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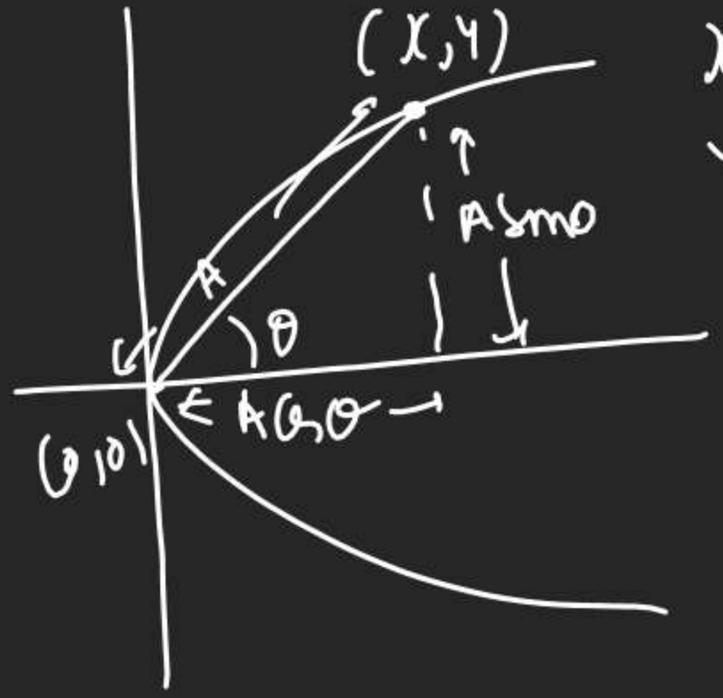
Q If one vertex to the chord of Parabola $y^2=4ax$ is $(0,0)$. If chord makes an angle θ with x Axis find length of chord.



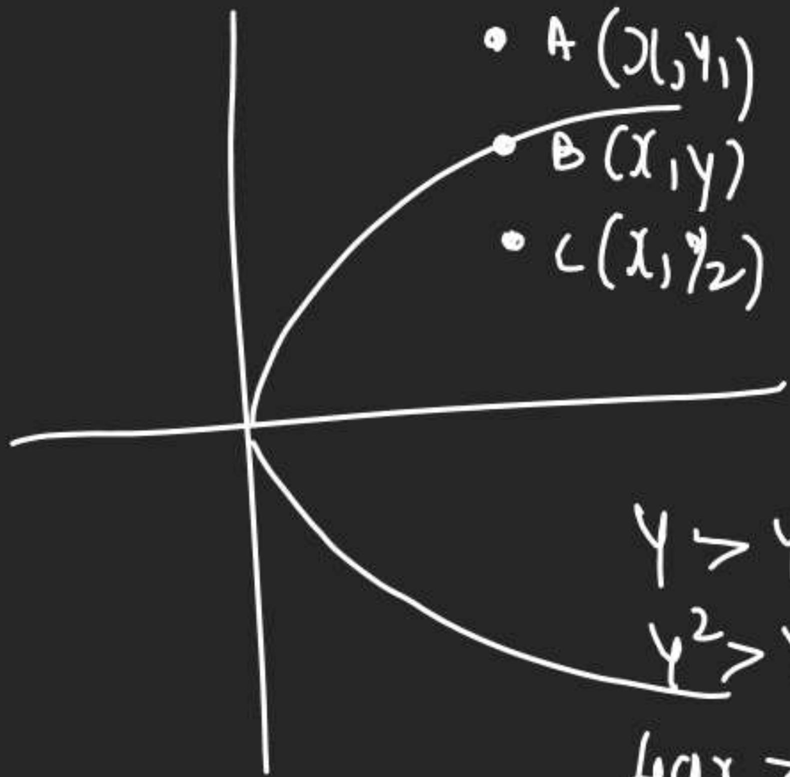
$$\left.\begin{array}{l} x = A\cos\theta \\ y = A\sin\theta \end{array}\right\} y^2 = 4ax$$

$$A^2\sin^2\theta = 4aA\cos\theta$$

$$A = \frac{4a\cos\theta}{\sin^2\theta}$$

$$A = 4a\cot\theta\cdot\csc\theta$$

Position of a Pt. WRT. Parabola



A) outside

B) on Par.

C) Inside Par.

$y > y_2$

$y^2 > y_2^2$

$4ax > y_2^2$

$y^2-4ax=0 \quad (x_1,y_2) \quad y_2^2-4ax<0$

$Par(x_1,y_2)<0$

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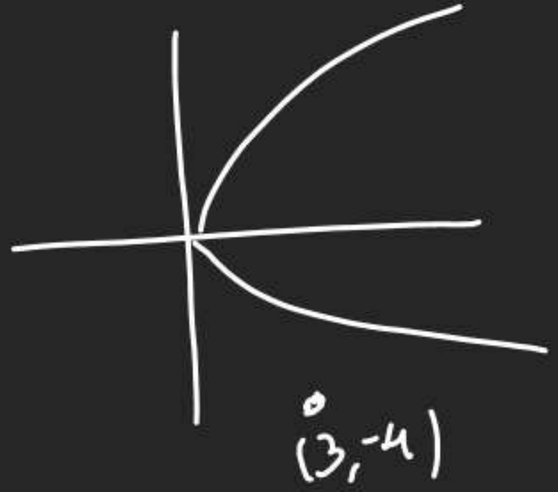
Par (Pt.) $> 0 \to$ outside
$= 0 \to$ on Par.
$< 0 \to$ Inside Par.

Q Find Position of $(3,-4)$ WRT. $y^2 = x$

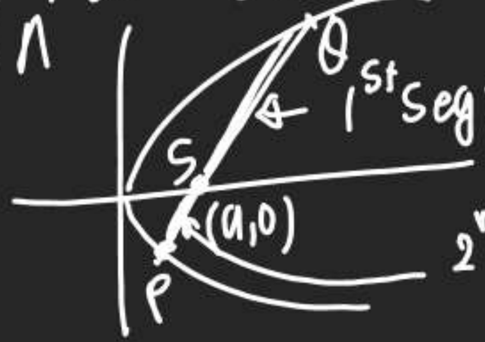


$y^2 - x = 0 \leftarrow (3,-4)$

$((-4)^2 - 3) = +ve$ (outside Par.)

Prop of Parabola

Semi LLR of $y^2 = 4ax$ is H.M. betn segment of any Focal chord



1^{st} segment $= K_1$

2^{nd} segment $= K_2$

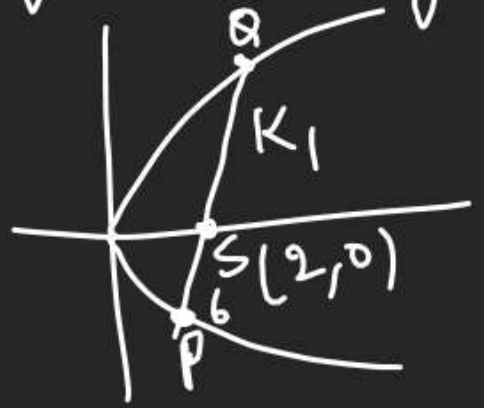
$\frac{1}{K_1} + \frac{1}{K_2} = \frac{2}{2a} \Rightarrow \boxed{\frac{1}{K_1} + \frac{1}{K_2} = \frac{1}{a}}$

b is HM of a & c $\to \frac{1}{a} + \frac{1}{c} = \frac{2}{b}$

$K_1, 2a, K_2$ are in H.P.

Q If PQ is F.C. of $y^2 = 8x$ & S is Focus, SP = 6, find SQ

$a = 2$



$\frac{1}{6} + \frac{1}{K_1} = \frac{1}{2}$

$\frac{1}{K_1} = \frac{1}{2} - \frac{1}{6} = \frac{1}{3}$

$SQ = K_1 = 3$

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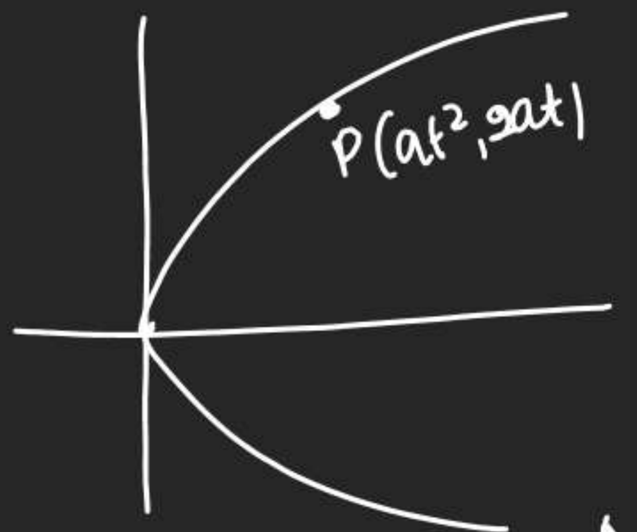
Parametric Coordinate

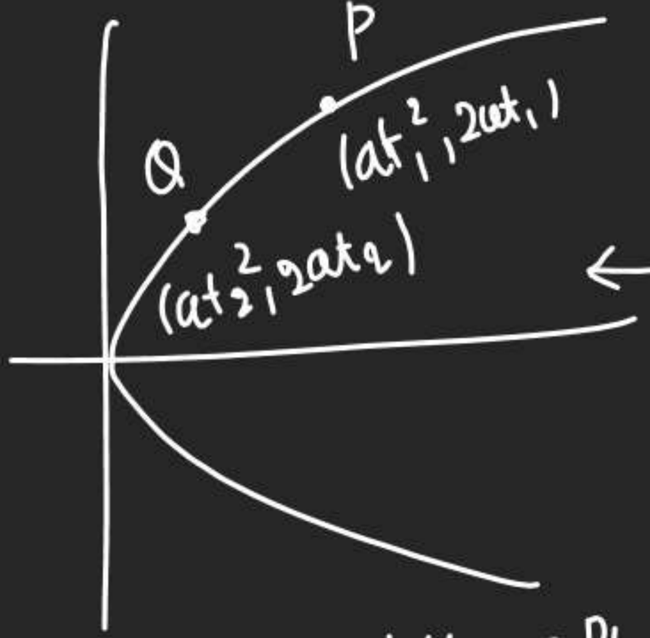
1) Parametric Coordinates of $y^2 = 4ax$ is $(at^2, 2at)$

2) ——— $x^2 = 4ay$ is $(2at, at^2)$

(3) We take P. Coord. instead of assuming (x_1, y_1) or (h, k) Whenever we need a pt. on Parabola.

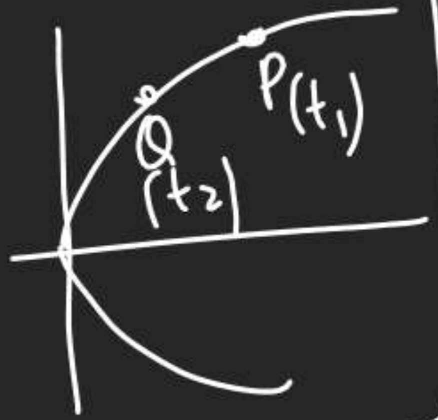


When 1 Point ↑

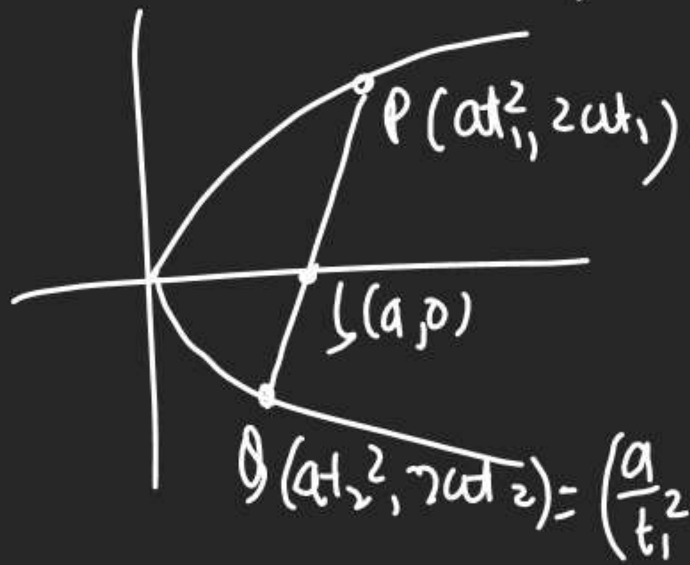


When 2 Pts on Par. Required

$\longleftrightarrow$



(4) Imp. Par. Coord. of Focal Chord



$\boxed{t_2 = -\frac{1}{t_1}}$

$P(at_1^2, 2at_1) = (at^2, 2at)$

$Q(at_2^2, 2at_2) = \left(\frac{a}{t_1^2}, \frac{-2a}{t_1}\right) = \left(\frac{a}{t^2}, \frac{-2a}{t}\right)$

Eqⁿ of PQ ⇒ $(y - 2at_1) = \frac{2at_2 - 2at_1}{at_2^2 - at_1^2}(x - at_1^2)$

$(y - 2at_1) = \frac{2a(\cancel{t_2 - t_1})}{\cancel{a}(\cancel{t_2 - t_1})(t_2 + t_1)}(x - at_1^2)$

$y(t_1 + t_2) - 2at_1(\cancel{t_1} + t_2) = 2x - \cancel{2at_1^2}$

$\Rightarrow y(t_1 + t_2) - 2at_1t_2 = 2x$ if Passes thru $(a, 0)$

$\Rightarrow 0 - 2at_1t_2 = 2a$

Focal Chord कि निशानी → $\boxed{t_1 t_2 = -1}$

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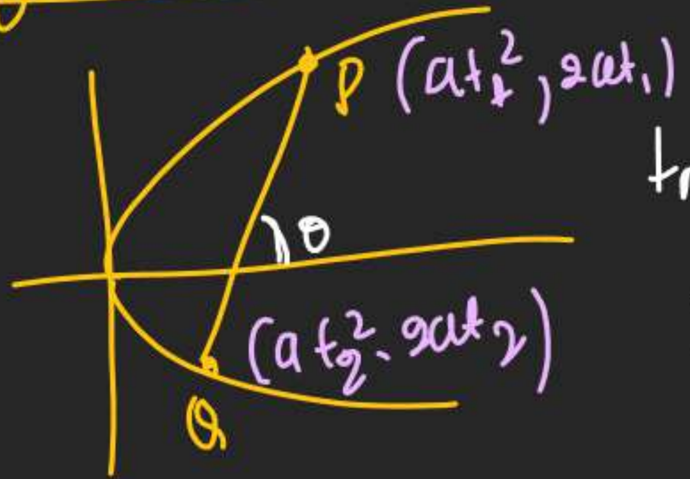
1) $y^2 = 4ax \rightarrow (at^2, 2at)$

$x^2 = 4ay \rightarrow (2at, at^2)$

$y^2 = -4ax \rightarrow (-at^2, 2at)$

$x^2 = -4ay \rightarrow (2at, -at^2)$

2) Eqn of chord PQ



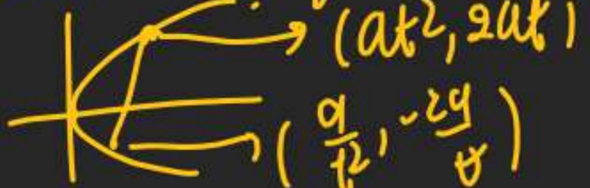
Slope $= \frac{2}{t_1+t_2}$

$\tan\theta = \frac{2}{t_1+t_2}$

$t_1+t_2 = 2\cot\theta$

$2x - y(t_1+t_2) + 2at_1t_2 = 0$

(3) In case of Focal chord $\rightarrow t_1 t_2 = -1$



Q Find Length of Chord PQ?

$PQ = \sqrt{(2at_2 - 2at_1)^2 + (at_2^2 - at_1^2)^2}$

$= a\sqrt{4(t_2-t_1)^2 + (t_2-t_1)^2(t_2+t_1)^2}$

$= a(t_2-t_1)\sqrt{4+(t_1+t_2)^2}$

Q Find length of Focal chord

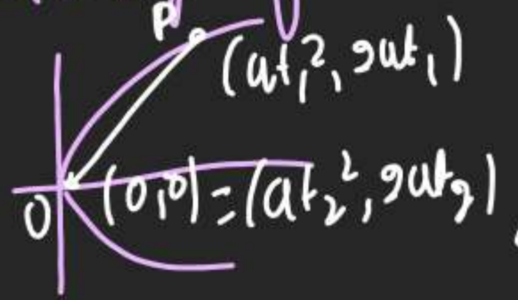
$PQ_{FC} = a(t_2-t_1)\sqrt{4 + t_1^2 + t_2^2 + 2t_1t_2}$

$= a(t_2-t_1)\sqrt{t_1^2+t_2^2-2t_1t_2}$

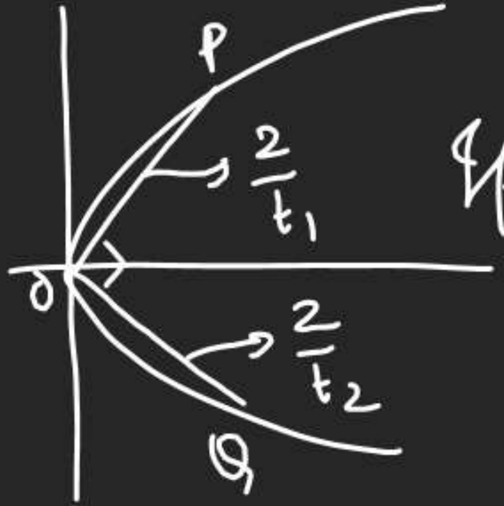
$= a(t_2-t_1)^2 = a\left(t+\frac{1}{t}\right)^2$

FC $\rightarrow \cot\theta = \frac{t-\frac{1}{t}}{2}$

Q Find slope of OP?



Slope $= \frac{2}{t_1+0} = \frac{2}{t_1}$



If 2 chords are ⊥ to each other.

$\frac{2}{t_1} \times \frac{2}{t_2} = -1$

$\boxed{t_1 t_2 = -4}$

Q Min Length of Focal Chord?

$L_{FC} = a\left(t+\frac{1}{t}\right)^2 \geq 4a$

≥ 2

Min length $= 4a$

Q length in terms of θ

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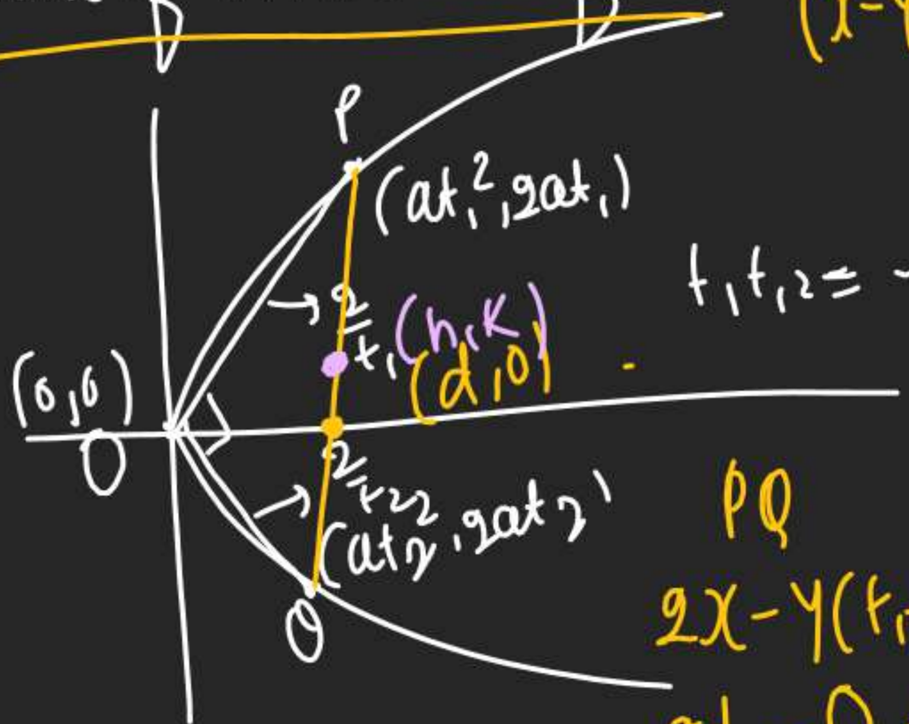
Q Thru Vertex O of Parabola $y^2=4x$ Chords OP & OQ are drawn at Rt Angle to one another. Show that for all Positions of P, PQ Cuts the axis of Parabola at a fixed Pt. Also find the Locus of mid Pt of PQ.

$(x-y)^2=(x+y)^2-4xy$



$t_1t_2=-4$

PQ

$2x-y(t_1+t_2)+2at_1t_2=0$ P.T. $(d,0)$

$2d-0+2\times1\cdot(-4)=0$

$\Rightarrow d=4$

So PQ P.T. (4,0) Which is a Fix pt.

(2) Mid Pt (h,k)

$h=\frac{(t_1^2+t_2^2)}{2}$ | $k=\frac{2t_2+2t_1}{2}$

$t_1^2+t_2^2=2h$ | $t_1+t_2=k$

$t_1^2+t_2^2+2t_1t_2=k^2$

$2h-8=k^2$

$\Rightarrow y^2=2x-8$ is Locus.

$-12a-0+2at_1t_2=0$

$\underline{t_1t_2=6}$

Q Find Eqn of chords of $y^2=4ax$ which Passes thru $(-6a,0)$ & which Subtends an angle of 45° at vertex.

$\tan 45^\circ=\left|\frac{\frac{2}{t_1}-\frac{2}{t_2}}{1+\frac{2}{t_1}\times\frac{2}{t_2}}\right|=\frac{2|t_2-t_1|}{|t_1t_2+4|}$

$1=\frac{2|t_2-t_1|}{10}\Rightarrow(t_2-t_1)^2=25$

$(t_1+t_2)^2-4t_1t_2=25\Rightarrow t_1+t_2=\pm7$

Eqn of PQ $\Rightarrow 2x\mp7y+12a=0$

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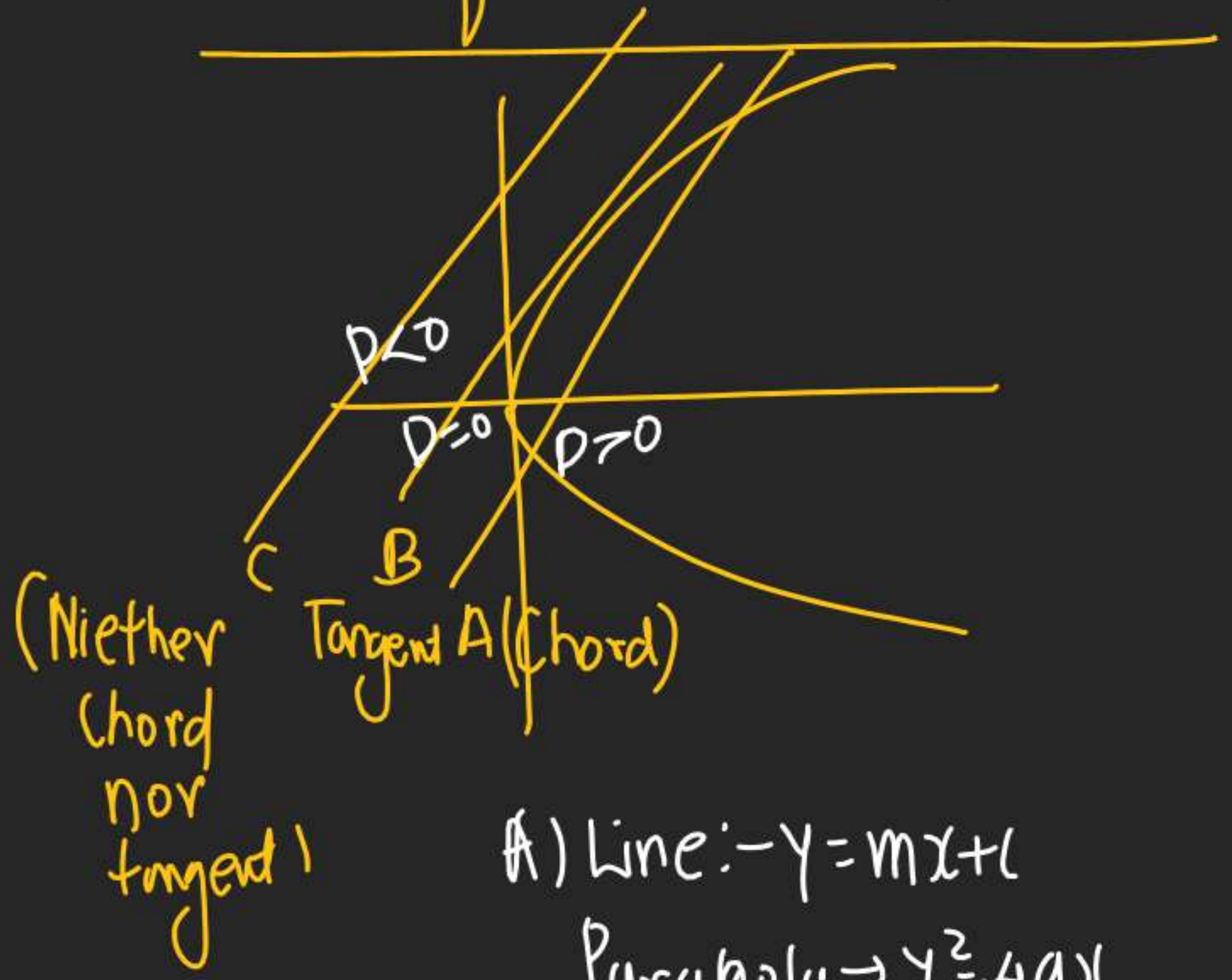
Position of a Line WRT a Parabola.



A) Line:- $y = mx + c$

Parabola → $y^2 = 4ax$

Mixture

$$(mx+c)^2 = 4ax$$

$$m^2x^2 + 2mcx - 4ax + c^2 = 0$$

$$m^2x^2 + x(2mc - 4a) + c^2 = 0$$

→ $D>0$ (A) Chord

→ $D=0$ (B) tangent

→ $D<0$ (C)

$$D = (2mc - 4a)^2 - 4m^2c^2$$

$$= \cancel{4m^2c^2} - 16amc + 16a^2 - \cancel{4m^2c^2}$$

$$= 16(a^2 - amc)$$

Chord	tangent	Door Ri
$D>0$	$D=0$	$D<0$
$a^2 > amc$	$mc = a$	$mc - a > 0$
$mc - a < 0$	$\boxed{c = \frac{a}{m}}$	×
×	Cond^n of tangency.	

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Eqn of tangent (3 Form)

Slope form

$$\boxed{Y = mx + \frac{a}{m}}$$

Par. form.

$(x_1, y_1) \rightleftharpoons (at^2, 2at)$

$y \cdot \cancel{2}at = \cancel{2}a(x + at^2)$

$$\boxed{tY = x + at^2}$$

(x_1, y_1)

Car. form.

$T = 0$

Change to

$y^2 \to yy_1$

$x^2 \to xx_1$

$2x \to x + x_1$

$2y \to y + y_1$

$y^2 = 4ax$

$yy_1 = 2a(x + x_1)$

EoT