Simulating Motion Of Charged Particle/Electron Through an Electric and Magnetic Field

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0.1 Aim

To simulate the motion of a charged particle inside an Electric and Magnetic Field.

0.2 Introduction

The project focuses on simulating/tracing the motion of a charged particle/electron under the influence of an external Electric and Magnetic Fieled. The electron trajectories can be given an desired shape by applying an Electric or Magnetic Field

0.3 Applications

- **Radio**
- **Television**
- **CRO**
- **Mass Spectrograph**
- **Particle Accelerators**

0.4 Motion of electron inside an uniform Electric Field

0.4.1 Types of motion exhibited by electron

- **Parallel to Electric Field**
- **Perpendicular to Electric Field**
- **Projected at an angle to the Electric Field**

0.4.2 Basic Calculation

When an electron is projected inside an electric field, sustained by 2 plates **A** and **B** with A being **positively** charged and B being **negatively** charged, having a potential **V** given by $[V_a$ **-** $V_b]$ at a distance of **d** the intensity of the electric field between the plate is given by

$$E = \frac{V}{d} \tag{1}$$

Force experienced by an electron at rest on plate **B**

$$F = -eE \tag{2}$$

The acceleration is given by

$$a = \frac{F}{m} = -\frac{eE}{m} \tag{3}$$

The corresponding code in the project is given in:

Basic_Calculations in *BasicCalc.h*

```
void Basic_Calculations(double PotentialDifference, double PlateDistance)
{
Energy_Electron = PotentialDifference / PlateDistance;  //Energy of the
    electron
Force_Electron = -1 * ELECTRON_ENERGY * Energy_Electron; //Force on
    electron at Plate B
Acceleration_Electron = fabs(Force_Electron) / ELECTRON_MASS;  //
    Acceleration of Electron
}
```

0.4.3 Parallel to Electric Field

The velocity **V** is given by :

$$V = V_0 + at \tag{4}$$

where V_0 is the **initial velocity** of the electron or charged particle
The displacement of the electron is given by

$$S = V_0 + \frac{1}{2}at^2 \tag{5}$$

Velocity at any given time denoted by V_t is given by

$$V_t = \frac{eE}{2m}t = \sqrt{\frac{2eE}{m}x} \tag{6}$$

parallel electric field used in project

$$X = \frac{eE}{2m}t^2 \tag{7}$$

The above equation gives the horizontal displacement of X which is depended on the value of **t**
The calculation is done by the following code :

```
void ElectronMovement_Parallel(int Identifier)
        {
        float count = Misc.count = 0;
        int Index = Misc.index =0;
        float Time = EField.Var.TimeEpoch;
        float StepSize = EField.Var.StepSize;
        unsigned int mem = Misc.MemAllocFactor;


        EField.CompArray.Xcomponent = (double *)calloc(mem, sizeof(double
            ));

        while (count <= Time)
        {
        EField.Parallel.X_Component = fabs(Force_Electron) / (2 *
            ELECTRON_MASS) * pow(count, 2);

        EField.CompArray.Xcomponent[Index] = EField.Parallel.X_Component;
        count += StepSize;
        Index++;
        }

        }
```

0.4.4 Perpendicular to Electric Field

If **A** and **B** are two metal plates placed horizontally parallel to each other at a length **l**
The velocity at **Y** axis would be 0 initially, and would be V_0 for **X** axis
Therefore, $V_y = 0$ and $V_x = V_0$.
The acceleration of the electron is given in y direction by

$$\alpha_y = \frac{eE}{m} \tag{8}$$

‘ As a result, the equation for velocity along the **Y** axis can be written as

$$V_y = \frac{eE}{m}t \tag{9}$$

The horizontal, displacement **X** is given by the following equation

$$x = v_0 t \tag{10}$$

The time **t** is known as ***transit time*** given by,

$$t = \frac{x}{v_0} \tag{11}$$

If V_A is the accelerating potential, then the initial velocity V_0 is given by

$$V_0 = \sqrt{\frac{2eV_a}{m}} \tag{12}$$

The vertical displacement can be written as

$$y = Dtan\theta = D.\left(\frac{V_y}{V_0}\right) \tag{13}$$

The angular displacement is given by :

$$\theta = tan^{-}1\left(\frac{V_y}{V_0}\right) \tag{14}$$

The code which processes the above information, is given in ***BasicCalc.h***

```
void ElectronMovement_Perpendicular(float PlateWidth, int Identifier)
{
Misc.count = 0;
Misc.index = 0;


EField.CompArray.Xcomponent = (double *)calloc(Misc.MemAllocFactor,
    sizeof(double));
EField.CompArray.Ycomponent = (double *)calloc(Misc.MemAllocFactor,
    sizeof(double));

while (Misc.count <= EField.Var.TimeEpoch)
{
EField.Perpendicular.HorizontalDisplacement_X = EField.Var.
    InitialVelocity * Misc.count;

EField.Perpendicular.VerticalDisplacement_Y = -1 * (Force_Electron / 2 *
    ELECTRON_MASS) * pow(Misc.count,2);

EField.Perpendicular.VerticalDisplacement_Leaving = (ELECTRON_ENERGY / 2
    * ELECTRON_MASS) * Energy_Electron * (PlateWidth / pow(EField.Var.
    InitialVelocity, 2));


EField.Perpendicular.AngularDisplacement = Energy_Electron/(2 *
    ELECTRON_MASS * EField.Var.InitialVelocity) * pow(EField.Perpendicular
    .HorizontalDisplacement_X,2);


EField.CompArray.Xcomponent[Misc.index] = EField.Perpendicular.
    HorizontalDisplacement_X;
EField.CompArray.Ycomponent[Misc.index] = EField.Perpendicular.
    VerticalDisplacement_Y;
Misc.index++;
Misc.count += EField.Var.StepSize;


}
```

0.4.5 Electron Projected at an angle

Consider an electron projected at an angle θ_0 to an electric field **E** directed vertically downwards, maintained between plates **A**(positively charged plate and **B**) (negativelycharged plate)

At the entry point P_0 the velocity V_0 has components

$$V_{X0} = V_0 sin\theta_0 \tag{15}$$

$$V_{Y0} = V_0 cos\theta_0 \tag{16}$$

When the electron is pushed or is projected into the this field, the electron experiences a force **f** which is directed to the positively charged plate. This causes the electron to have a projectile motion.

The force **f** causes the electron to change its velocity from v_1 to v_2 at point P_2.
At any instant the position of the electron canbe given by the following equations

$$x = V_{X0}.t = (V_0 cos\theta_0)\, t \tag{17}$$

$$y = V_{y0} + \frac{1}{2}at^2 = (V_0 sin\theta_0)t + \frac{1}{2}at^2 \tag{18}$$

The maximum vertical displacement denoted by **H** of the electron can be calculated by

$$H = \frac{{V_0}^2 sin^2\theta_0}{2a} \tag{19}$$

The time taken by the electron to achieve maximum displacement **t** is given by

$$t = \frac{v_0 sin\theta_0}{a} \tag{20}$$

The time of flight **T** gives the time taken by the electron to reach its final height

$$T = 2t = \frac{2V_0 sin\theta_0}{a} \tag{21}$$

The range **R** is gives the maximum horizontal displacement

$$R = V_0 cos\theta_0 \frac{2V_0(sin\theta_0)}{a} = \frac{(2sin\theta_0 cos\theta_0)\, {V_0}^2}{a} \tag{22}$$

$$R = \frac{{V_0}^2 sin2\theta_0}{a} \tag{23}$$

0.5 Motion Of an Electron inside a Magnetic Field

When a charge **q** enters a magnetic field at an angle θ to the field intensity, it is acted upon by a magnetic force.

$$\vec{F} = q\vec{v} \times \vec{B} \tag{24}$$

If an electron moves in an uniform magnetci field makin an angle θ to the field intensity, it is acted upon by magnetci force given by :

$$\vec{F} = e\vec{v} \times \vec{B} \tag{25}$$

If the velocity **v** and the intensity **B** are perpendicular to each other the magnetic force is given by

$$F = evB \tag{26}$$

Since the magntic force is always perpendicular on the velocity of the electron, the work done by the force on the electron is zero.
Thus, the magnetic field does not affect the velocity or kinetic energy of the electron.This can be proved mathematically as :

$$\vec{F} \cdot \vec{v} = 0 \tag{27}$$

$$m\vec{a} \cdot \vec{v} = 0 \tag{28}$$

$$m\frac{d\vec{v}}{dt} \cdot \vec{v} = 0 \tag{29}$$

$$m\frac{d}{dt}\left(v^2\right) = 0 \tag{30}$$

$$\frac{d}{dt}\left(\frac{1}{2}mv^2\right) = 0 \tag{31}$$

$$K.E = \frac{1}{2}mv^2 = constant \tag{32}$$

Hence,***a magnetic field can change only the direction of the electron motion but not the magnitude of its velocity***

0.5.1 Motion of an Electron in a Longitudinal Magnetic Field

When an electron enters an uniform magnetic field parallel to the field lines or anti-parallel to the field lines, the magnetic force acting on the electron is :

$$F = ev\ sin\theta = 0 \tag{33}$$

Hence, ***the electron moves through the longitudinal magnetic field with unaffected velocity and energy***

0.5.2 Motion of an Electron in a Transverse Magnetic Field

Consider a magnetic field acting into the page. An electron enters the magnetic field of intensity **B** perpendicularly, Since the velocity and the field are perpendicular to each other the electron experiences a magnetic force

$$F = evB \tag{34}$$

As the force changes the direction of v and F continously, the electron gets deflected and follows a curved path. The velocity, **v** of the electron is always tangential to the curved path and the force being perpendicular to the velocity is obviously a centripetal force whic his given by

$$F_c = \frac{mv^2}{R} \tag{35}$$

where **m** is the mass of the electron and **R** is the radius of the curved path.

If the centripetal force balances the magnetic force the electron continues to follow a circular path of radius R. So, from equation **34** and equation **35** it can be written as

$$evB = \frac{mv^2}{R} \tag{36}$$

$$R = \frac{mv}{eB} \tag{37}$$

The time period, **T** of the circular path is the time taken by the electron to complete one revolution. Hence,

$$T = \frac{2\pi R}{v} = \frac{2\pi}{v} \cdot \frac{mv}{eB} \tag{38}$$

$$T = \frac{2\pi m}{eB} \tag{39}$$

The frequency of revolution of the orbit is

$$f = \frac{1}{T} = \frac{eB}{2\pi m} \tag{40}$$