PROBLEM SET-01

Q.1 If $z + z^3 = 0$ then which of the following must be true on the complex plane?

(A) Re $(z) < 0$ (B) Re $(z) = 0$ (C) Im $(z) = 0$ (D) $z^4 = 1$

Q.2 Let $i = \sqrt{-1}$. The product of the real part of the roots of $z^2 - z = 5 - 5i$ is

(A) -25 (B) -6 (C) -5 (D) 25

Q.3 In the quadratic equation $x^2 + (p + iq)x + 3i = 0, p\&q$ are real. If the sum of the squares of the roots is 8 then

(A) $p = 3, q = -1$ (B) $p = -3, q = -1$

(C) $p = \pm 3, q = \pm 1$ (D) $p = -3, q = 1$

Q.4 The complex number z satisfying $z + |z| = 1 + 7i$ then the value of $|z|^2$ equals

(A) 625 (B) 169 (C) 49 (D) 25

Q.5 Number of values of z (real or complex) simultaneously satisfying the system of equations $1 + z + z^2 + z^3 + \cdots + z^{17} = 0$ and $1 + z + z^2 + z^3 + +z^{13} = 0$ is

(A) 1 (B) 2 (C) 3 (D) 4

Q.6 Number of complex numbers z satisfying $z^3 = \bar{z}$ is

(A) 1 (B) 2 (C) 4 (D) 5

Q.7 If $x = 9^{1/3}\, 9^{1/9} 9^{1/27}$ ….. ad inf $y = 4^{1/3}\, 4^{-1/9}\, 4^{1/27}$ … … . . ad inf and $z = \sum^{\infty}(1+i)^{-r}$ then, the argument of the complex number $w = x + yz$ is

(A) 0 (B) $\pi - \tan^{-1}\left(\frac{\sqrt{2}}{3}\right)$

$(C) - \tan^{-1}\left(\frac{\sqrt{2}}{3}\right)$ (D) $-\tan^{-1}\left(\frac{2}{\sqrt{3}}\right)$

Q.8 If z is a complex number satisfying the equation $|z - (1+i)|^2 = 2$ and $\omega = \frac{2}{z}$, then the locus traced by ' ω ' in the complex plane is

$(A)x - y - 1 = 0$ (B) $x + y - 1 = 0$ (C) $x - y + 1 = 0$ (D) $x + y + 1 = 0$

Q.9 If the expression $(1+ir)^3$ is of the form of $s(1+i)$ for some real ' s ' where ' r ' is also real and $i=\sqrt{-1}$, then the value of ' r ' can be

(A) $\cot\frac{\pi}{8}$ (B) $\sec\pi$ (C) $\tan\frac{\pi}{12}$ (D) $\tan\frac{5\pi}{12}$

PROBLEM SET-02

Q.10 The diagram shows several numbers in the complex plane. The circle is the unit circle centered at the origin. One of these numbers is the reciprocal of F, which is

(A) A (B) B (C) C (D) D

Q.11 (b) Let z be a complex number such that $\arg(z-2)=\frac{2\pi}{3}$ and $|z|=2$. Then principle value of the argument of z is

(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{6}$ (D) $\frac{\pi}{2}$

Q.12 Let $z=x+iy$, where $x,y\in R$ and $i=\sqrt{-1}$. If locus of $P(z)$ satisfying $\operatorname{Re}\left(\frac{1}{z}\right)=\frac{1}{2}$ represents a circle then maximum distance of a point on the circle from $M(-2,4)$, is equal to [Note: Re (z) denotes the real part of z.]

(A) 4 (B) 5 (C) 6 (D) 8

Q.13 For $Z_1=\sqrt[6]{\frac{1-i}{1+i\sqrt{3}}}$; $Z_2=\sqrt[6]{\frac{1-i}{\sqrt{3}+i}}$; $Z_3=\sqrt[6]{\frac{1+i}{\sqrt{3}-i}}$ which of the following holds good?

(A) $\sum|Z_1|^2=\frac{3}{2}$ (B) $|Z_1|^4+|Z_2|^4=|Z_3|^{-8}$

(C) $\sum|Z_1|^3+|Z_2|^3=|Z_3|^{-6}$ (D) $|Z_1|^4+|Z_2|^4=|Z_3|^8$

Q.14 A point ' z ' moves on the curve $|z-4-3i|=2$ in an argand plane. The maximum and minimum values of $|z|$ are

(A) 2,1 (B) 6,5 (C) 4,3 (D) 7,3

Q.15 If z is a complex number satisfying the equation $|z+i|+|z-i|=8$, on the complex plane then maximum value of $|z|$ is

(A) 2 (B) 4 (C) 6 (D) 8

Q.16 Let $z_r(1\le r\le 4)$ be complex numbers such that $|z_r|=\sqrt{r+1}$ and

$|30z_1 + 20z_2 + 15z_3 + 12z_4| = k|z_1z_2z_3 + z_2z_3z_4 + z_3z_4z_1 + z_4z_1z_2|$. Then the value of k equals

(A) $|z_1z_2z_3|$ (B) $|z_2z_3z_4|$ (C) $|z_3z_4z_1|$ (D) $|z_4z_1z_2|$

PROBLEM SET-03

Q.17 Let Z be a complex number satisfying the equation

$(Z^3 + 3)^2 = -16$ then $|Z|$ has the value equal to

(A) $5^{1/2}$ (B) $5^{1/3}$ (C) $5^{2/3}$ (D) 5

Q.18 If z_1, z_2, z_3 are 3 distinct complex numbers such that $\frac{3}{|z_2-z_3|} = \frac{4}{|z_3-z_1|} = \frac{5}{|z_1-z_2|}$, then the value of $\frac{9}{z_2-z_3} + \frac{16}{z_3-z_1} + \frac{25}{z_1-z_2}$ equals

(A) 0 (B) $\sqrt{5}$ (C) 5 (D) 25

Q.19 If $i = \sqrt{-1}$, then $4 + 5\left(-\frac{1}{2} + \frac{i\sqrt{3}}{2}\right)^{334} + 3\left(-\frac{1}{2} + \frac{i\sqrt{3}}{2}\right)^{365}$ is equal to

(A) $1 - i\sqrt{3}$ (B) $-1 + i\sqrt{3}$ (C) $i\sqrt{3}$ (D) $-i\sqrt{3}$

Q.20 Let Z is complex satisfying the equation

$z^2 - (3 + i)z + m + 2i = 0$, where $m \in R$. Suppose the equation has a real root.

The additive inverse of non-real root, is

(A) $1 - I$ (B) $1 + I$ (C) $-1 - I$ (D) -2

Q.21 The minimum value of $|z - 1 + 2i| + |4i - 3 - z|$ is

(A) $\sqrt{5}$ (B) 5 (C) $2\sqrt{13}$ (D) $\sqrt{15}$

Q.22 The area of the triangle whose vertices are the roots of $z^3 + iz^2 + 2i = 0$ is

(A) 2 (B) $\frac{3}{2}\sqrt{7}$ (C) $\frac{3}{4}\sqrt{7}$ (D) $\sqrt{7}$

Q.23 A particle starts from a point $z_0 = 1 + i$, where $i = \sqrt{-1}$. It moves horizontally away from origin by 2 units and then vertically away from origin by 3 units to reach a point z_1. From z_1 particle moves $\sqrt{5}$ units in the direction of $2\hat{i} + \hat{j}$ and then it moves through an angle of $\operatorname{cosec}^{-1}\sqrt{2}$ in anticlockwise direction of a circle with centre at origin to reach a point z_2. The arg z_2 is given by

(A) $\sec^{-1} 2$ (B) $\cot^{-10}$ (C) $\sin^{-1}\left(\frac{\sqrt{3}-1}{2\sqrt{2}}\right)$ (D) $\cos^{-1}\left(\frac{-1}{2}\right)$

Q.24 Let $z = x + iy$ then locus of moving point $P(z)$ such that $\frac{1+\bar{z}}{z} \in R$, is (where $i^2 = -1$)

(A) union of lines with equations $x = 0$ and $y = \frac{-1}{2}$ but excluding origin.

(B) union of lines with equations $x = 0$ and $y = \frac{1}{2}$ but excluding origin.

(C^*) union of lines with equations $x = \frac{-1}{2}$ and $y = 0$ but excluding origin.

(D) union of lines with equations $x = \frac{1}{2}$ and $y = 0$ but excluding origin.

Q.25 If P and Q are represented by the complex numbers z_1 and z_2 such that $\left|\frac{1}{z_1} + \frac{1}{z_2}\right| = \left|\frac{1}{z_1} - \frac{1}{z_2}\right|$, then the circumcentre of $\triangle$ OPQ (where O is the origin) is

(A) $\frac{z_1 - z_2}{2}$ (B) $\frac{z_1 + z_2}{2}$ (C) $\frac{z_1 + z_2}{3}$ (D) $z_1 + z_2$

Q.26 Number of complex numbers z such that $|z| = 1$ and $\left|\frac{z}{\bar{z}} + \frac{\bar{z}}{z}\right| = 1$ is

(A) 4 (B) 6 (C) 8 (D) more than 8

Q.27 Number of complex numbers satisfying the relation $|z + \bar{z}| + |z - \bar{z}| = 2$ and $|z + i| + |z - i| = 2$, is

(A) 1 (B) 2 (C) 3 (D) 4

Paragraph for question no. 28 to 30

Consider complex number z_1 and z_2 satisfying $|z_1| = 1$ and $|z_2 - 2| + |z_2 - 4| = 2$.

Q.28 Let m and M denotes minimum and maximum value of $|z_1 - z_2|$, then $(m + M)$ is equal to

(A) 5 (B) 6 (C) 7 (D) 8

Q.29 Re $(z_1 z_2)$ can never exceed

(A) 1 (B) 2 (C) 3 (D) 4

Q.30 If principal argument of z_1 = principal argument of z_2, then $|z_1 + 2|$ is equal to

(A) 0 (B) 1 (C) 2 (D) 3

ANSWER KEY

1.	(B)	2.	(B)	3.	(C)	4.	(A)	5.	(A)	6.	(D)	7.	(C)
8.	(A)	9.	(BCD)	10.	(C)	11.	(B)	12.	(C)	13.	(B)	14.	(D)
15.	(B)	16.	(D)	17.	(B)	18.	(A)	19.	(C)	20.	(C)	21.	(C)
22.	(A)	23.	(B)	24.	(C)	25.	(B)	26.	(C)	27.	(B)	28.	(B)
29.	(D)	30.	(D)										