EE 200 2018 Signals, Systems & Networks

Assignment 7

1 Prove that convolution has the following properties.

(a) associativity

(b) commutativity

(c) distribution over +

(d) $y(t) = x(t) * h(t) \Rightarrow \dot{x}(t) * h(t) = x(t) * \dot{h}(t) = \dot{y}(t)$ ($\dot{x}(t), \dot{h}(t)$ cont., $\dot{f}(t)) = df(t)/dt$)

(e) $y(t) = x(t) * h(t) \Rightarrow \dot{x}(t) * \dot{h}(t) = \ddot{x}(t) * h(t) = x(t) * \ddot{h}(t) = \ddot{y}(t)$ ($\ddot{x}(t), \ddot{h}(t)$ cont., $\ddot{f}(t)) = d^2 f(t)/dt^2$)

(f) Let $A(f) = \int_{-\infty}^{+\infty} f(t)dt$. Show that $y(t) = x(t) * h(t) \Rightarrow A(y) = A(x)A(h)$. State and prove the discrete version of this result.

2 A different interpretation of the convolution operation from the one discussed in the class is that $y(t) = x(t) * h(t)$ is the inner product of the *convolver* $x(\tau)$ with the *convolvend* $h(\tau)$ after the latter is time reversed and centered about t, *viz*, $h(t-\tau)$, where the inner product of any 2 functions $f(t), g(t)$ is given by $\int_{t=-\infty}^{t=+\infty} f(t)g(t)dt$. Write out the corresponding interpretation for the case of discrete convolution. Verify for the example solved in the lecture that this yields the same result.

3 We define the *support* of any signal $x(t)$ or $x[n]$ as the smallest interval outside which the signal is zero. Thus, the support of $\sin t$ is $(-\infty, \infty)$ and the support of $\delta[n+1] - 2\delta[n-3]$ is $[-1, 3]$. Let us denote the support of $x(t)$ by (t_{xL}, t_{xR}) or $(t_{xL}, t_{xR}]$ or $[t_{xL}, t_{xR})$ or $[t_{xL}, t_{xR}]$ as applicable, and that of $x[n]$ by $[n_{xL}, n_{xR}]$. The *support time* of $x(t)$ is denoted as $t_{xR} - t_{xL}$ and of $x[n]$ by $n_{xR} - n_{xL}$. Show that the convolution of two signals $x(t), x'(t)$ with finite support intervals T, T' has a finite support interval of $T + T'$. Similarly, prove the corresponding result for discrete signals: the convolution of two discrete signals having finite supports N, N' is $N + N' - 1$.

4 Develop relations between the boundary points $t_{xL}, t_{xR}, t_{hL}, t_{hR}$ and t_{yL}, t_{yR}. Similarly, develop the corresponding relations between $n_{xL}, n_{xR}, n_{hL}, n_{hR}$ and n_{yL}, n_{yR} for discrete convolution.

5 Under the new interpretation of convolution, the value of $y(t)$ is equal to the area under $x(\tau)h(t-\tau)$. Use this to construct two examples of $y(t) = x(t) * h(t)$ which is finite but not bounded, though $x(t), h(t)$ are both bounded. Find constraints on $x(t), h(t)$ that will ensure that $y(t)$ remains finite for all t. Find a sufficient constraint to be applied upon $x(t), h(t)$ to ensure that $y(t)$ remains bounded, and not just finite. Compare these constraints with those obtained above to keep $y(t)$ finite.

6 After the above problems, can you comment on $y(t)$ when $x(t)$ is periodic and $h(t)$ is finitely supported and both are bounded? What will happen when both $x(t), h(t)$ are non negative, periodic and bounded?

7 Following the consequences of the above, we seek a way out for the specific case of convolving periodic signals. The *periodic convolution* of two signals $x(t), y(t)$ of period T is defined as $x(t) \circledast y(t) = \int_{t=-T/2}^{t=T/2} x(\tau)y(t-\tau)d\tau$. The convolution is now bounded because the integration limits have been restricted to exactly one period. What if the indicated integration interval $(-T/2, T/2]$ is replaced by any other contiguous T-length time interval of the form $(\Delta, T+\Delta]$? Use this definition to convolve a T-periodic signal $x(t)$ with a constant $y(t) = y_0$, using T as the convolution interval. Next, use any finite convolution interval T to convolve two constant signals $x(t) = x_0$ and $y(t) = y_0$. Express your result in terms of T in both cases.

8 Let $x_i(t); i = 1, 2$ be periodic signals of the same period T, and let each cycle of $x_i(t); i = 1, 2$ be nonzero only over $t \leq T/2$ and zero over the remaining part of width $T/2 < t \leq T$ of the cycle. Define $x_i'(t) = \begin{cases} x_i(t); & t \leq T \\ 0; & t < 0, t > T \end{cases}$; $i = 1, 2$. Show that $x_1(t) \circledast x_2(t) = x_1'(t) * x_2'(t); t \leq T$.