

$$\begin{aligned}
\log P(S) &= \int_R f(y) dy \\
\frac{\partial}{\partial \theta} \log P(s) &= \frac{\partial}{\partial \theta} \log \int_R \varphi(y; \theta, \Sigma) dy = \frac{1}{P(S)} \frac{\partial}{\partial \theta} P(S) \\
\frac{\partial}{\partial \theta} P(S) &= \int_R (\Sigma^{-1} y - \Sigma^{-1} \theta) \varphi(y; \theta, \Sigma) dy \\
&= P(S) (\Sigma^{-1} E(y|S) - \Sigma^{-1} \theta) \\
\Rightarrow \frac{\partial}{\partial \theta} \log P(s) &= \Sigma^{-1} E(y|S) - \Sigma^{-1} \theta \\
\frac{\partial^2}{\partial \theta^2} \log P(S) &= -\Sigma^{-1} + \frac{\partial}{\partial \theta} \Sigma^{-1} E(y|S)
\end{aligned}$$

$$\begin{aligned}
\frac{\partial}{\partial \theta} E(y|S) &= \int_R \frac{y}{P(S)^2} (P(S)(\Sigma^{-1} y - \Sigma^{-1} \theta) \varphi(y; \theta, \Sigma) - \varphi(y; \theta, \Sigma) P(S)(\Sigma^{-1} E(y|S) - \Sigma^{-1} \theta)) \\
&= \int_R f(y|S) y (\Sigma^{-1} y - \Sigma^{-1} E(y|S)) dy = \text{Var}(y|S) \Sigma^{-1} \\
\Rightarrow \frac{\partial^2}{\partial \theta^2} \log P(S) &= \text{Var}(\Sigma^{-1} y|S) - \Sigma^{-1}
\end{aligned}$$