Nishant Jindal

1. Find 'r', $r>0$ so that circles

$$(x-1)^2+(y-3)^2=r^2 \quad \& \quad (x-4)^2+(y+1)^2=9$$

intersect at 2 distinct points.

$$\sqrt{9+16}=5$$

$$|r-3| < \boxed{5 < r+3}$$

$$r>2$$

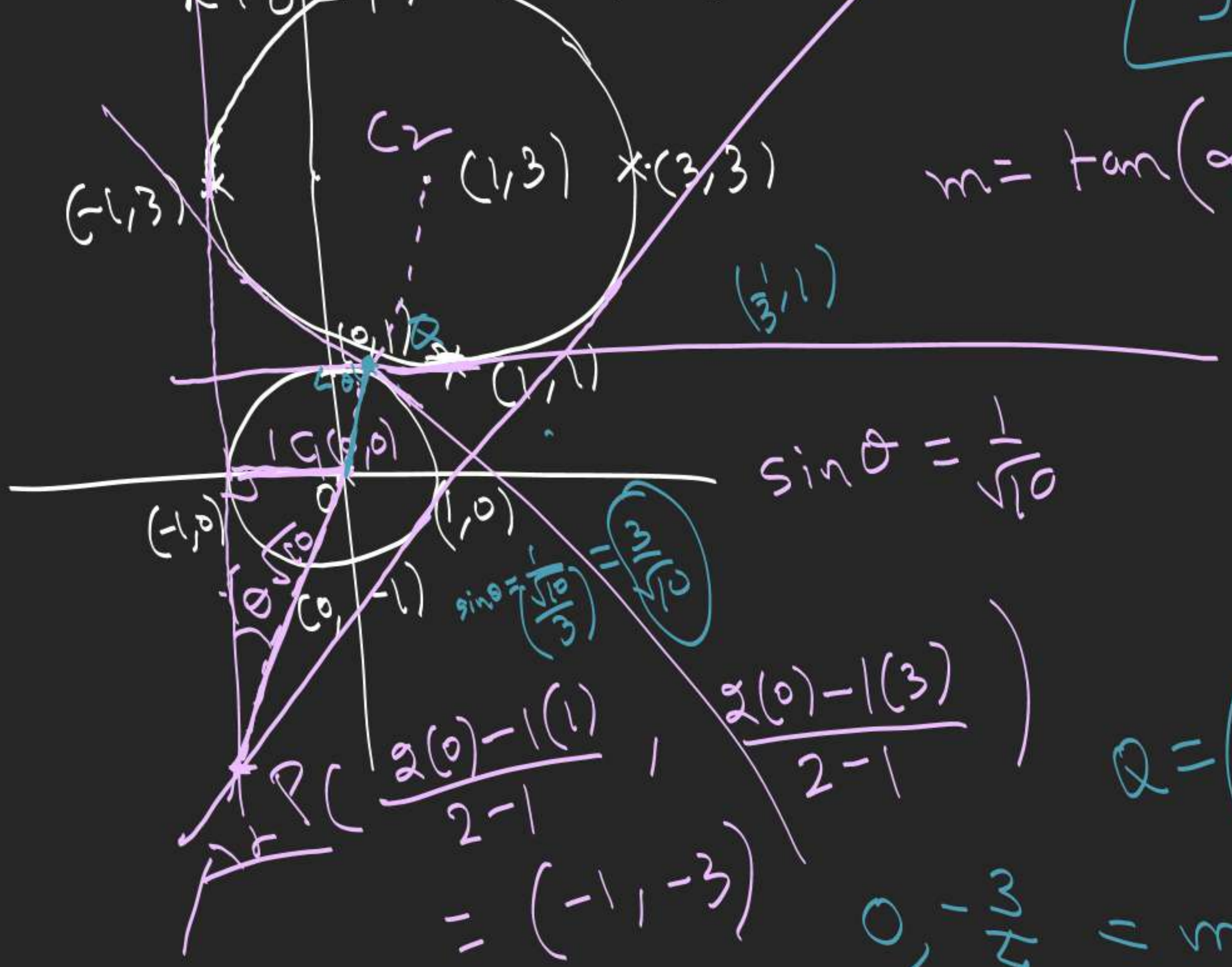
$$-5 < r-3 < 5$$

$$-2 < r < 8$$

$$\boxed{r \in (2,8)}$$

Nishant Jindal

2. Find eqn. of all common tangents to circles $x^2+y^2=1$ and $(x-1)^2+(y-3)^2=4$



$$P\left(\frac{2(0)-1(1)}{2-1}, \frac{2(0)-1(3)}{2-1}\right) = (-1,-3)$$

$\boxed{y=1}$

$$y-1=-\frac{3}{4}\left(x-\frac{1}{3}\right)$$

$$m=\tan(\alpha\pm\theta)=\frac{3+\frac{1}{3}}{1-3(\frac{1}{3})}, \frac{3-\frac{1}{3}}{1+3(\frac{1}{3})}$$

$$\infty, \frac{4}{3}$$

$$\sin\theta=\frac{1}{\sqrt{10}}$$

$$x=-1, \quad y+3=\frac{4}{3}(x+1)$$

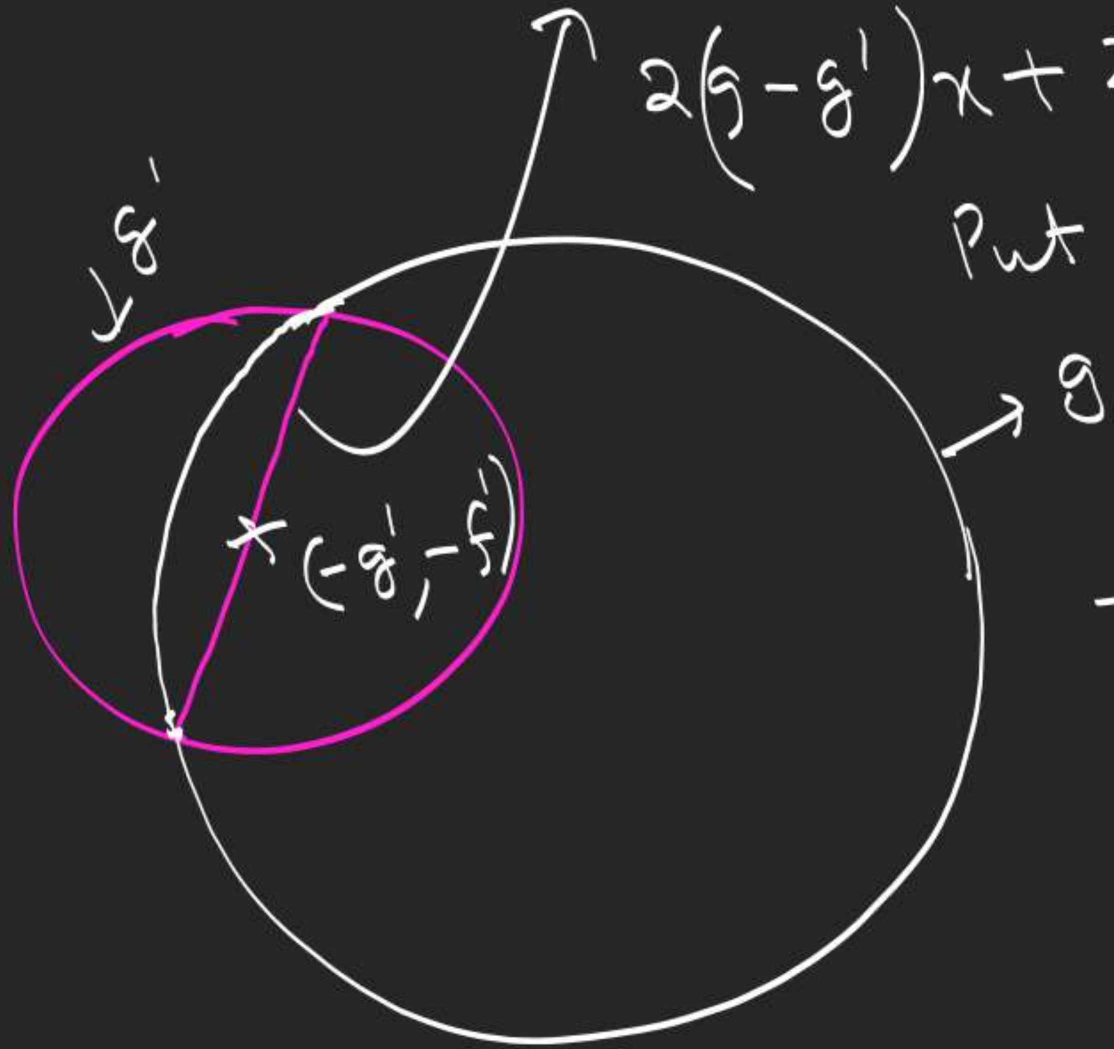
$$Q=\left(\frac{1\times1+2(0)}{1+2}, \frac{1\times3+2\times0}{1+2}\right)=\left(\frac{1}{3},1\right)$$

$$0, -\frac{3}{4} = m = \tan(\alpha\pm\theta)=\frac{3+3}{1-3(3)}, \frac{3-3}{1+3(3)}$$

Nishant Jindal

3: P.T. circle $x^2+y^2+2gx+2fy+c=0$ will bisect the circumference of the circle $x^2+y^2+2g'x+2f'y+c'=0$ if

$$2g'(g-g')+2f'(f-f')=c-c'.$$



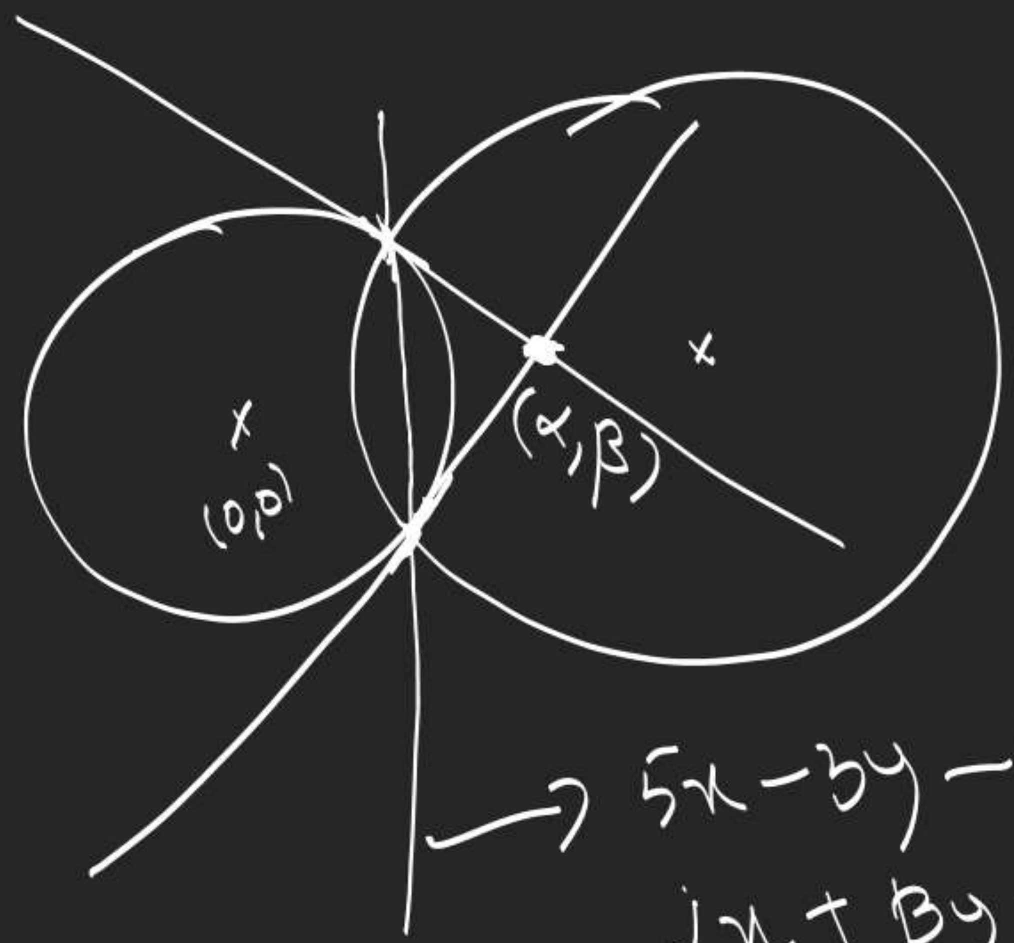
$$2(g-g')x+2(f-f')y+c-c'=0$$

Put $(-g', -f')$

$$-2g'(g-g')-2f'(f-f')+c-c'=0$$

Nishant Jindal

4. Tangents are drawn to the circle $x^2+y^2=12$ at the points where it is met by the circle $x^2+y^2-5x+3y-2=0$. Find the point of intersection of the tangents.



$$5x-3y-10=0$$
$$\alpha x+\beta y-12=0$$

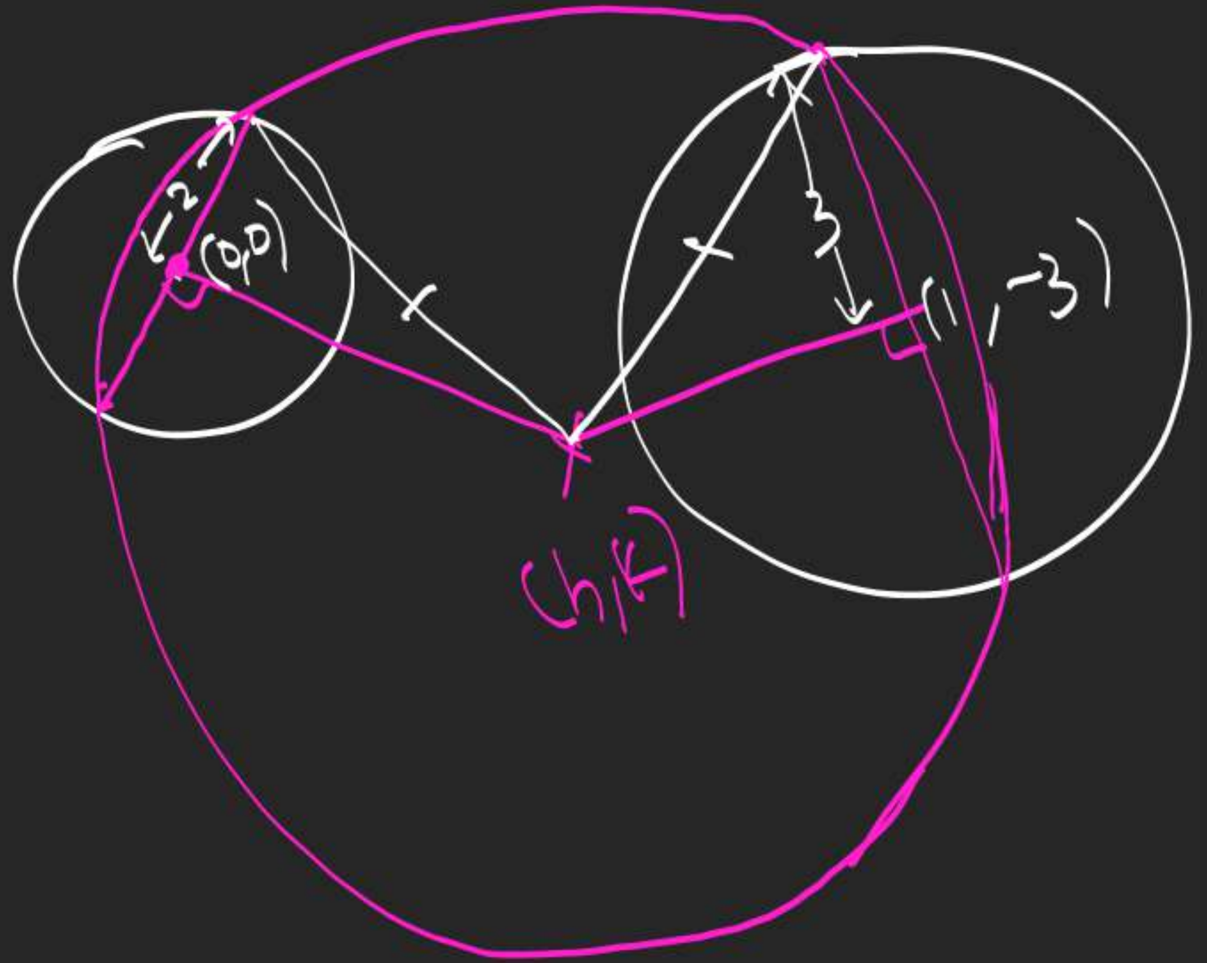
$$\frac{\alpha}{5}=\frac{\beta}{-3}=\frac{6}{5}$$

$$(\alpha,\beta)=\left(6,-\frac{18}{5}\right)$$

Nishant Jindal

5. Find the locus of centre of circles which bisect the circumference of the circles $x^2+y^2=4$ and $x^2+y^2-2x+6y+1=0$

$$h^2+k^2+4=(h-1)^2+(k+3)^2+9$$



$$\boxed{2x-6y=15}$$

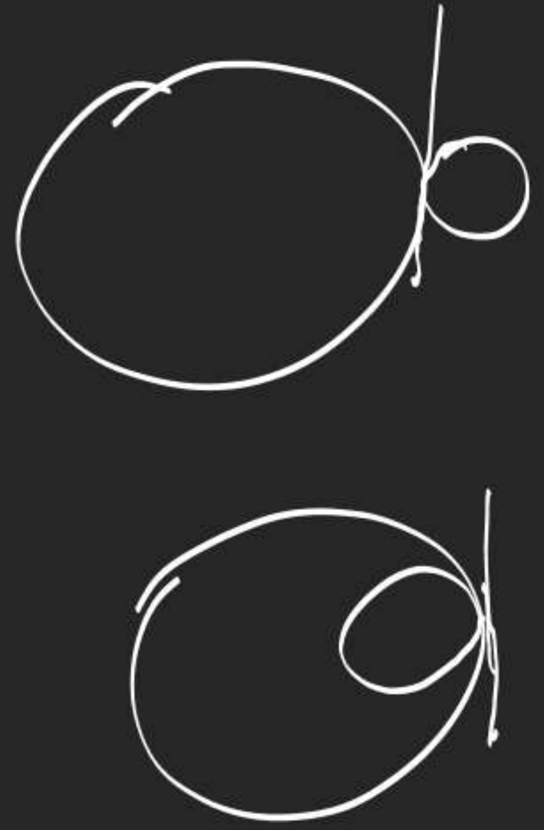
Nishant Jindal

6. P.T. circles $x^2+y^2+2ax+c^2=0$ and $x^2+y^2+2by+c^2=0$ touch each other if $\frac{1}{a^2}+\frac{1}{b^2}=\frac{1}{c^2}$.

$(-a,0)$, $r=\sqrt{a^2-c^2}$

$2ax-2by=0$

$ax-by=0$



$$\frac{|-a^2|}{\sqrt{a^2+b^2}}=\sqrt{a^2-c^2}$$

$$a^4=(a^2-c^2)(a^2+b^2)$$

$$0=-b^2c^2+a^2b^2-a^2c^2$$

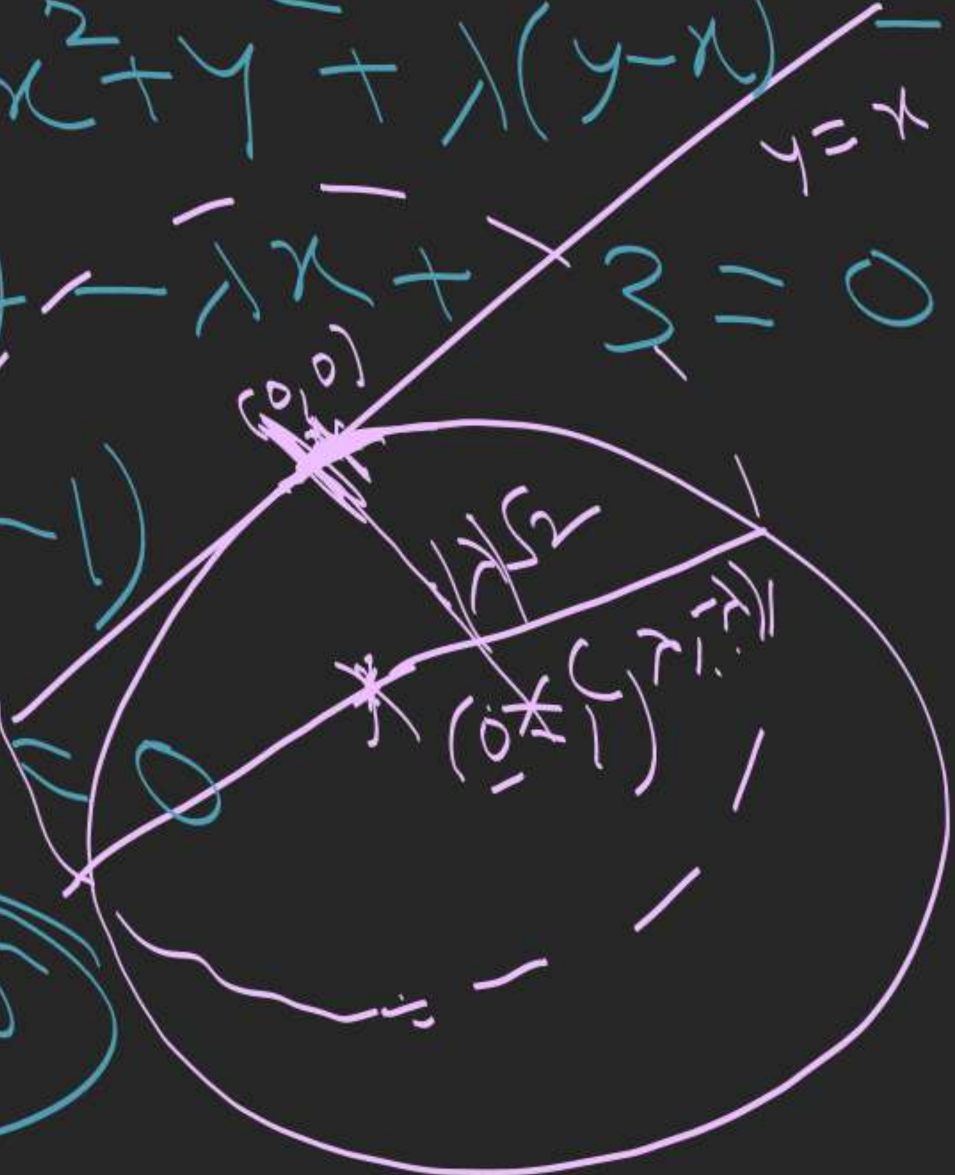
$$\frac{1}{c^2}=\frac{1}{a^2}+\frac{1}{b^2}$$

7: Find the eqn. of circle which bisects the circumference of circle $x^2+y^2+2y-3=0$ and touches the line $y=x$ at origin.

$(0,-1)$ $\quad r=2$

$x^2+y^2+\lambda(y-x)=0$

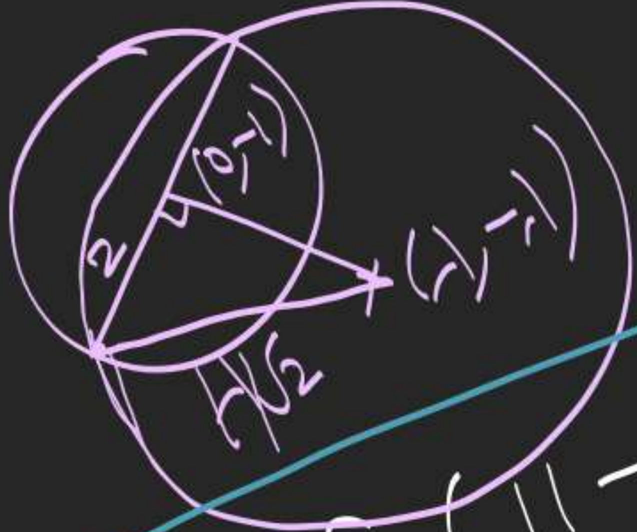
$(\lambda-2)y-\lambda x+3=0$



Put $(0,-1)$

$2-\lambda+3=0$

$\lambda=5$



$\lambda^2+(\lambda-1)^2+4=\lambda^2(2)$

$\lambda=?$

$\boxed{\text{Ex-I}\ (11-25)}$

$-2\lambda+5=0$

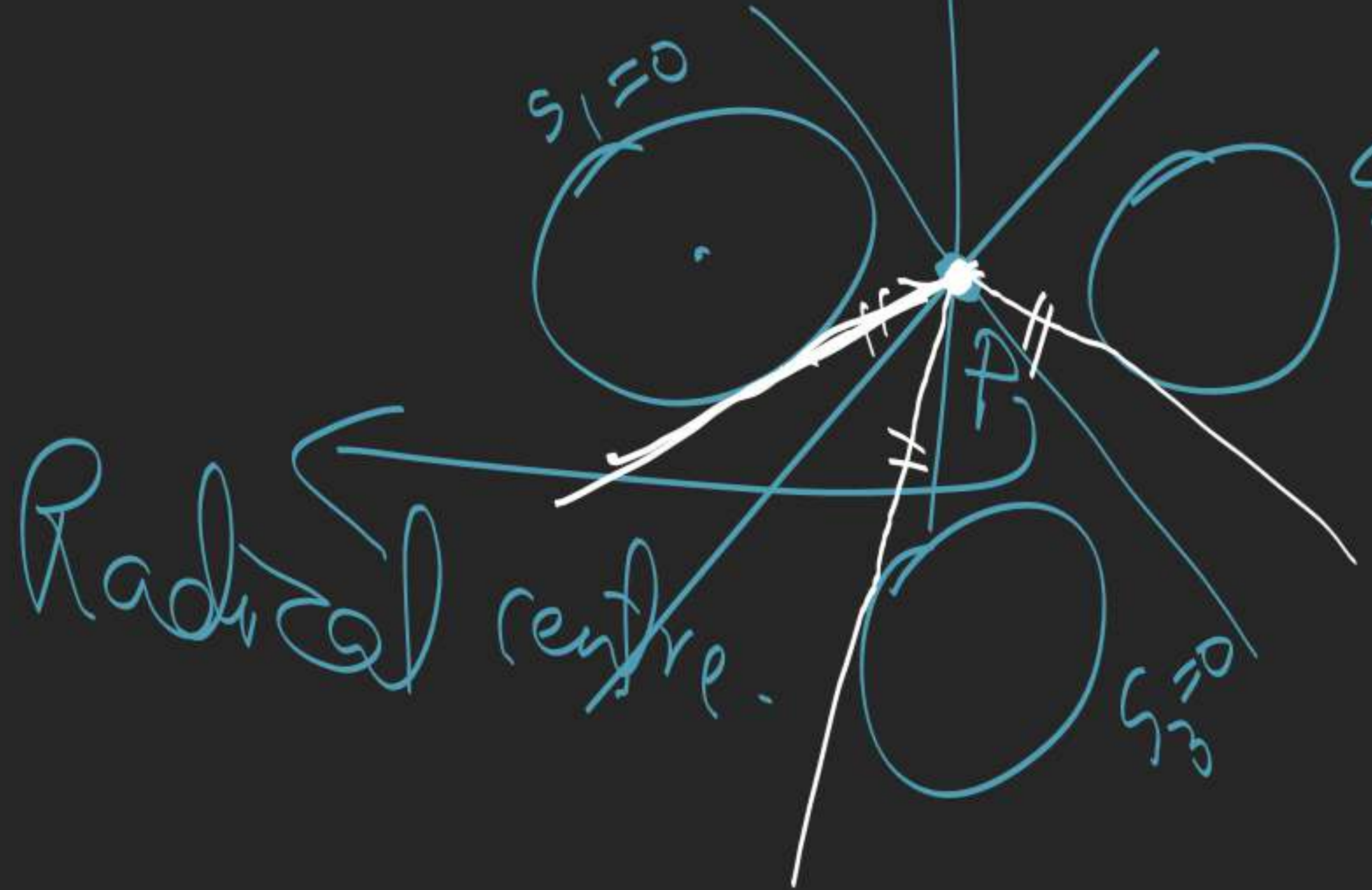
$\lambda=\frac{5}{2}$

$\left(x-\frac{5}{2}\right)^2+\left(y+\frac{5}{2}\right)^2=\left(\frac{5}{\sqrt{2}}\right)^2$

Nishant Jindal

Radical Centre of 3 Circles

RA of 3 circles taken pairwise are always concurrent and their point of concurrence is called radical centre of 3 circles.



$$0=\begin{vmatrix} 2(g_1-g_2) & 2(f_1-f_2) & c_1-c_2 \\ 2(g_2-g_3) & 2(f_2-f_3) & c_2-c_3 \\ 2(g_3-g_1) & 2(f_3-f_1) & c_3-c_1 \end{vmatrix}$$

$R_1 \to R_1+R_2+R_3$

Nishant Jindal