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Monotonocity.

1) Property of graph showing graph going in which direction

↑ up. — ↑ing

↓ down. — ↓ing

Strictly ↑
Monotonically ↑

Non decreasing

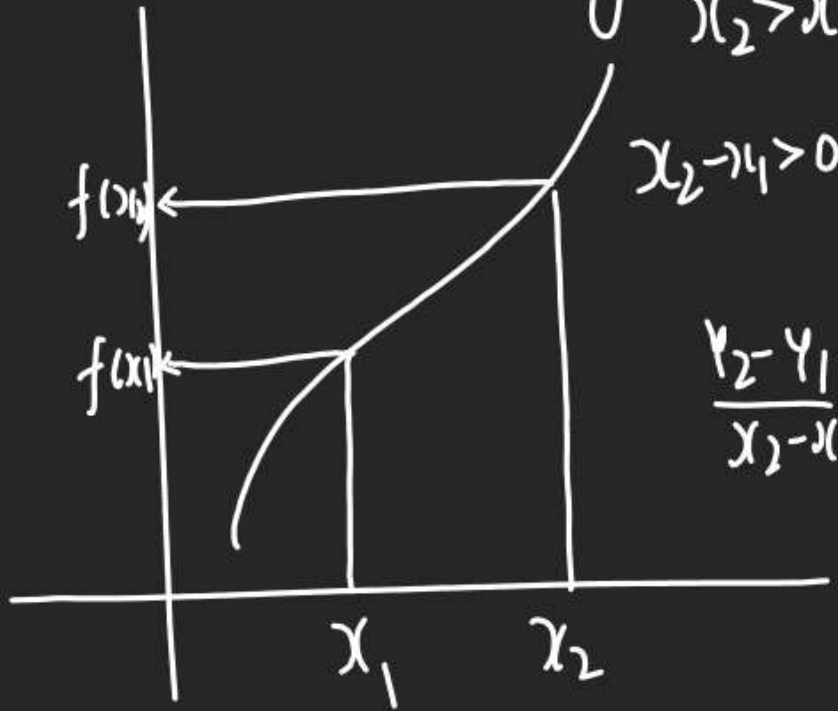
Constant

(2) fxn — Monotonic → ↑ing or ↓ing
— Non Monotonic → Sometime ↑ Sometime ↓ (ऊँचा नीचा Nature)

$Y = \sin x$

3) Monotonocity in Interval.

A) fxn is ↑ Increasing



$x_2 > x_1 \Rightarrow f(x_2) \geq f(x_1)$

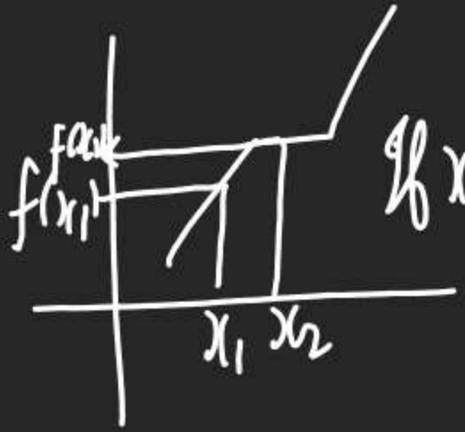
$x_2 - x_1 > 0$

$Y_2 \geq Y_1$

$Y_2 - Y_1 \geq 0$

$\frac{Y_2 - Y_1}{x_2 - x_1} \geq 0 \Rightarrow \frac{dY}{dx} \geq 0$

If $x_2 > x_1$ then $f(x_2) > f(x_1)$ then f is ↑ing

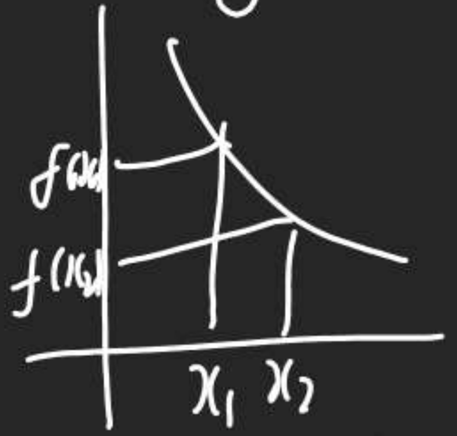


If $x_2 > x_1$ & $f(x_2) \geq f(x_1)$ then fxn is Nondecreasing

↑ing

- Basic defination Based
  $x_2 > x_1 \Rightarrow f(x_2) \geq f(x_1)$
- By derivation
  If $f(x)$ is ↑ in (a, b) then $f'(x) \geq 0$

④ Decreasing fxn



$x_2 > x_1 \Rightarrow f(x_2) \leq f(x_1)$ then f is ↓ing

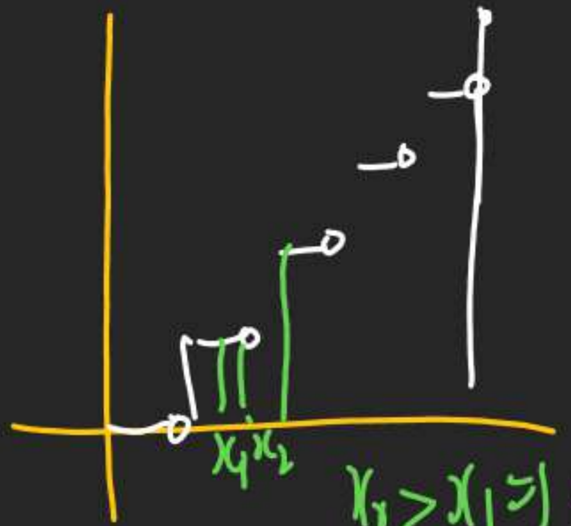
OR $f'(x) \leq 0$ then ↓ing fxn

Rk.

$$\boxed{\begin{array}{l} x_2 > x_1 \Rightarrow f(x_2) \geq f(x_1) \to \text{fxn is } \uparrow \\ x_2 > x_1 \Rightarrow f(x_2) \leq f(x_1) \to \text{fxn is } \downarrow \end{array}}$$

If fxn is ↓ing then whenever it is applied or Removed it change sign of Inequality

Q $f(x) = [x]$ in $[1, 5]$ ↑ or ↓

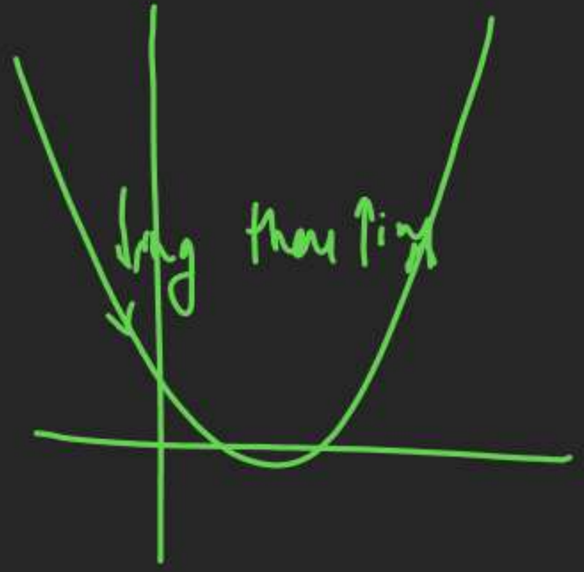


this is ↑ing

$x_2 > x_1 \Rightarrow f(x_2) \geq f(x_1)$

Q $f(x) = ax^2 + bx + c,\ a \neq 0$ is ↑ or ↓ ?

Non Monotonic



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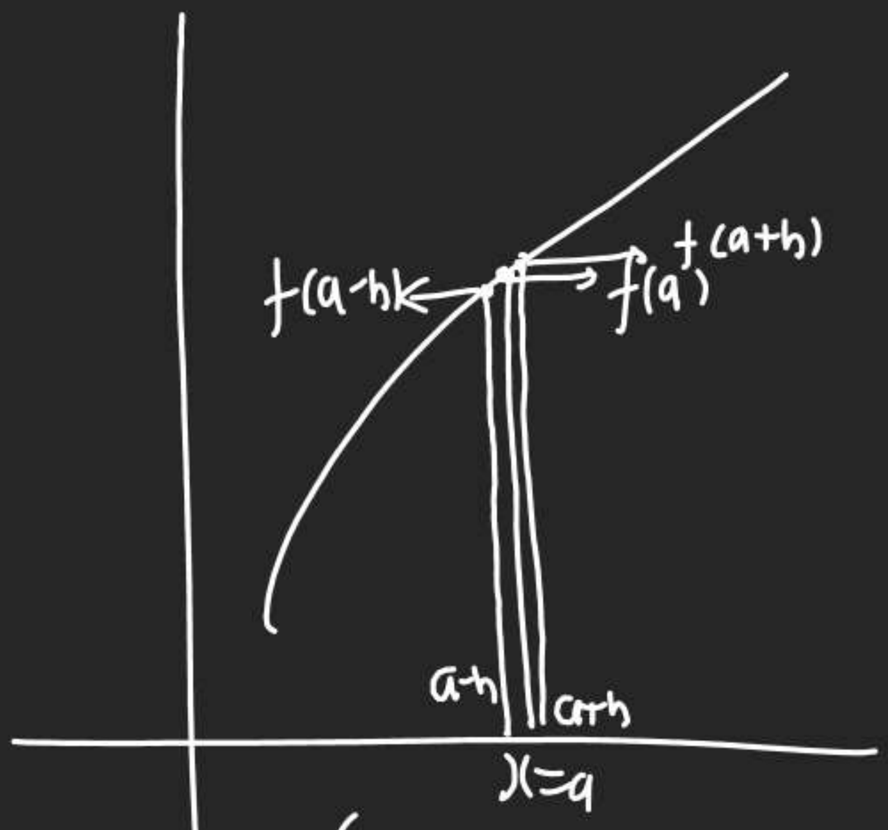
(5) Monotonocity at a Pt $x=a$



Comparing value at left neighbourhood & Rt. neighbourhood.

$f(a-h) < f(a) < f(a+h)$ value upr की ओर जा रहे

$\Rightarrow$ fxn is $\uparrow$ at $x=a$

Q $f(x) = x^2 - 2x - 3$ at $x = -1$

Check Monotonocity.

$f'(x) = 2x - 2$

$f'(-1) = 2(-1) - 2 = -4 < 0$

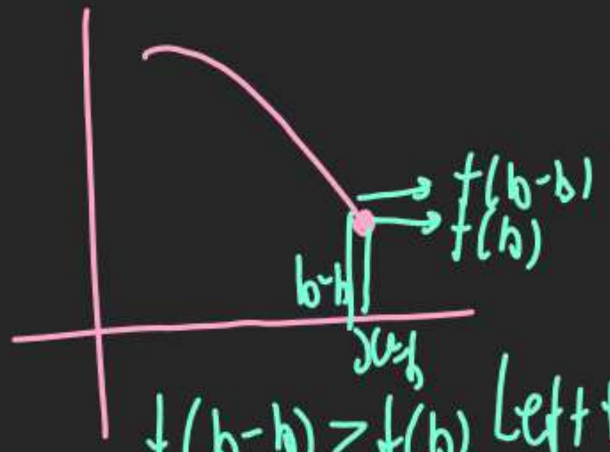
$f'(x) < 0 \therefore$ fxn is $\downarrow$ing at $x = -1$

(6) Monotonocity at Boudary Pt.

Compare Whatever avaible.

$f(a+h) > f(a)$

$\therefore x = a \uparrow$ing



$f(b-h) > f(b)$ Left to Rt

$\downarrow$ value decreasing

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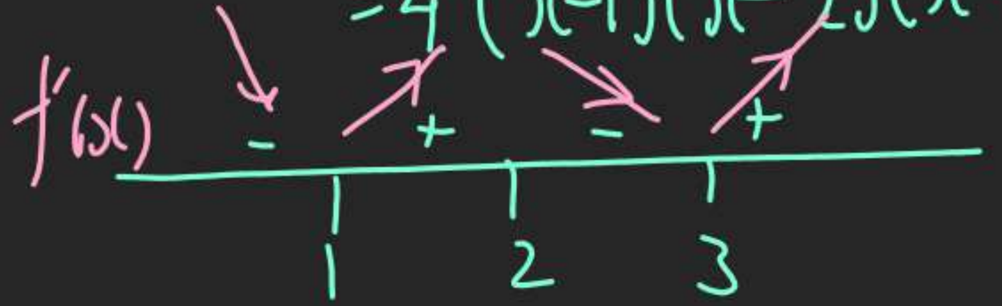
Finding Interval.

Q Find Interval in which $f(x) = x^4 - 8x^3 + 22x^2 - 24x + 21$ is ↑ing or ↓ing.

$$f'(x) = 4x^3 - 24x^2 + 44x - 24$$
$$= 4(x^3 - 6x^2 + 11x - 6)$$
$$= 4(x-1)(x-2)(x-3)$$



↑ing in $x \in [1,2] \cup [3,\infty)$

↓ing in $x \in (-\infty,1] \cup [2,3]$

Q Find Interval in which $f(x) = x^x$ is ↑ing/↓ing

$$f'(x) = x^x(1+\ln x)$$

Domain $x \in (0,\infty)$

$x = \frac{1}{e}$

f(x) is ↓ing in $x \in (0, 1/e)$

↑ing in $x \in (1/e, \infty)$