Introductory meteorology – Modellus activity

Gradient wind theory

The dynamics of atmospheric air is determined applying Newton's laws to each air particle. One of the simplest solutions of the equations of motion can be obtained when we consider steady state circular horizontal and frictionless atmospheric flow. In the northern hemisphere the balance between the pressure gradient force, the Coriolis force and the centrifugal force is given, respectively for low-pressure and high-pressure systems, by

$$-fv + \frac{1}{\rho} gradp - \frac{v^2}{R} = 0 \qquad (1)$$

and

$$fv - \frac{1}{\rho} gradp - \frac{v^2}{R} = 0 \qquad (2)$$

where v is the speed of the air particle, R is the curvature of the air particle trajectory, ρ is the air density, $gradp$ is the magnitude of the pressure gradient and f is the Coriolis parameter defined as $f = 2\omega_T \sin(\varphi)$ with ω_T denoting the Earth's rotation angular velocity and φ the latitude angle. At large scale synoptic low and high pressure systems, the natural solution of Eq. 1 is

$$v = \frac{R}{2}\left(-f + \sqrt{f^2 + \frac{4gradp}{R\rho}}\right) \qquad (3)$$

and the natural solution of Eq. 2 is

$$v = \frac{R}{2}\left(f - \sqrt{f^2 - \frac{4\, gradp}{R\rho}}\right). \qquad (4)$$

The solutions given in Eq. 3 and Eq. 4 describe the gradient wind approximation in which the air blows parallel to the isobars leaving the high pressures on the right (Buy-Ballot law). In this activity you will calculate the velocity and trajectory of an air particle around a low pressure and a high pressure system using the Modellus software. In addition you will analyze the force-balance that governs the gradient wind approximation.

PART I

Gradient wind motion (low pressure system)

1. Place the pressure chart image in the animation area.

In the Modellus menu bar top line select the **Objects** ribbon and then the **Image** icon. A left mouse click on the animation area will display the object definitions panel (see Fig. 1). Browse your computer to give the path to the image “PressureChart.jpg”. Set the horizontal and vertical scales to 0.2. Adjust the image to the animation area by dragging the image’s left lower corner.

2. Write the **Mathematical Model** equations

Write the expressions for the velocity (Eq. 3) and for the trajectory of the wind using polar coordinates $r = R$ and $\theta = \omega t$, where $\omega = v/R$ is the gradient wind angular velocity. Program both the x and y components of the velocity and trajectory (see Fig. 2).

3. Specify the parameters

Using the **Parameters** ribbon specify the values of the constants $R =$ 4.0E5 (in meters), f=1.031E-4 (in rad/s), $gradp$ =1.0E-3 (in Pa/m) and rho =1.290 (in kg/m^3).

4. Define the independent variable

Using the **Independent variable** ribbon chose time t as the independent variable. Specify the time Step as 1000.0 (in seconds) and define the time interval between 0 and 1.0E6 (see Fig. 2).

5. Place a particle object in the low pressure system

Select the **Object** ribbon and chose the **Particle** object. Use the mouse to insert the particle in the animation area and to place it halfway between the 996 and 992 mb isobars. While doing this you will see the particle object ribbon (see Fig. 1)

6. Place a vector object on top of the air particle

Select the **Object** ribbon and chose the **Vector** object. Use the mouse to place the vector on top of the particle. Assign the horizontal and vertical coordinates of the vector to the x and y components of the velocity and use 5.0 as the scale value. Finally, attach the vector to the particle using the "*attach object to*" option in the vector object definition panel (see Fig. 1).

7. Place a variable object at the bottom of the figure

Select the **Object** ribbon and chose the **Variable** object. Use the mouse to place this object in the workspace at the bottom of the pressure chart image (see Fig. 2). In the variable box select the variable velocity (v) (see Fig. 1)

8. Run the model

To run the model click on the **Play** button at the lower left corner of the Modellus window. The velocity and position of the air particle are computed at each time step throughout the specified time interval. You should see the particle describing a circular anti-clockwise motion around the low pressure system.

Gradient wind force-balance (low pressure system)

The acceleration vectors corresponding to the Coriolis force ($\vec{a}_{\mathrm{Cor}}$), the pressure gradient force ($\vec{a}_{\mathrm{p}}$) and the centrifugal force ($\vec{a}_{\mathrm{c}}$) are given by,

$$\vec{v} = v\overrightarrow{u_\theta}\,, \qquad \vec{a}_{Cor} = fv\overrightarrow{u_r}\,, \qquad \vec{a}_p = -\frac{gradp}{\rho}\overrightarrow{u_r}\,, \qquad \vec{a}_c = \frac{v^{\,2}}{R}\overrightarrow{u_r} \tag{5}$$

where $\overrightarrow{u_r} = (x, y)/R$ and $\overrightarrow{u_\theta} = (-y, x)/R$ are unit vector in polar coordinates. In the next exercise you will write the x and y components of these acceleration vectors. Next you will display them in connection with the air particle in the low pressure system.

1. Write the **Mathematical Model** equations

The x and y components of the acceleration vectors can be written as the product of the expressions in Eq. (5) by $x/R\,, y/R\; or\; -y/R$. For example, to get the x component of the pressure gradient acceleration write $\mathrm{Ap\_x} = -gradp/\rho \times x/R$. After writing the components of each vector write also the expression to compute the vector's modulus (see Fig. 2).

2. Place vector objects on top of the air particle

Select the **Object** ribbon and chose the **Vector** object. Proceed as in point 7 of the previous exercise to connect the Coriolis acceleration vector to the air particle. Repeat the procedure for the pressure gradient and the centrifugal acceleration vectors.

3. Place variable objects at the bottom of the figure

Select the **Object** ribbon and chose the **Variable** object. Use the mouse to place this object in the workspace at the bottom of the pressure chart image and assign it to the Coriolis acceleration modulus. Repeat the procedure for the pressure gradient and centrifugal acceleration modulus (see Fig. 2).

4. Run the model

Click on the **Play** button. You should see the vectors moving around the low pressure system in connection with the air particle.

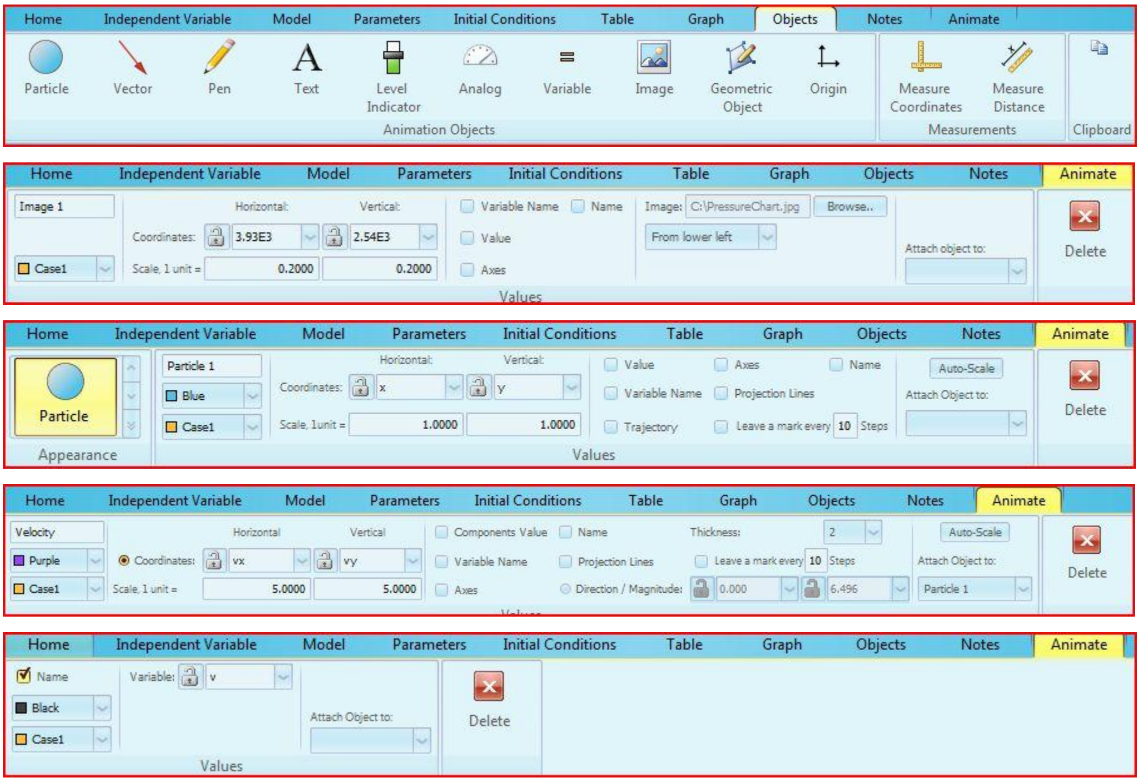

Fig. 1 The Object ribbon, at the top, displays the several kinds of objects available. The Image, the Particle, the Vector and the Variable panels shown below are displayed upon selection of the corresponding object. During the exercise fill the boxes and select the options as shown.

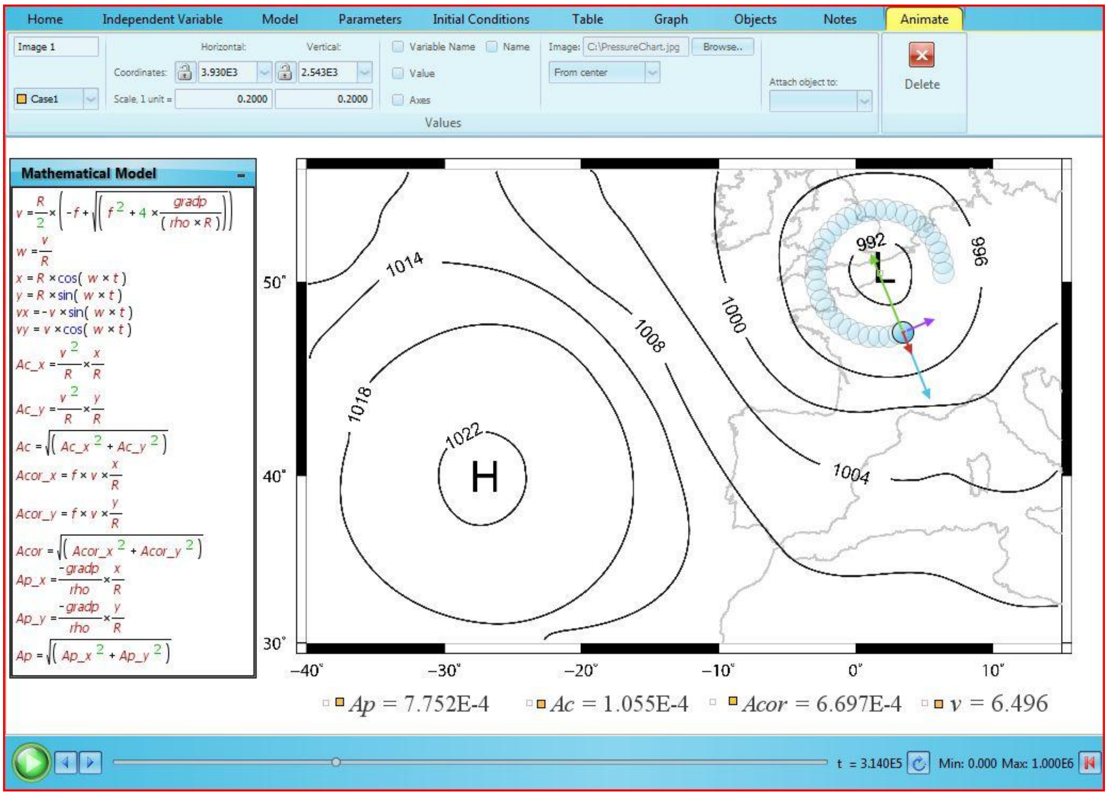

Fig. 2 Place the pressure chart image in the workspace area as shown. The Mathematical Model window shows how to write the *x* and *y* components of the velocity and the accelerations vectors. In the end of Part I exercise the particle's trajectories and the force vectors in balance should look as depicted (the green vector represents the pressure gradient force, the blue vector the Coriolis force and the red vector the centrifugal force).

PART II

Gradient wind motion (high pressure system)

In this part you will program and display the gradient wind solution for a high pressure system. You should follow the same steps as in part I with the following modifications:

1. Take into consideration that the natural solution of the gradient wind approximation in a high pressure system is given by Eq. 4. Change the mathematical model accordingly. Change also the Coriolis parameter value (f) since the high pressure system is now centred at the latitude of 40° N.

2. Insert an air particle in the high pressure system between the 1018 and 1022 mb isobars. Estimate the radius of curvature of this particle's trajectory.

3. Estimate the magnitude of the pressure gradient (pressure difference between two adjacent isobars divided by the distance between them). Replace the R and $gradp$ parameters by these new estimates remembering that the pressure gradient in the high pressure has the opposite direction (and sign) to the pressure gradient in the low pressure system.

4. Run the model and stop the animation at time t=3.14E5 s (this is the time frame of Fig. 2)

5. Select the **Object** ribbon and chose the **Clipboard** option to copy an image of the animation showing your model results (it should look similar to Fig.2). Past it below into the blue Multimedia Comments area of this document.

Multimedia Comments:

6. Compare the low pressure and high pressure solutions and answer to the questions: a) What is the direction of wind motion around a high and low pressure system in the northern hemisphere? b) What are the differences in the force-balance between the two systems? c) Which air particle moves faster? Why? Write the answers in this document in the blue Answers box below and save it.

Answers: