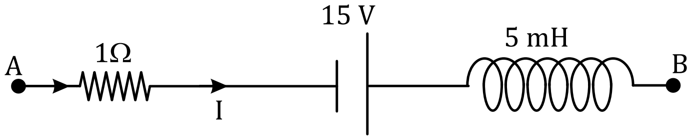
DPP 05

Solution

1. In accordance with law of potential distribution, for the given network,]

$$V_A - IR + E - L\frac{dI}{dt} = V_B$$



and as here I is decreasing (dI/dt) is negative.

So, $V_B - V_A = -5 \times 1 + 15 - 5 \times 10^{-3}(-10^3)$

i.e., $V_B - V_A = -5 + 15 + 5 = 15$ V

2. Inductance, $L = 10mH$

Voltage, $V = 20$ V, $e^{-1} = 0.37$

Resistance, $R = 10k\Omega$, Time $t = 1\mu s$ The maximum current is,

$$I_{max} = \frac{V}{R} = \frac{20}{10 \times 10^3} = 2 \times 10^{-3} \text{ A}$$

For $L - R$ decay circuit,

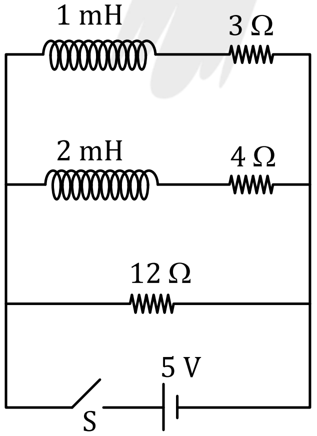
$$I = I_{max}e^{-R\frac{t}{L}} = 2 \times 10^{-3}e^{\frac{-10\times10^3\times10^{-6}}{10\times10^{-3}}}$$

$$I = 2 \times 10^{-3}e^{-1} = \frac{74}{100} \text{ mA}$$

3. Just after closing the switch S, inductor behaves like an open circuit

$I = 6/2 + 4 = 1$ A

4.



At $t = 0$, current will flow only in 12Ω resistance

$$\therefore\ I_{min} = \frac{5}{12}$$

At $t \to \infty$ both

L_1 and L_2 behave as conducting wires

$\therefore\ R_{eff} = \frac{3}{2}$

$I_{max} = \frac{10}{3}$

$\frac{I_{max}}{I_{min}} = 8$

5. Initially the inductor offers infinite resistance hence i_1 is 1 A. Finally, at steady state inductor offers zero resistance and current i_2 is 1.25 A in the battery.

6. $R = \frac{V}{I}$ $\tau = \frac{L}{R} = 1$ ms.

7. Current will have to increase in order to oppose the cause (decrease in field).

8. $I = I_1 + I_2$

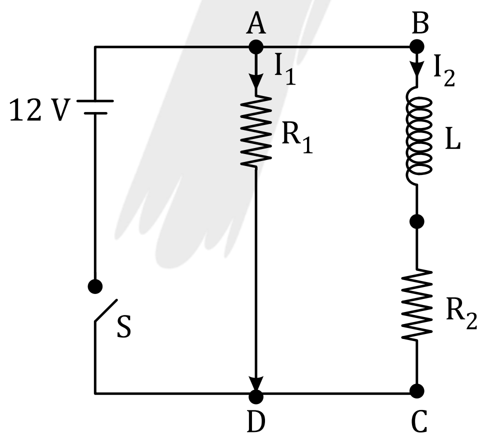
$I_1 = E/R$

$L\frac{dI}{dt} = E.\ I_2 = \frac{Et}{L}$

$I = E/R + \frac{Et}{L}$

$I = 12A.$

9.



For the given $R - L$ circuit the potential difference across $AD = V_{BC}$ as they are parallel.

$I_1 = E/R_1; I_2 = I_0\left(1 - e^{-t/\tau}\right)$ where

$\tau =$ mean life or L/R.

$\tau = t_0$ (given)

$E(\text{ across BC}) = L\frac{dI_2}{dt} + R_2 I_2$

$I_2 = I_0\left(1 - e^{-t/t0}\right)$

But $\quad I_0 = \dfrac{E}{R_2} = \dfrac{12}{2} = 6\text{ A}$

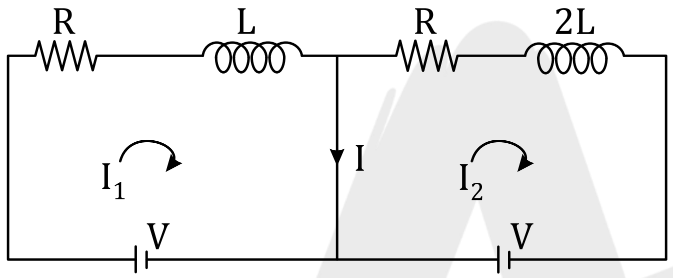
$\tau = t_0 = \dfrac{L}{R} = \dfrac{400 \times 10^{-3}H}{2\Omega} = 0.2\text{ s}$

$\therefore\ I_2 = 6\left(1 - e^{-t/0.2}\right)$

Potential drop across $L = E - R_2 I_2$

$= 12 - 2 \times 6\left(1 - e^{-t/0.2}\right) = 12e^{-t/0.2} = 12e^{-5t}\text{ V}$

10. Let I_1 and I_2 be the currents in both loops as shown in figure.



$I = (I_1 - I_2)$

$I = \dfrac{V}{R}\left[1 - e^{-\left(\frac{R}{L}\right)t}\right] - \dfrac{V}{R}\left[1 - e^{-\left(\frac{R}{2L}\right)t}\right]$

$I = \dfrac{V}{R}\left[e^{-\left(\frac{R}{2L}\right)t} - e^{-\left(\frac{R}{L}\right)t}\right] \qquad \ldots\text{(i)}$

For $I_{max}, \dfrac{dI}{dt} = 0$

$-\dfrac{V}{2L}e^{-\left(\frac{R}{2L}\right)t} + \dfrac{V}{L}e^{-\left(\frac{R}{L}\right)t} = 0$

$e^{-\left(\frac{R}{L}\right)t} = \dfrac{1}{2}e^{-\left(\frac{R}{2L}\right)t} \quad$ or $\quad e^{-\left(\frac{R}{2L}\right)t} = \dfrac{1}{2}$

$\Rightarrow\ \left(\dfrac{R}{2L}\right)t = \ln\ 2$

$\Rightarrow\ t = \dfrac{2L}{R}\ln 2 = \tau \rightarrow$ time when I is maximum.

Using t in equation (i),

$I_{max} = \dfrac{V}{R}\left[e^{-\frac{R}{2L}\left(\frac{2L}{R}\ln\ 2\right)} - e^{-\frac{R}{L}\left(\frac{2L}{R}\ln\ 2\right)}\right]$

or $\qquad I_{max} = \dfrac{V}{R}\left[\dfrac{1}{2} - \dfrac{1}{4}\right] = \dfrac{V}{4R}$