Assignment 6

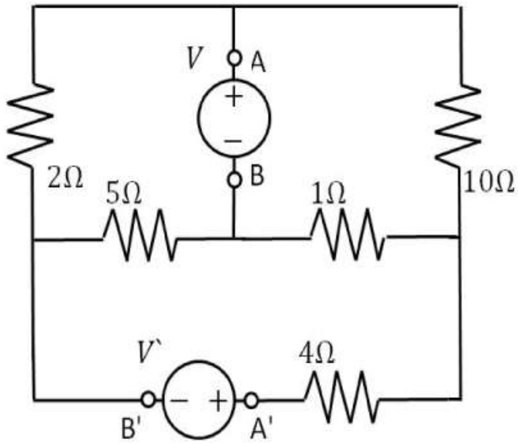

Fig.1

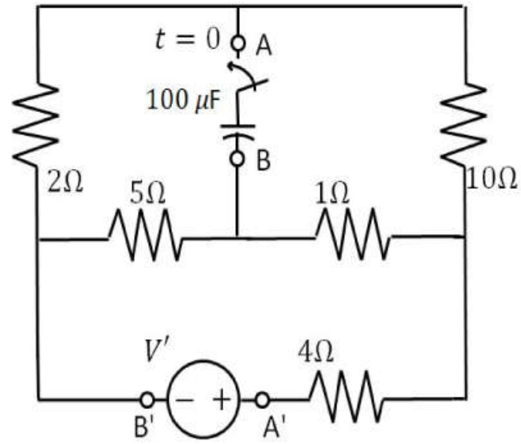

Fig.2

1 In Fig.1, with respect to the terminals AB, reduce the rest of the network (other than the source V) to its Thevenin and Norton equivalents. Repeat the exercise for the terminals A′B′. Choose $V = 5\,\text{V}$, $V' = 12\,\text{V}$.

2 In Fig.1, determine the powers in the two sources. Find out to how much you need to change V, keeping $V' = 5\,\text{V}$ for V' to develop positive power.

3 In Fig.1, find the total current in the $10\,\Omega$ resistor using superposition.

The first 3 problems are straightforward. In Problem 1, you have to carefully apply series parallel reduction for the 'rest of the circuit' in both the cases. Using the results of Problem 1 makes Problem 2 very easy. For Problem 3, however, it is better to reduce the rest of the circuit *with respect to* the $10\,\Omega$ resistor to get the reduced Thevenin network. After this one time exercise, kill V and V' one by one, and find respective E_{Th} and E'_{Th} separately. Find the current in each case through the $10\,\Omega$ 'load', and add them.

4 In Fig.2, find the total current for $t \geq 0$ in the $10\,\Omega$ resistor using generalized superposition, which accounts for both zero state- and zero input- responses, if the capacitor has an initial voltage of $10\,\text{V}$.

Using generalized superposition means allowing the superposition of zero state and zero input responses. Here, you just set V' to zero and evaluate the ZIR current in the $10\,\Omega$ resistor for $t \geq 0$. Nest set the initial capacitor voltage to zero, and find the ZSR current in the $10\,\Omega$ resistor with only V' present. Add the two.

5 Use the substitution theorem to replace just the capacitor alone in Fig.2 with a source $v_C(t)$, chosen so that currents and voltages in the rest of the network outside AB are unaffected for $t > 0$ by the replacement.

Find $v_C(t); t \geq 0$ by using the first of the Thevenin reductions found in Problem 1 (reduction of the 'rest of the circuit' with respect to AB) and putting this reduction across the capacitor. Use this $v_C(t); t \geq 0$ to define the substituting voltage source for the capacitor in Fig.2. The switch stays put both before and after substitution.

6 Suppose now, we remove the capacitor in Fig.2 entirely (replace it with a short), leaving behind only the switch that is open until $t = 0$ and closed at $t = 0$. Can we use the substitution theorem to replace just the switch with a source $i_S(t)$, chosen so that currents and voltages in the rest of the network outside AB are unaffected for $-\infty < t < \infty$ by the replacement? Is such a use of the substitution theorem allowed? Justify.

Such a use of the substitution theorem is certainly allowed. For $t < 0$ $i_S(t) = 0$ and for $t \geq 0$, $i_S(t)$ becomes the Norton equivalent source current obtained in Problem 1 (for reduction of the 'rest of the circuit' with respect to AB).

7 Find the zero state amd zero input components of the capacitor current $i_C(t)$ for $t > 0$ in Fig.2.

Use the Thevenin equivalent of the network outside of AB (already solved in Problem 1) in Fig.2 along with the 10 V pre-charged capacitor and switch connected across AB. Now note that the current in the capacitor is the sum of the Norton current due to V', (which would be the ZSR) and its own discharge (the ZIR) into the Thevenin equivalent circuit. This approach, it must be clear, is not an application of superposition, as it would work even of the capacitor had had a nonlinear $v - q$ characteristic. However, if the capacitor just *happens* to be linear, as in the present case, you can use superposition equally well.

8 FInd the Thevenin equivalent of just the charged capacitor using the theorem in its full generality. Consider the Thevenin 'load' to be the entire network outside AB. Find the capacitor current $i_C(t)$ in two different cases: (i) with the rest of the network outside AB retained fully as in Fig.1 (or Fig.2); (ii) with the source V' set to zero.

Here we are 'reducing' just the capacitor to its Thevenin equivalent, which is a simple matter of putting an $e_{Th}(t) = v_{C(OC)}(t)$, the open circuit (constant) capacitor voltage which will be equal to $v_C(0) = 10\,\text{V}$ in series with an uncharged capacitor. This is permitted as long as the capacitor is linear. The rest of the circuit is either entirely a passive resistence of value calculated in Problem 1 (for reduction of the 'rest of the circuit' with respect to AB) or that same resistor along with the Thevenin equivalent source at AB due to V' (also calculated already in Problem 1). However, this problem has an interesting twist: what if you were asked to find a *Norton* equivalent for the capacitor? Think about it!