Nishant Jindal

1) $\sum {}^nC_r (\mathrm{I})^{n-r} (\mathrm{II})^r$ type

$= (\mathrm{I} + \mathrm{II})^n$

2) $\sum (-1)^r\, {}^nC_r (\mathrm{I})^{n-r} (\mathrm{II})^r = \sum {}^nC_r (\mathrm{I})^{n-r} (-\mathrm{II})^r$

$= (\mathrm{I} - \mathrm{II})^n$

Q Find coff of x^{53} in $\sum_{m=0}^{100} {}^{100}C_m (x-3)^{100-m} (2)^m$

$= (x-3+2)^{100}$ coff of x^{53}.

$=$ coff x^{53} in $(x-1)^{100}$

${}^{100}C_r (x)^{\boxed{100-r=53}} \cdot (-1)^r \to r = 47$

coff $(-1)^{47} \times {}^{100}C_{47} = -1 \times {}^{100}C_{53}$

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Q $\displaystyle\sum_{r=0}^{n} (-1)^r\, {}^nC_r \left[\frac{1}{2^r} + \frac{3^r}{2^{2r}} + \frac{7^r}{2^{3r}} + \frac{15^r}{2^{4r}} + \cdots \text{ Upto } m \text{ terms}\right]$

$$\sum (-1)^r\, {}^nC_r \left(\frac{1}{2}\right)^r + \sum (-1)^r\, {}^nC_r \left(\frac{3}{2^2}\right)^r + \sum (-1)^r\, {}^nC_r \left(\frac{7}{2^3}\right)^r + \sum (-1)^r\, {}^nC_r \left(\frac{15}{2^4}\right)^r + \cdots m \text{ terms.}$$

$n-r$ वाली Party MISSING h. $\therefore$ तो [illegible]

$$\left(1-\frac{1}{2}\right)^n + \left(1-\frac{3}{2^2}\right)^n + \left(1-\frac{7}{2^3}\right)^n + \left(1-\frac{15}{2^4}\right)^n + \cdots m \text{ terms.}$$

$$\frac{1}{2^n} \times \frac{1}{2^n} = \frac{1}{4^n} \times \frac{1}{2^n} = \frac{1}{8^n}$$

$$\left(\frac{1}{2}\right)^n + \left(1-\frac{3}{4}\right)^n + \left(1-\frac{7}{8}\right)^n + \left(1-\frac{15}{16}\right)^n + \cdots m \text{ terms}$$

$$\left(\frac{1}{2}\right)^n + \left(\frac{1}{4}\right)^n + \left(\frac{1}{8}\right)^n + \left(\frac{1}{16}\right)^n + \cdots m \text{ term} \quad [\text{GP of } m \text{ terms}]$$

$$\frac{1}{2^n} \qquad \frac{1}{2^n}$$

$$\frac{\left(\frac{1}{2}\right)^n\left[1-\left(\frac{1}{2^n}\right)^m\right]}{\left(1-\frac{1}{2^n}\right)} = \frac{\frac{1}{\cancel{2^n}}\left(\frac{2^{mn}-1}{2^{mn}}\right)}{\frac{2^n-1}{\cancel{2^n}}} = \frac{\left(2^{mn}-1\right)}{2^{mn}\left(2^n-1\right)} \underline{\underline{A}}$$

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Q. Show that values of x for which 6th term in exp $\left[2^{\log_2\sqrt{9^{x-1}+7}} + \frac{1}{2^{\frac{1}{5}\log_2(3^{x-1}+1)}}\right]^7$ is 84, are 2 & 1.

$\log_5 3 + \log_5 5 = \log_5 3\times5 = \log_5 15$

$$\left[2^{\log_2\sqrt{9^{x-1}+7}} + \frac{1}{2^{\frac{1}{5}\log_2(3^{x-1}+1)}}\right]^7$$

$$\left[\sqrt{9^{x-1}+7} \times \frac{1}{(3^{x-1}+1)^{1/5}}\right]^7 \quad \text{6th term}$$

$$T_6 = {}^7C_5 \cdot \left(\sqrt{9^{x-1}+7}\right)^2 \cdot \left(\frac{1}{(3^{x-1}+1)^{1/5}}\right)^5$$

$$= \frac{\cancel{7}\cancel{6}}{1\cdot 2}\,\frac{9^{x-1}+7}{3^{x-1}+1} = 84 \Rightarrow \frac{t^2+7}{t+1} = 4$$

$t^2+7 = 4t+4$

$t^2-4t+3=0$

$(t-1)(t-3)=0$

$t=1$ or $t=3$

$3^{x-1}=3^0$	$3^{x-1}=3^1$
$x-1=0$	$x-1=1$
$x=1$	$x=2$

H.P.

Q. 9th term in exp of $\left[3^{\log_3\sqrt{25^{x-1}+7}} + 3^{-\frac{1}{8}\log_3(5^{x-1}+1)}\right]^{10}$ is 180 $(x>1)$ then S.T. $x=\log_5 15$?

$$\Rightarrow \left[\sqrt{25^{x-1}+7} + \frac{1}{(5^{x-1}+1)^{1/8}}\right]^{10} \quad \text{9th term}$$

$$T_9 = {}^{10}C_8 \cdot \left(\sqrt{25^{x-1}+7}\right)^2 \cdot \left(\frac{1}{(5^{x-1}+1)^{1/8}}\right)^8 = 180$$

$$= \frac{\cancel{10}\,9}{1\cdot 2}\,\frac{(25^{x-1}+7)}{(5^{x-1}+1)} = \cancel{180}\,2 \Rightarrow \frac{t^2+7}{t+1} = 4$$

$t=1$ or $t=3$

$5^{x-1}=5^0$	$5^{x-1}=3$
$x=1$ ✗	$\log_5 5^{x-1} = \log_5 3$
$x>1$	$x-1 = \log_5 3$

$x = \log_5 3 + 1$

$x = \log_5 15$

$\log_5 3$

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Q Find value of:

$a^3+b^3+3ab(a+b) = (a+b)^3$

$$\frac{18^3+7^3+3\cdot 18\cdot 7\cdot 25}{3^6+6\cdot 243\cdot 2+15\cdot 81\cdot 4+20\cdot 27\cdot 8+15\cdot 9\cdot 16+6\cdot 3\cdot 32+64} = ?$$

$$\frac{(18)^3+(7)^3+3\times 18\times 7\times (18+7)}{3^6+{}^6C_1\cdot 3^5\cdot 2^1+{}^6C_2\, 3^4\cdot 2^2+{}^6C_3\cdot 3^3\cdot 2^3+{}^6C_4\, 3^2\cdot 2^4+{}^6C_5\, 3^1\cdot 2^5+2^6}$$

$$= \frac{(18+7)^3}{(3+2)^6} = \frac{(5)^6}{5^6} = 1$$

Q Deg of Poly

6 terms तक जाएगा

$$\left[x+\sqrt{x^3-1}\right]^5+\left[x-\sqrt{x^3-1}\right]^5 \text{ is ?}$$

$$(a+x)^n+(a-x)^n = (P+Q)+(P-Q)$$

$$= 2P = 2\left[T_1+T_3+T_5\right]$$

$$= 2\left[{}^5C_0(x)^5\left(\sqrt{x^3-1}\right)^0+{}^5C_2(x)^3\left(\sqrt{x^3-1}\right)^2+{}^5C_4\, x\left(\sqrt{x^3-1}\right)^4\right]$$

$$= 2\left[\underbrace{x^5\times 1}_{\substack{\text{deg}\\5}}+\underbrace{10\cdot x^3\cdot(x^3-1)}_{\substack{\text{deg}\\6}}+5\cdot x\cdot\underbrace{(x^3-1)^2}_{\substack{\text{deg}\\7}}\right]$$

Highest deg = 7 $\therefore$ deg of Poly = 7

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Q Deg of Poly

$$\left[\sqrt{2y^2+1}+\sqrt{2y^2-1}\right]^6+\left[\frac{2}{\sqrt{2y^2+1}+\sqrt{2y^2-1}}\right]^6 \text{ is}$$

Sochhhhhoooo:.... (o_o)

$$\left[\sqrt{2y^2+1}+\sqrt{2y^2-1}\right]^6+\left[\sqrt{2y^2+1}-\sqrt{2y^2-1}\right]^6.$$

$$(a+x)^n+(a-x)^n=(P+Q)+(P-Q)$$

$$2P=2[T_1+T_3+T_5+T_7]$$

$$2\left[{}^6C_0\left(\sqrt{2y^2+1}\right)^6\left(\sqrt{2y^2-1}\right)^0+{}^6C_2\left(\sqrt{2y^2+1}\right)^4\left(\sqrt{2y^2-1}\right)^2+{}^6C_4\left(\sqrt{2y^2+1}\right)^2\left(\sqrt{2y^2-1}\right)^4+{}^6C_6\left(\sqrt{2y^2+1}\right)^0\left(\sqrt{2y^2-1}\right)^6\right]$$

$$2\left[\underbrace{(2y^2+1)^3}_{\text{deg } 6}+15\cdot\underbrace{(2y^2+1)^2}_{4}\underbrace{(2y^2-1)}_{+\,2}_{\text{deg } 6}+15\cdot\underbrace{(2y^2+1)}_{2}\underbrace{(2y^2-1)^2}_{+\,4}_{\text{deg}=6}+\underbrace{(2y^2-1)^3}_{\text{deg}=6}\right]$$

$$(2y^2+1)^2(2y^2-1)$$

$$(4y^4+4y^2+1)(2y^2-1)$$

$$(8y^6-4y^4+8y^4-4y^2+2y^2-1)$$

$\therefore$ Deg of Poly $=6$

Nishant Jindal

Divisibility Problems.

Q S.T. $(4^2)^n = (16)^n$

$4^{2n} - 15n - 1$ is divisible by $\boxed{225}$; $n \geq 3$, $n \in N$.

15^2

$(1+15)^n - 15n - 1$

$\left(\cancel{{}^nC_0 \cdot 15^0} + \cancel{{}^nC_1 \cdot 15} + {}^nC_2 \cdot 15^2 + {}^nC_3 \cdot 15^3 + \dots + {}^nC_n \cdot 15^n\right) - \cancel{15n} - \cancel{1}$

$\underline{15^2\left({}^nC_2 + 15 \cdot {}^nC_3 + 15^2 \cdot {}^nC_4 + \dots\right)}$

$\cancel{225}$

Q $11^n - 10n - 1$ is divisible by $\underbrace{100}_{10^2}$?

$(1+10)^n - 10n - 1$

$\left(\cancel{{}^nC_0 \cdot 10^0} + \cancel{{}^nC_1 \cdot 10^1} + {}^nC_2 \cdot 10^2 + {}^nC_3 \cdot 10^3 - \dots {}^nC_n \cdot 10^n\right) - \cancel{10n} - \cancel{1}$

$\underline{10^2\left({}^nC_2 + {}^nC_3 \cdot 10 + {}^nC_4 \cdot 10^2 - \dots\right)}$ is div. by 10^2

$\Rightarrow$ div by 100.

Q $2^{3n+3} - 7n - 8$ is div. by $\underbrace{49}_{7^2}$?

$2^{3n} \cdot 2^3 - 7n - 8$

$8 \cdot (2^3)^n - 7n - 8$

$8(1+7)^n - 7n - 8 = 8\{\cancel{1} + \underbrace{{}^nC_1 \cdot 7} + \underbrace{{}^nC_2 \cdot 7^2 + {}^nC_3 \cdot 7^3 + \dots +}_{\text{Rem.}}\} - 7n - \cancel{8}$

$56n + 8 \times 7^2\left({}^nC_2 + {}^nC_3 \cdot 7 + {}^nC_4 \cdot 7^2 - \dots\right) - 7n$

is div. by 49 $\leftarrow$ $49n + 8 \times 49\left({}^nC_2 + {}^nC_3 \cdot 7 + {}^nC_4 \cdot 7^2 - \dots\right)$

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Rk.

A) $x^n - y^n$ is always divisible by "$x-y$"

$\frac{x^2-y^2}{x-y}$ ✓, $\frac{x^3-y^3}{x-y}$ ✓, $\frac{x^4-y^4}{x-y}$ ✓, $\frac{x^5-y^5}{x-y}$ ✓

$(x^2-1) = (x-1)(x+1)$

$(x^3-1) = (x-1)(x^2+x+1)$

$(x^4-1) = (x-1)(x^3+x^2+x+1)$

$(x^5-1) = (x-1)(x^4+x^3+x^2+x+1)$

$\vdots$

$(x^{17}-1) = (x-1)(x^{16}+x^{15}+x^{14} - \cdots + x^2+x+1)$

(B) x^n+y^n is divisible by $x+y$ When $n =$ odd.

$\frac{x^3+y^3}{x+y}$ ✓, $\frac{x^5+y^5}{x+y}$ ✓, $\frac{x^7+y^7}{x+y}$ ✓

Advance Understanding

$25^n - 20^n \div (25-20)$

$\div 5$

Nishant Jindal

Q $N = 25^n - 20^n - 8^n + 3^n$, $n \in N$

then S.T. N is divisible by 85

$N = (25^n - 20^n) - (8^n - 3^n) \div$ div. by 5

&

$N = (25^n - 8^n) - (20^n - 3^n) \rightarrow$ div by 17

div by 85

Today H.W.

1, 3, 4, 5, 6, 7, 8, 9

10 - 18, 19 - 22, 23 - 28

58, 59 61, 64, 65, 67, 69

85, 91, 92