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Hyperbola

(1) $ax^2+by^2+2hxy+2gx+2fy+c=0$

$\Delta \neq 0,\ h^2>ab \rightarrow$ hyp

(2) $ecc. = e = \frac{SP}{PM} > 1$ (hyp.)

$SP = ePM$

(3) Ellipse $\rightarrow PF_1+PF_2=2a$ (Req.)

Hyperbola $\Rightarrow |PF_1-PF_2| = 2a$ (Req.)

$\{2c>2a\}$

$\left|\sqrt{(x+c)^2+y^2}-\sqrt{(x-c)^2+y^2}\right| = 2a$

After soln $\Rightarrow \frac{x^2}{a^2}-\frac{y^2}{c^2-a^2}=1$

(4) here $c^2-a^2=b^2$ Replace

$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

(5) $e=\frac{c}{a}$

$e^2=\frac{c^2}{a^2}=\frac{a^2+b^2}{a^2}$

$\boxed{e^2=1+\frac{b^2}{a^2}}$

Q If

$\frac{x^2}{1-r}-\frac{y^2}{1+r}=1$ (Hype)

find $r\in$?

$\rightarrow |r|<1$ as $1-r=+ve$

$\Rightarrow -1<r<1$

as $1-r>0$ & $1+r>0$

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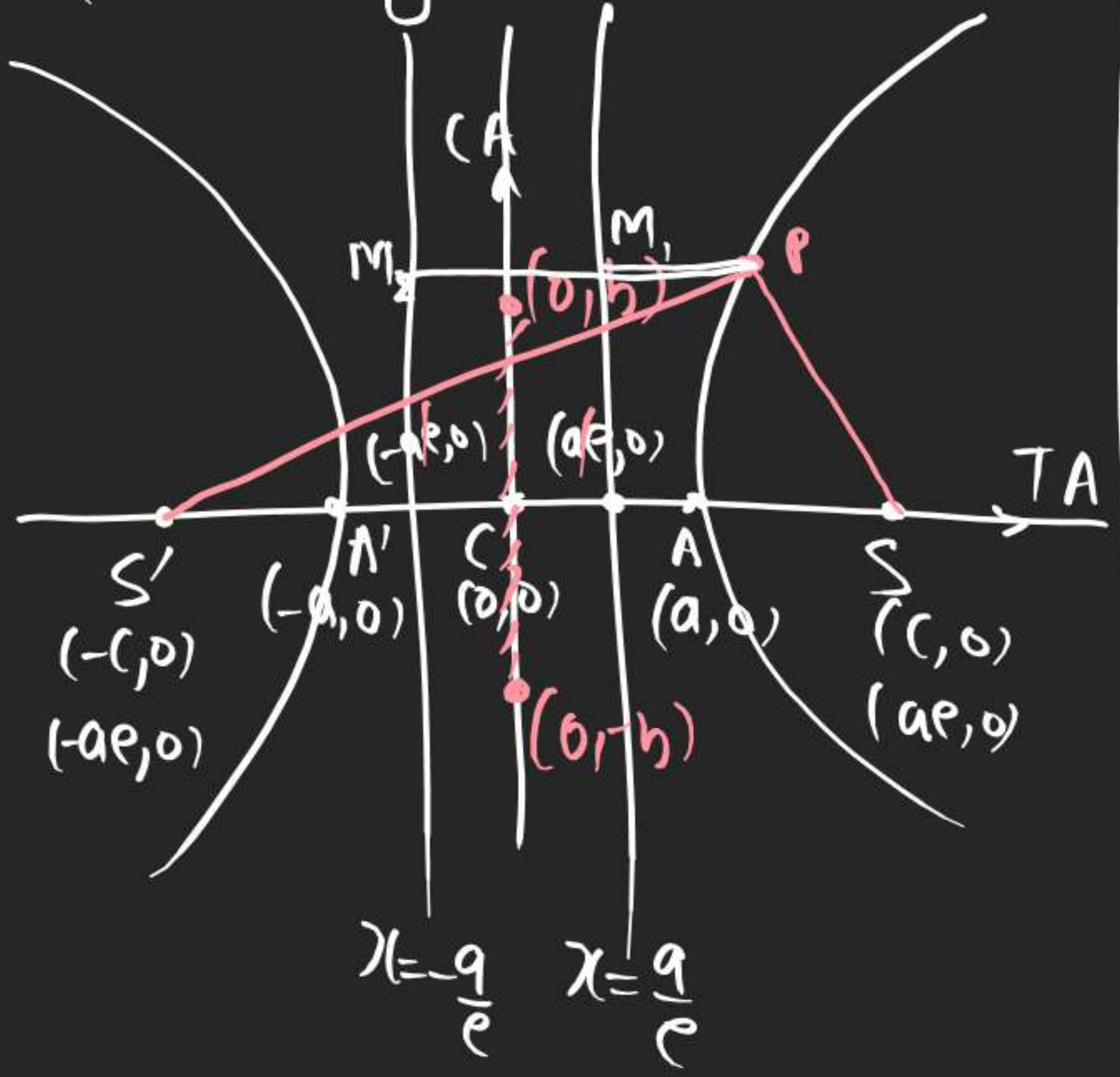
(6) Diagram.



TA = Transverse Axis
CA = Conjugate Axis

(7) A) Line joining S_1S_2 is called Transverse Axis & distance betn S_1S_2 is focal length.

(B) Distance betn both Vertex A & A' is 2a. known as Length of TA = 2a

(C) Sly Length of Conjugate Axis = 2b

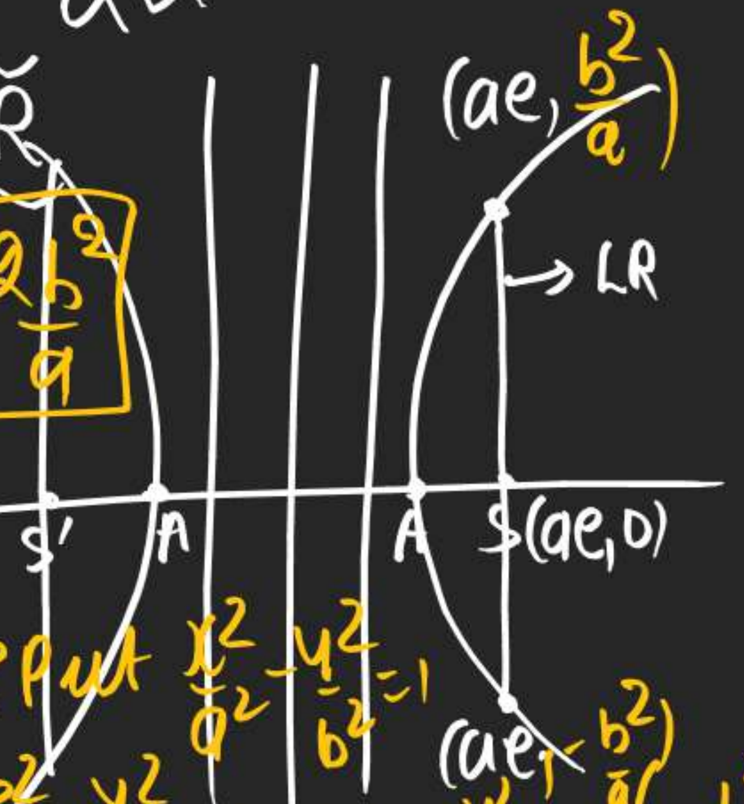
(8) $SP = ePM_1$
$S'P = ePM_2$ } Focal Rad.

(9) Focal dir. Prop.

$$|PF_1 - PF_2| = |ePM_1 - ePM_2|$$
$$= \left|e\left(x-\frac{a}{e}\right) - e\left(x+\frac{a}{e}\right)\right|$$
$$= 2a$$

(10) LR

$$\boxed{LR = \frac{2b^2}{a}}$$



$x = ae$ Put $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

$\frac{a^2e^2}{a^2} - \frac{y^2}{b^2} = 1 \Rightarrow -\frac{y^2}{b^2} = 1 - \left(1 + \frac{b^2}{a^2}\right)$

$\Rightarrow y^2 = \frac{b^4}{a^2} \Rightarrow y = \pm \frac{b^2}{a}$

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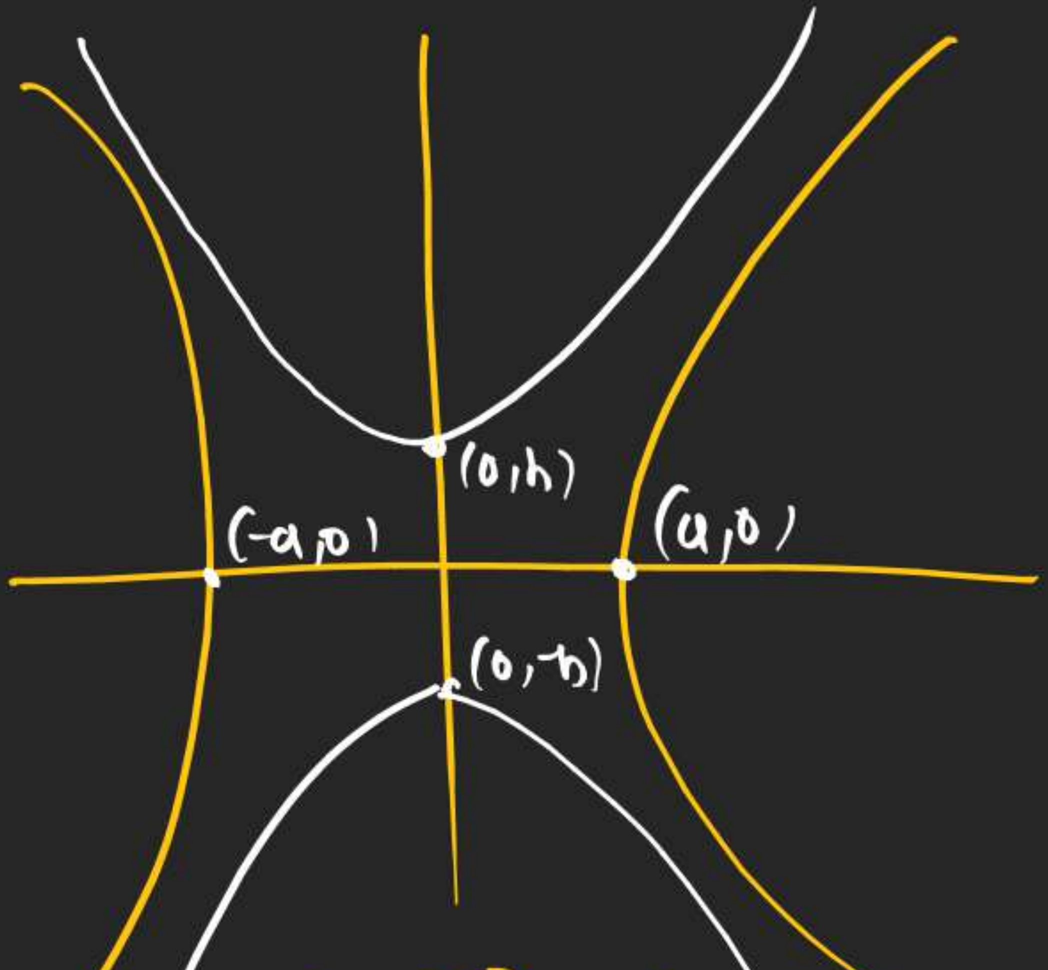
II) Conjugate Hyperbola.



$H: \frac{x^2}{a^2}-\frac{y^2}{b^2}=1 \Rightarrow e=1+\frac{b^2}{a^2}$

$CH: \frac{x^2}{a^2}-\frac{y^2}{b^2}=-1 \Rightarrow e'=1+\frac{a^2}{b^2}$

When CA → TA
& TA → CA
New hyp. becomes
Conjugate hype.

Q If ecc of Hyp $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is e_1 & e_2 is ecc. of its C.H. find $e_1^{-2}+e_2^{-2}=?$

$H: \frac{x^2}{a^2}-\frac{y^2}{b^2}=1 \Rightarrow e_1^2=1+\frac{b^2}{a^2}$

$CH: \frac{x^2}{a^2}-\frac{y^2}{b^2}=-1 \Rightarrow e_2^2=1+\frac{a^2}{b^2}$

$\Rightarrow \frac{1}{e_1^2}=\frac{a^2}{a^2+b^2}$ | $\frac{1}{e_2^2}=\frac{b^2}{a^2+b^2}$

$\frac{1}{e_1^2}+\frac{1}{e_2^2}=\frac{a^2}{a^2+b^2}+\frac{b^2}{a^2+b^2}$

$\boxed{e_1^{-2}+e_2^{-2}=1}$

Q If $\frac{x^2}{\cos^2\alpha}-\frac{y^2}{\sin^2\alpha}=1$ Rep. Hyp. Where α = varies then

~~A)~~ e Remains Const.

B) Abscissa of foci remains Const.

~~C)~~ Eqn of Dir = Const.

~~D)~~ Abs. of vertices = Const.

(A) $e^2=1+\frac{\sin^2\alpha}{\cos^2\alpha}$

$e^2=\sec^2\alpha$
variable

(C) Eq of Dir
$x=\frac{a}{e}=\frac{\cos\alpha}{\sec\alpha}$
$x=\cos^2\alpha$ (var.)

(B) foci Abscissa
$= a.e$
$=\cos\alpha.\sec\alpha=1$

(D) Vertices Abs. $=\pm a$
(var.) $=\pm\cos\alpha$

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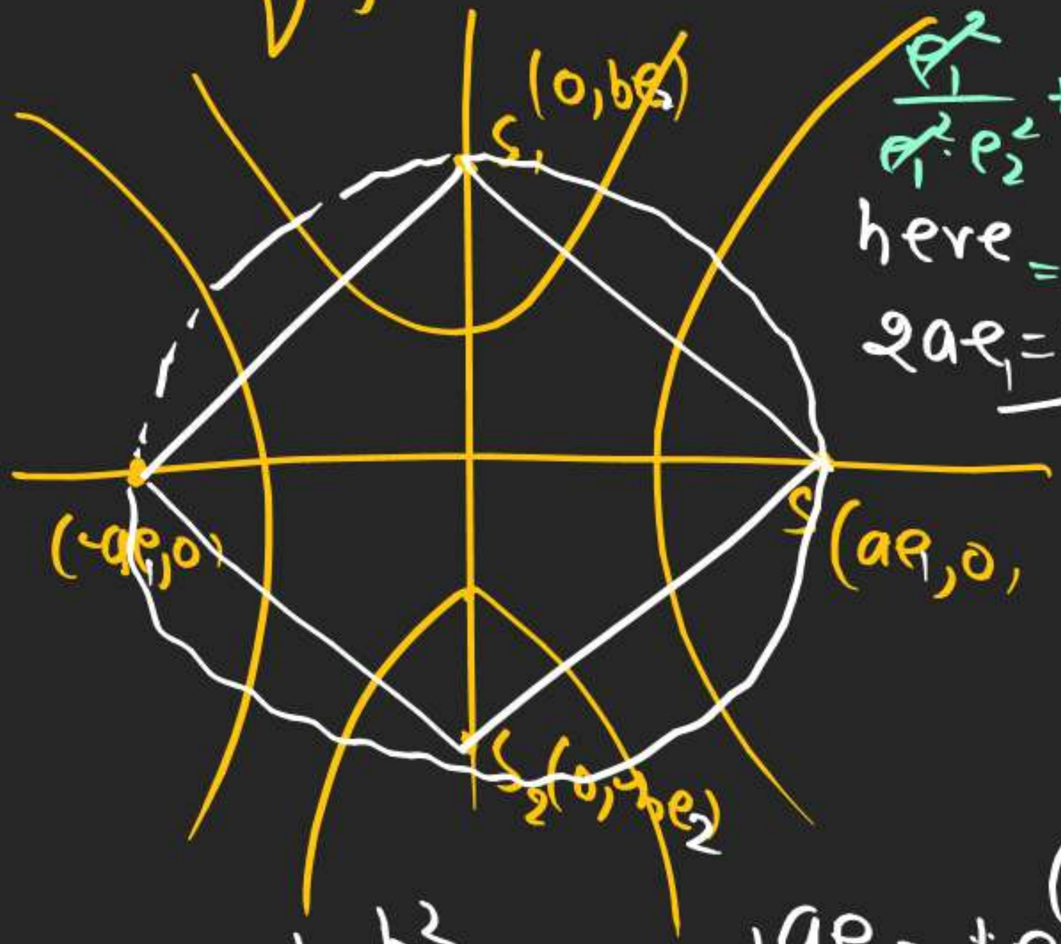
(12) Focii of Hyp. & its C.H. are Concyclic & form vertices of sqr.



$e_1^2+e_2^2=2e_1^2e_2^2$ $+e_1^2e_2^2$

$\frac{\cancel{e_1^2}}{e_1^2e_2^2}+\frac{\cancel{e_2^2}}{e_1^2e_2^2}=\frac{2e_1^2e_2^2}{e_1^2e_2^2}$

here $\Rightarrow \frac{1}{e_1^2}+\frac{1}{e_2^2}=2$

$2ae_1=2be_2$

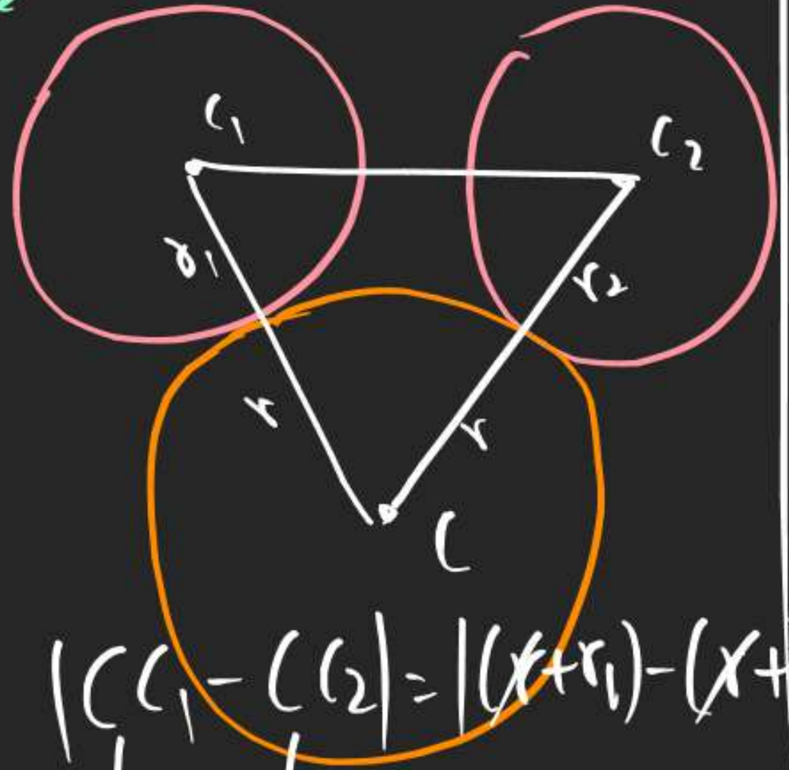
$ae_1=be_2$ (check)

$\frac{e_1}{e_2}=\frac{b}{a}\Rightarrow \frac{e_1^2}{e_2^2}=\frac{b^2}{a^2}$

$\frac{1+\frac{b^2}{a^2}}{1+\frac{a^2}{b^2}}=\frac{b^2}{a^2}$

LHS=RHS

Q P.T. Locus of centre of Circle which touches externally the given Circles is a Hyperbola.



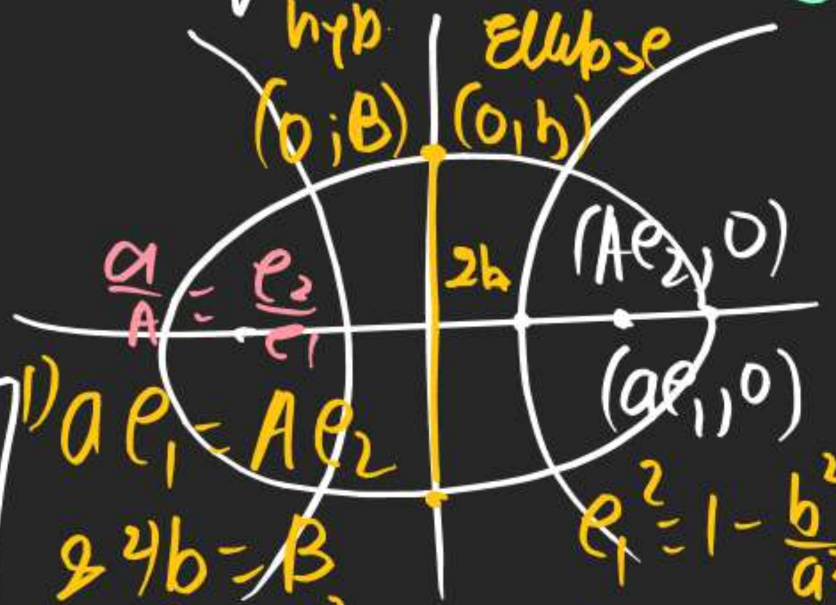
$|CC_1-CC_2|=|(r+r_1)-(r+r_2)|$

$=|r_1-r_2|$

$|PF_1-PF_2|=$ Constant

$\Rightarrow$ Locus of C is hyp.

Q An ellipse & an hyp. are Confocal & C.A of hyp. is equal to Minor axis of ellipse, if e_1 & e_2 are ecc. of ellipse & hyp. find value of $e_1^{-2}+e_2^{-2}$? $=2$



$\frac{a}{A}=\frac{e_2}{e_1}$

1) $ae_1=Ae_2$

& $2b=B$

$\Rightarrow b^2=B^2$

$a^2(1-e_1^2)=A^2(e_2^2-1)$

$e_1^2=1-\frac{b^2}{a^2}$

$e_2^2=1+\frac{B^2}{A^2}$

(3) $\frac{a^2}{A^2}(1-e_1^2)=e_2^2-1\Rightarrow\frac{e_2^2}{e_1^2}(1-e_1^2)=e_2^2-1$

$e_2^2-e_1^2e_2^2=e_1^2e_2^2-e_1^2$

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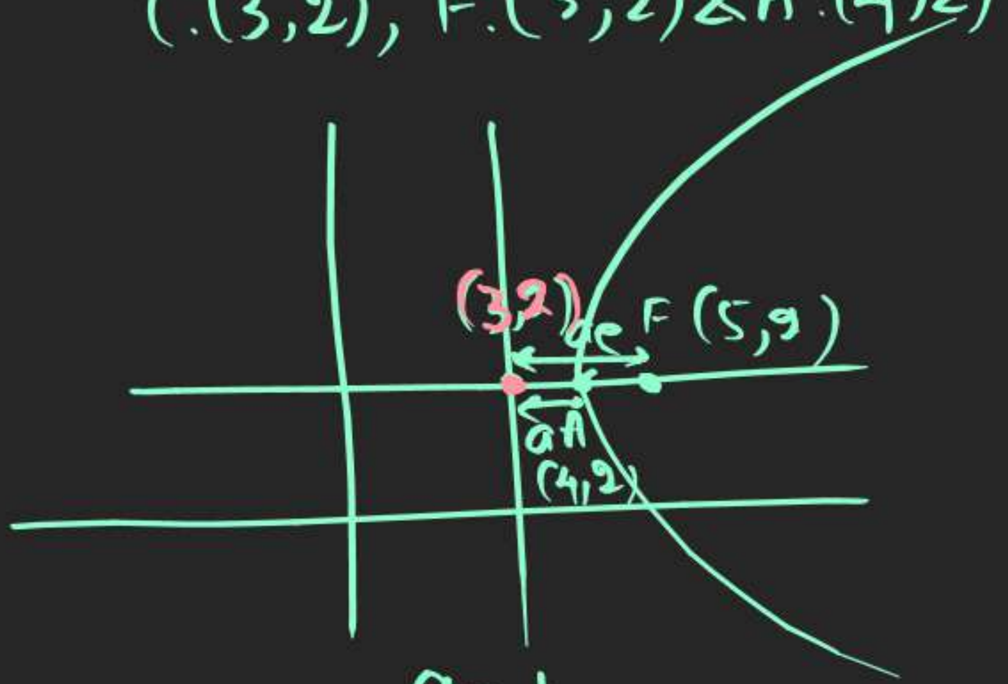
Q find hyp. having

$C:(3,2)$, $F:(5,2)$ & $A:(4,2)$



$a = 1$

$ae = 2$

$e = 2$

$e^2 = 1 + \frac{b^2}{a^2}$

$4 = 1 + \frac{b^2}{1}$

$b^2 = 3$

$$\frac{(x-3)^2}{1} - \frac{(y-2)^2}{3} = 1$$

13) Rectangular Hyperbola.

1) if $a = b$ then hyp is R.H

$$\frac{x^2}{a^2} - \frac{y^2}{a^2} = 1$$

R.H $\boxed{x^2 - y^2 = a^2}$

2) $e^2 = 1 + \frac{b^2}{a^2} = 1 + \frac{a^2}{a^2}$

$e^2 = 2 \Rightarrow e = \sqrt{2}$

	$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$	$x^2 - y^2 = a^2$
Ver	$(\pm a, 0)$	$(\pm a, 0)$
Focus	$(\pm ae, 0)$	$(\pm\sqrt{2}a, 0)$
Focal length =	$2ae$	$2\sqrt{2}a$
Dir	$x = \pm\frac{a}{e}$	$x = \pm\frac{a}{\sqrt{2}}$

CA =	$2b$	$2a$
TA =	$2a$	$2a$
LLR	$\frac{2b^2}{a}$	$2a$.

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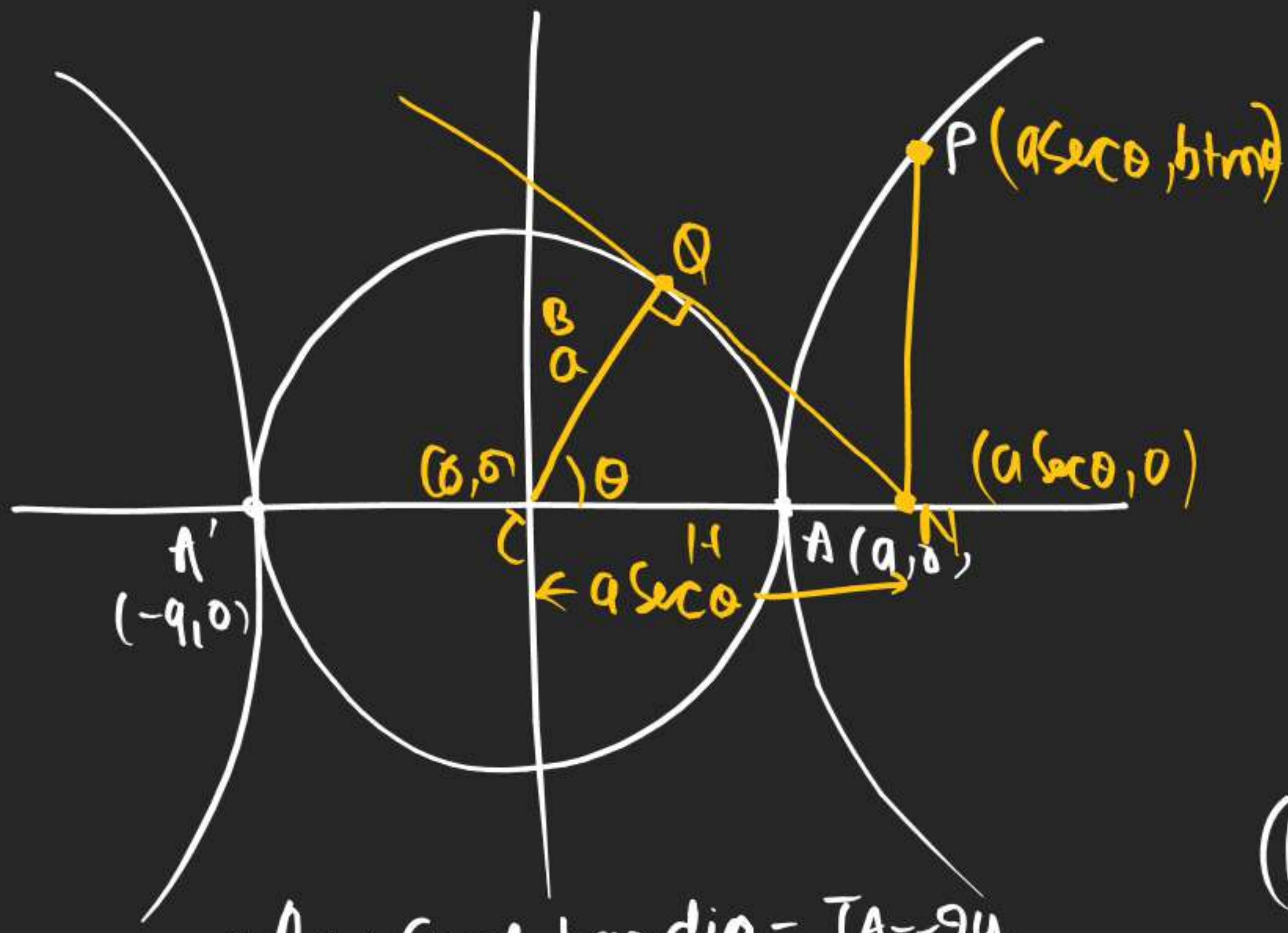
(14) Auxilliary Circle & Ecc. Angle



1) Aux. Circle has dia = TA = 2a

2) Aux Circle : → $x^2+y^2=a^2$

(3) $\cos\theta = \frac{B}{H} = \frac{a}{CN} \Rightarrow CN = a\sec\theta$

(4) $x = a\sec\theta$ in $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

$$\frac{\cancel{a^2}\sec^2\theta}{\cancel{a^2}} - \frac{y^2}{b^2} = 1$$

$$\Rightarrow \sec^2\theta - 1 = \frac{y^2}{b^2}$$

$$\Rightarrow y = b\tan\theta$$

(5) So $(a\sec\theta, b\tan\theta)$ are Parr. Coord of Hyp.

$$0 < \theta < 2\pi$$

(15) Position of Pt. (ULTA)

Hyp(pt.) > 0 inside Hyp
= 0 on Hyp.
< 0 outside Hype

(16) Line & Hyperbola.

LINE ⇒ $y = mx + c$

Hyp ⇒ $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

$$\frac{x^2}{a^2} - \frac{(mx+c)^2}{b^2} = 1$$

$$x^2(\quad) - x(\quad) + \text{Cost} = 0$$

D

D > 0 $\quad$ $\boxed{D = 0}$ $\quad$ D < 0

(OT

$$\boxed{c = \pm\sqrt{a^2m^2 - b^2}}$$