Nishant Jindal

Q $0\cdot{}^{n}C_0 + 1\cdot C_1 + 2\cdot C_2 + \cdots + n\,C_n = ?$

AP

$= \frac{(0+n)}{2}\cdot 2^n = n\cdot 2^{n-1}$

Q $\frac{1}{{}^{n}C_0} + \frac{1}{{}^{n}C_1} + \frac{1}{{}^{n}C_2} + \cdots = A$

find $\sum_{r=0}^{n} \frac{r}{{}^{n}C_r} = ?$

Demand $= \frac{0}{{}^{n}C_0} + \frac{1}{{}^{n}C_1} + \frac{2}{{}^{n}C_2} + \cdots + \frac{n}{{}^{n}C_n} = S$

$\frac{n}{{}^{n}C_n} + \frac{n-1}{{}^{n}C_{n-1}} + \frac{n-2}{{}^{n}C_{n-2}} + \cdots + \frac{0}{{}^{n}C_0} = S$

$2S = \left(\frac{0}{{}^{n}C_0} + \frac{n}{{}^{n}C_n}\right) + \left(\frac{1}{{}^{n}C_1} + \frac{n-1}{{}^{n}C_{n-1}}\right) + \left(\frac{2}{{}^{n}C_2} + \frac{n-2}{{}^{n}C_{n-2}}\right) + \cdots \left(\frac{n}{{}^{n}C_n} + \frac{0}{{}^{n}C_0}\right)$

$= \frac{n}{{}^{n}C_0} + \frac{\cancel{1}+n-\cancel{1}}{{}^{n}C_1} + \frac{\cancel{2}+n-\cancel{2}}{{}^{n}C_2} + \cdots + \frac{n+0}{{}^{n}C_n}$

$2S = n\left\{\frac{1}{{}^{n}C_0} + \frac{1}{{}^{n}C_1} + \frac{1}{{}^{n}C_2} + \cdots + \frac{1}{{}^{n}C_n}\right\} = nA \Rightarrow \boxed{S = \frac{nA}{2}}$

Q3 $\sum_{r=0}^{n} {}^{n}C_r\, x^{n-r}\cdot y^{r} = ?$ $(x+y)^n$

$$\boxed{\begin{aligned}(x+a)^n &= {}^{n}C_0\, x^n\cdot a^0 + {}^{n}C_1\, x^{n-1}\cdot a^1 + {}^{n}C_2\, x^{n-2}\cdot a^2 + \cdots + {}^{n}C_n\, x^0\cdot a^n \\ &= \sum_{r=0}^{n} {}^{n}C_r\,(x)^{n-r}\cdot(a)^r\end{aligned}}$$

Q $\sum_{r=0}^{n} {}^{n}C_r\,(x)^{n-r} = ?$ $(\text{I} + \text{II})^n = (x+1)^n$

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$(r+1)$

$r=1$

$^{50}C_1 + {}^{50}C_2 + {}^{50}C_3 + {}^{50}C_4 + \dots + {}^{50}C_{50} \} - {}^{50}C_0 - {}^{50}C_1$

$= 2^{50} - 1 - 50 = 2^{50} - 51$

$\Rightarrow \sum_{r=0}^{n} a\,{}^nC_r + \sum_{r=0}^{n} r\cdot d\,{}^nC_r$

IJUS

$\Rightarrow a\sum_{r=0}^{n} {}^nC_r + d\sum_{r=0}^{n} r\,{}^nC_r$

$\Rightarrow a\cdot 2^n + d\sum_{r=1}^{n} \cancel{r}\cdot\frac{n}{\cancel{r}}\,{}^{n-1}C_{r-1}$

$a\cdot 2^n + nd\sum_{r=1}^{n} {}^{n-1}C_{r-1}$

$= 39$ $a\cdot 2^n + n\cdot(2)^{n-1}d$

Q $\sum_{r=1}^{39} {}^{40}C_{r+1}(-1)^r$ ← (LFDA)

Suffix r+1 But (-1)'s Deg = r

No repeat

$\frac{1}{-1} \times \sum_{r=1}^{39} {}^{40}C_{r+1}(-1)^{r+1}$ ← Now suffix Same

$-1\{\ \cancel{(1-1)^{40}} + {}^{40}C_0 - {}^{40}C_1\} = -\{1-40\}$

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Q Find

$0^2\cdot {}^nC_0 + 1^2\cdot {}^nC_1 + 2^2\cdot {}^nC_2 + 3^2\cdot {}^nC_3 + \dots + n^2\cdot {}^nC_n = ?$

① $G.T = \sum_{r=1}^{n} r^2\cdot {}^nC_r$ D.U.S

$= \sum_{r=1}^{n} r^{\cancel{2}}\cdot \frac{n}{\cancel{r}}\cdot {}^{n-1}C_{r-1}$

$= n\sum_{r=1}^{n} r\cdot {}^{n-1}C_{r-1}$

$= n\sum ((r-1)+1)\, {}^{n-1}C_{r-1}$

$= n\sum_{r=1}^{n} (r-1)\cdot {}^{n-1}C_{r-1} + n\sum_{r=1}^{n} {}^{n-1}C_{r-1}$
DUS

Q. $1\cdot {}^nC_1 + 2\cdot {}^nC_2 + 3\cdot {}^nC_3 + \dots + n\cdot {}^nC_n = ?$

① $\sum_{r=1}^{n} r\cdot {}^nC_r$ DUS

$\Rightarrow \sum_{r=1}^{n} \cancel{r}\cdot \frac{n}{\cancel{r}}\cdot {}^{n-1}C_{r-1}$

$= n\sum_{r=1}^{n} {}^{n-1}C_{r-1}$

$= n\cdot (1+1)^{n-1} = n\cdot 2^{n-1}$

$n\cdot\sum_{r=2}^{n} \cancel{(r-1)}\frac{n-1}{\cancel{(r-1)}}\cdot {}^{n-2}C_{r-2} + n\sum_{r=1}^{n} {}^{n-1}C_{r-1}$

$(n)(n-1)\sum_{r=2}^{n} {}^{n-2}C_{r-2} + n\cdot (1+1)^{n-1}$

$(n)(n-1)\cdot (1+1)^{n-2} + n\cdot 2^{n-1}$

$(n)(n-1)\cdot 2^{n-2} + n\cdot 2^{n-1}$

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$$\sum_{r=1}^{n} r^2 \cdot {}^nC_r$$

$$\sum r^2 \cdot \frac{n}{\cancel{r}} \cdot {}^{n-1}C_{r-1}$$

$$n \sum_{r=1}^{n} ((r-1)+1) \cdot {}^{n-1}C_{r-1}$$

$$n\left\{\sum (r-1) \cdot {}^{n-1}C_{r-1} + \sum {}^{n-1}C_{r-1}\right\}$$

$$n\left\{\sum \cancel{(r-1)} \frac{n-1}{\cancel{r-1}} \cdot {}^{n-2}C_{r-2} + (2)^{n-1}\right\}$$

$$n\left\{(n-1)\sum {}^{n-2}C_{r-2} + 2^{n-1}\right\}$$

$$n \cdot (n-1) \cdot 2^{n-2} + n \cdot 2^{n-1} = 2^{n-2}\{n^2 - n + 2n\}$$

$$= (n)(n+1) \cdot 2^{n-2}$$

Result

$$\boxed{\begin{aligned} &\sum {}^nC_r = 2^n \\ &\sum r \cdot {}^nC_r = n \cdot 2^{n-1} \\ &\sum r^2 \cdot {}^nC_r = (n)(n+1) \cdot 2^{n-2} \end{aligned}}$$

Direct

Q. $1 \cdot 2 \cdot {}^nC_0 + 2 \cdot 3 \cdot {}^nC_1 + 3 \cdot 4 \cdot {}^nC_2 + \ldots + (n+1)(n+2)\,{}^nC_n = ?$

① $= \sum_{r=0}^{n} (r+1)(r+2) \cdot {}^nC_r$

② $\sum (r^2+3r+2) \cdot {}^nC_r$

$$\sum r^2 \cdot {}^nC_r + 3\sum r \cdot {}^nC_r + 2\sum {}^nC_r$$

$$(n)(n+1)2^{n-2} + 3 \cdot n \cdot 2^{n-1} + 2 \cdot 2^n$$

Q $\sum_{r=0}^{n} (2r+1) \cdot {}^nC_r = ?$

$$\Rightarrow 2\sum_{r=0}^{n} r \cdot {}^nC_r + \sum_{r=0}^{n} {}^nC_r$$

$$= 2 \cdot n \cdot 2^{n-1} + 2^n$$

$$= (n+1)2^n$$

Q $\sum_{r=0}^{n} (r)(r-1) \cdot {}^nC_r = ?$

$$\sum_{r=0}^{n} r^2 \cdot {}^nC_r - \sum_{r=0}^{n} r \cdot {}^nC_r$$

$$\Rightarrow (n)(n+1)2^{n-2} - n \cdot 2^{n-1}$$

$$= 2^{n-2}\{n^2 + n - 2n\} = (n)(n-1)2^{n-2}$$

Nishant Jindal

Q $\sum_{r=0}^{n} r \cdot {}^nC_r (-1)^r = ?$

DUS

$\sum r \cdot \frac{n}{r} \, {}^{n-1}C_{r-1} (-1)^r$

$n \sum_{r=1}^{n} {}^{n-1}C_{r-1} (-1)^{r-1} \times (-1)$

$-n \sum_{r=1} {}^{n-1}C_{r-1} \cdot (-1)^{r-1}$

$-n (1-1)^{n-1} = -n \times 0$

$= 0$

Q $\sum_{r=0}^{n} r^2 \cdot {}^nC_r (-1)^r$

$= 0$

Result $\boxed{\sum r^K \cdot {}^nC_r (-1)^r = 0 \quad K \in W}$

$(x-r)(y-r)(z-r)$

$(xy - yr - xr + r^2)(z-r)$

$xyz - r(yz+xz) + zr^2 \underbrace{- xyr + r^2(x+y) - r^3}$

$xyz - r(xy+yz+zx) + r^2(x+y+z)$

Q $xyz \cdot {}^nC_0 - (x-1)(y-1)(z-1) \cdot {}^nC_1 + (x-2)(y-2)(z-2) \cdot {}^nC_2 - \dots - (-1)^n (x-n)(y-n)(z-n) \cdot {}^nC_n = ?$

$\sum_{r=0}^{n} (x-r)(y-r)(z-r) \cdot {}^nC_r (-1)^r$

$\Rightarrow \sum \left(xyz + r^2(x+y+z) - r(xy+yz+zx) - r^3\right) \cdot {}^nC_r \cdot (-1)^r$

$\Rightarrow xyz \cancel{\sum {}^nC_r \cdot (-1)^r} + (x+y+z) \cancel{\sum r^2 \cdot {}^nC_r \cdot (-1)^r} - (xy+yz+zx) \cancel{\sum r \, {}^nC_r (-1)^r} - \cancel{\sum r^3 \cdot {}^nC_r (-1)^r}$

$= 0$

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Q $(a-1)^2 \cdot {}^nC_1 - (a-2)^2 \cdot {}^nC_2 + (a-3)^2 \cdot {}^nC_3 - - - - + (-1)^{n-1}(a-n)^2 \cdot {}^nC_n = ?$

$$-\sum_{r=1}^{n} (a-r)^2 \cdot {}^nC_r (-1)^r = -\left\{ \sum (a^2 - 2ar + r^2) \cdot {}^nC_r \cdot (-1)^n \right\}$$

$$= -\left\{ a^2 \sum_{r=1}^{n} {}^nC_r \cdot (-1)^r - 2a \cancel{\sum_{r=1}^{n} r \cdot {}^nC_r (-1)^r} + \cancel{\sum_{r=1}^{n} r^2 \, {}^nC_r \cdot (-1)^r} \right\}$$

$$= -a^2 \left(\underbrace{{}^nC_0 - {}^nC_1 + {}^nC_2 - {}^nC_3 + {}^nC_4 - - - {}^nC_n}_{} - {}^nC_0 \right)$$

$$= -a^2(0-1)$$

$$= a^2$$

Q $C_0 + \frac{C_1}{2} + \frac{C_2}{3} + - - - + \frac{C_n}{n+1} = ?$

$$\Rightarrow \sum_{r=0}^{n} \frac{{}^nC_r}{r+1} = \sum_{r=0}^{n} \frac{1}{r+1} \, {}^nC_r$$

DJS

$$\Rightarrow \frac{1}{n+1} \sum_{r=0}^{n} \frac{n+1}{r+1} \cdot {}^nC_r$$

$$= \frac{1}{n+1} \sum_{r=0}^{n} {}^{n+1}C_{r+1} \cdot 1 \cdot 1$$

$$= \frac{1}{n+1} \left\{ \underbrace{{}^{n+1}C_0 + {}^{n+1}C_1 + {}^{n+1}C_2 + - - - + {}^{n+1}C_{n+1}}_{} - {}^{n+1}C_0 \right\}$$

$$\frac{1}{n+1}\left(2^{n+1} - {}^{n+1}C_0\right) = \frac{2^{n+1}-1}{n+1}$$

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Q $C_0 - \frac{C_1}{2} + \frac{C_2}{3} - \frac{C_3}{4} + \dots + (-1)^n \cdot \frac{C_n}{n+1} = ?$

$$T_{r+1} = \sum_{r=0}^{n} \frac{{}^nC_r}{(r+1)}(-1)^r = \sum \frac{1}{(r+1)} {}^nC_r \cdot (-1)^r$$

DJS.

$$= \frac{1}{n+1} \sum \frac{(n+1)}{(r+1)} \cdot {}^nC_r \cdot (-1)^r$$

$$= \frac{1}{n+1} \sum_{r=0}^{n} {}^{n+1}C_{r+1} \cdot (-1)^r$$

$$= \frac{1}{n+1} \left\{ {}^{n+1}C_0 - \left({}^{n+1}C_0 - \left({}^{n+1}C_1 - {}^{n+1}C_2 + {}^{n+1}C_3 - {}^{n+1}C_4 - \dots \right) \right) \right\}$$

$$\frac{1}{n+1}(1-0) = \frac{1}{n+1}$$

Q $2 \cdot C_0 + \frac{2^2 \cdot C_1}{2} + \frac{2^3 \cdot C_2}{3} + \frac{2^4 \cdot C_4}{4} + \dots + \frac{2^{n+1} \cdot C_n}{n+1} = ?$

$$\Rightarrow T_{r+1} = \sum \frac{{}^nC_r}{r+1} \cdot 2^{r+1} = \frac{1}{n+1} \sum \frac{n+1}{r+1} {}^nC_r \cdot 2^{r+1}$$ DJS

$$\frac{1}{n+1} \sum_{r=0}^{n} {}^{n+1}C_{r+1} \cdot 2^{r+1}$$

$$\frac{1}{n+1} \left\{ (1+2)^{n+1} - {}^{n+1}C_0 \right\} = \frac{3^{n+1} - 1}{n+1}$$

Q $\sum_{i=0}^{20} i \cdot {}^{20}C_i = ? \Rightarrow 20 \cdot 2^{19}$ $(n \cdot 2^{n-1})$

Q $\sum_{i=1}^{20} i \cdot (i-1) {}^{20}C_i$

$$\sum_{i=1}^{20} i^2 \cdot {}^{20}C_i - \sum_{i=1}^{20} i \cdot {}^{20}C_i$$

$$= (20)(20+1) 2^{20-2} - 20 \cdot 2^{20-1}$$

$$= 20 \times 21 \times 2^{18} - 20 \times 2^{19}$$

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