Nishant Jindal

Q $\displaystyle\int_0^3 \sqrt{\frac{x^3}{3-x}}\,dx$

$x = 3\sin^2\theta$

$dx = 6\sin\theta\cos\theta\,d\theta$

| $x$ | $\theta$ |
|---|---|
| 0 | 0 |
| 3 | $\frac{\pi}{2}$ |

$3\sin^2\theta = 0 \Rightarrow \sin^2\theta = 0 \Rightarrow \sin\theta = 0$

$3\sin^2\theta = 3 \Rightarrow \sin\theta = 1 \Rightarrow \theta = \frac{\pi}{2}$

$\displaystyle\int_0^{\pi/2} \sqrt{\frac{27\sin^6\theta}{3-3\sin^2\theta}} \times 6\sin\theta\cos\theta\,d\theta$

$\displaystyle\int \sqrt{\frac{\cancel{27}^{9}\sin^6\theta}{\cancel{3}\cancel{\cos^2\theta}}} \times 6\sin\theta\cdot\cancel{\cos\theta}\,d\theta$

$\displaystyle 18\int_0^{\pi/2} \sin^4\theta\,d\theta$ Wallis.

$\cancel{18}^{9} \times \frac{3\cdot 1}{4\cdot 2} \times \frac{\pi}{\cancel{2}} = \frac{27\pi}{8}$

$\to \int e^x\,(f(x)+f'(x))\,dx$

Q $\displaystyle\int_0^{\pi/2} e^x\left\{\cos(\sin x)\cdot\cos^2\frac{x}{2} + \sin(\sin x)\cdot\sin^2\frac{x}{2}\right\}dx$

$\displaystyle \Rightarrow \int_0^{\pi/2} e^x\left\{\cos(\sin x)\left(\frac{1+\cos x}{2}\right) + \sin(\sin x)\cdot\left(\frac{1-\cos x}{2}\right)\right\}dx$

$\displaystyle \Rightarrow \frac{1}{2}\int_0^{\pi/2} e^x\Big\{\underbrace{\cos(\sin x)}_{f} + \underbrace{\sin(\sin x)}_{f} + \overbrace{\cos x\cdot\cos(\sin x)}^{f'} - \underbrace{\cos x\cdot\sin(\sin x)}_{f'}\Big\}$

$\displaystyle \frac{1}{2}\left\{e^x\left(\cos(\sin x) + \sin(\sin x)\right)\right\}\Big|_0^{\pi/2}$

$\displaystyle \frac{1}{2}\left\{e^{\pi/2}(\cos 1 + \sin 1) - 1\cdot(1+0)\right\}$

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Q Let $h(x) = f o g(x) + K$ if $\boxed{\dfrac{d}{dx} h(x) = -\dfrac{\sin x}{\cos^2(\cos x)}}$ Attacking.

then $\boxed{\text{find } j(0)}$ if $j(x) = \int_{g(x)}^{f(x)} \dfrac{f(t)}{g(t)} \cdot dt$ ; $f, g =$ Trigo fxn.

① $h(x) = -\int \dfrac{\sin x}{\cos^2(\cos x)} \cdot dx$ $\quad \cos x = t$

$-\sin x \cdot dx = dt$

$h(x) = \int \dfrac{dt}{\cos^2(t)} = \int \sec^2(t) \cdot dt = \tan(\cos x)$

② $\left.\begin{array}{l} h(x) = \tan(\cos x) + C \\ h(x) = f(g(x)) + C \end{array}\right\}$ $\begin{array}{l} f(x) = \tan x \\ g(x) = \cos x \end{array}$

③ $j(x) = \int_{\cos x}^{\tan x} \dfrac{\tan t}{\cos t} \cdot dt = \int_{\cos x}^{\tan x} \sec t \cdot \tan t \cdot dt = \sec t \Big|_{\cos x}^{\tan x}$

$j(x) = \sec(\tan x) - \sec(\cos x)$

④ $j(0) = \sec(\tan 0) - \sec(\cos 0)$

$= 1 - \sec 1$ Ans

Nishant Jindal

Q Let $\alpha, \beta$ be distinct +ve Roots of $\underline{\tan x = 2x}$ then.

Evaluate $\int_0^1 \sin\alpha x \cdot \sin\beta x \cdot dx$

① Prod → Sum/Diff

$$I = \frac{1}{2}\int_0^1 2\sin\alpha x \cdot \sin\beta x \cdot dx$$

$$= \frac{1}{2}\int_0^1 \cos(\alpha-\beta)x - \cos(\alpha+\beta)x \, dx$$

$$= \frac{1}{2}\left\{ \frac{\sin(\alpha-\beta)x}{(\alpha-\beta)} - \frac{\sin(\alpha+\beta)x}{(\alpha+\beta)} \right\}_0^1$$

$$I = \frac{1}{2}\left( \frac{\sin(\alpha-\beta)}{(\alpha-\beta)} - \frac{\sin(\alpha+\beta)}{(\alpha+\beta)} \right)$$

$$= \frac{1}{2}\left(\cancel{2\cos\alpha\cdot\cos\beta} - \cancel{2\cos\alpha\cdot\cos\beta}\right)$$

$$\underline{I = 0}$$

② $\alpha, \beta$ are Roots of

$$\tan x = 2x$$

Add

$$\tan\alpha = 2\alpha$$
$$\tan\beta = 2\beta$$

$$\tan\alpha + \tan\beta = 2(\alpha+\beta)$$

$$\frac{\sin(\alpha+\beta)}{\cos\alpha\cdot\cos\beta} = 2(\alpha+\beta)$$

$$\frac{\sin(\alpha+\beta)}{\alpha+\beta} = 2\cos\alpha\cos\beta$$

Sub

$$\tan\alpha = 2\alpha$$
$$-\tan\beta = -2\beta$$

$$\tan\alpha - \tan\beta = 2(\alpha-\beta)$$

$$\frac{\sin(\alpha-\beta)}{\cos\alpha\cdot\cos\beta} = 2(\alpha-\beta)$$

$$\frac{\sin(\alpha-\beta)}{\alpha-\beta} = 2\cos\alpha\cdot\cos\beta$$

③

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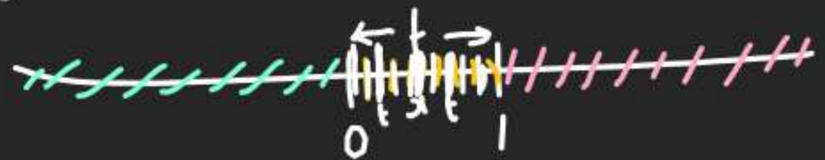
Q $\int_0^1 |x-t|\cdot \sin\pi t \, dt \,;\, x \in R$

Advance Level

① $t \in (0,1)$

Case 1 $x < 0 \rightarrow x = -\text{ve}$ Liya



$\rightarrow -t = -\text{ve}$

$x - t = -\text{ve}$

$|x-t| = -(x-t) = t - x$

$I = \int_0^1 (t-x)\cdot \sin\pi t \, dt = \int_0^1 t\cdot \sin\pi t - x\int_0^1 \sin\pi t \, dt = -\frac{2}{\pi}$

Case 2 When $x > 1 \rightarrow x = 2$

$-t = (-1, 0)$ $\quad -t = -0.5$

$x - t = +\text{ve}$ $\quad x - t = 1.5$ (+ve)

$|x-t| = (x-t)$

$I = \int_0^1 (x-t)\cdot \sin\pi t \, dt = x\int_0^1 \sin\pi t \, dt - \int_0^1 t\cdot \sin\pi t \, dt = \frac{2}{\pi^2}$

Case 3 When $x \in (0,1)$ & $t \in (0,1)$

$I = \int_0^x (x-t)\sin\pi t \, dt + \int_x^1 (t-x)\sin\pi t \, dt$

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# Properties of Definite Integral

**P1** Change of variable makes no difference in value of definite Integration.

$$\int_a^b f(x)\cdot dx = \int_a^b f(t)\cdot dt = \int_a^b f(z)\cdot dz$$

Ex:- $\int_0^{\pi/2} \sin x\cdot dx = -[\cos x]_0^{\pi/2} = -[\cos\frac{\pi}{2} - \cos 0] = +1$

$$\int_0^{\pi/2} \sin u\cdot du = -[\cos u]_0^{\pi/2} = -[\cos\frac{\pi}{2} - \cos 0] = +1$$

$$\int_0^{\pi/2} \sin x\cdot dx = \int_0^{\pi/2} \sin z\cdot dz$$

**P2** Interchange in Limit gives -ve value to Definite Integral.

$$\int_b^a f(x)\cdot dx = -\int_a^b f(x)\, dx$$

Q If $I_n = \int_0^\infty e^{-x}\cdot x^{n-1}\cdot dx$ then $\int_0^\infty e^{-\lambda x}\cdot x^{n-1}\cdot dx = ?$

| $t$ | $x$ |
|---|---|
| $0$ | $0$ |
| $\infty$ | $\infty$ |

$\lambda x = t$

$\lambda dx = dt$

$dx = \frac{dt}{\lambda}$

$$\int_0^\infty e^{-t}\,\frac{t^{n-1}}{\lambda^{n-1}}\cdot\frac{dt}{\lambda}$$

$$= \frac{1}{\lambda^n}\int_0^\infty e^{-t}\cdot t^{n-1}\cdot dt \xrightarrow{P_1} \frac{1}{\lambda^n}\boxed{\int_0^\infty e^{-x}\, x^{n-1}\, dx}$$

$$= \frac{I_n}{\lambda^n}$$

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Q If $\boxed{\frac{d F(x)}{dx} = \frac{e^{\sin x}}{x}}$ $(x>0)$ & $\int_1^4 \frac{3}{x} \cdot e^{\sin x^3} dx = F(K) - F(1)$

Attacking

Mains

then $K = ?$

① $F(x) = \int \frac{e^{\sin x}}{x} \cdot dx$

② $\int_1^4 \frac{3}{x} \cdot e^{\sin x^3} \cdot dx$

$= \int_1^4 \frac{3x^2 \cdot e^{\sin x^3} dx}{x^3}$

$\boxed{x^3 = t}$

$x^2 dx = \frac{dt}{3}$

| $x$ | $t$ |
|---|---|
| 1 | 1 |
| 4 | 64 |

$= \int_1^{64} \frac{e^{\sin t} dt}{t} \xrightarrow{P_1} \int_1^{64} \frac{e^{\sin x} dx}{x} = F(x)\Big|_1^{64} = F(64) - F(1)$

$\therefore K = 64$

Adv Q If $\frac{d(F(x))}{dx} = \frac{e^{\sin x}}{x}$ $(x>0)$

& $\int_1^4 \frac{2 e^{\sin x^2}}{x} dx =$

$F(K) - F(1)$

find K?

Ans = 16

See Qs (What's special???)

Main factor → $x, \frac{1}{x}$

$\frac{1}{x} = t$ maan lo. $\frac{1}{0}$ ∞

① Evaluate $\int_0^{\infty} f(x^n + x^{-n}) \cdot \frac{\log x \cdot dx}{x}$

$x^n + \frac{1}{x^n}$

② Evaluate $\int_0^{\infty} f(x^n + x^{-n}) \cdot \frac{\log x \cdot dx}{1+x^2}$

$x^n + \frac{1}{x^n}$

③ Evaluate $\int_{1/e}^{e} \frac{1}{x} \cdot \sin\left(x - \frac{1}{x}\right) \cdot dx$

Gsco

④ If $f\left(\frac{1}{x}\right) + x^2 \cdot f(x) = 0$ then find $\int_{\sin\theta}^{\operatorname{cosec}\theta} f(x) dx = ?$

(5) If $F(x) = f(x) + f\left(\frac{1}{x}\right)$ & $f(x) = \int_1^x \frac{\log t \, dt}{1+t}$

find $f(e)$?

Nishant Jindal

Q $I=\int_0^\infty f(x^n+x^{-n})\cdot\frac{\log x\cdot dx}{1+x^2}$

$\boxed{x=\frac{1}{t}}$

$dx=-\frac{1}{t^2}dt$

| $x$ | $t$ |
|---|---|
| 0 | $\infty$ |
| $\infty$ | 0 |

$$I=\int_\infty^0 f(t^{-n}+t^n)\frac{\left(\log\frac{1}{t}\right)\cdot -\frac{1}{t^2}dt}{1+\frac{1}{t^2}}$$

$$=+\int_\infty^0 f(t^n+t^{-n})\cdot\frac{(+\log t)\times\frac{1}{t^2}dt}{\frac{t^2+1}{t^2}}$$

P1 $\downarrow$ P2 $\downarrow$

$$=-\int_0^\infty f(x^n+x^{-n})\frac{\log x\cdot dx}{1+x^2}$$

$I=-I \Rightarrow 2I=0 \Rightarrow \boxed{I=0}$

Q. Evaluate $I_2\int_{1/e}^{e}\frac{1}{x}\cdot\sin(x-1/x)\cdot dx$.

$x=\frac{1}{t}$

$dx=-\frac{1}{t^2}dt$

| $x$ | $t$ |
|---|---|
| $e$ | $1/e$ |
| $\frac{1}{e}$ | $e$ |

$$I=\int_e^{1/e} x\cdot\sin\left(\frac{1}{t}-t\right)\times-\frac{1}{t^2}dt$$

$$I=\int_e^{1/e}\frac{1}{t}\sin\left(t-\frac{1}{t}\right)dt$$

P1 $\downarrow$ $\downarrow$ P2

$$=-\int_{1/e}^{e}\frac{1}{x}\cdot\sin\left(x-\frac{1}{x}\right)dx$$

$I=-I \Rightarrow 2I=0 \Rightarrow I=0$

Nishant Jindal

Q $f\left(\frac{1}{x}\right)+x^2\cdot f(x)=0$ then find $\int_{\sin\theta}^{\operatorname{cosec}\theta} f(x)\cdot dx = ?$

$$f(x) = -\frac{1}{x^2}\cdot f\left(\frac{1}{x}\right)$$

$$I = \int_{\sin\theta}^{\operatorname{cosec}\theta} -\frac{1}{x^2}\cdot f\left(\frac{1}{x}\right)\cdot dx$$

$x = \frac{1}{t}$

$dx = -\frac{1}{t^2}\,dt$

| $x$ | $t$ |
|---|---|
| $\sin\theta$ | $\operatorname{cosec}\theta$ |
| $\operatorname{cosec}\theta$ | $\sin\theta$ |

$$= \int_{\operatorname{cosec}\theta}^{\sin\theta} \cancel{t^2}\, f(t) + \frac{1}{\cancel{t^2}}\,dt$$

$P_1 \quad P_2$

$\operatorname{cosec}\theta \to 1$

$$I = -\int_{\sin\theta}^{\operatorname{cosec}\theta} f(x)\,dx = -I$$

$\Rightarrow 2I = 0$

$I = 0$

Nishant Jindal

Q If $F(x)=f(x)+f\left(\frac{1}{x}\right)$ & $f(x)=\int_1^x \frac{\log t\, dt}{1+t}$ then $F(e)=?$

Mains

$$F(x)=\int_1^x \frac{\log t\cdot dt}{1+t}+\int_1^{1/x}\frac{\log t\cdot dt}{1+t}$$

$\frac{F(x)}{2}$ चाहिए?

Kasshhh ye $\frac{1}{x}$ ki Jgh $x$ hota

Me to limit me $\frac{1}{x}$ ki Jgh $x$ Banaunga

Lekin Kese??

$\frac{1}{x}\to x$

Use $x=\frac{1}{t}$

$t=\frac{1}{z}$

$dt=-\frac{1}{z^2}\cdot dz$

| $t$ | $z$ |
|---|---|
| 1 | 1 |
| $1/x$ | $x$ |

$$+\int_1^x \frac{\log\frac{1}{z}\cdot x-\frac{1}{z^2}dz}{1+\frac{1}{z}}$$

$$+\int_1^x \frac{+\log z \times +\frac{1}{z^2}dz}{\frac{z+1}{z}}$$

$$\int_1^x \frac{\log t\cdot dt}{1+t}+\int_1^x \frac{\log z\, z\, dz}{z(z+1)} \quad \text{put}$$

$$I=\int_1^x \frac{\log t\, dt}{1+t}+\int_1^x \frac{\log t\, dt}{t(t+1)}=\int_1^x \frac{\log t}{t+1}\left(1+\frac{1}{t}\right)dt=\int_1^x \frac{\log t}{t}\,dt=-\frac{(\log t)^2}{2}\Big|_1^x$$

$$\therefore F(x)=-\frac{(\log x)^2}{2}$$

$$\Rightarrow F(e)=-\frac{(\log e)^2}{2}=-\frac{1}{2}$$

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$$\int_{-4}^{-5} e^{(x+5)^2} \cdot dx + 3\int_{1/3}^{2/3} e^{9\left(x-\frac{2}{3}\right)^2} dx$$