EE210: HW-5 Solution

Date: 05.02.2019

Unless stated otherwise, the BJT in the problems given below has the following characteristics
$I_S = 2.03 * 10^{-15}A;\ \beta_F = 100;\ \beta_R = 1;\ V_A = 100;\ r_{bb} = 200\Omega;\ V_T = 26mV;\ C_{je0} = 1pF;$
$C_{jc0} = 0.5pF;\ C_{js0} = 3pF; m = 0.5;\ V_{bi} = 0.85;\ \tau_F = 1ns$

Q.1 Design the amplifier shown in Fig.1 such that open circuit voltage gain is 100. What happens to the bias point if V_{BB} increases by 10%?

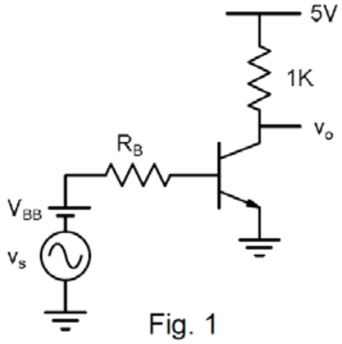


Fig. 1

Sol.:

$$A_V = -\frac{r_\pi}{R_B + r_\pi} * g_m R_C = -100$$

$$-(r_\pi * g_m) * \frac{R_C}{R_B + r_\pi} = -100$$

Given, $r_\pi * g_m = \beta = 100$
Thus,

$$R_B + r_\pi = R_C$$

$$R_B + \frac{V_T}{I_{BQ}} = R_B + \frac{V_T}{I_{CQ}} * \beta = R_B + \frac{2.6}{I_{CQ}} = 1k\Omega$$

From this relation, I_{CQ} must be larger than 2.6mA.
Also,

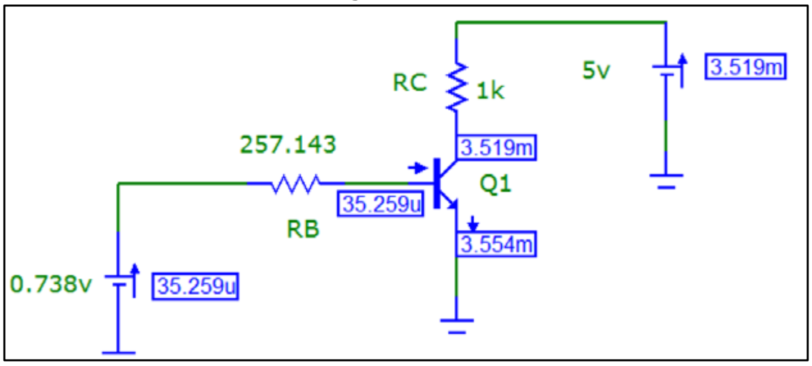
$$V_{CEQ} = V_{CC} - I_{CQ} * R_C = 5 - I_{CQ} * 1k\Omega \geq 0.2\text{V}$$

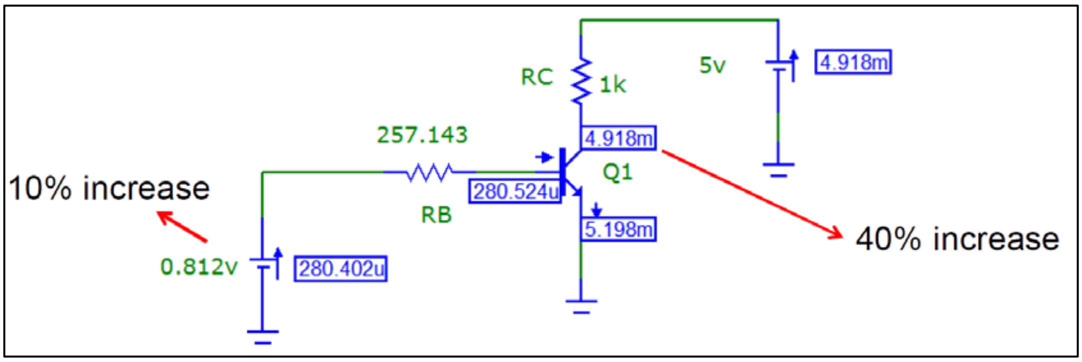
From this relation, I_{CQ} must be less than ~5mA to avoid BJT going into saturation. Let $I_{CQ} = 3.5\text{mA} \Rightarrow R_B = 257\Omega$.

$$V_{BB} = R_B I_B + V_{BE} = 0.009 + V_{BE}$$

We can not simply assume that V_{BE} ~ 0.7 due to the almost negligible value of term $I_B R_B$.

$$V_{BE} = V_T * \ln\left(1 + \frac{I_{CQ}}{I_S}\right) = 0.733 \Rightarrow V_{BB} = 0.742V$$





Q.2 Design the amplifier shown in Fig. 2 such that open circuit voltage gain is also 100. What happens to the bias point if V_{CC} increases by 10%? What would be the impact on the amplifier's characteristics if β were to become 200?

Sol.:

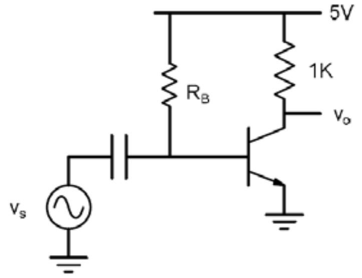


Fig. 2

$$A_V = -g_m R_C = -\frac{I_{CQ}}{V_T} R_C = -100$$
$$I_{CQ} = 2.6mA$$
$$R_B = \frac{V_{CC} - 0.7}{I_{BQ}} = \frac{V_{CC} - 0.7}{I_{CQ}} * \beta = 165k\Omega$$

Once design is fixed:

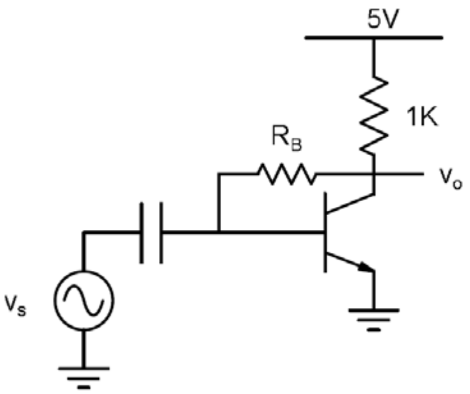
$$I_{CQ} = \beta * I_{BQ} = \beta * \frac{V_{CC} - 0.7}{R_B}$$

Change in bias point due to 10% increase in V_{CC},

$$\frac{\Delta I_C}{I_C} = \frac{I_{CQ2} - I_{CQ1}}{I_{CQ1}} = \frac{\Delta V_{CC}}{V_{CC} - 0.7} = \frac{0.5}{5 - 0.7} = 11.6\%$$

If β were doubled, then I_{CQ} would tend to double as well, but that would force the transistor into saturation.

Q.3 Two alternative bias schemes are shown below. Design these amplifiers also for an open circuit voltage gain of 100 and check their sensitivity to β. Also try to evaluate your design using circuit simulation.



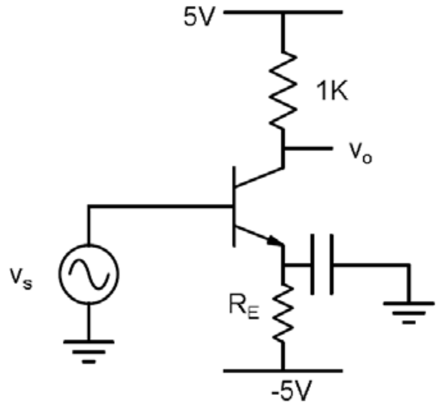
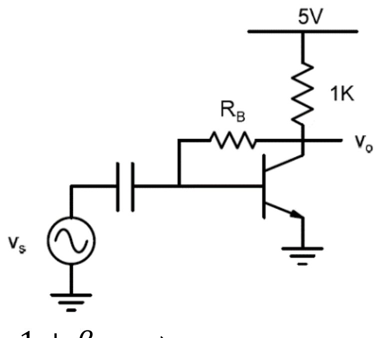


Fig. 3

Fig. 4

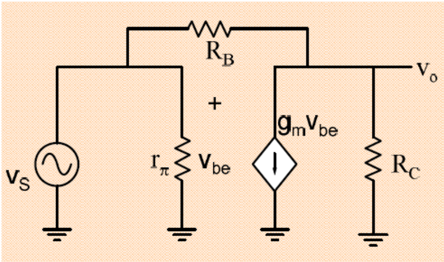
Sol.:

DC analysis of Fig. 3.



$$\frac{V_{CC} - V_C}{R_C} = I_E = I_B + I_C = \frac{1+\beta}{\beta} I_C$$
$$\frac{V_C - 0.7}{R_B} = I_B = \frac{I_C}{\beta}$$
$$I_C = \left(\frac{V_{CC} - 0.7}{R_C}\right) * \left(\frac{\beta}{1 + \beta + R_B/R_C}\right)$$
$$V_{CEQ} = V_{CC} - I_{EQ} R_C = V_{CC}\left(\frac{R_B/R_C}{1 + \beta + R_B/R_C}\right) + 0.7\left(\frac{1+\beta}{1 + \beta + R_B/R_C}\right)$$

Small-signal analysis of Fig. 3.



$$v_{be} = v_s$$

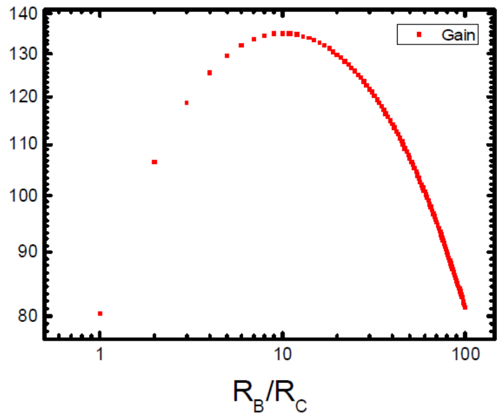
$$-\frac{v_0}{R_C} = g_m v_{be} + \left(\frac{v_0 - v_{be}}{R_B}\right)$$

$$v_0 = -\left(\frac{g_m - \frac{1}{R_B}}{\frac{1}{R_B} + \frac{1}{R_C}}\right) v_{be}$$

$$A_V = -\left(\frac{g_m - \frac{1}{R_B}}{1 + \frac{R_C}{R_B}}\right) R_C = -\left(\frac{g_m}{1 + \frac{R_C}{R_B}}\right) R_C + \left(\frac{\frac{1}{R_B}}{1 + \frac{R_C}{R_B}}\right) R_C$$

$$A_V = -\frac{1}{V_T} * \left(\frac{V_{CC} - 0.7}{1 + R_C/R_B}\right) * \left(\frac{\beta}{1 + \beta + R_B/R_C}\right) + \left(\frac{R_C/R_B}{1 + R_C/R_B}\right)$$

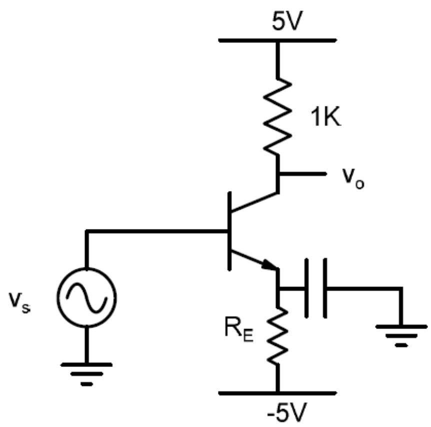
Gain (A_V) depends on the ratio R_B/R_C.



For R_B/R_C = 1.65, A_V = -100.
Let's choose R_C = 1kΩ, then R_B = 1.65kΩ.
For this design, I_{CQ} = 4.189mA and V_{CEQ}~0.8V.
If we double β then I_{CQ} ~ 4.244mA. This is a change of only 1.3%.

Another possibility:
For R_B/R_C = 62, A_V = -100.
Let's choose R_C = 1kΩ, then R_B = 62kΩ.
For this design, I_{CQ} = 2.64mA and V_{CEQ}~2.3V.
If we double β then I_{CQ} ~ 3.27mA, a change of 24%.

Analysis of Fig. 4.



$$A_V = -g_m R_C = -100$$

$$\frac{I_{CQ}}{V_T} R_C = \frac{I_{CQ}}{26mV} * 1k = 100$$

$$I_{CQ} = 2.6mA$$

$$\frac{-0.7 + 5}{R_E} = I_E = \frac{1 + \beta}{\beta} I_C$$

$$R_E = 1.64k\Omega$$

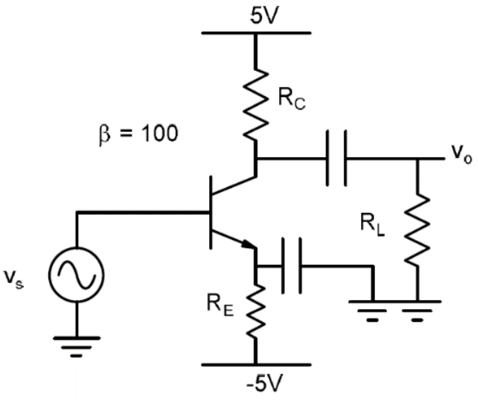
Once the design is fixed,

$$I_{CQ} = \frac{\beta}{1 + \beta} * \frac{-0.7 + 5}{R_E}$$

If we double β then I_{CQ} ~ 2.613mA, a change of only 0.5%.

Q.4 (a) Design the amplifier shown below in Fig. 5 such that: A_{V0} = -100; R_{in} = $1k\Omega$.

Sol:



$$R_{in} = r_\pi = \frac{V_T}{I_{CQ}}\beta = 1k\Omega$$
$$\Rightarrow I_{CQ} = 2.6mA$$

$$A_{VO} = -g_m R_c$$
$$-100 = -\frac{I_{CQ} \times R_c}{V_T}$$
$$\Rightarrow R_c = 1k\Omega$$

$$\frac{-0.7+5}{R_E} = I_E = \frac{1+\beta}{\beta} I_C \Rightarrow R_E = 1.64k\Omega$$
$$V_{CEQ} = V_{CC} - I_{CQ} \times R_c - (-0.7) = 3.1V$$

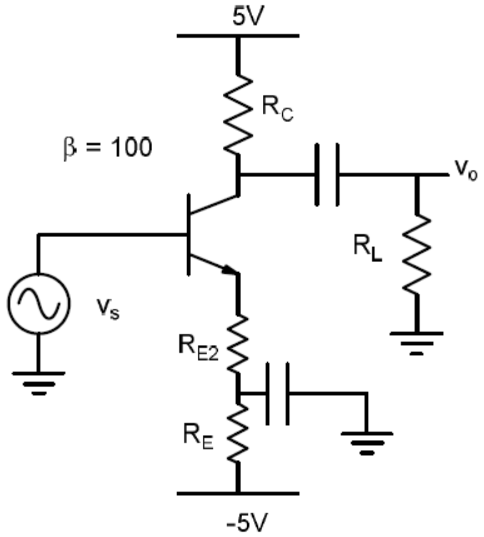
(b) Determine the voltage gain and maximum voltage swing with 10% HD_2 distortion for $R_L = 2k\Omega$. Assume a saturation voltage of ~0.2V.

Sol.:

$$A_V = -g_m R_c \parallel R_L = -66.67$$
$$v_{om} = Min\left\{\underbrace{V_{CEQ} - V_{CEsat}}_{2.9V}; \underbrace{I_{CQ}(R_c \parallel R_L) * \left(\frac{HD_2}{25}\right)}_{0.693V}\right\} = 0.69V$$

(c) Determine the value of an extra un-bypassed emitter resistance that may be required to reduce open circuit voltage gain by half. Determine the new value of R_{in}. Simulate the circuit to determine the swing for harmonic distortion of 10%.

Sol.:



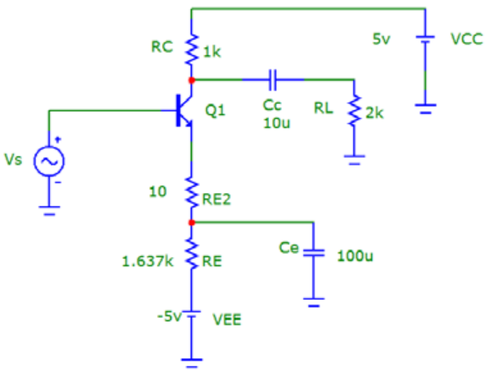
$$A_{VO} \cong -\frac{g_m R_c}{1 + g_m R_{E2}} = -50$$

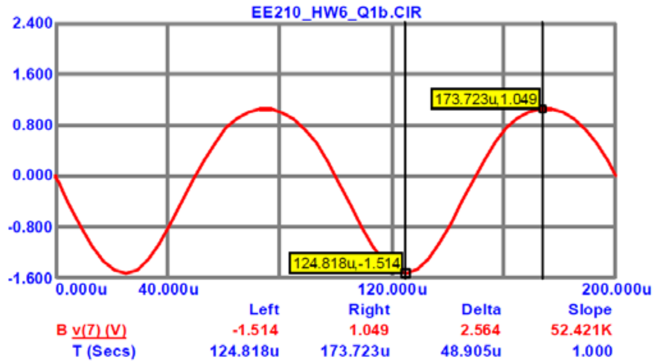
$$g_m R_{E2} = 0.1\Omega^{-1} * R_{E2} = 1 \Rightarrow R_{E2} = 10\Omega$$

Note that extra 10Ω resistor does not disturb the bias point much.
We may split the original R_E into 2 resistors and keep only 10Ω un-bypassed.

$$R_{in} \cong r_\pi \times (1 + g_m R_{E2}) = 2k\Omega$$

As expected, the input resistance has doubled when gain is halved for constant output resistance.



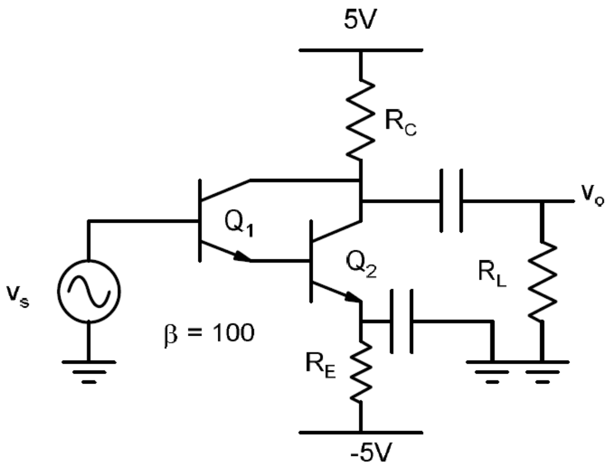


$$v_{opp} \sim 2.56V \; for \; THD \sim 10\%$$

Negative feedback reduces distortion, which allowed input voltage of 80mV p-p to be applied to obtain this swing.

Q.5 Suppose the amplifier shown in Fig. 6 is designed with same bias point (I_{CQ} and V_{CEQ}) calculated earlier in Q.4. What would be the open circuit voltage gain and input resistance for this amplifier? Will the output swing be similar?

$$I_{CQ} \sim I_S \times \exp\left(\frac{V_{BE}}{V_T}\right)$$



Sol: Bias point from previous question is $I_{CQ2} = 2.6mA, \; V_{CEQ2} = 3.1V$.

$$I_{CQ2} = 2.6mA \Rightarrow I_{CQ1} = \frac{2.6mA}{\beta} = 26\mu A$$

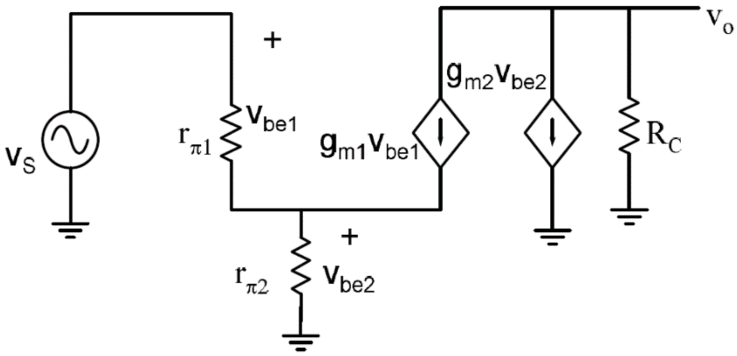
$$\frac{-1.3 + 5}{R_E} = I_{EQ2} = \frac{1+\beta}{\beta} I_{CQ2} \Rightarrow R_E = 1.41k\Omega$$

Although not required, note that if we assume 0.7V drop for Q_2, then drop for Q_1 (whose collector current is 100 times lower) would be ~0.1 V lower.

$$V_{CEQ2} = V_{C2} - V_{E2} = 3.1V$$

$$\Rightarrow V_C = 3.1 + (-1.3) = 1.8V \Rightarrow R_C = \frac{V_{CC} - V_C}{I_{CQ2}} = 1.23k\Omega$$

Small signal equivalent circuit:



$$I_{CQ2} = \beta I_{BQ2} = \beta I_{CQ1}$$

As $r_\pi = \frac{V_T}{I_{CQ}}\beta$ and $g_m = \frac{I_{CQ}}{V_T}$,

$$r_{\pi1} = \beta r_{\pi2}$$
$$g_{m2} = \beta * g_{m1}$$

$$v_s = v_{be1} + v_{be2}$$

$$v_{be2} = g_{m1}v_{be1} * r_{\pi2} = \frac{I_{CQ1}}{V_T} * \frac{V_T}{I_{CQ2}}\beta * v_{be1} = v_{be1}$$

$$v_{be1} \cong v_{be2} = \frac{v_s}{2}$$

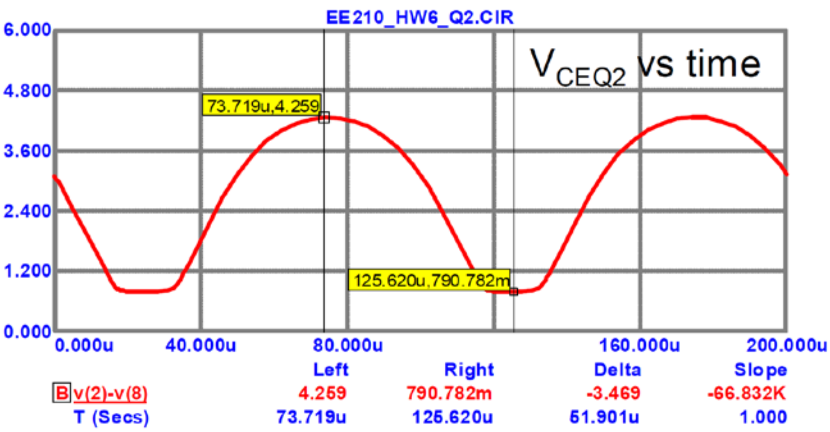
$$v_o = -(g_{m2}v_{be2} + g_{m1}v_{be1})R_C \cong -g_{m2}v_{be2}R_C$$

$$A_{Vo} \cong -\frac{g_{m2}R_C}{2} = -\frac{\frac{2.6mA}{26mV} * 1.23k\Omega}{2} = -61.5$$

$$R_{in} \cong r_{\pi1} * (1 + g_{m1}r_{\pi2}) = 2r_{\pi1} = 2\frac{V_T}{I_{CQ1}}\beta = 200k\Omega$$

$$\frac{|A_{Vo}| * R_{in}}{R_o} = \frac{\frac{g_{m2}R_C}{2} * 2r_{\pi1}}{R_C} = \frac{I_{CQ2}}{V_T} * \frac{V_T}{I_{CQ1}}\beta \cong \beta^2 = 10^4$$

Note that $V_{CEQ} = V_{CEQ1} + V_{BEQ2}$. Transistor Q₁ can go into saturation but not transistor Q₂. When Q₁ goes into saturation, distortion begins.



$$v_{om} = Min\left\{\underbrace{V_{CEQ2} - V_{CEsat1} - 0.7}_{2.2V}; \underbrace{I_{CQ2}(R_c \parallel R_L) * \left(\frac{HD_2}{25}\right)}_{0.693V}\right\} = 0.69V$$

Note that second term is lower and thus swing is about the same.