



DPP-01(Solution)

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1. $\int \sqrt{x} dx$

Ans. $\frac{2}{3}\sqrt{x^3} + C$

Sol. $\because \int x^n \cdot dx = \frac{x^{n+1}}{n+1} + C$

$$\int x^{1/2} \cdot dx \Rightarrow \frac{x^{1/2+1}}{\frac{1}{2}+1} + C \Rightarrow \frac{x^{3/2}}{3/2} + C \Rightarrow \frac{2}{3}x^{3/2} + C \Rightarrow \frac{2}{3}\sqrt{x^3} + C.$$

2. $\int \sqrt[m]{x^n} dx$

Ans. $\frac{mx^{\frac{n}{m}+1}}{n+m} + C$

Sol. $\int (x^n)^{1/m} \cdot dx \Rightarrow \int x^{\frac{n}{m}} \cdot dx = \frac{x^{\frac{n}{m}+1}}{\frac{n}{m}+1} + C$

$$= \frac{x^{\frac{n+m}{m}}}{\binom{n+m}{m}} + C \Rightarrow \frac{m}{n+m}(x^{n+m})^{1/m} + C \Rightarrow \frac{mx^{\frac{n}{m}+1}}{n+m} + C$$

3. $\int \frac{dx}{x^2}$

Ans. $C - \frac{1}{x}$

Sol. $\int \frac{1}{x^2} \cdot dx = \int x^{-2} \cdot dx = \frac{x^{-2+1}}{-2+1} + C = \frac{x^{-1}}{-1} + C = -\frac{1}{x} + C$

4. $\int 10^x dx$.

Ans. $\approx 0.4343 \times 10^x + C$

Sol. $\because \int a^x \cdot dx = \frac{a^x}{\log_e a} + C$
 $= \frac{10^x}{\log_e 10} + C$

5. $\int a^x e^x dx$.

Ans. $\frac{(ae)^x}{1+\ln a} + C$

Sol. $[\because m^a \cdot n^a = (m \cdot n)^a]$

$$= \int (ae)^x \cdot dx = \frac{(ae)^x}{\log_e (ae)} + C = \frac{a^x \cdot e^x}{\log_e a + \log_e e} + C = \frac{a^x \cdot e^x}{1 + \log_e a} + C = \frac{(ae)^x}{1 + \ln a} + C.$$

6. $\int \frac{dx}{2\sqrt{x}}$

Ans. $\sqrt{x} + C$



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Sol. $\frac{1}{2} \int \frac{dx}{x^{1/2}} = \frac{1}{2} \int x^{-1/2} \cdot dx = \frac{1}{2} \left(\frac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} \right) + c = \frac{1}{2} \cdot \frac{x^{1/2}}{\left(\frac{1}{2}\right)} + c = x^{1/2} + c = \sqrt{x} + c$

7. $\int \frac{dh}{\sqrt{2gh}}$.

Ans. $\sqrt{\frac{2h}{g}} + c$

Sol. $\int \frac{1}{\sqrt{2g}} \cdot \frac{dh}{\sqrt{h}} = \frac{1}{\sqrt{2g}} \int \frac{1}{\sqrt{h}} \cdot dh = \frac{1}{\sqrt{2g}} \cdot \sqrt{2} \sqrt{h} + c = \sqrt{\frac{2h}{g}} + c$

8. $\int 3.4x^{-0.17} dx$.

Ans. $\approx 4.1x^{0.83} + c$

Sol. $= 3.4 \int x^{-0.17} \cdot dx = 3.4 \left\{ \frac{x^{-0.17+1}}{-0.17+1} \right\} + C = \left(\frac{3.4}{0.83} \right) x^{0.83} + C \approx 4.1x^{0.83} + C$

9. $\int \frac{\sqrt{x}-x^3e^x+x^2}{x^3} dx$.

Ans. $c - \frac{2}{3x\sqrt{x}} - e^x + \ln|x|$

Sol. $\int \frac{x^{1/2}}{x^3} - \frac{x^3e^x}{x^3} + \frac{x^2}{x^3} \cdot dx = \int \left(\frac{1}{x^{5/2}} - e^x + \frac{1}{x} \right) \cdot dx$
 $\int x^{-5/2} \cdot dx - \int e^x \cdot dx + \int \frac{1}{x} \cdot dx = \frac{x^{-5/2+1}}{-\frac{5}{2}+1} - e^x + \ln x + c$
 $\frac{-2}{3}x^{-3/2} - e^x + \ln x + c = \frac{-2}{3x\sqrt{x}} - e^x + \ln x + c$

10. $\int \frac{(1-x)^2}{x\sqrt{x}} dx$.

Ans. $\frac{2x^2-12x-6}{3\sqrt{x}} + c$

Sol. $= \int \frac{1+x^2-2x}{x^{\frac{3}{2}}} \cdot dx = \int \left(\frac{1}{x^{3/2}} + \frac{x^2}{x^{3/2}} - \frac{2x}{x^{3/2}} \right) \cdot dx$
 $= \int (x^{-3/2} + x^{1/2} - 2x^{-1/2}) \cdot dx$
 $= \frac{x^{-3/2+1}}{-\frac{3}{2}+1} + \frac{x^{1/2+1}}{\frac{1}{2}+1} - 2 \left(\frac{x^{-1/2+1}}{-1/2+1} \right) + c = \frac{x^{-1/2}}{(-1/2)} + \frac{x^{3/2}}{3/2} - 2 \left(\frac{x^{1/2}}{1/2} \right) + c$
 $= \frac{-2}{\sqrt{x}} + \frac{2}{3}x\sqrt{x} - 4\sqrt{x} + c = \frac{2x^2-12x-6}{3\sqrt{x}} + c$

11. $\int \frac{\sqrt[3]{x^2}-\sqrt[4]{x}}{\sqrt{x}} dx$.

Ans. $\frac{6}{7}\sqrt[6]{x^7} - \frac{4}{3}\sqrt[4]{x^3} + c$



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Sol. $\int \frac{(x^2)^{1/3} - (x)^{1/4}}{x^{1/2}} \cdot dx = \int \frac{x^{2/3}}{x^{1/2}} - \frac{x^{1/4}}{x^{1/2}} \cdot dx = \int (x^{1/6} - x^{-1/4}) \cdot dx$

$$\frac{x^{7/6}}{\frac{1}{6}+1} - \frac{x^{-\frac{1}{4}+1}}{-\frac{1}{4}+1} + c = \frac{x^{7/6}}{(7/6)} - \frac{x^{3/4}}{(3/4)} + c = \frac{6}{7}x^{7/6} - \frac{4}{3}x^{3/4} + c = \frac{6}{7}\sqrt[6]{x^7} - \frac{4}{3}\sqrt[4]{x^3} + c$$

12. $\int \frac{dx}{\sqrt{3-3x^2}}$

Ans. $\frac{1}{\sqrt{3}} \sin^{-1}(x) + c$

Sol. $\because \int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1}(x) + c = \int \frac{dx}{\sqrt{3}\sqrt{1-x^2}} = \frac{1}{\sqrt{3}} \int \frac{dx}{\sqrt{1-x^2}} = \frac{1}{\sqrt{3}} \sin^{-1}(x) + c$

13. $\int \frac{3 \cdot 2^x - 2 \cdot 3^x}{2^x} dx$

Ans. $3x - \frac{2(1.5)^x}{\ln 1.5} + c$

Sol. $\int \frac{3 \cdot 2^x}{2^x} - \frac{2 \cdot 3^x}{2^x} \cdot dx = 3 \int 1 \cdot dx - 2 \int \left(\frac{3}{2}\right)^x \cdot dx = 3(x) - 2 \frac{\left(\frac{3}{2}\right)^x}{\log_e \left(\frac{3}{2}\right)} + c$
 $= 3x - \frac{2 \cdot 3^x}{2^x (\log_e 3 - \log_e 2)} + c = 3x - \frac{2(1.5)^x}{\ln 1.5} + C.$

14. $\int \frac{1+\cos^2 x}{1+\cos 2x} dx.$

Ans. $\frac{1}{2}(\tan x + x) + C.$

Sol. $\begin{bmatrix} \because \cos 2x = 2\cos^2 x - 1 \\ 1 + \cos 2x = 2\cos^2 x \end{bmatrix}$

$$\begin{aligned} \int \frac{1+\cos^2 x}{2\cos^2 x} \cdot dx &= \int \frac{1}{2\cos^2 x} + \frac{\cos^2 x}{2\cos^2 x} \cdot dx = \\ \frac{1}{2} \int \frac{1}{\cos^2 x} \cdot dx + \frac{1}{2} \int 1 \cdot dx &= \frac{1}{2} \int \sec^2 x \cdot dx + \frac{1}{2} \int 1 \cdot dx \\ \frac{1}{2}(\tan x) + \frac{1}{2}(x) + c &= \frac{1}{2}((\tan x + x) + c) \end{aligned}$$

15. $\int \frac{\cos 2x}{\cos^2 x \cdot \sin^2 x} dx.$

Ans. $c - \cot x - \tan x.$

Sol. $\because \cos 2x = \cos^2 x - \sin^2 x$

$$\begin{aligned} \int \frac{\cos^2 x - \sin^2 x}{\cos^2 x \cdot \sin^2 x} \cdot dx &= \int \frac{\cos^2 x}{\cos^2 x \cdot \sin^2 x} - \frac{\sin^2 x}{\cos^2 x \cdot \sin^2 x} \cdot dx \\ \int \csc^2 x \cdot dx - \int \sec^2 x \cdot dx &= (-\cot x) - (\tan x) + c \\ &= -(\tan x + \cot x) + c \end{aligned}$$



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16. $\int \tan^2 x dx.$

Ans. $\tan x - x + c$

Sol. $\sec^2 x - \tan^2 x = 1$

or $\sec^2 x - 1 = \tan^2 x$

$$\int (\sec^2 x - 1) \cdot dx = \int \sec^2 x \cdot dx - \int 1 \cdot dx = \tan x - x + c$$

17. $\int \cot^2 x dx.$

Ans. $c - \cot x - x$

Sol. $\cosec^2 x - \cot^2 x = 1$

$\operatorname{cosec}^2 x - 1 = \cos^2 x$

$$\int (\operatorname{cosec}^2 x - 1) \cdot dx = \int \operatorname{cosec}^2 x \cdot dx - \int 1 \cdot dx = -\cot x - x + c = -(\cot x + x) + c$$

18. $\int 2\sin^2 \frac{x}{2} dx$

Ans. $x - \sin x + c$

Sol. $\because \cos 2x = 1 - 2\sin^2 x$

$2\sin^2 x = 1 - \cos 2x$

or $2\sin^2 \frac{x}{2} = 1 - \cos x$

$$= \int 1 - \cos x \cdot dx = (x - \sin x) + c$$

19. $\int \sin x d(\sin x)$

Ans. $\frac{\sin^2 x}{2} + c$

Sol. let $\sin x = t$

$$\Rightarrow \int t \cdot dt = \frac{t^{1+1}}{1+1} + c = \frac{t^2}{2} + c = \frac{\sin^2 x}{2} + c$$

20. $\int \tan^3 x d(\tan x).$

Ans. $\frac{\tan^4 x}{4} + c$

Sol. let $\tan x = t$

$$\int t^3 dt = \frac{t^{3+1}}{3+1} + c = \frac{t^4}{4} + c = \frac{\tan^4 x}{4} + c$$

21. $\int \frac{d(1+x^2)}{\sqrt{1+x^2}}$

Ans. $2\sqrt{1+x^2} + c$



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Sol. let $1 + x^2 = t$

$$\int \frac{dt}{\sqrt{t}} = \int \frac{dt}{t^{1/2}} = \int t^{-1/2} dt = \frac{t^{\frac{1}{2}+1}}{\frac{-1}{2}+1} + c = 2t^{1/2} + c = 2\sqrt{1+x^2} + c$$

22. $\int \frac{dx}{(2x-3)^5}$

Ans. $c - \frac{1}{8(2x-3)^4}$

Sol. $= \int (2x-3)^{-5} \cdot dx = \frac{(2x-3)^{-5+1}}{(-5+1)\frac{d}{dx}(2x-3)} + c = \frac{(2x-3)^{-4}}{(-4)(2)} + c = -\frac{1}{8} \cdot \frac{1}{(2x-3)^4} + c$

23. $\int \sqrt[5]{(8-3x)^6} dx$

Ans. $c - \frac{5}{33}(8-3x)^{11/5}$

Sol. $\int \{(8-3x)^6\}^{6/5} \cdot dx$

$$\int (8-3x)^{6/5} \cdot dx = \frac{(8-3x)^{\frac{6}{5}+1}}{\left(\frac{6}{5}+1\right)\frac{d}{dx}(8-3x)} + c$$

$$= \frac{(8-3x)^{11/5}}{\left(\frac{11}{5}\right)(-3)} + c$$

$$= -\frac{5}{33}(8-3x)^{11/5} + c$$

24. $\int \sqrt{8-2x} dx$

Ans. $c - \frac{\sqrt{(8-2x)^3}}{3}$

Sol. $\int (8-2x)^{1/2} \cdot dx = \frac{(8-2x)^{\frac{1}{2}+1}}{\left(\frac{1}{2}+1\right)\frac{d}{dx}(8-2x)} + C = \frac{(8-2x)^{3/2}}{\frac{3}{2}(-x)} + c = -\frac{1}{3}(8-2x)^{3/2} + c$

25. $\int \frac{m}{\sqrt[3]{(a+bx)^2}} dx$

Ans. $\frac{3m}{b} \sqrt[3]{a+bx} + c$

Sol. $m \int \frac{dx}{[(a+bx)^2]^{1/3}} = m \int \frac{dx}{(a+bx)^{2/3}} = m \int (a+bx)^{-2/3} \cdot dx$

$$\left(\frac{-2}{3}+1\right)\frac{d}{dx}(a+bx) + c = \frac{m(a+bx)^{1/3}}{\frac{1}{3}(b)} + c = \frac{3m \sqrt[3]{a+bx}}{b} + c$$



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26. $\int \frac{\sin x dx}{\cos^2 x}$

Ans. $\sec x + c$

Sol. $\int \frac{\sin x}{\cos x} \cdot \frac{1}{\cos x} \cdot dx$

$$\int \tan x \cdot \sec x dx = \sec x + c$$

27. $\int (\cos \alpha - \cos 2x) dx.$

Ans. $x \cos \alpha - \frac{1}{2} \sin 2x + c$

Sol. $\int \cos \alpha \cdot dx - \int \cos 2x \cdot dx$

$$\cos \alpha \int 1 \cdot dx - \frac{\sin 2x}{2} + c = x \cos \alpha - \frac{1}{2} (\sin 2x) + c$$

28. $\int \sin (2x - 3) dx$

Ans. $c - \frac{1}{2} \cos (2x - 3).$

Sol. $= -\frac{\cos(2x-3)}{\frac{d}{dx}(2x-3)} + c = -\frac{1}{2} \cos (2x - 3) + c$

29. $\int \cos (1 - 2x) dx.$

Ans. $c - \frac{1}{2} \sin (1 - 2x).$

Sol. $= \frac{\sin(1-2x)}{\frac{d}{dx}(1-2x)} + c = \frac{\sin(1-2x)}{(-2)} + c = -\frac{1}{2} \sin (1 - 2x) + c$

30. $\int \frac{dx}{2x-1}$

Ans. $\frac{1}{2} \ln |2x - 1| + C.$

Sol. $\because \frac{1}{x} \cdot dx = \ln(x) + c = \frac{\ln(2x-1)}{\frac{d}{dx}(2x-1)} + c = \frac{1}{2} \ln (2x - 1) + c$

31. $\int \frac{dx}{cx+m}$

Ans. $\frac{1}{c} \ln |cx + m| + C.$

Sol. $= \frac{\ln(cx+m)}{\frac{d}{dx}(cx+m)} + c = \frac{1}{c} \ln (cx + m) + c$

32. $\int a^{3x} dx.$

Ans. $\frac{a^{3x}}{3 \ln a} + C.$

Sol. $\left(\because \int a^x \cdot dx = \frac{a^x}{\log_e a} + c \right) = \frac{a^{3x}}{(\log_e a)(\frac{d}{dx} 3x)} + c = \frac{a^{3x}}{(\log_e a) \cdot 3} = \frac{a^{3x}}{3 \log_e a} + c$



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33. $\int a^{-x} dx.$

Ans. $c - \frac{a^{-x}}{\ln a}$

Sol. $(\because a^x = \frac{a^x}{\log_c a} + c)$

$$= \frac{a^{-x}}{\log_e a \left(\frac{d(-x)}{dx} \right)} + c = \frac{a^{-x}}{-\log_e a} + c = -\frac{a^{-x}}{\log_e a} + c$$

34. $\int \frac{dx}{\sqrt{4-x^2}}.$

Ans. $\arcsin \frac{x}{2} + c$

Sol. $= \int \frac{1}{\sqrt{2^2-x^2}} \cdot dx = \sin^{-1} \left(\frac{x}{2} \right) + c (\because \int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1} \left(\frac{x}{a} \right) + c)$

35. $\int \frac{dx}{2x^2+9}.$

Ans. $\frac{1}{3\sqrt{2}} \arctan \frac{\sqrt{2}}{3} x + c$

Sol. $(\because \int \frac{1}{a^2+x^2} \cdot dx = \frac{1}{a} \tan^{-1} \left(\frac{x}{a} \right) + c)$

$$\frac{1}{2} \int \frac{dx}{x^2+\frac{9}{2}} = \frac{1}{2} \int \frac{dx}{x^2+(\frac{3}{\sqrt{2}})^2} = \frac{1}{2} \left[\frac{1}{(3/\sqrt{2})} \tan^{-1} \left(\frac{x}{3/\sqrt{2}} \right) \right] + c = \frac{1}{\sqrt{2}} \left[\frac{8}{3} \tan^{-1} \left(\frac{\sqrt{2}x}{3} \right) \right] + c$$

$$= \frac{1}{3\sqrt{2}} \tan^{-1} \left(\frac{\sqrt{2}x}{3} \right) + c$$

36. $\int \frac{dx}{\sqrt{4-9x^2}}.$

Ans. $\frac{1}{3} \arcsin \frac{3x}{2} + c$

Sol. $\int \frac{dx}{3\sqrt{\frac{4}{9}-x^2}} = \frac{1}{3} \int \frac{dx}{\sqrt{\left(\frac{2}{3}\right)^2-x^2}}$

$$= \frac{1}{3} \cdot \sin^{-1} \left(\frac{x}{2/3} \right) + C = \frac{1}{3} \sin^{-1} \left(\frac{3x}{2} \right) + c$$

Alter :

$$\int \frac{dx}{\sqrt{(2)^2-(3x)^2}} = \frac{\sin^{-1} \left(\frac{3x}{2} \right)}{\frac{d}{dx}(3x)} = \frac{1}{3} \sin^{-1} \left(\frac{3x}{2} \right) + c$$

37. $\int \frac{2^x dx}{\sqrt{1-4^x}}$

Ans. $\frac{\arcsin 2^x}{\ln 2} + c$



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Sol. Using Substitution

$$\text{let } 2^x = t$$

$$2^x \log_e \frac{2dx}{dt} = dt$$

$$2^x \cdot dx = \frac{1}{\log_e^2} \cdot dt$$

$$\int \frac{2^x \cdot dx}{\sqrt{1-(2^x)^2}}$$

$$\int \frac{\frac{1}{\log_e^2} dt}{\sqrt{1-t^2}}$$

$$= \frac{1}{\log_e^2} \int \frac{1}{\sqrt{1-t^2}} \cdot dt$$

$$= \frac{1}{\log_e^2} \sin^{-1}(t) + c \Rightarrow \frac{1}{\log_e^2} \sin^{-1}(2^x) + c$$

38. $\int \cos^2 x dx$

Ans. $\frac{x}{2} + \frac{\sin 2x}{4} + c$

Sol. $= \int \frac{1+\cos 2x}{2} \cdot dx = \frac{1}{2} \int (1 + \cos 2x) \cdot dx = \frac{1}{2} \left(x + \frac{\sin 2x}{2} \right) + c = \frac{x}{2} + \frac{\sin 2x}{4} + c$

39. $\int \sin^2 x dx$

Ans. $\frac{x}{2} - \frac{\sin 2x}{4} + c$

Sol. $= \int \frac{1-\cos 2x}{2} \cdot dx = \frac{1}{2} \int (1 - \cos 2x) dx = \frac{1}{2} \left(x - \frac{\sin 2x}{2} \right) + c = \frac{x}{2} - \frac{\sin 2x}{4} + c$

40. $\int \cos x \sin 3x dx$.

Ans. $c - \frac{1}{4} \left(\frac{\cos 4x}{2} + \cos 2x \right)$

Sol. ($\because 2\sin A \cos B = \sin(A+B) + \sin(A-B)$)

$$= \frac{1}{2} \int 2\sin 3x \cos x \cdot dx = \frac{1}{2} \int [\sin(3x+x) + \sin(3x-x)] \cdot dx$$

$$= \frac{1}{2} \int (\sin 4x + \sin 2x) \cdot dx = \frac{1}{2} \left(-\frac{\cos 4x}{4} - \frac{\cos 2x}{2} \right) + C = -\frac{1}{8} \cos 4x - \frac{1}{4} \cos 2x + c$$

41. $\int \cos 2x \cos 3x dx$.

Ans. $\frac{1}{10} \sin 5x + \frac{1}{2} \sin x + c$



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Sol. $[2\cos A \cos B = \cos(A + B) + \cos(A - B)]$

$$\begin{aligned} &= \frac{1}{2} \int 2\cos 2x \cdot \cos 3x \cdot dx = \frac{1}{2} \int \frac{1}{2}[(\cos(2x + 3x) + \cos(2x - 3x))] \cdot dx \\ &= \frac{1}{2} \left(\frac{\sin 5x}{5} + \sin x \right) + c = \frac{\sin 5x}{10} + \frac{\sin x}{2} + c \end{aligned}$$

42. $\int \sin 2x \sin 5x dx$

Ans. $\frac{1}{6} \sin 3x - \frac{1}{14} \sin 7x + c$

Sol. $[\because 2\sin A \cdot \sin B = \cos(A - B) - \cos(A + B)]$

$$\begin{aligned} &\frac{1}{2} \int (2\sin 2x \sin 5x) dx = \frac{1}{2} \int [\cos(2x - 5x) - \cos(2x + 5x)] dx \\ &= \frac{1}{2} \int (\cos 3x - \cos 7x) \cdot dx = \frac{1}{2} \left(\frac{\sin 3x}{3} - \frac{\sin 7x}{7} \right) + c \\ &= \frac{\sin 3x}{6} - \frac{\sin 7x}{14} + c \end{aligned}$$

43. $\int \cos x \cos 2x \cos 3x dx$

Ans. $\frac{1}{8} \left(2x + \sin 2x + \frac{1}{2} \sin 4x + \frac{1}{3} \sin 6x \right) + c$

Sol. $\int \frac{1}{2} (2\cos x \cdot \cos 2x) \cos 3x \cdot dx$

$$\begin{aligned} &\frac{1}{2} \{(\cos 3x + \cos x) \cos 3x\} \\ &\frac{1}{2} [\cos^2 3x + \cos x \cdot \cos 3x] \\ &\frac{1}{4} (2\cos^2 3x + 2\cos x \cdot \cos 3x) \quad (\because 2\cos^2 \theta = 1 + \cos 2\theta) \end{aligned}$$

$$\frac{1}{4} (1 + \cos 6x + \cos 4x + \cos 2x)$$

$$\frac{1}{4} \int (1 + \cos 6x + \cos 4x + \cos 2x) \cdot dx$$

$$\begin{aligned} &\frac{1}{4} \left(x + \frac{\sin 6x}{6} + \frac{\sin 4x}{4} + \frac{\sin 2x}{2} \right) + c \\ &\frac{x}{4} + \frac{\sin 6x}{24} + \frac{\sin 4x}{16} + \frac{\sin 2x}{8} + c \end{aligned}$$

44. $\int \frac{dx}{\cos x}$

Ans. $\ln \left| \tan \left(\frac{\pi}{4} + \frac{x}{2} \right) \right| + C$



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$$\text{Sol. } = \int \sec x \cdot dx \times \frac{\sec x + \tan x}{\sec x + \tan x}$$

$$= \int \frac{\sec^2 x + \sec x \cdot \tan x}{\sec x + \tan x} \cdot dx$$

Let $\sec x + \tan x = t$

$$(\sec x \cdot \tan x + \sec^2 x) \cdot dx = t$$

$$\int \frac{dt}{t} = \ln t + c$$

$$= \ln |\sec x + \tan x| + c$$

$$= \ln (\sec x + \tan x) + c = \ln \left| \frac{1 + \sin x}{\cos x} \right| + c = \ln \left| \frac{(\cos \frac{x}{2} + \sin \frac{x}{2})^2}{\cos^2 \frac{x}{2} - \sin^2 \frac{x}{2}} \right| + c$$

$$= \ln \left| \frac{(\cos \frac{x}{2} + \sin \frac{x}{2})^2}{(\cos \frac{x}{2} + \sin \frac{x}{2})(\cos \frac{x}{2} - \sin \frac{x}{2})} \right| + c = \ln \left| \frac{\cos \frac{x}{2} + \sin \frac{x}{2}}{\cos \frac{x}{2} - \sin \frac{x}{2}} \right| + c$$

$$= \ln \left| \tan \left(\frac{\pi}{4} + \frac{x}{2} \right) \right| + c$$