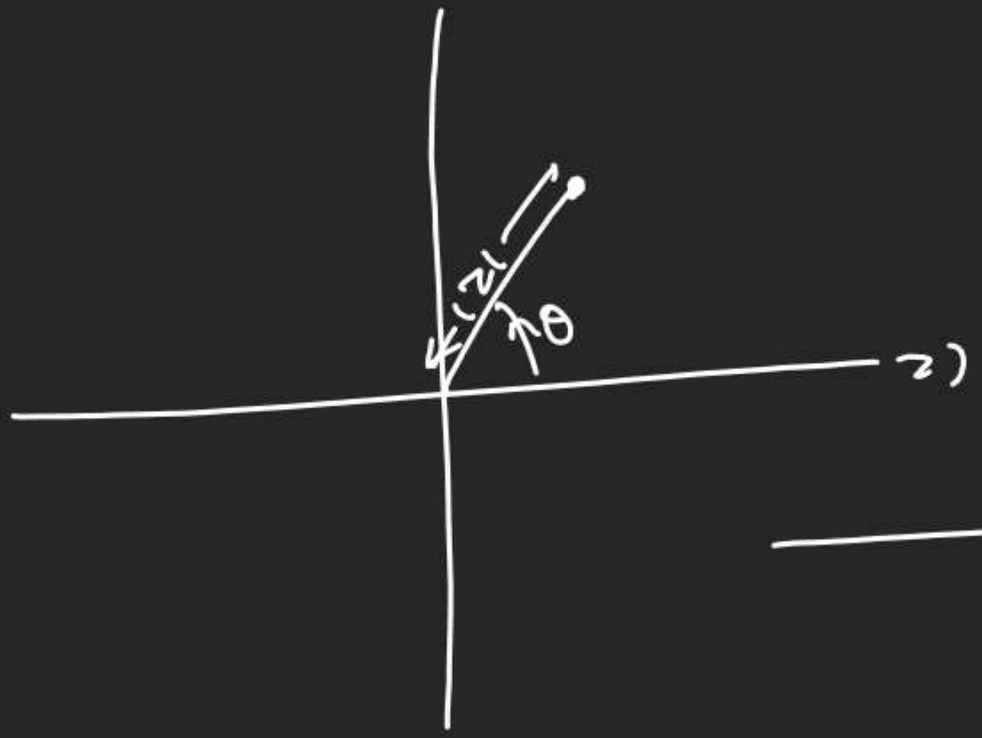
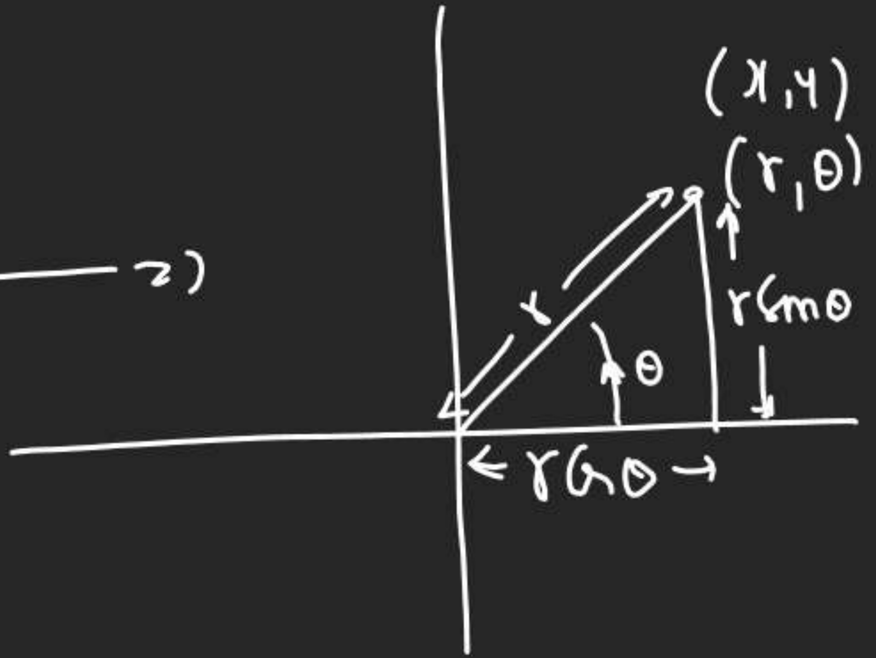
Nishant Jindal

Polar form





$$z = x + iy$$
$$= r\cos\theta + i\,r\sin\theta$$
$$= r(\cos\theta + i\sin\theta)$$
$$= r.\,\mathrm{cis}\,\theta = |z|\,\mathrm{cis}\,\theta$$

$r = |z|$, $\theta = \operatorname{Arg} z$.

Conversion:

1) $z = 1+i \rightarrow |z| = \sqrt{2}$, $\operatorname{Arg} z = \tan^{-1}\left|\frac{1}{1}\right| = \frac{\pi}{4}$

$$z = \sqrt{2}\,\mathrm{cis}\left(\frac{\pi}{4}\right)$$

2) $z = 1 - i \rightarrow |z| = \sqrt{2}$ $\quad \operatorname{Arg} = -\tan^{-1}\left|\frac{-1}{1}\right| = -\frac{\pi}{4}$

$(1,-1)$ 4th

$$z = \sqrt{2}\,\mathrm{cis}\left(-\frac{\pi}{4}\right) = \sqrt{2}\left(\cos\left(-\frac{\pi}{4}\right) + i\sin\left(-\frac{\pi}{4}\right)\right)$$
$$= \sqrt{2}\left(\cos\frac{\pi}{4} - i\sin\frac{\pi}{4}\right)$$

(3) $z = -1+i \rightarrow |z| = \sqrt{2}$, $\operatorname{Arg} = \pi - \tan^{-1}\left|\frac{1}{-1}\right| = \pi - \frac{\pi}{4}$

$(-1,1)$ 2nd

$$= \frac{3\pi}{4}$$

$$z = \sqrt{2}\,\mathrm{cis}\left(\frac{3\pi}{4}\right)$$

(4) $z = 5 \rightarrow |z| = 5$, $\operatorname{Arg}(z) = 0$

$$= 5\,\mathrm{cis}\,0$$

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5) $z = 5i \to |z| = 5,\ Arg(z) = \frac{\pi}{2}$

(0,5) +y

$z = 5\,Cis\left(\frac{\pi}{2}\right)$

(6) $z = -1-\sqrt{3}\,i \to |z| = \sqrt{-1^2 + -\sqrt{3}^2} = 2$

$(-1, -\sqrt{3})$ 3rd

$Arg(z) = -\pi + \tan^{-1}\left|\frac{-\sqrt{3}}{-1}\right|$

$= -\pi + \frac{\pi}{3} = -\frac{2\pi}{3}$

$z = 2\,Cis\left(-\frac{2\pi}{3}\right)$

z Real $\to \underline{z = \bar{z}}$

z Img $\to \underline{z = -\bar{z}}$

Q $Re(z_1\bar{z}_2) = ?$

$z_1 = |z_1|(\cos\theta_1 + i\sin\theta_1),\ z_2 = |z_2|(\cos\theta_2 + i\sin\theta_2)$

$\bar{z}_2 = |z_2|(\cos(-\theta_2) + i\sin(-\theta_2))$

$z_1\cdot\bar{z}_2 = |z_1|\cdot|z_2|\left((\cos\theta_1 + i\sin\theta_1)(\cos\theta_2 - i\sin\theta_2)\right)$

$z_1\bar{z}_2 = |z_1||z_2|\left(\cos\theta_1\cdot\cos\theta_2 - i\sin\theta_2\cdot\cos\theta_1 + i\sin\theta_1\cdot\cos\theta_2 + \sin\theta_1\cdot\sin\theta_2\right)$

$Re(z_1\bar{z}_2) = |z_1||z_2|(\cos\theta_1\cdot\cos\theta_2 + \sin\theta_1\cdot\sin\theta_2) = |z_1||z_2|\cos(\theta_1-\theta_2)$

$Re(z_1\cdot\bar{z}_2) = |z_1||z_2|\cos(Arg z_1 - Arg z_2)$

$Re(z_1\cdot\bar{z}_2) = |z_1||z_2|\cos(\theta_1-\theta_2)$

Result

$|z_1+z_2|^2 = |z_1|^2 + |z_2|^2 + 2Re(z_1\bar{z}_2)$ Can be Changed

into

$$\boxed{\begin{aligned}|z_1+z_2|^2 &= |z_1|^2 + |z_2|^2 + 2|z_1||z_2|\cos(\theta_1-\theta_2)\\ |z_1-z_2|^2 &= |z_1|^2 + |z_2|^2 - 2|z_1||z_2|\cos(\theta_1-\theta_2)\end{aligned}}$$

Sly

Q If $|z_1+z_2|^2 = |z_1|^2 + |z_2|^2$ then P.T. $\frac{z_1}{z_2}$ img $\Rightarrow \frac{z_1}{z_2} = -\frac{\bar{z}_1}{\bar{z}_2}$ $\Rightarrow z_1\bar{z}_2 = -z_2\bar{z}_1$ $z_1\bar{z}_2 = -z_2\bar{z}_1$ Gayab.

We know that

$|z_1+z_2|^2 = |z_1|^2 + |z_2|^2 + \underline{2|z_1||z_2|\cos(\theta_1-\theta_2)}$

$\Rightarrow 2|z_1||z_2|\cdot\cos(\theta_1-\theta_2) = 0$

$\Rightarrow \cos(\theta_1-\theta_2) = 0 \Rightarrow \theta_1 - \theta_2 = \pm\frac{\pi}{2}$

$Arg(z_1) - Arg(z_2) = \pm\frac{\pi}{2}$

$\Rightarrow Arg\left(\frac{z_1}{z_2}\right) = \pm\frac{\pi}{2}$

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Q If $|z_1+z_2|=|z_1|+|z_2|$ then P.T. $z_1\bar{z}_2=z_2\bar{z}_1$

$|z_1+z_2|=|z_1|+|z_2|$

Soln $|z_1+z_2|^2=(|z_1|+|z_2|)^2$

$\cancel{|z_1|^2}+\cancel{|z_2|^2}+\cancel{2}\cancel{|z_1|}|z_2|\cos(\theta_1-\theta_2)$

$=\cancel{|z_1|^2}+\cancel{|z_1|^2}+\cancel{2}\cancel{|z_1|}|z_2|$

$\Rightarrow \cos(\theta_1-\theta_2)=1$

$\theta_1-\theta_2=0$

$\text{Arg}(z_1)-\text{Arg}(z_2)=0$

$\text{Arg}\left(\frac{z_1}{z_2}\right)=0$

$\frac{z_1}{z_2}=\text{Real} \Rightarrow \frac{z_1}{z_2}=\frac{\bar{z}_1}{\bar{z}_2}$

$z_1\bar{z}_2=z_2\cdot\bar{z}_1$

Q $|z_1+z_2|=|z_1|-|z_2|$

then P.T. $\frac{z_1}{z_2}$ is -ve Real No.

$|z_1+z_2|^2=(|z_1|-|z_2|)^2$

$\cancel{|z_1|^2}+\cancel{|z_2|^2}+2\cancel{|z_1|}|z_2|\cos(\theta_1-\theta_2)$

$=\cancel{|z_1|^2}+\cancel{|z_2|^2}-2\cancel{|z_1|}|z_2|$

$\Rightarrow \cos(\theta_1-\theta_2)=-1$

$\theta_1-\theta_2=\pi$

$\text{Arg}\left(\frac{z_1}{z_2}\right)=\pi$

$\frac{z_1}{z_2}$ is -ve Real No.

Q If $|z|=|w|$ & $\text{Arg}(zw)=\pi$

then $z=?$

A) w B) $-w$ (C) $\bar{w}$ (D) $-\bar{w}$

(1)

$\text{Arg}\,z+\text{Arg}\,w=\pi$

$\text{Arg}\,z=\pi-\text{Arg}\,w$

$=\pi-\theta$

$-\theta=-\text{Arg}\,w$

$=\text{Arg}\,\bar{w}$

(2) $z=|z|\,\text{Cis}\,\text{Arg}\,z$

$=|w|(\cos(\pi-\theta)+i\sin(\pi-\theta))$

$=|w|(-\cos\theta+i\sin\theta)$

$=-|w|(\cos\theta-i\sin\theta)$

$=-|w|(\cos(-\theta)+i\sin(-\theta))$

$=-|\bar{w}|(\cos(\text{Arg}\,\bar{w})+i\sin(\text{Arg}\,\bar{w}))$

$z=-\bar{w}$

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Q If $|ZW| = 1$ & $\text{Arg } Z - \text{Arg } W = \frac{\pi}{2}$

then $\bar{Z} W = ?$

Objective: Writing $\bar{Z} W$ in polar form

$|ZW| = 1$

$|Z||W| = 1$

$|\bar{Z}||W| = 1$

$|\bar{Z} W| = 1$

$\text{Arg } \bar{Z} W$

$= \text{Arg } \bar{Z} + \text{Arg } W$

$= -\text{Arg } Z + \text{Arg } W$

$= -(\text{Arg } Z - \text{Arg } W)$

$= -\pi/2$

$\bar{Z} W = |\bar{Z} W| \left(\text{cis Arg } \bar{Z} W\right) = 1 \cdot \left(\cos\left(-\frac{\pi}{2}\right) + i \sin\left(-\frac{\pi}{2}\right)\right)$

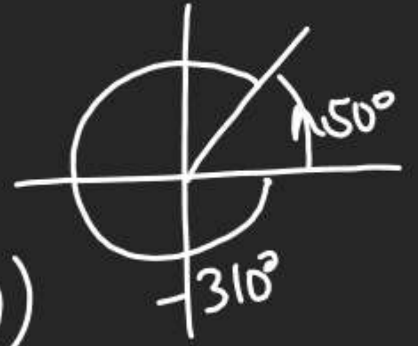
$= 1 \cdot (0 - i \times 1) = -i$

Q Simplify.

$Z = 6(\cos 310° - i \sin 310°)$

$= 6(\cos(-310°) + i \sin(-310°))$

$= 6(\cos 50° + i \sin 50°)$



original Arg = 50°

Q $Z = 6(\sin 310° - i \cos(310°))$

$\sin\theta = \cos(90 - \theta)$

$\cos\theta = \sin(90 - \theta)$

$= 6\{\cos(90 - 310°) - i \sin(90 - 310°)\}$

$= 6\{\cos(-220°) - i \sin(-220°)\}$

$= 6\{\cos 220° + i \sin 220°\}$

$= 6\{\cos(-140°) + i \sin(-140°)\}$

$= 6\,\text{cis}(-140°)$ Ans

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Exponantial form (Euler's form)

① $\text{cis}\,\theta \rightarrow e^{i\theta}$

$$\cos(\text{Arg}\,z) + i\sin(\text{Arg}\,z) = e^{i(\text{Arg}\,z)}$$

$$e^{x} = 1 + x + \frac{x^2}{\lfloor 2} + \frac{x^3}{\lfloor 3} + \cdots$$

$$e^{i\theta} = 1 + i\theta - \frac{\theta^2}{\lfloor 2} - \frac{i\theta^3}{\lfloor 3} + \frac{\theta^4}{\lfloor 4}$$

$$= \underbrace{\left(1 - \frac{\theta^2}{\lfloor 2} + \frac{\theta^4}{\lfloor 4} - \cdots\right)}_{} + i\left(\theta - \frac{\theta^3}{\lfloor 3} - \cdots\right)$$

$$e^{i\theta} = \cos\theta + i\sin\theta$$

② $z = |z|\,\text{cis}\,\theta$

$$\boxed{z = |z|\,e^{i\theta}}$$

(3) $z_1 = |z_1|\,\text{cis}(\theta_1)$, $z_2 = |z_2|\,\text{cis}\,\theta_2$

$$z_1 \cdot z_2 = |z_1||z_2| \cdot e^{i\theta_1} \cdot e^{i\theta_2}$$

$$= |z_1||z_2| \cdot e^{i(\theta_1+\theta_2)}$$

$$z_1 z_2 = |z_1||z_2| \cdot \text{cis}(\theta_1+\theta_2)$$

(4) $\text{Re}(z_1 \cdot \bar{z}_2) =$

$$z_1 \cdot \bar{z}_2 = |z_1||\bar{z}_2|\, e^{i\theta_1} \cdot e^{i(-\theta_2)}$$

$$= |z_1||\bar{z}_2| \cdot e^{i(\theta_1-\theta_2)}$$

$$= |z_1||\bar{z}_2| \cdot \text{cis}(\theta_1-\theta_2)$$

(5) Conversion

(A) $1 - i \rightarrow |z| = \sqrt{2}$ $\text{Arg}\,z = -\tan\left(\frac{-1}{1}\right)$

$(1,-1)$ 4^{th} $= -\frac{\pi}{4}$

$$(1-i) = \sqrt{2}\,e^{i\left(-\frac{\pi}{4}\right)}$$

(B) $-5i \rightarrow |z| = 5$, $\text{Arg}\,z = -\frac{\pi}{2}$

$$-5i = 5 \cdot e^{i\left(-\frac{\pi}{2}\right)}$$

(C) $-7 \rightarrow |z| = 7$, $\text{Arg}\,z = \pi$

$$-7 = 7 \cdot e^{i(\pi)}$$

Q $3e^{-i\left(\frac{\pi}{2}\right)} = 3\left(\cos\left(-\frac{\pi}{2}\right) + i\sin\left(-\frac{\pi}{2}\right)\right)$

$= 3(0 - i) = -3i$

Q $3e^{i(\pi)}$

$= -3$

$3(\cos\pi + i\sin\pi)$

$3(-1 + i \cdot 0) = -3$

Q If $(\cos\theta+i\sin\theta)(\cos2\theta+i\sin2\theta)\cdot(\cos3\theta+i\sin3\theta)\,\cdots\,(\cos n\theta+i\sin n\theta)=1$

$\theta=?$

$$e^{i\theta}\cdot e^{i2\theta}\cdot e^{i(3\theta)}\cdots e^{i(n\theta)}=1$$

$$e^{i(\theta+2\theta+3\theta+\cdots+n\theta)}=1$$

$$e^{i\theta(1+2+3+\cdots+n)}=1$$

$$e^{i\theta\frac{(n)(n+1)}{2}}=e^{i(0+2m\pi)}$$

$$\theta\cdot\frac{(n)(n+1)}{2}=2m\pi$$

$$\theta=\frac{4m\pi}{(n)(n+1)}$$

Q Find

$$\left[\frac{1+i\tan\alpha}{1-i\tan\alpha}\right]^{2n}-\left[\frac{1+i\tan2n\alpha}{1-i\tan2n\alpha}\right]=?$$

$$\Rightarrow\left[\frac{\cos\alpha+i\sin\alpha}{\cos\alpha-i\sin\alpha}\right]^{2n}-\left[\frac{\cos2n\alpha+i\sin2n\alpha}{\cos2n\alpha-i\sin2n\alpha}\right]$$

$$\Rightarrow\left[\frac{e^{i\alpha}}{e^{i(-\alpha)}}\right]^{2n}-\left[\frac{e^{i2n\alpha}}{e^{i(-2n\alpha)}}\right]$$

$$\left[e^{i\alpha+i\alpha}\right]^{2n}-\left[e^{i2n\alpha+i2n\alpha}\right]$$

$$\cancel{\left[e^{i4n\alpha}\right]}-\cancel{\left[e^{i4n\alpha}\right]}=0$$

Rest

1) D.M.T.
2) Rotation Thm.
3) Cube Root / n^{th} Root

40% Remaining.

CIRCLE

24. The tangent from the point of intersection of the lines $2x - 3y + 1 = 0$ and $3x - 2y - 1 = 0$ to the circle $x^2 + y^2 + 2x - 4y = 0$ is

(A) $x + 2y = 0, x - 2y + 1 = 0$

(B) $2x - y - 1 = 0$

(C) $y = x, y = 3x - 2$

(D) $2x + y + 1 = 0$

$2x - 3y + 1 = 0 \Rightarrow 4x - 6y + 2 = 0$

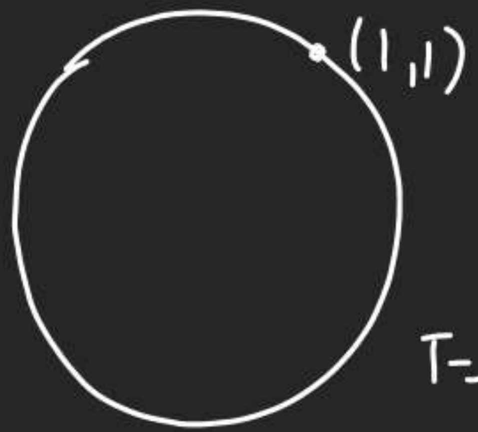
$3x - 2y - 1 = 0 \Rightarrow 9x - 6y - 3 = 0$

$- \quad + \quad +$

$-5x = -5$

$x = 1$

$y = 1$



$T = 0$

$x \cdot 1 + y \cdot 1 + (x+1) - 2(y+1) = 0$

CIRCLE

25. The equation of the circle having the lines $y^2 - 2y + 4x - 2xy = 0$ as its normals & passing through the point $(2, 1)$ is

(A) $x^2 + y^2 - 2x - 4y + 3 = 0$

(B) $x^2 + y^2 - 2x + 4y - 5 = 0$

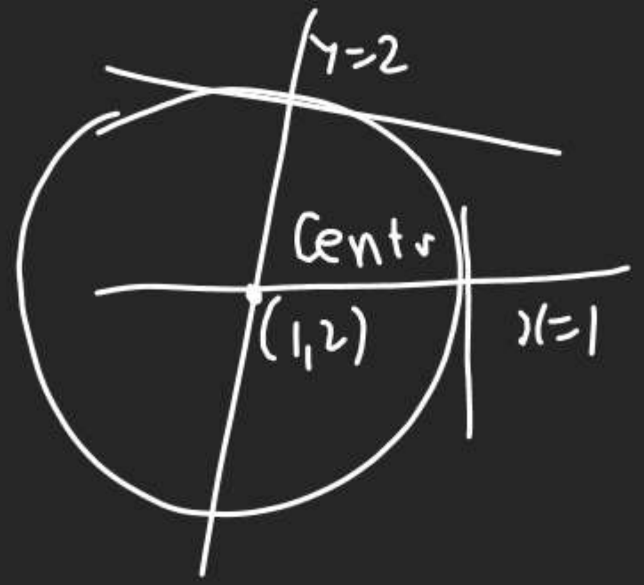
(C) $x^2 + y^2 + 2x + 4y - 13 = 0$

(D) none

$y(y-2) + 2x(2-y) = 0$

$y = 2x, y = 2$

$x = 1$ & $y = 2$



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CIRCLE

26. The equation of director circle to the circle $x^2 + y^2 = 8$ is-

(A) $x^2 + y^2 = 8$

(B) $x^2 + y^2 = 16$

(C) $x^2 + y^2 = 4$

(D) $x^2 + y^2 = 12$

$x^2 + y^2 = 16.$

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CIRCLE

27. Two perpendicular tangents to the circle $x^2 + y^2 = a^2$ meet at P. Then the locus of P has the equation-

(A) $x^2 + y^2 = 2a^2$

(B) $x^2 + y^2 = 3a^2$

(C) $x^2 = y^2 = 4a^2$

(D) None of these

$x^2+y^2=2a^2$ (D.C.)

28. The locus of the mid-points of the chords of the circle $x^2 + y^2 - 2x - 4y - 11 = 0$ which subtend 60° at the centre is

(A) $x^2 + y^2 - 4x - 2y - 7 = 0$

(B) $x^2 + y^2 + 4x + 2y - 7 = 0$

(C) $x^2 + y^2 - 2x - 4y - 7 = 0$

(D) $x^2 + y^2 + 2x + 4y + 7 = 0$

$\sqrt{1+4+11} = 4$

$\frac{\sqrt{h^2+k^2}}{4} = \cos 30$

CIRCLE

29. Find the locus of mid point of chords of circle $x^2 + y^2 = 25$ which subtends right angle at origin-

(A) $x^2 + y^2 = 25/4$

(B) $x^2 + y^2 = 5$

(C) $x^2 + y^2 = 25/2$

(D) $x^2 + y^2 = 5/2$

CIRCLE

30. The equation to the chord of the circle $x^2 + y^2 = 16$ which is bisected at $(2, -1)$ is-

(A) $2x + y = 16$
(B) $2x - y = 16$
(C) $x + 2y = 5$
(D) $2x - y = 5$

$T = S_1$

$2x - y = 4 + (1)$

$2x - y = 5$

CIRCLE

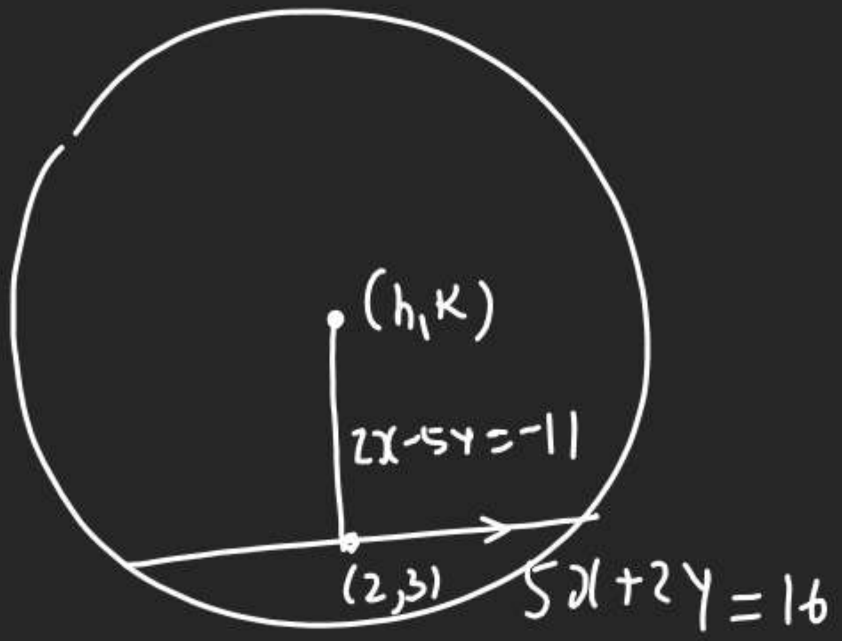
31. The locus of the centres of the circles such that the point $(2, 3)$ is the mid point of the chord $5x + 2y = 16$ is

(A) $2x - 5y + 11 = 0$

(B) $2x + 5y - 11 = 0$

(C) $2x + 5y + 11 = 0$

(D) none



$2h - 5K = -11$

$2x - 5y + 11 = 0$