DPP-01

AREA OF TRIANGLE

SOLUTION

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Q. Find the areas of the triangles the coordinates of whose angular points are respectively:

1. $(1,3), (-7,6)$ and $(5,-1)$.

Ans. 10

Sol. $\Delta = \frac{1}{2}\left|\begin{vmatrix} 1 & 3 & 1 \\ -7 & 6 & 1 \\ 5 & -1 & 1 \end{vmatrix}\right| = \frac{1}{2}|\{(6+1) - 3(-7-5) + 1(7-30)\}|$

$= \frac{1}{2}(7 + 36 - 23) = \frac{20}{2} = 10$ Sq. Units

2. $(0,4), (3,6)$ and $(-8,-2)$.

Ans. 1

Sol. $\Delta = \frac{1}{2}\left|\begin{vmatrix} 0 & 4 & 1 \\ 3 & 6 & 1 \\ -8 & -2 & 1 \end{vmatrix}\right| = \frac{1}{2}|\{0 - 4(3+8) + 1(-6+48)\}|$

$= \frac{1}{2}|(-44 + 42)| = \frac{1}{2} \times 2 = 1$

3. $(5,2), (-9,-3)$ and $(-3,-5)$.

Ans. 29

Sol. $\Delta = \frac{1}{2}\left|\begin{vmatrix} 5 & 2 & 1 \\ -9 & -3 & 1 \\ -3 & -5 & 1 \end{vmatrix}\right|$

$\Delta = \frac{1}{2}|\{5(-3+5) - 2(-9+3) + 1(45-9)\}|$

$\Delta = \frac{1}{2}(10 + 12 + 36) = \frac{1}{2} \cdot (58) = 29$ Sq. Units

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4. $(a, b + c), (a, b - c)$ and $(-a, c)$.

Ans. $2ac$

Sol. $\Delta = \frac{1}{2}\left\|\begin{matrix} a & b+c & 1 \\ a & b-c & 1 \\ -a & c & 1 \end{matrix}\right\|$

$R_1 \to R_1 - R_2$

$\Delta = \frac{1}{2}\left\|\begin{matrix} 0 & 2c & 0 \\ a & b-c & 1 \\ -a & c & 1 \end{matrix}\right\|$

$\Delta = \frac{1}{2}|\{-2c(a + a)\}|$

$\Delta = | - 2ac |$

$\Delta = 2ac$

5. $(a, c + a), (a, c)$ and $(-a, c - a)$.

Ans. a^2

Sol. $\Delta = \frac{1}{2}\left\|\begin{matrix} a & c+a & 1 \\ a & c & 1 \\ -a & c-a & 1 \end{matrix}\right\|$

$R_1 \to R_1 - R_2$

$\Delta = \frac{1}{2}\left\|\begin{matrix} 0 & a & 0 \\ a & c & 1 \\ -a & c-a & 1 \end{matrix}\right\|$

$\Delta = \frac{1}{2}|\{-a(a + a)\}|$

$\Delta = \frac{1}{2}| - 2a^2| = a^2$

6. $(a\cos\ \phi_1, b\sin\phi_1), (a\cos\ \phi_2, b\sin\ \phi_2)$ and $(a\cos\ \phi_3, b\sin\ \phi_3)$.

Ans. $2ab\sin\frac{\phi_2-\phi_3}{2}\sin\frac{\phi_3-\phi_1}{2}\sin\frac{\phi_1-\phi_2}{2}$

Sol. $\Delta = \frac{1}{2}\left\|\begin{matrix} a\cos\ \phi_1 & b\sin\ \phi_1 & 1 \\ a\cos\ \phi_2 & b\sin\ \phi_2 & 1 \\ a\cos\ \phi_3 & b\sin\ \phi_3 & 1 \end{matrix}\right\|$

$R_1 \to R_1 - R_2\ \ \&\ \ R_2 \to R_2 - R_3$

$\Delta = \frac{1}{2}ab \cdot \left\|\begin{matrix} \cos\ \phi_1 - \cos\ \phi_2 & \sin\ \phi_1 - \sin\ \phi_2 & 0 \\ \cos\ \phi_2 - \cos\ \phi_3 & \sin\ \phi_2 - \sin\ \phi_3 & 0 \\ \cos\ \phi_3 & \sin\ \phi_3 & 1 \end{matrix}\right\|$

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$$\Delta = \frac{ab}{2}\left\|\begin{matrix} -2\sin\left(\frac{\phi_1+\phi_2}{2}\right)\sin\left(\frac{\phi_1-\phi_2}{2}\right) & 2\cos\left(\frac{\phi_1+\phi_2}{2}\right)\sin\left(\frac{\phi_1-\phi_2}{2}\right) & 0 \\ -2\sin\left(\frac{\phi_2+\phi_3}{2}\right)\sin\left(\frac{\phi_2-\phi_3}{2}\right) & 2\cos\left(\frac{\phi_2+\phi_3}{2}\right)\sin\left(\frac{\phi_2-\phi_2}{2}\right) & 0 \\ \cos\phi_3 & \sin\phi_3 & 1 \end{matrix}\right\|$$

$$\Delta = \left|\frac{ab}{2}.2.\sin\left(\frac{\phi_1-\phi_2}{2}\right)\sin\left(\frac{\phi_2-\phi_3}{2}\right)\right|\begin{vmatrix} -\sin\left(\frac{\phi_1+\phi_2}{2}\right) & \cos\left(\frac{\phi_1+\phi_2}{2}\right) & 0 \\ -\sin\left(\frac{\phi_2+\phi_3}{2}\right) & \cos\left(\frac{\phi_2+\phi_3}{2}\right) & 0 \\ \cos\phi_3 & \sin\phi_3 & 1 \end{vmatrix}$$

$$\Delta = \left|2ab\sin\left(\frac{\phi_1-\phi_2}{2}\right)\sin\left(\frac{\phi_2-\phi_3}{2}\right)\left\{1\cdot\left(-\sin\left(\frac{\phi_1+\phi_2}{2}\right)\cos\left(\frac{\phi_2+\phi_3}{2}\right)+\sin\left(\frac{\phi_2+\phi_3}{2}\right)\cos\left(\frac{\phi_1+\phi_2}{2}\right)\right)\right\}\right|$$

$$\Delta = 2ab\sin\left(\frac{\phi_1-\phi_2}{2}\right)\sin\left(\frac{\phi_2-\phi_3}{2}\right)\cdot\sin\left\{\left(\frac{\phi_2+\phi_3}{2}\right)-\left(\frac{\phi_1+\phi_2}{2}\right)\right\}$$

$$\Delta = 2ab\sin\left(\frac{\phi_1-\phi_2}{2}\right)\sin\left(\frac{\phi_2-\phi_3}{2}\right)\sin\left(\frac{\phi_3-\phi_1}{2}\right)$$

7. $(am_1^2, 2am_1), (am_2^2, 2am_2)$ and $(am_3^2, 2am_3)$.

Ans. $a^2(m_2-m_3)(m_3-m_1)(m_1-m_2)$

Sol. $$\Delta = \frac{1}{2}\left\|\begin{matrix} am_1^2 & 2am_1 & 1 \\ am_2^2 & 2am_2 & 1 \\ am_3^2 & 2am_3 & 1 \end{matrix}\right\|$$

$R_1 \to R_1 - R_2$ & $R_2 \to R_2 - R_3$

$$\Delta = \frac{1}{R}(2a^2)\left\|\begin{matrix} m_1^2-m_2^2 & m_1-m_2 & 0 \\ m_2^2-m_3^2 & m_2-m_3 & 0 \\ m_3^2 & m_3 & 1 \end{matrix}\right\|$$

$$\Delta = a^2(m_1-m_2)(m_2-m_3)\begin{vmatrix} m_1+m_2 & 1 & 0 \\ m_2+m_3 & 1 & 0 \\ m_3^2 & m_3 & 1 \end{vmatrix}|$$

$$\Delta = a^2|(m_1-m_2)\cdot(m_2-m_3)\cdot(m_1+m_2-m_2-m_3)|$$

$$\Delta = a^2|(m_1-m_2)(m_2-m_3)(m_1-m_3)|$$

$$\Delta = a^2(m_1-m_2)(m_2-m_3)(m_3-m_1)$$

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8. $\{am_1m_2, a(m_1+m_2)\}, \{am_2m_3, a(m_2+m_3)\}$ and $\{am_3m_1, a(m_3+m_1)\}$

Ans. $\frac{1}{2}a^2(m_2-m_3)(m_3-m_1)(m_1-m_2)$

Sol. $\Delta = \frac{1}{2}\left\|\begin{vmatrix} am_1m_2 & a(m_1+m_2) & 1 \\ am_2m_3 & a(m_2+m_3) & 1 \\ am_3m_1 & a(m_3+m_1) & 1 \end{vmatrix}\right\|$

$R_1 \to R_1 - R_2$ & $R_2 \to R_2 - R_3$

$$\Delta = \frac{a^2}{2}\begin{vmatrix} m_2(m_1-m_3) & (m_1-m_3) & 0 \\ m_3(m_2-m_1) & (m_2-m_1) & 0 \\ m_3m_1 & (m_3+m_1) & 1 \end{vmatrix}$$

$$\Delta = \frac{a^2}{2}(m_1-m_3)(m_2-m_1)\cdot\begin{vmatrix} m_2 & 1 & 0 \\ m_3 & 1 & 0 \\ m_3m_1 & (m_3+m_1) & 1 \end{vmatrix}$$

$$\Delta = \frac{a^2}{2}(m_1-m_2)(m_3-m_1)[(m_2-m_3)]$$

$$\Delta = \frac{a^2}{2}(m_1-m_2)(m_2-m_3)(m_3-m_1)$$

9. $\left\{am_1, \frac{a}{m_1}\right\}, \left\{am_2, \frac{a}{m_2}\right\}$ and $\left\{am_3, \frac{a}{m_3}\right\}$.

Ans. $\frac{1}{2}a^2(m_2-m_3)(m_3-m_1)(m_1-m_2) \div m_2m_2m_3$

Sol. $\Delta = \frac{1}{2}\left\|\begin{vmatrix} am_1 & \frac{a}{m_1} & 1 \\ am_2 & \frac{a}{m_2} & 1 \\ am_3 & \frac{a}{m_3} & 1 \end{vmatrix}\right\|$

$R_1 \to R_1 - R_2$ & $R_2 \to R_2 - R_3$

$$D = \frac{a^2}{2}\begin{vmatrix} m_1-m_2 & \frac{1}{m_1}-\frac{1}{m_2} & 0 \\ m_2-m_3 & \frac{1}{m_2}-\frac{1}{m_3} & 0 \\ m_3 & \frac{1}{m_3} & 1 \end{vmatrix}$$

$$\Delta = \frac{a^2}{2}\begin{vmatrix} m_1-m_2 & \frac{m_2-m_1}{m_1m_2} & 0 \\ m_2-m_3 & \frac{m_3-m_2}{m_2m_3} & 0 \\ m_3 & \frac{1}{m_3} & 1 \end{vmatrix}$$

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$$\Delta = \frac{a^2}{2}(m_1 - m_2)(m_2 - m_3)\begin{vmatrix} 1 & \frac{-1}{m_1 m_2} & 0 \\ 1 & \frac{-1}{m_2 m_3} & 0 \\ m_3 & \frac{1}{m_3} & 1 \end{vmatrix}$$

$$\Delta = \frac{q^2}{2}(m_1 - m_2)(m_2 - m_3)\left(\frac{-1}{m_2 m_3} + \frac{1}{m_1 m_2}\right)$$

$$\Delta = \frac{a^2}{2}(m_1 - m_2)(m_2 - m_3)\left(\frac{-m_1 + m_3}{m_1 m_2 m_3}\right) = \frac{a^2}{2}\frac{(m_1 - m_2)(m_2 - m_3)(m_3 - m_1)}{m_1 m_2 m_3}$$

Q. Prove (by showing that the area of the triangle formed by them is zero) that the following sets of three points are in a straight line:

10. $(1,4), (3,-2)$, and $(-3,16)$.

Sol. $\Delta = \frac{1}{2}\begin{vmatrix} 1 & 4 & 1 \\ 3 & -2 & 1 \\ -3 & 16 & 1 \end{vmatrix} = \frac{1}{2}\{1(-2-16) - 4(3+3) + 1(48-6)\}$

$\Delta = \frac{1}{2}(-18 - 24 + 42) = 0$

Hence three given points are collinear

11. $\left(-\frac{1}{2}, 3\right), (-5,6)$ and $(-8,8)$.

Sol. $\Delta = \frac{1}{2}\begin{vmatrix} -\frac{1}{2} & 3 & 1 \\ -5 & 6 & 1 \\ -8 & 8 & 1 \end{vmatrix}$

$\Delta = \frac{1}{2}\left\{-\frac{1}{2}(6-8) - 3(-5+8) + 1(-40+48)\right\}$

$\Delta = \frac{1}{2}(1 - 9 + 8) = 0$, Hence three given points are collinear

12. $(a, b+c), (b, c+a)$, and $(c, a+b)$.

Sol. $\Delta = \frac{1}{2}\begin{vmatrix} a & b+c & 1 \\ b & c+a & 1 \\ c & a+b & 1 \end{vmatrix} = \frac{1}{2}\begin{vmatrix} a+b+c & b+c & 1 \\ a+b+c & c+a & 1 \\ a+b+c & a+b & 1 \end{vmatrix}$

$c_1 \to c_1 + c_2$

$\Delta = \frac{(a+b+c)}{2}\begin{vmatrix} 1 & b+c & 1 \\ 1 & c+a & 1 \\ 1 & a+b & 1 \end{vmatrix} = 0$, Hence three given points are collinear

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Q. Find the areas of the quadrilaterals the coordinates of whose angular points, taken in order, are:

13. $(1,1),(3,4),(5,-2)$ and $(4,-7)$.

Ans. $20\frac{1}{2}$

Sol. $\Delta=\frac{1}{2}\left|\left[\begin{vmatrix}1 & 1\\ 3 & 4\end{vmatrix}+\begin{vmatrix}3 & 4\\ 5 & -2\end{vmatrix}+\begin{vmatrix}5 & -2\\ 4 & -7\end{vmatrix}+\begin{vmatrix}4 & -7\\ 1 & 1\end{vmatrix}\right]\right|$

$\Delta=\frac{1}{2}|\{(4-3)+(-6-20)+(-35+8)+(4+7)\}|$

$\Delta=\frac{1}{2}|(1-26-27+11)|=\frac{41}{2}=20.5$

14. $(-1,6),(-3,-9),(5,-8)$, and $(3,9)$.

Ans. 96

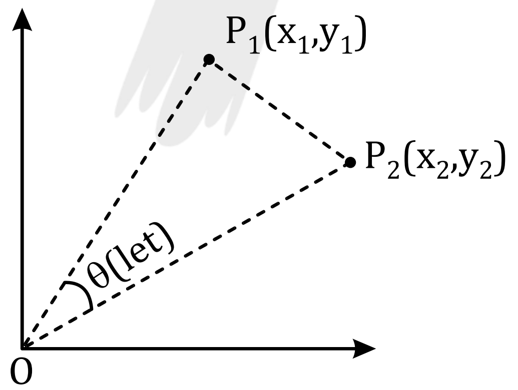
Sol. $\Delta=\frac{1}{2}\left|\left[\begin{vmatrix}-1 & 6\\ -3 & -9\end{vmatrix}+\begin{vmatrix}-3 & -9\\ 5 & -8\end{vmatrix}+\begin{vmatrix}5 & -8\\ 3 & 9\end{vmatrix}+\begin{vmatrix}3 & 9\\ -1 & 6\end{vmatrix}\right]\right|$

$\Delta=\dfrac{1}{2}[(9+18)+(24+45)+(45+24)+(18+9)]$

$\Delta=\dfrac{1}{2}\,.2.\;(27+69)=96$

15. If O be the origin, and if the coordinates of any two points P_1 and P_2 be respectively (x_1,y_1) and (x_2,y_2), prove that: $OP_1\cdot OP_2\cdot\cos\ P_1OP_2=x_1x_2+y_1y_2$.

Sol.



In ΔP_1OP_2

$$\cos\ \theta=\frac{(OP_1)^2+(OP_2)^2-(P_1P_2)^2}{2(OP_1)(OP_2)}$$

$$OP_1\cdot OP_2\cdot\cos\ \theta=\frac{(x_1^2+y_1^2)+(x_2^2+y_2)^2-\{(x_2-x_1)^2+(y_2-y_1)^2\}}{2}$$

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$OP_1 \cdot OP_2 \cdot \cos\ \theta = \frac{x_1^2+y_1^2+x_2^2+y_2^2-x_2^2-x_1^2+2x_1x_2-y_2^2-y_1^2+2y_1y_2}{2}$

$OP_1 . OP_2 \cdot \cos(\angle P_1OP_2) = x_1x_2 + y_1y_2$, Hence Proved.

16. Find the area of the pentagon whose vertices are $A(1,1), B(7,21), C(12,2), D(7,-3)$, and $E(0,-3)$.

Ans. **146 sq. units**

Sol. The required area is

$$\frac{1}{2}\begin{vmatrix} 1 & 1 \\ 7 & 21 \\ 12 & 2 \\ 7 & -3 \\ 0 & -3 \\ 1 & 1 \end{vmatrix} = \frac{1}{2}|(21+14-36-21)-(7+252+14-3)|$$

$= 146$ sq. units

17. Four points $A(6,3), B(-3,5), C(4,-2)$, and $D(x, 2x)$ are given in such a way that $\frac{(\text{Area of } \triangle DBC)}{(\text{Area of } \triangle ABC)} = 1/2$. Find x.

Ans. $\mathbf{\frac{11}{6}}$

Sol. $\frac{\text{Area of } \triangle DBC}{\text{Area of } \triangle ABC} = \frac{1}{2}$

or $\frac{\frac{1}{2}[x(5+2)-3(-2-2x)+4(2x-5)]}{\frac{1}{2}[6(5+2)-3(-2-3)+4(3-5)]} = \frac{1}{2}$

or $\frac{7x+6+6x+8x-20}{42+15-8} = \frac{1}{2}$

or $\frac{3x-2}{7} = \frac{1}{2}$ or $x = \frac{11}{6}$

18. Given three points $P(2,3), Q(4,-2)$ and $R(\alpha, 0)$.
(i) Find the value of α if $PR + RQ$ is minimum
(ii) Find the value of α if $|PR - RQ|$ is maximum.

Ans. **(i)** $\boldsymbol{\alpha = 16/5}$

(ii) $\boldsymbol{\alpha = 8}$

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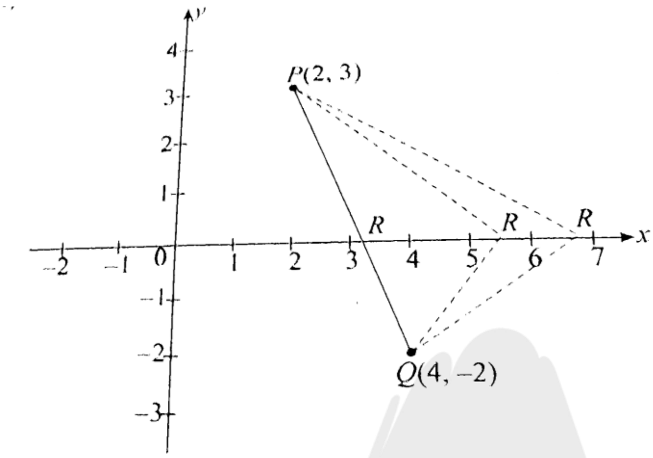
Sol. **(i)** Point R(α, 0) lies on x-axis.



Now, from triangular inequality, PR + RQ ≥ PQ

Thus, minimum value of PR + RQ is PQ, which occurs when points P, Q and R are collinear.

$$\Rightarrow \begin{vmatrix} 2 & 3 \\ 4 & -2 \\ \alpha & 0 \\ 2 & 3 \end{vmatrix} = 0$$

$\Rightarrow \; -4 - 12 + 2\alpha + 3\alpha = 0$

$\Rightarrow \; \alpha = 16/5$

(ii)

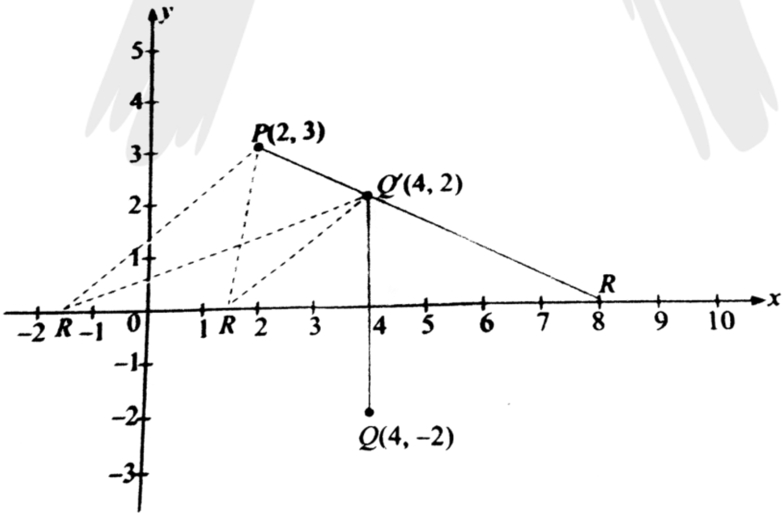


Image of Q in x-axis is Q(4,2).

Now, RQ = RQ′

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$\therefore |PR - RQ| = |PR - RQ|$

From triangular inequality,
$|PR - RQ'| \leq PQ'$.
$\therefore |PR - RQ'|_{max} = PQ'$, when P, Q′ and R are collinear

$$\therefore \begin{vmatrix} 2 & 3 \\ 4 & 2 \\ \alpha & 0 \\ 2 & 3 \end{vmatrix} = 0$$

$\Rightarrow 4 - 12 - 2\alpha + 3\alpha = 0$

$\Rightarrow \alpha = 8$

TRANSFORMATION OF AXES

19. At what point should the origin be shifted if the coordinates of a point $(4,5)$ become $(-3,9)$?

Ans. $\mathbf{(7, -4)}$.

Sol. Let (h, k) be the point to which the origin is shifted. Then, $x = 4, y = 5, X = -3, Y = 9$

$\therefore x = X + h$ and $y = Y + k$

or $4 = -3 + h$ and $5 = 9 + k$
or $h = 7$ and $k = -4$

Hence, the origin must be shifted to $(7, -4)$.

20. If the origin is shifted to the point $(1, -2)$ without the rotation of the axes, what do the following equations become?
(i) $2x^2 + y^2 - 4x + 4y = 0$
(ii) $y^2 - 4x + 4y + 8 = 0$

Ans. **(i)** $\mathbf{2X^2 + Y^2 = 6}$ **(ii)** $\mathbf{Y^2 = 4X}$

Sol. (i) Substituting $x = X + 1$ and $y = Y + (-2) = Y - 2$ in the equation $2x^2 + y^2 - 4x + 4y = 0$, we get

$2(X + 1)^2 + (Y - 2)^2 - 4(X + 1) + 4(Y - 2) = 0$

or $\quad 2X^2 + Y^2 = 6$

(ii) Substituting $x = X + 1$ and $y = Y - 2$ in the equation $y^2 - 4x + 4y + 8 = 0$, we get

$(Y - 2)^2 - 4(X + 1) + 4(Y - 2) + 8 = 0$ or $Y^2 = 4X$

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21. Shift the origin to a suitable point so that the equation $y^2 + 4y + 8x - 2 = 0$ will not contain a term in y and the constant term.

Ans. $(\mathbf{3/4, -2})$.

Sol. Let the origin be shifted to (h, k). Then, $x = X + h$ and $y = Y + k$
Substituting $x = X + h$ and $y = Y + k$ in the equation $y^2 + 4y + 8x - 2 = 0$, we get

$(Y + k)^2 + 4(Y + k) + 8(X + h) - 2 = 0$

$Y^2 + (4 + 2k)Y + 8X + (k^2 + 4k + 8h - 2) = 0$

For this equation to be free from the term containing Y and the constant term, we must have
$4 + 2k = 0$ and $k^2 + 4k + 8h - 2 = 0$
or $k = -2$ and $h = \frac{3}{4}$
Hence, the origin is shifted at the point $(3/4, -2)$.

22. Point $P(-2,3)$ goes through following transformations in succession:
(i) reflection in line $y = x$
(ii) translation of 4 units to the right
(iii) translation of 5 units up
(iv) reflection in y-axis
Find the coordinates of final position of the point.

Ans. **(–7, 3)**

Sol. Given point is $P(-2,3)$.
Changed position of point P after said transformations are:
(i) reflection in the line $y = x$: $(3, -2)$
(ii) translation of 4 units to the right: $(3 + 4, -2) \equiv (7, -2)$
(iii) translation of 5 units up: $(7, -2 + 5) \equiv (7,3)$
(iv) reflection in y-axis: $(-7,3)$

ROTATION OF AXES ABOUT ORIGIN

23. The axes are rotated through an angle of $\pi/3$ in the anticlockwise direction with respect to (0,0). Find the coordinates of point (4,2) (w.r.t. old coordinate system) in the new coordinates system.

Ans. $(\mathbf{2 + \sqrt{3}, -2\sqrt{3} + 1})$

Sol. Here, $(x, y) \equiv (4,2)$

and $\theta = \pi/3$

$\therefore\ X = x\cos\theta + y\sin\theta$

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$= 4\cos\frac{\pi}{3} + 2\sin\frac{\pi}{3}$

$= 4 \times \frac{1}{2} + 2 \times \frac{\sqrt{3}}{2} = 2 + \sqrt{3}$
and $Y = -x\sin\theta + y\cos\theta$

$= -4\sin\frac{\pi}{3} + 2\cos\frac{\pi}{3}$

$= -4 \times \frac{\sqrt{3}}{2} + 2 \times \frac{1}{2} = -2\sqrt{3} + 1$

24. The equation of a curve referred to a given system of axes is $3x^2 + 2xy + 3y^2 = 10$. Find its equation if the axes are rotated about the origin through an angle of 45°.

Ans. $\mathbf{2X^2 + Y^2 = 5}$

Sol. We have

$x = X\cos\theta - Y\sin\theta$

$= X\cos 45^\circ - Y\sin 45^\circ = \frac{X}{\sqrt{2}} - \frac{Y}{\sqrt{2}}$

$y = X\sin\theta + Y\cos\theta = X\sin 45^\circ + Y\cos 45^\circ = \frac{X}{\sqrt{2}} + \frac{Y}{\sqrt{2}}$

Hence, the equation $3x^2 + 2xy + 3y^2 = 10$ transforms to

$$3\left(\frac{X}{\sqrt{2}} - \frac{Y}{\sqrt{2}}\right)^2 + 2\left(\frac{X}{\sqrt{2}} - \frac{Y}{\sqrt{2}}\right)\left(\frac{X}{\sqrt{2}} + \frac{Y}{\sqrt{2}}\right) + 3\left(\frac{X}{\sqrt{2}} + \frac{Y}{\sqrt{2}}\right)^2 = 10$$

or $2X^2 + Y^2 = 5$

25. Without rotating the original coordinate axes, to which point should origin be transferred, so that the equation $x^2 + y^2 - 4x + 6y - 7 = 0$ is changed to an equation which contains no term of first degree?

Ans. **(2, – 3)**

Sol. Let origin be shifted at point (h, k) without rotating the coordinate axes.

Now, we replace x by $(x + h)$ and y by $(y + k)$ in the equation of given curve.
Then the transformed equation is

$(x + h)^2 + (y + k)^2 - 4(x + h) + 6(y + k) - 7 = 0$

$\Rightarrow x^2 + y^2 + x(2h - 4) + y(2k + 6) + h^2 + k^2 - 4h + 6k - 7 = 0$

Since, this equation does not contain the terms of first degree.

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$\therefore 2h - 4 = 0$ and $2k + 6 = 0$

$\Rightarrow h = 2$ and $k = -3$

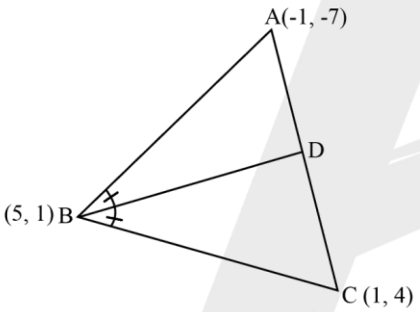
Hence, the point to which the origin should be shifted is $(2, -3)$.

CENTRES OF TRIANGLE

26. The vertices of a triangle are $A(-1, -7)$, $B(5,1)$ and $C(1,4)$. If the internal angle bisector of $\angle B$ meets the side AC in D, then find the length of AD.

Ans. $\frac{10\sqrt{2}}{3}$

Sol. The angle bisector of $\angle B$ meets the opposite side in D.



Using angle bisector theorem, we have

$AD: DC = AB: BC$

Now, $AB = \sqrt{(5+1)^2 + (1+7)^2} = 10$
and $BC = \sqrt{(5-1)^2 + (1-4)^2} = 5$

$$\therefore D \equiv \left(\frac{2(1) + 1(-1)}{2+1}, \frac{2(4) + 1(-7)}{2+1}\right) \equiv \left(\frac{1}{3}, \frac{1}{3}\right)$$

$$\therefore BD = \sqrt{\left(5 - \frac{1}{3}\right)^2 + \left(1 - \frac{1}{3}\right)^2} = \frac{10\sqrt{2}}{3}$$

27. The vertices of a triangle are $A(x_1, x_1 \tan \theta_1)$, $B(x_2, x_2 \tan \theta_2)$, and $C(x_3, x_3 \tan \theta_3)$. If the circumcenter of $\triangle ABC$ coincides with the origin and $H(a, b)$ is the orthocenter, show that

$$\frac{a}{b} = \frac{\cos \theta_1 + \cos \theta_2 + \cos \theta_3}{\sin \theta_1 + \sin \theta_2 + \sin \theta_3}$$

Ans. $\frac{\cos \theta_1 + \cos \theta_2 + \cos \theta_3}{\sin \theta_1 + \sin \theta_2 + \sin \theta_3}$

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Sol. Since the circumcenter is at the origin, the orthocenter is

$(x_1 + x_2 + x_3, x_1\tan\ \theta_1 + x_2\tan\ \theta_2 + x_3\tan\ \theta_3)$

$\therefore\ a = x_1 + x_2 + x_3$

and $\quad b = x_1\tan\ \theta_1 + x_2\tan\ \theta_2 + x_3\tan\ \theta_3$

Also $x_1^2 + x_1^2\tan\ \theta_1^2 = x_1^2 + x_2^2\tan\ \theta_2^2 = x_3^2 + x_3^2\tan\ \theta_3^2$

or $x_1\sec\ \theta_1 = x_2\sec\ \theta_2 = x_3\sec\ \theta_3 = \lambda$ (say)

Now, $\frac{a}{b} = \frac{x_1+x_2+x_3}{x_1\tan\ \theta_1+x_2\tan\ \theta_2+x_3\tan\ \theta_3}$

$= \frac{\lambda\cos\ \theta_1+\lambda\cos\ \theta_2+\lambda\cos\ \theta_3}{\lambda\cos\ \theta_1\tan\ \theta_1+\lambda\cos\ \theta_2\tan\ \theta_2+\lambda\cos\ \theta_3\tan\ \theta_3} \quad = \frac{\cos\ \theta_1+\cos\ \theta_2+\cos\ \theta_3}{\sin\ \theta_1+\sin\ \theta_2+\sin\ \theta_3}$

28. If $(x_i, y_i), i = 1,2,3$, are the vertices of an equilateral triangle such that

$(x_1 + 2)^2 + (y_1 - 3)^2 = (x_2 + 2)^2 + (y_2 - 3)^2 = (x_3 + 2)^2 + (y_3 - 3)^2$, then

find the value of $\frac{x_1+x_2+x_3}{y_1+y_2+y_3}$.

Ans. **–2/3**

Sol. $(x_1 + 2)^2 + (y_1 - 3)^2 = (x_2 + 2)^2 + (y_2 - 3)^2 = (x_3 + 2)^2 + (y_3 - 3)^2$. Therefore, the circumcenter of the triangle formed by points $(x_1, y_1), (x_2, y_2), (x_3, y_3)$ is $(-2,3)$. But the triangle is equilateral; so, the centroid is $(-2,3)$. Therefore,

$\frac{x_1+x_2+x_3}{3} = -2, \frac{y_1+y_2+y_3}{3} = 3 \qquad$ or $\frac{x_1+x_2+x_3}{y_1+y_2+y_3} = -\frac{2}{3}$