Applied Mechanism Design and Big Data

→ sub-topic: Statistical Data Analysis

Max Baak

(with many thanks to Wouter Verkerke)

Roadmap for this course

- 1. Statistics basics
 - Probability theory
 - Probability distributions
- 2. Parameter estimation

- 3. Pitfalls in (big) data analysis
 - Spurious correlations
 - Data-quality assessment
- 4. Hypothesis Testing

Correlation & covariance in >2 variables

 Concept of covariance, correlation is easily extended to arbitrary number of variables

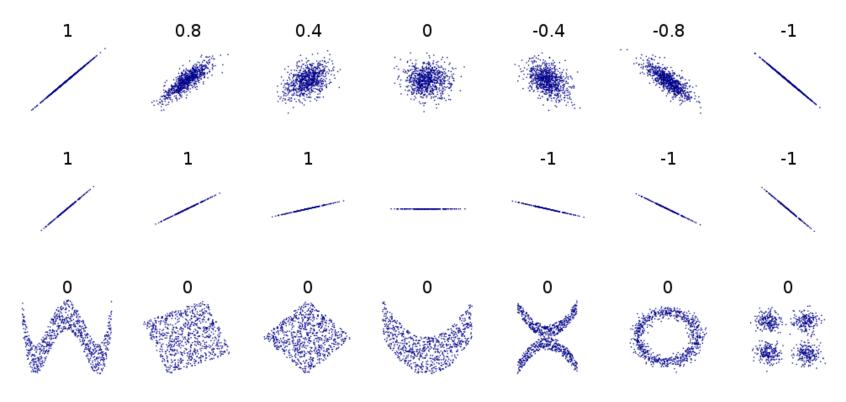
$$cov(x_{(i)}, x_{(j)}) = \overline{x_{(i)}x_{(j)}} - \overline{x}_{(i)}\overline{x}_{(j)}$$

- so that $V_{ij} = \text{cov}(x_{(i)}, x_{(j)})$ takes the form of a $n \times n$ symmetric matrix
- This is called the covariance matrix, or error matrix
- Similarly the correlation matrix becomes

$$\rho_{ij} = \frac{\text{cov}(x_{(i)}, x_{(j)})}{\sigma_{(i)}\sigma_{(j)}} \qquad \qquad V_{ij} = \rho_{ij}\sigma_i\sigma_j$$

Linear vs non-linear correlations

- Correlation coefficients used here are (linear)
 Pearson product-moment correlation coefficients
- Data can have more subtle (non-linear) correlations that contained in these coefficients



Always check correlation by eye!

Trivia Quiz!

Trivia quiz! (1/2)

 Which of the following coin-flip series is random, and which one made-up?

b. 00101101100111010110110110001101001110

Trivia quiz! (2/2)

 What percentage of the course material have you understood so far?

- a. More than 70%
- b. Less than 70%

Write down the number you think is right

Overconfidence bias!

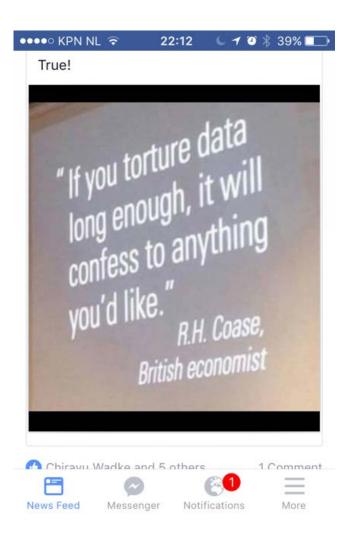
(From KPMG Audit test)

Group Tested	Information Type	(% Misses) Target/Observe			
Harvard MBAs	Trivia facts	2%	46%		
Computer Co. Managers	General business Company-specific	5% 5%	80% 58%		
Physicians	Probability of pneumonia	0-20%	82%		

- E.g. when physicians were asked to assess the likelihood of pneumonia, they were highly confident that they would only be wrong between 0-20% of the time
- Instead, they were wrong more than 80% of the time

Analysis Pitfalls

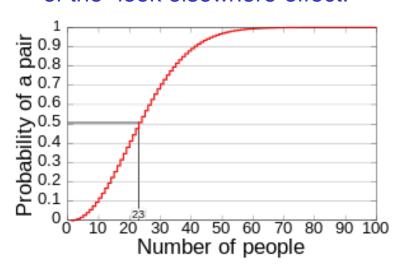
Just to show...



Look Elsewhere Effect - The Birthday Problem

- Humans have tendency to highlight unlikely events, but to ignore all likely events.
- The more experiments performed (on the same dataset), the less significant the result(s) found.
 - Bonferroni correction (conservative): multiply the observed pvalue by the number of tests performed.

"the birthday problem" as illustration of the "look elsewhere effect."

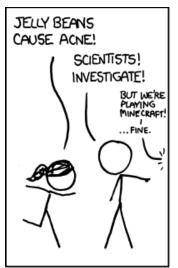


With 23 people the chance of a matching birthday is already 50%.

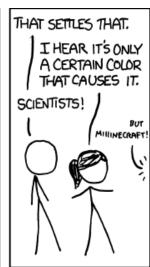
Example Look Elsewhere Effect

- A Swedish study in 1992 tried to determine whether or not power lines caused some kind of poor health effects.
- The researchers surveyed everyone living within 300 meters of high-voltage power lines over a 25-year period and looked for statistically significant increases in rates of over 800 ailments.
- The study found that the incidence of childhood leukaemia was four times higher among those that lived closest to the power lines.
- → It spurred calls to action by the Swedish government.
 (Wikipedia, LEE)

aka: Law of very large numbers "With a sample size large enough, any outrageous thing is likely to happen."

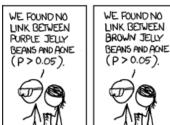














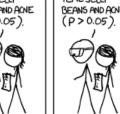
WE FOUND NO

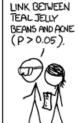
LINK BETWEEN

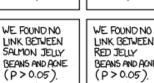
TURQUOISE JELLY

BEANS AND ACNE















WE FOUND NO LINK BETWEEN MAGENTA JELLY BEANS AND ACNE (P>0.05).



WE FOUND NO LINK BETWEEN YELLOW JELLY BEANS AND ACNE (P>0.05).



WE FOUND NO LINK BETWEEN GREY JELLY BEANS AND ACNE (P>0.05).



WE FOUND NO LINK BETWEEN TAN JELLY BEANS AND ACNE (P > 0.05).



WE FOUND NO LINK BETWEEN CYAN JELLY BEANS AND ACNE (P>0.05).



WE FOUND A LINK BETWEEN GREEN JELLY BEANS AND ACNE (P<0.05)



WE FOUND NO LINK BETWEEN MAUVE JELLY BEANS AND ACNE (P>0.05).



WE FOUND NO LINK BETWEEN BEIGE JELLY BEANS AND ACNE (P > 0.05).



WE FOUND NO LINK BETWEEN LILAC JELLY BEANS AND ACNE (P>0.05),



WE FOUND NO LINK BETWEEN BLACK JELLY BEANS AND ACNE (P>0.05)

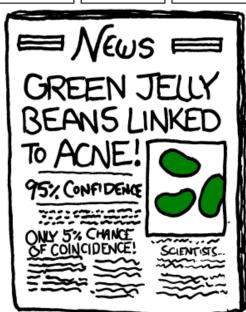


WE FOUND NO LINK BETWEEN PEACH JELLY BEANS AND ACNE (P>0.05)



WE FOUND NO LINK BETWEEN ORANGE JELLY BEANS AND ACNE (P > 0.05)





https://xkcd.com/882/

Curious correlation?

http://www.tylervigen.com/spurious-correlations

US spending on science, space, and technology

correlates with

Suicides by hanging, strangulation and suffocation



◆ Hanging suicides◆ US spending on science

tylervigen.com

Correlation and causality

- Correlation does not necessarily imply causality
- Spurious correlations are caused by dependence on "hidden" confounding variables

Example: Increased rates of drownings in a city's swimming pools in summers with high ice cream sales do not necessarily imply that eating ice cream causes drowning. There might be a common, (not so) confounding dependence on the weather.

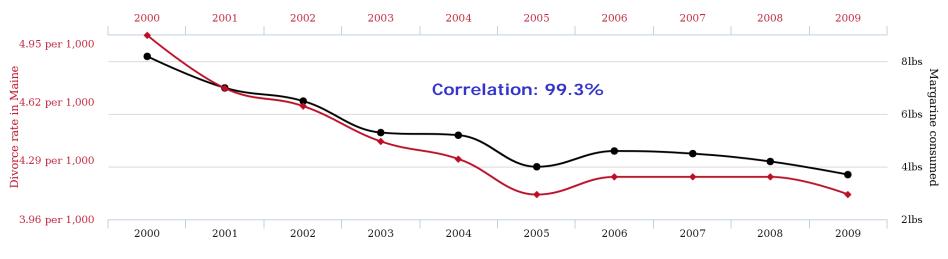
Spurious correlations - example 2

http://www.tylervigen.com/spurious-correlations

Divorce rate in Maine

correlates with

Per capita consumption of margarine



◆ Margarine consumed → Divorce rate in Maine

tylervigen.com

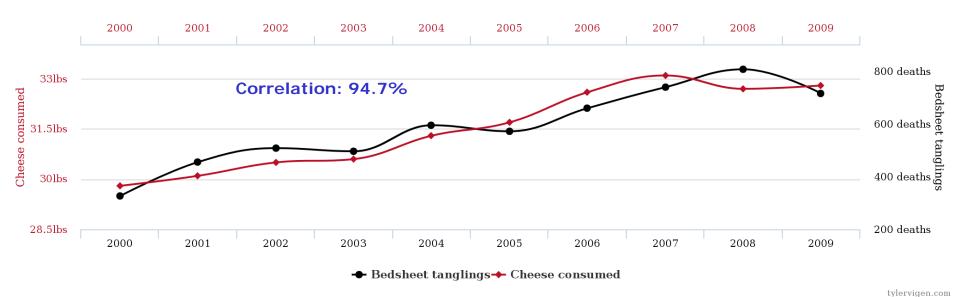
Spurious correlations - example 3

http://www.tylervigen.com/spurious-correlations

Per capita cheese consumption

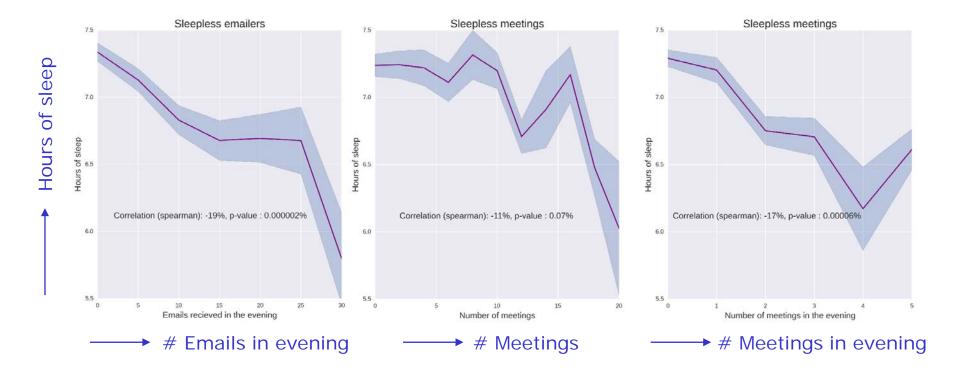
correlates with

Number of people who died by becoming tangled in their bedsheets



Spurious correlations - example 4

From internal KPMG study on correlation between fitness and health

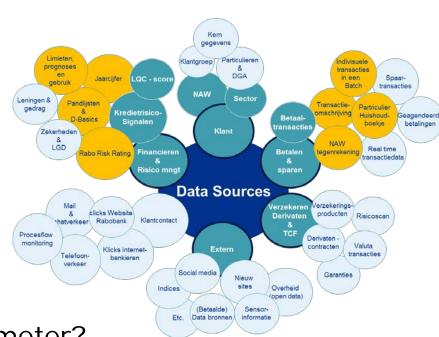


Spurious correlations

- Typical client question:
 "We have multiple (large) datasets [with many variables]
 Please extract any relevant insights [...]"
- With N variables, number of (spurious) correlations proportional to N²

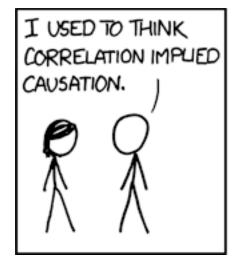
Debunking spurious correlations:

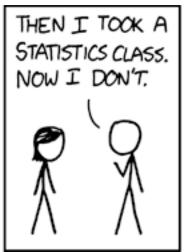
- Causal relationship?
- Correlation through other parameter?
- Awareness! Use your brains!

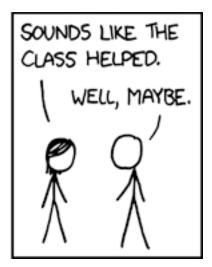


Correlation and causality

https://xkcd.com/552/





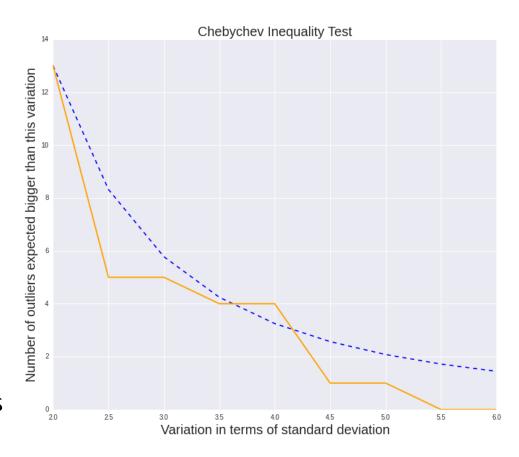


Chebyshev Inequality Test

 For arbitrary continuous distributions in x the Chebyshev inequality theorem holds:

$$P(|x - \mu| > k\sigma) \le \frac{1}{k^2}$$

- → Gives upper bound on size & significance of random fluctuations
- Clearly more strict bounds apply for Gaussian distribution



blue curve = Chebyshev prediction:

If the orange line is on top (or below) of the blue curve, the findings are not inconsistent with random fluctuations

Simpson's paradox

- Other (extreme) example of dependence on confounding variables: Simpson's paradox
- Conclusion of a study is reversed when confounding variables are taken into account

Example: study of success rate in removing kidney stones

Two methods were compared:

- Open surgery: 78% of treatments successful
- Small puncture: 83% of treatments successful

Conclusion: "Small puncture method" more successful. Or is it? →

Simpson's paradox

Example: study of success rate in removing kidney stones

Split success rates into cases with small stones and cases with large stones:

		Open surgery	Small puncture		
	Small stones	93% (81/87)	87% (234/270)	preference in treatment: dominates overall rates	
lower rates →	Large stones	73% (192/263)	69% (55/80)		
	Overall	78% (273/350)	83% (289/350)	•	

- Surgery more often successful for both small and large stones
- Treatment large stones less often successful for both surgery and puncture
- Large stones usually treated by surgery, small stones by puncture
- As a result, surgery less often successful overall

Note:

- Success probabilities in table are also conditional on the initial choice of treatment.
 I.e., success rate is 87%, given small stones, choice for puncture, and treatment by puncture. For (hypothetical) cases treated by puncture, where surgery would have been preferred choice, the success rate is likely to be smaller.
- Statistical significance of differences in surgery and puncture rates (binomial): small stones: 1.9 (σ_{Δ} =3%); large stones: 0.7 (σ_{Δ} =6%); overall: 1.5 (σ_{Δ} =3%) $\sigma_{r} = \sqrt{\frac{r(1-r)}{r}}$

Data Quality

What data did we get?

Where it goes wrong...

1999 - Mars Climate Orbiter: Crash caused by incorrect data from thruster software

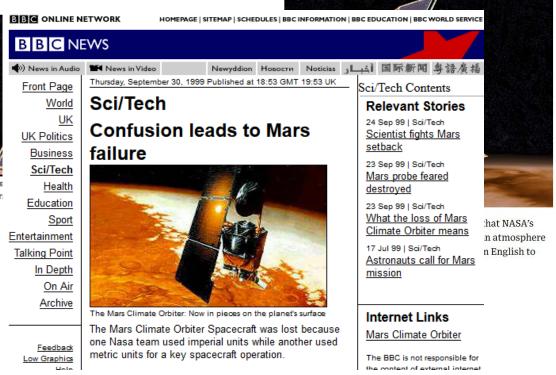
NOV. 10, 1999: METRIC MATH MISTAKE MUFFED MARS METEOROLOGY MISSION

LISA GROSSMAN 11.10.10 7:00 AM

Mystery of Orbiter Crash Solved

By Kathy Sawyer Washington Post Staff Writer Friday, October 1, 1999; Page A1

NASA's Mars Climate Orbiter was lost in space last week because engineers failed to make a simple conversion from English units to metric, an embarrassing lapse that sent the \$125 million craft fatally close to the Martian surface, investigators said yesterday.



Unit of thruster impulse in data was pound-seconds, where it should have been Newton-seconds according to specifications

Where it goes wrong...

Review of British hospital data: Pregnant men?!



Letters

Hospital episode statistics

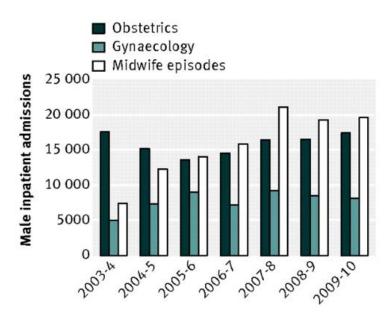
The importance of knowing context of hospital episode statistics when reconfiguring the NHS

BMJ 2012; 344 doi: http://dx.doi.org/10.1136/bmj.e2432 (Published 04 April 2012) Cite this as: BMJ 2012:344:e2432

Article Related content Metrics Responses

Lauren Brennan, specialty doctor and honorary clinical research fellow¹, Mando Watson, consultant paedia Robert Klaber, consultant paediatrician¹, Tagore Charles, consultant paediatrician¹

BMJ 2012; 344: e2432



Britain's 17,000 Pregnant Men Aren't Really Pregnant



Call it the mother of all medical coding errors.

Sarah Kliff of Ezra Klein's WonkBlog recently wrote about an interesting nugget that appeared in a letter published in the British Medical Journal.

Between 2009 and 2010, thousands of British men turned up at hospitals to be treated for many pregnancy-related services, things like obstetric exams and midwife services. All told, there were 17,000 of them.



"Junior" YouTube

Hospital data of men visiting gynaecologists and midwifes were probably entered with incorrect admission codes

Data quality (DQ)

Are the data in a dataset what one expects them to be?

- Missing files or records
 missing few hours in a daily log file (less activity that day or missing data?)
- Missing / inconsistent / incorrect data fields
 birth date in 2984, "John Dough" instead of "John Doe", "7" instead of "007",
 record for a car with a length of 1m
- Inconsistent / duplicate records
 mutations bank account don't add up to total change in balance,
 daily log files with overlap between 23:00 and 01:00,
 company details for both "Anderson Enterprises" and "Anderson Enterpr."
- Biased data sets
 manual preselection of "noteworthy" maintenance tasks on a train
- Distributions changing in time increasing temperature values over last month (indication of failing monitored machine or running calibration of sensor?)

Lead to systematic uncertainties in results of data analysis

Data-quality requirements

Example: data-quality levels large insurance company

- Strict risk-management regulations ("Solvency II"), especially after financial crisis
- Regulations also affect data requirements

The required data-quality level depends on the purpose of the data. Data-quality guide lines must be designed for each purpose. The goal is to "extend the project *DQ* in source systems beyond Solvency-II".

Financial reporting (Solvency requirement) minimal DQ: 100%

Risk and premium definition minimal DQ: 90%

Advanced analytics

Reporting, marketing & sales minimal DQ: 85%

Identify/monitor trends in data minimal DQ: 75%

The desired DQ definitions and levels are determined iteratively on the job and evolve in time

Central question

How to quantify level of data quality for any given dataset?

... and to design automated data-quality checks on our data?

Data-quality business rules

- DQ business rule: specific, well-defined test of data Results in a "traffic light": red, yellow, green
- DQ rules are applied at various levels
 - 1. Field level
 Is a certain data field filled? Does it have the right format? Does it have an allowed value?
 - 2. Record level Are all related elements filled? And filled correctly (wrt each other)?
 - 3. Cross-record level
 Are all records corresponding to one person consistent with each other?
 - Analysis level / Over time
 Detects anomalies or trends in data over time → Focus of this lecture
- Assess the DQ level: Apply DQ rules, count how often they work/fail



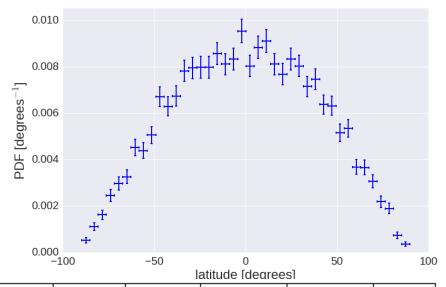
Data profiling

"Data profiling": collecting generic information on data

- About each data field (column) in a dataset
 length, #nans, #infs, #zeros, #unique, their fractions, mean, std, max, min, ...
- Histogram of distribution of each field
 For strings: count how often each string occurs
- Histograms of derived distributions
 - Distribution of most significant digit
 Check for Benford's law, e.g. for money values
 - Distribution of relative value counts
 Histogram of (normalized) value-histogram counts...
- Note: All quite easy evaluations!
 "Count how often something happens"

Data profiling

profile for single column in input data



	length	frac. unique	frac. NaN	frac. 0	frac. positive	minimum	maximum	mean	std	median
datetimeCol										
2015-01-31	8390	0.984148	0.010250	0	0.605006	-999.99	1499.74	257.213800	721.762423	264.405
2015-02-28	7653	0.983928	0.008624	0	0.598981	-999.94	1499.78	247.177364	716.717271	243.140
2015-03-31	8508	0.981782	0.010696	0	0.605783	-999.88	1499.50	259.871051	717.811026	266.865
2015-04-30	8282	0.981043	0.010746	0	0.600217	-999.63	1500.00	245.900256	721.913009	255.275
2015-05-31	8405	0.982867	0.010708	0	0.590720	-999.85	1499.88	237.254653	723.026722	226.310
2015-06-30	8117	0.981767	0.009363	0	0.601084	-999.93	1499.88	252.777563	720.835069	245.820
2015-07-31	8566	0.978403	0.008639	0	0.595844	-998.95	1499.64	244.516981	723.872802	227.695
2015-08-31	8430	0.983274	0.010202	0	0.599644	-999.91	1499.77	250.079751	718.966038	247.775
2015-09-30	7894	0.983785	0.010894	0	0.609704	-999.50	1500.00	264.444297	730.142341	273.130
2015-10-31	8344	0.983341	0.010187	0	0.603787	-999.62	1499.90	261.942858	723.093769	254.820
2015-11-30	8175	0.982997	0.008073	0	0.598532	-999.93	1499.93	251.151890	716.849333	240.870
2015-12-31	8258	0.983410	0.010051	0	0.597239	-999.76	1499.29	242.961905	721.729556	243.145

Define generic DQ rules based on profiling

def traffic_light(x):

```
if x < red_lo or x >= red_hi: return 'red'
if x < yellow_lo or x >= yellow_hi: return 'yellow'
return 'green'
```



def nan_traffic_light(num_nans)

```
if num_nans >= 3: return 'red'
if num_nans >= 1: return 'yellow'
return 'green'
```

- Setting fixed thresholds would be kind of annoying
 \omega
 - Could get ranges automatically from reference data
 - This also enables further comparison of test data and reference

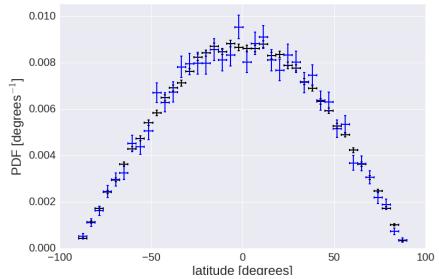
Compare with reference dataset

Key concept for generic DQ monitor

- Base DQ evaluation on comparison of test dataset with reference dataset
- For this example: records in datasets are time-stamped Useful, but not a necessary requirement
- DQ assessment done per (user-defined) unit time block e.g. per day/week/month
 Flag data if inconsistencies are found
- Possibly reject block of test data if DQ is insufficient (red)
- Apply data profiling to
 - Entire reference dataset
 If possible to the blocks of the reference dataset
 - All blocks of the test datasetIf not possible, to entire test dataset only

Compare with reference

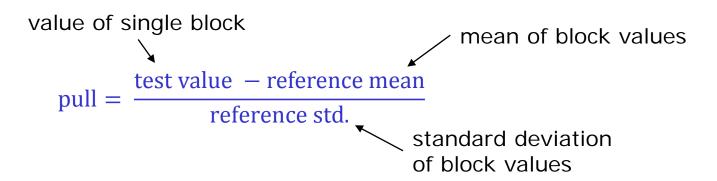
profile for single column in input data



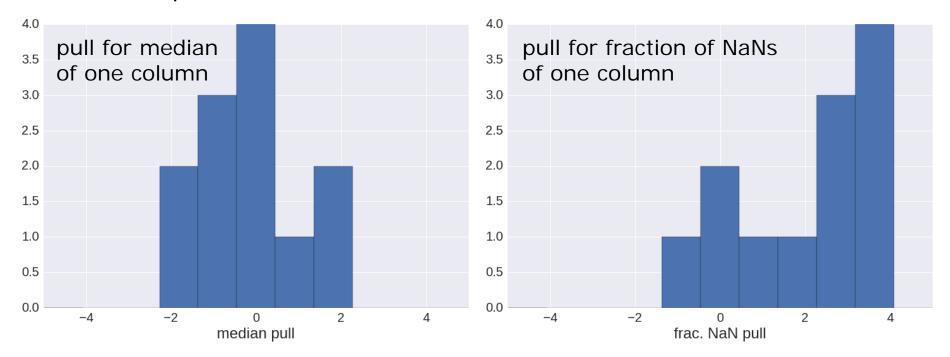
	length	frac. unique	frac. NaN	frac. 0	frac. positive	minimum	maximum	mean	std	median
period mean	8261.333333	0.983654	0.008721	0	0.599139	-999.769167	1499.694167	250.237557	722.372284	250.537500
period std.	297.886596	0.001208	0.000591	0	0.004021	0.251413	0.380513	6.570123	4.984437	11.410786

2015-02-28	7653	0.983928	0.008624	0	0.598981	-999.94	1499.78	247.177364	716.717271	243.140
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2015-12-31	8258	0.983410	0.010051	0	0.597239	-999.76	1499.29	242.961905	721.729556	243.145

Pulls with respect to reference data

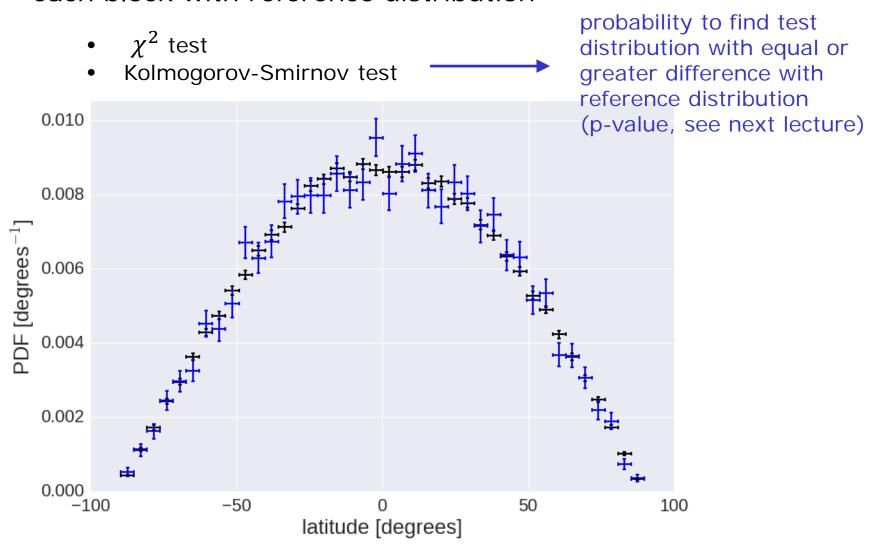


Expect normal distributions with $\mu = 0$ and $\sigma = 1$



Further tests of distributions

Compare distribution of test-column values in each block with reference distribution



Define additional DQ rules

Test all evaluated pulls for outliers

```
def pull_traffic_light(pull):
    if |pull| > 5: return 'red'
    if |pull| > 3: return 'yellow'
    return 'green'
```



Test distribution p-values for outliers:

```
def distr_test_traffic_light(p_value):
    if p_value < 0.001: return 'red'
    if p_value < 0.01: return 'yellow'
    return 'green'</pre>
```

Simply count how often the DQ rules work and fail...

Data-quality dashboard

Data Quality Summary

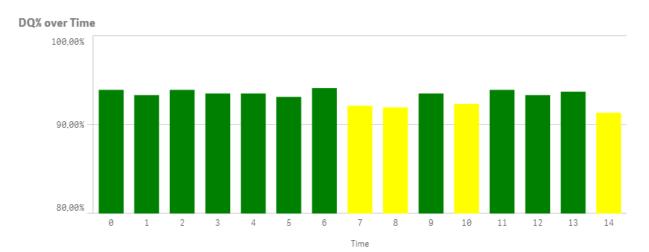
DQ% Overall

93,12%

Last Period

91,29% -2,38%

Change from last Period



Green

7,05k

Yellow

Red

Observable	Q DQ%	
Totals	93,12%	
cat	86,67%	
amount	88,70%	
f7	92,44%	
f2	92,74%	
f0	93,04%	
f4	93,19%	
f3	93,63%	
f8	93,93%	
f1	94,37%	

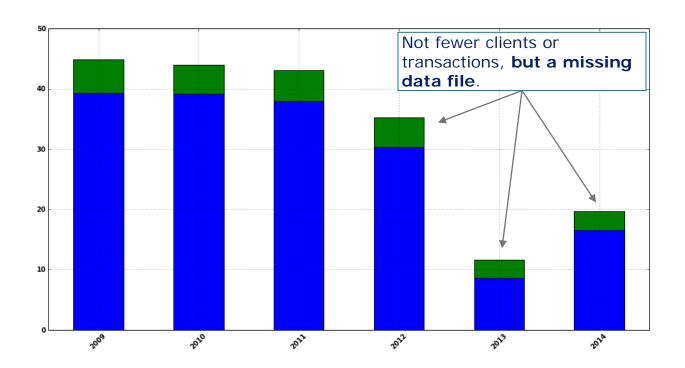
Backup

Data Errors

Types of data 'errors':

- Missing files or records
- Missing or incorrect fields

	Follow-up number		Date	Amoı	unt	Transaction type
	20150004	Jan 13	3015	-100.00) €	Pin withdrawal
	20150005	Jan 15	2015	-50.00) €	Pin payment
_	20150006	Jan 20	2015	2500.00) €	
	20150007	Jan 22	2015	-40.00) €	Pin payment
	20150008	Jan 29	2015	-1500.00) €	failed
	20150008	Jan 29	2015	-1500.00) €	Bank transfer
	20150009	Feb 3	2015	-250	.00	Pin payment
_	20150010	Jan 4	2015	-150	.00	Pin payment



Data Errors

Types of data 'errors':

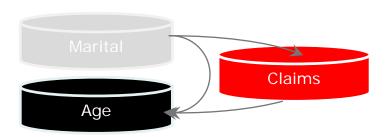
- Missing or ill understood relations between fields
- Missing or ill understood relations between records

Follow-up number	Date	Amount	Transaction type
20150004	Jan 13 3015	-100.00 €	Pin withdrawal
20150005	Jan 15 2015	-50.00 €	Pin payment
20150006	Jan 20 2015	2500.00 €	
20150007	Jan 22 2015	-40.00 €	Pin payment
20150008	Jan 29 2015	-1500.00 €	failed
20150008	Jan 29 2015	-1500.00 €	Bank transfer
20150009	Feb 3 2015	-250.00	Pin payment
20150010	Jan 4 2015	-150.00	Pin payment

Data Errors

Types of data 'errors':

Biased data



Enriching Age data with Marital status:

- 1. If name given in Age data, then get Marital status directly with name.
- 2. Else get name from Claims data through address, and then get Marital status with name.
- 3. Else marital status remains unknown.

Now what is the probability that a married person makes a claim?

But people with claims tend to have better known marital status than others!