Nishant Jindal

Q1 $P=\int_0^\infty \frac{x^2dx}{1+x^4}$, $Q=\int_0^\infty \frac{x\,dx}{1+x^4}$, $R=\int_0^\infty \frac{dx}{1+x^4}$

(1) $Q=\frac{\pi}{4}$ ✓ (2) $P=R$ ✓ (3) $P-\sqrt{2}Q+R=\frac{\pi}{2\sqrt{2}}$

$x=\frac{1}{t}$

$P=\int_\infty^0 \frac{\frac{1}{t^2}\times -\frac{1}{t^2}\cdot dt}{1+\frac{1}{t^4}}$

$=\int_0^\infty \frac{dt}{1+t^4}=R$

(2) $Q=\int_0^\infty \frac{x\,dx}{1+x^4}=\frac{1}{2}\int_0^\infty \frac{dt}{1+t^2}$

$x^2=t$

$=\frac{1}{2}\left(\tan^{-1}\infty-\tan^{-1}0\right)=\frac{\pi}{4}$

Q2 $\int_0^1 \frac{x^2dx}{\sqrt{1-x^2}}$

$x=\sin\theta$

Q3 $I=\int_0^{2\pi}\frac{dx}{2+\sin 2x} \to A$

$2x=t$

$P_4\int x\to 2\pi-x$

$=\int_0^{2\pi}\frac{dx}{2+\sin 2(2\pi-x)}$

$=\int_0^{2\pi}\frac{dx}{2+\sin(4\pi-2x)}$

$I=\int_0^{2\pi}\frac{dx}{2-\sin 2x} \to B$

$2I=\int_0^{2\pi}\left(\frac{1}{2+\sin 2x}+\frac{1}{2-\sin 2x}\right)dx$

$=\int_0^{2\pi}\frac{4}{4-\sin^2(2x)}dx$

$2x=t$

$=\frac{4}{2}\int_0^{4\pi}\frac{dt}{4-\sin^2 t}$

$2I=2\int_0^{4\pi}\frac{\sec^2 t\,dt}{4(1+\tan^2 t)-\tan^2 t}$

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Q $I = \int_0^{2\pi} e^x \cos\left(\frac{\pi}{4} + \frac{x}{2}\right) dx$

$= \int_0^{2\pi} e^x \left(\frac{1}{\sqrt2}\cos\frac{x}{2} - \frac{1}{\sqrt2}\sin\frac{x}{2}\right) dx$

$= \frac{1}{\sqrt2}\int_0^{2\pi} e^x \underbrace{\left(\cos\frac{x}{2} - \sin\frac{x}{2}\right)}_{\text{Google (IBP)}} dx$

Q5 $I = \int_0^1 \tan^{-1}x \cdot \left(\frac{\pi}{2} - x\right) dx$

$= \frac{\pi}{2}\int \underbrace{\tan^{-1}x \cdot 1\,dx}_{\checkmark} - \int_0^1 \underbrace{x\tan^{-1}x}_{(IBP)}$

Q6 $I = \int_{-1}^{1} \frac{\sin x \, dt}{1 - 2t\cos x + t^2}$ → const

$= \int_{-1}^{1} \frac{\sin x \cdot dt}{t^2 - 2t\cos x + (1 - \sin^2 x) + \sin^2 x}$

$= \int_{-1}^{1} \frac{\sin x \, dt}{(t^2 - 2t\cos x + \cos^2 x) + \sin^2 x}$

$= \sin x \int_{-1}^{1} \frac{dt}{(t - \cos x)^2 + \sin^2 x} \rightarrow \int \frac{dx}{x^2 + a^2}$

7) $\int (\ln x)^n dx$ → t → IFF

8) $I = \int_0^{\sqrt3} \sin^{-1}\frac{2x}{1+x^2} dx = \int_0^1 \underbrace{2\tan^{-1}x \, dx}_{IBP} + \int_1^{\sqrt3} \pi - 2\tan^{-1}x \, dx$

Q 9 Copy → Prob Q 10 Copy.

Q11 $I = \int_0^{\pi} |\sqrt2 \sin x + 2\cos x| dx$ → T.P → $\tan^{-1}\sqrt2$

$I = \int_0^{\pi - \tan^{-1}\sqrt2} \sqrt2\sin x + \cos x \cdot dx + -\int_{\pi - \tan^{-1}\sqrt2}^{\pi} \sqrt2\sin x + \cos x \, dx$

Sochiyega

Q12 Copy

Q13 $I = \int_{-\pi/3}^{\pi/3} \frac{\pi \cdot dx}{\underbrace{2 - \cos\left(|x| + \frac{\pi}{3}\right)}_{E.V.O}} + \cancel{\int_{-\pi/3}^{\pi/3} \frac{4x^3 dx}{2 - \cos\left(|x| + \frac{\pi}{3}\right)}} \; 0$

$2\pi \int_0^{\pi/3} \frac{dx}{2 - \cos\left(x + \frac{\pi}{3}\right)}$ * $x + \frac{\pi}{3} = t$

Done

Q14) Copy

Q15 $I=\int_{-2}^{2}\frac{x^2\,dx}{\sqrt{x^2+4}}+\cancel{\int_{-2}^{2}\frac{x\,dx}{\sqrt{x^2+4}}}$

$$=2\int_0^2\frac{x^2+4-4}{\sqrt{x^2+4}}\cdot dx$$

2 formulae

direct

Q16) Copy Mein

Q17) $I=\sqrt{2}\int_0^{\pi/2}\frac{a\sin x+b\cos x}{\cos x+\sin x}$ **

King & Add

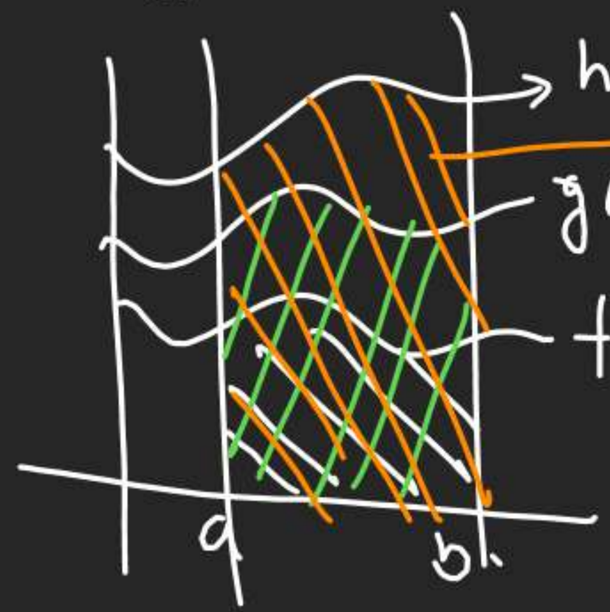
19) Copy

(20) $x=a\sin^2\theta$

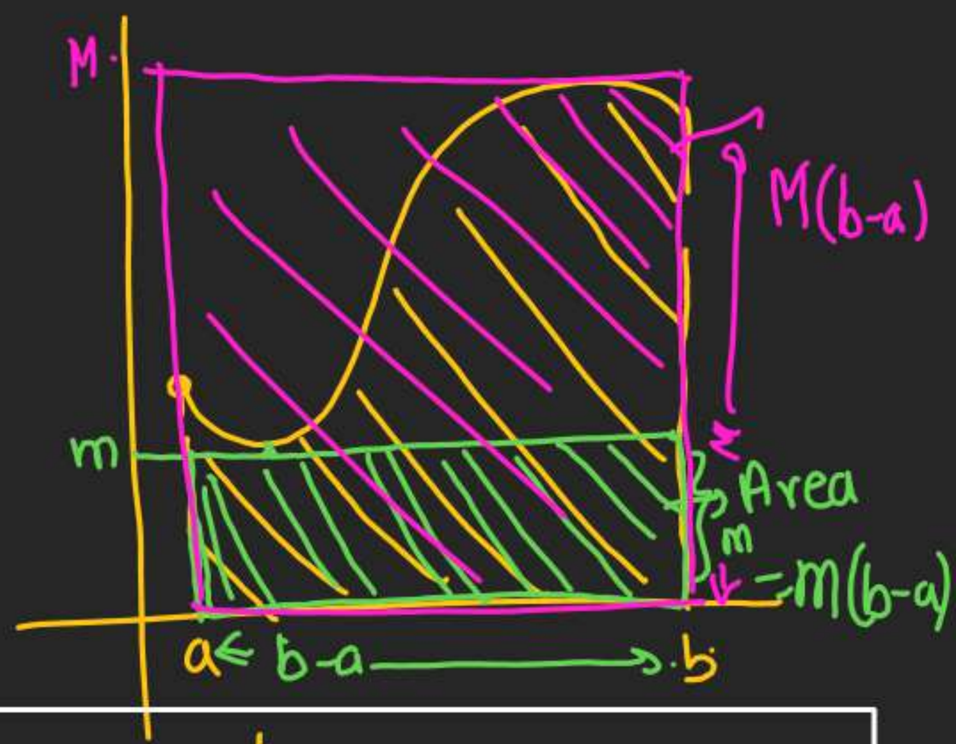
Prop: Inequality Based Prop.

1) If $f(x)<g(x)<h(x)$ in $[a,b]$

then $\int_a^b f(x)\cdot dx<\int_a^b g(x)\cdot dx<\int_a^b h(x)\cdot dx$



$\int_a^b h(x)dx>\int_a^b g(x)>\int_a^b f(x)$



$$\boxed{m(b-a)<\int_a^b f(x)\,dx<M(b-a)}$$

(2) If $f(x)$ is an Monotonic fxn in $[a,b]$ & max value of $f(x)$ is M, min. value of $f(x)=m$.

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Q $I_1=\int_0^1 \frac{dx}{\sqrt{1+x^2}}$, $I_2=\int_0^1\frac{dx}{x}$ then $I_1>I_2$ (T|F)

$x\in(0,1)$

$1+x^2>x^2$

$\sqrt{1+x^2}>x$

$\frac{1}{\sqrt{1+x^2}}<\frac{1}{x}$

$\int_0^1 \frac{dx}{\sqrt{1+x^2}} < \int_0^1 \frac{dx}{x}$

$I_1<I_2$ (False)

Q2

$I_1=\int_0^1 \frac{1+x^8}{1+x^4}dx$, $I_2=\int_0^1\frac{1+x^9}{1+x^3}dx$

$I_1<I_2$ (T|F)

$x\in(0,1)$

$x^8>x^9$ & $x^3>x^4$

$1+x^8>1+x^9$ & $1+x^3>1+x^4$

$B_1>C_1$ $\quad$ $B_2>C_2$

$\frac{1+x^8}{1+x^4}>\frac{1+x^9}{1+x^3}$

Aur Bda $\quad$ Aur chhota

$\int_0^1\frac{1+x^8}{1+x^4}dx > \int_0^1 \frac{1+x^9}{1+x^3}dx$

$I_1>I_2$ (False)

Q Adv $I_1=\int_0^{\pi/4} e^{x^2}dx$, $I_2=\int_0^{\pi/4}e^{x}dx$

$I_3=\int_0^{\pi/4} e^{x^2}\cos x\,dx$, $I_4=\int_0^{\pi/4}e^{x^2}\sin x\,dx$

then Relate I_1, I_2, I_3, I_4?

$x\in(0,\frac{\pi}{4}) \approx (0, .75)$

$x^2<x$ | $\cos x>\sin x$

$e^{x^2}<e^{x}$ | $e^{x^2}\cos x > e^{x^2}\sin x$

$\int_0^{\pi/4} e^{x^2}dx<\int_0^{\pi/4}e^{x}dx$ | $\int_0^{\pi/4}e^{x^2}\cos x > \int_0^{\pi/4} e^{x^2}\sin x$

$I_1<I_2$ (A) | $I_3>I_4$ (B)

$(0,\frac{\pi}{4})\to \cos x<1$

$e^{x^2}\cos x<e^{x^2}\cdot 1$

$\int_0^{\pi/4} e^{x^2}\cos x\,dx<\int_0^{\pi/4}e^{x^2}dx$ | $I_1>I_3$

$I_2>I_1>I_3>I_4$

Nishant Jindal

Q $I_1 = \int_0^1 \frac{\sin x\,dx}{\sqrt{x}}$, $I_2 = \int_0^1 \frac{\cos x}{\sqrt{x}}dx$

Adv

then $I_1 < \_\_\_$ & $I_2 < \_\_\_$?

V. Imp. $x \in (0,1)$ (Borrow from LIMIT)

$\sin x < x$

$\frac{\sin x}{\sqrt{x}} < \frac{x}{\sqrt{x}}$

$\int_0^1 \frac{\sin x}{\sqrt{x}}dx < \int_0^1 \sqrt{x}\,dx$

$I_1 < \frac{2}{3}(x)^{3/2}\Big|_0^1$

$I_1 < \frac{2}{3}$.

$\cos x < 1$

$\frac{\cos x}{\sqrt{x}} < \frac{1}{\sqrt{x}}$

$\int_0^1 \frac{\cos x}{\sqrt{x}}dx < \int_0^1 \frac{dx}{\sqrt{x}}$

$I_2 < 2\sqrt{x}\Big|_0^1$

$< 2(1-0)$

$\underline{\underline{I_2 < 2}}$

Q P.T. $\frac{\sqrt{3}}{8} < \int_{\pi/4}^{\pi/3} \frac{\sin x}{x}.dx < \frac{\sqrt{2}}{6}$.

$\rightarrow$ Int. नहीं निकलेगा only Estimation is PSBL

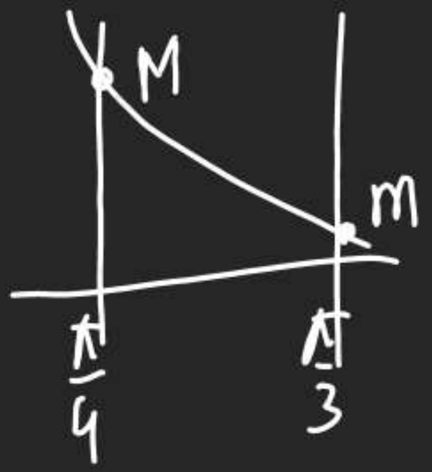
① $f(x) = \frac{\sin x}{x}$

Naline $f'(x) = \frac{x.\cos x - \sin x}{x^2}$

$= \frac{\cos x(x - \tan x)}{x^2}$ (−ve) $\tan x > x$

$x^2 \rightarrow (+)$

$f'(x) = $ (−ve) $f(x) \downarrow$

② 

$M = \frac{\sin\frac{\pi}{4}}{\frac{\pi}{4}} = \frac{4}{\sqrt{2}\pi}$

$m = \frac{\sin\frac{\pi}{3}}{\pi/3} = \frac{3\sqrt{3}}{2\pi}$

$b = \frac{\pi}{3}$, $a = \frac{\pi}{4}$

$b - a = \frac{\pi}{12}$

$m(b-a) < \int_{\pi/4}^{\pi/3} \frac{\sin x}{x}dx < M(b-a)$

$\frac{3\sqrt{3}}{2\pi}\left(\frac{\pi}{12}\right) < \int_{\pi/4}^{\pi/3} \frac{\sin x}{x}dx < \frac{4}{\sqrt{2}\pi}\left(\frac{\pi}{12}\right)$

$\frac{\sqrt{3}}{8} < I < \frac{\sqrt{2}}{6}$.

H.P.

Nishant Jindal

Q $I = \sum_{K=1}^{98} \int_K^{K+1} \frac{K+1}{x(x+1)}\,dx$ then S.T. $I < \ln 99$

Ans

$\boxed{K < x} < K+1 \Rightarrow x > K$

$\Rightarrow 1+x > 1+K \Rightarrow \frac{1}{(x+1)} < \frac{1}{(K+1)}$

$\Rightarrow \frac{1}{x(x+1)} < \frac{1}{x(K+1)}$

$\Rightarrow \int_K^{K+1} \frac{K+1}{x(x+1)} < \int_K^{K+1} \frac{\cancel{K+1}}{x(\cancel{K+1})}\,dx$

$\sum_{K=1}^{98} \int_K^{K+1} \frac{(K+1)\,dx}{x(x+1)} < \sum_{K=1}^{98} \int_K^{K+1} \frac{dx}{x}$

$< \sum \ln(x)\Big|_K^{K+1}$

$< \sum_{K=1}^{98} \ln\left(\frac{K+1}{K}\right) = \left(\ln\frac{2}{1} + \ln\frac{3}{2} + \ln\frac{4}{3} \cdots + \ln\frac{99}{98}\right) = \ln\left(\frac{2}{1} \times \frac{3}{2} \times \frac{4}{3} \times \frac{5}{4} \cdots \times \frac{99}{98}\right) = \ln 99.$

Prob. 9 Limit As a Sum (Theorey Kal)

In this Title We Solve Qs. of limit having Series With following Idententication

(1) Qs. of Series

(2) $\lim_{n\to\infty}$

(3) $+++$ Sign or $\times\times\times$ Sign

Method → (1) find T_n With Σ

(2) make change

(A) $\Sigma \to \int$ (B) $\frac{r}{n} \to x$ (C) $\frac{1}{n} \to dx$

(D) $\frac{UL}{n}$ = New UL (E) $\frac{LL}{n}$ = New LL

Q $\lim_{n\to\infty} \frac{1}{n} \sum_{r=1}^{2n} \frac{r}{\sqrt{n^2+r^2}} = ?$

$\lim_{n\to\infty} \sum_{r=1/n}^{2n/n} \frac{r}{n\sqrt{1+\left(\frac{r}{n}\right)^2}} \cdot \frac{1}{n}$

$= \int_0^2 \frac{x}{\sqrt{1+x^2}}\,dx$

$= \sqrt{1+x^2}\Big|_0^2$

$= \sqrt{5} - 1$ Ans

Nishant Jindal

Q $\lim_{n\to\infty}\left[\frac{1}{n+1}+\frac{1}{n+2}+\frac{1}{n+3}+\cdots+\frac{1}{6n}\right]$

$\lim_{n\to\infty}\left[\frac{1}{n+1}+\frac{1}{n+2}+\frac{1}{n+3}+\cdots\frac{1}{n+5n}\right]$

$\lim_{n\to\infty}\sum_{r=1}^{5n}\frac{1}{n+r} = \sum_{r=1/n}^{5n/n}\frac{1}{n\left(1+\frac{r}{n}\right)}$

$\int_0^5\frac{dx}{1+x} = \ln|1+x|\Big|_0^5$

$= \ln 6 - \ln 1$

$= \ln 6$

Q $\lim_{n\to\infty}\frac{(\lfloor n)^{\frac{1}{n}}}{n}$

Yad

$\lim_{n\to\infty}\left(\frac{\lfloor n}{n^n}\right)^{\frac{1}{n}}$

$\Rightarrow \lim_{n\to\infty}\left(\frac{1\cdot 2\cdot 3\cdot 4\cdot 5\cdots n}{n\cdot n\cdot n\cdot n\cdot n \quad n}\right)^{1/n}$

$y = \lim_{n\to\infty}\left(\left(\frac{1}{n}\right)\left(\frac{2}{n}\right)\left(\frac{3}{n}\right)\left(\frac{4}{n}\right)\left(\frac{5}{n}\right)\cdots\left(\frac{n}{n}\right)\right)^{1/n}$

$\log y = \lim_{n\to\infty}\frac{1}{n}\left\{\log\frac{1}{n}+\log\frac{2}{n}+\log\frac{3}{n}+\cdots+\log\frac{n}{n}\right\}$

$\lim_{n\to\infty}\sum_{r=1/n}^{n/n}\log\left(\frac{r}{n}\right)\cdot\frac{1}{n} = \int_0^1 \ln x\cdot dx$

$\log_e y = x\ln x - x\Big|_0^1 = -1$

$\Rightarrow y = e^{-1} = \frac{1}{e}$

Q $\lim_{n\to\infty}\frac{3}{n}\left[1+\sqrt{\frac{n}{n+3}}+\sqrt{\frac{n}{n+6}}+\sqrt{\frac{n}{n+9}}+\cdots+\sqrt{\frac{n}{n+3(n-1)}}\right]$

$\sqrt{\frac{n}{n+3\times 0}}$ $\sqrt{\frac{n}{n+3\times 1}}$ $\sqrt{\frac{n}{n+3\times 2}}$

$\lim_{n\to\infty}\frac{3}{n}\sum_{r=0/n}^{\frac{n}{n}-\frac{1}{n}}\sqrt{\frac{n}{n+3r}} = \sum\sqrt{\frac{\cancel{n}}{\cancel{n}\left(1+\frac{3r}{n}\right)}}\cdot\frac{3}{n}$

$1-0$

$\int_0^1\frac{3}{\sqrt{1+3x}}dx = \frac{\cancel{3}\times 2\sqrt{1+3x}}{\cancel{3}}\Bigg|_0^1$

$2\left(\sqrt{1+3\times 1}-\sqrt{1+0}\right)$

$2(2-1) = 2$

Nishant Jindal

Q If $f(x)$ is Integrable over $[1,2)$ then $\int_1^2 f(x)\,dx =$

A) $\lim_{n\to\infty} \frac{1}{n}\sum_{r=1/n}^{n/n} f\left(\frac{r}{n}\right) = \int_0^1 f(x)\,dx$ ×

B) $\lim_{n\to\infty} \frac{1}{n}\sum_{r=\frac{n+1}{n}}^{\frac{2n}{n}} f\left(\frac{r}{n}\right) = \int_{1+0}^{2} f(x)\,dx$ ✓

C) $\lim_{n\to\infty} \frac{1}{n}\sum_{r=1/n}^{n/n} f\left(\frac{r+n}{n}\right) = \int_0^1 f(x+1)\,dx$ ✓

$x+1=t$, $dx=dt$

x	t
0	1
1	2

$= \int_1^2 f(t)\,dt$

D) NOT.

Prop 10 Newton Liebnitz Theorem.

(NL)

$$\boxed{\frac{d}{dx}\left(\int_{\phi(x)}^{\psi(x)} f(t)\,dt\right) = f(UL)\cdot(UL)' - f(LL)\cdot(LL)'}$$

Q If $g(x) = \int_1^x \sqrt{t^4+1}\,dt$ then $g'(x)$?

$g'(x) = \sqrt{x^4+1}\cdot(x)' - \sqrt{1^4+1}\times(1)'$

$= 1\times\sqrt{x^4+1} - 0$

$g'(x) = \sqrt{x^4+1}$

Subjective:

Q 32, 33, 34

35, 45, 46

47, 48

52, 53, 54, 55

56, 57, 58, 59, 60

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