Charges Moving Through a Magnetic Field

Grant McNamara Robert Lando Nick Wertz

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1 Introduction

We attempted to simulate several charge configurations moving through Magnetic fields. To start, we made sure that we could accurately simulate the path taken by a point charge with an initial velocity in the plane orthogonal to the direction of the field. From there, we gave the particle a component of velocity in the direction opposite the field. More complex situations we attempted to create were the following: two and four charges moving together together, at a constant distance apart, as if fixed on a ring, with initial velocities in just the plane orthogonal to the field, as well as both orthogonal and anti-parallel to the field. With four charges, we envisioned them forming an infinitely thin ring, but due to computer capabilities we had to settle for a reasonable number of points on the ring. Our ultimate goal, which went unachieved due to the difficulty we faced in the infinitely thin case, was to extend the infinitely thin ring to 3 dimensions (torus).

2 Code Description

As a preliminary test, we picked an arbitrary length of time, 10 seconds, and chose time step of .1 seconds. We then, to make plots visible on general scales, chose the strength of the magnetic field to be equal to one Tesla. We then defined the differential equations necessary to solve this problem in the function called "uniform." Again, to make scales natural for the computer we selected the mass and charge to be one unit of mass and electron's charge and set them equal to 1. We then initialized the position and velocity arrays, respectively, by setting them equal to columns in an array of size 101 (number

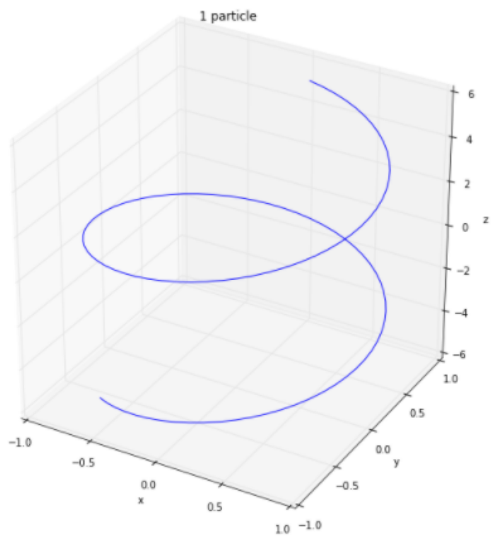


Figure 1: Trajectory of a single particle in 3 dimensions

of time steps) by 6 (number of spatial dimensions plus spatial velocities). We wanted the particles to always travel in circles around the z-axis, so we constrained the velocity and set the acceleration to be that which results in circular motion, given by

$$v = \frac{qRB}{m} \tag{1}$$

$$v = \frac{v^2}{r} = \frac{qvB}{m} \tag{2}$$

To have a proper differential equation we kept the velocity term in the equation for acceleration. We explicitly set $a_z = 0$ because the acceleration parallel to the field should be zero. It was the only way we could fix the acceleration at zero in the z-direction. We ran the function through scipy.odeint with the particle starting on the x-axis (1,0) and with $v_x = -v_z = 1$ to get a helical trajectory. The 3D plot of the particle's path is shown below.

The next function, "nonuniform," is essentially the same as "uniform" but it allows for the field to be a function of time, which, rethinking things, could have been accounted for in the "uniform" function, had we made the field an array of 1s. We did attempt to make a function that would take a

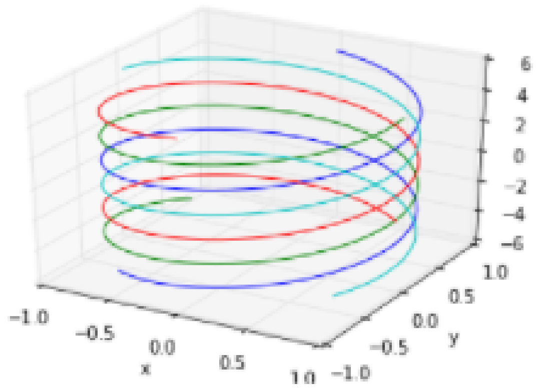


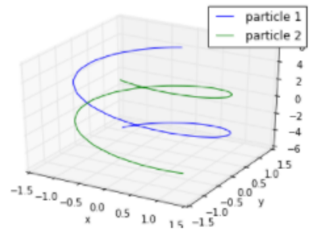
Figure 2: Trajectory in 3 dimensions of four particle system

field that varied randomly with time, using np.random.randint, but we soon scratched that effort when it didn't behave as the other varying fields did. Instead of having B in the function, we changed it to $B(t)$ to force it to accept varying fields. We ran the nonuniform function with the same initial conditions as the uniform function, and an exponential field.

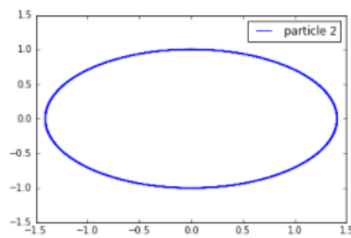
We then ran the uniform function with four sets of initial conditions for x, y, z, v_x, v_y, v_z: (0, 1, 5, 1, 0, -1); (1, 0, 5, 0, -1, -1); (0, -1, 5, -1, 0, -1); and (-1, 0, 5, 0, 1, -1). We ran them separately, so none of the particles interact with each other in any way, and their paths are shown below. (add plots) As a sanity check, we plotted the paths in x and y with respect to time, and they were sine waves, as expected. We then ran another "sanity check" and graphed the energy of the system, to test whether the field actually does any work on the system. The graph below shows, with a scale of $1x10^{-7}$ and an offset of .499999, that the change in energy is on the scale of $1.5x10^{-7}$ Joule.

We redefined the varying field function specifically for a field whose magnitude varies as $B = \sin(2\pi t)$ but its direction is constant (z-direction), which we named "oscillating." We ran the oscillating function with charges with initial conditions of charges 1 and 3 of the four charges through the uniform field. The result of this nonuniform field is shown below

We weren't sure exactly what to expect, but we did know that it should be a stable shape and we came to the conclusion that the results in the xy plane for both velocity and position should be, and are ellipses, with the semi-major axis being the x-axis. We then ran an exponential field function



(a) 3 Dimensional trajectory of particles in a sinusoidal magnetic field



(b) Trajectory in the xy plane of particles in a sinusoidal magnetic field

through "nonuniform" with the same initial conditions as the single particle in a uniform magnetic field had, and this was the result

All of the previous functions were essentially tests to make sure our fields worked properly, our particles behaved appropriately alone with different initial conditions, or just a curiosity (exponential field) to see what would happen. The real goal of the project starts with "uniform2" which was supposed to take two particles simultaneously, run them through "uniform" and output the paths, but we added a constraint to the system to simulate the two points on either side of a diameter of the ring, so we restricted the particles to be exactly 2 units of length apart in the xy plane. We ran this function with the charges that started on the positive and negative y-axis, and then with the charges at +45 degrees from the x-axis (angles 45 and 225 from the + x-axis). The resulting paths are shown on the graph below.

Note: the charges on starting on the y-axis will follow the same path, so there is only one circle, but it indeed represents both charges.

Finally, we tried to make a three-dimensional ring, but we soon realized that to do so, we would need arrays of inputs and outputs of size 101 by 192, and we decided that was not feasible. We settled for the infinitely thin ring, with 8 charges tracked through the process. We essentially quadrupled

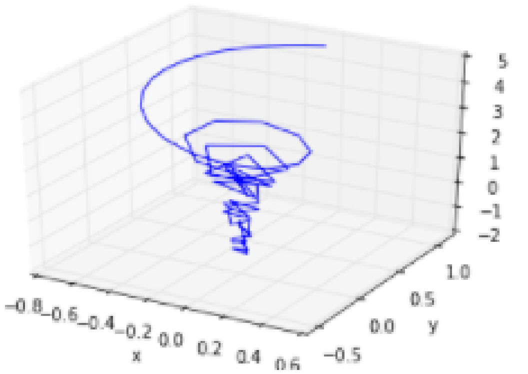
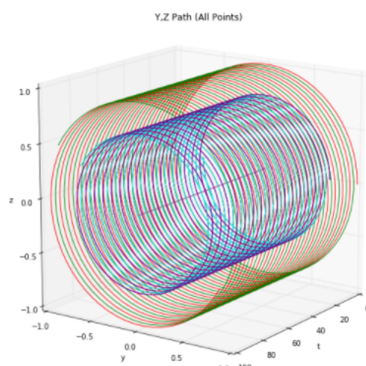
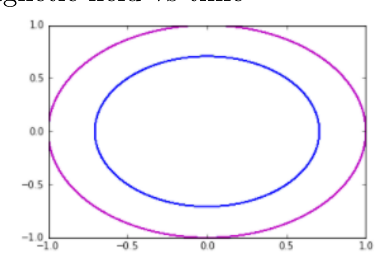


Figure 4: 3 Dimensional trajectory of a particle in an exponential magnetic field



(a) Trajectory in the yz plane of a ring in a uniform magnetic field vs time



(b) Trajectory of a ring in a uniform magnetic field in 3D what we did in "uniform2," restricting the particles opposite each other to maintain a distance of 2 units of length, and then, for each particle, we defined the velocity, radius of the circle it would move in, and the acceleration (magnitude and components). To generalize the acceleration equations, we added that $a_x \neq 0$ necessarily, but that it would be equal to $x - x_0$ which, paired with $v_x = 0$, would fix the x position for all charges. We did this so that we could easily alter the code to accomodate not fixing the ring's center of mass at the origin. We showed the graphs of the spatial positions of each charge, as well as the y-z positions vs time, and we were unable to reconcile the yellow and blue paths moving in the x-direction, despite acting according to expectations in the time graph. We also attempted to apply the ring to an oscillating field but were unable to get useful results.

3 Discussion

We were successful in getting a ring to rotate in a magnetic field, but we did want to try the3 dimensional ring as well. That said, getting a ring to flip, instead of spin in the circle it already occupied, was much more difficult than we anticipated. The single particle, two particle, and ring systems all behaved as we wanted them to, but, as far as the code is concerned, the nuance of adding to the function to force it to allow for a varying field was quite surprising. The math behind the ring was also extremely convoluted, and we were unable to fully and accurately incorporate it into the code as we had hoped.

4 Future Directions

If we had more time, we would definitely attempt the 32 charge tracking scenario, and probably crash our computers in the process. We were also hoping to be able to simulate a free moving ring (i.e. not fixed at the origin). If a future class were to take over this project I would suggest that they try to make the ring neutral, as we had a charged ring. We were also looking to do that here, but we could not manage to get it to spin if we did that. I think it would be an interesting extension of our work.