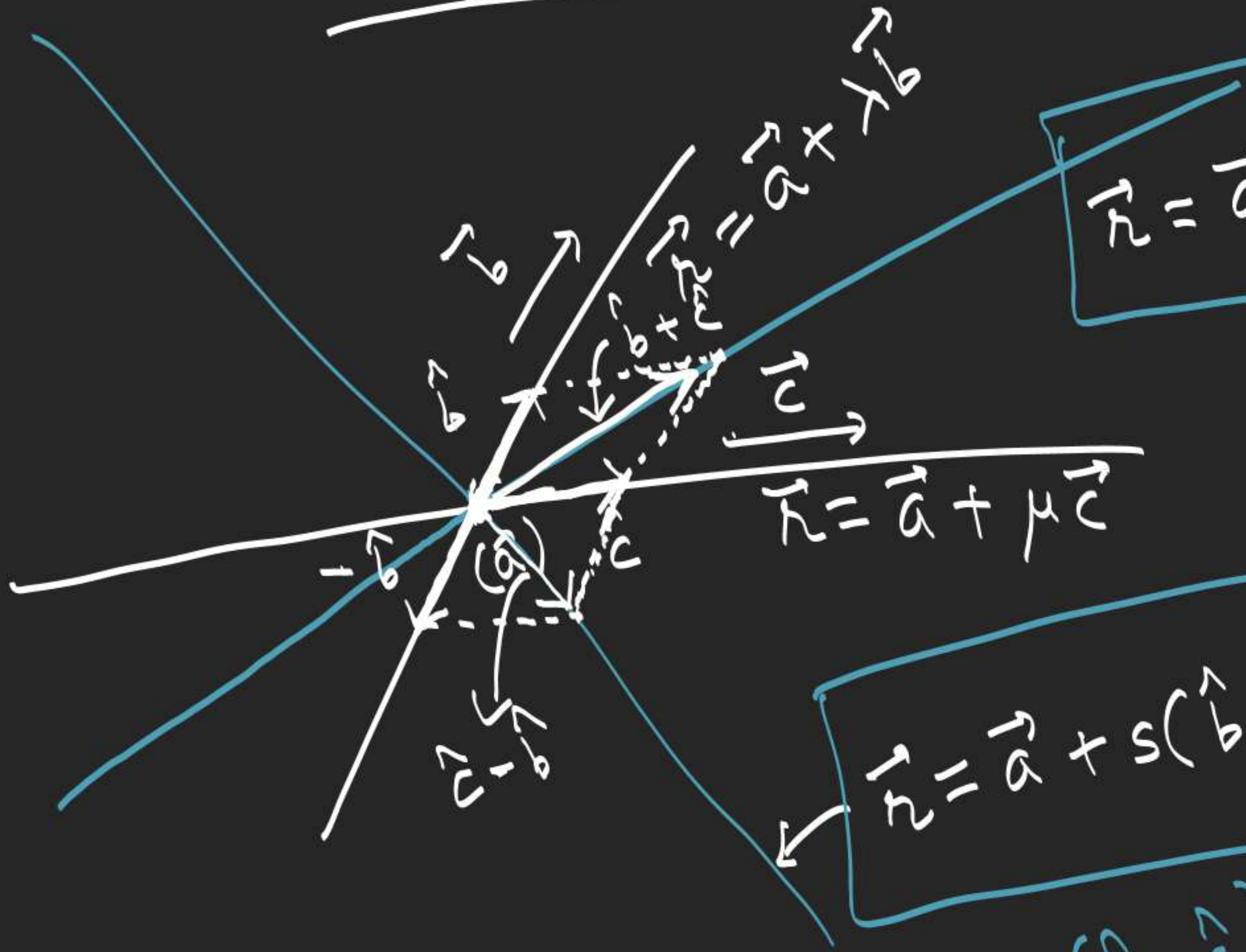
Nishant Jindal

Angle Bisector



$$\boxed{\vec{r} = \vec{a} + t(\hat{b}+\hat{c})}$$

$$\boxed{\vec{r} = \vec{a} + s(\hat{b}-\hat{c})}$$

$$(\hat{b}+\hat{c})\cdot(\hat{b}-\hat{c}) = 0$$

Condition of Collinearity of 3 points

3 points with p.v. $\vec{a}, \vec{b}, \vec{c}$ are collinear iff $\exists$ scalars x, y, z (not all zero) satisfying

$$x\vec{a} + y\vec{b} + z\vec{c} = \vec{0} \quad \text{and} \quad x + y + z = 0$$

$z \neq 0$

$$\frac{x\vec{a} + y\vec{b}}{-z} = \vec{c}$$

$$\frac{x\vec{a} + y\vec{b}}{x+y} = \vec{c}$$

$\times (\vec{a}) \quad \times (\vec{b}) \quad \times (\vec{c})$

$$\vec{b} - \vec{a} = \lambda(\vec{c} - \vec{a})$$

$$(\lambda - 1)\vec{a} + (1)\vec{b} + (-\lambda)\vec{c} = \vec{0}$$

$$(\lambda - 1) + (1) + (-\lambda) = 0$$

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1. Check whether points with p.v. given are collinear or not

$$\overset{A}{2\hat{i}+5\hat{j}-4\hat{k}}, \quad \overset{B}{\hat{i}+4\hat{j}-3\hat{k}}, \quad \overset{C}{4\hat{i}+7\hat{j}-6\hat{k}}.$$

$\vec{AB} = -\hat{i}-\hat{j}+\hat{k}$ $\quad -2\vec{AB} = \vec{AC}$

$\vec{AC} = 2\hat{i}+2\hat{j}-2\hat{k}$

2. ABC is a scalene triangle. AD, AD' are bisectors of angle A meeting BC in D, D' respectively. A' is the midpoint of DD'. B', C' are points on CA, AB similarly obtained. Show that A', B', C' are collinear.

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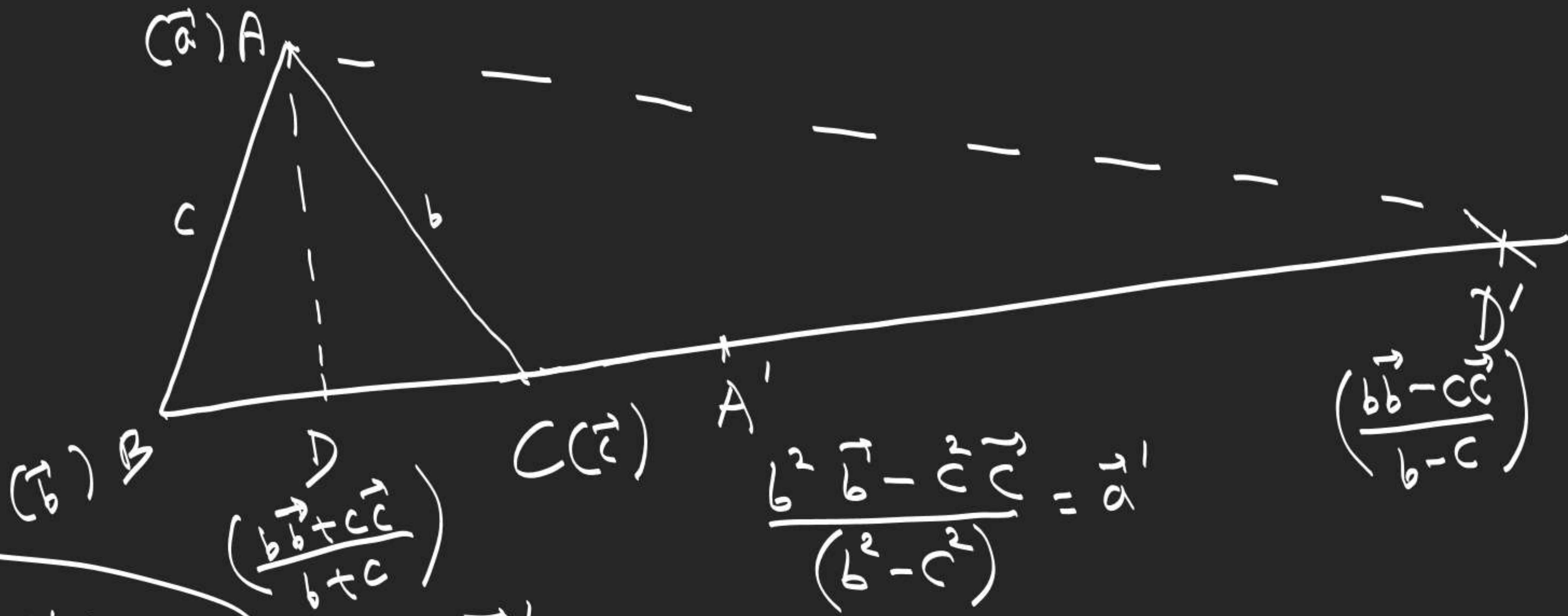

$$\frac{b^2\vec{b}-c^2\vec{c}}{(b^2-c^2)}=\vec{a}'$$

$$(b^2-c^2)\vec{a}'+(c^2-a^2)\vec{b}'+(a^2-b^2)\vec{c}'=\vec{0}$$

$$(b^2-c^2)\vec{a}'=b^2\vec{b}-c^2\vec{c}$$

$$(c^2-a^2)\vec{b}'=c^2\vec{c}-a^2\vec{a}$$

$$(a^2-b^2)\vec{c}'=a^2\vec{a}-b^2\vec{b}$$

$$(b^2-c^2)+(c^2-a^2)+(a^2-b^2)=0$$

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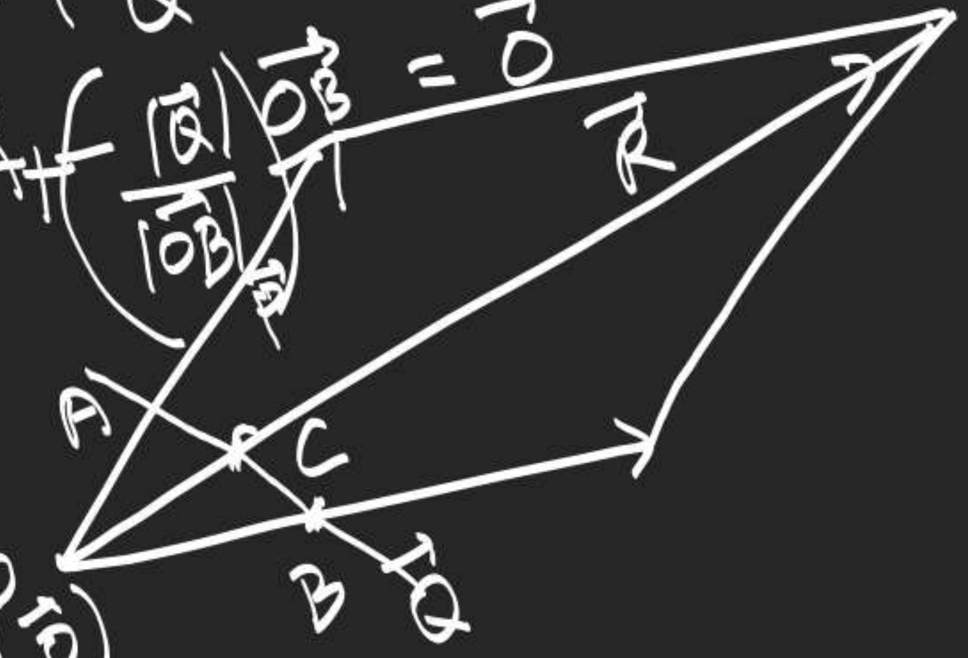
3: Vectors $\vec{P}, \vec{Q}$ act at 'O' (origin) have resultant $\vec{R}$.

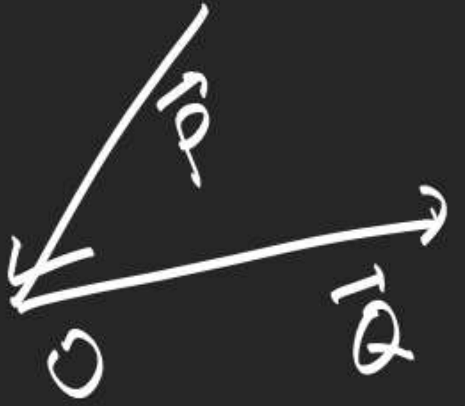
If any transversal line cuts their line of action at A, B, C respectively. Then P.T.
$$\frac{|\vec{P}|}{|\vec{OA}|} + \frac{|\vec{Q}|}{|\vec{OB}|} = \frac{|\vec{R}|}{|\vec{OC}|}$$

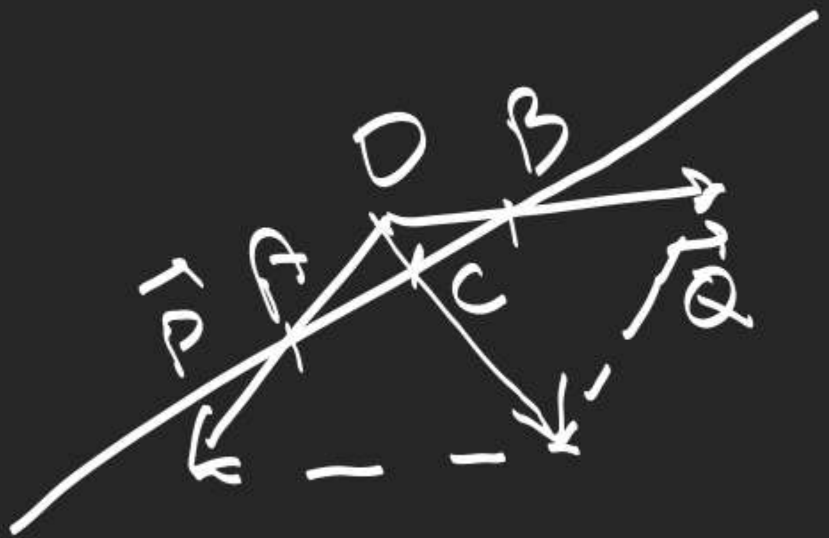
$$\vec{R} = \vec{P} + \vec{Q}$$

$$\frac{|\vec{R}|}{|\vec{OC}|}\vec{OC} + \left(-\frac{|\vec{P}|}{|\vec{OA}|}\right)\vec{OA} + \left(-\frac{|\vec{Q}|}{|\vec{OB}|}\right)\vec{OB} = \vec{0}$$



$$\frac{|\vec{R}|}{|\vec{OC}|} - \frac{|\vec{P}|}{|\vec{OA}|} - \frac{|\vec{Q}|}{|\vec{OB}|} = 0 \quad (\vec{0})$$





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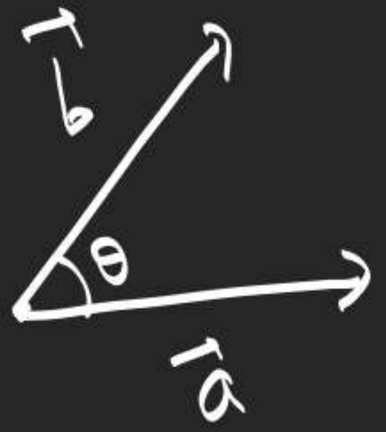
Scalar (Dot) Product

$$\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos\theta \quad , \quad \theta = \vec{a} \wedge \vec{b}$$



Find locus of P moving in space s.t.

$$\vec{PA}\cdot\vec{PB} < 0$$

where A, B are given fixed points.

$$\vec{a}\cdot\vec{b} < 0 \Rightarrow \theta \in \left(\frac{\pi}{2}, \pi\right]$$

$$\vec{a}\cdot\vec{b} > 0 \Rightarrow \theta \in \left[0, \frac{\pi}{2}\right)$$

$$\vec{a}\cdot\vec{b} = 0 \Rightarrow \vec{a} = \vec{0} \text{ or } \vec{b} = \vec{0} \text{ or } \theta = \frac{\pi}{2}$$

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* $\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}$
* $\vec{a} \cdot (\vec{b} + \vec{c}) = \vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c}$
* $\hat{i} \cdot \hat{i} = 1 = \hat{j} \cdot \hat{j} = \hat{k} \cdot \hat{k}$
* $\hat{i} \cdot \hat{j} = 0$
*
* $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

  $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

$$\sum_{i=1}^{n} (a_i + x b_i)^2 \geq 0$$

$$\Rightarrow \left(\sum_{i=1}^{n} b_i^2\right)x^2 + 2\left(\sum_{i=1}^{n} a_i b_i\right)x + \sum_{i=1}^{n} a_i^2 \geq 0 \quad \forall x \in R$$

$$\boxed{D \leq 0}$$

$$\boxed{(a_1b_1 + a_2b_2 + a_3b_3 + \cdots a_nb_n)^2 \leq (a_1^2 + a_2^2 + \cdots + a_n^2)(b_1^2 + b_2^2 + \cdots + b_n^2)}$$

Equality holds if $\frac{a_1}{b_1} = \frac{a_2}{b_2} = \cdots = \frac{a_n}{b_n}$

Cauchy's Inequality

$$\vec{a} \cdot \vec{b} = a_1b_1 + a_2b_2 + a_3b_3 = \sqrt{a_1^2 + a_2^2 + a_3^2}\sqrt{b_1^2 + b_2^2 + b_3^2}\cos\theta$$

$$\boxed{(a_1b_1 + a_2b_2 + a_3b_3)^2 \leq (a_1^2 + a_2^2 + a_3^2)(b_1^2 + b_2^2 + b_3^2)}$$

Equality holds if $\frac{a_1}{b_1} = \frac{a_2}{b_2} = \frac{a_3}{b_3}$

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$$|\vec{a}+\vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 + 2\vec{a}\cdot\vec{b}$$

$$|\vec{a}-\vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 - 2\vec{a}\cdot\vec{b}$$

$$|\vec{a}+\vec{b}+\vec{c}|^2 = |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a})$$

$$(\vec{a}+\vec{b})\cdot(\vec{a}-\vec{b}) = |\vec{a}|^2 - |\vec{b}|^2$$

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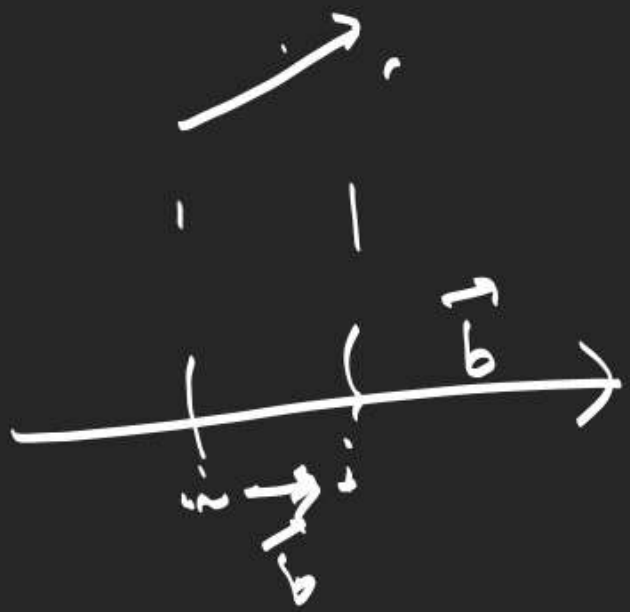
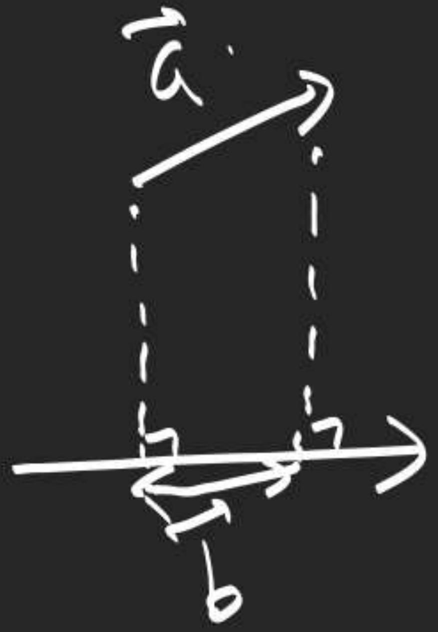
Projection of $\vec{a}$ on $\vec{b}$





$$|\vec{a}|\cos\theta = \boxed{\text{Projection of } \vec{a} \text{ on } \vec{b} = \frac{\vec{a}\cdot\vec{b}}{|\vec{b}|}}$$

Proj of $\vec{a}$ on $\vec{b}$ is scalar which if multiplied to $\hat{b}$ will give vector component of $\vec{a}$ $\parallel$ to $\vec{b}$.

Projection of $\vec{b}$ on $\vec{a}$ $= |\vec{b}|\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}|}$

$-l$

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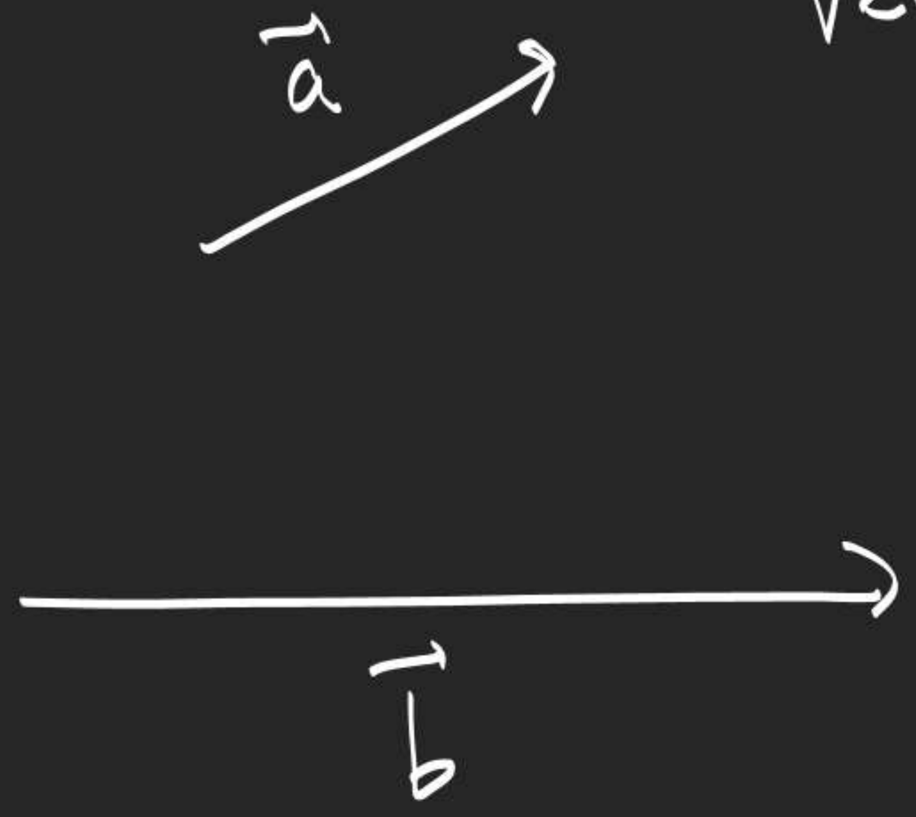
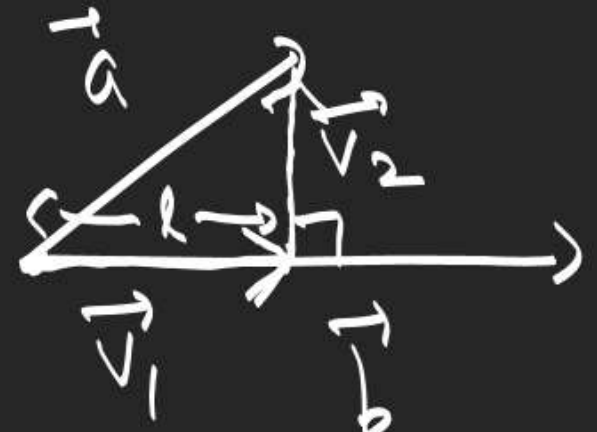
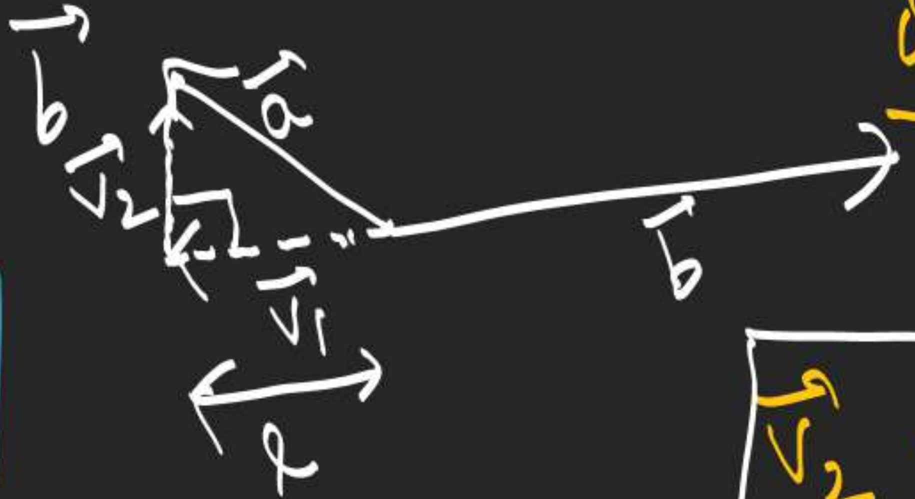
Vector Component of $\vec{a}$ parallel to $\vec{b}$





$$\vec{v}_1 = \left(\frac{\vec{a}\cdot\vec{b}}{|\vec{b}|}\right)\hat{b} = \frac{\vec{a}\cdot\vec{b}}{|\vec{b}|^2}\,\vec{b}$$



$$\boxed{\vec{v}_1 = \frac{\vec{a}\cdot\vec{b}}{|\vec{b}|^2}\,\vec{b}}$$

Vector Component of $\vec{a}$ perpendicular to $\vec{b}$

$$\boxed{\vec{v}_2 = \vec{a} - \frac{\vec{a}\cdot\vec{b}}{|\vec{b}|^2}\,\vec{b}}$$

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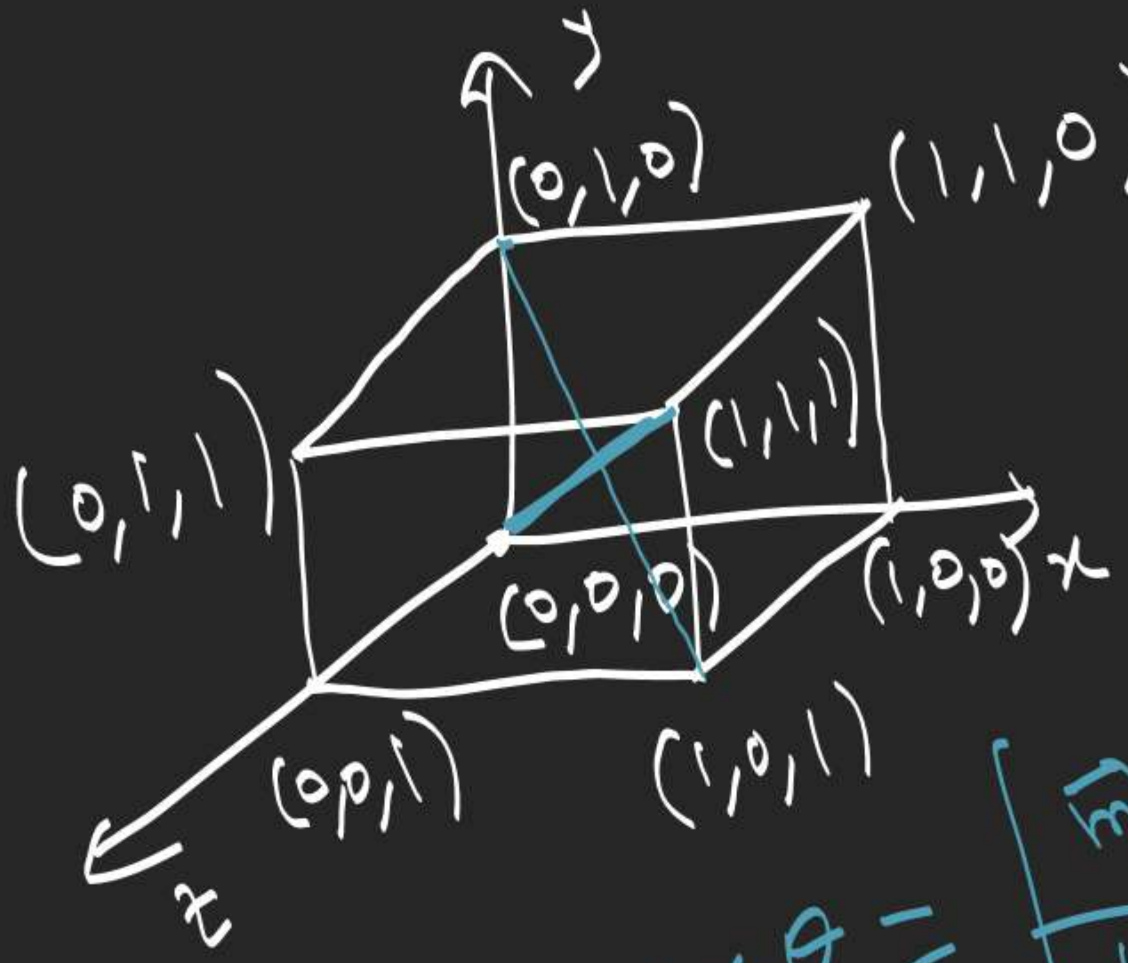
1: Find the acute angle b/n the diagonals of a cube.



$\vec{m}_1 = \hat{i} + \hat{j} + \hat{k}$

$\vec{m}_2 = \hat{i} - \hat{j} + \hat{k}$

$$\cos\theta = \left| \frac{\vec{m}_1 \cdot \vec{m}_2}{|\vec{m}_1||\vec{m}_2|} \right| = \frac{1}{\sqrt{3}\sqrt{3}} = \frac{1}{3}$$

$$\cos^{-1}\frac{1}{3} = \theta$$

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Q. Two vectors $\vec{e_1}$ and $\vec{e_2}$ with $|\vec{e_1}|=2$, $|\vec{e_2}|=1$ and angle between $\vec{e_1}$ and $\vec{e_2}$ is 60°. The angle between $2t\vec{e_1}+7\vec{e_2}$ and $\vec{e_1}+t\vec{e_2}$ belongs to interval $\left(\frac{\pi}{2},\pi\right)$, find t.

$$2t(4)+7t(1)+(2t^2+7)\left(2\times1\times\frac{1}{2}\right)<0$$

$$\boxed{t\in\left(-7,-\frac{1}{2}\right)-\left\{-\sqrt{\frac{7}{2}}\right\}}$$

$$2t\vec{e_1}+7\vec{e_2}=\lambda(\vec{e_1}+t\vec{e_2})$$

$$t=\pm\sqrt{\frac{7}{2}} \Leftarrow \frac{2t}{7}=\frac{1}{t}$$

$\lambda<0$

$2t=\lambda$

$7=\lambda t$

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3.

L: $\vec{r} = \vec{a} + \lambda \vec{b}$

* $P(\vec{p})$

Find p.v. of (i) foot of $\perp$ar of P on Line 'L'

(ii) image of P on line 'L'.

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4. Find the radius of sphere circumscribing and radius of sphere inscribed in a regular tetrahedron having length of edge 'k'.

5. Use Vectors to P.T. in ΔABC,

$$\cos 2A + \cos 2B + \cos 2C \geq -\frac{3}{2}$$

. Also Prove that the distance between circumcentre and centroid is $\sqrt{R^2 - \frac{1}{9}(a^2+b^2+c^2)}$.