Nishant Jindal

Trigonometry

Q P.T.

$\tan 20° + \tan 25° + \tan 20° \cdot \tan 25° = 1$ ← theorem 1 Kab Aata hai?

$\Rightarrow \tan 45° = 1$

$\tan 45° = 1$

$\tan(20+25°) = 1$

$$\frac{\tan 20° + \tan 25°}{1 - \tan 20° \tan 25°} = 1$$

$\tan 20° + \tan 25° = 1 - \tan 20° \cdot \tan 25°$

$\Rightarrow \tan 20° + \tan 25° + \tan 20° \cdot \tan 25° = 1$ H.P.

Result $A+B = 45°$

$(1+\tan A)(1+\tan B) = 2$

Q2 If $A+B = \frac{\pi}{4}$ then P.T.

$(1+\tan A)(1+\tan B) = ?$

$A+B = \frac{\pi}{4}$

$\tan(A+B) = 1$

$$\frac{\tan A + \tan B}{1 - \tan A \tan B} = 1$$

$\tan A + \tan B = 1 - \tan A \tan B$

$(1+\tan A) + \tan B + \tan A \cdot \tan B = 1+1$

$(1+\tan A) + \tan B(1+\tan A) = 2$

$(1+\tan A)(1+\tan B) = 2$

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Trigonometry

Q $(1+\tan 20^\circ)(1+\tan 25^\circ) = ?$

$20^\circ + 25^\circ = \frac{\pi}{4}$

$\Rightarrow (1+\tan 20^\circ)(1+\tan 25^\circ) = 2$ Ans

$\tan(-\theta) = -\tan\theta$

$-\tan\theta = \tan(-\theta)$

$(1+\tan A)(1+\tan B)$

$A+B = \frac{\pi}{4} \Rightarrow \tan A + \tan B = 2$

Q4 Find value of

$(1+\tan 245^\circ)(1+\tan 250^\circ)(1+\tan 260^\circ)(1-\tan 200^\circ)(1-\tan 205^\circ)(1-\tan 215^\circ) = ?$

$(1+\tan 245^\circ)(1-\tan 200^\circ)(1+\tan 250^\circ)(1-\tan 205^\circ)(1+\tan 260^\circ)(1-\tan 215^\circ)$

$(1+\tan 245^\circ)(1+\tan(-200^\circ))(1+\tan 250^\circ)(1+\tan(-205^\circ))(1+\tan 260^\circ)(1+\tan(-215^\circ))$

$245^\circ + (-200^\circ) = 45^\circ$ $\quad 250^\circ + -205^\circ = 45^\circ$ $\quad 260^\circ + -215^\circ = 45^\circ$

$2 \times 2 \times 2 = 8$

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$\sin(A+B)\sin(A-B) = \sin^2 A - \sin^2 B$ P.T.

Q $\sin^2 A + \sin^2(A-B) - 2\sin A\cos B \sin(A-B) = \sin^2 B$

$\sin^2 A + \sin(A-B)\{\sin(A-B) - 2\sin A\cos B\}$

$\sin^2 A + \sin(A-B)\{\sin A\cos B - \cos A\sin B - 2\sin A\cos B\}$

$\sin^2 A + \sin(A-B)\{-\sin A\cos B - \cos A\sin B\}$

$\sin^2 A - \sin(A-B)\{\sin A\cos B + \cos A\sin B\}$

$\sin^2 A - \sin(A-B)\sin(A+B)$

$\sin^2 A - (\sin^2 A - \sin^2 B) = \sin^2 B$ RHS

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Trigonometry

$\cos^2 A - \cos^2 B = \sin^2 B - \sin^2 A$

$= \sin(B+A) \cdot \sin(B-A)$

Q 6 P.T. $\cos(2x \pm 2y) = \cos 2x \cos 2y + \cos^2(x+y) - \cos^2(x-y)$

B A

RHS $\cos 2x \cdot \cos 2y + \sin(x+y+x-y) \cdot \sin(x-y-x+y)$

$\cos 2x \cos 2y + \sin(2x) \cdot \sin(-2y)$

$\cos 2x \cdot \cos 2y - \sin 2x \cdot \sin 2y$

$= \cos(2x+2y)$ LHS

$\sin(A+B) \cdot \sin(A-B)$

$\sin^2 A - \sin^2 B$

$\cos^2 B - \cos^2 A$

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Trigonometry

15°/75°

1) $\sin 15° = \sin(45° - 30°)$

$= \sin 45° \cdot \cos 30° - \cos 45° \cdot \sin 30°$

$= \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \cdot \frac{1}{2} = \frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}} = \frac{\sqrt{3}-1}{2\sqrt{2}} = \cos 75°$

$\frac{3+1-2\sqrt{3}}{2}$

$\Rightarrow \frac{4-2\sqrt{3}}{2}$

2) $\cos 15° = \cos(45° - 30°) = \sin 75°$

$= \cos 45° \cdot \cos 30° + \sin 45° \cdot \sin 30°$

$= \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \cdot \frac{1}{2} = \frac{\sqrt{3}+1}{2\sqrt{2}} = \sin 75°$

$= 2 - \sqrt{3}$

(3) $\tan 15° = \frac{\sin 15°}{\cos 15°} = \frac{\frac{\sqrt{3}-1}{2\sqrt{2}}}{\frac{\sqrt{3}+1}{2\sqrt{2}}} = \frac{\sqrt{3}-1}{\sqrt{3}+1}$ Ans

$= \frac{\sqrt{3}-1}{\sqrt{3}+1} \times \frac{\sqrt{3}-1}{\sqrt{3}-1} = \frac{(\sqrt{3}-1)^2}{3-1}$

$\tan 15° = 2 - \sqrt{3} = \cot 75°$

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Trigonometry

4) $\cot 15^\circ = \frac{\cos 15^\circ}{\sin 15^\circ} = \frac{\frac{\sqrt{3}+1}{2\sqrt{2}}}{\frac{\sqrt{3}-1}{2\sqrt{2}}} = \frac{\sqrt{3}+1}{\sqrt{3}-1}$ ✍

$= \frac{\sqrt{3}+1}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1} = \frac{(\sqrt{3}+1)^2}{3-1}$

$= \frac{3+1+2\sqrt{3}}{2}$

$= \frac{4+2\sqrt{3}}{2} = 2+\sqrt{3}.$

$\boxed{\cot 15^\circ = 2+\sqrt{3} = \tan 75^\circ}$

1) $\sin 15^\circ = \frac{\sqrt{3}-1}{2\sqrt{2}}$

2) $\cos 15^\circ = \frac{\sqrt{3}+1}{2\sqrt{2}}$

3) $\tan 15^\circ = 2-\sqrt{3}.$

4) $\tan 75^\circ = 2+\sqrt{3}.$

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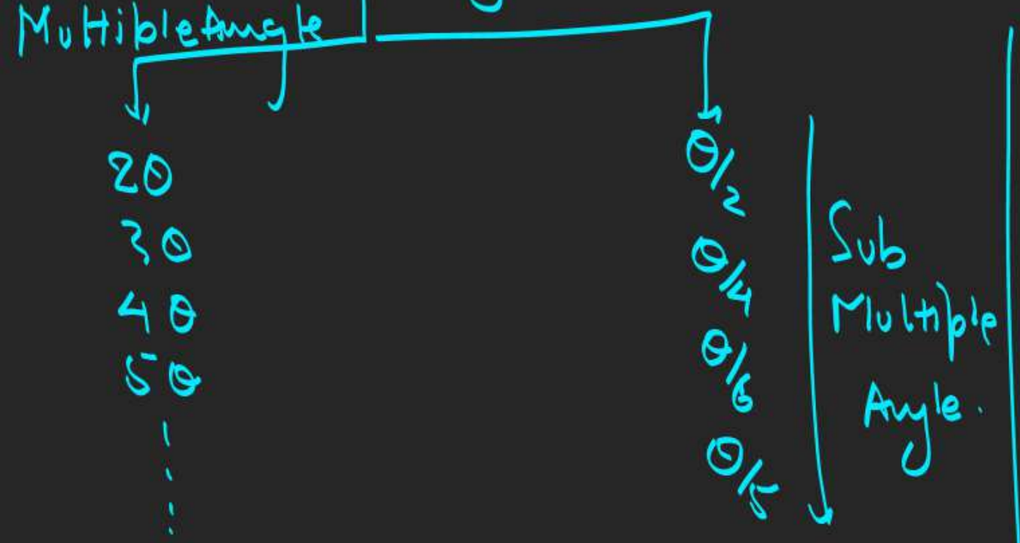
Trigonometry

Multiple / Submultiple Angle.

θ = Angle



$\cos 2\theta = \cos^2\theta - \sin^2\theta$

$= \cos^2\theta - (1 - \cos^2\theta) = 2\cos^2\theta - 1$

2θ $\boxed{\cos 2\theta = 2\cos^2\theta - 1}$

A) $\sin 2\theta$ B) $\cos 2\theta$ C) $\tan 2\theta$

$\sin(A+B) = \sin A \cos B + \cos A \sin B$

$A = \theta$, $B = \theta$

$\sin(\theta+\theta) = \sin\theta\cos\theta + \cos\theta\sin\theta$

$\boxed{\sin 2\theta = 2\sin\theta\cos\theta}$

(B) $\cos(A+B) = \cos A \cos B - \sin A \sin B$

$A = \theta, B = \theta$

$\cos(\theta+\theta) = \cos\theta\cdot\cos\theta - \sin\theta\cdot\sin\theta$

$\boxed{\cos 2\theta = \cos^2\theta - \sin^2\theta}$

$\cos 2\theta = 1 - \sin^2\theta - \sin^2\theta$

$\boxed{\cos 2\theta = 1 - 2\sin^2\theta}$

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Trigonometry

(() $\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$ $A = \theta$, $B = \theta$

$\tan(\theta+\theta) = \frac{\tan\theta + \tan\theta}{1 - \tan\theta \cdot \tan\theta}$

$$\boxed{\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}}$$

2θ

Imp

(1) $\sin 2\theta = 2\sin\theta\cos\theta$

(2) $\cos 2\theta = c^2 - s^2$

$= 1 - 2s^2$

$= 2c^2 - 1$

(3) $\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$

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Trigonometry

Apas Me Relations

$\cos 2\theta = 2\cos^2\theta - 1$ & $\cos 2\theta = 1 - 2\sin^2\theta$

$\boxed{1 + \cos 2\theta = 2\cos^2\theta}$ $\boxed{2\sin^2\theta = 1 - \cos 2\theta}$

Q $\dfrac{1-\cos 2A}{1+\cos 2A}$

$= \dfrac{2\sin^2 A}{2\cos^2 A} = \tan^2 A$

Q$_8$ $\dfrac{1+\cos 2A}{1-\cos 2A} = ?$

$\dfrac{2\cos^2 A}{2\sin^2 A} = \cot^2 A$

Q$_9$ $\dfrac{1-\cos 2A}{\sin 2A} = ?$

$\dfrac{2\sin^2 A}{2\sin A\cos A} = \tan A$

Q$_{10}$ $\dfrac{1+\cos 2A}{\sin 2A} = ?$

$\dfrac{2\cos^2 A}{2\sin A\cos A} = \cot A$

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Trigonometry

Dhyan Rahe....

1) $\sin 2A = 2 \sin A \cos A$

2) $\sin 4A = ?$

$2 \sin 2A \cos 2A$

3) $\sin 8A = ?$

$2 \sin 4A \cos 4A$ Ans

$2(2 \sin 2A \cdot \cos 2A) \cos 4A$

$4 \sin 2A \cos 2A \cos 4A$ Ans 2

$4(2 \sin A \cos A) \cos 2A \cdot \cos 4A$

$8 \sin A \cos A \cos 2A \cos 4A$ Ans 3

$1 - \cos 2A = 2 \sin^2 A$

Q11. $\frac{1 - \cos A}{\sin A} = ?$

$$\frac{2 \sin^2 A/2}{2 \sin \frac{A}{2} \cos A/2} = \frac{\sin A/2}{\cos A/2} = \tan \frac{A}{2}$$

Q12. $\frac{1 + \cos A}{\sin A}$

$$= \frac{2 \cos^2 A/2}{2 \sin \frac{A}{2} \cos \frac{A}{2}} = \frac{\cos \frac{A}{2}}{\sin A/2} = \cot \frac{A}{2}$$

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Trigonometry

Q13 $\frac{1+\cos A}{1-\cos A} = ?$

$$\frac{2\cos^2 A/2}{2\sin^2 A/2} = \cot^2 A/2$$

Q14 $\frac{\sin A}{1-\cos A}$

$$\frac{2\sin A/2 \cos A/2}{2\sin^2 A/2} = \cot \frac{A}{2}$$

Q15 $1+\sin 2A = ?$

$$1 + 2\sin A \cos A$$

$$\Rightarrow (\sin^2 A + \cos^2 A) + 2\sin A \cos A$$

$$\Rightarrow (\sin A + \cos A)^2$$

Q16 $(1-\sin 2A) = ?$

$$1 - 2\sin A \cos A$$

$$\sin^2 A + \cos^2 A - 2\sin A \cos A$$

$$(\sin A - \cos A)^2$$

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Trigonometry

Q $\sin\theta \cdot \sin(60-\theta) \cdot \sin(60+\theta)$

$\sin(A+B)\cdot\sin(A-B) = \sin^2A - \sin^2B$

$\sin\theta \cdot (\sin^2 60 - \sin^2\theta)$

$\sin\theta\left(\left(\frac{\sqrt{3}}{2}\right)^2 - \sin^2\theta\right)$

$\sin\theta\left(\frac{3}{4} - \sin^2\theta\right) = \sin\theta\frac{(3-4\sin^2\theta)}{4}$

$= \frac{3\sin\theta - 4\sin^3\theta}{4} = \frac{1}{4}(\sin 3\theta)$

3θ

① $\sin 3\theta = \sin(2\theta+\theta)$

$= \sin 2\theta\cos\theta + \cos 2\theta \sin\theta$

$= 2\sin\theta\cos\theta\cdot\cos\theta + (1-2\sin^2\theta)\sin\theta$

$= 2\sin\theta\cdot\cos^2\theta + \sin\theta - 2\sin^3\theta$

$= 2\sin\theta(1-\sin^2\theta) + \sin\theta - 2\sin^3\theta$

$= 2\sin\theta - 2\sin^3\theta + \sin\theta - 2\sin^3\theta$

$\boxed{\sin 3\theta = 3\sin\theta - 4\sin^3\theta}$

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Trigonometry

3 Imp Results

1) $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$

2) $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$

3) $\tan 3\theta = \dfrac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}$

$$\tan(2\theta+\theta) = \frac{\tan 2\theta + \tan\theta}{1 - \tan 2\theta \cdot \tan\theta}$$

$$= \frac{\frac{2\tan\theta}{1-\tan^2\theta} + \tan\theta}{1 - \frac{2\tan\theta}{1-\tan^2\theta}\cdot\tan\theta}$$

$$= \frac{2\tan\theta + \tan\theta - \tan^3\theta}{1 - \tan^2\theta - 2\tan^2\theta}$$

$$\tan 3\theta = \frac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}$$

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Trigonometry

Q $\cos\theta \cdot \cos(60-\theta) \cdot \cos(60+\theta) = ?$

$\cos\theta(\cos^2\theta - \sin^2 60^\circ)$

$\cos\theta\left(\cos^2\theta - \left(\frac{\sqrt{3}}{2}\right)^2\right)$

$\cos\theta\left(\cos^2\theta - \frac{3}{4}\right)$

$\cos\theta\left(\frac{4\cos^2\theta - 3}{4}\right)$

$\frac{4\cos^3\theta - 3\cos\theta}{4} = \frac{\cos 3\theta}{4}$

$\cos(A+B)\cdot\cos(A-B) =$

$(\cos A\cos B - \sin A\sin B)(\cos A\cos B + \sin A\sin B)$

$\cos^2 A\cos^2 B - \sin^2 A\sin^2 B$

$(1-\sin^2 A)\cos^2 B - \sin^2 A(1-\cos^2 B)$

$\cos^2 B - \sin^2 A\cos^2 B - \sin^2 A + \sin^2 A\cos^2 B$

$\cos^2 B - \sin^2 A$

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Trigonometry

Compli

$\cos 10^\circ = \sin 80^\circ$

(H.W)

Ex 16 → Q7

Ex 17 → Q16

$1 + \cos 2A = 2\cos^2 A$

$$\frac{1+\cos(20^\circ)}{\sin^2 80} = \frac{2\cos^2 10^\circ}{\cos^2 10^\circ} = 2$$

Result

(1) $\sin\theta \cdot \sin(60-\theta)\cdot\sin(60+\theta) = \frac{\sin 3\theta}{4}$

(2) $\cos\theta \cdot \cos(60-\theta)\cos(60+\theta) = \frac{\cos 3\theta}{4}$

(3) $\tan\theta \cdot \tan(60-\theta)\tan(60+\theta) = \tan 3\theta$

Q $\sin 20^\circ \cdot \sin 40^\circ \boxed{\sin 60} \sin 80$

$\frac{\sqrt{3}}{2}\ \sin 20^\circ \cdot \sin 40^\circ \sin 80$

$\frac{\sqrt{3}}{2}\ \sin 20^\circ \cdot \sin(60^\circ - 2\theta)\cdot\sin(60+2\theta)$

$\frac{\sqrt{3}}{2}\cdot\frac{\sin(3\times 20^\circ)}{4} = \frac{\sqrt{3}}{2}\times\frac{\sqrt{3}}{8} = \frac{3}{16}$

Q $\frac{\cos 20^\circ + 8\sin 70^\circ \cdot \sin 50^\circ \cdot \sin 10^\circ}{\sin^2 80^\circ} = ?$

$$\frac{\cos 20^\circ + 8\times \sin(60+10)\sin(60+0)\sin 10^\circ}{\sin^2 80^\circ}$$

$$\frac{\cos 20^\circ + 8^2 \times \frac{\sin 3\times 10^\circ}{4}}{\sin^2 80^\circ}$$

$$\frac{\cos 20^\circ + 2\cdot\times 1}{\sin^2 80^\circ} = \frac{1+\cos 20^\circ}{\sin^2 80^\circ}$$