ALGEBRA-COMPUTING SPECTRAL SEQUENCES

R. GIMÉNEZ CONEJERO

In the theory of singularities of maps, also called *Thom-Mather theory*, one of the commonly studied topics is the topology of the discriminants of maps and how they change after small perturbations. In close connection with this, the algebraic properties of the singularities and their deformations are often expressed in terms of certain *codimensions*, *versal unfoldings*, *bifurcations sets*, etc. Some modern references for this are [8] and [2, Chapter 2].

In the case that the dimension of the target space of a map germ, say p, is lower than the dimension of the source, say n, the discriminant coincides with the image. There is a tool to compute the homology of the image of a map that works in great generality, called the *Image-Computing Spectral Sequence* (ICSS). However, there is no such tool to help us to compute the algebra of the singularity of a map, something similar to an *Algebra-Computing Spectral Sequence* (ACSS).

The ICSS of a map $f : N \to P$ uses the *homological properties* of the multiple point spaces of f $D^k(f)$ to compute (to some extent) the homology of $f(N)$ and, for this reason, one hopes that there is an ACSS that uses the *algebraic information* of the multiple point spaces to compute the algebraic invariants we want. More precisely:

Problem 0.1. Find a spectral sequence that, for any map germ $f : (\mathbb{C}^n, 0) \to (\mathbb{C}^p, 0)$, computes the $\mathscr{A}_e$-codimension of f using algebraic information of the multiple point spaces $D^k(f)$.

The case of Problem 0.1 for corank one map germs should be simpler, since the multiple point spaces are isolated complete intersection singularities when the germ has finite $\mathscr{A}_e$-codimension. See [7, Proposition 2.14].

Problem 0.2. For corank one map germs $f : (\mathbb{C}^n, 0) \to (\mathbb{C}^p, 0)$, there is a spectral sequence that uses the Tjurina modules of the multiple point spaces $D^k(f)$ and computes the $\mathscr{A}_e$-codimension of f.

Some recommended references for the ICSS are [5, 6, 1] and [3, Chapter 2]. This problem was first stated in the Remark 7.7 of the related work [4].

References

[1] José Luis Cisneros-Molina and David Mond. Multiple points of a simplicial map and image-computing spectral sequences. *Journal of Singularities*, 24:190–212, 2022.

[2] José Luis Cisneros Molina, Lê Dũng Tráng, and José Seade, editors. *Handbook of geometry and topology of singularities III*. Springer Cham, 2022.

[3] R. Giménez Conejero. *Singularities of germs and vanishing homology*. PhD thesis, Universitat de València, 2021.

Key words and phrases. Singularities of maps, Thom-Mather theory, codimension, spectral sequence, computation.

[4] R. Giménez Conejero. Isotypes of ICIS and images of deformations of map germs. *arXiv:2207.05196*, July 2022.

[5] Victor Goryunov and David Mond. Vanishing cohomology of singularities of mappings. *Compositio Mathematica*, 89(1):45–80, 1993.

[6] Kevin Houston. A general image computing spectral sequence. In *Singularity theory*, pages 651–675. World Sci. Publ., Hackensack, NJ, 2007.

[7] Washington Luiz Marar and David Mond. Multiple point schemes for corank 1 maps. *Journal of the London Mathematical Society. Second Series*, 39(3):553–567, 1989.

[8] David Mond and J. J. Nuño-Ballesteros. *Singularities of mappings*, volume 357 of *Grundlehren der mathematischen Wissenschaften*. Springer, Cham, 2020.

Email address: `roberto.gimenez@uam.es`