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MAP 2302 Homework 7.4

Problem 1. Determine the inverse Laplace transform of

$$\frac{5}{s^2+25}.$$

Solution.

$$\mathscr{L}^{-1}\left\{\frac{5}{s^2+25}\right\} = \sin 5t$$

Problem 2. Determine the inverse laplace transform of

$$\frac{12}{s^2+25}.$$

Solution.

$$\mathscr{L}^{-1}\left\{\frac{12}{s^2+25}\right\} = \frac{12}{5}\sin 5t$$

Problem 3. Determine the inverse laplace transform of

$$\frac{5}{(3s+8)^3}.$$

Solution.

$$\begin{aligned}
\mathscr{L}^{-1}\left\{\frac{5}{(3s+8)^3}\right\} &= \mathscr{L}^{-1}\left\{\frac{5}{[3(s+8/3)]^3}\right\} \\
&= \frac{5}{27}\mathscr{L}^{-1}\left\{\frac{1}{(s+8/3)^3}\right\} \\
&= \frac{5}{54}\mathscr{L}^{-1}\left\{\frac{2!}{(s+8/3)^{2+1}}\right\} \\
&= \frac{5}{54}t^2e^{-(8/3)t}
\end{aligned}$$

Problem 4. Determine the inverse Laplace transform of

$$\frac{4s+32}{s^2+12s+40}.$$

Solution.

$$\begin{aligned}
\mathscr{L}^{-1}\left\{\frac{4s+32}{s^2+12s+40}\right\} &= 4\mathscr{L}^{-1}\left\{\frac{(s+6)+2}{(s+6)^2+2^2}\right\} \\
&= 4\mathscr{L}^{-1}\left\{\frac{s+6}{(s+6)^2+2^2}+\frac{2}{(s+6)^2+2^2}\right\} \\
&= 4\left(e^{-6t}\cos 2t + e^{-6t}\sin 2t\right)
\end{aligned}$$

Problem 5. Determine the inverse Laplace transform of

$$\frac{1}{s^5}.$$

Solution.

$$\mathscr{L}^{-1}\left\{\frac{1}{s^5}\right\} = \frac{t^4}{24}$$

Problem 6. Determine the partial fraction expansion for the rational function

$$\frac{10s+33}{(s+2)(s^2+9)}.$$

Solution.

$$\begin{aligned}
\frac{10s+33}{(s+2)(s^2+9)} &= \frac{As+B}{s^2+9}+\frac{C}{s+2} \\
10s+33 &= (As+B)(s+2)+C(s^2+9) \\
&= As^2+2As+Bs+2B+Cs^2+9C \\
&= (A+C)s^2+(2A+B)s+(2B+9C) \\
13 &= 13C, \quad s=-2; \quad C=1 \\
A+C &= 0; \quad 2A+B=10; \quad 2B+9C=33 \\
A &= -1; \quad B=12 \\
\frac{10s+33}{(s+2)(s^2+9)} &= \frac{12-s}{s^2+9}+\frac{1}{s+2}
\end{aligned}$$

Problem 7. Determine the partial fraction expansion for the rational function

$$\frac{s}{(s-5)(s^2-25)}.$$

Solution.

$$
\begin{aligned}
\frac{s}{(s-5)(s^2-25)} &= \frac{A}{(s-5)^2} + \frac{B}{s-5} + \frac{C}{s+5} \\
s &= A(s+5) + B(s^2-25) + C(s-5)^2 \\
s &= As + 5A + Bs^2 - 25B + Cs^2 - 10Cs + 25C \\
s &= (B+C)s^2 + (A-10C)s + (5A - 25B + 25C) \\
10A = 5, \quad A &= 1/2; \quad 100C = -5, \quad C = -1/20; \quad B = 1/20 \\
\frac{s}{(s-5)(s^2-25)} &= \frac{1}{2(s-5)^2} + \frac{1}{20(s-5)} - \frac{1}{20(s+5)}
\end{aligned}
$$

Problem 8. Determine $\mathscr{L}^{-1}\{F\}$.

$$F(s) = \frac{7s^2 - 16s + 5}{s(s-6)(s-7)}$$

Solution.

$$
\begin{aligned}
\frac{7s^2 - 16s + 5}{s(s-6)(s-7)} &= \frac{A}{s} + \frac{B}{s-6} + \frac{C}{s-7} \\
7s^2 - 16s + 5 &= A(s-6)(s-7) + Bs(s-7) + Cs(s-6) \\
&= As^2 - 13As + 42A + Bs^2 - 7Bs + Cs^2 - 6Cs \\
&= (A+B+C)s^2 + (-13A - 7B - 6C)s + 42A \\
-6B = 161, \quad B &= -161/6; \quad 7C = 236, \quad C = 236/7; \quad A = 5/42 \\
F(s) &= \frac{5}{42s} - \frac{161}{6(s-6)} + \frac{236}{7(s-7)} \\
\mathscr{L}^{-1}\{F\} &= \frac{5}{42} - \frac{161}{6}e^{6t} + \frac{236}{7}e^{7t}
\end{aligned}
$$

Problem 9. Determine $\mathscr{L}^{-1}\{F\}$.

$$F(s) = \frac{s^2 - 3s - 2}{(s+1)^2(s+3)}$$

Solution.

$$\begin{aligned}
\frac{s^2-3s-2}{(s+1)^2(s+3)} &= \frac{A}{(s+1)^2}+\frac{B}{s+1}+\frac{C}{s+3}\\
s^2-3s-2 &= A(s+3)+B(s+1)(s+3)+C(s+1)^2\\
&= As+3A+Bs^2+4Bs+3B+Cs^2+2Cs+C\\
&= (B+C)s^2+(A+4B+2C)s+(3A+3B+C)\\
4C=16,\quad C&=4;\quad 2A=2,\quad A=1;\quad B=-3\\
F(s) &= \frac{1}{(s+1)^2}-\frac{3}{(s+1)}+\frac{4}{(s+3)}\\
\mathscr{L}^{-1}\{F\} &= te^{-t}-3e^{-t}+4e^{-3t}
\end{aligned}$$

Problem 10. Determine $\mathscr{L}^{-1}\{F\}$.

$$F(s)=\frac{4s^2+92s+318}{(s-6)(s^2+12s+61)}$$

Solution.

$$\begin{aligned}
\frac{4s^2+92s+318}{(s-6)(s^2+12s+61)} &= \frac{A(s+6)+5B}{(s+6)^2+5^2}+\frac{C}{s-6}\\
4s^2+92s+318 &= [A(s+6)+5B](s-6)+C(s^2+12s+61)\\
169C=1014,\quad C=6;\quad -60B+150&=-90;\quad B=4\\
-36A-30B+61C=318;\quad -36A+246&=318,\quad A=-2\\
F(s) &= -\frac{2(s+6)-20}{(s+6)^2+5^2}+\frac{6}{s-6}\\
\mathscr{L}^{-1}\{F\} &= -2\left(\frac{s+6}{(s+6)^2+5^2}-\frac{10}{(s+6)^2+5^2}\right)+6e^{6t}\\
&= -2\left(e^{-6t}\cos 5t-2e^{-6t}\sin 5t-3e^{6t}\right)\\
&= -2e^{-6t}\left(\cos 5t-2\sin 5t-3e^{12t}\right)
\end{aligned}$$

Problem 11. Determine $\mathscr{L}^{-1}\{F\}$.

$$F(s)=\frac{5s^3-7s^2-9s+63}{s^3(s-7)}$$

Solution.

$$\begin{aligned}
\frac{5s^3-7s^2-9s+63}{s^3(s-7)} &= \frac{A}{s^3}+\frac{B}{s^2}+\frac{C}{s}+\frac{D}{s-7}\\
5s^3-7s^2-9s+63 &= A(s-7)+Bs(s-7)+Cs^2(s-7)+Ds^3\\
&= As-7A+Bs^2-7Bs+Cs^3-7Cs^2+Ds^3\\
&= (C+D)s^3+(B-7C)s^2+(A-7B)s-7A\\
343D=1372,\quad D &= 4;\quad C+4=5,\quad C=1;\\
B-7=-7,\quad B &= 0;\quad A=-9\\
F(s) &= -\frac{9}{s^3}+\frac{1}{s}+\frac{4}{s-7}\\
\mathscr{L}^{-1}\left\{F\right\} &= -\frac{9}{2}t^2+4e^{7t}+1
\end{aligned}$$