

# Analysing the Impact of Capacity Fluctuations on the Design of a Supply Chain Network

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  - ▶ low frequency, high economic impact (flooding, strikes);
  - ▶ may be correlated among facilities.
- ▶ Need of a better understanding of the impact of imperfect reliability on location patterns.

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- ▶ Perfect information: decision maker knows the state of the system and has direct influence on customers' choice;
- ▶ Imperfect information: customers visit a -probably unavailable facility- and are redirected to another one or lost.

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## Insight

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Berman et al. (2007, 2009) identify a *facility centralization* pattern:

*Facilities tend to locate closer to each other as the likelihood of failure increases. On the extreme, facilities are placed in the same location in order to provide effective back-up in case of failure.*

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- ▶ Effect of capacity on solution patterns (concentration) when disrupted facilities can take different capacity values instead of the traditional 0-1 approach;
- ▶ Sensitivity of solution patterns to over-capacity;
- ▶ Sensitivity of solution patterns to low levels of expected h-capacity (when the system is not working at planned capacity).

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- ▶  $w_i$  is the demand from node  $i$ ;
- ▶  $\mathfrak{L}$  is the set of all feasible solutions (locations).

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## Admissible Allocation Scheme

An allocation scheme

$$q = (q_{ij})_{|\mathcal{I}| \times |\mathcal{J}|}$$

is admissible for  $\mathcal{I} \in \mathfrak{L}$  if  $q \in Q_{\mathcal{I}}^k \cup Q_{\mathcal{I}}^k$ , where

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and  $q_{ij}$  is the quantity shipped from a facility in  $i \in \mathcal{I}$  to demand node  $j \in \mathcal{J}$ .

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For any given  $C^k \in \mathcal{C}$  and  $\mathcal{I} \in \mathfrak{L}$

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If  $Z(\mathcal{I}) = \sum_{k=1}^K p^k S_k(\mathcal{I})$  then the *Unreliable Capacitated Facility Location Problem* (UCFLP) becomes

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- ▶ Greedy heuristics can be applied to our problem;
- ▶ The solution returned by the greedy heuristic will not be more than 37% suboptimal;
- ▶ Improvement of **average gap** by facilities ordering may have a substantial positive impact on  $Z$ .

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- ▶ Effective capacity of a facility is a factor  $0 < \beta < 1$  of the planned or theoretical capacity, i.e.  $C_i^k = \beta C_i^1$ .

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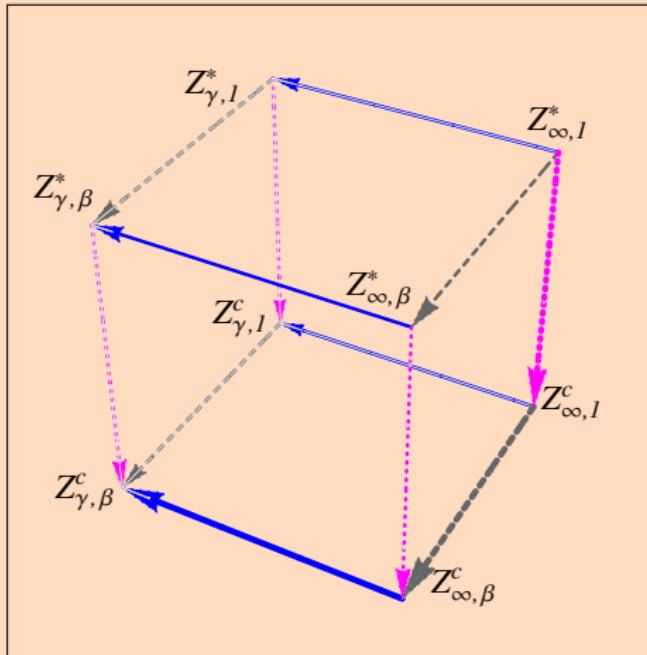
and define an ideal scenario

$$Z_{\infty,1}^*$$

where capacity is not a concern and the facility is perfectly reliable.

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## Loss Cube



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- ▶ ...

Thank You!!!!