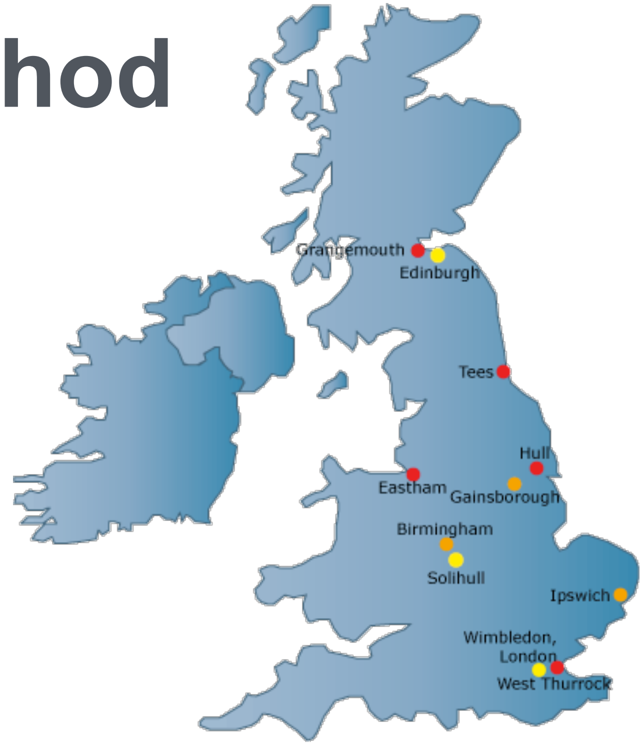
Topology Optimisation Using the Level Set Method

Dr H Alicia Kim



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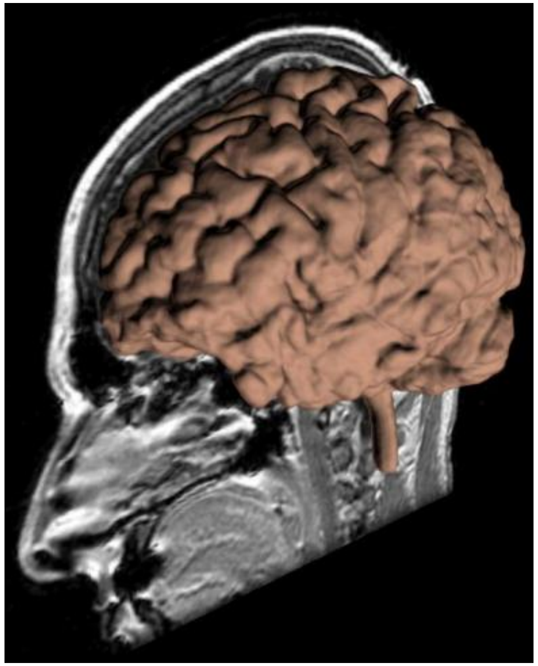
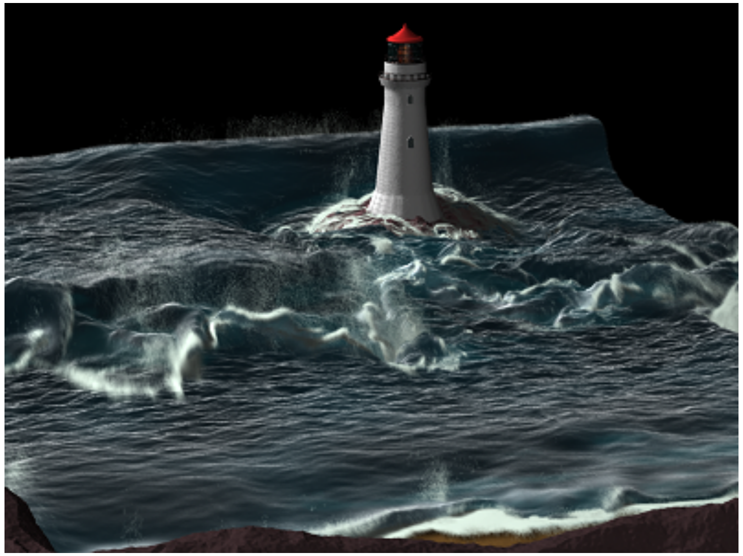
Researchers

- Chris Bowen, Chris Budd, Julian Padget
- Dave Betts, Peter Dunning, Yi-Zhe Song, Joao Duro
- Chris Brampton, Phil Browne, Kewei Duan, Caroline Edwards, Peter Giddings, Tom Makin, Vincent Seow, Phil Williams

The Level Set Method

- Front or boundary tracking method
- Commonly used in image processing, moving boundary problems, multiphase problems, movies, etc…
- Level set topology optimisation since 2000 (Sethian and Wiegmann), < 20,000 papers (Google Scholar)



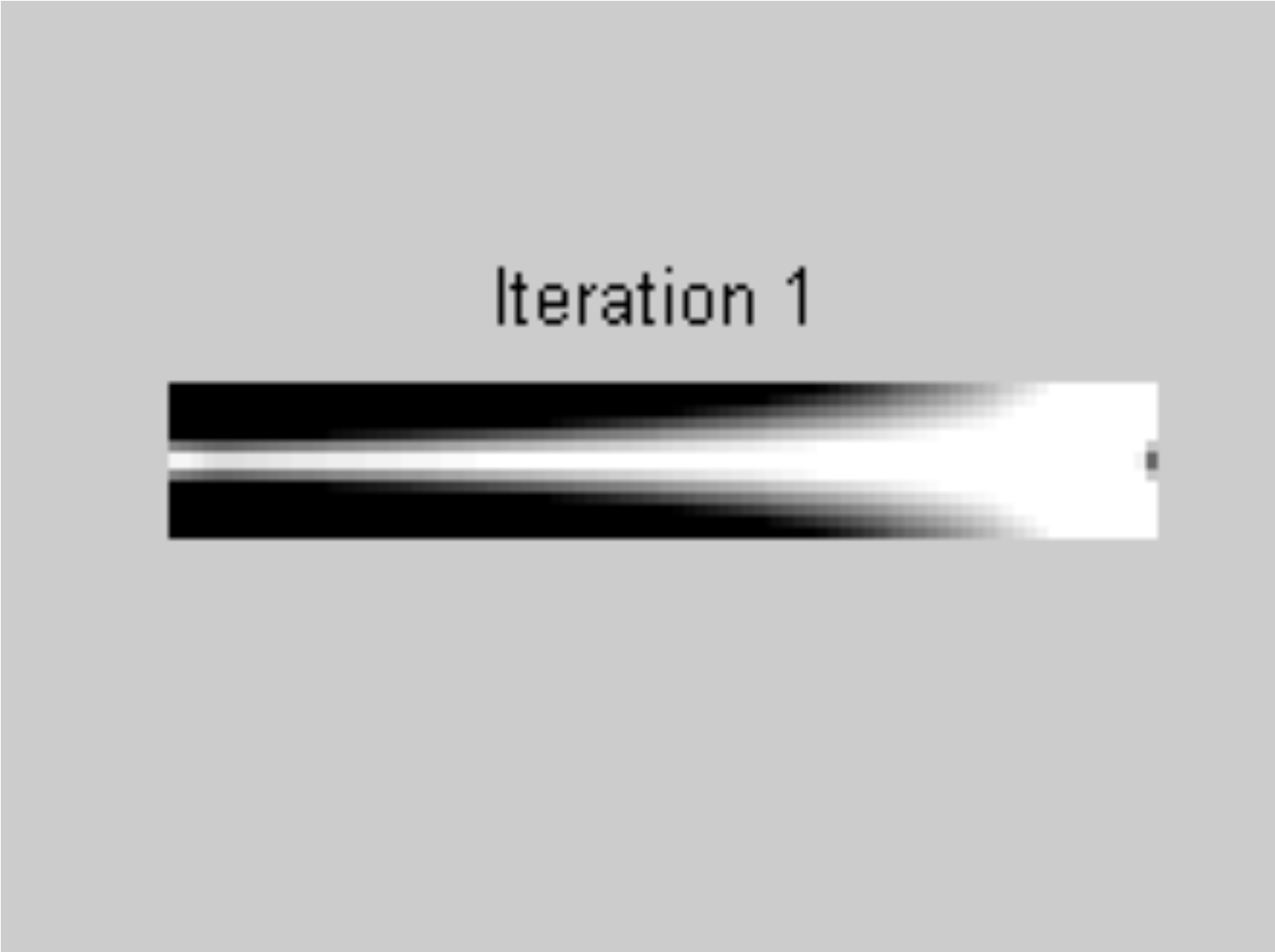


Example: Level-Set (3D)

Cantilever Beam with Vertical Load

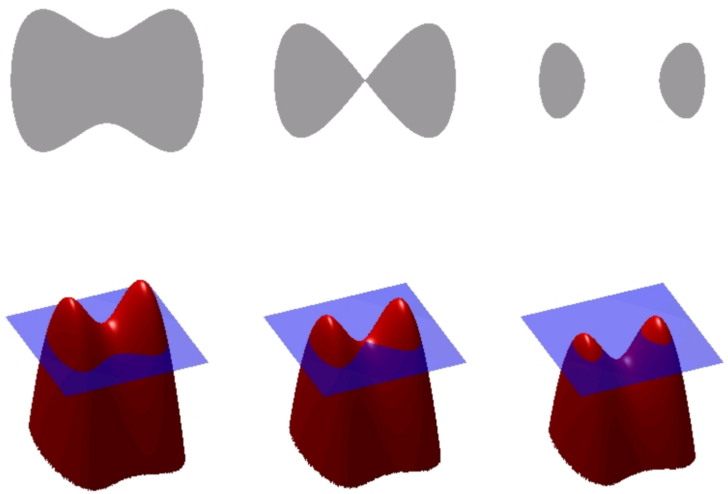
Element-Based Method (SIMP)

Cantilever Beam with Vertical Load



The Level Set Method

$$\begin{cases} \phi(x) > 0, & x \in \Omega_S \\ \phi(x) = 0, & x \in \Gamma_S \\ \phi(x) < 0, & x \notin \Omega_S \end{cases}$$



http://en.wikipedia.org/wiki/File:Level_set_method.jpg

- Update $\phi(x)$ by solving discrete Hamilton-Jacobi equation

$$\phi_i^{k+1} = \phi_i^k - \Delta t \left|\nabla \phi_i^k\right| V_{n,i}$$

- Naturally splits and merges holes

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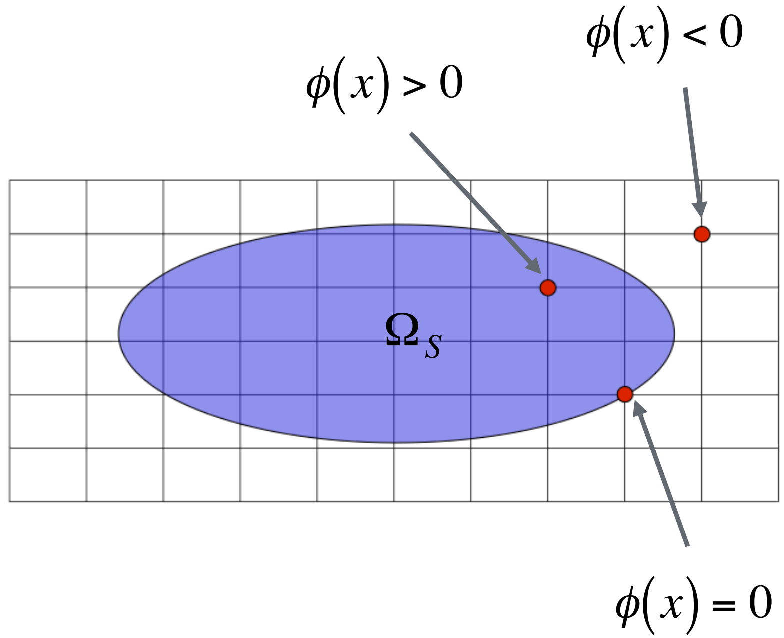
Level Set Topology Optimisation Method

1. Define the design problem.

2. Finite element analysis to compute boundary shape sensitivities, ς.

$$\varsigma(u) = A\varepsilon(u)\varepsilon(u)$$

Where A = material property, ε = strain



Dunning and Kim (2011) FEA&D

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Level Set Topology Optimisation Method

3. Level set functions updated using a Hamilton-Jacobi equation

$$\phi_i^{k+1} = \phi_i^k - \Delta t \left| \nabla \phi_i^k \right| V_{n,i}$$

   where $V_n = \lambda - \varsigma(u)$, λ = Lagrange multiplier for a constraint and Δt = iterative time step.

4. λ is determined by Newton's method.
5. Gradient, $\Delta\phi$ is computed using the upwind finite difference scheme and higher order weighted essentially non-oscillatory method (WENO).
6. Check for convergence and iterate.

How does it create a new hole?

Where to create a hole is not difficult,

when to create is!

Previous Hole Creation Approaches

Methodology	Challenge
Start with a random number of holes.	Does not create holes; solutions dependent on the initial design.
Topological derivative to create a hole.	Does not link to shape derivative so optimisation of boundaries and hole creation are unrelated.
Topological derivatives are exclusively used.	Convergence can be slow.
Holes are created at regular intervals.	The selection of the interval is arbitrary and can slow the convergence.
Hole creation criteria based on stress or strain energy.	Heuristic, fundamentally does not link to shape derivative.

Our Approach of Creating a Hole

- Introduce a secondary level set function, $\bar{\phi}$.
- Describes the additional fictitious dimension, bounded by

$$-\bar{h} \leq \bar{\phi} \leq +\bar{h}$$

- $\bar{\phi}$ is updated using the Hamilton-Jacobi equation

$$\bar{\phi}_i^{k+1} = \bar{\phi}_i^{k} - \Delta t \bar{V}_{n,i}$$

where $\bar{V}_n = \bar{\lambda} - \varsigma(u)$ → establishes link between shape and topological optimisation.

- Hole creation only when more optimal than shape optimisation.

Dunning and Kim (2013) IJNME

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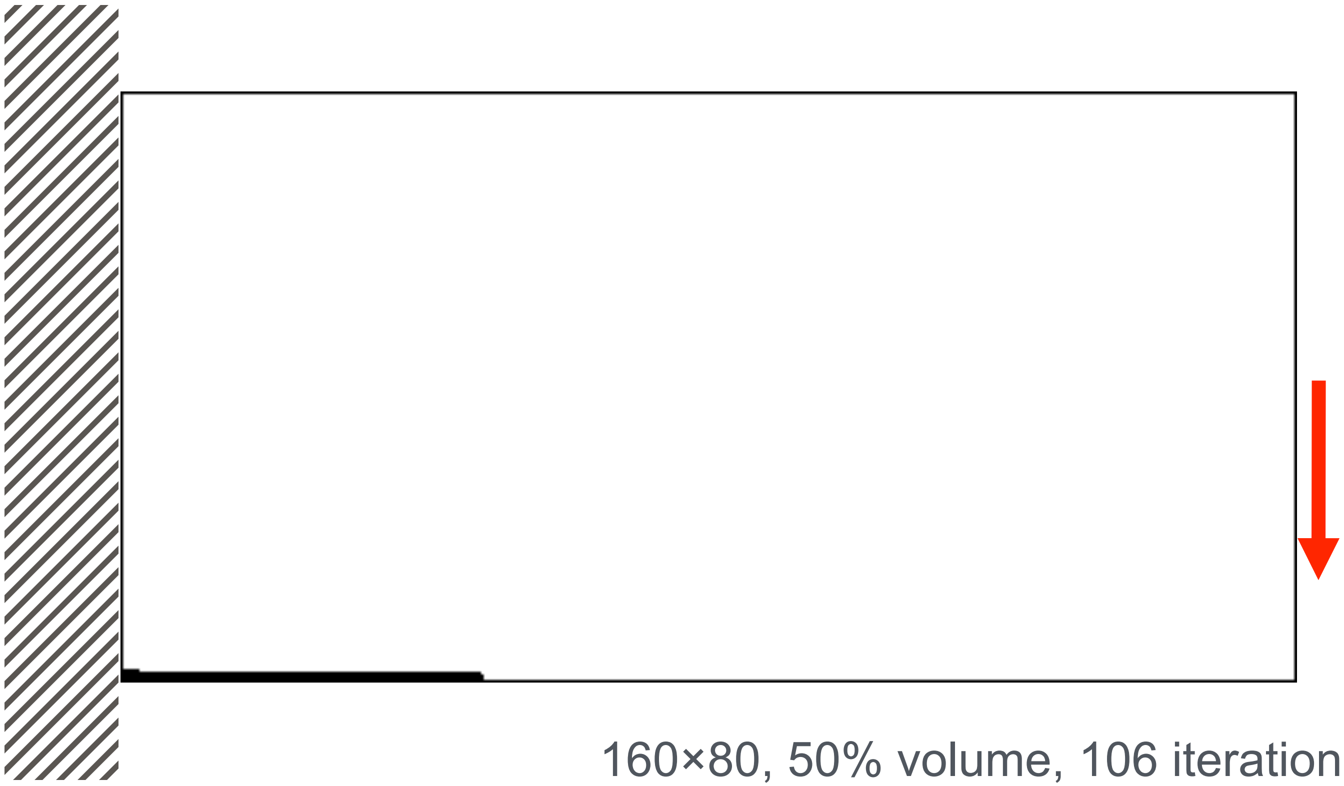
Cantilevered Beam in 2D



160×80, 50% volume, 106 iterations

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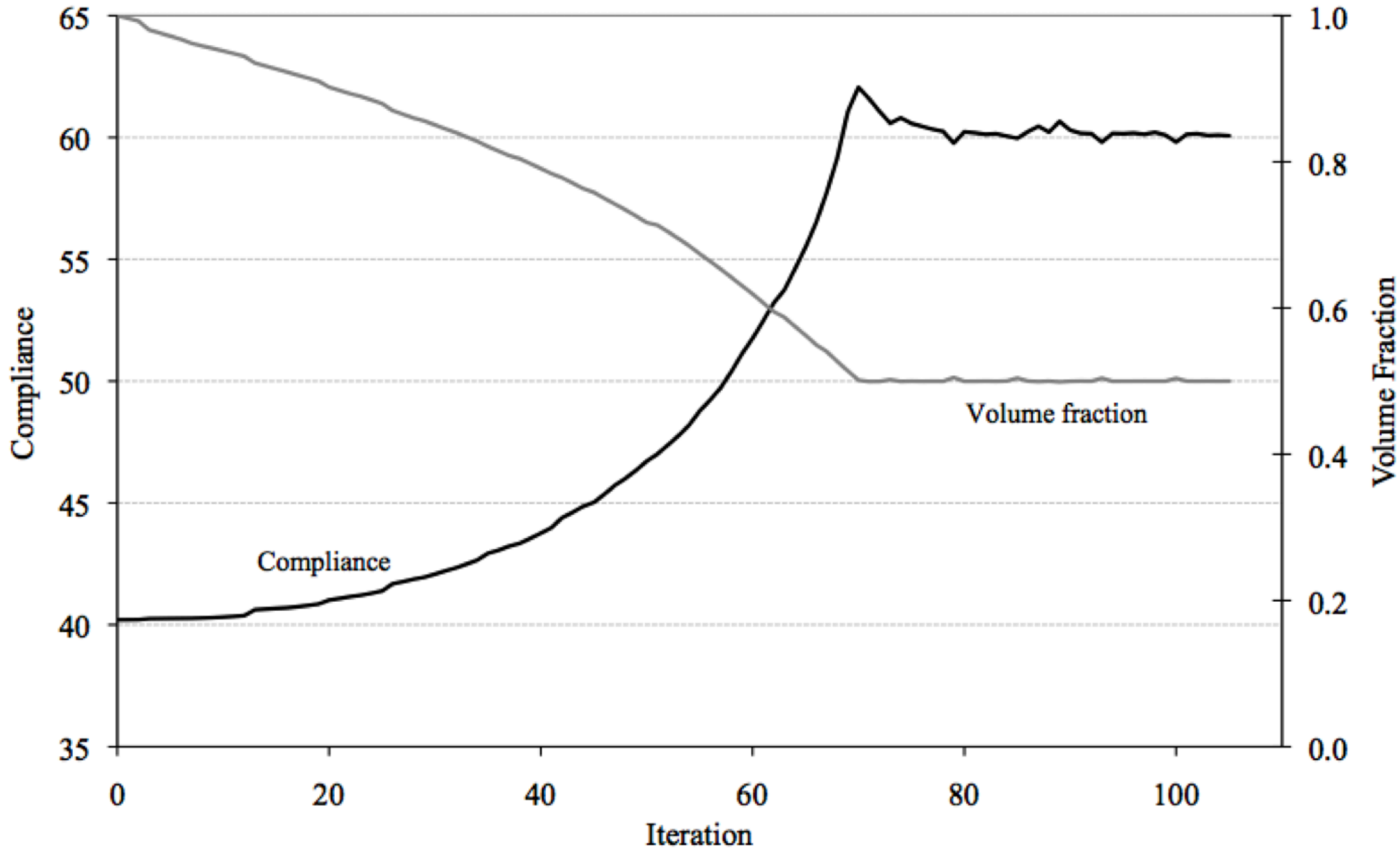
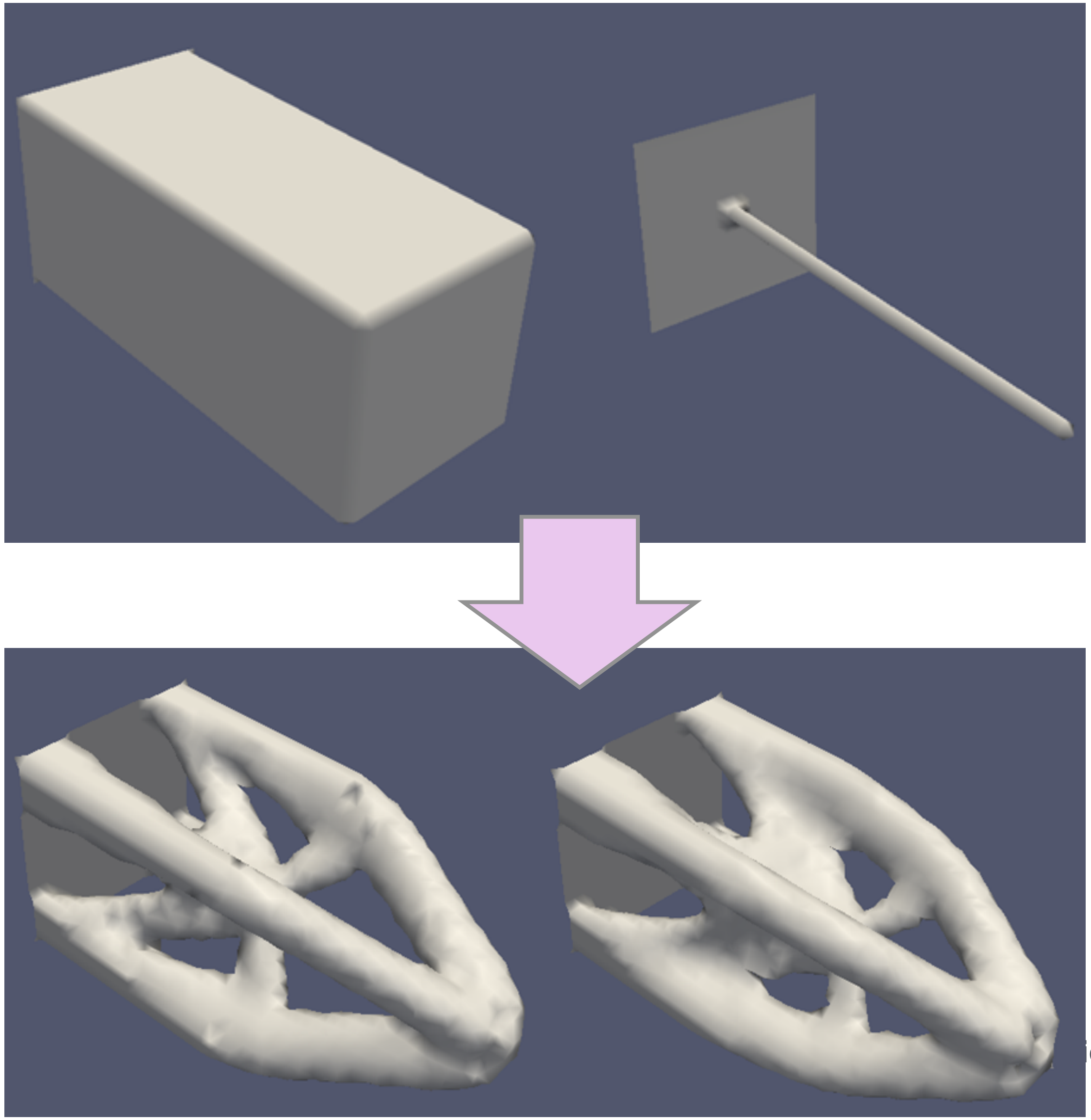
Cantilevered Beam in 2D



Cantilevered Beam



- 45% volume constraint
- 0.5% difference in compliance
- Robust with respect to the initial design

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Robust Topology Optimisation

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Robust Topology Optimisation

- Minimisation of expected and variance of performance

$$J = \left(\frac{\eta}{w}\right)E[C] + \left(\frac{1-\eta}{w^2}\right)Var[C]$$

where $E[C] = \int_f C(u,f)\prod_{i=1}^{n} P(f_i)df$

$$Var[C] = \int_f C(u,f)^2\prod_{i=1}^{n} P(f_i)df - E[C]^2$$

- Topology optimisation + Uncertainties = conventional methods are computationally intractable

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We have shown:

- The robust energy functional has an analytical minimum
- Can be solved by a small set of auxiliary problems
- Uncertainties in magnitude of loading
  - Expected compliance: (N+1) cases $E\left[C\right]=\sum_{i,j=1}^{m}\kappa_{ij}\mu_i\mu_j+\sum_{i=1}^{m}\kappa_{ii}\sigma_i^2$
  - Variance: (N+3) cases

$$\left\langle C\right\rangle = C_1(u,\bar{f},\theta)+\sum_{i=i}^{n}\left[w_{1,i}C_{2,i}(u,1,\mu_{\theta i})+w_{2,i}\left(C_{x,i}(u,1,\theta_x)+C_{y,i}(u,1,\theta_y)\right)\right]$$

- Uncertainties in direction of loading ($f_{ix}=f_i\cos\,\theta_i$ and $f_{iy}=f_i\sin\,\theta_i$)
  - Expected compliance: (N+1) cases

$$Var[C(f)]=4\sum_{i=1}^{m}\sum_{j=1}^{m}\sum_{k=1}^{m}\left(\kappa_{i.k}\kappa_{j,k}\mu_i\mu_j\sigma_k^2\right)+2\sum_{i=1}^{m}\sum_{j=1}^{m}\left(\kappa_{i.j}^2\sigma_i^2\sigma_j^2\right)$$

  - Variance: 256 → 23, on-going.

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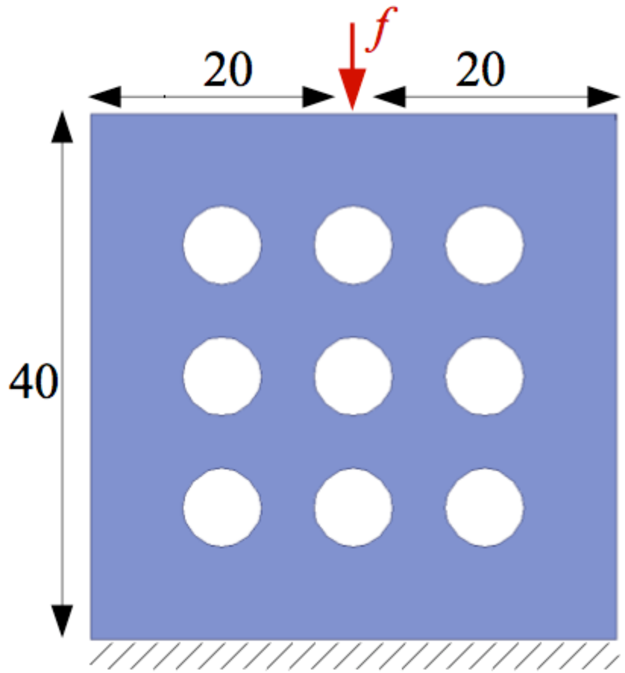
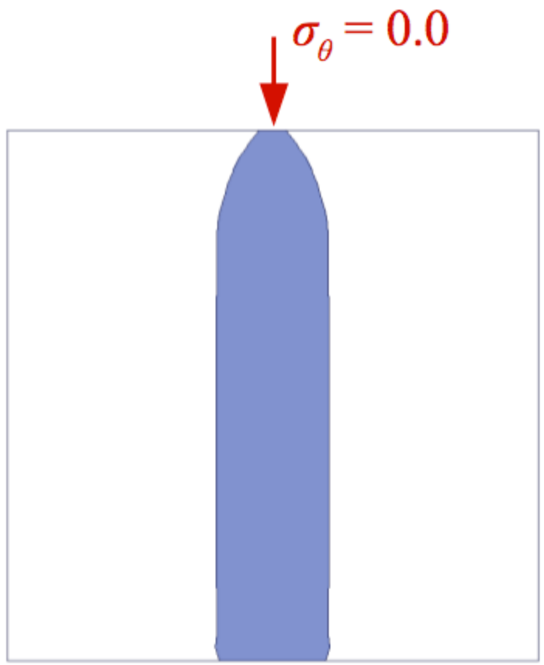
Example: Column under compression

A single load with uncertainty in direction, 20% volume

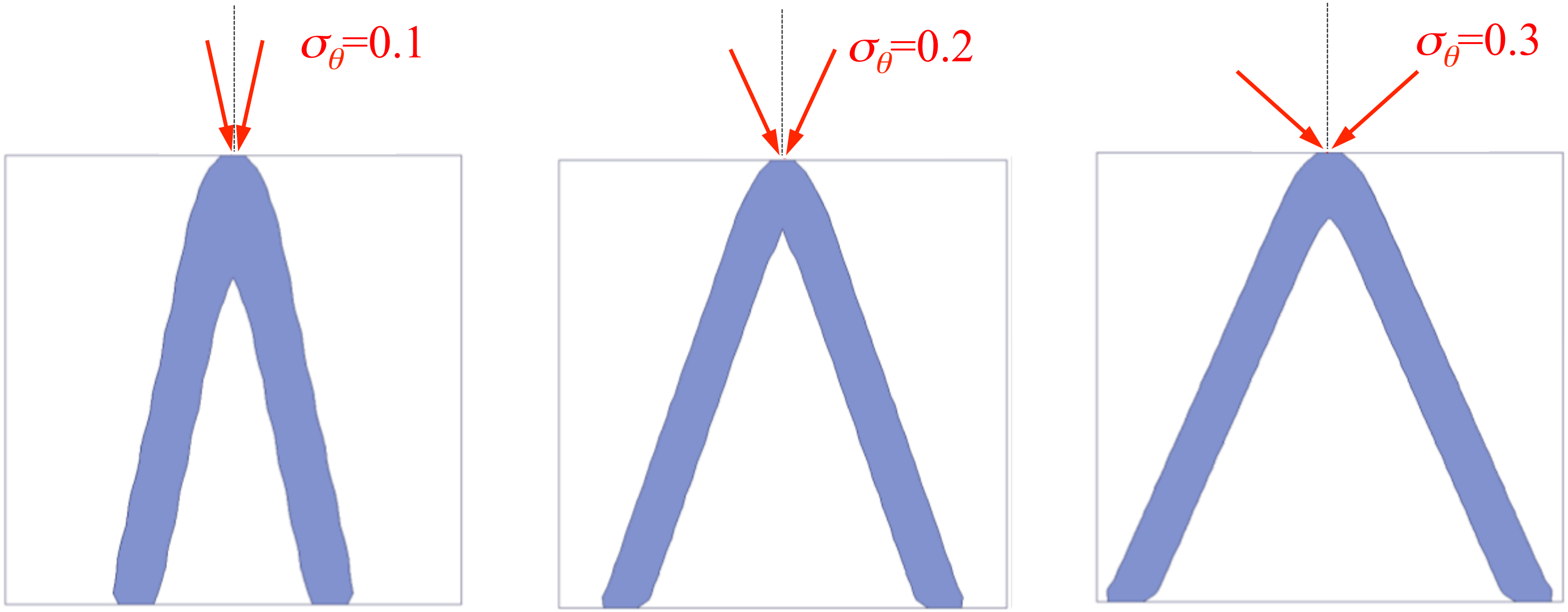


Initial design



Deterministic solution

Example: Column under compression



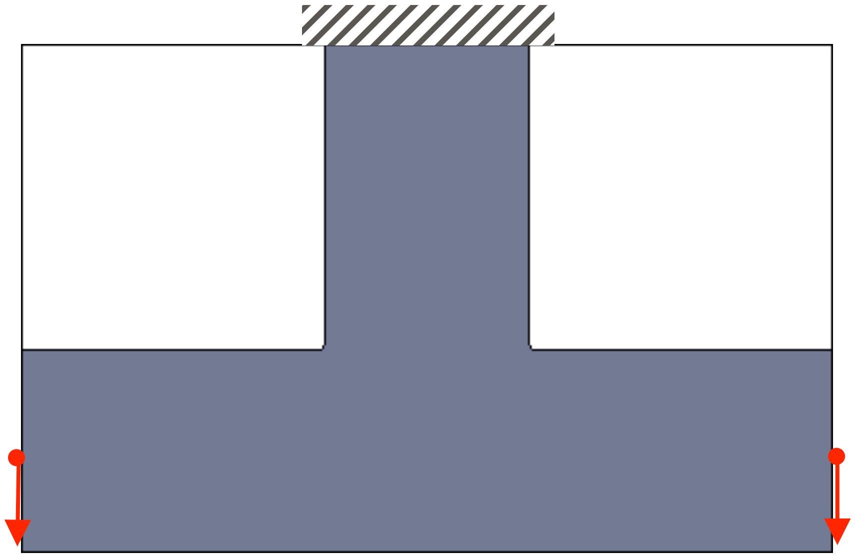
E[J]: Det. sol. = 525
Robust sol. = 377 (22%)

E[J]: Det. sol. = 1124
Robust sol. = 449 (60%)

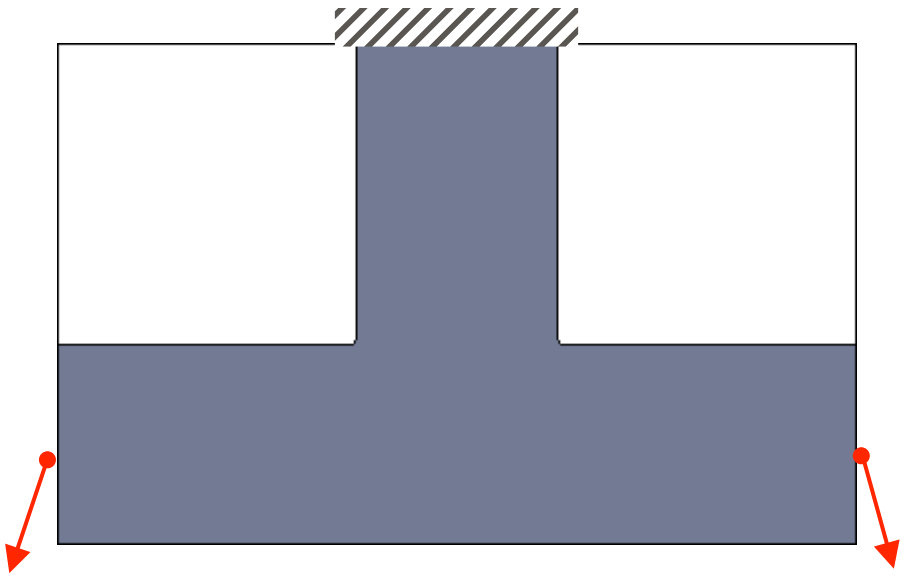
E[J]: Det. sol. = 2045
Robust sol. = 523 (74%)

Example: Double hook

Uncertainties in magnitude: $\mu = 5.0$, $\sigma = 0.5$
Uncertainties in direction of loading: $\mu = 3\pi/2$, $\sigma = 0.25$
50% volume



Deterministic solution
E[C] = 2.36



Robust solution
E[C] = 1.50

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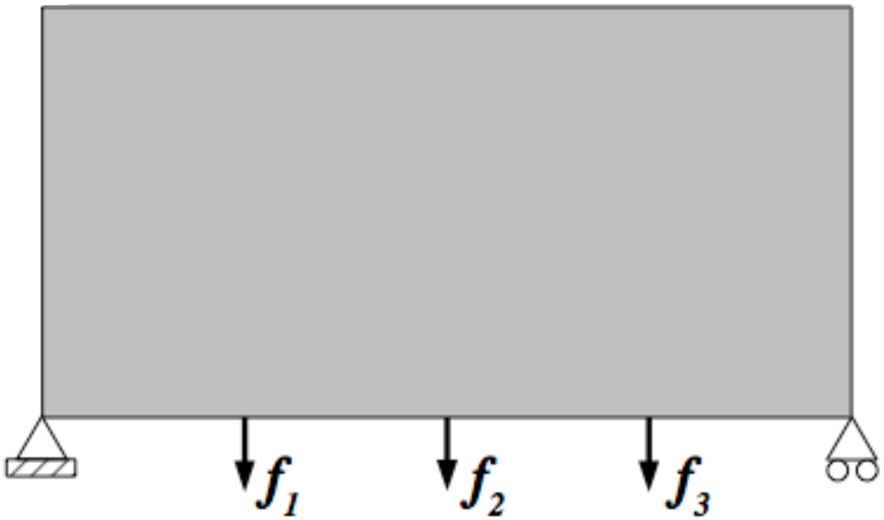
Example: Beam

160 x 80, 40% volume

$$J = \left(\frac{\eta}{w}\right)E\left[C\right] + \left(\frac{1-\eta}{w^2}\right)Var[C]$$



Initial Design

μ_1=1.0 μ_2=1.0 μ_3=1.0
σ_1=0.5 σ_2=0.1 σ_3=0.2

Deterministic Solution

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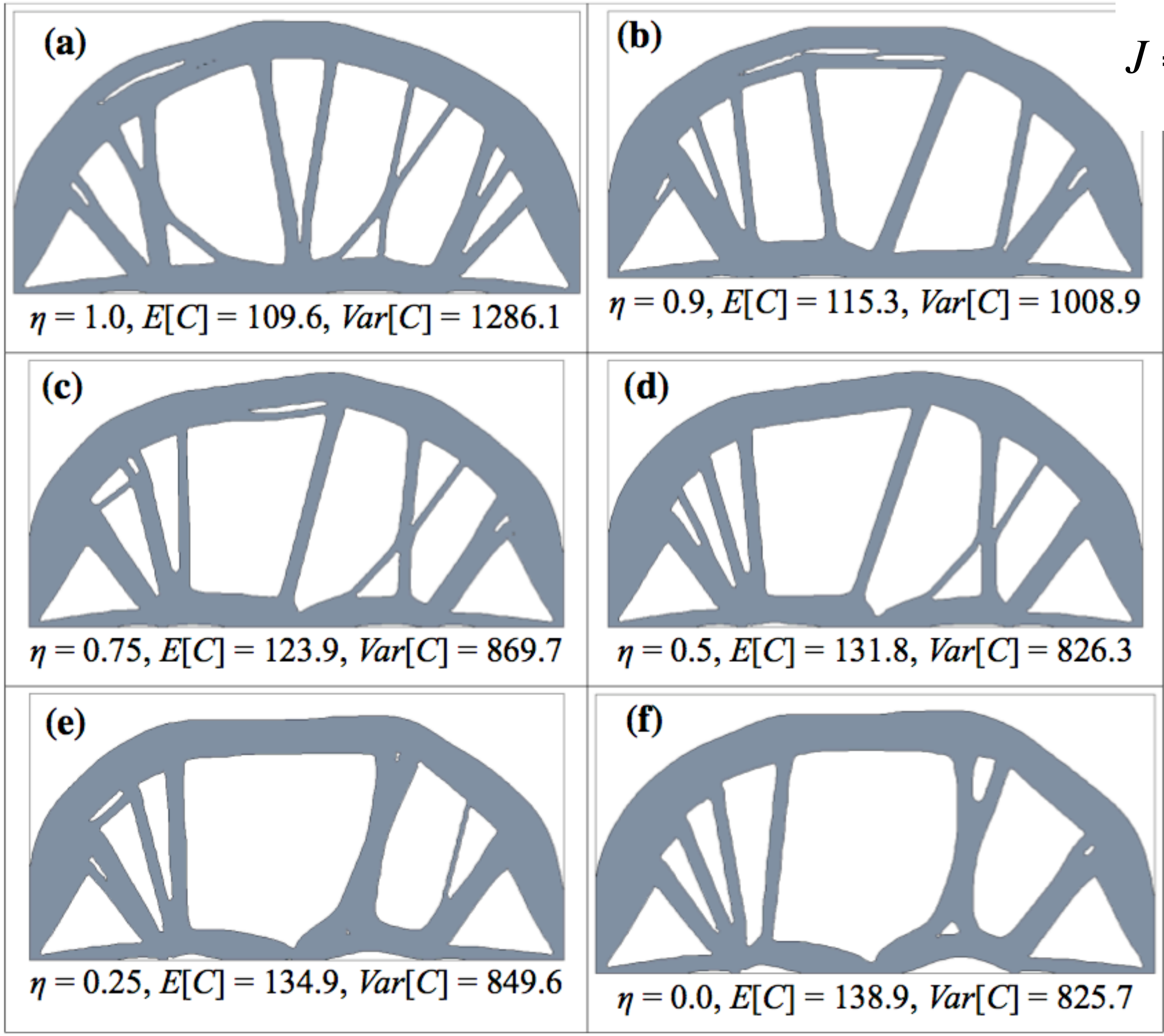
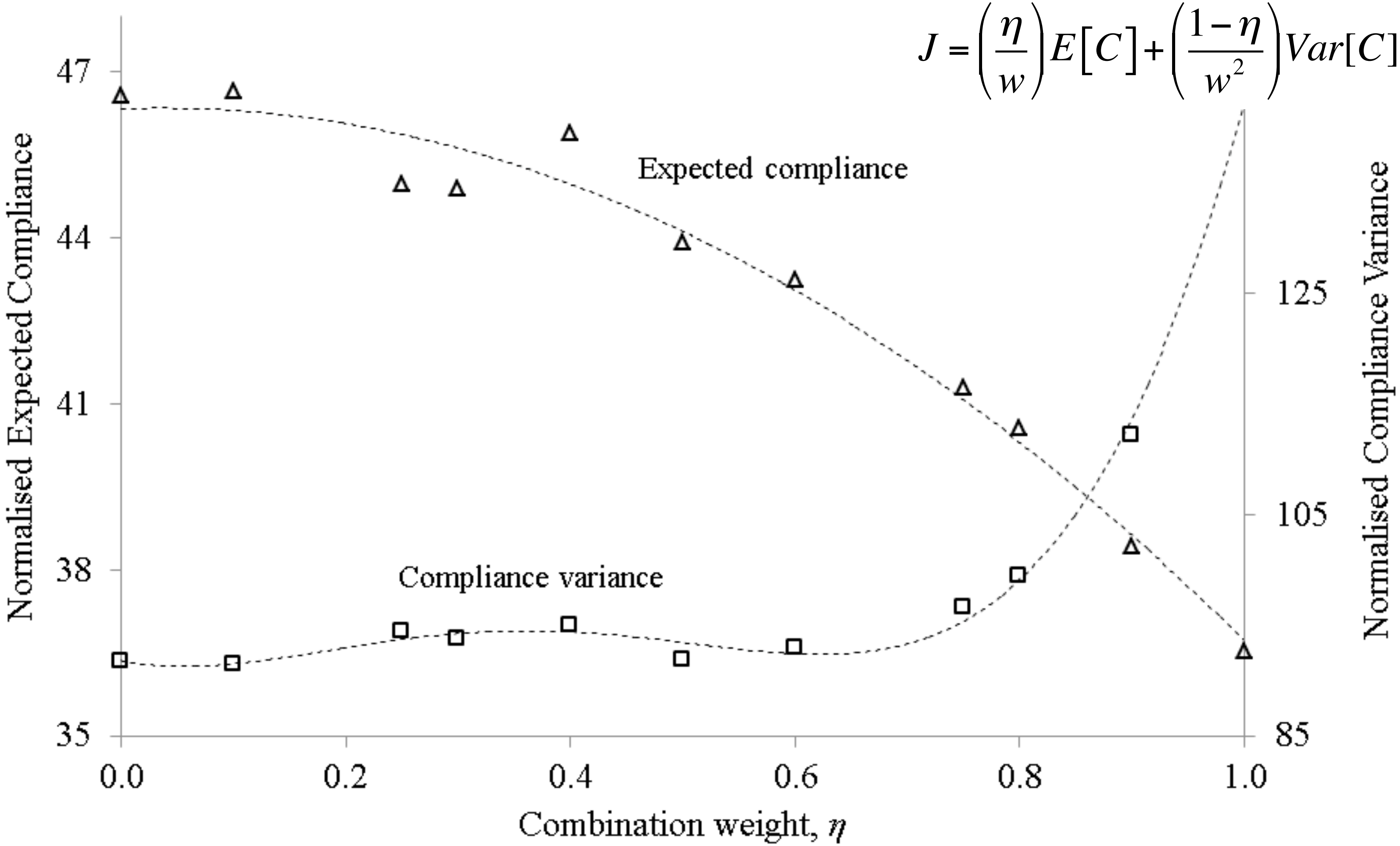
Example: Beam

$$J = \left(\frac{\eta}{w}\right)E[C] + \left(\frac{1-\eta}{w^2}\right)Var[C]$$



Robust Solutions for Varying Weights

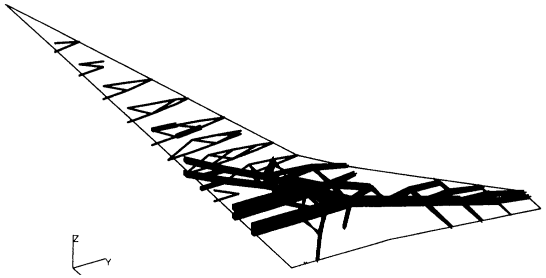
$$J = \left(\frac{\eta}{w}\right) E\left[C\right] + \left(\frac{1-\eta}{w^2}\right) Var[C]$$



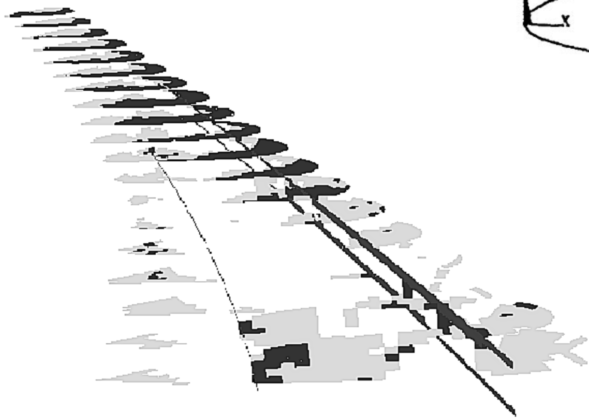
Multidisciplinary Topology Optimisation

Aircraft Wing
Aero-Structural Optimisation

- Topology optimization be used to explore alternative designs
- Mostly applied to a pre-determined layout or an individual component

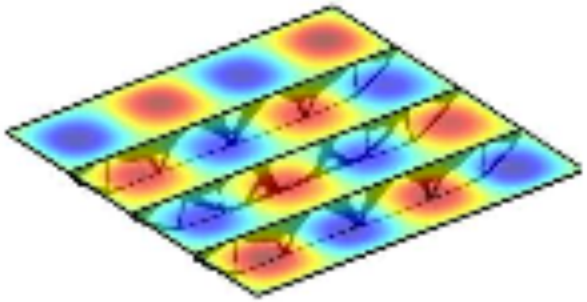


Balabanov & Haftka 1996

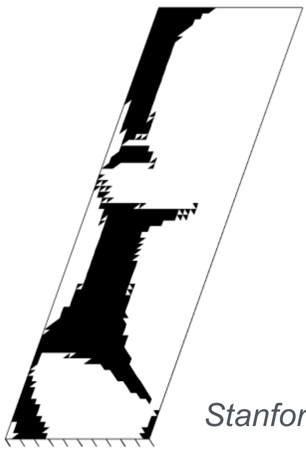


Maute & Allen 2004

Eschenauer, Becker & Schumacher 1998



Stanford, Beran & Bhatia, 2013



Stanford & Beran, 2010

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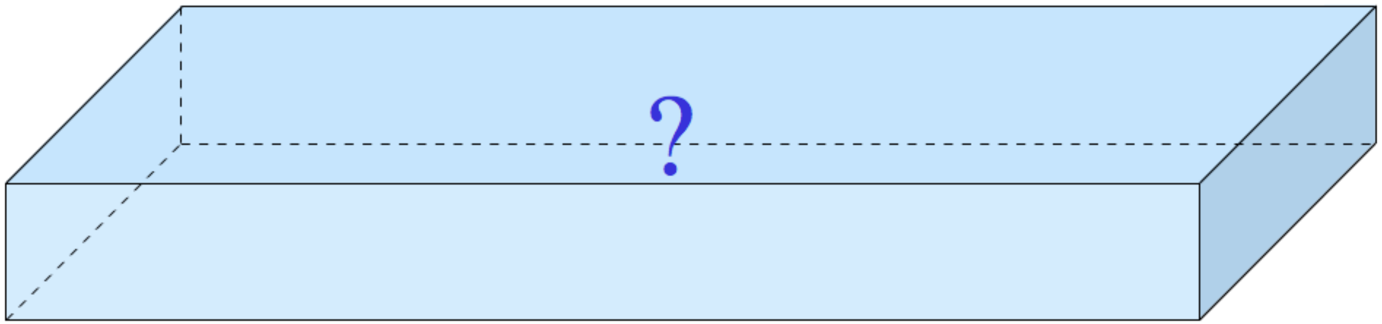
3D Level Set Topology Optimisation of a Wing

- Objective to optimize 3D topology of wing box domain
- Including aero-structural coupling is important:
  - Loading dependent on deformed shape of the wing
  - Analysis & sensitivity computation



Aerostructural Topology Optimisation

- Minimize: Total structural compliance
- Subject to: Lift ≥ Weight
- Aerodynamic loading from single flight condition
- Fixed angle of attack

$$\text{Minimize: } C(u) = f(u)^T u$$

$$\text{Subject to: } L(u) \geq W_c + W_b$$

$$Ku = f(u) = f_c + Qu$$

C = total compliance

u = structure displacement vector

f = total load vector

K = structure stiffness matrix

L = total lift

W_c = fixed aircraft weight

W_b = wing structure weight

f_c = fixed load vector

Q = aerodynamic stiffness matrix

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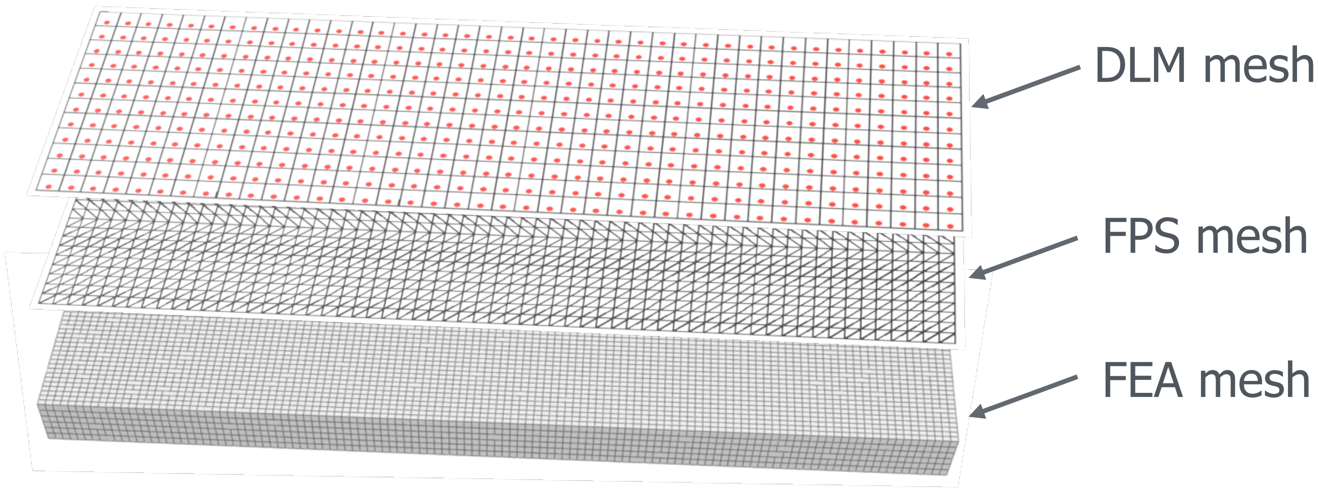
3D Level Set Topology Optimisation of a Wing

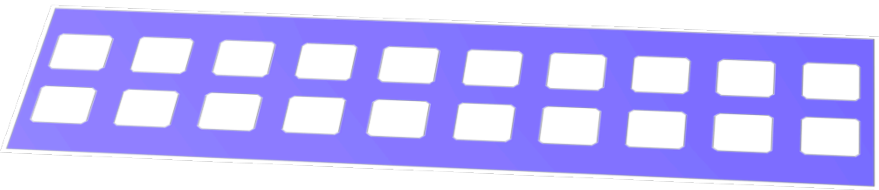
- Aerodynamics: Doublet Lattice Method
- Fluid-structure interaction: Finite Plate Spline (work conserved)
- Structures: Finite Element Analysis



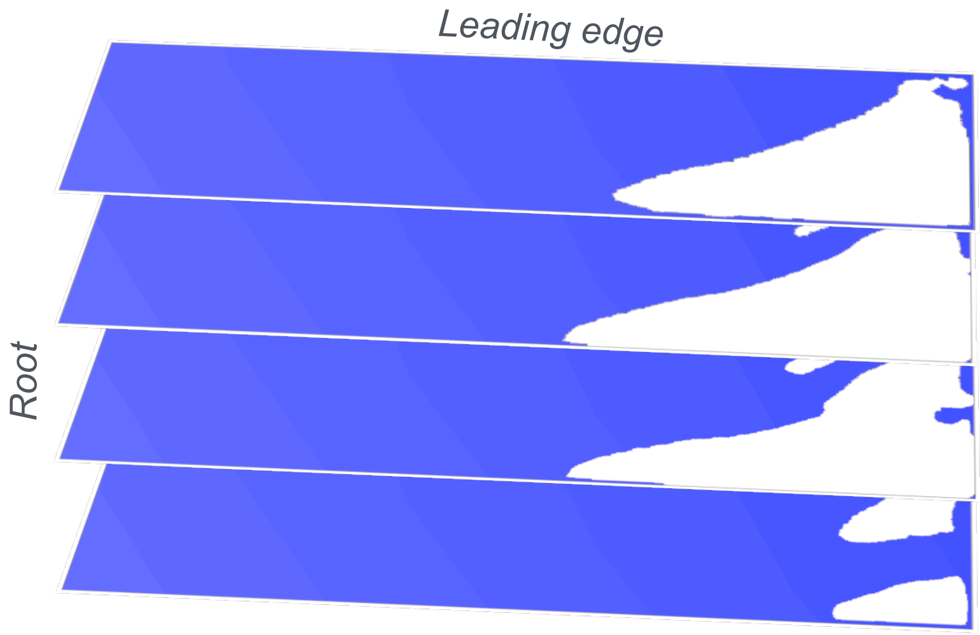
Example: 3D Wing Box

- Aspect ratio 6, Chord 2m
- Wing box from leading edge to 80% chord
- Wing box depth 15% Chord
- Discretization: 32 × 120 × 6 elements

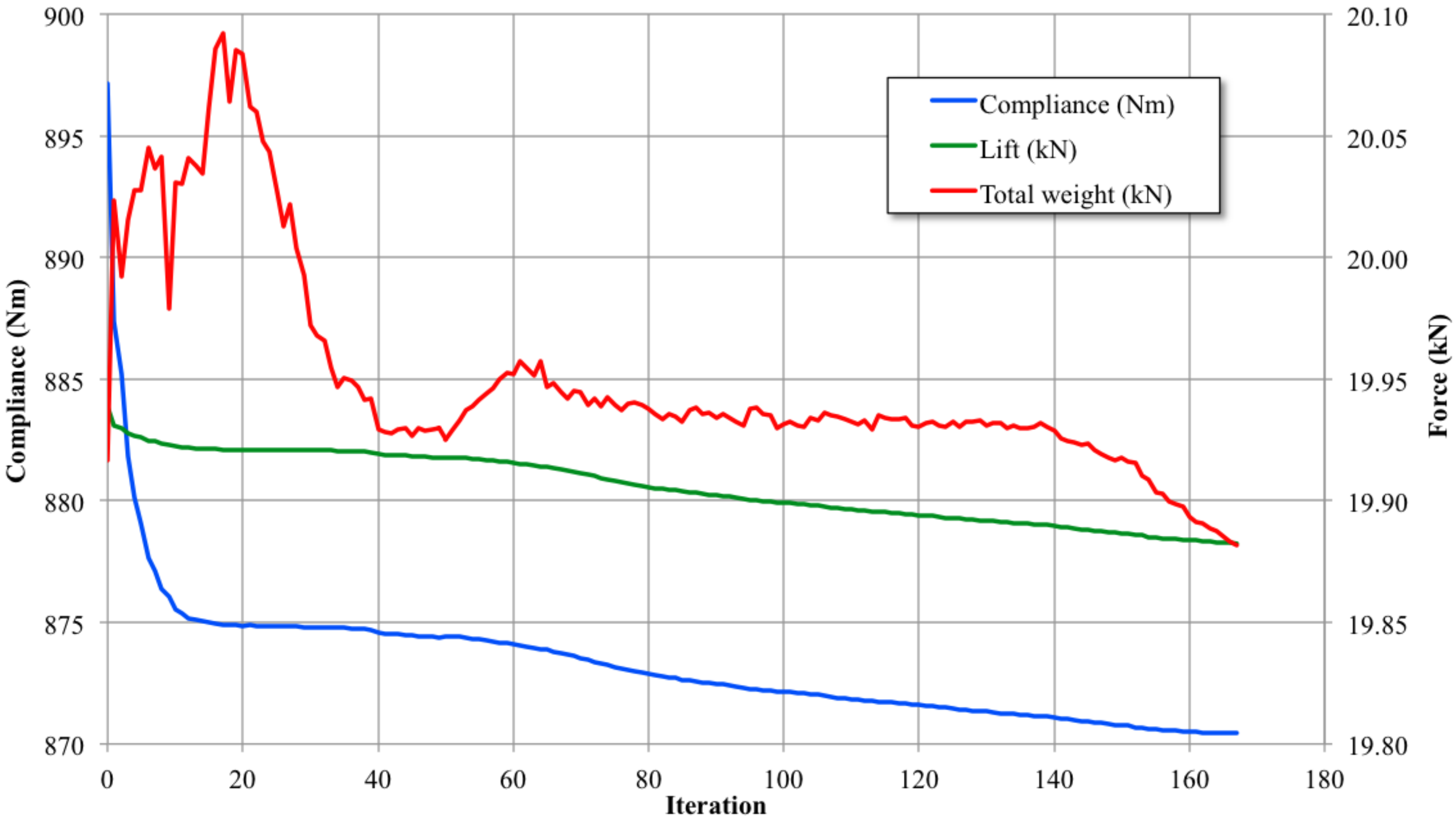
Design	Compliance (Nm)	Weight (kN)	Lift (kN)
Initial	897.1	19.92	19.94
Optimum	870.5	19.88	19.88



Initial design x-section

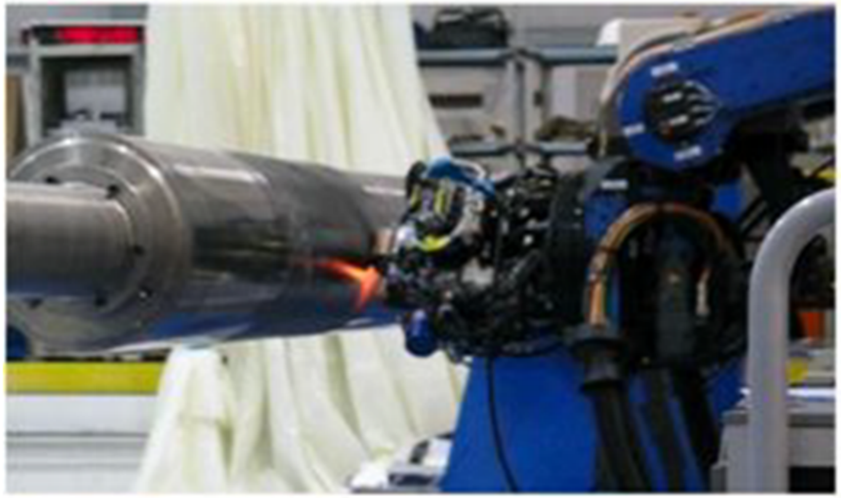


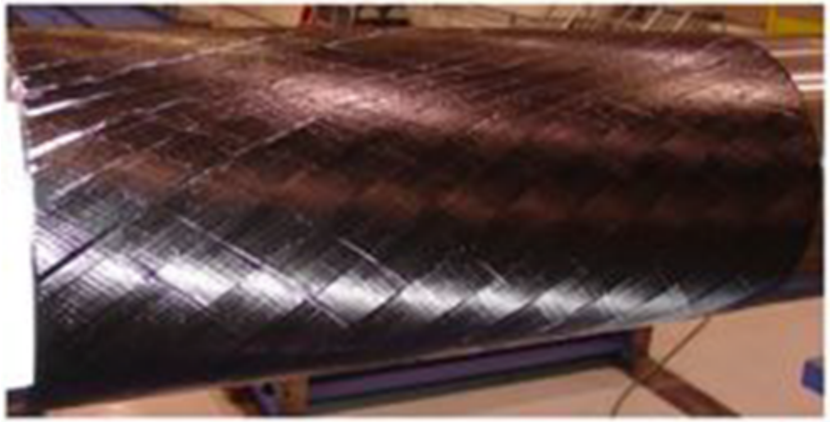
Example: 3D Wing Box

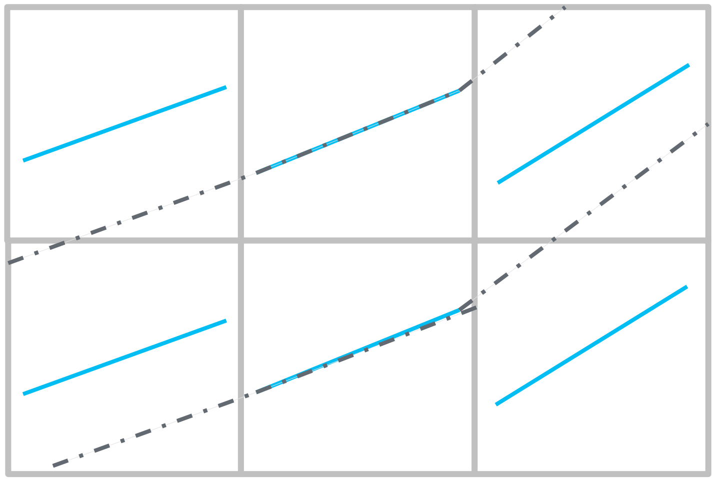


Composite Tow Paths Optimisation

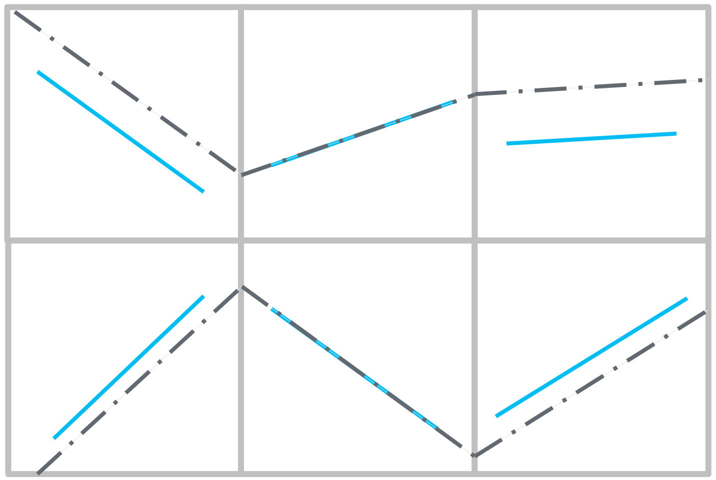
Advanced Composite Materials







Continuous Fiber Angles.



Discontinuous Fiber Angles.

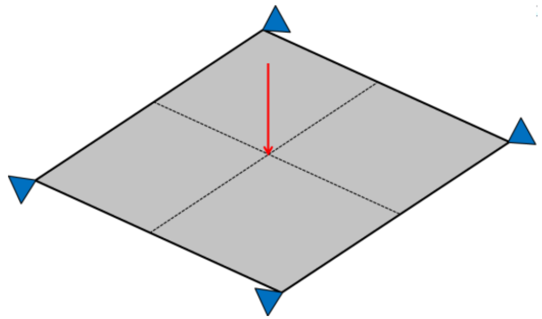
Composite Tow Paths Optimisation

- Our approach: use the level set method
- Optimise the tow paths not the fibre angles
- Ensures continuity of fibre angles
- Initial solution: topological optimum with isotropic material
- Single and multiple level set functions
- Minimise compliance

Test Models

Cantilever Beam:

Plate Loaded Out of Plane:



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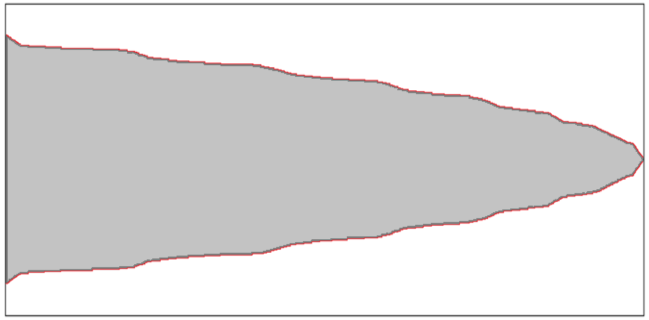
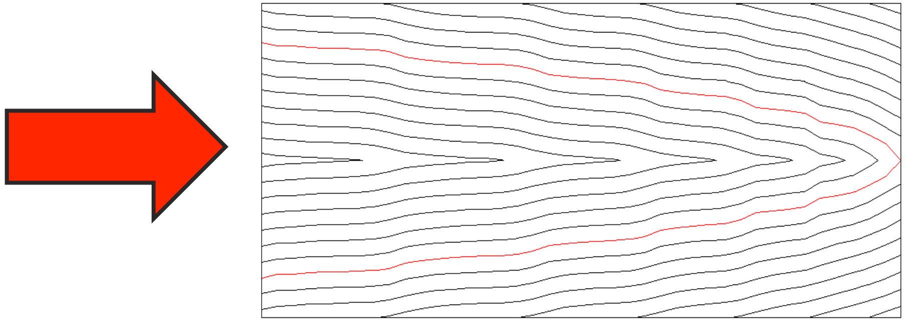
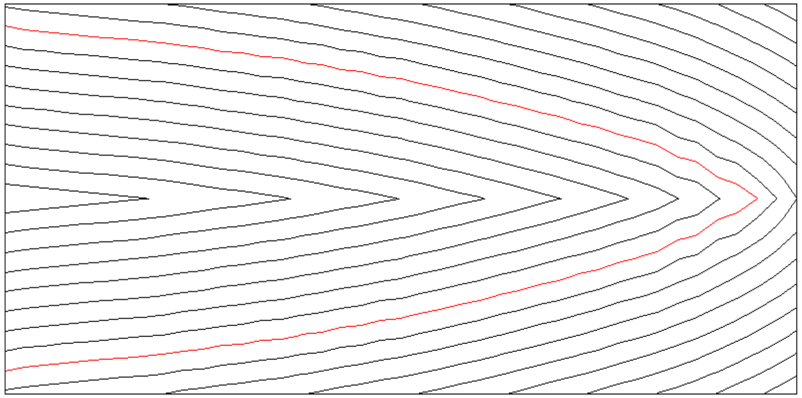
Cantilever Beam Result

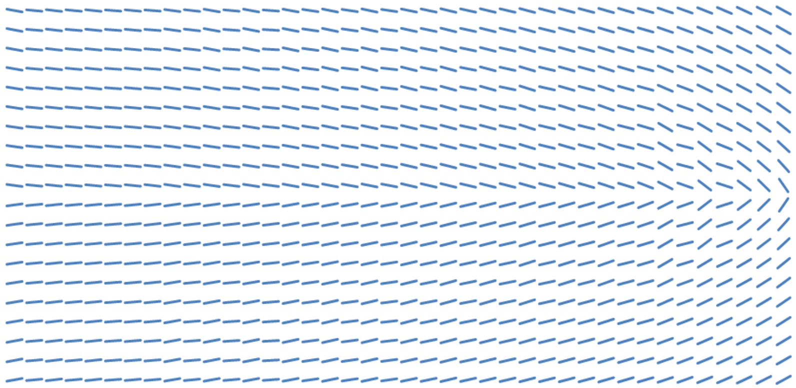
Initialisation



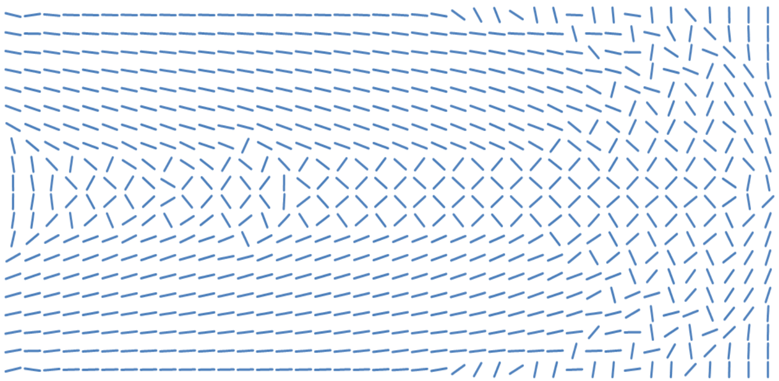


Final Solution





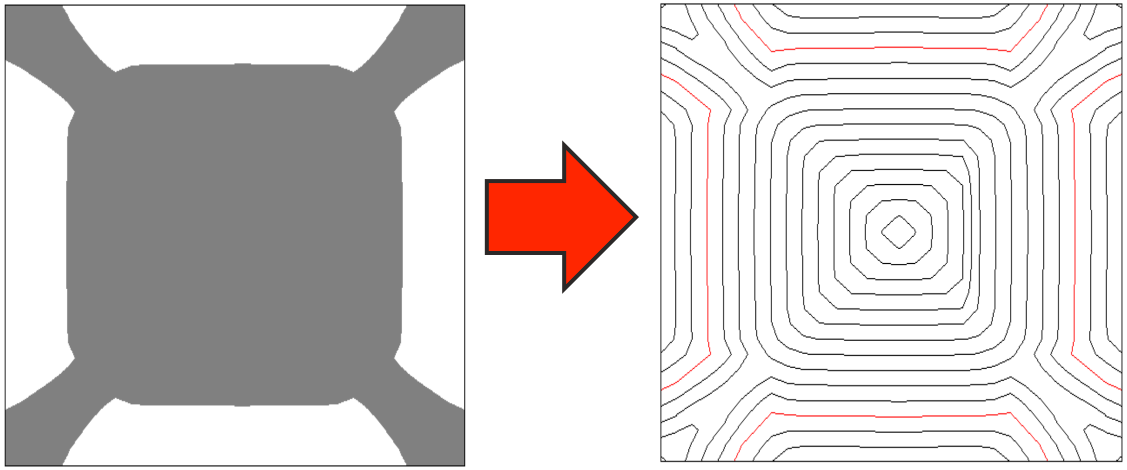
Elemental Solution



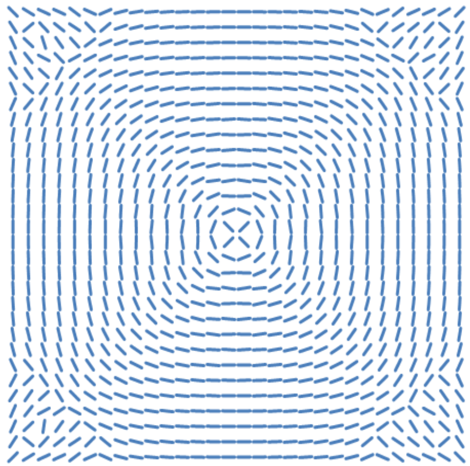
	Initial Compliance	Solution Compliance	% Difference	FCS
Level Set Method	26.94	19.30	18.99%	93.12%
Elemental Method	34.42	16.22	-	66.0%

Plate Loaded Out of Plane

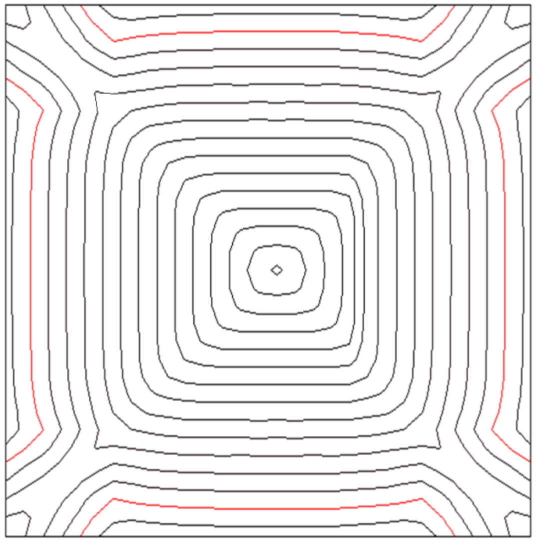
Initialisation

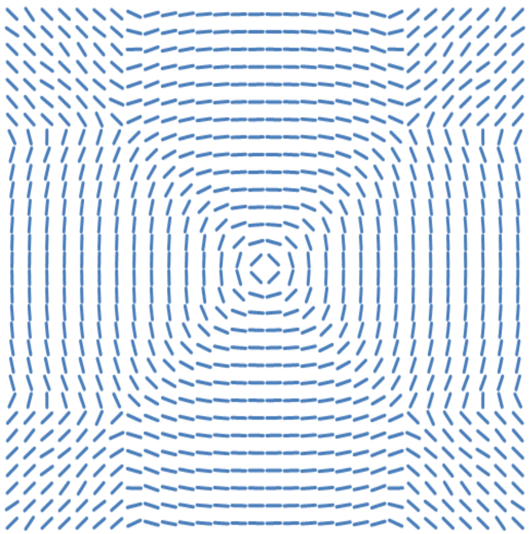


Elemental Solution



Final Solution

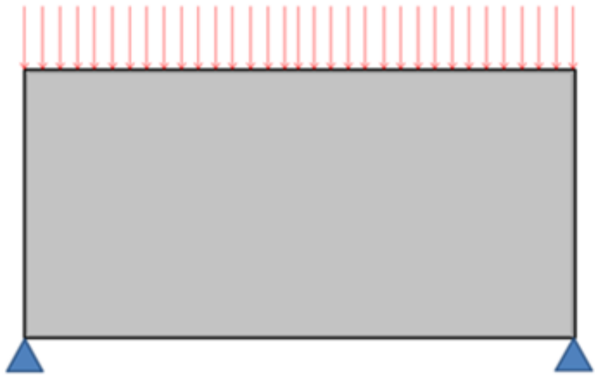




	Compliance	% Difference	FCS
Level Set Method	1.51	9.42%	76.31%
Elemental Method	1.38	-	68.71%

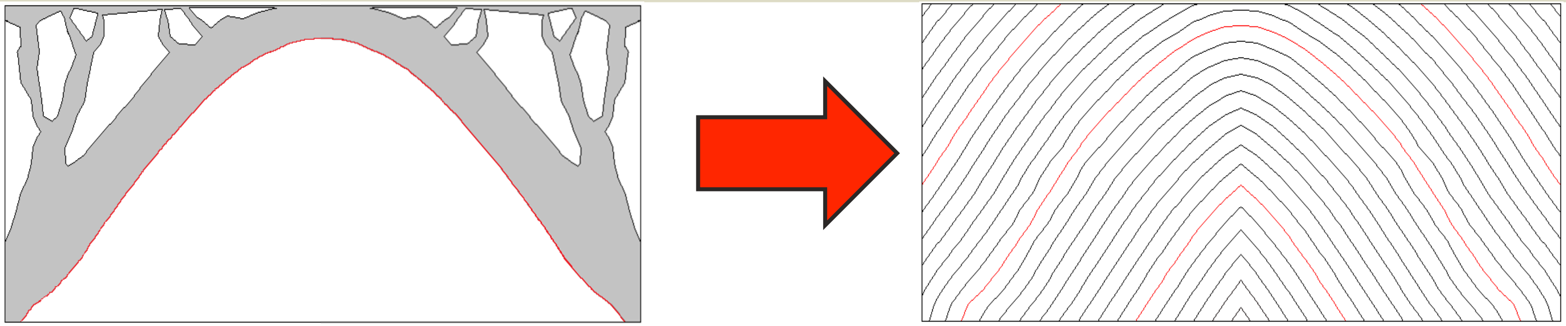
Multiple Level Set Function

Bridge Beam:

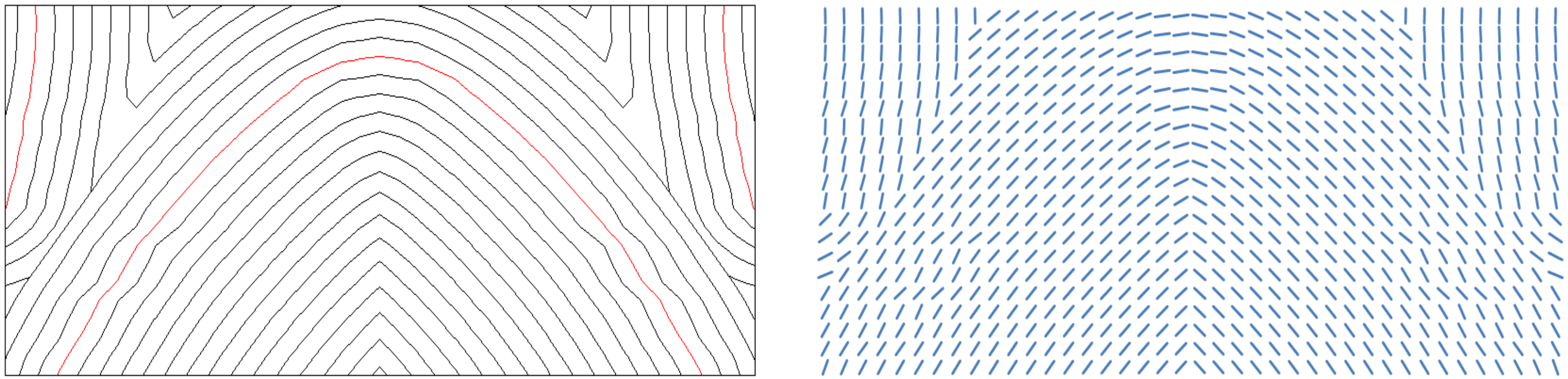


Bridge Beam Result

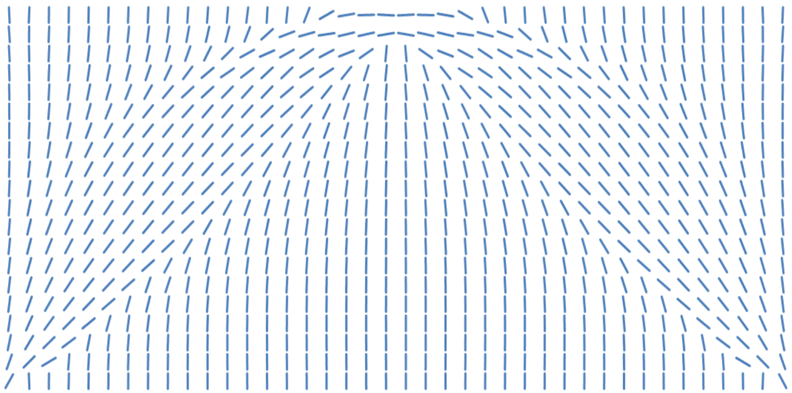
Initialisation



Final Solution



Elemental Solution



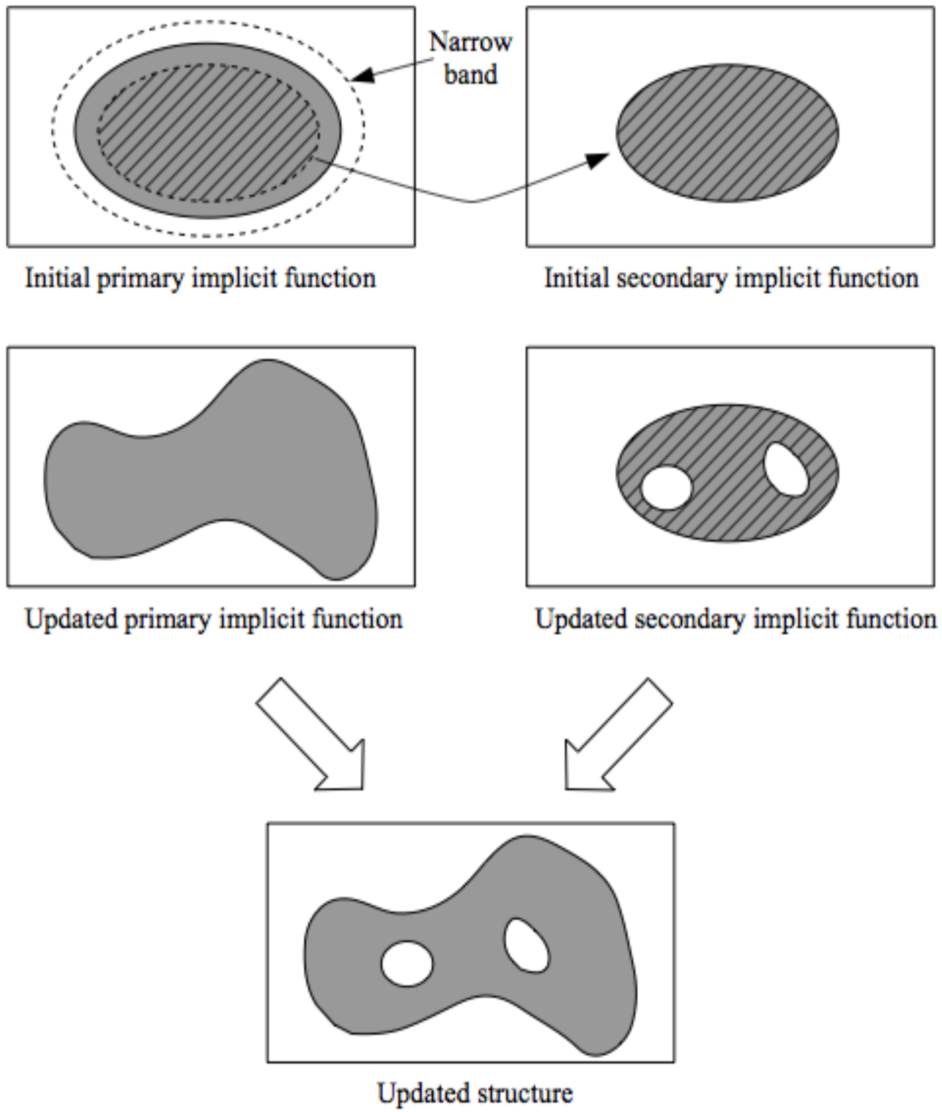
	Initial Compliance	Solution Compliance	% Difference	FCS
Level Set Method	2.54	1.86	18.47%	85.95%
Elemental Method	4.89	1.57	-	84.10%

Conclusion

- Level set topology optimisation method
  - Numerically stable
  - Reduced dependency on mesh and starting solutions
  - Robust with the starting solution
  - Well-defined boundaries guaranteed.
- Challenging and structural multidisciplinary problems
  - Aeroelasticity
  - Stress, buckling
  - Piezoelectric (actuated structures, energy harvesting)
  - Poro-elastic microstructures
  - Electromagnetic (antenna)
  - Composite materials
  - Resource oriented architecture
- Robust optimisation for uncertainties

Our Approach of Creating a Hole

- A new hole is created when $\bar{\phi} < 0$
- Minimum hole size threshold: not if only one $\bar{\phi}_i < 0$ and $\bar{\phi}_j > 0$.
- $\bar{\phi}$ is copied on to ϕ.
- $\phi, \bar{\phi}$ are reinitialised and continued for another hole creation.

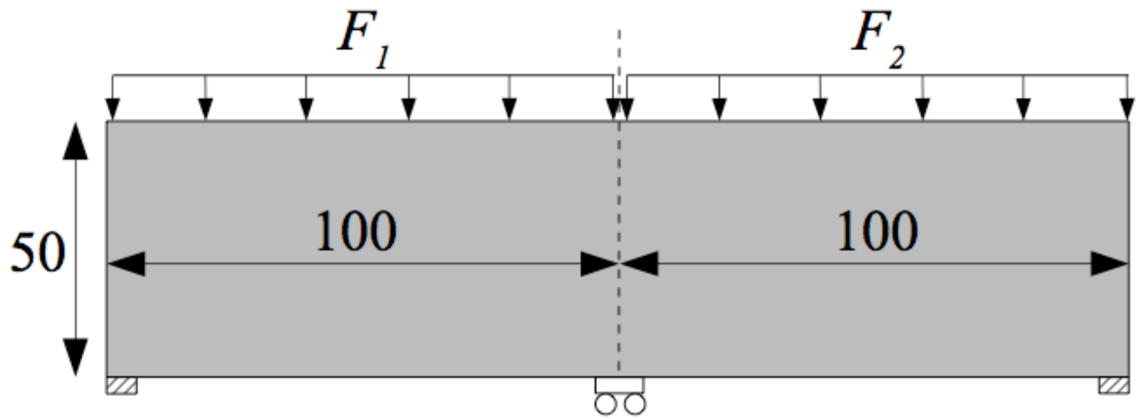


Dunning and Kim (2013) IJNME

Example: Bridge

200 x 50, 50% volume

$$J = \left(\frac{\eta}{w}\right)E\left[C\right] + \left(\frac{1-\eta}{w^2}\right)Var\left[C\right]$$



Initial Design
μ = 0.1 / unit length
σ = 0.04 / unit length

Deterministic Solution
Expectancy = 1321
Variance = 1326x10^3

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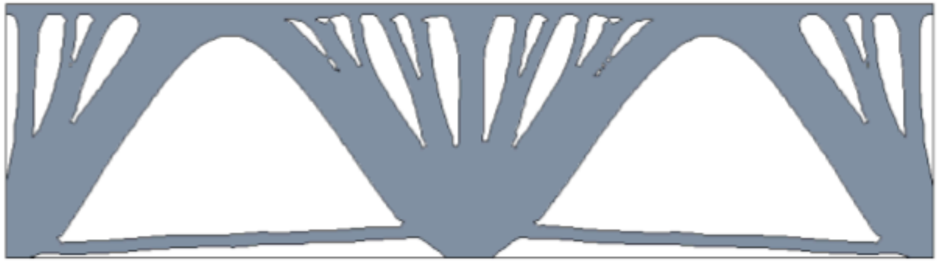
Example: Bridge

η = 1.0
Expectancy = 635.1
Variance = 100.7×10^3



η = 0.5
Expectancy = 633.7
Variance = 100.3×10^3

η = 0.0
Expectancy = 633.5
Variance = 100.2×10^3

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