Multidimensional Riemann problems

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1 Mathematical problem

The domain is defined by space coordinates $\boldsymbol{x} = (x, y) \in \Omega_{\boldsymbol{x}}$ and time $t \in \Omega_t$. The problem considers two-dimensional forms of the Euler's equations:

$$\frac{\partial}{\partial t}\begin{bmatrix}\rho \\ \rho u \\ \rho v \\ E_t\end{bmatrix} + \frac{\partial}{\partial x}\begin{bmatrix}\rho u \\ \rho u^2 + p \\ \rho u v \\ u(E_t + p)\end{bmatrix} + \frac{\partial}{\partial y}\begin{bmatrix}\rho v \\ \rho u v \\ \rho v^2 + p \\ v(E_t + p)\end{bmatrix} = 0, \boldsymbol{x} \in \Omega_{\boldsymbol{x}}, t \in \Omega_t \tag{1}$$

where the pressure is related through the equations of state for the two-dimensional case:

$$p = (\gamma - 1)(E_t - \frac{1}{2}\rho(u^2 + v^2)), \tag{2}$$

2 Numerical test

The input data for the Two-dimensional Euler's Equations of Gas Dynamics for the 3 cases are given in Table 1.

Table 1. *Input Data for the Two-dimensional Euler's Equations of Gas Dynamics*

Case	N [−]	CFL [−]	$\Omega_{\boldsymbol{x}}$	Ω_t	Parameter	Value
1	1500×1500	0.01	$[0, 1] \times [0, 1]$	$[0, 0.3]$	γ [−]	1.4
2	1500×1500	0.01	$[-1, 1] \times [-1, 1]$	$[0, 0.52]$	γ [−]	1.4
3	1500×1500	0.01	$[-1, 1] \times [-1, 1]$	$[0, 1.1]$	γ [−]	1.4

The first case consists of setting:

$$(\rho, u, v, p)|_{t=0} = \begin{cases} (0.1380, 1.2060, 1.2060, 0.0290), & 0 \le x \le 0.5, \quad 0 \le y \le 0.5 \\ (0.5323, 0.0000, 1.2060, 0.3000), & 0.5 \le x \le 1, \quad 0 \le y \le 0.5 \\ (0.5323, 1.2060, 0.0000, 0.3000), & 0 \le x \le 0.5, \quad 0.5 \le y \le 1 \\ (1.5000, 0.0000, 0.0000, 1.5000), & 0.5 \le x \le 1, \quad 0.5 \le y \le 1 \end{cases} \tag{3}$$

The second case consists of setting:

$$(\rho, u, v, p)|_{t=0} = \begin{cases} (0.5313, 0.0000, 0.0000, 0.4000), & 0 \le x \le 1, \quad 0 \le y \le 1 \\ (1.0000, 0.0000, 0.7276, 1.0000), & 0 \le x \le 1, \quad -1 \le y \le 0 \\ (1.0000, 0.7276, 0.0000, 1.0000), & -1 \le x \le 0, \quad 0 \le y \le 1 \\ (0.8000, 0.0000, 0.0000, 1.0000), & -1 \le x \le 0, \quad -1 \le y \le 0 \end{cases} \tag{4}$$

The third case consists of setting:

$$(\rho,\ u,\ v,\ p)|_{t=0} = \begin{cases} (1.5000,\ 0.0000,\ 0.0000,\ 1.5000), & 0 \le x \le 1, \quad 0 \le y \le 1 \\ (0.5323,\ 0.0000,\ 1.2060,\ 0.3000), & 0 \le x \le 1, \quad -1 \le y \le 0 \\ (0.5323,\ 1.2060,\ 0.0000,\ 0.3000), & -1 \le x \le 0, \quad 0 \le y \le 1 \\ (0.1379,\ 1.2060,\ 1.2060,\ 0.0290), & -1 \le x \le 0, \quad -1 \le y \le 0 \end{cases} \tag{5}$$

And for the boundary conditions Dirichlet boundary conditions is applied, meaning that the conserved quantities take on the values prescribed by the initial conditions at both boundaries.

3 Results

In Figure 1, there is presented contour plot for the density contours for the case 1 at a time $t = 0.3$. One can see a clear formation of the shock that was achieved by the numerical model.

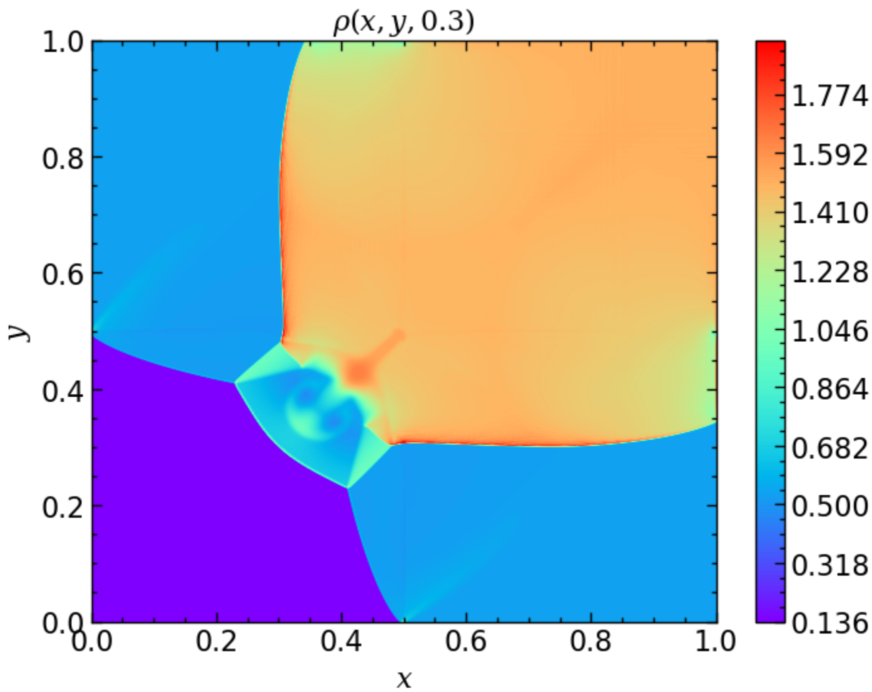

Figure 1. *density ρ contour plot for the time $t = 0.3$ for the case 1*

The contours for the density variable at the time $t = 0.52$ is shown in 2 for the case 2. The result may be described in terms of two contact surfaces created at the intersections of the four shocks, a density valley is visibly moving in the direction of the place where these four shocks pass through.

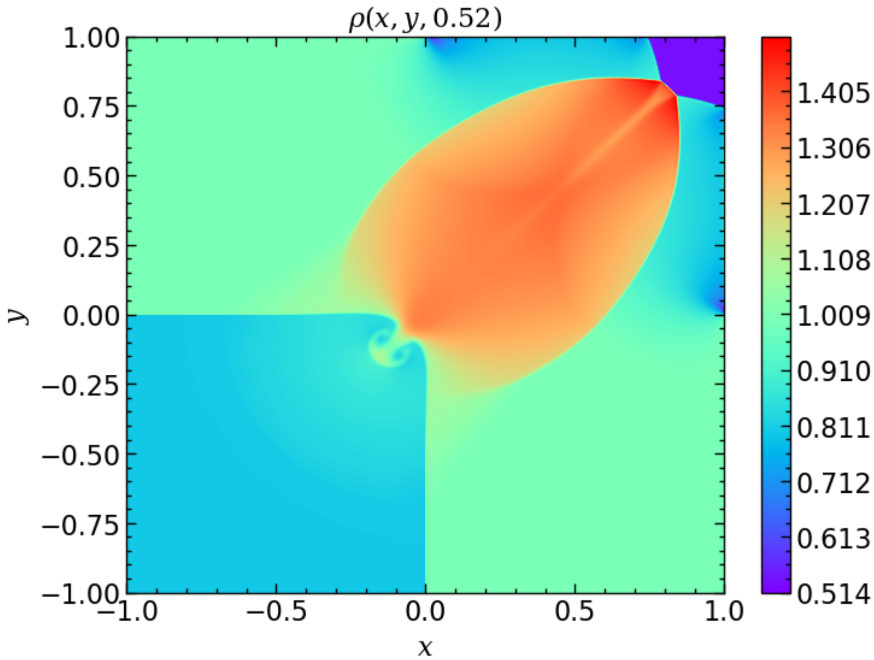

Figure 2. *density ρ contour plot for the time $t = 0.52$ for the case 2*

The density variable is shown in 3 at time $t = 1.1$ for the case 3. An accurate representation of the mushroom cap in this scenario is achieved through the use of the numerical model.

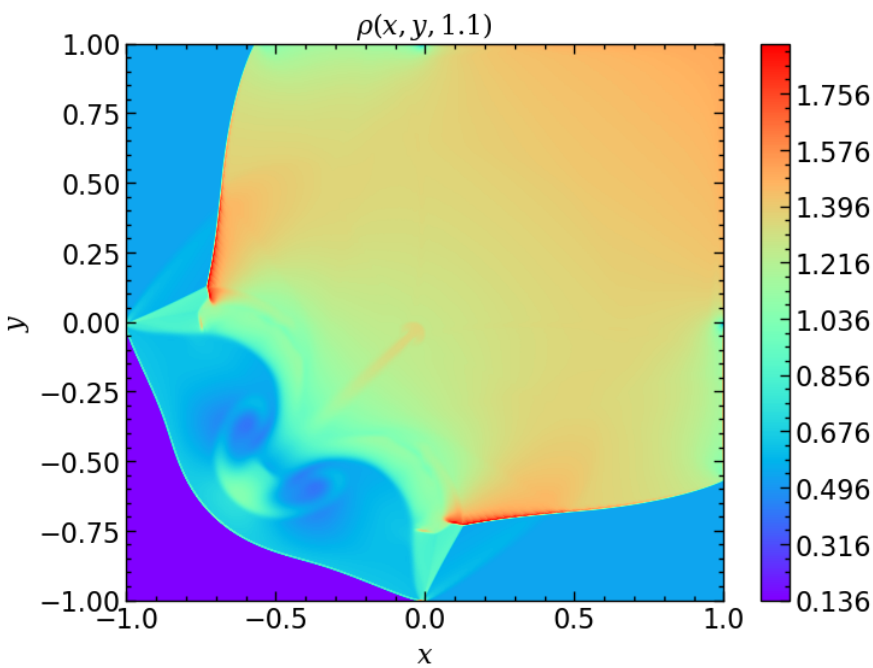

Figure 3. *density ρ contour plot for the time $t = 1.1$ for the case 3*