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Suite of Verification Test Problems for Edge Turbulence Simulations

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We present a suite of test problems that are used for verification of the edge turbulence code BOUT. BOUT is an electromagnetic fluid turbulence code for tokamak edge plasma that performs time integration of reduced Braginskii plasma fluid equations, using spatial discretization in realistic geometry and employing a standard ODE integration package PVODE. Recently the code underwent a substantial redesign and extensive verification testing. In the verification process, a series of linear and nonlinear test problems was applied to BOUT targeting different subgroups of physical terms. The tests include reproducing basic electrostatic and electromagnetic plasma modes in simplified geometry, axisymmetric benchmarks against the 2D edge code UEDGE in the actual DIII-D tokamak divertor geometry. Successful passing of these tests by BOUT gives strong evidence that equations in the code are solved correctly. Although the tests were developed specifically for the BOUT code they can well be applied for verification of other codes used for simulations of edge turbulence.

1 Introduction

Edge plasma turbulence in tokamak is a challenging subject. On the one hand the physics is very rich, with variety of physical phenomena that can interact with each other in complex ways. On the other hand the geometry is also very complicated, with underlying magnetic field curvature and shear, x-points and branch cuts. Due to this complexity it is virtually impossible to tackle the problem analytically, and in most cases one has to resort to numerical simulation. The BOUT code is a tool for simulation of edge physics, it is an electromagnetic fluid edge turbulence code [1, 2]. The physics model is based on the Braginskii equations for a collisional plasma, and the equations are solved numerically in the real geometry of a divertor tokamak. BOUT model supports a large variety of plasma modes: ideal and resistive ballooning, drift, shear-Alfven, sheath-driven modes and others.

With such a complex numerical model two questions naturally arise: (i) whether the equations form a valid physics model for the studied phenomena, and (ii) whether the equations are solved correctly by the code. The answer to the first question is that for sufficiently collisional plasma, which is reasonably well satisfied in many existing tokamaks, the collisional closure should hold and thus the model should be valid. To address the second question one needs to do thorough verification testing to make sure the numerical model can be trusted as a research tool.

For the time-integration method used in BOUT (method of lines) one needs to have a function representing the right-hand side of the equations being solved, which is a sum of differential operators, such as $\nabla_\perp^2$, $V_E \cdot \nabla$, $\vec{b} \times \vec{\kappa} \cdot \nabla$ etc, applied to plasma variables, such as N_i, ϕ etc. In a recent restructuring of the code individual differential operators were implemented as separate software modules. Such modular organization of the code greatly simplifies its maintenance, debugging, and further development.

This sets the strategy of verification testing: Since these operator functions are applied in same way to all plasma variables, it is sufficient to verify correctness of one such term resulting from application of a particular mathematical operator to a particular fluid variable, e.g. $(V_E \cdot \nabla) N_i$, rather than checking dozens of similar terms. In the tests isolated physical phenomena are reproduced using small subsets of terms in the fluid equations. Then

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the simplified equations are solved with the code and the results are compared with analytic solutions. The test problems are designed based on the requirements that they are: (i) relevant, i.e. contain physics that is important for edge turbulence, (ii) specific, i.e. each problem targets a specific group of terms and calculations, and (iii) unambiguous, i.e. for each problem an independent exact (or very accurate) solution can be found. Below is is a description of several such test problems.

2 Resistive drift instability

Resistive drift instability (RDI) is one of key players in edge turbulence. The underlying physics for RDI is ExB advection of density, charge conservation by balancing divergence of electron parallel current and ion polarization current, and balance of parallel forces exerted on the electrons: electric field, pressure gradient, and collisions with ions. Here is the minimal set of terms in the two-fluid equations for the linear RDI:

$$\frac{\partial N_i}{\partial t} = -\vec{V}_E \cdot \nabla N_{i0} \tag{1}$$

$$\frac{\partial \varpi}{\partial t} = N_{i0} Z_i e \frac{4\pi V_A^2}{c^2} \nabla_{||} j_{||} \tag{2}$$

$$0 = \frac{1}{m_e} \partial_{||}(e\phi - \frac{T_{e0}}{N_{i0}} N_i) + 0.51\nu_{ei}(-V_{||e}) \tag{3}$$

The current density is given by $j_{||} = -eN_{i0}V_{||e}$, the potential vorticity is defined as $\varpi = N_{i0}Z_i e \nabla_\perp^2 \phi$. Equations (1,2,3) can be combined in the dispersion relation

$$(\omega - \omega_*) i\sigma_{||} + \omega^2 = 0 \tag{4}$$

where $\omega_* = k_z T_{e0}/L_N$ and $\sigma_{||} = (k_{||}/k_\perp)^2(\Omega_{ci}\omega_{ce})/(0.51\nu_{ei})$. The unstable root of Eq. (4) is the RDI. The analytic solution of Eq. (4) for the complex ω is shown with BOUT data points in Fig. (1). The code run is done in toroidal geometry where magnetic field varies by 10%, that caused some mismatch. However the general level of qualitative and quantitative agreement appears to be good.

3 Shear Alfven wave

Coupling between drift waves and shear Alfven waves is another important process in edge turbulence. That is why a numerical model should faithfully reproduce shear-Alfven modes (SAM) and their coupling to plasma density. To reproduce SAM we use a few terms in equations for density, potential vorticity, and electron parallel momentum:

$$\frac{\partial N_i}{\partial t} = \nabla_{||}(j_{||}/e) \tag{5}$$

$$\frac{\partial \varpi}{\partial t} = N_{i0} Z_i e \frac{4\pi V_A^2}{c^2} \nabla_{||} j_{||} \tag{6}$$

$$m_e \frac{\partial V_{||e}}{\partial t} = e\left(\partial_{||}\phi + \frac{1}{c}\frac{\partial A_{||}}{\partial t}\right) - \frac{T_{e0}}{N_{i0}} \partial_{||} N_i \tag{7}$$

The parallel component of the vector potential is found via $\nabla_\perp^2 A_{||} = -(4\pi/c) j_{||}$. Combining Eqs. (5,6,7) one can derive the dispersion relation

$$V^2 = \frac{V_A^2 + V_{te}^2 \left(\frac{k_\perp^2 c^2}{\omega_{pe}^2}\right)}{1 + \left(\frac{k_\perp^2 c^2}{\omega_{pe}^2}\right)} \tag{8}$$

Test runs are conducted for three values of T_e, 0., 10., and 100 eV, for a set of $k_\perp$. In each run the wave velocity is calculated from BOUT by tracking a running wave pulse as a function of time. The measured values of the wave velocity are plotted vs. $k_\perp$ in Fig. (2), together with analytic curves based on the analytic dispersion relation Eq. (8).

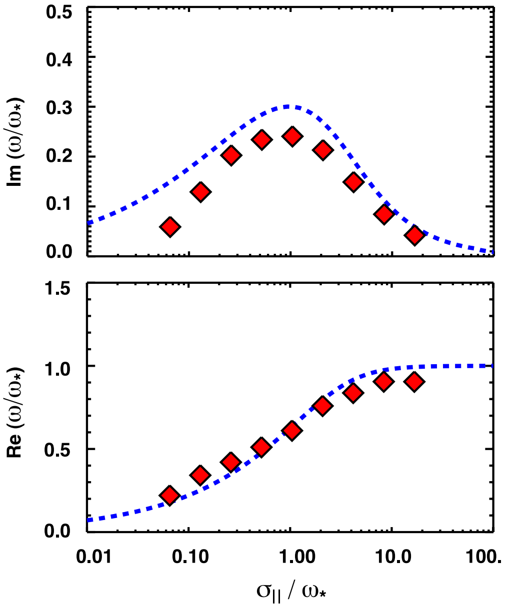

Fig. 1 Comparison of the real and imaginary parts of the dispersion relation for RDI with the exact analytic answer. In the code the imaginary part is measured from the wave growth rate and the real part is measured from the wave phase speed.

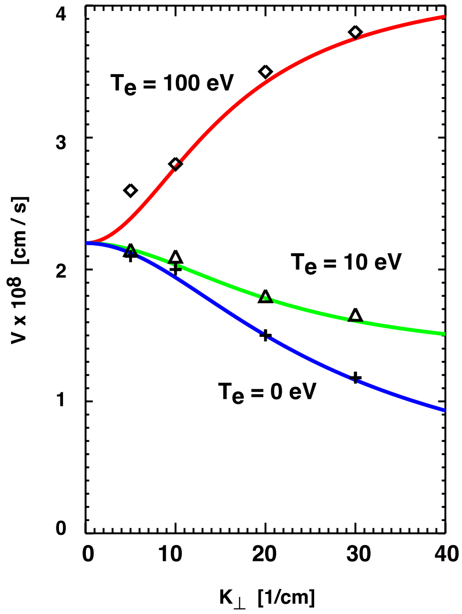

Fig. 2 Alfven pulse velocity from BOUT shown with diamonds, triangles, and crosses for T_e = 100, 10, and 0 eV respectively. Solid lines show analytic dispersion relation curves from Eq. (8.

4 Resistive interchange instability

Resistive interchange instability (RII) is the mechanism behind resistive ballooning modes in tokamak. It includes a nontrivial interplay of physics and geometry, including magnetic curvature and magnetic shear. Moreover, it is related to such important edge phenomena as ELMs.

We use the geometry of a sheared cylindrical slab. The radial coordinate is R, and R_0 is the resonant surface location. Normalized radial coordinate is $\xi = (R - R_0)/R_0$. We assume the slab is thin, $|\xi| \ll 1$, and at the resonance the safety factor is $\nu_0 = B_{t0}h_\theta/B_{p0}R_0 = m/n$. The toroidal field is $B_t = B_{t0}R_0/R$ and the poloidal field is $B_p = B_{p0}(R/R_0)^\sigma$, where σ is a given constant and $B_{p0} \ll B_{t0}$

For the physics model we use a set of dynamic equations similar to [3].

$$\frac{\partial\varpi}{\partial t} = (2\omega_{ci})\underline{0}\times\vec{\kappa}\cdot(\nabla P_{ei}) + N_{i0}Z_i e\frac{4\pi V_A^2}{c^2}\nabla_{||}j_{||} \tag{9}$$

$$\frac{\partial N_i}{\partial t} = -\vec{V}_E\cdot\nabla N_{i0} \tag{10}$$

$$j_{||} = -(1/\eta)\partial_{||}\phi \tag{11}$$

where η is the parallel resistivity. It is assumed that T_{i0}=0, T_{e0}=const, $Z_i = 1$.

After some algebra the system reduces to a second-order eigenvalue problem:

$$-\frac{\partial^2\hat{\phi}}{\partial\xi^2} + \frac{C_2}{\gamma}\xi^2\hat{\phi} = \frac{C_1}{\gamma^2}\hat{\phi}, \tag{12}$$

where

$$C_1 = 2\frac{R_0^2}{h_\theta^2}\frac{C_s^2}{L_N R_0}m^2, \qquad C_2 = \frac{4\pi V_A^2}{c^2}\frac{\hat{s}^2}{\eta}\frac{m^2}{\nu_0^2} \tag{13}$$

Noticing that Eq.(12) is equivalent to the Schrödinger equation for the quantum harmonic oscillator we find the dispersion relation and the eigenfunction

$$\gamma^3 = \frac{C_1^2}{C_2}, \qquad \hat{\phi} \propto \exp(-\xi^2/\Delta^2), \tag{14}$$

where $\Delta = \sqrt{2}\,(C_1/C_2^2)^{1/6}$. Comparison between the code and analytic answer is shown in Figs. (3) and (4).

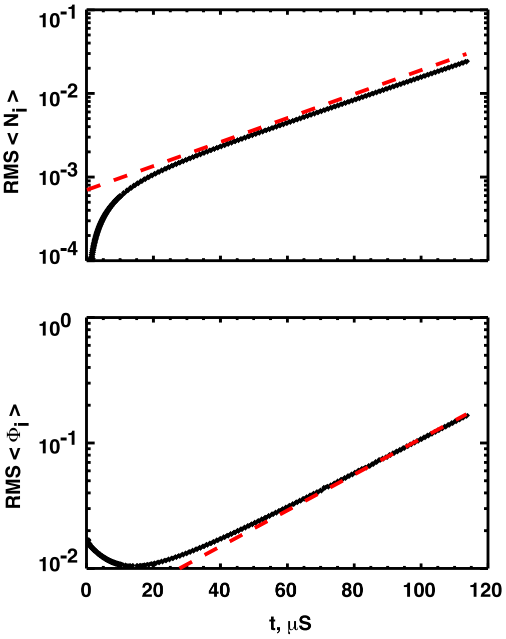

Fig. 3 Growth rate of RII is compared with the analytic answer. Amplitudes of N_i perturbations (top) and ϕ perturbations (bottom) are shown vs. time. Instability quickly settles on an eigenmode with exponential growth within a few percent of the analytic result (shown by the dotted line).

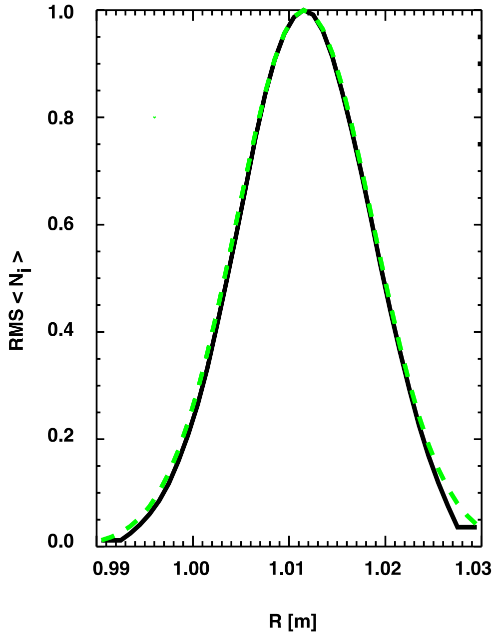

Fig. 4 Normalized radial profile of N_i perturbation in the RII exponential growth stage from the code (solid line) is compared with the analytic answer, a Gaussian profile $\exp(-\xi^2/\Delta^2)$, shown in dash. The profile from the code is very close to the Gaussian shape with the right width.

5 Axisymmetric benchmark with UEDGE

To test treatment of real divertor tokamak geometry we do an axisymmetric test comparing with edge transport code UEDGE. In this test we are solving the following system of equations:

$$\frac{\partial}{\partial t} n_i + \frac{\partial}{\partial \theta}(n_i u_\theta) + \frac{\partial}{\partial \psi}(-D_\perp \frac{\partial n_i}{\partial \psi}) = 0 \tag{15}$$

$$\frac{\partial}{\partial t}(m n_i u_{||}) + \frac{\partial}{\partial \theta}(m n_i u_\theta u_{||}) + \frac{\partial}{\partial \psi}(-m n_i \nu_\perp \frac{\partial u_{||}}{\partial \psi}) = B_\theta/B(-\frac{\partial P_{ei}}{\partial \theta}) \tag{16}$$

$$\frac{\partial}{\partial t}(3/2 n_i T_e) + \frac{\partial}{\partial \theta}(-\kappa_{||e} \frac{\partial T_e}{\partial \theta}) + \frac{\partial}{\partial \psi}(-n_i \chi_{\perp e} \frac{\partial T_e}{\partial \psi}) = 0 \tag{17}$$

$$\frac{\partial}{\partial t}(3/2 n_i T_i) + \frac{\partial}{\partial \theta}(-\kappa_{||i} \frac{\partial T_e}{\partial \theta}) + \frac{\partial}{\partial \psi}(-n_i \chi_{\perp i} \frac{\partial T_e}{\partial \psi}) = 0 \tag{18}$$

The geometry is based on an actual DIII-D shot, with x-point. For the transport coefficients we use $D_\perp = \chi_{\perp e} = \chi_{\perp i} = \nu_\perp = 0.6\ m^2/s$. Also we use the classical parallel heat conductivities without flux limiters, and $m_i = 2m_p$ (deuterium). The run starts from same flat profiles and same boundary conditions are set in both codes. Results for time history of plasma parameters at three location at outer mid-plane are shown in Fig. (5).

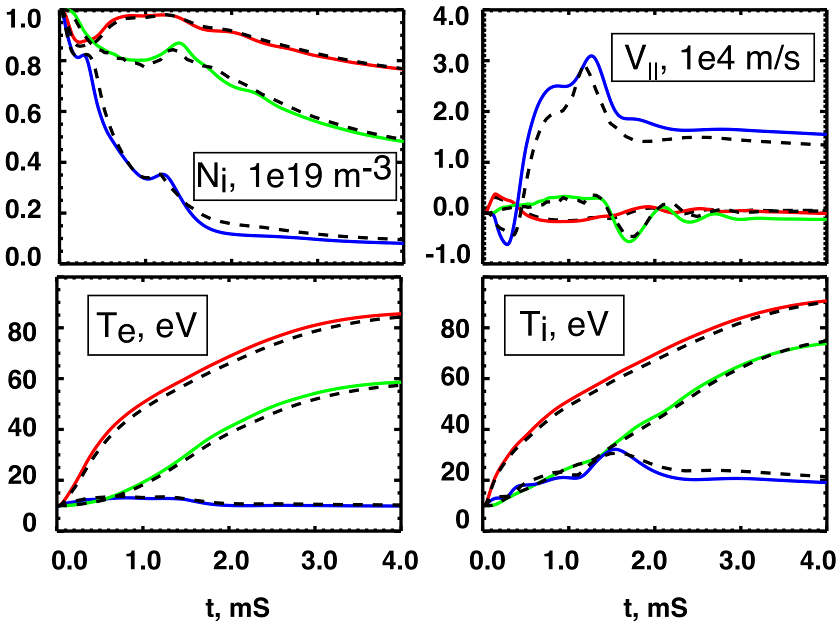

Fig. 5 Comparison of UEDGE (dash) and BOUT (solid) solutions for axisymmetric benchmark. The initial state is flat profiles, $T_e = T_i = 10$ eV, $N_i = 10^{19} m^{-3}$. The boundary conditions on core interface: $T_e = T_i = 100$ eV, $N_i = 10^{19} m^{-3}$, $\partial u_{||}/\partial r$ = 0; on outer wall and private-flux boundaries: $T_e = T_i = 10$ eV, $N_i = 10^{18} m^{-3}$, $\partial u_{||}/\partial r$ = 0; on target plates: $T_e = T_i$ = 10 eV, $\partial N_i/\partial||$ = 0, $u_{||} = 3 \times 10^4$ m/s. (Online colour: www.cpp-journal.org).

6 Summary and conclusions

We have described four test problems that have been used for verification of the BOUT code. These tests are designed to be relevant, specific, and unambiguous. Successful passing of these tests by the BOUT code gives a strong evidence that the equations in the code are solved correctly.

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References

[1] X. Q. Xu and R.H.Cohen, Contr. Plas. Phys. **36** 158 (1998).
[2] X.Q. Xu et al., Phys. Plasmas **7** 1951 (2000).
[3] Carreras et al., Phys. Fluids **30** (5) 1388 (1987).

