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MAP 2302 Homework 4.1

Problem 1. Verify that $y = \sin 2t + 6\cos 2t$ is a solution to the initial value problem $6y" + 24y = 0$; $y(0) = 6, y'(0) = 2$. Find the maximum of $|y(t)|$ for $\infty < t < \infty$.

Solution.

$$\begin{aligned} 6(-4\sin 2t - 24\cos 2t) + 24(\sin 2t + 6\cos 2t) &= 0 \\ \sin(2(0)) + 6\cos(2(0)) &= 6 \\ 2\cos(2(0)) - 12\sin(2(0)) &= 2 \\ |y(t)|_{\max} = \sqrt{1^2 + 6^2} &= \sqrt{37} \end{aligned}$$

Problem 2. Find a synchronous solution of the form $A\cos\Omega t + B\sin\Omega t$ to the given forced oscillator equation using the method of insertion, collecting terms, and matching coefficients to solve for A and B.

$$y" + 3y' + 2y = 3\sin 3t,\ \Omega = 3$$

Solution.

$$\begin{aligned} y' &= -3A\sin 3t + 3B\cos 3t \\ y" &= -9A\cos 3t - 9B\sin 3t \\ 3\sin 3t &= -9A\cos 3t - 9B\sin 3t - 9A\sin 3t + 9B\cos 3t + 2A\cos 3t + 2B\sin 3t \\ 3\sin 3t &= (-7A + 9B)\cos 3t + (-9A - 7B)\sin 3t \\ -7A + 9B &= 0 \quad -9A - 7B = 3; \quad A = -27/130,\ B = -21/130 \\ y &= -\frac{27}{130}\cos 3t - \frac{21}{130}\sin 3t \end{aligned}$$