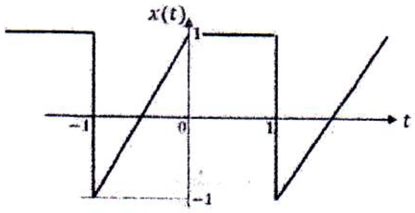
NAME: ____________________

SECTION ________ ROLL NO: [ ][ ][ ][ ][ ][ ][ ]

MARKS

Complete your answer within the 2 sides of this sheet. Additional sheets will neither be provided nor accepted.

Decompose the CTFS of the signal $x(t)$ shown into $x_c(t)$ and $x_s(t)$ that result respectively from accumulating only the cosine terms and only the sine terms in its trigonometric Fourier Series expansion. Sketch both. Likewise, we could also decompose $x(t)$ into a constant part $x_f(t)$ and a time varying part $x_v(t)$. Sketch both.



$$x(t) = \sum_k x_k e^{jk\omega_0 t} = a_0 + \sum_k a_k \cos k\omega_0 t + b_k \sin k\omega_0 t$$

Now, all the cosine terms yield even symmetric components and also a_0. On the other hand, the sine terms yield odd symmetric components. Hence

$$x(t) = x_c(t) + x_s(t) : x_c(t) = a_0 + \sum_k a_k \cos k\omega_0 t$$

$$x_s(t) = \sum_k b_k \sin k\omega_0 t.$$

We now know that $x_c(t)$ is even, $x_s(t)$ is odd. Already we also know,

$$x_c(t) = \frac{x(t) + x(-t)}{2}, \quad x_s(t) = \frac{x(t) - x(-t)}{2}$$

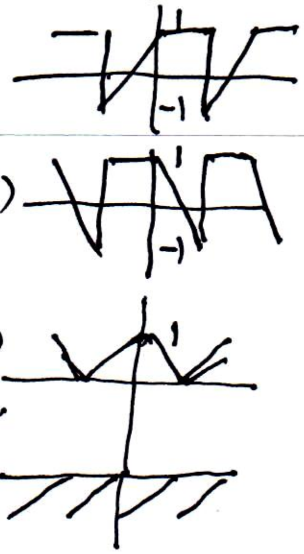
We next proceed to graphically obtain $x_c(t)$ and $x_s(t)$

$x(t)$

$x(-t)$

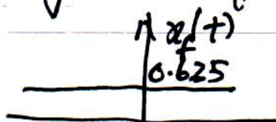
$x_c(t)$

$x_s(t)$

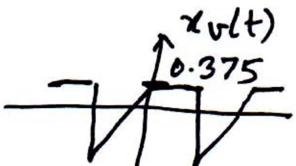


Constant part = avg value of $x(t)$

$x_f(t) = 1.25/2.$



$x_v(t) = x(t) - x_f(t)$



EE 200 | Signals, Systems & Networks | 2018

Miniquiz 9

25-10-2018
10 marks
10 min

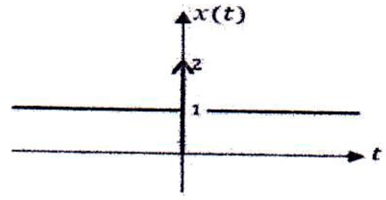
NAME: ____________________

SECTION ________ ROLL NO: ______

MARKS

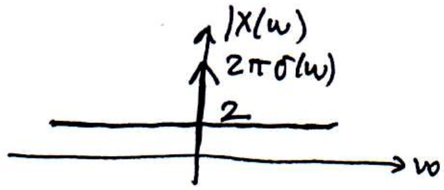
Complete your answer within the 2 sides of this sheet. Additional sheets will neither be provided nor accepted.

Find and sketch $|X(\omega)|, \angle X(\omega)$ for $x(t)$ shown. Next, find and sketch $|X'(\omega)|, \angle X'(\omega)$ if $x'(t) = \sum_n \delta(t - nT)$



$x(t)$ is even: hence $\angle X(\omega) = 0$.

$x(t) = 1 + 2\delta(t) \longleftrightarrow 2\pi\,\delta(\omega) + 2$



$x'(t)$ is also even symmetric so $|X'(\omega)|$ is real and $\angle X'(\omega) = 0$

To find $|X'(\omega)|$. $x'(t)$ is periodic and we first evaluate its CTFS:

$$x'_k = \frac{1}{T}\int_{-T/2}^{T/2} \delta(t)\, e^{-jk\frac{2\pi}{T}t}\, dt = 1/T \; : \; -\infty < k < \infty.$$

Using the CTFS we write the extended CTFT of $x'(t)$

$$X'(\omega) = |X'(\omega)| = 2\pi \sum_k x'_k\, \delta\left(\omega - \frac{2\pi k}{T}\right) = \frac{2\pi}{T} \sum_k \delta\left(\omega - \frac{2\pi k}{T}\right)$$