Nishant Jindal

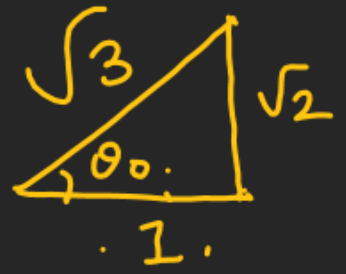
Projectile motion

H.W

$\sin\theta_0 = \sqrt{2/3}$

$\theta_0 = \sin^{-1}\sqrt{\frac{2}{3}}$

Q. A body is thrown at an angle θ_0 with the horizontal such that it attains a speed equal to $\sqrt{\frac{2}{3}}$ times the speed of projection when the body is at half of its maximum height. Find the angle θ_0.



Sol^n

$V = \sqrt{\frac{2}{3}}\,u$ (given)

$V^2 = \frac{2}{3}u^2$

$V_x^2 + V_y^2 = \frac{2}{3}(u_x^2 + u_y^2)$

$u_x^2 + \frac{u_y^2}{2} = \frac{2}{3}u_x^2 + \frac{2}{3}u_y^2$

$\left(u_x^2 - \frac{2}{3}u_x^2\right) = \left(\frac{2}{3}u_y^2 - \frac{u_y^2}{2}\right) \Rightarrow \frac{u_x^2}{3} = \frac{4u_y^2 - 3u_y^2}{6} \Rightarrow$

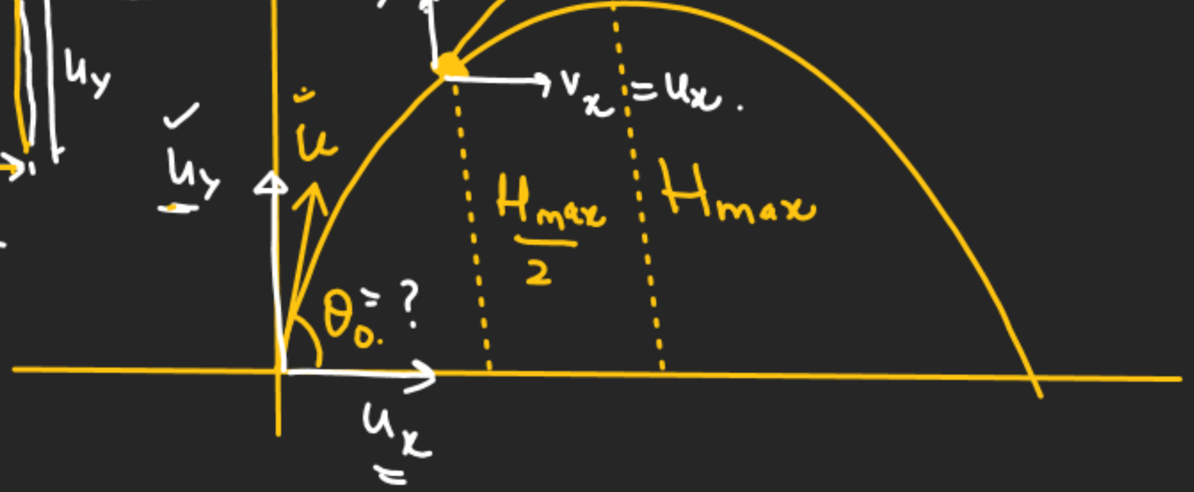
$H_{max} = \left(\frac{u_y^2}{2g}\right)$

$V_y^2 = u_y^2 - \cancel{2}g\left(\frac{H_{max}}{\cancel{2}}\right)$

$V_y^2 = u_y^2 - \cancel{g}\left(\frac{u_y^2}{2\cancel{g}}\right)$

$V_y^2 = \left(\frac{u_y^2}{2}\right)$

$\tan\theta_0 = \left(\frac{u_y}{u_x}\right)$



$\frac{u_x^2}{3} = \frac{u_y^2}{6} \Rightarrow \frac{u_y}{u_x} = \sqrt{2}$

$\tan\theta_0 = \sqrt{2}$

$\theta_0 = \tan^{-1}(\sqrt{2})$

Nishant Jindal

Projectile motion

$\theta = 30°$ ← $\theta \to$ Missing ??

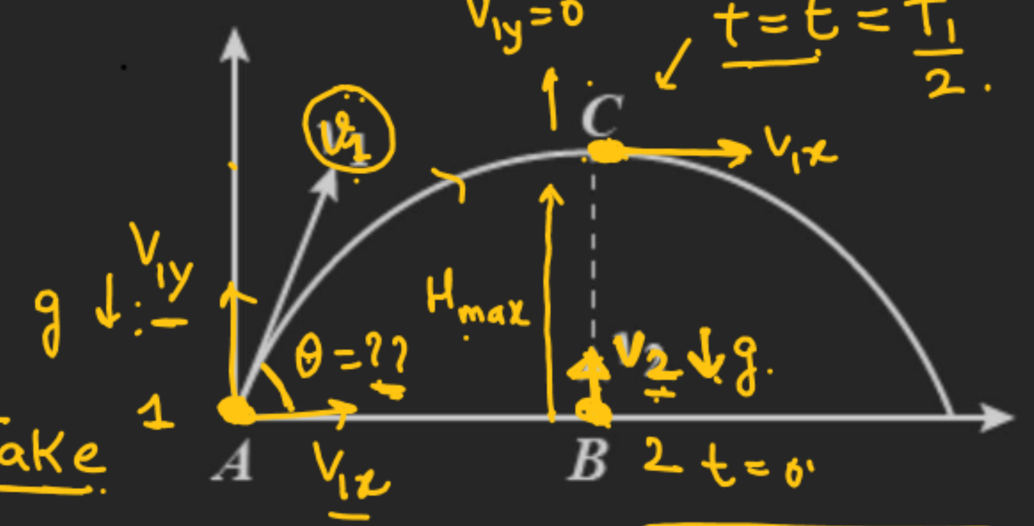
Q. A body is projected with velocity v_1 from the point A as shown in Fig. At the same time, another body is projected vertically upwards from B with velocity v_2. The point B lies vertically below the highest point of first particle. For both the bodies to collide, v_2/v_1 should be = ??

a. 2

b. $\sqrt{\frac{3}{2}}$

c. ~~0.5~~

d. 1



Collision time $T_1 = \left(\frac{2V_{1y}}{g}\right)$

$t = \frac{T_1}{2}$

$t = \left(\frac{V_{1y}}{g}\right)$ ✓

$H_{max} = \left(\frac{v_{1y}^2}{2g}\right)$

For particle-2

$H_{max} = V_2 t - \frac{1}{2} g t^2$

$\frac{V_{1y}^2}{2g} = V_2\left(\frac{V_{1y}}{g}\right) - \frac{1}{2} g \left(\frac{V_{1y}}{g}\right)^2$

$\frac{V_{1y}^2}{2\cancel{g}} + \frac{V_{1y}^2}{2\cancel{g}} = \frac{V_2 \cdot V_{1y}}{\cancel{g}} \Rightarrow \boxed{V_{1y} = v_2}$ ✓

Take

$\tan 30° = \frac{V_{1y}}{V_{1x}}$

$v_1 = \sqrt{(v_{1y}^2 + v_{1x}^2)}$

$V_{1x} = \sqrt{3}\, V_{1y} = \sqrt{3}\, v_2$

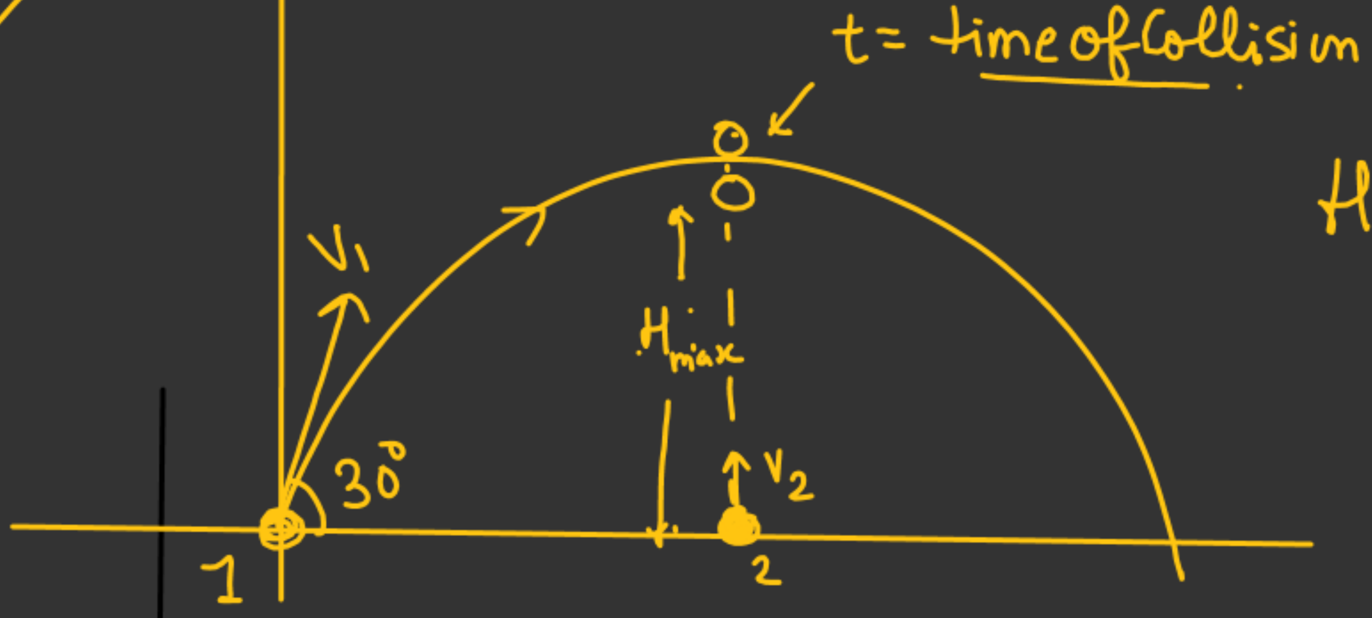
$v_1^2 = v_2^2 + 3v_2^2$

$v_1^2 = 4v_2^2 \Rightarrow (v_1/v_2) = \sqrt{4} = 2/1$

$\frac{v_2}{v_1} = \frac{1}{2}$

Nishant Jindal

Another Method



$$t = \frac{T_1}{2}, \quad T_1 \rightarrow \text{Time period of projectile-1}$$

$$t = \frac{\cancel{2} V_1 \sin 30^\circ}{\cancel{2} g} = \left(\frac{V_1}{2g}\right)$$

$$H_{max} = \left(\frac{V_1^2 \sin^2 30^\circ}{2g}\right)$$

$$= \frac{V_1^2}{2g} \times \frac{1}{4} = \left(\frac{V_1^2}{8g}\right)$$

For particle-2

$$H_{max} = V_2 t - \frac{1}{2} g t^2$$

$$\frac{V_1^2}{8g} = V_2\left(\frac{V_1}{2g}\right) - \frac{1}{2} g \left(\frac{V_1}{2g}\right)^2$$

$$\frac{\cancel{V_1^2}}{8\cancel{g}} = \frac{\cancel{V_1}}{2\cancel{g}}\left[V_2 - \frac{V_1}{4}\right]$$

$$\frac{V_1}{4} + \frac{V_1}{4} = V_2$$

$$\frac{V_1}{2} = V_2$$

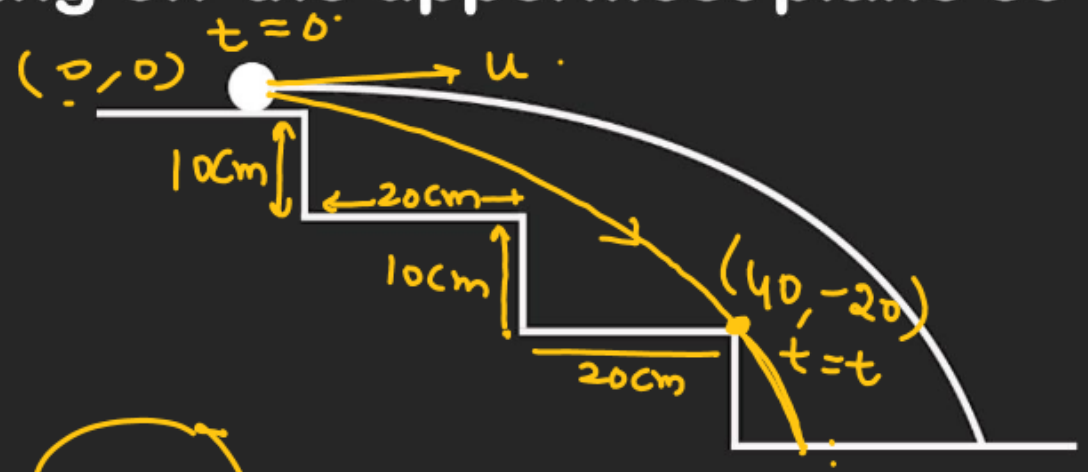
$$\frac{V_2}{V_1} = \frac{1}{2} = 0.5 \text{ Ans.}$$

Nishant Jindal

Projectile motion

H.W

Q. A staircase contains three steps each $10\ \mathrm{cm}$ high and $20\ \mathrm{cm}$ wide. What should be the minimum horizontal velocity of the ball rolling off the uppermost plane so as to hit directly the lowest plane? (in ms^{-1})



Sol^n In x-direction

$40 = ut$ — ①

In y-direction

$-20 = -\frac{1}{2} \times 10 \times t^2$

$t^2 = 4$

$t = 2\,\mathrm{sec}$

$u = \frac{40}{t}$

$u = \frac{40}{2} = 20\,\mathrm{m/s}$

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Projectile motion

H.W

Q. A student and his friend while experimenting for projectile motion with a stop-watch, taken some approximate readings. As one throws a stone in air at some angle, other observes that after 2.0 s it is moving at an angle 30° to the horizontal and after 1.0 s, it is travelling horizontally. Determine the magnitude and the direction of initial velocity of the stone.

Sol^n $\Rightarrow$ $T = 6\,sec$

$V_y = u_y - gt$

$0 = u\sin\theta - (10\times 3)$

$\underline{u\sin\theta = 30}$

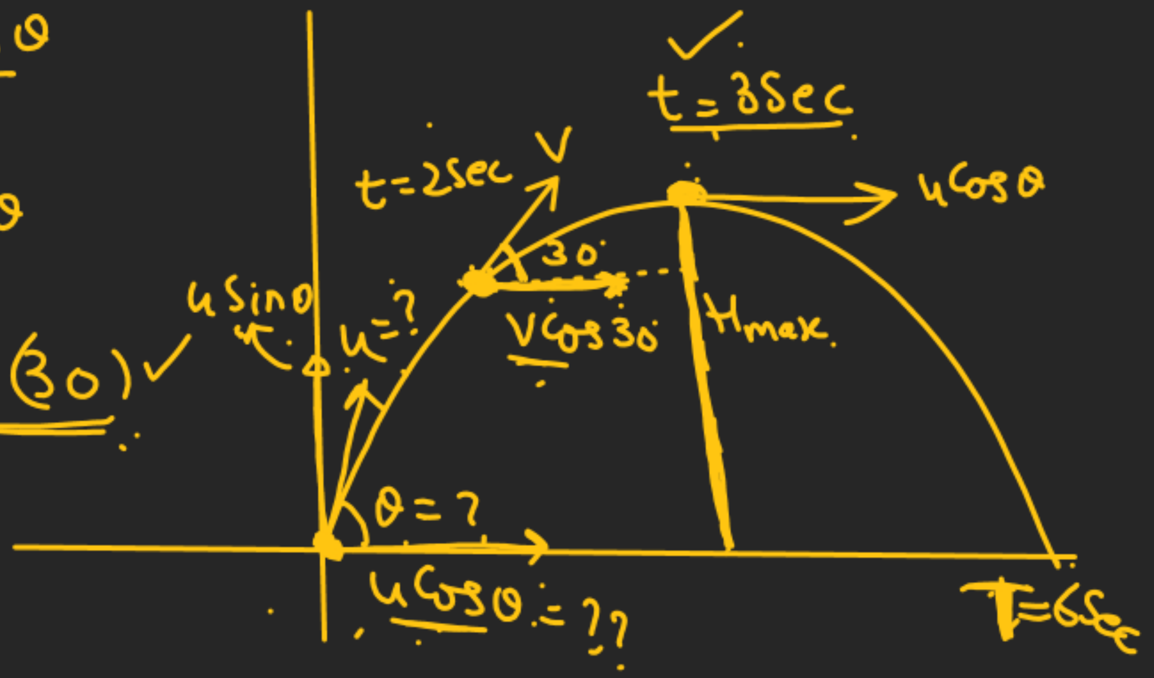
$V\cos 30^\circ = u\cos\theta$

$\boxed{\left(\frac{\sqrt{3}V}{2}\right) = (u\cos\theta)}$

$\frac{T}{2} = \frac{u\sin\theta}{g}$

$\frac{6}{2} = \frac{u\sin\theta}{g}$

$\underline{u\sin\theta = (30)}$



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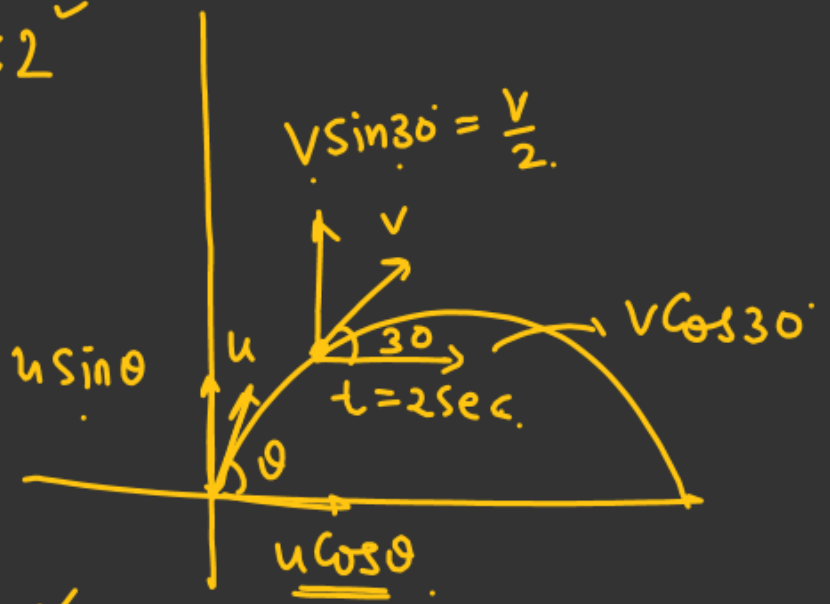
$(V_y = u_y - gt)$ From $t = 0$ to $t = 2\,sec$

$\frac{V}{2} = (u\sin\theta) - g \times 2$

$\frac{V}{2} = (30 - 20)$

$\frac{V}{2} = 10$

$\boxed{V = 20}$



$u\cos\theta = V\cos 30^\circ$

$= 20 \times \frac{\sqrt{3}}{2} = 10\sqrt{3}$

$u\sin\theta = 30$

$u^2(\cos^2\theta + \sin^2\theta)$

$= (300 + 900)$

$u^2 = 1200$

$u = \sqrt{1200}$

$= 10 \times 2\sqrt{3}$

$= 20\sqrt{3}$

$\tan\theta = \frac{30}{10\sqrt{3}}$

$\tan\theta = \sqrt{3} \Rightarrow \boxed{\theta = 60^\circ}$

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Projectile motion

H.W

Q. A particle moves in the plane xy with constant acceleration a directed along the negative y-axis. The equation of motion of the particle has the form $\boxed{y = k_1x - k_2x^2}$, where k_1 and k_2 are positive constants. Find the velocity of the particle at the origin of coordinates.

$y = K_1x - K_2x^2$

for Roots, $y = 0$

$x(K_1 - K_2x) = 0$

$x = 0$, $\quad x = \left(\frac{K_1}{K_2}\right) = R$

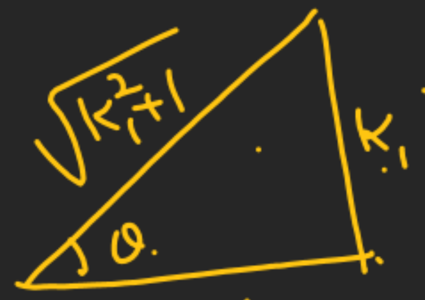
$y = K_1x - K_2x^2$

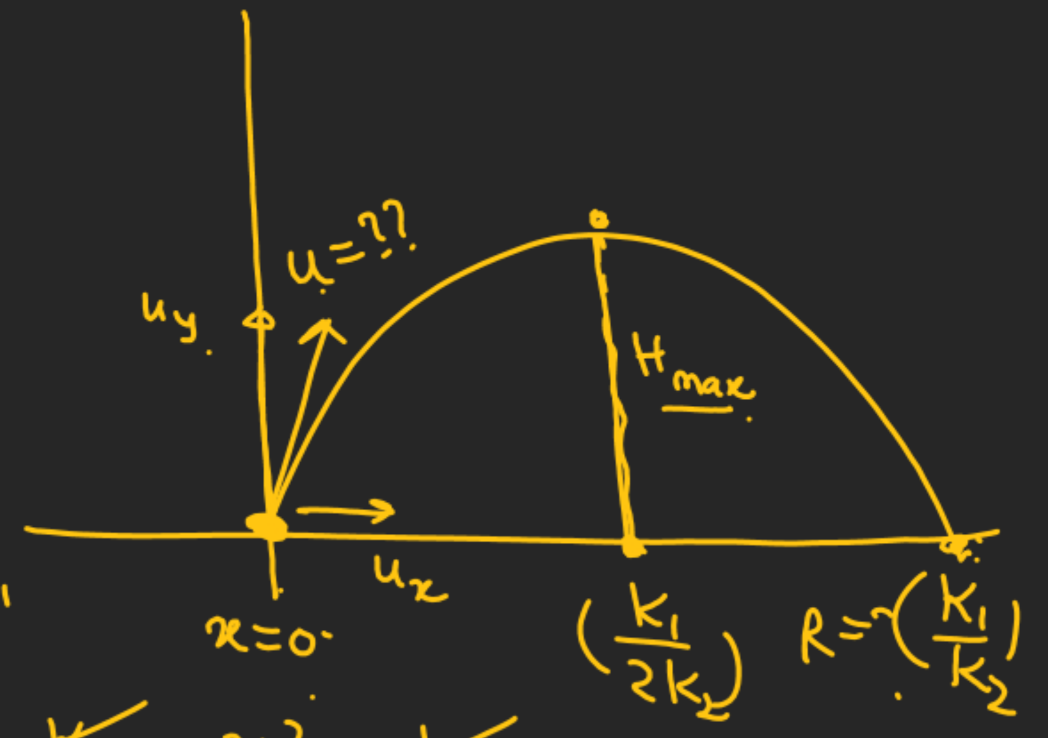
$y = K_1x\left[1 - \frac{K_2}{K_1}x\right]$

$$\boxed{y = x\tan\theta\left[1 - \frac{x}{R}\right]}$$

$\tan\theta_1 = \frac{K_1}{1}$

$R = \frac{K_1}{K_2}$





$R = \frac{u^2\sin 2\theta}{g}$

$\frac{K_1}{K_2} = \frac{u^2}{g} \times 2 \times \sin\theta \times \cos\theta$

$\frac{\cancel{K_1}}{K_2} = \frac{2u^2}{g} \times \frac{\cancel{K_1}}{\sqrt{K_1^2+1}} \times \frac{1}{\sqrt{K_1^2+1}}$

$\frac{1}{K_2} = \frac{2u^2}{g(K_1^2+1)}$

$u_2 = \sqrt{\frac{5}{K_2}(K_1^2+1)}$

Nishant Jindal

Projectile motion

H-W

5, 3, 4, 37°, 53°

Q. A particle is projected from point O on the ground with velocity $u = 5\sqrt{5}\ \mathrm{m/s}$ at angle $\alpha = \tan^{-1}(0.5)$. It strikes at a point C on a fixed smooth plane AB having inclination of 37° with horizontal as shown in Fig. If the particle does not rebound, calculate

(a) coordinates of point C in reference to coordinate system as shown in the figure.

(b) maximum height from the ground to which the particle rises. $(g = 10\ \mathrm{m/s^2})$.

$x_1 = \frac{10}{3} + x = \left(\frac{10}{3} + \frac{4y}{3}\right)$

$\left(\tan\alpha = 0.5 = \frac{1}{2}\right)$

$\sin\alpha = \frac{1}{\sqrt{5}}$

$\cos\alpha = \frac{2}{\sqrt{5}}$

$\sqrt{5}$, 1, α

$\frac{3}{4} = \tan 37^\circ = \frac{y}{x}$

$\boxed{x = \left(\frac{4y}{3}\right)}$

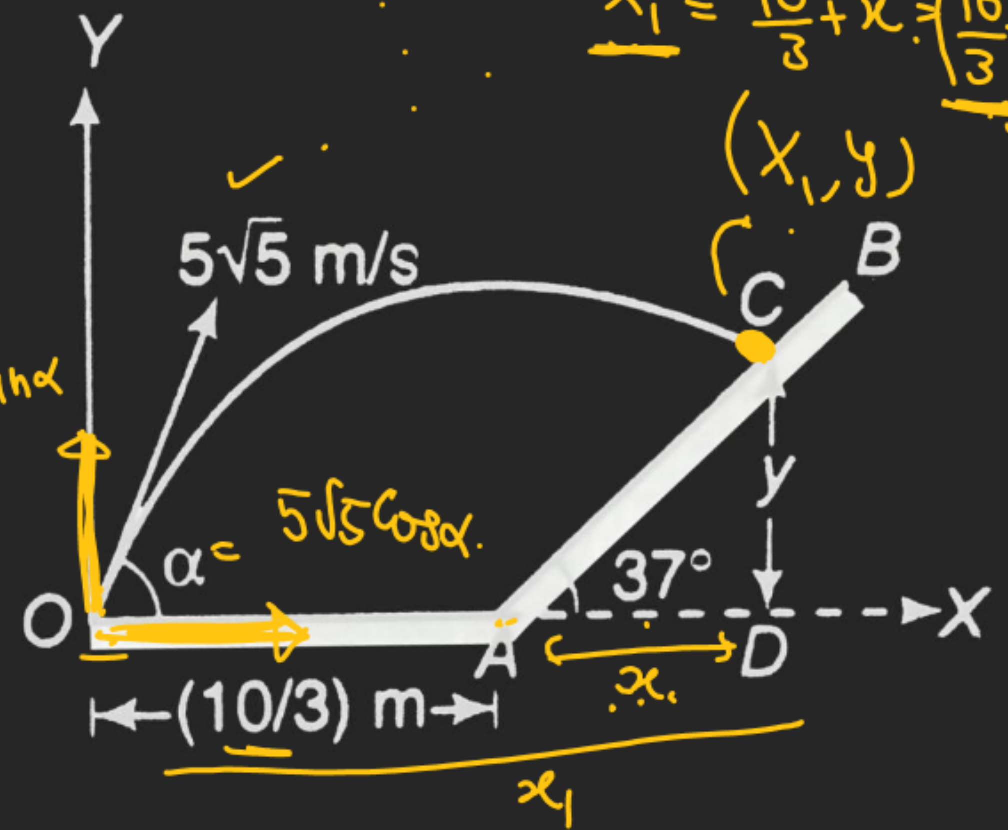
C, y, A, 37°, D, x

$\boxed{y = x_1 \tan\alpha \left[1 - \frac{x_1}{R}\right]}$

$R = \frac{2u^2 \sin\alpha \cdot \cos\alpha}{g} = \left[\frac{(5\sqrt{5})^2 \times 2 \times \frac{1}{\sqrt{5}} \times \frac{2}{\sqrt{5}}}{10}\right] = 10\,\mathrm{m}$

$x_1 = \frac{10}{3} + x = \left(\frac{10}{3} + \frac{4}{3}y\right)$

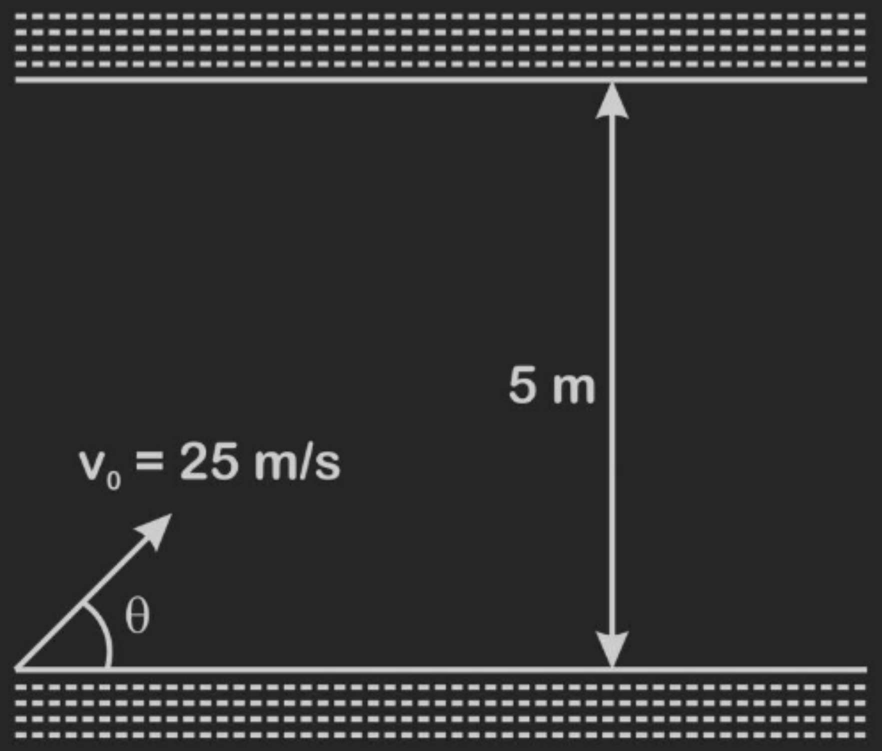
(x_1, y)



Projectile motion

H.W.

Q. A projectile is launched with a speed $v_B = 25 \text{ m/s}$ from the floor of a 5 m high tunnel as shown in figure. Determine the maximum horizontal range R of the projectile and the corresponding launch angle θ.



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Projectile motion

H.W.

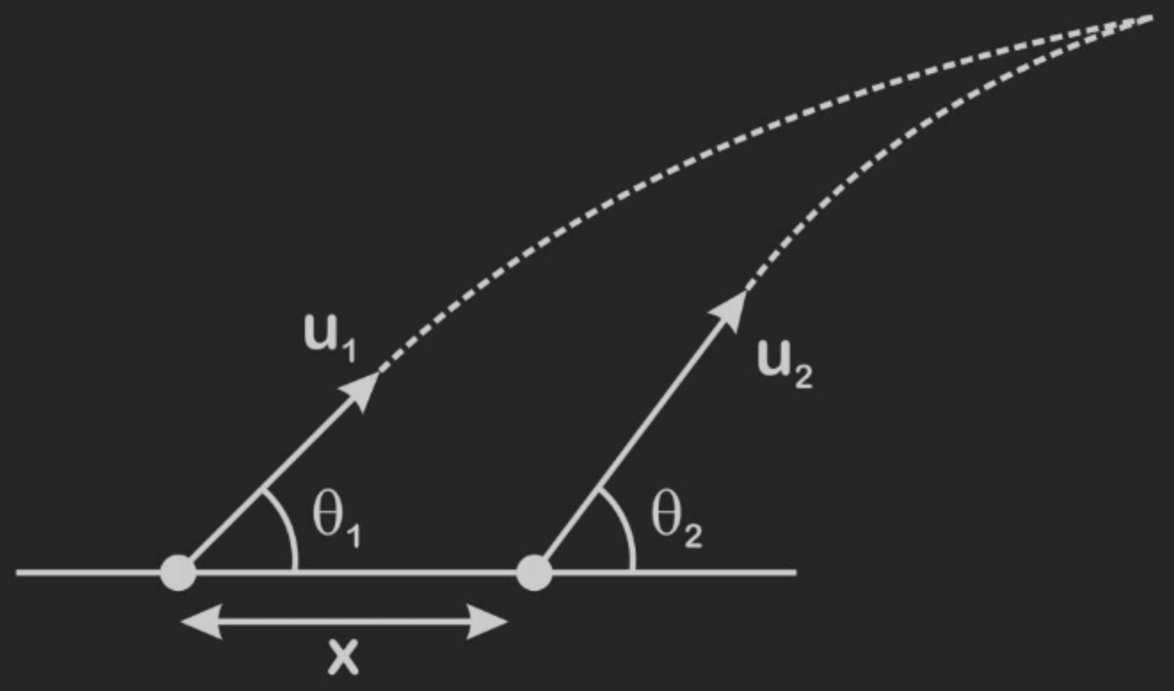
Q. Two particles are projected simultaneously from the level ground as shown in figure. They may collide after a time :

(a) $\frac{x\sin\theta_2}{u_1}$

(b) $\frac{x\cos\theta_2}{u_2}$

(c) $\frac{x\sin\theta_2}{u_1\sin(\theta_2-\theta_1)}$

(d) $\frac{x\sin\theta_1}{u_2\sin(\theta_2-\theta_1)}$



Projectile motion

HW

Q. A particle is projected from the ground. If the equation of the trajectory is $y = x - \frac{x^2}{2}$, then the time of flight is:

(a) $\frac{2}{\sqrt{g}}$

(b) $\frac{3}{\sqrt{g}}$

(c) $\frac{9}{\sqrt{g}}$

(d) $\sqrt{\frac{2}{g}}$

Nishant Jindal

Projectile motion

H.W

Q. A projectile moves from the ground such that its horizontal displacement is $x = Kt$ and vertical displacement is $y = Kt(1 - \alpha t)$, where K and α are constants and t is time. Find out total time of flight (T) and maximum height attained (Y_{max}) its

(a) $T = \alpha, Y_{max} = \frac{K}{2\alpha}$

(b) $T = \frac{1}{\alpha}, Y_{max} = \frac{2K}{\alpha}$

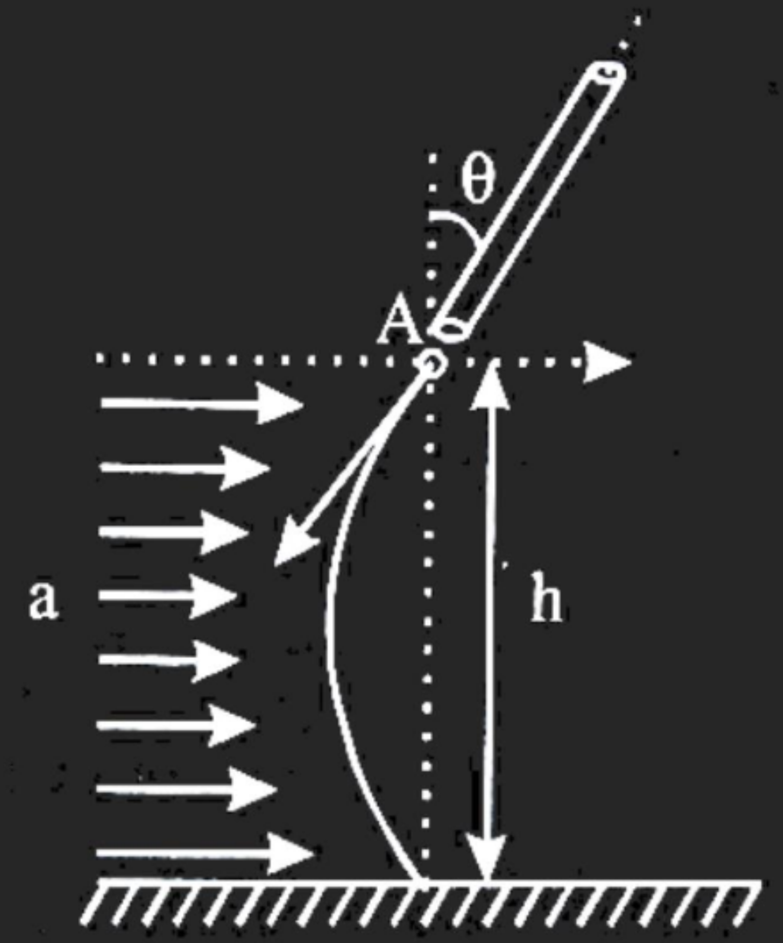
(c) $T = \frac{1}{\alpha}, Y_{max} = \frac{K}{6\alpha}$

(d) $T = \frac{1}{\alpha}, Y_{max} = \frac{K}{4\alpha}$

Projectile motion

H.W

Q. A particle is ejected from the tube at A with a velocity v at an angle θ with the vertical y-axis. A strong horizontal wind gives the particle a constant horizontal acceleration a in the x-direction. If the particle strikes the ground at a point directly under its released position and the downward y-acceleration is taken as g then



(a) $h = \frac{2v^2 \sin\theta \cos\theta}{a}$

(b) $h = \frac{2v^2 \sin\theta \cos\theta}{g}$

(c) $h = \frac{2v^2}{g} \sin\theta \left(\cos\theta + \frac{a}{g} \sin\theta\right)$

(d) $h = \frac{2v^2}{a} \sin\theta \left(\cos\theta + \frac{g}{a} \sin\theta\right)$

Projectile motion

HW :

Q. Trajectories are shown in figure are for three kicked footballs, ignoring the effect of the air on the footballs. If T_1, T_2 and T_3 are their respective time of flights then:

(a) $T_1 > T_3$

(b) $T_1 < T_3$

(c) $T_2 = \left(\frac{T_1 + T_3}{2}\right)$ ✓

(d) $T_1 = T_2 = T_3$ ✓

$T_1 = T_3 = T_2$

$$H_{max} = \frac{u_y^2}{2g}$$

$$(H_{max})_1 = (H_{max})_2 = (H_{max})_3$$

$$\boxed{(u_1)_y = (u_2)_y = (u_3)_y}$$

$$T = \left(\frac{2u_y}{g}\right) \longrightarrow T_1 = T_2 = T_3$$