Project 1 Particle Mover

The particle equations of motion are

$$m\frac{dv}{dt} = q(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B}) \qquad (1)$$

$$\frac{d\boldsymbol{x}}{dt} = \boldsymbol{v}$$

A centered-difference form of he Newton-Lorentz equation of motion is

$$\frac{v_{t+\Delta t/2} - v_{t-\Delta t/2}}{\Delta t} = \frac{q}{m}\left(\boldsymbol{E} + \frac{v_{t+\Delta t/2} + v_{t-\Delta t/2}}{2} \times \boldsymbol{B}\right) \qquad (2)$$

We can separate the electric and magnetic forces completely (Boris, 1970) by substituting

$$\boldsymbol{v}_{t-\Delta t/2} = \boldsymbol{v}^- - \frac{q\boldsymbol{E}}{m}\frac{\Delta t}{2} \qquad (3)$$

$$\boldsymbol{v}_{t+\Delta t/2} = \boldsymbol{v}^+ + \frac{q\boldsymbol{E}}{m}\frac{\Delta t}{2} \qquad (4)$$

into Eq. (2). Then, **E** cancels entirely, which leaves

$$\frac{v^+ - v^-}{\Delta t} = \frac{q}{2m}(v^+ + v^-) \times \boldsymbol{B} \qquad (5)$$

which is rotation.

The steps to compute are:

I. add half electric field electric field impulse to $\boldsymbol{v}_{t-\Delta t/2}$ to obtain $\boldsymbol{v}^-$

$$\boldsymbol{v}^- = \boldsymbol{v}_{t-\Delta t/2} + \frac{q\boldsymbol{E}}{m}\frac{\Delta t}{2}$$

II. rotate from $\boldsymbol{v}^-$ **to** $\boldsymbol{v}^+$ according to (5)

III. add half electric field electric field impulse to $\boldsymbol{v}^+$ to obtain $\boldsymbol{v}_{t+\Delta t/2}$

Regarding to II, when the directions of B and v are arbitrary, a convenient rotation in vector form is described by Boris (1970). First, $\boldsymbol{v}^-$ is increment to produce a vector $\boldsymbol{v}'$ which is perpendicular to $v^+ - v^-$ and **B** (see the Figure 1).

$$v' = v^- + v^- \times \boldsymbol{t} \qquad \textbf{(6)}$$

The angle between v^- and v' is just $\theta/2$, therefore the vector t is seen from the figure to be given by

$$\boldsymbol{t} \equiv -\hat{\boldsymbol{b}} \tan\frac{\theta}{2} = \frac{q\boldsymbol{B}}{m}\frac{\Delta t}{2} \qquad (7)$$

Finally, $v^+ - v^-$ is parallel to $v' \times \mathbf{B}$, so

$$v^+ = v^- + \overset{v'}{\cancel{v}} \times s \qquad \textbf{(8)}$$

where **s** is parallel to **B** and its magnitude is determined by the requirements $|v^-|^2 = |v^+|^2$.

$$s = \frac{2t}{1+t^2} \qquad (9)$$

Problem 1 Show that Eq. (5) is rotation.

Problem 2 Using $|v^-|^2 = |v^+|^2$, obtain $\boldsymbol{s}$

Project 1: Using Boris algorithm steps I-III (Eq. (6)-(9)), compute a particle trajectories starting from x=(10,12,1), v=(2,3,4) at T=0 with E=(1,2,1), B=(5,7,8) assuming q/m=100. Any program language can be used.

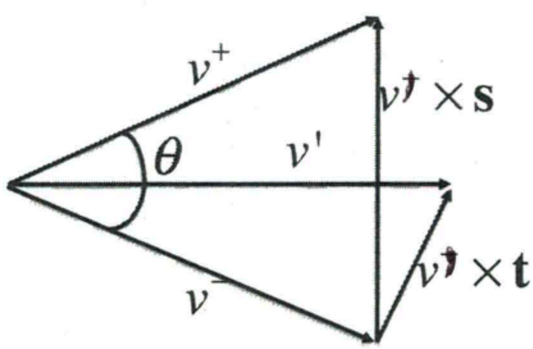

Figure 1: Velocity space showing the rotation from v^- to v^+. The shown velocities are the projection of the total velocities to the plane perpendicular to **B**.