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$$\lim_{n\to\infty} \prod_{r=2}^{n} \frac{r^3+1}{r^3-1} = \lim_{n\to\infty}\left(\prod_{r=2}^{n} \frac{r+1}{r-1} \prod_{r=2}^{n} \frac{r^2-r+1}{r^2+r+1}\right)$$

$$\begin{bmatrix} 2 & 1 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 2a+c & 2b+d \\ 2a+c & 2b+d \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 2 & 1 \end{bmatrix}$$

$$P(AP)^{-1} = PP^{-1}A^{-1} = \boxed{A^{-1}}$$

$$2a+c=2$$

$$2b+d=1$$

$$A^{-1}(A^{-1})^{T} = A^{-1}(A^{T})^{-1}$$

$$= (A^{T}A)^{-1} = I^{-1} = I$$

$$\boxed{A = \begin{bmatrix} a & b \\ 2-2a & 2-2b \end{bmatrix}}$$

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$$T = \underbrace{P^T P B P^T P B P^T P B P^T \cdots P B P^T P}_{} = B^K = (I+D)^K$$

$$BA = A$$

$$AB = B$$

$$= I + {}^KC_1 D + {}^KC_2 D^2 + \cdots + {}^KC_K D^K$$

$$\boxed{A^2 = I}$$

$$D^2 = \begin{bmatrix} a-1 & b \\ 0 & 0 \end{bmatrix}\begin{bmatrix} a-1 & b \\ 0 & 0 \end{bmatrix}$$

$$BAB = B^2$$

$$AB = B^2 = I + \frac{D}{(a-1)}\left((a-1)\,{}^KC_1 + {}^KC_2 (a-1)^2 + {}^KC_3 (a-1)^3 + \cdots {}^KC_K (a-1)^{1+K-1}\right)$$

$$D^2 = \begin{bmatrix} (a-1)^2 & (a-1)b \\ 0 & 0 \end{bmatrix} = (a-1)D$$

$$B^2 = B$$

$$D^3 = (a-1)D^2 = (a-1)^2 D$$

$$D^4 = (a-1)^3 D$$

$$2^2 |A^7| |B| = I + \frac{(1+a-1)^K - 1}{a-1} D$$

$$= 2^2 \frac{|A|^7}{|B|} = I + \left(\frac{a^K - 1}{a-1}\right) D$$

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$$\begin{bmatrix} 2A+1 & -5 \\ -4 & A \end{bmatrix}^{-1} \begin{bmatrix} A-5 & B \\ 2A-2 & C \end{bmatrix} = \begin{bmatrix} 14 & D \\ E & F \end{bmatrix}$$

$$\begin{bmatrix} A-5 & B \\ 2A-2 & C \end{bmatrix} = \begin{bmatrix} 2A+1 & -5 \\ -4 & A \end{bmatrix} \begin{bmatrix} 14 & D \\ E & F \end{bmatrix}$$

$$A^{-1}BA\,A^{-1}BA\,A^{-1}BA \cdots A^{-1}BA$$

$$A^{-1}B^{n}A$$

$$A-5 = 28A+14-5E \quad (1)$$

$$2A-2 = -56+AE \quad (2)$$

$$(1)\times A + (2)\times 5$$

Methods of Integration

Substitution

$$\int f(x)\,dx$$

Put $x = \phi(t)$

$$\frac{dx}{dt} = \phi'(t)$$

$$dx = \phi'(t)\,dt$$

$$= \int f(\phi(t))\,\phi'(t)\,dt$$

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$$\int 2^{f(x)} f'(x)\,dx$$

$f(x) = t$

$f'(x) = \frac{dt}{dx}$

$f'(x)\,dx = dt$

$$= \int 2^t\,dt$$

$$= \frac{2^t}{\ln 2} + C$$

$$= \frac{2^{f(x)}}{\ln 2} + C.$$

$f(x) = t$

$f'(x) = \frac{dt}{dx}$

$f'(x)dx = dt$

$$\int \frac{f'(x)\,dx}{f^2(x)+9}$$

$$= \int \frac{dt}{t^2+9} = \frac{1}{3}\tan^{-1}\frac{t}{3} + C$$

$$= \frac{1}{3}\tan^{-1}\left(\frac{f(x)}{3}\right) + C.$$

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$$\int \tan x \, dx = \int \frac{\sin x \, dx}{\cos x}$$

$\cos x = t$

$-\sin x \, dx = dt$

$$= \int \frac{-dt}{t}$$

$$= -\ln|t| + C$$

$$= -\ln|\cos x| + C$$

$$= \ln|\sec x| + C$$

$$\boxed{\begin{aligned} \int \tan x \, dx &= \ln|\sec x| + C \\ \int \cot x \, dx &= \ln|\sin x| + C \end{aligned}}$$

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$$\int \frac{dx}{(x^2-a^2)} = \int \frac{dx}{(x-a)(x+a)} = \frac{1}{2a}\int \frac{\big((x+a)-(x-a)\big)}{(x-a)(x+a)}\,dx$$

$$\boxed{\begin{aligned}\int \frac{dx}{x^2-a^2} &= \frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right|+C\\ \int \frac{dx}{a^2-x^2} &= \frac{1}{2a}\ln\left|\frac{a+x}{a-x}\right|+C\end{aligned}}$$

$$= \frac{1}{2a}\int\left(\frac{1}{x-a}-\frac{1}{x+a}\right)dx$$

$$= \frac{1}{2a}\Big(\ln|x-a| - \ln|x+a|\Big)+C$$

$$= \frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right|+C$$

$x-a=t$

$dx=dt$

$$\int \frac{dt}{t} = \ln|t|+c = \ln|x-a|+c.$$

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$$\int \sec x\, dx = \int \frac{\sec x(\sec x+\tan x)\,dx}{(\sec x+\tan x)} =$$

Put $\sec x+\tan x = t$.

$(\sec x\tan x+\sec^2 x)dx = dt$

$$= \int \frac{dt}{t} = \ln|t| + C$$

$$= \ln|\sec x+\tan x| + C$$

$$\frac{c+s}{c-s}$$

$$= \ln\left|\frac{1+\sin x}{\cos x}\right| + C = \ln\left|\frac{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)^2}{\cos^2\frac{x}{2}-\sin^2\frac{x}{2}}\right| + C = \ln\left|\frac{1+\tan\frac{x}{2}}{1-\tan\frac{x}{2}}\right| + C$$

$$= \ln\left|\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)\right| + C.$$

$$\int \frac{dx}{\cos x}$$

$$= \int \frac{\cos x\, dx}{1-\sin^2 x}$$

$\sin x = t$

$\cos x\, dx = dt$

$$= \int \frac{dt}{1-t^2} = \frac{1}{2}\ln\left|\frac{1+t}{1-t}\right| + C$$

$$= \frac{1}{2}\ln\left|\frac{1+\sin x}{1-\sin x}\right| + C$$

$$\ln\left|\frac{\cos\frac{x}{2}+\sin\frac{x}{2}}{\cos\frac{x}{2}-\sin\frac{x}{2}}\right| + C.$$

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$$\int \sec x\,dx = \ln\left|\sec x + \tan x\right| + C$$
$$= \ln\left|\tan\left(\frac{\pi}{4} + \frac{x}{2}\right)\right| + C$$

$$\int \operatorname{cosec} x\,dx = \ln\left|\operatorname{cosec} x - \cot x\right| + C$$
$$= \ln\left|\tan\frac{x}{2}\right| + C$$

Nishant Jindal

1. $$\int \frac{\tan(\sin^{-1}x)}{\sqrt{1-x^2}}\,dx$$

Put $\sin^{-1}x = t$

$$\frac{1}{\sqrt{1-x^2}}\,dx = dt$$

$$= \int \tan t\,dt$$

$$= \ln|\sec t| + C = \ln\left|\sec(\sin^{-1}x)\right| + C$$

$$\int \frac{x}{\sqrt{1-x^2}}\cdot\frac{1}{\sqrt{1-x^2}}\,dx = \int \frac{x\,dx}{1-x^2}$$

$$\boxed{1-x^2=t}$$

$$-2x\,dx = dt$$

$$= -\frac{1}{2}\int \frac{dt}{t} = -\frac{1}{2}\ln|t| + c = -\frac{1}{2}\ln|1-x^2| + c$$

2. $$\int \frac{x\cos x\,dx}{(x\sin x+\cos x)^2}$$

$x\sin x + \cos x = t \Rightarrow dt = (x\cos x + \sin x - \sin x)\,dx$

$dt = x\cos x\,dx$

$$= \int \frac{dt}{t^2} = -\frac{1}{t} + C = -\frac{1}{x\sin x+\cos x} + C$$

3. $$\int \frac{\cos x\,dx}{\cos(x-a)}$$

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$$= \frac{1}{3}\ln|\sin 3x| + c$$

$$= \frac{1}{3}\ln|\sin t| + C$$

$$\frac{1}{3}\int \cot t \, \cancel{dt} = \int \cot 3x \, dx$$

$3x = t$

$dx = \frac{1}{3}dt$

$$\int \frac{\cos x \, dx}{\cos(x-a)} = \int \frac{\cos(\underline{x-a} + \underline{a})\, dx}{\cos(x-a)}$$

$$= \int \left(\cos a - \sin a \tan(x-a)\right) dx$$

$$= x\cos a - \sin a \ln|\sec(x-a)| + C$$

$5x - 3x$

4. $$\int \frac{\sin 2x \, dx}{\sin 5x \sin 3x} = \int (\cot 3x - \cot 5x)\, dx = \frac{1}{3}\ln|\sin 3x| - \frac{1}{5}\ln|\sin 5x| + C$$

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5. $\displaystyle\int \frac{x^5\,dx}{1+x^{12}}$

$x^6 = t$

$6x^5\,dx = dt$

$= \frac{1}{6}\displaystyle\int \frac{dt}{1+t^2} = \frac{1}{6}\tan^{-1} t + C = \frac{1}{6}\tan^{-1}(x^6) + C.$

6. $\displaystyle\int \frac{x^2\,\tan^{-1}(x^3)\,dx}{(1+x^6)}$

$\boxed{\tan^{-1}x^3 = t} \rightarrow$

$= \frac{1}{3}\displaystyle\int \frac{(\tan^{-1}x^3)\,3x^2\,dx}{(1+x^6)}$

$= \frac{1}{3}\displaystyle\int t\,dt = \frac{t^2}{6} + C$

$= \frac{1}{6}\left(\tan^{-1}(x^3)\right)^2 + C.$

$\boxed{\text{Ex-4}}$ $\boxed{\text{Test Paper-1}}$

Nishant Jindal

G.N. Berman $\longrightarrow$ $\boxed{1690 - 1750}$

1703.

$$\int \sin x \, d(\sin x)$$

$$= \int t \, dt = \frac{t^2}{2} + c = \frac{\sin^2 x}{2} + c.$$

$$\frac{d}{dx}\sin x = \cos x \Rightarrow d(\sin x) = \cos x \, dx.$$

$$= \int \sin x \cos x \, dx.$$