

$$\lim_{x \rightarrow 1^-} \frac{h(x)+1}{3x+3} = \frac{h(1)+1}{6}$$

$$\lim_{x \rightarrow 1^+} \frac{f(x)}{2} = \frac{f(1)}{2}$$

$$\frac{f(x)}{g(x)} = \frac{\sin^2(2\pi(2^{x-1}-1))}{1 - \cos(2\pi(2^{x-1}-1))} = 2$$

$$x=a$$

$$\frac{f(x)}{f(1)} = \frac{f(x)f(1)}{f^2(1)} \Rightarrow \boxed{f(1)=1}$$

$$\lim_{x \rightarrow a} f(x) = \lim_{x \rightarrow a} f(x \cdot 1) = \underbrace{f(1)}_{f(a)} \lim_{x \rightarrow a} f(x)$$

$$\lim_{h \rightarrow 0} f(a+h) = f(a) \lim_{h \rightarrow 0} f\left(1+\frac{h}{a}\right)$$

$$f(0) = f^2(0)$$

$$f(0) = 0/1$$

$$\lim_{x \rightarrow a} f\left(\frac{a+x}{a}\right) = f(a) \lim_{x \rightarrow a} f\left(\frac{a+x}{a}\right)$$

$$\boxed{a \neq 0}$$

$$= f(a) \lim_{t \rightarrow 1} f(t)$$

$$= f(a) f(1)$$

$$= f(a)$$

$$\lim_{h \rightarrow 0} f(h) = RHL$$

$$\lim_{h \rightarrow 0} f(-h) = LHL$$

$$= LHL = \underline{f(-1)} \lim_{h \rightarrow 0} f(h)$$

$$\begin{aligned} f(-1) &= -1 \\ f(1) &= 1 \\ f(0) &= 0 \\ \boxed{f(-1) = \pm 1} \end{aligned}$$

$$LHL = RHL$$

$$= 0$$

$$\lim_{x \rightarrow \frac{\pi}{4}} - \lim_{n \rightarrow \infty}$$

$$\frac{\ln(\tan x)}{1 + (\tan x)^n} = \lim_{x \rightarrow \frac{\pi}{4}} \ln \tan x = 0$$

$$\lim_{x \rightarrow \frac{\pi}{4}} + \lim_{n \rightarrow \infty} \frac{\ln(\tan x)}{1 + (\tan x)^n}$$

$$\frac{\sin\left(\frac{x}{2^{n+1}} - \frac{x}{2^n}\right)}{\cos \frac{x}{2^n} \cos \frac{x}{2^{n+1}}}$$

$$\lim_{x \rightarrow \frac{\pi}{4}} + 0 = 0$$

$$\frac{(x+1)(x^2-2x-1)}{g(x) = (x+1) \dots}$$

$$(x+1)(x-2) \quad \times$$

$$x \rightarrow \infty \quad (x+1)^2 \quad \times$$

$$\lim_{x \rightarrow -1} = \infty$$

$$g(x) = k(x+1)$$

$$\lim_{x \rightarrow -1} \frac{(x+1)(\dots)}{k(x+1)} = \frac{1}{2}$$

$k = ?$

$$f_i(x) =$$

$$\sin|x| + |x|$$

$$\sin |x| = |x| \therefore$$

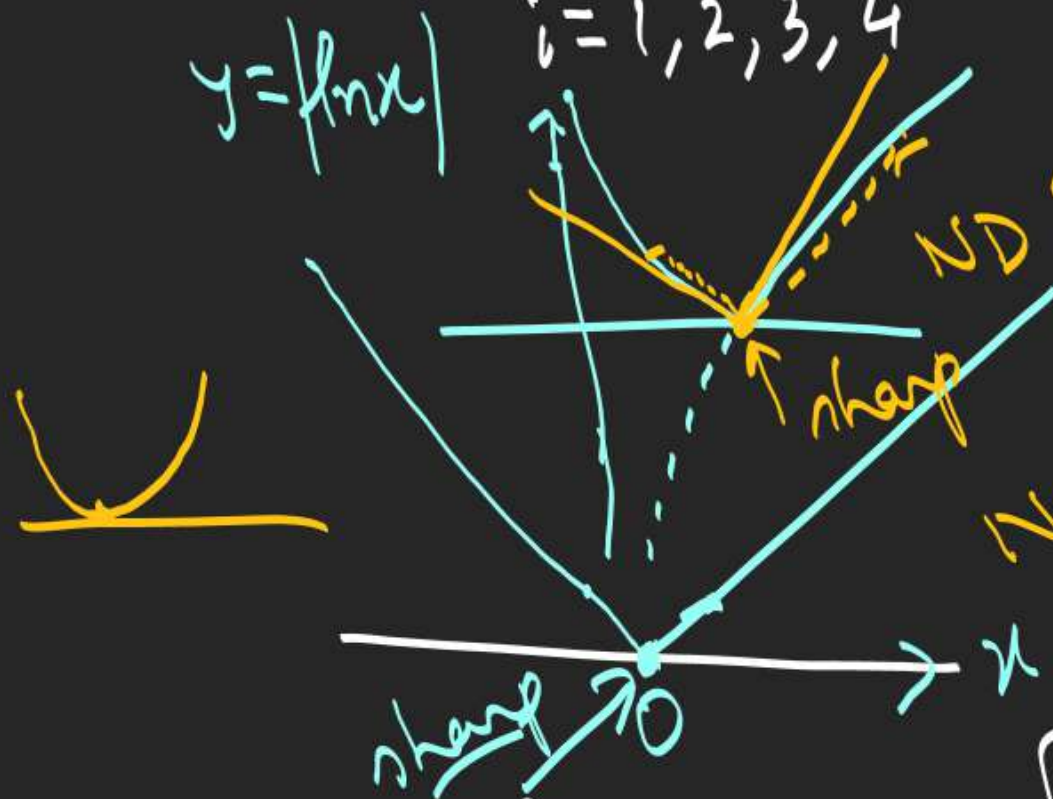
$$\cos |x| + |x| = \cos x + x \quad \checkmark$$

$$\cos |x| - |x| = \cos x - |x| \checkmark$$

$$\lim_{x \rightarrow 0} \frac{\sin|x| + |x| - 0}{x - 0} = 1$$

at $x=0$

$$= \lim_{x \rightarrow 0} \left(\frac{\sin nx + nx}{nx} \right) \frac{nx}{x}$$



$$\lim_{x \rightarrow 0} \frac{|x| - 0}{x - 0}$$

$$ALND = -1$$

$$RFD = 1$$

$(1+x+x^2) = 1+x^2+x^4$

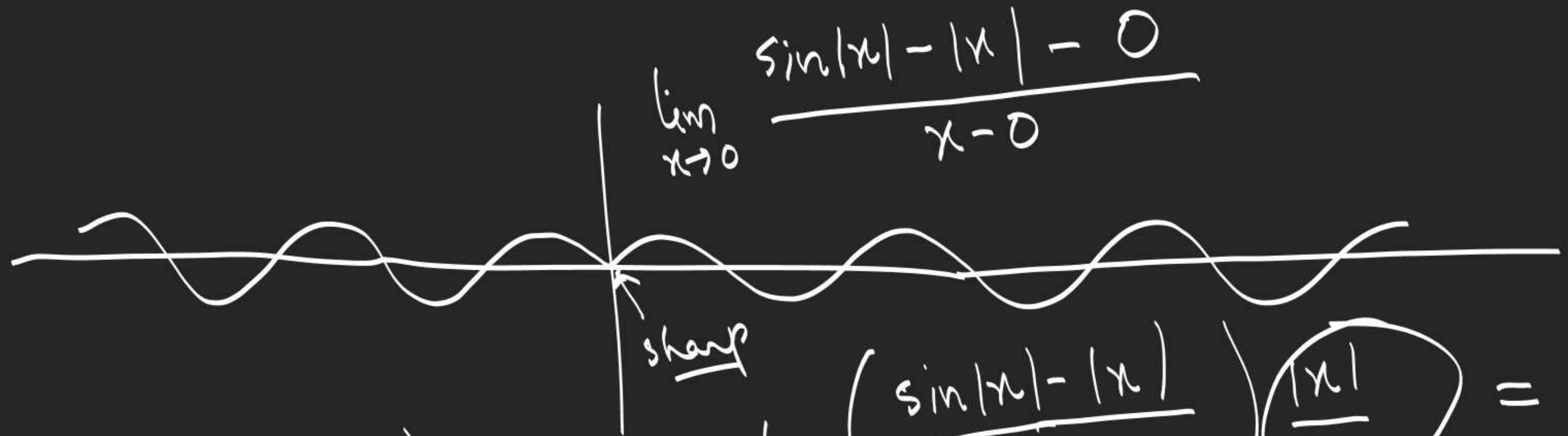
LHD & RHD both finite

\neq RHD

LHD & RHD both finite
LHD \neq RHD

$$\angle H_2 D = -2$$

$$R_{HD} \approx 2$$



$$\lim_{x \rightarrow 0} \frac{\sin|x| - |x|}{x - 0} = 0$$

$$\sin|x| \pm |x|$$

↓
LHD

$$y = \sin|x|$$

$$\lim_{x \rightarrow 0} \left(\frac{\sin|x| - |x|}{|x|} \right) \left(\frac{|x|}{x} \right) = 0$$

$$\lim_{x \rightarrow 0} \frac{\cos|x| + |x| - 1}{x} = \lim_{x \rightarrow 0} \left(\frac{\cos|x| - 1}{|x|^2} \cdot \frac{|x|^2}{x} + \frac{|x|}{x} \right)$$

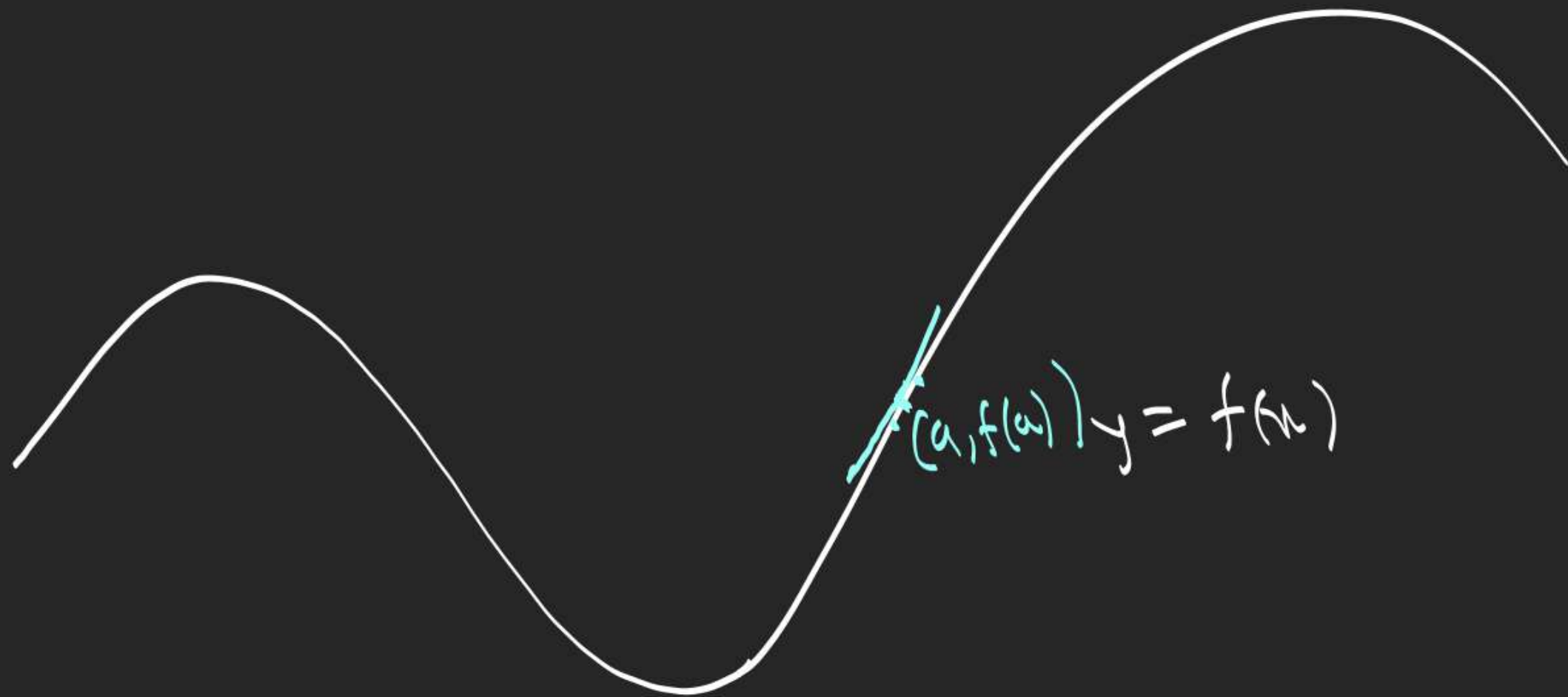
LHD = -1 RHD = 1

Note \rightarrow If $f(x)$ is diff. at $x=a$, doesn't necessarily imply that $f'(x)$ is continuous at $x=a$.

$$f'(x) = g(x)$$

$f'(a)$ exist \checkmark

$f'(x)$ may be cont. or discont.



$\cos \frac{1}{x}$

~~1/x~~

$$\lim_{x \rightarrow a} f'(x) = f'(a)$$

$f'(a)$ exist
 $f'(x)$ discont.

$$f(x) = \begin{cases} x^2 \sin \frac{1}{x} & x \neq 0 \\ 0 & x = 0 \end{cases}$$

P.T. $f(x)$ is diff. at $x=0$ & $f'(x)$

$$\lim_{x \rightarrow 0} f'(x) = \lim_{x \rightarrow 0} \left(\underbrace{2x \sin \frac{1}{x}}_0 - \underbrace{\cos \frac{1}{x}}_{[-1,1]} \right) = \text{not exist.} \text{ is discontinuous at } x=0.$$

$$f'(0) = \lim_{x \rightarrow 0} \frac{x^2 \sin \frac{1}{x} - 0}{x - 0} = \lim_{x \rightarrow 0} x \sin \frac{1}{x} = 0$$

$$f'(x) = \begin{cases} 2x \sin \frac{1}{x} + x^2 \cos \frac{1}{x} \left(-\frac{1}{x^2} \right), & x \neq 0 \\ 0, & x = 0 \end{cases} \Rightarrow f'(x) = \begin{cases} 2x \sin \frac{1}{x} - \cos \frac{1}{x} & x \neq 0 \\ 0 & x = 0 \end{cases}$$

$$f'(a) = \lim_{x \rightarrow a} \frac{x^2 \sin \frac{1}{x} - a^2 \sin \frac{1}{a}}{x - a} = \lim_{x \rightarrow a} \frac{x^2 \sin \frac{1}{x} - a^2 \sin \frac{1}{x} + a^2 \sin \frac{1}{x} - a^2 \sin \frac{1}{a}}{x - a}$$

$$\lim_{x \rightarrow a} \left((x+a) \sin \frac{1}{x} + a^2 \frac{2 \sin \left(\frac{\frac{1}{x} - \frac{1}{a}}{2} \right) \cos \left(\frac{\frac{1}{x} + \frac{1}{a}}{2} \right) (a-x)}{2ax} \right)$$

$$= 2a \sin \frac{1}{a} + a^2 \left(2 \cos \frac{1}{a} \right) \frac{\left(\frac{1}{x} - \frac{1}{a} \right)}{2} \frac{1}{2ax} (-1)$$

$$= 2a \sin \frac{1}{a} - \cos \frac{1}{a}$$

