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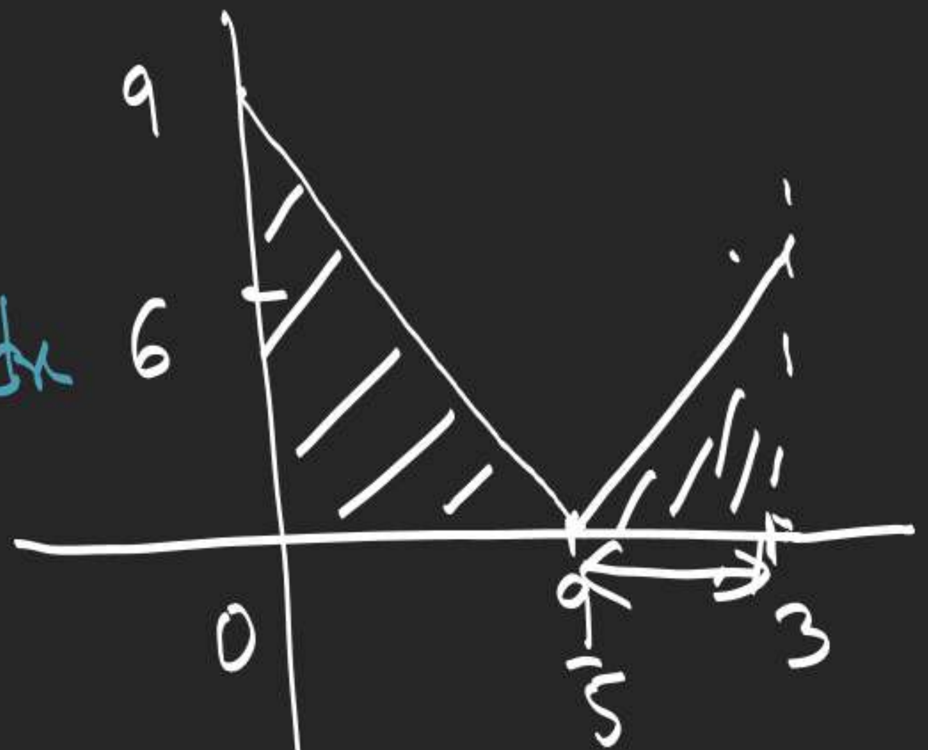
1. $\displaystyle\int_0^3 |5x-9|\,dx$

$\displaystyle\int_0^{9/5}(9-5x)\,dx+\int_{9/5}^{3}(5x-9)\,dx$



$\dfrac{1}{2}\times\dfrac{9}{5}\times 9+\dfrac{1}{2}\times 6\times\dfrac{6}{5}=\dfrac{117}{10}$

2. $\displaystyle\int_0^{2\pi}|1+2\cos x|\,dx$

$\displaystyle=\int_0^{\pi}\Big(|1+2\cos x|+|1+2\cos(2\pi-x)|\Big)dx$

$\displaystyle=2\int_0^{\pi}|1+2\cos x|\,dx=2\left[\int_0^{2\pi/3}(1+2\cos x)\,dx+\int_{2\pi/3}^{\pi}-(1+2\cos x)\,dx\right]=2\left[\frac{\pi}{3}+4\left(\frac{\sqrt{3}}{2}\right)\right]=\frac{2\pi}{3}+4\sqrt{3}$

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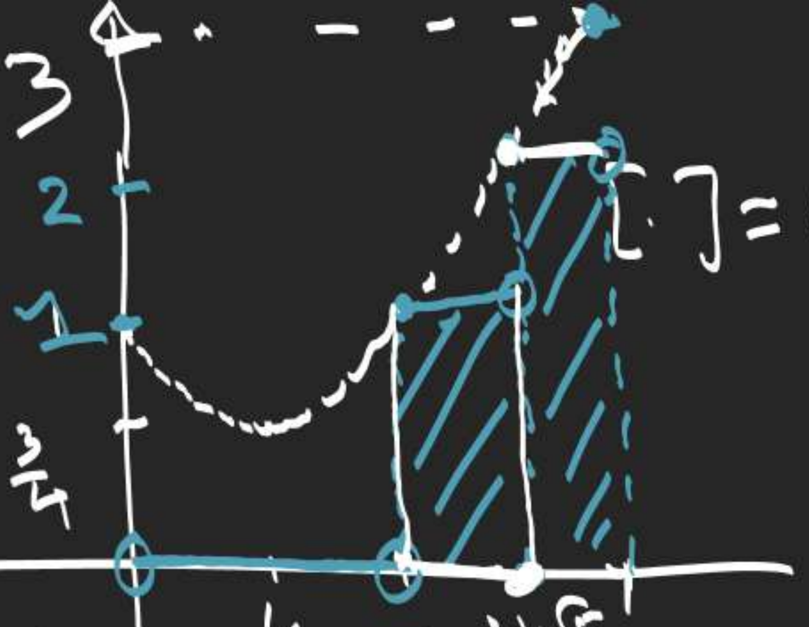
3. $\int_0^2 [x^2-x+1]\,dx$



$[\cdot] = G.I.F.$

$\int_0^1 0\,dx + \int_1^{\frac{1+\sqrt{5}}{2}} 1\,dx + \int_{\frac{1+\sqrt{5}}{2}}^2 2\,dx$

$\frac{5-\sqrt{5}}{2} = 1\left(\frac{1+\sqrt{5}}{2}-1\right) + 2\left(2-\frac{1+\sqrt{5}}{2}\right)$

$x^2-x+1=2$

$x^2-x-1=0$

$\frac{1\pm\sqrt{5}}{2}$

4. $\int_{-\frac{1}{2}}^{\frac{1}{2}} \left([x] + \ln\left(\frac{1+x}{1-x}\right)\right) dx$

$[\cdot] = G.I.F$

$= \int_0^{\frac{1}{2}} \left([x]+[-x] + \ln\left(\frac{1+x}{1-x}\right) + \ln\left(\frac{1-x}{1+x}\right)\right) dx$

$\int_0^{1/2} \left([x]+[-x]\right) dx = \int_0^{1/2} -1\,dx = -\frac{1}{2}.$

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5. $$\int_{-1}^{3}\left(\underbrace{\tan^{-1}\left(\frac{x}{x^2+1}\right)+\tan^{-1}\left(\frac{x^2+1}{x}\right)}\right)dx = \int_{-1}^{0}\left(\underbrace{\tan^{-1}\frac{x}{x^2+1}+\cot^{-1}\frac{x^2+1}{x}}_{-\pi}\right)dx+\int_{0}^{3}\left(\underbrace{\tan^{-1}\frac{x}{x^2+1}+\cot^{-1}\left(\frac{x^2+1}{x}\right)}\right)dx$$

$$\int_{-1}^{1} \;//\; \int_{0}^{1} 0\,dx + \int_{1}^{3}\frac{\pi}{2}dx = \boxed{\pi}$$

$$\pi = -\frac{\pi}{2}(1) + \frac{3\pi}{2}$$

6. $$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}}\left(\frac{x^7-3x^5+3x^3-x+1}{\cos^2 x}\right)dx = \int_{0}^{\frac{\pi}{4}}\frac{2\,dx}{\cos^2 x} = 2$$

$$I: \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left(\left(\frac{1}{(2023)^x + 1} \right) \left(\frac{\sin^{2024} x}{\sin^{2024} x + \cos^{2024} x} \right) \right) dx$$

$$= \int_{0}^{\frac{\pi}{2}} \frac{\sin^{2024} x}{\sin^{2024} x + \cos^{2024} x} \underbrace{\left(\frac{1}{(2023)^x + 1} + \frac{1}{\frac{1}{(2023)^x} + 1} \right)}_{=1} dx$$

$$= \int_{0}^{\frac{\pi}{4}} \left(\frac{\sin^{2024} x + \cos^{2024} x}{\sin^{2024} x + \cos^{2024} x} \right) dx = \frac{\pi}{4}$$

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8. $I = \int_{50}^{100} \frac{\ln x \, dx}{\ln x + \ln(150-x)}$ — ①

$I = \int_{50}^{100} \frac{\ln(150-x)\,dx}{\ln(150-x) + \ln(150-(150-x))}$ — ②

① + ② . $2I = \int_{50}^{100} 1 \, dx = 50$

$\boxed{I = 25}$

9. $\int_0^{\frac{\pi}{4}} \ln(1+\tan x)\,dx$

$= \int_0^{\frac{\pi}{8}} \ln\left((1+\tan x)\left(1+\tan\left(\frac{\pi}{4}-x\right)\right)\right) dx = \int_0^{\frac{\pi}{8}} \ln\left((1+\tan x)\left(1+\frac{1-\tan x}{1+\tan x}\right)\right) dx$

$= \frac{\pi}{8} \ln 2$

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10. $I = \int_2^3 \frac{x^2\,dx}{(2x^2-10x+25)}$ — ①

$= x^2 + (5-x)^2$

$I = \int_2^3 \frac{(5-x)^2\,dx}{(5-x)^2 + x^2}$ — ②

① + ②

$2I = \int_2^3 dx = 1$

$\boxed{I = \frac{1}{2}}$

11. $\int_0^2 \frac{dx}{\underbrace{(17+8x-4x^2)}\left(e^{6(1-x)}+1\right)} = \int_0^1 \frac{dx}{17+8x-4x^2} = \int_0^1 \frac{dx}{21-4(x-1)^2}$

$= \int_{-1}^{0} \frac{dx}{21-4x^2} = -\frac{1}{2}\int_2^0 \frac{dx}{21-x^2} = \frac{1}{2}\int_0^2 \frac{dx}{21-x^2}$

$x \to -\frac{x}{2}$

$\frac{2}{4\sqrt{21}} \ln\left(\frac{\sqrt{21}+2}{\sqrt{21}-2}\right)$

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12. $\displaystyle\int_0^{\pi/4} \frac{x\,dx}{(1+\cos 2x+\sin 2x)} = \frac{\pi}{4}\int_0^{\pi/8} \frac{dx}{\underbrace{1+\cos 2x}+\sin 2x}$

$\displaystyle = \frac{\pi}{8}\int_0^{\pi/8} \frac{dx}{\cos^2 x+\sin x\cos x} = \frac{\pi}{8}\int_0^{\pi/8} \frac{\sec^2 x\,dx}{1+\tan x}$

$\displaystyle = \frac{\pi}{8}\ln\left(\sqrt{2}\right)^{\circ} = \frac{\pi}{16}\ln 2.$

13. $\displaystyle\int_0^1 \cot^{-1}(1-x+x^2)\,dx$

$\displaystyle = \int_0^1 \tan^{-1}\left(\frac{x+(1-x)}{1-x(1-x)}\right)dx = \int_0^1 \tan^{-1}x\,dx + \int_0^1 \tan^{-1}(1-x)\,dx$

$\displaystyle = 2\int_0^1 1\cdot\tan^{-1}x\,dx = 2\left. x\tan^{-1}x - \ln(1+x^2)\right|_0^1$

$\displaystyle = \frac{\pi}{2} - \ln 2.$

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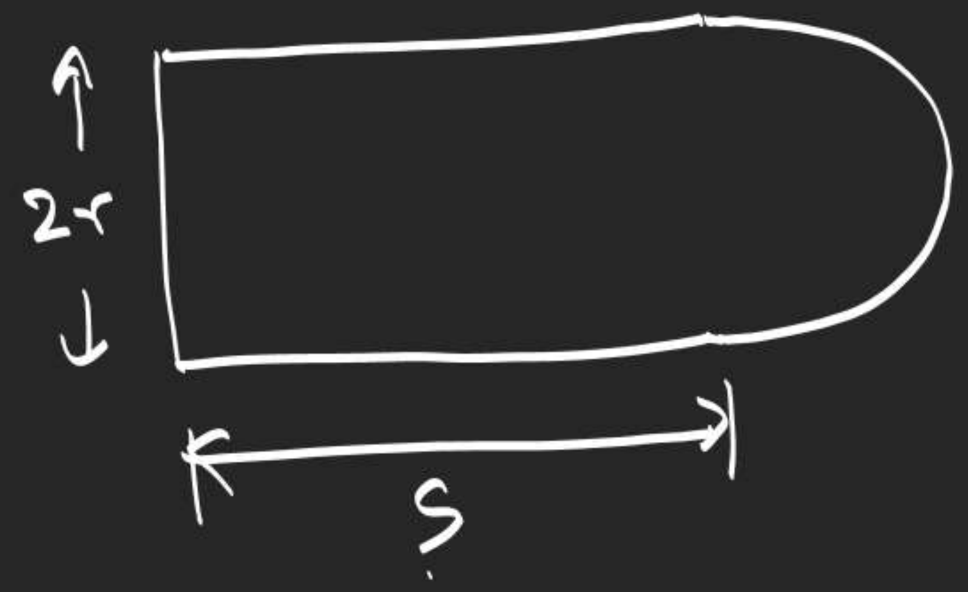
7.



$$\frac{A}{r} = \left(\frac{\pi}{2}+2\right)r$$

$$r = ?$$

$$A = s(2r) + \frac{\pi r^2}{2}$$

$$P = \pi r + 2s + 2r$$

$$= (\pi+2)r + \frac{1}{r}\left(A - \frac{\pi r^2}{2}\right) = \frac{A}{r} + \left(\frac{\pi}{2}+2\right)r$$

$$\geq 2\sqrt{A\left(\frac{\pi}{2}+2\right)}$$

$$\frac{dP}{dr} = 0 = ?$$

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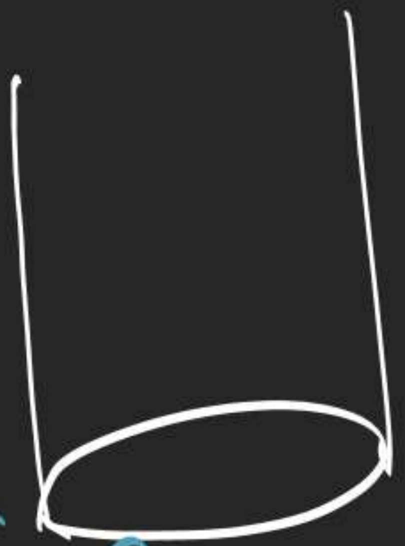
Ex-I (nem.)

Ex-II

maxima & minima



$2\pi r + h = 3$

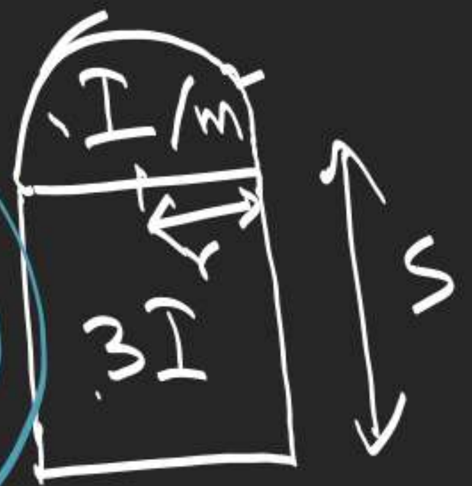
$V = \pi r^2 h$

$V = \pi r^2 \underbrace{(3 - 2\pi r)}$

$$1 = \left(\frac{3 - 2\pi r + \pi r + \pi r}{3}\right)^3 \geq \pi^2 (3 - 2\pi r) r^2$$

$$L = I\left(\frac{\pi r^2}{2} + 3r(P - 4r - \pi r)\right) \leq \frac{1}{\pi}$$

$3 - 2\pi r = \pi r$



$L = I\frac{\pi r^2}{2} + 3I(2rs)$

$P = 4r + 2s + \pi r$