Minimum Spanning Tree Algorithm (MST)

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Outline

Introduction

Algorithm explanation (Kruskal)

Applications [1] and Problem

- Usually relates to optimizations in network design
  - telephone
  - electrical
  - hydraulic
  - TV cable
  - road
- Also indirect applications
  - learning salient features for real-time face verification
  - max bottleneck paths

[1] https://www.geeksforgeeks.org/applications-of-minimum-spanning-tree/

Conditions

- graph G with *positive* edge weights
- *undirected* edges
- find a min weight set of *edges* that connects all of the *vertices*
- number of *edges* we want:

```
edges = number of vertices - 1
```

Few algorithms that implement MST [2]

1. Kruskal's algorithm
2. Prim's algorithm
3. Boruvka's algorithm

- Why did Kruskal?
  - easier to code and understand

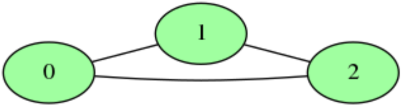
[2] https://www.ics.uci.edu/~eppstein/161/960206.html

Overview

1. order *edges* by weight (ascendant)
2. pick edge and check if it forms a cycle with the spanning tree
   - if cycle formed then discard edge
   - else include edge
3. repeat 2) until `e = v - 1`

How to check if graph has cycle?

- Union-find[3] algorithm
  - temporary vector to save parent of vertices being inserted



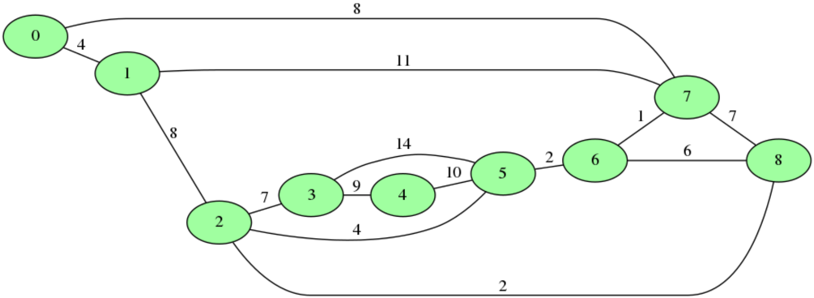
0	1	2
-1	-1	-1

0	1	2
1	-1	-1

0	1	2
1	2	-1

[3] https://www.geeksforgeeks.org/union-find/

Step by step with example[4]

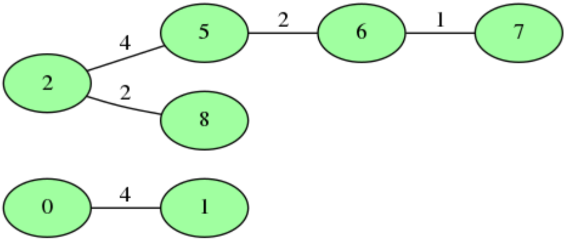


[4] https://www.geeksforgeeks.org/greedy-algorithms-set-2-kruskals-minimum-spanning-tree-mst/

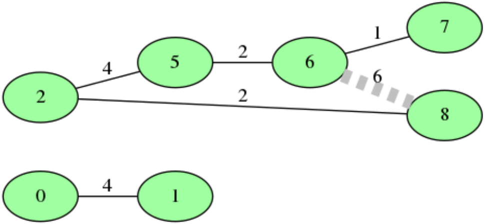
Graph sorted by edges

weight	src	dst
1	7	6
2	8	2
2	6	5
4	0	1
4	2	5
6	8	6
7	2	3
7	7	8
8	0	7
8	1	2
9	3	4
10	5	4
11	1	7
14	3	5

pick edges from sorted and try to include each one

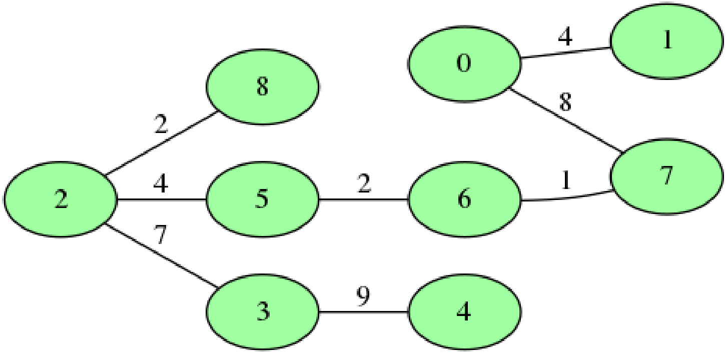


. . . no cycles so far.



. . . including 8-6 produces a cycle. Do not include it.

repeat until: `edges = vertices - 1`



Complexity

summing up:

1. sort graph ascendant (qsort $\rightarrow$ O(nlogn))
2. apply MST (Kruskal) algorithm
   2.1 for *each edge* in sorted list:
      2.1.1 include it in MST graph
      2.1.2 check if *formed cycle*; if so remove it
      (find-union algorithm)

Kruskal complexity

- The find-union is O(n) in the *present work*
  - we could improve it to O(logn) using *union by Rank or Height*[5]

[5] https://www.geeksforgeeks.org/union-find/

Complexity

Consider the worst case

- Everytime we add a new vertice in a graph, we can have:

`edges + = number of vertices - 1.`

- Thus, number of edges in the worst case is:

$$\text{edges} = v^2$$
$$\text{when edges} \rightarrow \infty$$

This implementation:

$$O(e \times \log e) + O(e \times O(v))$$
$$\mathrm{O}(\mathrm{e} \times \log e + v^2 \times v)$$
$$\mathrm{O}(\mathrm{e} \times \log v + v^3)$$

Complexity

Implementation with: `union-find = O(logn)`

$$O(e \times \log e) + O(e \times \log v)$$
$$\mathrm{e} = \mathrm{v}^2 \to \log e = \log v^2 = 2 \times \log v \approx \log v$$
$$\therefore$$
$$\mathrm{O}(\mathrm{e} \times (\log v + \log v)) = O(e \times 2 \times \log v)$$
$$\therefore$$
$$\mathrm{O}(\mathrm{e} \times \log v)$$

Code

Structs

- grafo
- vertice
- lista
- nó

New functions

- hasCycle()
- union()
- find()

Conclusion

- Instead of creating new *structs* or *files*, I add *members* to structs and add new *functions* to files

How to avoid checking double vertice addition without O(n)?

- With this issue, the complexity has one more v factor:

$$O(e \times \log v + v^4)$$

Kruskal algorithm is *much easier* to implement than other

- Tradeoffs: Complexity vs. Time of coding