H- atom problem

Exactly solvable

2-Body problem

$1e^-$, 1 nucleus (1 proton)
$\qquad\qquad\qquad\qquad\quad e^+$

m_e $\qquad\qquad\qquad\qquad m_p$

$$\hat{H} = (\hat{KE})_{e^-} + (\hat{KE})_{nu} + \hat{V}$$

$$V(r) = -\frac{e^2}{4\pi\epsilon_0 r}$$

CM frame
Reduced mass frame

$$\frac{1}{\mu} = \frac{1}{m_e} + \frac{1}{m_n}$$

$$\mu = \frac{m_e m_n}{m_e + m_n} \simeq m_e$$

$m_n \gg m_e$

$$\boxed{\hat{H} = -\frac{\hbar^2}{2\mu}\nabla^2 - \frac{e^2}{4\pi\varepsilon_0 r}}$$

$\Psi(r, \theta, \phi)$ fixed

radial

spherical coordinates

$$\Psi(r,\theta,\phi) = N\, R(r) \cdot \underbrace{Y(\theta,\phi)}$$

Normalization

Additional part

angular part

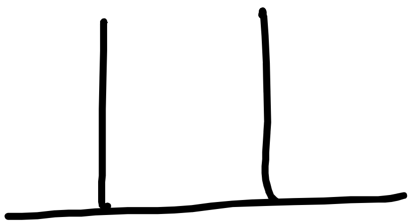
$R(r)$: radial part $\longrightarrow$ n Quantum No.

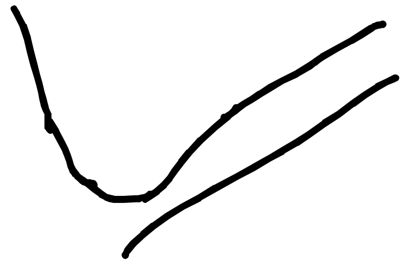
$\uparrow$ Energy

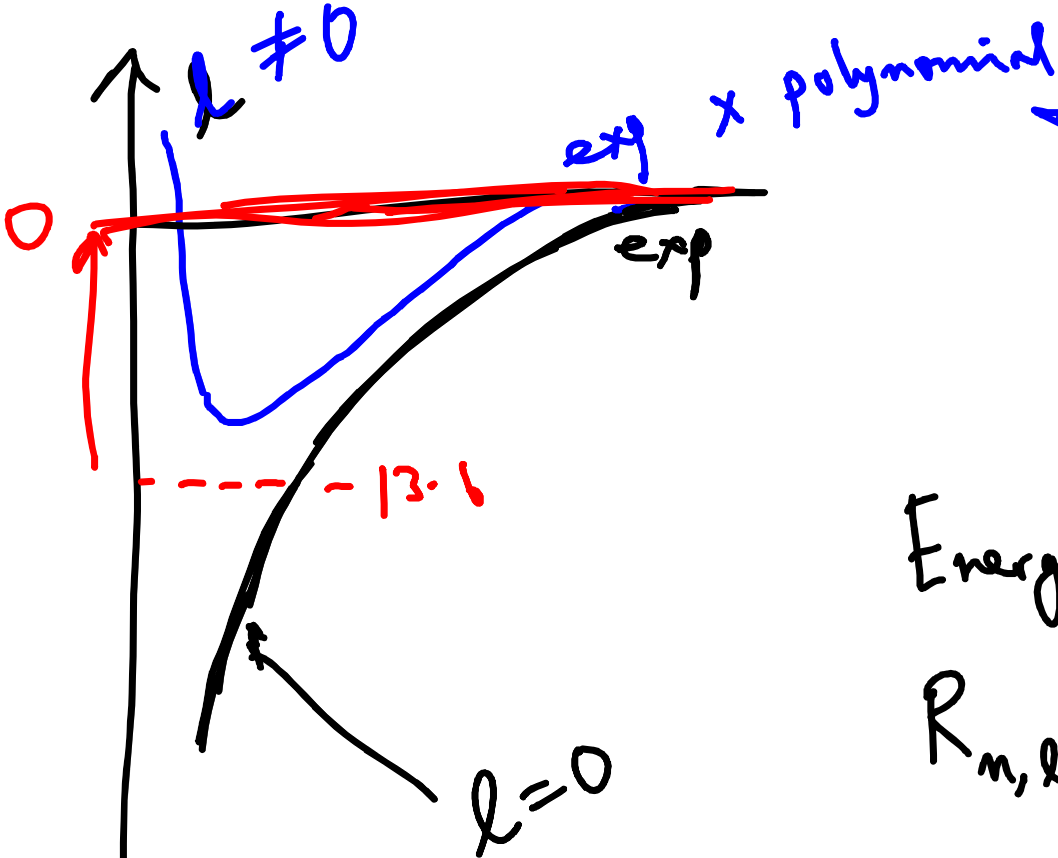
$Y(\theta, \phi)$: Angular part $\longrightarrow$ l, m_l $\longleftarrow$ Quantum #

$$\boxed{\psi_{n,l,m_l}(r, \theta, \phi)}$$

$R(r) \longrightarrow \exp(\;)^{r} \cdot$ (Polynomial in r)







$l \neq$

$$E_l = \frac{l(l+1)\hbar}{2\mu r^2}$$

Energy is quantized

$R_{n,l}(r)$

$l \neq 0$ $$V_{eff} = -\frac{e^2}{4\pi\varepsilon_0 r} + \frac{\hbar l(l+1)}{2\mu r^2}$$

$\boxed{R = 109676\ \text{cm}^{-1}}$

$$E_n = \frac{-\mu e^4}{32\pi^2\varepsilon_0^2\hbar}\cdot\frac{1}{n^2}\ ; \quad n = 1, 2, \ldots$$

at T

Energy $\rightarrow$ in K $\rightarrow$ $k_B T$

cm^{-1} $\leftarrow$ hc

$$R_{n,l}(r) = N_{n,l,m_l} \left(\frac{2r}{n a_0}\right)^l L_{nl} \exp\left(-r/n a_0\right)$$

Bohr radius

$$\left(-\frac{\hbar^2}{2\mu}\nabla^2 - \frac{e^2}{4\pi\varepsilon_0 r}\right)\psi(r,\theta,\phi) = E\,\psi(r,\theta,\phi)$$

$$\langle r \rangle_n \simeq n^2 a_0$$

Quantum number

principal Quantum #