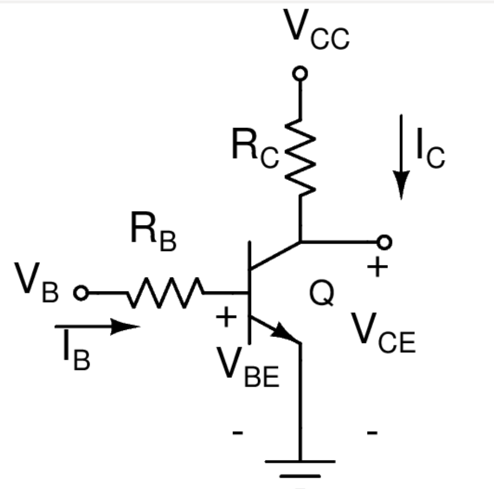
EE210: HW-8Z Solution

Date: 05/03/2019

Q1. Consider the BJT circuit show in the Fig. 1. Determine the values of R_C and R_B that would make the transistor Q operate at the bias point of I_C = 0.5 mA and V_{CE} = 3 V. Assume $V_{CC} = V_B$ = 5 V, and β = 100. Keeping the value of R_B unchanged, determine the new value of R_c that would make the transistor operate at the onset of saturation. Now, assume that this value of R_C is further doubled. What is the new mode of operation of Q, and what is its degree of saturation (DoS) under this condition?



Solution:

$$R_C = \frac{V_{CC} - V_{CE}}{I_C} = \frac{5-3}{0.5\ mA} = 4\ \text{k}\Omega$$

$$I_B = \frac{I_C}{\beta} = \frac{0.5\ mA}{100} = 5\ \mu A$$

$$R_B = \frac{V_B - V_{BE}}{I_B} = \frac{5-0.7}{5\ \mu A} = 860\ \text{k}\Omega$$

At onset of saturation, V_{CE} = 0.7 V

With I_B unchanged, I_C will remain unchanged, then

$$R_C = \frac{V_{CC} - V_{CE}}{I_C} = \frac{5-0.7}{0.5\ mA} = 8.6\ \text{k}\Omega$$

With R_C further doubled to 17.2 kΩ, the potential drop across it will drive of into hard saturation, with V_{CE} (sat) = 0.2 V (assumed).

$$I_{C,sat} = \frac{5-0.2}{17.2\ k\Omega} = 279\ \mu A$$

(Note the reduction in I_C).

V_{BE} will adjust itself to 0.8 V in hard saturation (Why? – Because of increase in base current as discussed in the class.).

Therefore,

$$I_{B,sat} = \frac{5-0.8}{860\ k\Omega} = 4.88\ \mu A$$

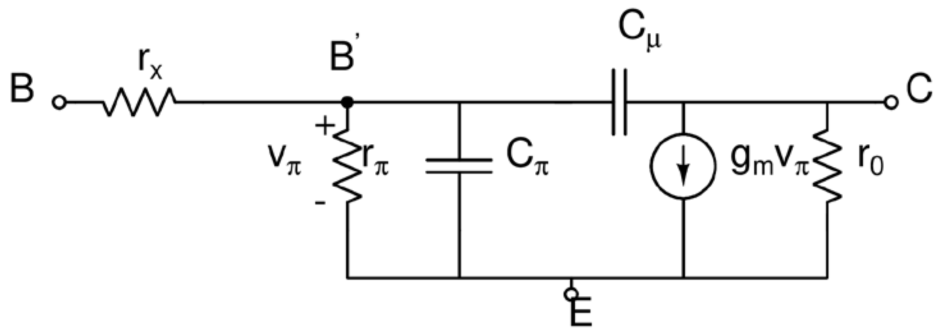
$$\beta_{sat} = \frac{I_{C,sat}}{I_{B,sat}} = \frac{279}{4.88} = 57$$

Degree of saturation (DoS),

$$DoS = \frac{\beta}{\beta_{sat}} = \frac{100}{57} = 1.75$$

Q2. The total emitter-base capacitance C_π for an npn transistor under forward active mode of operation is measured to be 6 pF and 8 pF at dc bias current I_C of 1 mA and 2mA respectively. Determine the zero-bias emitter-base junction capacitance C_{je0} (using the thumb rule given in class for forward biased junctions), and the base transit time τ_F, assuming that both of these are constants.

Solution:



$$C_\pi = C_{je} + g_m\tau_F$$

$\because (C_{je1} = C_{je2})$, we have

$$C_{\pi2} - C_{\pi1} = \tau_F(g_{m2} - g_{m1})$$

$$8pF - 6pF = \tau_F\left(\frac{2mA}{26mV} - \frac{1mA}{26mV}\right)$$

We get,

$$\tau_F = 52\ pSec$$

$$C_{je} = 6\ pf - 52\ pSec \times \frac{1}{26} = 4\ pf$$

$\because$ EB junction is forward biased, apply thumb rule (empircal),

$$C_{je} = 2C_{je0}$$

Which, gives C_{je0}=2 pf.

Q3. An integrated-circuit npn transistor has $\beta_0 = 100$, and $r_0 = 50\ k\Omega$ at $I_C = 1$ mA. With V_{CB} held constant at 10 V, $C_\mu = 0.15$ pF, and $f_T = 600$ MHz and 1 GHz for $I_C = 1$ mA and 10 mA respectively. Assume $V_{bi} = 0.55$ V for all junctions, and C_{je} is constant in the forward-bias region. Use $r_\mu = 5\ \beta_0 r_0$. Form the complete small-signal equivalent circuits for this transistor at $I_C = 0.1$ mA, 1 mA and 5 mA, all with V_{CB} held constant at 2 V.

Solution:

For both $I_C = 1$ mA & 10 mA, V_{CB} is held constant at 10 V. $\Longrightarrow$ C_μ remains constant,

$$f_T = \frac{g_m}{2\pi(C_\pi + C_\mu)} = \frac{1}{2\pi\tau}$$

Where, τ is an effective time constant = $\frac{C_\pi + C_\mu}{g_m}$.

For I_C = 1 mA,

$$\tau_1 = \frac{1}{2\pi f_{\tau 1}} = \frac{1}{2\pi * 600MHz} = 0.265\ nSec$$

For I_C = 10 mA,

$$\tau_2 = \frac{1}{2\pi f_{\tau 2}} = 0.159\ nSec.$$

Thus,

$$0.265\ nsec = \frac{C_\pi + C_\mu}{g_{m1}} = \tau_f + \frac{C_{je} + C_\mu}{g_{m1}}$$

∵ $\tau_{F,}$ Cje & C_μ are constants, with $g_{m1} = \frac{1}{26}$ ℧.

$$0.159\ nsec = \tau_f + \frac{C_{je} + C_\mu}{g_{m2}}$$

With $g_{m2} = \frac{1}{2.6}$℧, $C_\mu = 0.15\ pf$ is given. Solving, we get

$$C_{je} = 4.38\ pf$$

$$\tau_f = 147.22\ pSec$$

Assuming that the junction is abrupt, $C_\mu = \frac{C_{\mu 0}}{\left(1 - \frac{V_{BC}}{V_{bi}}\right)^{0.5}}$

With V_{BC} = -10 V, C_μ = 0.15 pf, & V_{bi} = 0.55 V (given), we get $C_{\mu 0}$= 0.657 pf

So, with V_{BC} = -2 V,

$C_\mu = \frac{0.657\text{ pf}}{\left(1 - \frac{2}{0.55}\right)^{0.5}} = 0.305pF$, r_0 = 50 KΩ at I_C = 1 mA then V_A= I_C r_0 = 50 V.

Now, we have all the required parameters to obtain the small-signal model.

I_C = 0.1 mA: g_m = 3.846 m℧, r_π = 26 KΩ, r_0 = 500 KΩ, r_μ = 250 MΩ,

$C_b = \tau_f \cdot g_m$= 0.566 pf, $C_\pi = C_{je}+C_b$=4.946 pf, C_μ = 0.305 pf.

I_C = 1 mA: g_m = 38.46 m℧, r_π = 2.6 KΩ, r_0 = 50 KΩ, r_μ = 25 MΩ,

$C_b = \tau_f \cdot g_m$= 5.66 pf, $C_\pi = C_{je} + C_b$ = 10.04 pf, C_μ = 0.305 pf.

I_C = 5 mA: g_m = 192.3 m℧, r_π = 520 Ω, r_0 = 10 KΩ, r_μ = 5 MΩ,

$C_b = \tau_f \cdot g_m$= 28.31 pf, $C_\pi = C_{je}+C_b$ = 32.69 pf, C_μ = 0.305 pf.

Check all these numbers for their correctness

The small signal model will be identical to that given in class.

Q4. An integrated-circuit npn transistor has the following parameters: τ_F = 0.25 nsec, small-signal short-circuit common-emitter current gain is 9 with I_C = 1 mA at frequency f = 50 MHz, V_A = 40 V, β_0 = 100, and C_μ = 0.6 pF at the bias voltage used. Determine all elements in the small-signal equivalent circuit at I_C = 2 mA, assuming that V_{CB} is held constant (as that for I_C = 1 mA), and τ_F remains constant.

Solution:

$$\beta = \frac{\beta_0}{1 + j\frac{f}{f_\beta}}$$

Now, with β_0 =100 & β of only 9 at f = 50 MHz, let's assume that f >> f_β.

$$|\beta| = \frac{\beta_0 \cdot f_\beta}{f} = \frac{f_T}{f} \Rightarrow f_T = f|\beta| = 50MHz * 9 = 450\ MHz$$

Now,

$$f_T = \frac{g_m}{2\pi(C_\pi + C_\mu)} \Rightarrow C_{je} + \tau_f g_m + C_\mu = \frac{g_m}{2\pi f_T}$$

It is given that I_C = 1mA. Thus

$$g_m = \frac{1}{26}\mho, \tau_f = 0.25\ nSec\ \&\ C_\mu = 0.6\ pf$$

$$C_{je} = \frac{1}{26 \times 2\pi \times 450 \times 10^6} - 0.25\ nS \times \frac{1}{26} - 0.6\ pf = 3.4\ pf$$

Small-signal model parameters for I_C = 2 mA:

$$g_m = \frac{I_C}{V_T} = \frac{2\ mA}{26\ mV} = 76.923\ m\mho$$

$$r_\pi = \frac{\beta_0}{g_m} = 1.3\ K\Omega$$

$$r_0 = \frac{V_A}{I_C} = 20\ K\Omega$$

$$C_{je} = 3.4\ pf$$

$$C_b = \tau_f g_m = 19.23\ pf$$

$$C_\pi = C_{je} + \tau_F g_m = 22.63\ pf$$

$$C_\mu = 0.6\ pf$$

Check all these numbers for their correctness

Q5. An npn transistor has the following specifications: $\beta_0 = 100$, $\tau_F = 26$ psec, $C_{je} = 5$ pF, and $C_\mu = 0.5$ pF at a particular bias point with $I_C = 2$ mA. Determine the three important characteristic frequencies f_T (unity-gain cutoff frequency), f_β (beta-cutoff frequency), and f_α (alpha-cutoff frequency) of the transistor at this bias point. Also, estimate f_{max} (absolute maximum operable frequency) of the transistor.

Solution:

$$C_\pi = C_{je} + \tau_F g_m = 5\,pf + 26\,ps \times \frac{2}{26} = 7\,pf$$

$$C_\mu = 0.5\,pf\ (given)$$

$$f_T = \frac{g_m}{2\pi(C_\pi + C_\mu)} = \frac{2}{26} \times \frac{1}{2\pi(7\,pf + 0.5\,pf)} = 1.63\,GHz$$

$$f_\beta = \frac{f_T}{\beta_0} = 16.32\,MHz$$

(Note: how small f_β is as compared to f_T, of course, the difference depends on β_0)

$$f_\alpha = (\beta_0 + 1)f_\beta = 1.65\,GHz$$

(Note: that f_α is so very slightly larger than f_T)

$$f_{max} = \frac{1}{2\pi\tau_f} = 6.12\,GHz$$

(way higher than f_β, and almost 4 times that of f_T and f_α)

The transistor cannot be operated for any frequency higher than f_{max}.