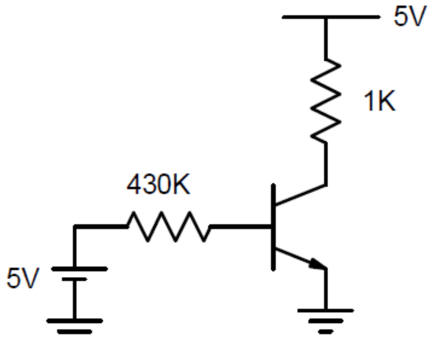
EE210: HW-4 Solution

Date: 29.01.2019

Unless stated otherwise, the BJT in the problems given below has the following characteristics

$$I_S = 2.03 * 10^{-15} A; \beta_F = 100; \ \beta_R = 1; \ V_A = 100; \ r_{bb} = 200\Omega \ V_T = 26mV; \ C_{je0} = 1pF;$$
$$C_{jc0} = 0.5pF; \ C_{js0} = 3pF; m = 0.5; \ V_{bi} = 0.85; \ \tau_F = 1ns$$

Q.1: For the circuit shown below in Fig. 1, determine the dc collector current. Using small signal analysis, determine the change in collector current if collector supply voltage changes by 10%. Determine next the change in collector current if temperature increases by 1^oC. Neglect change in β due to temperature.

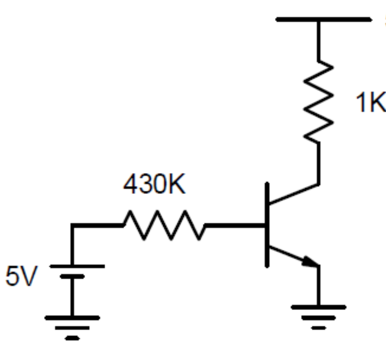


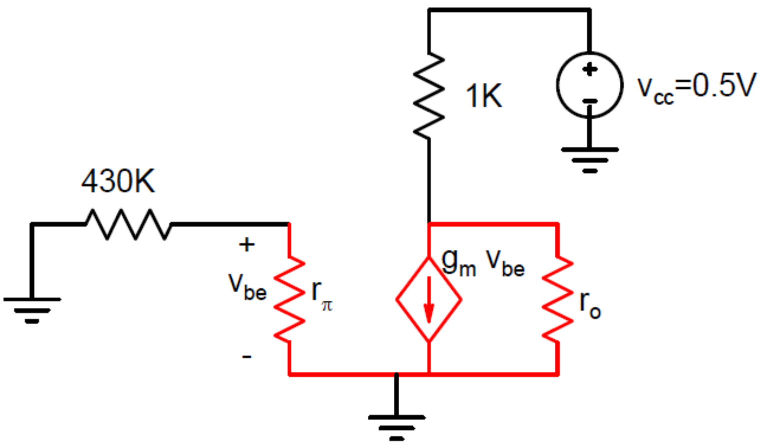
Sol.:

$$I_B = \frac{5 - 0.7}{430 * 10^3} = 10\ \mu A$$
$$I_C = \ \beta * \ I_B \ = 1mA$$

This assumes forward active mode which must be verified.

$$V_{CE} = V_{CC} - I_C * R_C = 5 - I_C * 10^3 = 4V > 0.2V$$



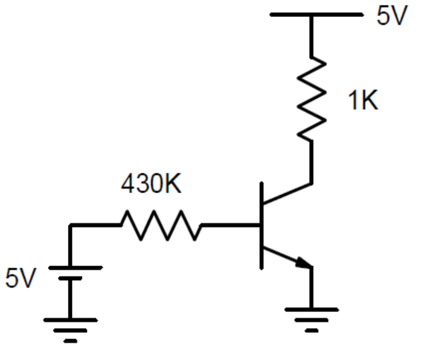


$$r_o = \frac{V_A}{I_C} = 100k\Omega$$

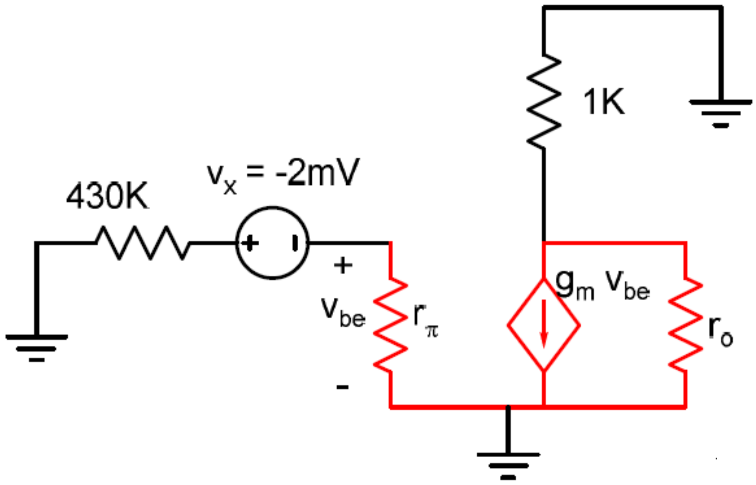
$$V_{be} = 0 \ \Rightarrow \ i_C = \frac{v_{cc}}{r_o + 1k} \cong \frac{0.5}{100k} = 5\mu A$$

$$\frac{\Delta I_C}{\Delta V_{CC}} = 1 * 10^{-5}\ \Omega^{-1}$$

Now, determine next the change in collector current if temperature increases by 1°C. Neglect change in β due to temperature.



We know that as temperature increases by 1°C, the base emitter diode voltage will decrease by -2mV. Its effect on collector current can again be determined using small signal analysis.



$$r_\pi = \frac{V_T}{I_B} = 2.6k\Omega;\ g_m = \frac{I_C}{V_T} = 0.038\ \Omega^{-1}$$

$$v_{be} = 2 * 10^{-3} * \frac{2.6k}{2.6 + 430}$$

$$i_C \cong g_m v_{be} = 0.46\ \mu A$$

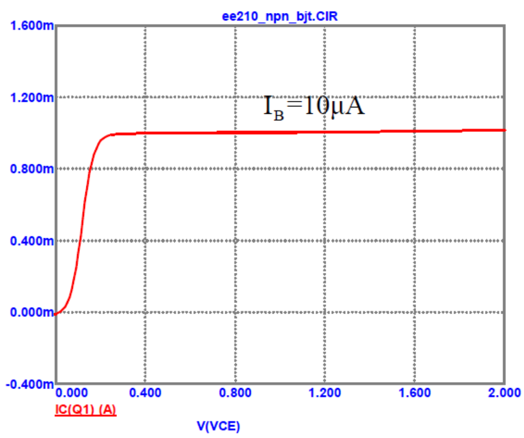
$$\frac{\Delta I_C}{\Delta V_{BE}} = \frac{0.46\ \mu A}{2mV} = 2.3 * 10^{-4}\ \Omega^{-1}$$

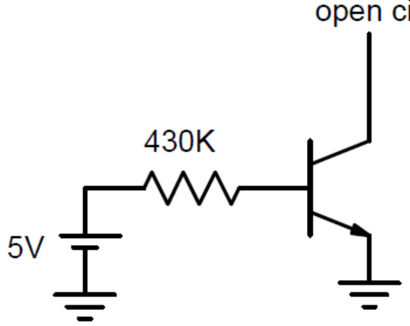
$$\frac{\Delta I_C}{\Delta T} = 0.46\ \mu A/°\text{C}$$

Q.2: Determine the mode of operation of the transistor shown in Fig. 2.
Sol.: Note that $I_C = 0$ while $I_\beta > 0$. Thus,

$$\frac{I_C}{\beta I_B} < 1$$

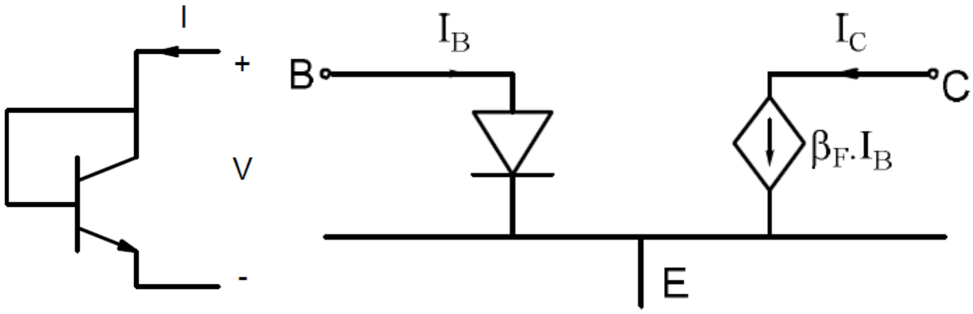
Transistor is in saturation.





Q.3: Sketch the qualitative I-V characteristics of the circuit shown in Fig. 3. What does the transistor behave as in this circuit? Determine the low frequency small signal model of the circuit assuming that dc current I = 1mA. At what frequency, will this circuit ceases to behave as a resistor under small signal condition?

Sol.:

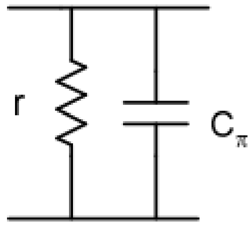


$$I = I_E = I_C + I_B = I_S\left(exp\left(\frac{V_{BE}}{V_T}\right) - 1\right) * \left(\frac{\beta + 1}{\beta}\right)$$

- The above equation shows that it is a diode behavior.
- Small signal model at low frequency is a resistor of value $r = \frac{V_T}{I} = 26\Omega$.

At higher frequency, capacitance become important and equivalent becomes,

$$C_\pi = \frac{C_{jeo}}{\sqrt{1 - \frac{0.7}{0.85}}} + \frac{I}{V_T} * \tau_F = 2.38pF + 38.47pF = 40.85\ pF$$



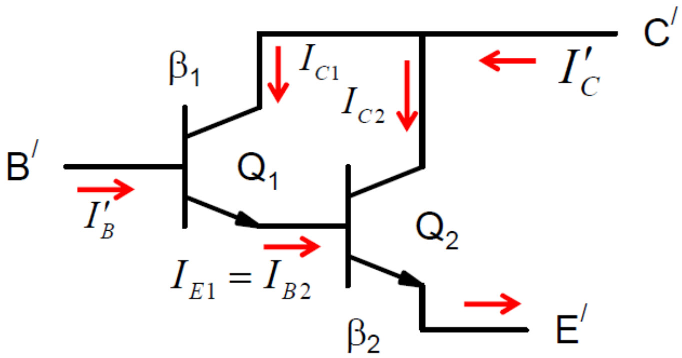
It will stop behaving as resistor when,

$$\frac{r}{1 + jwC_\pi r} \neq r \Longrightarrow jwC_\pi r \geq 10\% = 0.1$$

$$\frac{1}{2\pi f * C_\pi r} = 10 \Longrightarrow f = 15\ MHz$$

Q.4: The two-transistors connected in the manner shown in Fig. 4 can be considered as a single transistor with an effective current gain β_{eff}. Determine this gain in terms of gains of transistors Q_1 and Q_2.

Sol.:



$$I'_C = I_{C1} + I_{C2}$$

$$I_{C1} = \beta_1 * I'_B$$

$$I_{C2} = \beta_2 * I_{B2} = \beta_2 * I_{E1} = \beta_2 * (\beta_1 + 1) * I'_B$$

$$\beta_{eff} = \frac{I'_C}{I'_B} = \beta_1 + \beta_2 * (\beta_1 + 1) \cong \beta_1\beta_2$$

Q.5: Assuming that one has the freedom to choose V_{B2} in the circuit shown in Fig.5, determine the minimum supply voltage V_{CC} required to ensure that transistors Q_1 and Q_2 operate in forward active mode. Determine appropriate value for bias voltage V_{B2} under this condition.

Sol.:

$$I_{C1} = \beta * I_{B1} = 1mA$$

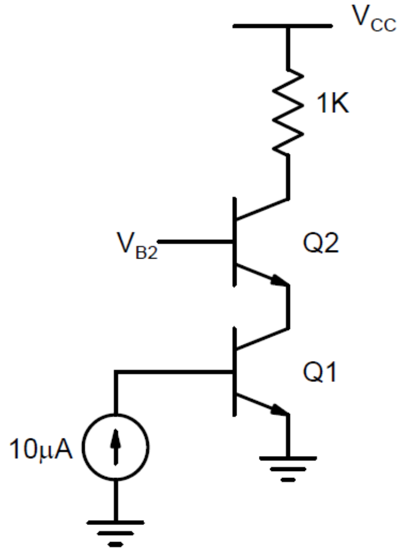
$$V_{CE1} \geq 0.2V$$

$$V_{CE2} \geq 0.2V$$

$$V_{CC} = I_{C1} * R + V_{CE2} + V_{CE1}$$

$$V_{CC} \geq 1 + 0.2 + 0.2 = 1.4V$$

$$V_{B2} = 0.7 + V_{CE1} = 0.9V$$



Q.6: The circuit shown in Fig. 6 is called a current mirror, where it is desired that current through Q_2 be equal to I_{REF}. Taking base currents into account but neglecting Early effect, determine the collector currents of the two transistors assuming that they have identical characteristics (in other words are 'matched'). Use Early effect next to determine the % difference in the two collector currents.

Sol.:

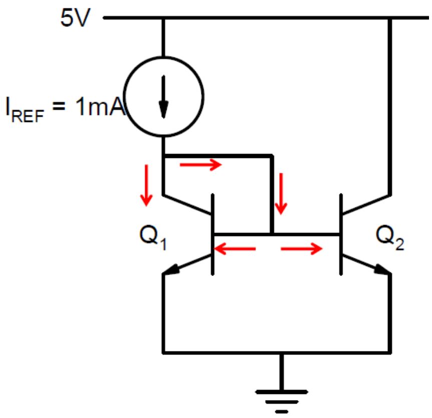
$$I_{ref} = I_{C1} + 2 * I_B = I_{C1} + 2 * \frac{I_{C1}}{\beta}$$

$$I_{C1} = \frac{I_{ref}}{1 + 2/\beta} = 0.98mA$$

$$\frac{I_{C2}}{I_{C1}} = \frac{I_S * exp\left(\frac{V_{BE}}{V_T}\right) * \left(1 + \frac{V_{CE2}}{V_A}\right)}{I_S * exp\left(\frac{V_{BE}}{V_T}\right) * \left(1 + \frac{V_{CE1}}{V_A}\right)}$$

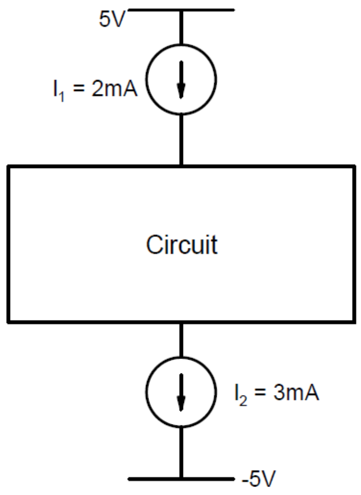
$$\frac{I_{C2}}{I_{C1}} \cong 1 + \frac{V_{CE2} - V_{CE1}}{V_A}$$

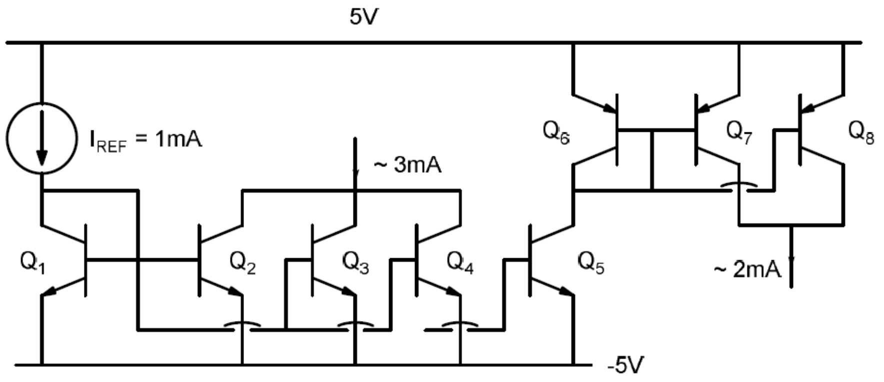
$$\frac{\Delta I_C}{I_{C1}} \cong \frac{\Delta V_{CE}}{V_A} = \frac{5 - 0.7}{100} = 0.043$$



If no Early effect, then collector current will be same in both transistors.

Q.7: There is a circuit shown in Fig. 7, where current sources of 2mA and 3mA are required. Using the idea of current mirror shown in Fig.6 and assuming that a reference current of 1mA is available, design appropriate circuit to generate these current sources. For simplicity, neglect base current and Early effect in your calculations. You have any number of matched NPN and PNP transistors to carry out the design. (This example illustrates that from one reference current, biasing currents for different parts of the circuit can be easily generated.)





Actual currents are ~2.85mA instead of 3mA and 1.85mA instead of 2mA due to losses caused by base current.