

Calculus Warm-Up Quiz — Day 1

Review of Probability and Linear Algebra

*Complete before lecture. 10 minutes.***Name:** _____

Today we will minimize prediction error using derivatives, compute expectations and marginal densities using integrals, and take gradients to derive OLS. This quiz checks those prerequisite skills.

1. Power Rule. Compute the following derivatives.

(a) $\frac{d}{dx}x^2 =$

(b) $\frac{d}{dx}3x^3 =$

(c) $\frac{d}{d\mu}(\mu^2 - 4\mu) =$

2. Optimization. Consider $f(\mu) = \mu^2 - 6\mu + 5$.

(a) Find $f'(\mu)$ and set it equal to zero to find the critical point μ^* .

(b) Compute $f''(\mu)$. Is μ^* a minimum or a maximum?

3. Definite Integrals. Evaluate the following.

(a) $\int_0^1 2x \, dx =$

(b) $\int_0^1 (1 - y) \, dy =$

$$(c) \int_0^{1-x} 6(1-x-y) dy = \quad (\text{Treat } x \text{ as a constant.})$$

4. Partial Derivatives. Let $f(x, y) = 3x^2y + 2xy^2 - y$.

$$(a) \frac{\partial f}{\partial x} =$$

$$(b) \frac{\partial f}{\partial y} =$$

5. Preview: Minimizing Mean Squared Error.

Suppose we want to choose μ to minimize $M(\mu) = \mathbb{E}[(Y - \mu)^2]$. Expanding: $M(\mu) = \mathbb{E}[Y^2] - 2\mu \mathbb{E}[Y] + \mu^2$.

(a) Treating $\mathbb{E}[Y^2]$ and $\mathbb{E}[Y]$ as constants, find $\frac{dM}{d\mu}$.

(b) Set the derivative equal to zero and solve for μ^* .

Answer Key — Day 1

1. (a) $2x$
(b) $9x^2$
(c) $2\mu - 4$
2. (a) $f'(\mu) = 2\mu - 6 = 0 \implies \mu^* = 3.$
(b) $f''(\mu) = 2 > 0$, so $\mu^* = 3$ is a minimum.
3. (a) $[x^2]_0^1 = 1.$
(b) $[y - y^2/2]_0^1 = 1 - 1/2 = 1/2.$
(c) $6[(1-x)y - y^2/2]_0^{1-x} = 6[(1-x)^2 - (1-x)^2/2] = 6 \cdot (1-x)^2/2 = 3(1-x)^2.$
4. (a) $6xy + 2y^2$
(b) $3x^2 + 4xy - 1$
5. (a) $\frac{dM}{d\mu} = -2\mathbb{E}[Y] + 2\mu$
(b) $-2\mathbb{E}[Y] + 2\mu^* = 0 \implies \mu^* = \mathbb{E}[Y].$