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MAP 2302 Homework 7.3

Problem 1. Use the accompanying table of Laplace transforms and properties of Laplace transforms to find the Laplace transform of

$$e^{-4t}\cos 4t + e^{6t} - 1.$$

Solution.

$$\mathscr{L}\left\{e^{-4t}\cos 4t + e^{6t} - 1\right\} = \frac{s+4}{(s+4)^2+16} + \frac{1}{s-6} - \frac{1}{s}$$

Problem 2. Use the accompanying table of Laplace transforms and properties of Laplace transforms to find the Laplace transform of

$$7t^2e^{-6t} - t^5 + \cos 3t.$$

Solution.

$$\mathscr{L}\left\{7t^2e^{-6t} - t^5 + \cos 3t\right\} = \frac{14}{(s+6)^3} - \frac{120}{s^6} + \frac{s}{s^2+9}$$

Problem 3. Use the accompanying table of Laplace transforms and properties of Laplace transforms to find the Laplace transform of

$$(t+1)^4.$$

Solution.

$$\begin{aligned}\mathscr{L}\left\{(t+1)^4\right\} &= \mathscr{L}\left\{t^4 + 4t^3 + 6t^2 + 4t + 1\right\} \\ &= \frac{24}{s^5} + \frac{24}{s^4} + \frac{12}{s^3} + \frac{4}{s^2} + \frac{1}{s}\end{aligned}$$

Problem 4. Use the accompanying table of Laplace transforms and properties of Laplace transforms to find the Laplace transform of

$$(5 + e^{-4t})^2.$$

Solution.

$$\begin{aligned}\mathscr{L}\left\{(5+e^{-4t})^2\right\} &= \mathscr{L}\left\{25 + 10e^{-4t} + e^{-8t}\right\} \\ &= \frac{25}{s} + \frac{10}{s+4} + \frac{1}{s+8}\end{aligned}$$

Problem 5. Use the accompanying table of Laplace transforms and properties of Laplace transforms to find the Laplace transform of

$$te^t \cos t.$$

Solution.

$$\begin{aligned}\mathscr{L}\left\{te^t \cos t\right\} &= (-1)^1 \frac{d}{ds}\left(\mathscr{L}\left\{e^t \cos t\right\}\right) \\ &= -\frac{d}{ds}\left(\frac{s-1}{(s-1)^2+1}\right) \\ &= -\frac{(s-1)^2+1-2(s-1)^2}{[(s-1)^2+1]} \\ &= -\frac{1-(s-1)^2}{[(s-1)^2+1]^2}\end{aligned}$$

Problem 6. Use the accompanying table of Laplace transforms and properties of Laplace transforms to find the Laplace transform of

$$\cosh bt.$$

Solution.

$$\begin{aligned}\mathscr{L}\left\{\cosh bt\right\} &= \mathscr{L}\left\{\frac{e^{bt}+e^{-bt}}{2}\right\} \\ &= \frac{1}{2}\left(\frac{1}{s-b}+\frac{1}{s+b}\right) \\ &= \frac{s}{s^2-b^2}\end{aligned}$$

Problem 7. Use the accompanying table of Laplace transforms and properties of Laplace transforms to find the Laplace transform of

$$13\sin^2 6t.$$

Solution.

$$\begin{aligned}\mathscr{L}\left\{13\sin^2 6t\right\} &= \mathscr{L}\left\{\frac{1-\cos 12t}{2}\right\} \\ &= \frac{13}{2}\left(\frac{1}{s}-\frac{s}{s^2+144}\right)\end{aligned}$$

Problem 8. Use the accompanying table of Laplace transforms and properties of Laplace transforms to find the Laplace transform of

$$\sin 3t \sin 6t.$$

Solution.

$$\begin{aligned}\mathscr{L}\left\{\sin 3t \sin 6t\right\} &= \mathscr{L}\left\{\frac{\cos\left(-3t\right) - \cos\left(9t\right)}{2}\right\} \\ &= \frac{1}{2}\left(\frac{s}{s^2+9} - \frac{s}{s^2+81}\right)\end{aligned}$$