

$$2 = 2, 4 = 2, 2 = 1$$

A large, hand-drawn oval containing the numbers 1, 2, 3, and 4. The numbers are written in a simple, slightly irregular style. The number 1 is at the top right, 2 is at the bottom left, 3 is in the center, and 4 is at the top left. The oval is drawn with a single continuous line.

$$\sum_{r=1}^{1000} \frac{1}{r^3}$$

$$\sqrt[2]{1000}^{1/3} < \sqrt[3]{1000}$$

~~$f'(u) \leq 0$~~

~~$(0, 2) - \{ \sqrt{2} \}$~~

~~999 1000~~

Handwritten graph of  $y = 2^x$  showing the area under the curve from  $x=0$  to  $x=3$ , approximated by rectangles. The rectangles have widths of 1 unit. The heights are labeled as  $2^0$ ,  $2^1$ ,  $2^2$ ,  $2^3$ . The area is approximated by the sum of the areas of these rectangles:  $1 \cdot 2^0 + 1 \cdot 2^1 + 1 \cdot 2^2 + 1 \cdot 2^3 = 1 + 2 + 4 + 8 = 15$ . The curve is labeled  $y = 2^x$ . The x-axis is labeled 0, 1, 2, 3. The y-axis is labeled 1, 2, 4, 8.

$$\frac{1}{2} \times \left( 1 + (1+2^{\frac{1}{3}})^{1+2} \right) + \dots$$

$$\begin{array}{r} 1 \\ 2 \end{array} \times \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

$$\frac{1}{2} \times \left( 1 + \frac{1+2^3}{1+2} \right)$$

$\frac{1}{x^2} = x^{-2}$

2<sup>5</sup> 3<sup>2</sup> 5<sup>1</sup> 7<sup>1</sup>

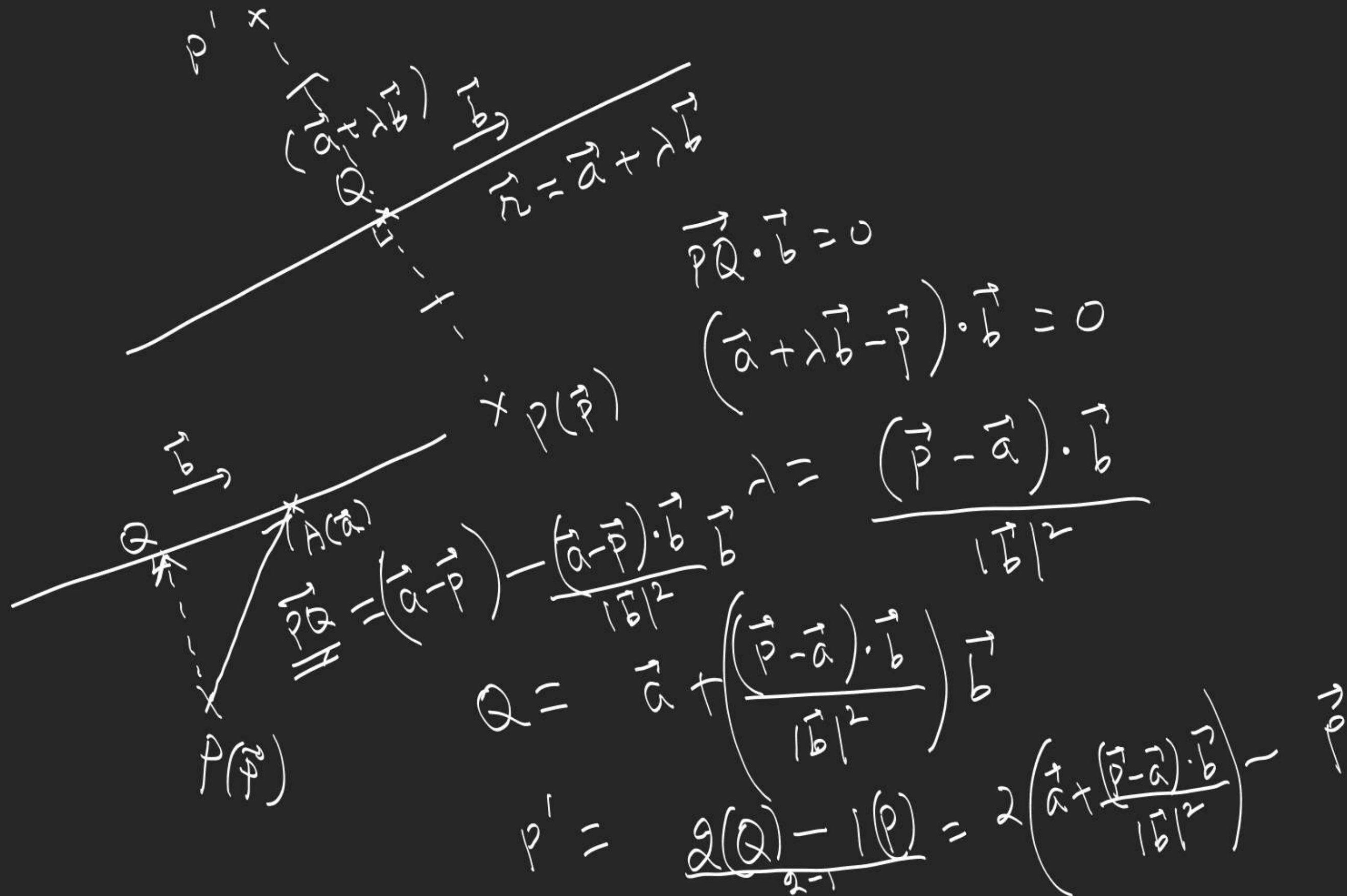
~~$f'(n) < 0$~~

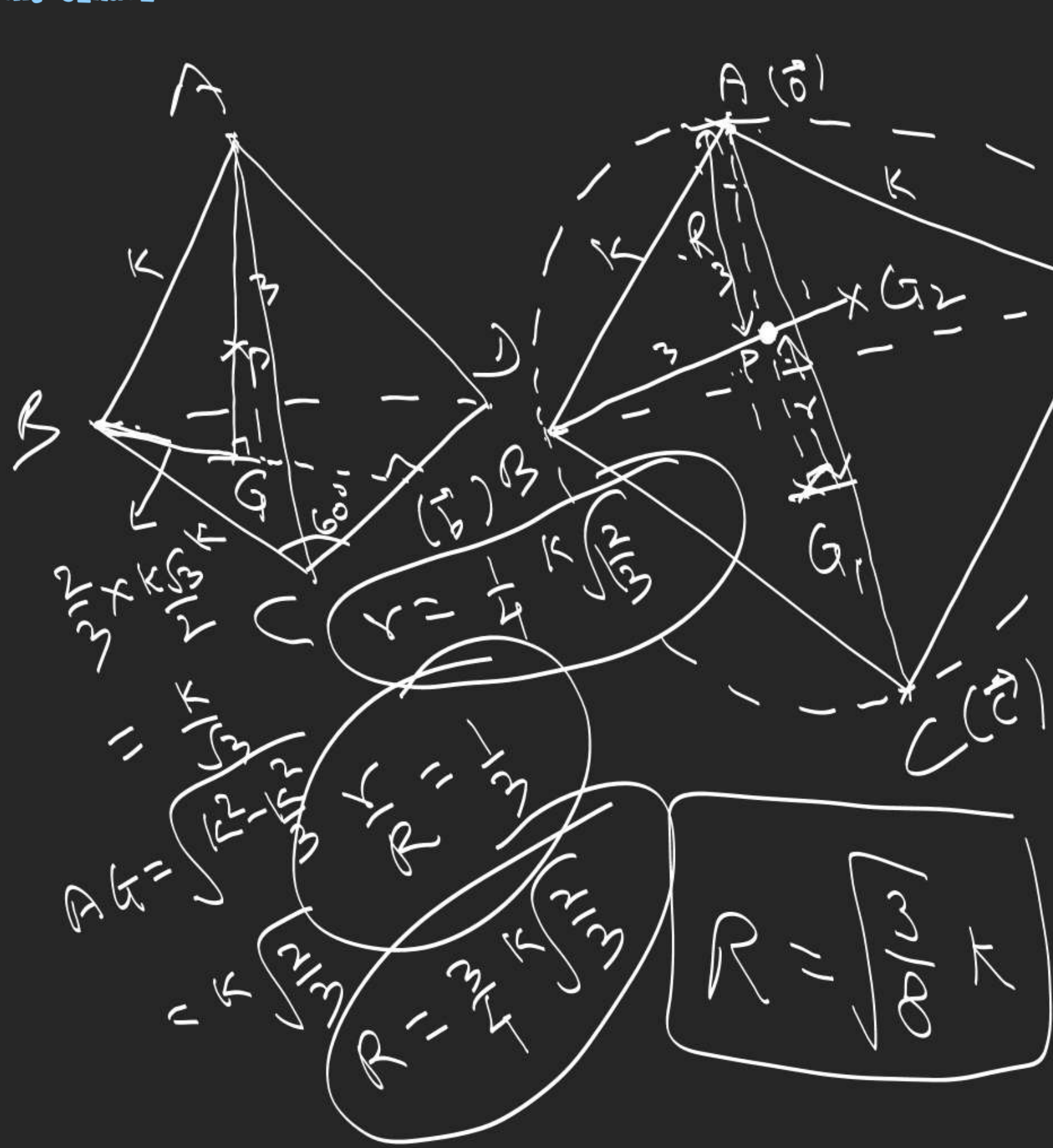
$$\left( \frac{1}{3} + 1000^{\frac{1}{3}} \right) < \frac{3}{4} (10000)$$

$$f'\left(\frac{x+y}{2}\right) = \frac{f'(x) + f'(y)}{2}$$

$$f(y+1) - f(y) = f'\left(y + \frac{1}{2}\right)$$

Diagram illustrating the Mean Value Theorem (MVT) for the function  $f$  on the interval  $[y, y+1]$ . The expression  $f(y+1) - f(y)$  is shown with arrows pointing down to the interval  $[y, y+1]$ . The derivative  $f' is shown in a circle, representing the value of the derivative at the midpoint of the interval.$





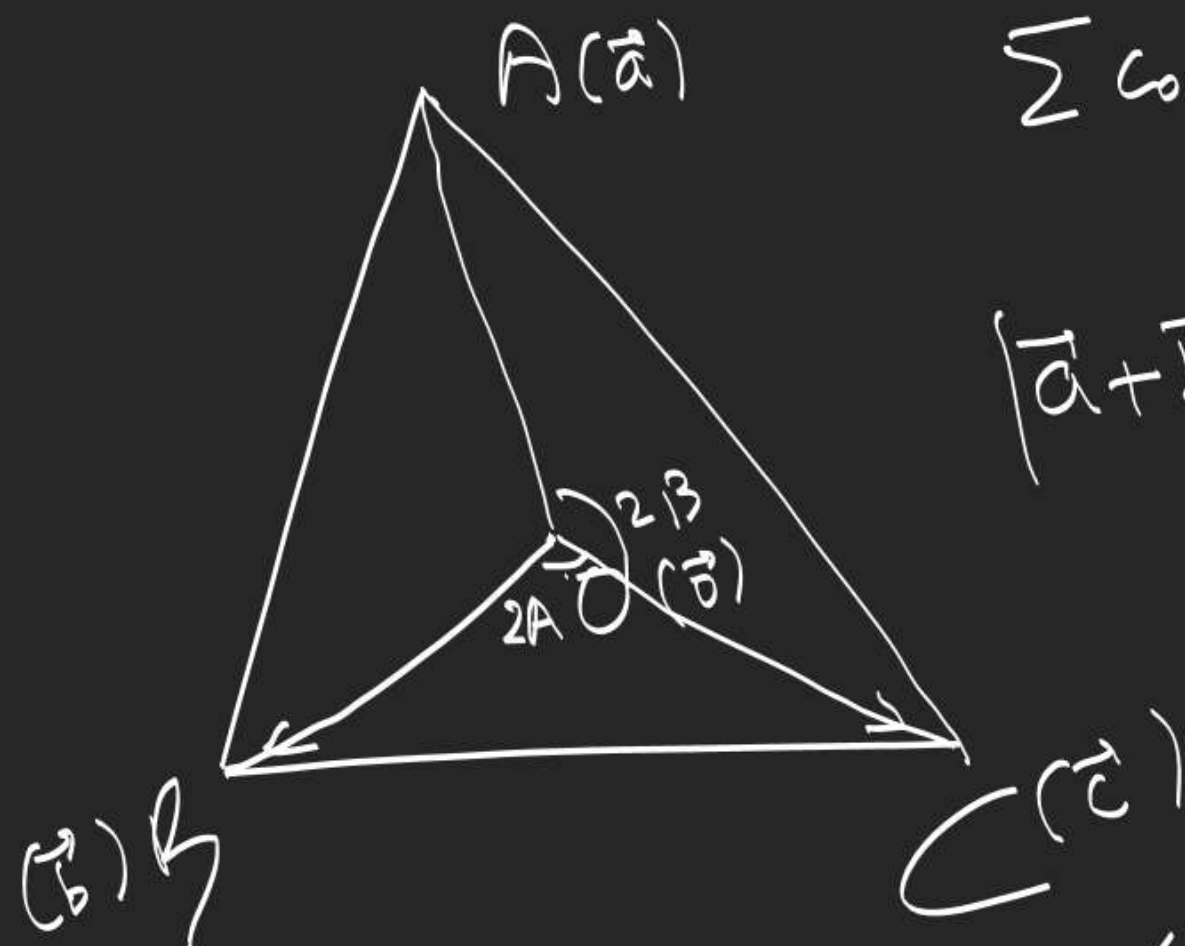
$$\vec{P} = \frac{\vec{b} + \vec{c} + \vec{d}}{4}$$

$$R = |\vec{AP}| = \frac{|\vec{b} + \vec{c} + \vec{d}|}{4}$$

$$R^2 = \frac{1}{16} \left[ \sum |\vec{b}|^2 + 2 \sum \vec{b} \cdot \vec{c} \right]$$

$$= \frac{1}{16} \left[ 3K^2 + 2 \left( K \times K \times \frac{1}{2} \times 3 \right) \right]$$

$$= \frac{6}{8} K^2$$



$$\sum \cos 2A$$

$$|\vec{a} + \vec{b} + \vec{c}|^2 = \sum |\vec{a}|^2 + 2 \sum \vec{a} \cdot \vec{b} \geq 0$$

$$3R^2 + 2R^2 \sum \cos 2A \geq 0$$

$$\begin{aligned} |\vec{OG}|^2 &= \frac{1}{9} |\vec{a} + \vec{b} + \vec{c}|^2 = \frac{1}{9} (3R^2 + 2R^2 \sum (1 - 2\sin^2 A)) \\ &= \frac{1}{9} (9R^2 - (a^2 + b^2 + c^2)) \end{aligned}$$

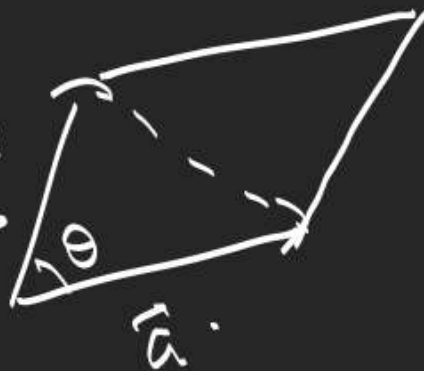
# Vector (Cross) Product

$$\vec{a} \times \vec{b} = |\vec{a}| |\vec{b}| \sin \theta \hat{n}$$

$\hat{n}$  = unit vector along  $\vec{a} \times \vec{b}$

$$\theta = \vec{a} \wedge \vec{b}$$

$|\vec{a} \times \vec{b}|$  = Area of  $\parallel$  gm with  $\vec{a}, \vec{b}$  as adjacent sides



$$\vec{a} \times \vec{b} = -\vec{b} \times \vec{a}$$

$$\vec{a} \times (\vec{b} + \vec{c}) = \vec{a} \times \vec{b} + \vec{a} \times \vec{c}$$

$$\hat{i} \times \hat{i} = \vec{0} = \hat{j} \times \hat{j} = \hat{k} \times \hat{k}$$

$$\hat{i} \times \hat{j} = \hat{k}$$

$$\hat{j} \times \hat{i} = -\hat{k}$$

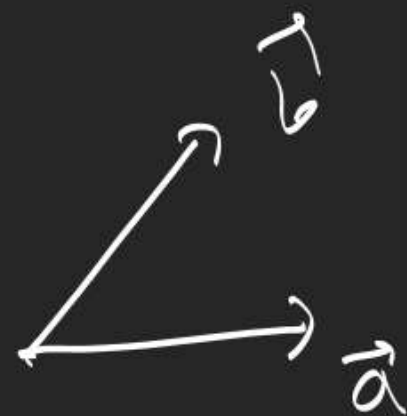


$$\vec{a} = a_1 \hat{i} + a_2 \hat{j} + a_3 \hat{k}$$

$$\vec{b} = b_1 \hat{i} + b_2 \hat{j} + b_3 \hat{k}$$

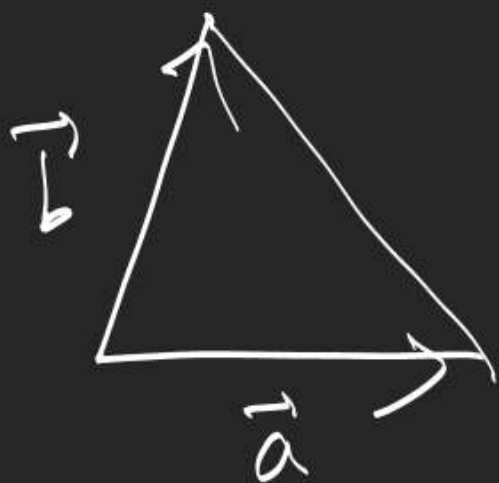
$$\vec{a} \times \vec{b} =$$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$



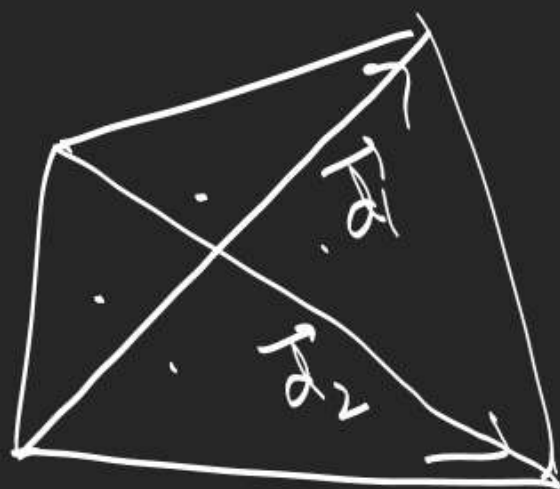
$$\vec{a} \times \vec{b} = \vec{0} \Rightarrow \vec{a} = \vec{0} \text{ or } \vec{b} = \vec{0}$$

$$\text{or} \vec{a} = \lambda \vec{b}$$



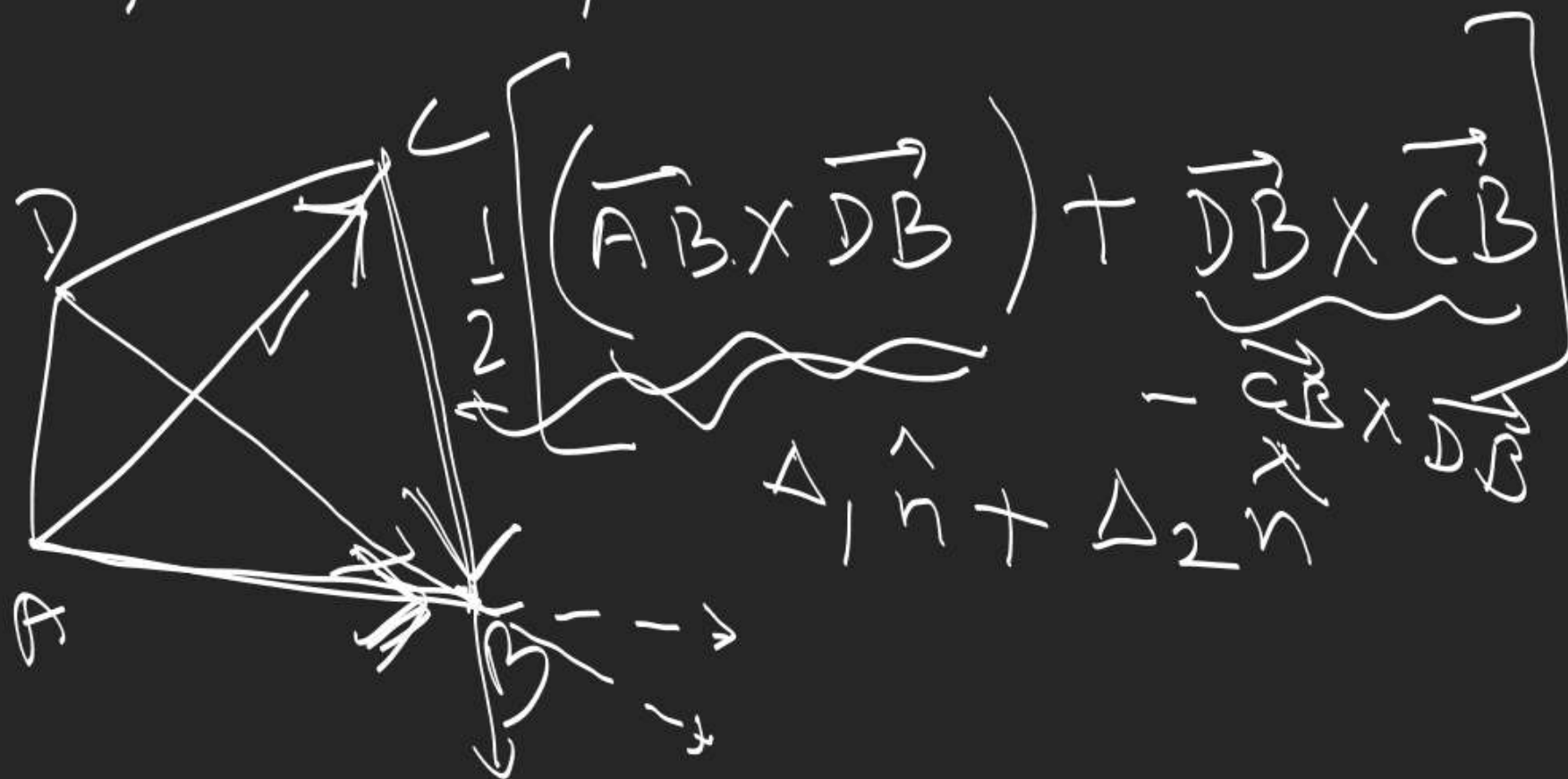
$$\text{Area of } \Delta = \frac{1}{2} |\vec{a} \times \vec{b}|$$

$\vec{a} \times \vec{b}$  is  $\perp$  to plane containing  $\vec{a}, \vec{b}$



$$\frac{1}{2} |\vec{d}_1 \times \vec{d}_2| = \text{Area of quadrilateral}$$

$$\begin{aligned} \frac{1}{2} (\vec{AB} - \vec{CB}) \times \vec{DB} &= \\ = \frac{1}{2} (\vec{AC} \times \vec{DB}) \end{aligned}$$



\* Unit vector  $\perp$  to plane containing two non collinear vectors  $\vec{a}, \vec{b}$

$$= \pm \frac{\vec{a} \times \vec{b}}{|\vec{a} \times \vec{b}|}$$

$$\frac{1}{2} |(\vec{b} - \vec{a}) \times (\vec{c} - \vec{a})| = \frac{1}{2} |\vec{b} \times \vec{c} + \vec{c} \times \vec{a} + \vec{a} \times \vec{b}|$$

\* 

$$\text{Area of } \triangle ABC = \frac{1}{2} |\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}|$$

Condition of collinearity  
 of 3 points with p.v.  $\vec{a}, \vec{b}, \vec{c}$   
 $\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a} = \vec{0}$

- Lagnangian Identity

$$|\vec{a} \times \vec{b}|^2 = |\vec{a}|^2 |\vec{b}|^2 - (\vec{a} \cdot \vec{b})^2$$

1. Find the eqn. of line thru the point with p.v.  
 $2\hat{i} + 3\hat{j}$  and  $\perp$  to vectors  $\vec{A} = \hat{i} + 2\hat{j} + 3\hat{k}$  and

$$\vec{B} = 3\hat{i} + 4\hat{j} + 5\hat{k}$$

$$\vec{r} = 2\hat{i} + 3\hat{j} + \lambda(\hat{i} - 2\hat{j} + \hat{k})$$

$$\begin{aligned}\vec{R} &= 4\hat{i} - 3\hat{j} + 7\hat{k} - 5(\hat{i} + \hat{j} + \hat{k}) \\ &= -\hat{i} - 8\hat{j} + 2\hat{k}\end{aligned}$$

2. Find the unknown vector  $\vec{R}$  satisfying

$$\vec{R} \times \vec{B} = \vec{C} \times \vec{B}$$

and  $\vec{R} \cdot \vec{A} = 0$ , where  $\vec{A} = 2\hat{i} + \hat{k}$ ,

$$\vec{B} = \hat{i} + \hat{j} + \hat{k}, \quad \vec{C} = 4\hat{i} - 3\hat{j} + 7\hat{k}$$

$$\begin{aligned}(\vec{R} - \vec{C}) \times \vec{B} &= \vec{0} \\ \vec{R} - \vec{C} &= \lambda \vec{B}\end{aligned}$$

$$(\vec{C} + \lambda \vec{B}) \cdot \vec{A} = 0 \quad \lambda = -5$$

$$\vec{C} \cdot \vec{A} + \lambda \vec{B} \cdot \vec{A} = 0$$

$$\Rightarrow 15 + \lambda(3) = 0$$

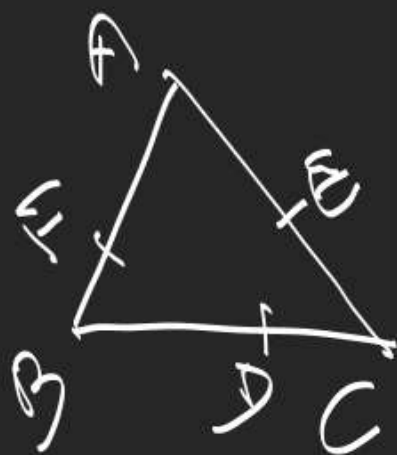
3. In  $\triangle ABC$ , M is midpoint of AB and D be foot of internal bisector of  $\angle C$  on AB.

(i) P.T.  $\frac{\text{area of } \triangle CDM}{\text{area of } \triangle ABC} = \frac{|a-b|}{2(a+b)}$

(ii) P.T.  $\cos \angle DCM = \frac{(a+b) \cos \frac{C}{2}}{\sqrt{a^2 + b^2 + 2ab \cos C}}$

Vector  
 $\underline{\underline{\sum x - 1}} (1-10)$

4.



$$\frac{BD}{DC} = \frac{CE}{EA} = \frac{AF}{FB} = n$$

find the ratio

$$\frac{\triangle DEF}{\triangle ABC}$$