EE 200 Signals, Systems & Networks 2018

Assignment 8

1 Given two complex exponentials of radian frequency ω_1, ω_2, find the product of the two signals. Is it also a complex exponential? What is its period? How does its period relate to the periods of the constituents? Now consider two periodic signals $x_1(t), x_2(t)$ with respective periods T_1, T_2. that posess an FS expansion. Generalize your earlier result to comment on the period of $x_1(t)x_2(t)$ in terms of T_1 and T_2. If $T_1 = T_2$, find an expression for the FS coefficients of the product in terms of the respective FS coefficients x_{1k}, x_{2k}.

2 We call $p(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT)$ an *impulse train*. Given a continuous signal $x(t)$, consider the system H that produces output $y(t) = x(t)p(t)$ which is a so-called *modulated* impulse train. Answer the following questions.

(a) If $x(t)$ is periodic with period T_0, will $y(t)$ be periodic in general? If not, are there special conditions under which $y(t)$ can be periodic?

(b) Is the system H a time-invariant system, *ie,* do shifts in the input appear as equal shifts in the output? If this is not generally true, are there special conditions under which certain shifts int the input will indeed appear as equal shifts in the output?

(c) Also examine whether H is linear, causal, stable and whether it posesses memory.

3 Consider some given time period T. How many different complex exponentials $e^{j\omega t}$ exist that are harmonic with T? Consider next the corresponding discrete situation. How many different discrete complex exponentials $e^{j2\pi nk/N}$ can be found that have a given period of N?

4 Let $x(t)$ be of period T with a Fourier Series x_k; $-\infty < k < \infty$. Find the Fourier Series of the following signals.

(a) $y(t) = x(\alpha t); 0 < \alpha < 1$. Repeat for $\alpha > 1$.

(b) $\sum_{k=-\infty}^{\infty} w(t - k\alpha T)$ where $w(t) = x(t)(u(t + T/2) - u(t - T/2))$ and $\alpha > 1$. Repeat for $0 < \alpha < 1$

5 Suppose f is some function. Let $f(t) \leftrightarrow F(\omega)$. Find the time function $f'(t)$ that is the inverse FT of $f(\omega)$. Similarly, find the FT of $F'(t)$ of $F(\omega)$.

6 (a) Find the Fourier Transform of the unit step $u(t)$. (**Hint:** Carry out the even-odd decomposition $u(t) = u_e(t) + u_o(t)$ and find the FTs of each term separately. Finally use the linearity of the FT.)

(b) Often, we wish to study the *step response* $s(t)$ of a system, which is the output of the system when $u(t)$ is the input. Using (a) above, devise a means of finding the step response of a system whose impulse response is $h(t)$. How does the step response throw light on the stability of the system?

(c) Using (a) above, find the FT of $\int_{-\infty}^{t} x(t')dt'$ if $x(t) \leftrightarrow X(\omega)$

7 (a) Use the previous problem to obtain the FT of the signum function, $\mathrm{sgn}(t) = \begin{cases} -1 & t < 0 \\ 0 & t = 0 \\ 1 & t > 0 \end{cases}$.

(b) From the above, find the FT of $x(t) \star (\pi t)^{-1}$ and of $x(t) \star (\pi t)^{-1} \star (\pi t)^{-1}$.

8 A certain linear system has the property that if $x(t) \rightarrow y(t)$, then $x(-t) \rightarrow y(-t)$. Show that the following is valid for any input-output pair:

$$x(t) = x_e(t) + x_o(t) \rightarrow y(t) = y_e(t) + y_o(t) \quad iff \quad x(t) = \alpha x_e(t) + \beta x_o(t) \rightarrow y(t) = \alpha y_e(t) + \beta y_o(t).$$

Does the expression above remind you of the definition of linearity? Figure out why. Then figure out how the present statement is actually different.