HOME WORK –1

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1. The first term of an A.P. of consecutive integer is $\mathbf{p^2 + 1}$. The sum of $(\mathbf{2p + 1})$ terms of this series can be expressed as

(A) $(\mathbf{p + 1})^2$ (B) $(\mathbf{2p + 1})(\mathbf{p + 1})^2$ (C) $(\mathbf{p + 1})^3$ (D) $\mathbf{p^3 + (p + 1)^3}$

Ans. **(D)**

Sol. $\mathbf{a = p^2 + 1}$

Since series is of consecutive integer

Sum of $(2p + 1)$ **terms** $= \frac{n}{2}(2a + (n - 1)d)$

$= \frac{(2p+1)}{2}[2(p^2 + 1) + (2p + 1 - 1)1] = \frac{(2p+1)}{2}[2p^2 + 2 + 2p] = (2p + 1)(p^2 + p + 1)$

$= 2p^3 + 2p^2 + 2p + p^2 + p + 1 = p^3 + p^3 + 1 + 3p^2 + 3p = p^3 + (p + 1)^3$

2. If $\mathbf{a_1, a_2, a_3}, \dots \dots$ are in A.P. such that $\mathbf{a_1 + a_5 + a_{10} + a_{15} + a_{20} + a_{24} = 225}$, then $\mathbf{a_1 + a_2 + a_3 + \cdots .. + a_{23} + a_{24}}$ is equal to

(A) 909 (B) 75 (C) 750 (D) 900

Ans. **(D)**

Sol. **Let say** $a_1 = a,\ a_2 = a + d$ **and** $a_n = a + (n - 1)d$

So, $a_1 + a_5 + a_{10} + a_{15} + a_{20} + a_{24} = 225$

$a + a + 4d + a + 9d + a + 14d + a + 19d + a + 23d = 225$

$6a + 69d = 225$ $3(2a + 23d) = 225$

$2a + 23d = \frac{22r}{3} - (i)$

Now sum of 24 terms $\Rightarrow$

$S_n = \frac{n}{2}[2a + (n - 1)d] = \frac{24}{2}[2a + (2u - 1)d]$ $S_{2u} = 12(2a + 23d)$

Now from the equation (i)

$S_{24} = 12\left[\frac{225}{3}\right] = 4 \times 225$ $S_{24} = 900 = 9 \times 10^2$

3. The sum of the series $\frac{1}{\log_2 4} + \frac{1}{\log_4 4} + \frac{1}{\log_8 4} + \cdots \dots . + \frac{1}{\log_{2^n} 4}$ is

(A) $\frac{1}{2}n(n + 1)$ (B) $\frac{1}{12}n(n + 1)(2n + 1)$

(C) $\frac{1}{n(n+1)}$ (D) $\frac{1}{4}n(n + 1)$.

Ans. **(D)**

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Sol. **The sum of the series**

$$=\frac{1}{\log_2^4}+\frac{1}{\log_4^4}+\cdots+\frac{1}{\log_{2n}^4}=\frac{\log_e^2}{\log_e^4}+\frac{\log_e^2}{\log_e^4}+..+\frac{\log_e^{2^{2^n}}}{\log_e^4}=\frac{\log_e^2+\log_e^{2^2}+\log_e^{2^3}+\cdots\log_e^{2^n}}{\log_e^4}$$

$$=\frac{\log_e\left[2\cdot 2^2\cdot 2^3\ldots 2^n\right]}{\log_e^4}=\frac{\log_e\left[2^{1+2+3+\cdots+n}\right]}{\log_e^4}=\frac{\log_e^{2^{2^2}(n+1)2}}{\log_e^{2^{2^2}}}\quad=\frac{\frac{n(n+1)}{2}\log_e^2}{2\log_e^2}=\frac{n(n+1)}{4}$$

4. If **a** and **b** are $\mathbf{p}^{\text{th}}$ and $\mathbf{q}^{\text{th}}$ terms of an AP, then the sum of its $(\mathbf{p}+\mathbf{q})$ terms is

(A) $\frac{p+q}{2}\left[a-b+\frac{a+b}{p-q}\right]$ (B) $\frac{p+q}{2}\left[a+b+\frac{a-b}{p-q}\right]$

(C) $\frac{p-q}{2}\left[a+b+\frac{a+b}{p+q}\right]$ (D) $\frac{p-q}{2}[a+b]$

Ans. **(B)**

Sol. **Let the first term be A and common difference D.**

$S_{p+1}=\frac{p+q}{2}[2A+(p+q-1)D]$ **It is given that** $T_p=a$ **and** $T_q=b$

$$\therefore \begin{cases} A+(p-1)D=a \\ A+(q-1)D=b \end{cases}$$

Subtracting, $(p-q)D=a-b$ $\therefore D=\frac{a-b}{p-q}$

Adding, $2A+(p+q-2)D=a+b$ **or** $2A+(p+q-1)D=a+b+D=a+b+\frac{a-b}{p-q}$**, by** (2)

Hence from (1) and (3) , $S_{p+q}=\frac{p+q}{2}\left[a+b+\frac{a-b}{p-q}\right]$.

5. The sum of integers from 1 to 100 that are divisible by 2 or 5 but not by both is

(A) 2550 (B) 1050 (C) 3050 (D) 2050

Ans. **(C)**

Sol. **The numbers between 1 to 100 that are divisible by 2 are,** $2,4,6,8,\ldots,100$

There sum is $2+4+6+8+\cdots.+100=\frac{50}{2}(2+100)=2550$

The numbers between 1 to 100 that are divisible by 5 are, $5,10,15,20,\ldots\ldots,100$

There sum is $5+10+15+\cdots..+100=\frac{20}{2}(5+100)=1050$

But, the number which are divisible by both 2 and 5 have been counted twice, So we have deduct their sum once, $10,20,\ldots,100$

There sum is $10+20+\cdots.+100=\frac{10}{2}(10+100)=550$

Thus, Final sum $=2550+1050-550=3050$

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6. Consider an A.P. with first term '**a**' and the common difference '**d**'. Let S_k denote the sum of its first **K** terms. If $\frac{S_{kx}}{S_x}$ is independent of **x**, then

(A) $a = d/2$ (B) $a = d$ (C) $a = 2d$ (D) $a = d/4$

Ans. (A)

Sol. Given, first term $= a$, common difference $= d$

By using $S_n = \frac{n}{2}[2a + (n-1)d]$ we have,

$\therefore \frac{Skx}{Sx} = \frac{\frac{kx}{2}(2a+(kx-1)d)}{\frac{x}{2}(2a+(x+1)d)} \Rightarrow \frac{Skx}{Sx} = \frac{k(2a+(kx-1)d)}{(2a+(x+1)d)} \Rightarrow \frac{Skx}{Sx} = \frac{k(2a-d+kxd)}{2a+xd-d}$

Now this will be independent of x only when $2a = d$ or $a = \frac{d}{2}$

7. The common difference **d** of the A.P. in which $T_7 = 9$ and $T_1T_2T_7$ is least is

(A) $\frac{33}{2}$ (B) $\frac{5}{4}$ (C) $\frac{33}{20}$ (D) $\frac{33}{10}$

Ans. (C)

Sol. $T_1 = a\, T_2 = a + d$ $\quad T_7 = a + 6d = 9 - 6d$ $\quad T_1\, T_2\, T_7 = a(a + d)(a + 6\,d) = N$

$N = (9 - 6\,d)(9 - 5\,d)9$ $\quad N = (6\,d - 9)(5\,d - 9)9\frac{dN}{dd} = 0$ $\quad d = \frac{33}{20}$

8. If $1, 2, 3$ ... are first terms; $1, 3, 5$ .... are common differences and S_1, S_2, S_3 ... are sums of **n** terms of given **p AP**'s; then $S_1 + S_2 + S_3 + \cdots + S_p$ is equal to

(A) $\frac{np(np+1)}{2}$ (B) $\frac{n(np+1)}{2}$ (C) $\frac{np(p+1)}{2}$ (D) $\frac{np(np-1)}{2}$

Ans. (A)

Sol. We know, $\sum a = 1 + 2 + 3 + \cdots + p = \frac{p(p+1)}{2}$

$\sum d = 1 + 3 + 5 + \cdots + (2p - 1) = \frac{p}{2} \cdot [2 \cdot 1 + (p - 1)2] = p^2$, $S_p = \frac{n}{2} \cdot [2a_p + (n - 1)d_p]$

$\therefore S_1 + S_2 + \cdots + S_p = \frac{n}{2} \cdot \left[2\sum a + (n-1)\sum d\right]$

$= \frac{n}{2} \cdot \left[2\frac{p(p+1)}{2} + (n-1) \cdot p^2\right] = \frac{n}{2} \cdot [p^2 + p + np^2 - p^2] = \frac{np(np+1)}{2}$

9. If $a_1, a_2 \ldots a_n$ are in A.P. with common difference $d \neq 0$, then the sum of the series

(sind) $[\operatorname{cosec} a_1 \operatorname{cosec} a_2 + \operatorname{cosec} a_2 \operatorname{cosec} a_3 + \cdots .. + \operatorname{cosec} a_{n-1} \operatorname{cosec} a_n]$

(A) $\sec a_1 - \sec a_n$ (B) $\operatorname{cosec} a_1 - \operatorname{cosec} a_n$

(C) $\cot a_1 - \cot a_n$ (D) $\tan a_1 - \tan a_n$

Ans. (C)

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Sol. Given $a_1, a_2, a_3 \dots\dots$ an be A.P with d as common difference, then

$d = a_2 - a_1 = a_3 - a_2 \dots\dots.. = a_n - a_{n-1}$

$\sin d[\csc a_1 \csc a_2 + \csc a_2 \csc a_3 + \cdots . + \csc a_{n-1} \csc a_n]$

$$= \frac{\sin d}{\sin a_1 \sin a_2} + \frac{\sin d}{\sin a_2 \sin a_3} \dots . + \frac{\sin d}{\sin a_{n-1} \sin a_n} = \frac{\sin (a_2 - a_1)}{\sin a_1 \sin a_2} + \frac{\sin (a_3 - a_2)}{\sin a_2 \sin a_3} \dots + \frac{\sin (a_n - a_{n-1})}{\sin a_{n-1} \sin a_n}$$

$$= \frac{\sin a_1 \cos a_2 - \cos a_1 \sin a_2}{\sin a_1 \sin a_2} + \frac{\sin a_2 \cos a_3 - \cos a_2 \sin a_3}{\sin a_2 \sin a_3} \dots + \frac{\sin a_n \cos a_{n-1} - \cos a_{n-1} \sin a_n}{\sin a_{n-1} \sin a_n}$$

$= \cot a_1 - \cot a_2 + \cot a_2 - \cot a_3 + \cdots .. + \cot a_{n-1} - \cot a_n = \cot a_1 - \cot a_n$

10. The third term of an A.P. is 18 , and the seventh term is 30 ; find the sum of 17 terms.

Ans. **612**

Sol. **Let the first term of an AP be a and common difference be d The third term of an AP is a +2 d, given that it is equal to 18 $\Rightarrow a + 2d = 18$**

The seventh term of an AP is a +6 d, given that it is equal to 30 $\Rightarrow a + 6d = 30$

By subtracting first equation from second equation, we get $4d = 12 \Rightarrow d = 3$

By substituting the value of d it in first equation, we get $a = 12$

Therefore sum of first 17 terms in AP is $\frac{17}{2}(2 \times 12 + (17-1) \times 3) = \frac{17}{2} \times 72 = 612$

11. Find the number of integers between **100** & **1000** that are

(i) divisible by 7

(ii) not divisible by 7

Ans. **(i)** 128 **(ii)** 771

Sol. **(i)**

Next number after 100 which is divisible by 7 is 105 and biggest three digit number divisible by 7 is

$1000 \div 7$ has 6 as reminder $\therefore 1000 - 6 = 994$ $994 = 7^* 142$ $105 = 7^* 15$

Therefore between 100 and 1000 there will be

142-15=127 numbers divisible by 7 and considering 105 there are 128 such numbers.

(ii)There are total 889 numbers between 100 and 1000 out of which 128 numbers are divisible by 7 $\therefore$ Remaining numbers i.e. $899 - 128 = 771$ are not divisible by 7 .

12. Find the sum of all those integers between 100 and 800 each of which on division by 16 leaves the remainder 7 .

Ans. **19668**

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Sol. $N = 16k + 7(k\text{ - integer})$ $N = 100 \Rightarrow k = \frac{93}{16} = 5.81 \Rightarrow k = 6$

$N = 16 \times 6 + 7 = 103$ $N = 800 \Rightarrow k = \frac{793}{16} = 49.56 \Rightarrow k = 6$

$N = 16 \times 49 + 7 = 791$

from $k = 6$ (included) to $k = 49$ (included), total numbers $= n = 44$

Therefore this is an AP, 103, 119,------, 791, with $a = 103, d = 16, 1 = 791$ and $n = 44$

We have, $S_n = \frac{n}{2}[a + 1]$ $S_n = \frac{44}{2}[103 + 791] = 19668$

13. The sum of three numbers in A.P. is 27, and their product is 504 , find them.

Ans. 4 , 9, 14 OR 14, 9, 4

Sol. let 3 numbers in A.P. be $(a - d), a, (a + d) \Rightarrow$ Sum of 3 numbers $= 27$

$\Rightarrow a - d + a + a + d = 27$ $\Rightarrow 3a = 27$ $a = 9 \Rightarrow$ Product of numbers $= 504$

$\Rightarrow (a - d)a(a + d) = 504 \Rightarrow (a^2 - d^2)a = 504 \Rightarrow [9^2 - d^2]9 = 504$

$\Rightarrow 81 - d^2 = 56$ $\Rightarrow d^2 = 81 - 56 \Rightarrow d^2 = 25$ $\Rightarrow d = \pm 5$

If $d = 5$, series $= 4, 9, 14$ If $d = -5$, series $= 14, 9, 4$

$\therefore$ These numbers are $4, 9, 14$

14. If a, b, c are in A.P., then show that

(i) $a^2(b + c), b^2(c + a), c^2(a + b)$ are also in A.P.

(ii) $b + c - a, c + a - b, a + b - c$ are in A.P.

Sol. $\because a, b, c$ are in AP

hence, $(b - a) = (c - b) \rightarrow (1)$

(i) $\because b^2(c + a) - a^2(b + c) = (b - a)(ab + bc + ca)$ and $c^2(a + b) - b^2(c + a)$

$= (c - b)(ab + bc + ca)$ $= (b - a)(ab + bc + ca)$ (from (1))

hence, $b^2(c + a) - a^2(b + c) = c^2(a + b) - b^2(c + a)$

$\therefore a^2(b + c), b^2(c + a), c^2(a + b)$ are also in AP

(ii) $\because (c + a - b) - (b + c - a) = 2(a - b) = -2(b - a)$ and $(a + b - c) - (c + a - b)$

$= 2(b - c) = -2(c - b)$ $= -2(b - a)$ (from (1))

hence $(c + a - b) - (b + c - a) = (a + b - c) - (c + a - b)$

$\therefore (b + c - a), (c + a - b), (a + b - c)$ are in AP

15. In an A.P. of which ' a ' is the 1^{st} term, if the sum of the 1^{st} ' p ' terms is equal to zero, show that the sum of the next ' q ' terms is $\frac{-(a)(p+q)q}{p-1}$.

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Sol. $S_p = 0 \therefore \frac{p}{2}[2a + (p-1)d] = 0 \qquad \therefore d = \frac{-2a}{p-1}$

Sum of next q terms = sum of an A.P. whose first term will be $T_{p+1} = a + pd$

$\therefore S = \frac{q}{2}[2(a+pd) + (q-1)d] = \frac{q}{2}[2a + (p-1)d + (p+q)d]$

$= \frac{q}{2}\left[0 - (p+q)\frac{2a}{p-1}\right] = -a\frac{(p+q)q}{p-1}$, by(1).

16. If the p^{th}, q^{th} and r^{th} terms of an A.P. are **a**, **b**, **c** respectively, show that $(q-r)a + (r-p)b + (p-q)c = 0$.

Sol. Let A be the first term and D the common difference of A.P.

$T_p = a = A + (p-1)D = (A-D) + pD$,$T_q = b = A + (q-1)D = (A-D) + qD$

$T_r = c = A + (r-1)D = (A-D) + rD$

Here we have got two unknowns A and D which are to be eliminated.

We multiply (1), (2) and (3) by $q-r, r-p$ and $p-q$ respectively and add:

$a(q-r) + b(r-p) + c(p-q) \quad = (A-D)[q - r + r - p + p - q] + D[p(q-r) + q(r-p) + r(p-q)] = 0$

17. A person is to count 4500 currency notes. Let a_n denote the number of notes he counts in the n^{th} minute. If $a_1 = a_2 = \cdots = a_{10} = 150$ and a_{10}, a_{11} ... are in an AP with common diference -2 , then the time taken by him to count all notes is - **[AIEEE 2010]**

(A) 24 minutes (B) 34 minutes (C) 125 minutes (D) 135 minutes

Ans. (B)

Sol. The number of notes counted in first 10 minutes $= 150 \times 10 = 1500$

Suppose, the person counts the remaining 3000 currency notes in n minutes.

Then, 3000 = Sum of n terms of an A.P. with first term 148 and common difference -2

$\Rightarrow 3000 = \frac{n}{2}\{2 \times 148 + (n-1) \times (-2)\} \Rightarrow 3000 = n(149 - n) \Rightarrow n^2 - 149n + 3000 = 0$

$\Rightarrow (n-125)(n-24) = 0 \quad \Rightarrow n = 125, 24$ Clearly, $n = 125$ is not possible.

Total, time taken $= (10 + 24) = 34$ min.

18. If 100 times the 100^{th} term of an AP with non zero common difference equals the 50 times its 50^{th} term, then the 150^{th} term of this AP is: **[AIEEE 2012]**

(A) 150 (B) zero (C) -150 (D) 150 times its 50^{th} term

Ans. (B)

Sol. $100(a + 99d) = 50(a + 49d) \Rightarrow 2a + 198d = a + 49d \qquad a + 149d = 0$

$\therefore T_{150} = a + 149 = 0$

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19. Let α and β be the roots of equation $\mathbf{px^2 + qx + r = 0, p \neq 0}$. If $\mathbf{p, q, r}$ are in A.P. and $\frac{1}{\alpha} + \frac{1}{\beta} = 4$, then the value of $|\alpha - \beta|$ is: **[JEE Main 2014]**

(A) $\frac{\sqrt{61}}{9}$ (B) $\frac{2\sqrt{17}}{9}$ (C) $\frac{\sqrt{34}}{9}$ (D) $\frac{2\sqrt{13}}{9}$

Ans. **(D)**

Sol. **GIven α, β are the roots of the equation $px^2 + qx + r = 0, p \equiv 0$**

$\Rightarrow \alpha + \beta = -\frac{q}{p}$ and $\Rightarrow \alpha\beta = \frac{r}{p} \Rightarrow |\alpha - \beta|^2 = \alpha^2 + \beta^2 - 2\alpha\beta = (\alpha + \beta)^2 - 4\alpha\beta = \frac{q^2}{p^2} - 4\frac{r}{p}$

$= \left(\frac{q}{p}\right)^2 - 4\left(\frac{r}{p}\right) \ldots (1)$ **Also,** $\frac{2q}{p} = 1 + \frac{r}{p} \ldots (2)$ **Given that,** $\frac{1}{\alpha} + \frac{1}{\beta} = 4$

$\Rightarrow \frac{\alpha+\beta}{\alpha\beta} = 4 \Rightarrow -\frac{q}{p} \cdot \frac{p}{r} = 4 \quad \Rightarrow \frac{q}{r} = -4$

Given p, q, r are in A.P.

$\Rightarrow 2q = p + r \Rightarrow \frac{2q}{r} = \frac{p}{r} + 1 \Rightarrow \frac{p}{r} = \frac{2q}{r} - 1 \Rightarrow \frac{p}{r} = 2(-4) - 1 \ldots \left(\because \frac{q}{r} = -4\right)$

$\Rightarrow \frac{p}{r} = -9 \quad \Rightarrow \frac{r}{p} = -\frac{1}{9}$

Substitute tha value of $\frac{r}{p}$ in equation (2), we get,

$\frac{2q}{p} = 1 - \frac{1}{9} \Rightarrow \frac{q}{p} = \frac{1}{2}\left(1 - \frac{1}{9}\right) \Rightarrow \frac{q}{p} = \frac{1}{2}\left(\frac{9-1}{9}\right) \Rightarrow \frac{q}{p} = \frac{1}{2} \cdot \frac{8}{9} \quad \Rightarrow \frac{q}{p} = \frac{4}{9}$

Now, substitute the values of $\frac{q}{p}$ and $\frac{r}{p}$ in the equation (1), we get

$\Rightarrow |\alpha - \beta|^2 = \left(\frac{4}{9}\right)^2 - 4\left(-\frac{1}{9}\right) \Rightarrow |\alpha - \beta|^2 = \left(\frac{4}{9}\right)^2 + \frac{4}{9} \Rightarrow |\alpha - \beta|^2 = \frac{4}{9}\left(\frac{13}{9}\right)$

$\Rightarrow |\alpha - \beta|^2 = \frac{52}{81} \quad \Rightarrow |\alpha - \beta| = \frac{2\sqrt{13}}{9}$

20. Let $\mathbf{a_1, a_2, a_3, \ldots, a_{49}}$ be in A.P. such that $\sum_{k=0}^{12} \mathbf{a_{4k+1}} = \mathbf{416}$ and $\mathbf{a_9 + a_{43} = 66}$.
If $\mathbf{a_1^2 + a_2^2 + \cdots \ldots a_{17}^2 = 140m}$, then $\mathbf{m}$ equal to : **[JEE Main2018]**

(A) 33 (B) 66 (C) 68 (D) 34

Ans. **(D)**

Sol. $\Rightarrow n^{th}$ **term** $a_n = a + (n-1)d \Rightarrow a_9 + a_{43} = 66 \therefore a + 8d + a + 42d = 66 \therefore a + 25d = 33$

Now, $\sum_{k=0}^{12} a_{4k+1} = 416$

$\therefore 13a + 312\,d = 416$ **(using sum of AP on the common difference parts)**

$\therefore a + 24d = 32 \ldots (2)$ **from (1) and (2), we get $d = 1$ and $a = 8$**

$\therefore \sum_{k=1}^{17} a_k^2 = 8^2 + 9^2 \cdots + 24^2 \quad = (1^2 + 2^2 \cdots + 24^2) - (1^2 + 2^2 \cdots + 7^2)$

$= \frac{24\times25\times49}{6} - \frac{7\times8\times15}{6}$ **(using sum of squares of n natural numbers is**

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$$\frac{n(n+1)(2n+1)}{6} = 4760 = 140 \times 34$$

Hence, the required answer is 34 .

21. If the sum of first n terms of an A.P. is cn^2, then the sum of squares of these n terms is

(A) $\frac{n(4n^2-1)c^2}{6}$ (B) $\frac{n(4n^2+1)c^2}{3}$ (C) $\frac{n(4n^2-1)c^2}{3}$ (D) $\frac{n(4n^2+1)c^2}{6}$ **[JEE 2009, 3]**

Ans. **(C)**

Sol. $S_n = cn^2 \quad S_{n-1} = c(n-1)^2 = cn^2 + c - 2cnT_n = 2cn - cT_n^2 = (2cn-c)^2 = 4c^2n^2 + c^2 - 4c^2n$

Required sum

$$= \sum T_n^2 = 4c^2 \sum n^2 + nc^2 - 4c^2 \sum n = \frac{4c^2n(n+1)(2n+1)}{6} + nc^2 - 2c^2n(n+1)$$

$$= \frac{2c^2n(n+1)(2n+1) + 3nc^2 - 6c^2n(n+1)}{3} = \frac{nc^2[4n^2 + 6n + 2 + 3 - 6n - 6]}{3}$$

$$= \frac{nc^2(4n^2-1)}{3}$$

22. Let $a_1, a_2, a_3, \ldots, a_{11}$ be real numbers satisfying $a_1 = 15, 27 - 2a_2 > 0$ and

$a_k = 2a_{k-1} - a_{k-2}$ for $k = 3, 4, \ldots, 11$. If $\frac{a_1^2 + a_2^2 + \cdots + a_{11}^2}{11} = 90$, then the value of $\frac{a_1 + a_2 + \cdots + a_{11}}{11}$ is equal to **[JEE 2010]**

Ans. **0**

Sol. $a_1 = 15 \; \frac{a_k + a_{k-2}}{2} = a_{k-1}$ **fork** $= 3, 4, \ldots \ldots \ldots 11$

$\Rightarrow a_1, a_2, a_3, \ldots \ldots \ldots a_{11}$ **are in A.P.** $\frac{a_1^2 + a_2^2 + \cdots \ldots + a_{11}^2}{11} = 90$

$\Rightarrow 7d^2 + 30d + 27 = 0 \Rightarrow d = -3$ **or** $-\frac{9}{7}$**Since** $27 - 2a_2 > 0 \Rightarrow a_2 < \frac{27}{2} \Rightarrow d = -3$

$$\frac{a_1 + a_2 + \cdots \ldots \ldots + a_{11}}{11}$$

$$= \frac{11}{2} \frac{[(30 + 10(-3))]}{11} = 0$$

23. Let $a_1, a_2, a_3, \ldots, a_{100}$ be an arithmetic progression with $a_1 = 3$ and

$S_p = \sum_{i=1}^{p} a_i, 1 \le p \le 100$. For any integer n with $1 \le n \le 20$, let $m = 5n$. If $\frac{S_m}{S_n}$ does not depend on n, then a_2 is **[JEE 2011]**

Ans. **9**

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Sol. **We know that sum of n terms of an A.P. is given by**

$S_k = \frac{k}{2}\{2a_1 + (k-1)d\}$ **So,** $\frac{S_m}{S_n} = \frac{S_{5n}}{S_n} = \frac{\frac{5n}{2}\{6+(5n-1)d\}}{\frac{n}{2}\{6+(n-1)d\}} = \frac{5n\{6+(5n-1)d\}}{n\{6+(n-1)d\}} = \frac{5\{(6-d)+5nd\}}{\{(6-d)+n\}}$

Since, $\frac{S_m}{S_n}$ **is independent of** n**, so** $d = 6$**. If** $d = 6$ **then** $a_2 = a_1 + d = 3 + 6 = 9$

24. Suppose that all the terms of an arithmetic progression (A.P.) are natural numbers. If the ratio of the sum of the first seven terms to the sum of first eleven terms is $6:11$ and the seventh term lies in between 130 and 140 , then the common difference of this A.P. is.

[JEE Adv. 2015]

Ans. 9

Sol. **By using** $S_n = \frac{n}{2}[2a + (n-1)d]$ **we have,** $\frac{\frac{7}{2}(2a+6d)}{\frac{11}{2}(2a+10d)} = \frac{6}{11}$ …..[Given]

$$\Rightarrow \frac{7a+21d}{a+5d} = 6 \Rightarrow 7a + 21\,d = 6a + 30\,d \Rightarrow a = 9\,d - - - - - - (1)$$

But, $130 < a + 6\,d < 140$ **from (1)** $\Rightarrow 130 < 15\,d < 140$ **As all terms in AP are natural.**

$\therefore d = 9$

25. Let X be the set consisting of the first 2018 terms of the arithmetic progression $1, 6, 11, \ldots$ and Y be the set consisting of the first 2018 terms of the arithmetic progression $9, 16, 23$, Then, the number of elements in the set X U Y is______.

Ans. 3748

Sol. $X: 1, 6, 11,$ __________, 10086

$Y: 9, 16, 23$__________,14128

$X \cap Y: 16, 51, 86$____________

Let $m = n(X \cap Y)$

$\therefore 16 + (m-1) \times 35 \le 10086 \Rightarrow m \le 288.71 \Rightarrow m = 288$

$\therefore n(X \cup Y) = n(X) + n(Y) - n(X \cap Y) = 2018 + 2018 - 288 = 3748$

26. Let $l_1, l_2, \ldots, l_{100}$ be consecutive terms of an arithmetic progression with common difference d_1 and let $w_1, w_2 \ldots, w_{100}$ be consecutive terms of another arithmetic progression with common difference d_2, where $d_1 d_2 = 10$. For each $i = 1, 2, \ldots, 100$, let R_i be a rectangle with length l_i width w_i and area A_i. If $A_{51} - A_{50} = 1000$, then the value of $A_{100} - A_{90}$ is

[JEE Adv. 2022]

Ans. 18900

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Sol. $A_{51} - A_{50} = 1000 \quad l_{51}w_{51} - l_{50}w_{50} = 100 \quad (l_1 + 50\,d_1)(w_1 + 50\,d_2) - (l_1 + 49\,d_1)$
$(w_1 + 49\,d_2) = 1000$

$l_1w_1 + 50l_1\,d_2 + 50\,d_1w_1 + 2500\,d_1\,d_2 - l_1w_1 - 49l_1\,d_2 - 49\,d_1w_1 - 240\,d_1\,d_2 = 1000$

$\Rightarrow l_1\,d_2 + d_1w_1 + 99\,d_1\,d_2 = 1000 \Rightarrow l_1\,d_2 + d_1w_1 = 10$

$A_{100} - A_{90} = (l_{100}w_{100}) - (l_{90}w_{90})$

$= (l_1 + 99\,d_1)(w_1 + 99\,d_2) - (l_1 + 89\,d_1)(w_1 + 89\,d_2)$

$= 10\,d_1w_1 + 10l_1\,d_2 + 1880\,d_1\,d_2 \quad = 10 \times 10 + 18800 = 18900$

27. Let $a_1, a_2, a_3, \ldots$ be an arithmetic progression with $a_1 = 7$ and common difference 8 . Let $T_1, T_2, T_3, \cdots$ be such that $T_1 = 3$ and $T_{n+1} - T_n = a_n$ for $n \geq 1$. Then, which of the following is/are TRUE ? **[JEE Adv. 2022]**

(A) $T_{20} = 1604$

(B) $\sum_{k=1}^{20} T_k = 10510$

(C) $T_{30} = 3454$

(D) $\sum_{k=1}^{3a} T_k = 35610$

Ans. **(BC)**

Sol. **(B)** $\sum_{k=1}^{20} t_k = 10510$ **Paste Options:**

(C) $T_{30} = 3454$

$$\sum_{n=1}^{n} (T_{n+1} - T_n) = \sum a_n \Rightarrow T_{n+1} - T_1 = \sum a_n = \frac{n}{2}[2 \times 7 + (n-1)8]$$

$T_{n+1} = n(4n+3) + T_1 \qquad T_{n+1} = 4n^2 + 3n + 3$

Set Default Paste [Copy More from $\Rightarrow T_{n+1} - T_1 = \sum a_n = \frac{n}{2}[2 \times 7 + (n-1)8]$

$\mathrm{T}_{n+1} = n(4n+3) + \mathrm{T}_1\mathrm{T}_{n+1} = 4n^2 + 3n + 3$

(A) $\mathrm{T}_{20} = 4 \times 19^2 + 3 \times 9 + 3 = 1444 + 27 + 3 = 1474$

(B) $\sum_{k=0}^{19} \mathrm{T}_{n+1} = \sum_{k=0}^{19} k(4k+3) + 3 = \sum_{k=0}^{19} (4k^2 + 3k + 3) = 10510$

(C) $\mathrm{T}_{30} = 29(4 \times 29 + 3) + 3 = 3454$

(D) $\sum_{k=1}^{30} \mathrm{T}_k = \sum_{n=0}^{29} \mathrm{T}_{n+1} = \sum_{n=0}^{29} n(4n+3) + 3 = 4 \times \left(\frac{29 \times 30 \times 59}{6}\right) + \frac{3(29 \times 30)}{2} + 90$

$= 35615$