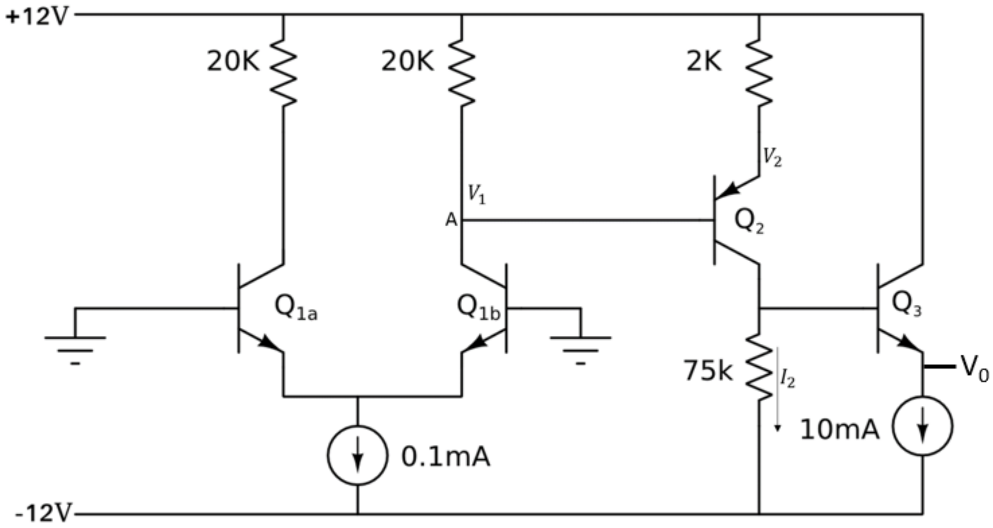
EE210: HW-13 Solution

Date: 11/04/2019

Q.1 For the BJT op-amp shown below, determine the bias voltage at the output and then determine the open circuit gain, input and output resistances. Determine the input voltage that is required to bring voltage at the output to zero. $\beta = 100$.



Sol:

(a) Bias voltage determination

$$I_{E3} = 10mA, \beta = 100,\ I_{B3} = \frac{I_{E3}}{\beta+1} = 0.099mA \cong 0.1mA$$

$$I_{C2} = I_{B3} + I_2 = 0.1m + \frac{V_0 + 0.7 + 12}{75K}$$

$$I_{C2}(mA) = 0.1 + \frac{V_0 + 0.7 - (-12)}{75} = 0.27 + 0.013V_0$$

$$I_{E2}(mA) = \frac{\beta + 1}{\beta} I_{C2} = \frac{101}{100}(0.27 + 0.013V_0) \approx 0.27 + 0.013V_0$$

$$I_{B2}(mA) = \frac{I_{C2}}{\beta} = \frac{1}{100}(0.27 + 0.013V_0)$$

Voltage V_2 at the emitter of transistor Q_2,

$$12 - V_2 = I_{E2}(mA) * 2K \rightarrow V_2 = 12 - 2(0.27 + 0.013V_0) = 11.46 - 0.026V_0$$

Voltage V_1 at the collector of transistor Q_1,

$$V_1 = V_2 - 0.7$$

$$V_1 = 11.46 - 0.026V_0 - 0.7 = 10.76 - 0.026V_0$$

$$I_{E1b}(mA) = \frac{0.1}{2} = 0.05 \approx I_{C1b}(mA)$$

Now, KCL at node A

$$I_{C1b} = I_{B2} + \frac{12 - V_1}{20K}$$

$$0.05\text{mA} = \frac{1}{100}(0.27 + 0.013V_0)mA + \frac{12 - 10.76 + 0.026V_0}{20K}$$

$$5 = 0.27 + 0.013V_0 + 5(1.24 + 0.026V_0)$$

$$-1.47 = 0.143V_0$$

Solving for V_0

$$V_0 \approx -10.3V$$

(b) Finding open circuit voltage gain

$$(\, g_{m1a,b} \cong \frac{2mA}{V},\; g_{m2} \cong \frac{5mA}{V}, r_{\pi 1b} \cong 52K, r_{\pi 2} \cong 19K\Omega)$$

$$A_{V1} \cong -0.5 * g_{m1a} * \left(20K \,\|(r_{\pi 2} + \beta * 2K)\right) = -18$$

$$A_{V2} \cong \frac{g_{m2} * (75K)}{1 + g_{m2} * (2K)} \cong -34$$

Assuming no loading between the stages we can write overall gain as

$$A_V = A_{V1} * A_{V2} \approx 612$$

(c) Finding input resistance

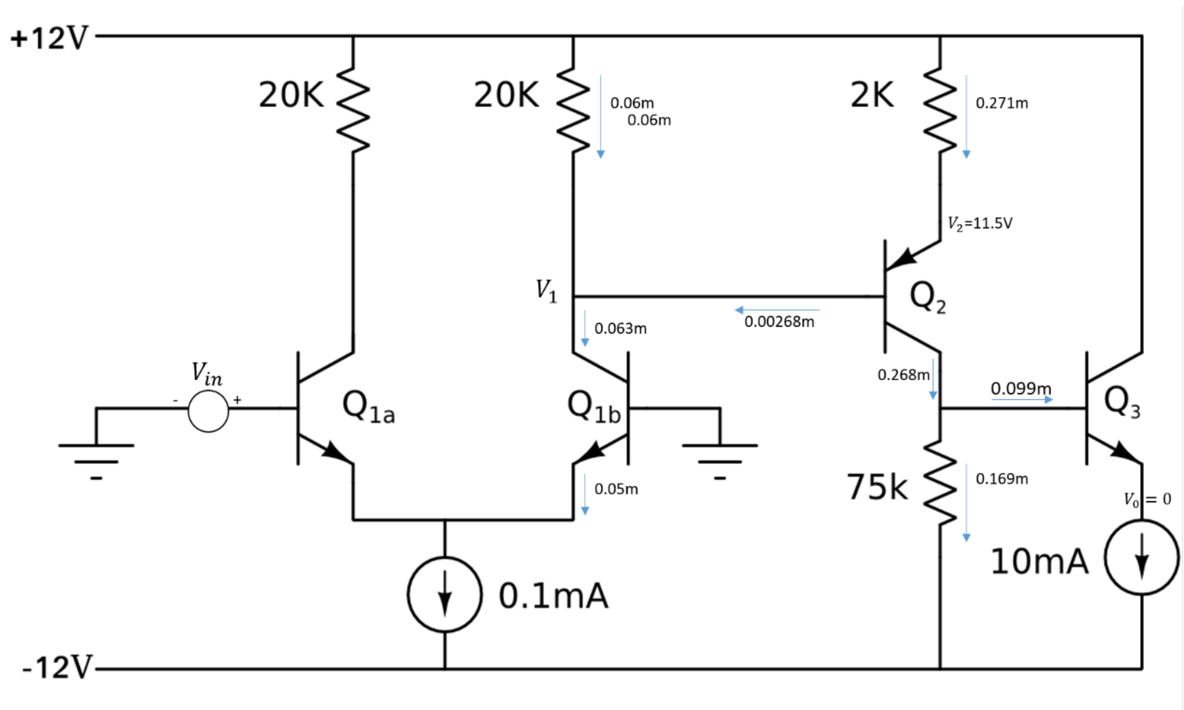
$$R_{in} = 2 * r_{\pi 1b} = 104\text{K}\Omega$$

(d) Finding output resistance

$$R_{out} \approx \frac{75K + r_{\pi 3}}{\beta}$$

$$R_{out} \approx 752\Omega$$

(e) Finding V_{in} such that $V_0 = 0$.



From the figure shown above for transistor Q_{1b},

$$I_s = 0.244 * 10^{-15}A;\ V_T = 0.026V;\ I_C = 0.063mA$$

$$I_C = I_S * \exp(V_{BE}/V_T)$$

$$V_E = -I_S \ln\left(\frac{I_C}{I_S}\right)$$

$$V_S = -0.763V$$

In transistor Q_{1a},

$$V_{in} - V_E = 0.7$$

$$V_{in} = 0.7 + V_E = 0.7 - V_T \ln\left(\frac{I_C}{I_S}\right) = 0.7 - 0.026 * \ln\left(\frac{0.063mA}{0.244 * 10^{-15}A}\right)$$

$$V_{in} \approx\ 16.8mV$$

Alternate solution: Use small-signal analysis.

Currently $V_0 \approx -10.3V$. To reduce it to zero, apply V_{in} to inverting terminal of differential stage:

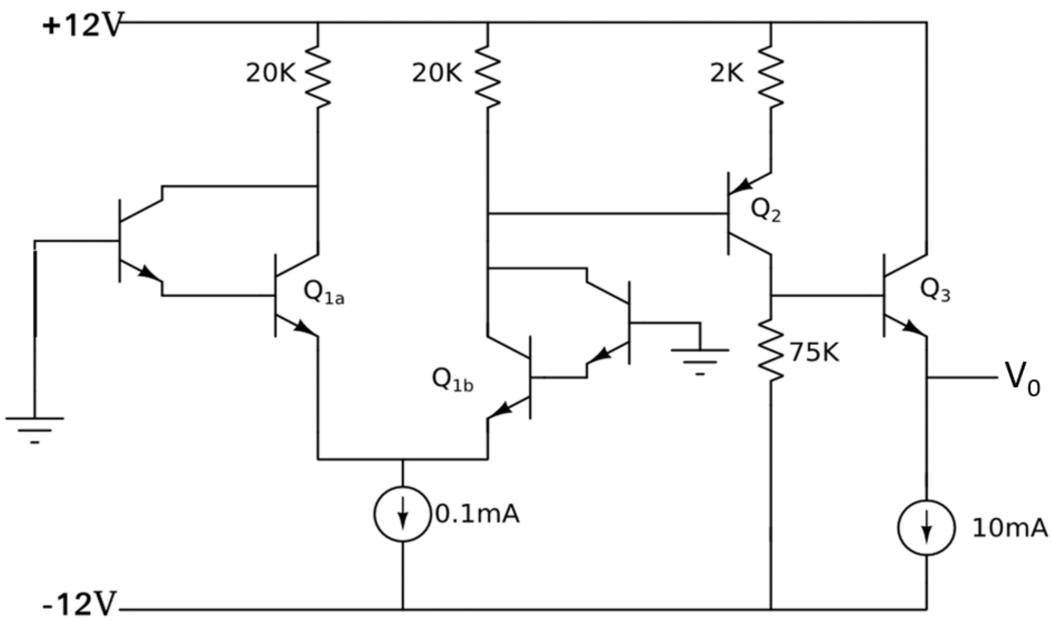
$$V_0 = A_V V_{in}$$

$$10.3=612*V_{in}$$

$$V_{in} = 16.8\text{mV}$$

As $V_{in} < V_T$, small-signal analysis gives good result.

Q.2 For the BJT op-amp shown below, determine the bias voltage at the output and then determine the open circuit gain, input and output resistances.



Sol:

(a) Finding bias voltage

Approximately same as the previous one because current source is a very low value and β is quite high.

(b) Finding open loop gain

$$A_{V1} \cong -0.25 * g_{m1b} * \left(20K \,\|(r_{\pi 1b} + \beta * 2K)\right) \cong -9$$

$$A_{V2} \cong \frac{g_{m2} * (75K)}{1 + g_{m2} * (2K)} \cong -34$$

Assuming no loading between the stages we can write overall gain as,

$$A_V = A_{V1} * A_{V2} \cong 306$$

(c) Finding input resistance

$$R_{in} = 4 * (\beta + 1) r_{\pi 1a} = 20\text{M}\Omega$$

(d) Finding output resistance

$$R_{out} \cong \frac{75K + r_{\pi 3}}{\beta} \cong 752\Omega$$