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$(\ )^{\frac{1}{x}}$ $\sqrt[x]{\ }$ $x \in N$.

$\frac{6}{5} \cdot 5^{\log x} = \frac{10}{3} 3^{\log x}$.

$\log_2(\underline{2(x-4)}) = \log_2\left(\underline{\sqrt{x+3}-\sqrt{x-3}}\right)^2$

$=$

$\sqrt{N}$

$\sqrt[3]{N}$

$\sqrt[4]{N}$

$\sqrt[\frac{1}{4}]{\ }$

$\left(\frac{5}{3}\right)^{\log x} = \frac{25}{9}$

$1 - t = 24t$.

$5^{-x-3} = 5^{-2}$.

$$\frac{(120) \cdot 5^{x-3}}{(1-5^{x-3})^2} = \frac{1}{0.2-5^{x-3}}$$

$\log x = \frac{1}{\log_4 x} = x$.

$x^{\log_4 x}$

$\log_x 4 = x$.

$$\frac{\cancel{120}^{24} \cdot 5^{x-3}}{(1-5^{x-3})^{\cancel{2}}} = \frac{\cancel{5}}{\cancel{1-5^{x-3}}}$$

$\boxed{x=2}$

$x^x = 4 = 2^2$.

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$\log_{10}\frac{200}{y} = \frac{1}{10}$

$\log_{10} 2y = 1$

$= \cancel{\log_{10} 200y = 2\log_{10} y = 1}$

$\boxed{x = 1,\ z = 1,\ y = 5 \quad \text{or} \quad x = 100,\ z = 100,\ y = 20}$

$\log_{10} x = 0, 2$

$\log_{10}(2xy) - \log_{10} x \log_{10} y = 1$

$\log_{10}(2yz) - \log_{10} y \log_{10} z = 1$

$\log_{10}(zx) - \log_{10} z \log_{10} x = 0$

$\log_{10} x - \log_{10} z - \log_{10} y(\log_{10} x - \log_{10} z) = 0$

$(\log_{10} x - \log_{10} z)(1 - \log_{10} y) = 0$

$\Rightarrow y = 10$ or $\boxed{x = z}$

$\boxed{\log_{10} x^2 - (\log_{10} x)^2 = 0}$

If $y = 10$ $\quad \log_{10}(20x) - \log_{10} x = 1$ $\quad$ X

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$\log_a abc$

$\log_b abc$

$\log_c abc$

$\log_{abc} a$

$\log_{abc} b$

$$\boxed{\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=1}$$

$p+1$

$$\frac{1}{p+1}+\frac{1}{q+1}+\frac{1}{r+1}=1$$

$$\alpha^3+\beta^3+\gamma^3-3\alpha\beta\gamma$$

$$=(\alpha+\beta+\gamma)(\alpha^2+\beta^2+\gamma^2-\alpha\beta-\beta\gamma-\gamma\alpha)$$

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slope of tangent to $f(x)$ at $x=a$

$[t_1, t_2]$

Av. rate of vel. w.r.t. time in $[t_1, t_2]$ $= \dfrac{v_2 - v_1}{t_2 - t_1}$

$\lim_{t_2 \to t_1} \dfrac{v_2 - v_1}{t_2 - t_1}$

↓

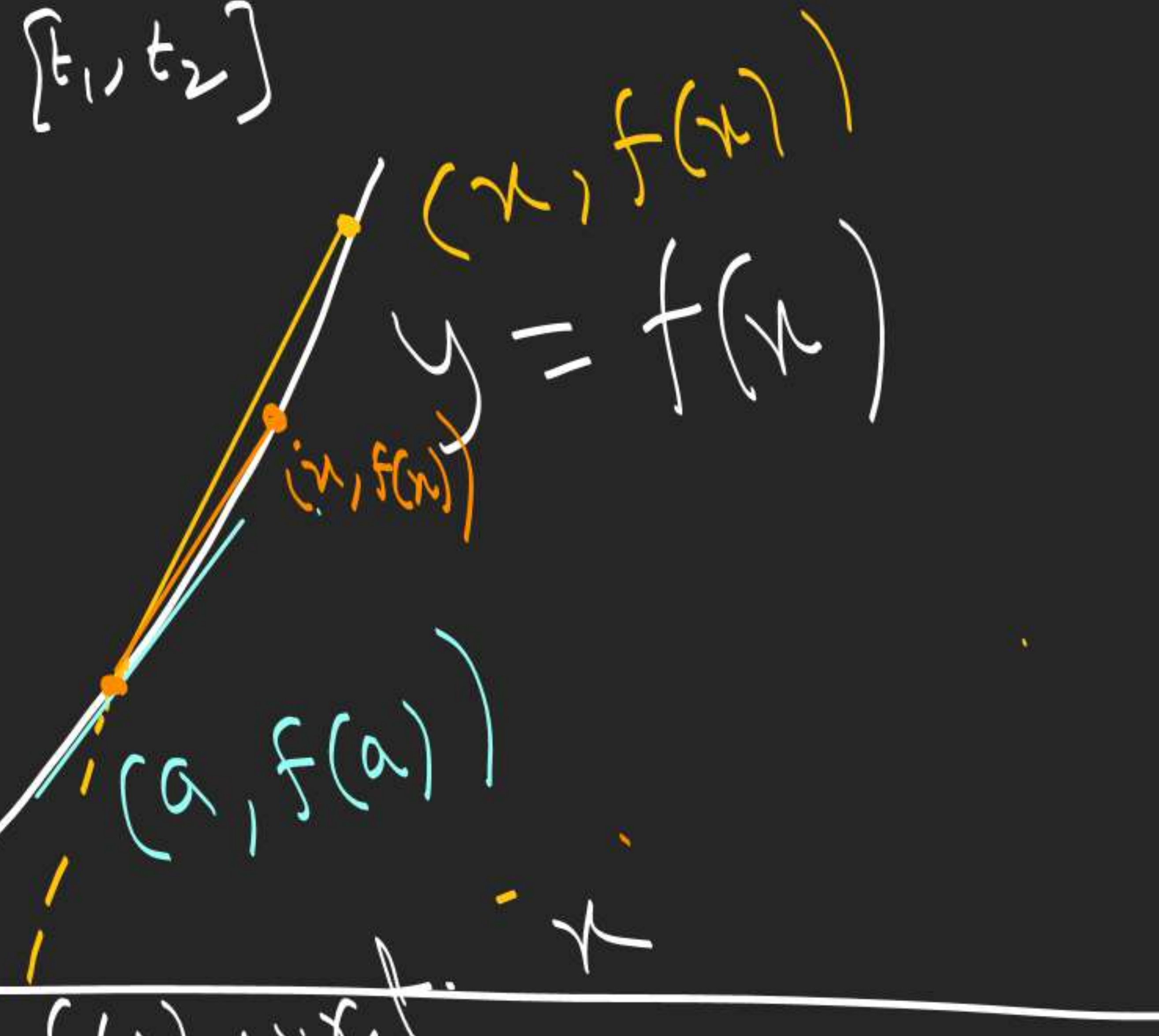
Instantaneous rate of change of vel. w.r.t. time at $t = t_1$

$\dfrac{f(x) - f(a)}{x - a}$

$$\boxed{\frac{d}{dx} f(x)\Big|_{x=a} = f'(a) = \lim_{x \to a} \left(\frac{f(x) - f(a)}{x - a} \right)}$$

↓

Instantaneous rate of change of $f(x)$ w.r.t. x at $x = a$:



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$f(x)$

$$\frac{d}{dx}\left(x^3\right) \text{ at } x=a$$

$$f'(a) = \lim_{x\to a}\left(\frac{x^3-a^3}{x-a}\right) = \lim_{x\to a}\frac{\cancel{(x-a)}(x^2+xa+a^2)}{\cancel{x-a}} = 3a^2.$$

$$\lim_{x\to 0}\frac{x}{2x} = \lim_{x\to 0}\frac{1}{2} = \frac{1}{2} \quad \to \frac{0}{0}$$

$$n\in N, \quad a^n-b^n = (a-b)(a^{n-1}+a^{n-2}b+a^{n-3}b^2+a^{n-4}b^3+\cdots ab^{n-2} + b^{n-1})$$

$$n \text{ is odd}, \quad a^n+b^n = (a+b)(a^{n-1}-a^{n-2}b+a^{n-3}b^2-a^{n-4}b^3+\cdots+b^{n-1})$$

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Increase and decrease of function

If x increases, & $f(x)$ increases

$\Rightarrow f$ is increasing.

$$\frac{d}{dx}\left(f(x)\,g(x)\right) = f'g + fg'$$

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{gf' - fg'}{g^2}$$

$$\frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x$$

$$\frac{d}{dx}(\sec x) = \sec x \tan x$$

$$\frac{d}{dx}(\operatorname{cosec} x) = -\operatorname{cosec} x \cot x$$

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If $f'(x) > 0$

$\Rightarrow$ $f(x)$ is increasing function

If $f'(x) < 0$, then $f(x)$ is decreasing function

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$f(x) = \ln x \quad , x > 0 .$

$f'(x) = \frac{1}{x} > 0$

$\Rightarrow f(x) = \ln x$ is increasing

$f(x) = \sin x \qquad x \in [0, 2\pi)$

$f'(x) = \cos x$

$f(x)$ is increasing in $\left(0, \frac{\pi}{2}\right) \cup \left(\frac{3\pi}{2}, 2\pi\right)$

decreasing in $\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$

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Concavity of function

$f''(x)$

$f''(x) > 0 \Rightarrow \boxed{f'(x) \text{ increasing}}$ If $f''(x) > 0$, then $f(x)$ is concave up

$\frac{dy}{dx} = f'(x)$

$\frac{d^2y}{dx^2} = \frac{d}{dx} f'(x) = f''(x)$

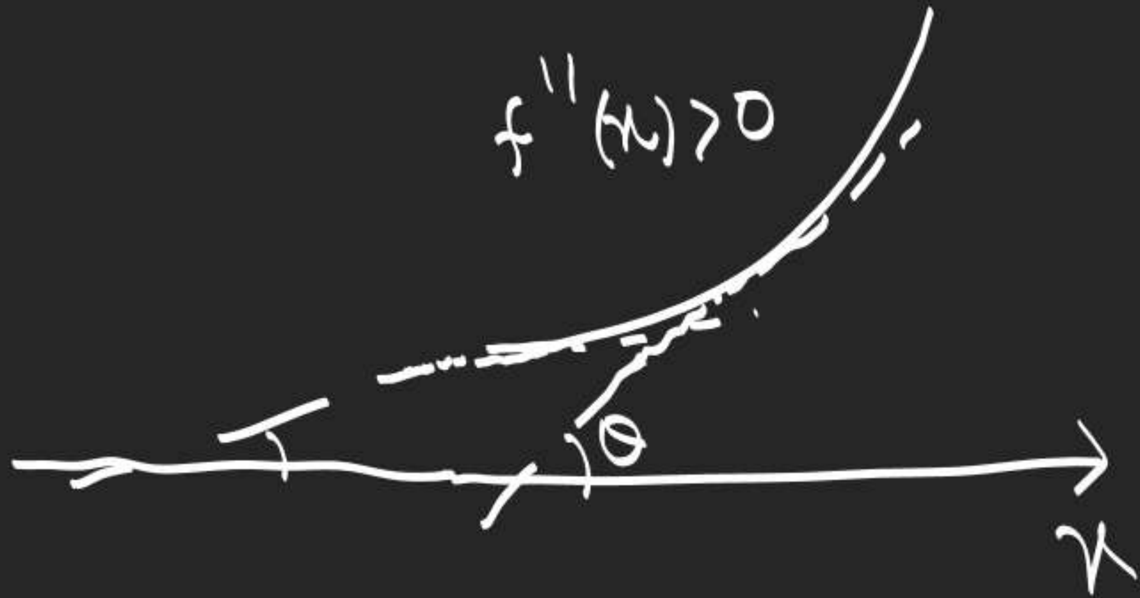
$y = f(x)$

If $f''(x) < 0$, then $f(x)$ is concave down.

If $f''(x) < 0$

$\Rightarrow f'(x)$ is decreasing.

$y = f(x)$

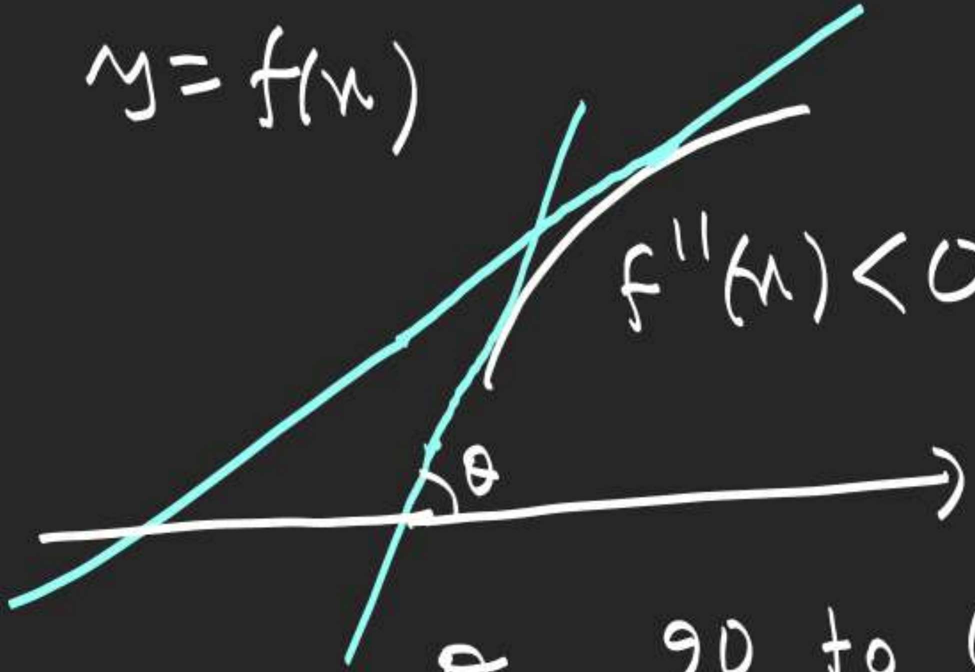


$f''(x) > 0$

θ 0° to 90°

$\tan\theta$ 0 to ∞

slope is increasing

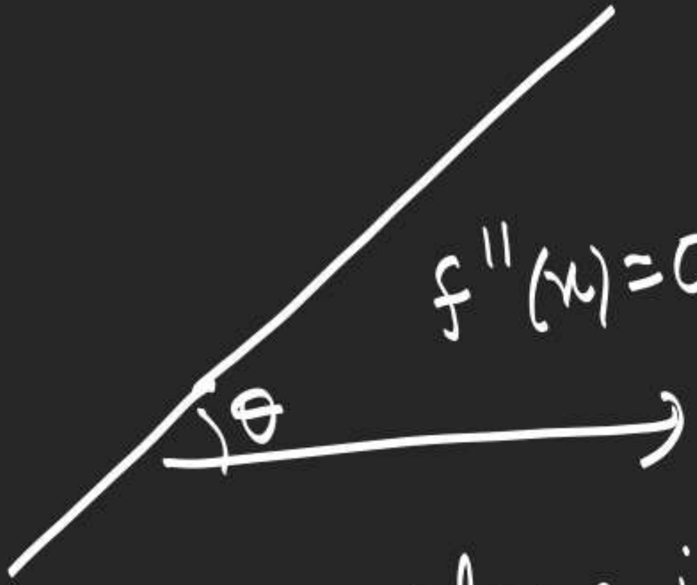


$f''(x) < 0$

θ 90 to 0

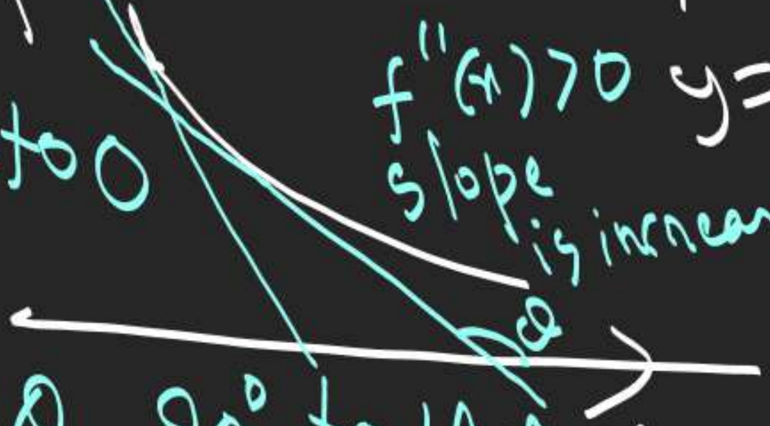
$\tan\theta$ ∞ to 0

$f'(x)$ = slope is decreasing



$f''(x) = 0$

slope is const.

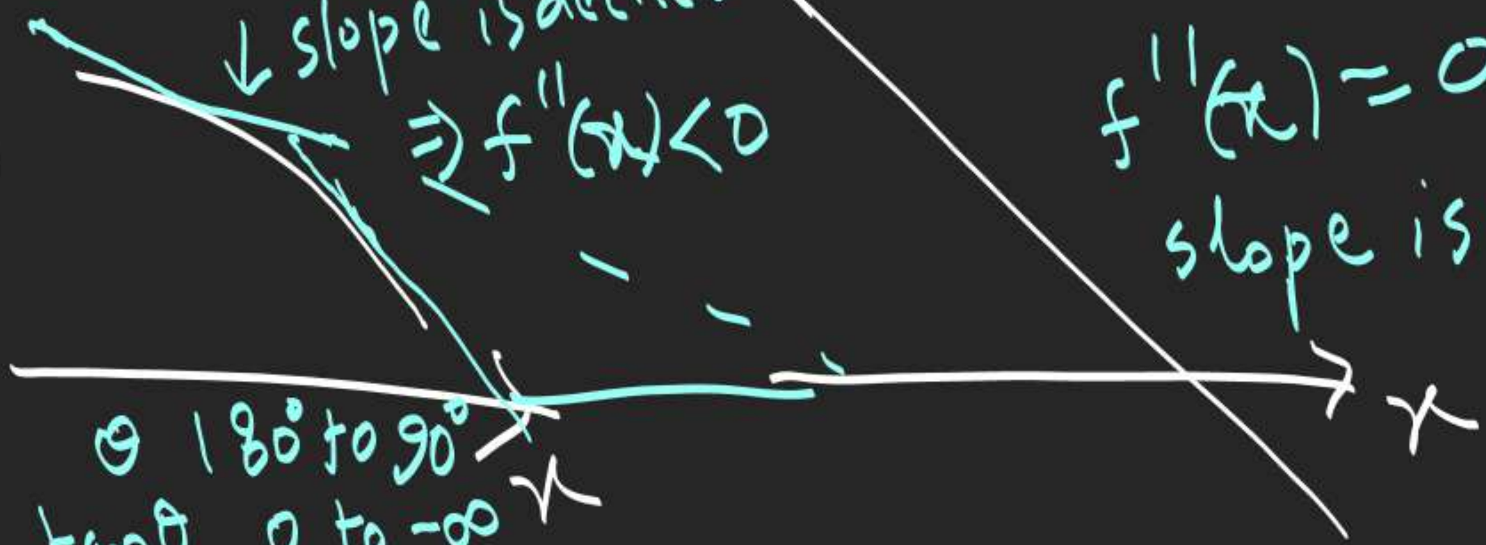


$\tan\theta$ $-\infty$ to 0

$f''(x) > 0$ $y = f(x)$

slope is increasing

θ 90° to 180°



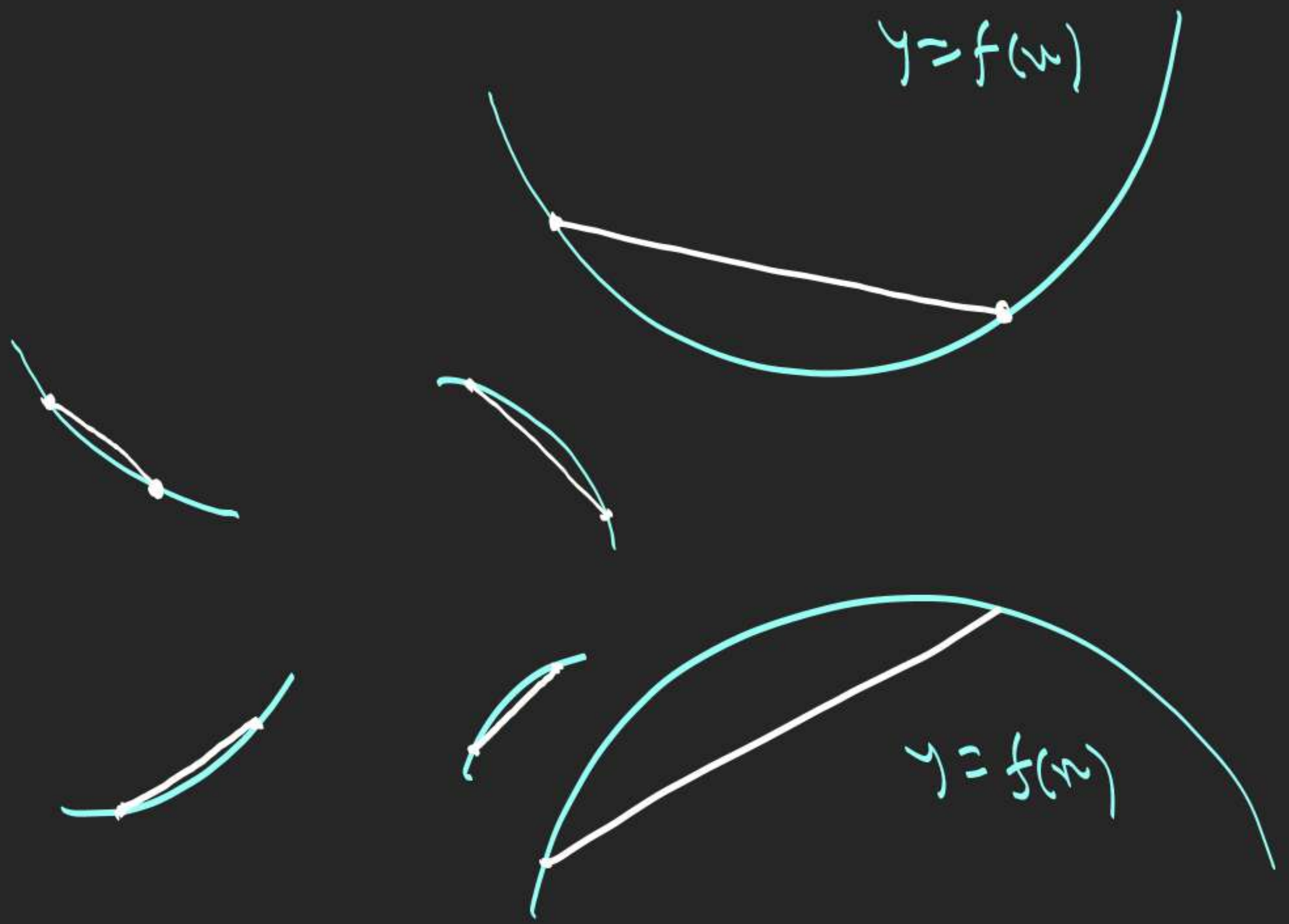
slope is decreasing $\Rightarrow f''(x) < 0$

θ 180° to 90°

$\tan\theta$ 0 to $-\infty$

$f''(x) = 0$

slope is const.



$\rightarrow f''(x) > 0$

$\rightarrow f''(x) < 0$

$\boxed{\text{Ex-II} \rightarrow 14 \text{ to } 20}$