Simulating Motion Of Charged Particle/Electron Through an Electric and Magnetic Field

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0.1 Aim

To simulate the motion of a charged particle inside an Electric and Magnetic Field.

0.2 Introduction

The project focuses on simulating/tracing the motion of a charged particle/electron under the influence of an external Electric and Magnetic Fieled. The electron trajectories can be given an desired shape by applying an Electric or Magnetic Field

0.3 Applications

- **Radio**
- **Television**
- **CRO**
- **Mass Spectrograph**
- **Particle Accelerators**

0.4 Motion of electron inside an uniform Electric Field

0.4.1 Types of motion exhibited by electron

- **Parallel to Electric Field**
- **Perpendicular to Electric Field**
- **Projected at an angle to the Electric Field**

0.4.2 Basic Calculation

When an electron is projected inside an electric field, sustained by 2 plates **A** and **B** with A being **positively** charged and B being **negatively** charged, having a potential **V** given by [V_a **-** V_b] at a distance of **d** the intensity of the electric field between the plate is given by

$$E = \frac{V}{d} \tag{1}$$

Force experienced by an electron at rest on plate **B**

$$F = -eE \tag{2}$$

The acceleration is given by

$$a = \frac{F}{m} = -\frac{eE}{-eE}m \tag{3}$$

The corresponding code in the project is given in:

Basic_Calculations in *BasicCalc.h*

```
void Basic_Calculations(double PotentialDifference, double PlateDistance)
{
Energy_Electron = PotentialDifference / PlateDistance;  //Energy of the electron
Force_Electron = -1 * ELECTRON_ENERGY * Energy_Electron; //Force on electron at Plate B
Acceleration_Electron = fabs(Force_Electron) / ELECTRON_MASS;  //Acceleration of Electron
}
```

0.4.3 Parallel to Electric Field

The velocity **V** is given by :

$$V = V_0 + at \tag{4}$$

where V_0 is the **initial velocity** of the electron or charged particle
The displacement of the electron is given by

$$S = V_0 + \frac{1}{2}at^2 \tag{5}$$

Velocity at any given time denoted by V_t is given by

$$V_t = \frac{eE}{2m}t = \sqrt{\frac{2eE}{m}x} \tag{6}$$

parallel electric field used in project

$$X = \frac{eE}{2m}t^2 \tag{7}$$

The above equation gives the horizontal displacement of X which is depended on the value of **t**
The calculation is done by the following code :

```
void ElectronMovement_Parallel(int Identifier)
{
float count = Misc.count = 0;
int Index = Misc.index =0;
float Time = EField.Var.TimeEpoch;
float StepSize = EField.Var.StepSize;
unsigned int mem = Misc.MemAllocFactor;


EField.CompArray.Xcomponent = (double *)calloc(mem, sizeof(double));

while (count <= Time)
{
EField.Parallel.X_Component = fabs(Force_Electron) / (2 * ELECTRON_MASS) * pow(count, 2);

EField.CompArray.Xcomponent[Index] = EField.Parallel.X_Component;
count += StepSize;
Index++;
}

}
```

0.4.4 Perpendicular to Electric Field

If **A** and **B** are two metal plates placed horizontally parallel to each other at a length **l**
The velocity at **Y** axis would be 0 initially, and would be V_0 for **X** axis
Therefore, $V_y = 0$ and $V_x = V_0$.
The acceleration of the electron is given in y direction by

$$\alpha_y = \frac{eE}{m} \tag{8}$$

‘ As a result, the equation for velocity along the **Y** axis can be written as

$$V_y = \frac{eE}{m}t \tag{9}$$

The horizontal, displacement **X** is given by the following equation

$$x = v_0 t \tag{10}$$

The time **t** is known as ***transit time*** given by,

$$t = \frac{x}{v_0} \tag{11}$$

If V_A is the accelerating potential, then the initial velocity V_0 is given by

$$V_0 = \sqrt{\frac{2eV_a}{m}} \tag{12}$$

The vertical displacement can be written as

$$y = Dtan\theta = D.\left(\frac{V_y}{V_0}\right) \tag{13}$$

The angular displacement is given by :

$$\theta = tan^{-}1\left(\frac{V_y}{V_0}\right) \tag{14}$$

0.4.5 Electron Projected at an angle

Consider an electron projected at an angle θ_0 to an electric field **E** directed vertically downards, maintained between plates **A**(positively charged plate and **B**) (negativelycharged plate)

At the entry point P_0 the velocity V_0 has components

$$V_{X0} = V_0 sin\theta_0 \tag{15}$$

$$V_{Y0} = V_0 cos\theta_0 \tag{16}$$

When the electron is pushed or is projected into the this field, the electron experiences a force **f** which is directed to the positively charged plate. This causes the electron to have a projectile motion.

The force **f** causes the electron to change its velocity from v_1 to v_2 at point P_2.
At any instant the position of the electron canbe given by the following equations

$$x = V_{X0}.t = (V_0 cos\theta_0)\, t \tag{17}$$

$$y = V_{y0} + \frac{1}{2}at^2 = (V_0 sin\theta_0)t + \frac{1}{2}at^2 \tag{18}$$

The maximum vertical displacement denoted by **H** of the electron can be calculated by

$$H = \frac{{V_0}^2 sin^2\theta_0}{2a} \tag{19}$$

The time taken by the electron to achieve maximum displacement **t** is given by

$$t = \frac{v_0 sin\theta_0}{a} \tag{20}$$

The time of flight **T** gives the time taken by the electron to reach its final height

$$T = 2t = \frac{2V_0 sin\theta_0}{a} \tag{21}$$

The range **R** is gives the maximum horizontal displacement

$$R = V_0 cos\theta_0 \frac{2V_0(sin\theta_0)}{a} = \frac{(2sin\theta_0 cos\theta_0)\, {V_0}^2}{a} \tag{22}$$

$$R = \frac{{V_0}^2 sin2\theta_0}{a} \tag{23}$$

0.5 Motion Of an Electron inside a Magnetic Field

When a charge **q** enters a magnetic field at an angle θ to the field intensity, it is acted upon by a magnetic force.

$$\vec{F} = q\vec{v} \times \vec{B} \tag{24}$$

If an electron moves in an uniform magnetci field makin an angle θ to the field intensity, it is acted upon by magnetci force given by :

$$\vec{F} = e\vec{v} \times \vec{B} \tag{25}$$

If the velocity **v** and the intensity **B** are perpendicular to each other the magnetic force is given by

$$F = evB \tag{26}$$