EE 200 Signals, Systems & Networks 2018

Assignment 4

There are two circuits containing resistors, inductors, capacitors and ideal independent sources given below. The sources are sinusoidal, and the circuits are assumed to be in the steady state. Use phasor analysis to study the networks. Assign node and branch numbers and directions to the circuit that you will use for the rest of this assignment.

1 Write the reduced node incidence matrix $\mathbf{A}$. Choose a reference node. Get the node admittance matrix $\mathbf{Y}_n$ by inspection.

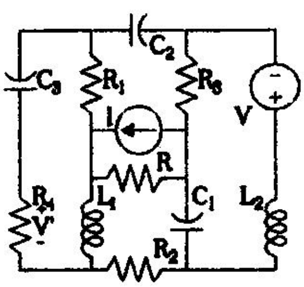
2 Write the mesh matrix $\mathbf{M}$. Get the mesh impedance matrix $\mathbf{Z}_m$ by inspection.

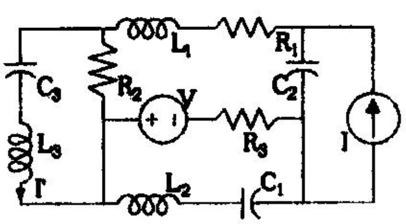
3 Select a tree for the graph. Write the fundamental cutset matrix $\mathbf{Q}$. Get the cutset admittance matrix $\mathbf{Y}_q$ by inspection.

4 For the same tree selected above, write the fundamental loop matrix $\mathbf{B}$. Get the loop impedance matrix $\mathbf{Z}_l$ by inspection.

5 Now replace the independent current source I in the first circuit by a dependent current source given by $I = V'$. Now write all the node KCL equations manually, and get the modified $\mathbf{Y}_n$. Write all the cutset KCL equations manually, and get the modified $\mathbf{Y}_q$.

6 Now replace the independent voltage source V in the second circuit by a dependent voltage source given by $V = I'$. Now write all the mesh KVL equations manually, and get the modified $\mathbf{Z}_m$. Write all the loop KVL equations manually, and get the modified $\mathbf{Z}_l$.





Assignment 4

1. Given Circuit

(a)

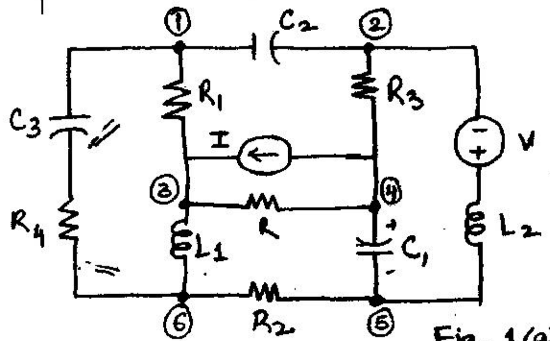


Fig. 1(a)

$\rightarrow$ To get reduced note incidence matrix A, moving to tree structure.

$\downarrow$

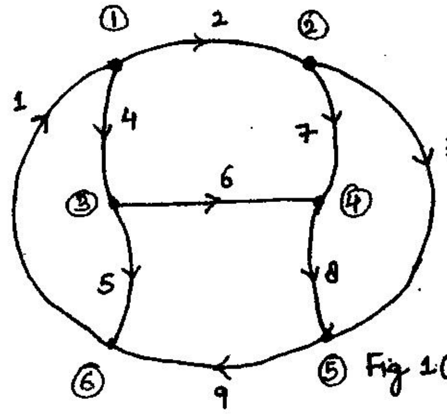


Fig 1(b)

$\rightarrow$ Considering Node 6 as datum Node

The following will represent Reduced Node Incidence Matrix

$$A i = 0$$

At node ① $-J_1 + J_2 + J_4 = 0$

② $-J_2 + J_7 + J_3 = 0$

③ $-J_4 + J_5 + J_6 = 0$

④ $-J_6 - J_7 + J_8 = 0$

⑤ $-J_8 - J_3 + J_9 = 0$

⑥ Datum Node.

According to book (desoer & Kuh)

J represents the branch current

Branches $\rightarrow$

Nodes $\downarrow$

$$\begin{array}{c} \begin{array}{ccccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \end{array} \\ \begin{bmatrix} -1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & -1 & 1 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & -1 & 1 \end{bmatrix} \end{array} \begin{bmatrix} J_1 \\ J_2 \\ J_3 \\ J_4 \\ J_5 \\ J_6 \\ \vdots \\ J_9 \end{bmatrix} = 0$$

The desired matrix.

(b)

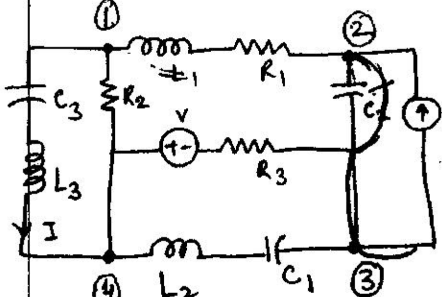


Fig 2(a)

$\longrightarrow$

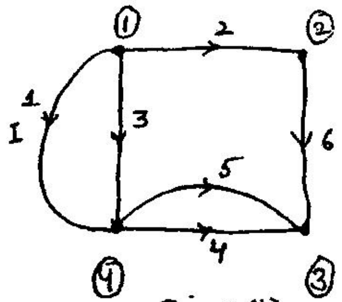


Fig 2(b).

At node ① we have $+i_1 + i_2 + i_3 = 0$

② $-i_2 + i_6 = 0$

③ $-i_4 - i_5 - i_6 = 0$

④ — Datum Node.

Reduced Node Incidence matrix $Ai = 0$ is represented as:

Branches $\longrightarrow$

nodes $\downarrow$

$$\begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -1 & -1 & -1 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ \vdots \\ i_6 \end{bmatrix} = 0$$

Node admittance matrix for part (a)

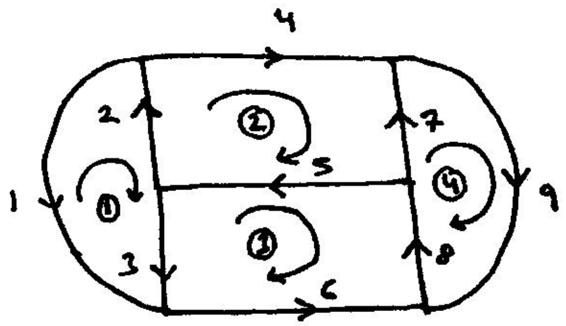
$$Y_n = \begin{bmatrix} \frac{1}{R_4+\frac{1}{J\omega C_3}} + J\omega C_2 + \frac{1}{R_1} & -J\omega C_2 & -\frac{1}{R_1} & 0 & 0 \\ -J\omega C_2 & \frac{1}{R_3} + J\omega C_2 + \frac{1}{J\omega L_2} & 0 & -\frac{1}{R_3} & -\frac{1}{J\omega L_2} \\ -\frac{1}{R_1} & 0 & \frac{1}{R_1} + \frac{1}{J\omega L_1} + \frac{1}{R} & -\frac{1}{R} & 0 \\ 0 & -\frac{1}{R_3} & -\frac{1}{R} & \frac{1}{R} + \frac{1}{R_3} + J\omega C_1 & -J\omega C_1 \\ 0 & -\frac{1}{J\omega L_2} & 0 & -J\omega C_1 & \frac{1}{R_2} + J\omega C_1 + \frac{1}{J\omega L_2} \end{bmatrix}$$

Node admittance matrix for part (b)

$$Y_n = \begin{bmatrix} \frac{1}{J\omega L_3 + \frac{1}{J\omega C_3}} + \frac{1}{R_1 + J\omega L_1} + \frac{1}{R_2} & -\frac{1}{R_1 + J\omega L_1} & 0 \\ -\frac{1}{R_1 + J\omega L_1} & \frac{1}{R_1 + J\omega L_1} + J\omega C_2 & -J\omega C_2 \\ 0 & -J\omega C_2 & \frac{1}{J\omega L_2 + \frac{1}{J\omega C_1}} + \frac{1}{R_3} (+ J\omega C_2) \end{bmatrix}$$

Question - 2 (a)

Ideal voltage sources are short ckt and ideal current sources are open ckt



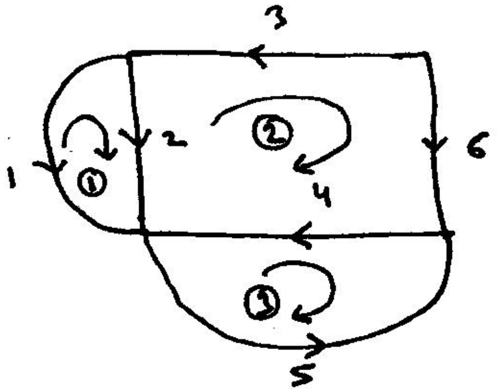
Mesh incident Matrix

$$M = \begin{array}{c} \\ ① \\ ② \\ ③ \\ ④ \end{array} \begin{array}{c} \begin{array}{ccccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \end{array} \\ \begin{bmatrix} -1 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 & -1 & -1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \end{bmatrix} \end{array}$$

Mesh impedance Matrix,

$$Z_m = \begin{bmatrix} R_4 + \frac{1}{j\omega C_3} + R_1 + j\omega L_1 & -R_1 & -j\omega L_1 & 0 \\ -R_1 & R + R_1 + \frac{1}{j\omega C_2} + R_3 & -R & -R_3 \\ -j\omega L_1 & -R & R + R_2 + j\omega L_1 + \frac{1}{j\omega C_1} & -\frac{1}{j\omega C_1} \\ 0 & -R_3 & -\frac{1}{j\omega C_1} & R_3 + j\omega L_2 + \frac{1}{j\omega C_4} \end{bmatrix}$$

Question - 2. (b)



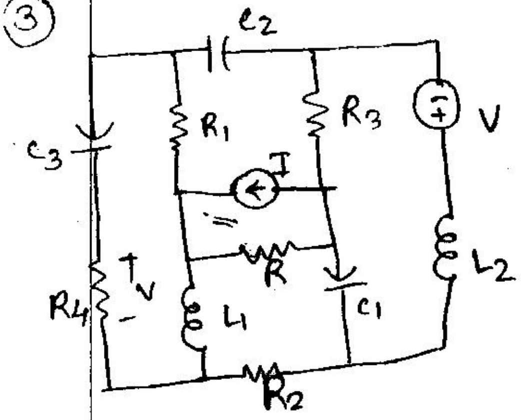
Mesh - incident Matrix

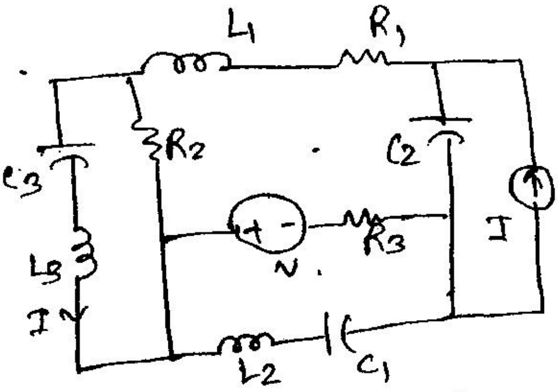
$$
M = \begin{array}{c} \\ ① \\ ② \\ ③ \end{array}
\begin{array}{c}
\begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\
\begin{bmatrix} -1 & 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & -1 & 1 & 0 & 1 \\ 0 & 0 & 0 & -1 & -1 & 0 \end{bmatrix}
\end{array}
$$

Mesh Impedence Matrix,

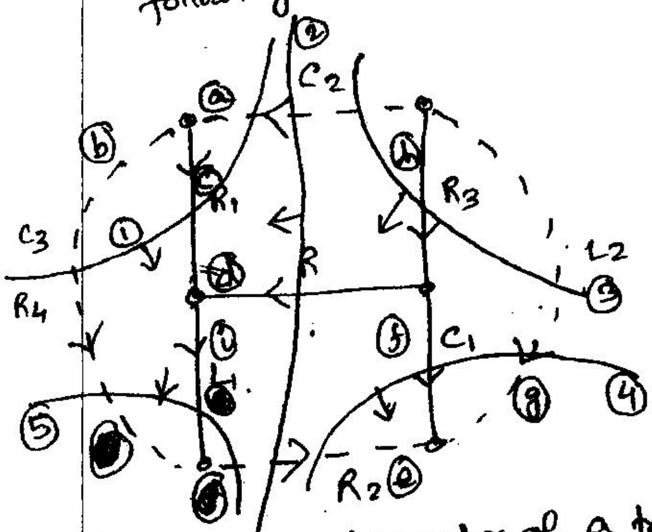
$$Z_m = \begin{bmatrix} j\omega L_3 + \frac{1}{j\omega C_3} + R_2 & -R_2 & 0 \\ -R_2 & j\omega L_1 + R_1 + R_2 + R_3 + \frac{1}{j\omega C_2} & -R_3 \\ 0 & -R_3 & j\omega L_2 + \frac{1}{j\omega C_1} + R_3 \end{bmatrix}$$

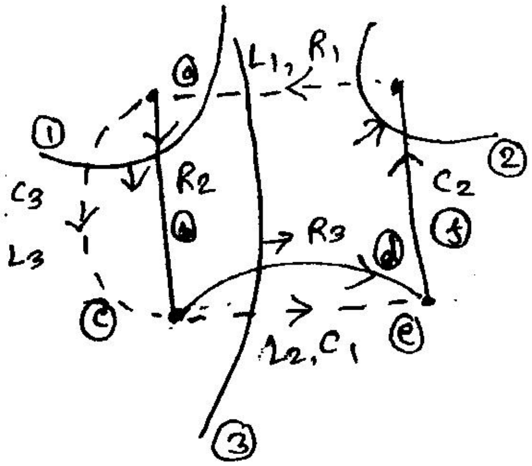
③





For the given graphs consider a tree in each of the following.





where —— branches of a tree.

- - - links of the graph

The fundamental cut-set matrix Q is defined as follows:-

if $q_{ik} \in Q$. Then.

$$q_{ik} = \begin{cases} 1 & \rightarrow \text{if branch } k \text{ belongs to cut set } i \text{ and direction agrees} \\ -1 & \rightarrow \text{" " but in "opposite" directions "} \\ 0 & \rightarrow \text{if branch } k \text{ does not belong to cut set } i \end{cases}$$

The branch number has been given in the original ~~ckt~~ ckt.

$$
1 = \begin{array}{c}
\begin{matrix} a & b & c & d & e & f & g & h & i \end{matrix} \\
\left[\begin{matrix}
-1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\
0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 \\
0 & 1 & 0 & 0 & -1 & 0 & 0 & 0 & 1
\end{matrix}\right]
\end{array}
$$

$$
Q_2 = \begin{array}{c|c}
 & \begin{matrix} a & b & c & d & e & f \end{matrix} \\
\begin{matrix} ① \\ ② \\ ③ \\ ④ \end{matrix} &
\left[\begin{matrix}
-1 & 1 & 1 & 0 & 0 & 0 \\
-1 & 0 & 0 & 0 & 0 & 1 \\
-1 & 0 & 0 & 1 & 1 & 0 \\
 & & & & &
\end{matrix}\right]
\end{array}
$$

The cut-set admittance matrix Y_q by inspection for the given circuits are as follows:-

$$
Y_{q1} = \begin{array}{c}
\begin{matrix} ① \\ \\ ③ \\ ④ \\ ⑤ \end{matrix}
\end{array}
\left[\begin{matrix}
G_a+G_b+G_c & -G_a & -G_a & 0 & G_b \\
-G_a & G_a+G_d+G_e & G_a & -G_e & +G_e \\
-G_a & G_a & G_a+G_g+G_h & G_g & 0 \\
0 & -G_e & G_g & G_e+G_f+G_g & -G_e \\
G_b & +G_e & 0 & -G_e & G_b+G_e+G_i
\end{matrix}\right]
$$

$$
Y_{q2} = \begin{array}{c}
\begin{matrix} ① \\ ② \\ ③ \\ ④ \end{matrix}
\end{array}
\left[\begin{matrix}
G_a+G_b+G_c & +G_a & +G_a \\
+G_a & G_a+G_f & +G_a \\
+G_a & +G_a & G_a+G_d+G_e \\
 & &
\end{matrix}\right]
$$

Where,

For circuit 1

$G_a = j\omega C_2$

$G_b = \dfrac{1}{R_4 + \frac{1}{j\omega C_3}}$

$G_c = \dfrac{1}{R_1}$

$G_d = \dfrac{1}{R}$

$G_e = \dfrac{1}{R_2}$

$G_f = j\omega C_1$

$G_g = \dfrac{1}{j\omega L_2}$

$G_h = \dfrac{1}{R_3}$

$G_i = \dfrac{1}{j\omega L_1}$

For circuit 2

$G_a = \dfrac{1}{R_1 + j\omega L_1}$

$G_b = \dfrac{1}{R_2}$

$G_c = \dfrac{1}{j\omega L_3 + \frac{1}{j\omega C_3}}$

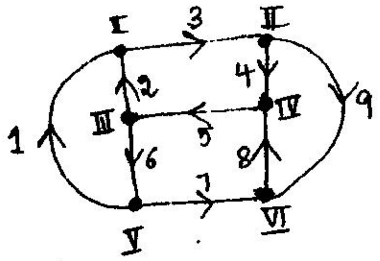
$G_d = \dfrac{1}{R_3}$

$G_e = \dfrac{1}{j\omega L_2 + \frac{1}{j\omega C_1}}$

$G_f = j\omega C_2$

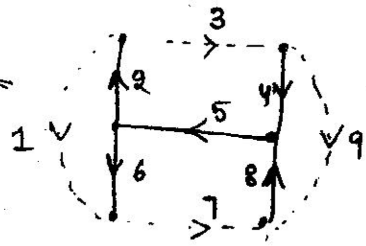
EE 200 Assignment-4

(4.1) The Graph for the given circuit is below:



given graph consists of branches $b = 9$ & vertices $n_t = 6$

Selecting one tree:



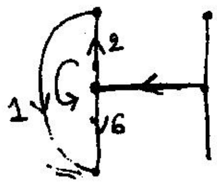
The branches $(n_t - 1 = 6-1 = 5)$ 2, 4, 5, 6, 8 are twigs & 1, 3, 9, 7 are links.

Every link defines ~~the~~ a fundamental loop of the network.

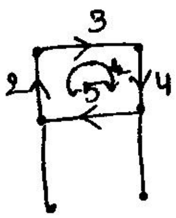
The no. of fundamental loops of this graph will be $b - n_t + 1$

$= 9-6+1 = 4$

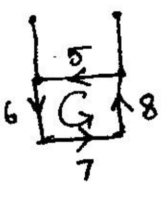
Let l_1, l_2, l_3 & l_4 be the f-loops.



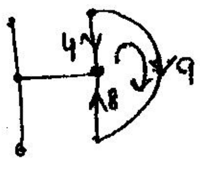
$l_1: \quad V_1 - V_6 + V_2 = 0$



$l_2: \quad V_3 + V_4 + V_5 + V_2 = 0$



$l_3: \quad V_8 + V_7 + V_5 + V_6 = 0$



$l_4: \quad V_9 + V_8 - V_4 = 0$

Required f-loop matrix:

branches $\longrightarrow$

loops $\downarrow$

$$= \begin{array}{c} \\ l_1 \\ l_2 \\ l_3 \\ l_4 \end{array} \begin{array}{c} \begin{array}{ccccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \end{array} \\ \left[\begin{array}{ccccccccc} 1 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 & 1 \end{array}\right] \end{array}$$

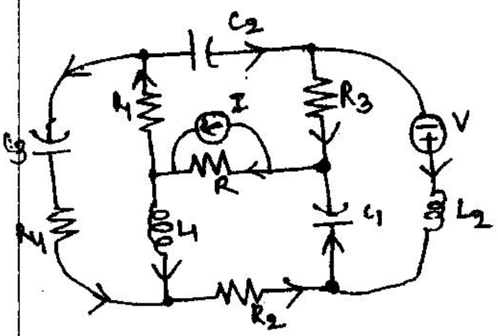
B is $l \times b$ matrix

$$BV = 0$$

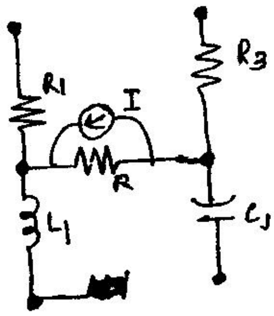
$$B \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \\ V_7 \\ V_8 \\ V_9 \end{bmatrix} = 0$$

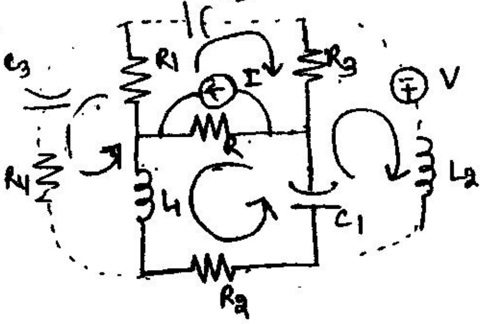
$$b_{iK} = \begin{cases} 1 & \text{if brank K is in loop i \& same as ref direction} \\ -1 \rightarrow & \text{" " \& opp ref direction} \\ 0 \rightarrow & \text{if brank K is not in loop i.} \end{cases}$$

For Loop impedance matrix Z_l:



Tree :

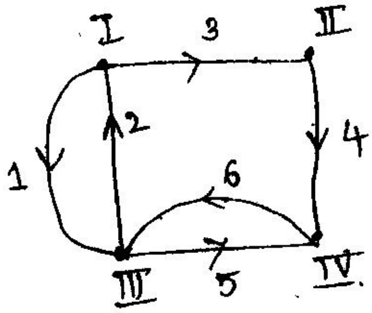




(2)

$$Z_l = \begin{bmatrix} R_1+R_4+j\omega L_1+\frac{1}{j\omega C_3} & +R_1 & -j\omega L_1 & 0 \\ +R_1 & \frac{1}{j\omega C_2}+R_1+R_3+R & R & -R_3 \\ -j\omega L_1 & R & R+j\omega L_1+R_2+\frac{1}{j\omega C_1} & \frac{1}{j\omega C_1} \\ 0 & -R_3 & \frac{1}{j\omega C_1} & R_3+\frac{1}{j\omega C_1}+j\omega L_2 \end{bmatrix}$$

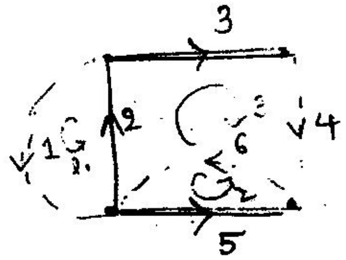
For the second circuit, the graph is



No. of branches $b = 6$

Nodes $n_t = 4$

considering one tree for the above graph :



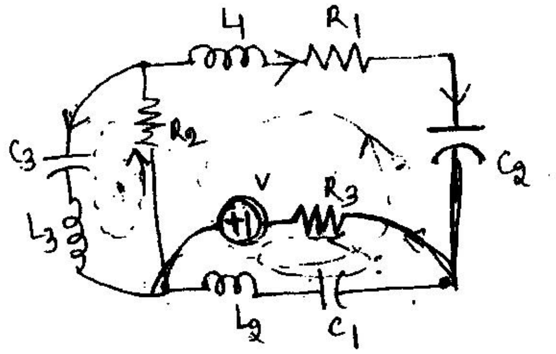
Twigs = 2, 3, 5

links = 1, 4, 6

No. of fundamental loops we get are three.

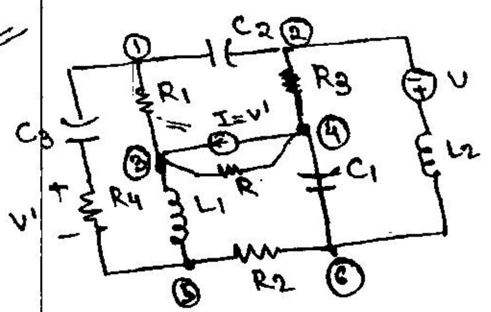
$$B = \begin{array}{c} \\ l_1 \\ l_2 \\ l_3 \end{array} \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 & -1 & 0 \end{bmatrix} \end{array}$$

Loop impedance matrix Z_l:



$$Z_l = \begin{bmatrix} \frac{1}{j\omega C_3} + j\omega L_3 + R_2 & 0 & -R_2 \\ 0 & R_3 + j\omega L_2 + \frac{1}{j\omega C_1} & j\omega L_2 + \frac{1}{j\omega C_1} \\ -R_2 & j\omega L_2 + \frac{1}{j\omega C_1} & j\omega L_1 + R_1 + \frac{1}{j\omega C_2} + R_2 + j\omega L_2 + \frac{1}{j\omega C_1} \end{bmatrix}$$

J) ①



each node i has voltage e_i

$$V' = \frac{(e_1-e_5)R_4}{R_4+\frac{1}{j\omega C_3}}.$$

node ①:

$$\frac{1}{R_4+\frac{1}{j\omega C_3}}(e_1-e_5) + \frac{1}{R_1}(e_1-e_3) + (e_1-e_2)j\omega C_2 = 0$$

$$\boxed{e_1\left(\frac{1}{R_4+\frac{1}{j\omega C_3}} + \frac{1}{R_1} + j\omega C_2\right) - j\omega C_2 e_2 - \frac{1}{R_1}e_3 - \frac{1}{R_4+\frac{1}{j\omega C_3}}e_5 = 0} \rightarrow ①$$

node ②: $$j\omega C_2(e_2-e_1) + \frac{1}{R_3}(e_2-e_4) + \frac{1}{j\omega L_2}(e_2+v-e_6) = 0$$

$$\boxed{-j\omega C_2 e_1 + \left(j\omega C_2 + \frac{1}{R_3} + \frac{1}{j\omega L_2}\right)e_2 - \frac{1}{R_3}e_4 - \frac{e_6}{j\omega L_2} = -\frac{v}{j\omega L_2}} \rightarrow ②$$

node ③

$$\frac{1}{R_1}(e_3-e_1) + \frac{1}{j\omega L_1}(e_3-e_5) + \frac{1}{R}(e_3-e_4) = V' = \frac{(e_1-e_5)R_4}{R_4+\frac{1}{j\omega C_3}}$$

$$\boxed{-\left(\frac{1}{R_1} + \frac{R_4}{R_4+\frac{1}{j\omega C_3}}\right)e_1 + e_3\left(\frac{1}{R_1} + \frac{1}{j\omega L_1} + \frac{1}{R}\right) - \frac{e_4}{R} + e_5\left[\frac{R_4}{R_4+\frac{1}{j\omega C_3}} - \frac{1}{j\omega L_1}\right] = 0} \rightarrow ③$$

node ④: $$(e_4-e_2)\frac{1}{R_3} + j\omega C_1(e_4-e_6) + \frac{(e_4-e_3)}{R} = -V' = \frac{-(e_1-e_5)R_4}{R_4+\frac{1}{j\omega C_3}}$$

$$\boxed{\frac{R_4}{R_4+\frac{1}{j\omega C_3}}e_1 - \frac{1}{R_3}e_2 - \frac{1}{R}e_3 + \left(\frac{1}{R_3} + \frac{1}{R} + j\omega C_1\right)e_4 - \frac{R_4}{R_4+\frac{1}{j\omega C_3}}e_5 - j\omega C_1 e_6 = 0} \rightarrow ④$$

node ⑤

$$\frac{(e_5-e_1)}{R_4+\frac{1}{j\omega C_3}} + \frac{(e_5-e_3)}{j\omega L_1} + \frac{(e_5-e_6)}{R_2} = 0$$

$$\boxed{-\frac{1}{R_4+\frac{1}{j\omega C_3}}e_1 - \frac{1}{j\omega L_1}e_3 + \left(\frac{1}{R_4+\frac{1}{j\omega C_3}} + \frac{1}{j\omega L_1} + \frac{1}{R_2}\right)e_5 - \frac{1}{R_2}e_6 = 0} \rightarrow ⑤$$

node ⑥

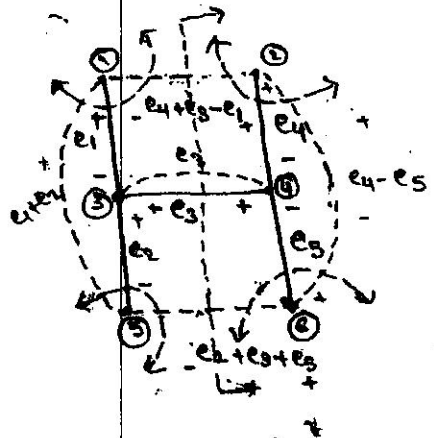
$$j\omega C_1(e_6-e_4) + \frac{1}{R_2}(e_6-e_5) + \frac{1}{j\omega L_2}(e_6-e_2-V) = 0$$

$$\boxed{-\frac{1}{j\omega L_2}e_2 - j\omega C_1 e_4 - \frac{1}{R_2}e_5 + \left(j\omega C_1 + \frac{1}{R_2} + \frac{1}{j\omega L_2}\right)e_6 = \frac{V}{j\omega L_2}} \rightarrow ⑥$$

$$\begin{bmatrix}
\left(\frac{1}{R_4+\frac{1}{j\omega C_3}}+\frac{1}{R_1}+j\omega C_2\right) & -j\omega C_2 & -\frac{1}{R_1} & 0 & -\frac{1}{R_4+\frac{1}{j\omega C_3}} & 0 \\
-j\omega C_2 & \left(j\omega C_2+\frac{1}{R_3}+\frac{1}{j\omega L_2}\right) & 0 & -\frac{1}{R_3} & 0 & -\frac{1}{j\omega L_2} \\
-\left(\frac{1}{R_1}+\frac{R_4}{R_4+\frac{1}{j\omega C_3}}\right) & 0 & \left(\frac{1}{R_1}+\frac{1}{R}+\frac{1}{j\omega L_1}\right) & -\frac{1}{R} & \left(\frac{R_4}{R_4+\frac{1}{j\omega C_3}}-\frac{1}{j\omega L_1}\right) & 0 \\
\frac{R_4}{R_4+\frac{1}{j\omega C_3}} & -\frac{1}{R_3} & -\frac{1}{R} & \left(\frac{1}{R_3}+\frac{1}{R}+j\omega C_1\right) & -\frac{R_4}{R_4+\frac{1}{j\omega C_3}} & -j\omega C_1 \\
-\frac{1}{R_4+\frac{1}{j\omega C_3}} & 0 & -\frac{1}{j\omega L_1} & 0 & \left(\frac{1}{R_4+\frac{1}{j\omega C_3}}+\frac{1}{j\omega L_1}+\frac{1}{R_2}\right) & -\frac{1}{R_2} \\
0 & -\frac{1}{j\omega L_2} & 0 & -j\omega C_1 & -\frac{1}{R_2} & \left(j\omega C_1+\frac{1}{R_2}+\frac{1}{j\omega L_2}\right)
\end{bmatrix}$$

$$= Y_n$$

$$\times [e_1 \ e_2 \ e_3 \ e_4 \ e_5 \ e_6]^T = \left[0 \ \frac{-V}{j\omega L_2} \ 0 \ 0 \ 0 \ \frac{V}{j\omega L_2}\right]^T$$



choosing Tree and branch voltages as shown in figure.

$$V' = \frac{(e_1+e_2)\,R_4}{R_4+\frac{1}{j\omega C_3}}$$

Applying KCL at cutsets

at ①

$$\frac{1}{R_4+\frac{1}{j\omega C_3}}(e_1+e_2) + \frac{1}{R_1}e_1 - (e_4+e_3-e_1)\,j\omega C_2 = 0$$

$$\boxed{e_1\left(\frac{1}{R_4+\frac{1}{j\omega C_3}}+\frac{1}{R_1}+j\omega C_2\right) + e_2\left(\frac{1}{R_4+\frac{1}{j\omega C_3}}\right) - e_3\,j\omega C_2 - e_4\,j\omega C_2 = 0} \rightarrow ①$$

cutset around node ②

$$(e_4+e_3-e_1)\,j\omega C_2 + e_4\left(\frac{1}{R_3}\right) + (e_4-e_5)\frac{1}{j\omega L_2} = \frac{-V}{j\omega L_2}$$

$$\boxed{-j\omega C_2\,e_1 + j\omega C_2\,e_3 + e_4\left(j\omega C_2+\frac{1}{R_3}+\frac{1}{j\omega L_2}\right) - \frac{1}{j\omega L_2}e_5 = \frac{-V}{j\omega L_2}} \rightarrow ②$$

cutset around node ⑤

$$\frac{+e_2}{j\omega L_1} + (e_1+e_2)\left(\frac{1}{R_4+\frac{1}{j\omega C_3}}\right) + (e_2+e_3+e_5)\left(\frac{1}{R_2}\right) = 0$$

$$e_1\left(\frac{1}{R_4+\frac{1}{j\omega C_3}}\right) + e_2\left(\frac{1}{j\omega L_1}+\frac{1}{R_4+\frac{1}{j\omega C_3}}+\frac{1}{R_2}\right) + \frac{1}{R_2}e_3 + \frac{1}{R_2}e_5 = 0$$

=0

→①

$\frac{1}{\omega L_2}$ →②

3

$\frac{1}{j\omega L_1}$]

→③

4

3

Cutset around node 6

$$(e_2+e_3+e_5)\frac{1}{R_2} + (+e_5)\,j\omega c_1 - \frac{(e_4-e_5)}{j\omega L_2} = \frac{v}{j\omega L_2}$$

$$e_2\frac{1}{R_2} + \frac{1}{R_2}e_3 - \frac{1}{j\omega L_2}e_4 + e_5\left(\frac{1}{R_2}+j\omega c_1+\frac{1}{j\omega L_2}\right) = \frac{v}{j\omega L_2} \longrightarrow (4)$$

applying KCL at middle cutset

Fatin.

$$\frac{e_3}{R} + (e_4+e_3-e_1)\,j\omega c_2 + (e_2+e_3+e_5)\left(\frac{1}{R_2}\right) = -I$$

$$= -v' = -\frac{(e_1+e_2)R_4}{R_4+\frac{1}{j\omega c_3}}$$

$$\left(\frac{R_4}{R_4+\frac{1}{j\omega c_3}} - j\omega c_2\right)e_1 + \left(\frac{1}{R_2}+\frac{R_4}{R_4+\frac{1}{j\omega c_3}}\right)e_2$$

$$+ e_3\left(\frac{1}{R}+j\omega c_2+\frac{1}{R_2}\right) + j\omega c_2\, e_4$$

$$+ \frac{1}{R_2}e_5 = 0 \longrightarrow (5)$$

Now from eq

Proper order

$$\begin{bmatrix} \left(\frac{1}{R_4+\frac{1}{j\omega c_3}}+\frac{1}{R_1}+\right. \\ \vdots \\ \frac{1}{R_4+\frac{1}{j\omega c_3}} \\ \vdots \\ \left(\frac{R_4}{R_4+\frac{1}{j\omega c_3}}-j\omega c_2\right) \\ \vdots \\ -j\omega c_2 \\ \vdots \\ 0 \end{bmatrix}$$

$\times\ [e_1$

$=$

Now from eq. 1, 2, 3, 4 & 5 and arranging in proper order:

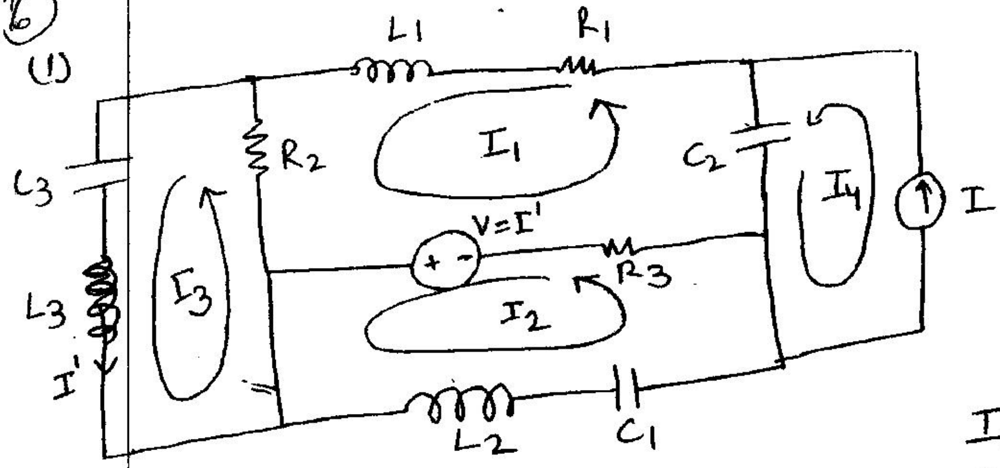
$$\underbrace{\begin{bmatrix} \left(\frac{1}{R_4+\frac{1}{J\omega C_3}}+\frac{1}{R_1}+J\omega C_2\right) & \frac{1}{R_4+\frac{1}{J\omega C_3}} & -J\omega C_2 & -J\omega C_2 & 0 \\ \frac{1}{R_4+\frac{1}{J\omega C_3}} & \left(\frac{1}{J\omega L_1}+\frac{1}{R_4+\frac{1}{J\omega C_3}}+\frac{1}{2}\right) & \frac{1}{R_2} & 0 & \frac{1}{R_2} \\ \left(\frac{R_4}{R_4+\frac{1}{J\omega C_3}}-J\omega C_2\right) & \left(\frac{1}{R_2}+\frac{R_4}{R_4+\frac{1}{J\omega C_3}}\right) & \left(\frac{1}{R}+J\omega C_2+\frac{1}{R_2}\right) & J\omega C_2 & \frac{1}{R_2} \\ -J\omega C_2 & 0 & J\omega C_2 & \left(J\omega C_2+\frac{1}{R_3}+\frac{1}{J\omega L_2}\right) & -\frac{1}{J\omega L_2} \\ 0 & \frac{1}{R_2} & \frac{1}{R_2} & \frac{-1}{J\omega L_2} & \frac{1}{R_2}+J\omega C_1+\frac{1}{J\omega L_2} \end{bmatrix}}_{Y_{av}}$$

$$\times \begin{bmatrix} e_1 & e_2 & e_3 & e_4 & e_5 \end{bmatrix}^T$$

$$= \begin{bmatrix} 0 & 0 & 0 & \frac{-V}{J\omega L_2} & \frac{V}{J\omega L_2} \end{bmatrix}^T$$

(b)

(1)



$I_4 = I$

$V = I_3 = I'$

mesh-1

$$\left(j\omega L_1 + R_1 + \frac{1}{j\omega C_2} + R_3 + R_2\right) I_1 - R_3 I_2 - R_2 I_3 + I_3 - \frac{1}{j\omega C_2} I = 0$$

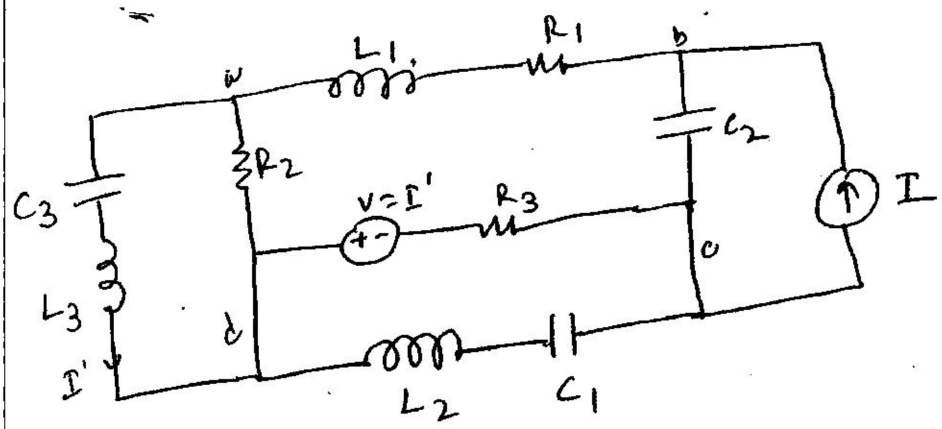
mesh-2

$$\left(R_3 + \frac{1}{j\omega C_1} + j\omega L_2\right) I_2 + (-R_3 I_1) - I_3 = 0$$

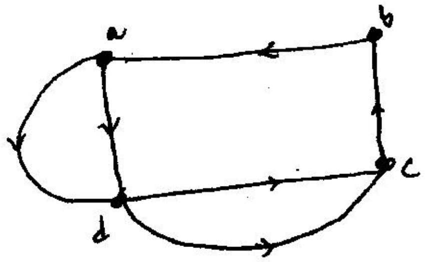
mesh-3 $\left(R_2 + j\omega L_3 + \frac{1}{j\omega C_3}\right) I_3 - R_2 I_1 = 0$

$$\begin{bmatrix} R_1 + R_2 + R_3 + j\omega L_1 + \frac{1}{j\omega L_2} & -R_3 & -R_2 + 1 \\ -R_3 & R_3 + \frac{1}{j\omega C_1} + j\omega L_2 & -1 \\ -R_2 & 0 & R_2 + j\omega L_3 + \frac{1}{j\omega C_3} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} \frac{I}{j\omega C_2} \\ 0 \\ 0 \end{bmatrix}$$

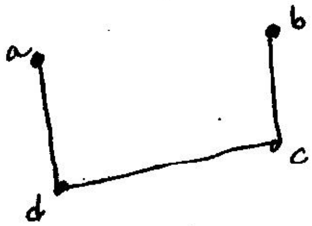
(ii)



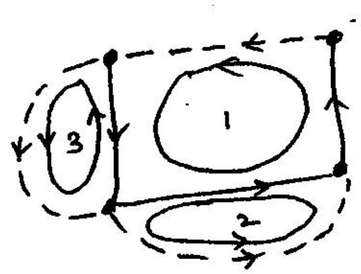
The graph of the above network is



Tree



Fundamental Loops for the chosen tree of above graph



--- Links

— tree branches

There are three fundamental loops.

$V = I' = I_3$,

Loop-1

$$I_1(R_1 + j\omega L_1) + (I_1 - I_3)R_2 + V + (I_1 - I_2)R_3 + (I_1 - I_4)\frac{1}{j\omega C_2} = 0$$

since
$V = I' = I_3$

$$I_1\left(R_1 + j\omega L_1 + R_2 + R_3 + \frac{1}{j\omega C_2}\right) + I_2(-R_3) + I_3(-R_2) + I_4\left(-\frac{1}{j\omega C_2}\right) + I_3 = 0$$

$$I_1\left(R_1 + R_2 + R_3 + \frac{1}{j\omega C_2} + j\omega L_1\right) + I_2(-R_3) + (1 - R_2)I_3 = \frac{I_4}{j\omega C_2}$$

Loop-2

$$I_2\left(j\omega L_2 + \frac{1}{j\omega C_1}\right) + (I_2 - I_1)R_3 - I_3 = 0$$

$$I_1(-R_3) + I_2\left(R_3 + j\omega L_2 + \frac{1}{j\omega C_1}\right) - I_3 = 0$$

Loop-3

$$I_3\left(j\omega L_3 + \frac{1}{j\omega C_3}\right) + (I_3 - I_1)R_2 = 0$$

$$I_1(-R_2) + \left(R_2 + j\omega L_3 + \frac{1}{j\omega C_3}\right)I_3 = 0$$

Loop equations are

$$\underbrace{\begin{bmatrix} R_1+R_2+R_3+j\omega L_1+\frac{1}{j\omega L_2} & -R_3 & 1-R_2 \\ -R_3 & R_3+j\omega L_2+\frac{1}{j\omega C_1} & -1 \\ -R_2 & 0 & R_2+j\omega L_3+\frac{1}{j\omega C_3} \end{bmatrix}}_{Z_l} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} \frac{I}{j\omega L_2} \\ 0 \\ 0 \end{bmatrix}$$