EE210: Microelectronics-I

Lecture-18 : BJT Amplifier-part-7
Frequency Response

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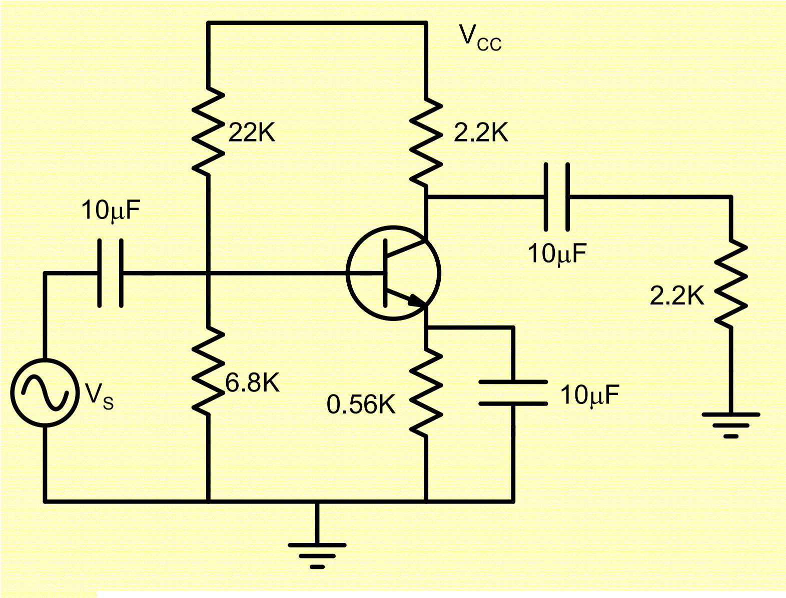
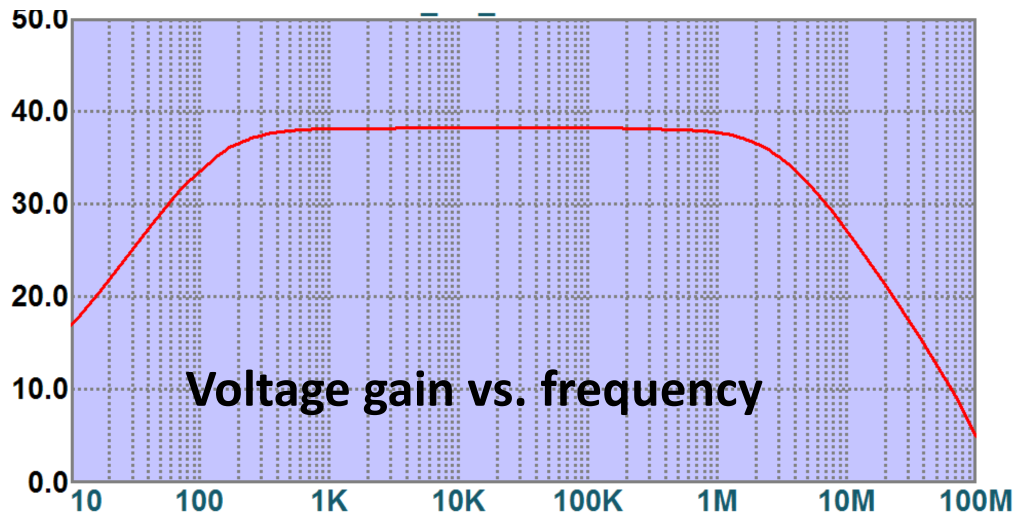
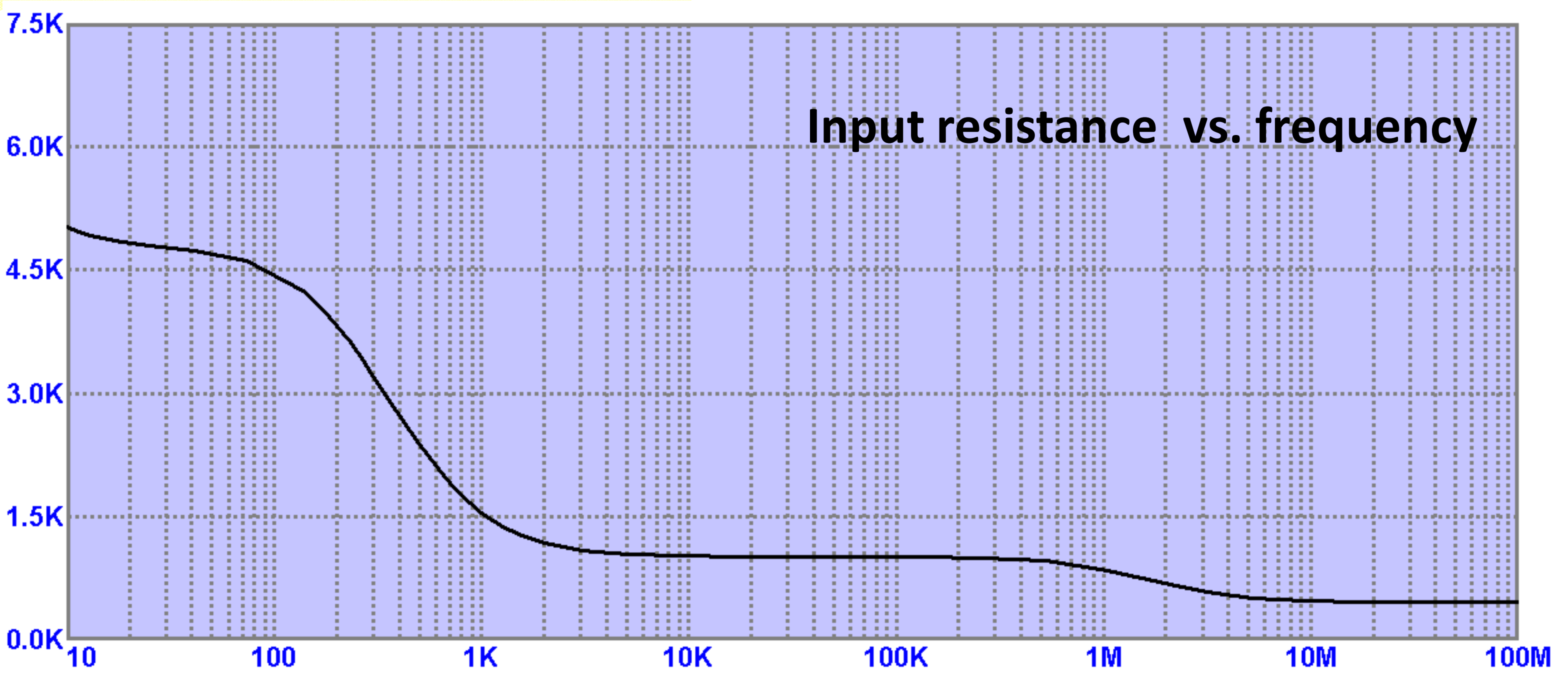
Characteristics of Amplifiers are frequency dependent

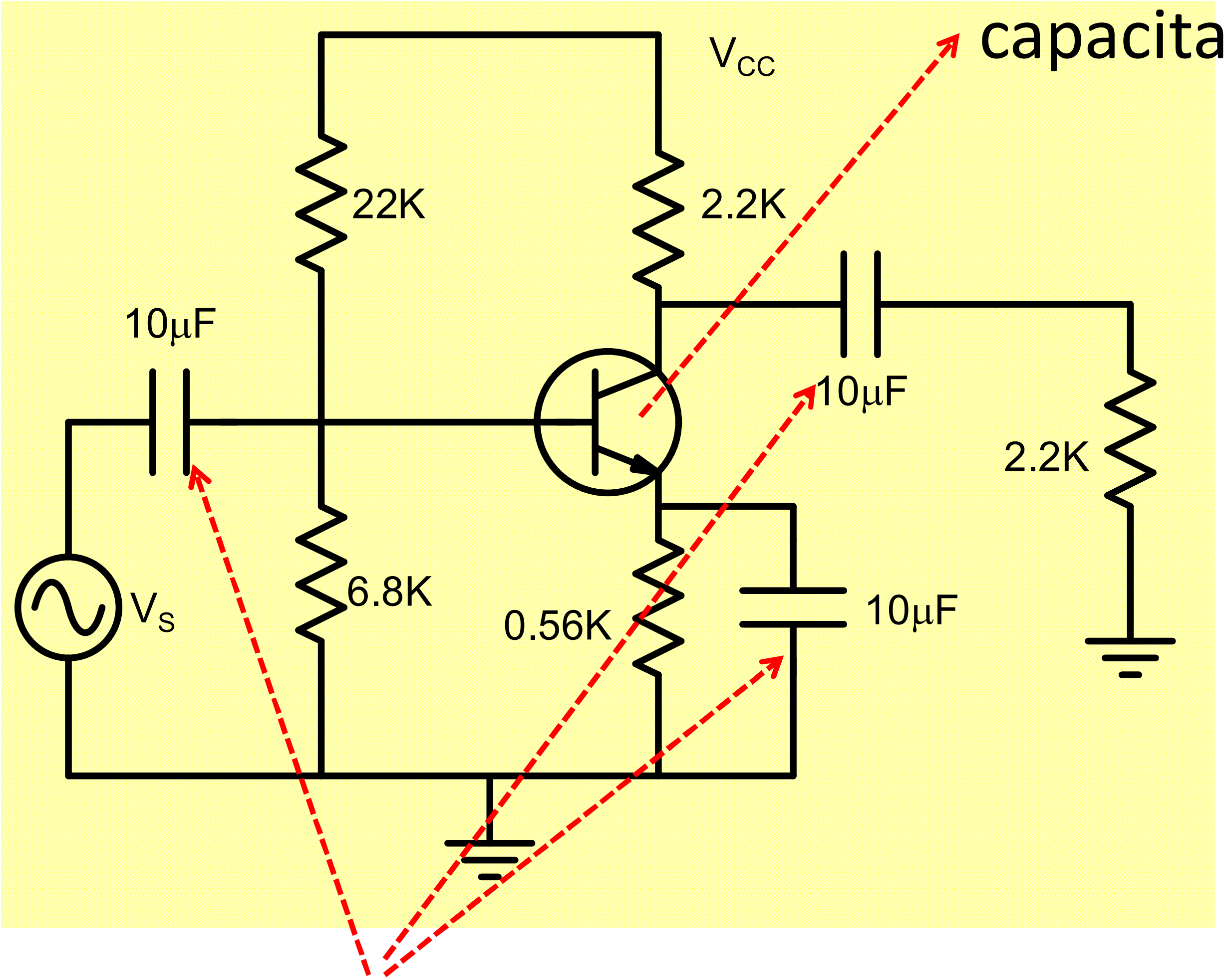






Frequency dependent behavior occurs because of resistances and capacitances

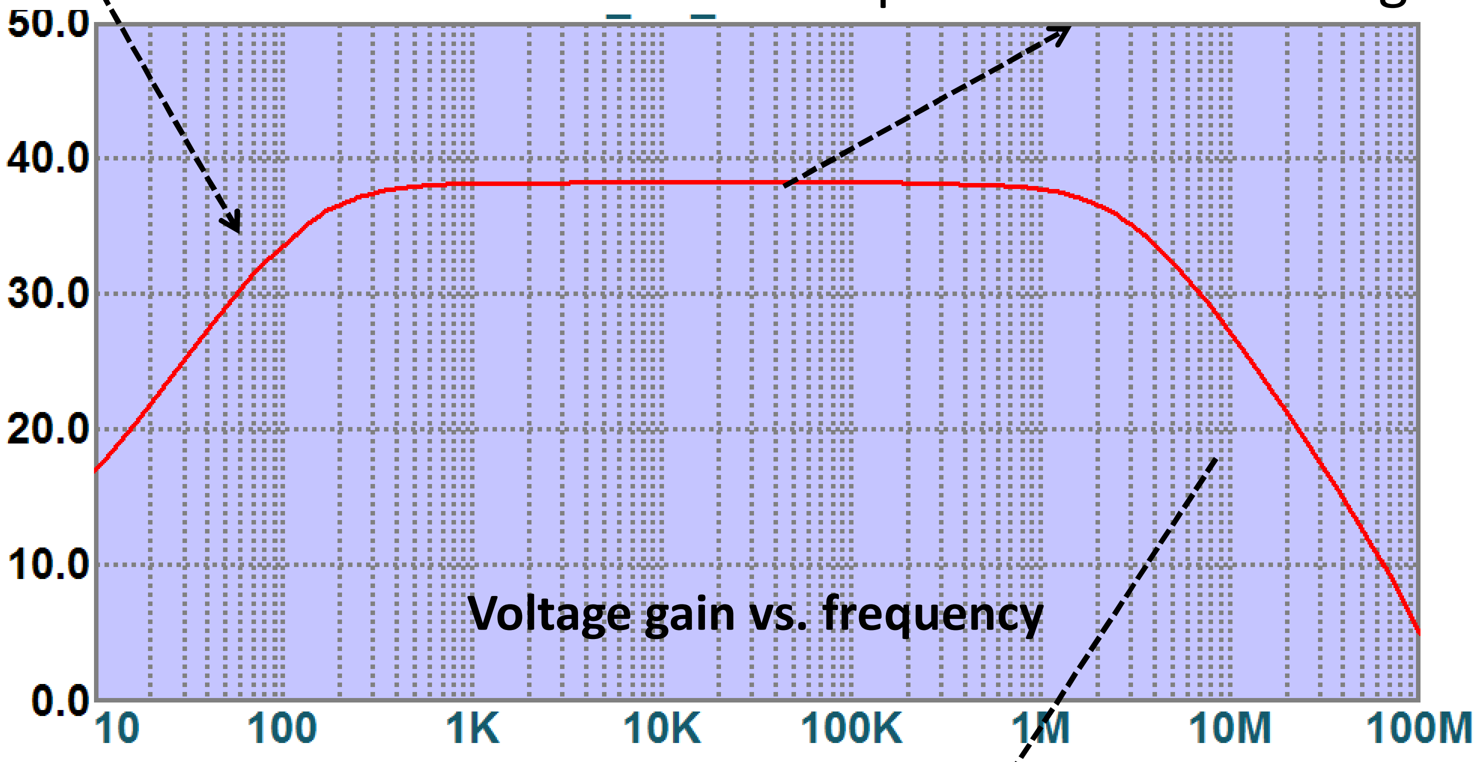
Internal transistor capacitances



External coupling and bypass capacitors

Low frequency behavior is caused by external capacitances

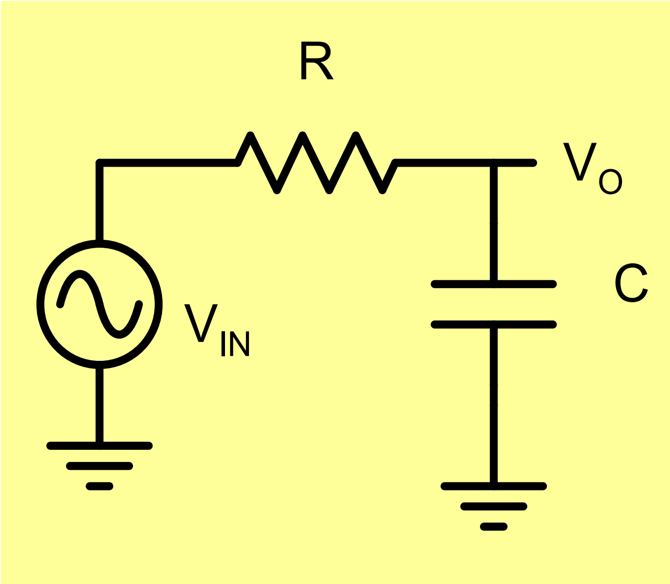
Mid-frequency region: all capacitances can be ignored



High frequency behavior is caused by internal transistor capacitances

Background : Analysis of RC Circuits

Simple Low pass RC filter circuit



Time domain

Frequency domain

Frequency domain:

$$H(s) = \frac{v_O(s)}{v_{in}(s)} = \frac{1/sC}{R + 1/sC} = \frac{1}{sCR + 1}$$

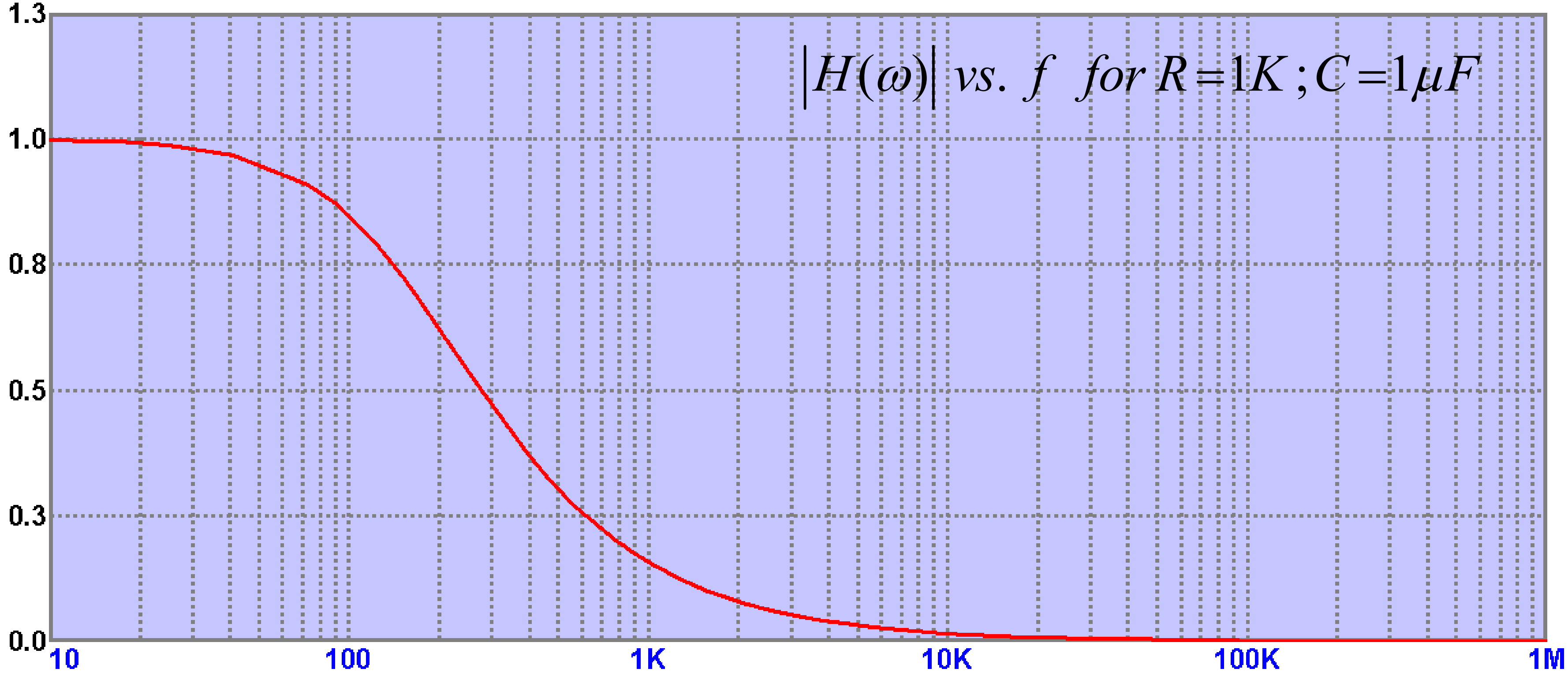
Substitute s = jω

$$H(\omega) = \frac{1}{1 + j\omega RC}$$

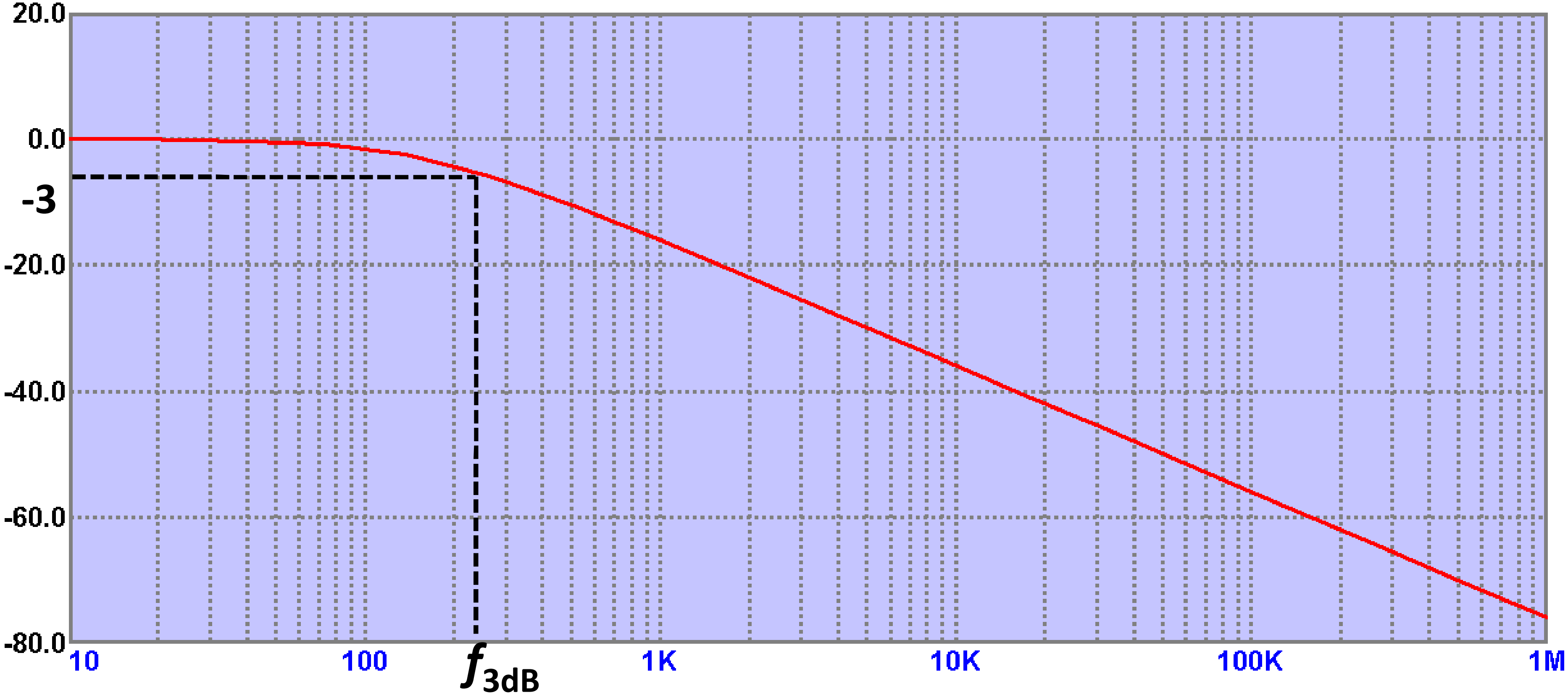
$$H(\omega) = \frac{1}{1 + j\omega RC}$$

$$|H(\omega)| = \frac{1}{\sqrt{1+\omega^2 R^2 C^2}}$$

$$Phase(H) = -\tan^{-1}(\omega RC)$$



Magnitude plot is often plotted in dB vs. frequency



$$20Log_{10}(|H(\omega)|) = -10Log_{10}(1+\omega^2R^2C^2)$$

3dB frequency: $\omega_{3dB}RC = 1 \Rightarrow |H| = -3dB$

$$f_{3dB} = \frac{1}{2\pi RC}$$

3dB frequency gives complete information about the behavior of RC circuit

$$H(\omega) = \frac{1}{1 + j\omega RC}$$

$$f_{3dB} = \frac{1}{2\pi RC}$$

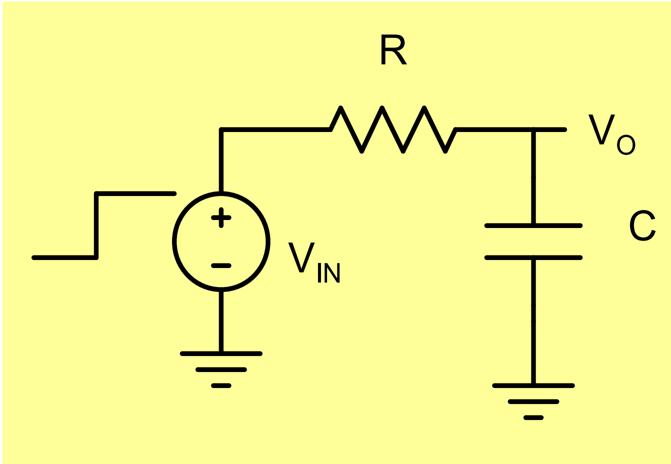
$$H(\omega) = \frac{1}{1 + j(\frac{f}{f_{3dB}})}$$

$$\left|H(\omega)\right| = \frac{1}{\sqrt{1 + (\frac{f}{f_{3dB}})^2}}$$

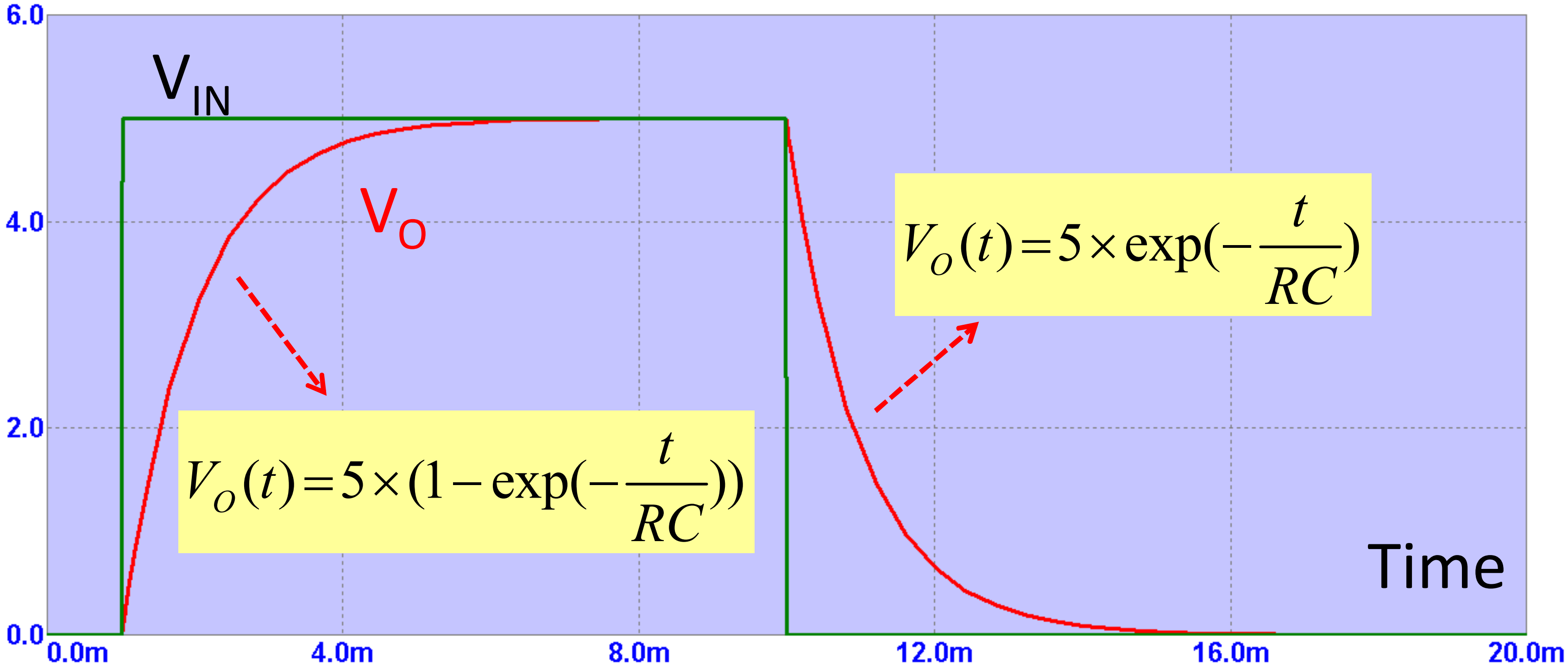
$$Phase(H) = -\tan^{-1}(\frac{f}{f_{3dB}})$$

Time Domain (or Transient) Analysis

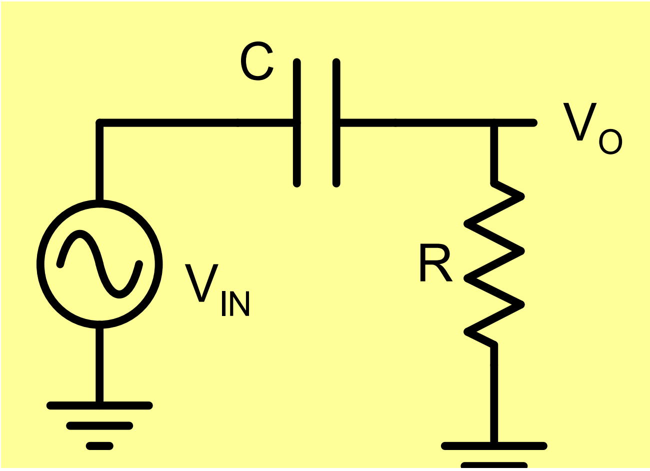
Step Response



$$\tau_R = \tau_F = 2.2RC = \frac{0.35}{f_{3dB}}$$



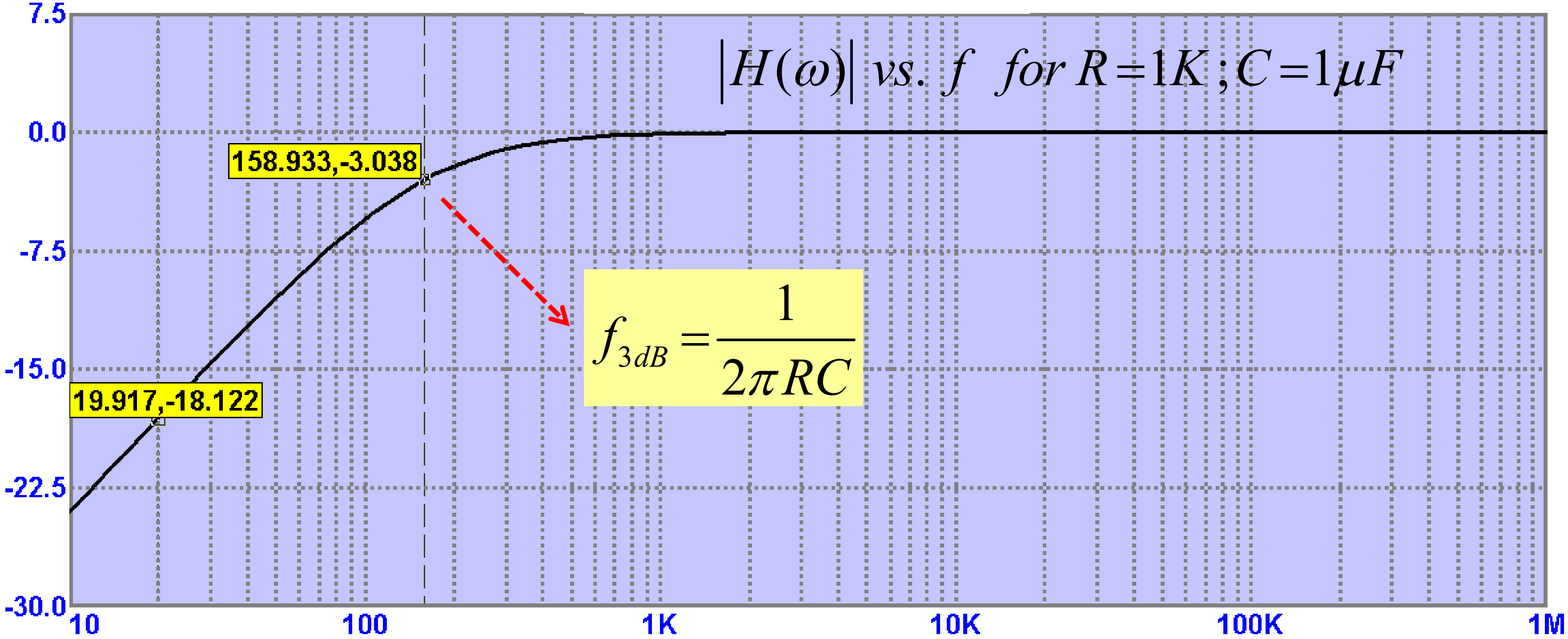
Simple High pass RC filter circuit



$$H(s)=\frac{v_O(s)}{v_{in}(s)}=\frac{R}{R+1/sC}=\frac{sCR}{sCR+1}$$

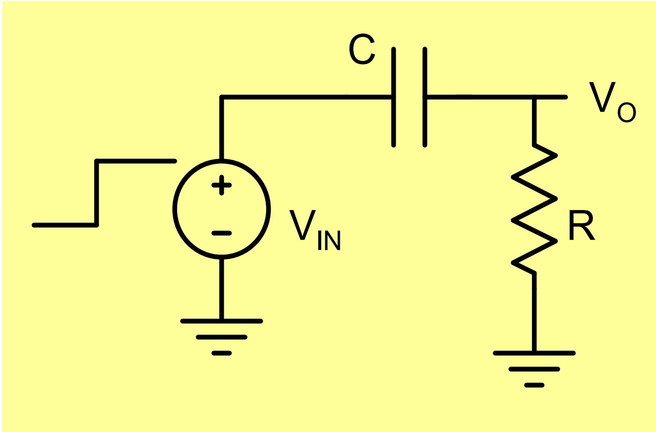
$$H(\omega)=\frac{j\omega RC}{1+j\omega RC}$$

$$|H(\omega)|=\frac{\omega RC}{\sqrt{1+\omega^2R^2C^2}}$$

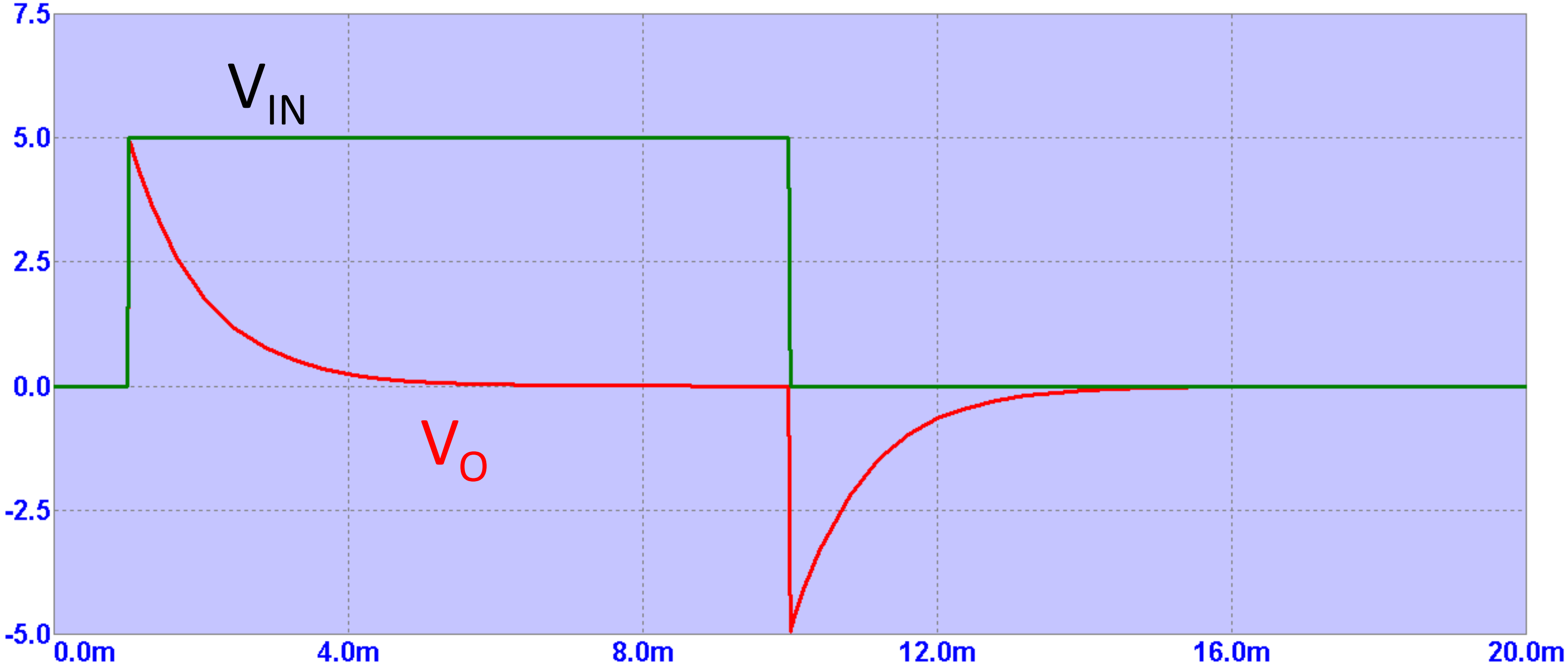


Simple High pass RC filter circuit: Transient Response

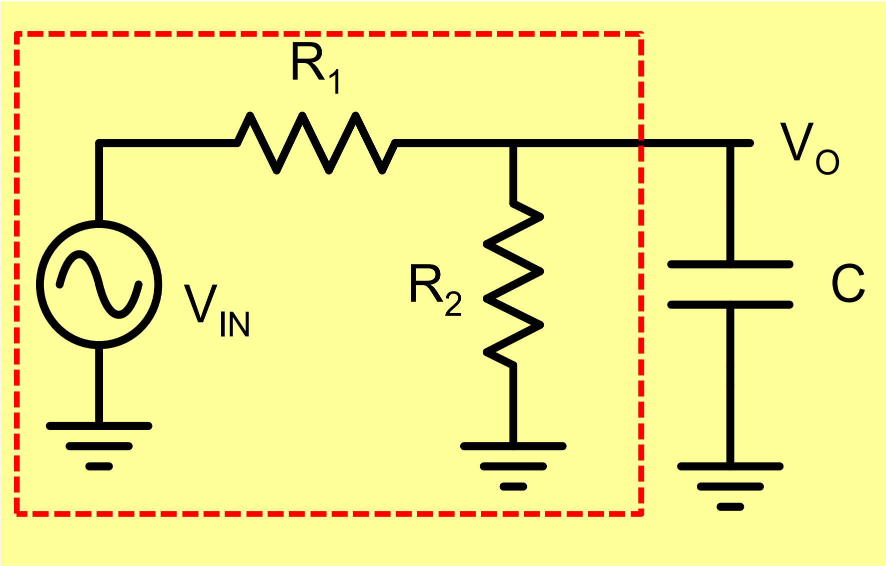
Step Response



$$\tau_R = \tau_F = 2.2RC = \frac{0.35}{f_{3dB}}$$

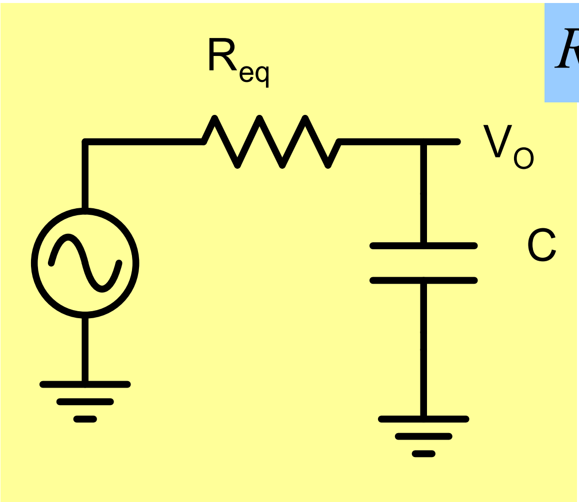


Circuits with one capacitor but many Resistors



$$f_{3dB} = ?$$

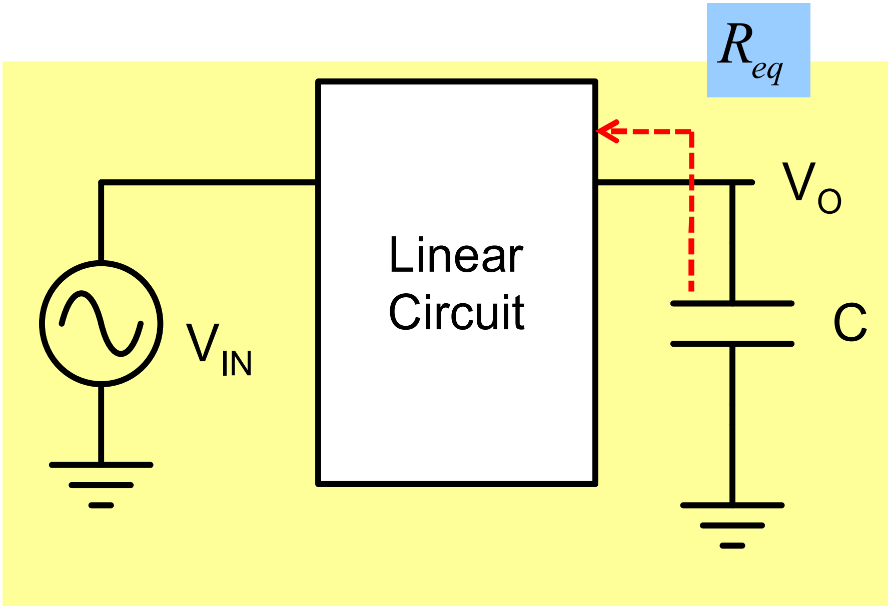
Apply Thevenin's theorem:



$$R_{eq} = R_1 \| R_2$$

$$f_{3dB} = \frac{1}{2\pi R_{eq} C}$$

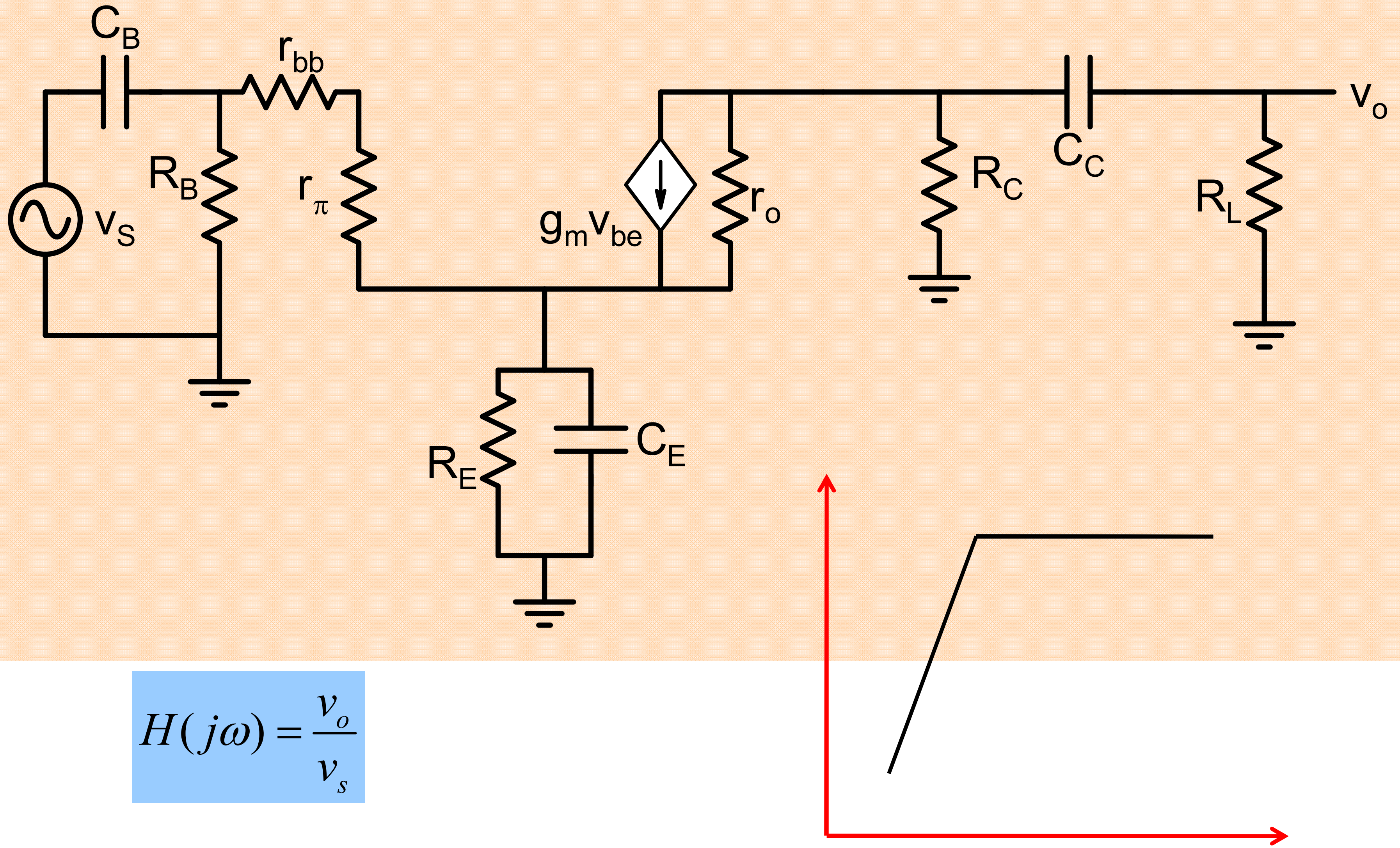
Generalization



$$f_{3dB} = \frac{1}{2\pi R_{eq} C}$$

How do we know if the capacitor causes high or low pass behavior?

Lower Cutoff Frequency



$$H(j\omega) = \frac{v_o}{v_s}$$

Transfer Function

(- Hgm_q1 Cc CB RB RC RL) s^2

(- Hgm_q1 Cc CE CB RB RC RE RL) s^3

(+ Gbe_q1 RE + Gbe_q1 RB + Hgm_q1 RE +1)

(+ Gbe_q1 CB RE RS + Gbe_q1 CB RB RS + Gbe_q1 CB RB RE +
Hgm_q1 CB RE RS + Hgm_q1 CB RB RE + CB RS + CB RB + Gbe_q1 CE RB RE +
CE RE + Gbe_q1 Cc RE RL + Gbe_q1 Cc RC RE + Gbe_q1 Cc RB RL +
Gbe_q1 Cc RB RC + Hgm_q1 Cc RE RL + Hgm_q1 Cc RC RE + Cc RL + Cc RC) s

(+ Gbe_q1 CE CB RB RE RS + CE CB RE RS + CE CB RB RE +
Gbe_q1 Cc CB RE RL RS + Gbe_q1 Cc CB RC RE RS +
Gbe_q1 Cc CB RB RL RS + Gbe_q1 Cc CB RB RE RL +
Gbe_q1 Cc CB RB RC RS + Gbe_q1 Cc CB RB RC RE +
Hgm_q1 Cc CB RE RL RS + Hgm_q1 Cc CB RC RE RS +
Hgm_q1 Cc CB RB RE RL + Hgm_q1 Cc CB RB RC RE +
Cc CB RL RS + Cc CB RC RS + Cc CB RB RL + Cc CB RB RC +
Gbe_q1 Cc CE RB RE RL + Gbe_q1 Cc CE RB RC RE + Cc CE RE RL + Cc CE RC RE) s^2

(+ Gbe_q1 Cc CE CB RB RE RL RS + Gbe_q1 Cc CE CB RB RC RE RS +
Cc CE CB RE RL RS + Cc CE CB RC RE RS + Cc CE CB RB RE RL + Cc CE CB RB RC RE) s^3

Simplified Transfer Function

R_B, R_S neglected

(- Hgm_q1 Cc CB RC RL) s^2

(- Hgm_q1 Cc CE CB RC RE RL) s^3

(+ Gbe_q1)

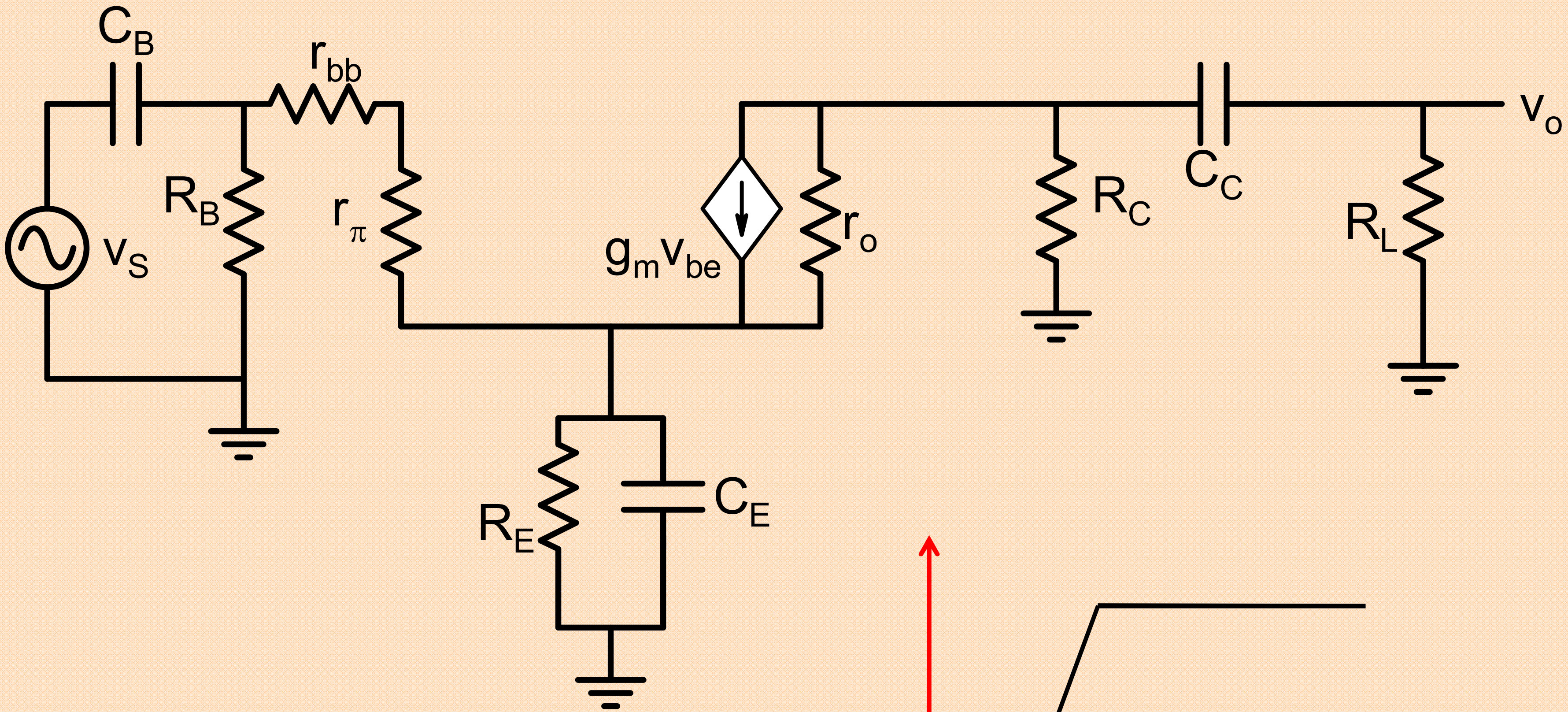
(+ Gbe_q1 CB RE + Hgm_q1 CB RE + CB + Gbe_q1 CE RE +
Gbe_q1 Cc RL + Gbe_q1 Cc RC) s

(+ CE CB RE + Gbe_q1 Cc CB RE RL + Gbe_q1 Cc CB RC RE +
Hgm_q1 Cc CB RE RL + Hgm_q1 Cc CB RC RE + Cc CB RL +
Cc CB RC + Gbe_q1 Cc CE RE RL + Gbe_q1 Cc CE RC RE) s^2

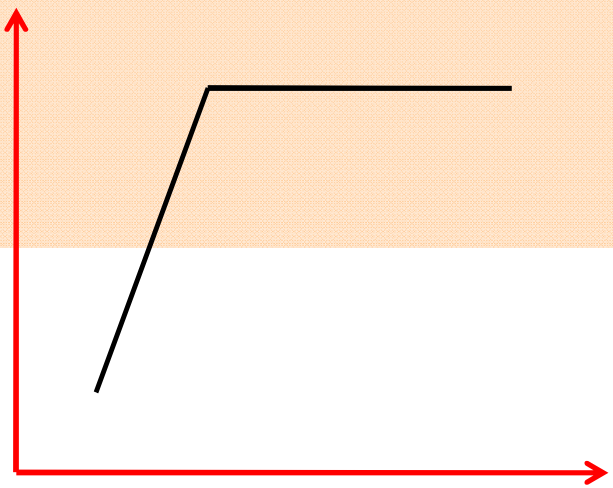
(+ Cc CE CB RE RL + Cc CE CB RC RE) s^3

Determine impact of each capacitor independently

Lower Cutoff Frequency



$$H(j\omega) = \frac{v_o}{v_s}$$



Simplified Procedure Short-circuit time constant approach

-We consider one capacitor (C_j) at a time assuming that it plays a dominant role and short-circuit the remaining.

-We then determine time constant $R_{eqj}C_j$ and 3dB frequency f_j due to this capacitor.

-Like this we determine for each of the capacitors.

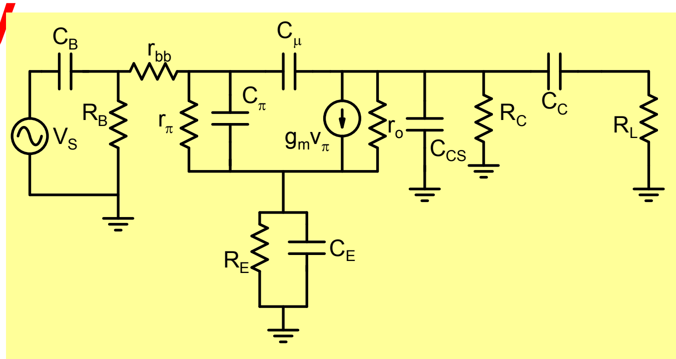
-If one of the 3dB frequencies is larger by a factor of 4 or so compared to others, we take this frequency as the 3dB frequency of the circuit

-otherwise

$$f_L \cong \frac{1}{2\pi}\sum \frac{1}{R_{eqj}C_j} = \sum f_j$$

$$f_L \leq \sqrt{\sum f_{pi}^2 - 2\sum f_{zi}^2}$$

Determination of upper cutoff frequency



First classify the capacitors into 2 classes

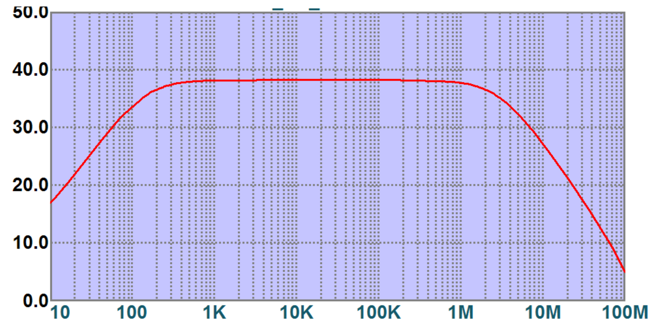
$(C_B\ ,\ C_C,\ C_E,\ C_\pi\ ,\ C_\mu,\ C_{CS}\ )$

HP

LP

$(C_B\ ,\ C_C,\ C_E)$

$(C_\pi\ ,\ C_\mu,\ C_{CS}\ )$



Short circuit these capacitors

We still have three capacitors while we know only how to estimate 3 dB frequency when only one capacitor is present. So we use a simplified method to obtain an approximate value.

Simplified Procedure Open-circuit time constant approach

-We consider one capacitor (C_j) at a time assuming that it plays a dominant role and open-circuit the remaining.

-We then determine time constant $R_{eqj}C_j$ and 3dB frequency f_j due to this capacitor.

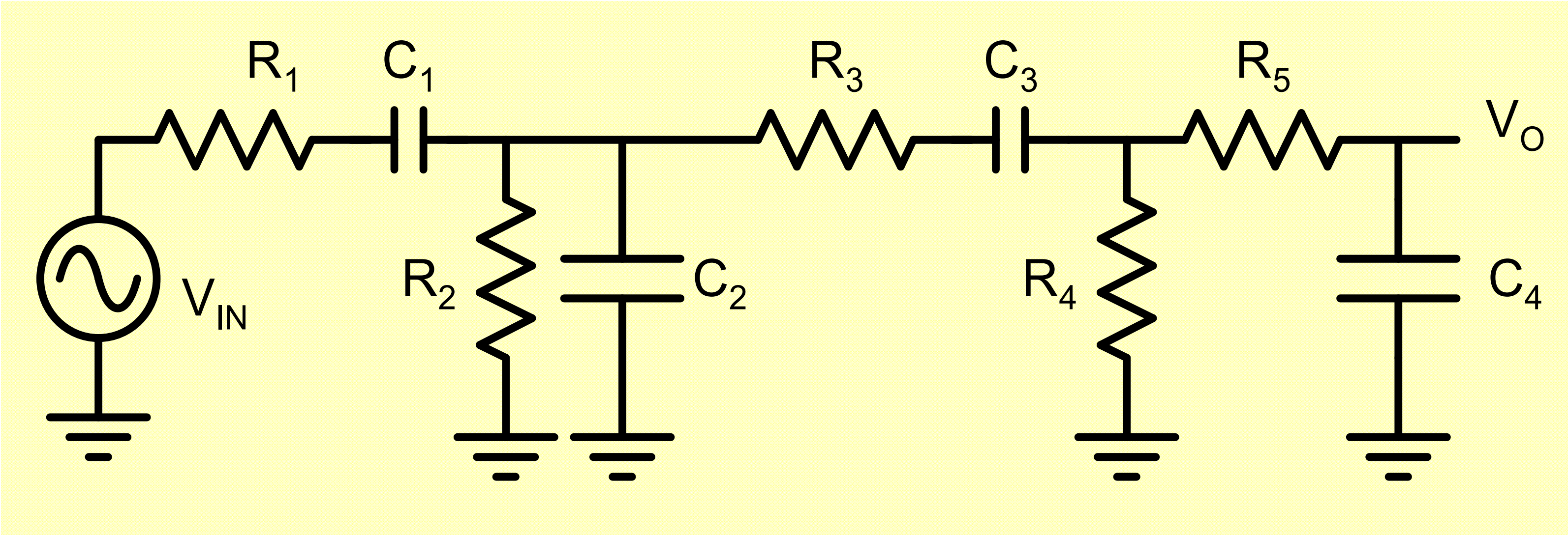
-Like this we determine for each of the capacitors.

-If one of the 3dB frequencies is **smaller** by a factor of 4 or so compared to others, we take this frequency as the 3dB frequency of the circuit

-otherwise

$$f_H \cong \frac{1}{2\pi \sum R_{eqj} C_j}$$

Example



(C_1 , C_2, C_3, C_4)

HP

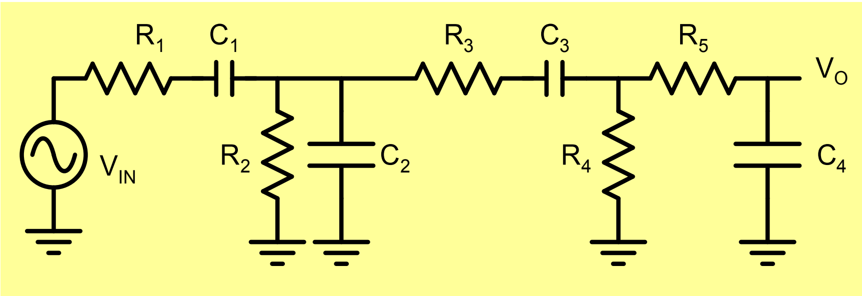
LP

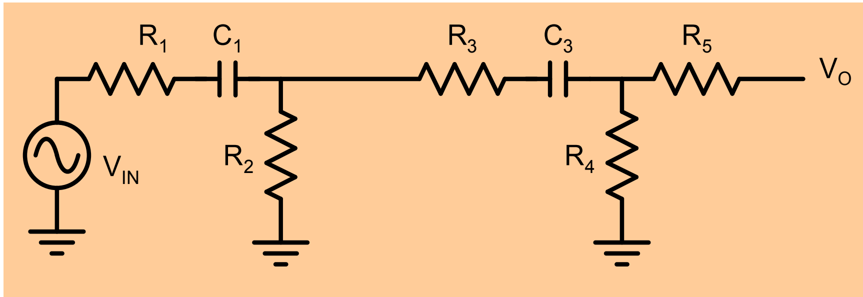
(C_1 , C_3)

(C_2 , C_4)

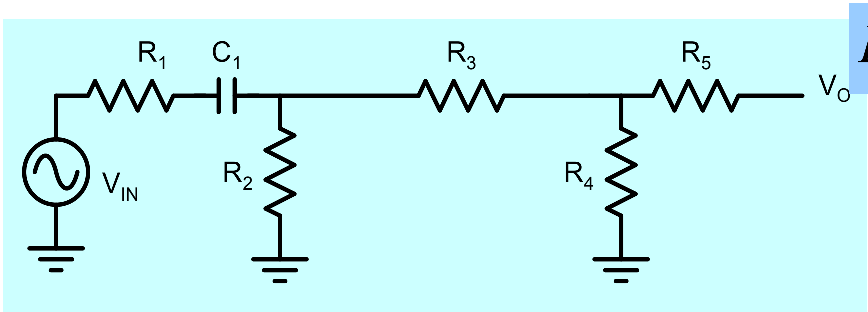
Determination of lower cutoff frequency

Open circuit capacitors C_2 and C_4





Consider C_1 first .

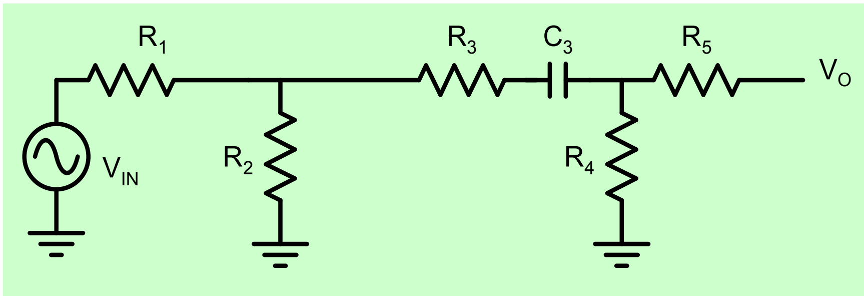


$$R_{eq1} = R_1 + R_2 \| (R_3 + R_4)$$

$$\tau_1 = C_1 R_{eq1}$$

$$f_1 = \frac{1}{2\pi C_1 R_{eq1}}$$

Consider C_3 next.



$$R_{eq3} = R_4 + (R_2 \| R_1) + R_3$$

$$\tau_3 = C_3 R_{eq3}$$

$$f_3 = \frac{1}{2\pi C_3 R_{eq3}}$$

Suppose all resistors are 1K and C_1 = 1uF , C_3 = 10uF C_2 = 1pF , C_4 = 0.5pF

$$R_{eq1} = R_1 + R_2 \| (R_3 + R_4) = 1.67K$$

$$\tau_1 = C_1 R_{eq1} = 1.67ms$$

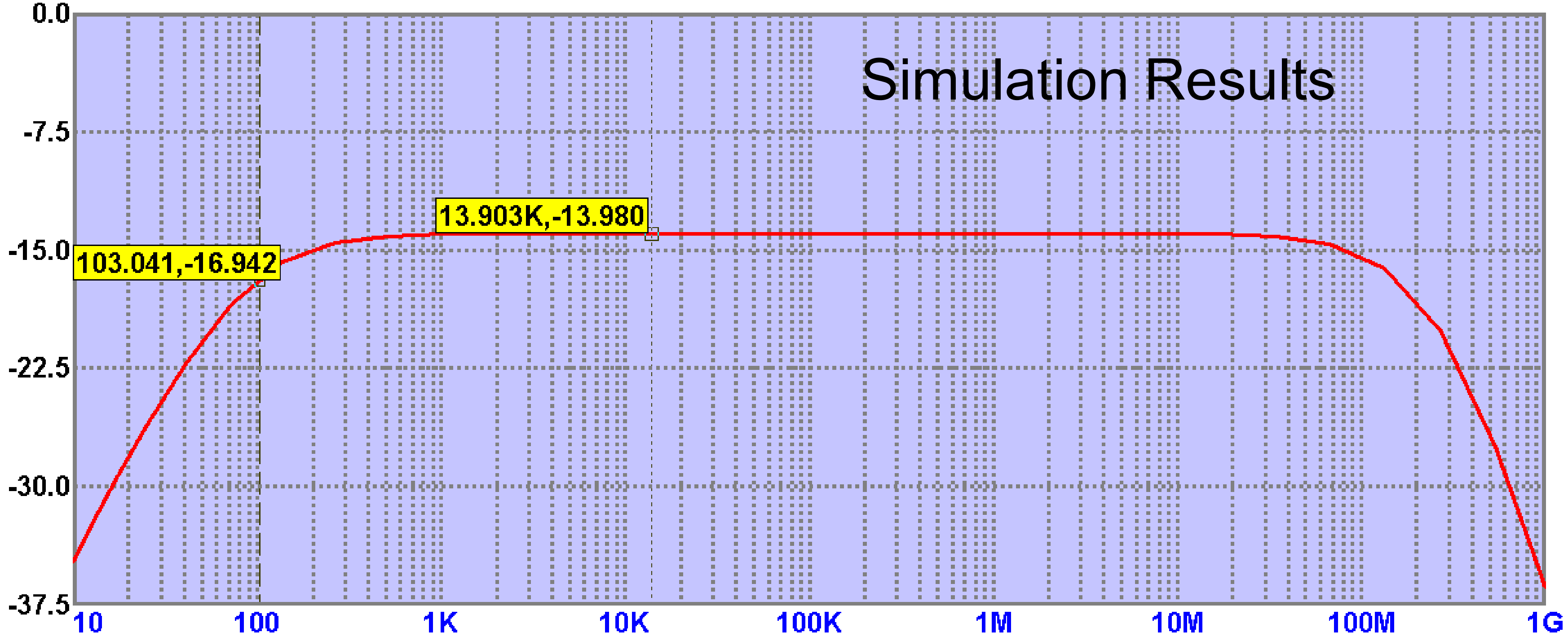
$$f_1 = \frac{1}{2\pi C_1 R_{eq1}} = 95.3Hz$$

$$R_{eq3} = R_4 + (R_2 \| R_1) + R_3 = 2.5K$$

$$\tau_3 = C_3 R_{eq3} = 25ms$$

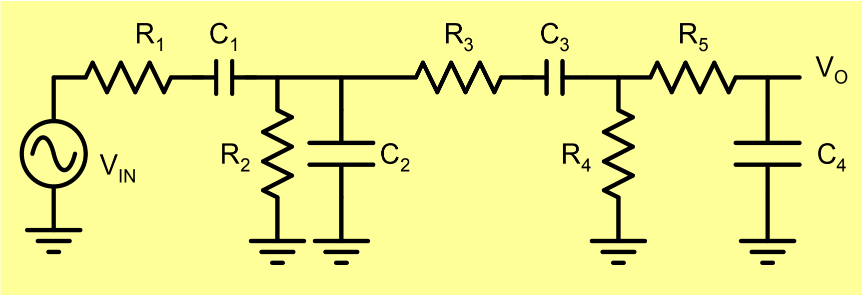
$$f_3 = \frac{1}{2\pi C_3 R_{eq3}} = 6.36Hz$$

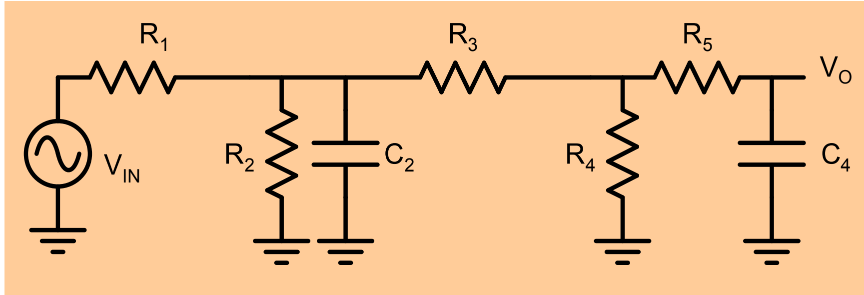
$$f_L \cong f_1 = 95.3Hz$$



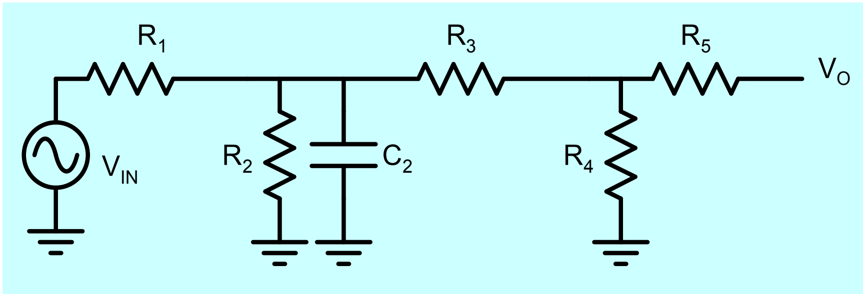
Determination of upper cutoff frequency

Short circuit capacitors C_1 and C_3





Consider C_2 first .

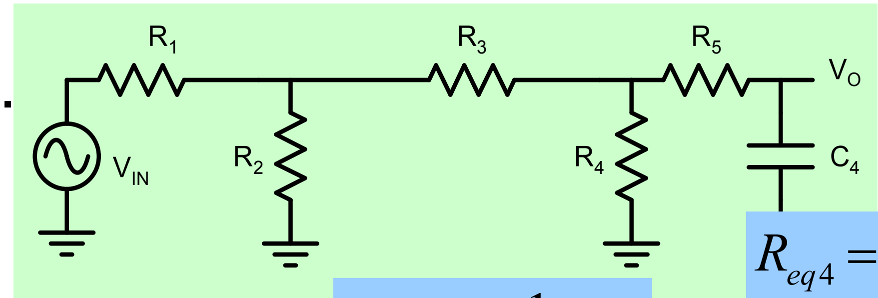


$$R_{eq2} = R_1 \| R_2 \| (R_3 + R_4)$$

$$\tau_2 = C_2 R_{eq2}$$

$$f_2 = \frac{1}{2\pi C_2 R_{eq2}}$$

Consider C_4 next .



$$R_{eq4} = [\{(R_1 \| R_2) + R_3\} \| R_4] + R_5$$

$$f_4 = \frac{1}{2\pi C_4 R_{eq4}}$$

$$\tau_4 = C_4 R_{eq4}$$

$$R_{eq2} = R_1 \| R_2 \| (R_3 + R_4) = 0.4K$$

$$\tau_2 = C_2 R_{eq2} = 0.4ns$$

$$f_2 = \frac{1}{2\pi C_2 R_{eq2}} = 0.39GHz$$

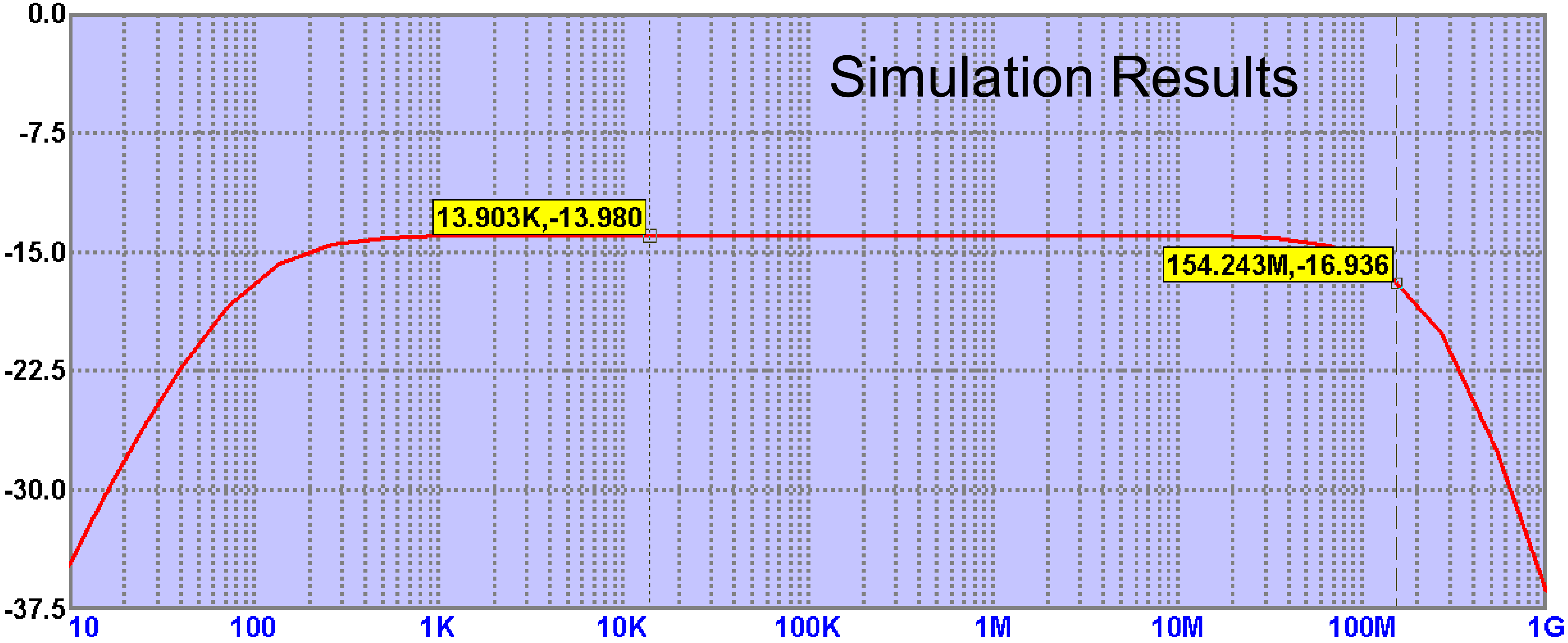
$$R_{eq4} = [\{(R_1 \| R_2) + R_3\} \| R_4] + R_5 = 1.6K$$

$$\tau_4 = C_4 R_{eq4} = 0.8ns$$

$$f_4 = \frac{1}{2\pi C_4 R_{eq4}} = 0.2GHz$$

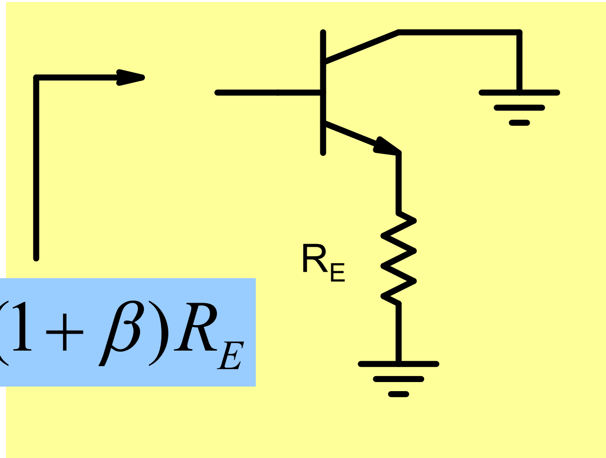
Since no frequency is dominant

$$f_H \cong \frac{1}{2\pi \sum R_{eqj} C_j} = 0.13GHz$$

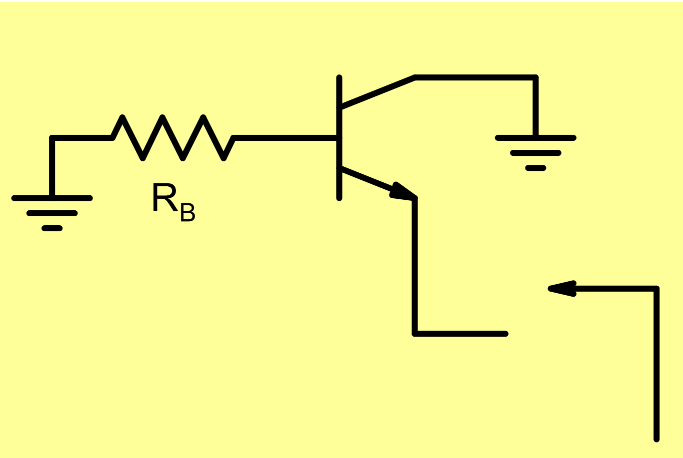


Lower Cutoff Frequency

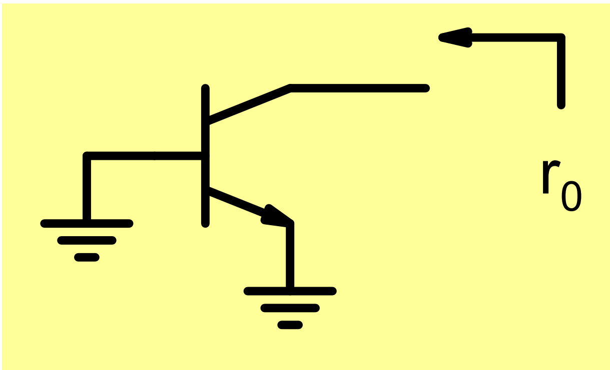
In the estimation of 3dB frequency, we have to determine equivalent resistance seen by a capacitance. The results given below can be very useful for determination of these equivalent resistances



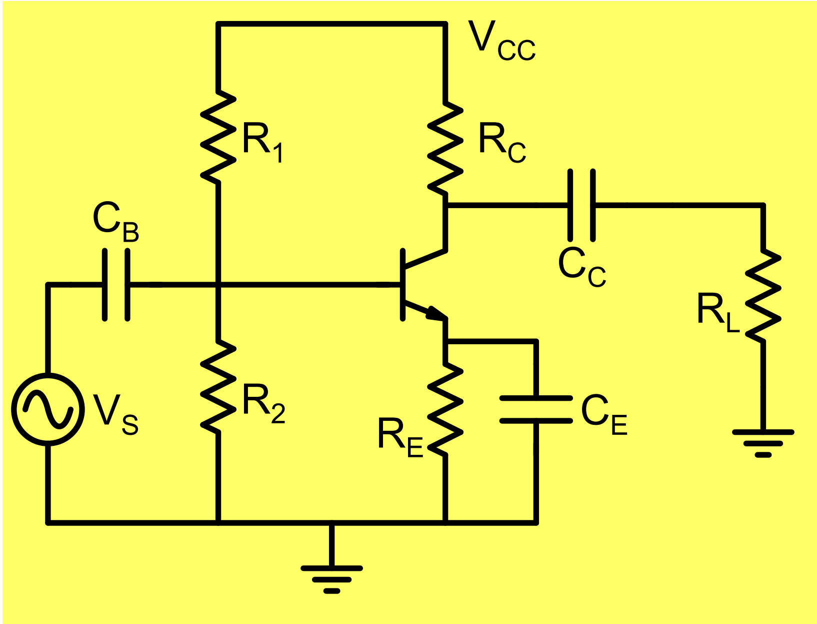
$$r_\pi + (1+\beta)R_E$$

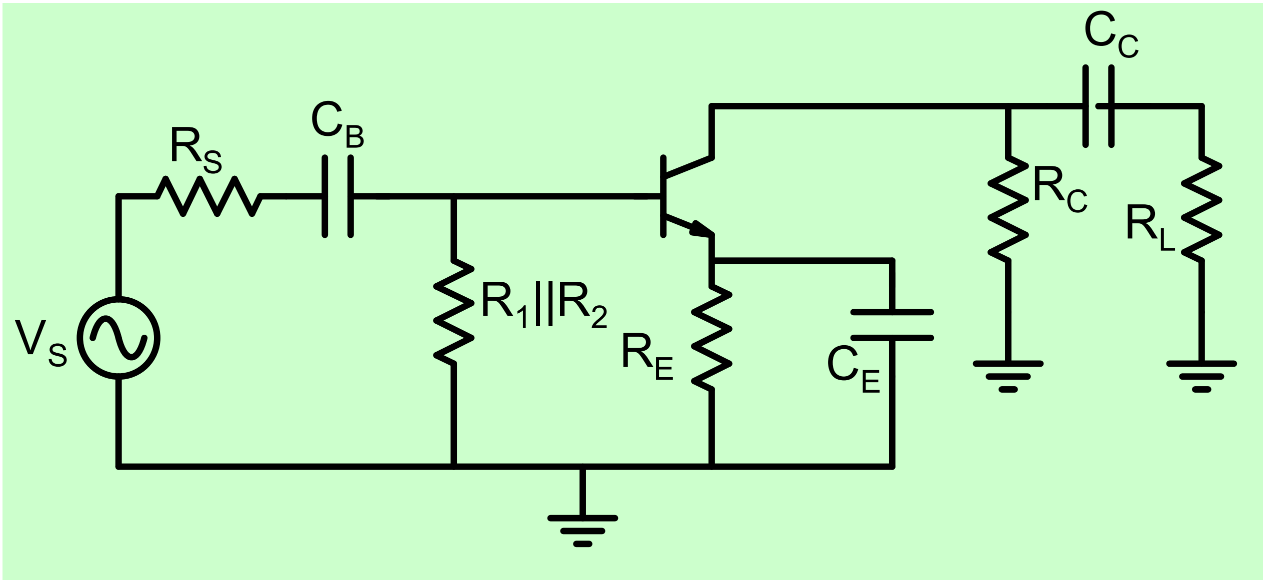


$$\frac{r_\pi + R_B}{1+\beta}$$

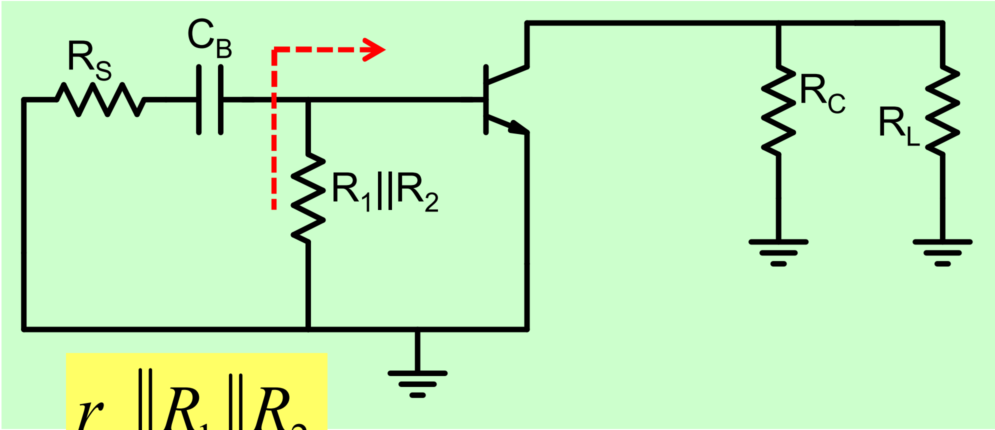


CE Amplifier





Let us calculate 3dB frequency due to each capacitor alone

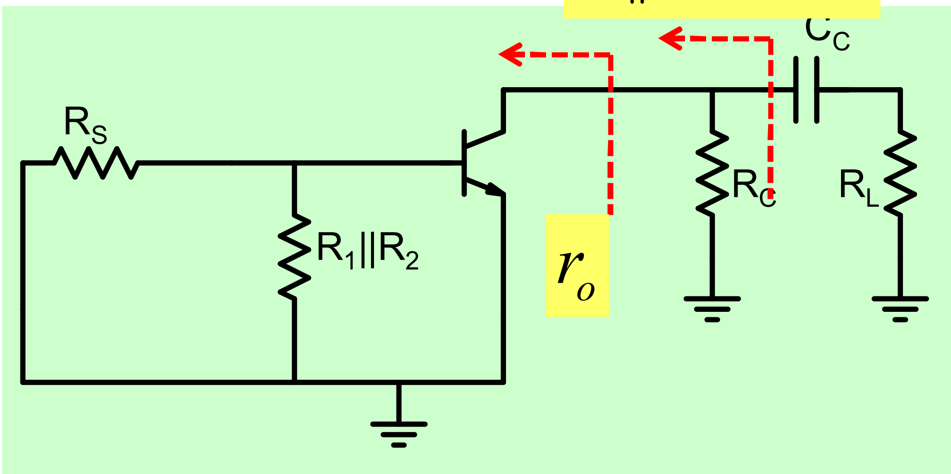


$r_\pi \| R_1 \| R_2$

$$R_{eqB} = R_S + (r_\pi \| R_1 \| R_2)$$

$$f_B = \frac{1}{2\pi C_B \{R_S + (r_\pi \| R_1 \| R_2)\}}$$

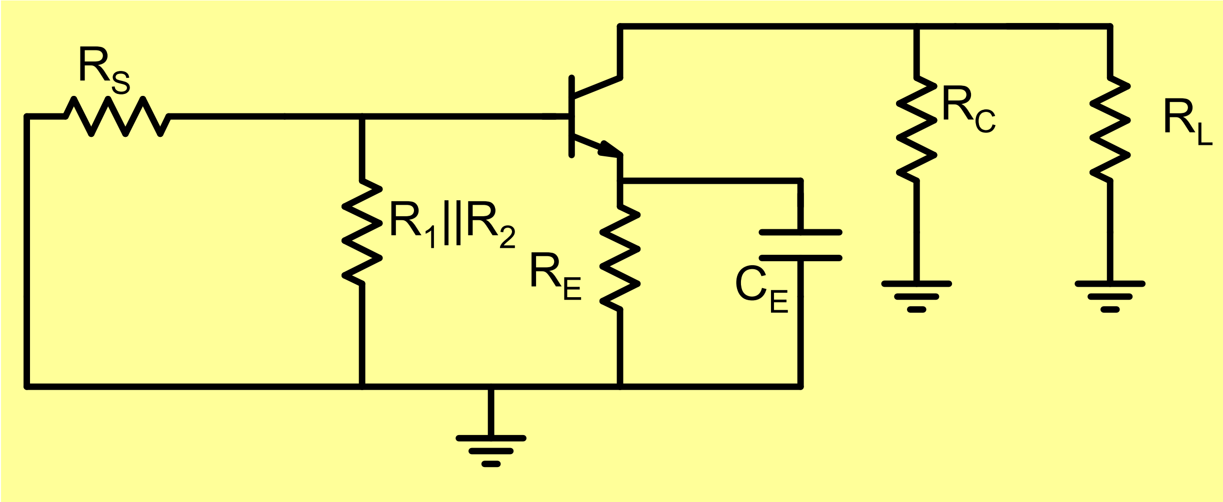
$r_o \| R_C \cong R_C$

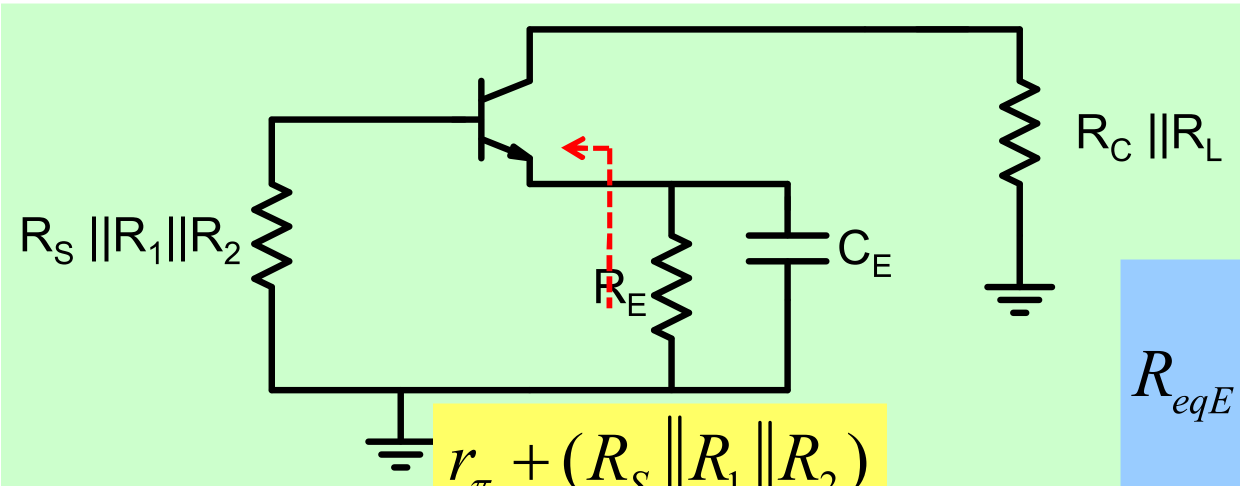


r_o

$$R_{eqC} = R_C + R_L$$

$$f_C = \frac{1}{2\pi C_C \{R_C + R_L\}}$$



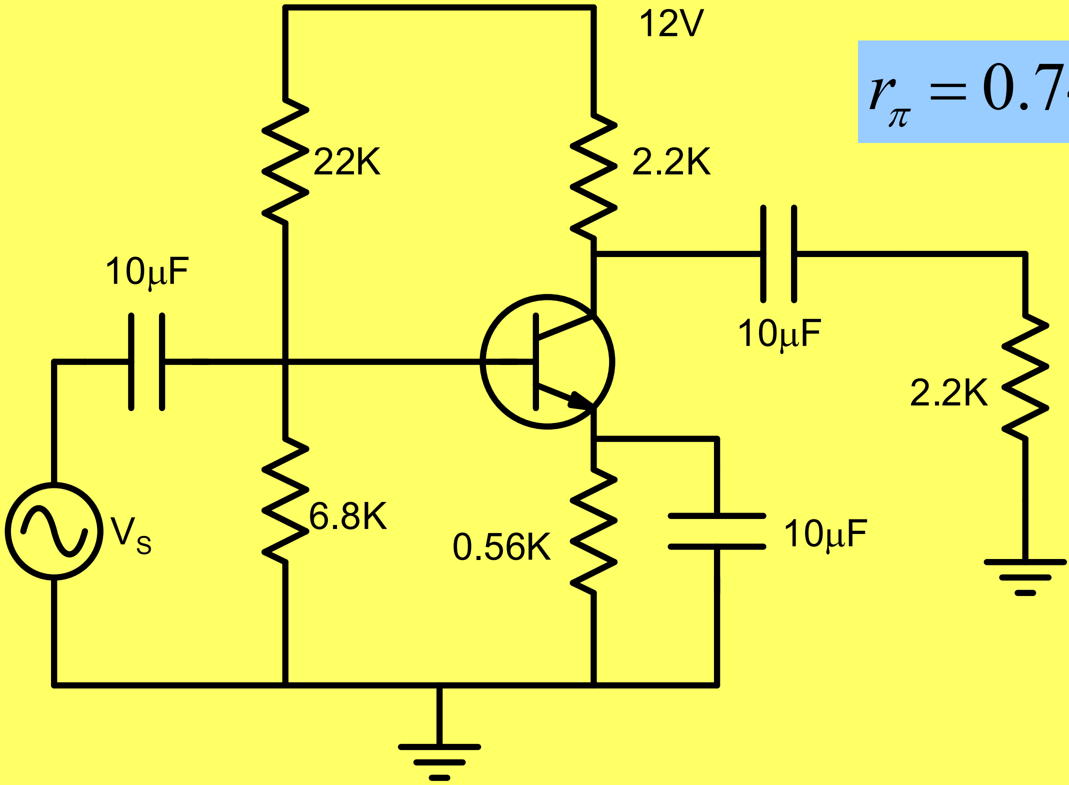


$$\frac{r_\pi + (R_S \| R_1 \| R_2)}{1+\beta}$$

$$R_{eqE} = R_E \left\| \frac{r_\pi + (R_S \| R_1 \| R_2)}{1+\beta} \right.$$

$$f_E = \frac{1}{2\pi C_E R_{eqE}}$$

Example



$$I_{CQ} = 3.48mA$$

$$r_\pi = 0.74K \; ; \; g_m = 0.13\mho \; ; \; A_V = -147.5$$

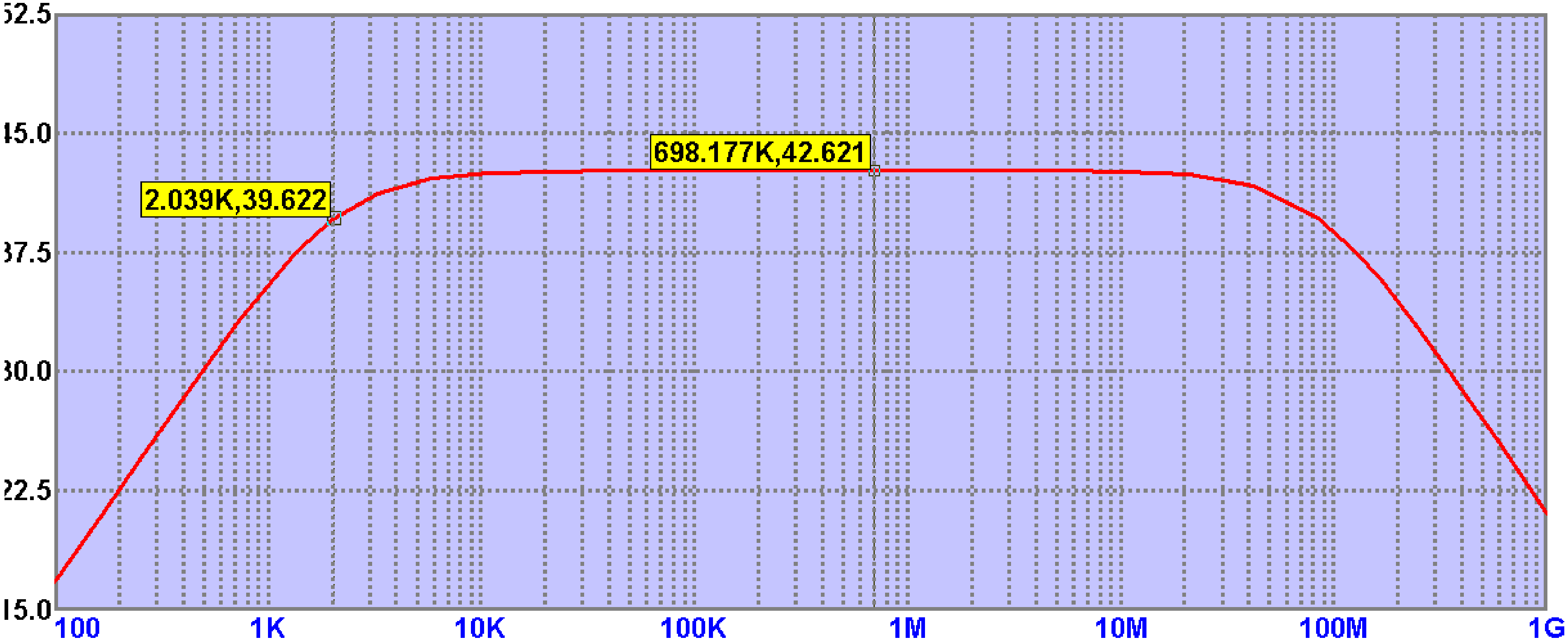
$$f_B = 24.4Hz$$

$$f_C = 3.6Hz$$

$$f_E = 2.18KHz$$

$$f_L \cong 2.18KHz$$

Simulation Results



Typically $f_E >> f_B$ and f_c

Consider a case where R_S ~0 , no load and all capacitors are of equal magnitude

$$f_B = \frac{1}{2\pi C_B \times (R_S + R_B \| r_\pi)}$$

$$f_B \cong \frac{1}{2\pi C_B \times r_\pi}$$

$$f_c = \frac{1}{2\pi C_c \times (R_C + R_L)}$$

$$f_c \le \frac{1}{2\pi C_c \times R_C}$$

$$f_E = \frac{1}{2\pi C_E \times (R_E \| \frac{(R_S \| R_B) + r_\pi}{\beta})}$$

$$f_E \cong \frac{\beta}{2\pi C_E \times r_\pi}$$

Note that : $I_{CQ} R_E >> V_T \Rightarrow R_E >> \frac{r_\pi}{\beta}$

$$f_B \cong \frac{1}{2\pi C \times r_\pi}$$

$$f_c \le \frac{1}{2\pi C \times R_C}$$

$$f_E \cong \frac{\beta}{2\pi C \times r_\pi}$$

$$f_E = \beta \times f_B$$

$$\frac{f_E}{f_c} \ge \frac{I_{CQ} R_C}{V_T} = A_V >> 1$$

This shows that typically, emitter bypass capacitor determines lower cutoff frequency !

EE 210

BJT Amplifier Analysis

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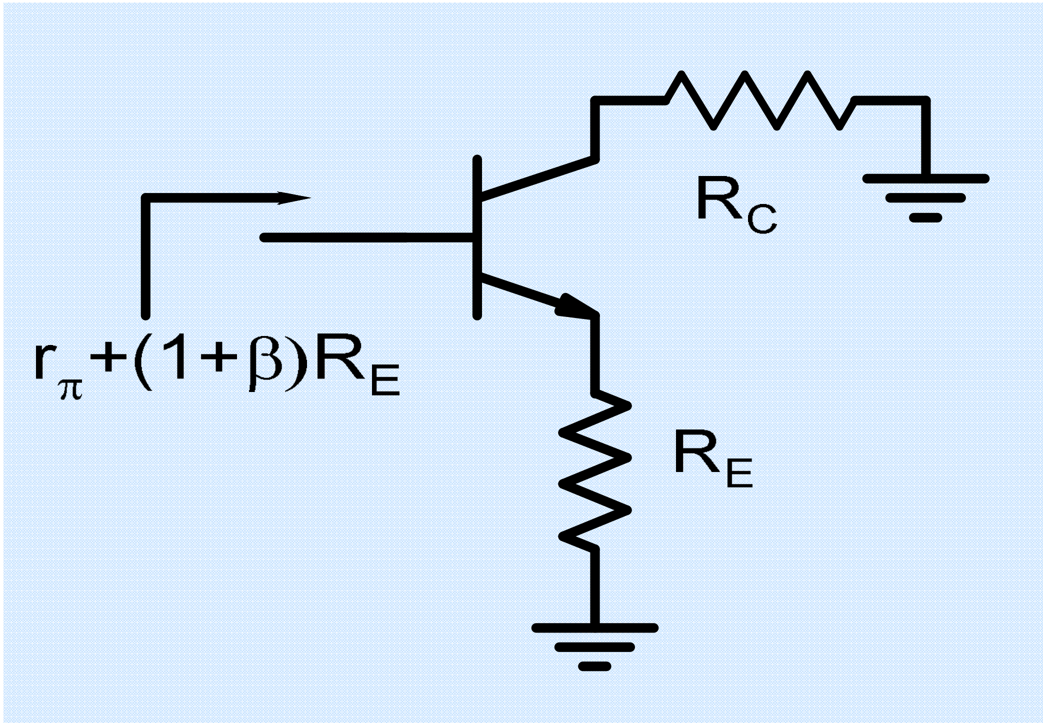
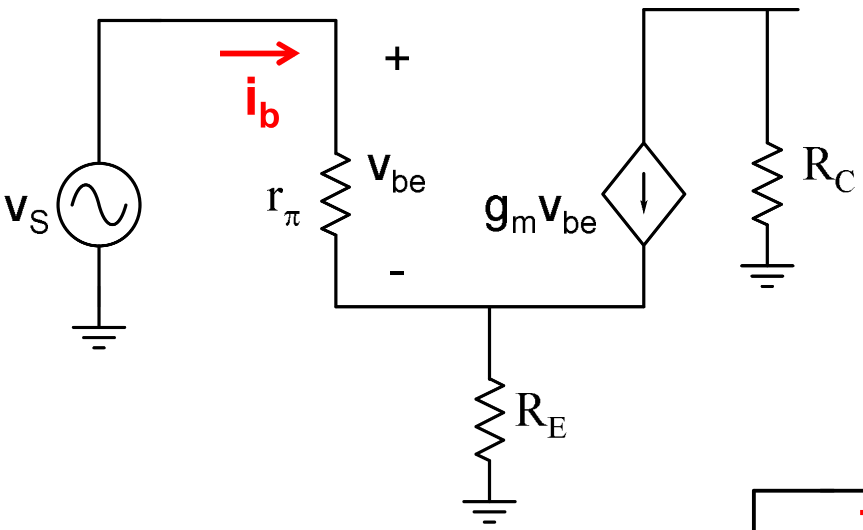
Dept. of EE, IIT Kanpur

https://youtu.be/HGqLEM8gaRM

- One useful result in small signal analysis of BJT amplifiers is that "looking from the base" the emitter resistance gets multiplied by the current gain β of the transistor.



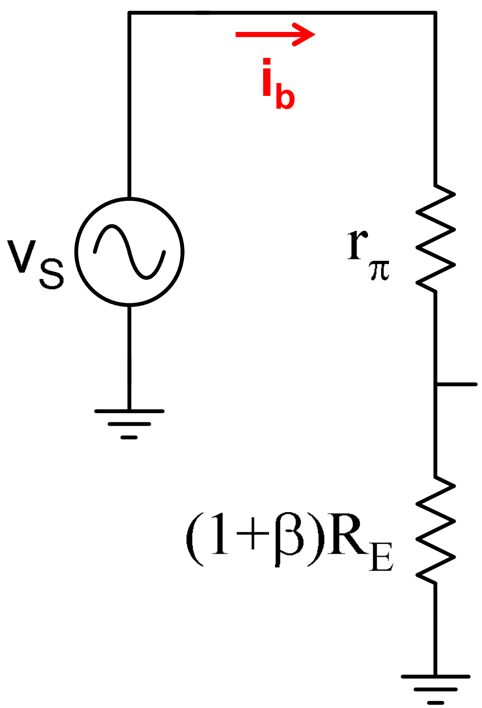
Simple Derivation



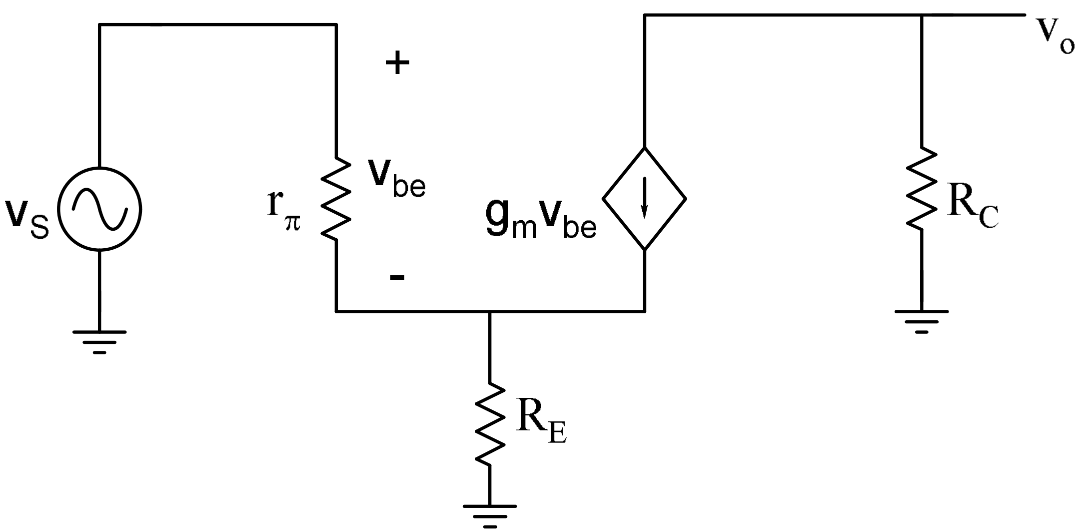
Circuit to Equation

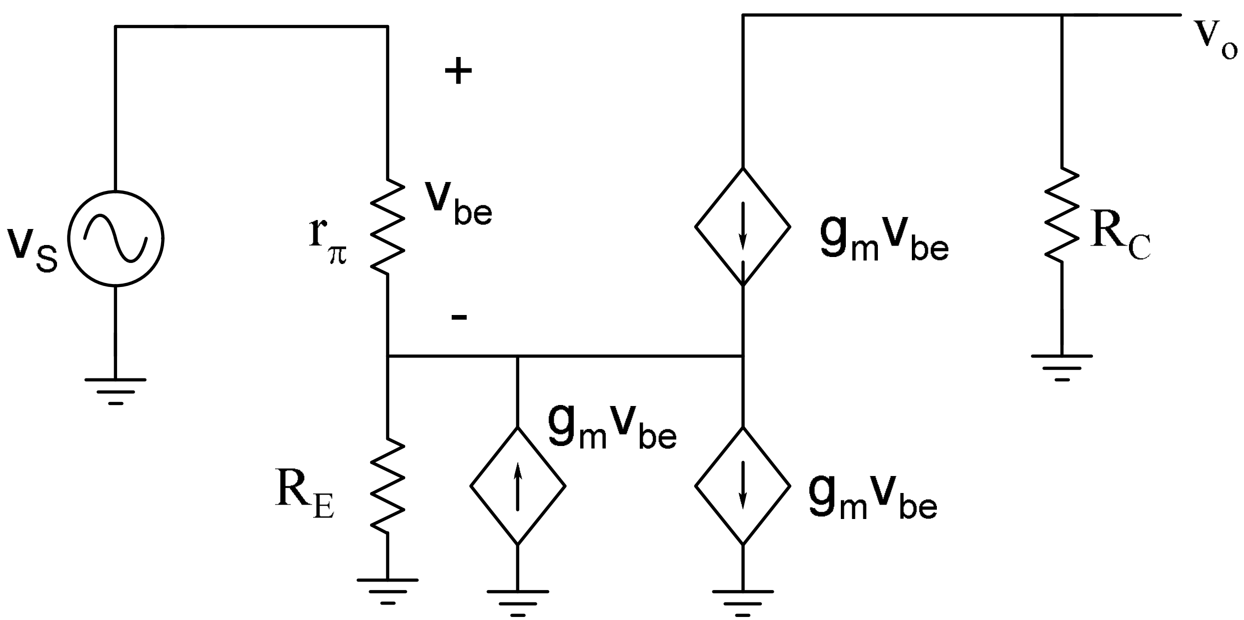
$$v_s = i_b r_\pi + \left(i_b + g_m \times \left(i_b r_\pi\right)\right) R_E$$
$$= i_b \times r_\pi + i_b \times \left(1 + \beta\right) R_E$$

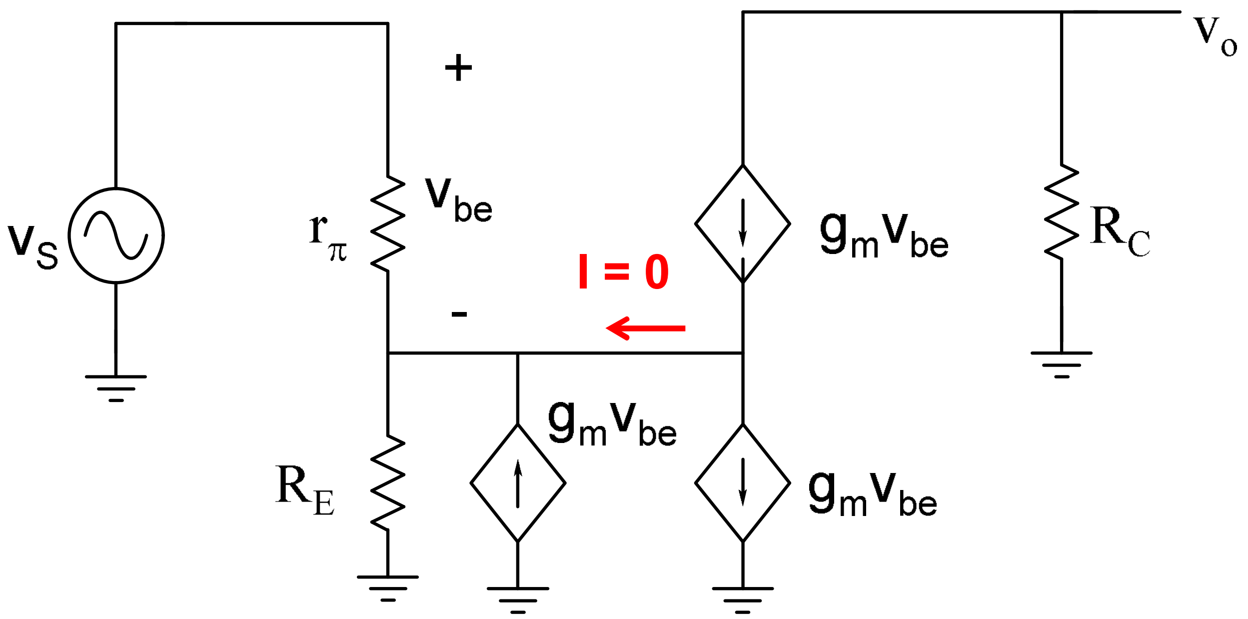
Equation to Circuit



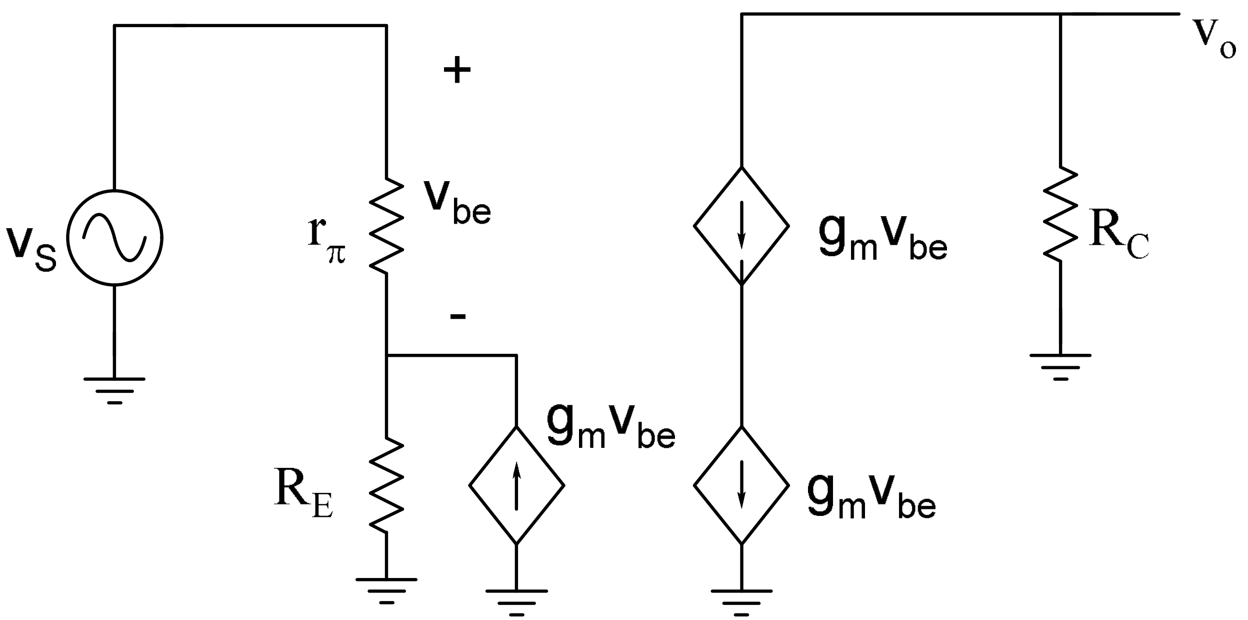
Alternative derivation based on circuit transformation



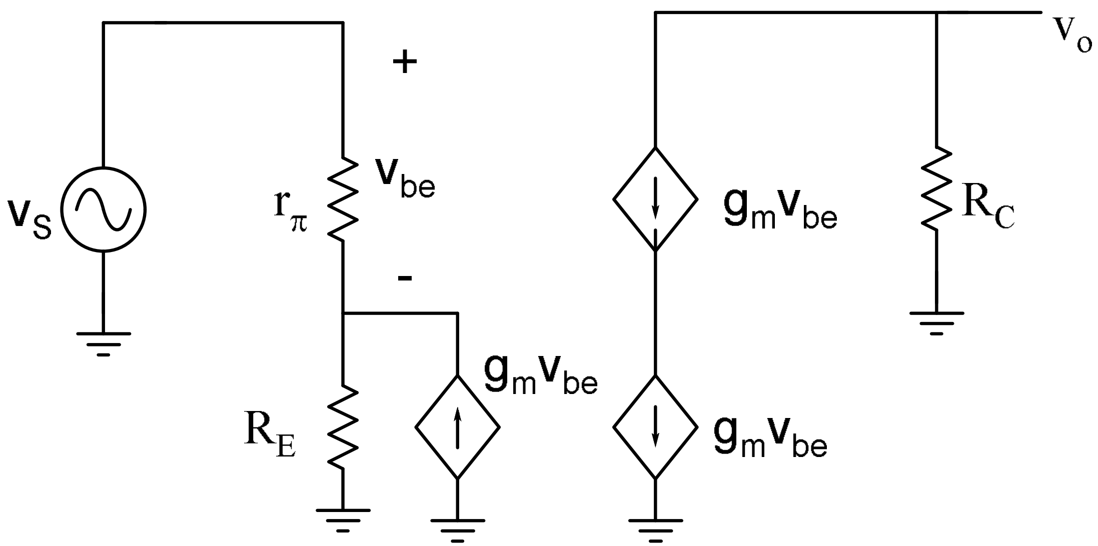


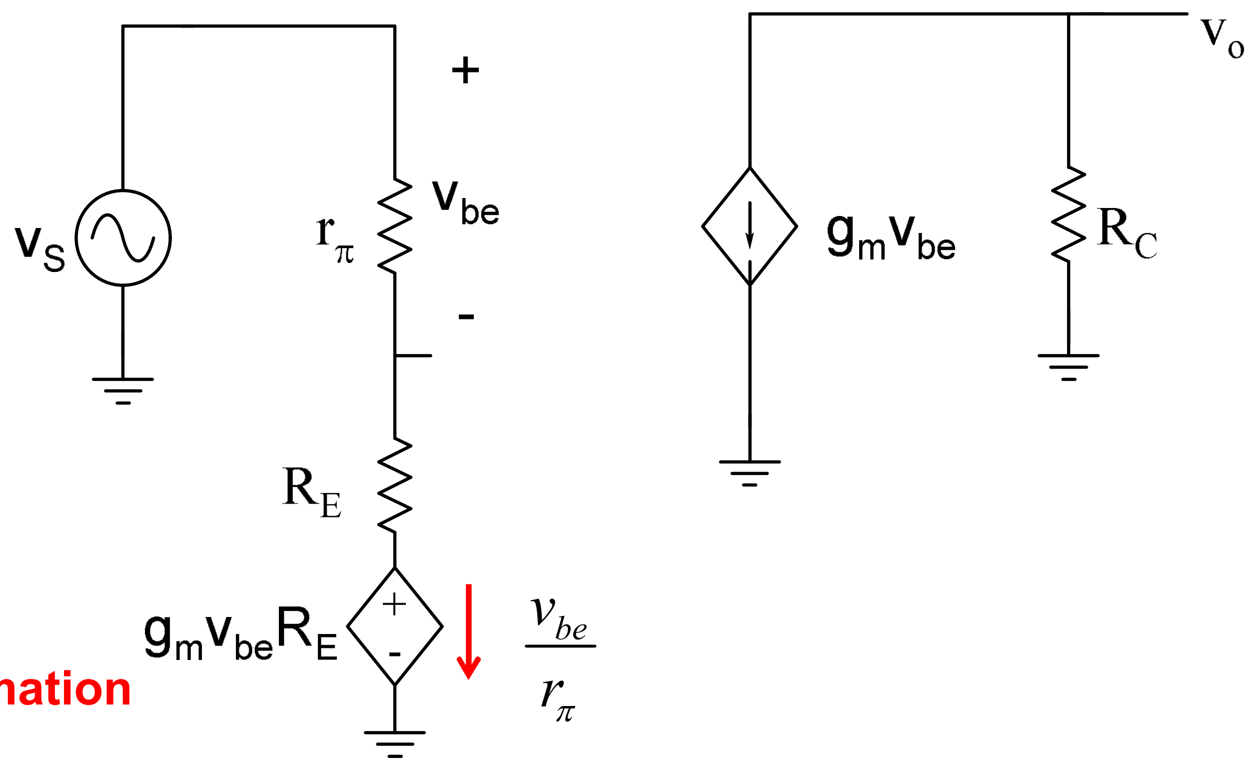


v_S
r_π
+
v_{be}
-
$g_m v_{be}$
I = 0
$g_m v_{be}$
$g_m v_{be}$
R_E
R_C
v_o

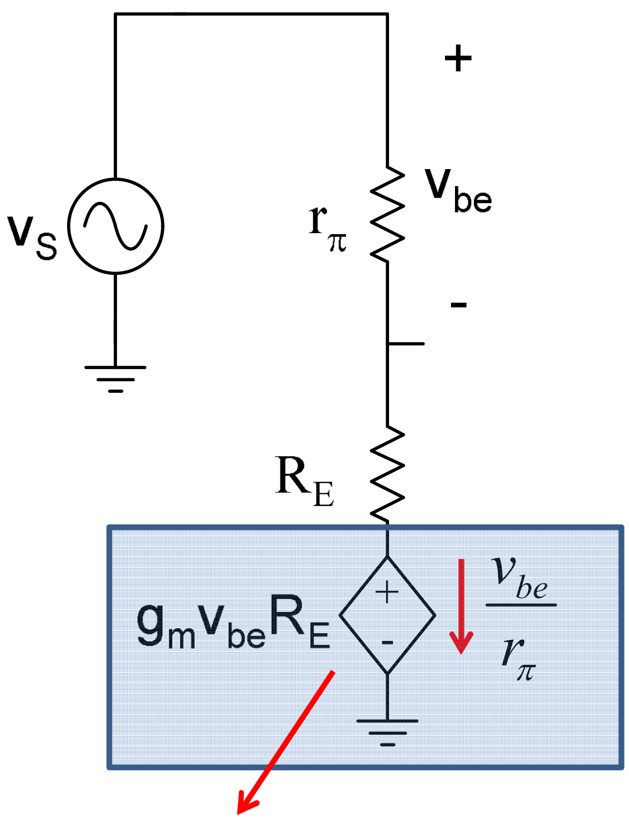


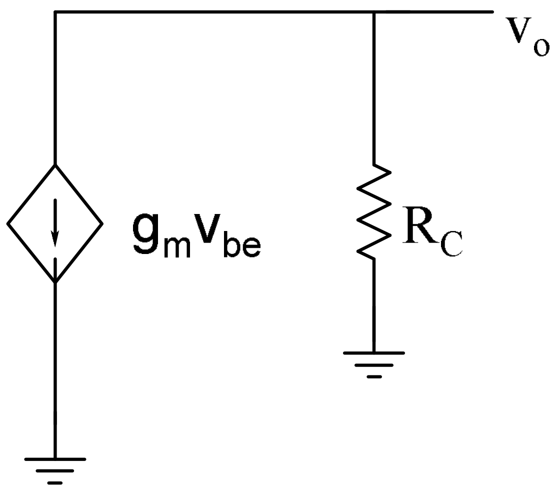
v_S
r_π
+
v_{be}
-
$g_m v_{be}$
$g_m v_{be}$
$g_m v_{be}$
R_E
R_C
v_o





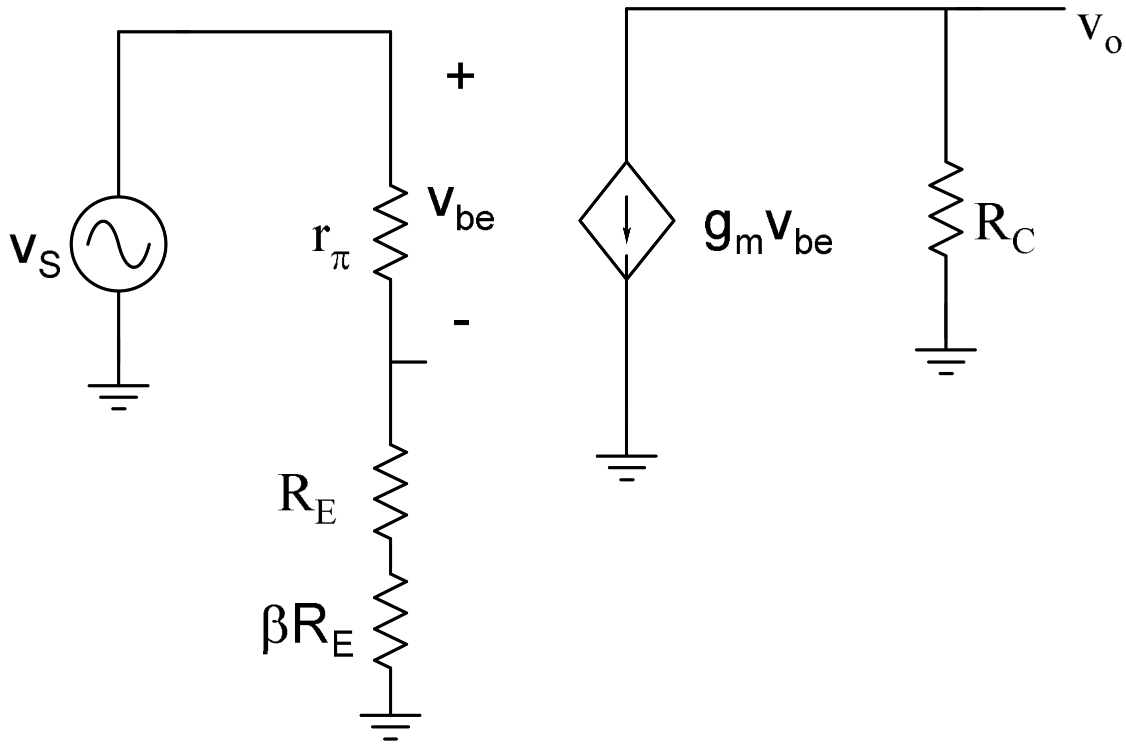
Source transformation

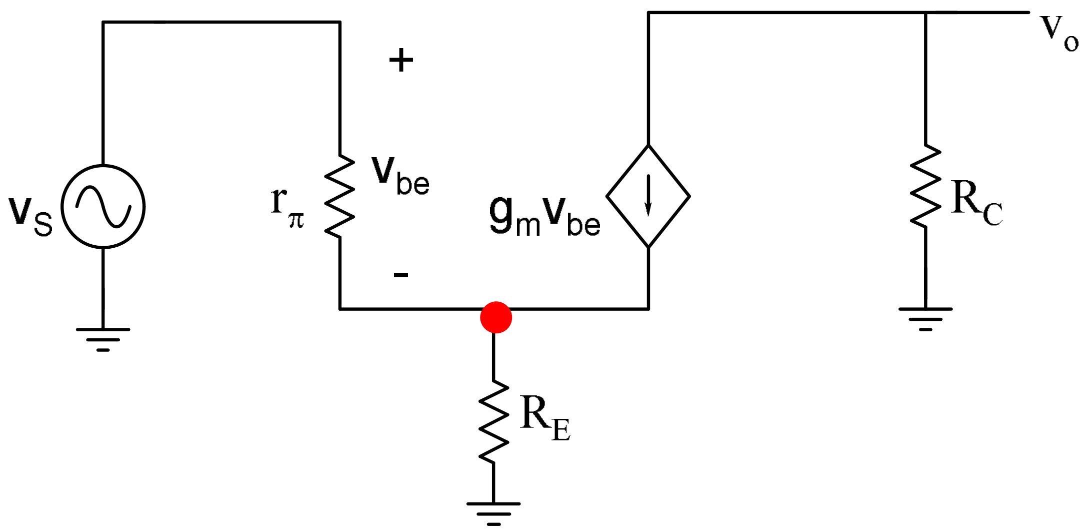




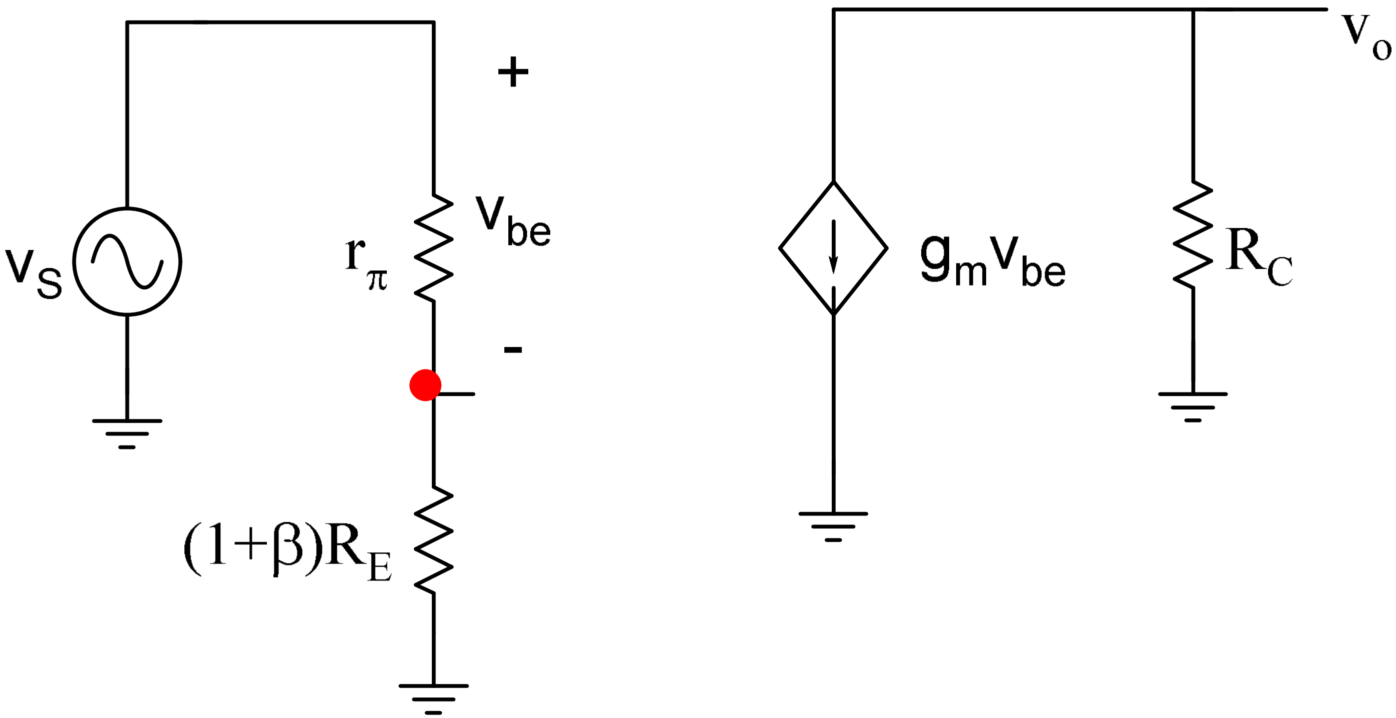
$$R = \frac{g_m v_{be} R_E}{\frac{v_{be}}{r_\pi}}$$

$$= g_m r_\pi R_E = \beta R_E$$



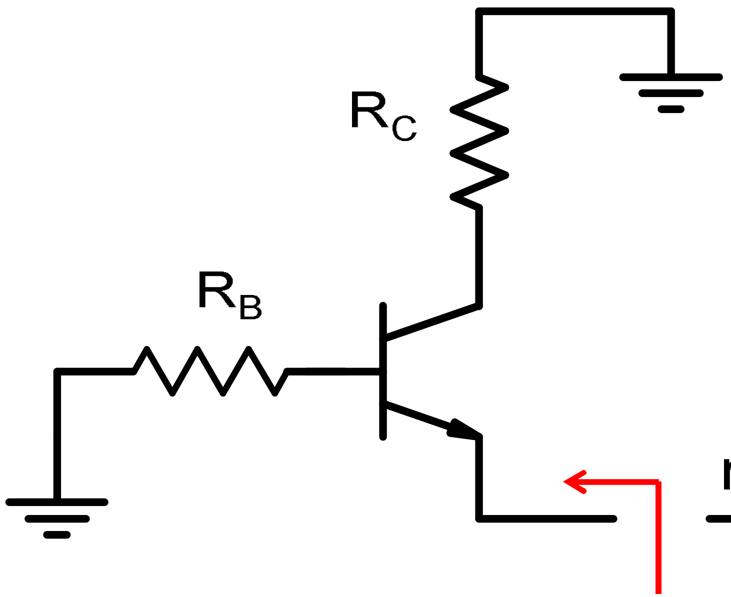


v_o
+
v_{be}
r_π
v_S
$g_m v_{be}$
R_C
-
R_E



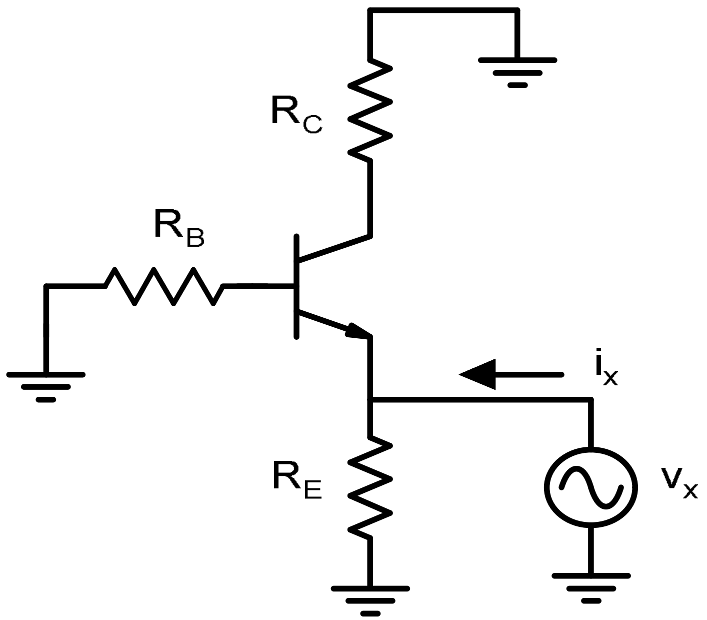
v_o
+
v_{be}
r_π
v_S
$g_m v_{be}$
R_C
-
$(1+\beta)R_E$

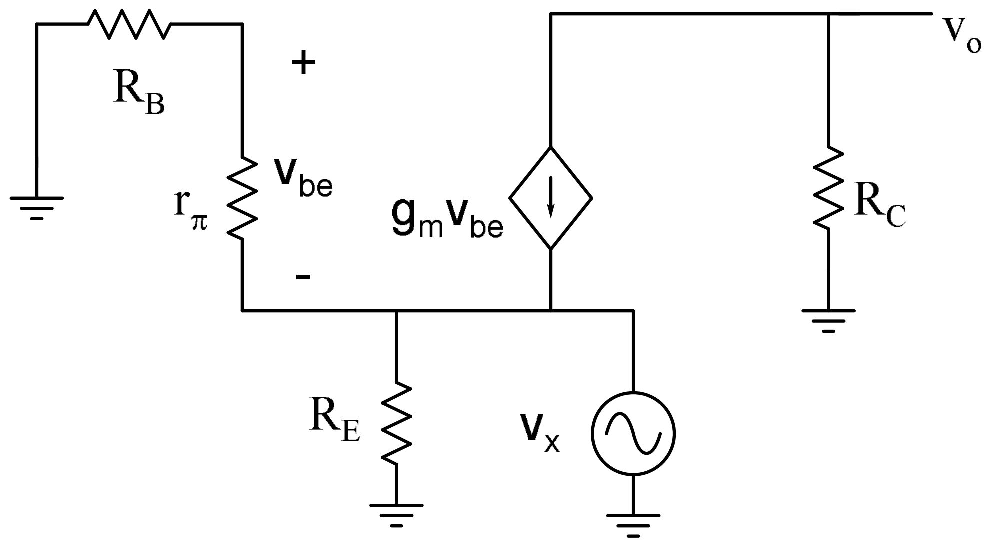
- Another useful result in small signal analysis of BJT amplifiers is that "looking from the emitter" any resistance in the base gets divided by the current gain β of the transistor.

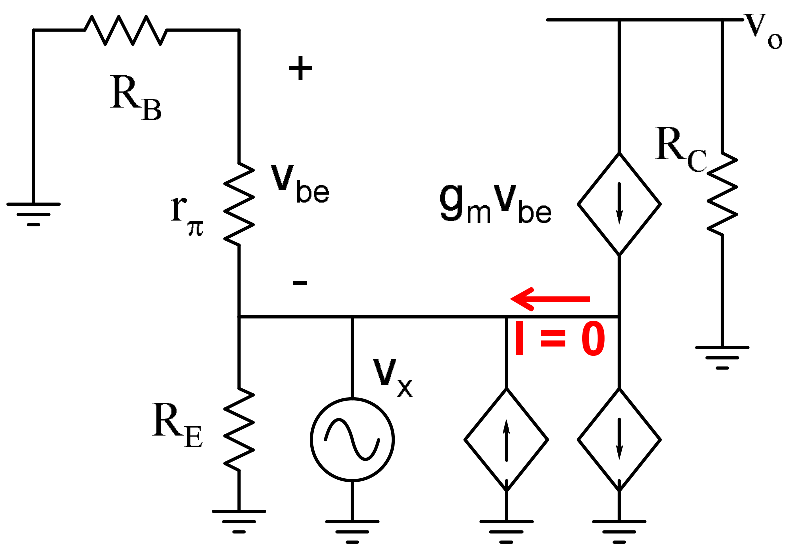


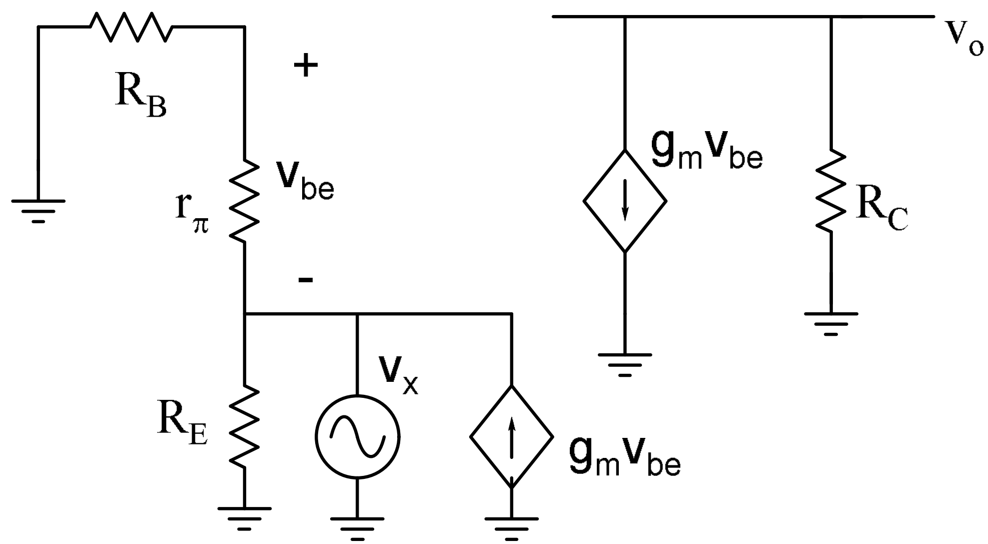
$$\frac{r_\pi + R_B}{1+\beta}$$

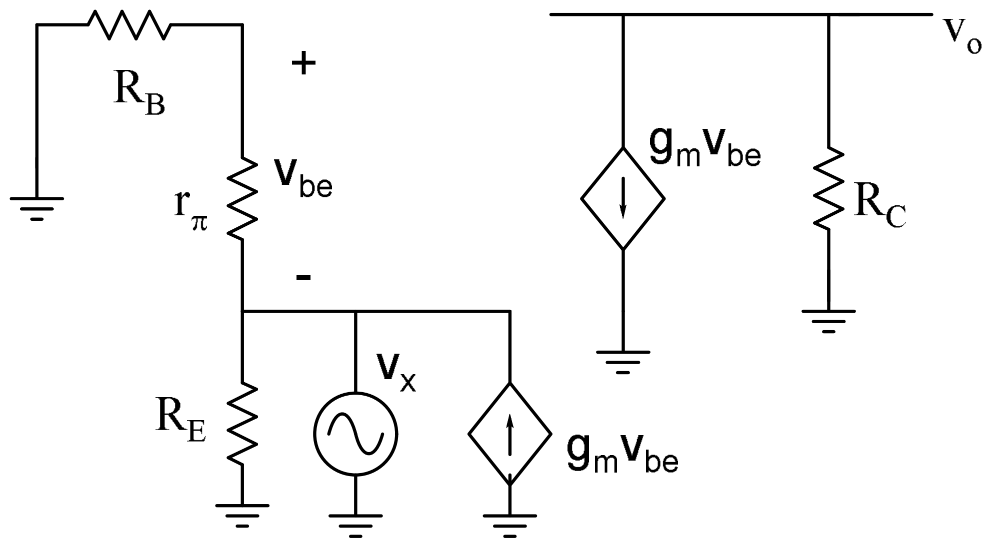
Derivation based on circuit transformation



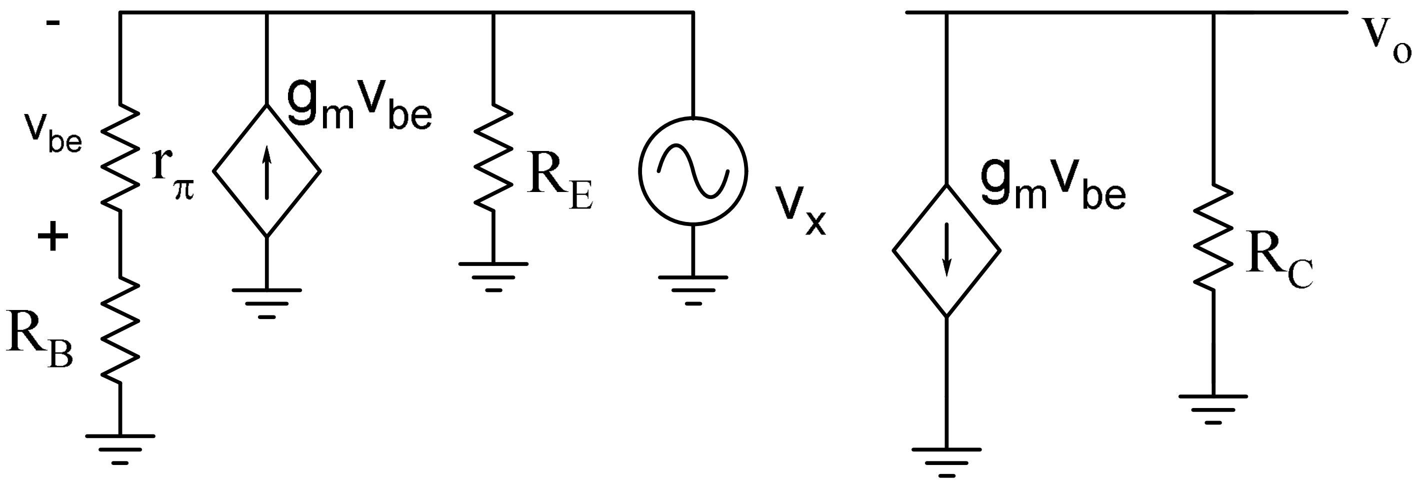




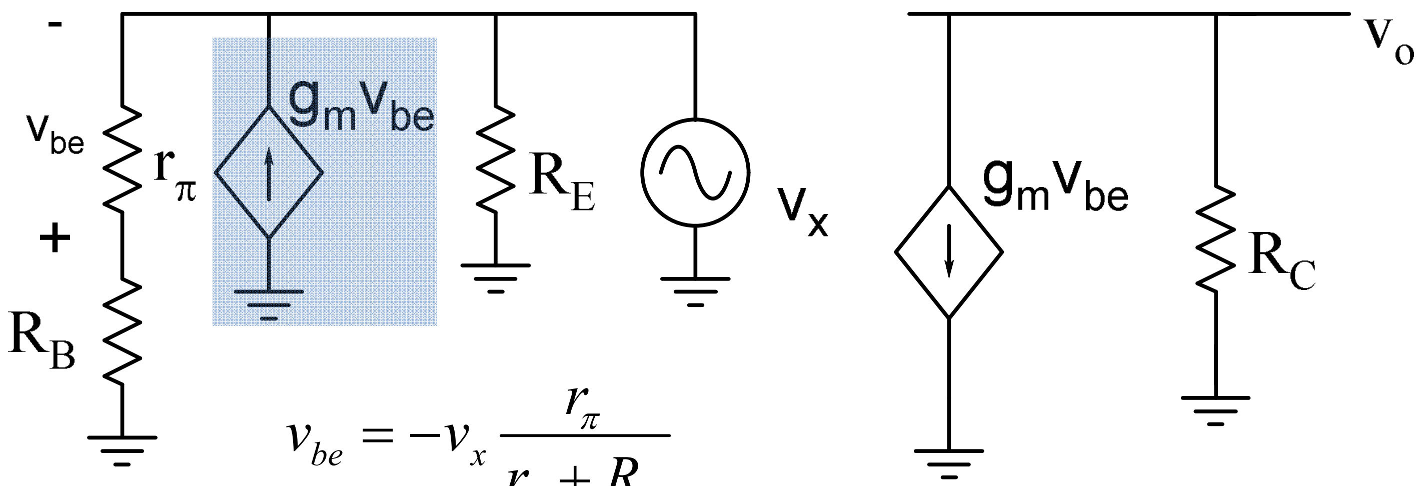




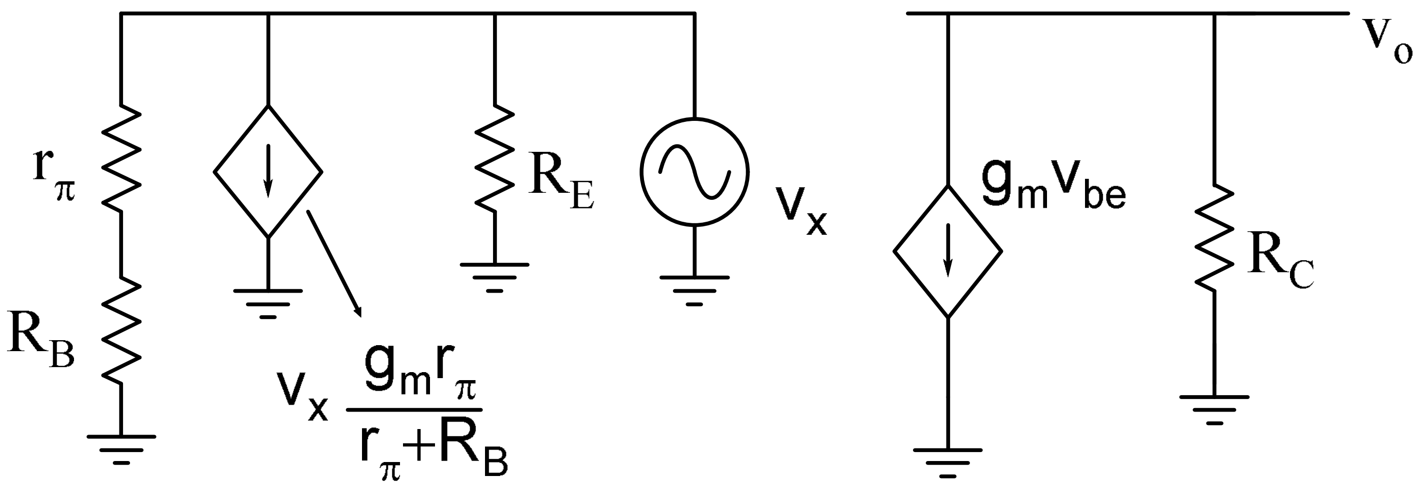
R_B
+
v_{be}
r_π
-
R_E
v_x
$g_m v_{be}$
$g_m v_{be}$
R_C
v_o

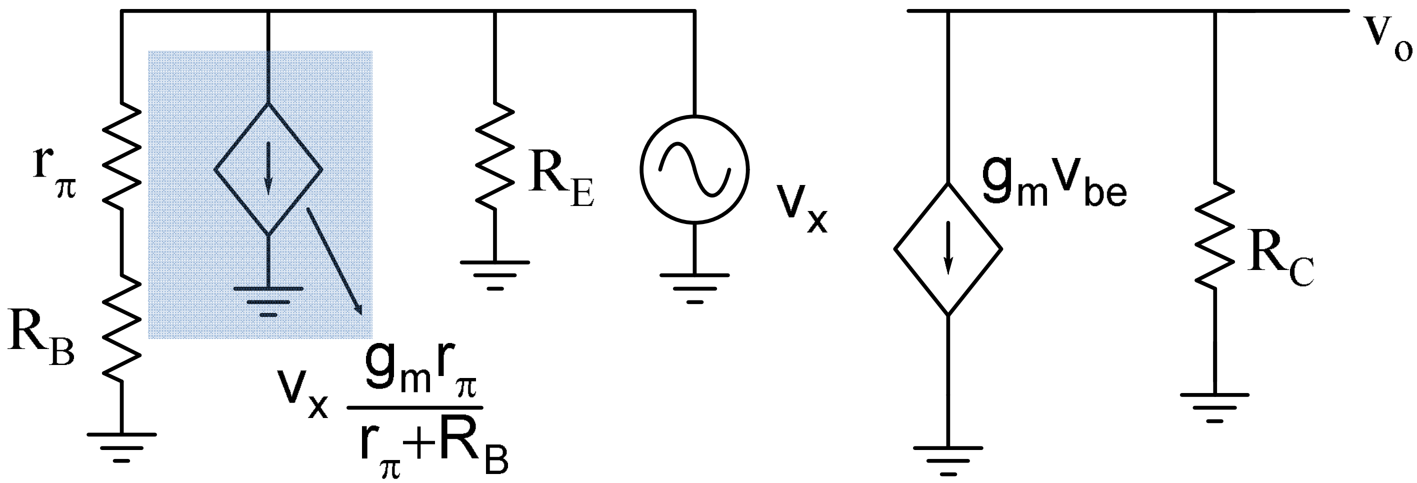


-
v_{be}
r_π
+
R_B
$g_m v_{be}$
R_E
v_x
$g_m v_{be}$
R_C
v_o

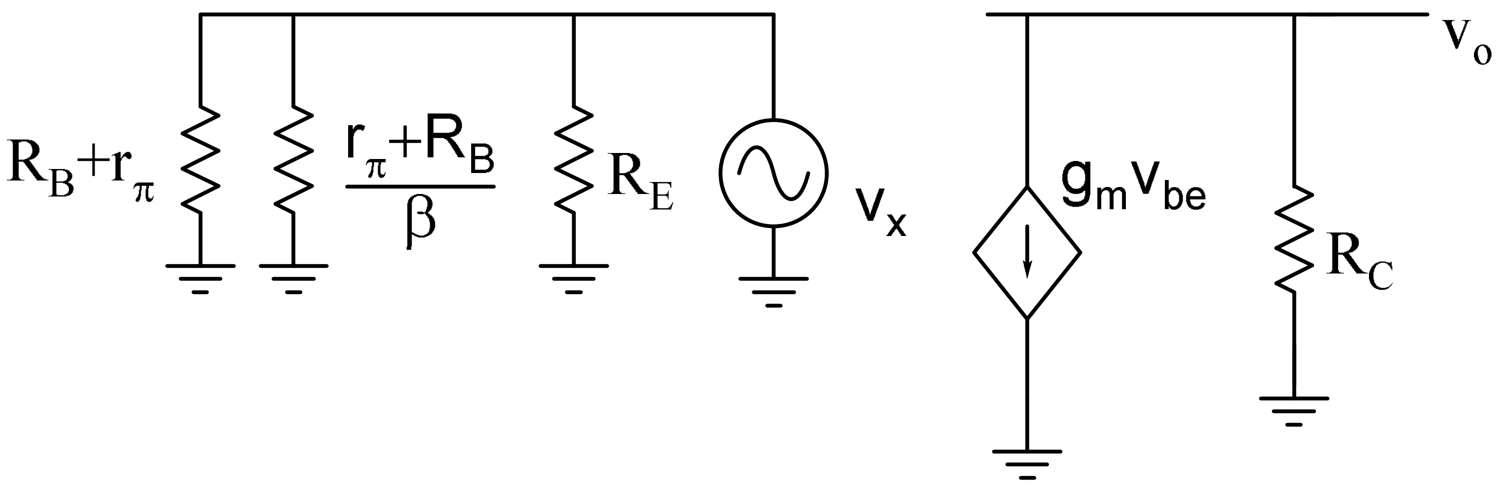


$$v_{be} = -v_x \frac{r_\pi}{r_\pi + R_B}$$

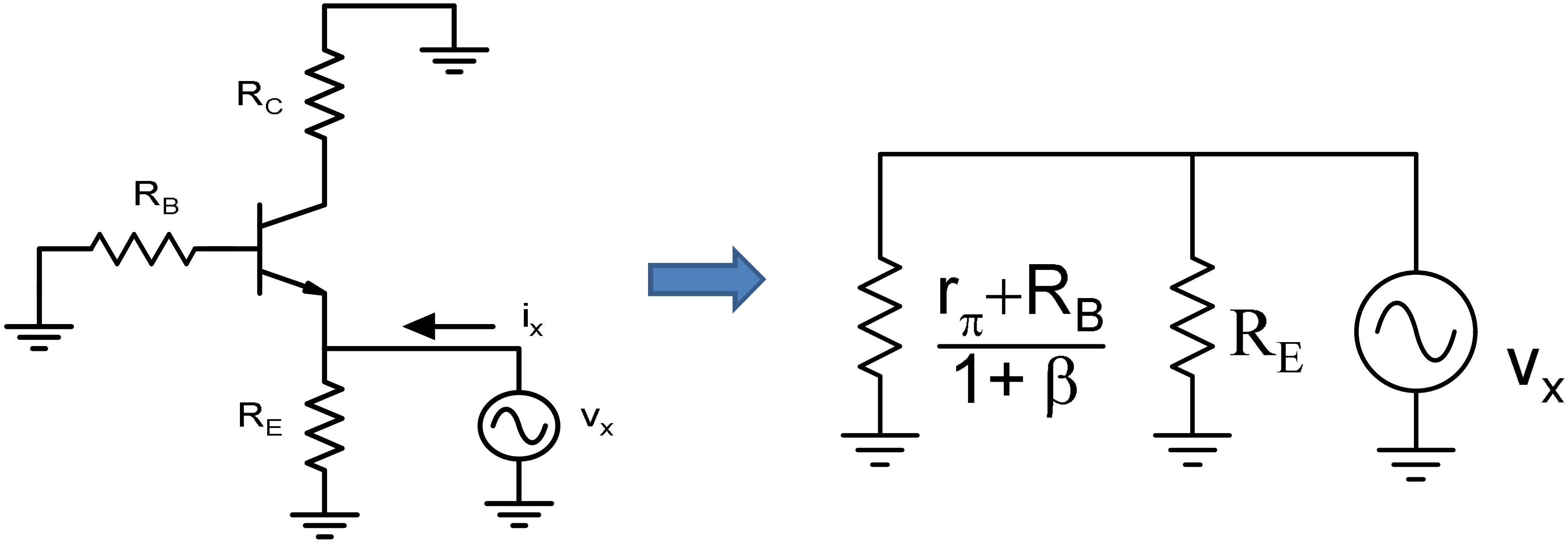




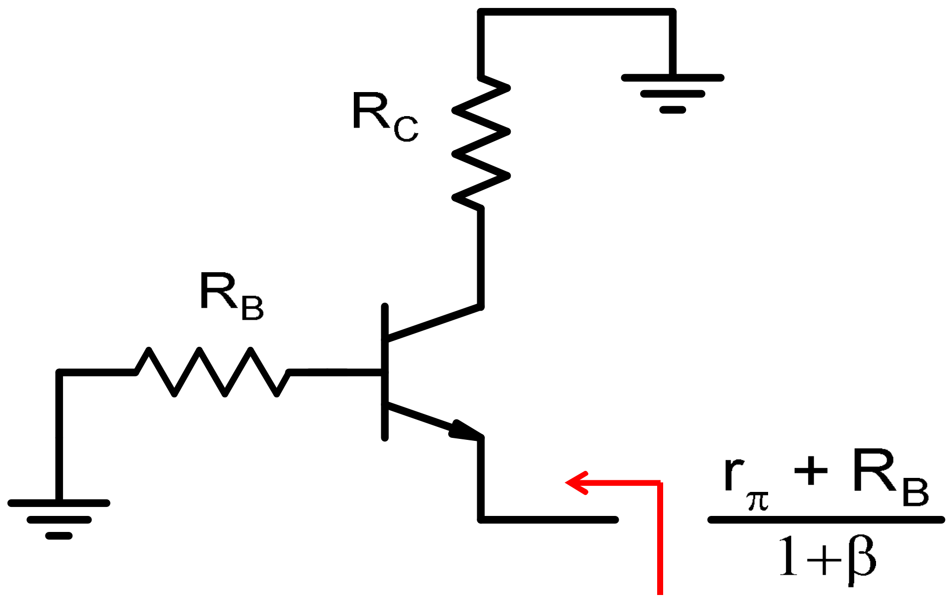
r_π
R_B
R_E
v_x
$v_x \frac{g_m r_\pi}{r_\pi + R_B}$
$g_m v_{be}$
R_C
v_o



$R_B + r_\pi$
$\frac{r_\pi + R_B}{\beta}$
R_E
v_x
$g_m v_{be}$
R_C
v_o



R_C
R_B
i_x
R_E
v_x
$\frac{r_\pi + R_B}{1+\beta}$
R_E
v_x



R_C
R_B
$\frac{r_\pi + R_B}{1+\beta}$