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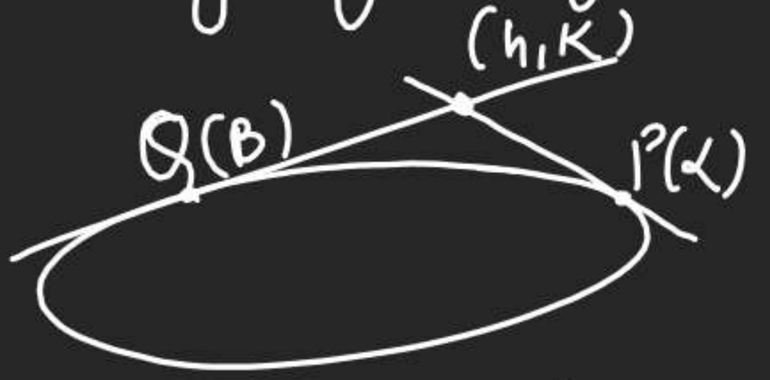
Q Find Locus of Pt. of Int. of tangents if diff. of ecc. angle of Pt. of Contact is $\frac{2\pi}{3}$.



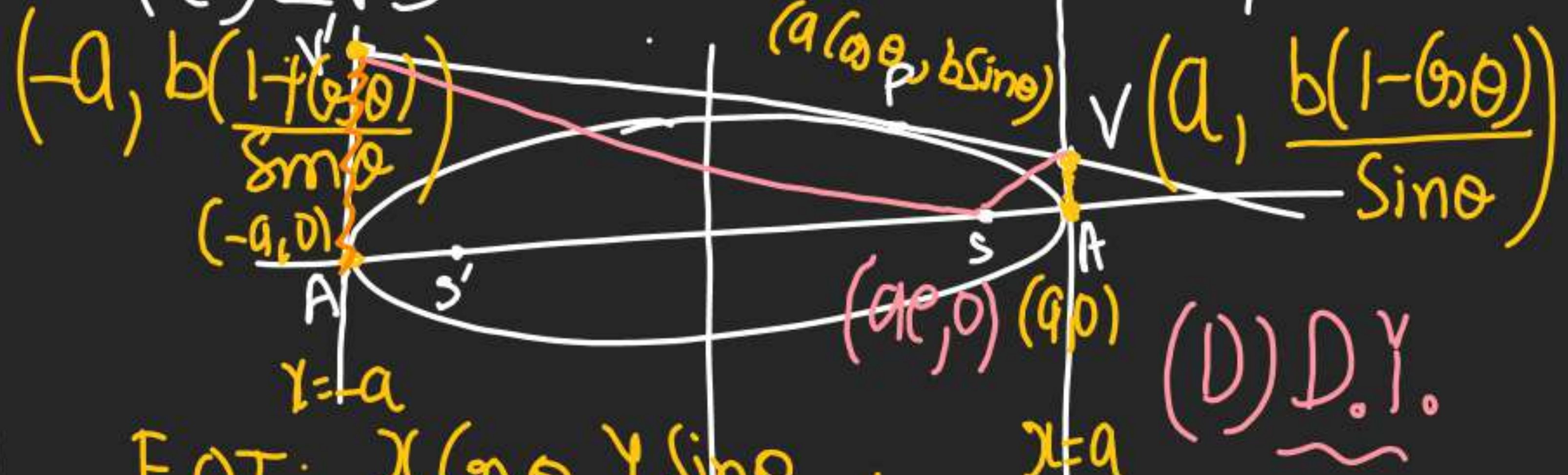
then $|\alpha-\beta| = \frac{2\pi}{3}$

$$h = \frac{a\cos\left(\frac{\alpha+\beta}{2}\right)}{\cos\left(\frac{\pi}{3}\right)}, \quad k = \frac{b\sin\left(\frac{\alpha+\beta}{2}\right)}{\cos\left(\frac{\pi}{3}\right)}$$

$$\frac{h}{2a} = \cos\left(\frac{\alpha+\beta}{2}\right), \quad \frac{k}{2b} = \sin\left(\frac{\alpha+\beta}{2}\right)$$

$$c^2+s^2=1 \Rightarrow \frac{h^2}{4a^2}+\frac{k^2}{4b^2}=1 \Rightarrow \frac{x^2}{4a^2}+\frac{y^2}{4b^2}=1$$ is Locus

Q Standard Ellipse with foci S & S', tangent at Pt. P meets T.V at A & A' at V & V' then (A) $AV\cdot A'V' = b^2$ (B) $AV\cdot A'V' = a^2$ (C) $\angle VSV' = 90°$ (D) $V'SS'V$ is Cy. Quad.



(D) D.Y.

EOT: $\frac{x\cos\theta}{a}+\frac{y\sin\theta}{b}=1$

$AV = \frac{b(1-\cos\theta)}{\sin\theta}$, $A'V' = \frac{b(1+\cos\theta)}{\sin\theta}$

$$A\cdot V\cdot A'V' = \frac{b(1-\cos\theta)}{\sin\theta}\times\frac{b(1+\cos\theta)}{\sin\theta} = b^2$$

(2) $$m_{SV}\times m_{SV'} = \frac{\frac{b(1-\cos\theta)}{\sin\theta}-0}{a(1-e)}\times\frac{\frac{b(1+\cos\theta)}{\sin\theta}}{-a(1+e)} = \frac{\frac{b^2\times\cancel{\sin^2\theta}}{\cancel{\sin^2\theta}}}{-a^2(1-e^2)}$$

$$= -\frac{b^2}{a^2}\times\frac{a^2}{b^2} = -1$$

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EON

EOT $\Rightarrow \frac{xx_1}{a^2}+\frac{yy_1}{b^2}=1$ at (x_1,y_1)

$$\Rightarrow (Sl)_T = \frac{-\frac{x_1}{a^2}}{\frac{y_1}{b^2}} = -\frac{x_1 \cdot b^2}{y_1 a^2}$$

$$(Sl)_N = \frac{y_1 a^2}{x_1 b^2}$$

$$\Rightarrow \text{EON} \Rightarrow (y-y_1) = \frac{y_1 a^2}{x_1 b^2}(x-x_1)$$

$$\boxed{\frac{a^2x}{x_1}-\frac{b^2y}{y_1}=a^2-b^2}$$

(2) EOT.

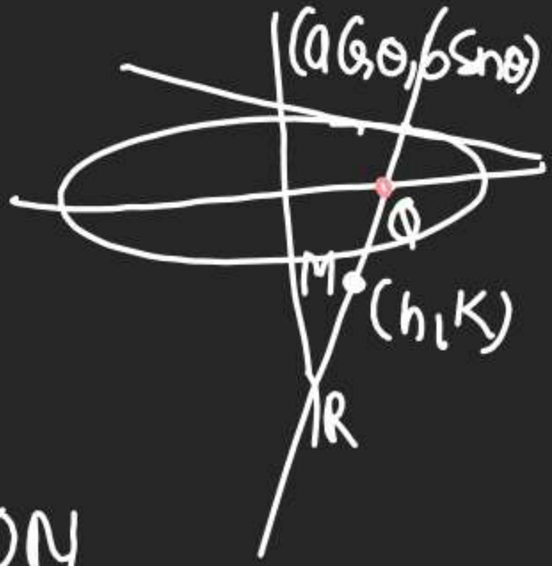
$$\frac{x\cos\theta}{a}+\frac{y\sin\theta}{b}=1 \text{ at } (\underset{x_1}{a\cos\theta},\ \underset{y_1}{b\sin\theta})$$

$$\text{EON} \Rightarrow \frac{a^2x}{x_1}-\frac{b^2y}{y_1}=a^2-b^2$$

$$\frac{a^{\cancel{2}}x}{\cancel{a}\cos\theta}-\frac{b^{\cancel{2}}y}{\cancel{b}\sin\theta}=a^2-b^2$$

$$\boxed{ax\sec\theta - by\csc\theta = a^2-b^2}$$

Q Normal at a var. pt. of $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ meets axes of Ellipse at Q & R find locus of Mid pt of QR.



EON

$$ax\sec\theta - by\csc\theta = a^2-b^2$$

$$Q \equiv \left(\frac{a^2-b^2}{a\sec\theta},\ 0\right)$$

$$R \equiv \left(0,\ \frac{b^2-a^2}{b\csc\theta}\right)$$

$$h = \frac{a^2-b^2}{2a}\cos\theta$$

$$K = \frac{b^2-a^2}{2b}\sin\theta$$

$$C^2+S^2=1$$

$$\frac{4a^2x^2}{(a^2-b^2)^2}+\frac{4b^2y^2}{(a^2-b^2)^2}=1$$

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Eqn of Chord having Mid PT.

$$T = S_1$$

$$\frac{xx_1}{a^2} + \frac{yy_1}{b^2} - \cancel{1} = \frac{x_1^2}{a^2} + \frac{y_1^2}{b^2} - \cancel{1}$$

Pair of tangents:

$$SS_1 = T^2$$

Q Find locus of Mid Pt of Focal Chord of $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

Mid PT. $\equiv (h, k)$

Mid Pt's Chord $T = S_1$

$$\frac{x \cdot h}{a^2} + \frac{yk}{b^2} = \frac{h^2}{a^2} + \frac{k^2}{b^2}$$

P. T. $(ae, 0)$

$$\frac{ae \cdot h}{a^2} + 0 = \frac{h^2}{a^2} + \frac{k^2}{b^2}$$

$$\Rightarrow \frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{xe}{a}$$

Q Find length of chord having Mid Pt. $\left(\frac{1}{2}, \frac{2}{5}\right)$ for E: $\frac{x^2}{2} + \frac{y^2}{4} = 1$

Mid Pt. Chord

1) $$\frac{\frac{1}{2} \cdot x}{2} + \frac{\frac{2}{5} \cdot y}{4} = \frac{1}{8} + \frac{1}{25}$$

$$\frac{x}{4} + \frac{y}{10} = \frac{33}{200} \Rightarrow$$

$$50x + 20y = 33 \Rightarrow y = \frac{33 - 50x}{20}$$

Curve में

$$\frac{x^2}{2} + \frac{(33 - 50x)^2}{1600} = 1$$

$$800x^2 + (33 - 50x)^2 = 1600$$

x_1, x_2 Ans को $y = \frac{33 - 50x}{20} \to y_1, y_2$

(x_1, y_1) & (x_2, y_2)

Dis $= \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

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Q Find Prod of ⊥r from focii of $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ at tangent at any pt P:

$$y=mx\pm\sqrt{a^2m^2+b^2}$$

$$p_2\times p_1=\frac{\left|aem\pm\sqrt{a^2m^2+b^2}\right|}{\sqrt{m^2+1}}\times\frac{\left|-aem\pm\sqrt{a^2m^2+b^2}\right|}{\sqrt{m^2+1}}$$

$$=\frac{(a^2m^2+b^2)-a^2e^2m^2}{m^2+1}=\frac{a^2m^2(1-e^2)+b^2}{m^2+1}$$

$$=\frac{\cancel{a^2}m^2\times\frac{b^2}{\cancel{a^2}}+b^2}{m^2+1}=\frac{b^2\cancel{(m^2+1)}}{\cancel{(m^2+1)}}=b^2$$

Q If Normal at upper end of LR P.T. $(0,-b)$ find $e^4+e^2=?$

$1-e^2=\frac{b^2}{a^2}$

$1-\frac{b^2}{a^2}=e^2$

$a^2-b^2=a^2e^2$

$$\text{E O N:}\to\frac{a^2x}{x_1}-\frac{b^2y}{y_1}=a^2-b^2$$

$$\frac{a^{\cancel{2}}x}{\cancel{a}e}-\frac{\cancel{b^2}y}{\cancel{b^2}/a}=a^2-b^2$$

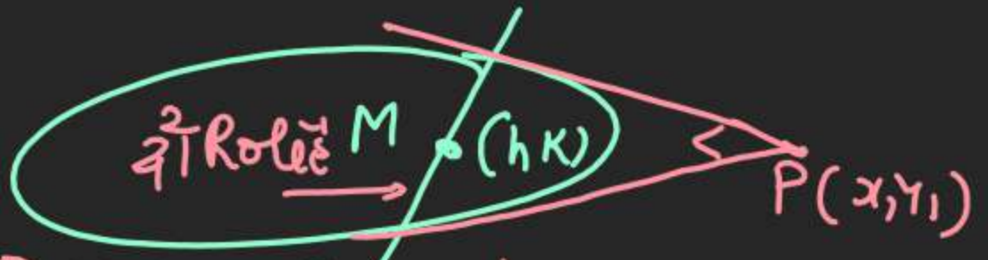
$$\frac{ax}{e}-ay=a^2-b^2\quad \text{P.T.}(0,-b)$$

$$\Rightarrow 0+ab=a^2-b^2$$

$$ab=a^2e^2$$

$$\frac{b^2}{a^2}=e^4\Rightarrow 1-e^2=e^4\Rightarrow 1=e^2+e^4$$

Q Find Locus of mid Pt. of chord of an ellipse, the tangent at the Pt. of which Intersect at 90°.



① Mid Pt of chord

② यहाँ COC आता है।

$\Rightarrow$ Mid Pt of chord $\Rightarrow \frac{hx}{a^2}+\frac{Ky}{b^2}=\frac{h^2}{a^2}+\frac{K^2}{b^2}$

COC $\Rightarrow \frac{xx_1}{a^2}+\frac{yy_1}{b^2}=1$

$$\frac{\frac{h}{\cancel{a^2}}}{\frac{x_1}{\cancel{a^2}}}=\frac{\frac{K}{\cancel{b^2}}}{\frac{y_1}{\cancel{b^2}}}=\frac{\frac{h^2}{a^2}+\frac{K^2}{b^2}}{1}$$

$$\frac{h}{x_1}=\frac{K}{y_1}=\left(\frac{h^2}{a^2}+\frac{K^2}{b^2}\right)$$

$\Rightarrow x_1=\frac{h}{\frac{h^2}{a^2}+\frac{K^2}{b^2}}$ | $y_1=\frac{K}{\frac{h^2}{a^2}+\frac{K^2}{b^2}}$

$$\frac{h^2a^4b^4}{(h^2b^2+K^2a^2)^2}+\frac{K^2a^4b^4}{(b^2h^2+a^2K^2)^2}=a^2+b^2$$

Whenever tangent to any Curve Intersect at $\frac{\pi}{2}$ they lie on D.C.

& if (x_1,y_1) lies on D.C.

$\Rightarrow x_1^2+y_1^2=a^2+b^2$