Nishant Jindal

Binomial Theorem [Khushiyan hi Khusiyan]

Basic → ① Factorial

$n! = \underline{|n}$

$3! = 3 \cdot 2 \cdot 1 = 6$

$4! = 4 \cdot 3 \cdot 2 \cdot 1 = 24$

$5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$

$6! = 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 720$

$7! = 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 5040$

$8! = 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 40320$

② $\underline{|n} = n \, \underline{|n-1}$

$= (n)(n-1)\, \underline{|n-2}$

$8! = 8 \, \underline{|7}$

$= 8 \cdot 7 \, \underline{|6}$

$\underline{|8} = 56 \cdot \underline{|6}$

③ $\underline{|n} = (n)(n-1)(n-2) \cdots 4 \cdot 3 \cdot 2 \cdot 1$

$\underline{|2n} = (2n)(2n-1)(2n-2) \cdots 4 \cdot 3 \cdot 2 \cdot 1$

(4) $\underline{|1} = 1$

$\underline{|2} = 2$

$\underline{|0} = 1$

(5) ${}^nC_r$, ${}^nP_r$ (Notation)

$${}^nC_r = \frac{\lfloor n}{\lfloor r\ \lfloor n-r}$$

$${}^4C_2 = \frac{\lfloor 4}{\lfloor 2\ \lfloor 2}$$

$${}^{15}C_3 = \frac{\lfloor 15}{\lfloor 3\ \lfloor 12}$$

$$= \frac{15\cdot 14\cdot 13\cdot \cancel{\lfloor 12}}{\lfloor 3\ \cancel{\lfloor 12}}$$

$$= \frac{\overset{5}{\cancel{15}}\cdot \overset{7}{\cancel{14}}\cdot 13}{\cancel{3}\cdot \cancel{2}\cdot 1} = 35\times 13$$

$$= \cancel{395}\ 455$$

Q. ${}^7C_3 = \frac{\lfloor 7}{\lfloor 3\ \lfloor 4}$ M1

$$= \frac{7\cdot 6\cdot 5\ \cancel{\lfloor 4}}{\lfloor 3\ \cancel{\lfloor 4}}$$

$$= \frac{7\cdot \cancel{6}\cdot 5}{\cancel{3\cdot 2\cdot 1}} = 35$$

M2 (Direct)

$${}^7C_3 = \frac{\text{7 se 3 terms in decreasing order}}{\text{1 se 3 terms inc. order}}$$

$$= \frac{7\cdot \cancel{6}\cdot 5}{\cancel{1\cdot 2\cdot 3}} = 35$$

Q ${}^{15}C_4 = ?$

$${}^{15}C_4 = \frac{15\cdot \overset{7}{\cancel{14}}\cdot 13\cdot \cancel{12}}{1\cdot \cancel{2}\cdot \cancel{3\cdot 4}}$$

$$= 105\times 13$$

$\boxed{\text{Rk :- } {}^nC_r \text{ is known as Binomial Cofficient}}$

$\boxed{2)\ n\in N,\ \&\ r\in W,\ n\geq r.}$

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Q $^nC_2 = ?$

M 2) $^nC_2 = \frac{n\cdot(n-1)}{1\cdot 2} = \frac{n(n-1)}{2}$

M1 $^nC_2 = \frac{\lfloor n}{\lfloor 2 \lfloor n-2}$

Q $^nC_3 = ?$

M2 $\frac{n\cdot(n-1)(n-2)}{1\cdot 2\cdot 3}$

M1. $^nC_3 = \frac{\lfloor n}{\lfloor 3 \lfloor n-3}$

Rk:-

1) ^nC_r = No of ways to select r different things out of n diff. things

2) $^{10}C_3$ = No of ways to select 3 things out of 10 things

$^{10}C_3 = {}^{10}C_7$. $3+7=10$

$^{10}C_3$: 10 mese 3 chijo ko chun na

$^{10}C_7$: 10 mese 7 chijo ko chhodna.

(6) $^nC_r = {}^nC_{n-r}$

(7) $^nC_a = {}^nC_b \rightarrow a=b$ or $a+b=n$

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Q $^nC_{10} = {^nC_{15}}$ then $^{27}C_n = ?$

$10+15=n$

$\Rightarrow n=25$

So $^{27}C_n = {^{27}C_{25}} = {^{27}C_2}$

$= \frac{\lfloor 27}{\lfloor 25 \lfloor 2}$

$= \frac{27 \cdot \cancel{26}^{13} \cdot \cancel{\lfloor 25}}{\cancel{\lfloor 25} \cdot \times \cancel{2}}$

$= 27 \times 13$

$= 351$

$= \frac{27 \cdot \cancel{26}^{13}}{1 \cdot \cancel{2}}$

$= 351$

$$\frac{\lfloor n}{\lfloor r-1 \lfloor n-r}\left\{\frac{(n+1)}{(r)(n-r+1)}\right\}$$

$$\frac{\lfloor n+1}{\lfloor r \lfloor n+1-r} = {^{n+1}C_r}$$

$= RHS$

Q If $^{15}C_{3r} = {^{15}C_{r+3}}$ then $r = ?$

$\underline{^nC_a = {^nC_b}}$

$3r + (r+3) = 15$

$4r = 12$

$r = 3$

$5 \times \lfloor 4 = \lfloor 5$

See it again

Q $^nC_r + {^nC_{r-1}} = {^{n+1}C_r}$ (P.T.)

LHS $\frac{\lfloor n}{\lfloor r \lfloor n-r} + \frac{\lfloor n}{\lfloor r-1 \lfloor n-r+1}$

$\lfloor 4 = 4 \lfloor 3$

$$\frac{\lfloor n}{r \lfloor r-1 \lfloor n-r} + \frac{\lfloor n}{\lfloor r-1 (n-r+1) \lfloor n-r}$$

$$= \frac{\lfloor n}{\lfloor r-1 \lfloor n-r}\left\{\frac{1}{r} + \frac{1}{n-r+1}\right\} = \frac{\lfloor n}{\lfloor r-1 \lfloor n-r}\left\{\frac{n-\cancel{r}+1+\cancel{r}}{(r)(n-r+1)}\right\}$$