



Competition Effects and Transfers in Rail Rapid Transit Network Design

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Introduction

- The problem of increasing traffic congestion has raised the concerns about energy constraints and greenhouse emissions.
- Increasing mobility and longer journeys caused by the growth of cities have stimulated the construction and expansion of rail transit systems (metro, urban rail, light rail).
- The strategic and tactical railway planning problems may be summarized by the two following steps:
 - the railway network design problem;
 - and the line planning problem.

Rail rapid transit network design

- Designing a Rapid Transit Network: strategic.
It may reduce traffic congestion, passenger travel, time and pollution.
- Main goals:
 - location decisions;
 - and the maximum coverage of the demand for the new public network.
- List of potential rapid transit corridors and stations,
- Topology design
- and budget availability.

Line planning

- The following step: planning lines (tactical planning level).
- It consists of designing a line system such that all travel demands are satisfied and certain objectives are met:
 - maximizing the service towards the passengers;
 - and minimizing the operating costs of the railway system.
- In this phase the system capacity is considered.

Contributions

We present a mixed integer non-linear programming model for the rail rapid transit network design problem. Our major contributions include:

1. introduction of competition effects through a logit model;
2. introduction of transfers in the modeling approach; a decisive attribute for attracting passengers;
3. computational experiments in real networks.

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Infrastructure

- The rapid transit network consists of arcs and nodes.
- We assume that the location of the potential stations are given.
- Each node has an associated construction cost and each arc a construction cost and a distance.
- Lines support the design: but neither frequencies nor capacity.
- The new infrastructure: not isolated from the current network.
- We consider the existence of a current transport network formed by different modes of transport.

Passenger demand I

- Passenger groups: origin centroid, destination centroid, and passenger group size.
- The number of potential passengers from each origin to each destination is in average given.
- Passengers choose a path (new or current network).

Passenger demand II

- Demand will choose its path based on the generalized travel cost (distance).

$$P_{new}^w = \frac{e^{-(v^{new} + \beta u_w^{new})}}{e^{-(v^{new} + \beta u_w^{new})} + e^{-(v^{cur} + \beta u_w^{cur})}}$$

- Generalized costs for the current network are not well known.
- Congestion: different scenarios in the current network.

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Input Notation

- Parameters:

d_{ij} length of arc (ij).

c_{ij} cost of constructing an arc (ij).

c_{\max} upper budget bound.

g_w number of passengers in passenger group w = (o(w); d(w)).

u_{cur}^w generalized cost for passenger group w through the current network.

A chalkboard with several mathematical expressions written in blue chalk. At the top left, there is a summation symbol with a large bracket underneath, followed by $f(x) dx$. To the right of this is another expression involving $f(x_{t-1}) + f(x_t)$. Below these, there is a large bracket spanning across the board, and at the bottom right, the number '2' is written.

Output Notations

x_{ij}^l binary variable: = 1 if line l is located using the arc (ij); = 0, otherwise.

y_i^l binary variable: = 1 if station i is located; = 0, otherwise.

f_{ij}^w binary variable: = 1 if demand w uses arc (ij) in the new network; = 0, otherwise.

P_{new}^w probability for a passenger in demand w of selecting the new network.

u_w^{new} the generalized cost in the new network for passenger group w.

ϑ_w binary variable: = 1 if a path within the new network exists for demand w; 0, otherwise.

τ_w^l binary variable: =1 if demand w uses line l, and 0 otherwise.

Model formulation

$$\min z = \alpha z_{cur} + \beta z_{loc} + \gamma z_{route}$$

Objective function

$$z_{cur} = \sum_{w \in W} g_w (1 - P_{new}^w),$$

Demand coverage

$$z_{loc} = \sum_{l \in L} \sum_{(ij) \in A_r, i < j} c_{ij} x_{ij}^l + \sum_{i \in N_r} c_i \psi_i,$$

Location costs

$$z_{route} = \sum_{w \in W} u_w^{new}.$$

Routing costs

Model formulation

$$z_{loc} \leq c_{\max}$$

$$x_{ij}^l \leq y_i^l \quad \forall (ij) \in A_r : i < j, \forall l \in L$$

$$x_{ij}^l \leq y_j^l \quad \forall (ij) \in A_r : i < j, \forall l \in L$$

$$x_{ij}^l = x_{ji}^l \quad \forall (ij) \in A_r : i < j, \forall l \in L$$

$$y_i^l \leq \psi_i \quad \forall i \in N_r, \forall l \in L$$

Budget

$$\sum_{j \in N_r(i): i < j} x_{ij}^l + \sum_{j \in N_r(i): i < j} x_{ji}^l \leq 2 \quad \forall i \in N_r, \forall l \in L$$

$$\sum_{(ij) \in B: i < j} x_{ij}^l \leq |B| - 1 \quad \forall l \in L, \forall B \subset N_r, |B| \geq 2$$

Line location

Line paths

+ other: location constraints, line constraints, etc.

Model formulation

$$u_w^{new} = \sum_{(ij) \in A_r \cup A_d \cup A_o} d_{ij} f_{ij}^w + \nu_w \left(\sum_{l \in L} \tau_w^l - \vartheta_w \right) + \varphi_w (1 - \vartheta_w) \quad \forall w \in W$$

$$\vartheta_w \leq \sum_{(ij) \in A_r \cup A_d \cup A_o} d_{ij} f_{ij}^w \quad \forall w \in W$$

$$P_{new}^w = \frac{e^{-(v^{new} + \beta u_w^{new})}}{e^{-(v^{new} + \beta u_w^{new})} + e^{-(v^{cur} + \beta u_w^{cur})}} \quad \forall w \in W$$

$$o_{o(w)}^w \geq P_{new}^w \quad \forall w \in W$$

$$d_{d(w)}^w \geq P_{new}^w \quad \forall w \in W$$

$$\sum_{k \in N(i)} f_{ki}^w - \sum_{j \in N(i)} f_{ij}^w = -o_{o(w)}^w + d_{d(w)}^w \quad \forall i \in N, w \in W$$

$$M_\tau \tau_w^l \geq \sum_{(i,j) \in A_r} f_{ij}^w x_{ij}^l \quad \forall w \in W, l \in L$$

Generalized cost

Mode choice

Flow conservation

Transfers

+ other: location-allocation constraints, etc.

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Lagrangian relaxation

- The objective function in the Lagrangian relaxation approach is as follows:

$$\min \quad \alpha z_{cur} + \beta z_{loc} + \gamma z_{route} + \sum_{ijw} (f_{ij}^w + f_{ji}^w - \chi_{ij}) \mu_{ij}^w$$

- The dual function is the function defined by:

$$\mathcal{R}^+ \ni \mu \rightarrow \theta(\mu) := \min_{x \in X} \mathcal{L}(x, \mu).$$

- The dual problem is then:

$$\max \theta(\mu), \mu \in \mathcal{R}^+.$$

Lagrangian relaxation

- Two submodels:

$$\min \beta z_{loc} - \sum_{ijw} \chi_{ij} \mu_{ij}^w \quad \text{Location submodel}$$

$$\min \alpha z_{cur} + \gamma z_{route} + \sum_{ijw} (f_{ij}^w + f_{ji}^w) \mu_{ij}^w \quad \text{Passenger submodel}$$

Lagrangian relaxation

- For given values of the duals we are able to solve problem easily.
- We use a cutting plane method to estimate the value of the duals at each iteration.

$$\max r$$

$$r \leq \theta(\mu_{it}) + g^{it,T}(\mu - \mu_{it}) \quad \forall it \in IT$$

$$r \in \mathcal{R}$$

$$\mu \in \mathcal{R}^+.$$

- We subtract the following stabilization term to the objective function:

$$\frac{1}{2t} \|\mu - \hat{\mu}\|^2$$

Recovering the solution

- Store the location variables for each iteration in: χ_{ij}^{it} .
- Construct a linear combination of the solutions at each iteration: $\chi_{ij} = \sum \lambda_{it} \chi_{ij}^{it}$.
- Solve the original model using λ_{it} as decision variables, imposing $\sum_{it} \lambda_{it} = 1$.

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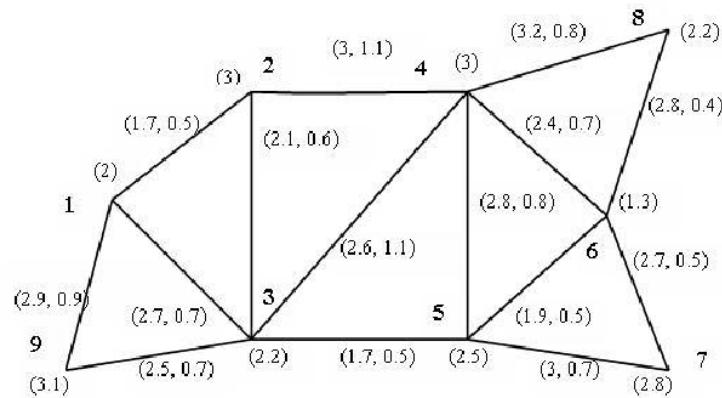
2. Problem description

3. Optimisation model

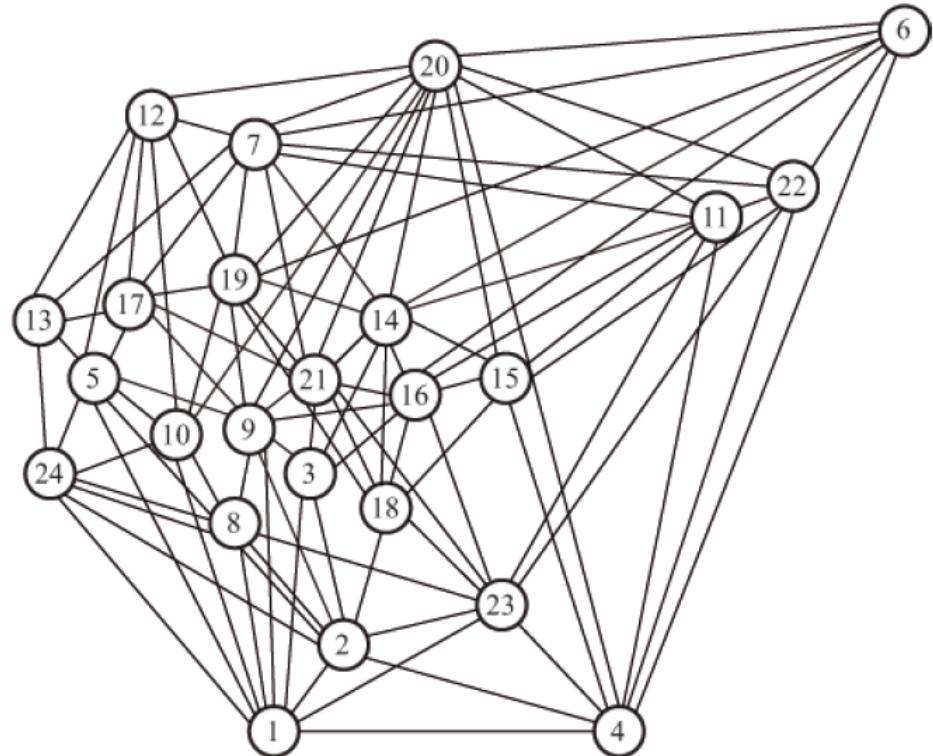
4. Solution approach

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Case study



Network R1



Seville network

Case study

Network R1

101 iterations

2 minutes



Case study

Competition effects & Transfers

Total demand: 1044

Captured demand: 646.56

Transfers: 199.14

Competition effects & No Transfers

Total demand: 1044

Captured demand: 654,47

Transfers: 457.71

No Competition effects & No Transfers

Total demand: 1044

Captured demand: 980

Transfers: 648

Conclusions

- We have proposed a new formulation for the rapid transit network design problem:
 - We have integrated the network design and the line planning.
 - We have modeled the transfers of passengers and minimized them;
 - And we have introduced competition effects in order to better estimate demand coverage.
- Future research: robust solutions. Further research needs to redefine the concept of robustness: recoverable robustness.

THANK YOU FOR YOUR ATTENTION

Any question, comment?

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