

EXERCISE - 1

Objective Problems | JEE Main

SECTION - A

DISTANCE BETWEEN TWO POINTS

- If the sum of the squares of the distances of a point from the three coordinate axes be 36, then its distance from the origin is
(A) 6 (B) $3\sqrt{2}$
(C) $2\sqrt{3}$ (D) $6\sqrt{2}$
- The locus of a point P which moves such that $PA^2 - PB^2 = 2k^2$ where A and B are (3, 4, 5) and (-1, 3, -7) respectively is
(A) $8x + 2y + 24z - 9 + 2k^2 = 0$
(B) $8x + 2y + 24z - 2k^2 = 0$
(C) $8x + 2y + 24z + 9 + 2k^2 = 0$
(D) None of these

SECTION - B

DIRECTION COSINES AND DIRECTION RATIOS OF A LINE

- A line makes angles α, β, γ with the coordinates axes. If $\alpha + \beta = 90^\circ$, then γ equal to
(A) 0 (B) 90°
(C) 180° (D) None of these
- If a line makes an angle of $\frac{\pi}{4}$ with the positive directions of each of x-axis and y-axis, then the angle that the line makes with the positive direction of the z-axis is
(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{3}$
(C) $\frac{\pi}{4}$ (D) $\frac{\pi}{2}$
- A line makes the same angle θ with each of the x and z-axis. If the angle β , which it makes with y-axis is such that $\sin^2 \beta = 3 \sin^2 \theta$, then $\cos^2 \theta$ equals
(A) $\frac{2}{3}$ (B) $\frac{1}{5}$
(C) $\frac{3}{5}$ (D) $\frac{2}{5}$
- Let O be the origin and P be the point at a distance 3 units from origin. If D.r.'s of OP are (1, -2, -2), then co-ordinates of P is given by
(A) 1, -2, -2 (B) 3, -6, -6
(C) $\frac{1}{3}, -\frac{2}{3}, -\frac{2}{3}$ (D) $\frac{1}{9}, -\frac{2}{9}, -\frac{2}{9}$

SECTION - D

PROJECTION PROBLEMS

- The coordinates of the point A, B, C, D are (4, α , 2), (5, -3, 2), (β , 1, 1) & (3, 3, -1). Line AB would be perpendicular to line CD when
(A) $\alpha = -1, \beta = -1$ (B) $\alpha = 1, \beta = 2$
(C) $\alpha = 2, \beta = 1$ (D) $\alpha = 2, \beta = 2$

SECTION - C / E

ANGLE BETWEEN TWO LINES / EQUATION OF A LINE AND ANGLE BETWEEN THEM

- The straight lines $\frac{x-1}{1} = \frac{y-2}{2} = \frac{z-3}{3}$ and $\frac{x-1}{2} = \frac{y-2}{2} = \frac{z-3}{-2}$ are
(A) Parallel lines
(B) intersecting at 60°
(C) Skew lines
(D) Intersecting at right angle
- Let the points A(a, b, c) and B(a', b', c') be at distances r and r' from origin. The line AB passes through origin when
(A) $\frac{a'}{a} = \frac{b'}{b} = \frac{c'}{c}$
(B) $aa' + bb' + cc' = rr'$
(C) $aa' + bb' + cc' = r^2 + r'^2$
(D) None of these
- Angle between the pair of lines $\frac{x-2}{1} = \frac{y-1}{5} = \frac{z+3}{-3}$ and $\frac{x+1}{-1} = \frac{y-4}{8} = \frac{z-5}{4}$
(A) $\cos^{-1} \left(\frac{13}{9\sqrt{38}} \right)$ (B) $\cos^{-1} \left(\frac{3}{\sqrt{35}} \right)$
(C) $\cos^{-1} \left(\frac{4}{\sqrt{38}} \right)$ (D) $\cos^{-1} \left(\frac{2\sqrt{2}}{\sqrt{19}} \right)$
- If the lines $\frac{x-1}{-3} = \frac{y-2}{2k} = \frac{z-3}{2}$ and $\frac{x-1}{3k} = \frac{y-5}{1} = \frac{z-6}{-5}$ are perpendicular to each other then k =
(A) $\frac{5}{7}$ (B) $\frac{7}{5}$ (C) $\frac{-7}{10}$ (D) $\frac{-10}{7}$

12. The angle between the lines, whose direction ratios are 1, 1, 2 and $\sqrt{3} - 1, -\sqrt{3} - 1, 4$, is-

(A) 45° (B) 30°
(C) 60° (D) 90°

13. A tetrahedron has vertices at $O(0, 0, 0)$, $A(1, 2, 1)$, $B(2, 1, 3)$ and $C(-1, 1, 2)$. Then the angle between the faces OAB and ABC will be-

(A) 90° (B) $\cos^{-1}\left(\frac{19}{35}\right)$

(C) $\cos^{-1}\left(\frac{17}{31}\right)$ (D) 30°

14. Two systems of rectangular axes have the same origin. If a plane makes intercepts a, b, c and a', b', c' on the two systems of axes respectively, then

(A) $a^2 + b^2 + c^2 = a'^2 + b'^2 + c'^2$

(B) $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = \frac{1}{a'} + \frac{1}{b'} + \frac{1}{c'}$

(C) $\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} = \frac{1}{a'^2} + \frac{1}{b'^2} + \frac{1}{c'^2}$

(D) $\frac{1}{a^2 - a'^2} + \frac{1}{b^2 - b'^2} + \frac{1}{c^2 - c'^2} = 0$

15. If the straight lines $x = 1 + s, y = -3 - \lambda s, z = 1 + \lambda s$ and $x = \frac{t}{2}, y = 1 + t, z = 2 - t$, with parameters s and t respectively are coplanar then λ equals

(A) -2 (B) -1
(C) $-1/2$ (D) 0

SECTION - F

PERPENDICULAR DISTANCE OF A POINT FROM A LINE, FOOT OF THE PERPENDICULAR

16. A line with direction cosines proportional to 2, 1, 2 meets each of the lines $x = y + a = z$ and $x + a = 2y = 2z$. The co-ordinates of each of the points of intersection are given by

(A) $(3a, 3a, 3a), (a, a, a)$
(B) $(3a, 2a, 3a), (a, a, a)$
(C) $(3a, 2a, 3a), (a, a, 2a)$
(D) $(2a, 3a, 3a), (2a, a, a)$

SECTION - G

DISTANCE BETWEEN TWO LINES AND INTERSECTION POINT

17. If the lines $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}, \frac{x-1}{3} = \frac{y-2}{-1} = \frac{z-3}{4}$ and

$\frac{x+k}{3} = \frac{y-1}{2} = \frac{z-2}{h}$ are concurrent then

(A) $h = -2, k = -6$ (B) $h = \frac{1}{2}, k = 2$

(C) $h = 6, k = 2$ (D) $h = 2, k = \frac{1}{2}$

18. The lines $\frac{x-2}{1} = \frac{y-3}{1} = \frac{z-4}{-k}$ and

$\frac{x-1}{k} = \frac{y-4}{2} = \frac{z-5}{1}$ are coplanar if

(A) $k = 0$ or -1 (B) $k = 1$ or -1

(C) $k = 0$ or -3 (D) $k = 3$ or -3

SECTION - H

DIFFERENT FORM OF THE PLANE

19. The equation of plane which passes through $(2, -3, 1)$ & is normal to the line joining the points $(3, 4, -1)$ & $(2, -1, 5)$ is given by

(A) $x + 5y - 6z + 19 = 0$

(B) $x - 5y + 6z - 19 = 0$

(C) $x + 5y + 3z + 19 = 0$

(D) $x - 5y - 6z - 19 = 0$

20. The equation of the plane passing through the point $(1, -3, -2)$ and perpendicular to planes $x + 2y + 2z = 5$ and $3x + 3y + 2z = 8$, is

(A) $2x - 4y + 3z - 8 = 0$

(B) $2x - 4y - 3z + 8 = 0$

(C) $2x - 4y + 3z + 8 = 0$

(D) None of these

21. If plane cuts off intercepts $OA = a, OB = b, OC = c$ from the coordinate axes, then the area of the triangle ABC equal to

(A) $\frac{1}{2}\sqrt{b^2c^2 + c^2a^2 + a^2b^2}$

(B) $\frac{1}{2}(bc + ca + ab)$

(C) $\frac{1}{2}abc$

(D) $\frac{1}{2}\sqrt{(b+c)^2(c-a)^2 + (a-b)^2}$



22. The equation of plane which meet the co-ordinate axes whose centroid is (a, b, c)

(A) $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ (B) $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 0$
(C) $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 3$ (D) $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = \frac{1}{3}$

23. A variable plane is at a constant distance p from the origin and meets the axes in A, B and C . The locus of the centroid of the tetrahedron $OABC$ is

(A) $x^{-2} + y^{-2} + z^{-2} = 16p^{-2}$ (B) $\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = \frac{16}{p}$
(C) $\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = 16$ (D) None of these

24. The acute angle between the planes $2x - y + z = 6$ and $x + y + 2z = 3$ is-

(A) 30° (B) 45°
(C) 60° (D) 75°

SECTION - I

INTERSECTION OF TWO PLANES

25. The locus represented by $xy + yz = 0$ is
(A) A pair of perpendicular lines
(B) A pair of parallel lines
(C) A pair of parallel planes
(D) A pair of perpendicular planes
26. A variable plane passes through a fixed point (a, b, c) and meets the coordinate axes in A, B, C . Locus of the point common to the planes through A, B, C and parallel to coordinate plane, is

(A) $\frac{a}{x} + \frac{b}{y} + \frac{c}{z} = 1$ (B) $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$
(C) $ax + by + cz = 1$ (D) None of these

27. The angle between the plane $2x - y + z = 6$ and a plane perpendicular to the planes $x + y + 2z = 7$ and $x - y = 3$ is

(A) $\pi/4$ (B) $\pi/3$
(C) $\pi/6$ (D) $\pi/2$

28. The direction ratios of a normal to the plane through $(1, 0, 0), (0, 1, 0)$, which makes an angle of $\pi/4$ with the plane $x + y = 3$ are

(A) $(1, \sqrt{2}, 1)$ (B) $(1, 1, \sqrt{2})$
(C) $(1, 1, 2)$ (D) $(\sqrt{2}, 1, 1)$

29. Distance between two parallel planes $2x + y + 2z = 8$ and $4x + 2y + 4z + 5 = 0$ is

(A) $3/2$ (B) $5/2$
(C) $7/2$ (D) $9/2$

SECTION - G LENGTH & FOOT OF PERPENDICULAR & IMAGE OF THE POINT W.R.T. PLANE

30. A variable plane passes through a fixed point $(1, 2, 3)$. The locus of the foot of the perpendicular drawn from origin to this plane is

(A) $x^2 + y^2 + z^2 - x - 2y - 3z = 0$
(B) $x^2 + 2y^2 + 3z^2 - x - 2y - 3z = 0$
(C) $x^2 + 4y^2 + 9z^2 + x + 2y + 3 = 0$
(D) $x^2 + y^2 + z^2 + x + 2y + 3z = 0$

31. The reflection of the point $(2, -1, 3)$ in the plane $3x - 2y - z = 9$ is

(A) $\left(\frac{26}{7}, \frac{15}{7}, \frac{17}{7}\right)$ (B) $\left(\frac{26}{7}, -\frac{15}{7}, \frac{17}{7}\right)$
(C) $\left(\frac{15}{7}, \frac{26}{7}, -\frac{17}{7}\right)$ (D) $\left(\frac{26}{7}, \frac{15}{7}, -\frac{15}{7}\right)$

32. Two systems of rectangular axes have same origin. If a plane cuts them at distances a, b, c and a_1, b_1, c_1 from the origin, then

(A) $\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} = \frac{1}{a_1^2} + \frac{1}{b_1^2} + \frac{1}{c_1^2}$
(B) $\frac{1}{a^2} - \frac{1}{b^2} + \frac{1}{c^2} = \frac{1}{a_1^2} - \frac{1}{b_1^2} + \frac{1}{c_1^2}$
(C) $a^2 + b^2 + c^2 = a_1^2 + b_1^2 + c_1^2$
(D) $a^2 - b^2 + c^2 = a_1^2 + b_1^2 + c_1^2$

33. Distance between two parallel planes $2x + y + 2z = 8$ and $4x + 2y + 4z + 5 = 0$ is

(A) $3/2$ (B) $5/2$
(C) $7/2$ (D) $9/2$

SECTION - K : LINE AND PLANE

34. The distance of the point $(-1, -5, -10)$ from the point of intersection of the line, $\frac{x-2}{3} = \frac{y+1}{4} = \frac{z-2}{12}$ and the plane, $x - y + z = 5$, is

(A) 10 (B) 11
(C) 12 (D) 13

35. The distance of the point $(1, -2, 3)$ from the plane $x - y + z = 5$ measured parallel to the line,

$\frac{x}{2} = \frac{y}{3} = \frac{z}{-6}$ is

(A) 1 (B) $6/7$
(C) $7/6$ (D) None of these

36. The non zero value of 'a' for which the lines $2x - y + 3z + 4 = 0 = ax + y - z + 2$ and $x - 3y + z = 0 = x + 2y + z + 1$ are co-planar is
(A) -2 (B) 4
(C) 6 (D) 0
37. Let L be the line of intersection of the planes $2x + 3y + z = 1$ and $x + 3y + 2z = 2$. If L makes an angle α with the positive x-axis, the $\cos \alpha$ equals
(A) $\frac{1}{\sqrt{3}}$ (B) $\frac{1}{2}$
(C) 1 (D) $\frac{1}{\sqrt{2}}$
38. If the angle θ between the line $\frac{x+1}{1} = \frac{y-1}{2} = \frac{z-2}{2}$ and the plane $2x - y + \sqrt{\lambda}z + 4 = 0$ is such that $\sin \theta = \frac{1}{3}$. The value of λ is
(A) $-\frac{4}{3}$ (B) $\frac{3}{4}$
(C) $-\frac{3}{5}$ (D) $\frac{5}{3}$
39. The shortest distance between the z-axis and the line, $x + y + 2z - 3 = 0$, $2x + 3y + 4z - 4 = 0$ is
(A) 1 (B) 2
(C) 3 (D) None of these
42. The line, $\frac{x-2}{3} = \frac{y+1}{2} = \frac{z-1}{-1}$ intersects the curve $xy = c^2$, $z = 0$ then c is equal to
(A) ± 1 (B) $\pm \frac{1}{3}$
(C) $\pm \sqrt{5}$ (D) None of these
43. The lines $\frac{x-2}{1} = \frac{y-3}{1} = \frac{z-4}{-k}$ and $\frac{x-1}{k} = \frac{y-4}{2} = \frac{z-5}{1}$ are coplanar if -
(A) $k = 3$ or -3 (B) $k = 0$ or -1
(C) $k = 1$ or -1 (D) $k = 0$ or -3

SECTION - L : MIXED PROBLEMS

40. A point moves so that the sum of the squares of its distances from the six faces of a cube given by $x = \pm 1$, $y = \pm 1$, $z = \pm 1$ is 10 units. The locus of the point is
(A) $x^2 + y^2 + z^2 = 1$
(B) $x^2 + y^2 + z^2 = 2$
(C) $x + y + z = 1$
(D) $x + y + z = 2$
41. The coplanar points A, B, C, D are $(2-x, 2, 2)$, $(2, 2-y, 2)$, $(2, 2, 2-z)$ and $(1, 1, 1)$ respectively. Then
(A) $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 1$
(B) $x + y + z = 1$
(C) $\frac{1}{1-x} + \frac{1}{1-y} + \frac{1}{1-z} = 1$
(D) None of these

EXERCISE - 2 (Level-I)

Objective Problems | JEE Main

DIRECTION COSINES AND DIRECTION RATIOS OF A LINE

1. A mirror and a source of light are situated at the origin O and at a point on OX, respectively. A ray of light from the source strikes the mirror and is reflected. If the D.r.'s of the normal to the plane are 1, -1, 1, then D.C.'s of the reflected ray are

(A) $\frac{1}{3}, \frac{2}{3}, \frac{2}{3}$ (B) $-\frac{1}{3}, \frac{2}{3}, \frac{2}{3}$
(C) $-\frac{1}{3}, -\frac{2}{3}, -\frac{2}{3}$ (D) $-\frac{1}{3}, -\frac{2}{3}, \frac{2}{3}$

2. If the line joining the origin and the point (-2, 1, 2) makes angle θ_1, θ_2 and θ_3 with the positive direction of the coordinate axes, then the value of

$\cos 2\theta_1 + \cos 2\theta_2 + \cos 2\theta_3$ is

- (A) -1 (B) 1
(C) 2 (D) -2

3. The square of the perpendicular distance of point P(p, q, r) from a line through A(a, b, c) and whose direction cosine are ℓ, m, n is

- (A) $\Sigma\{(q-b)n-(r-c)m\}^2$
(B) $\Sigma\{(q+b)n-(r+c)m\}^2$
(C) $\Sigma\{(q-b)n+(r-c)m\}^2$
(D) None of these

ANGLE BETWEEN TWO LINES

4. The two lines $x = ay + b, z = cy + d$ and $x = a'y + b', z = c'y + d'$ will be perpendicular, iff

- (A) $aa' + bb' + cc' + 1 = 0$
(B) $aa' + bb' + cc' = 0$
(C) $(a + a')(b + b') + (c + c') = 0$
(D) $aa' + cc' + 1 = 0$

5. ABC is a triangle where A = (2, 3, 5), B = (-1, 2, 2) and C($\lambda, 5, \mu$). If the median through A is equally inclined to the axes then

- (A) $\lambda = \mu = 5$ (B) $\lambda = 5, \mu = 7$
(C) $\lambda = 6, \mu = 9$ (D) $\lambda = 0, \mu = 0$

6. The direction cosines of a line equally inclined to three mutually perpendicular lines having D.C.'s as

$\ell_1, m_1, n_1; \ell_2, m_2, n_2; \ell_3, m_3, n_3$ are

- (A) $\ell_1 + \ell_2 + \ell_3, m_1 + m_2 + m_3, n_1 + n_2 + n_3$
(B) $\frac{\ell_1 + \ell_2 + \ell_3}{\sqrt{3}}, \frac{m_1 + m_2 + m_3}{\sqrt{3}}, \frac{n_1 + n_2 + n_3}{\sqrt{3}}$
(C) $\frac{\ell_1 + \ell_2 + \ell_3}{3}, \frac{m_1 + m_2 + m_3}{3}, \frac{n_1 + n_2 + n_3}{3}$
(D) None of these

LINE AND PLANE

7. Equation of plane which passes through the point

of intersection of lines $\frac{x-1}{3} = \frac{y-2}{1} = \frac{z-3}{2}$ and

$\frac{x-3}{1} = \frac{y-1}{2} = \frac{z-2}{3}$ and at greatest distance

from the point (0, 0, 0) is

- (A) $4x + 3y + 5z = 25$ (B) $4x + 3y + 5z = 50$
(C) $3x + 4y + 5z = 49$ (D) $x + 7y - 5z = 2$

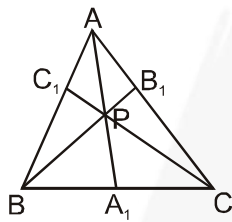
8. The co-ordinates of the point where the line joining the points (2, -3, 1), (3, -4, -5) cuts the plane $2x + y + z = 7$ are

- (A) (2, 1, 0) (B) (3, 2, 5)
(C) (1, -2, 7) (D) None of these

MIXED PROBLEMS

9. In the adjacent figure 'P' is any arbitrary interior point of the triangle ABC such that the lines AA_1, BB_1, CC_1 are concurrent at P. Value of

$$\frac{PA_1}{AA_1} + \frac{PB_1}{BB_1} + \frac{PC_1}{CC_1} \text{ is always equal to}$$



- (A) 1
(B) 2
(C) 3
(D) None of these

10. The equation of motion of a point in space is $x = 2t$, $y = -4t$, $z = 4t$ where t measured in hours and the co-ordinates of moving point in kilometers. The distance of the point from the starting point $O(0, 0, 0)$ in 10 hours is
(A) 20 km (B) 40 km
(C) 60 km (D) 55 km

11. Minimum value of $x^2 + y^2 + z^2$ when $ax + by + cz = p$ is
(A) $\frac{p}{\Sigma a}$ (B) $\frac{p^2}{\Sigma a^2}$
(C) $\frac{\Sigma a^2}{p}$ (D) 0

EXERCISE - 2 (Level-II) Multiple

Correct | JEE Advanced

ANGLE BETWEEN TWO LINES

1. The direction cosines of the lines bisecting the angle between the lines whose direction cosines are ℓ_1, m_1, n_1 and ℓ_2, m_2, n_2 and the angle between these lines is θ , are

- (A) $\frac{\ell_1 + \ell_2}{\cos \frac{\theta}{2}}, \frac{m_1 + m_2}{\cos \frac{\theta}{2}}, \frac{n_1 + n_2}{\cos \frac{\theta}{2}}$
 (B) $\frac{\ell_1 + \ell_2}{2 \cos \frac{\theta}{2}}, \frac{m_1 + m_2}{2 \cos \frac{\theta}{2}}, \frac{n_1 + n_2}{2 \cos \frac{\theta}{2}}$
 (C) $\frac{\ell_1 + \ell_2}{\sin \frac{\theta}{2}}, \frac{m_1 + m_2}{\sin \frac{\theta}{2}}, \frac{n_1 + n_2}{\sin \frac{\theta}{2}}$
 (D) $\frac{\ell_1 + \ell_2}{2 \sin \frac{\theta}{2}}, \frac{m_1 + m_2}{2 \sin \frac{\theta}{2}}, \frac{n_1 + n_2}{2 \sin \frac{\theta}{2}}$

EQUATION OF A LINE AND ANGLE BETWEEN THEM

2. Consider the lines $\frac{x}{2} = \frac{y}{3} = \frac{z}{5}$ and $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ the

equation of the line which

- (A) bisects the angle between the lines is

$$\frac{x}{3} = \frac{y}{3} = \frac{z}{8}$$

- (B) bisects the angle between the lines is

$$\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$$

- (C) passes through origin and is perpendicular to the given lines is $x = y = -z$

- (D) None of these

DISTANCE BETWEEN TWO LINES AND INTERSECTION POINT

3. Let a perpendicular PQ be drawn from P(5, 7, 3) to the line $\frac{x-15}{3} = \frac{y-2}{8} = \frac{z-6}{-5}$ when Q is the foot.

Then

- (A) Q is (9, 13, -15)

- (B) PQ = 14

- (C) the equation of plane containing PQ and the given line is $9x - 4y - z - 14 = 0$

- (D) None of these

DIFFERENT FORMS OF THE PLANE

4. The acute angle that the vector $2\hat{i} - 2\hat{j} + \hat{k}$ makes with the plane contained by the two vectors $2\hat{i} + 3\hat{j} - \hat{k}$ and $\hat{i} - \hat{j} + 2\hat{k}$ is given by

(A) $\cos^{-1} \left(\frac{1}{\sqrt{3}} \right)$ (B) $\sin^{-1} \left(\frac{1}{\sqrt{3}} \right)$

(C) $\tan^{-1} (\sqrt{2})$ (D) $\cot^{-1} (\sqrt{2})$

INTERSECTION OF TWO PLANES

5. The planes $2x - 3y - 7z = 0$, $3x - 14y - 13z = 0$ and $8x - 31y - 33z = 0$

- (A) pass through origin (B) intersect in a common line
 (C) form a triangular prism (D) None of these

LENGTH & FOOT OF PERPENDICULAR & IMAGE OF THE POINT W.R.T. PLANE

6. If the length of perpendicular drawn from origin on a plane is 7 units and its direction ratios are -3, 2, 6, then that plane is

- (A) $-3x + 2y + 6z - 7 = 0$ (B) $-3x + 2y + 6z - 49 = 0$
 (C) $3x - 2y - 6z - 49 = 0$ (D) $-3x + 2y - 6z - 49 = 0$

LINE AND PLANE

7. Equation of the plane passing through A(x_1, y_1, z_1)

and containing the line $\frac{x-x_2}{d_1} = \frac{y-y_2}{d_2} = \frac{z-z_2}{d_3}$

is

(A) $\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ d_1 & d_2 & d_3 \end{vmatrix} = 0$

(B) $\begin{vmatrix} x-x_2 & y-y_2 & z-z_2 \\ x_1-x_2 & y_1-y_2 & z_1-z_2 \\ d_1 & d_2 & d_3 \end{vmatrix} = 0$

(C) $\begin{vmatrix} x-d_1 & y-d_2 & z-d_3 \\ x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \end{vmatrix} = 0$

(D) $\begin{vmatrix} x & y & z \\ x_1-d_1 & y_1-d_2 & z_1-d_3 \\ x_2 & y_2 & z_2 \end{vmatrix} = 0$

8. The equation of the line $x + y + z - 1 = 0$, $4x + y - 2z + 2 = 0$ written in the symmetrical form is
- (A) $\frac{x+1}{1} = \frac{y-2}{-2} = \frac{z-0}{1}$
- (B) $\frac{x}{1} = \frac{y}{-2} = \frac{z}{1}$
- (C) $\frac{x+1/2}{1} = \frac{y-1}{-2} = \frac{z-1/2}{1}$
- (D) $\frac{x-1}{2} = \frac{y+2}{-1} = \frac{z-2}{2}$
9. The equations of the planes through the origin which are parallel to the line $\frac{x-1}{2} = \frac{y+3}{-1} = \frac{z+1}{-2}$ and distance $\frac{5}{3}$ from it are
- (A) $2x + 2y + z = 0$ (B) $x + 2y + 2z = 0$
- (C) $2x - 2y + z = 0$ (D) $x - 2y + 2z = 0$
10. The equation of line AB is $\frac{x}{2} = \frac{y}{-3} = \frac{z}{6}$. Through a point $P(1, 2, 5)$, line PN is drawn perpendicular to AB and line PQ is drawn parallel to the plane $3x + 4y + 5z = 0$ to meet AB is Q. Then
- (A) co-ordinate of N is $\left(\frac{52}{49}, -\frac{78}{49}, \frac{156}{49}\right)$
- (B) the equation of PN is $\frac{x-1}{3} = \frac{y-2}{-176} = \frac{z-5}{-89}$
- (C) the co-ordinates of Q is $\left(3, -\frac{9}{2}, 9\right)$
- (D) the equation of PQ is $\frac{x-1}{4} = \frac{y-2}{-13} = \frac{z-5}{8}$
11. **MIXED PROBLEMS**
- If the edges of a rectangular parallelopiped are 3, 2, 1 then the angle between a pair of diagonals is given by
- (A) $\cos^{-1} \frac{6}{7}$ (B) $\cos^{-1} \frac{3}{7}$
- (C) $\cos^{-1} \frac{2}{7}$ (D) None of these

EXERCISE - 3

Subjective | JEE Advanced

1. Show that points $(0, 7, 10)$, $(-1, 6, 6)$ and $(-4, 9, 6)$ form an isosceles right angled triangle.
2. Prove that the tetrahedron with vertices at the points $(0, 0, 0)$, $(0, 1, 1)$, $(1, 0, 1)$, $(1, 1, 0)$ is a regular tetrahedron. Find also the co-ordinates of its centroid.
3. Find the coordinates of the point equidistant from the point $(a, 0, 0)$, $(0, b, 0)$, $(0, 0, c)$ and $(0, 0, 0)$.
4. What are the direction cosines of a line that passes through the points $P(6, -7, -1)$ and $Q(2, -3, 1)$ and is so directed that it makes an acute angle α with the positive direction of x-axis.
5. Find the angle between the lines whose direction cosines are given by $\ell + m + n = 0$ and $\ell^2 + m^2 = n^2$.
6. Find the acute angle between the lines $\frac{x-1}{\ell} = \frac{y+1}{m} = \frac{z}{n}$ & $\frac{x+1}{m} = \frac{y-3}{n} = \frac{z-1}{\ell}$ where $\ell > m > n$ and ℓ, m, n are the roots of the cubic equation $x^3 + x^2 - 4x = 4$.
7. P and Q are the points $(-1, 2, 1)$ and $(4, 3, 5)$. Find the projection of PQ on a line which makes angles of 120° and 135° with y and z axes respectively and an acute angle with x-axis.
8. Find the equations of the two lines through the origin which intersect the line $\frac{x-3}{2} = \frac{y-3}{1} = \frac{z}{1}$ at an angle of $\frac{\pi}{3}$.
9. Find the equation to the line passing through the point $(1, -2, -3)$ parallel to the line $2x + 3y - 3z + 2 = 0 = 3x - 4y + 2z - 4$.
10. Find the equation to the line which can be drawn from the point $(2, -1, 3)$ perpendicular to the lines $\frac{x-1}{2} = \frac{y-2}{2} = \frac{z-3}{2}$ and $\frac{x-4}{3} = \frac{y}{2} = \frac{z+3}{1}$.
11. Show that the foot of the perpendicular from the origin to the join of $A(-9, 4, 5)$ and $B(11, 0, -1)$ is the mid point of AB.
12. If $2d$ be the shortest distance between the lines $\frac{y}{b} + \frac{z}{c} = 1$; $x = 0$ & $\frac{x}{a} - \frac{z}{c} = 1$; $y = 0$ then prove that $\frac{1}{d^2} = \frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2}$.
13. Find the distance between points of intersection of
(i) Lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ & $\frac{x-4}{5} = \frac{y-1}{2} = z$
(ii) Lines $\vec{r} = (\hat{i} + \hat{j} - \hat{k}) + \lambda(3\hat{i} - \hat{j})$ & $\vec{r} = (4\hat{i} - \hat{k}) + \mu(2\hat{i} + 3\hat{k})$
14. If the distance between point $(\alpha, 5\alpha, 10\alpha)$ from the point of intersection of the lines $\vec{r} = (2\hat{i} - \hat{j} + 2\hat{k}) + \lambda(2\hat{i} + 4\hat{j} + 12\hat{k})$ and plane $\vec{r} \cdot (\hat{i} - \hat{j} + \hat{k}) = 5$ is 13 units. Find the possible values of α .
15. (i) Find the equation of the plane passing through the points $(2, 1, 0)$, $(5, 0, 1)$ and $(4, 1, 1)$.
(ii) If P is the point $(2, 1, 6)$ then find the point Q such that PQ is perpendicular to the plane in (i) and the mid point of PQ lies on it.

16. Find the equation of the planes passing through points $(1, 0, 0)$ and $(0, 1, 0)$ and making an angle of 0.25π radians with plane $x + y - 3 = 0$.
17. Find the angle between the plane passing through point $(1, 1, 1)$, $(1, -1, 1)$, $(-7, -3, -5)$ & x - z plane.
18. Find the point where the line of intersection of the planes $x - 2y + z = 1$ and $x + 2y - 2z = 5$, intersect the plane $2x + 2y + z + 6 = 0$.
19. Find the plane π passing through the points of intersection of the planes $2x + 3y - z + 1 = 0$ and $x + y - 2z + 3 = 0$ and is perpendicular to the plane $3x - y - 2z = 4$. Find the image of point $(1, 1, 1)$ in plane π .
20. Feet of the perpendicular drawn from the point $P(2, 3, -5)$ on the axes of coordinates are A , B and C . Find the equation of the plane passing through their feet and the area of $\triangle ABC$.
21. Find the ratio in which the line joining the points $(3, 5, -7)$ and $(-2, 1, 8)$ is divided by the y - z plane. Find also the point of intersection on the plane and the line.
22. Find the equation of the plane containing parallel lines $(x-4) = \frac{3-y}{4} = \frac{z-2}{5}$ and $(x-3) = \lambda(y+2) = \mu z$.
23. Find the equation of image of the line $\frac{x-1}{9} = \frac{y-2}{-1} = \frac{z+3}{-3}$ in the plane $3x - 3y + 10z = 26$.
24. Find the equation of the straight line which passes through the point $(2, -1, -1)$; is parallel to the plane $4x + y + z + 2 = 0$ and is perpendicular to the line of intersection of the planes $2x + y = 0$, $x - y + z$.
25. Find the equation of the projection of line $3x - y + 2z - 1 = 0$, $x + 2y - z - 2 = 0$ on the plane $3x + 2y + z = 0$.
26. Prove that the line $\frac{x-1}{2} = \frac{y-2}{-3} = \frac{z+3}{1}$ lies in the plane $3x + 4y + 6z + 7 = 0$. If the plane is rotated about the line till the plane passes through the origin then find the equation of the plane in the new position.
27. Prove that the line $\frac{x}{1} = \frac{y}{1} = \frac{z-1}{-2}$ lies in the plane $x + y + z = 1$. Find the lines in the plane through the point $(0, 0, 1)$ which are inclined at an angle $\cos^{-1}\left(\frac{1}{\sqrt{6}}\right)$ with the line.
28. Find the equations of the straight line passing through the point $(1, 2, 3)$ to intersect the straight line $x + 1 = 2(y - 2) = z + 4$ and parallel to the plane $x + 5y + 4z = 0$.
29. Find the distance of the point $P(-2, 3, -4)$ from the line $\frac{x+2}{3} = \frac{2y+3}{4} = \frac{3z+4}{5}$ measured parallel to the plane $4x + 12y - 3z + 1 = 0$.
30. Find the equation of the line passing through the point $(4, -14, 4)$ and intersecting the line of intersection of the planes $3x + 2y - z = 5$ and $x - 2y - 2z = -1$ at right angles.
31. Find the equation of the plane containing the straight line $\frac{x-1}{2} = \frac{y+2}{-3} = \frac{z}{5}$ and perpendicular to the plane $x - y + z + 2 = 0$.
32. Find the value of p so that the lines $\frac{x-1}{-3} = \frac{y-p}{2} = \frac{z+2}{1}$ and $\frac{x}{1} = \frac{y-7}{-3} = \frac{z+7}{2}$ are in the same plane. For this value of p , find the coordinates of their point of intersection and the

equation of the plane containing them.

33. Find the equations to the line of greatest slope through the point $(7, 2, -1)$ in the plane $x - 2y + 3z = 0$ assuming that the axes are so placed that the plane $2x + 3y - 4z = 0$ is horizontal.
34. Find the equation of the line which is reflection of the line $\frac{x-1}{9} = \frac{y-2}{-1} = \frac{z+3}{-3}$ in the plane $3x - 3y + 10z = 26$.
35. Find the equation of the plane containing the line $\frac{x-1}{2} = \frac{y}{3} = \frac{z}{2}$ and parallel to the line $\frac{x-3}{2} = \frac{y}{5} = \frac{z-2}{4}$. Find the also the S.D. between the two lines.
36. Let $P(1, 3, 5)$ and $Q(-2, 1, 4)$ be two points from which perpendiculars PM and QN are drawn to the x - z plane. Find the angle that the line MN makes with the plane $x + y + z = 5$.
37. A line $\frac{x+2}{1} = \frac{y-3}{2} = \frac{z-k}{3}$ cuts the y - z plane and the x - y plane at A and B respectively. If $\angle AOB = \frac{\pi}{2}$, then find k , where O is the origin.
38. Find the volume of the tetrahedron with vertices $P(2, 3, 2)$, $Q(1, 1, 1)$, $R(3, -2, 1)$ and $S(7, 1, 4)$.
39. Let PM be the perpendicular from the point $P(1, 2, 3)$ to the x - y plane. If OP makes an angle θ with the positive direction of the z -axis and OM makes an angle ϕ with the positive direction of the x -axis, where O is the origin, then find θ and ϕ .
40. Let $P = (1, 0, -1)$; $Q = (1, 1, 1)$ and $R = (2, 1, 3)$ are three points.
- (a) Find the area of the triangle having P , Q and R as its vertices.
- (b) Given the equation of the plane through P , Q and R in the form $ax + by + cz = 1$.
- (c) Where does the plane in part (b) intersect the y -axis.
- (d) Give parametric equations for the line through R that is perpendicular to the plane in part (b).
41. The line $\frac{x+6}{5} = \frac{y+10}{3} = \frac{z+14}{8}$ is the hypotenuse of an isosceles right angled triangle whose opposite vertex is $(7, 2, 4)$. Find the equation of the remaining sides.

COMPREHENSION

Let two planes $P_1 : 2x - y + z = 2$ and $P_2 : z + 2y - z = 3$ are given.

On the basis of the above information, answer the following questions.

42. The equation of the plane through the intersection of P_1 and P_2 and the point $(3, 2, 1)$ is
 (A) $3x - y + 2z - 9 = 0$
 (B) $x - 3y + 2z + 1 = 0$
 (C) $2x - 3y + z - 1 = 0$
 (D) $4x - 3y + 2z - 8 = 0$
43. Equation of the plane which passes through the point $(-1, 3, 2)$ and is perpendicular to each of the planes P_1 and P_2 is
 (A) $x + 3y - 5z + 2 = 0$ (B) $x + 3y + 5z - 18 = 0$
 (C) $x - 3y - 5z + 20 = 0$ (D) $x - 3y + 5z = 0$
44. The equation of the acute angle bisector of planes P_1 and P_2 is
 (A) $x - 3y + 2z + 1 = 0$ (B) $3x + y - 5 = 0$

(C) $x + 3y - 2z + 1 = 0$ (D) $3x + z + 7 = 0$

MATRIX MATCH TYPE

45. Column - I

Column - II

(A) If acute and obtuse angle (P) A: $32x + 13y - 3 = 0$

bisectors of the planes

$2x - y + 2z + 3 = 0$ and

$3x - 2y + 6z + 8 = 0$

are represented by A and O, (Q) O: $x - 5y - 4z - 45 = 0$

then

(B) If acute and obtuse angle

bisectors of the planes

(R) A: $23x - 13y + 32z + 45 = 0$

$x - 2y + 2z - 3 = 0$ and

$2x - 3y + 6z + 8 = 0$

are represented by A and O, (S) O: $4x - y + 5z - 45 = 0$

then

(C) The acute and obtuse angle

bisectors of the planes

(T) A: $13x - 23y + 32z + 3 = 0$

$2x + y - 2z + 3 = 0$ and

$6x + 2y - 3z - 8 = 0$

are represented by A and O

EXERCISE - 4 | Level-I

Previous Year | JEE Main

1. Let the line $\frac{x-2}{3} = \frac{y-1}{-5} = \frac{z+2}{2}$ lie in the plane

$x + 3y - \alpha z + \beta = 0$. then (α, β) equals :

[AIEEE-2009]

- (A) $(-6, 7)$ (B) $(5, -15)$
(C) $(-5, 5)$ (D) $(6, -17)$

2. The projections of a vector on the three coordinate axis are 6, -3, 2 respectively. The direction cosines of the vector are :

[AIEEE-2009]

- (A) $\frac{6}{5}, \frac{-3}{5}, \frac{2}{5}$ (B) $\frac{6}{7}, \frac{-3}{7}, \frac{2}{7}$
(C) $\frac{-6}{7}, \frac{-3}{7}, \frac{2}{7}$ (D) 6, -3, 2

3. A line AB in three dimensional space makes angles 45° and 120° with the positive x - axis and the positive y - axis respectively. If AB makes an acute angle θ with the positive z - axis, then θ equals-

[AIEEE-2010]

- (A) 30° (B) 45°
(C) 60° (D) 75°

4. If the angle between the line $x = \frac{y-1}{2} = \frac{z-3}{\lambda}$

and the plane $x + 2y + 3z = 4$ is $\cos^{-1}\left(\sqrt{\frac{5}{14}}\right)$, then

λ equals ;

[AIEEE-2011]

- (A) $2/3$ (B) $3/2$
(C) $2/5$ (D) $5/2$

5. Statement - 1 :

The point A(1, 0, 7) is the mirror image of the point B(1, 6, 3) in the line :

$$\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3} \quad [\text{AIEEE-2011}]$$

Statement - 2 : The line : $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$

bisects the line segment joining A(1, 0, 7) and B(1, 6, 3).

(A) Statement (1) is true and statement (2) is true and statement (2) is correct explanation for Statement (1)

(B) Statement (1) is true and statement (2) is true and statement (2) is NOT a correct explanation for Statement (1)

(C) Statement (1) is true but (2) is false

(D) Statement (1) is false but (2) is true

6. An equation of a plane parallel to the plane $x - 2y + 2z - 5 = 0$ and at a unit distance from the origin is :

[AIEEE-2012]

- (A) $x - 2y + 2z - 1 = 0$ (B) $x - 2y + 2z + 5 = 0$
(C) $x - 2y + 2z - 3 = 0$ (D) $x - 2y + 2z + 1 = 0$

7. If the lines $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4}$ and

$\frac{x-3}{1} = \frac{y-k}{2} = \frac{z}{1}$ intersect, then k is equal to:

[AIEEE-2012]

- (A) $9/2$ (B) 0
(C) -1 (D) $2/9$

8. If the lines $\frac{x-2}{1} = \frac{y-3}{1} = \frac{z-4}{-k}$ and

$\frac{x-1}{k} = \frac{y-4}{2} = \frac{z-5}{1}$ are coplaner, then k can

have :

[JEE-MAIN-2013]

- (A) exactly two values (B) exactly three values
(C) any value (D) exactly one value

9. Distance between two parallel planes $2x+y+2z=8$ and $4x+2y+4z+5=0$ is : [JEE-MAIN-2013]
 (A) $\frac{7}{2}$ (B) $\frac{9}{2}$
 (C) $\frac{3}{2}$ (D) $\frac{5}{2}$
10. The image of the line $\frac{x-1}{3} = \frac{y-3}{1} = \frac{z-4}{-5}$ in the plane $2x-y+z+3=0$ is the line : [JEE-MAIN-2014]
 (A) $\frac{x+3}{3} = \frac{y-5}{1} = \frac{z-2}{-5}$
 (B) $\frac{x+3}{-3} = \frac{y-5}{-1} = \frac{z+2}{5}$
 (C) $\frac{x-3}{3} = \frac{y+5}{1} = \frac{z-2}{-5}$
 (D) $\frac{x-3}{-3} = \frac{y+5}{-1} = \frac{z-2}{5}$
11. The distance of the point $(1, 0, 2)$ from the point of intersection of the line $\frac{x-2}{3} = \frac{y+1}{4} = \frac{z-2}{12}$ and the plane $x-y+z=16$, is [JEE-MAIN-2015]
 (A) $3\sqrt{21}$ (B) 13
 (C) $2\sqrt{14}$ (D) 8
12. If the line, $\frac{x-3}{2} = \frac{y+2}{-1} = \frac{x+4}{3}$ lies in the plane, $lx+my-z=9$, then l^2+m^2 is equal to : [JEE - MAIN-2016]
 (A) 18 (B) 5
 (C) 2 (D) 26
13. The distance of the point $(1, -5, 9)$ from the plane $x-y+z=5$ measured along the line $x=y=z$ is : [JEE - MAIN-2016]
 (A) $10\sqrt{3}$ (B) $\frac{10}{\sqrt{3}}$
 (C) $\frac{20}{3}$ (D) $3\sqrt{10}$
14. If the image of the point $P(1, -2, 3)$ in the plane, $2x+3y-4z+22=0$ measured parallel to the line, $\frac{x}{1} = \frac{y}{4} = \frac{z}{5}$ is Q, then PQ is equal to : [JEE - MAIN-2017]
 (A) $3\sqrt{5}$ (B) $2\sqrt{42}$
 (C) $\sqrt{42}$ (D) $6\sqrt{5}$
15. The distance of the point $(1, 3, -7)$ from the plane passing through the point $(1, -1, -1)$ having normal perpendicular to both the lines $\frac{x-1}{1} = \frac{y+2}{-2} = \frac{z-4}{3}$ and $\frac{x-2}{2} = \frac{y+1}{-1} = \frac{z+7}{-1}$, is : [JEE - MAIN-2017]
 (A) $\frac{20}{\sqrt{74}}$ (B) $\frac{10}{\sqrt{83}}$
 (C) $\frac{5}{\sqrt{83}}$ (D) $\frac{10}{\sqrt{74}}$
16. If L_1 is the line of intersection of the plane $2x-2y+3z-2=0$, $x-y+z+1=0$ and L_2 is the line of intersection of the plane $x+2y-z-3=0$, $3x-y+2z-1=0$, then the distance of the origin from the plane containing the lines L_1 and L_2 is : [JEE-MAIN 2018]
 (A) $\frac{1}{\sqrt{2}}$ (B) $\frac{1}{4\sqrt{2}}$
 (C) $\frac{1}{3\sqrt{2}}$ (D) $\frac{1}{2\sqrt{2}}$
17. The length of the projection of the line segment joining the points $(5, -1, 4)$ and $(4, -1, 3)$ on the plane, $x+y+z=7$ is : [JEE-MAIN 2018]
 (A) $\sqrt{\frac{2}{3}}$ (B) $\frac{2}{\sqrt{3}}$
 (C) $\frac{2}{3}$ (D) $\frac{1}{3}$

EXERCISE - 4 | Level-II

Previous Year | JEE Advanced

1. Equation of the plane containing the straight line $\frac{x}{2} = \frac{y}{3} = \frac{z}{4}$ and perpendicular to the plane containing the straight lines $\frac{x}{3} = \frac{y}{4} = \frac{z}{2}$ and $\frac{x}{4} = \frac{y}{2} = \frac{z}{3}$ is
[JEE 2010]
(A) $x + 2y - 2z = 0$ (B) $3x + 2y - 2z = 0$
(C) $x - 2y + z = 0$ (D) $5x + 2y - 4z = 0$
2. If the distance between the plane $Ax - 2y + z = d$ and the plane containing the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{5}$ is $\sqrt{6}$, then $|d|$ is
[JEE 2010]
3. If the distance of the point $P(1, -2, 1)$ from the plane $x + 2y - 2z = \alpha$, where $\alpha > 0$ is 5, then the foot of the perpendicular from P to the plane is
[JEE 2010]
(A) $\left(\frac{8}{3}, \frac{4}{3}, -\frac{7}{3}\right)$ (B) $\left(\frac{4}{3}, -\frac{4}{3}, \frac{1}{3}\right)$
(C) $\left(\frac{1}{3}, \frac{2}{3}, \frac{10}{3}\right)$ (D) $\left(\frac{2}{3}, -\frac{1}{3}, \frac{5}{2}\right)$
4. Match the statements in Column I with the values in Column II
[JEE 2010]
Column-I **Column-II**
(A) A line from the origin meets the lines $\frac{x-2}{1} = \frac{y-1}{-2} = \frac{z+1}{1}$ & $\frac{x-8}{2} = \frac{y+3}{-1} = \frac{z-1}{1}$ at P and Q respectively. If length PQ = d, then d^2 is (P) - 4
(B) The values of x satisfying $\tan^{-1}(x+3) - \tan^{-1}(x-3) = \sin^{-1}\left(\frac{3}{5}\right)$ are (Q) 0
(C) Non-zero vectors \vec{a} , \vec{b} and \vec{c} satisfy $\vec{a} \cdot \vec{b} = 0$, $(\vec{b} - \vec{a}) \cdot (\vec{b} + \vec{c}) = 0$ and $2|\vec{b} + \vec{c}| = |\vec{b} - \vec{a}|$. If $\vec{a} = \mu\vec{b} + 4\vec{c}$, then the possible values of μ are (R) 4
(D) Let f be the function on $[-\pi, \pi]$ given by $f(0) = 9$ and $f(x) = \sin\left(\frac{9x}{2}\right) / \sin\left(\frac{x}{2}\right)$ for $x \neq 0$. Then the value of $\frac{2}{\pi} \int_{-\pi}^{\pi} f(x) dx$ is (S) 5
5. The point P is the intersection of the straight line joining the points $Q(2, 3, 5)$ and $R(1, -1, 4)$ with the plane $5x - 4y - z = 1$. If S is the foot of the perpendicular drawn from the point $T(2, 1, 4)$ to QR, then the length of the line segment PS is
[JEE 2012]
(A) $\frac{1}{\sqrt{2}}$ (B) $\sqrt{2}$
(C) 2 (D) $2\sqrt{2}$
6. The equation of a plane passing through the line of intersection of the planes $x + 2y + 3z = 2$ and $x - y + z = 3$ and at a distance $\frac{2}{\sqrt{3}}$ from the point $(3, 1, -1)$ is
[JEE 2012]
(A) $5x - 11y + z = 17$ (B) $\sqrt{2}x + y = 3\sqrt{2} - 1$
(C) $x + y + z = \sqrt{3}$ (D) $x - \sqrt{2}y = 1 - \sqrt{2}$
7. If the straight lines $\frac{x-1}{2} = \frac{y+1}{k} = \frac{z}{2}$ and $\frac{x+1}{5} = \frac{y+1}{2} = \frac{z}{k}$ are coplanar, then the plane(s) containing these two lines is (are)
[JEE 2012]
(A) $y + 2z = -1$ (B) $y + z = -1$
(C) $y - z = -1$ (D) $y - 2z = -1$
8. Perpendiculars are drawn from points on the line $\frac{x+2}{2} = \frac{y+1}{-1} = \frac{z}{3}$ to the plane $x + y + z = 3$. The feet of perpendiculars lie on the line
[JEE 2013]
(A) $\frac{x}{5} = \frac{y-1}{8} = \frac{z-2}{-13}$
(B) $\frac{x}{2} = \frac{y-1}{3} = \frac{z-2}{-5}$
(C) $\frac{x}{4} = \frac{y-1}{3} = \frac{z-2}{-7}$
(D) $\frac{x}{2} = \frac{y-1}{-7} = \frac{z-2}{5}$

9. Two lines $L_1 : x = 5, \frac{y}{3-\alpha} = \frac{z}{-2}$ and $L_2 : x = \alpha, \frac{y}{-1} = \frac{z}{2-\alpha}$ are coplanar. Then α can take value(s) [JEE 2013]
(A) 1 (B) 2
(C) 3 (D) 4
10. Consider the lines $L_1 : \frac{x-1}{2} = \frac{y}{-1} = \frac{z+3}{1}$, $L_2 : \frac{x-4}{1} = \frac{y+3}{1} = \frac{z+3}{2}$ and the planes $P_1 : 7x+y+2z=3, P_2 : 3x+5y-6z=4$. Let $ax+by+cz=d$ be the equation of the plane passing through the point of intersection of lines L_1 and L_2 , and perpendicular to planes P_1 and P_2 . Match List-I with List-II and select the correct answer using the code given below the lists : [JEE 2013]
- | | | |
|---------------|--|----------------|
| List-I | | List-II |
| P. $a =$ | | 1. 13 |
| Q. $b =$ | | 2. -3 |
| R. $c =$ | | 3. 1 |
| S. $d =$ | | 4. -2 |
- Codes :**
- | | | | | |
|-----|---|---|---|---|
| | P | Q | R | S |
| (A) | 3 | 2 | 4 | 1 |
| (B) | 1 | 3 | 4 | 2 |
| (C) | 3 | 2 | 1 | 4 |
| (D) | 2 | 4 | 1 | 3 |
11. From a point $P(\lambda, \lambda, \lambda)$, perpendiculars PQ and PR are drawn respectively on the lines $y = x, z = 1$ and $y = -x, z = -1$. If P is such that $\angle QPR$ is a right angle, then the possible value (s) of λ is(are) [JEE 2014]
(A) $\sqrt{2}$ (B) 1 (C) -1 (D) $-\sqrt{2}$
12. In R^3 , let L be a straight line passing through the origin. Suppose that all the points on L are at a constant distance from the two planes $P_1 : x + 2y - z + 1 = 0$ and $P_2 : 2x - y + z - 1 = 0$. Let M be the locus of the feet of the perpendiculars drawn from the points on L to the plane P_1 . Which of the following points lie(s) on M? [JEE 2015]
(A) $(0, -\frac{5}{6}, -\frac{2}{3})$ (B) $(-\frac{1}{6}, -\frac{1}{3}, \frac{1}{6})$
(C) $(-\frac{5}{6}, 0, \frac{1}{6})$ (D) $(-\frac{1}{3}, 0, \frac{2}{3})$
13. Suppose that \vec{p}, \vec{q} and \vec{r} are three non-coplanar vectors in R^3 . Let the components of a vector \vec{s} along \vec{p}, \vec{q} and \vec{r} be 4, 3 and 5, respectively. If the components of this vector \vec{s} along $(-\vec{p} + \vec{q} + \vec{r}), (\vec{p} - \vec{q} + \vec{r})$ and $(-\vec{p} - \vec{q} + \vec{r})$ are x, y and z, respectively, then the value of $2x + y + z$ is [JEE 2015]
14. The equation of the plane passing through the point (1,1,1) and perpendicular to the planes $2x + y - 2z = 5$ and $3x - 6y - 2z = 7$, is [JEE Adv. 2017]
(A) $14x + 2y - 15z = 1$
(B) $14x - 2y + 15z = 27$
(C) $-14x + 2y + 15z = 3$
(D) $14x + 2y + 15z = 31$
15. Let $P_1 : 2x + y - z = 3$ and $P_2 : x + 2y + z = 2$ be two planes. Then, which of the following statement(s) is (are) TRUE? [JEE Adv. 2018]
(A) The line of intersection of P_1 and P_2 has direction ratios 1, 2, -1
(B) The line $\frac{3x-4}{9} = \frac{1-3y}{9} = \frac{z}{3}$ is perpendicular to the line of intersection of P_1 and P_2
(C) The acute angle between P_1 and P_2 is 60°
(D) If P_3 is the plane passing through the point (4, 2, -2) and perpendicular to the line of intersection of P_1 and P_2 , then the distance of the point (2, 1, 1) from the plane P_3 is $\frac{2}{\sqrt{3}}$
16. Let P be a point in the first octant, whose image Q in the plane $x + y = 3$ (that is, the line segment PQ is perpendicular to the plane $x + y = 3$ and the mid-point of PQ lies in the plane $x + y = 3$) lies on the z-axis. Let the distance of P from the x-axis be 5. If R is the image of P in the xy-plane, then the length of PR is [JEE Adv. 2018]

17. Let L_1 and L_2 denote the lines

$$\vec{r} = \hat{i} + \lambda(-\hat{i} + 2\hat{j} + 2\hat{k}), \lambda \in \mathbb{R} \text{ and}$$

$$\vec{r} = \mu(2\hat{i} - \hat{j} + 2\hat{k}), \mu \in \mathbb{R}$$

respectively. If L_3 is a line which is perpendicular to both L_1 and L_2 and cuts both of them, then which of the following options describe(s) L_3 ? [JEE Adv. 2019]

(A) $\vec{r} = \frac{2}{9}(2\hat{i} - \hat{j} + 2\hat{k}) + t(2\hat{i} + 2\hat{j} - \hat{k}), t \in \mathbb{R}$

(B) $\vec{r} = \frac{2}{9}(4\hat{i} + \hat{j} + \hat{k}) + t(2\hat{i} + 2\hat{j} - \hat{k}), t \in \mathbb{R}$

(C) $\vec{r} = \frac{1}{3}(2\hat{i} + \hat{k}) + t(2\hat{i} + 2\hat{j} - \hat{k}), t \in \mathbb{R}$

(D) $\vec{r} = t(2\hat{i} + 2\hat{j} - \hat{k}), t \in \mathbb{R}$

18. Three lines are given by

$$\vec{r} = \lambda \hat{i}, \lambda \in \mathbb{R}$$

$$\vec{r} = \mu(\hat{i} + \hat{j}), \mu \in \mathbb{R}$$

$$\vec{r} = \nu(\hat{i} + \hat{j} + \hat{k}), \nu \in \mathbb{R}$$

Let the lines cut the plane $x + y + z = 1$ at the points A, B and C respectively. If the area of the triangle ABC is Δ then value of $(6\Delta)^2$ equals _____. [JEE Adv. 2019]

19. Three lines

$$L_1 : \vec{r} = \lambda \hat{i}, \lambda \in \mathbb{R}$$

$$L_2 : \vec{r} = \hat{k} + \mu \hat{i}, \mu \in \mathbb{R}$$

$$L_3 : \vec{r} = \hat{i} + \hat{j} + \nu \hat{k}, \nu \in \mathbb{R}$$

are given. For which point(s) Q on L_2 can we find a point P on L_1 and a point R on L_3 so that P, Q and R are collinear? [JEE Adv. 2019]

(A) $\hat{k} + \hat{j}$ (B) $\hat{k} - \frac{1}{2}\hat{j}$

(C) \hat{k} (D) $\hat{k} + \frac{1}{2}\hat{j}$

ANSWER KEY

EXERCISE - I

JEE Main

- | | | | | | | |
|-------|---------|-------|-------|-------|-------|-------|
| 1. B | 2. C | 3. B | 4. D | 5. C | 6. A | 7. A |
| 8. D | 9. A,B, | 10. B | 11. D | 12. C | 13. B | 14. C |
| 15. A | 16. B | 17. D | 18. C | 19. A | 20. A | 21. A |
| 22. C | 23. A | 24. C | 25. D | 26. A | 27. D | 28. B |
| 29. C | 30. A | 31. B | 32. A | 33. C | 34. D | 35. A |
| 36. A | 37. A | 38. D | 39. B | 40. B | 41. A | 42. C |
| 43. D | | | | | | |

EXERCISE - II

JEE Advance

(Level - I) Single correct Option - type Questions

- | | | | | | | |
|------|------|-------|-------|------|------|------|
| 1. D | 2. A | 3. A | 4. D | 5. C | 6. B | 7. B |
| 8. C | 9. A | 10. C | 11. B | | | |

(Level - II) Multiple correct Option - type Questions

- | | | | | | | |
|--------|--------|-------------|-----------|--------|--------|--------|
| 1. B,D | 2. C | 3. D | 4. B,D | 5. A,B | 6. B,C | 7. A,B |
| 8. A,B | 9. A,D | 10. A,B,C,D | 11. A,B,C | | | |

EXERCISE - III

Subjective - type Questions

2. $(1/2, 1/2, 1/2)$ 3. $(a/2, b/2, c/2)$ 4. $(2/3, -2/3, -1/3)$ 5. 60°
6. $\frac{x+1}{11} = \frac{y-1}{9} = \frac{z-1}{-15}$ 7. $2 - 2\sqrt{3}$ 8. $\frac{x}{1} = \frac{y}{2} = \frac{z}{-1}$ or $\frac{x}{-1} = \frac{y}{1} = \frac{z}{-2}$
9. $\frac{x-1}{6} = \frac{y+2}{13} = \frac{z+3}{17}$ 10. $\frac{x-2}{1} = \frac{y+1}{-2} = \frac{z-3}{1}$ 11. $(1, 2, 2)$ 12. $\frac{1}{d^2} = \frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2}$
13. $\sqrt{26}$ 14. $\alpha = -1, \frac{80}{63}$ 15. (i) $x + y - 2z = 3$; (ii) $(6, 5, -2)$ 16. $-x - y \pm \sqrt{2}z + 1 = 0$ 17. $\pi/2$
18. $(1, -2, -4)$
19. $7x + 13y + 4z - 9 = 0$ 20. $\frac{19}{2}$ 21. $\left(0, \frac{13}{5}, 2\right)$ 22. $11x - y - 3z = 35$
23. $\frac{x-4}{9} = \frac{y+1}{-1} = \frac{z-7}{-3}$ 24. $2x + y = 0$ & $x - y + z = 0$ 26. $x + y + z = 0$
27. (a, b, c) 28. $\frac{x-1}{2} = \frac{y-2}{2} = \frac{z-3}{-3}$ 29. $\frac{17}{2}$ 30. $\frac{x-4}{3} = \frac{y+14}{10} = \frac{z-4}{4}$
31. $2x + 3y + z + 4 = 0$ 32. $\left(\frac{16}{7}, \frac{1}{7}, \frac{-17}{7}\right), x + y + z = 0$ 33. $\frac{x-7}{22} = \frac{y-2}{5} = \frac{z+1}{-4}$

34. $\frac{x-4}{9} = \frac{y+1}{-1} = \frac{z-7}{-3}$

35. $x - 2y + 2z = 1$

36. $\sin^{-1} \frac{4}{\sqrt{30}}$

37. $\frac{9}{2}$

38. $\frac{1}{2}$

39. $\theta = \cos^{-1} \frac{3}{\sqrt{14}}$ and $\phi = \cos^{-1} \frac{1}{\sqrt{5}}$

40. (a) $\frac{3}{2}$, (b) $\frac{2x}{3} + \frac{2y}{3} + \frac{z}{3} = 1$, (c) $\left(0, \frac{3}{2}, 0\right)$, (d) $\frac{x-2}{11} = \frac{y+1}{-10} = \frac{z-3}{2}$

Comprehension - based Questions

42. B

43. C

44. A

Matrix Match - type Questions

45. (A)-R; (B)-Q,T; (C)-P,S

EXERCISE - IV

Previous Year's Question

JEE Main

- | | | | | | | |
|-------|-------|-------|-------|-------|-------|-------|
| 1. D | 2. B | 3. C | 4. A | 5. B | 6. C | 7. A |
| 8. A | 9. A | 10. A | 11. B | 12. C | 13. A | 14. B |
| 15. B | 16. C | 17. A | | | | |

JEE Advanced

- | | | | | |
|-------|---------|----------|---------------------------------|---------|
| 1. C | 2. 6 | 3. A | 4. (A)-T; (B)-P,R; (C)-Q; (D)-R | 5. A |
| 6. A | 7. B,C | 8. D | 9. A,D | 10. A |
| 11. C | 12. A,B | 13. 9 | 14. D | 15. C,D |
| 16. 8 | 17. A,B | 18. 0.75 | 19. B,D | |