



# Competition Effects and Transfers in Rail Rapid Transit Network Design

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# Introduction

- The problem of increasing traffic congestion has raised the concerns about energy constraints and greenhouse emissions.
- Increasing mobility and longer journeys caused by the growth of cities have stimulated the construction and expansion of rail transit systems (metro, urban rail, light rail).
- The strategic and tactical railway planning problems may be summarized by the two following steps:
  - the railway network design problem;
  - and the line planning problem.

# Rail rapid transit network design

- Designing a Rapid Transit Network: strategic.

It may reduce traffic congestion, passenger travel, time and pollution.

- Main goals:
  - location decisions;
  - and the maximum coverage of the demand for the new public network.
- List of potential rapid transit corridors and stations,
- Topology design
- and budget availability.

# Line planning

- The following step: planning lines (tactical planning level).
- It consists of designing a line system such that all travel demands are satisfied and certain objectives are met:
  - maximizing the service towards the passengers;
  - and minimizing the operating costs of the railway system.
- In this phase the system capacity is considered.

# Contributions

We present a mixed integer non-linear programming model for the rail rapid transit network design problem. Our major contributions include:

1. introduction of competition effects through a logit model;
2. introduction of transfers in the modeling approach; a decisive attribute for attracting passengers;
3. computational experiments in real networks.

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# Infrastructure

- The rapid transit network consists of arcs and nodes.
- We assume that the location of the potential stations are given.
- Each node has an associated construction cost and each arc a construction cost and a distance.
- Lines support the design: but neither frequencies nor capacity.
- The new infrastructure: not isolated from the current network.
- We consider the existence of a current transport network formed by different modes of transport.



# Passenger demand I

- Passenger groups: origin centroid, destination centroid, and passenger group size.
- The number of potential passengers from each origin to each destination is in average given.
- Passengers choose a path (new or current network).

# Passenger demand II

- Demand will choose its path based on the generalized travel cost (distance).

$$P_{new}^w = \frac{e^{-(v^{new} + \beta u_w^{new})}}{e^{-(v^{new} + \beta u_w^{new})} + e^{-(v^{cur} + \beta u_w^{cur})}}$$

- Generalized costs for the current network are not well known.
- Congestion: different scenarios in the current network.

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# Input Notation

- Parameters:

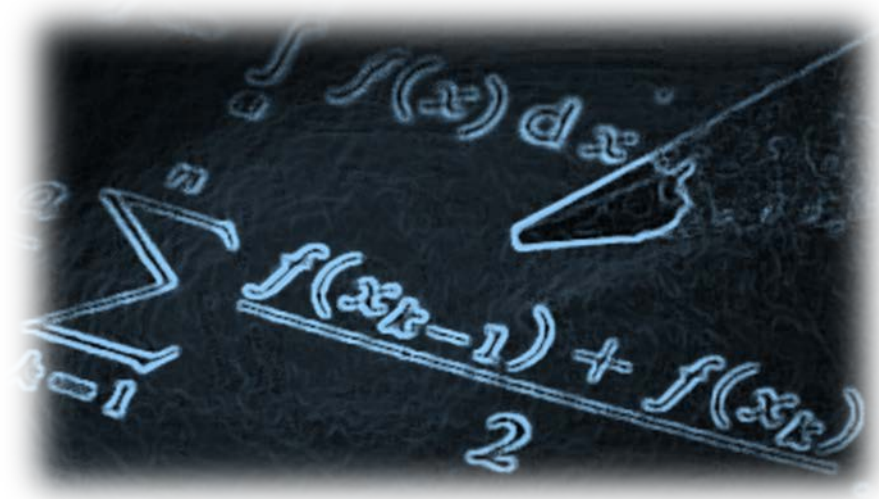
$d_{ij}$  length of arc (ij).

$c_{ij}$  cost of constructing an arc (ij).

$c_{\max}$  upper budget bound.

$g_w$  number of passengers in passenger group  $w = (o(w); d(w))$ .

$u_{cur}^w$  generalized cost for passenger group  $w$  through the current network.



# Output Notations

$x_{ij}^l$  binary variable: = 1 if line  $l$  is located using the arc  $(ij)$ ; = 0, otherwise.

$y_i^l$  binary variable: = 1 if station  $i$  is located; = 0, otherwise.

$f_{ij}^w$  binary variable: = 1 if demand  $w$  uses arc  $(ij)$  in the new network; = 0, otherwise.

$P_{new}^w$  probability for a passenger in demand  $w$  of selecting the new network.

$u_w^{new}$  the generalized cost in the new network for passenger group  $w$ .

$\vartheta_w$  binary variable: = 1 if a path within the new network exists for demand  $w$ ; 0, otherwise.

$\tau_w^l$  binary variable: = 1 if demand  $w$  uses line  $l$ , and 0 otherwise.



# Model formulation

$$\min z = \alpha z_{cur} + \beta z_{loc} + \gamma z_{route}$$

Objective function

$$z_{cur} = \sum_{w \in W} g_w (1 - P_{new}^w),$$

Demand coverage

$$z_{loc} = \sum_{l \in L} \sum_{(ij) \in A_r, i < j} c_{ij} x_{ij}^l + \sum_{i \in N_r} c_i \psi_i,$$

Location costs

$$z_{route} = \sum_{w \in W} u_w^{new}.$$

Routing costs

# Model formulation

$$z_{loc} \leq c_{\max}$$

Budget

$$x_{ij}^l \leq y_i^l$$

$$\forall (ij) \in A_r : i < j, \forall l \in L$$

$$x_{ij}^l \leq y_j^l$$

$$\forall (ij) \in A_r : i < j, \forall l \in L$$

$$x_{ij}^l = x_{ji}^l$$

$$\forall (ij) \in A_r : i < j, \forall l \in L$$

$$y_i^l \leq \psi_i$$

$$\forall i \in N_r, \forall l \in L$$

Line location

$$\sum_{j \in N_r(i) : i < j} x_{ij}^l + \sum_{j \in N_r(i) : i < j} x_{ji}^l \leq 2$$

$$\forall i \in N_r, \forall l \in L$$

$$\sum_{(ij) \in B : i < j} x_{ij}^l \leq |B| - 1$$

$$\forall l \in L, \forall B \subset N_r, |B| \geq 2$$

Line paths

+ other: location constraints, line constraints, etc.

# Model formulation

$$u_w^{new} = \sum_{(ij) \in A_r \cup A_d \cup A_o} d_{ij} f_{ij}^w + \nu_w \left( \sum_{l \in L} \tau_w^l - \vartheta_w \right) + \varphi_w (1 - \vartheta_w) \quad \forall w \in W$$

Generalized cost

$$\vartheta_w \leq \sum_{(ij) \in A_r \cup A_d \cup A_o} d_{ij} f_{ij}^w \quad \forall w \in W$$

$$P_{new}^w = \frac{e^{-(v^{new} + \beta u_w^{new})}}{e^{-(v^{new} + \beta u_w^{new})} + e^{-(v^{cur} + \beta u_w^{cur})}} \quad \forall w \in W$$

$$o_{o(w)}^w \geq P_{new}^w \quad \forall w \in W$$

Mode choice

$$d_{d(w)}^w \geq P_{new}^w \quad \forall w \in W$$

$$\sum_{k \in N(i)} f_{ki}^w - \sum_{j \in N(i)} f_{ij}^w = -o_{o(w)}^w + d_{d(w)}^w \quad \forall i \in N, w \in W$$

Flow conservation

$$M_\tau \tau_w^l \geq \sum_{(i,j) \in A_r} f_{ij}^w x_{ij}^l \quad \forall w \in W, l \in L$$

Transfers

+ other: location-allocation constraints, etc.

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# Lagrangian relaxation

- The objective function in the Lagrangian relaxation approach is as follows:

$$\min \quad \alpha z_{cur} + \beta z_{loc} + \gamma z_{route} + \sum_{ijw} (f_{ij}^w + f_{ji}^w - \chi_{ij}) \mu_{ij}^w$$

- The dual function is the function defined by:

$$\mathcal{R}^+ \ni \mu \rightarrow \theta(\mu) := \min_{\chi \in \chi} \mathcal{L}(\chi, \mu).$$

- The dual problem is then:

$$\max \theta(\mu), \mu \in \mathcal{R}^+.$$



# Lagrangian relaxation

- Two submodels:

$$\min \quad \beta z_{loc} - \sum_{ijw} \chi_{ij} \mu_{ij}^w$$

Location submodel

$$\min \quad \alpha z_{cur} + \gamma z_{route} + \sum_{ijw} (f_{ij}^w + f_{ji}^w) \mu_{ij}^w$$

Passenger submodel

# Lagrangian relaxation

- For given values of the duals we are able to solve problem easily.
- We use a cutting plane method to estimate the value of the duals at each iteration.

$$\begin{aligned} \max r \\ r &\leq \theta(\mu_{it}) + g^{it,T}(\mu - \mu_{it}) && \forall it \in IT \\ r &\in \mathcal{R} \\ \mu &\in \mathcal{R}^+. \end{aligned}$$

- We subtract the following stabilization term to the objective function:

$$\frac{1}{2t} \|\mu - \hat{\mu}\|^2$$

# Recovering the solution

- Store the location variables for each iteration in:  $\chi_{ij}^{it}$ .
- Construct a linear combination of the solutions at each iteration:  $\chi_{ij} = \sum \lambda_{it} \chi_{ij}^{it}$ .
- Solve the original model using  $\lambda_{it}$  as decision variables, imposing  $\sum_{it} \lambda_{it} = 1$ .

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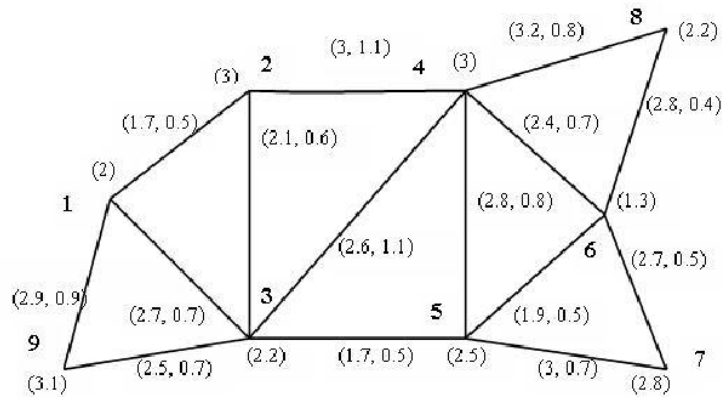
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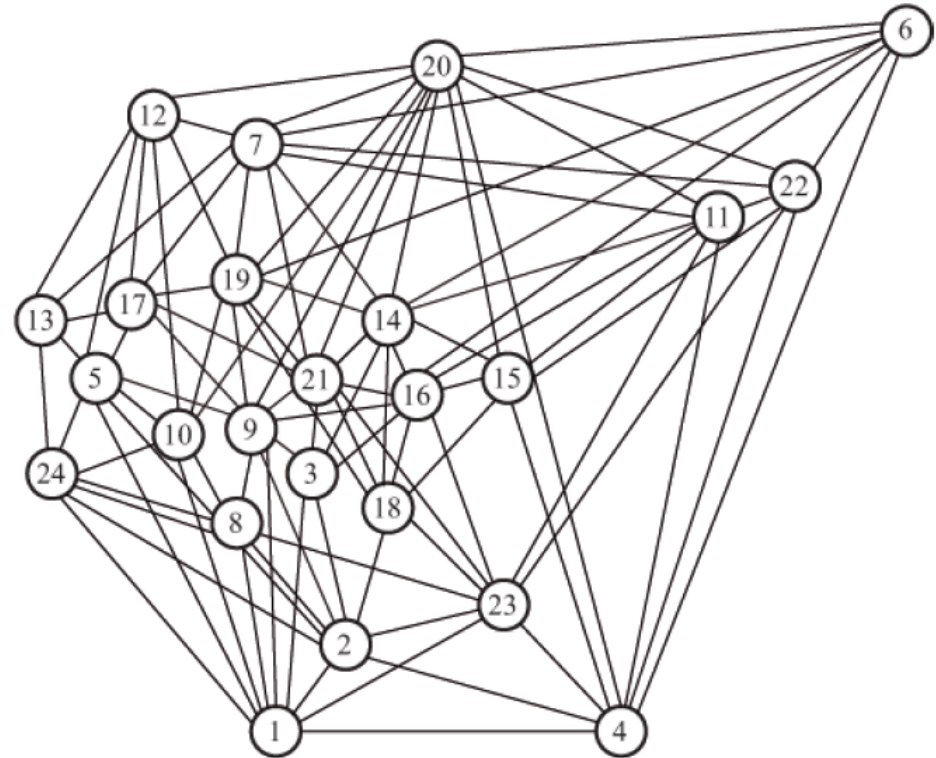
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# Case study



Network R1



Seville network



# Case study

**Network R1**

101 iterations

2 minutes



# Case study

## Competition effects & Transfers

Total demand: 1044  
Captured demand: 646.56  
# Transfers: 199.14

## Competition effects & No Transfers

Total demand: 1044  
Captured demand: 654.47  
# Transfers: 457.71

## No Competition effects & No Transfers

Total demand: 1044  
Captured demand: 980  
# Transfers: 648

# Conclusions

- We have proposed a new formulation for the rapid transit network design problem:
  - We have integrated the network design and the line planning.
  - We have modeled the transfers of passengers and minimized them;
  - And we have introduced competition effects in order to better estimate demand coverage.
- Future research: robust solutions. Further research needs to redefine the concept of robustness: recoverable robustness.

# THANK YOU FOR YOUR ATTENTION

## Any question, comment?

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