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$$\sin 1^\circ \sin 3^\circ \sin 5^\circ \sin 7^\circ \cdots \sin 85^\circ \sin 87^\circ \sin 89^\circ$$

$$= \frac{\left(\sin 1^\circ \sin 3^\circ \sin 5^\circ \cdots \sin 87^\circ \sin 89^\circ\right)\left(\sin 2^\circ \sin 4^\circ \cdots \sin 88^\circ\right)}{\sin 2^\circ \sin 4^\circ \cdots \sin 88^\circ}$$

$\boxed{\sin 44^\circ \sin 46^\circ}$ $\cos 2^\circ$ $\cos 1^\circ$

$$= \frac{\sin 1^\circ \sin 2^\circ \sin 3^\circ \sin 4 \cdots \sin 88^\circ \sin 89^\circ}{\sin 2^\circ \sin 4^\circ \sin 6^\circ \cdots \sin 88^\circ} = \frac{\sin 2^\circ \sin 4^\circ \cdots \sin 88^\circ}{2^{44}\sqrt{2}\,\sin 2^\circ \sin 4^\circ \cdots \sin 88^\circ}$$

$$= \frac{1}{2^{44+1/2}}$$

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5. $\sqrt{p^2+q^2}\left(\frac{p}{\sqrt{p^2+q^2}}\cos 2\beta - \frac{q}{\sqrt{p^2+q^2}}\sin 2\beta\right)$ over $2\sin 2\beta\cos 2\beta$ $= \dfrac{\sqrt{p^2+q^2}\left(\sin\alpha\cos 2\beta - \cos\alpha\sin 2\beta\right)}{\cancel{\sin 4\beta}}$

$\cancel{\phi}\sin(\phi-\theta) = \cancel{\phi}$

$\phi - \theta = \frac{\pi}{2} + \boxed{2n\pi}, \; n \in I$

$ac = 3\cos\theta\, 2\cos\phi$

$= 6\cos\theta\cos\left(\frac{\pi}{2}+\theta\right) = -6\cos\theta\sin\theta$

$= -3\sin 2\theta$.

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6. $\dfrac{\tan(x+100)}{\tan x} = \tan(x+50)\tan(x-50)$

$= \dfrac{\sin(x+50)\sin(x-50)}{\cos(x+50)\cos(x-50)}$

$\sin(4x+100) = -2\sin 150^\circ \cos 50^\circ = -\cos 50^\circ$

$= \sin(270-50)$

$\dfrac{\tan(x+100)-\tan x}{\tan(x+100)+\tan x} = \dfrac{\sin(x+50)\sin(x-50)-\cos(x+50)\cos(x-50)}{\sin(x+50)\sin(x-50)+\cos(x+50)\cos(x-50)}$

$\dfrac{\sin 100}{\sin(2x+100)} = -\dfrac{\cos 2x}{\cos 100} \Rightarrow -\sin 200 = \sin(4x+100)+\sin 100$

$$\frac{2\cos\left(\frac{3\theta+3\phi}{2}\right)\cancel{\cos\frac{3\theta-3\phi}{2}}}{\cancel{2\left(2\cos^2\frac{\theta-\phi}{2}-1\right)-1=\left(4\cos^2\left(\frac{\theta-\phi}{2}\right)-3\right)}}\times\frac{\cos\left(\frac{\theta-\phi}{2}\right)}{\cancel{\cos\left(\frac{\theta-\phi}{2}\right)}}$$

$$= 2\cos\frac{3\theta+3\phi}{2}\cos\frac{\theta-\phi}{2}$$

$$= \cos\left(\underbrace{2\theta+\phi}_{(\theta+\phi)+\theta}\right)+\cos\left(\underbrace{\theta+2\phi}_{(\theta+\phi)+\phi}\right)$$

Nishant Jindal

10.

$(1+\sin t)(1+\cos t) = \frac{5}{4}$ ①

$(1-\sin t)(1-\cos t) = S$ ②

$S < 4$

$\frac{5}{4} S = \cos^2 t \sin^2 t$

① + ② $\Rightarrow 2 + \underline{2\sin t \cos t} = \frac{5}{4} + S \Rightarrow \sin t \cos t = \frac{1}{2}\left(S - \frac{3}{4}\right)$

$\frac{13}{4} - \sqrt{10}$

$\cancel{\frac{5}{4}} S = \cancel{\frac{1}{4}}\left(S - \frac{3}{4}\right)^2$

$S = ?$

$\frac{13}{4} \pm \sqrt{10}$

$\boxed{S = \frac{13}{4} + \sqrt{10} \text{ reject}}$

Nishant Jindal

$\sin 2t \quad , \quad \sin t + \cos t.$

$2x^2 + 4x - 3 = 0$

$x = ? \quad x \in [-\sqrt{2}, \sqrt{2}]$

$\sin 2t \quad \sin t - \cos t$

$1 + \sin 2t = (\sin t + \cos t)^2$

$= \frac{1}{2} - x + \frac{3}{4} - x = \frac{5}{4} - 2x$

$1 + \sin t + \cos t + \frac{\sin 2t}{2} = \frac{5}{4} = 1 + (\sin t + \cos t) + \frac{(\sin t + \cos t)^2 - 1}{2}$

$\frac{x^2}{2} + x = \frac{3}{4} \Leftarrow \frac{x^2}{2} + \frac{1}{2} + x = \frac{5}{4}$

$= \frac{1}{2} - x + \left(\frac{x^2}{2}\right) = 1 - (\sin t + \cos t) + \frac{\sin 2t}{2} = 1 - (\sin t + \cos t) + \frac{(\sin t + \cos t)^2 - 1}{2}$

Nishant Jindal

1. P.T. $2^{\sqrt{\log_2 3}} = 3^{\sqrt{\log_3 2}}$

$$2^{\sqrt{\log_2 3}} = \left(2^{\log_2 3}\right)^{\frac{1}{\sqrt{\log_2 3}}} = (3)^{\sqrt{\frac{1}{\log_2 3}}}$$

$$2^{\log_2 3} = 3$$

$$= 3^{\sqrt{\log_3 2}}$$

$$\sqrt{x} = \frac{x}{\sqrt{x}}$$

$$a^{mn} = (a^m)^n$$

Nishant Jindal

Q. If $\boxed{b=\sqrt{ac}}$, then P.T.

$$\frac{\log_a N - \log_b N}{\log_b N - \log_c N} = \frac{\log_a N}{\log_c N}$$

$b^2 = ac$

$\Rightarrow \frac{b}{a} = \frac{c}{b}$

$\log_N \frac{b}{a} = \log_N \frac{c}{b}$

$$\frac{\frac{1}{\log_N a} - \frac{1}{\log_N b}}{\frac{1}{\log_N b} - \frac{1}{\log_N c}} = \frac{\left(\frac{\log_N b - \log_N a}{\log_N a \cancel{\log_N b}}\right)}{\left(\frac{\log_N c - \log_N b}{\cancel{\log_N b} \log_N c}\right)} = \frac{\cancel{\log_N \left(\frac{b}{a}\right)} \log_N c}{\cancel{\log_N \left(\frac{c}{b}\right)} \log_N a}$$

$$= \frac{\log_a N}{\log_c N}$$

Nishant Jindal

3: Which is bigger $\log_3 5$ or $\log_{17} 25$

$\log_3 5 = x \Rightarrow 3^x = 5 \Rightarrow (3^x)^2 = 25 = 9^x$

$\log_{17} 25 = y \Rightarrow 17^y = 25$

$9^x = 17^y$

$\boxed{x > y}$

$\boxed{\log_3 5 > \log_{17} 25}$

P.T. $\log_2 7$ is irrational.

$\log_2 7 = \frac{p}{q}$ $\quad p, q \in I, q \neq 0$

$2^{p/q} = 7 \Rightarrow 2^p = 7^q$

Contradiction $\Rightarrow$

$3^p = 7^q$

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# Logarithmic equations

Check.

1. $x^2 + 7^{\log_7 x} - 2 = 0$

$x^2 + x - 2 = 0$

$(x+2)(x-1) = 0$

$x = -2, 1$

$x = -2 \rightarrow$ reject

$\boxed{x = 1}$

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2. $\log_{(x-1)} 4^{=2^2} = 1 + \log_2 (x-1)$

$\log_2 (x-1) = t$

$2\log_{(x-1)} 2 = 1 + \log_2 (x-1)$

$\log_2 (x^2) = 2\log_2 x$

$\frac{2}{t} = 1 + t$

$t^2 + t - 2 = 0$

$\Rightarrow (t+2)(t-1) = 0$

$\log_2 (x-1) = t = -2, 1$

$x - 1 = \frac{1}{4}, 2 \Rightarrow \boxed{x = \frac{5}{4}, 3}$

Careful.

$\log_a b^n = n \log_a b$

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3. $5^{1+(\log_4 x)} + 5^{(\log_{\frac{1}{4}} x)-1} = \frac{26}{5}$

4. $\log_5\left(5^{\frac{1}{x}}+125\right) = \log_5(6) + 1 + \frac{1}{2x}$

5. $\log_4\left(2\log_3\left(1+\log_2\left(1+3\log_2 x\right)\right)\right) = \frac{1}{2}$

$\boxed{\text{Ex-II} \quad \text{Q } 11-15}$