Nishant Jindal

QUADRATIC EQUATION

HW

(1) Draw graph.

A) $y = x^2 + 2x + 5$

2) $y = -2x^2 + 3x - 5$

3) $y = 2x^2 - x + 1$

Q. If α, β are Roots of Eqn. $2x^2 - 5x + 3 = 0$ then $\alpha^2\beta + \beta^2\alpha = ?$

$\frac{15}{2}$, $-\frac{15}{4}$, $\frac{15}{4}$, $-\frac{15}{2}$

Q. α, β are Roots of $p(x^2 + n^2) + pnx + qn^2x^2 = 0$ then value of $p(\alpha^2 + \beta^2) + p\alpha\beta + q\alpha^2\beta^2 = ?$

$\alpha + \beta$ $\quad$ 0 $\quad$ $p + q$ $\quad$ $\alpha + \beta + p + q$

Q. α & β are Roots of $ax^2 - bx + c = 0$ then value of $(\alpha + 1)(\beta + 1) = ?$

$\frac{a - b + c}{a}$, $\frac{a + b - c}{a}$, $\frac{a + b + c}{a}$, $\frac{b - a + c}{a}$

Q. If DOR of $x^2 - px + q = 0$ is 1 then $p^4 + 4q^2 = ?$

$2q + 3$ $\quad$ $(1 - 2q)^2$ $\quad$ $(1 + 2q)^2$ $\quad$ $2q - 3$

Nishant Jindal

QUADRATIC EQUATION

Q If Roots α & β of Eq^n $x^2+px+q=0$

are Such that $3\alpha+4\beta=7$ & $5\alpha-\beta=4$

then $(p,q)=?$

Nishant Jindal

QUADRATIC EQUATION

① $x+y>0$

2 Comments Psbl.

+ve No. will be having greater Magnitude

$x=-2, y=5$

⊕ ≥ Mag ≥

$x=-5, y=2$ ✗

Atleast one of them has to be +ve

② $x+y<0$

atleast one of them should be -ve

-ve No. should be of gotr Mag.

③ $x \cdot y>0$

Either both +ve or Both -ve

4) $x \cdot y<0$

One has to be -ve & other should be +ve

Nishant Jindal

QUADRATIC EQUATION

2 Eqn given.

4) D_1 & D_2 both Carries Responsibility of 2 Root each

5) We made sure that one of them is +ve it implies that atleast 2 Roots are Real.

$a_1x^2+b_1x+c_1$
↓
D_1

$a_2x^2+b_2x+c_2$
↓
D_2

$D_1+D_2>0$	$D_1+D_2<0$	$D_1 \cdot D_2>0$	$D_1 \cdot D_2<0$
1) Atleast one of them must be +ve 2) Hr. May +ve चाहे ना	1) Atleast one of them is -ve 2) Hr. May -ve दोनों ना	1) Same Sign. 2) Either both +ve or both -ve	1) opp sign. 2) one of them +ve other -ve

$D_1+D_2=b^2+d^2>0$

➤ atleast one of D_1 or D_2 Should be +ve

➤ Whosoever in D_1 or D_2 is +ve that gives 2 Real Roots

$\boxed{H.P.}$

Demand.

1) ये 4 deg Eqn वाला Qs है।

2) इसके 4 Roots Possible हैं।

3) He wants us to make sure that out of 4 Roots, 2 Roots Will be Real

Q P.7. Biquad Eqn $(ax^2+bx+c)(ax^2+dx-c)=0$ must have atleast 2 Real Roots & $a,b,c,d \in R$.

$$(ax^2+bx+c)(ax^2+dx-c)=0$$

$$D_1=b^2-4ac\ ,\ D_2=d^2+4ac \xrightarrow[\text{Aa Rhahi}]{\text{सोच से}} D_1+D_2=(b^2-4ac)+d^2+4ac$$

Nishant Jindal

QUADRATIC EQUATION

Roots Under Particular Condition.

$$ax^2+bx+c=0$$

1) $b=0$

$ax^2+c=0$

$x^2=-\frac{c}{a} \Rightarrow x=\pm\sqrt{\frac{-c}{a}}$

$\Rightarrow$ Roots are Equal in Mag but opp in Sign

2) $c=0$

$ax^2+bx=0$

$x(ax+b)=0$

$x=0$ or $x=-b/a$

$\rightarrow$ one Root Zero (sure)

3) $b=0=c$

$ax^2=0 \Rightarrow x^2=0$

$x=0$

$\rightarrow$ Both Roots Zero.

4) $\boxed{a=c}$

$\alpha\cdot\beta=\frac{c}{a}=\frac{a}{a}=1$

$\boxed{\beta=\frac{1}{\alpha}}$

$\rightarrow$ Both Roots Reciprocal.

5) a & c opp Sign.

$\alpha\cdot\beta=\frac{c}{a}=\frac{+}{-}=-ve$

α & β are also of opp Sign.

$+, +, +$

(6) a, b, c Same Sign.

$\alpha+\beta=-\frac{b}{a}=-\frac{(+)}{(+)}=-ve$

$\alpha\cdot\beta=\frac{c}{a}=\frac{+}{+}=+ve$

$\rightarrow$ Both α, β (Roots) $=-ve$

$+ \; - \; +$

(7) a, b, c alternate Sign.

$\alpha+\beta=-\frac{b}{a}=-\frac{(-)}{+}=+ve$

$\alpha\cdot\beta=\frac{c}{a}=\frac{+}{+}=+ve$

α, β both Root $+ve$

$+ \quad + \quad -$

8) Sign of a = Sign of b ≠ Sign of c

$\alpha+\beta=-\frac{b}{a}=-\frac{+}{+}=-ve$

$\alpha\cdot\beta=\frac{c}{a}=\frac{-}{+}=-ve$

gr in Mag should be $-ve$

$\alpha=-5, \; \beta=3$

$\alpha+\beta=-2$

$\alpha\times\beta=-15$

$+ \quad + \quad -$

9) Sign of b = Sign of c ≠ Sign of a

$\alpha+\beta=-\frac{b}{a}=-\frac{+}{-}=+ve \;\Big|\; \alpha\cdot\beta=\frac{c}{a}=\frac{+}{-}=-$

Both Roots opp in Sign & Gr. $+ve$

Nishant Jindal

QUADRATIC EQUATION

Q If α, β ($\boxed{\alpha<\beta}$) are Roots of $x^2+bx+c=0$ $a=1$

$\boxed{c<0<b}$ then P.T. $\alpha<0<\beta<|\alpha|$

Sign of $c = -ve$
Sign of $b = +ve$
Sign of $a = 1 = +ve$

$\alpha+\beta = -\frac{b}{a} = \frac{-(+)}{(+)} = -ve$

$\alpha \cdot \beta = \frac{c}{a} = \frac{(-)}{(+)} = -ve$

One Root +ve, other Root -ve & Hr. Root is -ve

$\alpha=-5, \beta=2$?? $\alpha<\beta$

α, β से छोटा है Par Mag में बड़ा है।

$\text{Mag} = |\alpha| = |-5| = 5$

$\alpha<\underbrace{0<\beta}_{\beta\ +ve}<|\alpha|$

α -ve

Mag of α

Mag of β से Bda

$\boxed{\text{V.I.P}}$

Nishant Jindal

QUADRATIC EQUATION

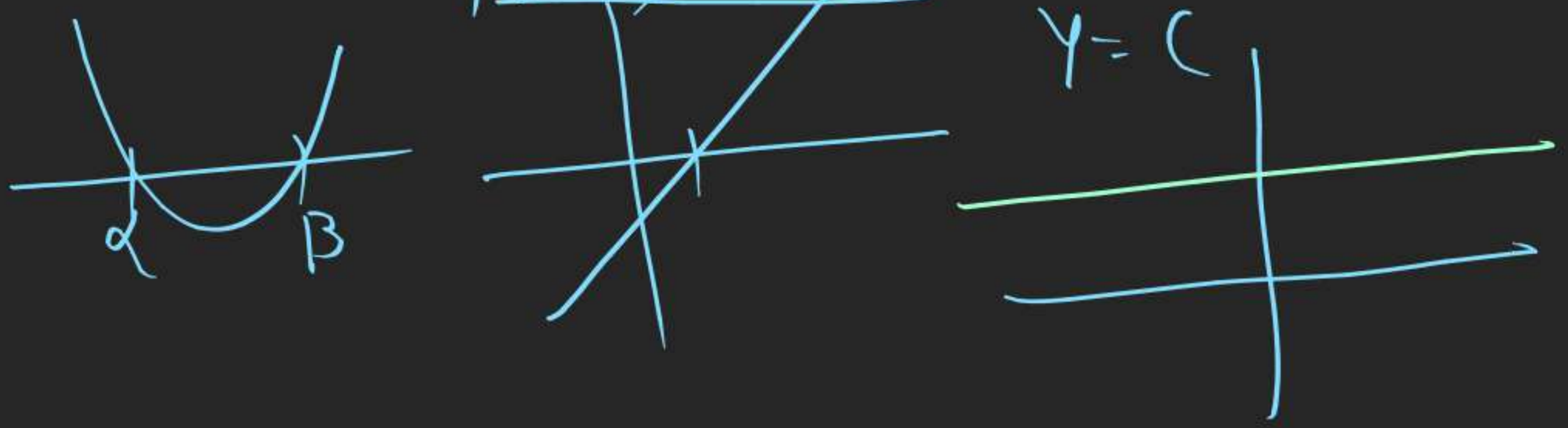
4 Imp points to Remember

1) If Eqn $ax^2+bx+c=0$ has Exactly one Root Zero then $c=0$ & $b\neq 0$

2) If Eqn $ax^2+bx+c=0$ has both Roots Zero then $b=c=0$ & $a\neq 0$

3) If Eqn $\boxed{ax^2+bx+c=0}$ has exactly One Root ∞ then $a=0$ & $b\neq 0$

4) If Eqn $\boxed{\cancel{a x^2}+\cancel{b x}+c=0}$ has both Roots ∞ then $a=0$ & $b=0$ but $c\neq 0$



Nishant Jindal

QUADRATIC EQUATION

Q. $(m^2-3)x^2+3mx+3m+1=0$ & Roots are Reciprocal to each other then $m=?$

$\alpha \cdot \beta = 1$

$\frac{c}{a} = 1$

$\frac{3m+1}{m^2-3} = 1$

$m^2-3 = 3m+1$

$m^2-3m-4=0$

$(m-4)(m+1)=0$

$m=4,-1$

Nishant Jindal

QUADRATIC EQUATION

Q $f(x) = ax^2+bx+c$, $a \neq 0$, $\boxed{a, b, c \in Int}$ Suppose $f(1)=0$ & $50 < f(7) < 60$, $70 < f(8) < 80$ then $f(10) = ?$

$$\boxed{a+b+c=0}$$

$$50 < 49a+7b+c < 60$$

$48a + a + 6b + b + c$

$$50 < 48a+6b < 60 \quad \div 6$$

$$8.\ldots < 8a+b < 10$$

$$8.\ldots < \text{Integer} < 10$$

$$\Rightarrow 8a+b = 9$$

$$70 < 64a+8b+c < 80$$

$$70 < 63a+7b < 80$$

$$10 < 9a+b < 11.\ldots$$

$$10 < \text{Int} < 11.\ldots$$

$$\Rightarrow 9a+b = 11$$

$$\begin{aligned} 9a+b &= 11 \\ 8a+b &= 9 \\ \hline a = 2,\ b &= -7 \end{aligned}$$

$$a+b+c=0$$

$$2-7+c=0$$

$$c=5$$

$$f(x) = 2x^2-7x+5$$

$$f(10) = 2\times10^2-7\times10+5$$

$$= 135$$

Nishant Jindal

QUADRATIC EQUATION

Q a, b, c = Integer & $\boxed{b^2 = 4(ac + 5d^2)}$, $d \in N$

then Roots of $ax^2 + bx + c = 0$ are.

A) Irr. B) Rational & Equal C) Conjugate Complex D) Rational & Unequal

$$b^2 = 4ac + 20d^2$$

$$b^2 - 4ac = 20d^2$$

$$D = 20d^2$$

Irr. No.

$$x = \frac{-b \pm \sqrt{D}}{2a} = \frac{-b \pm \sqrt{20d^2}}{2a} = \frac{-b \pm 2\sqrt{5}\,d}{2a} = \underline{\text{Irr. No.}}$$