

Q Eqn of Ellipse

In whose Major Axis = 8 & ecc =  $\frac{1}{2}$

$$\begin{array}{l|l} 2a=8 & e=\frac{1}{2} \\ a=4 & \end{array}$$

$$b^2=a^2(1-e^2)$$

$$= 16 \left(1 - \frac{1}{4}\right)$$

$$b^2=12$$

$$\frac{x^2}{16} + \frac{y^2}{12} = 1$$

$$\frac{x^2}{4^2} + \frac{y^2}{a^2} = 1$$

Q Ecc of Ellipse is  $\frac{5}{8}$   
8 distance betn focii = 16  
find L.R.

$$\begin{array}{l|l} e = \frac{5}{8} & 2ae = 16 \\ & 2a \times \frac{5}{8} = 16 \end{array}$$

$$b^2 = a^2(1-e^2) = 16 \left(1 - \frac{25}{64}\right) = 39$$

Q An ellipse is such that  
its L.R. = Sum of length  
of its P.T. Principle Axis  
Then Ellipse becomes  
Circle.

2 Pr. Axis  $\rightarrow$  Maj./Min

Sum of Pr. Axis =  $a+b$

$$\frac{2b^2}{a} = a+b$$

$$2b^2 = a^2 + ab \quad \div b^2$$

$$\left(\frac{a}{b}\right)^2 + \left(\frac{a}{b}\right) - 2 = 0$$

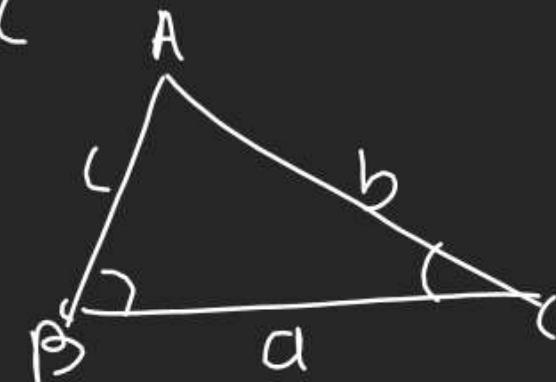
$$\left(\frac{a}{b}\right)^2 + 2\left(\frac{a}{b}\right) - 3 = 0$$

$$\left(\frac{a}{b}\right)^2 + 2\left(\frac{a}{b}\right) - 1 = 0 \Rightarrow \left(\frac{a}{b}\right)^2 = 1 \Rightarrow a=b$$

Q In  $\triangle ABC$  with fixed base BC

Verte A moves such that

$$\csc B + \csc C = 4 \sin^2 \left( \frac{A}{2} \right)$$



If  $a, b, c$  denotes length of sides  $\angle A + \angle B + \angle C = \pi$

of  $\triangle ABC$  then

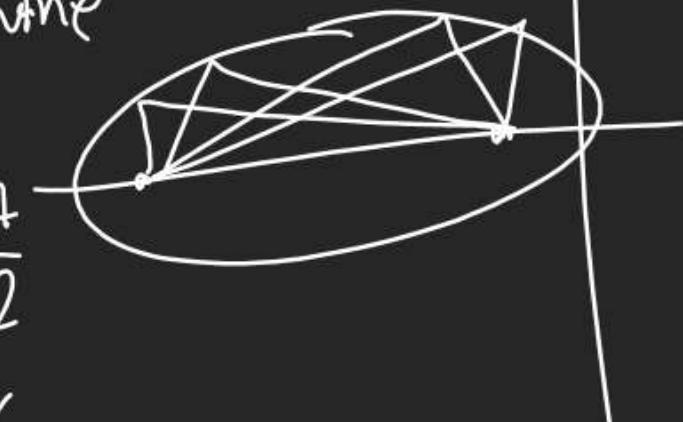
$$\textcircled{1} \quad b+c = 4a \quad \Rightarrow \quad b+c = 2a$$

(3) Locus of A is Ellipse (2) A in pair of St. line

$$\csc B + \csc C = 1 \sin^2 \frac{A}{2}$$

$$\csc \left( \frac{B+C}{2} \right) \cdot \csc \left( \frac{B-C}{2} \right) = 4 \sin^2 \frac{A}{2}$$

$$\sin \frac{A}{2} \cdot \csc \left( \frac{B-C}{2} \right) = 2 \sin^2 \frac{A}{2}$$



$$\csc \left( \frac{B-C}{2} \right) = 2 \sin \left( \frac{A}{2} \right)$$

$$\csc \frac{A}{2} \cdot \csc \left( \frac{B-C}{2} \right) = 2 \sin \frac{A}{2} \csc \frac{A}{2}$$

$$2 \left( \pi - (B+C) \right) \csc \left( \frac{B-C}{2} \right) = 2 \sin A$$

$$2 \sin \left( \frac{B+C}{2} \right) \cdot \csc \left( \frac{B-C}{2} \right) = 2 \sin A$$

$$\sin B + \sin C = 2 \sin A$$

$$\frac{\sin B + \sin C}{\sin A} = 2$$

$$\frac{b+c}{a} = 2 \Rightarrow \underbrace{b+c}_{2a}$$

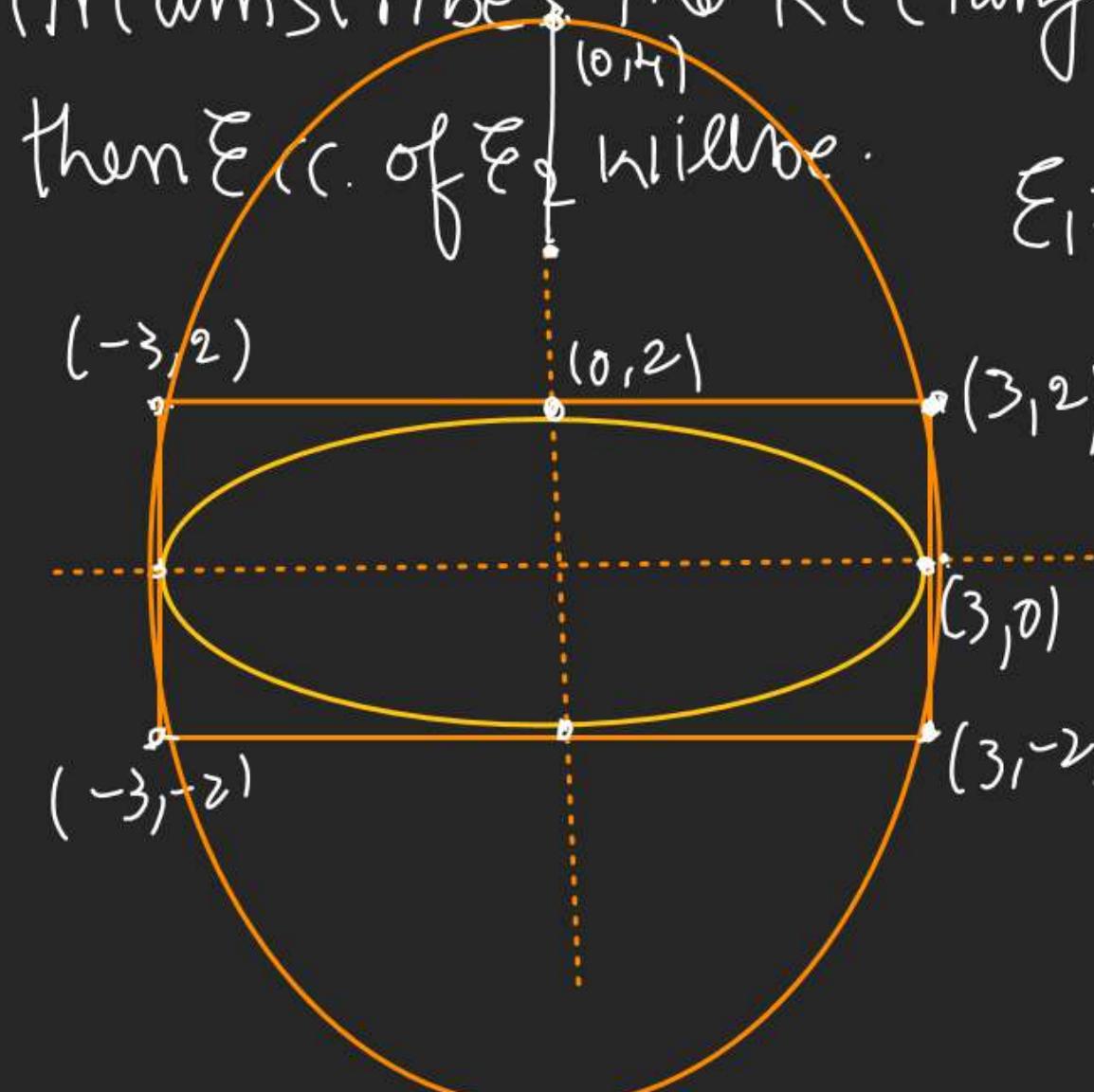
$\emptyset \mathcal{E}_1: \frac{x^2}{9} + \frac{y^2}{4} = 1$  is Inscribed in a Rect.

Whose sides are  $\parallel$  to  $(0\text{-axis})$ .

Another Ellipse  $\mathcal{E}_2$  P.T.  $(0, 4)$

(circumscribes the Rectangle

then Ecc. of  $\mathcal{E}_2$  will be



New Ellipse

$$\mathcal{E}_2: \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad P.T. (3, 2)$$

$$\frac{9}{a^2} + \frac{4}{b^2} = 1$$

$$\frac{30y}{a^2} = \frac{y^2}{4}$$

$$a^2 = 12$$

Normally

$$b^2 = a^2(1-e^2)$$

$$12 = 16(1-e^2)$$

$$\Rightarrow 1-e^2 = \frac{3}{4} \Rightarrow e^2 = \frac{1}{4}$$

$$e = \frac{1}{2}$$

Q P(x, y), F<sub>1</sub> = (3, 0)

F<sub>2</sub> = (-3, 0)

|6x<sup>2</sup> + 25y<sup>2</sup> = 400 then PF<sub>1</sub> + PF<sub>2</sub> = 2

$$\frac{x^2}{25} + \frac{y^2}{16} = 1$$

$$a=5, b=4$$

$$\frac{2a}{b} = 10$$

$$\begin{array}{c} 2x^2 + 24y^2 + 28 = 0 \\ -2x - y + 3 = 0 \\ \hline 25y = -25 \end{array}$$

$$\left. \begin{array}{l} y = -1 \\ x = -2 \end{array} \right\} \begin{array}{l} (-2, -1) \\ \text{Centre} \end{array}$$

$$\frac{x^2}{(6-a)} + \frac{y^2}{a-2} = 1$$

$$\begin{array}{l|l} 6-a > 0 & a-2 > 0 \\ a < 6 & a > 2 \end{array}$$

$$a \in (2, 6) - \{4\}$$

(a ≠ b)

Q Centre of  $x^2 + 24xy - 6y^2 + 28x + 36y + 10 = 0$  in  
Non-Homogeneous then Centre

$$\left. \begin{array}{l} \frac{\partial f}{\partial x} = 0 \\ \frac{\partial f}{\partial y} = 0 \end{array} \right\} \begin{array}{l} \text{Eqn 1} \\ \text{Eqn 2} \end{array}$$

$$\begin{array}{l} y \text{ (const)} \\ x \text{ (const)} \end{array}$$

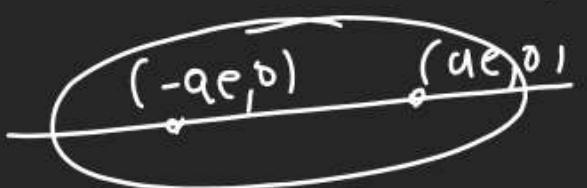
$$\begin{array}{c} 2x + 24y - 0 + 28 = 0 \\ x + 12y + 14 = 0 \rightarrow ① \\ \hline 0 + 24x - 12y + 36 = 0 \\ 2x - 4 + 3 = 0 \rightarrow ② \end{array}$$

Q Find Area of figure Bounded betw.

$$\text{Focii of } \mathcal{E}_1: \frac{x^2}{25} + \frac{y^2}{16} = 1 \quad a > b$$

$$\mathcal{E}_2: \frac{x^2}{24} + \frac{y^2}{49} = 1 \quad a < b$$

$$a = 5, b = 4$$



$$b^2 = a^2(1 - e^2)$$

$$16 = 25(1 - e^2)$$

$$\frac{25}{25} = 1 - e^2$$

$$e = \frac{3}{5}$$

$$\text{Focii} = (3, 0), (-3, 0)$$

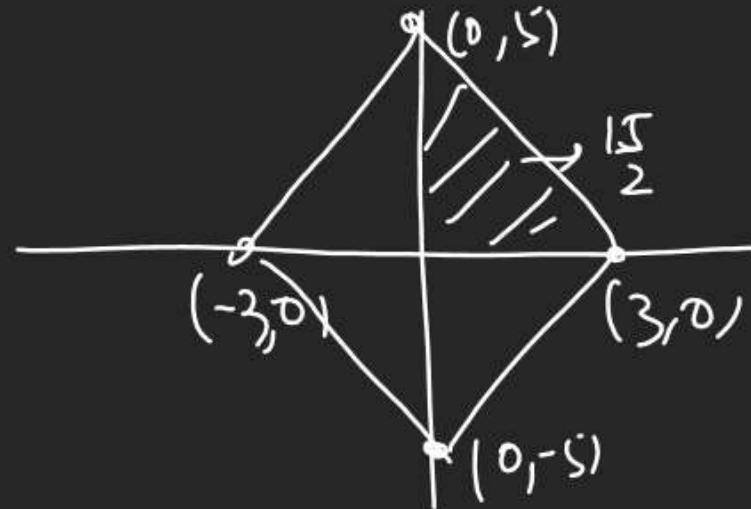
$$\begin{aligned} a &= \sqrt{24}, b = 7 \\ \bullet (0, be) &= (0, 5) \\ \bullet (0, -be) &= (0, -5) \end{aligned}$$

$$\begin{aligned} a^2 &= b^2(1 - e^2) \\ 24 &= 49(1 - e^2) \end{aligned}$$

$$\frac{24}{49} = 1 - e^2$$

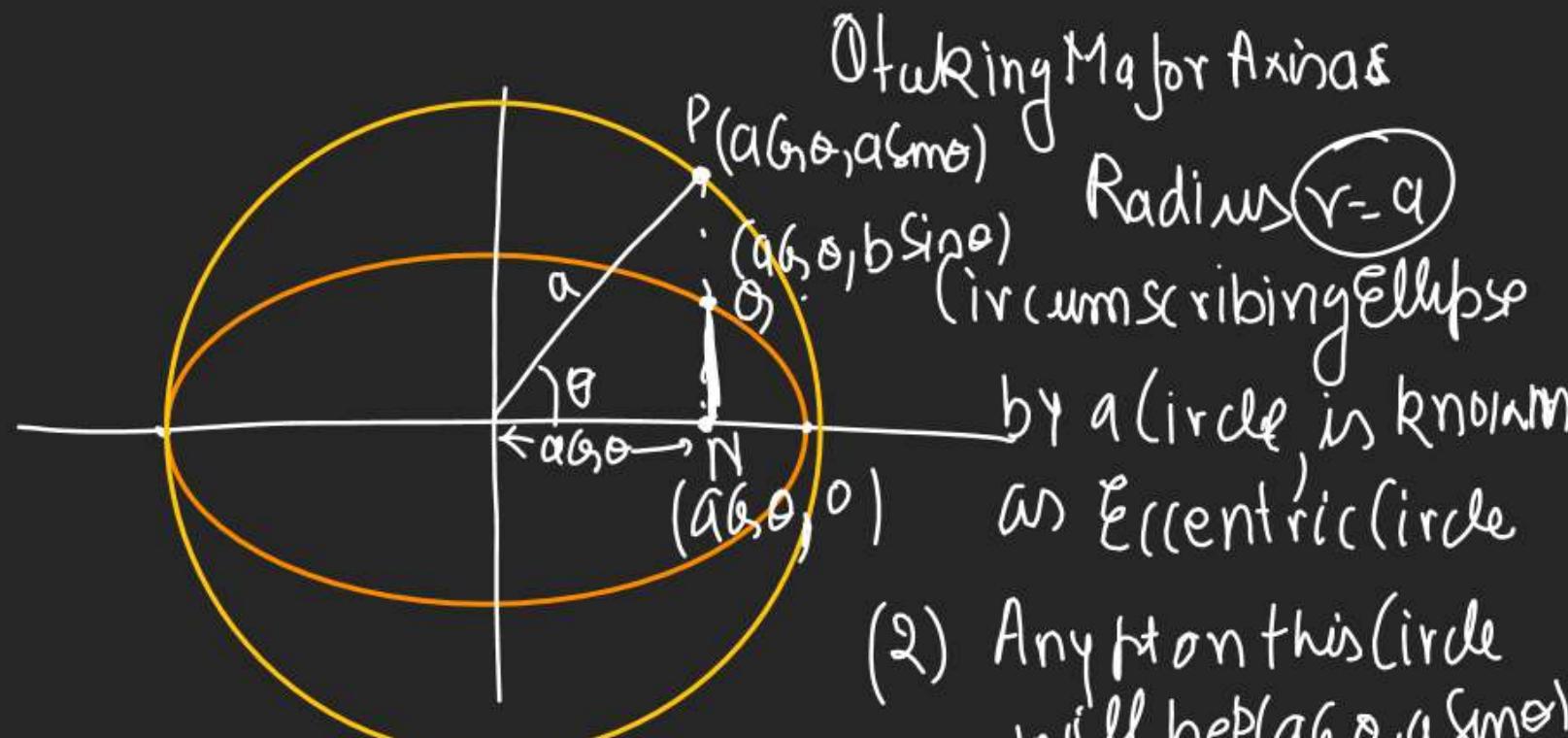
$$e = \frac{5}{7}$$

$$be = 5$$



$$\begin{aligned} \text{Area} &= 4 \times \frac{15}{2} \\ &= 30 \end{aligned}$$

# Eccentric Angle & Eccentric circle:



Q.P.T.  $\boxed{\frac{PN}{PQ} = \text{const}}$

$$\frac{PN}{PQ} = \frac{a \sin \theta}{a \sin \theta - b \cos \theta} = \frac{a}{a-b}$$

$\frac{PN}{PQ} = \frac{a \sin \theta}{a \sin \theta - b \cos \theta} = \frac{a}{a-b}$

$$\frac{x^2 \cos^2 \theta}{a^2} + \frac{y^2}{b^2} = 1$$

$$\frac{y^2}{b^2} = \sin^2 \theta \Rightarrow y = b \sin \theta$$

So we are getting Q  $(a \cos \theta, b \sin \theta)$   
on ellipse

& This Par. Coord of Ellipse.

(5) Par. Coord of Ellipse

$$\begin{cases} x = a \cos \theta \\ y = b \sin \theta \end{cases}$$

where  $\theta$  - Ecc. Angle

$$0 \leq \theta < 2\pi$$

Q) find (coord. of any pt on E ellipse)

whose focii are  $(-1, 0)$  &  $(7, 0)$   
 $\& E\text{cc} = \frac{1}{2}$ ?

$2ae$

$$2ae = 8$$

$$ae = 4$$

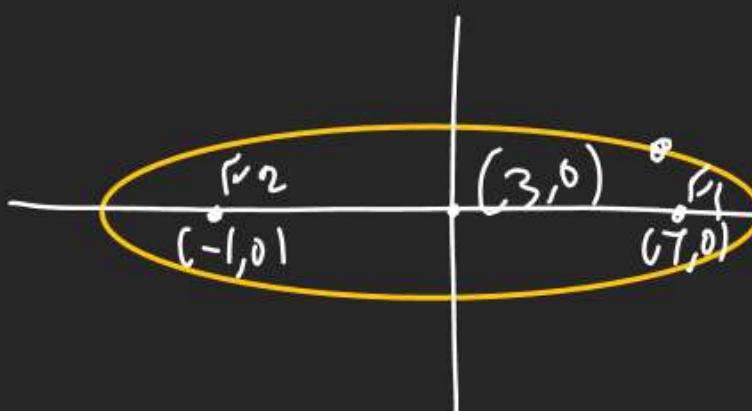
$$ax_1 = 4$$

$$a = 8$$

$$b^2 = a^2(1 - e^2)$$

$$b^2 = 64\left(1 - \frac{1}{4}\right)$$

$$= 48$$



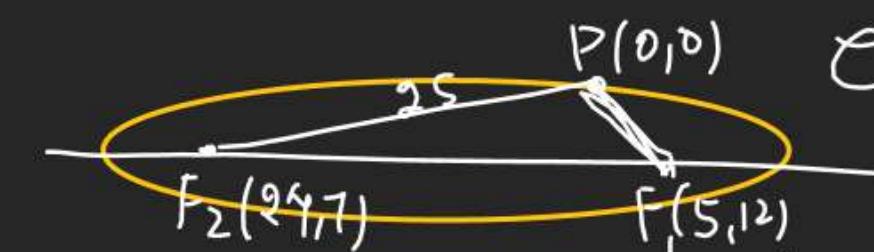
$$\frac{(x-3)^2}{64} + \frac{(y-0)^2}{48} = 0$$

$$\begin{cases} x - 3 = 8 \cos \theta \\ y - 0 = 4\sqrt{3} \sin \theta \end{cases}$$

$$( \text{coord} = (3 + 8 \cos \theta, 4\sqrt{3} \sin \theta) )$$

$$R_K : \boxed{\begin{aligned} e &= \frac{2a e}{2a} \\ e &= \frac{F_1 F_2}{PF_1 + PF_2} \end{aligned}}$$

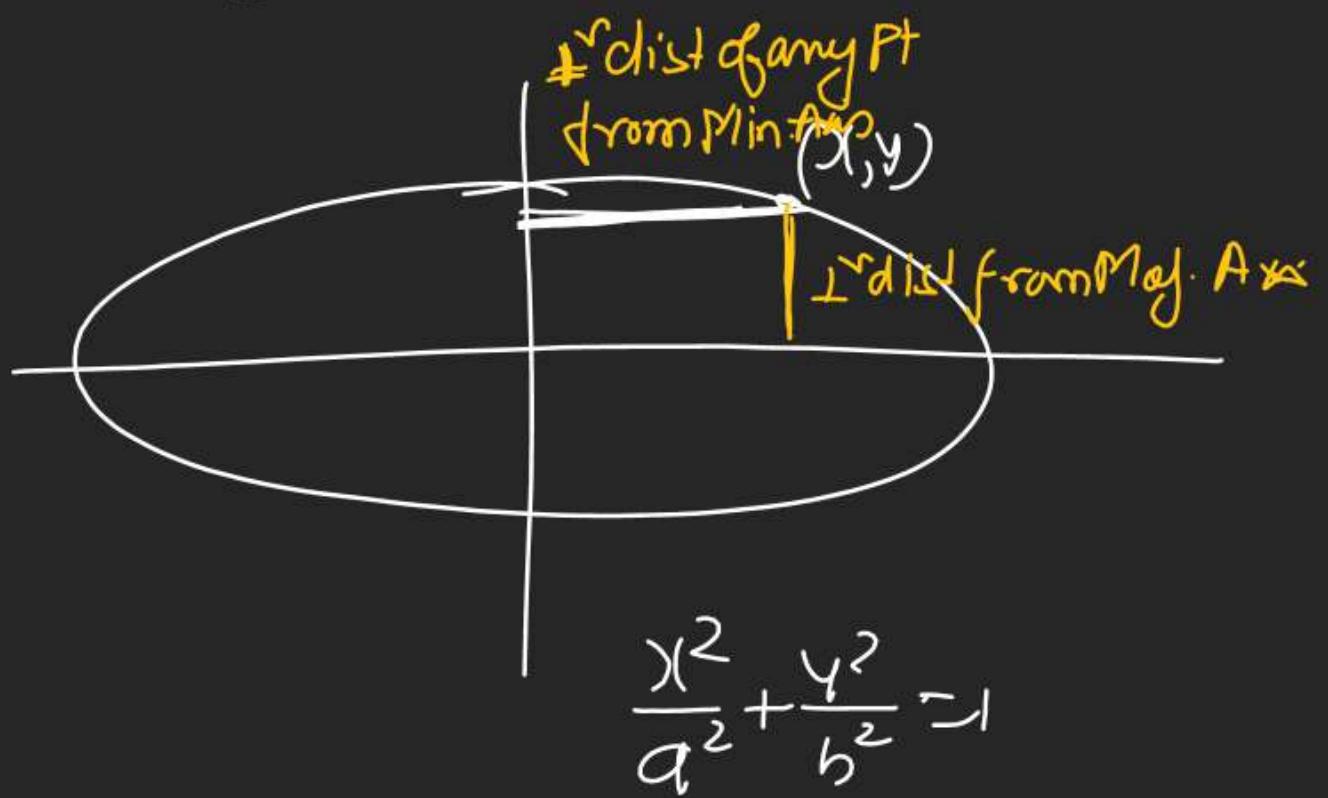
Q) If  $(5, 12)$  &  $(-1, 0)$  are focii of Ellipse P.T. Origin.  
then find E.C. of Ellipse



$$e = \frac{\sqrt{386}}{13+25} = \frac{\sqrt{386}}{38}$$

$$\begin{aligned} F_1 F_2 &= \sqrt{(24-5)^2 + (7-12)^2} \\ &= \sqrt{361 + 25} \\ &= \sqrt{386} \end{aligned}$$

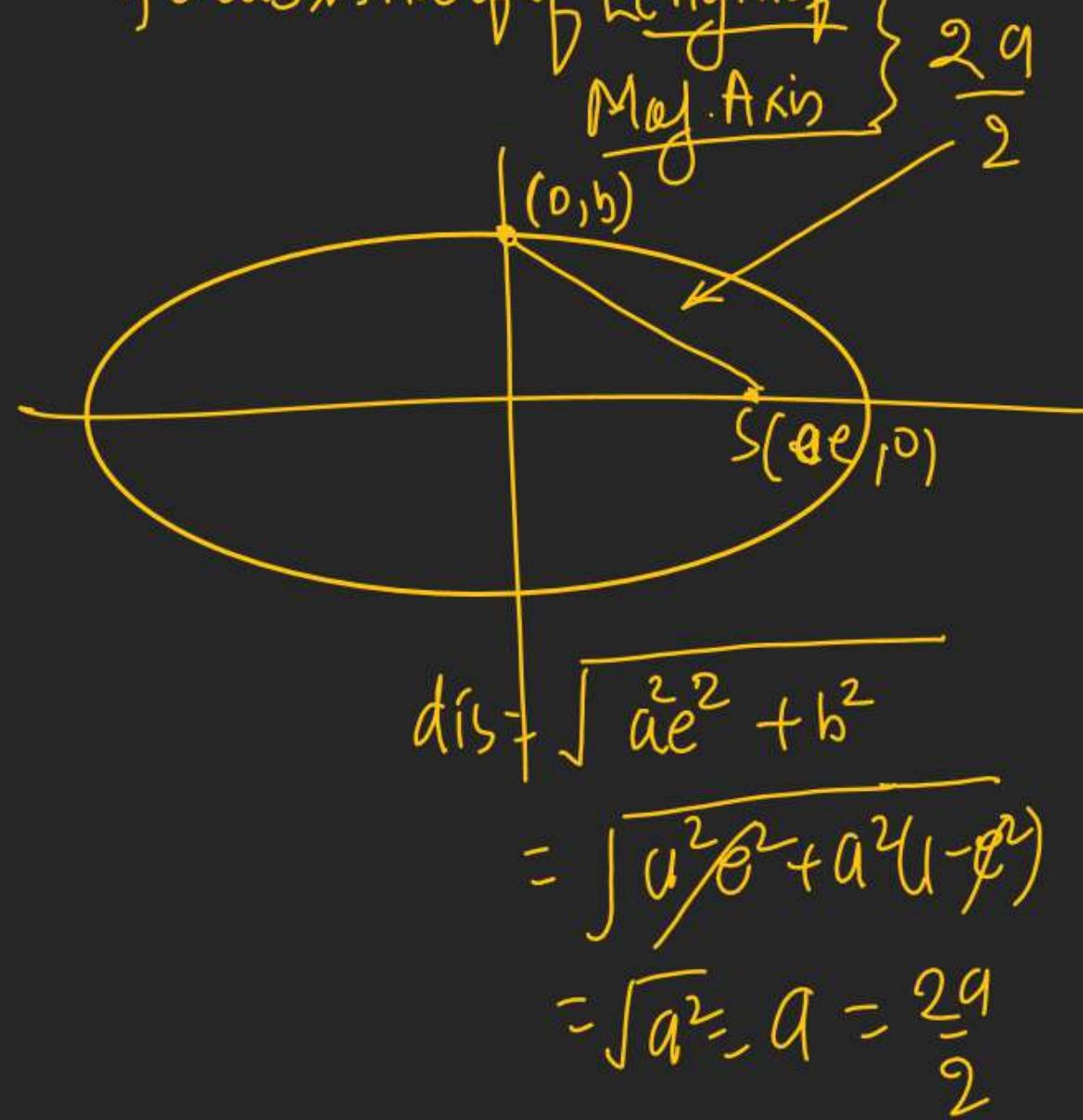
Teda Ellipse hai Per Mera Ellipse hai



$$\left(\frac{\text{Min Axis}}{a^2}\right)^2 + \left(\frac{\text{Maj Axis}}{b^2}\right)^2 = 1$$

P.T. distance of Extremity

of Minor Axis from  
foci is half of Length of  
Maj. Axis



Q If distance of any pt. on  $\frac{x^2}{12} + \frac{y^2}{4} = 1$   
from centre is  $2\sqrt{2}$  find eccentric angle.



$$(2\sqrt{3}\cos\theta)^2 + (2\sin\theta)^2 = 2^2 \cdot 2$$

$$4(3\cos^2\theta + 4\sin^2\theta) = 8$$

$$4 + 8\cos^2\theta = 8$$

$$8\cos^2\theta - 4 = 0 \Rightarrow \cos^2\theta = \frac{1}{2}$$

$$\cos\theta = \pm\frac{1}{\sqrt{2}}$$

$$\left| \begin{array}{l} \theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4} \\ \theta = \frac{\pi}{4} \end{array} \right.$$

Q Eqn of ellipse in whose focus  $= (3,4)$

$$(3) ae = \sqrt{-1^2 + -1^2} = \sqrt{2} \quad (\text{centre } (2,3), e = \frac{1}{2})$$

$$a \times \frac{1}{2} = \sqrt{2} \Rightarrow a = 2\sqrt{2}$$

$$(4) b^2 = 8\left(1 - \frac{1}{4}\right) + 6$$

$$(y-3) = \frac{4-3}{3-2}(x-2)$$

$$x - y + 1 = 0$$

1) Major Axis

$$x + y + K = 0$$

$$2+3+K=0$$

$$K = -5$$

P.T.(23)

$$\text{Min } x + y - 5 = 0$$

2) Minor Axis