

$\text{Q}_1 \vee 7 \vee 8 \vee$

$$\text{Q}_2 \quad A_1 A_2 \rightarrow a_1, A_1 A_2 b \rightarrow AP \Rightarrow A_1 + A_2 = a + b$$

$$G_1 G_2 \rightarrow g_1, G_1 G_2 b \rightarrow GP \rightarrow ab = g_1 g_2$$

$$H_1 H_2 \rightarrow h_1, H_1 H_2 b \rightarrow HP \rightarrow$$

$$\frac{1}{a}, \frac{1}{H_1}, \frac{1}{H_2}, \frac{1}{b} \rightarrow AP \rightarrow \frac{1}{H_1} + \frac{1}{H_2} = \frac{1}{a} + \frac{1}{b}$$

$$\frac{A_1 + A_2}{H_1 H_2} = ?$$

$$\frac{H_1 + H_2}{H_1 H_2} = \frac{a+b}{ab}$$

$$\frac{H_1 + H_2}{H_1 H_2} \xleftarrow{\text{?}} \frac{A_1 + A_2}{g_1 g_2}$$

$$\text{P: } \frac{g_1 g_2}{H_1 H_2} = \frac{A_1 + A_2}{H_1 + H_2} \quad \boxed{B}$$

$\text{Q}_3 \quad \text{Chhad Dena}$   
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$$\text{Q}_5 \quad x^2 - 2AMx + GM^2 = 0$$

$$x^2 - 18x + 16 = 0 \quad \&$$

$$\text{Q}_9 \quad \frac{HM}{GM} = \frac{4}{5} \quad \text{Then } \frac{a}{b} = ? \quad \boxed{A}$$

$$\frac{2ab\sqrt{ab}}{a+b} = \frac{4}{5}$$

$$\frac{2\sqrt{ab}}{a+b} = \frac{4}{5}$$

$$(2x-y)(x-2y) = 0$$

$$2x = y \quad \text{OR} \quad l = 2y$$

$$2a + 2b - 5\sqrt{ab} = 0$$

$$\frac{x}{y} = \frac{1}{2} \quad \frac{y}{x} = 2$$

$$2x^2 + 2y^2 - 5xy = 0$$

$$2x^2 - 4xy - xy + 2y^2 = 0 \quad \frac{\sqrt{a}}{\sqrt{b}} = \frac{1}{2} \quad \sqrt{\frac{a}{b}} = \frac{1}{2}$$

$$2((x-2y) - y(x-2y)) = 0$$

$$\text{Q10} \quad \frac{A}{h} = \frac{m}{n}$$

$$A = \frac{m}{n} h.$$

$$\frac{\alpha}{\beta} = \frac{A + \sqrt{A^2 - h^2}}{A - \sqrt{A^2 - h^2}}$$

$$= \frac{\frac{m}{n} h + \sqrt{\frac{m^2}{n^2} h^2 - h^2}}{\frac{m}{n} h - \sqrt{\frac{m^2}{n^2} h^2 - h^2}}$$

$$\frac{\alpha}{\beta} = \frac{m + \sqrt{m^2 - n^2}}{m - \sqrt{m^2 - n^2}}$$

$$\text{Q11} \quad A = \frac{75}{4}, h = 15$$

$$\alpha = \frac{75}{4} + \sqrt{\left(\frac{75}{4}\right)^2 - (15)^2} = \frac{75}{4} + (15) \sqrt{\left(\frac{5}{4}\right)^2 - 1^2} = \frac{75}{4} + 15 \times \frac{3}{4} \\ = \frac{120}{4} = 30$$

$$\beta = \frac{75}{4} - \sqrt{\left(\frac{75}{4}\right)^2 - (15)^2}$$

12)  $\checkmark$ , 13 (opy 15) 1,  $A_1 A_2 A_3 \dots A_n$  51

$$\frac{A_4}{A_7} = \frac{3}{5} \quad \text{then } n=9$$

$$d = \frac{5l-1}{n+1} = \frac{50}{n+1} = 2 \quad \begin{array}{l} n+l=26 \\ \downarrow \\ n=24 \end{array}$$

$$\frac{l+4d}{l+7d} = \frac{3}{5} \Rightarrow 5+20d = 3+21d \\ 2=d$$

16) to by  $\rightarrow$ 

$$\stackrel{?}{=} a, A, b \rightarrow AP$$

$$a, P, q, b \rightarrow HP$$

$$P^3 + q^3 = 2APq \quad (\text{Q सिक्या})$$

$$\frac{P^3 + q^3}{2APq} = 2A \quad \textcircled{A} \checkmark$$

$$Q18) \quad a, b, c \rightarrow GP \Rightarrow \boxed{b^2 = ac}$$

$$a, x, b \& \quad b, y, c \rightarrow AP$$

$$\frac{2x = a+b}{a = 2x-b} \quad \frac{2y = b+c}{c = 2y-b}$$

$$a = 2x - b \quad (= 2y - b)$$

$$b^2 = (2x-b)(2y-b)$$

$$b^2 = 4xy - 2by - 2xb + b^2$$

$$2b(x+y) = 4xy$$

$$\frac{x+y}{xy} = \frac{2}{b}$$

$$\frac{1}{x} + \frac{1}{y} = \frac{2}{b} \quad \boxed{D}$$

$$(9) \quad A = \frac{3}{2}, \quad H = \frac{4}{3}$$

$$\frac{\alpha+\beta}{x} = \frac{3}{2} \quad \left| \begin{array}{l} \frac{H+M}{\alpha+\beta} = \frac{4}{3} \\ \frac{2\alpha\beta}{3} = \frac{4}{3} \end{array} \right.$$

$$\alpha+\beta=3 \quad \alpha\beta=2$$

$$x^2 - 3x + 2 = 0 \quad \boxed{B}$$

20 ✓

21 Chhod Dena.  
22

$$23) \quad \text{G GM of } x, y.$$

$$G = \sqrt{xy}$$

$$\text{Demand} \rightarrow \frac{1}{G^2 - x^2} + \frac{1}{G^2 - y^2} = \frac{1}{(x^2 - y^2)^2} + \frac{1}{x^2 y^2 - y^2} = \frac{1}{x^2(y-x)(y+x)} + \frac{1}{y^2(x-y)(x+y)}$$

$$\frac{1}{x+y} \left\{ \frac{1}{x^2(y-x)} - \frac{1}{y^2(y-x)} \right\}$$

$$\frac{1}{x+y} \left\{ \frac{y^2 - x^2(y+x)}{x^2 y^2 (x-y)} \right\} = \frac{1}{x^2 y^2}$$

 $\boxed{F.P.}$

$$Q4) \alpha = A + \sqrt{A^2 - h^2}$$

$$\beta = A - \sqrt{A^2 - h^2}$$

26 ✓

27 ✓

H P  
1 ✓ 2 ✓

$$Q5) a = b + \frac{c}{2}$$

$$b, \zeta_1, \zeta_2, c \rightarrow \mathbb{P}^1$$

$$\zeta_1 = b \cdot r = b \cdot \left(\frac{c}{b}\right)^{\frac{1}{2+1}} = b \cdot \cancel{c}^{\frac{1}{3}} = b^{\frac{2}{3}} \cdot c^{\frac{1}{3}}$$

$$\zeta_2 = b \cdot r^2 = b \cdot \left(\frac{c}{b}\right)^{\frac{2}{2+1}} = \cancel{b}^{\frac{1}{3}} \cdot \cancel{c}^{\frac{2}{3}} = b^{\frac{1}{3}} \cdot c^{\frac{2}{3}}$$

$$\zeta_1^3 + \zeta_2^3$$

## Basic Relation betn AM, GM & HM.

$$\text{And } AM = \frac{a+b}{2}, \quad GM = \sqrt{ab}, \quad HM = \frac{2ab}{a+b}$$

$$2A = a+b \quad G^2 = ab$$

$$H = \frac{2ab}{a+b}$$

$$H = \frac{2G^2}{2A}$$

$$G^2 = AH$$

$$b^2 = ac$$

Jesi  
jeet

A, G, H  $\rightarrow$  GP.

1) If A & G are AM, GM of 2 No then No's Are

$$\alpha = A + \sqrt{A^2 - G^2}$$

$$\beta = A - \sqrt{A^2 - G^2}$$

2) If A, H are AM & HM of 2 Nos then No's Are

$$\alpha = A + \sqrt{A^2 - AH}$$

$$\beta = A - \sqrt{A^2 - AH}$$

Q If H is HM betn P & q then  $\frac{H}{P} + \frac{H}{q} = ?$

$$H = \frac{2Pq}{P+q}$$

$$\text{Demand} = H \left( \frac{1}{P} + \frac{1}{q} \right) = H \left( \frac{P+q}{Pq} \right)$$

$$= \frac{2Pq}{P+q} \times \frac{P+q}{Pq} = 2.$$

Q If  $a, a_1, a_2, a_3, \dots, a_{2n}, b$  are in AP

$a, g_1, g_2, g_3, \dots, g_{2n}, b$  are in HP

&  $h$  is HM of  $a$  &  $b$  then P.T.

$$\frac{a_1 + a_{2n}}{g_1 g_{2n}} + \frac{a_2 + a_{2n-1}}{g_2 g_{2n-1}} + \frac{a_3 + a_{2n-2}}{g_3 g_{2n-2}} + \dots + \frac{a_n + a_{n+1}}{g_n g_{n+1}}$$

$$a+b = a_1 + a_{2n} = a_2 + a_{2n-1} = a_3 + a_{2n-2} = \dots$$

$$ab = g_1 g_{2n} = g_2 g_{2n-1} = g_3 g_{2n-2} = \dots$$

$$\text{LHS } \frac{a+b}{ab} + \frac{a+b}{ah} + \frac{a+b}{an} + \dots + \frac{a+b}{ab} = \frac{2n(a+b)}{2ab} = \frac{2n}{h} \text{ RHS.}$$

Q If  $a, x, y, z, b$  are in AP then  $x+y+z=15$   
 If  $a, x, y, z, b$  are in HP then  $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = \frac{5}{3}$   
 then  $a, b$  are

$$1, 9, \quad 2, 8 \quad 3, 7 \quad 4, 6$$

1)  $a, x, y, z, b \rightarrow \text{AP} \rightarrow \text{AM}$

$$\underline{x+y+z} = 3 \frac{(a+b)}{2} = 15$$

$$a+b=10$$

2)  $a, x, y, z, b \rightarrow \text{HP} \rightarrow \text{HM}$

$$\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = \frac{3}{\frac{2ab}{a+b}} \Rightarrow$$

$$\frac{5}{3} = \frac{3 \times 10}{2ab} \Rightarrow ab=9$$

$$1, 9 \text{ or } 9, 1,$$

Q  $a_1, a_2, a_3, \dots, a_{10}$  are in AP.

A.S. &  $h_1, h_2, h_3, \dots, h_{10}$  are in HP.

$$\text{If } a_1 = h_1 = 2 \text{ & } a_{10} = h_{10} = 3$$

$$\text{then } a_4 \cdot h_7 = ?$$

$$a_4 = a_1 + 3d$$

$$= 2 + 3d$$

$$= 2 + \frac{3}{9}$$

$$a_4 = 2 + \frac{1}{3} = \frac{7}{3}.$$

$$a_{10} = a_1 + 9d = 3$$

$$2 + 9d = 3$$

$$d = \frac{1}{9}$$

S.C.

$$a_1 h_{10} = a_2 h_9 = a_3 h_8 = a_4 h_7 \\ 2 \times 3 = a_4 h_7$$

h7

$$h_1 = 2 \text{ & } h_{10} = 3, \quad h_7$$

$$A_1 = \frac{1}{2}$$

$$A_{10} = \frac{1}{3}, \quad \downarrow$$

$$A_1 + 9d = \frac{1}{3}$$

$$\frac{1}{2} + 9d = \frac{1}{3}$$

$$9d = \frac{1}{3} - \frac{1}{2}$$

$$9d = -\frac{1}{6}$$

$$d = -\frac{1}{54}$$

$$a_4 \times h_7 = \frac{7}{3} \times \frac{18}{1} = 6$$

$$A_7 = A_1 + 6d$$

$$= \frac{1}{2} + 6 \times \frac{1}{54}$$

$$= \frac{27 - 6}{54}$$

$$A_7 = \frac{2+7}{54} \frac{1}{18}$$

$$h_7 = \frac{18}{7}$$

Q If  $\alpha, \beta, \gamma, H$  are AM, GM & HM of Roots of cubic

Eqn then find which Eqn?

1) Cubic Eqn in the form of  $\alpha, \beta, \gamma$ .

$$\lambda^3 - (\alpha + \beta + \gamma)\lambda^2 + (\alpha\beta + \beta\gamma + \gamma\alpha)\lambda - \alpha\beta\gamma = 0$$

$$\lambda^3 - 3Ax^2 + 3\frac{h^3}{H} - h^3 = 0$$

2)

$A = \text{AM of } \alpha, \beta, \gamma \Rightarrow \alpha + \beta + \gamma = 3A$	$G = \text{GM of } \alpha, \beta, \gamma \Rightarrow G = (\alpha \cdot \beta \cdot \gamma)^{\frac{1}{3}}$ $h^3 = \alpha\beta\gamma$	$H = \text{HM of } \alpha, \beta, \gamma$ $H = \frac{3}{\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}}$ $H = \frac{3\alpha\beta\gamma}{\alpha\beta + \beta\gamma + \gamma\alpha}$
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$$\alpha + \beta + \gamma = 3A$$

$$G = (\alpha \cdot \beta \cdot \gamma)^{\frac{1}{3}}$$

$$h^3 = \alpha\beta\gamma$$

$$H = \frac{3}{\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}}$$

$$H = \frac{3\alpha\beta\gamma}{\alpha\beta + \beta\gamma + \gamma\alpha}$$

$$\alpha + \beta + \gamma = \frac{3h^3}{H}$$

# Arithmetico Geometric Progression - AHP

1) Sum of n AHP:

$$S = a + (a+d)r + (a+2d)r^2 + (a+3d)r^3 + \dots + (a+(n-1)d)r^{n-1}$$

$$S \cdot r = \frac{a \cdot r + (a+d) \cdot r^2 + (a+2d) \cdot r^3 + \dots + (a+(n-2)d) \cdot r^{n-1} + (a+(n-1)d) \cdot r^n}{-}$$

$$S(1-r) = a + \{dr^1 + dr^2 + dr^3 + \dots + dr^{n-1}\} - (a+(n-1)d)r^{n-1}$$

$\leftarrow$  HP of  $(n-1)$  terms

Q

$$1 + 2 \cdot 2 + 3 \cdot 2^2 + 4 \cdot 2^3 + \dots + 100 \cdot 2^{99} = ?$$

$$S = 1 + 2 \cdot 2 + 3 \cdot 2^2 + 4 \cdot 2^3 + \dots + 100 \cdot 2^{99}$$

$$2S = 2 \cdot 1 + 2 \cdot 2^2 + 3 \cdot 2^3 + \dots + 99 \cdot 2^{99} + 100 \cdot 2^{100}$$

$$-S = 1 + 2(2-1) + 2^2(3-2) + 2^3(4-3) + \dots + 2^{99}(100-99) - 100 \cdot 2^{100}$$

$$-S = 1 + \{2^1 + 2^2 + 2^3 + \dots + 2^{99}\} - 100 \cdot 2^{100}$$

$\leftarrow$  99 terms of HP

$$-S = 1 + 2 \cdot \frac{2^{99} - 1}{(2-1)} - 100 \cdot 2^{100}$$

$$-S = 1 + 2^{100} - 2 - 100 \cdot 2^{100}$$

$$= 2^{100}(1 - 100) - 1$$

$$S = + 99 \cdot 2^{100} + 1$$

$$S = 1 + 99 \cdot 2^{100}$$

Ans.

$$Q) 3 + \frac{1}{4}(3+d) + \frac{1}{4^2}(3+2d) + \dots + \infty = 8 \text{ then } d = ?$$

$$S = 3 + \frac{1}{4}(3+d) + \frac{1}{4^2}(3+2d) + \dots - \infty$$

$$\frac{S}{4} = -\frac{3}{4} + \frac{1}{4^2}(3+d) + \dots - \infty$$

$$\frac{3S}{4} = 3 + \left\{ \frac{d}{4} + \frac{d}{4^2} + \frac{d}{4^3} + \dots - \infty \right\}$$

$\leftarrow \infty \text{ HP} \longrightarrow$

AHP  
HM

$$\frac{3S}{4} = 3 + \frac{\frac{d}{4}}{1 - \frac{1}{4}} = 3 + \frac{d}{\frac{3}{4}}$$

$$\frac{3 \times 8^2}{4} = 3 + \frac{d}{3} \Rightarrow 6 = 3 + \frac{d}{3}$$

$$\frac{d}{3} = 3 \Rightarrow d = 9$$