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Differential Eqn.

Adv → 2-3 Qs.
Main → 1 Qs.
} → Branch → Calculus

① Diff' Eqn is a Eqn which Contains —
→ Independent variable (x)
→ Dependent variable (y)
→ Diff' Coefficient $\left(\frac{dy}{dx}, \frac{d^2y}{dx^2} \ldots\right)$ (Mandatory)

(1) $\frac{dy}{dx} + 3x = 2y$ Diff Eqn ✓

(2) $\frac{dy}{dx} + 3\sin x = 2\cos y$ ✓

(3) $\frac{dy}{dx} = 3\sin x$ ✓

(4) $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + y = 0$ ✓

(5) $\frac{dy}{dx} = 4$ ✓

(6) $\frac{dy}{dx} = a$ ✗ Not a Diff' Eqn as a is arbitrary Constant

(2) Meaning of diff Eqn.

Diff Eqn gives family of Curves.

(3) 2 terms in D. Eqn are very Popular.

(A) Order of DE

(B) Degree of DE

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(4) Order of D.E

A) $\frac{dy}{dx} \rightarrow$ Order = 1

$\frac{d^2y}{dx^2} \rightarrow$ Order = 2

$\frac{d^3y}{dx^3} \rightarrow$ order 3

(13) Highest derivative present in D.Eqn is Order of DE.

$$\boxed{\frac{d^2y}{dx^2}} + y\cdot\sin x = \frac{4}{x}$$

सबसे बड़ा derivative

$\therefore$ Order of DE = 2

(5) Degree of DE

Highest derivative's deg is degree of DE

$$\left(\frac{d^3y}{dx^3}\right)^3 + y\left(\frac{dy}{dx}\right)^{19} - \sin x = 0$$

Order = 3 Deg = 3.

Q $\left(\frac{dy}{dx}\right) = \frac{2}{\left(\frac{dy}{dx}\right)}$ O/D?

$$\left(\frac{dy}{dx}\right)^2 = 2$$

Order = 1

Deg = 2

Q. $\left(\frac{d^2y}{dx^2}\right) - 4\left(\frac{dy}{dx}\right)^2 + 8 = 0$ O/D?

Order = 2

Deg = 1

Q $\left(\frac{d^2y}{dx^2}\right)^3 - 3\left(\frac{dy}{dx}\right)^5 - e^{2x} = \frac{y}{4}$

Or = 2

deg = 3.

RK① While Calculating O/D. We remove fractional deg from Differential Coff.

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Q $\dfrac{d^2y}{dx^2} = \left\{1 + \left(\dfrac{dy}{dx}\right)^4\right\}^{5/3}$ * O/D ?

making it Radical free.

Cube both Side.

$$\left(\frac{d^2y}{dx^2}\right)^3 = \left(1 + \left(\frac{dy}{dx}\right)^4\right)^5$$

O = 2 D = 3.

RK2 A) If derivative in given $\sin\left(\frac{dy}{dx}\right), \cos\left(\frac{dy}{dx}\right), e^{\frac{dy}{dx}}, \log\left(\frac{dy}{dx}\right)$ form then degree of DE in undefined.

(B) as these terms can be expanded Using taylor series which left deg undefined

Q $\dfrac{d^2y}{dx^2} = x \cdot \ln\left(\dfrac{dy}{dx}\right)$

Ord = 2

Deg = Undefined

(P) $\left(\left(1 + \left(\dfrac{dy}{dx}\right)^2\right)^{\frac{1}{2}}\right)^{\times 6}{}^{3} = \left(K \cdot \dfrac{d^2y}{dx^2}\right)^{\frac{1}{3} \times 6}{}^{2}$

Ord = 2

(Q) $\dfrac{d^3y}{dx^3} + 5 \cdot \dfrac{d^2y}{dx^2} + y = 2x$ Order 3

Q List 1

P. $\sqrt{1 + \left(\dfrac{dy}{dx}\right)^2} = \left(K \cdot \dfrac{d^2y}{dx^2}\right)^{1/3}$

Q. $\dfrac{d^2y}{dx^2} + 5\dfrac{dy}{dx} + \int y \cdot dx = x^2$

R. $\sqrt{\sin x}\,(dx + dy) + \sqrt{\cos x}\,(dx - dy) = 0$ $\div dx$

S. $\left(\dfrac{d^2y}{dx^2}\right)^5 + 4\left(\dfrac{d^4y}{dx^4}\right)^2 + \left(\dfrac{dy}{dx}\right)^3 + 2y + 1 = 0$

Ord = 4

List 2 (order of List 1)

A) 1

B) 2

C) 3

D) 4

P→B, Q→C, R→A, S→D

(R) $\sqrt{\sin x}\left(1 + \dfrac{dy}{dx}\right) + \sqrt{\cos x}\left(1 - \dfrac{dy}{dx}\right) = 0$

$\Rightarrow \sqrt{\sin x} + \sqrt{\cos x} = \dfrac{dy}{dx}\left(\sqrt{\cos x} - \sqrt{\sin x}\right)$

$$\boxed{\frac{dy}{dx} = \frac{\sqrt{\sin x} + \sqrt{\cos x}}{\sqrt{\cos x} - \sqrt{\sin x}}}$$

Ord = 1

D = 1

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Q List 1

P) $\sqrt{\frac{dy}{dx}} - 4\left(\frac{dy}{dx}\right) - 7x = 0$

Q) $y = x\cdot\left(\frac{dy}{dx}\right)^2 + \left(\frac{dx}{dy}\right)^2$

R) $\frac{d^2y}{dx^2} = \left[1 + \left(\frac{dy}{dx}\right)^2\right]^{2/3}$

S) $\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{1/5 \times 15} = \left(\lambda\cdot\frac{d^2y}{dx^2}\right)^{1/3 \times 15}$

List 2 (Deg of List 1.)

5

4

3

2

P → 2

Q → 4

R → 3

S → 5

(P) $\frac{dy}{dx} = \left(4\left(\frac{dy}{dx}\right) + 7x\right)^2$

O = 1, D = 2

(Q) $y = x\cdot\left(\frac{dy}{dx}\right)^2 + \frac{1}{\left(\frac{dy}{dx}\right)^2}$

$y\cdot\left(\frac{dy}{dx}\right)^2 = x\cdot\left(\frac{dy}{dx}\right)^4 + 1$

O = 1, D = 4

(R) $\left(\frac{d^2y}{dx^2}\right)^3 = \left(1 + \left(\frac{dy}{dx}\right)^2\right)^2$

O = 2, D = 3

(S) $\left(1 + \left(\frac{dy}{dx}\right)^2\right)^3 = \left(\lambda\cdot\frac{d^2y}{dx^2}\right)^5$

O = 2, D = 5

(5) Meaning of order.

$\frac{d^4y}{dx^4} = 0$

$\frac{d^3y}{dx^3} = 1$

$\frac{d^2y}{dx^2} = x$ in a DE

Order = 2

Int ↓

$\frac{dy}{dx} = \frac{x^2}{2} + C_1$

Int ↓

$y = \frac{x^3}{6} + \boxed{C_1}x + \boxed{C_2}$

Jitne Arb. Constant Utni bar diff^n Krna is Safe to Return to basic Eq^n.

लौटते हैं

$\frac{d^3y}{dx^3} = C_1$

$\frac{d^2y}{dx^2} = C_1x + C_2$

$\frac{dy}{dx} = \frac{C_1x^2}{2} + C_2x + C_3$

$y = \frac{C_1x^3}{6} + \frac{C_2x^2}{2} + C_3x + C_4$

* Conclusion → No of Arb. Constant Present in Basic Eq^n = Order of DE

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Formation of DE.

(A) No. of Arb. Const.

Count Krna Ana Chahiye.

(1) $ax+by+c=0$ has ... No. of Arb. Const.

$ax+by+c=0 \quad \div$ by c

$\boxed{\frac{a}{c}}x+\boxed{\frac{b}{c}}y+1=0$

(1) (2)

2 Arb. Const

(2) $y=ax+b$ has ... Arb. Const.

2 Arb.

Q $y=cx-c^2-c^3$ has ... Arb Const.?

If Arb. Const. are linked to each other we count them one.

$\therefore$ Arb Const $=1$

Q $y^2=2cx(1+\sqrt{c})$

1 Arb.

Q. $y=(c_1+c_2)e^{x+c_3}+c_4 e^{x+c_5}$ ... Arb Const?

$=(c_1+c_2)e^x \cdot e^{c_3}+c_4 \cdot e^x \cdot e^{c_5}$

$K_1e^x+K_2e^x=(K_1+K_2)e^x=Ke^x$

1 Arb. Const.

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(B) Formation of DE

* Count No of Arb. Const & diff. as many times as No of Arb. Const

Q Find DE of $y = Ae^x + Be^{-x}$?

Arb. Const. = 2

2 or R diff'e

$$\frac{dy}{dx} = Ae^x - Be^{-x}$$

$$\frac{d^2y}{dx^2} = Ae^x + Be^{-x}$$

$$\frac{d^2y}{dx^2} = y \text{ is DE.}$$

Q DE of $Ax^2 + By^2 = C$?

2 Arb → 2 or R diff

diff^n $2Ax + 2By \cdot \frac{dy}{dx} = 0$

$$Ax + B\left(y \cdot \frac{dy}{dx}\right) = 0$$

$$B\left(y\frac{dy}{dx}\right) = -Ax \Rightarrow \frac{y}{x} \cdot \frac{dy}{dx} = -\frac{A}{B}$$

diff^n $A + B\left(y \cdot \frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2\right) = 0$

$$\left(y \cdot \frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2\right) = -\frac{A}{B}$$

$$y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = \frac{y}{x} \cdot \left(\frac{dy}{dx}\right)$$

O = 2

D = 1

Q DEq^n of $x^2 + y^2 - 2ay = 0$

Arb. Const = 1

$x^2 + y^2 = 2ay \Rightarrow a = \frac{x^2 + y^2}{2y}$

diff^n $2x + 2y\frac{dy}{dx} - 2a\frac{dy}{dx} = 0$

$$\frac{dy}{dx}(y - a) = -x$$

$$\frac{dy}{dx}\left(y - \frac{x^2 + y^2}{2y}\right) = -x$$

$$\frac{dy}{dx}\left(\frac{y^2 - x^2}{2y}\right) = -x$$

DE

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Q Find DE of $y = e^{cx}$

Adv

$y = e^{cx}$

Arb = 1

$\ln y = cx$

$c = \frac{\ln y}{x}$

$y' = e^{cx} c$

$y' = cy$

$y' = \frac{\ln y}{x} \cdot y$

$xy' = y \ln y$

Q DE of $y = c_1 \cdot e^{c_2 x}$ (c_1, c_2)

Main

Ans ⇒ $xy' = y \ln y$

Arb

$(yy'' = y'^2)$

2 Arb

$y' = c_1 \cdot e^{c_2 x} \cdot c_2$

$y' = c_2 y \longrightarrow c_2 = \frac{y'}{y}$

$y'' = c_2 \cdot y'$

$y'' = \frac{y'}{y} \cdot y'$

$y \cdot y'' = (y')^2$

Q DE of $y = cx - c^2 - c^3$

Arb = 1

$\frac{dy}{dx} = c$

$y = \left(\frac{dy}{dx}\right) \cdot x - \left(\frac{dy}{dx}\right)^2 - \left(\frac{dy}{dx}\right)^3$

O = 1, D = 3

Q. Find DE of all line P.T. origin.

line P.T. origin → $y = mx$

$\frac{dy}{dx} = m$

$y = \left(\frac{dy}{dx}\right) \cdot x$

⇒ $\boxed{y\,dx - x\,dy = 0}$

Q DE of all St. line P.T. $(-1,-1)$

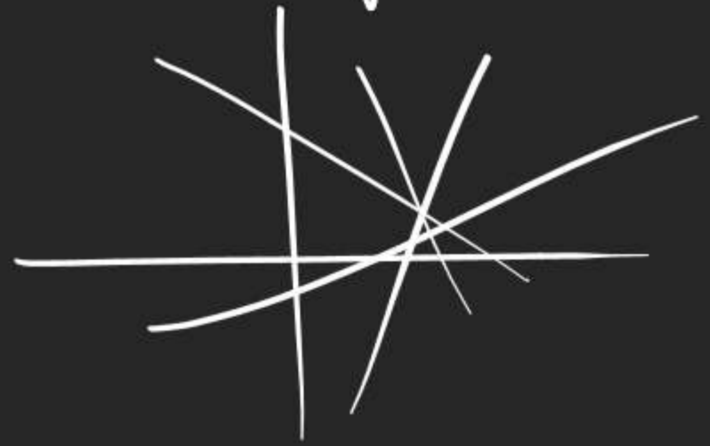
line $\rightarrow (y+1) = m(x+1)$

$\uparrow$ 1 Arb.

$\frac{dy}{dx} = m$

$(y+1) = \frac{dy}{dx}(x+1)$

Q DE of all lines in xy Plane?



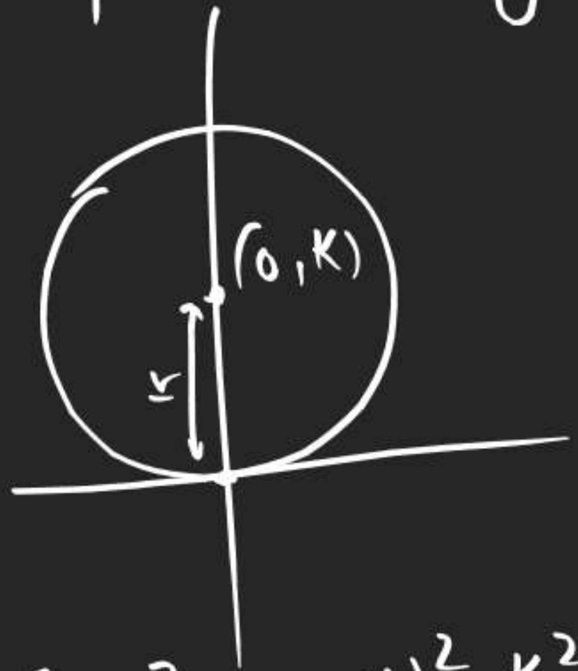
$y = mx + c$

2 Arb.

$\frac{dy}{dx} = m$

$\boxed{\frac{d^2y}{dx^2} = 0}$ A

Q D.E of all Circles having Centre at y Axis & P.T. origin.



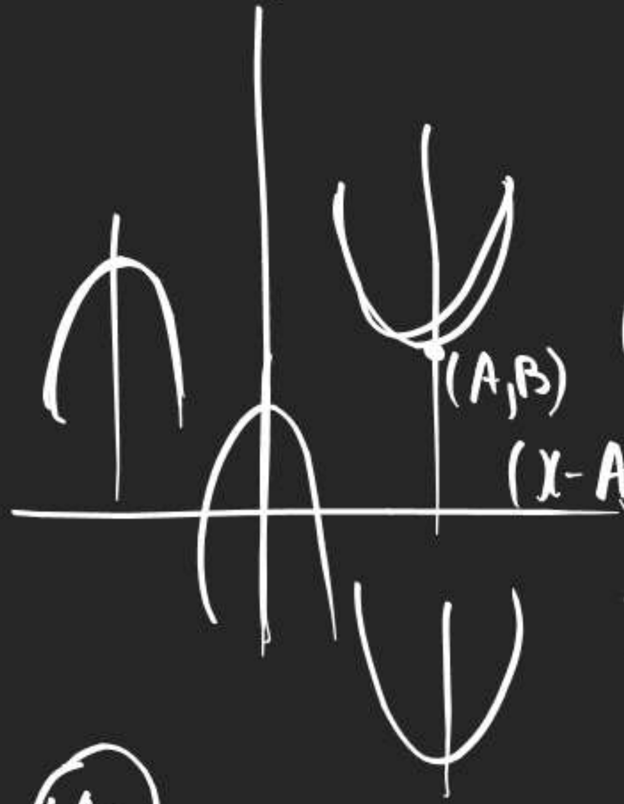
$(x-0)^2 + (y-K)^2 = K^2$

$x^2 + y^2 = 2Ky + \cancel{K^2} = \cancel{K^2}$

$\underline{x^2 + y^2 - 2Ky = 0}$

$\frac{dy}{dx}\left(\frac{y^2 - x^2}{2y}\right) = -x$

Q DE of all Parabolas having their axes $\parallel^r$ to y Axis?



M_1

$(x-A)^2 = 4a(y-B)$

3 Arb

M2 Parabola $\rightarrow y = ax^2 + bx + c$

$\frac{dy}{dx} = 2ax + b$

$\frac{d^2y}{dx^2} = 2a$

$\frac{d^3y}{dx^3} = 0$