Analytical solution:

Consider only electron transport.

$$\frac{1}{r}\frac{\partial}{\partial r}\left(rn\chi_{\perp}\frac{\partial k_B T}{\partial r}\right) = 0$$

$$rn\chi_{\perp}\frac{\partial k_B T}{\partial r} = C_1$$

$$\frac{\partial T}{\partial r} = \frac{C_1}{rn\chi_{\perp}k_B} \qquad (1)$$

$$\therefore T(r) = \frac{C_1}{n\chi_{\perp}k_B}\ln r + C_2$$

At the inner and outer boundaries r=a, b (a<b)), respectively,

$$T(a) = \frac{C_1}{n\chi_{\perp}k_B}\ln a + C_2 \qquad (2a)$$

$$T(b) = \frac{C_1}{n\chi_{\perp}k_B}\ln b + C_2 \qquad (2b)$$

Boundary condition:

r=a

$$n\chi_{\perp}\frac{\partial k_B T}{\partial r} = \frac{C_1}{a} = -q_{up} \rightarrow\ C_1 = -aq_{up} \qquad \text{(3a)}$$

r=b

$$-n\chi_{\perp}\frac{\partial k_B T}{\partial r} = \frac{aq_{up}}{b} = \beta n\sqrt{\frac{2k_B T(b)}{M}}(\gamma T(b) + 31)k_B$$

$$\therefore 1 = \frac{b}{aq_{up}}\beta n\sqrt{\frac{2k_B T(b)}{M}}(\gamma T(b) + 31)k_B \qquad \text{(3b)}$$

T(b) can be obtained from the above equation (numerically). On the other hand,

$$-n\chi_{\perp}\frac{\partial k_B T}{\partial r} = -n\chi_{\perp}k_B\frac{T(b)}{\lambda}$$

$$\therefore T(b) = \frac{aq_{up}}{b}\frac{\lambda}{n\chi_{\perp}k_B} \qquad (4)$$

Substitute (4) and (3a) for (2b), then one gets,

$$C_2 = T(b) = \frac{aq_{up}}{n\chi_{\perp}k_B}\left(\frac{\lambda}{b} - \ln\frac{1}{b}\right)$$

The analytical solution is, therefore,

$$\therefore T(r) = \frac{a q_{up}}{n \chi_{\perp} k_B}\left(\frac{\lambda}{b} + \ln\frac{b}{r}\right) \qquad (5a)$$

Removing λ using (4), it reads,

$$\therefore T(r) = T(b) + \frac{a q_{up}}{n \chi_{\perp} k_B}\ln\frac{b}{r}. \qquad (5b)$$

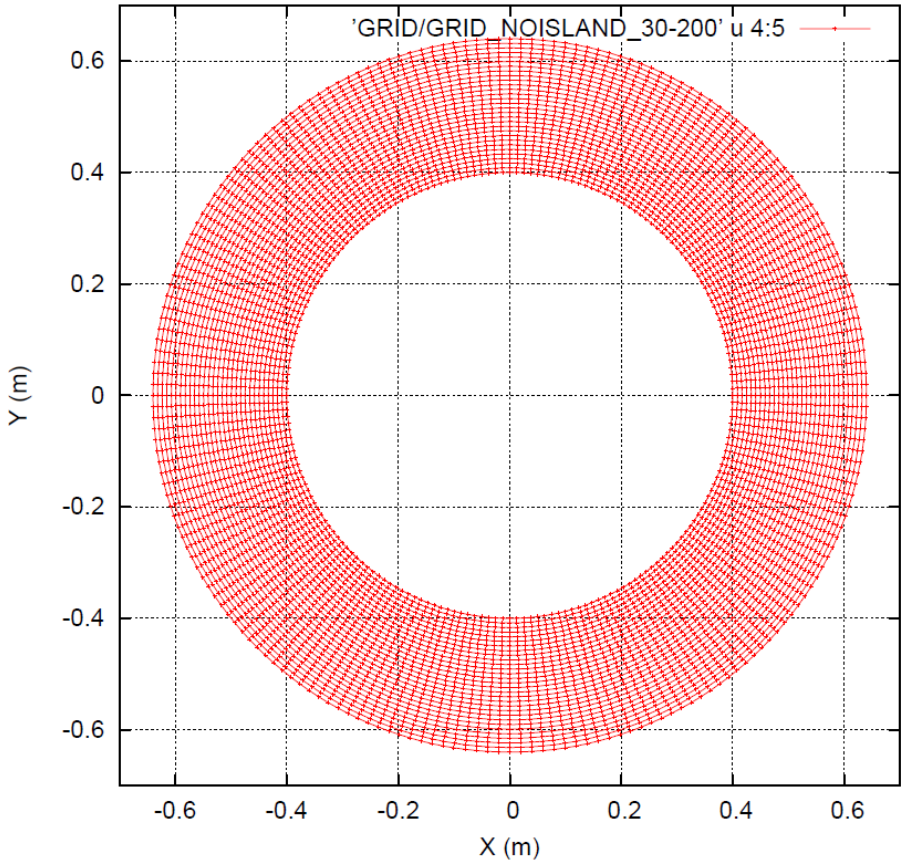
Parameters in the simulation;

Psol = 2 MW, a = 0.40 m, b = 0.64 m, n_e = 3e19 m^{-3}, γ = 5.5, β = 1e-3, q_{up} = Psol/($4\pi^2$ R_0 a), R_0 = 3.90 m. T(b) ~ 11.06230 eV.

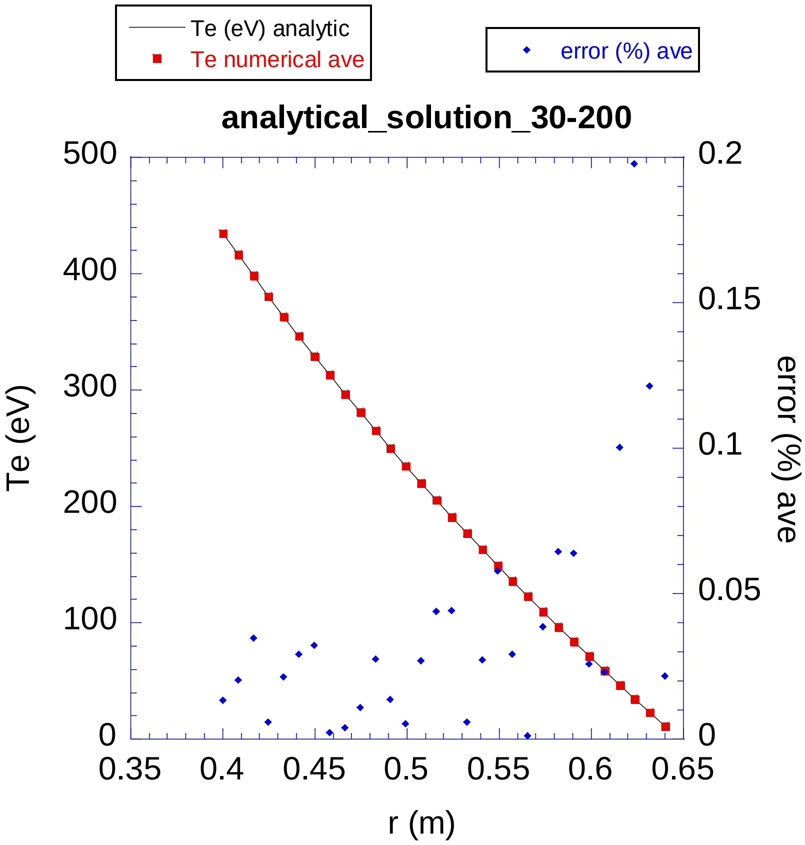
Iota profile:

$$\iota = \frac{d\theta}{d\phi} = 0.45 - 1.04r + 9.70r^2 - 29.07r^3 + 33.70r^4$$

Grid points: radial 30, poloidal 200.



Results:


Te (eV) analytic
Te numerical ave
error (%) ave
analytical_solution_30-200
Te (eV)
error (%) ave
r (m)
500
400
300
200
100
0
0.2
0.15
0.1
0.05
0
0.35
0.4
0.45
0.5
0.55
0.6
0.65