(MATHEMATICS)   **TRIGONOMETRIC EQUATIONS**

KEY CONCEPTS

1. If $\sin\theta = \sin\alpha \Rightarrow \theta = n\pi + (-1)^n\alpha$ where $\alpha \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right], n \in I$.

2. If $\cos\theta = \cos\alpha \Rightarrow \theta = 2n\pi \pm \alpha$ where $\alpha \in [0, \pi], n \in I$.

3. If $\tan\theta = \tan\alpha \Rightarrow \theta = n\pi + \alpha$ where $\alpha \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right), n \in I$.

4. If $\sin^2\theta = \sin^2\alpha \Rightarrow \theta = n\pi \pm \alpha$.

5. $\cos^2\theta = \cos^2\alpha \Rightarrow \theta = n\pi \pm \alpha$.

6. $\tan^2\theta = \tan^2\alpha \Rightarrow \theta = n\pi \pm \alpha$.   [Note: α is called the principal angle]

7. **TYPES OF TRIGONOMETRIC EQUATIONS :**

(a) Solutions of equations by factorising. Consider the equation ;

$(2\sin x - \cos x)(1 + \cos x) = \sin^2 x; \cot x - \cos x = 1 - \cot x \cos x$

(b) Solutions of equations reducible to quadratic equations. Consider the equation :

$3\cos^2 x - 10\cos x + 3 = 0$ and $2\sin^2 x + \sqrt{3}\sin x + 1 = 0$

(c) Solving equations by introducing an Auxilliary argument. Consider the equation:

$\sin x + \cos x = \sqrt{2}; \sqrt{3}\cos x + \sin x = 2; \sec x - 1 = (\sqrt{2} - 1)\tan x$

(d) Solving equations by transforming a sum of trigonometric functions into a product.
Consider the example: $\cos 3x + \sin 2x - \sin 4x = 0$;

$\sin^2 x + \sin^2 2x + \sin^2 3x + \sin^2 4x = 2; \sin x + \sin 5x = \sin 2x + \sin 4x$

(e) Solving equations by transforming a product of trigonometric functions into a sum. Consider the equation :

$$\sin 5x \cdot \cos 3x = \sin 6x \cdot \cos 2x \ ; \ 8\cos x \cos 2x \cos 4x = \frac{\sin 6x}{\sin x} \ ; \ \sin 3\theta = 4\sin\theta \sin 2\theta \sin 4\theta$$

(f) Solving equations by a change of variable :

(i) Equations of the form of $a\sin x + b\cos x + d = 0$, where a, b & d are real numbers & $a, b \neq 0$ can be solved by changing $\sin x$ & $\cos x$ into their corresponding tangent of half the angle. Consider the equation $3\cos x + 4\sin x = 5$.
(ii) Many equations can be solved by introducing a new variable.

e.g. the equation $\sin^4 2x + \cos^4 2x = \sin 2x \cdot \cos 2x$ changes to
$2(y + 1)\left(y - \frac{1}{2}\right) = 0$ by substituting, $\sin 2x \cdot \cos 2x = y$.

(g) Solving equations with the use of the Boundness of the functions sinx & cosx or by making two perfect squares. Consider the equations :

$$\sin x \left(\cos \frac{x}{4} - 2\sin x \right) + \left(1 + \sin \frac{x}{4} - 2\cos x \right) \cdot \cos x = 0;$$

$$\sin^2 x + 2\tan^2 x + \frac{4}{\sqrt{3}}\tan x - \sin x + \frac{11}{12} = 0$$

8. **TRIGONOMETRIC INEQUALITIES :** There is no general rule to solve a Trigonometric inequations and the same rules of algebra are valid except the domain and range of trigonometric functions should be kept in mind.

Consider the examples : $\log_2\left(\sin\frac{x}{2}\right) < -1$; $\sin x\left(\cos x + \frac{1}{2}\right) \le 0$; $\sqrt{5 - 2\sin 2x} \ge 6\sin x - 1$

PROFICIENCY TEST-01

Solve the following equations :

1. $\sin x = \frac{1}{2}$

2. $\sqrt{2}\cos^2 7x - \cos 7x = 0$

3. $2\sin x + \tan x = 0$

4. $(2\sin x - \cos x)(1 + \cos x) = \sin^2 x$

5. $1 + \sin x \cos 2x = \sin x + \cos 2x.$

6. $\sin 3x + \sin x = \sin 2x.$

7. $\sin^4 x = 1 - \cos^4 x.$

8. $\sqrt{3}\sin x - \tan x + \tan x \sin x - \sqrt{3} = 0.$

9. $\cos 2x + 3\sin x = 2.$

10. $\cos x + \sec x = 2.$

11. $\sin x \sin 7x = \sin 3x \sin 5x.$

PROFICIENCY TEST-02

1. $\tan^3 x - 1 + \frac{1}{\cos^2 x} - 3\cot\left(\frac{\pi}{2} - x\right) = 3.$

2. $1 - \sin 2x = \cos x - \sin x.$

3. $\cos 2x - \cos 8x + \cos 6x = 1.$

4. $\sin x + \sin 2x + \sin 3x + \sin 4x = 0.$

5. $\sin x + \cos x = \sqrt{2}\sin 5x.$

6. $\tan x + \frac{\tan\frac{\pi}{4} + \tan x}{1 - \tan\frac{\pi}{4}\tan x} = 2.$

7. $1 + 2\cos 3x \cos x - \cos 2x = 0.$

8. $2\cos^2 x - 1 = \sin 3x.$

9. $\cos 3x \tan 5x = \sin 7x.$

10. $\sin^4 x + \cos^4 x = \sin x \cos x.$

11. $\sin^2 x + \sin^2 2x - \sin^2 3x - \sin^2 4x = 0.$

PROFICIENCY TEST-03

1. $\sin^2 x + \sin^2 2x + \sin^2 3x = \frac{3}{2}$

2. $\sin^2 x(1 + \tan x) = 3\sin x(\cos x - \sin x) + 3.$

3. $(1 + \cos 4x)\sin 2x = \cos^2 2x$

4. $\sin 2x = \cos 2x - \sin^2 x + 1.$

5. $(\cos 2x - 1)\cot^2 x = -3\sin x$

6. $\cos 3x - \cos 2x = \sin 3x.$

7. $\sec x + \csc x = 2\sqrt{2}$

8. $\cos^2 x - 2\cos x = 4\sin x - \sin 2x.$

9. $\cos 9x - 2\cos 6x = 2$

10. $4\sin 3x + 3 = \sqrt{2\sin 3x + 2}$

11. $\frac{1 + \sin x + \cos x + \sin 2x + \cos 2x}{\tan 2x} = 0$

EXERCISE–I

1. Solve the equality: $2 \sin 11x + \cos 3x + \sqrt{3} \sin 3x = 0$

2. Find all value of θ, between 0 & π, which satisfy the equation; $\cos\theta \cdot \cos 2\theta \cdot \cos 3\theta = 1/4$.

3. Solve for x, the equation $\sqrt{13 - 18\tan x} = 6\tan - 3$, where $-2\pi < x < 2\pi$.

4. Determine the smallest positive value of x which satisfy

   the equation, $\sqrt{1 + \sin 2x} - \sqrt{2}\cos 3x = 0$.

5. Find the general solution of the trigonometric equation $3^{\left(\frac{1}{2} + \log_3 (\cos x + \sin x)\right)} - 2^{\log_2 (\cos x - \sin x)} = \sqrt{2}$.

6. Find the solution set of the equation, $\log_{\frac{-x^2-6x}{10}} (\sin 3x + \sin x) = \log_{\frac{-x^2-6x}{10}} (\sin 2x)$.

7. Find the value of θ, which satisfy $3 - 2\cos\theta - 4\sin\theta - \cos 2\theta + \sin 2\theta = 0$.

8. Find the general values of θ for which the quadratic function $(\sin\theta)x^2 + (2\cos\theta)x + \frac{\cos\theta+\sin\theta}{2}$ is the square of a linear function.

9. Let $f(x) = \sin^6 x + \cos^6 x + k(\sin^4 x + \cos^4 x)$ for some real number k. Determine
   (a) all real numbers k for which f(x) is constant for all values of x.
   (b) all real numbers k for which there exists a real number ' c ' such that f(c) = 0.
   (c) If k = −0.7, determine all solutions to the equation f(x) = 0.

10. If α and β are the roots of the equation, $a\cos\theta + b\sin\theta = c$ then match the entries of column-I with the entries of Column-II.

    Column-I | **Column-II**

    (A) $\sin\alpha + \sin\beta$ (P) $\frac{2b}{a+c}$

    (B) $\sin\alpha \cdot \sin\beta$ (Q) $\frac{c-a}{c+a}$

    (C) $\tan\frac{\alpha}{2} + \tan\frac{\beta}{2}$ (R) $\frac{2bc}{a^2+b^2}$

    (D) $\tan\frac{\alpha}{2} \cdot \tan\frac{\beta}{2} =$ (S) $\frac{c^2-a^2}{a^2+b^2}$

11. Solve the inequality $\sin 2x > \sqrt{2}\sin^2 x + (2 - \sqrt{2})\cos^2 x$.

12. Find the set of values of ' a ' for which the equation, $\sin^4 x + \cos^4 x + \sin 2x + a = 0$ possesses solutions. Also find the general solution for these values of 'a'.

13. $\sin^4 x + \cos^4 x - 2\sin^2 x + \frac{3}{4}\sin^2 2x = 0$.

14. Solve: $\tan^2 x \cdot \tan^2 3x \cdot \tan 4x = \tan^2 x - \tan^2 3x + \tan 4x$.

15. Find the set of values of x satisfying the equality
    $\sin\left(x - \frac{\pi}{4}\right) - \cos\left(x + \frac{3\pi}{4}\right) = 1$ and the inequality $\frac{2\cos 7x}{\cos 3 + \sin 3} > 2^{\cos 2x}$

16. Solve: $\sin\left(\frac{\sqrt{x}}{2}\right) + \cos\left(\frac{\sqrt{x}}{2}\right) = \sqrt{2}\sin\sqrt{x}$.

17. Let S be the set of all those solutions of the equation,

    $(1 + k)\cos x \cos(2x - \alpha) = (1 + k\cos 2x)\cos(x - \alpha)$ which are independent of k & α. Let H be the set of all such solutions which are dependent on k & α. Find the condition on k & α such that H is a non-empty set, state S. If a subset of H is $(0, \pi)$ in which k = 0, then find all the permissible values of α.

18.   Solve for x & y $x\cos^3 y + 3x\cos y \sin^2 y = 14$,

$x\sin^3 y + 3x\cos^2 y \sin y = 13$

19.   Find all values of 'a' for which every root of the equation, $a\cos 2x + |a|\cos 4x + \cos 6x = 1$ is also a root of the equation, $\sin x \cos 2x = \sin 2x \cos 3x - \frac{1}{2}\sin 5x$, and conversely, every root of the second equation is also a root of the first equation.

20.   Solve the equations for ' x ' given in **column-I** and match with the entries of **column-II**.

Column-I	Column-II
(A) $\cos 3x \cdot \cos^3 x + \sin 3x \cdot \sin^3 x = 0$	(P) $n\pi \pm \frac{\pi}{3}$
(B) $\sin 3\alpha = 4\sin\alpha \sin(x+\alpha)\sin(x-\alpha)$ where α is a constant $\neq n\pi$.	(Q) $n\pi + \frac{\pi}{4}, n \in I$
(C) $|2\tan x - 1| + |2\cot x - 1| = 2$.	(R) $\frac{n\pi}{4} + \frac{\pi}{8}, n \in I$
(D) $\sin^{10}x + \cos^{10}x = \frac{29}{16}\cos^4 2x$.	(S) $\frac{n\pi}{2} \pm \frac{\pi}{4}$

21.   Find the value(s) of θ in $[0,2\pi]$ for which the expression $y = \frac{\tan(x-\theta) + \tan x + \tan(x+\theta)}{\tan(x-\theta)\tan x \tan(x+\theta)}$ is independent of x.

EXERCISE–II

1. The value of θ satisfying : $3\cos^2\theta - 2\sqrt{3}\sin\theta\cos\theta - 3\sin^2\theta = 0$ are :
    (A) $n\pi - \frac{2\pi}{3}, n\pi + \frac{\pi}{6}$ (B) $n\pi - \frac{\pi}{3}, n\pi + \frac{\pi}{6}$
    (C) $2n\pi - \frac{\pi}{3}, n\pi - \frac{\pi}{6}$ (D) $2n\pi + \frac{\pi}{3}, n\pi - \frac{\pi}{6}$

2. The general solution of the equation $\tan2\theta \cdot \tan\theta = 1$ for $n \in I$ is, θ is equal to :
    (A) $(2n + 1)\frac{\pi}{4}$ (B) $n\pi \pm \frac{\pi}{6}$
    (C) $(2n + 1)\frac{\pi}{2}$ (D) $(2n + 1)\frac{\pi}{3}$

3. The number of distinct solutions of $\sin5\theta \cdot \cos3\theta \cdot \cos7\theta = 0$ in $[0, \pi/2]$ is :
    (A) 4   (B) 5   (C) 9   (D) 8

4. The number of solutions of the equation
    $\log_2(\sin x + \cos x) - \log_2(\sin x) + 1 = 0$ in $x \in [0, 2\pi]$ is
    (A) 3   (B) 2   (C) 1   (D) 0

5. Total number of solution of $16^{\cos^2 x} + 16^{\sin^2 x} = 10$ in $x \in [0, 3\pi]$ is equal to :
    (A) 4   (B) 8   (C) 12   (D) 16

6. The set of values of x for which $\sin x \cdot \cos^3 x > \cos x \cdot \sin^3 x, 0 \le x \le 2\pi$, is NOT satisfied is :
    (A) $\left(0, \frac{\pi}{4}\right)$ (B) $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$ (C) $\left(\frac{\pi}{2}, \frac{3\pi}{4}\right)$ (D) $\left(\frac{3\pi}{2}, \frac{7\pi}{4}\right)$

7. Find the general solution of x , $\cos^2 2x + \cos^2 3x = 1$
    (A) $\frac{k\pi}{5} + \frac{\pi}{10}, k \in I$ (B) $(k\pi + 1)\frac{\pi}{10}; k \in I$
    (C) $\frac{k\pi}{5} - \frac{\pi}{10}, k \in I$ (D) Both (A) and (C)

8. If $\tan\theta + \tan4\theta + \tan7\theta = \tan\theta\tan4\theta\tan7\theta$, then number of solutions in $\theta \in [0, \pi]$
    (A) 10   (B) 11   (C) 12   (D) 13

9. The roots of the equation, $\cot x - \cos x = 1 - \cot x \cdot \cos x$ are (where $n \in I$)
    (A) $n\pi + \frac{\pi}{4}$ (B) $2n\pi + \frac{\pi}{4}$ (C) $n\pi + \frac{\pi}{4}$ or $n\pi$ (D) $(4n + 1)\frac{\pi}{4}$ or $(2n + 1)\pi$

10. Number of solutions of the equation $\sin7\theta = \sin\theta + \sin3\theta$ in $0 < \theta < \frac{\pi}{2}$ is equal to
    (A) 1   (B) 2   (C) 3   (D) 4

11. Statement-1: The equation $\sin(\cos x) = \cos(\sin x)$ has no real solution.
    Statement-2: $\sin x \pm \cos x$ is bounded in $[-\sqrt{2}, \sqrt{2}]$
    (A) Statemetn-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.
    (B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1.
    (C) Statement- 1 is true, statement-2 is false.
    (D) Statement- 1 is false, statement- 2 is true.

12. Consider the system of equations $\sin x \cos 2y = (a^2 - 1)^2 + 1$ and $\cos x \sin 2y = a + 1$.
    Which of the following ordered pairs (x, y) of real numbers does not satisfy the given system of equations for permissible real values of a ?
    (A) $\left(\frac{-\pi}{2}, \frac{-\pi}{2}\right)$ (B) $\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$ (C) $\left(\frac{3\pi}{2}, \frac{-\pi}{2}\right)$ (D) $\left(\frac{-\pi}{2}, \frac{3\pi}{2}\right)$

13. Sum of all the solutions in $[0,4\pi]$ of the equation $\tan x + \cot x + 1 = \cos\left(x + \frac{\pi}{4}\right)$ is $k\pi$ then the value of k is

(A) 3.5   (B) 2.5   (C) 2   (D) 2.25

14. If the quadratic equation $x^2 + (2 - \tan\theta)x - (1 + \tan\theta) = 0$ has two integral roots, then sum of all possible values of θ in interval $(0,2\pi)$ is $k\pi$. Find the value of k.

(A) 1   (B) 2   (C) 3   (D) 4

15. Find the number of solutions of the equation $2\cos 3x\,(3 - 4\sin^2 x) = 1$ in $[0,2\pi]$.

(A) 6   (B) 8   (C) 10   (D) 12

EXERCISE–III

1. Solve the following system of equations for x and y   **[REE 2001 (Mains), 3]**

    $5^{(\csc^2 x - 3\sec^2 y)} = 1$ and $2^{(2\csc x + \sqrt{3}|\sec y|)} = 64$

2. The number of solutions of $\tan x + \sec x = 2\cos x$ in $[0, 2\pi)$ is :   **[JEE 2002]**
    (A) 2   (B) 3   (C) 0   (D) 1

3. The number of integral values of k for which the equation $7\cos x + 5\sin x = 2k + 1$ has a solution is   **[JEE 2002 (Screening), 3]**
    (A) 4   (B) 8   (C) 10   (D) 12

4. $\cos(\alpha - \beta) = 1$ and $\cos(\alpha + \beta) = 1/e$, where $\alpha, \beta \in [-\pi, \pi]$, numbers of pairs of α, β which satisfy both the equations is   **[JEE 2005 (Screening)]**
    (A) 0   (B) 1   (C) 2   (D) 4

5. The number of values of x in the interval $[0, 3\pi]$ satisfying the equation

    $2\sin^2 x + 5\sin x - 3 = 0$ is   **[JEE 2006 (Screening)]**
    (A) 4   (B) 6   (C) 1   (D) 2

6. If $0 \le x < 2\pi$, then the number of real values of x, which satisfy the equation

    $\cos x + \cos 2x + \cos 3x + \cos 4x = 0$, is :   **[JEE Main 2016]**
    (A) 3   (B) 5   (C) 7   (D) 9

EXERCISE–IV

1. Find the general values of x and y satisfying the equations

    $5 \sin x \cos y = 1$, $4 \tan x = \tan y$ **[REE '98, 6]**

2. Find real values of x for which, $27^{\cos 2x} \cdot 81^{\sin 2x}$ is minimum. Also find this minimum value.

    [REE 2000, 3]

3. If $0 < \theta < 2\pi$, then the intervals of values of θ for which $2\sin^2\theta - 5\sin\theta + 2 > 0$, is

    (A) $\left(0, \frac{\pi}{6}\right) \cup \left(\frac{5\pi}{6}, 2\pi\right)$ (B) $\left(\frac{\pi}{8}, \frac{5\pi}{6}\right)$ **[JEE 2006, 3]**

    (C) $\left(0, \frac{\pi}{8}\right) \cup \left(\frac{\pi}{6}, \frac{5\pi}{6}\right)$ (D) $\left(\frac{41\pi}{48}, \pi\right)$

4. The number of solutions of the pair of equations   **[JEE 2007, 3]**
    $2\sin^2\theta - \cos 2\theta = 0$

    $2\cos^2\theta - 3\sin\theta = 0$

    in the interval $[0, 2\pi]$ is
    (A) zero   (B) one   (C) two   (D) four

5. The number of all possible values of θ, where $0 < \theta < \pi$, for which the system of equations

    $(y + z)\cos 3\theta = (xyz)\sin 3\theta$; $x \sin 3\theta = \frac{2\cos 3\theta}{y} + \frac{2\sin 3\theta}{z}$

    $(xyz)\sin 3\theta = (y + 2z)\cos 3\theta + y \sin 3\theta$

    have a solution (x_0, y_0, z_0) with $y_0 z_0 \neq 0$, is   **[JEE 2010]**

6. The number of values of θ in the interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ such that $\theta \neq \frac{n\pi}{5}$ for $n = 0, \pm1, \pm2$ and
    $\tan\theta = \cot 5\theta$ as well as $\sin 2\theta = \cos 4\theta$ is   **[JEE 2010]**

7. Let $P = \{\theta : \sin\theta - \cos\theta = \sqrt{2}\cos\theta\}$ and $Q = \{\theta : \sin\theta + \cos\theta = \sqrt{2}\sin\theta\}$ be two sets. Then
    (A) $P \subset Q$ and $Q - P \neq Q$ (B) $Q \not\subset P$
    (C) $P \not\subset Q$ (D) $P = Q$ **[JEE 2011]**

8. Let $\theta, \varphi \in [0, 2\pi]$ be such that $2\cos\theta (1 - \sin\varphi) = \sin^2\theta \left(\tan\frac{\theta}{2} + \cot\frac{\theta}{2}\right)\cos\varphi - 1$,

    $\tan(2\pi - \theta) > 0$ and $-1 < \sin\theta < -\frac{\sqrt{3}}{2}$. Then φ cannot satisfy   **[JEE 2012]**
    (A) $0 < \varphi < \frac{\pi}{2}$ (B) $\frac{\pi}{2} < \varphi < \frac{4\pi}{3}$
    (C) $\frac{4\pi}{3} < \varphi < \frac{3\pi}{2}$ (D) $\frac{3\pi}{2} < \varphi < 2\pi$

9. For $x \in (0, \pi)$, the equation $\sin x + 2\sin 2x - \sin 3x = 3$ has :   **[JEE Advanced 2014]**
    (A) infinitely many solutions   (B) three solutions
    (C) one solution   (D) no solution

10. The number of distinct solutions of the equation   **[JEE Advanced 2015]**

$\frac{5}{4}\cos^2 2x + \cos^4 x + \sin^4 x + \cos^6 x + \sin^6 x = 2$ in the interval $[0, 2\pi]$ is

11. Let $S = \left\{x \in (-\pi, \pi) : x \neq 0, \pm\frac{\pi}{2}\right\}$. The sum of all distinct solutions of the equation

$\sqrt{3} \sec x + \operatorname{cosec} x + 2(\tan x - \cot x) = 0$ in the set S is equal to   **[JEE Advanced 2016]**

(A) $-\frac{7\pi}{9}$ (B) $-\frac{2\pi}{9}$ (C) 0   (D) $\frac{5\pi}{9}$

12. Let $f : [0, 2] \to R$ be the function defined by   **[JEE Advanced 2020]**

$f(x) = (3 - \sin(2\pi x)) \sin\left(\pi x - \frac{\pi}{4}\right) - \sin\left(3\pi x + \frac{\pi}{4}\right)$

If $\alpha, \beta \in [0, 2]$ are such that $\{x \in [0, 2] : f(x) \geq 0\} = [\alpha, \beta]$, then the value of $\beta - \alpha$ is_____

TRIGONOMETRIC EQUATIONS

Answer Key

PROFICIENCY TEST-01

1. $x = (-1)^n \frac{\pi}{6} + \pi n \ (n \in Z).$

2. $\frac{\pi(8n\pm1)}{28}, \frac{\pi(2k+1)}{14} (n, k \in Z)$

3. $\pi n, \frac{2\pi(3k\pm1)}{3} (n, k \in Z)$

4. $\pi + 2\pi n, \frac{\pi(6k+(-1)^k)}{6} (n, k \in Z)$

5. $\frac{\pi(4n+1)}{2}, \pi k, (n, k \in Z)$

6. $\frac{\pi n}{2}, \frac{\pi(6k\pm1)}{3} (n, k \in Z)$

7. $\pi n/2, (n \in N)$

8. $\frac{\pi(4k+1)}{2}, \frac{\pi(3k-1)}{3} (n, k \in Z)$

9. $\frac{\pi(4n+1)}{2}, \frac{\pi(6k+(-1)^k)}{6} (n, k \in Z)$

10. $2\pi n (n \in Z)$

11. $\frac{\pi n}{4} (n \in Z)$

PROFICIENCY TEST-02

1. $\frac{\pi(4n-1)}{4}, \frac{\pi(3k\pm1)}{3} (n, k \in Z)$

2. $2\pi n, \frac{\pi(4\,m-1)}{2}, \frac{\pi(4k+1)}{4} (n, m, k \in Z)$

3. $\frac{\pi n}{3}, \frac{\pi(2k+1)}{8} (n, k \in Z)$

4. $\frac{2\pi n}{5}, \frac{\pi(2k+1)}{2}, \pi(2\,m+1) (n, k, m \in Z)$

5. $\frac{\pi(8n+3)}{24}, \frac{\pi(8k+1)}{16} (n, k \in Z)$

6. $\arctan(2 \pm \sqrt{3}) + \pi n \equiv \frac{\pi(6n+(-1)^n)}{12} (n \in Z)$

7. $\frac{\pi(2n+1)}{4} (n \in Z)$

8. $\frac{\pi(4k+1)}{10} (k \in Z)$

9. $\pi n, \frac{\pi(2k+1)}{20} (n, k \in Z)$

10. $\frac{\pi(4n+1)}{4} (n \in Z)$

11. $\frac{\pi n}{5}, \frac{\pi(2k+1)}{2} (n, k \in Z)$

PROFICIENCY TEST-03

1. $\frac{\pi(2n+1)}{8}, \frac{\pi(3k\pm1)}{3} (n, k \in Z)$

2. $\frac{\pi(4k-1)}{4}, \frac{\pi(3n\pm1)}{3} (n, k \in Z)$

3. $\frac{\pi(2n+1)}{4}, \frac{\pi(6k+(-1)^k)}{12}$

4. $\pi n + \arctan(-1 \pm \sqrt{3}) (n \in Z)$

5. $\frac{\pi(6n+(-1)^n)}{6} (n \in Z)$

6. $\frac{\pi(4n-1)}{4}, 2\pi m, \frac{\pi(4k-1)}{2}, \frac{\pi(4\ell+1)}{4} + (-1)^\ell \arcsin\left(\frac{\sqrt{2}}{4}\right) (n, m, k, \ell \in Z)$

7. $\frac{\pi(8k+1)}{4}, \frac{\pi(8n+3)}{12} (n, k \in Z)$

8. $\pi n - \arctan(1/2) (n \in Z)$

9. $\frac{\pi(2n+1)}{6}, \frac{2\pi(3k\pm1)}{9} (n, k \in Z)$ **10.** $\frac{\pi(6n-(-1)^n)}{18} (n \in Z)$.

11. $\frac{2\pi(3n\pm1)}{3} (n \in Z)$.

EXERCISE-I

1. $x = \frac{n\pi}{7} - \frac{\pi}{84}$ or $x = \frac{n\pi}{4} + \frac{7\pi}{48}, n \in I$ **2.** $\frac{\pi}{8}, \frac{\pi}{3}, \frac{3\pi}{8}, \frac{5\pi}{8}, \frac{2\pi}{3}, \frac{7\pi}{8}$

3. $\alpha - 2\pi; \alpha - \pi, \alpha, \alpha + \pi$, where $\tan \alpha = \frac{2}{3}$ **4.** $x = \pi/16$

5. $x = 2n\pi + \frac{\pi}{12}, n \in I$ **6.** $x = -\frac{5\pi}{3}$

7. $\theta = 2n\pi$ or $2n\pi + \frac{\pi}{2}; n \in I$ **8.** $2n\pi + \frac{\pi}{4}$ or $(2n+1)\pi - \tan^{-1} 2, n \in I$

9. (a) $-\frac{3}{2}$ (b) $k \in \left[-1, -\frac{1}{2}\right]$ (c) $x = \frac{n\pi}{2} \pm \frac{\pi}{6}$

10. (A) R ;(B) S ;(C) P ; (D) Q   **11.** $n\pi + \frac{\pi}{8} < x < n\pi + \frac{\pi}{4}$

12. $\frac{1}{2}\left[n\pi + (-1)^n \sin^{-1}(1 - \sqrt{2a+3})\right]$ where $n \in I$ and $a \in \left[-\frac{3}{2}, \frac{1}{2}\right]$

13. $n\pi \pm \frac{1}{2}\cos^{-1}(2 - \sqrt{5})$ **14.** $\frac{(2n+1)\pi}{4}, k\pi$, where $n, k \in I$

15. $x = 2n\pi + \frac{3\pi}{4}, n \in I$

16. $x = \left(4n\pi + \frac{\pi}{2}\right)^2$ or $x = \left(\frac{4m\pi}{3} + \frac{\pi}{2}\right)^2$ where $m, n \in W$.

17. (i) $|k\sin \alpha| \le 1$ (ii) $S = n\pi, n \in I$ (iii) $\alpha \in (-m\pi, 2\pi - m\pi) m \in I$

18. $x = \pm 5\sqrt{5}$ & $y = n\pi + \tan^{-1}\frac{1}{2}$ **19.** $a = 0$ or $a < -1$

20. (A) S ;(B) P ; (C) Q ;(D) R   **21.** $\theta = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

EXERCISE-II

1. B   **2.** B   **3.** D   **4.** C   **5.** C   **6.** B   **7.** D

8. C   **9.** A   **10.** C   **11.** A   **12.** B   **13.** A   **14.** D

15. C

EXERCISE–III

1. $x = n\pi + (-1)^n \frac{\pi}{6}$ and $y = m\pi \pm \frac{\pi}{6}$ where m & n are integers.

2. A   **3.** B   **4.** D   **5.** A   **6.** C

EXERCISE–IV

1. $y = (n-m)\frac{\pi}{2} + (-1)^n\frac{\pi}{4} - (-1)^m\frac{\alpha}{2}$; $x = (m+n)\frac{\pi}{2} + (-1)^n\frac{\pi}{4} + (-1)^m\frac{\alpha}{2}$

where $\alpha = \sin^{-1}\left(-\frac{3}{5}\right)$, $m, n \in I$

2. Min. value $= 3^{-5}$ for $x = (4n-1)\frac{\pi}{4} - \frac{1}{2}\tan^{-1}\frac{3}{4}$, $n \in I$; max.

value $= 3^5$ for $x = (4n+1)\frac{\pi}{4} - \frac{1}{2}\tan^{-1}\frac{3}{4}$, $n \in I$

3. A   **4.** C   **5.** 3   **6.** 3   **7.** D   **8.** A, C, D

9. D   **10.** 8   **11.** C   **12.** 1.00