

Problem Set 3
CHM102A

1. Assuming particle in a 1-D box model, calculate the energy separation between the lowest two levels for a particle confined in a box of length 3 nm (consider the particle to be the H₂ molecule). In this model, at what quantum number, n , does the energy of the molecule equal $k_B T$ when $T=300\text{K}$. Compare the results to the case of nitrogen molecule whose mass is 14-times higher.

2. Consider a system of two non-interacting particles of mass m_1 and m_2 confined along the x -axis such that:

$$V(x_1, x_2) = \begin{cases} 0 & \text{if } 0 < x_1 < L, \text{ and } 0 < x_2 < L \\ \infty & \text{otherwise} \end{cases}$$

where x_1 and x_2 are coordinates of particles 1 and 2.

- Solve the Schrodinger equation for this problem.
 - How many quantum numbers are required to specify a state of this system?
 - Can degenerate states appear for this problem as special case? Explain.
 - Sketch the ground state wave function.
 - For the ground state, sketch the probability density for finding both particle 1 and particle 2 simultaneously at the same point for $0 < x < L$?
3. For the ground state of a quantum mechanical 1-D simple harmonic oscillator, compute the expectation value of Kinetic Energy $\langle T \rangle$.
4. Assuming a particle in a 1-D box model, calculate the separation between the lowest two energy levels for a ¹⁴N₂ molecule in a box of length 3 nm. In this model, at what quantum number, n , does the energy of the molecule equal $k_B T$ when $T = 300\text{K}$.
5. Consider the wavefunction $\psi = A \cos\left(\frac{n\pi x}{a}\right)$, where n is a nonzero positive integer and ' a ' is the length of the box as a possible solution for a particle in a 1D box with infinite potential. Under what condition, if any, will ψ be an allowed wavefunction for this 'particle in a 1D box'.
6. For a 3D rigid rotor with quantum number $l = 1$, what are the possible angles (θ) the angular momentum vector with the z -axis?