Nishant Jindal

Capacitor

(*) Energy analysis in Capacitive Ckt :→

↳ **General result**

**

$$\boxed{W_{battery} + W_{extagent} = \Delta U + heat}$$

(V_b = Potential of battery) $W_{battery} = \Delta q \cdot V_b$

$\Delta q = (q_f - q_i)$

if $(q_f < q_i) \Rightarrow \Delta q \rightarrow (-ve)$

$\Rightarrow$ (Work done on the battery)

if $q_f > q_i \Rightarrow \Delta q \rightarrow +ve$

$\Rightarrow$ Work done by the battery.

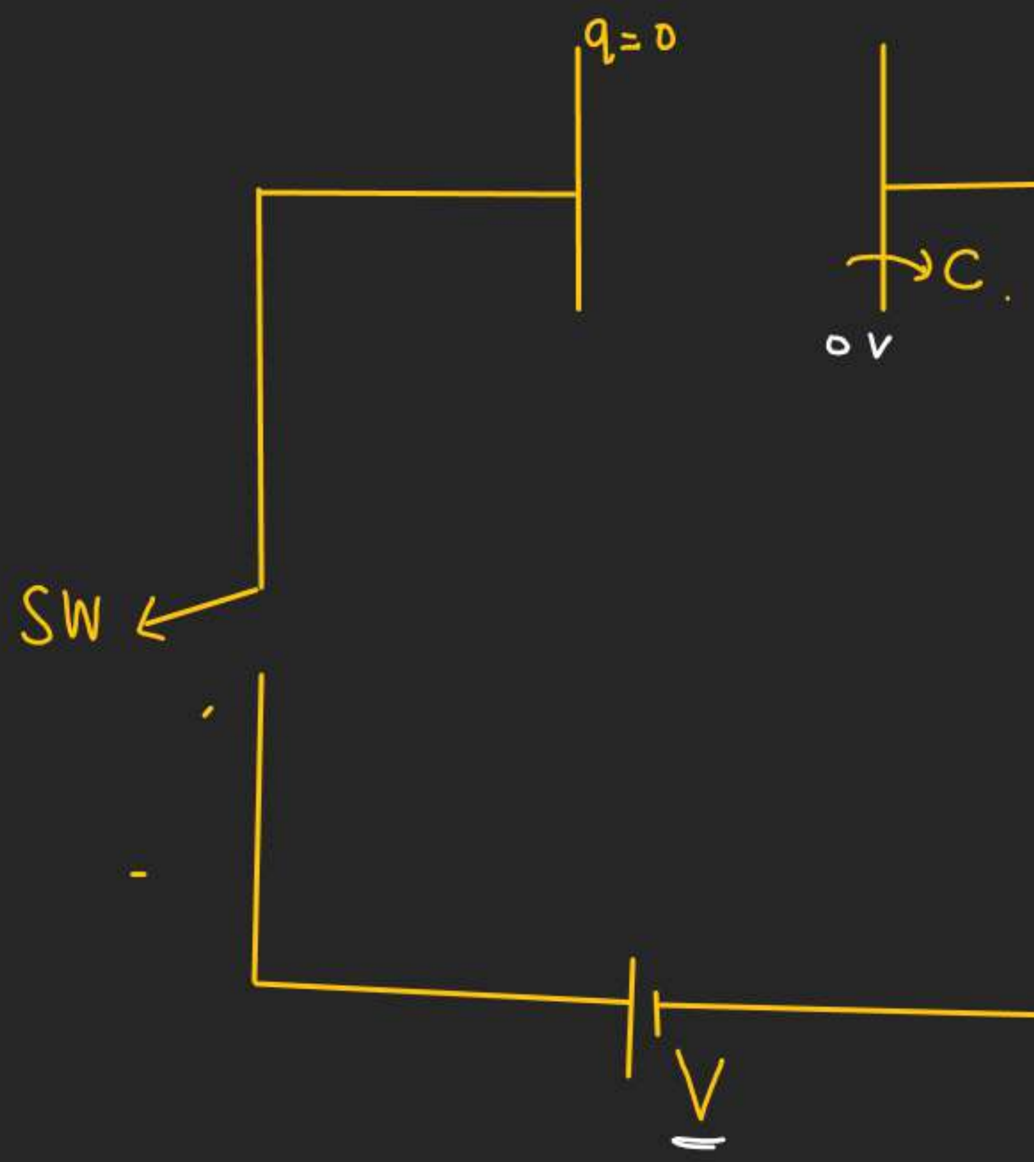
$\Delta U = U_f - U_i$

$U_f \rightarrow$ Final P.E Stored in Capacitor

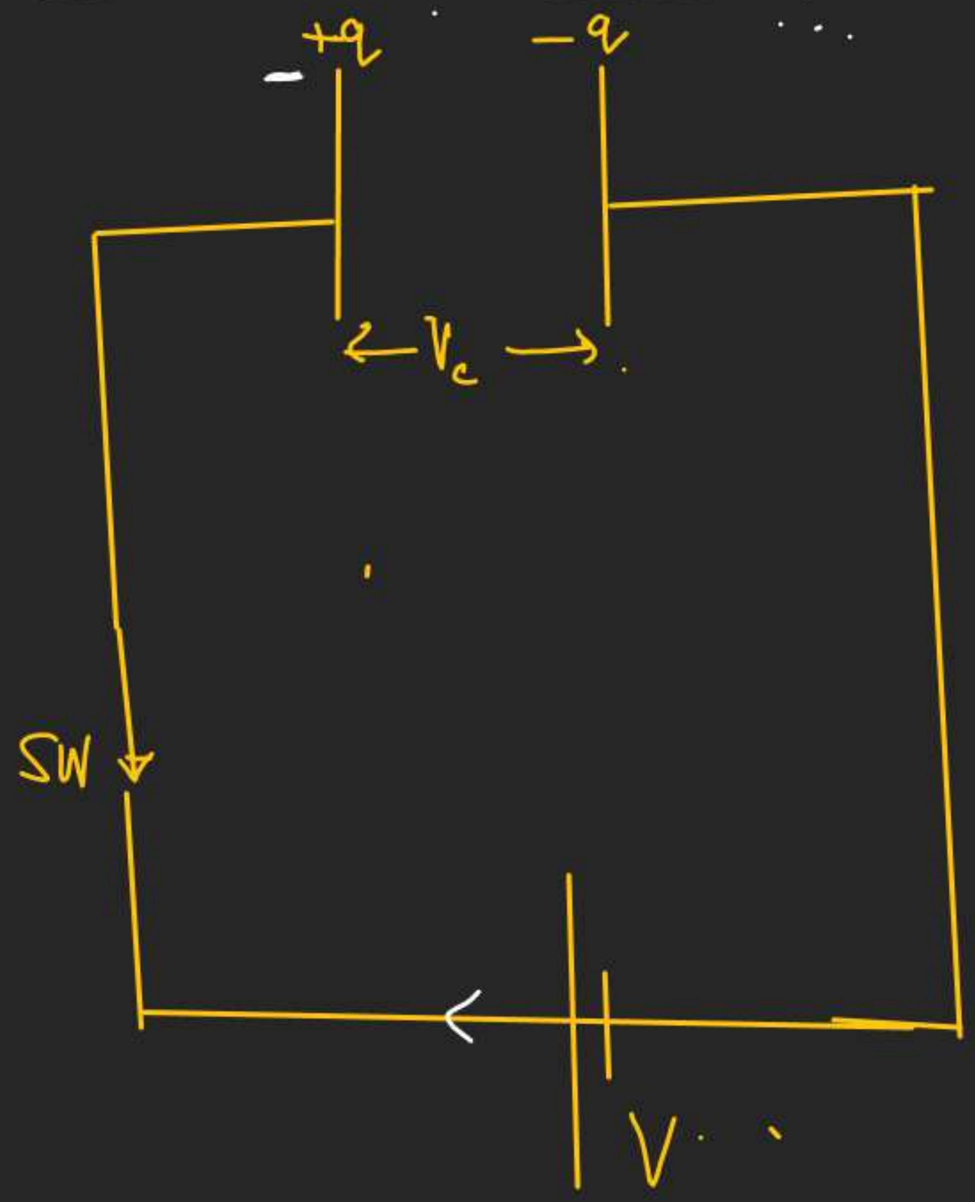
$U_i \rightarrow$ Initial P.E stored in Capacitor

Capacitor

(*) Charging of a neutral Capacitor:→.



At t=0 Switch is Closed



At the time when Capacitor is fully Charged:-

$$V_c = V$$

$$\frac{q}{c} = V$$

$$\boxed{q = cV}$$

$$W_b + \cancel{W_{ext\,agent}}^{\,0} = \Delta U + \text{heat}$$

$$W_b = \Delta q \cdot V_b$$

$$= (q_f - q_i)V_b$$

$$= cV^2$$

$q_f = q = cV$

$q_i = 0$

$V_b = V$

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$$\Delta U = U_f - U_i$$

$$\begin{cases} U_f = \frac{1}{2}CV^2 \\ U_i = 0 \end{cases}$$

$\Delta U =$

$$W_b = \Delta U + \text{heat}$$

$$\Downarrow$$

$$CV^2 = \frac{CV^2}{2} + \text{heat}$$

$$\text{heat} = \left[CV^2 - \frac{CV^2}{2}\right]$$

$$\boxed{\text{heat} = \frac{CV^2}{2} = \frac{q^2}{2C}}$$

(*) During Charging of a neutral Capacitor. Only half of the work done by battery is Stored in the form of P.E and. rest half of work done is dissipiated in the form of heat

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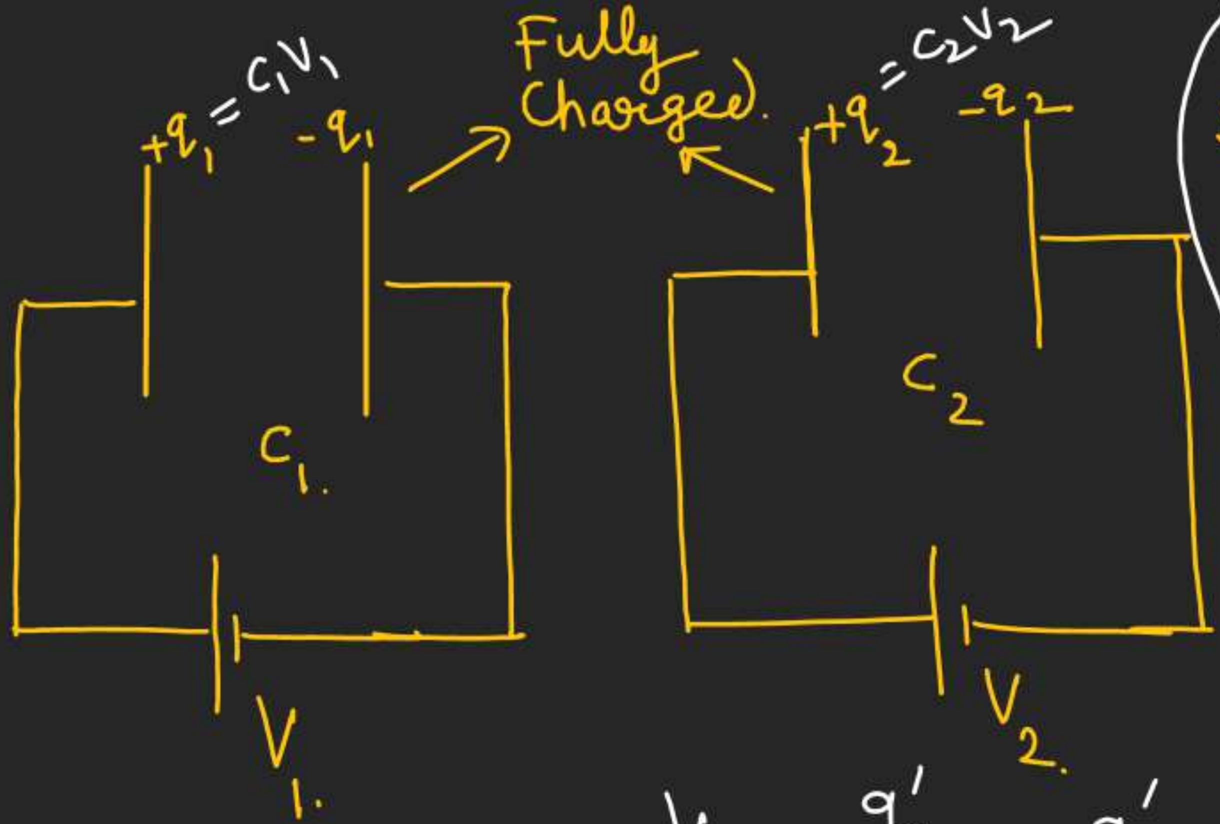
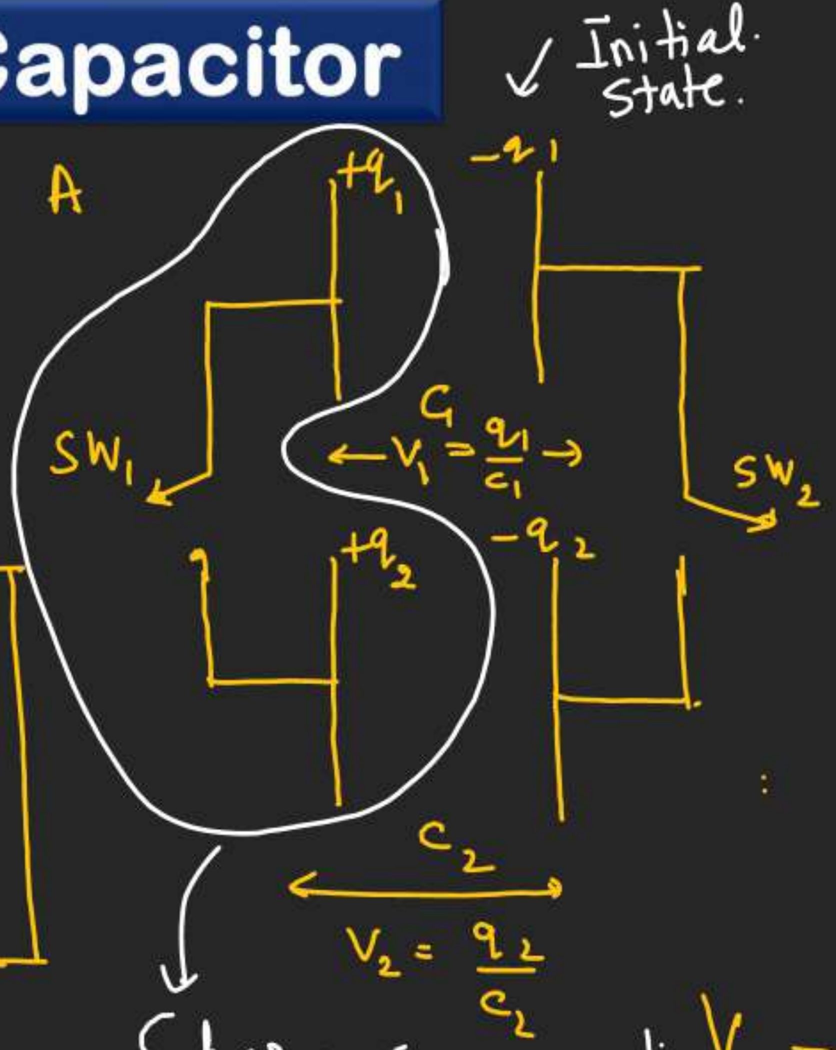
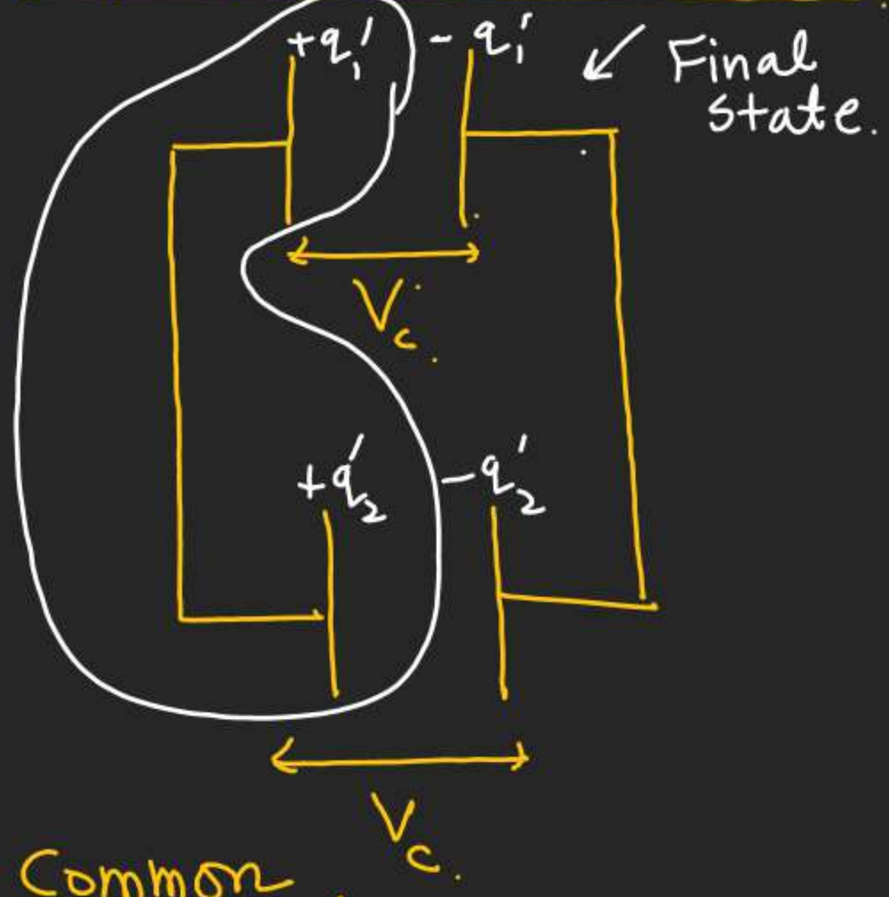
Inter Connection of two Charge Capacitors:-





At $t=0$, Sw_1 and Sw_2 Closed



Charge conservation

V_c = Common potential

$$V_c = \frac{q_1'}{C_1} = \frac{q_2'}{C_2}$$

$$\frac{q_1'}{q_2'} = \frac{C_1}{C_2} \quad -(1)$$

$$q_1 + q_2 = q_1' + q_2'$$

$$C_1V_1 + C_2V_2 = q_1' + \frac{C_2}{C_1}q_1'$$

$$\boxed{\begin{aligned} &\Rightarrow q_1' = \left(\frac{C_1V_1 + C_2V_2}{C_1 + C_2}\right) \times C_1 \\ &\Rightarrow q_2' = \left(\frac{C_1V_1 + C_2V_2}{C_1 + C_2}\right) \times C_2 \end{aligned}}$$

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Capacitor

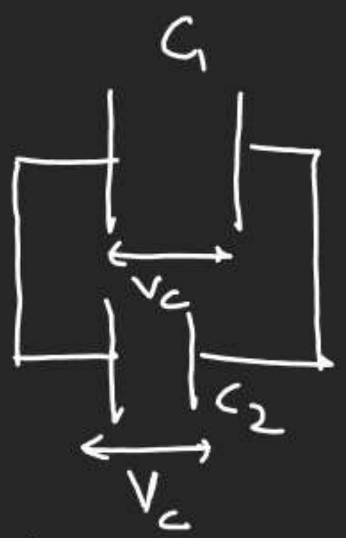
Common potential :-

$$V_c = \frac{q_1'}{C_1}$$

$$V_c = \left(\frac{C_1V_1 + C_2V_2}{C_1+C_2}\right) \times \frac{C_1}{C_1}$$

$$\boxed{V_c = \frac{C_1V_1 + C_2V_2}{C_1+C_2}}$$

$$\cancelto{0}{W_b} + \cancelto{0}{W_{ext\ agent}} = \Delta U + heat.$$



$$heat = -[\Delta U]$$

$$|heat| = (U_i - U_f)$$

$$= \left[\frac{1}{2}C_1V_1^2 + \frac{1}{2}C_2V_2^2\right] - \frac{1}{2}(C_1+C_2)V_c^2$$

$$|heat| = \left[\frac{1}{2}C_1V_1^2 + \frac{1}{2}C_2V_2^2\right] - \frac{1}{2}(C_1+C_2)\left(\frac{C_1V_1+C_2V_2}{C_1+C_2}\right)^2$$

$$|heat| = \frac{1}{2}C_1V_1^2 + \frac{1}{2}C_2V_2^2 - \left[\frac{C_1^2V_1^2 + C_2^2V_2^2 + 2C_1C_2V_1V_2}{2(C_1+C_2)}\right]$$

$$|heat| = \frac{C_1V_1^2(C_1+C_2) + C_2V_2^2(C_1+C_2) - C_1^2V_1^2 - C_2^2V_2^2 - 2C_1C_2V_1V_2}{2(C_1+C_2)}$$

$$|heat| = \frac{\cancel{C_1^2V_1^2} + C_1C_2V_1^2 + C_2C_1V_2^2 + \cancel{C_2^2V_2^2} - \cancel{C_1^2V_1^2} - \cancel{C_2^2V_2^2} - 2C_1C_2V_1V_2}{2(C_1+C_2)} = \boxed{\frac{C_1C_2(V_1-V_2)^2}{2(C_1+C_2)}}$$

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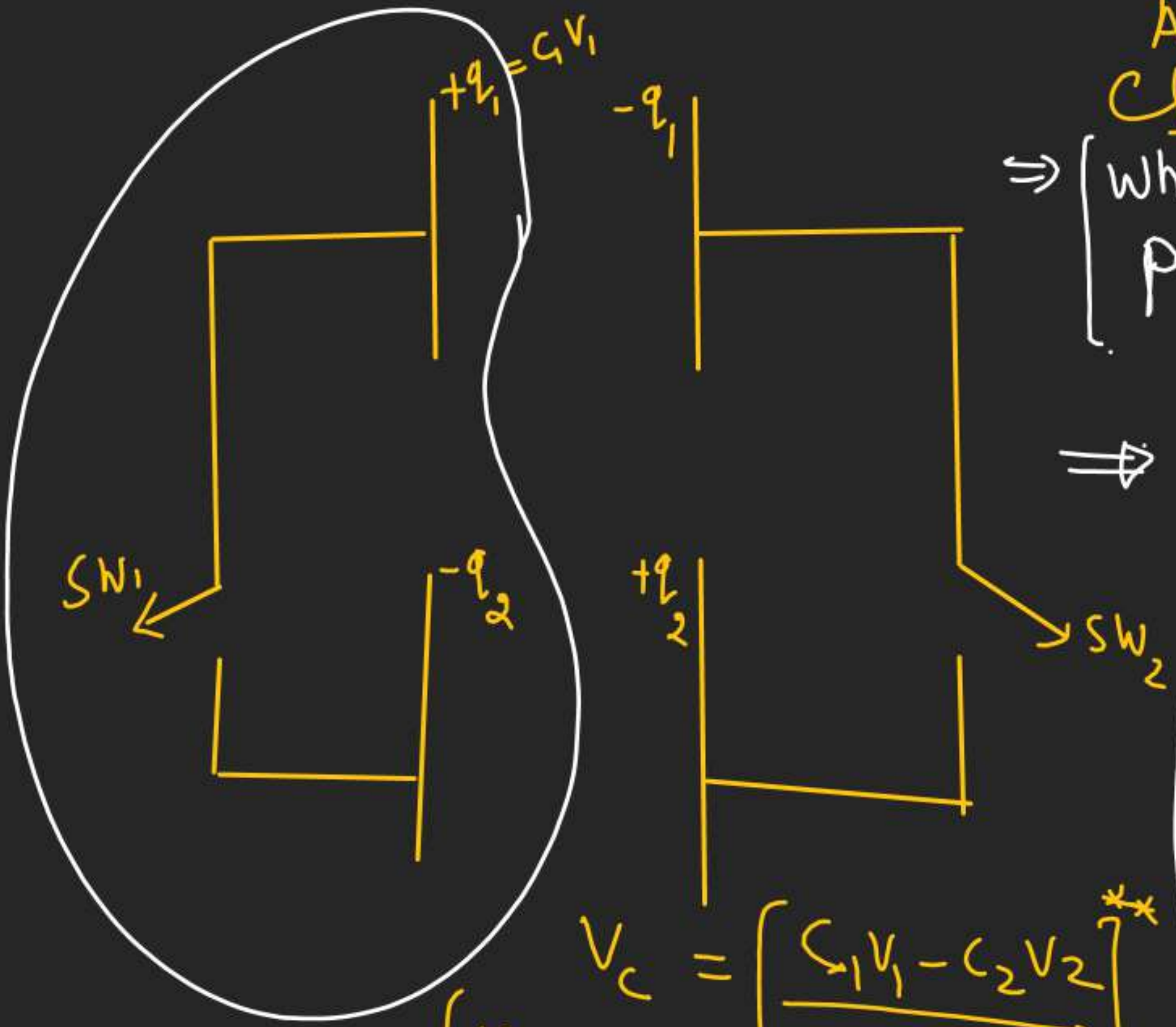
Capacitor

Case-2 If plates of Opposite polarity are interconnected:-



At $t=0$, SW_1 and SW_2 Closed.

⇒ When steady state (Both capacitor acquire same potential) the plates which are interconnected are of same polarity.

⇒

$$V_c = \frac{q_1'}{C_1} = \frac{q_2'}{C_2}$$

$$\frac{q_1'}{q_2'} = \frac{C_1}{C_2} \quad \text{— (1)}$$

Charge Conservation

$$q_1 - q_2 = q_1' + q_2' \quad \text{— (2)}$$

$$C_1V_1 - C_2V_2 = q_1' + \frac{C_2}{C_1}q_1'$$

$$q_1' = \frac{(C_1V_1 - C_2V_2)}{(C_1+C_2)} \times C_1$$

$$q_2' = \left(\frac{C_1V_1 - C_2V_2}{C_1+C_2}\right) \times C_2$$

$$V_c = \left[\frac{C_1V_1 - C_2V_2}{C_1+C_2}\right] **$$

$$\text{Heat} = \left[\frac{C_1C_2}{C_1+C_2}(V_1+V_2)^2\right] **$$

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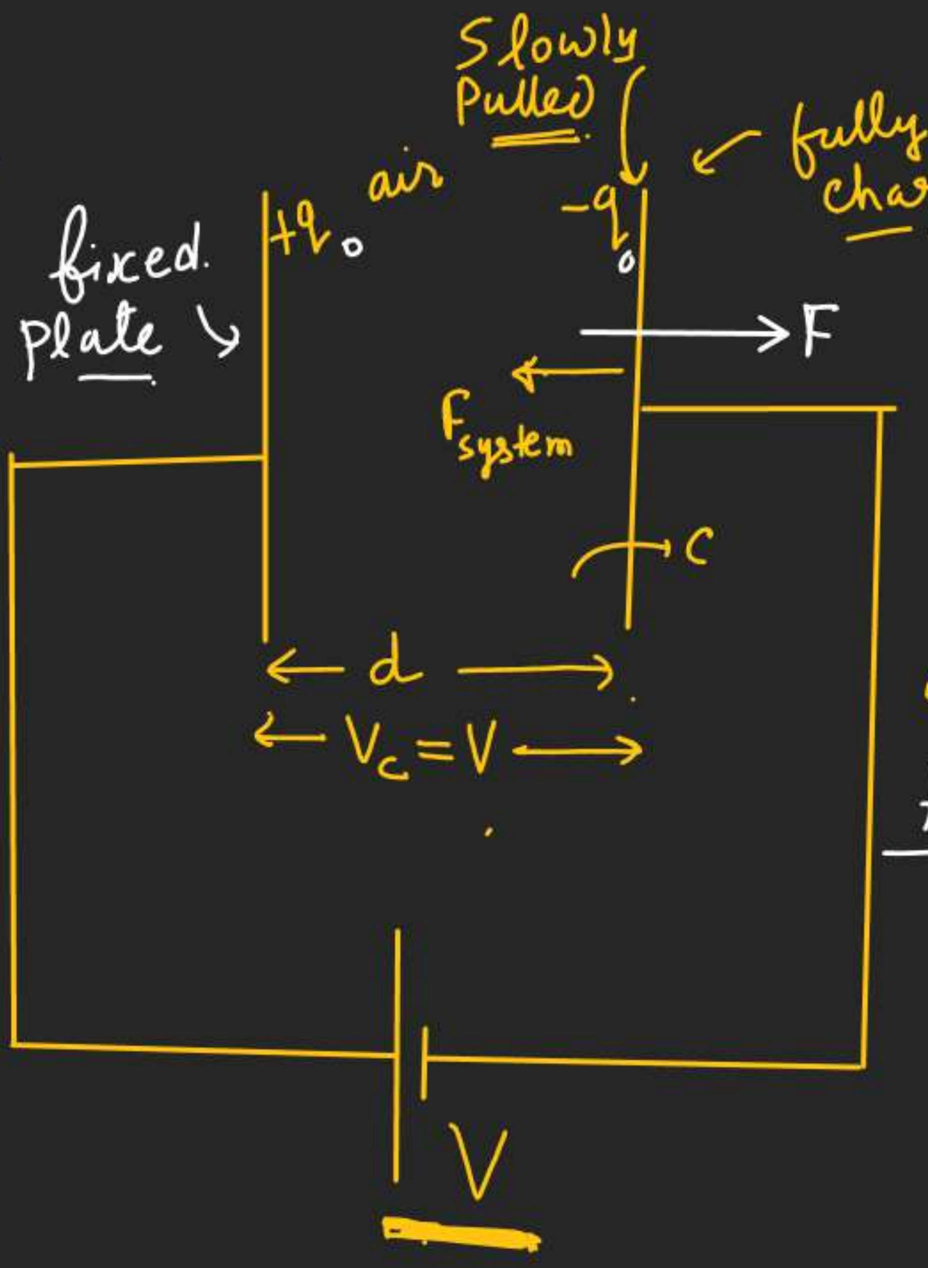
Capacitor

(A).

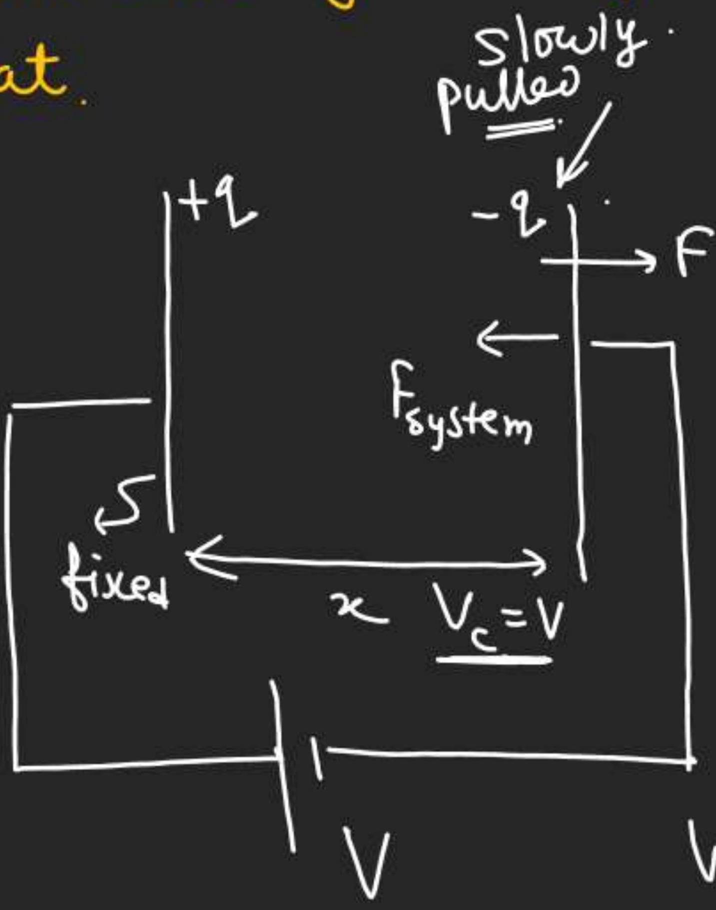


Find workdone by external agent when Seperation b/w the plate become 2d.

b) Work done by battery

c) Heat.

At t = t



If plate is Slowly pulled,

$$F = F_{system} = \frac{q^2}{2\varepsilon_0 A}$$

$$q = CV = \frac{\varepsilon_0 A V}{x}$$

$$F = \frac{1}{2\varepsilon_0 A} \times \left(\frac{\varepsilon_0 A V}{x}\right)^2 = \frac{\varepsilon_0 A V^2}{2} \cdot \frac{1}{x^2}$$

$$\int_0^{W_F} dW_F = \frac{\varepsilon_0 A V^2}{2} \int_d^{2d} \frac{dx}{x^2}$$

$$= \frac{\varepsilon_0 A V^2}{2} \left[\frac{-1}{x}\right]_d^{2d}$$

$$= \frac{\varepsilon_0 A V^2}{2} \left[\frac{-1}{2d} + \frac{1}{d}\right]$$

$$W_F = \frac{\varepsilon_0 A V^2}{4d}$$

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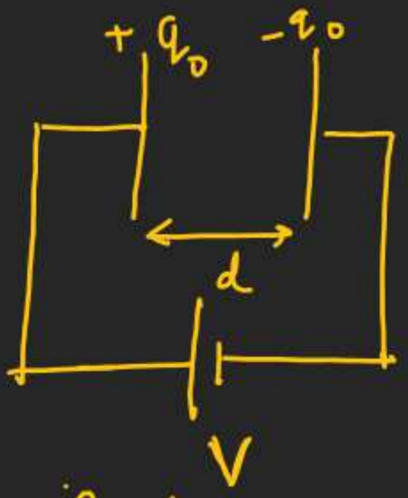
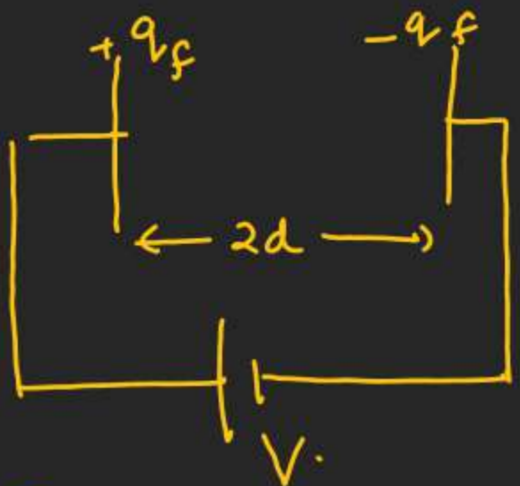
$W_{battery} = ??$





$q = \left(\frac{\varepsilon_0 A}{d}\right)V$

$q_f = \left(\frac{\varepsilon_0 A V}{2d}\right)$

$$W_b + W_f = \Delta U + \text{heat}$$

$$-\frac{\varepsilon_0 A V^2}{2d} + \frac{\varepsilon_0 A V^2}{4d} = -\frac{\varepsilon_0 A V^2}{4d} + \text{heat}$$

$$\boxed{\text{heat} = 0}$$

$$W_{battery} = (\Delta q)V$$

$$= (q_f - q_i)V$$

$$= \left(\frac{\varepsilon_0 A V^2}{2d} - \frac{\varepsilon_0 A V^2}{d}\right)$$

$$= -\left(\frac{\varepsilon_0 A V^2}{2d}\right)$$

(Charge flow from Capacitor to battery)

$$U_i = \frac{1}{2}\left(\frac{\varepsilon_0 A}{d}\right)V^2$$

$$U_f = \frac{1}{2}\left(\frac{\varepsilon_0 A}{2d}\right)V^2$$

$$\Delta U = U_f - U_i = \frac{\varepsilon_0 A V^2}{4d} - \frac{\varepsilon_0 A V^2}{2d}$$

$$= \left(-\frac{\varepsilon_0 A V^2}{4d}\right)$$