CGT 520 : Matrix-vector Ops

- Dot product
- Matrix-vector multiplication
- See Week 1 notes for other operations, description of *scalar*, *vector*, *matrix*

- *We will use libraries to compute these things, but to understand how the machine works you need to think like the machine.*
- *So we will work some math problems...*

Dot product

- Sometimes called “inner product” or “scalar product”
- Operates on two **vectors**
- The result is a **scalar**
  - In 3D:

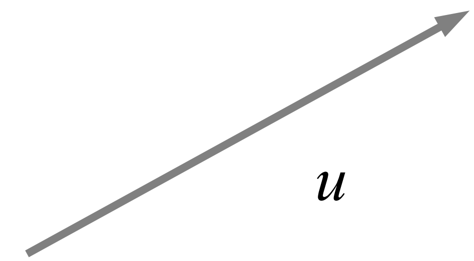
$$u = \begin{bmatrix} u_x \\ u_y \\ u_z \end{bmatrix}, \quad v = \begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix}$$

$$u \cdot v = u_x v_x + u_y v_y + u_z v_z$$

Dot product uses

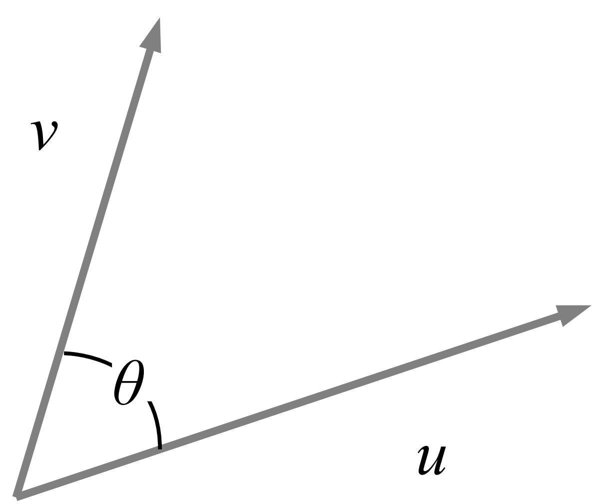
- Can use it to compute vector magnitude (length)
  - $\|u\| = \sqrt{u_x^2 + u_y^2 + u_z^2}$

    $\|u\| = \sqrt{u \cdot u}$



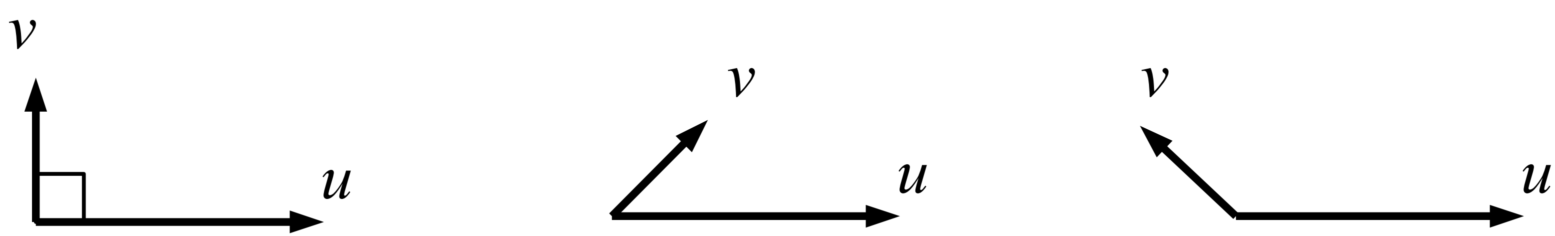
- Can use it to compute angles
  - $u \cdot v = \|u\| \|v\| \cos\theta$

    $\cos\theta = \frac{u \cdot v}{\|u\| \|v\|}$



Dot product uses

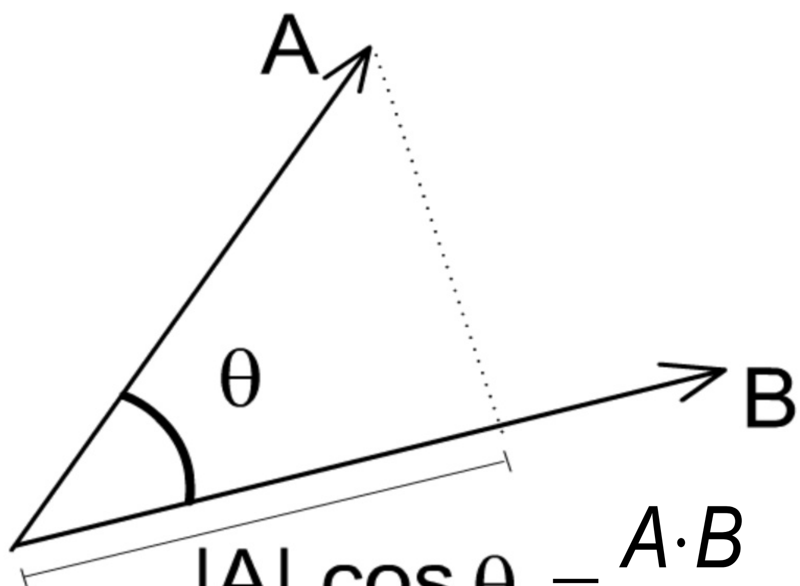
- Relative orientation of vectors



$u \cdot v = 0$ $u \cdot v > 0$ $u \cdot v < 0$

Dot product uses

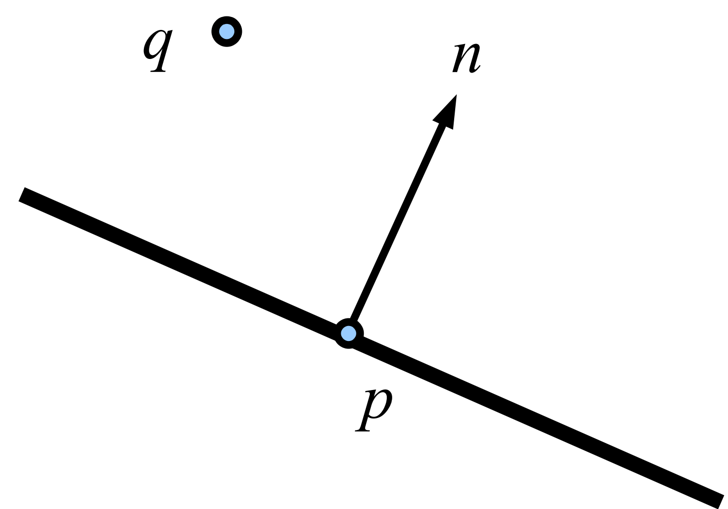
- Projections
  - How long is the “shadow” that A casts onto B?



$$|A| \cos\theta = \frac{A \cdot B}{|B|}$$

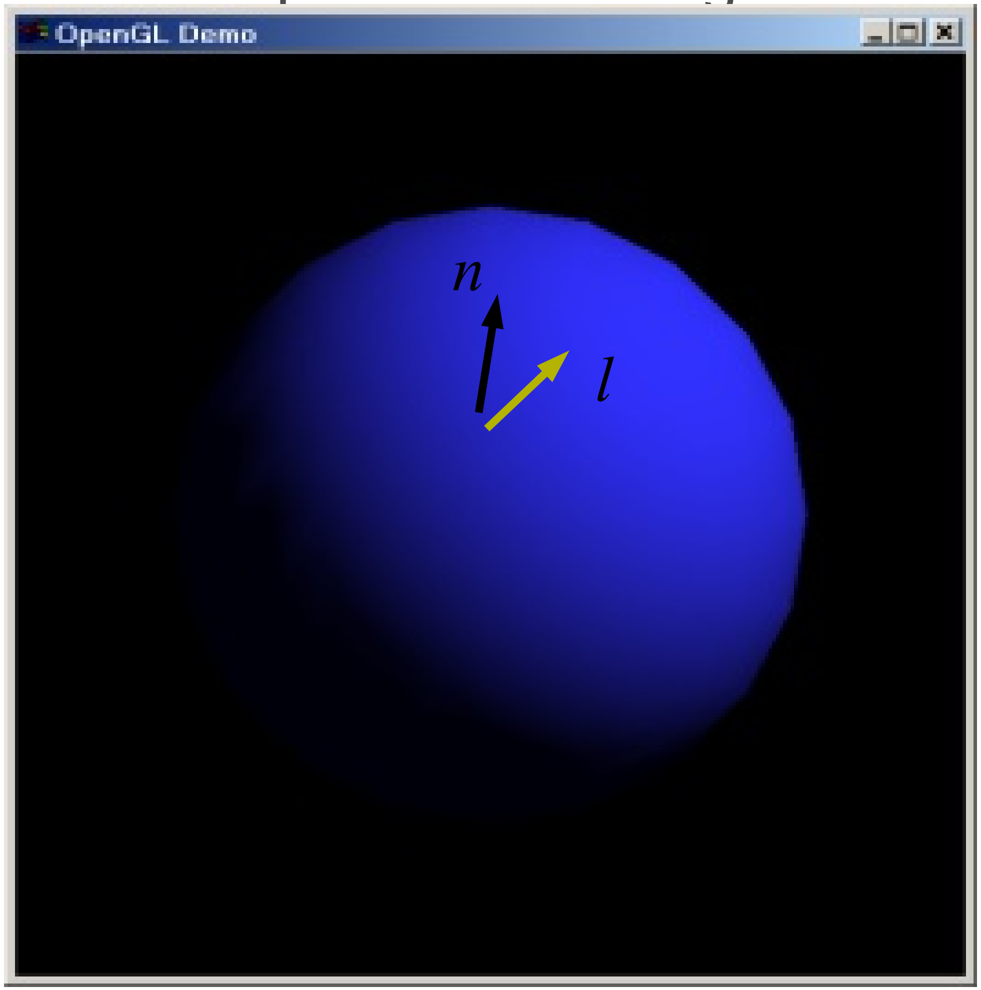
Dot product uses

- Collision detection
  - A plane can be characterized by a point on the plane, p, and the normal vector, n (perpendicular to the plane)
  - What side of the plane is point q on?
  - Depends on sign of $(q-p)\cdot n$
    - if > 0 then q is on the side n points to
    - if < 0 then it is on the opposite side
    - if =0 then q is on the plane



Dot product uses

- Diffuse lighting
  - Vector l points toward light source



$$Intensity = max(0.0, n \cdot l)$$

Dot product

- What is $u \cdot v$ in each case?

$$u = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \quad v = \begin{bmatrix} 2 \\ 1 \\ 4 \end{bmatrix}$$

$$u = \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}, \quad v = \begin{bmatrix} 2 \\ 1 \\ -4 \end{bmatrix}$$

$$u = \begin{bmatrix} 0 \\ 2 \\ 3345 \end{bmatrix}, \quad v = \begin{bmatrix} 298 \\ 1 \\ 0 \end{bmatrix}$$

Dot product

- What is $u \cdot v$ in each case?

$$u = \begin{bmatrix} x \\ y \\ 0 \end{bmatrix}, \quad v = \begin{bmatrix} -y \\ x \\ 0 \end{bmatrix}$$

$$u = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad v = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

Matrix-vector multiplication

- The result is a vector

$$M = \begin{bmatrix} M_{11} & M_{12} & M_{13} \\ M_{21} & M_{22} & M_{23} \\ M_{31} & M_{32} & M_{33} \end{bmatrix}$$

$$Mv = \begin{bmatrix} M_{11} & M_{12} & M_{13} \\ M_{21} & M_{22} & M_{23} \\ M_{31} & M_{32} & M_{33} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} M_{11} v_1 + M_{12} v_2 + M_{13} v_3 \\ M_{21} v_1 + M_{22} v_2 + M_{23} v_3 \\ M_{31} v_1 + M_{32} v_2 + M_{33} v_3 \end{bmatrix}$$

Matrix-vector multiplication

- The result is a vector

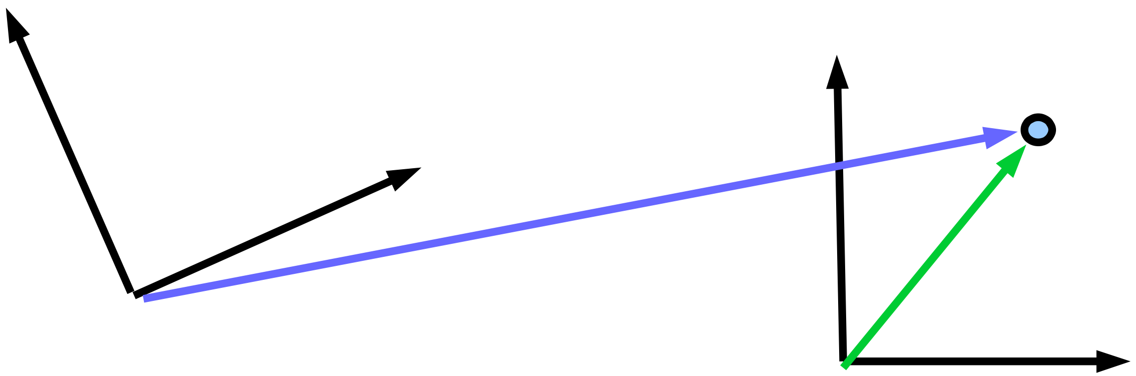
$$Mv = \begin{bmatrix} M_{11} & M_{12} & M_{13} \\ M_{21} & M_{22} & M_{23} \\ M_{31} & M_{32} & M_{33} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} M_{11}v_1 + M_{12}v_2 + M_{13}v_3 \\ M_{21}v_1 + M_{22}v_2 + M_{23}v_3 \\ M_{31}v_1 + M_{32}v_2 + M_{33}v_3 \end{bmatrix}$$

- It looks like 3 dot products

$$Mv = \begin{bmatrix} R_1 \\ R_2 \\ R_3 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} R_1 \cdot v \\ R_2 \cdot v \\ R_3 \cdot v \end{bmatrix}$$

Matrix-vector mult uses

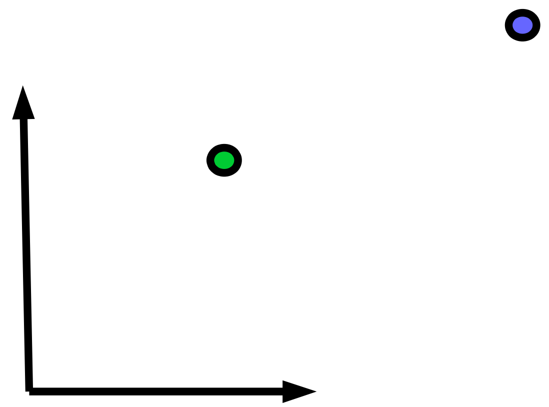
- Compute coordinates of a point in a different coordinate system
  - Projecting a point onto the basis vectors of another coordinate system



$$\boldsymbol{q} = \boldsymbol{M}\boldsymbol{p}$$

Matrix-vector mult uses

- Compute coordinates of a point after a geometric transformation
  - Can be considered as the same thing as the previous slide



$$\boldsymbol{q} = \boldsymbol{M}\boldsymbol{p}$$

Matrix-vector multiplication

- What is the result in each case?

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} =$$

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} =$$

$$\begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} =$$

Matrix-vector multiplication

- What is the result in each case?

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} =$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 2 \\ 5 & 1 & 2 \end{bmatrix}\begin{bmatrix} 2 \\ 5 \\ 1 \end{bmatrix} =$$

$$\begin{bmatrix} 1 & 0 & a \\ 0 & 1 & b \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} =$$

Practice for next week

- For Quiz 1
  - What is the result of the dot product of two vectors?
    - Scalar, vector or matrix?
  - Be able to evaluate the dot product and get a numerical or symbolic answer
  - What is the result of a matrix-vector multiplication?
    - Scalar, vector or matrix?
  - Be able to evaluate the matrix-vector mult and get a numerical or symbolic answer
  - Also topics from notes, textbook readings and lab
    - glut, OpenGL, graphics pipeline, etc...