

# EMERGENCY VEHICLE LOCATION MODEL

## CONSIDERING UNCERTAINTY AND THE HIERARCHICAL STRUCTURE OF THE RESOURCES

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# Outline

1. Description of the Portuguese Organization of Emergency Vehicles
2. Mathematical Model
3. Work in progress
4. Conclusions and future work

# 1. Emergency services in Portugal


1. The work of emergency services in Portugal is framed in the National Integrated Medical Emergency System.
2. There are different vehicles with different characteristics:
  - Motorcycles, assistance ambulances, medical emergency ambulances, immediate life support ambulances, medical emergency and resuscitation vehicles, helicopters.
3. These vehicles differ from the point of view of the kind of support they can guarantee (immediate, basic or advanced life support) and by having or not the capacity of transporting patients.
4. These vehicles belong to the National Medical Emergency Institute and to other institutions (Red Cross, Firemen, and so on).
5. All the resources are managed by national Urgent Patient Guidance Centers.



# 1. Example of the geographical area under study



 Advanced Support

 Basic Support

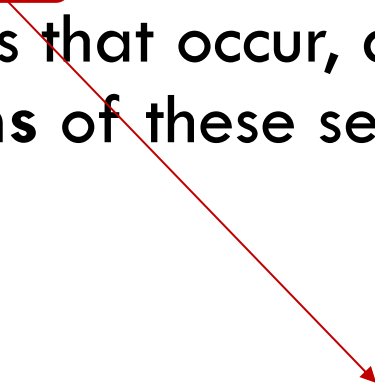
# 1. IMPORTANT CHARACTERISTICS

1. Different types of vehicles capable of providing different levels of emergency healthcare;
2. The possibility of substitutability between emergency vehicles of different types;
3. The possibility of more than one vehicle, of similar or different types, being needed in a given emergency episode, and possibly not arriving all at the same time.

## 2.MATHEMATICAL MODELLING

### Main objective:

To assess whether the available resources are **adequate** considering the pattern of emergency episodes that occur, and what should be the **optimal locations** of these services.



Ensure the necessary care in the appropriate time

## 2. MATHEMATICAL MODELLING

$t \in \{1, \dots, T\}$ : planning horizon

$i \in I$ : set of possible bases for emergency vehicles

$j \in J$ : set of locations where emergency episodes may occur

$N_i$ : maximum number of vehicles that can be positioned in  $i$

$k \in K$ : set of different types of vehicles, where each type  $k$  also determines the level of assistance associated with

$l_k \in \{BLS, ILS, ALS\}$

$v \in V$ : set of existing vehicles. Each vehicle is characterized by being of a certain type ( $k_v$ ).



## 2. MATHEMATICAL MODELLING

$e \in E$ : set of emergency episodes that occur during the considered time horizon. Each emergency episode is characterized by:

- $j_e \in J$ : where the episode takes place
- $n_{ek}$ : number of vehicles of type  $k$  that will need to be deployed
- $TSt_{ek}$ : time period in which the assistance begins with vehicles of type  $k$
- $TSt_{ek}$ : time period in which the assistance ends with vehicles of type  $k$

## 2.MATHEMATICAL MODELLING

Coverage Matrix:

$$a_{ijl} = \begin{cases} 1, & \text{if vehicles located at } i \text{ are within the maximum} \\ & \text{times defined for level of assistance } l \text{ regarding location } j \\ & (j \text{ is covered by vehicles in } i, \text{ for assistance level } l) \\ 0, & \text{otherwise} \end{cases} \quad \begin{matrix} i \in I, j \in J, \\ l \in \{ILS, ALS, BLS\} \end{matrix}$$

## 2. MATHEMATICAL MODELLING

Substitution Matrix:

$$c_{kk'} = \begin{cases} 1, & \text{if a vehicle of type } k' \text{ can substitute other vehicle of type } k \\ 0, & \text{otherwise} \end{cases}, k, k' \in K$$

## 2.MATHEMATICAL MODELLING

Incompatibilities Matrix

$$b_{ee'k} = \begin{cases} 1, & \text{if episodes } e \text{ and } e' \text{ have intersection time periods respecting} \\ & \text{the assistance by vehicles of type } k, \text{ so that a vehicle cannot be} \\ & \text{simultaneously assigned to both} \\ 0, & \text{otherwise} \end{cases}, e, e' \in E, k \in K$$

## 2.MATHEMATICAL MODELLING

### Decision variables

$$y_i = \begin{cases} 1, & \text{if location } i \text{ is going to be prepared to receive vehicles} \\ 0, & \text{otherwise} \end{cases}, i \in I$$

$$h_{vi} = \begin{cases} 1, & \text{if vehicle } v \text{ is located at } i \\ 0, & \text{otherwise} \end{cases}, v \in V, i \in I$$

$$z_e = \begin{cases} 1, & \text{if episode } e \text{ receives all the needed vehicles within the maximum time limit} \\ 0, & \text{otherwise} \end{cases}, e \in E$$

$$x_{viek} = \begin{cases} 1, & \text{if vehicle } v, \text{ located at } i, \text{ is assigned to episode } e \\ & \text{to fulfill level } k \text{ needs (even if it is of a higher level)} \\ 0, & \text{otherwise} \end{cases}, v \in V, i \in I, e \in E, k \in K$$

*The idea is not optimizing the dispatchment of vehicles, but to include the unavailability of vehicles in case they are already assigned to other emergency episodes.*

## 2.MATHEMATICAL MODELLING

$$\textit{Max } Z = \sum_{e \in E} z_e$$

## 2.MATHEMATICAL MODELLING

- An episode is considered "covered" if and only if it receives all the necessary vehicles of the needed types.

$$n_{ek}z_e \leq \sum_{i \in I} \sum_{v \in V} c_{kk_v} x_{viek}, \forall e, k$$

- An emergency vehicle can only contribute to one given level of assistance, in each episode.

$$\sum_{i \in I} \sum_{k \in K} x_{viek} \leq 1, \forall e, v$$

## 2. MATHEMATICAL MODELLING

- The emergency vehicle  $v$  can only attend episode  $e$  from  $i$  if it is located there, and if  $e$  occurs within the coverage radius of  $i$  for that level of care, and it is available.

$$x_{viek} \leq d_{ev} a_{ij_e l_k} h_{vi}, \forall v, i, e, k$$

- An emergency vehicle can only be assigned to two episodes if their occurrence time periods do not intersect.

$$\sum_{k' \in K} x_{viek'} + \sum_{k' \in K} x_{vie'k'} \leq 2 - b_{ee'k}, \forall v, k, i, e, e': e < e'$$



## 2. MATHEMATICAL MODELLING

- There is a maximum number of vehicles that can be located at each base, and vehicles can only be assigned to a base that has been prepared.

$$\sum_{v \in V} h_{vi} \leq N_i y_i, \forall i$$

- Each emergency vehicle can only be located at one base.

$$\sum_{i \in I} h_{vi} \leq 1, \forall v$$

## 2. MATHEMATICAL MODELLING



## 2. MATHEMATICAL MODELLING



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$$\begin{aligned} \sum_{i \in I} \sum_{k' \in K} x_{vie'k'} \leq & 1 - \left( b_{ee'k} - \sum_{i \in I} x_{viek} - z_e \right) + \frac{\sum_{v' \in V} \sum_{i \in I} x_{v'iek}}{n_{ek}} + M(1 - c_{kk_v}) \\ & + \sum_{i \in I} \sum_{k' \in K} \sum_{e'' : e'' < e} b_{e''ek'} x_{vie''k'}, \forall v, e < e', k : n_{ek} \geq 1 \end{aligned}$$

## 2.MATHEMATICAL MODELING

This problem is inherently stochastic, since it is not possible to know *a priori* what are the emergency episodes that will occur.

The stochastic version of the model considers uncertainty represented by scenarios, that differ in the number and features of the emergency episodes.

## 2. MATHEMATICAL MODELLING

### Decision variables

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Are not scenario dependent

Are now scenario dependent,  
needing an extra index  $s$

### 3. WORK IN PROGRESS

We are working on building a case study based on real data. The results will be compared with the present real situation.

We are also developing a new model that will not only assess whether the available resources are adequate but will also consider the possibility of acquiring new resources and locate them in an optimal way.

We are studying the compromises between maximum allowed coverage times and the need for resources, as well as differences between the service in regional and urban areas.

Computational results are under way to understand what is the maximum number of scenarios that it will be possible to incorporate.

## 4. CONCLUSIONS AND FUTURE WORK

1. Including in the model the possibility of partial coverage.
2. Considering multiobjective approaches.
3. Developing a metaheuristic.



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