Target : JEE (Main + Advanced)

Fundamentals of Mathematics-II

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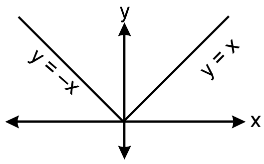
He is unworthy of the name of man who is ignorant of the fact that the diagonal of square is incommensurable with its side ....**Plato**

Absolute value function / modulus function :

The symbol of modulus function is f (x) = $|x|$ and is defined as: y = $|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$.



y = |x|

Properties of modulus : For any a, b $\in$ R

(i) $|a| \geq 0$ (ii) $|a| = |-a|$

(iii) $|a| \geq a$, $|a| \geq -a$ (iv) $|ab| = |a|\,|b|$

(v) $\left|\dfrac{a}{b}\right| = \dfrac{|a|}{|b|}$ (vi) $|a + b| \leq |a| + |b|$; Equality holds when $ab \geq 0$

(vii) $|a - b| \geq ||a| - |b||$; Equality holds when $ab \geq 0$

Example # 1 : Solve the following linear equations

(i) $x|x| = 4$ (ii) $|x - 3| + 2|x + 1| = 4$

Solution : (i) $x|x| = 4$

If $x > 0$

$\therefore$ $x^2 = 4$ $\Rightarrow$ $x = \pm 2$

$\therefore$ $x = 2$ $(\because x \geq 0)$

If $x < 0$ $\Rightarrow$ $-x^2 = 4$ $\Rightarrow$ $x^2 = -4$ which is not possible

(ii) $|x - 3| + 2|x + 1| = 4$

<u>case I</u> : If $x \leq -1$

$\therefore$ $-(x - 3) - 2(x + 1) = 4$

$\Rightarrow$ $-x + 3 - 2x - 2 = 4$ $\Rightarrow$ $-3x + 1 = 4$

$\Rightarrow$ $-3x = 3$ $\Rightarrow$ $x = -1$

<u>case II</u> : If $-1 < x \leq 3$

$\therefore$ $-(x - 3) + 2(x + 1) = 4$

$\Rightarrow$ $-x + 3 + 2x + 2 = 4$ $\Rightarrow$ $x = -1$ which is not possible

<u>case III</u> : If $x > 3$

$x - 3 + 2(x + 1) = 4$

$3x - 1 = 4$ $\Rightarrow$ $x = 5/3$ which is not possible $\therefore$ $x = -1$ **Ans.**

Rational function :

A rational function is a function of the form, y = f (x) = $\dfrac{g(x)}{h(x)}$, where g (x) & h (x) are polynomial functions.

Irrational function :

An irrational function is a function y = f(x) in which the operations of addition, substraction, multiplication, division and raising to a fractional power are used.

For example $y = \dfrac{x^3 + x^{1/3}}{2x + \sqrt{x}}$ is an irrational function

(a) The equation $\sqrt{f(x)} = g(x)$, is equivalent to the following system

$f(x) = g^2(x)$ & $g(x) \geq 0$

(b) The inequation $\sqrt{f(x)} < g(x)$, is equivalent to the following system
$f(x) < g^2(x)$ & $f(x) \ge 0$ & $g(x) \ge 0$

(c) The inequation $\sqrt{f(x)} > g(x)$, is equivalent to the following system
$g(x) \le 0$ & $f(x) \ge 0$ or $g(x) \ge 0$ & $f(x) > g^2(x)$

Example # 2 : **Solve :** $x + 2 > 2\sqrt{1-x^2}$

Solution : $4(1 - x^2) < (x + 2)^2$ and $x + 2 \ge 0$ & $1 - x^2 \ge 0$

$x \in \left(-\infty, \dfrac{-4}{5}\right) \cup (0, \infty)$ ...(1)

$x \in [-2, \infty)$ ...(2)

$x \in [-1, 1]$ ...(3)

$(1) \cap (2) \cap (3)$

$\left[-1, -\dfrac{4}{5}\right) \cup (0, 1]$

Self Practice Problem :

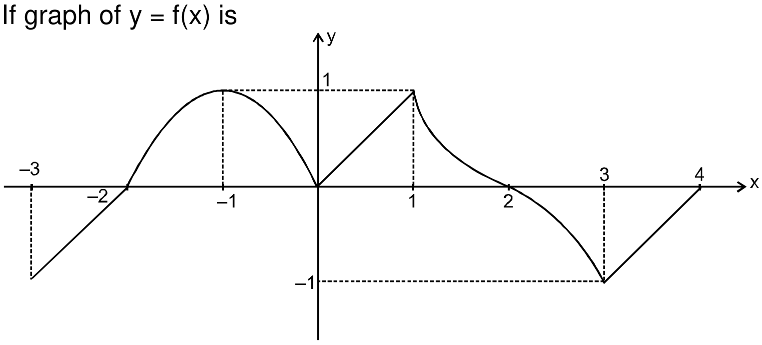
(1) $\sqrt{2x^2 + x - 6} < x$ (2) $\sqrt{5-x} > x + 1$

(3) $x + 3 + \sqrt{x^2 + 4x - 5} > 0$ (4) $\sqrt{x} - \sqrt{4-x} \ge 1$

Ans. (1) $\left[\dfrac{3}{2}, 2\right)$ (2) $(-\infty, 1)$ (3) $(-\infty, -1] \cup [5, \infty)$ (4) $\left[\dfrac{4+\sqrt{7}}{2}, 4\right]$

Graphs Related to modulus :

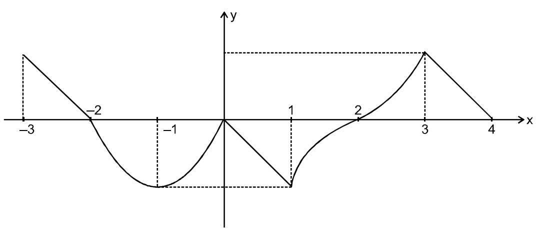
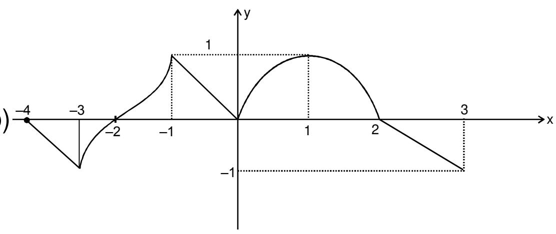
If graph of y = f(x) is

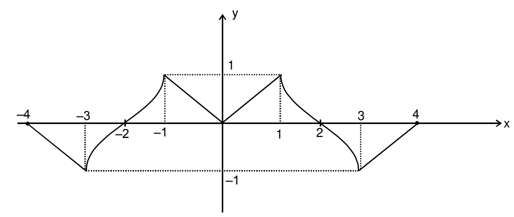
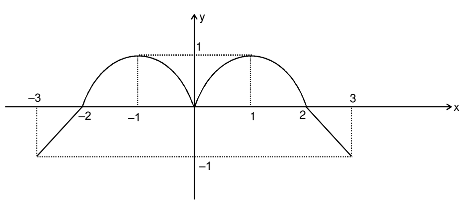


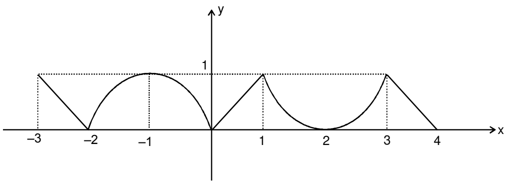
then draw graph of

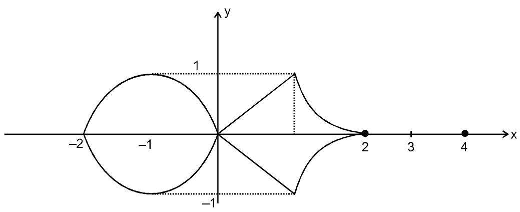
(a) $y = -f(x)$ (b) $y = f(-x)$ (c) $y = f(|x|)$ (d) $y = f(-|x|)$

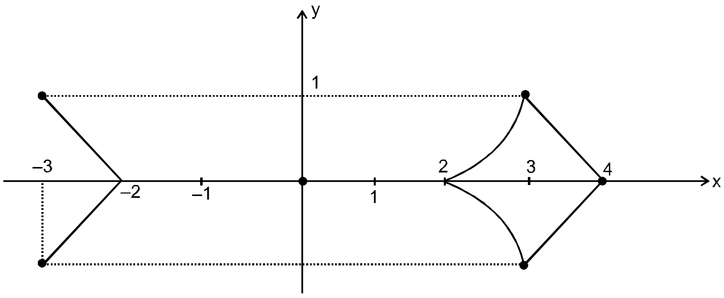
(e) $y = |f(x)|$ (f) $|y| = f(x)$ (g) $|y| = -f(x)$

(a)  (b) 

(c)  (d) 

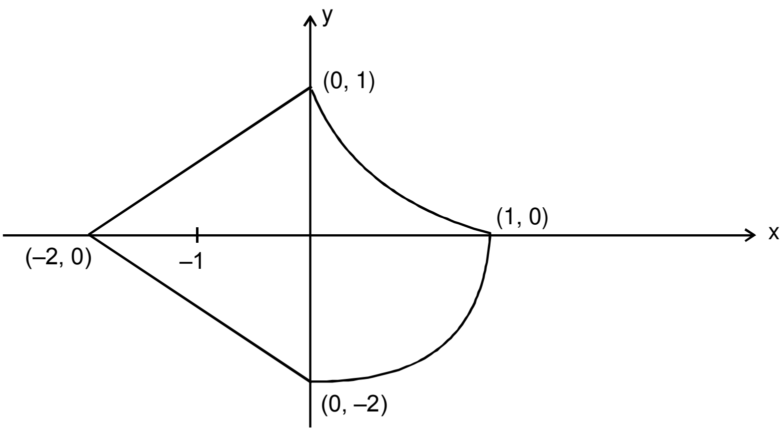
(e)


(f)


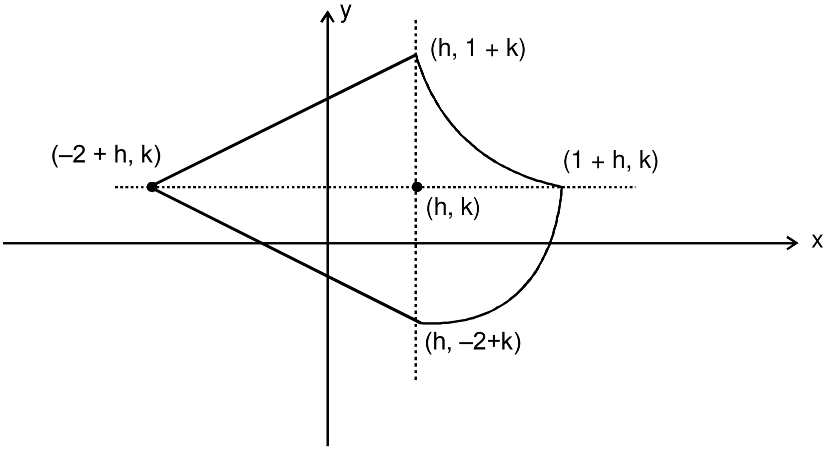
(g)


Graphical Trasformation :

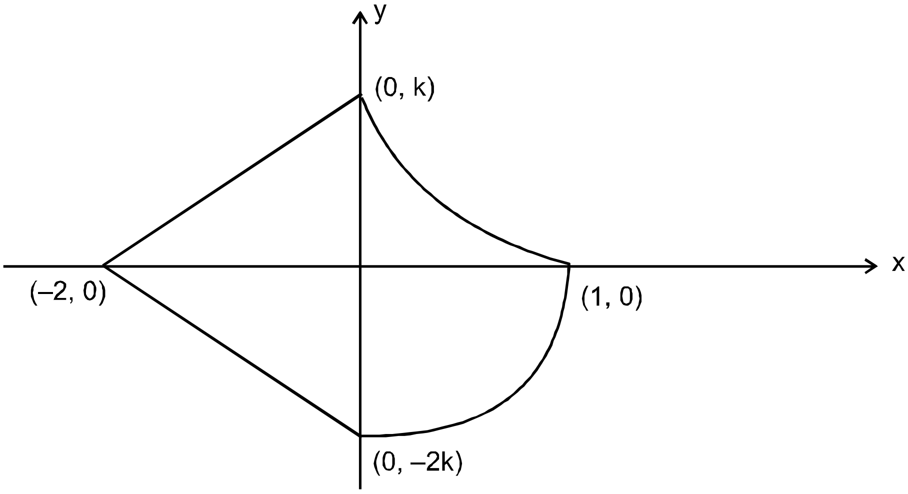
If graph of y = f(x) is



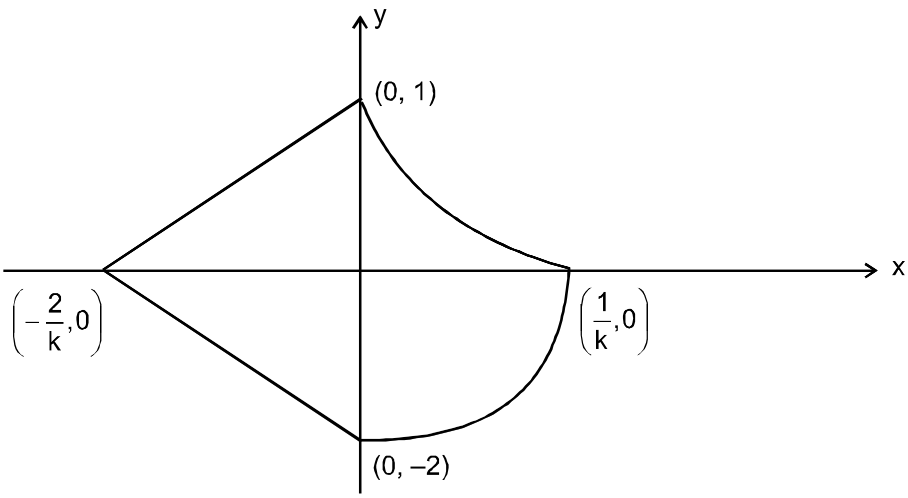
then graph of (a) y – k = f(x – h)   (b) y = kf(x), (k > 0)   (c) y = f(kx), (k > 0)

(a)


(b)

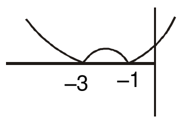


(c)



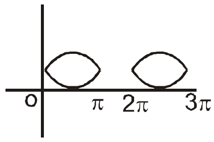
Example # 3 : $y = |x^2 + 4x + 3|$

Solution :



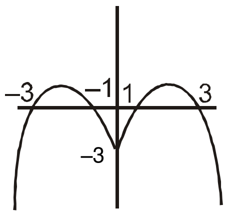
Example # 4 : $|y–1| = \sin x$

Solution :



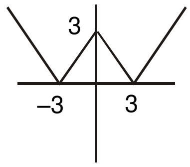
Example # 5 : $y = –x^2 + 4|x| – 3$

Solution :



Example # 6 : $y = ||x| - 3|$

Solution :



Example # 7 : $y = \sin\left(\frac{x}{3}\right)$

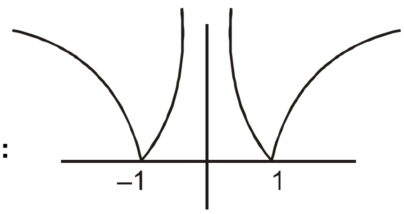
Solution : period is 6π

Example # 8 : $y = |\sin x - 3|$

Solution : Graphical Transformation

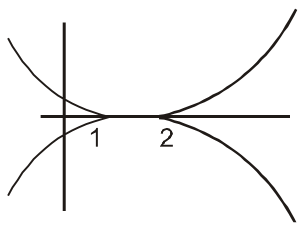
Example # 9 : $y = |-\ell n\, |-x||$

Solution :



Example # 10 : $|y| = x^2 - 3x + 2$

Solution :



Exercise-1

✍ Marked questions are recommended for Revision.

PART - I : SUBJECTIVE QUESTIONS

Section (A) : Modulus Function & Equation

A-1. Write the following expression in appropriate intervals so that they are bereft of modulus sign
(i) $|x^2 - 7x + 10|$ (ii) $|x^3 - x|$ (iii) $|2^x - 2|$
(iv) $|x^2 - 6x + 10|$ (v) $|x - 1| + |x^2 - 3x + 2|$ (vi) $\sqrt{x^2 - 6x + 9}$
(vii) $2^{(x-1)} + |x + 2| - 3^{|x+1|}$

A-2. Draw the labled graph of following
(i) $y = |7 - 2x|$ (ii) $y = |x - 1| - |3x - 2|$
(iii) $y = |x - 1| + |x - 4| + |x - 7|$ (iv) $y = |4x + 5|$ (v) $y = |2x - 3|$

A-3. Solve the following equations
(i) $|x| + 2\,|x - 6| = 12$ (ii) $||x + 3| - 5| = 2$
(iii)✍ $|||x - 2| - 2\,| - 2| = 2$ (iv) $|4x + 3| + |3x - 4| = 12$

A-4. Solve the following equations :
(i) $x^2 - 7|x| - 8 = 0$ (ii) $|x^2 - x + 1| = |x^2 - x - 1|$
(iii) $|x^3 - 6x^2 + 11x - 6| = 6$ (iv) $|x^2 - 2x| + x = 6$
(v)✍ $|x^2 - x - 6| = x + 2.$

A-5. Find the number of real roots of the equation
(i) $|x|^2 - 3|x| + 2 = 0$ (ii) $||x - 1| - 5| = 2$ (iii) $|2x^2 + x - 1| = |x^2 + 4x + 1|$

A-6. Find the sum of solutions of the following equations :
(i) $x^2 - 5|x| - 4 = 0$
(ii) $(x - 3)^2 + |x - 3| - 11 = 0$ (iii) $|x|^3 - 15x^2 - 8|x| - 11 = 0$
(iv) $||x - 3| - 4| = 1$ (v) $2^{|x|} + 3^{|x|} + 4^{|x|} = 9$

A-7. Find number of solutions of the following equations
(i) $|x - 1| + |x - 2| + |x - 3| = 9$ (ii) $|x - 1| + |x - 2| + |x - 3| + |x - 4| = 4$
(iii)✍ $|x| + |x + 2| + |x - 2| = p,\ p \in R$

A-8. Find the minimum value of $f(x) = |x - 1| + |x - 2| + |x - 3|$

A-9 If $x^2 - |x - 3| - 3 = 0$, then $|x|$ can be

A-10.✍ If $|x^3 - 6x^2 + 11x - 6|$ is a prime number then find the number of possible integral values of x.

Section (B) : Modulus Inequalities

B-1. Solve the following inequalities :
(i) $|x - 3| \geq 2$ (ii) $|\,|x - 2| - 3| \leq 0$ (iii) $||3x - 9| + 2\,| > 2$
(iv) $|2x - 3| - |x| \leq 3$ (v) $|x - 1| + |x + 2| \geq 3$ (vi)✍ $||x - 1| - 1| \leq 1$

B-2. Solve the following inequalities :

(i) $\left|1+\frac{3}{x}\right| > 2$ (ii) $\left|\frac{3x}{x^2-4}\right| \le 1$ (iii) $\frac{|x+3|+x}{x+2} > 1$

(iv) $|x^2 + 3x| + x^2 - 2 \ge 0$ (v) $|x + 3| > |2x - 1|$

B-3. Solve the following inequalities

(i) $|x^3 - 1| \ge 1 - x$ (ii) $\left|x^2 - 4x + 4\right| \ge 1$ (iii) $\frac{|x+2|-x}{x} < 2$

(iv) $\frac{|x-2|}{x-2} > 0$ (v) $|x - 2| > |2x - 3|$ (vi) $|x + 2| + |x - 3| < |2x + 1|$

B-4. Solve the following equations

(i) $|x^3 + x^2 + x + 1| = |x^3 + 1| + |x^2 + x|$

(ii) $|x^2 - 4x + 3| + |x^2 - 6x + 8| = |2x - 5|$

(iii) $|x^2 + x + 2| - |x^2 - x + 1| = |2x + 1|$

(iv) $|x^2 - 2x - 8| + |x^2 + x - 2| = 3\,|x + 2|$

(v) $|2x - 3| + |x + 5| \le |x - 8|$

B-5. Find the solution set of the inequalities $|x^2 + x - 2| \le 0$ and $|x^2 - x + 2| \ge 0$

Section (C) : Miscellaneous Modulus Equations & Inequations

C-1. Write the following expression in appropriate intervals so that they are bereft of modulus sign

(i) $|\log_{10}x| + |2^{x-1} - 1|$ (ii) $|(\log_2x)^2 - 3(\log_2x) + 2|$ (iii) $|5^{x^2-4x+5} - 25|$

C-2. Solve the equations $\log_{100}|x + y| = 1/2$, $\log_{10}y - \log_{10}|x| = \log_{100}4$ for x and y.

C-3. Solve the inequality

(i) $(\log_2x)^2 - |(\log_2x) - 2| \ge 0$

(ii) $2\,|\log_3x| + \log_3x \ge 3$

(iii). Find the complete solution set of $2^x + 2^{|x|} \ge 2\sqrt{2}$

C-4. Find the number of real solution(s) of the equation $|x-3|^{3x^2-10x+3} = 1$

C-5. If x, y are integral solutions of $2x^2 - 3xy - 2y^2 = 7$, then find the value of $|x + y|$

C-6. If x, $|x + 1|$, $|x - 1|$ are three terms of an A.P., then find the number of possible values of x

Section (D) : Irrational Inequations

D-1. Solve the following inequalities :

(i) $\frac{\sqrt{2x-1}}{x-2} < 1$ (ii) $x - \sqrt{1-|x|} < 0$ (iii) $\sqrt{x^2-x-6} < 2x - 3$

(iv) $\sqrt{x^2-6x+8} \le \sqrt{x+1}$ (v) $\sqrt{x^2-7x+10} + 9\log_4\left(\frac{x}{8}\right) \ge 2x + \sqrt{14x-20-2x^2} - 13$

(vi) $x - 3 < \sqrt{x^2+4x-5}$ (vii) $\sqrt{x^2-5x-24} > x + 2$ (viii) $\sqrt{4-x^2} \ge \frac{1}{x}$

(ix) $\frac{\sqrt{x+7}}{x+1} > \sqrt{3-x}$

D-2. Solve the equation $\sqrt{a(2^x-2)+1} = 1 - 2^x$ for every value of the parameter a.

Section (E) : Transformation of curves

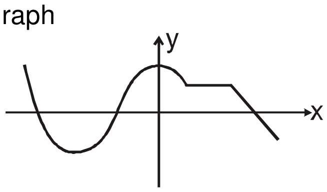
E-1. Draw the graph of followings —
(i) $y = -|x + 2|$ (ii) $y = ||x - 1| - 2|$
(iii) $y = |x + 2| + |x - 3|$ (iv)✍ $|y| + x = -1$

E-2. Draw the graphs of the following curves :
(i) $y = -\dfrac{1}{|2x+1|}$ (ii) $\dfrac{y}{|x|-1} = -1$ (iii) $|y - 3| = |x - 1|$
(iv) $y = \dfrac{|x^2-1|}{(x^2-1)}\ell nx$

E-3. Draw the graph of $y = \log_{1/2}(1 - x)$.

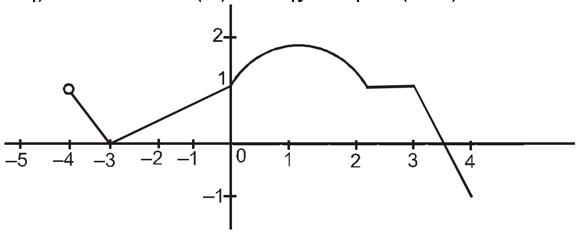
E-4.✍ Find the set of values of λ for which the equation $|x^2 - 4|x| - 12| = \lambda$ has 6 distinct real roots.

E-5. If y = f(x) has following graph



Then draw the graph of
(i) $y = |f(x)|$ (ii) $y = f(|x|)$ (iii)✍ $y = f(-|x|)$ (iv) $y = |f(|x|)|$

E-6. If y = f(x) is shown in figure given below, then plots the graph for
(A) $y = f(|x + 2|)$ (B) $|y - 2| = f(-3x)$.



E-7.✍ Find the number of roots of equation
(i) $3^{|x|} - |2 - |x|| = 1$ (ii) $x + 1 = x \cdot 2^x$

E-8 Find values of k for which the equation $|x^2 - 1| + x = k$ has
(i) 4 solution (ii) 3 solutions (iii) 1 solution (iv) 2 solutions

Exercise-2

✍ **Marked questions are recommended for Revision.**
*** Marked Questions may have more than one correct option.**

1.✍ Number of integral values of 'x' satisfying the equation $3^{|x+1|} - 2.3^x = 2.|3^x - 1| + 1$ are
(A) 1 (B) 2 (C) 3 (D) 4

2. $|x^2 + 6x + p| = x^2 + 6x + p \ \forall \ x \in R$ where p is a prime number then least possible value p is
(A) 7 (B) 11 (C) 5 (D) 13

3. If $(\log_{10}x)^2 - 4|\log_{10}x| + 3 = 0$, the product of roots of the equation is :
(A) 3 (B) 10^4 (C) 10^8 (D) 1

4. The equaiton $||x - 1| + a| = 4$ can have real solutions for x if a belongs to the interval
(A) $(-\infty, 4]$ (B) $(4, \infty)$ (C) $(-4, \infty)$ (D) $(-\infty, -4) \cup (4, \infty)$

5. The number of values of x satisfying the equation $|2x + 3| + |2x - 3| = 4x + 6$, is
(A) 1 (B) 2 (C) 3 (D) 4

6. Number of prime numbers satisfying the inequality $\log_3 \frac{|x^2 - 4x| + 3}{x^2 + |x - 5|} \geq 0$ is equal to
(A) 1 (B) 2 (C) 3 (D) 4

7. If $|x + 2| + y = 5$ and $x - |y| = 1$ then the value of $(x + y)$ is
(A) 1 (B) 2 (C) 3 (D) 4

8. The number of value of x satisfying the equation $|x - 1|^A = (x - 1)^7$, where $A = \log_3 x^2 - 2 \log_x 9$
(A) 1 (B) 2 (C) 0 (D) 3

9. The number of integral value of x satisfying the equation $\left|\log_{\sqrt{3}} x - 2\right| - \left|\log_3 x - 2\right| = 2$
(A) 1 (B) 2 (C) 3 (D) 4

10. The sum of all possible integral solutions of equation
$$||x^2 - 6x + 5| - |2x^2 - 3x + 1|| = 3|x^2 - 3x + 2| \text{ is}$$
(A) 10 (B) 12 (C) 13 (D) 15

11. The complete solution set of the inequality $(|x - 1| - 3)(|x + 2| - 5) < 0$ is $(a, b) \cup (c, d)$ then the value of $|a| + |b| + |c| + |d|$ is
(A) 14 (B) 15 (C) 16 (D) 17

12. The product of all the integers which do not belong to the solution set of the inequality $\left|\frac{3|x| - 2}{|x| - 1}\right| \geq 2$ is
(A) –1 (B) –4 (C) 4 (D) 0

13. Let $f(x) = |x - 2|$ and $g(x) = |3 - x|$ and
A be the number of real solutions of the equation $f(x) = g(x)$
B be the minimum value of $h(x) = f(x) + g(x)$
C be the area of triangle formed by $f(x) = |x - 2|$, $g(x) = |3 - x|$ and x-axis and $\alpha < \gamma < \beta < \delta$ where $\alpha < \beta$ are the roots of $f(x) = 4$ and $\gamma < \delta$ are the roots of $g(x) = 4$, then the value of sum of digits of $\frac{\alpha^2 + \beta^2 + \gamma^2 + \delta^2}{ABC}$.
(A) 7 (B) 8 (C) 11 (D) 9

14*. If $f(x) = |x + 1| - 2|x - 1|$ then
(A) maximum value of f(x) is 2.
(B) there are two solutions of f(x) = 1.
(C) there is one solution of f(x) = 2.
(D) there are two solutions of f(x) = 3.

15*. The solution set of inequality $|x| < \frac{a}{x}$, $a \in R$, is
(A) $\left(-\sqrt{-a}, 0\right)$ if $a < 0$ (B) $\left(0, \sqrt{a}\right)$ if $a > 0$ (C) ϕ if $a = 0$ (D) $(0, a)$ if $a > 0$

16*. If a and b are the solutions of equation : $\log_5\left(\log_{64}|x| - \frac{1}{2} + 25^x\right) = 2x$, then
(A) $a + b = 0$ (B) $a^2 + b^2 = 128$ (C) $ab = 64$ (D) $a - b = 8$

17. The number of solution of the equation $\log_3|x-1| \cdot \log_4|x-1| \cdot \log_5|x-1|$
$= \log_5|x-1| + \log_3|x-1| \cdot \log_4|x-1|$ are

(A) 3 (B) 4 (C) 5 (D) 6

18. Find the number of all the integral solutions of the inequality $\dfrac{(x^2+2)(\sqrt{x^2-16})}{(x^4+2)(x^2-9)} \le 0$

(A) 1 (B) 2 (C) 3 (D) 4

19. Find the complete solution set of the inequality $\dfrac{1-\sqrt{21-4x-x^2}}{x+1} \ge 0$

(A) $\left[2\sqrt{6}-2,\ 3\right]$ (B) $\left[-2\ -2\sqrt{6},\ -1\right)$

(C) $\left[-2-2\sqrt{6}, -1\right] \cup \left[2\sqrt{6}-2,\ 3\right]$ (D) $\left[-2\ -2\sqrt{6},\ -1\right) \cup \left[2\sqrt{6}-2,\ 3\right]$

20. The solution set of the inequality $\dfrac{|x+2|-|x|}{\sqrt{4-x^3}} \ge 0$ is

(A) $[-1,\ \sqrt[3]{4})$ (B) $[1,\ \sqrt[3]{4})$ (C) $[-1,\ \sqrt[3]{2})$ (D) $[0,\ \sqrt[3]{4})$

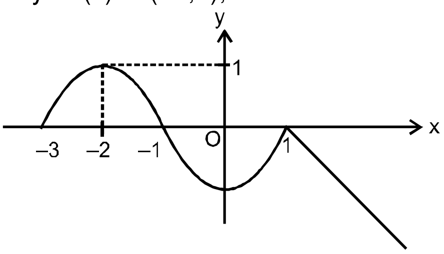
21. The number of integers satisfying the inequality $\sqrt{\log_{1/2}^2 x + 4\log_2\sqrt{x}} < \sqrt{2}\ (4 - \log_{16}x^4)$ are

(A) 2 (B) 3 (C) 4 (D) 5

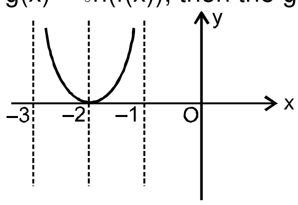
22. If $f_1(x) = ||x|-2|$ and $f_n(x) = |f_{n-1}(x) - 2|$ for all $n \ge 2$, $n \in N$, then number of solution of the equation $f_{2015}(x) = 2$ is

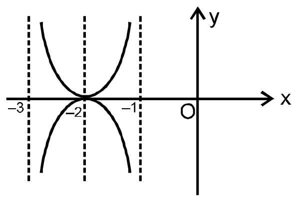
(A) 2015 (B) 2016 (C) 2017 (D) 2018

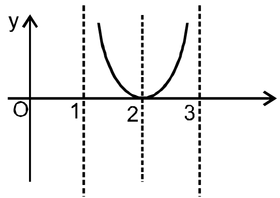
23. If graph of $y = f(x)$ in $(-3,1)$, is as shown in the following figure

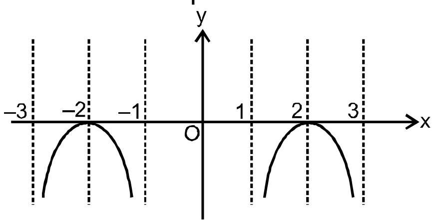


and $g(x) = \ell n(f(x))$, then the graph of $y = g(-|x|)$ is

(A)


(B)


(C)


(D)


24*. Solution set of inequality $||x| - 2| \le 3 - |x|$ consists of :

(A) exactly four integers
(B) exactly five integers
(C) Two prime natural number
(D) One prime natural number

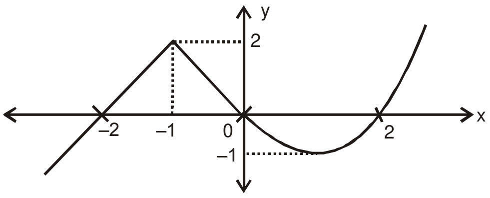
25*. If $a \ne 0$, then the inequation $|x - a| + |x + a| < b$

(A) has no solutions if $b \le 2|a|$
(B) has a solution set $\left(\frac{-b}{2}, \frac{b}{2}\right)$ if $b > 2|a|$
(C) has a solution set $\left(\frac{-b}{2}, \frac{b}{2}\right)$ if $b < 2|a|$
(D) All above

26. The equation $||x - a| - b| = c$ has four distinct real roots, then

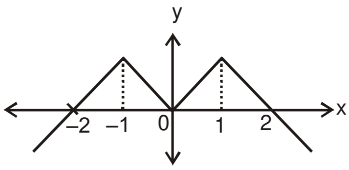
(A) $a > b - c > 0$
(B) $c > b > 0$
(C) $a > c + b > 0$
(D) $b > c > 0$

27*. If graph of $y = f(x)$ is as shown in figure

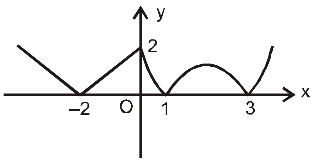


then which of the following options is/are correct ?

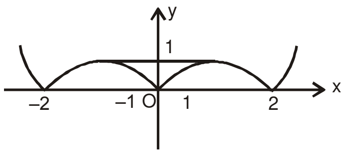
(A) Graph of $y = f(-|x|)$ is



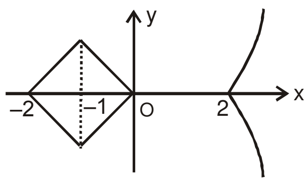
(B) Graph of $y = f(|x|)$ is



(C) Graph of $y = |f(|x|)|$ is



(D) Graph of $|y| = f(x)$ is



28*. Consider the equation $\left|x^2 - 4|x| + 3\right| = p$

(A) for p = 2 the equation has four solutions
(B) for p = 2 the equation has eight solutions
(C) there exists only one real value of p for which the equation has odd number of solutions
(D) sum of roots of the equation is zero irrespective of value of p

29*. Consider the equation $|\ell nx| + x = 2$, then

(A) The equation has two solutions (B) Both solutions are positive

(C) One root exceeds one and other in less than one (D) Both roots exceed one

30*. Consider the equation $||x - 1| - |x + 2|| = p$. Let p_1 be the value of p for which the equation has exactly one solution. Also p_2 is the value of p for which the equation has infinite solution. Let α be the sum of all the integral values of p for which this equation has solution then

(A) $p_1 = 0$ (B) $p_2 = 3$ (C) $\alpha = 6$ (D) $p_1 + p_2 = 4$

31. Number of the solution of the equation $2^x = |x - 1| + |x + 1|$ is

(A) 0 (B) 1 (C) 2 (D) ∞

32. Number of the solution of the equation $x^2 = |x - 2| + |x + 2| - 1$ is

(A) 0 (B) 3 (C) 2 (D) 4

33. f(x) is polynomial of degree 5 with leading coefficient = 1, f(4) = 0. If the curve $y = |f(x)|$ and $y = f(|x|)$ are same, then the value of f(5) is

(A) 405 (B) –405 (C) 45 (D) –45

34. The area bounded by the curve $y \geq |x - 2|$ and $y \leq 4 - |x - 3|$ is

(A) $\frac{13}{2}$ (B) 7 (C) $\frac{15}{2}$ (D) 8

Exercise-3

PART - I : JEE (ADVANCED) / IIT-JEE PROBLEMS (PREVIOUS YEARS)

✍ Marked questions are recommended for Revision.

*** Marked Questions may have more than one correct option.**

1. Draw the graph of $y = |x|^{1/2}$ for $-1 \le x \le 1$.

2. The number of real solutions of the equation $|x|^2 - 3|x| + 2 = 0$ is :
(A) 4 (B) 1 (C) 3 (D) 2

3.✍ If p, q, r are any real numbers, then
(A) max (p, q) < max (p, q, r) (B) min (p, q) = $\frac{1}{2}$ (p + q – |p – q|)
(C) max (p, q) < min(p, q, r) (D) None of these

4. Let $f(x) = |x - 1|$. Then
(A) $f(x^2) = (f(x))^2$ (B) $f(x + y) = f(x) + f(y)$ (C) $f(|x|) = |f(x)|$ (D) None of these

5. If x satisfies $|x - 1| + |x - 2| + |x - 3| \ge 6$, then
(A) $0 \le x \le 4$ (B) $x \le -2$ or $x \ge 4$ (C) $x \le 0$ or $x \ge 4$ (D) None of these

6. Solve $|x^2 + 4x + 3| + 2x + 5 = 0$.

7. If p, q, r are positive and are in A.P., then roots of the quadratic equation $px^2 + qx + r = 0$ are real for
(A) $\left|\frac{r}{p} - 7\right| \ge 4\sqrt{3}$ (B) $\left|\frac{r}{p} - 7\right| < 4\sqrt{3}$
(C) all p and r (D) no p and r

8. The function $f(x) = |ax - b| + c\,|x|\ \forall\ x \in (-\infty, \infty)$, where a > 0, b > 0, c > 0, assumes its minimum value only at one point if
(A) $a \ne b$ (B) $a \ne c$ (C) $b \ne c$ (D) a = b = c

9.✍ Find the set of all solutions of the equation $2^{|y|} - |2^{y-1} - 1| = 2^{y-1} + 1$

10. The sum of all the real roots of the equation $|x - 2|^2 + |x - 2| - 2 = 0$ is ______.

11. If α & β ($\alpha < \beta$) are the roots of the equation $x^2 + bx + c = 0$, where c < 0 < b, then
(A) $0 < \alpha < \beta$ (B) $\alpha < 0 < \beta < |\alpha|$ (C) $\alpha < \beta < 0$ (D) $\alpha < 0 < |\alpha| < \beta$

12. If $f(x) = x^2 + 2bx + 2c^2$ and $g(x) = -x^2 - 2cx + b^2$ are such that min f(x) > max g(x), then the relation between b and c, is
(A) no relation (B) 0 < c < b/2 (C) $|c| < \sqrt{2}\,|b|$ (D) $|c| > \sqrt{2}\,|b|$

PART - II : JEE (MAIN) / AIEEE PROBLEMS (PREVIOUS YEARS)

1. Product of real roots of the equation $t^2x^2 + |x| + 9 = 0$
(1) is always positive (2) is always negative (3) does not exist (4) none of these

2. The number of real solutions of the equation $x^2 - 3|x| + 2 = 0$ is
(1) 3 (2) 2 (3) 4 (4) 1

3. The sum of the roots of the equation, $x^2 + |2x - 3| - 4 = 0$, is :

(1) $-\sqrt{2}$ (2) $\sqrt{2}$ (3) -2 (4) 2

4. The equation $\sqrt{3x^2+x+5} = x - 3$, where x is real , has :

(1) exactly four solutions (2) exactly one solutions

(3) exactly two solutions (4) no solution

5. The domain of the function $f(x) = \frac{1}{\sqrt{|x|-x}}$ is :

(1) $(-\infty, \infty)$ (2) $(0, \infty)$ (3) $(-\infty, 0)$ (4) $(-\infty, \infty) - \{0\}$

6. If x is a solution of the equation, $\sqrt{2x+1} - \sqrt{2x-1} = 1$, $\left(x \ge \frac{1}{2}\right)$, then $\sqrt{4x^2-1}$ is equal to

(1) 2 (2) $\frac{3}{4}$ (3) $2\sqrt{2}$ (4) $\frac{1}{2}$

7. Let α and β be the roots of equation $px^2 + qx + r = 0$, $p \ne 0$. If p, q ,r are in the A.P. and $\frac{1}{\alpha} + \frac{1}{\beta} = 4$, then the value of $|\alpha - \beta|$ is :

(1) $\frac{\sqrt{34}}{9}$ (2) $\frac{2\sqrt{13}}{9}$ (3) $\frac{\sqrt{61}}{9}$ (4) $\frac{2\sqrt{17}}{9}$

8. Let $S = \{x \in R : x \ge 0 \text{ and } 2|\sqrt{x} - 3| + \sqrt{x}(\sqrt{x} - 6) + 6 = 0\}$. Then S :

(1) contains exactly two elements. (2) contains exactly four elements.

(3) is an empty set. (4) contains exactly one element

Answers

EXERCISE # 1

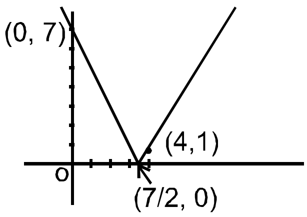
PART-I

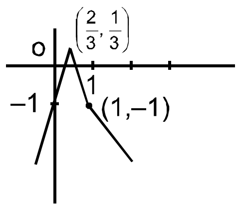
Section (A) :

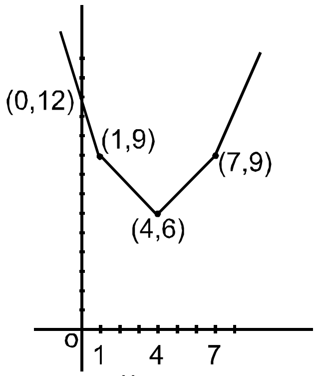
A-1. (i) $x^2 - 7x + 10,\ x > 5$ or $x \le 2$; $-(x^2 - 7x + 10),\ 2 < x \le 5$
(ii) $x^3 - x,\ x \in [-1, 0] \cup [1, \infty)$; $x - x^3,\ x \in (-\infty, -1) \cup (0, 1)$

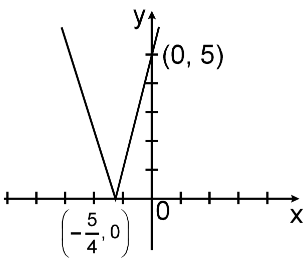
(iii) $2^x - 2,\ x \ge 1$; $2 - 2^x,\ x < 1$ (iv) $x^2 - 6x + 10,\ x \in R$

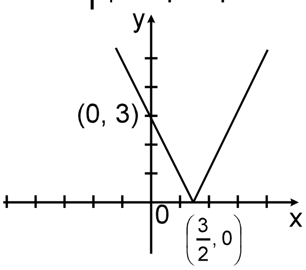
(v) $x^2 - 2x + 1,\ x \ge 2$; $4x - x^2 - 3,\ 1 \le x < 2$; $x^2 - 4x + 3,\ x < 1$
(vi) $x - 3,\ x \ge 3$; $3 - x,\ x < 3$
(vii) $2^{x-1} + x + 2 - 3^{x+1}$ $x \ge -1$; $2^{x-1} + x + 2 - 3^{-(x+1)}$ $-2 \le x < -1$
$2^{x-1} - x - 2 - 3^{-(x+1)}$ $x < -2$

A-2. (i)


(ii)


(iii)


(iv)


(v)


A-3. (i) $x = 0, 8$ (ii) $x = -10, -6, 0, 4$ (iii) $x = 0, \pm 4, 8$ (iv) $x = -\frac{11}{7}, \frac{13}{7}$

A-4. (i) ± 8 (ii) 0, 1 (iii) 0, 4 (iv) –2, 3 (v) $x \in \{-2, 2, 4\}$

A-5. (i) 4 (ii) 4 (iii) 4

A-6. (i) 0 (ii) 6 (iii) 0 (iv) 12 (v) 0

A-7. (i) 2 (ii) Infinite
(iii) $p < 4$ no solution $p = 4$ one solution $p > 4$ Two solution

A-8. 2 **A-9** 2, 3 **A-10.** 0

Section (B) :

B-1. (i) $x \in (-\infty, 1] \cup [5, \infty)$ (ii) $x = 5$ or $x = -1$ (iii) $x \in R - \{3\}$ (iv) $x \in [0, 6]$
(v) R (vi) $[-1, 3]$

B-2. (i) $x \in (-1, 0) \cup (0, 3)$ (ii) $x \in (-\infty, -4] \cup [-1, 1] \cup [4, \infty)$

(iii) $x \in (-5, -2) \cup (-1, \infty)$ (iv) $x \in \left(-\infty, -\frac{2}{3}\right] \cup \left[\frac{1}{2}, \infty\right)$ (v) $x \in \left(-\frac{2}{3}, 4\right)$

B-3. (i) $x \in (-\infty, -1] \cup [0, \infty)$ (ii) $x \in (-\infty, 1] \cup [3, \infty)$ (iii) $x \in (-\infty, 0) \cup (1, \infty)$

(iv) $x \in (2, \infty)$ (v) $(1, 5/3)$ (vi) $(2, \infty)$

B-4. (i) $\{-1\} \cup [0, \infty)$ (ii) $[1, 2] \cup [3, 4]$ (iii) $x \in \left[-\frac{1}{2}, \infty\right)$

(iv) $[1, 4] \cup \{-2\}$ (v) $\left[-5, \frac{3}{2}\right]$

B-5. $\{-2, 1\}$

Section (C) :

C-1. (i) $\log_{10}x + 2^{x-1} - 1$ $\quad x \ge 1$

$-(\log_{10}x + 2^{x-1} - 1)$ $\quad 0 < x < 1$

(ii) $(\log_2 x)^2 - 3(\log_2 x) + 2$ $\quad x \in (0, 2] \cup [4, \infty)$

$-((\log_2 x)^2 - 3(\log_2 x) + 2)$ $\quad x \in (2, 4)$

(iii) $5^{x^2-4x+5} - 25$ $\quad x \in (-\infty, 1] \cup [3, \infty)$

$25 - 5^{x^2-4x+5}$ $\quad x \in (1, 3)$

C-2. $x = 10/3, y = 20/3$ & $x = -10, y = 20$

C-3. (i) $x \in \left(0, \frac{1}{4}\right] \cup [2, \infty)$ (ii) $\left(0, \frac{1}{27}\right] \cup [3, \infty)$ (iii) $(-\infty, \log_2(\sqrt{2} - 1)] \cup \left[\frac{1}{2}, \infty\right)$

C-4. 3 **C-5.** 4 **C-6.** 2

Section (D) :

D-1. **(i)** $\left[\frac{1}{2}, 2\right) \cup (5, \infty)$ (ii) $[-1, (\sqrt{5} - 1)/2)$ (iii) $x \in [3, \infty)$

(iv) $x \in \left[\frac{7-\sqrt{21}}{2}, 2\right] \cup \left[4, \frac{7+\sqrt{21}}{2}\right]$ (v) $x = 2$

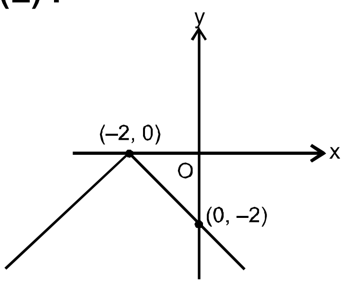
(vi) $(-\infty, -5]$ U $[1, \infty)$ (vii) $(-\infty, -3]$ (viii) $[-2, 0) \cup [\sqrt{2-\sqrt{3}}, \sqrt{2+\sqrt{3}}]$

(ix) $(-1, 1) \cup (2, 3)$

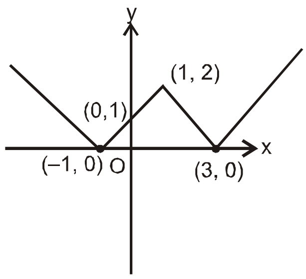
D-2. $x = \log_2 a$ where, $a \in (0, 1]$

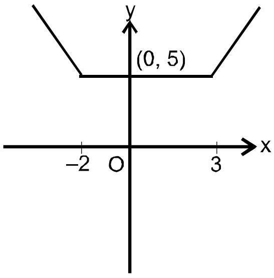
Section (E) :

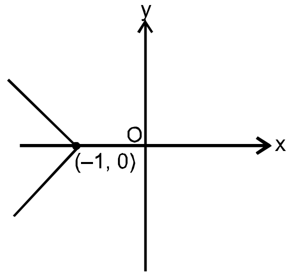
E-1. (i)

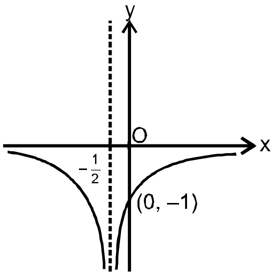


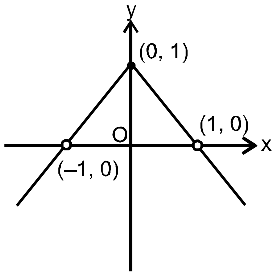
(ii)

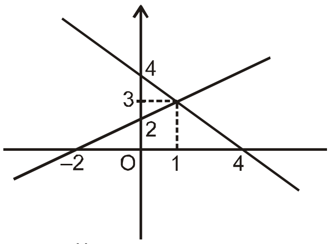


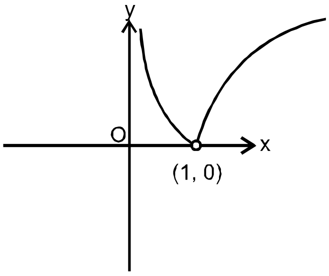
(iii) 

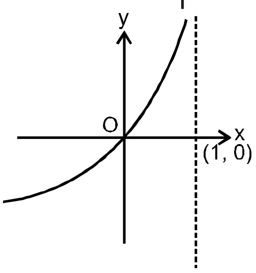
(iv) 

E-2. (i) 

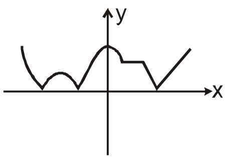
(ii) 

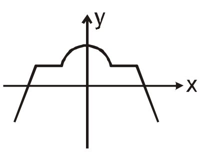
(iii) 

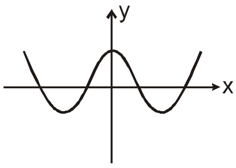
(iv) 

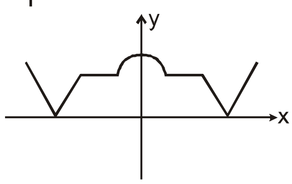
E-3. 

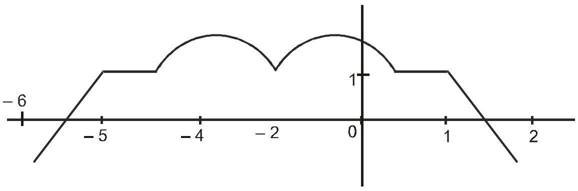
E-4. $\lambda \in (12, 16)$

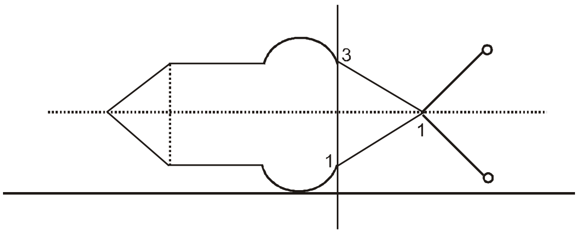
E-5. (i) 

(ii) 

(iii) 

(iv) 

E-6. (A) 

(B) 

E-7. (i) 2 (ii) 2

E-8 (i) $k \in \left(1, \frac{5}{4}\right)$ (ii) $k = 1, \frac{5}{4}$ (iii) $k = -1$ (iv) $k \in \left(\frac{5}{4} \infty\right) \cup (-1, 1)$

EXERCISE # 2

1.	(B)	**2.**	(B)	**3.**	(D)	**4.**	(A)	**5.**	(A)	**6.**	(A*)	**7.**	(C)
8.	(B)	**9.**	(A)	**10.**	(D)	**11.**	(C)	**12.**	(A)	**13.**	(D)	**14.**	(ABC)
15.	(ABC)	**16.**	(AB)	**17.**	(D)	**18.**	(B)	**19.**	(D)	**20.**	(A)	**21.**	(B)
22.	(C)	**23.**	(D)	**24.**	(BD)	**25.**	(AB)	**26.**	(D)	**27.**	(ACD)		
28.	(ACD)	**29.**	(ABC)	**30.**	(ABC)	**31.**	(C)	**32.**	(C)	**33.**	(A)	**34.**	(C)

EXERCISE # 3

PART-I

2.	(A)	**3.**	(B)	**4.**	(D)	**5.**	(C)	**6.**	$x = -1 - \sqrt{3}$	or	-4
7.	(A)	**8.**	(B)	**9.**	$\{-1\} \cup [1, \infty)$	**10.**	4	**11.**	(B)	**12.**	(D)

PART-I

1.	(3)	**2.**	(3)	**3.**	(2)	**4.**	(4)	**5.**	(3)	**6.**	(2)	**7.**	(2)
8.	(1)												