Nishant Jindal

① Q. $\dfrac{1-\cos\theta}{\sin\theta}$

$\dfrac{2\sin^2\theta/2}{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}$

$=\tan\frac{\theta}{2}$

Q $\dfrac{1+\cos\theta}{\sin\theta}$

$\dfrac{2\cos^2\frac{\theta}{2}}{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}=\cot\frac{\theta}{2}$

Q P.T. $\dfrac{(1-\sin 2\theta)}{\cos 2\theta}=\tan\left(\frac{\pi}{4}-\theta\right)$

$(1-\sin 2\theta)=(\cos\theta-\sin\theta)^2$

$(1+\sin 2\theta)=(\cos\theta+\sin\theta)^2$

LHS $=$ $\dfrac{(\cos\theta-\sin\theta)^2}{\cos^2\theta-\sin^2\theta}$

$\dfrac{(\cancel{\cos\theta-\sin\theta})(\cos\theta-\sin\theta)}{(\cancel{\cos\theta-\sin\theta})(\cos\theta+\sin\theta)}$ $\div\cos\theta$

$\dfrac{\frac{\cancel{\cos\theta}}{\cancel{\cos\theta}}-\frac{\sin\theta}{\cos\theta}}{\frac{\cancel{\cos\theta}}{\cancel{\cos\theta}}+\frac{\sin\theta}{\cos\theta}}$

$\dfrac{1-\tan\theta}{1+\tan\theta}=\tan\left(\frac{\pi}{4}-\theta\right)$

RHS

Q $\dfrac{2\tan\theta}{1-\tan^2\theta}=?$

$=\tan 2\theta$ ← Direct

Q $\dfrac{2\tan\theta}{1+\tan^2\theta}$

$\dfrac{2\,\frac{\sin\theta}{\cos\theta}}{1+\frac{\sin^2\theta}{\cos^2\theta}}=\dfrac{2\sin\theta}{\cancel{\cos\theta}}\times\dfrac{\cos^2\theta}{\cos^2\theta+\sin^2\theta}$

$=\dfrac{2\sin\theta\cos\theta}{1}=\sin 2\theta$

Nishant Jindal

Q $\dfrac{1-\tan^2\theta}{1+\tan^2\theta} = ?$

$$\frac{1-\dfrac{\sin^2\theta}{\cos^2\theta}}{1+\dfrac{\sin^2\theta}{\cos^2\theta}}$$

$$= \frac{\dfrac{\cos^2\theta-\sin^2\theta}{\cancel{\cos\theta}}}{\dfrac{(\cos^2\theta+\sin^2\theta)}{\cancel{\cos\theta}}} =$$

$$= \frac{(\cos^2\theta-\sin^2\theta)}{1}$$

$$= \cos 2\theta$$

$$\boxed{\begin{aligned} &1)\ \sin 2\theta = 2\sin\theta\cdot\cos\theta = \frac{2\tan\theta}{1+\tan^2\theta} \\ &2)\ \cos 2\theta = C^2 - S^2 \qquad (3)\ \tan 2\theta = \frac{2T}{1-T^2} \\ &\qquad = 2C^2-1 \\ &\qquad = 1-2S^2 \\ &\qquad = \frac{1-T^2}{1+T^2} \end{aligned}}$$

Q Given that $\tan\alpha = \dfrac{16}{63}$ find $\sin 2\alpha$?

$$\sin 2\alpha = \frac{2\tan\alpha}{1+\tan^2\alpha} = \frac{\dfrac{32}{63}}{1+\dfrac{16^2}{63^2}}$$

$$= \frac{32}{\cancel{63}} \times \frac{63^{\cancel{2}}}{65^2} = \frac{2016}{4225}$$

Q P.T.

$$\frac{(\sin^2 A - \sin^2 B)}{\sin A\cdot\cos A - \sin B\cos B} = \tan(A+B)!$$

LHS $\dfrac{\sin(A+B)\cdot\sin(A-B)}{\dfrac{1}{2}[2\sin A\cos A - 2\sin B\cos B]}$

$$\frac{2\sin(A+B)\cdot\sin(A-B)}{\sin 2A - \sin 2B}$$

$$\frac{\cancel{2}\sin(A+B)\cdot\cancel{\sin(A-B)}}{\cancel{2}\cos(A+B)\cdot\cancel{\sin(A-B)}} = \tan(A+B)$$

RHS

Nishant Jindal

Q. Special

$\boxed{E = \cos^2\frac{\pi}{7} + \cos^2\frac{2\pi}{7} + \cos^2\frac{3\pi}{7} \text{ then } E = ?}$ $\frac{5}{4}$

4Qs / याद

$E = \frac{1+\cos\frac{2\pi}{7}}{2} + \frac{1+\cos\frac{4\pi}{7}}{2} + \frac{1+\cos\frac{6\pi}{7}}{2}$

$2E = 3 + \left(\cos\frac{2\pi}{7} + \cos\frac{4\pi}{7} + \cos\frac{6\pi}{7}\right) \Rightarrow 2E = 3 - \frac{1}{2}$

$\Rightarrow 2E = \frac{5}{2} \Rightarrow \boxed{E = \frac{5}{4}}$

$S = \cos 2\theta + \cos 4\theta + \cos 6\theta \} \times 2\sin\theta$

$2S\sin\theta = 2\sin\theta\cos 2\theta + 2\sin\theta\cdot\cos 4\theta + 2\sin\theta\cos 6\theta$

$= \cancel{\sin(3\theta)} + \sin(-\theta) + \cancel{\sin(5\theta)} + \cancel{\sin(-3\theta)}$

$+ \sin(7\theta) + \cancel{\sin(-5\theta)}$

$2S\sin\frac{\pi}{7} = \sin 7\theta - \sin\theta = \sin 7\times\frac{\pi}{7} - \sin\frac{\pi}{7}$

$\rightarrow 2S\cancel{\sin\frac{\pi}{7}} = 0 - \cancel{\sin\frac{\pi}{7}} \longrightarrow S = -\frac{1}{2}$

Ans $= 2\sin\frac{x}{2}\cdot\sin\frac{5x}{2} = \cos(2x) - \cos(3x)$

Q. Special / याद

Simplify: $\frac{\cos 7x - \cos 8x}{1 + 2\cos 5x} = \frac{2\sin\frac{x}{2}\cdot\sin\frac{5x}{2}\{\cancel{1+2\cos 5x}\}}{(\cancel{1+2\cos 5x})}$

Nr: $\cos 7x - \cos 8x$

$= +2\sin\left(\frac{15x}{2}\right)\cdot\sin\left(+\frac{x}{2}\right)$

$= 2\sin\frac{x}{2}\cdot\sin\left(3\cdot\left(\frac{5x}{2}\right)\right)$ $\quad \sin 3\theta = 3\sin\theta - 4\sin^3\theta$

$= 2\sin\frac{x}{2}\left\{3\sin\frac{5x}{2} - 4\sin^3\frac{5x}{2}\right\}$

$= 2\sin\frac{x}{2}\cdot\sin\frac{5x}{2}\left\{3 - 4\sin^2\frac{5x}{2}\right\}$

$= 2\sin\frac{x}{2}\cdot\sin\frac{5x}{2}\left\{3 - \cancel{4}^{2}\left(\frac{1-\cos 5x}{\cancel{2}}\right)\right\}$

$= 2\sin\frac{x}{2}\cdot\sin\frac{5x}{2}\{1 + 2\cos 5x\}$

Nishant Jindal

$\sin C + \sin D$

Q3 $X = \left\{\sin\left(\theta + \frac{7\pi}{12}\right) + \sin\left(\theta - \frac{\pi}{12}\right)\right\} + \sin\left(\theta + \frac{3\pi}{4}\right)$

$Y = \left\{\cos\left(\theta + \frac{7\pi}{12}\right) + \cos\left(\theta - \frac{\pi}{12}\right)\right\} + \cos\left(\theta + \frac{3\pi}{4}\right)$ then $\frac{X}{Y} - \frac{Y}{X} = ?$

$\hookrightarrow \cos C + \cos D.$

$$X = 2\sin\left(\frac{2\theta + \frac{7\pi}{12} - \frac{\pi}{12}}{2}\right)\cos\left(\frac{\frac{7\pi}{12} + \frac{\pi}{12}}{2}\right) + \sin\left(\theta + \frac{\pi}{4}\right)$$

$$= 2\sin\left(\theta + \frac{\pi}{4}\right)\cos\left(\frac{4\pi}{12}\right) + \sin\left(\theta + \frac{\pi}{4}\right) = \cancel{2}\sin\left(\theta + \frac{\pi}{4}\right) \times \frac{1}{\cancel{2}} + \sin\left(\theta + \frac{\pi}{4}\right) = 2\sin\left(\theta + \frac{\pi}{4}\right)$$

$$Y = 2\cos\left(\frac{2\theta + \frac{7\pi}{12} - \frac{\pi}{12}}{2}\right)\cos\left(\frac{\frac{7\pi}{12} + \frac{\pi}{12}}{2}\right) + \cos\left(\theta + \frac{\pi}{4}\right)$$

$$= \cancel{2}\cos\left(\theta + \frac{\pi}{4}\right)\cancel{\cos\left(\frac{4\pi}{12}\right)} + \cos\left(\theta + \frac{\pi}{4}\right) = 2\cos\left(\theta + \frac{\pi}{4}\right)$$

$$\frac{X}{Y} - \frac{Y}{X} = \frac{\cancel{2}\sin\left(\theta + \frac{\pi}{4}\right)}{\cancel{2}\cos\left(\theta + \frac{\pi}{4}\right)} - \frac{\cancel{2}\cos\left(\theta + \frac{\pi}{4}\right)}{\cancel{2}\sin\left(\theta + \frac{\pi}{4}\right)} = \tan\left(\theta + \frac{\pi}{4}\right) - \frac{1}{\tan\left(\theta + \frac{\pi}{4}\right)} = \frac{1 + \tan\theta}{1 - \tan\theta} - \frac{1 - \tan\theta}{1 + \tan\theta}$$

$$= \frac{4\tan\theta}{1 - \tan^2\theta} = 2\left(\frac{2\tan\theta}{1 - \tan^2\theta}\right) = \underline{2\tan 2\theta}$$

Nishant Jindal

half Angle

$22\frac{1}{2}^\circ / 7\frac{1}{2}^\circ$

$$\boxed{\cos\theta = \sqrt{\frac{1+\cos 2\theta}{2}} \;;\; \sin\theta = \sqrt{\frac{1-\cos 2\theta}{2}} \qquad \tan\theta = \frac{1-\cos 2\theta}{\sin 2\theta}}$$

Q $\tan\frac{\pi}{8} = ?$

$$\tan\frac{\cancel{180}^{\cancel{90}\,45}}{\cancel{8}_{\cancel{4}\,2}} = \tan 22\frac{1}{2}^\circ = ?$$

$$\tan 22\frac{1}{2}^\circ = \frac{1-\cos 45^\circ}{\sin 45^\circ}$$

$$= \frac{1 - \frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}} = \frac{\sqrt{2}-1}{1} = \sqrt{2}-1 = \cot 67.5^\circ$$

Q $\cos\frac{\pi}{8} = ?$

$$\cos\frac{\pi}{8} = \sqrt{\frac{1+\cos 45^\circ}{2}} = \sqrt{\frac{1+\frac{1}{\sqrt{2}}}{2}}$$

$$= \sqrt{\frac{\sqrt{2}+1}{2\sqrt{2}}}$$

Q $\cot\frac{\pi}{8} = ?$

$$\cot\frac{\pi}{8} = \frac{1}{\tan\frac{\pi}{8}} = \frac{1}{\sqrt{2}-1} = \frac{1}{\sqrt{2}-1} \times \frac{\sqrt{2}+1}{\sqrt{2}+1}$$

$$= \frac{\sqrt{2}+1}{2-1} = \sqrt{2}+1$$

$$\cot 22\frac{1}{2} = \sqrt{2}+1$$

Nishant Jindal

1) $\tan 0^\circ = 0$

2) $\tan 15^\circ = 2-\sqrt{3}$.

3) $\tan 22\tfrac{1}{2}^\circ = \sqrt{2}-1$

4) $\tan 30^\circ = \frac{1}{\sqrt{3}}$

5) $\tan 45^\circ = 1$

$\cos 15^\circ = \frac{\sqrt{3}+1}{2\sqrt{2}}$

6) $\tan 60^\circ = \sqrt{3}$

7) $\tan 75^\circ = 2+\sqrt{3}$.

8 $\tan 90^\circ \to \infty$

Q. $\tan 7\tfrac{1}{2}^\circ = \frac{1-\cos 15^\circ}{\sin 15^\circ}$

$\tan\theta = \frac{1-\cos 2\theta}{\sin 2\theta}$

Bde Bachho Ka Khel

Adv level.

$$= \frac{1-\frac{\sqrt{3}+1}{2\sqrt{2}}}{\frac{\sqrt{3}-1}{2\sqrt{2}}}$$

$$= \frac{2\sqrt{2}-(\sqrt{3}+1)}{(\sqrt{3}-1)} \times \frac{(\sqrt{3}+1)}{\sqrt{3}+1}$$

$$= \frac{(2\sqrt{6}+2\sqrt{2})-(\sqrt{3}+1)^2}{3-1}$$

$$= \frac{2\sqrt{6}+2\sqrt{2}-(3+1+2\sqrt{3})}{2}$$

$$= \frac{\cancel{2}\sqrt{6}+\cancel{2}\sqrt{2}-\cancel{4}^{2}-\cancel{2}\sqrt{3}}{\cancel{2}}$$

$\sqrt{6}-\sqrt{4}-\sqrt{3}+\sqrt{2}$

Q If $\cot \boxed{\frac{\pi}{24}} = \sqrt{a}+\sqrt{b}+\sqrt{c}+\sqrt{d}$ & $a>b>c>d$

then find $a-b+c-d=?$

$(6-4)+(3-2)=3$

$$\cot\frac{\pi}{24} = \cot 7\frac{1}{2}^\circ = \frac{1+\cos 15^\circ}{\sin 15^\circ}$$

$$= \frac{1+\frac{\sqrt{3}+1}{2\sqrt{2}}}{\frac{\sqrt{3}-1}{2\sqrt{2}}} = \frac{2\sqrt{2}+\sqrt{3}+1}{(\sqrt{3}-1)} \times \frac{\sqrt{3}+1}{(\sqrt{3}+1)}$$

$$= \frac{2\sqrt{6}+3+\sqrt{3}+2\sqrt{2}+\sqrt{3}+1}{2}$$

$$= \frac{2\sqrt{6}+2\sqrt{2}+2\sqrt{3}+4}{2} = \sqrt{6}+\sqrt{3}+\sqrt{2}+2$$

$$= \underset{a}{\sqrt{6}}+\underset{b}{\sqrt{4}}+\underset{c}{\sqrt{3}}+\underset{d}{\sqrt{2}}$$

<u>half angle.</u>

$$\cos\theta = \sqrt{\frac{1+\cos 2\theta}{2}}$$

$$\sin\theta = \sqrt{\frac{1-\cos 2\theta}{2}}$$

$$\tan\theta = \frac{1-\cos 2\theta}{\sin 2\theta}$$

$$\cot\theta = \frac{1+\cos 2\theta}{\sin 2\theta}$$

Nishant Jindal

Q $\cos 15^\circ = ?$

$$\cos\theta = \sqrt{\frac{1+\cos 2\theta}{2}}$$

$$\cos 15^\circ = \sqrt{\frac{1+\cos 30^\circ}{2}}$$

$$= \sqrt{\frac{1+\frac{\sqrt{3}}{2}}{2}} = \sqrt{\frac{2+\sqrt{3}}{4}}$$

$$= \frac{\sqrt{2+\sqrt{3}}}{2} \times \frac{\sqrt{2}}{\sqrt{2}}$$

$$= \frac{\sqrt{4+2\sqrt{3}}}{2\sqrt{2}} = \frac{\sqrt{(\sqrt{3})^2+1^2+2\sqrt{3}\cdot 1}}{2\sqrt{2}} = \frac{\sqrt{(\sqrt{3}+1)^2}}{2\sqrt{2}} = \frac{\sqrt{3}+1}{2\sqrt{2}}$$

$$\cos 15^\circ = \frac{\sqrt{3}+1}{2\sqrt{2}}$$

$$\boxed{\begin{array}{c} 7\frac{1}{2} \\ 22\frac{1}{2} \\ 67\frac{1}{2}^\circ \end{array}}$$

$\cos 15^\circ = \cos(45^\circ - 30^\circ)$

Nishant Jindal

$$\boxed{\begin{aligned}\sin^4\theta+\cos^4\theta &= 1-2\sin^2\theta\cdot\cos^2\theta\\ \sin^6\theta+\cos^6\theta &= 1-3\sin^2\theta\cdot\cos^2\theta\end{aligned}}$$

Q $\cos^4\frac{\pi}{8}+\cos^4\left(\frac{3\pi}{8}\right)+\cos^4\left(\frac{5\pi}{8}\right)+\cos^4\left(\frac{7\pi}{8}\right)=?$

Nr = 5, Dr's half = 4

Nr > Dr's half

$$+\left(\cos\left(\frac{8\pi-3\pi}{8}\right)\right)^4+\left(\cos\frac{8\pi-\pi}{8}\right)^4$$

$$\cos^4\left(\frac{\pi}{8}\right)+\cos^4\left(\frac{3\pi}{8}\right)+\left(\cos\left(\pi-\frac{3\pi}{8}\right)\right)^4+\left(\cos\left(\pi-\frac{\pi}{8}\right)\right)^4$$

$$\cos^4\left(\frac{\pi}{8}\right)+\cos^4\left(\frac{3\pi}{8}\right)+\left(-\cos\frac{3\pi}{8}\right)^4+\left(-\cos\frac{\pi}{8}\right)^4$$

$$\cos^4\left(\frac{\pi}{8}\right)+\cos^4\left(\frac{3\pi}{8}\right)+\cos^4\left(\frac{3\pi}{8}\right)+\cos^4\left(\frac{\pi}{8}\right)=2\left[\cos^4\frac{\pi}{8}+\cos^4\left(\frac{3\pi}{8}\right)\right]=$$

$$2\left[\cos^4\frac{\pi}{8}+\left(\cos\left(\frac{4\pi-\pi}{8}\right)\right)^4\right]$$

$$2\left[\cos^4\frac{\pi}{8}+\left(\cos\left(\frac{\pi}{2}-\frac{\pi}{8}\right)\right)^4\right]$$

$$\Rightarrow 2\left[\cos^4\left(\frac{\pi}{8}\right)+\sin^4\frac{\pi}{8}\right]$$

$$=2\left[1-2\sin^2\left(\frac{\pi}{8}\right)\cdot\cos^2\frac{\pi}{8}\right]$$

$$=2\left[1-\left(\frac{4}{2}\sin^2\left(\frac{\pi}{8}\right)\cos^2\left(\frac{\pi}{8}\right)\right)\right]$$

$$=2\left[1-\frac{1}{2}\left(2\sin\frac{\pi}{8}\cdot\cos\frac{\pi}{8}\right)^2\right]$$

$$2\left[1-\frac{1}{2}\left(\sin\frac{\pi}{4}\right)^2\right]=2\left[1-\frac{1}{2}\times\frac{1}{2}\right]$$

$$=2\left[\frac{3}{4}\right]=\frac{3}{2}$$

Nishant Jindal

Q $\sin^6\frac{\pi}{8}+\sin^6\frac{3\pi}{8}+\sin^6\left(\frac{5\pi}{8}\right)+\sin^6\left(\frac{7\pi}{8}\right)=?$