

6.  $x, 2x$

$$3x = -(2x-1) \checkmark$$

$$2x^2 = x^2 + 2$$

$$\frac{2}{9} = \frac{(2x-1)^2}{x^2+2}$$

$$x = ?$$

$$\frac{9}{10} \cdot \frac{10x^2}{9} = \frac{3x+2}{x^2}$$

0  $x, x^2$

$$x + x^2 = \frac{15}{4}$$

$$4x^2 + 4x - 15 = 0$$

$$-6x + 10x$$

$$x^3 = x \checkmark$$

$$(2x+5)(2x-3) = 0$$

$$x = -\frac{5}{2}, \frac{3}{2}$$

$$x = -\frac{125}{8}, \frac{27}{8}$$

1.

$$p^2 - 4(12) = 1$$

4.

$$5x_1 + 2x_2 = 1$$

$$x_1 + x_2 = -\frac{6}{5}$$

$$\textcircled{1} - \textcircled{2} \times 2$$

$$3x_1 = 1 + \frac{2b}{5}$$

$$x^2 - 2a(x-1) - 1 = 0$$

$$x + \beta = \alpha^2 + \beta^2$$

$$\underline{8.} \quad x^2 + px + q = 0 \quad \begin{matrix} p \\ q \end{matrix}$$

$$p + q = -p$$

$$pq = q$$

$$q = 0 \text{ or } p = 1$$

$$\boxed{\text{I} \mid q = 0, p = 0} \text{ or } \boxed{p = 1, q = -2}$$

$$\frac{b}{5} \left( \frac{5+2b}{5} \right)^2 + \frac{b}{5} \left( \frac{5+2b}{5} \right) - 28 = 0$$

∴ Form a cubic whose roots are cubes of  
the roots of the equation  $x^3 + 3x^2 + 2 = 0$   $\left\{ \begin{array}{l} \alpha \\ \beta \\ \gamma \end{array} \right\} = x$

$$x^3 + 33x^2 + 12x + 8 = 0$$

$$y + 2 = -3y^{2/3}$$

$$(y+2)^3 = -27y^2$$

$$y^3 + 33y^2 + 12y + 8 = 0$$

$$y = \alpha^3, \beta^3, \gamma^3$$

$$y = x^3 \Rightarrow x = \sqrt[3]{y}$$

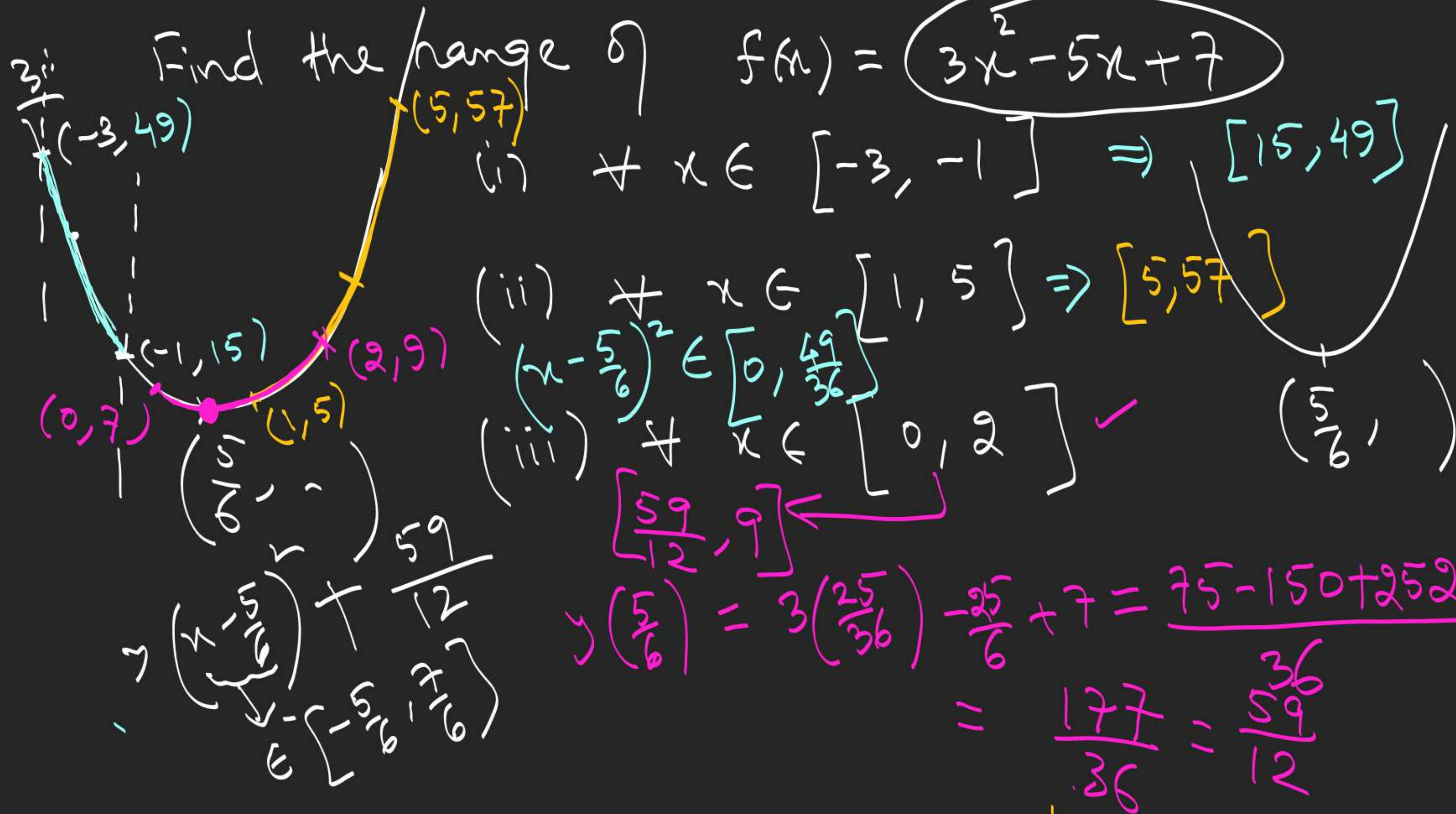
2.  $\sqrt{3(3-a)(3-b)(3-c)} = \sqrt{s(s-a)(s-b)(s-c)} = \Delta = \frac{3}{2}$

$$4(x-a)(x-b)(x-c) = 4x^3 - 24x^2 + 47x - 30 = 0$$

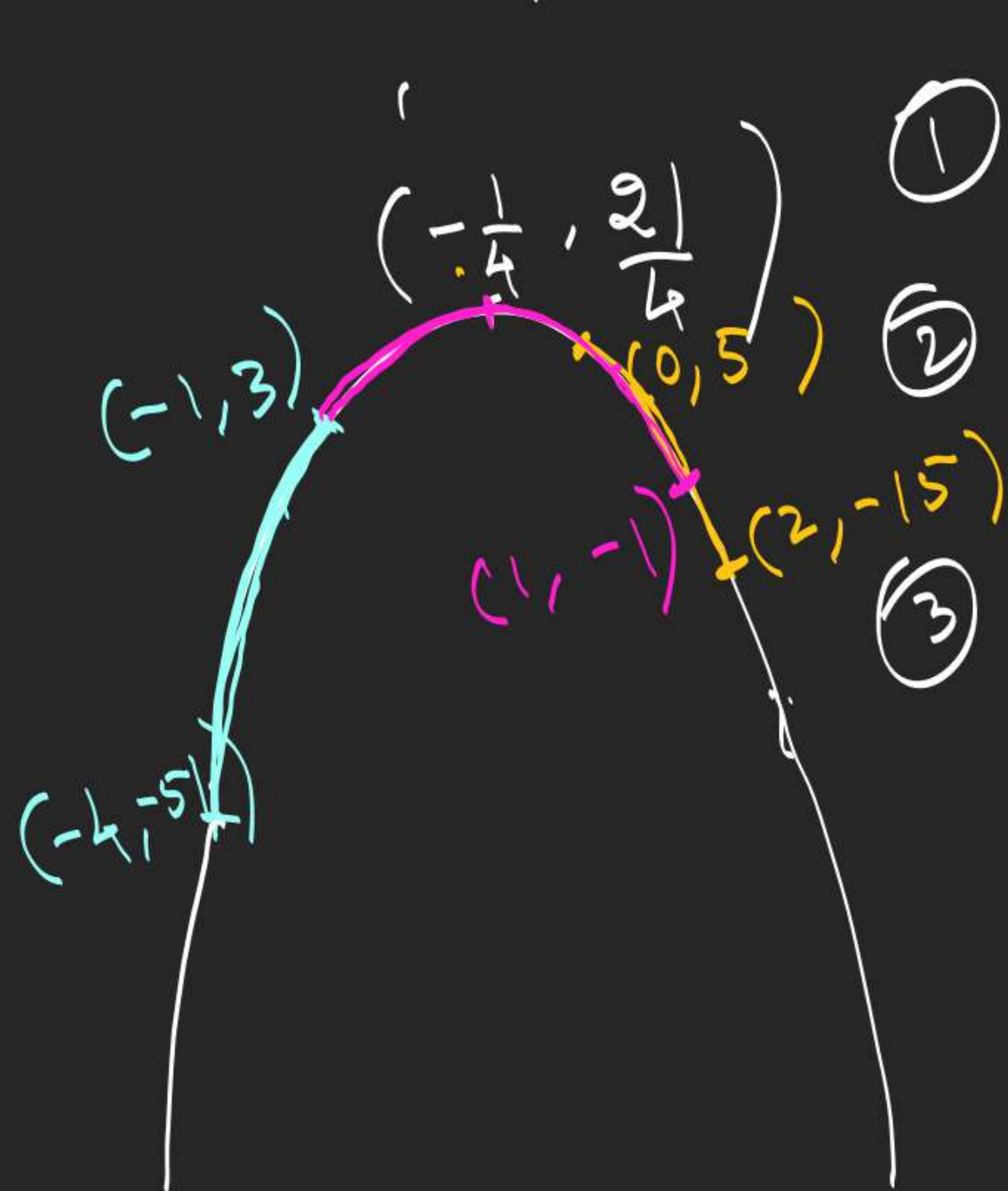
$$a+b+c = 6 \Rightarrow s = 3$$

$$4(27) - 24(9) + 47(3) - 30$$

$$= 4(3-a)(3-b)(3-c)$$



4. Find <sup>range of</sup>  $f(x) = 5 - 2x - 4x^2$



①

$$\forall x \in [-4, -1] \Rightarrow [-5, 3]$$

②

$$\forall x \in [0, 2] \Rightarrow [-15, 5]$$

③

$$\forall x \in [-1, 1] \Rightarrow [-1, \frac{21}{4}]$$

$$\Rightarrow [-5, 3]$$

$$\Rightarrow [-15, 5]$$

$$\Rightarrow [-1, \frac{21}{4}]$$

$$5 + \frac{1}{2} - \frac{1}{4} = \frac{20 + 2 - 1}{4}$$

5. Find range of function,  $f(x) = \frac{x^2 - 3x + 4}{x^2 + 3x + 4}$

$$y = \frac{x^2 - 3x + 4}{x^2 + 3x + 4}$$

$$y = ?$$

$$R_f = \left[ \frac{1}{7}, 7 \right] \checkmark$$

$$(y-1)x^2 + (3y+3)x + 4(y-1) = 0$$

OR

$$y \neq 1 \checkmark$$

$$D \geq 0$$

$\Rightarrow$

$$9(y+1)^2 - 16(y-1)^2 \geq 0$$

$$(3y+3-4y+4)(3y+3+4y-4) \geq 0$$

$$(7y-1)(y-7) \leq 0$$

$$y \in \left[ \frac{1}{7}, 1 \right) \cup (1, 7] \checkmark$$

is  $y=1 \checkmark$   
 $6x=0$   
 $x=0$

$$y = \frac{x^2 - 3x + 4}{x^2 + 3x + 4} = \frac{(x^2 + 3x + 4)(1) - 6x}{x^2 + 3x + 4} = 1 - \frac{6x}{x^2 + 3x + 4}$$

$$1 - \frac{6x}{x^2 + 3x + 4} \in \left[1 - \frac{6}{7}, 1 + 6\right] \quad \frac{6}{x + \frac{4}{x} + 3}$$

$$\frac{x}{x^2 + 3x + 4} \in \left[-1, \frac{1}{7}\right] \quad x + \frac{4}{x} + 3 \in (-\infty, -1] \cup [7, \infty)$$

$$\frac{-6x}{x^2 + 3x + 4} \in \left[-\frac{6}{7}, 6\right] \quad \frac{1}{x + \frac{4}{x} + 3} \in \left[-1, 0\right) \cup \left(0, \frac{1}{7}\right]$$

$$x + \frac{4}{x} \geq \left(x - \frac{2}{x}\right)^2 + 4 \geq 4 \quad x > 0$$

$$\leq -4 \quad x < 0$$

$$f(x) = y = \frac{x^2 - 3x + 4}{x^2 + 3x + 4} = 1 - \frac{6x}{x^2 + 3x + 4}$$

$$\textcircled{1} \mathcal{D}_f = \mathbb{R}$$

$$\textcircled{2} f'(x) = -6 \frac{(x^2 + 3x + 4) - x(2x + 3)}{(x^2 + 3x + 4)^2} = \frac{6(x^2 - 4)}{(x^2 + 3x + 4)^2} = \frac{6(x-2)(x+2)}{(x^2 + 3x + 4)^2}$$

$$f \uparrow (-\infty, -2) \cup (2, \infty)$$

$$f \downarrow (-2, 2)$$

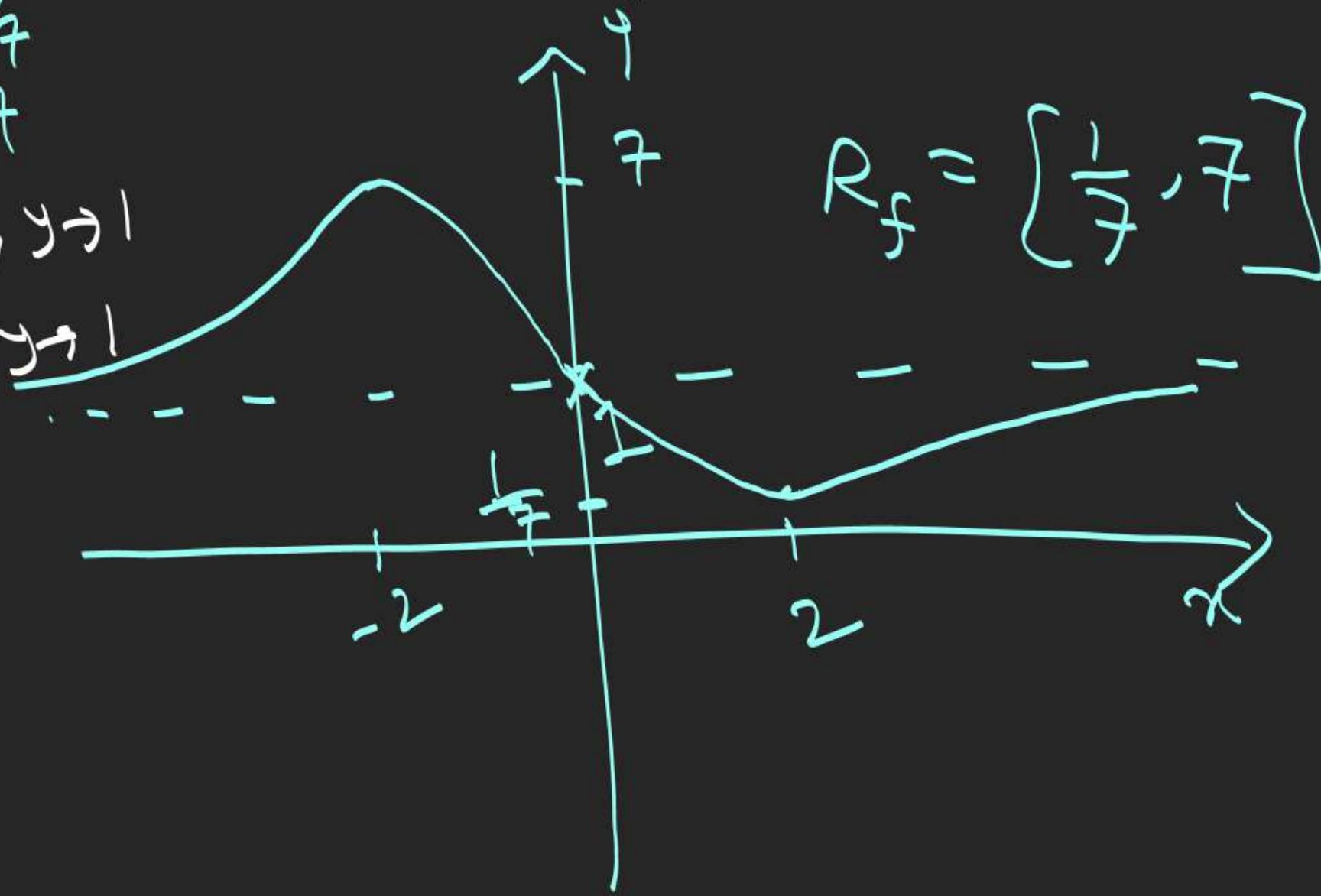
$$y(2) = \frac{1}{7}$$

$$y(-2) = 7$$

$$x \rightarrow -\infty, y \rightarrow 1$$

$$x \rightarrow \infty, y \rightarrow 1$$

$$y = \frac{\left(1 - \frac{3}{x} + \frac{4}{x^2}\right)}{\left(1 + \frac{3}{x} + \frac{4}{x^2}\right)}$$



1. Find the range and sketch the graph of

$$(i) \quad f(x) = \frac{x^2 + 2x - 11}{2(x-3)}$$

$$(ii) \quad f(x) = \frac{(x+1)(x-2)}{x(x+3)}$$

$$(iii) \quad f(x) = \frac{x^2 + 14x + 9}{x^2 + 2x + 3}$$

$$\begin{array}{l} \text{H\&K} \rightarrow \\ \hline \frac{x - \sqrt{17}}{2} (b) \\ 2, 3, 4, 6, 8, \\ 9 \end{array}$$

