

EE210: Microelectronics-I

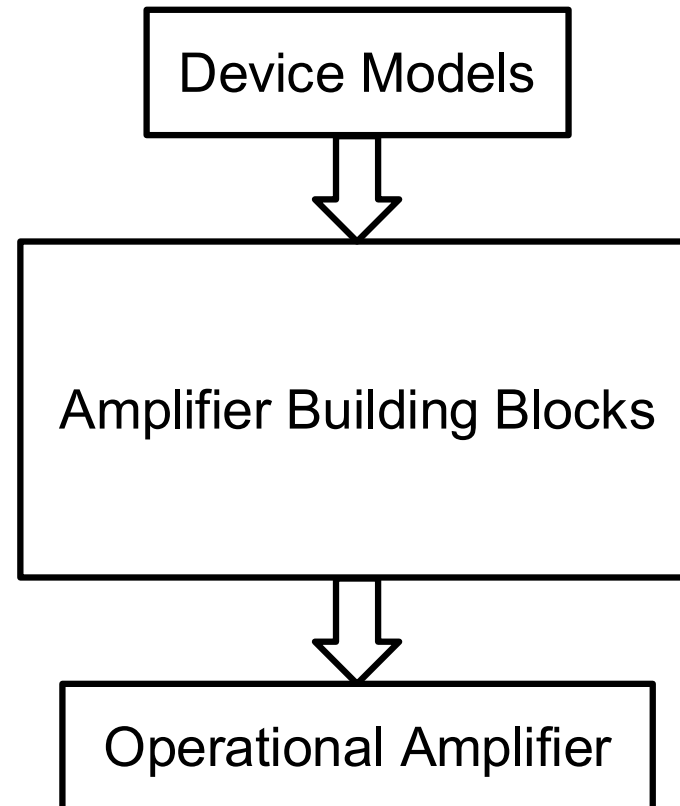
Lecture-3 Small Signal Device Model-1

Instructor - Y. S. Chauhan

Slides from: B. Mazhari
Dept. of EE, IIT Kanpur

EE210

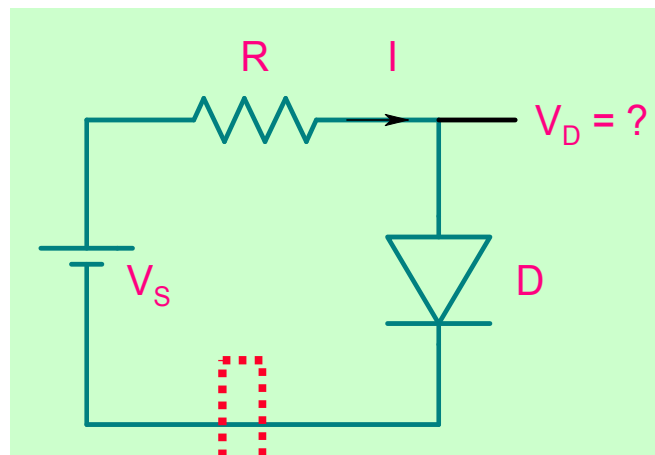
Topics



Both BJT and MOS analog circuits will be studied

Circuit Analysis

Analysis of a circuit involves **transformation** of it into a set of mathematical equations and their solution



KVL, KCL

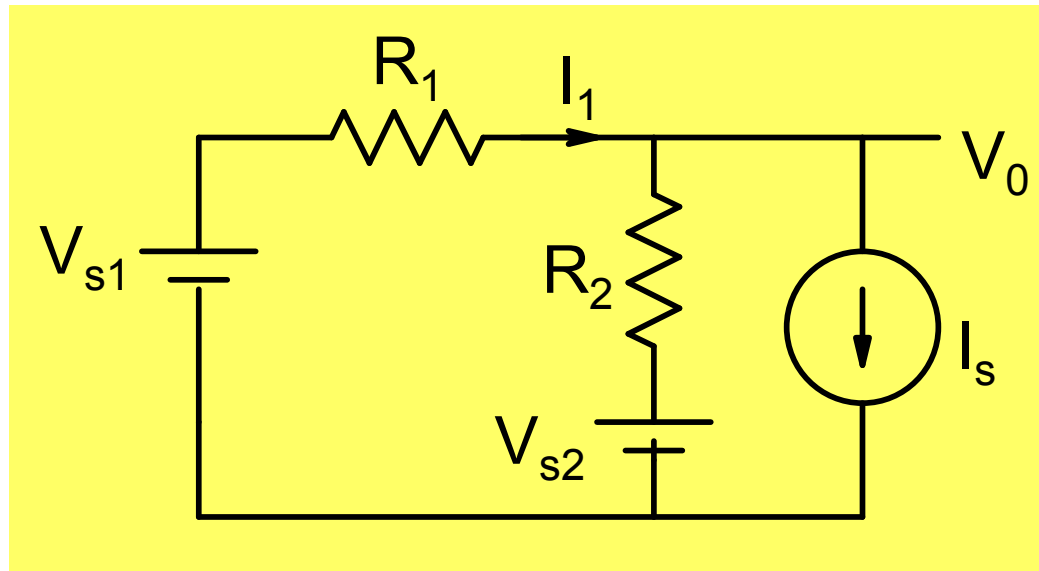
$$V_S = I \times R + V_D$$

models

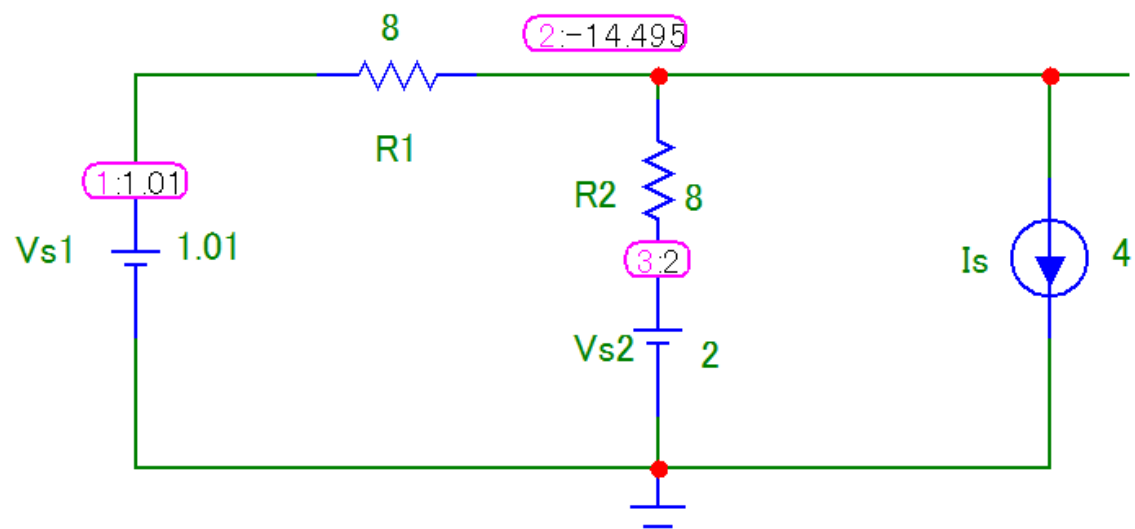
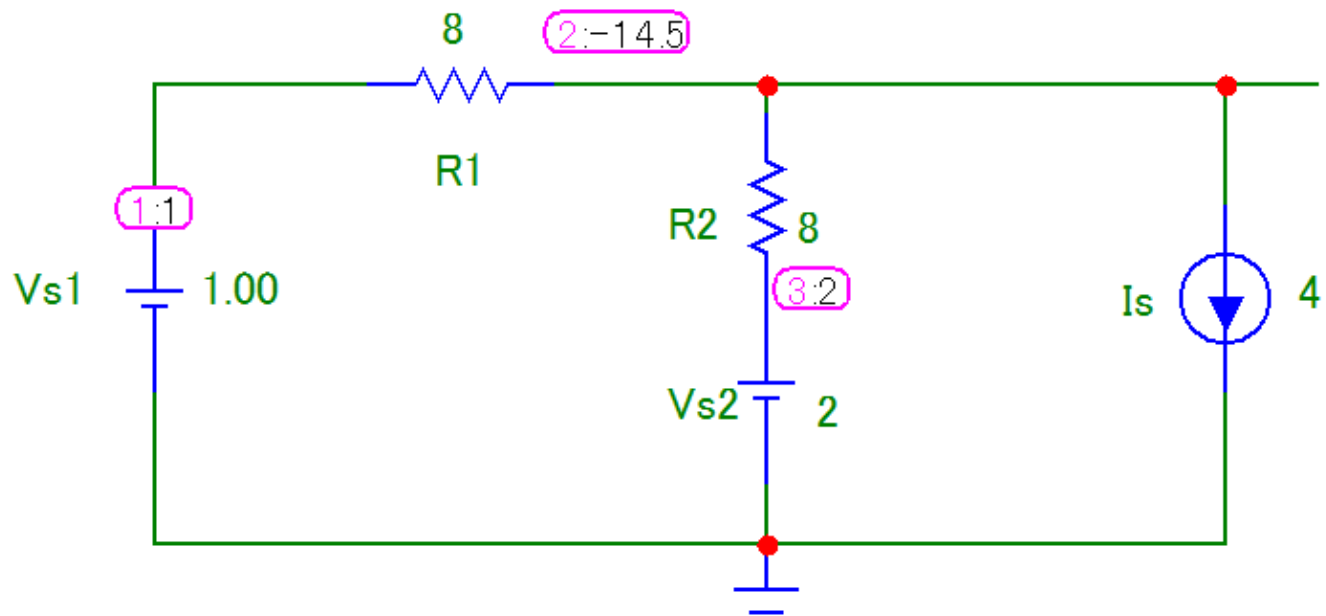
$$I_D = I_S \times \left\{ \exp\left(\frac{V_D}{n V_T}\right) - 1 \right\}$$

(Mathematical equations can be transformed into a circuit as well)

Incremental Circuit Analysis

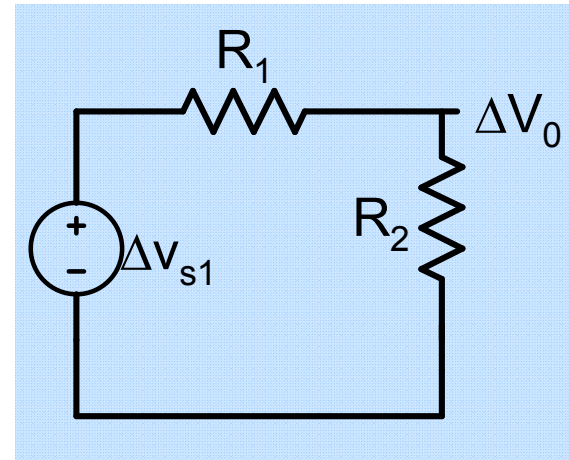
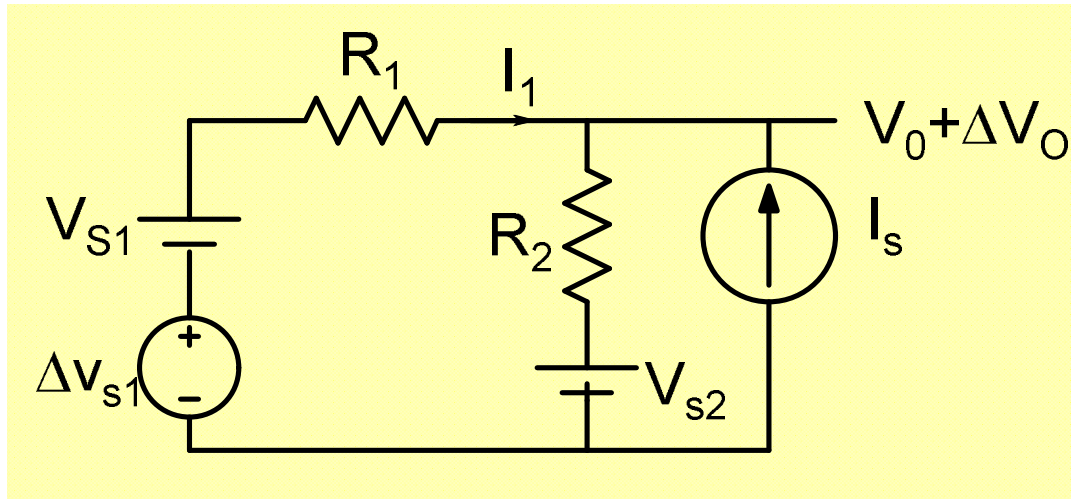


$$\Delta V_{s1} = 10\text{mV volts}; \Delta V_0 = ?$$



$$\Delta V_o = 5\text{mV}$$

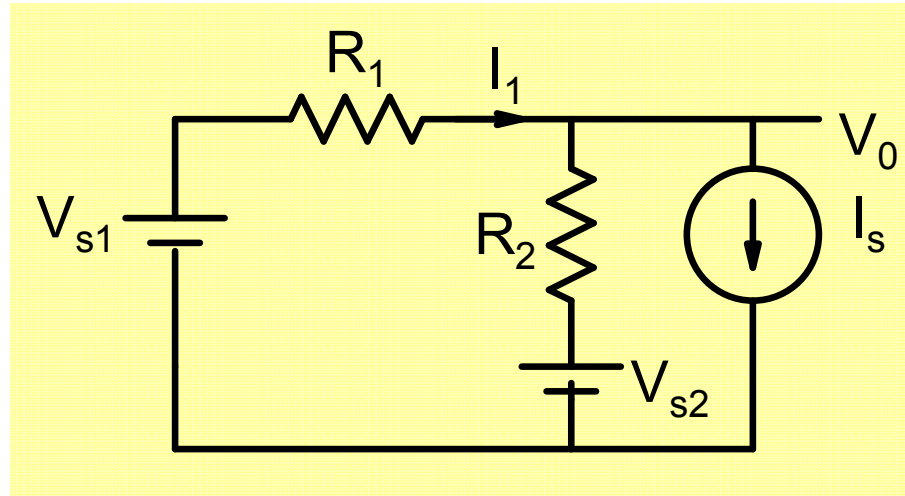
One Method: Superposition Theorem



$$\Delta V_o = \Delta v_{s1} \times \frac{R_2}{R_1 + R_2} \quad 5\text{mV}$$

But this requires the circuit to be linear !

Alternative perspective



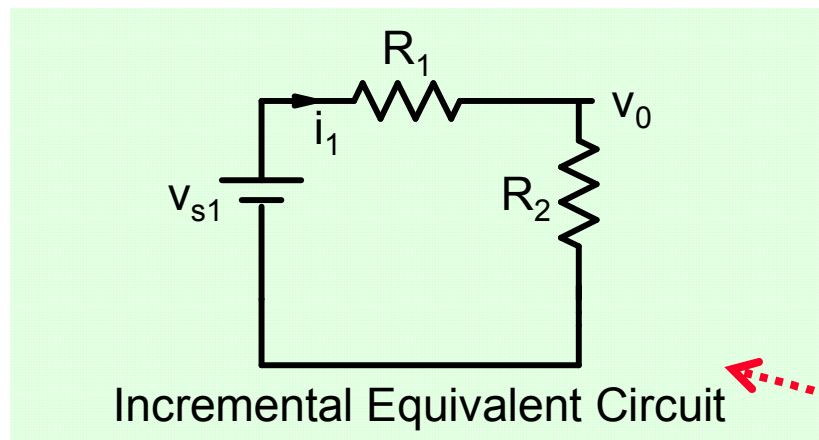
$$V_{S1} = I_1 R_1 + V_0 \quad (1)$$

$$I_1 = I_S + (V_0 - V_{S2}) / R_2 \quad (2)$$

Let $\Delta V_{s1} = v_{s1}$

$$V_{S1} + v_{s1} = (I_1 + i_1) \times R_1 + V_0 + v_o \quad (3)$$

$$I_1 + i_1 = I_S + (V_0 + v_o - V_{S2}) / R_2 \quad (4)$$



$$(3)-(1): v_{s1} = i_1 R_1 + v_o$$

$$(4)-(2): i_1 = v_o / R_2$$

$$\Rightarrow v_{s1} = i_1 R_1 + i_1 R_2$$

Increment equivalent circuit can be obtained by building incremental device model for each circuit element.

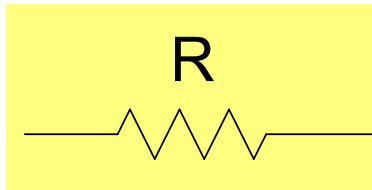
Terminology

V_X : Nominal or base Value Normally is a dc

v_x : incremental Value Often ac but could be dc as well

$V_x = V_X + v_x$: Net Value

Incremental Models: Resistor



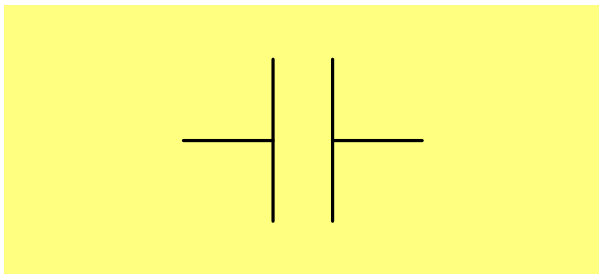
$$V = I \times R$$

$$V + v = (I + i) \times R$$

$$\Rightarrow v = i \times R$$

Incremental model of a resistor is a resistor of the Same magnitude

Capacitor



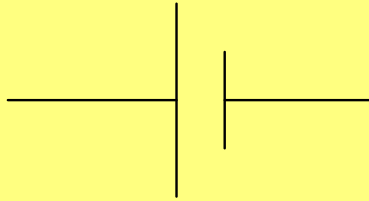
$$I = C \frac{dV}{dt}$$

$$I + i = C \frac{d(V + v)}{dt}$$

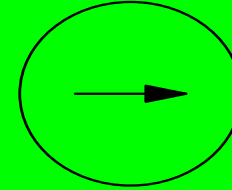
$$i = C \times \frac{dv}{dt}$$

Incremental model of a capacitor is a capacitor of the same magnitude. The same holds for an inductor as well.

Incremental Models



$$V = \text{constant}$$
$$v = 0$$

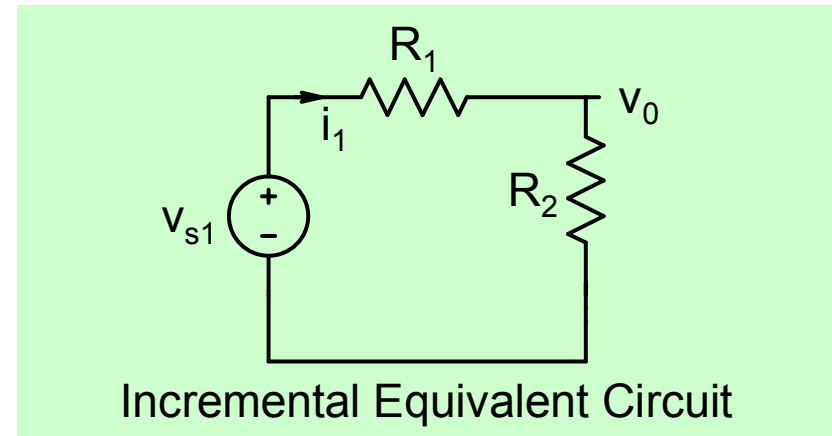
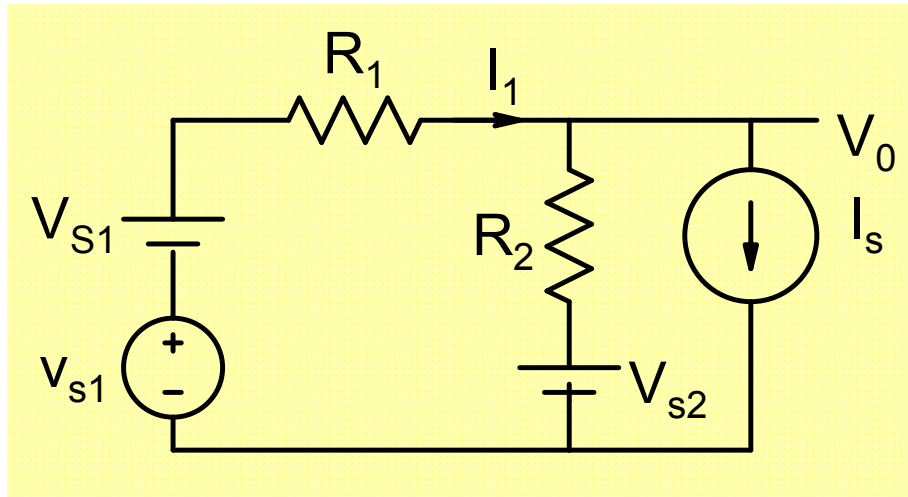


$$I = \text{constant}$$
$$i = 0$$

Incremental model of a constant Voltage Source is a short circuit

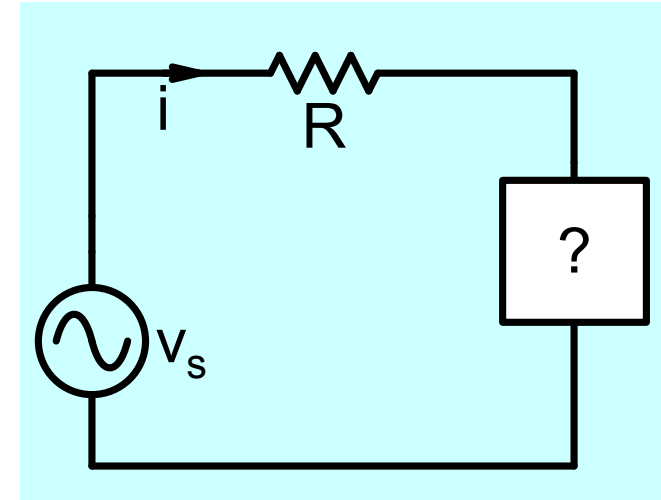
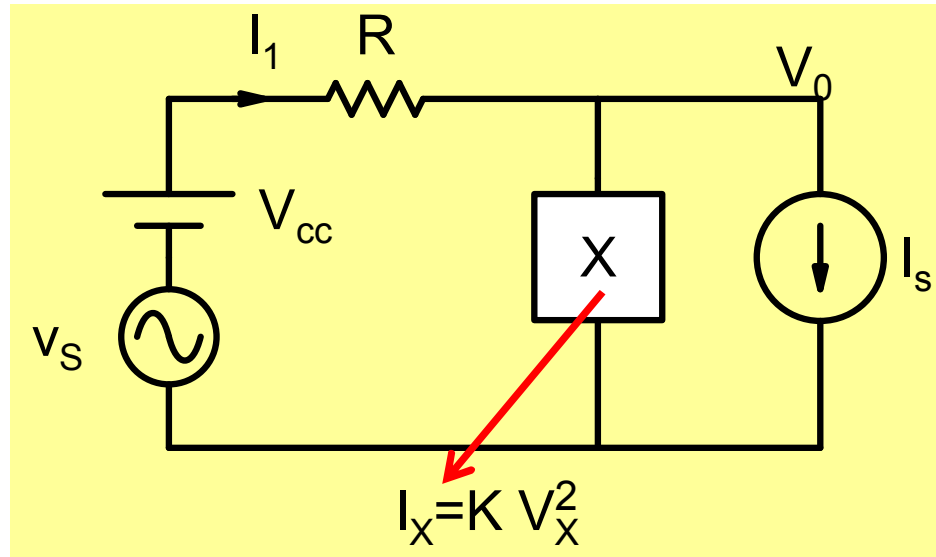
Incremental model of a constant current Source is an open circuit

Solution using incremental equivalent device models



$$v_o = v_{s1} \times \frac{R_2}{R_1 + R_2}$$

Nonlinear element



$$I_X = k \times V_X^2$$

$$I_X + i_x = k \times (V_X + v_x)^2$$

$$i_x = k V_X^2 \{ (1 + v_x / V_X)^2 - 1 \}$$

Non-linearity makes the model difficult to use so approximations are used to make it **linear**

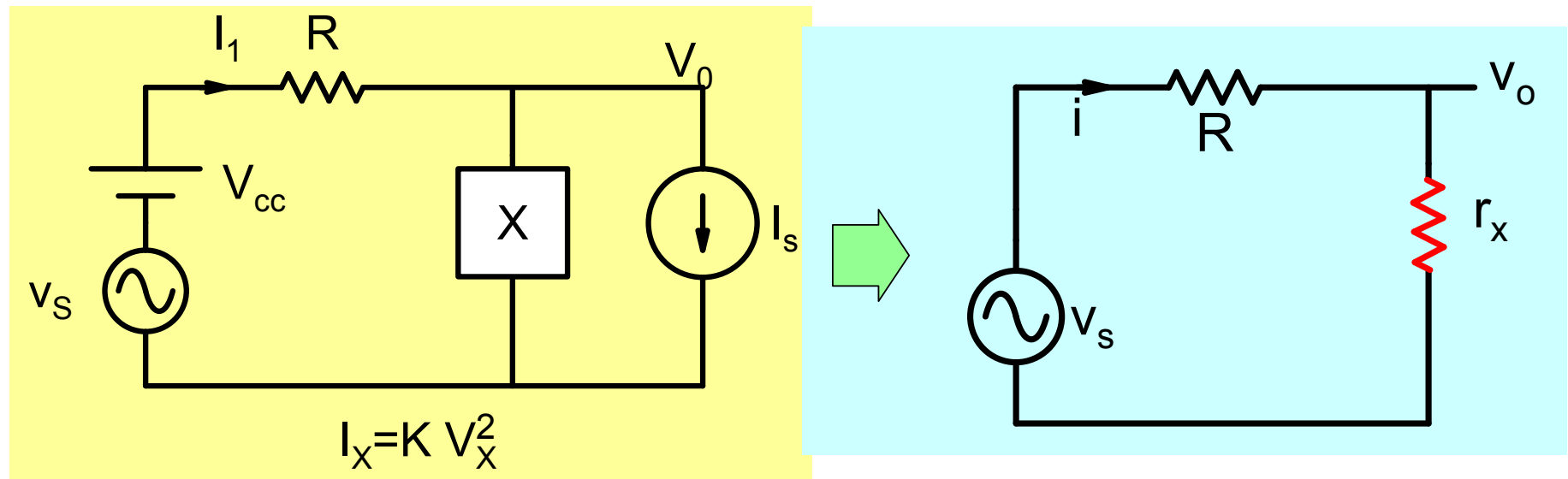
$$i_x = k V_X^2 \{ (1 + v_x / V_X)^2 - 1 \}$$

Small signal approximation: $v_x / V_X \ll 1$

$$i_x \cong k V_X^2 \times \left\{ \left(1 + \frac{2 v_x}{V_X} \right) - 1 \right\} = 2 k \times V_X \times v_x$$

$$i_x = v_x / r_x \quad ; \quad r_x = \frac{1}{2 k V_X}$$

Nonlinear element



$$r_x = \frac{1}{2kV_o}$$

$$v_o = v_s \times \frac{r_x}{R + r_x}$$

How small is small?

Depends on how much error we can tolerate!

$$i_x = kV_X^2 \{(1 + v_x / V_X)^2 - 1\}$$

$$i'_x \cong 2k \times V_X \times v_x$$

$$V_X = 1V$$

v_x (V)	Error (%)
0.02	1
0.22	10
0.5	20
1.0	33.3

$$I_x = k \times V_x^4$$

$$V_X = 1V$$

v_x (V)	Error (%)
0.007	1
0.071	10
0.5	50.77
1.0	73.3

Stronger nonlinearity implies smaller voltage for same error