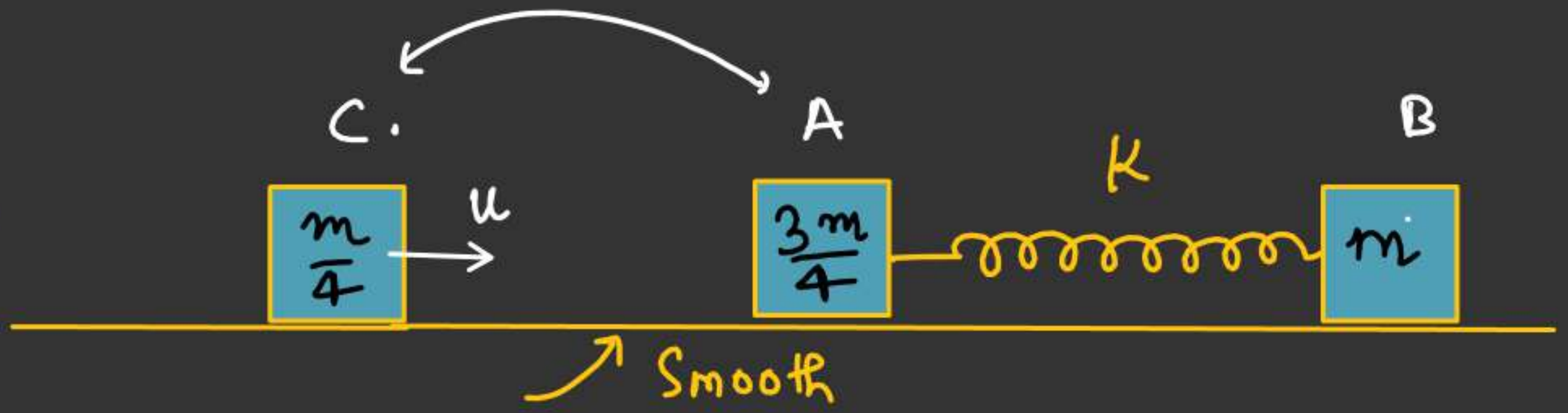
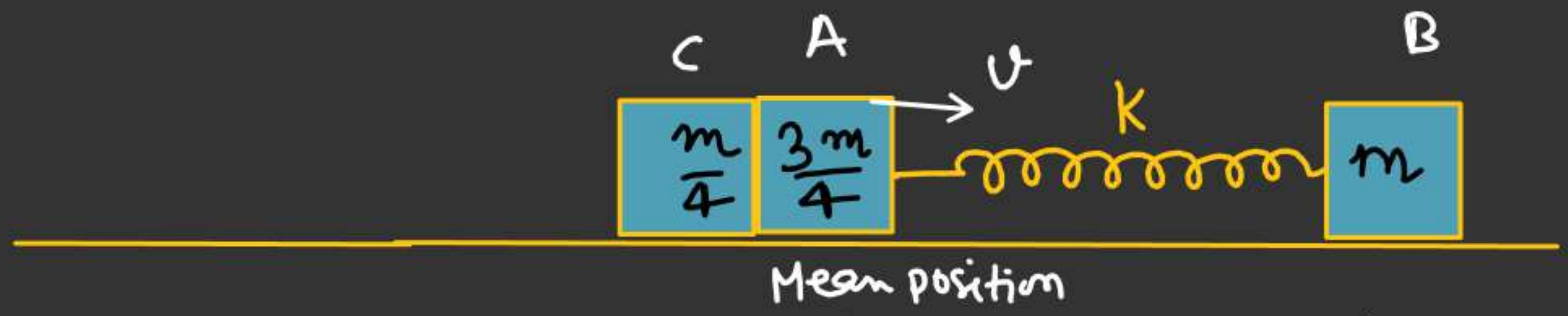
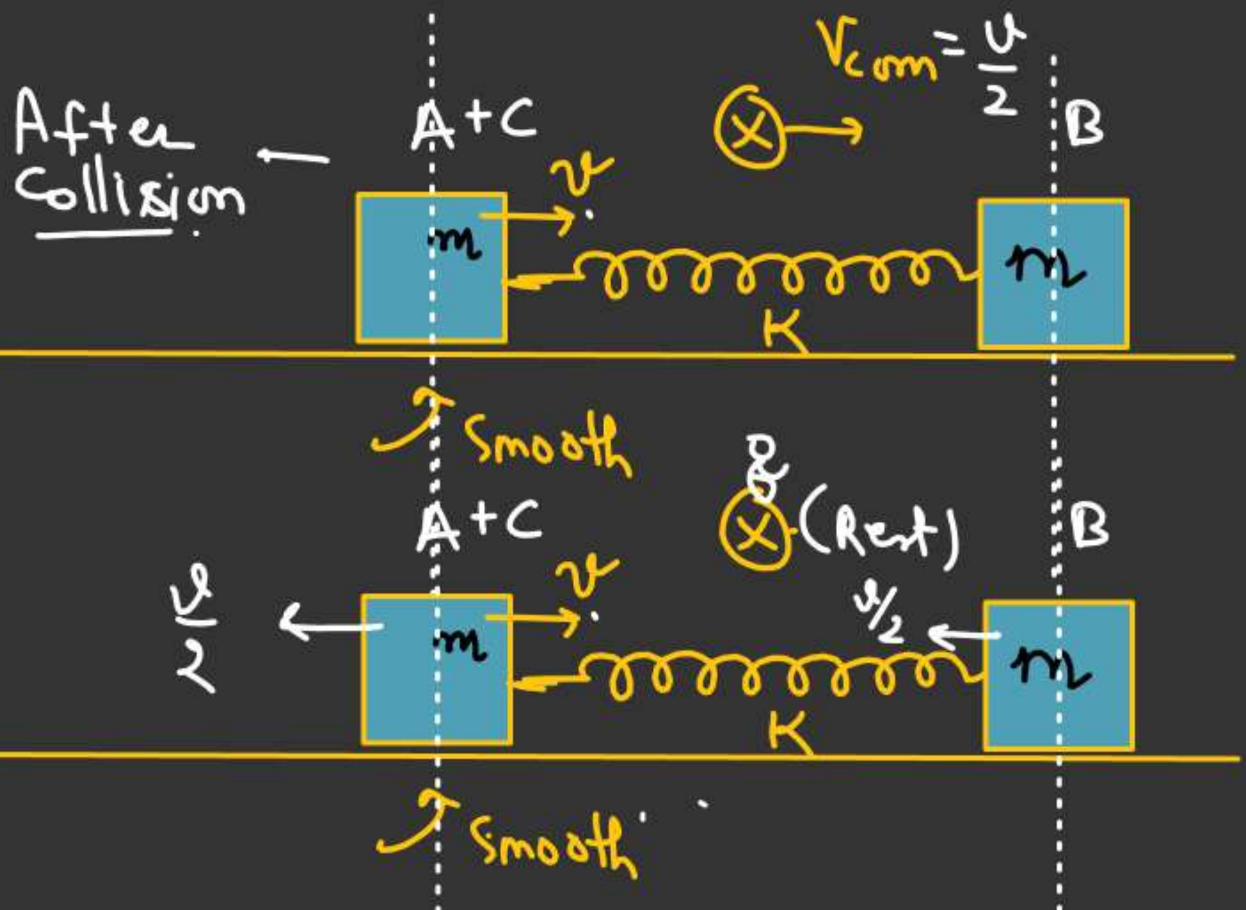
Collision b/w block C & A is perfectly inelastic. Find Time period and amplitude after Collision.







L.M.C

$$\frac{m}{4}u = m \cdot v$$

$$v = \left(\frac{u}{4}\right)$$

$$T = 2\pi\sqrt{\frac{\mu}{K}} = \left(2\pi\sqrt{\frac{m}{2K}}\right) \checkmark$$

$$\mu = \frac{m}{2} \qquad V_{com} = \frac{mv}{2m} = \frac{v}{2}$$

$$\left|V_{blocks/com}\right|_{max} = \left(\frac{v}{2}\right)$$

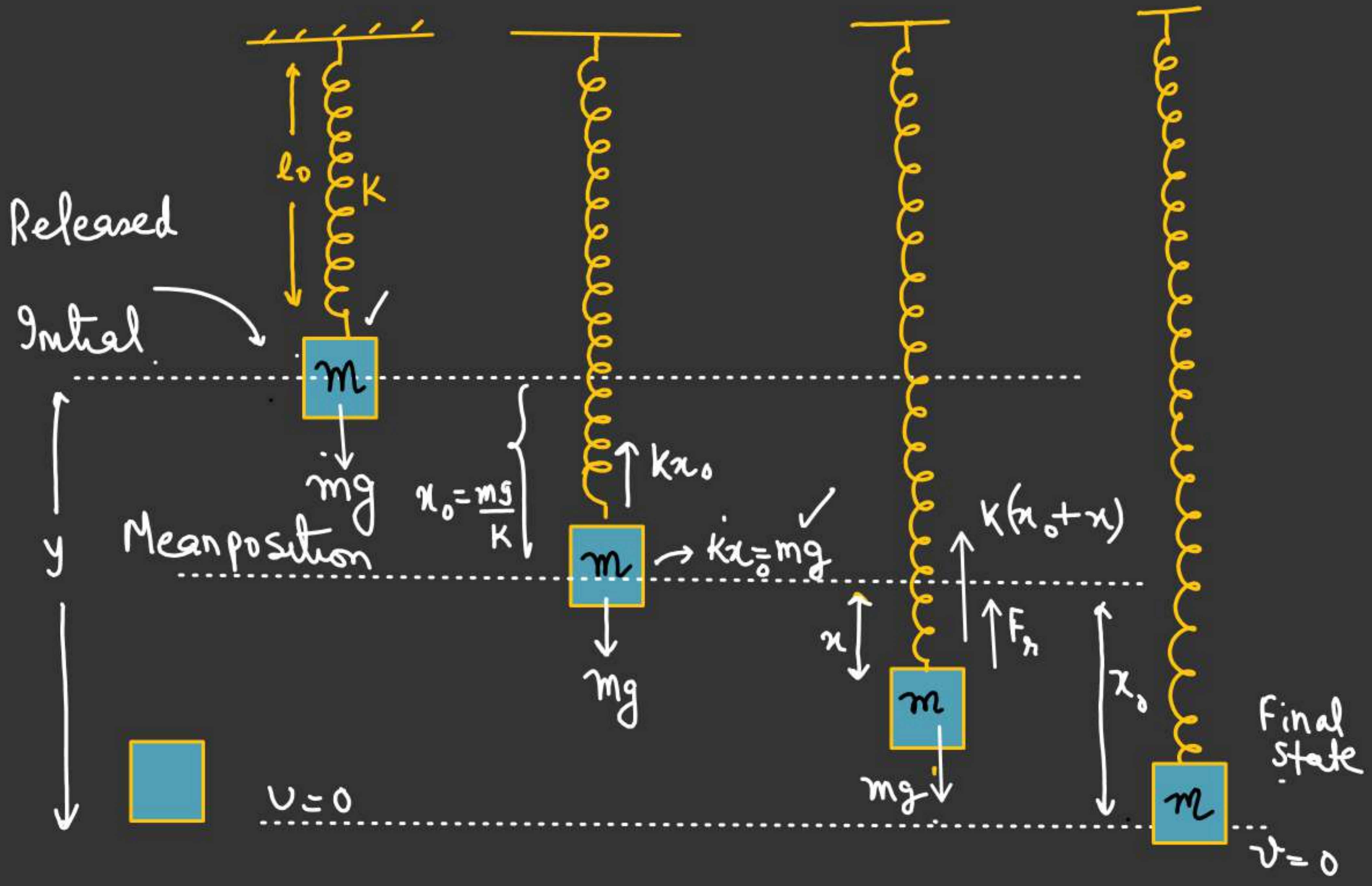
$$\frac{v}{2} = A\omega$$

$$A = \frac{v}{2\omega} = \frac{u}{8\omega} = \frac{uT}{8\times 2\pi} = \frac{u}{16\pi} \times 2\pi\sqrt{\frac{m}{2K}}$$

$$A = \frac{u}{8}\sqrt{\frac{m}{2K}} \checkmark$$

Nishant Jindal

Case of vertical spring



$$F_r = -\left[K(x_0+x) - mg\right]$$

$$F_r = -\left[\cancel{(Kx_0 - mg)}^{\,0} + Kx\right]$$

$$F_r = -Kx$$

$$a = -\frac{K}{m}x$$

$$a = -\omega^2 x$$

$$\omega = \sqrt{\frac{K}{m}}$$

$$\boxed{T = 2\pi\sqrt{\frac{m}{K}}}$$

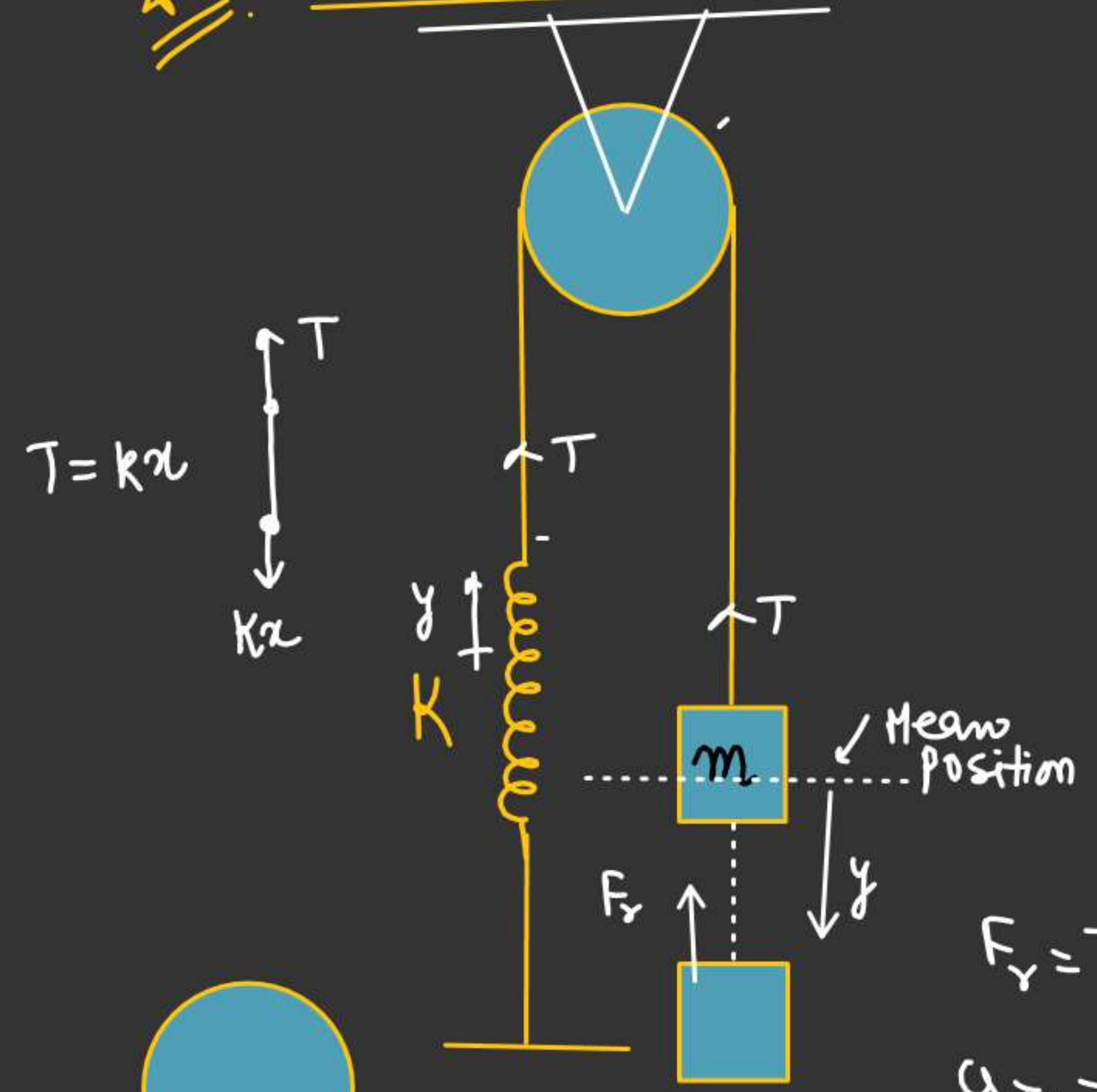
$$mgy = \frac{1}{2}Ky^2$$

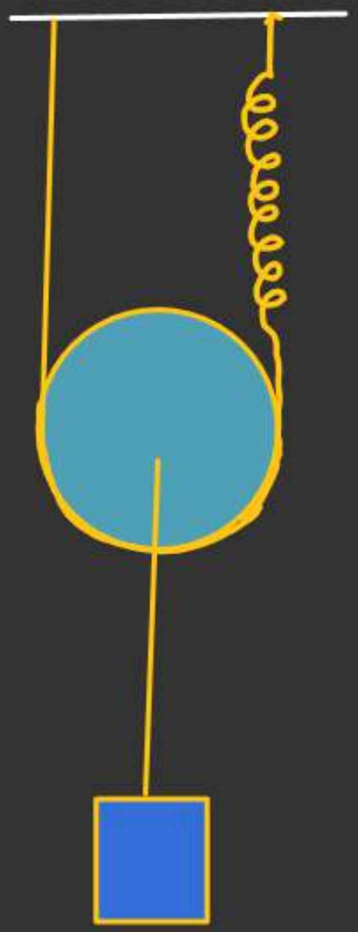
$$y = \left(\frac{2mg}{K}\right)$$

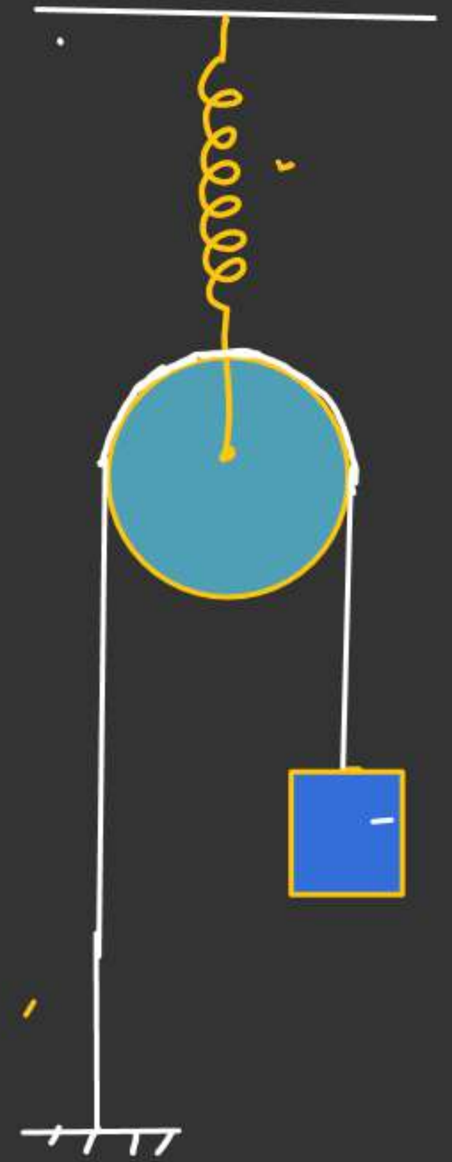
Nishant Jindal

Extra force Method

Pully Massless, String & Spring Massless.



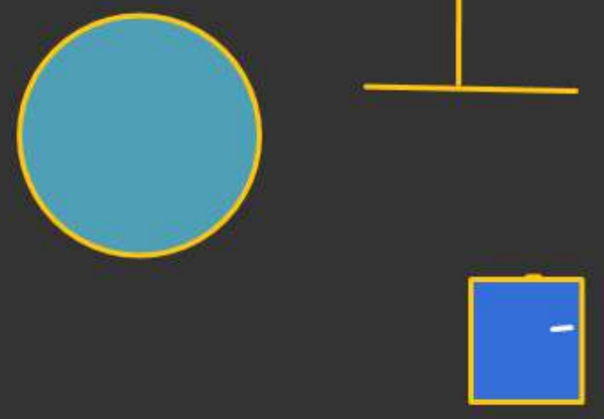




$$F_r = T = -ky$$

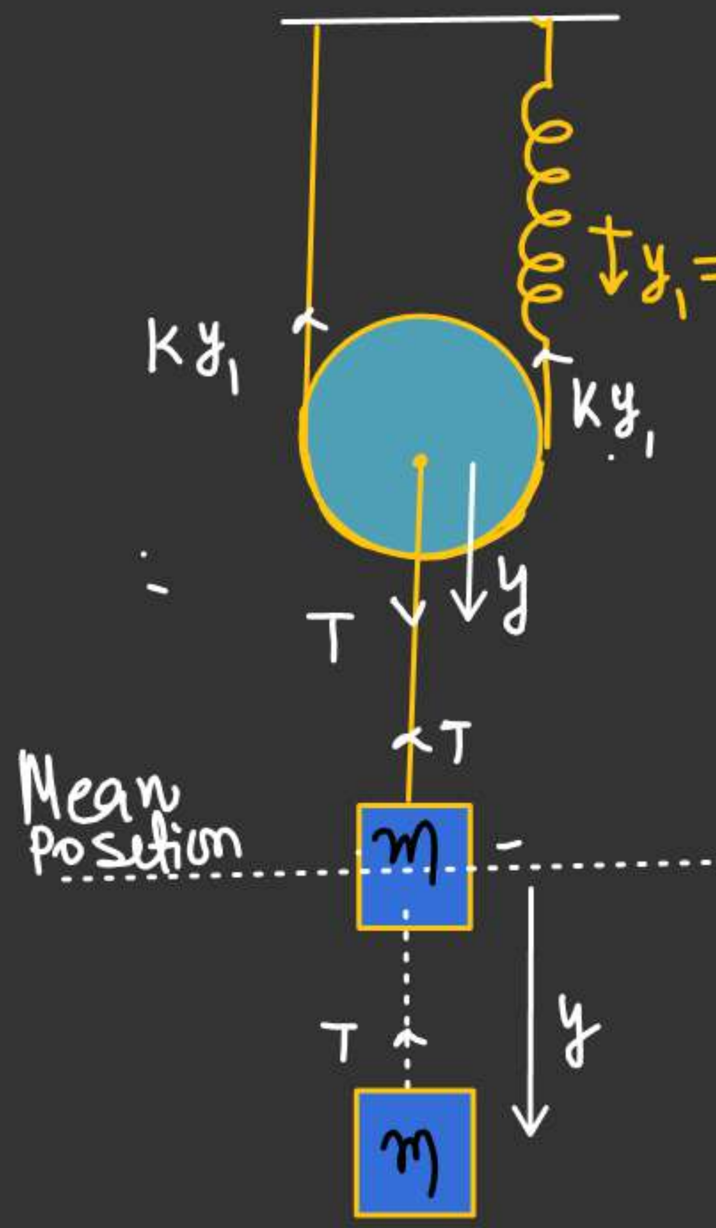
$$a = -\frac{K}{m}y$$

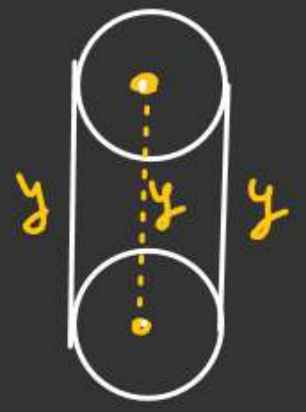
$$T = \left(2\pi\sqrt{\frac{m}{K}}\right)$$



Nishant Jindal

Extra force Method — Pully Massless, String & Spring Massless.





$T = 2Ky_1$ (Pully Masseless)

$T = -2K(2y)$

$T = -4Ky$

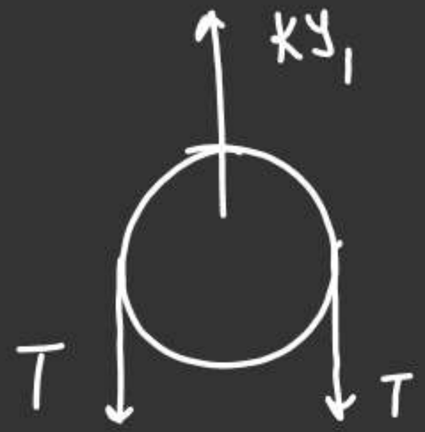
$a = -\frac{4K}{m} y$

$a = -\omega^2 y$

$T = 2\pi\sqrt{\frac{m}{4K}}$

$T = \pi\sqrt{\frac{m}{K}}$

$y = 2y_1$



$2T = Ky_1$

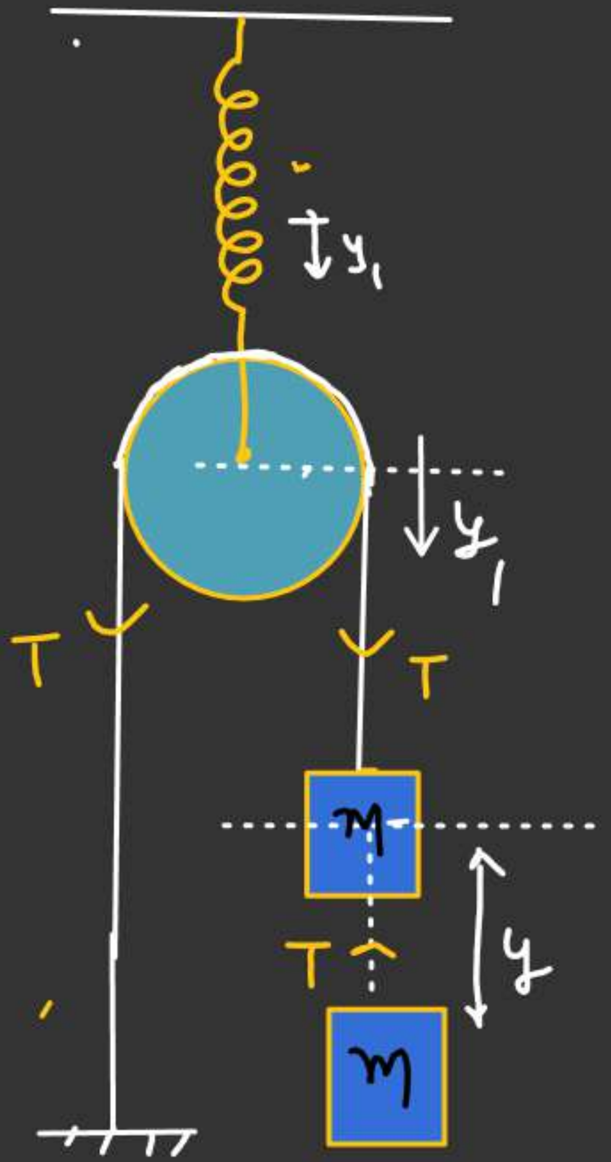
$T = \frac{K}{2} y_1 = \frac{Ky}{4}$

$F_r = -T = -\frac{K}{4} y$

$a = -\frac{K}{4m} y$

$a = -\omega^2 y$

$\omega = \sqrt{\frac{K}{4m}}$



$T = 2\pi\sqrt{\frac{4m}{K}}$

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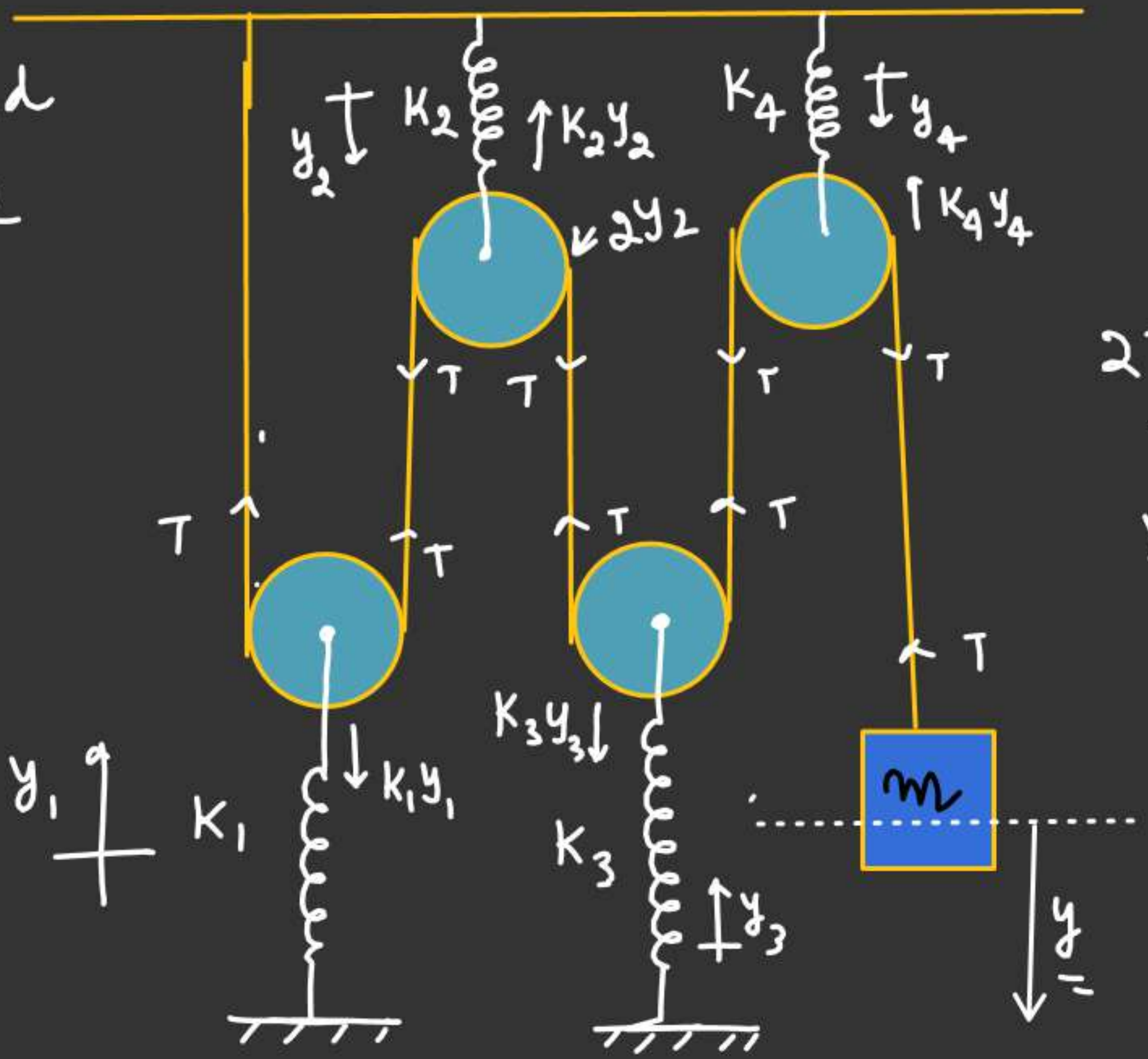
All pully and Strings are massless.



$$y = 2(y_1 + y_2 + y_3 + y_4) \qquad T \propto y.$$

$$2T = K_1 y_1 = K_2 y_2 = K_3 y_3 = K_4 y_4 \Rightarrow \text{Series Combination.}$$

$$y_1 = \frac{2T}{K_1}, \quad y_2 = \frac{2T}{K_2}, \quad y_3 = \frac{2T}{K_3}, \quad y_4 = \frac{2T}{K_4}$$

$$y = 4T\left[\frac{1}{K_1} + \frac{1}{K_2} + \frac{1}{K_3} + \frac{1}{K_4}\right]$$

$$T = \frac{y}{4\left[\frac{1}{K_1} + \frac{1}{K_2} + \frac{1}{K_3} + \frac{1}{K_4}\right]}$$

$$a = \frac{y}{4m\left[\frac{1}{K_1} + \frac{1}{K_2} + \frac{1}{K_3} + \frac{1}{K_4}\right]}$$

$$\omega^2 = \frac{1}{4m\left[\frac{1}{K_1} + \frac{1}{K_2} + \frac{1}{K_3} + \frac{1}{K_4}\right]}$$

$$T = 2\pi\sqrt{4m\Big(\underbrace{\frac{1}{K_1} + \frac{1}{K_2} + \frac{1}{K_3} + \frac{1}{K_4}}_{1/K_{eq}}\Big)} \Rightarrow T = 2\pi\sqrt{4m\left(\frac{1}{K_{eq}}\right)}$$

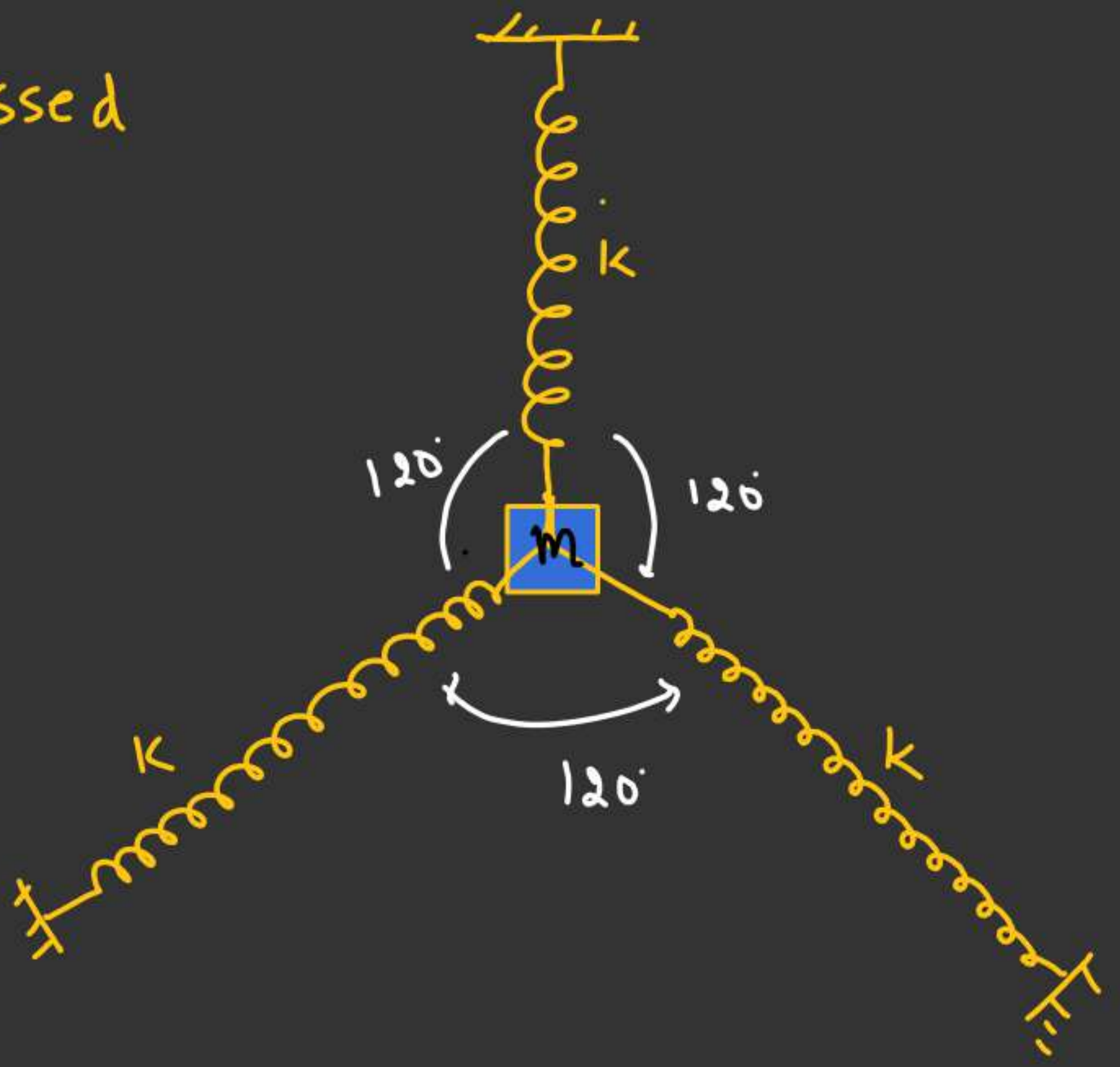
A.

Nishant Jindal

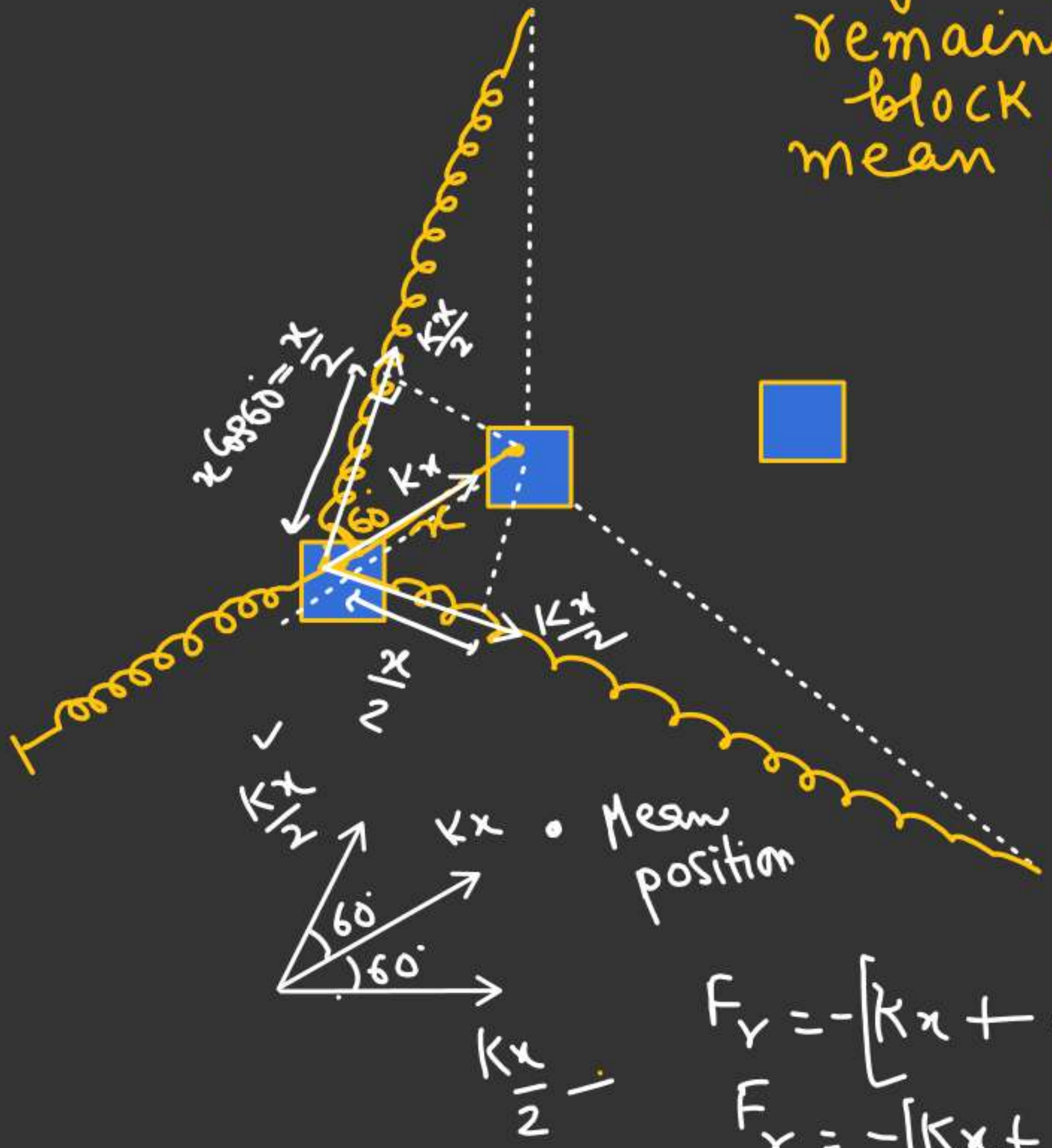
The whole System is placed on smooth horizontal table.

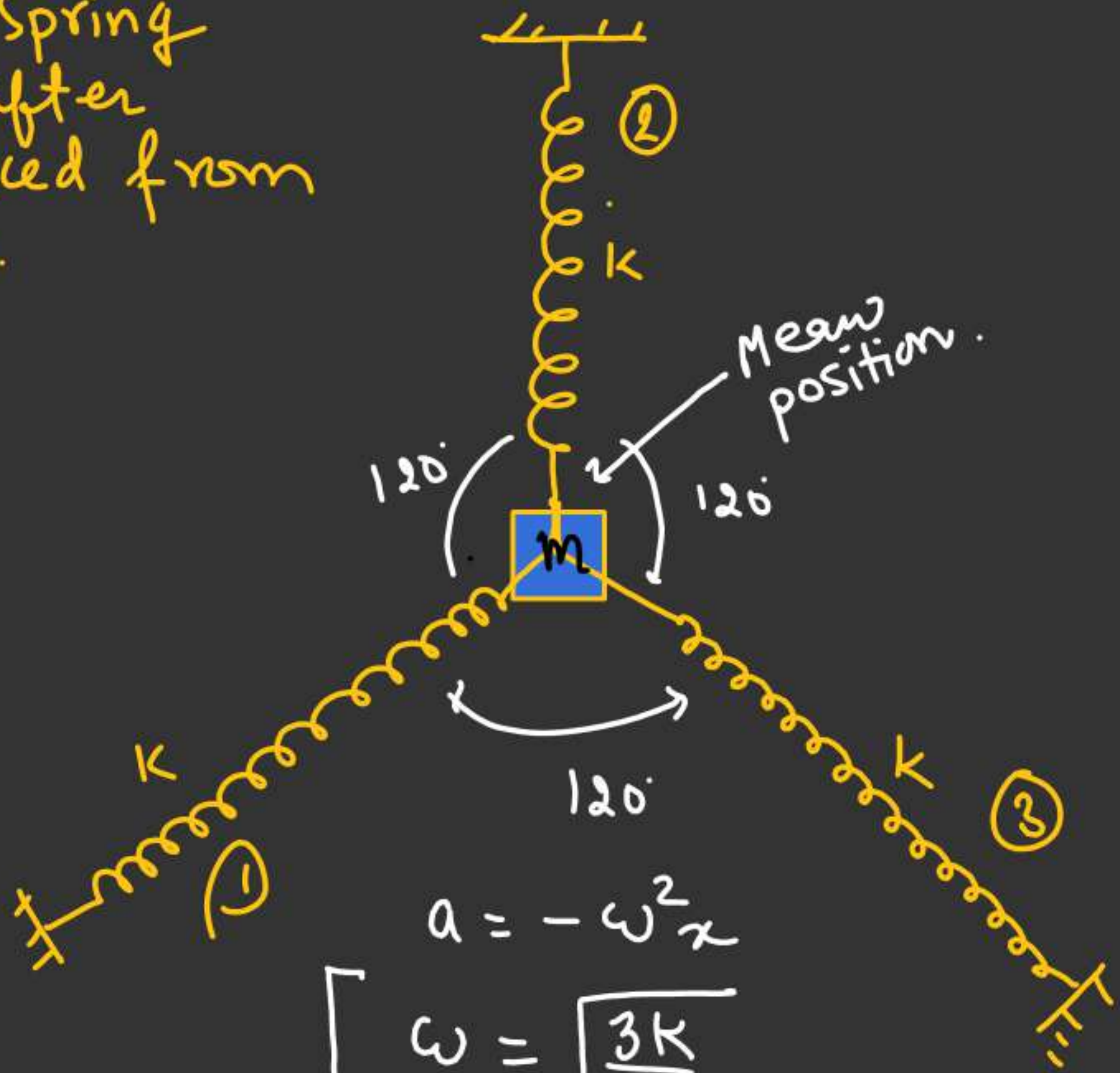
Find time period of block if block is Compressed along one of the Spring and released.

All the Springs are at its natural length.



x is Very Small So.
angle b/w two Spring
remain Same after
block is displaced from
mean position.





$$F_r = -\left[Kx + 2\cdot\left(\frac{Kx}{2}\cos 60^\circ\right)\right]$$

$$F_r = -\left[Kx + \frac{Kx}{2}\right] = -\left[\frac{3Kx}{2}\right]$$

$$a = -\frac{3K}{2m}x$$

$$a = -\omega^2 x$$

$$\omega = \sqrt{\frac{3K}{2m}}$$

$$T = 2\pi\sqrt{\frac{2m}{3K}}$$

Nishant Jindal

Find the time period of Cylinder.
Cylinder roll's without slipping.

M-1 $\quad E_T = C, \quad \frac{dE_T}{dt} = 0.$

→ Write total energy of the System at instant t=t at any intermediate position i.e b/w mean or extreme position.

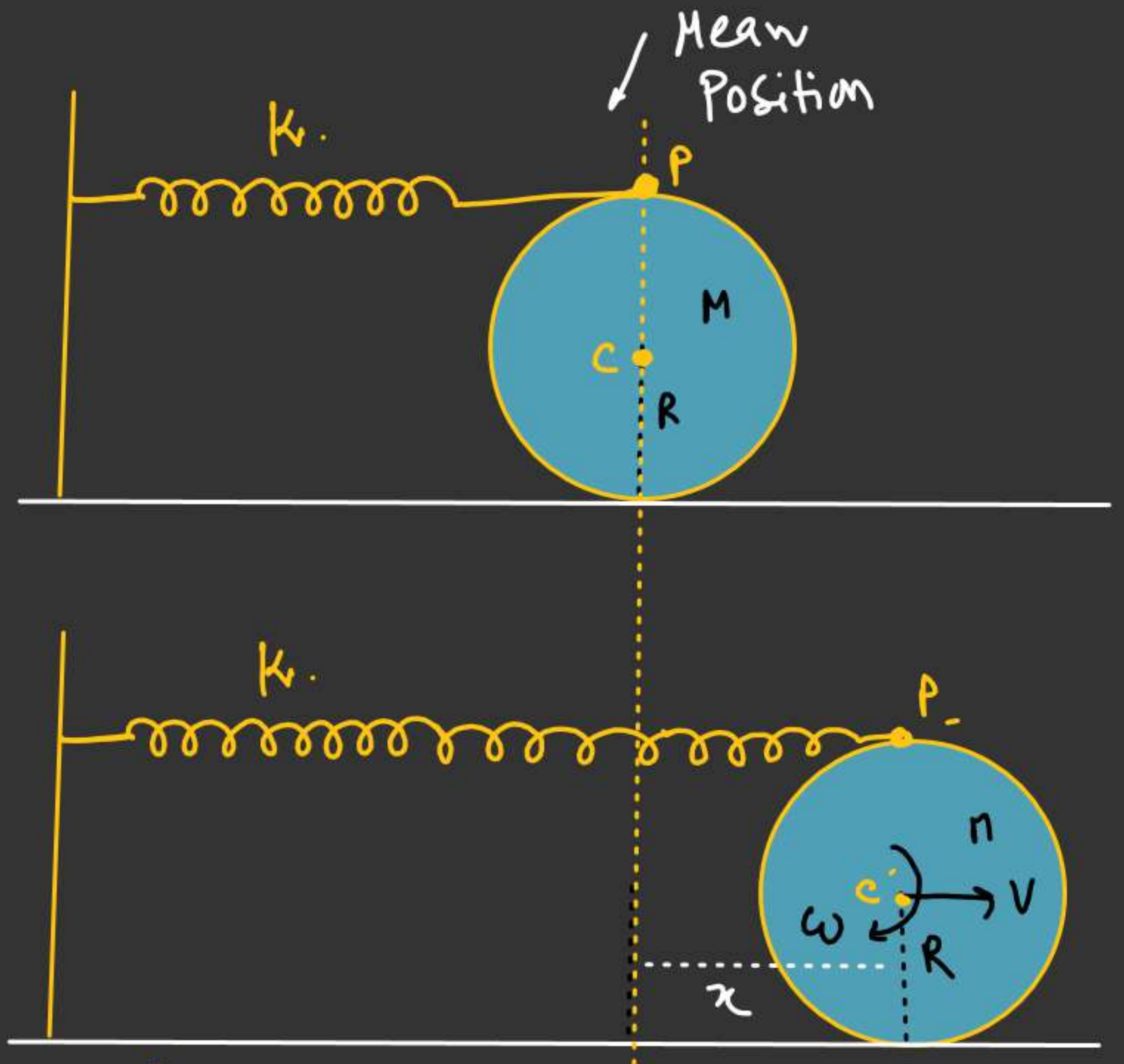
$$E_T = \frac{1}{2}MV^2 + \frac{1}{2}\left(\frac{MR^2}{2}\right)\omega^2 + \frac{1}{2}K(2x)^2$$

$$v = R\omega \Rightarrow \omega = \frac{v}{R}$$

$$E_T = \frac{1}{2}MV^2 + \frac{M\cancel{R^2}}{4} \times \frac{v^2}{\cancel{R^2}} + 2Kx^2$$

$$E_T = \frac{MV^2}{2} + \frac{MV^2}{4} + 2Kx^2$$

$$E_T = \frac{3}{4}MV^2 + 2Kx^2$$



$$\vec{X}_{P/E} = \vec{X}_{P/com} + \vec{X}_{com/E}$$

$$= R\theta + X$$

$$= \underline{2x}$$

$X = R\theta$

Nishant Jindal

$$E_T = \frac{3}{4}Mv^2 + 2Kx^2$$

$$\frac{dE_T}{dt} = \frac{3}{4}M\frac{d}{dt}(v^2) + 2K\frac{d}{dt}(x^2)$$

$$0 = \frac{3}{4}M\left(2v\frac{dv}{dt}\right) + 2K(2x)\left(\frac{dx}{dt}\right)$$

$$0 = \frac{3Mv}{2}\left(\frac{dv}{dt}\right) + 4Kx\left(\frac{dx}{dt}\right)$$

$$\frac{3M\cancel{v}}{2}\left(\frac{dv}{dt}\right) = -4Kx\cancel{\left(\frac{dx}{dt}\right)}$$

$$\Downarrow$$

$$\left(\frac{3M}{2}\right)a = -4Kx$$

$$a = -\frac{8K}{3M}x$$

$$a = -\frac{8K}{3M}x$$

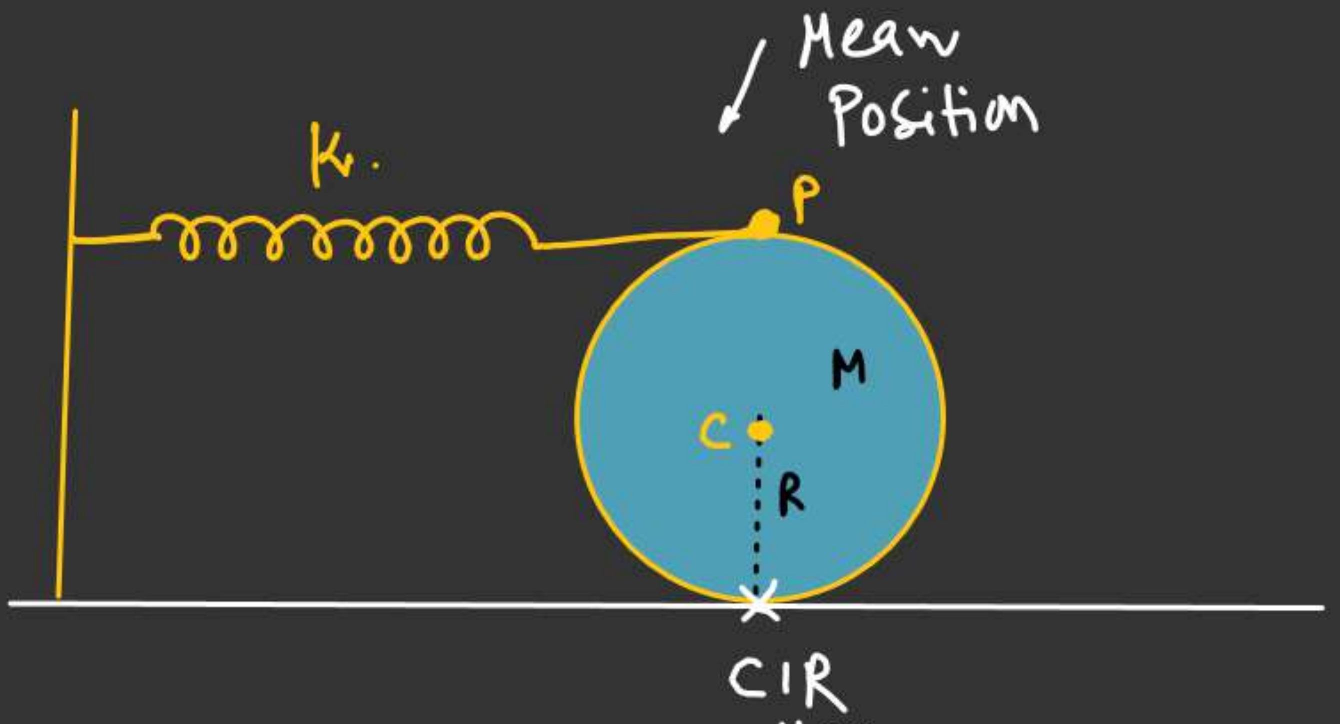
$$a = -\omega^2 x$$

$$\omega = \sqrt{\frac{8K}{3M}}$$

$$T = 2\pi\sqrt{\frac{3M}{8K}} \checkmark$$

Find the time period of cylinder. Cylinder roll's without slipping.

M-2:



$\tau_{res} = \_\_\_$.

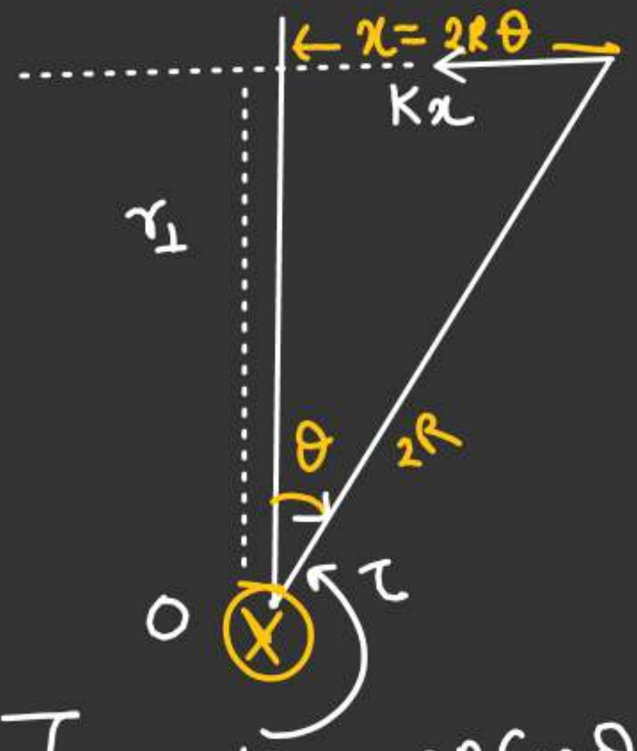
$\alpha = \dfrac{\tau_{res}}{I}$

$\alpha = -\omega^2\theta$

$\sin\theta = \dfrac{x}{2R}$

$x = 2R\sin\theta$

$x = 2R(\theta)$



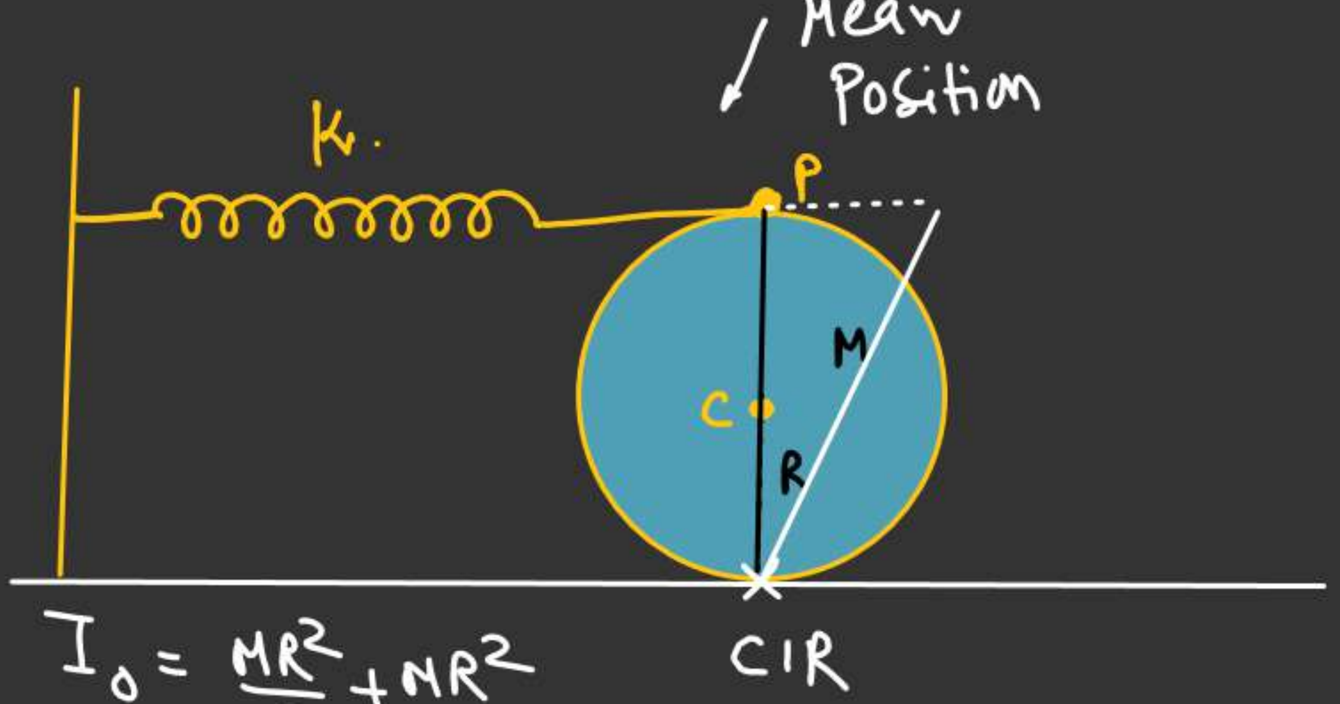
$\theta \to 0$

$\cos\theta \to 1$

$\tau_r = -(Kx)\,2R\underbrace{\cos\theta}_{\to 1}$

$\tau_r = -K(2R\theta)\times 2R$

$\tau_r = -4KR^2\theta$.



$I_0 = \dfrac{MR^2}{2} + MR^2 = \dfrac{3}{2}MR^2$

$\alpha = -\dfrac{4KR^2\theta}{I_0} = \dfrac{-4KR^2\theta}{\left(\frac{3MR^2}{2}\right)} = -\dfrac{8K}{3M}\theta$

$\omega^2 = \dfrac{8K}{3M}$

$T = 2\pi\sqrt{\dfrac{3M}{8K}}$