Nishant Jindal

Sheet 2 → 80Qs.

Q1 No. of C.N. z Satisfying $z^3=\bar{z}$ is?

$|z^3|=|\bar{z}|$

$|z|^3=|z|$

$|z|(|z|^2-1)=0$

$|z|=0$

$|z|^2=1$

B.S.

$z^3=\bar{z}$ $\quad \times z$

$z^4=z\bar{z}$

$z^4=|z|^2$

1) $z^4=1$

$z=1,-1,i,-i$

2) $z^4=|z|^2=0$

$\underline{z=0}$

total 5 sol.

Q2 $z=x+iy$ & $w=\dfrac{1-iz}{z-i}$

& $|w|=1$ then z lies in

① $|w|=1 \Rightarrow \left|\dfrac{1-iz}{z-i}\right|=1$ $\quad$ Pr -i com

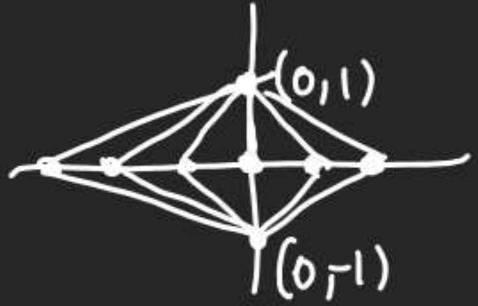
$\left|\dfrac{-i\left(\frac{1}{-i}+z\right)}{z-i}\right|=1$

$|-i|\left|\dfrac{z+i}{z-i}\right|=1$ $\qquad |z-z_1|$

$\Rightarrow \left|\dfrac{z+i}{z-i}\right|=1$

$\Rightarrow |z+i|=|z-i|$

dist. from $(0,-1)$ $\qquad$ dist from $(0,1)$



z lies on Real Axis

Q3 C.N z Satisfies $z+|z|=2+8i$

find $|z|=?$

let $z=a+ib$

$(a+ib)+\sqrt{a^2+b^2}=2+8i$

$(a+\sqrt{a^2+b^2})+ib=2+8i \Rightarrow \boxed{b=8}$

$a+\sqrt{a^2+64}=2 \Rightarrow \sqrt{a^2+64}=2-a$

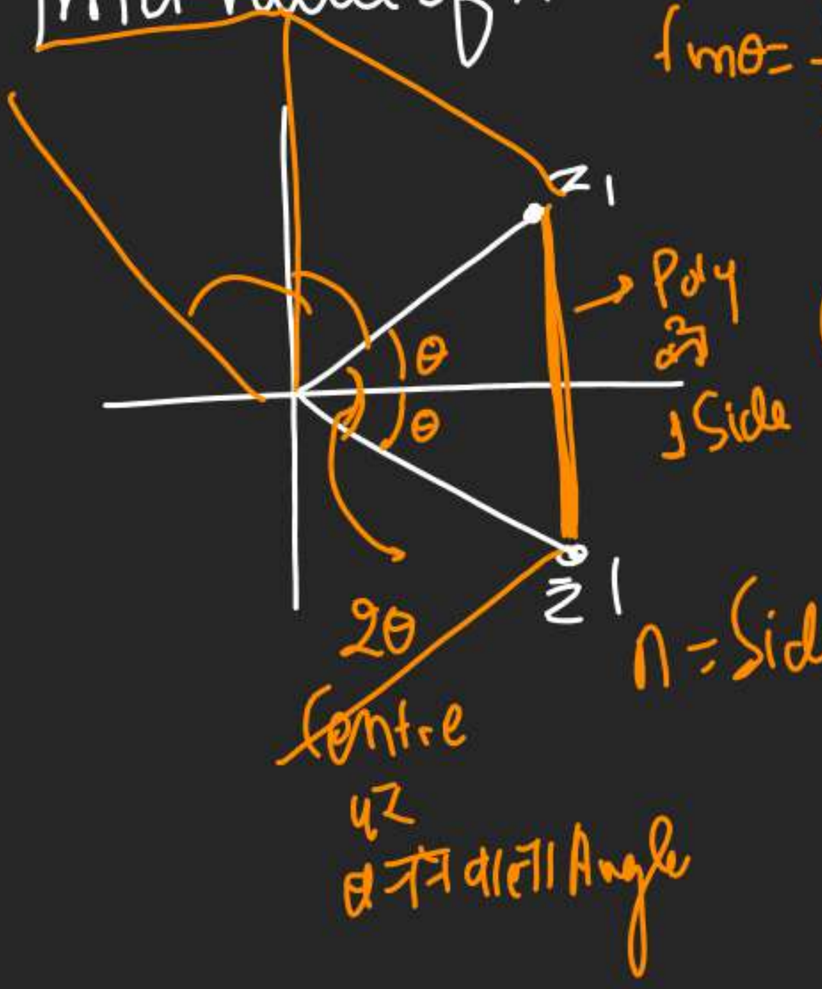
$a^2+64=4+a^2-4a \Rightarrow 4a=-60$

$a=-15$

$\therefore z=a+ib=-15+8i$

$|z|=\sqrt{15^2+8^2}=\underline{17}$

Q 1 If z_1 & $\bar{z}_1$ Rep. Adjacent vertices of a Regular Polygon of n Sides with Centre at Origin & if $\frac{Im(z)}{Re(z)} = \sqrt{2}-1$ find Value of n.



$\tan\theta = \frac{y}{x} = \sqrt{2}-1$

$\theta = 22.5^\circ$

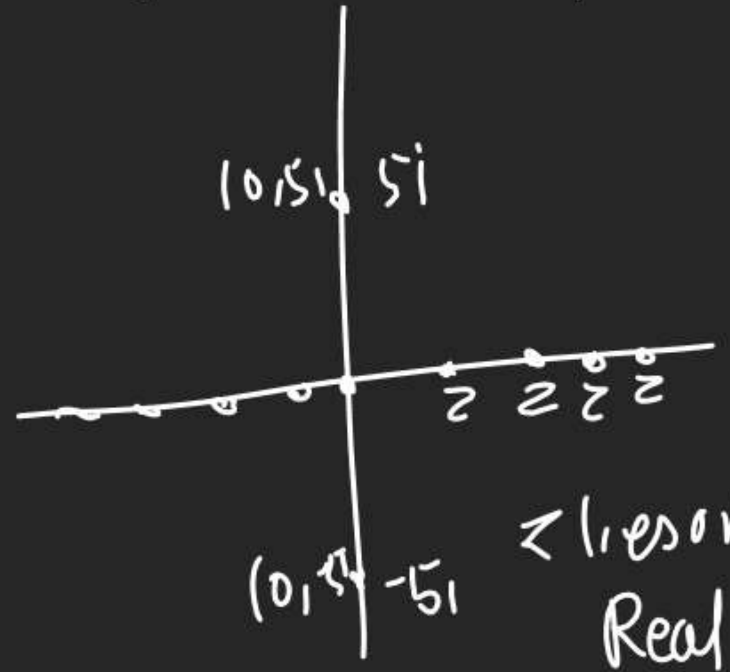
Centre से Angle $= 45^\circ$

$n = \text{Side} = \frac{360^\circ}{45^\circ} = 8$

Q 2 C.N z Satisfying $\left|\frac{z-5i}{z+5i}\right| = 1$ lying on . . .

$|z-5i| = |z+5i|$

dist of z from $(0,5)$ — dist of z from $(0,-5)$



z lies on Real Axis

Rk:- 1) $|z-1| = 2$ → Locus = Circle

⇒ dist of z from $(1,0)$ is 2 (Cen, Rad)

2) $|z-i| = 2$ → Circle.

dist of z from $(0,1)$ is 2 (Cent, Rad)

3) $|z-2+3i| = 2$

dist of z from $(2,-3) = 2$ (Cent, Rad)

Nishant Jindal

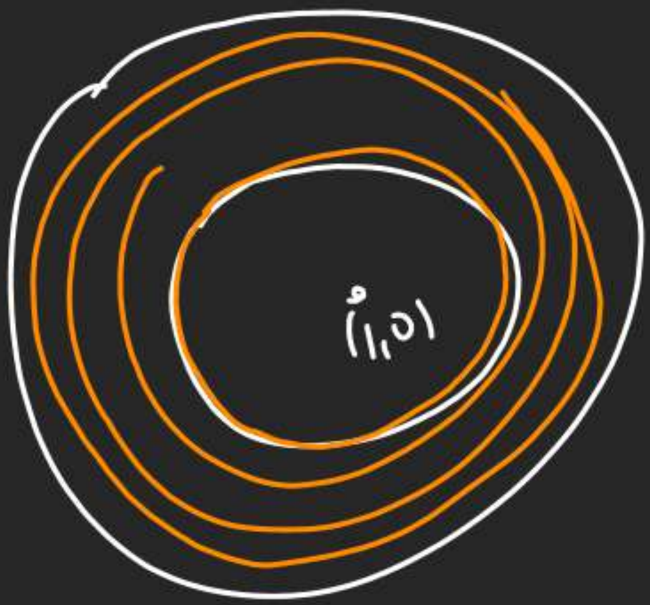
Q 3 z Satisfying

$z - z_1 \rightarrow z_1 = 1 \Rightarrow (1,0)$

$2 \le |z - 1| < 3$ denotes?

Circle Centre $(1,0)$

$z_1 = 1 + 0i$



z lies betⁿ Concentric circles of Rad 2 & 3

touching circle 2 Rad But not touching 3.

Q 4 All Real No. Satisfying

$|1+4i-2^{-x}| \le 5$ is

$|(1-2^{-x})+4i| \le 5$

$\sqrt{(1-2^{-x})^2+4^2} \le 5$

$(1-2^{-x})^2+16 \le 25$

$(1-2^{-x})^2-9 \le 0$

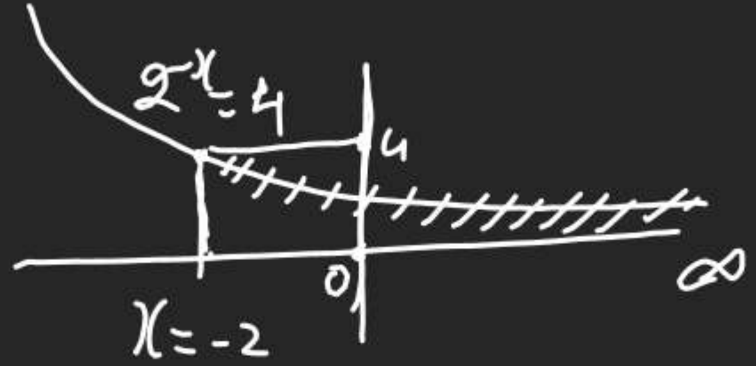
$(1-2^{-x})^2-3^2 \le 0$

$(1-2^{-x}-3)(1-2^{-x}+3) \le 0$

$(+2^{-x}+2)(2^{-x}-4) \le 0$

$-2 \le 2^{-x} \le 4$

$0 \le 2^{-x} \le 4$



$\therefore x \in [-2, \infty)$

Q 5 z be a C.N Satisfying

$(z^3+3)^2 = -16$ then $|z| = ?$

$z^3+3 = 4i$

$z = (-3+4i)^{1/3}$

$|z| = |(-3+4i)^{1/3}|$

$= |-3+4i|^{1/3} = (5)^{1/3}$

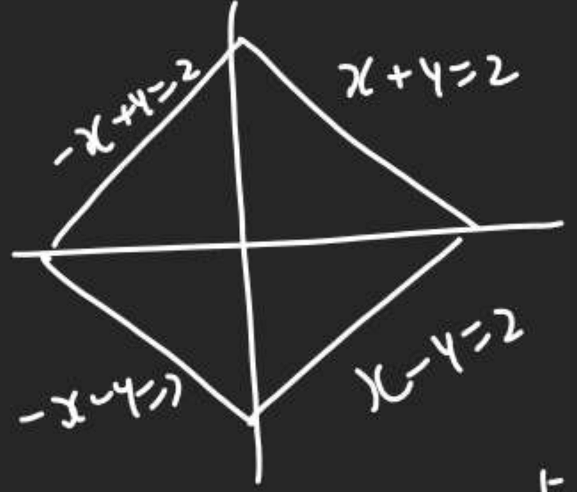
Nishant Jindal

Q 6 $|z+\bar{z}|+|z-\bar{z}|=4$ find locus of z?

$|z+\bar{z}| \to 2\mathrm{Re}(z)$, $|z-\bar{z}| \to 2\mathrm{Im}\,z$

$|2x|+|2iy|=4$

$|x|+|y|=2$



Q 7 $\left(\frac{\sqrt{3}}{2}+\frac{i}{2}\right)^5+\left(\frac{\sqrt{3}}{2}-\frac{i}{2}\right)^5$ is Purely Img [T/F]

If $z=\frac{\sqrt{3}}{2}+\frac{i}{2}$

then $\frac{\sqrt{3}}{2}-\frac{i}{2}=\bar{z}$

$z^5+(\bar{z})^5$

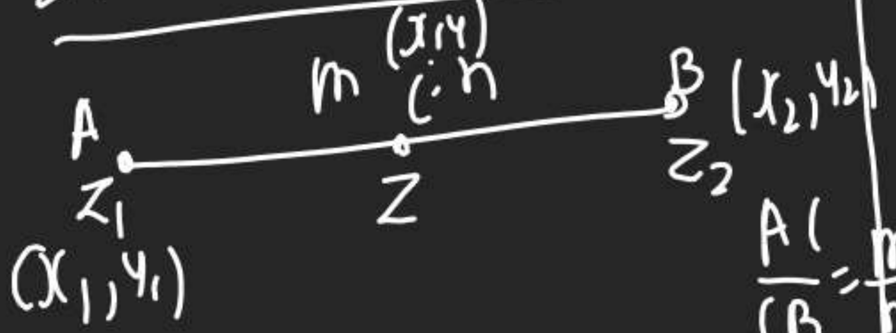
$z^5+(\overline{z^5})$

$=2\,\mathrm{Re}(z^5)$

$\therefore$ it is Purely Real

False.

Section Formula in C.N.



$\frac{AC}{CB}=\frac{m}{n}$

$x=\frac{mx_2+nx_1}{m+n}$, $y=\frac{my_2+ny_1}{m+n}$

$C=z=(x,y)=\left(\frac{mx_2+nx_1}{m+n},\frac{my_2+ny_1}{m+n}\right)$

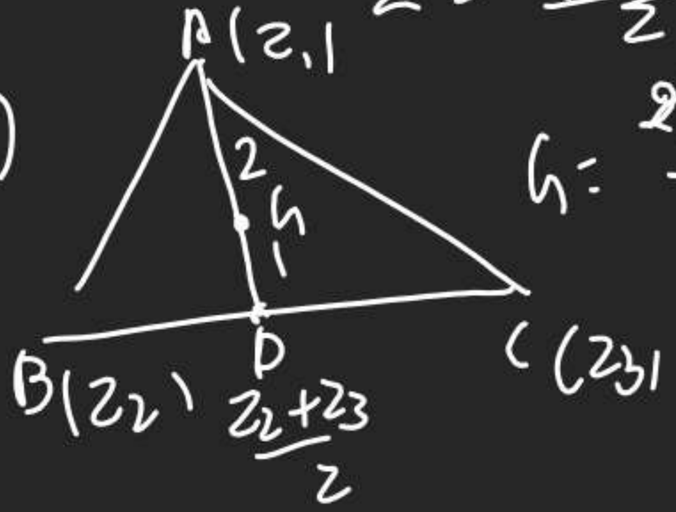
$z=\frac{mx_2+nx_1}{m+n}+i\,\frac{my_2+ny_1}{m+n}$

$z=\frac{m(x_2+iy_2)+n(x_1+iy_1)}{m+n}=\frac{nz_1+mz_2}{m+n}$

(A) Mid Pt. $\to$ z is MP of z_1 & z_2

$z=\frac{z_1+z_2}{2}$

(B)

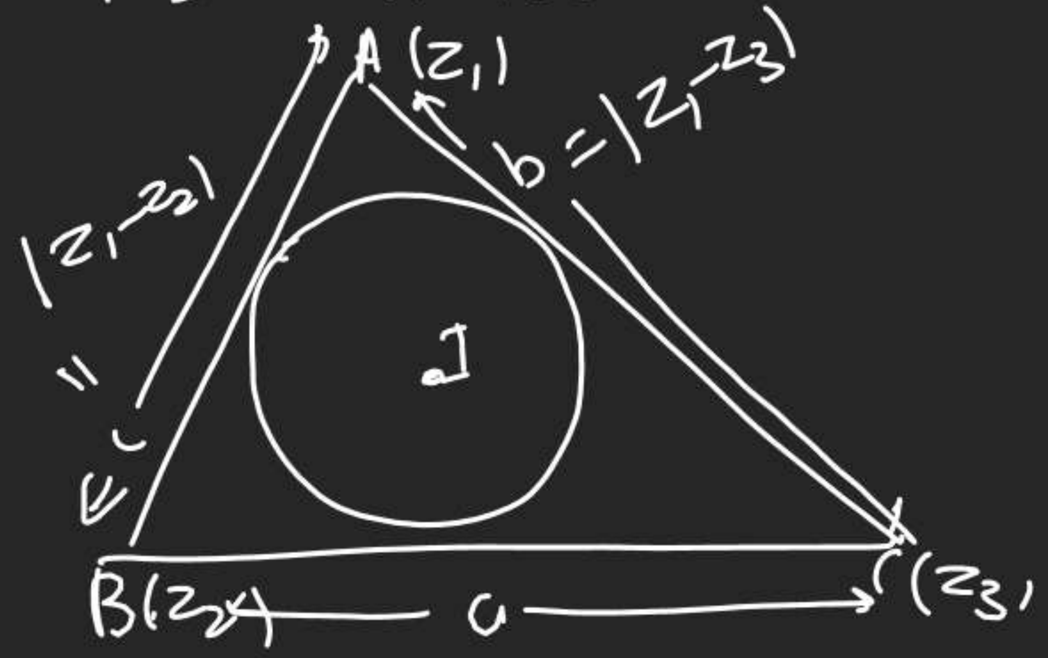


$G=\frac{2\left(\frac{z_2+z_3}{2}\right)+1\cdot z_1}{2+1}$

$\therefore G=\left(\frac{z_1+z_2+z_3}{3}\right)$

Nishant Jindal

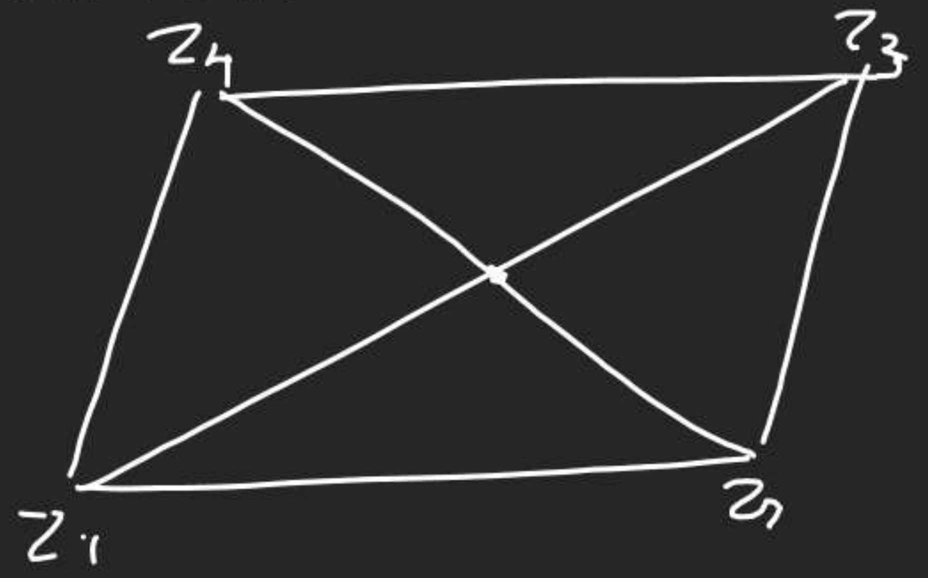
(C) Incentre.



$a = |z_2 - z_3|$

$$I = \frac{a z_1 + b z_2 + c z_3}{a+b+c}.$$

Q 8 Find Relation betⁿ $z_1 z_2 z_3 z_4$ affixes of vertices of ||gm taken in order.



$$\frac{z_1 + z_3}{2} = \frac{z_2 + z_4}{2}$$

$$z_1 + z_3 = z_2 + z_4$$

Relation

Q 9 If a, b, c are Real No & z_1, z_2, z_3 C.N. Such that

$a + b + c = 0$ & $a z_1 + b z_2 + c z_3 = 0$

then P.T. z_1, z_2, z_3 are collinear.

$c = -a - b$

Putting in this

$$a z_1 + b z_2 - (a+b) z_3 = 0$$

$$z_3 = \frac{a z_1 + b z_2}{a+b}$$

Section Formula ⇒ Collinear.

Nishant Jindal

Q $z_1 z_2 z_3$ 3 distinct
10
C.N. Satisfying

$|z_1-1| = |z_2-1| = |z_3-1|$

& $z_1+z_2+z_3=3$ then

z_1, z_2, z_3 must be Vertices of

- - - Δ.

A) $|z_1-1| = |z_2-1| = |z_3-1|$

z_1 & $(1,0)$ dis. = z_2 & $(1,0)$ dis = z_3 & $(1,0)$ dis

$\Rightarrow$ $(1,0)$ is Circumcentre

(B) $z_1+z_2+z_3=3$

$$\frac{z_1+z_2+z_3}{3} = 1 = z \quad (1+0i)$$

z is Centroid, in $(1,0)$

(C) $(1,0)$ is Centroid as well as Circumcentre.

$\Rightarrow$ Δ is equilateral Δ.

Q Define a Seq $z_1 = 0$
11
$z_{n+1} = z_n^2 + i$; $n \geq 1$

then how far from the origin is z_{111} = ?

$1, \sqrt{2}, \sqrt{3}, \sqrt{10}$

$z_1 = 0$

$z_2 = z_1^2 + i = 0 + i = i$

$z_3 = z_2^2 + i = -1+i$

$z_4 = z_3^2 + i = (-1+i)^2 + i$

$= 1 - 1 - 2i + i = -i$

$z_5 = z_4^2 + i = (-i)^2 + i = -1+i$

$z_6 = z_5^2 + i = (-1+i)^2 + i = -i$

$z_7 = z_6^2 + i = (-i)^2 + i = -1+i$

⋮

$z_{111} = -1+i$

$|z_{111}| = |-1+i| = \sqrt{2}$

Nishant Jindal

Q z Rep. . . .

if $|z+5|^2 +$

Nishant Jindal

Q 12 Z Rep. . . .

if $|z+5|^2-|z-5|^2=10$?

$|x+iy+5|^2-|x+iy-5|^2=10$

$|(x+5)+iy|^2-|(x-5)+iy|^2=10$

$\left(\sqrt{(x+5)^2+y^2}\right)^2-\left(\sqrt{(x-5)^2+y^2}\right)^2=10$

$\cancel{x^2}+\cancel{y^2}+10x+\cancel{25}-(\cancel{x^2}+\cancel{y^2}-10x+\cancel{25})=10$

$20x=10$

$x=\frac{1}{2} \Rightarrow 2x-1=0$

St. line.

Q 13 $z_1=\sqrt[6]{\frac{1-i}{1+i\sqrt{3}}}$, $z_2=\sqrt[6]{\frac{1-i}{\sqrt{3}+i}}$

$z_3=\sqrt[6]{\frac{1+i}{\sqrt{3}-i}}$

(1) $\sum |z_1|^2=\frac{3}{2}$ (2) $|z_1|^4+|z_2|^4=|z_3|^{-8}$

(3) $|z_1|^4+|z_2|^4=|z_3|^8$ (D) NOT.

all of tim have Mod

$|z_1|=\left|\left(\frac{1-i}{1+i\sqrt{3}}\right)^{1/6}\right|=\left|\frac{1-i}{1+i\sqrt{3}}\right|^{1/6}$

$=\left(\frac{|1-i|}{|1+i\sqrt{3}|}\right)^{1/6}=\left(\frac{\sqrt{2}}{2}\right)^{1/6}=2^{-1/12}$

$|z_2|=\left|\frac{1-i}{\sqrt{3}+i}\right|^{1/6}=\left(\frac{\sqrt{2}}{2}\right)^{1/6}=2^{-1/12}$

$|z_3|=2^{-1/12}$

(2) $(2^{-1/12})^4+(2^{-1/12})^4=2^{-1/3}+2^{-1/3}$

$=\frac{1}{2^{1/3}}+\frac{1}{2^{1/3}}=\frac{2}{2^{1/3}}$

$=2^{2/3}$

RHS $\rightarrow |z_3|^{-8}=\left(2^{-1/12}\right)^{-8}=2^{2/3}$

Nishant Jindal

Q 14 C.N Whose Real & Img Parts are Integer Satisfying → $x, y = \text{Int}$

$z\bar{z}^3 + z^3\bar{z} = 350$

form a Rectangle, then length of its diagonal is?

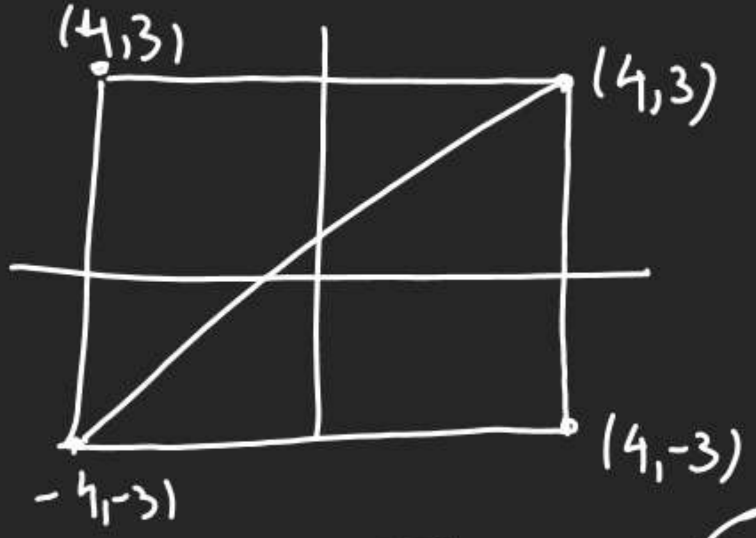
$z\bar{z}^3 + z^3\bar{z} = 350$

$\overset{|z|^2}{z\bar{z}}(\bar{z}^2 + z^2) = 350$

$(x^2+y^2)(\cancel{2}(x^2-y^2)) = \cancel{350}\ 175$

$25 \times 7 = 175$

$(x, y) = (4,3), (-4,-3)$
$(4,-3)\ (-4,3)$



$\sqrt{8^2+6^2} = 10$

$\frac{1}{z} = \frac{\bar{z}}{|z|^2}$

Imp Q 15 $|z_1| = 1, |z_2| = 2, |z_3| = 3.$

$|z_1+z_2+z_3| = 1$

find $|z_2z_3 + 4z_1z_3 + 9z_1z_2| = ?$

Fix Style $\left|z_1z_2z_3\left(\frac{1}{z_1}+\frac{4}{z_2}+\frac{9}{z_3}\right)\right|$

$= |z_1||z_2||z_3|\left|\frac{1}{z_1}+\frac{4}{z_2}+\frac{9}{z_3}\right|$

$= 1\times2\times3\left|\frac{\bar{z}_1}{|z_1|^2}+\frac{4\bar{z}_2}{|z_2|^2}+\frac{9\bar{z}_3}{|z_3|^2}\right|$

$6\left|\frac{\bar{z}_1}{1}+\frac{4\bar{z}_2}{4}+\frac{9\bar{z}_3}{9}\right|$

$= 6|\bar{z}_1+\bar{z}_2+\bar{z}_3|$ $\quad |\bar{z}| = |z|$

$= 6|\overline{z_1+z_2+z_3}|$

$= 6|z_1+z_2+z_3| = 6\times1 = 6$

Nishant Jindal

Q16 Find Roots of

$$z^2+2(1+2i)z-(11+2i)=0$$

Values of z → $SOR = -2-4i$

$$z=\frac{-2(1+2i)\pm\sqrt{4(1+2i)^2+4(11+2i)}}{2}$$

$$=\frac{-2(1+2i)\pm\sqrt{4(1-4+4i)+44+8i}}{2}$$

$$=\frac{-2(1+2i)\pm2\sqrt{-3+4i+11+2i}}{2}$$

$$z=-1-2i\pm\sqrt{8+6i}$$

$$z=-1-2i+3+i \;/\; -1-2i-3-i$$

$$=2-i \;/\; -4-3i$$

$$(2-i)+(-4-3i)=-2-4i$$

$$\sqrt{8+6i}=\pm\left\{\sqrt{\frac{10+8}{2}}+i\sqrt{\frac{10-8}{2}}\right\}$$

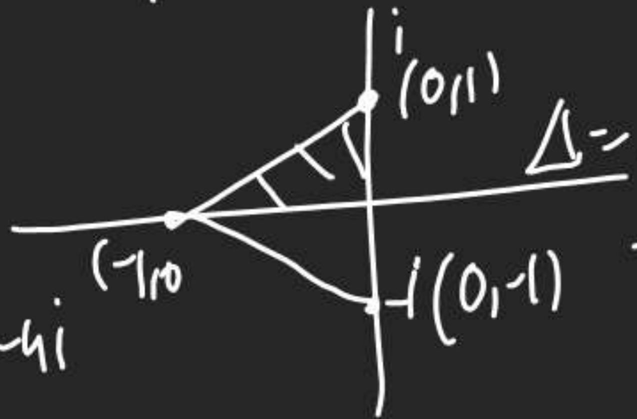
$$=\pm\{3+i\}$$

Q17 Roots of $z^3+z^2+z+1=0$ form a Δ find Area?

$$z^2(z+1)+1(z+1)=0$$

$$(z+1)(z^2+1)=0$$

$$z=-1 \;\Big|\; z^2=-1 \Rightarrow z=i,-i$$



$$\Delta=2\times\frac{1}{2}\times1\times1$$

$$=1$$

Q18 If Eqn $2z^2+2(i-1)=z-10$ has a Purely Img. Root find other Root?

$$SOR=-\frac{b}{a}=\frac{-(-1)}{2}=\frac{1}{2}$$

$$2z^2-z+10+2(i-1)=0$$

① z is Purely Img $\Rightarrow z=ib$.

$$-2b^2-2+2i=ib-10$$

$$\boxed{b=2}$$

$$-2b^2-2=-10$$

② 1 Root is $2i$

Other Root $\Rightarrow 2i+x=\frac{1}{2}$

$$x=\frac{1}{2}-2i$$

Nishant Jindal

Some Standard Locus.

(1) $|z-z_1|$ = distance of z from z_1

(2) $|z-z_1| = r$

then It Rep. Circle with Centre z_1 & Rad $= r$

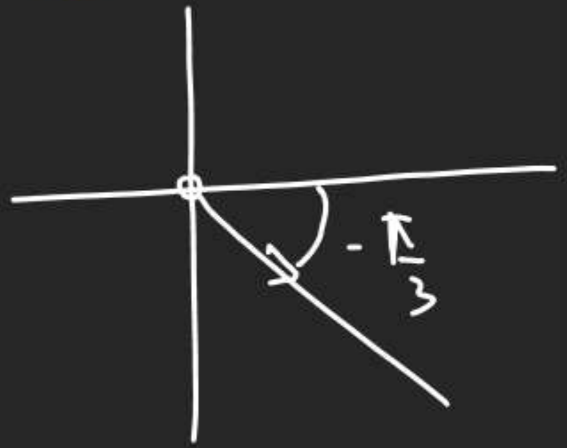
(3) $\text{Arg}(z) = \theta$

then it Rep. Locus of Ray Starting from origin at angle θ

Q $\text{Arg}(z) = -\frac{\pi}{3}$ Rep. Ray.

$x<0, y>0$?

False Stmt.



$m = \tan(-\frac{\pi}{3})$

$= -\sqrt{3}$.

EOL: $y = mx$

$y = -\sqrt{3}x$

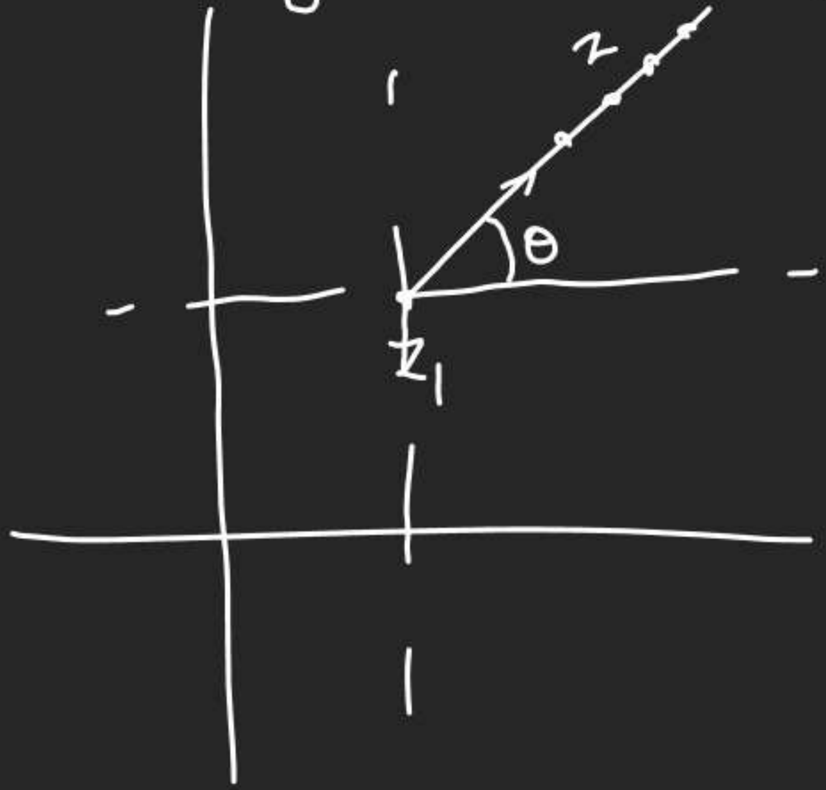
$\sqrt{3}x + y = 0$

4th Q way

$x>0, y<0$

(4) $\text{Arg}(z-z_1) = \theta$

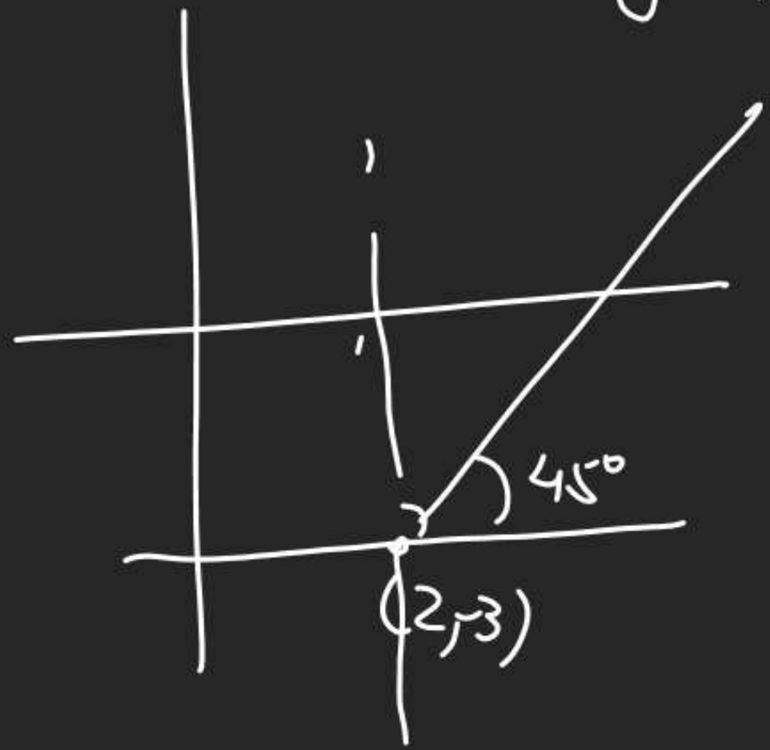
then it Rep Locus of Ray Starting from z_1 at angle θ



Q 20 $Arg(z-2+3i)=\frac{\pi}{4}$.

$\Rightarrow$ Ray starting from

$z-(2-3i)$ $(2,-3)$ at angle $\frac{\pi}{4}$



Next Class

$\boxed{\text{Sunday}}$

$Arg(x+iy-2+3i)=\frac{\pi}{4}$

$Arg((x-2)+i(y+3))=\frac{\pi}{4}$

$\tan^{-1}\left(\frac{y+3}{x-2}\right)=\frac{\pi}{4}$

$\frac{y+3}{x-2}=1$

$y+3=x-2$

$\underline{\underline{x-y=5}}$

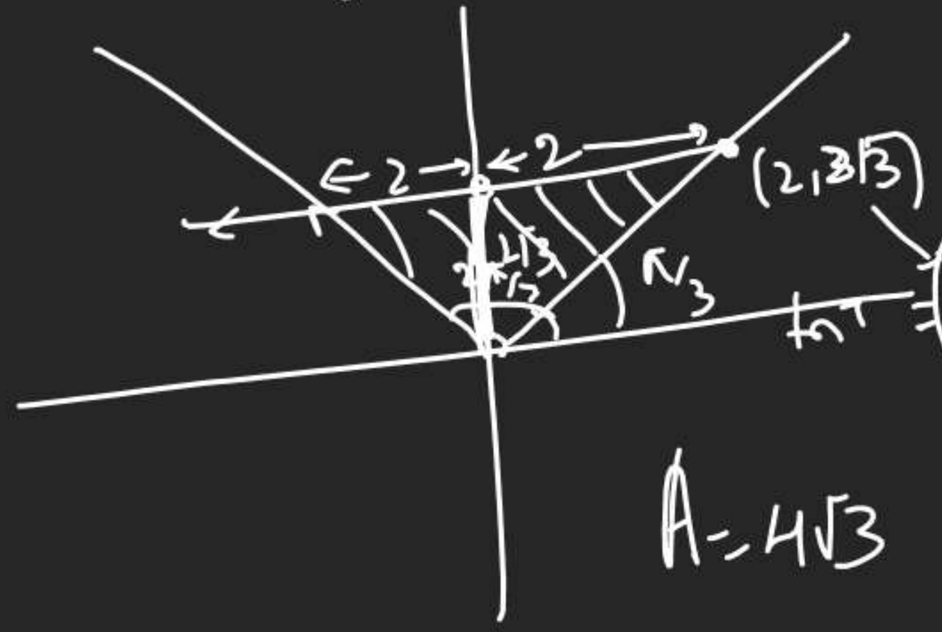
Q ABB 21

$Arg(z)=\frac{\pi}{3}$

$Arg(z)=\frac{2\pi}{3}$

$Arg(z-2-2\sqrt{3}i)=\pi \longrightarrow Arg(z-(2+2\sqrt{3}i))=\pi$

Ray at angle π



$\tan^{-1}\left(\frac{y}{x}\right)=\frac{2\sqrt{3}}{2}=\frac{\pi}{3}$

St. pt $(2,2\sqrt{3})$

$A=4\sqrt{3}$

$\Delta=\frac{1}{2}\times 4\times 2\sqrt{3}$