Nishant Jindal

Q $f(x) = \dfrac{1-\cos x}{1-\sin x}$ find $f'\left(\frac{\pi}{2}\right) = ?$

$f'(x) = \dfrac{(1-\sin x)(+\sin x) - (1-\cos x)(-\cos x)}{(1-\sin x)^2}\Bigg|_{x=\frac{\pi}{2}} = \dfrac{0}{0}$

$f(x)$ at $x = \frac{\pi}{2}$ not defined

$\Rightarrow$ Domain $\neq \frac{\pi}{2}$

$\therefore f'(x) = $ Not defined

Q $f(x) = \log_e(\sin x - 2)$ $\dfrac{dy}{dx} = ?$

$\sin x - 2$ is always $-ve$

$f(x) =$ Not defined

$\Rightarrow f'(x)$ DNE

Q $y = x[x]$ $[x] = n$ (let) $\dfrac{dy}{dx} = ?$ $\{x \notin Z\}$

$y = nx$

$\dfrac{dy}{dx} = n = [x]$

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Q $f(x)=\begin{cases} x^2 \cdot \sin\frac{1}{x} & x\neq 0 \\ 0 & x=0 \end{cases}$

defined fxn.

find $f'(0)=?$

K.K.F

$$\boxed{f'(a)=\lim_{x\to a}\frac{f(x)-f(a)}{x-a}}$$

$$f'(0)=\lim_{x\to 0}\frac{x^2\cdot\sin\frac{1}{x}-0}{x-0}$$

$$=0\cdot\sin(\infty)=0\times[-1 \text{ to } 1]$$

$$f'(0)=0$$

Q $f(x)=\begin{cases} \dfrac{x-1}{2x^2-7x+5} & x\neq 1 \\ -\frac{1}{3} & x=1 \end{cases}$ find $f'(1)$?

$\left(\frac{1}{x}\right)' \to -\frac{1}{x^2}$

$$f'(1)=\lim_{x\to 1}\frac{f(x)-f(1)}{x-1}$$

$$=\lim_{x\to 1}\frac{\frac{\cancel{(x-1)}}{\cancel{(x-1)}(2x-5)}-\left(-\frac{1}{3}\right)}{(x-1)}\quad \frac{0}{0}\text{ form.}$$

DL

$$=\lim_{x\to 1}\frac{-\frac{1}{(2x-5)^2}\times 2+0}{1}=-\frac{2}{9}$$

Nishant Jindal

Q $\frac{d}{dx}\left(\frac{x^4+x^2+1}{x^2+x+1}\right) = ax+b$ find $\frac{a}{b}=?$

$$\left(\frac{(\cancel{(x^2+x+1)})(x^2-x+1)}{(\cancel{x^2+x+1})}\right)' = (x^2-x+1)'$$

$$= 2x-1$$

$$a=2,\ b=-1$$

$$\therefore \frac{a}{b} = \frac{2}{-1} = -2$$

Proof

$$x^4+x^2+1 = (x^4+2x^2+1)-x^2$$
$$= (x^2+1)^2 - x^2$$
$$= (\qquad)(\qquad)$$

Q If $y=\sqrt{\frac{1-x}{1+x}}$ then $\frac{dy}{dx}=?$

A) $\frac{y}{1-x^2}$ B) $\frac{y}{x^2-1}$ C) $\frac{y}{1+x^2}$ D) NOT.

all 3 option have y in Nr $\Rightarrow$ log has been Used

$$\log_e(y) = \frac{1}{2}\log\left(\frac{1-x}{1+x}\right) = \frac{1}{2}\{\log(1-x) - \log(1+x)\}$$

$$\frac{1}{y}\cdot\frac{dy}{dx} = \frac{1}{2}\left\{\frac{-1}{1-x} - \frac{1}{(1+x)}\right\}$$

$$\frac{1}{y}\cdot\frac{dy}{dx} = \frac{1}{2}\left\{\frac{-1-\cancel{x}-1+\cancel{x}}{(1-x^2)}\right\} = \frac{1}{x^2-1}$$

$$\frac{dy}{dx} = \frac{y}{x^2-1}$$

Q $\sin y = x \cdot \sin(a+y)$ then $\frac{dy}{dx} = ?$

A) $\frac{\cos^2(a+y)}{\cos a}$ B) $\frac{\cos y}{\cos^2(a+y)}$ (C) $\frac{\sin^2(a+y)}{\sin a}$ (D) $\frac{\sin y}{\sin^2(a+y)}$

No option has term of "x"

$\Rightarrow \frac{dx}{dy}$ has been Used

$$x = \frac{\sin y}{\sin(a+y)}$$

$$\frac{dx}{dy} = \frac{\sin(a+y)\cos y - \sin y(\cos(a+y))}{\sin^2(a+y)}$$

$$\frac{dx}{dy} = \frac{\sin((a+\cancel{y})-\cancel{y})}{\sin^2(a+y)} \Rightarrow \frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}$$

Q $y = \sec x^\circ$ then $\frac{dy}{dx} = ?$

$$y = \sec \frac{\pi x}{180}$$

$$\frac{dy}{dx} = \sec\left(\frac{\pi x}{180}\right)\tan\left(\frac{\pi x}{180}\right) \times \frac{\pi}{180} \times 1$$

$180^\circ = \pi$

$1^\circ = \frac{\pi}{180}$

$x^\circ = \frac{\pi x}{180}$

Q $y = \log_{\sin x} \cos x$ find $\frac{dy}{dx} = ?$

Base हटाओ

$$y = \frac{\log_e \cos x}{\log_e \sin x} \Rightarrow \frac{u}{v} \text{ Use}$$

$$y' = \frac{(\log \sin x \times -\tan x) - (\log \cos x \cdot \cot x)}{(\log \sin x)^2}$$

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Q $y = \left(\log_{\cos x} \sin x\right)\left(\log_{\sin x} \cos x\right)^{-1} + \sin^{-1} x^2$ find $\frac{dy}{dx} = ?$

$$y = \left(\frac{\log \sin x}{\log \cos x}\right) \times \left(\frac{\log \sin x}{\log \cos x}\right) + \sin^{-1} x^2$$

$$y = \left(\frac{\log \sin x}{\log \cos x}\right)^2 + \sin^{-1} x^2$$

$$\frac{dy}{dx} = 2\left(\frac{\log \sin x}{\log \cos x}\right) \times \frac{\cot x \log \cos x + \tan x \cdot \log \sin x}{(\log \cos x)^2}$$

$$+ \frac{1}{\sqrt{1-(x^2)^2}} \times 2x$$

$$\log_a b = \frac{\log b}{\log a}$$

$$\log_{\cos x} \sin x = \frac{\log \sin x}{\log \cos x}$$

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Q $y = \ln\left(x+\sqrt{x^2+a^2}\right)$ then $\frac{dy}{dx} = ?$

$$y' = \frac{1}{\left(x+\sqrt{x^2+a^2}\right)} \times \left\{1 + \frac{\cancel{2}x}{\cancel{2}\sqrt{x^2+a^2}}\right\}$$

$$= \frac{1}{\cancel{\left(x+\sqrt{x^2+a^2}\right)}} \left\{\frac{\cancel{x+\sqrt{x^2+a^2}}}{\sqrt{x^2+a^2}}\right\}$$

$$\frac{dy}{dx} = \frac{1}{\sqrt{x^2+a^2}}$$

$$\boxed{\int \frac{dx}{\sqrt{x^2+a^2}} = \ln\left(x+\sqrt{x^2+a^2}\right) + C}$$

Q $f(x) = \sin x,\ g(x) = x^2,\ h(x) = \ln(x)$ ; $\frac{d}{dx}(hogof(x)) = ?$

$$hogof(x) = h(g(f(x))) = h(g(\sin x)) = h(\sin^2 x)$$

$$= \ln \sin^2 x$$

$$= 2 \ln \sin x$$

$$\frac{dy}{dx} = 2 \times \frac{1}{\sin x} \times (\cos x)$$

$$y' = 2\cot x$$

Q If $\underbrace{f(x) \text{ is an odd fxn in } (-\infty,\infty)}$ & $f'(-1) = 5$ then $f'(1) = ?$

$$f(-x) = -f(x)$$

diff$^n$ $\quad f'(-x) \times -1 = -f'(x) \times 1$

$x = 1 \quad +f'(-x) = +f'(x)$

$$f'(-1) = f'(1)$$

$$5 = f'(1)$$

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Q $f(x)$ is an diff^ble^ fxn $\forall x \in R$

$f(x^3) = x^5$ then $f'(8) = ?$

$f'(x^3) \times 3\cancel{x^2} = 5\cancel{x^4}x^2$

$x = 2$ $\quad f'(x^3) = \frac{5x^2}{3}$

$f'(8) = \frac{20}{3}.$

Q If $f$ is a 2 deg Poly fxn & $f(1) = f(-1)$ & $A, B, C$ are in AP

Mains.

then $f'(A), f'(B), f'(C)$ are in ?

AP.

$f(x) = ax^2 + bx + c$

$f(1) = a + b + c$

$f(-1) = a - b + c$

$a + b + c = a - b + c$

$b = -b$

$2b = 0$

$\boxed{b = 0}$

$f(x) = ax^2 + c$

$f'(x) = 2ax$

$2aA, 2aB, 2aC \rightarrow$ AP.

$A, B, C \rightarrow$ AP $\rightarrow$ given.

Aisa fxn $f(x) = x^2$

$f'(x) = 2x$

$f'(A), f'(B), f'(C) \rightarrow$ AP

$2A \quad 2B \quad 2C \rightarrow$ AP

Nishant Jindal

Q If $f(x) = \underset{\text{I}}{(1+x)}\ \underset{\text{II}}{(3+x^2)^{1/2}}\ \underset{\text{III}}{(9+x^3)^{1/3}}$ then $f'(-1) = ?$

$$f'(x) = 1\cdot(3+x^2)^{1/2}(9+x^3)^{1/3} + (1+x)\cdot \text{II}'(9+x^3)^{1/3} + (1+x)(3+x^2)^{1/2}\ \text{III}'$$

$$f'(-1) = (3+1)^{1/2}(9-1)^{1/3} + 0 + 0$$

$$= 2\times 2$$

$$= 4$$

Q If $1, 1, \alpha, \beta, \gamma$ are Roots of $x^5 + ax + b = 0$ then $(1-\alpha)(1-\beta)(1-\gamma) = ?$

T.P. for $(1-\alpha)(1-\beta)(1-\gamma)$ $\xrightarrow{}$ Put $x=1$

$$x^5 + ax + b = (x-1)^2(x-\alpha)(x-\beta)(x-\gamma)$$

diff $5x^4 + a = 2(x-1)(x-\alpha)(x-\beta)(x+\gamma) + \{(x-1)^2 + (x-1)^2 + (x-1)^2\}$

But $(x-1)^2$ will give Zero

Now $(x-1)^2$ Can be Removed by diff^ing 2 time

diff $20x^3 = 2\times 1(x-\alpha)(x-\beta)(x-\gamma) + 2(x-1)\,\text{II}'\,\text{III}(1)+ \cdots$

$x=1$ $20 = 2(1-\alpha)(1-\beta)(1-\gamma) + 0 + 0 + 0 + 0 + 0$ $\longrightarrow$ $(1-\alpha)(1-\beta)(1-\gamma) = 10$

Nishant Jindal

## $(U \cdot V \cdot W)'$ Redefined.

$(fgh)' = f'gh + f \cdot g'h + fgh'$ ← Jaante hain.

$(fgh)' = \frac{1}{2}\{2f'gh + 2fg'h + 2fgh'\}$

$= \frac{1}{2}\{f'gh + f'gh + fg'h + fg'h + fgh' + fgh'\}$

$= \frac{1}{2}\{h(f'g + fg') + g(f'h + fh') + f(g'h + gh')\}$

$(fgh)' = \frac{1}{2}\{(fg)'h + g(fh)' + f(gh)'\}$

Q Let $f, g, h$ are diff$^{ble}$ fxn, $f(0) = 1$, $g(0) = 2$, $h(0) = 3$, $(fg)'0 = 6$, $(gh)'0 = 4$, $(hf)'0 = 5$ then $(fgh)'0 = ?$

$(fgh)'0 = \frac{1}{2}\{f(0) \cdot (gh)'0 + g(0) \cdot (f \cdot h)'0 + h(0) \cdot (fg)'0\}$

$= \frac{1}{2}\{1 \times 4 + 2 \times 5 + 3 \times 6\}$

$= \frac{1}{2}\{32\} = 16$

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Q If $f''(x) = -f(x)$, $g(x) = f'(x) \rightarrow g'(x) = f''(x) = -f(x)$

& if $F(x) = \left(f\left(\frac{x}{2}\right)\right)^2 + \left(g\left(\frac{x}{2}\right)\right)^2$; $F(5) = 10$

find $F(10) = ?$

$$F'(x) = \cancel{2} f\left(\frac{x}{2}\right) \times f'\left(\frac{x}{2}\right) \times \frac{1}{\cancel{2}} + 2 g\left(\frac{x}{2}\right) \times g'\left(\frac{x}{2}\right) \times \frac{1}{2}$$

$$= \cancel{f\left(\frac{x}{2}\right) \times g\left(\frac{x}{2}\right)} + \cancel{g\left(\frac{x}{2}\right) \times -f\left(\frac{x}{2}\right)}$$

$F'(x) = 0 \Rightarrow F(x) =$ Constant fxn.

$F(5) = 10$

$F(10) = 10$

Q If $f(x)$ is a diff$^{b'e}$ fxn S.T.

$$\boxed{f(x+2y) = f(x) + f(2y) + 6xy(x+2y)}$$

$f(x+h) = f(x) + f(h) + 3hx(x+h)$ $\forall x, y \in R$

then $f''(0), f''(1), f''(2)$ are in? AP

Use RHD for $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$$f'(x) = \lim_{h \to 0} \frac{\cancel{f(x)} + f(h) + 3hx(x+h) - \cancel{f(x)}}{h}$$

Non $x$

$$= \left\{\lim_{h \to 0} \frac{f(h)}{h}\right\} + \lim_{h \to 0} \frac{3\cancel{h}x(x+h)}{\cancel{h}}$$

$f'(x) = K + 3x^2$

$f''(x) = 6x \Rightarrow$

0, 6, 12

AP

Nishant Jindal

# Modulus Based Qs.

First define mod then diff$^{te}$

Q $f(x) = |x-1|$ then $\left.\frac{dy}{dx}\right|_{x=1/2}$

at $x = \frac{1}{2} \to \frac{1}{2} - 1 = -ve$

$f(x) = -(x-1)$

$f'(x) = -1$

$f'\left(\frac{1}{2}\right) = -1$

Q $f(x) = |x-1|$ $\left.\frac{dy}{dx}\right|_{x=2}$

$x = 2$ पर $2-1$ +ve

$f(x) = (x-1)$

$f'(x) = 1$

$f'(2) = 1$

Q $y = \log_e|x|$ then $\left.\frac{dy}{dx}\right|_{x \neq 0} = ?$

$y' = \frac{1}{|x|} \times \frac{|x|}{x} = \frac{1}{x}$

Q $y = \sqrt{1 - \sin 2x}$ $\left.\frac{dy}{dx}\right|_{x=\frac{\pi}{6}}$

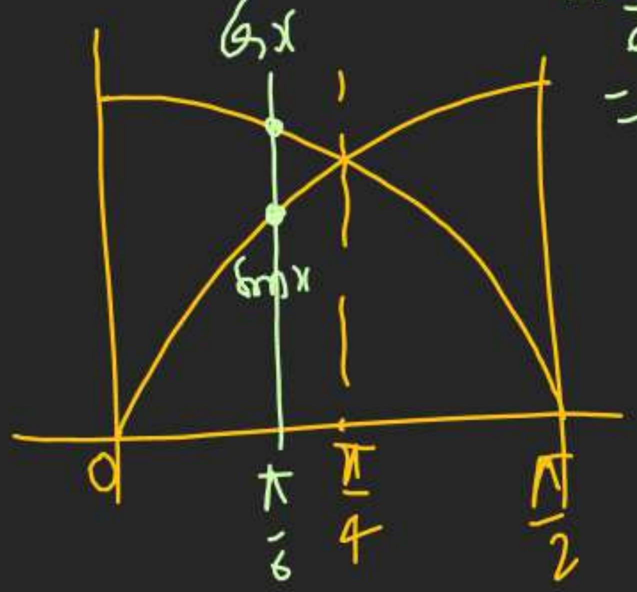
$y = \sqrt{(\cos x - \sin x)^2}$

$y = |\underbrace{\cos x - \sin x}_{+}| \Rightarrow y = \cos x - \sin x$

$\left.\frac{dy}{dx}\right|_{x=\frac{\pi}{6}} = -\sin x - \cos x$

$= -\frac{1}{2} - \frac{\sqrt{3}}{2}$

$= -\frac{(\sqrt{3}+1)}{2}$




$\cos x > \sin x$

$\cos x - \sin x > 0$