EE 200 Assignment 9

1 The energy of a continuous-time signal $x(t)$ is defined as $\int_{-\infty}^{\infty} |x(t)|^2 dt$.

(*a*) Show that in general, $\int_{-\infty}^{\infty} x(t)y^*(t)dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)Y^*(\omega)d\omega$.

(*b*) From this, show that the energy of $X(\omega)$ is 2π times the energy of $x(t)$: $\int_{-\infty}^{\infty} |x(t)|^2 dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} |X(\omega)|^2 d\omega$.

(*c*) Similarly prove the following. For the continuous time Fourier Series (CTFS): $\frac{1}{T}\int_T |x(t)|^2 = \sum_k |x_k|^2$. For the discrete Fourier Transform (DFT/DTFS): $\frac{1}{N}\sum_{n=0}^{N-1} |x[n]|^2 = \sum_{k=0}^{N-1} |X[k]|^2$. And, finally, for the discrete time Fourier Transform, (DTFT): $\sum_n |x[n]|^2 = \frac{1}{2\pi}\int_{2\pi} |X\Omega|^2$.

2 The following are the modulation property of the CTFT and the DTFT repectively. Prove them.

$$x(t)y(t) \leftrightarrow \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\theta) \star Y(\omega-\theta)d\theta; \qquad x[n]y[n] \leftrightarrow \frac{1}{2\pi}\oint_{2\pi} X(\Theta)Y(\Omega-\Theta)d\Theta$$

3 A system with zero initial conditions satisfies the difference equation $y[n] = x[n] - \alpha y[n-1]$. If $x[n] \leftrightarrow X(\Omega)$ is applied to it as input, find $Y(\Omega)$ in terms of $X(\Omega)$. Find also the sufficient conditions under which $Y(\Omega)$ will exist. -

4 Consider the DFT $X[k]$ of an N-length signal - $x[n]$. Now treat $X[n]$ as a time sequence' $y[n]$ and again obtain its DFT $Y[k]$. Compare $x[n]$ and $Y[n]$. -

5 (*A*) Find the N-point DFT of the following N-length - sequences: (*a*) $x[n] = (-1)^n;\ 0 \le n < N$, (*b*) $-x[n] = 1 + (-1)^n;\ 0 \le n < N$, (*c*) $x[n] = j^n;\ 0 \le n < N$, (*d*) $x[n] = 1, 0, \ldots 0$.

(*B*) Find the DTFT of the following: (*a*) $x[n] = (-1)^n;-$ (*b*) $x[n] = 1 + (-1)^n$; (*c*) $x[n] = j^n$; (*d*) $x[n] = \ldots, 0, 1, 0, \ldots = \delta[n]$.

6 If $x[n] \leftrightarrow X(\Omega)$, find the DTFT of the following signals in terms of $X(\Omega)$.

(*a*)	$\ldots, 0, x[-2], 0, x[0], 0, x[2], \ldots,$	odd members replaced by zero.
(*b*)	$\ldots, 0, x[-1], 0, x[1], 0 \ldots,$	even members replaced by zero.
(*c*)	$\ldots, -x[-3], x[-2], -x[-1], x[0], -x[1], x[2] \ldots,$	odd members inverted.
(*d*)	$\ldots, x[-3], -x[-2], x[-1], -x[0], x[1], -x[2] \ldots,$	even members inverted.

7 Consider the N-length sequence $x[n]$ and its $2N$-length extension $x'[n]$ obtained by *padding* $x[n]$ with N consecutive zeros at the end. Let their respective DFTs of lengths N and $2N$ be $X[k];\ 0 \le k < N-1$ and $X'[k];\ 0 \le k < 2N-1$. Show that every member of $X[k]$ is to be found at a specific place in $X'[k]$ and find its exact location. As to the remaining N 'new' members in $X'[k]$, show that their computation can be - economized in certain ways, instead of merely applying - direct approach through the formula.

Assignment-

1 a) To prove: $\int_{-\infty}^{\infty} x(t)\, y^*(t)\, dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega) Y^*(\omega)\, d\omega$

$$y(t) = \mathcal{F}^{-1}[Y(\omega)] = \int_{-\infty}^{\infty} \frac{1}{2\pi} Y(\omega)\, e^{j\omega t}\, d\omega$$

$$y^*(t) = \int_{-\infty}^{\infty} \frac{1}{2\pi} Y^*(\omega)\, e^{-j\omega t}\, d\omega$$

$$\int_{-\infty}^{\infty} x(t)\, y^*(t)\, dt = \int_{-\infty}^{\infty} x(t) \left[\int_{-\infty}^{\infty} \frac{1}{2\pi} Y^*(\omega)\, e^{-j\omega t}\, d\omega \right] dt$$

$$= \int_{-\infty}^{\infty} Y^*(\omega) \left[\frac{1}{2\pi} \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt \right] d\omega$$

$$= \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega)\, Y^*(\omega)\, d\omega$$

b) If $y(t)$ is same as $x(t)$,

$$\int_{-\infty}^{\infty} x(t)\, x^*(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega)\, X^*(\omega)\, d\omega$$

$$\Rightarrow \int_{-\infty}^{\infty} |x(t)|^2 dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} |X(\omega)|^2 d\omega.$$

c) To prove:

$$\cancel{\int_{-\infty}^{\infty} x(t)} \quad \frac{1}{T} \int_{T} |x(t)|^2 = \sum_k |x_k|^2$$

$$P = \frac{1}{T}\int_{-T/2}^{T/2} |x(t)|^2\, dt.$$

$$= \frac{1}{T}\int_{-T/2}^{T/2} |x(t)|^2\, dt \quad \text{for periodic signal}$$

$$= \frac{1}{T}\int_{-T/2}^{T/2} x(t)x^*(t)\, dt.$$

$$x(t) = \sum_{k=-\infty}^{\infty} X(k)\, e^{jk\omega_0 t}$$

$$x^*(t) = \sum_{k=-\infty}^{\infty} X^*(k)\, e^{-jk\omega_0 t}$$

$$P = \sum_{k=-\infty}^{\infty} X^*(k) \frac{1}{T}\int_{\langle T\rangle} x(t) e^{-jk\omega_0 t}\, dt.$$

$$= \sum_{k=-\infty}^{\infty} X^*(k)\, X(t) = \sum_{k=-\infty}^{\infty} |X(k)|^2$$

DFT:

$$\frac{1}{N}\sum_{n=0}^{N-1} |x[n]|^2 = \frac{1}{N}\sum_{n=0}^{N-1} x[n]\, x^*[n]$$

$$= \frac{1}{N}\sum_{n=0}^{N-1} x^*[n] \sum_{k=0}^{N-1} X[k] e^{j2\pi k/N}$$

$$= \frac{1}{N}\sum_{k=0}^{N-1} X[k] \sum_{n=0}^{N-1} x^*[n]\, e^{j2\pi k/N}$$

$$= \frac{1}{N}\sum_{k=0}^{N-1} X[k] \left[\sum_{n=0}^{N-1} x[n]\, e^{-j2\pi k/N}\right]^*$$

$$= \cancel{\frac{1}{N}} \sum_{k=0}^{N-1} X[k]\, X^*[k]$$

$$= \sum_{k=0}^{N-1} |X[k]|^2$$

② ⓐ

$$\frac{1}{2\pi}\int_{-\infty}^{\infty} X(\theta)Y(w-\theta)d\theta \longleftrightarrow \frac{1}{2\pi}\int_{w=-\infty}^{\infty}\frac{1}{2\pi}\int_{\theta=-\infty}^{\infty} X(\theta)Y(w-\theta)e^{+jwt}d\theta dw$$

$$= \frac{1}{2\pi}\int_{\theta=-\infty}^{\infty} X(\theta)\left[\frac{1}{2\pi}\int_{w=-\infty}^{\infty} Y(w-\theta)e^{+jwt}dw\right]d\theta$$

$$= \frac{1}{2\pi}\int_{\theta=-\infty}^{\infty} X(\theta)\, y(t)e^{j\theta t}d\theta = y(t)\cdot\frac{1}{2\pi}\int_{-\infty}^{\infty} X(\theta)e^{j\theta t}d\theta$$

$$= y(t)\cdot x(t)$$

2 b

Modulation property:

To prove: $x[n]\, v[n] \longleftrightarrow \frac{1}{2\pi} \oint X(\Omega)\, V(\theta-\Omega)\, d\Omega$

Let $y(n) = x[n]\, v[n]$.

$$Y(\Omega) = \sum_{n=-\infty}^{\infty} y[n] e^{-j\Omega n} = \sum_{n=-\infty}^{\infty} x[n] v[n] e^{-j\Omega n}$$

$$x[n] = \frac{1}{2\pi} \int_{2\pi} X(\theta) e^{j\theta n} \cdot d\theta.$$

$$Y(\Omega) = \sum_{n=-\infty}^{\infty} v[n] \sum \left[\frac{1}{2\pi} \int_{2\pi} X(\theta) e^{j\theta n} d\theta \right] e^{-j\Omega n}.$$

$$= \frac{1}{2\pi} \int_{2\pi} X(\theta) \left[\sum_{n=-\infty}^{\infty} v[n] e^{-j(\Omega-\theta)n} \right] d\theta.$$

$$= \frac{1}{2\pi} \int_{2\pi} X(\theta)\, V(\Omega-\theta)\, d\theta.$$

3

$y[n] = x[n] - \alpha y[n-1]$.

$Y(\Omega) = X(\Omega) - \alpha e^{-j\Omega} Y(\Omega)$

$Y(\Omega) \{ 1 + \alpha e^{-j\Omega} \} = X(\Omega)$

$$Y(\Omega) = \frac{X(\Omega)}{1+\alpha e^{-j\Omega}}$$

$$H(\Omega) = \frac{Y(\Omega)}{X(\Omega)} = \frac{1}{1+\alpha e^{-j\Omega}}$$

For all values of $\alpha \neq -1$, $H(\Omega)$ is stable.

For forward time calculation,

when $|\alpha| > 1$,

$y[n] = x[n] - \alpha y[n-1]$.

Here $y[n]$ keeps increases as n increases.

$\therefore$ For stability, in forward-time calculation, α must be less than 1.

For reverse-time,

when $|\alpha| < 1$,

$$y[n-1] = \frac{x[n] - y[n]}{\alpha}$$

~~As, For n~~,

$\because |\alpha| < 1$, $y[n-1]$ keeps increasing.

For stability, in reverse-time calculation, α must be greater than 1

$$\int_{-\infty}^{\infty} x(t)\, y^*(t)\, dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega) Y^*(\omega)\, d\omega$$

$$y(t) = \int_{-\infty}^{\infty} \frac{1}{2\pi} Y(\omega)\, e^{j\omega t}\, d\omega$$

$$y^*(t) = \int_{-\infty}^{\infty} \frac{1}{2\pi} Y^*(\omega)\, e^{-j\omega t}\, d\omega$$

$$\int_{-\infty}^{\infty} x(t)\, y^*(t)\, dt = \int_{-\infty}^{\infty} x(t) \left[\int_{-\infty}^{\infty} \frac{1}{2\pi} Y^*(\omega)\, e^{-j\omega t}\, d\omega\right] dt$$

$$= \int_{-\infty}^{\infty} Y^*(\omega) \left[\frac{1}{2\pi}\int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt\right] d\omega$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)\, Y^*(\omega)\, d\omega$$

If $y(t)$ is same as $x(t)$,

$$\int_{-\infty}^{\infty} x(t)\, x^*(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)\, X^*(\omega)\, d\omega$$

$$\Rightarrow \int_{-\infty}^{\infty} |x(t)|^2\, dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} |X(\omega)|^2\, d\omega.$$

To prove:

$$\int_{-\infty}^{\infty} x(t) \quad \frac{1}{T}\int_{T} |x(t)|^2 = \sum_{k} |x_k|^2$$

4

$x[n] \leftrightarrow X[k]$

$$X[k] = \sum_{n=0}^{N-1} x[n] e^{-j\Omega_0 kn}$$

$$\text{DFT}\{X[n]\} = \sum_{n=0}^{N-1} X[n] e^{-j\Omega_0 kn}$$

$$= \sum_{n=0}^{N-1} X[k] e^{j\Omega_0 (-k) n}$$

$$= N x[-k]$$

5

(A) N-point DFT of the following N-length sequences ①

(a) $x[n] = (-1)^n \; ; \; 0 \le n < N$

$\Rightarrow x[n] = e^{j\pi n}$

$$X[k] = \sum_{n=0}^{N-1} x[n] W_N^{kn} \quad , \; 0 \le k < N \quad , \text{where } W_N^k = e^{-j\frac{2\pi}{N}k}$$

$$= \sum_{n=0}^{N-1} (-1)^n W_N^{kn} \quad , \; 0 \le k < N$$

$$= \sum_{n=0}^{N-1} e^{j\pi n} W_N^{kn} \quad , \; 0 \le k < N$$

$$= 1 + e^{j\pi} W_N^{k} + e^{j2\pi} W_N^{2k} + e^{j3\pi} W_N^{3k} + \ldots + e^{j\pi(N-1)} W_N^{(N-1)k}$$

$$= \frac{1\left[(e^{j\pi} W_N^k)^N - 1\right]}{e^{j\pi} W_N^k - 1}$$

$$= \frac{e^{j\pi N} W_N^{Nk} - 1}{e^{j\pi} W_N^k - 1}$$

$$= \frac{(-1)^N W_N^{Nk} - 1}{-1 \cdot W_N^k - 1} = \frac{1 - W_N^{Nk}}{1 + W_N^k} \quad , \; 0 \le k < N$$

(b) $x[n] = 1 + (-1)^n \; ; \; 0 \le n < N$

Ans: $$X[k] = \sum_{n=0}^{N-1} x[n] W_N^{nk} \quad , \; 0 \le k < N$$

$$= \sum_{n=0}^{N-1} \left[1 + (-1)^n\right] W_N^{nk} \quad , \; 0 \le k < N$$

$$= \sum_{n=0}^{N-1} W_N^{nk} + \sum_{n=0}^{N-1} (-1)^n W_N^{nk}$$

$$= 1 + W_N^k + W_N^{2k} + W_N^{3k} + \ldots + W_N^{(N-1)k} + \sum_{n=0}^{N-1} e^{j\pi n} W_N^{nk} \quad , \; 0 \le k < N$$

$$= \frac{1(W_N^{Nk}-1)}{W_N^k-1} + \sum_{n=0}^{N-1} e^{j\pi n} W_N^{nk}, \quad 0 \le k < N$$

$$= \frac{W_N^{Nk}-1}{W_N^k-1} + 1 + e^{j\pi} W_N^k + e^{j2\pi} W_N^{2k} + e^{j3\pi} W_N^{3k} + \dots + e^{j(N-1)\pi} W_N^{(N-1)k}, \quad 0 \le k < N$$

$$= \frac{W_N^{Nk}-1}{W_N^k-1} + \frac{(e^{j\pi} W_N^k)^N - 1}{e^{j\pi} W_N^k - 1}, \quad 0 \le k < N$$

$$= \frac{W_N^{Nk}-1}{W_N^k-1} + \frac{e^{jN\pi} W_N^{kN} - 1}{-W_N^k - 1}, \quad 0 \le k < N$$

$$= \frac{W_N^{Nk}-1}{W_N^k-1} \mp \frac{(-1)^N W_N^{kN} - 1}{W_N^k + 1}, \quad 0 \le k < N$$

(c) $x[n] = j^n; \ 0 \le n < N$

Sol^n: $x[n] = e^{jn\pi/2} \quad ; 0 \le n < N$

$$X[k] = \sum_{n=0}^{N-1} x[n] W_N^{nk} \ ; \quad 0 \le k < N$$

$$= \sum_{n=0}^{N-1} e^{jn\pi/2} W_N^{nk} \ ; \quad 0 \le k < N$$

$$= 1 + e^{j\pi/2} W_N^k + e^{j2\pi/2} W_N^{2k} + \dots + e^{j(N-1)\pi/2} W_N^{(N-1)k} \ ; 0 \le k <$$

$$= \frac{1(e^{jN\pi/2} W_N^{Nk} - 1)}{e^{j\pi/2} W_N^k - 1}$$

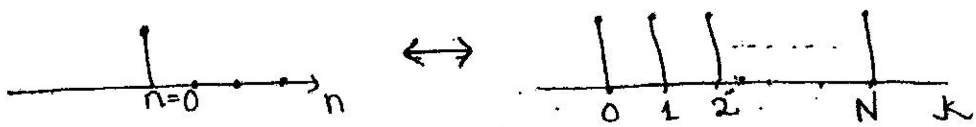
$$= \frac{j^N W_N^{Nk} - 1}{j W_N^k - 1} \ ; \quad 0 \le k < N$$

(d) $x[n] = 1, 0, \ldots 0 \quad 0 \leq n < N$

Sol^n: $X[k] = \sum_{n=0}^{N-1} x[n]\, e^{-j(2\pi/N)nk} \quad ; 0 \leq k < N$

$= \sum_{n=0}^{N-1} x[n]\, W_N^{nk} \quad ; 0 \leq k < N$

$= 1 \quad , 0 \leq k < N$



(B) The DTFT of the signals are given below

(a) $x[n] = (-1)^n$

Sol^n: $X(\Omega) = \sum_{n=-\infty}^{\infty} x[n]\, e^{-j\Omega n}$

$= \sum_{n=-\infty}^{\infty} (-1)^n e^{-j\Omega n}$

$= \ldots + (-1)^{-3} e^{j\Omega 3} + (-1)^{-2} e^{j\Omega 2} + (-1)^{-1} e^{j\Omega} + 1$
$+ (-1) e^{-j\Omega} + (-1)^2 e^{-j2\Omega} + (-1)^3 e^{-j3\Omega} + \ldots$

$= \ldots - e^{j3\Omega} + e^{j2\Omega} - e^{j\Omega}$
$+ 1 - e^{-j\Omega} + e^{-j2\Omega} - e^{-j3\Omega} + \ldots$

$= \frac{-e^{j\Omega}}{1-(-e^{j\Omega})} + \frac{1}{1-(-e^{-j\Omega})}$

$= \frac{-e^{j\Omega}}{1+e^{j\Omega}} + \frac{1}{1+e^{-j\Omega}}$

$= \frac{e^{-j\Omega}}{1+e^{j\Omega}} + \frac{e^{j\Omega}}{1+e^{j\Omega}} = \frac{e^{j\Omega}+e^{-j\Omega}}{1+e^{j\Omega}}$

6

Given,

$x[n] \leftrightarrow X(\Omega)$

(a) $\ldots 0, x[-2], 0, x[0], 0, x[2], \ldots$

Let $z(n) = \ldots, 0, x[-2], 0, x[0], 0, x[2], \ldots$

$= x[n] \cdot \{\ldots 0, 1, 0, \underset{\uparrow}{1}, 0, 1, \ldots\}$

$= x[n] \cdot v[n]$.

$= x[n] \cdot \left(\frac{1+(-1)^n}{2}\right)$

From the modulation property, we have

$$Z(\Omega) = \frac{X(\Omega) \circledast V(\Omega)}{2\pi}$$

Where, $V(\Omega) = \left(2\pi\delta(\Omega) + \frac{e^{j\Omega} + e^{-j\Omega}}{1+e^{j\Omega}}\right)\left(\frac{1}{2}\right)$

(from 4(B)(b))

(b) $z[n] = \ldots, 0, x[-1], 0, x[1], 0, \ldots,$

$= x[n] \times \{\ldots 0, 1, \underset{\uparrow}{0}, 1, 0, \ldots\}$

$= x[n] \times \left\{\frac{1-(-1)^n}{2}\right\}$

$= x[n] \cdot v[n]$

From the modulation property, we have

$$Z(\Omega) = \frac{X(\Omega) \circledast V(\Omega)}{2\pi}$$

(b) $x[n] = 1+(-1)^n$ ④

Sol^n: $X(\Omega) = \sum_{n=-\alpha}^{\alpha} x[n]\, e^{-j\Omega n}$

$$= \sum_{n=-\alpha}^{\alpha} \left[1+(-1)^n\right] e^{-j\Omega n}$$

$$\Rightarrow X(\Omega) = \sum_{n=-\alpha}^{\alpha} e^{-j\Omega n} + \sum_{n=-\alpha}^{\alpha} (-1)^n e^{-j\Omega n}$$

$$\Rightarrow X(\Omega) = 2\pi\delta(\Omega) + \frac{e^{j\Omega}+e^{-j\Omega}}{1+e^{j\Omega}} \quad , \quad |\Omega| \leq \pi$$

(c) $x[n] = j^n$

Sol^n: $x[n] = j^n = e^{jn\pi/2}$

$$X(\Omega) = \sum_{n=-\alpha}^{\alpha} x[n]\, e^{-j\Omega n}$$

$$= \ldots + (j)^{-3} e^{-j3\Omega} + (j)^{-2} e^{-j2\Omega} + (j)^{-1} e^{-j\Omega}$$

$$+ 1 + j e^{j\Omega} + j^2 e^{j2\Omega} + j^3 e^{j3\Omega} + \ldots$$

$$= \frac{j^{-1} e^{-j\Omega}}{1-j^{-1}e^{-j\Omega}} + \frac{1}{1-je^{j\Omega}}$$

$$= \frac{e^{-j\Omega}}{j-e^{-j\Omega}} + \frac{1}{1-je^{j\Omega}}$$

(d) $x[n] = \ldots, 0, 1, 0, \ldots = \delta[n]$

Sol^n: $X(\Omega) = \sum_{n=-\alpha}^{\alpha} x[n]\, e^{j\Omega n} = 1$

(c) ~~4B~~ ⑧

$z[n] = \ldots, -x[-3], x[-2], -x[-1], x[0], -x[1], x[2], \ldots$

$= x[n] \cdot \{\ldots, -1, 1, -1, \underset{\uparrow}{1}, -1, 1, \ldots\}$

$= x[n] \cdot (-1)^n$

~~$= x[n] \cdot (-1$~~

$= x[n] \cdot v(n)$

Now, using the modulation property, we have

$$Z(\Omega) = \frac{X(\Omega) \circledast V(\Omega)}{2\pi}$$

Where $V(\Omega) = \dfrac{e^{j\Omega} + e^{-j\Omega}}{1 + e^{j\Omega}}$

from 4(B)(a)

d) $z[n] = \{\ldots, x[-3], -x[-2], x[-1], -x[0], x[1], -x[2], \ldots\}$

$\Rightarrow z[n] = x[n] \cdot \{\ldots, 1, -1, 1, \underset{\uparrow}{-1}, 1, -1, \ldots\}$

$= x[n] \cdot \{-(-1)^n\}$

$= x[n] \cdot v[n]$

~~Where~~
Now, using modulation property, we have

$$Z(\Omega) = \frac{X(\Omega) \circledast V(\Omega)}{2\pi}$$

Where, $V(\Omega) = -\dfrac{e^{j\Omega} + e^{-j\Omega}}{1 + e^{j\Omega}}$

7

⑤

Given,

$x[n] \rightarrow$ N-length sequence

$x'[n] \rightarrow$ 2N-length sequence

$$x'[n] = \begin{cases} x[n] & 0 \leq n < N \\ 0 & N \leq n < 2N \end{cases}$$

$X[k] \Rightarrow$ N-length DFT of $x[n]$, $0 \leq k < N-1$

$X'[k]$; $0 \leq k < 2N-1 \rightarrow$ 2N-length DFT of $x'[n]$

$$\cancel{X[k] = \sum_{n=0}^{N-1} x[n] W_N^{kn} \quad 0 \leq k \leq N-1}$$

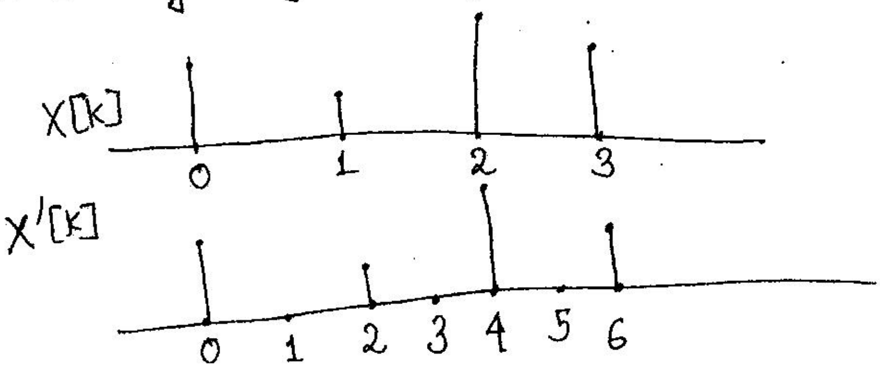
$$\cancel{= \sum_{n=0}^{N-1} x[n] W_{2N}^{2kn} + \sum_{n=N}^{2N-1} 0\, W_{2N}^{2kn}}$$

$$2N X'[k] = \sum_{n=0}^{2N-1} x'[n] W_{2N}^{kn}, \quad 0 \leq k \leq (2N-1)$$

$$= \sum_{n=0}^{N-1} x[n] W_{2N}^{kn}$$

$$= \sum_{n=0}^{N-1} x[n] W_N^{(k/2)n}, \quad 0 \leq k \leq (N-1)$$

$\Rightarrow$ So, $NX[k]$ can be obtained by decimating $2NX'[k]$ by a factor of 2.



⑥

When K is odd, Let n = 3

~~X [k=3] =~~

$$N X[3] = \sum_{n=0}^{N-1} x[n]\, e^{-j\frac{2\pi}{N}kn}$$

$$= \sum_{n=0}^{N-1} x[n]\, e^{-j\frac{2\pi}{N}3n}$$

$$2N X'[3] = \sum_{n=0}^{2N-1} x'[n]\, e^{-j\frac{2\pi}{2N}\cdot 3n}$$

$$= \sum_{n=\text{even}} x'[n]\, e^{-j\frac{2\pi}{2N}\cdot 3.2n'}$$

$$= \sum_{n=\text{even}} x'[n]\, e^{-j\frac{2\pi}{N}\cdot 3n'}$$

$$= N X[3].$$