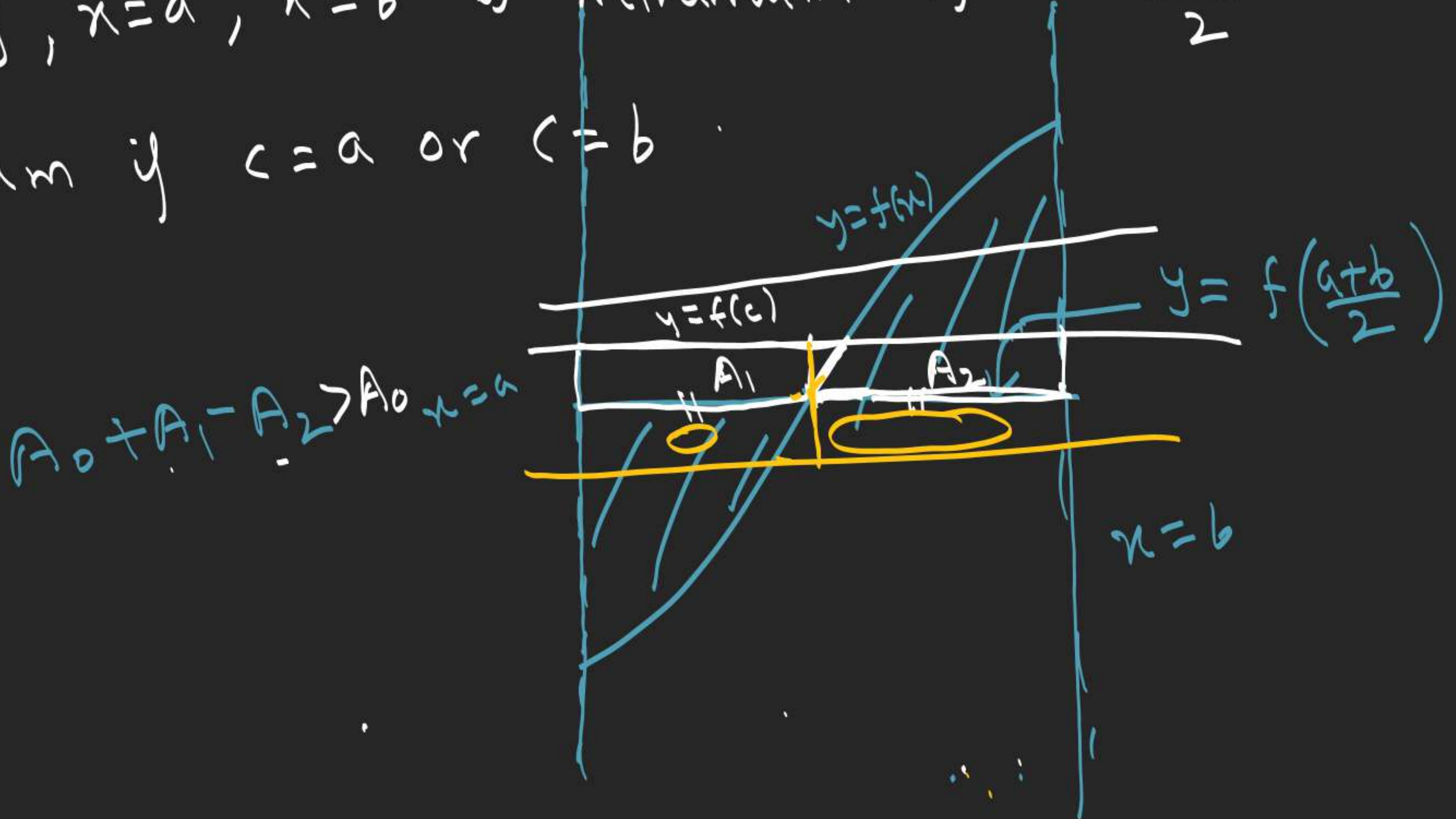
Nishant Jindal

Note ⇒ Let $f(x)$ be continuous and strictly monotonic in $[a,b]$, then area bounded by $y=f(x)$, $y=f(c)$, $c \in [a,b]$, $x=a$, $x=b$ is minimum if $c=\frac{a+b}{2}$ and maximum if $c=a$ or $c=b$.



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$y = x^2+2x-3$, $\boxed{y = kx+1}$

$x^2+(2-k)x-4=0$ — x_1, x_2

$$\int_{x_1}^{x_2}\Big((k-2)x+4-x^2\Big)\,dx$$

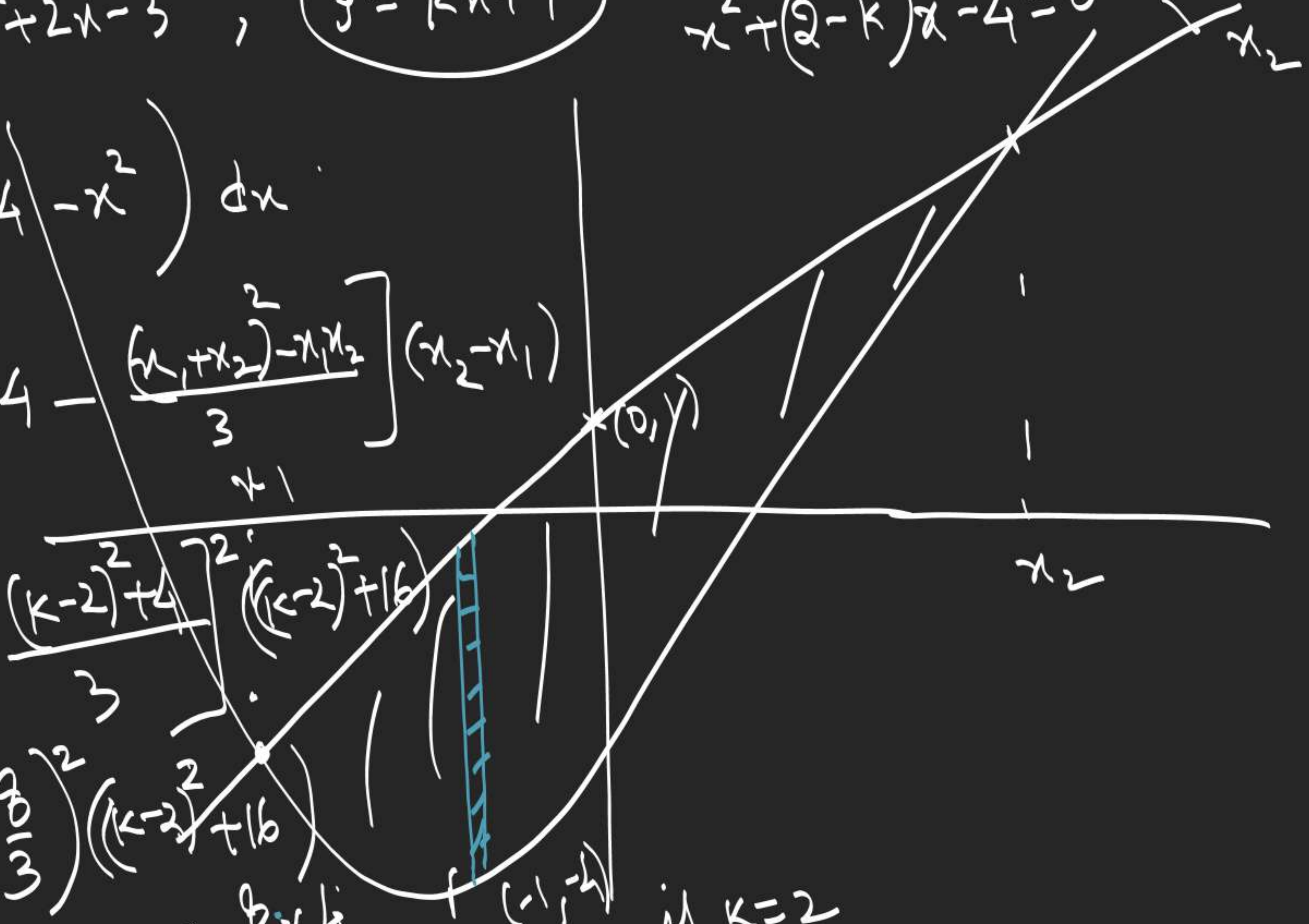
$$= \left[\frac{(k-2)}{2}(x_2+x_1)+4-\frac{(x_1+x_2)^2-x_1x_2}{3}\right](x_2-x_1)$$

$$A^2 = \left[\frac{(k-2)^2}{2}+4-\frac{(k-2)^2+4}{3}\right]^2\Big((k-2)^2+16\Big)$$

$$A^2 = \left(\frac{(k-2)^2}{6}+\frac{8}{3}\right)^2\Big((k-2)^2+16\Big)$$

$$A_{min} = \frac{8}{3}\times 4$$

if $\underline{k=2}$



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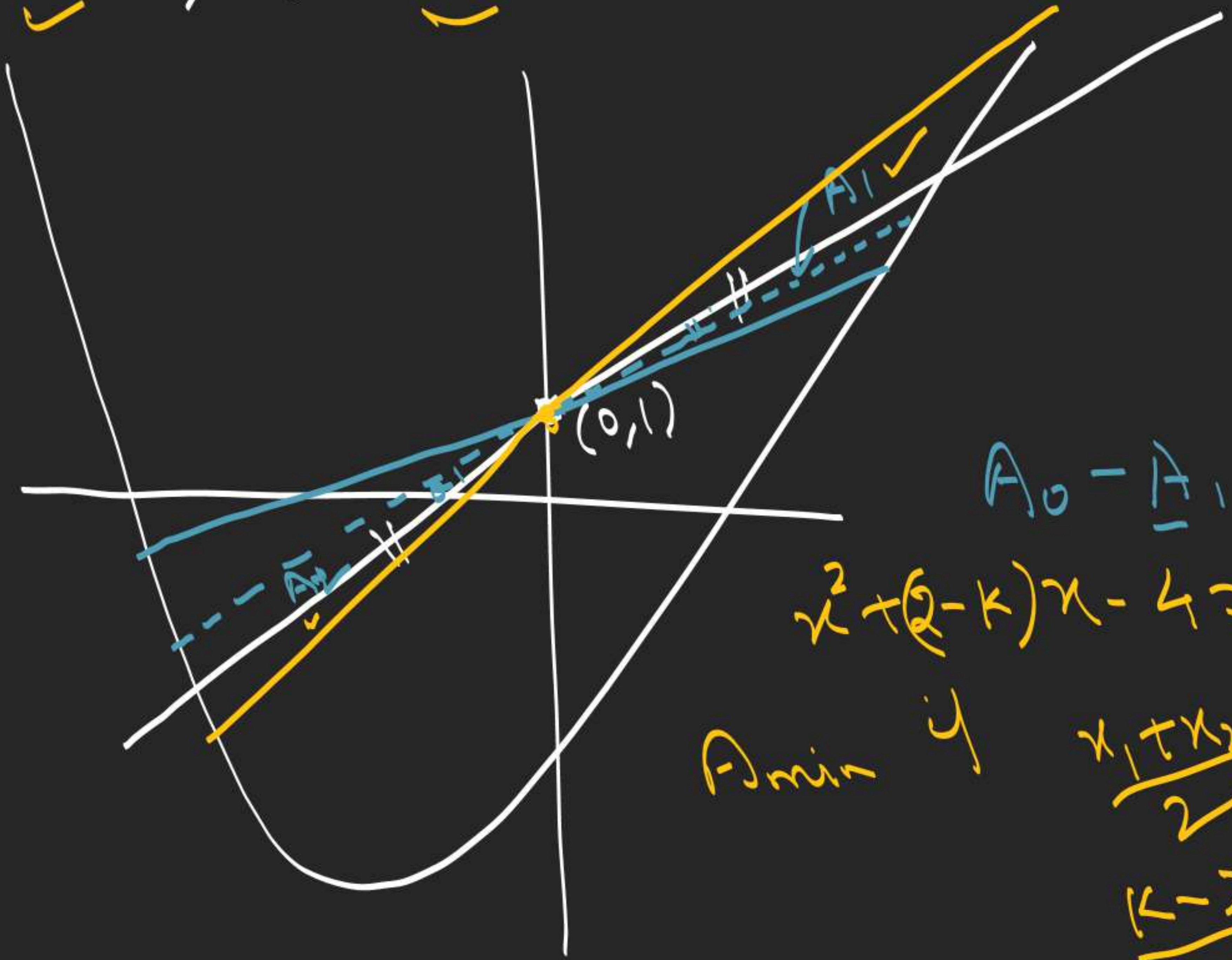
$y = x^2 + 2x - 3$, $y = kx + 1$



$A_0 - A_1 + A_2 > A_0$

$x^2 + (2-k)x - 4 = 0 \; < \begin{matrix} x_1 \\ x_2 \end{matrix}$

A_{min} if $\dfrac{x_1 + x_2}{2} = 0$

$\dfrac{k-2}{2} = 0$

$\boxed{k = 2}$

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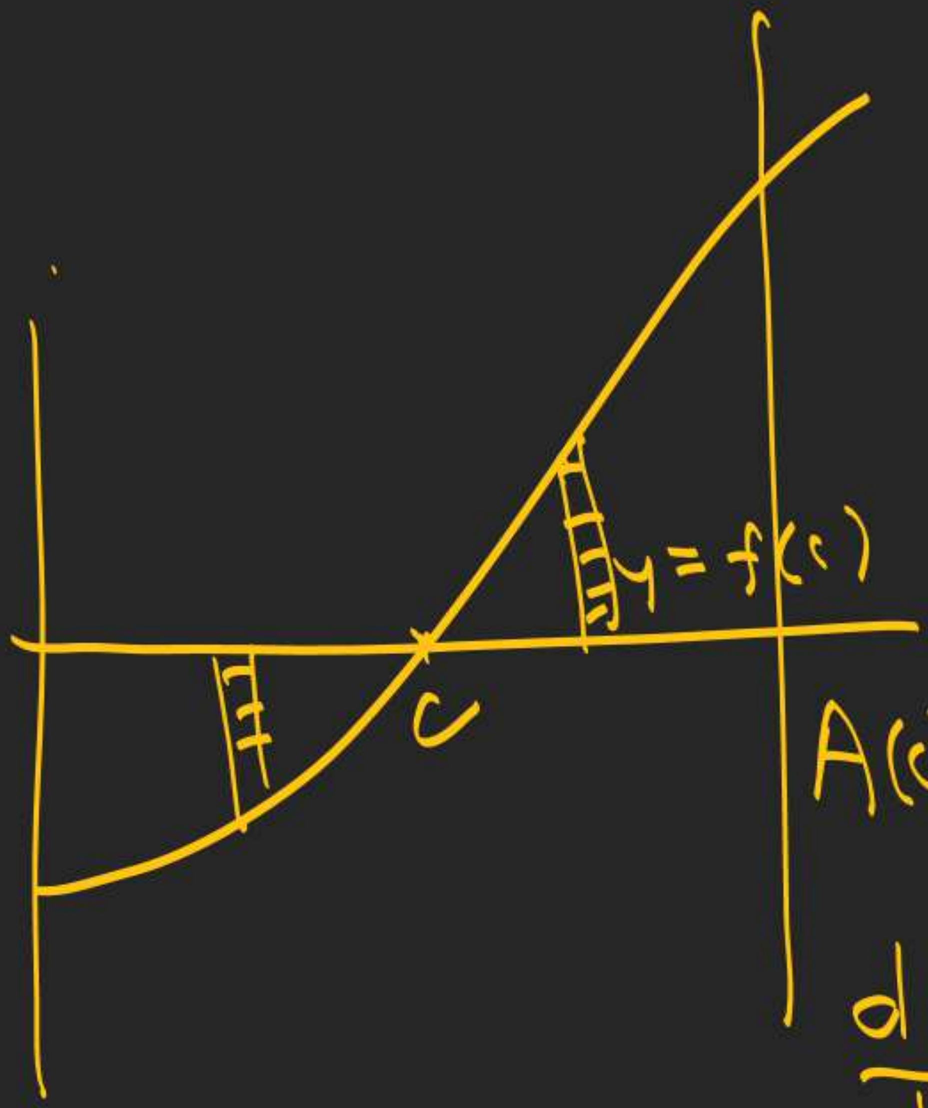

$$\int_a^c \left(f(c) - f(x)\right)dx + \int_c^b \left(f(x) - f(c)\right)dx$$

$$A(c) = f(c)\underbrace{(2c-a-b)}_{>0} - \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$$

$$\frac{d}{dc}A(c) = \cancel{f'(c)(2c-a-b)} + 2\cancel{f(c)} - \left(\cancel{f(c)} - 0\right)$$
$$+ \left(0 - \cancel{f(c)}\right)$$

$-$ $+$

$c = \frac{a+b}{2}$

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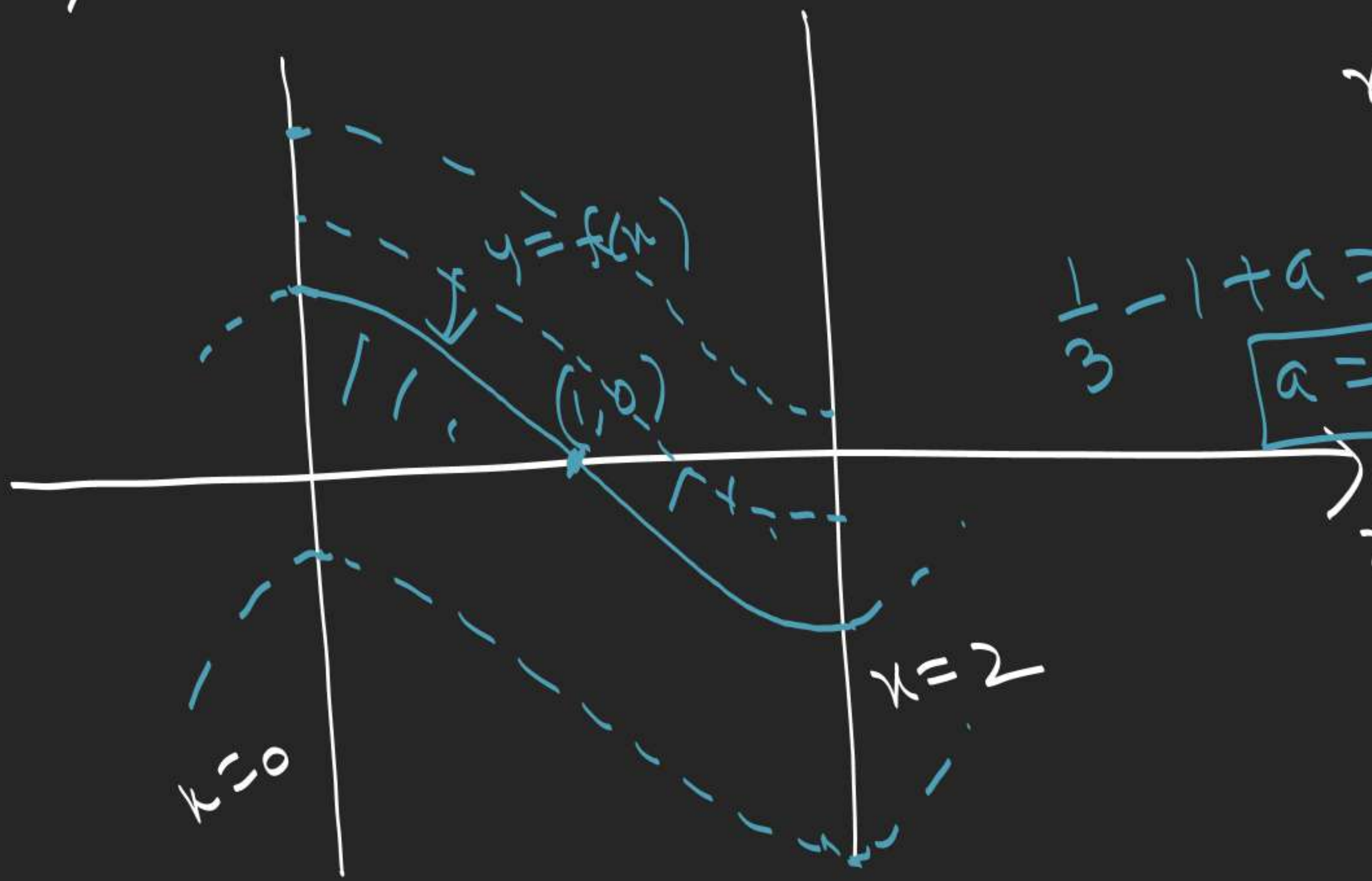
1: If the area bounded by $f(x) = \frac{x^3}{3} - x^2 + a$ and $x=0$, $x=2$ and x-axis is minimum, find 'a'.

$x^2 - 2x < 0 \quad x \in (0,2)$



$\frac{1}{3} - 1 + a = 0$

$\boxed{a = 2/3}$

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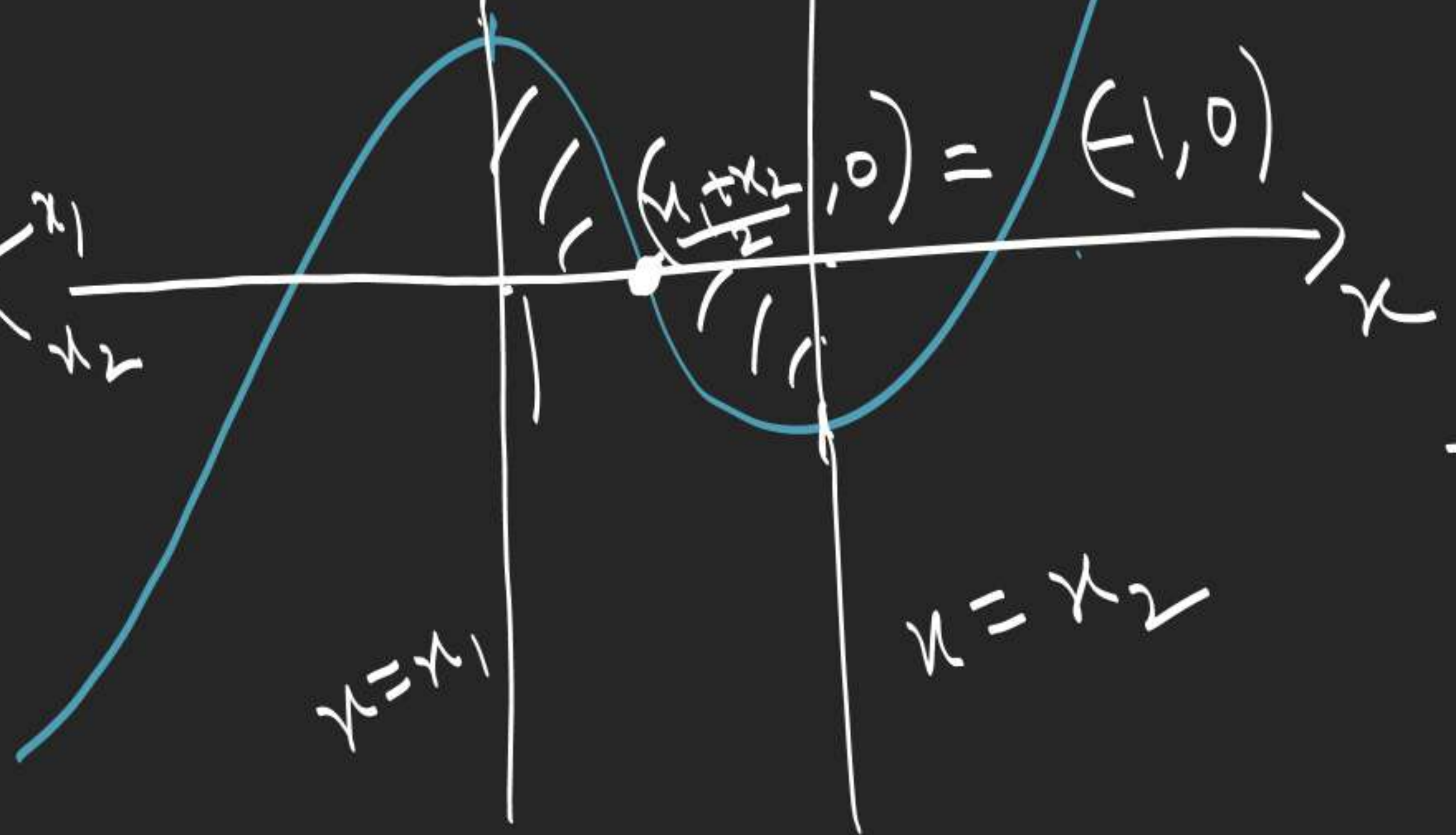
2. Find 'a' for which area bounded by x-axis, $f(x) = x^3 + 3x^2 + x + a$, straight lines which are parallel to y-axis and cut x-axis at the point of extremum of $y = f(x)$ is the least.



$3x^2 + 6x + 1 = 0 \begin{cases} x_1 \\ x_2 \end{cases}$

$-1 + 3 - 1 + a = 0$

$\boxed{a = -1}$

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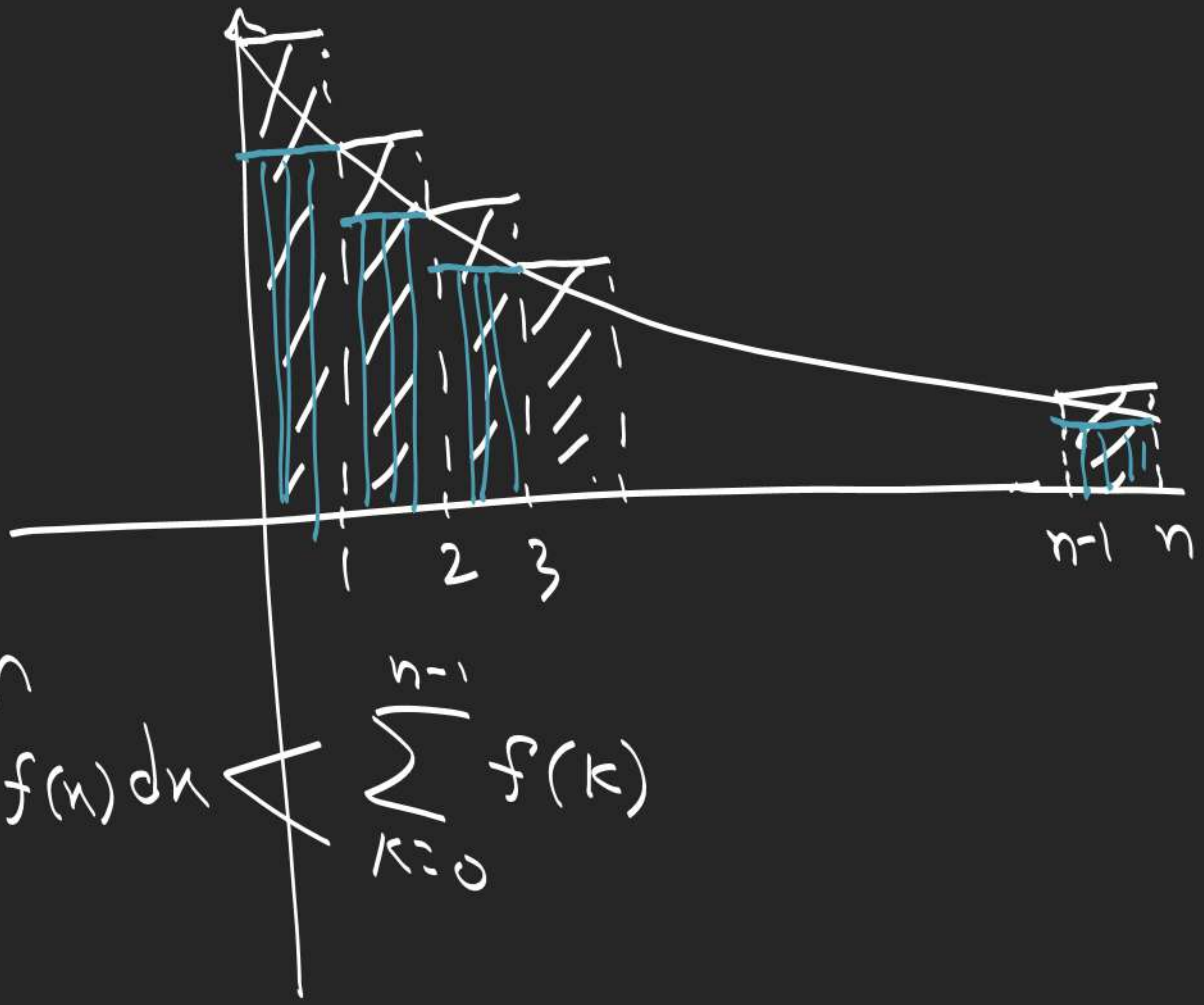
2 (b, c) , 5 (b).

$$f(x) = \frac{n}{n^2 + nx + x^2}$$

$\left(x + \frac{n}{2}\right)$ $x > 0$

$$\sum_{k=1}^{n} f(k) < \int_0^n f(x)\,dx < \sum_{k=0}^{n-1} f(k)$$



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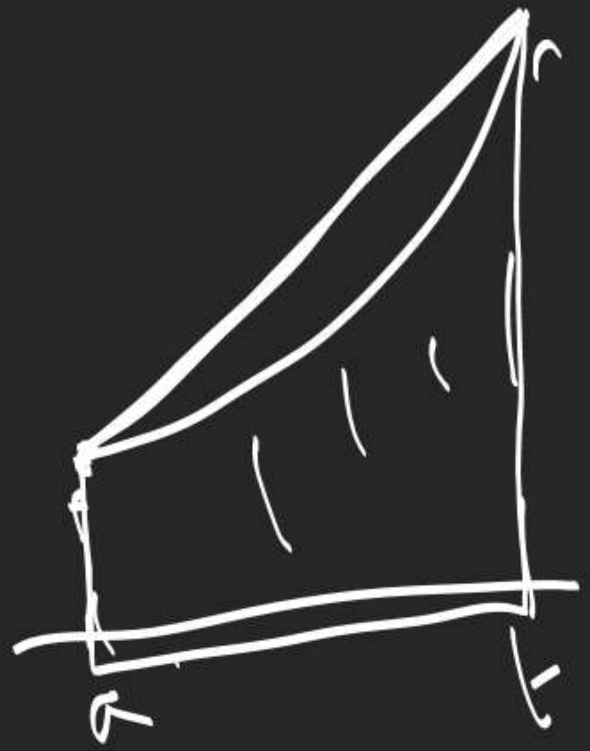
$$\lim_{t\to a} \frac{f(t) - \frac{1}{2}\left(f(t)+f(a)\right) - \frac{(t-a)}{2} f'(t)}{3(t-a)^2} = \lim_{t\to a} \frac{\left(f(t) - f(a)\right) - (t-a)f'(t)}{6(t-a)^2}$$



$$\frac{\cancel{f'(t)} - \cancel{f'(t)} - (t-a)f''(t)}{12(t-a)}$$

$$0 = \frac{-f''(a)}{12}$$