➢ GENERAL DEFINITION(S):

1. $\sin^{-1} x, \cos^{-1} x, \tan^{-1} x$ etc. denote angles or real numbers whose sine is x, whose cosine is x and whose tangent is x, provided that the answers given are numerically smallest available. These are also written as arcsin x, arc cos x etc. If there are two angles one positive & the other negative having same numerical value, then positive angle should be taken.

2. PRINCIPAL VALUES AND DOMAINS OF INVERSE CIRCULAR FUNCTIONS:

(i) $y = \sin^{-1} x$ where $-1 \le x \le 1; -\frac{\pi}{2} \le y \le \frac{\pi}{2}$ and $\sin y = x$

(ii) $y = \cos^{-1} x$ where $-1 \le x \le 1; 0 \le y \le \pi$ and $\cos y = x$.

(iii) $y = \tan^{-1} x$ where $x \in R; -\frac{\pi}{2} < x < \frac{\pi}{2}$ and $\tan y = x$.

(iv) $y = \operatorname{cosec}^{-1} x$ where $x \le -1$ or $x \ge 1; -\frac{\pi}{2} \le y \le \frac{\pi}{2}, y \ne 0$ and $\operatorname{cosec} y = x$

(v) $y = \sec^{-1} x$ where $x \le -1$ or $x \ge 1; 0 \le y \le \pi; y \ne \frac{\pi}{2}$ and $\sec y = x$

(vi) $y = \cot^{-1} x$ where $x \in R, 0 < y < \pi$ and $\cot y = x$.

Note That:

(a) 1st quadrant is common to all the inverse functions.

(b) 3rd quadrant is not used in inverse functions.

(c) 4th quadrant is used in the clockwise direction i.e. $-\frac{\pi}{2} \le y \le 0$.

3. PROPERTIES OF INVERSE CIRCULAR FUNCTIONS:

P–1

(i) $\sin(\sin^{-1} x) = x, -1 \le x \le 1$

(ii) $\cos(\cos^{-1} x) = x, -1 \le x \le 1$

(iii) $\tan(\tan^{-1} x) = x, x \in R$

(iv) $\sin^{-1}(\sin x) = x, -\frac{\pi}{2} \le x \le \frac{\pi}{2}$

(v) $\cos^{-1}(\cos x) = x; 0 \le x \le \pi$

(vi) $\tan^{-1}(\tan x) = x; -\frac{\pi}{2} < x < \frac{\pi}{2}$

P–2

(i) $\operatorname{cosec}^{-1} x = \sin^{-1}\frac{1}{x}$; $x \le -1, x \ge 1$

(ii) $\sec^{-1} x = \cos^{-1}\frac{1}{x}$; $x \le -1, x \ge 1$

(iii) $\cot^{-1} x = \tan^{-1}\frac{1}{x}$; $x > 0 = \pi + \tan^{-1}\frac{1}{x}$; $x < 0$

P–3

(i) $\sin^{-1}(-x) = -\sin^{-1} x$, $-1 \le x \le 1$

(ii) $\tan^{-1}(-x) = -\tan^{-1} x$, $x \in R$

(iii) $\cos^{-1}(-x) = \pi - \cos^{-1} x$, $-1 \le x \le 1$

(iv) $\cot^{-1}(-x) = \pi - \cot^{-1} x$, $x \in R$

P–4

(i) $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$ $-1 \le x \le 1$

(ii) $\tan^{-1} x + \cot^{-1} x = \frac{\pi}{2}$ $x \in R$

(iii) $\operatorname{cosec}^{-1} x + \sec^{-1} x = \frac{\pi}{2}$ $|x| \ge 1$

P–5 $\tan^{-1} x + \tan^{-1} y = \tan^{-1}\frac{x+y}{1-xy}$ where $x > 0$, $y > 0$ & $xy < 1$

$= \pi + \tan^{-1}\frac{x+y}{1-xy}$ where $x > 0, y > 0$ & $xy > 1$

$\tan^{-1} x - \tan^{-1} y = \tan^{-1}\frac{x-y}{1+xy}$

where $x > 0, y > 0$

P–6 **(i)** $\sin^{-1}x+\sin^{-1}y=\sin^{-1}\left[x\sqrt{1-y^2}+y\sqrt{1-x^2}\right]$ where $x\ge 0, y\ge 0 \& (x^2+y^2)\le 1$

Note that: $x^2+y^2\le 1 \Rightarrow 0\le \sin^{-1}x+\sin^{-1}y\le \frac{\pi}{2}$

(ii) $\sin^{-1}x+\sin^{-1}y=\pi-\sin^{-1}\left[x\sqrt{1-y^2}+y\sqrt{1-x^2}\right]$ where $x\ge 0, y\ge 0 \& x^2+y^2>1$

Note that: $x^2+y^2>1\Rightarrow \frac{\pi}{2}<\sin^{-1}x+\sin^{-1}y<\pi$

(iii) $\sin^{-1}x-\sin^{-1}y=\sin^{-1}\left[x\sqrt{1-y^2}-y\sqrt{1-x^2}\right]$ where $x\ge 0, y\ge 0$

(iv) $\cos^{-1}x\pm\cos^{-1}y=\cos^{-1}\left|xy\mp\sqrt{1-x^2}\sqrt{1-y^2}\right|$ where $x\ge 0, y\ge 0$

P–7 If $\tan^{-1}x+\tan^{-1}y+\tan^{-1}z=\tan^{-1}\left[\frac{x+y+z-xyz}{1-xy-yz-zx}\right]$ if, $x>0, y>0, z>0 \& xy+yz+zx<1$

Note: **(i)** If $\tan^{-1}x+\tan^{-1}y+\tan^{-1}z=\pi$ then $x+y+z=xyz$

(ii) If $\tan^{-1}x+\tan^{-1}y+\tan^{-1}z=\frac{\pi}{2}$ then $xy+yz+zx=1$

P–8 $2\tan^{-1}x=\sin^{-1}\frac{2x}{1+x^2}=\cos^{-1}\frac{1-x^2}{1+x^2}=\tan^{-1}\frac{2x}{1-x^2}$

Note very carefully that:

$$\sin^{-1}\frac{2x}{1+x^2}=\begin{cases}2\tan^{-1}x & \text{if } |x|\le 1\\ \pi-2\tan^{-1}x & \text{if } x>1\\ -(\pi+2\tan^{-1}x) & \text{if } x<-1\end{cases}\qquad \cos^{-1}\frac{1-x^2}{1+x^2}=\begin{cases}2\tan^{-1}x & \text{if } x\ge 0\\ -2\tan^{-1}x & \text{if } x<0\end{cases}$$

$$\tan^{-1}\frac{2x}{1-x^2}=\begin{cases}2\tan^{-1}x & \text{if } |x|<1\\ \pi+2\tan^{-1}x & \text{if } x<-1\\ -(\pi-2\tan^{-1}x) & \text{if } x>1\end{cases}$$

➢ **REMEMBER THAT:**

(i) $\sin^{-1}x+\sin^{-1}y+\sin^{-1}z=\frac{3\pi}{2}\Rightarrow x=y=z=1$

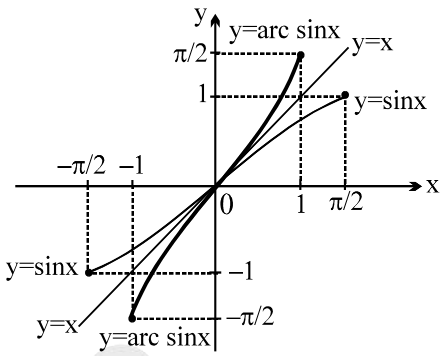
(ii) $\cos^{-1}x+\cos^{-1}y+\cos^{-1}z=3\pi\Rightarrow x=y=z=-1$

(iii) $\tan^{-1}1+\tan^{-1}2+\tan^{-1}3=\pi$ and $\tan^{-1}1+\tan^{-1}\frac{1}{2}+\tan^{-1}\frac{1}{3}=\frac{\pi}{2}$

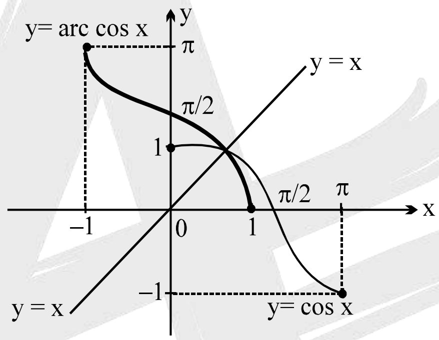
INVERSE TRIGONOMETRIC FUNCTIONS

SOME USEFUL GRAPHS

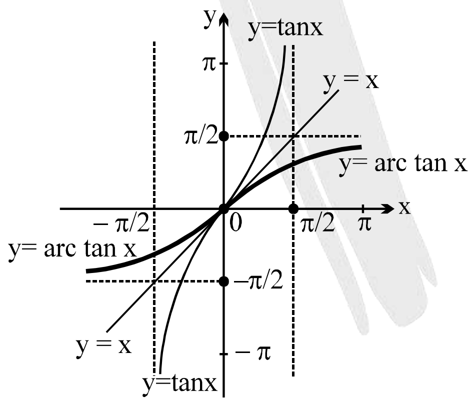
1. $y = \sin^{-1} x, |x| \le 1, y \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$



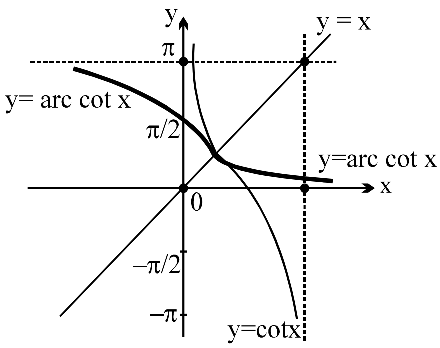
2. $y = \cos^{-1} x, |x| \le 1, y \in [0, \pi]$



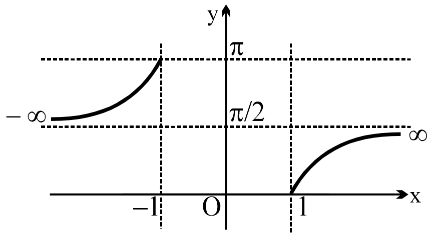
3. $y = \tan^{-1} x, x \in R, y \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$



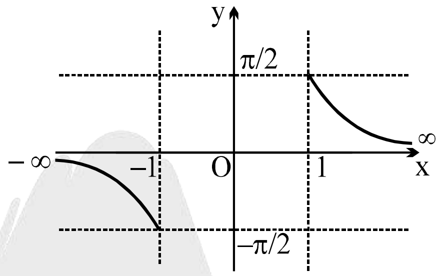
4. $y = \cot^{-1} x, x \in R, y \in (0, \pi)$



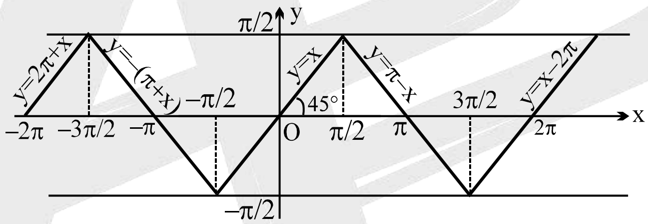
5. $y = \sec^{-1} x, |x| \geq 1, y \in \left[0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \pi\right]$



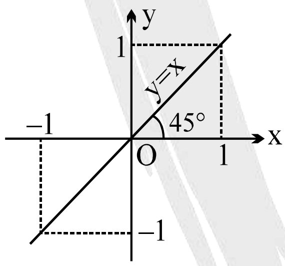
6. $y = \operatorname{cosec}^{-1} x, |x| \geq 1, y \in \left[-\frac{\pi}{2}, 0\right) \cup \left(0, \frac{\pi}{2}\right]$



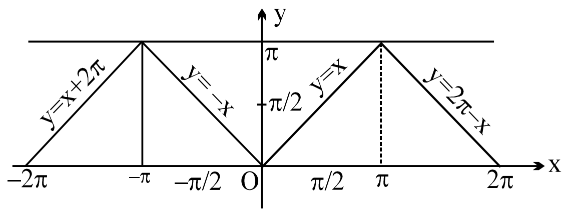
7. **(a)** $y = \sin^{-1}(\sin x), x \in R, y \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, Periodic with period 2π



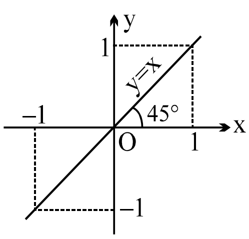
(b) $y = \sin(\sin^{-1} x), = x; x \in [-1,1], y \in [-1,1]$, y is aperiodic



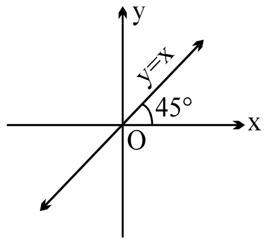
8. **(a)** $y = \cos^{-1}(\cos x), x \in R, y \in [0, \pi]$, periodic with period $2\pi = x$



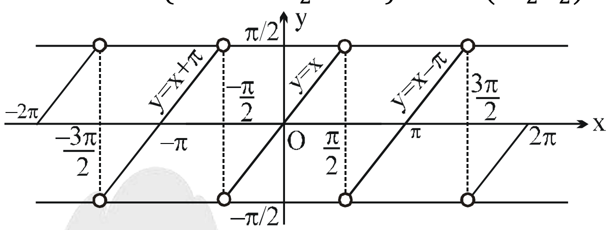
(b) $y = \cos(\cos^{-1} x), = x;\ x \in [-1,1], y \in [-1,1]$, y is aperiodic



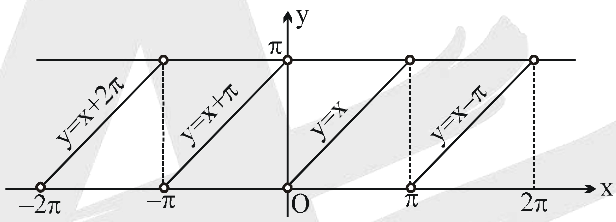
9. **(a)** $y = \tan(\tan^{-1} x), x \in R, y \in R$, y is aperiodic $= x$



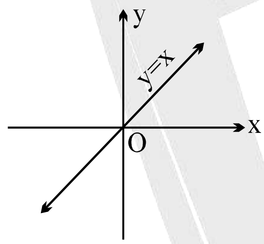
(b) $y = \tan^{-1}(\tan x) = x; x \in R - \left\{(2n-1)\frac{\pi}{2} n \in I\right\},\ y \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, periodic with period π



10. **(a)** $y = \cot^{-1}(\cot x) = x;\ x \in R - \{n\pi\}, y \in (0, \pi)$, periodic with π



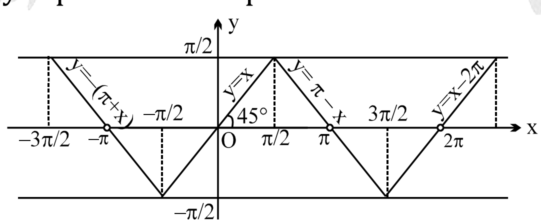
(b) $y = \cot(\cot^{-1} x), = x; x \in R, y \in R$, y is aperiodic



11. **(a)** $y = \mathrm{cosec}^{-1}(\mathrm{cosec}\ x), = x$

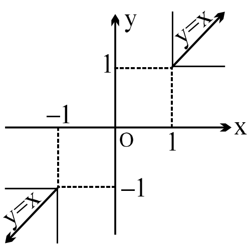
$x; x\varepsilon R - \{n\pi, n\varepsilon\ I\}, y \in \left[-\frac{\pi}{2}, 0\right) \cup \left(0, \frac{\pi}{2}\right]$

y is periodic with period 2π



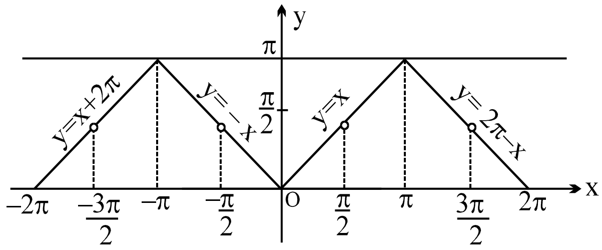
(b) $y = \mathrm{cosec}(\mathrm{cosec}^{-1} x), = x$

$|x| \geq 1,\ |y| \geq 1$, y is aperiodic



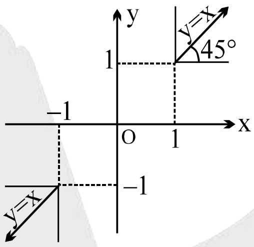
12. **(a)** $y = \sec^{-1}(\sec\ x), =\ x$; y is periodic with period 2π;

$x \in R - \left\{(2n-1)\frac{\pi}{2} n \in I\right\}$ $y \in \left[0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \pi\right]$



(b) $y = \sec(\sec^{-1}\ x), = x$

$|x| \geq 1; |y| \geq 1]$, y is aperiodic



PROFICIENCY TEST-01

1. Find value of:

(i) $\sin\left[\cot^{-1}\left(\cot\frac{17\pi}{3}\right)\right]$

(ii) $\sin^{-1}(\sin(-600°))$

(iii) $\sin\left[2\cos^{-1}\left(-\frac{3}{5}\right)\right]$

(iv) $\tan^{-1}\tan\left(\frac{5\pi}{7}\right)$

(v) $\sin^{-1}\left(\cos\frac{33\pi}{5}\right)$

(vi) $\cos^{-1}\left(\cos\frac{2\pi}{3}\right)+\sin^{-1}\left(\sin\frac{2\pi}{3}\right)$

(vii) $\sin^2\left(\cos^{-1}\frac{1}{2}\right)+\cos^2\left(\sin^{-1}\frac{1}{3}\right)$

(viii) $\cos^{-1}\left[\cos\left(-\frac{17}{15}\pi\right)\right]$

(ix) $\sin\left[\frac{\pi}{2}-\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right]$

(x) $\sec^2(\tan^{-1}2)+\mathrm{cosec}^2(\cot^{-1}3)$

2. If cos–1(a) + cos–1(b) + cos–1(c) = 3π and f(1) = 2, f(x + y) = f(x) f(y) for all x, y ;

then $a^{2f(1)}+b^{2f(2)}+c^{2f(3)}+\frac{(a+b+c)}{a^{2f(1)}+b^{2f(2)}+c^{2f(3)}}$ is equal to:

(A) 0 (B) 1 (C) 2 (D) 3

3. If $\sin^{-1}x+\tan^{-1}x=y(-1<x<1)$, then which is not possible:

(A) $y=\frac{3\pi}{2}$ (B) y = 0 (C) $y=\frac{\pi}{2}$ (D) $y=-\frac{\pi}{2}$

4. The trigonometric equation sin–1x = 2sin–1 a has a solution for:

(A) $\frac{1}{2}<|a|<\frac{1}{\sqrt{2}}$ (B) All real values of a

(C) $|a|<\frac{1}{2}$ (D) $|a|\le\frac{1}{\sqrt{2}}$

5. If $3\cos^{-1}\left(x^2-7x+\frac{25}{2}\right)=\pi$, then x =

(A) only 3 (B) only 4 (C) 3 or 4 (D) None of these

6. The value of $\sin^2\left(\cos^{-1}\frac{1}{2}\right)+\cos^2\left(\sin^{-1}\frac{1}{3}\right)$ is:

(A) $\frac{17}{36}$ (B) $\frac{59}{36}$ (C) $\frac{36}{59}$ (D) None

7. If $\sum_{i=1}^{20}\sin^{-1}x_i=10\pi$ then $\sum_{i=1}^{20}x_i$ is equal to:

(A) 20 (B) 10 (C) 0 (D) None of these

8. If $x+\frac{1}{x}=2$, the principal value of $\sin^{-1}x$ is:

(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{2}$ (C) π (D) $\frac{3\pi}{2}$

9. $\sin^{-1}\sin\frac{23\pi}{7}+\cos^{-1}\cos\frac{39\pi}{7}$

(A) $\frac{\pi}{7}$ (B) $\frac{2\pi}{7}$ (C) $\frac{3\pi}{7}$ (D) $\frac{4\pi}{7}$

10. $\cos^{-1}\sqrt{\frac{1+\cos x}{2}}$; $\forall 0<x<\pi$ is :

(A) x (B) $\frac{x}{2}$ (C) 2x (D) None of these

PROFICIENCY TEST-02

1. $\sec(\operatorname{cosec}^{-1} x)$ is equal to: (where $|x| \geq 1$)

(A) $\operatorname{cosec}(\sec^{-1} x)$
(B) $1/x$
(C) π
(D) Depends on sign of x

2. If $\sin^{-1} x - \cos^{-1} x = \frac{\pi}{6}$, then x is:

(A) $\frac{1}{2}$
(B) $\frac{\sqrt{3}}{2}$
(C) $-\frac{1}{2}$
(D) None of these

3. Solution of equation $\tan(\cos^{-1} x) = \sin\left(\cot^{-1} \frac{1}{2}\right)$ is:

(A) $x = \frac{\sqrt{7}}{3}$
(B) $x = \frac{\sqrt{5}}{3}$
(C) $x = \frac{3\sqrt{5}}{2}$
(D) None of these

4. $\sin^{-1} x + \sin^{-1} \frac{1}{x} + \cos^{-1} x + \cos^{-1} \frac{1}{x} =$

(A) π
(B) $\frac{\pi}{2}$
(C) $\frac{3\pi}{2}$
(D) None of these

5. If $x > 0, \sin^{-1}(2\pi + x) + \cos^{-1}(2\pi + x)$

(A) $2\pi + \frac{\pi}{2}$
(B) $\frac{\pi}{2}$
(C) $x + \frac{\pi}{2}$
(D) None of these

6. If $\sin^{-1} x + \sin^{-1} y = \frac{2\pi}{3}$, then $\cos^{-1} x + \cos^{-1} y =$

(A) $\frac{2\pi}{3}$
(B) $\frac{\pi}{3}$
(C) $\frac{\pi}{6}$
(D) π

7. The value of $\tan\left\{\cos^{-1}\left(\frac{4}{5}\right) + \sin^{-1}\left(\frac{2}{\sqrt{13}}\right)\right\}$ is:

(A) $\frac{7}{16}$
(B) $\frac{17}{6}$
(C) $\frac{6}{17}$
(D) $\frac{16}{7}$

8. The value of $\cos[\tan^{-1} \tan 2]$ is:

(A) $\frac{1}{\sqrt{5}}$
(B) $-\frac{1}{\sqrt{5}}$
(C) $\cos 2$
(D) $-\cos 2$

9. $\cos[\tan^{-1}\{\sin(\cot^{-1} x)\}]$ is equal to -

(A) $\sqrt{\frac{x^2+2}{x^2+3}}$
(B) $\sqrt{\frac{x^2+2}{x^2+1}}$
(C) $\sqrt{\frac{x^2+1}{x^2+2}}$
(D) None of these

10. If $a \leq \tan^{-1} x + \cot^{-1} x + \sin^{-1} x \leq b$. Then:

(A) $a = 0, b = \pi$
(B) $b = \frac{\pi}{2}$
(C) $a = \frac{\pi}{4}$
(D) None of these

PROFICIENCY TEST-03

1. $\tan^{-1} n + \cot^{-1}(n+1)$ is equal to $(n > 0)$:

(A) $\cot^{-1}(n^2 + n + 1)$
(B) $\cot^{-1}(n^2 - n + 1)$
(C) $\tan^{-1}(n^2 + n + 1)$
(D) None of these

2. If $\sin^{-1}(\sin x) = \pi - x$ then x belongs to:

(A) R
(B) $[0, \pi]$
(C) $\left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$
(D) $[\pi, 2\pi]$

3. If $x = 3\tan^{-1}\left(\frac{1}{2}\right) + 2\tan^{-1}\left(\frac{1}{5}\right)$ then,

(A) $\frac{\pi}{4} < x < \frac{\pi}{2}$
(B) $\frac{\pi}{2} < x < \pi$
(C) $\pi < x < \frac{3\pi}{2}$
(D) $0 < x < \frac{\pi}{4}$

4. The principal value of $\cos^{-1}\left\{\frac{1}{\sqrt{2}}\left(\cos\frac{9\pi}{10} - \sin\frac{9\pi}{10}\right)\right\}$ is:

(A) $\frac{3\pi}{20}$
(B) $\frac{7\pi}{20}$
(C) $\frac{7\pi}{10}$
(D) $\frac{17\pi}{20}$

5. $\tan^{-1}\frac{a-b}{1+ab} + \tan^{-1}\frac{b-c}{1+bc} =$ $(a, b, c > 0)$

(A) $\tan^{-1} a - \tan^{-1} b$
(B) $\tan^{-1} a - \tan^{-1} c$
(C) $\tan^{-1} b - \tan^{-1} c$
(D) $\tan^{-1} c - \tan^{-1} a$

6. If $\tan^{-1} 2x + \tan^{-1} 3x = \frac{\pi}{4}$ then $x =$

(A) -1
(B) $\frac{-1}{6}$
(C) $-1, \frac{1}{6}$
(D) $\frac{1}{6}$

7. If $\cos^{-1} x > \sin^{-1} x$, then:

(A) $x < 0$
(B) $-1 < x < 0$
(C) $0 \le x < \frac{1}{\sqrt{2}}$
(D) $-1 \le x < \frac{1}{\sqrt{2}}$

8. $\sin^{-1}\sin 15 + \cos^{-1}\cos 20 + \tan^{-1}\tan 25 =$

(A) $19\pi - 60$
(B) $30 - 9\pi$
(C) $19 - 60\pi$
(D) $60\pi - 19$

9. $\tan\left[\frac{1}{2}\cos^{-1}\left(\frac{\sqrt{5}}{3}\right)\right] =$

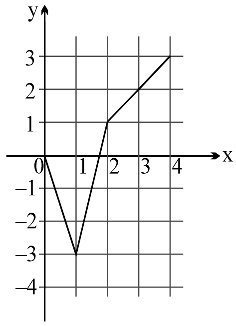
(A) $\frac{3-\sqrt{5}}{2}$
(B) $\frac{3+\sqrt{5}}{2}$
(C) $\frac{2}{3-\sqrt{5}}$
(D) $\frac{2}{3+\sqrt{5}}$

10. If α and β are the roots of the equation $x^2 + 5x - 49 = 0$ then find the value of $\cot^{t}(\cot^{-1}\alpha + \cot^{-1}\beta)$.

EXERXISE – I

1. Given is a partial graph of an even periodic function f whose period is 8. If [*] denotes greatest integer function then find the value of the expression.

$$f(-3) + 2|f(-1)| + \left[f\left(\frac{7}{8}\right)\right] + f(0) + \arccos\,(f(-2)) + f(-7) + f(20)$$



2. **(a) Find the following**

(i) $\tan\left[\cos^{-1}\frac{1}{2} + \tan^{-1}\left(\frac{-1}{\sqrt{3}}\right)\right]$ **(ii)** $\cos^{-1}\left(\cos\frac{7\pi}{6}\right)$

(iii) $\cos\left(\tan^{-1}\frac{3}{4}\right)$ **(iv)** $\tan\left(\sin^{-1}\frac{3}{5} + \cot^{-1}\frac{3}{2}\right)$

(b) Find the following:

(i) $\sin\left[\frac{\pi}{2} - \sin^{-1}\left(\frac{-\sqrt{3}}{2}\right)\right]$ **(ii)** $\cos\left[\cos^{-1}\left(\frac{-\sqrt{3}}{2}\right) + \frac{\pi}{6}\right]$

(iii) $\tan^{-1}\left(\tan\frac{3\pi}{4}\right)$ **(iv)** $\sin\left(\frac{1}{4}\arcsin\frac{\sqrt{63}}{8}\right)$

3. Find the domain of definition the following functions.

(Read the symbols [*] and { * } as greatest integers and fractional part functions respectively.)

(i) $f(x) = \arccos\frac{2x}{1+x}$

(ii) $f(x) = \sqrt{\cos\,(\sin x)} + \sin^{-1}\frac{1+x^2}{2x}$

(iii) $f(x) = \sin^{-1}\left(\frac{x-3}{2}\right) - \log_{10}(4-x)$

(iv) $f(x) = \sin^{-1}(2x + x^2)$

(v) $f(x) = \frac{\sqrt{1-\sin x}}{\log_5(1-4x^2)} + \cos^{-1}(1-\{x\})$, where $\{x\}$ is the fractional part of x.

(vi) $f(x) = \sqrt{3-x} + \cos^{-1}\left(\frac{3-2x}{5}\right) + \log_6(2|x|-3) + \sin^{-1}(\log_2 x)$

(vii) $f(x) = \log_{10}\left(1 - \log_7(x^2 - 5x + 13)\right) + \cos^{-1}\left(\frac{3}{2+\sin\frac{9\pi x}{2}}\right)$

(viii) $f(x) = e^{\sin^{-1}\left(\frac{x}{2}\right)} + \tan^{-1}\left[\frac{x}{2} - 1\right] + \ell n\left(\sqrt{x - [x]}\right)$

(ix) $f(x) = \sqrt{\sin\,(\cos x)} + \ln(-2\cos^2 x + 3\cos x + 1) + e^{\cos^{-1}\left(\frac{2\sin x+1}{2\sqrt{2}\sin x}\right)}$

4. Identify the pair(s) of functions which are identical. Also plot the graphs in each case.

(a) $y = \tan(\cos^{-1} x);\ y = \frac{\sqrt{1-x^2}}{x}$

(b) $y = \tan(\cot^{-1} x);\ y = \frac{1}{x}$

(c) $y = \sin(\arctan x);\ y = \frac{x}{\sqrt{1+x^2}}$

(d) $y = \cos(\arctan x);\ y = \sin(\operatorname{arccot} x)$

5. Find the domain and range of the following functions .

(Read the symbols [*] and { *} as greatest integers and fractional part functions respectively.)

(i) $f(x) = \cot^{-1}(2x - x^2)$ **(ii)** $f(x) = \sec^{-1}(\log_3 \tan x + \log_{\tan x} 3)$

(iii) $f(x) = \cos^{-1}\left(\frac{\sqrt{2x^2+1}}{x^2+1}\right)$ **(iv)** $f(x) = \tan^{-1}\left(\log_{\frac{4}{5}}(5x^2 - 8x + 4)\right)$

6. Let $y = \sin^{-1}(\sin 8) - \tan^{-1}(\tan 10) + \cos^{-1}(\cos 12) - \sec^{-1}(\sec 9) + \cot^{-1}(\cot 6)$ $- \operatorname{cosec}^{-1}(\operatorname{cosec}^7)$. If y simplifies to $a\pi + b$ then find $(a - b)$.

7. Show that : $\sin^{-1}\left(\sin\frac{33\pi}{7}\right) + \cos^{-1}\left(\cos\frac{46\pi}{7}\right) + \tan^{-1}\left(-\tan\frac{13\pi}{8}\right) + \cot^{-1}\left(\cot\left(-\frac{19\pi}{8}\right)\right) = \frac{13\pi}{7}$

8. Let $\alpha = \sin^{-1}\left(\frac{36}{85}\right)$, $\beta = \cos^{-1}\left(\frac{4}{5}\right)$ and $\gamma = \tan^{-1}\left(\frac{8}{15}\right)$, find $(\alpha + \beta + \gamma)$ and hence prove that

(i) $\sum\cot\alpha = \prod\cot\alpha$, **(ii)** $\sum\tan\alpha \cdot \tan\beta = 1$

9. Prove that: $\sin\cot^{-1}\tan\cos^{-1}x = \sin\operatorname{cosec}^{-1}\cot^{-1}x = x$ where $x \in (0,1]$

10. Prove that:

(a) $2\cos^{-1}\frac{3}{\sqrt{13}} + \cot^{-1}\frac{16}{63} + \frac{1}{2}\cos^{-1}\frac{7}{25} = \pi$ **(b)** $\cos^{-1}\left(\frac{5}{13}\right) + \cos^{-1}\left(-\frac{7}{25}\right) + \sin^{-1}\frac{36}{325} = \pi$

(c) $\arccos\sqrt{\frac{2}{3}} - \arccos\frac{\sqrt{6}+1}{2\sqrt{3}} = \frac{\pi}{6}$

11. If $a > b > c > 0$ then find the value of :

$\cot^{-1}\left(\frac{ab+1}{a-b}\right) + \cot^{-1}\left(\frac{bc+1}{b-c}\right) + \cot^{-1}\left(\frac{ca+1}{c-a}\right)$.

12. Find the simplest value of

(a) $f(x) = \arccos x + \arccos\left(\frac{x}{2} + \frac{1}{2}\sqrt{3 - 3x^2}\right), x \in \left(\frac{1}{2}, 1\right)$

(b) $f(x) = \tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right), x \in R - \{0\}$

13. Prove that: $\tan^{-1}\left(\frac{3\sin 2\alpha}{5+3\cos 2\alpha}\right) + \tan^{-1}\left(\frac{\tan\alpha}{4}\right) = \alpha$ (where $-\frac{\pi}{2} < \alpha < \frac{\pi}{2}$)

14. If $\arcsin x + \arcsin y + \arcsin z = \pi$ then prove that $(x, y, z > 0)$

$x\sqrt{1 - x^2} + y\sqrt{1 - y^2} + z\sqrt{1 - z^2} = 2xyz$

15. Prove the identities.

(a) $\sin^{-1}\cos(\sin^{-1}x) + \cos^{-1}\sin(\cos^{-1}x) = \frac{\pi}{2}, |x| \le 1$

(b) $2\tan^{-1}(\operatorname{cosec}\tan^{-1}x - \tan\cot^{-1}x) = \tan^{-1}x\ (x \ne 0)$

(c) $\tan^{-1}\left(\frac{2mn}{m^2-n^2}\right) + \tan^{-1}\left(\frac{2pq}{p^2-q^2}\right) = \tan^{-1}\left(\frac{2MN}{M^2-N^2}\right)$

where $M = mp - nq, N = np + mq, \left|\frac{n}{m}\right| < 1; \left|\frac{q}{p}\right| < 1$ and $\left|\frac{N}{M}\right| < 1$

(d) $\tan(\tan^{-1}x + \tan^{-1}y + \tan^{-1}z) = \cot(\cot^{-1}x + \cot^{-1}y + \cot^{-1}z)$

16. Find all values of k for which there is a triangle whose angles have measure

$\tan^{-1}\left(\frac{1}{2}\right), \tan^{-1}\left(\frac{1}{2}+k\right)$, and $\tan^{-1}\left(\frac{1}{2}+2k\right)$

17. **(a)** Solve the inequality: $(\operatorname{arcsec} x)^2 - 6(\operatorname{arcsec} x) + 8 > 0$

(b) If $\sin^2 x + \sin^2 y < 1, x, y \in R$ then prove that $\sin^{-1}(\tan x \cdot \tan y) \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

18. Let $f(x) = \cot^{-1}(x^2 + 4x + \alpha^2 - \alpha)$ be a function defined $R \rightarrow \left(0, \frac{\pi}{2}\right]$ then find the complete set of real values of α for which $f(x)$ is onto.

19. If $X = \operatorname{cosec} \cdot \tan^{-1} \cdot \cos \cdot \cot^{-1} \cdot \sec \cdot \sin^{-1} a \&\& = \sec \cot^{-1} \sin \tan^{-1} \operatorname{cosec}^{-\cos^{-1} a}$; where $0 \leq a \leq 1$. Find the relation between X & Y. Express them in terms of ' a '.

20. Prove that the equation, $(\sin^{-1} x)^3 + (\cos^{-1} x)^3 = \alpha\pi^3$ has no roots for $\alpha < \frac{1}{32}$ and $\alpha > \frac{7}{8}$

21. Solve the following inequalities:

(a) $(\operatorname{arccot} x)^2 - 5\operatorname{arccot} x + 6 > 0$

(b) $\arcsin x > \arccos x$

(c) $\tan^2(\arcsin x) > 1$

EXERXISE – II

1. If $\alpha = 2\arctan\left(\frac{1+x}{1-x}\right)$ & $\beta = \arcsin\left(\frac{1-x^2}{1+x^2}\right)$ for $0 < x < 1$, then prove that $\alpha + \beta = \pi$. What will the value of $\alpha + \beta$ be if $x > 1$.

2. If $x \in \left[-1, -\frac{1}{2}\right]$ then express the function $f(x) = \sin^{-1}(3x - 4x^3) + \cos^{-1}(4x^3 - 3x)$ in the form of $a\cos^{-1} x + b\pi$, where a and b are rational numbers.

3. Find the sum of the series:

(a) $\cot^{-1} 7 + \cot^{-1} 13 + \cot^{-1} 21 + \cot^{-1} 31 + \cdots$ ... to n terms.

(b) $\tan^{-1}\frac{1}{3} + \tan^{-1}\frac{2}{9} + \cdots .. + \tan^{-1}\frac{2^{n-1}}{1+2^{2n-1}} + \cdots .. \infty$

(c) $\tan^{-1}\frac{1}{x^2+x+1} + \tan^{-1}\frac{1}{x^2+3x+3} + \tan^{-1}\frac{1}{x^2+5x+7} + \tan^{-1}\frac{1}{x^2+7x+13}$ to n terms.

(d) $\sin^{-1}\frac{1}{\sqrt{2}} + \sin^{-1}\frac{\sqrt{2}-1}{\sqrt{6}} + \cdots .. + \sin^{-1}\frac{\sqrt{n}-\sqrt{n-1}}{\sqrt{n(n+1)}} + \cdots .. \infty$

(e) $\text{Lim}_{n\to\infty} \sum_{k=2}^{n} \cos^{-1}\left(\frac{1+\sqrt{(k-1)k(k+1)(k+2)}}{k(k+1)}\right)$

4. Solve the following equations/system of equations:

(a) $\sin^{-1} x + \sin^{-1} 2x = \frac{\pi}{3}$

(b) $\tan^{-1}\frac{1}{1+2x} + \tan^{-1}\frac{1}{1+4x} = \tan^{-1}\frac{2}{x^2}$

(c) $\tan^{-1}(x-1) + \tan^{-1}(x) + \tan^{-1}(x+1) = \tan^{-1}(3x)$

(d) $3\cos^{-1} x = \sin^{-1}\left(\sqrt{1-x^2}(4x^2-1)\right)$

(e) $\tan^{-1}\frac{x-1}{x+1} + \tan^{-1}\frac{2x-1}{2x+1} = \tan^{-1}\frac{23}{36}$

(f) $\sin^{-1} x + \sin^{-1} y = \frac{2\pi}{3}$ & $\cos^{-1} x - \cos^{-1} y = \frac{\pi}{3}$

(g) $2\tan^{-1} x = \cos^{-1}\frac{1-a^2}{1+a^2} - \cos^{-1}\frac{1-b^2}{1+b^2}$ $(a > 0, b > 0)$.

(h) $\cos^{-1}\frac{x^2-1}{x^2+1} + \tan^{-1}\frac{2x}{x^2-1} = \frac{2\pi}{3}$

5. Find the integral values of K for which the system of equations;

$$\left[\begin{array}{l} \arccos x + (\arcsin y)^2 = \frac{K\pi^2}{4} \\ L(\arcsin y)^2 \cdot (\arccos x) = \frac{\pi^4}{16} \end{array}\right.$$ possesses solutions & find those solutions.

6. Find all the positive integral solutions of,

$$\tan^{-1} x + \cos^{-1}\frac{y}{\sqrt{1+y^2}} = \sin^{-1}\frac{3}{\sqrt{10}}.$$

7.	Column-I		Column-II
(A)	$f(x)=\sin^{-1}\left(\frac{2}{\lvert\sin x-1\rvert+\lvert\sin x+1\rvert}\right)$	(P)	f (x) is many one
(B)	$f(x)=\cos^{-1}(\lvert x-1\rvert-\lvert x-2\rvert)$	(Q)	Domain of f (x) is R
(C)	$f(x)=\sin^{-1}\left(\frac{\pi}{\lvert\sin^{-1}x-(\pi/2)\rvert+\lvert\sin^{-1}x+(\pi/2)\rvert}\right)$	(R)	Range contain only irrational Number
(D)	$f(x)=\cos(\cos^{-1}\lvert x\rvert)+\sin^{-1}(\sin x)$ $-\operatorname{cosec}^{-1}(\operatorname{cosec} x)+\operatorname{cosec}^{-1}\lvert x\rvert$	(S)	f (x) is even.

8. Solve the following system of inequations

$4(\arctan x)^2-8\arctan x+3<0\ \&\ 4\operatorname{arccot} x-(\operatorname{arccot} x)^2-3\geq 0$

9. Consider the two equations in x :

(i) $\sin\left(\frac{\cos^{-1}x}{y}\right)=1$ **(ii)** $\cos\left(\frac{\sin^{-1}x}{y}\right)=0$

The sets $X_1,X_2\subseteq[-1,1]$; $Y_1,Y_2\subseteq 1-\{0\}$ are such that

X_1 : the solution set of equation (i) **X_2 :** the solution set of equation (ii)

Y_1 : the set of all integral values of y for which equation (i) possess a solution

Y_2 : the set of all integral values of y for which equation (ii) possess a solution

Let : C_1 be the correspondence : $X_1\to Y_1$ such that xC_1y for $x\in X_1, y\in Y_1\&(x,y)$ satisfy (i).

C_2 be the correspondence: $X_2\to Y_2$ such that xC_2y for $x\in X_2, y\in Y_2\&(x,y)$ satisfy (ii). State with reasons if C_1 & C_2 are functions? If yes, state whether they are bijective or not?

10. If the sum $\sum_{n=1}^{10}\sum_{m=1}^{10}\tan^{-1}\left(\frac{m}{n}\right)=k\pi$, find the value of k.

11. Show that the roots r, s, and t of the cubic $x(x-2)(3x-7)=2$, are real and positive. Also compute the value of $\tan^{-1}(r)+\tan^{-1}(s)+\tan^{-1}(t)$.

12. Solve for x: $\sin^{-1}\left(\sin\left(\frac{2x^2+4}{1+x^2}\right)\right)<\pi-3$.

13. Find the set of values of ' a ' for which the equation $2\cos^{-1}x=a+a^2(\cos^{-1}x)^{-1}$ posses a solution.

14. Let $f:\left[\frac{\pi}{2},\frac{3\pi}{2}\right]\to[-1,1], f(x)=\sin x$; $g:[\pi,2\pi]\to[-1,1], g(x)=\cos x$; $h:\left(\frac{\pi}{2},\frac{3\pi}{2}\right)\to R, h(x)=\tan x$; $u:(\pi,2\pi)\to R, u(x)=\cot x$

	Column I		Column II
(A)	Let $f^{-1}(x)+g^{-1}(x)=k\pi$, then k =	(P)	$\frac{3}{4}$
(B)	The value of x satisfying the equation	(Q)	1
(C)	Let complete range of function $h^{-1}(x)+u^{-1}(x)-g^{-1}(x)$ is $[m\pi,n\pi]$, then m + n =	(R)	2
(D)	The greatest value of function $h^{-1}(x)-f^{-1}(x)$ is $m\pi$, then m =	(S)	$\frac{5}{2}$

EXERXISE – III

1. $\cot^{-1}(\sqrt{\cos\alpha}) - \tan^{-1}(\sqrt{\cos\alpha}) = x$, then $\sin x$ is equal to **[AIEEE 2002]**

(A) $\tan^2\frac{\alpha}{2}$ (B) $\cot^2\frac{\alpha}{2}$ (C) $\tan\alpha$ (D) $\cot\frac{\alpha}{2}$

2. If $\cos^{-1}x - \cos^{-1}\frac{y}{2} = \alpha$, then $4x^2 - 4xy\cos\alpha + y^2$ is equal to - **[AIEEE 2005]**

(A) $2\sin 2\alpha$ (B) 4 (C) $4\sin^2\alpha$ (D) $-4\sin^2\alpha$

3. Let $f: (-1,1) \to B$, be a function defined by $f(x) = \tan^{-1}\frac{2x}{1-x^2}$, then f is both one-one and onto when B is the interval - **[AIEEE 2005]**

(A) $\left(0, \frac{\pi}{2}\right)$ (B) $\left[0, \frac{\pi}{2}\right)$ (C) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ (D) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

4. If $\sin^{-1}\left(\frac{x}{5}\right) + \operatorname{cosec}^{-1}\left(\frac{5}{4}\right) = \frac{\pi}{2}$ then a value of x is : **[AIEEE 2007]**

(A) 1 (B) 3 (C) 4 (D) 5

5. The largest interval lying in $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$ for which the function $f(x) = 4^{-x^2} + \cos^{-1}\left(\frac{x}{2} - 1\right) + \log(\cos x)$, is defined, is- **[AIEEE 2007]**

(A) $[0, \pi]$ (B) $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$ (C) $\left[-\frac{\pi}{4}, \frac{\pi}{2}\right)$ (D) $\left[0, \frac{\pi}{2}\right)$

6. The vale of $\cot\left(\operatorname{cosec}^{-1}\frac{5}{3} + \tan^{-1}\frac{2}{3}\right)$ is: **[AIEEE 2008]**

(A) $\frac{3}{17}$ (B) $\frac{2}{17}$ (C) $\frac{5}{17}$ (D) $\frac{6}{17}$

7. If x, y, z are in A.P. and $\tan^{-1}x, \tan^{-1}y$ and $\tan^{-1}z$ are also in A.P., then : **[IIT Mains 2013]**

(A) $6x = 4y = 3z$ (B) $x = y = z$ (C) $2x = 3y = 6z$ (D) $6x = 3y = 2z$

8. Let $\tan^{-1}y = \tan^{-1}x + \tan^{-1}\left(\frac{2x}{1-x^2}\right)$ where $|x| < \frac{1}{\sqrt{3}}$. Then a value of y is: **[IIT Mains 2015]**

(A) $\frac{3x+x^3}{1+3x^2}$ (B) $\frac{3x-x^3}{1-3x^2}$ (C) $\frac{3x+x^3}{1-3x^2}$ (D) $\frac{3x-x^3}{1+3x^2}$

EXERXISE – IV

1. The number of real solutions of $\tan^{-1}\sqrt{x(x+1)} + \sin^{-1}\sqrt{x^2+x+1} = \frac{\pi}{2}$ is:

[JEE '99, 2 (out of 200)]

(A)zero (B)one (C)two (D) infinite

2. Using the principal values, express the following as a single angle :

$3\tan^{-1}\left(\frac{1}{2}\right) + 2\tan^{-1}\left(\frac{1}{5}\right) + \sin^{-1}\frac{142}{65\sqrt{5}}$. **[REE '99, 6]**

3. Solve, $\sin^{-1}\frac{ax}{c} + \sin^{-1}\frac{bx}{c} = \sin^{-1} x$, where $a^2 + b^2 = c^2, c \neq 0$.

[REE 2000(Mains), 3 out of 100]

4. Solve the equation:

$\cos^{-1}(\sqrt{6}x) + \cos^{-1}\left(3\sqrt{3}x^2\right) = \frac{\pi}{2}$ **[REE 2001 (Mains), 3 out of 100]**

5. If $\sin^{-1}\left(x - \frac{x^2}{2} + \frac{x^3}{4} - \cdots .....\right) + \cos^{-1}\left(x^2 - \frac{x^4}{2} + \frac{x^6}{4} - \cdots ....\right) = \frac{\pi}{2}$ for $0 < |x| < \sqrt{2}$ then x equals to **[JEE 2001(screening)]**

(A) ½ (B) 1 (C) $-1/2$ (D) -1

6. Prove that $\cos\ \tan^{-1}\ \sin\ \cot^{-1}\ x = \sqrt{\frac{x^2+1}{x^2+2}}$ **[JEE 2002 (mains) 5]**

7. Domain of $f(x) = \sqrt{\sin^{-1}(2x) + \frac{\pi}{6}}$ is **[JEE 2003 (Screening) 3]**

(A) $\left(-\frac{1}{2}, \frac{1}{2}\right]$ (B) $\left[-\frac{1}{4}, \frac{3}{4}\right)$ (C) $\left[-\frac{1}{4}, \frac{1}{4}\right]$ (D) $\left[-\frac{1}{4}, \frac{1}{2}\right]$

8. If $\sin(\cot^{-1}(x+1)) = \cos(\tan^{-1} x)$, then $x =$ **[JEE 2004 (Screening)]**

(A) $-\frac{1}{2}$ (B) $\frac{1}{2}$ (C) 0 (D) $\frac{9}{4}$

9. Let (x, y) be such that $\sin^{-1}(ax) + \cos^{-1}(y) + \cos^{-1}(bxy) = \frac{\pi}{2}$

Match the statements in Column I with statements in Column II and indicate your answer by darkening the appropriate bubbles in the 4×4 matrix given in the ORS. **[JEE 2007, 6]**

	Column I		**Column II**
(A)	If $a = 1$ and $b = 0$, then (x, y)	(P)	lies on the circle $x^2 + y^2 = 1$
(B)	If $a = 1$ and $b = 1$, then (x, y)	(Q)	lies on $(x^2 - 1)(y^2 - 1) = 0$
(C)	If $a = 1$ and $b = 2$, then (x, y)	(R)	lies on $y = x$
(D)	If $a = 2$ and $b = 2$, then (x, y)	(S)	lies on $(4x^2 - 1)(y^2 - 1) = 0$

10. If $0 < x < 1$, then $\sqrt{1+x^2}[\{x\cos(\cot^{-1} x) + \sin(\cot^{-1} x)\}^2 - 1]^{1/2} =$ **[JEE 2008, 3]**

(A) $\frac{x}{\sqrt{1+x^2}}$ (B) x

(C) $x\sqrt{1+x^2}$ (D) $\sqrt{1+x^2}$

11. The value of $\cot\left(\sum_{n=1}^{23}\cot^{-1}\left(1+\sum_{k=1}^{n}2k\right)\right)$ is **[JEE Advanced 2013]**

(A) $\frac{23}{25}$ (B) $\frac{25}{23}$ (C) $\frac{23}{24}$ (D) $\frac{24}{23}$

12. Match List I with List II and select the correct answer using the codes given below the lists:

[JEE Advanced 2013]

	List-I		**List-II**
(P)	$\left(\frac{1}{y^2}\left(\frac{\cos(\tan^{-1}y)+y\sin(\tan^{-1}y)}{\cot(\sin^{-1}y)+\tan(\sin^{-1}y)}\right)+y^4\right)^{1/2}$ takes value	(1)	$\frac{1}{2}\sqrt{\frac{5}{3}}$
(Q)	If $\cos x+\cos y+\cos z=0 \sin x+\sin y+\sin z$ then possible value of $\cos\frac{x-y}{2}$ is	(2)	$\sqrt{2}$
(R)	If $\cos\left(\frac{\pi}{4}-x\right)\cos 2x+\sin x\sin 2x\sec x=\cos x\sin 2x\sec x$ $+\cos\left(\frac{\pi}{4}+x\right)\cos 2x$ then possible value of $\sec x$ is	(3)	$1/2$
(S)	If $\cot\left(\sin^{-1}\sqrt{1-x^2}\right)=\sin\left(\tan^{-1}\left(x\sqrt{6}\right)\right), x\neq 0,$ then possible value of x is	(4)	1

Codes:

	(P)	(Q)	(R)	(S)
(A)	4	3	1	2
(B)	4	3	2	1
(C)	3	4	2	1
(D)	3	4	1	2

13. Let $f:[0,4\pi]\to[0,\pi]$ be defined by $f(x)=\cos^{-1}(\cos x)$. The number of points $x\in[0,4\pi]$ satisfying the equation $f(x)=\frac{10-x}{10}$ is _______ . **[JEE Advanced 2014]**

14. If $\alpha=3\sin^{-1}\left(\frac{6}{11}\right)$ and $\beta=3\cos^{-1}\left(\frac{4}{9}\right)$, where the inverse trigonometric functions take only the principal values, then the correct option(s) is (are) **[JEE Advanced 2015]**

(A) $\cos\beta>0$ (B) $\sin\beta<0$ (C) $\cos(\alpha+\beta)>0$ (D) $\cos\alpha<0$

15. Let $E_1=\left\{x\in R: x\neq 1 \text{ and } \frac{x}{x-1}>0\right\}$ and $E_2=\left\{x\in E_1: \sin^{-1}\left(\log_e\left(\frac{x}{x-1}\right)\right) \text{ is a real number}\right\}$

(Here, the inverse trigonometric function $\sin^{-1}x$ assumes values in $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$).

Let $f: E_1\to R$ be the function defined by $f(x)=\log_e\left(\frac{x}{x-1}\right)$ and

$g: E_2\to R$ be the function defined by $g(x)=\sin^{-1}\left(\log_e\left(\frac{x}{x-1}\right)\right)$. **[JEE Advanced 2018]**

	List - I		List – II
(P)	The range of f is	(1)	$\left(-\infty, \frac{1}{1-e}\right] \cup \left[\frac{e}{e-1}, \infty\right)$
(Q)	The range of g contains	(2)	$(0,1)$
(R)	The domain of f contains	(3)	$\left[-\frac{1}{2}, \frac{1}{2}\right]$
(S)	The domain of g is	(4)	$(-\infty, 0) \cup (0, \infty)$
		(5)	$\left(-\infty, \frac{e}{e-1}\right]$
		(6)	$(-\infty, 0) \cup \left(\frac{1}{2}, \frac{e}{e-1}\right]$

The correct option is :

(A) $P \to 4; Q \to 2; R \to 1; S \to 1$

(B) $P \to 3; Q \to 3; R \to 6; S \to 5$

(C) $P \to 4; Q \to 2; R \to 1; S \to 6$

(D) $P \to 4; Q \to 3; R \to 6; S \to 5$

16. The number of real solutions of the equation $\sin^{-1}\left(\sum_{i=1}^{\infty} x^{i+1} - x\sum_{i=1}^{\infty}\left(\frac{x}{2}\right)^i\right) = \frac{\pi}{2} - \cos^{-1}\left(\sum_{i=1}^{\infty}\left(-\frac{x}{2}\right)^i - \sum_{i=1}^{\infty}(-x)^i\right)$ lying in the interval $\left(-\frac{1}{2}, \frac{1}{2}\right)$ is ________ .

(Here, the inverse trigonometric functions $\sin^{-1} x$ and $\cos^{-1} x$ assume values in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ and $[0, \pi]$, respectively.) **[JEE Advanced 2018]**

ANSWER KEY

PROFICIENCY TEST-01

1. **(i)** $\frac{\sqrt{3}}{2}$ **(ii)** $\frac{\pi}{3}$ **(iii)** $-\frac{24}{25}$ **(iv)** $-\frac{2\pi}{7}$ **(v)** $\frac{-\pi}{10}$ **(vi)** π **(vii)** $\frac{59}{36}$

(viii) $\frac{13\pi}{15}$ **(ix)** $\frac{1}{2}$ **(x)** 15

2. C **3.** A **4.** D **5.** C **6.** B **7.** A **8.** B

9. A **10.** B

PROFICIENCY TEST-02

1. A **2.** B **3.** B **4.** A **5.** D **6.** B **7.** B

8. D **9.** C **10.** A

PROFICIENCY TEST-03

1. C **2.** C **3.** B **4.** D **5.** B **6.** D **7.** D

8. B **9.** A **10.** 10

EXERCISE–I

1. 5

2. **(a)** **(i)** $\frac{1}{\sqrt{3}}$, **(ii)** $\frac{5\pi}{6}$, **(iii)** $\frac{4}{5}$, **(iv)** $\frac{17}{6}$;

(b) **(i)** $\frac{1}{2}$, **(ii)** -1, **(iii)** $-\frac{\pi}{4}$, **(iv)** $\frac{\sqrt{2}}{4}$

3. **(i)** $-1/3 \le x \le 1$ **(ii)** $\{1, -1\}$ **(iii)** $1 \le x < 4$

(iv) $[-(1+\sqrt{2}), (\sqrt{2}, -1)]$ **(v)** $x \in (-1/2, 1/2), x \neq 0$ **(vi)** $(3/2, 2]$

(vii) $\{7/3, 25/9\}$ **(viii)** $(-2,2) - \{-1,0,1\}$ **(ix)** $\left\{x \mid x = 2n\pi + \frac{\pi}{6}, n \in I\right\}$

4. (a), (b), (c) and (d) all are identical.

5. **(i)** $D: x \in R$ $R: [\pi/4, \pi)$

(ii) $D: x \in \left(n\pi, n\pi + \frac{\pi}{2}\right) - \left\{x \mid x = n\pi + \frac{\pi}{4}\right\}$ $n \in I$; $R: \left[\frac{\pi}{3}, \frac{2\pi}{3}\right] - \left[\frac{\pi}{2}\right]$

(iii) $D: x \in R$ $R: \left[0, \frac{\pi}{2}\right)$ **(iv)** $D: x \in R$ $R: \left(-\frac{\pi}{2}, \frac{\pi}{4}\right]$

6. 53 **8.** $\pi/2$ **11.** π **12.** **(a)** $\frac{\pi}{3}$; **(b)** $\frac{1}{2}\tan^{-1} x$

16. $k = \frac{11}{4}$ **17.** **(a)** $(-\infty, \sec 2) \cup [1, \infty)$ **18.** $\frac{1 \pm \sqrt{17}}{2}$

19. $X = Y = \sqrt{3 - a^2}$

21. **(a)** $(\cot 2, \infty) \cup (-\infty, \cot 3)$

(b) $\left(\frac{\sqrt{2}}{2}, 1\right]$

(c) $\left(\frac{\sqrt{2}}{2}, 1\right) \cup \left(-1, -\frac{\sqrt{2}}{2}\right)$

EXERCISE–II

1. $-\pi$

2. $6\cos^{-1}x-\frac{9\pi}{2}$, so $a=6, b=-\frac{9}{2}$

3. **(a)** $\text{arccot}\left[\frac{2n+5}{n}\right]$, **(b)** $\frac{\pi}{4}$, **(c)** $\arctan(x+n)-\arctan x$, **(d)** $\frac{\pi}{2}$, (e) $\frac{\pi}{6}$

4. **(a)** $x=\frac{1}{2}\sqrt{\frac{3}{7}}$; **(b)** $x=3$; **(c)** $x=0,\frac{1}{2},-\frac{1}{2}$ **(d)** $\left[\frac{\sqrt{3}}{2},1\right]$

   (e) $x=\frac{4}{3}$; **(f)** $x=\frac{1}{2}, y=1$; **(g)** $x=\frac{a-b}{1+ab}$ **(h)** $x=2-\sqrt{3}$ or $\sqrt{3}$

5. $K=2; \cos\frac{\pi^2}{4}, 1$ & $\cos\frac{\pi^2}{4}, -1$

6. $x=1; y=2$ & $x=2; y=7$

7. **(A)** P, Q, R, S;
   (B) P, Q;
   (C) P, R, S;
   (D) P, R, S

8. $\left(\tan\frac{1}{2}, \cot 1\right]$ Q.9 C_1 is a bijective function, C_2 is many to many correspondence, hence it is not a function

10. $k=25$ **11.** $\frac{3\pi}{4}$ **12.** $x\in(-1,1)$

13. $a\in[-2\pi,\pi]-\{0\}$ **14.** **(A)** –S; **(B)** –Q; **(C)** –R; **(D)** –P

EXERCISE–III

1. A **2.** C **3.** D **4.** B **5.** D **6.** D **7.** B

8. B

EXERCISE-IV

1. C **2.** π **3.** $x\in\{-1,0,1\}$ **4.** $x=1/3$ **5.** B

7. D **8.** A

9. **(A)** P; **(B)** Q; **(C)** P; **(D)** S

10. C **11.** B **12.** B **13.** 3 **14.** BCD **15.** A **16.** 2