Nishant Jindal

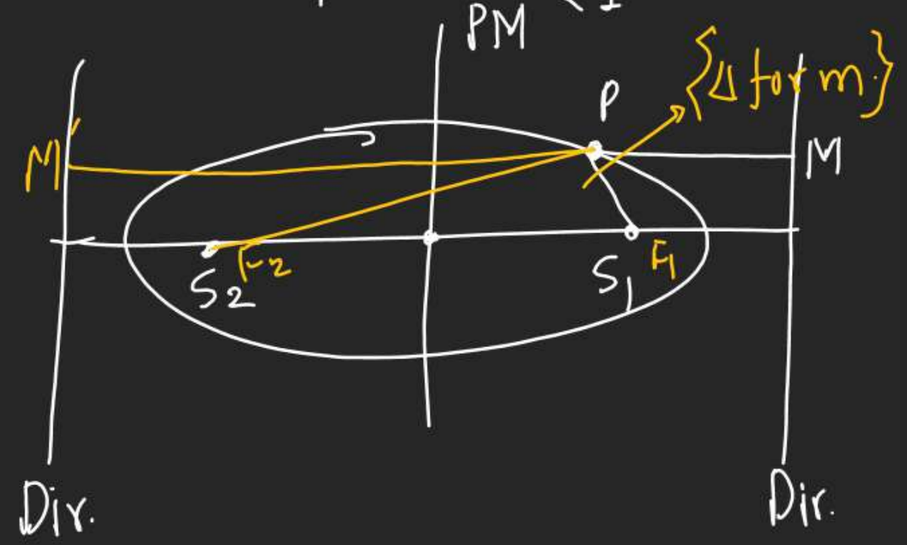
Ellipse

(1) Non Hom Eqn

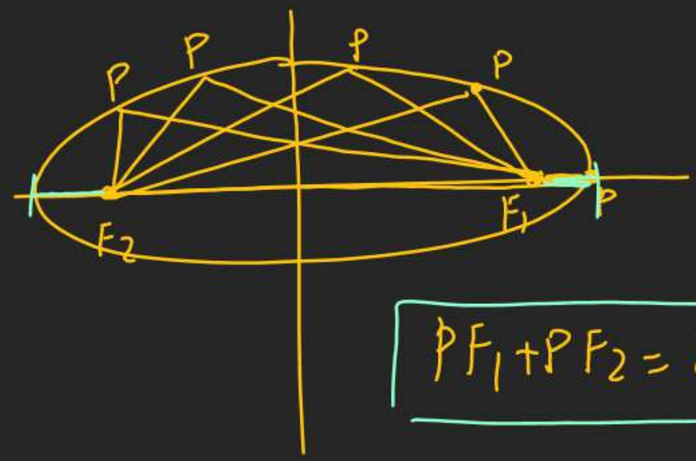
$$ax^2+2hxy+by^2+2gx+2fy+c=0$$

(i) $\Delta \neq 0$ (ii) $h^2 < ab$

(2) Eccentricity : $e = \frac{SP}{PM} < 1 \rightarrow SP < PM$



(3) Basic Defination

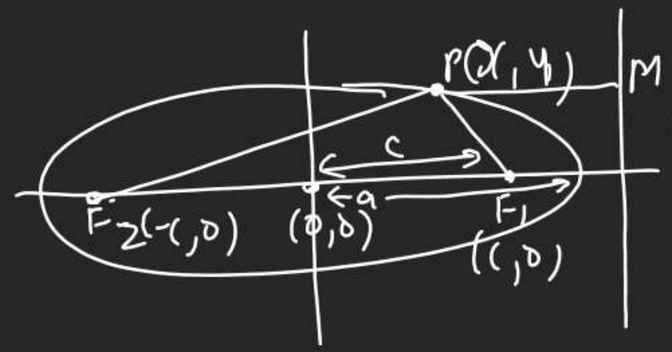


$$PF_1 + PF_2 = 2a = \text{Const}$$

If Sum of distance of a var. pt. from 2 fix pt. Remains constant then Locus of variable pt = Ellipse

Nishant Jindal

(2) Standard Eqn of Ellipse.



$a > c$

$a^2 > c^2$

$a^2 - c^2 > 0$

$PF_1 + PF_2 = 2a$ 3, 17, 2

$\sqrt{(x-c)^2+(y-0)^2} + \sqrt{(x+c)^2+(y-0)^2} = (2a)$

$\frac{x^2}{a^2} + \frac{y^2}{(a^2-c^2)} = 1$

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$b^2 = a^2 - c^2$

$c^2 = a^2 - b^2$

RK

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$\frac{x^2}{a^2} = 1 - \frac{y^2}{b^2}$

$1 - \frac{y^2}{b^2} \geq 0$

$b^2 - y^2 \geq 0$

$y^2 - b^2 \leq 0$

$(y-b)(y+b) \leq 0$

$-b \leq y \leq b$

$\frac{y^2}{b^2} = 1 - \frac{x^2}{a^2}$

$1 - \frac{x^2}{a^2} \geq 0$

$a^2 - x^2 \geq 0$

$x^2 - a^2 \leq 0$

$(x-a)(x+a) \leq 0$

$-a \leq x \leq a$

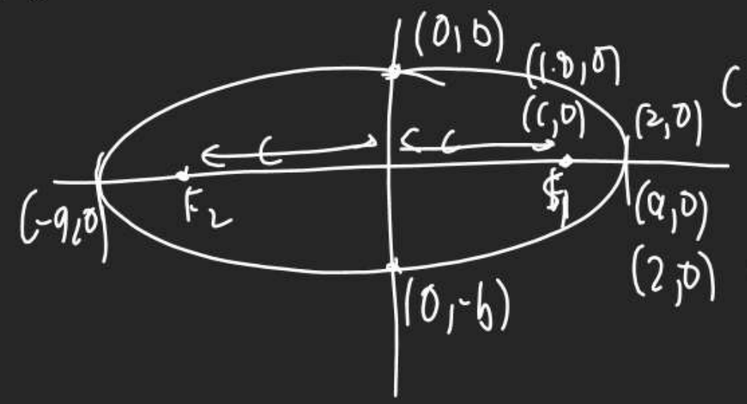
$e = \frac{F_1F_2}{PF_1 + PF_2}$

$e = \frac{2c}{PF_1 + PF_2}$

$e = \frac{2c}{2a}$

$e = \frac{c}{a}$

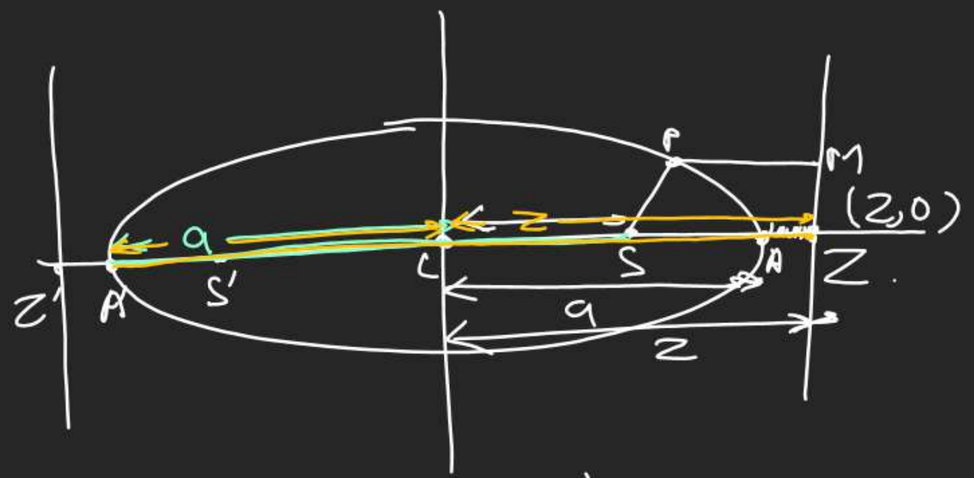
$c = a \times e$



$c = 2 \times 9$

$= 1.8$

Nishant Jindal



$$e = \frac{SP}{PM}$$

$$\Rightarrow SP = e\,PM$$

① $SA = e(AZ)$

$a - CS = e(z - a)$

② $SA' = e(ZA')$

$a + CS = e(z + a)$

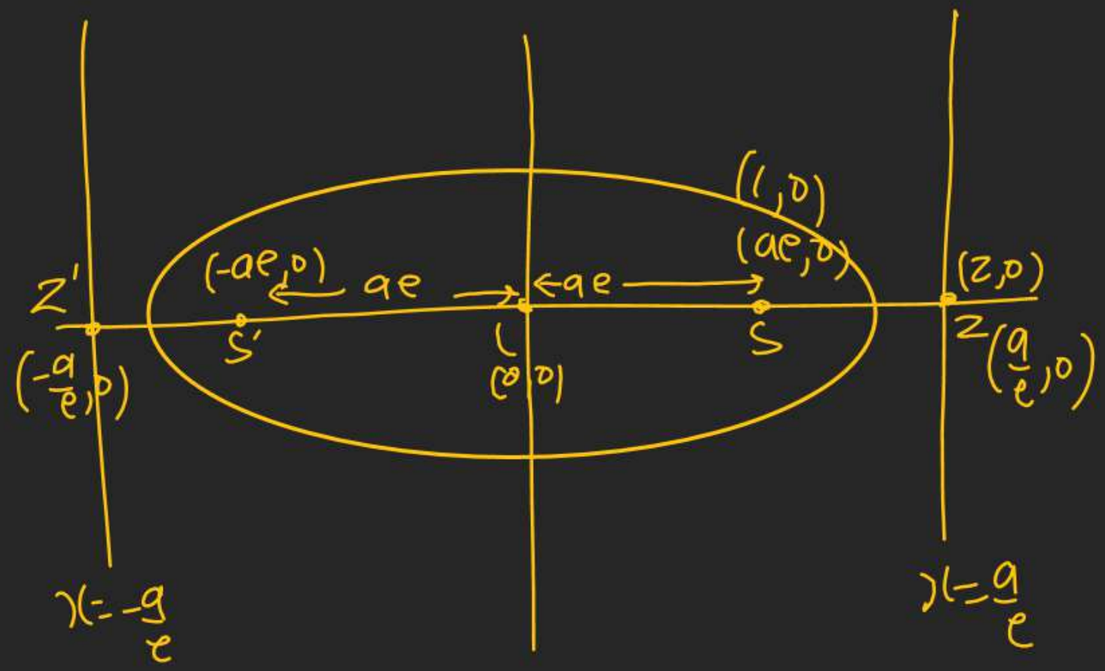
add $2a = e(2z)$

$e = \frac{a}{z} \Rightarrow z = \frac{a}{e}$

Sub → $-2CS = e(z - a - z - a)$

$+2CS = e(+2a)$

$CS = ae$



$= ae$

Nishant Jindal

Eccentricity

1) $e = \frac{c}{a} = \frac{\text{dist. of Focus from Centre}}{\text{dist of Vertex from Centre.}}$

$$e = \frac{2c}{2a}$$

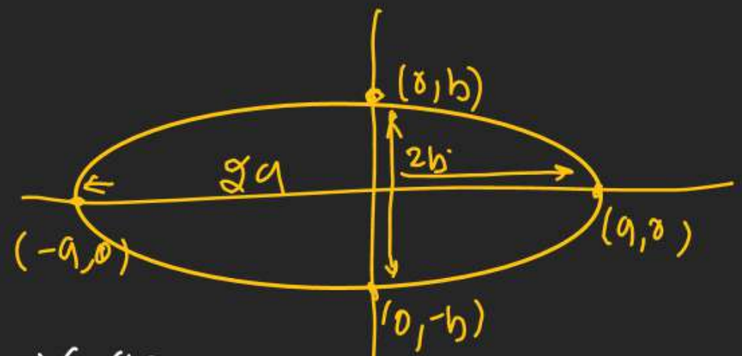
2) $$e = \frac{F_1F_2}{PF_1 + PF_2}$$

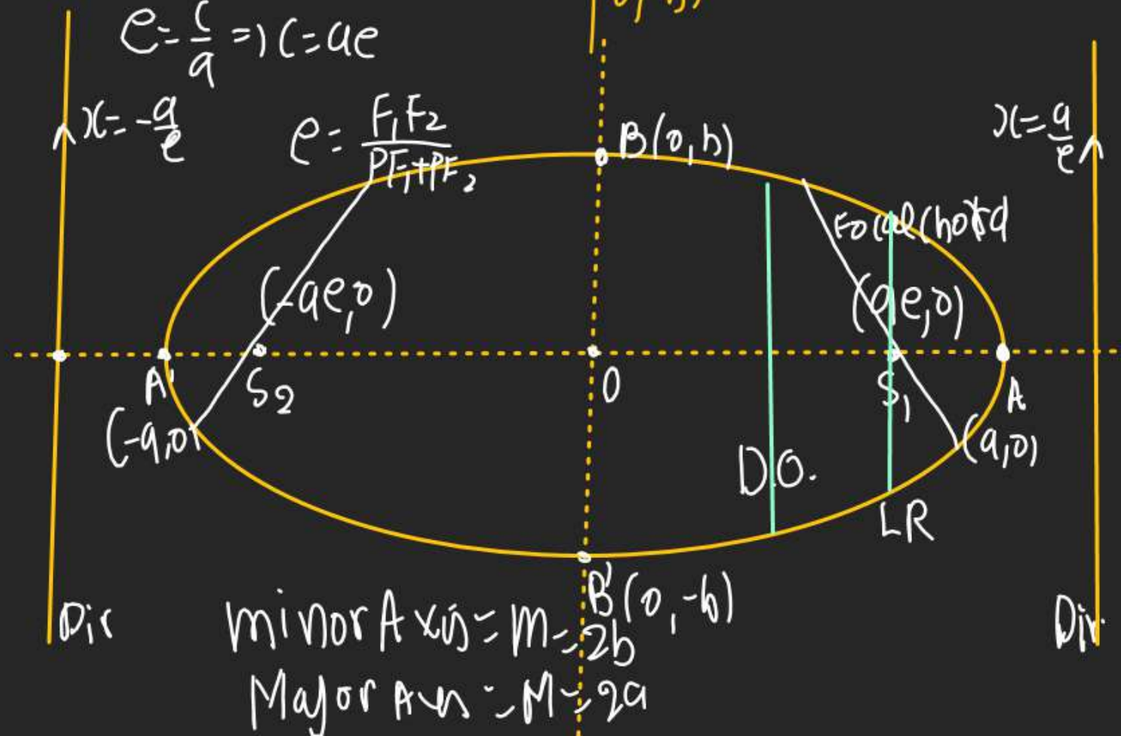
(3) $$e = \frac{c}{a} \Rightarrow e^2 = \frac{c^2}{a^2}$$

$$1 - e^2 = 1 - \frac{c^2}{a^2} = \frac{(a^2 - c^2)}{a^2}$$

$$1 - e^2 = \frac{b^2}{a^2} \Rightarrow b^2 = a^2(1 - e^2)$$

(4) $$e^2 = 1 - \frac{b^2}{a^2} = 1 - \left(\frac{2b}{2a}\right)^2 = 1 - \left(\frac{m}{M}\right)^2$$





Nishant Jindal

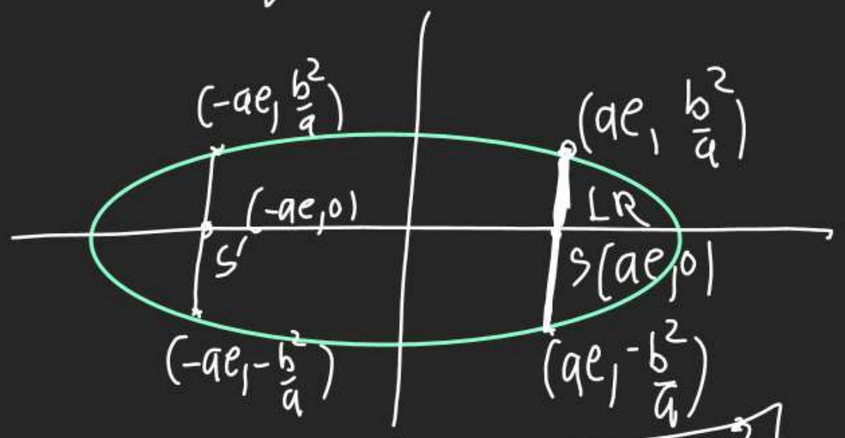
(4) Focal length = dist. bet^n F_1 & F_2

$F_1F_2 = 2ae$

Focal distance = PF_1 या PF_2

$= ePM$ या ePM'

(5) Vertex of L.R.



$\text{L.L.R} = \frac{2b^2}{a}$

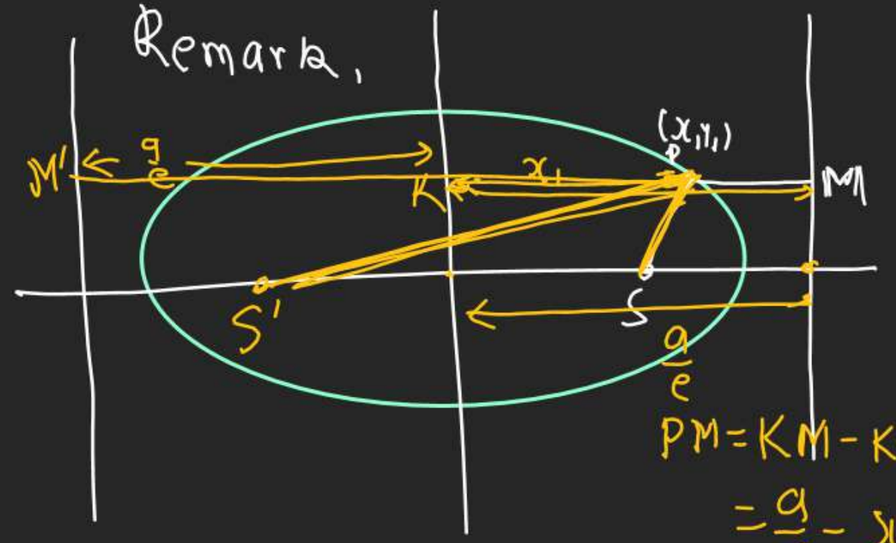
$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$\frac{a^2e^2}{a^2} + \frac{y^2}{b^2} = 1$

$\frac{y^2}{b^2} = 1 - e^2 = \frac{b^2}{a^2}$

$y^2 = \frac{b^4}{a^2} \Rightarrow y = \frac{b^2}{a}, -\frac{b^2}{a}$

Remark,



$PM = KM - KP$

$= \frac{a}{e} - x_1$

$SP = ePM$

$= e\left(\frac{a}{e} - x_1\right)$

$SP = a - ex_1$

$S'P = ePM'$

$S'P = e\left(\frac{a}{e} + x_1\right)$

$S'P = a + ex_1$

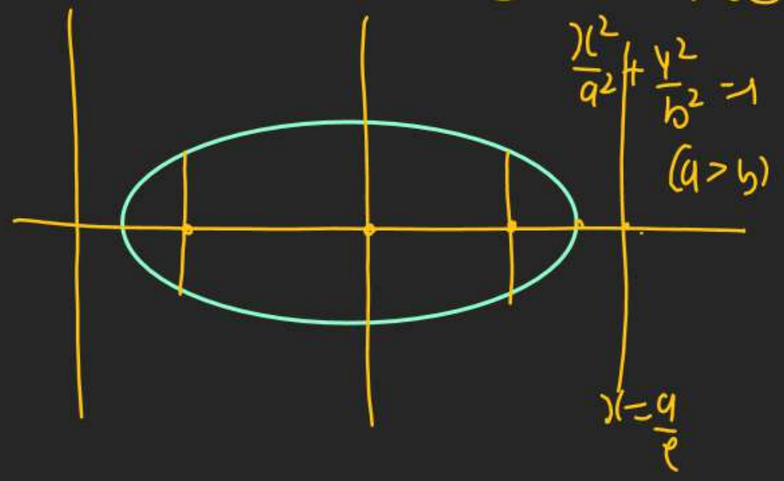
$S'P + SP = 2a$

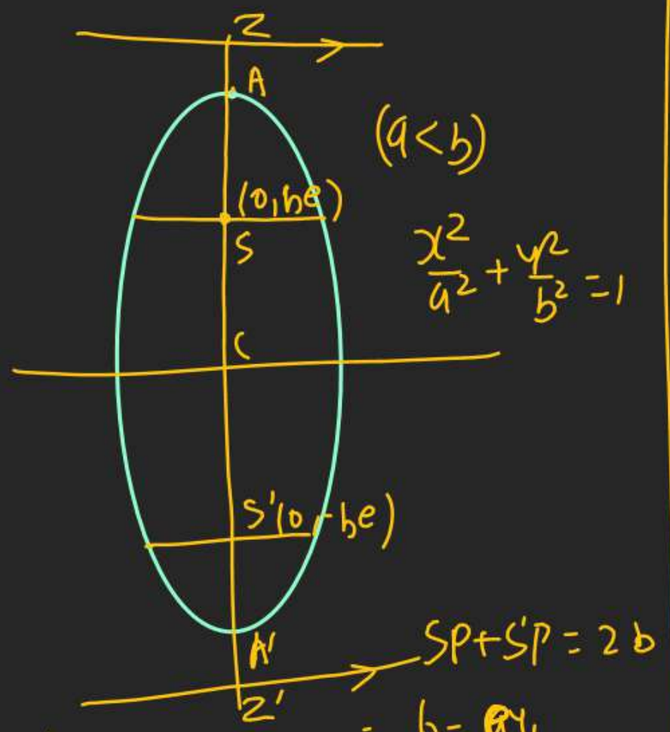
Sum of focal radii of any pt. = length of Major Axis

Nishant Jindal

6) Comparison of Ellipse.

When $a > b$ & $a < b$.





$SP + S'P = 2a$

Focal Radii $= a - ex_1$

Centre $(0,0)$

Focus $(ae, 0)$ $(-ae, 0)$

Dir. → $x = \frac{a}{e}$, $x = -\frac{a}{e}$

LR → $(ae, \frac{b^2}{a})$ $(ae, -\frac{b^2}{a})$

$LR = \frac{2b^2}{a}$

$(0,0)$

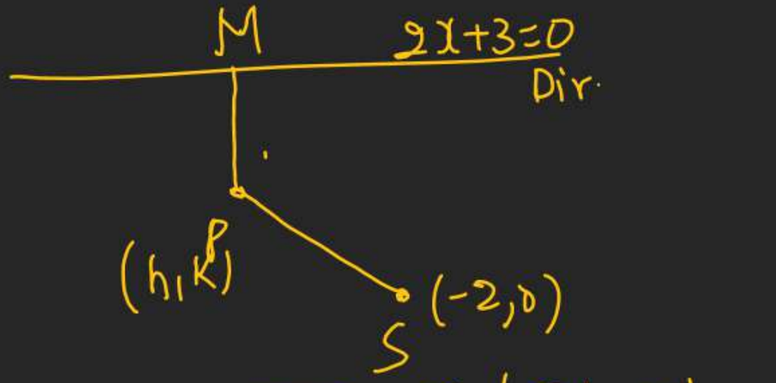
$(0, be)$ $(0, -be)$

$y = \frac{b}{e}$, $y = -\frac{b}{e}$

$(\frac{a^2}{b}, be)$ $(-\frac{a^2}{b}, be)$ $LR = \frac{2a^2}{b}$

Q A Pt. is moving such a way that its dist. from variable fixed $(-2,0)$ is $\frac{2}{3}$ times of its dist. from $x = -\frac{3}{2}$ find LOCUS?

$x = -\frac{3}{2}$ → fix Line



$$\sqrt{(h+2)^2 + K^2} = \frac{2}{3} \times \frac{|2h+3|}{\sqrt{2^2+0}}$$

$$9((h+2)^2 + K^2) = (2h+3)^2$$

$$9((x+2)^2 + y^2) = 4x^2 + 9 + 12x$$

Locus $5x^2 + 9y^2 + 24x + 27 = 0$

Nishant Jindal

Q If $(5x-1)^2+(5y-2)^2 = (\lambda^2-2\lambda+1)(3x+4y-1)^2$ is ellipse find λ?

$$\left(x-\frac{1}{5}\right)^2+\left(y-\frac{2}{5}\right)^2 = (\lambda-1)^2\left(\frac{3x+4y-1}{\sqrt{25}}\right)^2$$

dist of (x,y) from $\left(\frac{1}{5},\frac{2}{5}\right)$ $= (\lambda-1)^2$ (dist. of (x,y) from line $3x+4y-1=0$)

$SP^2 = (e^2)\ PM^2$

if it is ellipse $e<1$

$0 \le e^2 < 1$

$0 \le (\lambda-1)^2 < 1$

$0 \le |\lambda-1| < 1$

$-1 < \lambda-1 < 1 \Rightarrow 0 < \lambda < 2$

Q Find eqn of Ellipse whose ecc $= 1/2$ & Focii $(\pm 1, 0)$?

$1-e^2 = \frac{b^2}{a^2}$

$(-ae,0)$ $(ae,0)$ $(-1,0)$ S_2 S_1 $(1,0)$

$S_1S_2 = 2$

$2ae = 2$

$ae = 1$

$a \times \frac{1}{2} = 1$

$\boxed{a = 2}$

$e = 1/2$

$b^2 = a^2(1-e^2)$

$= 4\left(1-\frac{1}{4}\right)$

$b^2 = 3$

$\frac{x^2}{4}+\frac{y^2}{3} = 1$

Nishant Jindal

Q Find Eqn of Ellipse whose length of Minor axis = dist bet^n foci & LLR = 10

We know $b^2 = a^2(1-e^2)$

$2b = 2ae$

$b = ae$

$a^2e^2 = a^2 - a^2e^2$

$2a^2e^2 = a^2$

$e^2 = \frac{1}{2}$

$e = \frac{1}{\sqrt{2}}$

$b = \frac{a}{\sqrt{2}}$

$\frac{2b^2}{a} = 10$

$b^2 = 5a$

$\frac{a^2}{2} = 5a$

$a = 10$

$b = \frac{10}{\sqrt{2}} = 5\sqrt{2}$

$\frac{x^2}{100} + \frac{y^2}{50} = 1$