Problem Set 1
CHM102A

1. Assuming that we know the position of an electron in its orbit to an accuracy of 1% of the Bohr radius, calculate the uncertainty in the velocity of the electron.

2. When a beam of electrons is directed through an accelerating electrostatic field (two parallel plates with a potential difference of V volts), the potential energy gained by each electron, eV, can be equated to its kinetic energy as: $eV = \frac{1}{2}mv^2$. What is the wavelength (in Å) of an electron when it is accelerated by 1.00×10^3 V?

3. Calculate the ratio of deBroglie wavelengths for a cricket ball of mass 0.4 kg to that of a ^{4}He atom, both of which are travelling at 1 km/s.

4. Is $(\sin\theta.\cos\theta)$ an eigenfunction of the operator $(\sin\theta\frac{d}{d\theta}\left(\sin\theta\frac{d}{d\theta}\right)+6\sin^2\theta)$? If so, what is the eigenvalue?

5. A light source of wavelength λ illuminates a metal and ejects photoelectrons with a maximum kinetic energy of 1.00 eV. A second light source with half the wavelength of the first ejects photoelectrons with a maximum kinetic energy of 4.00 eV. However, any light source with more than double the wavelength of the first does not eject any photoelectrons. From these statements, what characteristics of the metal be arrived at by using Einstein's photoelectric effect?

6. If $\hat{A} = x, \hat{B} = \frac{d}{dx}$, then compute $\hat{A}\hat{B}x^3$ and $\hat{B}\hat{A}x^3$. Do the operators: $\hat{A}$ and $\hat{B}$ form a commutative pair for the function x^3?

ADDITIONAL PRACTICE PROBLEMS FROM the SUGGESTED BOOK (QUANTUM MECHANICS by Levine: solved problems)

1. Exponentials of matrices arise directly from Schrodinger equation. We will use them frequently this semester. They can be defined by the Taylor series:

   exp(**A**) = **1** + **A** + $\mathbf{A}^2/2 + \mathbf{A}^3/6 + \ldots$

   This implies:

   (a) The exponential of a diagonal matrix **Λ** is also a diagonal matrix, and $(\exp(\mathbf{\Lambda}))_{ii} = \exp(\mathbf{\Lambda}_{ii})$.

   (b) If **U** is the unitary matrix (i.e., $\mathbf{U}^\dagger\mathbf{U}=1$) which diagonalizes the Hermitian matrix **A**, then **U** also diagonalizes exp(**A**), exp(-**A**) or exp(i**A**). ($i = \sqrt{-1}$)

   (c) If **A** is Hermitian then $\exp(\mathbf{A})^{-1} = \exp(-\mathbf{A})$.

   (NOTE: a **Hermitian matrix** (or self-adjoint **matrix**) is a complex square **matrix** that is equal to its own conjugate transpose—that is, the element in the i-th row and j-th column is equal to the complex conjugate of the element in the j-th row and i-th column, for all indices i and j)

2. Show that the following operators **U** are unitary, i.e., $\mathbf{U}^\dagger\mathbf{U}=1$.

   (a) $\mathbf{U} = \exp(i\mathbf{A})$ where **A** is Hermitian. ($i = \sqrt{-1}$)

   (b) $\mathbf{U} = (1+i\mathbf{B})(1-i\mathbf{B})^{-1}$ where **B** is Hermitian.

   (c) Show that **A** and **B** can related as $\mathbf{B} = \tan(\mathbf{A}/2)$.

   (d) Use the theorem $\det(\mathbf{AB})=\det(\mathbf{A})\det(\mathbf{B})$ to show that the determinant of an orthogonal matrix must equal to +1 or -1. What is true about the determinant of a unitary matrix?