DPP 02

SOLUTION

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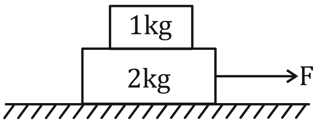
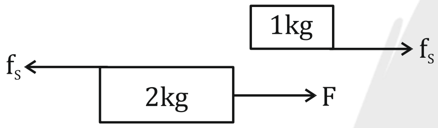
1. $\mu_s = 0.5$
Apply the force F
suppose a is acceleration





Newton's second law.

$F - f_s = 2a$ ...(i)

$f_s = 1 \times a$ ...(ii)

$f_s = m \times 1 \times g; f_s = \mu Mg$

$\mu \times 1 \times g = 1a$

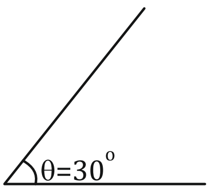
$0.5 \times 10 = a \Rightarrow a = 5 \text{ m/s}^2$

$\therefore F - 5 = 2 \times 5 = 15 \text{ N}$ $(\because \text{ using (ii)})$

2. $m = \frac{\sqrt{x}}{5}$

Time of ascent $= \frac{1}{2}$ time of descent

$$t_a = \frac{1}{2} t_d$$



suppose L is the length of the incline, a_a is acceleration of ascent and a_d is acceleration of descent.

So, $\sqrt{\frac{2L}{a_a}} = \frac{1}{2}\sqrt{\frac{2L}{a_d}}; \frac{2L}{a_a} = \frac{1}{4} \times \frac{2L}{a_d}$

$4a_d = a_a$ ...(i)

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Acceleration of ascent,

$a_a = g(\sin\theta + \mu\cos\theta)$

Acceleration of descent,

$a_d = g(\sin\theta - \mu\cos\theta)$

Using values of a_a and a_d in equation (i), we get

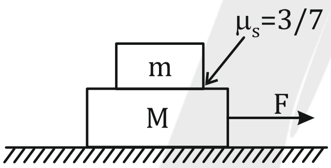
$\Rightarrow\ 4 \cdot g(\sin\theta - \mu\cos\theta) = g(\sin\theta + \mu\cos\theta)$

$$\Rightarrow\ 4\left[\frac{1}{2} - \mu\frac{\sqrt{3}}{2}\right] = \left[\frac{1}{2} + \mu\frac{\sqrt{3}}{2}\right]$$

$\Rightarrow\ 4 - 4\mu\sqrt{3} = 1 + \mu\sqrt{3}; 3 = 5\mu\sqrt{3}$

$\Rightarrow\ \mu = \frac{\sqrt{3}}{5}$

3. Given, $m = 0.5$ kg, $M = 4.5$ kg



Coefficient of static friction between two blocks,

$\mu_s = \frac{3}{7}$

On 0.5 kg, only horizontal force is acting which is friction.
Minimum acceleration of this block,

$$a_{max} = \frac{f}{m} = \frac{\mu mg}{m} = \mu g = \frac{3}{7} \times 9.8\ \text{m/s}^2$$

[If there is no friction between m and M, then M will not move on application of force on M]
Since both the blocks are moving together,

$F_{max} = (m + M)a_{max}$

$= (0.5 \times 4.5) \times \frac{3}{7} \times 9.8 = 21\ \text{N}$

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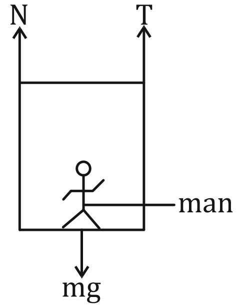
4. Mass of boy = 4 kg Mass of wood = 5 kg

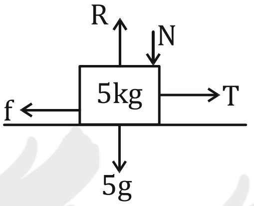


Coefficient of friction between wood and floor = 0.5

Free body diagram of man

$N + T = mg = 40$ ...(i)

Free body diagram of wood



$T = f$

Also, $R = N + 5g = N + 50$

$T = \mu R = \mu(N + 50);$

$T = 0.5(N + 50)$

$N = 2T - 50$

$40 - T = 2T - 50$ [Using (i)]

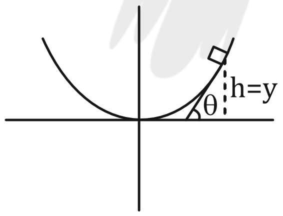
$3T = 90$ or $T = 30$ N

5. Given, $y = \frac{x^2}{4}$ and $\mu = 0.5$

suppose the maximum height is h.

So, the slope of tangent at height h is angle of repose.

$\therefore \tan\theta = \frac{dy}{dx} = \frac{1}{4} \cdot 2x = \frac{x}{2}$



Slope, $m = \tan\theta$

For no slipping, $\tan\theta \le \mu$

$\Rightarrow 0.5 \ge \frac{x}{2} \Rightarrow x \le 1$ m

So, $y = h = \frac{x^2}{4} \le 0.25$ m ≤ 25 cm

$\Rightarrow h \le 25$ cm

$\therefore$ Maximum height, $h_{max} = 25$ cm.

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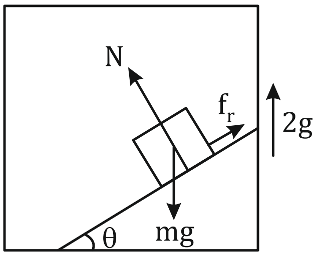
6. Pseudo = 2mg.

Normal (N) = 3mgcosθ



$F_r = \mu N$

$F_r = 3\mu_m mg\cos\theta$

In balanced condition

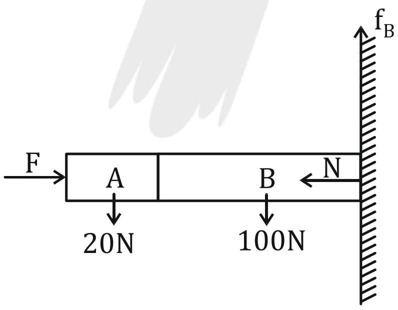
$3mg\sin\theta = 3\mu_m mg\cos\theta$

$\sin\theta = \mu_m\cos\theta.$

$\tan\ \theta = \mu_m$

$\mu_m = \tan\theta$

7. Various forces acting on the system are shown in the figure.



For vertical equilibrium of the system,

$f_B = 100\text{ N} + 20\text{ N} = 120\text{ N}$

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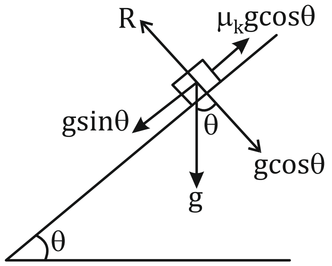
8. Component of g down the plane = $g\sin\theta$



$\therefore$ For smooth plane, $d = \frac{1}{2}(g\sin\theta)t^2$

For rough plane,

Frictional retardation up the plane = $\mu_k(g\cos\theta)$

$\therefore d = \frac{1}{2}(g\sin\theta - \mu_k g\cos\theta)(nt)^2$

$\therefore \frac{1}{2}(g\sin\theta)t^2 = \frac{1}{2}(g\sin\theta - \mu_k g\cos\theta)n^2t^2$

Or $\sin\theta = n^2(\sin\theta - \mu_k\cos\theta)$

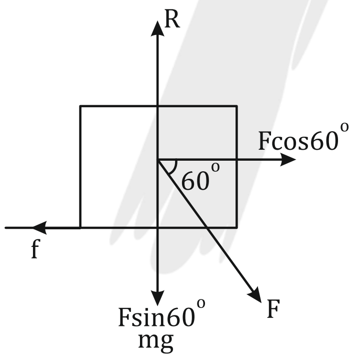
Putting $\theta = 45°$

or $\sin 45° = n^2(\sin 45° - \mu_k\cos 45°)$

or $\frac{1}{\sqrt{2}} = \frac{n^2}{\sqrt{2}}(1 - \mu_k)$

or $\mu_k = 1 - \frac{1}{n^2}$

9. suppose F be the maximum value of force applied when the block of $m = \sqrt{3}$ kg does not move on the rough surface.



R = normal reaction

Normal to surface.

Or $R = F\sin 60° + mg$

f = force of friction

$\mu R = F\cos 60°$

$\mu(F\sin 60° + mg) = F\cos 60°$

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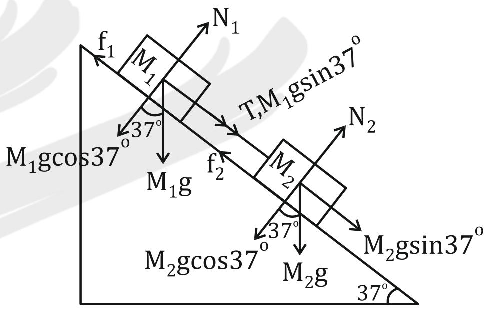
or $\mu F\sin 60^\circ + \mu mg = F\cos 60^\circ$

or $F = \dfrac{\mu mg}{\cos\ 60^\circ - \mu\sin\ 60^\circ}$

$$= \frac{\frac{1}{2\sqrt{3}} \times \sqrt{3} \times 10}{\frac{1}{2} - \left(\frac{1}{2\sqrt{3}} \times \frac{\sqrt{3}}{2}\right)} = \frac{5}{\frac{1}{2} - \frac{1}{4}} = 5 \times 4 = 20\ \text{N}$$

$\therefore$ Maximum value of force = 20 N.

10. suppose a be the common acceleration of the two masses M_1 and M_2 and T be tension of the string.



Equation of motion for M_1 moving down

$M_1 g\sin\ \theta + T - f_1 = M_1 a$

Since $f_1 = \mu_1 M_1 g\cos\ \theta$

$\therefore M_1 g\sin\theta - \mu_1 M_1 g\cos\ \theta + T = M_1 a$ ...(i)

Equation of motion for M_2 moving down

$M_2 g\sin\theta - f_2 - T = M_2 a$

Since $f_2 = \mu_2 M_2 g\cos\ \theta$

$\therefore\ M_2 g\sin\ \theta - \mu_2 M_2 g\cos\theta - T = M_2 a$ ...(ii)

Solving (i) and (ii)

$(M_1 + M_2)\ g\sin\theta$

or $a = \dfrac{-g\cos\theta(\mu_1 M_1 + \mu_2 M_2)}{(M_1 + M_2)}$

$$= \frac{(4+2) \times 9.8 \times 0.6 - 9.8 \times 0.8(0.75 \times 4 + 0.25 \times 2)}{6}$$

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$$= \frac{3.6 \times 9.8 - 9.8 \times 0.8(3 + 0.5)}{6}$$

$$= \frac{35.28 - 27.44}{6} = \frac{7.84}{6} = 1.3\ \mathrm{m/s^2}$$

$$\mathrm{T} = \mathrm{M_2 g \sin\,\theta} - \mu_2 \mathrm{M_2 g \cos\,\theta} - \mathrm{M_2 a}$$

$$= 2 \times 9.8 \times 0.6 - 0.25 \times 2 \times 9.8 \times 0.8 - 2 \times 1.3$$

$$= 11.76 - 3.92 - 2.6 = 5.24\ \text{newton}$$