Nishant Jindal

HW-4.

12

$A = 5 + G.$

$G = 4 + H$

$G^2 = AH$

$G^2 = (5+G)(G-4)$

$G = 20$

$A = 25, \quad H = 16.$

(13) $\alpha = A + \sqrt{A^2 - G^2}$

$\beta = A - \sqrt{A^2 - G^2}$

(13) ① $H = 4$

$\frac{2ab}{a+b} = 4$

$\frac{2ab}{9} = 4$

$ab = 2 \times 9$

$a \cdot b = 18$

$2A + G^2 = 27$

$2A + 4A = 27$

$A = \frac{27}{6} = \frac{9}{2}$

$\frac{a+b}{\cancel{2}} = \frac{9}{\cancel{2}}$

$G^2 = A \cdot H$

$a, b = (6,3)$ or $(3,6)$

14) $ax^2 + bx + c = 0$

$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$

$x^2 + 2Ax + G^2 = 0$

$G_1 = \sqrt{\frac{c}{a}}$

$lx^2 + mx + n = 0$

$x^2 + \frac{m}{l}x + \frac{n}{l} = 0$

$G_2 = \sqrt{\frac{n}{l}}$

$\frac{G_1}{G_2} = \frac{\sqrt{\frac{c}{a}}}{\sqrt{\frac{n}{l}}} = \sqrt{\frac{cl}{an}}$

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① $A_1 > G_1 > H_1$

A_2 is AM of A_1 & H_1

$A_1 > A_2 > H_1$

A_3 is AM of A_2 & H_1

$A_2 > A_3 > H_1$

$A_1 > A_2 > A_3 > A_4 - \cdots$

24) (rgby.

Q25 $1\cdot\lfloor 1 + 2\cdot\lfloor 2 + 3\cdot\lfloor 3 + \cdots$

$(2-1)\lfloor 1 + (3-1)\lfloor 2 + (4-1)\lfloor 3 + (5-1)\lfloor 4 - \cdots$

$(2\lfloor 1 + 3\lfloor 2 + 4\lfloor 3 + 5\lfloor 4 \quad) - (\lfloor 1 + \lfloor 2 + \lfloor 3 + \lfloor 4 + \cdots)$

$(\lfloor 2 + \lfloor 3 + \lfloor 4 + \lfloor 5 \quad) - (\lfloor 1 + \lfloor 2 + \lfloor 3 + \lfloor 4 \cdots)$

$\cancel{\lfloor 2} - \lfloor 1 + \cancel{\lfloor 3} - \cancel{\lfloor 2} + \cancel{\lfloor 4} - \cancel{\lfloor 3} + \lfloor 5 - \cancel{\lfloor 4} + \cdots \cdot \lfloor n+1 - \cancel{\lfloor n}$

$= \lfloor n+1 - \lfloor 1$

Q26 $S_n = \sum T_r = \sum (2r+1)\cdot 2^r = \sum 2\cdot 2^r\cdot r + \sum 2^r$

$= \sum 2^{r+1}\cdot r + \sum 2^r$

$= \{\underbrace{1\cdot 2^2 + 2\cdot 2^3 + 3\cdot 2^4 + \cdots + n\cdot 2^{n+1}}_{AGP}\} + \{\underbrace{2^1 + 2^2 + 2^3 + \cdots + 2^n}_{GP}\}$

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29) $H_n = 1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\cdots+\frac{1}{n}$.

$S = 1+\frac{3}{2}+\frac{5}{3}+\frac{7}{4}+\frac{9}{5}+\cdots+\left(\frac{2n-1}{n}\right)$

$S_n = \sum T_n = \sum \frac{2n-1}{n}$

$= \sum 2 - \sum \frac{1}{n}$

$= 2\sum 1 - \left\{\frac{1}{1}+\frac{1}{2}+\frac{1}{3}+\cdots+\frac{1}{n}\right\}$

$S_n = 2n - H_n$

Q 30 20 $1,2,3,4,5\cdots n$

$1\cdot2+1\cdot3+1\cdot4+1\cdot5\cdots\cdots 1\cdot n$

$2\cdot3+2\cdot4+2\cdot5+2\cdot6+\cdots 2\cdot n$

$3\cdot4+3\cdot5+3\cdot6\cdots+3\cdot n$

$4\cdot5+4\cdot6 \quad +4\cdot n$

$5\cdot6 \quad +5\cdot n$

$\cdots (n-1)\,n$

$1\cdot2+3\cdot3+(1+2+3)\cdot4+(1+2+3+4)\cdot5\cdots\cdots+(1+2+\cdots(n-1))n$

$S = \sum T_n = \sum(1+2+\cdots(n-1))\cdot n$

Q 33 $S = 1^2-\frac{2^2}{5}+\frac{3^2}{5^2}-\frac{4^2}{5^3}+\frac{5^2}{5^4}-\frac{6^2}{5^5}+\cdots$

$-\frac{S}{5} = -\frac{1^2}{5}+\frac{2^2}{5^2}-\frac{3^2}{5^3}+\cdots$

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34) $\sum T_n = \sum \frac{n}{1+n^2+n^4}$

$$= \frac{1}{2} \sum \frac{2n}{(n^2+n+1)(n^2-n+1)}$$

$$= \frac{1}{2} \sum \frac{(n^2+n+1)-(n^2-n+1)}{(n^2+n+1)(n^2-n+1)}$$

$$= \frac{1}{2} \sum \frac{1}{n^2-n+1} - \frac{1}{n^2+n+1}$$

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37) $\sum \sqrt{1+\frac{1}{n^2}+\frac{1}{(n+1)^2}} = \sqrt{\frac{n^4+2n^3+n^2+n^2+2n+1+n^2}{(n)^2(n+1)^2}}$

$$= \sqrt{\frac{(n^2+n+1)^2}{(n)^2(n+1)^2}}$$

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(1) ${}^{n}C_{14} = {}^{n}C_{16}$ then ${}^{32}C_{n} = ?$

(2) $2 \cdot {}^{n}C_5 = 9 \cdot {}^{n-2}C_5$ then $n = ?$

(3) Value of
${}^{5}C_3, {}^{12}C_4, {}^{15}C_{13}, {}^{25}C_{22}$?

(4) ${}^{19}C_{r-1} = {}^{19}C_{3r}$ then $r = ?$

(5) ${}^{2n}C_3 : {}^{n}C_2 = 44 : 3$ then n

$(n-\cancel{r})-(r-\cancel{r})$

Q. ${}^{n}C_r + {}^{n-1}C_{r-1} = ?$ (P.T.)

Wrong

$$\frac{\lfloor n}{\lfloor r \lfloor n-r} + \frac{\lfloor n-1}{\lfloor r-1 \lfloor n-r}$$

$$\frac{n \lfloor n-1}{r \lfloor r-1 \times \lfloor n-r} + \frac{\lfloor n-1}{\lfloor r-1 \lfloor n-r}$$

$$\frac{\lfloor n-1}{\lfloor r-1 \lfloor n-r} \left\{ \frac{n}{r} + 1 \right\}$$

$$\frac{\lfloor n-1}{\lfloor r-1 \lfloor n-r} \left\{ \frac{n+r}{r} \right\}$$

$$\frac{\lfloor n-1}{\lfloor r \lfloor n-r} (n+r)$$

$n = 4, r = 2$

${}^{4}C_2 + {}^{4-1}C_{2-1} = {}^{4}\!/_{2}$

$\frac{\cancel{4}^{2} \cdot 3}{1 \cdot \cancel{2}} + {}^{3}C_1$

$6 + 3 \neq 2$

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Q. P.T. $^{2n}C_n = \dfrac{1\cdot3\cdot5\cdots(2n-1)}{1\cdot2\cdot3\cdots n}\times 2^n$.

LHL $^{2n}C_n = \dfrac{\lfloor 2n}{\lfloor n\ \lfloor n} \rightarrow$ $\lfloor 2n, \lfloor n$ open

2 Com.

$$\boxed{(2\cdot4\cdot6\cdot8\cdot10) = 2^5(1\cdot2\cdot3\cdot4\cdot5)}$$

$$= \frac{1\cdot2\cdot3\cdot4\cdot5\cdot6\cdot7\cdot8\cdot9\cdot10 \cdots (2n-2)(2n-1)(2n)}{(1\cdot2\cdot3\cdot4\cdots(n-1)(n))(\lfloor n)}$$

odd odd Alg

$$= \frac{(1\cdot3\cdot5\cdot7\cdots(2n-1))(2\cdot4\cdot6\cdot8\cdot10\cdots(2n-2)(2n))}{(1\cdot2\cdot3\cdots(n-1)(n))(\lfloor n)} = \frac{(1\cdot3\cdot5\cdot7\cdots(2n-1))(2^n)\cancel{(1\cdot2\cdot3\cdot4\cdots n)}}{\cancel{(1\cdot2\cdot3\cdots n)}\lfloor n}$$

$$= \frac{(1\cdot3\cdot5\cdots(2n-1))\cdot 2^n}{(1\cdot2\cdot3\cdots n)} = \text{RHS}.$$

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Q P.T. ${}^{n}C_{r} + {}^{n}C_{r-1} = {}^{n+1}C_{r}$ → Padhte Kese.

Upar wala Same & niche wale me 1 ka diff. } Ans. Upar 1 Bda do Niche jo Bda hai use Likhdo.

LHS

$$\frac{\lfloor n}{\lfloor r \lfloor n-r} + \frac{\lfloor n}{\lfloor r-1 \lfloor n-r+1}$$

$$\frac{\lfloor n}{r \lfloor r-1 \lfloor n-r} + \frac{\lfloor n}{\lfloor r-1 \cdot (n-r+1) \lfloor n-r}$$

Kal Ho chuka.

Q $${}^{8}C_{3} + {}^{8}C_{2} = ?$$

$$= {}^{9}C_{3} = \frac{\cancel{9}^{3} \cdot \cancel{8}^{4} \cdot 7}{1 \cdot \cancel{2} \cdot \cancel{3}} = 84$$

Q ${}^{n}C_{r} = \frac{n}{r} \cdot {}^{n-1}C_{r-1}$ (P.T.)

RHS $\frac{n}{r} \cdot {}^{n-1}C_{r-1}$

$$\frac{n}{r} \times \frac{\lfloor n-1}{\lfloor r-1 \lfloor n-r}$$

$$= \frac{\lfloor n}{\lfloor r \lfloor n-r} = {}^{n}C_{r}.\ \text{LHS}$$

Q ${}^{20}C_{4} = \frac{20}{4} \cdot {}^{19}C_{3}$ Dant Ukhado

$4 \lfloor 3 = \lfloor 4$

$n \lfloor n-1 = \lfloor n$

Q $\frac{{}^{15}C_{4}}{{}^{13}C_{2}} = ?$

$$= \frac{\frac{15}{4} \times \frac{14}{3} \cdot \cancel{{}^{13}C_{2}}}{\cancel{{}^{13}C_{2}}}$$

$$= \frac{35}{2}$$

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HW:

Q $\dfrac{^nC_r}{^nC_{r-1}} = \dfrac{n-r+1}{r}$ (P.T.)

Rk. (Note)

1) $^nC_r = \dfrac{\lfloor n}{\lfloor r \lfloor n-r}$

2) $^nC_r = \dfrac{n}{r} \cdot {}^{n-1}C_{r-1}$

3) $^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$

4) $\dfrac{^nC_r}{^nC_{r-1}} = \dfrac{n-r+1}{r}$

(5) $^nC_r = {}^n_rC = \binom{n}{r}$ ← Notation

$^{13}C_2 = \binom{13}{2}$

Q If $2 \le r \le n$ then

$$\binom{n}{r} + 2\binom{n}{r-1} + \binom{n}{r-2} = ?$$

$$\underbrace{\binom{n}{r} + \binom{n}{r-1}}_{1} + \underbrace{\binom{n}{r-1} + \binom{n}{r-2}}_{1}$$

$$\binom{n+1}{r} + \binom{n+1}{r-1}$$

$$= \binom{n+2}{r} \text{ A}$$

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Q $^{50}C_4 + \sum_{i=1}^{5} {}^{55-i}C_3$

Strike होना चाहिए

${}^2C_2 = {}^3C_3 = {}^4C_4$

$\| \quad \| \quad \|$

$1 \quad 1 \quad 1$

$^{50}C_4 + {}^{55-1}C_3 + {}^{55-2}C_3 + {}^{55-3}C_3 + {}^{55-4}C_3 + {}^{55-5}C_3$

$^{50}C_4 + \{ {}^{54}C_3 + {}^{53}C_3 + {}^{52}C_3 + {}^{51}C_3 + {}^{50}C_3 \}$

$\underbrace{\underbrace{{}^{50}C_4 + {}^{50}C_3}_{①}}_{{}^{51}C_4} + {}^{51}C_3 + {}^{52}C_3 + {}^{53}C_3 + {}^{54}C_3$

${}^{51}C_4 \to {}^{52}C_4 \to {}^{53}C_4 \to {}^{54}C_4 \to {}^{55}C_4$ Ans

Decreasing:

Q $\binom{n}{m} + \binom{n-1}{m} + \binom{n-2}{m} + \binom{n-3}{m} + \cdots + \binom{m+1}{m} + \binom{m}{m} = ?$

$\binom{m}{m} + \binom{m+1}{m} + \binom{m+2}{m} + \binom{m+3}{m} + \cdots + \binom{n-1}{m} + \binom{n}{m}$

$\|$

$\underbrace{\underbrace{\binom{m+1}{m+1} + \binom{m+1}{m}}_{①}}_{\binom{m+2}{m+1}} + \binom{m+2}{m} + \binom{m+3}{m} + \cdots + \binom{n-1}{m} + \binom{n}{m}$

$\binom{m+2}{m+1} \to \binom{m+3}{m+1} \to \binom{m+4}{m+1} \cdots \to \binom{n+1}{m+1}$

Ans

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Monomial = Single term.
Binomial = 2 terms.
Trinomial = 3 term

$\underbrace{a^2}_{\text{Mon.}}$, $\underbrace{a+2x}_{\text{Bin.}}$, $\underbrace{a+2x+3z}_{\text{Trin.}}$

Binomial Expansion.

Expansion of $(x+a)^n$

x की deg Reduce

1) $(x+a)^n = {}^nC_0 (x)^{n-0} \cdot (a)^0 + {}^nC_1 (x)^{n-1} \cdot (a)^1 + {}^nC_2 (x)^{n-2} a^2 + {}^nC_3 (x)^{n-3} \cdot a^3 + \dots + {}^nC_n \cdot (x)^0 \cdot (a)^n$

2) $(x-a)^n = {}^nC_0 (x)^{n-0} (-a)^0 + {}^nC_1 (x)^{n-1} (-a)^1 + {}^nC_2 (x)^{n-2} (-a)^2 + {}^nC_3 (x)^{n-3} (-a)^3 + {}^nC_n \cdot (x)^0 \cdot (-a)^n$

3) $(1+x)^n = {}^nC_0 \cdot (1)^n (x)^0 + {}^nC_1 (1)^{n-1} (x)^1 + {}^nC_2 x^2 + {}^nC_3 x^3 - + {}^nC_n \cdot x^n$

Q $(2x+3y)^5$ Expand?

$= {}^5C_0 \cdot (2x)^5 (3y)^0 + {}^5C_1 (2x)^4 (3y)^1 + {}^5C_2 (2x)^3 (3y)^2 + {}^5C_3 (2x)^2 (3y)^3 + {}^5C_4 (2x)^1 (3y)^4 + {}^5C_5 (2x)^0 (3y)^5$

$= (2x)^5 + 5 (2x)^4 (3y) + 10 \cdot (2x)^3 (3y)^2 + 10 \cdot (2x)^2 \cdot (3y)^3 + 5 (2x)(3y)^4 + 1 \cdot 1 \cdot (3y)^5$

H.W. Q $(3x-y)^5$, $(x+y)^4$, $(x-y)^4$, $(1-2x)^6$ Expand?