

$$\frac{1+z+\bar{z}^2}{1-\bar{z}+z^2} = \frac{1+\bar{z}+\bar{z}^2}{1-z+\bar{z}^2}$$

$$(z-\bar{z})(1-\bar{z}\bar{z}) = 0$$

value

$$\delta_{\min}^{\max} = \frac{\bar{a}^2 - 3\bar{b}^2}{3\bar{a}^2 - \bar{b}^2}$$

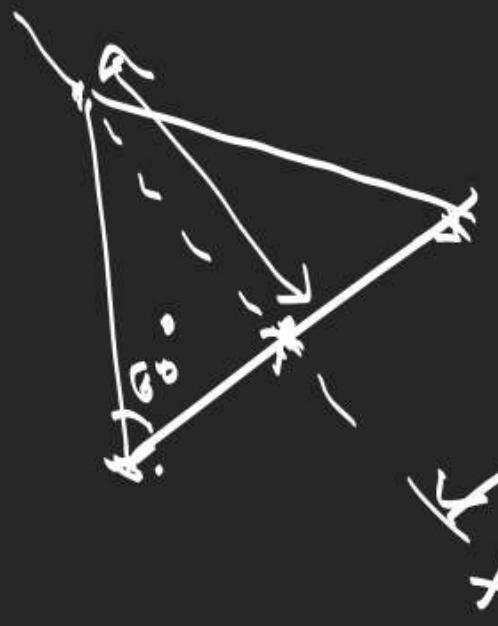
$$\min |z| \quad \max |z|$$

$$2\sqrt{3} - \sqrt{3}$$

$$(1+z)(1+\bar{z}) (1-z)(1-\bar{z})$$

$$= (1+z_1/z_2) (z_1 - z_2)^2 = (z_1/z_2)^2 + (z_1 - z_2)^2$$

$$\frac{(1-z_1)(1-\bar{z}_2)}{(1-z_2)(1-\bar{z}_1)} = \frac{(1-z_1)(1-\bar{z}_2)}{(1-z_2)(1-\bar{z}_1)} = \frac{1+z_1^2}{1+z_2^2} = \frac{z_1 + 1}{z_2 + 1} \in \mathbb{R}.$$



$$-i\bar{z} = z^2$$

$$|z| = |\bar{z}| \Rightarrow |z| = 0, 1$$

$$-i|z|^2 = z^3$$

$$z^3 = 0 \text{ or}$$

$$z = 0$$

$$z^3 = -i$$

$$z = e^{i\left(\frac{-\frac{\pi}{2} + 2k\pi}{3}\right)}$$

$$k=0, 1, 2$$

$$z = \{$$

$$\eta = 4k$$

$$4k+1$$

$$4k+2$$

$$\sqrt{i} = e^{i \left( \frac{\pi + 2k\pi}{2} \right)} \quad k=0, 1$$

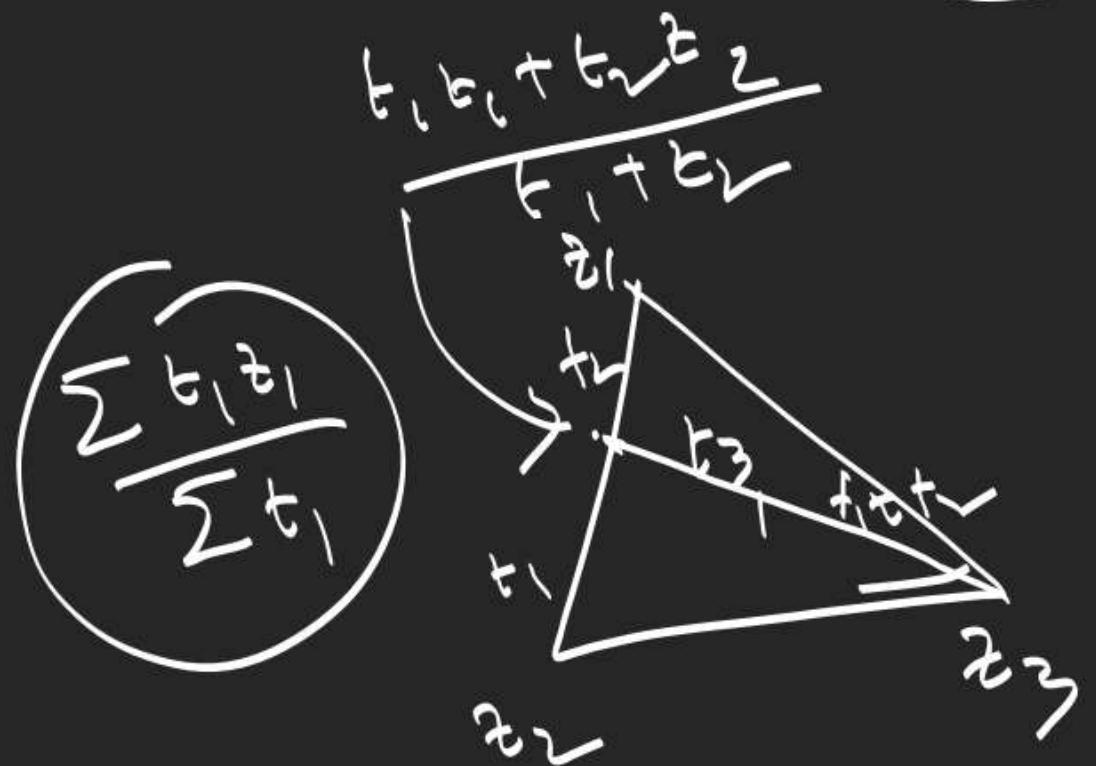
$$\sqrt{-i} = e^{i \left( \frac{-\pi + 2k\pi}{2} \right)} \quad k=0, 1$$

$$\frac{i-1}{i+1} = \frac{2\sin^2 \frac{\pi}{5} + 2\sin \frac{\pi}{5} \cos \frac{\pi}{5}}{2\sin \frac{\pi}{5}}$$



$$\frac{\sqrt{2} e^{i \left( \frac{3\pi}{5} - \frac{\pi}{5} \right)}}{2 \sin \frac{\pi}{5}}$$

$$|z_1 + 1| + |z_2 + 1| + |-z_1 z_2 - 1| \geq |z_1 + 1| + |z_2 - z_1 z_2|$$



$$= |\underline{1+z_1}| + |\underline{1-z_1}|$$

$$\geq 2.$$

$$a^2 = 3, \quad a = \sqrt{3} - \sqrt{3}$$

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$$z \in P.I. \quad z^5 + 1 = (z+1) \left( z^2 - 2z\omega \cos \frac{\pi}{5} + 1 \right) \left( z^2 - 2z\omega \sin \frac{3\pi}{5} + 1 \right)$$

$|z| = 1$

$z^2 \leq 0$

$z = i$

$$= \frac{i+1}{i+2\omega \cos \frac{\pi}{5}} = \frac{-2i\omega \cos \frac{\pi}{5}}{1 - e^{ix}}$$

$$= \frac{-2i\omega \sin \frac{3\pi}{5}}{1 - e^{ix}}$$

$$= \frac{1}{1 - e^{-ix}}$$

$$= 1 - e^{-ix}$$

$$(a-1)x^2 - 4x + a+2 = 0$$

both roots  $\leq 0$

$$\frac{az+b}{z-c} = z$$

$$|c|= \sqrt{\frac{c\bar{c}+b\bar{b}}{z-a}} = 1$$

$$\frac{z+\bar{b}}{z-\bar{a}} = \frac{1}{\bar{c}}$$

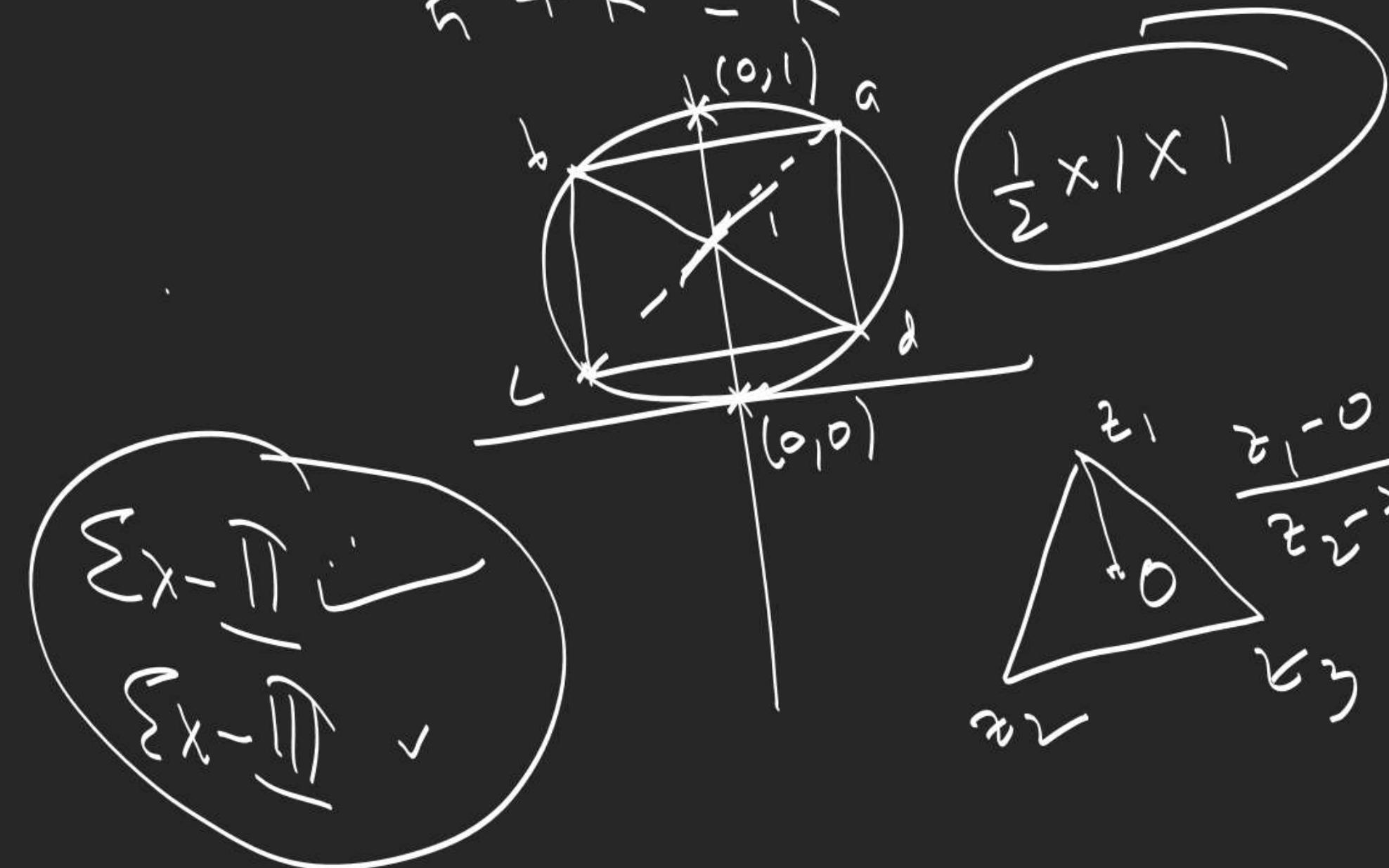
$$a = ?$$

$$e^{inx} - e^{-inx}$$

$$2i \cdot 3^n = \frac{1}{2i} \left[ \left( \frac{e^{ix}}{1/3} \right)^n - \left( \frac{e^{-ix}}{1/3} \right)^n \right]$$

$$f(z) = \frac{z+i}{z^2+1} = \left( \frac{z}{z^2+1}, \frac{1}{z^2+1} \right) = (h, k)$$

$$h^2 + k^2 = 1$$



$$\frac{z_1 - 0}{z_2 - z_3} + \frac{\bar{z}_1}{\bar{z}_2 - \bar{z}_3} = 0$$