EE200 Assignment 1

1 You know that the signal in a conductor travels at a finite velocity less than that of light in vaccum. We call this the propogation velocity, v Let v for a given conductor be 0.4c. As engineers, we accept that a long conductor can be considered a lumped element if the phase difference between the two ends is less than a milliradian.

(a) If we apply a sinusoid at 200 MHz, what is the maximum possible length of the conductor?

(b) If the conductor is 50km long, what is the highest frequency sinusoid we can apply?

(c) We join two equal lengths of different conductors with v of 0.4c and 0.25c. Considering the length of the complete conductor, answer (a) and (b) above.

(d) A signal transmitted on a conductor is a complex signal with multiple Fourier components (harmonics) at $f, 2f, 3f, \ldots$ A ready example is a square wave. What criteria should be used to determine whether the conductor behaves as a lumped or distributed element?

2 In the circuit shown in Fig.2, let the current and voltage in branch $k : k = 1, 2, \ldots, 7$ be i_k and v_k respectively. Assume that the associated direction convention is followed to define branch voltage polarities. What is the minimum number of branch voltages you need to be given (and which ones) in order to find the rest using KVL? For the minimum number, is the set of branch voltages to be given unique? Or can there be multiple solutions?

3 A loop is any closed path without self intersections. A mesh is a closed path inside which no element/branches are present. List all meshes and loops in the graph of the previous problem.

4 A time-invariant inductor has an $i - \phi$ characteristic given by $i = \phi^2 \,\mathrm{sgn}(\phi)$.

(a) Sketch the characteristics.

(b) Is it active or passive? Linear or nonlinear?

(c) How would you assign L_{dc} at $\phi = 0$? What is L_{ac} at $\phi = 0$?

(d) Show that inverse of L_{ac} increases linearly as ϕ.

5 (a) Is the resistor in Fig.5 voltage or current controlled (or both or neither controlled?)

(b) Obtain the characteristics of the resistor which, when put in parallel with the given resistor will make the combination have a linear characteristic of 1Ω.

(c) Put this resistor in series with an ideal diode in the two configurations, (i) parallel, (ii) series and sketch the combined characteristic over $-4 < v < 4$. Next reverse the diode in the two configurations and repeat (sketch the combined characteristic).

6 A cylindrical core has an iron rod core that executes SHM $x(t) = \sin(100\pi t)$ along the coil axis. When rod is fully out (when $100\pi t = 2\pi n + 3\pi/2$), the coil inductance: $L_{min} = 0.5\,\mathrm{H}$ and when the rod is fully in (when $100\pi t = 2\pi n + \pi/2$), the coil inductance: $L_{max} = 2\,\mathrm{H}$.

(a) Find the coil terminal voltage when $i = 1\,\mathrm{A}$ dc.

(b) Find the coil terminal voltage when $i = 5\cos 240\pi t\,\mathrm{A}$.

(c) Find the coil terminal voltage if we set $i(t) = -L(t)\,\mathrm{A}$.

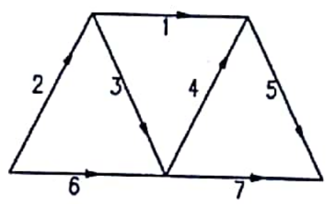


Fig.2

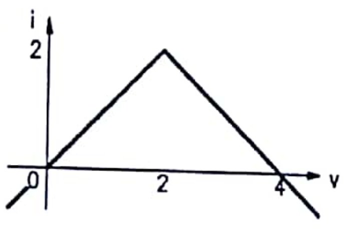


Fig.5

Assignment - 1

Q.1. Propagation velocity $v_p = 0.4C$

Conductor is lumped if

Phase diff. $\Delta\phi < 0.001$ rad

Phase difference between conductor of length L

$$\Delta\phi = \frac{2\pi L}{\lambda} \text{ radian}$$

(Taking $c = 3\times10^8$ m/sec)

(a) we have sinusoid of 200 MHZ

$$\lambda = \frac{v}{f} = \frac{0.4C}{200 \text{ MHZ}} = \frac{0.4\times3\times10^8 \text{ m/sec}}{200\times10^6 \text{ HZ}}$$

$$\lambda = .6 \text{ m}$$

$$\frac{2\pi L}{.6} < .001 \text{ rad}$$

$$L < \frac{.001\times.6}{2\pi} \text{ mt} = 9.55\times10^{-5} \text{ mt.}$$

$$\boxed{L < 9.55\times10^{-2} \text{ mm}}$$

(b) $L = 50$ km

$$\frac{2\pi\times50\times10^3}{\lambda} < .001$$

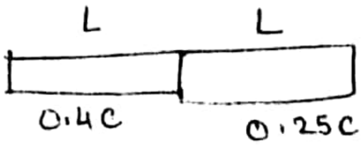
$$\lambda > \frac{2\pi\times50\times10^3}{.001} = 3.14\times10^8 \text{ m}$$

$$f_{max} < \frac{0.4c}{\lambda_{min}}$$

$$f_{max} < \frac{0.4 \times 3 \times 10^{8}\ m/sec}{3.14 \times 10^{8}\ m} = 0.382\ Hz$$

$$\boxed{f_{max} = 0.382\ Hz}$$

(c) (i)



$$\lambda_1 = \frac{0.4c}{200\times10^6} = .6\,m$$

$$\lambda_2 = \frac{0.25c}{200\times10^6} = 0.375\,m$$

$$\frac{2\pi L}{\lambda_1} + \frac{2\pi L}{\lambda_2} < 0.001$$

$$L < \frac{0.001}{2\pi} \Big/ \left[\frac{1}{\lambda_1} + \frac{1}{\lambda_2}\right]$$

$\boxed{\cancel{L < 0.0272\,m}}$ $\boxed{L < 6.9\times10^{-4}\,m}$

(ii) $L = 50\,km$

$$\frac{2\pi L}{\lambda_1} + \frac{2\pi L}{\lambda_2} < 0.001$$

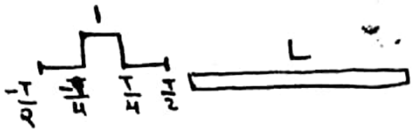
$$f_1 = \frac{0.4c}{\lambda_1}, \quad f_2 = \frac{0.25c}{\lambda_2}$$

$$f\left(\frac{2\pi L}{0.4c} + \frac{2\pi L}{0.25c}\right) < 0.001$$

$$f_{max} = \frac{0.001}{2\pi\, 50\times10^3 \times 3\times10^8}\,[4+2.5]$$

$$\boxed{f_{max} = 6.8967\times10^{-17}\ Hz}$$

(a)



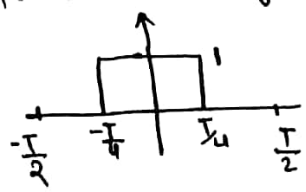
So for given wave we can calculate the frequency components present.

~~Since harmonics~~

Take example of square wave

using Fourier Analysis we can find out

Fourier components as follows



$$\boxed{a_k = \frac{1}{T}\int_{-T/4}^{T/4} e^{-jk\frac{2\pi}{T}t}\,dt = \frac{\sin(\pi k/2)}{\pi k} \qquad k \neq 0}$$

$a_0 = \frac{1}{2}$

From above relation

$a_1 = a_{-1} = \frac{1}{\pi}$

$a_2 = a_{-2} = 0$

$a_3 = a_{-3} = -\frac{1}{3\pi}$

$a_4 = a_{-4} = 0$

$a_5 = a_{-5} = \frac{1}{5\pi}$

Now we can choose frequency upto 90% of energy is stored

$$\frac{1}{T}\int_{-T/4}^{T/4} 1^2\,dt = \frac{1}{2}$$

Using parsival Relation

$$\frac{1}{2} \times .9 < \left(\frac{1}{2}\right)^2 + \frac{2}{\pi^2}\left[\frac{1}{1} + \frac{1}{9} + \frac{1}{25} + \cdots\right]$$

$$.45 < .25 + \frac{2}{\pi^2}\left[1 + \frac{1}{9} + \frac{1}{25} + \cdots + \frac{1}{(2k+1)^2}\right]$$

$$.20 < \frac{2}{\pi^2}\left[1 + \frac{1}{9} + \cdots + \frac{1}{(2k+1)^2}\right]$$

$$.1 \times \pi^2 < \left[1 + \frac{1}{9} + \cdots + \frac{1}{(2k+1)^2}\right]$$

$$.986 < \left[1 + \frac{1}{9} + \cdots + \frac{1}{(2k+1)^2}\right]$$

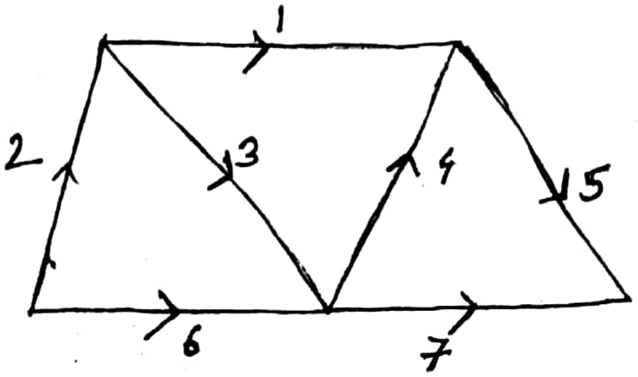
So ~~for k = 0,~~ energy greater than 90% is concentrated in ~~~~, f component

so we take Frequency to be f.

Now we know frequency, so can proceed as in part a.

Assignment-1

(2)



Where, i_k, v_k are the branch currents & branch voltages. where $k = 1, 2, \dots 7$

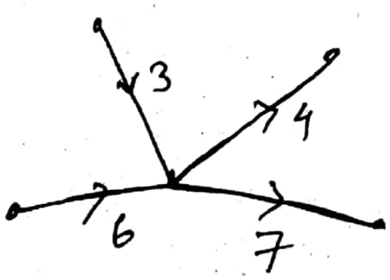
Since, there are 5 nodes, so we required a minimum no. of $(5-1) = 4$ branch voltage [redacted] in order to find the rest using KVL.

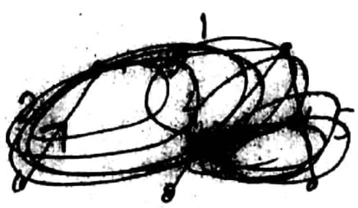
The minimum no. of branch voltage polarities required is 4

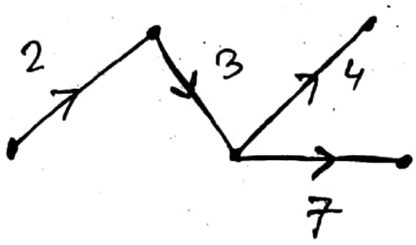
For finding the set of branch voltages we have to consider a tree for the given graph.

Hence, the branch voltages are not unique but it depends on the TREE which we have considered.

For, example.







Q. 3 The Meshes are :
branches consisting of :

2 - 3 - 6, 3 - 4 - 1, 4 - 5 - 7.

and the loops are :-

2 - 1 - 4 - 6, 1 - 5 - 7 - 3, 1 - 5 - 7 - 6 - 2

Q4) ⇒ a) →

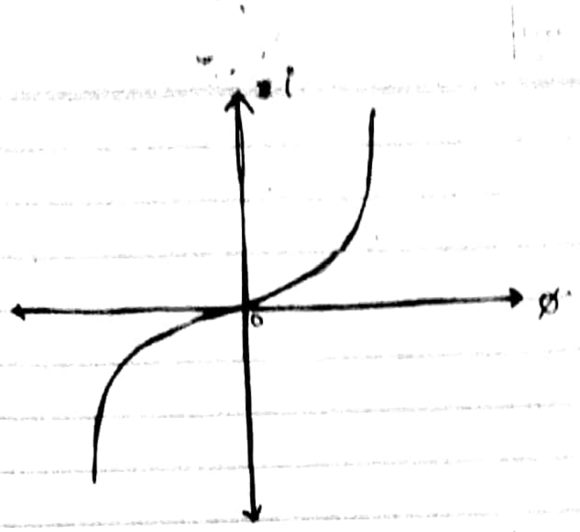


fig :- i-ø characteristics of inductor.

b) → The inductor is a passive as the graph of characteristics lies in ~~1st~~ first and third quadrant. The characteristic does not follow ~~the~~ homogeneous theorem. Hence it is non-linear.

c) → $$L_{ac} = \frac{d\phi}{di}\Big|_{\phi=0} = \cancel{\frac{1}{d\phi/i}} = \frac{1}{2\phi}$$

$$L_{dc} = \lim_{\phi_0 \to 0} \frac{\phi_0}{i_0} = \cancel{\lim_{\phi_0 \to 0} \frac{1}{\phi_0}} = \frac{\phi_0}{\phi_0^2}\Big|_{\lim \phi_0 \to 0} = \lim_{\phi_0 \to 0} \frac{1}{\phi_0}$$

d) $$L_{ac} = \frac{d\phi}{di}$$

$$\frac{1}{L_{ac}} = \frac{di}{d\phi} = \frac{d}{d\phi}(\phi^2) = 2\phi$$

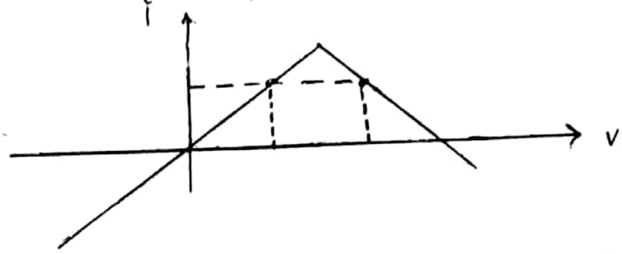
$$\therefore \frac{1}{L_{ac}} \propto \phi$$

inverse of Lac increases linearly as ø.

Q5

(a) The resistor is voltage controlled

Since for each voltage value in the x-axis there is only one corresponding current value in the y-axis.

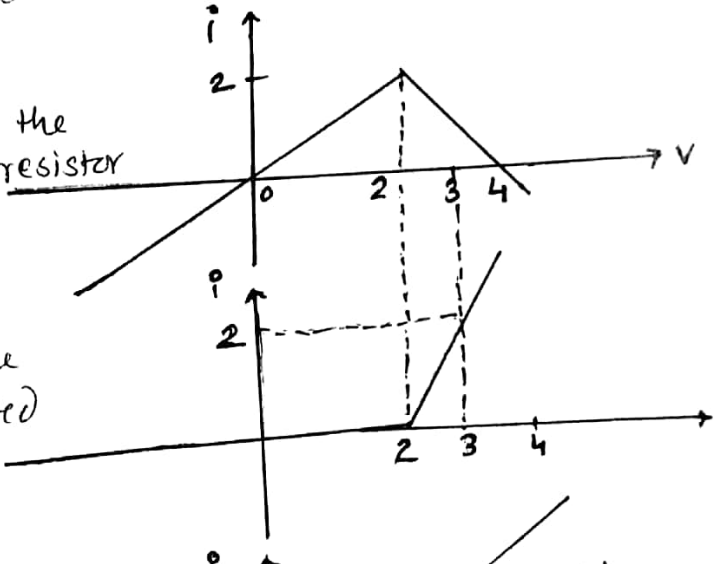


But for each current value in y-axis there are multiple voltage values in x-axis. (two)

(b)

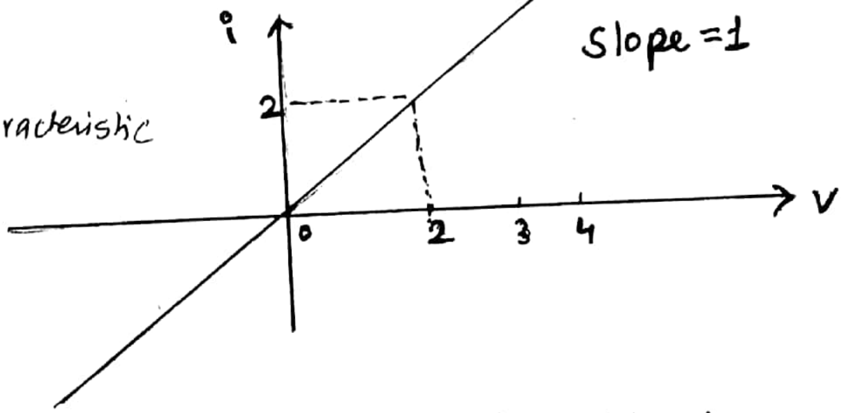
char. of the given resistor

char. of the resistor connected in parallel



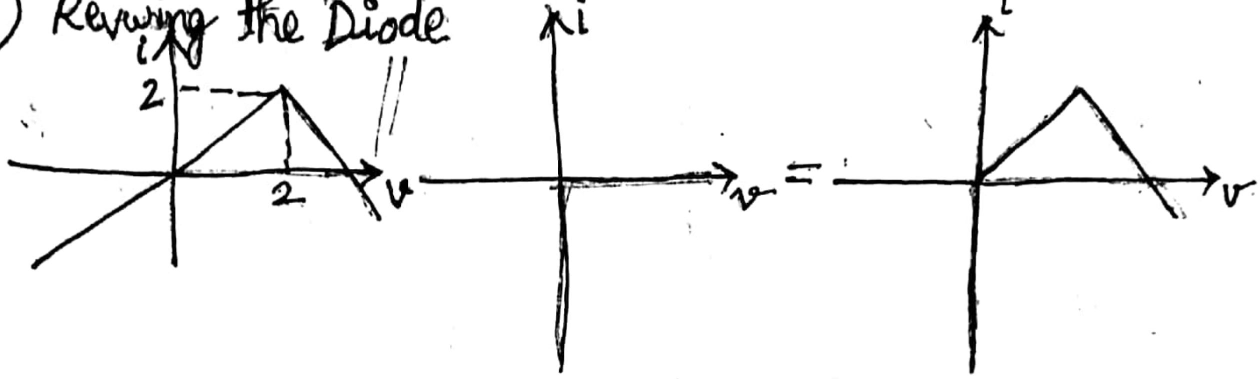
In Parallel

Linear characteristic

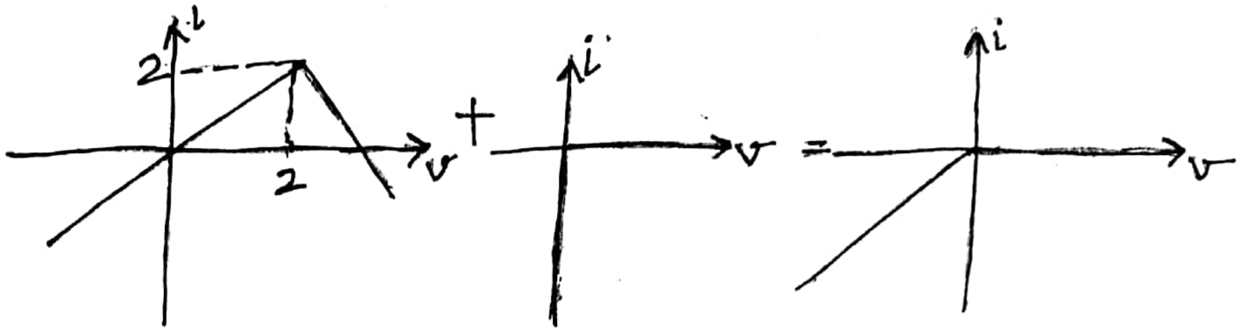


In parallel connection, Currents add up.

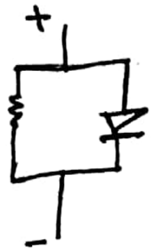
c) Reversing the Diode

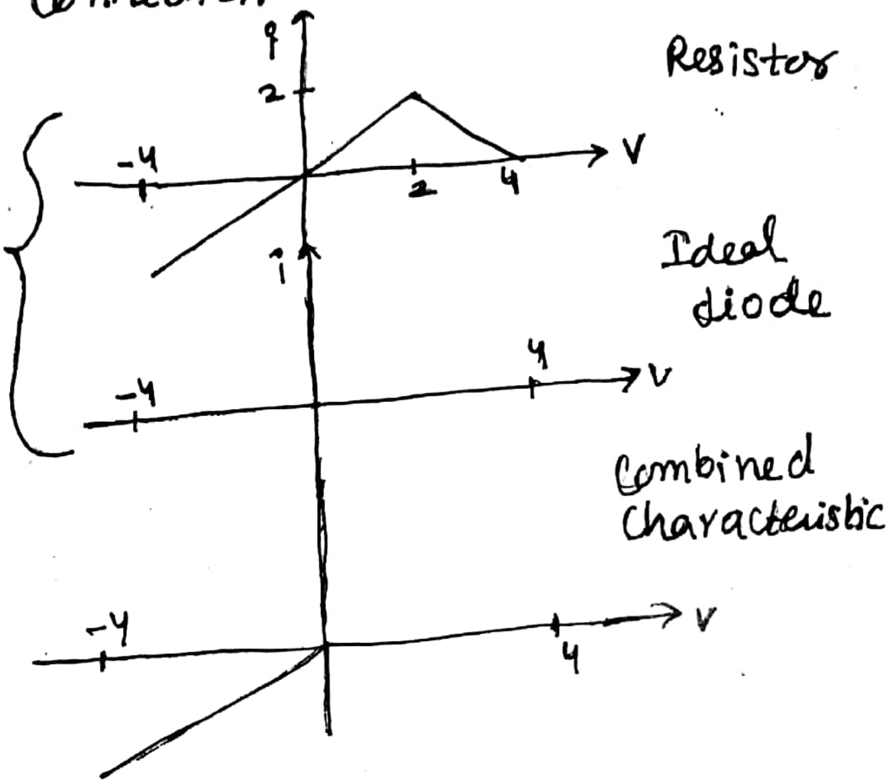


For Series connection



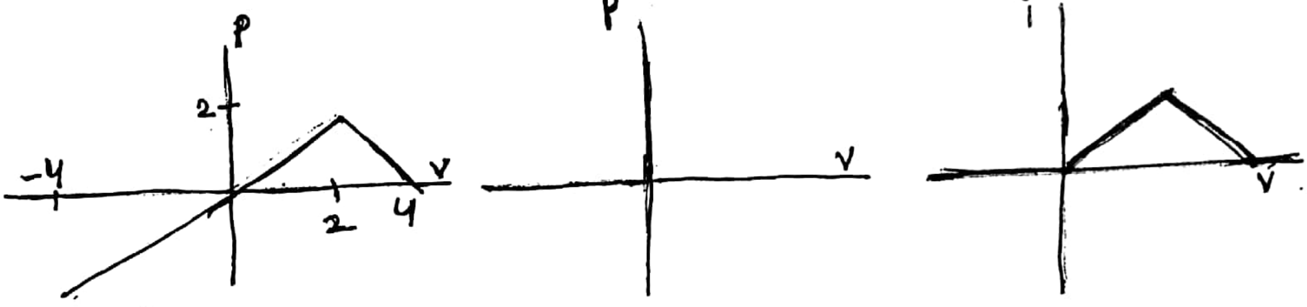
(c) (i) Parallel Connection





In parallel, currents add-up

(ii) Series Connection



Resistor Char.

Ideal Diode Char.

Combined Char.

In Series, voltages addup.

$$= 0.75 \times 100\pi \times \cos(100\pi t)$$

$$V = 75\pi \cos 100\pi t \quad V$$

(b) when $i = 5 \cos 240\pi t$

$$\phi(t) = L(t)\, i(t)$$

$$V = L(t) \frac{di(t)}{dt} + i(t) \frac{dL(t)}{dt}$$

$$= \left[1.25 + 0.75 \sin(100\pi t)\right] - 5 \times 240\pi \sin 240\pi t$$

$$+ 5 \cos 240\pi t \times 0.75 \cos 100\pi t \times 100\pi$$

$$= \Big[-1500\pi \sin 240\pi t - 900 \sin 100\pi t \sin 240\pi t$$

$$+ 375\pi \cos 240\pi t \cos 100\pi t\Big] V$$

(c)

$$i(t) = -L(t)$$

$$V = L(t) \frac{di(t)}{dt} + i(t) \frac{dL(t)}{dt}$$

$$= L(t) - \frac{dL(t)}{dt} + \frac{dL(t)}{dt} (-L(t))$$

$$= -2 L(t) \frac{dL(t)}{dt}$$

$$= -2 (1.25 + 0.75 \sin 100\pi t)\, 75\pi \cos 100\pi t$$

$$V = \Big[-187.5\pi \cos 100\pi t - 112.5\pi \sin 100\pi t \times \cos 100\pi t\Big]$$

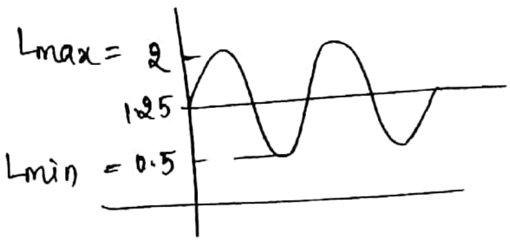
V

Assignment - 1

6). A cylindrical solenoid, has an iron rod core that executes SHM $x(t) = \sin(100\pi t)$ along the coil axis. when rod is fully out (when $100\pi t = 2\pi n + \frac{3\pi}{2}$), the coil inductance: $L_{min} = 0.5H$ and when rod is fully in (when $100\pi t = 2\pi n + \frac{\pi}{2}$), the coil inductance: $L_{max} = 2H$.

ⓐ Find the coil terminal voltage when $i = 1A$ dc
ⓑ Find the coil terminal voltage when $i = 5\cos 240\pi t$
ⓒ Find the coil terminal voltage if we set $i(t) = -L(t)$.

Sol: Finding $L(t)$: By satisfying ~~the~~ given two conditions,



$$L(t) = 1.25 + 0.75\sin(100\pi t)$$

when $100\pi t = 2\pi n + \frac{3\pi}{2}$, $L(t) = L_{min} = 1.25 - 0.75$
$= 0.5$

when $100\pi t = 2\pi n + \frac{\pi}{2}$, $L(t) = L_{max} = 1.25 + 0.75$
$= 2$.

ⓐ For coil terminal voltage,

$$\phi(t) = L(t)\, i(t)$$
$$= 1.25 + 0.75\sin(100\pi t)$$

$$v(t) = \frac{d\phi(t)}{dt}$$

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$= 0.75 \times 100\pi \times \cos(100\pi t)$

$V = 75\pi \cos 100\pi t$ V

(b) when $i = 5\cos 240\pi t$

$\therefore \phi(t) = L(t)\, i(t)$

$$V = L(t)\frac{di(t)}{dt} + i(t)\frac{dL(t)}{dt}$$

$$= [1.25 + 0.75\sin(100\pi t)] - 5 \times 240\pi \sin 240\pi t$$

$$+ 5\cos 240\pi t \times 0.75 \cos 100\pi t \times 100\pi$$

$$= [-1500\pi \sin 240\pi t - 900 \sin 100\pi t \sin 240\pi t$$

$$+ 375\pi \cos 240\pi t \cos 100\pi t]\ \text{V}$$

(c)

$i(t) = -L(t)$

$$V = L(t)\frac{di(t)}{dt} + i(t)\frac{dL(t)}{dt}$$

$$= L(t) - \frac{dL(t)}{dt} + \frac{dL(t)}{dt}(-L(t))$$

$$= -2L(t)\frac{dL(t)}{dt}$$

$$= -2(1.25 + 0.75\sin 100\pi t)\, 75\pi \cos 100\pi t$$

$$V = [-187.5\pi \cos 100\pi t - 112.5\pi \sin 100\pi t \times \cos 100\pi t]\ \text{V}$$