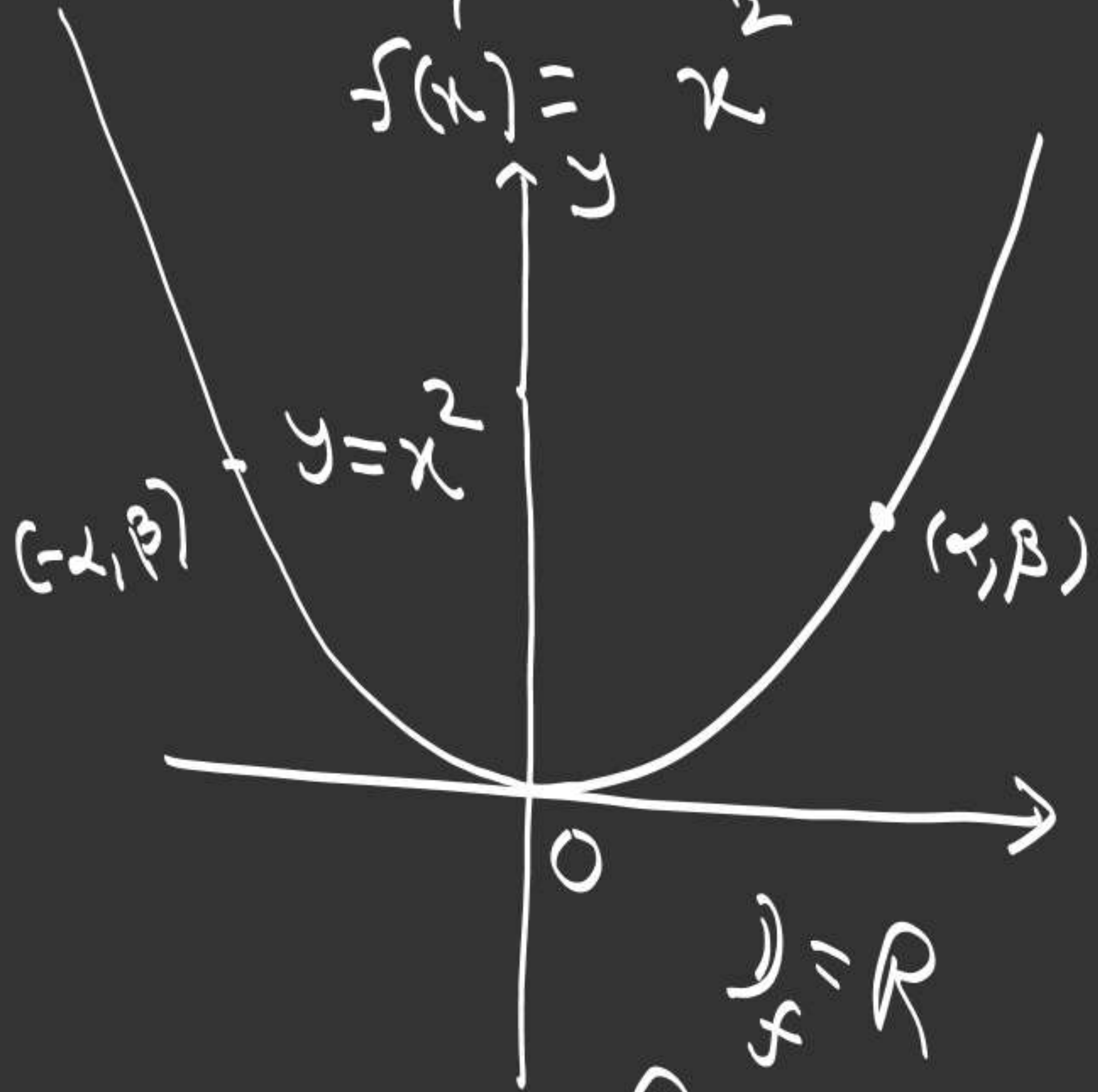
$\boxed{(\alpha, \beta), (-\alpha, \beta)}$

$f(x) = x^2$

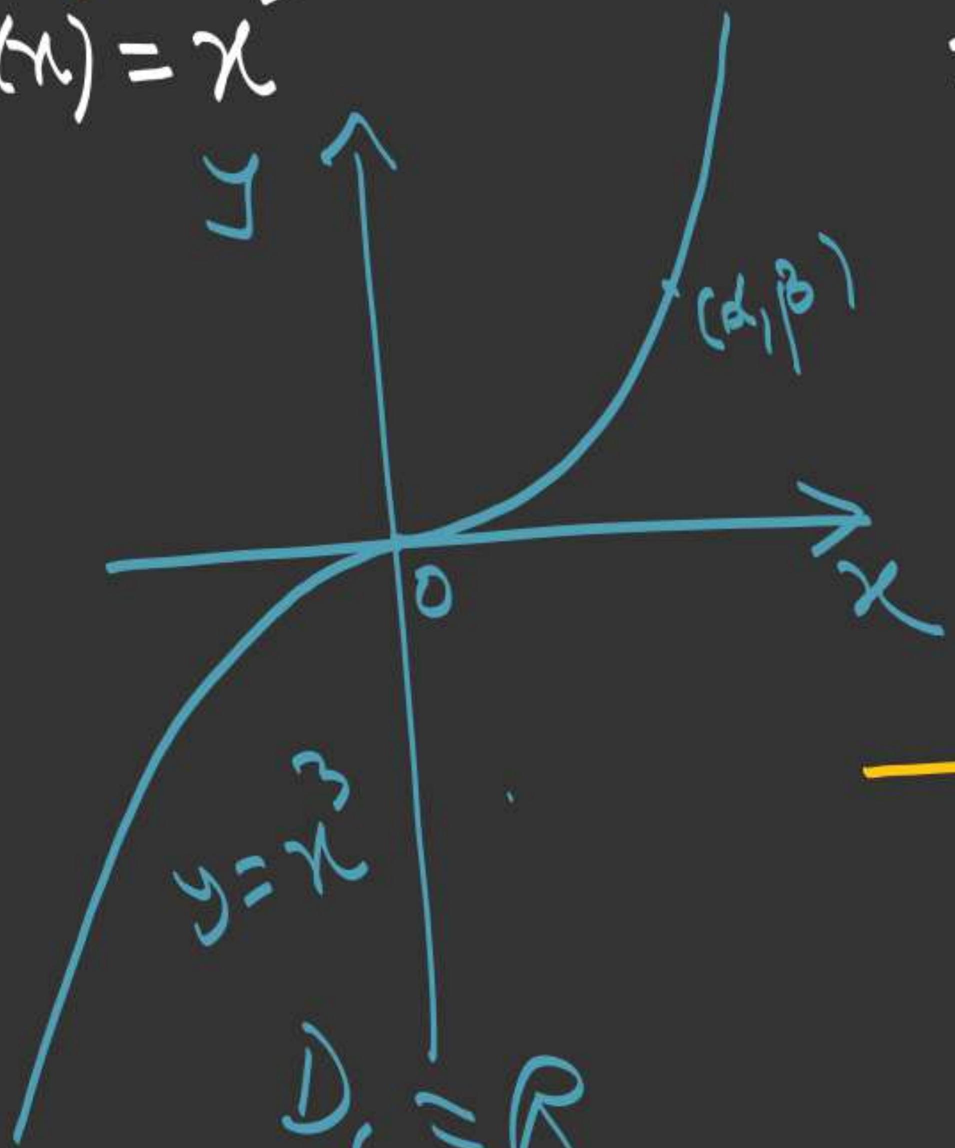


$D_f = R$

$R_f = [0, \infty)$

$(\alpha, \beta), (-\alpha, -\beta)$

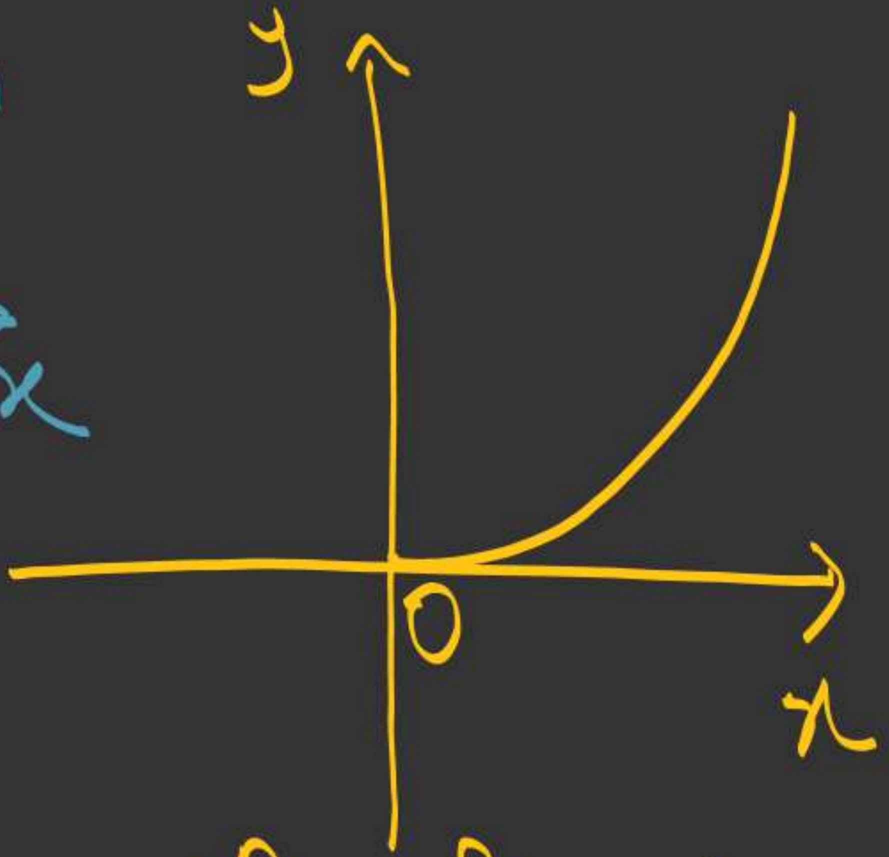
$f(x) = x^3$



$D_f = R$

$R_f = R$

$f(x) = x^{3/2}$



$D_f = [0, \infty)$

$R_f = [0, \infty)$

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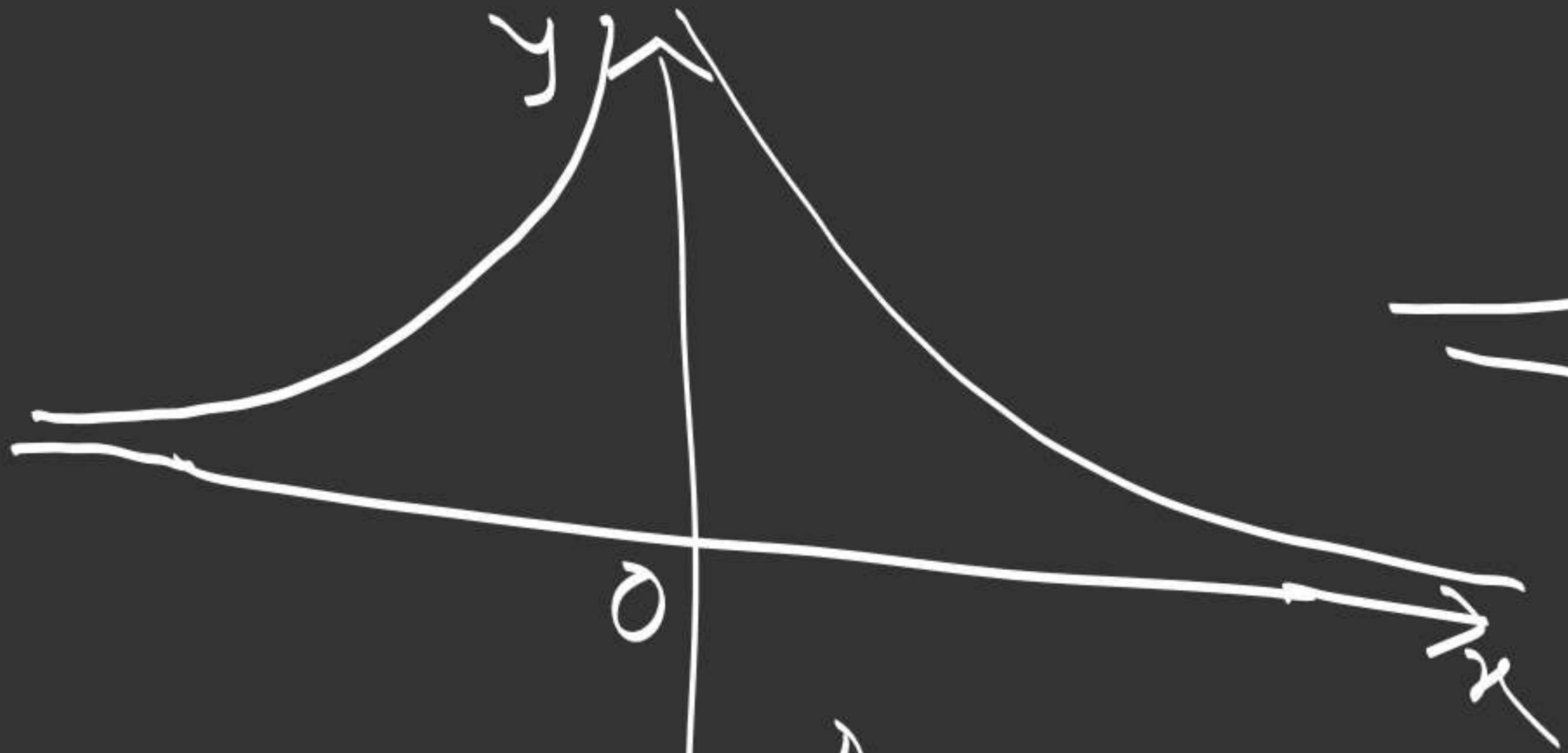
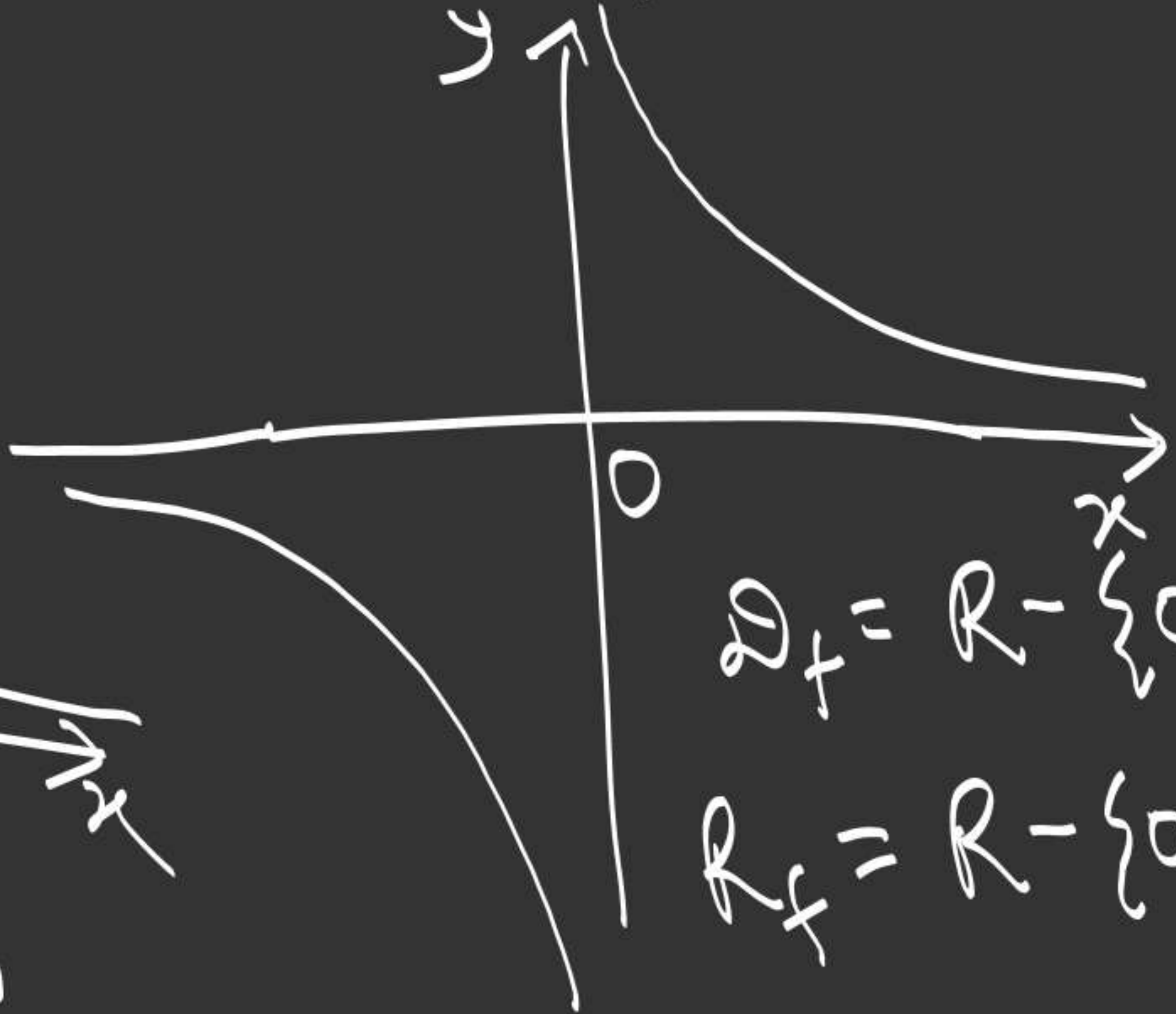
$$f(x) = \frac{1}{x^4}$$



$$D_f = R - \{0\}$$

$$R_f = (0, \infty)$$

$$f(x) = \frac{1}{x^7}$$

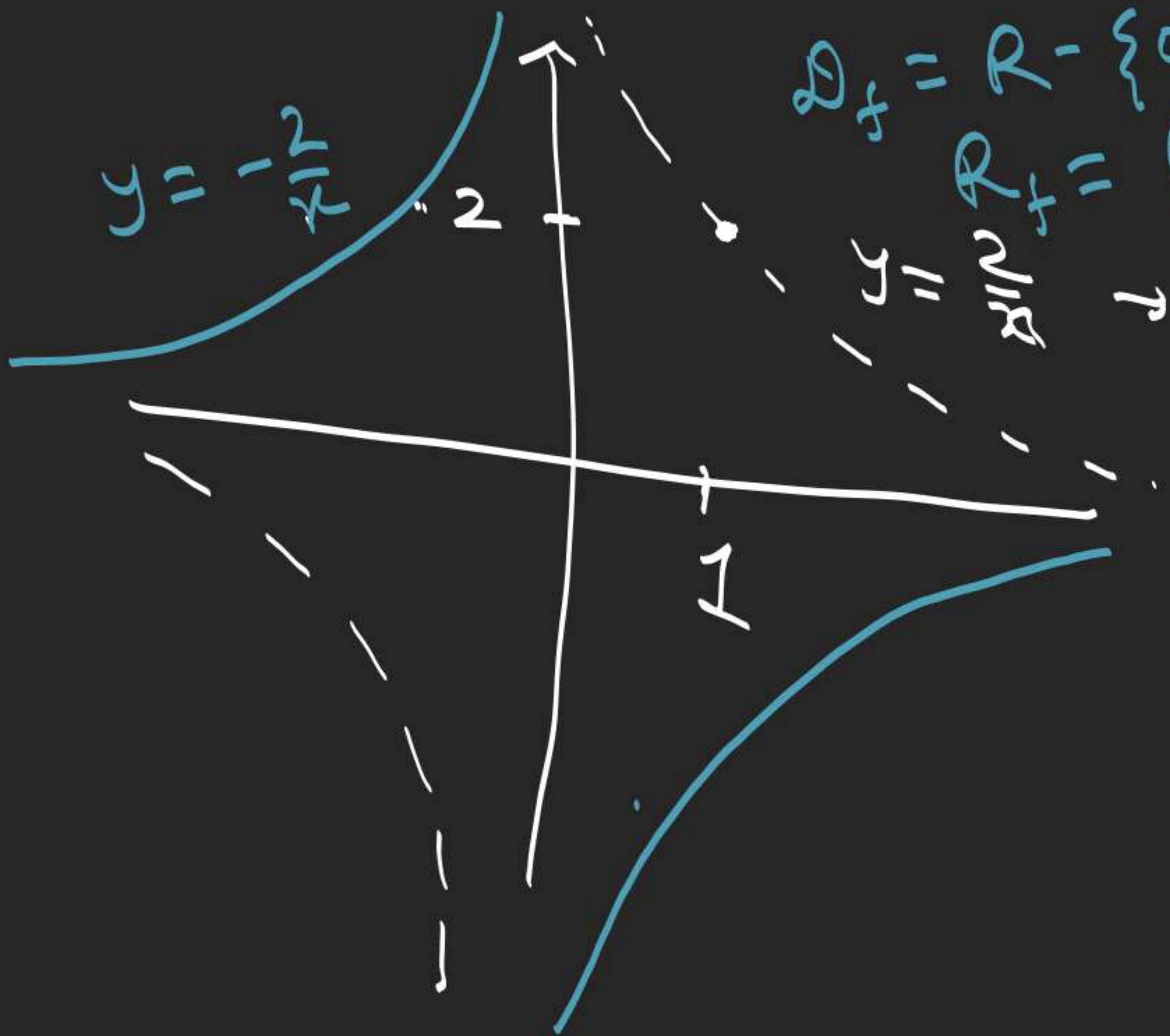


$$D_f = R - \{0\}$$

$$R_f = R - \{0\}$$

FUNCTIONS

$$y = -\frac{2}{x} \longrightarrow (-\alpha, \beta)$$



$$D_f = R - \{0\}$$
$$R_f = R - \{0\}$$

$$y = \frac{2}{x} \rightarrow (\alpha, \beta)$$

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$$y = \frac{3-5x}{x+2} = \frac{-5(x+2)+13}{x+2}$$

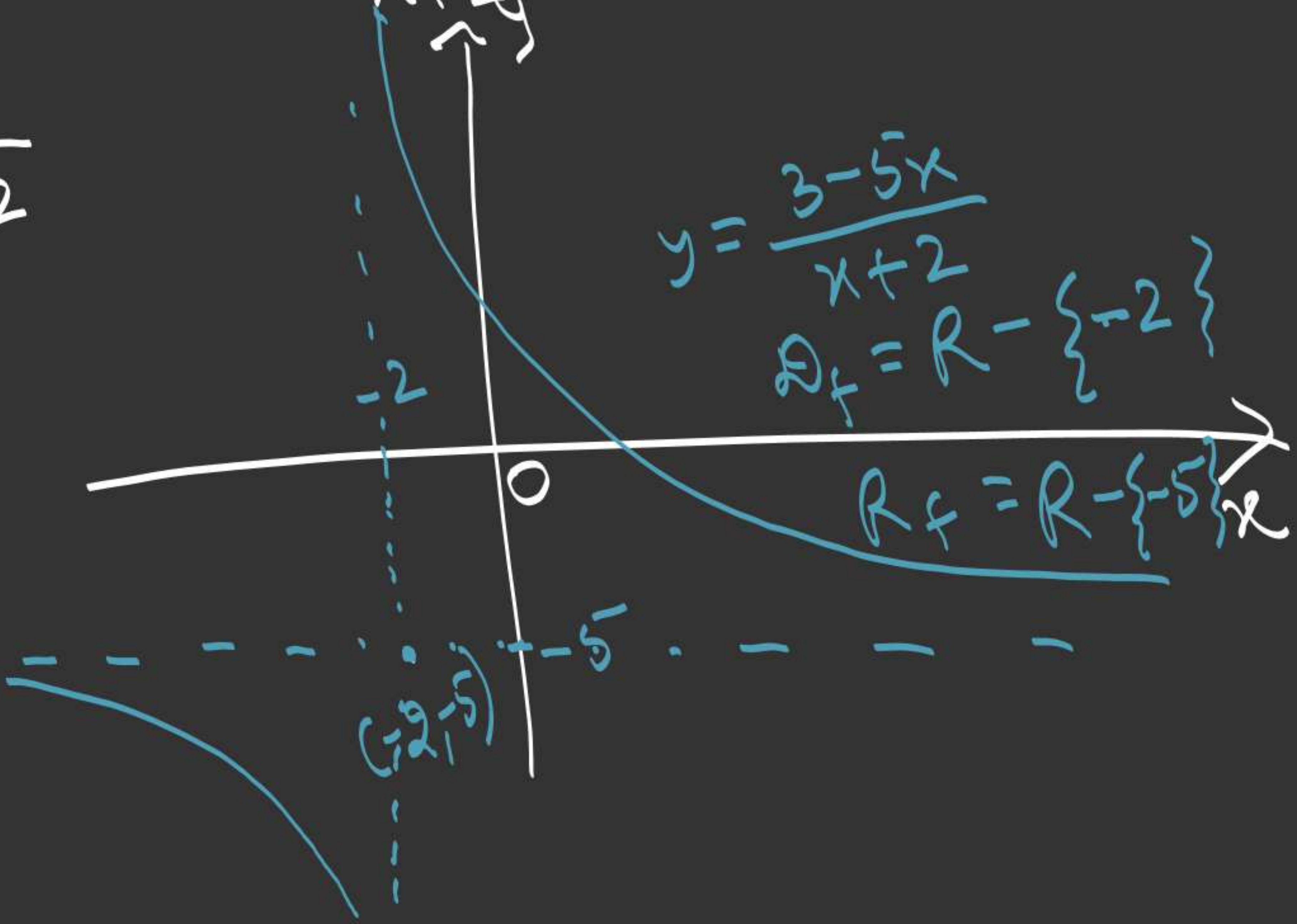
$$y = -5 + \frac{13}{x+2}$$

$$y+5 = \frac{13}{x+2}$$

$$\boxed{Y = \frac{13}{X}}$$

$$y+5 = Y$$

$$x+2 = X$$



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FUNCTIONS

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$\underline{\underline{13}}, \underline{12}, 11$

Find the domain of

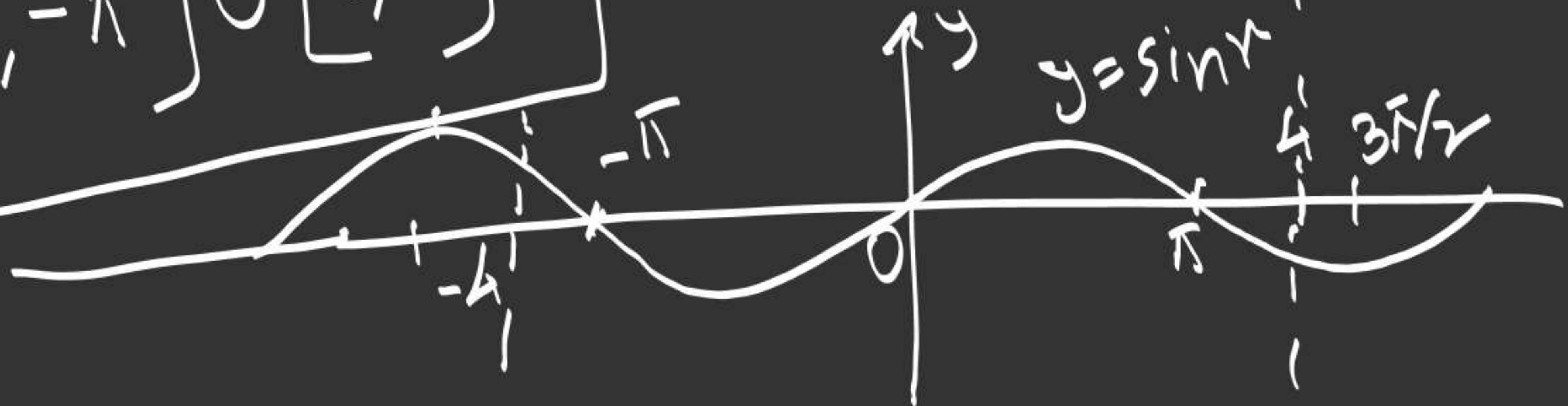
① $f(x) = \sqrt{\sin x} + \sqrt{16 - x^2}$

$\sin x \geq 0$ & $16 - x^2 \geq 0$

$\downarrow$

$x \in [-4, 4]$

$$\boxed{D_f = [-4, -\pi] \cup [0, \pi]}$$



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FUNCTIONS

2 $\quad f(x) = \dfrac{\sqrt{\cos x - \frac{1}{2}}}{\sqrt{6 + 35x - 6x^2}}$

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FUNCTIONS

11. (a) 11, 1 (b) $\frac{13}{3}$, -1 (c) 10, -4

9. $\boxed{23}$

11. (a) $10\cos^2 x - 6\sin x\cos x + 2\sin^2 x$

$= 8\cos^2 x - 6\sin x\cos x + 2$

$= 4(1+\cos 2x) - 3\sin 2x$

$= 4 + \underbrace{4\cos 2x - 3\sin 2x}$

$\in [4-5,\ 4+5]$

(b) $1 + 2\sin x + 3\cos^2 x$

$= 4 + 2\sin x - 3\sin^2 x$

$= 4 - 3\left(\sin^2 x - \frac{2}{3}\sin x\right)$

$= \frac{13}{3} - 3\left(\underbrace{\sin x - \frac{1}{3}}_{\left[-\frac{4}{3},\ \frac{2}{3}\right]}\right)^2$

$\left(\sin x - \frac{1}{3}\right)^2 \in \left[0, \frac{16}{9}\right]$

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FUNCTIONS

12.

$$\sum_{r=1}^{5} \cos^2 \frac{r\pi}{11} = \frac{1}{2}\sum_{r=1}^{5}\left(1+\cos\frac{2r\pi}{11}\right)$$

$$= \frac{1}{2}\left(5 + \frac{\sin\left(\frac{5}{2}\left(\frac{2\pi}{11}\right)\right)}{\sin\left(\frac{2\pi}{11\times 2}\right)}\cos\left(\left(\frac{2\pi}{11}+\frac{10\pi}{11}\right)\frac{1}{2}\right)\right)$$

13.

$$\cos^2\alpha + \cancel{\cos^2(\alpha+\beta)} - \left(\underbrace{\cos(\cancel{\alpha+\beta})+\cos(\alpha-\beta)}\right)\underbrace{\cos(\cancel{\alpha+\beta})}$$

$$= \cos^2\alpha - \left(\cos^2\alpha - \sin^2\beta\right) = \sin^2\beta.$$

Ans → 13