

# Distribution of alike objects

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Aim → To find  $(x_1, x_2, x_3) = \binom{n}{n+p-1}$  over 'p' persons

$- , (0,0,6), (0,6,0), (6,0,0)$   
 $- , - , -$

Method →

$$\frac{(n + (p-1))!}{n! (p-1)!} = C_{p-1}$$

$0|00|000$

$6+2C_2$

$0|0|00$

$0|0|00$

$0|0|00$

$0|0|00$

$0|0|00$

$0|0|00$

alike  
6 objects over 3 persons

6 objects  
over 3 persons

$x_1 + x_2 + x_3 = 6$   
 $x_1, x_2, x_3 \in \text{Whole no.s}$

$0|00|000$

$0|0|00|00$

$0|0|00|00$

$0|00|00|00$

To distribute 'n' alike objects over 'p' persons

$(n > p)$  so that every person gets atleast one object.

$$\text{Distribute 1 object to each person}$$

$$\begin{aligned} & \text{Total objects left: } n-p \\ & \text{Number of ways to distribute: } C_{p-1}^{(n-p)+p-1} \\ & \text{Condition: } x+y+z=6, x,y,z \in \mathbb{N} \\ & \text{Simplifying: } 6-3+2 = 5 \\ & \text{Number of ways: } C_2^{n-1} \end{aligned}$$

$x \quad (n-p)+p-1 \quad C_{p-1}^{p-1}$

$x+y+z=6, x,y,z \in \mathbb{N}$

$\boxed{5 \quad C_2^{n-1}}$

$0x0x0x\dots x0x\dots x0$

$$Q_1 \quad Q_2 \quad Q_3 \quad Q_8$$

3

$$30 - 2(8) + 7 = C_7 = \boxed{21}$$

Find no. of solns of  $x_1 + x_2 + x_3 + \dots + x_8 = 30$   
 s.t.  $x_i \geq 2, x_i \in \mathbb{I}$ .

$\uparrow$

$Q_1 + Q_2$

$\downarrow$

$x_1 + x_2 + x_3 + \dots + x_8 = 30$

$\uparrow$

$Q_1 + Q_2 + Q_3$

2: Find no. of natural number soln.

of eqn:

$$\begin{matrix} \geq 1 & \geq 1 & \geq 1 \\ x+y+z+t = 102 \end{matrix}$$

DPP-9 (remaining)  
DPP-10 (1-7)

$$x+y+z+t+k = 0, 1, 2, 3, \dots, 30$$

$$102 - 3+2 \binom{101}{2} = \binom{4(45)}{3} + \binom{5(45)}{3} + \binom{6(45)}{3} + \dots + \binom{33(3)}{3}$$

$$= \boxed{\binom{34}{4}}$$

3: Find no. of non negative integral solution

of (i) inequality

$$x+y+z+t \leq 30$$

(ii) inequality  $23 < x+y+z+t \leq 30$

$$\boxed{35\binom{4}{4} - 27\binom{4}{4}}$$

$$24 \leq \leq 30 \text{ w.r.t. } 0, 1$$

$$30+4\binom{4}{4}$$

$$23 < x + y + z + t \leq 30$$

$$x + y + z + t = 24, 25, 26, 27, 28, 29, 30$$

$$- {}^{27}C_4 \left( {}^{27}C_5 + {}^{27}C_3 \right) {}^{20}C_3 + {}^{29}C_2 + {}^{30}C_3 + {}^{31}C_3 + {}^{32}C_3 + {}^{33}C_3$$

$\swarrow$        $\searrow$   
 ${}^{28}C_4$        ${}^{29}C_4$

$$= - {}^{27}C_4 {}^{34}C_4$$