PROBLEM SET-03

SOLUTION

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Q.1 Find the number of ways in which letters of the word VALEDICTORY be arranged so that the vowels may never be separated.

Ans. **(967680)**

Sol. [Hint: $\boxed{A\ E\ I\ O}$ VLDCTRY or $8! \times 4! = 40320 \times 24 = 967680$ Ans]

['Valediction means farewell after graduation from a college. Valedictory : to take farewell]

Q.2 How many numbers between 400 and 1000 (both exclusive) can be made with the digits 2,3,4,5,6,0 if

(a) repetition of digits not allowed. (b) repetition of digits is allowed.

Ans. **(a) 60; (b) 107**

Sol. [Hint: (a) $\boxed{\ }\boxed{\ }\boxed{\ }$ 4/ 5/ 6 $3\times5\times4 = 60$; (b) $\boxed{\ }\boxed{\ }\boxed{\ }$ 4/ 5/ 6 $3\times6\times6 = 108 - 1 = 107$]

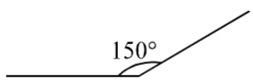
Q.3 The interior angles of a regular polygon measure 150° each. The number of diagonals of the polygon is

(A) 35 (B) 44 (C) 54 (D) 78

Ans. **(C)**

Sol. [Hint: exterior angle = 30°

Hence number of sides $= \dfrac{360°}{30} = 12$



$\therefore$ number of diagonals $= \dfrac{12(12-3)}{2} = 54$]

[Note that sum of all exterior angles of a polygon $= 2\pi$ and sum of all the interior angles of a polygon $=(2n-4)\dfrac{\pi}{2}$]

Q.4 The number of ways in which 5 different books can be distributed among 10 people if each person can get at most one book is:

(A) 252 (B) 10^5 (C) 5^{10} (D) ${}^{10}C_5 \cdot 5\,!$

Ans. **(D)**

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Sol. [Hint:$_{206/perm/SC}$ Select 5 boys in ${}^{10}C_5$ and distribute 5 books in 5! ways hence ${}^{10}C_5 \cdot 5!$]

Q.5 The 9 horizontal and 9 vertical lines on an 8 × 8 chessboard form 'r' rectangles and 's' squares. The ratio $\frac{s}{r}$ in its lowest terms is

(A) $\frac{1}{6}$ (B) $\frac{17}{108}$ (C) $\frac{4}{27}$ (D) none

Ans. **(B)**

Sol. [Sol. Number of squares are $S = 1^2 + 2^2 + 3^2 + \dots\dots + 8^2 = \dfrac{8(9)(17)}{6} = 204$

no. of rectangles $r = {}^9C_2 \cdot {}^9C_2 = 1296$

hence $\dfrac{s}{r} = \dfrac{204}{1296} = \dfrac{51}{324} = \dfrac{17}{108}$]

Q.6 There are 720 permutations of the digits 1,2,3,4,5,6. Suppose these permutations are arranged from smallest to largest numerical values, beginning from 123456 and ending with 654321.

(a) What number falls on the 124th position?

(b) What is the position of the number 321546?

Ans. **((a) 213564, (b) 267th)**

Sol. [Sol.$_{81015/perm/SUB}$ **(a)** digits 1, 2, 3, 4, 5, 6

starting with 1, number of numbers = 120 $\boxed{1\,|\;\,|\;\,|\;\,|\;\,|\;\,}$

starting with 2, 1, 3, 4 number of numbers = 2 $\boxed{2\,|\,1\,|\,3\,|\,4\,|\;\,|\;\,}$

123rd $\boxed{2\,|\,1\,|\,3\,|\,5\,|\,4\,|\,6}$

finally 124th is = 213564

(b) N = 321546

number of numbers beginning with 1 = 120 $\boxed{1\,|\;\,|\;\,|\;\,|\;\,|\;\,}$

number of numbers beginning with 2 = 120 $\boxed{2\,|\;\,|\;\,|\;\,|\;\,|\;\,}$

starting with 31 .... = 24 $\boxed{3\,|\,1\,|\;\,|\;\,|\;\,|\;\,}$

starting with 3214 .... = 2 $\boxed{3\,|\,2\,|\,1\,|\,4\,|\;\,|\;\,}$

finally = 1 $\boxed{3\,|\,2\,|\,1\,|\,5\,|\,4\,|\,6}$

hence N has 267th position]

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Q.7 Number of 4-digit numbers of the form N = abcd which satisfy following three conditions

(i) $4000 \leq N < 6000$ (ii) N is a multiple of 5 (iii) $3 \leq b < c \leq 6$ is equal to

(A) 12 (B) 18 (C) 24 (D) 48

Ans. **(C)**

Sol. [Sol.$_{223/perm/SC}$ We have N = | a | b | c | d | **[12th, 03-01-2010, P-1]**

First place a can be filled in 2 ways i.e. 4, 5 $(4000 \leq N < 6000)$

For b and c, total possibilities are '6' $(3 \leq b < c \leq 6)$

i.e. 34, 35, 36, 45, 46, 56

Last place d can be filled in 2 ways i.e. 0, 5 (N is a multiple of 5)

Hence total numbers $= 2 \times 6 \times 2 = 24$ **Ans.**]

Q.8 Find the number of 3-digit numbers in which the digit at hundredth's place is greater than the other two digit.

Ans. **(285)**

Sol. [Sol. When all 3 are distinct

1st	2nd	3rd
x	y	z

[11th, 16-11-2008]

$^{10}C_3 \cdot 2$ (largest being at the 1st place and the remaining two can be arranged in two ways)

$= 240$

when $y = z$

$^{10}C_2 \cdot 1$ (largest on the 1st place and remaining two being equal on the 2nd and 3rd place) e.g. (211, 100)]

$= 45$

Total $= 285$ **Ans.**]

Q.9 Find the number of 6-digit numbers of the form a b c d e f if the digits satisfy the condition

$a + b + c + d + e + f = a^2 + b^2 + c^2 + d^2 + e^2 + f^2$.

Ans. **(32)**

Sol. [Sol. Possible only if all six digit are either 0 or 1.

$\square\square\square\square\square\square \rightarrow 1 \cdot 2^5$

0 can not be filled

$= 32$]

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Q.10 Number of permutations of 1,2,3,4,5,6,7,8 and 9 taken all at a time, such that the digit

1 appearing somewhere to the left of 2

3 appearing to the left of 4 and

5 somewhere to the left of 6 , is

(e.g. 815723946 would be one such permutation)

(A) $9 \cdot 7\,!$ (B) 8! (C) 5! 4! (D) 8! 4!

Ans. **(A)**

Sol. [Sol.$_{214/\text{perm/SC}}$ Number of digits are 9 **[12th, 28-09-2008]**

select 2 places for the digit 1 and 2 in ${}^9C_2$ ways

from the remaining 7 places select any two places for 3 and 4 in ${}^7C_2$ ways

and from the remaining 5 places select any two for 5 and 6 in ${}^5C_2$ ways

now, the remaining 3 digits can be filled in 3! ways

$\therefore$ Total ways = ${}^9C_2 \cdot {}^7C_2 \cdot {}^5C_2 \cdot 3!$

$$= \frac{9!}{2!\cdot 7!} \cdot \frac{7!}{2!\cdot 5!} \cdot \frac{5!}{2!\cdot 3!} \cdot 3! = \frac{9!}{8} = \frac{9\cdot 8\cdot 7!}{8} = 9 \cdot 7!$$ **Ans.**]

Q.11 A convex polygon has 44 diagonals. The polygon is

(A) nonagon (B) decagon (C) undecagon (D) Dodecagon

Ans. **(C)**

Sol. [Sol.$_{64/\text{perm/SC}}$ If number of sides is n, then

total number of diagonals of a convex polygon = ${}^nC_2 - n = 44$ (given) **[11th, 28-11-2009, P-2]**

$\Rightarrow \quad n(n-1) - 2n = 88 \quad \Rightarrow \quad n^2 - 3n - 88 = 0 \quad \Rightarrow \quad (n-11)(n+8) = 0$

$\Rightarrow \quad n = 11 \quad \Rightarrow$ undecagon **Ans.**]

Q.12 The number of ways in which 10 candidates $A_1, A_2, \dots\dots, A_{10}$ can be ranked, so that A_1 is always above A_2 is

(A) $\frac{10!}{2}$ (B) $8! \times {}^{10}C_2$ (C) ${}^{10}P_2$ (D) ${}^{10}C_2$

Ans. **(AB)**

Sol. [Hint:$_{1558/\text{pc}}$ $\frac{(10)!}{2!}$] **[11th, 14-12-2008]**