Nishant Jindal

$$(4+3\sqrt{3})(2-\sqrt{3}) = -1+2\sqrt{3}.$$

$$4x^2+2x-1=0 \qquad \frac{-1\pm\sqrt{5}}{4}$$

$$4\alpha^3-3\alpha = \cos 216^\circ = -\cos 36^\circ = -\frac{1+\sqrt{5}}{4}$$

$$\boxed{\alpha = \frac{-1+\sqrt{5}}{4} = \cos 72^\circ}$$

$$\left(\frac{k-1}{k}\right)^2 - 2\left(\frac{5}{k}\right) = \frac{4}{5} \qquad \frac{5}{k}$$

$$(\ )k^2+(\ )k+(\ )=0 \quad k_1,\ k_2$$

Symmetric function of roots

$$f(\alpha,\beta) = f(\beta,\alpha)$$

$$\alpha^2\beta+\alpha\beta^2$$

(i) $f(\alpha,\beta) = \alpha\beta^2+\beta\alpha^2$

$f(\beta,\alpha) = $

$$\frac{c}{a}+\frac{c^2}{a^2}+3\frac{c}{a}\left(-\frac{b}{a}\right)$$

$$-\frac{b^3}{a^3} =$$

$$\boxed{4\alpha^3-3\alpha} = (4\alpha^2+2\alpha-1)\left(\alpha-\frac{1}{2}\right)-\alpha-\frac{1}{2}$$

$$= \boxed{-\alpha-\frac{1}{2}}.$$

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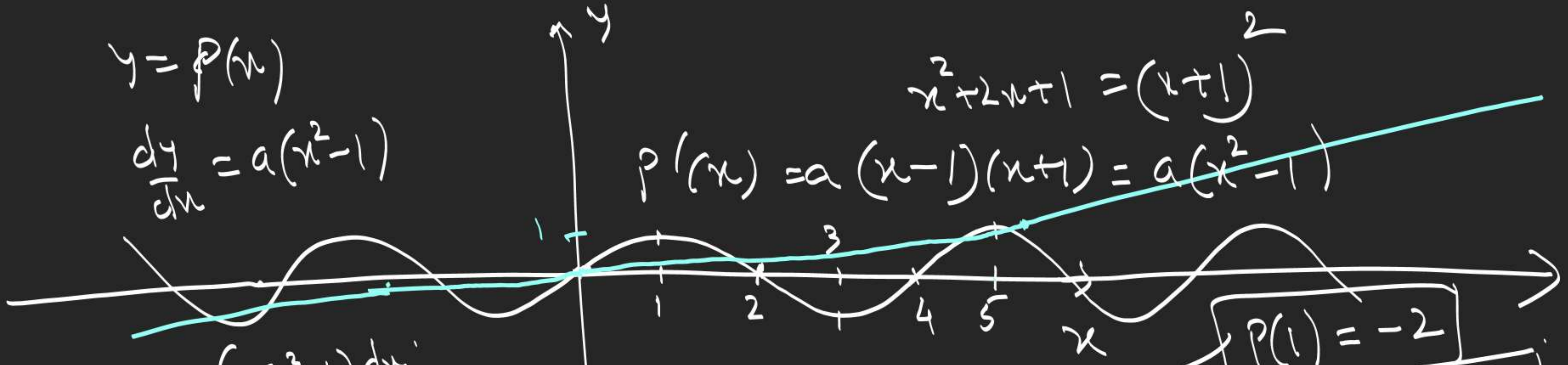
$y = p(x)$

$\frac{dy}{dx} = a(x^2-1)$

$x^2+2x+1 = (x+1)^2$

$p'(x) = a(x-1)(x+1) = a(x^2-1)$



$\int dy = \int a(x^2-1)\,dx$

7

$y = a\left(\frac{x^3}{3}-x\right)+b$

$\boxed{p(x) = a\left(\frac{x^3}{3}-x\right)+b}$

$\sin \pi x$

$a, b = ?$

$\boxed{p(1) = -2}$

$\boxed{p(-1) = 2}$

$p(x)+2 = (x-1)^2 Q_1(x)$

$\boxed{p(x)-2 = (x+1)^2 Q_2(x)} \Rightarrow \boxed{p'(x) = (x+1)h(x)}$

$p'(x) = 2(x-1)Q_1(x) + (x-1)^2 Q_1'(x)$

$p'(x) = (x-1)g(x)$

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Polynomial

- $f(x) = (x-\alpha)^2 g(x)$ ✓ $\qquad f(\alpha) = 0 = f'(\alpha)$

$$f'(x) = (x-\alpha)\left(2g(x) + (x-\alpha)g'(x)\right)$$

$$\boxed{P(x) = a_3x^3 + a_2x^2 + a_1x + a_0}$$

- $f(x) = (x-\alpha)^3 g(x) \Rightarrow \boxed{f(\alpha) = f'(\alpha) = f''(\alpha) = 0}$

$\boxed{P(x)+2} = (x-1)^2 g(x)$

$P(1)+2 = 0 = P'(1)$

$$f'(x) = 3(x-\alpha)^2 g(x) + (x-\alpha)^3 g'(x)$$

$$f''(x) = (x-\alpha)h(x)$$

$\boxed{a^2 - 4b \geq 0}$

$\boxed{D \geq 0}$

$|\alpha-\beta| < c$

$\boxed{a^2 - 4b < c^2}$

$$\boxed{\begin{gathered} f(x) = (x-\alpha)^n g(x), \\ f(\alpha) = f'(\alpha) = f''(\alpha) = \dots = f^{(n-1)}(\alpha) \\ = 0 \end{gathered}}$$

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17. $f(x) = x^{12} - x^9 + x^4 - x + 1$

$x \le 0$ ✓

$f(x) \geq 1 > 0$

$x \geq 1$

$\underbrace{(x^{12} - x^9)}_{\geq 0} + \underbrace{(x^4 - x)}_{\geq 0} + 1 \geq 1 > 0$

$\boxed{3\left(x + \frac{1}{x}\right)}$

$0 < x < 1$

$x^{12} + \underbrace{(x^4 - x^9)}_{>0} + \underbrace{(1 - x)}_{>0} > 0$

$$\frac{\left(x+\frac{1}{x}\right)^6 - \left(x^3+\frac{1}{x^3}\right)^2}{\left(x+\frac{1}{x}\right)^3 + \left(x^3+\frac{1}{x^3}\right)} = \frac{\left(\left(x+\frac{1}{x}\right)^3 - \left(x^3+\frac{1}{x^3}\right)\right)\left(\left(x+\frac{1}{x}\right)^3 + \cancel{\left(x^3+\frac{1}{x^3}\right)}\right)}{\cancel{\left[\left(x+\frac{1}{x}\right)^3 + \left(x^3+\frac{1}{x^3}\right)\right]}}$$

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$$(x-16)P(2x) = 16(x-1)P(x)$$

$\boxed{x=16}$

$P(x)$	$P(2x)$
$x-16$	$2(x-8)$
$x-8$	$2(x-4)$
$x-4$	$2(x-2)$
$x-2$	$2(x-1)$

$$\boxed{P(x) = a(x-16)(x-8)(x-4)(x-2)}$$

$P(7) = 135 \Rightarrow a = ?$

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Arithmetic Mean of 2 numbers a, c.

b is the A.M. of a, c

$\Rightarrow a, \underline{b}, c$ are in A.P.

$$a + c = b + b$$

$$\boxed{b = \frac{a+c}{2}}$$

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Arithmetic mean of 'n' numbers $x_1, x_2, x_3, \ldots, x_n$
$= A$

$$\boxed{A = \frac{x_1 + x_2 + x_3 + \cdots + x_n}{n}}$$

Inserting 'n' Arithmetic Means between 2 numbers
a, b

let A.Ms are $A_1, A_2, A_3, \ldots, A_n$

$\Rightarrow$ $\boxed{a, A_1, A_2, A_3, \ldots, A_n, b \text{ form A.P.}}$

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1. Insert 20 A.Ms between 4 & 67.

$4, A_1, A_2, \ldots, A_{20}, 67$ $\qquad 4+(21)d=67$, $d=3$

$4, \boxed{7, 10, 13, \ldots, 64}, 67$

2. If 'p' A.Ms $5, A_1, A_2, A_3, \ldots, A_p, 41$ are inserted between 5 and 41, so that $\frac{A_3}{A_{p-1}} = \frac{2}{5}$, find p.

$A_1 \to 5+d$, $A_2 \to 5+2d$

$41 = 5+(p+1)d$

$d = \frac{36}{p+1}$

$$\frac{5+3\left(\frac{36}{p+1}\right)}{5+(p-1)\left(\frac{36}{p+1}\right)} = \frac{2}{5} \Rightarrow \boxed{p=11}$$

3. If $\log_3 2$, $\log_3(2^x-5)$ & $\log_3\left(2^x-\frac{7}{2}\right)$ are in A.P. find $\boxed{x=3}$

$2\log_3(2^x-5) = \log_3 2 + \log_3\left(2^x - \frac{7}{2}\right)$

$t^2-10t+25 = 2t-7$

$t^2-12t+32=0$

$2^x = 4, 8$

$x = 2, 3$

2 reject

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4. If the sum of roots of eqn. $ax^2+bx+c=0$ is equal to the sum of squares of their reciprocals, then P.T. bc^2, ca^2, ab^2 are in A.P.

$$-\frac{b}{a} = \frac{\left(-\frac{b}{a}\right)^2 - 2\frac{c}{a}}{\left(\frac{c}{a}\right)^2} = \frac{b^2-2ac}{c^2}$$

$$-bc^2 = ab^2 - 2a^2c$$

$$\boxed{2a^2c = ab^2 + bc^2}$$

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5. Find the condition that the roots of equation $x^3 - px^2 + qx - r = 0$ may be in A.P.

$\alpha \quad \beta \quad \gamma$

$\alpha, \beta, \gamma \to$ A.P.

$2\beta = \alpha + \gamma$

$3\beta = \alpha + \beta + \gamma = p \Rightarrow \boxed{\beta = \frac{p}{3}}$

$$\frac{p^3}{27} - p\left(\frac{p^2}{9}\right) + q\left(\frac{p}{3}\right) - r = 0$$

$\boxed{\begin{matrix} \text{Ex - II } (1-10) \\ \text{Ex - III } (1-10) \end{matrix}}$

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Q. Given that $a_1, a_2, a_3, a_4, \ldots, a_n$ are in A.P.

Then P.T. $\frac{1}{a_1 a_n} + \frac{1}{a_2 a_{n-1}} + \frac{1}{a_3 a_{n-2}} + \ldots + \frac{1}{a_n a_1} = \frac{2}{(a_1+a_n)}\left(\frac{1}{a_1} + \frac{1}{a_2} + \ldots + \frac{1}{a_n}\right)$

$$\frac{1}{(a_1+a_n)}\left[\frac{a_1+a_n}{a_1 a_n} + \frac{a_{n-1}+a_2}{a_2 a_{n-1}} + \frac{a_3+a_{n-2}}{a_3 a_{n-2}} + \ldots + \frac{a_n+a_n}{a_n a_1}\right]$$

$$\frac{1}{a_1+a_n}\left\{\frac{1}{a_1} + \frac{1}{a_n} + \frac{1}{a_2} + \frac{1}{a_{n-1}} + \frac{1}{a_3} + \frac{1}{a_{n-2}} + \ldots + \frac{1}{a_n} + \frac{1}{a_1}\right\}$$

$$\frac{2}{(a_1+a_n)}\left(\frac{1}{a_1} + \frac{1}{a_2} + \frac{1}{a_3} + \ldots \frac{1}{a_n}\right).$$

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7. P.T. $\sqrt{2}, \sqrt{3}, \sqrt{5}$ can not be the terms of an A.P.

$\sqrt{2} \to p^{th}$, $\sqrt{3} \to q^{th}$, $\sqrt{5} \to r^{th}$

$$a+(p-1)d = \sqrt{2}$$
$$a+(q-1)d = \sqrt{3}$$
$$a+(r-1)d = \sqrt{5}$$

$$(q-p)d = \sqrt{3}-\sqrt{2}$$
$$(r-q)d = \sqrt{5}-\sqrt{3}$$

$$\frac{r-q}{q-p} = \frac{\sqrt{5}-\sqrt{3}}{\sqrt{3}-\sqrt{2}}$$

rational

irrational

$\boxed{\text{Contradiction}}$