

MATH 2418: Linear Algebra

Assignment #9

Due : 11/04, Tuesday, 11:59pm

Term : Fall 2025

[Last Name]	[First Name]	[Net ID]	[Lab Section]
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Recommended Problems:(Do not turn in)

Sec 4.3: 1, 2, 3, 4, 5, 6, 10, 17, 18, 21, 22, 23; Sec 4.4: 1, 2, 3, 4, 5, 6, 7, 9, 16, 17, 18, 21, 22, 24, 31.

1. Find the values of C and D so that the straight line $b = C + Dt$ gives the least squares approximation to the data below. Calculate the predicted values \hat{b} which lie on the resulting least squares regression line.

t	0	1	2	3
b	1	1	-2	4
\hat{b}				

2. Find the best parabola to fit the data points: $(-1, -1), (0, 1), (1, 0), (2, 3)$.

3. Find orthonormal vectors \mathbf{q}_1 and \mathbf{q}_2 in the plane spanned by $\mathbf{a} = (3, 2, 5, 1)$ and $\mathbf{b} = (4, 3, 8, 3)$. Which vector in this plane is closest to $(0, 1, 2, 1)$?

4. Find orthonormal vectors \mathbf{q}_1 , \mathbf{q}_2 , and \mathbf{q}_3 such that \mathbf{q}_1 and \mathbf{q}_2 span the column space of the matrix

$$A = \begin{bmatrix} 1 & 2 \\ 2 & -3 \\ -1 & 4 \end{bmatrix}.$$

Which of the four fundamental subspaces contains \mathbf{q}_3 ?

Solve $A\mathbf{x} = \mathbf{b}$, where $\mathbf{b} = (1, 2, 4)$ by least squares.

5. Find orthonormal vectors \mathbf{q}_1 , \mathbf{q}_2 , and \mathbf{q}_3 as combinations of the columns of matrix A, where

$$A = \begin{bmatrix} 2 & 2 & 4 \\ 0 & 0 & 3 \\ 0 & 3 & 6 \end{bmatrix}.$$

Then write A as QR .

6. Find a basis for the subspace \mathbf{S} in \mathbb{R}^4 spanned by all solutions of

$$2x_1 + x_2 + x_3 - 4x_4 = 0.$$

Find a basis for the orthogonal complement of \mathbf{S} .

Find \mathbf{b}_1 in \mathbf{S} and \mathbf{b}_2 in the orthogonal complement of \mathbf{S} so that $\mathbf{b}_1 + \mathbf{b}_2 = \mathbf{b} = (1, 1, 2, 4)$.