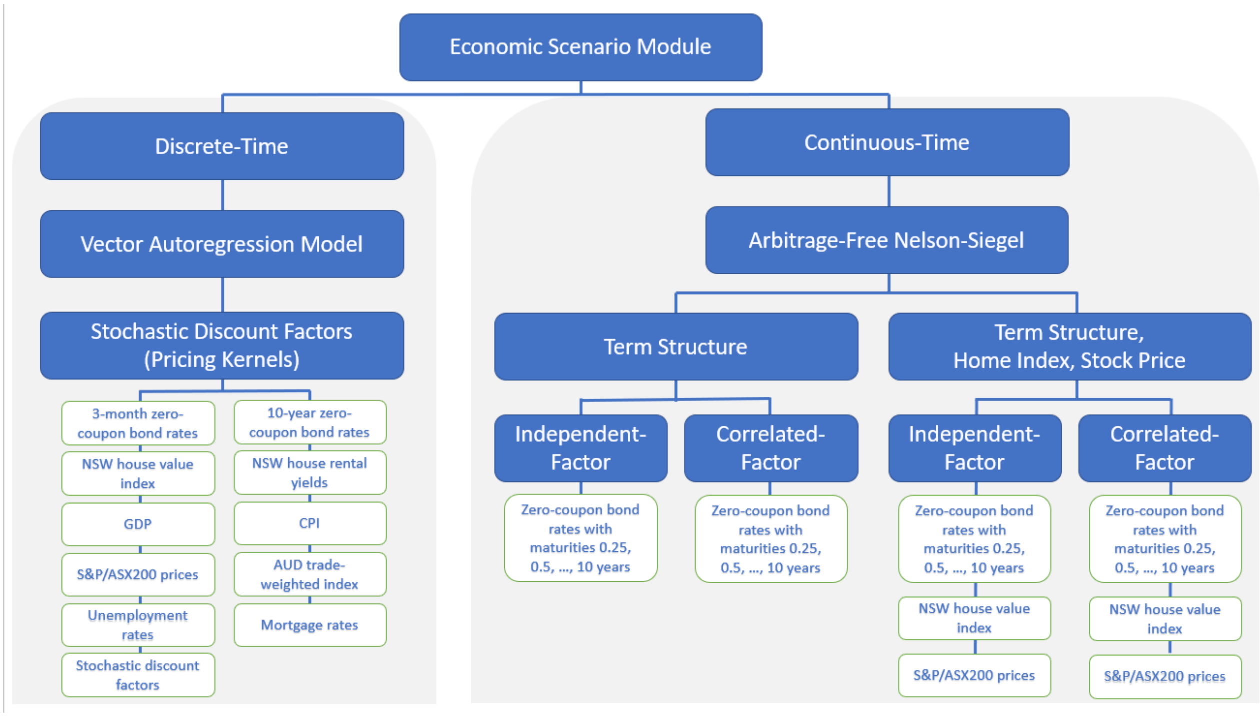
Pseudocodes for ESG Module

1 Overview



2 Vector Autoregression

`get_var_simulations` (`num_years`, `num_paths`, `frequency`, `perc_change`, `return_sdf`), stored in the file “`VAR_simulatio`

- `num_years` is a positive integer, denoted by T.
- `num_paths` is a positive integer.
- `frequency` is a character string, being one of “year”, “quarter”, and “month”, denoted by $\Delta t = 1, \frac{1}{4}, \frac{1}{12}$ respectively.
- `perc_change` is logical. `TRUE` if the user prefers percentage change instead of absolute values.
- `return_sdf` is logical. `TRUE` if the user wants stochastic discount factors.

2.1 Pseudocodes

2.1.1 Vector Autoregression.

Inputs:

- Model parameters: $\widehat{\boldsymbol{\mu}} \in \mathbb{R}^8$, $\widehat{\Phi} \in \mathbb{R}^{8\times 8}$, $\widehat{\Sigma} \in \mathbb{R}^{8\times 8}$.
- Initial value: $\boldsymbol{z}_0 \in \mathbb{R}^8$ for original series and $\tilde{\boldsymbol{z}}_{-1}, \tilde{\boldsymbol{z}}_0 \in \mathbb{R}^8$ for stationary series.

Output: Simulated paths $\{\boldsymbol{z}_t\} \subset \mathbb{R}^{10}$ and noises $\{\boldsymbol{\epsilon}_t\} \subset \mathbb{R}^8$

2.1.2 Stochastic Discount Factors.

Inputs:

- Parameters $\boldsymbol{\lambda}_0 \in \mathbb{R}^8, \lambda_1 \in \mathbb{R}^{8\times 8}$.
- Vector Autoregression stationary series $\{\tilde{\boldsymbol{z}}_t\}$, noises $\{\boldsymbol{\epsilon}_t\}$.

Output: simulated stochastic discount factors $\{s_t\} \subset \mathbb{R}$.

Algorithm 1 Vector Autoregression & Stochastic Discount Factors.

```
Initialise z̃_{t-1} ← z̃_0; z̃_{t-2} ← z̃_{-1}; Δt ← 1/4
for t = 1, 2, ··· , TΔt do          ▷ Simulate the stationary series {z̃_t} and obtain errors {ε_t}
    ε_t ← random N(0, I) variables
    z̃_t ← μ̂ + Φ̂_1 z̃_{t-1} + Φ̂_2 z̃_{t-2} + Σ̂^{1/2} ε_t          ▷ VAR formula
    z̃_{t-2} ← z̃_{t-1}; z̃_{t-1} ← z̃_t
end for
Repeat for all paths.

if return_sdf == TRUE then
    for t = 1, 2, ··· , TΔt do          ▷ Risk premiums {λ_t}
        λ_t ← λ_0 + λ_1 z̃_t
    end for
    for t = 1, 2, ··· , TΔt do          ▷ Stochastic discount factors {s_t}
        s_t ← exp(−z̃_{t-1}^⊤ e_1 − 1/2 λ_{t-1}^⊤ λ_{t-1} − λ_{t-1}^⊤ ε_t)          ▷ e_1 is the first basis vector
    end for
end if

for t = 1, 2, ··· , TΔt do          ▷ Convert {z̃_t} to original units {z_t}
    e_3^⊤ z_t ← e_3^⊤ z_{t-1} exp(e_3^⊤ z̃_t); e_5^⊤ z_t ← e_5^⊤ z_{t-1} exp(e_5^⊤ z̃_t)
    e_6^⊤ z_t ← e_6^⊤ z_{t-1} exp(e_6^⊤ z̃_t); e_7^⊤ z_t ← e_7^⊤ z_{t-1} exp(e_7^⊤ z̃_t); e_8^⊤ z_t ← e_8^⊤ z_{t-1} exp(e_8^⊤ z̃_t)
    e_9^⊤ z_t ← e_1^⊤ z_t + 2.825; e_10^⊤ z_t ← e_2^⊤ z̃_t + 4.956
end for

if frequency == "year" then          ▷ Convert the frequencies.
    Average
else if frequency == "quarter" then
    Does not change
else if frequency == "month" then
    Interpolate
end if
```

2.2 Constraints

- Output is a list of 10 data frames (11 if `return_sdf == T`) containing the simulations for each of the 10 variables.

- Each data frame has (`num_years` × `frequency`) rows.
- Each data frame has `num_paths` columns.

- Does not contain any `NA`'s.
- All values are numeric.
- Results are not reproducible (need to `set.seed()`).

2.3 Sample Cases

`simulations = get_var_simulations (10,10000, \year",return_sdf = T).`

Output should be a list of 11 dataframes (corresponding to 10 economic variables + stochastic discount factors), each data frame is 10x10000:

- 10 rows of the years (2021, 2022, ...)
- 10000 columns of the trajectories (`trajectory_1`, `trajectory_2`, ...).

3 Arbitrage-Free Nelson-Siegel

`get_afns_simulation` (`num_years`, `num_paths`, `frequency`), contained in the file "`AFNS_simulation.R`".

- `num_years` is a positive integer.
- `num_paths` is a positive integer.
- `frequency` is a character string, being one of "year", "quarter", and "month".
- `type` is a character string, being either "independent" or "correlated".
- `model` is a character string, being either "interest_rate" or "interest_house_stock".

3.1 Pseudocodes

3.1.1 Term Structure Arbitrage-Free Nelson-Siegel.

Inputs:

- Model-specific parameters: $\widehat{K} \in \mathbb{R}^{3\times 3}, \widehat{\exp\_K} = I - \exp(-\Delta t\widehat{K}) \in \mathbb{R}^{3\times 3}, \widehat{\boldsymbol{\theta}} \in \mathbb{R}^3, \widehat{\lambda} \in \mathbb{R}, B \in \mathbb{R}^{40\times 3}, A \in \mathbb{R}^{40}$.
- Noise parameters: $\widehat{Q}^{\text{month}} \in \mathbb{R}^{3\times 3}, \widehat{Q}^{\text{qtr}} \in \mathbb{R}^{3\times 3}, \widehat{Q}^{\text{year}} \in \mathbb{R}^{3\times 3}$.
- Initial value: $\boldsymbol{x}_0 \in \mathbb{R}^3$.

Outputs: simulated zero-coupon bond rates $\{\boldsymbol{y}_t\} \subset \mathbb{R}^{40}$.

3.1.2 Term Structure, Home Index, Stock Prices Arbitrage-Free Nelson-Siegel.

Inputs:

- Model-specific parameters: $\widehat{K} \in \mathbb{R}^{5\times 5}, \widehat{\exp\_K} = \int_0^{\Delta t} \exp(-\widehat{K}s)ds, \widehat{\boldsymbol{\theta}} \in \mathbb{R}^5, \widehat{\lambda} \in \mathbb{R}, B \in \mathbb{R}^{42\times 5}, A \in \mathbb{R}^{42}$.
- Noise parameters: $\widehat{Q}^{\text{month}} \in \mathbb{R}^{5\times 5}, \widehat{Q}^{\text{qtr}} \in \mathbb{R}^{5\times 5}, \widehat{Q}^{\text{year}} \in \mathbb{R}^{5\times 5}$.
- Initial value: $\boldsymbol{x}_0 \in \mathbb{R}^5$.

Outputs: simulated zero-coupon bond rates, home value indexes, and S&P/ASX200 closing prices: $\{\boldsymbol{y}_t\} \subset \mathbb{R}^{42}$.

Note: if you require simulations for house index/stock price, use also the term structure simulations from the SAME model!!

Algorithm 2 Arbitrage-Free Nelson-Siegel.

```
if model == "interest_rate" then                                  ▷ Specify the model-specific parameters
    if type == "independent" then                                 ▷ Specify the factor-specific parameters
        K ← K^{afns_indep}; θ ← θ^{afns_indep}; λ ← λ^{afns_indep}; B ← B^{afns_indep}; A ← A^{afns_indep}
        x_0 ← x_0^{afns}
        if frequency == "month" then                              ▷ Specify frequency-specific parameters
            Δt ← 1/12; Q̂ ← Q̂^{afns_indep_month}; exp_K ← exp_K^{afns_indep_month}
        else if frequency == "quarter" then
            Δt ← 1/4; Q̂ ← Q̂^{afns_indep_qtr}; exp_K ← exp_K^{afns_indep_qtr}
        else if frequency == "year" then
            Δt ← 1; Q̂ ← Q̂^{afns_indep_year}; exp_K ← exp_K^{afns_indep_year}
        end if
    else if type == "correlated" then                             ▷ Specify the factor-specific parameters
        K ← K^{afns_corr}; θ ← θ^{afns_corr}; λ ← λ^{afns_corr}; B ← B^{afns_corr}; A ← A^{afns_corr}
        x_0 ← x_0^{afns}
        if frequency == "month" then                              ▷ Specify frequency-specific parameters
            Δt ← 1/12; Q̂ ← Q̂^{afns_corr_month}; exp_K ← exp_K^{afns_corr_month}
        else if frequency == "quarter" then
            Δt ← 1/4; Q̂ ← Q̂^{afns_corr_qtr}; exp_K ← exp_K^{afns_corr_qtr}
        else if frequency == "year" then
            Δt ← 1; Q̂ ← Q̂^{afns_corr_year}; exp_K ← exp_K^{afns_corr_year}
        end if
    end if
else if model == "interest_house_stock" then                      ▷ Specify the model-specific parameters
    Follow similarly for K, θ, λ, B, A, x_0                       ▷ Specify the factor-specific parameters
    Follow similarly for exp_K, Δt, Q                             ▷ Specify frequency-specific parameters
end if

Initialise x_{t−1} ← x_0                                          ▷ Simulate state variables {x_t}.
for t = 1, ···, TΔt do
    η ← random N(0, Q̂) variables
    x_t ← exp_K θ̂ + exp(−Δt K̂) x_{t−1} + η                       ▷ State transition equation
    x_{t−1} ← x_t
end for
for t = 1, ···, TΔt do                                            ▷ Calculate zero-coupon bond rates {y_t}
    y_t ← B x_t − A
end for

Repeat for each path.
```

3.2 Constraints

- Output is a list of 40 (resp.42) dataframes, each represents zero-coupon bond rates of certain maturities (resp. zcp rates, home indexes, stock prices).
- Each data frame has (`num_years` * `frequency`) rows and `num_paths` columns.
- Does not contain any `NA`'s.
- All values are numerics.
- Results are not reproducible (need to `set.seed()`).

3.3 Sample Cases

```
simulations = get_zcp_simulations (10,10000, "year", "interest_house_stock", "correlated").
```

Output should be a list of 42 dataframe, each dataframe is 10x10000:

- Each dataframe represents (1-40) zero-coupon yields with maturities from 1 quarter to 40 quarters, (41) NSW home value index, and (42) stock prices.
- 10 rows of the years (2021-06-01, 2022-06-01, ...).
- 10000 columns of the trajectories (`trajectory_1`, `trajectory_2`, ...).