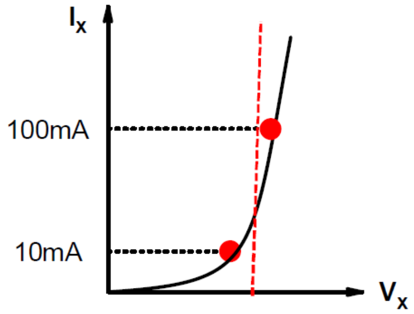
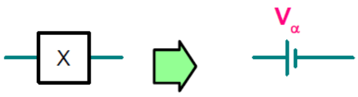
EE210: HW-2 Solution

Date: 17.01.19

Q.1 (a) A two terminal element X has I-V characteristics given by the relation $I_x = V_x{}^6$. A simple model for this device is a constant voltage of V_α. Determine this voltage, if the model is to be used in the current range 10mA-100mA. Determine the % error in the use of such a model over the specified current range. Will the error reduce if the non-linearity in the device became stronger (a higher order polynomial)?

Sol.





For 10mA current: $V_x = (10^{-2})^{1/6} = 0.464V$

For 100mA current: $V_x = (10^{-1})^{1/6} = 0.681V$

One estimate for V_α is the average of these two voltage values,

$$V_\alpha = \frac{0.464 + 0.681}{2} = 0.572V$$

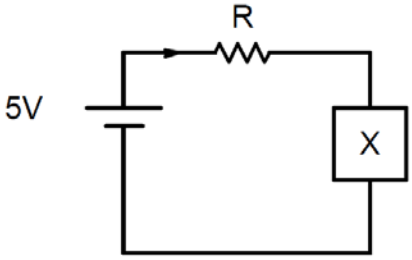
Error at the two extreme ends of current range:

$$\varepsilon_1 = \frac{0.464 - 0.572}{0.464} \times 100 = -23.38\%$$

$$\varepsilon_2 = \frac{0.681 - 0.572}{0.681} \times 100 = 15.93\%$$

Yes, error will reduce for higher order polynomial as slope will be steeper closely resembling to a constant voltage source.

(b) A battery of 5V is in series with a resistor and the element X. Determine an approximate value for the maximum % error in current, when the above model is used. (Note that error would increase as supply voltage gets closer to the voltage V_α.)



$$I = \frac{5 - V_x}{R} \Rightarrow \Delta I = \frac{-\Delta V_x}{R} \Rightarrow \frac{\Delta I}{I} = \frac{-\Delta V_x}{5 - V_x}$$

Maximum error:

$$\frac{\Delta I}{I} \cong \frac{0.464 - 0.572}{5 - 0.572} = 2.4\%$$

(c) What would be a new suitable value of V_α, if the current range is changed to 1µA-10µA? (Note that a model is a representation for a purpose).

For 1µA current:

$$V_x = (10^{-6})^{1/6} = 0.1V$$

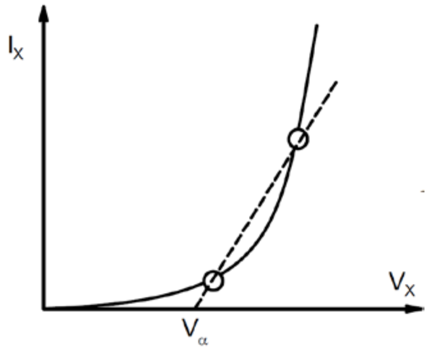
For 10µA current:

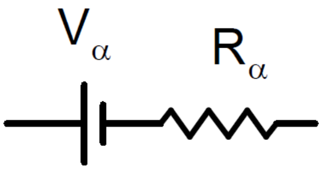
$$V_x = (10^{-5})^{1/6} = 0.147V$$

$$V_\alpha = \frac{0.1 + 0.147}{2} = 0.123V$$

Q.2 An improved model for the element X is a constant voltage source V_α in series with a resistor R_α. Determine values of these elements for the current range $10mA$-$100mA$.

Sol.

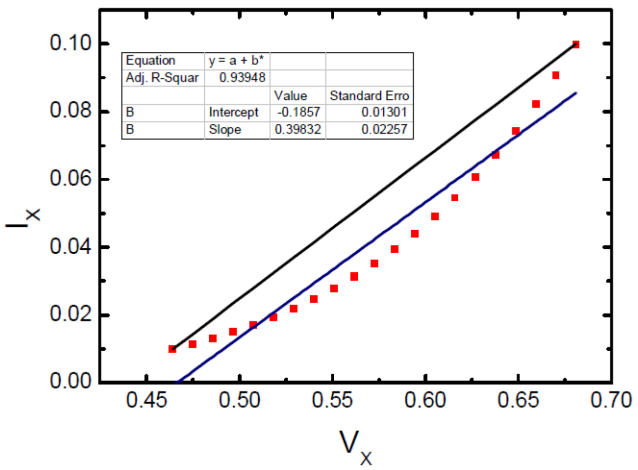


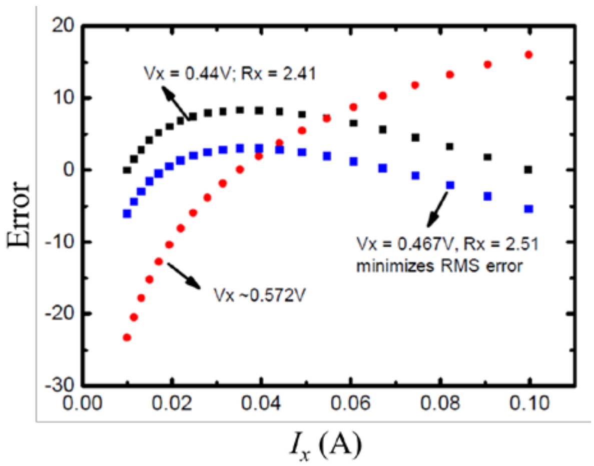


$$R_\alpha = \frac{V_{x2} - V_{x1}}{I_{x2} - I_{x1}} = \frac{0.681 - 0.464}{100mA - 10mA} = 2.41\Omega$$

$$V_\alpha = V_{x1} - I_{x1} * R_\alpha = 0.44V$$

This approach is accurate at the two extreme ends and error peaks somewhere in the middle. There are other ways of developing this model which may result in less overall error.





Minimum rms error:

$$I_x = \frac{(V - 0.467)}{2.51}$$

Q.3 (a) Determine the small signal model for element X, when it is biased at a voltage of 0.5V. Show that the small signal voltage across X has to be less than or equal to 21mV for error in small signal current to be less than or equal to 10%.

Sol.

Small signal model is a resistor of value:

$$\frac{1}{r_x} = \left(\frac{\partial I_x}{\partial V_x}\right)_{|V_x=0.5V} = 6{V_x}^5{}_{|V_x=0.5V} \Rightarrow r_x = 5.33\Omega$$

$$I_x + i_x = (V_x + v_x)^6 = {V_x}^6 * \left(1 + \frac{v_x}{V_x}\right)$$

$$i_x = I_x * \left\{\left(1 + \frac{v_x}{V_x}\right)^6 - 1\right\}$$

Small signal approximation:

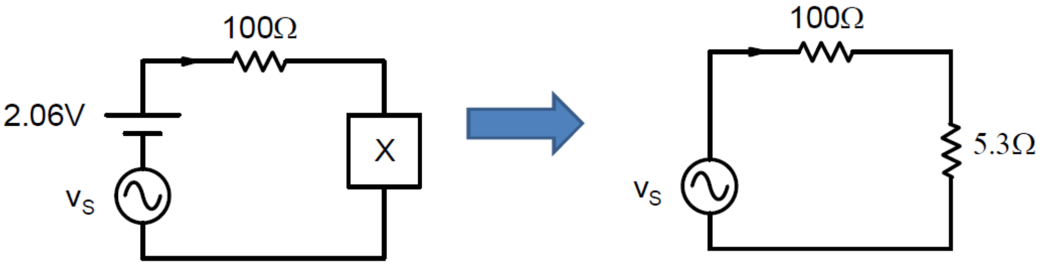
$$i'_x \cong I_x * \left\{\left(1 + 6\frac{v_x}{V_x}\right) - 1\right\} = 6I_x \times \frac{v_x}{V_x}$$

Error:

$$\frac{i_x - i'_x}{i_x} = \frac{\left(1 + \frac{v_x}{V_x}\right)^6 - 1 - \frac{6v_x}{V_x}}{\left(1 + \frac{v_x}{V_x}\right)^6 - 1}$$

Substitution of v_x = 21mV and V_x =0.5V gives error of 10%.

(b) How large can the ac voltage in the circuit be for the small signal model to be valid (with error < 10%) in the following circuit? (Note that X has a dc bias of 0.5V).

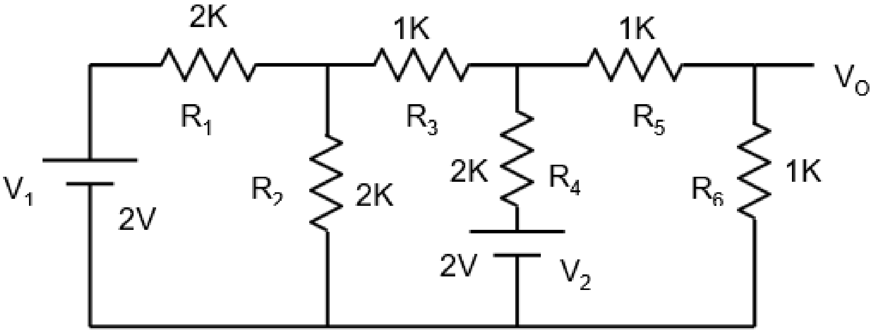


Sol.

$$v_x = v_s \times \frac{r_x}{100 + r_x} \le 21mV$$

$$v_s \le 0.415V$$

Q.4 For the circuit shown below, V_o = 0.5V and I_{R1} = 0.5mA for the given values of components. Using the small signal analysis technique, determine the approximate change in output voltage, if R_1 changes by 10%.

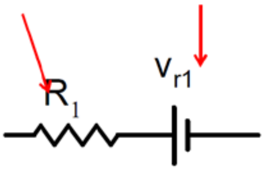


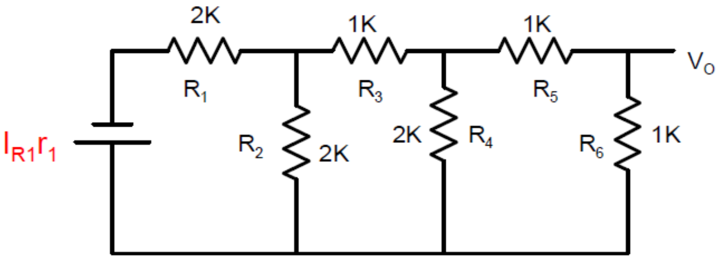
Sol.

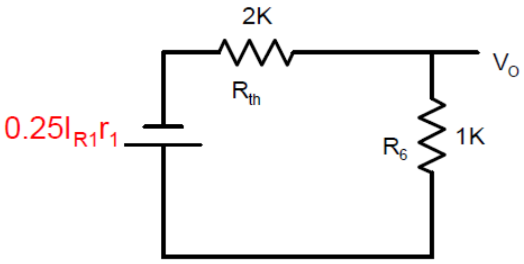
As a result of change in R1, current would change to $I_{R1}+i_{r1}$.

$$V_{R1}+v_{r1}=(I_R+i_{r1})*(R_1+r_1)$$
$$v_{r1}\cong i_{r1}*R_1+I_R*r_1$$

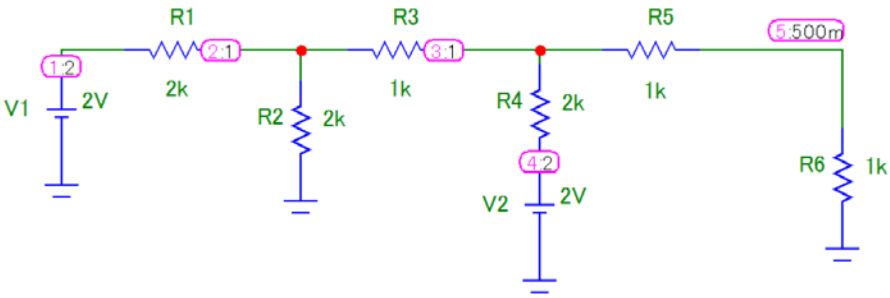
Small signal equivalent circuit:

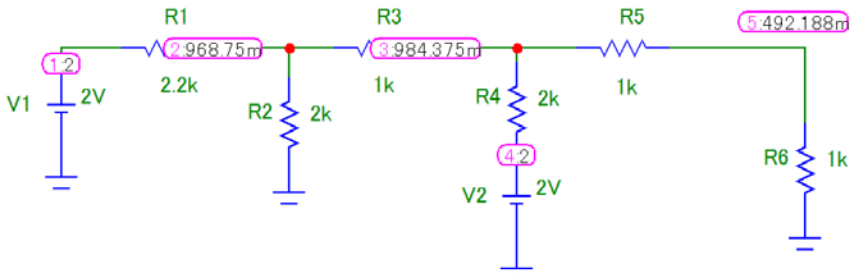






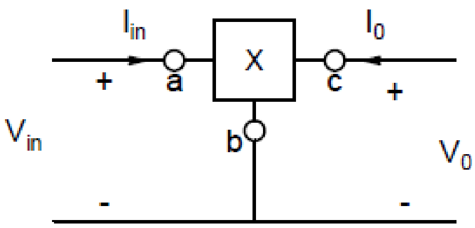
$$r_1=10\%\ of\ 2K=200\Omega$$
$$v_o=\frac{I_{r1}r_1}{12}=-\frac{0.5\times10^{-3}\times200}{12}=-8.33mV$$

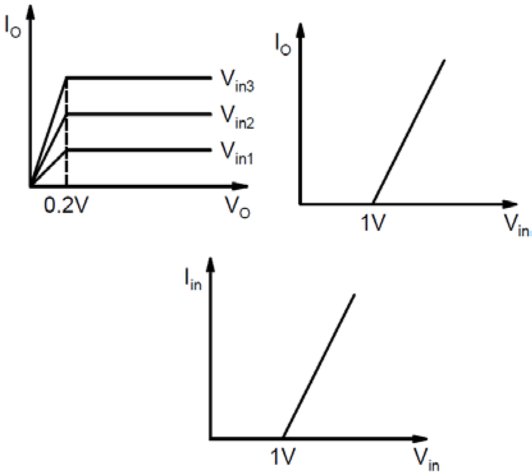




Difference in our hand calculation vs. simulation = 0.5 – 0.4921 =7.9 x 10^{-3}

Q.5 A hypothetical 3-terminal (a, b & c) unilateral device (shown below) has the following characteristics.





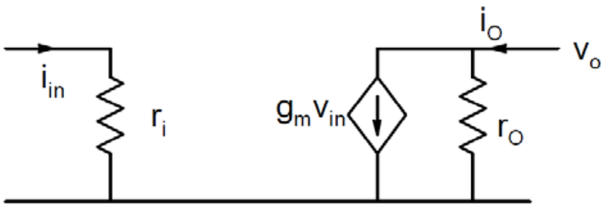
$$
\begin{aligned}
I_o &= 0 \text{ for } V_{in} \le 1 \text{ for all values of } V_o \\
&= 0.05 \times (V_{in} - 1) \;\text{ for } V_{in} > 1V, V_o \ge 0.2\text{V} \\
&= 0.05 \times (V_{in} - 1) \times \frac{V_o}{0.2} \;\text{ for } V_{in} > 1V, 0 \le V_o \le 0.2\text{V} \\
&= 0 \text{ for } V_o \le 0
\end{aligned}
$$

$$I_{in} = 0.001 \times (V_{in} - 1) \;\text{ for } V_{in} > 1V \text{ and } 0 \text{ for } V_{in} \le 0$$

Determine the small signal model of the device around the bias point V_{in} = 1.5V and two values of output voltage V_o = 0.1V and V_o = 2V.

Sol.

The general small signal model for a 3-terminal device is:

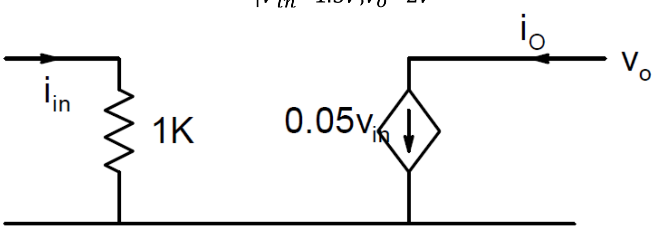


For V_o = 2V and V_{in} = 1.5V,

$$\frac{1}{r_i} = \left(\frac{\partial I_{in}}{\partial V_{in}}\right)_{|V_{IN}=1.5V} = 10^{-3}\Omega^{-1} \Rightarrow r_i = 1K\Omega$$

$$\frac{1}{r_o} = \left(\frac{\partial I_o}{\partial V_o}\right)_{|V_o=1V} = 0 \Rightarrow r_o = \infty$$

$$g_m = \left(\frac{\partial I_o}{\partial V_{in}}\right)_{|V_{in}=1.5V, V_o=2V} = 0.05\Omega^{-1}$$



For $V_o = 0.1$ V and $V_{in} = 1.5$V

$$\frac{1}{r_i} = \left(\frac{\partial I_{in}}{\partial V_{in}}\right)_{|V_{IN}=1.5V} = 10^{-3}\Omega^{-1} \Longrightarrow r_i = 1K\Omega$$

$$\frac{1}{r_o} = \left(\frac{\partial I_o}{\partial V_o}\right)_{|V_o=0.1V} = 0.05 * (1.5 - 1) * \frac{1}{0.2} = 0.125\Omega^{-1}$$

$$g_m = \left(\frac{\partial I_o}{\partial V_{in}}\right)_{|V_{in}=1.5V, V_o=0.1V} = 0.05 * \frac{0.1}{0.2} = 0.025\Omega^{-1}$$