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THERMAL EXPANSION

Case of bimettalic strip

$(\alpha_1 > \alpha_2)$

$\alpha_1 < \alpha_2$

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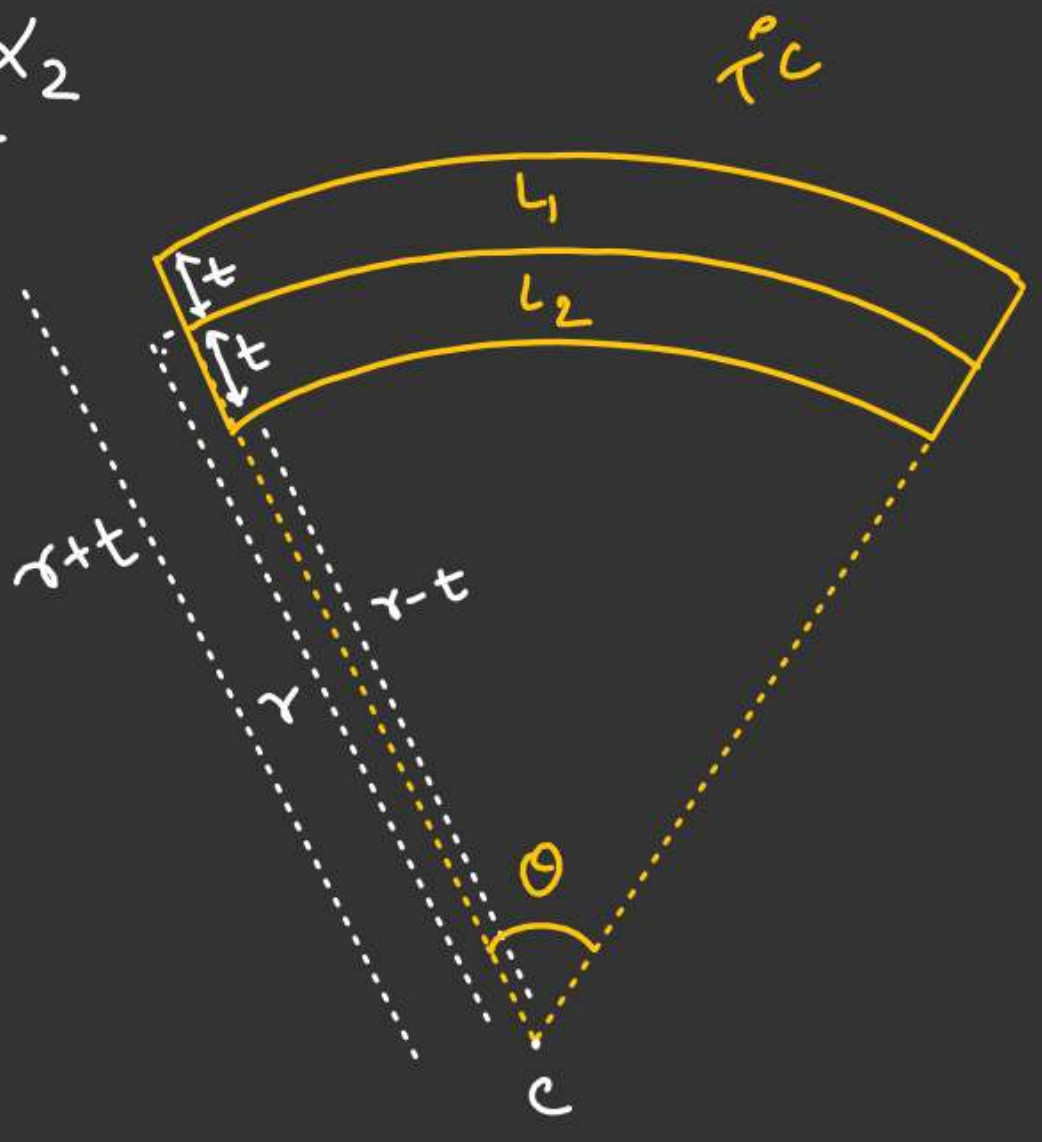

r = Mean radius

$$(r-t)\cdot\theta = L_2$$

$$(r+t)\,\theta = L_1$$

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$$\frac{r+t}{r-t} = \frac{L_1}{L_2}$$

$$L_1 = L(1+\alpha_1 \Delta T)$$

$$L_2 = L(1+\alpha_2 \Delta T)$$

$$\frac{r+t}{r-t} = \frac{\cancel{L}(1+\alpha_1 \Delta T)}{\cancel{L}(1+\alpha_2 \Delta T)}$$

$$\frac{(r+t)}{(r-t)} = \left(\frac{1+\alpha_1 \Delta T}{1+\alpha_2 \Delta T}\right)$$

By componendo & dividendo

$$\frac{\cancel{2}r}{\cancel{2}t} = \frac{(1+\alpha_1 \Delta T)+(1+\alpha_2 \Delta T)}{(1+\alpha_1 \Delta T)-(1+\alpha_2 \Delta T)} \qquad \left[\frac{a}{b} = \frac{a+b}{a-b}\right]$$

$$\frac{r}{t} = \frac{2 + \underbrace{(\alpha_1+\alpha_2)\Delta T}_{\to 0}}{(\alpha_1-\alpha_2)\Delta T} \qquad (\alpha_1 \Delta T, \alpha_2 \Delta T) << 1$$

$$\left[r = \frac{2t}{(\alpha_1-\alpha_2)\Delta T}\right]$$

$\downarrow$

Mean radius

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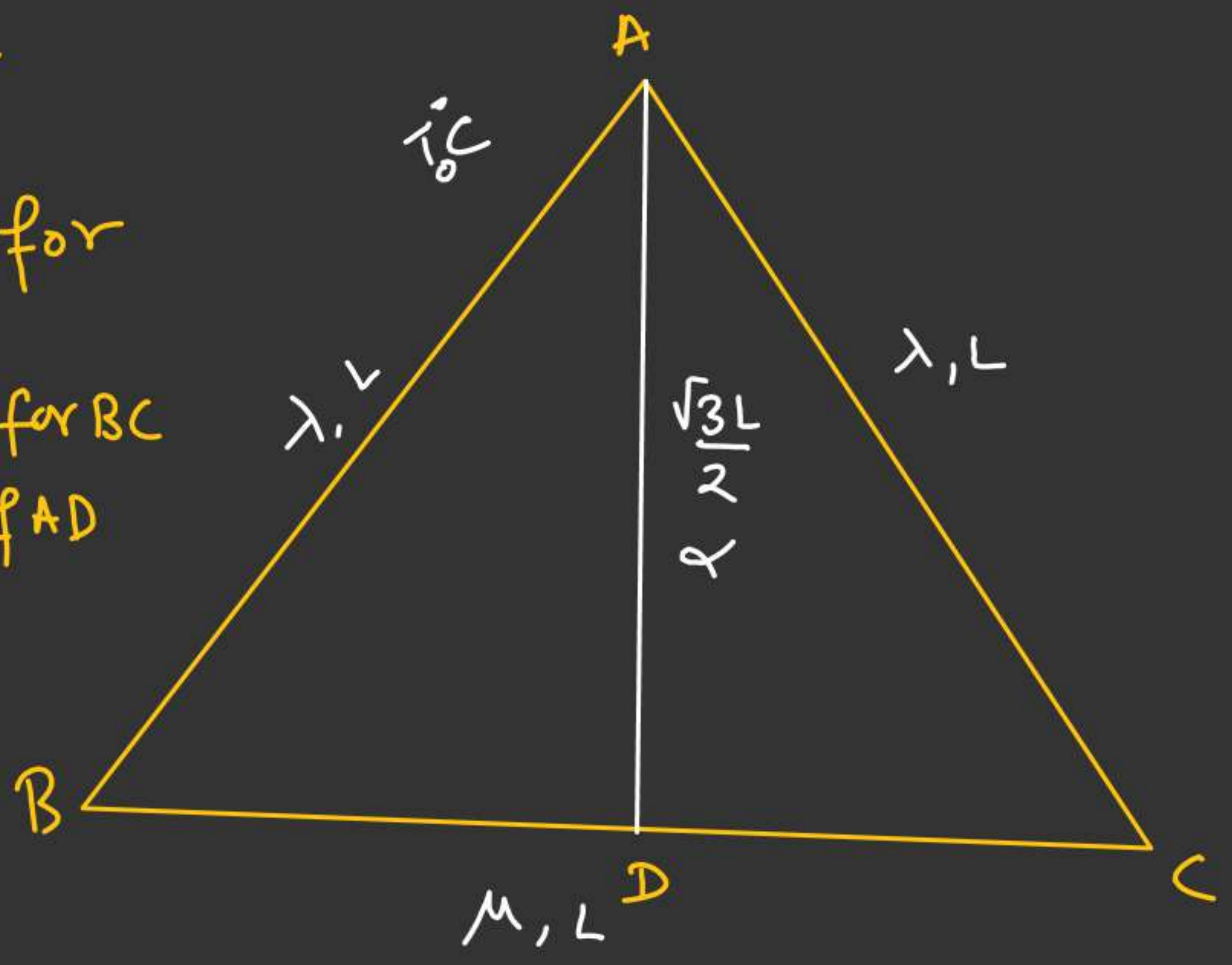
ABC equilaterial triangle form by 3-rods AB, BC & CA

λ is the coff^n of linear expansion for AB and AC

μ be the coff^n of linear expansion for BC

α be the coff^n of linear expansion of AD

Find α so that the frame will not deformed if heated from $T_0^\circ C$ to $T^\circ C$



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For AB & AC

$$L' = L(1+\lambda\Delta T)$$

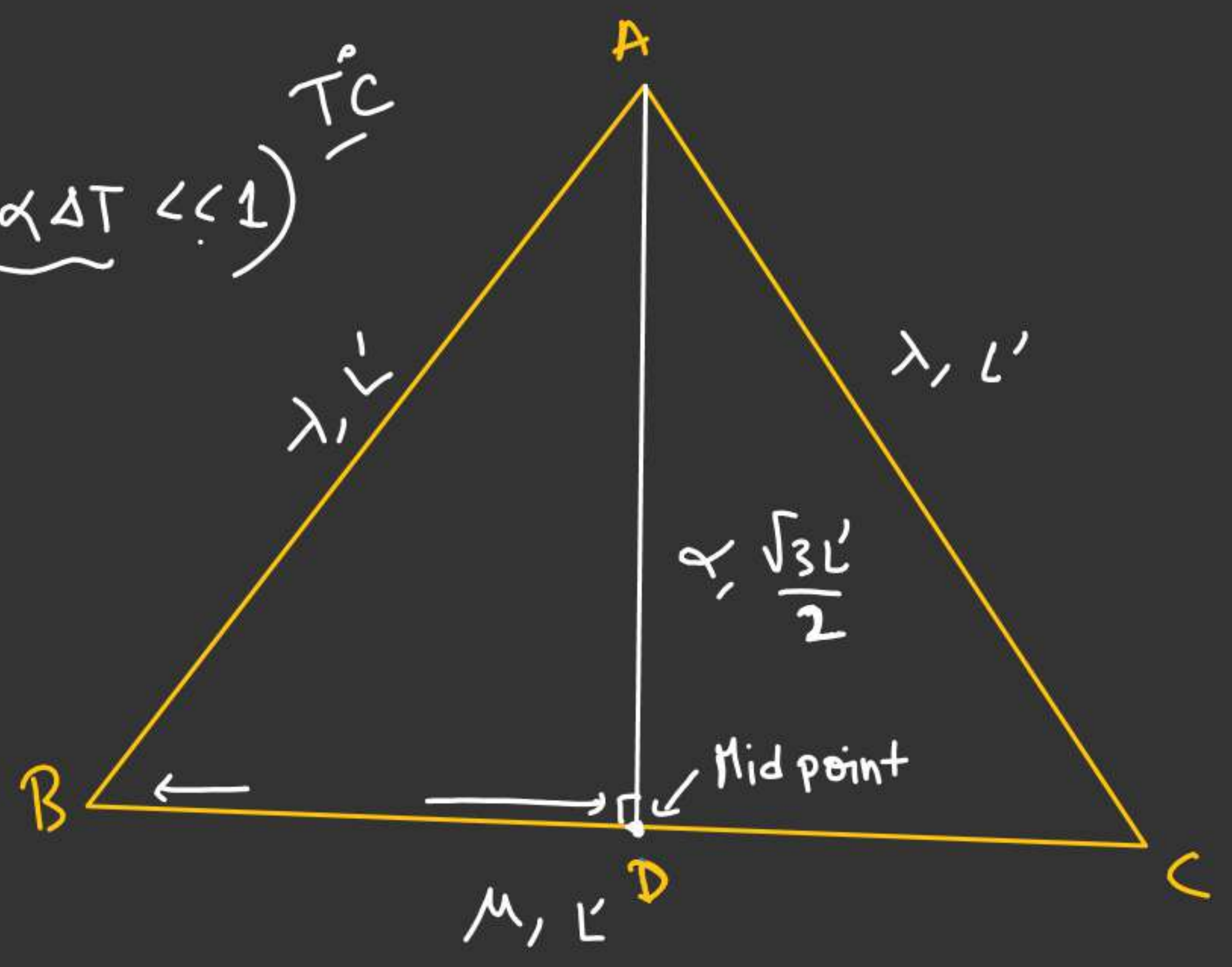
For BC

$$L' = L(1+\mu\Delta T)$$

For AD

$$\frac{\sqrt{3}L'}{2} = \frac{\sqrt{3}L}{2}(1+\alpha\Delta T)$$

$(\lambda\Delta T,\ \mu\Delta T,\ \alpha\Delta T \ll 1)$



$$(L')^2 = \left(\frac{L'}{2}\right)^2 + \left(\frac{\sqrt{3}L'}{2}\right)^2$$

$$\cancel{L^2}(1+\lambda\Delta T)^2 = \frac{\cancel{L^2}}{4}(1+\mu\Delta T)^2 + \frac{3\cancel{L^2}}{4}(1+\alpha\Delta T)^2$$

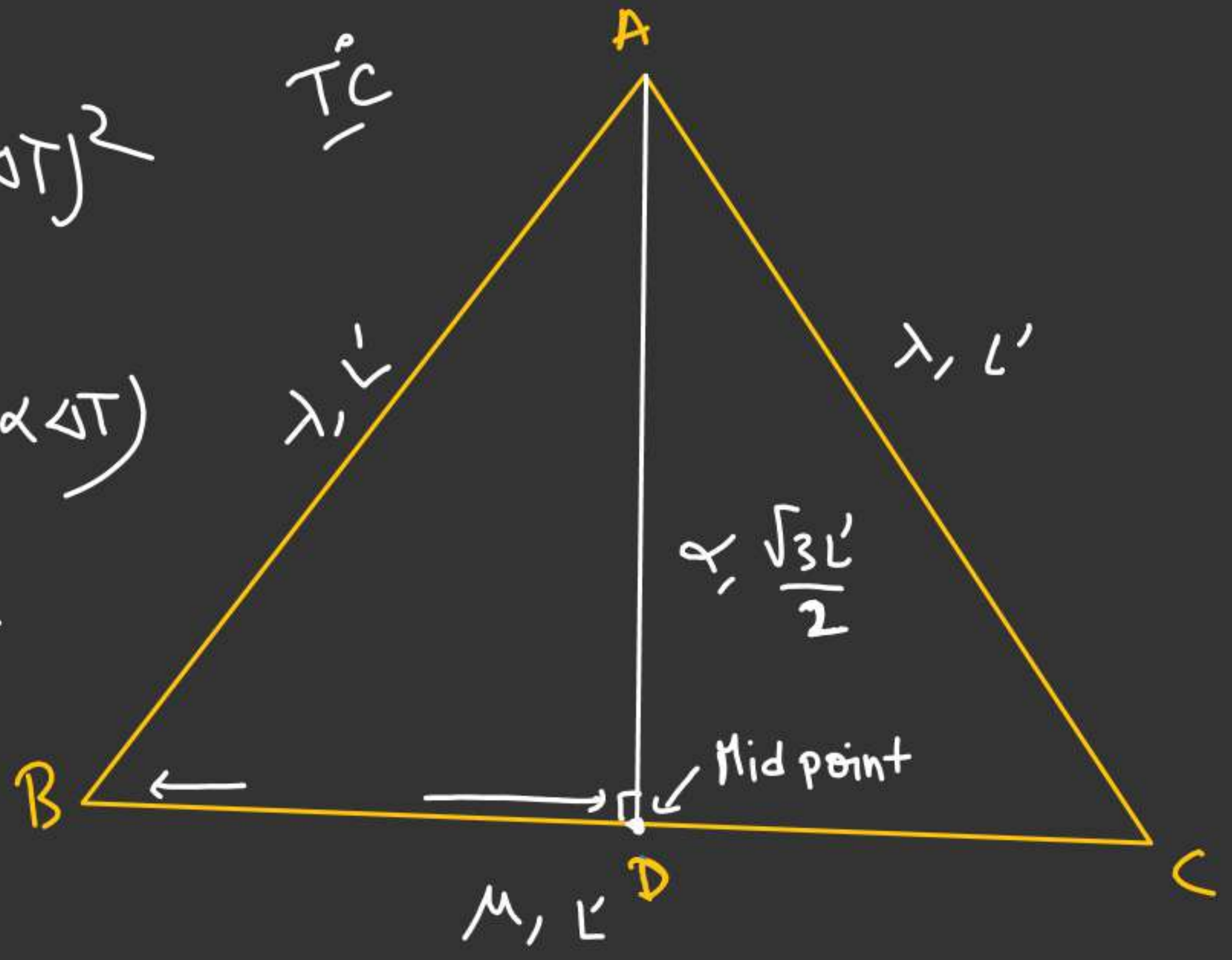
$$\cancel{l'^2}(1+\lambda\Delta T)^2 = \frac{\cancel{l'^2}}{4}(1+\mu\Delta T)^2 + \frac{3\cancel{l'^2}}{4}(1+\alpha\Delta T)^2$$

$$1+2\lambda\Delta T = \frac{1}{4}(1+2\mu\Delta T) + \frac{3}{4}(1+2\alpha\Delta T)$$

$$\cancel{1}+2\lambda\Delta T = \cancel{\left(\frac{1}{4}+\frac{3}{4}\right)} + \left(\frac{\mu}{2}+\frac{3\alpha}{2}\right)\Delta T$$

$$2\lambda = \left(\frac{\mu+3\alpha}{2}\right)$$

$$\lambda = \left(\frac{\mu+3\alpha}{4}\right) \checkmark$$



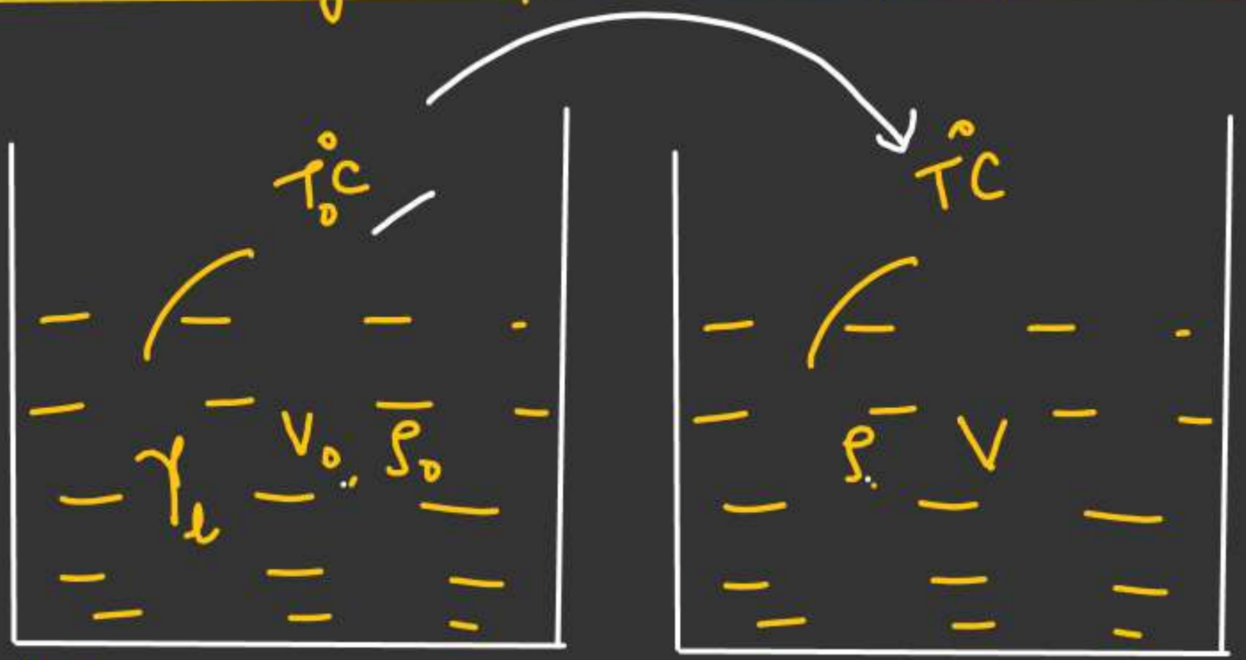
Thermal Expansion in liquid.

Case when only Expansion of liquid not vessel

$\rho = \frac{m}{V}$

$\left(\rho \propto \frac{1}{V}\right)$

$m = C$



$\rho_0 \rightarrow$ Density of liquid at $T_0\,°C$

$\rho \rightarrow$ Density of liquid at $T°C$

$$\rho \propto \frac{1}{V}$$

$$\frac{\rho}{\rho_0} = \frac{V_0}{V}$$

$$\frac{\rho}{\rho_0} = \frac{\cancel{V_0}}{\cancel{V_0}(1+\gamma_l \Delta T)}$$

$$\boxed{\rho = \frac{\rho_0}{(1+\gamma_l \Delta T)}}$$

$$\rho = \rho_0 (1+\gamma_l \Delta T)^{-1}$$

$$\boxed{\rho = \rho_0 (1-\gamma_l \Delta T)}$$

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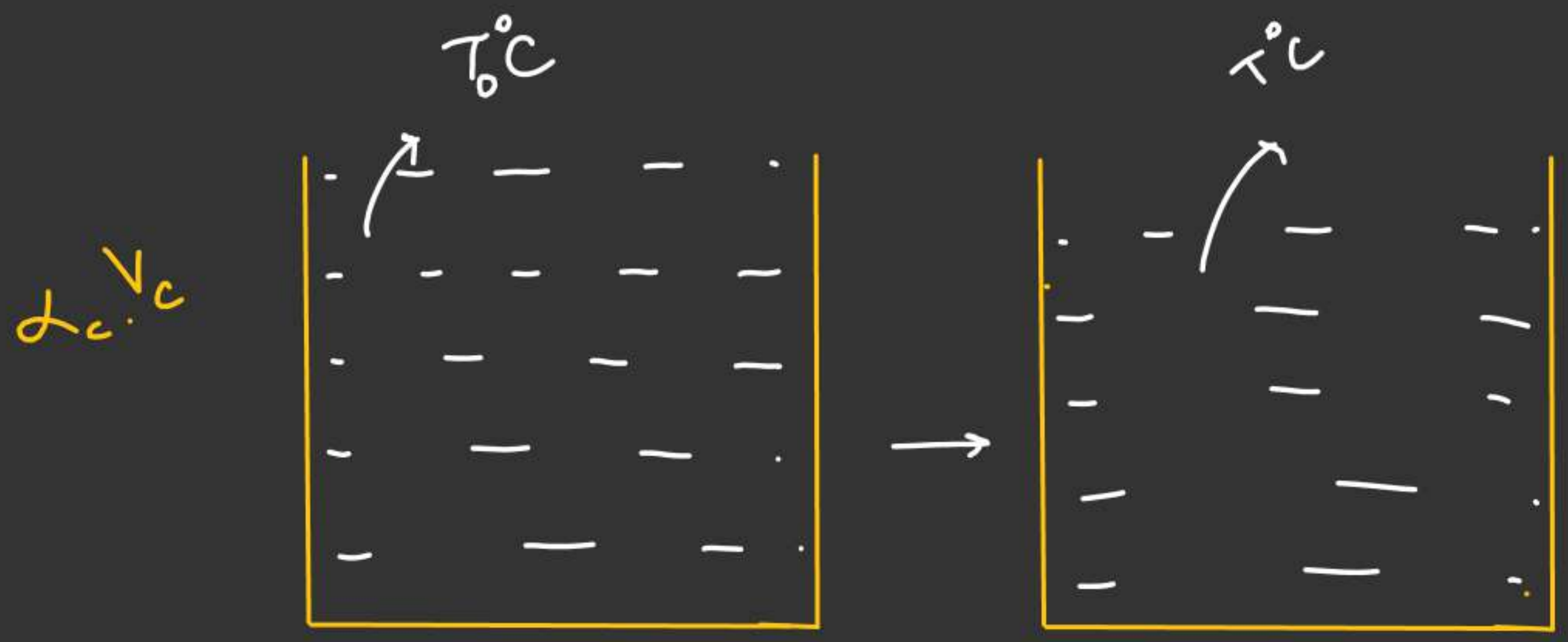
Expansion of vessel as well as liquid



At $T_0°C$

✓ V_ℓ = Volume of liquid

✓ V_c = Volume of container

γ_c = Volume expansion coffⁿ of container

γ_ℓ = coffⁿ of Volume expansion of liquid.

Initially liquid is completely filled

$\underline{V_c = V_L \text{ at } T_0°C}$

let $V_c = V_L = V_0$

For Container

$$V_c' = V_c(1+\gamma_c \Delta T)$$

For liquid.

$$V_\ell' = \underline{V_\ell}(1+\gamma_\ell \Delta T)$$

$$\underline{\underline{\Delta V}} = (V_c' - V_\ell')$$

$$= V_c(1+\gamma_c \Delta T) - V_\ell(1+\gamma_\ell \Delta T)$$

$$= \cancel{V_0} + V_0\gamma_c \Delta T - \cancel{V_0} - V_0\gamma_\ell \Delta T$$

$$= \underline{V_0(\gamma_c - \gamma_\ell)\Delta T}$$

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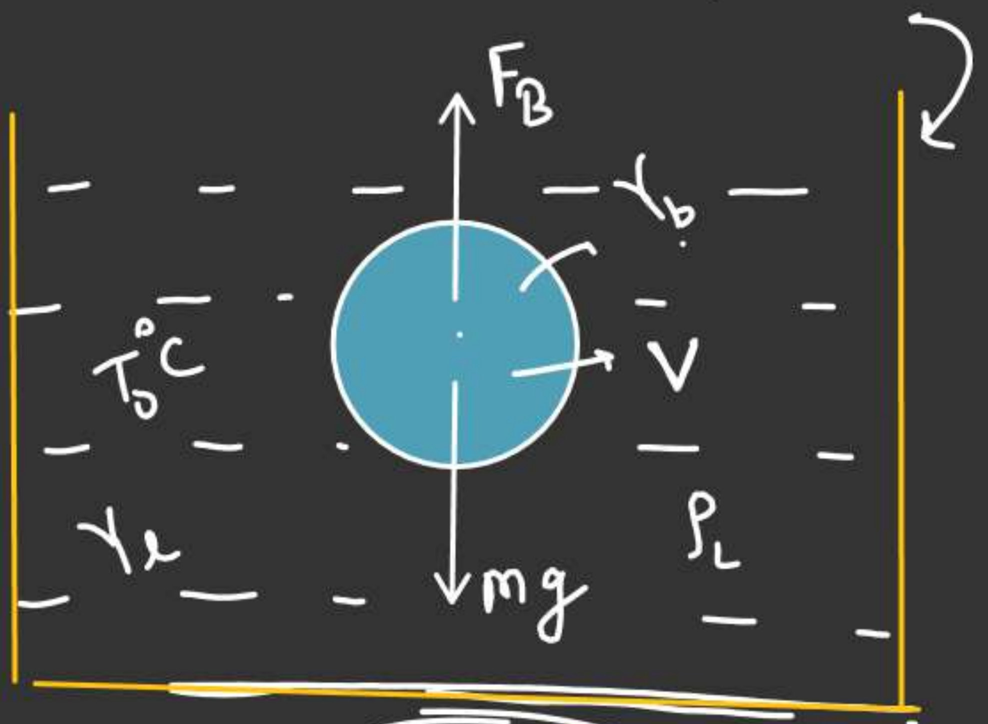
$\Delta V > 0$	$\Delta V < 0$
$r_c > r_e$	$r_c < r_e$
↳ liquid level decreases	↳ liquid level increases.

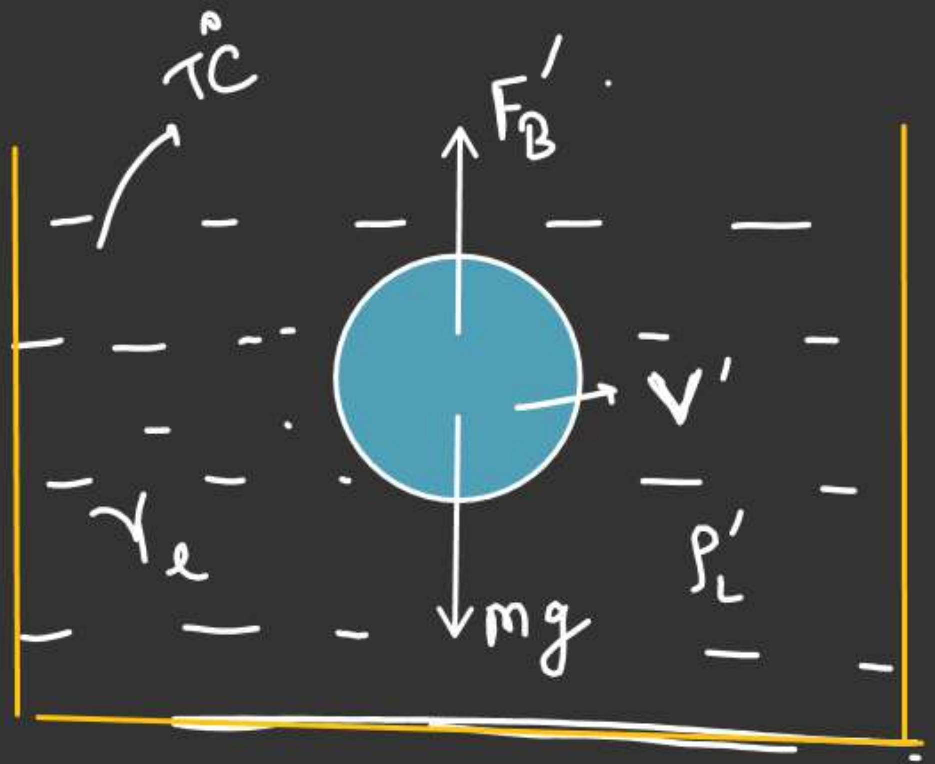
Effect of Temperature on the Apparent weight of the body fully Submerged

No Expansion of vessel



$$F_B = V\rho_L g$$

$$N_{app} = (mg - F_B)$$



γ_b = Coff^n of volume expansion of body

γ_l = Coff^n of Volume expansion of liquid

$$F_B' = V'\rho_L' g$$

$$V' = V(1+\gamma_b \Delta T)$$

$$\rho_L' = \rho_0(1-\gamma_l \Delta T)$$

$$F_B' = V\rho_0 g(1+\gamma_b\Delta T)(1-\gamma_l\Delta T)$$

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$$W_{app} = (mg - F_B) \rightarrow \text{at } T_0^\circ C$$

$$W'_{app} = mg - F'_B \rightarrow \text{at } T^\circ C$$

$$W'_{app} = mg - \left[V\rho_0 g(1+\gamma_b \Delta T)(1-\gamma_l \Delta T)\right]$$

$$W'_{app} = mg - \left[V\rho_0 g\left[1 - \gamma_l \Delta T + \gamma_b \Delta T - \underbrace{\gamma_l \gamma_b \Delta T^2}_{0}\right]\right]$$

$$W'_{app} = \underbrace{(mg - V\rho_0 g)}_{W_{app}} + (\gamma_l - \gamma_b)\Delta T\, V\rho_0 g$$

$$(W'_{app} - W_{app}) = (\gamma_l - \gamma_b)V\rho_0 g \cdot \Delta T$$

$$F'_B = V\rho_0 g(1+\gamma_b \Delta T)(1-\gamma_l \Delta T)$$

Case when apparent weight increases

$$W'_{app} > W_{app}.$$
$$\gamma_l > \gamma_b.$$

Case when apparent weight decreases.

$$(\gamma_b > \gamma_l)$$

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SCALE ERROR

$$\boxed{l_a = l_o\left[1+\alpha(T-T_o)\right]}$$

l_a = Acutual length

l_o = Obsered or Measured length

T_o = Temperature at which Scale gives correct reading

T = Temperature at which Observation is made.

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Heat Transfer