DPP – 04

Solution

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1. (A) Prove that $\sin 65^\circ + \cos 65^\circ = \sqrt{2}\cos 20^\circ$.

   (B) Prove that $\sin 47^\circ + \cos 77^\circ = \cos 17^\circ$.

Sol. (A) $\sin 65^\circ + \cos 65^\circ = \sin(45^\circ + 20^\circ) + \cos(45^\circ + 20^\circ)$

$= \sin 45^\circ \cos 20^\circ + \cos 45^\circ \sin 20^\circ + \cos 45^\circ \cos 20^\circ - \sin 45^\circ \sin 20^\circ = \sqrt{2}\cos 20^\circ$

(B) $\sin 47^\circ + \cos 77^\circ = \sin 47^\circ + \cos(90^\circ - 13^\circ) = \sin 47^\circ + \sin 13^\circ$

$= 2\sin\left(\frac{47^\circ+13^\circ}{2}\right)\cos\left(\frac{47^\circ-13^\circ}{2}\right)\left\{\because \sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)\right\}$

$= 2\sin 30^\circ \cos 17^\circ = 2 \times \frac{1}{2}\cos = \cos 17^\circ =$ RHS

Hence, LHS = RHS.

2. Prove that $\cos 80^\circ + \cos 40^\circ - \cos 20^\circ = 0$.

Sol. $\text{Cos } 80^\circ + \cos 40^\circ - \cos 20^\circ = 2\cos\frac{80^\circ+40^\circ}{2}\cos\frac{80^\circ-40^\circ}{2} - \cos 20^\circ$

$\left[\because \cos C^\circ + \cos D^\circ = 2\cos\left(\frac{C^\circ+D^\circ}{2}\right)\cos\left(\frac{C^\circ-D^\circ}{2}\right)\right] = 2\cos 60^\circ \cos 20^\circ - \cos 20^\circ = 0$

3. Prove that $\sin 10^\circ + \sin 20^\circ + \sin 40^\circ + \sin 50^\circ = \sin 70^\circ + \sin 80^\circ$

Sol. LHS $= (\sin 10^\circ + \sin 50^\circ) + (\sin 20^\circ + \sin 40^\circ)$

$= (2\sin 30^\circ \cos 20^\circ) + (2\sin 30^\circ \cos 10^\circ) = 2\sin 30^\circ(\cos 10^\circ + \cos 20^\circ)$

$= 2 \times \frac{1}{2}(\sin 80^\circ + \sin 70^\circ) = \sin 80^\circ + \sin 70^\circ =$ RHS

4. Prove that $\cos\frac{\pi}{5} + \cos\frac{2\pi}{5} + \cos\frac{6\pi}{5} + \cos\frac{7\pi}{5} = 0$.

Sol. $\cos\frac{\pi}{5} + \cos\frac{2\pi}{5} + \cos\frac{6\pi}{5} + \cos\frac{7\pi}{5}$ rearranging $= \cos\frac{\pi}{5} + \cos\frac{6\pi}{5} + \cos\frac{2\pi}{5} + \cos\frac{7\pi}{5}$

$= \cos\frac{\pi}{5} + \cos\left(\pi + \frac{\pi}{5}\right) + \cos\frac{2\pi}{5} + \cos\left(\pi + \frac{2\pi}{5}\right)$

as $\cos(\pi + \theta) = -\cos\theta = \cos\frac{\pi}{5} - \cos\left(\frac{\pi}{5}\right) + \cos\frac{2\pi}{5} - \cos\left(\frac{2\pi}{5}\right) = 0$

5. If $\sin\alpha - \sin\beta = \frac{1}{3}$ and $\cos\beta - \cos\alpha = \frac{1}{2}$, show that $\cot\frac{\alpha+\beta}{2} = \frac{2}{3}$.

Sol. $\sin\alpha - \sin\beta = \frac{1}{3} \Rightarrow 2\sin\left(\frac{\alpha-\beta}{2}\right)\cos\left(\frac{\alpha+\beta}{2}\right) = \frac{1}{3}$

$\cos\beta - \cos\alpha = \frac{1}{2} \Rightarrow 2\sin\left(\frac{\alpha-\beta}{2}\right)\sin\left(\frac{\alpha+\beta}{2}\right) = \frac{1}{2}$

Dividing Eqs. (i) by (ii), we have $\cot\frac{\alpha+\beta}{2} = \frac{2}{3}$.

6. If $\operatorname{cosec} A + \sec A = \operatorname{cosec} B + \sec B$, prove that $\tan A \tan B = \cot\frac{A+B}{2}$.

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Sol. Given : $\mathbf{cosec\ A + sec\ A = cosec\ B + sec\ B}$ ... (1)

From (1), $\mathbf{sec\ A - sec\ B = cosec\ B - cosec\ A}$

$$\therefore \left(\frac{1}{\cos A}\right) - \left(\frac{1}{\cos B}\right) = \left(\frac{1}{\sin B}\right) - \left(\frac{1}{\sin A}\right)$$

$$\therefore \frac{\cos B - \cos A}{\cos A \cdot \cos B} = \frac{\sin A - \sin B}{\sin A \cdot \sin B}$$

∴ rearranging the terms on two sides,

$$\left(\frac{\sin A}{\cos A}\right)\left(\frac{\sin B}{\cos B}\right) = \frac{\sin A - \sin B}{\cos B - \cos B} \qquad \therefore (\tan A)(\tan B)$$

$$= [2 \cdot \cos (A + B)/2 \cdot \sin (A - B)/2]/[2 \cdot \sin \left(\frac{A+B}{2} \cdot \frac{\sin(A-B)}{2}\right]]$$

$\therefore (\tan\ A)(\tan\ B) = \cot [(A + B)/2]$ Hence Proved.

7. Prove that $\mathbf{\sin 25^\circ \cos\ 115^\circ = \frac{1}{2}(\sin 40^\circ\ - 1)}$.

Sol. $\mathbf{\sin 25^\circ \cos 115^\circ}$

$$\frac{1}{2}(2\ \text{Sin}25^\circ\ \text{Cos}115^\circ) = \frac{1}{2}[\sin(25^\circ + 115^\circ) + \sin(25^\circ - 115^\circ)]$$

$$\because 2\ \text{Sin A Cos}\ \ B = \text{Sin}\ (A + B) + \text{Sin}\ (A - B) = \frac{1}{2}[\sin 140^\circ - 1]$$

8. If $\mathbf{\cos\ A = \frac{3}{4}}$, then find the value of $\mathbf{32 \sin\left(\frac{A}{2}\right) \sin\left(\frac{5A}{2}\right)}$

Sol. $16\left(2 \sin\left(\frac{A}{2}\right) \sin\left(\frac{5\,A}{2}\right)\right) = 16(\cos 2\,A - \cos 3\,A) = 16(2 \cos^2\ A - 1 - 4 \cos^3\ A + 3 \cos A)$

$$= 16\left(2 \times \frac{9}{16} - 1 - 4 \times \frac{27}{64} + 3 \times \frac{3}{4}\right) = 18 - 16 + 27 + 36 = 11$$

9. If $\mathbf{x \cos\theta = y \cos\left(\theta + \frac{2\pi}{3}\right) = z \cos\left(\theta + \frac{4\pi}{3}\right)}$, prove that $\mathbf{xy + yz + zx = 0}$.

Sol. Let $\mathbf{x \cos\theta = y \cos\left(\theta + \frac{2\pi}{3}\right) = z \cos\left(\theta + \frac{4\pi}{3}\right) = k}$ (say)

$$\frac{K}{x} = \cos\theta \Rightarrow \frac{K}{y} = \cos\left(\theta + \frac{2\pi}{3}\right) \Rightarrow \frac{k}{z} = \cos\left(\theta + \frac{4\pi}{3}\right)$$

$$\frac{k}{x} + \frac{k}{y} + \frac{k}{z} = \cos\ \theta + \cos\left(\theta + \frac{2\pi}{3}\right) + \cos\left(\theta + \frac{4\pi}{3}\right) \Rightarrow \frac{k}{x} + \frac{k}{y} + \frac{k}{z} = 0$$

$$k\left[\frac{yz + xz + xy}{xyz}\right] = 0 \Rightarrow xy + yz + zx = 0$$

10. If $\mathbf{y \sin\varphi = x \sin(2\theta + \varphi)}$, show that $\mathbf{(x + y) \cot (\theta + \varphi) = (y - x)\cot\theta}$.

Sol. $\mathbf{y \sin\phi = x \sin(2\theta + \phi) \Rightarrow \frac{x}{y} = \frac{\sin\phi}{\sin(2\theta+\phi)}}$

Applying componendo and dividendo $\frac{x + y}{x - y} = \frac{\sin\phi + \sin(2\theta + \phi)}{\sin\phi - \sin\ (2\theta + \phi)}$

$$\frac{x\ + y}{x - y} = \frac{2\sin\left(\frac{2\theta\ +\ 2\phi}{2}\right)\cos\left(\frac{\phi - 2\theta - \phi}{2}\right)}{2\cos\left(\frac{2\theta\ +\ 2\phi}{2}\right)\sin\left(\frac{\phi - 2\theta - \phi}{2}\right)} = -\frac{\cot\theta}{\cot(\theta\ + \phi)} \cdots\cdots \frac{\cos A}{\sin A} = \cot A$$

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$$\therefore (x + y)(\cot(\theta + \phi)) = (y - x)\cot\theta$$

11. If $\cos(A + B)\sin(C + D) = \cos(A - B)\sin(C - D)$, Prove that $\cot A \cot B \cot C = \cot D$

Sol. $\cos(A + B)\sin(C + D) = \cos(A - B)\sin(C - D)$

or $\frac{\cos(A + B)}{\cos(A - B)} = \frac{\sin(C - D)}{\sin(C + D)}$

or $\frac{\cos(A + B) + \cos(A - B)}{\cos(A + B) - \cos(A - B)} = \frac{\sin(C - D) + \sin(C + D)}{\sin(C - D) - \sin(C + D)}$

or $\frac{2\cos A \cos B}{-2\sin A \sin B} = \frac{2\sin c \cos D}{-2\sin D \cos C} \Rightarrow \cot A \cot B = \tan C \cot D$ or $\cot A \cot B \cot C = \cot D$

12. If $\tan(A + B) = 3\tan A$, Prove that

(A) $\sin(2A + B) = 2\sin B$ (B) $\sin 2(A + B) + \sin 2A = 2\sin 2B$

Sol. (A) $\tan(A + B) = 3\tan A \frac{\sin(A+B)}{\cos(A+B)} = \frac{3\sin A}{\cos A}$

$\sin(A + B)\cos A = 3\sin A\cos(A + B)$

$\sin(A + B)\cos A - \sin A(A + A + B) + \sin(A - A - B)$

$\sin(A + B - A) = \sin(A + A + B) + \sin(A - A - B)$

$\sin B = \sin(2A + B) - \sin B$

(B) $\tan(A + B) = 3\tan A \frac{\sin(A + B)}{\cos(A + B)} = \frac{3\sin A}{\cos A}$

$\sin(A + B)\cos A = 3\sin A\cos(A + B)$

$\sin(A + B)\cos A - \sin A(A + A + B) + \sin(A - A - B)$

$\sin(A + B - A) = \sin(A + A + B) + \sin(A - A - B)$

$\sin B = \sin(2A + B) - \sin B$

$2\sin B - \sin(2A + B)$ not LHS $\sin(2A + 2B) + \sin 2A \Rightarrow 2\sin(2A + B)GB$

$2 \times 2\sin BGB = 2\sin 2B$