Nishant Jindal

Q If tngent to $y^2 = 16x$ makes angle of $45°$ with $+ve$ x-Axis then Pt. of Contact is?

$m = 1$ $\quad \left(\frac{a}{m^2}, \frac{2a}{m}\right)$

$\left(\frac{4}{1^2}, \frac{2\times 4}{1}\right) = (4, 8)$

Q Eqn of Gm. tngent touching Circle $(x-3)^2 + y^2 = 9$ & $y^2 = 4x$ above x-Axis?

tngent to $y^2 = 4x \to y = mx + \frac{1}{m}$.

$$\frac{\left|3m - 0 + \frac{1}{m}\right|}{\sqrt{m^2+1^2}} = 3$$

$$|3m^2+1| = 3m\sqrt{m^2+1}$$

$$9m^4 + 6m^2 + 1 = 9m^4 + 9m^2$$

$$3m^2 = 1 \Rightarrow m = \frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}$$

$\therefore$ EOT $\to y = \frac{x}{\sqrt{3}} + \sqrt{3}$.

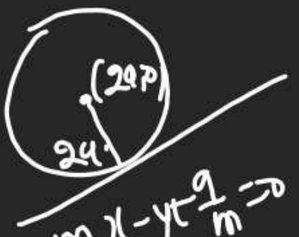
$x - \sqrt{3}y + 3 = 0$

Q Find Eqn of Gm. tngent to $x^2 + y^2 = 4ax$ & $y^2 = 4ax$

Circle $\Rightarrow x^2 + y^2 - 4ax = 0$

Centre $= (2a, 0)$

Rad $= (2a)$



$$\frac{\left|2am - 0 + \frac{a}{m}\right|}{\sqrt{m^2+1^2}} = 2a$$

$$\left|2am + \frac{a}{m}\right| = 2a\sqrt{m^2+1}$$

$$\left|2am + \frac{a}{m}\right|^2 = \left(2a\sqrt{m^2+1}\right)^2$$

$$4a^2m^2 + \frac{a^2}{m^2} + 4a^2 = 4a^2m^2 + 4a^2$$

$$\frac{a^2}{m^2} = 0 \Rightarrow \frac{1}{m} = 0$$

$m \to \infty$

EOT $y = mx + \frac{a}{m}$.

$\times m$ $\quad \frac{y}{m} = x + \frac{a}{m^2}$

$0 = x + 0 \Rightarrow x = 0$ is Gm. I.

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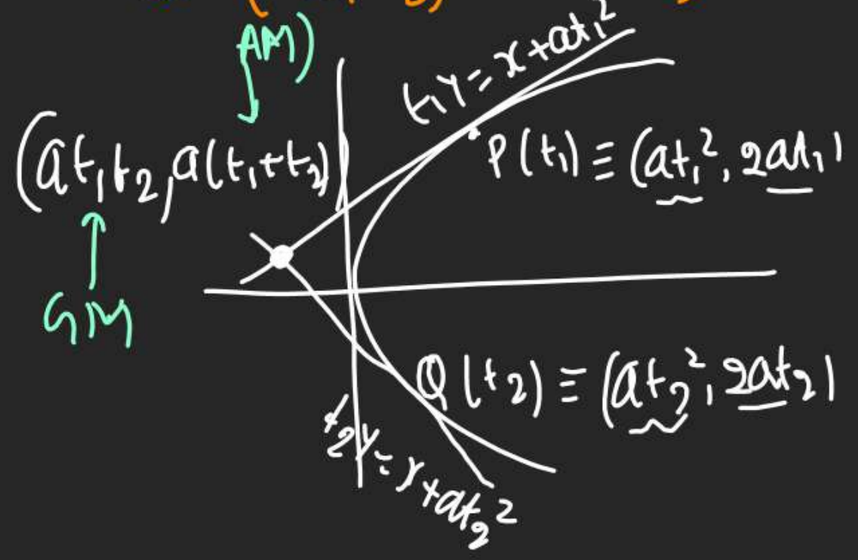
Prop. of Tangents

1) Tangents at $P(t_1)$ & $Q(t_2)$ Intersects at $(at_1t_2, a(t_1+t_2))$



(2) abscissa of Pt. of Intersection is GM of abscissas of Pt. of contacts

$$GM = \sqrt{at_1^2 \times at_2^2} = at_1t_2 = \text{abscissa}$$

(3) AM of Ordinates of Pt of Contact is ordinate of Pt of Intersection

$$AM = \frac{2at_1 + 2at_2}{2} = a(t_1+t_2)$$

$= y$(coord.

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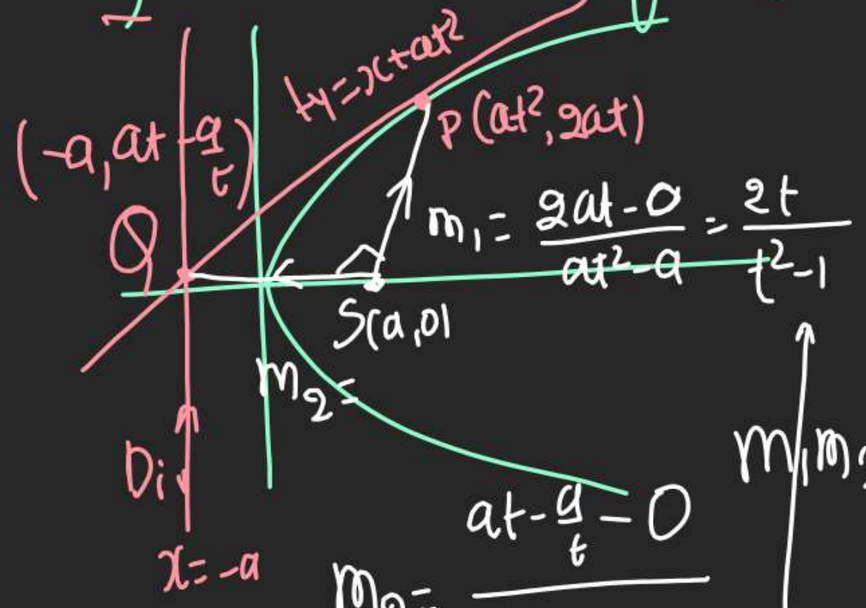
(4) Portion of tmgent betⁿ Pt of Contact P & the Pt. where it meets the Directrix subtends Rt. angle at focus.



$$m_1 = \frac{2at-0}{at^2-a} = \frac{2t}{t^2-1}$$

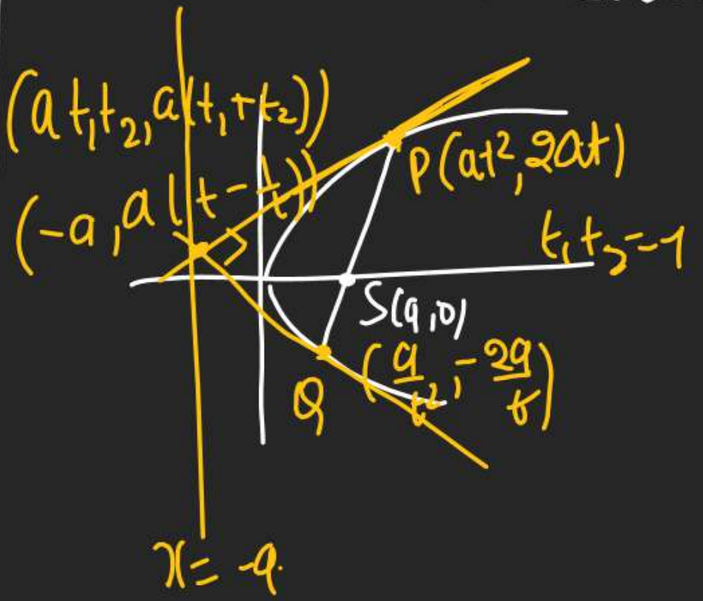
$$m_2 = \frac{at-\frac{a}{t}-0}{-a-a}$$

$$m_2 = \frac{\frac{1}{t}-t}{2} = \frac{1-t^2}{2t}$$

$$m_1 m_2 = \frac{2t}{t^2-1} \times \frac{1-t^2}{2t} = -1$$

Rt angle

(5) Tangents drawn at Extremity of Focal chord are ⊥ʳ & Intersect on directrix.



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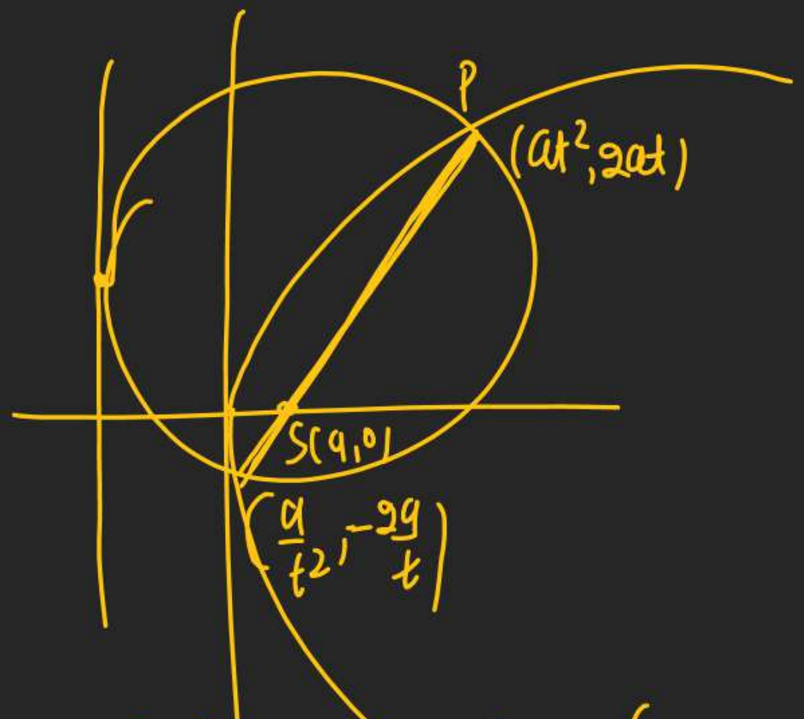
(6) Circle with diameter Focal chord touches Directrix.



$$S: (x - at^2)\left(x - \frac{a}{t^2}\right) + (y - 2at)\left(y + \frac{2a}{t}\right) = 0$$

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Example xxvi

① $y^2 = 9x$ $(4,6)$ $m = \frac{3}{4}$

EOT → $y \cdot 6 = \frac{9}{2}(x+4) \rightarrow y = \frac{9}{2\times 6}(x+4)$

EON → $(y-6) = -\frac{4}{3}(x-4)$

Q2 ord = $6 = y$ in $y^2 = 6x \Rightarrow x = 6$

$\therefore$ Pt. $(6,6)$

$y \cdot 6 = 3(x+6) \Rightarrow y = \frac{1}{2}(x+6)$

EON $\Rightarrow (y-6) = -2(x-6)$

Q3 $y^2 = 12x \rightarrow a = 3$ LR $= (9,2a)$ & $(9,-2a)$

2T 2N

$= (3,6)$ & $(3,-6)$

at $(3,6)$ $m = 1$

$y \cdot 6 = 6(x+3) \Rightarrow y = x+3$

EON $\Rightarrow y - 6 = -1(x-3)$

Q4 $y^2 = 4a(x-a)$

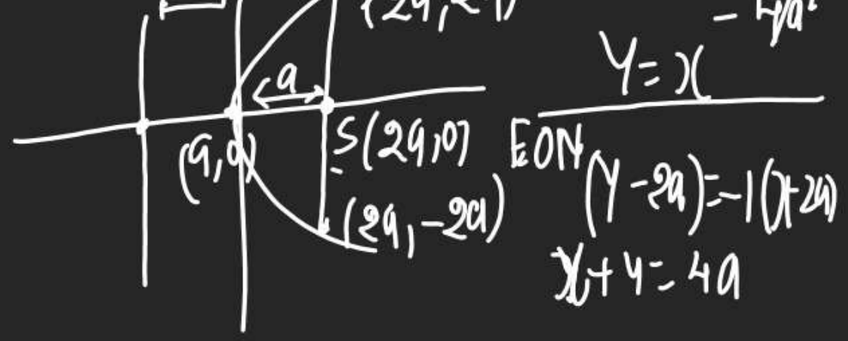
$y^2 = 4a(a)$

$2a \times 2x - 4a^2 = 2at(2a, 2a)$

$(2a, 2a)$

$y - 2a = 2a(x+2a) - 4a^2$

$y = x$



EON $(y-2a) = -1(x+2a)$

$x + 4 = 4a$

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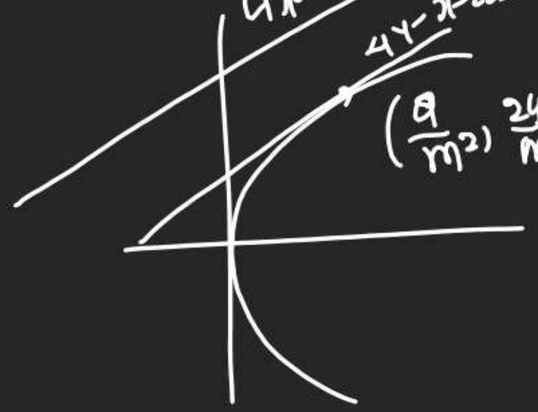
Q5: $y^2 = 7x$ $\quad a = \frac{7}{4}$ $\quad L: 4y - x + 3 = 0$

$y = mx + \frac{7}{4m}$ $\quad m = -\frac{(-1)}{4} = \frac{1}{4}$

$y = \frac{x}{4} + \frac{7}{4 \times \frac{1}{4}} \Rightarrow y = \frac{x}{4} + 7$

$4y - x - 28 = 0$



$\left(\frac{a}{m^2}, \frac{2a}{m}\right) \equiv \left(\frac{7}{\frac{1}{16}}, \frac{7}{\frac{1}{4}}\right)$

$= (28, 14)$ //

Q6: $y^2 = 4ax$, $m = \sqrt{3}$.

$y = \sqrt{3}x + \frac{a}{\sqrt{3}}$ $\quad \left(\frac{a}{m^2}, \frac{2a}{m}\right) \equiv \left(\frac{a}{3}, \frac{2a}{\sqrt{3}}\right)$

$3x - \sqrt{3}y + a = 0$

Q7: $y^2 = 8x$

Line $\xrightarrow{m}$ which makes $45°$ with $y = 3x + 5$

$\tan 45° = \frac{m-3}{1+3m} = 1 \Rightarrow 1 + 3m = m - 3$

$2m = -4 \Rightarrow m = -2$

$m' = \frac{1}{2}$

EOT $\rightarrow y = -2x + \frac{2}{-2} \Rightarrow 2x + y = -1$

Point $\equiv \left(\frac{2}{4}, \frac{4}{-2}\right) \equiv \left(\frac{1}{2}, -2\right)$

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Q8 $y^2 = 4ax$, A) tangent is Inclined to 30°

$y = mx + \frac{a}{m}$

$y = \frac{x}{t} + at$

$tt = 1 + at^2$

$(at^2, 2at)$

tangent की Side से

$m = \frac{1}{t}$

tangent का Case में

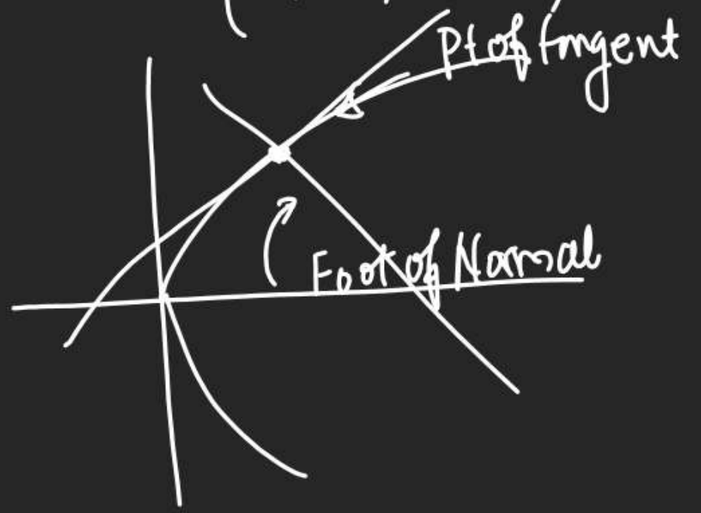
$(at^2, 2at) \equiv \left(\frac{a}{m^2}, \frac{2a}{m}\right)$

Normal की Side से

$(a(-m)^2, 2a(-m))$

$(am^2, -2am)$

Normal $m' = -t$



Pt. of Contact → tangent का $m = \frac{1}{\sqrt{3}}$

$\left(\frac{a}{m^2}, \frac{2a}{m}\right) \equiv (3a, 2\sqrt{3}a)$

Pt of Contact → Normal का $m = \frac{1}{\sqrt{3}}$

$(am^2, -2am) \equiv \left(\frac{a}{3}, \frac{-2a}{\sqrt{3}}\right)$

Q9 $y^2 = 9x$ → Tangent P.T. $(4, 10)$ $y = mx + \frac{9}{4m}$

$10 = 4m + \frac{9}{4m} \Rightarrow 4m^2 - 40m + 9 = 0$

$(4m - 9)(4m - 1) = 0 \Rightarrow m = \frac{9}{4}, \frac{1}{4}$

$y - 10 = \frac{1}{4}(x - 4)$ & $y - 10 = \frac{9}{4}(x - 4)$

Q $x+y=1$ touches $y=x-x^2$

$1-x=x-x^2$

$x^2-2x+1=0 \Rightarrow x=1, y=0$

$\in$ $(1,0)$ is only pt

Q11 P.T. $y=mx+c$ touches $y^2=4a(x+a)$ if $c=am+\frac{a}{m}$

$(mx+c)^2=4ax+4a^2$

$m^2x^2+2mcx-4ax+c^2-4a^2=0$

$D=0 \Rightarrow (2mc-4a)^2-4m^2(c^2-4a^2)=0$

$4m^2c^2-16amc+16a^2-4m^2c^2+16a^2m^2=0$

$-mc+a+am^2=0$

$cm=a+am^2$

$c=\frac{a}{m}+am$