Problem Set 1

1. Assuming that we know the position of an electron in its orbit to an accuracy of 1% of the Bohr radius (0.529 Å), calculate the uncertainty in the velocity of the electron.

   Since, Bohr radius = 0.529 Å, positional accuracy (Δx) of the electron = 1% of 0.529 Å.
   From Heisenberg uncertainty principle:

   $$\Delta p = \frac{h}{4\pi\Delta x} = \frac{6.626\times10^{-34}Js}{4\times3.14\times0.529\times10^{-12}m} = 9.97\times10^{-23}\text{ kg ms}^{-1}$$

   Now, $\Delta p = m_e\Delta v$.
   So, the uncertainty in velocity of the electron is:

   $$\Delta v = \frac{9.97\times10^{-23}kg\ ms^{-1}}{9.1\times10^{-31}kg} = 1.1\times10^{8}\text{ ms}^{-1}$$

   That is 36% of the speed of light.

2. When a beam of electrons is directed through an accelerating electrostatic field (two parallel plates with a potential difference of V volts), the potential energy gained by each electron, eV, can be equated to its kinetic energy as: $eV = \frac{1}{2}mv^2$. What is the wavelength (in Å) of an electron when it is accelerated by 1.00×10^3 V?

   $$eV = \frac{1}{2}m_e v^2$$

   $$\Rightarrow v = \sqrt{\frac{2eV}{m_e}} \qquad \text{-----(1)}$$

   Now, from deBroglie equation: $p = m_e v = \frac{h}{\lambda}$, we get, $\lambda = \frac{h}{m_e v}$ -----(2)

   Substituting (1) in (2), we get: $\lambda = \frac{h}{\sqrt{2m_e eV}}$

   Putting in the values: $h = 6.626\times10^{-34}$ Js, $m_e = 9.1\times10^{-31}$ kg, $e = 1.602\times10^{-19}$ C, V = 1000V, we get, $\lambda = 3.88\times10^{-11}m$ =0.388 Å.

3. Calculate the ratio of deBroglie wavelengths for a cricket ball of mass 0.4 kg to that of a ^{4}He atom, both of which are travelling at 1 km/s.

   deBroglie equation: $\lambda = \frac{h}{p} \Rightarrow \frac{\lambda_{cricket\ ball}}{\lambda_{He\ atom}} = \frac{1\frac{km}{s}\times m_{He\ atom}}{1\frac{km}{s}\times m_{cricket\ ball}} = \frac{4\times1.67\times10^{-27}kg}{0.4\ kg} = \frac{1.67\times10^{-26}}{1}$

   $$\therefore \lambda_{cricket\ ball}:\lambda_{He\ atom} :: 1.67\times10^{-26}:1$$

   $$\frac{\lambda_{cricket-Ball}}{\lambda_{He-atom}} = \frac{1.66\times10^{-26}}{1}$$

4. Is $(\sin\theta.\cos\theta)$ an eigenfunction of the operator $(\sin\theta\frac{d}{d\theta}\left(\sin\theta\frac{d}{d\theta}\right)+6\sin^2\theta)$? If so, what is the eigenvalue?

We have, Operator: $(\sin\theta\frac{d}{d\theta}\left(\sin\theta\frac{d}{d\theta}\right)+6\sin^2\theta)$ & Function: $(\sin\theta.\cos\theta)$

So, $\left[\sin\theta\frac{d}{d\theta}\left(\sin\theta\frac{d}{d\theta}\right)+6\sin^2\theta\right]\sin\theta\cos\theta = \sin\theta\frac{d}{d\theta}\sin\theta\frac{d}{d\theta}\sin\theta\cos\theta + 6\sin^3\theta\cos\theta$

$$= \sin\theta\frac{d}{d\theta}\{\sin\theta\,(\cos^2\theta-\sin^2\theta)\} + 6\sin^3\theta\cos\theta$$

$$= \sin\theta\frac{d}{d\theta}(\sin\theta - 2\sin^3\theta) + 6\sin^3\theta\cos\theta$$

$$= \sin\theta\,(\cos\theta - 6\sin^2\theta\cos\theta) + 6\sin^3\theta\cos\theta$$

$$= \sin\theta\cos\theta - 6\sin^3\theta\cos\theta + 6\sin^3\theta\cos\theta$$

$$= 1.\sin\theta\cos\theta$$

Therefore, $(\sin\theta.\cos\theta)$ is an Eigenfunction with an Eigenvalue of 1 for the given operator.

5. A light source of wavelength λ illuminates a metal and ejects photoelectrons with a maximum kinetic energy of 1.00 eV. A second light source with half the wavelength of the first ejects photoelectrons with a maximum kinetic energy of 4.00 eV. However, any light source with more than double the wavelength of the first does not eject any photoelectrons. From these statements, what characteristics of the metal be arrived at by using Einstein's photoelectric effect?

Einstein's photoelectric effect relates the maximum kinetic energy K_{max} of a photoelectron with the wavelength λ of the light producing the photoelectron and the work function ϕ of the metal: $K_{max} = \frac{hc}{\lambda} + \phi$

It was with this equation that Einstein introduced the notion of light quanta in 1905 and for which he received the Nobel Prize for Physics in 1922.

Let λ_1 and λ_2 be the wavelengths of the light emitted by the first and second sources, respectively, and let K_1 and K_2 be the maximum kinetic energies of the corresponding photoelectrons. Then, it follows from the above equation:

$$K_1 = \frac{hc}{\lambda_1} + \phi$$

$$K_2 = \frac{hc}{\lambda_2} + \phi$$

Since, $\lambda_2 = 0.5\lambda_1$, we get: $K_2 - 2K_1 = \phi$. Thus, ϕ = 2 eV when K_1 = 1 eV and K_2 = 4eV. This also confirms as to the fact that any light source with more than double the wavelength of the first does not eject any photoelectrons.

6. If $\hat{A} = x, \hat{B} = \frac{d}{dx}$, then compute $\hat{A}\hat{B}x^3$ and $\hat{B}\hat{A}x^3$. Do the operators: $\hat{A}$ and $\hat{B}$ form a commutative pair for the function x^3?

$$\hat{A}\hat{B}x^3 = 3x^3$$
$$\hat{B}\hat{A}x^3 = 4x^3$$
$$\Rightarrow [\hat{A}\hat{B}]x^3 = \hat{A}\hat{B}x^3 - \hat{B}\hat{A}x^3 = x^3 \neq 0$$

Operators $\hat{A}$ and $\hat{B}$ do not form a commutative pair for the function x^3
