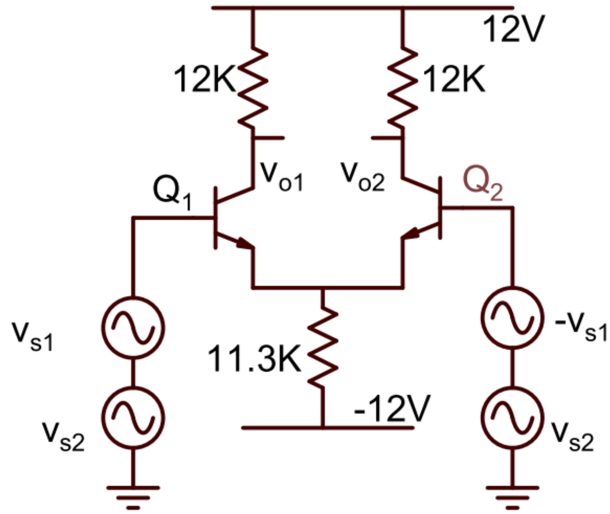
EE210: HW-12 Solution

Date: 04/04/2019

Q.1 For the differential pair shown in Fig. 1, determine the net voltage including dc and small-signal components at output nodes 1 and 2. $V_{s1} = 0.01sin(\omega_1 t)$; $V_{s2} = 1sin(\omega_2 t)$.

Sol.:



$$I_{EE} = \frac{-0.7 - V_{EE}}{R_{EE}} = 1mA$$

$$I_{CQ} = \frac{I_{EE}}{2} = 0.5\ mA$$

$$V_{O1Q} = V_{O2Q} = V_{CC} - I_{CQ}R_C = 6V$$

$$A_{dm} = -0.5\ g_m R_c = -115.4$$

$$A_{cm} = \frac{-g_m R_c}{1 + 2g_m R_{EE}} = -0.53$$

$$V_{O1} = V_{O1Q} + V_{O1d} + V_{O1c}$$

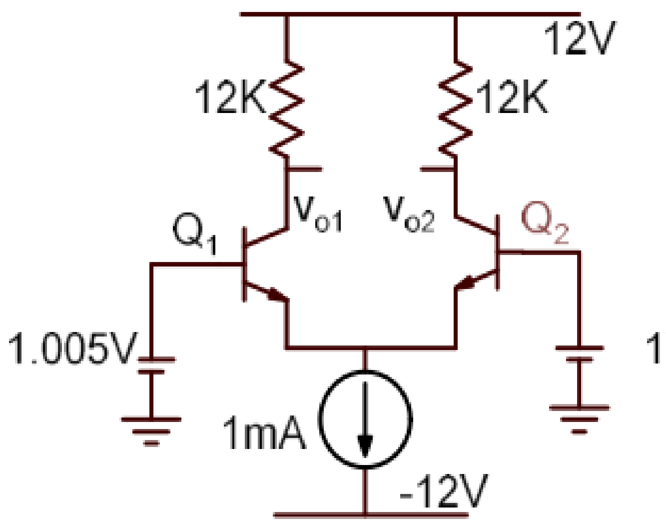
$$V_{O1} = 6 + A_{dm} * (2v_{s1}) + A_{cm} * (v_{s2}) = 6 - 115.4 * (2v_{s1}) - 0.53 * (v_{s2})$$

$$V_{O2} = V_{O2Q} + V_{O2d} + V_{O2c}$$

$$V_{O2} = 6 - A_{dm} * (2v_{s1}) + A_{cm} * (v_{s2}) = 6 + 115.4 * (2v_{s1}) - 0.53 * (v_{s2})$$

Q.2 For the amplifier shown in Fig. 2, determine dc value of collector currents of transistors Q1 and Q2. Assume that current source is ideal.

Sol.:



Common mode gain is zero.

$$V_{id} = 0.005V$$

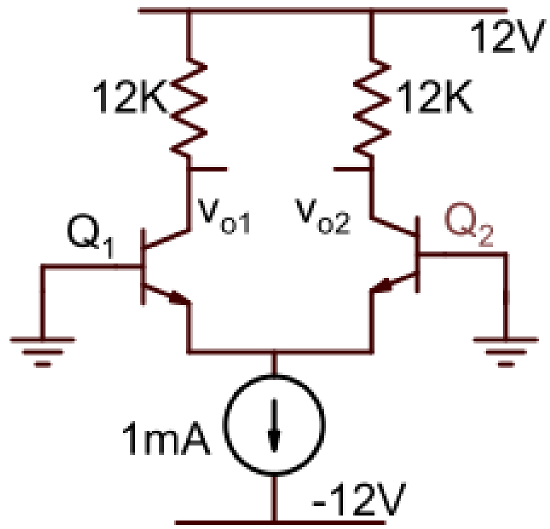
As V_{id} is small, small signal analysis can be applied.

$$I_{C1} = 0.5 + g_m \frac{v_{id}}{2}$$

$$I_{C2} = 0.5 - g_m \frac{v_{id}}{2}$$

Q.3 In the differential pair shown in Fig. 3, there is a mismatch between transistors Q1 and Q2: $I_{s1} = 1.5I_{s2}$. Determine collector currents of transistors Q1 and Q2.

Sol.



$$I_{c1} = I_{S1} \times \exp\left(\frac{V_{BE1}}{V_T}\right)$$

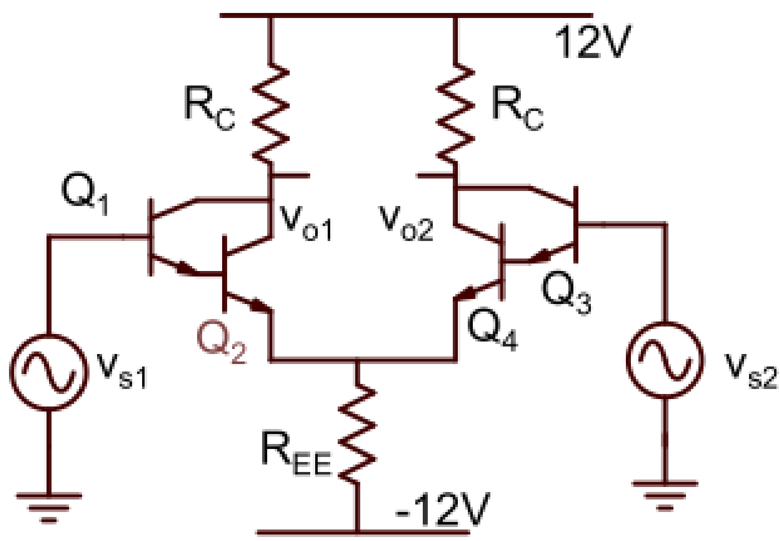
$$I_{c2} = I_{s2} \times \exp\left(\frac{V_{BE2}}{V_T}\right)$$

$$\frac{I_{c1}}{I_{c2}} = \frac{I_{s1}}{I_{s2}} = 1.5\,;\ I_{c1} + I_{c2} = 1\,mA$$

$$I_{c2} = 0.4\,mA\,;\ I_{c1} = 0.6\,mA$$

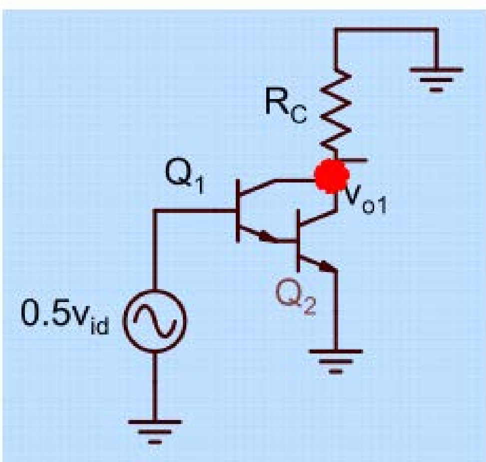
Q.4 Determine expressions for differential mode gain and input resistance for the amplifier shown in Fig. 4.

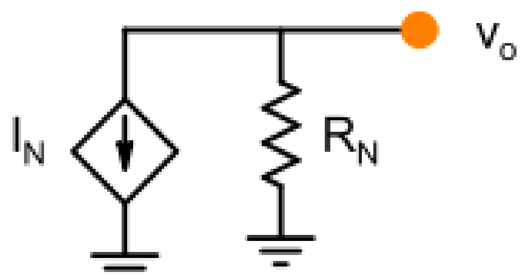
Sol.:



$$I_{c2} = \beta * I_{B2} = \beta * I_{E1} \sim \beta * I_{c1}$$

$$g_{m2} = \beta * g_{m1};\ r_{\pi 2} = \frac{r_{\pi 1}}{\beta}$$

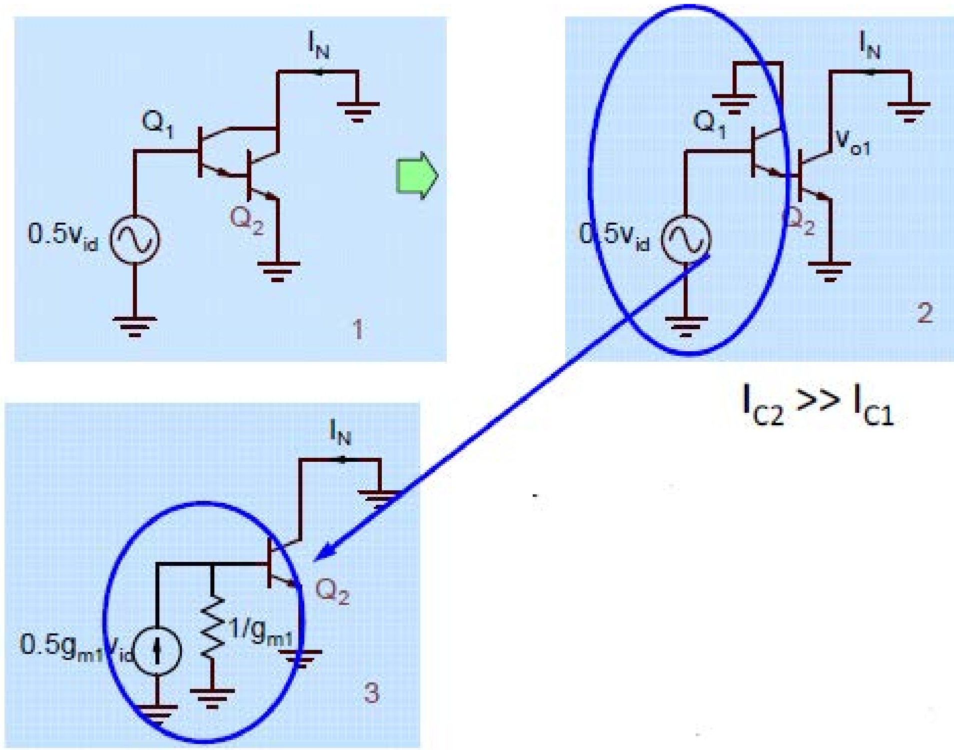




$$A_v = \frac{-i_N \times R_N}{v_s}$$

$$R_N = R_C$$

Norton's current



$$I_N = \beta * 0.5g_{m1}v_{id} * \frac{1/g_{m1}}{1/g_{m1} + r_{\pi 2}} = \frac{\beta}{1 + g_{m1} * r_{\pi 2}} * 0.5g_{m1}v_{id} = 0.25g_{m2}v_{id}$$

$$A_{dm} = -\frac{I_N R_N}{v_{id}} = -0.25g_{m2}R_C$$

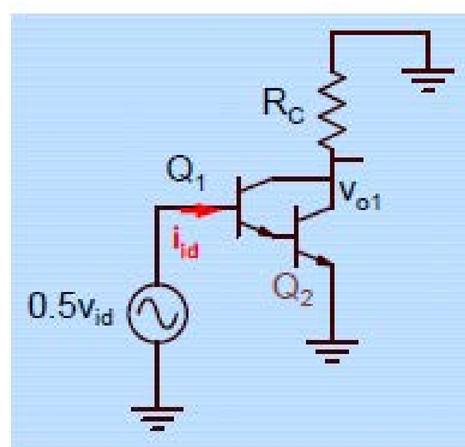
Input Resistance

$$R_{id} = \frac{v_{id}}{i_{id}}$$

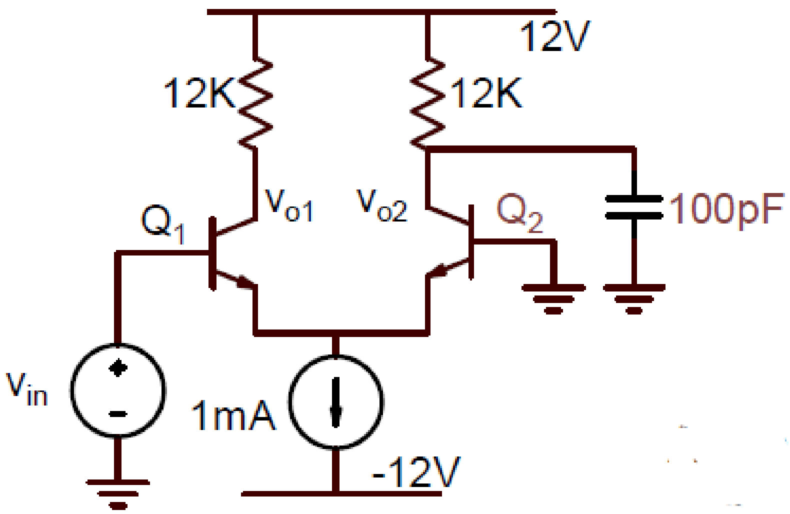
$$\frac{0.5v_{id}}{i_{id}} = r_{\pi 1} + (1+\beta)r_{\pi 2} \cong 2\beta r_{\pi 2}$$

$$R_{id} = 4\beta r_{\pi 2}$$

$$\frac{A_{dm} \times R_{id}}{R_o} \cong \beta^2$$



Q.5 For the differential amplifier shown in Fig. 5, a step input of 1V is applied to transistor Q1. Sketch the qualitative variation of output voltage. Determine the time taken for output voltage to reach 10V. Assume that transistors respond instantaneously to changes in voltage.



Sol.: A step input of 1V will shift the whole of current 1mA to Q1 and I_{C2} will be zero. The capacitor will charge through the 12K resistor from 6V to eventually 12V.

$$V_C(t) = V_C(\infty) - [V_C(\infty) - V_C(0)]e^{-t/\tau}$$

$$V_C(t) = 12 - 6 * \exp(-t/R_C C)$$

$$T = R_C C \times \ln(3) = 1.3\mu s$$

Q.6 (a) In the differential amplifier shown in Fig. 6a, determine the values of resistors R_C and R_{EE} such that differential input resistance is 52kΩ and differential mode gain is 100.

Sol.:

Use following relations:

$$A_{dm} = -0.5 g_m R_C$$

$$R_{id} = 2r_\pi$$

$$r_\pi = \frac{V_T}{I_{CQ1}} \times \beta;\ g_m = \frac{I_{CQ1}}{V_T}$$

$$I_{EE} = \frac{-0.7 - V_{EE}}{R_{EE}}$$

$$I_{CQ1} = I_{CQ2} = 0.5 I_{EE}$$

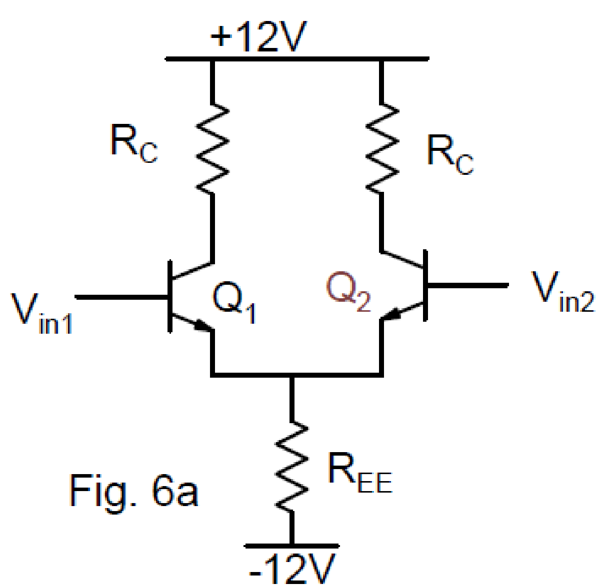
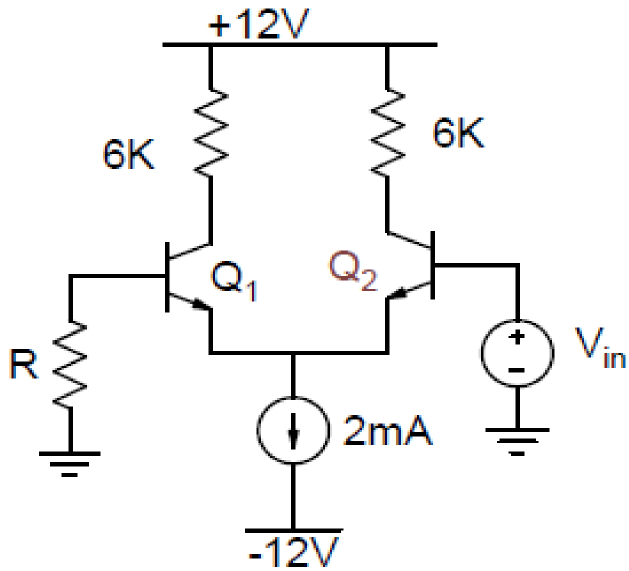
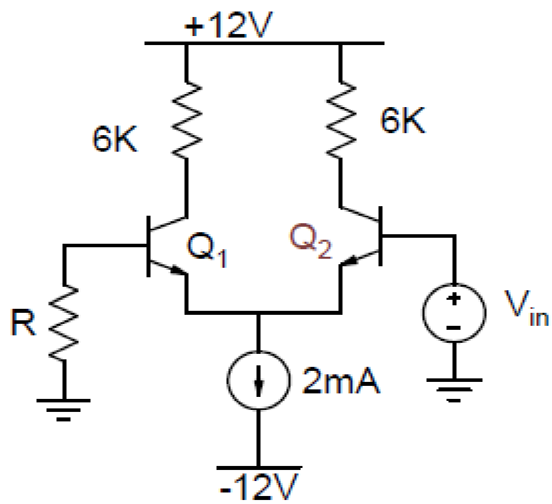


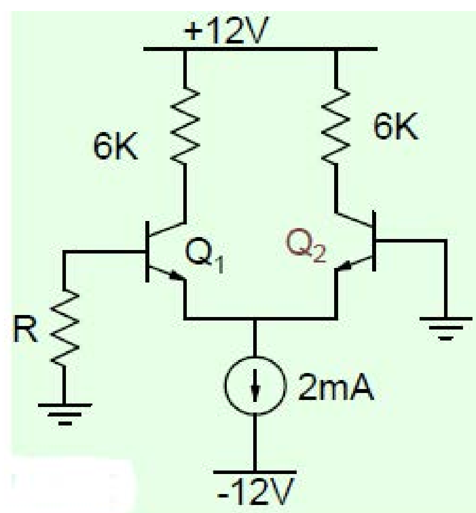
Fig. 6a

(b) In the differential amplifier shown in Fig. 6(b), for zero input voltage the dc value of collector voltages of the two transistors were measured to be 5.5V and 6.5V. Assign these values to appropriate transistors and justify your answer. Determine the approximate value of small input voltage (dc) that needs to be applied to make both collector voltages equal to 6V. Current source is ideal.



Sol.:





$$V_{B1} = -I_{B1} \times R < 0$$

$$I_{C1} < I_{C2}\,;\; V_{C1} > V_{C2}$$

$$V_{C1} = 6.5 = 6 + 0.5;\; V_{C2} = 5.5 = 6 - 0.5$$

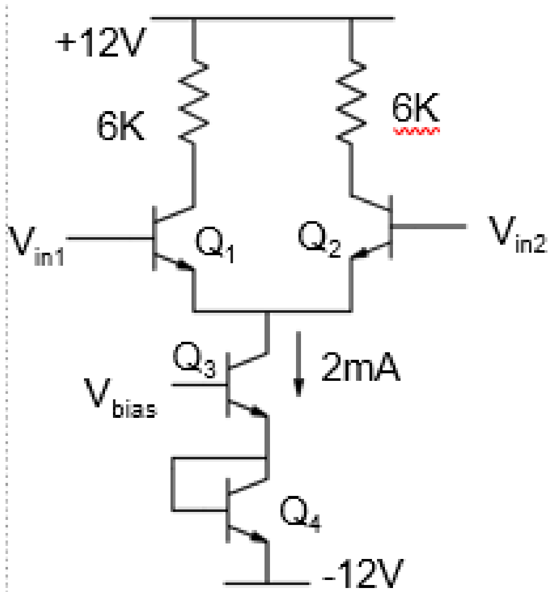
$$v_{id} = -v_{in}$$

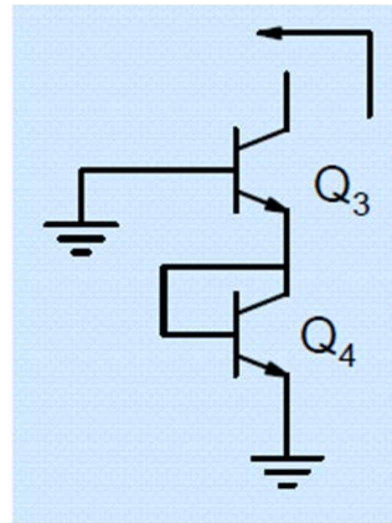
$$V_{C1} = 6.5 - A_{dm} \times v_{id}; V_{C2} = 5.5 + A_{dm} \times v_{id}$$

$$A_{dm} = -0.5 g_m R_C = -115.39$$

$$V_{C1} = V_{C2} \rightarrow v_{id} = -v_{in} = 8.7\text{mV}$$

Q.7 (a) Determine the common mode gain of the differential amplifier shown in Fig. 7(a).





Sol.:

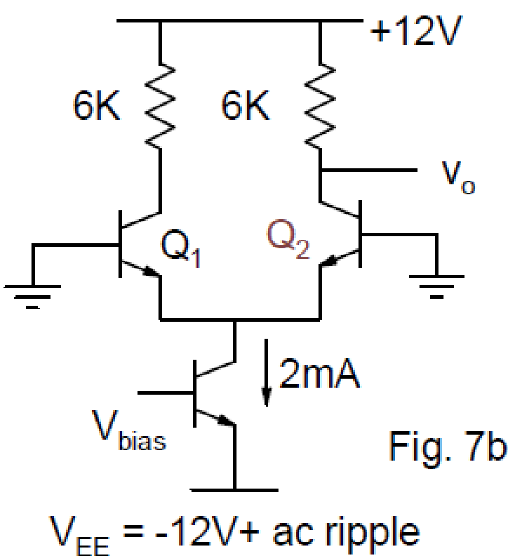
$$A_{cm} = -\frac{g_m}{1 + 2g_m R_{EE}} R_C$$

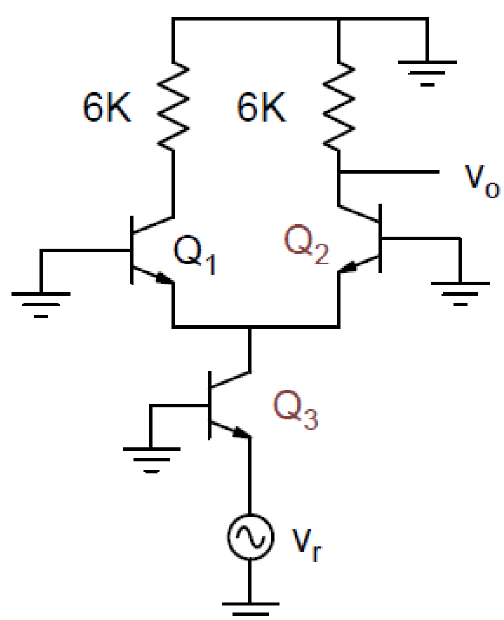
$$R_{EE} = r_{o3} * \left(1 + g_{m3} * \frac{1}{g_{m4}}\right) = 2r_{o3}$$

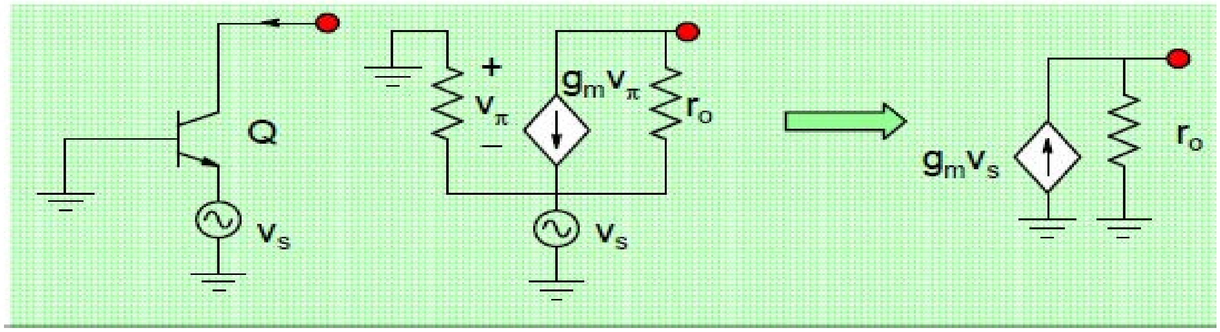
$$A_{cm} = -\frac{g_m}{1 + 4g_m r_{o3}} R_C$$

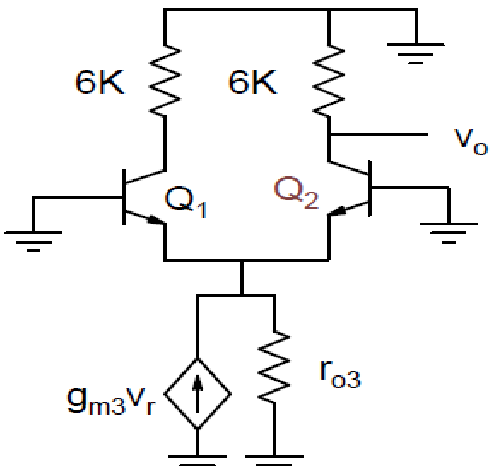
(b) In Fig. 7(b), the negative supply voltage V_{EE} has an ac ripple of 1mV magnitude. Determine the resulting ac ripple at the output.

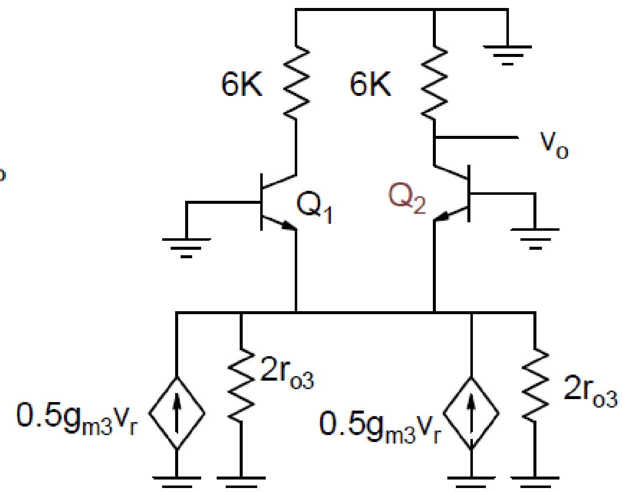
Sol.:







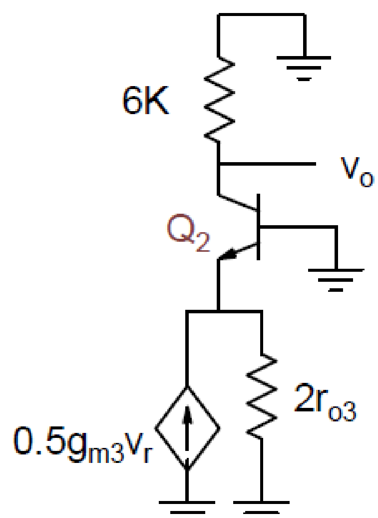




$$i_N = -0.5 g_{m3} v_r$$

$$R_N = 6k$$

$$v_o = -i_N \times R_N$$



Q.8 Determine the differential and common mode gain of the circuit shown below.

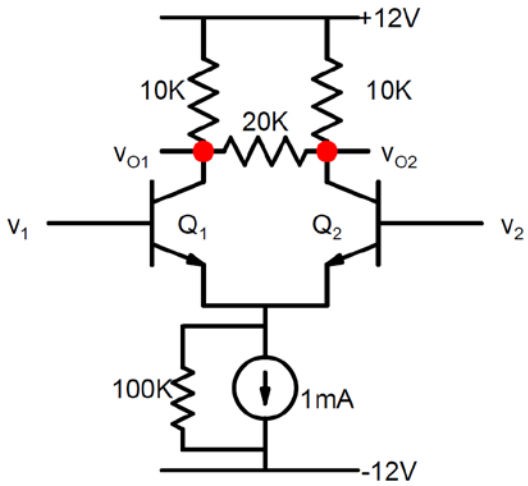
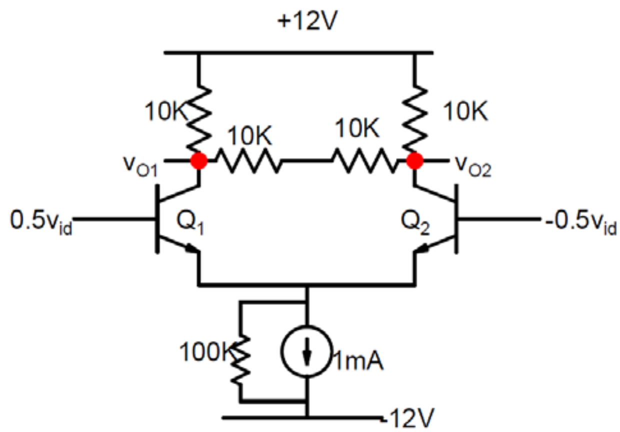
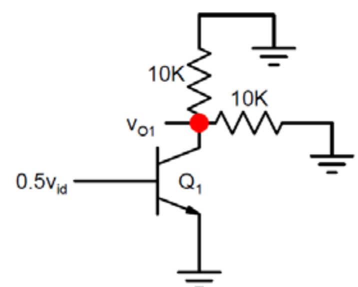


Fig. 8

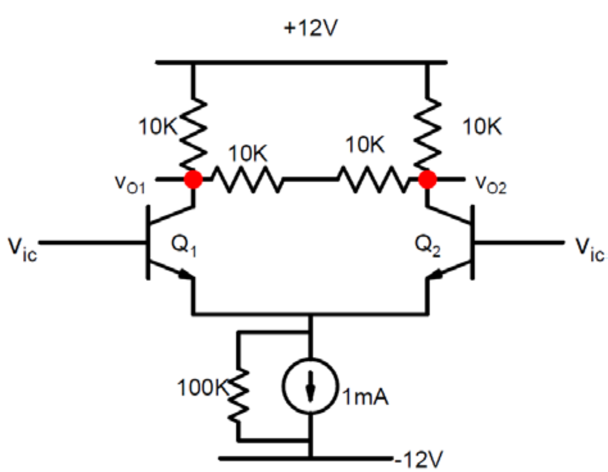
Sol.:

Differential mode analysis





Common Model analysis



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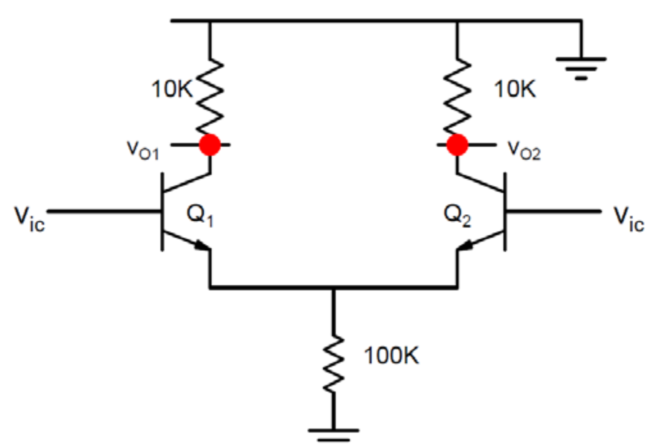
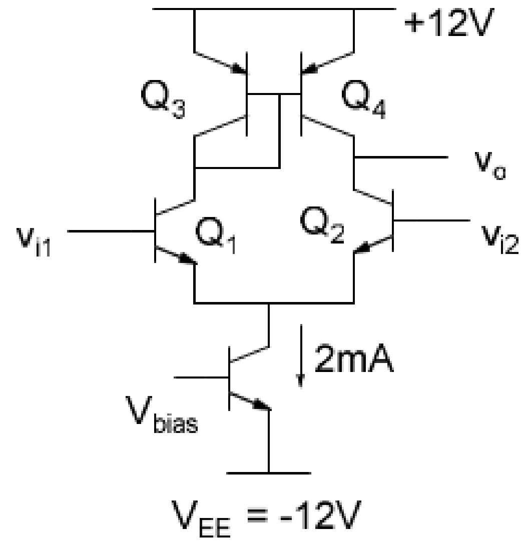


Remaining analysis is like that of conventional diff. amp.

Q.9 For the differential pair with current mirror load shown in Fig. 9, determine value of differential mode gain.



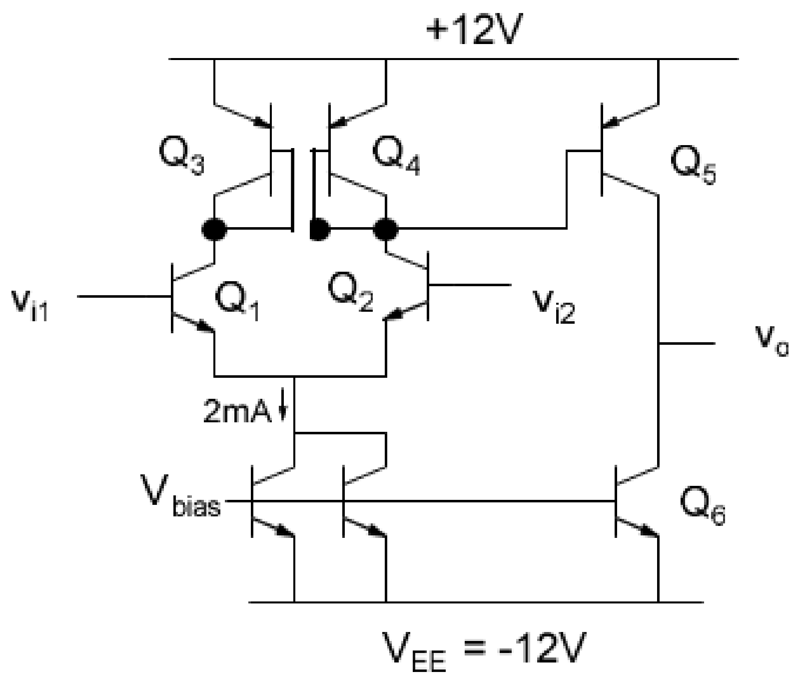
Sol.:

$$g_m = \frac{1}{26} = 0.038\Omega^{-1}$$

$$r_{o2} = r_{o4} = \frac{100}{10^{-3}} = 100k\Omega$$

$$A_{dm} = g_m \times r_{o2}||r_{o4} = 1.9 \times 10^3$$

Q.10 For the two-stage amplifier shown in Fig. 10, determine net differential voltage gain.



Sol.:

$$A_{dm} = A_{dm1} \times A_{v2}$$

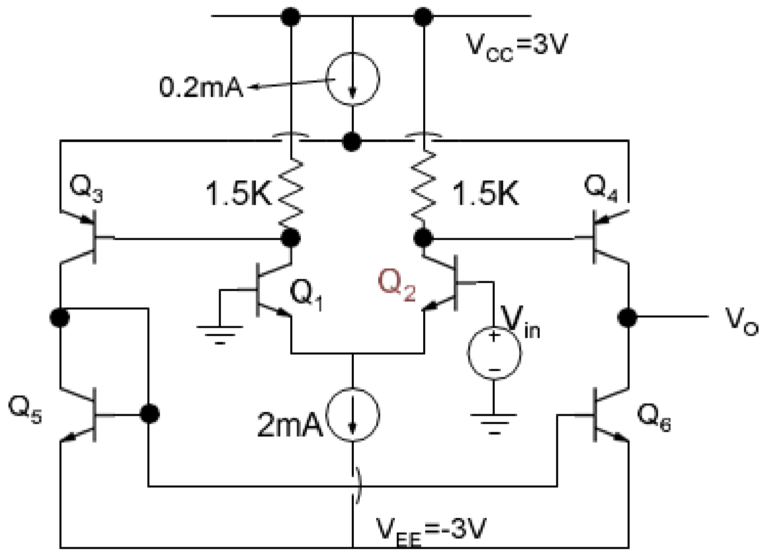
Because $r_{\pi 5} \gg \frac{1}{g_{m4}}$, there is little loading of first stage by second.

$$A_{dm1} = 0.5 g_{m2} \times \frac{1}{g_{m4}} \sim 0.5$$

$$A_{v2} = -g_{m5} \times r_{o5}||r_{o6}$$

Note that $I_{CQ5} = I_{CQ6} = 1mA$.

Q.11 Determine the small signal voltage gain of the amplifier shown in Fig. 11. Assume that current sources are ideal.



Sol.:

First stage is a diff. amp. with resistive load and current source biasing.

$$A_{dm1} = -0.5 g_m R_C || R_L$$

$$A_{cm1} = 0$$

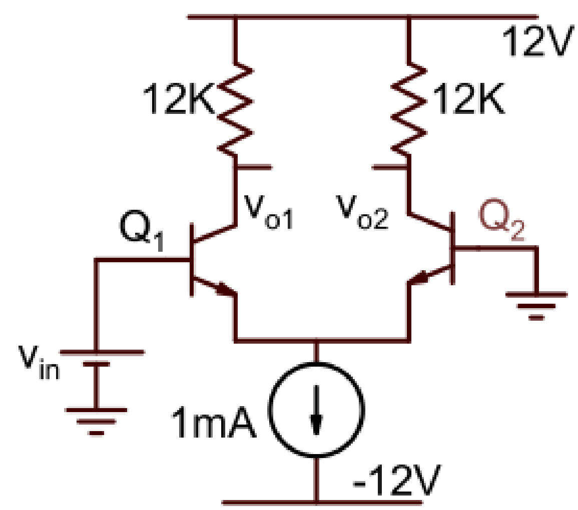
$$R_L = r_{\pi 3} = r_{\pi 4} = \frac{V_T}{0.1mA} \times \beta = 26K\Omega$$

The two outputs of first stage serve as inputs to second differential pair which has a current mirror load.

$$A_{dm2} = -g_{m4} \times r_{o4} || r_{o6}$$

$$v_o = A_{dm1} \times A_{dm2} \times v_{in}$$

Q.12 Determine the extra voltage V_{in} that needs to be applied to transistor Q1 shown in Fig. 12, in order to make collector currents of both the transistors identical, if $I_{s2} = 1.5 I_{s1}$.



Sol.:

$$\frac{I_{S1} \times \exp\left(\frac{V_{BE1}}{V_T}\right)}{I_{S2} \times \exp\left(\frac{V_{BE2}}{V_T}\right)} = 1$$

$$v_{in} = V_{BE1} - V_{BE2}$$

$$v_{in} = V_T \times \ln\left(\frac{I_{s2}}{I_{s1}}\right) = 10.54\ mV$$