Project 2 Field Solver 1 (1D FDTD)

We have to solve the normalized Maxwell's equations as follows

$$\frac{\partial}{\partial t}\mathbf{E}(\mathbf{x},t) = \nabla \times \mathbf{B}(\mathbf{x},t) - \mathbf{J}(\mathbf{x},t), \qquad \nabla \cdot \mathbf{E}(\mathbf{x},t) = \rho(\mathbf{x},t)$$
$$\frac{\partial}{\partial t}\mathbf{B}(\mathbf{x},t) = -\nabla \times \mathbf{E}(\mathbf{x},t), \qquad \nabla \cdot \mathbf{B}(\mathbf{x},t) = 0. \tag{1}$$

We consider plane waves that move along the x-direction (1D) and simplify the two vector equations to five scalars

$$\frac{\partial}{\partial t}E_x(x,t) = -J_x(x,t)$$
$$\frac{\partial}{\partial t}E_y(x,t) = -\frac{\partial}{\partial x}B_z(x,t) - J_y, \qquad \frac{\partial}{\partial t}B_z(x,t) = -\frac{\partial}{\partial x}E_y(x,t),$$
$$\frac{\partial}{\partial t}E_z(x,t) = \frac{\partial}{\partial x}B_y(x,t) - J_z, \qquad \frac{\partial}{\partial t}B_y(x,t) = \frac{\partial}{\partial x}E_z(x,t). \tag{2}$$

The upper equation describes electrostatic processes. The two pairs of equations describe linearly polarized EM waves. Here, we ignore for simplicity the response of the medium and set J = 0 and

$$\tfrac{\partial}{\partial t}E_x(x,t) = 0. \tag{3}$$

Here, we solve one pair of wave equations

$$\frac{\partial}{\partial t}E_y(x,t) = -\frac{\partial}{\partial x}B_z(x,t), \qquad \frac{\partial}{\partial t}B_z(x,t) = -\frac{\partial}{\partial x}E_y(x,t). \tag{4}$$

We want to use the leapfrog scheme for spatial- and time differences. Thus, we have two grids. The time axis is denoted by n and j denotes space. The black grid has nodes at integer values of (x,t) = j,n ⇔ it is defined at full time and space steps. The blue grid is defined at (x, t) = (j + 1/2, n + 1/2); it is shifted by Δx /2 and Δt /2.

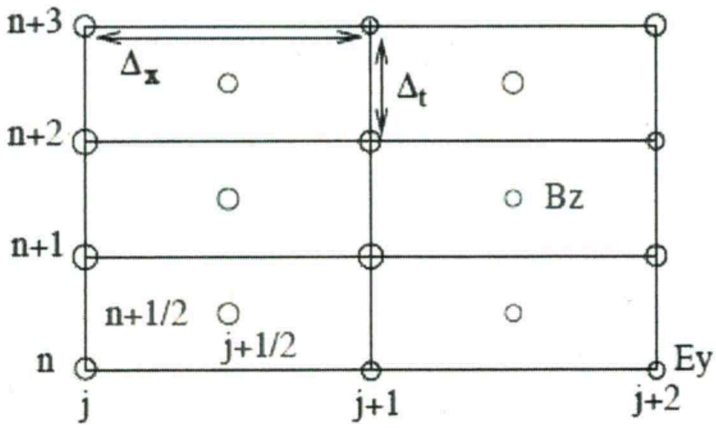

Figure 1: 1D FDTD grid system

The electric fields Ey and Ez are defined on the black nodes. The magnetic fields Bz and By are defined on the blue nodes. The magnetic field Bz(j+1/2,n+1/2) is defined on the shifted grid, while Ey (j , n) is defined on the original grid. The discretized form of the differential equations is:

$$\frac{E_y(j,n+1)-E_y(j,n)}{\Delta_t}=-\frac{B_z(j+1/2,n+1/2)-B_z(j-1/2,n+1/2)}{\Delta_x},$$
$$\frac{B_z(j+1/2,n+1/2)-B_z(j+1/2,n-1/2)}{\Delta_t}=-\frac{E_y(j,n)-E_y(j+1,n)}{\Delta_x}. \tag{5}$$

Project: Using Eq. (5) to implement 1D FDTD (Finite Difference Time Domain) program. You can consider to implement a periodic boundary. At t=0, you can put a impulse, for example, Ey(L/2, 0)=1, and see what happen in time. Later, you have to implement full set of Eq. (2) to move particle. Jx can be calculated from particle motions.

Take Grid number 128.