EE 200 2018 Signals, Systems & Networks

Assignment 5

1 2 inductors of 2H and 5H are in series.

(a) What is the range of values their collective impedance can take as their mutual inductive coupling coefficient k varies from 0 to 1 and M is allowed to be both positive and negative at $\omega = 100\pi$rad/s.

(b) Is it possible, at any frequency, for any valid value of k and for any sign for M that the system can exhibit negative (capacitive) reactance?

2 Repeat all the parts of Q.1 if the same inductors are in parallel.

3 Can mutual inductance be handled and studied as dependent sources? Take Fig.3 below as an example. $V_1 = 40\underline{/30°}$V; $V_2 = 65\underline{/-75°}$V; $\omega = 100$rad/s for both, $L_1 = 0.2$H, $L_2 = 0.8$H, $R_1 = 50\Omega$, $R_2 = 600\Omega$, $C_1 = 0.8\mu$F, $C_2 = 2\mu$F. Suppose L_1, L_2 are mutually coupled by $k = 3/4$ Instead of treating this as a mutual inductance phenomenon, we want to depict the observed behaviour as the result of the presence of dependent sources.

(a) What sort of dependent sources are to be used? V dependent I sources? I dependent V sources? I dependent I sources? V dependent V sources? Justify. (confine your arguments to the context of sunisoidal steady state 'phasor' analysis at a known frequency)

(b) Suppose the dots on both L_1 and L_2 are to the left side. Redraw the circuit with the dots removed and with the proper dependent sources introduced. Repeat for the other three cases for different dot locations (both dots to the right, dot on L_1 to the right, dot on L_2 to the left and dot on L_1 to the left, dot on L_2 to the right).

(c) Take $V_2 = 0$. Find the choice of dot convention (dot positions) for the mutual inductance between L_1, L_2 that will ensure the maximum magnitude of impedance seen from the source.

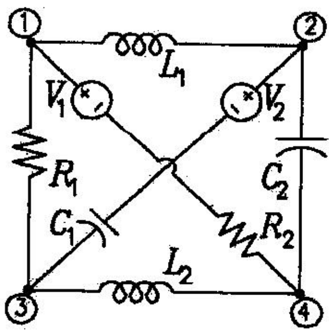


Fig.3

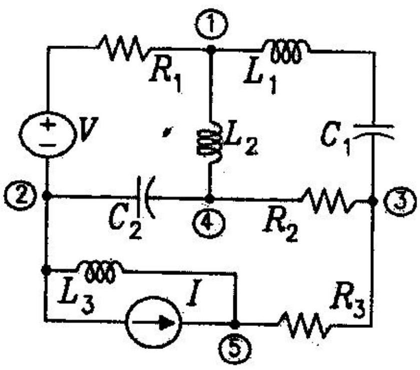


Fig.4

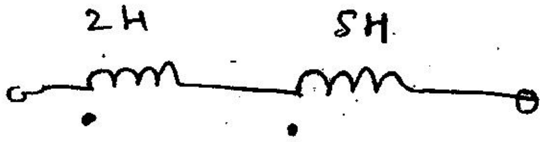
4 (a) For Fig.4, write out the three mesh equations taking all inner mesh current clockwise. The self and mutual inductances between L_1, L_2, L_3 are given in the form of the following matrix where all entries are in mH:

$$\mathbf{L} = \begin{bmatrix} 500 & -200 & -500 \\ -200 & 400 & 250 \\ -500 & 250 & 1000 \end{bmatrix}$$

(b) For the given inductance matrix, evaluate the different coupling coefficients and the appropriate dot conventions and draw the circuit to show the couplings and the dot positions.

Q. 1)

(a)



M is positive in this case.

$$M = K\sqrt{L_1 L_2}$$

$K = 0 \Rightarrow M = 0$

$K = 1 \Rightarrow M = \sqrt{10} = 3.162\ H$

$\rightarrow Z_{eq} = j \times 100\pi\,[L_1 + L_2 + 2M]$ $\left[\begin{matrix}\omega = 100\pi \\ rad/s\end{matrix}\right]$

(K = 1) $= j \times 100\pi\,[2 + 5 + 2(3.162)]$

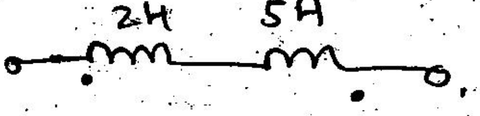
$= j \times 100\pi \times 13.324\ \Omega$

$\therefore Z_{eq} = 4183.91\,j\ \Omega$

(K = 0) $\rightarrow Z_{eq} = j \times 100\pi\,[L_1 + L_2]$

$= j \times 100\pi\,(2 + 5)$

$\therefore Z_{eq} = 700\pi j\ \Omega$



M is negetive in this case $M = -3.162\,H$

$\rightarrow Z_{eq} = j \times 100\pi\,[L_1 + L_2 - 2M]$

$= j \times 100\pi\,[2 + 5 - 2(3.162)]$

$= j \times 100\pi \times 0.676\ \Omega = 212.264\,j\ \Omega$

(b) No, it is not possible.

k's valid range [0, 1]

So, $Z_{eq} = 700\pi j\ \Omega$ $(k=0)$
$\Rightarrow m = 0$

$Z_{eq} = 4183.91 j\ \Omega$ [$k=1$ & m is +ve]

$Z_{eq} = 212.264 j\ \Omega$ [$k=1$ & m is ~~+~~ -ve]

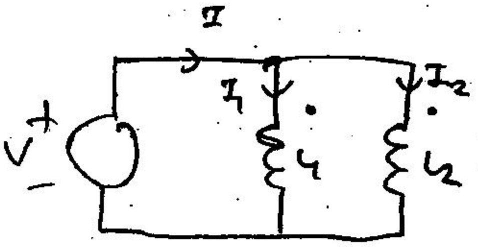
(b) $$Z_{min} = L_1 + L_2 - 2K\sqrt{L_1 L_2}$$

$$\Rightarrow 2\left(\frac{L_1+L_2}{2} - K\sqrt{L_1 L_2}\right)$$

$$AM > GM$$

Hence always +ive

2 Derivation of parallel Inductor's ckt:



$$V = j\omega L_1 I_1 + j\omega M I_2$$

$$V = j\omega L_2 I_2 + j\omega M I_1$$

→ Comparing Both the equation.

$$L_1 I_1 + M I_2 = L_2 I_2 + M I_1$$

$$\therefore (L_1 - M) I_1 = (L_2 - M) I_2$$

$$\therefore \frac{I_1}{I_2} = \frac{L_2 - M}{L_1 - M} \quad \text{--- (1)}$$

→ $I = I_1 + I_2$

$$= \frac{L_2 - M}{L_1 - M} I_2 + I_2$$

$$\therefore \frac{I}{I_2} = \frac{L_1 + L_2 - 2M}{L_1 - M} \quad \text{--- (2)}$$

$\rightarrow \quad V = j\omega L_1 I_1 + j\omega M I_2$

$$\therefore V = j\omega L_1 \left[\frac{L_2 - M}{L_1 - M}\right] I_2 + j\omega M I_2$$

[using (1)]

$$\therefore \frac{V}{I_2} = j\omega \left[\frac{L_1 L_2 - L_1 M + L_1 M - M^2}{L_1 - M}\right]$$

$$\therefore \frac{V}{I_2} = j\omega \left[\frac{L_1 L_2 - M^2}{L_1 - M}\right] \quad -(3)$$

Take the ratio of (3) & (2)

$$\frac{V/I_2}{I/I_2} = \frac{j\omega\left(\frac{L_1 L_2 - M^2}{L_1 - M}\right)}{\left(\frac{L_1 + L_2 - 2M}{L_1 - M}\right)}$$

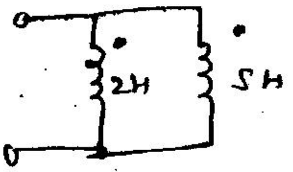
$$\therefore \boxed{\frac{V}{I} = Z_{eq} = j\omega \left[\frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M}\right]}$$

When the coupling is opposing,

$$\frac{V}{I} = Z_{eq} = j\omega \left[\frac{L_1 L_2 - M^2}{L_1 + L_2 + 2M}\right]$$

Q.2

(a)



m is positive in this case.

k = 0

$$L_{eq} = \frac{L_1 L_2}{L_1 + L_2} = \frac{2 \times 5}{2+5} = \frac{10}{7}\ H$$

$$Z_{eq} = j \times 100\pi \times \frac{10}{7}\ \Omega = 448.57j\ \Omega$$

k = 1

$$Z_{eq} = j\omega \left[\frac{L_1 L_2 - m^2}{L_1 + L_2 - 2m}\right] \qquad \left[\begin{array}{c} m = 3.162\ H \\ @\ k = 1 \end{array}\right]$$

$$\therefore Z_{eq} = j \times 100\pi \left[\frac{10 - 10}{7 - 2(3.162)}\right] = 0\ \Omega$$

k = 1/2

$$m = \frac{1}{2}\sqrt{L_1 L_2} = 1.581\ H$$

$$Z_{eq} = j \times 100\pi \left[\frac{10 - (1.581)^2}{7 - 2(1.581)}\right]$$

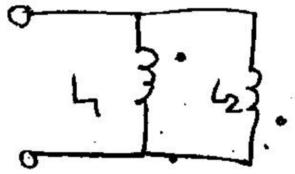
$$= j \times 100\pi \times 1.954$$

$$\therefore Z_{eq} = 613.63\ \Omega$$

To get the range, we differentiate w.r.t k and get

$k = \sqrt{L_1/L_2}, \sqrt{L_2/L_1}$, So we get $k = \sqrt{0.4}$ as $k \geq 0$ and $k \leq 1$

$Z_{max} = 628.32\ \Omega$. and $k_{max} = 0.63$.



Here M is negetive

$M = -3.162$

$\underline{\underline{k=1}}$ $Z_{eq} = j \times 100\pi \left[\frac{10 - (3.162)^2}{7 + 2(3.162)} \right]$

$\therefore Z_{eq} = 0 \ \Omega$

$\underline{\underline{k = \frac{1}{2}}}$ $M = -1.581 \ H$

$Z_{eq} = j \times 100\pi \left[\frac{10 - (1.581)^2}{7 + 2(1.581)} \right]$

$= j \times 100\pi \times 0.738$

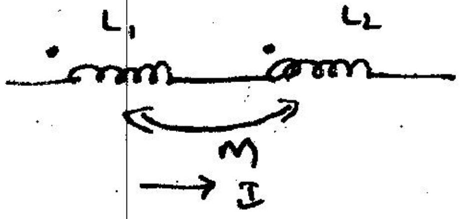
$\therefore \underline{Z_{eq} = 231.76 j \ \Omega}$

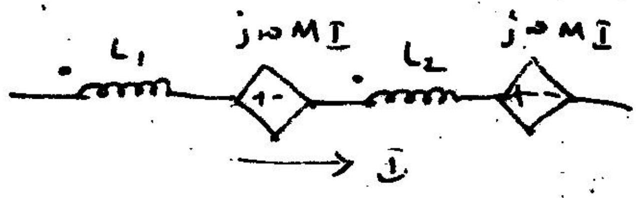
$\boxed{b}$ No, it's not possible that system can exhibit negetive reactance k's valid range $[0, 1]$

So, $Z_{eq} = \begin{cases} 448.57 j \ \Omega & [k=0] \\ 0 \ \Omega & [k=1, \ M \ +ve] \\ 613.63 j \ \Omega & (k = \frac{1}{2}, \ M \ +ve) \\ 0 \ \Omega & [k=1, \ M \ -ve] \\ 231.76 j \ \Omega & (k = \frac{1}{2}, \ M \ -ve) \end{cases}$

I dependent voltage source.

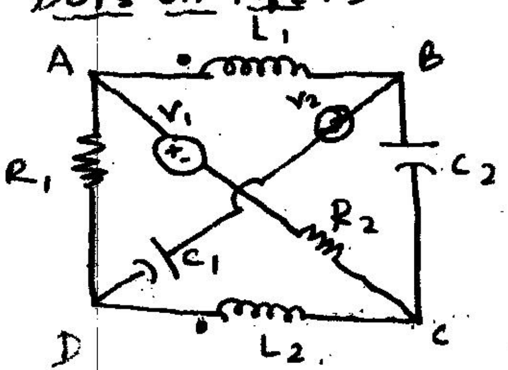
M is replaced by the source

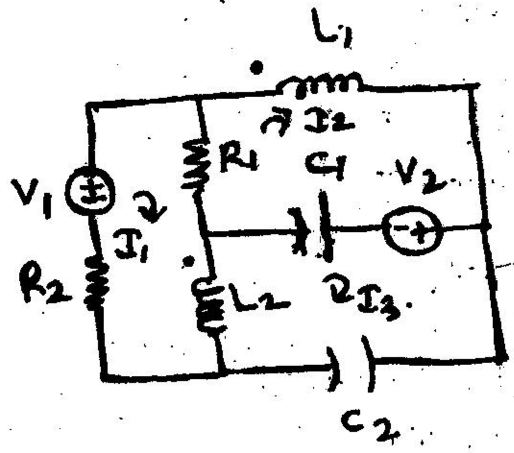


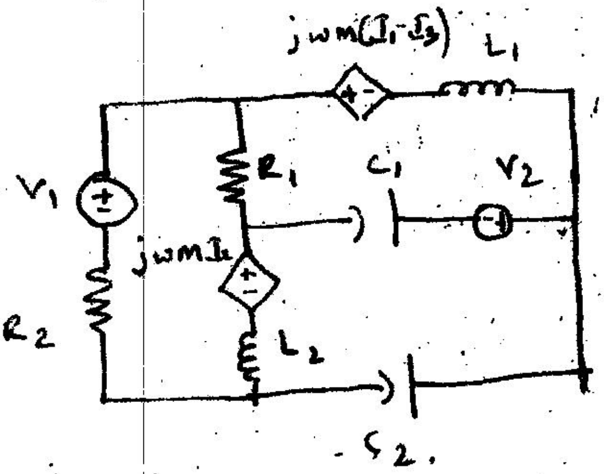


Dots on left :→

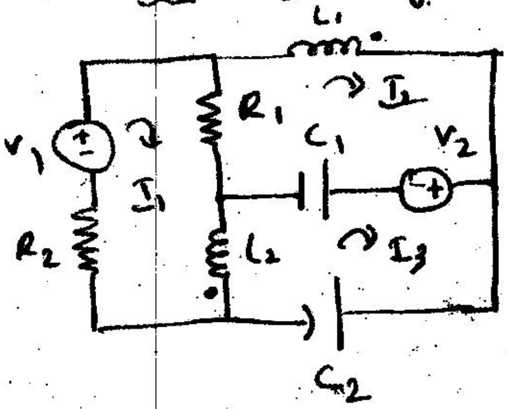
i)

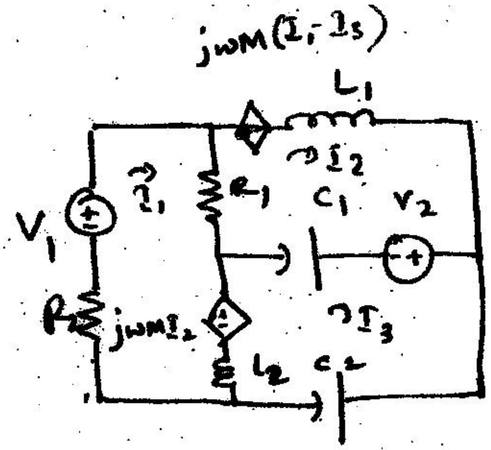




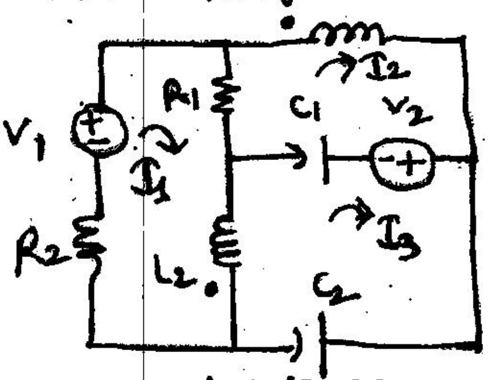


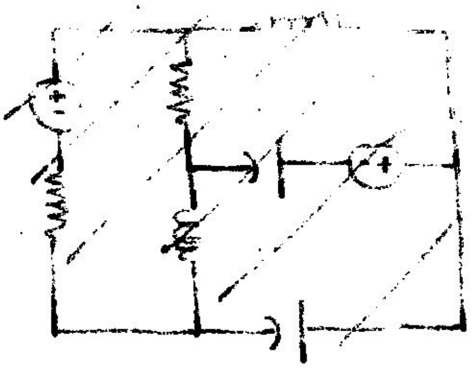
ii) Dots on right

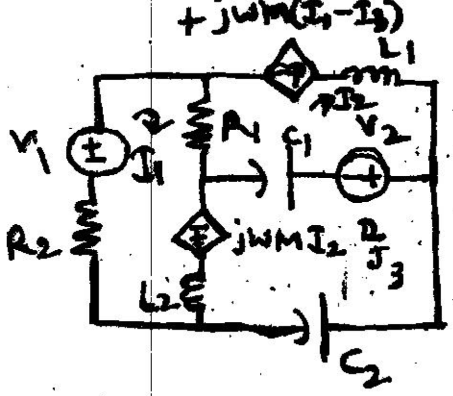




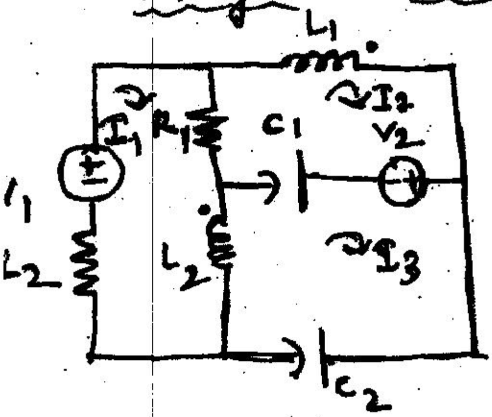
ii) Left & Right :-

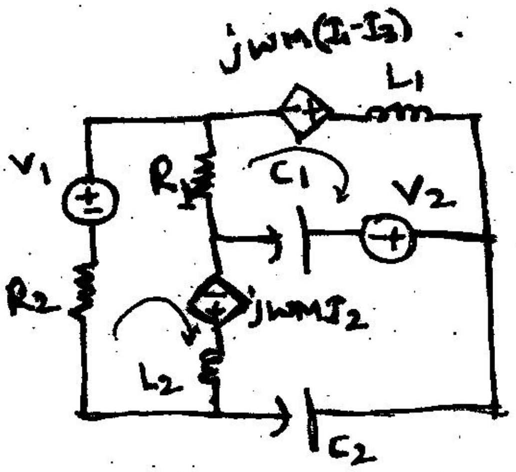




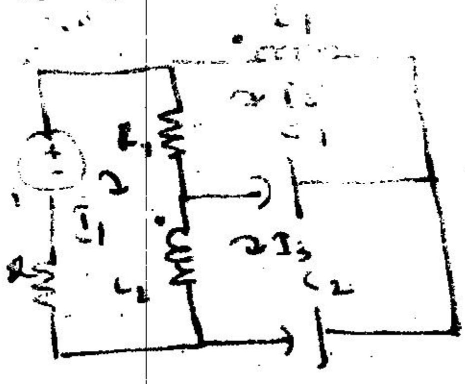


v) L1 Right & L2 left :->





$\therefore V_2 = 0$



$M = k\sqrt{L_1 L_2} = 0.3\,H$

$$V_1 = R_1(I_1 - I_2) + j\omega L_2(I_1 - I_3) + I_1 R_2 + j\omega M I_2$$

$$= R_1 I_1 - R_1 I_2 - j\omega L_2 I_3 + j\omega L_2 I_1 + j\omega M I_2 + R_2 I_1$$

$$= (R_1 + R_2 + j\omega L_2) I_1 + (j\omega M - R_1) I_2 - j\omega L_2 I_3 \rightarrow ①$$

$$j\omega L_1 I_2 + \frac{1}{j\omega C_1}(I_2 - I_3) + R_1(I_2 - I_1) + j\omega M(I_1 - I_3) = 0.$$

$$\therefore j\omega L_1 I_2 + \frac{1}{j\omega C_1} I_2 - \frac{1}{j\omega C_1} I_3 - j\omega M I_3 + R_1 I_2 - R_1 I_1$$
$$+ j\omega M I_1$$

$$\therefore (j\omega M - R_1) I_1 + \left(j\omega L_1 + \frac{1}{j\omega C_1} + R_1\right) I_2 - \left(\frac{1}{j\omega C_1} + j\omega M\right) I_3$$
$$= 0$$
$$\rightarrow ②$$

$$-j\omega L_2 I_1 - \left(\frac{1}{j\omega C_1} + j\omega M\right) I_2 + \left(\frac{1}{j\omega C_1} + \frac{1}{j\omega C_2} + j\omega L_2\right) I_3 = 0$$
$$\rightarrow ③$$

$$\begin{pmatrix} V_1 \\ 0 \\ 0 \end{pmatrix} = \begin{bmatrix} R_1 + R_2 + j\omega L_2 & j\omega M - R_1 & -j\omega L_2 \\ j\omega M - R_1 & j\omega L_1 + \frac{1}{j\omega C_1} + R_1 & -\frac{1}{j\omega C_1} - j\omega M \\ -j\omega L_2 & -\frac{1}{j\omega C_1} - j\omega M & \frac{1}{j\omega C_1} + \frac{1}{j\omega C_2} + j\omega L_2 \end{bmatrix} \begin{pmatrix} I_1 \\ I_2 \\ I_3 \end{pmatrix}$$

$I_1 = 0.0562 + 0.0238i$
$I_2 = -0.0008 + 0.0006i$
$I_3 = -0.0008 + 0.0003i$

$$Z = \frac{V}{I_1} = \frac{40\angle 30}{(0.0562+0.0238i)} = 655.39\angle 7^\circ$$

Ily for L_1 left & L_2 right (L_1 right & L_2 left) only the sign of M changes. So writing same matrix with sign change.

$$\begin{bmatrix} V_1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} R_1+R_2+j\omega L_2 & -j\omega M - R_1 & -j\omega L_2 \\ -j\omega M - R_1 & j\omega L_1 + \frac{1}{j\omega C_1} + R_1 & -\frac{1}{j\omega C_1} + j\omega M \\ -j\omega L_2 & -\frac{1}{j\omega C_1} + j\omega M & \frac{1}{j\omega C_1} + \frac{1}{j\omega C_2} + j\omega L_2 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}$$

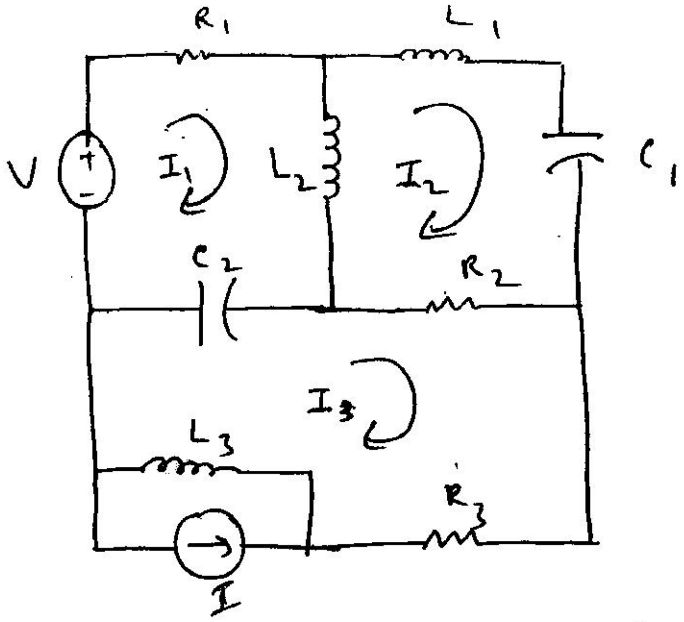
$I_1 = 0.0560 + 0.0236i$
$I_2 = -0.0018 + 0.0002i$
$I_3 = -0.0015 + 0.0001i$

$$Z = \frac{V}{I_1} = \frac{40\angle 30^\circ}{(0.0560+0.0236i)} = 658.22\angle 7^\circ$$

Hence max. impedence is there for the ckts with dot for L_1 at left (right) and dot for L_2 at right (left).

4.

(a)

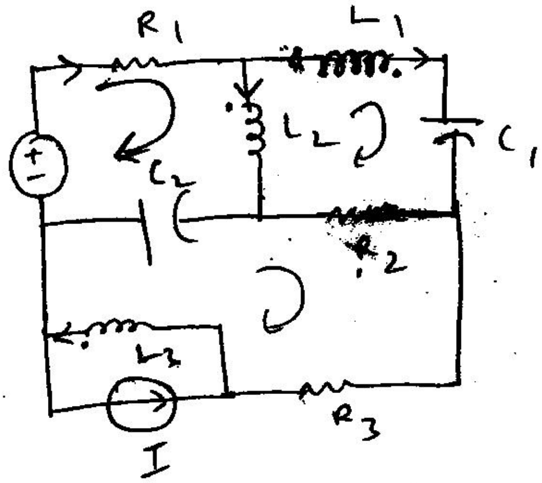


$$V = I_1 R_1 + (j\omega L_2)(I_1 - I_2) + (j\omega L_{12}) I_2 + (j\omega L_{13})(I_3 + I) + \frac{(I_1 - I_3)}{j\omega C_2}$$

$$0 = (j\omega L_1) I_2 + (j\omega L_{21})(I_1 - I_2) + (j\omega L_{31})(I_3 + I) + \frac{I_2}{j\omega C_1} + (I_2 - I_3)R_2$$
$$+ j\omega L_2 (I_2 - I_1) + j\omega L_{12}(I_2) + (j\omega L_{23})(I_3 + I)$$

$$0 = \frac{I_3 - I_2}{j\omega C_2} + (I_3 - I_2)R_2 + I_3 R_3 + j\omega L_3 (I_3 + I) + (j\omega L_{13}) I_1$$
$$+ (j\omega L_{32})(I_2 - I_1)$$

(b) $L_{12} = -ve$ $\therefore$ if current enters L_2 at dot then it [redacted] L_1 at dot and llly for L_3



$k = \underline{0.45}$

$k_{23} = \underline{0.40}$

$k_{13} = \underline{0.71}$