Principles of Parallel Algorithm Design: Concurrency and Mapping

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Last Thursday

- Introduction to parallel algorithms
  - —tasks and decomposition
  - —threads and mapping
  - —threads versus cores
- Decomposition techniques - part 1
  - —recursive decomposition
  - —data decomposition

Topics for Today

- Decomposition techniques - part 2
  - —exploratory decomposition
  - —hybrid decomposition
- Characteristics of tasks and interactions
- Mapping techniques for load balancing
  - —static mappings
  - —dynamic mappings
- Methods for minimizing interaction overheads
- Parallel algorithm design templates

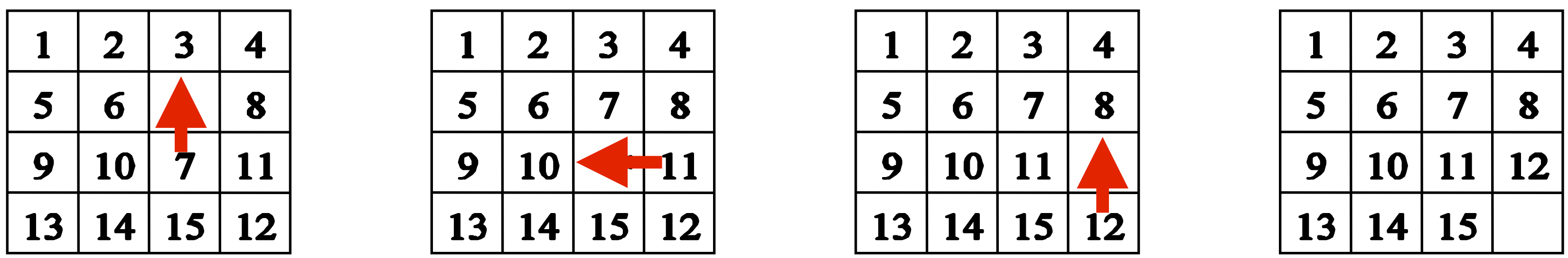
Exploratory Decomposition

- Exploration (search) of a state space of solutions
  - —problem decomposition reflects shape of execution
- Examples
  - —discrete optimization
    - – 0/1 integer programming
  - —theorem proving
  - —game playing

Exploratory Decomposition Example

Solving a 15 puzzle

- **Sequence of three moves from state (a) to final state (d)**



(a) (b) (c) (d)

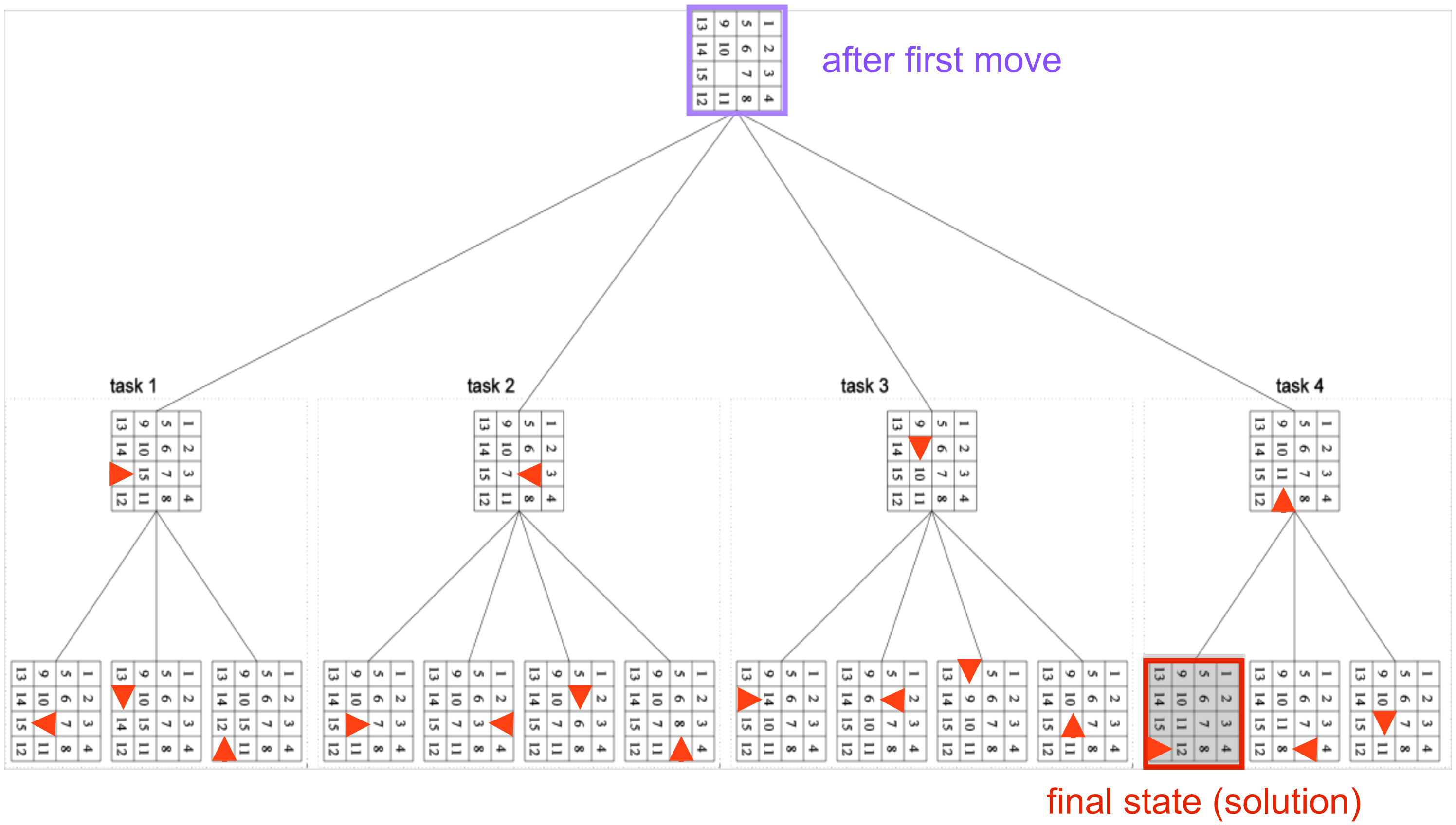
- **From an arbitrary state, must search for a solution**

Exploratory Decomposition: Example

Solving a 15 puzzle

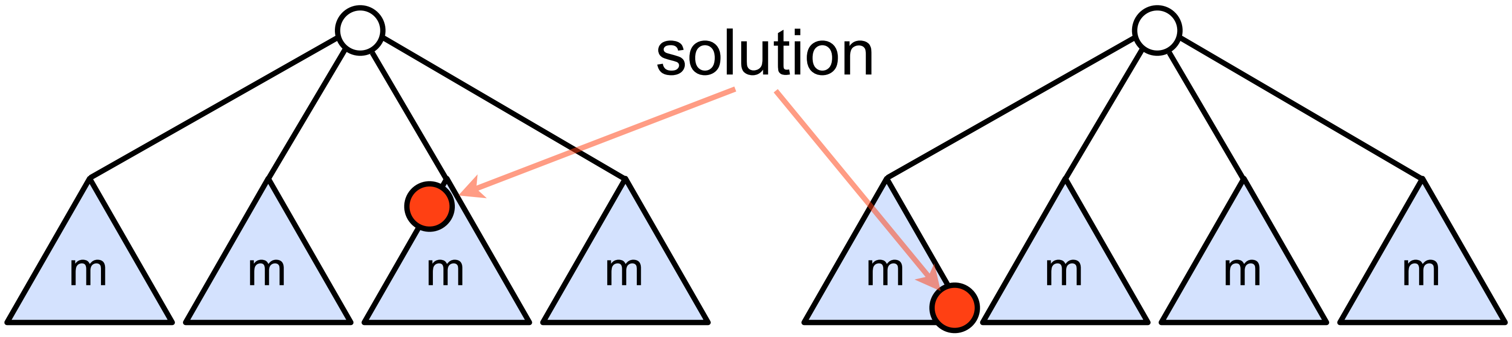
Search

- —generate successor states of the current state
- —explore each as an independent task



Exploratory Decomposition Speedup

- **Parallel formulation may perform a different amount of work**



total serial work = 2m + 1
total parallel work = 4

total serial work = m
total parallel work = 4m

- **Can cause super- or sub-linear speedup**

Speculative Decomposition

- Dependencies between tasks are not always known *a-priori*
  —makes it impossible to identify independent tasks
- Conservative approach
  —identify independent tasks only when no dependencies left
- Optimistic (speculative) approach
  —schedule tasks even when they may potentially be erroneous
- Drawbacks for each
  —conservative approaches
    – may yield little concurrency
  —optimistic approaches
    – may require a roll-back mechanism if a dependence is encountered

Speculative Decomposition In Practice

Discrete event simulation

- **Data structure: centralized time-ordered event list**
- **Simulation**
  - — extract next event in time order
  - — process the event
  - — if required, insert new events into the event list
- **Optimistic event scheduling**
  - — assume outcomes of all prior events
  - — speculatively process next event
  - — if assumption is incorrect, roll back its effects and continue

Time Warp

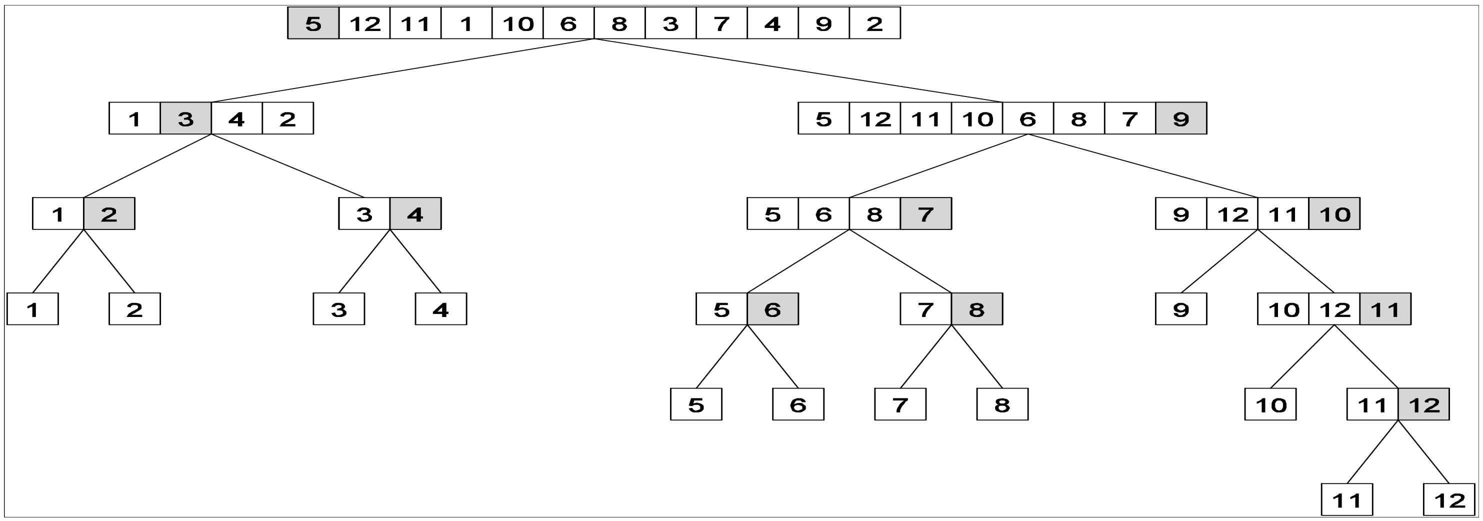
David Jefferson. “Virtual Time,”
ACM TOPLAS, 7(3):404-425, July 1985

Hybrid Decomposition

Use multiple decomposition strategies together

Often necessary for adequate concurrency

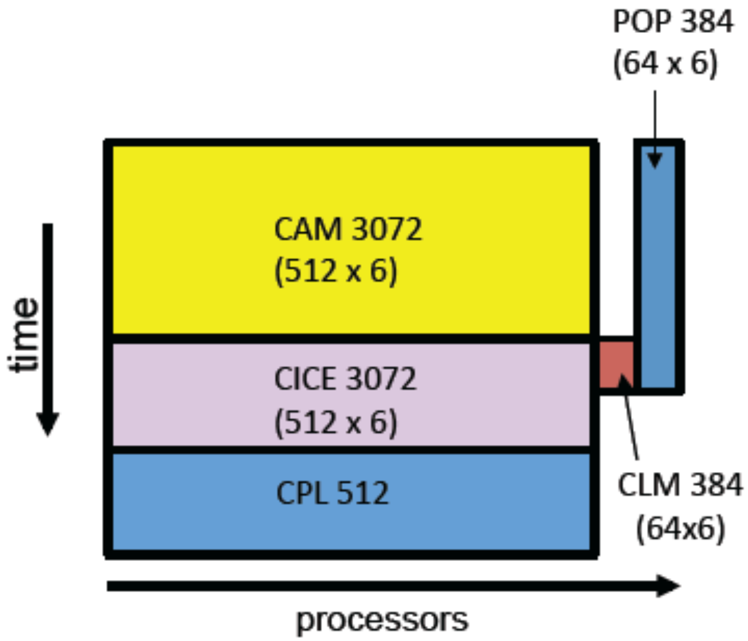
- **Quicksort**
  - recursive decomposition alone limits concurrency (why?)



- **Climate simulation**
  - data parallelism can be applied within atmosphere, ocean, land, and sea-ice simulations

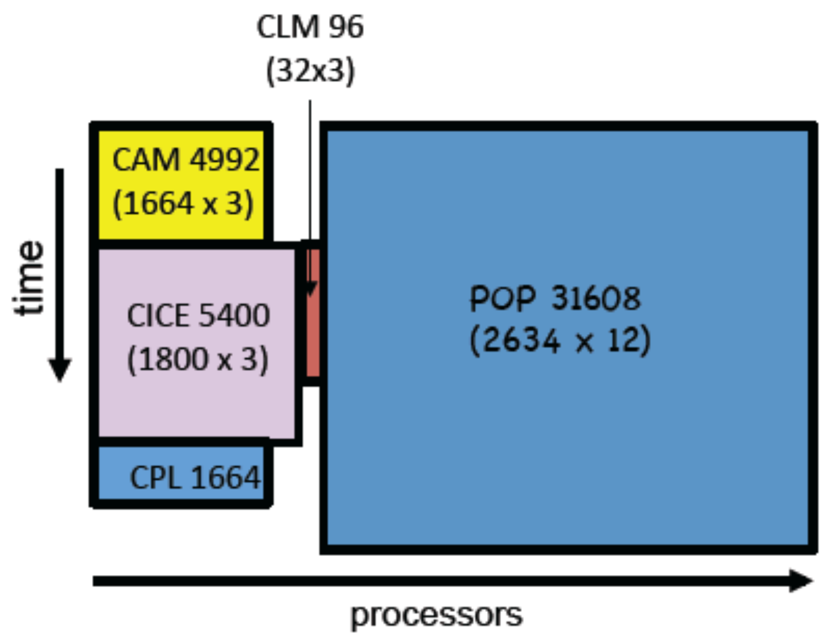
CESM Simulations on a Cray XT

140 km ATM/LND, CCSM4 physics
100 km OCN/ICE



32.7 SYPD on 3,844 cores
(P processes x T threads)

70 km ATM/LND, CCSM4 physics
10 km OCN/ICE



3.9 SYPD on 37,104 cores
(P processes x T threads)

Performance Limiters: Left is CAM; Right is POP.

Figure courtesy of Pat Worley (ORNL)

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  - —static mappings
  - —dynamic mappings
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Characteristics of Tasks

- **Key characteristics**
  - —generation strategy
  - —associated work
  - —associated data size
- **Impact choice and performance of parallel algorithms**

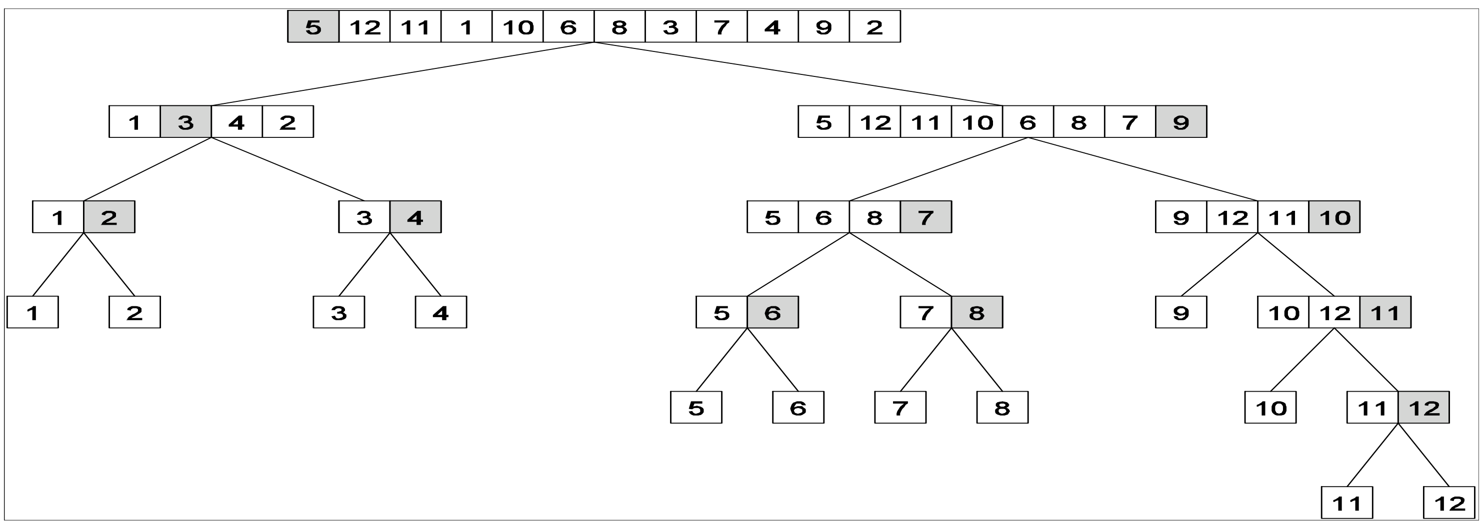
Task Generation

- **Static task generation**
  - identify concurrent tasks a-priori
  - typically decompose using data or recursive decomposition
  - examples
    - matrix operations
    - graph algorithms
    - image processing applications
    - other regularly structured problems
- **Dynamic task generation**
  - identify concurrent tasks as a computation unfolds
  - typically decompose using exploratory or speculative decompositions
  - examples
    - puzzle solving
    - game playing

Task Size

- **Uniform: all the same size**
- **Non-uniform**
  - **—sometimes sizes known or can be estimated *a-priori***
  - **—sometimes not**
    - **example: tasks in quicksort**

      size of each partition depends upon pivot selected



Size of Data Associated with Tasks

- **Data may be small or large compared to the computation**
  - —size(input) < size(computation), *e.g.*, 15 puzzle
  - —size(input) = size(computation) > size(output), *e.g.*, min
  - —size(input) = size(output) < size(computation), *e.g.*, sort

- **Implications**
  - —small data: task can easily migrate to another thread
  - —large data: ties the task to a thread
    - – possibly can avoid communicating the task context

      reconstruct/recompute the context elsewhere

Characteristics of Task Interactions

Orthogonal classification criteria

- **Static vs. dynamic**
- **Regular vs. irregular**
- **Read-only vs. read-write**
- **One-sided vs. two-sided**

Characteristics of Task Interactions

- **Static interactions**
  - —tasks and interactions are known a-priori
  - —simpler to code
- **Dynamic interactions**
  - —timing or interacting tasks cannot be determined a-priori
  - —harder to code
    - – especially using two-sided message passing APIs

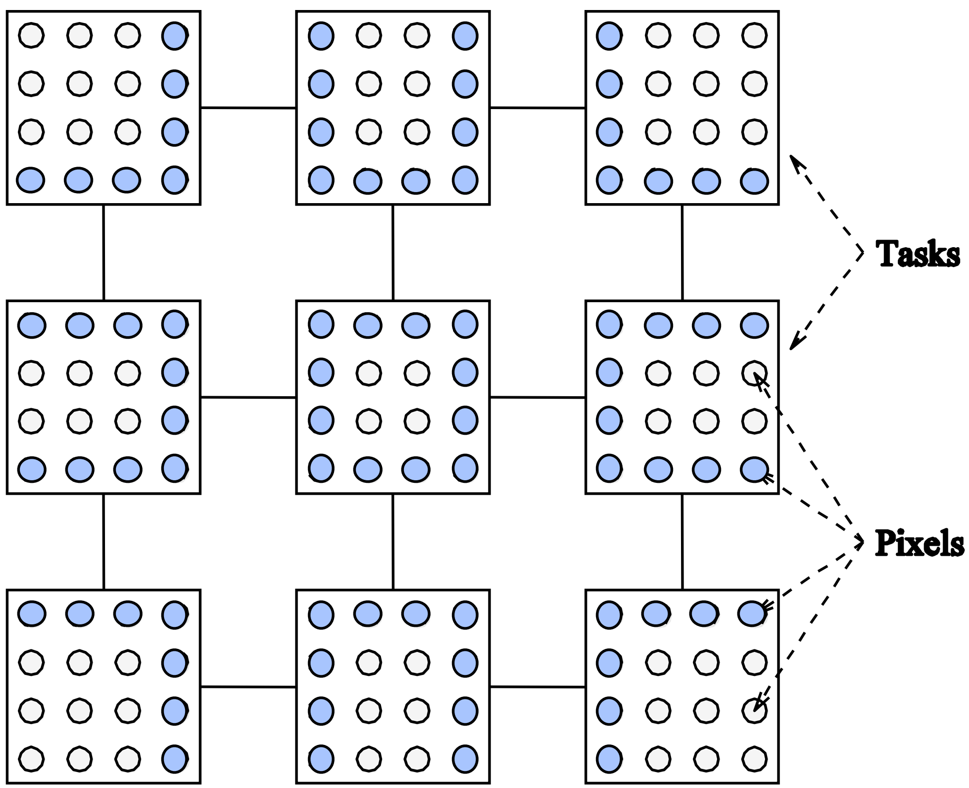
Characteristics of Task Interactions

- **Regular interactions**
  - —interactions have a pattern that can be described with a function
    - – e.g. mesh, ring
  - —regular patterns can be exploited for efficient implementation
    - – e.g. schedule communication to avoid conflicts on network links
- **Irregular interactions**
  - —lack a well-defined topology
  - —modeled by a graph

Static Regular Task Interaction Pattern

Image operations, e.g., edge detection

Nearest neighbor interactions on a 2D mesh



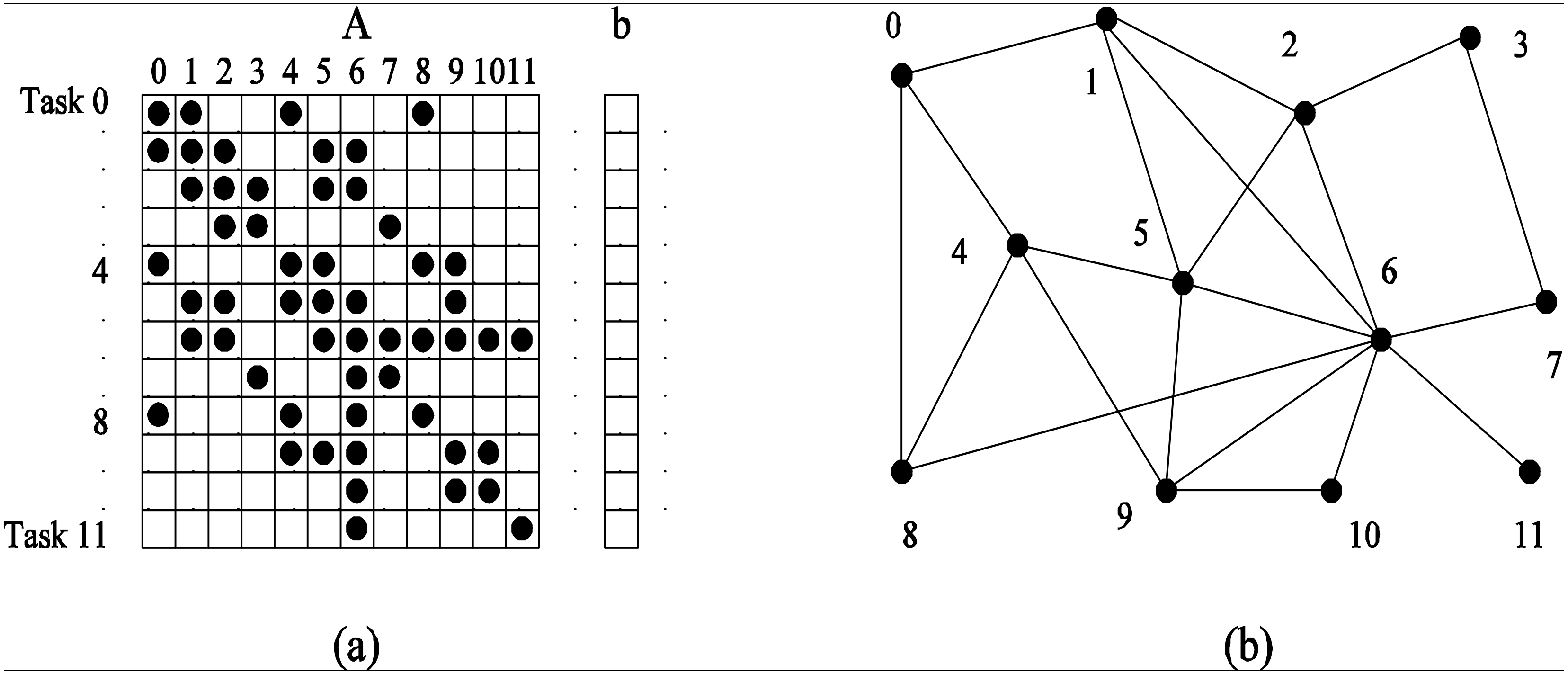
Sobel Edge Detection Stencils

$$\mathbf{G}_x = \begin{bmatrix} -1 & 0 & +1 \\ -2 & 0 & +2 \\ -1 & 0 & +1 \end{bmatrix}$$

$$\mathbf{G}_y = \begin{bmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ +1 & +2 & +1 \end{bmatrix}$$

Static Irregular Task Interaction Pattern

Sparse matrix-vector multiply



Characteristics of Task Interactions

- **Read-only interactions**
  - —tasks only read data associated with other tasks
- **Read-write interactions**
  - —read and modify data associated with other tasks
  - —harder to code: requires synchronization
    - – need to avoid read-write and write-write ordering races

Characteristics of Task Interactions

- **One-sided**
  - —initiated & completed independently by 1 of 2 interacting tasks
    - READ or WRITE
    - GET or PUT
- **Two-sided**
  - —both tasks coordinate in an interaction
    - SEND and RECV

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  —exploratory decomposition
  —hybrid decomposition
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  —static mappings
  —dynamic mappings
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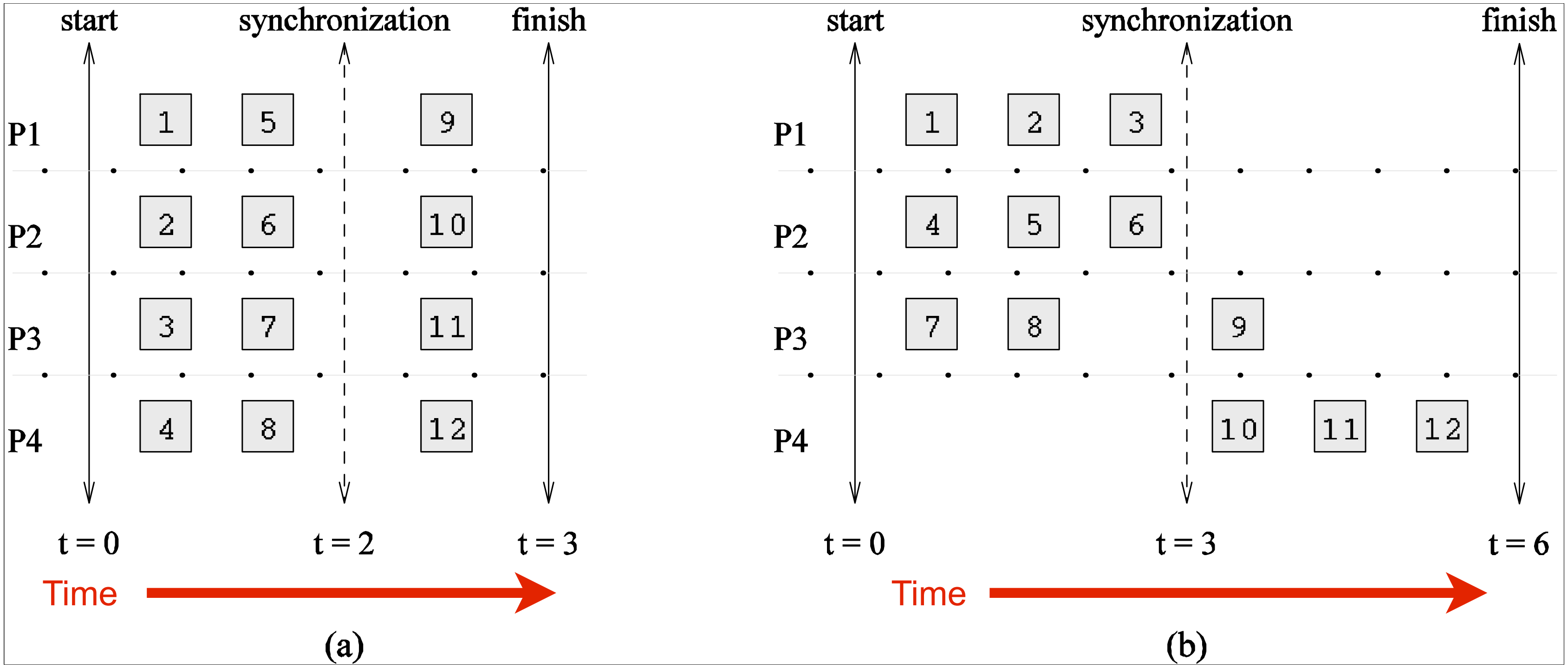
Mapping Techniques

Map concurrent tasks to processes for execution

- **Overheads of mappings**
  - —serialization (idling)
  - —communication
- **Select mapping to minimize overheads**
- **Conflicting objectives: minimizing one increases the other**
  - —assigning all work to one processor
    - minimizes communication
    - significant idling
  - —minimizing serialization introduces communication

Mapping Techniques for Minimum Idling

- Must simultaneously minimize idling and load balance
- Balancing load alone does not minimize idling



Mapping Techniques for Minimum Idling

Static vs. dynamic mappings

- **Static mapping**
  - —*a-priori* mapping of tasks to processes
  - —requirements
    - – a good estimate of task size
    - – even so, optimal mapping may be NP complete
      - e.g., multiple knapsack problem
- **Dynamic mapping**
  - —map tasks to processes at runtime
  - —why?
    - – tasks are generated at runtime, or
    - – their sizes are unknown

Factors that influence choice of mapping

- size of data associated with a task
- nature of underlying domain

Schemes for Static Mapping

- Data partitionings
- Task graph partitionings
- Hybrid strategies

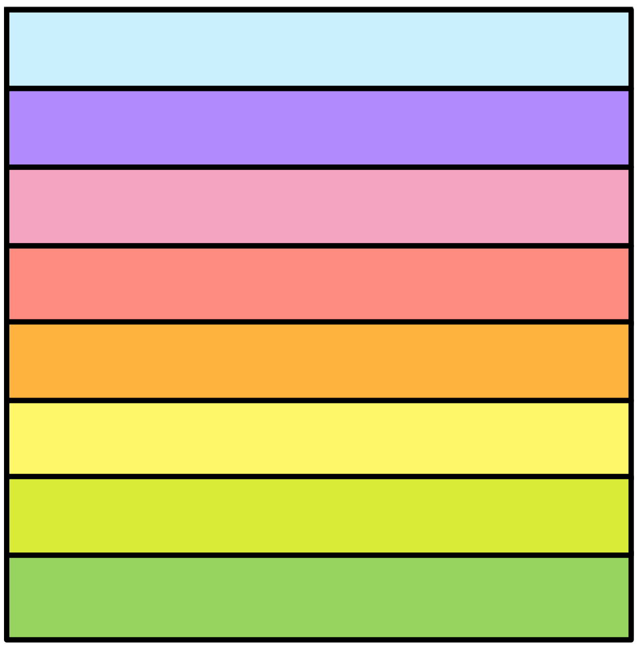
Mappings Based on Data Partitioning

Partition computation using a combination of

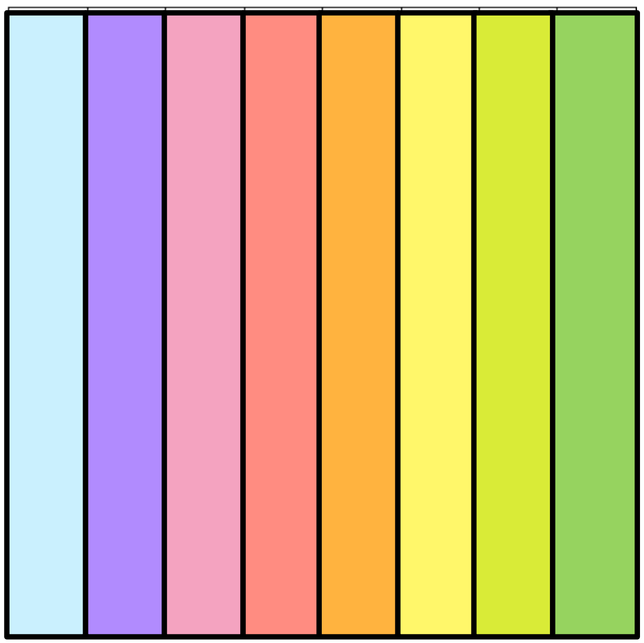
- **data partitioning**
- ***owner-computes* rule**

Example: 1-D block distribution for dense matrices

row-wise distribution

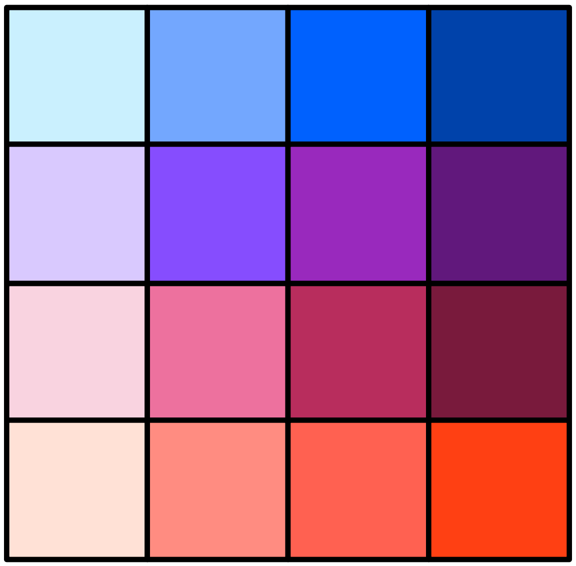


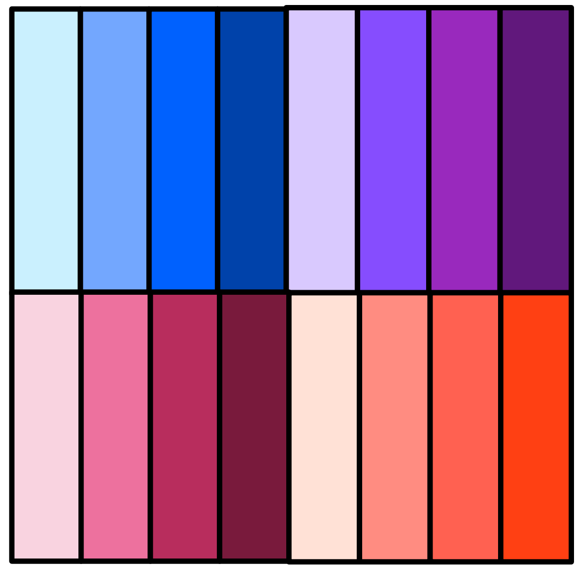
column-wise distribution



Block Array Distribution Schemes

Multi-dimensional block distributions





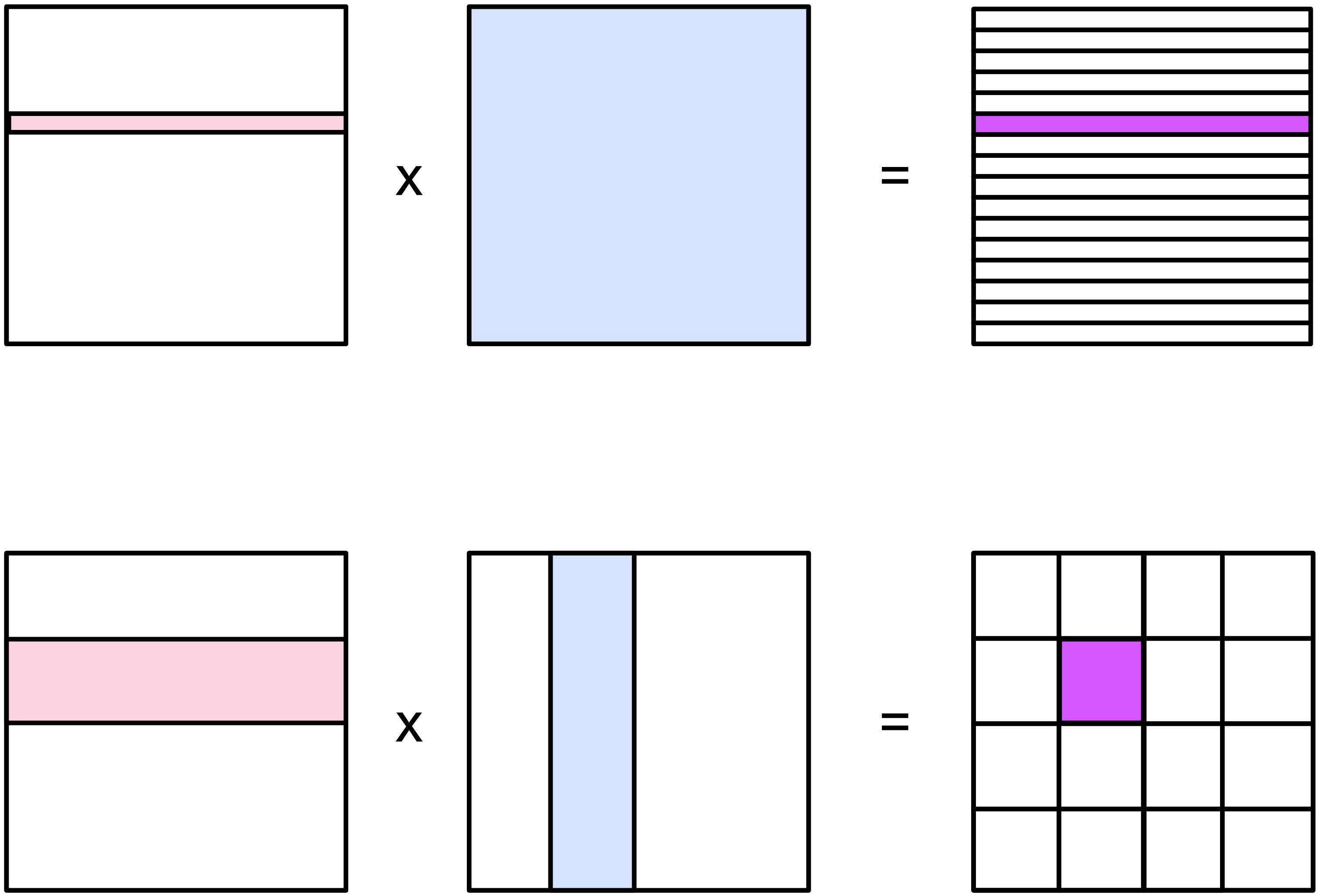
Multi-dimensional partitioning enables larger # of processes

Block Array Distribution Example

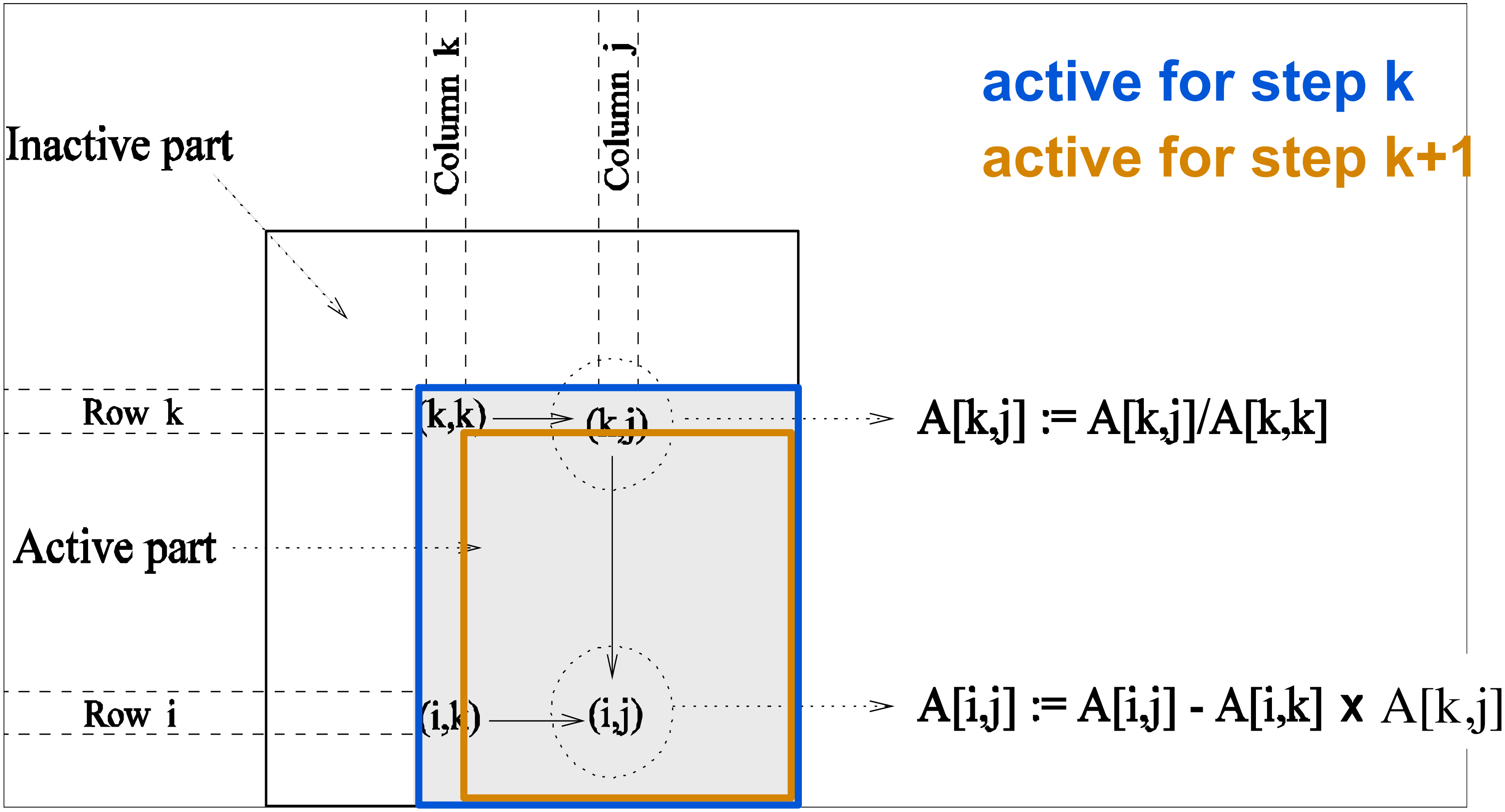
Multiplying two dense matrices C = A x B

- **Partition the output matrix C using a block decomposition**
- **Give each task the same number of elements of C**
  - —each element of C corresponds to a dot product
  - —even load balance
- **Obvious choices: 1D or 2D decomposition**
- **Select to minimize associated communication overhead**

Data Usage in Dense Matrix Multiplication



Consider: Gaussian Elimination



Active submatrix shrinks as elimination progresses

Imbalance and Block Array Distributions

- **Consider a block distribution for Gaussian Elimination**
  - —amount of computation per data item varies
  - —a block decomposition would lead to significant load imbalance

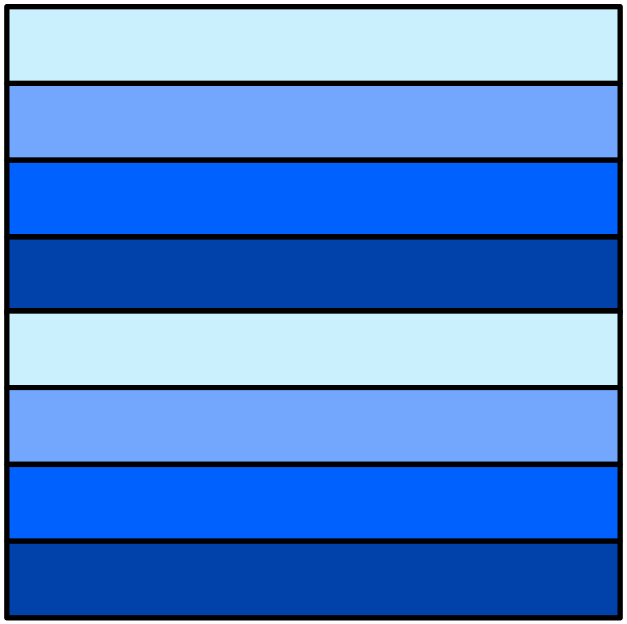
Block Cyclic Distribution

Variant of the block distribution scheme that can be used to alleviate the load-imbalance and idling

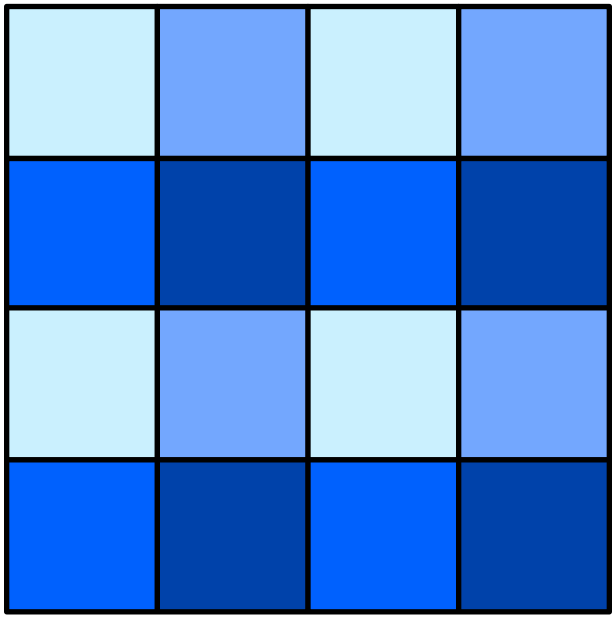
Steps

1. **partition an array into many more blocks than the number of available processes**
2. **assign blocks to processes in a round-robin manner**
   - **each process gets several non-adjacent blocks**

Block-Cyclic Distribution



1D block-cyclic



2D block-cyclic

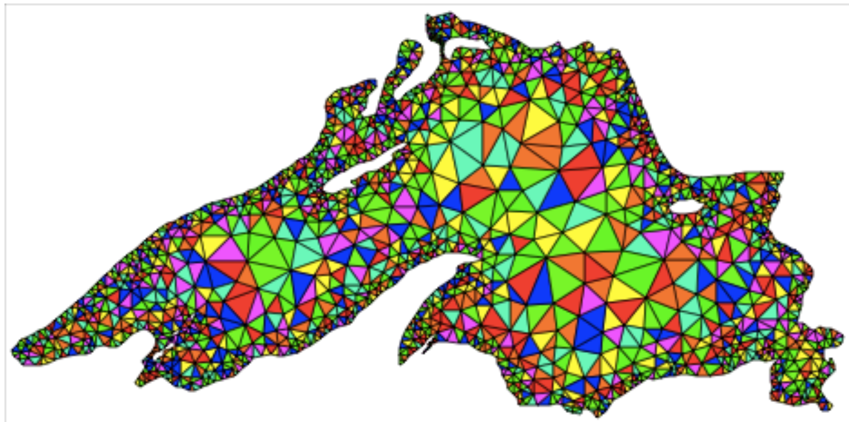
- Cyclic distribution: special case with block size = 1
- Block distribution: special case with block size is *n/p*
  — *n* is the dimension of the matrix; *p* is the # of processes

Decomposition by Graph Partitioning

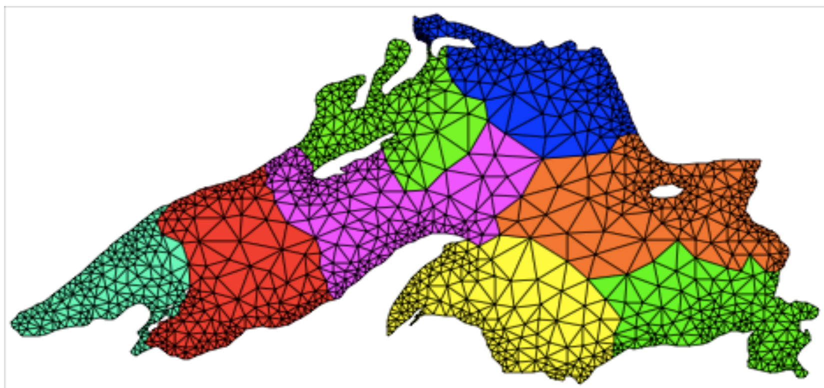
Sparse-matrix vector multiply

- **Graph of the matrix is useful for decomposition**
  - —work ~ number of edges
  - —communication for a node ~ node degree
- **Goal: balance work & minimize communication**
- **Partition the graph**
  - —assign equal number of nodes to each process
  - —minimize edge count of the graph partition

Partitioning a Graph of Lake Superior



Random Partitioning



Partitioning for minimum edge-cut

Mappings Based on Task Partitioning

Partitioning a task-dependency graph

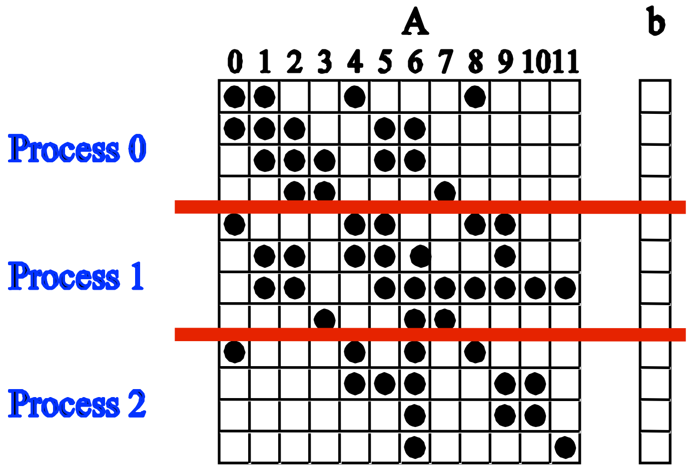
- **Optimal partitioning for general task-dependency graph**

  —NP-complete problem

- **Excellent heuristics exist for structured graphs**

Mapping a Sparse Matrix

Sparse matrix-vector product



17 items to communicate

C0 = (4,5,6,7,8)

C1 = (0,1,2,3,8,9,10,11)

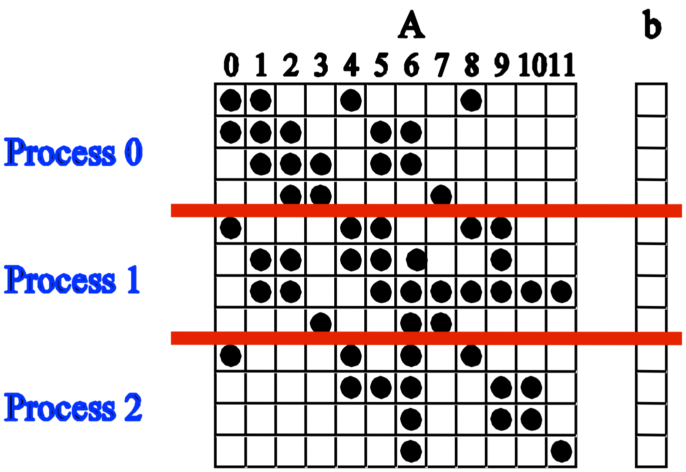
C2 = (0,4,5,6)

sparse matrix structure

mapping
partitioning

Mapping a Sparse Matrix

Sparse matrix-vector product



sparse matrix structure

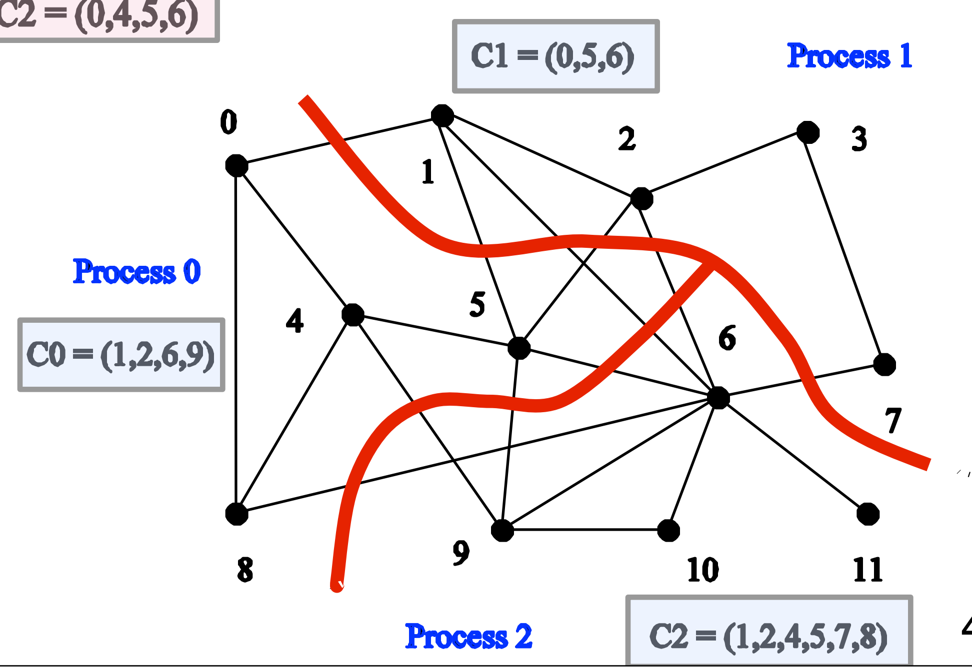
17 items to communicate

C0 = (4,5,6,7,8)

C1 = (0,1,2,3,8,9,10,11)

C2 = (0,4,5,6)

13 items to communicate



mapping

partitioning

Hierarchical Mappings

- **Sometimes a single mapping is inadequate**
  - **e.g., task mapping of quicksort binary tree cannot readily use a large number of processors.**
- **Hierarchical approach**
  - **use a task mapping at the top level**
  - **data partitioning within each task**

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  - ☛—dynamic mappings
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Schemes for Dynamic Mapping

- **Dynamic mapping AKA dynamic load balancing**
  - —load balancing is the primary motivation for dynamic mapping
- **Styles**
  - —centralized
  - —distributed

Centralized Dynamic Mapping

- Processes = masters or slaves
- General strategy
  - —when a slave runs out of work → request more from master
- Challenge
  - —master may become bottleneck for large # of processes
- Approach
  - —chunk scheduling: process picks up several of tasks at once
  - —however
    - large chunk sizes may cause significant load imbalances
    - gradually decrease chunk size as the computation progresses

Distributed Dynamic Mapping

- All processes as peers
- Each process can send or receive work from other processes
  —avoids centralized bottleneck
- Four critical design questions
  —how are sending and receiving processes paired together?
  —who initiates work transfer?
  —how much work is transferred?
  —when is a transfer triggered?
- Ideal answers can be application specific
- Cilk uses a distributed dynamic mapping: “work stealing”

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Minimizing Interaction Overheads (1)

"Rules of thumb"

- **Maximize data locality**
  - —don't fetch data you already have
  - —restructure computation to reuse data promptly
- **Minimize volume of data exchange**
  - —partition interaction graph to minimize edge crossings
- **Minimize frequency of communication**
  - —try to aggregate messages where possible
- **Minimize contention and hot-spots**
  - —use decentralized techniques (avoidance)

Minimizing Interaction Overheads (2)

Techniques

- **Overlap communication with computation**
  - —use non-blocking communication primitives
    - overlap communication with <u>your own</u> computation
    - one-sided: prefetch remote data to hide latency
  - —multithread code on a processor
    - overlap communication with <u>another thread's</u> computation
- **Replicate data or computation to reduce communication**
- **Use group communication instead of point-to-point primitives**
- **Issue multiple communications and overlap their latency**

  (reduces exposed latency)

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Parallel Algorithm Model

- **Definition: ways of structuring a parallel algorithm**
- **Aspects of a model**
  - —decomposition
  - —mapping technique
  - —strategy to minimize interactions

Common Parallel Algorithm Templates

- **Data parallel**
  - —each task performs similar operations on different data
  - —typically statically map tasks to processes
- **Task graph**
  - —use task dependency graph relationships to
    - – promote locality, or reduce interaction costs
- **Master-slave**
  - —one or more master processes generate work
  - —allocate it to worker processes
  - —allocation may be static or dynamic
- **Pipeline / producer-consumer**
  - —pass a stream of data through a sequence of processes
  - —each performs some operation on it
- **Hybrid**
  - —apply multiple models hierarchically, or
  - —apply multiple models in sequence to different phases

References

- **Adapted from slides "Principles of Parallel Algorithm Design" by Ananth Grama**
- **Based on Chapter 3 of "Introduction to Parallel Computing" by Ananth Grama, Anshul Gupta, George Karypis, and Vipin Kumar. Addison Wesley, 2003**