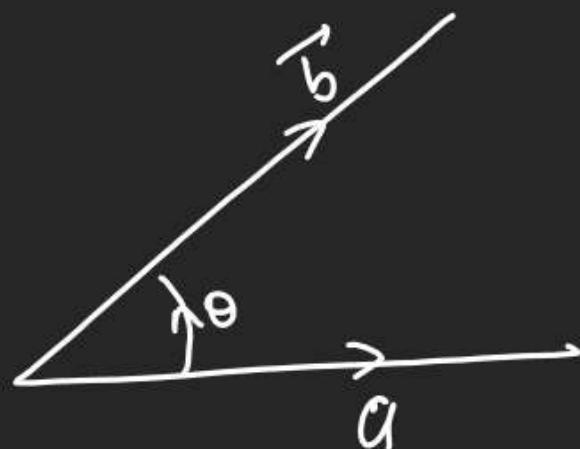


# ① Dot Product



$$1) \vec{a} \cdot \vec{b} = |\vec{a}| |\vec{b}| \cos \theta$$

$$0 \leq \theta < \pi$$

$$2) \vec{a} \cdot \vec{b} = |\vec{a}| |\vec{b}| (\cos \theta \text{ is scalar})$$

$\downarrow$  Sc.     $\downarrow$  Sc.     $\downarrow$  Sc.

$$(3) \text{ Angle b/w } \vec{a} \text{ & } \vec{b} = \theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| |\vec{b}|}$$

$$(4) \vec{a} = a_1 \hat{i} + a_2 \hat{j} + a_3 \hat{k}$$

$$\vec{b} = b_1 \hat{i} + b_2 \hat{j} + b_3 \hat{k}$$

$$\boxed{\vec{a} \cdot \vec{b} = a_1 b_1 + a_2 b_2 + a_3 b_3}$$

$$(5) \cos \theta = \frac{a_1 b_1 + a_2 b_2 + a_3 b_3}{\sqrt{a_1^2 + a_2^2 + a_3^2} \sqrt{b_1^2 + b_2^2 + b_3^2}}$$

$$\boxed{(\cos \theta) = \frac{\sum a_i b_i}{\sqrt{\sum a_i^2} \sqrt{\sum b_i^2}}}$$

$$(6) |\vec{a}|^2 = ?$$

$$\vec{a}^2 = \vec{a} \cdot \vec{a}$$

$$= |\vec{a}| |\vec{a}| \cos 0^\circ$$

$$|\vec{a}|^2 = (a_1^2 + a_2^2 + a_3^2) = |\vec{a}|^2$$

Satur.

$$(7) (\vec{a} + \vec{b})^2 = ?$$

$$(\vec{a} + \vec{b}) \cdot (\vec{a} + \vec{b})$$

$$|\vec{a}|^2 + |\vec{b}|^2 + \underline{\vec{a} \cdot \vec{b}} + \underline{\vec{b} \cdot \vec{a}}$$

$$|\vec{a}|^2 + |\vec{b}|^2 + 2 \vec{a} \cdot \vec{b}$$

$$(8) \vec{a} \cdot \vec{b} = |\vec{a}| |\vec{b}| \cos \theta$$

|   |  |                          |                           |
|---|--|--------------------------|---------------------------|
|   | $\downarrow$                                   | $\downarrow$             | $\downarrow$              |
| $\theta = 0$                                  | $\theta = \pi$                                 | $\theta = \frac{\pi}{2}$ | $\theta = \frac{3\pi}{2}$ |
| $\vec{a} \cdot \vec{b} =  \vec{a}   \vec{b} $ | $\vec{a} \cdot \vec{b} = - \vec{a}   \vec{b} $ | $\vec{a} \perp \vec{b}$  | $\vec{a} \perp \vec{b}$   |
| Max.  |  |                          |                           |

$$-|\vec{a}| |\vec{b}| \leq \vec{a} \cdot \vec{b} \leq |\vec{a}| |\vec{b}|$$

Min.

(9)  $(\vec{a} \cdot \vec{b})^2 \in ?$

$$\vec{a} \cdot \vec{b} = |a| |b| \cos \theta$$

$$(\vec{a} \cdot \vec{b})^2 = |a|^2 |b|^2 \cos^2 \theta$$

$$0 \leq \cos^2 \theta \leq 1$$

$$0 \leq |a|^2 |b|^2 \cos^2 \theta \leq |a|^2 |b|^2$$

$$0 \leq (\vec{a} \cdot \vec{b})^2 \leq |a|^2 |b|^2$$

Rwye.

(10)  $\theta = \text{acute angle.}$ 

$$\vec{a} \cdot \vec{b} > 0$$

obtuse angle  $\Rightarrow 0 > \theta < 0$ 

$$\vec{a} \cdot \vec{b} < 0$$

11)  $\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}$   
 $\vec{a} \cdot (\vec{b} + \vec{c}) = \vec{b} \cdot \vec{a} + \vec{a} \cdot \vec{c}$   
 $\vec{a} \perp \vec{b} \Rightarrow \vec{a} \cdot \vec{b} = 0$

But opposite is not true.  
 $\vec{a} \cdot \vec{b} = 0$   
 $\downarrow \quad \downarrow \quad \downarrow$   
 $a_1 = 0 \quad b_1 = 0 \quad \vec{a} \perp \vec{b}$

Q.  $\vec{a} \cdot \hat{i} = ?$   
 $(a_1 \hat{i} + a_2 \hat{j} + a_3 \hat{k}) \cdot \hat{i}$

$$a_1 + 0 + 0 = a_1$$

Q.  $\vec{a} \parallel \vec{b}$  then.

$$\frac{a_1}{b_1} = \frac{a_2}{b_2} = \frac{a_3}{b_3}$$

Q.  $(\vec{a} \cdot \hat{i}) \hat{i} + (a_2 \hat{j}) \hat{j} + (a_3 \hat{k}) \hat{k} = ?$

$$a_1 \hat{i} + a_2 \hat{j} + a_3 \hat{k} = \vec{a}$$

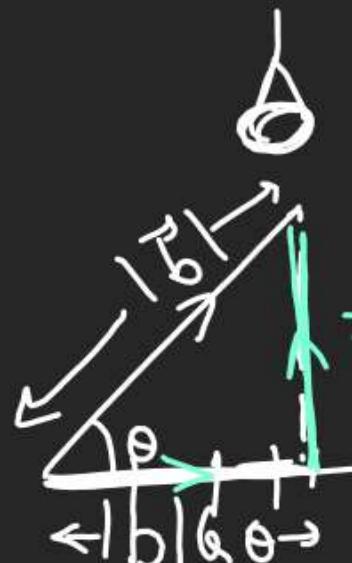
Q.  $(\vec{a} \cdot \hat{i})^2 + (\vec{a} \cdot \hat{k})^2 + (\vec{a} \cdot \hat{j})^2 = ?$

$$a_1^2 + a_2^2 + a_3^2 = |\vec{a}|^2$$

$$\text{as } |\vec{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$$

# Projection of $\vec{b}$ on $\vec{a}$

$\leftarrow$   
Scalar value & Vector value can be asked



$$\vec{a} \cdot \vec{b} = |\vec{a}| |\vec{b}| \cos \theta$$

$$|\vec{b}| \cos \theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}|}$$

$$|\vec{b}| \cos \theta \hat{a} + \vec{r} = \vec{b}$$

$$\vec{r} = \vec{b} - |\vec{b}| \cos \theta \hat{a}$$

$$5\vec{a}$$

$$\text{Proj of } \vec{b} \text{ on } \vec{a} = |\vec{b}| \cos \theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}|}$$

$$\text{Proj vector of } \vec{b} \text{ on } \vec{a} = |\vec{b}| \cos \theta \cdot \hat{a} = \left( \frac{\vec{a} \cdot \vec{b}}{|\vec{a}|} \right) \hat{a} = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}|^2} \vec{a}$$

$$\text{Component of } \vec{b} \text{ along } \vec{a} = |\vec{b}| \cos \theta \cdot \hat{a} = \left( \frac{\vec{a} \cdot \vec{b}}{|\vec{a}|^2} \right) \vec{a}$$

$$\text{Component of } \vec{b} \perp \text{ to } \vec{a} = \vec{b} - \left( \frac{\vec{a} \cdot \vec{b}}{|\vec{a}|^2} \right) \vec{a}$$

$$\text{Proj. vector of } \vec{P} \text{ on } \vec{m} = \frac{\vec{P} \cdot \vec{m}}{|\vec{m}|} \cdot \hat{m}$$

| Q If  $\vec{m}$  is decomposed into  $\parallel^r$  &  $\perp^r$   
vector of  $\vec{n}$  find vectors.

$$(1) \parallel^r = \left( \frac{\vec{m} \cdot \vec{n}}{|\vec{n}|} \right) \hat{n}$$

$$(2) \perp^r = \vec{m} - \left( \frac{\vec{m} \cdot \vec{n}}{|\vec{n}|} \right) \hat{n}$$

Q Angle b/w  $\overrightarrow{1-2j+k}$  &  $3\hat{i} - 2\hat{j} + \hat{k}$

$$|\vec{a}| = \sqrt{1+4+1} = \sqrt{6}$$

$$|\vec{b}| = \sqrt{9+4+1} = \sqrt{14}$$

$$\cos \theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| |\vec{b}|} = \frac{3 \times 1 + -2 \times -2 + 1 \times 1}{\sqrt{6} \sqrt{14}}$$

$$\cos \theta = \frac{8}{2\sqrt{21}}$$

$$\theta = \cos^{-1}\left(\frac{4}{\sqrt{21}}\right)$$

Q  $|a| = |b| = |a+b| = 1$  & find  $|a-b| = ?$

Qs already  $\rightarrow$  Lemni Thm (Sine Rule)

$$|\vec{a} + \vec{b}|^2 = |a|^2 + |b|^2 + 2\vec{a} \cdot \vec{b}$$

$$|\vec{a} - \vec{b}|^2 = |a|^2 + |b|^2 - 2\vec{a} \cdot \vec{b}$$

$$|\vec{a} + \vec{b}|^2 + |\vec{a} - \vec{b}|^2 = 2(|a|^2 + |b|^2)$$

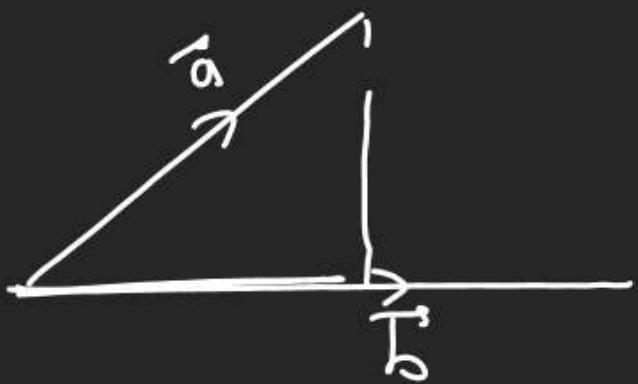
$$1^2 + |a-b|^2 = 2(1^2 + 1^2)$$

$$|a-b|^2 = 3$$

$$|a-b| = \sqrt{3}$$

$$\text{Q. } \vec{a} = 4\hat{i} + 6\hat{j}, \vec{b} = 3\hat{j} + 4\hat{k} \text{ fnd.}$$

Component of  $\vec{a}$  along  $\vec{b}$



$$\left( \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|^2} \right) \cdot \vec{b}$$

$$\vec{a} \cdot \vec{b} = 4 \times 0 + 6 \times 3 + 0 \times 4 = 18$$

$$|\vec{b}| = \sqrt{0+9+16} = 5$$

$$= \left( \frac{18}{25} \right) \cdot (3\hat{j} + 4\hat{k})$$

$$\vec{a} \cdot \vec{b} > 0$$

$$\text{Q. If } \vec{r} = ((a^2 - 4)\hat{i} + 2\hat{j} - (a^2 - 9)\hat{k})\hat{r}$$

has acute angle with o axes  
then interval of  $a$ ?

$$\vec{r} \text{ is Acute}$$

Acute

$$\vec{r} \cdot \hat{i} > 0$$

$$(a^2 - 4) > 0$$

$$(a-2)(a+2) > 0$$

$$a < -2 \cup a > 2$$

$$\vec{r} \text{ is Acute}$$

Acute

$$\vec{r} \cdot \hat{j} > 0$$

$$2 > 0$$

$$R$$

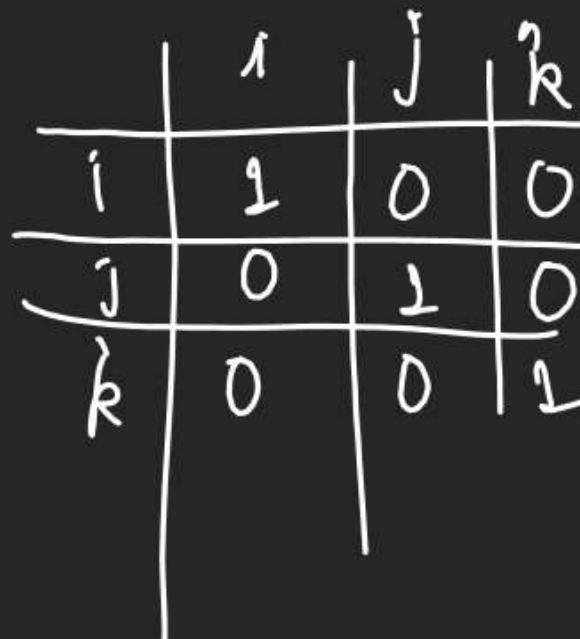
$$\vec{r} \cdot \hat{k} > 0$$

$$-(a^2 - 9) > 0$$

$$a^2 - 9 < 0$$

$$(a-3)(a+3) < 0$$

$$-3 < a < 3$$



Q If  $\vec{a}, \vec{b}, \vec{c}$  are non zero vectors.

Such that  $\boxed{\vec{a} + \vec{b} + \vec{c} = 0}$

then if  $m = \vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$  then

$m < 0$      $m > 0$      $m = 0$     NOT.

$$(\vec{a} + \vec{b} + \vec{c})^2 = 0$$

$$|\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) = 0$$

$$2m = -(|\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2)$$

$$m = -\frac{(|\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2)}{2}$$

$$m = -V_0 \Rightarrow m < 0$$

Q If  $\vec{a}, \vec{b}, \vec{c}$  are vectors such that

$$|\vec{a}| = 3, |\vec{b}| = 4, |\vec{c}| = 5$$

$$\begin{aligned} & (\vec{a} + \vec{b}) \text{ is } \perp \text{ to } \vec{c}, \\ & (\vec{b} + \vec{c}) \text{ is } \perp \text{ to } \vec{a} \\ & (\vec{c} + \vec{a}) \text{ is } \perp \text{ to } \vec{b} \end{aligned}$$

$$|\vec{a} + \vec{b} + \vec{c}|^2 = (\vec{a} + \vec{b} + \vec{c})^2$$

$$= |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a})$$

$$|\vec{a} + \vec{b} + \vec{c}|^2 = 9 + 16 + 25$$

$$|\vec{a} + \vec{b} + \vec{c}| = \sqrt{50} = 5\sqrt{2}$$

$$\begin{aligned} & (\vec{a} + \vec{b}) \cdot \vec{c} = 0 \Rightarrow \vec{a} \cdot \vec{c} + \vec{b} \cdot \vec{c} = 0 \\ & (\vec{b} + \vec{c}) \cdot \vec{a} = 0 \Rightarrow \vec{b} \cdot \vec{a} + \vec{c} \cdot \vec{a} = 0 \\ & (\vec{c} + \vec{a}) \cdot \vec{b} = 0 \Rightarrow \vec{c} \cdot \vec{b} + \vec{a} \cdot \vec{b} = 0 \end{aligned}$$

$$2(a \cdot b + b \cdot c + c \cdot a) = 0$$

Q If  $\vec{r} \cdot \hat{i} = \vec{r} \cdot \hat{j} = \vec{r} \cdot \hat{k}$  and  $|\vec{r}| = 3$

find  $\vec{r}$ ?

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

$$\vec{r} \cdot \hat{i} = x$$

$$\vec{r} \cdot \hat{j} = y, \vec{r} \cdot \hat{k} = z$$

$$x = y = z = K$$

$$\textcircled{2} \quad |\vec{r}| = \sqrt{x^2 + y^2 + z^2}$$

$$= \sqrt{K^2 + K^2 + K^2} = K\sqrt{3}$$

$$K\sqrt{3} = 3 \Rightarrow K = \sqrt{3}$$

$$\therefore \vec{r} = \sqrt{3}\hat{i} + \sqrt{3}\hat{j} + \sqrt{3}\hat{k}$$

$$= \sqrt{3}(\hat{i} + \hat{j} + \hat{k})$$

Q)  $\vec{a}, \vec{b}, \vec{c}$  are unit vectors.

Q)  $\vec{a} + \vec{b} + \vec{c} = 0$  find value of  
 $\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a} = ?$

$$(\vec{a} + \vec{b} + \vec{c})^2 = 0$$

$$|a|^2 + |b|^2 + |c|^2 + 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) = 0$$

$$1 + 1 + 1^2 + 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) = 0$$

$$\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a} = -\frac{3}{2}$$

Shortcut  
use i, j, k

Q)  $\vec{a}, \vec{b}, \vec{c}$  are mutually  $\perp$   
 vectors of equal Magnitude.  
 find angle betw  $\vec{a}$  &  $\vec{a} + \vec{b} + \vec{c}$ ?

$$(1) \vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{c} = \vec{c} \cdot \vec{a} = 0$$

$$(2) |a| = |b| = |c| = K.$$

$$(3) |\vec{a} + \vec{b} + \vec{c}|^2 \\ = |a|^2 + |b|^2 + |c|^2 + 2(0)$$

$$|\vec{a} + \vec{b} + \vec{c}|^2 = K^2 + K^2 + K^2$$

$$|\vec{a} + \vec{b} + \vec{c}|^2 = 3K^2$$

$$|\vec{a} + \vec{b} + \vec{c}| = \sqrt{3}K$$

$$\vec{a} \cdot \vec{b}$$

$$\theta = \frac{\vec{a} \cdot \vec{b}}{|a||b|}$$

$$\theta = \frac{\vec{a} \cdot (\vec{a} + \vec{b} + \vec{c})}{|a||a+b+c|}$$

$$= \frac{|a|^2 + 0 + 0}{|a||a+b+c|}$$

$$\theta = \frac{K\sqrt{3}K}{K \times \sqrt{3}K}$$

$$\theta = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

Q If  $|a|=|b|=|c|=k$

&  $\vec{a} + \vec{b} = \vec{c}$  find angle between  $\vec{a}$  &  $\vec{b}$ .

Sqr.

$$(\vec{a} + \vec{b})^2 = (\vec{c})^2$$

$$|a|^2 + |b|^2 + 2(a \cdot b) = |c|^2$$

$$k^2 + k^2 + 2(a \cdot b) = k^2$$

$$a \cdot b = -\frac{k^2}{2}$$

$$\vec{a} + \vec{b} - \vec{c} = 0$$

$$G) \theta = \frac{\vec{a} \cdot \vec{b}}{|a||b|}$$

$$G) \theta = \frac{-\frac{k^2}{2}}{k \times k}$$

$$\theta = 120^\circ$$

Q If  $\theta$  is angle between Unit vectors

$\vec{a}$  &  $\vec{b}$  then find  $\frac{1}{2} |\vec{a} + \vec{b}|$  &  $\frac{1}{2} |\vec{a} - \vec{b}|$ ?

$$① |a+b|^2 = |a|^2 + |b|^2 + 2|a||b|\cos\theta$$

$$② |\vec{a} - \vec{b}|^2 =$$

$$|\vec{a} + \vec{b}|^2 = 1 + 1 + 2 \times 1 \times 1 \times \cos 60^\circ$$

$$= 2 + 2 \cos 60^\circ$$

$$= 2(1 + \cos 60^\circ)$$

$$|a+b|^2 = 2 \times 2 \cos^2 \frac{\theta}{2}$$

$$\frac{1}{4} |a+b|^2 = \cos^2 \frac{\theta}{2}$$

$$G) \frac{\theta}{2} : \frac{1}{2} |\vec{a} + \vec{b}|$$

$$|\vec{a} - \vec{b}|^2 = 1 + 1 - 2 \times 1 \times 1 \cdot \cos 60^\circ$$

$$= 2 - 2 \cos 60^\circ$$

$$= 2(1 - \cos 60^\circ)$$

$$|\vec{a} - \vec{b}|^2 = 2 \times 2 \sin^2 \frac{\theta}{2}$$

$$G) \frac{\theta}{2} = \frac{1}{2} |a - b|$$

Q Find gr. value of

$$|\vec{a} + \vec{b}| + |\vec{a} - \vec{b}| \text{ if } \vec{a}, \vec{b} \text{ are Unit Vectors}$$

$$= 26\frac{0}{2} + 28m\frac{0}{2}$$

$$\left| \begin{array}{l} \text{Result की रूपरेखा} \\ \frac{1}{2}|\vec{a} + \vec{b}| = 6\frac{0}{2} \\ \frac{1}{2}|\vec{a} - \vec{b}| = 8m\frac{0}{2} \end{array} \right.$$

$$-\sqrt{2^2+2^2} \leq 28m\frac{0}{2} + 26\frac{0}{2} \leq \sqrt{2^2+2^2}$$

$$-2\sqrt{2} \leq 2\sqrt{2}$$

$$\therefore \text{gr. value} = 2\sqrt{2}$$

Saturday Vector 5:45 PM

Q  $\vec{a}, \vec{b}, \vec{c}$  are Unit vector

$$\text{then } |\vec{a} - \vec{b}|^2 + |\vec{b} - \vec{c}|^2 + |\vec{c} - \vec{a}|^2$$

does not exceed?

$$|\vec{a} - \vec{b}|^2 + |\vec{b} - \vec{c}|^2 + |\vec{c} - \vec{a}|^2$$

$$= 2(|a|^2 + |b|^2 + |c|^2) - 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a})$$

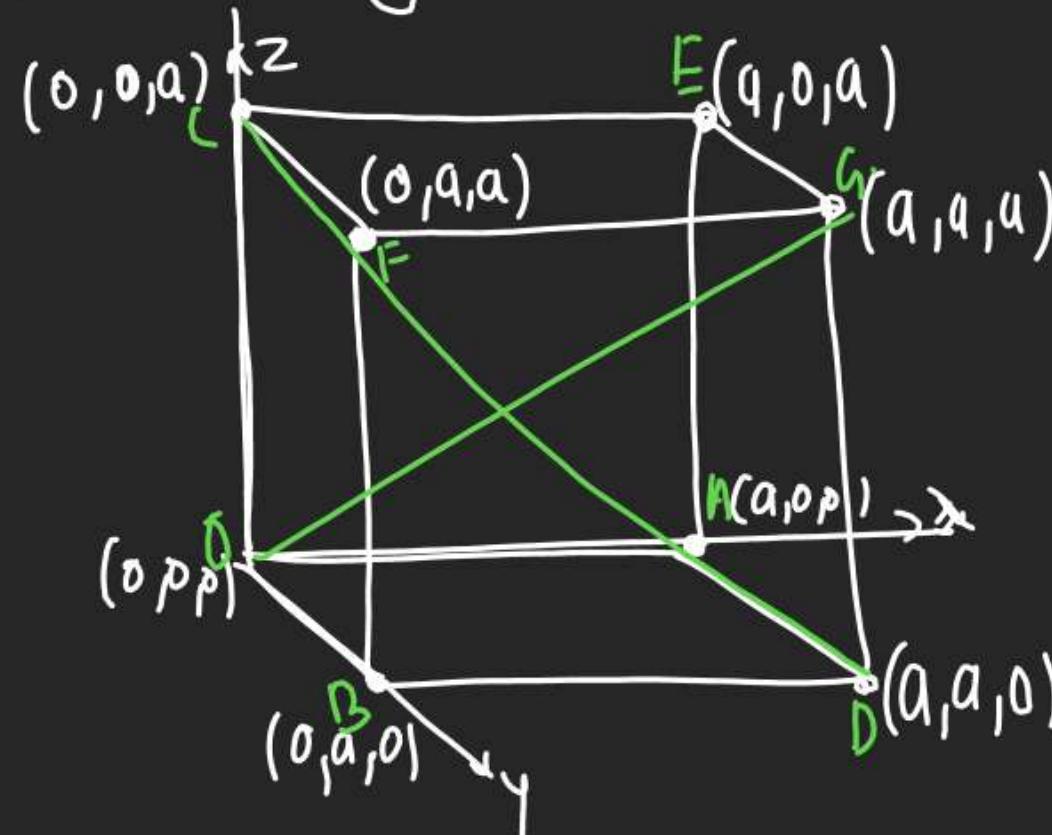
$$= 3(|a|^2 + |b|^2 + |c|^2) - \left\{ |a|^2 + |b|^2 + |c|^2 + 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) \right\}$$

$$Ex_{lb} = 3(1+1+1) - |a+b+c|^2$$

$$Ex_{gr} = 9 - |a+b+c|^2 \text{ Min.}$$

$$= 9 - 0 = 9$$

Q Find angle bet<sup>n</sup> diagonal of cube?



$$\vec{OB} = \langle a, a, a \rangle - \langle 0, 0, 0 \rangle = a\hat{i} + a\hat{j} + a\hat{k}$$

$$\vec{CD} = \langle a, a, 0 \rangle - \langle 0, 0, a \rangle = a\hat{i} + a\hat{j} - a\hat{k}$$

$$(g) \theta = \frac{\vec{OB} \cdot \vec{CD}}{|\vec{OB}| |\vec{CD}|} = \frac{a^2 + a^2 - a^2}{\sqrt{3a^2} \sqrt{3a^2}} = \frac{a^2}{3a^2} \\ \theta = 61\frac{1}{3}$$

Q  $\vec{U}, \vec{V}, \vec{W}$  are 3 vectors s.t. Proj. of  $\vec{V}$  on  $\vec{U}$

is equal to Proj. of  $\vec{W}$  on  $\vec{U}$  &  $\vec{V}, \vec{W}$  are  $\perp$  to each other  
find  $|\vec{U} - \vec{V} + \vec{W}| = ?$

$$① |\vec{U} - \vec{V} + \vec{W}|^2 = |\vec{U}|^2 + |\vec{V}|^2 + |\vec{W}|^2 - 2\vec{U} \cdot \vec{V} - 2\vec{V} \cdot \vec{W} + 2\vec{U} \cdot \vec{W}$$

$$\therefore |\vec{U} - \vec{V} + \vec{W}| = \sqrt{|\vec{U}|^2 + |\vec{V}|^2 + |\vec{W}|^2}$$

$$\vec{U} \cdot \vec{V} = |\vec{U}| |\vec{V}| \cos \theta$$

$$|\vec{W} \cdot \vec{V}| = |\vec{W}| |\vec{V}| \cos \theta$$

$$② |\vec{V}| \cos \theta = |\vec{W}| \cos \theta$$

$$\frac{\vec{U} \cdot \vec{V}}{|\vec{U}|} = \frac{\vec{W} \cdot \vec{U}}{|\vec{W}|}$$

$$③ \vec{V} \perp \vec{W} \\ \vec{V} \cdot \vec{W} = 0$$

Q 1-15 118-19, 21-22