Principles of Parallel Algorithm Design: Concurrency and Decomposition

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Parallel Algorithm

Recipe to solve a problem on multiple processors

Typical steps for constructing a parallel algorithm

- —identify what pieces of work can be performed concurrently
- —partition concurrent work onto independent processors
- —distribute a program's input, output, and intermediate data
- —coordinate accesses to shared data: avoid conflicts
- —ensure proper order of work using synchronization

Why "typical"? Some of the steps may be omitted.

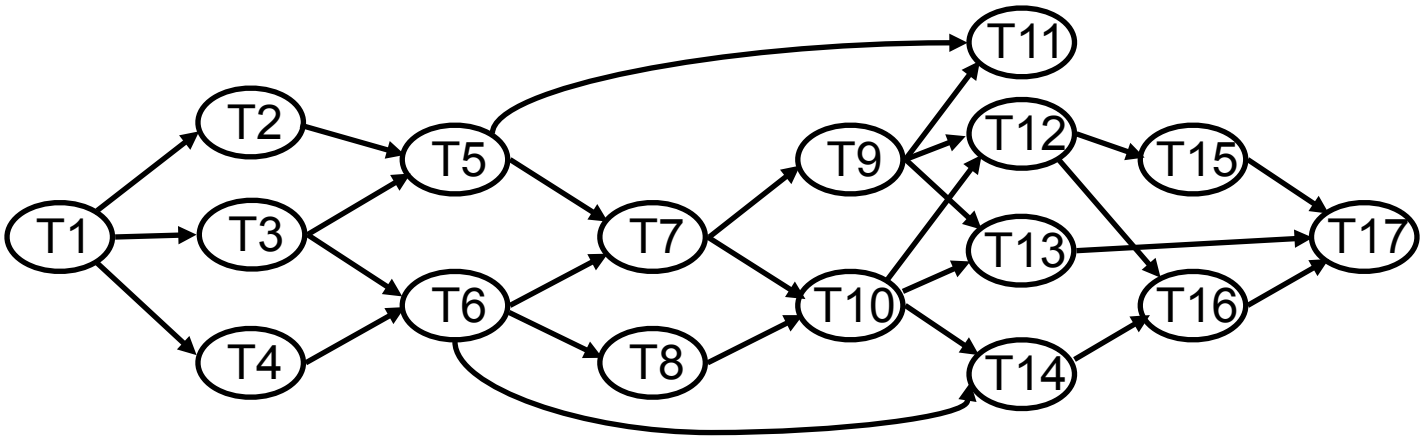
- —if data is in shared memory, distributing it may be unnecessary
- —if using message passing, there may not be shared data
- —the mapping of work to processors can be done statically by the programmer or dynamically by the runtime

Topics for Today

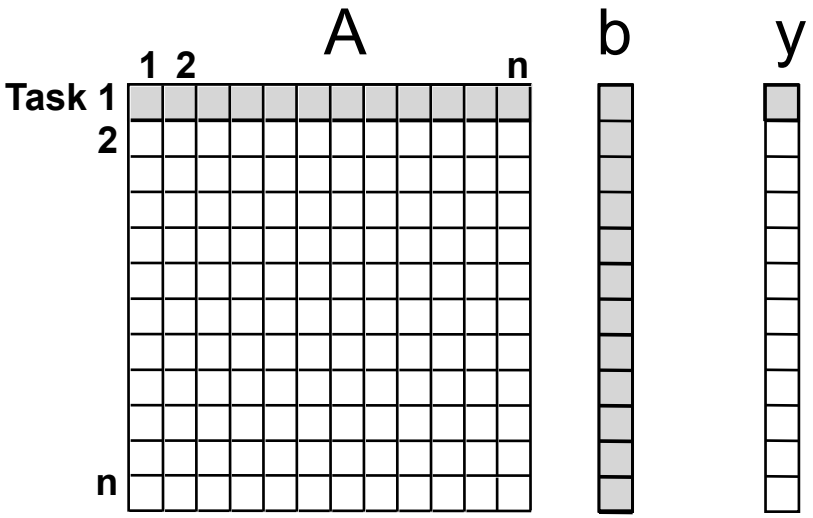
- Introduction to parallel algorithms
  - —tasks and decomposition
  - —threads and mapping
  - —threads versus cores
- Decomposition techniques - part 1
  - —recursive decomposition
  - —data decomposition

Decomposing Work for Parallel Execution

- Divide work into tasks that can be executed concurrently
- Many different decompositions possible for any computation
- Tasks may be same, different, or even indeterminate sizes
- Tasks may be independent or have non-trivial order
- Conceptualize tasks and ordering as *task dependency DAG*
  - —node = task
  - —edge = control dependence



Example: Dense Matrix-Vector Product



- Computing each element of output vector y is independent
- Easy to decompose dense matrix-vector product into tasks
  - —one per element in y
- Observations
  - —task size is uniform
  - —no control dependences between tasks
  - —tasks share *b*

Example: Database Query Processing

Consider the execution of the query:

MODEL = ``CIVIC'' AND YEAR = 2001 AND

(COLOR = ``GREEN'' OR COLOR = ``WHITE)

on the following database:

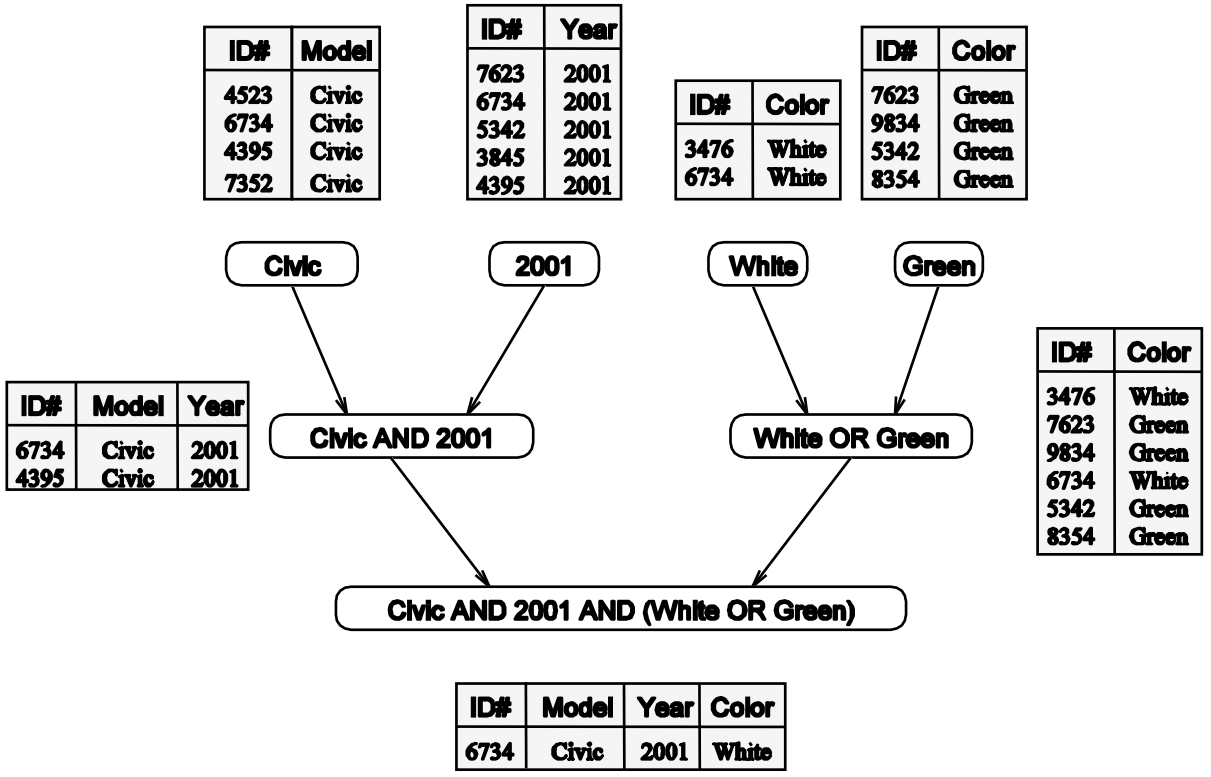
ID#	Model	Year	Color	Dealer	Price
4523	Civic	2002	Blue	MN	$18,000
3476	Corolla	1999	White	IL	$15,000
7623	Camry	2001	Green	NY	$21,000
9834	Prius	2001	Green	CA	$18,000
6734	Civic	2001	White	OR	$17,000
5342	Altima	2001	Green	FL	$19,000
3845	Maxima	2001	Blue	NY	$22,000
8354	Accord	2000	Green	VT	$18,000
4395	Civic	2001	Red	CA	$17,000
7352	Civic	2002	Red	WA	$18,000

Example: Database Query Processing

- Task: compute set of elements that satisfy a predicate
  - —task result = table of entries that satisfy the predicate
- Edge: output of one task serves as input to the next

MODEL = ``CIVIC'' AND YEAR = 2001 AND

(COLOR = ``GREEN'' OR COLOR = ``WHITE)

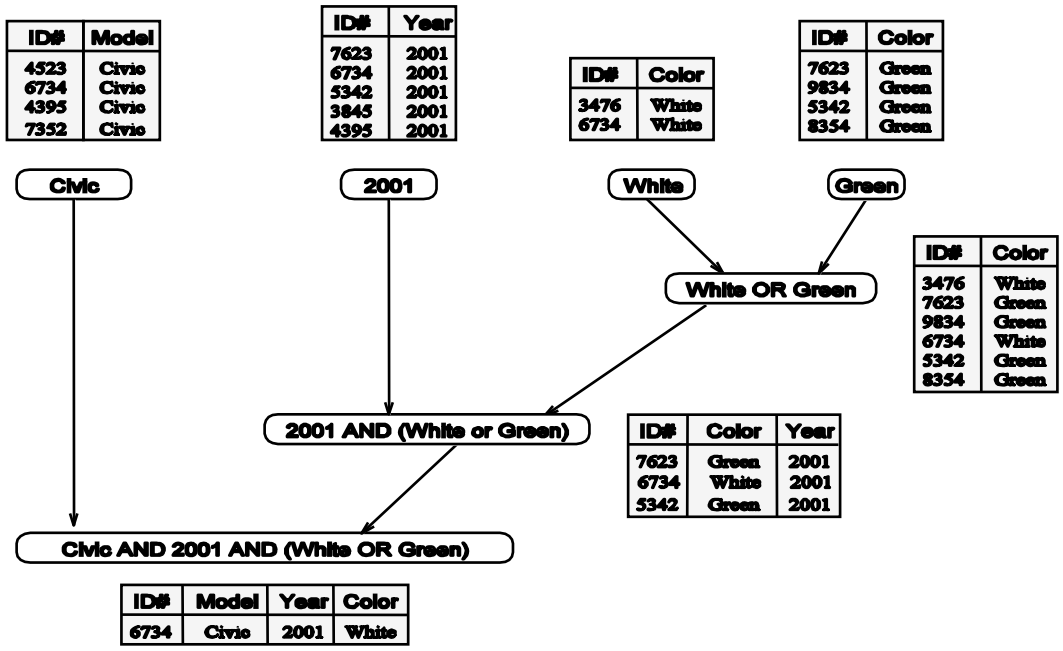


Example: Database Query Processing

- Alternate task decomposition for query

MODEL = ``CIVIC'' AND YEAR = 2001 AND

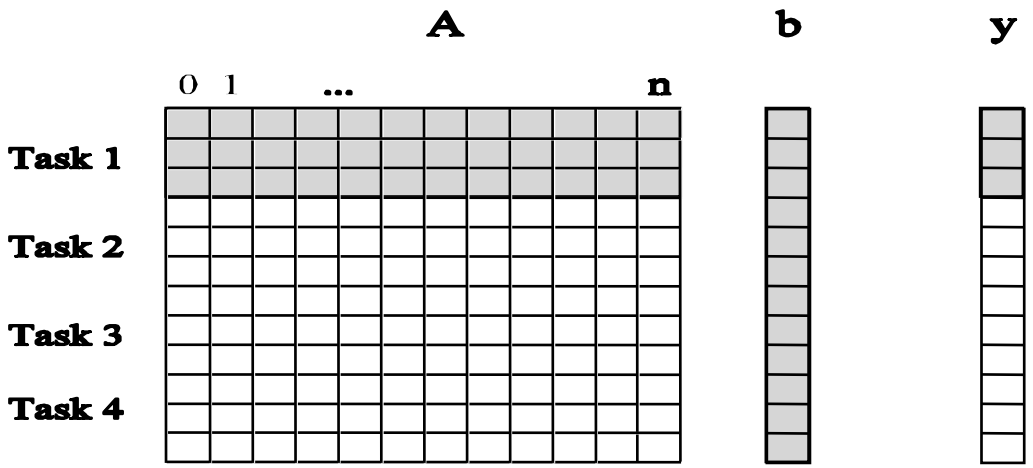
(COLOR = ``GREEN'' OR COLOR = ``WHITE)



- Different decompositions may yield different parallelism and different amounts of work

Granularity of Task Decompositions

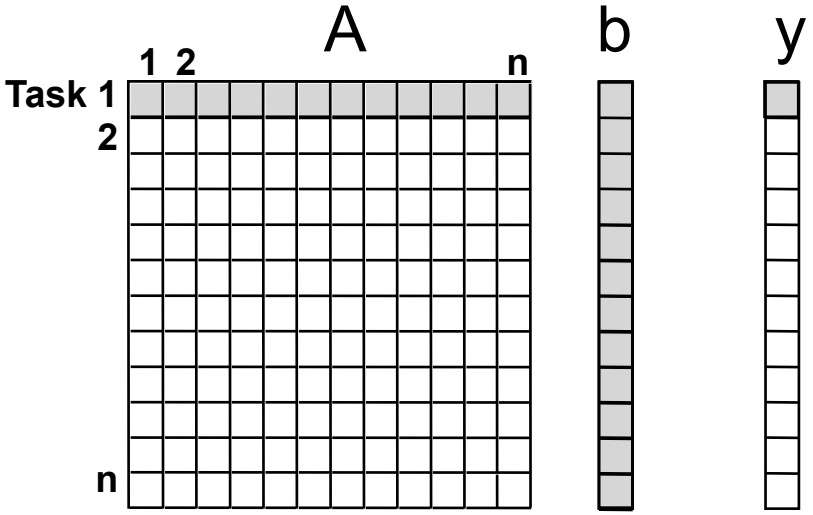
- Granularity = task size
  - —depends on the number of tasks
- Fine-grain = large number of tasks
- Coarse-grain = small number of tasks
- Granularity examples for dense matrix-vector multiply
  - —fine-grain: each task represents an individual element in y
  - —coarser-grain: each task computes 3 elements in *y*



Degree of Concurrency

- **Definition: number of tasks that can execute in parallel**
- **May change during program execution**
- **Metrics**
  - ***—maximum degree of concurrency***
    - ***largest # concurrent tasks at any point in the execution***
  - ***—average degree of concurrency***
    - **average number of tasks that can be processed in parallel**
- **Degree of concurrency vs. task granularity**
  - **—inverse relationship**

Example: Dense Matrix-Vector Multiplication



- Computing each element of output vector y is independent
- Easy to decompose dense matrix-vector product into tasks
  - —one per element in y
- Observations
  - —task size is uniform
  - —no control dependences between tasks
  - —tasks share *b*

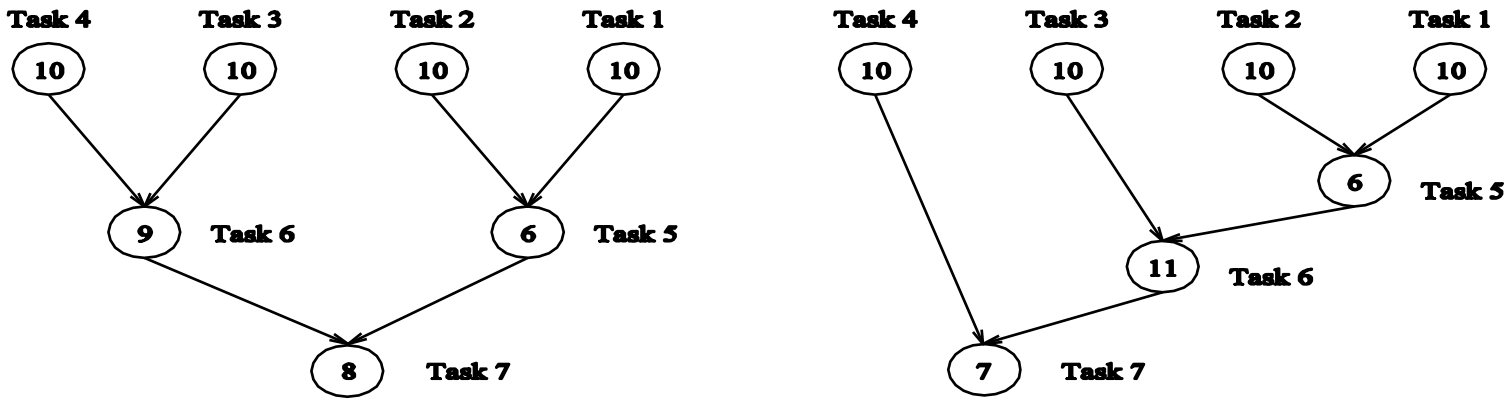
Question: Is *n* the maximum number of tasks possible?

Critical Path

- Edge in task dependency graph represents task serialization
- Critical path = longest weighted path though graph
- Critical path length = lower bound on parallel execution time

Critical Path Length

Examples: database query task dependency graphs



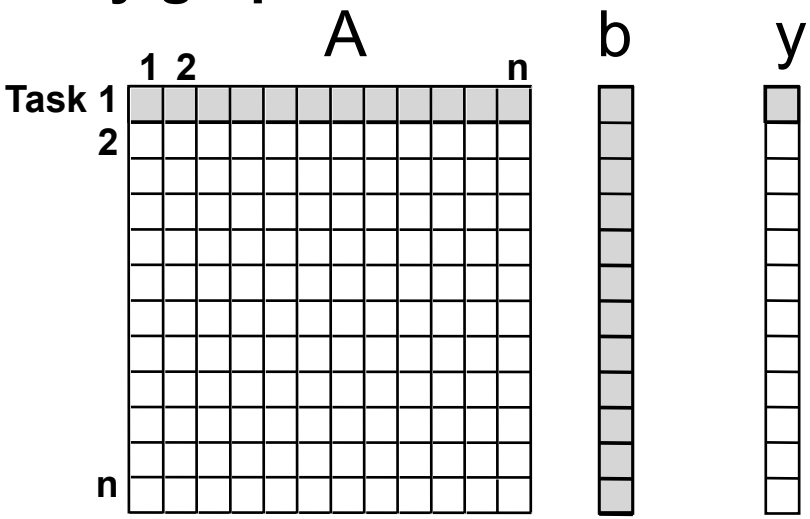
Note: number in vertex represents task cost

Questions:

- What are the tasks on the critical path for each dependency graph?
- What is the shortest parallel execution time for each decomposition?
- How many processors are needed to achieve the minimum time?
- What is the maximum degree of concurrency?
- What is the average parallelism?

Critical Path Length

Example: dependency graph for dense-matrix vector product



Questions:

- **What does a task dependency graph look like for DMVP?**
- **What is the shortest parallel execution time for the graph?**
- **How many processors are needed to achieve the minimum time?**
- **What is the maximum degree of concurrency?**
- **What is the average parallelism?**

Limits on Parallel Performance

- **What bounds parallel execution time?**
  - —**minimum task granularity**
    - – ***e.g. dense matrix-vector multiplication ≤ n^2 concurrent tasks***
  - —**dependencies between tasks**
  - —**parallelization overheads**
    - – **e.g., cost of communication between tasks**
  - —**fraction of application work that can't be parallelized**
    - – **more about Amdahl's law in a later lecture ...**
- **Measures of parallel performance**
  - —**speedup = T_1/T_p**
  - —**parallel efficiency = $T_1/(pT_p)$**

Task Interaction Graphs

- **Tasks generally exchange data with others**
  - **—example: dense matrix-vector multiply**
    - **if vector b is not replicated in all tasks, tasks will have to communicate elements of b**
- **Task interaction graph**
  - **—node = task**
  - **—edge = interaction or data exchange**
- **Task interaction graphs vs. task dependency graphs**
  - **—*task interaction graphs* represent data dependences**
  - **—*task dependency graphs* represent control dependences**

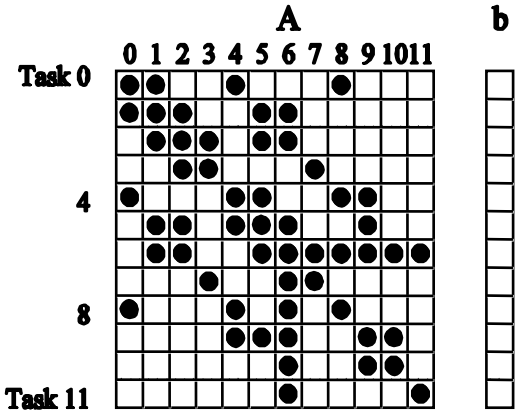
Task Interaction Graph Example

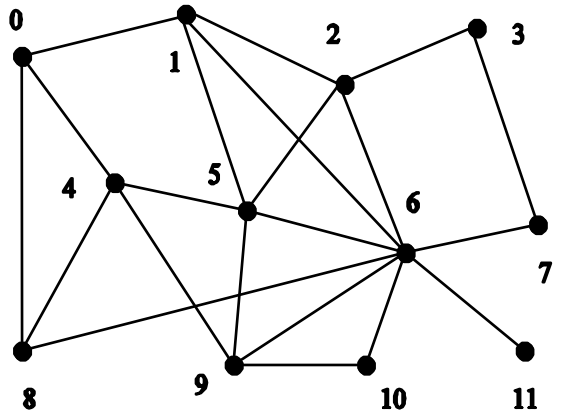
Sparse matrix-vector multiplication

- Computation of each result element = independent task
- Only non-zero elements of sparse matrix *A* participate
- If, b is partitioned among tasks …

  —structure of the task interaction graph = graph of the matrix *A*

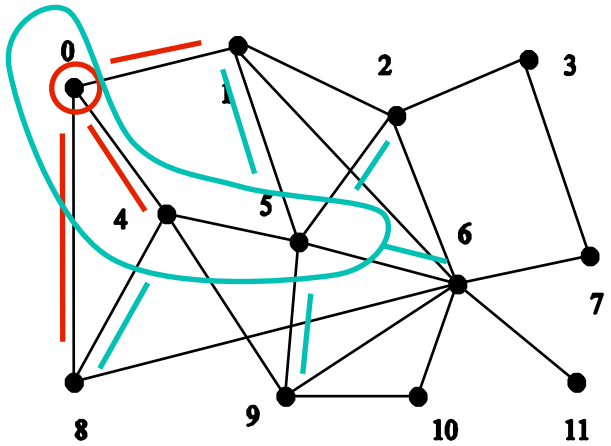
  (i.e. the graph for which *A* represents the adjacency structure)





Interaction Graphs, Granularity, & Communication

- Finer task granularity increases communication overhead
- Example: sparse matrix-vector product interaction graph



- Assumptions:
  - —each node takes unit time to process
  - —each interaction (edge) causes an overhead of a unit time
- If node 0 is a task: communication = 3; computation = 4
- If nodes 0, 4, and 5 are a task: communication = 5; computation = 15
  - —coarser-grain decomposition → smaller communication/computation ratio

Tasks, Threads, and Mapping

- Generally
  - —# of tasks > # cores available
  - —parallel algorithm must map tasks to threads
- Why threads rather than cores?
  - —aggregate tasks into threads
    - thread = processing or computing agent that performs work
    - assign collection of tasks and associated data to a thread
  - —operating system maps threads to physical cores
    - some operating systems allow one to bind a thread to a core

Tasks, Threads, and Mapping

- Mapping tasks to threads is critical for parallel performance
- On what basis should one choose mappings?
  - —using task dependency graphs
    - schedule independent tasks on separate threads
      - minimum idling
      - optimal load balance
  - —using task interaction graphs
    - want threads to have minimum interaction with one another
      - minimum communication

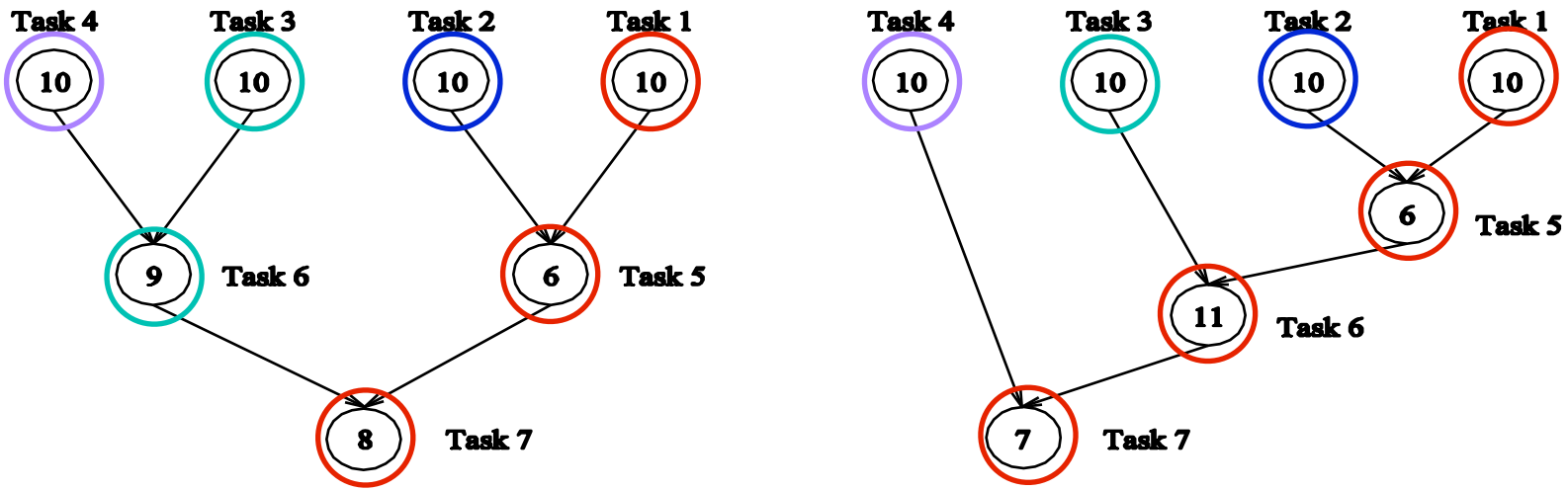
Tasks, Threads, and Mapping

A good mapping must minimize parallel execution time by

- **Mapping independent tasks to different threads**
- **Assigning tasks on critical path to threads ASAP**
- **Minimizing interaction between threads**
  - **—map tasks with dense interactions to the same thread**

- **Difficulty: criteria often conflict with one another**
  - **—e.g. no decomposition minimizes interactions but no speedup!**

Threads and Mapping Example



Example: mapping database queries to threads

- **Consider the dependency graphs in levels**
  - **—no nodes in a level depend upon one another**
  - **—compute levels using topological sort**
- **Assign all tasks within a level to different threads**

Topics for Today

- Introduction to parallel algorithms
  - —tasks and decomposition
  - —processes and mapping
  - —processes versus processors
- **Decomposition techniques - part 1**
  - **—recursive decomposition**
  - **—data decomposition**

Decomposition Techniques

How should one decompose a task into various subtasks?

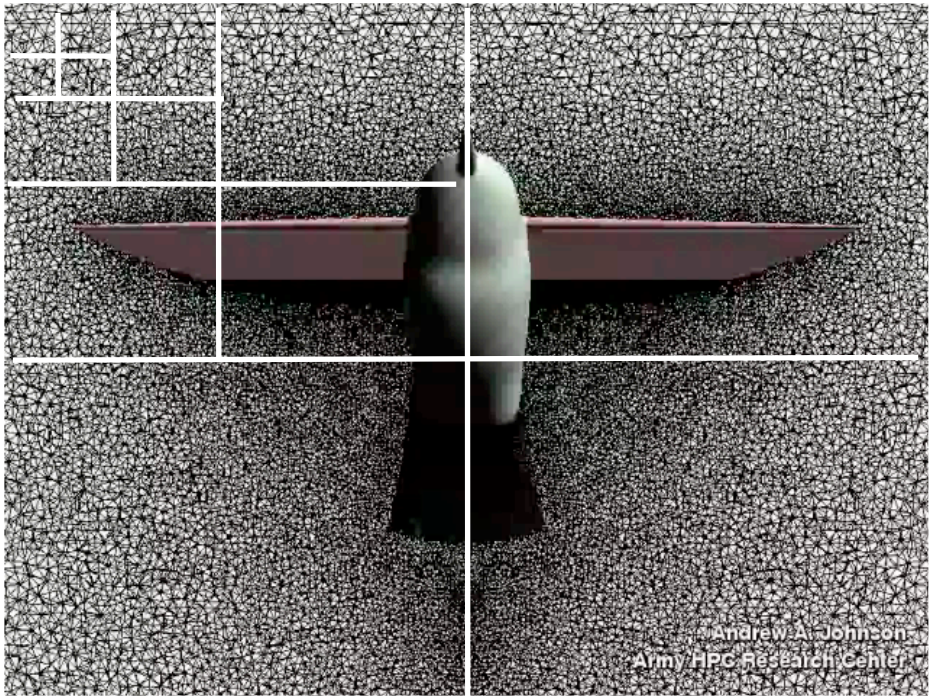
- **No single universal recipe**
- **In practice, a variety of techniques are used including**
  - **—recursive decomposition**
  - **—data decomposition**
  - **—exploratory decomposition**
  - **—speculative decomposition**

Recursive Decomposition

Suitable for problems solvable using divide-and-conquer

Steps

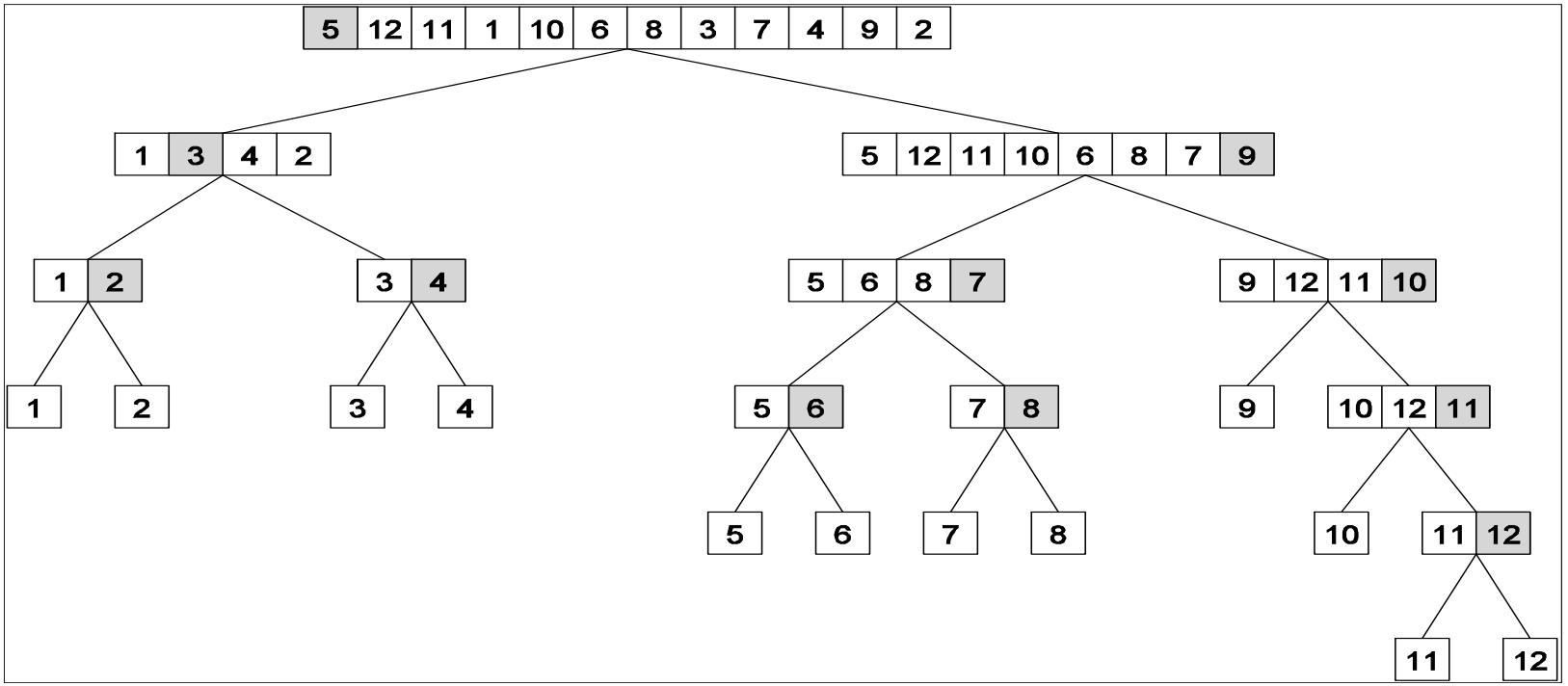
1. decompose a problem into a set of sub-problems
2. recursively decompose each sub-problem
3. stop decomposition when minimum desired granularity reached



Recursive Decomposition for Quicksort

Sort a vector v:

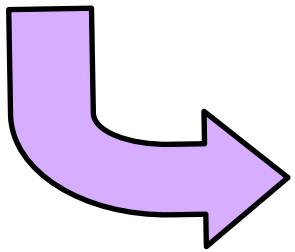
1. **Select a pivot**
2. **Partition v around pivot into vleft and vright**
3. **In parallel, sort vleft and sort vright**



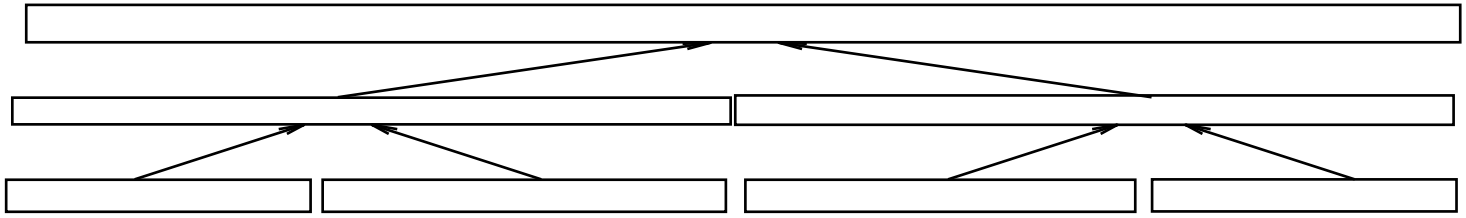
Recursive Decomposition for Min

Finding the minimum in a vector using divide-and-conquer

```
procedure SERIAL_MIN(A, n)
begin
   min = A[0];
   for i := 1 to n − 1 do
      if (A[i] < min) min := A[i];
   return min;
```



```
procedure RECURSIVE_MIN (A, n)
begin
   if ( n = 1 ) then
      min := A[0];
   else
      lmin := spawn RECURSIVE_MIN(&A[0], n/2 );
      rmin := spawn RECURSIVE_MIN(&A[n/2], n-n/2);
      if (lmin  < rmin) then
         min := lmin;
      else
         min := rmin;
    return min;
```



Applicable to other associative operations, e.g. sum, AND …

Data Decomposition

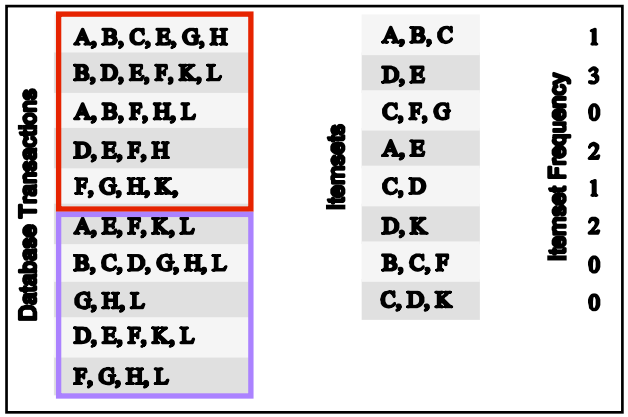
- **Steps**
  1. identify the data on which computations are performed
  2. partition the data across various tasks
     - partitioning induces a decomposition of the problem
- **Data can be partitioned in various ways**
  - appropriate partitioning is critical to parallel performance
- **Decomposition based on**
  - input data
  - output data
  - input + output data
  - intermediate data

Decomposition Based on Input Data

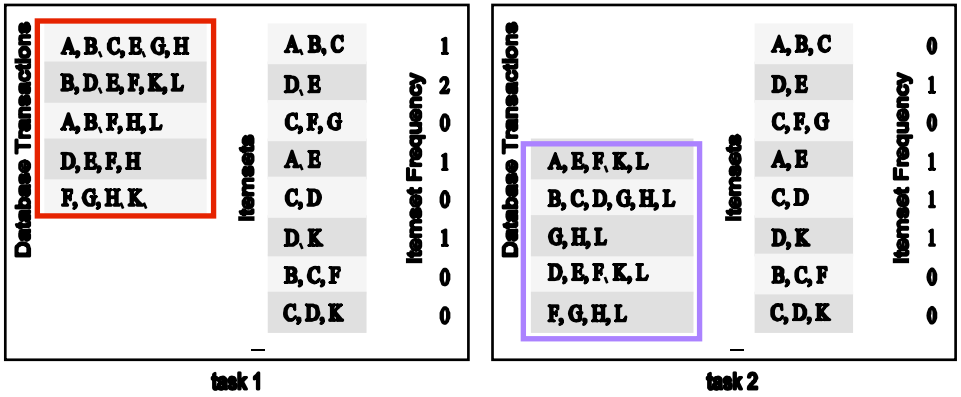
- Applicable if each output is computed as a function of the input
- May be the only natural decomposition if output is unknown
  - —examples
    - finding the minimum in a set or other reductions
    - sorting a vector
- Associate a task with each input data partition
  - —task performs computation on its part of the data
  - —subsequent processing combines partial results from earlier tasks

Example: Decomposition Based on Input Data

Count the frequency of item sets in database transactions



- **Partition computation by partitioning the set of transactions**
  - —a task computes a local count for each item set for its transactions

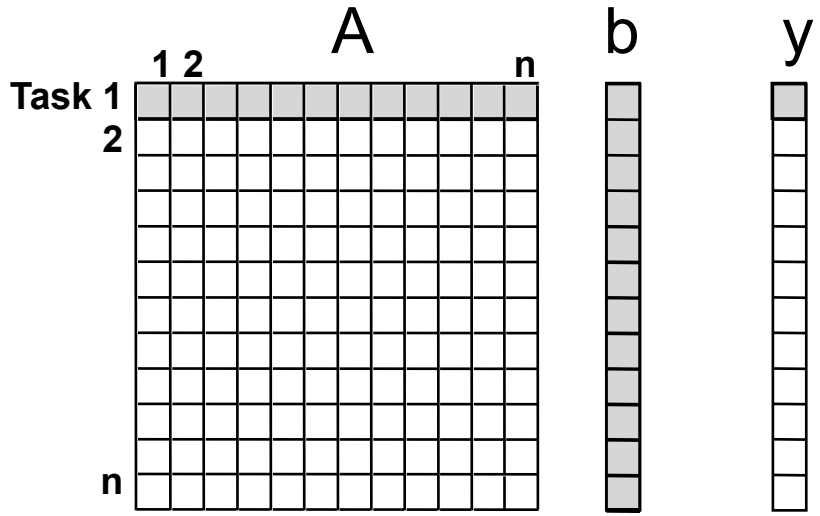


  - —sum local count vectors for item sets to produce total count vector

Decomposition Based on Output Data

- If each element of the output can be computed independently
- Partition the output data across tasks
- Have each task perform the computation for its outputs

Example:
dense matrix-vector multiply



Output Data Decomposition: Example

- **Matrix multiplication:** *C* **=** *A* **x** *B*
- **Computation of** *C* **can be partitioned into four tasks**

$$\begin{pmatrix} A_{1,1} & A_{1,2} \\ A_{2,1} & A_{2,2} \end{pmatrix} \cdot \begin{pmatrix} B_{1,1} & B_{1,2} \\ B_{2,1} & B_{2,2} \end{pmatrix} \rightarrow \begin{pmatrix} C_{1,1} & C_{1,2} \\ C_{2,1} & C_{2,2} \end{pmatrix}$$

Task 1: $C_{1,1} = A_{1,1}B_{1,1} + A_{1,2}B_{2,1}$

Task 2: $C_{1,2} = A_{1,1}B_{1,2} + A_{1,2}B_{2,2}$

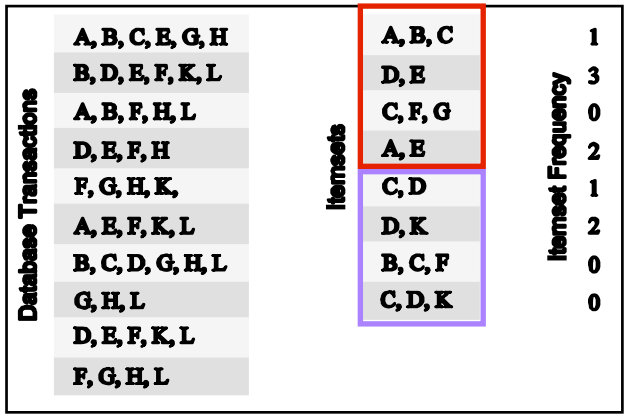
Task 3: $C_{2,1} = A_{2,1}B_{1,1} + A_{2,2}B_{2,1}$

Task 4: $C_{2,2} = A_{2,1}B_{1,2} + A_{2,2}B_{2,2}$

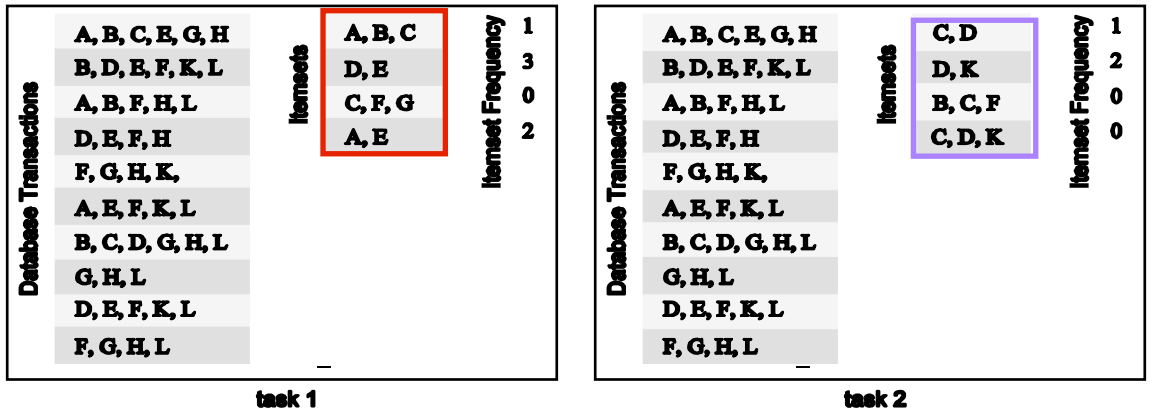
Other task decompositions possible

Example: Decomposition Based on Output Data

Count the frequency of item sets in database transactions



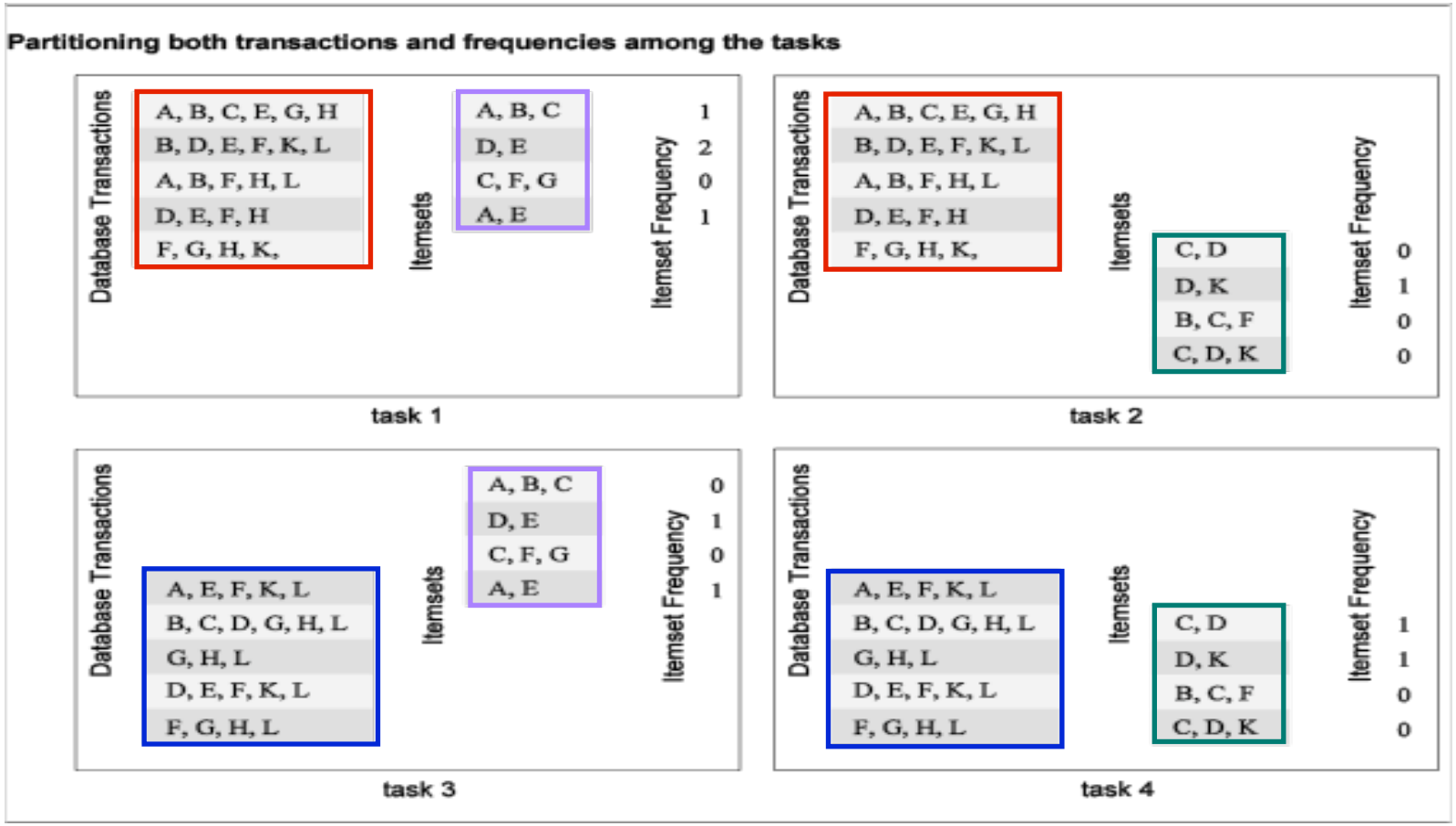
- **Partition computation by partitioning the item sets to count**
  - —each task computes total count for each of its item sets



  - —append total counts for item subsets to produce result

Partitioning Input *and* Output Data

- **Partition on both input and output for more concurrency**
- **Example: item set counting**



Intermediate Data Partitioning

- **If computation is a sequence of transforms**

  (from input data to output data)

- **Can decompose based on data for intermediate stages**

Example: Intermediate Data Partitioning

Dense Matrix Multiply

Decomposition of intermediate data: yields 8 + 4 tasks

Stage I

$$\begin{pmatrix} A_{1,1} & A_{1,2} \\ A_{2,1} & A_{2,2} \end{pmatrix} \cdot \begin{pmatrix} B_{1,1} & B_{1,2} \\ B_{2,1} & B_{2,2} \end{pmatrix} \rightarrow \begin{pmatrix} \begin{pmatrix} D_{1,1,1} & D_{1,1,2} \\ D_{1,2,1} & D_{1,2,2} \end{pmatrix} \\ \begin{pmatrix} D_{2,1,1} & D_{2,1,2} \\ D_{2,2,1} & D_{2,2,2} \end{pmatrix} \end{pmatrix}$$

Stage II

$$\begin{pmatrix} D_{1,1,1} & D_{1,1,2} \\ D_{1,2,2} & D_{1,2,2} \end{pmatrix} + \begin{pmatrix} D_{2,1,1} & D_{2,1,2} \\ D_{2,2,2} & D_{2,2,2} \end{pmatrix} \rightarrow \begin{pmatrix} C_{1,1} & C_{1,2} \\ C_{2,1} & C_{2,2} \end{pmatrix}$$

Task 01: $D_{1,1,1} = A_{1,1}\, B_{1,1}$ **Task 02:** $D_{2,1,1} = A_{1,2}\, B_{2,1}$

Task 03: $D_{1,1,2} = A_{1,1}\, B_{1,2}$ **Task 04:** $D_{2,1,2} = A_{1,2}\, B_{2,2}$

Task 05: $D_{1,2,1} = A_{2,1}\, B_{1,1}$ **Task 06:** $D_{2,2,1} = A_{2,2}\, B_{2,1}$

Task 07: $D_{1,2,2} = A_{2,1}\, B_{1,2}$ **Task 08:** $D_{2,2,2} = A_{2,2}\, B_{2,2}$

Task 09: $C_{1,1} = D_{1,1,1} + D_{2,1,1}$ **Task 10:** $C_{1,2} = D_{1,1,2} + D_{2,1,2}$

Task 11: $C_{2,1} = D_{1,2,1} + D_{2,2,1}$ **Task 12:** $C_{2,2} = D_{1,2,2} + D_{2,2,2}$

Intermediate Data Partitioning: Example

Tasks: dense matrix multiply decomposition of intermediate data

Task 01: $D_{1,1,1} = A_{1,1} B_{1,1}$

Task 02: $D_{2,1,1} = A_{1,2} B_{2,1}$

Task 03: $D_{1,1,2} = A_{1,1} B_{1,2}$

Task 04: $D_{2,1,2} = A_{1,2} B_{2,2}$

Task 05: $D_{1,2,1} = A_{2,1} B_{1,1}$

Task 06: $D_{2,2,1} = A_{2,2} B_{2,1}$

Task 07: $D_{1,2,2} = A_{2,1} B_{1,2}$

Task 08: $D_{2,2,2} = A_{2,2} B_{2,2}$

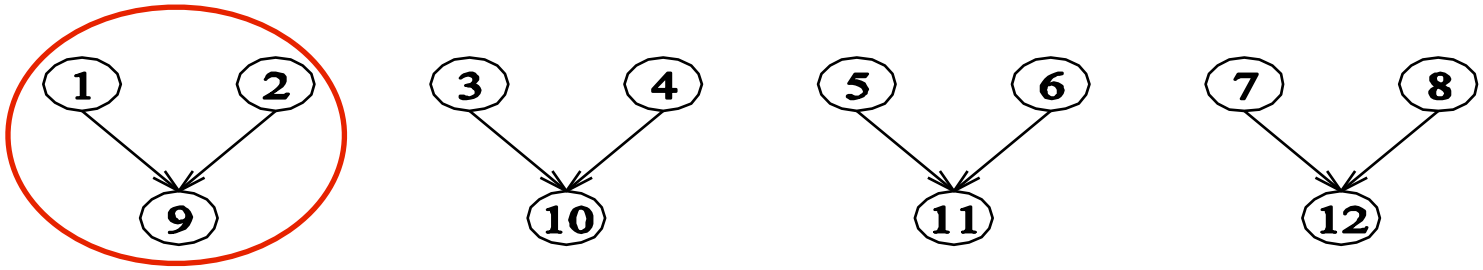
Task 09: $C_{1,1} = D_{1,1,1} + D_{2,1,1}$

Task 10: $C_{1,2} = D_{1,1,2} + D_{2,1,2}$

Task 11: $C_{2,1} = D_{1,2,1} + D_{2,2,1}$

Task 12: $C_{2,,2} = D_{1,2,2} + D_{2,2,2}$

Task dependency graph



Owner Computes Rule

- Each datum is assigned to a thread
- Each thread computes values associated with its data
- Implications
  - —input data decomposition
    - all computations using an input datum are performed by its thread
  - —output data decomposition
    - an output is computed by the thread assigned to the output data

Topics for Next Class

- Decomposition techniques - part 2
  - —exploratory decomposition
  - —hybrid decomposition
- Characteristics of tasks and interactions
  - —task generation, granularity, and context
  - —characteristics of task interactions
- Mapping techniques for load balancing
  - —static mappings
  - —dynamic mappings
- Methods for minimizing interaction overheads
- Parallel algorithm design templates

References

- Adapted from slides “Principles of Parallel Algorithm Design” by Ananth Grama
- Based on Chapter 3 of “Introduction to Parallel Computing” by Ananth Grama, Anshul Gupta, George Karypis, and Vipin Kumar. Addison Wesley, 2003