

## Calculus Warm-Up Quiz — Day 1

Review of Probability and Linear Algebra

*Complete before lecture. 10 minutes.*

**Name:** \_\_\_\_\_

Today we will minimize prediction error using derivatives, compute expectations and marginal densities using integrals, and take gradients to derive OLS. This quiz checks those prerequisite skills.

**1. Power Rule.** Compute the following derivatives.

(a)  $\frac{d}{dx}x^2 =$

(b)  $\frac{d}{dx}3x^3 =$

(c)  $\frac{d}{d\mu}(\mu^2 - 4\mu) =$

**2. Optimization.** Consider  $f(\mu) = \mu^2 - 6\mu + 5$ .

(a) Find  $f'(\mu)$  and set it equal to zero to find the critical point  $\mu^*$ .

(b) Compute  $f''(\mu)$ . Is  $\mu^*$  a minimum or a maximum?

**3. Definite Integrals.** Evaluate the following.

(a)  $\int_0^1 2x \, dx =$

(b)  $\int_0^1 (1 - y) \, dy =$

(c)  $\int_0^{1-x} 6(1-x-y) dy =$  (Treat  $x$  as a constant.)

**4. Partial Derivatives.** Let  $f(x, y) = 3x^2y + 2xy^2 - y$ .

(a)  $\frac{\partial f}{\partial x} =$

(b)  $\frac{\partial f}{\partial y} =$

**5. Preview: Minimizing Mean Squared Error.**

Suppose we want to choose  $\mu$  to minimize  $M(\mu) = \mathbb{E}[(Y - \mu)^2]$ . Expanding:  $M(\mu) = \mathbb{E}[Y^2] - 2\mu \mathbb{E}[Y] + \mu^2$ .

(a) Treating  $\mathbb{E}[Y^2]$  and  $\mathbb{E}[Y]$  as constants, find  $\frac{dM}{d\mu}$ .

(b) Set the derivative equal to zero and solve for  $\mu^*$ .

**Answer Key — Day 1**

1. (a)  $2x$   
(b)  $9x^2$   
(c)  $2\mu - 4$
2. (a)  $f'(\mu) = 2\mu - 6 = 0 \implies \mu^* = 3$ .  
(b)  $f''(\mu) = 2 > 0$ , so  $\mu^* = 3$  is a minimum.
3. (a)  $[x^2]_0^1 = 1$ .  
(b)  $[y - y^2/2]_0^1 = 1 - 1/2 = 1/2$ .  
(c)  $6[(1-x)y - y^2/2]_0^{1-x} = 6[(1-x)^2 - (1-x)^2/2] = 6 \cdot (1-x)^2/2 = 3(1-x)^2$ .
4. (a)  $6xy + 2y^2$   
(b)  $3x^2 + 4xy - 1$
5. (a)  $\frac{dM}{d\mu} = -2\mathbb{E}[Y] + 2\mu$   
(b)  $-2\mathbb{E}[Y] + 2\mu^* = 0 \implies \mu^* = \mathbb{E}[Y]$ .