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DIMENSION ANALYSIS

Fundamental quantities

Mass → M

length → L

Time → T

Temperature → K

Current → A

No of moles → mol

lumenious Intensity → Cd

Derived quantities

Which are derived from fundamental quantities.

Dimensionless

↳ All Numerical values are dimensionless.

1, 2, 3, θ (radian)

↳ log, expontial & trignomentric functions are always dimensionless.

$$y \to \sin\theta$$

$$y \to e^x$$

$$y \to \log x$$

$$y \to [M^0 L^0 T^0]$$

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Prinapal of homogeneity

$$A = B \pm C$$

$$[A] = [B] = [C]$$

$\Rightarrow$ Quantities having same dimension can be added or Subtracted & resultant quantity is also of same dimension.

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Application

① To derive any formula.

$$F \propto m^a A^b.$$

$$F = \underline{K}\,\underline{m}^a\,\underline{A}^b$$

K → can be found experimentally.

$$[MLT^{-2}] = [M]^a [LT^{-2}]^b$$

$$MLT^{-2} = M^a L^b T^{-2b}$$

$a = 1, \; b = 1$

② To Check any expression is dimensionally correct or not.

L.H.S (dimension) = R.H.S (Dimension)

$$v = \underline{u} + at$$

$$\text{L.H.S} \rightarrow [LT^{-1}]$$

$$\text{R.H.S} = u + at$$

$$[LT^{-1}] + [LT^{-2} \cdot T]$$

L.H.S = R.H.S

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III Conversion of Unit of any physical quantity

$$[\text{Old Unit}] = [\text{New Unit}]$$

Ex:- Unit of mass taken as 5 kg, unit of length 20 m and Unit of time as 10 sec.
Convert 1 J of energy according to new unit

$$W = E = F \times L$$
$$= MLT^{-2}(L)$$
$$= ML^2T^{-2}$$

$$E' = E\left[\frac{M_1}{M_2}\right]^1\left[\frac{L_1}{L_2}\right]^2\left[\frac{T_1}{T_2}\right]^{-2}$$

M_2, L_2, T_2 New Unit

$$= 1J\left[\frac{1}{5}\right]\left[\frac{1}{20}\right]^2\left[\frac{1}{10}\right]^{-2}$$

$$E' = \frac{1}{5} \times \frac{1}{4\times\cancel{10^2}} \times \cancel{10^2}$$

$$E' = \left(\frac{1}{20}\right)$$

New unit

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DIMENSION ANALYSIS

Q.1 Expression for time in terms of G (universal gravitational constant), h (Planck constant) and c (speed of light) is proportional to **[JEE (Main)-2019]**

(A) $\sqrt{\frac{Gh}{c^5}}$ (B) $\sqrt{\frac{c^3}{Gh}}$ (C) $\sqrt{\frac{Gh}{c^3}}$ (D) $\sqrt{\frac{hc^5}{G}}$

$t \propto G^x h^y c^z$

$F = \frac{Gm_1m_2}{r^2}$

$G = \frac{Fr^2}{m_1m_2}$

$= \frac{MLT^{-2} \cdot L^2}{M^2}$

$= M^{-1}L^3T^{-2}$

$E = h\nu$

$h = \frac{E}{\nu}$

$[h] = \frac{ML^2T^{-2}}{T^{-1}}$

$[h] = ML^2T^{-1}$

$c = LT^{-1}$

$[M^0L^0T] = [M^{-1}L^3T^{-2}]^x [ML^2T^{-1}]^y [LT^{-1}]^z$

$M^0L^0T = M^{-x+y} L^{3x+2y+z} T^{-2x-y-z}$

$-x+y=0$ — (1)

$3x+2y+z=0$ — (2)

$-2x-y-z=0$ — (3)

DIMENSION ANALYSIS

Q.2 The force of interaction between two atoms is given by $F = \alpha\beta\exp\left(-\frac{x^2}{\alpha kT}\right)$; where x is the distance, k is the Boltzmann constant and T is temperature and α and β are two constants. The dimension of β is **[JEE (Main)-2019]**

(A) $M^0L^2T^{-4}$ (B) M^2LT^{-4} (C) MLT^{-2} (D) $M^2L^2T^{-2}$

$$F = \alpha\beta\, e^{\left(\frac{-x^2}{\alpha KT}\right)}$$

Dimension Same

Dimensionless

Boltzmann Constant $k = \frac{\text{Energy}}{\text{Temp.}} = \frac{ML^2T^{-2}}{K}$

$= [ML^2T^{-2}K^{-1}]$

$[F] = [\alpha\beta]$

$MLT^{-2} = [M^{-1}T^2][\beta]$

$[\beta] = \underline{M^2LT^{-4}}$

$[\alpha KT] = [x^2]$

$\alpha\,[ML^2T^{-2}K^{-1}\cdot K] = L^2$

$\alpha = \underline{M^{-1}T^2}$

DIMENSION ANALYSIS

Q.3 If speed (V), acceleration (A) and force (F) are considered as fundamental units, the dimension of Young's modulus will be **[JEE (Main)-2019]**

(A) $V^{-2}A^{2}F^{-2}$ (B) $V^{-2}A^{2}F^{2}$

(C) $V^{-4}A^{2}F$ (D) $V^{-4}A^{-2}F$

$Y = \frac{\text{Stress}}{\text{Strain}}$, Stress $\rightarrow \frac{F}{A}$

Strain $\downarrow$ Dimensionless

$[Y] = \left[\frac{MLT^{-2}}{L^2}\right] = (ML^{-1}T^{-2})$

$Y \propto V^x A^y F^z$

$[Y] = [LT^{-1}]^x [LT^{-2}]^y [MLT^{-2}]^z$

$\downarrow$

$ML^{-1}T^{-2} = L^{x+y+z}\, T^{-x-2y-2z} \cdot M^z$

$z = 1$ — (1)

$x+y+z = -1$ — (2) —— $x+y = -2$

$x+2y+2z = 2$ — (3) $\quad x+2y = 0$

$-y = -2$

$y = 2,\ x = -4,\ z = 1$

DIMENSION ANALYSIS

Q.4 If Surface tension (S), Moment of Inertia (I) and Planck's constant (h), were to be taken as the fundamental units, the dimensional formula for linear momentum would be: **[JEE (Main)-2019]**

(A) $S^{1/2}I^{3/2}h^{-1}$

(B) $S^{3/2}I^{1/2}h^{0}$

(C) $S^{1/2}I^{1/2}h^{-1}$

(D) $S^{1/2}I^{1/2}h^{0}$ ✓

$p \propto S^x I^y h^z$

$S = \frac{F}{l} = \frac{MLT^{-2}}{l} = MT^{-2}$

$I = mr^2 = ML^2$

$h = \frac{ML^2T^{-2}}{T^{-1}} = ML^2T^{-1}$

$[MLT^{-1}] = [MT^{-2}]^x [ML^2]^y [ML^2T^{-1}]^z$

$MLT^{-1} = M^{x+y+z} L^{2y+2z} T^{-2x-z}$

$x+y+z = 1$

$2y+2z = 1 \Rightarrow y = \frac{1}{2} - z$

$-2x-z = -1 \Rightarrow x = \left(\frac{-1+z}{-2}\right)$

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DIMENSION ANALYSIS

Q.5 In the formula $X = 5YZ^2$, X and Z have dimensions of capacitance and magnetic field, respectively. What are the dimensions of Y in SI units? **[JEE (Main)-2019]**

Dimension less.

(A) $[M^{-2}L^{-2}T^{6}A^{3}]$ (B) $[M^{-1}L^{-2}T^{4}A^{2}]$

(C) $[M^{-2}L^{0}T^{-4}A^{-2}]$ (D) $[M^{-3}L^{-2}T^{8}A^{4}]$

$X =$ Capacitance.

$Q = CV$

$C = \frac{Q}{V} = \frac{AT}{\frac{ML^2T^{-2}}{AT}}$

$C = \underline{M^{-1}L^{-2}A^2}T^4$

$V = \frac{W}{q} = \frac{ML^2T^{-2}}{AT}$

$F = I\ell B$

$B = \frac{F}{I\ell} = \frac{MLT^{-2}}{AL}$

$B = MT^{-2}A^{-1}$

$Y = \frac{X}{Z^2}$

$[Y] = \frac{M^{-1}L^{-2}A^2T^4}{M^2T^{-4}A^{-2}}$

$[Y] = M^{-3}L^{-2}T^{8}A^{4}$

DIMENSION ANALYSIS

Q.6 A quantity f is given by $f = \sqrt{\frac{hc^5}{G}}$ where c is speed of light, G universal gravitational constant and h is the Planck's constant. Dimension of f is that of

[JEE (Main)-2020]

ML^2T^{-2}

(A) Energy (B) Area (C) Volume (D) Momentum

L^2 L^3 MLT^{-1}

$[h] = ML^2T^{-1}$

$[c] = LT^{-1}$

$[G] = \frac{MLT^{-2} \cdot L^2}{M^2}$

$= M^{-1}L^3T^{-2}$

$f = \left[\frac{(ML^2T^{-1})(LT^{-1})^5}{M^{-1}L^3T^{-2}}\right]^{\frac{1}{2}}$

$f = \sqrt{\frac{hc^5}{G}}$

$f =$

DIMENSION ANALYSIS

Q.7 If momentum (P), area (A) and time (T) are taken to be the fundamental quantities then the dimensional formula for energy is [JEE (Main)-2020]

(A) $\left[\mathrm{P}^{\frac{1}{2}}\mathrm{AT}^{-1}\right]$ (B) $\left[\mathrm{P}^{2}\mathrm{AT}^{-2}\right]$

(C) $\left[\mathrm{PA}^{\frac{1}{2}}\mathrm{T}^{-1}\right]$ (D) $\left[\mathrm{PA}^{-1}\mathrm{T}^{-2}\right]$

DIMENSION ANALYSIS

Q.8 Dimensional formula for thermal conductivity is (here K denotes the temperature)

[JEE (Main)-2020]

(A) $MLT^{-2} K^{-2}$

(B) $MLT^{-3} K^{-1}$

(C) $MLT^{-3} K$

(D) $MLT^{-2} K$

$P = \frac{\text{Energy}}{\text{time}}$ $[P] = \frac{ML^2T^{-2}}{T}$

$= ML^2T^{-3}$

$\frac{dQ}{dt} = K' A \left(\frac{dT}{dx}\right)$

Power = K (Area) ×

$K = \frac{\text{Power} \times \text{length}}{\text{Area} \times \text{Temp}}$

$K' = \left[\frac{ML^2T^{-3} \cdot L}{L^2 \cdot K}\right]$

$K' = [MLT^{-3}K^{-1}]$

DIMENSION ANALYSIS

Q.9 A quantity x is given by (IFv^2/WL^4) in terms of moment of inertia I, force F, velocity v, work W and length L. The dimensional formula for x is same as that of

[JEE (Main)-2020]

(A) Coefficient of viscosity

(B) Force constant ×

(C) Energy density ✓

(D) Planck's constant ×

$$x = \left(\frac{IFv^2}{WL^4}\right)$$

$W = F \cdot L$

$I = ML^2$

$$x = \frac{ML^2 \times \cancel{F} \times v^2}{\cancel{F} \times L \times L^4}$$

$$x = ML^{-3}(LT^{-1})^2$$

$$x = ML^{-1}T^{-2}$$

$$\text{Energy density} = \frac{ML^2T^{-2}}{L^3} = ML^{-1}T^{-2}$$

$F = Kx$

$$K = \frac{MLT^{-2}}{L} = MT^{-2}$$

DIMENSION ANALYSIS

Q.10 If e is the electronic charge, c is the speed of light in free space and h is Planck's constant, the quantity $\frac{1}{4\pi\varepsilon_0}\frac{|e|^2}{\hbar c}$ has dimensions of : **[JEE (Main)-2021]**

$\to X$

Fr^2

(A) $[MLT^{-1}]$ (B) $[MLT^0]$

(C) $[LC^{-1}]$ (D) $[M^0 L^0 T^0]$

$$F = \frac{1}{4\pi\varepsilon_0}\frac{|e|^2}{r^2}$$

$$E = \frac{hc}{\lambda}$$

$$Fr^2 = \frac{e^2}{4\pi\varepsilon_0}$$

$$E \cdot \lambda = hc$$

$$X = \frac{Fr^2}{\hbar c}$$

$$X = \frac{Fr^2}{E\lambda}$$

Dimen.

$$X = \frac{\cancel{F}\, r^2}{\cancel{F}\, L\, \lambda} \to (\text{length})^2$$

$(\text{length})^2$

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DIMENSION ANALYSIS

Q.11 In a typical combustion engine the work done by a gas molecule is given by $W = \alpha^2 \beta e^{-\frac{\beta x^2}{kT}}$, where x is the displacement, k is the Boltzmann constant and T is the temperature, If α and β are constants, dimensions of α will be : **[JEE (Main)-2021]**

(A) $[MLT^{-1}]$ ~~(B)~~ $[M^0LT^0]$

(C) $[MLT^{-2}]$ (D) $[M^2LT^{-2}]$

$W = \alpha^2 \beta e^{\frac{-\beta x^2}{KT}}$

$\frac{-\beta x^2}{KT} \Rightarrow$ Dimensionless $(M^0L^0T^0)$

$\alpha^2\beta$ ⇓ Dimension of work

$K = \frac{\text{Energy}}{\text{temp}} = ML^2T^{-2}K^{-1}$

$\frac{\beta x^2}{KT} = 1$

$\beta = \frac{KT}{x^2} = \frac{ML^2T^{-2}K^{-1}\,K}{L^2}$

$\beta = MT^{-2}$

$[\alpha^2\beta] = [ML^2T^{-2}]$

$[\alpha^2] = \frac{ML^2T^{-2}}{MT^{-2}} = L^2$

$[\alpha] = [L]$

$= [M^0 L T^0]$

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DIMENSION ANALYSIS

Q.12 If ' C ' and ' V ' represent capacity and voltage respectively then what are the dimensions of λ where $C/V = \lambda$? **[JEE (Main)-2021]**

Capacitance.

??

(A) $[M^{-1} L^{-3} I^{-2} T^{-7}]$

(B) $[M^{-2} L^{-4} I^{3} T^{7}]$

(C) $[M^{-2} L^{-3} I^{2} T^{6}]$

(D) $[M^{-3} L^{-4} I^{3} T^{7}]$

A

$V = \frac{W}{q}$

$\lambda =$

$q = CV$

$C = \left(\frac{q}{V}\right)$

$\lambda = \frac{C}{V}$

$\lambda = \frac{q}{V^2}$

DIMENSION ANALYSIS

Q.13 The entropy of any system is given by

W

$$S = \alpha^2 \beta \ln \left[\frac{\mu k R}{J \beta^2} + 3 \right]$$

Dimensionless

where α and β are the constants, μ, J, k and R are no. of moles, mechanical equivalent of heat, Boltzmann constant and gas constant respectively.

[Take $S = \frac{dQ}{T}$]

[JEE (Main)-2021]

Choose the incorrect option from the following

(A) S, β, k and μR have the same dimensions

(B) α and k have the same dimensions

(C) α and J have the same dimensions

(D) S and α have different dimensions

$\frac{\mu K R}{J \beta^2} = M^0 L^0 T^0$

$[\beta] = [\mu K R]^{\frac{1}{2}}$

Entropy $= \frac{\text{Energy}}{\text{Temp}} = ML^2T^{-2}K^{-1}$

$[\alpha] = \left[\frac{ML^2T^{-2}K^{-1}}{[\beta]}\right]^{\frac{1}{2}}$

DIMENSION ANALYSIS

Q.14 If time (t), velocity (v), and angular momentum (l) are taken as the fundamental units. Then the dimension of mass (m) in terms of t, v, and l is: **[JEE (Main)-2021]**

(A) $[t^{-1}v^{1}t^{-2}]$

~~(B)~~ $[t^{-1}v^{-2}i]$

(C) $[t^{1}v^{2}r^{-1}]$

(D) $[t^{-2}V^{-1}p^{1}]$

$L = pr_{\perp}$

Angular Momentum $= (MLT^{-1}) \cdot L$

$(I) = ML^2T^{-1}$

$m \propto t^x V^y I^z$

$[ML^0T^0] = [T]^x [LT^{-1}]^y [ML^2T^{-1}]^z$

$ML^0T^0 = M^z L^{y+2z} T^{-y-z+x}$

$z = 1,\quad -y-z+x = 0$

$y + 2z = 0 \quad x = y + z = 0 \Rightarrow x = -1$

$\therefore y = -2,$

DIMENSION ANALYSIS

Q.15 The force is given in terms of time t and displacement x by the equation

$F = A\cos Bx + C\sin Dt$

The dimensional formula of $\frac{AD}{B}$ is

[JEE (Main)-2021]

(A) $[M^1 L^1 T^{-2}]$

(B) $[M^0 L T^{-1}]$

(C) $[M^2 L^2 T^{-3}]$

(D) $[ML^2 T^{-3}]$

Dt & Bx → Dimensionless

$[D] = [T^{-1}]$

$[B] = [L^{-1}]$

$[A] = [C] = [F]$

$[A] = [MLT^{-2}]$

$\left[\frac{AD}{B}\right] = \frac{MLT^{-2} \cdot T^{-1}}{L^{-1}}$

$= [ML^2 T^{-3}]$

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DIMENSION ANALYSIS

Q.16 Match List-I with List-II. **[JEE (Main)-2021]**

List-I	List-II
(a) Capacitance, C	(i) $M^1L^1T^{-3}A^{-1}$
(b) Permittivity of free space, ε_0	(ii) $M^{-1}L^{-3}T^4A^2$
(c) Permeability of free space, μ_0	(iii) $M^{-1}L^{-2}T^4A^2$
(d) Electric field, E	(iv) $M^1L^1T^{-2}A^{-2}$

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DIMENSION ANALYSIS

Q.17 If E, L, M and G denote the quantities as energy, angular momentum, mass and constant of gravitation respectively, then the dimensions of P in the formula $P = EL^2M^{-5}G^{-2}$ are: **[JEE (Main)-2021]**

(A) $[M^{-1}\ L^{-1}\ T^2]$

(B) $[M^1\ L^1\ T^{-2}]$

(C) $[M^0\ L^1\ T^0]$

(D) $[M^0\ L^0\ T^0]$

$$[G] = \frac{ML^3T^{-2}}{M^2} = M^{-1}L^3T^{-2}$$

$$P = EL^2M^{-5}G^{-2}$$

$$[P] = [ML^2T^{-2}][ML^2T^{-1}]^2[M^{-5}][M^{-1}L^3T^{-2}]^{-2}$$

DIMENSION ANALYSIS

Q.18 Which of the following equations is dimensionally incorrect?

Where $t =$ time, $h =$ height, $s =$ surface tension, $\theta =$ angle, $\rho =$ density, $a, r =$ radius, $g =$ acceleration due to gravity, $v =$ volume, $p =$ pressure, $W =$ work done, $\tau =$ torque $e =$ permittivity, $\varepsilon =$ electric field, $J =$ current density, $L =$ length

[JEE (Main)-2021]

(A) $h = \frac{2s\cos\theta}{prg}$ correct $h = \frac{2T\cos\theta}{\rho g r}$

(B) $W = \tau\theta$ correct

(C) $v = \frac{\pi p a^4}{8\eta L}$

(D) $J = \varepsilon\frac{\partial E}{\partial t}$

If formula is correct then it must be dimensionally correct

if expression dimensionally correct then formula may or may not correct