Nishant Jindal

JEE ADVANCED

1. The number of integral values of k for which the equation $7\cos x + 5\sin x = 2k + 1$ has a solution is **[JEE 2002 (Screening), 3]**

(A) 4 (B) 8 (C) 10 (D) 12

$$-1 \le \frac{2k+1}{\sqrt{7^2+5^2}} \le 1$$

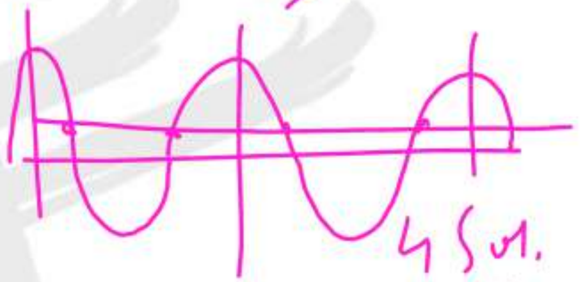
2. $\cos(\alpha - \beta) = 1$ and $\cos(\alpha + \beta) = 1/e$, where $\alpha, \beta \in [-\pi, \pi]$, numbers of pairs of α, β which satisfy both the equations is **[JEE 2005 (Screening)]**

(A) 0 (B) 1 (C) 2 (D) 4

$\alpha - \beta = 0$

$\alpha = \beta$

$\cos 2\alpha = \frac{1}{e}$



4 sol.

3. If $0 < \theta < 2\pi$, then the intervals of values of θ for which $2\sin^2\theta - 5\sin\theta + 2 > 0$, is **[JEE-2006, 3]**

(A) $\left(0, \frac{\pi}{6}\right) \cup \left(\frac{5\pi}{6}, 2\pi\right)$ (B) $\left(\frac{\pi}{8}, \frac{5\pi}{6}\right)$ (C) $\left(0, \frac{\pi}{8}\right) \cup \left(\frac{\pi}{6}, \frac{5\pi}{6}\right)$ (D) $\left(\frac{41\pi}{48}, \pi\right)$

$\cos(\alpha+\beta) = \frac{1}{e} = \frac{1}{2.7}$

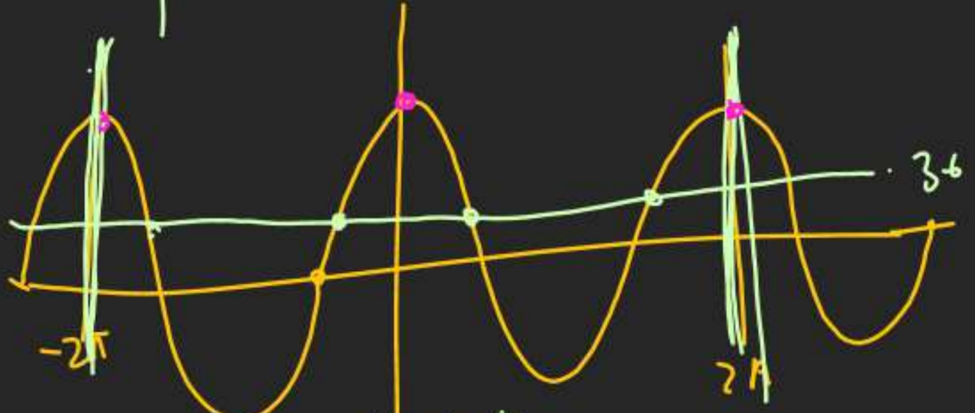
$\cos(\alpha+\beta) = .36$

$\alpha, \beta \in [-\pi, \pi]$

$-\pi \le \alpha \le \pi$

$-\pi \le \beta \le \pi$

$-2\pi \le \alpha + \beta \le 2\pi$



$-\pi \le \alpha \le \pi$

$-\pi \le -\beta \le \pi$

$$\frac{1}{\sin\frac{\pi}{n}}-\frac{1}{\sin\frac{3\pi}{n}}=\frac{1}{\sin\frac{2\pi}{n}}$$

$$\frac{\sin\frac{3\pi}{n}-\sin\frac{\pi}{n}}{\sin\frac{\pi}{n}\cdot\sin\frac{3\pi}{n}}=\frac{1}{\sin\frac{2\pi}{n}}$$

$$\frac{2\cos\left(\frac{2\pi}{n}\right)\cdot\cancel{\sin\left(\frac{\pi}{n}\right)}}{\cancel{\sin\frac{\pi}{n}}\cdot\sin\frac{3\pi}{n}}=\frac{1}{\sin\frac{2\pi}{n}}$$

$$2\sin\left(\frac{2\pi}{n}\right)\cdot\cos\frac{2\pi}{n}=\sin\frac{3\pi}{n}$$

$$\sin\frac{4\pi}{n}=\sin\frac{3\pi}{n}$$

$\sin\theta=\sin(\pi-\theta)$ | $\frac{4\pi}{n}=\pi-\frac{3\pi}{n}$

$$\frac{7\cancel{\pi}}{n}=\cancel{\pi}$$

$$\boxed{n=7}$$

4. The number of solutions of the pair of equations **[JEE 2007, 3]**

$2\sin^2\theta-\cos 2\theta=0$

$2\cos^2\theta-3\sin\theta=0$

in the interval $[0,2\pi]$ is

(A) zero (B) one (C) two (D) four

5. The number of values of θ in the interval $\left(\frac{-\pi}{2},\frac{\pi}{2}\right)$ such that $\theta\neq\frac{n\pi}{5}$ for $n=0,\pm1,\pm2$ and $\tan\theta=\cot 5\theta$ as well as $\sin 2\theta=\cos 4\theta$, is **[JEE 2010, 3]**

6. The positive integer value of $n>3$ satisfying the equation **[JEE 2011, 4]**

$\frac{1}{\sin\left(\frac{\pi}{n}\right)}=\frac{1}{\sin\left(\frac{2\pi}{n}\right)}+\frac{1}{\sin\left(\frac{3\pi}{n}\right)}$ is

$n=7$

1) $\tan\theta=\cot 5\theta$

$\tan\theta=\tan\left(\frac{\pi}{2}-5\theta\right)$

$\theta=n\pi+\left(\frac{\pi}{2}-5\theta\right)$

$6\theta=n\pi+\frac{\pi}{2}$

$\theta=\frac{n\pi}{6}+\frac{\pi}{12}$

2) $\sin 2\theta=\cos 4\theta$

$\cos\left(\frac{\pi}{2}-2\theta\right)=\cos 4\theta$

$4\theta=2n\pi\pm\left(\frac{\pi}{2}-2\theta\right)$

$4\theta=2n\pi+\frac{\pi}{2}-2\theta$ | $4\theta=2n\pi-\frac{\pi}{2}+2\theta$

$6\theta=2n\pi+\frac{\pi}{2}$ | $2\theta=2n\pi-\frac{\pi}{2}$

$\theta=\frac{n\pi}{3}+\frac{\pi}{12}$ | $\theta=n\pi-\frac{\pi}{4}$

Nishant Jindal

Q8 $\sin x + 2\sin 2x - \sin 3x = 3$

$\sin x + 4\sin x \cos x - 3\sin x + 4\sin^3 x =$

$\sin x\{4\cos x - 2 + 4\sin^2 x\} = 3$

$4\cos x - 2 + 4(1-\cos^2 x) = 3\operatorname{cosec} x$

$2 - 4\cos^2 x + 4\cos x = 3\operatorname{cosec} x$

$3 - (4\cos^2 x - 4\cos x + 1) = 3\operatorname{cosec} x$

$3 - (2\cos x - 1)^2 = 3\operatorname{cosec} x$

$\cos x = \frac{1}{2}$ RHS > 3

3-0 Min, 3-1, 3-1, 3-9

LHS ≤ 3

$\boxed{3}$ Max, 2, 2, -6

No sol

7. Let $\theta, \varphi \in [0,2\pi]$ be such that

$2\cos\theta(1-\sin\varphi) = \sin^2\theta\left(\tan\frac{\theta}{2} + \cot\frac{\theta}{2}\right)\cos\varphi - 1$, $\tan(2\pi - \theta) > 0$ and $-1 < \sin\theta < -\frac{\sqrt{3}}{2}$. Then φ cannot satisfy -

(A) $0 < \varphi < \frac{\pi}{2}$ (B) $\frac{\pi}{2} < \varphi < \frac{4\pi}{3}$ (C) $\frac{4\pi}{3} < \varphi < \frac{3\pi}{2}$ (D) $\frac{3\pi}{2} < \varphi < 2\pi$.

$\theta \in \left(\frac{3\pi}{2}, \frac{5\pi}{3}\right) = (270, 300)$

$\phi \in \left(\frac{4\pi}{3}, \frac{3\pi}{2}\right) = (240°, 270°)$

$(510°, 570°)$

8. For $x \in (0,\pi)$, the equation $\sin x + 2\sin 2x - \sin 3x = 3$ has **[JEE(Advanced)-2014, 3(-1)]**

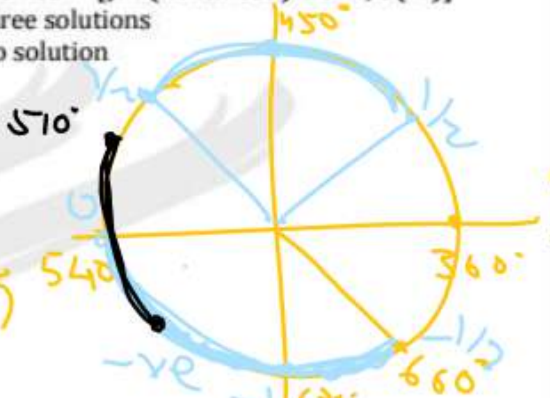
(A) infinitely many solutions (B) three solutions

(C) one solution (D) no solution

$\theta \in \left(\frac{3\pi}{2}, \frac{5\pi}{3}\right)$

$\phi \in \left(\frac{3\pi}{2}, 2\pi\right)$

$\theta + \phi \in (540°, 660°)$



9. The number of distinct solutions of equation $\frac{5}{4}\cos^2 2x + \cos^4 x + \sin^4 x + \cos^6 x + \sin^6 x = 2$ in the interval $[0,2\pi]$ is **[JEE 2015, 4M,-0M]**

10. Let a, b, c be three non-zero real numbers such that the equation

$\sqrt{3}a\cos x + 2b\sin x = c, x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ has two distinct real roots α and β with $\alpha + \beta = \frac{\pi}{3}$. Then, the value $\frac{b}{a}$ is.......... **[JEE (Advanced)-2018, 3(0), P- 1]**

(7)

$(1-\sin\phi) = \sin^2\theta\left(\tan\frac{\theta}{2} + \cot\frac{\theta}{2}\right)\cos\phi - 1$

$270°, 300°$

$\theta \in \left(\frac{3\pi}{2}, \frac{5\pi}{3}\right)$

$= \sin^2\theta\left(\frac{\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}} + \frac{\cos\frac{\theta}{2}}{\sin\frac{\theta}{2}}\right)\cos\phi - 1$

$\phi \in \left(0, \frac{\pi}{2}\right)$

$\left(\frac{3\pi}{2}, 390°\right)$

$= 2\sin^2\theta\left(\frac{1}{2\sin\frac{\theta}{2}\cdot\cos\frac{\theta}{2}}\right)\cdot\cos\phi - 1$

$= 2\sin^2\theta \cdot \times \frac{1}{\sin\theta}\cos\phi - 1$

$-2\cos\theta\sin\phi = 2\sin\theta\cdot\cos\phi - 1$

$\theta + 1 = 2(\sin\theta\cos\phi + \cos\theta\sin\phi)$

$\theta + 1 = \boxed{2\sin(\theta+\phi)}$

$0 < \cos\theta < \frac{1}{2}$

$0 < 2\cos\theta < 1$

$1 < 1 + 2\cos\theta < 2$

$1 < 2\sin(\theta+\phi) < 2$

$\frac{1}{2} < \sin(\theta+\phi) < 1$

~~$\frac{\pi}{6} < \theta+\phi < \frac{\pi}{2}$~~

~~$2\pi + \frac{\pi}{6} < \theta+\phi < 2\pi + \frac{\pi}{2}$~~

Nishant Jindal

(9) $\frac{5}{4}\cos^2(2x)+(\cos^4x+\sin^4x)+(\cos^6x+\sin^6x)=2$

$\frac{5}{4}\cos^2(2x)+(1-2\sin^2x\cos^2x)+(1-3\sin^2x\cos^2x)=2$

$\frac{5}{4}\cos^2(2x)-\frac{5}{4}4\sin^2x\cos^2x=0$

$\frac{5}{4}\left\{\cos^2(2x)-\sin^2(2x)\right\}=0$

$\sin^2(2x)=\cos^2(2x)$

$\tan^2(2x)=1=\tan^2\frac{\pi}{4}$

$2x=n\pi\pm\frac{\pi}{4}$

$x=\frac{n\pi}{2}\pm\frac{\pi}{8}$

$\frac{\pi}{8},-\frac{\pi}{8},\frac{\pi}{2}+\frac{\pi}{8},\frac{\pi}{2}-\frac{\pi}{8},\pi\pm\frac{\pi}{8},\frac{3\pi}{2}\pm\frac{\pi}{8},2\pi-\frac{\pi}{8}$

8 Sol

$\tan^2\theta=\tan^2\alpha$

$\theta=n\pi\pm\alpha$

7. Let $\theta, \varphi \in [0,2\pi]$ be such that

$2\cos\theta(1-\sin\varphi)=\sin^2\theta\left(\tan\frac{\theta}{2}+\cot\frac{\theta}{2}\right)\cos\varphi-1, \tan(2\pi-\theta)>0$ and $-1<\sin\theta<-\frac{\sqrt{3}}{2}$. Then φ cannot satisfy -

(A) $0<\varphi<\frac{\pi}{2}$ (B) $\frac{\pi}{2}<\varphi<\frac{4\pi}{3}$ (C) $\frac{4\pi}{3}<\varphi<\frac{3\pi}{2}$ (D) $\frac{3\pi}{2}<\varphi<2\pi$.

8. For $x \in (0,\pi)$, the equation $\sin x+2\sin 2x-\sin 3x=3$ has [JEE(Advanced)-2014, 3(-1)]

(A) infinitely many solutions (B) three solutions

(C) one solution (D) no solution

9. The number of distinct solutions of equation $\frac{5}{4}\cos^2 2x+\cos^4 x+\sin^4 x+\cos^6 x+\sin^6 x=2$ in the interval $[0,2\pi]$ is [JEE 2015, 4M,-0M]

10. Let a, b, c be three non-zero real numbers such that the equation

$\sqrt{3}a\cos x+2b\sin x=c, x\in\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$ has two distinct real roots α and β with $\alpha+\beta=\frac{\pi}{3}$. Then, the value $\frac{b}{a}$ is.......... [JEE (Advanced)-2018, 3(0), P- 1]

$\frac{1}{2}$

2018

(10)

$\sqrt{3}a\cos x+2b\sin x=c$

$\sqrt{3}\cos x+\frac{2b}{a}\sin x=\frac{c}{a}$ α, β

$\sqrt{3}\cos\alpha+\frac{2b}{a}\sin\alpha=\frac{c}{a}$

$\sqrt{3}\cos\beta+\frac{2b}{a}\sin\beta=\frac{c}{a}$

$\sqrt{3}(\cos\alpha-\cos\beta)+\frac{2b}{a}(\sin\alpha-\sin\beta)=0$

$-\sqrt{3}\left(2\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)\right)+\frac{2b}{a}\left(2\cos\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)\right)=0$

30° 30°

$-\sqrt{3}\sin\left(\frac{\alpha-\beta}{2}\right)+\frac{2\sqrt{3}b}{a}\sin\left(\frac{\alpha-\beta}{2}\right)=0$

$\sin\left(\frac{\alpha-\beta}{2}\right)\left\{\frac{2\sqrt{3}b}{a}-\sqrt{3}\right\}=0$

$\frac{2\sqrt{3}b}{a}=\sqrt{3}$ $\boxed{\frac{b}{a}=\frac{1}{2}}$

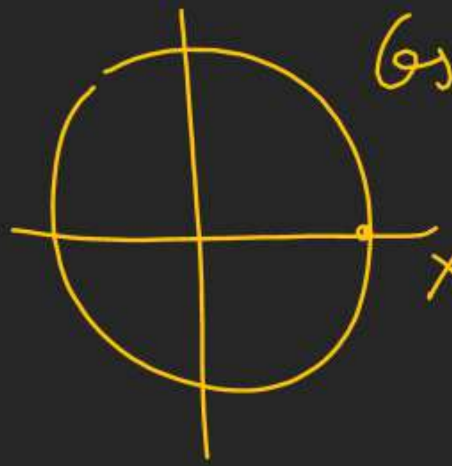
① $f(x) = \sin(\pi \cos x)$

$X = \{ \sin(\pi \cos x) = 0 \}$

$\sin(\pi \cos x) = \sin(n\pi)$

$\pi \cos x = n\pi$

$[-1, 1] \leftarrow \cos x = n = \{\cancel{-1}, \cancel{0}, 1\}$



$\cos x = 1$

$x = 2n\pi$

$X = \{\cancel{0}, 2\pi, 4\pi, 6\pi \ldots\}$

11. Answer the following appropriately matching the list based on the information given in the paragraph.
Let $f(x) = \sin(\pi \cos x)$ and $g(x) = \cos(2\pi \sin x)$ be two functions defined for $x > 0$. Define the following sets whose elements are written in the increasing order.

$X = \{x: f(x) = 0\}$, $Y = \{x: f'(x) = 0\}$
$Z = \{x: g(x) = 0\}$, $W = \{x: g'(x) = 0\}$ **[JEE (Advanced)-2019]**

List I contains the sets X, Y, Z and W. List II contains some information regarding these sets.

List I	List II
(I) X	(P) $\supseteq \left\{\frac{\pi}{2}, \frac{3\pi}{2}, 4\pi, 7\pi\right\}$
(II) Y	(Q) an arithemetic progression
(III) Z	(R) NOT an arithemetic progression
(IV) W	(S) $\supseteq \left\{\frac{\pi}{6}, \frac{7\pi}{6}, \frac{13\pi}{6}\right\}$
	(T) $\supseteq \left\{\frac{\pi}{3}, \frac{2\pi}{3}, \pi\right\}$
	(U) $\supseteq \left\{\frac{\pi}{6}, \frac{3\pi}{4}\right\}$ ×

Which of the following is the only CORRECT combination ?
(A) (I), (Q), (U) ×
(B) (II), (Q), (T)
(C) (I), (P), (R)
(D) (II), (R), (S)

Nishant Jindal

① $\cos x + \sin x = 1$

A. A. $= \sqrt{2}$

$\cos\left(x - \frac{\pi}{4}\right) = \frac{1}{\sqrt{2}} = \cos\frac{\pi}{4}$

$x - \frac{\pi}{4} = 2n\pi \pm \frac{\pi}{4}$

$x = 2n\pi + \frac{\pi}{4} + \frac{\pi}{4},\ 2n\pi - \cancel{\frac{\pi}{4}} + \cancel{\frac{\pi}{4}}$

$x = 2n\pi + \frac{\pi}{2},\ 2n\pi$

$\frac{\pi}{2},\ 0$

12. Consider the following lists: $[-120, 120]$

List - I	List - II
(I) $\left\{x \in \left[-\frac{2\pi}{3}, \frac{2\pi}{3}\right] : \cos x + \sin x = 1\right\}$	(P) has two elements
(II) $\left\{x \in \left[-\frac{5\pi}{18}, \frac{5\pi}{18}\right] : \sqrt{3}\tan 3x = 1\right\}$	(Q) has three elements
(III) $\left\{x \in \left[-\frac{6\pi}{5}, \frac{6\pi}{5}\right] : 2\cos(2x) = \sqrt{3}\right\}$	(R) has four elements
(IV) $\left\{x \in \left[-\frac{7\pi}{4}, \frac{7\pi}{4}\right] : \sin x - \cos x = 1\right\}$	(S) has five elements
	(T) has six elements

The correct option is: **[JEE (Advanced)-2022]**

(A) (I) → (P); (II) → (S); (III) → (P); (IV) → (S)

(B) (I) → (P); (II) → (P); (III) → (T); (IV) → (R)

(C) (I) → (Q); (II) → (P); (III) → (T); (IV) → (S)

(D) (I) → (Q); (II) → (S); (III) → (P); (IV) → (R)

(2) $\sqrt{3}\tan 3x = 1$

$\tan 3x = \frac{1}{\sqrt{3}} = \tan\frac{\pi}{6}$

$3x = n\pi + \frac{\pi}{6}$ $\quad x \in \left[-\frac{5\pi}{18}, \frac{5\pi}{18}\right]$ ✓

$x = \frac{n\pi}{3} + \frac{\pi}{18}$

$x = \frac{\pi}{18},\ \frac{\pi}{3} + \frac{\pi}{18},\ -\frac{\pi}{3} + \frac{\pi}{18}$

$\frac{7\pi}{18}$ ✗ $\quad -\frac{5\pi}{18}$ ✓