Projectile motion

Syllabus for jee Advance Test :—

→ Vector

→ Kinematic

- Distance, Displacement.
- Avg velocity, Inst velocity.
- Avg acceleration, Inst Acceleration
- Kinematics equations.
- graph.
- Motion under gravity.

Nishant Jindal

Projectile motion

↳ [Particle move under the influence (प्रभाव) of gravitational force]

* If particle is projected obliquely (at a certain angle 'θ') the the path or trajectory of the projectile motion is parabolic.

(*) Assumption.

- 'g' → Constant.
- No resistive force.

Nishant Jindal

Projectile motion

2-dimensional Motion

$\theta \rightarrow$ Angle of projection.

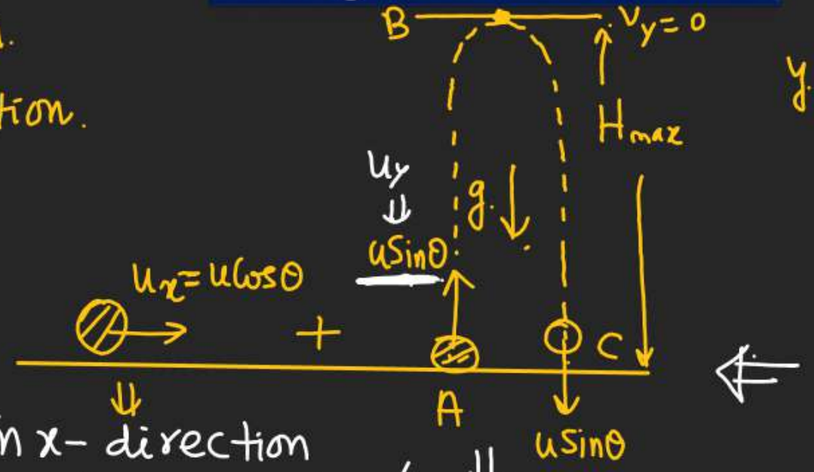
$u \rightarrow$ Speed of projection.

Time of flight

(↳ Time taken by projectile for the whole journey)

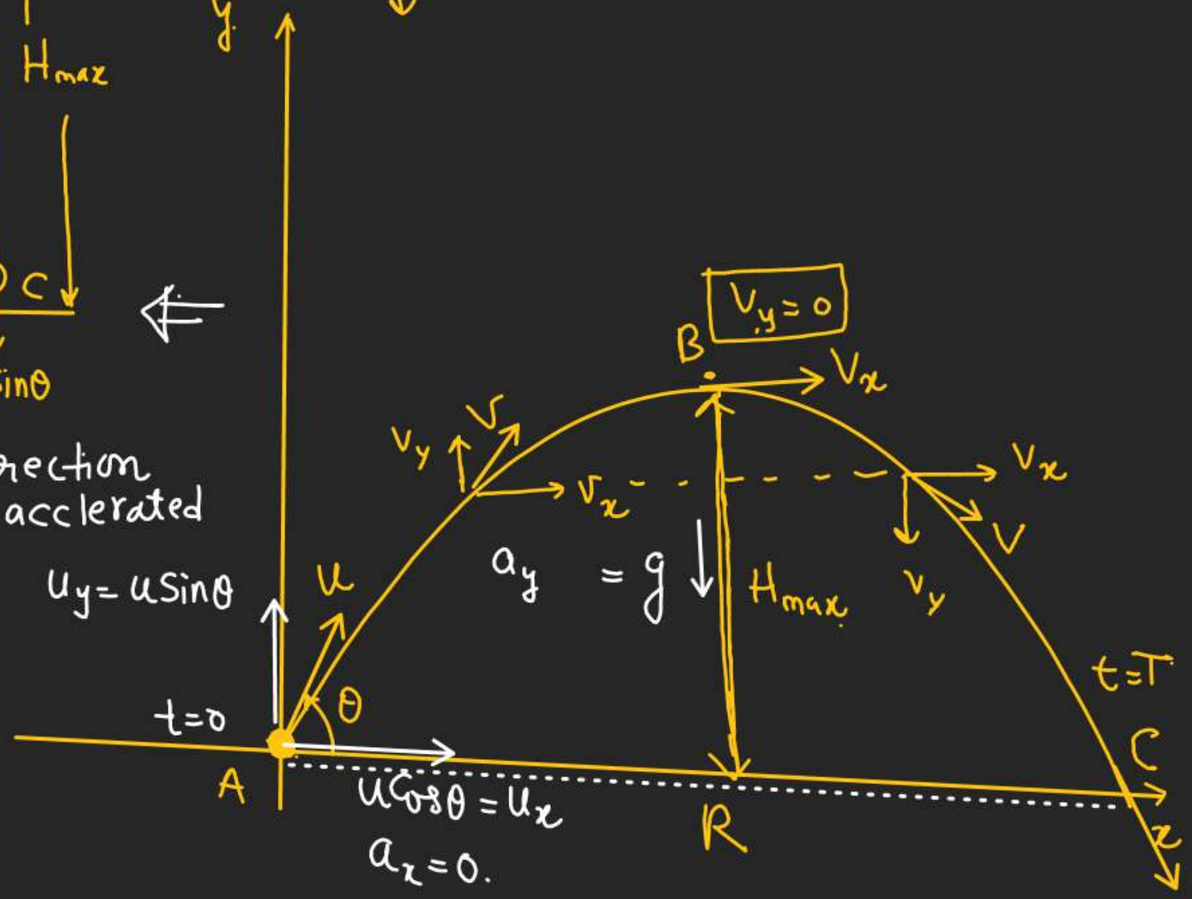
⇒ Depends on Vertical Motion.

$$** \quad \boxed{T = \frac{2u_y}{g}} \Rightarrow \boxed{T = \frac{2u\sin\theta}{g}} \checkmark$$



(In x-direction Uniform velocity Motion)

(In y-direction Uniform accelerated Motion)



Nishant Jindal

Projectile motion

Maximum height :-

↳ [Depends on vertical motion]

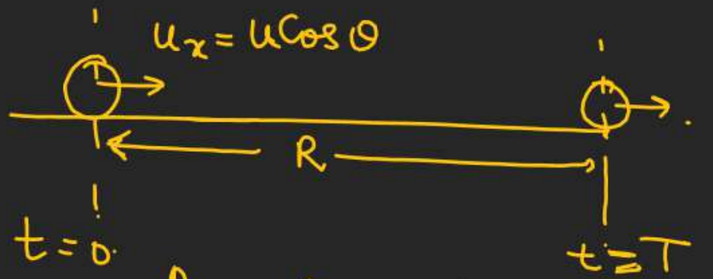
↳ [For maximum height $V_y = 0$]

$$\Rightarrow \boxed{H_{max} = \frac{u_y^2}{2g}}$$

$$\Rightarrow \text{☁}\ H_{max} = \frac{u^2 \sin^2\theta}{2g} \ \text{☁}\ **$$

Range :-

[Total horizontal distance covered by the particle in x– direction]



$$R = u_x \times T$$

$$** \quad R = u_x \times \frac{2u_y}{g}$$

$$\boxed{R = \frac{2u_x u_y}{g}}$$

$$R = \frac{u\cos\theta \times (2u\sin\theta)}{g}$$

$$R = \frac{2u^2 \sin\theta\cos\theta}{g}$$

$$** \quad R = \frac{u^2}{g}(2\sin\theta\cos\theta)$$

$$\boxed{R = \frac{u^2 \sin 2\theta}{g}}$$

Nishant Jindal

Projectile motion

(A)

Motion in x-direction

$x = u_x t$

$x = (u\cos\theta)t$

Motion in y-direction

$V_y = u_y - gt$

$V_y = u\sin\theta - gt$ ... ①

$y = u_y t - \frac{1}{2}gt^2$

$y = (u\sin\theta)t - \frac{1}{2}gt^2$ ... ②

$V_y^2 = u_y^2 - 2gy$.

$V_y^2 = u^2\sin^2\theta - 2gy$ ... ③

Time of Flight

At $t = T$, $y = 0$

Put $y = R$ in ② equation.

$(u\sin\theta)t - \frac{1}{2}gt^2 = 0$

$t\left[u\sin\theta - \frac{1}{2}gt\right] = 0$

$t = 0$, $t = \frac{2u\sin\theta}{g}$

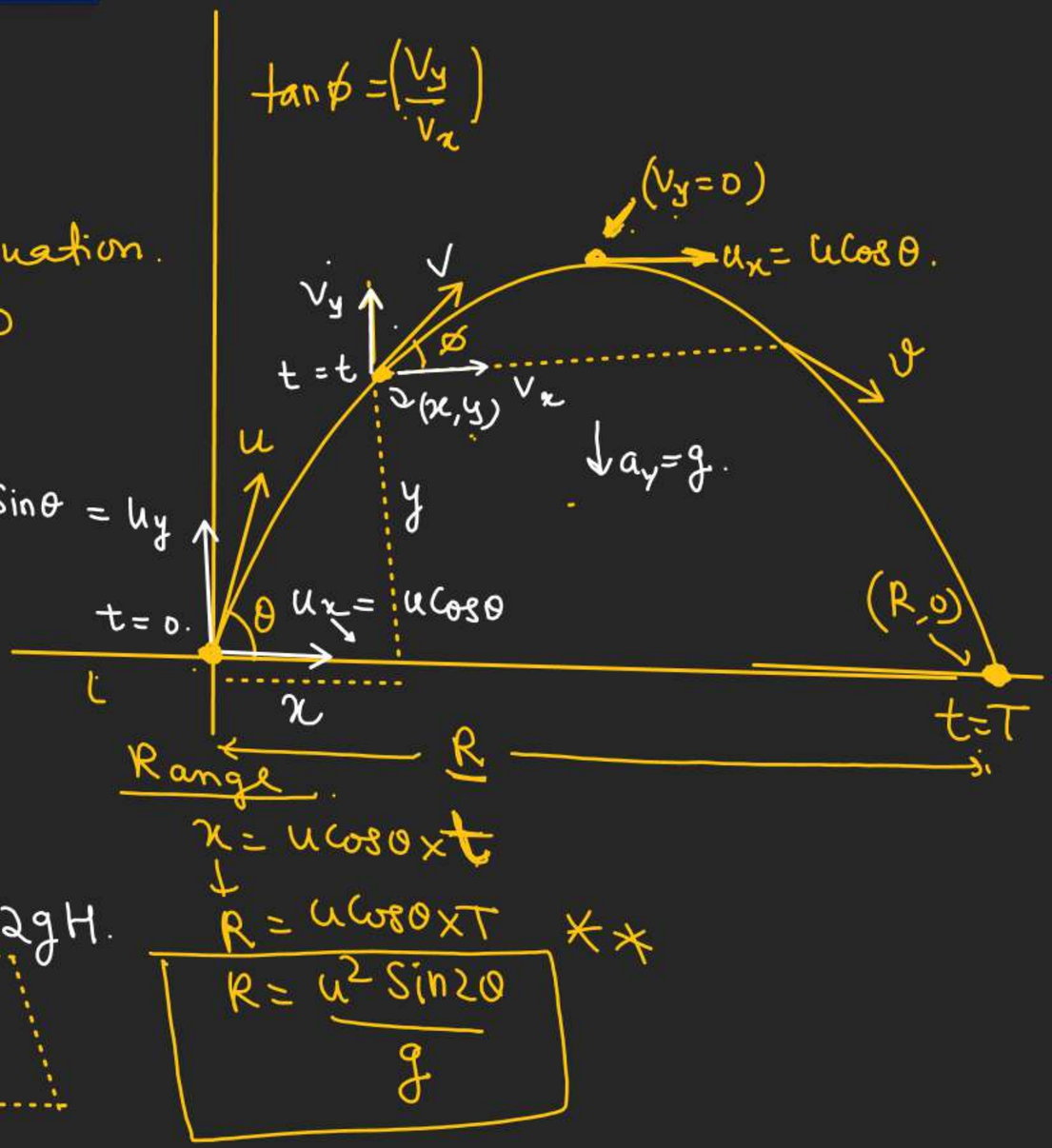
Maximum height

From ③

$y = H$, $V_y = 0$

$0 = u^2\sin^2\theta - 2gH$

$H = \frac{u^2\sin^2\theta}{2g}$



Range

$x = u\cos\theta \times t$

$R = u\cos\theta \times T$ **

$R = \frac{u^2\sin 2\theta}{g}$

Nishant Jindal

Vector form in projectile Motion

$$\vec{g} = -g\hat{j}$$

$$\vec{v} = \vec{u} + \vec{a}t$$

$$\vec{s} = \vec{u}t + \frac{1}{2}\vec{a}t^2$$

$$\vec{v}\cdot\vec{v} = \vec{u}\cdot\vec{u} + 2\vec{a}\cdot\vec{s}$$

$$\vec{AB} = (\vec{u}T)$$

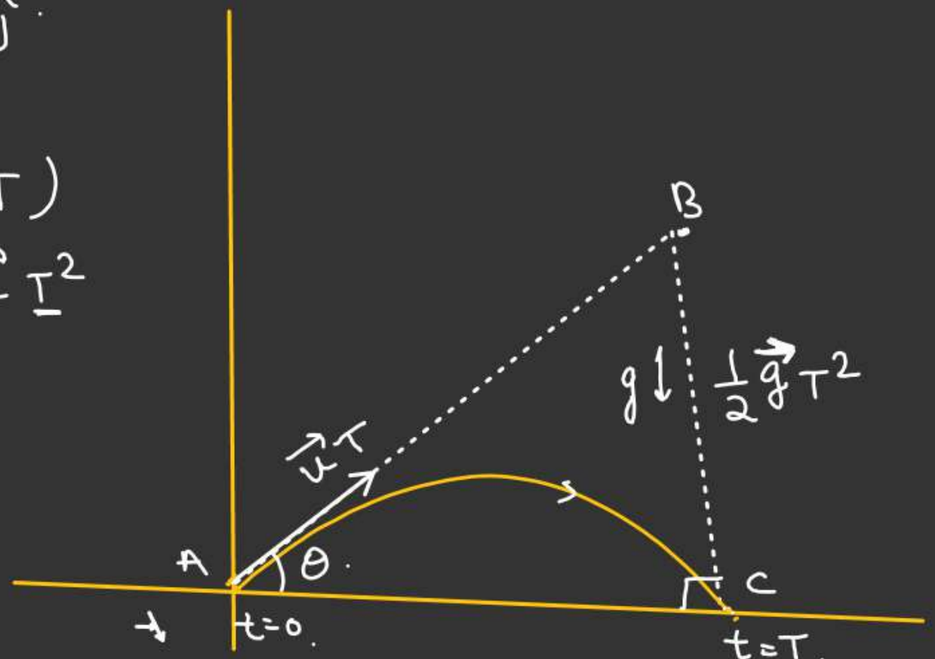
$$\vec{BC} = \frac{1}{2}\vec{g}T^2$$

In $\triangle ABC$

$$\sin\theta = \frac{BC}{AB}$$

$$\sin\theta = \frac{\frac{1}{2}gT^2}{uT}$$

$$* \quad \boxed{T = \frac{2u\sin\theta}{g}}$$



Nishant Jindal

Projectile motion

(8)

Maximum height

$\vec{AB} = \vec{u}\left(\frac{T}{2}\right)$

$\vec{BC} = \frac{1}{8}\vec{g}T^2$

$AB\sin\theta = BC + CD$ (ABSinθ)

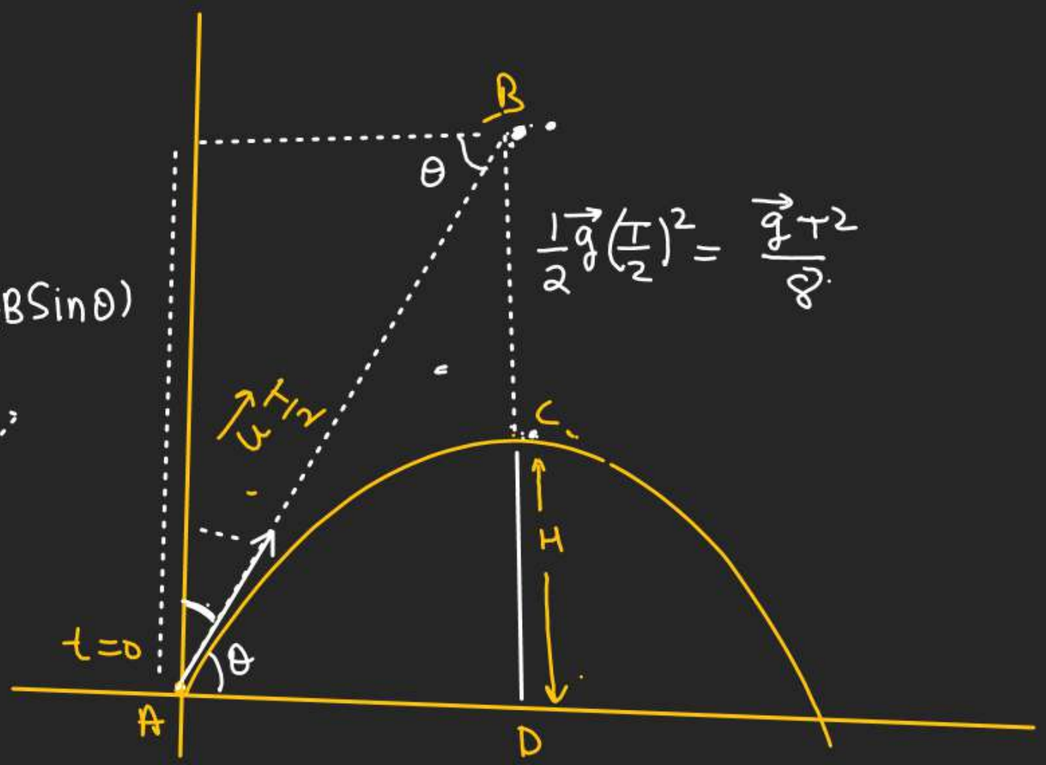
$\frac{uT}{2}\sin\theta = \frac{gT^2}{8} + H$ ....

$H = \frac{uT}{2}\sin\theta - \frac{gT^2}{8}$

$H = \frac{u\sin\theta}{2} \times \left(\frac{2u\sin\theta}{g}\right) - \frac{g}{8}\left(\frac{2u\sin\theta}{g}\right)^2$

$H = \left(\frac{u^2\sin^2\theta}{g} - \frac{u^2\sin^2\theta}{2g}\right)$ ⇒

$$\boxed{H = \frac{u^2\sin^2\theta}{2g}}$$



$\frac{1}{2}\vec{g}\left(\frac{T}{2}\right)^2 = \frac{\vec{g}T^2}{8}$

** $t = \frac{T}{2}$ ∴