Nishant Jindal

Built in Limit (Limit me Limit)

- One Limit insure $x \to \infty$
- other Limit can be any Const. No.

Q Check Cont^y of $\lim\limits_{t \to \infty} \dfrac{(1+\sin \pi x)^t - 1}{(1+\sin \pi x)^t + 1}$ at $x = 1, 2, 3$.

$x=1$ पर check करके दिखा रहा हूं।

$$f(x) = \begin{cases} \lim\limits_{t\to\infty} \dfrac{(1+\sin x)^t - 1}{(1+\sin \pi x)^t + 1} = \dfrac{(\text{Exact }1)^{\infty} - 1}{(\text{Exact }1)^{\infty} + 1} = \dfrac{1-1}{1+1} = 0 \\[2ex] \lim\limits_{t\to\infty} \dfrac{(1+\sin \pi x)^t - 1}{(1+\sin \pi x)^t + 1} = \dfrac{(1-\sin \pi h)^{\infty} - 1}{(1-\sin \pi h)^{\infty} + 1} = \dfrac{0-1}{0+1} = -1 \\[2ex] \lim\limits_{t\to\infty} \dfrac{(1+\sin \pi x)^t - 1}{(1+\sin \pi x)^t + 1} = \dfrac{(1+\sin \pi h)^{\infty} - 1}{(1+\sin \pi h)^{\infty} + 1} = \dfrac{\cancel{(1+\sin \pi h)^t}\left(1 - \frac{1}{(1+\sin \pi h)^t}\right)}{\cancel{(1+\sin \pi h)^t}\left(1 + \frac{1}{(1+\sin \pi h)^t}\right)} = \dfrac{1-0}{1+0} = 1 \end{cases}$$

less than 1

(Greater than 1) $^{\infty}$

$\boxed{x=1}$

$\pi \cdot x = \pi$

$\sin \pi x = \sin \pi = 0$

$1 + \sin \pi x = 1 + 0 = 1$

D.C. at $x = 1$

$\boxed{x = 1+h} \to \pi x = \pi(1+h) = \pi + \pi h$

$1 + \sin(\pi x) = 1 + \sin(\pi + \pi h) = (1 - \sin \pi h)$

$\boxed{x = 1-h}$

$\pi x = \pi(1-h) = \pi - \pi h$

$1 + \sin \pi x = 1 + \sin(\pi - \pi h) = (1 + \sin \pi h)$

Q $\lim\limits_{t \to \infty} \dfrac{(1+\sin \pi x)^t - 1}{(1+\sin \pi x)^t + 1}$ $\quad x = 1, 2, 3$

$(1 + \sin \pi x)^t \to 1 + \sin \pi(1) = 1$

$1 + \sin \pi(2) = 1$

$1 + \sin \pi(3) = 1$

Trick

$\boxed{\text{D.C. wherever } (f(x))^n \to f(x) = 1}$

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Q let a fxn $f(x) = \lim_{n\to\infty} (\sin x)^{2n}$ $n \in N$

Then fxn becomes D.C ?

$f(x) = \lim_{n\to\infty} (\sin^2 x)^n$ is D.C.

Whenever $\sin^2 x = 1$

$$\boxed{x = \frac{(2m+1)\pi}{2}}$$

Q $f(x) = \lim_{n\to\infty} \frac{\log_e(2+x) - x^{2n}\sin x}{1 + x^{2n}}$ is D.C. at $x=1$?

$(x^2)^n$ is D.C

When $x^2 = 1$

$x = 1, -1$

$$LHL = \frac{\log(2+1-h) - (1-h)^{2n}\sin x}{(1-h)^{2n}} = \frac{\log(3-h)}{0}$$

$$RHL \Rightarrow \frac{\log(2+(1+h)) - (1+h)^{2n}\sin x}{(1+h)^{2n}}$$

$$\frac{\cancel{(1+h)^{2n}}\left(\frac{\log(3+h)}{\cancel{(1+h)^{2n}}} - \sin(1+h)\right)}{\cancel{(1+h)^{2n}}}$$

$$-\sin(1+h)$$

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Ex1 (Contd)

(1) $f(x) = \begin{cases} ax+1 & x<1 \\ 3 & x=1 \\ bx^2+1 & x>1 \end{cases}$

$a+1 = 3 = b+1$

$a=2,\ b=2$

$a-b=0$

(2) $f(x) = \lim_{x\to 2} \dfrac{1}{x+(2)^{\frac{1}{x-2}}}$ $x \neq 2$

$f(2) = \lim_{x\to 2} \dfrac{1}{x+(2)^{\frac{1}{x-2}}} \to$ (hor)

$\dfrac{1}{2-h+(2)^{\frac{1}{\cancel{2}-h-\cancel{2}}}}$, $2^{-\infty}$

$\dfrac{1}{2-h} = \dfrac{1}{2}$

$\dfrac{1}{2+h+(2)^{\frac{1}{\cancel{2}+h-\cancel{2}}}} = \dfrac{1}{\infty} = 0$, $2^{\infty} = \infty$

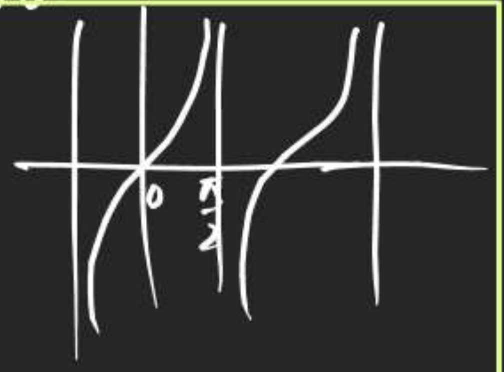
(3) $f(x) = \dfrac{4-x^2}{4x-x^3}$ all 4 obtain

Dr $\neq 0$ domain

$4x - x^3 = 0$ ✗

Cont^s everywhere

Concept: $f(x) = \tan x$



D.C. at every odd $\frac{\pi}{2}$

in Domain $\to$ $x = R - (2n+1)\frac{\pi}{2}$

in Domain it is Cont^s

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Q $f(x) = \lim_{x \to 5} \dfrac{x^2 - bx + 25}{x^2 - 7x + 10}$ $\dfrac{\cancel{\ \ }}{0}$ $25 - 5b + 25 = 0$ $\Rightarrow b = 10$

$\dfrac{x^2 - 10x + 25}{x^2 - 7x + 10} = \dfrac{(x-5)^2}{\cancel{(x-5)}(x-2)} = \dfrac{0}{3} = 0$

Q5 Same.

Q6 $y = f(x) = \tan^3 x \rightarrow \underset{\sim\sim}{(a, 0)}$

$f(a) = 0$

$f(a) = \lim_{x \to a} \dfrac{\log(1 + 3f(x))}{2f(x)} = \dfrac{\log(1 + 3f(x))}{3f(x)} \times \dfrac{3f(x)}{2\cancel{f(x)}}$

$1 \times \dfrac{3}{2}$

Q7 $f(x) = [x^2 + 1]$ in $x \in [1, 3]$.

$= [x^2] + 1$

$x \in [1, 3)$

$x^2 \in [1, 9]$

$x^2 = 1, \underbrace{2, 3, 4, 5, 6, 7, 8, 9}$

8 pt

Q8 $f(x) = [2x^3] - 5$ $x \in [1, 2)$

$x^3 \in [1, 8)$

$2x^3 \in [2, 16)$

13 pts

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$$f(x)=\begin{cases}|x+1|=1 & x<-2\\ 2x+3 & -2\le x<0\\ x^2+3 & 0\le x<3\\ x^3-15 & x\ge 3\end{cases}$$

$-2\times 2+3=-1=|-2+1|$

$-1=+1$

-2

$2\times 0+3=0^2+3$

3

$9+3=3^3-15$

$12=12$

12 $f(x)=[x]^2-[x^2]$

$f(0)=[0]^2-[0^2]=0$

$f(0^+)=[h]^2-[h^2]=0$

$f(0^-)=[-h]^2-[(-h)^2]=1$

DL

Q13 $f:R\to R$ Cont's fxn $f(x)=5\ \forall x\in R$ ans

$$f(x)=\begin{cases}5 & x\in Q\\ 5 & x\in Q'\end{cases}$$

$f(x)=5$

$f(3)=5$

(10) $\lim\limits_{x\to 0} f\left(\frac{1-\cos 3x}{x^2}\right)=f\left(\lim\limits_{x\to 0}\frac{1-\cos 3x}{x^2}\right)=f\left(\frac{9}{2}\right)=\frac{2}{9}$

(11) $x^2+(f(x)-2)x-\sqrt{3}f(x)+2\sqrt{3}-3=0$

$f(x)\,(x\ne\sqrt{3})=\dfrac{3-2\sqrt{3}+2x-x^2}{(x-\sqrt{3})}=-\dfrac{(x^2-3)+2(x-\sqrt{3})}{(x-\sqrt{3})}=-\dfrac{(x+\sqrt{3})+2}{1}$

$=2-2\sqrt{3}$

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Q14 $f(x)=\dfrac{1}{(x-1)(x-2)}$ $\quad g(x)=\dfrac{1}{x^2} \rightarrow x^2=0$

$x=0$

$$f(g(x))=\frac{1}{(x-1)(x-2)}$$

$x=1$	$x=2$
$g(x)=1$	$g(x)=2$
$\dfrac{1}{x^2}=1$	$\dfrac{1}{x^2}=2$
$x^2=1$	$x^2=\dfrac{1}{2}$
$x=\pm 1$	$x=\pm\dfrac{1}{\sqrt{2}}$

(15)

$$f(x)=\begin{cases}\dfrac{x}{[x]} & 1\le x<2\\ 1 & x=2\\ \sqrt{6-x} & 2<x\le 3.\end{cases}$$

T.P $\Rightarrow x=2$

$f(2^-)=\dfrac{2-h}{[2-h]}=\dfrac{2-h}{1}=\dfrac{2}{1}=2$

$f(2)=1$

$f(2^+)=\sqrt{6-(2+h)}=\sqrt{4}=2$

LHL $=2=$ RHL

Removab

Isolated

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16) $f(0^-) = \lim_{x\to 0^-} (-h)[-h]^2 \log_{(1+h)} 2$

$= \lim_{h\to 0} 1 \times \left[\frac{\log 2 \times -h}{\log(1-h)}\right] = \ln 2$

$f(0^+) = \frac{\ln(1-(1+e^{h^2}+2\sqrt{h}))}{\tan\sqrt{h}}$

$\frac{\ln(1-(1+e^{h^2}+2\sqrt{h}))}{(1-e^{h^2}+2\sqrt{h})} \times \frac{(1-e^{h^2}-2\sqrt{h})}{\sqrt{h}}$

L HITLERIAL
Irr.

$\frac{\cancel{1}-(\cancel{1}+h^2)-2\sqrt{h}}{\sqrt{h}}$

$= \frac{-\sqrt{h}(2-h^{3/2})}{\sqrt{h}} = -2$

21) $\begin{cases} \frac{(1+\tan x)^{1/x}-e}{x} & x\neq 0 \\ K & x=0 \end{cases}$ $(1+x)^{1/x}$

$= e - \frac{ex}{2} + \frac{11e}{24}x$

$\frac{(e-\frac{ex}{2})-e}{x} = -\frac{e}{2} = K$

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22) $f(x) = \begin{cases} px^2 - px + q & x < 1 \\ x - 1 & 1 \le x \le 3 \\ lx^2 + mx + 2 & x > 3 \end{cases}$

$\frac{ql - m}{l} = ?$

$x = 1$

$p - p + q = 1 - 1$

$q = 0$

$x = 3$

$3 - 1 = 9l + 3m + 2$

$9l + 3m = 0$

$m = -3l$

$\Rightarrow -\frac{(-3l)}{l} = 3.$

2?) $f : [0,1] \to R$ be $\underline{\text{Cont}^s \text{ fxn}}$. assume Rational value only $f(0) = 2$

$f(x) = \begin{cases} 2 & x \in Q \\ 2 & x \in Q' \end{cases}$

$f(x) = 2$

Value of $\tan^{-1}\left(f\left(\frac{1}{2}\right)\right) + \tan^{-1}\left(\frac{3}{2} f\left(\frac{1}{2}\right)\right)$

$\tan^{-1}(2) + \tan^{-1}\left(\frac{3}{\cancel{2}} \times \cancel{2}\right)$

$\pi + \tan^{-1}\left(\frac{2+3}{1-2\times 3}\right) = \pi + \tan^{-1}(-1)$

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Differentiabity.

Unq tangency → one tangent at 1 pt.

Break → Discont.
Sharp → Non Differentiable

→

No Break → Conts.
No Break / No sharp = Differentiable

① 

θ = Inclination

Derivative = Slope of tangent.

$$\left.\frac{dy}{dx}\right|_{(x,y)} = m = \tan\theta$$

(2) Defination of tangent

tangent is limiting case of secant

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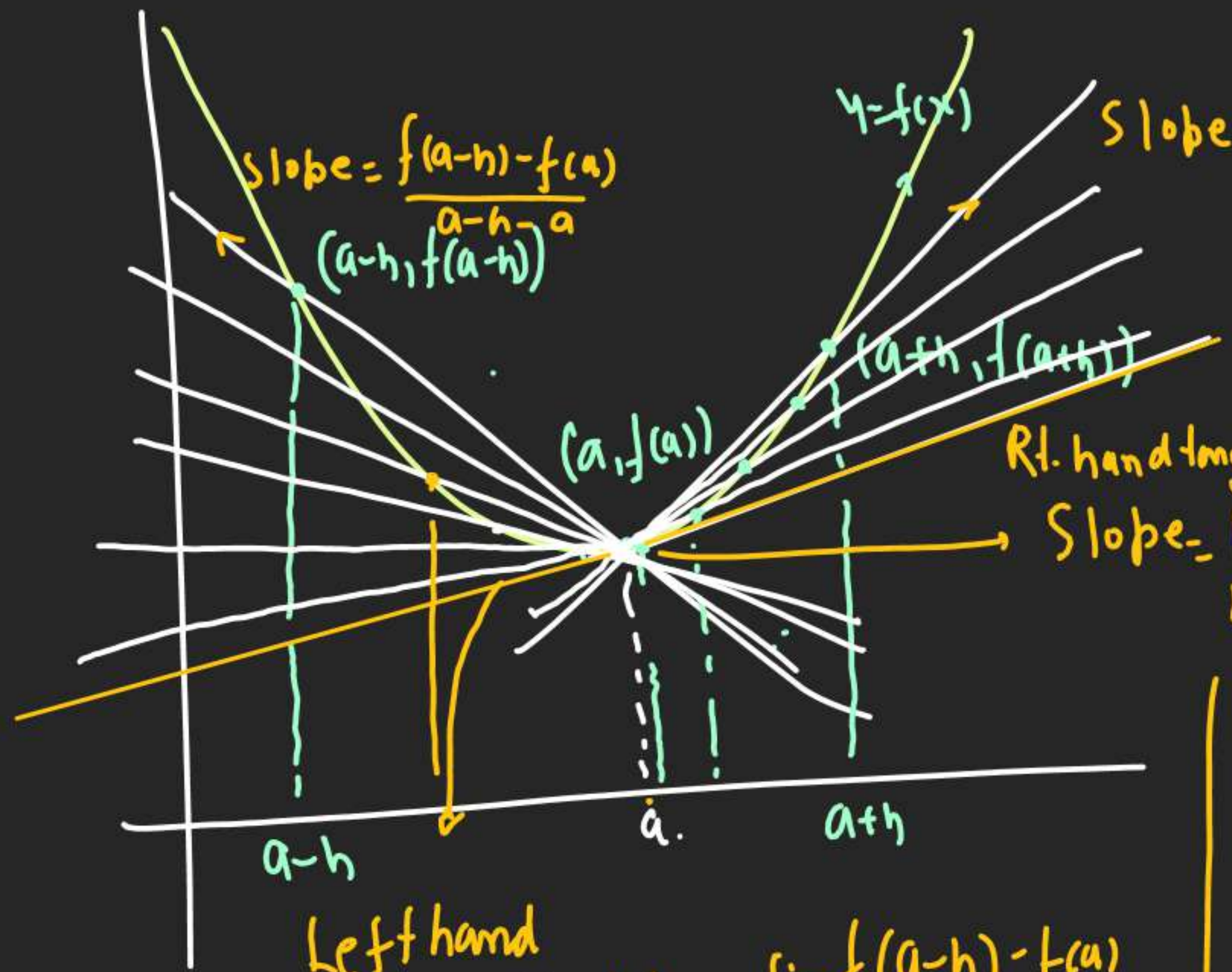
(3) Classical Defination of Derivability.



Slope = $\dfrac{f(a+h)-f(a)}{a+h-a}$

Rt. hand tangents

Slope = $\lim\limits_{h\to 0} \dfrac{f(a+h)-f(a)}{a+h-a}$

Left hand tangent's slope = $\lim\limits_{h\to 0} \dfrac{f(a-h)-f(a)}{a-h-a}$

Sharp hone pr

LHD $\neq$ RHD

$\Rightarrow$ N.D

If fxn differentiable at $x=a$

LH Tangent's Sl = RH tangent's Sl.

LHD = RHD $\Rightarrow$ Unique tangent

$f'(a^-) = f'(a^+)$

$$\lim_{h\to 0} \frac{f(a-h)-f(a)}{-h} = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$$

RK

1) LHD = left hand Derivative = $f'(a^-)$

RHD = Right hand Derivative = $f'(a^+)$

2) If fxn is diff^{ble} at $x = a$.

then $f'(a^-) = f'(a^+)$

3) If fxn has LHD = RHD at $x = a$.

$\Rightarrow f'(a^-) = f'(a^+)$

$\Rightarrow f'(a)$ will exist

(4) Differential coff at $x = a$ is denoted

by $f'(a)$ & it takes value of $f'(a^+) / f'(a^-)$

$$\boxed{f'(a^+) = f'(a^-) = f'(a)}$$

Ex.

if $f'(2)$ Exists.

$f'(2) = f'(2^+)$ या $f'(2) = f'(2^-)$

Practice

1) $f'(1^+) = \lim_{h \to 0} \frac{f(1+h) - f(1)}{h}$

2) $f'(5^-) = \lim_{h \to 0} \frac{f(5-h) - f(5)}{-h}$

3) $f'(0^+) = \lim_{h \to 0} \frac{f(0+h) - f(0)}{h}$

4) $f'(3^-) = \lim_{h \to 0} \frac{f(3-h) - f(3)}{-h}$

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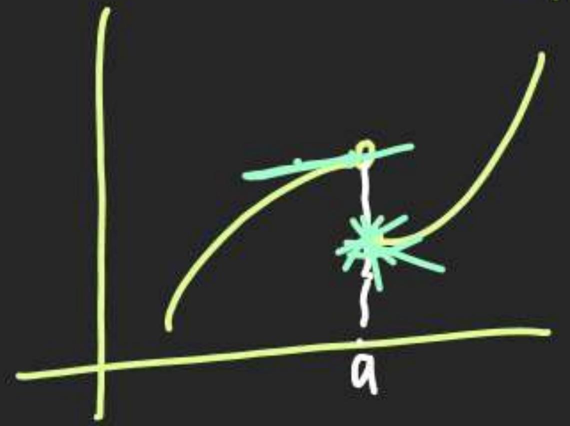
$$f'(7^-) = LHD = \lim_{h\to 0} \frac{f(7-h)-f(7)}{-h}$$

$$f'(a^+) = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$$

$$f'(a^-) = \lim_{h\to 0} \frac{f(a-h)-f(a)}{-h}$$

$$f'(b^-) = \lim_{h\to 0} \frac{f(b-h)-f(b)}{-h}$$



D.C.

$\Rightarrow$ N.D.

Rk

(5) If fxn is D.C. at $x=a$ then it is N.D. at $x=a$

for checking differtiability we check Cont^n first.

(6) LCD Rule

If fxn is diff^ble $\Rightarrow$ fxn is Cont^s.

If fxn is Cont^s $\Rightarrow$ fxn may/may not diff^ble

Q Check diff^{ty} of $f(x)$

$$f(x) = \begin{cases} x + [2x] & x < 1 \\ \{x\} + 1 & x \geq 1 \end{cases} \quad \text{at } x = 1$$

Cont^{y}

$f(1^+) = \{1+h\} + 1 = \{h\} + 1 = h + 1 = 1$

$f(1^-) = (1-h) + [2(1-h)]$

$= 1 - h + [2 - 2h]$

$= 1 - h + 1 = 2 - h = 2$

fxn is D. C. $\Rightarrow$ N.D.

Q Check diff^{ty} of $f(x) = e^{-|x|}$ at $x = 0$?

$C(C) =$ Cont^{y}

$$\text{LHD} = f'(0^-) = \lim_{h \to 0} \frac{f(0-h) - f(0)}{-h} = \lim_{h \to 0} \frac{e^{-|-h|} - e^{-|0|}}{-h}$$

$$= \lim_{h \to 0} \frac{e^{-h} - 1}{-h} = 1$$

$$f'(0^+) = \lim_{h \to 0} \frac{f(0+h) - f(0)}{h}$$

$$= \lim_{h \to 0} \frac{e^{-|h|} - e^{-|0|}}{h} = \frac{e^{-h} - 1}{-h} \times -1 = -1$$

fxn is N.D. at $x = 0$

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① Check diffᵞ of $y = |\ln x|$ at $x=1$

$$f'(1^+) = \lim_{h\to 0} \frac{f(1+h)-f(1)}{h} = \lim_{h\to 0} \frac{|\ln(1+h)| - \overbrace{|\ln 1|}^{=0}}{h}$$

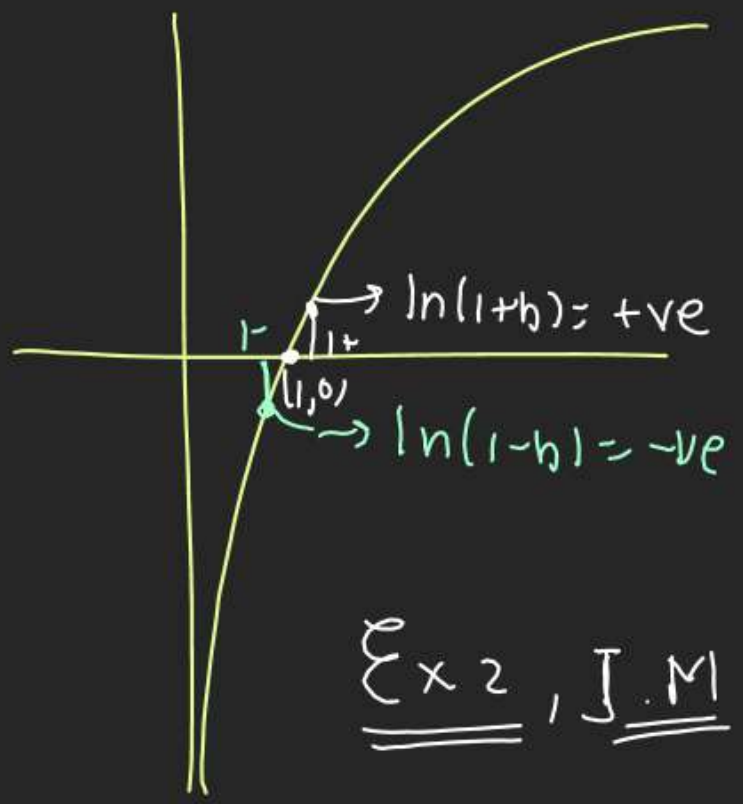
$$= \lim_{h\to 0} \frac{\ln(1+h)}{h} = 1$$

$$f'(1^-) = \lim_{h\to 0} \frac{f(1-h)-f(1)}{-h} = \lim_{h\to 0} \frac{|\ln(1-h)| - \overbrace{|\ln 1|}^{=0}}{-h}$$

$$= \lim_{h\to 0} \frac{-(\ln(1-h))}{-h} = -1$$

$f'(1^+) \neq f'(1^-)$

$\Rightarrow$ fxn is N.D. at $x=1$



Ex 2, J.M