DPP–01 (AREA UNDER THE CURVE)

CALCULATING AREA BY USING

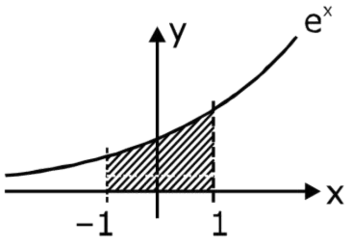
HORIZONTAL STRIP

1. The area between the curve $y = e^x$ and x-axis which lies between x = −1 and x = 1 is-

(A) $e^2 - 1$ (B) $\frac{(c^2-1)}{e}$ (C) $\frac{(1-e)}{c}$ (D) $\frac{(e-l)}{e^2}$

Ans. **(B)**

Sol.



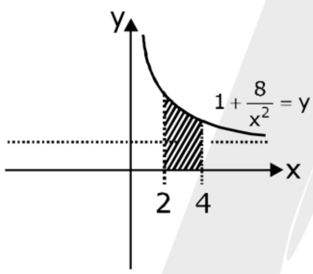
Area $= \int_{-1}^{1} e^x dx = \frac{e^2-1}{e}$

2. The area bounded by the curve $y = 1 + \frac{8}{x^2}$, x-axis, $x = 2$ and $x = 4$ is-

(A) 2 (B) 3 (C) 4 (D) 5

Ans. **(C)**

Sol.



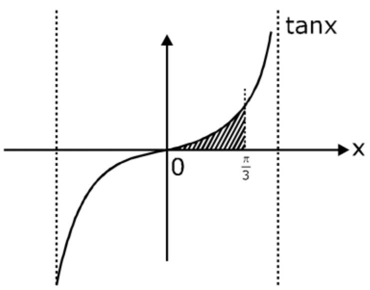
Area $= \int_{2}^{4} \left(1 + \frac{8}{x^2}\right) dx = 4$

3. The area bounded by curves $y = \tan x$, x-axis and $x = \frac{\pi}{3}$ is

(A) 2log2 (B) log2 (C) $\log\left(\frac{2}{\sqrt{3}}\right)$ (D) 0

Ans. **(B)**

Sol.



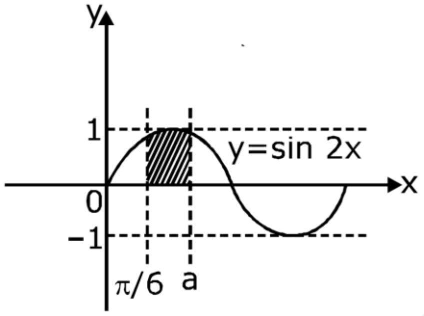
Area $\int_{0}^{\pi/3} \tan x dx = (\log \sec x)_0^{\pi/3} = \log 2$

4. The value of a for which the area of the region bounded by the curve $y = \sin 2x$, the straight lines $x = \frac{\pi}{6}, x = a$ and x-axis is equal to $\frac{1}{2}$ is-

(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{3}$ (C) $\frac{4}{3}$ (D) $\frac{\pi}{6}$

Ans. **(B)**

Sol.



Given that $\int_{\pi/6}^{a} \sin 2x dx = \frac{1}{2}$ so $a = \frac{\pi}{3}$

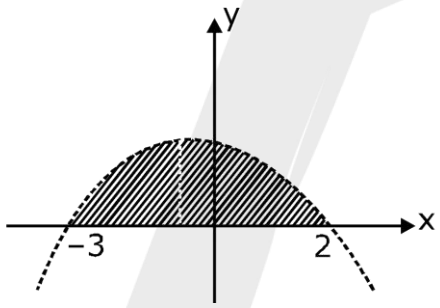
5. The area between the curves $y = 6 - x - x^2$ and x-axis is -

(A) $\frac{125}{6}$ (B) $\frac{125}{2}$ (C) $\frac{25}{6}$ (D) $\frac{25}{2}$

Ans. **(A)**

Sol. Curve $y = 6 - x - x^2$

$\Rightarrow y = (2 - x)(3 + x)$



The shaded area

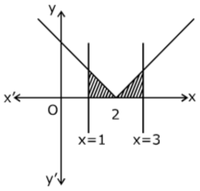
$$= \int_{-3}^{2} (6 - x - x^2) dx = 125/6$$

6. The area of the region bounded by the curves $y = |x - 2|, x = 1, x = 3$ and the x-axis is

(A) 3 (B) 2 (C) 1 (D) 4

Ans. **(C)**

Sol.

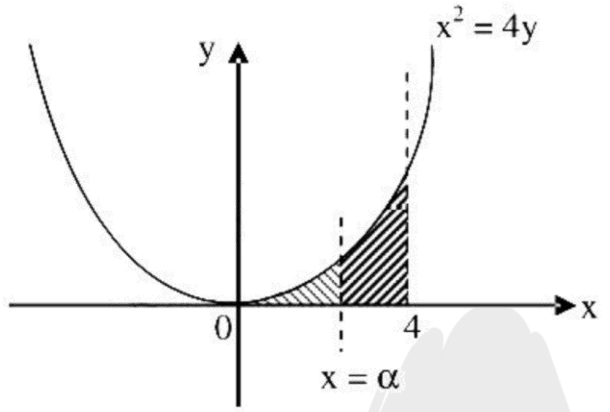


$A = \frac{1}{2} + \frac{1}{2} = 1$

7. The area bounded by the parabola $x^2 = 4y$, the x-axis and the line x = 4 is divided into two equal area by the line $x = \alpha$, then the value of α is-

(A) $2^{1/3}$ (B) $2^{2/3}$ (C) $2^{4/3}$ (D) $2^{5/3}$

Ans. **(D)**



Sol.

$\int_0^\alpha \frac{x^2}{4}dx + \int_\alpha^4 \frac{x^2}{4}dx \Rightarrow \alpha = 2^{5/3}$

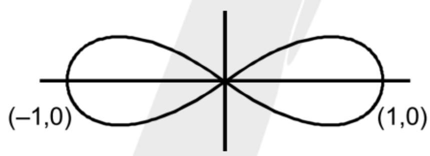
CALCULATING AREA BY USING VERTICAL STRIP

8. The area enclosed by the curve $y^2 + x^4 = x^2$ is

(A) $\frac{2}{3}$ (B) $\frac{4}{3}$ (C) $\frac{8}{3}$ (D) $\frac{10}{3}$

Ans. **(B)**

Sol. curve is symmetric about both the axes & cuts x-axis at $(-1,0)(0,0)\&(1,0)$



Area of loop $= 2\int_0^1 x\sqrt{1-x^2}dx$

$= 2 \cdot \frac{2}{3} = \frac{4}{3}$

9. If $(a,0); a > 0$ is the point where the curve $y = \sin 2x - \sqrt{3}\sin x$ cuts the x-axis first, A is the area bounded by this part of the curve, the origin and the positive $x = axis$, then

(A) $4A + 8\cos a = 7$ (B) $4A + 8\sin a = 7$

(C) $4A - 8\sin a = 7$ (D) $4A - 8\cos a = 7$

Ans. **(A)**

Sol. $\sin 2x - \sqrt{3}\sin x = 0 \Rightarrow \sin x\left(\cos x - \frac{\sqrt{3}}{2}\right) = 0$

$x = 0$ on $\pi/6$

so $A = \int_0^a (\sin 2x - \sqrt{3}\sin x)dx$ $\Rightarrow 4A + 8\cos a = 7$

10. The area of the region for which $0 < y < 3 - 2x - x^2$ & $x > 0$ is

(A) $\int_1^3 (3 - 2x - x^2)dx$ (B) $\int_0^3 (3 - 2x - x^2)dx$

(C) $\int_0^1 (3 - 2x - x^2)dx$ (D) $\int_1^3 (3 - 2x - x^2)dx$

Ans. **(C)**

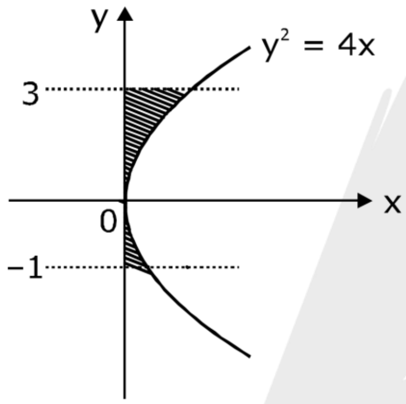
Sol. $A = \int_0^1 (3 - 2x - x^2)dx$

11. The area between the curve $y^2 = 4x$, y-axis, and $y = -1$ and $y = 3$ is-

(A) $\frac{7}{3}$ (B) $\frac{9}{4}$ (C) $\frac{1}{12}$ (D) $\frac{1}{4}$

Ans. **(A)**

Sol.



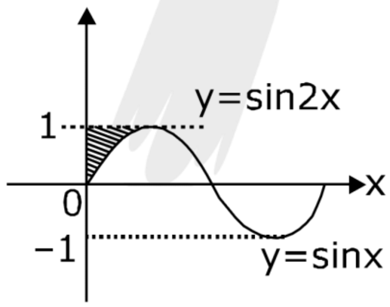
The area $= \int_{-1}^{3} \frac{y^2}{4} dx = 7/3$

12. The area bounded by the curve $y = \sin 2x$, y-axis and the line $y = 1$ is-

(A) 1 (B) $\frac{1}{4}$ (C) $\frac{\pi}{4}$ (D) $\left(\frac{\pi}{4}\right) - \left(\frac{1}{2}\right)$

Ans. **(D)**

Sol.



Area $= \int_0^1 \frac{1}{2}\sin^{-1} y\,dy = \frac{\pi}{4} - \frac{1}{2}$

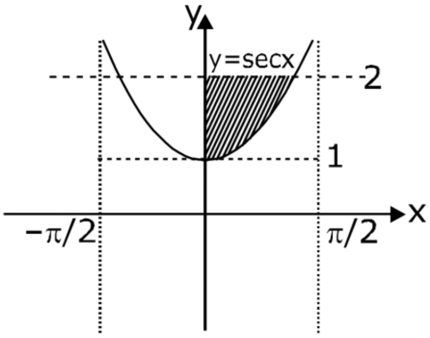
13. The area between the curve $y = \sec x$ and y-axis when $I \le y \le 2$ is-

(A) $\frac{2\pi}{3} - \log(2 + \sqrt{3})$ (B) $\frac{2\pi}{3} + \log(2 + \sqrt{3})$

(C) $\frac{\pi}{3} - \frac{1}{2}\log(2 + \sqrt{3})$ (D) $\frac{\pi}{3} + \log(2 + \sqrt{3})$

Ans. **(A)**

Sol.



Area $= \int_1^2 \sec^{-1} y\,dy = \frac{2\pi}{3} - \log(2+\sqrt{3})$

14. If the area bounded by the x-axis, curve $y = f(x)$ and the lines $x = 1, x = b$ is equal to $\sqrt{b^2+1} - \sqrt{2}$ for all $b > 1$, then $f(x)$ is-

(A) $\sqrt{(x-1)}$ (B) $\sqrt{(x+1)}$ (C) $\sqrt{(x^2+1)}$ (D) $\frac{x}{\sqrt{1+x^2}}$

Ans. **(D)**

Sol. Given $\int_1^b f(x)dx = \sqrt{b^2+1} - \sqrt{2}$

Differentiating it, $f(b) = \frac{b}{\sqrt{b^2+1}}$

So, $f(x) = \frac{x}{\sqrt{x^2+1}}$

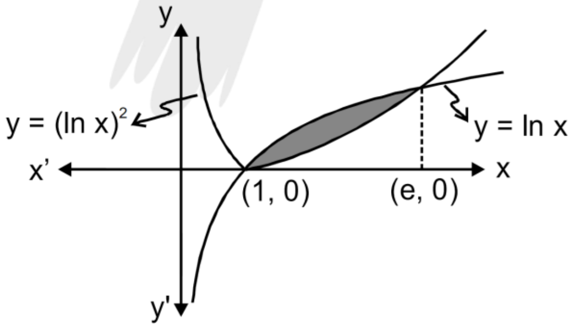
AREA BETWEEN TWO CURVES AND CURVE SKETCH

15. The area of the figure bounded by the curves $y = \ln x$ & $y = (\ln x)^2$ is

(A) $e+1$ (B) $e-1$ (C) $3-e$ (D) 1

Ans. **(C)**

Sol.



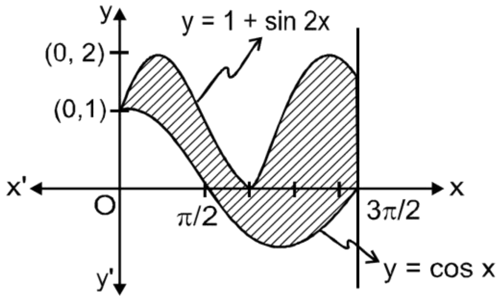
$A = \int_1^e (\ln^2 x - \ln x)dx = 3 - e$

16. The area enclosed by the curves $y = \cos x, y = 1 + \sin 2x$ and $x = \frac{3\pi}{2}$ as x varies from 0 to $\frac{3\pi}{2}$, is

(A) $\frac{3\pi}{2} - 2$ (B) $\frac{3\pi}{2}$ (C) $2 + \frac{3\pi}{2}$ (D) $1 + \frac{3\pi}{2}$

Ans. **(C)**

Sol.



$A = \int_0^{3\pi/2} (1 + \sin 2x - \cos x)dx$

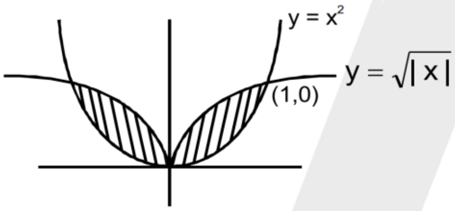
$A = \int_0^{3\pi/2} (1 + \sin 2x - \cos x)dx = 2 + \frac{3\pi}{2}$

17. The area of the region (s) enclosed by the curves $y = x^2$ and $y = \sqrt{|x|}$ is

(A) 1/3 (B) 2/3 (C) 1/6 (D) 1

Ans. (B)

Sol.



$A = 2\int_0^1 \left(\sqrt{|x|} - x^2\right)dx = \frac{2}{3}$

18. The area bounded by the curves $y = -\sqrt{-x}$ and $x = -\sqrt{-y}$ where $x, y \leq 0$

(A) cannot be determined

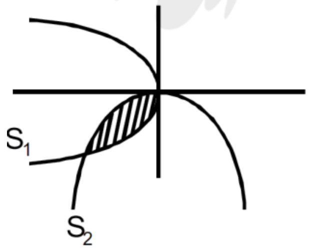
(B) is 1/3

(C) is 2/3

(D) is same as that of the figure bounded by the curves $y = \sqrt{-x}; x \leq 0$ and $x = \sqrt{-y}; y \leq 0$

Ans. (B)

Sol. Area of shaded region $= \frac{1}{3}$

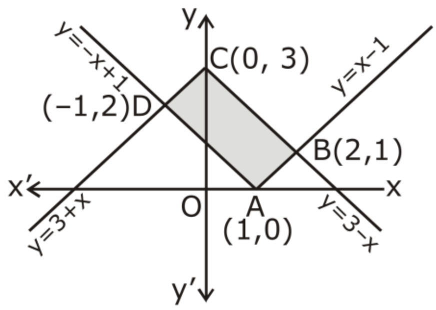


19. The area of the region bounded by the curves $y = |x - 1|$ and $y = 3 - |x|$ is-

(A) 6 s units (B) 2 s units (C) 3 s units (D) 4 s units

Ans. (D)

Sol. Since, $y = |x - 1| = \begin{cases} x - 1, & x > 1 \\ -x + 1, & x \leq 1 \end{cases}$ and $y = 3 - |x| = \begin{cases} 3 + x, & x \leq 0 \\ 3 - x, & x > 0 \end{cases}$



On solving $y = x - 1$ and $y = 3 - x$

$\Rightarrow x - 1 = 3 - x$

$\Rightarrow x = 2$

and $y = 3 - 2 \Rightarrow y = 1$

Now, $AB^2 = (0-2)^2 + (3-1)^2$

$= 4 + 4 = 8$

$\Rightarrow BC = 2\sqrt{2}$

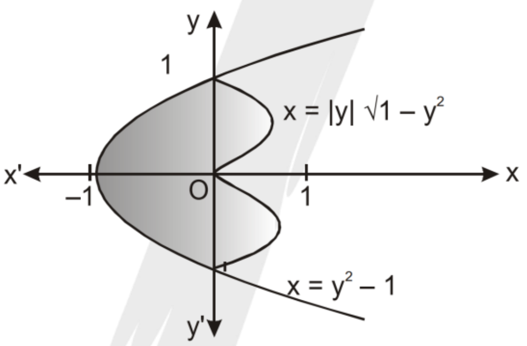
Area of rectangle $ABCD = AB \times BC = \sqrt{2} \times 2\sqrt{2} = 4$ sq unit

20. Area of the region enclosed between the curves $x = y^2 - 1$ and $x = |y|\sqrt{1-y^2}$ is

(A) 1 (B) $\frac{4}{3}$ (C) $\frac{2}{3}$ (D) 2

Ans. (D)

Sol.



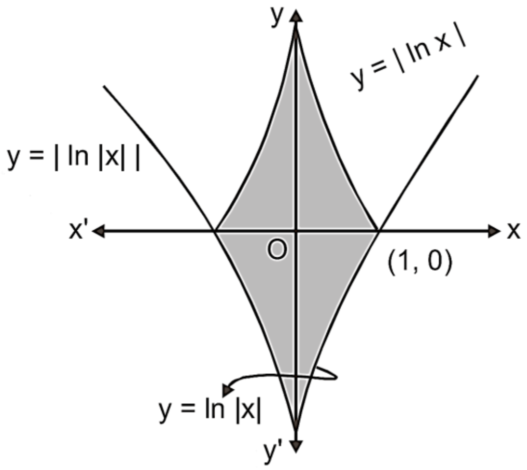
Area $= 2\int_0^1 y\sqrt{1-y^2}dy + 2\int_0^1 (y^2-1)dy$

$\Rightarrow A = 2$ Ans.

21. Area enclosed by the curves $y = \ell nx$, $y = \ell n|x|$; $y = |\ell nx|$ and $y = |\ell n|x||$ is equal to

(A) 2 (B) 4 (C) 8 (D) 6

Ans. (B)



Sol.

Area enclosed by the curves $y = \ln x, y = \ln|x|$ $y = |\ln x|$ and $y = |\ln|x||$ is

$4\int_0^1 |\ln x| dx = 4[x\ln x - x]_0^1 = 4$

AREA BOUNDED BY INVERSE OF A FUNCTION WITH Y - AXIS

22. The parabolas $y^2 = 4x$ and $x^2 = 4y$ divide the square region bounded by the lines $x = 4, y = 4$ and the coordinate axes. If S_1, S_2, S_3 are respectively the areas of these parts numbered from top to bottom; then $S_1 : S_2 : S_3$ is-

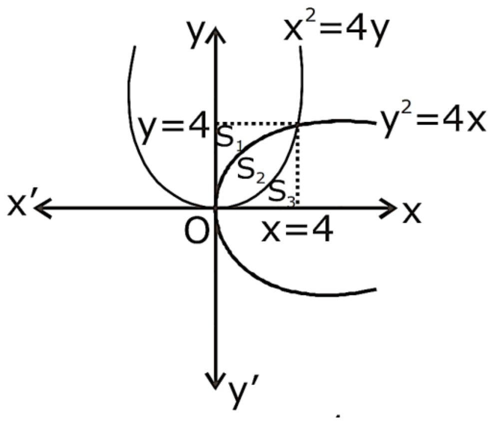
(A) 1: 2: 1 (B) 1: 2: 3 (C) 2: 1: 2 (D) 1: 1: 1

Ans. **(D)**

Sol. It is clear from the figure, that

$$S_1 = S_3 = \int_0^4 y dx$$

$$= \int_0^4 \frac{x^2}{4} dx = \frac{1}{4}\left[\frac{x^3}{3}\right]_0^4$$



$\Rightarrow S_1 = S_3 = \frac{1}{12} \times 64$

$= \frac{16}{3}$ sq unit

and $S_2 + S_3 = \int_0^4 \sqrt{4x}dx$

$= 2\left[\frac{x^{3/2}}{3/2}\right]_0^4 = \frac{4}{3} \times 8$

$\Rightarrow\ S_2 = \frac{32}{3} - \frac{16}{3}$ [from Eq. (i)]

$\Rightarrow\ S_2 = \frac{16}{3}$ sq. unit

$\therefore\ S_1 : S_2 : S_3 = \frac{16}{3} : \frac{16}{3} : \frac{16}{3} = 1:1:1$

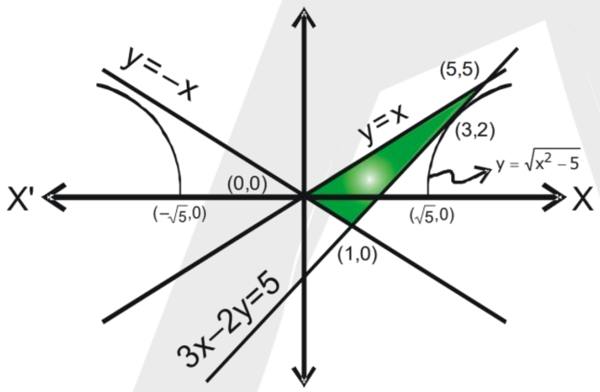
MIXED PROBLEM

23. The area of the closed figure bounded by $y = x, y = -x$ & the tangent to the curve $y = \sqrt{x^2 - 5}$ at the point (3,2) is

(A) 5 (B) $2\sqrt{5}$ (C) 10 (D) $\frac{5}{2}$

Ans. **(A)**

Sol.



Equation of tangent area of shaded region

$= \frac{1}{2}|5(-1) - 5(1)| = 5$

24. The area bounded by the curve $y = f(x)$, the x-axis & the ordinates $x = 1$ & $x = b$ is $(b-1)\sin(3b+4)$. Then $f(x)$ is

(A) $(x-1)\cos(3x+4)$ (B) $\sin(3x+4)$

(C) $\sin(3x+4) + 3(x-1)\cdot\cos(3x+4)$ (D) none

Ans. **(C)**

Sol. $\int_1^b f(x)dx = (b-1)\sin(3b+4)$

differentiate w.r.t ' b '

$f(b)\cdot 1 = 3(b-1)\cos(3b+4) + \sin(3b+4)$

so, $f(x) = 3(x-1)\cos(3x+4) + \sin(3x+4)$

25. Let $f(x)$ be a non-negative continuous function such that the area bounded by the curve $y = f(x)$, x-axis and the ordinates $x = \frac{\pi}{4}$ and $x = \beta > \frac{\pi}{4}$ is $\left(\beta\sin\beta + \frac{\pi}{4}\cos\beta + \sqrt{2}\beta\right)$. Then $f\left(\frac{\pi}{2}\right)$ is

(A) $\left(\frac{\pi}{4} + \sqrt{2} - 1\right)$ (B) $\left(\frac{\pi}{4} - \sqrt{2} + 1\right)$

(C) $\left(1 - \frac{\pi}{4} - \sqrt{2}\right)$ (D) $\left(1 - \frac{\pi}{4} + \sqrt{2}\right)$

Ans. **(D)**

Sol. According to the given condition

$$\int_{\pi/4}^{\beta} f(x)dx = \beta\sin\beta + \frac{\pi}{4}\cos\beta + \sqrt{2}\beta$$

On differentiating w.r.t β on both sides, we get

$$f(\beta) = \sin\beta + \beta\cos\beta = -\frac{\pi}{4}\sin\beta + \sqrt{2}$$

$\therefore\ f\left(\frac{\pi}{2}\right) = 1 + 0 - \frac{\pi}{4} + \sqrt{2}$

$= 1 - \frac{\pi}{4} + \sqrt{2}$

26. $y = f(x)$ is a function which satisfies

(i) $f(0) = 0$

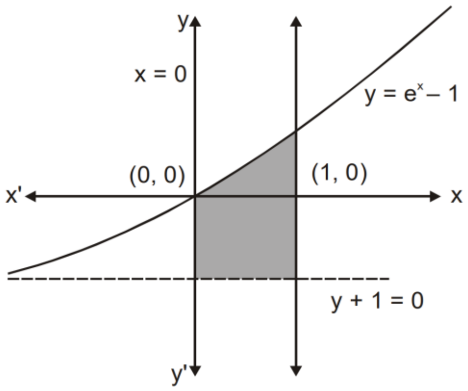
(ii) $f''(x) = f'(x)$ and

(iii) $f(0) = 1$

then the area bounded by the graph of $y = f(x)$, the lines $x = 0, x - 1 = 0$ and $y + 1 = 0$, is

(A) e (B) $e - 2$ (C) $e - 1$ (D) e + 1

Ans. **(C)**



Sol.

According to questions $f''(x) = f'(x)$

$\Rightarrow \int f''(x)dx = \int f'(x)dx$

$f'(x) = f(x) + 4 \Rightarrow f'(0) = f(0) + 4 \Rightarrow 4 = 1$

Now $f'(x) = f(x) + 1$

$\Rightarrow \frac{f'(x)}{f(x)+1} = 1 \Rightarrow \int \frac{f'(x)}{f(x)+1} dx = \int dx$

In $|f(x) + 1| = x + C_2 \Rightarrow \ln|f(0) + 1| = 0 + C_2 \Rightarrow C_2 =$

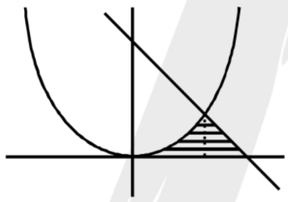
In $\mid f(x) + 1 = x \Rightarrow f(x) + 1 = e^x \Rightarrow f(x) = e^x - 1$

$A = \int_0^1 (e^x - 1)dx + 1 \times 1 \Rightarrow A = e - 2 + 1 \Rightarrow A = e - 1$

SUBJECTIVE PROBLEM

27. Find the area bounded by the curves $y = x^2, x + y = 2, x \geq 0, y \geq 0$.

Sol. $A = \int_0^1 x^2 dx + \frac{1}{2} = \frac{5}{6}$



28. Find the value of c for which the area of the figure bounded by the curves $y = \sin 2x$, the straight lines $x = \frac{\pi}{6}, x = c$ and the abscissa axis is equal to $\frac{1}{2}$.

Sol. $\left|\int_{\pi/6}^{c} \sin 2x dx\right| = \frac{1}{2}$

on solving $c = -\frac{\pi}{6}$ or $\frac{\pi}{3}$

29. Find the value of ' c ' for which the area of the figure bounded by the curve, $y = 8x^2 - x^5$, the straight lines x = I and x = c and the abscissa axis is equal to $\frac{16}{3}$.

Sol. For $c < 1, \int_c^1 (8x^2 - x^5)dx = \frac{16}{3}$ (given)

$$\Rightarrow \left[\frac{8x^3}{3} - \frac{x^6}{6}\right]_c^1$$

$$\Rightarrow \frac{8}{3} - \frac{1}{6} - \frac{8c^3}{3} + \frac{c^6}{6} = \frac{16}{3}$$

$$\Rightarrow c^3\left[-\frac{8}{3} + \frac{c^3}{6}\right] = \frac{16}{3} - \frac{8}{3} + \frac{1}{6} = \frac{17}{6}$$

$$\Rightarrow -\frac{8}{3}c^3 + \frac{c^6}{6} = \frac{17}{6}$$

$$\Rightarrow c^6 - 16c^3 - 17 = 0$$

On factorization, we get $(c^3 + 1)(c^3 - 17) = 0$

$\Rightarrow c^3 = -1, 17$

$\Rightarrow c = -1, 17^{\frac{1}{3}}$

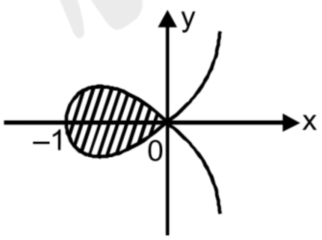
$\Rightarrow$ c = −1 satisfy the above equation.
For c ≥ 1, none of the values of c satisfies the required condition that

$$\int_1^c (8x^2 - x^5)dx = \frac{16}{3}$$

$\therefore c = -1$

30. Compute the area of the loop of the curve

$$y^2 = x^2\left[\frac{(1+x)}{(1-x)}\right]$$



Sol. Area $= 2\left|\int_1^0 \left(x\sqrt{\frac{1+x}{1-x}}\right)dx\right|$

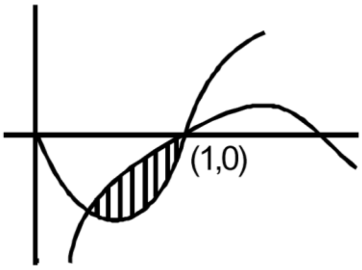
$= 2 - \frac{\pi}{2}$

31. Compute the area of the region bounded by the curves $y = ex.\ln x$ and $y = \frac{\ell nx}{\text{e.x}}$ where $\ell ne = 1$.

Sol. Intersecting points are $\frac{1}{e}$ & 1

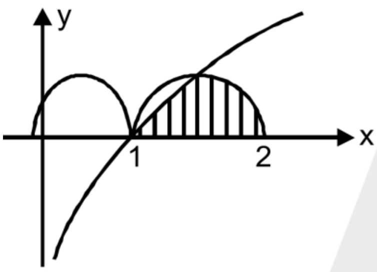
so $A = \int_{1/e}^{1}\left(\frac{\ln x}{ex} - ex\ln x\right)dx$

$= \frac{e^2-5}{4e}$ sq. units



32. Find the area of the region bounded by the curves, $y = \log_e x, y = \sin^4 \pi x$ and $x = 0$

Sol. $y = \log_e\ x, y = \sin^4\ (\pi x)$ and $x = 0$



Hence, the required area

$= \left|\int_0^1\ \log\ xdx\right| + \int_0^1\ \sin^4\ xdx$

$= |x(\log\ x - 1)|_0^1 + \int_0^1\ \left(\frac{1-\cos\ 2\pi x}{2}\right)^2\ dx$

$= 1 + \int_0^1\ \left(\frac{1-\cos\ 2\pi x}{2}\right)^2\ dx$

$= 1 + \frac{1}{4}\int_0^1\ (1 - 2\cos\ (2\pi x) + \cos^2\ (2\pi x))dx$

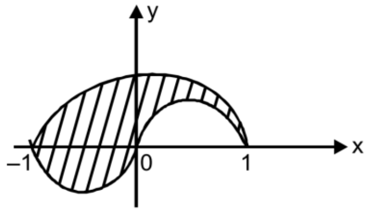
$$= 1 + \frac{1}{4}\int_0^1 \left(1 - 2\cos\ (2\pi x) + \left(\frac{1 + \cos\ (4\pi x)}{2}\right)\right) dx$$

$$= 1 + \frac{1}{4}\left(\frac{3x}{2} - \frac{2\sin\ (2\pi x)}{2\pi} + \frac{\sin\ (4\pi x)}{8\pi}\right)_0^1$$

$= 1 + \frac{1}{4}\left(\frac{3}{2} - 0\right) = 1 + \frac{3}{8} = \frac{11}{8}$

33. Find the area bounded by the curves $y = \sqrt{1 - x^2}$ and $y = x^3 - x$. Also find the ratio in which the y-axis divided this area.

Sol.



The two curves are $y=\sqrt{1-x^2}$
and $y=x^3-x$
The point of intersection are $\mathrm{P}(-1,0); \mathrm{Q}(1,0)$
Consider $\mathrm{y}=\sqrt{1-\mathrm{x}^2}$
On squaring both sides, we get

$\mathrm{x}^2+\mathrm{y}^2=1$

But $y=\sqrt{1-x^2} \geq 0$ by the definition of square root which is a semi-circle with center (0,0) and radius 1 and above X-axis.
Consider $\mathrm{y}=\mathrm{x}^3-\mathrm{x}=\mathrm{x}(\mathrm{x}-1)(\mathrm{x}+1)$
Now for $\mathrm{x} \leq -1, 0 \leq \mathrm{x} \leq 1; \mathrm{y} \leq 0$
and for $-1 \leq \mathrm{x} \leq 0, \mathrm{x} \geq 1; \mathrm{y} \geq 0$
Taking into account the oddness of the function and the intervals of constant sign.
(We can construct its graph by finding the maxima and minima at $x=\pm\frac{1}{\sqrt{3}}$
Thus the required Area $=\mathrm{A}_1+\mathrm{A}_2$
where $A_1=\int_{-1}^{0}\left[\sqrt{1-x^2}-x^3+x\right]dx=\frac{\pi}{4}-\frac{1}{4}$
and $A_2=\int_0^1\left[\sqrt{1-x^2}-x^3+x\right]dx=\frac{\pi}{4}+\frac{1}{4}$
$\Rightarrow$ Required area $=\frac{\pi}{2}$ and required ratio $=\frac{A_1}{A_2}=\frac{\pi-1}{\pi+1}$

34. If the area enclosed by the parabolas $y=a-x^2$ and $y=x^2$ is $18\sqrt{2}$ sq. units. Find the value of 'a'.

Sol. $A=2\int_0^{\sqrt{a/2}}(2x^2-a)dx=18\sqrt{2}$

$\Rightarrow a^{3/2}=9\times 3 \Rightarrow a=9$

35. Find the area of the region enclosed by the curve $y=x^4-2x^2$ and $y=2x^2$.

Sol. Intersecting points $=0, \pm 2$

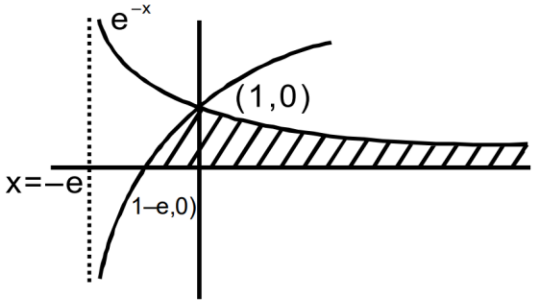
so area $=2\int_0^2 (2x^2-x^4+2x^2)dx$

$=\frac{128}{15}$

36. Find the area enclosed between the curves $y=\log_e(x+e), x=\log_e(1/y)$ and the x-axis.

Sol. $x=\operatorname{loge}(1/y)$

$\Rightarrow y=e^{-x}$



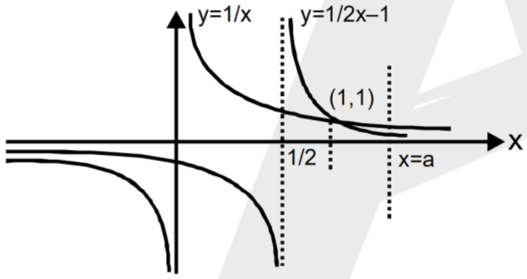
$A=\int_{1-e}^{0}\log(x+c)dx+\int_{0}^{\infty}e^{-x}dx=2$

37. For what value of ' a ' is the area of the figure bounded by the lines, $y=\frac{1}{x}, y=\frac{1}{2x-1}, x=2$ and $x=a$ equal to $\ln\frac{4}{\sqrt{5}}$?

Sol. $A=\left|\int_{1}^{a}\left(\frac{1}{x}-\frac{1}{2x-1}\right)dx\right|$

$=\ln\frac{4}{\sqrt{5}}$

$a=8$ on $\frac{2}{5}(6-\sqrt{2})$



38. For the curve $f(x)=\frac{1}{1+x^2}$, let two points on it are $A(\alpha, f(\alpha)), B\left(-\frac{1}{\alpha}, f\left(-\frac{1}{\alpha}\right)\right)(\alpha>0)$. Find the minimum area bounded by the line segments OA, OB and $f(x)$, where $'O'$ is the origin.

Sol. The shaded area in the graph is the required area that has to be maximized

Point A $\equiv\left(\alpha,\frac{1}{1+\alpha^2}\right)$

Point $B\equiv\left(-\frac{1}{\alpha},\frac{\alpha^2}{1+\alpha^2}\right)$

Thus the required area to be maximised is,

$A=\int_{-1/\alpha}^{\alpha}f(x)dx$-(Area of triangle under segment OB) - (Area of triangle under segment OA)

$\int f(x)dx=\tan^{-1}x$

$$\therefore \mathrm{A}=\tan^{-1}\mathrm{x}|_{-1/\alpha}^{\alpha}-\left(\frac{1}{2}\times\frac{1}{\alpha}\times\frac{\alpha^2}{1+\alpha^2}\right)-\left(\frac{1}{2}\times\alpha\times\frac{1}{1+\alpha^2}\right)$$

$$\therefore \mathrm{A}=\tan^{-1}\alpha+\tan^{-1}\frac{1}{\alpha}-\frac{\alpha}{1+\alpha^2}$$

$$\tan^{-1}\alpha+\tan^{-1}\frac{1}{\alpha}=\frac{\pi}{2}$$

$$\therefore \mathrm{A}=\frac{\pi}{2}-\frac{\alpha}{1+\alpha^2}$$

This is maximum when $\frac{\alpha}{1+\alpha^2}$ is minimum.

$\therefore \frac{d}{d\alpha}\left(\frac{\alpha}{1+\alpha^2}\right) = 0$

$\therefore \frac{1+\alpha^2-\alpha(2\alpha)}{1+\alpha^2} = 0$

$\therefore \alpha = \pm 1$

Maximum value occurs at $\alpha = 1$

Therefore minimum value of bounded area $= \frac{\pi}{2} - \frac{1}{2}$

PREVIOUS YEAR QUESTION

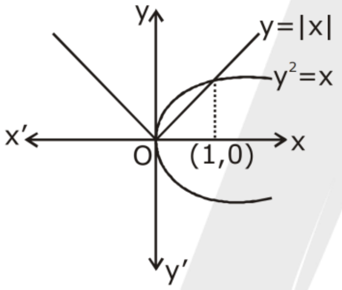
39. The area enclosed between the curves $y^2 = x$ and $y = |x|$ is

(A) $\frac{2}{3}$ (B) 1 (C) $\frac{1}{6}$ (D) $\frac{1}{3}$

Ans. **(C)**

Sol. Required area,

$A = \int_0^1 (\sqrt{x} - x)dx$



$= \left[\frac{2}{3}x^{3/2} - \frac{x^2}{2}\right] = \frac{2}{3} - \frac{1}{2}$

$= \frac{1}{6}$ sq unit

40. The area of the plane region bounded by the curves $x + 2y^2 = 0$ and $x + 3y^2 = 1$ is equal to

(A) $\frac{1}{3}$ (B) $\frac{2}{3}$ (C) $\frac{4}{3}$ (D) $\frac{5}{3}$

Ans. **(C)**

Sol. Given curves are

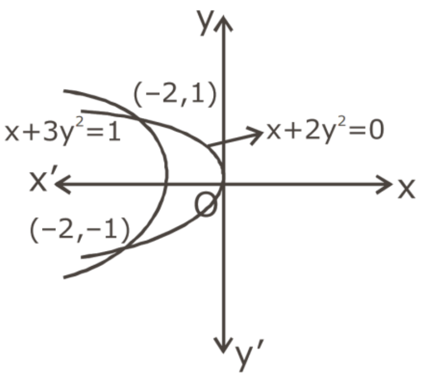
$x + 3y^2 = 1$

and

$x + 2y^2 = 0$

On solving Eqs. (i) and (ii), we get

$y = \pm 1$ and $x = -2$



$\therefore$ Required area $= \left|\int_{-1}^{1} (x_1 - x_2)dy\right|$

$= \left|\int_{-1}^{1} (1 - 3y^2 + 2y^2)dy\right|$

$= \left|\int_{-1}^{1} (1 - y^2)dy\right|$

$= \left|2\int_{0}^{1} (1 - y^2)dy\right|$

$= \left|2\left[y - \frac{y^3}{3}\right]_0^1\right| = \left|2\left(1 - \frac{1}{3}\right)\right|$

$= \frac{4}{3}$ sq unit

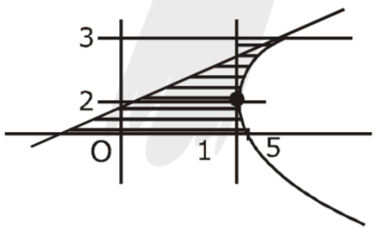
41. The area of the region bounded by the parabola $(y-2)^2 = x - 1$, the tangent to the parabola at the point $(2,3)$ and the x-axis is

(A) 3 (B) 6 (C) 9 (D) 12

Ans. **(C)**

Sol. Eq. of tangent at (2,3)

$\equiv x - 2y + 4 = 0$



Required area

$\int_0^3 [(y-2)^2 + 1 - 2y + 4]dy$

$= \int_0^3 [(y-2)^2 - 2y + 5]dy$

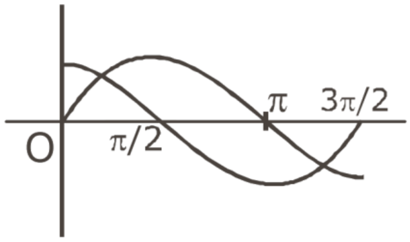
$= 9$ sq. unit.

42. The area bounded by the curves $y = \cos x$ and $y = \sin x$ between the ordinates $x = 0$ and $x = \frac{3\pi}{2}$ is

(A) $4\sqrt{2} - 2$ (B) $4\sqrt{2} + 2$ (C) $4\sqrt{2} - 1$ (D) $4\sqrt{2} + 1$

Ans. (A)

Sol.



Required area

$= \int_0^{\pi/4} (\cos x - \sin x)dx + \int_{\pi/4}^{5\pi/4} (\sin x - \cos x)dx + \int_{5\pi/4}^{3\pi/2} (\cos x - \sin x)dx$

$= (\sin x + \cos x)_0^{\pi/4} + (-\cos x - \sin x)_{\pi/4}^{5\pi/4} + (\sin x + \cos x)_{5\pi/4} 3\pi/2$

$= (4\sqrt{2} - 2)$ sq unit

43. The area of the region enclosed by the curves $y = x, x = e, y = \frac{1}{x}$ and the positive x-axis is:

(A) $\frac{1}{2}$ square units

(B) 1 square unit

(C) $\frac{3}{2}$ square units

(D) $\frac{5}{2}$ square units

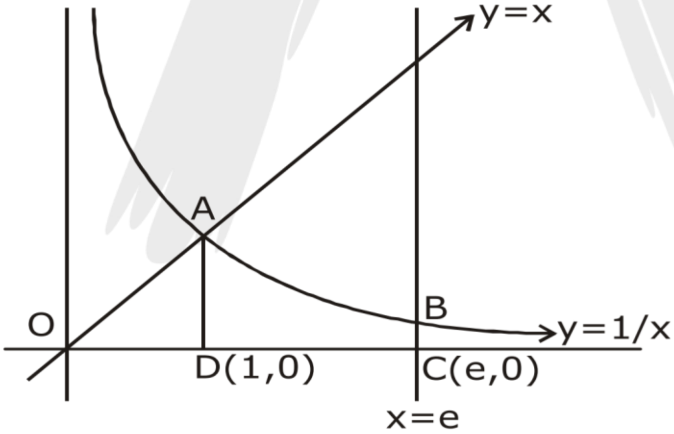
Ans. (C)

Sol. Given: $y = x, x = e$ and $y = \frac{1}{x}, x \leq 0$

Since, $y = x$ and $x \leq 0$

$\Rightarrow \; y \geq 0$

$\therefore$ Area to be calculated in I quadrant shown as



$\therefore$ Area $=$ Area of $\triangle ODA +$ Area of DABCD

$= \frac{1}{2}(1 \times 1) + \int_a^e \frac{1}{x} dx$

$= \frac{1}{2} (\log|x|)_1^e$

$= \frac{1}{2} + \{\log|e| - \log 1\}$

$= \frac{1}{2} + 1 = \frac{3}{2}$ sq unit

44. The area bounded between the parabola $x^2 = \frac{y}{4}$ and $x^2 = 9y$ and the straight line $y = 2$ is:

(A) $\frac{20\sqrt{2}}{3}$ (B) $10\sqrt{2}$ (C) $20\sqrt{2}$ (D) $\frac{10\sqrt{2}}{3}$

Ans. **(A)**

Sol. $x^2 = \frac{y}{4}, x^2 = 9y, y = 2$

Area $= 2\int_0^2 \left|\frac{\sqrt{y}}{2} - 3\sqrt{y}\right| dy$

$= \left|2\left[\frac{1}{2}\frac{y^{3/2}}{3/2} - \frac{3(y)^{3/2}}{3/2}\right]_0^2\right|$

$= \left|2\left[\frac{1}{3}\cdot 2^{3/2} - 2\left(2^{3/2}\right)\right]\right|$

$= \left|2\cdot 2^{3/2}\left[\frac{1}{3} - 2\right]\right|$

$= 2^{5/2}\cdot\frac{5}{3} \Rightarrow \frac{20\sqrt{2}}{3}$

45. The area (in square units) bounded by the curves $y = \sqrt{x}$, $2y - x + 3 = 0$, x-axis, and lying in the first quadrant is:

(A) 18 (B) $\frac{27}{4}$ (C) 9 (D) 36

Ans. **(C)**

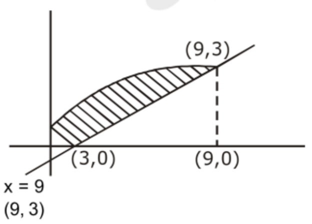
Sol. $y = \sqrt{x}$ $\quad x = 2y + 3$

$y^2 = x$ $\quad y^2 = 2y + 3$

$y^2 = 2y - 3 = 0$

$y = 3, -1$

(reject)



$A = \int_0^9 \sqrt{x} - \frac{1}{2}\times 6\times 3$

$= \frac{2}{3}\left(x^{3/2}\right)_0^9 - 9$

$= 18 - 9 \Rightarrow 9$

46. The area of the region described by $A = \{(x, y): x^2 + y^2 \le 1 \text{ and } y^2 \le 1 - x\}$ is :

(A) $\frac{\pi}{2} + \frac{4}{3}$ (B) $\frac{\pi}{2} - \frac{4}{3}$ (C) $\frac{\pi}{2} - \frac{2}{3}$ (D) $\frac{\pi}{2} + \frac{2}{3}$

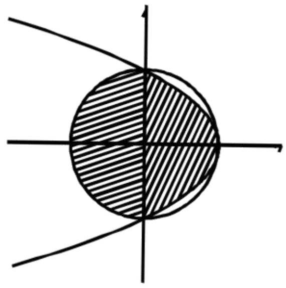
Ans. **(A)**

Sol. $A_1 = 2\left|\int_0^1 \sqrt{1-x}\right| dx$

$A_1 = 2\left|\int_1^0 2t^2 dt\right|$

$A_1 = 4 \cdot \frac{1}{3}$

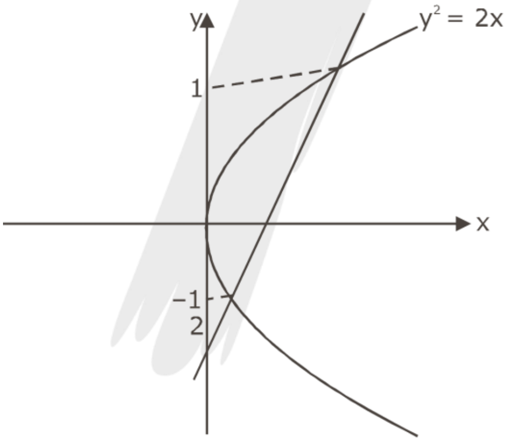
$\boxed{Area = \frac{\pi}{2} + \frac{4}{3}}$



47. The area (in sq. units) of the region described by $\{(x, y): y^2 \le 2x \text{ and } y \ge 4x - 1\}$ is

(A) $\frac{15}{64}$ (B) $\frac{9}{32}$ (C) $\frac{7}{32}$ (D) $\frac{5}{64}$

Ans. **(B)**



Sol.

$y^2 = 2x$

$\frac{y^2}{2} = \frac{y+1}{4}$

$2y^2 - y - 1 = 0$

$2y^2 - 2y + y - 1 = 0$

$(2y + 1)(y - 1)$

$A = \int_{\frac{-1}{2}}^{1} \left(\frac{y+1}{4} - \left(\frac{y^2}{2}\right)\right) d$

$$A = \left(\frac{\frac{y^2}{2}+y}{4}\right)_{-\frac{1}{2}}^{1} - \left(\frac{y^3}{6}\right)_{-\frac{1}{2}}^{1}$$

$$A = \left(\frac{y^2+2y}{8}\right)_{-\frac{1}{2}}^{1} - \left(\frac{y^3}{6}\right)_{-\frac{1}{2}}^{1}$$

$$A = \left(\frac{3}{8} - \frac{\left\{\frac{1}{4}-1\right\}}{8}\right) - \left(\frac{1}{6}+\frac{1}{48}\right)$$

$$A = \left(\frac{3}{8}+\frac{3}{32}\right) - \left(\frac{8+1}{48}\right) = \frac{12+3}{32} - \frac{9}{48}$$

$$A = \frac{15}{32} - \frac{9}{48} = \frac{3}{16}\left(\frac{5}{2}-\frac{3}{3}\right)$$

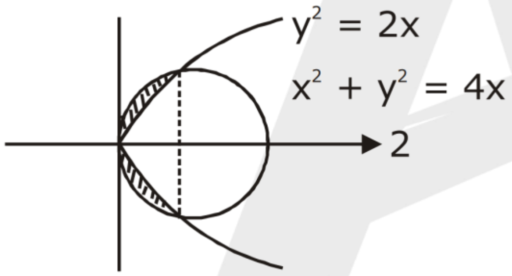
$$= \frac{3}{16} \times \left(\frac{15-6}{6}\right) = \frac{3\times 9}{16\times 6} = \frac{9}{32}$$

48. The area (in sq. units) of the region $\{(x, y): y^2 \geq 2x \text{ and } x^2 + y^2 \leq 4x, x \geq 0, y \geq 0\}$ is :

(A) $\pi - \frac{8}{3}$ (B) $\pi - \frac{4\sqrt{2}}{3}$ (C) $\frac{\pi}{2} - \frac{2\sqrt{2}}{3}$ (D) $\pi - \frac{4}{3}$

Ans. **(A)**

Sol.



$x^2 = 2x$

$x = 0, x = 2$

Area $= \sqrt{2}\int \sqrt{x}dx$

$\sqrt{2}\left\{\frac{x^{3/2}}{3/2}\right\}_0^2$

$= \sqrt{2}\left(2^{3/2}\right)\frac{2}{3}$

$= 8/3$

Area of shaded region

$= \frac{\pi\times 4}{2} - \frac{16}{3}$

$= 2\pi - \frac{16}{3}$

if $n \geq 0, y \geq 0$

$\therefore$ Area $= \pi - 8/3$

49. The area (in sq, units) of the region $\{(x,y): x \geq 0, x+y \leq 3, x^2 \leq 4y \text{ and } \}$ is :

(A) $\frac{59}{12}$ (B) $\frac{3}{2}$ (C) $\frac{7}{3}$ (D) $\frac{5}{2}$

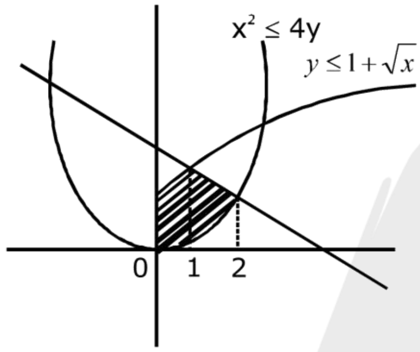
Ans. **(D)**

Sol. $x \geq 0$

$x + y \leq 3$

$x^2 \leq 4y$

$y \leq 1 + \sqrt{x}$



$\Delta = \int_0^1 \left(1 + \sqrt{x} - \frac{x^2}{4}\right) dx + \int_1^2 \left(3 - x - \frac{x^2}{4}\right) dx$

$\Delta = x + \frac{\lambda^{3/2}}{3/2} - \frac{x^3}{12}\Big|_1^2 + 3x - \frac{x^2}{2} - \frac{x^3}{12}\Big|_1^2$

$\Delta = \left(1 + \frac{2}{3} - \frac{1}{12}\right) + \left(6 - 2 - \frac{8}{12}\right) -$

$\left(3 - \frac{1}{2} - \frac{1}{12}\right)$

$\Delta = \frac{5}{3} - \frac{1}{12} + 4 - \frac{8}{12} - 3 + \frac{1}{2} + \frac{1}{12}$

$\Delta = \frac{5}{3} + 1 + \frac{1}{2} - \frac{2}{3}$

$\Delta = 1 + 1 + \frac{1}{2}$

$\Delta = 2 + \frac{1}{2} = \frac{5}{2}$

50. Let $g(x) = \cos x^2, f(x) = \sqrt{x}$, and $\alpha, \beta (\alpha < \beta)$ be the roots of the quadratic equation $18x^2 - 9\pi x + \pi^2 = 0$. Then the area (is sq. units) bounded by the curve $y = (g \circ f)(x)$ and the lines $x = \alpha, x = \beta$ and $y = 0$, is:

(A) $\frac{1}{2}(\sqrt{2} - 1)$ (B) $\frac{1}{2}(\sqrt{3} - 1)$ (C) $\frac{1}{2}(\sqrt{3} + 1)$ (D) $\frac{1}{2}(\sqrt{3} - \sqrt{2})$

Ans. **(B)**

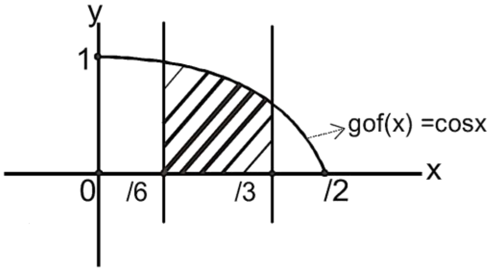
Sol. $g(x) = \cos(x^2), f(x) = \sqrt{x}$

$18x^2 - 9\pi x + \pi^2 = 0$ (α and β are roots of equ.)

$\Rightarrow (6x - \pi)(3x - \pi) = 0$

$\alpha = \frac{\pi}{6}$ and $\beta = \frac{\pi}{3}$

$y = gof(x) = g(f(x)) = g(\sqrt{x}) = \cos(x)$



$A = \int_{\pi/6}^{\pi/3} \cos x dx = (\sin x)_{\pi/6}^{\pi/3} = \frac{\sqrt{3}}{2} - \frac{1}{2} = \frac{\sqrt{3}-1}{2}$

∴ correct option is (B).