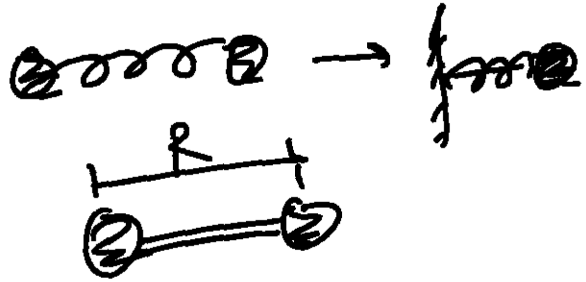
Recap: Particle 1D box — Zero point energy nonzero
Position has to finite



SHO — -same-

~~Angular momentum~~ — ϕ can be indefinite
So Energy can be 'zero'

<u>3D Rotor</u> (spherical motion)

$$E_{m_l} = \frac{m_l^2 \hbar^2}{2I}$$

$$E_l = l(l+1)\frac{\hbar^2}{2I}$$

$$\hat{L}^2 = -\hbar^2 \left\{ \frac{1}{\sin\theta} \frac{\partial}{\partial\theta}\left(\sin\theta \frac{\partial}{\partial\theta}\right) + \frac{1}{\sin^2\theta} \frac{\partial^2}{\partial\phi^2} \right)$$

$\hat{L}_z^2$ part

$$L^2 = L_x^2 + L_y^2 + L_z^2$$

$$-\frac{\hbar^2}{2I}\left[\frac{1}{\sin\theta}\frac{d}{d\theta}\left(\sin\theta \frac{d}{d\theta}\right) + \frac{1}{\sin^2\theta}\frac{d^2}{d\phi^2}\right]\Psi(\theta,\phi) = E\Psi(\theta,\phi)$$

$P_{l|m_l|}(\cos\theta)$

$$\Psi(\theta,\phi) \equiv Y_{l m_l}(\theta,\phi) = S(\theta)\cdot T(\phi)$$

Associated Lagendre Polynomials $\leftarrow$ Polynomial soln.

$\exp(i m_l \phi)$

Spherical Harmonics $Y_{\ell m_\ell}$ Satisfy:

$$\hat{H} Y_{\ell m_\ell}(\theta, \phi) = \left(\frac{\hbar^2 \ell(\ell+1)}{2I}\right) Y_{\ell m_\ell}(\theta, \phi)$$

$E_{\ell m_\ell} = \frac{\hbar^2}{2I}\ell(\ell+1)$

independent of m_ℓ.

$$\hat{L}^2 Y_{\ell m_\ell}(\theta, \phi) = \left[\hbar^2 \ell(\ell+1)\right] Y_{\ell m_\ell}(\theta, \phi) \longrightarrow \ell = 0, 1, 2, 3 \ldots$$

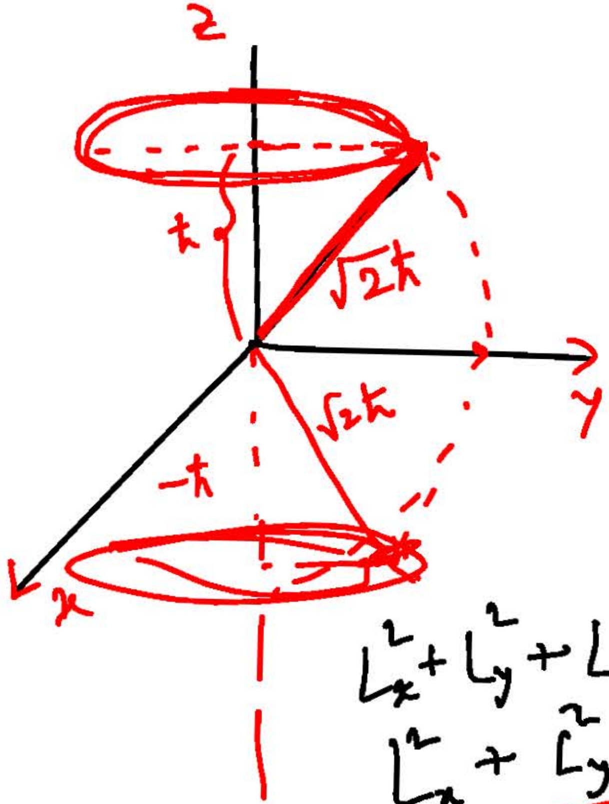
$$\hat{L}_z \underbrace{Y_{\ell m_\ell}(\theta, \phi)}_{\text{function is an eigen } f^n \text{ of } \hat{H},\ \hat{L}^2\ \&\ \hat{L}_z} = (\hbar \cdot m_\ell) Y_{\ell m_\ell}(\theta, \phi) \qquad m_\ell = 0, \pm 1, \pm 2, \ldots, \pm \ell$$

i.e. $|m_\ell| \leq \ell$

Choice of z-axis is arbitrary

$\Delta E = 0$, $\Delta L_z = 0$ & L^2 is known with certainty.

$Y_{\ell m_\ell}$ is not an eigen f^n of $\hat{L}_x$ and $\hat{L}_y$.



$\underbrace{l=1}$ $\quad m_l = 0, \pm 1 \quad$ degeneracy = 3

$$E_{1,0} = E_{1,+1} = E_{1,-1} = \frac{\hbar^2}{I}$$

$$L^2 = L_x^2 + L_y^2 + L_z^2 \longleftarrow$$

$$\hat{L}_z Y_{10}(\theta, \phi) = 0 \cdot Y_{10}(\theta, \phi)$$

$$\hat{L}_z Y_{1,\pm 1}(\theta, \phi) = (\pm \hbar) Y_{1,\pm 1}(\theta, \phi)$$

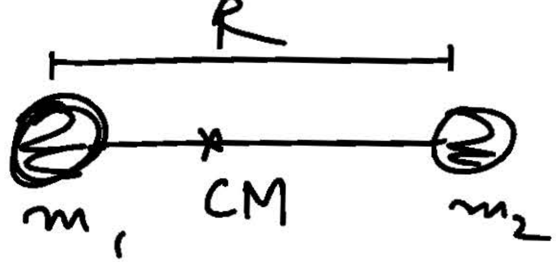
$$L_x^2 + L_y^2 + L_z^2 = 2\hbar^2$$

$$L_x^2 + L_y^2 = \hbar^2$$

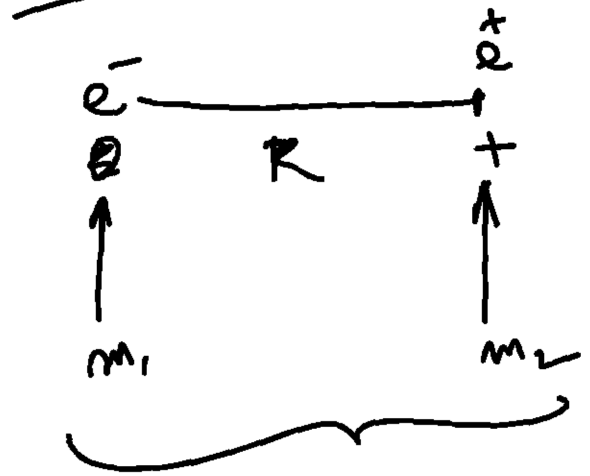
$\hat{L}^2$ has eigenvalue of $2\hbar^2$.

Angular momentum vector precessing about "z"-axis

$$|L| = \sqrt{2}\,\hbar$$



H-atom Problem



V = Coulomb Potential

H = KE of e^- + KE of the nucleus + Coulomb potential (attraction)

Radial coordinates

$$\hat{H} = \frac{1}{2m_e}\hat{p}_e^2 + \frac{1}{2m_N}\hat{p}_N^2 - \frac{e^2}{4\pi\varepsilon_0 r}$$

$$\boxed{\mu = \frac{m_e m_{nu}}{m_e + m_{nu}} \simeq m_e}$$

Laplacians $\longrightarrow$ ∇_e^2 $\qquad$ ∇_{nu}

(vectors)

i j k unit vectors.

$$\hat{H}_{relative} = -\frac{\hbar^2}{2\mu}\nabla^2 - \frac{e^2}{4\pi\varepsilon_0 r}$$

$$\boxed{E_n = -\frac{13.6}{n^2}\,eV}$$