EE 200 Signals, Systems & Networks 2018 Assignment 11

1 Sketch these signals, find their Laplace Transforms and sketch the respective ROCs and pole-zero plots. $\delta(t-t_0)$, $u(t-1)$, $u(t+1)-u(t-1)$, $e^{-|t|}$, $\sin\omega_0 t$, $\cos\omega_0 t$, $e^{-at}u(t)\sin\omega_0 t;\ a>0$, $e^{-at}u(t)\cos\omega_0 t;\ a>0$, $e^{-a|t|}\sin\omega_0 t;\ a>0$, $e^{-a|t|}\cos\omega_0 t;\ a>0$, $tu(t)$, $t^k u(t)$, $tu(t)-2(t-2)u(t-2)$, $[tu(t)-2(t-2)u(t-2)+(t-4)u(t-4)]e^{-at}$.

2 Prove your answer. When $h(t)$ is real, its LT $H(s) =:$ $(a)\ -H(s)$ $(b)\ H^*(s)$ $(c)\ H(-s)$ $(d)\ H^*(s^*)$.

3 Sketch the pole-zero plots of the given transfer functions and *by direct inspection alone,* and no calculations, answer the following questions. First, is the causal time response that of a stable system? Second, find whether the stable time response is causal, anticausal or acausal.
$(a)\ H_1(s) = \frac{(s-1)(s+3)}{(s+1-j2)(s+1+j2)}$ $(b)\ H_2(s) = \frac{(s-2-j1)(s-2+j1)}{(s+3)(s-1)}$ $(c)\ H_3(s) = \frac{(s-j4)(s+j4)}{(s-1-j1)(s-1+j1)}$
$(d)\ H_4(s) = \frac{(s-2)}{(s+2)}$ $(e)\ H_5(s) = H_1(s) + H_4(s)$ $(f)\ H_6(s) = H_1(s)H_2(s)$.

4 The value of $H(s)$ at any point s is best understood in terms of the factorized form of the rational polynomial: $H(s) = \frac{\text{product of distances of } s \text{ from different zeroes}}{\text{product of distances of } s \text{ from different poles}}$. From this visualize what happens when s approaches a pole, and what happens when s approaches a zero. As an example, sketch the magnitude and phase for $H(s) = 1/s$ at points $|s| = 1$ lying on a circle of unit radius around the origin of the s-plane as well as for $H'(s) = (s+5)/(s-5)$ at points $\mathrm{Re}[s] = 0$ which is the $j\omega$ axis.

5 What is the LT of an impulse $\delta(t)$? From this, find the LT of the inverse system $g(t)$ of a system $h(t)$ when $H(s)$ is in rational polynomial form. Relate the pole/zero positions of $H(s)$ and $G(s)$.

6 Construct at least two examples of systems $H_1(s) : R_1\ H_2(s) : R_2$ where the ROC R of the sum of the two systems is larger than the intersection $R_1 \cap R_2$. Deduce the principle underlying such examples.