Let $f(x)$ be periodic with period 'T',

* $\int_0^{nT} f(x)\,dx = n\int_0^T f(x)\,dx$, $n \in I$.

$= \int_0^T + \int_T^{2T} + \int_{2T}^{3T} + \dots \int_{(n-1)T}^{nT}$

$\int_0^T + \int_0^T f(x+T)\,dx + \int_0^T f(x+2T)\,dx + \dots$

* $\int_{n_1T}^{n_2T} f(x)\,dx = (n_2-n_1)\int_0^T f(x)\,dx$, $n_1, n_2 \in I$.

$= \int_{n_1T}^0 + \int_0^{n_2T}$

* $\int_a^{a+T} f(x)\,dx = \int_0^T f(x)\,dx$, $a \in R$.

$\int_a^0 f(x)\,dx + \int_0^T f(x)\,dx + \int_T^{a+T} f(x)\,dx$.

$\cancel{\int_a^0 f(x)\,dx} + \int_0^T f(x)\,dx + \cancel{\int_0^a f(x+T)\,dx}$

* $\int_a^{a+nT} f(x)\,dx = n\int_0^T f(x)\,dx$, $a \in R$, $n \in I$.

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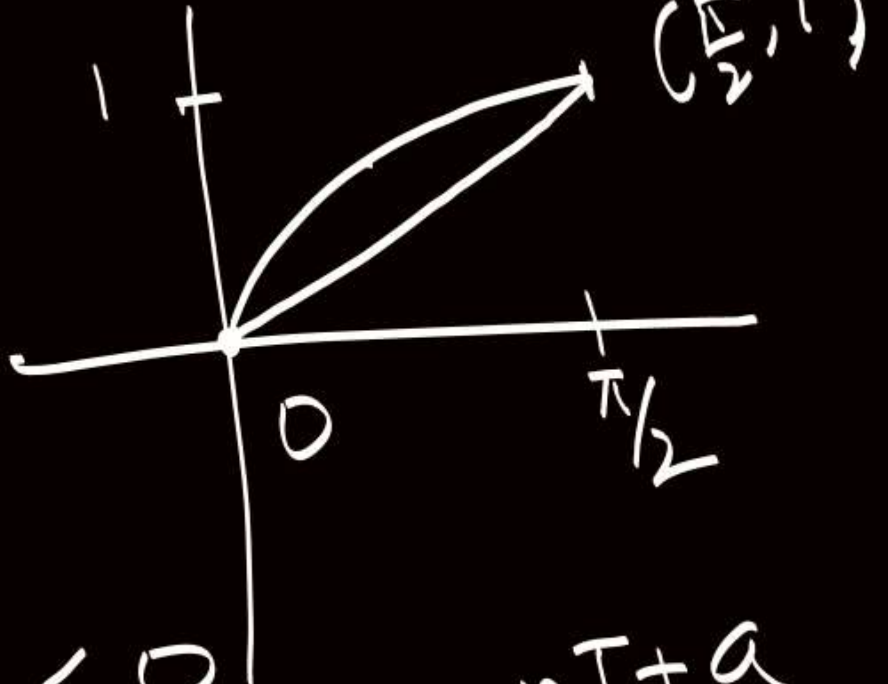
$f(x) \le f(0)$

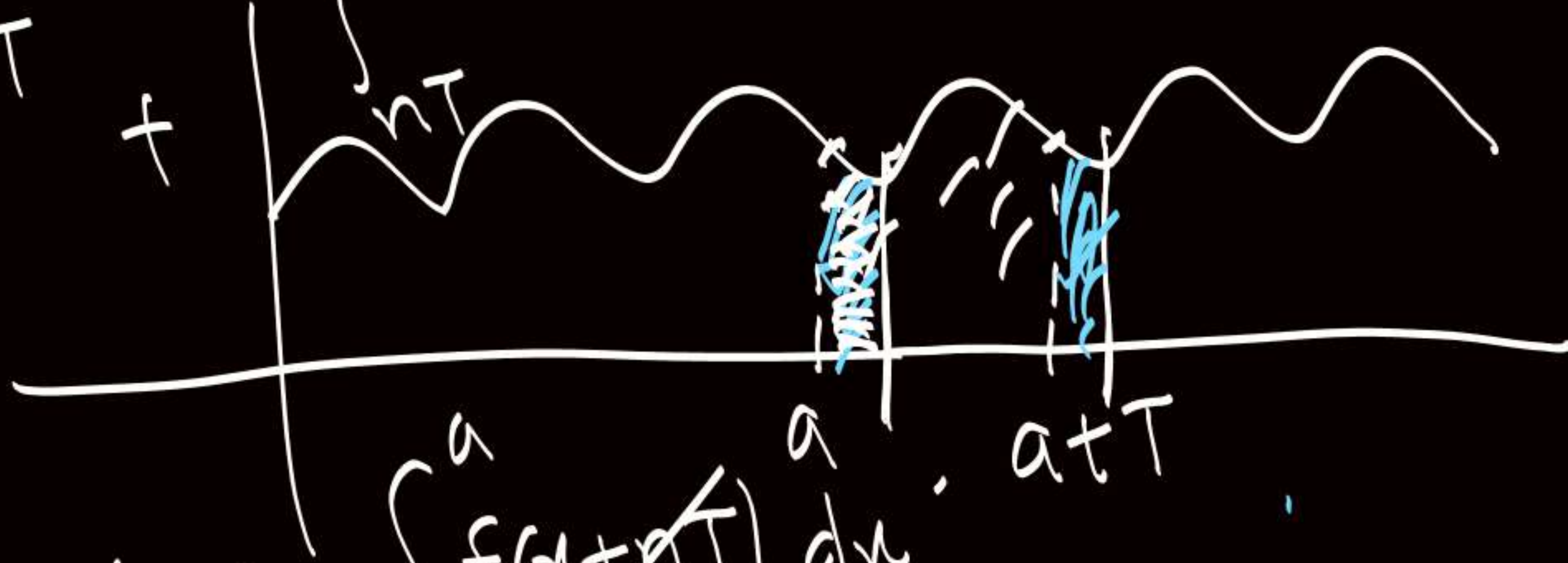
$\sin x \ge \frac{2x}{\pi}$



$f(x) = \cos x + \frac{x^2}{\pi} - 1$

$f'(x) = -\sin x + \frac{2x}{\pi} \le 0$

$$\int_a^{a+nT} f(x)\,dx = \int_a^0 + \int_0^{nT} + \int_{nT}^{nT+a}$$



$$\int_a^0 f(x)\,dx + n\int_0^T f(x)\,dx + \int_0^a \underbrace{\boxed{f(x+nT)}}_{=f(x)}\,dx$$

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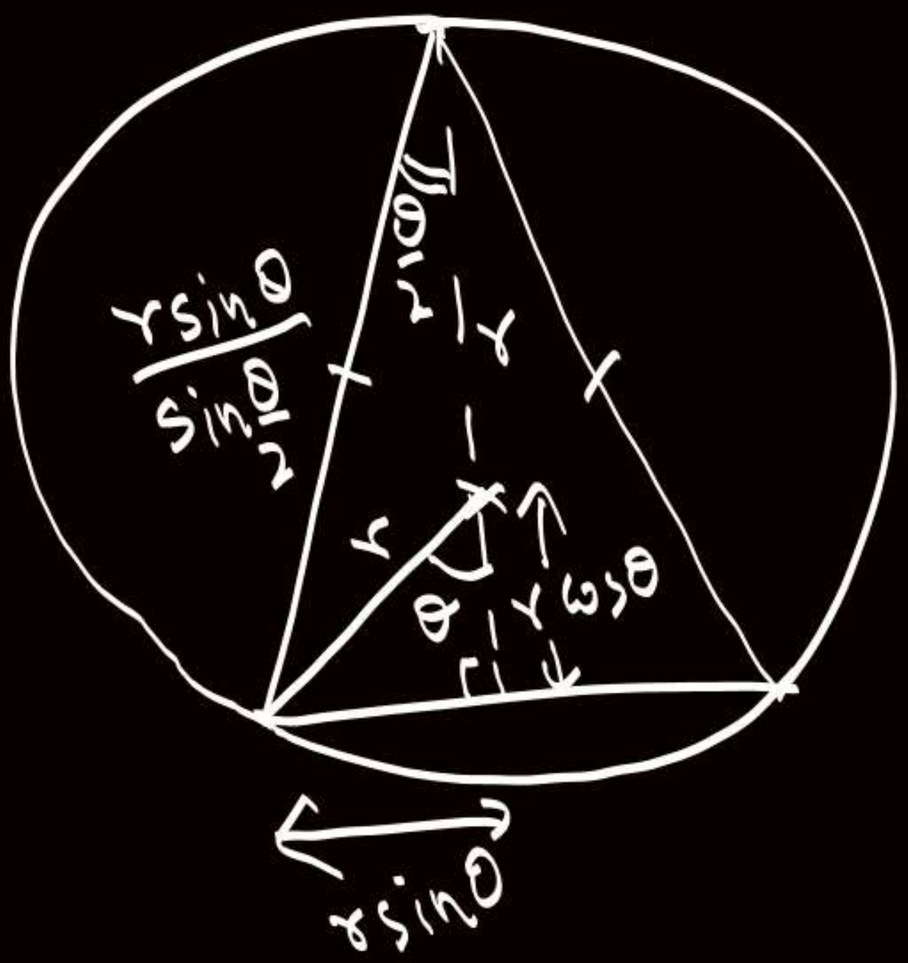

$$2r\sin\theta + \frac{2r\sin\theta}{\sin\frac{\theta}{2}} = L$$

$$r = \frac{L}{2\sin\theta + 4\cos\frac{\theta}{2}}$$

$$\theta \in \left(0, \frac{\pi}{2}\right]$$

$r_{max} = ?$

$r(0)$

$r\left(\frac{\pi}{2}\right)$

$r(\ )$

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1. $$\int_0^{2000\pi} \frac{dx}{1+e^{\sin x}} = \int_0^{1000\pi}\left(\frac{1}{1+e^{\sin x}}+\frac{1}{1+e^{-\sin x}}\right)dx$$

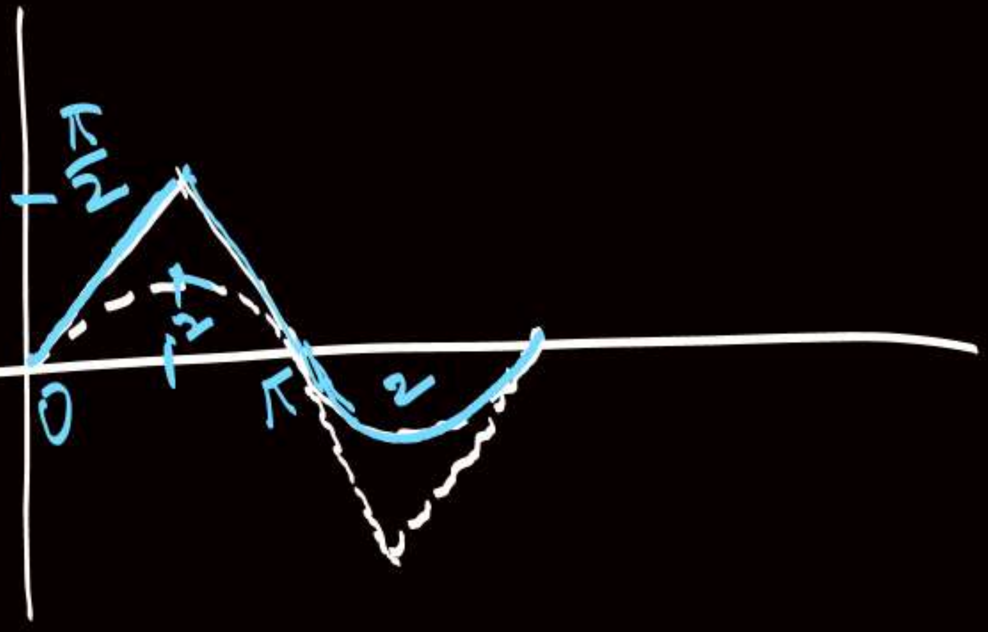
$$= 1000\pi$$

$$1000\int_0^{2\pi}\frac{dx}{1+e^{\sin x}} = 1000\int_0^{\pi}\left(\frac{1}{1+e^{\sin x}}+\frac{1}{1+e^{-\sin x}}\right)dx$$

$$= 1000\pi$$

2. $$n\int_0^{2\pi}\max\left(\sin x,\ \sin^{-1}(\sin x)\right)dx \quad , n\in I.$$

$$= n\left(\frac{1}{2}\pi\frac{\pi}{2}-2\right)$$



3. $\displaystyle\int_0^{n\pi+v} |\cos x|\,dx$, where $v \in \left(\frac{\pi}{2}, \pi\right)$, $n \in N$.

$$\int_0^{v} + \int_v^{n\pi+v}$$

$$= \int_0^{n\pi} |\cos x|\,dx + \int_{n\pi}^{n\pi+v} |\cos x|\,dx.$$

$$= \int_0^{v} |\cos x|\,dx + n\int_0^{\pi} |\cos x|\,dx$$

$$= n\int_0^{\pi} |\cos x|\,dx + \int_0^{v} |\cos x|\,dx.$$

$$= 2n + \int_0^{\frac{\pi}{2}} \cos x\,dx + \int_{\frac{\pi}{2}}^{v} -\cos x\,dx = 2n+1-(\sin v - 1)$$

$$= 2n+2-\sin v.$$

Nishant Jindal

Derivative of Anti Derivative (Newton, Leibnitz rule)

$$\frac{d}{dx}\left(\int_{g(x)}^{h(x)} f(t)\,dt\right) = h'(x)\, f(h(x)) - g'(x)\, f(g(x))$$

$$F'(x) = f(x)$$

$$\int_{g(x)}^{h(x)} f(t)\,dt = F(h(x)) - F(g(x))$$

$$\frac{d}{dx}\left(\int_{g(x)}^{h(x)} f(t)\,dt\right) = F'(h(x))\, h'(x) - F'(g(x))\, g'(x)$$

$$= f(h(x))\, h'(x) - f(g(x))\, g'(x)$$

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$$\boxed{\frac{d}{dx}\underbrace{\left(\int_a^b f(t,x)\,dt\right)}_{=I(x)} = \int_a^b \left(\frac{\partial}{\partial x}\left(f(t,x)\right)\right)dt \quad , \; a, b \cdot \text{are constants}}$$

$$I'(x) = \lim_{\Delta x \to 0}\left(\frac{I(x+\Delta x) - I(x)}{\Delta x}\right) = \lim_{\Delta x \to 0} \frac{\int_a^b f(t, x+\Delta x)\,dt - \int_a^b f(t,x)\,dt}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \int_a^b \left(\frac{f(t, x+\Delta x) - f(t,x)}{\Delta x}\right) dt = \int_a^b \left(\lim_{\Delta x \to 0} \frac{f(t,x+\Delta x) - f(t,x)}{\Delta x}\right) dt$$

Nishant Jindal

1. Find derivative of $f(x) = \int_{e^{2x}}^{e^{3x}} \frac{t}{\ln t}\, dt$ w.r.t. $\ln x$ at $x = \ln 2$.

$$\frac{f'(x)}{g'(x)} \qquad \frac{3e^{3x}\frac{e^{3x}}{\ln e^{3x}} - 2e^{2x}\frac{e^{2x}}{\ln e^{2x}}}{\frac{1}{x}}$$

$$= e^{6x} - e^{4x} \Big|_{x=\ln 2}$$

$$= 2^6 - 2^4 = \boxed{48}$$

Ex-4 | Ex-5 (1-6)