Nishant Jindal

Ex-5 (remaining) / Ex-3

Differentiation

3:00-3:45 pm

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$$\int \sin^{-1}\sqrt{\frac{x}{a+x}}\,dx \quad = a\int \theta \quad 2\tan\theta\sec^2\theta\ d\theta$$

$$x = a\tan^2\theta$$

$$a\left[\theta\sec^2\theta - \int \theta\sec^2\theta\right]$$

$$\frac{f'(c)}{g'(c)} = \frac{f(b)-f(a)}{g(b)-g(a)}$$

$$h(x) = \big(f(b)-f(a)\big)g(x) - \big(g(b)-g(a)\big)f(x)$$

$$h(b)-h(a) = \big(f(b)-f(a)\big)\big(g(b)-g(a)\big) - \big(g(b)-g(a)\big)\big(f(b)-f(a)\big) = 0$$

$$\exists\, c \in (a,b),\ h'(c) = 0$$

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1. Verify Rolle's theorem for

$$f(x) = x(x+3)e^{-\frac{x}{2}} \quad \text{in} \quad [-3, 0]$$

- f is cont.
- —"— diff.
- $f(0) = f(-3) = 0$

$$f'(c) = e^{-\frac{c}{2}}\left(2c+3 - \frac{1}{2}(c^2+3c)\right) = 0$$

$$c^2+3c-4c-6=0 = c^2-c-6 = (c-3)(c+2)$$

$$\boxed{c=-2}$$

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Q. Given $a_0, a_1, a_2, a_3, \ldots, a_n \in R$ satisfying

$$\frac{a_0}{n+1} + \frac{a_1}{n} + \frac{a_2}{n-1} + \frac{a_3}{n-2} + \cdots + \frac{a_{n-1}}{2} + a_n = 0 \text{. Then}$$

P.T. $\exists\, x \in (0,1)$, s.t. $a_0 x^n + a_1 x^{n-1} + a_2 x^{n-2} + \cdots + a_n = 0$.

$$f(x) = \frac{a_0 x^{n+1}}{n+1} + \frac{a_1 x^n}{n} + \frac{a_2 x^{n-1}}{n-1} + \cdots + a_n x$$

$f(0) = 0$

$f(1) = 0$

$\exists\, c \in (0,1)$, $f'(c) = 0$

3. Suppose $f''(x)$ exists $\forall x \in R$ and $f(x_1) = f(x_2) = f(x_3) = 0$ where $x_1 < x_2 < x_3$. P.T. $f''(c) = 0$ for some number 'c', $c \in (x_1, x_3)$.

$\exists\, c_1 \in (x_1, x_2)$, $f'(c_1) = 0$

$\exists\, c_2 \in (x_2, x_3)$, $f'(c_2) = 0$

Rolle's over $f'(x)$ in $[c_1, c_2]$

$\exists\, c \in (c_1, c_2)$, $f''(c) = 0$

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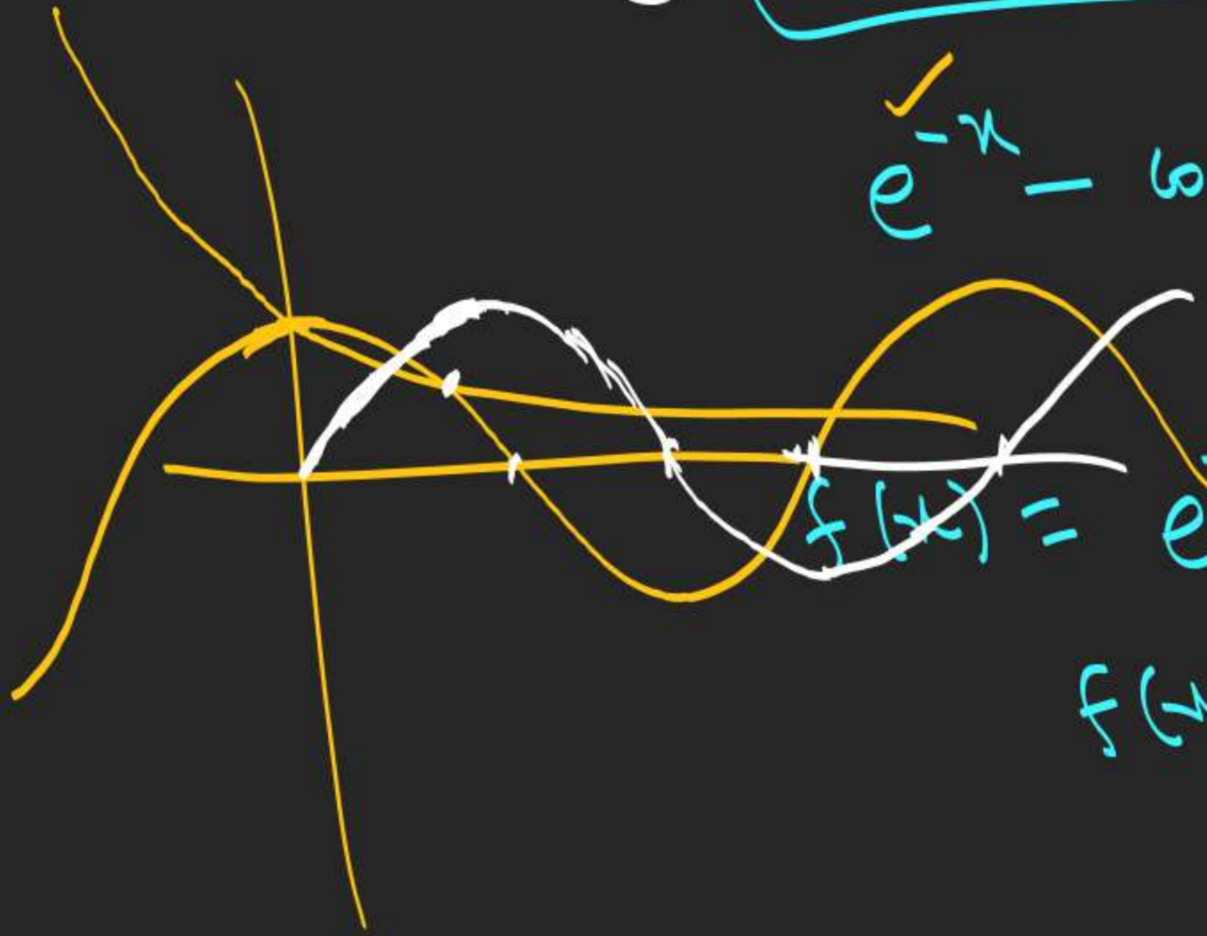
4. Show that between any two roots of eqn. $e^x \cos x = 1$, there exists atleast one root of $\boxed{e^x \sin x = 1}$.

$$f(x) = e^x \cos x - 1 \quad \begin{cases} x_1 \\ x_2 \end{cases}$$

$$f(x_1) = f(x_2) = 0$$

$$\exists\, c \in (x_1, x_2), \quad f'(c) = e^c(\cos c - \sin c) = 0.$$



$$e^{-x} - \cos x = 0 \quad \begin{cases} x_1 \\ x_2 \end{cases}$$

$$f(x) = e^{-x} - \cos x$$

$$f(x_1) = f(x_2) = 0$$

$$\exists\, c \in (x_1, x_2), \quad f'(c) = 0$$

$$-e^{-c} + \sin c = 0$$

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5: Let $P(x)$ be a polynomial function. Let $a, b \in \mathbb{R}$, $a < b$, be two consecutive roots of $P(x)$. P.T. $\exists\, c \in (a,b)$ s.t. $P'(c) + 100\,P(c) = 0$.

$$\boxed{P'(c)+100P(c)=0}$$

$$\Uparrow$$

$$0 = e^{100c}\left(P'(c)+100P(c)\right)$$

$P(x) = 0 < {a \atop b}$

$$f(x) = e^{100x}\,P(x)$$

$f(a) = f(b) = 0$

$\exists\, c \in (a,b)$, $f'(c) = 0$

$$\text{I.F} = e^{100x}$$

$$\frac{dy}{dx} + P(x)\,y$$

$$\frac{d}{dx}\left(e^{100x}\,P(x)\right)$$

$$\int P(x)\,dx = \phi(x)$$

$$\phi'(x) = P(x)$$

$$\frac{d}{dx}\left(e^{\phi(x)}\,y\right) = e^{\phi(x)}\left(\frac{dy}{dx} + P(x)\,y\right)$$

$$= e^{\phi(x)}\frac{dy}{dx} + y\,e^{\phi(x)}\,\phi'(x) = e^{\phi(x)}\frac{dy}{dx} + y\,e^{\phi(x)}P(x)$$

$e^{\phi(x)}$ = Integrating factor.

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P.T. $p'(c) + \frac{1}{1+c^2} p(c) = 0$.

$\underline{\underline{f'(x)}} + \phi(x) \underline{\underline{f(x)}}$

$f(x) = e^{\tan^{-1}x} \, p(x)$

$\exists c \in (a,b)$, $f'(c) = 0$

$e^{\tan^{-1}c}\left(p'(c) + \frac{1}{1+c^2} p(c) \right) = 0$

$\Rightarrow p'(c) + \frac{1}{1+c^2} p(c) = 0$