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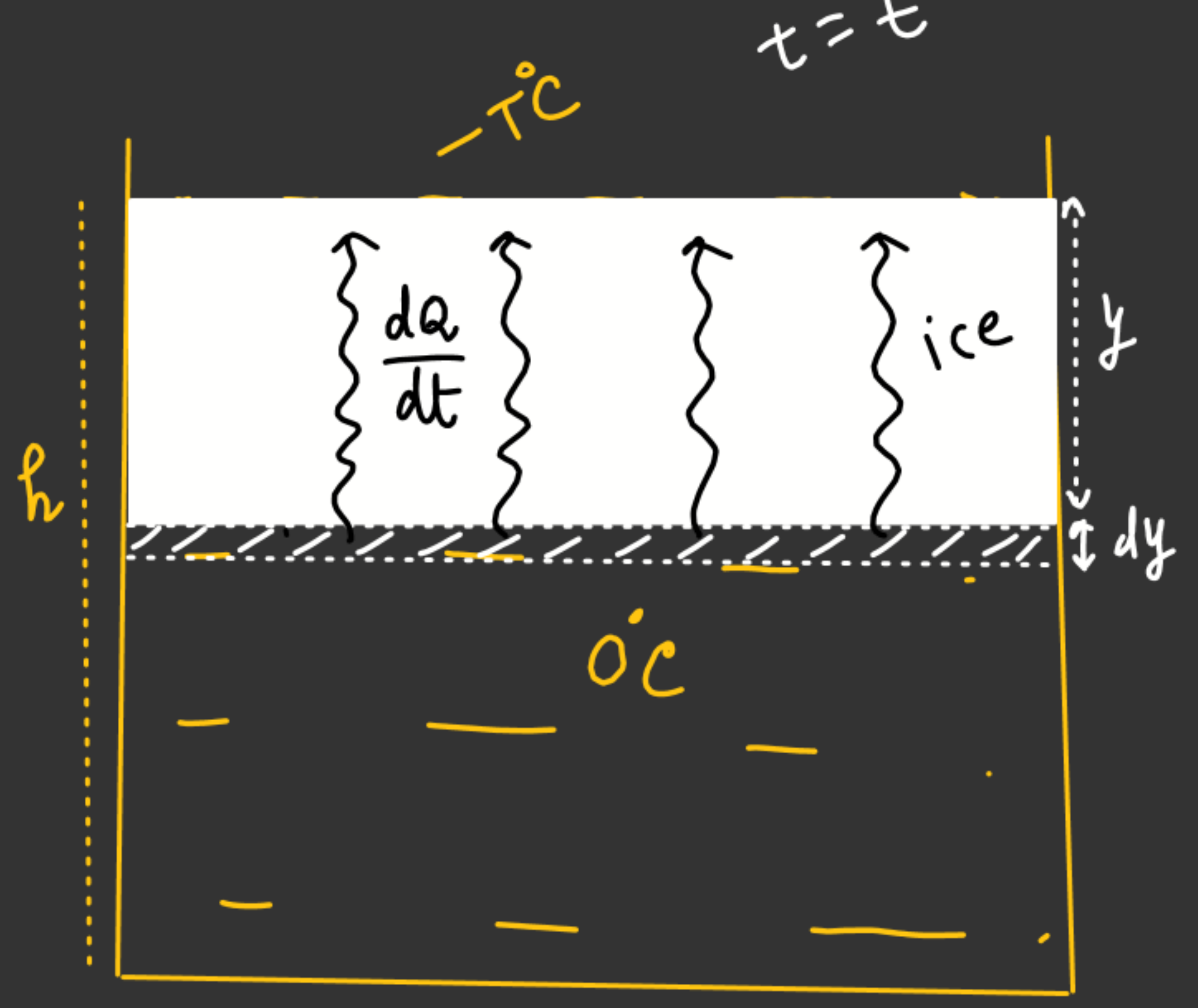
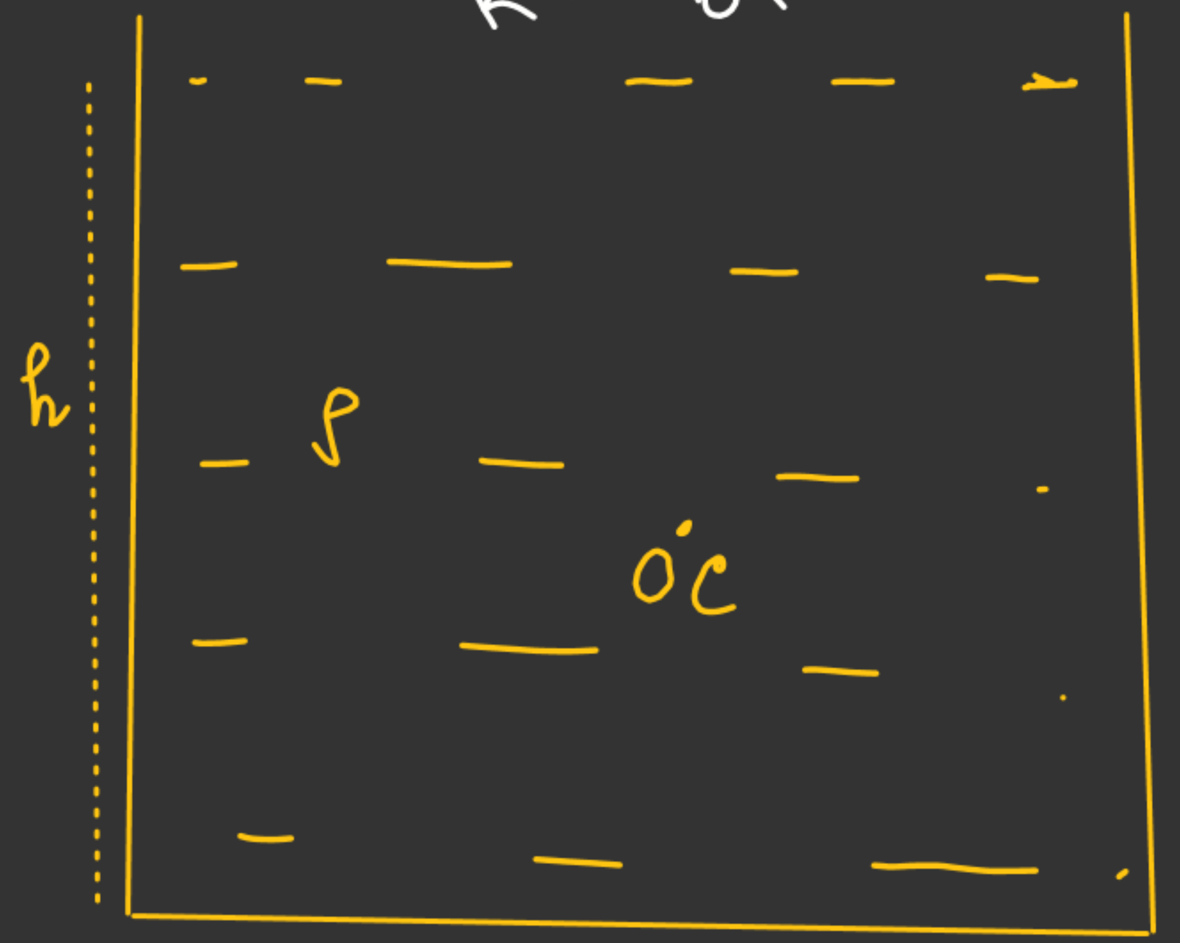
Case of freezing of water



At $t = 0$.
A = Crossectional area of vessel
K = Conductivity of ice



$0^\circ C$ water $\rightarrow$ $0^\circ C$ ice

$dQ = dm\,L_f$

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Let, in 'dt' time 'dy' thickness of water freezes and dQ be the heat released

$$dQ = dm\,L_f$$

dm = mass of dy thickness of water

$$= \rho(A\,dy)$$

$$dQ = \rho A L_f\,dy$$

$$\left(\frac{dQ}{dt}\right) = \rho A L_f\left(\frac{dy}{dt}\right) \quad -(1)$$

dQ conduct through y length of ice.

$$\frac{dQ}{dt} = \frac{KA}{y}\left(0-(-T)\right) \quad -(2)$$

$$\left(\frac{\Delta Q}{\Delta t} = \frac{dQ}{dt}\right)$$

Same.

$$\rho \cancel{A} L_f\left(\frac{dy}{dt}\right) = \frac{K\cancel{A}T}{y}$$

$$\int_0^y y\cdot dy = \frac{KT}{\rho L_f}\int_0^t dt$$

$$\frac{y^2}{2} = \frac{KT}{\rho L_f}\,t \Rightarrow \boxed{t = \frac{\rho L_f y^2}{2KT}}$$

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Radiation

** Emissive power

Energy radiate per second per unit area.

$$E = \left(\frac{\Delta U}{\Delta A \, \Delta t} \right)$$

Energy

Power

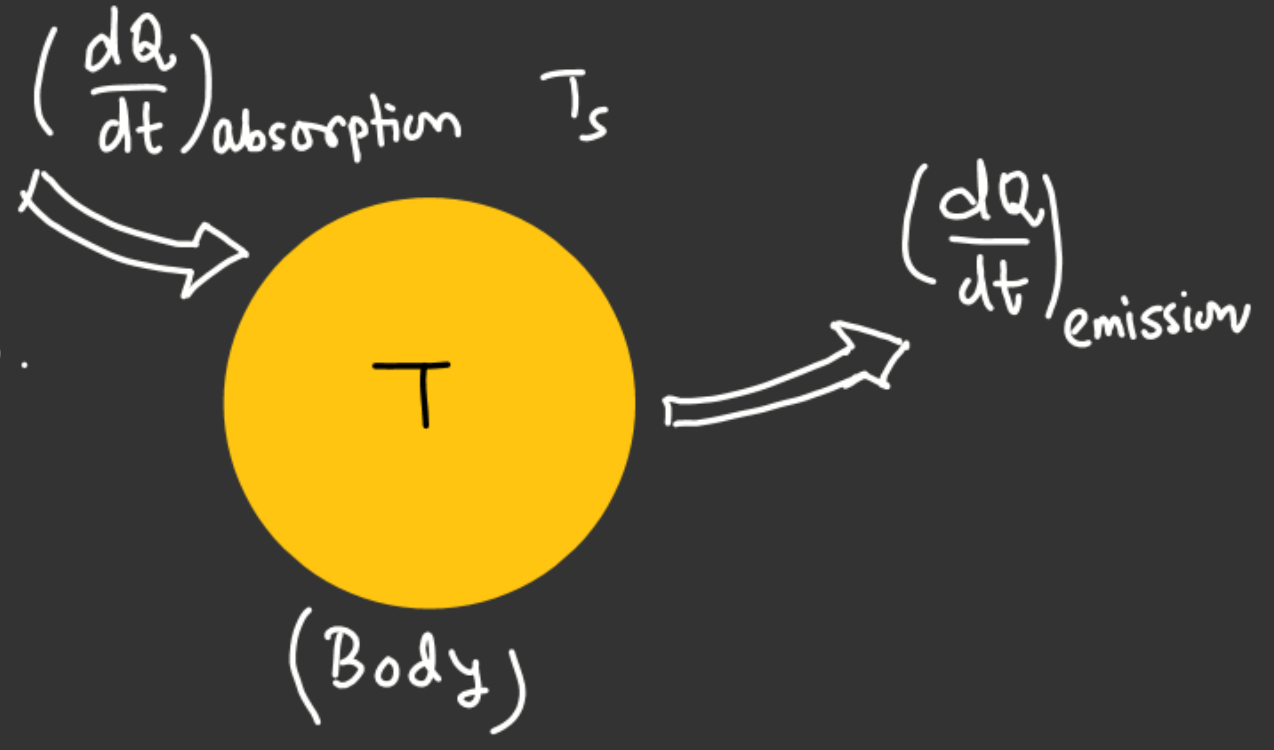
** Absorptive power

$$a = \left(\frac{\text{Energy absorb}}{\text{Energy incident}} \right)$$

(Dimensionless)

PREVOST THEORY OF HEAT EXCHANGE

- According to this every body emit and absorb radiation simultaneously at all temp.



- if $\left(\frac{dQ}{dt}\right)_{absorption} > \left(\frac{dQ}{dt}\right)_{emission}$

  $\Rightarrow$ Temperature of the body increases.

- if $\left(\frac{dQ}{dt}\right)_{absorption} < \left(\frac{dQ}{dt}\right)_{emission}$

  $\Rightarrow$ Temperature of the body decreases

- If $\left(\frac{dQ}{dt}\right)_{absorption} = \left(\frac{dQ}{dt}\right)_{emission} \Rightarrow T = \text{Constant}$

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☆☆ Black-body

- $a_{\text{black body}} = 1.$

- A good emitter is a good absorber.

$$a = \left(\frac{\text{Energy absorb}}{\text{Energy incident}}\right)$$

For black body

$$(\text{Energy absorb}) = (\text{Energy incident})$$

$$a_{\text{black body}} = 1.$$

- **Krichhoff's Law**

Ratio of emissive power to absorptive power of any body is Constant and is equal to emissive power of a black body.

$$\boxed{\frac{E_{\text{body}}}{a_{\text{body}}} = E_{\text{black body}}}$$

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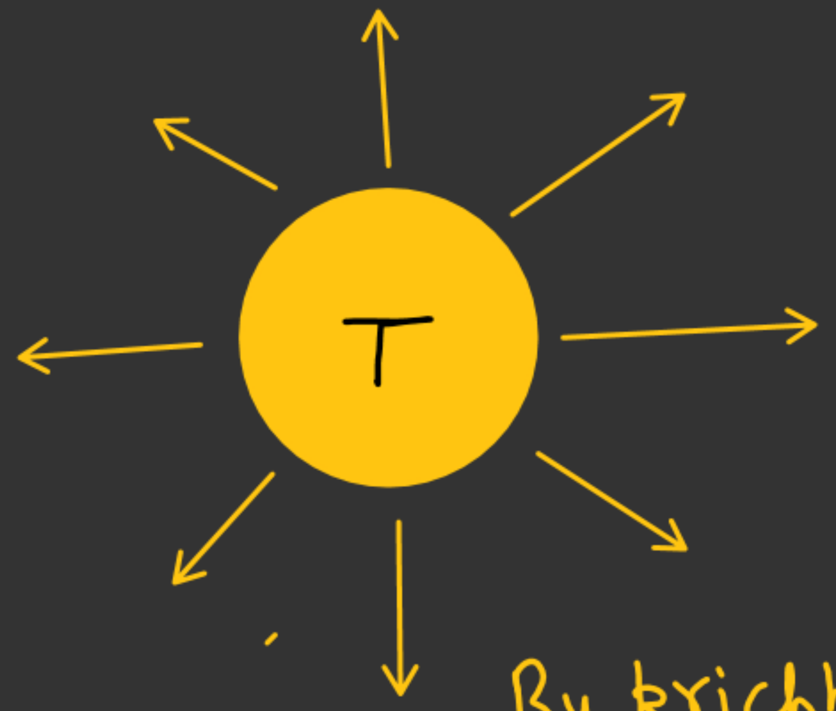
STEFAN'S LAW

$$E_{\text{black body}} \propto A T^4$$

$$E_{\text{body}} = e \cdot E_{\text{black body}}$$



$$\text{Emissive power} = \left(\frac{dQ}{dt}\right)$$

$$\boxed{E_{\text{body}} = e \sigma A T^4}$$

e = emissivity

↳ a constant whose value lie b/w 0 to 1

$$\boxed{E_{\text{black body}} = \sigma A T^4}$$

$$\boxed{0 \leq e \leq 1}$$

σ = Stefan's Constant

↳ $\underline{5.67 \times 10^{-8}\ W/m^2K^4}$

A = Surface area

T = Temp of body

By Krichhoff's law

$$\frac{E_{\text{body}}}{a_{\text{body}}} = E_{\text{black body}}.$$

$$\frac{e \sigma A T^4}{a} = \sigma A T^4$$

Dimension of body & blackbody same

$$\boxed{e = a},$$

Emissivity is equal to absorptive power.

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$$\left(\frac{dQ}{dt}\right)_{\text{net emission}} = \left(\frac{dQ}{dt}\right)_{\text{emission}} - \left(\frac{dQ}{dt}\right)_{\text{absorption}}$$

$$\swarrow \quad = \left(e\sigma A T^4 - e\sigma A T_s^4\right)$$

$$\boxed{E_{net} = e\sigma A\left(T^4 - T_s^4\right)}$$

$$\left(\frac{dQ}{dt}\right)_{\underset{\sim\sim\sim}{\text{absorption}}} = \left(\frac{dQ}{dt}\right)_{\text{emission } \underset{\sim\sim\sim}{\text{of surrounding}}}$$

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$$\boxed{E_{net} = e\sigma A(T^4 - T_s^4)}$$

$$\downarrow$$

$$\frac{dQ}{dt} = e\sigma A(T^4 - T_s^4)$$

$$\Downarrow$$

$$ms\left(\frac{dT}{dt}\right) = e\sigma A(T^4 - T_s^4)$$

$$\boxed{\frac{dT}{dt} = \frac{e\sigma A}{ms}(T^4 - T_s^4)}$$

$$Q = msT$$

$$\frac{dQ}{dt} = ms\left(\frac{dT}{dt}\right)$$

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Newton's Law of Cooling

$$\boxed{\frac{dT}{dt} = \frac{e\sigma A}{ms}\left(T^4 - T_s^4\right)}$$

Stefan's law

if $T = (T_s + \Delta T)$ $\quad (\Delta T << T_s \text{ or } T)$ (Temp of body)

then $\frac{dT}{dt} = -ve$ as temp of body decreases with time.

$$-\frac{dT}{dt} = \frac{e\sigma A}{ms}\left[(T_s + \Delta T)^4 - T_s^4\right]$$

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$\Delta T << T_s$ or T_b

Newton's Law of Cooling

$$-\frac{dT}{dt} = \frac{e\sigma A}{ms}\left[(T_s+\Delta T)^4 - T_s^4\right]$$

$$-\frac{dT}{dt} = \frac{e\sigma A}{ms}\left[T_s^4\left(1+\frac{\Delta T}{T_s}\right)^4 - T_s^4\right]$$

$$-\frac{dT}{dt} = \frac{e\sigma A T_s^4}{ms}\left[\left(1+\frac{\Delta T}{T_s}\right)^4 - 1\right]$$

$$-\frac{dT}{dt} = \frac{e\sigma A T_s^{\cancel{4}}}{ms}\left[\cancel{1}+\frac{4\Delta T}{\cancel{T_s}}-\cancel{1}\right]$$

Constant

$$-\frac{dT}{dt} = \boxed{\left(\frac{4e\sigma A T_s^3}{ms}\right)}\Delta T$$

$\Downarrow$

Constant

$$-\frac{dT}{dt} \propto \Delta T$$

&&

$$\boxed{-\frac{dT}{dt} \propto (T-T_s)}$$

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Newton's Law of Coolin

$$\boxed{\left(\frac{4e\sigma A T_s^3}{ms}\right)}$$

$\Downarrow$

K = proportionality Constant

$$\boxed{-\frac{dT}{dt} \propto (T - T_s)}$$

Rate of decrease of temp of any body w.r.t time is directly proportional to temp difference b/w body & Surrounding at any instant