



Design variables (Parameters to be updated) are the hyperparameters (E, N1, N2, F1, F2, S1, S2, and H1) of SCNN model.

- Encode representation of a particle: we represent a particle by an array or vector in which each element corresponds to one of the design variables (E, N1, N2, F1, F2, S1, S2, and H1) for SCNN.
- The objective function, denoted as $fitness(N1, F1, S1, H1, N2, F2, S2)$, represents the mathematical expression that measures the classification accuracy of the groundwater quality classification model based on the given hyperparameters for the Spatial Convolutional Neural Network (SCNN).
- we have a multi-class classification problem, we commonly used objective function is the categorical cross-entropy loss.

$$fitness(N1, F1, S1, H1, N2, F2, S2) = -\frac{1}{N} \sum_{i=1}^N \sum_{c=1}^C y_{ic} \cdot \log(p_{ic})$$
 where $p_{ic} = SCNN(N1, F1, S1, H1, N2, F2, S2)(X_i)$

we use using categorical cross-entropy loss (multiclass) not binary cross-entropy loss (2 classes)

where N is the number of data samples, y_{ic} is an indicator that is 1 if the true label of the i th sample is in class c (e.g., 1 for "Excellent" if the true label is "Excellent"), and 0 otherwise. $SCNN(N1, F1, S1, H1, N2, F2, S2)$ use the $SCNN$ model trained using (E, N1, F1, S1, H1, N2, F2, S2) hyperparameters to predict the categorical labels of the sample X_i in dataset.

The fitness function is the categorical cross-entropy loss of SCNN model $fitness(N1, F1, S1, H1, N2, F2, S2) = fitness(N1, F1, S1, H1, N2, F2, S2)$

Evolution of Pbest and gbest:

- Update Personal Best (pbest):** For each particle i , compare its fitness to its personal best fitness (p_{best}_i).
 - If the fitness of the current position is better than the previous personal best, update p_{best}_i with the current position and fitness.
 - If $fitness_i < p_{best}_i$, then $p_{best}_i = fitness_i$ and $p_{best}_i = [N1_i, F1_i, S1_i, H1_i, N2_i, F2_i, S2_i]$
- Update Global Best (gbest):** Compare the fitness of all particles in the current iteration t .
 - If this particle has a better fitness than the current global best fitness ($gbest$), update $gbest$ with the fitness and position of the best particle.
 - If $min(fitness_i) < gbest$, then $gbest = min(fitness_i)$ and $gbest = [N1_i, F1_i, S1_i, H1_i, N2_i, F2_i, S2_i]$
- Stopping conditions:**
 - The algorithm is terminated when the **convergence criteria** are met or the maximum iteration **maxIter** is reached.
- Convergence criteria:**
 - If $fitness > ConsecutiveIterations$ (example 50):
 - Train $SCNN = SCNN(gbest, N1_i, F1_i, S1_i, H1_i, N2_i, F2_i, S2_i)$
 - Validation Loss $ValidationLoss$ using $SCNN_{gbest}$ on $X_{validation}$ and $Y_{validation}$
 - If $ValidationLoss$ doesn't improve by more than 0.001 over $ConsecutiveIterations$ iterations or (or the maximum iteration **maxIter** is reached) Terminate PSO

