Topic: Risk-Management Strategies from an Evolutionary Game Theory Perspective

Risk-taking is an important part of strategic interactions; objective is to incorporate the study of risk-taking behavior into the game theory framework. Consider the case of the iterated Prisoners Dilemma, where players choose their desired risk level for each individual encounter. Categorize resulting strategies and determine Nash equilibria. This model generalizes the current IPD framework: the standard model can be viewed as a special subcase where there is only 1 risk-level choice for each individual encounter.

Alternative Interpretation: Trust formation instead of risk management.

Players explore each other's trustworthiness in a combination of low-stakes and high-stakes settings; how should players tradeoff exploration of the counterparty's trustworthiness and exploitation of that knowledge?

Question

- Will new strategies appear in this expanded framework, or will it simplify to single-game TfT or WSLS?
- Will the threshold for cooperation be the same as that of the low-stakes game, the high-stakes game, or will it be a function of the two?

Model Overview

Consider a binary option of a high-risk Game 1 enviroment or a low-risk (regular) Game 2 environment. If both players opt for a high-risk environment, they play Game 1 where the reward structure magnifies the benefit of cooperation - higher rewards for mutual cooperation, higher temptation to defect. Otherwise, the environment is the de- fault low-risk environment, with the standard benefit of cooperation/temptation to defect.

Additional avenues of investigation:

- high-risk environment by default (instead of low-risk default environment)
- limit case of continuous risk-preference level (not just binary high/low risk)

- interplay between the players risk environment preference and stochastic nature

Model Specifics

We model the choice between high-stakes and low-stakes encounters in an iterated Prisoner's Dilemma as a choice between two possible payoff matrices for each round: a "high-stakes" Game 1 matrix $\begin{bmatrix} b_1-c & -c \\ b_1 & 0 \end{bmatrix}$ and a "low-stakes" Game 2 matrix $\begin{bmatrix} b_2-c & c \\ b_2 & 0 \end{bmatrix}$. In R^{16}, a player's strategy is given by

$$[p_{1cc}, p_{1cd}, p_{1dc}, p_{1dd}, \quad p_{2cc}, p_{2cd}, p_{2dc}, p_{2dd}, \quad x_{1cc}, x_{1cd}, x_{1dc}, x_{1dd} \qquad x_{2cc}, x_{2cd}, x_{2dc}, x_{2dd}] \in [0,1]^{16},$$

where

- $p_{1cc}, p_{1cd}, p_{1dc}, p_{1dd}$ describe the probability of cooperating, given that we are in Game 1, for the previous round results of CC, CD, DC, and DD, respectively.
- $p_{2cc}, p_{2cd}, p_{2dc}, p_{2dd}$ describe the probability of cooperating, given we are in the Game 2, for the previous round results of CC, CD, DC, and DD, respectively,
- $x_{1cc}, x_{1cd}, x_{1dc}, x_{1dd}$ describe the probability of preferring Game 1 given the previous round result of (Game 1, CC), (Game 1, CD), (Game 1, DC), and (Game 1, DD).
- $x_{2cc}, x_{2cd}, x_{2dc}, x_{2dd}$ describe the probability of preferring Game 1 given the previous round result of (Game 2, CC), (Game 2, CD), (Game 2, DC), and (Game 2, DD).

Thus, strategies can react differently to a previous round outcome of $CC, .., DD$ if they are in a low-stakes situation or in a high-stakes situation.

Prospective Research Outline

- Categorize successful memory 1 strategies in R^{16}.
  - first: pure strategies
  - second: epsilon-noisy
  - stretch: fully mixed strategies
- Prove Nash equilibria conditions for the successful strategies.
- Check robustness of top strategies to different initial conditions. Initial conditions include parameters (e.g. imitation probability) and initial population, e.g. (starting with uniform ALL-D population, ALL-C population, or a mix of strategies).

- Check robustness of convergence time to different initial conditions. Convergence time can be defined as the number of timesteps to when a stable cooperative strategy takes root (e.g. first reaches more than 10 % of the population). Given worse initial conditions, does it take polynomial or exponentially more time for a stable cooperative strategy to take root?
- Stretch: develop visualizations for how the strategies evolve over time as a function of the parameters.
- Repeat analysis in R^{12}, looking at whether a stable cooperative strategy in R16 implies a stable cooperative strategy in R^{12}.
- Compare the conditions for the emergence of stable cooperative strategies in the two risk level environment vs. single risk environment (i.e. the standard IPD framework).

Feasibility

To make it computationally tractable to handle strategies in R^{16}, perhaps use Monte-Carlo to approximate average payoff between two strategies. Since new strategies are invented relatively rarely, I only need to do MC to compute the sample average payoff of the newly invented strategy vs. the other strategies in the population (at most 99) for a fraction of the timesteps.

Prior Work

Last semester, I investigated the 12-dimensional model with Dr. Hilbe, where strategies could not distinguish between preferring to cooperate in Game 1 and preferring to cooperate in Game 2. Specifically, a strategy vector looked like:

$$[p_{1cc}, p_{1cd}, p_{1dc}, p_{1dd}, \quad p_{2cc}, p_{2cd}, p_{2dc}, p_{2dd}, \quad x_{cc}, x_{cd}, x_{dc}, x_{dd}] \in [0,1]^{12},$$

where

- $p_{1cc}, p_{1cd}, p_{1dc}, p_{1dd}$ describe the probability of cooperating, given that we are in Game 1, for the previous round results of CC, CD, DC, and DD, respectively.
- $p_{2cc}, p_{2cd}, p_{2dc}, p_{2dd}$ describe the probability of cooperating, given we are in the Game 2, for the previous round results of CC, CD, DC, and DD, respectively,
- $x_{cc}, x_{cd}, x_{dc}, x_{dd}$ describe the probability of preferring Game 1 given the previous round results of CC, CD, DC, and DD, respectively.

The motivation for doing this was (a) lower computational complexity and (b) intuitively, successful strategies would spend most of their time in Game 1 anyways, so the additional specificity of tailoring responses to a prior CC in Game 1 or Game 2 would probably not be important in the long-run.

In this framework, we found that strategy A = [1000, 0001, 1000] is a subgame perfect equilibrium if $2b_1 - b_2 > c$.

This was surprising because this requirement is easier to meet than the cooperation threshold for a single high-stakes IPD game: $2b_1 - b_2 > 2c$ vs. $b_1 > 2c$. So the existence of lower-stakes interactions appears to lower the barrier for mutually cooperative strategies to become Nash equilibria.