Nishant Jindal

RELATION FUNCTION

2) **Algebric fxn** → Fxn which consist Sum, diffrence, Product, quotient Roots of a variable is called Algebric fxn. y is algebric fxn of x

$y^2 - x^2 = 0$, $y = |x|$, $y = \sqrt{x^4 + 5x^2} + 2x + 3$

$y = \sqrt{x} + \frac{1}{\sqrt[3]{x}}$

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RELATION FUNCTION

(3) Rational fxn / Irr. fxn.

Ratio of 2 Polynomial fxn is $\boxed{\text{Ratio}}$nal fxn.

$h(x) = \dfrac{f(x)}{g(x)}$; $h(x)$ is Rational fxn.

Ex:→ $h(x) = \dfrac{x^2-3x+2}{2x+1}$

Rational fxn

$2x+1 \neq 0$ for Define.

$h(x) = \dfrac{x^{2/5}-3x^{1/3}+2}{2x^{1/7}+2}$ Irr. fxn.

gndi gndi deg

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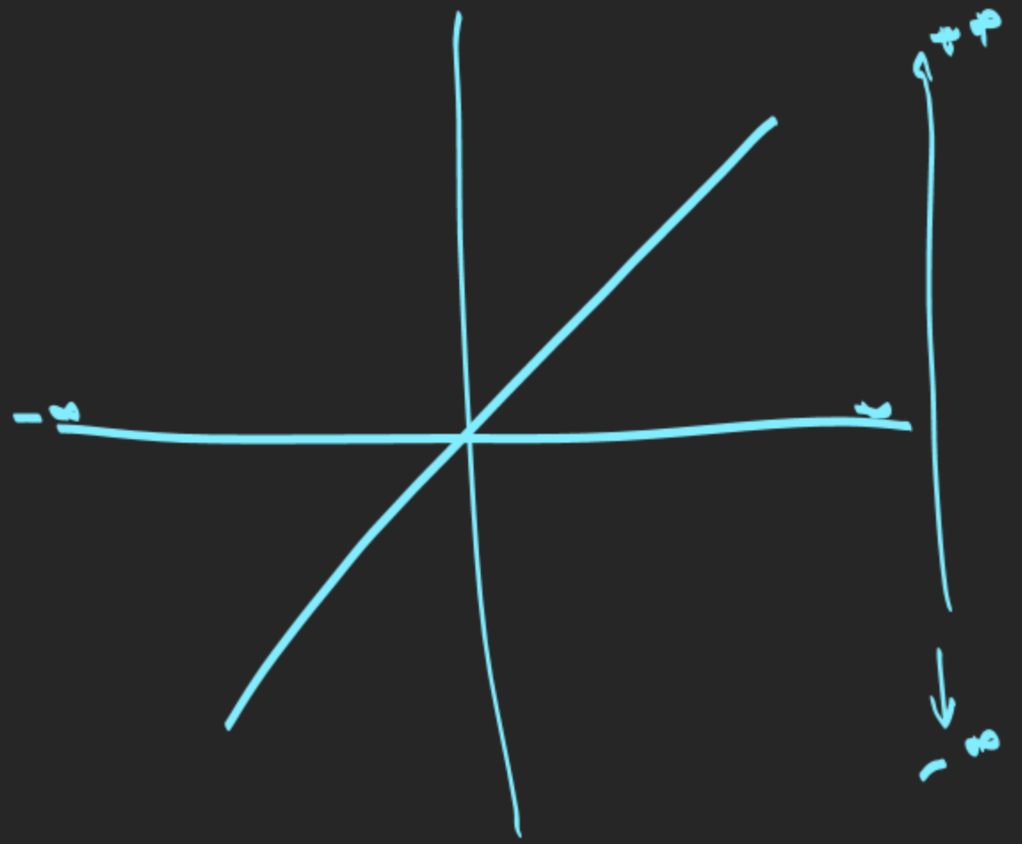
RELATION FUNCTION

(4) Identity fxn.

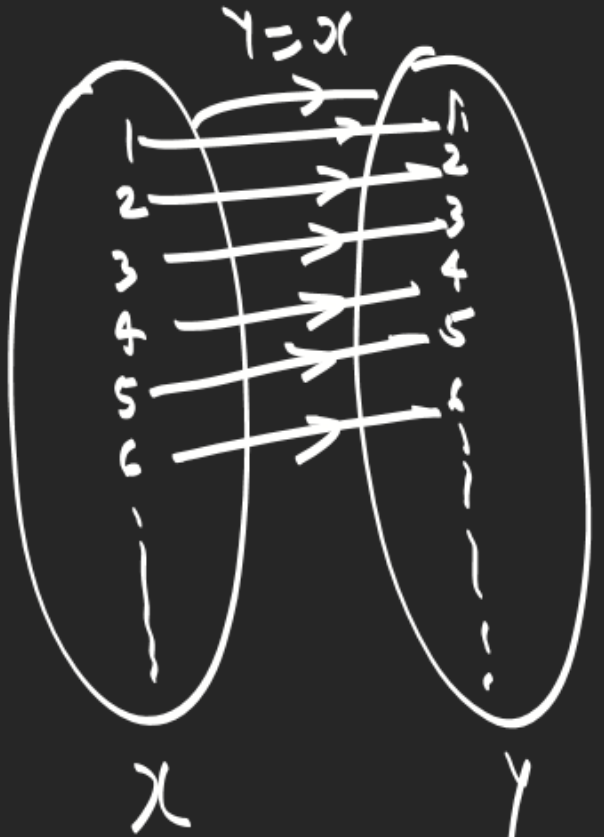
A) $f(x) = x$ OR $\boxed{y = x}$ is an Identity fxn.



R_f $y \in R$

D_f $x \in R$

B)



(C) It is Rep. by I_A or I_B or $\boxed{I}$

(5) Constant fxn ★

A) Any fxn in the form of $\boxed{f(x) = K}$ is constant fxn.

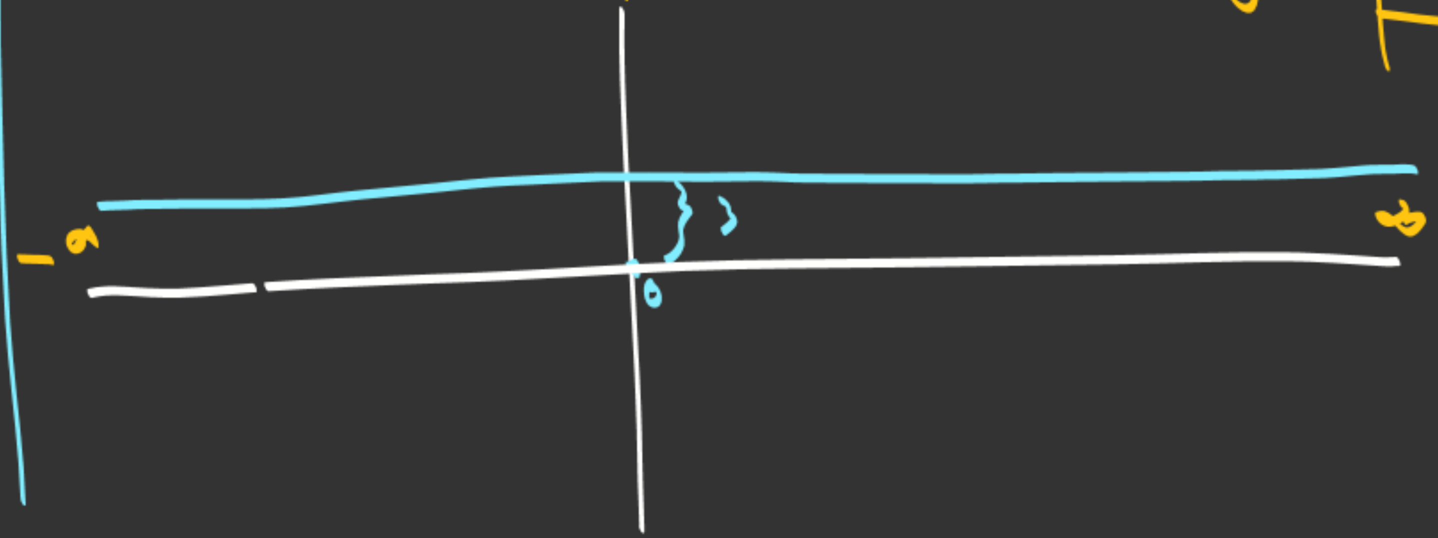
Ex:- $f(x) = a$, $f(x) = b$, $f(x) = c$, $f(x) = 2$, $f(x) = 0$, $f(x) = 3$, $f(x) = -\frac{1}{4}$

$\boxed{\text{Range} = y = \text{Answer} = \text{hieght}}$

(B) Graph of a constant fxn is Line $\parallel^{\text{rl}}$ to X Axis

$Y = 3$ / $\boxed{f(x) = 3} \rightarrow R_f \rightarrow \boxed{y \in \{3\}}$

$D_f \rightarrow \boxed{x \in R}$



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RELATION FUNCTION

Q $Y = |x-1| + |x-3|$ is a constant fxn? $\{x \in (1,3)\}$

$1 < x < 3 \rightarrow x = 2$

$|2-1| + |2-3|$

$+ \quad -$

$Y = (\cancel{x}-1) - (\cancel{x}-3)$

$Y = 2$ yes it is a Const fxn in $x \in (1,3)$

Q $Y = |x-1| + |x-2| + |x-3|$ is an Identity fxn? $\{2 < x < 3\}$

$2 < x < 3 \rightarrow x = 2.5$

$|2.5-1| + |2.5-2| + |2.5-3|$

$+ \quad + \quad -$

$Y = (x-\cancel{1}) + (\cancel{x}-\cancel{2}) - (\cancel{x}-\cancel{3})$

$Y = x$ yes it is an Identity fxn. in $x \in (2,3)$

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Q $f(x) = \dfrac{\cos^2 x + \sin^4 x}{\cos^4 x + \sin^2 x}$ find $f(2023)$?

$Nr = \cos^2 x + \sin^4 x$

$= \cos^2 x + (1 - \cos^2 x)^2$

$= \cos^2 x + 1 + \cos^4 x - 2\cos^2 x$

$= \cos^4 x + (1 - \cos^2 x)$

$= \cos^4 x + \sin^2 x = Dr.$

$f(x) = \dfrac{\cancel{\cos^2 x + \sin^4 x}}{\cancel{\cos^4 x + \sin^2 x}} \to 1$

$f(x) = 1$ ← ye to const fxn

$f(2023) = 1$

Q If $f(x) = \sin^2 x + \sin^2\left(x + \frac{\pi}{3}\right) + \cos x \cdot \cos\left(x + \frac{\pi}{3}\right)$

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Q If $f(x) = \sin^2 x + \sin^2\left(x+\frac{\pi}{3}\right) + \cos x \cdot \cos\left(x+\frac{\pi}{3}\right)$ & $\boxed{g\left(\frac{5}{4}\right) = 1}$ then $g \circ f(x) = ?$

AIEEE

$g \circ f(x) = g(f(x)) = g\left(\frac{5}{4}\right) = 1$ Ans

$$\boxed{\begin{aligned} &\sin^2 x + \sin^2\left(x+\frac{\pi}{3}\right) + \cos x \cdot \cos\left(x+\frac{\pi}{3}\right) = \frac{5}{4} \\ &\cos^2 x + \cos^2\left(x+\frac{\pi}{3}\right) - \cos x \cdot \cos\left(x+\frac{\pi}{3}\right) = \frac{3}{4} \end{aligned}}$$

Result

$f(x) = \sin^2 x + \left(\sin\left(x+\frac{\pi}{3}\right)\right)^2 + \cos x \cdot \cos\left(x+\frac{\pi}{3}\right)$

$= \sin^2 x + \left(\sin x \cdot \frac{1}{2} + \cos x \cdot \frac{\sqrt{3}}{2}\right)^2 + \cos x \cdot \left(\boxed{\cos x \cdot \frac{1}{2}} - \sin x \cdot \frac{\sqrt{3}}{2}\right)$

Hmmt

$= \sin^2 x + \left(\frac{\sin^2 x}{4} + \frac{3\cos^2 x}{4} + \cancel{2} \times \frac{1}{\cancel{2}} \times \frac{\sqrt{3}}{2} \sin x \cdot \cos x\right) + \frac{\cos^2 x}{2} - \frac{\sqrt{3}}{2} \sin x \cdot \cos x$

$f(x) = \sin^2 x\left(1 + \frac{1}{4}\right) + \cos^2 x\left(\frac{3}{4} + \frac{1}{2}\right) = \frac{5}{4}\left(\sin^2 x + \cos^2 x\right) = \frac{5}{4}.$

(6) **Exponantial fxn.** → 1) $f(x) = \boxed{a}^{x}$ is Exp. fxn.

2) $f(x) = (\text{Constant})^{\text{variable}}$ is an Exp. fxn.

Exp. 2^{x}, 2^{-x}, $2^{\frac{x^2}{2}}$, $2^{1/x}$, 10^{x}, π^{-x^2}, $(-2)^{x}$ ✗

(3) here $\boxed{\text{Constant} > 0}$

(4) Whenever we have variable deg. $\boxed{\text{Base must be} > 0}$

Q $y = x^{\boxed{x}}$ ← variable, find Dom?

$x > 0$

Dom:- $x \in (0, \infty)$

Q ✱ $f(x) = (\sin x)^{\boxed{x}}$ ← var. find Dom

$\sin x > 0 \rightarrow x \in (0, \pi) \Rightarrow \boxed{x \in (2n\pi, 2n\pi + \pi)}$

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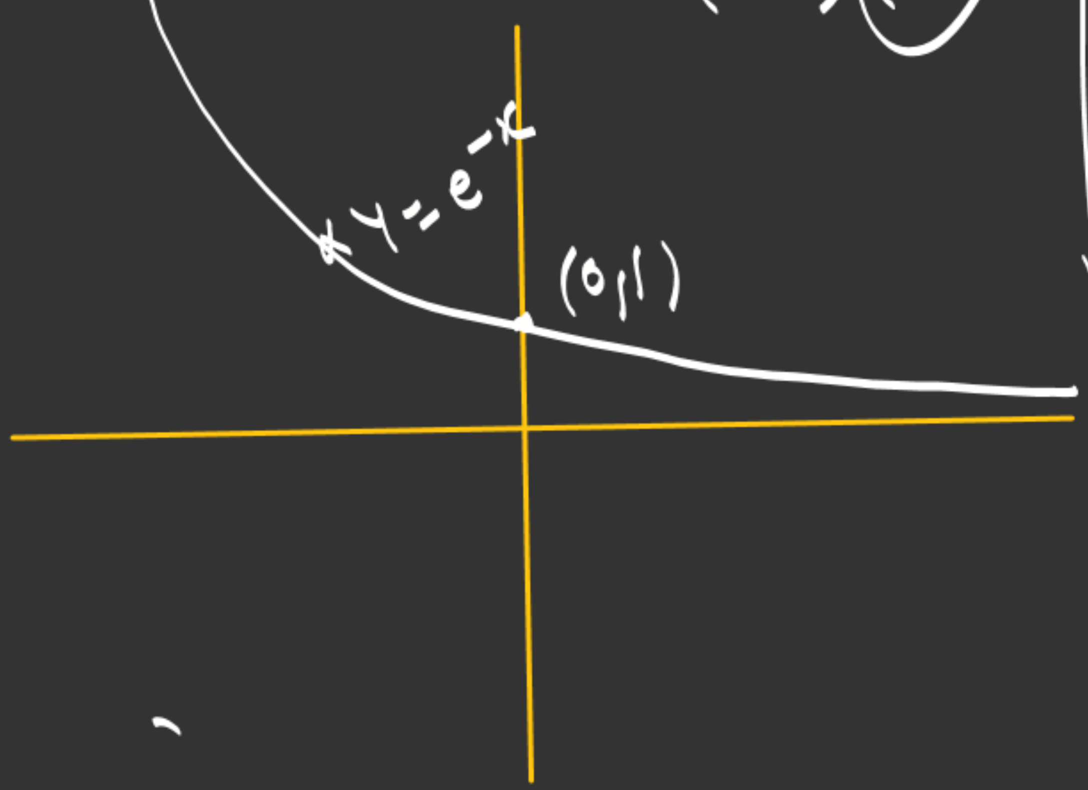
(5) $f(x) = a^x \longrightarrow a > 0 \ \& \ a \neq 1$

$\boxed{f(x) = 1^x = 1}$

$0 < a < 1$

$0 < \text{Base} < 1$

$(.2)^x, (.3)^x, \left(\frac{1}{2}\right)^x, \left(\frac{1}{e}\right)^x$



$a > 1$

$\text{Base} > 1$

$2^x, 3^x, \pi^x, 10^x, (2023)^x, \boxed{e^x}$

Ulta e mundi Cut

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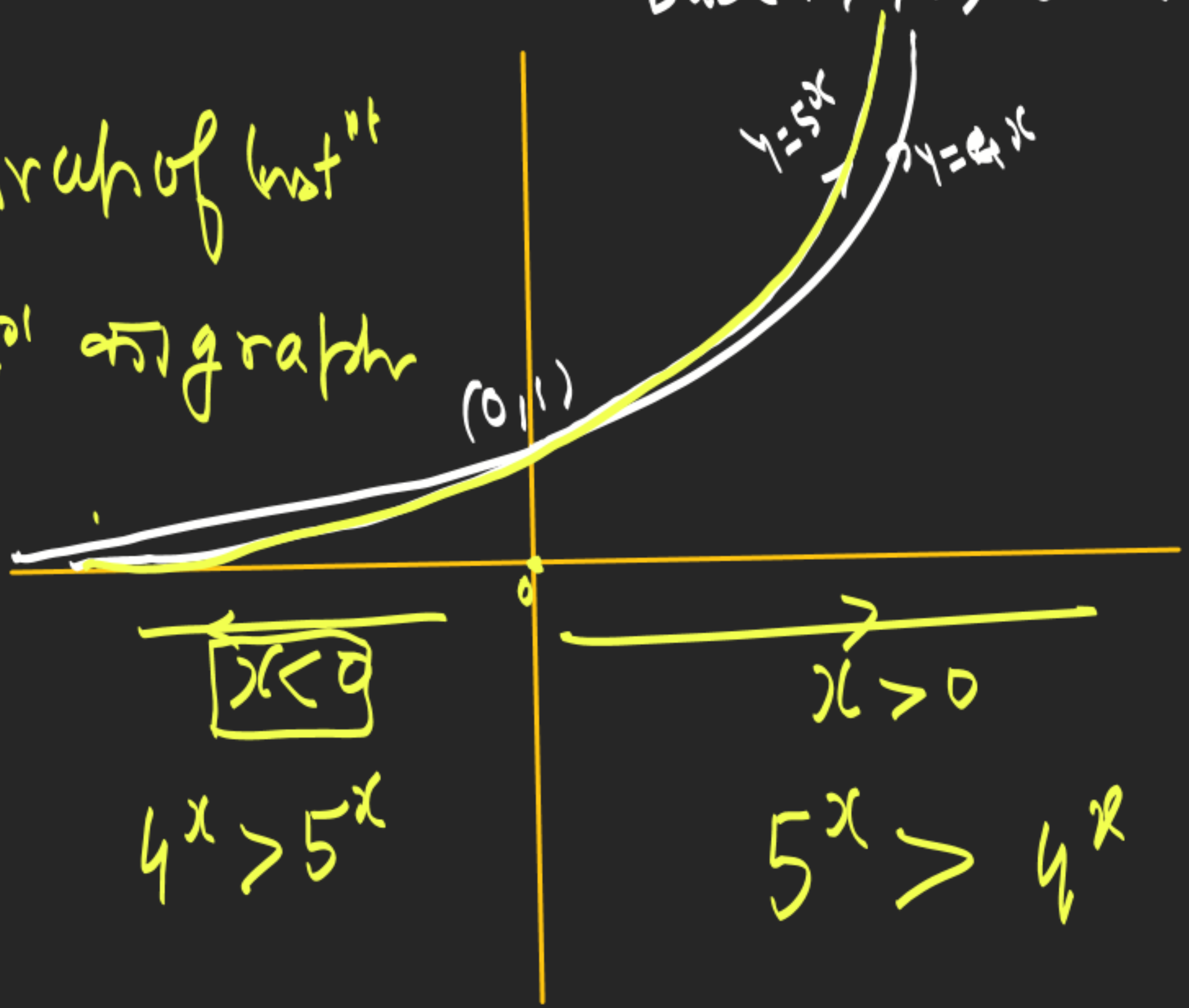
RELATION FUNCTION

$y=4^x$, $y=5^x$ → Base $4>1$ → Ulta e mundi Cut

① Graph of funct"

② e^x का graph



$4^x > 5^x$ $5^x > 4^x$

Dhyan

Q. $y=\sqrt{12^x-14^x}$ find D_f? $\sqrt{f(x)}$

$12^x-14^x \geq 0$

$12^x \geq 14^x$

Chh.   Bde

$x \leq 0$

D_f $\underline{x \in (-\infty, 0]}$

RELATION FUNCTION

(8) logarthimic fxn

1) $f(x) = \log_a x$ is log fxn.

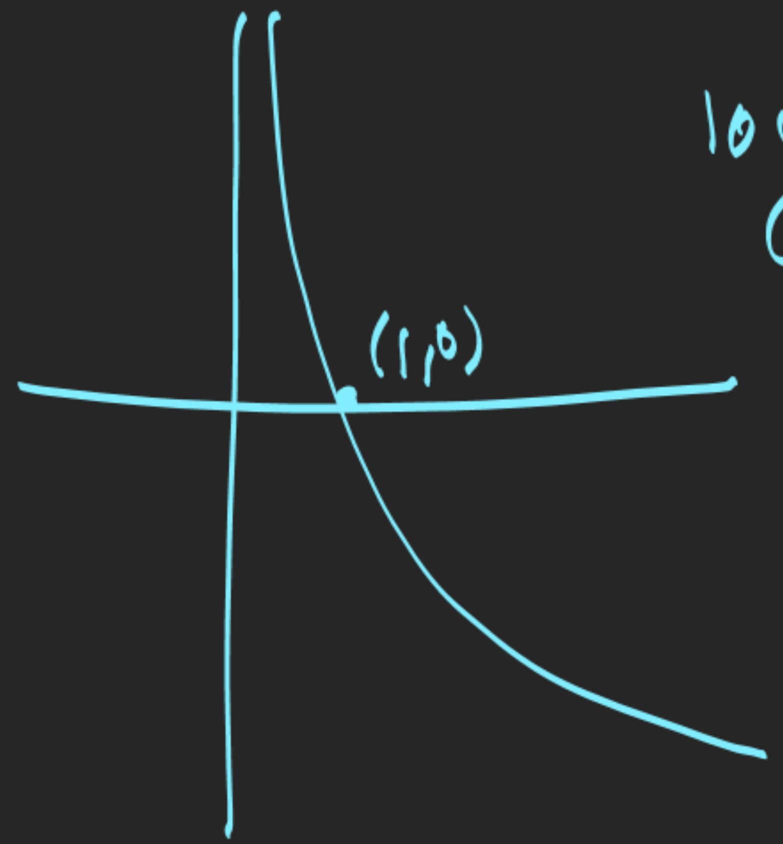
$a \neq 1,\ a > 0$

log = Ped KaTna.

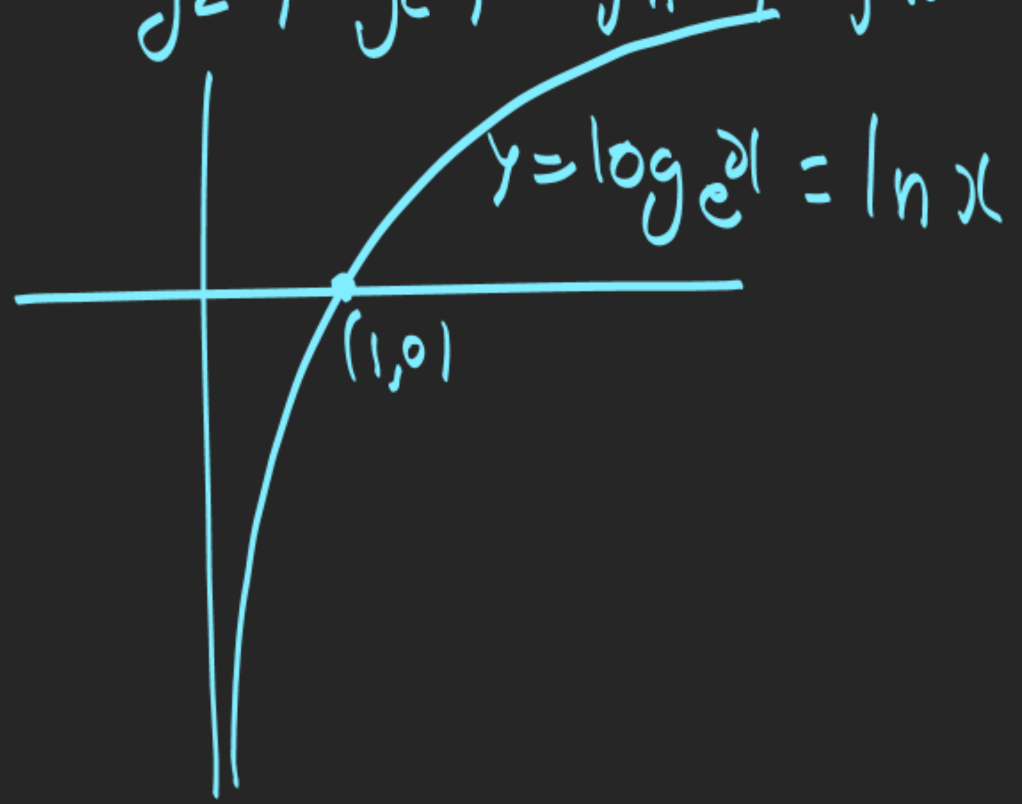
$0 < a < 1$

$a > 1$ (Popular)

$\log_{1/2} x,\ \log_{1/e} x$



$\log_2 x,\ \log_e x,\ \log_\pi x,\ \log_{10} x$ - -



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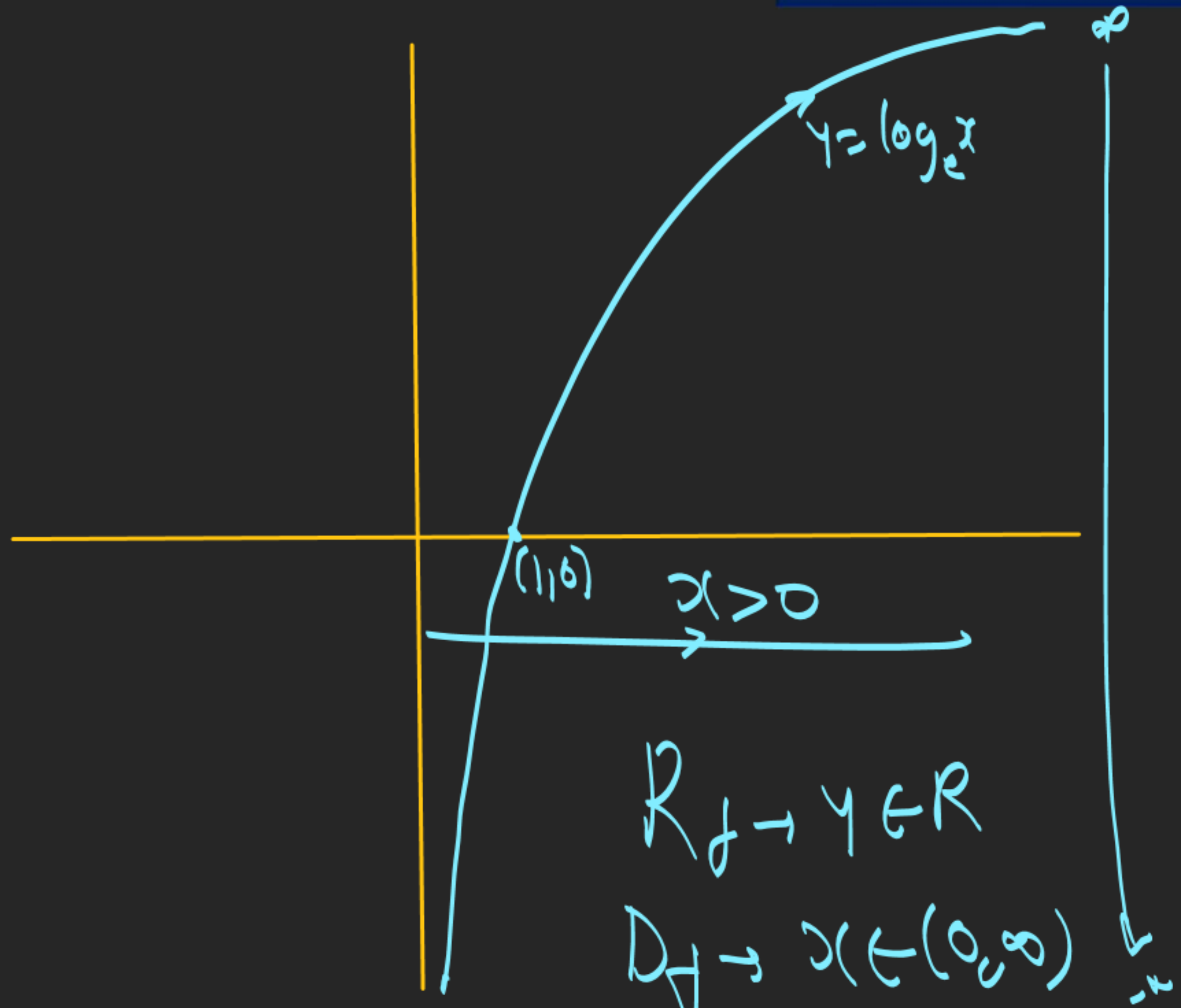
RELATION FUNCTION

Range = y = ht = Answer



$R_f \to y \in R$

$D_f \to x \in (0, \infty)$

(2) $y = \log_a f(x)$ Defined

3 Cond^n

$\boxed{\begin{matrix} a > 0 \\ f(x) > 0 \\ a \neq 1 \end{matrix}}$

$\boxed{\begin{matrix} \text{Base} > 0 \\ \text{Base} \neq 1 \\ f(x) > 0 \end{matrix}}$

Q $y = \log_x (x^2 - 1)$ find Dom? $x \in (1, \infty)$

$\boxed{x > 0}$ | $x^2 - 1 > 0$ | $\boxed{x \neq 1}$

$(x-1)(x+1) > 0$

$\boxed{x < -1} \cup \boxed{x > 1}$

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NO CUT ⇒ NO Solutions

Q $\left(\frac{9}{10}\right)^x = -x^2+x-3$ find $\boxed{\text{No. of Sol.}}$?

Graph बनाना पड़ेगा

$y = \left(\frac{9}{10}\right)^x$, $y = -x^2+x-3$

Short Notes

$\boxed{\text{No of Qs} = 200}$

Excersise

$Y = (.9)^x$

Base $= .9 < 1$

$Y = -x^2+x-3.$

$\boxed{a=-1}$, $b=1$, $c=-3$

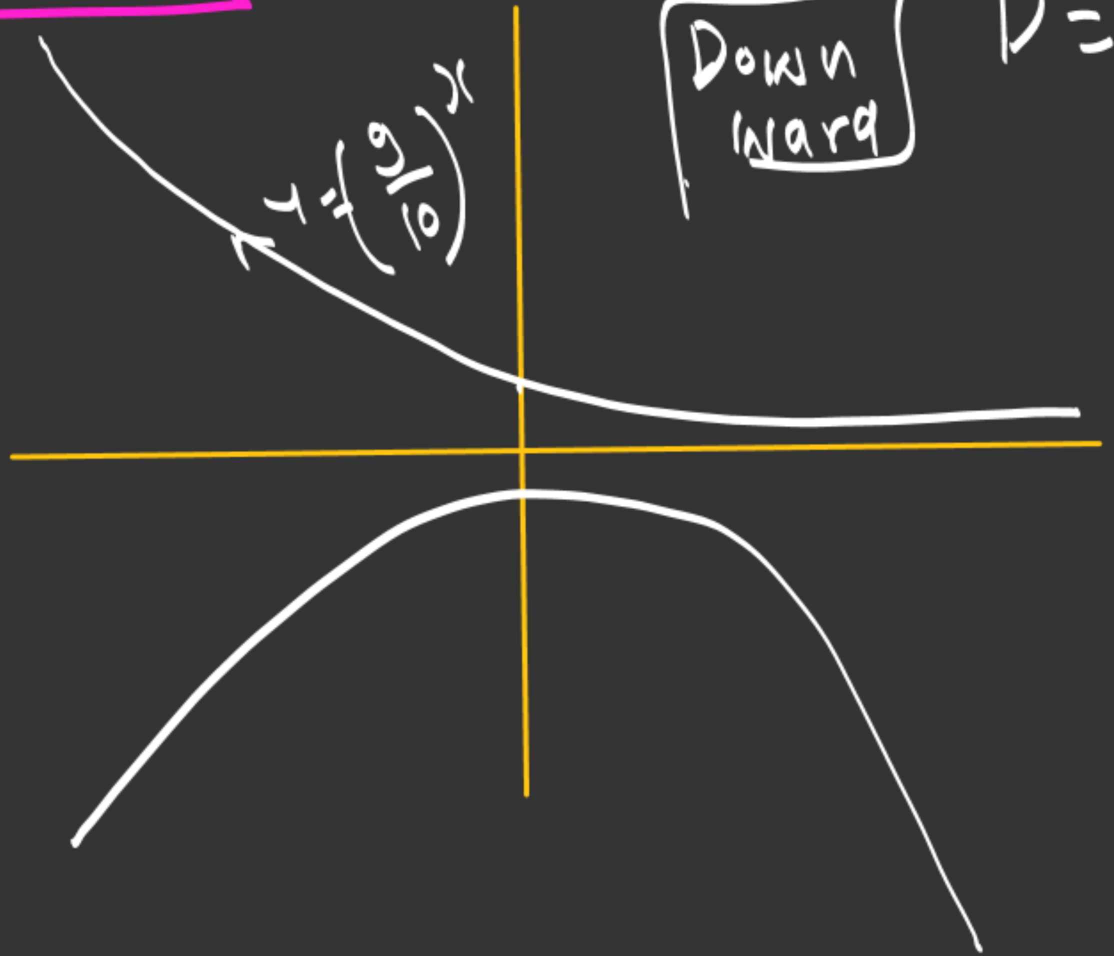
$\boxed{\text{Down ward}}$

$D = b^2 - 4ac$

$= (1)^2 - 4\times -1\times -3$

$= 1 - 12 = -11$

$= -ve$



Graph x Axis को cut नहीं करेगा

Q $f(x) = 6^x + 6^{-x} + 2^x + 2^{-x} + 3$ find Range?

$f(x) = \left(6^x + \frac{1}{6^x}\right) + \left(2^x + \frac{1}{2^x}\right) + 3$

$> 2 \quad > 2$

> 4

> 7

Answer

$y \in (7, \infty)$

Concept

Sum of +ve fxn & its Reciprocal is always gr. than or Equal to 2

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Q $y = \log_{10}(1+x^3)$ find Dom?

$\boxed{\text{Base} > 0,\ \text{Base} \neq 1,\ f(x) > 0}$

$10 > 0$

$10 \neq 1$

$1 + x^3 > 0$

$(1+x)(\underbrace{1+x^2-x}_{+}) > 0$

factorise नहीं होगा

$D = -ve$

$1 + x > 0$

$\boxed{x > -1}$

$x \in (-1, \infty)$

Q $y = \log_2(\log_3(\log_4 x))$ find Dom?

$2 > 0$

$2 \neq 1$

$\log_3 \log_4 x > 0$

$\log_4 x > 3^0$

$\log_4 x > 1$

$x > 4^1$

$\boxed{x > 4}$

$3 > 0$

$3 \neq 1$

$\log_4 x > 0$

$x > 4^0$

$\boxed{x > 1}$

$4 > 0$

$4 \neq 1$

$\boxed{x > 0}$

$\boxed{x \in (4, \infty)}$

RELATION FUNCTION

Q. If $\log_{12}27 = a$, then $\log_6 16 =$

(A) $2\left(\frac{3-a}{3+a}\right)$ (B) $3\left(\frac{3-a}{3+a}\right)$ (C) $4\left(\frac{3-a}{3+a}\right)$ (D) None of these

$$\frac{\log_3 27}{\log_3 12} = a \Rightarrow \frac{3}{\log_3(2^2\times 3)} = a \Rightarrow \frac{3}{\log_3 2^2 + \log_3 3^3} = a$$

$$\frac{3}{2\log_3 2 + 1} = a \Rightarrow 3 = 2\log_3 2\, a + a \Rightarrow \frac{3-a}{2a} = \log_3 2$$

$$\log_2 3 = \frac{2a}{3-a}$$

$$\frac{\log_2 16}{\log_2 6} = \frac{4}{\log_2 2 + \log_2 3} = \frac{4}{1+\frac{2a}{(3-a)}} = \frac{4(3-a)}{3-a+2a} = \frac{4(3-a)}{(3+a)}$$

RELATION FUNCTION

Q. Suppose that a and b are positive real numbers such that $\log_{27}a + \log_{9}b = \frac{7}{2}$ and $\log_{27}b + \log_{9}a = \frac{2}{3}$. Then the value of $a \cdot b$ is :

(A) 81 (B) 243 (C) 27 (D) 729

$\log_{3^3} b + \log_{3^2} a = \frac{2}{3}$

$\frac{1}{3}\log_3 b + \frac{1}{2}\log_3 a = \frac{2}{3}$

$\log_{3^3} a + \log_{3^2} b = \frac{7}{2}$

$\frac{1}{3}\log_3 a + \frac{1}{2}\log_3 b = \frac{7}{2}$

$\frac{1}{2}\log_3 a + \frac{1}{3}\log_3 b = \frac{2}{3}$

find a & b & $\boxed{a \times b}$

RELATION FUNCTION

Q. $\log_{(x-1)}(3) = 2$

(A) $\sqrt{3}$ (B) $1-\sqrt{3}$ (C) 1 (D) None of these

$3 = (x-1)^2$

$(x-1)^2 - (\sqrt{3})^2 = 0$

$(x-1-\sqrt{3})(x-1+\sqrt{3}) = 0$

$x = 1+\sqrt{3},\ 1-\sqrt{3}$

$\log_{(\cancel{1}+\sqrt{3}-\cancel{1})}(3) = 2$

$\log_{(\cancel{1}-\sqrt{3}-\cancel{1})}3 = 2$

$\log_{(-\sqrt{3})}3$

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RELATION FUNCTION

Q. $\log_2[\log_4(\log_{10} 16^4 + \log_{10} 25^8)]$ simplifies to

(A) an irrational ~~(B)~~ an odd prime (C) a composite ~~(D)~~ unity

$$\log_2\left[\log_4\left(\log_{10}(4^2)^4 + \log_{10}(5^2)^8\right)\right]$$

$$\log_2\left[\log_4\left(\log_{10}(4)^8 + \log_{10}(5)^{16}\right)\right]$$

$$\log_2\left[\log_4\left(\log_{10}(2)^{16} \times (5)^{16}\right)\right]$$

$$\log_2\left[\log_4\left(\log_{10}(10)^{16}\right)\right] = \log_2 \log_4 (16) = \log_2 2 = 1$$