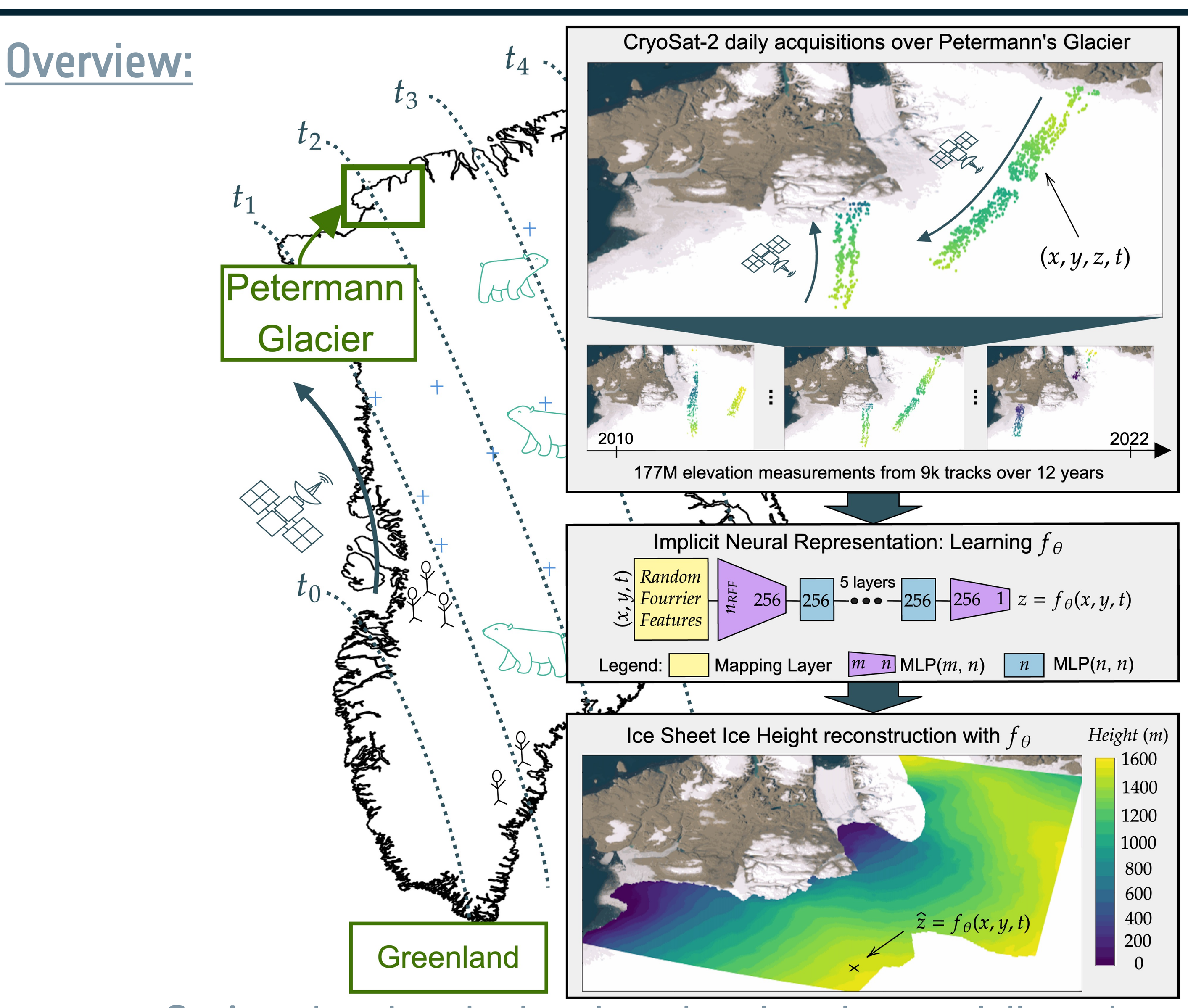
Altimeter and velocity data fusion for enhanced spatiotemporal resolution of the Ice Sheet elevation

P. Naylor[1], A. Stokholm[2]

1. Φ-lab, ESRIN, European Space Agency
2. DTU Space, Department of Space Research and Space Technology, Technical University of Denmark

Overview:



Goal: estimating the ice sheet ice elevation spatially and temporally

INR [1]:

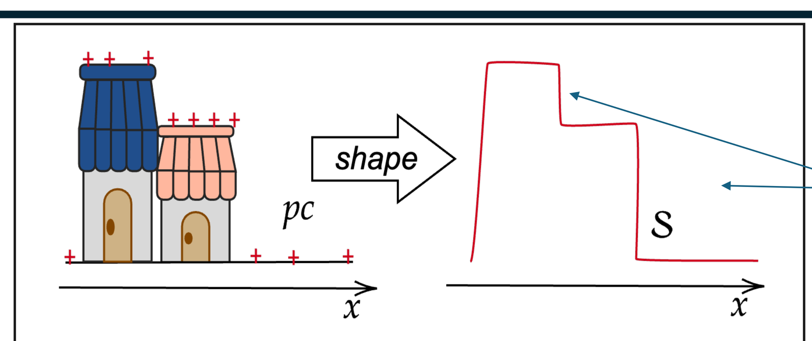


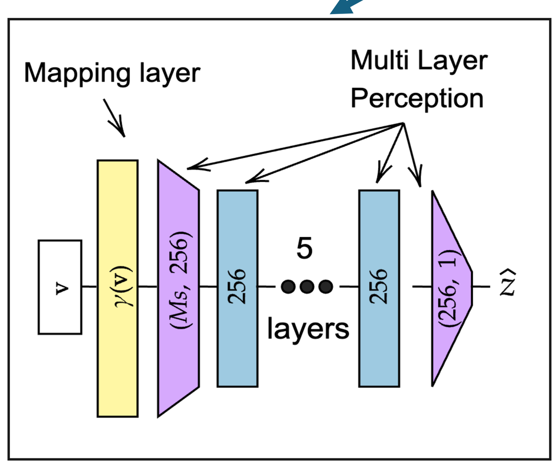
Fig. Surface reconstruction

Like an interpolation but with physical and geometric priors

$$f_\theta : \mathbb{R}^3 \to \mathbb{R}$$
$$(x, y, t) \mapsto z$$

Ice elevation $\iff f_\theta$

Optimiser



Data-fit loss $\mathcal{L}_{MSE} = ||z - \hat{z}||$

Temporal regularisation $\mathcal{L}_t = ||\frac{\partial f_\theta}{\partial t}||_2$

$$\mathcal{L} = \mathcal{L}_{MSE} + \lambda_t \cdot \mathcal{L}_t + \lambda_{xy} \cdot \mathcal{L}_{xy} + \lambda_B \cdot \mathcal{L}_B$$

Eq. PINNs loss

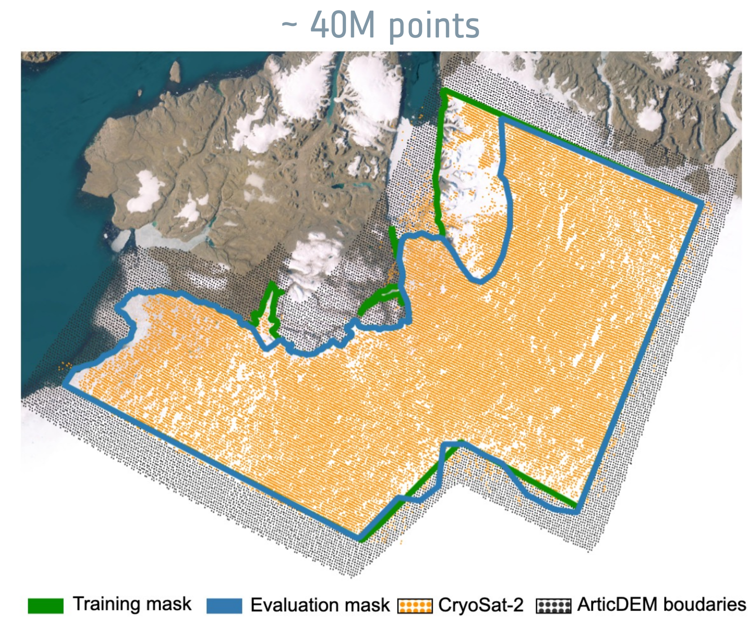
Boundary condition loss with articDEM

Spatial regularisation $\mathcal{L}_{x,y} = ||\frac{\partial f_\theta}{\partial x}||_2 + ||\frac{\partial f_\theta}{\partial y}||_2$

Elevation Training data (from CryoSat-2):

Temporal period: 2017-2021

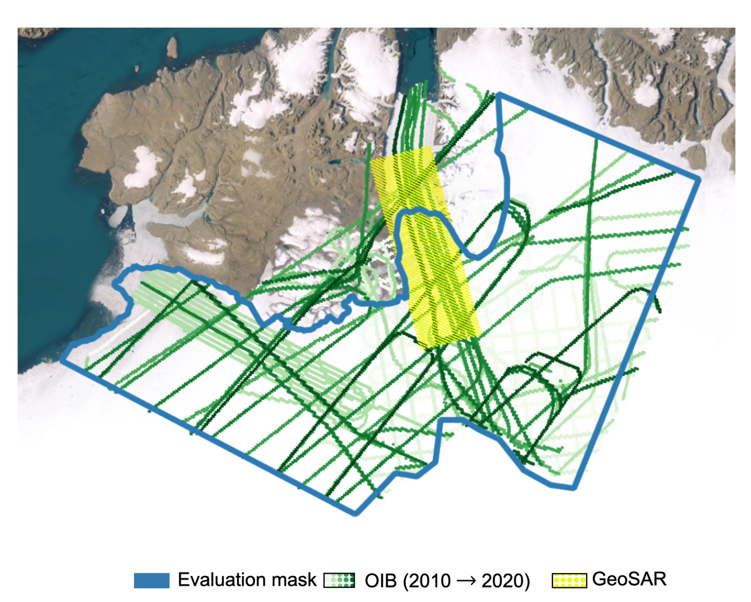
~ 40M points



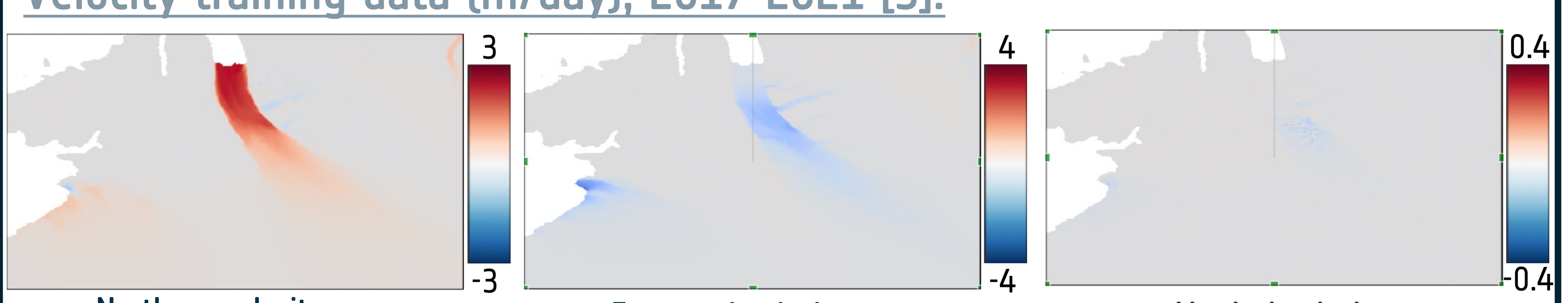
Evaluation data [2]:

CryoSat-2: Altimeter Data (Random orbit sampling)

Operation Ice Bridge: Airborne Topographic Mapper



Velocity training data (m/day), 2017-2021 [3]:



Integrating the velocity product:

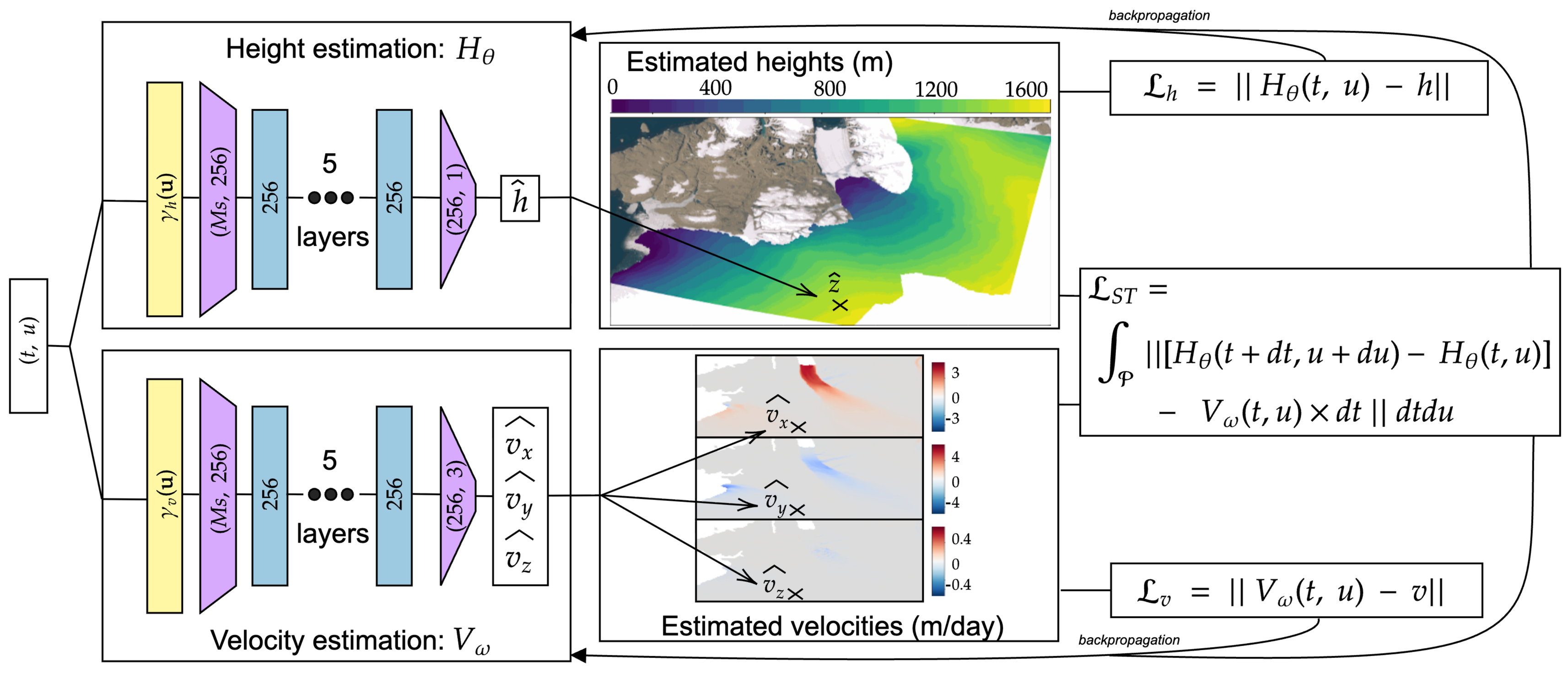


Fig. Method overview

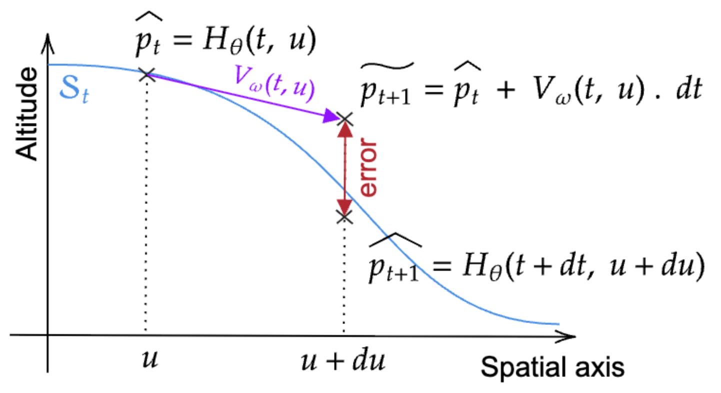


Fig. Deriving the physical penalty

As the velocity product are superior at estimating horizontal displacement, we propose to use the derivative of the elevation with respect to time to substitute the vertical velocity product. We named this version: Velocity(2).

Elevation loss — Velocity loss — Velocity/Elevation aggrement loss — Spatial/Temporal regularisation

$$\mathcal{L} = \mathcal{L}_H + \lambda_V \cdot \mathcal{L}_V + \lambda_{ST} \cdot \mathcal{L}_{ST} + \lambda_S \cdot \mathcal{L}_S + \lambda_T \cdot \mathcal{L}_T$$

Metrics:

$$\text{MAE} = ||z - \hat{z}||_1 \qquad T_{STD} = \mathbb{E}[\sqrt{\mathbb{V}_{x,y}(z)}]$$

$$\text{MED} = \text{median}\{(z_i - z)_i\} \qquad STD = \sqrt{\frac{1}{N}\sum(z_i - \bar{z})}$$

Allows for monitoring the time dynamics. Daily variation of the elevation.

Results:

Datasets (→)		T_{STD}	CS2						OIB		
(↓) HP / Metrics (→)			Train + Test			Test					
Model	Option		MAE	MED	STD	MAE	MED	STD	MAE	MED	STD
RFF	RS	1.58 ± 0.10	3.20 ± 0.07	-0.11 ± 0.14	4.32 ± 0.06	**3.19 ± 0.07**	-0.17 ± 0.13	4.42 ± 0.09	5.49 ± 0.15	-0.09 ± 0.14	9.69 ± 0.21
	ROS	0.52 ± 0.02	3.52 ± 0.25	-0.10 ± 0.23	4.77 ± 0.23	3.43 ± 0.27	-0.13 ± 0.25	4.74 ± 0.26	4.81 ± 0.32	-0.15 ± 0.38	8.83 ± 0.28
	Velocity (1)	0.06 ± 0.02	3.73 ± 0.05	-0.27 ± 0.24	4.93 ± 0.04	3.63 ± 0.05	-0.10 ± 0.22	4.96 ± 0.06	4.31 ± 0.10	-0.00 ± 0.29	7.36 ± 0.10
	Velocity (2)	0.04 ± 0.05	4.03 ± 0.42	-0.51 ± 0.55	5.15 ± 0.32	3.91 ± 0.39	-0.34 ± 0.54	5.16 ± 0.30	4.64 ± 0.45	-0.36 ± 0.71	7.61 ± 0.36
SIREN	RS	0.23 ± 0.35	3.43 ± 0.23	0.06 ± 0.32	4.74 ± 0.21	3.37 ± 0.22	0.14 ± 0.35	4.77 ± 0.22	4.13 ± 0.32	0.33 ± 0.40	7.78 ± 0.77
	ROS	0.32 ± 0.29	3.98 ± 0.82	-0.78 ± 1.20	5.10 ± 0.60	3.88 ± 0.73	-0.69 ± 1.12	5.14 ± 0.54	4.77 ± 1.07	-0.71 ± 1.38	8.33 ± 0.97
	Velocity (1)	0.10 ± 0.11	3.25 ± 0.04	-0.14 ± 0.10	4.64 ± 0.04	3.23 ± 0.04	-0.07 ± 0.08	4.73 ± 0.05	3.71 ± 0.07	0.02 ± 0.41	7.48 ± 0.17
	Velocity (2)	0.01 ± 0.003	3.26 ± 0.06	-0.23 ± 0.19	4.64 ± 0.05	3.23 ± 0.06	-0.18 ± 0.19	4.71 ± 0.06	3.69 ± 0.11	0.15 ± 0.17	7.65 ± 0.17
ISRIN [1]		0.29 ± 0.11	3.19 ± 0.09	-0.35 ± 0.41	4.24 ± 0.14	3.23 ± 0.05	-0.31 ± 0.47	4.26 ± 0.13	**3.67 ± 0.12**	-0.07 ± 0.15	6.63 ± 0.12
Bilinear interpolation		×	**1.90**	0.00	13.42	12.41	0.00	34.16	6.99	-0.023	14.06

-> The ST terms reduces the daily variation and ensures physical elevation maps
-> The bilinear interpolation overfits on the training data
-> The best performing model (RFF and RS) on Test is not physically possible, the high daily is too high.
-> ISRIN and Velocity perform the best on OIB
-> The Elevation/Velocity agreement term allows us to reach very similar performance to ISRIN, the state of the art on this topic without validating on random left out orbits (ROS)

Take-away:

- Achieve similar performance to ISRIN (state of the art on this task) without Random Orbit Sampling (ROS).
- The ST agreement terms allows to produce physically possible elevation maps.

Next-steps:

- Expand the temporal range (from 2012) and the spatial scope.(whole of Greenland)
- Validate the produced velocity maps and check if we improve the performance for that product as well!

References:

[1] Naylor. P., et al. "Implicit neural representation for ice sheet surface elevation reconstruction to assess elevation change in high-spatiotemporal resolution. Preprint, 2025.

[2] Andersen, N. H., et al. "Regional assessments of surface ice elevations from swath-processed CryoSat-2 SARIn data." Remote Sensing 13.11 (2021): 2213.

[3] Copernicus Climate Change Service. Greenland ice sheet annual gridded velocity data from 2017 to present derived from satellite observation, 2020.

GitHub repos:
- Main: https://github.com/PeterJackNaylor/IceSheetReconstruction
- INR package: https://github.com/PeterJackNaylor/INR4torch