Nishant Jindal

Ex 2 [Cont'd]

Q1 $f(x) = \dfrac{3x^2 + ax + a + 3}{x^2 + x - 2}$ [Copy]

Q2 $f(x) = \begin{cases} |ax+3| & x \le -1 \\ |3x+a| & -1 < x \le 0 \\ \dfrac{b\sin 2x}{x} - 2b & 0 < x < \pi \\ a^2 x - 3 & x \ge \pi \end{cases}$

$x = -1$

$|-a+3| = |a-3|$

$x = 0$

$|a| = \lim_{x \to 0} b \left(\dfrac{\sin 2x}{2x}\right) 2 - 2b$

$|a| = 2b - 2b$

$|a| = 0$

$a = 0$

$x = \pi$

$\dfrac{b\sin 2\pi}{\pi} - 2b = a^2\pi - 3$

$= (-1)^2 - 3$

$-2b = -2$

$\boxed{b = 1}$

Q3. Copy

Q4

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$(3-2)(3+2)=5$

$(\cancel{x-3})(x-2)(x+2)$

$$h(x)=\begin{cases}\lim\limits_{x\to 3}\dfrac{x^3-3x^2-4x+12}{\cancel{x-3}} & x\neq 3.\\ K & x=3.\end{cases}$$

① $f(x)=x^3-3x^2-4x+12$ Zeroes

$= x^2(x-3)-4(x-3)$

$=(x-3)(x-2)(x+2)$

Zeroes $= 3, 2, -2$

(2) K h is cont at $x=3$

$K=5$

Q5

$$f(x)=\begin{cases}\lim\limits_{x\to\frac{1}{2}}\dfrac{1-\sin\pi x}{1+\cos 2\pi x}\ \frac{0}{0}\ \text{DL} & x<\frac{1}{2}\\ p & x=\frac{1}{2} \quad p=\phi\\ \dfrac{\sqrt{2x-1}}{\sqrt{4+\sqrt{2x-1}}-2} & x>\frac{1}{2}\end{cases}$$

$$\frac{\sqrt{\cancel{2x-1}}\times(\sqrt{4+\sqrt{2x-1}}+2)}{(\cancel{4}+\cancel{\sqrt{2x-1}}-\cancel{4})}$$

④

$$\lim_{x\to\frac{1}{2}}\frac{+\pi\cos\pi x}{+2\pi\sin 2\pi x}=\lim_{x\to\frac{1}{2}}\frac{\cos\pi x}{2\sin 2\pi x}=\frac{0}{0}\ \text{DL}$$

$$\lim_{x\to\frac{1}{2}}\frac{-\pi\sin\pi x}{4\pi\cdot\cos 2\pi x}=\frac{-\cancel{\pi}}{-4\cancel{\pi}}=\frac{1}{4}$$

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$g(x) = \sqrt{6-2x}$ $h(x) = 2x^2 - 3x + a$

$h(g(2)) = 2(\sqrt{6-2x})^2 - 3\sqrt{6-2x} + a$

$h = 2\times 2 - 3\sqrt{2} + a.$

(b) $f(x) = \begin{cases} g(x) & x \le 1 \\ h(x) & x > 1 \end{cases}$

$f(x) = \begin{cases} \sqrt{6-2x} & x \le 1 \\ 2x^2 - 3x + a & x > 1 \end{cases}$

$\sqrt{6-2} = \sqrt{4}$
2

$2 - 3 + a$
$a - 1 = 2$
$a = 3$

Q 7 (Copy)

Q 8 Copy

9)

$\boxed{10)}$

J.M
Q 2

$f(x) = \begin{cases} \lim\limits_{x\to 0^-} \dfrac{\sin(a+1)x + \sin x}{x} & x < 0 \\ c & x = 0 \\ \dfrac{x^{1/2}(1+bx)^{1/2} - x^{1/2}}{bx^{3/2}} & x > 0 \quad b \in R - \{0\} \end{cases}$

$\lim\limits_{x\to 0^-} \dfrac{\sin(a+1)x}{x} + \dfrac{\sin x}{x}$

$\dfrac{(a+1)x}{x} + \dfrac{x}{x} 1 = a+2$

$\dfrac{x^{1/2}(\cancel{1} + \frac{bx}{2} - \cancel{1})}{bx\,x} = \dfrac{1}{2}$

$= \boxed{c = \frac{1}{2}}$

$a + 2 = \frac{1}{2}$

$\boxed{a = -\frac{3}{2}}$

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JM.

$$f(x) = \lim_{x\to 0} \frac{1}{x} - \frac{2}{e^{2x}-1}$$

$$= \lim_{x\to 0} \frac{e^{2x}-1-2x}{(x)(e^{2x}-1)}$$

$$= \lim_{x\to 0} \frac{\left(\cancel{1}+\cancel{\frac{(2x)}{\lfloor 1}}+\frac{(2x)^2}{\lfloor 2}\right)-\cancel{1}-\cancel{2x}}{(x)\left(1+\frac{(2x)}{\lfloor 1}-1\right)}$$

$$= \lim_{x\to 0} \frac{\cancel{2x^2}}{\cancel{2x^2}} = 1$$

$$f(x) = \begin{cases} 5 & x \le 1 \\ a+bx & 1 < x < 3 \\ b+5x & 3 \le x < 5 \\ 30 & x \ge 5 \end{cases}$$

$x=1$

$a+b=5$

$3b+a = b+15$

$2b+a = 15$

$b+a = 5$

$b = 10, a = -5$

Q3 Drop करें Coming time

$f(x) \to f\left(\frac{9}{2}\right) = \frac{2}{9}$

$$\lim_{x\to 0} f\left(\frac{1-\cos 3x}{x^2}\right)$$

$$f\left(\lim_{x\to 0} \frac{1-\cos 3x}{x^2}\right) = f\left(\frac{3^2}{2}\right) = f\left(\frac{9}{2}\right)$$

$= \frac{2}{9}$

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Differentiability

Q Check diff^y of $f(x)=\sin|x|^3$ at $x=0$

$$LHD=f'(0^-)=\lim_{h\to 0}\frac{f(0-h)-f(0)}{-h}=\lim_{h\to 0}\frac{\sin|-h|^3-\sin|0|^3}{-h}$$

$$=\lim_{h\to 0}\frac{\sin h^3-0}{-h}=\frac{h^{\cancel{3}2}}{-\cancel{h}}=0$$

$$RHD=f'(0^+)=\lim_{h\to 0}\frac{f(0+h)-f(0)}{h}=\lim_{h\to 0}\frac{\sin|h|^3-\underbrace{\sin|0|^3}_{=0}}{h}$$

$$=\lim_{h\to 0}\frac{\sin h^3-0}{h}=\frac{h^{\cancel{3}2}}{\cancel{h}}=0$$

$LHD=RHD \Rightarrow$ fxn is diff

Q Check diff^y of

$$f(x)=\begin{cases}\dfrac{\sin x^2}{x} & x\neq 0\\ 0 & x=0\end{cases}\quad \text{at } x=0$$

$$LHD=f'(0^-)=\lim_{h\to 0}\frac{f(0-h)-f(0)}{-h}$$

$$=\lim_{h\to 0}\frac{\frac{\sin(-h)^2}{-h}-0}{-h}=\lim_{h\to 0}\frac{\sin h^2}{h^2}=1$$

$$RHD=f'(0^+)=\lim_{h\to 0}\frac{f(0+h)-f(0)}{h}$$

$$=\lim_{h\to 0}\frac{\frac{\sin h^2}{h}-0}{h}=\lim_{h\to 0}\frac{\sin h^2}{h^2}=\frac{\cancel{h^2}}{\cancel{h^2}}=1$$

$LHD=RHD$

$\Rightarrow$ fxn is Diff at $x=0$

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Q $f(x) = |x-a| \times \phi(x)$ & $\boxed{\phi(x) = \text{Const}^s \text{ fxn}}$ find $f'(a^+) = ?$

$$f'(a^+) = RHD = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$$

$$= \lim_{h\to 0} \frac{|a+h-a| \cdot \phi(a+h) - \cancel{|a-a| \cdot \phi(a)}^{=0}}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{h} \cdot \phi(a+h)}{\cancel{h}} = \phi(a) = f'(a^+)$$

Adv

Q $f(x) = [x] \sin \pi x$ find LHD at $x = K$; $K = I = n$

$$LHD = f'(K^-) = \lim_{h\to 0} \frac{f(K-h)-f(K)}{-h} = \lim_{h\to 0} \frac{[K-h] \sin \pi(K-h) - \cancel{[K] \sin K\pi}^{0}}{-h}$$

$[K-h]$

K-1 K K+1

$\sin n\pi = 0$

$$= (K-1) \lim_{h\to 0} \frac{\sin(K\pi - \pi h)}{-h} = (K-1) \lim_{h\to 0} \frac{\cancel{\sin K\pi \cdot \cos \pi h}^{0} - \cos K\pi \cdot \sin \pi h}{-h}$$

$$(K-1) \lim_{h\to 0} \frac{\overset{(-1)^K}{\cos K\pi} \sin \pi h}{h}$$

$$(K-1)(-1)^K \cdot \lim_{h\to 0} \frac{\sin \pi h}{h}$$

$$(-1)^K (K-1) \lim_{h\to 0} \frac{\pi K}{K}$$

$$(-1)^K (K-1) \cdot \pi$$

$\cos \pi = -1$
$\cos 2\pi = +1$
$\cos 3\pi = -1$
$\cos 4\pi = +1$
$\cos 5\pi = -1$
$\cos n\pi = (-1)^n$

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Q $f(x) = [x]\tan \pi x$ find RHD at $x = K$, $K \in I$

$$\text{RHD} = f'(K^+) = \lim_{h \to 0} \frac{f(K+h) - f(K)}{h}$$

$$= \lim_{h \to 0} \frac{[K+h]\tan \pi (K+h) - \cancel{[K]\tan K\pi}}{h}$$

$\tan n\pi = 0$ $[K+h]$ Int. आये

$$= K \lim_{h \to 0} \frac{\tan(K\pi + \pi h)}{h}$$

$$= K \lim_{h \to 0} \frac{\frac{\cancel{\tan K\pi} + \tan \pi h}{1 - \cancel{\tan K\pi \tan \pi h}}}{h}$$

$$= K \lim_{h \to 0} \frac{\frac{0 + \tan \pi h}{1}}{h} = K \lim_{h \to 0} \frac{\tan \pi h}{h} = K \cdot \lim_{h \to 0} \frac{\pi \cancel{h}}{\cancel{h}}$$

$$= K\pi$$

Q $f(x)$ is a Real Valued fxn such that $\boxed{|f(x) - f(y)| \le (x-y)^2}$ & $f(0) = 0$ find $f\left(\frac{22}{7}\right) = ?$

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① Diff Coff at $x=a$ is Rep by $f'(a)$

(2) $f'(a)=?$ (3 formulae)

① $f'(a)=$ RHD

$$f'(a^+)=\lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$$

(2) $f'(a)=$ LHD

$$f'(a)=\lim_{h\to 0}\frac{f(a-h)-f(a)}{-h}$$

$$\boxed{(3)\quad f'(a)=\lim_{x\to a}\frac{f(x)-f(a)}{x-a}}$$

Kranti Kari formula

$$\boxed{f\left(\tfrac{22}{7}\right)=0} \Leftarrow f(x)=0 \Leftarrow \underline{f(x)=\text{Const}} \Leftarrow f'(x)=0 \Leftarrow |f'(x)|=0$$

as $f(0)=0$

Q f(x) is a Real Valued fxn Such that $\boxed{|f(x)-f(y)|\le(x-y)^2}$ & $\underline{f(0)=0}$ find $f\left(\frac{22}{7}\right)=?$

$(\ )^2=|\ |^2$

$$|f(x)-f(y)|\le|x-y|^2$$

$$\frac{|f(x)-f(y)|}{|x-y|}\le|x-y|$$

$$\left|\frac{f(x)-f(y)}{(x-y)}\right|\le|x-y|$$

$$\Rightarrow \left|\lim_{y\to x}\frac{f(y)-f(x)}{y-x}\right|\le\left|\lim_{y\to x}y-x\right|$$

$$|f'(x)|\le|0|$$

$$\Rightarrow |f'(x)|\le 0$$

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Q. $f(x) = \begin{cases} \log_a\left(a\left|[x]+[-x]\right|\right)^{x}\left(\dfrac{a^{\left(\frac{2}{\frac{[x]+[-x]}{|x|}}\right)-5}}{3+a^{\frac{1}{|x|}}}\right) & x>0 \\ & x<0 \end{cases}$

$|x| \neq 0$

$a>1$

$x=0$

fxn is diff'ble & Cont's

Check Cont'y & diff'y at $x=0$

$f(x) = \begin{cases} \log_a(a|-1|)^{x}\left(\dfrac{a^{\frac{2}{\left(\frac{-1}{x}\right)}-5}}{3+a^{\frac{1}{x}}}\right) & x>0 \\ \log_a(a|-1|)^{x}\left(\dfrac{a^{\frac{2}{\left(\frac{+1}{+x}\right)}-5}}{3+a^{\frac{1}{-x}}}\right) & x<0 \\ & x=0 \end{cases}$

$f(x) = \begin{cases} x\cancel{\log_a a}\left(\dfrac{a^{-2x-5}}{3+a^{1/x}}\right) & x>0 \\ x\cancel{\log_a a}\left(\dfrac{a^{2x-5}}{3+a^{-1/x}}\right) & x<0 \\ 0 & x=0 \end{cases} \Rightarrow f(x) = \begin{cases} \dfrac{x\cdot(a^{-2x-5})}{3+a^{1/x}} & x>0 \\ \dfrac{(x)(a^{2x-5})}{3+a^{-1/x}} & x<0 \\ 0 & x=0 \end{cases}$

$f'(0^+) = \lim\limits_{h\to 0} \dfrac{f(0+h)-f(0)}{h}$

$= \lim\limits_{h\to 0} \dfrac{\dfrac{\cancel{h}(a^{-2h-5})}{3+a^{1/h}} - 0}{\cancel{h}} = \dfrac{a^{-5}}{3+(>1)^{\infty}}$

$= 0$

$f'(0^-) = \lim\limits_{h\to 0} \dfrac{f(0-h)-f(0)}{-h}$

$= \lim\limits_{h\to 0} \dfrac{\dfrac{(\cancel{-h})(a^{-2h-5})}{3+a^{\frac{1}{h}}} - 0}{(\cancel{-h})} =$

$= \dfrac{a^{-5}}{3+a^{\infty}} = 0$

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Q Check diff of $f(x) = \begin{cases} x + \{x\} + x \cdot \sin\{x\} & \boxed{x \neq 0} \\ 0 & x = 0 \end{cases}$ at $x = 0$?

$$f'(0^+) = \lim_{h\to 0} \frac{f(0+h) - f(0)}{h} = \lim_{h\to 0} \frac{h + \{h\} + h \cdot \sin\{h\} - 0}{h} = \lim_{h\to 0} \frac{h + h + h \cdot \sin h}{h}$$

$$= \lim_{h\to 0} \frac{2h + h\sin h}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{h}(2 + \sin h)}{\cancel{h}} = 2 + \sin 0$$

$$= 2$$

LHD $$f'(0^-) = \lim_{h\to 0} \frac{f(0-h) - f(0)}{-h}$$

$$= \lim_{h\to 0} \frac{-h + \{-h\} + (-h)\sin\{-h\} - 0}{-h}$$

Kalega Nhi

$$= \lim_{h\to 0} \frac{-h + 1 - h + -h\sin(1-h)}{-h} = \lim_{h\to 0} \underbrace{\left(\frac{1-2h}{-h}\right)}_{\infty} + \frac{-\cancel{h}\sin(1-h)}{-\cancel{h}} = \text{Not a finite Quantity}$$

fxn is not diff^ble

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Q $f(x) = \begin{cases} x \cdot \dfrac{\ln(1+x)+\ln(1-x)}{\sec x - \cos x} & x \in (-1, 0) \\ (a^2-3a+1)x + x^2 & x \in [0, \infty) \end{cases}$

diff$^{b/e}$ at $x=0$ then $a_1^2 + a_2^2 = ?$

$f(x) = \begin{cases} x \cdot \dfrac{\ln(1-x^2) \cdot \cos x}{1-\cos^2 x} & -1 < x < 0 \\ (a^2-3a+1)x + x^2 & \underline{0 \le x} < \infty \end{cases}$

$f'(0^-) = \lim_{h \to 0} \dfrac{f(0-h) - f(0)}{-h} = \lim_{h \to 0} \dfrac{\dfrac{(\cancel{-h}) \ln(1-(-h)^2) \cdot \cos(-h)}{\sin^2(-h)} - ((a^2-\cancel{3a+1})^{7} \cdot 0 + 0^2)}{\cancel{-h}} = \dfrac{\ln(1-h^2)}{-(-h^2)} = -1$

$f'(0^+) = \lim_{h \to 0} \dfrac{f(0+h) - f(0)}{h} = \lim_{h \to 0} \dfrac{(a^2-3a+1)h + h^2 - ((a^2-\cancel{3a+1}) \cdot 0 + 0^2)}{h} = \lim_{h \to 0} \dfrac{\cancel{h}(a^2-3a+1+h)}{\cancel{h}}$

$= a^2 - 3a + 1$

as f is diff $\Rightarrow f'(0^+) = f'(0^-)$

$\Rightarrow a^2 - 3a + 1 = -1$

$a^2 - 3a + 2 = 0 \Rightarrow a = 1, 2$

$a_1^2 + a_2^2 = 1^2 + 2^2 = 5$

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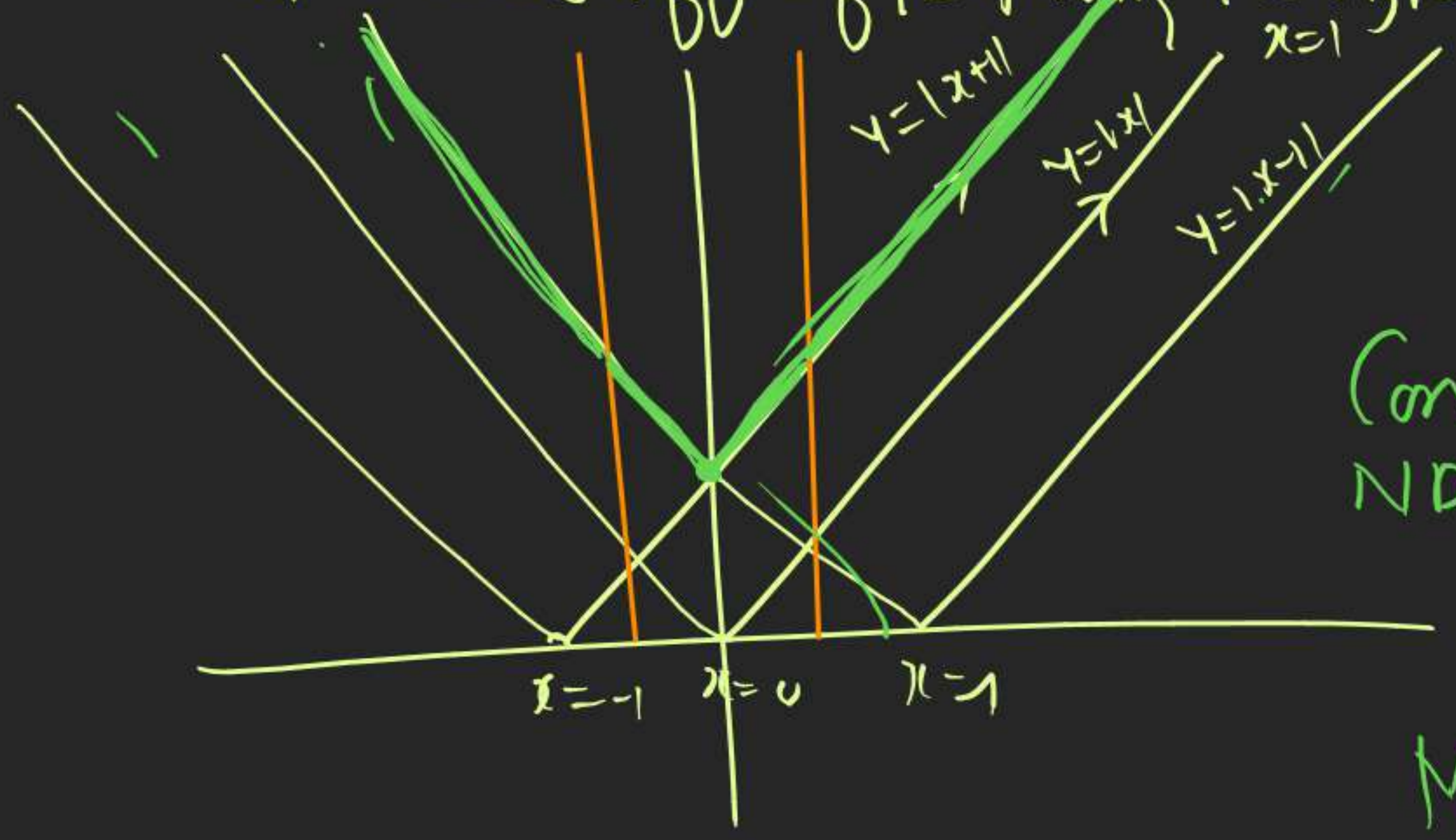
Q Check diff^y of $y = \text{Max}\{|x-1|, |x|, |x+1|\}$



Cont^s

N.D. at $x=0$

Max$\{|x|-$

Σx

20, 18, 11, 12, 15, 17, 6, 7, 8, 9, 1