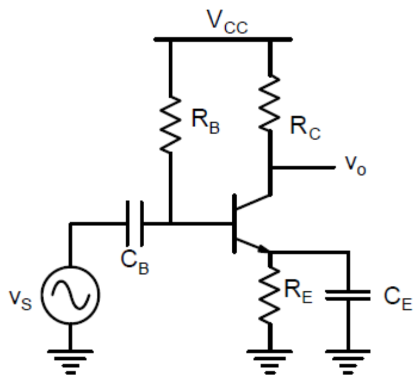
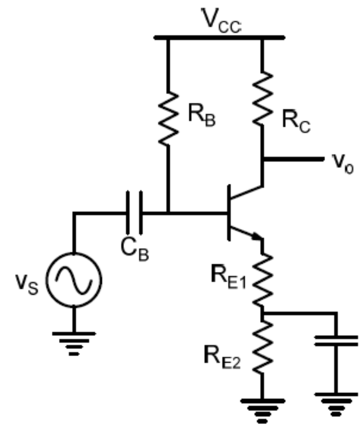
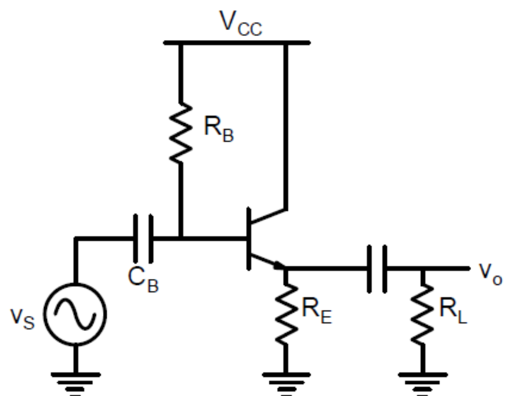
EE210: HW-8 Solution

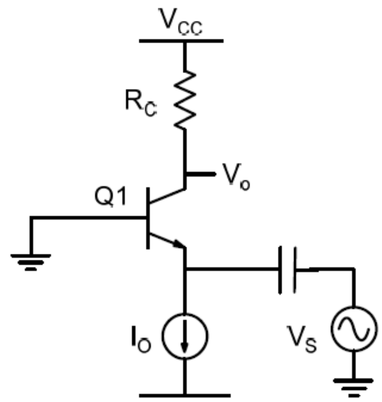
Date: 28-02-2019

Q1. Analyze the following circuits from first principles to determine voltage gain and input resistance.

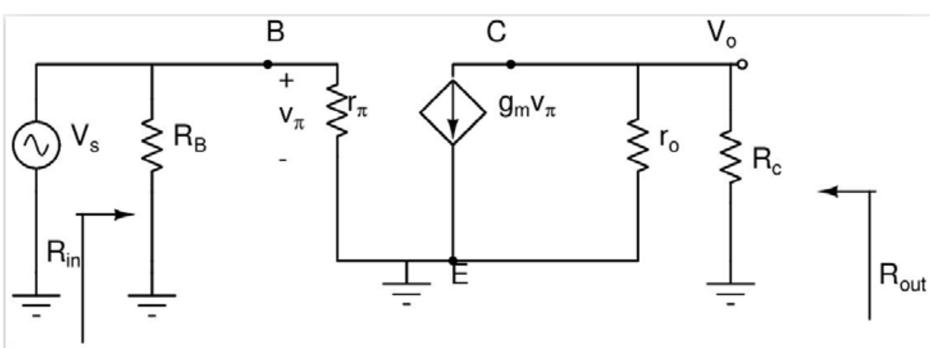








Sol.
(a)



$$R_{in} = R_B \parallel r_\pi$$

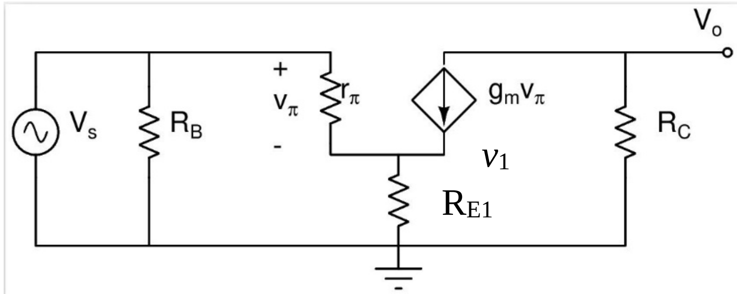
$$v_\pi = v_s$$

$$R_{out} = R_C \parallel r_0$$

$$v_0 = -g_m(R_C \parallel r_0)v_\pi$$

$$A_V = \frac{v_0}{v_s} = -g_m(R_C \parallel r_0) \cong -g_m R_C$$

(b)



$$v_s = v_\pi + v_1$$

$$\frac{v_1 - v_s}{r_\pi} + \frac{v_1}{R_{E1}} = g_m v_\pi$$

$$\frac{v_s - v_\pi - v_s}{r_\pi} + \frac{v_s - v_\pi}{R_{E1}} = g_m v_\pi$$

$$\frac{v_s}{R_{E1}} - v_\pi \left(\frac{1}{r_\pi} + \frac{1}{R_{E1}}\right) = g_m v_\pi$$

$$\frac{v_s}{R_{E1}} = v_\pi \left(\frac{1}{r_\pi} + \frac{1}{R_{E1}} + g_m\right)$$

$$v_\pi = \frac{r_\pi}{R_{E1} + r_\pi + g_m R_{E1} r_\pi} v_s$$

$$v_\pi = \frac{r_\pi}{r_\pi + R_{E1}(1+\beta)} v_s$$

$$v_O = -g_m v_\pi R_c$$

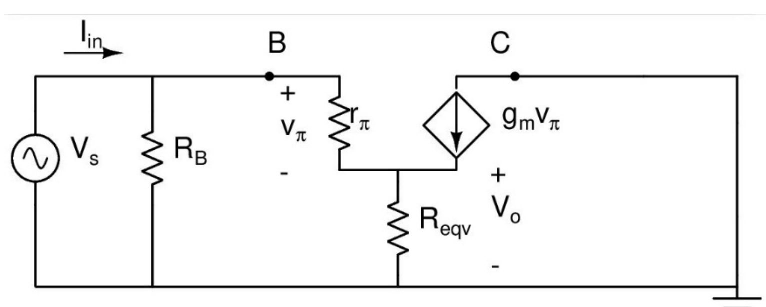
Gain

$$A_v = \frac{v_O}{v_s} = \frac{-g_m r_\pi R_c}{r_\pi + R_{E1}(1+\beta)}$$

Input resistance

$$R_{in} = R_B \parallel (r_\pi + R_{E1}(1+\beta))$$

(c)



C_B, C_E = short circuit

$$R_{eq} = R_E \parallel R_L$$

Assuming $r_0 \rightarrow \infty$,

$$v_s = v_\pi + v_0$$

$$\frac{v_0}{R_{eq}} = g_m v_\pi + \frac{v_\pi}{r_\pi}$$

$$v_0 = R_{eq} v_\pi \left(g_m + \frac{1}{r_\pi}\right)$$

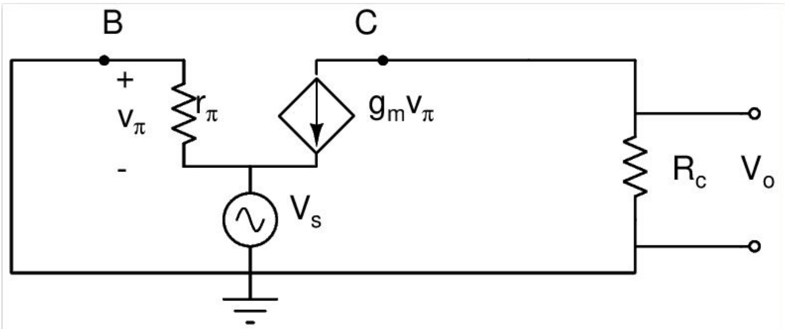
$$v_0 = R_{eq}(v_s - v_0)\left(g_m + \frac{1}{r_\pi}\right)$$

$$A_v = \frac{v_0}{v_S} = \frac{R_{eq}\,(\mathrm{gm} + \frac{1}{r_\pi})}{1 + R_{eq}\,(\mathrm{gm} + \frac{1}{r_\pi})}$$

Input Resistance,

$$R_{in} = R_B \parallel (r_\pi + R_{eq}(\beta + 1))$$

(d)



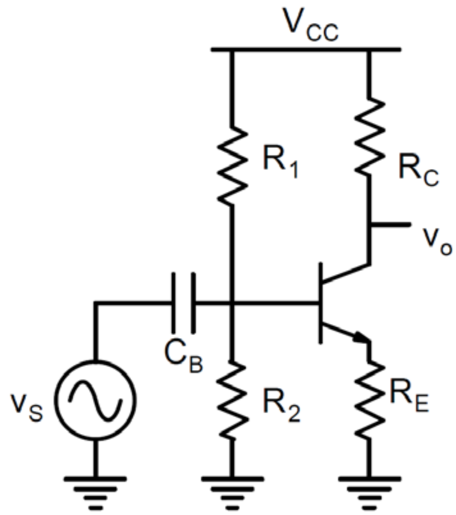
$$v_\pi + v_S = 0$$

$$v_0 = g_m R_C v_\pi$$

$$\frac{v}{v_S} = -g_m R_C$$

$$R_{in} = \frac{r_\pi}{1 + \beta}$$

Q2. Check the claim that “it is not possible to design the single stage BJT amplifier shown below such that magnitude of open circuit voltage gain is ≥ 100, input resistance is ≥ 1kΩ and the design is such that voltage gain stability against variation in current gain β is better than 5% and supply voltage is less than 12V”.



Sol:

Let’s assume

$$V_{CC} = 12V$$

$$A_V = 100$$

$$R_{in} = 1K\Omega$$

Stability

$$S = \frac{\Delta I_{CQ}/I_{CQ}}{\Delta\beta/\beta} = \frac{1}{1 + \frac{\beta R_E}{R_B}} \leq 0.05$$

Thus

$$\frac{\beta R_E}{R_B} \geq 19$$

$$\beta R_E \geq 19 * R_B$$

$$R_{in} = R_B \parallel (r_\pi + \beta R_E) \geq 1k$$

Thus

$$R_B \geq 1k$$

For R_B = 1k, let’s assume R_2=1k and R_1=11k. This gives,

$$V_B = \frac{1k}{1k + 11k} * V_{CC} \cong 1V$$

$$V_E = V_B - 0.7V = 0.3V = I_{CQ} R_E$$

Also,

$$|A_V| \geq \frac{g_m R_C}{1 + g_m R_E} \geq 100$$

$$R_C \geq 100 * R_E$$

Thus

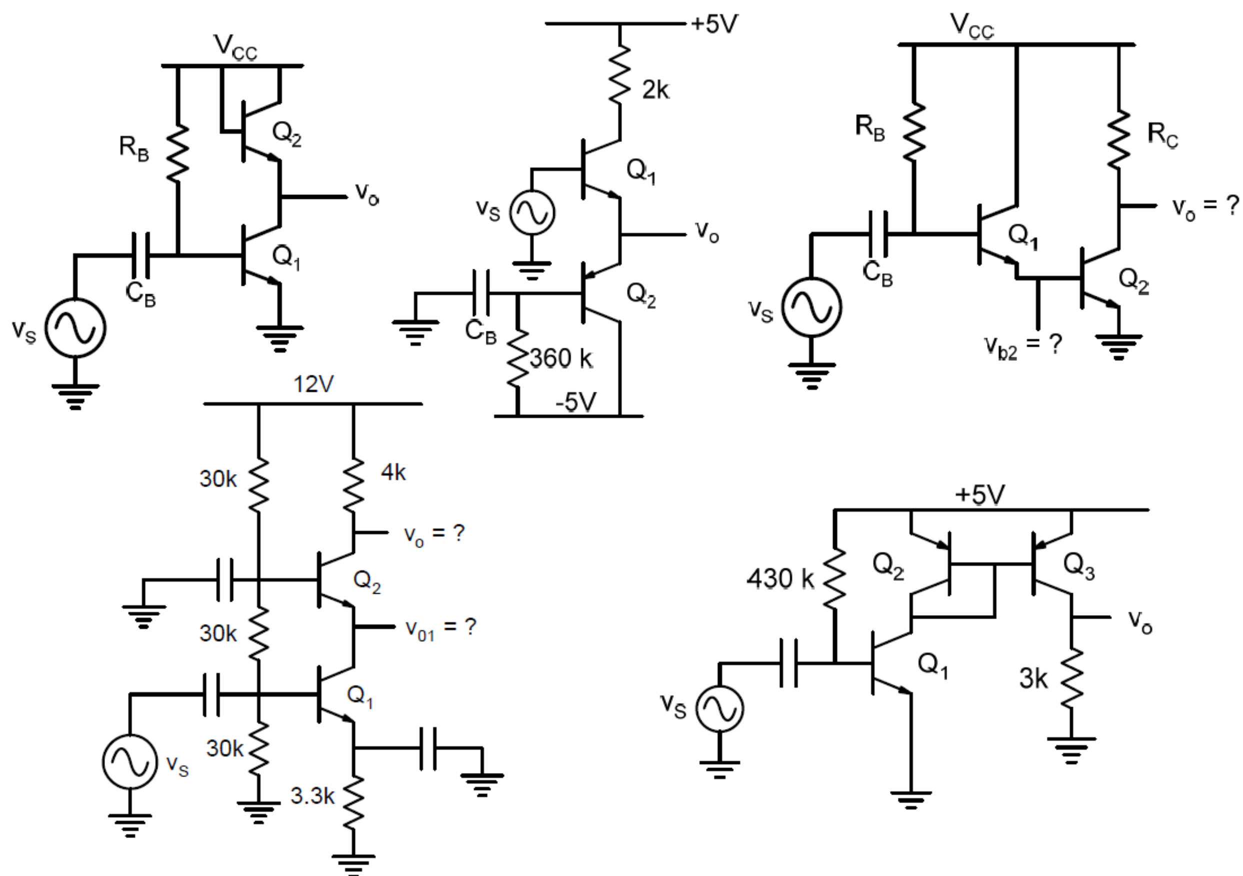
$$I_{CQ} R_C \geq 100 * I_{CQ} * R_E = 30V$$

This gives us

$$V_{CC} \geq I_{CQ} R_C + v_{om} + I_{CQ} R_E \geq 30V + v_{om} + 0.3V$$

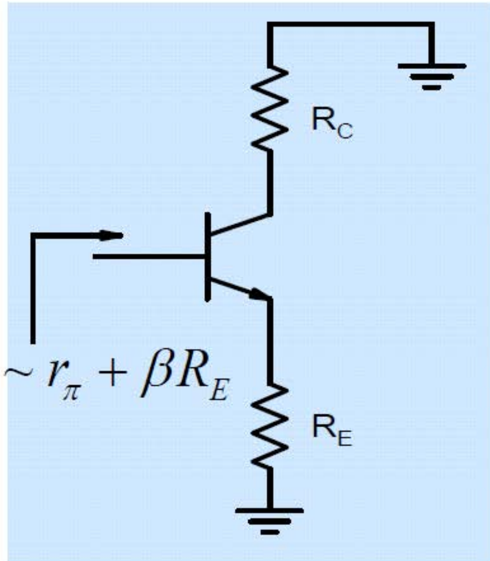
Thus $V_{CC} = 12V$ cannot satisfy the input resistance and gain conditions.

Q3. Analyze the following circuit through use of “divide & reuse “methodology.

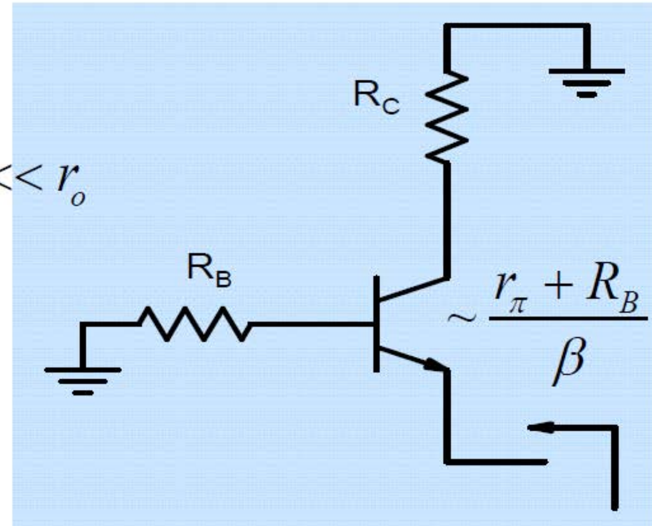


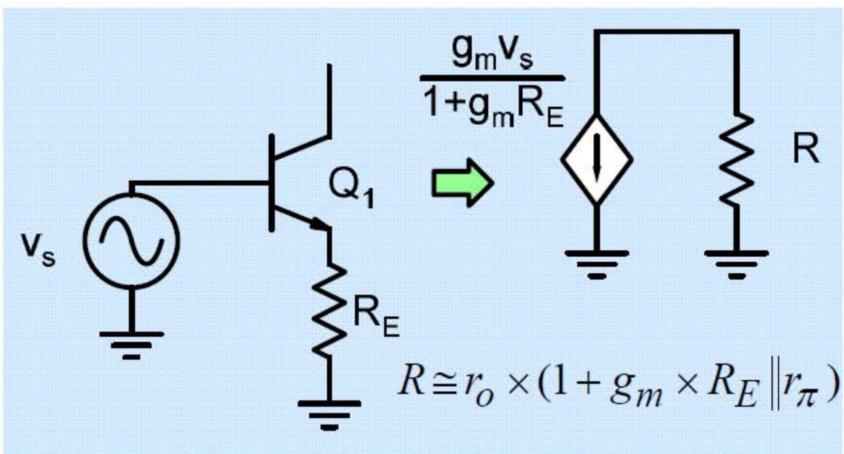
Sol.

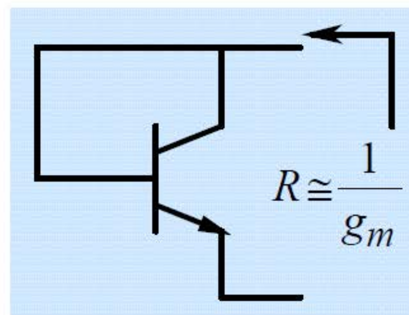
Keep in mind following relations.



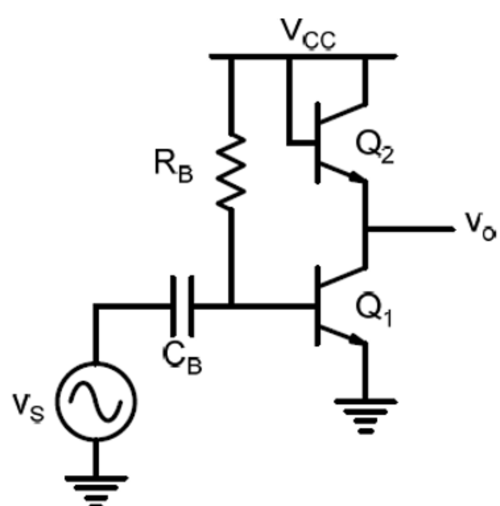
$R_C, R_E, R_B << r_o$







Input Resistance:



$$R_i = r_\pi || R_B$$

Output Resistance:

$$R_o = R_{01} \| R_{02} = r_{01} \| r_{02} \| \left(1/g_{m2} \right) \cong 1/g_{m2}$$

Gain (A_V):

$$A_V = -g_m \times R_{out} = -g_{m1} \times \left(\frac{1}{g_{m2}} \right) \| r_{01} \| r_{02} \cong -g_{m1} \times \left(\frac{1}{g_{m2}} \right)$$
$$= -1$$

Note that $g_{m1} = g_{m2}$, as I_{CQ} is same in both transistors.

Input Resistance:

$$R_{in} = r_{\pi1} + \beta_1 * \frac{1}{g_{m2}}$$

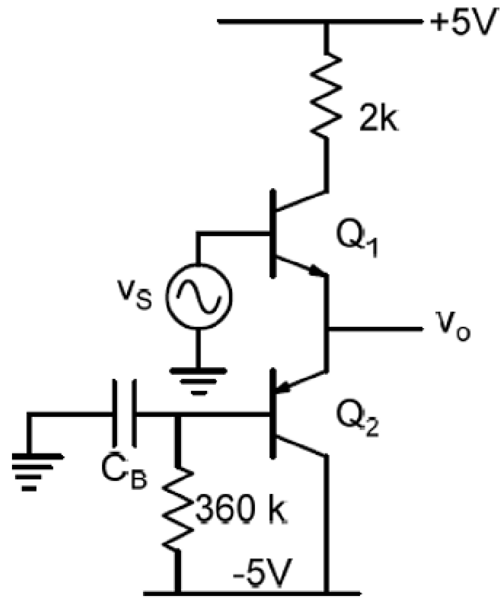
Output Resistance:

$$R_{out} = R_{01} \| R_{02} = \frac{1}{g_{m1}} \| \frac{1}{g_{m2}}$$

<u>Gain:</u>

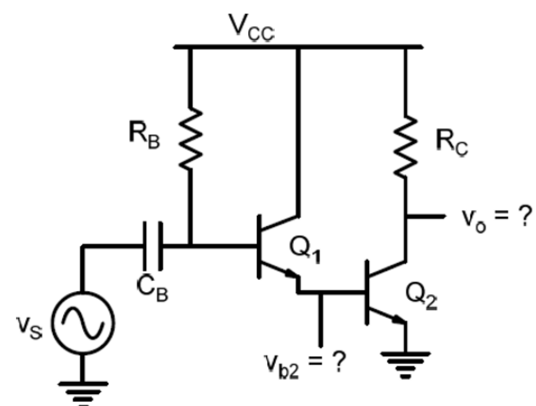
$$v_0 = v_s \frac{\beta_1 * \frac{1}{g_{m2}}}{r_{\pi1} + \beta_1 * \frac{1}{g_{m2}}}$$

$$A_V = \frac{\beta_1 * \frac{1}{g_{m2}}}{r_{\pi1} + \beta_1 * \frac{1}{g_{m2}}}$$



$$v_{b2} = v_s \frac{\beta_1 * r_{\pi2}}{r_{\pi1} + \beta_1 * r_{\pi2}} \cong v_s$$

$$v_0 = -g_{m2} \times R_c \times v_{b2} \cong -g_{m2} \times R_c \times v_s$$



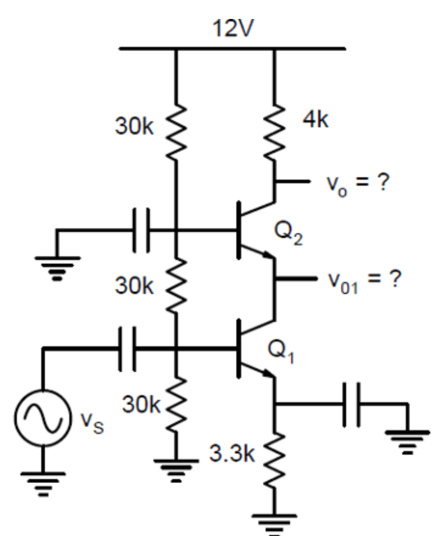
$$v_{01} = -g_{m1} * R_{out} * v_s$$

$$R_{out} = \left(\frac{1}{g_{m2}}\right) \| r_{02}$$

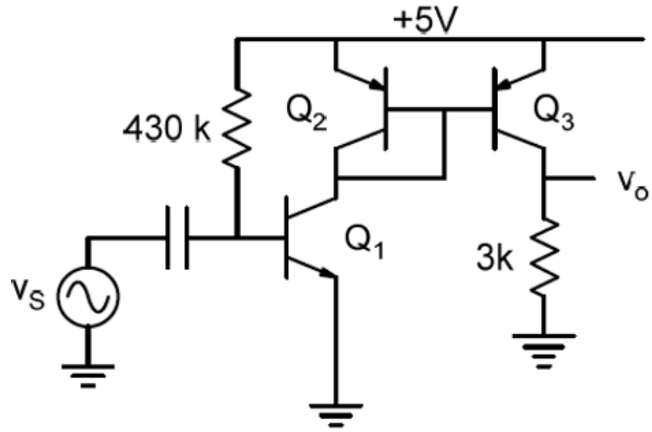
$$v_{01} = -g_{m1} * \left(\frac{1}{g_{m2}} \| r_{02}\right) * v_s \cong v_s$$

Current is same in Q1 and Q2. So, $g_{m1} = g_{m2}$.

$$v_0 = -g_{m1} R_c * v_s$$



Resistance seen at the collector of Q1 is parallel combination of r_{01}, $\frac{1}{g_{m2}}$ and $(r_{\pi 3} + \beta R_E)$. Thus,



$$\frac{v_{c1}}{v_s} = -g_{m1} * \left[(r_{01}) \middle\| \left(\frac{1}{g_{m2}}\right) \middle\| (r_{\pi 3})\right]$$

$$\frac{v_{c1}}{v_s} \cong -g_{m1} * \left(\frac{1}{g_{m2}}\right) \cong -1$$

$$v_0 = -g_{m1} R_C * v_{c1}$$

$$\frac{v_0}{v_s} = g_{m1} R_C$$