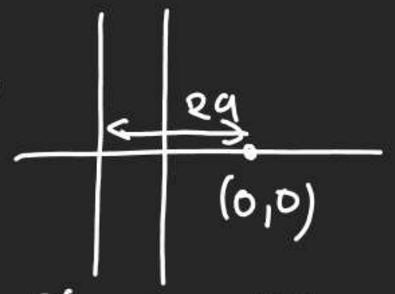
Nishant Jindal

Parabola Ex 1

Q2



4a का है।

$2a = \frac{|-2|}{\sqrt{1^2+1^2}}$

$2a = \sqrt{2}$

LLR = $4a = 2\sqrt{2}$

Q3 (A) $\cos t = \frac{x}{3}$ | $\sin t = \frac{y}{4}$

$\frac{x^2}{9} + \frac{y^2}{16} = 1$ ⊗

(B) $x^2 - 2 = -\cos t$

$= -(2\cos^2\frac{t}{2} - 1)$

$x^2 - 2 = -(\frac{y}{2} - 1)$

$x^2 - 2 = -\frac{(y-2)}{2} + 2$

Parabola

$y = 4\cos^2\frac{t}{2}$

(4) $x = t^2+1$, $y = 2t$ | $x = 25$, $y = \frac{2}{5}$

$t = \frac{y}{2}$

$x = \frac{y^2}{4} + 1$

$\frac{x}{2} = 5$ → $y = \frac{2}{\frac{x}{2}} = \frac{4}{x}$

$x = \frac{16}{x^2 \times y} + 1 \Rightarrow \frac{4+x^2}{x^2} = x$

$x^3 = x^2 + 4$ (x=2)

$y = 2$

(2,2)

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Q5

$y^2 - Kx + 8 = 0$

$y^2 = Kx - 8$

$y^2 = K\left(x - \frac{8}{K}\right)$

$Y^2 = 4AX$

$X = x - \frac{8}{K} \mid 4A = K$

$X = -A$ $\quad x = (1)$

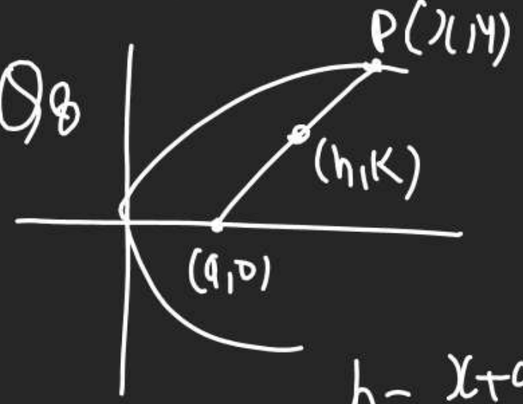
$x - \frac{8}{K} = -\frac{K}{4} \Rightarrow x = \frac{8}{K} - \frac{K}{4}$

$\frac{8}{K} - \frac{K}{4} = 1$ $\quad 32 - K^2 = 4K$

$K^2 + 4K - 32 = 0$ $\quad K = -8, 4$ //

Q7 Qty. Q9 Qty.

Q8



$h = \frac{x+a}{2} \mid K = \frac{y}{2}$

$x = 2h - a \mid y = 2K$

$Y^2 = 4AX$

$4K^2 = 4A(2h - a)$

$y^2 = a(2x - a)$

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Ex 2(1)

Q 1 Wait, 2 Wait

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Lecture.

Q Find EOT at Pt. $(\underset{x_1}{\frac{a}{t^2}}, \underset{y_1}{\frac{2a}{t}})$ on Parabola $y^2 = 4ax$.

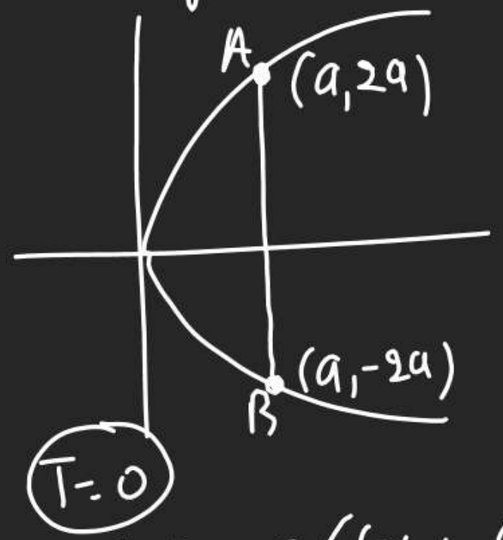
As Pt. lying on Parabola

$T = 0$ is EOT.

$$y\left(\frac{2a}{t}\right) = 2a\left(x + \frac{a}{t^2}\right)$$

EOT: $\boxed{y = xt + \frac{a}{t}}$

Q Find EOT at end Pts of LR of $y^2 = 4ax$



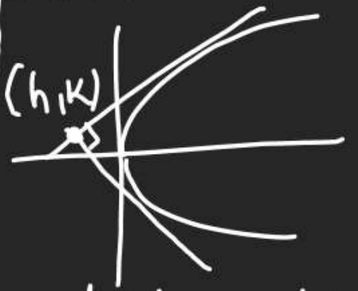
(at A) $y(2a) = 2a(x + a)$

$y = x + a$

(at B) $-y(2a) = 2a(x + a)$

$x + y + a = 0$

Q Find Locus of Pt of Intersection of $\perp^r$ tangents to the Par. $y^2 = 4ax$



(2) (h, k) is outside & making $\perp^r$ tangent.

$y = mx + \frac{a}{m}$ at (h, k)

$k = mh + \frac{a}{m}$

$\Rightarrow m^2h - mk + a = 0 \rightarrow m_1, m_2$

$m_1 + m_2 = \frac{k}{h} \mid m_1 m_2 = \frac{a}{h}$

(3) $m_1 m_2 = -1 \Rightarrow \frac{a}{h} = -1 \Rightarrow \underline{h = -a}$

$\boxed{x = -a}$ is Locus of $\perp^r$ tangents

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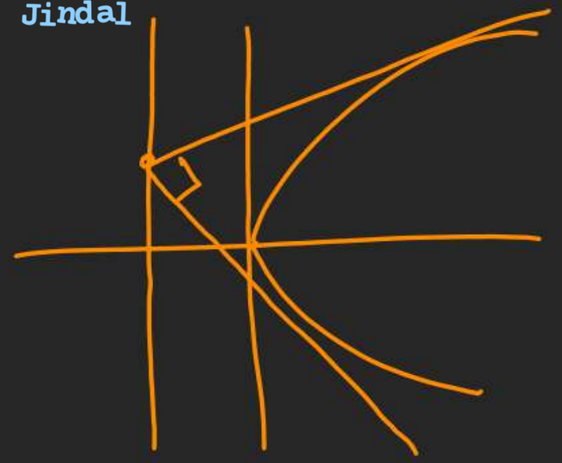

Directrix = ~~Director circle~~ Axis

Directrix is locus of $\perp^r$ tmgents

Q If Line $\sqrt{3}y = 3x+2$ is tmgent to Par. $y^2 = 8x$ find their Point of Contact?

let Line is touching at (x_1,y_1)

$T=0$

EOT: $yy_1 = 4(x+x_1)$

Hmari $\rightarrow 4x - 4y_1 + 4x_1 = 0$

Sarkari $\rightarrow 3x - \sqrt{3}y + 2 = 0$

$\frac{4}{3} = \frac{+4y_1}{+\sqrt{3}} = \frac{4x_1}{2}$

$x_1 = \frac{2}{3}$ $\quad y_1 = \frac{4}{\sqrt{3}}$

Pt of Cont. $\left(\frac{2}{3}, \frac{4}{\sqrt{3}}\right)$

Q5 Find EOT to Par. $y^2 = 12x$ which P.T. $a=3$ $(2,5)$

(1) Position of $(2,5)$

$25 - 12\times 2 > 0$

Pt is outside.

(2) $y = mx + \frac{3}{m}$ P.T. $(2,5)$

$5 = 2m + \frac{3}{m}$

$2m^2 - 5m + 3 = 0$

$2m^2 - 2m - 3m + 3 = 0$

$(2m-3)(m-1) = 0$

$m = \frac{3}{2}, 1$

(3) $(y-5) = \frac{3}{2}(x-2)$ EOT1

$(y-5) = 1(x-2)$ → EOT2

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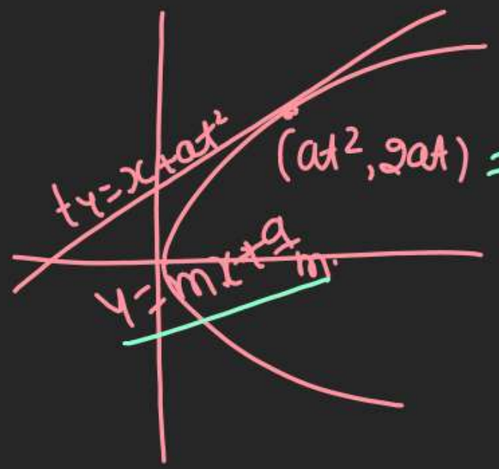
Relation betⁿ Slope form / Parform.



$ty = x + at^2$

$y = mx + \frac{a}{m}$

$\boxed{\frac{t}{1} = \frac{1}{m}} = \frac{at^2}{a/m}$

$m = \frac{1}{t}$

Pt. of Contact for $y = mx + \frac{a}{m}$ is $\left(\frac{a}{m^2}, \frac{2a}{m}\right)$

Q If tangent to $y^2 = 16x$ makes an angle of 45° with x Axis then Pt. of Contact is?

$\left(\frac{a}{m^2}, \frac{2a}{m}\right) = \left(\frac{4}{1^2}, \frac{2\times4}{1}\right)$

$m = \tan 45° = 1$

$a = 4$

$= (4, 8)$

Q EOT to $x^2 = 8y$ which makes angle θ with +ve dir. of x Axis

A) $x = y\cot\theta - 2\tan\theta$

B) $y = x\tan\theta - 2\cot\theta$

(C) $x = y\cot\theta + 2\tan\theta$ //

(D) $y = x\tan\theta + 2\cot\theta$

Inclination $= \theta$

Slope $= m = \tan\theta$

$x^2 = 8y$ → Pt. $(4t, 2t^2)$

$xx_1 = 4(y + y_1)$

$x(4t) = 4(y + 2t^2)$

$tx = y + 2t^2$

$y = \boxed{t}x - 2t^2$ } $m = t$

$y = mx + c$

$mx = y + 2m^2$

$\tan\theta\, x = y + 2\tan^2\theta$

$x = y\cot\theta + 2\tan\theta$

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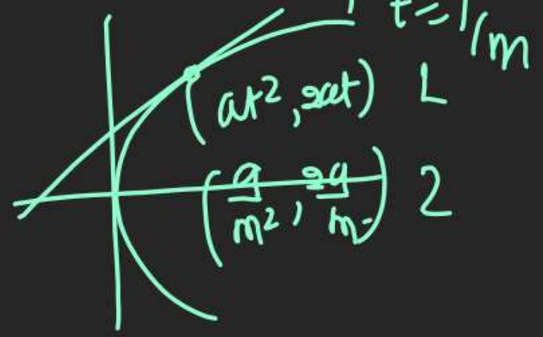
$y^2 = 4ax$

Sl → $y = mx + \frac{a}{m}$

Cart → $yy_1 = 2a(x + x_1)$

Parat → $t x = x + at^2$ → $t = 1/m$



⊥ tangents ka POI = dir.

$x^2 = 4ay$

EOT. → $(2at, at^2)$

$x(x) = 2a(y + y_1)$

$x(2at) = 2a(y + at^2)$

$t x = y + at^2$ → $t = m$

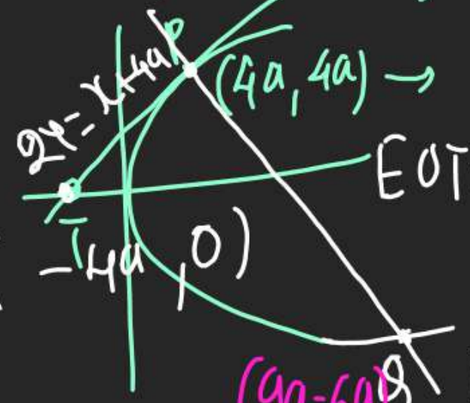
$mx = y + am^2$

tangents ka POI = $at_1t_2, a(t_1+t_2)$

Q. In Parabola $y^2 = 4ax$, tangent at P ∈ P, whose abscissa = latus rectum ($4a$) is equal to the LR, meets axis in T & Normal at P cuts Parabola again in Q

$y^2 = 4a \times 4a = 16a^2 \Rightarrow y = \pm 4a$

P.T. PT : PQ = 4 : 5 ✓H.P



$(4a, 4a) \rightarrow 4(4a) = 2a(x + 4a)$

EOT: $2y = x + 4a \rightarrow Sl = \frac{1}{2}$

$(Sl)_N = -2$

EON → $(y - 4a) = -2(x - 4a)$

$2x + y = 12a$

$PT = \sqrt{(4a+4a)^2 + (4a)^2} = a\sqrt{80} = 4\sqrt{5}a$

$PQ = \sqrt{(9a-4a)^2 + (-6a-4a)^2} = a\sqrt{125} = 5\sqrt{5}a$

$\frac{4\sqrt{5}a}{5\sqrt{5}a} = \frac{4}{5}$

For Q Solving Par & Nor. → $(12a - 2x)^2 = 4ax$

$36a^2 + x^2 - 12ax = ax$

$x = 4a, 9a$ ← $x^2 - 13ax + 36a^2 = 0$

Com. tangent:

Q Find Eqn of Com. tangent to Parabola $y^2 = 8x$ & $xy = -1$

$a = 2$

EOT will be $\Rightarrow y = mx + \frac{2}{m}$

This Tangent also touches $xy = -1$

$x(mx + \frac{2}{m}) = -1$

$A = m^2$
$B = 2$
$C = m$

$\Rightarrow m^2x^2 + m + 2x = 0$ → Q Eqn in x

$D = 0$

$4 - 4m^3 = 0 \Rightarrow m^3 = 1$

$m = 1$

$\therefore$ EOT $\Rightarrow \boxed{y = x + 2}$

XXVI Q 15

the negative end of the axis

EXAMPLES XXVI

Write down the equations to the tangent and normal

1. at the point (4, 6) of the parabola $y^2 = 9x$.
2. at the point of the parabola $y^2 = 6x$ whose ordinate is 12.
3. at the ends of the latus rectum of the parabola $y^2 = 12x$.
4. at the ends of the latus rectum of the parabola $y^2 = 4a(x - a)$.
5. Find the equation to that tangent to the parabola $y^2 = 7x$ which is parallel to the straight line $4y - x + 3 = 0$. Find also its point of contact.
6. A tangent to the parabola $y^2 = 4ax$ makes an angle of $60°$ with the axis; find its point of contact.
7. A tangent to the parabola $y^2 = 8x$ makes an angle of $45°$ with the straight line $y = 3x + 5$. Find its equation and its point of contact.
8. Find the points of the parabola $y^2 = 4ax$ at which (i) the tangent, and (ii) the normal is inclined at $30°$ to the axis.
9. Find the equation to the tangents to the parabola $y^2 = 9x$ which goes through the point (4, 10).
10. Prove that the straight line $x + y = 1$ touches the parabola $y = x - x^2$.
11. Prove that the straight line $y = mx + c$ touches the parabola $y^2 = 4a(x + a)$ if $c = ma + \frac{a}{m}$.
12. Prove that the straight line $lx + my + n = 0$ touches the parabola $y^2 = 4ax$ if $ln = am^2$.
13. For what point of the parabola $y^2 = 4ax$ is (1) the normal equal to twice the subtangent, (2) the normal equal to the difference between the subtangent and the subnormal?

Find the equations to the common tangents of

14. the parabolas $y^2 = 4ax$ and $x^2 = 4by$,
15. the circle $x^2 + y^2 = 4ax$ and the parabola $y^2 = 4ax$.