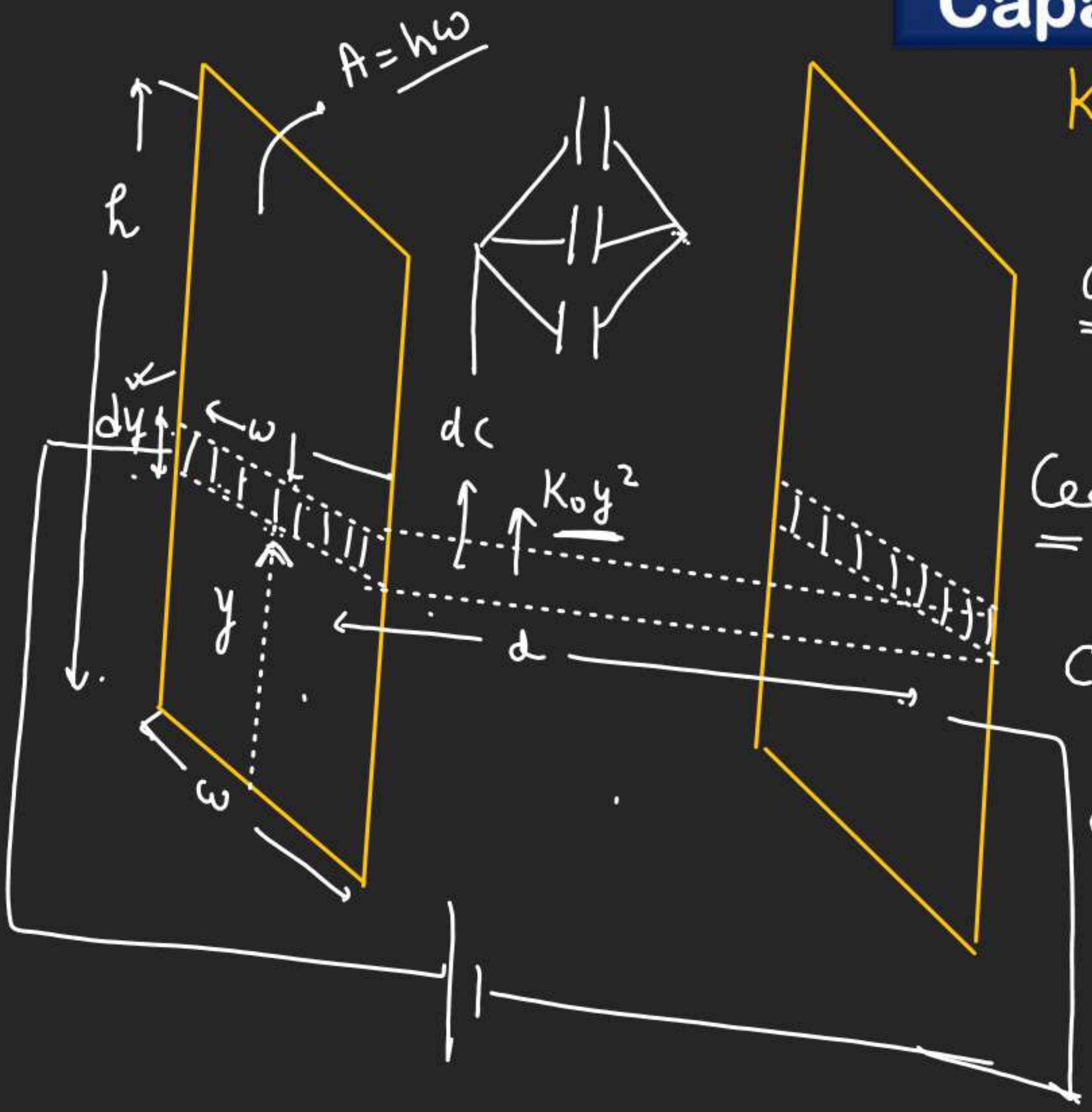
Nishant Jindal

Capacitor

$C_{air} = \frac{\varepsilon_0 A}{d}$, $C_{dielectric} = \frac{K \varepsilon_0 A}{d}$



$K = K_0 y^2$

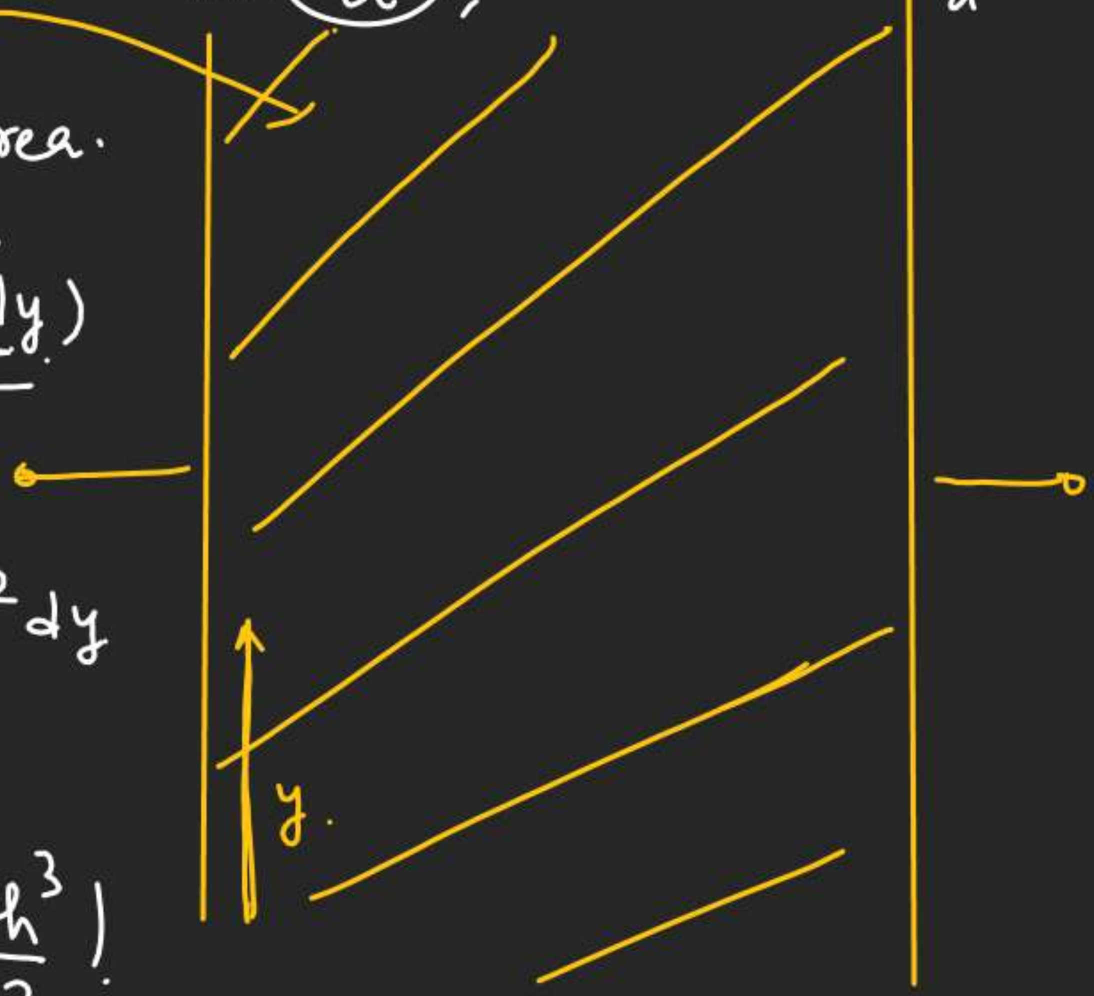
Area

$$dC = \frac{K_y \cdot \varepsilon_0 (w\,dy)}{d}$$

$$C_{eq} = \frac{K_0 \varepsilon_0 w}{d} \int_0^h y^2\,dy$$

$$C_{eq} = \frac{K_0 \varepsilon_0 w}{d} \left(\frac{h^3}{3} \right)$$

$$C_{eq} = \frac{K_0 \varepsilon_0}{3d} (wh)(h^2) = \left(\frac{K_0 \varepsilon_0 A h^2}{3d} \right)$$

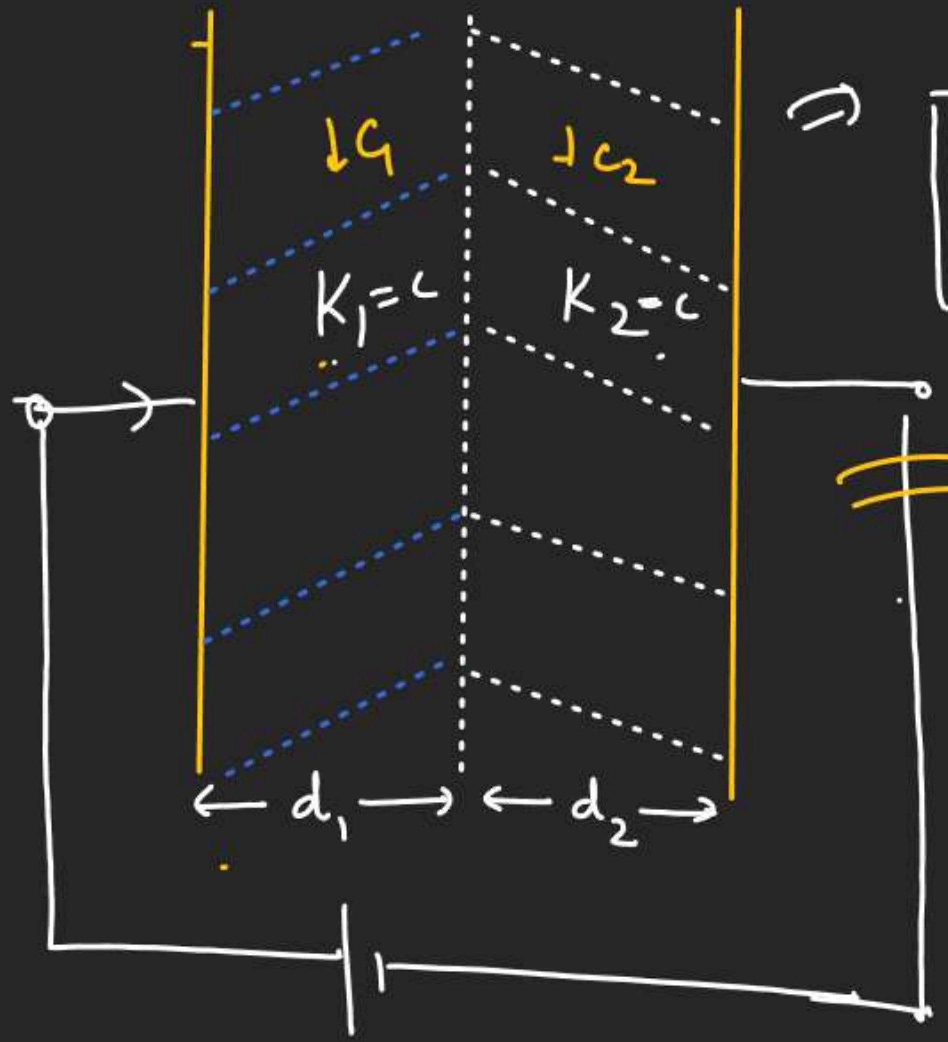


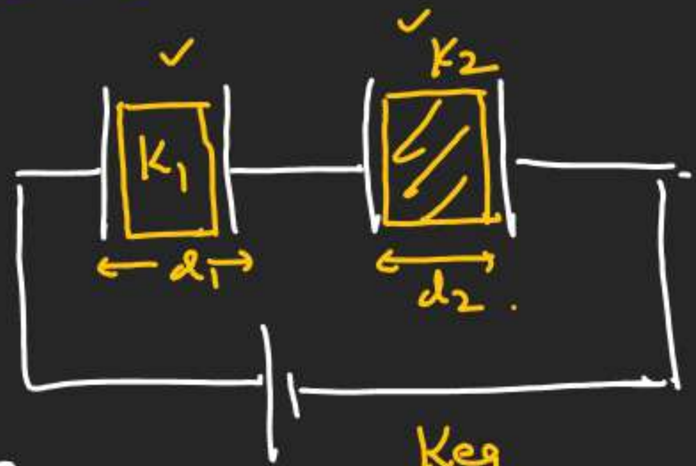
Nishant Jindal

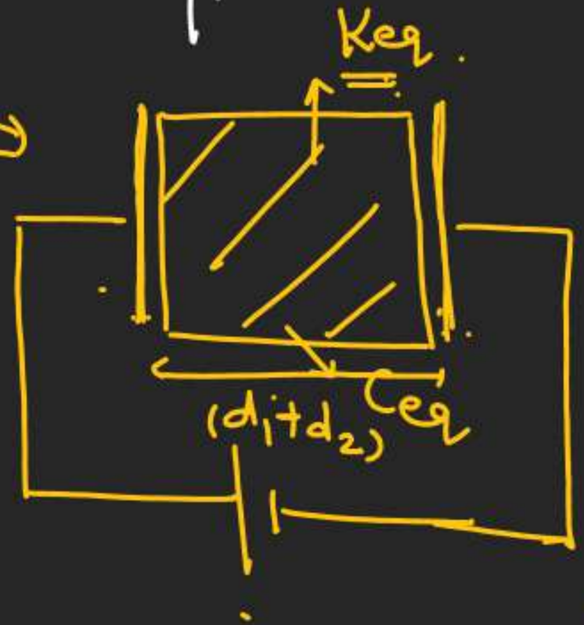
Capacitor

\# Find. $K_{eq} = ??$

↳ Equivalent dielectric constant.







$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$\frac{1}{\frac{K_{eq}\varepsilon_0 A}{(d_1+d_2)}} = \frac{1}{\frac{K_1\varepsilon_0 A}{d_1}} + \frac{1}{\frac{K_2\varepsilon_0 A}{d_2}}$$

$$\frac{d_1+d_2}{K_{eq}} = \frac{d_1}{K_1} + \frac{d_2}{K_2} = \frac{d_1K_2+d_2K_1}{K_1K_2}$$

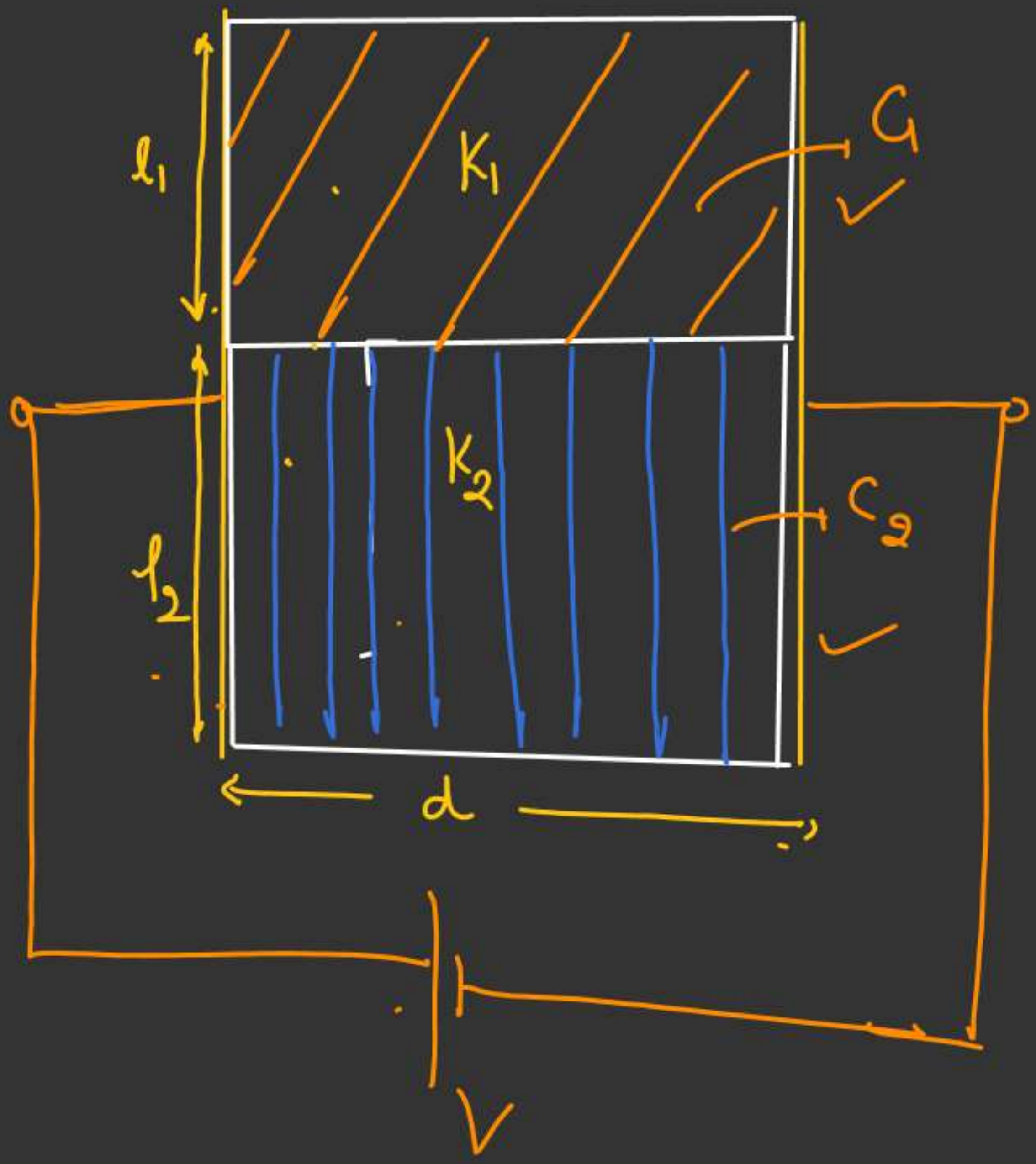
$$K_{eq} = \left[\frac{(d_1+d_2)K_1K_2}{d_1K_2+d_2K_1}\right] ✓$$

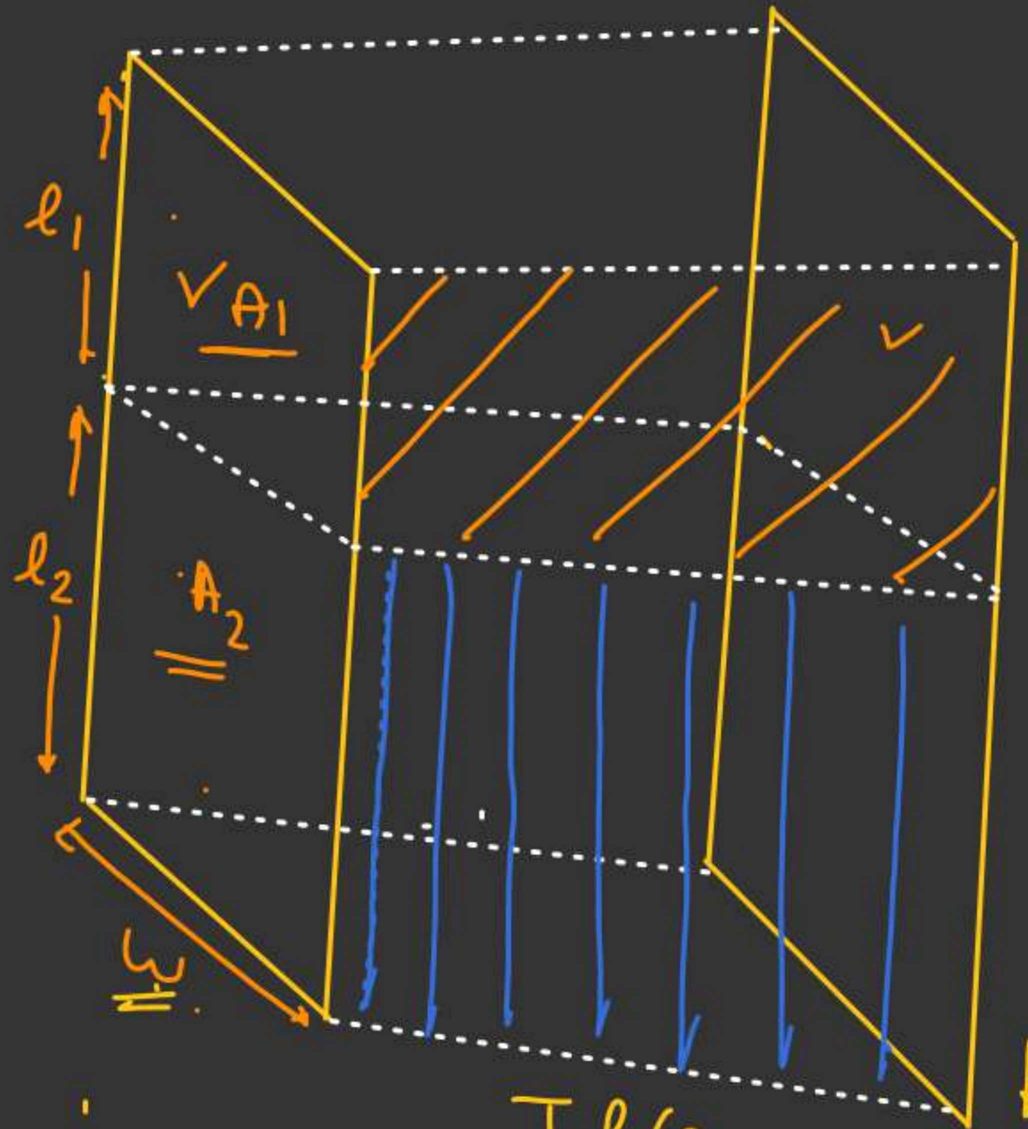
If $\boxed{d_1 = d_2 = d}$

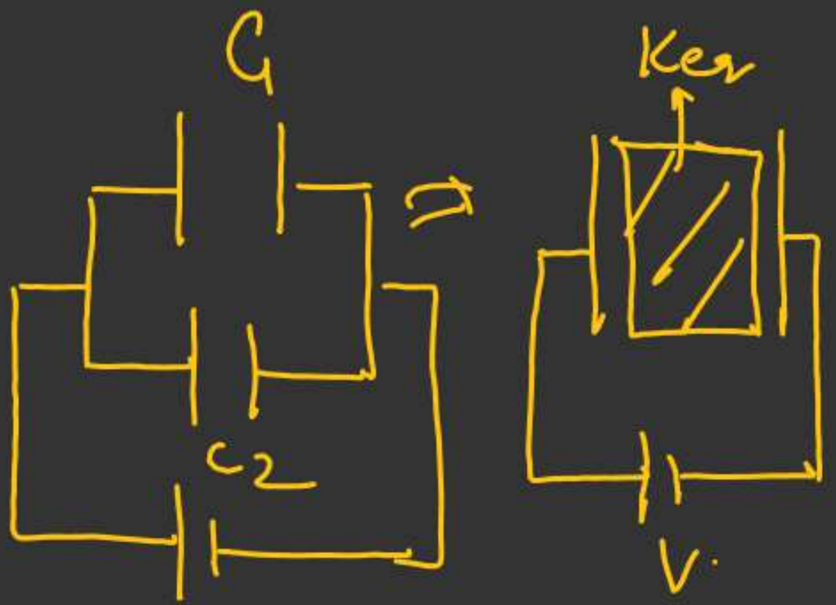
$$K_{eq} = \left(\frac{2K_1K_2}{K_1+K_2}\right)$$

Nishant Jindal

Find $K_{eq} = ??$







$$C_{eq} = C_1 + C_2$$

$$\frac{K_{eq}\,\varepsilon_0\,\underbrace{(l_1+l_2)w}_{A}}{d} = \frac{K_1\varepsilon_0 l_1 w}{d} + \frac{K_2\varepsilon_0 l_2 w}{d}$$

$$K_{eq} = \left[\frac{K_1 l_1 + K_2 l_2}{l_1+l_2}\right]$$

If $(l_1 = l_2)$

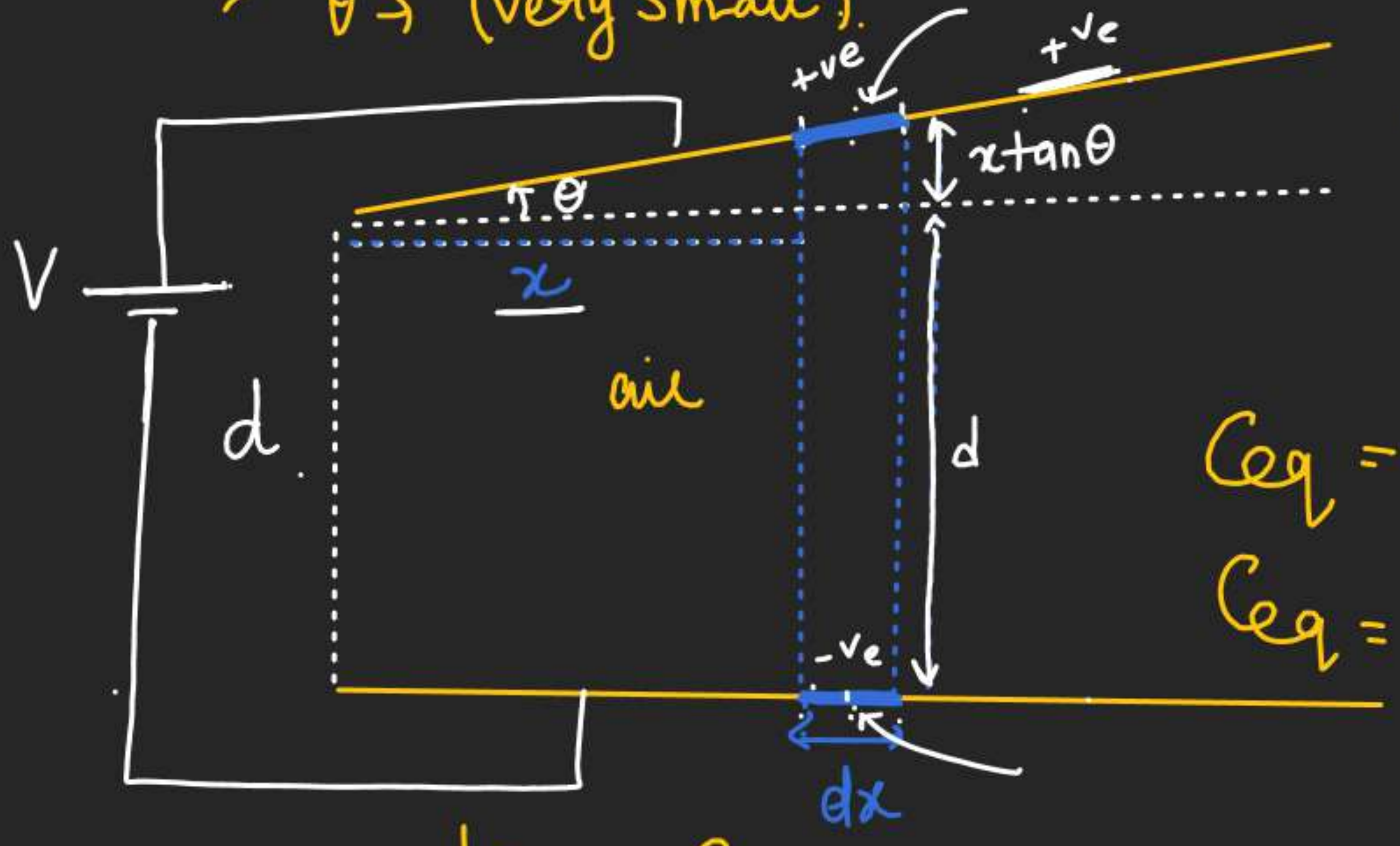
$$\left(K_{eq} = \frac{K_1+K_2}{2}\right)$$

Capacitor

$\boxed{\tan\theta \simeq \sin\theta \simeq \theta}$

Q. Find Capacitance of this Capacitor.

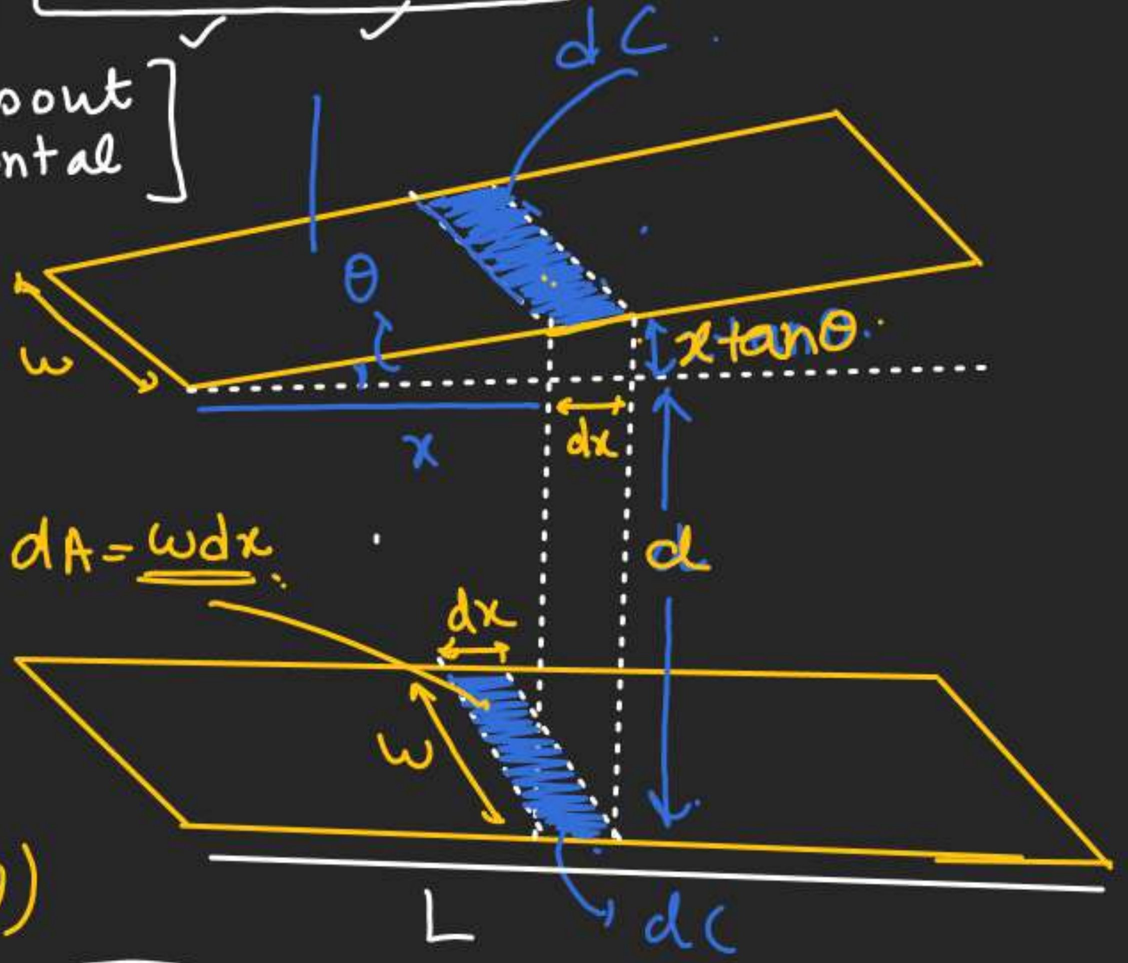
$\theta \to$ (very Small).



dx — [Near about horizontal]

$dx\cos\theta \to dx$

$\theta \to 0$



$dA = w\,dx$

$dc = \varepsilon_0$

$$C_{eq} = \int dC$$

$$C_{eq} = \int_0^L \frac{\varepsilon_0 (w\,dx)}{(d + x\tan\theta)}$$

$d \to a, \quad \tan\theta \to b$

$$\left[\int \frac{dx}{a+bx} = \frac{\ln(a+bx)}{b}\right]$$

$$C_{eq} = \frac{\varepsilon_0 w}{\tan\theta} \ln\left(d + x\tan\theta\right)\Big|_0^L$$

$$\frac{\varepsilon_0 w}{\tan\theta}\left\{\ln[d + L\tan\theta] - \ln(d)\right\}$$

$$C_{eq} = \frac{\varepsilon_0 w}{\tan\theta} \ln\left[\frac{d + L\tan\theta}{d}\right]$$

Capacitor

$$C_{eq} = \frac{\varepsilon_0 w}{\tan\theta} \ln\left(\frac{d + L\tan\theta}{d}\right)$$

$$\ln[1+x] = x - \frac{x^2}{2} \left(- - - \rightarrow \text{neglected if } x \ll 1 \right)$$

$$C_{eq} = \frac{\varepsilon_0 w}{\theta} \ln\left[1 + \underbrace{\frac{L}{d}\theta}_{x}\right]$$

$$C_{eq} = \frac{\varepsilon_0 w}{\theta} \left[\frac{L}{d}\theta - \frac{L^2}{2d^2}\theta^2\right]$$

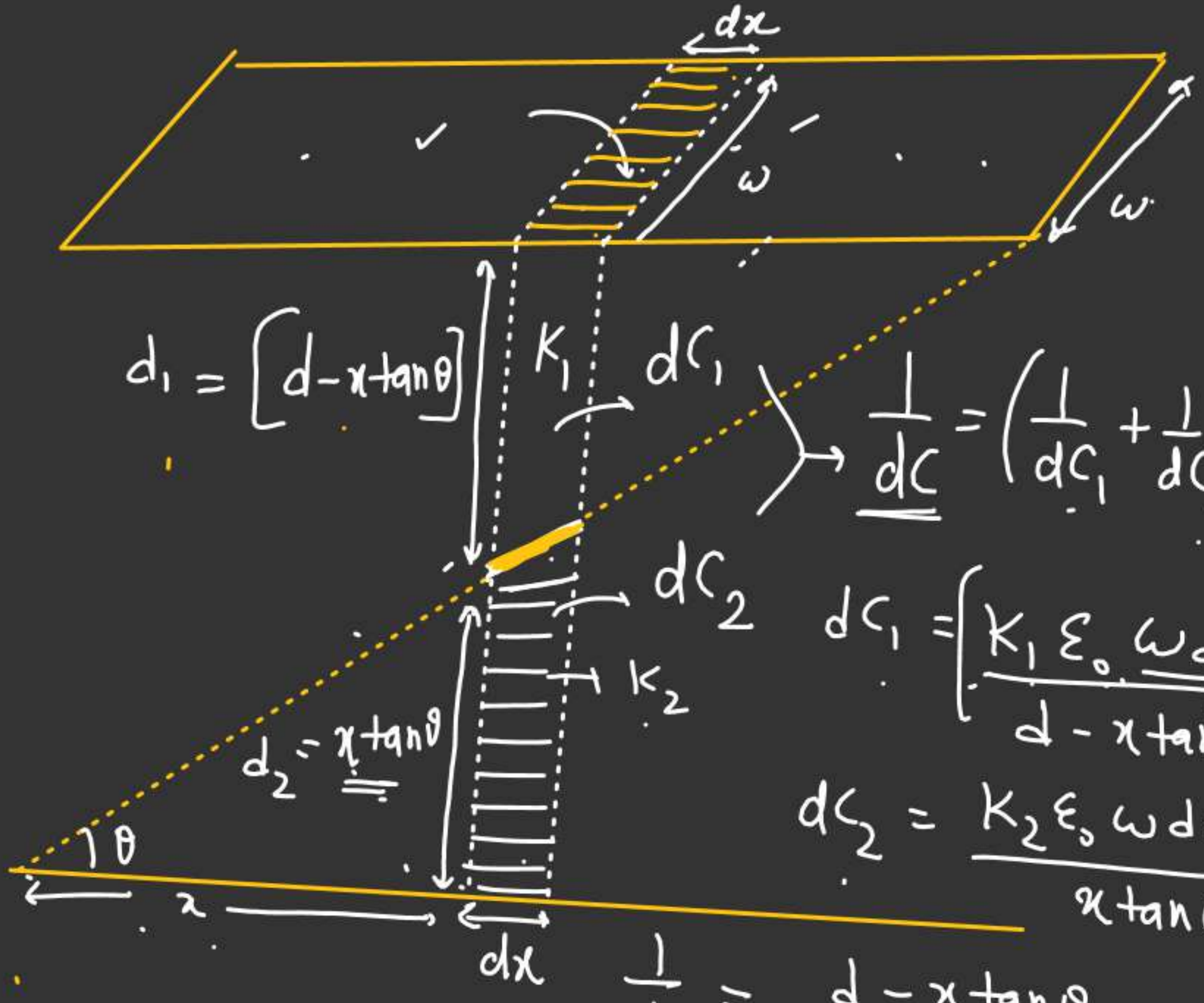
$$C_{eq} = \frac{\varepsilon_0 w L \cancel{\theta}}{\cancel{\theta} \cdot d} \left[1 - \frac{L\theta}{2d}\right] \Rightarrow \boxed{C_{eq} = \frac{\varepsilon_0 w \cdot L}{d} \left[1 - \frac{L\theta}{2d}\right]}$$

Nishant Jindal

$\omega \rightarrow$ width of the plate
$L \rightarrow$ length of the plate

Find $C_{eq} = ??$

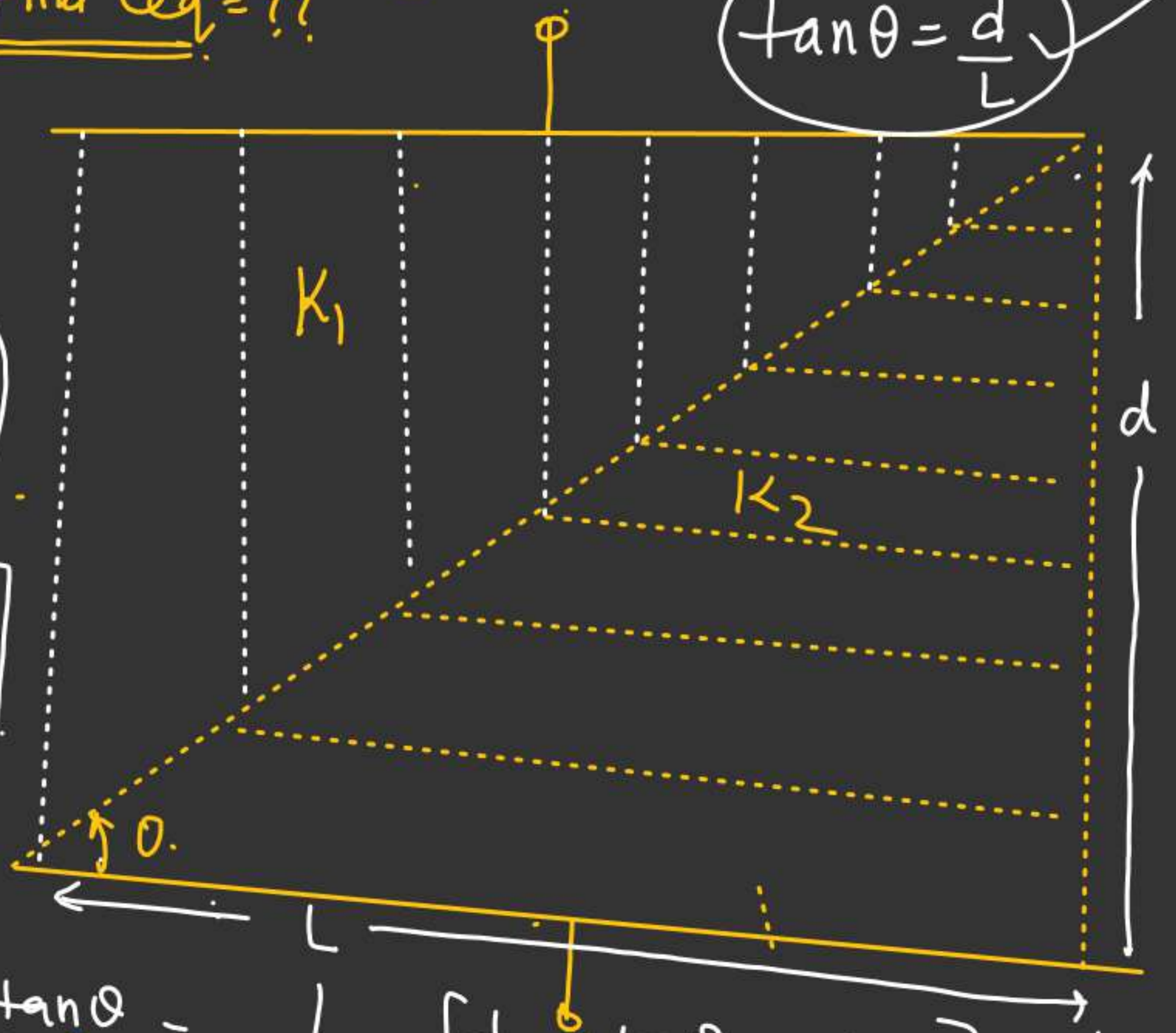
$\tan\theta = \dfrac{d}{L}$



$$\frac{1}{dC} = \left(\frac{1}{dC_1} + \frac{1}{dC_2}\right)$$

$$dC_1 = \left[\frac{K_1 \varepsilon_0 \omega dx}{d - x\tan\theta}\right]$$

$$dC_2 = \frac{K_2 \varepsilon_0 \omega dx}{x\tan\theta}$$



$$\frac{1}{dC} = \frac{d - x\tan\theta}{K_1 \varepsilon_0 \omega dx} + \frac{x\tan\theta}{K_2 \varepsilon_0 \omega dx} = \frac{1}{\varepsilon_0 \omega dx}\left[\frac{d - x\tan\theta}{K_1} + \frac{x\tan\theta}{K_2}\right]$$

Nishant Jindal

$$\frac{1}{dC} = \frac{1}{\varepsilon_0 \omega dx}\left[\frac{d - x\tan\theta}{K_1} + \frac{x\tan\theta}{K_2}\right]$$

$$\frac{1}{dC} = \frac{1}{\varepsilon_0 \omega dx}\left[\frac{K_2 d - K_2 x\tan\theta + K_1 x\tan\theta}{K_1K_2}\right]$$

$$\frac{1}{dC} = \frac{1}{\varepsilon_0 \omega K_1K_2 dx}\left[K_2 d + (K_1 - K_2)\tan\theta\, x\right]$$

$$\underline{dC} = \left[\frac{\varepsilon_0 \omega K_1K_2\, dx}{\underbrace{K_2 d}_{a} + \underbrace{(K_1-K_2)\tan\theta}_{b}\, x}\right]$$

$$C_{eq} = \int_0^L dC$$

$$C_{eq} = \varepsilon_0 \omega K_1K_2 \int_0^L \frac{dx}{a+bx}$$

$$\Rightarrow C_{eq} = \frac{\varepsilon_0 \omega K_1K_2}{b} \ln\left[a+bx\right]_0^L$$

$$C_{eq} = \frac{\varepsilon_0 \omega K_1K_2}{b}\left[\ln(a+bL) - \ln(a)\right]$$

$$C_{eq} = \frac{\varepsilon_0 \omega K_1K_2}{b} \ln\left[\frac{a+bL}{a}\right]$$

$$C_{eq} = \frac{\varepsilon_0 \omega K_1K_2}{(K_1-K_2)\underbrace{\tan\theta}_{(d/L)}} \ln\left[\frac{K_2 d + (K_1-K_2)\underbrace{\tan\theta}_{(d/L)}\cdot L}{K_2 d}\right]$$

$$C_{eq} = \frac{\varepsilon_0 K_1K_2 \overbrace{(\omega L)}^{A}}{d(K_1-K_2)} \ln\left[\frac{\cancel{K_2 d} - \cancel{K_2 d} + K_1 d}{K_2 d}\right]$$

$$\boxed{C_{eq} = \frac{\varepsilon_0 K_1K_2 A}{d(K_1-K_2)} \ln\left(\frac{K_1}{K_2}\right)}$$