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$$\lim_{x\to 1}\left(\frac{(1-x)}{|1-x|}\cos\frac{1}{1-x} - x\cos\frac{1}{1-x}\right)$$

$$\lim_{x\to 1^+}\left(-1-x\right)\cos\frac{1}{x}$$

$= \text{not exist.}$

$$\lim_{x\to 1^-}\left((1-x)\cos\frac{1}{1-x}\right)$$

$= 0.$

$x = \frac{n}{n+1}$

$$f_{min} = \frac{n^2}{n+1} - \frac{2n^2}{n+1} + 1$$

$= 1 - \frac{n^2}{n+1} = \frac{n+1-n^2}{n+1}$

$$\frac{nx}{1+(n+1)x^2} \le \frac{1}{2}$$

$(n+1)x^2 - 2nx + 1 \geq 0 \quad \forall \; x > 0, \; n \in N.$

$$\boxed{\tan^{-1}(n+1)x - \tan^{-1}x}$$

$f_n(x) = \tan^{-1}(x+n) - \tan^{-1}x.$ $\quad x > 0$

$$\lim_{\cancel{x}\to\infty}\tan(f_n(x)) = \lim_{x\to\infty}\frac{n}{1+x(n+x)} = \lim 0.$$

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2: $\lim_{x\to 0} y_n(x) = \lim_{x\to 0}\left(x^2+\frac{x^2}{1+x^2}+\cdots+\frac{x^2}{(1+x^2)^{n-1}}\right) = 0.$

$y_n(0) = 0+0+\cdots 0 = 0.$

$y(x) = \lim_{n\to\infty}\left(x^2+\cdots\right)$

$$y(x) = \begin{cases} \dfrac{x^2}{1-\frac{1}{1+x^2}} = \dfrac{x^2}{\left(\frac{x^2}{1+x^2}\right)} = 1+x^2 & x \neq 0. \\ 0 & x = 0 \end{cases}$$

$\lim_{x\to 0} y(x) = 1$

$y(0) = 0$

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4. $\lim_{x \to 0^+} \left[\dfrac{e^x - 1}{x} \right] = 1$ ($\dfrac{e^x-1}{x} \to 1^+$)

$ax+b \quad -1 \le x \le 0$

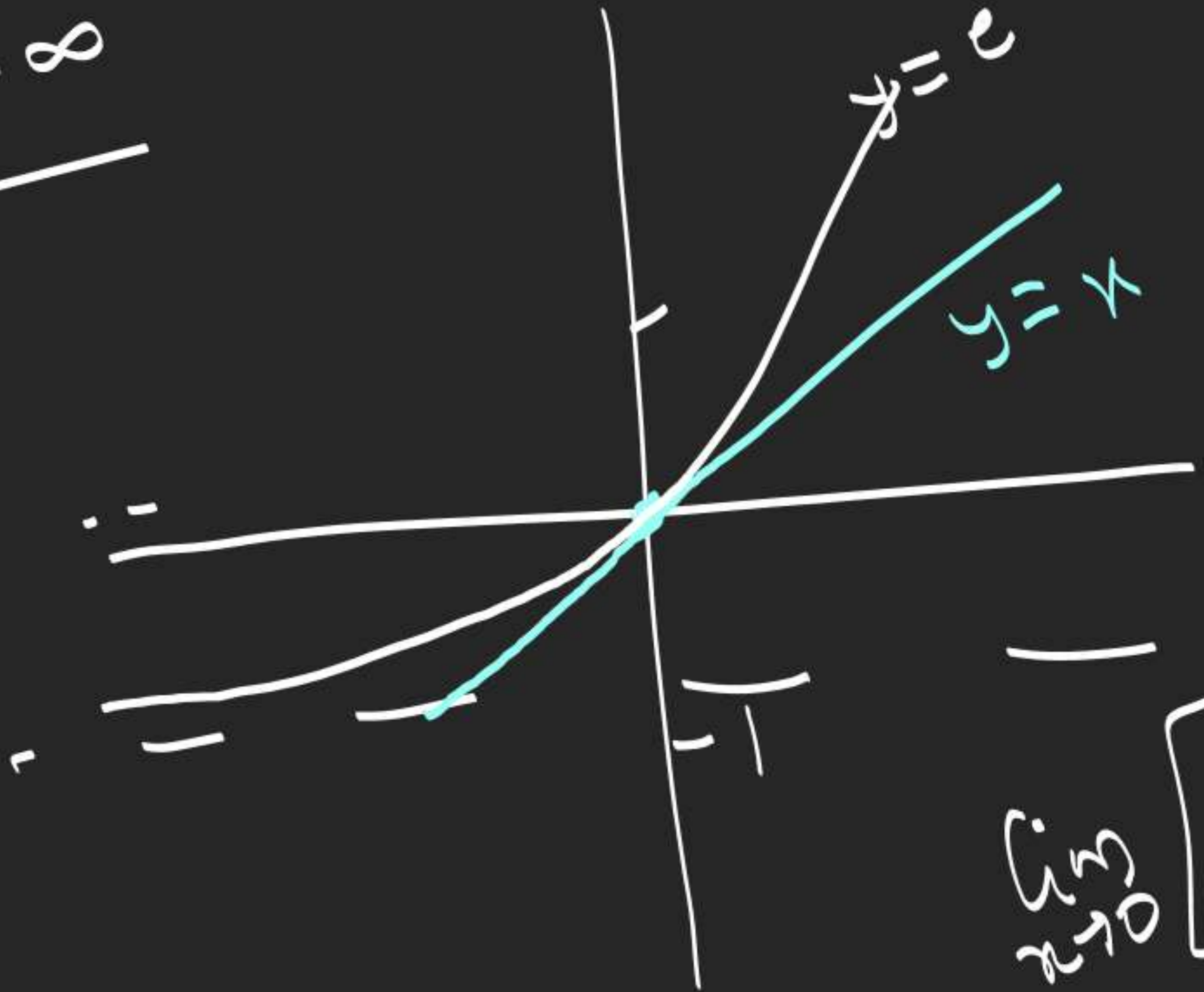
$x < 0, \quad e^x - 1 > x$

$\dfrac{e^x - 1}{x} < 1$

$x > 0 \quad e^x - 1 > x$

$\dfrac{e^x-1}{x} > 1$

$1 < \dfrac{\frac{\cancel{x}}{1!} + \frac{x^{\cancel{2}}}{2!} + \dots \infty}{\cancel{x}}$



$\lim_{x \to 0} \left[\dfrac{e^x - 1}{x} \right]$ → $LHL = 0$, → $RHL = 1$

↳ not exist.

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$g(f(x))$

$\underline{x=a}$

- $a=0$

- $f(a)=0=1+a^3$ or $a^2-1 \Rightarrow a=-1,1$

$\underline{x=0,1,-1}$

$\underline{x=-1}$

$LHL=\lim_{x\to -1^-} g(\underbrace{f(x)}_{0^-}) = -1$

$RHL=\lim_{x\to -1^+} g(\underbrace{f(x)}_{0^+}) = 1$

$g(f(-1))=g(0)=1$

$\lim_{t\to 1}\frac{t^{1/2}-1^{1/2}}{t-1}$

$\lim_{x\to 0^+}\frac{(\underbrace{1+bx}_{t})^{1/2}-1}{bx}=\frac{1}{2}.$

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<u>8.</u>

$$\lim_{x \to 0} \frac{\frac{3x}{1!} - \frac{(3x)^3}{3!} + \frac{(3x)^5}{5!} - \cdots + A\left(\frac{2x}{1!} - \frac{(2x)^3}{3!} + \frac{(2x)^5}{5!} - \cdots\right) + B\left(\frac{x}{1!} - \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots\right)}{x^5}$$

$$l = \frac{3^5 + 2^5 A + B}{5!}$$

$$3 + 2A + B = 0$$

$$-\frac{27}{6} - \frac{8A}{6} - \frac{B}{6} = 0$$

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$$\lim_{x\to 0} \frac{\sin x}{x}\left(\frac{3-4\sin^2 x+2A\cos x+B}{x^4}\right)$$

$$\boxed{3+2A+B=0}$$

$$\frac{\sin x}{x}\left(\frac{-4\sin^2 x+2A\cos x-2A}{x^4}\right)$$

$$\frac{\sin x}{x}\left(\frac{-4\sin^2 x+8(1-\cos x)}{x^4}\right)$$

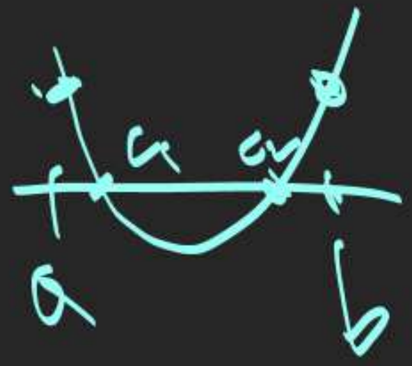
$$(1-c)(1+c)$$

$$\frac{\sin x}{x}\left(\frac{4(1-\cos x)(-1-\cos x+2)}{x^4}\right)$$

$$\frac{-4\frac{\sin^2 x}{x^2}+2A\frac{(\cos x-1)}{x^2}}{x^2}=1$$

$$\boxed{-A-4=0}$$

$$B=5$$



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$$\lim_{n\to\infty} f\left(\underset{\to 0}{\frac{1}{4^n}}\right) = \sin e^n\, e^{-n^2} + \frac{1}{1+\frac{1}{n^2}}$$

$$0 + 1 = f(0).$$

$\ln(1+x)$

$$\lim_{x\to 0} \underline{g(x)} \sin\frac{5}{x} = 0 \qquad \lim_{x\to 0} f(x)$$

$$(p-1)x^2 + (q-p)x + r - 1 = 0 < \begin{matrix} 1 \\ 2 \end{matrix}$$

$$\frac{p-2}{p-1} = 3$$

$$\frac{x}{2} - 1 = \frac{\pi}{2}$$

$$x = \frac{\pi}{4} + \frac{1}{2}$$

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Derivability / Differentiability of function at point $x=a$.

$$f'(a) = \lim_{x \to a} \frac{f(x)-f(a)}{x-a} = \frac{d}{dx} f(x) \Big|_{x=a}.$$

Derivative of $f(x)$ at $x=a$

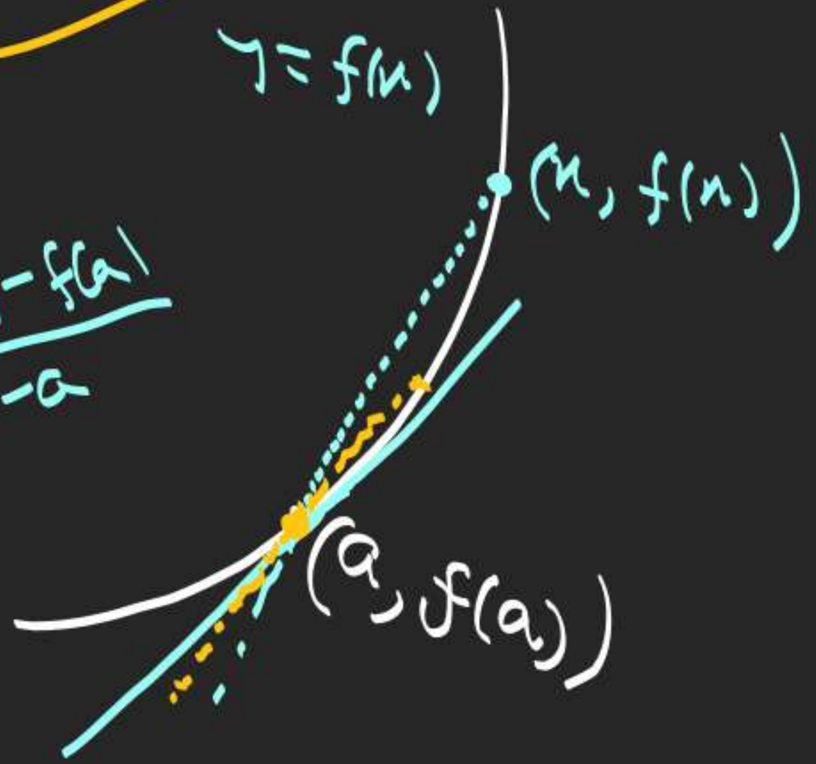
$\frac{v_2 - v_1}{t_2 - t_1}$ Av. rate of change of vel. w.r.t. time in $[t_1, t_2]$

Slope of tangent to $y=f(x)$ at $x=a$

$\lim_{t_2 \to t_1} \frac{v_2 - v_1}{t_2 - t_1}$

||

Instantaneous rate of change of vel. w.r.t. time at $t=t_1$



Instantaneous rate of change of $f(x)$ w.r.t. x at $x=a$.

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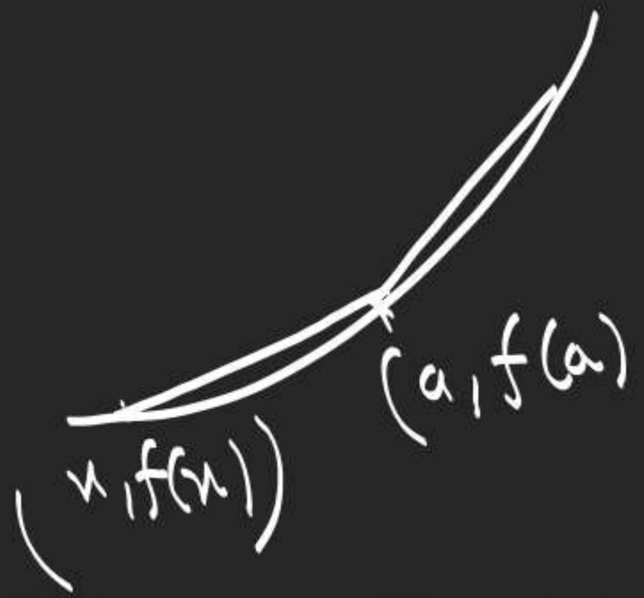

LHD = Left hand derivative

$$= \lim_{x \to a^-} \frac{f(x) - f(a)}{x - a}$$

$$= \lim_{h \to 0} \frac{f(a-h) - f(a)}{-h} \qquad h > 0$$

If LHD = RHD = finite = l

$\Rightarrow$ $f(x)$ is differentiable at $x = a$ & $f'(a) = l$.

RHD = Right hand derivative $= \lim_{x \to a^+} \frac{f(x) - f(a)}{x - a}$

$$= \lim_{h \to 0} \left(\frac{f(a+h) - f(a)}{h} \right)$$

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$f(x) = |\ln x|$ at $x = 1$

$\leftarrow$ not differentiable at $x=1$

$f(x) = \ln^2 x$ at $x = 1$ $\rightarrow$ $f(x)$ is derivable at $x=1$ & $\boxed{f'(1) = 0}$

$$\lim_{x\to 1} \frac{\ln^2 x - \cancel{\ln^2 1}}{x-1} = \lim_{x\to 1} \frac{\ln^2(1+(x-1))}{(x-1)^2}(x-1) = 0$$

$$\lim_{x\to 1} \frac{|\ln x| - |\ln 1|}{x-1} = \lim_{x\to 1} \frac{|\ln(1+(x-1))|}{|x-1|} \frac{|x-1|}{(x-1)}$$

$$= \lim_{x\to 1} \left|\frac{\ln(1+(x-1))}{x-1}\right| \frac{|x-1|}{(x-1)}$$

LHD $= -1$

RHD $= 1$

$\boxed{Ex-3,4}$