Thermal Properties Of Matter

Heat and Temperature Heat :

- Heat, as a type of energy, gives rise to the sensation of hotness.

Concept of Temperature :

- Temperature is a physical parameter that gauges the level of hotness or coldness exhibited by a body.
- Temperature is a scalar quantity.

Measurement of Temperature

- To measure temperature accurately, two essential steps are involved:
  (a) Building a thermometer, which serves as the measuring instrument.
  (b) Calibrating the thermometer to ensure its accuracy and reliability.
- These fixed reference points are known as the "ice point" (the temperature of the melting point of ice) and the "steam point" (the temperature of water's boiling point under normal atmospheric pressure).

Thermometric Scales :

- The distance between the LFP and UFP of a thermometer is called the fundamental interval.
  Fundamental interval $=(\text{UFP})-(\text{LFP})$.
- The fundamental temperature interval is divided into uniform segments, and each segment is assigned arbitrary numerical values, creating what is known as a thermometric scale.
- The Celsius $(^\circ\text{C})$, Fahrenheit $(^\circ\text{F})$, Kelvin (K), Reaumur (R), Rankine (R_e) are commonly used thermometric scales.

Different Thermometric Scales :

Thermometric scale	LFP	UFP	Total No. of divisions
Celsius scale	0°C	100°C	100
Fahrenheit scale	32°F	212°F	180
Kelvin scale (or) Absolute scale	273.15K	373.15K	100
Reaumur scale	0°R	80°R	80

- On any thermometric scale

$$\frac{\text{Reading}-\text{LFP}}{\text{UFP}-\text{LFP}}=\text{constant}\ (\text{or})\ \frac{X-L}{U-L}=\text{constant}.S$$

Relation Between Temperatures of different Scales :

- $\frac{C-0}{100-0}=\frac{F-32}{212-32}=\frac{K-273}{373-273}=\frac{R-0}{80-0}$ (or)

  $\frac{C}{100}=\frac{F-32}{180}=\frac{K-273}{100}=\frac{R}{80}$ (or)

  $\frac{C}{5}=\frac{F-32}{9}=\frac{K-273}{5}=\frac{R}{4}$

- Temperature difference on different scales is $\frac{\Delta C}{5}=\frac{\Delta F}{9}=\frac{\Delta K}{5}=\frac{\Delta R}{4}$
- Common reading on Celsius and Fahrenheit scales is $-40°$ i.e., $-40°C=-40°F$.

  Since, $\frac{C}{5}=\frac{F-32}{9}\Rightarrow\frac{X}{5}=\frac{X-32}{9}$

  $X=-40°$.

- Temperature of the core of the sun is $10^7 K$ while of its surface 6000 K. Normal temperature of human body is $310.15K(=37°C=98.6°F)$ while NTP implies $273.15K(=0°C=32°F)$.

Faulty Thermometer :

- If the reading on a faulty thermometer is 'X' and its lower and upper fixed points are L and U respectively then
- Correct reading on Celsius scale is $\frac{C-0}{100}=\frac{X-L}{U-L}$
- Correct reading on Fahrenheit scale is $\frac{F-32}{180}=\frac{X-L}{U-L}$
- Correct reading on Kelvin scale is $\frac{K-273}{100}=\frac{X-L}{U-L}$.
- Error in measurement by faulty thermometer = measured value – true value

  Correction = - Error

Principle of Thermometry :

If X is a property that varies linearly with temperature T as $X=aT+b$, where 'a' and 'b' are constants then $t=\left(\frac{X_t-X_0}{X_{100}-X_0}\right)\times 100°C$ is general equation used to measure temperature t.

or $X_t=X_0(1+\alpha t)$

Measurement of Temperature Based on Triple point

- If the value of thermometric property at 0K, 273.16K and $T_K K$ is 0, X_{T_r} and X respectively
- $\frac{T_K}{273.16} = \frac{X}{X_{T_r}}$, i.e., $T_K = (273.16)\left|\frac{X}{X_{T_r}}\right| K$
- When a constant volume gas thermometer is used to measure temperature of a body then

  $T_K = (273.16)\left|\frac{P}{P_{T_r}}\right| K$

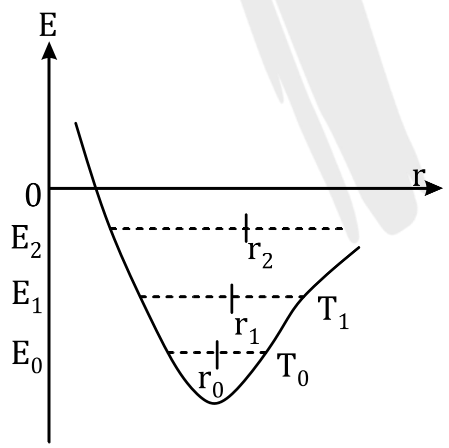
  Where P_{T_r} is pressure of a given amount of gas at triple point of water and P is the pressure at a temperature which is to be determined

Thermal Expansion of Solids :

- The thermal expansion of solids varies depending on whether they are isotropic or anisotropic.
- Isotropic solids exhibit identical thermal expansion in all directions.

Potential Energy Curve :

- Intermolecular attractions in solids result in the accumulation of potential energy (PE) within the molecules.
- The relationship between interatomic distance and potential energy is depicted by a curve known as the potential energy curve.



Coefficient of Linear Expansion :

- The ratio of increase in length of a solid per degree rise in temperature to its original length is callec coefficient of linear expansion $(\propto)$

  $\alpha = \frac{\ell_2 - \ell_1}{\ell_1 \times (t_2 - t_1)} / °C$

Coefficient of Areal (or) Superficial expansion :

The ratio of increase in its area per degree rise in temperature to its original area is called coefficient of areal expansion (β).

$$\beta = \frac{A_2 - A_1}{A_1 \times (t_2 - t_1)} / {}^\circ C$$

Final area $A_2 = A_1[1 + \beta(t_2 - t_1)]$

Coefficient of volume expansion :

The coefficient of volume expansion is defined as the ratio of the increase in volume per degree rise in temperature to its original volume.

$$\gamma = \frac{V_2 - V_1}{V_1(t_2 - t_1)} / {}^\circ C; V_2 = V_1[1 + \gamma(t_2 - t_1)]$$

Relation among α, β, γ **:**

- $\beta = 2\alpha, \quad \gamma = 3\alpha$
- $\alpha : \beta : \gamma = \alpha : 2\alpha : 3\alpha = 1 : 2 : 3$
- $\frac{\alpha}{1} = \frac{\beta}{2} = \frac{\gamma}{3}$

Variation of density of substance with temperature

If ρ_1 and ρ_2 are densities of a solid at $t_1{}^\circ C$ and $t_2{}^\circ C$, and as $m_1 = m_2$; $\rho_1 V_1 = \rho_2 V_2$

$$\rho_1 V_1 = \rho_2 V_1[1 + \gamma(t_2 - t_1)]$$

$$\rho_1 = \rho_2[1 + \gamma(t_2 - t_1)]$$

- For anisotropic materials γ is the sum of linear coefficient in three mutually perpendicular directions.

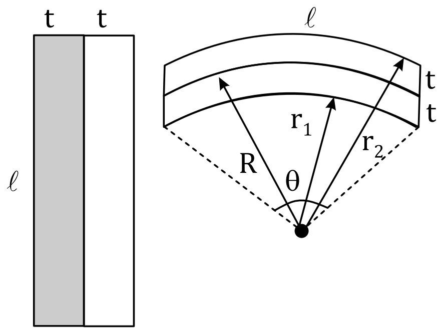
$$\gamma = \alpha_x + \alpha_y + \alpha_z.$$

For isotropic solids $\gamma = 3\alpha$

Bimetallic Strip :

Where t is thickness of each strip

- Radius of curvature of a bimetallic strip.



$$\theta = \frac{\ell}{R} \text{ (or) } \theta = \frac{d\ell}{dr} = \frac{\ell_2 - \ell_1}{r_2 - r_1}; \frac{\ell}{R} = \frac{\ell(\alpha_2 - \alpha_1)\Delta T}{2t}$$

$$\therefore R = \frac{2t}{(\alpha_2 - \alpha_1)\Delta T} \quad (\because \alpha_2 > \alpha_1)$$

Thermal Stress :

- It is developed due to preventation of expansion of a solid when it is heated.
- A rod of length ℓ_0 clamped between two fixed walls.

  For Δt change in temperature

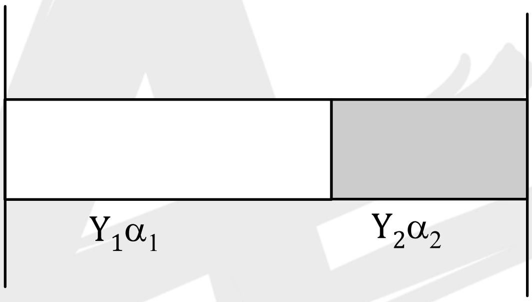
  Young's modulus

  $$Y = \frac{F/A}{\Delta\ell/\ell_0} = \frac{F\ell_0}{F\Delta\ell} = \frac{F}{A\alpha\Delta t} \quad (\because \; \Delta\ell = \ell_0\alpha\Delta t)$$

  or $\dfrac{F}{A} = Y\alpha\Delta t$

- Thermal force $F = YA\alpha\Delta t$.

  Thermal force is independent of length of rod.



Total length prevented from expansion

$$\Delta\ell_1 + \Delta\ell_2 = \frac{F\times\ell_1}{Y_1\times A} + \frac{F\times\ell_2}{Y_2\times A}$$

Thermal force $= F = \dfrac{At\left[(\ell_1\alpha_1)+(\ell_2\alpha_2)\right]}{\left(\dfrac{\ell_1}{Y_1}+\dfrac{\ell_2}{Y_2}\right)}$

$$\left(\because \quad \Delta\ell = \ell\alpha t = \frac{F\ell}{YA}\right)$$

Thermal stress $= \dfrac{F}{A} = \dfrac{t\left[(\ell_1\alpha_1)+(\ell_2\alpha_2)\right]}{\left(\dfrac{\ell_1}{Y_1}+\dfrac{\ell_2}{Y_2}\right)}$

Thermal Expansion of Liquids

Introduction :

- Liquids lack a specific shape and take on the shape of the container they are placed in, with their volume being equal to the volume of the container.

Coefficient of Real Expansion (γ_R) :

- The real increase in volume per unit original volume per $1°C$ rise in temperature is called coefficient of real expansion

  $$\gamma_R = \frac{V_2 - V_1}{V_1(t_2 - t_1)} / °C \Rightarrow V_2 = V_1\left[1 + \gamma_R(t_2 - t_1)\right]$$

- Percentage change in volume of a liquid is given by $\left(\frac{\Delta V}{V}\right) \times 100 = \gamma_R(t_2 - t_1) \times 100$.

Coefficient of Apparent Expansion (γ_A) :

- The coefficient of apparent expansion of a liquid refers to the apparent increase in volume per unit original volume per degree rise in temperature.

  $$\gamma_A = \frac{V_2 - V_1}{V_1(t_2 - t_1)} / °C \Rightarrow V_2 = V_1\left[1 + \gamma_A(t_2 - t_1)\right]$$

Relation between (γ_R) and (γ_A) :

- The coefficient of real expansion of a liquid is equivalent to the combined sum of the coefficient of apparent expansion of the liquid and the coefficient of volume expansion of the vessel.

  $\gamma_R = \gamma_a + \gamma_g$

ΔV	on heating Level of liquid
$\gamma_R > \gamma_g => \gamma_A > 0$	Rises
$\gamma_R < \gamma_g => \gamma_A < 0$	Falls
$\gamma_R = \gamma_g => \gamma_A = 0$	Remains same
$\gamma_g = 0 => \gamma_A = \gamma_R$	Rises
$\gamma_R = -ve => \gamma_A = \gamma_R$	Rises

Variation of Density of Liquid With Temperature :

- For calculating the change in density, the coefficient of real expansion of the liquid is to be considered.

  $d_0 = d_t(1 + \gamma_R t)$ or $d_t = d_0(1 - \gamma_R t)$

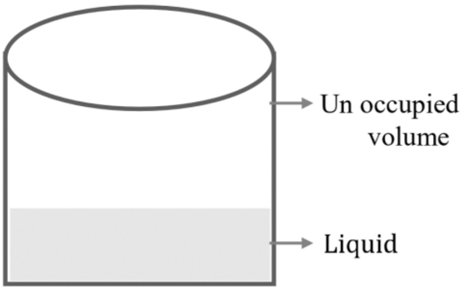
  where, d_0 = density of liquid at $0°C$

  d_t = density of liquid at $t°C$

  γ_R = Coefficient of real expansion of liquid

Condition for constant volume of unoccuped space in the container at all temperatures :

- The unoccupied volume or volume of air present in the vessel will be constant only when both container liquid have same thermal expansion. (or) same increase in volume

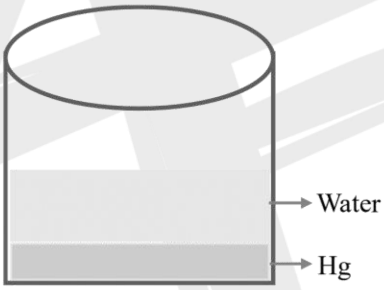


To compensate expansion of container using mercury (Diatometer)

A small amount of Hg is taken in a container such that its expansion is equal to expansion of container made of glass

Let V_c and V_ℓ are volumes of container and liquid.

Also γ_c and γ_ℓ are coefficient of container and liquid respectively.



$$\Delta V_c = \Delta V_\ell \,;\; V_c\gamma_c\Delta t = V_\ell\gamma_\ell\Delta t$$

$$V_c\gamma_c = V_\ell\gamma_\ell \,;\; \frac{V_{Hg}}{V_c} = \frac{\gamma_g}{\gamma_{Hg}} = \frac{1}{7}$$

Volume of Liquid Expelled :

- A container of volume V_c at temperature $t_1{}^\circ C$ is completely filled with a liquid. If the container is heated to $t_2{}^\circ C$, then volume of liquid over flown is V_1 = initial volume of the liquid = initial volume of the container

$$V_{2(\text{liquid})} = V_1\left[1+\gamma_\ell\left(t_2-t_1\right)\right]$$

$$V_{2(\text{container})} = V_1\left[1+\gamma_c\left(t_2-t_1\right)\right]$$

Volume of liquid over flow is

$$\left(V_2\right)_\ell - \left(V_2\right)_c = V_1\left(\gamma_\ell-\gamma_c\right)\left(t_2-t_1\right)$$

Apparent Weight of a Solid Immersed in a Liquid :

- The apparent weight of the body, $W = V\left(d_{body} - d_{liq}\right)g$; (V = volume of body).
- If d_1 and d_2 are densities of a liquid at $t_1{}^\circ C$ and $t_2{}^\circ C$ and volume of a solid body at $t_1{}^\circ C$ and $t_2{}^\circ C$ are V_1 and V_2 respectively then

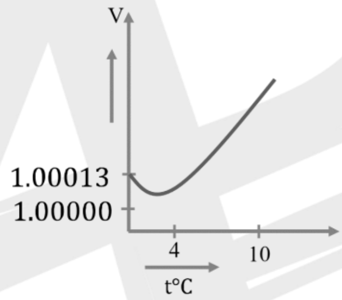
  Upthrust at $t_1{}^\circ C$ is $F_1 = V_1 d_1 g$

  Upthrust at $t_2{}^\circ C$ is $F_2 = V_2 d_2 g$

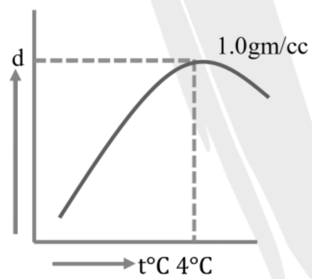
  $$\frac{F_1}{F_2} = \left(\frac{V_1}{V_2}\right)\left(\frac{d_1}{d_2}\right); \frac{F_1}{F_2} = \frac{1+\gamma_R\left(t_2 - t_1\right)}{1+\gamma_s\left(t_2 - t_1\right)} \Rightarrow \frac{F_1}{F_2} = 1 + \left(\gamma_R - \gamma_s\right)\left(t_2 - t_1\right)$$

Anomalous Expansion of Water :

- At $4^\circ C$ water occupies minimum volume and hence density becomes maximum (1gm/cc.)



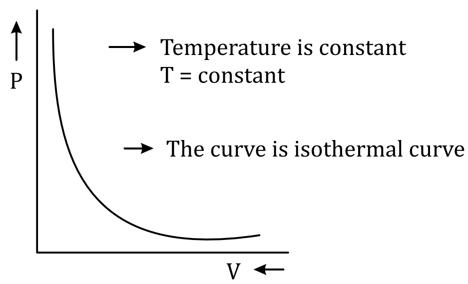
- The density of water increases from $0^\circ C$ to $4^\circ C$ and decreases with rise in temperature from $4^\circ C$



Boyle's Law:

- Statement: At constant temperature, the volume of given mass of a gas is inversely proportional to its pressure.

  For a given mass of a gas and at a given temperature. $P_1V_1 = P_2V_2$



Charles Law:

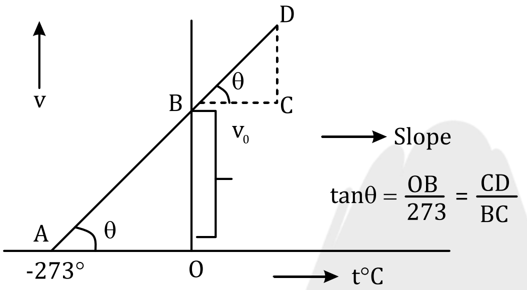
Charles constant pressure law

- The volume of a given mass of gas at constant pressure is directly proportional to the absolute temperature. $V \propto T$ (at constant pressure)

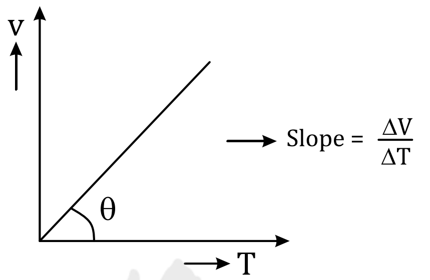
$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

Graphs between volume and temperature of gas in different cases:

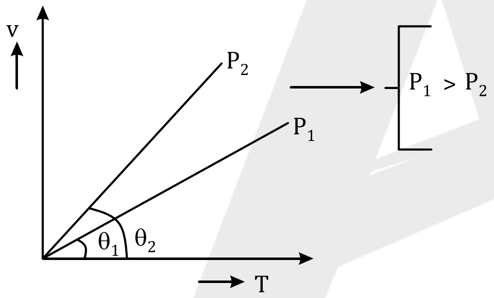
(a)



(b)



(c)



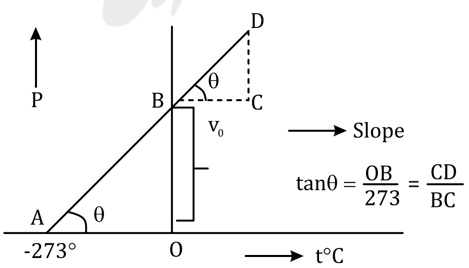
Charles law of constant volume:

- The pressure of a given mass of a gasses is directly proportional to the absolute temperature at constant volume.

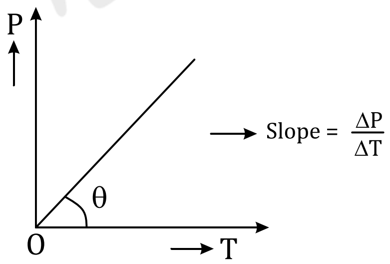
$$\Rightarrow \frac{P_1}{T_1} = \frac{P_2}{T_2}$$

Graphs between pressure and temperature of a gas in different cases:

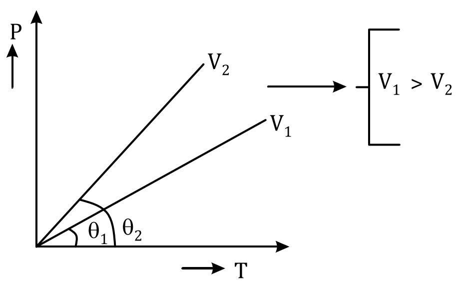
(a)



(b)



(c)



Ideal Gas

A gas which obeys Boyle's law and Charle's laws strictly at all temperatures and pressures is called a perfect or an ideal gas.

Real gas

The gas which obeys Boyle's law at low pressure and high temperature is called a real gas.

Ideal gas (or Perfect gas) Equation:

An ideal gas of mass 'm' having pressure 'P' volume 'V', at temperature 'T'

$$\therefore \frac{PV}{T} = \text{constant}$$

Ideal gas equation in terms of Boltzmann's constant:

- Boltzmann's constant (K) is defined as universal gas constant per molecule.

  i.e., $K = \frac{R}{N_A}$ where N_A is Avogadro's number of molecules.

  We know, $PV = nRT$ and

  $$n = \frac{\text{Totalnumberofmolecules}(N)}{\text{Avagadronumberofmolecules}(N_A)}$$

  $\therefore PV = \frac{N}{N_A}RT$ (or) $PV = NKT$

CALORIMETRY

Introduction:

- The quantity of heat required to warm a given substance depends on its mass (m), the change in temperature $(\Delta\theta)$ and nature of the substance.

  i.e., $\Delta Q = mS\Delta\theta$

Specific Heat

- The amount of heat required to rise the temperature of unit mass of a substance through $1°C$ is called specific heat of the material of the body.

$$S = \frac{1}{m}\frac{\Delta Q}{\Delta\theta}$$

Thermal capacity or Heat capacity

- It is the amount of heat required to rise the temperature of the body by $1°C$

$$H = \frac{\Delta Q}{\Delta\theta}$$

Water equivalent

- The mass of water that possesses an equivalent thermal capacity to a given substance is referred to as its "water equivalent."
- If m_w, m_s are masses of water and substance and S_w, S_s are their specific heats respectively then,

  $\Rightarrow m_w \times 1 = m_S S_S$,

  $\therefore m_w = m_S S_S$

Calorimetry

- Calorimetry refers to the process of measuring heat.
- A device designed for the purpose of measuring heat is known as a "calorimeter."

Law of method of mixtures (or) Principle of Calorimetry

- When three substances of different masses m_1, m_2 and m_3, specific heats S_1, S_2, S_3 and at different temperatures θ_1, θ_2 and θ_3 respectively are mixed, then the resultant temperature is

  $$\theta = \frac{m_1S_1\theta_1 + m_2S_2\theta_2 + m_3S_3\theta_3}{m_1S_1 + m_2S_2 + m_3S_3}$$

  (when state of contents does not change)

Change of State

Melting:

- The transition of matter from a solid state to a liquid state is known as "melting" (and its reverse process is referred to as "solidification" or "fusion").

Vaparisation (Boiling):

- Vaporization is the transition of matter from a liquid to a vapor state at a specific temperature (and the reverse process is called condensation).

Evaporation:

- Evaporation is the process where molecules escape from the free surface of a liquid.

Effect of pressure on melting point and boiling point

- Effect of pressure on M.P and B.P can be explained with Clausius – Clapeyron relation which can be derived on the basis of thermodynamics

  i.e., $\dfrac{dP}{dT} = \dfrac{L}{T\left(V_f - V_i\right)}$

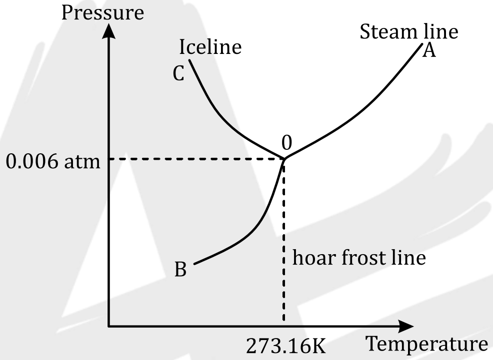
Regelation of ice:

- Regelation is the process of ice melting under applied pressure and then re-solidifying upon pressure removal. It is responsible for snowball preparation.

Triple point:

- The point at which the solid, liquid, and vapor states of a substance coexist simultaneously under specific temperature and pressure conditions is referred to as the "triple point."
- The triple point of water is $273.16K(0.01°C)$ and pressure 0.006 atm. $(0.459 cm of Hg)$.

Phase diagram of water



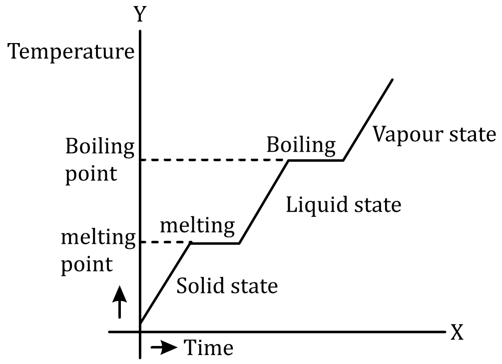
- Under normal pressure conditions, certain solids such as camphor, iodine, arsenic, etc., do not undergo melting when heated. Instead, they undergo a process known as "sublimation," where they transform directly from a solid state to a vapor state.

Latent heat:

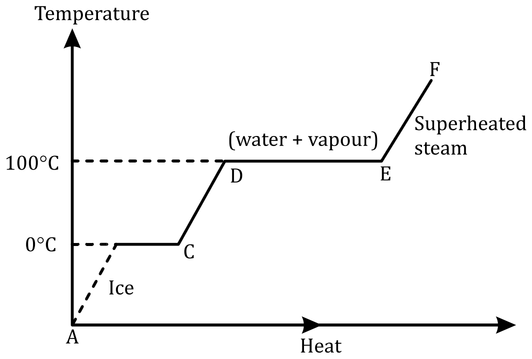
- The term "latent heat" refers to the amount of heat required to change one unit mass of a substance from one state to another state without any accompanying change in emperature.

$$L = \frac{Q}{m}(J/kg \text{ or } Cal/g)$$

$$\therefore Q = mL;$$



Total heat required $Q = Q_1 + Q_2 + Q_3 + Q_4$



Super cooling:

- Supercooling, also referred to as superfusion, is a phenomenon where most liquids, under carefully controlled conditions of being pure, in a pristine container, and with minimal disturbance, can be cooled to a temperature significantly below their regular freezing point without undergoing solidification.

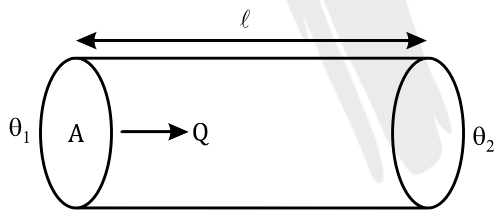
Saturated and Unsaturated Vapours:

A vapour is termed "saturated vapour" when the pressure it exerts is at its maximum, whereas it is known as "unsaturated vapour" when the pressure it exerts is not at its maximum.

TRANSMISSION OF HEAT

Conduction

- Conduction is the transmission of heat without the actual movement of the particles of the medium.
- Consider a good conductor in the shape of uniform rod of length ℓ, whose opposite paralle faces are maintained at different steady temperatures θ_1 and θ_2 such that $\theta_1 > \theta_2$.



- Under steady state conditions the amount of heat Q flowing from hot face to the cold face of the block is $Q \propto \frac{A(\theta_1 - \theta_2)t}{\ell}$ or $Q = \frac{KA(\theta_1 - \theta_2)t}{\ell}$

Temperature gradient :

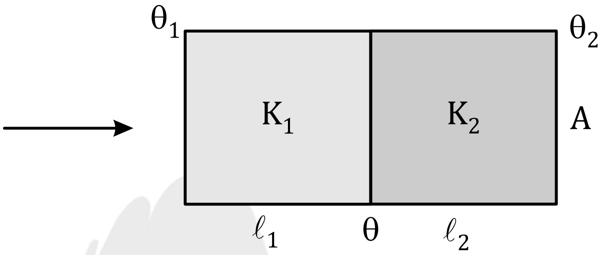
- If $d\theta$ is small change in temperature in the direction of heat flow, across small element of length dx then, temperature gradient $= -\frac{d\theta}{dx}$. Here the negative sign indicates the decreases in temperaure gradient is same along the length of the conductor.

Thermal resistance : (R)

- Thermal resistance R of a conductor of length ℓ, cross-section A and conductivity K is given by $R = \frac{\ell}{KA}$

Conduction of heat through a Composite slab

(a) When different rods of same cross sections are connected in series.



From equation (iii) and (iv) $R_{eff} = R_1 + R_2$

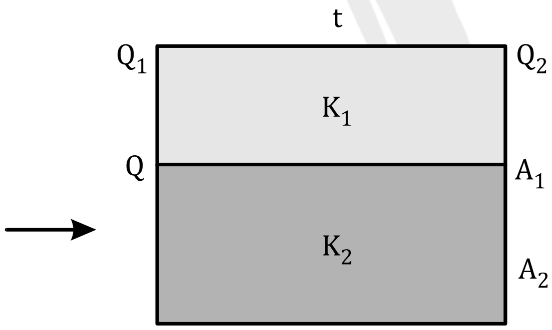
Hence, the effective thermal resistance for rods in series the sum of thermal resistances of each rod.

- **In series,** $R_{eff} = R_1 + R_2$

  If $\ell_1 = \ell_2$ then $K_{eff} = \frac{2K_1K_2}{K_1 + K_2}$

- If n rods of different materials and same area of cross sections are connected in series then, effective thermal conductivity is $\frac{\ell_1 + \ell_2 + ... + \ell_n}{K_{eff}} = \frac{\ell_1}{K_1} + \frac{\ell_2}{K_2} + ... + \frac{\ell_n}{K_n}$

(b) When different slabs of same thickness are connected in parallel



- The quantity of heat supplied is distributed between the two rods ; i.e

$$\frac{Q}{t} = \frac{Q_1}{t} + \frac{Q_2}{t} \ (\text{or}) \ P = P_1 + P_2$$

$$\therefore \ \frac{1}{R_{eff}} = \frac{1}{R_1} + \frac{1}{R_2} \ (\text{or}) \ R_{eff} = \frac{R_1R_2}{R_1 + R_2}$$

- **In parallel,** $\frac{1}{R_{eff}} = \frac{1}{R_1} + \frac{1}{R_2}$ $\therefore$ $K_{eff} = \frac{K_1A_1 + K_2A_2}{A_1 + A_2}$

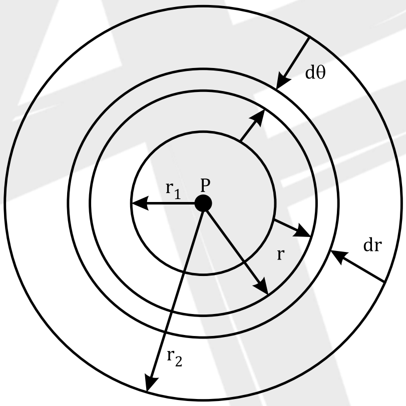
  If n rods of same length and different area of cross sections of different materials are connected in parallel then, effective thermal conductivity is

$$\frac{K_{eff}\left(A_1 + A_2 + \ldots.. + A_n\right)}{\ell} = \frac{K_1A_1}{\ell} + \frac{K_2A_2}{\ell} + \ldots.. + \frac{K_nA_n}{\ell}$$

$$K_{eff} = \frac{K_1A_1 + K_2A_2 + \ldots + K_nA_n}{A_1 + A_2 + \ldots. + A_n}$$

Radial flow of heat

- Consider two thin spherical shells of radii r_1 and r_2. A medium with thermal conductivity 'K' is enclosed between these shells, with a heater positioned at the center. Heat is conducted radially through the medium, flowing from the inner shell to the outer shell. Let the temperatures of the inner and the outer shells be θ_1 and θ_2 at steady state.



- The rate of flow of heat throught he element

$$H = KA\left(\frac{d\theta}{dr}\right) = K\pi r^2\left(\frac{d\theta}{dr}\right) \text{ (or) } \int_{r_1}^{r_2}\frac{dr}{r^2} = -\frac{4\pi K}{H}\int_{\theta_1}^{\theta_2} d\theta \Rightarrow H = \frac{4\pi K r_1 r_2\left(\theta_1 - \theta_2\right)}{\left(r_2 - r_1\right)}$$

Thickness fo the shell, $\left(r_2 - r_1\right) = \frac{4\pi K r_1 r_2\left(\theta_1 - \theta_2\right)}{H}$

Ingen bausz experiment

- In a steady state, when several identical rods made of different metals are coated with wax and one end of each rod is placed in boiling water, the square of the length over which the wax melts is directly proportional to the thermal conductivity of the metal. i.e.,

$$L^2 \propto K \Rightarrow \frac{L^2}{K} = \text{constant}$$

Convection

- Convection is the heat transfer process where heat moves from one place to another due to the actual movement of particles in the medium, caused by differences in density.

(a) Natural or Free convection:

- Natural convection occurs when fluid particles move solely due to temperature differences (density variations).

(b) Forced convection:

- Forced convection involves fluid particles being compelled to move by an external agent, such as a fan, blower, pump, etc.

Convection co-efficient

- In forced convection, the rate of heat flow is proportional to the surface area and the temperature difference between the surface and the fluid. $P = Q/t = hA\Delta\theta$.

  Where h represents the coefficient of convection, A is the surface area over which the fluid moves, and $\Delta\theta$ is the temperature difference between the surface and the fluid.

Radiation :

- Radiation is the process of heat transmission without any material medium.

Thermal radiation :

Thermal radiation is the transfer of heat energy through the means of electromagnetic waves.

Prevost's theory of heat exchange :

- All objects emit and absorb radiant energy at all temperatures, except at absolute zero.
- The amount of radiant energy emitted by a body is influenced by the nature of its surface, the surface area of the body, and the body's temperature.

Emissive power (e_λ) :

- The amount of energy emitted per second per unit surface area of a body at a given temperature for a given wavelength range $(\lambda \text{ and } \lambda + d\lambda)$ is called emissive power.

Emissivity (e) :

- Emissivity is defined as the ratio of the radiant energy emitted by a surface to the radiant energy emitted by a black body under the same conditions.
- For a perfect black body emissivity e = 1 and e = 0 (for perfect reflector); e = 0.97 (for human skin)

Absorptive power $\left(a_\lambda\right)$:

At a specific temperature, within a given wavelength range, the absorptive power of a body is determined as the ratio of the energy absorbed to the energy incident on the body.

$$\therefore \quad a_\lambda = \frac{\text{Amount of radiant energy absorbed}}{\text{Amount of radiant energy incident}}$$

- $\alpha_\lambda = 1$. (for black body)

Reflection power (r) : $r = \dfrac{\text{Amount of radiant energy reflected}}{\text{Amount of radiant energy incident}}$

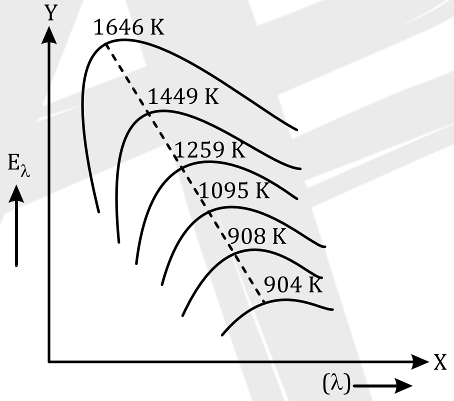
Transmitting power (t) : $r = \dfrac{\text{Amount of radiant energy reflected}}{\text{Amount of radiant energy incident}}$

$\alpha_\lambda + r + t = 1$, $'\alpha_\lambda'$ is absorptive power, 'r' is reflecting power & 't' is the transmitting power.

Black body radiation :

- A perfect black body is a body that completely absorbs all the incident heat radiation.

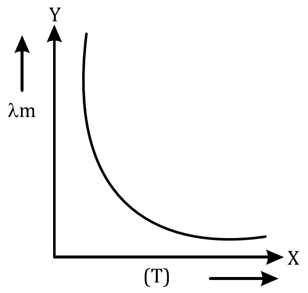
  Distribution of energy in black body spectrum

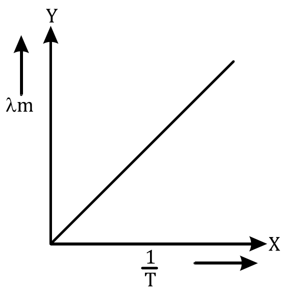


Wien's displacement law :

- Wavelength corresponding to the maximum intensity $\left(\lambda_m\right)$ shift towards left (or smaller wavelength side) along the axis (i.e., decreases) as the temperature of the body is increased. So the wavelength $\left(\lambda_m\right)$ varies inversely as the absolute temperature of the body.

  $\lambda_m \propto \dfrac{1}{T}$ or $\lambda_m T = b$ where 'b' is known as Wien's constant $b = 2.9 \times 10^{-3}$ m-K





$$\frac{\lambda_{m_1}}{\lambda_{m_2}} = \frac{T_2}{T_1} = \frac{\upsilon_{m_2}}{\upsilon_{m_1}}$$

Stefan's law :

The amount of heat radiated by a black body per second per unit area is directly proportional to the fourth power of its absolute temperature.

$$E \propto T^4 \Rightarrow E = \sigma T^4$$

(σ = Stefan's constant)

$$\sigma = 5.67 \times 10^{-8}\ Wm^{-2}K^{-4}$$

Stefan - Boltzmann's law :

- If a black body at absolute temperature T is surrounded by an enclosure at absolute temperature T_0, then the rate of loss of heat energy per unit area by radiation is given by

  $$E = \sigma\left(T^4 - T_0^4\right)$$

Kirchhoff's law :

At a specific temperature and within a given wavelength range, the ratio of emissive power to absorptive power of a body remains constant, and this constant value is equivalent to the emissive power of a perfect black body at the same temperature and wavelength.

i.e., $\frac{e_\lambda}{a_\lambda}$ = constant = E_λ

Green house effect

- The warming of earth atmosphere and surface due to greenhouse gases $(CO_2, N_2O, C.F.C, O_3)$ is called green house effect.

Newton's law of cooling

- Rate of loss of heat $\propto (\theta - \theta_0)$

  Rate of loss of heat of a hot body due to cooling

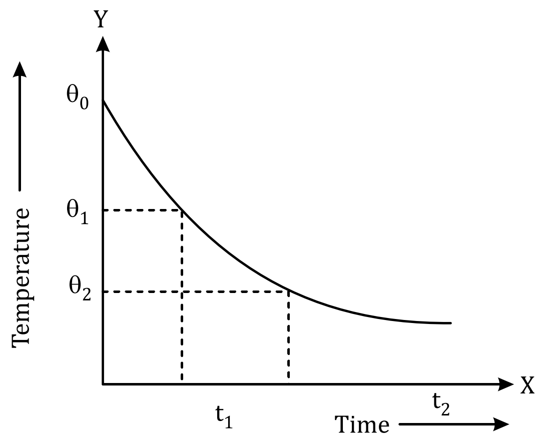
  $$\frac{dQ}{dt} = ms\frac{d\theta}{dt}$$

  Here 'm', 's' are mass and specific heat of the body.

- Cooling curve

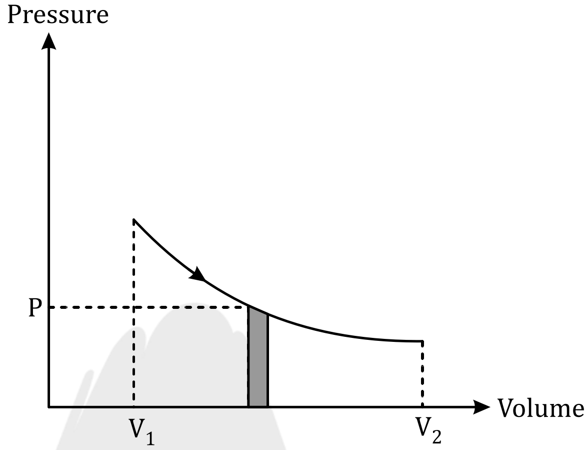
  $\theta_2 = \theta_0 + (\theta_1 - \theta_0)e^{-Kt+C}$, θ_0

  = temperature of the surroundings



THERMODYNAMICS

- Thermodynamics:Thermodynamics, as a scientific discipline, focuses on the transfer of heat energy between different bodies and the conversion of this heat energy into mechanical energy, and vice versa.



If the volume changes from V_1 to V_2,

then the total external work done by the system is

$$W = \int_{V_1}^{V_2} PdV$$ = Area under P-V curve.

Internal energy:

- The internal energy of a system refers to the energy possessed by the system resulting from molecular motion and molecular configuration. The energy associated with molecular motion is known as internal kinetic energy U_k, while the energy linked to molecular configuration is called internal potential energy U_p, i.e,

$$U = U_k + U_p$$

First Law of Thermodynamics:

- When heat is added to a system, a portion of it is utilized to perform external work, while the remainder is used to increase the system's internal energy.
- The differential form of first law of thermodynamics is

$$dQ = dU + dW,$$

where dQ = heat added,

For bulk changes

$$\Delta Q = \Delta U + \int PdV$$

Molar specific heats $\left(C_P, C_V\right)$ of a gas

- When the above specific heats c_p, c_v are defined per 1 mole of gas, then they are said to be molar specific heats and represented by C_P, C_V

  These are, $C_P = \frac{1}{n}\left(\frac{dQ}{dT}\right)_P \Rightarrow C_V = \frac{1}{n}\left(\frac{dQ}{dT}\right)_v = \frac{1}{n}\left(\frac{dU}{dT}\right), \left(\because \Delta W = 0\right)$

  SI unit of both molar specific heats is $J/mol-K$

- C_P is greater than C_V and, $\frac{C_P}{C_V} = \gamma$

  (C_P, C_V are molar specific heats)

  $C_P - C_V = R$, where R is universal gas constant

- $c_p - c_v = r = \frac{R}{M}$ (but $C = Mc$)

  $\Rightarrow M\left(c_p - c_v\right) = R \Rightarrow J/Kg-K$

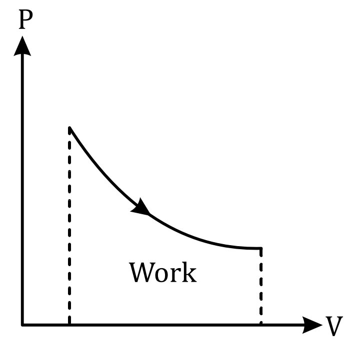
- For a gas having 'f' degrees of freedom,

  $C_v = \left(\frac{f}{2}\right)R, C_p = \left(1 + \frac{f}{2}\right)R$

  $\gamma = 1 + \frac{2}{f}$ $\left(\therefore \gamma = \frac{C_p}{C_v}\right)$

Thermodynamic processes:

- **Quasi-static process:** A quasi-static process refers to an extremely slow process where the system maintains thermal and mechanical equilibrium with its surrounding at every intermediate stage.
- **Isothermal process:** An isothermal process ivolves changes in pressure and volume of a gas while keeping the temperature constant. **Indicator diagram**



- Slope of isothermal curve, $\tan\theta = \frac{dP}{dV} = -\frac{P}{V}$
- Isothermal bulk modulus $-\frac{dP}{dV/V} = P$
- The workdone during the isothermal change at temperature T for n moles of gas is

$$W = 2.303nRT\log_{10}\left(\frac{V_2}{V_1}\right) = 2.303nRT\log_{10}\left(\frac{P_1}{P_2}\right)$$

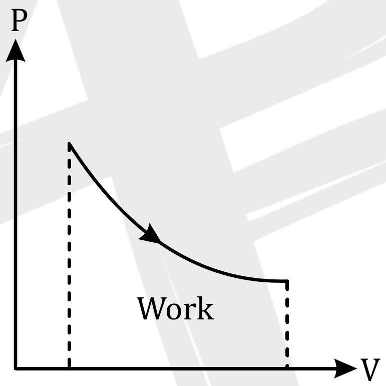
- In the adiabatic process, P, V and T are related as

(i) PV^{γ} = constant

(ii) $TV^{\gamma-1}$ = constant

(iii) $P^{1-\gamma}T^{\gamma}$ = constant

- Indicator diagram is

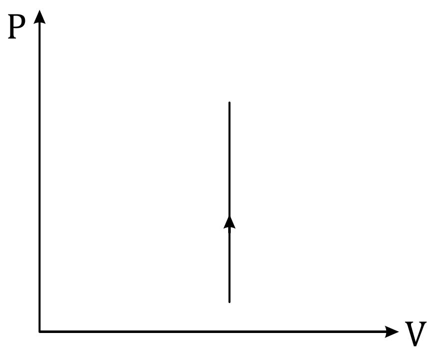


- The workdone by the system during the adiabatic expansion is

$$W = \frac{nR}{\gamma-1}(T_1 - T_2) = nC_v(T_1 - T_2) = -nC_v\Delta T$$

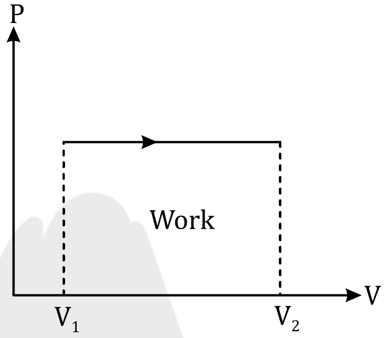
$$= n\frac{C_p}{\gamma}(T_1 - T_2) = \frac{P_1V_1 - P_2V_2}{\gamma - 1}$$

Isochoric process (or) isometric process



- Slope of isometric curve, $\frac{dP}{dV} = \infty$
- Specific heat is $C_V = \frac{f}{2}R$
- Bulk modulus of elasticity $K = \infty$

Isobaric process:



- Slope of isobaric curve, $\frac{dP}{dV} = 0$
- Specific heat is $C_P = \left(\frac{f}{2} + 1\right)R$
- Bulk modulus of elasticity K = 0

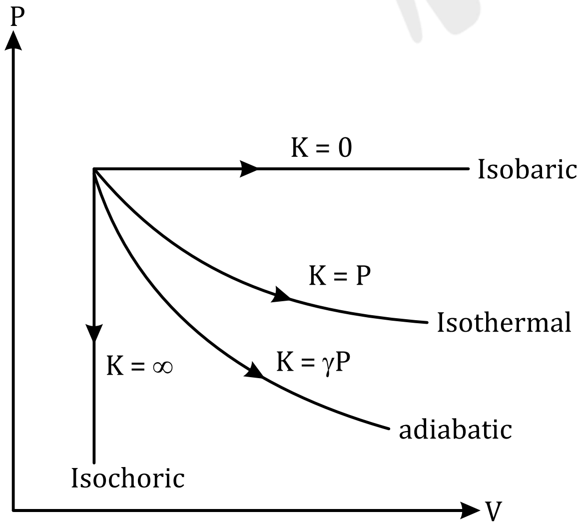
Cyclic process:

- A process is considered cyclic if a system, after undergoing a series of changes, returns to its initial state.

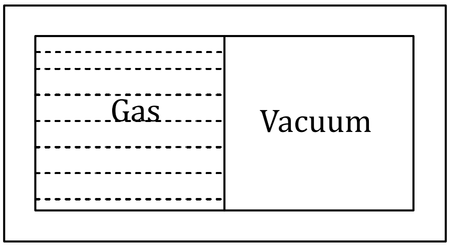
Polytropic process:

- In this process the gas obeys an additional law in the form of PV^x = constant,

<u>Comparison of P-V curves of various processes:</u>



Free expansion:



During this expansion process, no heat is exchanged with the surroundings as the walls are thermally insulated.

$\therefore \Delta Q = 0$

Second law of thermodynamics:

- Calusius' statement asserts that a self-acting machine, without assistance from any external source, cannot transfer heat from a cold reservoir to a hot reservoir. In simpler terms, heat cannot spontaneously flow on its own from a colder body to a hotter one.
- The Kelvin-Planck Statement states that no heat engine can achieve the complete conversion of all the heat it absorbs from a reservoir into useful work. In other words, it is impossible to achieve 100% efficiency in converting heat into work within a heat engine.

Reversible process:

- A reversible process is one that can be retraced in such a manner that the system passes through the same states as in the direct process. Eventually, the system returns to its initial conditions, leaving no net changes elsewhere.

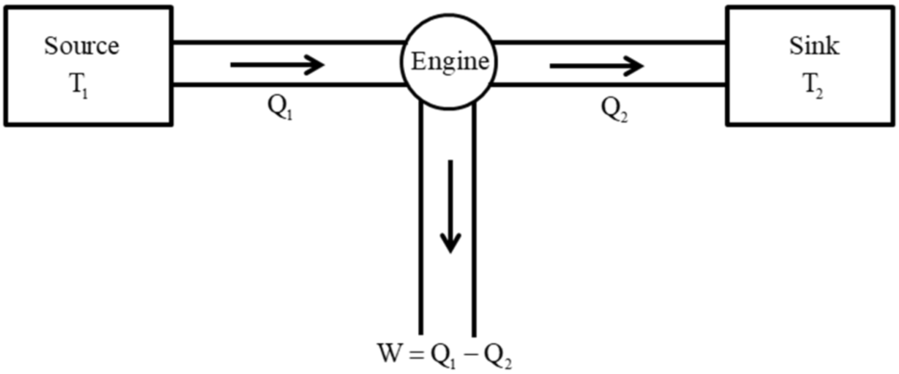
Irreversible process:

- In this process, the system does not follow the same intermediate states as observed in the direct process.

Heat engine:

- The device responsible for converting heat energy into mechanical energy is commonly referred to as a heat engine.

  Schematic diagram of heat engine



Efficiency of heat engine:

- The efficiency of a heat engine (η) is defined as the ratio of the work produced by the engine to the total heat supplied to it.

  Mathematically,

$$\eta = \frac{W}{Q_1} = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1}$$

Carnot Cycle:

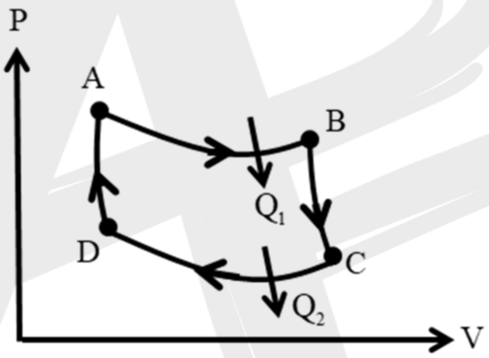
Carnot cycle consists of the following four stages:

(i) Isothermal expansion (process AB)

(ii) Adiabatic expansion (process BC)

(iii) Isothermal compression (process CD) and

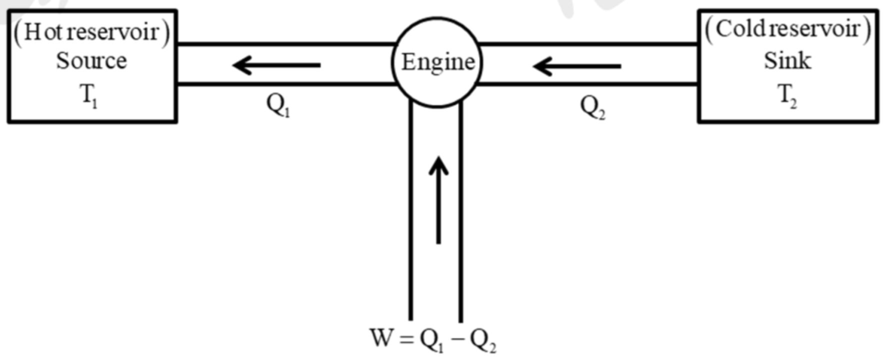
(iv) Adiabatic compression (process DA)



Efficiency of the Carnot engine is

$$\eta = 1 - \frac{Q_2}{Q_1} = 1 - \frac{T_2}{T_1}$$

Refrigerator:



Coefficient of performance of a refrigerator

$$\beta = \frac{Q_2}{W} = \frac{Q_2}{Q_1 - Q_2} \qquad [\because W = Q_1 - Q_2]$$

Entropy (s):

- Entropy is the thermodynamic quantity or parameter that quantifies the level of disorder in a system.

$$ds = \frac{dQ}{T}$$

Where dQ is exchange of heat between system and surroundings at temperature T.

Now the total change in entropy is $\Delta s = \int \frac{dQ}{T}$

KINETIC THEORY OF GASES

The perfect gas equation is given by $PV = \mu RT \to (4)$ where μ is number of moles and

$$R = N_A K_B = 8.314\,J\,mol^{-1}\,K^{-1}$$

$$PV = \frac{N}{N_A} \cdot N_A K_B T = NK_B T$$

Relation between pressure and kinetic energy :

From the pressure equation $P = \frac{1}{3}\rho V_{rms}^2$ ...(1)

The kinetic energy per unit volume is $E = \frac{1}{2}\left(\frac{M}{V}\right)v_{rms}^2 = \frac{1}{2}\rho v_{rms}^2$ ...(2)

From (1) and (2) $P = \frac{2}{3}E$

KINETIC INTERPRETATION OF TEMPERATURE

We know that pressure of an ideal gas is $P = \frac{1}{3}\frac{mN}{V}v_{rms}^2 ; P = \frac{1}{3}nmV_{rms}^2$

$\frac{2}{3}E = N_A K_B T \ (\text{or}) \ E = (3/2)K_B NT$ ...(3)

Root mean square speed : It is defined as the square root of mean of squares of the speed of different molecules.

i.e., $v_{rms} = \sqrt{\frac{v_1^2 + v_2^2 + v_3^2 + v_4^2 + \ldots}{N}} = \sqrt{\left(\bar{v}\right)^2}$

From the expression of pressure $P = \frac{1}{3}\rho v_{rms}^2$

$$\Rightarrow \quad v_{rms} = \sqrt{\frac{3P}{\rho}} = \sqrt{\frac{3PV}{\text{Mass of gas}}} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3kT}{m}}$$

[where $\rho = \frac{\text{Mass of gas}}{V}$ = Density of the gas]

Most probable speed:

$$v_{mp} = \sqrt{\frac{2P}{\rho}} = \sqrt{\frac{2RT}{M}} = \sqrt{\frac{2kT}{m}}$$

$$v_{mp} = \sqrt{\frac{2}{3}} v_{rms} = 0.816\, v_{rms}$$

Average speed : It is the arithmetic mean of the speeds of molecules in a gas at given temperature.

$$v_{av} = \frac{v_1 + v_2 + v_3 + v_4 + ...}{N}$$ and according to kinetic theory of gases

$$v_{av} = \sqrt{\frac{8P}{\pi\rho}} = \sqrt{\frac{8}{\pi}\frac{RT}{M}} = \sqrt{\frac{8}{\pi}\frac{kT}{m}}$$

Note : $v_{rms} > v_{av} > v_{mp}$

SPECIFIC HEAT CAPACITY (IN TERMS OF DEGREE OF FREEDOM)

$$C_p = C_v + R = \frac{f}{2}R + R = \left(\frac{f}{2} + 1\right)R$$

Ratio of C_p and C_v (γ) :

$$\gamma = \frac{C_p}{C_v} = \frac{\left(\frac{f}{2}+1\right)R}{\frac{f}{2}R} = 1 + \frac{2}{f}$$

MEAN FREE PATH

The distance travelled by a gas molecule between two successive collisions is known as free path.

$$\lambda = \frac{\text{Total distance travelled a gas molecule between successive collisions}}{\text{Total number of collision}}$$

$$\therefore \quad \lambda = \frac{1}{\sqrt{2}n\pi d^2}$$

$$\lambda = \frac{1}{\sqrt{2}\pi n d^2} = \frac{m}{\sqrt{2}\pi(mn)d^2} = \frac{m}{\sqrt{2}\pi d^2 \rho}$$

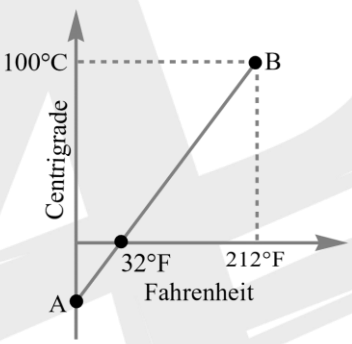
EXCERCISE-I

1. 20 gm of ice at $0°C$ is mixed with 'm' gm of water at $50°C$. What is minimum value of m so that ice melts completely? $\left(L_f = 80 \text{ cal/gm and } s_w = 1 \text{ cal/gm}-°C\right)$

(A) 32 gm (B) 20 gm (C) 40 gm (D) 16 gm

2. Three liquids A, B and C are at temperatures of $60°C$, $55°C$ and $50°C$ and 2 g of A mixed with 3 g of B gives $57°C$. The temperature of the mixture when equal masses of B and C are mixed is $\frac{521}{n}°C$. Find n ?

3. The graph AB shown in figure is a plot of temperature of a body in degree Celsius and degree Fahrenheit. Then :



(A) slope of line AB is $\frac{9}{5}$ (B) slope of line AB is $\frac{5}{9}$

(C) slope of line AB is $\frac{1}{9}$ (D) slope of line AB is $\frac{3}{9}$

4. We have half a bucket (6 litre) of water at $30°C$. If we want water at $50°C$, how much steam at $100°C$ should be added to it?

(A) 200 g (B) $\frac{2000}{9}$g (C) 2 kg (D) $\frac{200}{3}$g

5. An ice block at $0°C$ is dropped from height 'h' above the ground. What should be the value of 'h' so that it melts completely by the time it reaches the bottom assuming the loss of whole gravitational potential energy is used as heat by the ice?

[Given: $L_f = 80 \text{ cal/gm}$]

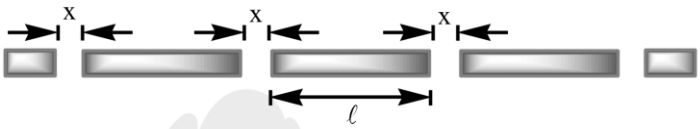
(A) 33.6 m (B) 33.6 km (C) 8 m (D) 8 km

6. A 5 litre glass vessel is filled to its brim with water. The coefficient of cubical expansion of water is $1.3\times10^{-4}/°C$ and coefficient of volume expansion of glass is $80\times10^{-6}/°C$. The temperature is increased by $20°C$. The amount of water over how (in cc) is

7. The density of steel at $0°C$ is 8.0 g/cc. At what temperature is density lesser by 1%? Coefficient of linear thermal linear expansion of steel is 10^{-5} / °C.

(A) 337.2°C (B) 333.3°C (C) 554.3°C (D) 409.3°C

8. The gap between any two rails, each of length ℓ laid on a railway track equals x at $27°C$. When the temperature rises to $40°C$, the gap closes up. The coefficient of linear expansion of the material of the rail is α. The length ℓ of a rail at $27°C$ is $\frac{Px}{Q\alpha}$. Find P + Q?

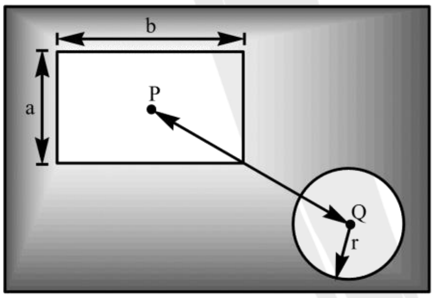


9. If a steel band were to fit snugly around the Earth's equator at $30°C$, but then was heated to $50°C$, how high above the Earth would the band be (assume equal everywhere)?

(Take: α for steel to be 12×10^{-6} / °C)

(A) 1.028 km (B) 1.536 km (C) 2.062 km (D) None of these

10. There is a rectangular metal plate in which two cavities in the shape of rectangle and circle are made, as shown with dimensions. P and Q are centres of these cavities. On heating the plate, which of the following quantities increase?



(A) Area (πr^2) of circular cavity

(B) Area (ab) of rectangular cavity

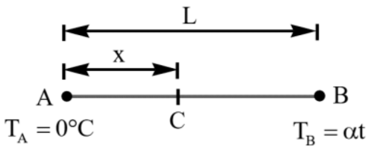
(C) Distance (R) between centre of both cavities

(D) Width (b) of rectangular cavity

11. Earth receives $1400\ Wm^{-2}$ of solar power. If all the solar energy falling on a lens of area $0.2\ m^2$ is focused on to a block of ice of mass 280 g, the time taken to melt the ice will be : (Take latent heat of fusion of ice $= 3.3\times10^5\ Jkg^{-1}$)

(A) 303 s (B) 330 s (C) 33 s (D) 5.5 s

12. The temperature of end A of a rod is maintained at $0°C$. The temperature of end B is changing slowly such that the rod may be considered in steady state at all time and is given by $T_B = \alpha t$; where α is positive constant and t is time. Temperature of point C, at a distance x from end A, at any time is:

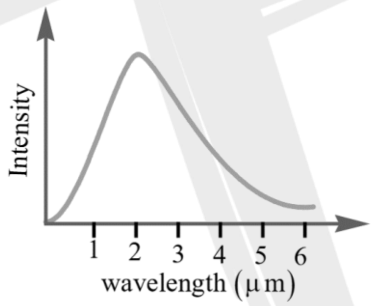


(A) $\frac{\alpha x t}{L}$ (B) $\frac{\alpha x t^2}{2L}$ (C) $\frac{\alpha x^2 t}{2L}$ (D) $\frac{\alpha(L-x)}{L}t$

13. A hot liquid is kept in a big room. The logarithm of the numerical value of the tempeature difference between the liquid and the room is plotted against time. The plot will be very nearly.

(A) a straight line (B) a circular arc (C) a parabola (D) exponential decay

14. The distribution of relative intensity $I(\lambda)$ of black body radiation from a solid body versus the wavelength λ is shown in the figure. If the Wien displacement law constant is 2.9×10^{-3} mK, what is the approximate temperature (in K) of the object?



15. The solar constant of earth is $2\,cal/min-cm^2$. If distance of mercury planet from sun is 0.4 times the distance of earth from the sun, the solar constant of mercury planet is $\frac{125}{n}$ cal / min -cm^2. Find n

16. Two spheres A and B are identical except that sphere A is white whereas sphere B is black. After they have been in thermal contact for long time with each other and their surroundings, in the visible range:

(A) A radiates less than B

(B) Both emit the same amount of radiation

(C) A radiates more than B

(D) A radiates more than B only if its temperature is high enough

17. A black body radiates maximum energy around the wavelength λ_0. If the temperature of the black body is changed so that it radiates maximum energy around the wavelength $2\lambda_0$, the ratio of final to initial power radiated by it will be:

(A) $\frac{1}{2}$ (B) 2 (C) $\frac{1}{16}$ (D) 16

18. Select the correct statement(s):

(A) The empty space in a beaker containing a liquid, partly filling it, on heating both beaker and liquid through the same temperature rise, does not change in volume, only if coefficients of cubical expansion of the two are different.

(B) Two copper spheres (one solid and the other hollow) have same initial radius and temperature. They are heated to same higher temperature. At the end of the heating, the radius of the hollow one is greater than that of the solid.

(C) Two spheres of the same material have radii 1m and 4m and temperatures 4000K and 2000K respectively. The energy radiated per second by the second sphere is greater than that by the first.

(D) When you touch two bodies, the body felt warmer on the touch must be of higher temperature than the other.

19. A solid sphere of copper of radius R and a hollow sphere of the same material of inner radius r and outer radius R are heated to the same temperature and allowed to cool in the same environment. Hollow sphere:

(A) will cool faster than solid sphere

(B) will cool slower than solid sphere

(C) will cool at the same rate as the solid sphere

(D) data is not sufficient to decide

20. There are two thin spheres A and B of the same material and same thickness. They emit like black bodies. Radius of A is double that of B. A and B have same temperature T. When A and B are kept in a room of temperature $T_0(<T)$, the ratio of their rates of cooling (rate of fall of temperature) is: [assume negligible heat exchange between A and B]

(A) 2:1

(B) 1:1

(C) 4:1

(D) 8:1

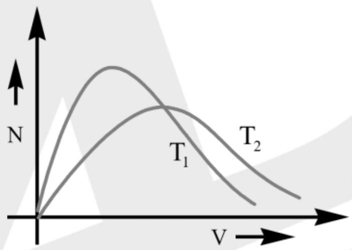
21. A black body emits radiation at the rate P when its temperature is T. At this temperature the wavelength at which the radiation has maximum intensity is λ_0. If at another temperature T' the power radiated is P' and wavelength at maximum intensity is $\frac{\lambda_0}{2}$ then:

(A) $T'=2T$ (B) $T'=\frac{T}{2}$ (C) $P'=16P$ (D) $P'=\frac{P}{16}$

22. Four molecules have speed u, 3u, 9u, and 15u. The ratio of their rms speed to average speed will be

(A) $\sqrt{\frac{66}{56}}$ (B) $\sqrt{\frac{342}{225}}$ (C) $\sqrt{\frac{3\pi}{8}}$ (D) $\sqrt{\frac{316}{196}}$

23. Maxwell's velocity distribution curve is given for two different temperatures. For the given curves,



(A) $T_1 > T_2$ (B) $T_1 < T_2$ (C) $T_1 \leq T_2$ (D) $T_1 = T_2$

24. At ordinary temperatures, the molecules of a diatomic gas have only translational and rotational kinetic energies. At high temperatures, they may also have vibrational energy. As a result of this compared to lower temperatures, a diatomic gas at higher temperatures will have

(A) Lower molar heat capacity

(B) Higher molar heat capacity

(C) Lower isothermal compressibility

(D) Higher isothermal compressibility

25. The rms speed of helium gas at $27°C$ and 1 atm pressure is $1800\,ms^{-1}$. Then the rms speed of helium molecules at temperature $27°C$ and 2 atm pressure is

(A) 450m/s (B) 900m/s (C) 1800m/s (D) 750m/s

26. The respective speeds of five molecules are 2, 1.5, 1.6, 1.6 and 1.2 km/s. The most probable speed in km/s will be

(A) 2 (B) 2.56 (C) 1.6 (D) 1.91

27. At a pressure of 24×10^5 dyne/cm^2, the volume of O_2 is 10 litre and mass is 20g. The rms velocity (in m/s) will be

28. In an H_2 gas process, PV^2= constant. The ratio of work done by gas to change in its internal energy is $-4/n$. Find n?

29. Which of the following statement(s) is/are true for a fixed amount of ideal gas?

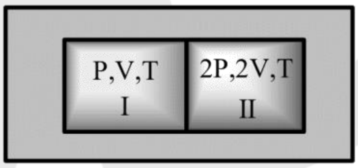
(A) If we double pressure, T may be double

(B) If we double volume, T will be double

(C) If we double volume, T may be halved

(D) If the double pressure, T may be doubled

30. A partition divides a container having insulated walls into two compartments I and II. The same gas fills the two compartments whose initial parameters are given. The partition is a conducting wall which can move freely without friction. Which of the following statements is/are correct, with reference to the final equilibrium position?



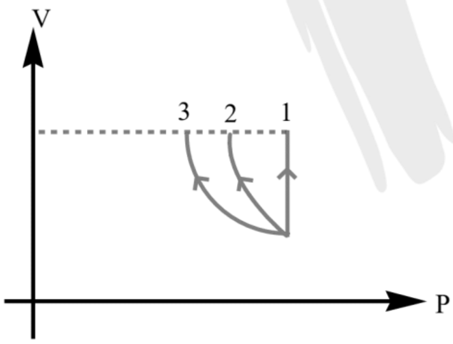
(A) The pressure in the two compartments are equal

(B) Volume of compartment I is $\frac{3V}{5}$

(C) Volume of compartment II is $\frac{8V}{5}$

(D) Final pressure in compartment I is $\frac{5P}{3}$

31. The graph aside shows V-P curve for three processes. Choose the correct statement(s).



(A) Work done is maximum in process 1

(B) Temperature must be increase in process 2 and 3

(C) Heat must be supplied in process 1

(D) If final volume of gas in process 1, 2 and 3 are same the temperature must be same

EXCERCISE-II

1. A body of mass 25 kg is dragged on a rough horizontal floor for one hour with a speed of 2 km/h. The coefficient of friction between the body and the surface in contact is 0.5 and half the heat produced is absorbed by the body. If specific heat of body is $0.1 \text{calg}^{-1}(°\text{C})^{-1}$, then the rise in temperature of body is $\frac{119}{n}$k. Find n

2. A body of mass 25 kg dragged on a rough horizontal floor for one hour with a speed of $4\ \text{kmh}^{-1}$. The coefficient of friction for the surface in contact is 0.5 and half the heat produced is absorbed by the body. If specific heat of body is 0.1 cal $g^{-1}\left(°C^{-1}\right)$ and $g = 9.8\ \text{ms}^{-2}$, then the rise in temperature of body is :

(A) 39 K (B) 59.5 K (C) 84.5 K (D) 23.3 K

3. A block of ice of mass 20 gm is kept in a steel container of water equivalent 10 gm. The temperature of both – ice and the container is $-30°C$. Now 30 gm water at $80°C$ is poured in this container. The final common temperature $(\text{in } °C)$ is

$(S_{ice} = 0.5\text{cal/g°C}, S_{water} = 1\text{cal/g°C}, L_{ice} = 80\text{cal/g})$

4. Imagine a hypothetical material whose average temperature coefficient of linear expansion is $0.1/°C$. Find the fractional increase in the area of a thin square plate of above material when temperature increase by $20°C$.

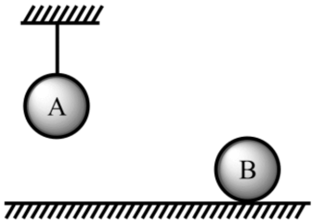
(A) 1 (B) 2 (C) 3 (D) 8

5. A thermometer with an arbitrary scale has the ice point at $-20°$ and the steam point at $180°$. When the thermometer reads $5°$, a centigrade thermometer will read $\frac{125}{n}$. Find 'n'

6. A solid sphere of radius r and coefficient of linear expansion α rotates about an axis passing through it's centre in absence of any external torque. What should be the change in temperature so that it's angular velocity increases by 1%?

(A) $\frac{1}{100\alpha}$ increase (B) $\frac{1}{200\alpha}$ increase (C) $\frac{1}{100\alpha}$ decrease (D) $\frac{1}{200\alpha}$ decrease

7. Consider two identical homogeneous ball, A and B, with same initial temperatures. One of them is at rest on fixed horizontal surface, while the second one hangs on a thread as shown. Now the same amount of heat has been supplied to both the balls. We assume that such quantities as specific heat and coefficient of thermal expansion are constant (i.e., do not depend on temperature) and positive. Also we neglect expansion in the length of string and take all kinds of heat losses from balls to be negligible. Considering loss in gravitational potential energy is converted into heat energy and vice versa, the final temperature of the balls, T_A and T_B will obey the relation:



(A) $T_A > T_B$ (B) $T_A < T_B$ (C) $T_A = T_B$ (D) cannot be determined

8. A uniform metallic spherical shell of inner radius R has a thickness t, such that $\frac{R}{t} = 1000$. The shell is kept in vaccum and also, there is vacuum inside the shell. If absolute temperature of metallic spherical shell is doubled, then the ratio of new radius to new thickness $\frac{R'}{t'}$ will be :

(A) 2000 (B) 500 (C) 1000 (D) $\sqrt{2000}$

9. A vessel is partly filled with liquid. When the vessel is cooled to a lower temperature, the space in the vessel, unoccupied by the liquid remains constant. Then the volume of the liquid (V_L), volume of the vessel (V_v), the coefficients of cubical expansion of the material of the vessel (γ_v) and of the liquid (γ_L) are related as:

(A) $\gamma_L > \gamma_v$ (B) $\gamma_L < \gamma_v$

(C) $\frac{\gamma_v}{\gamma_L} = \frac{V_v}{V_L}$ (D) $\frac{\gamma_v}{\gamma_L} = \frac{V_L}{V_v}$

10. The coefficient of thermal expansion of a rod is temperature depends and is given by the formula $\alpha = aT$, where a is a positive constant and T in °C. If the length of the rod is ℓ at temperature 0°C, then the temperature at which the length will be 2ℓ is :

(A) $\sqrt{\frac{\ln 2}{\alpha}}$ (B) $\sqrt{\frac{\ln 4}{\alpha}}$

(C) $\frac{1}{\alpha}$ (D) $\frac{2}{\alpha}$

11. 50 gm ice at −10°C is mixed with 20 gm steam at 100°C. When the mixture finally reaches its steady state inside a calorimeter of water equivalent 1.5 gm then : [Assume calorimeter was initially at 0°C, Take latent heat of vaporization of water = 540 cal/gm, Latent heat of fusion of water = 80 cal/gm, Specific heat capacity of water = 1cal/gm-°C, Specific hat capacity of ice = 0.5 cal/gm-°C]

(A) Mass of water remaining is : 67.4 gm

(B) Mass of water remaining is : 67.87 gm

(C) Mass of steam remaining is : 2.6 gm

(D) Mass of steam remaining is : 2.13 gm

12. A uniform cylinder of steel of mass M, radius R is placed on frictionless bearings and set to rotate about its axis with angular velocity ω_0. After the cylinder has reached the specified state of rotation, it is heated from temperature T_0 to $(T_0 + \Delta T)$ without any mechanical contact. If $\frac{\Delta I}{I}$ is the fractional change in moment of inertia of the cylinder and $\frac{\Delta\omega}{\omega_0}$ be the fractional change in the angular velocity of the cylinder and α be the coefficient of linear expansion, then:

(A) $\frac{\Delta I}{I} = \frac{2\Delta R}{R}$

(B) $\frac{\Delta I}{I} = \frac{2\Delta\omega}{\omega_0}$

(C) $\frac{\Delta\omega}{\omega_0} = -2\alpha\Delta T$

(D) $\frac{\Delta I}{I} = -\frac{2\Delta R}{R}$

13. Block A is a 50 g aluminium block originally at $90°C$. Block B is a 100 g aluminium block originally at $45°C$. The blocks are placed in two separate 1.0 liter containers of water that were originally at $20°C$. When the systems reach thermal equilibrium, which aluminium block will have the higher final temperature?

(A) Block A

(B) Block B

(C) The blocks will have the same final temperature

(D) The answer depends on the specific heat of water

14. Two long, thin, solid cylinders are identical in size, but they are made of different substances with two different thermal conductivities. The two cylinders are connected in series between a reservoir at temperature T_{hot} and a reservoir at temperature T_{cold}. The temperature at the boundary between the two cylinders is T_b. One can conclude that:

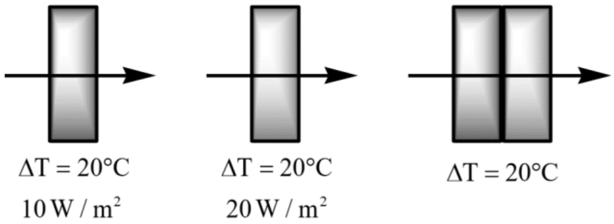
(A) T_b is closer to T_{hot} than it is to T_{cold}

(B) T_b is closer to T_{cold} than it is to T_{hot}

(C) T_b is closer to the temperature of the reservoir that is connected through the cylinder of lower thermal conductivity

(D) T_b is closer to the temperature of the reservoir that is connected through the cylinder of higher thermal conductivity

15. For a temperature difference $\Delta T = 20°C$, one slab of material conducts $10\,W/m^2$, another of same shape conducts $20\,W/m^2$. What is the rate of heat flow per m^2 of surface area when the slabs are placed side by side with same $\Delta T = 20°C$



(A) $10W/m^2$ (B) $30W/m^2$ (C) $\frac{20}{3}W/m^2$ (D) None of these

16. Two cylindrical rods of the same material have the same temperature difference between their ends. The ratio of the rates of flow of heat through them is 1 : 8. The radii of the rods are in the ratio 1 : 2. What is the ratio of their lengths?

(A) 2 : 1 (B) 4 : 1 (C) 1 : 8 (D) 1 : 32

17. When a body is placed in surroundings at a constant temperature of 20°C, and heated by a 10-W heater, its temperature remains constant at 40°C. If the temperature of the body is now raised from 20°Cto 80°Cin 5 minutes at a uniform rate, the total heat it will lose to the surroundings will be approximately:

(A) 3000 J (B) 3600 J (C) 4500 J (D) 5400 J

18. A hot black body emits the energy at the rate of $16Jm^{-2}s^{-1}$ and its most intense radiation corresponds to 20,000$\overset{o}{A}$. When the temperature of this body is further increased and its most intense radiation corresponds to 10,000$\overset{o}{A}$, then the energy radiated in $Jm^{-2}s^{-1}$ will be:

19. A black body in shape of a cube radiates 567 W. Its temperature is 127$°C$. What is length of its side?

(A) $\frac{5}{8}$m (B) $\frac{5}{2\sqrt{6}}$m (C) $\frac{5}{2\sqrt{3}}$m (D) $\frac{5}{8\sqrt{6}}$m

20. A body cools from 50°C to 40°C in 5 minutes. The surrounding temperature is 20°C. In what further time (in minutes) will it cool to $30°C$:

(A) 5 (B) $\frac{25}{3}$ (C) $\frac{15}{2}$ (D) 10

21. A hollow copper sphere and a hollow copper cube, of same surface area and negligible thickness, are filled with warm water of same temperature and placed in an enclosure of constant temperature, a few degrees below that of the bodies. Then in the beginning:

(A) the rate of energy lost by the sphere is greater than that by the cube

(B) the rate of fall of temperature for sphere is greater than that for the cube

(C) the rate of energy lost by the sphere is less than that by the cube

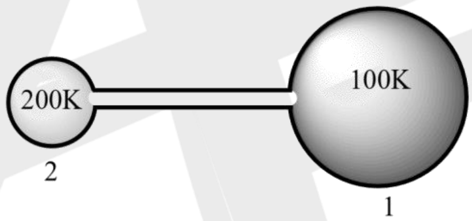
(D) the rate of fall of temperature for sphere is less than that for the cube

22. Two bodies A and B have emissivities 0.5 and 0.8 respectively. At some temperatures the two bodies have maximum spectral emissive powers at wavelength $8000\overset{o}{A}$ and $4000\overset{o}{A}$ respectively. The ratio of their emissive powers at these temperatures is:

(A) $\frac{5}{128}$ (B) 10 (C) $\frac{5}{16}$ (D) none of these

23. A gas mixture consists of 2 moles of oxygen and 4 moles of argon at temperature T. Neglecting all vibrational modes, the total internal energy of the system is αRT. Find α ?

24. Figure shows two flasks connected to each other. The volume of the flask 1 is twice that of flask 2. The system is filled with an ideal gas at temperature 200K and 400K respectively in the flasks. If the mass of the gas in 1 be m, then what is the mass of the gas in flask 2 in equilibrium?



(A) m (B) $\frac{m}{2}$ (C) $\frac{m}{4}$ (D) $\frac{m}{8}$

25. A container has an ideal gas at pressure P. Assuming the mass of a molecule is m and all the molecules are moving with same speed v in random directions. The expression for number of collisions per second which the molecules make with unit area of container wall is $\frac{\sqrt{\alpha}P}{\beta m v}$. Find $\alpha + \beta$

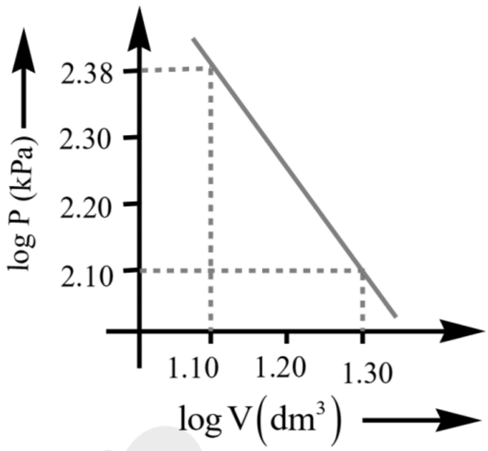
26. One mole of an ideal gas under goes a process whose molar heat capacity is 4R and in which work done by gas for small change in temperature is given by the relation $dW = 2RdT$, then the ratio $\frac{C_P}{C_V}$ is

(A) $\frac{7}{5}$ (B) $\frac{5}{3}$ (C) $\frac{3}{2}$ (D) $\frac{4}{3}$

27. When water is boiled at 2atm pressure, the latent heat of vaporization is $2.2\times10^6 J/kg$ and the boiling point is $120°C$. At 2atm pressure, 1kg of water has volume of $10^{-3}m^3$ and 1kg of steam has a volume of $0.824m^3$. The increase in internal energy of 1kg of water when it is converted into steam at 2atm pressure and $120°C$ is [1atm pressure $= 1.013 \times 10^5 N/m^2$]

(A) 2.033J (B) 2.033×10^6J (C) 0.167×10^6J (D) 2.267×10^6J

28. Logarithms of readings of pressure and volume for an ideal gas were plotted on a graph as shown in figure. By measuring the gradient it can be shown that the gas may be



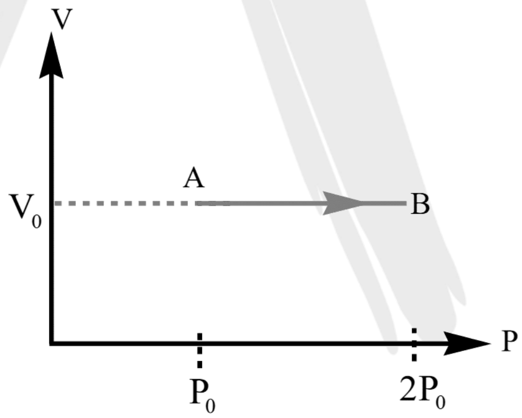
(A) Monoatomic and undergoing an adiabatic change

(B) Monoatomic and undergoing an isothermal change

(C) Diatomic and undergoing an adiabatic change

(D) Triatomic and undergoing an isothermal change

29. 2 moles of a monoatomic ideal gas, initially at P_0, V_0 and T_0, is taken through the process $A \rightarrow B$ as shown in the P-V diagram. The total heat supplied to the system is $\frac{\alpha}{\beta}P_0V_0$. Find $\alpha + \beta$?



30. From the following statements concerning ideal gas at any given temperature T, select the correct one(s)

(A) The coefficient of volume expansion at constant pressure is the same for all ideal gases

(B) The average translational kinetic energy per molecule of oxygen gas is 3kT, k being Boltzmann constant

(C) The mean free path of molecules increases with increases in the pressure

(D) In a gaseous mixture, the average translational kinetic energy of the molecules of each component is the same

31. At ordinary temperature the molecules of an ideal gas have only translational and rotational energy. At high temperature, they may also have vibrational energy. As a result of this at high temperature.

(A) $C_V = \frac{3R}{2}$ for monoatomic gas

(B) $C_V > \frac{3R}{2}$ for monoatomic gas

(C) $C_V < \frac{5R}{2}$ for diatomic gas

(D) $C_V > \frac{5R}{2}$ for diatomic gas

32. 2 moles of He are mixed with 2 moles of H_2 in a closed adiabatic container. Initially the mixture occupies 3 litres at $27°C$. The volume is suddenly decreased to $\left(\frac{3}{2}\right)$ litres. Choose the correct option(s) (H_2 and He can be treated as an ideal gas)

(A) γ for mixture is $\frac{3}{2}$

(B) Final temperature is $200\sqrt{2}K$

(C) C_P for mixture is 2R

(D) Work done in compression is totally converted into internal energy

EXCERCISE-III

1. 10 gm of ice at $-20°C$ is dropped into a calorimeter containing 10 gm of water at $10°C$. The specific hat of water is twice that of ice. Neglect heat capacity of the calorimeter. When equilibrium is reached, the calorimeter will contain :

(A) 10 gm ice and 10 gm of water (B) 20 gm of water

(C) 5 gm ice and 15 gm of water (D) 20 gm ice

2. A metal vessel of negligible heat capacity contains 1kg of water (of specific heat capacity $4200\,J\,kg^{-1}K^{-1}$). It is heated from $15°C$ to $85°C$ by 1500 W immersion heater in 4 minutes. The average loss of heat in watts to the surroundings from the vessel during this time is nearly :

(A) 70 W (B) 123 W (C) 275 W (D) 212 W

3. In an industrial process 10 kg of water per hour is to be heated from $20°C$ to $80°C$. To do this steam at $150°C$ is passed from a boiler into a copper coil immersed in water. The steam condenses in the coil and is returned to the boiler as water at $90°C$. How many kg of steam is required per hour (in kg) is (Specific heat of steam $=1\,cal/g°C$, Latent heat of vaporization = 540 cal/g)

4. A clock pendulum made of invar has a period of 0.5 sec at $20°C$. If the clock is used in a climate where average temperature is $30°C$, approximately how much fast or slow will the clock run in 10^6 sec? $\left[\alpha_{invar}=1\times10^{-6}/°C\right]$

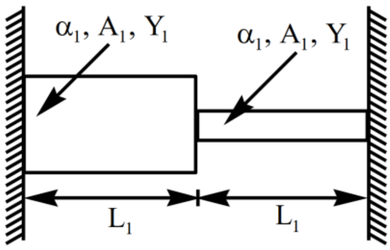
(A) 5 sec fast (B) 20 sec fast (C) 20 sec slow (D) 5 sec slow

5. The density of a uniform rod with cross-section A is d, its specific heat capacity is c and the coefficient of its linear expansion is α. Calculate the amount of heat that should be added in order to increase the length of the rod by ℓ.

(A) $\frac{dAc\ell}{\alpha}$ (B) $\frac{\alpha Ac\ell}{d}$ (C) $\frac{dc\ell}{A\alpha}$ (D) $\frac{c\ell\alpha d}{A}$

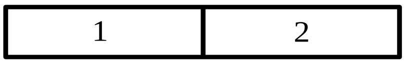
6. Two rods are joined between fixed supports as shown in the figure. Condition for no change in the lengths of individual rods with the increase of temperature will be :

(α_1, α_2 = linear expansion coefficient; A_1, A_2 = Area of rods; Y_1, Y_2 = Young modulus)



(A) $\frac{A_1}{A_2}=\frac{\alpha_1 Y_1}{\alpha_2 Y_2}$ (B) $\frac{A_1}{A_2}=\frac{L_1\alpha_1 Y_1}{L_2\alpha_2 Y_2}$ (C) $\frac{A_1}{A_2}=\frac{L_2\alpha_2 Y_2}{L_1\alpha_1 Y_1}$ (D) $\frac{A_1}{A_2}=\frac{\alpha_2 Y_2}{\alpha_1 Y_1}$

7. A composite bar of length $L = L_1 + L_2$ is made up from a rod of material 1 and of length L_1 attached to a rod of material 2 and of length L_2 as shown. If α_1 and α_2 are their respective coefficients of linear expansion, then equivalent coefficient of linear expansion for the composite rod is :



(A) $\frac{\alpha_1 L_2 - \alpha_2 L_1}{L}$ (B) $\frac{\alpha_1 L_2 - \alpha_2 L_2}{L}$ (C) $\frac{\alpha_1 L_1 - \alpha_2 L_2}{L}$ (D) $\frac{\alpha_1 \alpha_1 (L_1^2 - L_2)}{(\alpha_1 L_1 - \alpha_2 L_2)}$

8. 5 g of steam at $100°C$ is mixed with 10 g of ice at $0°C$. Choose correct alternative(s).

(Given $s_{water} = 1\ cal/g°C,\ L_F = 80\ cal/g,\ L_V = 540\ cal/g$)

(A) Equilibrium temperature of mixture is $160°C$

(B) Equilibrium temperature of mixture is $100°C$

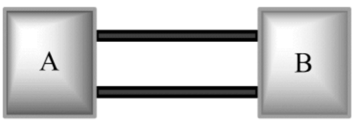
(C) At equilibrium, mixture contains $13\frac{1}{3}g$ of water

(D) At equilibrium, mixture contains $1\frac{2}{3}g$ of steam

9. Four different liquids, each of mass 1 kg, are separately heated at the same rate. The initial temperatures of the liquids are all $20°C$. The boiling points, specific heat capacities and latent heat of the liquids are shown below. Which one of them will completely evaporate first? (All are in SI units)

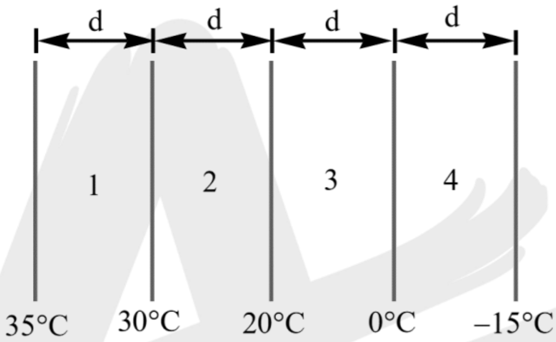
	Liquid	**Boiling point**	**Specific heat**	**Latent heat**
(A)	P	$50°C$	1000	30000
(B)	Q	$60°C$	900	25000
(C)	R	$70°C$	700	25000
(D)	S	$80°C$	500	39000

10. The container A is constantly maintained at $100°C$ and insulated container B of the figure initially contains ice at $0°C$. Different rods are used to connect them. For a rod made of copper, it takes 30 minutes for the ice to melt and for a rod of steel of same cross-section taken in different experiment it takes 60 minutes for ice to melt. When these rods are simultaneously connected in parallel, the ice melts in (min)



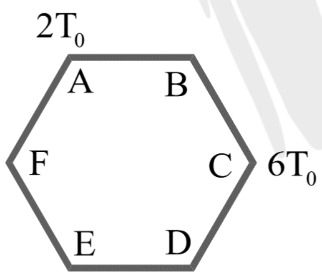
11. Three identical metal rods A, B and C are placed end to end and a temperature difference is maintained between the free ends of A and C. If the thermal conductivity of B (K_B) is twice that of C (K_C) and half that of A (K_A), then the effective thermal conductivity of the system is $\frac{\alpha K_B}{\beta}$. Find $\alpha+\beta$?

12. The diagram shows four slabs of different materials with equal thickness, placed side by side. Heat flows from left to right and the steady-state temperatures of the interfaces are given. Rank the materials according to their thermal conductivities, smallest to largest.



(A) 3, 4, 2, 1 (B) 2, 1, 3, 4 (C) 3, 4, 1, 2 (D) 4, 3, 2, 1

13. Six identical conducting rods having uniform cross-section area are fixed to make a regular hexagon as shown. The ends A and C are maintained at constant temperature $2T_0$ and $6T_0$. After the steady state is reached, picked up the correct relation between temperatures T_B, T_D, T_E and T_F of ends B, D, E and F respectively: (Neglect convection and radiation losses)



(A) $T_D > T_E = T_B > T_F$ (B) $T_D = T_B > T_E > T_F$
(C) $T_D > T_E > T_F = T_B$ (D) $T_B > T_F = T_E > T_D$

14. Two metal rods A and B, having equal cross-sectional areas, are joined end to end to form a composite bar, one end of which is heated. After a certain period of time, the temperature gradient along each rod is found to be uniform, but greater in A than that in B. thus, we can conclude that:

(A) both the rods are of equal lengths

(B) the heat current is the same in both the rods

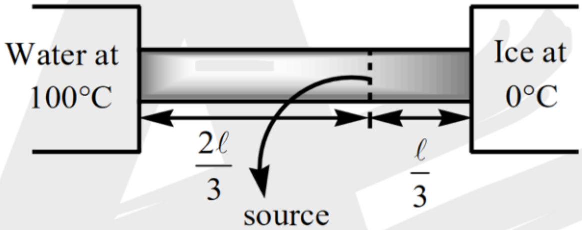
(C) A is a better conductor of heat than B

(D) None of the above

15. A rod of length ℓ and cross-section area A has a variable thermal conductivity given by $k = \alpha T$, where α is a positive constant and T is temperature in Kelvin. Two ends of the rod are maintained at temperatures T_1 and $T_2 (T_1 > T_2)$. Heat current flowing through the rod will be:

(A) $\frac{A\alpha(T_1^2 - T_2^2)}{\ell}$ (B) $\frac{A\alpha(T_1^2 + T_2^2)}{\ell}$ (C) $\frac{A\alpha(T_1^2 + T_2^2)}{3\ell}$ (D) $\frac{A\alpha(T_1^2 - T_2^2)}{2\ell}$

16. The rod connecting two reservoir and connected to a source is as shown in diagram. The rod is in steady state. Find the temperature of source, so that rate of melting of ice is 16 times that rate of vaporization. (Latent heat of vaporization is 540 Cal/gm and latenst heat of fusion is 80 Cal/gm)



(A) 540°C (B) 580°C (C) 640°C (D) 740°C

17. The room heater can provide only 16°Cin the room when the temperature outside is −20°C. If is not warm and comfortable, that is why the electric stove with power of 1 kW is also plugged in. together these two devices maintain the room temperature of 22°C. Determine the thermal power (in kW) of the heater.

18. Two identical heaters are coated with paint. In 1st case $e_1 = 1.0$ and in 2nd case $e_2 = 0.5$. Both are kept in identical chambers which are in similar surroundings. If the heaters are switched on, in steady state 1st heater has temperature T_1 on surface and θ_1 of its chamber. 2nd heater has temperature T_2 on surface and θ_2 of its chamber.

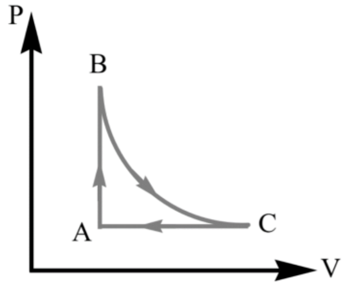
(A) $\theta_1 = \theta_2; T_1 < T_2$ (B) $\theta_1 > \theta_2; T_1 = T_2$

(C) $\theta_1 = \theta_2; T_1 > T_2$ (D) $\theta_1 < \theta_2; T_1 = T_2$

19. A planet is at an average distance 'd' from the sun and its average surface temperature is T. Assume that the planet receives energy only from sun and loses energy only through radiation, from surface. Neglect atmospheric effects. Temperature T is proportional to:

(A) d^2 (B) $d^{-1/2}$ (C) $d^{1/2}$ (D) $d^{1/4}$

20. One mole of diatomic ideal gas undergoes a cyclic process ABCA as shown in figure. Process BC is adiabatic. The temperatures at A, B and C are 300, 600 and 450K respectively. Choose the correct statement(s)



(A) In process CA change in internal energy is 225R

(B) In process AB change in internal energy is -150R

(C) In process BC change in internal energy is -225R

(D) Change in internal energy during the whole cyclic process is +150R

21. One mole of a diatomic gas undergoes a process $P = \frac{P_0}{1+\left(\frac{V}{V_0}\right)^3}$, where P_0, V_0 are constants. The translational kinetic energy of the gas when $V = V_0$ is given by $\frac{\alpha P_0 V_0}{\beta}$. Find $\alpha + \beta$

22. Let $\bar{v}$, v_{rms} and v_p respectively denote the mean speed, the root – mean – square speed, and the most probable speed of the molecules in an ideal monoatomic gas at absolute temperature T. The mass of a molecule is m

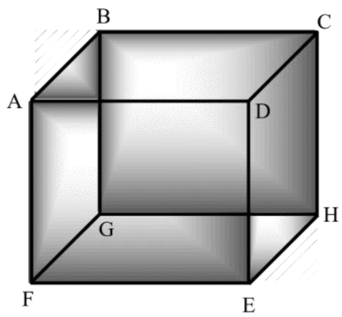
(A) No molecule can have speed greater than v_{rms}

(B) No molecule can have speed less than $\frac{v_p}{\sqrt{2}}$

(C) The average kinetic energy of a molecule is $\frac{3}{4}mv_p^2$

(D) None of the above

23. 1 mole of an ideal gas is contained in a cubical volume V, ABCDEFGH at 300K. One face of the cube (EFGH) is made up of a material which totally absorbs any gas molecule incident on it. At any given time



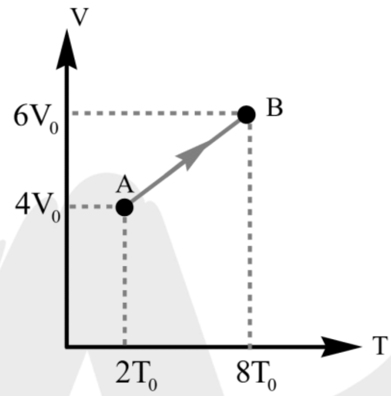
(A) The pressure on EFGH would be zero

(B) The pressure on all the faces will be equal

(C) The pressure on EFGH would be double the pressure on ABCD

(D) The pressure on EFGH would be half that on ABCD

24. One mole of an ideal gas undergoes an expansion from state A to state B as shown in the V – T diagram. The pressure of gas



(A) Remains constant during the process

(B) Increases from A to B, $\frac{8}{3}$ times

(C) is same at A and B, but increases in between

(D) Decreases from A to B, $\frac{3}{8}$ times

25. The heat capacity of a fixed amount of a gas is measured by heating the gas in a well insulated vessel and measuring the rise in temperature. The heat capacity is found to be H_V when the volume of the gas is kept constant and H_P when the pressure is kept constant. Which one of the following statements concerning the values of H_V and H_P is correct?

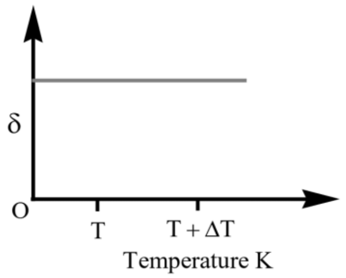
(A) H_V and H_P should be the same because heat capacity depends on the properties of the gas only

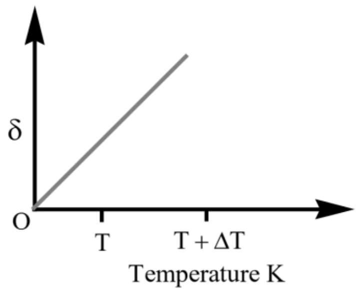
(B) H_V should be greater than H_P because energy must be supplied to keep the volume constant

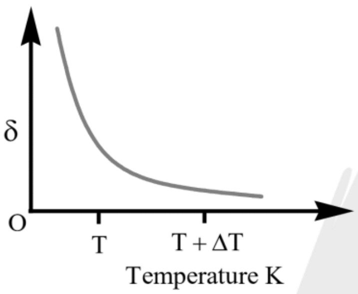
(C) H_V should be smaller than H_P because work has to be done by the gas to keep the pressure constant

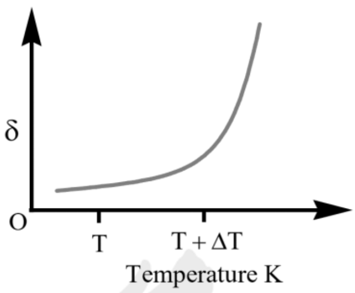
(D) H_V can either be greater or smaller than H_P depending on the gas used

26. An ideal gas has temperature T and volume V. The quantity $\delta = \frac{dV}{Vdt}$ varies with temperature under isobaric conditions as

(A)


(B)


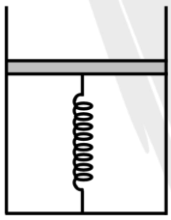
(C)


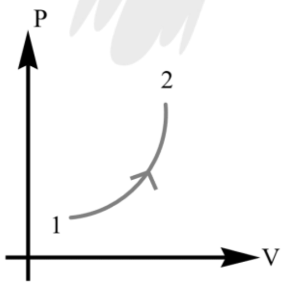
(D)


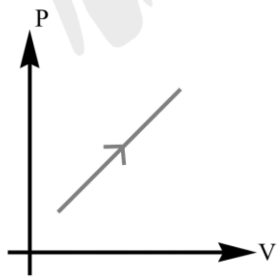
27. An equimolar mixture of a monoatomic and a diatomic ideal gas is suddenly compressed to $\frac{1}{8}$th of its original volume. The ratio of final temperature to the initial temperature

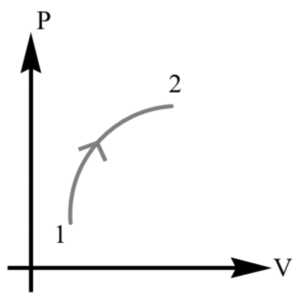
(A) $8^{0.53}$ (B) $8^{0.5}$ (C) $8^{1.53}$ (D) $8^{2/3}$

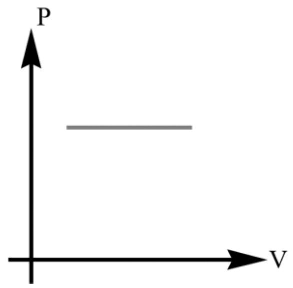
28. The piston is massless and the spring is ideal and initially stretched. The piston cylinder arrangement encloses an ideal gas. If the gas is heated quasistatically, the P-V graph is



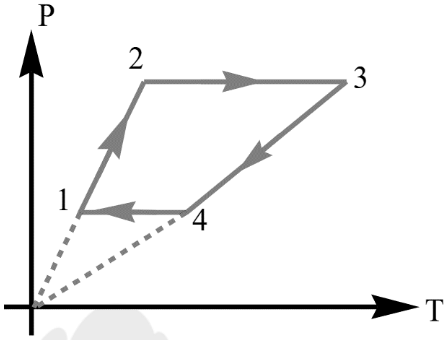
(A)


(B)


(C)


(D)


29. Three moles of an ideal monoatomic gas performs a cyclic process as shown in the figure. The temperatures in different states are $T_1 = 400K$, $T_2 = 800K$, $T_3 = 2400K$ and $T_4 = 1200K$. The work done by the gas during the cycle is $\frac{1994}{n} kJ$ .Find n? [Given $R = 8.31 J - mole^{-1} K^{-1}$]



30. A closed vessel contains a mixture of two diatomic gases A and B. Molar mass of A is 16 times that of B and mass of gas A, contained in the vessel is 2 times that of B

(A) Average kinetic energy per molecule of gas A is equal to that of gas B

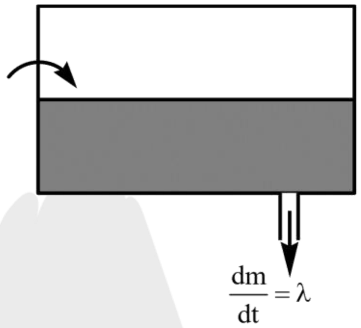
(B) Root mean square value of translational velocity of gas B is four times that of A

(C) Pressure exerted by gas B is eight times of that exerted by gas A

(D) Number of molecules of gas B in the cylinder is four times that of gas A

EXCERCISE-IV

1. A well insultanted box contains water (specific heat = C and very large conductivity) of mass m_0 and temperature T_0 at time t = 0. If heat is being added to it uniformly at a constant rate $\frac{dQ}{dt} = R$ and water is leaking from it at a constant rate $\frac{dm}{dt} = \lambda$ then the temperature of the water at time $t\left(t < \frac{m_0}{\lambda}\right)$ will be :

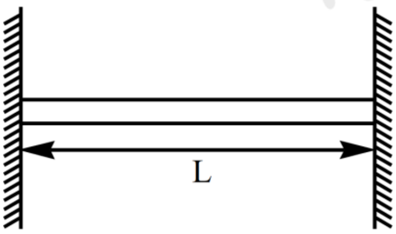


(A) $T = \frac{R}{2\lambda C}\ln\left(\frac{m_0}{m_0 - \lambda t}\right) + T_0$ (B) $T = \frac{R}{\lambda C}\ln\left(\frac{m_0}{m_0 - \lambda t}\right) + T_0$

(C) $T = \frac{R}{C\lambda}e^{(m_0 - \lambda t)}$ (D) $T = \frac{R}{2C\lambda}e^{(m_0 - \lambda t)}$

2. A steel rod is 4.000 cm in diameter at $30°C$. A brass ring has an interior diameter of 3.992 cm at $30°C$. In order that the ring just slides onto the steel rod, the common temperature (in°C) of the two should be nearly is ($\alpha_{steel} = 11 \times 10^{-6}/°C$ and $\alpha_{brass} = 19 \times 10^{-6}/°C$) :

3. A uniform rod having length L, is made of material having density ρ, coefficient of thermal expansion α, and Young's modulus Y. If it placed between two vertical walls having separation L and coefficient of friction for one wall is 2μ and other wall is μ, the minimum rise in temperature so that the rod is in equilibrium is $\frac{aPg}{b\alpha y\mu}$. Find a + b = ?



4. A hollow steel sphere, weighing 300 kg is floating on water. A weight of 10 kg is to be placed on it in order to just submerge it at a temperature of $20°C$. Now the weight that should be decreased when temperature increases to $25°C$ is :

$(\gamma_{water} = 1.5 \times 10^{-4}/°C, \alpha_{steel} = 1 \times 10^{-5}/°C)$

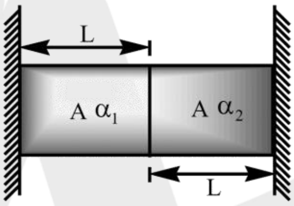
(A) 126 gram (B) 252 gram (C) 186 gram (D) 500 gram

5. A steel scale is calibrated at 0°C to measure accurately. If this scale measures the length of a wooden rod at $0°C$ and 20°C as 10 cm and 9.975 cm respectively, find the scale reading at $50°C$, by this scale :

(A) 9.9425 cm (B) 9.9375 cm

(C) 10.0625 cm (D) 9.875 cm

6. Figure shows two rods A and B of same length L and same cross sectional area S but of different material having coefficient of linear expansion α_1 and α_2 respectively. They are clamped between two rigid walls, separated by a distance 2L. This all refers to temperature $t°C$. (Take the Young's modulus for the two rods to be Y_1 and Y_2 respectively).



(A) $\frac{S(Y_1\alpha_1+Y_2\alpha_2)t}{Y_1+Y_2}$ (B) $\frac{SY_1Y_2(\alpha_1+\alpha_2)t}{Y_1+Y_2}$

(C) $\frac{St\alpha_1\alpha_2(Y_1+Y_2)t}{Y_1Y_2}$ (D) $\frac{StY_1Y_2(\alpha_1+\alpha_2)}{2(Y_1+Y_2)}$

7. We have three pieces of materials and we are required to heat each of them from 15°C to 65°C. Choose the correct amount of heat required for each of the case. The specific heats, in cal/g°C, for aluminum, pyrex and platinum are 0.21, 0.20 and 0.032, respectively.

(A) 3.0 g of aluminium requires 31.5 cal

(B) 5.0 g of pyrex glass requires 50 cal

(C) 20 g of platinum requires 32 cal

(D) 5.0 g of pyrex glass requires 5 cal

8. In a heat insulated vessel there is tap water of 15°C. We place in it 'n' ice cubes of −15°C taken out of the deep-freezer. $S_{ice} = \frac{S_{water}}{2}$. Each ice cube has same mass as mass of water in vessel.

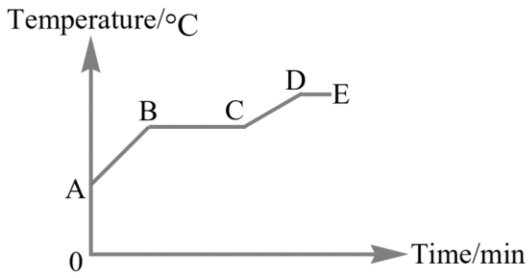
(A) If n = 1, some ice will melt

(B) If n = 1, some water will freeze

(C) If n = 2, some water will freeze

(D) If n = 2, final temperature is $0°C$

9. A well insulated substance in solid state is heated at a constant rate until it vaporizes completely. The temperature-time graph of the substance is shown below. Which of the following statements is/are true?



(A) The specific heat capacity of the substance in solid state is greater than that of the substance in liquid state.

(B) The specific latent heat of fusion of the substance is greater than the specific latent heat of vaporization of the substance

(C) If the rate of heating increases, the slop of AB will be increased but the length of BC remains unchanged

(D) If the mass of the substance is doubled, the length of DE is also doubled but the slope of CD is halved

10. When the temperature of a copper coin is raised by $80°C$, its diameter increases by 0.2%.

(A) Percentage rise in the area of a face is 0.4%

(B) Percentage rise in the thickness is 0.4%

(C) Percentage rise in the volume is 0.6%

(D) Coefficient of linear expansion of copper is $0.25\times10^{-4}\,C^{\circ\,-1}$

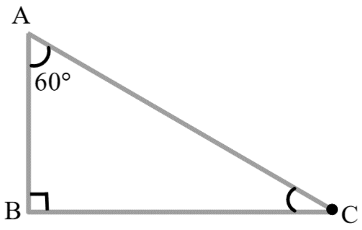
11. The ends of copper rod of length 1 m and area of cross-section 1cm^2 are maintained at $0°C$ and $100°C$. At the center of the rod there is a source of heat of power 25 W. Thermal conductivity of copper is 400 W/m-K. In steady state, the temperature at the section on rod at which source is supplying heat, will be:

(A) 150.50°C (B) 325.75°C (C) 206.25°C (D) 126.25°C

12. Two thin walled spheres of different materials, one with double the radius and one-fourth wall thickness of the other, are filled with ice. If the time taken for complete melting of ice in the sphere of larger radius is 25 minutes and that for smaller one is 16 minutes, the ratio of thermal conductivities of the materials of larger sphere to the smaller sphere is:

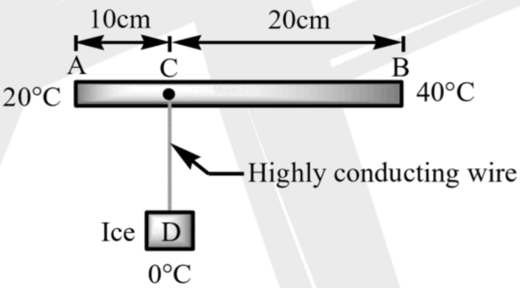
(A) 4 : 5 (B) 25 : 1 (C) 1 : 25 (D) 8 : 25

13. Three rods of the same cross-section and made of the same material form the sides of a triangle ABC as shown. The points A and B are maintained at temperatures T and $\sqrt{2}T$ respectively in the steady state. Assuming that only heat conduction takes plane, the temperatures at point C is:

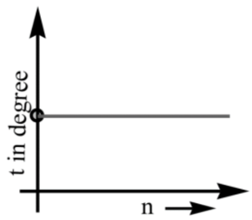


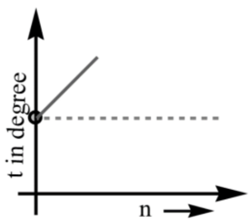
(A) $\left[\frac{2\sqrt{2}T+\sqrt{3}}{2+\sqrt{3}}\right]T$ (B) $\left[\frac{3\sqrt{2}}{2+\sqrt{3}}\right]T$ (C) $\left[\frac{2}{\sqrt{3}}\right]T$ (D) $\left[\frac{\sqrt{5}}{2}\right]T$

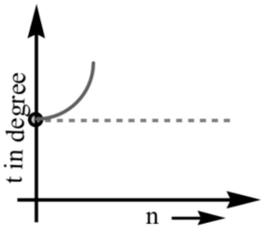
14. In the figure shown AB is a rod of length 30 cm and area of cross-section $1.0\,cm^2$ and thermal conductivity 336 S.I. units. The ends A and B are maintained at temperatures $20°C$ and $40°C$ respectively. A point C of this rod is connected to a box D, containing ice at $0°C$, through a highly conducting wire of negligible heat capacity. The rate (in mg/s) at which ice melts in the box is: [Assume latent heat of fusion for ice $L_f = 80\,cal/gm$]



15. A slab X of thickness 't', thermal conductivity 'K' and area of cross-section 'A' is placed in thermal contact with another slab Y which is $2n^2$ times thicker, 4n times conductive and having n times larger cross-section area. If the outside face of X is maintained at $100°C$, the outside face of Y at $0°C$, then the temperature of the junction is represented by the graph $(n > 0)$:

(A)


(B)


(C)


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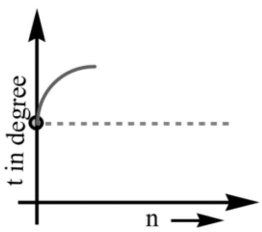
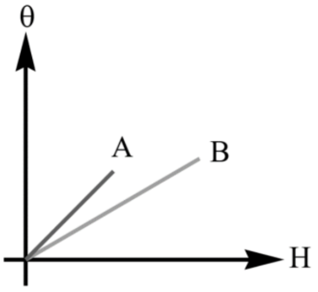
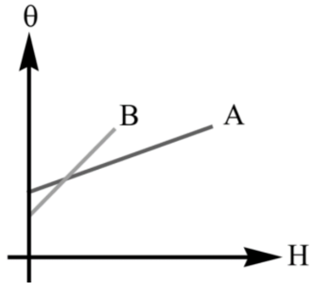
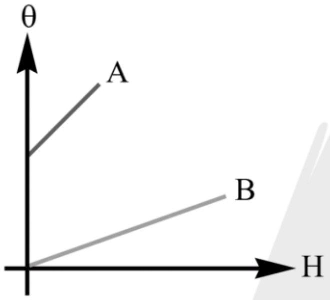
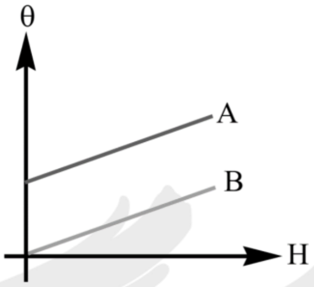


16. Two uniform brass rods A and B of length ℓ and 2ℓ and radii 2r and r respectively are heated through same temperature difference. Which of the graphs between temperature θ and heat H is correctly representing the heating?

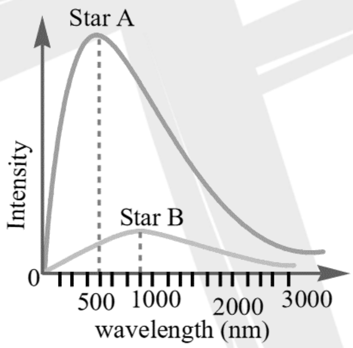
(A)


(B)


(C)


(D)


17. The spectra of radiation emiited by two distant stars are shown below.



The ratio of the surface temperature of star A to that of star B, $T_A : T_B$, is approximately:

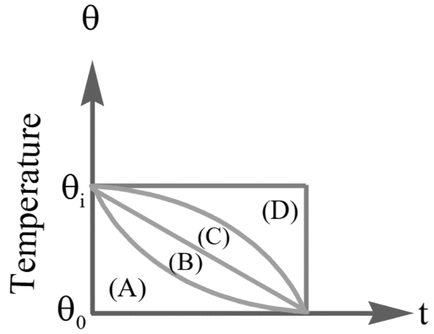
(A) 2:1 (B) 4:1 (C) 1:2 (D) 1:1

18. There are two thin spheres A and B of the same material and same thickness. They emit like black bodies. Radius of A is double that of B. A and B have same temperature T. When A and B are kept in a room of temperature $T_0 (< T)$, the ratio of their rates of cooling (rate of fall of temperature) is: [assume negligible heat exchange between A and B]

(A) 2:1 (B) 1:1 (C) 4:1 (D) 8:1

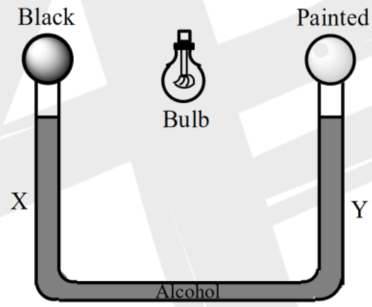
19. A long solid cylinder is radiating power. It is remoulded into a number of smaller cylinders, each of which has the same length as original cylinder. Each small cylinder has the same temperature as the original cylinder. How many smaller cylinders are there? Neglect the energy emitted by the flat faces of cylinder.

20. The temperature (θ) of "cup of coffee" in a "BARISTA" Restaurant was plotted as a function of time. Which of the following curves may represent the plot?



(A) curve A (B) curve B (C) curve C (D) curve D

21. The following figure shows two air-filled bulbs connected by a U-tube partly filled with alcohol. What happened to the levels of alcohol in the limbs X and Y when an electric bulb placed midway between the bulbs is lighted?



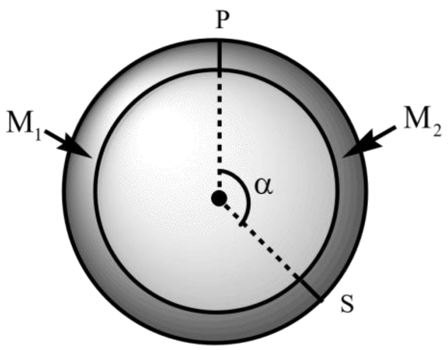
(A) The level of alcohol in limb X falls while that in limb Y rises

(B) The level of alcohol in limb X rises while that in limb Y falls

(C) The level of alcohol falls in both limbs

(D) The level of alcohol rises in both limbs

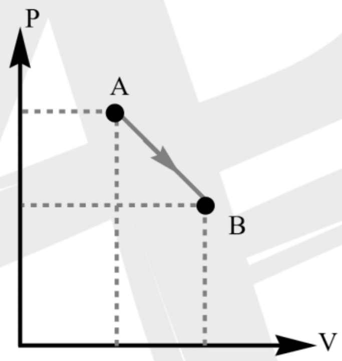
22. A ring shaped tube contains two ideal gases with equal masses and atomic mass numbers $M_1 = 32$and $M_2 = 28$. The gases are separated by one fixed partition P and another movable conducting partition S which can move freely without friction inside the ring. The angle α as shown in the figure in equilibrium is $\frac{\alpha\pi}{\beta}$. Find $\alpha + \beta$?



23. For a gas sample with N_0 number of molecules, function N(V) is given by $N(V) = \frac{dN}{dV} = \left(\frac{3N_0}{V_0^3}\right)$ for $0 \le V \le V_0$ and $N(V) = 0$ for $V > V_0$. Where dN is number of molecules in speed range V to V + dV. The rms speed of the molecules is $\sqrt{\frac{\alpha}{\beta}}V_0$. Find $\alpha + \beta$?

24. Two Carnot engines A and B are operated in succession. The first one, A receives heat from a source at $T_1 = 800K$ and rejects to sink at T_2K. The second engine B receives heat rejected by the first engine and rejects to another sink at $T_3 = 300K$. If the work outputs of two engines are equal, then the value of T_2 (in K) is

25. One mole of an ideal gas undergoes a process AB where $A(2P_0, V_0)$ and $B(1.5P_0, 1.5V_0)$. The process is shown in the PV diagram. Which of the following is incorrect regarding temperature of the gas?



(A) Only increases
(B) Increases and then decreases
(C) Has a maximum value of $\frac{9}{4}\frac{P_0V_0}{R}$
(D) is more at B than at A

26. Certain amount of an ideal gas are contained in a closed vessel. The vessel is moving with a constant velocity v. The molecular mass of gas is M. The raise in temperature of the gas when the vessel is suddenly stopped is $\left[\gamma = \frac{C_P}{C_V}\right]$

(A) $\frac{Mv^2}{2R(\gamma+1)}$
(B) $\frac{Mv^2(\gamma-1)}{2R}$
(C) $\frac{Mv^2}{2R\gamma}$
(D) $\frac{Mv^2\gamma}{2R(\gamma-1)}$

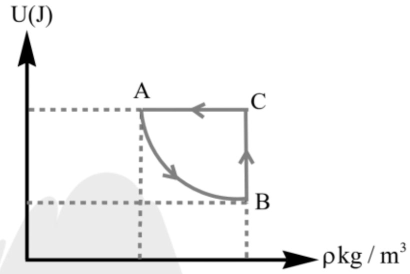
27. A heat engine is having a source at temperature 527°C and sink at temperature 127°C. If the useful work is required to be done by the engine at the rate of 750 watt, then the amount of heat absorbed by the sink per second from the source in calories and the efficiency of heat engine are

(A) 482.2 cal/sec, 50%
(B) 482.2 cal/sec, 25%
(C) 357.14 cal/sec, 50%
(D) 357.14 cal/sec, 25%

28. An ideal diatomic gas goes through a cycle consisting of two isochoric and two isobaric lines. The absolute temperature of the gas rises 5 times both in the isobaric heating and in the isobaric expansion. Efficiency of such a cycle is

(A) 0.1 (B) 0.2 (C) 3 (D) 4

29. Figure shows the variation of the internal energy U with density ρ of one mole of an ideal monoatomic gas for thermodynamics cycle ABCA. Here process AB is a part of rectangular hyperbola.



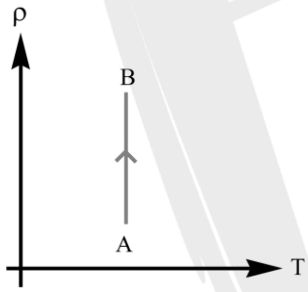
(A) Process AB is isothermal and network in cycle is done by gas

(B) Process AB is isobaric and network in cycle is done by gas

(C) Process AB is isobaric and network in cycle is done on the gas

(D) Process AB is adiabatic and network in cycle is done by gas

30. The density (ρ) of and ideal gas varies with temperature T as shown in figure. Then



(A) The product of P and V at A is equal to the product of P and V at B

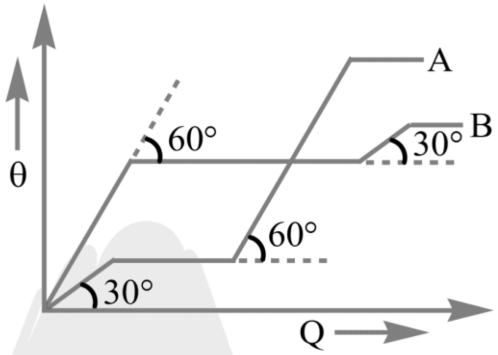
(B) Pressure at B is greater than the pressure at A

(C) Work done by the gas during the process AB is negative

(D) The change in internal energy from A to B is zero

EXCERCISE-V

1. The temperature (θ) versus heat transfer (Q) plot for two substances A and B is given in the figure. If some quantity of substance A is liquid phase at temperature $30°C$ is mixed with substance B at $20°C$ then the temperature of the mixture will be : (Given that boiling point for substance A is more than $30°C$)



(A) $22.5°C$ (B) $27.5°C$ (C) $25°C$ (D) $20°C$

2. A thermally insulated vessel contains some water at $0°C$. The vessel is connected to a vacuum pump to pump out water vapour. This results in some water getting frozen. It is given Latent heat of vaporization of water at $0°C = 21\times10^5\ J/kg$ and latent heat of freezing of water $= 3.36\times10^5\ J/kg$. The maximum percentage amount of water that will be solidified in this manner will be :

(A) 86.2% (B) 33.6% (C) 21% (D) 24.36%

3. Heat is being supplied at a constant rate to a sphere of ice which is melting at the rate of 0.1 gm/sec. It melts completely in 100 sec. The rate of rise of temperature thereafter will be : (Assume no loss of heat.)

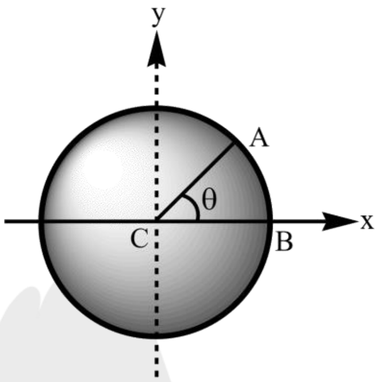
(A) $0.8\ °C/sec$ (B) $5.4\ °C/sec$ (C) $3.6\ °C/sec$ (D) change with time

4. A continuous flow water heater (geyser) has an electrical power rating = 2 kW and efficiency of conversion of electrical power into heat = 80%. If water is flowing through the device at the rate of 100 cc/sec, and the inlet temperature is $10°C$, the outlet temperature is $\frac{138}{n}$. Find n?

5. 4 gms of steam at $100°C$ is added to 20 gms of water at $46°C$ in a container of negligible mass. Assuming no heat is lost to surrounding. What is the mass of water in container at thermal equilibrium? (Latent heat of vaporization = 540 cal/gm, Specific heat of water = 1cal/gm – °C)

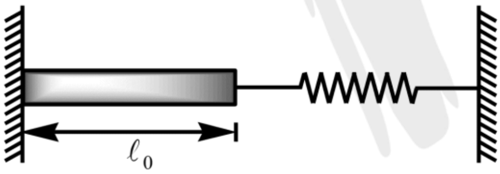
(A) 18 gm (B) 20 gm (C) 22 gm (D) 24 gm

6. Two lines AC and BC are drawn on a circular disc as shown in figure. The disc is made of a material with linear expansion coefficients $\alpha_x > \alpha_y$. Here α_x and α_y are linear expansion coefficient for expansion in x-direction and y-direction respectively. C is the centre of disc. If the disc is heated uniformly. Angle $ACB(=\theta)$ will :



(A) increase
(B) decrease
(C) remains constant
(D) may increase or decreases depending on value of temperature increment

7. A rod of mass M, area of cross section A and length ℓ_0 is connected with a spring as shown in figure. If coefficient of linear expansion of rod is α and initially no extension was there in the spring of spring constant k, then the stress developed in rod when its temperature is increased by ΔT, is :
[Young's modulus of material of the rod is Y]



(A) $\dfrac{k[Y\alpha\Delta T]\ell_0}{[k\ell_0 + YA]}$ (B) $Y\alpha\Delta T$ (C) $\left[\dfrac{k\ell_0 + YA}{k\ell_0}\right]Y\alpha\Delta T$ (D) $\left[\dfrac{k\ell_0 + Y}{k\ell_0}\right]Y\alpha\Delta T$

8. A liquid of volumetric thermal expansion coefficient = γ and bulk modulus B is filled in a spherical tank of negligible heat expansion coefficient. Its radius is R and wall thickness is 't' $(t << R)$. When the temperature of the liquid is raised by θ, the tensile stress developed in the walls of the tank is :

(A) $\dfrac{B\gamma\theta R}{2t}$ (B) $\dfrac{B\gamma\theta R}{t}$ (C) $\dfrac{2B\gamma\theta R}{t}$ (D) $\dfrac{B\gamma\theta R}{4t}$

9. In three experiments, a material A at a particular low temperature T_C and a material B at a particular high temperature T_H are placed in an isolated and insulated container. When they reach thermal equilibrium with each other (No phase change occurs), their final temperature T_f is measured. The masses m_A and m_B and specific heats C_A and C_B of the material are given in the table. Assume that heat transferred is Q in the experiment. Then which of the following is/are correct?

Experiment	m_A	C_A	m_B	C_B
1	m	c	m	c
2	m	c	2m	c
3	m	2c	m	c

(A) $(T_f)_1 > (T_f)_2 > (T_f)_3$

(B) $Q_3 > Q_1, Q_2 > Q_1$

(C) $Q_2 > Q_1 > Q_3$

(D) $(T_f)_2 > (T_f)_1 > (T_f)_3$

10. Due to thermal expansion, with rise in temperature :

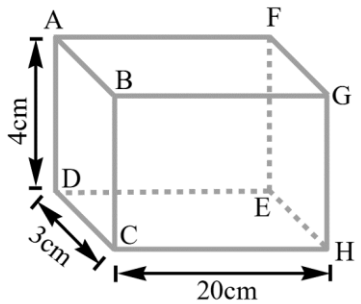
(A) metallic scale reading becomes lesser than true value (α of the metal is greater than α of the object)

(B) Pendulum clock becomes slower

(C) A floating body sinks a little more (assuming temperature of liquid remains unchanged)

(D) The apparent weight of a body in a liquid may decrease (assuming temperature of liquid remains unchanged)

11. A rectangular block of lead has dimensions 4 cm × 3 cm × 20 cm. A temperature difference of 100°C can be applied to any pair of opposite faces that we choose.



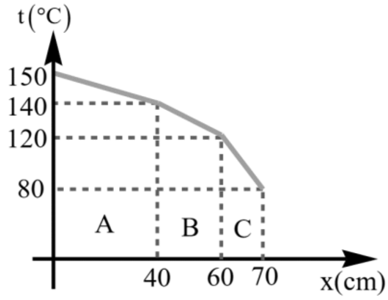
(A) The largest amount of heat flows if it flows parallel to line BG

(B) The largest amount of heat flows if it flows parallel to line AD

(C) The smallest amount of heat flows if it flows parallel to AF

(D) The smallest amount of heat flows if it flows parallel to AB

12. The graph shown gives the temperature along an x axis that extends directly through a wall consisting of three layers A, B and C. The air temperature on one side of the wall is $150°C$ and on the other side is $80°C$. Thermal conduction through the wall is steady. Out of the three layers A, B and C, thermal conductivity is greatest of the layer:



(A) A

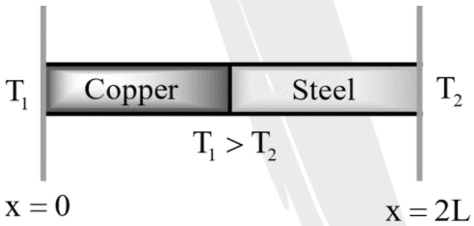
(B) B

(C) C

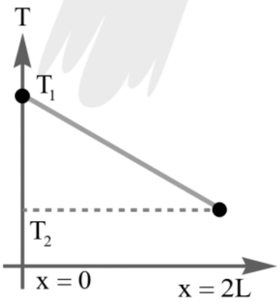
(D) Thermal conductivity of A = Thermal conductivity of B

13. A copper rod and a steel rod of equal cross-sections and lengths (L) are joined side by side and connected between two heat baths as shown in the figure.

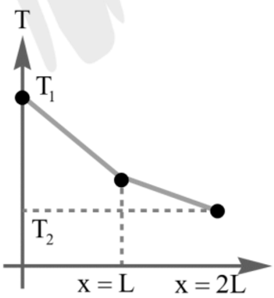
If heat flows through them from x = 0 to x = 2L at a steady rate and conductivities of the metals are K_{cu} and K_{steel} $(K_{cu} > K_{steel})$, then the temperature varies as: (convection and radiation are negligible)



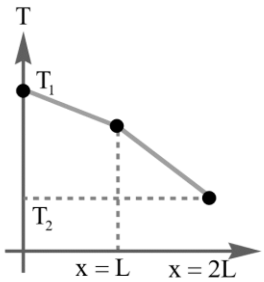
(A)



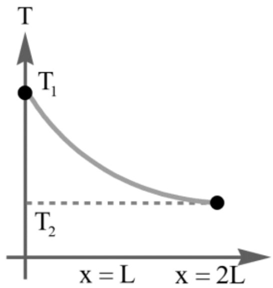
(B)



(C)



(D)



14. Let the wavelength at which the spectral emissive power of a black body at a temperature T is maximum, be denoted by λ_{max}. As the temperature of the body is increased by 1K, λ_{max} decreases by 1 percent. The temperature T of the black body (in K) is:

15. On a cold winter day, the atmospheric pressure is $-\theta$ (on Celsius scale) which is below $0°C$. A cylindrical drum of height 'h' made of a bad conductor is completely filled with water at $0°C$ and is kept outside without and lid. Calculate the time taken for the whole mass of water to freeze. Thermal conductivity of ice is k and its latent heat of fusion is L. Neglect expansion of water on freezing.

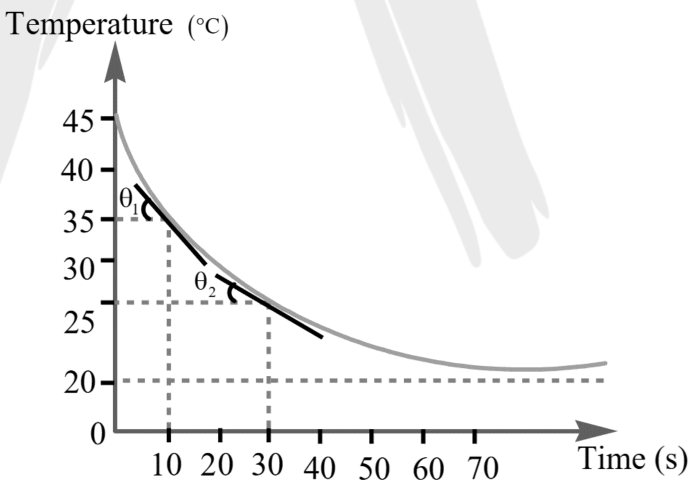
(A) $T=\dfrac{\rho Lh^2}{k\theta}$

(B) $T=\dfrac{\rho L^2h^2}{2k\theta}$

(C) $T=\dfrac{2k}{\rho L^2 h}$

(D) $T=\dfrac{\rho Lh^2}{2k\theta}$

16. The temperature of a well stirred liquid kept open to a cold surrounding is plotted against time. The value of $\sec^2\theta_1$ is:



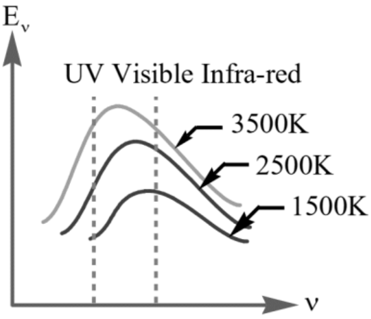
(A) $1+9\tan^2\theta_2$

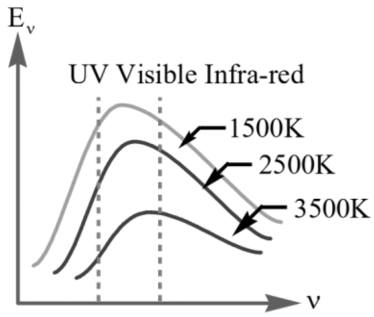
(B) $1+\tan^2\theta_2$

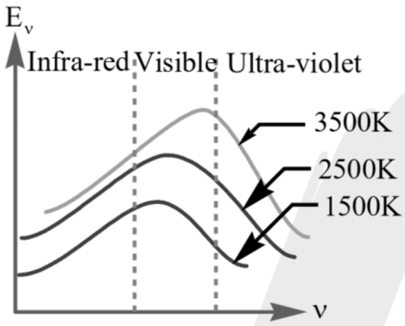
(C) $1+3\tan^2\theta_2$

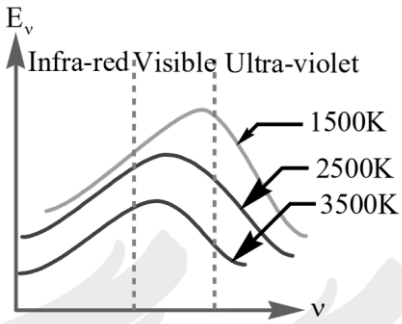
(D) $3+\tan^2\theta_2$

17. Which of the following graphs shows the correct variation in intensity of heat radiations by black body and frequency at a fixed temperature?

(A)


(B)


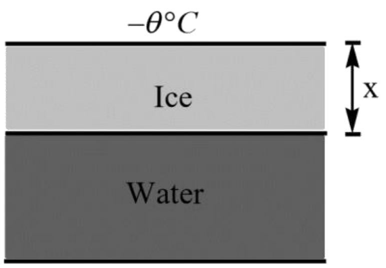
(C)


(D)


18. A copper sphere is suspended in an evacuated chamber maintained at 300 K. The sphere is maintained at a constant temperature of 500 k by heating it electrically. A total of 300 W of electric power is needed to do it. When **half** of the surface of the copper sphere is completely blackened, 600 W is needed to maintain the same temperature of the sphere. Calculate the emissivity of copper.

(A) $e=\frac{1}{3}$ (B) $e=\frac{2}{3}$ (C) $e=\frac{1}{2}$ (D) $e=\frac{1}{6}$

19. Consider the shown case of a freezing lake due to negative environmental temperature $(-\theta°C)$. Thickness (x) of ice layer is small in comparison to depth of lake. Rate of increase in x will be greater:



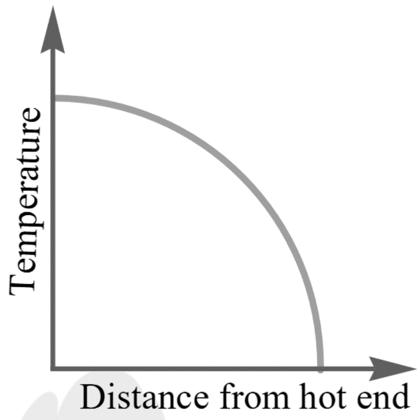
(A) if environmental temperature increases

(B) for larger thickness of ice layer

(C) if environmental temperature decreases

(D) for smaller thickness of ice layer

20. The ends of a long homogeneous bar are maintained at different temperatures and there is no loss of heat from the sides of the bar due to conduction or radiation. The graph of temperature against distance of the bar when it has attained steady state is shown here. The graph shows:



(A) the temperature gradient is not constant

(B) the bar has uniform cross-sectional area

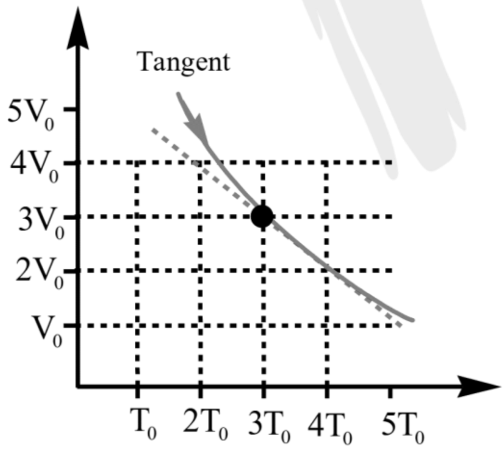
(C) the cross-sectional area of the bar increases as the distance from the hot end increases

(D) the cross-sectional area of the bar decreases as the distance from the hot end increases

21. In a sealed vessel of constant volume, diatomic gas is present. Due to significant increase in temperature, certain molecules dissociated into atoms as a result of which specific heat capacity of gas decreased by 8%. What fraction of moles got dissociated?

(A) 0.1 (B) 0.5 (C) 0.6 (D) 0.3

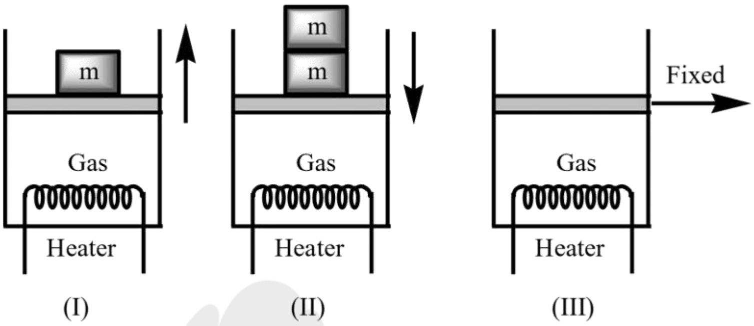
22. Figure shows the adiabatic curve for n moles of an ideal gas, the bulk's modulus for the gas corresponding to the point P will be (Symbols have usual meaning)



(A) $\frac{nRT_0}{V_0}$ (B) $\frac{2nRT_0}{V_0}$

(C) $\frac{nR}{2}\frac{T_0}{V_0}$ (D) $\frac{3nRT_0}{2V_0}$

23. A sample of gas is heated by three different methods from same initial state as shown. in each methods heat supplied is the same. In (I) piston moves up by some amount. In (II) piston moves down and in (III) piston does not move. Specific heat of the gas is calculated in each of the methods to be C_I, C_{II} and C_{III}



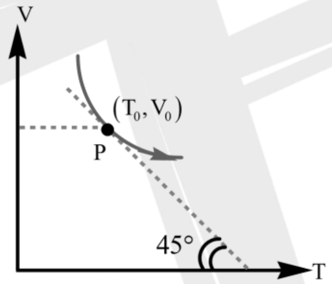
(A) $C_I = C_{III} < C_{II}$

(B) $C_I = C_{II} = C_{III}$

(C) $C_{III} > C_{II} > C_I$

(D) $C_I > C_{III} > C_{II}$

24. Figure shows the adiabatic curve for 2moles of an ideal gas. The bulk modulus (i.e., $B = \frac{dP}{-dV/V}$) at the point P will be



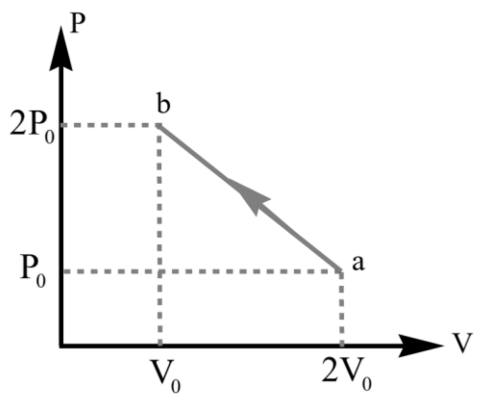
(A) $R\left(1 + \frac{T_0}{V_0}\right)$

(B) $2R\left(1 + \frac{T_0}{V_0}\right)$

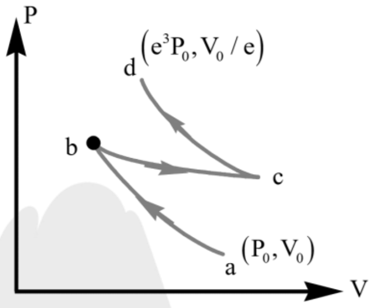
(C) $\frac{2RT_0}{V_0}$

(D) $R\left(1 - \frac{T_0}{V_0}\right)$

25. One mole of mono-atomic ideal gas undergoes the thermodynamics process $a \rightarrow b$ shown in the P-V diagram. During this process maximum temperature will occur when gas volume will be $V = \frac{\alpha V_0}{\beta}$n. Find $\alpha + \beta$?

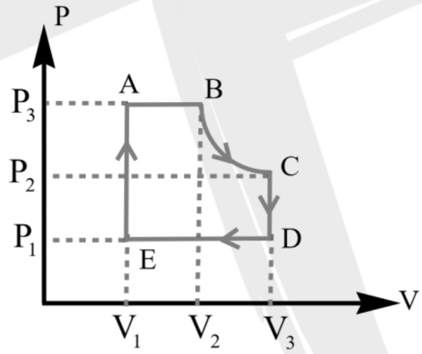


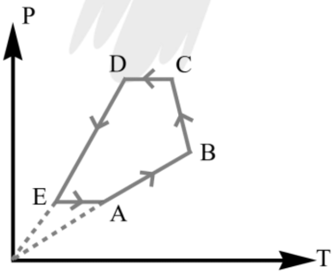
26. One mole of ideal monoatomic gas at initial state a with pressure P_0 and volume V_0 is taken to a final state 'd' with pressure e^3P_0 and volume $\frac{V_0}{e}$ through the path $a \to b \to c \to d$ (figure) $a \to b$ and $c \to d$ are adiabatic paths where as $b \to c$ is isothermal with temperature T_0. Find the work done by the gas in the process $b \to c$. (Given $\ln(e) = 1$)

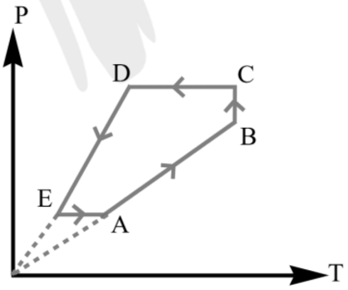


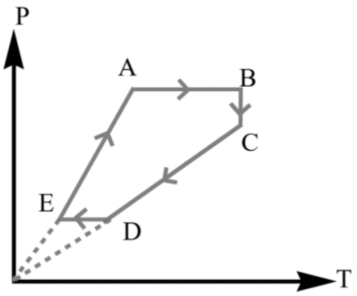
(A) $2RT_0$ (B) $\frac{5}{3}RT_0$ (C) $\frac{3}{2}RT_0$ (D) $\frac{7}{2}RT_0$

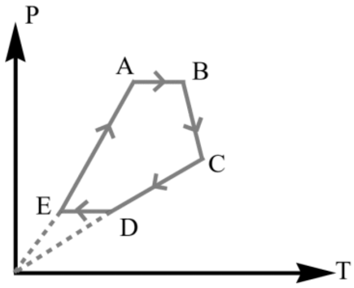
27. A monoatomic ideal gas undergoes a cyclic process ABCDEA as shown in the P-V diagram. (BC is isothermal and $P_3V_1 < P_1V_3$). Then which of the following curve(s) is/are correctly converted?



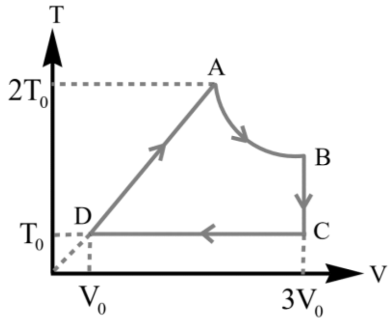
(A)


(B)


(C)


(D)


28. Consider the cyclic process ABCDA, as shown in figure, performed on a sample of n moles of an ideal gas. Net heat supplied to the gas during the process is $4nRT_0$, then the work done during the process AB will be



(A) $3nRT_0$ (B) $nRT_0(4 + \ln 3)$ (C) $nRT_0(3 + \ln 3)$ (D) $nRT_0(2 + \ln 3)$

29. Two moles of $O_2\left(\gamma = \frac{7}{5}\right)$ at temperature T_0 and 3 moles of $CO_2\left(\gamma = \frac{4}{3}\right)$ at temperature $2T_0$ are allowed to mix together in a closed adiabatic vessel. The resulting mixture finally comes in thermal equilibrium. Then

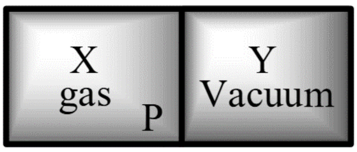
(A) Final temperature of the mixture is $\frac{23T_0}{14}$

(B) Final temperature of the mixture is $\frac{31T_0}{19}$

(C) Adiabatic exponent of the mixture formed is $\frac{14}{5}$

(D) Adiabatic exponent of the mixture formed is $\frac{14}{19}$

30. A closed container is fully insulated from outside. One half of it is filled with an ideal gas X separated by a plate P from the other half Y which contains a vacuum as shown in figure. When P is removed, X moves into Y. Which of the following statement(s) is/are correct?



(A) No work is done by X

(B) X decreases in temperature

(C) X increases in internal energy

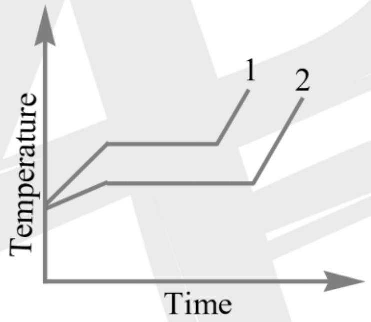
(D) X doubles in pressure

PROFICIENCY TEST-I

1. Suppose that thermal energy released by cooling water of volume $1\ m^3$ by $1\ ^\circ C$ is used to raise temperature of air by $1\ ^\circ C$ at constant volume. The volume of air that can be heated by $1^\circ C$ (in m^3) is ______

(Given: $1\ cal = 4.2\ J;\ S_{air} = 1\ J/gm^\circ C;\ S_{water} = 1\ cal/gm^\circ C;\ d_{air} = 1.2\ kg/m^3;\ d_{water} = 1\ gm/cc$)

2. Two solid objects of the same mass are supplied with heat at the same rate $\frac{\Delta Q}{\Delta t}$. The temperature of the first object with latent heat L_1 and specific heat capacity c_1 changes according to graph 1 on the diagram. The temperature of the second object with latest heat L_2 and specific heat capacity c_2 changes according to graph 2 on the diagram.



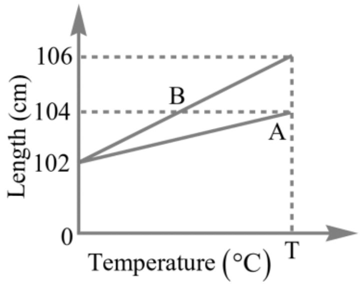
Based on what is shown on the graph, the latest heats L_1 and L_2 and the specific heat capacities c_1 and c_2 in solid state obey which of the following relationships ?

(A) $L_1 < L_2;\ c_1 < c_2$
(B) $L_1 < L_2;\ c_1 > c_2$
(C) $L_2 < L_1;\ c_1 > c_2$
(D) $L_2 < L_1;\ c_2 > c_1$

3. The specific heat of alcohol is about half that of water. Suppose you have identical masses of alcohol and water. The alcohol is initially at temperature T_A. The water is initially at a different temperature T_W. Now the two fluids are mixed in the same container and allowed to come into thermal equilibrium, with no loss of heat to the surroundings. The final temperature of the mixture will be :

(A) closer to T_A than T_W
(B) closer to T_W than T_A
(C) exactly halfway between T_A and T_W
(D) dependent on the volume of alcohol used

4. The variation of lengths of two metal rods A and B with change in temperature are shown in figure. The ratio of $\frac{\alpha_A}{\alpha_B}$ is :



(A) $\frac{1}{2}$ (B) $\frac{1}{3}$ (C) $\frac{2}{3}$ (D) $\frac{3}{2}$

5. 50 gm ice at $-10°C$ is mixed with 10 gm steam at $100°C$. When the mixture finally reaches its steady state inside a calorimeter of water equivalent 1.5 gm then : [Assume calorimeter was initially at $0°C$, Take latent heat of vaporization of water = 540 cal/gm, Latent heat of fusion of water = 80 cal/gm and specific heat capacity of water = 1 cal/gm- $°C$, specific heat of ice = 0.5 cal/gm $°C$]

(A) Mass of water remaining is 60 gm

(B) Mass of ice remaining is 3 gm

(C) Mass of steam remaining is 1.20 gm

(D) Final temperature is between $0°C$ and $100°C$

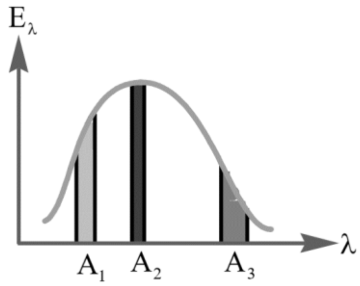
6. In a 10 meter deep lake, the bottom is at a constant temperature of $4°C$. The air temperature is constant at $-4°C$. The thermal conductivity of ice is 3 times that water. Neglecting the expansion of water on freezing, the maximum thickness of ice will be $\frac{75}{n}$ m. Find n?

7. The area of cross-section of rod is given by $A = A_0(1+\alpha x)$ where A_0 and α are constant and x is the distance from one end. If the thermal conductivity of the material is K, what is the thermal resistance of the rod if its length is ℓ_0?

(A) $KA_0\alpha \ln(1+\alpha\ell_0)$ (B) $\frac{1}{KA_0\alpha}\ln(1+\alpha\ell_0)$

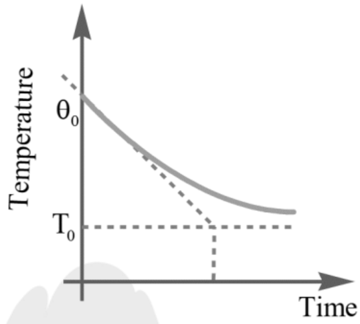
(C) $\frac{\alpha}{KA_0}\ln(\ell+\alpha\ell_0)$ (D) $\frac{KA_0}{\alpha}\ln(1+\alpha\ell_0)$

8. Three separate segments of equal area A_1, A_2 and A_3 are shown in the energy distribution curve of a black body radiation. If n_1, n_2 and n_3 are number of photons emitted per unit time corresponding to each area segment respectively then:



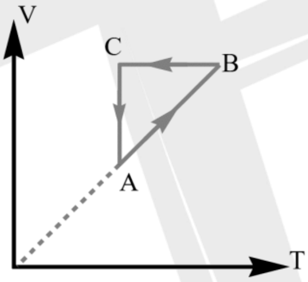
(A) $n_2 > n_1 > n_3$ (B) $n_3 > n_1 > n_2$ (C) $n_1 = n_2 = n_3$ (D) $n_3 > n_2 > n_1$

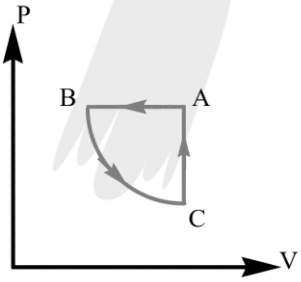
9. A body at temperature θ_0 having Newton's cooling constant K is placed in a surrounding having temperature T_0 at time t = 0. The graph of temperature of the body as a function of time t is shown in the adjacent figure. A tangent on the curve is drawn at t = 0 as shown in the figure. Find the value of τ.

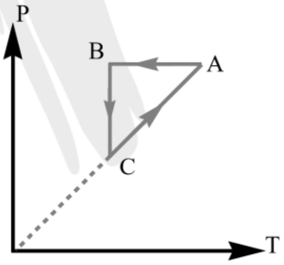


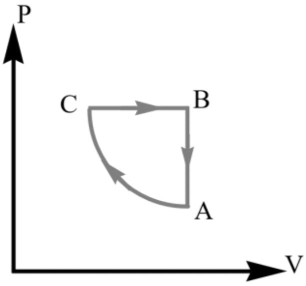
(A) K (B) $\frac{1}{K}$ (C) $K(\theta_0 - T_0)$ (D) data insufficient

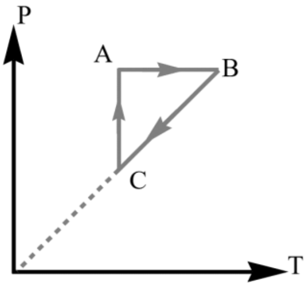
10. A cyclic process of an enclosed gas of constant mass is represented by volume (V) against absolute temperature (T) as shown. if P represents pressure, the graph representing the same process can be



(A)


(B)


(C)


(D)


11. A gas of temperature T is enclosed in a container whose walls are (initially) at temperature T_1.

(A) The gas exert a higher pressure on the walls of the container when $T_1 < T$

(B) The gas exert a higher pressure on the walls of the container when $T_1 > T$

(C) The gas exert same pressure in both cases

(D) None of the above

12. C_P for an ideal gas is $\frac{5}{2}R$. 2 moles of this gas is taken in a thermodynamically insulated system and 300 joules is supplied to the gas. The increase in temperature, is $\frac{10N}{R}$ K. Find N?

13. A Carnot engine works between the temperature of 1092K to 273K is used as heat pump. It consumes 1260 watt of electrical energy and heat is extracted from the sink containing water at $0°C$. Find the rate of freezing of water

(A) 1.25 gm/sec (B) 2 gm/sec (C) 3 gm/sec (D) 5 gm/sec

14. Number of collisions of molecules of a gas on the wall of a container per m^2 will

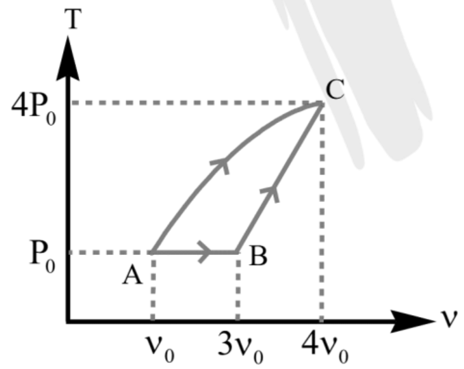
(A) Increase if temperature and volume both are doubled

(B) Increase if temperature and volume both are halved

(C) Increase if pressure and temperature both are doubled

(D) Increase if pressure and temperature both are halved

15. A certain quantity of ideal gas mixture takes up 300J of heat in process AB and its internal energy increases by 1200J in process AC. It's adiabatic constant (γ) is $\frac{\alpha 5}{\beta 2}$. Find $\alpha + \beta$?



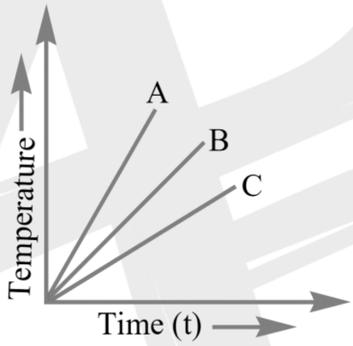
PROFICIENCY TEST-II

1. A thermally insulated vessel contains two liquids A and B with initial temperatures T_1 and T_2 and specific heats S_1 and S_2 respectively. They are separated by a non-conducting piston. When the partition is removed the final equilibrium temperature is average of the initial temperatures of the liquids. The ratio of masses of two liquids $\frac{m_1}{m_2}$ is :

(A) $\frac{s_2}{s_1}$ (B) $\frac{s_1}{s_2}$

(C) $\sqrt{\frac{s_1}{s_2}}$ (D) $\sqrt{\frac{s_2}{s_1}}$

2. Which of the substances A, B or C has the highest specified heat ? The temperature vs time graph is shown. Equal heat is supplied to all substances and the masses of all substances are equal.



(A) A

(B) B

(C) C

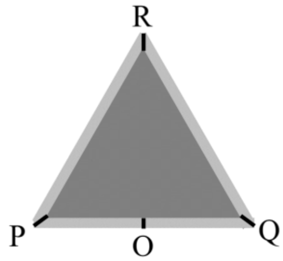
(D) All have equal specific heat

3. 2 kg of metal at $100°C$ is cooled by 1 kg of water at $0°C$. If specific heat capacity of metal is $\frac{1}{2}$ of specific heat capacity of water, final temperature of mixture would be : $(\text{in } °C)$

4. In a calorimeter, 1 kg water is filled at the room temperature. If 12 kcal heat is given to the water-calorimeter system by a heater, the temperature of the system increase by $8°C$. Now an unknown liquid of mass 1 kg is filled in the another identical calorimeter at room temperature. If we give the same amount of heat to this system, the temperature increases by $15°C$. Specific heat capacity of the unknown liquid (in cal/gm-$°C$) is $\frac{3}{n}$. Find n?

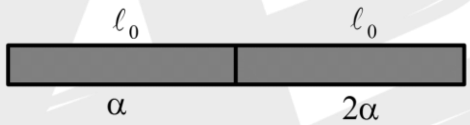
($S_{water} = 1\ \text{cal/gm}°C$, neglect any heat loss to the surrounding)

5. Three rods of equal length ℓ are joined to form an equilateral triangle PQR. O is the midpoint of PQ. Distance OR remains same for small change in temperature. Coefficient of linear expansion for PR and RQ is same, i.e., α_2 but that for PQ is α_1. Then :



(A) $\alpha_2 = 4\alpha_1$ (B) $\alpha_2 = 2\alpha_1$ (C) $\alpha_1 = 4\alpha_2$ (D) $\alpha_1 = 2\alpha_2$

6. Two uniform rods of equal lengths (ℓ_0) and equal masses have coefficient of linear expansion α and 2α are placed in contact on a smooth horizontal surface as shown. The temperature system is $\theta°C$. Now the temperature is increased by $\Delta\theta°C$. The junction of the rods will shift from its initial position by :

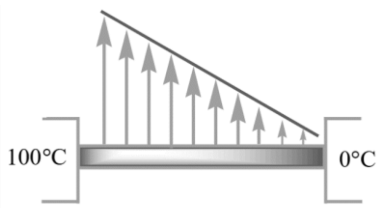


Initial temperature $= \theta°C$

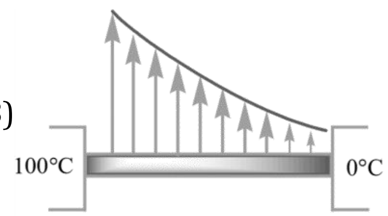
(A) $\frac{\ell_0\alpha\Delta\theta}{2}$ (B) $\frac{\ell_0\alpha\Delta\theta}{3}$ (C) $\frac{\ell_0\alpha\Delta\theta}{4}$ (D) $\frac{\ell_0\alpha\Delta\theta}{6}$

7. A conducting cylindrical rod of uniform cross-sectional area is kept between two large chambers which are at temperatures $100°C$ and $0°C$ respectively. The conductivity of the rod increases with x, where x is distance from $100°C$ end. The temperature profile of the rod in steady-state will be as:

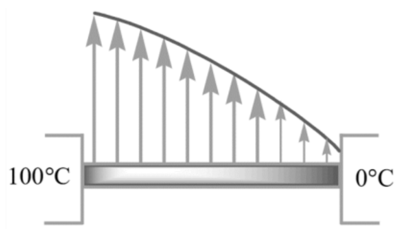
(A)



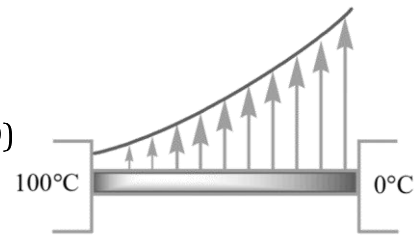
(B)



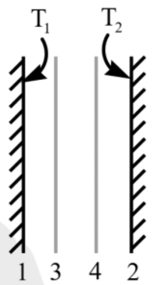
(C)



(D)



8. Two large black plane surfaces are maintained at constant temperature T_1 and $T_2 (T_1 > T_2)$. Two thin black plates are placed between the two surfaces and in parallel to these. After some time, steady conditions are obtained. What is the ratio of heat transfer rate between plate-1 and plate-3 to the ratio of original (when plate-3 and plate-4 was not present) heat transfer rate between plate-1 and plate-2 (η) in steady state?



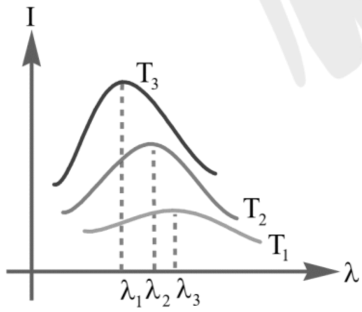
(A) $\eta = \frac{1}{2}$ (B) $\eta = \frac{1}{3}$ (C) $\eta = 1$ (D) $\eta = 0$

9. A hollow cylindrical shell of inner radius $R_1 = 1m$ and outer radius $R_2 = 2m$ is placed inside a heat reservoir of temperature $T_0 = 100°C$. The cylindrical shell is initially filled with water at $50°C$. The thermal conductivity of the material $K = 4200 \ln 2 W/mK$ and its heat capacity is negligible. The time required to raise the temperature of water to $75°C$ is $\alpha \ln 2$. Find α?

[Take specific heat of water $S = 4.2 kJ/kg°C$. Density of water is $1000 kg/m^3$.]

10. Assuming Newton's law of cooling to be valid. The temperature of body changes from $60°C$ to $40°C$ in 7 minutes. Temperature of surroundings being $10°C$, its temperature after next 7 minutes, is (in degrees)

11. According to Wien's displacement law



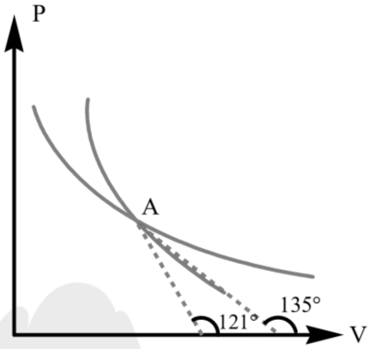
(A) total area under the three graphs A_1, A_2, A_3 are equal

(B) areas are in the ratio: $A_1 : A_2 : A_3 = T_1^2 : T_2^2 : T_3^2$

(C) areas are in the ratio: $A_1 : A_2 : A_3 = T_1^4 : T_2^4 : T_3^4$

(D) $T_3 > T_2 > T_1$

12. A gas undergoes an adiabatic process and an isothermal process. The two processes are plotted on a $P-V$ diagram. The resulting curves intersect at a point A. Tangents are drawn to the two curves at A. These make angles of 135° and 121° with the positive V – axis. The gas is likely to be $\left(\cos 59^\circ = \frac{3}{\sqrt{34}}\right)$

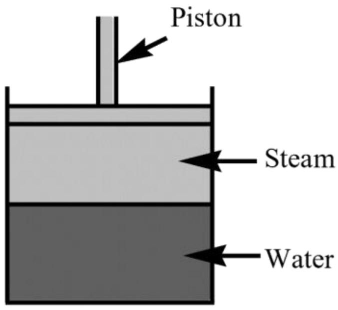


(A) Monoatomic (B) Diatomic
(C) Triatomic (D) A mixture of monoatomic and diatomic gases

13. An ideal gas can be expanded from an initial state to a certain volume through two different processes. (i) $PV^2 = K$ and (ii) $P = KV^2$, where K is a positive constant. Then, choose the correct option from the following

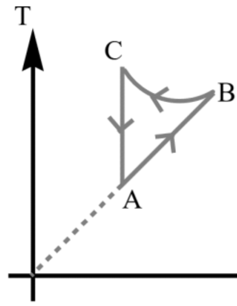
(A) Final temperature in (i) will be greater than in (ii)
(B) Final temperature in (ii) will be greater than in (i)
(C) Work done by the gas in both the processes would be equal
(D) Total heat given to the gas in (i) is greater than in (ii)

14. A cylinder fitted with a metal piston contains water and steam at constant temperature as shown in figure. The cross – sectional area of the cylinder is 2.5cm^2. The piston is falling slowly at a rate of 0.25cms^{-1}. The density of steam under the piston is $6.0 \times 10^{-4}\text{gcm}^{-3}$. Find the rate with which heat is leaving the chamber. (Take: Latent heat of vaporization for water $= 2250 \times 10^3 \text{Jkg}^{-1}$)

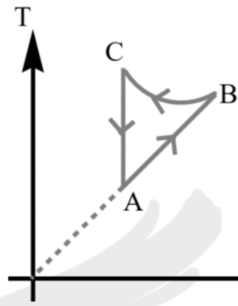


(A) $\frac{13}{16}$ J / s (B) $\frac{16}{13}$ J / s (C) $\frac{32}{27}$ J / s (D) $\frac{27}{32}$ J / s

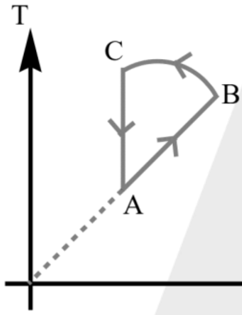
15. Two moles of an ideal gas is initially in state A having pressure $1.01\times10^{5}\,N/m^{2}$ and temperature 300K. Keeping pressure constant, the gas is taken to state B. Temperature at B is 500K. The gas is then taken to state C in such a way that its temperature increases and volume decreases. Also from B to C the magnitude of $\frac{dT}{dV}$ increases. The volume of gas at C is equal to volume of gas at A. Now the gas is taken to initial state A keeping volume constant. A total of 1200J of heat is with drawn from the sample in the cyclic process. The T-V graph for the cyclic process and work done in path B to C are respectively. (Take $R=8.3\,J/k\,mol$)

(A) 

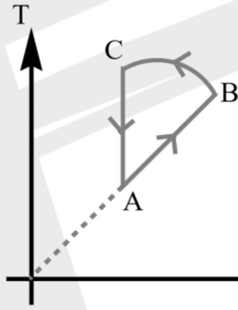
and -4520J

(B) 

and -460J

(C) 

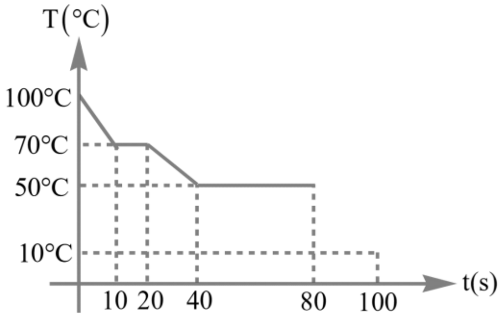
and -4520J

(D) 

and -460J

PROFICIENCY TEST-III

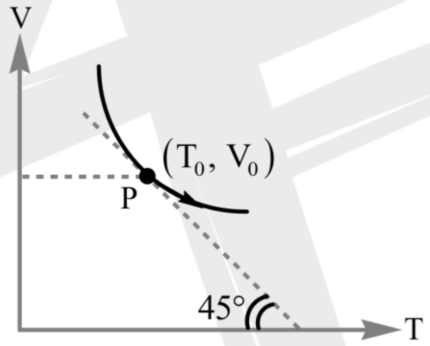
1. A substance is heated by a constant power its temperature vs time graph is shown. The value of $S_{solid} : S_{liquid} : S_{gas}$ is :



(A) 2:3:3
(B) 3:6:2
(C) 2:6:3
(D) 1:3:2

2. Figure shows the adiabatic curve for 2 moles of an ideal gas.

The Bulk modulus $\left(\text{i. e.}, B = \frac{dp}{-dV/V}\right)$ at the point P will be :

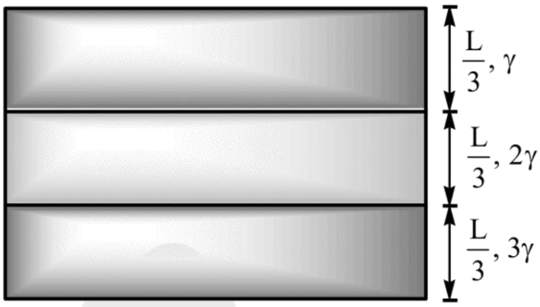


(A) $R\left[1-\frac{T_0}{V_0}\right]$
(B) $2R\left[1+\frac{T_0}{V_0}\right]$
(C) $R\left[1+\frac{T_0}{V_0}\right]$
(D) None of these

3. Two rods of length ℓ (placed on left) and $n\ell$ of different metals having same area of cross-section are placed between two unyielding supports. The coefficients of thermal expansion for two rods are α_1 and α_2 respectively. If the temperature of rods is raised by $t°C$ and the ratio of strains of two rods is ε_r, find the distance by which joint of the two rods may move towards right, if the supports unyield :

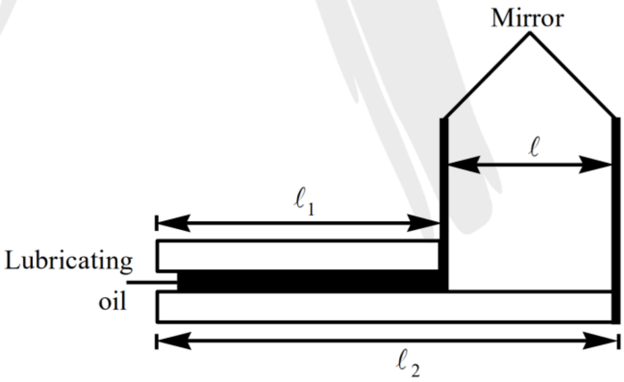
(A) $\frac{\ell t(\alpha_1-\varepsilon_r\alpha_2)}{\left(1+\frac{\varepsilon_r}{n}\right)}$
(B) $\frac{\ell t(\alpha_2-\varepsilon_r\alpha_1)}{\left(1+\frac{\varepsilon_r}{n}\right)}$
(C) $\frac{\ell t(\alpha_1-\varepsilon_r\alpha_2)}{\left(1-\frac{\varepsilon_r}{n}\right)}$
(D) $\frac{\ell t(\alpha_2+\varepsilon_r\alpha_1)}{\left(1-\frac{\varepsilon_r}{n}\right)}$

4. Three immiscible liquids are filled in a container as shown. The base area of the container is A and coefficient of cubical expansion of the material of the container is $\frac{3\gamma}{2}$ while the coefficient of cubical expansion of the liquids are shown in the figure. The temperature of the system is increased by ΔT. The volume of the liquid flown out of the container is :



(A) $\frac{A\ell\gamma\Delta T}{5}$ (B) $\frac{A\ell\gamma\Delta T}{4}$ (C) $\frac{2A\ell\gamma\Delta T}{3}$ (D) $\frac{A\ell\gamma\Delta T}{2}$

5. An optical engineering firm needs to ensure that the separation between two mirrors is unaffected by temperature changes. The mirrors are attached to the ends of two bars of different materials that we welded together at one end as shown in figure. The surfaces of the bars in contact are lubricated. The distance ℓ does not change with temperature change. ℓ_1 and ℓ_2 are the length of the bars α_1 and α_2 are the respective thermal coefficients of temperature. Which of the following options is/are correct?

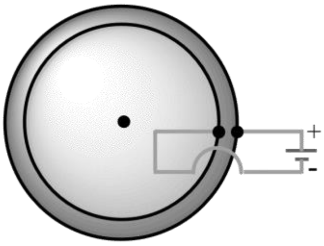


(A) $\ell_1 = \frac{\ell\alpha_2}{\alpha_2 - \alpha_1}$ (B) $\ell_2 = \frac{\ell\alpha_1}{\alpha_2 - \alpha_1}$

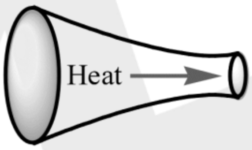
(C) $\alpha_1\ell_1 = \alpha_2\ell_2$ (D) $\alpha_1\ell_2 = \alpha_2\ell_1$

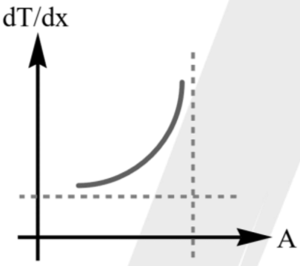
6. Ice starts forming in a lake with water at $0°C$ when the atmospheric temperature is $-10°C$. If the time taken for the first 1 cm of ice to be formed is 7 hour, then the time (in hour) taken for the thickness of ice to change from 1 cm to 2 cm is:

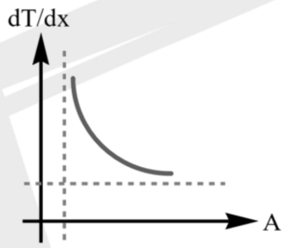
7. A shell, made of material of electrical conductivity $\frac{10^9}{\pi}(\Omega - m)^{-1}$, has thickness t = 2mm and radius R = 10cm. in an arrangement, its inside surface is kept at a lower potential than its outside surface. The resistance offered by the shell is equal to $n \times 10^{-11}\Omega$. Find n?

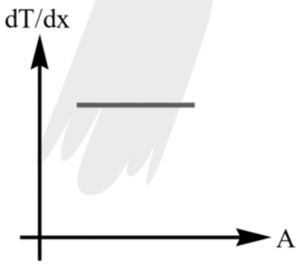


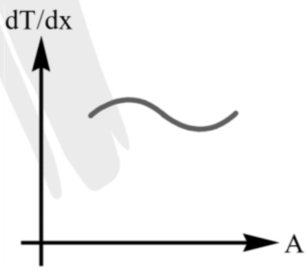
8. An irregular rod of same uniform material as shown in figure is conducting heat at a steady rate. The temperature gradient at various sections versus area of cross-section graph will be:



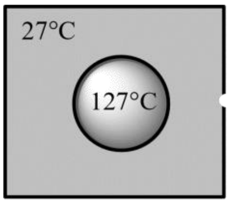
(A)


(B)


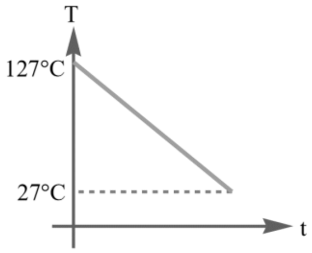
(C)


(D)


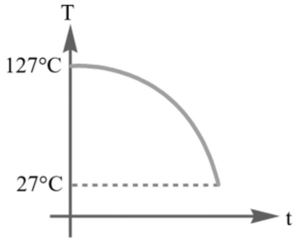
9. A silver ball, painted black is kept inside a box which is maintained at a temperature of $27°C$. The ball is maintained initially at a constant temperature of $127°C$ by making the radiation to fall on it through a small hole in the box. Latter on due to some chemical reaction between silver and paint, the paint uniformly evaporates from the surface of ball exposing the silver. If same amount of radiation continues to fall on ball, then temperature of ball as a function of time is shown as: (Assume emissivity of silver is zero and paint to be black body also assume radiation to be the only mode of heat transfer)



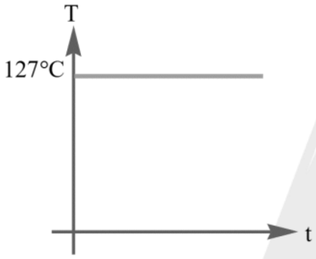
(A)



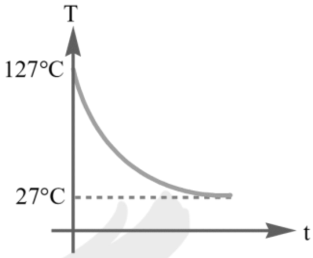
(B)



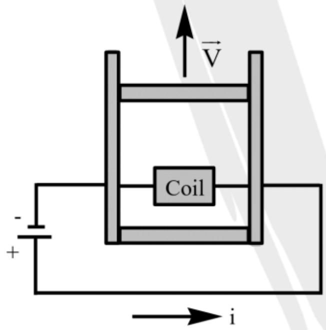
(C)



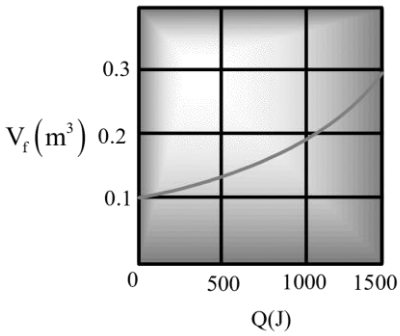
(D)



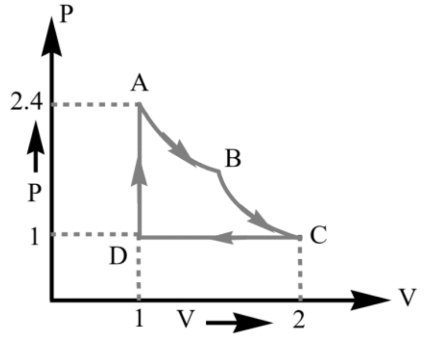
10. In figure, a resistance coil, wired to an external battery, is placed inside a thermally insulated cylinder fitted with a frictionless piston and containing an ideal diatomic gas. A current i = 200mA exists in the coil, which has a resistance$R = 350\Omega$. If the pressure of the gas remains constant, what should be the speed V (in cm/s) of the piston, of mass$m = 10kg$? (Take$p_{atm} = 10^5 Pa, A = 10cm^2$)



11. Suppose 0.5 mole of an ideal gas undergoes an isothermal expansion as energy is added to it as heat Q. Graph shows the final volume $V_f$versus Q. The temperature of the gas (in K) is (use $\ln 9 = 2$and $R = \frac{25}{3} J/mol - K$)



12. 100 mole of an ideal monoatomic gas undergoes the following thermodynamic process as shown in the figure. (P-V or pressure – Volume plots are shown)



A → B: Isothermal expansion

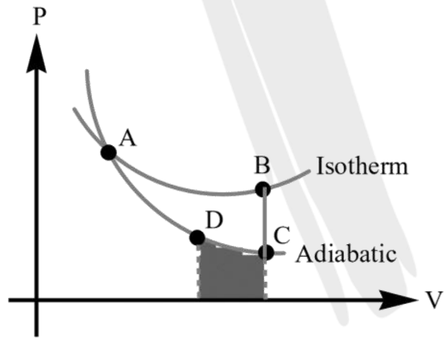
B → C: Adiabatic expansion

C → D: Isobaric compression

D → A: Isochoric process

The heat transfer along the process AB is 9×10^4J. The net work done by the gas during the cycle is $5 \times 10^{\alpha}$J. Find α? (Take $R = 8 J.K^{-1} mole^{-1}$)

13. Figure shows various processes in the PV – plane for an ideal gas. The process ADC is an adiabatic, AB is an isotherm, and BC is a constant – volume process. The heat added to the gas along AB is 400cal. The change in internal energy from C to A is +1000cal, and the work done from D to C is 150cal



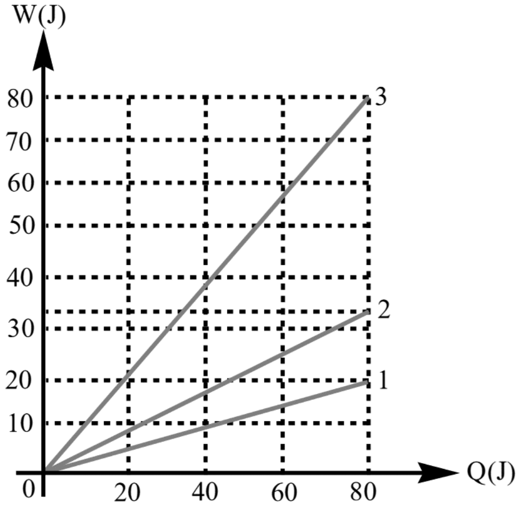
(A) Work done by the gas from A to B is 400cal

(B) Heat added to the system from B to C is -1000cal

(C) The change in internal energy from C to D is 850cal

(D) Work done by the gas from D to A is 850cal

14. In the figure shown, the amount of heat supplied to one mole of an ideal gas is plotted on the horizontal axis and the amount of work performed by the gas is drawn on the vertical axis. One of the straight lines in the figure is an isotherm and the other two are isobars of two gases. The initial states of both gases are same. Mark the correct statement(s).



(A) Curve 3 corresponds to isothermal process

(B) Curve 1 corresponds to a polyatomic gas

(C) Curve 2 corresponds to a monoatomic gas

(D) Process 1 and 2 are adiabatic process

15. A monoatomic gas is kept in a vessel at some finite temperature. Choose the correct statement(s)

(A) Number of atoms moving with speed equal to half of RMS speed will be more than number of atoms moving with speed equal to one third of RMS speed

(B) Average velocity of atoms of gas has magnitude equal to $\sqrt{\frac{8RT}{\pi M}}$

(C) Number of atoms moving with average speed will be more than number of atoms moving with RMS speed

(D) If its temperature (in degree Celsius) is increased to 4 times then its RMS speed will increase by a factor less than 2

ANSWERKEY

EXERCISE-I_KEY

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
A	10	B	A	B	5	B	14	B	ABCD
11	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
B	A	A	1450	10	A	C	A	A	B
21	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
AC	D	B	B	C	C	600	10	AC	ABD
31									
AC									

EXERCISE-II_KEY

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
10	D	4	D	10	D	A	C	AD	B
11	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
ACD	AC	A	D	C	A	C	256	D	B
21	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
D	A	11	C	5	C	B	C	5	AD
31	**32**								
AD	AD								

EXERCISE-III_KEY

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
A	C	1	D	A	D	C	BCD	AC	20
11	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
13	A	A	B	D	C	6	A	B	C
21	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
7	C	D	B	C	C	B	B	100	ABC

EXERCISE-IV_KEY

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
B	280	3	C	C	B	ABC	AD	BD	ACD
11	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
C	D	A	40	A	B	A	B	4	A
21	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
A	31	8	550	B	B	C	B	B	ABCD

EXERCISE-V_KEY

1	2	3	4	5	6	7	8	9	10
A	A	A	10	C	B	A	A	BD	ABD
11	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
C	A	C	99	D	A	C	A	CD	AD
21	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
A	B	D	B	5	A	C	C	A	A

PROFICIENCY TEST-I_KEY

1	2	3	4	5	6	7	8	9	10
3500	A	B	C	AD	10	B	D	B	D
11	**12**	**13**	**14**	**15**					
B	10	A	BC	7					

PROFICIENCY TEST-II_KEY

1	2	3	4	5	6	7	8	9	10
A	C	50	10	C	C	B	B	500	28
11	**12**	**13**	**14**	**15**					
D	A	B	D	A					

PROFICIENCY TEST-III_KEY

1	2	3	4	5	6	7	8	9	10
B	B	B	D	C	21	5	D	C	2
11	**12**	**13**	**14**	**15**					
360	4	AB	ABC	ACD					