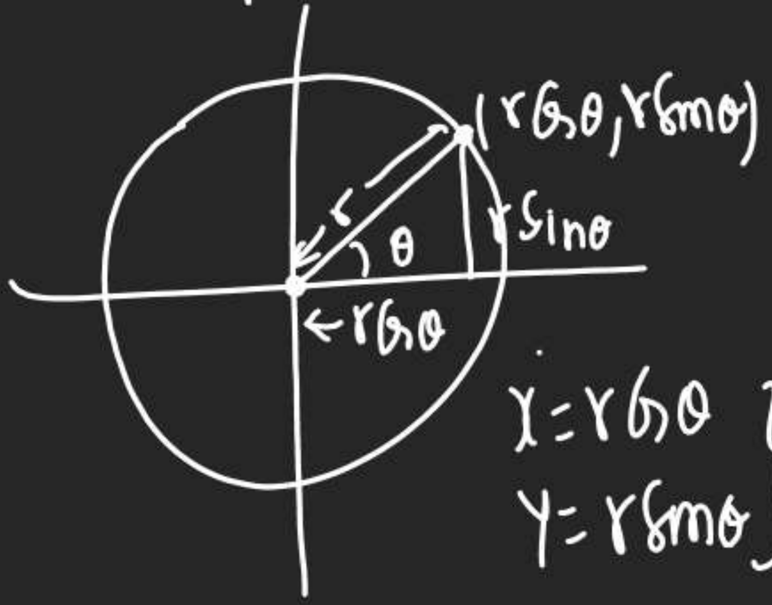
Nishant Jindal

Parametric Eqⁿ of Circle.

When Circle's Centre Origin

$$x^2+y^2=r^2$$



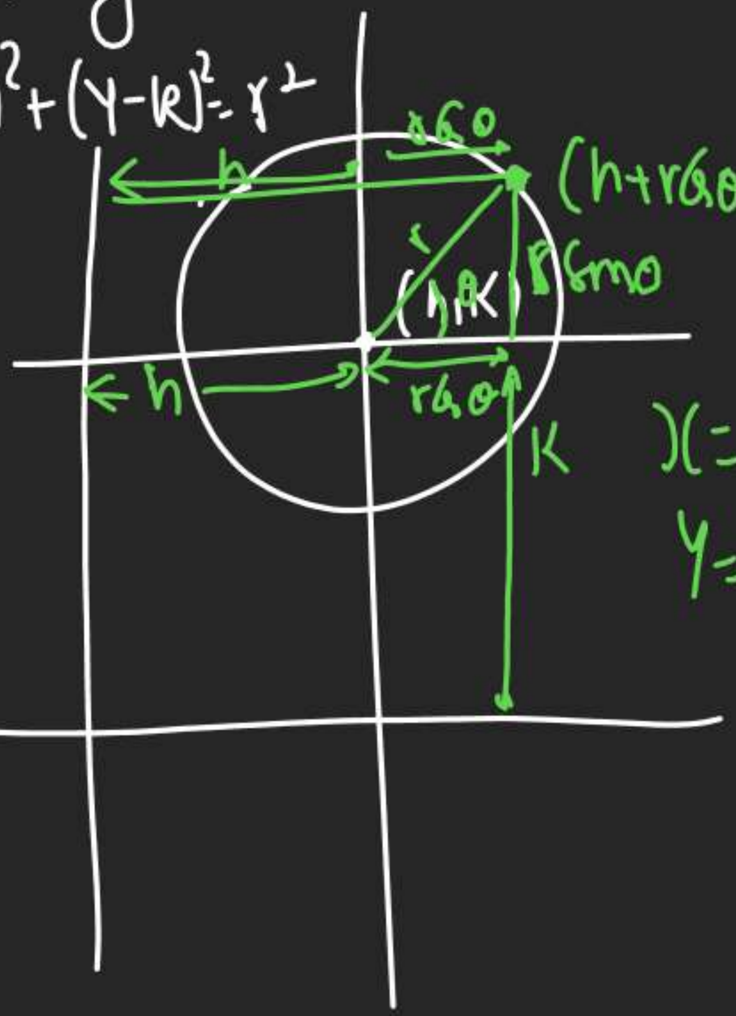
$$\left.\begin{aligned} x &= r\cos\theta \\ y &= r\sin\theta \end{aligned}\right\} \text{Par. form}$$

$$0 \le \theta < 2\pi$$

r remain same

When circle is centre other than origin.

$$(x-h)^2+(y-k)^2=r^2$$



$$\left.\begin{aligned} x &= h+r\cos\theta \\ y &= K+r\sin\theta \end{aligned}\right\} \boxed{\text{Par. EOC}}$$

Q1 Find Par. form of $x^2+y^2=9$

$r=3$

$\therefore$ Par EOC $=\{3\cos\theta, 3\sin\theta\}$

Q2 Find Par. Eqⁿ of circle for $(x-3)^2+y^2=9$

$r=3$

$$\left.\begin{aligned} x-3 &= 3\cos\theta \\ y &= 3\sin\theta \end{aligned}\right\} \left.\begin{aligned} x &= 3+3\cos\theta \\ y &= 3\sin\theta \end{aligned}\right| \leftarrow \text{Par. EOC}$$

Q3 Find Par. EOC for $x^2+y^2-6x+4y=0$

$$(x^2-\underset{2\times3}{6x}+3^2)+(y^2+\underset{2\times2}{4y}+2^2)=3^2+2^2$$

$$(x-3)^2+(y+2)^2=\sqrt{13}^2$$

Rad $=\sqrt{13}$

$\therefore$ Par E $\rightarrow$ $x-3=\sqrt{13}\cos\theta$, $y+2=\sqrt{13}\sin\theta$

$\{3+\sqrt{13}\cos\theta, -2+\sqrt{13}\sin\theta\}$

Q4 Par. EOC of $(x+2)^2+(y-4)^2=16$.

$r=4$

$x+2=4\cos\theta,\ y-4=4\sin\theta$

$$\left.\begin{aligned} x&=-2+4\cos\theta \\ y&=4+4\sin\theta \end{aligned}\right\}\text{Par. fn.}$$

Q5 Find EOC if Par. form is $x=\sqrt{3}\cos\theta,\ y=\sqrt{3}\sin\theta$

$\cos\theta=\dfrac{x}{\sqrt{3}},\quad \sin\theta=\dfrac{y}{\sqrt{3}}$

$s^2+c^2=1$

$$\left(\frac{x}{\sqrt{3}}\right)^2+\left(\frac{y}{\sqrt{3}}\right)^2=1$$

$$x^2+y^2=3$$

Q6

$$\left.\begin{aligned} x&=-1+2\cos\theta \\ y&=3+2\sin\theta \end{aligned}\right\}\text{find EOC}$$

$\cos\theta=\dfrac{x+1}{2},\quad \sin\theta=\dfrac{y-3}{2}$

$$\left(\frac{x+1}{2}\right)^2+\left(\frac{y-3}{2}\right)^2=1$$

$$(x+1)^2+(y-3)^2=2^2$$

$(-1,3)$

Q7 Find Circumcentre of ΔABC

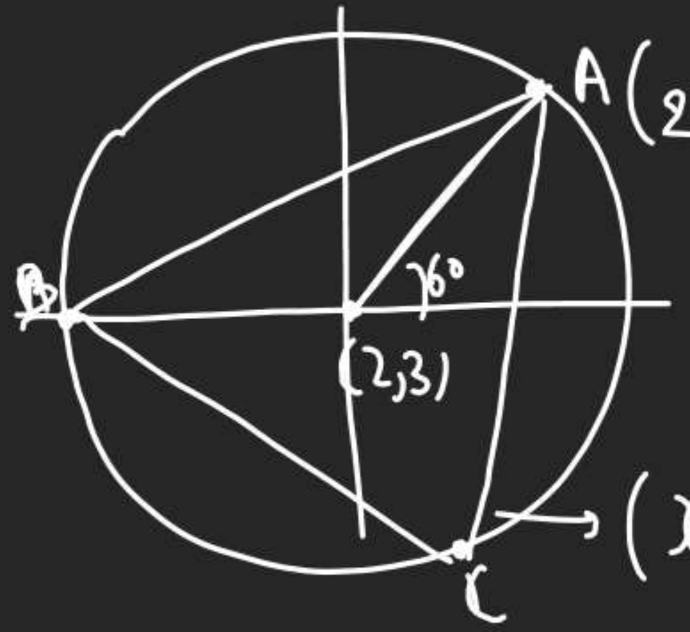
if $A\equiv\left\{2+\cos\frac{\pi}{3},\ 3+\sin\frac{\pi}{3}\right\}$

$B=\{2+\cos\pi,\ 3+\sin\pi\}$

$C=\left\{2+\cos\frac{4\pi}{3},\ 3+\sin\frac{4\pi}{3}\right\}$

A, B, C are on same circle

Centre $=(2,3)$



$A\left(2+\cos\frac{\pi}{3},\ 3+\sin\frac{\pi}{3}\right)$

$(x-2)^2+(y-3)^2=1^2$

Q8 If $x^2+y^2-2x-4y-4=0$ then find Max & Min Value of $3x+4y=?$

① $(x^2-\underset{2\times1}{2x}+1^2)+(y^2-\underset{2\times2}{4y}+2^2)-4=1^2+2^2$

$$(x-1)^2+(y-2)^2=3^2 \qquad \begin{aligned} x&=1+3\cos\theta \\ y&=2+3\sin\theta \end{aligned}$$

② $Z=3x+4y=3(1+3\cos\theta)+4(2+3\sin\theta)$

$=11+9\cos\theta+12\sin\theta$

$-\sqrt{81+144}\le 9\cos\theta+12\sin\theta\le\sqrt{81+144}$

$-15+11\le 9\cos\theta+12\sin\theta+11\le 15+11$

Min $=-4$

Max $=26$

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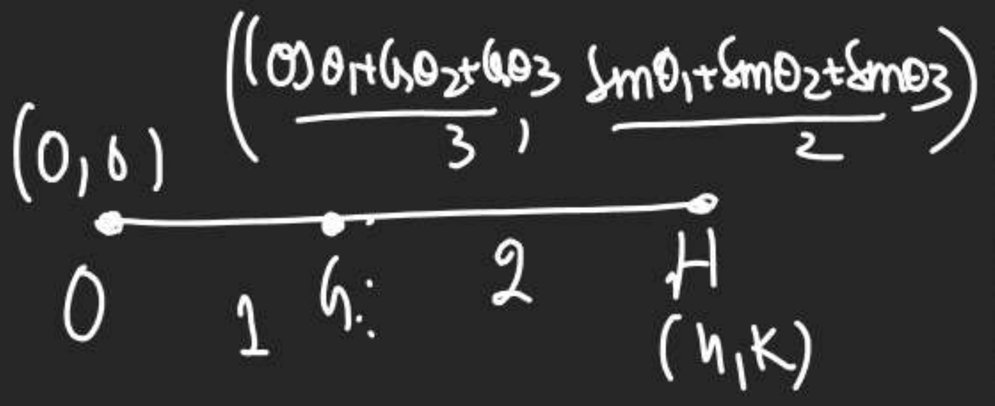
Q 9 If $A=(1\cos\theta_1, 1\sin\theta_1)$

$B=(\cos\theta_2, \sin\theta_2)$

$C=(\cos\theta_3, \sin\theta_3)$

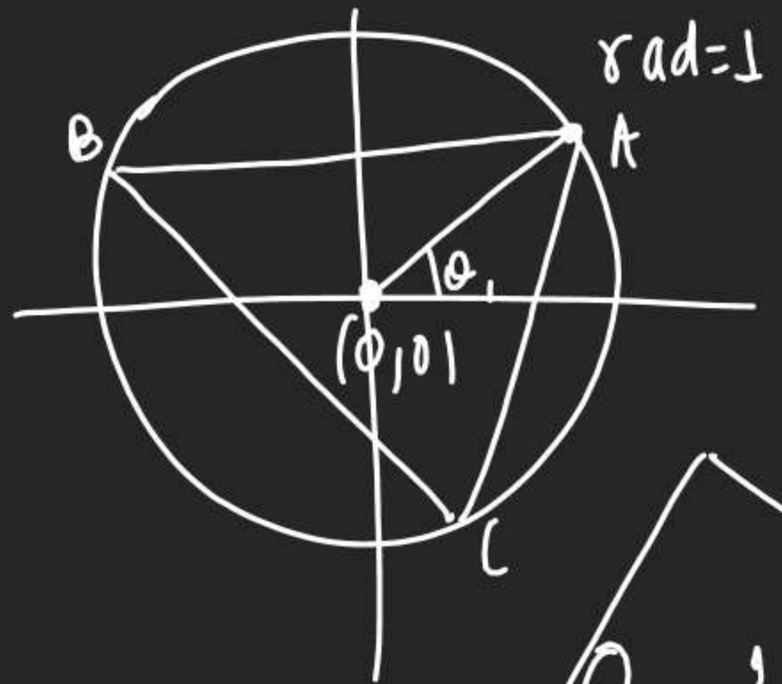
are vertices of Δ. find Orthocentre.

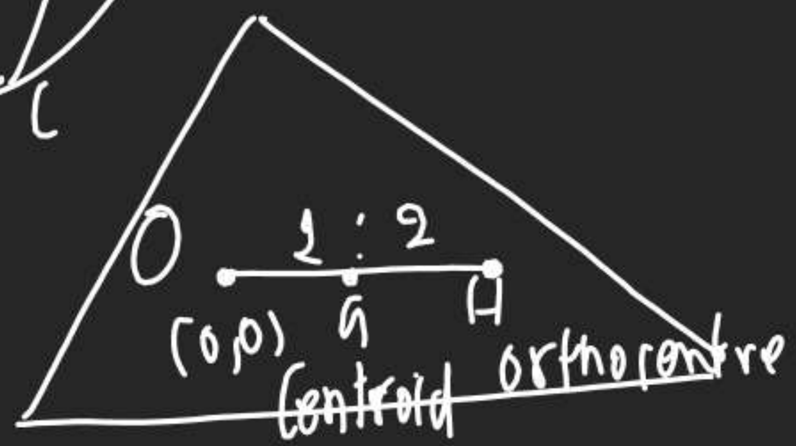


$\sum \frac{\cos\theta_1}{3} = \frac{1\times h+2\times 0}{3} = \frac{h}{3}$

$\sum \frac{\sin\theta_1}{3} = \frac{1\times K+2\times 0}{3} = \frac{K}{3}$

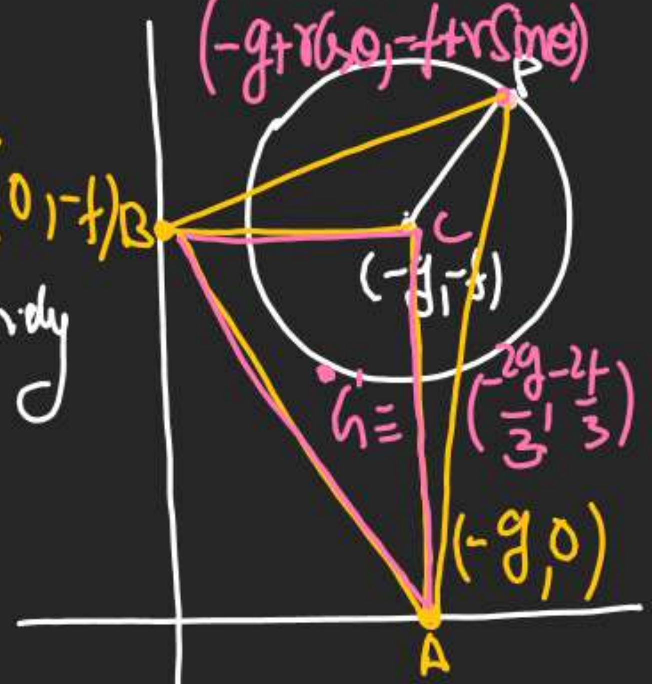
$H=(h,K)\equiv\{\cos\theta_1+\cos\theta_2+\cos\theta_3, \sin\theta_1+\sin\theta_2+\sin\theta_3\}$





Q 10 If Pt. P is a variable Pt on circle $x^2+y^2+2gx+2fy+c=0$. If centre of circle is C & CA, CB are respectively $\perp^r$ on x Axis & y Axis then S.T

$\boxed{\text{Locus of centroid}}$ of ΔPAB is a circle whose centre is also centre of ΔCAB & radius is $\frac{1}{3}$ of Radius of given circle.



radius = r

① let centroid $=(h,K)$

$h=\frac{-g+r\cos\theta-g+0}{3}$, $K=\frac{-f+r\sin\theta-f+0}{3}$

$\frac{(3h+2g)}{r}=\cos\theta$, $\frac{3K+2f}{r}=\sin\theta$

$S^2+C^2=1$

$(3h+2g)^2+(3K+2f)^2=r^2$

$\Rightarrow (x+\frac{2}{3}g)^2+(y+\frac{2}{3}f)^2=\left(\frac{r}{3}\right)^2 \Leftarrow$ CIRCLE

$(x-a)^2+(y-b)^2=R^2$

Centre $=\left(\frac{-2g}{3}, \frac{-2f}{3}\right)$

Centroid locus Rad $=\frac{r}{3}=\frac{\text{given circle's Radius}}{3}$

ΔCAB's Centroid is Same as Centre of Circle

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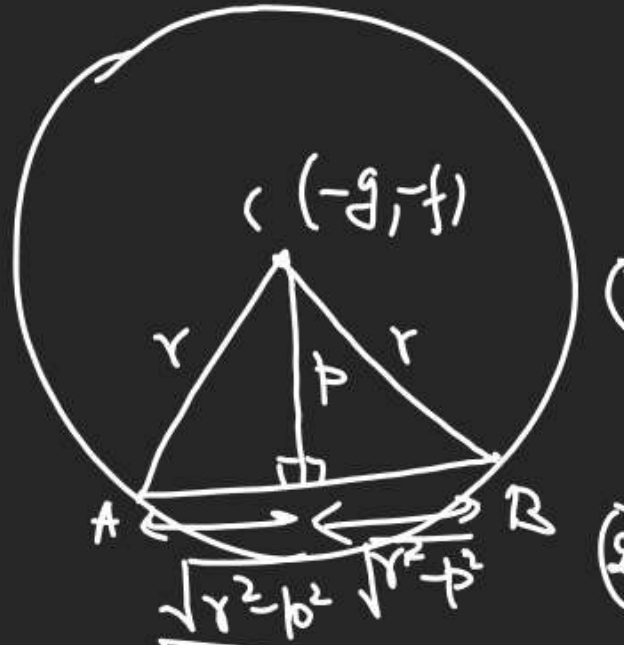
Length of chord.

When Circle is Intercepted by a Line then we get a Chord

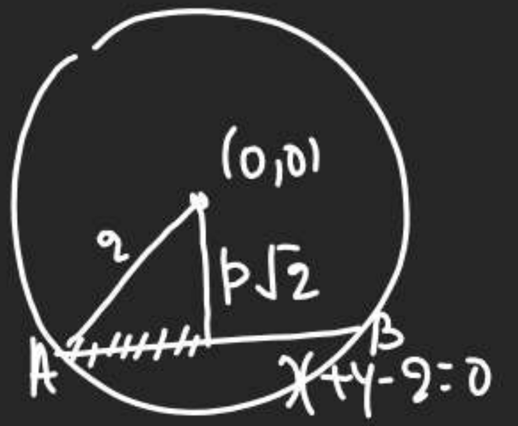


① Chord $lx+my+n=0$

② $\perp^r$ distance from Centre of chord

$$\boxed{AB = 2\sqrt{r^2-p^2}}$$

Q 11 Find length of chord Intercepted by a Line $x+y=2$ on Circle $x^2+y^2=4$



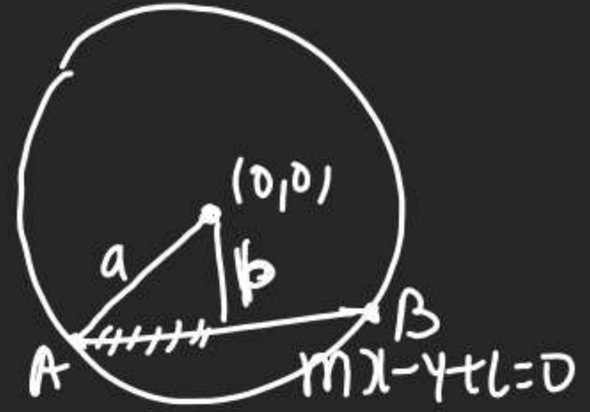
$p = \perp^r$ distance from $(0,0)$ to Line $x+y-2=0$

$$p = \frac{|0+0-2|}{\sqrt{1^2+1^2}} = \sqrt{2}$$

$$AB = 2\sqrt{2^2-(\sqrt{2})^2} = 2\sqrt{2}$$

Q 12 If Circle $x^2+y^2=a^2$ makes Chord of length $2b$ on Line $y=mx+c$ then P.T.

$$c^2 = (1+m^2)(a^2-b^2)$$



$$p = \frac{|m\times 0-0+c|}{\sqrt{m^2+(-1)^2}} = \frac{|c|}{\sqrt{m^2+1}}$$

$$\cancel{2}b = AB = \cancel{2}\sqrt{a^2-\frac{c^2}{(m^2+1)}}$$

$$b^2 = a^2 - \frac{c^2}{(m^2+1)}$$

$$\Rightarrow \frac{c^2}{m^2+1} = (b^2-a^2) \Rightarrow \boxed{c^2 = (1+m^2)(b^2-a^2)}$$

(H.P)

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Reverse (75% se upar ki Boddhik Kshmta vale logo k liye)

Q 13. If $4l^2-5m^2+6l+1=0$ then S.T.

Line $lx+my+1=0$ touches a fix Circle & Find Radius & Centre of that Circle. $\rightarrow p=r$

$$4l^2-5m^2+6l+1=0$$

Self

$$\Rightarrow 4l^2+6l+1=5m^2$$

2×3

$$5l^2+4l^2+6l+1=5m^2+5l^2$$

$$9l^2+6l+1=5(l^2+m^2)$$

$$(3l+1)^2=5(l^2+m^2)$$

$\sqrt{\ }$

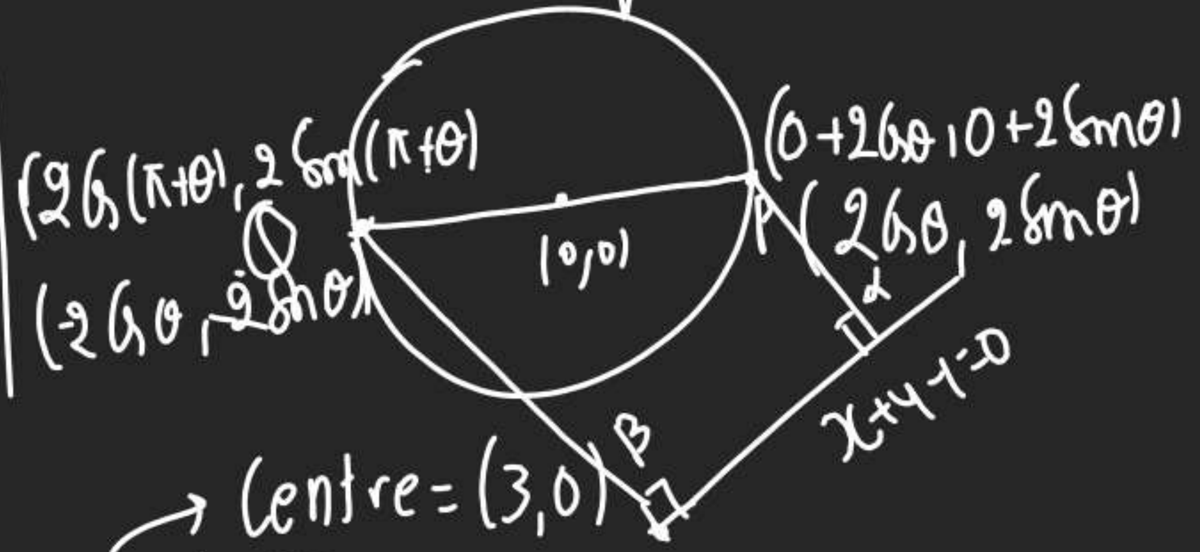
$$|3l+1|=\sqrt5\sqrt{l^2+m^2}$$

$$\frac{|3l+1|}{\sqrt{l^2+m^2}}=\sqrt5 \Rightarrow \frac{|l\times3+m\times0+1|}{\sqrt{l^2+m^2}}=\sqrt5 \rightarrow \text{Circle's Rad}$$

Centre $=(3,0)$

Circle $\rightarrow (x-3)^2+(y-0)^2=5$

Q 14. P & Q are 2 Pts on Circle $x^2+y^2=4$ Such that PQ is diametre If α,β are length of $\perp^r$ from P & Q on Line $x+y=1$ then max^m value of $\alpha\cdot\beta$ is?



$$\alpha\cdot\beta=\left|\left(\frac{2\cos\theta+2\sin\theta-1}{\sqrt2}\right)\left(\frac{-2\cos\theta-2\sin\theta-1}{\sqrt2}\right)\right|$$

$$=\frac{|4(\sin\theta+\cos\theta)^2-1|}{2}$$

$$=\frac{|3+4\sin2\theta|}{2}=\frac{7}{2}$$