Nishant Jindal

QUADRATIC EQUATION

Funda:- Sum of any +ve No or fxn with its Reciprocal is always gr than or equal to 2

$$f(x) + \frac{1}{f(x)} \geq 2 \Rightarrow \text{Equality holds when } f(x) = 1$$

Q (Com. Qs in Kota) Min. value of $\dfrac{\left(x+\frac{1}{x}\right)^6 - \left(x^6+\frac{1}{x^6}\right) - 2}{\left(x+\frac{1}{x}\right)^3 + \left(x^3+\frac{1}{x^3}\right)}$; $x > 0$?

$$\Rightarrow \frac{\left(x+\frac{1}{x}\right)^6 - \left(x^6+\frac{1}{x^6}+2\right)}{\left(x+\frac{1}{x}\right)^3 + \left(x^3+\frac{1}{x^3}\right)} \Rightarrow \frac{\left(x+\frac{1}{x}\right)^6 - \left(x^3+\frac{1}{x^3}\right)^2}{\left(x+\frac{1}{x}\right)^3 + \left(x^3+\frac{1}{x^3}\right)} \Rightarrow \frac{\left(\left(x+\frac{1}{x}\right)^3\right)^2 - \left(x^3+\frac{1}{x^3}\right)^2}{\left(x+\frac{1}{x}\right)^3 + \left(x^3+\frac{1}{x^3}\right)}$$

$A^2 - B^2$

$$\Rightarrow \frac{\left(\left(x+\frac{1}{x}\right)^3 - \left(x^3+\frac{1}{x^3}\right)\right)\cancel{\left(\left(x+\frac{1}{x}\right)^3 + \left(x^3+\frac{1}{x^3}\right)\right)}}{\cancel{\left(\left(x+\frac{1}{x}\right)^3 + \left(x^3+\frac{1}{x^3}\right)\right)}} = \left(x+\frac{1}{x}\right)^3 - \left(x^3+\frac{1}{x^3}\right)$$

Br. के Andr ही देखना।

No + Rec. No + Rec.

$$\geq (2)^3 - (2)$$

$$\geq 6$$

$\therefore$ **Min = 6**

Nishant Jindal

QUADRATIC EQUATION

Q $P(x) = 4x^2+6x+4$, $Q(y) = 4y^2-12y+25$. Find Unique Pair of Real No. (x, y) Satisfying $P(x)\cdot Q(y) = 28$.

Per Sqr Bnana Ana Chahiye

$$4\left\{x^2+\frac{3}{2}x+1\right\} \times 4\left\{y^2-3y+\frac{25}{4}\right\}$$

$3/4$, $3/2$

$$4\left\{\left(x+\frac{3}{4}\right)^2-\left(\frac{3}{4}\right)^2+1\right\} \times 4\left\{\left(y-\frac{3}{2}\right)^2-\left(\frac{3}{2}\right)^2+\frac{25}{4}\right\}$$

$1-\frac{9}{16}$

$\frac{7}{16}$

$\frac{25}{4}-\frac{9}{4}=\frac{16}{4}$

$$4\left\{\left(x+\frac{3}{4}\right)^2+\frac{7}{16}\right\} \times 4\left\{\left(y-\frac{3}{2}\right)^2+\frac{16}{4}\right\}$$

$$\left\{\left(\cancel{4\left(x+\frac{3}{4}\right)^2}+\frac{7}{4}\right)\right\} \times \left\{\cancel{4\left(y-\frac{3}{2}\right)^2}+16\right\}$$

$\frac{7}{4}\times 16$

$\left(-\frac{3}{4}, \frac{3}{2}\right)$

$P(x) \times Q(y) = 28$ Lana h!!

Ye 28 dega if $\left(x+\frac{3}{4}\right)=0$, $\left(y-\frac{3}{2}\right)=0$ (हो गया) $\Rightarrow x=-\frac{3}{4}, y=\frac{3}{2}$

Nishant Jindal

QUADRATIC EQUATION

Q Kota Prashan

Find Product of the real Roots of eqn $\boxed{x^2+18x}+30 = 2\sqrt{\boxed{x^2+18x}+45}$

let $x^2+18x = t$

$t+30 = 2\sqrt{t+45}$

$(t+30)^2 = 4(t+45)$

$t^2+60t+900 = 4t+180$

$t^2+56t+720 = 0$

$(t+36)(t+20) = 0$

$t = -36 \quad t = -20$

$-36+30 = 2\sqrt{-36+45}$

$-6 = 2\sqrt{9}$ ✗

$\ominus = \oplus$

$-20+30 = 2\sqrt{-20+45}$

$10 = 2\sqrt{25}$

$\oplus = \oplus$ ✓

$x^2+18x = -36$ (Reject)

$x^2+18x = -20$

$x^2+18x+20 = 0 \; \begin{matrix} \alpha \\ \beta \end{matrix}$

POR $\alpha \cdot \beta = \frac{20}{1} = 20$

Nishant Jindal

Q Sheet: Find Value of a for which $-3 < \dfrac{x^2+ax-2}{x^2+x+1} < 2$ is valid for all Real x?

$a \in (-2, 1)$

$\dfrac{x^2+ax-2}{x^2+x+1} > -3$ (denominator $\oplus$) Cross Mult

And

$\dfrac{x^2+ax-2}{(x^2+x+1)_{+ve}} < 2$ ($D = -3$) Cross Mult.

$x^2+ax-2 > -3x^2-3x-3$

$\boxed{4}x^2+x(a+3)+1 > 0$ QE > 0

$D < 0$

$(a+3)^2 - 4\times 4\times 1 < 0$

$a^2+6a-7 < 0$

$(a+7)(a-1) < 0$

$\underline{-7 < a < 1}$

$x^2+ax-2 < 2x^2+2x+2$

$x^2+x(2-a)+4 > 0$ QE > 0

$a = 1$ $\oplus$ Check

$D < 0$

$(2-a)^2 - 4\times 1\times(4) < 0$

$4-4a+a^2-16 < 0$

$a^2-4a-12 < 0$

$(a-6)(a+2) < 0$

$-2 < a < 6$

Nishant Jindal

QUADRATIC EQUATION

Q let a, b be arbitrary Real No. Find the Smallest Natural No. b for which eq^n $x^2+2(a+b)x+(a-b+8)=0$ has $\boxed{\text{Unequal Real Roots}}$ for all $a \in R$.

→ Distinct Roots

$D>0$

$D>0$

$4(a+b)^2-4(1)(a-b+8)>0$

$(a+b)^2-(a-b+8)>0$

$a^2+b^2+2ab-a+b-8>0$

$a^2+a(2b-1)+(b^2+b-8)>0$

Q Eq^n in a >0

$D<0$

$(2b-1)^2-4(1)(b^2+b-8)<0$

$\cancel{4b^2}-4b+1-\cancel{4b^2}-4b+32<0$

$-8b+33<0 \Rightarrow 8b>33$

$b>\frac{33}{8}$

$b>4.125$

b

Smallest Natural No $=5$

Jese hi b, $\left(\frac{33}{8}\right)$ se Bda hoga

$D<0$ Bnega

Q Eq^n in a +ve hogi

$\Rightarrow$ D +ve hoga $\Rightarrow$ Roots Unequal Milenge

Vahi to Q Ki demand hai!!

Nishant Jindal

Q If Range of fxn $f(x) = \boxed{\dfrac{x^2+ax+b}{x^2+2x+3}}$ Rational is $[-5, 4]$, $a, b \in N$ then find $a^2+b^2 = ?$

Y yahan Pr −5 se 4 ke Bich Chhupa hua hai

$-5 \le y \le 4$ ULTA

$(y+5)(y-4) \le 0$

$y^2 + y - 20 \le 0$

$-y^2 - y + 20 \ge 0$

Method

① Aap apni Range Nikalo.

② Use Qs ki Range se Compare Kardo.

$y = \dfrac{x^2+ax+b}{x^2+2x+3}$ (C.M.) & find Eqn in x

$x^2(y-1) + x(2y-a) + (3y-b) = 0 \rightarrow$ QEqn in x

$D \ge 0$

$(2y-a)^2 - 4(y-1)(3y-b) \ge 0$

$4y^2 + a^2 - 4ay - 4(3y^2 - 3y - by + b) \ge 0$

$-8y^2 - 4ay + 12y + 4by + a^2 - 4b \ge 0$

$-8y^2 - y(\underline{4a - 4b - 12}) + (a^2 - 4b) \ge 0$

$-y^2 - y + 20 \ge 0$

$\dfrac{+8}{+1} = \dfrac{4a-4b-12}{1} = \dfrac{a^2-4b}{20}$

$4a - 4b - 12 = 8$

$a - b - 3 = 2$

$b = a - 5$

$\dfrac{a^2-4b}{20} = 8$

$\Rightarrow a^2 - 4b = 160$

$a^2 - 4(a-5) = 160$

$a^2 - 4a - 140 = 0$

$(a-14)(a+10) = 0$

$a = -10, 14$

$b = a - 5$

$b = -15, 9$

~~$(-10, -15)$~~ & $(14, 9)$

$a^2 + b^2 = 14^2 + 9^2 = 277$

Nishant Jindal

QUADRATIC EQUATION

Q If $y=\frac{2x}{1+x^2}$ where $x\in R$ then Range of expression.

$y=\frac{\text{Linear}}{\text{Quad}}$ Ko $\frac{Q}{Q}$ K Method Se hi Solve Kurte chalte

$\boxed{C.M. \;\&\; Q.E}$

$x^2y+y=2x$

$x^2y-2x+y=0 \rightarrow$ Q Eq^n in x.

$D\geq 0$

$(-2)^2-4\times(y)(y)\geq 0$

$4-4(y)^2\geq 0$

$y^2-1\leq 0$

$(y-1)(y+1)\leq 0$

$-1\leq y\leq 1$

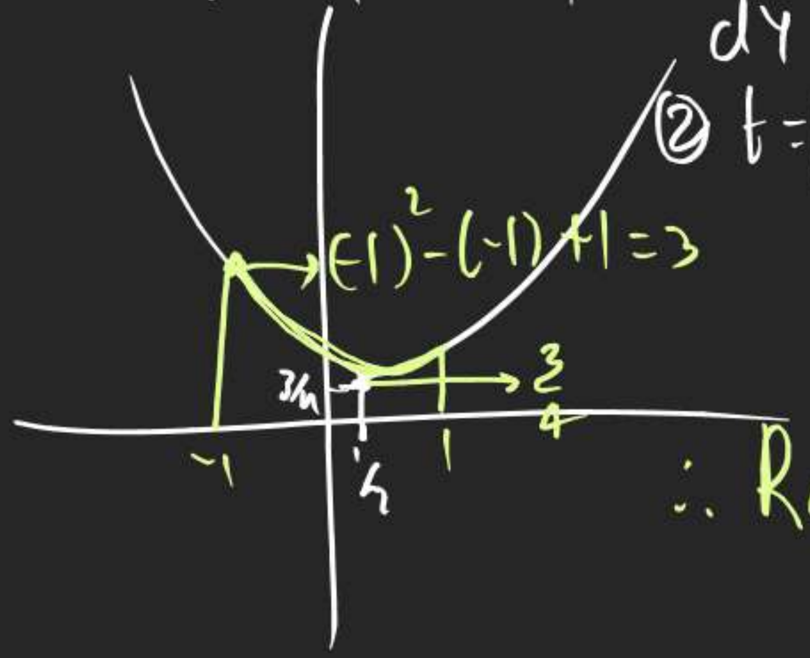
$y^2-y+1=?$

↳ Q. Eq^n Ka y Kahin aur connected h. then Quad Ka graph Banayenge & use connection Ko Mark Karenge

$t=y^2-y+1 \Rightarrow$ ① $\frac{dt}{dy}=2y-1=0 \Rightarrow y=\frac{1}{2}$

② $t=\left(\frac{1}{2}\right)^2-\frac{1}{2}+1=\frac{3}{4}$

Vertex $\left(\frac{1}{2},\frac{3}{4}\right)$



$\therefore$ Range $=\left[\frac{3}{4},3\right]$

QUADRATIC EQUATION

Q $y = \dfrac{\sin^2 x + 4\sin x + 5}{2\sin^2 x + 8\sin x + 8}$ find R_f ?

Sbki Sheet

$y = \dfrac{\sin^2 x + 4\sin x + 5}{2(\sin^2 x + 4\sin x + 4)}$

$y = \dfrac{(\sin^2 x + 4\sin x + 4) + 1}{2(\sin^2 x + 4\sin x + 4)}$

$= \dfrac{\cancel{\sin^2 x + 4\sin x + 4}}{2(\cancel{\sin^2 x + 4\sin x + 4})} + \dfrac{1}{2(\sin^2 x + 4\sin x + 4)}$

$y = \dfrac{1}{2} + \dfrac{1}{2(\sin x + 2)^2}$

$y = \dfrac{1}{2}\left(1 + \dfrac{1}{(\sin x + 2)^2}\right)$

$\sin x = 0$ | $\sin x = 1$ | $\sin x = -1$

$y = \frac{1}{2}\left(1 + \frac{1}{(0+2)^2}\right)$ | $y = \frac{1}{2}\left(1 + \frac{1}{(1+2)^2}\right)$ | $y = \frac{1}{2}\left(1 + \frac{1}{(-1+2)^2}\right)$

$y = \frac{5}{8}$ | $y = \frac{5}{9}$ | $y = 1$

Sbse Chhota | Sbse Bda

$y \in \left[\frac{5}{9}, 1\right]$

Nishant Jindal

QUADRATIC EQUATION

Q If α, β, γ Satisfies

PUMA

इस type का Qs हो तो पर
मैंने Ex 6 (Copy में बनाना है)

$$\left.\begin{aligned} a\alpha^2+b\alpha+c &= \sin\theta\,\alpha^2+\cos\theta\cdot\alpha \\ a\beta^2+b\beta+c &= \sin\theta\cdot\beta^2+\cos\theta\cdot\beta \\ a\gamma^2+b\gamma+c &= \sin\theta\cdot\gamma^2+\cos\theta\cdot\gamma \end{aligned}\right\}$$

$a\sin\theta+b\cos\theta$ (1) find Max value of $\dfrac{a^2+b^2}{a^2+3ab+5b^2}$

$3\sin2\theta+4\cos2\theta$

$\in\left[-\sqrt{3^2+4^2}, \sqrt{3^2+4^2}\right]$

$\in[\underset{\text{Min}}{-5}, 5]$

By observation

$$ax^2+bx+c=\sin\theta\cdot x^2+\cos\theta\cdot x \begin{matrix}\nearrow \alpha\\ \rightarrow \beta\\ \searrow \gamma\end{matrix}$$

$$(a-\sin\theta)x^2+x(b-\cos\theta)+c=0 \begin{matrix}\nearrow \alpha\\ \rightarrow \beta\\ \searrow \gamma\end{matrix}$$

$a-\sin\theta=0, \quad b-\cos\theta=0, \quad c=0$

$a=\sin\theta, \quad b=\cos\theta$

Expression $=\dfrac{a^2+b^2}{a^2+3ab+5b^2}$

$2\sin^2\theta=1-\cos2\theta$

$2\cos^2\theta=1+\cos2\theta$

$$=\frac{\sin^2\theta+\cos^2\theta}{\sin^2\theta+3\sin\theta\cos\theta+5\cos^2\theta}$$

$$=\frac{1}{\sin^2\theta+\frac{3}{2}\times\boxed{2\sin\theta\cos\theta}+5\cos^2\theta}$$

$$=\frac{1}{\frac{3}{2}\times\sin2\theta+\frac{1-\cos2\theta}{2}+\frac{5(1+\cos2\theta)}{2}}$$

$$\text{Exp}_{\text{Max}}=\frac{2}{\underbrace{(3\sin2\theta+4\cos2\theta)}_{\text{Min}}+6}=\frac{2}{-5+6}=\frac{2}{1}=2$$

Q Eq. के 2 Roots होते हैं

But here 3 Roots are given

as More Roots are Satisfying

$\Rightarrow$ this Eq^n is an Identity

$Ax^2+Bx+C=0$

$\boxed{A=B=C=0}$