Proper use of the commutation diagram for TIME INVARIANCE

Sincere thanks to Shashi Kant and Maneet!

This set of examples is intended to clarify the confusion that arose in the class due to some poor communication on my part. Essential points to keep in mind:

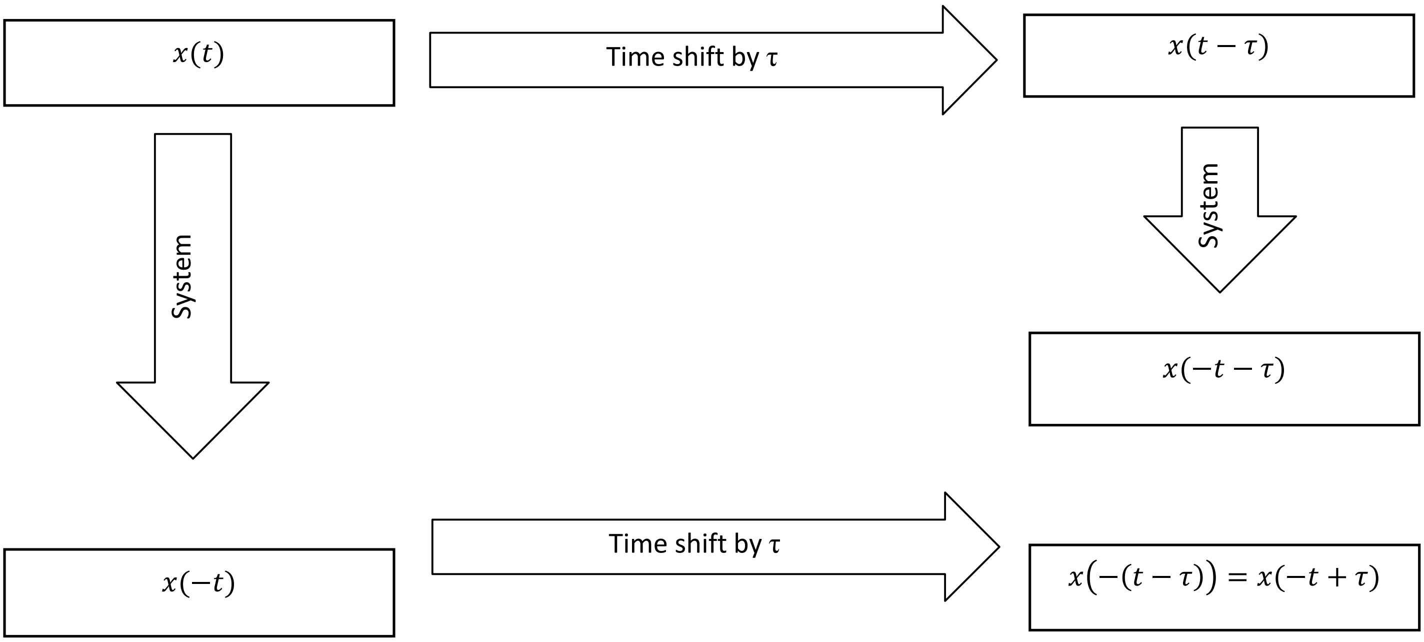
1. The time shift can also be understood as another system: call it the time shifter T.
2. So we have two systems, one is the time shifter, and the other is the given system H
3. The question to ask to check for time invariance of the 'given system' is whether the result (final output) is the same in the two cases, $T \to H$ and $H \to T$.
4. First point to keep in mind. There is just one absolute time axis as far as T or any H is concerned. No 'modified time axis'.
5. Each system T or H works in isolation: H does not 'know' that the signal it received for processing has been passed through T just before: it just got an input signal, and it blindly processes it. Same applies the other way around equally.
6. So, suppose you start with $x(t)$ and apply it to T to get $x(t-\tau)$. The latter is just another signal; it is best to rename it to say $p(t)$ to avoid any confusion. Do not look upon it as "$x(t)$ with modified time". Suppose H produces $y(t) = tx(t)$ when the input is $x(t)$. Then, given $p(t)$ as input, it will simply produce $tp(t) = tx(t-\tau)$. **This is written by simply replacing every occurrence of $\boldsymbol{p(t)}$ by $\boldsymbol{tp(t)}$, because $\boldsymbol{H}$ always just multiplies any input by $\boldsymbol{t}$.** Going on the other path, you would first apply H on $x(t)$ to get $tx(t) = q(t)$ and then pass this on to T which delays it to $q(t-\tau) = (t-\tau)x(t-\tau)$. **This is written by simply replacing every occurrence of $\boldsymbol{t}$ by $\boldsymbol{t-\tau}$ because $\boldsymbol{T}$ simply delays any applied input by $\boldsymbol{\tau}$.**
7. Since the two results are different, H is not time invariant.

Likewise, for H given by $y(t) = x(-t)$. Here, $T[x(t)] = p(t) = x(t-\tau)$, $H[x(t)] = q(t) = x(-t)$. Then $H\big[T[x(t)]\big] = H[p(t)] = p(-t) = x(-t-\tau)$ You get this by replacing every occurrence of t in $x(t-\tau)$ by $-t$. On the other path, $T\big[H[x(t)]\big] = T[q(t)] = q(t-\tau) = x\big(-(t-\tau)\big) = x(-t+\tau)$. You get this by replacing every occurrence of t in $x(-t)$ by $t-\tau$. Once again, the two results are different, so, H is not time invariant.

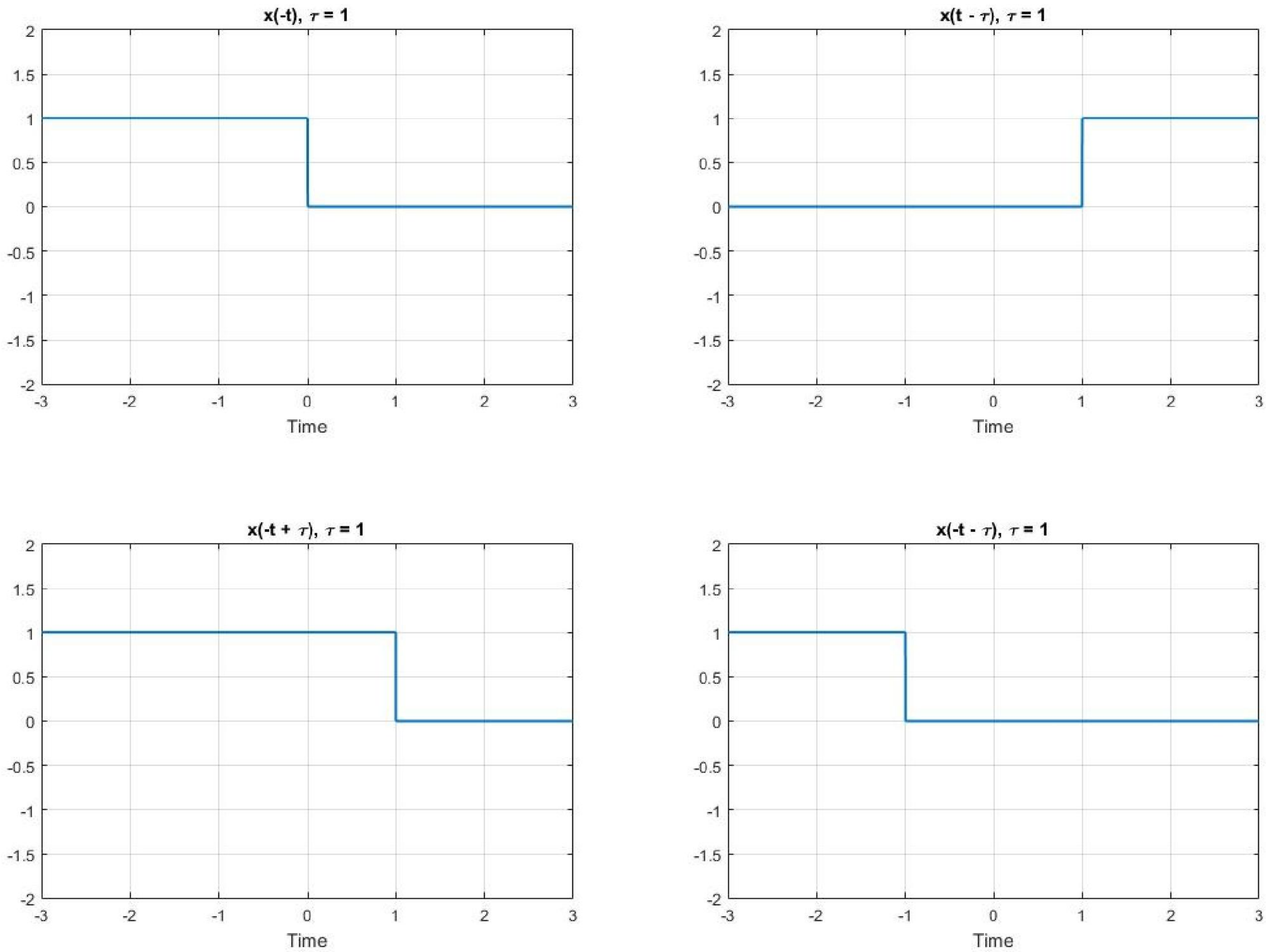
In the final example, for H given by $y(t) = x(2t)$. Here, $T[x(t)] = p(t) = x(t-\tau)$, $H[x(t)] = q(t) = x(2t)$. Then $H\big[T[x(t)]\big] = H[p(t)] = p(2t) = x(2t-\tau)$ You get this by replacing every occurrence of t in $x(t-\tau)$ by $2t$. On the other path, $T\big[H[x(t)]\big] = T[q(t)] = q(2(t-\tau)) = x\big(2(t-\tau)\big) = x(2t-2\tau)$. You get this by replacing every occurrence of t in $x(2t)$ by $t-\tau$. Once again, the two results are different, so, H is not time invariant.

Hope this finally settles the confusion.

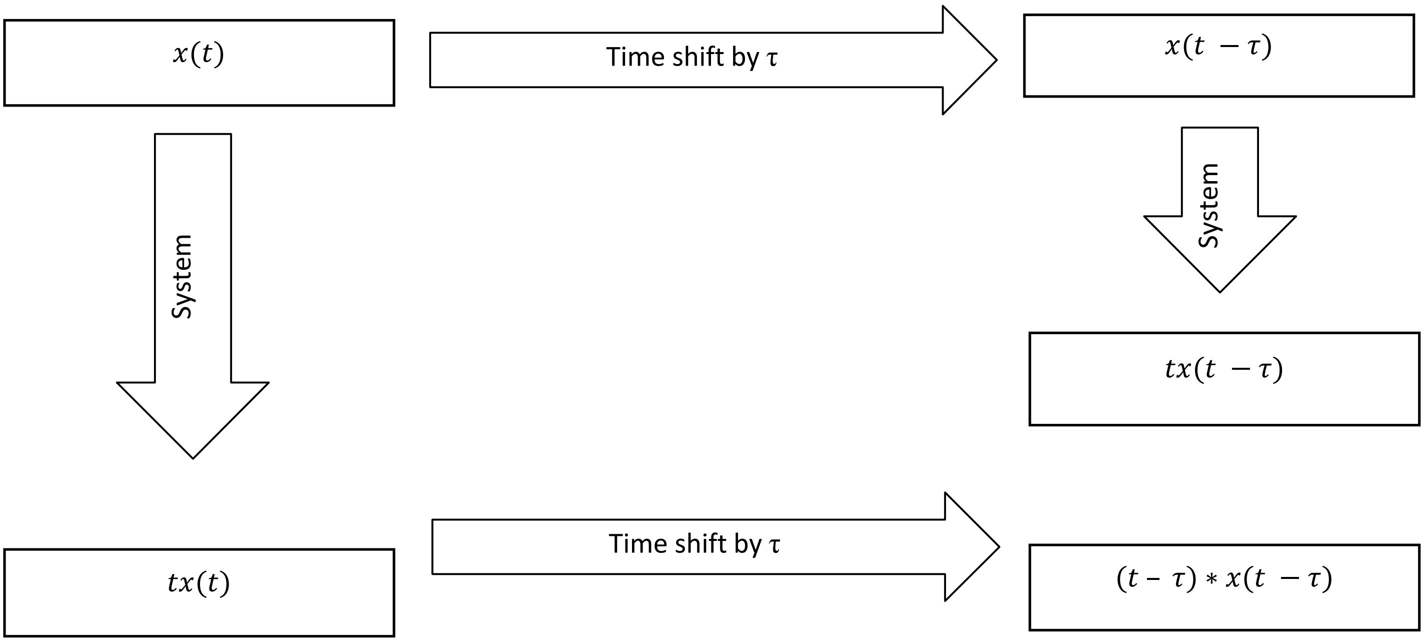
System: $y(t) \;=\; x(-t)$



Plots:



System: $y(t) \;=\; tx(t)$



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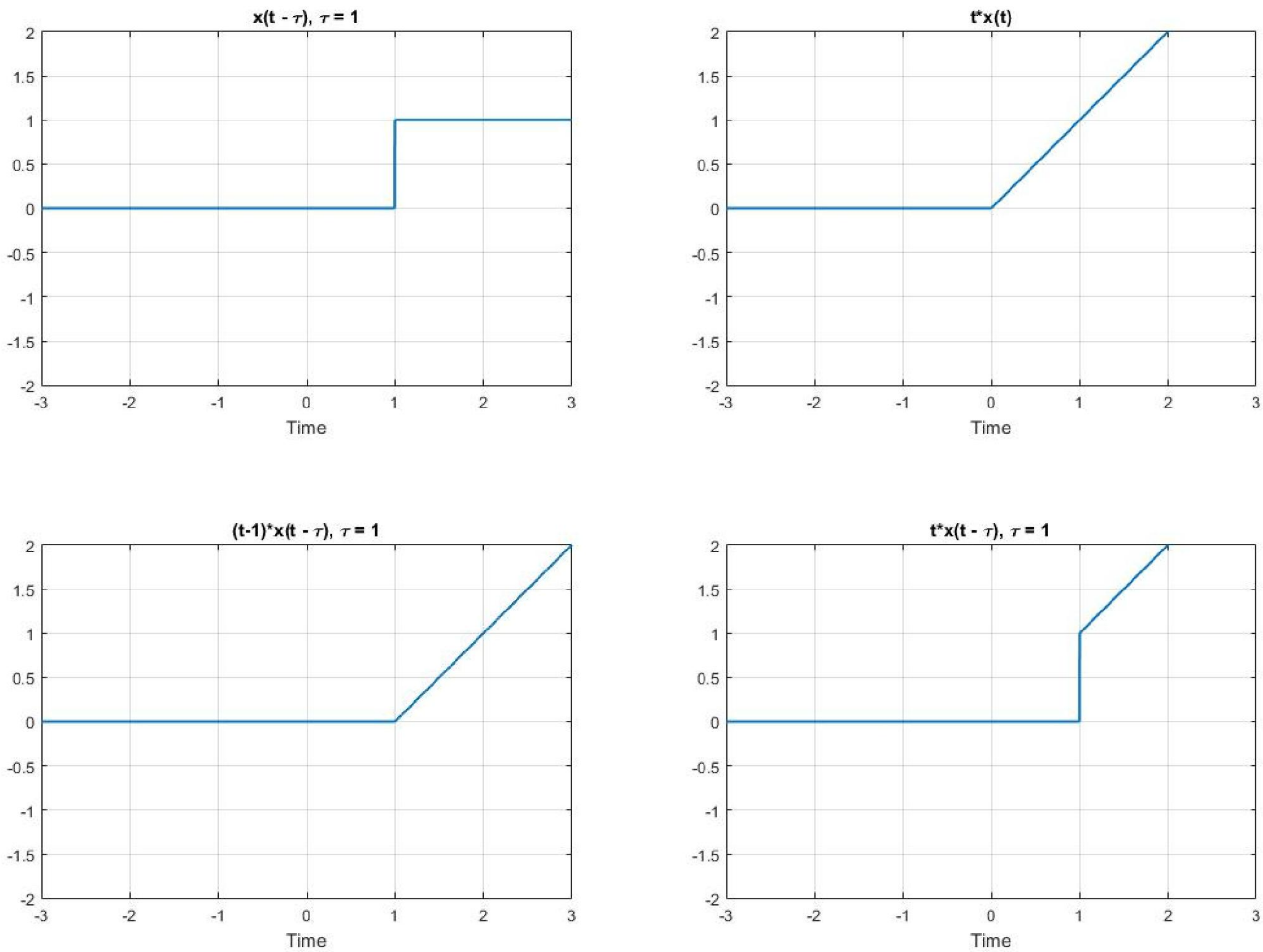
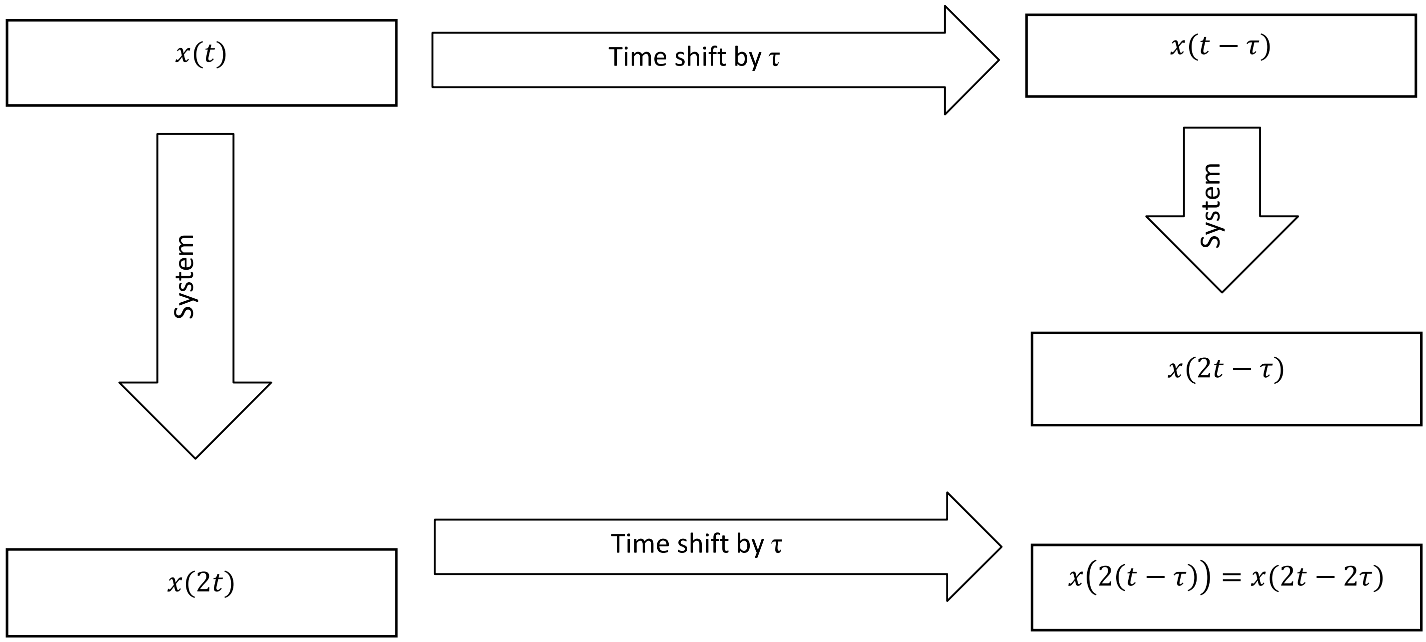


System: $y(t) \ = \ x(2t)$



Plots: