61A Extra Lecture 1

Thursday, January 29

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- John's office hours: 10am-12pm Wednesday & Friday by appt. (denero.org/meet) in 781 Soda

Lambda Expressions

(Demo)

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Lambda expressions in Python cannot contain statements at all!

Newton's Method

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Quickly finds accurate approximations to zeroes of differentiable functions!

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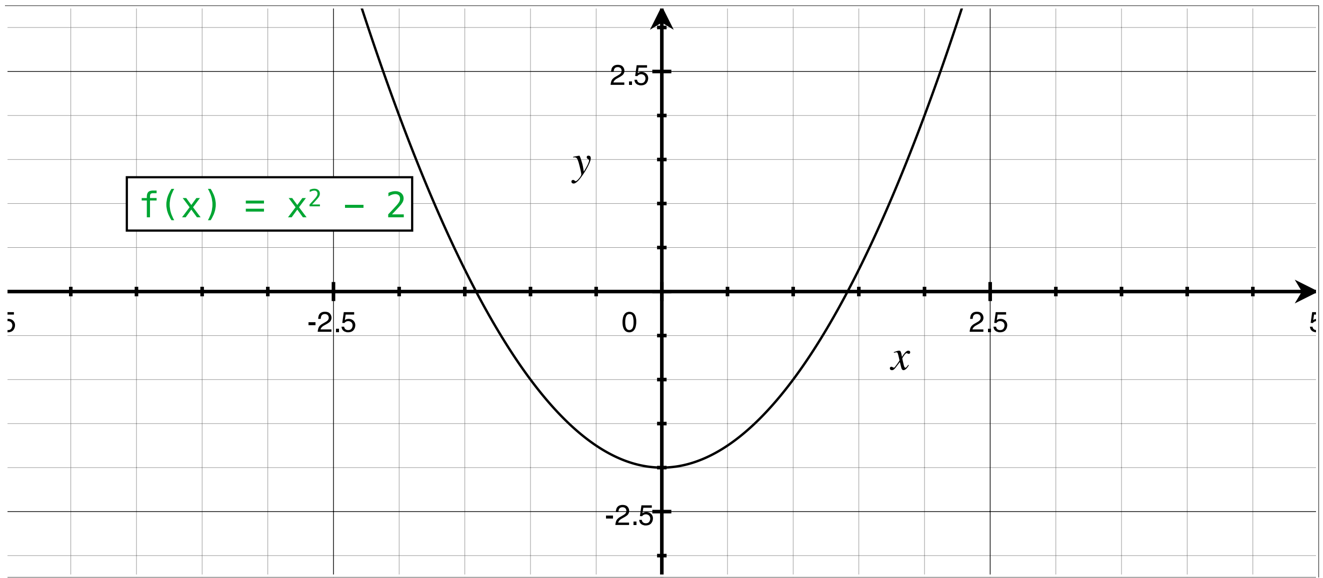

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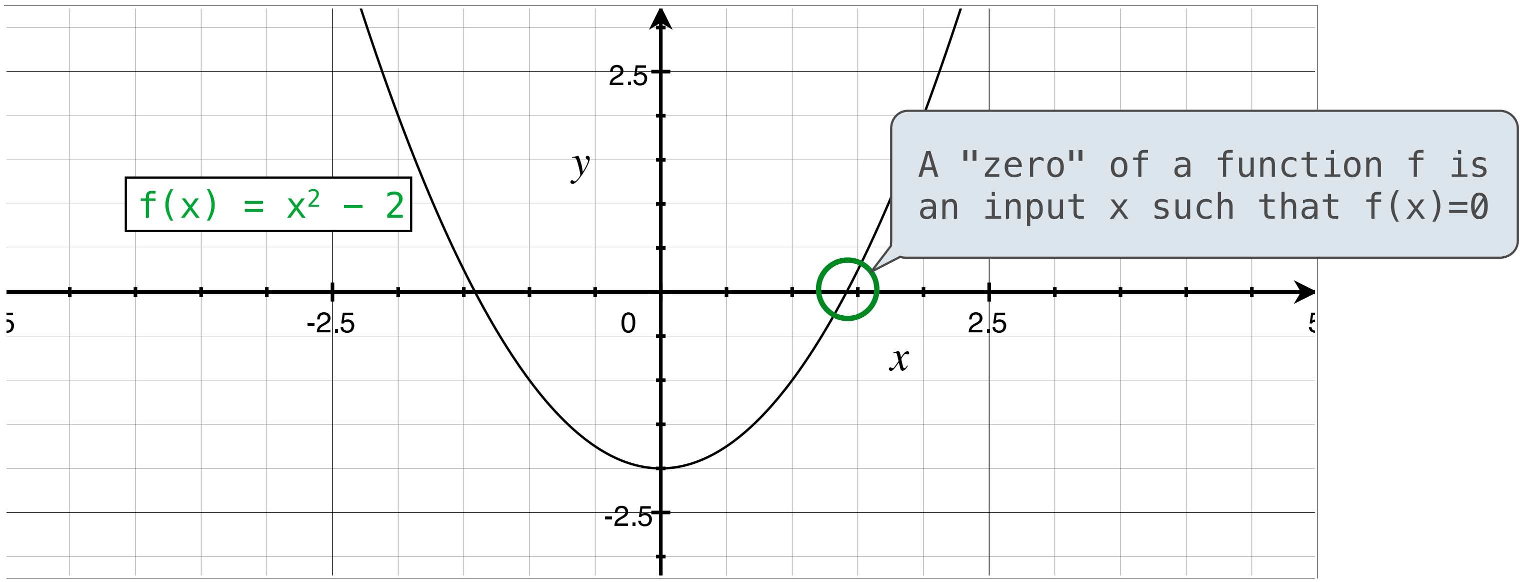

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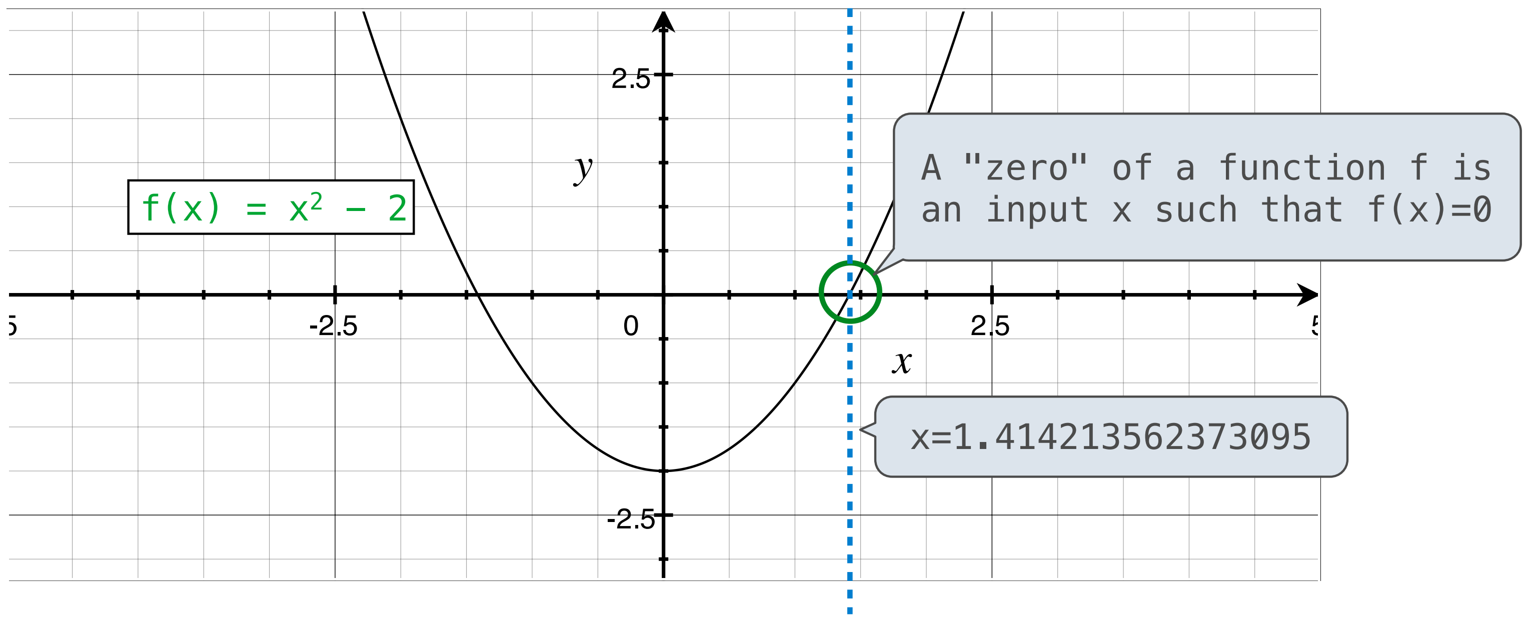

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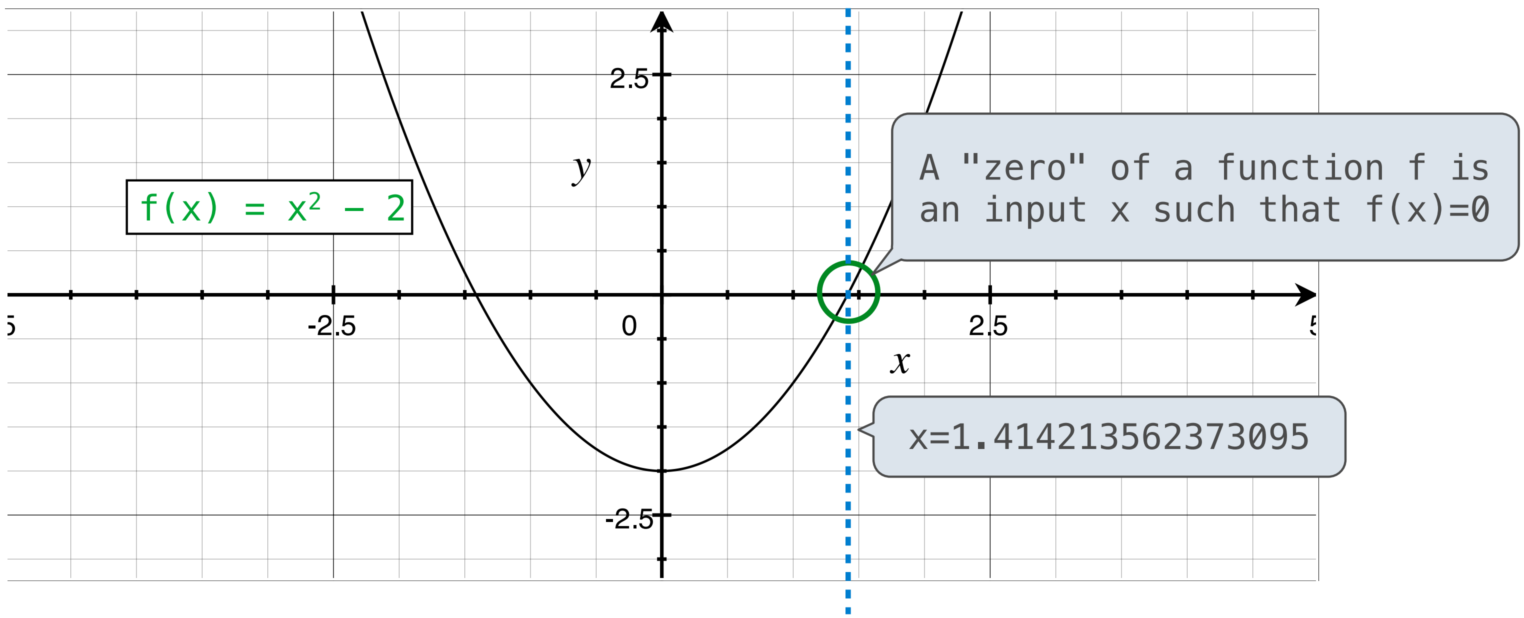


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Application: a method for computing square roots, cube roots, etc.
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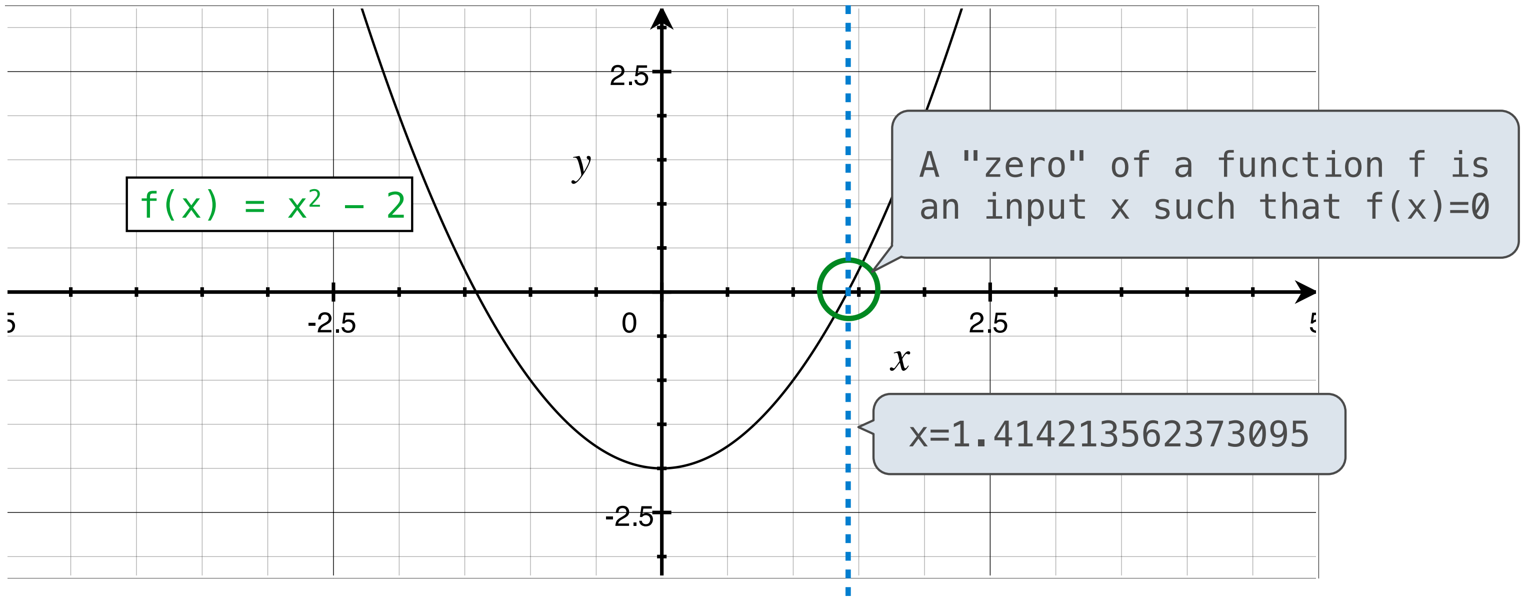

Application: a method for computing square roots, cube roots, etc.

The positive zero of $f(x) = x^2 - a$ is $\sqrt{a}$. (We're solving the equation $x^2 = a$.)

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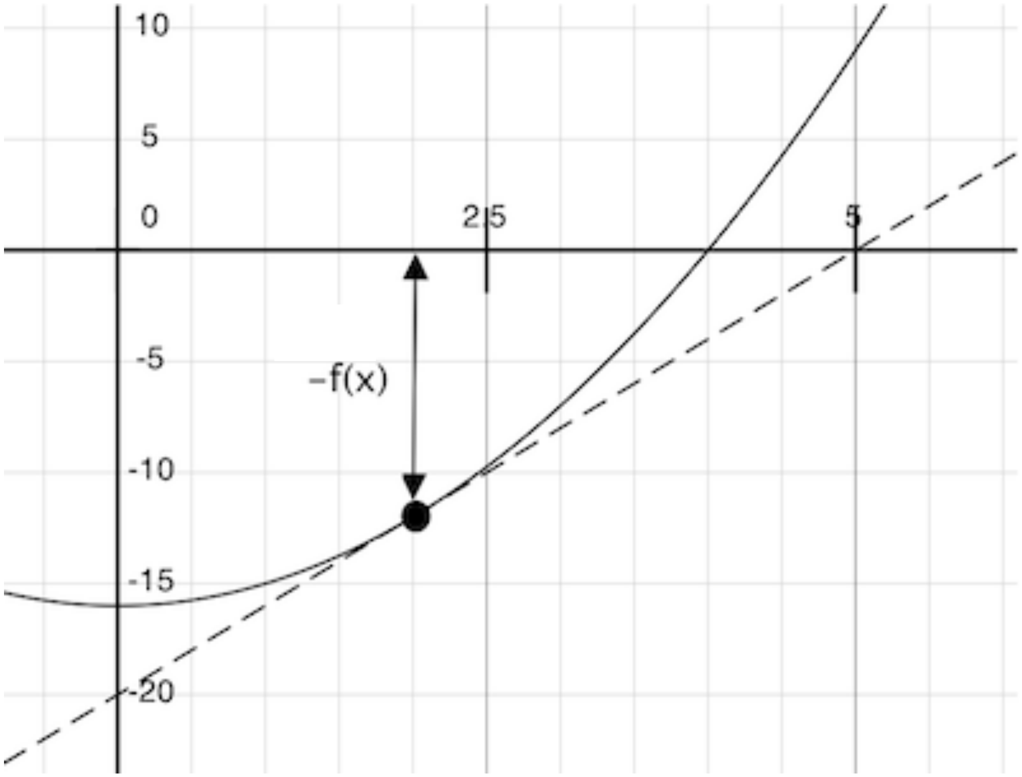

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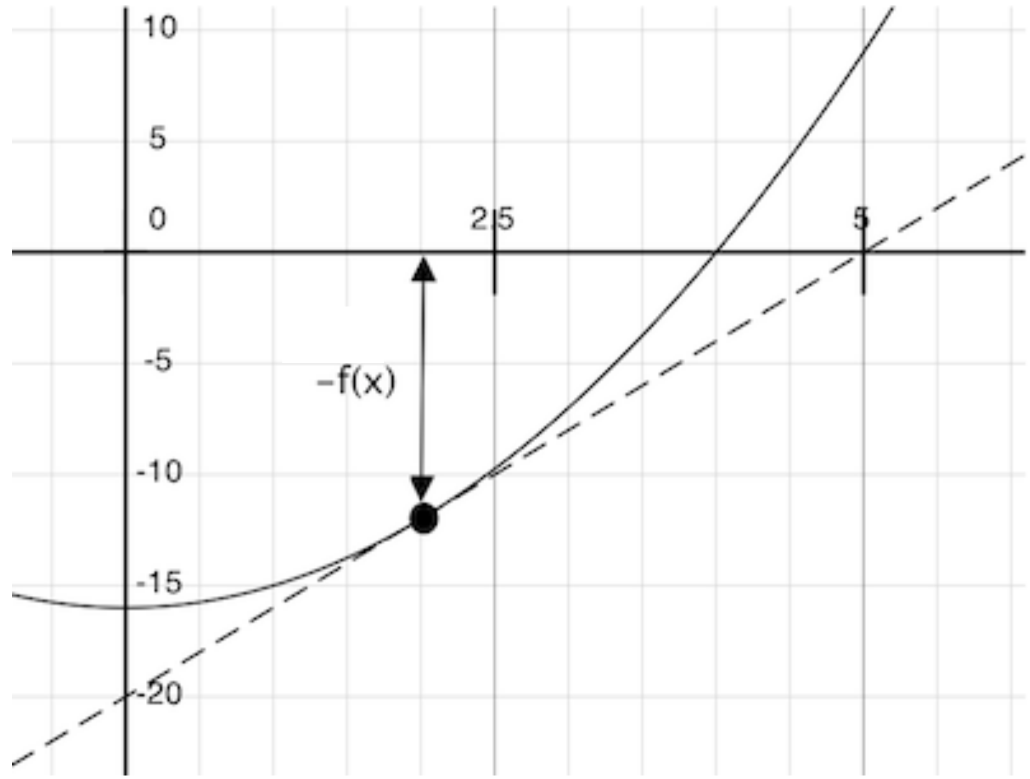

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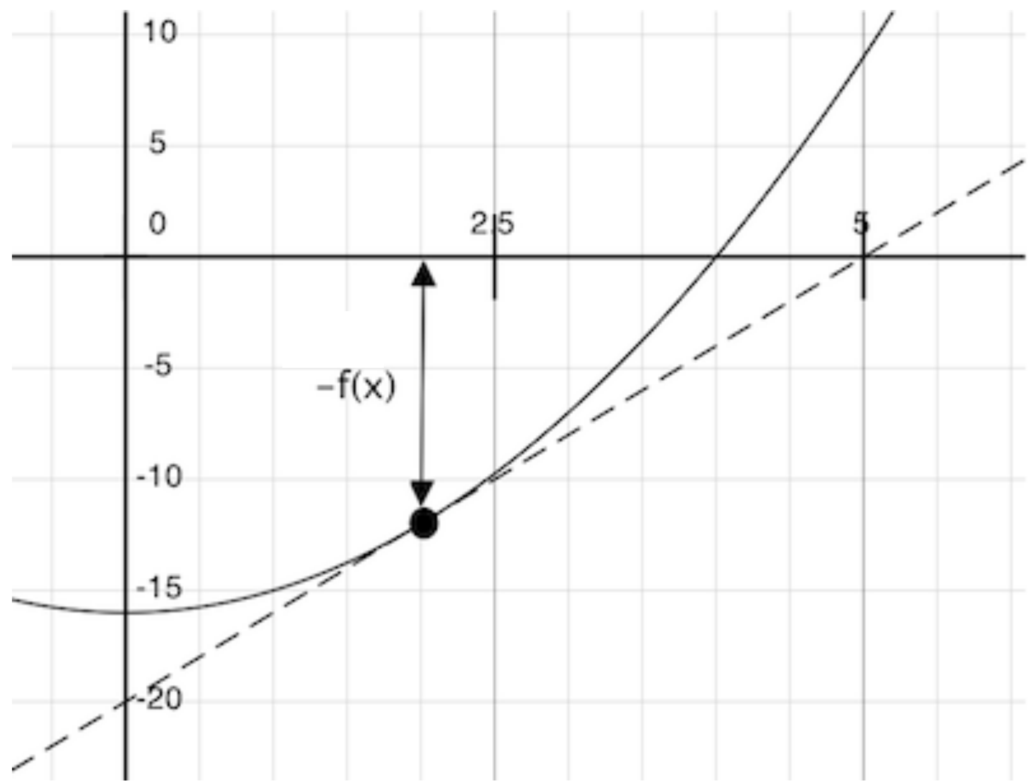

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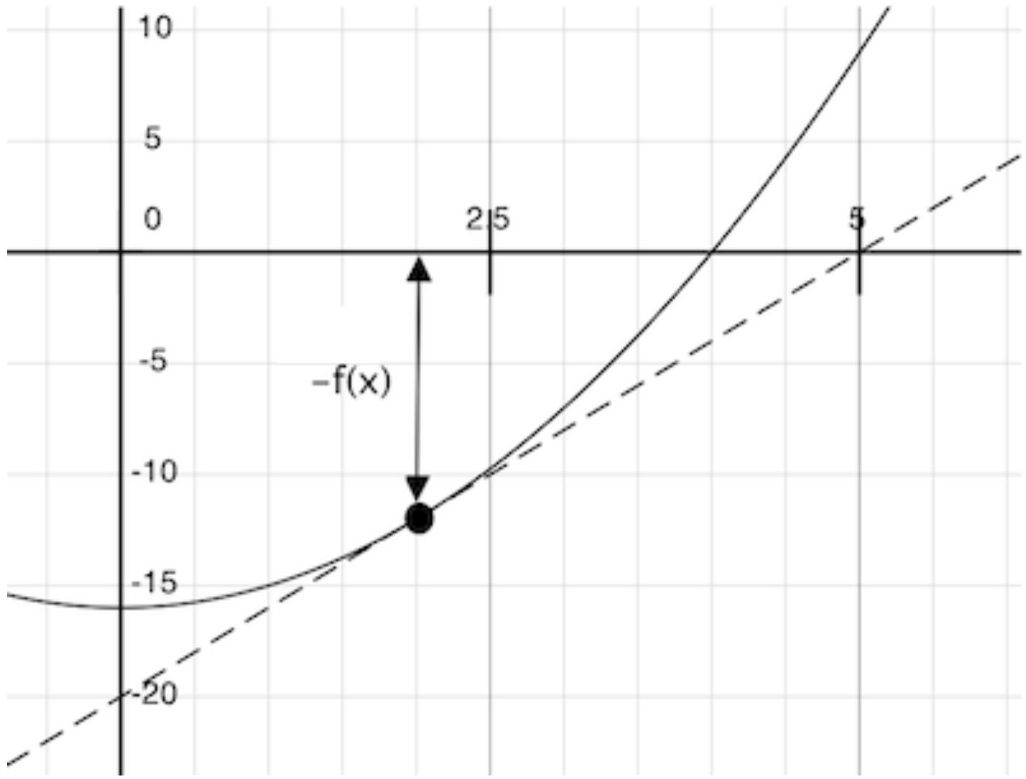

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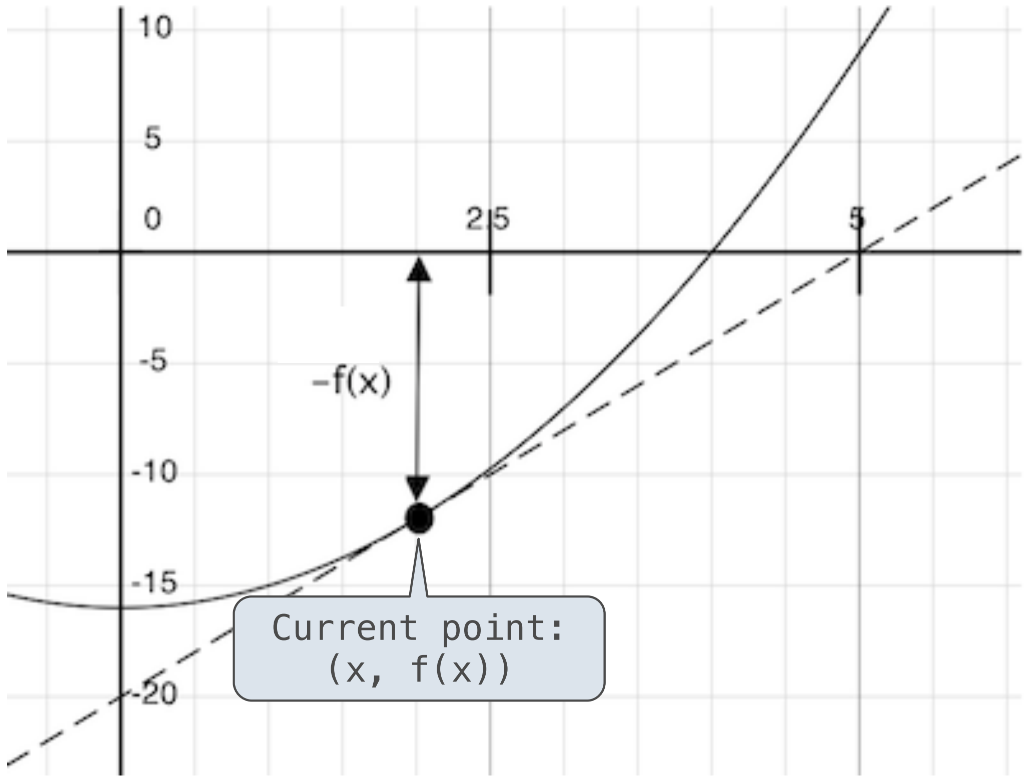

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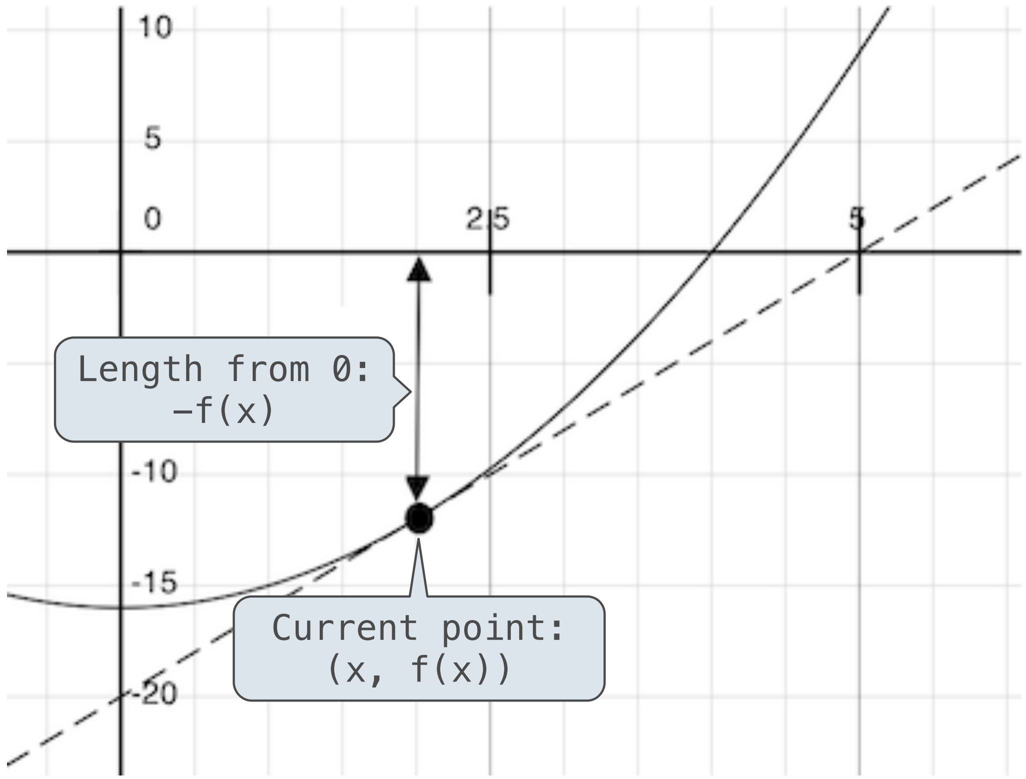

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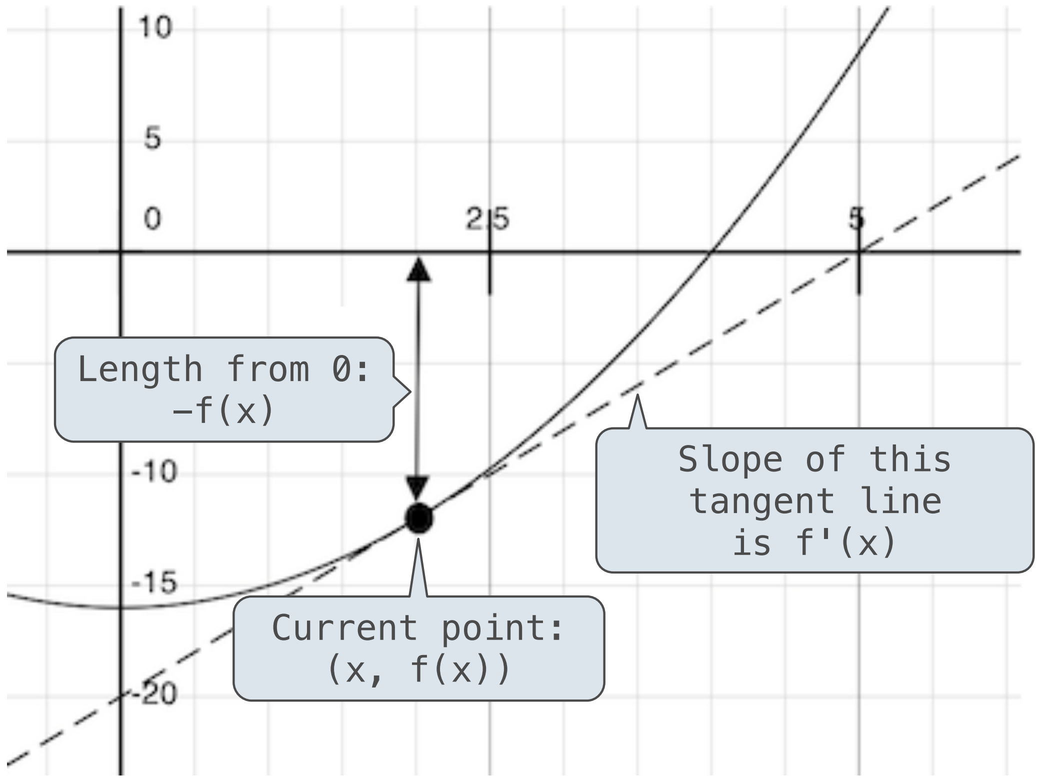

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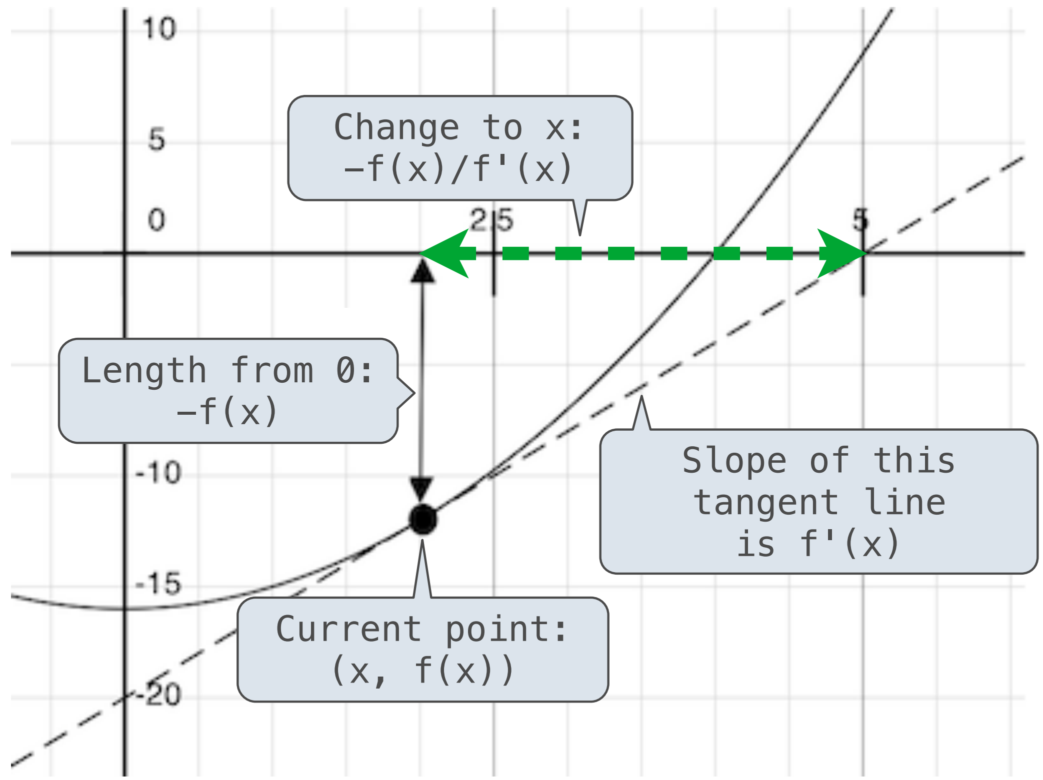

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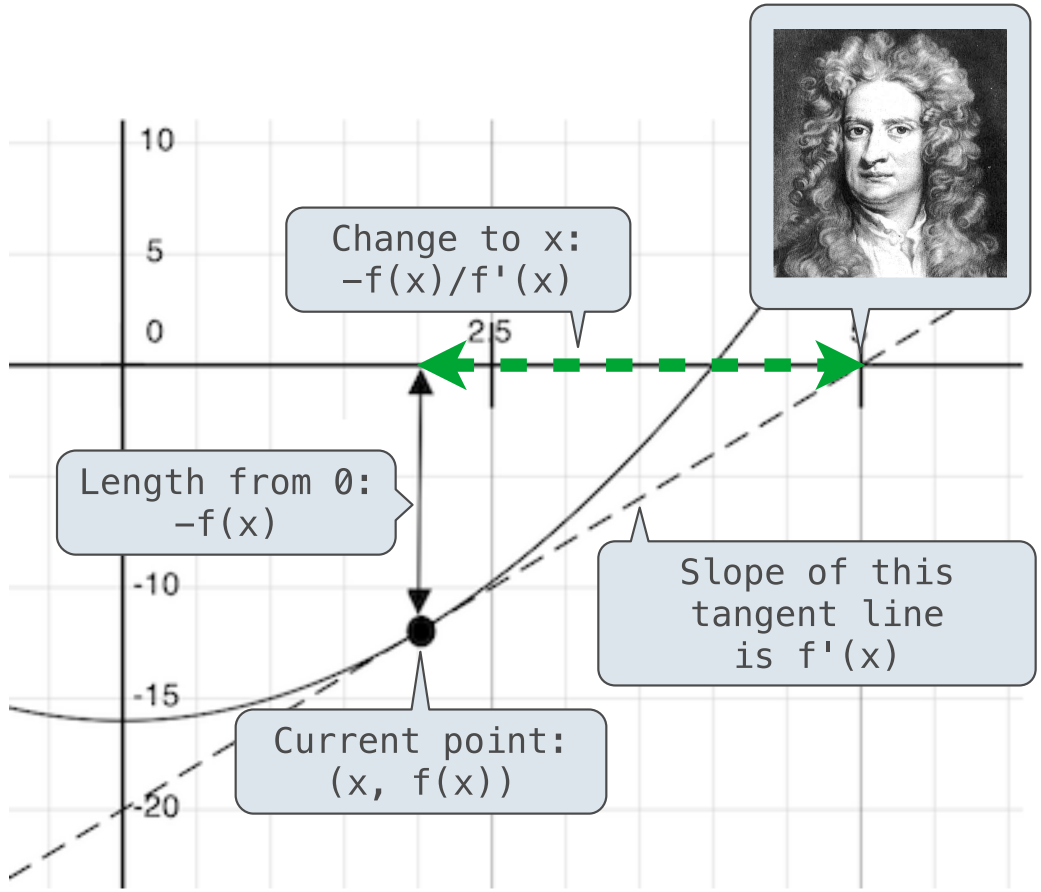

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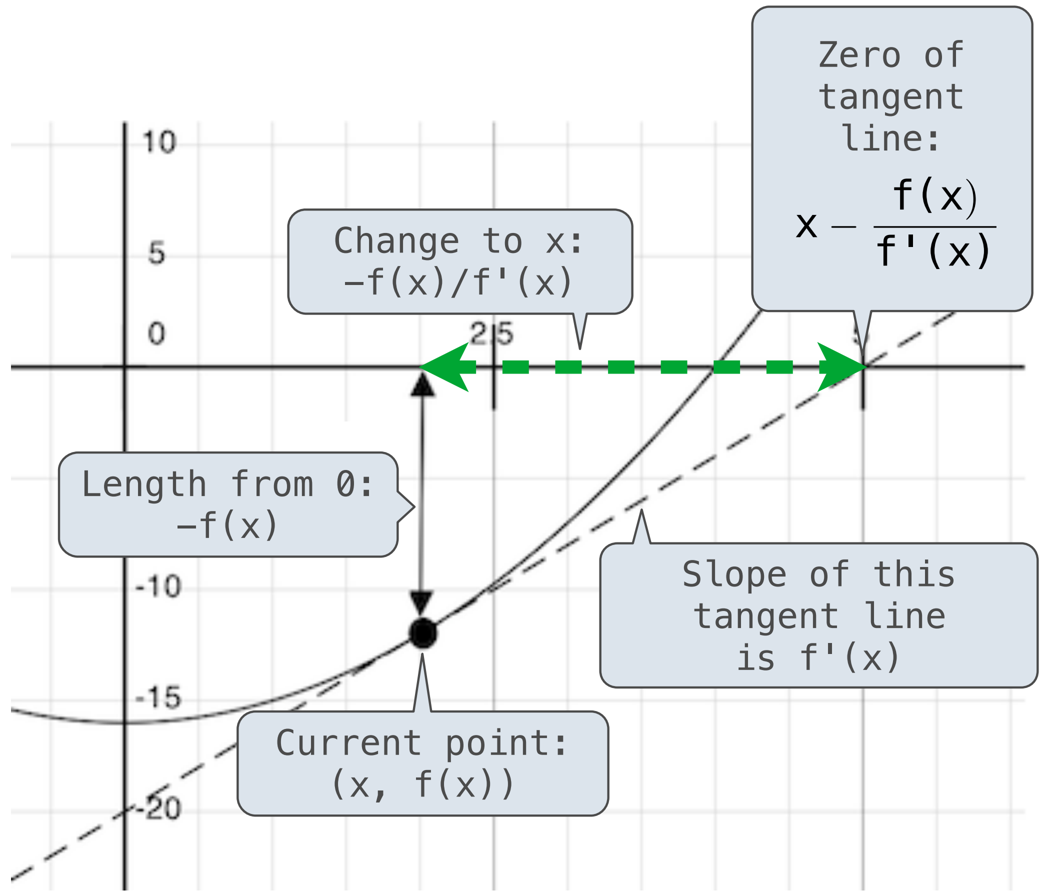

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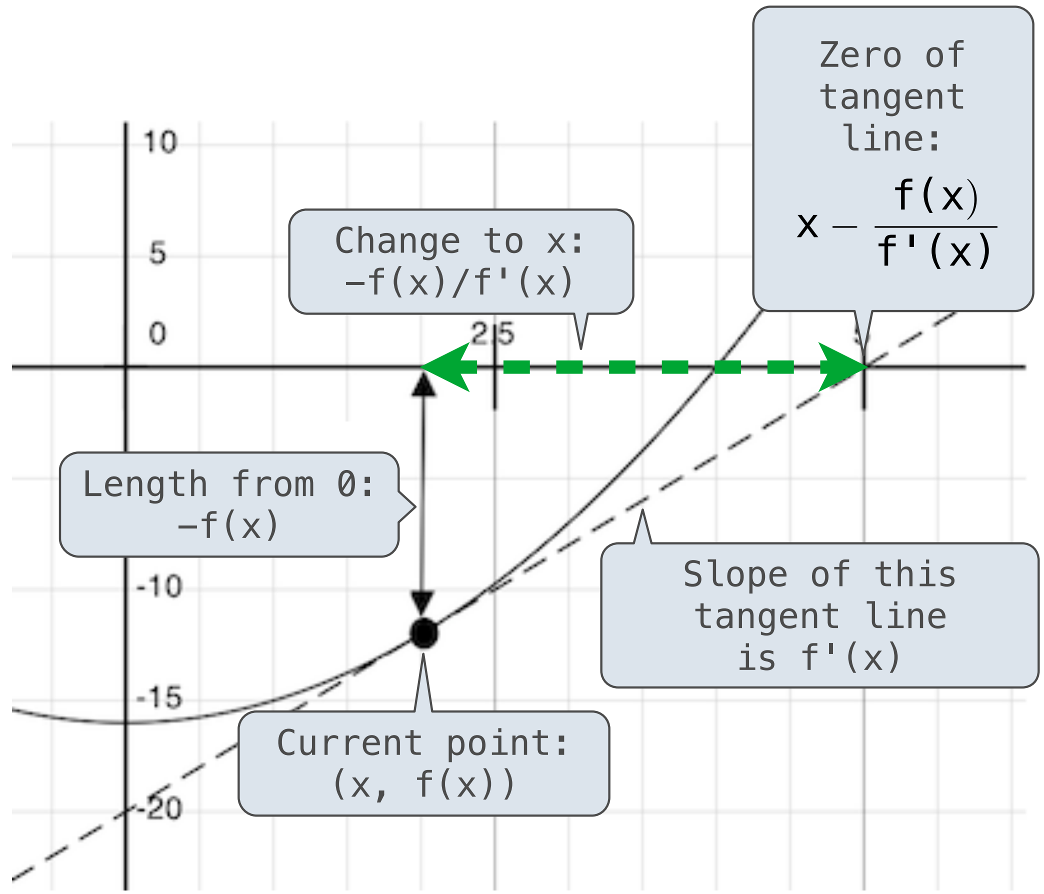

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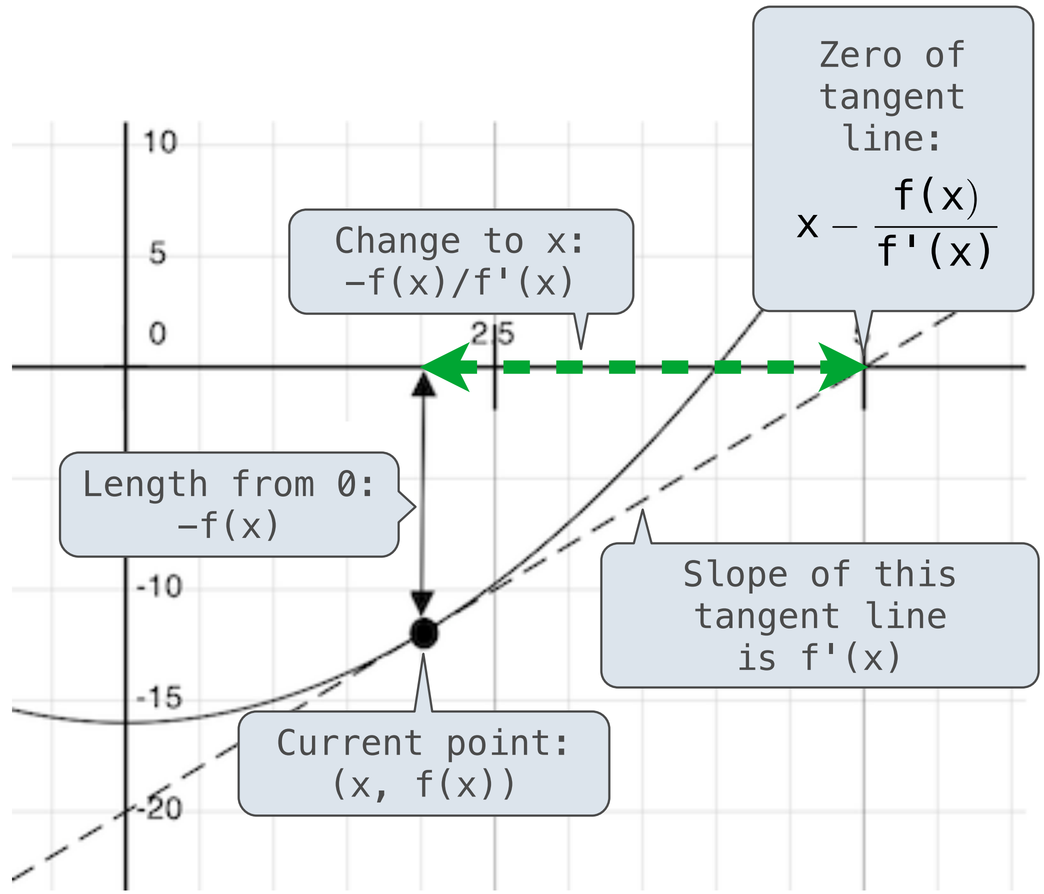

http://en.wikipedia.org/wiki/File:NewtonIteration_Ani.gif

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How to find the square root of 2?
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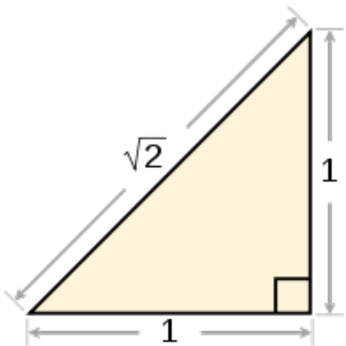


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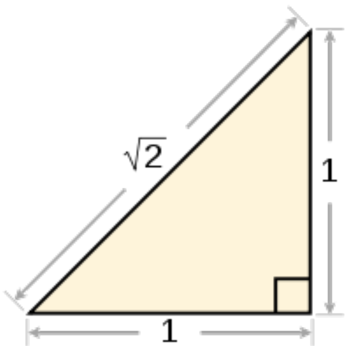


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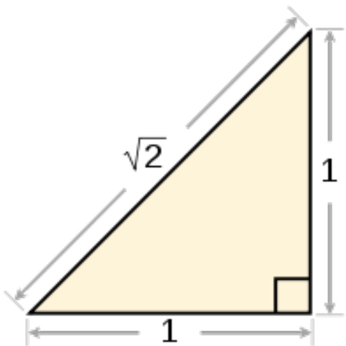


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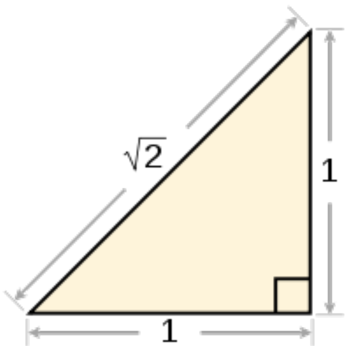


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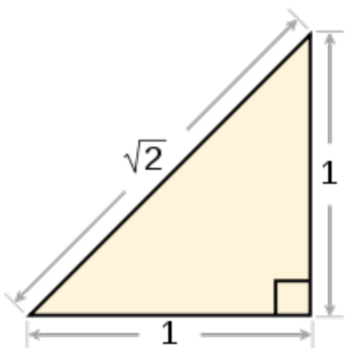


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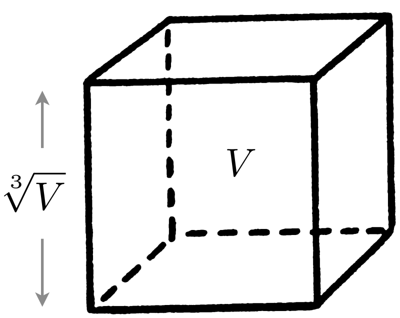

$\sqrt[3]{V}$

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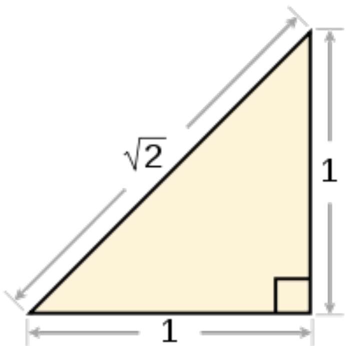


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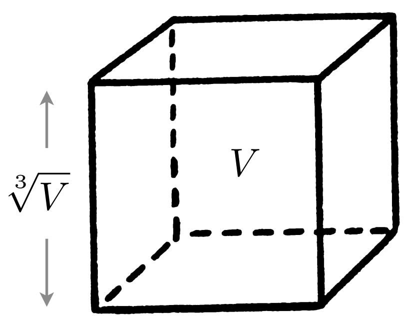


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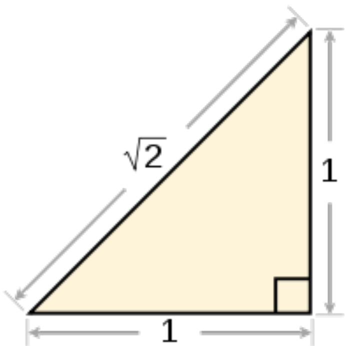


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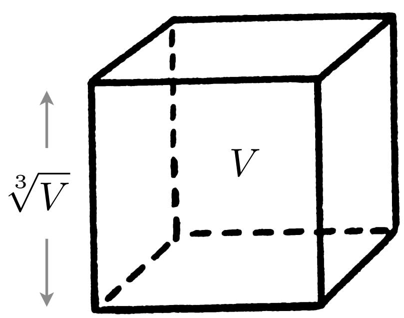


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Iterative Improvement

Special Case: Square Roots

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(Demo)

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Implementing Newton's Method

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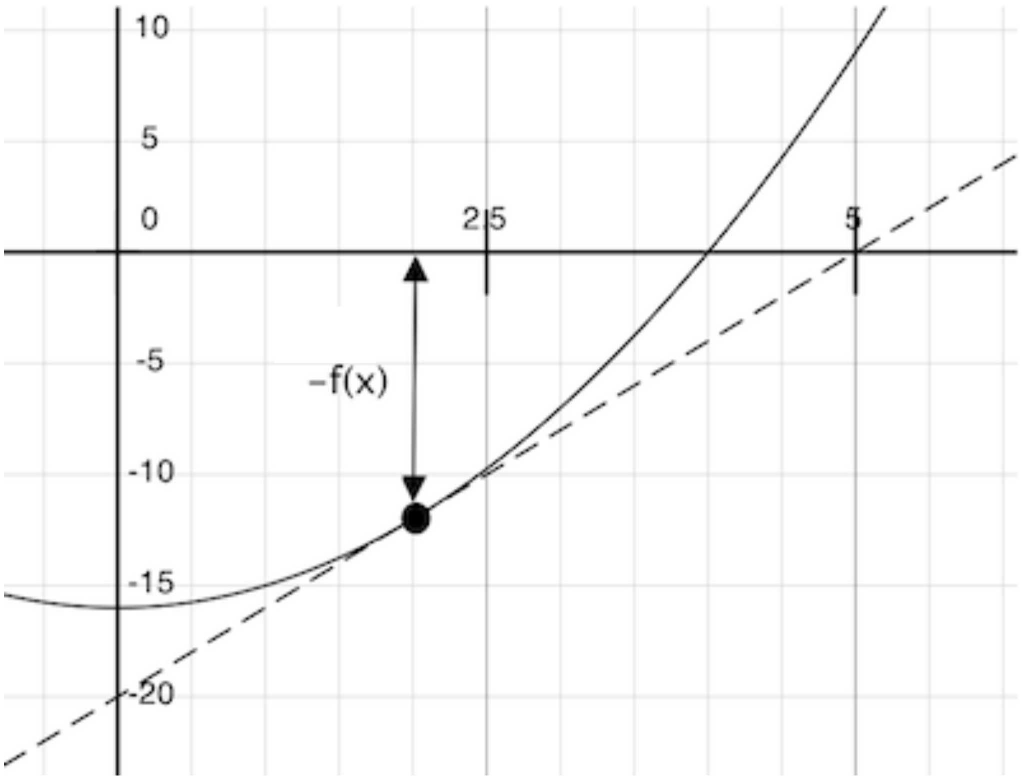
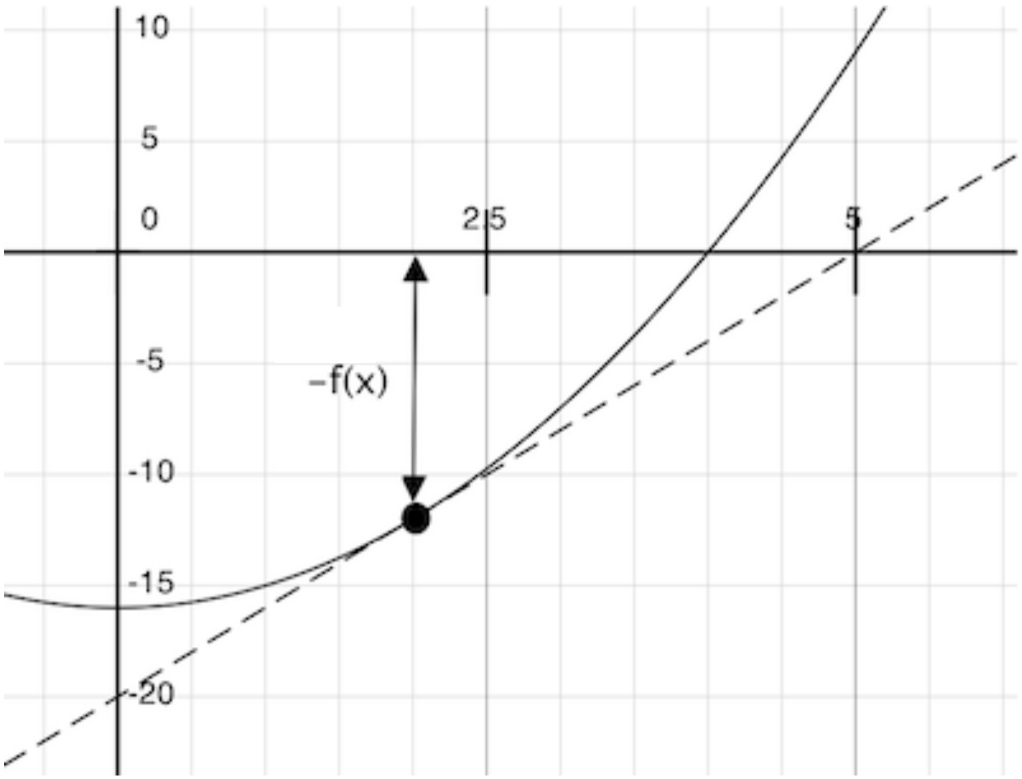


Approximate Differentiation

```
Differentiation can be performed
symbolically or numerically
```



Approximate Differentiation

```
Differentiation can be performed
symbolically or numerically
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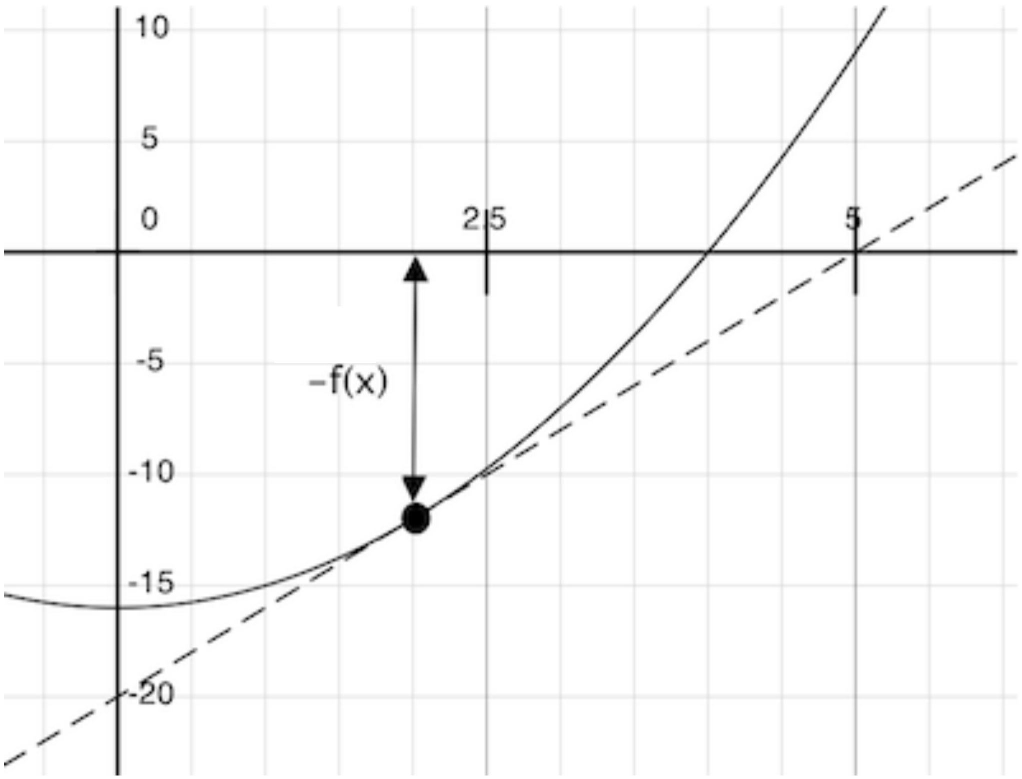
f(x) = $x^2 - 16$



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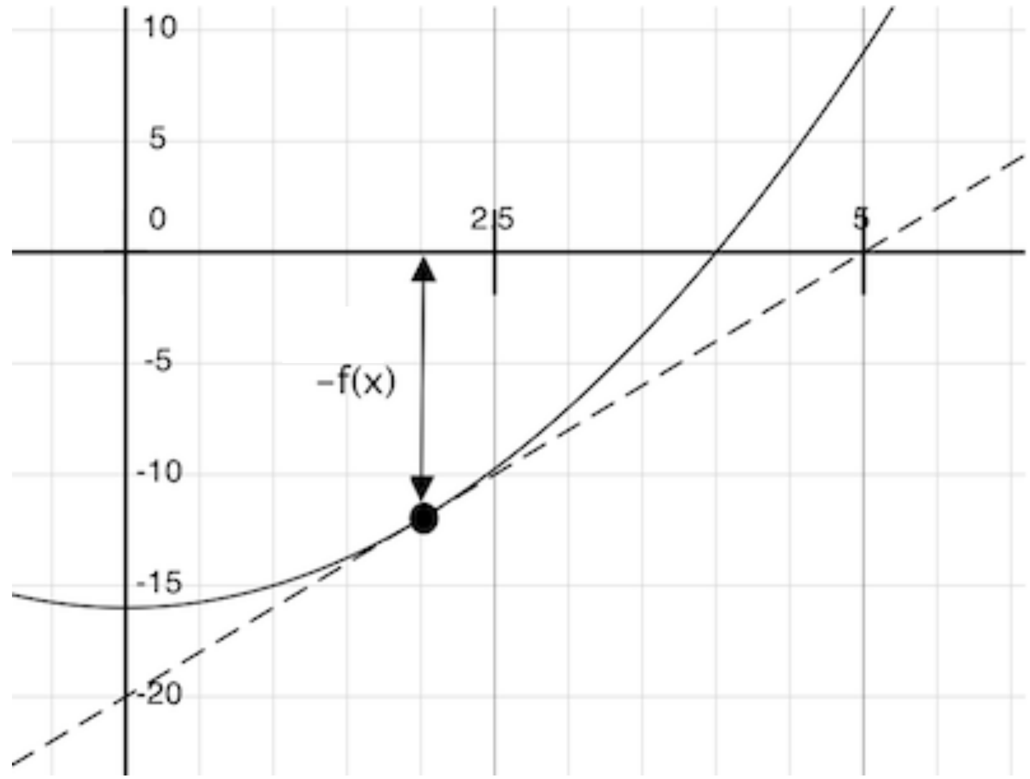

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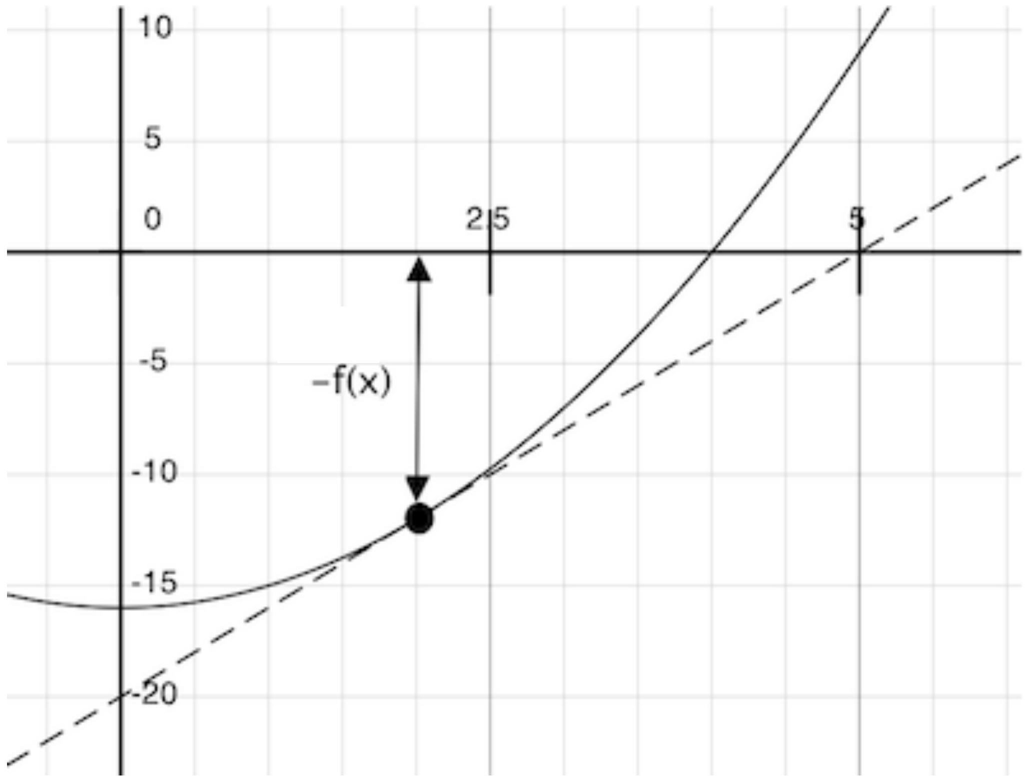

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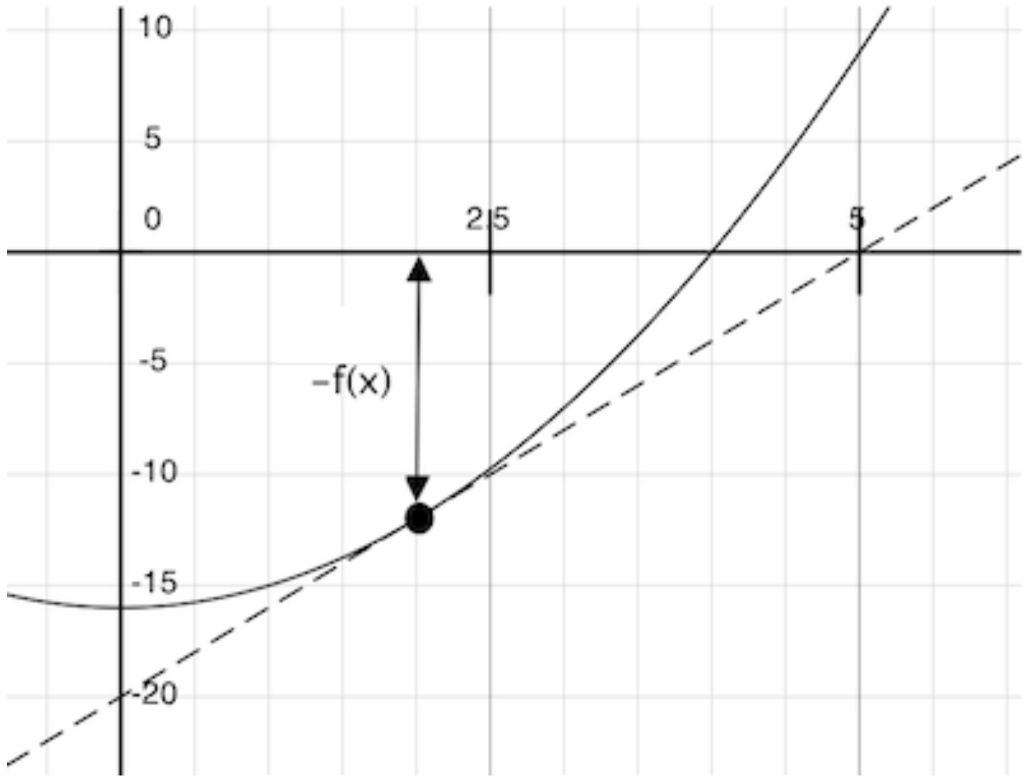

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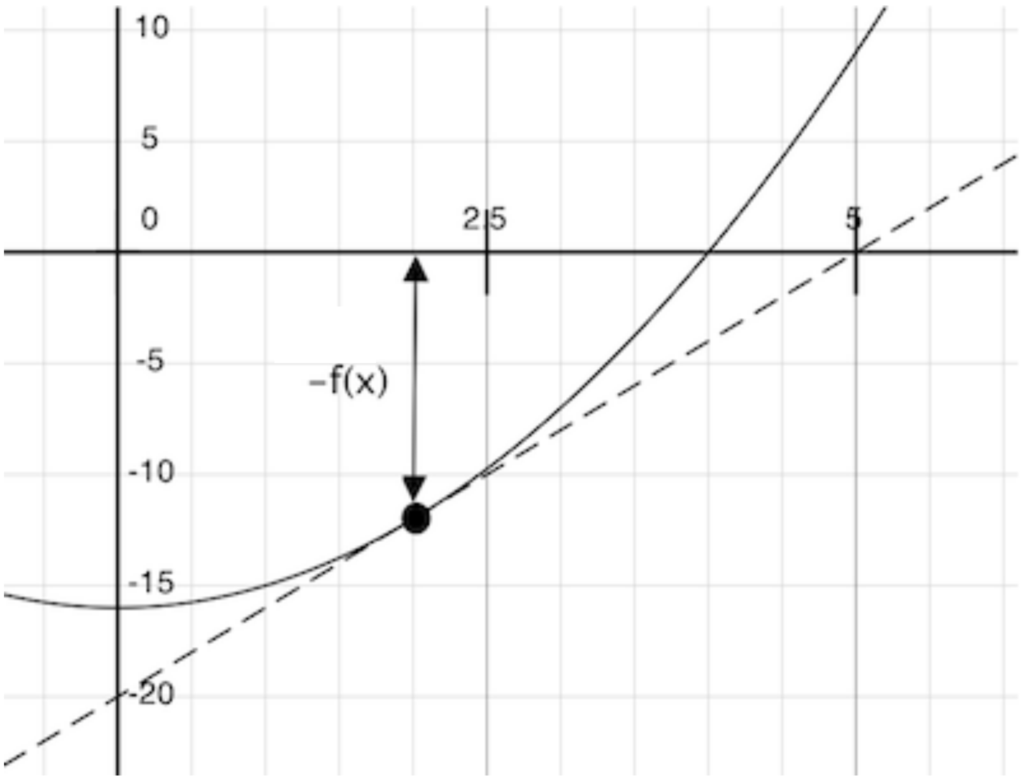

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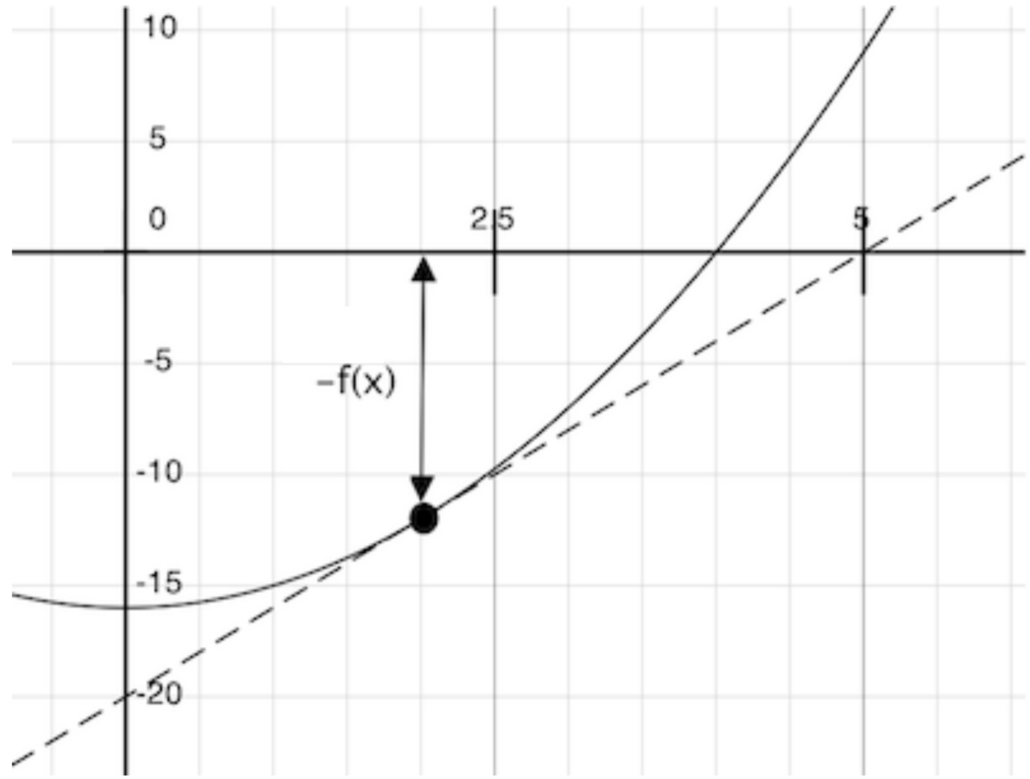

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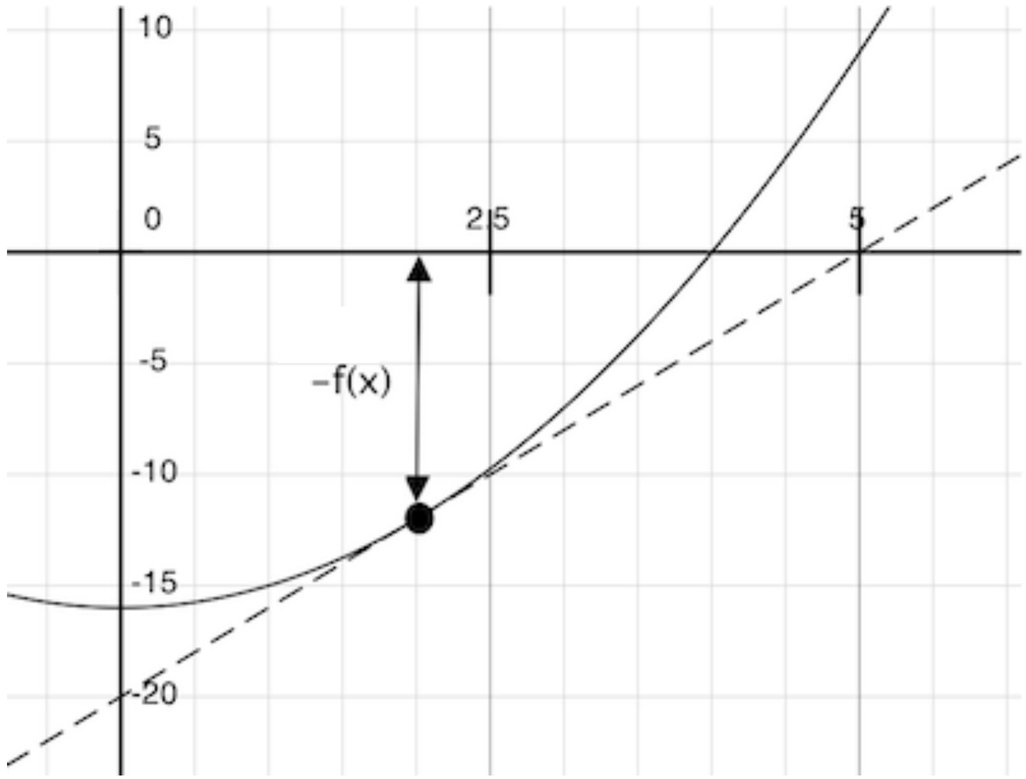

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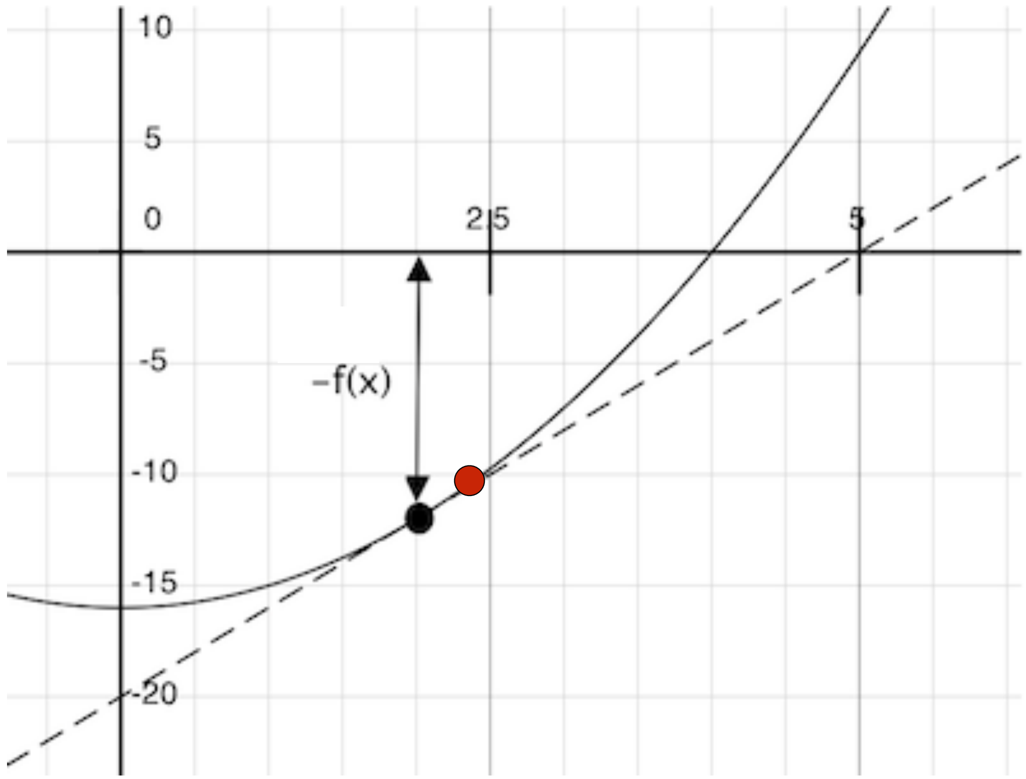

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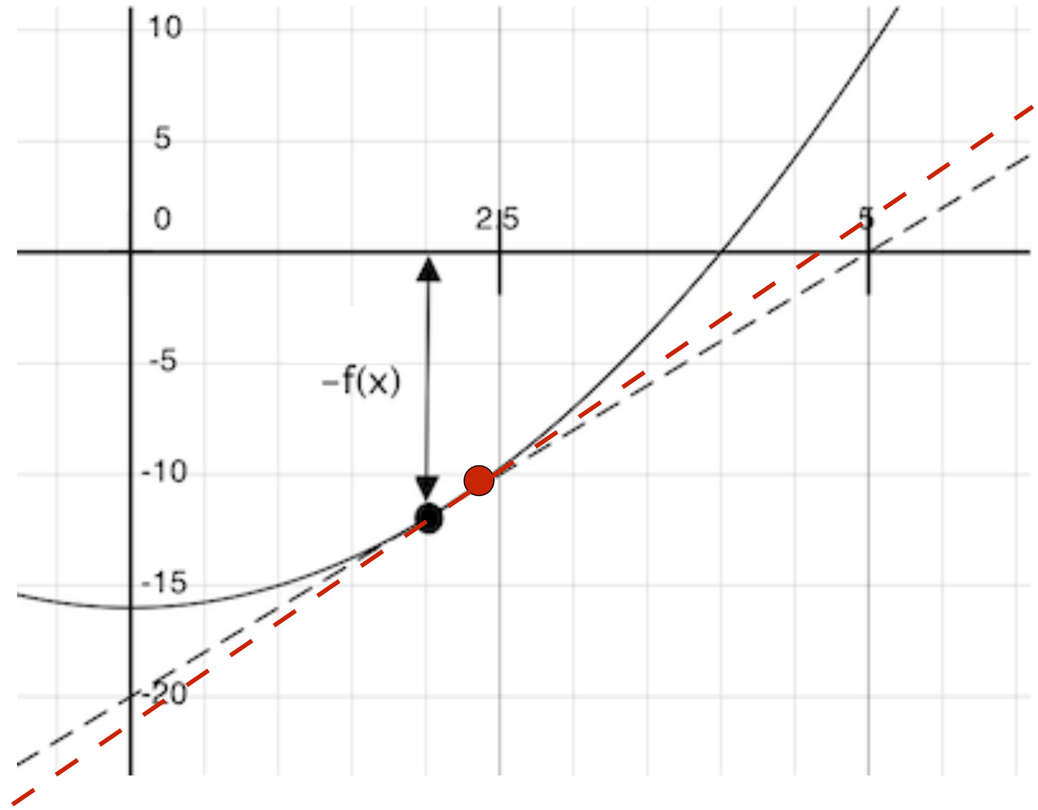

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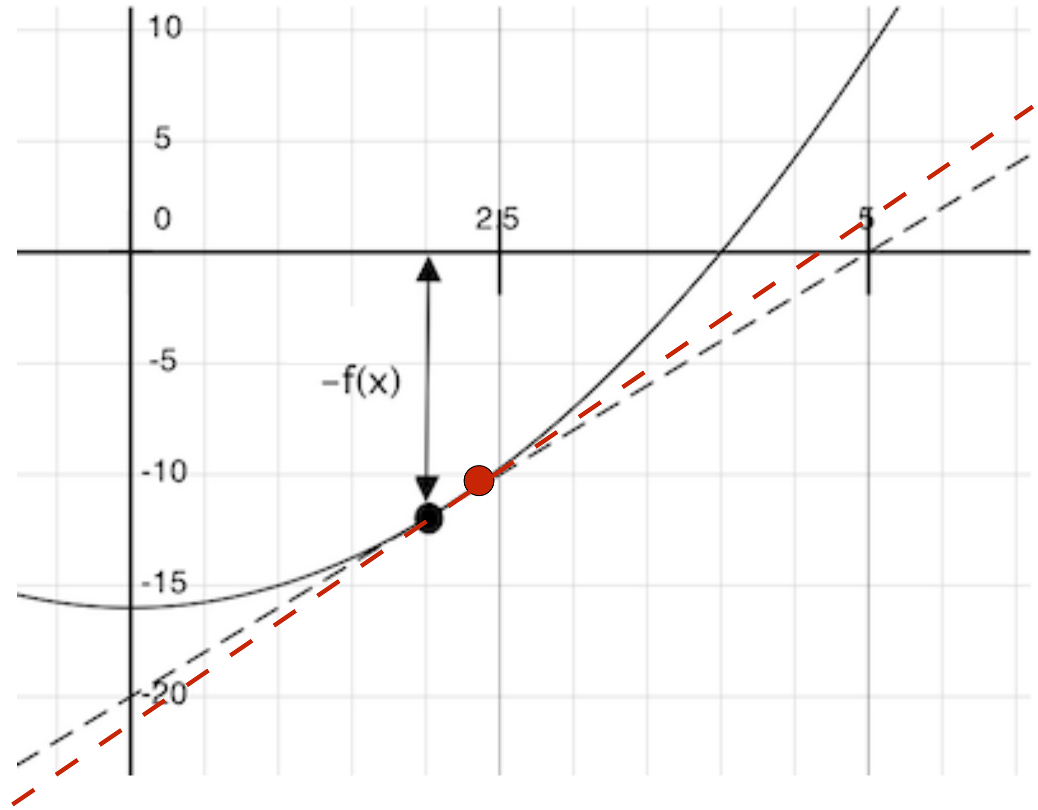
(Demo)



Critical Points and Inverses

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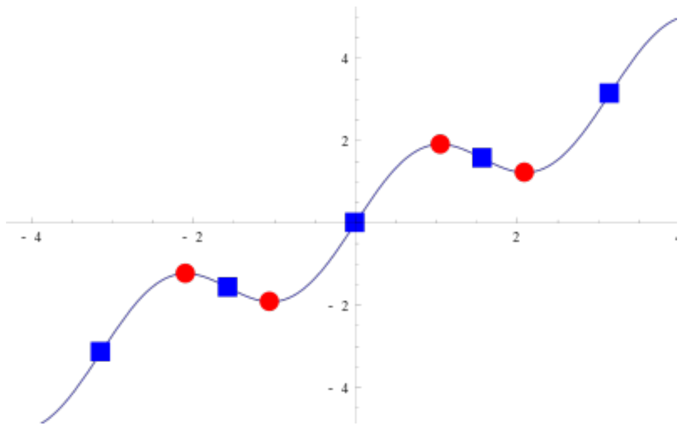

http://upload.wikimedia.org/wikipedia/commons/f/fd/Stationary_vs_inflection_pts.svg

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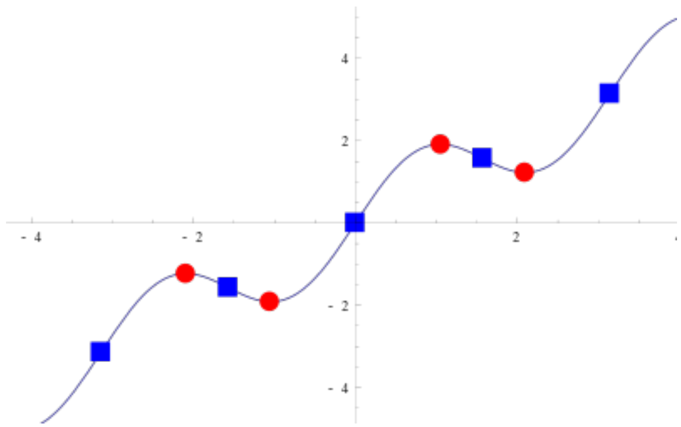

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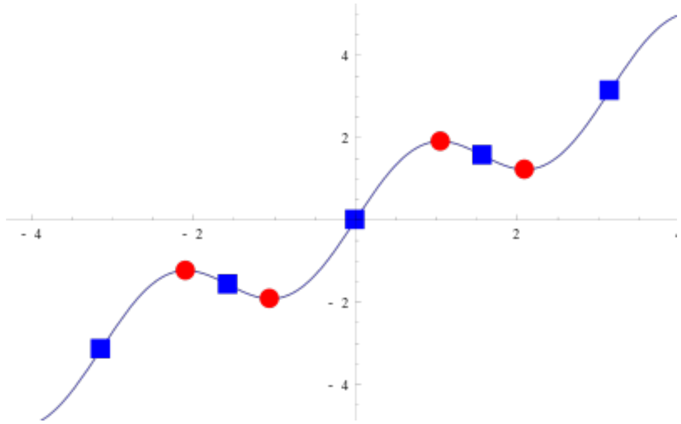

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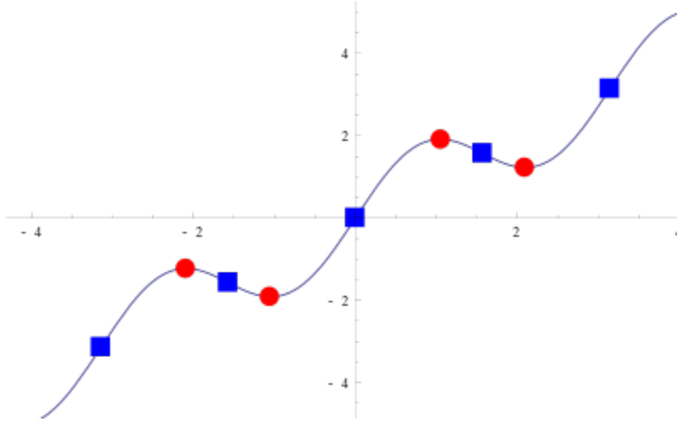

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