**Assignment 1**

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## Part 1

a)

$$\boldsymbol{T(n) = 2T\left(\frac{n}{2}\right) + 3n + 7}$$
$$a = 2, b = 2, \log_2 2 = 1, k = 1, p = 0$$
$$= \theta(nlogn)$$

b)

$$\boldsymbol{T(n) = 7T\left(\frac{n}{2}\right) + n^2}$$
$$a = 7, b = 2, k = 2, p = 0$$
$$\log_2 7 = 2.81$$
$$f(n) = n = O(n^{2-\epsilon}) for\ some\ \epsilon > 0?\ Yes!$$
$$Case\ 1. applies\ and\ T(n) = \Theta(n^{\log_2 7}) = \Theta(n^3)$$

c)

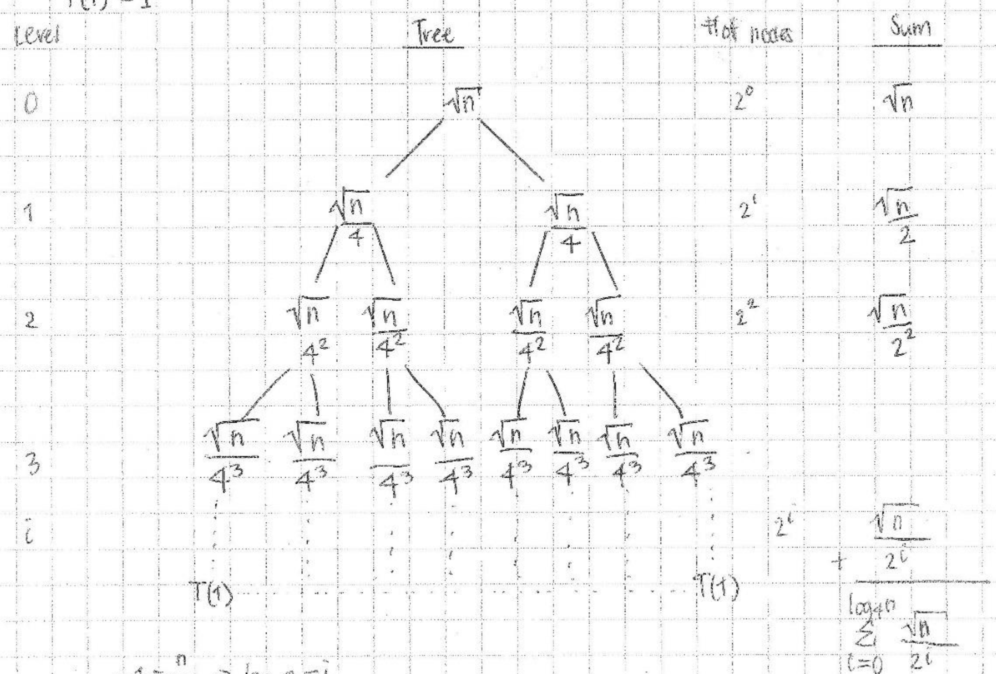
| Solution | Work |
|---|---|
| $T(n) = T\left(\frac{n}{2}\right) + T\left(\frac{n}{4}\right) + T\left(\frac{n}{8}\right) + n$ | |
| $= 3T\left(\frac{n}{2}\right) + n$ | |
| $= 3\left[3T\left(\frac{n}{2^2}\right) + \frac{n}{2}\right] + n$ | $T\left(\frac{n}{2}\right) = 3T\left(\frac{n}{2^2}\right) + \frac{n}{2}$ |
| $= 3^2 T(\frac{n}{2^2}) + 3\left(\frac{n}{2}\right) + n$ | |
| $= 3^2 T(\frac{n}{2^2}) + 3\left(\frac{n}{2}\right) + n$ | |
| $= 3^2\left[3T\ \left(\frac{n}{2^3}\right) + \left(\frac{n}{2^2}\right)\right] + 3(\frac{n}{2}) + n$ | $T\left(\frac{n}{2^2}\right) = 3T\ \left(\frac{n}{2^3}\right) + \left(\frac{n}{2^2}\right)$ |
| $= 3^3 T\left(\frac{n}{2^3}\right) + 3\left(\frac{n}{2}\right) + 3\left(\frac{n}{2}\right) + n$ | |

| | |
|---|---|
| $= 3^k T\left(\frac{n}{2^k}\right) + k\left(\frac{3n}{2}\right) + n$ | |
| $= 3^n T(0) + n\left(\frac{3n}{2}\right) + n$ | |
| $= 3^n + n\left(\frac{3n}{2}\right) + n$ | |
| $= O(3^n)$ | |

d)

d) $T(n) = 2T\left(\frac{n}{4}\right) + \sqrt{n}$

$T(1) = 1$




$\rightarrow 1 = \frac{n}{4^i} \rightarrow \log_4 n = i$

$$= \sum_{i=0}^{\log_4 n} \frac{\sqrt{n}}{2^i} \rightarrow \sum_{i=0}^{\log_4 n} \sqrt{n}\left(\frac{1}{2^i}\right) \rightarrow \sum_{i=0}^{\log_4 n} \sqrt{n}\left(\frac{1}{2}\right)^i$$

NOTE: $\sum_{i=m}^{n} ar^i = a\left[\frac{r^m - r^{n+1}}{1-r}\right]$ when $r \neq 1$

$$= \sqrt{n} \cdot \frac{\left(\frac{1}{2}\right)^0 - \left(\frac{1}{2}\right)^{\log_4 n + 1}}{1 - \left(\frac{1}{2}\right)} \rightarrow 2\sqrt{n}\left(1 - \left(\frac{1}{2}\right)^{\log_4 n + 1}\right)$$

$$= 2\sqrt{n}\left(1 - \left(\frac{1}{2n}\right)\left(\frac{1}{2}\right)\right)$$

$$= 2\sqrt{n}\left(1 - \frac{1}{4n}\right)$$

$$= 2\sqrt{n} - \frac{2\sqrt{n}}{4} \Rightarrow \frac{8\sqrt{n} - 2\sqrt{n}}{4} \Rightarrow \frac{6\sqrt{n}}{4} \Rightarrow \frac{3}{2}\sqrt{n}$$

$$= \boxed{O(n)}$$

**Part 2**

| Category | Function | |
|---|---|---|

| | | |
|---|---|---|
| n! | n! | |
| $c^n$ | $4^n$ | $2^n$ |
| $c^{logn}$ | $2^{log_2 n}$ | |
| $n^{71}$ | $n^{71}+5^n+17n$ | |
| $cn^4$ | $¾n^4$ | |
| $n^3$ | $n^3$-logn | $n^3$ |
| $n^2$ | $3n^2+7n+15$ | $n^2$ |
| √n3 | √n3 | |
| n | 18n | |
| $clog_b n$ | $Log_{10} n$ | $3log_2 n$ |

**Part 3**

T(n) = T(n/2+cs) + T(n/2) + 3n

O(n) = 4n