SYNOPSIS

Periodic Motion: It refers to a repetitive motion occurring at regular time intervals.

Ex: **1.** The planets orbiting the sun.

Harmonic Motion: When the displacement of a particle in periodic motion is described using harmonic functions such as "sine" or "cosine," it is commonly referred to as Harmonic Motion.

Oscillatory or Vibratory motion: This type of motion involves repetitive back-and-forth movement along the same path, centered around the equilibrium (mean) position.

Simple Harmonic Motion (SHM):

An object is considered to be undergoing Simple Harmonic Motion (SHM) when it moves back and forth along a straight line, centered around its mean position. In this motion, the acceleration at any point is directly proportional to the displacement from the mean position, with equal magnitude but opposite direction, and always directed towards the mean position.

Conditions for a body to be in SHM:

- If 'a' is the acceleration of the body at any given displacement 'y' from the mean position, then for the body in SHM, $a \propto -y$

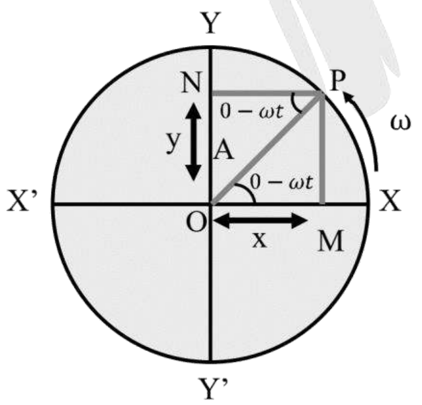
SHM is one of the basic concepts in physics:

Example 1: A particle in uniform circular motion projecting onto any diameter.

Example 2: A simple pendulum oscillating with small amplitudes.

Simple Harmonic Motion and Uniform Circular Motion:

Reference Circle: The projection of a particle engaged in a uniform circular motion onto any diameter serves as a mathematical representation of Simple Harmonic Motion (SHM). Therefore, the circle in this context is referred to as the "Reference circle."



<u>Quantities that characterize a SHM are:</u>

Displacement: A particle executing uniform circular motion (ω = constant) on a circle of radius 'A', at any instant 't', its projection 'N' on the vertical axis has a displacement $y = A\sin(\omega t)$and on the horizontal axis has a displacement $x = A\cos(\omega t)$.

Amplitude(A):

- It represents the maximum displacement of the particle from its mean position.

Time period (T):

- It refers to the time required by the particle to complete one full oscillation.
- $y' = A\sin(\omega t' \pm \phi) = A\sin\left[\omega\left(t + \frac{2\pi}{\omega}\right) \pm \phi\right]$

  $= A\sin(\omega t \pm \phi)$
- In other words, the displacement repeats after a time interval of $\frac{2\pi}{\omega}$ so that $T = \frac{2\pi}{\omega}$

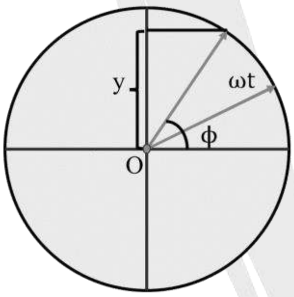
Frequency (f):

- Frequency is the term used to describe the number of oscillations made by a vibrating body in one second.
- Reciprocal of time period is the frequency $f = \frac{1}{T}$
- The SI unit of frequency is hertz or (cycle/sec).
- If T is the time period of oscillation, 'f' is the frequency of SHM. then $T = 2\pi\sqrt{\frac{y}{a}} \Rightarrow f = \frac{1}{2\pi}\sqrt{\frac{a}{y}}$

  where 'a' is the acceleration of SHM at a displacement 'y' from its mean position.

Phase:

- The phase of an oscillating system at any given moment represents its condition concerning both its position and direction of motion.

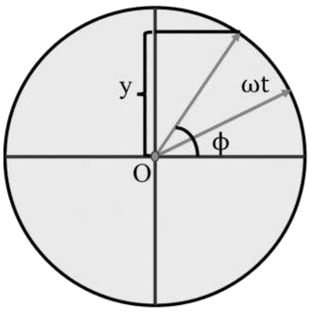


Displacement of SHO: -

At any time t, displacement $y = A\sin(\omega t \pm \phi)$ The argument $(\omega t \pm \phi)$ of the above function is called the phase of motion.

<u>Phase Constant (or) Initial Phase (or) Epoch:</u>

- The constant 'ϕ ' in the phase $\omega t \pm \phi$, is called the initial phase.



Phase Difference:

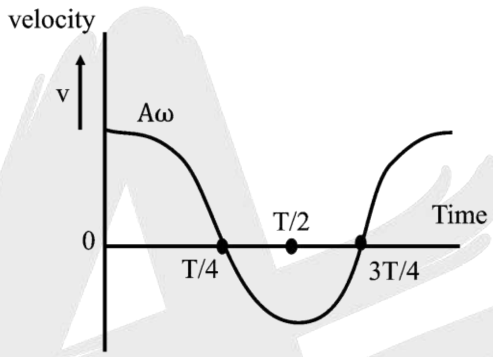
- The phase difference between two SHOs of different frequencies and different initial phases in the time 't' is equal to $(\omega_2 - \omega_1)t + (\phi_2 - \phi_1)$

Velocity of SHO: $y = A\sin(\omega t + \phi)$

$v = v_{max}\sqrt{1-\frac{y^2}{A^2}}$; $v = \omega\sqrt{A^2 - y^2}$,

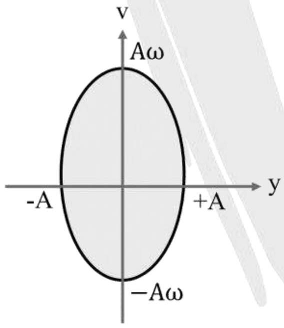
at Mean position v is maximum, $v_{max} = A\omega = \frac{2\pi A}{T}$ at extreme position v is minimum, $v_{min} = 0$

v - t graph: If $y = A\sin\omega t$ then $v = A\omega\cos\omega t$



v - y graph: $v = v_{max}\sqrt{1-\frac{y^2}{A^2}} \Rightarrow \frac{v^2}{v_{max}^2} + \frac{y^2}{A^2} = 1$

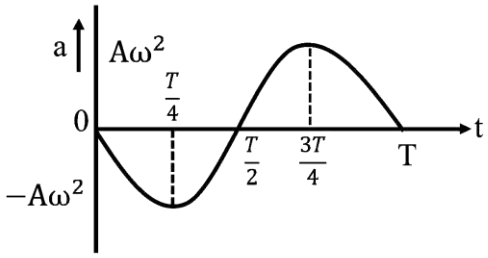
so the v - y graph is an ellipse



Acceleration of SHO: - $a = \frac{dv}{dt}$

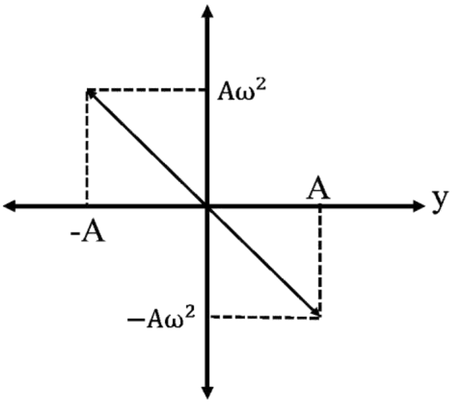
$a = A\omega^2 \sin(\omega t + \phi) \Rightarrow a = -\omega^2 y$

(1) a - t graph : If $y = A\sin\omega t, a = -A\omega^2 \sin\omega t$



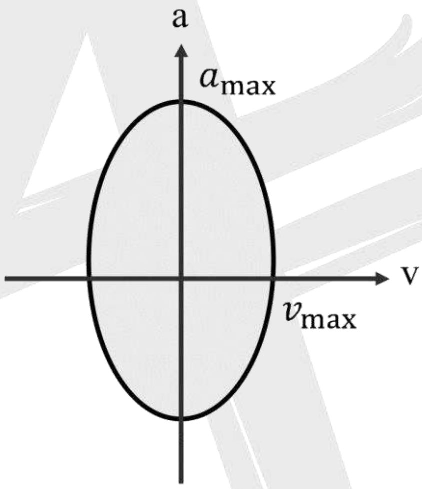
Acceleration – Time curve

(2) a - y graph: $a=-\omega^2 y$ It is a straight line through the origin



➢ Slope of a - y graph gives the square of angular frequency (ω^2)

(3) a - v graph :- $a^2=\omega^4 y^2$ From So, the a - v graph is an ellipse



Restoring force :-

The restoring force, a resultant force, always points towards the mean position, opposite in direction to the displacement, and directly proportional to the displacement of the object.

Force law :- $F \alpha - x \Rightarrow F = -Kx$ here 'K' is called force constant.

Energy in Simple Harmonic Motion

Potential Energy (PE or U): The energy expended to displace a simple harmonic oscillator is converted and stored as potential energy.

➢ If a body is displaced through 'x' from mean position then

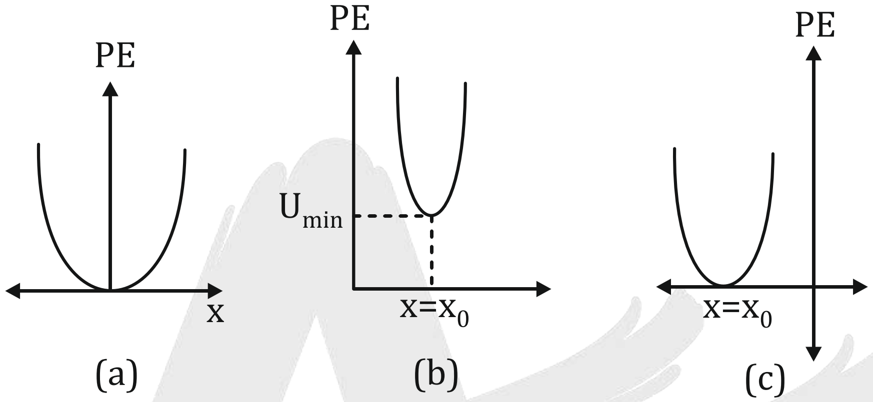
$$U=\int_0^x dW=\int_0^x Kxdx=\frac{1}{2}Kx^2=\frac{1}{2}m\omega^2x^2=\frac{1}{2}m\omega^2A^2\sin^2\omega t$$

Where m - mass of the particle **w** - angular velocity *x* - displacement from the mean position at any instant 't'.

- The reference PE of the SHO can be taken as zero at the mean position. $(U_{min} = 0)$ PE of the SHO is maximum at the extreme position.

$$U = \frac{1}{4}mA^2\omega^2\left[1 - \cos 2(\omega t + \phi)\right]$$

- This function exhibits periodicity with an angular frequency of 2ω.



Kinetic Energy (KE):

$$KE = \frac{1}{2}mv^2 = \frac{1}{2}m\omega^2\left(A^2 - x^2\right) = \frac{1}{2}m\omega^2A^2\cos^2\omega t$$

- $KE_{avg} = \frac{KE_{max} + KE_{min}}{2} = \frac{1}{4}m\omega^2A^2$

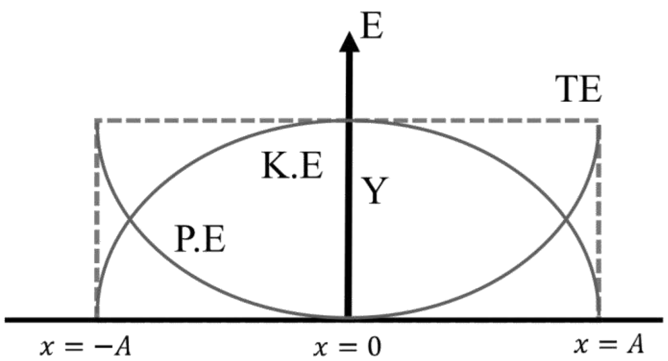
- KE versus time equation can also be written as $KE = \frac{1}{4}mA^2\omega^2\left[1 + \cos^2(\omega t + \phi)\right]$

Total energy (TE): The total mechanical energy of SHO is given by TE = PE + KE

$$TE = \frac{1}{2}m\omega^2x^2 + \frac{1}{2}m\omega^2\left(A^2 - x^2\right)$$

$TE = \frac{1}{2}m\omega^2A^2$ is constant and is independent of displacement 'x'

Energy Displacement Curve:



Some Systems Executing Simple Harmonic Motion

- **Oscillations Due to a Spring:**

  In the case of a spring mass system, the restoring force 'F' acting on the mass when displaced from its mean position by '*x*' is F = -K*x*, where K is spring constant (or) force constant (or) stiffness constant.

- The spring-block system consists of a block of mass M attached to a spring with mass m and force constant K. The time period of oscillation of this system is denoted as $T = 2\pi\sqrt{\frac{\left(M+\frac{m}{3}\right)}{K}}$.

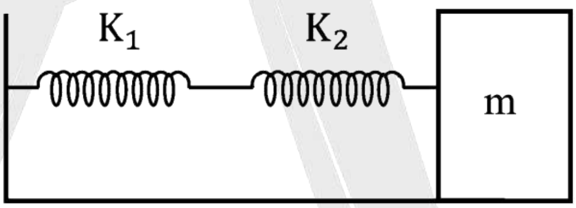
- For a spring the force constant is inversely proportional to its length. i.e., $K \propto \frac{1}{\ell}$

  $\Rightarrow K\ell = \text{constant} \Rightarrow K_1\ell_1 = K_2\ell_2$

- When a spring of force constant K is cut into two parts of lengths ℓ_1 and ℓ_2 having force constants K_1 and K_2 then $K(\ell_1 + \ell_2) = K_1\ell_1 = K_2\ell_2$

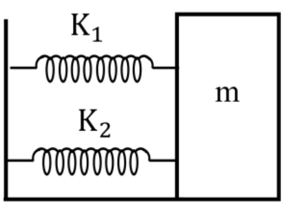
  $K_1 = \frac{K(\ell_1 + \ell_2)}{\ell_1}; K_2 = \frac{K(\ell_1 + \ell_2)}{\ell_2}$

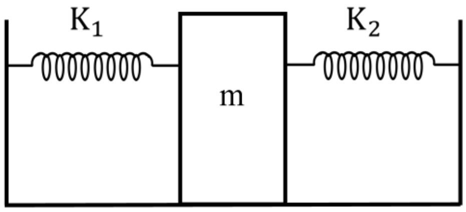
- When two springs of force constants K_1 and K_2 respectively are connected in series then effective force constant K is related as



Series combination $\frac{1}{K_S} = \frac{1}{K_1} + \frac{1}{K_2}$

- When two springs of force constants K_1 and K_2 respectively are connected in parallel then effective force constant K is related as

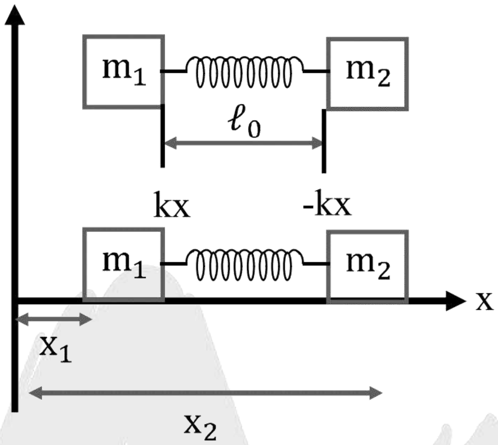




The parallel combination of springs $K_p = K_1 + K_2$

Coupled Oscillator:

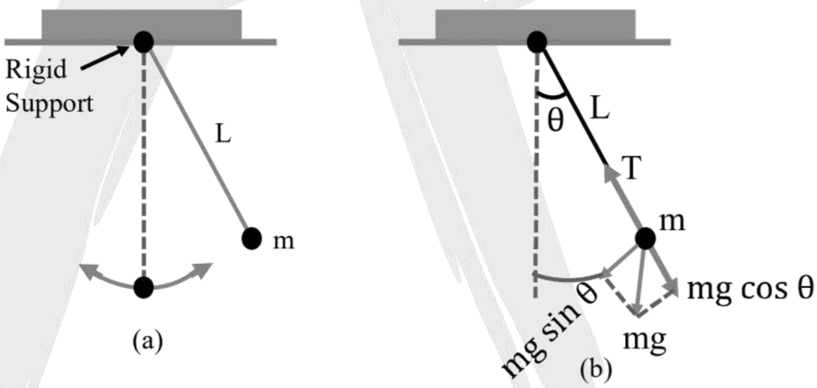
A coupled oscillator is a system of two bodies connected by a spring, allowing both bodies to oscillate with simple harmonic motion along the length of the spring.



For coupled oscillator $\omega = \sqrt{\frac{K}{\mu}}$ and $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{\mu}{K}}$, $\mu = \frac{m_1 m_2}{m_1+m_2}$

Therefore, a two-body oscillator is equivalent to a single-body oscillator with a reduced mass 'm'.

Simple Pendulum:



- Time period of a simple pendulum $T = 2\pi\sqrt{\frac{\ell}{g}}$ for small amplitudes
- If the angular amplitude of the pendulum is not small then

$$T = 2\pi\sqrt{\frac{\ell}{g}}\left[1 + \frac{1}{2^2}\sin^2\left(\frac{\theta_m}{2}\right) + \frac{1}{2^2}\frac{3^2}{4^2}\sin^4\left(\frac{\theta_m}{2}\right) + \ldots\ldots\right]$$

Law of length

- If the length of the pendulum is comparable to the radius of the earth then

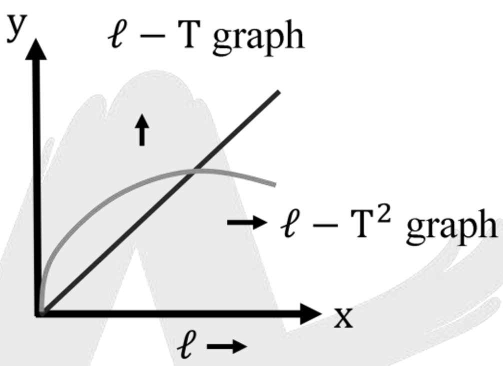
$$T = 2\pi\sqrt{\frac{1}{g\left(\frac{1}{\ell} + \frac{1}{R}\right)}}$$

where R= radius of the earth,

l = length of the pendulum

Law of gravity

- $T \alpha \frac{1}{\sqrt{g}}$ (when 'l' is constant)
- l - T^2 graph of a simple pendulum is a **straight line** passing through the origin.
- l -T graph of a simple pendulum is a **parabola.**



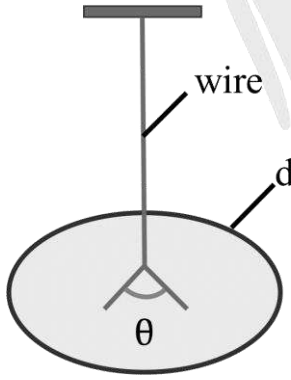
Seconds Pendulum:

The pendulum with a time period equal to 2 seconds is referred to as the "seconds pendulum."

- Length of seconds pendulum is $\ell = \frac{g}{\pi^2}$

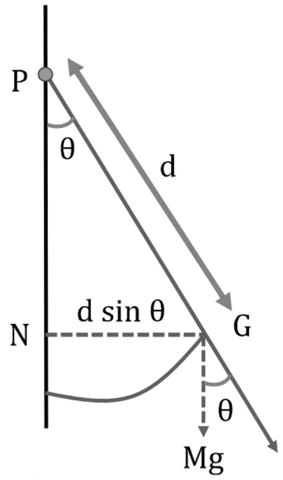
Angular SHM - Torsional Pendulum

When a body is rotated from its equilibrium position, a restoring torque proportional to the angle of rotation arises, causing the body to perform angular (or rotational) Simple Harmonic Motion (SHM).



Time period of oscillation of the torsional pendulum is $T = 2\pi\sqrt{\frac{I}{C}}$ where C is called the torsional constant.

Physical Pendulum (or) Compound Pendulum:



the time period of oscillation of the compound pendulum is $T = 2\pi\sqrt{\frac{I}{Mgd}}$

Damped simple harmonic oscillations, Forced oscillations and resonance

Free oscillation:

- Free oscillations refer to the oscillations of a body with its fundamental frequency under the influence of a restoring force.
- Free oscillations have an infinite quality factor and relaxation time.

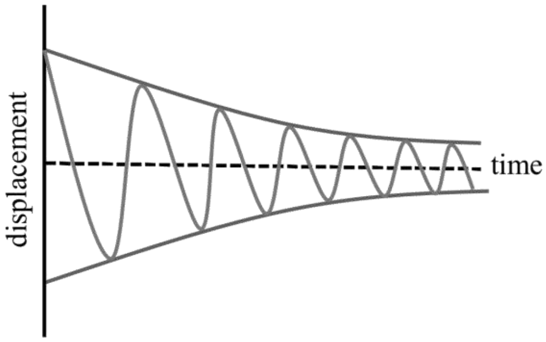
Damped oscillations:

- Damped oscillations are characterized by the oscillations of a body whose amplitude gradually decreases with time.
- Differential equation of damped harmonic oscillator is

  $\frac{d^2x}{dt^2} + 2\gamma\frac{dx}{dt} + \omega_0^2 x = 0$ where $2\gamma = \frac{b}{m}, \omega_0^2 = \frac{K}{m}$
- Solution to the above differential equation is

  $x = A\cos(\omega' t + \phi) = x_m e^{-\gamma t}\cos(\omega' t + \phi)$
- In these oscillations the amplitude of oscillations decreases exponentially due to damping forces like frictional force, viscous force, hysteresis etc.

  $A = x_m e^{-\gamma t}$ where $\gamma = \frac{b}{2m}$.



- Time period of the oscillator $T = \frac{2\pi}{\sqrt{\omega_0^2 - \gamma^2}}$, this is greater than the time period of the harmonic oscillator $T_0 = \frac{2\pi}{\omega_0}$
- Due to the decrease in amplitude, the energy of the oscillator also goes on decreasing exponentially,

$E_K = E_{K_0} e^{-2\gamma t} = \frac{1}{2} K{x^2}_m e^{-bt/m}$

Relaxation time for velocity (τ_v): The time interval, during which the velocity of the harmonic oscillator reduces to 1/e of its initial velocity, is defined as the relaxation time of velocity (t_v).

$V = V_0 e^{-\gamma t}$

Quality Factor

$$Q = 2\pi \times \frac{\text{average energy stored}}{\text{energy loss in one cycle}} = \omega_0 \tau$$

Forced oscillations:

- Forced oscillations refer to the oscillations of a body under the influence of an external periodic force (driver).

The resultant force acting on the oscillator

$F = F_{\text{damping}} + F_{\text{restoring}} + F_{\text{external}}$

$\Rightarrow F = -bv - Kx + F_m \cos \omega_d t$

Where ω_d is the frequency of driven (external) periodic force

- Differential equation of the oscillator

$\frac{d^2x}{dt^2} + 2\gamma \frac{dx}{dt} + \omega_0^2 x = F_m \cos \omega_d t$

Where $2\gamma = \frac{b}{m}$ and $\omega_0^2 = \frac{K}{m}$

- Solution to the above differential equation

$x = A \sin(\omega_d t + \phi)$ with amplitude

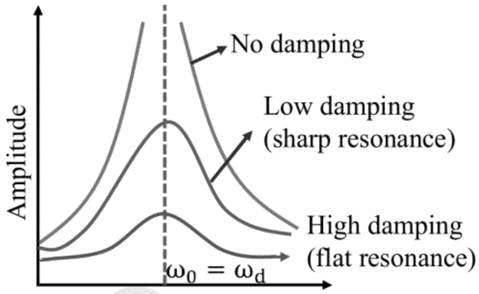
$$A = \frac{F_m/m}{\sqrt{(\omega_0^2 - \omega_d^2)^2 + \left(\frac{b\omega_d}{m}\right)^2}}$$

or $$A = \frac{F_m/m}{\sqrt{(\omega_0^2 - \omega_d^2)^2 + 4\gamma^2 \omega_d^2}}$$

where $\frac{b}{m} = 2\gamma$ and phase $\phi = \tan^{-1}\left(\frac{b\omega_d/m}{\omega_0^2 - \omega_d^2}\right)$

Resonance:

- Resonance occurs when the frequency of the external periodic force (driver) is equal to the natural frequency of the oscillator (driven), resulting in the amplitude increasing to its maximum value.
- The time period of the oscillator, in resonance is $T = \frac{2\pi}{\omega_d} = \frac{2\pi}{\omega_0} = 2\pi\sqrt{\frac{m}{K}}$.



Amplitude resonance:

The amplitude $A = \frac{F_m/m}{\sqrt{(\omega_0^2-\omega_d^2)^2+4\gamma^2\omega_d{}^2}}$ of the forced oscillator becomes maximum when

$(\omega_0^2 - \omega_d^2)^2 + 4\gamma^2\omega_d^2$ is minimum.

Thus $\frac{d}{d\omega_d}\left[(\omega_0^2 - \omega_d^2)^2 + 4\gamma^2\omega_d{}^2\right] = 0$

$\Rightarrow 2(\omega_0^2 - \omega_d^2)(-2\omega_d) + 4\gamma^2(2\omega_d) = 0$

$\Rightarrow \omega_0^2 - \omega_d^2 = 2\gamma^2 \Rightarrow \omega_d = \sqrt{\omega_0^2 - 2\gamma^2}$

Velocity (or energy) at resonance:

Velocity $V = \frac{dx}{dt} \Rightarrow V = A\omega_d \cos(\omega_d t + \phi)$

maximum velocity $V_o = A\omega_d$

$\Rightarrow V_o = \frac{F_m\omega_d}{m\sqrt{(\omega_0^2-\omega_d^2)^2+4\omega^2{}_d\gamma^2}}$

$V_o = \frac{F_m}{m\sqrt{\left(\frac{\omega_0^2-\omega_d^2}{\omega_d}\right)^2+4\gamma^2}}$

this becomes maximum when the denominator is minimum. i.e.,

$\left(\frac{\omega_0^2-\omega_d^2}{\omega_d}\right)^2 = 0 \Rightarrow \omega_0^2 - \omega_d^2 = 0 \Rightarrow \omega_0 = \omega_d$

Composition of Two SHMs of Equal Frequency in Mutually perpendicular Directions:

Let the two SHMs be

(i) $x = a\sin t \Rightarrow \frac{x}{a} = \sin t$

(ii) $y = b\sin(t +) \Rightarrow \frac{y}{b} = \sin(t +)$

$\sin t \cos + \cos t \sin = \frac{y}{b}$

$\frac{x}{a}\cos + \sqrt{1-\frac{x^2}{a^2}}\sin = \frac{y}{b}$

$\sqrt{1-\frac{x^2}{a^2}}\sin = \left(\frac{y}{b} - \frac{x}{a}\cos\right)$

Squaring on both sides

$\left(1-\frac{x^2}{a^2}\right)\sin^2 = \frac{y^2}{b^2} + \frac{x^2}{a^2}\cos^2 - \frac{2xy}{ab}\cos$

$\therefore \left(\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{2xy}{ab}\cos\right) = \sin^2$

This is the equation representing the resultant SHM. The path traversed by the particle, depends on the values of a, b & ϕ.

Note: The expression $T = 2\pi\sqrt{\frac{4m}{k}}$ can be written as $T = 2\pi\sqrt{\frac{m}{k_{eff}}}$, where $k_{eff} = \frac{k}{4}$. The given device is equivalent to a block of mass m connected to a spring of force constant k_{eff}.

EXCERCISE-I

1. The displacement of a particle in SHM is $x = 3\sin(20\pi t) + 4\cos(20\pi t)\,cm$. Its amplitude of oscillation is

(A) 3 cm (B) 4 cm (C) 5 cm (D) 25 cm

2. A particle moves according to the equation $x = a\cos\left(\frac{\pi t}{2}\right)$. The distance covered by it in the time interval between $t = 0$ to $t = 3$ s is

(A) 2a (B) 3a (C) 4a (D) a

3. The frequency of a particle performing SHM is 12 Hz. Its amplitude is 4 cm. Its initial displacement is 2 cm towards positive extreme position. Its equation for displacement is

(A) $x = 0.04\cos\left(24\pi t + \frac{\pi}{4}\right)$ m (B) $x = 0.04\sin(24\pi t)$ m

(C) $x = 0.04\sin\left(24\pi t + \frac{\pi}{6}\right)$ m (D) $x = 0.04\cos(24\pi t)$ m

4. A particle is executing simple harmonic motion between extreme positions given by $(-1,-2,-3)$cm and $(1,2,1)$cm. Its amplitude of oscillation is

(A) 6 cm (B) 4 cm (C) 2 cm (D) 3 cm

5. The velocity of a particle in SHM at the instant when it is 0.6 cm away from the mean position is 4 cm/s. If the amplitude of vibration is 1 cm then its velocity at the instant when it is 0.8 cm away from the mean position is (in cm/s)

(A) 2.25 (B) 2.5 (C) 3.0 (D) 3.5

6. A simple harmonic oscillator is of mass 0.100 kg. It is oscillating with a frequency of $\frac{5}{\pi}$ Hz. If its amplitude of vibration is 5 cm, then force acting on the particle at its extreme position is

(A) 2 N (B) 1.5 N (C) 1 N (D) 0.5 N

7. A small body of mass 10 gram is making harmonic oscillations along a straight line with a time period of $\frac{\pi}{4}$ and the maximum displacement is 10cm. The energy of oscillator is

(A) 0.32×10^{-2} J (B) 0.16×10^{-2} J (C) 0.48×10^{-2} J (C) 0.56×10^{-2} J

8. At what displacement is the KE of a particle performing SHM of ampligutde 10 cm is three times its PE ?

(A) 2.5 cm (B) 5 cm (C) 7.5 cm (D) 10 cm

9. A spring has length ℓ and force constant k it is cut into two springs of length ℓ_1 and ℓ_2 such that $\ell_1 = n\ell_2$ (n = an integer). Mass 'm' suspended from ℓ_1 oscillates with time period ...

(A) $T = 2\pi\sqrt{\frac{m}{(n+1)K}}$ (B) $T = 2\pi\sqrt{\frac{nm}{(n+1)K}}$

(C) $T = 2\pi\sqrt{\frac{m}{K}}$ (D) $T = 2\pi\sqrt{\frac{(n+1)m}{K}}$

10. A spring when loaded has a potential energy 'E'. Then 'm' turns out of 'n' turns are removed from the spring. If the same load is suspended, then the energy stored in the spring is

(A) $\frac{n}{(n-m)}E$ (B) $\frac{mE}{n}$ (C) $\frac{(n-m)}{m}E$ (D) $\frac{(n-m)}{n}E$

11. A body is executing SHM. If the force acting on the body is 6N when the displacement is 2cm, then the force acting on the body at displacement of 3 cm is

(A) 6N (B) 9N (C) 4N (D) $\sqrt{6}N$

12. The acceleration due to gravity on a planet is 3/2 times that on the earth. If length of a seconds pendulum on earth is 1m, length of seconds pendulum on surface of planet is

(A) 0.7m (B) 1m (C) 1.7m (D) 1.5m

13. A seconds pendulum is shifted from a place where $g = 9.8 m/s^2$ to another place where $g = 9.78 m/s^2$. To keep period of oscillation constant its length should be

(A) decreased by $\frac{2}{\pi^2}$ cm (B) increased by $\frac{2}{\pi^2}$ cm

(C) increased by $\frac{2}{\pi}$ cm (D) decreased by $\frac{2}{\pi}$ cm

14. A pendulum of length L swings from rest to rest n times in one second. The value of acceleration due to gravity is

(A) $4\pi^2 n^2 L$ (B) $2\pi^2 n^2 L$ (C) $\pi^2 n^2 L$ (D) $\frac{\pi^2 n^2 L}{2}$

15. A simple pendulum has a time period T_1 when on the earth's surface and T_2 when taken to height R above the earth's surface, where R is the radius of the earth. The value of $\frac{T_2}{T_1}$ is

(A) 2 (B) 1 (C) $\sqrt{2}$ (D) 4

EXCERCISE-II

1. A cabin is moving in a gravity free space vertically with an acceleration a. What is the time period of oscillation of particle of mass m attached with an inextensible string of length ℓ, in this cabin?

(A) $2\pi\sqrt{\frac{\ell}{g}}$ (B) $2\pi\sqrt{\frac{\ell}{a}}$ (C) $2\pi\sqrt{\frac{\ell}{a+g}}$ (D) $2\pi\sqrt{\frac{\ell}{g-a}}$

2. A simple pendulum has some time period T. What will be the percentage change in its time period if its amplitudes in decreased by 5%?

(A) 6% (B) 3% (C) 1.5% (D) 0%

3. Length of a simple pendulum is increased by 2% then the time period:

(A) increased by 2% (B) decreased by 2%

(C) increased by 1% (D) decreased by 1%

4. Four types of oscillatory systems; a simple pendulum; a physical pendulum; a torsional pendulum and a spring-mass system, each of same time period are taken to the Moon. If the time periods are measured on the moon, which system or systems will have it unchanged?

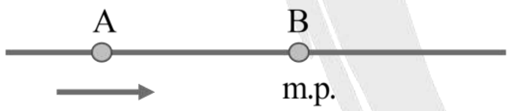
(A) Only spring-mass system

(B) Spring-mass system and torsional pendulum

(C) Spring-mass system and physical pendulum

(D) None of these

5. A particle is oscillating simple harmonically with angular frequency ω and amplitude A. It is at a point (A) at certain instant (shown in figure). At this instant it is moving towards mean position (B). It takes time t to reach mean position (B). If time period of oscillation is T, the average speed between A and B is:



(A) $\frac{A\sin\omega t}{t}$ (B) $\frac{A\cos\omega t}{t}$ (C) $\frac{A\sin\omega t}{T}$ (D) $\frac{A\cos\omega t}{T}$

6. A particle is performing simple harmonic motion

(P) its velocity – displacement graph is parabolic in nature

(Q) its velocity – time graph is sinusoidal in nature

(R) its velocity – acceleration graph is elliptical in nature

Correct answer is:

(A) (P), (Q) and (R) (B) (Q) and (R)

(C) (P) and (Q) (D) (P) and (R)

7. The total energy of a particle executing SHM is proportional to:

(A) displacement from equilibrium position (B) frequency of oscillation

(C) velocity at equilibrium position (D) square of amplitude of motion

8. A pendulum has time period T in air. When it is made to oscillate in water, it acquired a time period $T' = \sqrt{2}T$. The specific gravity of the pendulum bob is equal to:

9. The period of oscillation of a simple pendulum of length ℓ suspended from the roof of a vehicle which moves without friction down an inclined plane of inclination α, is given by :

(A) $2\pi\sqrt{\frac{\ell}{g\cos\alpha}}$ (B) $2\pi\sqrt{\frac{\ell}{g}}$ (C) $2\pi\sqrt{\frac{\ell}{g\sin\alpha}}$ (D) $2\pi\sqrt{\frac{\ell}{g\tan\alpha}}$

10. A simple harmonic motion has an amplitude A and time period T. The time required by it to travel from $x = A$ to $x = \frac{A}{2}$ is $\frac{T}{n}$. Find n?

11. Two bodies M and N of equal masses are suspended from two separate springs of the spring constants k_1 and k_2 respectively. If the two bodies oscillate vertically such that their maximum velocities are equal, the ratio of the amplitude of vibration of M to that on N is:

(A) $\frac{k_2}{k_1}$ (B) $\sqrt{\frac{k_2}{k_1}}$ (C) $\frac{k_1}{k_2}$ (D) $\sqrt{\frac{k_1}{k_2}}$

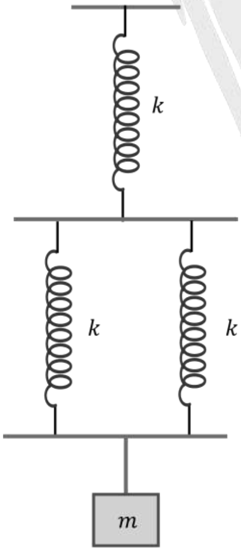
12. A particle executes SHM with an frequency f. The frequency with which it's KE oscillates is:

(A) $\frac{f}{2}$ (B) f (C) 2f (D) 4f

13. A particle executes SHM with an amplitude of 20 cm and time period of 12 sec. The minimum time (in sec) required for it to move between two points 10 cm on either side of the mean position:

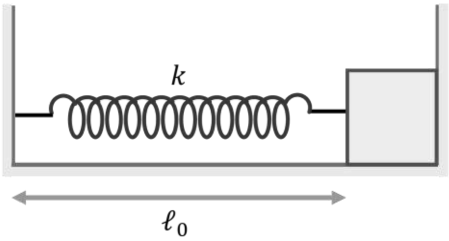
14. Two simple pendulum of length 1m and 16 m respectively are both given small displacement in the same direction at the same instant. They will again be in the same phase after shorter pendulum has completed η vibrations, when η is:

15. Calculate the period of oscillations of block of mass m attached with a set of springs as shown:



(A) $2\pi\sqrt{\frac{m}{3k}}$ (B) $2\pi\sqrt{\frac{3m}{2k}}$ (C) $2\pi\sqrt{\frac{2m}{3k}}$ (D) $2\pi\sqrt{\frac{3m}{k}}$

16. The surface of the arrangement shown in the figure is smooth. Natural length of spring is ℓ_0 and its spring constant is k. If collision between mass m and wall is elastic then period of small oscillation of the system is (neglecting time elapsed in collision):



(A) $2\pi\sqrt{\frac{m}{k}}$ (B) $2\pi\sqrt{\frac{k}{m}}$ (C) $\pi\sqrt{\frac{k}{m}}$ (D) $\pi\sqrt{\frac{m}{k}}$

17. A particle moves on x-axis according to the equation $x = x_0 \sin^2 \omega t$, the motion is simple harmonic:

(A) with amplitude $\left(\frac{x_0}{2}\right)$

(B) with amplitude $2x_0$

(C) with time period $\left(\frac{2\pi}{\omega}\right)$

(D) with time period $\left(\frac{\pi}{\omega}\right)$

18. A linear harmonic oscillator of force constant $2\times10^6\ \text{Nm}^{-1}$ and amplitude 0.01 m has a total mechanical energy of 160 J. Its:

(A) maximum potential energy is 100 J

(B) maximum kinetic energy is 100 J

(C) maximum potential energy is 160 J

(D) minimum potential energy is zero

19. If a simple harmonic motion is given by $y = \sin\omega t + \cos\omega t$, where y is in cm. Which of the following statement(s) is/are true?

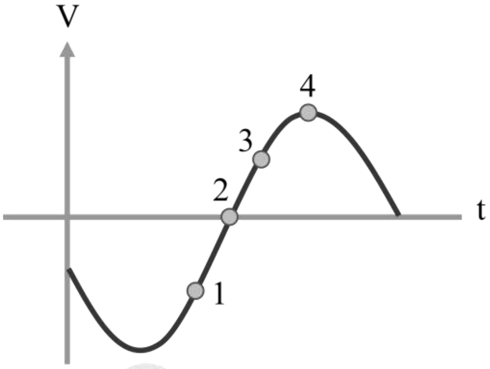
(A) The amplitude is 1cm (B) The amplitude is $\sqrt{2}$ cm

(C) Initial phase is $\frac{\pi}{6}$ (D) Initial phase is $\frac{\pi}{4}$

20. Equation of SHM is $x = 10\sin 10\pi t$. Find the distance between the two points where speed is $50\pi\,\text{cm/sec}$. x is in cm and t is in seconds:

(A) zero (B) 20 cm (C) 17.32 cm (D) 8.66 cm

21. Velocity – time graph of a particel executing SHM is shown in the figure. Select the correct alternative(s):



(A) At position 1 displacement of particle may be positive or negative

(B) At position 2 displacement of particle is negative

(C) At position 3 acceleration of particle is positive

(D) At position 4 acceleration of particle is positive

22. In simple harmonic motion:

(A) Potential energy and kinetic energy may not be equal in mean position

(B) Potential energy and kinetic energy may be equal in extreme position

(C) Potential energy may be zero at extreme position

(D) Kinetic energy plus potential energy oscillates simple harmonically

EXCERCISE-III

1. A block is placed on a horizontal platform vibrating up and down, simple harmonically. It is observed that the block loses its contact with the platform when its angular frequency is 5rad/s. The amplitude of vibration can not be less than 'A', then find the value of A.

(A) 10 cm (B) 20 cm (C) 30 cm (D) 40 cm

2. A particle executes SHM along a striaght line with mean position at x = 0, period 20 sec and amplitude 5cm. Find the shortest time taken by the particle to go from x = 4 cm to x = -3 cm

(A) 5 sec (B) 10 sec (C) 7 sec (D) 4 sec

3. If a simple pendulum of length ℓ has maximum angular displacement θ, then the maximum velocity of the bob is:

(A) $\sqrt{\frac{L}{g}}$ (B) $2\sqrt{g\ell}\sin\left(\frac{\theta}{2}\right)$ (C) $\sqrt{2g\ell}\sin\theta$ (D) $\sqrt{2g\ell}$

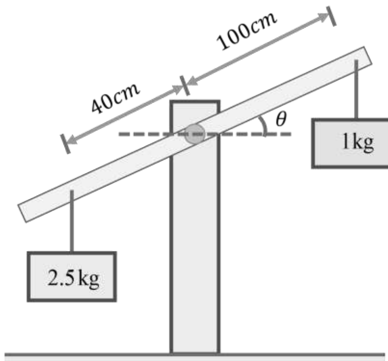
4. A simple pendulum with a brass bob has a time period T. The bob is now immersed in a non-viscous liquid and made to oscillate. The density of the liquid is (1/8) th that of the brass. The time period of pendulum will be $\sqrt{\frac{\alpha}{\beta}}T$. Find $\alpha+\beta$?

5. A particle of mass m executes SHM according to equation $x = A\cos\omega t$. The average velocity and average kinetic energy over a time interval 0 to T/2 (T = Time period) are, respectively:

(A) $0, \frac{mA^2\omega^2}{2}$ (B) $\frac{A\omega}{\pi}, \frac{mA^2\omega^2}{4}$ (C) $A\omega, 0$ (D) $\frac{2A\omega}{\pi}, \frac{mA^2\omega^2}{4}$

6. A 25 kg uniform solid sphere with a 20 cm radius is suspended by a vertical wire such that the point of suspension is vertically above the centre of the sphere. A torque of 0.10 N-m is required to rotate the sphere through an angle of 1.0 rad and then maintain the orientation. If the sphere is then released, its time period of the oscillation will be $n\pi$ second. Find n?

7. A straight rod of negligible mass is mounted on a frictionless pivot and masses 2.5 kg and 1 kg are suspended at distances 40 cm and 100 cm respectively from the pivot as shown. The rod is held at an angle θ with the horizontal and released.



(A) The rod executes periodic motion about horizontal position after the release

(B) The rod remains stationary after the release

(C) The rod comes to rest in vertical position with 2.5 kg mass at the lowest point

(D) The rod executes periodic motion about vertical position after the release

8. The amplitude of a particle due to superposition of following SHMs along the same line is:

$X_1 = 2\sin 50\pi t$

$X_2 = 10\sin(50\pi t + 37°)$

$X_3 = -4\sin 50\pi t$

$X_4 = -12\cos 50\pi t$

(A) $4\sqrt{2}$ (B) 4 (C) $6\sqrt{2}$ (D) None of these

9. The angular amplitude of a simple pendulum is θ_0. The maximum tension in its string will be:

(A) $mg(1-\theta_0)$ (B) $mg(1+\theta_0)$ (C) $mg(1+\theta_0^2)$ (D) $mg(1-\theta_0^2)$

10. A simple pendulum of length 1 m with a bob of mass m swings with an angular amplitude $30°$. Then: $(g = 9.8\,m/s^2)$

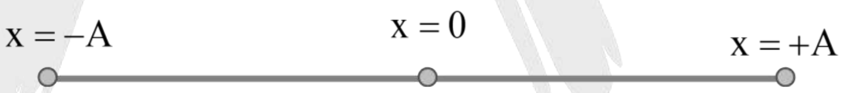
(A) time period of pendulum is 2 sec

(B) tension in the string is greater than $mg\cos 15°$ at angular displacement 15°

(C) rate of change of speed at an angular displacement 15° is $g\sin 15°$

(D) tension in the string is $mg\cos 15°$ at angular displacement 15°

11. Two particles undergo SHM along the same line with the same time period (T) and equal amplitudes (A). At a particular instant one particle is at x = −A and the other is at x = 0. They move in the same direction. They will cross each other at:



(A) $t = \frac{4T}{3}$ (B) $t = \frac{3T}{8}$ (C) $x = \frac{A}{2}$ (D) $x = \frac{A}{\sqrt{2}}$

12. A 20 gm particle is subjected to two simple harmonic motions $x_1 = 2\sin 10t$, $x_2 = 4\sin\left(10t + \frac{\pi}{3}\right)$

Where x_1 and x_2 are in meter and t is in sec.

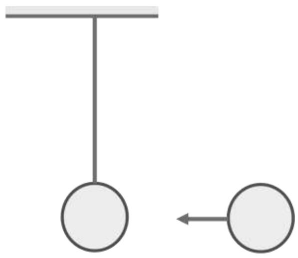
(A) The displacement of the particle at t = 0 will be $2\sqrt{3}\,m$

(B) Maximum speed of the particle will be $20\sqrt{7}\,m/s$

(C) Magnitude of maximum acceleration of the particle will be $200\sqrt{7}\,m/s^2$

(D) Energy of the resultant motion will be 28J

13. A ball is swinging on a swing like a simple pendulum. Its time period of oscillation is T and amplitude is A. When it is at the bottom of the swing, another ball of equal mass strikes and sticks to it while both of them are travelling in same direction. Choose the correct option(s).



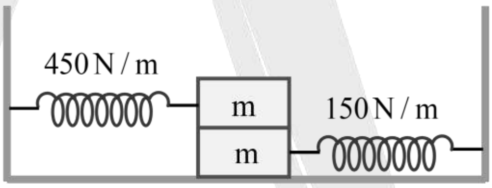
(A) The time period of oscillation remains same

(B) The amplitude increases

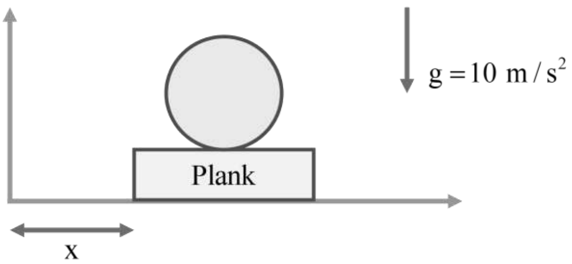
(C) The time period of oscillation increases

(D) The time period of oscillation decreases

14. When the system shown in the diagram is in equilibrium, the right spring is stretched by 1 cm. The coefficient of static friction between the blocks is 0.3. There is no friction between the bottom block and the supporting surface. The force constants of the springs are 150 N/m and 450 N/m (refer figure). The blocks have equal mass of 2 kg each. Find the maximum amplitude (in cm) of the oscillations of the system shown in the figure that does not allow the top block to slide on the bottom.



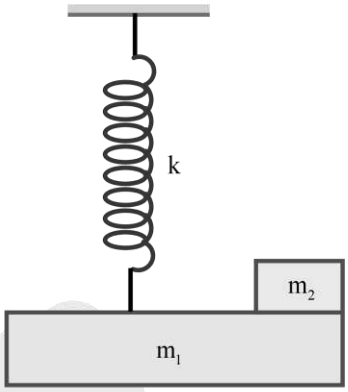
15. A solid cylinder is kept over a rough plank which is oscillating along x-axis according to equation $x = A\cos(10)t$, where x is in meter and t is in second. If coefficient friction between cylinder and plank is 0.3, then find maximum amplitude of plank possible, so that cylinder never slips on plank.



(A) 3cm (B) 6cm (C) 9cm (D) 12cm

EXCERCISE-IV

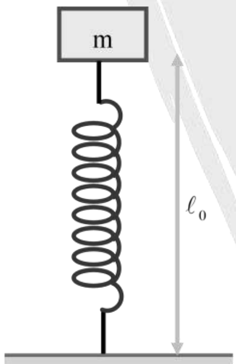
1. A spring is loaded with two blocks m_1 and m_2 where m_1 is rigidly fixed with the spring and m_2 is just kept on the block m_1 as shown in the figure. The maximum energy of oscillation that is possible for the system having the block m_2 in contact with m_1 is:



(A) $\frac{m_1^2g^2}{2k}$ (B) $\frac{m_2^2g^2}{2k}$ (C) $\frac{(m_1+m_2)^2g^2}{2k}$ (D) None of these

2. An object of mass 0.2 kg executes simple harmonic motion along x-axis with frequency of $25/\pi$ Hz. At the position x = 0.04 m, the object has kinetic energy of 0.5 J and potential energy of 0.4J. The amplitude of oscillation is equal to $\frac{6}{n}$ meters. Find n? (Assuming PE is zero at equilibrium)

3. A block of mass m attached with a spring and held by a person such that the spring is in natural length ℓ_0. Now the man releases the block, the ratio of maximum compression in the spring in the given situation, to that of the compression at equilibrium position:



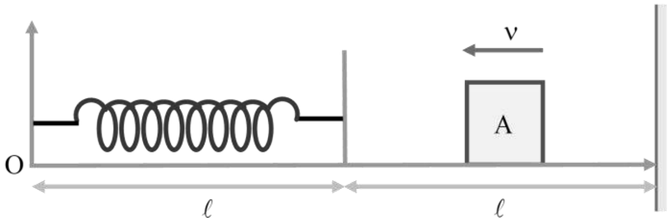
(A) 2 : 1 (B) 1 : 1 (C) 1 : 2 (D) $\sqrt{2}:1$

4. A particle is executing an SHM along the x-axis given by $x = A\sin\omega t$. What is the magnitude of the average acceleration of the particle between t = 0 and t = $\frac{T}{4}$s? Where T is the time period of oscillation.

(A) $\frac{2\omega^2A}{\pi}$ (B) $\frac{\omega^2A}{\pi}$

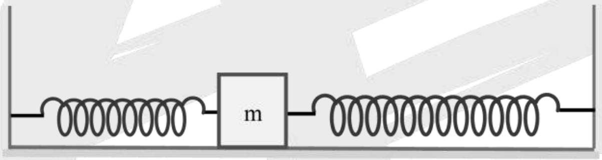
(C) $\frac{4\omega^2A}{\pi}$ (D) None of these

5. A block A of mass M is placed in contact with a fixed spring of length ℓ and force constant k and fixed from a smooth wall at a distance 'ℓ' on the other side. If the block is given a velocity v as shown, the time period of the motion will be:



(A) $\pi\sqrt{\frac{M}{k}}+\frac{\ell}{v}$ (B) $\pi\sqrt{\frac{M}{k}}$ (C) $2\pi\sqrt{\frac{M}{k}}+\frac{2\ell}{v}$ (D) $\pi\sqrt{\frac{M}{k}}+\frac{2\ell}{v}$

6. A uniform spring has a certain block suspended from it and its period for vertical oscillation is T_1. The spring is now cut into two parts of lengths $\frac{1}{3}$rd and $\frac{2}{3}$rd of original length and these springs are connected to the same block as shown in the figure. If time period of oscillation now is T_2 then $\frac{T_1}{T_2}$:



(A) $\sqrt{\frac{9}{2}}$ (B) 1 : 1 (C) 1 : 3 (D) $\sqrt{\frac{2}{9}}$

7. A simple pendulum with length L and mass m of the bob is vibrating with an amplitude 'a'. The tension in the string at the lowest point is:

(A) mg (B) $mg\left[1+\left(\frac{a}{L}\right)^2\right]$

(C) $mg\left[1+\frac{a}{2L}\right]^2$ (D) $mg\left[1+\left(\frac{a}{L}\right)\right]^2$

8. A particle starts SHM at time t = 0. Its amplitude is A and angular frequency is ω. At time t = 0 its kinetic energy is $\frac{E}{4}$. Assuming potential energy to be zero at mean position, then displacement – time equation of the particle can be written as:

(A) $x = A\cos\left[\omega t+\left(\frac{\pi}{6}\right)\right]$

(B) $x = A\sin\left[\omega t+\left(\frac{\pi}{3}\right)\right]$

(C) $x = A\cos\left[\omega t-\left(\frac{2\pi}{3}\right)\right]$

(D) $x = A\cos\left[\omega t-\left(\frac{\pi}{6}\right)\right]$

9. A particle is executing SHM on a straight line. A and B are two points at which its velocity is zero. It passes through a certain point P $(AP < BP)$ at successive intervals of 0.5 sec and 1.5 sec with a speed of 3 m/s:

(A) The maximum speed of particle is $3\sqrt{2}\,m/s$

(B) The maximum speed of particle is $\sqrt{2}\,m/s$

(C) The ratio $\frac{AP}{BP}$ is $\frac{\sqrt{2}-1}{\sqrt{2}+1}$

(D) The ratio $\frac{AP}{BP}$ is $\frac{1}{\sqrt{2}}$

10. Acceleration of a particle which is at rest at x = 0 is $\vec{a} = (4-2x)\hat{i}$. Select the correct alternative(s):

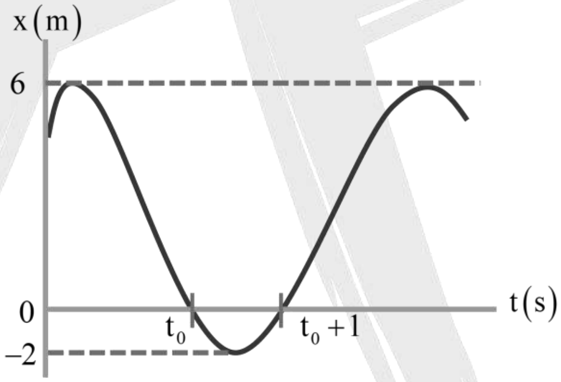
(A) particle further comes to rest at x = 4

(B) particle oscillates about x = 2

(C) maximum speed of particle is 4 units

(D) all of the above

11. A particle executes SHM about a point other than x = 0 as shown in the graph.



Choose the correct option(s):

(A) Amplitude is equal to 4 m

(B) Equilibrium position is at x = 0

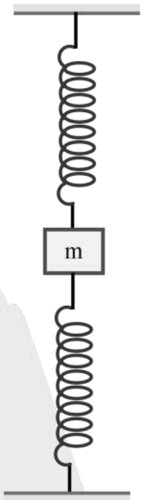
(C) Equilibrium position is at x = 2 m

(D) Angular frequency $= \frac{2\pi}{3}$

12. Two particles A and B are performing SHM along X-axis and Y-axis respectively with equal to amplitude and frequency of 2 cm and 1 Hz respectively. Equilibrium positions for the particles A and B are at the coordinate (3,0) and (0,4) respectively. At t = 0, B is at its equilibrium position and moving toward the origin, while A is nearest to the origin and moving away from the origin. Find the sum of maximum and minimum distances between A and B is s_1 and s_2. Find the $s_1 + s_2$?

(A) 2.5 cm (B) 5 cm (C) 7.5 cm (D) 10 cm

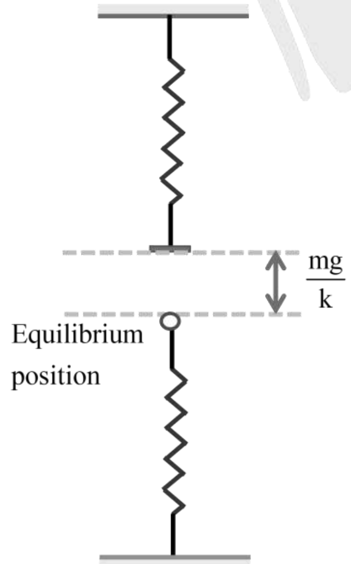
13. One end of a spring is fixed to the ceiling and other end is attached to a block. The block is released when spring is relaxed. The product of time period and amplitude is 8 S.I. units. If spring is cut in two equal parts and the two springs are attached to the block as shown in figure. The block is released when both springs are relaxed. Now find the product of time period and amplitude in S.I. units.



14. When we hang a 3kg body on each of a 12 cm and a 9 cm long spring it makes both springs extend by one third of their original lengths. Half of the long spring and two thirds of the short spring are cut off, therefore fastening the ends of the remaining springs together gives a 9 cm – long spring. At what angular frequency will a 30 kg boy oscillate on this spring?

(A) 2.5 rad/s (B) 5 rad/s (C) 7.5 rad/s (D) 10 rad/s

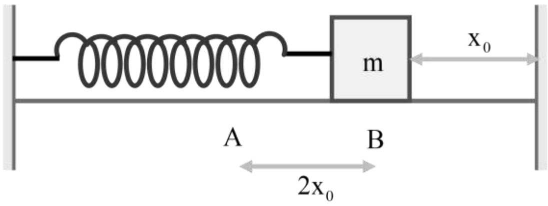
15. Figure shows a small block of mass m attached to a spring of force constant k and an identical spring hangs from ceiling. Initially lower spring is in compressed state with compression equal to $\frac{3mg}{k}$ from natural length of spring. When block is released, it strikes upper spring and sticks to it. What is the amplitude of oscillation after sticking given $mg = 10\,N$, $k = 100\sqrt{7}$ N/m?



(A) 10 cm (B) 15 cm (C) 5 cm (D) 20 cm

EXCERCISE-V

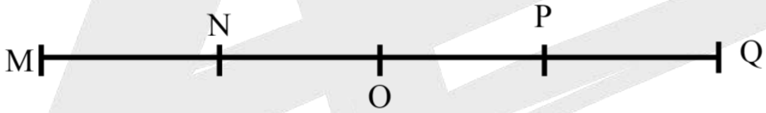
1. One end of a spring of force constant k is fixed to a vertical wall and the other to a block of mass m resting on a smooth horizontal surface. There is another wall at a distance x_0 from the block. The spring is then compressed by $2x_0$ and released. The time taken to strike the wall is:



(A) $\frac{1}{6}\pi\sqrt{\frac{k}{m}}$ (B) $\sqrt{\frac{k}{m}}$ (C) $\frac{2\pi}{3}\sqrt{\frac{m}{k}}$ (D) $\frac{\pi}{4}\sqrt{\frac{k}{m}}$

2. A particle of mass 10 gm lies in a potential field $v = 50x^2 + 100$. The value of frequency of oscillations is $\frac{n}{\pi}$ Hz. Find n?

3. A body performs SHM along the straight line MNOPQ. It's kinetic energy at N and at P is half of its peak value of O. If the time period is T, then the time taken to travel from N to P directly along MOP, is:



(A) $\frac{T}{2}$ (B) $\frac{T}{4}$ (C) $\frac{T}{2\sqrt{2}}$ (D) $\frac{T}{4\sqrt{2}}$

4. Two SHMs. $s_1 = a\sin\omega t$ and $s_2 = b\sin\omega t$ are superimposed on a particle. The s_1 and s_2 are along the directions which make angle $37°$ with each other:

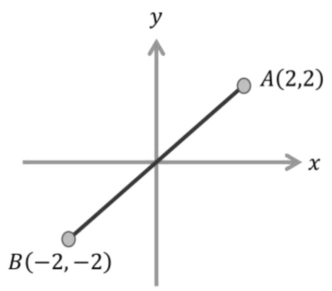
(A) the particle will perform SHM

(B) the particle will not perform SHM

(C) the particle will perform periodic motion but not SHM

(D) the motion will not be oscillatory

5. A particle of mass m = 2 kg executes SHM in xy – plane between points A and B under action of force $\vec{F} = F_x\hat{i} + F_y\hat{j}$. Minimum time taken by particle to move from A to B is 1 sec. At t = 0 the particle is at x = 2 and y = 2. Then F_x as function of time t is:

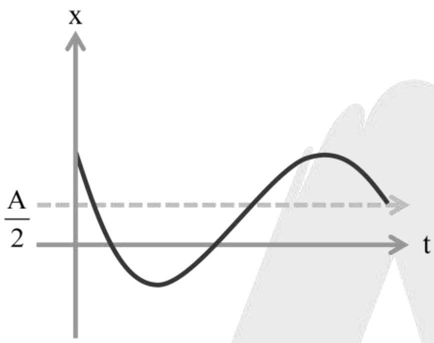


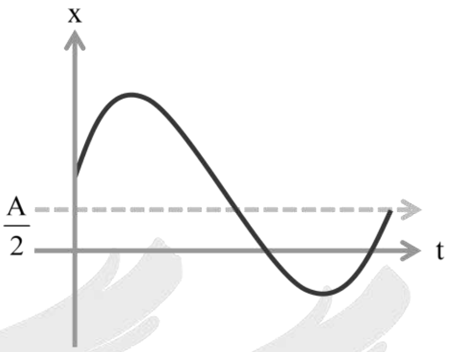
(A) $-4\pi^2 \sin \pi t$ (B) $-4\pi^2 \cos \pi t$ (C) $4\pi^2 \cos \pi t$ (D) None of these

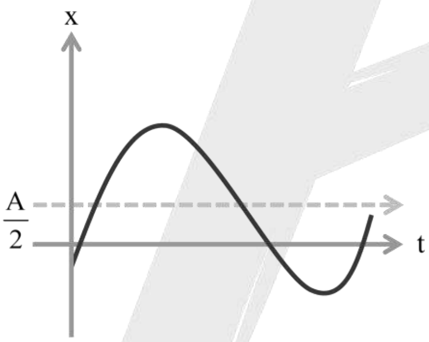
6. A point mass is subjected to two simultaneous sinusoidal displacements in x-direction, $x_1(t) = A\sin\omega t$ and $x_2(t) = A\sin\left(\omega t + \frac{2\pi}{3}\right)$. Adding a third sinusoidal displacement $x_3(t) = B\sin(\omega t + \phi)$ brings the mass to a complete rest. The values of B and ϕ are:

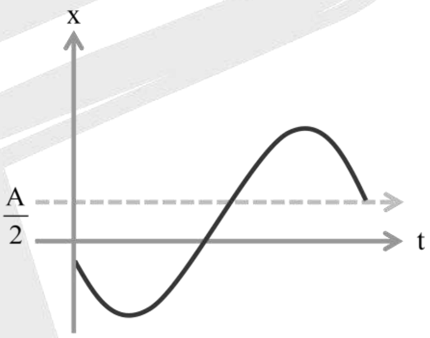
(A) $\sqrt{2}A, \frac{3\pi}{4}$ (B) $A, \frac{4\pi}{3}$ (C) $\sqrt{3}A, \frac{5\pi}{6}$ (D) $A, \frac{\pi}{3}$

7. A particle performing SHM about mean position $x = \frac{A}{2}$ and at t = 0. It has displacement $\frac{-3A}{4}$ and moving away from the origin. Then which of the following is its possible graph between position (x) and time (t)?

(A)


(B)


(C)


(D)


8. The displacement of a particle varies according to the relation $x = 3\sin 100t + 8\cos^2 50t$. Which of the following is incorrect about this motion?

(A) The motion of the particle is SHM

(B) The amplitude of the SHM of the particle is 5 units

(C) The amplitude of the resultant SHM is $\sqrt{73}$ units

(D) The maximum displacement of the particle from the origin is 9 units

9. The period of a particle executing SHM is T. There is a point P, at a distance x from the mean position O. When the particle passes P moving away from mean position, it has speed v then find the time in which it returns to P again

(A) T (B) $\frac{T}{\pi}\tan^{-1}\left(\frac{vT}{2\pi x}\right)$

(C) $T\sin^{-1}\left(\frac{vT}{x}\right)$ (D) $\frac{T}{2\pi}\cot^{-1}\left(\frac{vT}{2\pi x}\right)$

10. A simple pendulum of length L and mass M is oscillating in a plane about a vertical line between angular limits $-\phi$ and $+\phi$. For an angular displacement $\phi(|\theta|<\phi)$ the tension in the string and velocity of the bob are T and v respectively. The following relations hold good under the above condition:

(A) $T\cos\theta = Mg$

(B) $T + Mg\cos\theta = \frac{Mv^2}{L}$

(C) The magnitude of tangential acceleration of the bob $|a_T| = g\sin\theta$

(D) $T = Mg(3\cos\theta - 2\cos\phi)$

11. A horizontal spring – block system of mass 1 kg executes SHM of amplitude 10 cm. When the block is passing through its equilibrium position another mass of 1 kg is put on it and the two move together:

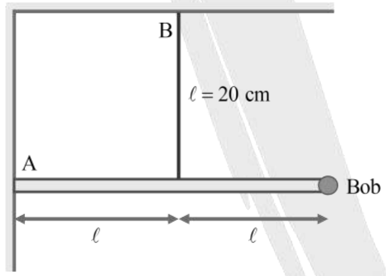
(A) amplitude will remain unchanged

(B) amplitude will become $5\sqrt{2}$cm

(C) the frequency of oscillations will remain same

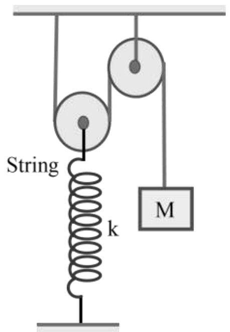
(D) the frequency of oscillations will decrease

12. A weightless rigid rod with a small iron bob at the end is hinged at point A to the wall so that it can rotate in all directions. The rod is kept in the horizontal position by a vertical inextensible string of length 20 cm, fixed at its mid point. The bob is displaced slightly, perpendicular to the plane of the rod and string. Find period of small oscillations of the system is $(g = 10\,m/s^2)$



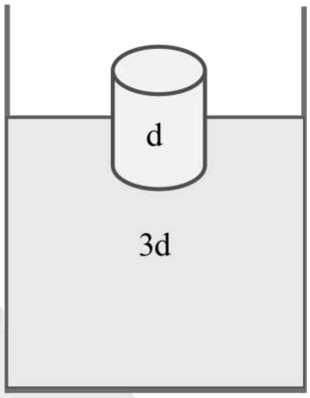
(A) $\frac{2\pi}{5}$sec (B) $\frac{\pi}{5}$sec (C) $\frac{5\pi}{2}$sec (D) $\frac{\pi}{2}$sec

13. Consider the given system. All string, pulleys and spring are ideal, mass M is in equilibrium. Find Maximum amplitude of oscillation of block in vertical direction is



(A) $\frac{mg}{k}$ (B) $\frac{3mg}{k}$ (C) $\frac{4mg}{k}$ (D) $\frac{2mg}{k}$

14. A cylindrical block of height 1m is in equilibrium in a beaker as shown. Cross – sectional area of cylindrical block is one fourth of cross – sectional area of beaker. Density of cylindrical block is one third of liquid. Determine the time period of small oscillation. (Given: $g = \pi^2\ m/s^2$)



(A) 1 sec (B) 2 sec (C) 3 sec (D) 4 sec

15. Two identical metal balls connected at the ends of a light sprig of force constant k form a dumbbell like structure. The dumbbell rests on a frictionless horizontal floor and third identical ball is placed at distance ℓ from the right ball of the dumbbell. All the three balls are in a line. A fourth identical ball moving with velocity u collides with left ball of the dumbbell. If all collisions are elastic and rightmost ball acquires a velocity u. The minimum value of ℓ is $\pi u\sqrt{\frac{m}{xk}}$. Find x?

PROFICIENCY TEST-I

1. The displacement of the particle varies with time $x = 12\sin\omega t - 16\sin^3\omega t$. If its motion is SHM, then maximum acceleration is:

(A) $12\omega^2$ (B) $36\omega^2$ (C) $144\omega^2$ (D) $\sqrt{192}\omega^2$

2. The motion of a particle is given by $x = A\sin\omega t + B\cos\omega t$. The motion of the particle is:

(A) not simple harmonic

(B) simple harmonic with amplitude A + B

(C) simple harmonic with amplitude $\left(\frac{A+B}{2}\right)$

(D) simple harmonic with amplitude $\sqrt{A^2 + B^2}$

3. A particle is subjected to two simple harmonic motions along x and y directions according to, $x = 3\sin 100\pi t$; $y = 4\sin 100\pi t$

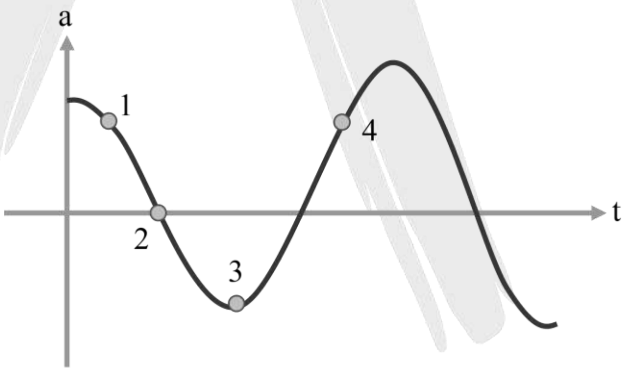
(A) Motion of particle will be on ellipse traversing it in clockwise direction

(B) Motion of particle will be on a straight line with slope $\frac{4}{3}$

(C) Motion will be a simple harmonic motion along x-axis with amplitude 5

(D) Phase difference between two motions is $\frac{\pi}{2}$

4. Acceleration – time graph of a particle executing SHM is as shown in the figure. Select the correct alternative(s):



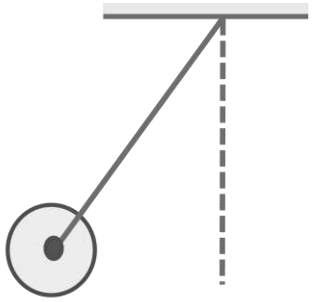
(A) Displacement of particle at 1 is negaitve

(B) Velocity of particle at 2 is positive

(C) Potential energy of particle at 3 is maximum

(D) Speed of particle at 4 is decreasing

5. A metal rod of length 'L' and mass 'm' is pivoted at one end. A thin disc of mass 'M' and radius 'R' (< L) is attached at its centre to the free end of the rod. Consider two ways the disc is attached: (case A). The disc is not free to rotate about its centre and (case B) the disc is free to rotated about its centre. The rod-disc system performs SHM in vertical plane after being released from the same displaced position. Which of the following statement(s) is (are) true?



(A) Restoring torque in case A = Restoring torque in case B

(B) Restoring torque in case A < Restoring torque in case B

(C) Angular frequency for case A > Angular frequency for case B

(D) Angular frequency for case A < Angular frequency for case B

6. A particle of mass m is executing oscillations about the origin on the x-axis. Its potential energy is $U(x)=k|x|^3$, where k is a positive constant. If the amplitude of oscillations is a, then its time period T is:

(A) Proportional to $\frac{1}{\sqrt{a}}$

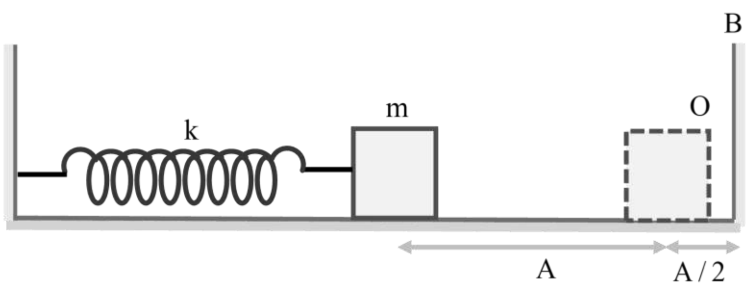
(B) independent of a

(C) Proportional to $\sqrt{a}$

(D) Proportional to $a^{3/2}$

7. A disc of radius R and mass M is pivoted at the rim and is set for small oscillations in vertical plane. If simple pendulum has to have the same period as that of the disc, the length of the simple pendulum should be $\frac{\alpha}{\beta}R$. Find $\alpha+\beta$?

8. A block of mass m rigidly attached with a spring k is compressed through a distance A. If the block is released, the period of oscillation of the block for a complete cycle is equal to:



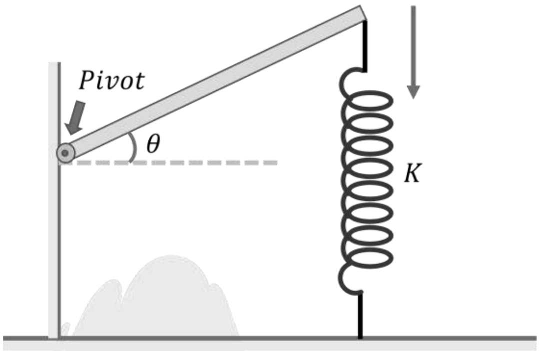
(A) $\frac{4\pi}{3}\sqrt{\frac{m}{k}}$

(B) $\frac{\pi}{\sqrt{2}}\sqrt{\frac{m}{k}}$

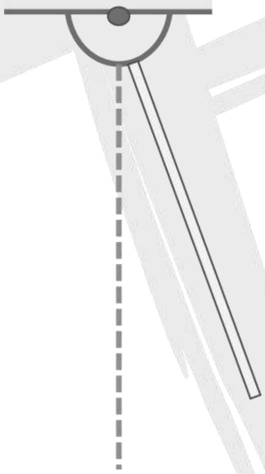
(C) $\frac{2\pi}{3}\sqrt{\frac{m}{k}}$

(D) None of these

9. A horizontal rod of mass m and length L is pivoted smoothly at one end. The rod's other end is supported by a spring of force constant k. The rod is rotated (in vertical plane) by a small angle θ from its horizontal equilibrium position and released. The angular frequency of the subsequent simple harmonic motion is $\sqrt{\frac{nk}{m}}$. Find n?



10. A meter stick swinging in vertical plane about an fixed horizontal axis passing through its one end undergoes small oscillation of frequency f_0. If the bottom half of the stick were cut off, then its new frequency of small oscillation would become $\sqrt{n}\,f_0$. Find n?

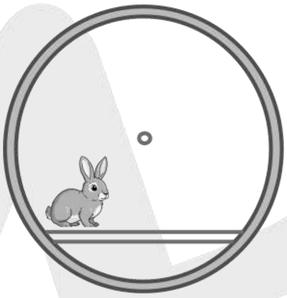


PROFICIENCY TEST-II

1. A particle of mass m moves in a potential field given by $U = U_0(1-\cos ax)$ where U_0 and a are positive constant. The period of small oscillations is:

(A) $2\pi\sqrt{\frac{ma}{U_0}}$ (B) $2\pi\sqrt{\frac{U_0}{ma}}$ (C) $\frac{2\pi}{a}\sqrt{\frac{m}{U_0}}$ (D) $2\pi\sqrt{\frac{m}{aU_0}}$

2. A small Rabbit is put into a circular wheel-cage, which has a frictionless central pivot. A horizontal platform is fixed to the wheel below the pivot. Initially, the Rabbit is at rest at one end of the platform. When the platform is released, Rabbit starts running, but, because of the Rabbit motion, the platform and wheel remains stationary. Determine how the Rabbit moves?



(A) Its motion is simple harmonic

(B) Its motion is uniform

(C) Its motion is uniformly accelerated motion

(D) Situation is not possible

3. Choose the correct statement from following given k is real positive constant:

(A) Function $F(t) = \sin kt + \cos kt$ is SHM having period $\frac{2\pi}{3}$

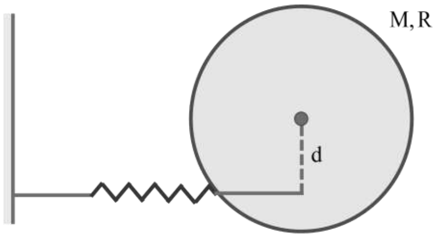
(B) $F(t) = 4\sin^2 \pi t + 2\cos 2\pi t$ is SHM

(C) $F(t) = \cos kt + 2\sin^2 kt$ is SHM having period $\frac{2\pi}{k}$

(D) $F(t) = e^{-kt}$ is not periodic function

4. A very large (uniform disk) is connected to a very stiff spring as shown in figure. Spring is attached at distance d below the axle of the disc. System is in equilibrium in configuration shown. If disc is rotated through a small angle $\theta_0 = 3^\circ$. Find the maximum kinetic energy of disc

Given $K = 900\,N/m$; $R = 3m$; $d = 2m$; $\pi^2 = 10$



(A) 5 Joule (B) 10 Joule (C) 15 Joule (D) 20 Joule

5. Two blocks A and B each of mass m are connected by a massless spring of natural length L and spring constant k. The blocks are initially resting on a smooth horizonal floor with the spring as its natural length, as shown in figure.

A third identical block C, also of mass m moves on the floor with a speed v along the line joining A and B collides with A, elastically, then:

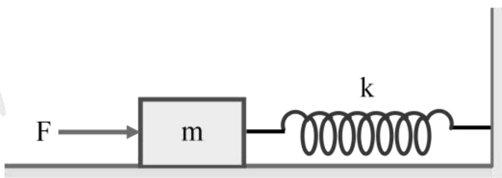
(A) the kinetic energy of AB system at maximum compression of the spring is zero

(B) the kinetic energy of the AB system at maximum compression of the spring is $\frac{mv^2}{4}$

(C) The maximum compression of the spring is $v\sqrt{\frac{m}{k}}$

(D) The maximum compression of the spring is $v\sqrt{\frac{m}{2k}}$

6. A constant force F is applied on a spring block system as shown in the figure. The mass of the block is m and spring constant is k. The block is placed over a smooth surface. Initially the spring was unstrethed. Choose the correct alternative(s):



(A) The block will execute SHM

(B) Amplitude of oscillation is $\frac{F}{2k}$

(C) Time period of oscillation is $2\pi\sqrt{\frac{m}{k}}$

(D) The maximum speed of block is $\sqrt{\frac{2Fx-kx^2}{m}}$

7. Tick the correct alternative(s):

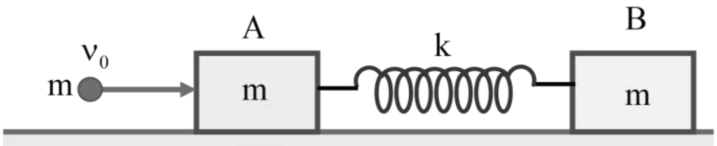
(A) The displacement of the particle varies with time as $x = 12\sin\omega t - 16\sin^3\omega t$. The motion of particle is SHM with amplitude 4 units

(B) A particle oscillates according to equation $x = 7\cos\frac{\pi t}{2}$ where t is in seconds. The point moves from the point of equilibrium to maximum displacement in 1 second

(C) If a simple pendulum of length ℓ_0 has maximum angular displacement θ_0, then the maximum speed of the bob is $2\sqrt{g\ell_0}\sin\frac{\theta}{2}$

(D) None of the above

8. Two blocks A and B, each of mass m, are connected by an ideal spring of stiffness k and placed on a smooth horizontal surface. A ball of mass m moving with a velocity v_0 strikes the block A and gets embedded to it. Then:



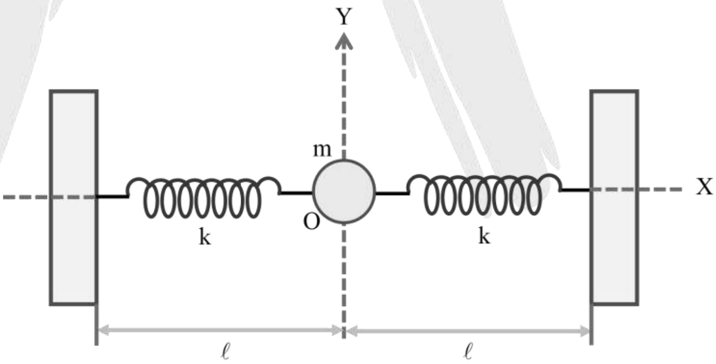
(A) velocity of block A just after collision is $\frac{v_0}{2}$

(B) velocity of block B just after collision is zero

(C) the maximum compression produced in the spring is $v_0\sqrt{\frac{m}{6k}}$

(D) the kinetic energy lost during collision is $\frac{1}{4}mv_0^2$

9. Figure shows a smooth horizontal table in x-y plane between two identical fixed walls. Two identical springs are connected to the small ball. The length of the springs in the free state is ℓ. The ball is shifted slightly from the equilibrium position in two different ways once along the axis OX and second along the y-axis and it begins to perform vibrations. The time period for these motions is T_x and T_y respectively:



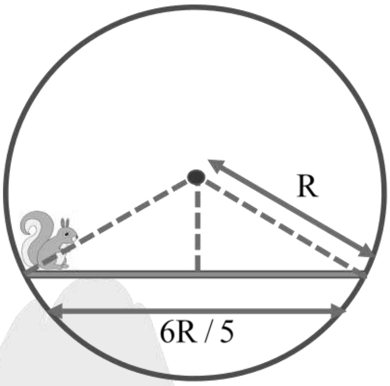
(A) Motion along x – axis is simple harmonic

(B) Motion along y – axis is simple harmonic

(C) $T_x = 2\pi\sqrt{\frac{m}{2k}}$

(D) $T_y = 2\pi\sqrt{\frac{m}{2k}}$

10. A small squirrel is put into a circular wheel cage of radius R which has a frictionless central pivot. A horizontal platform of length $\frac{6R}{5}$ is fixed to the wheel below the pivot as shown. Initially squirrel is at rest at one end of the platform. When the platform is released squirrel starts running but platform and wheel remain stationary. Choose the correct options:



(A) Maximum speed of squirrel is $\sqrt{\frac{Rg}{5}}$

(B) Maximum speed of squirrel is $\sqrt{\frac{9Rg}{20}}$

(C) Maximum acceleration of squirrel is $\frac{4g}{3}$

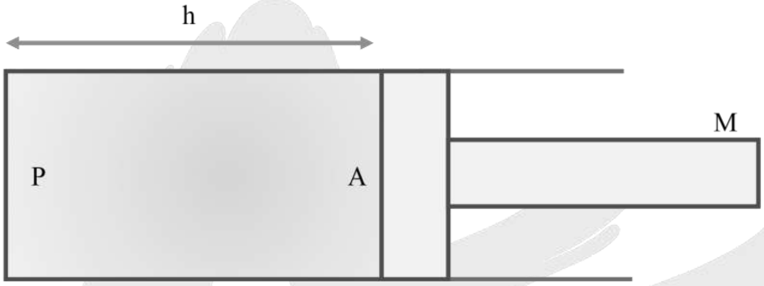
(D) Maximum acceleration of squirrel is $\frac{3g}{4}$

PROFICIENCY TEST-III

1. A particle moves in x-y plane according to the equation $\vec{r} = (\hat{i} + 2\hat{j})A\cos\omega t$ the motion of the particle is:

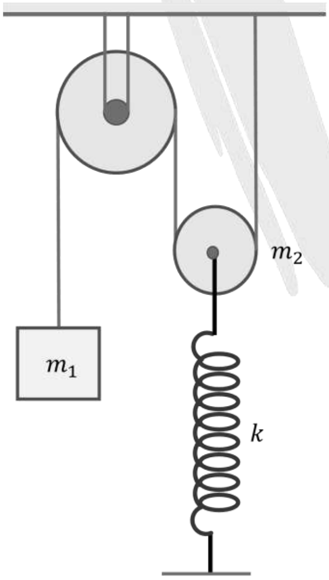
(A) on a straight line
(B) on an ellipse
(C) periodic
(D) simple harmonic

2. A cylindrical piston of mass M slides smootly inside a long cylinder closed at one end, enclosing a certain mass of gas. The cylinder is kept with its axis horizontal. If the piston is slightly compressed isothermally from its equlibrium position, it oscillates simple harmonically, the period of oscillation will be:



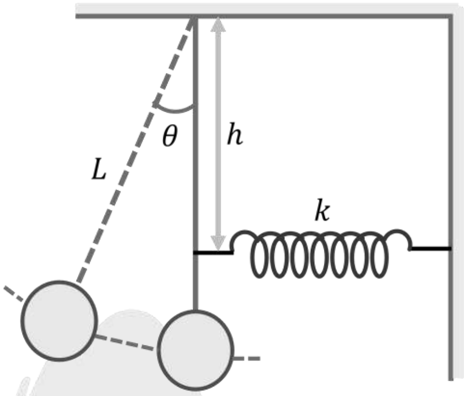
(A) $T = 2\pi\sqrt{\frac{Mh}{PA}}$
(B) $T = 2\pi\sqrt{\frac{MA}{Ph}}$
(C) $T = 2\pi\sqrt{\frac{M}{PAh}}$
(D) $T = 2\pi\sqrt{MPhA}$

3. The period of the free oscillation of the system shown here if mass m_1 is pulled down a little and force constant of the spring is k and masses of the fixed pulleys are negligible, is:



(A) $T = 2\pi\sqrt{\frac{m_1 + m_2}{k}}$
(B) $T = 2\pi\sqrt{\frac{m_1 + 4m_2}{k}}$
(C) $T = 2\pi\sqrt{\frac{4m_1 + m_2}{k}}$
(D) $T = 2\pi\sqrt{\frac{3m_1 + m_2}{k}}$

4. A pendulum of length L and bob of mass M has a spring of force constant k connected horizontally to it at a distance h below its point of suspension. The rod is in equilibrium in vertical position. The rod of length L used for vertical suspension is rigid and massless. The frequency of vibration of the system for small values of θ is:



(A) $\frac{1}{2\pi L}\sqrt{gL+\frac{kh}{m}}$ (B) $\frac{1}{2\pi L}\sqrt{\frac{mgL+k}{m}}$

(C) $2\pi\sqrt{\frac{mL^2}{mgL+kh}}$ (D) $\frac{1}{2\pi L}\sqrt{gL+\left(\frac{kh^2}{m}\right)}$

5. Density of liquid varies with depth as $\rho=\alpha h$. A small ball of density ρ_0 is released from the free surface of the liquid. Then:

(A) the ball will execute SHM of amplitude $\frac{\rho_0}{\alpha}$

(B) the mean position of the ball will be at a depth $\frac{\rho_0}{2\alpha}$ from the free surface

(C) the ball will sink to a maximum depth of $\frac{2\rho_0}{\alpha}$

(D) all of the above

6. A particle moves along the x-axis according to the equation $x=4+3\sin(2\pi t)$, here x is in cm and t in second. Select the correct alternative(s):

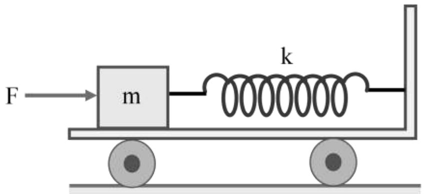
(A) The motion of the particle is simple harmonic with mean position at x = 0

(B) The motion of the particle is simple harmonic with mean position at x = 4 cm

(C) The motion of the particle is simple harmonic with mean position at x = - 4 cm

(D) Amplitude of oscillation is 3 cm

7. A block of mass m is attached to a massless spring of force constant k, the other end of which is fixed from the wall of a truck as shown in the figure. The block is placed over a smooth surface and initially the spring is unstretched. Suddenly the truck starts moving towards right with a constant acceleration a_0. As seen from the truck:



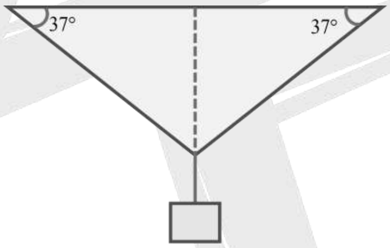
(A) the particle will execute SHM

(B) the time period of oscillations will be $2\pi\sqrt{\frac{m}{k}}$

(C) the amplitude of oscillations will be $\frac{ma_0}{k}$

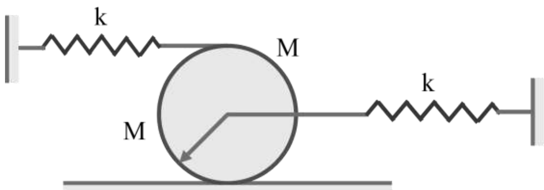
(D) the energy of oscillations will be $\frac{m^2a_0^2}{k}$

8. A block is hung by means of two identical wires having cross section area A $\left(1mm^2\right)$ as shown in the diagram. If temperature is lowered by $\Delta T\left(10°C\right)$, find the mass to be added to hanging mass such that junction remains at initial position. Given that co-efficient of linear expansion $\alpha = 2\times10^{-5}/°C$ and Young's modulus $Y = 5\times10^{11}N/m^2$ for the wire.



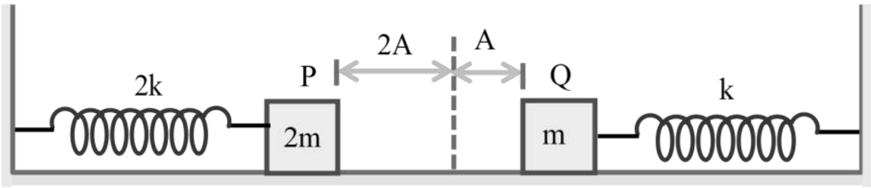
(A) 3kg (B) 6kg (C) 9kg (D) 12kg

9. A solid uniform cylinder of mass M performs small oscillations due to the action of two springs, each having stiffness k. If time period of these oscillations in absence of any sliding is $T = 2\pi\sqrt{\frac{aM}{10k}}$, then find the value of a. (Springs have their neutral length initially)



(A) 3 (B) 2 (C) 1 (D) 4

10. Two objects P and Q of masses 2m and m are connected to two springs of spring constant 2k and k respectively. Other ends of springs are connected to rigid walls as shown. Initially springs are compressed by lengths 2A and A respectively, and then released:



Smooth surface

(A) The period of P and Q remains same before and after the collision irrespective of the fact whether collision between masses is perfectly elastic or perfectly inelastic

(B) The period of P and Q remains same before and after the collision only if collision between the objects is perfectly elastic

(C) Time period of P and Q remains same before and after the collision only if collision between the objects is perfectly inelastic.

(D) If collision is perfectly inelastic, amplitude of SHM of combined mass is A.

ANSWER KEY

EXERCISE-I_KEY

1	2	3	4	5	6	7	8	9	10	11	12	13
C	B	C	D	C	D	A	B	B	D	B	D	A
14	**15**											
C	A											

EXERCISE-II_KEY

1	2	3	4	5	6	7	8	9	10	11
B	D	C	B	A	B	ABCD	2	A	6	B
12	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**
C	2	4	B	D	AD	BC	BD	C	BC	ABC

EXERCISE-III_KEY

1	2	3	4	5	6	7	8	9	10	11
D	A	B	15	D	4	B	C	C	BC	BD
12	**13**	**14**	**15**							
ABCD	AB	4	C							

EXERCISE-IV_KEY

1	2	3	4	5	6	7	8	9	10	11
C	100	A	A	D	A	B	ABCD	AC	AB	ACD
12	**13**	**14**	**15**							
10	1	B	C							

EXERCISE-V_KEY

1	2	3	4	5	6	7	8	9	10
C	5	B	A	B	B	D	C	B	CD
11	**12**	**13**	**14**	**15**					
BD	A	C	A	8					

PROFICIENCY TEST-I_KEY

1	2	3	4	5	6	7	8	9	10
B	D	BC	ABCD	AD	A	5	A	3	2

PROFICIENCY TEST-II_KEY

1	2	3	4	5	6	7	8	9	10
C	A	AD	A	BD	ACD	ABC	ABCD	AC	BD

PROFICIENCY TEST-III_KEY

1	2	3	4	5	6	7	8	9	10
ACD	A	C	D	AC	BD	ABC	D	A	AD