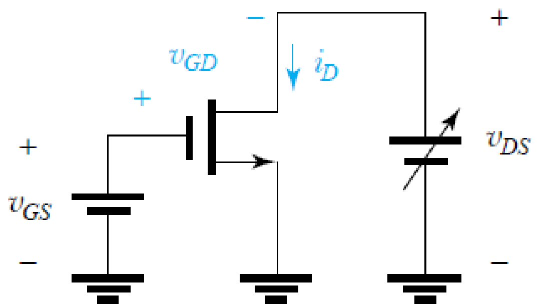
EE210: HW-14 Solution

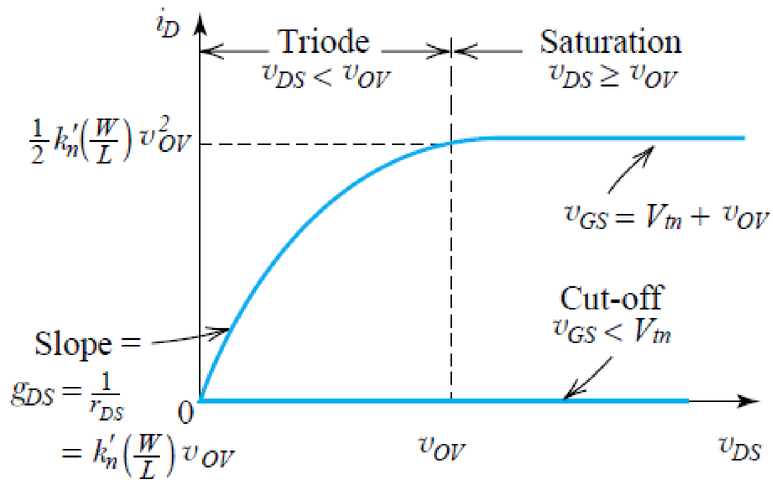
Date: 11/04/2019

Q.1 What are different regions of operation of the Enhancement mode NMOS Transistor?

Sol.:

Table 5.1 Regions of Operation of the Enhancement NMOS Transistor





- $v_{GS} < V_{tn}$: no channel; transistor in cut-off; $i_D = 0$
- $v_{GS} = V_{tn} + v_{OV}$: a channel is induced; transistor operates in the triode region or the saturation region depending on whether the channel is continuous or pinched-off at the drain end;

Triode Region

Continuous channel, obtained by:

$$v_{GD} > V_{tn}$$

or equivalently:

$$v_{DS} < v_{OV}$$

Then,

$$i_D = k_n' \left(\frac{W}{L}\right)\left[(v_{GS} - V_{tn})v_{DS} - \frac{1}{2}v_{DS}^2\right]$$

or equivalently,

$$i_D = k_n' \left(\frac{W}{L}\right)\left(v_{OV} - \frac{1}{2}v_{DS}\right)v_{DS}$$

Saturation Region

Pinched-off channel, obtained by:

$$v_{GD} \leqslant V_{tn}$$

or equivalently:

$$v_{DS} \geqslant v_{OV}$$

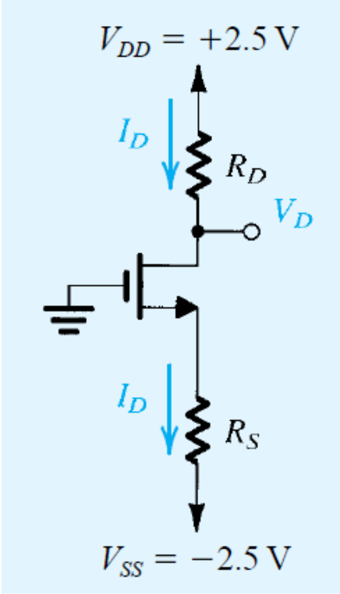
Then

$$i_D = \frac{1}{2}k_n' \left(\frac{W}{L}\right)(v_{GS} - V_{tn})^2$$

or equivalently,

$$i_D = \frac{1}{2}k_n' \left(\frac{W}{L}\right)v_{OV}^2$$

Q.2 Design the circuit of Fig. shown below, that is, determine the values of R_D and R_S, so that the transistor operates at $I_D = 0.4$ mA and $V_D = +0.5$ V. The NMOS transistor has $Vt = 0.7$ V, $\mu_n C_{ox} = 100\ \mu A/V^2$, $L = 1\ \mu m$, and $W = 32\ \mu m$. Neglect the channel-length modulation effect (i.e., assume that $\lambda = 0$).



Sol.:

To establish a dc voltage of +0.5 V at the drain, we must select R_D as follows:

$$R_D = \frac{V_{DD} - V_D}{I_D}$$

$$= \frac{2.5 - 0.5}{0.4} = 5\ k\Omega$$

To determine the value required for R_S, we need to know the voltage at the source, which can be easily found if we know V_{GS}. This in turn can be determined from V_{OV}. Toward that end, we note that since V_D = 0.5 V is greater than V_G, the NMOS transistor is operating in the saturation region, and we can use the saturation-region expression of i_D to determine the required value of V_{OV},

$$I_D = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}V_{OV}^2$$

Then substituting $I_D = 0.4$ mA $= 400\ \mu$A, $\mu_n C_{ox} = 100\ \mu A/V^2$, and $W/L = 32/1$ gives

$$400 = \frac{1}{2} \times 100 \times \frac{32}{1} V_{OV}^2$$

which results in

$$V_{OV} = 0.5\ V$$

Thus,

$$V_{GS} = V_t + V_{OV} = 0.7 + 0.5 = 1.2\ V$$

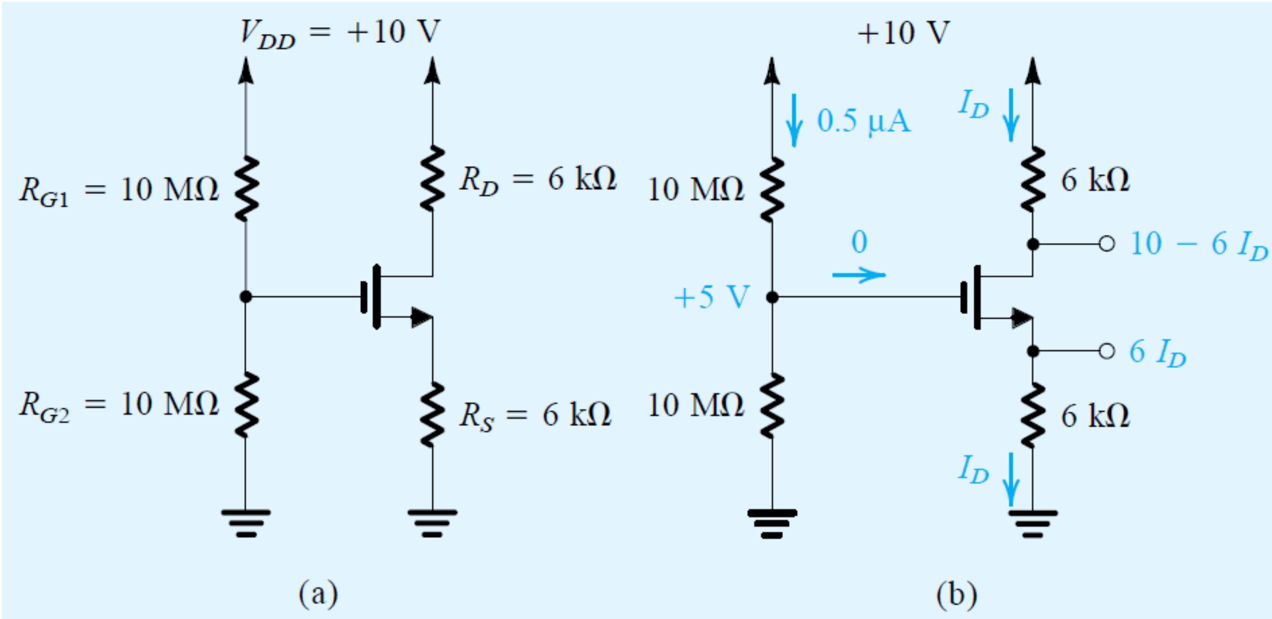
Referring to Fig. 5.21, we note that the gate is at ground potential. Thus, the source must be at –1.2 V, and the required value of R_S can be determined from

$$R_S = \frac{V_S - V_{SS}}{I_D}$$

$$= \frac{-1.2 - (-2.5)}{0.4} = 3.25\ k\Omega$$

Q.3 Analyze the circuit shown in Fig. below to determine the voltages at all nodes and the currents through all branches. Let $V_{tn} = 1$ V and $k_n'(W/L) = 1$ mA/V^2 and Neglect the channel-length modulation effect.

Sol.:



Since the gate current is zero, the voltage at the gate is simply determined by the voltage divider formed by the two 10-MΩ resistors,

$$V_G = V_{DD}\frac{R_{G2}}{R_{G2}+R_{G1}} = 10 \times \frac{10}{10+10} = +5 \text{ V}$$

With this positive voltage at the gate, the NMOS transistor will be turned on. We do not know, however, whether the transistor will be operating in the saturation region or in the triode region. We shall assume saturation-region operation, solve the problem, and then check the validity of our assumption. Obviously, if our assumption turns out not to be valid, we will have to solve the problem again for triode-region operation.

Refer to Fig. 5.24(b). Since the voltage at the gate is 5 V and the voltage at the source is $I_D\,(\text{mA}) \times 6\,(\text{k}\Omega) = 6I_D$, we have

$$V_{GS} = 5 - 6I_D$$

Thus, I_D is given by

$$I_D = \frac{1}{2}k_n'\frac{W}{L}(V_{GS} - V_{tn})^2$$

$$= \frac{1}{2} \times 1 \times (5 - 6I_D - 1)^2$$

which results in the following quadratic equation in I_D:

$$18I_D^2 - 25I_D + 8 = 0$$

Q.4 The NMOS and PMOS transistors in the circuit shown below are matched, with $k_n'(W_n/L_n) = k_p'(W_p/L_p) = 1\ mA/V^2$ and, $V_{tn} = -V_{tp} = 1$ V. and Assuming $\lambda = 0$ for both devices, find the drain currents i_{DN} and i_{DP}, as well as the voltage v_O, for $v_I = 0$ V, +2.5 V, –2.5 V.

Sol.:

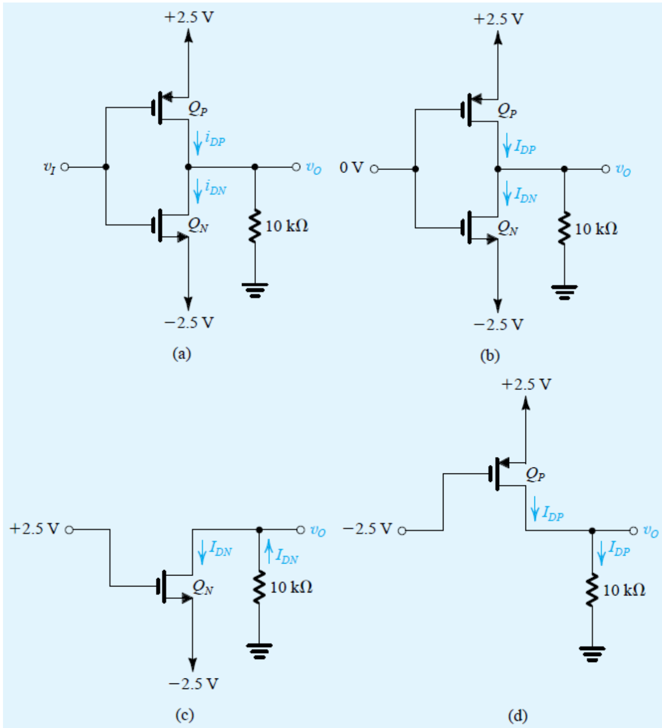


Figure 5.26(b) shows the circuit for the case $v_I = 0$ V. We note that since Q_N and Q_P are perfectly matched and are operating at equal values of $|V_{GS}|$ (2.5 V), the circuit is symmetrical, which dictates that $v_O = 0$ V. Thus both Q_N and Q_P are operating with $|V_{DG}| = 0$ and, hence, in saturation. The drain currents can now be found from

$$I_{DP} = I_{DN} = \tfrac{1}{2} \times 1 \times (2.5 - 1)^2 = 1.125 \text{ mA}$$

Next, we consider the circuit with $v_I = +2.5$ V. Transistor Q_P will have a V_{SG} of zero and thus will be cut off, reducing the circuit to that shown in Fig. 5.26(c). We note that v_O will be negative, and thus v_{GD} will be greater than V_{tn}, causing Q_N to operate in the triode region. For simplicity we shall assume that v_{DS} is small and thus use

$$\begin{aligned} I_{DN} &\simeq k_n'(W_n/L_n)(V_{GS} - V_{tn})V_{DS} \\ &= 1[2.5 - (-2.5) - 1][v_O - (-2.5)] \end{aligned}$$

From the circuit diagram shown in Fig. 5.26(c), we can also write

$$I_{DN}(\text{mA}) = \frac{0 - v_O}{10\ (\text{k}\Omega)}$$

These two equations can be solved simultaneously to yield

$$I_{DN} = 0.244 \text{ mA} \qquad v_O = -2.44 \text{ V}$$

Note that $V_{DS} = -2.44 - (-2.5) = 0.06$ V, which is small as assumed.

Finally, the situation for the case $v_I = -2.5$ V [Fig. 5.26(d)] will be the exact complement of the case $v_I = +2.5$ V: Transistor Q_N will be off. Thus $I_{DN} = 0$, Q_P will be operating in the triode region with $I_{DP} = 2.44$ mA and $v_O = +2.44$ V.

Q.5 For the Common Drain circuit shown in Fig. below, assume that the MOSFET M has W/L = 10 and λ = 0. Other parameters are: $k_n` = 40\ \mu A/V^2$, $\gamma = 0.4\ V^{1/2}$, $2\phi_F = 0.6V$, and $V_{TN0} = 0.7V$. Find the dc output voltage V_0, and the ac small-signal midband voltage gain v_0/v_i under the following conditions:

(i) Ignoring body effect and with R → ∞.

(ii) Including body effect and with R → ∞.

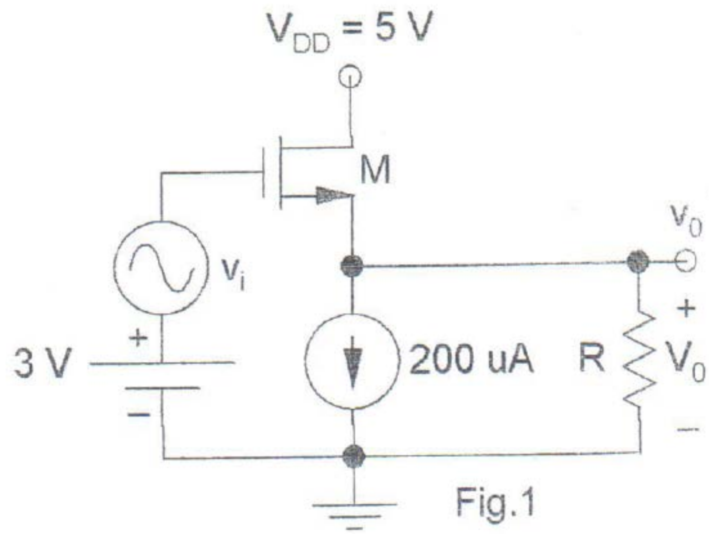


Fig.1

Sol.:

(i) Ignoring body effect and with R → ∞.

$$I_D = 200\mu A, \qquad V_{TN} = V_{TN0} = 0.7V$$

$$I_D = \frac{k_n`}{2}\frac{W}{L}(V_{GS} - V_{TN0})^2 = 200\mu A$$

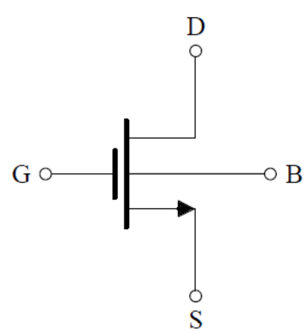
$$\rightarrow V_{GS} - V_{TN0} = 1V \rightarrow V_{GS} = 1.7V$$

$$V_0 = 3 - V_{GS} = 1.3V$$

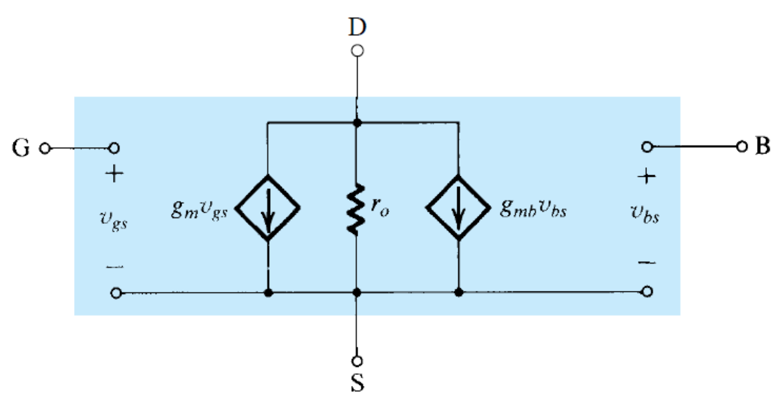
$$V_{DS} = 5 - V_0 = 3.7V$$

$$V_{DS} > V_{GS} - V_T \ \rightarrow Saturated.$$

Small-signal analysis:



(a)



(b)

$$A_v = \frac{g_m R_S}{1 + (g_m + g_{mb})R_S}$$

As R_S → ∞, g_{mb} does not exist (no body effect),

$$A_v = 1$$

(ii) Including body effect and with R → ∞.

$$I_D = 200\mu A$$

$$V_{BS} = -V_0, \qquad V_{TN} = V_{TN0} + \gamma\left(\sqrt{2\varphi_F + V_0} - \sqrt{2\varphi_F}\right)$$

Easiest way is to iterate these simultaneous equations, with initial guess $V_0 = 1.3V$ from first part. After a few iterations, it converges to,

$$V_0 = 1.09V, \quad V_{TN} = 0.91V, \quad V_{GS} = 1.91V, \quad V_{DS} = 3.91V \; (Saturated)$$

$$\chi = \frac{\gamma}{2\sqrt{2\varphi_F + V_0}} = 0.154$$

$$A_v = \frac{g_m R_S}{1 + (g_m + g_{mb})R_S} \cong \frac{g_m}{g_m + g_{mb}} = \frac{1}{1+\chi} = 0.867 \; (Reduced)$$

Q.6 Repeat Q.5 under the following conditions:

(i) Ignoring body effect and with R → 100kΩ.

(ii) Including body effect and with R → 10kΩ.

Sol.:

(i) Ignoring body effect and with R → 100kΩ. R is finite now (100kΩ).

$$I_D = 200\mu A + \frac{V_0}{100K}$$

$$V_{GS} - V_{TN} = \sqrt{\frac{I_D}{\frac{k_n`}{2}\frac{W}{L}}}, \quad V_0 = 3 - V_{GS}, \quad V_{TN} = V_{TN0} + \gamma\left(\sqrt{2\varphi_F + V_0} - \sqrt{2\varphi_F}\right)$$

This set of equations need to be iterated with intimal guess $V_0 = 1.3V$ (first part of last question). After a few iterations, it converges to,

$$V_0 = 1.067V, \quad I_{DS} = 210.67\mu A, \quad V_{TN} = 0.907V, \quad V_{GS} = 1.933V, \quad V_{DS} = 3.933V \; (Saturated)$$

$$\chi = \frac{\gamma}{2\sqrt{2\varphi_F + V_0}} = 0.155, \quad g_m = \sqrt{2k_n`\frac{W}{L}I_D} = 4.1 * 10^{-4}\Omega^{-1}$$

$$A_v = \frac{g_m R_S}{1 + (g_m + g_{mb})R_S} \cong \frac{g_m R}{1 + (1+\chi)g_m R} = 0.866$$

(ii) Including body effect and with R → 10kΩ. For this case, R is 10kΩ, which changes I_D to,

$$I_D = 200\mu A + \frac{V_0}{10K}$$

Rest of the equations remain same. A few iterations give,

$$V_0 = 0.911V, \quad I_{DS} = 291.1\mu A, \quad V_{TN} = 0.882V, \quad V_{GS} = 2.089V, \quad V_{DS} = 4.089V \; (Saturated)$$

$$\chi = \frac{\gamma}{2\sqrt{2\varphi_F + V_0}} = 0.163, \quad g_m = \sqrt{2k_n`\frac{W}{L}I_D} = 4.826 * 10^{-4}\Omega^{-1}$$

$$A_v = \frac{g_m R_S}{1 + (g_m + g_{mb})R_S} \cong \frac{g_m R}{1 + (1+\chi)g_m R} = 0.73$$

(Note the drastic reduction in A_v as R is reduced.)