The Supplementary Materials

I. Proof Of the calculation of Θ

Proof. We first list the following axioms that are assumed to be true.

- $\pi \vDash \neg\mu \implies \pi \nvDash \mu$
- if $\Phi = x \wedge y, \pi \vDash \neg x \implies \pi \nvDash \Phi, \pi \vDash \neg y \implies \pi \nvDash \Phi$
- if $\Phi = x \vee y, \pi \vDash (\neg x \wedge \neg y) \implies \pi \nvDash \Phi$
- $N(\neg\theta_1) = \Theta(\theta_1), \Theta(\neg\theta_1) = N(\theta_1)$

If Φ is μ, we have $\pi \vDash \neg\mu \implies \pi \nvDash \mu$, so $\Theta(\mu) = \{\neg\mu\}$ is reasonable.

If Φ is $x \wedge y$, we have $\pi \vDash \neg x \implies \pi \nvDash \Phi, \pi \vDash \neg y \implies \pi \nvDash \Phi$, so $\Theta(x \wedge y) = \Theta(x) \cup \Theta(y)$ is reasonable.

If Φ is $x \vee y$, we have $\pi \vDash (\neg x \wedge \neg y) \implies \pi \nvDash \Phi$, so $\Theta(x \vee y) = \{x \wedge y \mid x \in \Theta(a) \wedge y \in \Theta(b)\}$ is reasonable.

If Φ is $\bigcirc x$, we have $(\pi, t) \vDash \bigcirc\Phi \iff (\pi, t+1) \vDash \Phi$. Given $\pi \vDash \neg\mu \implies \pi \nvDash \mu$, we can get $(\pi, t) \vDash \bigcirc\neg\Phi \iff (\pi, t+1) \nvDash \Phi$. Hence, $\Theta(\bigcirc x) = \{\bigcirc x' \mid x' \in \Theta(x)\}$ is reasonable.

If Φ is $x \; \mathcal{U}_{\mathcal{I}} \; y$, in our definition, there are two parts of $\Theta(x \, \mathcal{U}_{\mathcal{I}} \, y)$. The first part is set Θ_1: $\{x' \wedge y' \mid x' \in \Theta(x) \wedge y' \in \Theta(y)\}$ which implies these equations are satisfies: $\pi \vDash x' \implies \pi \nvDash x, \pi \vDash y' \implies \pi \nvDash y$. Given the definition of $\mathcal{U}_{\mathcal{I}}$: $(\pi, t) \vDash x \, \mathcal{U}_{\mathcal{I}} \, y \iff \exists t' \in t + \mathcal{I}$ such that $(\pi, t') \vDash y \wedge \forall t'' \in [t, t'], (x, t'') \vDash x$, we can easily get:

$$\forall \xi \in \Theta_1.\ \pi \vDash \xi \implies \pi \nvDash x \, \mathcal{U}_{\mathcal{I}} \, y$$

The second part is set Θ_2: $\{x' \, \mathcal{U}_{\mathcal{I}} \, y' \mid x' \in \Theta(\neg x \vee y) \wedge y' \in \Theta(x \vee y)\}$. In order to obtain a contradiction, assume that there is an element ξ of set Θ_2 that satisfies $\pi \vDash \xi \implies \pi \vDash \Phi$. Then, for $\pi \vDash \Phi$, we get $\exists t' \in t + \mathcal{I}$ such that $(\pi, t') \vDash y \wedge \forall t'' \in [t, t'], (x, t'') \vDash x$, which means $(x \wedge \neg y)$ is satisfied until y is satisfied. For $\pi \vDash \xi$, we get $\exists t' \in t + \mathcal{I}$ such that $(\pi, t') \vDash (\neg x \wedge \neg y) \wedge \forall t'' \in [t, t'], (x, t'') \vDash (x \wedge \neg y)$, which means $(x \wedge \neg y)$ is satisfied until $(\neg x \wedge \neg y)$ is satisfied. Hence, at time step t, if x is satisfied in $[t, t']$ and violated at time step t'' after t', we should get that y is satisfied before t'' and y is violated before t'' at the same time. This is a contradiction, and so the assumption that there is an element ξ of set Θ_2 satisfies $\pi \vDash \xi \implies \pi \vDash \Phi$ must be false. We can get:

$$\forall \xi \in \Theta_2.\ \pi \vDash \xi \implies \pi \nvDash x \, \mathcal{U}_{\mathcal{I}} \, y$$

Hence, $\Theta(x \; \mathcal{U}_{\mathcal{I}} \; y) = \{x' \; \mathcal{U}_{\mathcal{I}} \; y' \mid x' \in \Theta(\neg x \vee y) \wedge y' \in \Theta(x \vee y)\} \cup \{x' \wedge y' \mid x' \in \Theta(x) \wedge y' \in \Theta(y)\}$ is reasonable.

Since the temporal operators $\mathcal{U}_{\mathcal{I}}$ and $\bigcirc$ are functionally complete, we omit the proof of the remaining temporal operators. $\square$