EE 200 2018 Signals, Systems & Networks — Assignment 9

1 The energy of a continuous-time signal $x(t)$ is defined as $\int_{-\infty}^{\infty}|x(t)|^2dt$.

(a) Show that in general, $\int_{-\infty}^{\infty}x(t)y^*(t)dt = \frac{1}{2\pi}\int_{-\infty}^{\infty}X(\omega)Y^*(\omega)d\omega$.

(b) From this, show that the energy of $X(\omega)$ is 2π times the energy of $x(t)$: $\int_{-\infty}^{\infty}|x(t)|^2dt = \frac{1}{2\pi}\int_{-\infty}^{\infty}|X(\omega)|^2d\omega$.

(c) Similarly prove the following. For the continuous time Fourier Series (CTFS): $\frac{1}{T}\int_T|x(t)|^2 = \sum_k|x_k|^2$. For the discrete Fourier Transform (DFT/DTFS): $\frac{1}{N}\sum_{n=0}^{N-1}|x[n]|^2 = \sum_{k=0}^{N-1}|X[k]|^2$. And, finally, for the discrete time Fourier Transform, (DTFT): $\sum_n|x[n]|^2 = \frac{1}{2\pi}\int_{2\pi}|X\Omega|^2$.

2 The following are the modulation property of the CTFT and the DTFT repectively. Prove them.

$$x(t)y(t) \leftrightarrow \frac{1}{2\pi}\int_{-\infty}^{\infty}X(\theta)\star Y(\omega-\theta)d\theta; \qquad x[n]y[n] \leftrightarrow \frac{1}{2\pi}\oint_{2\pi}X(\Theta)Y(\Omega-\Theta)d\Theta$$

3 A system with zero initial conditions satisfies the difference equation $y[n] = x[n] - \alpha y[n-1]$. If $x[n] \leftrightarrow X(\Omega)$ is applied to it as input, find $Y(\Omega)$ in terms of $X(\Omega)$. Find also the sufficient conditions under which $Y(\Omega)$ will exist. -

4 Consider the DFT $X[k]$ of an N-length signal - $x[n]$. Now treat $X[n]$ as a time sequence' $y[n]$ and again obtain its DFT $Y[k]$. Compare $x[n]$ and $Y[n]$. -

5 (A) Find the N-point DFT of the following N-length - sequences: (a) $x[n] = (-1)^n;\ 0 \le n < N$, ($b$) $-\ x[n] = 1+(-1)^n;\ 0 \le n < N$, ($c$) $x[n] = j^n;\ 0 \le n < N$, (d) $x[n] = 1, 0, \ldots 0$.

(B) Find the DTFT of the following: (a) $x[n] = (-1)^n; -$ (b) $x[n] = 1+(-1)^n$; (c) $x[n] = j^n$; (d) $x[n] = \ldots, 0, 1, 0, \ldots = \delta[n]$.

6 If $x[n] \leftrightarrow X(\Omega)$, find the DTFT of the following signals in terms of $X(\Omega)$.

(a)	$\ldots, 0, x[-2], 0, x[0], 0, x[2], \ldots,$	odd members replaced by zero.
(b)	$\ldots, 0, x[-1], 0, x[1], 0 \ldots,$	even members replaced by zero.
(c)	$\ldots, -x[-3], x[-2], -x[-1], x[0], -x[1], x[2] \ldots,$	odd members inverted.
(d)	$\ldots, x[-3], -x[-2], x[-1], -x[0], x[1], -x[2] \ldots,$	even members inverted.

7 Consider the N-length sequence $x[n]$ and its $2N$-length extension $x'[n]$ obtained by *padding* $x[n]$ with N consecutive zeros at the end. Let their respective DFTs of lengths N and $2N$ be $X[k];\ 0 \le k < N-1$ and $X'[k];\ 0 \le k < 2N-1$. Show that every member of $X[k]$ is to be found at a specific place in $X'[k]$ and find its exact location. As to the remaining N 'new' members in $X'[k]$, show that their computation can be - economized in certain ways, instead of merely applying - direct approach through the formula.