DPP – 01

Solution

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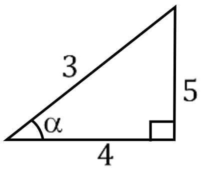
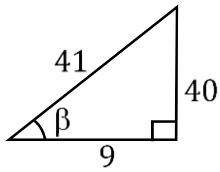
1. If $\sin\ \alpha = \frac{3}{5}$ and $\cos\ \beta = \frac{9}{41}$, find the value of $\sin(\alpha - \beta)$ and $\cos(\alpha + \beta)$.

Sol. $\sin\ \alpha = \frac{3}{5},\ \cos\ \beta = \frac{9}{41}$





$\Rightarrow \cos\ \alpha = \frac{4}{5},\ \sin\ \beta = \frac{40}{41}$

Now, value of $\sin(\alpha - \beta)$

$$\sin(\alpha - \beta) = \sin\ \alpha \cdot \cos\ \beta - \cos\ \alpha \cdot \sin\ \beta = \left(\frac{3}{5} \times \frac{9}{41}\right) - \left(\frac{4}{5} \times \frac{40}{41}\right) = \frac{27}{205} - \frac{160}{205}$$

$$\sin(\alpha - \beta) = -\frac{133}{205}$$

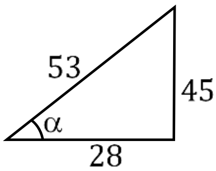
Now, value of $\cos(\alpha + \beta)$

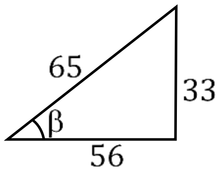
$$\cos(\alpha + \beta) = \cos\ \alpha \cdot \cos\ \beta - \sin\ \alpha \cdot \sin\ \beta = \left(\frac{4}{5} \times \frac{9}{41}\right) - \left(\frac{3}{5} \times \frac{40}{41}\right) = \frac{36}{205} - \frac{120}{205}$$

$$\cos(\alpha + \beta) = -\frac{84}{205}$$

2. If $\sin\alpha = \frac{45}{53}$ and $\sin\beta = \frac{33}{65}$, find the values of $\sin(\alpha - \beta)$ and $\sin(\alpha + \beta)$.

Sol. $\sin\alpha = \frac{45}{53}\quad ,\ \sin\beta = \frac{33}{65}$





$\Rightarrow \cos\alpha = \frac{28}{53}\quad ,\ \cos\beta = \frac{56}{65}$

Now, value of $\sin(\alpha - \beta)$

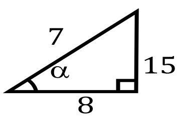
$$\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta = \left(\frac{45}{53} \times \frac{56}{65}\right) - \left(\frac{28}{53} \times \frac{33}{65}\right) = \frac{2520}{3445} - \frac{924}{3445} = \frac{1596}{3445}$$

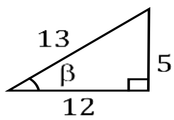
Now, value of $\sin(\alpha + \beta)$

$$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta = \left(\frac{45}{53} \times \frac{56}{65}\right) + \left(\frac{28}{53} \times \frac{33}{65}\right) = \frac{2520}{3445} + \frac{924}{3445} = \frac{3444}{3445}$$

3. If $\sin\ \alpha = \frac{15}{17}$ and $\cos\ \beta = \frac{12}{13}$, find the values of $\sin(\alpha + \beta)$, $\cos(\alpha - \beta)$, and $\tan(\alpha + \beta)$.

Sol. $\sin\ \alpha = \frac{15}{17},\ \cos\ \beta = \frac{12}{13}$





$\Rightarrow \cos\ \alpha = \frac{8}{17},\ \sin\ \beta = \frac{5}{13}$

$$\sin(\alpha + \beta) = \sin\ \alpha \cdot \cos\ \beta + \cos\ \alpha \cdot \sin\ \beta = \left(\frac{15}{17} \times \frac{12}{13}\right) + \left(\frac{8}{17} \times \frac{5}{13}\right) = \frac{180}{221} + \frac{40}{221}$$

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$\sin(\alpha+\beta) = \frac{220}{221}$

$\rightarrow$ value of $\cos(\alpha-\beta)$

$\cos(\alpha-\beta) = \cos\alpha\cdot\cos\beta + \sin\alpha\cdot\sin\beta = \left(\frac{8}{17}\times\frac{12}{13}\right) + \left(\frac{15}{17}\times\frac{5}{13}\right) = \frac{96}{221}+\frac{75}{221}$

$\cos(\alpha-\beta) = \frac{171}{221}$

$\tan\alpha = \frac{15}{8}$, $\tan\beta = \frac{5}{12}$

$\rightarrow$ value of $(\tan(\alpha+\beta))$: $\tan(\alpha+\beta) = \frac{\tan\alpha+\tan\beta}{1-\tan\alpha\cdot\tan\beta} = \frac{15/8+5/12}{1-15/8\cdot5/12} = \frac{229/96}{21/96}$

$\tan(\alpha+\beta) = \frac{220}{21}$

4. Prove that $\cos(45^\circ-A)\cos(45^\circ-B)-\sin(45^\circ-A)\sin(45^\circ-B) = \sin(A+B)$

Sol. $\cos(45-A)\cos(45-B)\sin(45-A)\sin(45-B)$

$\because \cos x\cos y - \sin x\sin y = \cos(x+y) = \cos[(45-A)+(45-B)] = \cos\{90-(A-B)\}$

$= \sin(A+B)$

5. Prove that $\sin(45^\circ+A)\cos(45^\circ-B)+\cos(45^\circ+A)\sin(45^\circ-B) = \cos(A-B)$

Sol. LHS $= \sin(45^\circ+A)\cdot\cos(45^\circ-B)+\cos(45^\circ+A)\cdot\sin(45^\circ-B)$

we know that, $\sin(\alpha+\beta) = \sin\alpha\cdot\cos\beta+\cos\alpha\cdot\sin\beta$

$\therefore$ LHS $= \sin(45^\circ+A+45^\circ-B) = \sin(90^\circ+(A-B))$

$= \cos(A-B)$ $\quad(\because \sin(90^\circ+\theta) = \cos\theta)$

6. Prove that $\frac{\sin(A-B)}{\cos A\cos B}+\frac{\sin(B-C)}{\cos B\cos C}+\frac{\sin(C-A)}{\cos C\cos A} = 0$

Sol. $\frac{\sin(A-B)}{\cos A\cos B}+\frac{\sin(B-C)}{\cos B\cos C}+\frac{\sin(C-A)}{\cos C\cos A}$

$= \frac{\sin A\cos B-\cos A\sin B}{\cos A\cos B}+\frac{\sin B\cos C-\cos B\sin C}{\cos B\cos C}+\frac{\sin C\cos A-\cos C\sin A}{\cos A\cos C}$

$= \tan A-\tan B+\tan B-\tan C+\tan C-\tan A = 0$

7. Prove that $\sin 105^\circ+\cos 105^\circ = \cos 45^\circ$

Sol. $\sin 105^\circ+\cos 105^\circ$

$\sin(60^\circ+45^\circ)+\cos(60^\circ+45^\circ)$

$(\because \sin(A^\circ+B^\circ) = \sin A\cos B+\cos A\sin B,\ \cos(A^\circ+B^\circ) = \cos A\cos B-\sin A\sin B)$

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$$= \cos 45^\circ(\sin 60^\circ + \cos 60^\circ) + \sin 45^\circ\,(\cos\ 60^\circ - \sin\ 60^\circ)$$

$$= \cos 45^\circ(\sin 60^\circ + \cos 60^\circ) + \cos 45^\circ\,(\cos\ 60^\circ - \sin\ 60^\circ) = \cos 45^\circ$$

8. Prove that $\sin 75^\circ - \sin 15^\circ = \cos 105^\circ + \cos 15^\circ$

Sol. RHS $\Rightarrow$ we have; $\cos 105 + \cos 15 \Rightarrow \cos(90+15) + \cos(90-75)$

$\Rightarrow -\sin 15 + \sin 75 \Rightarrow -\sin 15 + \sin 75 \Rightarrow \sin 75 - \sin 15$

Hence, RHS = LHS

9. Prove that $\cos\alpha\cos(\gamma-\alpha) - \sin\alpha\sin(\gamma-\alpha) = \cos\gamma$

Sol. $\text{LHS} = \cos\alpha\cdot\cos(\gamma-\alpha) - \sin\alpha\cdot\sin(\gamma-\alpha)$

we know that: $\cos A\cdot\cos B - \sin A\sin B = \cos(A+B)$

Let: $A = \alpha, B = (\gamma-\alpha)$, putting values in LHS

$\therefore \text{LHS} = \cos(\alpha + (\gamma-\alpha)) = \cos\gamma = \text{RHS}$

10. Prove that $\cos(\alpha+\beta)\cos\gamma - \cos(\beta+\gamma)\cos\alpha = \sin\beta\sin(\gamma-\alpha)$

Sol. $\text{LHS} = \cos(\alpha+\beta)\cos\gamma - \cos(\beta+\gamma)\cos\alpha$

we know that : $\cos(A+B) = \cos A\cos B - \sin A\sin B$

$\text{LHS} = (\cos\alpha\cos\beta - \sin\alpha\sin\beta)\cos\gamma - (\cos\beta\cos\gamma - \sin\beta\sin\gamma)\cos\alpha$

$= \sin\beta(\sin\gamma\cos\alpha - \cos\gamma\sin\alpha) = \sin\beta\sin(\gamma-\alpha) = \text{RHS}$

11. Prove that $\sin(n+1)A\sin(n-1)A + \cos(n+1)A\cos(n-1)A = \cos 2A$

Sol. $\text{LHS} = \sin(n+1)A\sin(n-1)A + \cos(n+1)A\cos(n-1)A$

$= \sin(nA+A)\sin(nA-A) + \cos(nA+A)\cos(nA-A)$

$= \cos(nA+A)\cos(nA-A) + \sin(nA+A)\sin(nA-A)$

$= \cos[(nA+A)-(nA-A)]$ $\quad [\because \cos A\cos B + \sin A\sin B = \cos(A-B)]$

$= \cos(nA + A - nA + A) = \cos 2A = \text{RHS}$

12. Prove that $\sin(n+1)A\sin(n+2)A + \cos(n+1)A\cos(n+2)A = \cos A$

Sol. We have, $\sin(n+1)A\sin(n+2)A + \cos(n+1)A\cos(n+2)A$

$\cos(n+2)A\cos(n+1)A + \sin(n+2)A\sin(n+1)A$

$\cos[(n+2)A - (n+1)A]$

$\cos(nA + 2A - nA - A)$
$\cos A$