position $x \longleftrightarrow \hat{x} \rightarrow x$

momentum $p_x \longleftrightarrow \hat{p}_x \rightarrow -i\hbar \frac{d}{dx}$

Operator - function - result

Dirac.

$$\hat{x}.f(x) = x f(x)$$

position

If $f(x) = x+1$

$$\hat{x} f(x) = x^2 + x$$

Uncertainty eqn.

Older Bohr model

$$x(x+1) = x f(x)$$

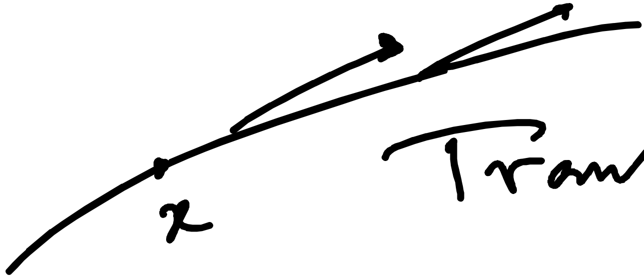
position

$$p_x f(x) = -i\hbar \frac{d}{dx} f(x)$$

$$= -i\hbar$$

Motivation (Dirac) in defining $\hat{p}_x$

$\hat{p}_x \longrightarrow$



Translation $\longrightarrow \boxed{\dfrac{d}{dx}}$

$\hbar$

$\hbar = \dfrac{h}{2\pi}$

de Broglie $\lambda = \dfrac{h}{p}$

Real $\longrightarrow$ "i" $\longrightarrow \sqrt{-1}$

(measurements)

$\pm i\hbar \dfrac{d}{dx}$

$$\hat{H} = \widehat{KE} + \hat{V}$$

Energy

$$\frac{p_x^2}{2m} \longrightarrow \frac{\hat{p}_x^2}{2m}$$

$$\hat{H} = \frac{\hat{p}_x^2}{2m} + \hat{V} \qquad \hat{p}_x = -i\hbar \frac{d}{dx}$$

$$\left(\frac{d}{dx}\right)^2 \times \qquad \frac{d}{dx}\left(\frac{d}{dx}\right) = \frac{d^2}{dx^2}$$

$$\hat{H} = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + \hat{V}(x)$$

Time Integrated QM $\longrightarrow$ $\boxed{\hat{H}\Psi = E\Psi}$

$$\frac{d^2}{dx^2} \longleftrightarrow f^n. \longrightarrow \text{Curvature of } f^n.$$

High KE

Low KE

$V(x) = 0 \longrightarrow$ Free particle

$$-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} = E\,\psi(x)$$

$$\frac{d^2\psi}{dx^2} = -\left(\sqrt{\frac{2mE}{\hbar}}\right)^2\psi(x) = -k^2\psi(x)$$

$$\psi(x) = A\exp(ikx) + Be^{-ikx}$$

Left

right

Plane waves

$A = 0$ right side

$$\boxed{\psi(x) = B\exp(-ikx)}$$

Complex No.?

Max Born (1926) $\longrightarrow$ psi Ψ

$$|\psi^*(x) \cdot \psi(x)| = |\psi(x)|^2$$ Probability

x & $x + dx$ Probability $|\psi(x)|^2 dx$

$$\int |\psi(x)|^2 \, dx = 1 \longrightarrow \text{Total Probability is } 1$$

(1) $\psi(x) \longrightarrow$ single valued

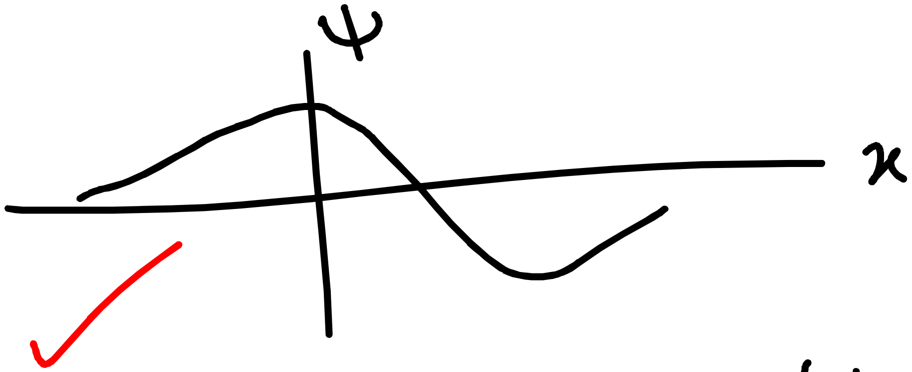
(2) $\psi(x)$ should be finite

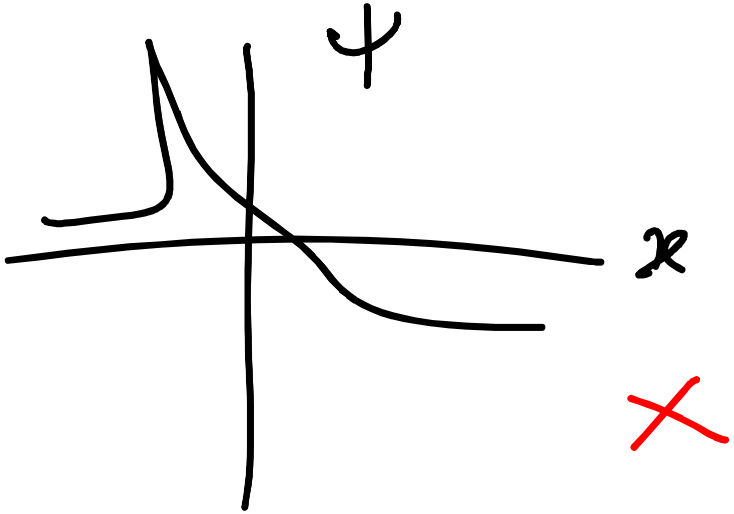
(3) $\psi(x)$ & $\dfrac{d\psi(x)}{dx}$ should be continuous

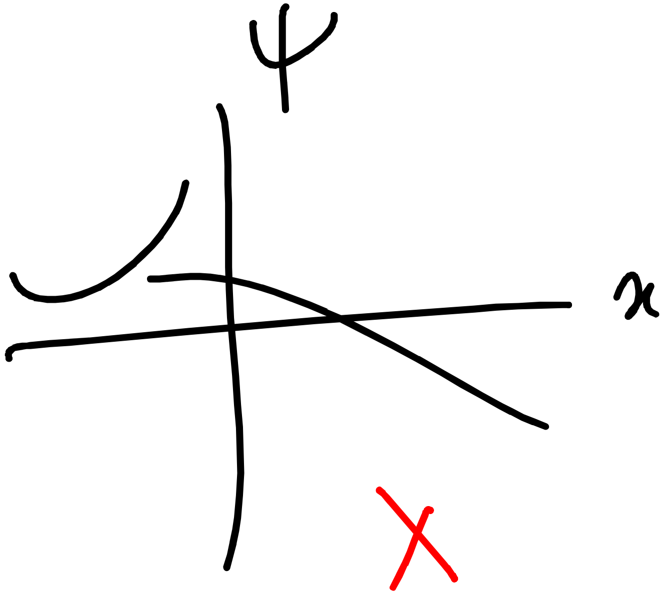
Probability

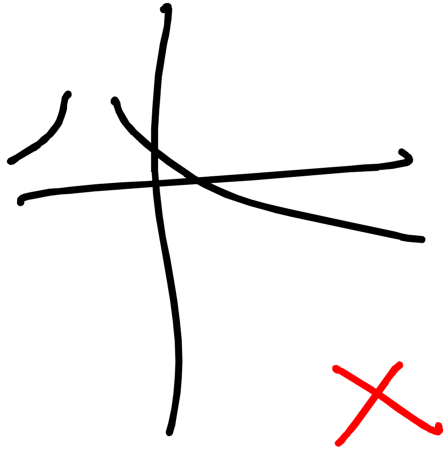
momentum

donot vanish.


ψ
x


ψ
x


ψ
x



Plane Wave Soln. for free particle

$$\psi(x) = A \exp(ikx) \quad \text{Left}$$

$$\psi(x) = B \exp(-ikx) \quad \text{Right}$$

$$\hat{H}\psi = E\psi$$

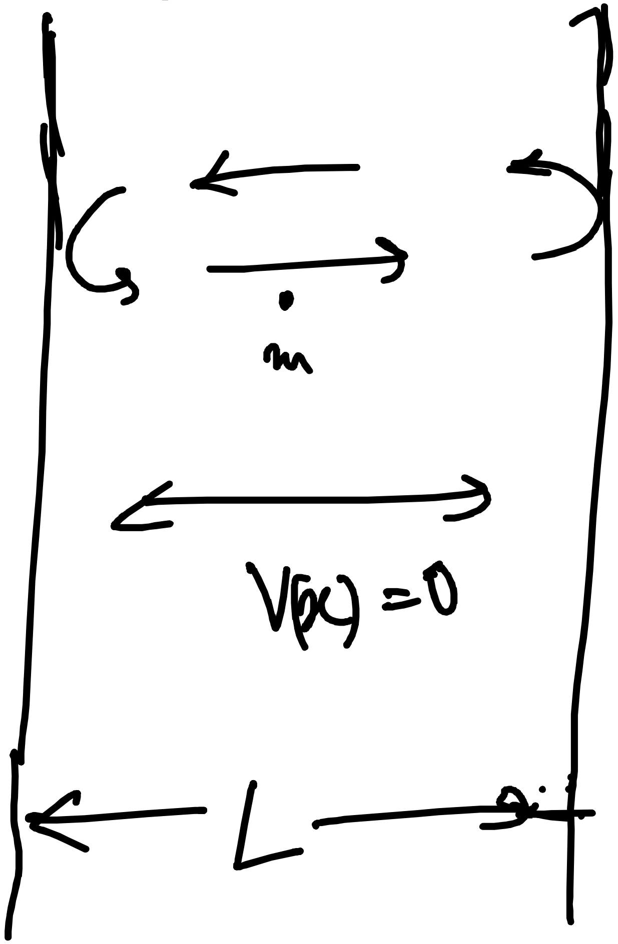
$$-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\left(B\exp(-ikx)\right) = \left(\frac{\hbar^2 k^2}{2m}\right)B\exp(-ikx)$$

$$\frac{p_x^2}{2m} \quad E \longleftrightarrow p_x = \pm\hbar k$$

$$\hat{H} = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + \underbrace{V(x)}_{0}$$

Particle in a Box

$V(x) = \infty$



$V(x) = \infty$

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} = E\psi \qquad k = \sqrt{\frac{2mE}{\hbar^2}}$$

$$\psi(x) = A\exp(ikx) + B\exp(-ikx)$$

(i) What is A & B ?

(ii) Is the energy going to be continuous or not ?

$\boxed{\text{Restriction or Constrain}}$ $\boxed{\text{Boundary Conditions}}$