HOME WORK –2(Solution)

Link to View Video Solution: Click Here

1. For a sequence $\{a_n\}, a_1 = 2$ and $\frac{a_{n+1}}{a_n} = \frac{1}{3}$. Then $\sum_{r=1}^{20} a_r$ is

(A) $\frac{20}{2}[4 + 19 \times 3]$

(B) $3\left(1 - \frac{1}{3^{20}}\right)$

(C) $2(1 - 3^{20})$

(D) 3

Ans. **(B)**

Sol. $\frac{a_{n+1}}{a_n} = \frac{1}{3}$

$$n = 1 \Rightarrow \frac{a_2}{a_1} = \frac{1}{3} \Rightarrow a_2 = \frac{a_1}{3} = \frac{2}{3}$$

$$n = 3 \Rightarrow \frac{a_3}{a_2} = \frac{1}{3} \Rightarrow a_3 = \frac{a_2}{3} = \frac{\left(\frac{2}{3}\right)}{3} = \frac{2}{3^2}$$

$$n = 4 \Rightarrow \frac{a_4}{a_3} = \frac{1}{3} \Rightarrow a_4 = \frac{a_3}{3} = \frac{\frac{2}{3^2}}{3} = \frac{2}{3^3} \dots \dots$$

$$\therefore \sum_{r=1}^{20} a_r = a_1 + a_2 + a_3 + \dots + a_{19} + a_{20}$$

$$= 2 + \frac{2}{3} + \frac{2}{3^2} + \dots - \text{ Upto } 20 \text{ terms}$$

$$= 2\left(1 + \frac{1}{3} + \frac{1}{3^2} + \dots \text{upto } 20 \text{ terms}\right)$$

$$S_{20} = 2\frac{1 - \left\{1 - \left(\frac{1}{3}\right)^{20}\right\}}{\left(1 - \frac{1}{3}\right)} \quad \because S_n = \frac{a(1 - r^n)}{1 - r}, r < 1$$

$$= \frac{x\left(1 - \frac{1}{3^{20}}\right)}{x/3} = 3\left(1 - \frac{1}{3^{20}}\right)$$

Link to View Video Solution: Click Here

2. The third term of a G.P. is 4. The product of the first five terms is

(A) 4^3 (B) 4^5 (C) 4^4 (D) 4^{10}

Ans. **(B)**

Sol. Let G.P. is $\Rightarrow$ $a, ar, ar^2, ar^3, ar^4, \ldots$

$T_3 = ar^2 = 4$

$\therefore$ Product of $= a \cdot ar \cdot ar^2 \cdot ar^3 \cdot ar^4$

1^{st} Five Terms $= a^5 r^{10} = (ar^2)^5$

$= 4^5$

3. If S is the sum of infinity of a G.P. whose first term is ' a ', then the sum of the first n terms is

(A) $s\left(1-\frac{a}{s}\right)^n$ (B) $S\left[1-\left(1-\frac{a}{s}\right)^n\right]$ (C) $a\left[1-\left(1-\frac{a}{s}\right)^n\right]$ (D) $s\left(\frac{a}{s}\right)^n$

Ans. **(B)**

Sol. $\left.\begin{matrix} S_\infty = S \\ I^{st} \text{ term } = a \end{matrix}\right\} \Rightarrow S_\infty = \frac{a}{1-r} = S$

$\Rightarrow \frac{a}{s} = 1 - r$

$\Rightarrow r = \left(1 - \frac{a}{s}\right)$

Let Common Ratio $= r$

Now $S_n = \frac{a(1-r^n)}{1-r}$

$\therefore S_n = \frac{a\left\{1-\left(1-\frac{a}{s}\right)^n\right\}}{1-\left(1-\frac{a}{s}\right)} = \frac{a\left\{1-\left(1-\frac{a}{s}\right)^n\right\}}{a/s}$

$\therefore S_n = S\left\{1 - \left(1 - \frac{a}{s}\right)^{n}\right\}$

4. α, β be the roots of the equation $x^2 - 3x + a = 0$ and γ, δ the roots of $x^2 - 12x + b = 0$ and numbers $\alpha, \beta, \gamma, \delta$ (in this order) form an increasing G.P., then

(A) $a = 3, b = 12$ (B) $a = 12, b = 3$

(C) $a = 2, b = 32$ (D) $a = 4, b = 16$

Ans. **(C)**

Link to View Video Solution: Click Here

Sol. Let $\alpha, \beta, r, \varepsilon = A, AR, AR^2, AR^3$

$x^2 - 3x + a = 0 \begin{cases} \alpha \\ \beta \end{cases}$ $\qquad$ $x^2 - 12x + b = 0 \begin{cases} \gamma \\ \delta \end{cases}$

$\alpha + \beta = \frac{-(-3)}{1} = 3$ $\qquad$ $\alpha \cdot \beta = a$

Or $A + AR = 3$ $\qquad$ $A \cdot (AR) = a$

$A(1 + R) = 3$ ......(i) $\qquad$ $A^2R = a$ .....(ii)

$\square + \delta = -\frac{(-12)}{1} = 12$ $\qquad$ $\square \cdot \delta = b$

$AR^2 + AR^3 = 12$ $\qquad$ $(AR^2 \cdot AR^3) = b$

$AR^2(1 + R) = 12$ .....(iii) $\qquad$ $A^2R^5 = b$ .....(iv)

$(3) \div (1) \Rightarrow \frac{A^2(1+R)}{A(1+R)} = \frac{12}{3}$

$R^2 = 4 \Rightarrow R = 2(\because R > 0)$

Put $R = 2$ in (1)

$A(1 + 2) = 3$

$A = 1$

From (2)

$A^2R = a$

$a = (1)^7(2) = 2$

From (4)

$b = A^2R^5$

$b = (1)^2(2)^5$

$b = 32$

5. In a G.P. of positive terms, any term is equal to the sum of the next two terms. The common ratio of the G.P. is

(A) $2\cos 18^\circ$ (B) $\sin 18^\circ$ (C) $\cos 18^\circ$ (D) $2\sin 18^\circ$

Link to View Video Solution: Click Here

Ans. (D)

Sol. let $\mathbf{G \cdot P}$ be $\mathbf{a, ar, ar^2, \cdots}$

$$a = ar + ar^2 \quad \Rightarrow \quad 1 = r + r^2$$

$$r^2 + r - 1 = 0$$

$$r = \frac{-1 \pm \sqrt{1+4}}{2(1)} \quad \Rightarrow \quad r = \frac{-1 \pm \sqrt{5}}{2}$$

$$r = \frac{\sqrt{5}-1}{2}$$

$$\because \sin 18^\circ = \frac{\sqrt{5}-1}{4}$$

$$2\sin 18^\circ = \frac{\sqrt{5}-1}{2}$$

$$\therefore r = 2\sin 18^\circ$$

6. Find three numbers in G.P. such that their sum is 14 and the sum of their squares is 84 –

(A) $\mathbf{3, 6, 12}$ (B) $\mathbf{2, 6, 18}$ (C) $\mathbf{1, 3, 9}$ (D) $\mathbf{2, 4, 8}$

Ans. (D)

Sol. let $\mathbf{a, ar, ar^2, ar^3}, \ldots$

$$a + ar + ar^2 = 14 \quad \& \quad (a)^2 + (ar)^2 + (ar^2)^2 = 84$$

$$a(1 + r + r^2) = 14 \quad \& \quad a^2(1 + r^2 + r^4) = 84$$

$$a^2(1 + r + r^2)^2 = 196 \text{....(i)} \quad a^2(r^2 + r + 1)(r^2 - r + 1) = 84 \text{ ....(ii)}$$

(2) ÷ (1)

$$\frac{a^2(r^2+r+1)(r^2-r+1)}{a^2(r^2+r+1)^2} = \frac{84}{196}$$

$$7r^2 - 7r + 7 = 3r^2 + 3r + 3$$

$$4r^2 - 10r + 4 = 0 \quad \Rightarrow \quad 2r^2 - 5r + 2 = 0$$

$$(2r - 1)(r - 2) = 0$$

$$r = \frac{1}{2} \Rightarrow a = 8$$

8, 4, 2

Link to View Video Solution: Click Here

& $\mathbf{r = 2, \Rightarrow a = 2}$

$\mathbf{2, 4, 8}$

7. If the sum of an infinite GP is 3 and the sum of the squares of its term is also 3, then its first term and common ratio are –

(A) $\mathbf{3/2, 1/2}$ (B) $\mathbf{1/2, 3/2}$ (C) $\mathbf{1, 1/2}$ (D) $\mathbf{3/2, 3/2}$

Ans. **(A)**

Sol. Let G.P. be $\Rightarrow \mathbf{a, ar, ar^2}, \ldots$

$\mathbf{S_\infty = 3}$

$\frac{\mathbf{a}}{\mathbf{1-r}} = \mathbf{3} \ldots..(\mathbf{1})$

$\mathbf{a^2 + (ar)^2 + (ar^2)^2 + \cdots + \infty = 3}$

$\mathbf{S_\infty^1 = 3}$

$\frac{\mathbf{a^2}}{\mathbf{1-r^2}} = \mathbf{3}$ ....(2)

$$\frac{(\mathbf{1})^2}{(2)} \Rightarrow \frac{\mathbf{a^2/(1-r)^2}}{a/\mathbf{1-r^2}} = \frac{\mathbf{9}}{\mathbf{3}}$$

$$\frac{(\mathbf{1-r})^2}{(\mathbf{1+r})(\mathbf{1-r})} = \mathbf{3}$$

$\frac{\mathbf{1-r}}{\mathbf{1+r}} = \mathbf{3} \Rightarrow \quad \mathbf{1 - r = 3 + 3r}$

$\mathbf{2 = 4r}$

$\mathbf{r} = \frac{\mathbf{1}}{\mathbf{2}} \Rightarrow \mathbf{a = 3/2}$

8. The continued product of three numbers in G.P. is 216, and the sum of the products of them in pairs is 156, find the numbers.

Ans.

Sol. Let Numbers be $\Rightarrow \frac{\mathbf{a}}{\mathbf{r}}, \mathbf{a, ar}$

$$\Rightarrow \frac{\mathbf{a}}{\mathbf{r}} \cdot \mathbf{a \cdot ar = 216 \Rightarrow a^3 = 216 \Rightarrow a = 6}$$

and $\left(\frac{\mathbf{a}}{\mathbf{r}} \cdot \mathbf{a}\right) + (\mathbf{a \cdot ar}) + \left(\frac{\mathbf{a}}{\mathbf{r}} \cdot \mathbf{ar}\right) = \mathbf{156}$

Link to View Video Solution: Click Here

$a^2\left(\frac{1}{r}+r+1\right)=156$ $\Rightarrow$ $\left(\frac{r^2+r+1}{r}\right)=\frac{156}{36}$

$\frac{r^2+r+1}{r}=\frac{13}{3}$ $\Rightarrow$ $3r^2+3r+3=13r$

$3r^2-10r+3=0$ $\Rightarrow$ $(3r-1)(r-3)=0$

$\Rightarrow r=\frac{1}{3},3$

$a=6, r=\frac{1}{3}$ $\quad$ $a=6, r=3$

$\Rightarrow 18,6,2$ or $\Rightarrow 2,6,18$

9. If the p^{th}, q^{th}, r^{th} terms of a G.P. be a, b, c respectively, prove that $a^{q-r}b^{r-p}c^{p-q}=1$.

Ans.

Sol. $T_n=ar^{n-1}$

$(m\cdot n)^a=m^a\cdot n^a$

$(m^n)^a=m^{na}$

$m^n\cdot m^l=m^{n+l}$

In a G.p. $\Rightarrow T_p=a$ $\quad$ $T_q=b$ $\quad$ $T_r=c$

$AR^{p-1}=a$ $\quad$ $AR^{q-1}=b$ $\quad$ $AR^{r-1}=c$

let I^{st} Term $=A$ & Common Ratio $=R$

Now $a^{q-r}\cdot b^{r-p}\cdot c^{p-q}=(AR^{p-1})^{q-r}\cdot(AR^{q-1})^{r-p}\cdot(AR^{r-1})^{p-q}$

$=A^{q-r}(R^{p-1})^{q-r}\cdot A^{r-p}\cdot(R^{q-1})^{r-p}\cdot A^{p-q}\cdot(R^{r-1})^{p-q}$

$=A^{q-r}\cdot A^{r-p}\cdot A^{p-q}\cdot R^{pq-pr-q+r}\cdot R^{qr-pq-r+p}\cdot R^{pr-qr-p+q}$

$=A^{q-r+r-p+p-q}\cdot R^{pq-pr-q+r+qr-pq-r+p+pr-qr-p+q}$

$=A^0\cdot R^0=1$

10. Find the sum in the n^{th} group of sequence,

(i) $1,(2,3);(4,5,6,7);(8,9,\ldots\ldots,15);\ldots\ldots\ldots$

(ii) $(1),(2,3,4),(5,6,7,8,9),\ldots\ldots\ldots$

Ans.

Link to View Video Solution: Click Here

Sol. $S_n = \frac{n}{2}[2a + (n-1)d]$

$I \to 2^{\circ} = 1$ $II \to 2^{2-1} = 2$ $III \to 2^{3-1} = 2^2 = 4$ $IV \to 2^{4-1} = 2^3 = 8$ n^{th} bracket

(i) $\left(\frac{1}{1}\right), (2,3), (4,5,6,7), (8,9, \ldots .15) \ldots \ldots (2, \ldots \ldots)$

11. Circles are inscribed in the acute angle α so that every neighbouring circles touch each other. If the radius of the first circle is R then find the sum of the radii of the first n circles in terms of R and α.

12. If the first 3 consecutive terms of a geometrical progression are the roots of the equation $2x^3 - 19x^2 + 57x - 54 = 0$, then find the sum to infinite number of terms of G.P.

13. The sum of three numbers which are consecutive terms of an A.P. is 21. If the second number is reduced by 1& the third is increased by 1, we obtain three consecutive terms of a G.P., find the numbers.

14. The sum of infinite number of terms of a G.P. is 4 and the sum of their cubes is 192. Find the series.

15. Sum the following series

(i) $1 + \frac{2}{2} + \frac{3}{2^2} + \frac{4}{2^3} + \cdots \ldots$ to n terms.

(ii) $1 + \frac{3}{4} + \frac{7}{16} + \frac{15}{64} + \frac{31}{256} + \cdots$.. to infinity.

16. The sum of the first ten terms of an AP is 155 & the sum of first two terms of a GP is 9. The first term of the AP is equal to the common ratio of the GP & the first term of the GP is equal to the common difference of the AP. Find the two progressions.

17. In a set of four numbers, the first three are in GP & the last three are in AP, with common difference 6. If the first number is the same as the fourth, find the four numbers.

18. Find the 10th term of the GP $\sqrt{2}, \frac{1}{\sqrt{2}}, \frac{1}{2\sqrt{2}}, \ldots$

19. Which term of the GP $\sqrt{2}, \frac{1}{\sqrt{2}}, \frac{1}{2\sqrt{2}}, \frac{1}{4\sqrt{2}}, \ldots$ is $\frac{1}{512\sqrt{2}}$?

20. Which term of the GP $2, 1, \frac{1}{2}, \frac{1}{4}, \ldots$ is $\frac{1}{128}$?

21. If the 2nd and 5th terms of a GP are 24 and 81, respectively, find the GP.

22. If the first term of a GP is 1 and the sum of the 5th and 1st term is 82, find the common ratio.

23. If the first and the nth terms of a GP are a and b, respectively and if P is the product of the first n terms, prove that $P^2 = (ab)^n$.

Link to View Video Solution: Click Here

24. If $\mathbf{a}, \mathbf{b}, \mathbf{c}, \mathbf{d}$ and $\mathbf{p}$ are different real numbers such that $(\mathbf{a^2+b^2+c^2})\mathbf{p^2} - 2(\mathbf{ab+bc+cd})\mathbf{p} + (\mathbf{b^2+c^2+d^2}) = \mathbf{0}$, show that $\mathbf{a}, \mathbf{b}, \mathbf{c}$ and $\mathbf{d}$ are in GP.

25. Find the sum of $\mathbf{1+3+9+27+\cdots}$ to $\mathbf{n}$ terms.

26. Find the sum of $\mathbf{1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\cdots}$ to $\mathbf{n}$ terms

27. How many terms of the series $\mathbf{1+3+3^2+3^3+\cdots}$ must be taken to make 3280?

28. Find the value of $\sum_{\mathbf{k=1}}^{\mathbf{10}}(\mathbf{2+3^k})$.

29. Evaluate $\sum_{\mathbf{n=1}}^{\mathbf{n}}(\mathbf{2^{n-1}+3^n})$.

30. Find the sum of the following series

(i) $\mathbf{5+55+555+\cdots}$ to $\mathbf{n}$ terms

(ii) $\mathbf{7+77+777+\cdots}$ to $\mathbf{n}$ terms

(iii) $\mathbf{9+99+999+\cdots}$ to $\mathbf{n}$ terms

31. Find the sum of $(\mathbf{6666\ldots6})^2 + (\mathbf{8888\ldots8})^2$ (upto $\mathbf{n}$ digits).

32. Prove that the sum to $\mathbf{n}$ terms of the series $\mathbf{11+103+1005+\cdots}$ is $\mathbf{\frac{10}{9}(10^n-1)+n^2}$

33. Find the sum of $\mathbf{n}$ terms of the series $\mathbf{\frac{1}{3}+\frac{5}{9}+\frac{19}{27}+\frac{65}{81}+\cdots}$ to $\mathbf{n}$ terms

34. If $\mathbf{S=\frac{2}{3}+\frac{8}{9}+\frac{26}{27}+\frac{30}{81}+\cdots}$ to $\mathbf{n}$ terms, find the value of $\mathbf{S}$.

35. If $\mathbf{S}$ be the sum, $\mathbf{P}$ the product and $\mathbf{R}$ the sum of the reciprocals of $\mathbf{n}$ terms of a GP, prove that $\left(\mathbf{\frac{S}{R}}\right)^{\mathbf{n}} = \mathbf{P^2}$.

36. Find the sum of $\mathbf{\frac{1}{2}+\frac{1}{3^2}+\frac{1}{2^3}+\frac{1}{3^4}+\frac{1}{2^5}+\frac{1}{3^6}+\cdots\infty}$

37. If $\mathbf{x=1+a+a^2+\cdots\infty}$, where $|\mathbf{a}|<\mathbf{1}$ and $\mathbf{y=1+b+b^2+\cdots\infty}$, where $|\mathbf{b}|<\mathbf{1}$. Prove that
$$\mathbf{1+ab+(ab)^2+\cdots\infty=\frac{xy}{x+y-1}}$$

38. If $\mathbf{x}=\sum_{\mathbf{n=0}}^{\infty}\mathbf{\cos^{2n}\ \theta}, \mathbf{y}=\sum_{\mathbf{n=0}}^{\infty}\mathbf{\sin^{2n}\ \varphi z}=\sum_{\mathbf{n=0}}^{\infty}\mathbf{\cos^{2n}\ \theta\sin^{2n}\ \varphi}$, where $\mathbf{0<\theta,\varphi<\frac{\pi}{2}}$, prove that $\mathbf{xz+yz-z=xy}$.

39. Let $\mathbf{S\subset(-\pi,\pi)}$ denotes the set of values of $\mathbf{x}$ satisfying the equation $\mathbf{8^{1+|\cos\ x|+|\cos\ x|^2+|\cos\ x|^3+\cdots\text{ to }\infty}=4^3}$, prove that $\mathbf{S}=\left(-\frac{\pi}{3},-\frac{2\pi}{3}\right)$.

40. If $\mathbf{exp}\ \{(\mathbf{\sin^2\ x+\sin^4\ x+\sin^6\ x+\cdots\infty})$ satisfies the equation $\mathbf{x^2-9x+8=0}$, prove that the value of $\left(\frac{\cos\ x}{\cos\ x+\sin\ x}\right)=\frac{1}{2}(\sqrt{3}-1)$

Link to View Video Solution: Click Here

ANSWER KEY

1. (B) **2.** (B) **3.** (B) **4.** (C) **5.** (D) **6.** (D) **7.** (A)

8. 2, 6,18 and 18,6,2 **10.** (i) $2^{n-2}(2^n + 2^{n-1} - 1)$ (ii) $(n-1)^3 + n^3$

11. $\frac{R\left(1-\sin\frac{\alpha}{2}\right)}{2\sin\frac{\alpha}{2}}\left[\left(\frac{1+\sin\frac{\alpha}{2}}{1-\sin\frac{\alpha}{2}}\right)^n - 1\right]$ **12.** $\frac{27}{2}$ **13.** $3, 7, 11$ or $12, 7, 2$

14. $6, -3, 3/2, \dots\dots$ **15.** (i) $4 - \frac{2+n}{2^{n-1}}$ (ii) $8/3$

16. $(3 + 6 + 12 + \cdots)$; $(2/3 + 25/3 + 625/6 + \cdots$.

17. $(8, -4, 2, 8)$ **18.** $2^{-\frac{17}{2}}$ **19.** 11th term **20.** 9th term

21. $(16, 24, 36, \dots)$ **22.** $(r = \pm 3)$ **25.** $\left(\frac{3^n-1}{2}\right)$ **26.** $2\left(1 - \frac{1}{2^n}\right)$

27. $n = 8$ **28.** $\frac{1}{2}(3^{11} + 37)$ **29.** $\frac{1}{2}(2^{n+1} + 3^{n+1} - 5)$

30. (i) $\frac{5}{81}(10^{n+1} - 9n - 10)$ (ii) $\frac{7}{81}(10^{n+1} - 9n - 10)$ (iii) $\frac{1}{9}(10^{n+1} - 9n - 10)$

31. $\frac{4}{3}(10^n - 1)^2$ **33.** $n + 2\left(1 - \left(\frac{2}{3}\right)^n\right)$ **34.** $n + \frac{\left(1-\left(\frac{1}{3}\right)^n\right)}{2}$

36. $\left(\frac{19}{24}\right)$