HOMEWORK 4

FSR

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Index

All the screenshots are in the folder.

1 First Exercise

The buoyancy is an hydrostatic effect, as it is not dependent on the relative movement between the body and the fluid. It is determined by the density of the fluid and the mass of the drone and it follows the rule that an object whose average density is greater than that of the fluid in which it is immersed will tend to sink. Conversely, if the object is less dense than the fluid, the force can keep the object afloat. In the context of underwater robotics, the density of the fluid is typically around $1000Kg/m^3$ (comparable with the density of the robot), which is about three times greater than the density of air ($1.204Kg/m^3$). Given this difference in density, the buoyancy effect can be considered negligible in the case of aerial robotics. The buoyancy force formula is:

$$f_b^b = -R_b^T \begin{bmatrix} 0 \\ 0 \\ b \end{bmatrix} = -R_b^T \begin{bmatrix} 0 \\ 0 \\ \rho \Delta g \end{bmatrix}$$

The buoyancy force acts at the centre of buoyancy, which is defined as $r_b^b \in \mathbb{R}$. It is not equal to the centre of mass and if it is not on the same line, the application of a buoyancy force results in the development of a moment on the body. It is preferable to have the centre of gravity below the centre of buoyancy on the same axis, as this configuration allows the weight of the vehicle to create a restoring moment, thereby adding stability to the vehicle. It is important to consider the following factors when designing the robot. In some instances, unmanned underwater vehicles (UUVs) utilise buoyancy to ascend and descend within the water column. This is achieved through the use of a tank oil system: while descending, hydraulic fluid moves from the external inflatable bladder, which produces a high pressure in the internal reservoir, which is at a low pressure through a valve. The decrease in volume of the bladder creates an increase in density, causing negative buoyancy. During ascent, hydraulic fluid is transferred from the internal accumulator to the external inflatable bladder via the pump. This results in an increase in volume, which in turn causes a decrease in density, thereby creating positive buoyancy. Additionally, the seawater flushes out the open hydrodynamic fairing of the vehicle, aiding its ascent to the surface. For neutral buoyancy, the vehicle must have a density equal to that of seawater. [Additional Source]

2 Second Exercise

a. ***FALSE***, the added mass is not a quantity of fluid to add to the system such that it has a bigger mass. It is a function of the body's geometry and it has also an added Coriolis and centripetal contribution. In certain circumstances, added mass forces can arise in one direction due to motion in a different direction, which can result in a 6 x 6 matrix of added mass coefficients (M_a). For symmetric geometries, the added mass tensor can be simplified significantly. Summing up, it is necessary to consider the additional effect (force) resulting from the fluid acting on the structure when formulating the system equation of motion but this additional effect is not an effective mass. [Additional Source]

b. ***TRUE***, it is necessary to consider the additional inertia of the fluid, given that the density of the fluid and the robot is the same. Consequently, these reaction forces, known as added mass contributions, must be taken into account. It is recommended that this contribution be disregarded in the case of a fluid as air, given that its low density renders it a relatively insignificant factor.

c. ***TRUE***, the damping effect is beneficial in stability analysis, as it introduces a positive definite kinetic energy term that facilitates the demonstration of the negativity of the time derivative of the Lyapunov candidate function.

d. ***FALSE***, it is true that the ocean current is usually considered as constant but it is expressed in the world frame; the related effect can be incorporated into the dynamic model of a rigid body moving in a fluid by considering the relative velocity in the body-fixed frame during the derivation of the Coriolis, centripetal, and damping terms. The ocean currents is also irrotational and are mainly caused by

- tidal movements
- atmospheric wind system over the sea
- heat exchnage at the sea's surface
- salinity change
- Coriolis forces due to the Earth's rotation

3 Third Exercise

Firstly, the code should be completed in the following manner.

```
%%
% Considering the matrices for the QP obtained from function fcn_get_QP_form_eta, use the QP solver qpSWIFT to
%  solve the quadratic problem with the following form
%  min. 0.5 * x' * H *x + g' * x
%  s.t. Aineq *x <= bineq
%      Aeq * x <= beq
%
% The result of the QP problem should be stored in a variable called zval in order to be used in the following

[zval,basic_info,adv_info] = qpSWIFT(sparse(H),g,sparse(Aeq),beq,sparse(Aineq),bineq);

```

Figure 1: Code point 3.a

The quadruped simulation allows for the testing of six different gaits:

- **trot**, is the only gait with the stance phase and it has an alternative phase of motion (LF with RH and RF with LH) with a major time of ground contact.
- **bound**, it has an alternative phase of motion (LF with RF and RH with LH), the worst motion tracking and some high oscillations in the velocity plot.
- **pacing**, it has an alternative phase of motion (LF with LH and RF with RH) and it has the major oscillations in the y axis of the velocity plot and some important oscillations on the angular velocity plot.
- **gallop**, it has the less ground contact time, with high value of Fz and the high oscillations in the velocity plot. It has an oscillation in the position plot and is the less stable gait.
- **trot run**, it has a big spike on the Fz plot, with low oscillations on the x axis of the velocity plot and a good tracking of heading velocity.
- **crawl**, it has an alternative phase of motion (LF, RF, LH and RH) and is the best gait for tracking position and tracking of heading velocity. It is the more stable gait.

The following behaviours are common:

- Modifying the reference z height of CoM of a $\pm 10\%$ resulted in the robot losing tracking and falling.
- Modifying the gravity with the value of the moon resulted in the robot falling.

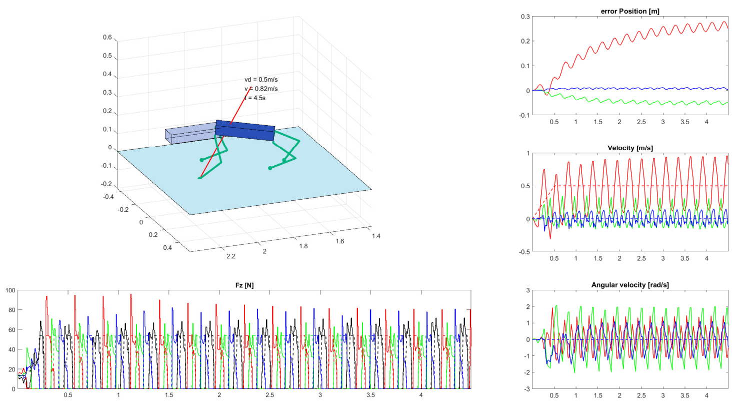

Figure 2: Gait 3: $\mu = 0.75$

- Modifying the friction value resulted in a change in the evolution of ground reaction forces, which was unstable and, in certain cases, resulted in the falling of the robot.
- An increase in the desired velocity results in a proportional increase in the amplitude of the ground reaction forces and in angular oscillations.

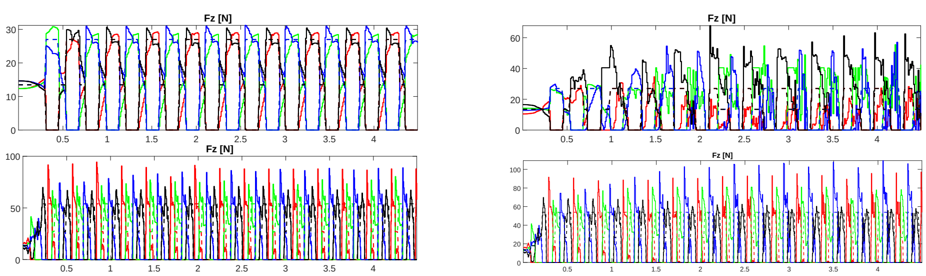

Figure 3: First row: Gait 0 with base and Veld=[0.5;0.75]; second row: gait 3 with base and Veld=[1;0]

3.1 Velocity Desired in Gait 5

A noteworthy phenomenon emerges when the desired velocity in gait 5 exceeds 1.4 m/s. This is characterised by a phase of the movement where only two legs are on the ground, rather than the usual three (more like a gallop). *Video in `Homework4/quadruped_simulation/Screenshot/5-veld-1.4-0/test.mp4`*
The desired velocity of 2.0 m/s results in a change of phase in the movement, with a period during which only one leg is on the ground. *Video in `Homework4/quadruped_simulation/Screenshot/5-veld-2-0/test.mp4`*

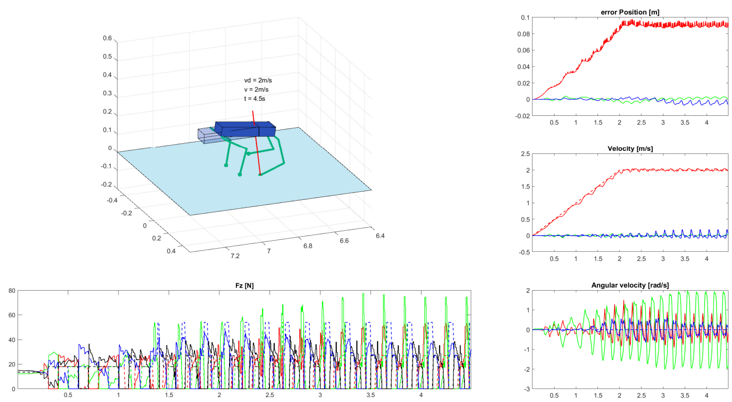

Figure 4: Gait 5: $vel_d = [2.0;0]$ m/s

The previous analysis of the amplitude of the ground reaction forces remains valid, and with this velocity, the ground reaction forces become more impulsive. Additionally, the amplitude of the angular velocity is augmented by a factor of four, particularly on the y-axis.

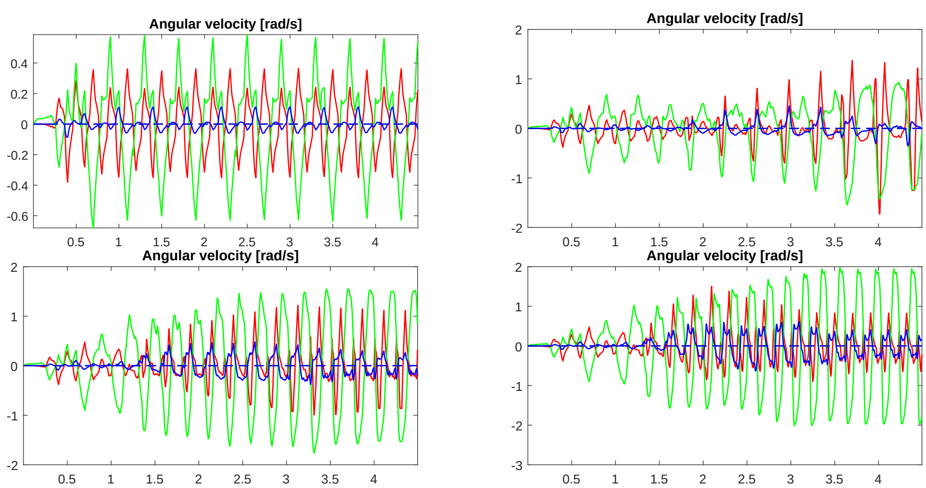

Figure 5: Gait 5 - Angular Velocity with vel_d: [0.5;0.0],[0.75;0.0],[1.4;0.0],[2.0;0.0] (from L to R)

3.2 Mass

Modifying the mass of the robot results in a distinct behavioural change.

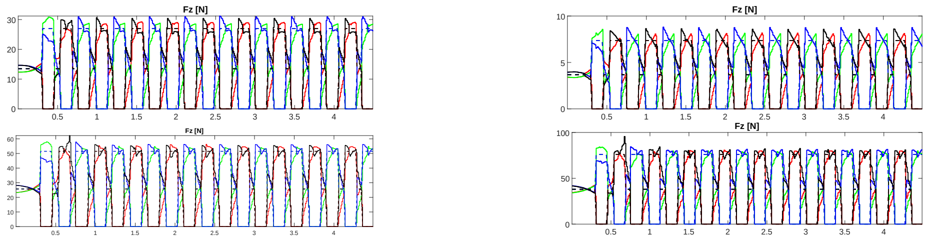

Figure 6: Gait 0 - GRFs with 5.5Kg, 1.5Kg, 10.5Kg, 15.5Kg (from L to R)

In this instance, it can be observed that the ground reaction forces exhibit a change in amplitude as a consequence of an increase in the mass of the robot, which also affects the performance of the controller.

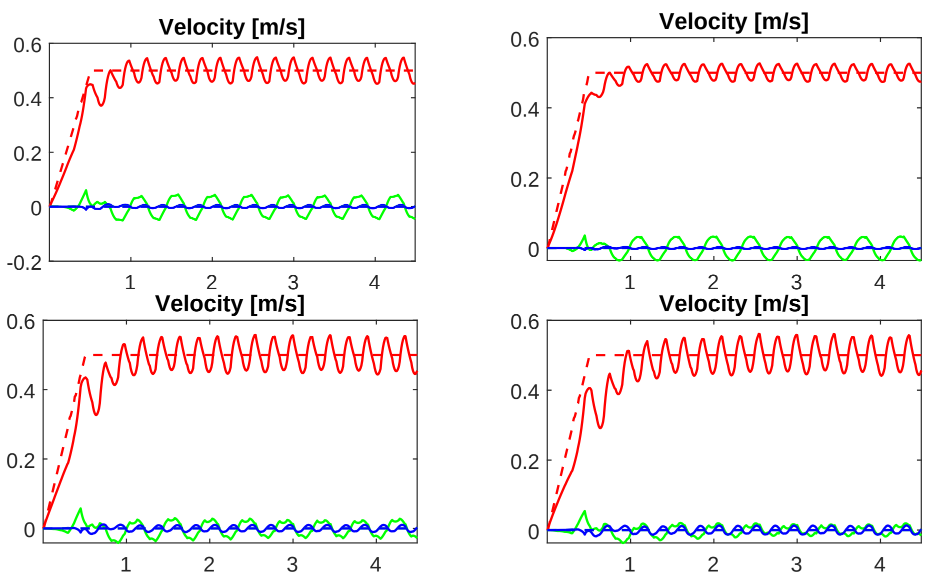

Figure 7: Gait 0 - Velocity with 5.5Kg, 1.5Kg, 10.5Kg, 15.5Kg (from L to R)

As can be observed, the transient response time increases in direct proportion to the mass. It can be noted that the oscillations of the velocity on the z-axis increase with the mass, while the oscillations of the velocity on the y-axis decrease. In this case, can be concluded that with an increase in mass, the stability of the robot is enhanced. This is corroborated by the angular velocity plot. The same behaviour cannot be observed in all gaits. Ad example, in gait 4 and in gait 5, there are no benefits in increasing the weight, instead, the amplitude of the angular velocity increases with a worst stability.

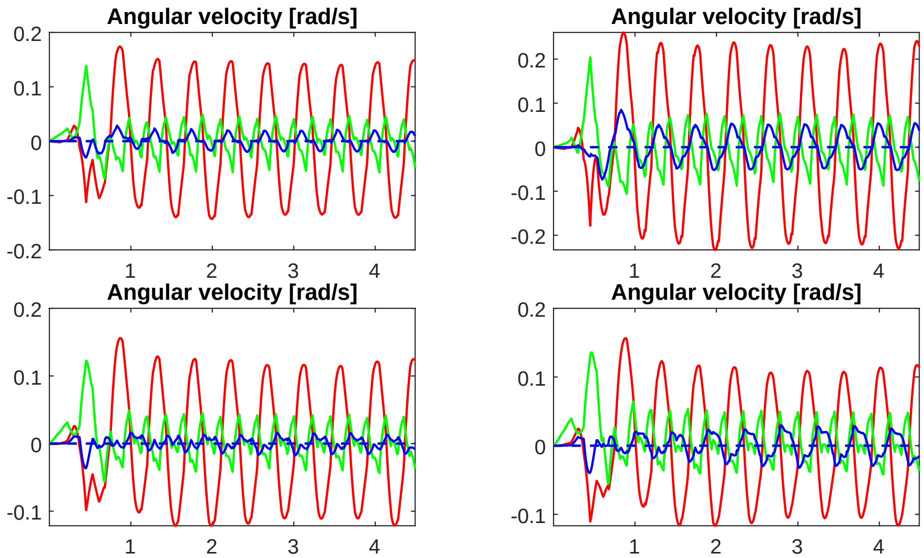

Figure 8: Gait 0 - Angular Velocity with 5.5Kg, 1.5Kg, 10.5Kg, 15.5Kg (from L to R)

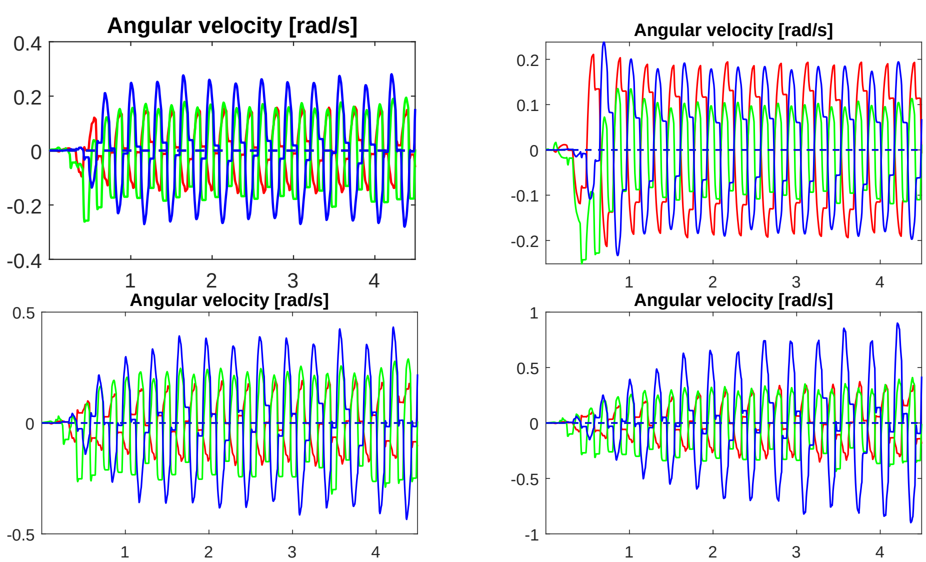

Figure 9: Gait 4 - Angular Velocity with 5.5Kg, 1.5Kg, 10.5Kg, 15.5Kg (from L to R)

4 Fourth Exercise

a. In the absence of an actuator at the point P, the system is unstable at the angle θ. The point $\theta_0 = \pi/2$ represents an unstable equilibrium point, and a small perturbation can result in the loss of this point. The system can be described as an inverse pendulum.

b. In light of the below axis definition, we may now proceed with the analysis. The ZMP formula can be expressed as follows:

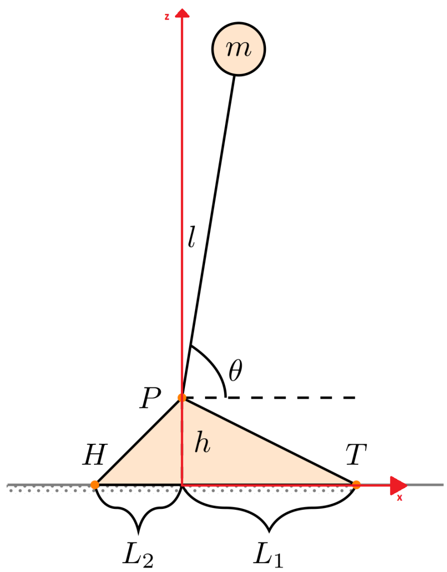

Figure 10: Axis

$$ZMP_x = p_c^x - \frac{p_c^z}{\ddot{p_c^z} - g_0^z}(\ddot{p_c^x} - g_0^x) - \frac{1}{m(\ddot{p_c^z} - g_0^z)}\dot{(L)}^y$$

with:

- $p_c^x = lcos(\theta)$
- $p_c^z = h + lsin(\theta)$
- $\ddot{p_c^x} = -l\ddot{\theta}sin(\theta) - l\dot{\theta}^2 cos\theta$
- $\ddot{p_c^z} = l\ddot{\theta}cos(\theta) - l\dot{\theta}^2 sin\theta$
- $g_0^x = 0$
- $g_0^z = -g = -9.81$
- $\dot{L}^y = m(l^2 + h^2)\ddot{\theta}$

become:

$$ZMP_x = lcos(\theta) - \frac{h + lsin(\theta)}{l\ddot{\theta}cos(\theta) - l\dot{\theta}^2 sin\theta + g}(-l\ddot{\theta}sin(\theta) - l\dot{\theta}^2 cos\theta) +$$

$$-\frac{1}{m(l\ddot{\theta}cos(\theta) - l\dot{\theta}^2 sin\theta + g)} m(l^2 + h^2)\ddot{\theta} =$$

$$= lcos(\theta) - \frac{h + lsin(\theta)}{l\ddot{\theta}cos(\theta) - l\dot{\theta}^2 sin\theta + g}(-l\ddot{\theta}sin(\theta) - l\dot{\theta}^2 cos\theta) - \frac{(l^2 + h^2)\ddot{\theta}}{(l\ddot{\theta}cos(\theta) - l\dot{\theta}^2 sin\theta + g)} =$$

$$= lcos(\theta) + \frac{-(h + lsin(\theta))(-l\ddot{\theta}sin(\theta) - l\dot{\theta}^2 cos\theta) - (l^2 + h^2)\ddot{\theta}}{l\ddot{\theta}cos(\theta) - l\dot{\theta}^2 sin\theta + g} =$$

$$= lcos(\theta) + \frac{(h + lsin(\theta))(l\ddot{\theta}sin(\theta) + l\dot{\theta}^2 cos\theta) - (l^2 + h^2)\ddot{\theta}}{l\ddot{\theta}cos(\theta) - l\dot{\theta}^2 sin\theta + g}$$

c. In the case $\dot{\theta} = 0, \ddot{\theta} = 0$ the ZMP formula is simplified in that way:

$$ZMP_x = p_c^x - \frac{p_c^z}{0 - g_0^z}(0 - g_0^x) - \frac{1}{m(0 - g_0^z)} \times 0 =$$

$$= p_c^x - \frac{p_c^z}{0 - g_0^z}(0 - g_0^x) =$$

$$= p_c^x - \frac{p_c^z}{g_0^z}(g_0^x) =$$

since:

- $p_c^x = lcos(\theta)$
- $p_c^z = h + lsin(\theta)$
- $\ddot{p}_c^x = 0$
- $\ddot{p}_c^z = 0$
- $g_0^x = 0$
- $g_0^z = -g = -9.81$
- $\dot{L}^y = 0$

So, at last, we have:

$$ZMP_x = p_c^x = lcos(\theta)$$

In this case, the ZMP represents the CoM. Therefore, if the ZMP is positioned outside the limits of the support polygon, the robot will fall. In this case, the limits of the support polygon are L1 (for H) and L2 (for T). It is necessary to calculate the angle θ since the projection of the mass is in a position with a length greater than L2 or L1:

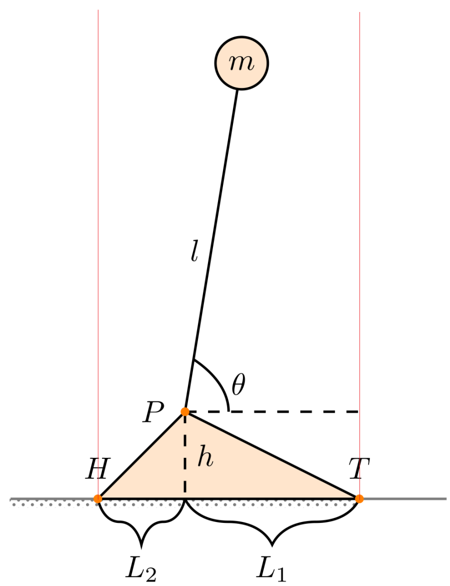

Figure 11: Limits

- for the point H $\longrightarrow ZMP_x = lcos(\theta) = -L2 \iff \theta_2 = cos^{-1}(\frac{-L2}{l})$
- for the point T $\longrightarrow ZMP_x = lcos(\theta) = L1 \iff \theta_1 = cos^{-1}(\frac{L1}{l})$

In considering the frame as a point, the relation can be written as follows:

$$\theta \in (\theta_2, \theta_1) = \left(cos^{-1}\left(\frac{L1}{l}\right), cos^{-1}\left(\frac{-L2}{l}\right)\right)$$

for not falling down.