Q $f(x) = \text{Sgn}\,x,\ g(x) = x(x^2-5x+6)$

$f(g(x))$ is cont^y ?

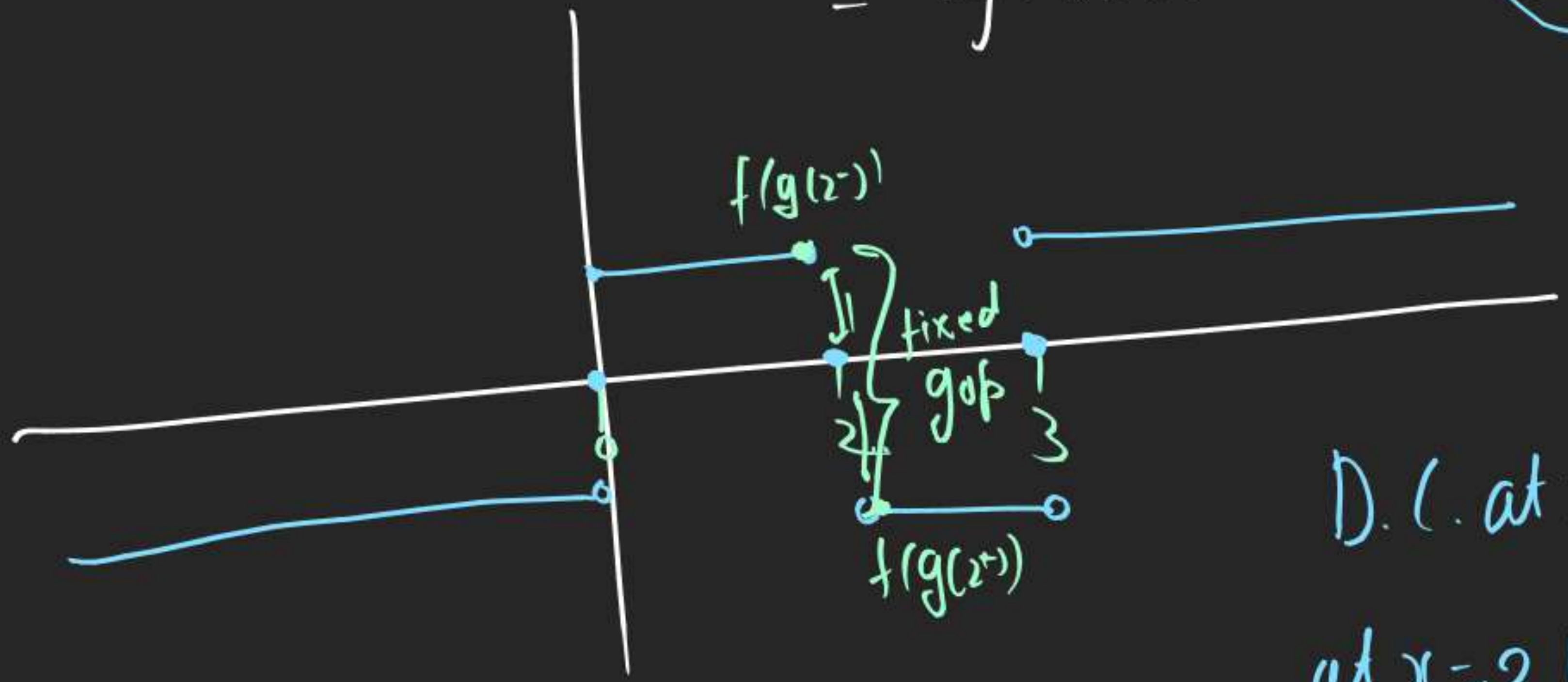
$f(x(x^2-5x+6)) = \text{Sgn}(x)(x^2-5x+6)$

$= \text{Sgn}(x)(x-2)(x-3)$

$\to \textcircled{>3}\,(4)(4-2)(4-3)\ \oplus$

$\to (3)(3-2)(3-3) = 0$

$\to (2,3) \to \underset{\oplus}{(2.5)}\,\underset{\oplus}{(2.5-2)}\,\underset{\ominus}{(2.5-3)}$



D.C. at $x = 0, 2, 3$

at $x = 2$ What kind of D.C. $f(g(x))$ is showing?

$f(g(2^-)) \neq f(g(2^+))$ ①

LHL ≠ RHL } Non Removable D.C

(2) finite D.C

(3) Jump $= |\text{LHL} - \text{RHL}|$

$= 2$

Q $f(x) = \text{Sgn}(\cos 2x - 2\sin x + 3)$ Make graph.

D.C ?

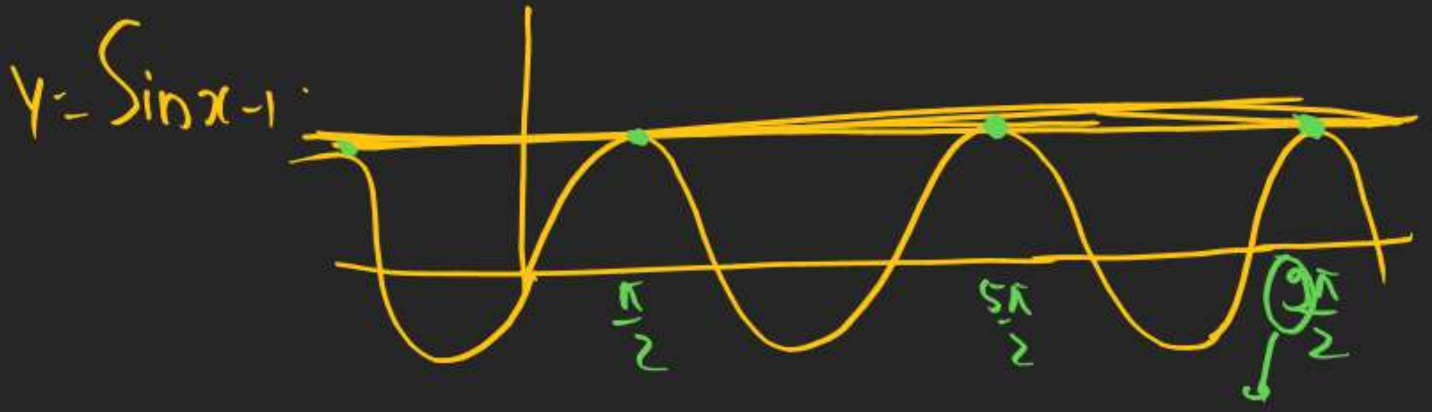
$$= \text{Sgn}(1 - 2\sin^2 x - 2\sin x + 3)$$

$$= \text{Sgn}(-2\sin^2 x - 2\sin x + 4)$$

$$= \text{Sgn}\left((-2)(\sin^2 x + \sin x - 2)\right)$$

$$= \text{Sgn}\left(\underbrace{(-2)\underbrace{(\sin x + 2)}_{+}}_{-ve}\underbrace{(\sin x - 1)}_{-ve}\right)$$

$$= \begin{cases} 1 & x \in R - (4n+1)\frac{\pi}{2} \\ 0 & (4n+1)\frac{\pi}{2} \end{cases}$$



Y = sinx-1 's graph is

Below x Axis always => it is -ve

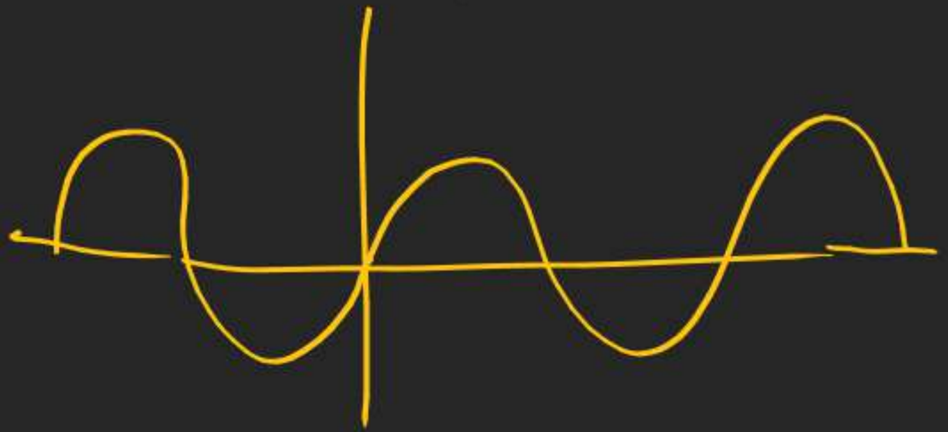
$Y = \sin x - 1 \leq 0$

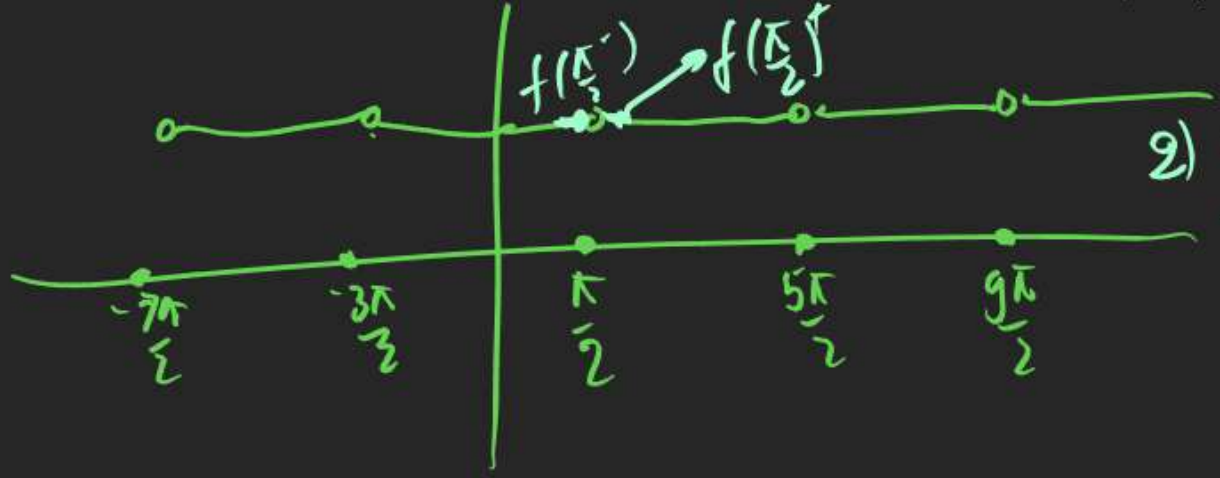
Kind of D.C?

1) LHL = RHL

Removable D.C (1st kind)

2) Isolated D.C.





Q $f(x) = \begin{cases} 1+x & 0 \le x \le 2 \\ 3-x & 2 < x \le 3. \end{cases}$

Defined fxn में Composite fxn के QS कैसे करेंगे?

$g(x) = \begin{cases} 1-x & 0 \le \underline{x \le 1} \\ 3-x & 1 < \underline{x \le 3} \end{cases}$

Check Cont^y of $\boxed{g(f(x))}$ at $x = \underline{2}$

$$g(f(2)) = g(1+2) = g(3) = 3-3 = 0$$

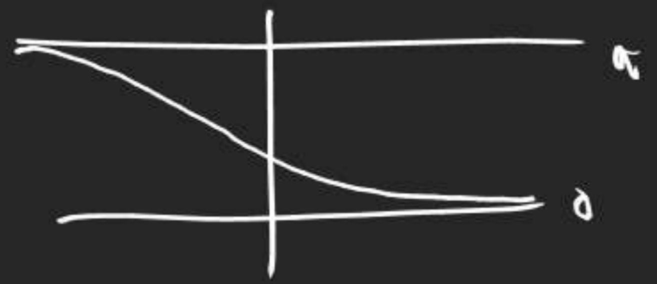
$$g(f(2^+)) = g(3-(2+h)) = g(1-h) = 1-(1-h) = h = 0$$

$$g(f(2^-)) = g(1+(2-h)) = g(3-h) = 3-(3-h) = h = 0$$

Cont^s at $x=2$

Nishant Jindal

Q $g(x) = \tan^{-1}|x| - \cot^{-1}|x|$



$f(x) = \frac{[x]\{x\}}{[x+1]}$; $h(x) = |g(f(x))|$ (check cont^n at $x=0$)

$$h(0) = |g(f(0))| = \left|g\left(\frac{[0]\{0\}}{[1]}\right)\right| = |g(0)| = |\tan^{-1}|0| - \cot^{-1}|0|| = \left|0 - \frac{\pi}{2}\right| = \frac{\pi}{2}$$

$$h(0^+) = |g(f(0^+))| = \left|g\left(\frac{[h]\{h\}}{[1+h]}\right)\right| = |g(0)| = |\tan^{-1}|0| - \cot^{-1}|0|| = \left|0 - \frac{\pi}{2}\right| = \frac{\pi}{2}$$

$$h(0^-) = |g(f(0^-))| = \left|g\left(\frac{[0-h]\{-h\}}{\underset{0}{[1-h]}}\right)\right| = \left|g\left(\frac{\overset{-ve}{\overleftrightarrow{\overset{\ominus}{(-1)}\,\overset{\oplus}{(1-h)}}}}{0}\right)\right| = |g(-\infty)|$$

$$= |\tan^{-1}|-\infty| - \cot^{-1}|-\infty|| =$$

$$= |\tan^{-1}\infty - \cot^{-1}\infty| = \left|\frac{\pi}{2} - 0\right| = \frac{\pi}{2}$$

$\underline{f\text{xn is cont}^s}$

Nishant Jindal

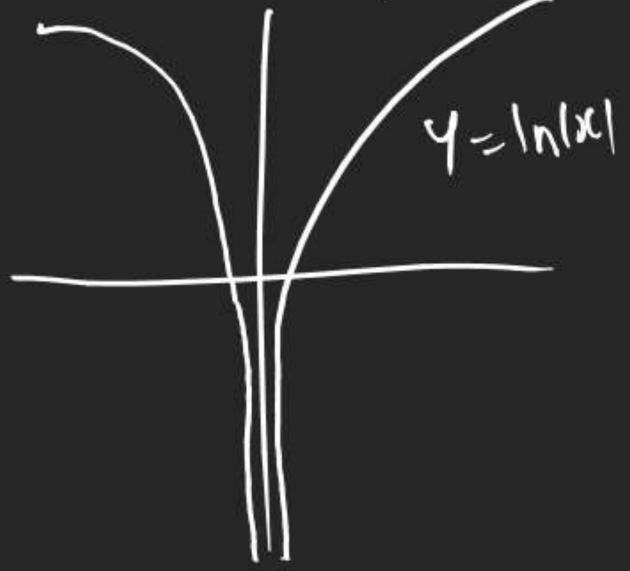
Q $f(x) = \frac{1}{\underset{\sim}{\ln}|x|}$ is D.C. at ?

1) D.C. where $\ln|x| = 0$

$|x| = e^0 = 1$

$x = 1, -1$

2) for $\ln\boxed{|x|}$, $\ln|x|$ is D.C. at $x = 0$ also



D.C. $\underbrace{x = 0, 1, -1}_{3\text{ Pt.}}$

$\frac{1}{\ln|x|}$

Q $f(x) = \boxed{\frac{1}{1-x}}$ ① $f(f(x)) = ?$ ② $f(f(f(x)))$

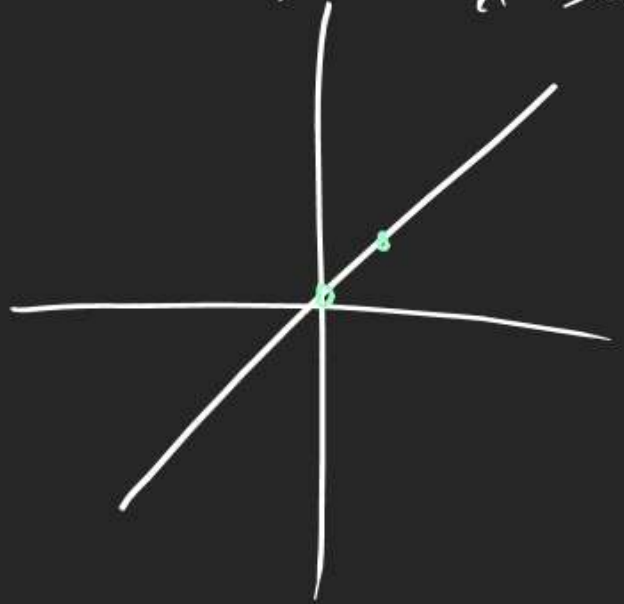
$\Rightarrow 1 - x \neq 0 \Rightarrow x \neq 1$

1) $f(f(x)) = f\left(\frac{1}{1-x}\right) = \frac{1}{1 - \frac{1}{1-x}} = \frac{1-x}{1-x-1} = \boxed{\frac{x-1}{x}}$

$x \neq 0$

2) $f(f(f(x))) = f\left(\frac{x-1}{x}\right) = \frac{1}{1 - \frac{x-1}{x}}$

$f(f(f(x))) = \frac{x}{\cancel{x} - \cancel{x} + 1} \rightarrow x$



$x = 0$ पर D.C.

$x = 1$ पर D.C.

Q Find Pt. of D.C of $y = f(u)$

When $f(u) = \dfrac{3}{2u^2+5u-3}$ & $\boxed{u = \dfrac{1}{x+2}}$

$\downarrow$

$x = -2$ ⓓ DC

$$\frac{3}{2\left(\frac{1}{x+2}\right)^2+5\left(\frac{1}{x+2}\right)-3}$$

D.C. at

$2u^2+5u-3=0$

$2u^2+6u-u-3=0$

$(2u-1)(u+3)=0$

$u = \frac{1}{2}$ or $u=-3$

$\boxed{x=0} \leftarrow \dfrac{1}{x+2} = \dfrac{1}{2}$

$\dfrac{1}{x+2} = -3 \Rightarrow x+2 = -\dfrac{1}{3}$

$x = -\frac{1}{3}-2 = -\frac{7}{3}$

DC

$\downarrow$

-2

0

$-\frac{7}{3}$

Theorems on D.C.

①

$f(x)$	$g(x)$	$f(x)+g(x)$	$f(x)\times g(x)$
C	C	C	C
C	D	D	Ⓜ
D	D	M	M

(2) $f(g(x))$ Behaviour.

$C(C) = C$

$C(D) = M$

$D(C) = M$

$D(D) = M$

Q $f(x) = \sin(\cos x)$ is Conts/D.C?

$\downarrow \quad \downarrow$

$C(C) = $ Conts.

Q $f(x) = \sin|x| + e^{x^2-3} + |x^2-2x-1|$ is C/D.C?

$C(C)$

C + C + C(C)

e^x Me x Ki

Jagah x^2-3

C + C + C = Conts always.

★

Sgn f(x) & log f(x) is D.C. When.

$f(x) = 0$

Q $h: R \to R$ is a fxn defined by at Integer.

Mains

$h(x) = \underbrace{[x]}_{} \overset{\text{D.C.} \times}{} \underbrace{\overset{\text{Conts}}{\cos\left((2x-1)\frac{\pi}{2}\right)}}_{}$

~~(A)~~ Conts for all x (B) D.C. at $x=0$

(C) D.C. at Non Zero Integral x

(4) Conts at $x=0$

$f(n) = [n]\cos\left((2n-1)\frac{\pi}{2}\right) = n \times 0 = 0$

$f(n^+) = [n+h] \times \cos(2(n+h)-1)\frac{\pi}{2} = n \times 0 = 0$

$f(n^-) = [n-h] \times \cos(2(n-h)-1)\frac{\pi}{2} = (n-1) \times 0$

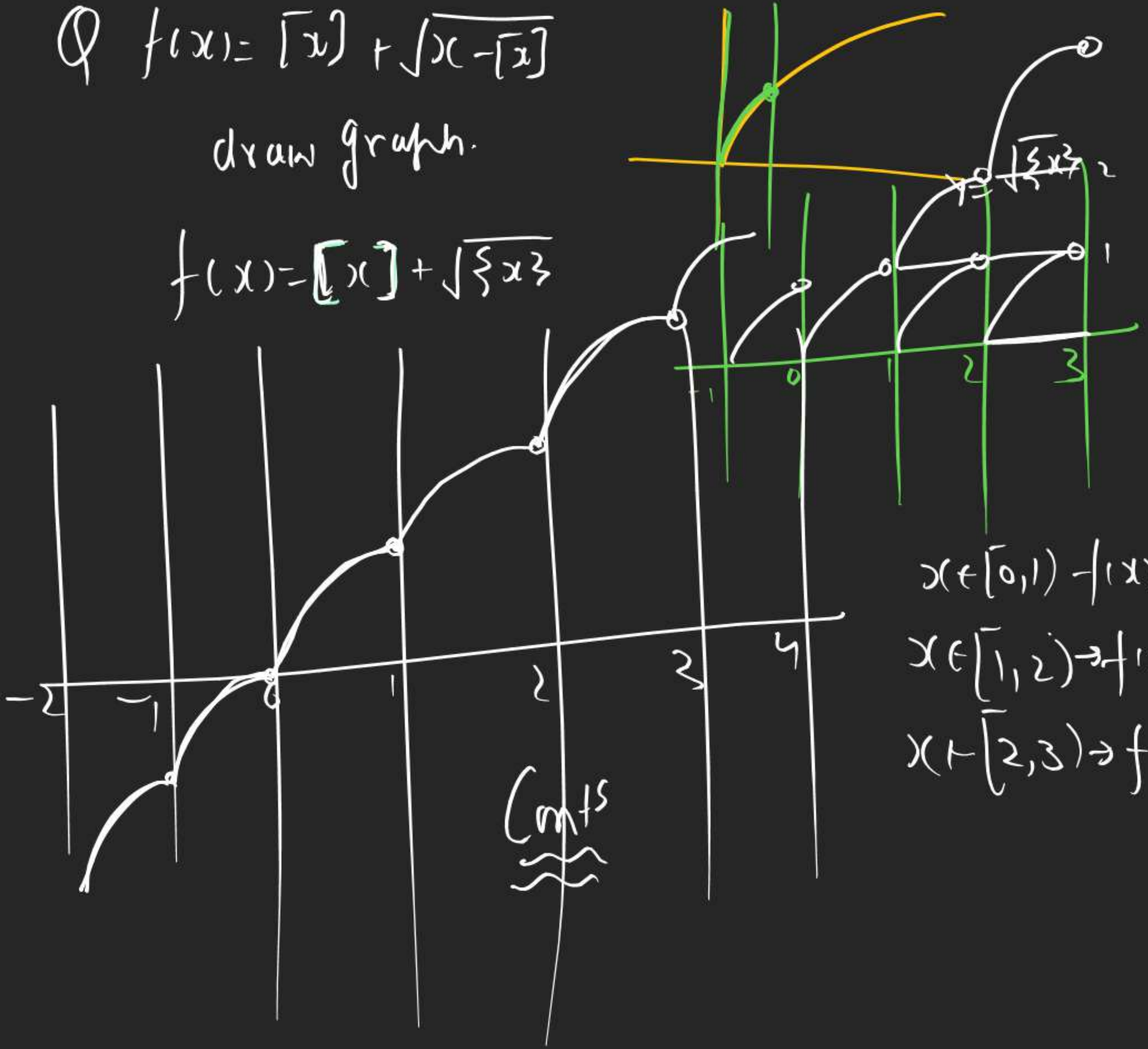
$= 0$

fxn is Conts at Integ.

Q $f(x) = [x] + \sqrt{x-[x]}$

draw graph.

$f(x) = [x] + \sqrt{\{x\}}$



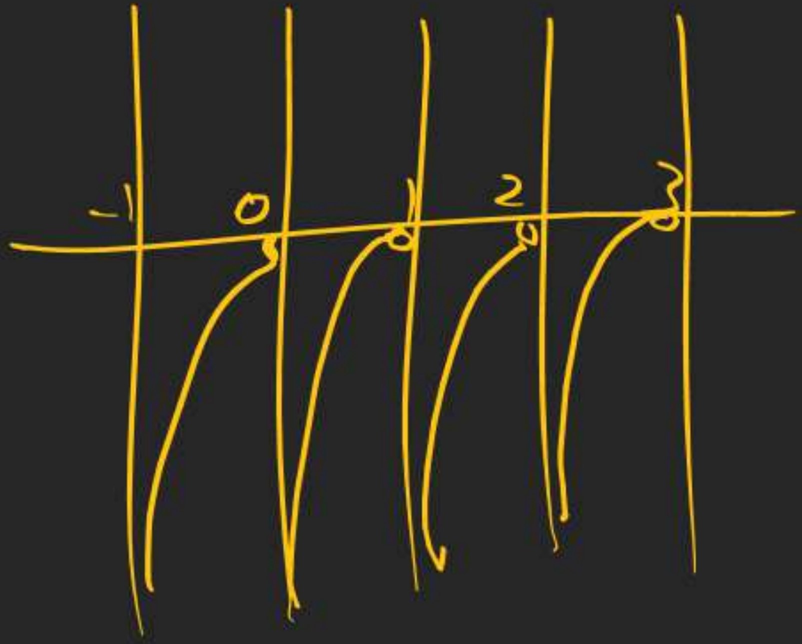
$\sqrt{\{-2.9\}} = \sqrt{1-.9} = \sqrt{.1}$

$y = \log\{x\}$

$x \in [0,1) \rightarrow f(x) = 0 + \sqrt{\{x\}}$

$x \in [1,2) \rightarrow f(x) = 1 + \sqrt{\{x\}}$

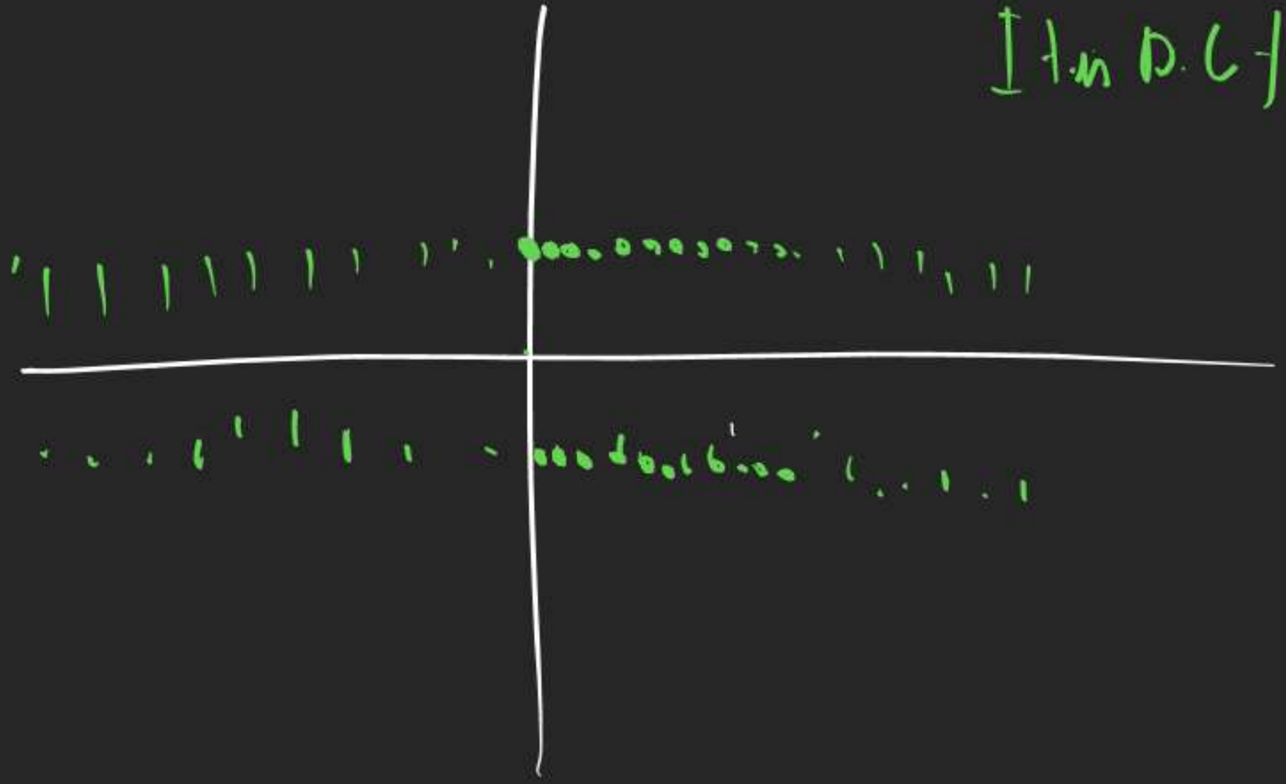
$x \in [2,3) \rightarrow f(x) = 2 + \sqrt{\{x\}}$



Single Pt. Cont^y.

fxn those who are Cont^s at one pt & defined Everywhere are Single Pt. Cont^s fxn.

Q $f(x) = \begin{cases} 1 & x \in Q \\ -1 & x \in Q' \end{cases}$



It is D.C fxn

$f(x) = \begin{cases} 1 & x \in Q \\ 1 & x \in Q' \end{cases}$

It would have been Cont^s when both values are same.

Q $f(x) = \begin{cases} x & x \in Q \\ 1-x & x \in Q' \end{cases}$ fxn can be Cont^s at $x = ?$

Cont^s होगा → $x = 1 - x$

$2x = 1$ at $\boxed{x = \frac{1}{2}}$

Q $f(x) = \begin{cases} \sin x & x \in Q \\ \cos x & x \in Q' \end{cases}$ is cont^s in $[0, 2\pi)$

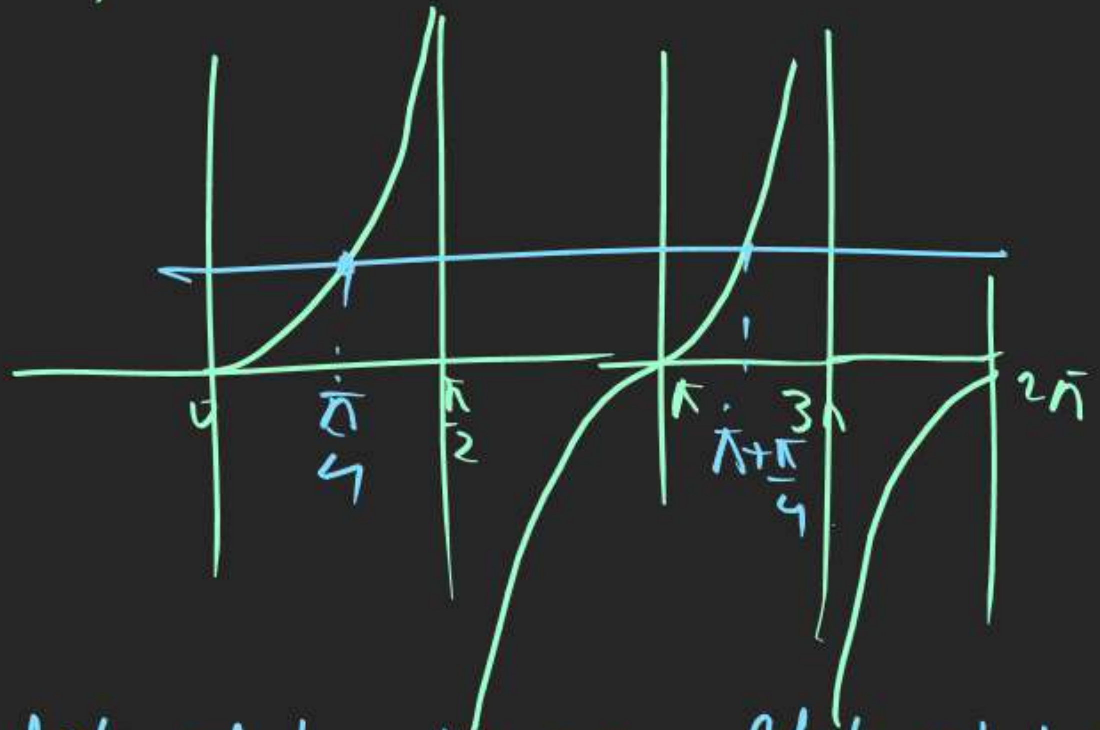
find $x = ?$

It can be cont^s if

$\sin x = \cos x$

$\tan x = 1$

$x = \frac{\pi}{4}, \frac{5\pi}{4}$



Q let f(x) be a cont^s fxn defined for $1 \le x \le 3$. If f(x) takes Rational values for all x & $f(2) = 10$ then value of $f(1.5) = ?$ → $f(x) = 10 \Rightarrow \boxed{f(1.5) = 10}$

$f(2) = 10$ ↓ Q

$f(x) = \begin{cases} 10 & x \in Q \\ 10 & x \in Q' \end{cases}$ It can be cont^s when at $x = Q'$ $f(x) = 10$

Built in Limit (Limit me Limit)

$\downarrow$ one Limit insure $x \to \infty$

$\downarrow$ other Limit can be any Const. No.

Q Check cont^y of $\displaystyle\lim_{t\to\infty} \frac{(1+\sin \pi x)^t - 1}{(1+\sin \pi x)^t + 1}$ at $x=1,2,3$.

$x=1$ पर check करके दिखा रहा हूँ।

$$f(x) = \begin{cases} \displaystyle\lim_{t\to\infty} \frac{(1+\sin x)^t - 1}{(1+\sin \pi x)^t + 1} = \frac{(\text{Exact }1)^{\infty} - 1}{(\text{Exact }1)^{\infty} + 1} = \frac{1-1}{1+1} = 0 \\ \displaystyle\lim_{t\to\infty} \frac{(1+\sin \pi x)^t - 1}{(1+\sin \pi x)^t + 1} = \frac{\overset{\text{less than}}{(1-\sin \pi h)}^{\infty} - 1}{(1-\sin \pi h)^{\infty} + 1} = \frac{0-1}{0+1} = -1 \\ \displaystyle\lim_{t\to\infty} \frac{(1+\sin \pi x)^t - 1}{(1+\sin \pi x)^t + 1} = \frac{\overset{\text{Gr than}}{(1+\sin \pi h)}^{\infty} - 1}{(1+\sin \pi h)^{\infty} + 1} = \frac{\cancel{(1+\sin \pi h)^t}\left(1 - \frac{1}{(1+\sin \pi h)^t}\right)}{\cancel{(1+\sin \pi h)^t}\left(1 + \frac{1}{(1+\sin \pi h)^t}\right)} = \frac{1-0}{1+0} = 1 \end{cases}$$

$\boxed{x=1} \rightarrow \pi \cdot x = \pi$

$\sin \pi x = \sin \pi = 0$

$1+\sin \pi x = 1+0 = 1$

D.C. at $x=1$

$\boxed{x=1+h} \rightarrow \pi x = \pi(1+h)$

$= \pi + \pi h$

$1+\sin(\pi x) = 1+\sin(\pi+\pi h)$

$= (1-\sin \pi h)$

$x = 1-h$

$\pi x = \pi(1-h) = \pi - \pi h$

$1+\sin \pi x = 1+\sin(\pi - \pi h) = \boxed{1+\sin \pi h}$