DPP – 1

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1. The angles α and β are such that tan α = m + 2 and tan β = m where m is a constant. If $\sec^2\alpha - \sec^2\beta = 16$ then the value of cot (α − β) is equal to

(A) 2 (B) 4 (C) 6 (D) 8

Ans. **(D)**

Sol. $\sec^2\alpha - \sec^2\beta = 16$

$\tan^2\alpha - \tan^2\beta = 16$

$(m+2)^2 - m^2 = 16$

$4m + 4 = 16 \Rightarrow m = 3$

$\therefore \tan\alpha = 5; \tan\beta = 3$

$$\tan(\alpha - \beta) = \frac{5-3}{1+15} = \frac{2}{16} = \frac{1}{8}$$

$\cot(\alpha - \beta) = 8$

2. The equation $|x|^2 + |x| - 6 = 0$ has

(A) only one root (B) four roots

(C) the sum of the roots is zero. (D) the product of the roots is – 6.

Ans. **(C)**

Sol. Let $|x| = t \rightarrow t^2 + t - 6 = 0 \Rightarrow (t+3)(t-2) = 0 \Rightarrow t = -3$ (rejected) or $t = 2$

$t = 2 \Rightarrow |x| = 2 \Rightarrow x = 2$ or $-2 \Rightarrow$ sum $= 0$ Ans.]

3. If $(n+3)^2 = a(n+2)^2 + b(n+1)^2 + cn^2$ holds true for every positive integer n then the quadratic equation $ax^2 + bx + c = 0$ has

(A) both positive roots (B) both negative roots

(C) one positive and one negative root (D) no real roots.

Ans. **(D)**

Sol. $a = 3; b = -3$ and $c = 1$

$\therefore 3x^2 - 3x + 1 = 0$

$D < 0 \Rightarrow$ no real roots]

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4. The real number x and y satisfy the equation $xy = \sin(2t)$ and $\frac{x}{y} = \tan(t)$ where $0 < t < \frac{\pi}{2}$.

The value of $x^2 + y^2$, is

(A) $\sqrt{2}$ (B) 1 (C) 2 (D) 4

Ans. **(C)**

Sol. $xy = \sin 2t$ ... (1)

$\frac{x}{y} = \tan t$ ... (2)

$(1) \times (2), x^2 = 2\sin^2 t$

$(1) \div (2), y^2 = 2\cos^2 t$

$\Rightarrow x^2 + y^2 = 2$

5. How many distinct real numbers belongs to the following collection

$$\left\{\ln(4-\sqrt{15});\ln(4+\sqrt{15});-\ln(4-\sqrt{15});-\ln(4+\sqrt{15});\ln\left(\frac{4+\sqrt{15}}{4-\sqrt{15}}\right);\ln(31+8\sqrt{15})\right\}$$

(A) 2 (B) 3 (C) 4 (D) 5

Ans. **(B)**

Sol. $\therefore (4-\sqrt{15})\cdot(4+\sqrt{15}) = 1$

$\therefore 4-\sqrt{15} = \frac{1}{4+\sqrt{15}}$ & $4+\sqrt{15} = \frac{1}{4-\sqrt{15}}$

$\therefore \ln(4-\sqrt{15}) = \ln\left(\frac{1}{4+\sqrt{15}}\right) = -\ln(4+\sqrt{15})$

$\therefore \ln(4-\sqrt{15}) = -\ln(4+\sqrt{15})$

$\ln\left\{\left(\frac{4+\sqrt{15}}{4-\sqrt{15}}\right)\times\left(\frac{4+\sqrt{15}}{4+\sqrt{15}}\right)\right\} = \ln\left[\frac{(4+\sqrt{15})^2}{1}\right] \quad = \ln(31+8\sqrt{15})$

6. In the range $0 \le x < 2\pi$, the equation $\cos(\sin x) = \frac{1}{2}$ has

(A) no solution (B) one solution

(C) two solutions (D) three solutions

Ans. **(A)**

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Sol. $\sin x \in [-1,1]$; θ i.e. $(-57°, 57°)$ nearly

$$\cos\theta \in \left(\frac{1}{2}, 1\right] \Rightarrow \cos(\sin x) = \frac{1}{2} \text{ has no solutions}$$

7. Suppose that ' a ' and ' b ' are two roots of the equation $x^2 + 3x + 5 = 0$. If $t = \frac{a+2}{b+2}$ and $s = \frac{b+2}{a+2}$ are the two roots of $x^2 - mx + 1 = 0$ then the value of ' m ' is

(A) $-5/2$ (B) $-5/3$ (C) $5/2$ (D) $5/3$

Ans. **(B)**

Sol. $a + b = -3$; $ab = 5$; $s + t = m$

$$s = \frac{b+2}{a+2}; t = \frac{a+2}{b+2}; \ s + t = \frac{(b+2)^2 + (a+2)^2}{(a+2)(b+2)} = \frac{(a^2+b^2) + 4(a+b) + 8}{ab + 2(a+b) + 4}$$

$$m = \frac{(a+b)^2 - 2ab + 4(a+b) + 8}{ab + 2(a+b) + 4}$$

$$s + t = -\frac{5}{3} \equiv m$$

8. The value of 'a' for which $\frac{\log_a 7}{\log_6 7} = \log_\pi 36$ holds good, is

(A) $1/\pi$ (B) π^2 (C) $\sqrt{\pi}$ (D) 2

Ans. **(C)**

Sol. $\frac{\log_a^7}{\log_6^7} = \frac{\log_7^6}{\log_7^a} \ \because \left(\log_b^a = \frac{1}{\log_a^b}\right)$

$= \log_a^6$

Now, $\log_a^6 = \log_\pi^{36}$

$\log_a^6 = \log_\pi^{6^2} = 2\log_\pi^6$

$\log_a^6 = \log_{\pi^{1/2}}^6 \Rightarrow a = \pi^{1/2} = \sqrt{\pi}$

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9. The product $\tan(\ln x)\cdot\tan\left(\ln\frac{x}{2}\right)\cdot\tan(\ln 2)$ wherever defined, is also equal to

(A) $\tan(\ln x)+\tan\left(\ln\frac{x}{2}\right)+\tan(\ln 2)$ (B) $\tan(\ln x)+\tan\left(\ln\frac{x}{2}\right)-\tan(\ln 2)$

(C) $\tan(\ln x)-\tan\left(\ln\frac{x}{2}\right)+\tan(\ln 2)$ (D) $\tan(\ln x)-\tan\left(\ln\frac{x}{2}\right)-\tan(\ln 2)$

Ans. **(D)**

Sol. $\ln x-\ln\left(\frac{x}{2}\right)=\ln 2$

or $\tan\left(\ln x-\ln\frac{x}{2}\right)=\tan(\ln 2)$ 1

$\left(\because \tan(A-B)=\dfrac{\tan A-\tan B}{(1+\tan A\cdot\tan B)}\right)$

$\therefore$ From (1)

$$\frac{\tan(\ln x)-\tan\left(\ln\frac{x}{2}\right)}{1+\tan(\ln x)\cdot\tan\left(\ln\frac{x}{2}\right)}=\tan(\ln 2)$$

$$\tan(\ln x)-\tan\left(\ln\frac{x}{2}\right)=\tan(\ln 2)+\tan(\ln 2).\tan(\ln x).\tan\left(\ln\frac{x}{2}\right)$$

$$\text{or }\tan(\ln x).\tan\left(\ln\frac{x}{2}\right).\tan(\ln 2)=\tan(\ln x)-\tan\frac{x}{2}-\tan(\ln 2)$$