Nishant Jindal

GRAVITATION

cir

$+q_1 \xrightarrow{F_{1/2}}$ $\xleftarrow{F_{2/1}} q_2$

$|\leftarrow r \rightarrow|$

$$|F_{1/2}| = |F_{2/1}| = \frac{1}{4\pi\varepsilon_0}\frac{|q_1||q_2|}{r^2}$$

$$\frac{1}{4\pi\varepsilon_0} \rightarrow G$$

$$q_1, q_2 \rightarrow m_1, m_2$$

m_1 $\quad$ m_2

$\circ \xrightarrow{} F_{1/2}$ $\quad$ $F_{2/1} \xleftarrow{} \circ$

$|\leftarrow r \rightarrow|$

$$|F_{1/2}| = |F_{2/1}| = \frac{G m_1 m_2}{r^2}$$

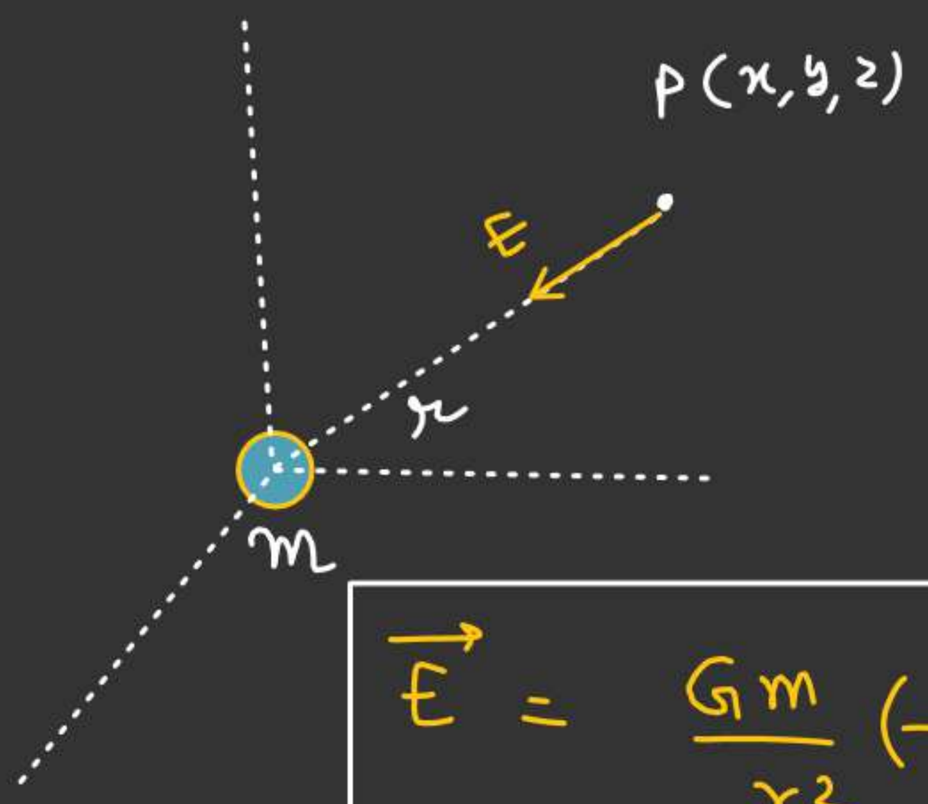
Medium independent

G = Universal gravitational Constant

$$6.67 \times 10^{-11} \frac{N\text{-}m^2}{Kg^2}$$

Nishant Jindal

Gravitational field due to point mass



$$\boxed{\vec{E} = \frac{\vec{F}}{m}}$$

Force acting per unit mass

(N/kg)

$$\boxed{\vec{E} = \frac{Gm}{r^2}(-\hat{r})}$$

$$\boxed{\vec{E} = -\frac{Gm}{r^2}\frac{\vec{r}}{|\vec{r}|} = \left(-\frac{Gm}{r^3}\right)\vec{r}}$$

Nishant Jindal

Gravitational potential due to a point mass

$$V = \left(\frac{U}{m_0}\right)$$

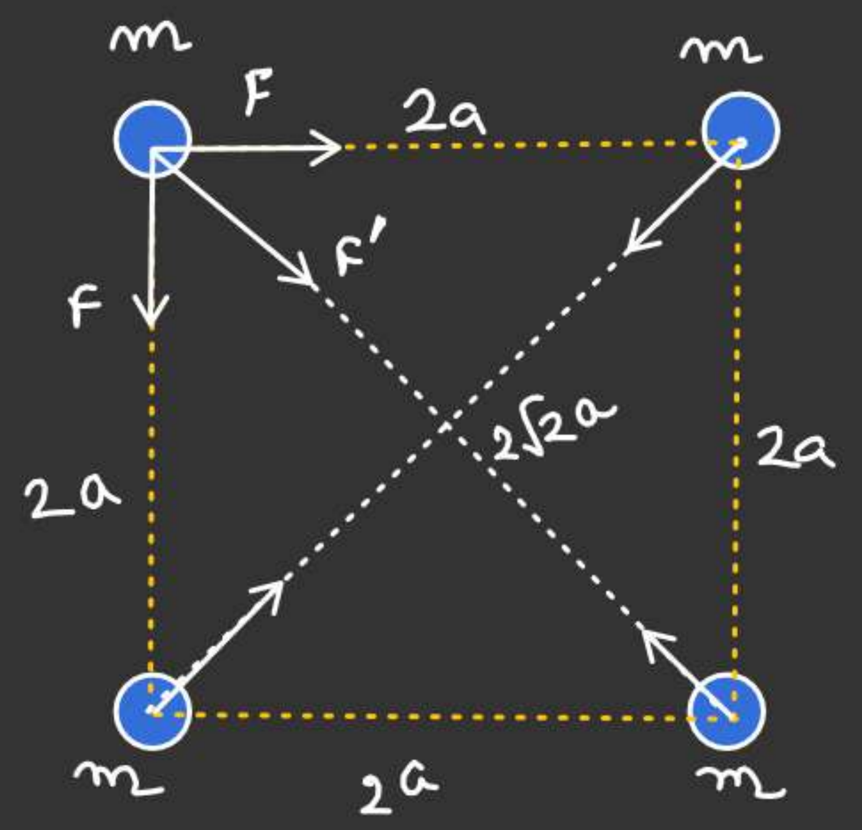
$$\boxed{V_m = -\frac{Gm}{r}}$$

(Gravitation potential energy b/w two point masses)

$$U_{1-2} = (V_{m_1})\, m_2$$

$$\boxed{U_{1-2} = -\frac{Gm_1m_2}{r}}$$

Whole System is kept on a Smooth horizontal table and released from the position shown in fig. Find Speed of each masses if their seperation is a.

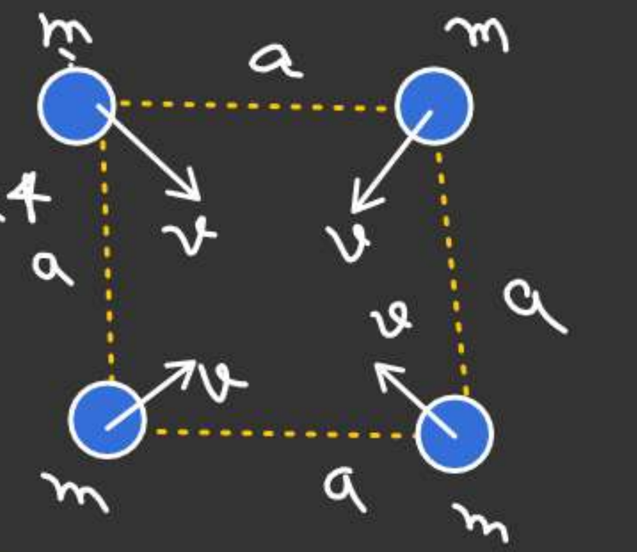


Energy Conservation

$$U_i + K.E_i = U_f + K.E_f$$

$$\downarrow$$

$$-\left(\frac{Gm^2}{2a}\times 4\right) - \frac{Gm^2}{2\sqrt{2}a}\times 2 + 0 = \left(-\frac{Gm^2}{a}\times 4 - \frac{Gm^2}{\sqrt{2}a}\times 2\right) + \frac{1}{2}mv^2\times 4$$



$$\Downarrow$$

$$\underline{v = ??}$$

Nishant Jindal

\# m_1 & m_2 released from very large distance. Find Speed of approach of each masses when they are at a seperation d

Initial

Final

L.M.C

$$p_i = p_f$$

$$0 = m_1 v_1 - m_2 v_2 \quad (1)$$

Energy Conservation

$$U_i + K.E_i = U_f + K.E_f$$

$$0 + 0 = -\frac{Gm_1m_2}{d} + \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 \quad (2)$$

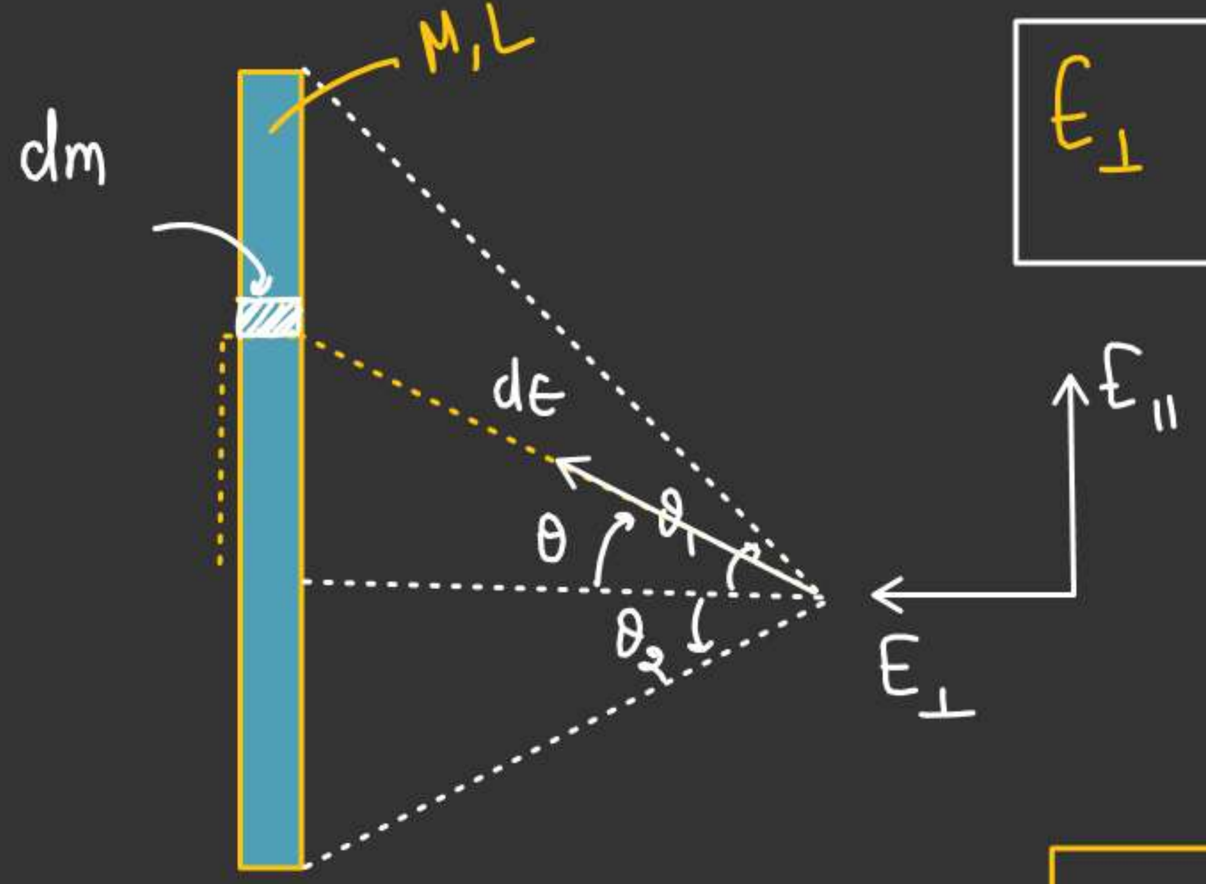
$$\left\{ \begin{aligned} v_1 &= \sqrt{\frac{2Gm_2}{d(m_1+m_2)}} \\ v_2 &= \sqrt{\frac{2Gm_1}{d(m_1+m_2)}} \end{aligned} \right.$$

$$v_{1/2} = (v_1 + v_2) = \sqrt{\frac{2G(m_1+m_2)}{d}} \checkmark$$

Nishant Jindal

Gravitation field and potential due to Continious mass distribution

Gravitational field due to a finite Rod



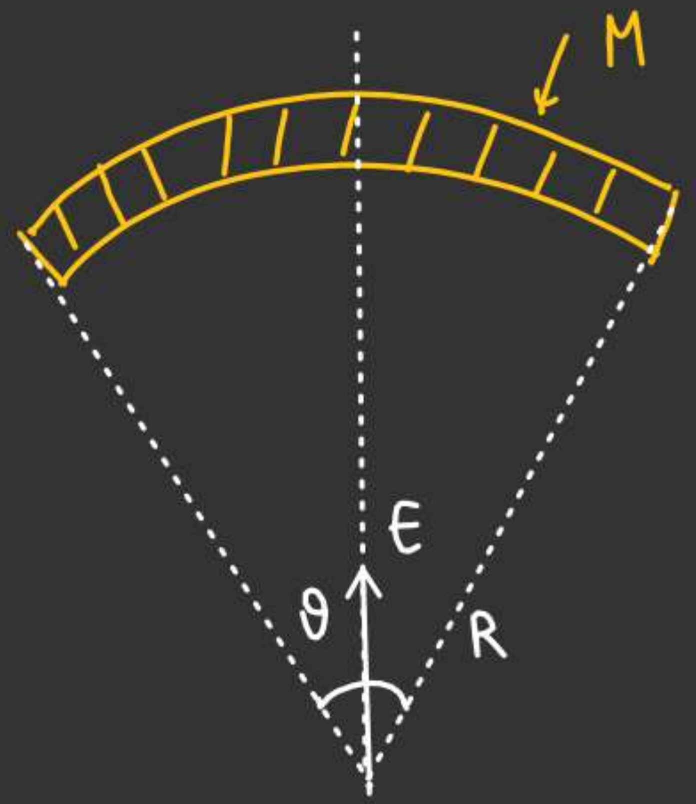
$$\boxed{E_{\perp} = \frac{GM}{Lr}\left(\sin\theta_1 + \sin\theta_2\right)}$$

$$\boxed{E_{\parallel} = \frac{GM}{Lr}\left(\cos\theta_2 - \cos\theta_1\right)}$$

$\theta_1 = \theta_2 = 90^\circ$

$$\boxed{E_{\text{infinitely long}} = \frac{2GM}{Lr}}$$

Gravitational field due to an arc at its center.



$$\boxed{E = \frac{2GM}{R^2}\,\frac{\sin(\theta/2)}{(\theta)}}$$

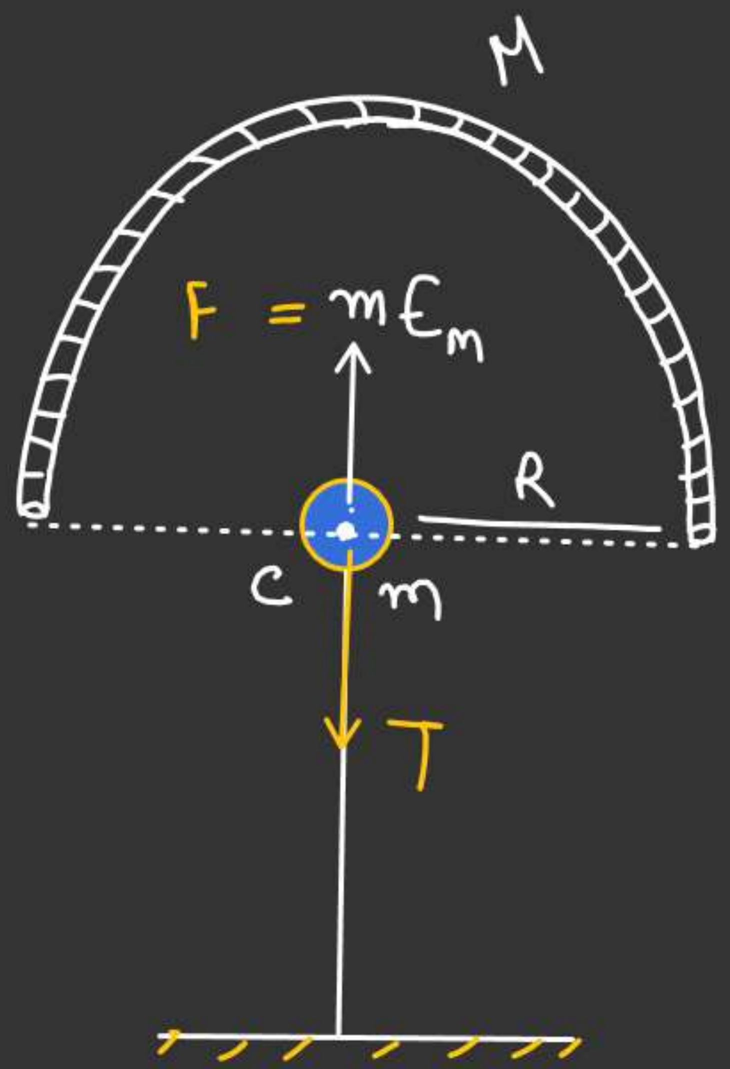
Potential due to an arc at its center

$$\boxed{V = -\frac{GM}{R}}$$

Nishant Jindal

Find tension is the String.

(gravity neglected)



$$mE_m = T$$

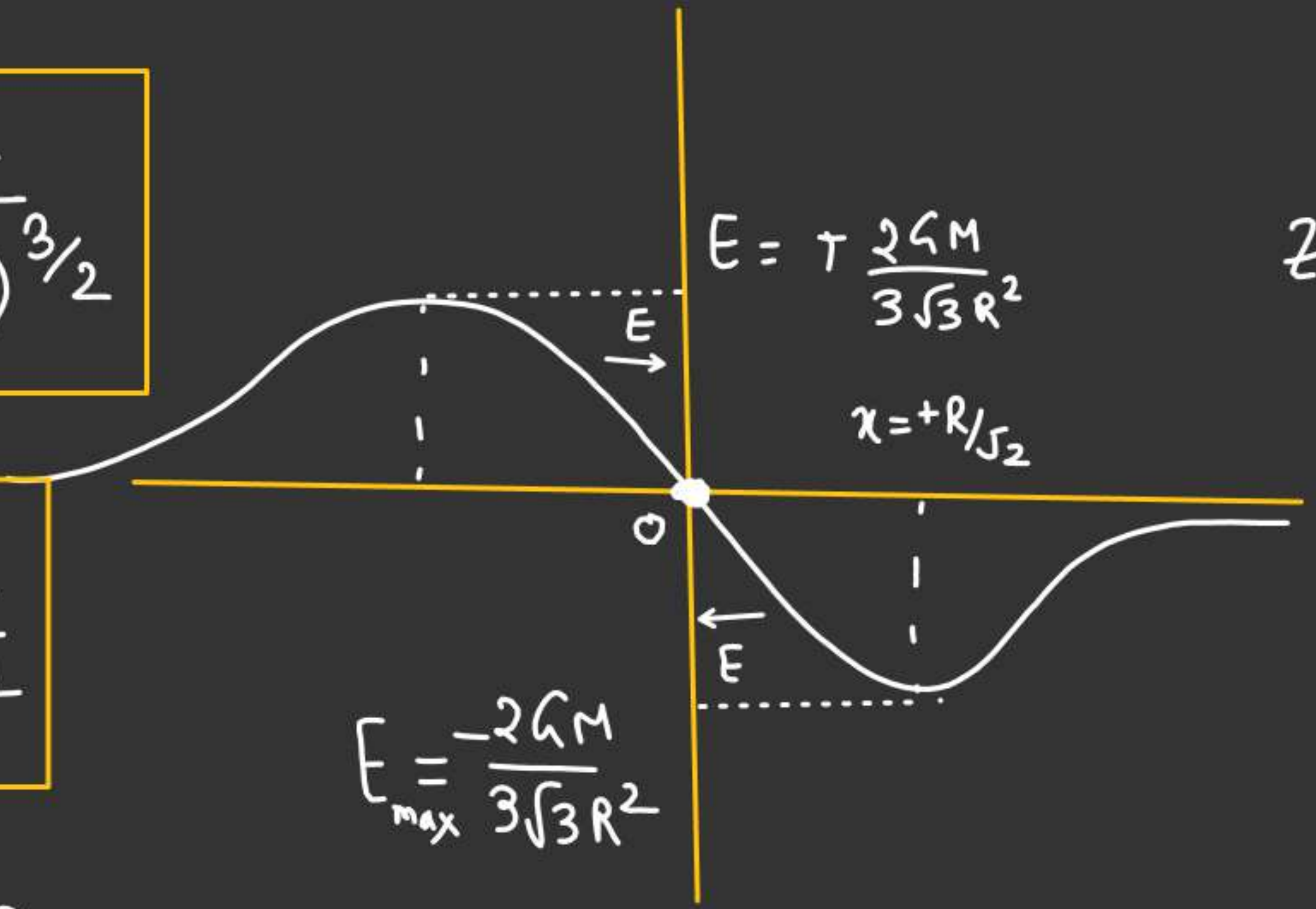
$$m\left(\frac{2GM}{\pi R^2}\right) = T$$

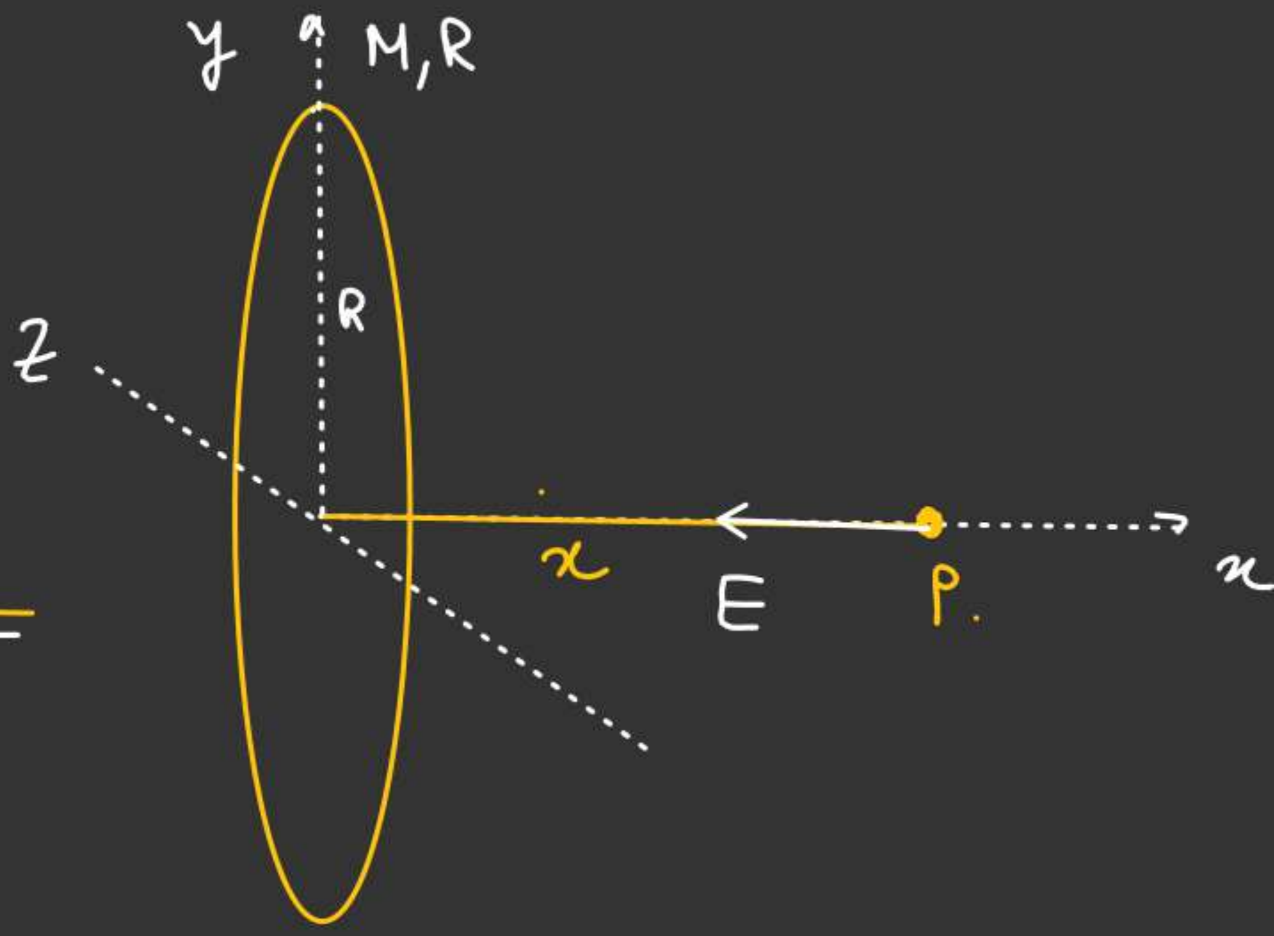
$$\frac{2GMm}{\pi R^2} = T$$

Gravitational field & potential due to a ring at its axis

$$|E| = \frac{GMx}{(x^2+R^2)^{3/2}}$$

$$V = \frac{-GM}{\sqrt{x^2+R^2}}$$

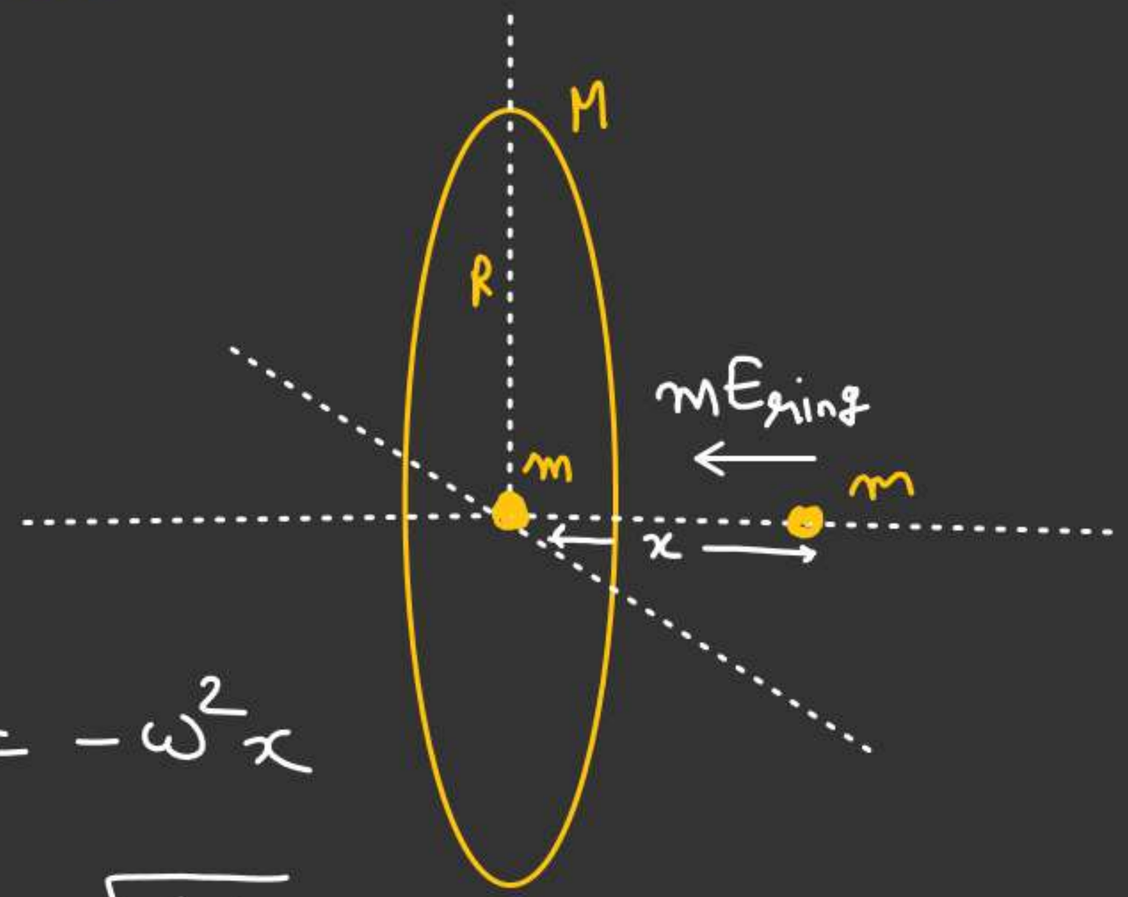




(Point o is a stable equilibrium)

Nishant Jindal

\# m is Slightly displaced along x-axis by a distance x and released. Find it's time period if $x << R$.



$$F_r = -mE_{ring}$$

$$F_r = -m\frac{GMx}{(x^2+R^2)^{3/2}}$$

$$a = \frac{F_r}{m} = -\frac{GMx}{(x^2+R^2)^{3/2}}$$

$$a = -\frac{GMx}{R^3\left(1+\underset{\downarrow\, 0}{\frac{x^2}{R^2}}\right)^{3/2}} = -\frac{GM}{R^3}x$$

$$a = -\omega^2 x$$

$$\omega = \sqrt{\frac{GM}{R^3}}$$

$$T = \left(2\pi\sqrt{\frac{R^3}{GM}}\right) ✓$$

Nishant Jindal

Gravitational field & potential due to a uniform disc

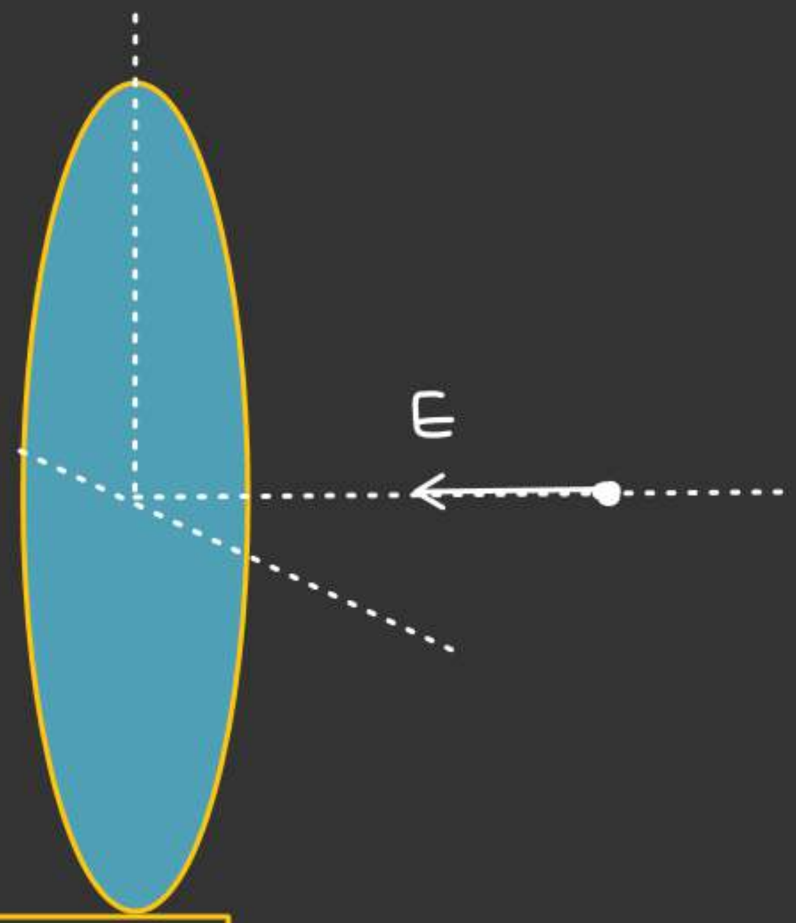
$$\left(\sigma = \frac{M}{\pi R^2} = \text{constant}\right)$$

$$\boxed{E = \frac{2GM}{R^2}\left[1 - \frac{x}{\sqrt{x^2+R^2}}\right]}$$

$$E = \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{x}{\sqrt{x^2+R^2}}\right]$$

$$\frac{1}{\varepsilon_0} = 4\pi G$$

$$\sigma = \left(\frac{M}{\pi R^2}\right)$$



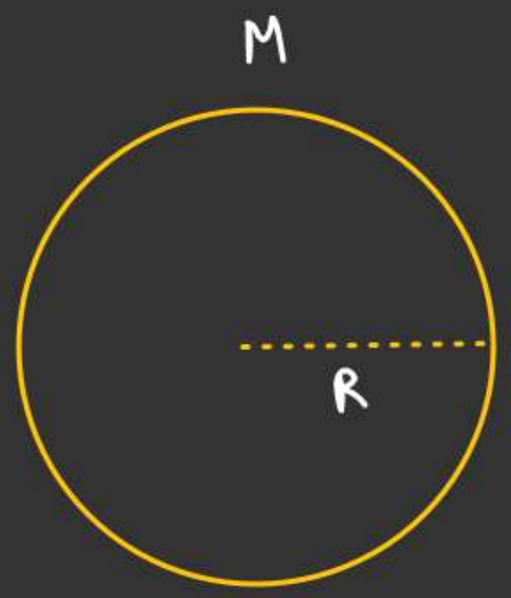
$$\boxed{V = -\frac{2GM}{R^2}\left[\sqrt{x^2+R^2} - x\right]}$$

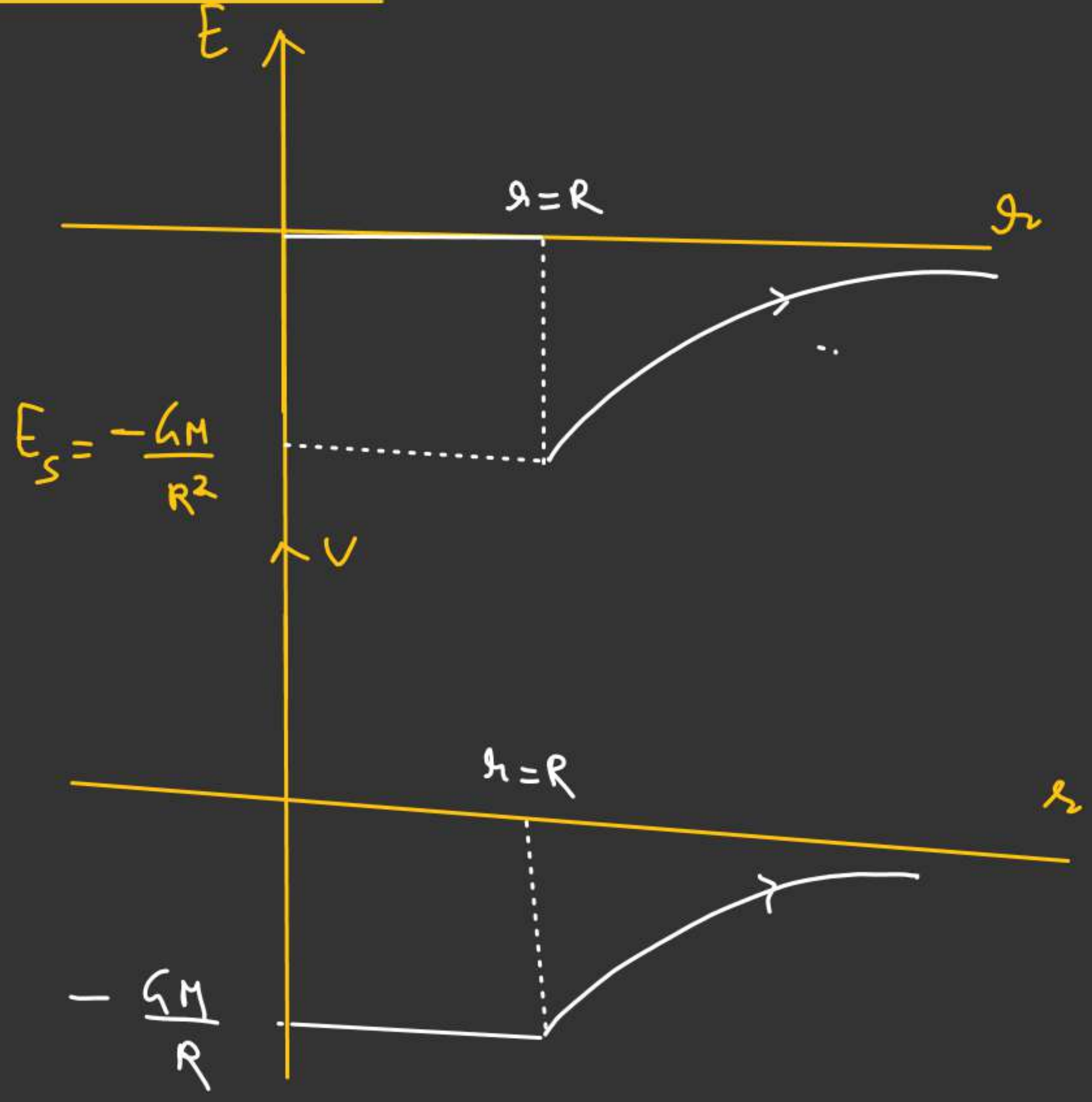
$$E_{center} = \left(\frac{2GM}{R^2}\right)$$

$$V_{center} = -\frac{2GM}{R}$$

Nishant Jindal

Field & potential due to a Spherical Shell





r < R

$$E = 0$$

$$V = -\frac{GM}{R}$$

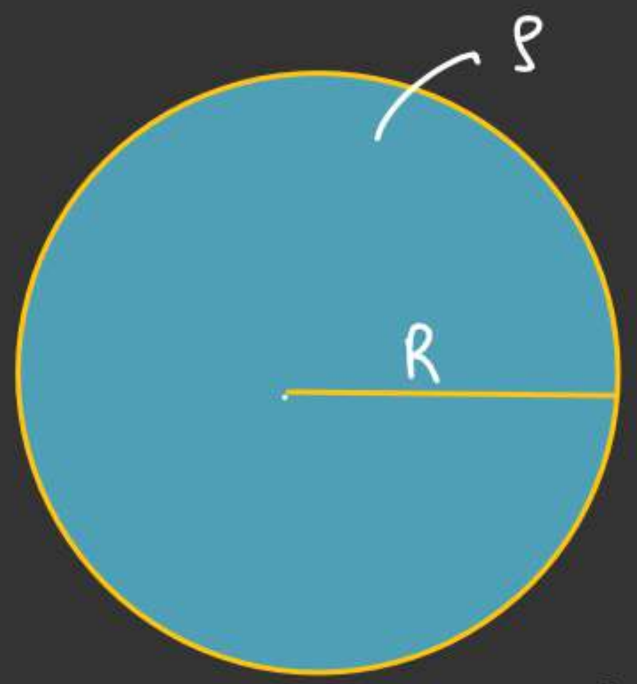
r > R (Acts as a point mass)

$$\vec{E} = -\frac{GM}{r^2}\hat{r}$$

$$V = -\frac{GM}{r}$$

Nishant Jindal

Electric field & potential due a Solid Sphere (uniform density)



$$E = \frac{\rho r}{3\varepsilon_0}$$

$$\frac{1}{4\pi\varepsilon_0} = G$$

$$\frac{1}{\varepsilon_0} = 4\pi G.$$

$$\rho = \left(\frac{M}{\frac{4}{3}\pi R^3}\right)$$

1) $r < R$.

$$\boxed{E = \frac{\rho\, 4\pi G}{3} r}$$

$$\boxed{E = \frac{GM}{R^3} r}$$

(ii) $r > R$

Acts as a point Mass

$$|E| = \frac{GM}{r^2}$$

Potential.

$r < R$

$$\boxed{V = -\frac{GM}{2R^3}\left(3R^2 - r^2\right)}$$

$r > R$

$$\boxed{V = -\frac{GM}{r}}$$

For Solid Sphere

$\rho = C$