

Simulating Motion Of Charged Particle/Electron Through an Electric and Magnetic Field

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0.1 Aim

To simulate the motion of a charged particle inside an Electric and Magnetic Field.

0.2 Introduction

The project focuses on simulating/tracing the motion of a charged particle/electron under the influence of an external Electric and Magnetic Field. The electron trajectories can be given an desired shape by applying an Electric or Magnetic Field

0.3 Applications

- Radio
- Television
- CRO
- Mass Spectrograph
- Particle Accelerators

0.4 Motion of electron inside an uniform Electric Field

0.4.1 Types of motion exhibited by electron

- Parallel to Electric Field
- Perpendicular to Electric Field
- Projected at an angle to the Electric Field

0.4.2 Basic Calculation

When an electron is projected inside an electric field, sustained by 2 plates **A** and **B** with A being **positively** charged and B being **negatively** charged, having a potential **V** given by $[V_a - V_b]$ at a distance of **d** the intensity of the electric field between the plate is given by

$$E = \frac{V}{d} \quad (1)$$

Force experienced by an electron at rest on plate **B**

$$F = -eE \quad (2)$$

The acceleration is given by

$$a = \frac{F}{m} = -\frac{eE}{m} \quad (3)$$

The corresponding code in the project is given in:

Basic_Calculations in *BasicCalc.h*

```
void Basic_Calculations(double PotentialDifference , double PlateDistance
{
    Energy_Electron = PotentialDifference / PlateDistance; //Energy of the
    Force_Electron = -1 * ELECTRON_ENERGY * Energy_Electron; //Force on elec
    Acceleration_Electron = fabs(Force_Electron) / ELECTRON_MASS; //Acceler
}
```

0.4.3 Parallel to Electric Field

The velocity \mathbf{V} is given by :

$$V = V_0 + at \quad (4)$$

where V_0 is the **initial velocity** of the electron or charged particle

The displacement of the electron is given by

$$S = V_0 t + \frac{1}{2}at^2 \quad (5)$$

Velocity at any given time denoted by V_t is given by

$$V_t = \frac{eE}{2m}t = \sqrt{\frac{2eE}{m}x} \quad (6)$$

parallel electric field used in project

$$X = \frac{eE}{2m}t^2 \quad (7)$$

The above equation gives the horizontal displacement of X which is depended on the value of t

The calculation is done by the following code :

```
void ElectronMovement_Parallel(int Identifier)
{
    float count = Misc.count = 0;
    int Index = Misc.index = 0;
    float Time = EField.Var.TimeEpoch;
    float StepSize = EField.Var.StepSize;
    unsigned int mem = Misc.MemAllocFactor;

    EField.CompArray.Xcomponent = (double *)calloc(mem, sizeof(double));

    while (count <= Time)
    {
        EField.Parallel.X_Component = fabs(Force_Electron) / (2 * ELECTRON_MASS);

        EField.CompArray.Xcomponent[Index] = EField.Parallel.X_Component;
        count += StepSize;
        Index++;
    }
}
```

0.4.4 Perpendicular to Electric Field

If **A** and **B** are two metal plates placed horizontally parallel to each other at a length **l**

The velocity at **Y** axis would be 0 initially, and would be V_0 for **X** axis

Therefore, $V_y = 0$ and $V_x = V_0$.

The acceleration of the electron is given in y direction by

$$\alpha_y = \frac{eE}{m} \quad (8)$$

As a result, the equation for velocity along the **Y** axis can be written as

$$V_y = \frac{eE}{m}t \quad (9)$$

The horizontal, displacement **X** is given by the following equation

$$x = v_0 t \quad (10)$$

The time t is known as *transit time* given by,

$$t = \frac{x}{v_0} \quad (11)$$

If V_A is the accelerating potential, then the initial velocity V_0 is given by

$$V_0 = \sqrt{\frac{2eV_a}{m}} \quad (12)$$

The vertical displacement can be written as

$$y = D \tan \theta = D \cdot \left(\frac{V_y}{V_0} \right) \quad (13)$$

The angular displacement is given by :

$$\theta = \tan^{-1} \left(\frac{V_y}{V_0} \right) \quad (14)$$

The code which processes the above information, is given in *BasicCalc.h*

```

void ElectronMovement_Perpendicular(float PlateWidth, int Identifier)
{
    Misc.count = 0;
    Misc.index = 0;

    EField.CompArray.Xcomponent = (double *)calloc(Misc.MemAllocFactor,
        sizeof(double));
    EField.CompArray.Ycomponent = (double *)calloc(Misc.MemAllocFactor,
        sizeof(double));

    while (Misc.count <= EField.Var.TimeEpoch)
    {
        EField.Perpendicular.HorizontalDisplacement_X = EField.Var.
            InitialVelocity * Misc.count;

        EField.Perpendicular.VerticalDisplacement_Y = -1 * (Force_Electron / 2 *
            ELECTRONMASS) * pow(Misc.count, 2);

        EField.Perpendicular.VerticalDisplacement_Leaving = (ELECTRONENERGY / 2
            * ELECTRONMASS) * Energy_Electron * (PlateWidth / pow(EField.Var.
            InitialVelocity, 2));

        EField.Perpendicular.AngularDisplacement = Energy_Electron / (2 *
            ELECTRONMASS * EField.Var.InitialVelocity) * pow(EField.Perpendicular.
            HorizontalDisplacement_X, 2);

        EField.CompArray.Xcomponent[Misc.index] = EField.Perpendicular.
            HorizontalDisplacement_X;
        EField.CompArray.Ycomponent[Misc.index] = EField.Perpendicular.
            VerticalDisplacement_Y;
        Misc.index++;
        Misc.count += EField.Var.StepSize;
    }
}

```

0.4.5 Electron Projected at an angle

Consider an electron projected at an angle θ_0 to an electric field \mathbf{E} directed vertically downwards, maintained between plates **A**(positively charged plate and **B**) (negatively charged plate)

At the entry point P_0 the velocity V_0 has components

$$V_{X0} = V_0 \sin \theta_0 \quad (15)$$

$$V_{Y0} = V_0 \cos \theta_0 \quad (16)$$

When the electron is pushed or is projected into the this field, the electron experiences a force \mathbf{f} which is directed to the positively charged plate. This causes the electron to have a projectile motion.

The force \mathbf{f} causes the electron to change its velocity from v_1 to v_2 at point P_2 .
At any instant the position of the electron can be given by the following equations

$$x = V_{X0} \cdot t = (V_0 \cos \theta_0) t \quad (17)$$

$$y = V_{y0} + \frac{1}{2} a t^2 = (V_0 \sin \theta_0) t + \frac{1}{2} a t^2 \quad (18)$$

The maximum vertical displacement denoted by \mathbf{H} of the electron can be calculated by

$$H = \frac{V_0^2 \sin^2 \theta_0}{2a} \quad (19)$$

The time taken by the electron to achieve maximum displacement \mathbf{t} is given by

$$t = \frac{v_0 \sin \theta_0}{a} \quad (20)$$

The time of flight \mathbf{T} gives the time taken by the electron to reach its final height

$$T = 2t = \frac{2V_0 \sin \theta_0}{a} \quad (21)$$

The range \mathbf{R} is gives the maximum horizontal displacement

$$R = V_0 \cos \theta_0 \frac{2V_0 (\sin \theta_0)}{a} = \frac{(2 \sin \theta_0 \cos \theta_0) V_0^2}{a} \quad (22)$$

$$R = \frac{V_0^2 \sin 2\theta_0}{a} \quad (23)$$

0.5 Motion Of an Electron inside a Magnetic Field

When a charge \mathbf{q} enters a magnetic field at an angle θ to the field intensity, it is acted upon by a magnetic force.

$$\vec{F} = q\vec{v} \times \vec{B} \quad (24)$$

If an electron moves in an uniform magnetci field makin an angle θ to the field intensity, it is acted upon by magnetci force given by :

$$\vec{F} = e\vec{v} \times \vec{B} \quad (25)$$

If the velocity \mathbf{v} and the intensity \mathbf{B} are perpendicular to each other the magnetic force is given by

$$F = evB \quad (26)$$