

A Find Coord. of Point

§ Intersection of Line

$$2x - 4 + 1 = 0 \text{ & circle}$$

$$\hookrightarrow x^2 + y^2 = 2$$

Solving Line with circle.

POE

$$\{P_0\} \cup x^2 + (2x+1)^2 = 9$$

Needed 929

$$\{ \text{Needed} \} \quad 15x^2 + 4y + 1 = 2$$

John Smith

Line with $y^2 - x - y - 1 = 0$

$$5x(x+1) - 1(x+1) = 0$$

$$x = -1 \quad y = 2 \frac{1}{5}$$

$$-2 - y + 1 = 0 \quad | \quad \frac{2}{5} - y + \frac{1}{7} = 0$$

$$Y = \begin{pmatrix} -1 \\ -1 \end{pmatrix}, \quad Y = \begin{pmatrix} \frac{1}{5} \\ \frac{7}{5} \end{pmatrix}$$

$$\text{Line } 4x + 3y + k = 0 \text{ is tangent to } x^2 + y^2 = 4$$

Conc \rightarrow Line $y = mx + c$ is
tangent to $x^2 + y^2 = a^2$ then
 $c = \pm a\sqrt{1+m^2}$

$$3y = -4x - 1 \quad | \quad m = -\frac{4}{3}$$

$$y = -\frac{4}{3}x - \frac{1}{3}$$

$$\text{then } -\frac{K}{2} = \pm 2 \sqrt{1 + \frac{16}{9}}$$

$$-\frac{K}{3} = t^2 \times \frac{5}{3}$$

$$B - K = \pm 10$$

Q Find value of c for which Line $y = 2x + c$ is tangent to $x^2 + y^2 = 5 \rightarrow a = \sqrt{5}$ $m = 2$

Q E.O.T. to circle $x^2 + y^2 = 4$ which is inclined to 60° at x axis? $a=2$

$$m = \sqrt{3}$$

$$y = \sqrt{3}x + 2\sqrt{1+3^2} \Rightarrow y = \sqrt{3}x + 4$$

Find EOT for circle $x^2 + y^2 = 9$ |||^r to

$$\text{Line } 2x+y-3=0 \text{ is tangent to } y^2 = 4x \text{ at point } (1, 2).$$

$$y = -9 + 3\sqrt{(1+2)^2} = y = -9 + 3\sqrt{5}$$

$$2x+4 \neq 3\sqrt{5}-10$$

Q Find EOT. to circle.

$$6 \quad 4x^2 + 4y^2 - 25 = 0 \text{ L.R. to}$$

$$\text{Line } 12x - 5y + 3 = 0 \quad m = \frac{12}{5}$$

$$x^2 + y^2 = \frac{25}{4} \quad \text{Tangent's Slope} = -\frac{5}{12} = m$$

$$a = \frac{5}{2} \quad y = -\frac{5}{12}x \pm \frac{5}{2}\sqrt{1 + \left(\frac{5}{12}\right)^2}$$

$$y = -\frac{5}{12}x \pm \frac{5}{2} \times \frac{13}{12}$$

$$12y + 5 = \pm 65/2$$

Q Find EOT. to circle $x^2 + y^2 = 9$

at $(1, 1)$?

$(1, 1)$ lying on circle

So Using Cart. form:

$$(x_1 + y_1) = 2 \\ x + y - 2 = 0$$

$$\begin{array}{l} Q \text{ EOT to circle } x^2 + y^2 = 25 \text{ at } (-3, -4) \\ 8 \\ \begin{array}{r} 2x - 3 + y - 4 - 25 \\ 3x + 4y + 25 = 0 \end{array} \end{array}$$

$$\begin{array}{l} x^2 \rightarrow xx_1, y^2 \rightarrow yy_1, 2x \rightarrow x+x_1 \\ 2y \rightarrow y+y_1 \end{array}$$

$$\begin{array}{l} Q \text{ EOT to circle } x^2 + y^2 - 30(x+6y+109) = 0 \\ g \quad \text{at } (4, -1) \quad -15x_2x \quad 3x_2y \\ 16 + (-1)^2 - 120 - 6 + 109 = 0 \quad \text{satisfying} \end{array}$$

Cart form EOT

$$\begin{array}{l} x \cdot 4 + y \cdot (-1) - 15(x+4) + 3(y-1) + 109 = 0 \\ -11x + 2y - 60 - 3 + 109 = 0 \\ -11x + 2y + 46 = 0 \quad \text{EOT} \end{array}$$

$$\begin{array}{l} Q \text{ Find EOT. to circle } x^2 + y^2 - 26x - 9 \\ 10 \quad \text{at } (2, 3)? \quad 4 + 9 - 52 - 6 + 45 = 0 \quad \text{EOT} \end{array}$$

$$\begin{array}{l} \text{EOT} \quad 2x + 3y - 13(x+2) - (y+3) + 45 = 0 \\ -11x + 2y - 26 - 3 + 45 \\ -11x + 2y + 16 = 0 \end{array}$$

Q Find EOT. to circle $x^2 + y^2 - 24x = 0$

11 at $(a(1+6s), a(8m))$?

$$a^2(1+6s)^2 + a^2(8m)^2 - 2a^2(1+6s) = 0$$

$$a^2 + a^2 + 2a^2(6s) - 2a^2 - 2a^2(6s) = 0$$

Satisfy

$$\begin{array}{l} \text{EOT} \rightarrow x \cdot a(1+6s) + y \cdot a(8m) - a(x + a(1+6s)) = 0 \\ \Rightarrow ax(1+6s) + ay(8m) - a^2(1+6s) = 0 \end{array}$$

$$\begin{array}{l} \text{Find EOT. to circle } x^2 + y^2 - 26x - 9 \\ 10 \quad \text{at } (2, 3)? \quad 4 + 9 - 52 - 6 + 45 = 0 \quad \text{EOT} \end{array}$$

$$x(6s) + y(8m) - a(1+6s) = 0$$

Q. S.T. $x^2 + y^2 - 4x + 6y + 8 = 0$

$$\& x^2 + y^2 - 10x - 6y + 14 = 0$$

to touch at $(3, -1)$?

Concept if 2 circles touches each other then they must have common tangent.



$$1) 9 + 1 - 12 - 6 + 8 = 0 \checkmark$$

$$9 + 1 - 30 + 6 + 14 = 0 \checkmark$$

$$2) 3x - y - 2(x+3) + 3(y+1) + 8 = 0 \text{ at } (3, -1)$$

$$(E.O.T.)_{(1)}: x + 2y - 6 - 3 + 8 = 0$$

$$x + 2y - 1 = 0 \checkmark$$

$$3) x - y - 5(x+3) - 3(y+1) + 14 = 0$$

$$-2x - 4y - 15 + 3 + 14 = 0$$

$$x + 2y - 1 = 0 \checkmark$$

Both circles have

some common tangent

\Rightarrow touching

each other.

Q. P.T. tangent to circle:

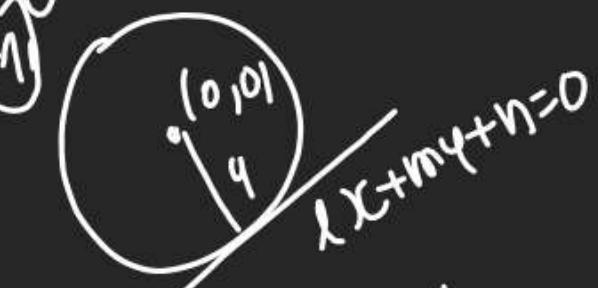
$$x^2 + y^2 = 16 \text{ g at } (5, 12) \& (12, -5)$$

are \perp to each other.

$$l=0 \quad 5x + 12y - 16g = 0 \rightarrow m = -\frac{5}{12}$$

$$l=0 \quad 12x - 5y - 16g = 0 \rightarrow m = \frac{12}{5}$$

Q. Find (m, n) that line $lx + my + n = 0$ is tangent to $x^2 + y^2 = a^2$?



$$b = \frac{|0+0+n|}{\sqrt{l^2+m^2}} = a$$

$$n^2 = a^2(l^2+m^2)$$

is required (and)

$$M^2 \quad l*x + n = -my \Rightarrow y = \frac{l*x + n}{-m}$$

$$x^2 + y^2 = a^2$$

$$x^2 + \frac{(lx+n)^2}{m^2} = a^2$$

$$m^2 x^2 + l^2 x^2 + 2nlx + n^2 - a^2 m^2 = 0$$

$$x^2(m^2 + l^2) + 2nlx + n^2 - a^2 m^2 = 0$$

Touch $\rightarrow D = 0$

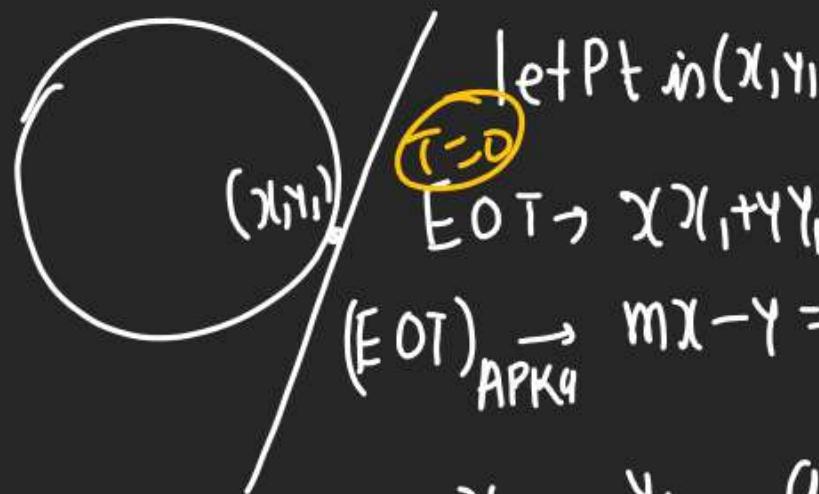
$$4n^2 l^2 = 4(l^2 + m^2)(n^2 - a^2 m^2)$$

$$n^2 l^2 = l^2 m^2 - a^2 m^2 l^2 + m^2 n^2 - a^2 m^4$$

$$= 4n^2 m^2 = a^2 m^2 (l^2 + m^2)$$

$$\therefore [n^2 = a^2(l^2 + m^2)]$$

Q Find Co-ord of Pt of contact
if $y = mx + c$ touches $x^2 + y^2 = a^2$



$$\frac{y_1}{m} = \frac{y_1}{-1} = \frac{a^2}{-c}$$

$$x_1 = \frac{a^2 m}{-c} \quad \left| \begin{array}{l} y_1 = \frac{a^2}{c} \end{array} \right.$$

$$\therefore \text{Pt of contact} = \left(-\frac{a^2 m}{c}, \frac{a^2}{c} \right)$$

$$= \left(\frac{-am}{\sqrt{1+m^2}}, \frac{a^2}{\sqrt{1+m^2}} \right)$$

Q (i) $x^2 + y^2 = 16$ has tangent
 $y = 2x + 4\sqrt{5}$ find Pt of
(contact) $m = 2 \rightarrow m_N = -\frac{1}{2}$
let Pt of contact = (x_1, y_1)

$$\text{EOT} \rightarrow x_1 x_1 + y_1 y_1 = 16$$
 $2x_1 - y_1 = -4\sqrt{5}$

$$\frac{x_1}{2} - \frac{y_1}{-1} = \frac{4\sqrt{5}}{4\sqrt{5}}$$

$$x_1 = -\frac{8}{\sqrt{5}}, y_1 = \frac{4}{\sqrt{5}}$$

$$\left(-\frac{8}{\sqrt{5}}, \frac{4}{\sqrt{5}} \right)$$

$N \rightarrow y - 0 = -\frac{1}{2}(x - 0)$

$$\begin{aligned} 2y + x &= 0 \times 2 \\ 4y + 2x &= 0 \\ -4y - 2x &= -4\sqrt{5} \\ 5y &= +4\sqrt{5} \end{aligned}$$

Q (ii) $x^2 + y^2 - 2x - 4y + 1 = 0$ & External
Pt (7, 4)

① Find EOT from (7, 4)

In the region pt is sum
outside of circle we use
 $y = mx + a\sqrt{1+m^2}$ (slope form) $\sqrt{1+m^2} = 2$

② Changing circle in central form:

$$(x-1)^2 + (y-2)^2 = 9^2$$

EOT $y - 2 = m(x - 1) \pm 2\sqrt{1+m^2}$ in P.T. (7, 4)

$$12 = m \times 6 \pm 2\sqrt{1+m^2} \Rightarrow 1 - 3m = \pm \sqrt{1+m^2}$$

$$\Rightarrow 9m^2 + 1 - 6m = x + m^2 \Rightarrow 8m^2 - 6m = 0$$

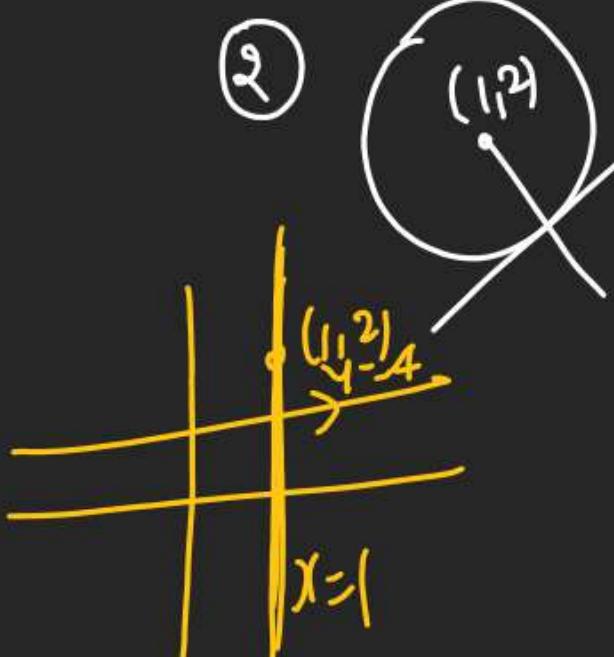
$$(3) \quad m = 0 \quad m = \frac{3}{4}$$

$$\begin{aligned} (1) & \text{ EOT } (y-4) = 0(x-7) \rightarrow 3x - 4y = 5 \\ (2) & \text{ EOT } y = \frac{1}{\sqrt{5}}(y-4) = 3y/(\sqrt{5}) \end{aligned}$$

Q18 If tangent is drawn from External Pt $(7, 4)$ to circle $x^2 + y^2 - 2x - 4y + 1 = 0$ find Pt of contact?

① From Prev.Qs, EOT at $(1, 2)$ on

$$Y = 4, \underline{3x - 4y = 5} \rightarrow m = \frac{3}{4}$$

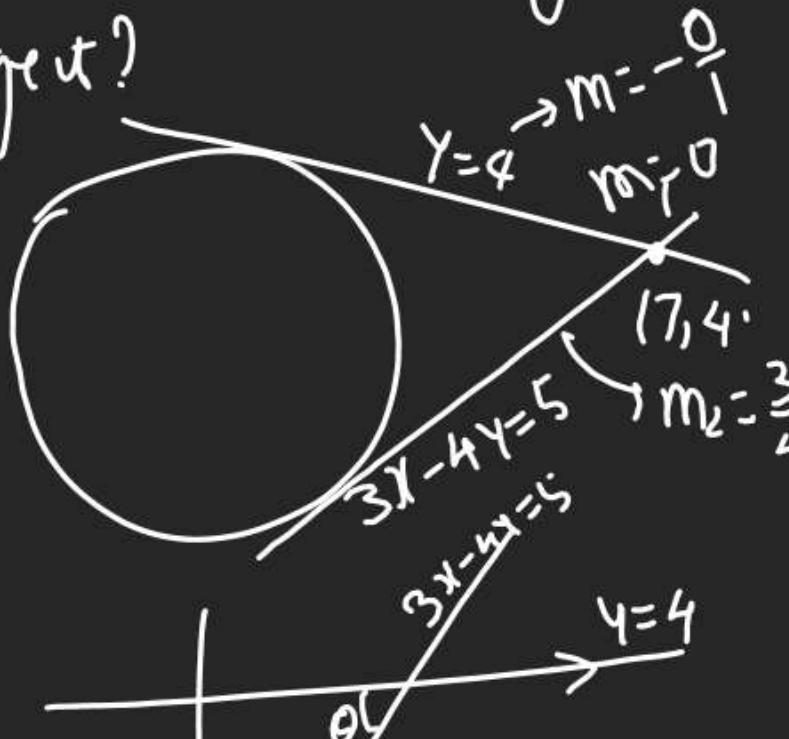


$$\begin{aligned} \text{EON} \\ (Y-2) &= -\frac{4}{3}(X-1) \\ 3Y-6 &= -4X+4 \\ 4X+3Y &= 10 \end{aligned}$$

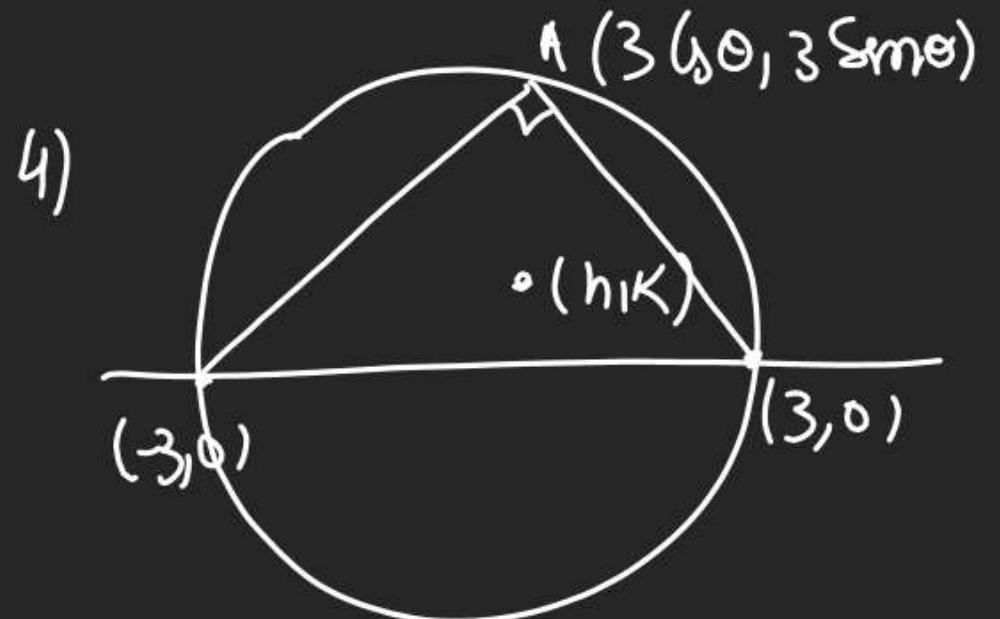
$$\begin{aligned} 9X-12Y &= 15 \\ 16X+12Y &= 40 \\ 25X &= 55 \Rightarrow X = \frac{11}{5} \end{aligned}$$

$$\begin{aligned} Y-4 &\rightarrow \perp \\ \text{Line P.T. } (1, 2) \text{ is} \\ X &= 1 \\ \therefore \text{Another Pt of contact} \\ \text{in } (1, 4) \end{aligned}$$

Q If tangents are drawn from External Pt $(7, 4)$ to circle $x^2 + y^2 - 2x - 4y + 1 = 0$ find Angle betn tangents?



$$\begin{aligned} \text{tang} &= \frac{|0 - \frac{3}{4}|}{|1 + 0 \times \frac{3}{4}|} \\ \theta &= \tan^{-1} \left| \frac{3}{4} \right| \end{aligned}$$



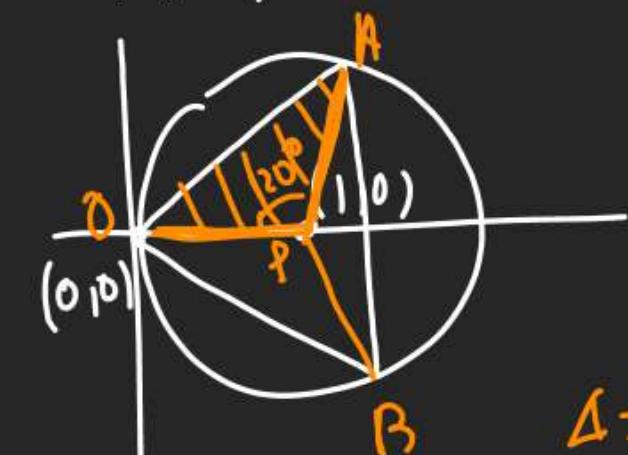
$$h = \frac{3\alpha - \beta + 3\cos \theta}{3} \quad K = \frac{0 + 0 + 3\sin \theta}{3}$$

$$h^2 + K^2 = 1$$

$$x^2 + y^2 = 1$$

$$(5) x^2 + y^2 - 2x - 0$$

$$(1, 0), R = \sqrt{1^2 + 0^2} = 1$$



$$\Delta = \frac{1}{2} ab \sin \theta$$

$$\Delta OAB = 3 \Delta OAP$$

$$= 3 \times \frac{1}{2} \times 1 \times 1 \cdot \sin 120^\circ$$

$$= \frac{3}{2} \times \frac{\sqrt{3}}{2} = \frac{3\sqrt{3}}{4}$$