DPP-02

SOLUTION

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1. If $f(x) = \begin{cases} x+\lambda & , x<3 \\ 4, & , x=3 \\ 3x-5 & , x>3 \end{cases}$ is continuous at $x = 3$, then the value of λ is

(A) 4 (B) 3 (C) 2 (D) 1

Ans. **(D)**

Sol. $\text{L.H.L} = f(3)$

$\lim_{h\to 0} f(3-h) = 4$

$\Rightarrow \lim_{h\to 0}(3-h)+\lambda = 4$

$\Rightarrow \lambda = 4-3 = 1$

2. If $f(x) = \begin{cases} kx+1, & \text{if} \quad x \le \pi \\ \cos x, & \text{if} \quad x > \pi \end{cases}$ is continuous at $x = \pi$, then k is equal to

(A) $2/\pi$ (B) $-2/\pi$ (C) $1/\pi$ (D) $-1/\pi$

Ans. **(B)**

Sol. $f(\pi) = \text{R.H.L}$

$\Rightarrow k\pi + 1 = \lim_{h\to 0} f(\pi+h)$

$\Rightarrow k\pi + 1 = \lim_{h\to 0} \cos(\pi+h)$

$\Rightarrow k\pi + 1 = -1$

$\Rightarrow k = \frac{-2}{\pi}$

3. If $f(x) = \begin{cases} ax+1, & \text{if } x \le 3 \\ bx+3, & \text{if} \quad x > 3 \end{cases}$ is continuous at $x = 3$, then $a - b$ is equal to

(A) 1/3 (B) 1/2 (C) 2/3 (D) 3/2

Ans. **(C)**

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Sol. $f(3) = R.H.L$

$3a + 1 = \lim_{h\to 0} b(3 + h) + 3$

$\Rightarrow 3a + 1 = 3b + 3$

$\Rightarrow 3(a - b) = 2$

$\Rightarrow a - b = \frac{2}{3}$

4. Function $f(x) = \begin{cases} -1, & \text{when } x < -1 \\ -x, & \text{when } -1 \le x \le 1 \\ 1, & \text{when } x > 1 \end{cases}$ is continuous

(A) only at $x = 1$ (B) only at $x = -1$

(C) at both $x = 1$ and $x = -1$ (D) neither at $x = 1$ nor at $x = -1$

Ans. **(D)**

Sol. $x = -1$

$L.H.L = -1$

$f(-1) = -(-1)$

$= 1$

$L.H.L \neq R.H.L = f(-1)$

f is not cont.

$x = 1$

$L.H.L = \lim_{h\to 0} f(1 - h)$

$= \lim_{h\to 0} -(1 - h) = -1$

$RHL = 1$

$L.H.L \neq R.H.L$

f is not cont.

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5. If $f(x)=\begin{cases}-x^2 & , x\le 0\\ 5x-4 & , 0<x\le 1\\ 4x^2-3x & , 1<x<2\\ 3x+4 & , x\ge 2\end{cases}$, then f(x) is

(A) continuous at $x=0$ but not at $x=1$ (B) continuous at $x=2$ but not at $x=0$

(C) continuous at $x=0,1,2$ (D) discontinuous at $x=0,1,2$

Ans. **(B)**

Sol. $x=0$

$f(0)=0$

$\text{R.H.L.} = \lim_{h\to 0} f(h)$

$=\lim_{h\to 0} 5h\cdot 4=4$

$f(10)\neq \text{R.H.L}$

$\text{L.H.L} = \lim_{h\to 0} 5(1-h)-y=1$ $f(1)=1$

$\text{R.H.L} = \lim_{h\to 0} 4(1+h)^2-3(1+h)=1$

$\text{L.H.L} = \text{R.H.L} = f(1)$

f is cont at $x=1$

6. If $f(x)=\begin{cases}x+2, & \text{when} & x<1\\ 4x-1, & \text{when} & 1\le x\le 3\\ x^2+5, & \text{when} & x>3\end{cases}$, then correct statement is

(A) $\lim_{x\to 1} f(x)=\lim_{x\to 3} f(x)$ (B) f(x) is continuous at $x=3$

(C) f(x) is continuous at $x=1$ (D) f(x) is continuous at $x=1$ and 3

Ans. **(C)**

Sol. $x=1$

$\text{L.H.L} = \lim_{h\to 0}(1-h)+2=3$

$\text{R.H.L} = \lim_{h\to 0} 4(1+h)-1=4-1=3$

$f(1) = 4-1=3$

$\text{L}\cdot\text{H}\cdot\text{L}=\text{R}\cdot\text{H}\cdot\text{L}=f(1)$

$x=3$

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L.H.L $= \lim_{h\to 0} 4(3-h) - 1 = 11$

R.H.L $= \lim_{h\to 0} (3+h)^2 + 5 = 14$

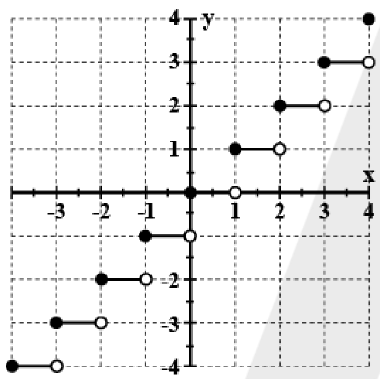
7. Function $f(x) = [x]$ is discontinuous at

(A) every real number (B) every rational number

(C) every integer (D) no where

Ans. **(C)**

Sol. function is break at integer point

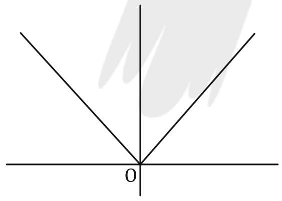


8. In the following, discontinuous function is

(A) $|x|$ (B) $x + |x|$ (C) $x|x|$ (D) $|x|/x$

Ans. **(D)**

Sol. (A)



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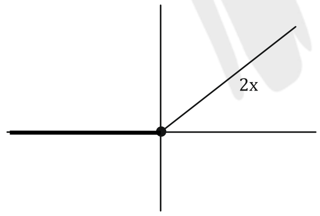

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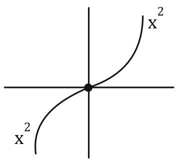

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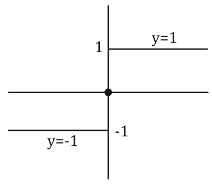

9. If $f(x) = \frac{\tan(\pi/4 - x)}{\cot 2x}\left(x \neq \frac{\pi}{4}\right)$ is everywhere continuous then $f(\pi/4)$ is equal to

(A) 1 (B) -1 (C) 1/2 (D) 2

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Ans. **(C)**

Sol. $f\left(\frac{\pi}{4}\right) = L \cdot HL = R \cdot H \cdot L$

$$f\left(\frac{\pi}{4}\right) = \lim_{h\to 0} f\left(\frac{\pi}{h} + h\right) = \lim_{h\to 0} \frac{\tan\left(\frac{\pi}{4} - \left(\frac{\pi}{4} + h\right)\right)}{\cot\ 2\left(\frac{\pi}{4} + h\right)}$$

$$= \lim_{h\to 0} \frac{\tan\ h}{\tan\ 2h}\left(\frac{0}{0}\right)$$

$$= \lim_{h\to 0} \frac{\sec^2 h}{2\sec^2 2h} = \frac{1}{2}$$

10. If $f(x) = [x/2]$ is discontinuous at $x = a$, then

(A) $a \in N$ (B) $a \in W$ (C) $(a/2) \in Z$ (D) $a \in Q$

Ans. **(C)**

Sol. $\because$ [x] is discontinuous only at integral point

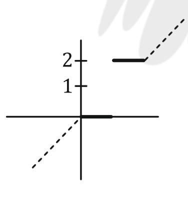
$x \in \mathbb{Z}$

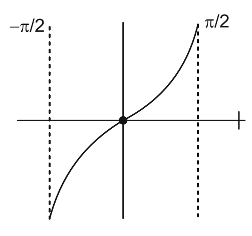
$\frac{a}{2} \in \mathbb{Z}$

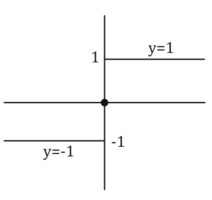
11. Which of the following functions has finite number of points of discontinuity

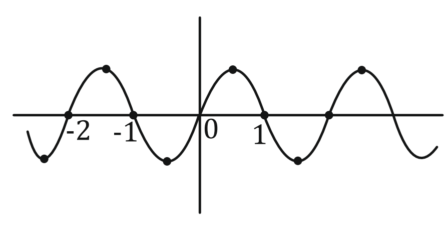
(A) $x + [x]$ (B) $\tan\ x$ (C) $|x|/x$ (D) $\sin\ [\pi x]$

Ans. **(C)**

Sol. (A) 

(B) 

(C) 

(D) 

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12. If $f(x) = \begin{cases} \frac{\sin (a+1)x+\sin x}{x} & ,x < 0 \\ 1/2 & ,x = 0 \\ \frac{x^{3/2}+1}{2} & ,x > 0 \end{cases}$

is continuous at $x = 0$, then the value of a is

(A) $1/2$ (B) $-1/2$ (C) $3/2$ (D) $-3/2$

Ans. **(D)**

Sol. L.H.L = f(0)

$$\Rightarrow \lim_{h\to 0} f(-h) = \frac{1}{2} \Rightarrow \lim_{h\to 0}\left(\left[\frac{\sin (a+h)h}{(a+1)h}\right](a+1) + \frac{\sin\ h}{h}\right) = \frac{1}{2}$$

$$\Rightarrow a + 1 + 1 = \frac{1}{2} \Rightarrow a = \frac{1}{2} - 2 = -\frac{3}{2}$$

13. If $f(x) = \begin{cases} x^a \sin 1/x & ,x \neq 0 \\ 0, & x = 0 \end{cases}$ is continuous at $x = 0$, then

(A) $a < 0$ (B) $a > 0$ (C) $a = 0$ (D) $a \geq 0$

Ans.

Sol. L.H.L=RH.L = f(0)

R.H.L = 0

$\sin (x) \in [-1,1]$

$\Rightarrow \lim_{h\to 0} h^a \cdot \sin \frac{1}{h} = 0$

$a > 0$

$a = 0$

$\sin \frac{1}{h}, h \neq 0$

14. If $f(x) = \begin{cases} x\cos 1/x, & x \neq 0 \\ k & ,x = 0 \end{cases}$ is continuous at $x = 0$, then

(A) $k > 0$ (B) $k < 0$ (C) $k = 0$ (D) $k \geq 0$

Ans. **(C)**

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Sol. L.H.L = R.H.L = f(0)

$\Rightarrow \lim_{h\to 0} h\cos\left(\frac{1}{h}\right) = k$

Note: - cos $\infty \in [-1, 1]$ $\mathbf{k = 0}$

$0 \cdot \cos x = 0$

$k = 0$

15. Function $f(x) = |\sin x| + |\cos x| + |x|$ is discontinuous at

(A) $x = 0$ (B) $x = \pi/2$ (C) $x = \pi$ (D) no where

Ans. **(D)**

Sol. $x = 0$

$\text{L. H. L} = \lim_{h\to 0} |\sin(h)| + |\cosh| + |h| = 1$

$\text{R. H. L} = \lim_{h\to 0} |\sinh| + |\cosh| + |h| = 1$

$f(0) = 1$, f is cont. at $x = 0$

$x = \frac{\pi}{2}$

$\text{L.H.L} = \lim_{h\to 0} \left|\sin\left(\frac{\pi}{2} - h\right)\right| + \left|\cos\left(\frac{\pi}{2} - h\right)\right| + \left|\frac{\pi}{2} - h\right|$

$= 1 + 0 + \frac{\pi}{2} = \frac{\pi}{2} + 1$

$\text{R. H. L.} = \lim_{h\to 0} \left|\sin\left(\frac{\pi}{2} + h\right)\right| + \left|\cos\left(\frac{\pi}{2} + h\right)\right| + \left|\frac{\pi}{2} + h\right|$

$= 1 + 0 + \frac{\pi}{2} = \frac{\pi}{2} + 1$

$f\left(\frac{\pi}{2}\right) = \frac{\pi}{2} + 1$

f is cont at $x = \frac{\pi}{2}$

$x = \pi$

$\text{L. H. L} = \lim_{h\to 0} |\cos(\pi - h)| + |\sin(\pi - h)| + |\pi - h|$

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$= 1 + 0 + \pi = \pi + 1$

$\text{R.H.L} = \pi + 1$

$f(\pi) = \pi + 1$

f is cont. at $x = \pi$

16. If $f(x) = \begin{cases} 1, & x \le 2 \\ ax + b, & 2 < x < 4 \\ 7, & x \ge 4 \end{cases}$ is continuous at $x = 2$ and $x = 4$, then

(A) $a = 3, b = 5$ (B) $a = 3, b = -5$

(C) $a = 0, b = 3$ (D) $a = 0, b = 5$

Ans. **(B)**

Sol. $f(2) = 1$

$\text{R.H.L} = \lim_{h\to 0} a(2 + h) + b$

$= 2a + b$

$2a + b = 1$ .....(i)

$x = 4$

$f(4) = 7$

$\text{L.H.L} = \lim_{h\to 0} a(4 + h) + b = 4a + b$

$4a + b = 7$ .....(ii)

From(i), (ii)

$2a + b = 1$

$4a + b = 7$

$-2a = -6$

$a = 3$

$b + b = 1$

$b = -5$

17. If $f(x) = \frac{\sqrt{a^2 - ax + x^2} - \sqrt{a^2 + ax + x^2}}{\sqrt{a + x} - \sqrt{a - x}}$ is continuous for all values of x, then $f(0)$ is equal to

(A) $a\sqrt{a}$ (B) $\sqrt{a}$ (C) $-\sqrt{a}$ (D) $-a\sqrt{a}$

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Ans. **(C)**

Sol. $f(x) = \left(\sqrt{a^2 - 4x + x^2} - \sqrt{a^2 + ax + x^2}\right) \times \left(\sqrt{a^2 - ax + x^2} + \sqrt{a^2 + ax + x^2}\right)\right) \quad \left(\sqrt{a + x}\right) + \sqrt{a - x}\right)$

$(\sqrt{a + x} + \sqrt{a - x})(\sqrt{a + a} - \sqrt{a - x}) \qquad \left(\sqrt{a^2 - ax + n^2} + \sqrt{a^2 + an + n^2}\right)$

$= \frac{(a^2-ax+x^2)-(a^2+ax+x^2)}{(a+x)-(a-x)} \text{ x } \frac{(\sqrt{a+x}+\sqrt{a-x})}{(\sqrt{a^2-ax+x^2}+\sqrt{a^2+ax+x^2})}$

$f(0) = \frac{-2ax}{2x} \times \frac{2\sqrt{a}}{2a} = -\sqrt{a}$

18. $f(x) = \begin{cases} \frac{x-4}{|x-4|} + a & , \quad x < 4 \\ a + b & , \quad x = 4 \\ \frac{x-4}{|x-4|} + b & , \quad x > 4 \end{cases}$ is continuous at $x = 4$, if

(A) $a = 0, b = 0$
(B) $a = 1, b = 1$
(C) $a = 1, b = -1$
(D) $a = -1, b = 1$

Ans. **(B)**

Sol. L.H.L $= \lim_{h\to 0} \frac{(4-h)-4}{|y-h-4|} + a = \lim_{h\to 0} \frac{-h}{h} + a = a + 1$

R.H.L $= \lim_{h\to 0} \frac{4 + h - 4}{|4 + h - 4\ |} + b = \lim_{h\to 0} \frac{h}{h} + b = b + 1$

$f(4) = a + b \quad a + 1 = a + b \quad b + 1 = a + b$

$\Rightarrow b = 1 \qquad a = 1$

19. If $f(x) = \lim_{n\to\infty} (\sin\ x)^{2n}$, then $f(x)$ is

(A) continuous at $x = \pi$
(B) discontinuous at $x = \pi/2$
(C) discontinuous at $x = -\pi/2$
(D) discontinuous at an infinite number of points.

Ans. **(BCD)**

Sol. **Note: -** $\lim\limits_{x\to\infty} x^{2n} = \begin{cases} 0, & |x| < 1 \\ 1, & |x| = 1 \end{cases}$

$f(x) = \lim\limits_{n\to\infty} (\sin\ x)^{2n} = \begin{cases} 0, & \sin\ x\ |< 1 \\ 1, & |\sin\ x| = 1 \end{cases}$

$f(x)$ is continuous' all except ' 1 '

$|\sin x| = 1 \Rightarrow x = (2n + 1)\frac{\pi}{2}$ is point of discontinuity