Nishant Jindal

QUADRATIC EQUATION

$a, b, c \in \mathbb{Q}$.

D

D = 0

(1) Persqr.

(2) $\left(x + \frac{b}{2a}\right)^2$

(3) Identical Roots
Same Root

D ≠ 0

D > 0 (Distinct)

D = Persqr

(1) Roots Rational

(2) Root distinct

D ≠ Persqr

(1) Root Irrational

(2) $\alpha + \sqrt{\beta}$ then other $\alpha - \sqrt{\beta}$.

D < 0

(1) Imaginar Roots

(2) $\alpha + i\beta$ form.

(3) $\alpha + i\beta$ then other root $\alpha - i\beta$.

$i = \sqrt{-1}$

$\sqrt{-4} = 2i$

$\sqrt{-9} = 3i$

Nishant Jindal

QUADRATIC EQUATION

Q $f(x)=ax^2+bx+c$ & $a-b+c=0$ given

$f(-1)=a(-1)^2+b(-1)+c$

$=a-b+c=0$ (दिया है)

$\Rightarrow ax^2+bx+c=0$ → $\alpha=-1$, $\beta=-\frac{c}{a}$

$\alpha\cdot\beta=\frac{c}{a} \Rightarrow -1\cdot\beta=\frac{c}{a} \Rightarrow \boxed{\beta=-\frac{c}{a}}$

Q $f(x)=ax^2+bx+c$ & $4a+2b+c=0$

Ex-2 Put $f(2)=a(2)^2+b\cdot 2+c$

$4a+2b+c=0$ (दिया है) $\Rightarrow$ 1 Root $=2=\alpha$

$\alpha\cdot\beta=\frac{c}{a} \Rightarrow 2\cdot\beta=\frac{c}{a} \Rightarrow \boxed{\beta=\frac{c}{2a}}$

Q Poly. $f(x)=ax^2+bx+c$ & $a+b+c=0$

$f(1)=a\cdot 1^2+b\cdot 1+c$

$=a+b+c=0$

$f(1)=0 \Leftarrow$ जब x की जगह 1 रखने पर 0

Q. Poly has $\boxed{x=1}$ as a Zero.

$ax^2+bx+c=0$ → $\alpha=1$, $\beta=\frac{c}{a}$

$\alpha\cdot\beta=\frac{c}{a}$

$1\cdot\beta=\frac{c}{a} \Rightarrow \beta=\frac{c}{a}$

Nishant Jindal

QUADRATIC EQUATION

Note:-

1) In case of Rational Roots $D>0$ & Perfect sq^r.

2) Irr. Roots & Imaginary Roots Comes in Pair.

3)* A) If $a+b+c=0$ for ax^2+bx+c then $\alpha=1$ & $\beta=\frac{c}{a}$.

B) If $a-b+c=0$ ———— then $\alpha=-1$ & $\beta=-\frac{c}{a}$

C) If $4a+2b+c=0$ ———— then $\alpha=2$ & $\beta=\frac{c}{2a}$

D) If $4a-2b+c=0$ ———— then $\alpha=-2$ & $\beta=-\frac{c}{2a}$

RK:- Poly has Zeroes
Eq^n has Roots

Q Poly.

$f(x)=ax^2+bx+c$ & $a+b+c=0$

$f(1)=a\cdot1^2+b\cdot1+c$

$=a+b+c=0$

$f(1)=0$ ← fxn में 1 रखने पर 0

Q. Poly has $\boxed{x=1}$ as a Zero.

$ax^2+bx+c=0$ → $\alpha=1$, $\beta=\frac{c}{a}$

$\alpha\cdot\beta=\frac{c}{a}$

$1\cdot\beta=\frac{c}{a} \Rightarrow \beta=\frac{c}{a}$

QUADRATIC EQUATION

$(x-b)(x-c)$

$x^2-(x-b)x+bc = x^2-x(b+c)+bc$

Q $x^2-(m+2)x+(m^2-4m+4)=0$ has Coincident Roots then $m=?$

Already Done

Q If roots of Eq^n $(b-c)x^2+(c-a)x+(a-b)=0$ be equal then a, b, c are in - - - ?

1) Coefficients are Cyclic $\Rightarrow$ 1 Root $= 1 = \alpha$

2) Prod $= \alpha\beta = \frac{a-b}{b-c}$

$\therefore \beta = \frac{a-b}{b-c}$

3) Roots are Eq^l $\Rightarrow \alpha = \beta$

$1 = \frac{a-b}{b-c} \Rightarrow a-b = b-c$

$a+c = 2b$ A.P.

Q $(x-b)(x-c)+(x-c)(x-a)+(x-a)(x-b)=0$

has Eq^n Roots then $a=b=c$ [TNF]

① $x^2-(b+c)x+bc + x^2-(c+a)x+ca + x^2-(a+b)x+ab = 0$

$3x^2-(b+c+c+a+a+b)x+bc+ca+ab=0$

$3x^2-2(a+b+c)x+(ab+bc+ca)=0$

② Eq^l Root $\rightarrow D=0 \Rightarrow B^2-4AC=0$

$(-2(a+b+c))^2-4\times3\times(ab+bc+ca)=0$

$4(a+b+c)^2-4\times(3ab+3bc+3ca)=0$

$a^2+b^2+c^2+2ab+2bc+2ca-3ab-3bc-3ca=0$

a

QUADRATIC EQUATION

$(x-b)(x-c)$

$x^2-(x-b)x+bc = x^2-x(b+c)+bc$

आपको

$a^2+b^2+c^2+2ab+2bc+2ca-3ab-3bc-3ca=0$

$a^2+b^2+c^2-ab-bc-ca=0$

नहीं पता क्या ??

$\frac{1}{2}\{(a-b)^2+(b-c)^2+(c-a)^2\}=0$

$(a-b)^2$ (+) $(b-c)^2$ (+) $(c-a)^2$ (+)

(Non-ve)

Sum of 3 +ve No = 0 is Psbl only when all of them are giving Zero.

$a-b=0$ & $b-c=0$ & $c-a=0$

$a=b$ & $b=c$ & $c=a$

$\Rightarrow \boxed{a=b=c}$

Q $(x-b)(x-c)+(x-c)(x-a)+(x-a)(x-b)=0$

has Eq^n Roots then $a=b=c$ [TIF]

① $x^2-(b+c)x+bc + x^2-(c+a)x+ca + x^2-(a+b)x+ab = 0$

$3x^2-(b+c+c+a+a+b)x+bc+ca+ab=0$

$3x^2-2(a+b+c)x+(ab+bc+ca)=0$

② Eq^l Roots $\rightarrow D=0 \Rightarrow B^2-4AC=0$

$(-2(a+b+c))^2-4\times3\times(ab+bc+ca)=0$

$4(a+b+c)^2-4\times(3ab+3bc+3ca)=0$

$\Rightarrow a^2+b^2+c^2+2ab+2bc+2ca-3ab-3bc-3ca=0$

a

Nishant Jindal

QUADRATIC EQUATION

Q For what value of K

$(4-K)x^2+(2K+4)x+(2K+1)$ is a Per. Sq.

Already Done

Q If Roots of Eq^n $(a^2+b^2+c^2)x^2-2(ab+bc+cd)x+b^2+c^2+d^2=0$ are Eq^l then P.T. $\frac{b}{a}=\frac{c}{b}=\frac{d}{c}$

$(a^2x^2-2abx+b^2)+(b^2x^2-2bcx+c^2)+(c^2x^2-2cdx+d^2)=0$

$(ax-b)^2+(bx-c)^2+(cx-d)^2=0$ (+) (+) (+)

3 +ve No of Sum = 0

Psbl only when all 3 are zero

$ax-b=0$ & $bx-c=0$ & $cx-d=0$

$ax=b$ $\quad bx=c$ $\quad cx=d$

$x=\frac{b}{a}$ $\quad x=\frac{c}{b}$ $\quad x=\frac{d}{c}$

40s Specially

$x=\frac{b}{a}, \frac{c}{b}=\frac{d}{c}$

Q Sh. that Roots of

$(b+(-2a)x^2+((+a-2b)x+(a+b-2c)=0$ are Rational?

Roots are Rational In them $D=$ Per sq.

Sum of roots $= b+(-2a+(+a-2b+a+b-2c$

$= 0 \rightarrow$ Ek Root $\alpha=1=$ Rational

Ek Root Rational then other has to be Rational

Both Roots are Rational

Nishant Jindal

Q $f(x) = 2x^3 + mx^2 - 13x + n$ has 2 & 3 as Roots of Eqn $f(x) = 0$ then find m, n?

① as 2 & 3 are Roots $\Rightarrow f(2) = 0$

$f(3) = 0$

② $f(2) = 2 \times 2^3 + 2^2 m - 13 \times 2 + n = 0$

$4m + n = 10$

$f(3) = 2 \times 3^3 + 3^2 m - 13 \times 3 + n = 0$

$9m + n = -15$

$4m + n = 10$

$9m + n = -15$

$-5m = 25$

$m = -5$

$n = 30$

54
-39

Nishant Jindal

QUADRATIC EQUATION

Q Roots of $6x^2 - 7x + K = 0$ are Rational than K?

A) -1 ✗

B) -2 ✗

C) -1, -2 ✗

D) 1, 2 ✓

D = Perfect sqr

$D = (-7)^2 - 4 \times 6 \times K$

$= 49 - 24K$ (can be Per Sqr.

$K = 0 \rightarrow D = 49 - 24 \times 0 = 49$ ✓

$K = 1 \rightarrow D = 49 - 24 \times 1 = 25$ ✓

$K = 2 \rightarrow D = 49 - 24 \times 2 = 1$ ✓

$K = -1 \Rightarrow D = 49 - 24 \times -1 = 73$ ✗

$49 + 24$

$K = -2 \Rightarrow D = 49 - 24 \times -2 = 97$ ✗

$49 + 48$

Q Roots of

$(a^2+b^2)x^2-2(bc+ad)x+(c^2+d^2)=0$ are Equal

if A) $ab=cd$ (B) $ac=bd$ (C) $ad+bc=0$ (D) NOT.

(M1) $D=0$ (check)

$$(-2(bc+ad)^2-4(a^2+b^2)(c^2+d^2)=0$$

$$4(bc+ad)^2-4(a^2c^2+a^2d^2+b^2c^2+b^2d^2)=0$$

$$b^2c^2+a^2d^2+2abcd-a^2c^2-a^2d^2-b^2c^2-b^2d^2=0$$

$$a^2c^2+b^2d^2-2abcd=0$$

$$(ac-bd)^2=0$$

$$\boxed{ac=bd}$$

Nishant Jindal

QUADRATIC EQUATION

Q Find value of m for which Roots of Eqn $x^2 - x + m = 0$ are not Real?

A) $\left(\frac{1}{4}, \infty\right)$

B) $\left(-\infty, \frac{1}{4}\right)$

C) $\left(-\frac{1}{4}, \frac{1}{4}\right)$

D) NOT

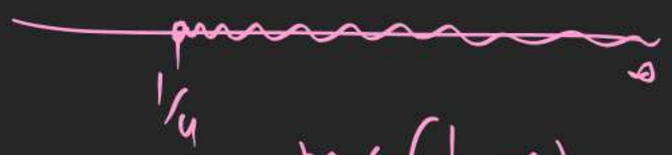
$D < 0$

$(-1)^2 - 4 \times 1 \times m < 0$

$1 - 4m < 0$

$4m > 1$

$m > \frac{1}{4}$



$m \in \left(\frac{1}{4}, \infty\right)$

Sum of coff $= a + b - c - 2a + a - b + c = 0$

Q Nature of Roots of $\alpha = 1$ (Rahu)

$\beta =$ Ratio

$(a+b-c)x^2 - 2ax + (a-b+c) = 0$

$D = (-2a)^2 - 4(a+b-c)(a-b+c)$

$= 4a^2 - 4[a^2 - ab + ac + ba - b^2 + bc - ac + bc - c^2]$

$= 4a^2 - 4a^2 + 4b^2 - 8bc + 4c^2$

$= 4(b^2 - 2bc + c^2)$

$D = 4(b-c)^2$ Per sq^r

Roots = Rational

Q1. The roots of the equation $(x+2)^2 = 4(x+1) - 1$ are-
(A) \pm 1 (B) $\pm$I (C) 1,2 (D) $-1, -2$

Q2. The roots of quadratic equation $x^2 + 14x + 45 = 0$ are -
(A) $-9,5$ (B) 5,9 (C) $-5,9$ (D) $-5, -9$

Q3. The roots of the equation $x^4 - 8x^2 - 9 = 0$ are-
(A) $\pm 3, \pm 1$ (B) $\pm 3, \pm$I (C) $\pm 2, \pm$I (D) None of these

Q4. Which of the following equations has 1 and -2 as the roots -
(A) $x^2 - x - 2 = 0$ (B) $x^2 + x - 2 = 0$ (C) $x^2 - x + 2 = 0$ (D) $x^2 + x + 2 = 0$

Q5. Roots of $3^x + 3^{-x} = 10/3$ are-
(A) 0,1 (B) $1, -1$ (C) $0, -1$ (D) None of these

Q6. If $f(x) = 2x^3 + mx^2 - 13x + n$ and 2 and 3 are roots of the equations $f(x) = 0$, then values of m and n are-
(A) 5,30 (B) $-5,30$ (C) $-5, -30$ (D) $5, -30$

Q7. The number of roots of the quadratic equation $8\sec^2\theta - 6\sec\theta + 1 = 0$ is -
(A) Infinite (B) 1 (C) 2 (D) 0

Q8. If roots of the equation $ax^2 + 2(a+b)x + (a+2b+c) = 0$ are imaginary, then roots of the equation $ax^2 + 2bx + c = 0$ are -
(A) rational (B) irrational (C) equal (D) complex

Q9. If a and b are the odd integers, then the roots of the equation $2ax^2 + (2a + b)x + b = 0, a \neq 0$, will be-

(A) rational (B) irrational (C) non-real (D) equal

Q10. If the roots of the equation $6x^2 - 7x + k = 0$ are rational then k is equal to -

(A) -1 (B) $-1, -2$ (C) -2 (4) 1,2

Q11. The roots of the equation

$(a^2 + b^2)x^2 - 2(bc + ad)x + (c^2 + d^2) = 0$ are equal, if -

(A) $ab = cd$ (B) $ac = bd$ (C) $ad + bc = 0$ (D) None of these

Q12. For what value of m, the roots of the equation $x^2 - x + m = 0$ are not real-

(A) $]\frac{1}{4}, \infty[$ (B) $]-\infty, \frac{1}{4}[$ (C) $]-\frac{1}{4}, \frac{1}{4}[$ (4) None of these

Q13. Roots of the equation $(a + b - c)x^2 - 2ax + (a - b + c) = 0$, $(a, b, c \in Q)$ are -
(A) rational (B) irrational (C) complex (D) none of these

Q14. The roots of the equation $x^2 - x - 3 = 0$ are-
(A) Imaginary (B) Rational (C) Irrational (D) None of these

Q15. The roots of the equation $x^2 + 2\sqrt{3}x + 3 = 0$ are-
(A) Real and equal (B) Rational and equal
(C) Irrational and equal (D) Irrational and unequal

Q16. If the roots of the equation $ax^2 + x + b = 0$ be real, then the roots of the equation $x^2 - 4\sqrt{ab}x + 1 = 0$ will be -
(A) Rational (B) Irrational (C) Real (D) Imaginary

$x = \frac{-2\sqrt{3} \pm \sqrt{0}}{2 \times 1}$

$x = -\sqrt{3}$ (Irrational)

$\alpha = -\sqrt{3} = \beta$

$x^2 - (\alpha+\beta)x + \alpha\beta = 0$

$x^2 - (-\sqrt{3}-\sqrt{3})x + (-\sqrt{3})(-\sqrt{3}) = 0$

$x^2 + 2\sqrt{3}x + 3 = 0$

$x^2 + 2\sqrt{3}x + 3 = 0$

Roots

Fail $D = (2\sqrt{3})^2 - 4 \times 1 \times 3$

$= 12 - 12 = 0$

Roots Identical

Rational + Equal

Sridhacharya Works When $a, b, c \in Q$

Q.17 If one root of equation $x^2 + px + 12 = 0$ is 4 , while the equation $x^2 + px + q = 0$ has equal roots then the value of q is-

(A) 49/4 (B) 4/49 (C) 4 (D) None of these

Q.18 If roots of the equation $(a - b)x^2 + (c - a)x + (b - c) = 0$ are equal, then a, b, c are in -

(A) A.P. (B) H.P. (C) G.P. (D) None of these

Q.19 If the roots of $x^2 - 4x - \log_2 a = 0$ are real, then-

(A) $a \geq \frac{1}{4}$ (B) $a \geq \frac{1}{8}$ (C) $a \geq \frac{1}{16}$ (D) None of these

Q.20 If the roots of both the equations $px^2 + 2qx + r = 0$ and $qx^2 - 2\sqrt{prx} + q = 0$ are real, then -

(A) $p = q, r \neq 0$ (B) $2q = \pm\sqrt{pq}$ (C) $p/q = q/r$ (D) None of these

Q.21 The roots of the equation $(p-2)x^2 + 2(p-2)x + 2 = 0$ are not real when-

(A) $p \in [1,2]$ (B) $p \in [2,3]$ (C) $p \in (2,4)$ (D) $p \in [3,4]$

Q.22 If the roots of the equation $x^2 - 10x + 21 = m$ are equal then m is-

(A) 4 (B) 25 (C) -4 (D) 0

Q.23 For what value of a, the difference of roots of the equation $(a-2)x^2 - (a-4)x - 2 = 0$ is equal to 3

(A) 3,3/2 (B) 3,1 (C) 1,3/2 (D) None of these

Q.24 If α, β are roots of the equation $x^2 + px - q = 0$ and γ, δ are roots of $x^2 + px + r = 0$, then the value of $(\alpha - \gamma)(\alpha - \delta)$ is-

(A) $p + r$ (B) $p - r$ (C) $q - r$ (D) $q + r$

Q.25 If α, β are roots of the equation $2x^2 - 35x + 2 = 0$, then the value of $(2\alpha - 35)^3 . (2\beta - 35)^3$ is equal to-

(A) 1 (B) 8 (C) 64 (D) None of these

Q.26 If α, β are roots of the equation $px^2 + qx - r = 0$, then the value of $\frac{\alpha}{\beta^2} + \frac{\beta}{\alpha^2}$ is equal to-

(A) $-\frac{p}{qr^2}(3pr + q^2)$ (B) $-\frac{q}{pr^2}(3pr + q^2)$

(C) $-\frac{q}{pr^2}(3pr - q^2)$ (D) $\frac{p}{pr^2}(3pr + q)$

Q.27 If product of roots of the equation $mx^2 + 6x + (2m - 1) = 0$ is -1 , then m equals-

(A) -1 (B) 1 (C) 1/3 (D) $-1/3$

Q.28 For what value of a the sum of roots of the equation $x^2 + 2(2 - a - a^2)x - a^2 = 0$ is zero -

(A) 1,2 (B) $1, -2$ (C) $-1,2$ (D) $-1, -2$

Q.29 The difference between the roots of the equation $x^2 - 7x - 9 = 0$ is –

(A) 7 (B) $\sqrt{85}$ (C) 9 (D) $2\sqrt{85}$

Q.30 The HM of the roots of the equation $x^2 - 8x + 4 = 0$ is –

(A) 1 (B) 2 (C) 3 (D) None of these

Q.31 If the sum of the roots of the equation $ax^2 + 4x + c = 0$ is half of their difference, then the value of ac is-

(A) 4 (B) 8 (C) 12 (D) -12

Q.32 If the sum of the roots of the equation $(a + 1)x^2 + (2a + 3)x + (3a + 4) = 0$ is -1 , then the product of the roots is –

(A) 0 (B) 1 (C) 2 (D) 3

Q.33 Sum of roots is -1 and sum of their reciprocals is $\frac{1}{6}$, then equation is –

(A) $x^2 + x - 6 = 0$ (B) $x^2 - x + 6 = 0$

(C) $6x^2 + x + 1 = 0$ (D) $x^2 - 6x + 1 = 0$