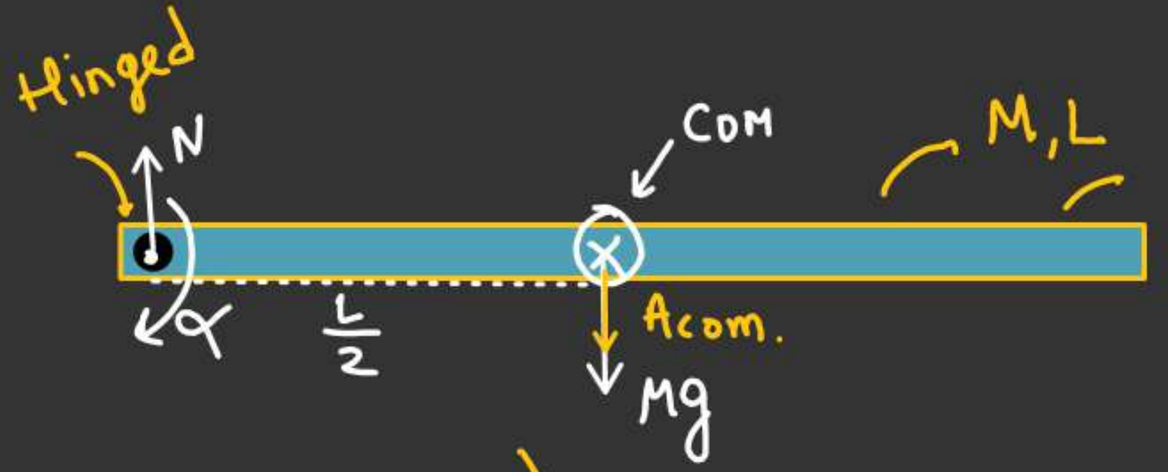
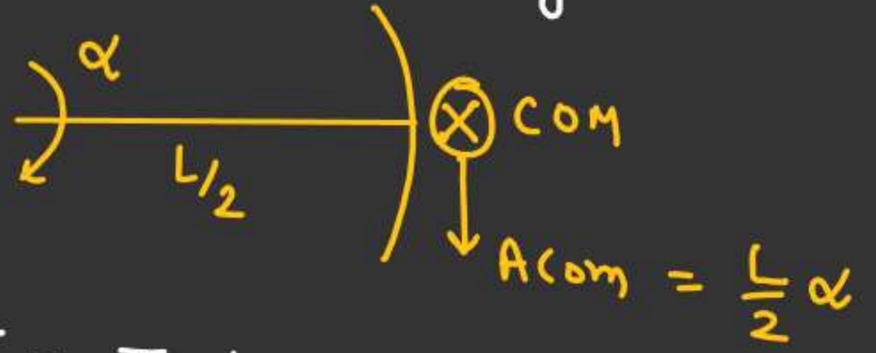
Nishant Jindal

Motion of Hinged Rod

Case-1 :- Rod released from horizontal position.

Hinged reaction just after rod is released





Sol^n: Just released means $\omega = 0$.

Newton's 2nd law in y-direction

$$Mg - N = M A_{com}$$

$$Mg - N = M\left(\frac{3g}{4}\right)$$

$$N = \left(Mg - \frac{3Mg}{4}\right) = \left(\frac{Mg}{4}\right) \text{ Newton}$$

$$\tau = I\alpha.$$

$$Mg\frac{L}{2} = \frac{ML^2}{3}\alpha$$

$$\alpha = \left(\frac{3g}{2L}\right)$$

$$A_{cm} = \frac{L}{2}\alpha = \frac{L}{2} \times \frac{3g}{2L} = \left(\frac{3g}{4}\right)$$

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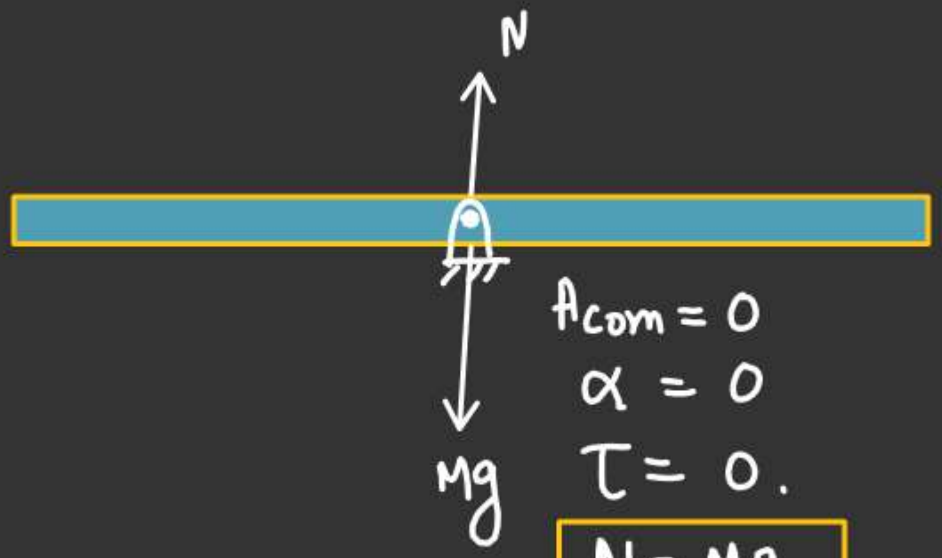

$a_{com} = 0$

$\alpha = 0$

$\tau = 0.$

$\boxed{N = Mg}$

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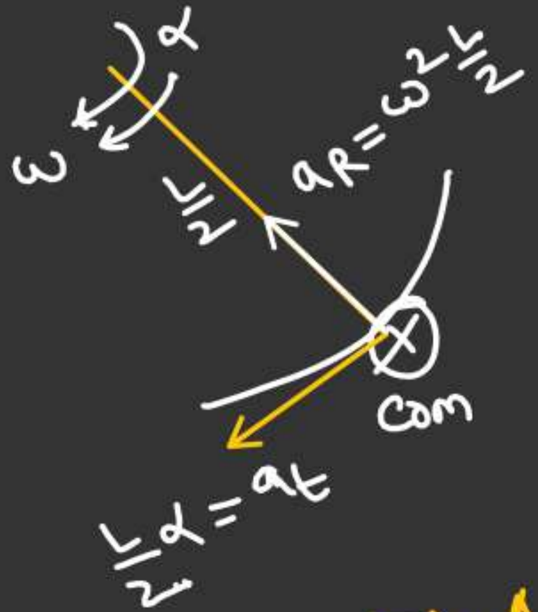
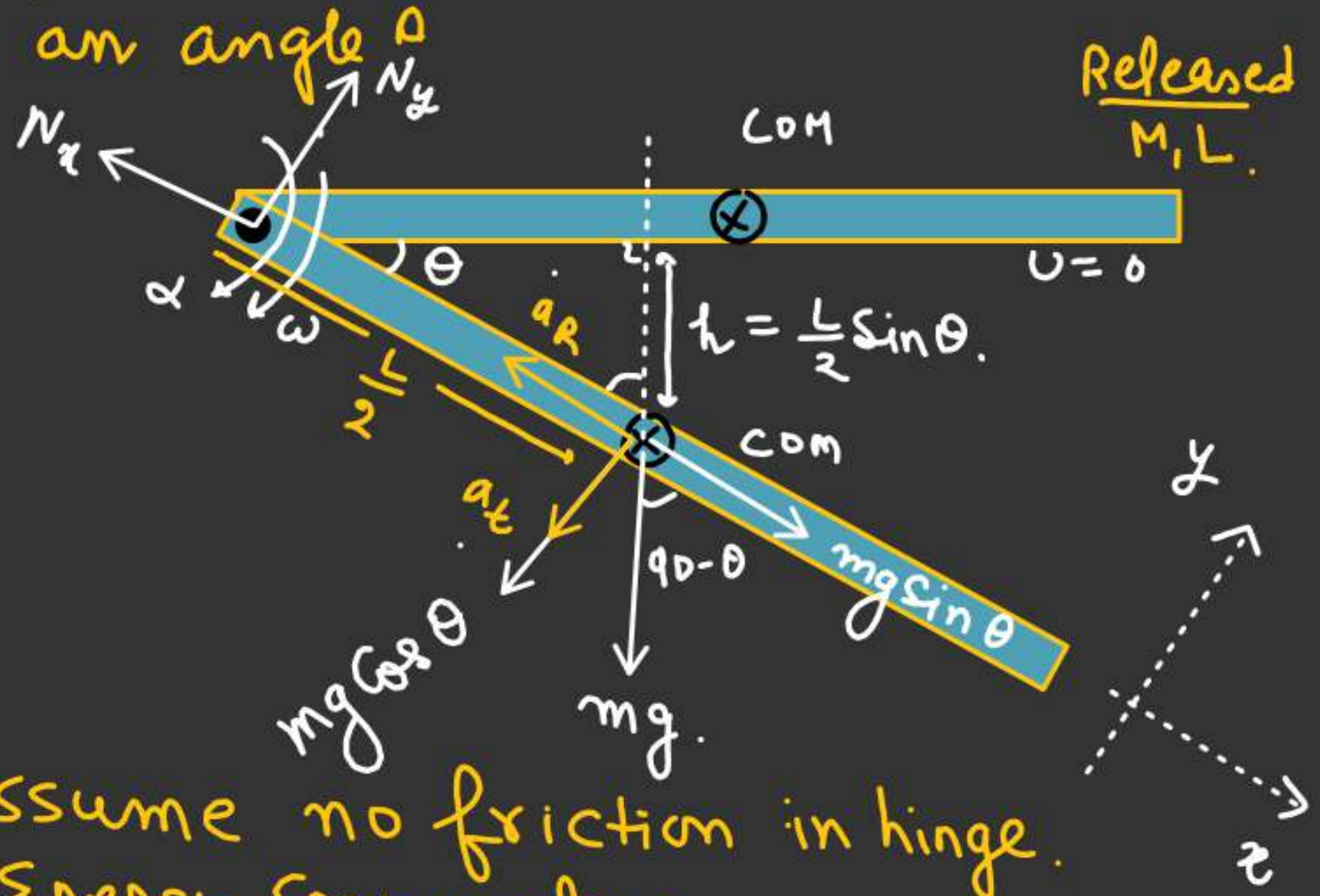
Case :- Rod is released from horizontal position.
Hinged reaction when rod make's an angle θ from horizontal.





$\Rightarrow$ τ about axis of rotation

$$\tau = I\alpha$$

$$mg\frac{L}{2}\cos\theta = \frac{ML^2}{3}\alpha$$

$$\boxed{\alpha = \frac{3g}{2L}\cos\theta}$$

$$a_t = \frac{L}{2}\alpha$$

$$\boxed{a_t = \frac{3g}{4}\cos\theta}$$

Along the Rod

$$N_x - mg\sin\theta = ma_R$$

$$N_x - mg\sin\theta = m\omega^2\frac{L}{2}$$

$$N_x = mg\sin\theta + m\frac{\cancel{L}}{2}\times\frac{3g}{\cancel{L}}\sin\theta$$

$$\boxed{N_x = \frac{5}{2}mg\sin\theta}$$

$\Rightarrow$ Assume no friction in hinge.
Energy Conservation.

$$U_i + K.E_i = U_f + K.E_f$$

$$0 + 0 = -mg\frac{L}{2}\sin\theta + \frac{1}{2}\left(\frac{ML^2}{3}\right)\omega^2$$

$$\omega = \sqrt{\frac{3g}{L}\sin\theta}$$

Case :- Rod is released from horizontal position.
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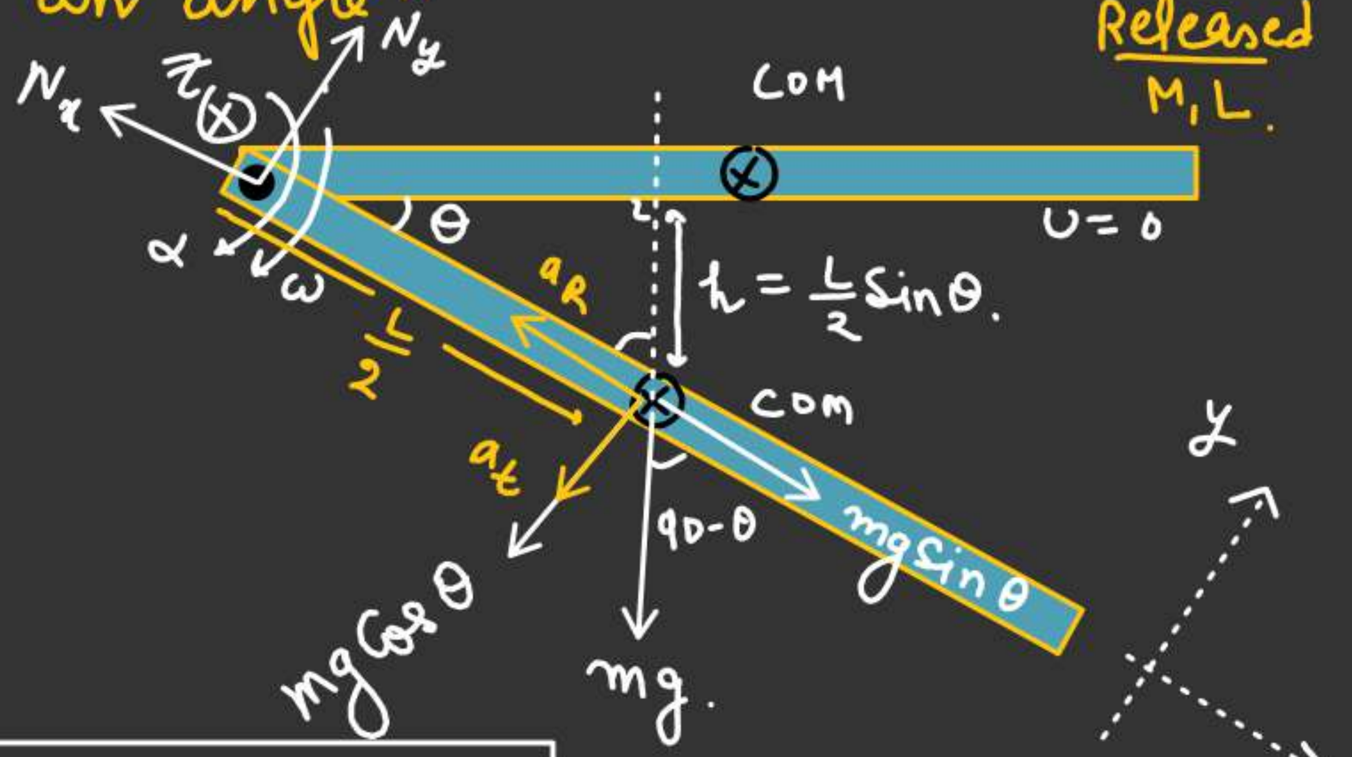


$N_y = ??$

$\Rightarrow$

$$mg\cos\theta - N_y = ma_t$$

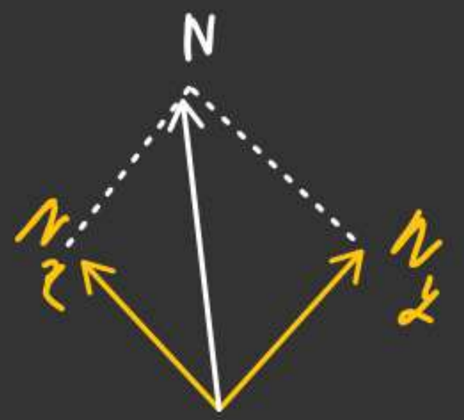
$$mg\cos\theta - N_y = m\left(\frac{3g}{4}\cos\theta\right)$$

$$N_y = mg\cos\theta - \frac{3mg}{4}\cos\theta$$

$$\boxed{N_y = \frac{mg}{4}\cos\theta}$$

$$\boxed{A_{com} = \sqrt{a_t^2 + a_R^2}}$$

$$N = \sqrt{N_x^2 + N_y^2}$$



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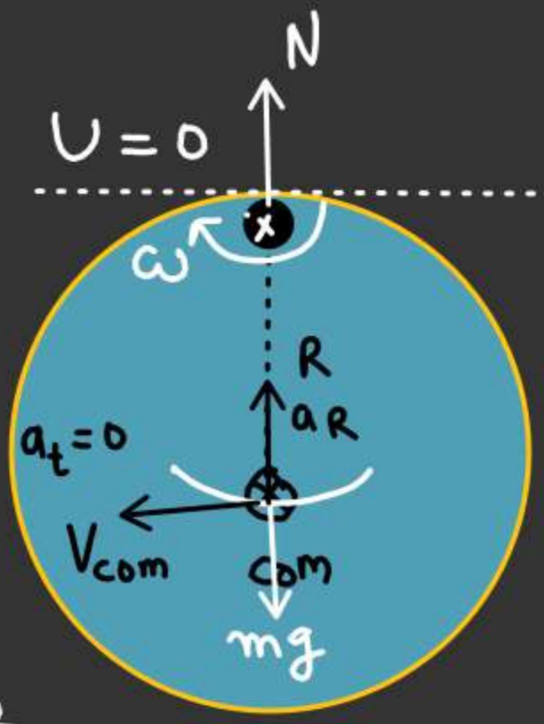
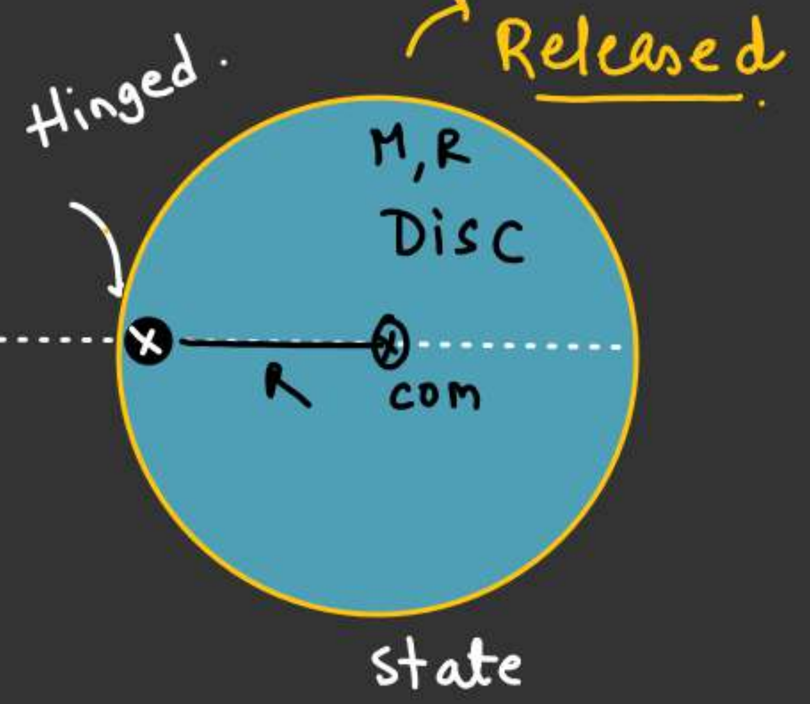
Disc Released from the position shown in fig. Find hinge reaction whe disc rotated by 90°

$$N - mg = m a_R \quad \text{—(1)}$$

$$a_R = \omega^2 R$$





$$U_i + K.E_i = U_f + K.E_f$$

$$0 + 0 = -mgR + \frac{1}{2}\left(\frac{3}{2}MR^2\right)\omega^2$$

$$\cancel{m}g\cancel{R} = \frac{3}{4}\cancel{MR^2}\omega^2$$

$$\sqrt{\frac{4g}{3R}} = \omega$$

From (1)

$$N = mg + m a_R$$

$$= mg + m\omega^2 R$$

$$N = mg + mR\,\frac{4g}{3R} \Rightarrow N = \frac{7Mg}{3} \quad \underline{\text{Ans}}.$$

$$K.E = \frac{1}{2}mV_{com}^2 + \frac{1}{2}I_{com}\omega^2$$

$$= \frac{1}{2}\left(I_{\text{axis of Rotation}}\right)\omega^2$$

$$I_{\text{axis of Rotation}} = I_{com} + MR^2$$

$$= \frac{MR^2}{2} + MR^2$$

$$= \frac{3}{2}MR^2$$

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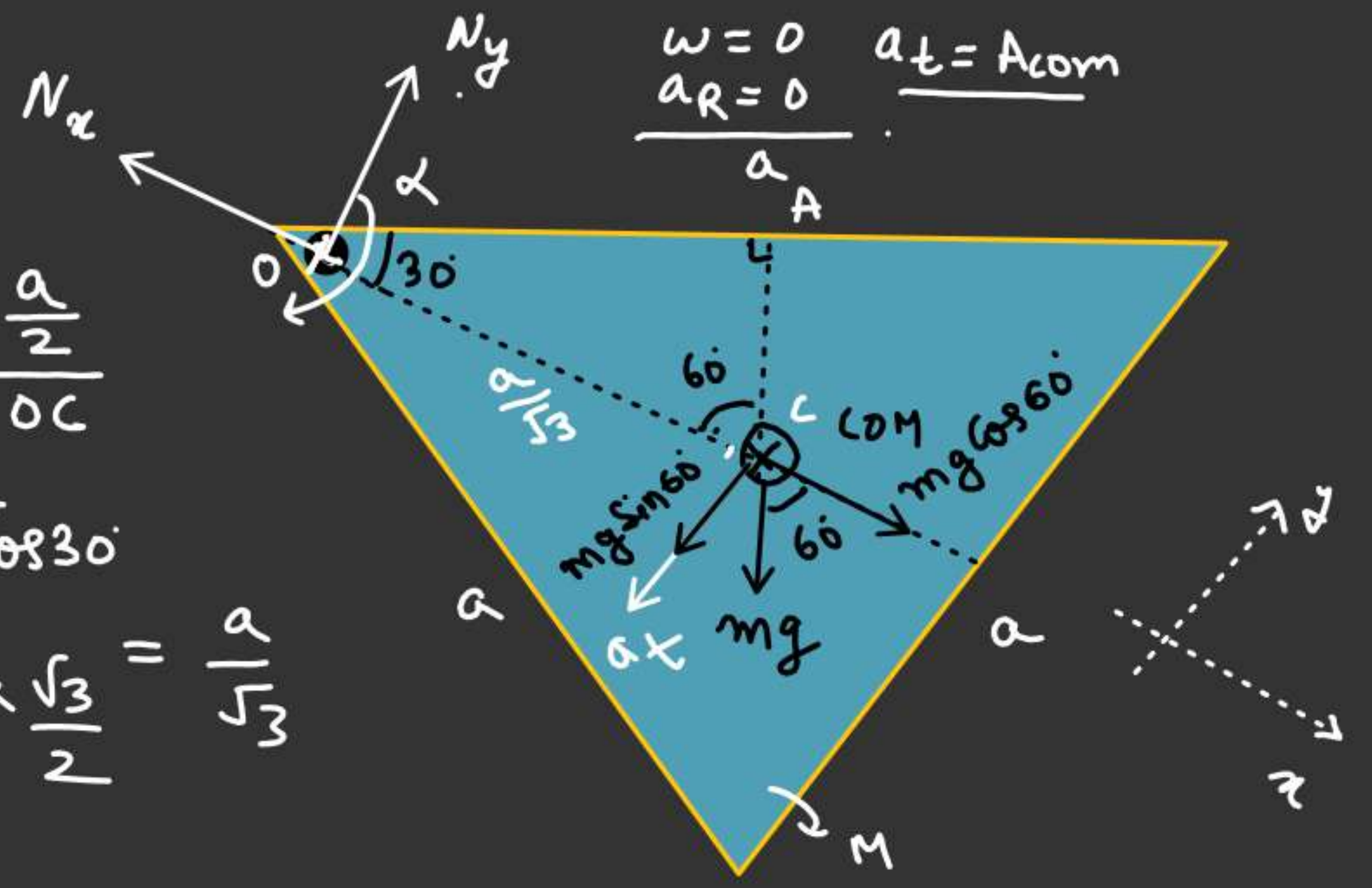
\# Find hinged reaction just after triangular lamina is released



$$\cos 30° = \frac{\frac{a}{2}}{OC}$$

$$OC = \frac{a}{2\cos 30}$$

$$OC = \frac{a}{2 \times \frac{\sqrt{3}}{2}} = \frac{a}{\sqrt{3}}$$

$$I_0 = I_{com} + m\left(\frac{a}{\sqrt{3}}\right)^2$$

$$I_0 = \frac{Ma^2}{12} + \frac{Ma^2}{3}$$

$$I_0 = \frac{Ma^2 + 4Ma^2}{12} = \frac{5Ma^2}{12}$$

$$\tau = I\alpha$$

$$(mg\sin 60)\frac{a}{\sqrt{3}} = \frac{5Ma^2}{12}\cdot\alpha$$

$$\frac{mga}{2} = \frac{5Ma^2}{12}\alpha$$

$$\alpha = \left(\frac{6g}{5a}\right)$$

$$a_t = \frac{a}{\sqrt{3}}\cdot\alpha$$

$$= \frac{a}{\sqrt{3}} \times \frac{6g}{5a} = \frac{6g}{5\sqrt{3}}$$

$$N_x = mg\cos 60 = \frac{mg}{2}$$

$$mg\sin 60 - N_y = ma_t$$

$$\frac{\sqrt{3}mg}{2} - N_y = \left(m\frac{6g}{5\sqrt{3}}\right)$$

$$N_y = \frac{\sqrt{3}mg}{2} - \frac{6mg}{5\sqrt{3}}$$

$$N_y = \frac{15mg - 12mg}{10\sqrt{3}} = \frac{\sqrt{3}mg}{10}$$

$$N = \sqrt{N_x^2 + N_y^2} = \sqrt{\frac{m^2g^2}{4} + \frac{3m^2g^2}{10}}$$