

## MAP 2302 Homework 7.4

**Problem 1.** Determine the inverse laplace transform of

$$\frac{5}{s^2 + 25}.$$

**Solution.**

$$\mathcal{L}^{-1} \left\{ \frac{5}{s^2 + 25} \right\} = \sin 5t$$

**Problem 2.** Determine the inverse laplace transform of

$$\frac{12}{s^2 + 25}.$$

**Solution.**

$$\mathcal{L}^{-1} \left\{ \frac{12}{s^2 + 25} \right\} = \frac{12}{5} \sin 5t$$

**Problem 3.** Determine the inverse laplace transform of

$$\frac{5}{(3s + 8)^3}.$$

**Solution.**

$$\begin{aligned} \mathcal{L}^{-1} \left\{ \frac{5}{(3s + 8)^3} \right\} &= \mathcal{L}^{-1} \left\{ \frac{5}{[3(s + 8/3)]^3} \right\} \\ &= \frac{5}{27} \mathcal{L}^{-1} \left\{ \frac{1}{(s + 8/3)^3} \right\} \\ &= \frac{5}{54} \mathcal{L}^{-1} \left\{ \frac{2!}{(s + 8/3)^{2+1}} \right\} \\ &= \frac{5}{54} t^2 e^{-(8/3)t} \end{aligned}$$

**Problem 4.** Determine the inverse Laplace transform of

$$\frac{4s + 32}{s^2 + 12s + 40}.$$

**Solution.**

$$\begin{aligned}
\mathcal{L}^{-1} \left\{ \frac{4s + 32}{s^2 + 12s + 40} \right\} &= 4\mathcal{L}^{-1} \left\{ \frac{(s + 6) + 2}{(s + 6)^2 + 2^2} \right\} \\
&= 4\mathcal{L}^{-1} \left\{ \frac{s + 6}{(s + 6)^2 + 2^2} + \frac{2}{(s + 6)^2 + 2^2} \right\} \\
&= 4 \left( e^{-6t} \cos 2t + e^{-6t} \sin 2t \right)
\end{aligned}$$

**Problem 5.** Determine the inverse Laplace transform of

$$\frac{1}{s^5}.$$

**Solution.**

$$\mathcal{L}^{-1} \left\{ \frac{1}{s^5} \right\} = \frac{t^4}{24}$$

**Problem 6.** Determine the partial fraction expansion for the rational function

$$\frac{10s + 33}{(s + 2)(s^2 + 9)}.$$

**Solution.**

$$\begin{aligned}
\frac{10s + 33}{(s + 2)(s^2 + 9)} &= \frac{As + B}{s^2 + 9} + \frac{C}{s + 2} \\
10s + 33 &= (As + B)(s + 2) + C(s^2 + 9) \\
&= As^2 + 2As + Bs + 2B + Cs^2 + 9C \\
&= (A + C)s^2 + (2A + B)s + (2B + 9C) \\
13 &= 13C, \quad s = -2; \quad C = 1 \\
A + C &= 0; \quad 2A + B = 10; \quad 2B + 9C = 33 \\
A &= -1; \quad B = 12 \\
\frac{10s + 33}{(s + 2)(s^2 + 9)} &= \frac{12 - s}{s^2 + 9} + \frac{1}{s + 2}
\end{aligned}$$

**Problem 7.** Determine the partial fraction expansion for the rational function

$$\frac{s}{(s - 5)(s^2 - 25)}.$$

**Solution.**

$$\begin{aligned}\frac{s}{(s-5)(s^2-25)} &= \frac{A}{(s-5)^2} + \frac{B}{s-5} + \frac{C}{s+5} \\ s &= A(s+5) + B(s^2-25) + C(s-5)^2 \\ s &= As + 5A + Bs^2 - 25B + Cs^2 - 10Cs + 25C \\ s &= (B+C)s^2 + (A-10C)s + (5A-25B+25C) \\ 10A &= 5, \quad A = 1/2; \quad 100C = -5, \quad C = -1/20; \quad B = 1/20 \\ \frac{s}{(s-5)(s^2-25)} &= \frac{1}{2(s-5)^2} + \frac{1}{20(s-5)} - \frac{1}{20(s+5)}\end{aligned}$$

**Problem 8.** Determine  $\mathcal{L}^{-1}\{F\}$ .

$$F(s) = \frac{7s^2 - 16s + 5}{s(s-6)(s-7)}$$

**Solution.**

$$\begin{aligned}\frac{7s^2 - 16s + 5}{s(s-6)(s-7)} &= \frac{A}{s} + \frac{B}{s-6} + \frac{C}{s-7} \\ 7s^2 - 16s + 5 &= A(s-6)(s-7) + Bs(s-7) + Cs(s-6) \\ &= As^2 - 13As + 42A + Bs^2 - 7Bs + Cs^2 - 6Cs \\ &= (A+B+C)s^2 + (-13A-7B-6C)s + 42A \\ -6B &= 161, \quad B = -161/6; \quad 7C = 236, \quad C = 236/7; \quad A = 5/42 \\ F(s) &= \frac{5}{42s} - \frac{161}{6(s-6)} + \frac{236}{7(s-7)} \\ \mathcal{L}^{-1}\{F\} &= \frac{5}{42} - \frac{161}{6}e^{6t} + \frac{236}{7}e^{7t}\end{aligned}$$

**Problem 9.** Determine  $\mathcal{L}^{-1}\{F\}$ .

$$F(s) = \frac{s^2 - 3s - 2}{(s+1)^2(s+3)}$$

**Solution.**

$$\begin{aligned}
\frac{s^2 - 3s - 2}{(s+1)^2(s+3)} &= \frac{A}{(s+1)^2} + \frac{B}{s+1} + \frac{C}{s+3} \\
s^2 - 3s - 2 &= A(s+3) + B(s+1)(s+3) + C(s+1)^2 \\
&= As + 3A + Bs^2 + 4Bs + 3B + Cs^2 + 2Cs + C \\
&= (B+C)s^2 + (A+4B+2C)s + (3A+3B+C) \\
4C &= 16, \quad C = 4; \quad 2A = 2, \quad A = 1; \quad B = -3 \\
F(s) &= \frac{1}{(s+1)^2} - \frac{3}{s+1} + \frac{4}{s+3} \\
\mathcal{L}^{-1}\{F\} &= te^{-t} - 3e^{-t} + 4e^{-3t}
\end{aligned}$$

**Problem 10.** Determine  $\mathcal{L}^{-1}\{F\}$ .

$$F(s) = \frac{4s^2 + 92s + 318}{(s-6)(s^2 + 12s + 61)}$$

**Solution.**

$$\begin{aligned}
\frac{4s^2 + 92s + 318}{(s-6)(s^2 + 12s + 61)} &= \frac{A(s+6) + 5B}{(s+6)^2 + 5^2} + \frac{C}{s-6} \\
4s^2 + 92s + 318 &= [A(s+6) + 5B](s-6) + C(s^2 + 12s + 61) \\
169C &= 1014, \quad C = 6; \quad -60B + 150 = -90; \quad B = 4 \\
-36A - 30B + 61C &= 318; \quad -36A + 246 = 318, \quad A = -2 \\
F(s) &= -2 \frac{(s+6) - 10}{(s+6)^2 + 5^2} + \frac{6}{s-6} \\
\mathcal{L}^{-1}\{F\} &= 6e^{6t}
\end{aligned}$$