Target : JEE (Main + Advanced)

Straight Line

CONTENT

▸ Straight Line

JEE (ADVANCED) SYLLABUS

Straight Line : Two dimensions: Cartesian coordinates, distance between two points, section formulae, shift of origin. Equation of a straight line in various forms, angle between two lines, distance of a point from a line. Lines through the point of intersection of two given lines, equation of the bisector of the angle between two lines, concurrency of lines, centroid, orthocentre, incentre and circumcentre of a triangle.

JEE (MAIN) SYLLABUS

Straight Line : Various forms of equations of a line, intersection of lines, angles between two lines, conditions of for concurrence of three lines, distance of a point from a line, equations of internal and external bisectors of angles between two lines, coordinates of centroid, orthocenter and circumcentre of a triangle, equation of family of lines passing through the point of intersection of two lines.

Straight Line

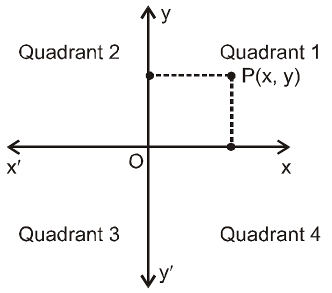
The knowledge of which geometry aims is the knowledge of the eternal...... Plato

A french mathematician and a greatest philosopher named Rene Descartes, pioneered the use of algebra in Geometry. He suggested methods to study geometry by algebraic methods without making direct reference to the actual figures

This geometry was called co-ordinate geometry or analytical geometry and it is the branch of geometry in which algebraic equations are used to denote points, lines and curves.

Rectangular cartesian co-ordinate systems :

We shall right now focus on two-dimensional co-ordinate geometry in which two perpendicular lines called co-ordinate axes (x-axis and y-axis) are used to locate a point in the plane.



O is called origin. Any point P in this plane can be represented by a unique ordered pair (x, y), which are called co-ordinates of that point. x is called x co-ordinate or abscissa and y is called y co-ordinate or ordinate. The two perpendicular lines xox′ and yoy′ divide the plane in four regions which are called quadrants, numbered as shown in the figure.

Let us look at some of the formulae linked with points now.

Distance Formula :

In rectangular Cartesian coordinate system

The distance between the points $A(x_1,y_1)$ and $B(x_2,y_2)$ is = $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$.

Example # 1 : Find the value of x, if the distance between the points (x, 8) and (4, 3) is 13

Solution : Let P(x ,8) and Q(4, 3) be the given points. Then PQ = 13 (given)

$\sqrt{(x-4)^2 + (8-3)^2} = 13 \Rightarrow (x-4)^2 + 25 = 169 \Rightarrow x = 16$ or $x = -8$

Self practice problems :

(1) Show that four points (0, –1), (6, 7) (–2, 3) and (8, 3) are the vertices of a rectangle.

(2) Find the co-ordinates of the circumcentre of the triangle whose vertices are (8, 6), (8, –2) and (2, –2). Also find its circumradius.

Ans. (2) (5, 2), 5

Section Formula :

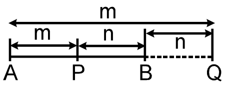
If P(x, y) divides the line joining $A(x_1, y_1)$ & $B(x_2, y_2)$ in the ratio m : n, then;

$x = \dfrac{mx_2 + nx_1}{m + n}$; $y = \dfrac{my_2 + ny_1}{m + n}$.

Notes : (i) If $\frac{m}{n}$ is positive, the division is internal, but if $\frac{m}{n}$ is negative, the division is external.

(ii) If P divides AB internally in the ratio m : n & Q divides AB externally in the ratio m : n then P & Q are said to be harmonic conjugate of each other w.r.t. AB.

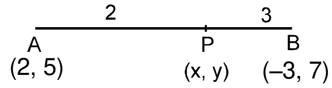
Mathematically, $\frac{2}{AB} = \frac{1}{AP} + \frac{1}{AQ}$ i.e. AP, AB & AQ are in H.P.



Example # 2 : Find the co-ordinates of the point which divides the line segment joining the points (2, 5) and (– 3, 7) in the ratio 2 : 3 (i) internally and (ii) externally.

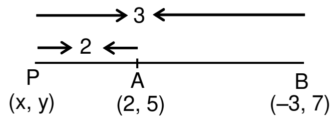
Solution : Let P (x, y) be the required point.

(i) For internal division :



$$(x, y) = \left(\frac{-6+6}{2+3}, \frac{14+15}{2+3}\right) = \left(0, \frac{29}{5}\right)$$

(ii) For external division



$$(x, y) = \left(\frac{-6-6}{2-3}, \frac{14-15}{2-3}\right) = (12, 1)$$

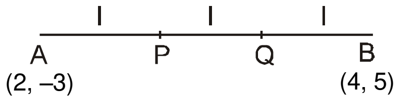
Example # 3 : Find the co-ordinates of points which trisect the line segment joining (2, – 3) and (4, 5).

Solution : Let A (2, –3) and B(4, 5) be the given points. Let the points of trisection be P and Q. Then AP = PQ = QB = λ (say)

$\therefore$ PB = PQ + QB = 2λ and AQ = AP + PQ = 2λ

$\Rightarrow$ AP : PB = λ : 2λ = 1 : 2 and AQ : QB = 2λ : λ = 2 : 1

So P divides AB internally in the ratio 1 : 2 while Q divides internally in the ratio 2 : 1



So P divides AB internally in the ratio 1:2 while Q divides internally in the ratio 2: 1

$\therefore$ the co-ordinates of P are $\left(\frac{4+4}{1+2}, \frac{5-6}{1+2}\right) = \left(\frac{8}{3}, -\frac{1}{3}\right)$

and the co-ordinates of Q are $\left(\frac{8+2}{1+2}, \frac{10-3}{1+2}\right) = \left(\frac{10}{3}, \frac{7}{3}\right)$

Hence, the points of trisection are $\left(\frac{8}{3}, -\frac{1}{3}\right)$ and $\left(\frac{10}{3}, \frac{7}{3}\right)$

Self practice problems :

(3) In what ratio does the point (4, 1) divide the line segment joining the points (1, –2) and (5, 2).

(4) The three vertices of a parallelogram taken in order are (–2, 0), (4, 2) and (5, 3) respectively. Find the co-ordinates of the fourth vertex.

Ans. (3) 3 : 1 internally (4) (–1, 1)

The ratio in which a given line divides the line segment joining two points :

Let the given line ax + by + c = 0 divide the line segment joining $A(x_1, y_1)$ & $B(x_2, y_2)$ in the ratio m : n, then $\frac{m}{n} = -\frac{ax_1 + by_1 + c}{ax_2 + by_2 + c}$. If A & B are on the same side of the given line then m/n is negative but if A & B are on opposite sides of the given line, then m/n is positive

Example#4 : Find the ratio in which the line joining the points A (1, 2) and B(– 3, 4) is divided by the line $x + y - 5 = 0$.

Solution : Let the line $x + y = 5$ divides AB in the ratio k : 1 at P

$\therefore$ co-ordinate of P are $\left(\frac{-3k+1}{k+1}, \frac{4k+2}{k+1}\right)$

Since P lies on $x + y - 5 = 0$

$\therefore$ $\frac{-3k+1}{k+1} + \frac{4k+2}{k+1} - 5 = 0 \quad \Rightarrow \quad k = -\frac{1}{2}$

$\therefore$ Required ratio is 1 : 2 externally. .

Aliter : Let the ratio is m : n

$\therefore$ $\frac{m}{n} = -\frac{(1\times1+1\times2-5)}{1\times(-3)+1\times4-5} = -\frac{1}{2}$ $\therefore$ ratio is 1 : 2 externally.

Self practice problem :

(5) If the line $2x - 3y + \lambda = 0$ divides the line joining the points A (– 1, 2) & B(– 3, – 3) internally in the ratio 2 : 3, find λ.

Ans. $\frac{18}{5}$

Slope Formula :

If θ is the angle at which a straight line is inclined to the positive direction of x–axis, & $0° \le \theta < 180°$, $\theta \ne 90°$, then the slope of the line, denoted by m, is defined by $m = \tan\theta$. If θ is 90°, m does not exist, but the line is parallel to the y–axis. If $\theta = 0$, then m = 0 & the line is parallel to the x-axis. If A (x_1, y_1) & B (x_2, y_2), $x_1 \ne x_2$, are points on a straight line, then the slope m of the line is given by : $m = \left(\frac{y_1 - y_2}{x_1 - x_2}\right)$.

Example # 5 : What is the slope of a line whose inclination with the positive direction of x-axis is :

(i) 30º (ii) 90º (iii) 135º

Solution : (i) Here $\theta = 30º$

Slope = $\tan\theta = \tan 30º = \frac{1}{\sqrt{3}}$.

(ii) Here $\theta = 90º$

$\therefore$ The slope of line is not defined.

(iii) Here $\theta = 135º$

$\therefore$ Slope = $\tan\theta = \tan 135º = \tan(180º - 45º) = -\tan 45º = -1$.

Example # 6 : Find the slope of the line passing through the points :

(i) (2, 7) and (– 3, 4) (ii) (6, 9) and (–2, 7)

Solution : (i) Let A = (2, 7) and B = (– 3, 4)

$\therefore$ Slope of AB = $\frac{4-7}{-3-2} = \frac{3}{5}$ $\left(\text{Using slope} = \frac{y_2 - y_1}{x_2 - x_1}\right)$

(ii) Let A = (6, 9), B = (–2, 7) $\therefore$ Slope of AB = $\frac{7-9}{-2-6} = \frac{1}{4}$

Self practice problems :

(6) Find the value of x, if the slope of the line joining (1, 5) and (x, –7) is 4.

(7) What is the inclination of a line whose slope is

(i) 0 (ii) 1 (iii) –1 (iv) $-1/\sqrt{3}$

Ans. (6) – 2 (7) (i) 0º, (ii) 45º, (iii) 135º, (iv) 150º

Condition of collinearity of three points :

Points A (x_1, y_1), B (x_2, y_2), C(x_3, y_3) are collinear if

(i) $m_{AB} = m_{BC} = m_{CA}$ i.e. $\left(\dfrac{y_1 - y_2}{x_1 - x_2}\right) = \left(\dfrac{y_2 - y_3}{x_2 - x_3}\right)$

(ii) Δ ABC = 0 i.e. $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$

(iii) AC = AB + BC or AB ~ BC

(iv) A divides the line segment BC in some ratio.

Example#7 : Show that the points (–2, –1), (2, 7) and (5, 13) are collinear.

Solution : Let (–2, –1) (2, 7) and (5, 13) be the co-ordinates of the points A, B and C respectively.

Slope of AB = $\dfrac{7+1}{2+2}$ = 2 and Slope of BC = $\dfrac{13-7}{5-2}$ = 2

$\therefore$ Slope of AB = slope of BC

$\therefore$ AB & BC are parallel

$\therefore$ A, B, C are collinear because B is on both lines AB and BC.

Self practice problem :

(8) Prove that the points (a, 0), (0, b) and (1, 1) are collinear if $\dfrac{1}{a} + \dfrac{1}{b} = 1$

Area of a Triangle :

If $A(x_1, y_1)$, $B(x_2, y_2)$, $C(x_3, y_3)$ are the vertices of triangle ABC, then its area is equal to

$\Delta \text{ ABC} = \dfrac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$, provided the vertices are considered in the counter clockwise sense. The above formula will give a $(-)$ve area if the vertices (x_i, y_i), i = 1, 2, 3 are placed in the clockwise sense.

Note: Area of n-sided polygon formed by points (x_1, y_1) ; (x_2, y_2); ....;(x_n, y_n) is given by

$$\frac{1}{2}\left(\begin{vmatrix} x_1 & x_2 \\ y_1 & y_2 \end{vmatrix} + \begin{vmatrix} x_2 & x_3 \\ y_2 & y_3 \end{vmatrix} + \ldots\ldots\ldots\ldots \begin{vmatrix} x_{n-1} & x_n \\ y_{n-1} & y_n \end{vmatrix} + \begin{vmatrix} x_n & x_1 \\ y_n & y_1 \end{vmatrix}\right).$$

Here vertices are taken in order.

Example # 8 : If the co-ordinates of two points A and B are (2, 1) and (4, –3)respectively. Find the co-ordinates of any point P if PA = PB and Area of ΔPAB = 6.

Solution : Let the co-ordinates of P be (x, y). Then

PA = PB $\Rightarrow$ $PA^2 = PB^2$

$\Rightarrow$ $(x-2)^2 + (y-1)^2 = (x-4)^2 + (y+3)^2$ $\Rightarrow$ x – 2y = 5 ....(i)

Now, Area of ΔPAB = 6 $\Rightarrow$ $\dfrac{1}{2}\begin{vmatrix} x & y & 1 \\ 2 & 1 & 1 \\ 4 & -3 & 1 \end{vmatrix} = \pm 6$ $\Rightarrow$ 4x + 2y – 10 = ± 12

$\Rightarrow$ 4x + 2y = 22 or 4x + 2y = –2 $\Rightarrow$ 2x + y = 11 or 2x + y = –1

Solving 2x + y = 11 and x – 2y = 5 we get $x = \dfrac{27}{5}$, $y = \dfrac{1}{5}$.

Solving 2x + y = –1 and x – 2y = 5, we get $x = \dfrac{3}{5}$, $y = -\dfrac{11}{5}$.

Thus, the co-ordinates of P are $\left(\dfrac{27}{5}, \dfrac{1}{5}\right)$ or $\left(\dfrac{3}{5}, -\dfrac{11}{5}\right)$

Self practice problems :

(9) The area of a triangle is 5. Two of its vertices are (2, 1) and (3, –2). The third vertex lies on y = x + 3. Find the third vertex.

(10) The coordinates of A, B, C are (6, 3), (–3, 5) & (4,–2) respectively and p is any point (x, y).Show that the ratio of the areas of the triangles PBC and ABC is $\left|\frac{x+y-2}{7}\right|$

Ans. (9) $\left(\frac{7}{2}, \frac{13}{2}\right)$ or $\left(-\frac{3}{2}, \frac{3}{2}\right)$

Equation of a Straight Line in various forms :

Now let us understand, how a line can be represented with the help of an algebraic equation. A moving point (point with variable co-ordinates) is assumed on the line and a link is established between its co-ordinates with the help of some given parameters. There are various forms of lines depending on the specified parameter

Point - Slope form :

$y - y_1 = m (x - x_1)$ is the equation of a straight line whose slope is m & which passes through the point (x_1, y_1).

Example # 9 : Find the equation of a line passing through (3, –4) and inclined at an angle of 150º with the positive direction of x-axis.

Solution : Here, m = slope of the line = tan 150º = tan (90º + 60º) =– cot 60º = $-\frac{1}{\sqrt{3}}$, $x_1 = 3$, $y_1 = -4$

So, the equation of the line is $y - y_1 = m (x - x_1)$

i.e. $y + 4 = -\frac{1}{\sqrt{3}} (x - 3)$

$x + \sqrt{3}\, y + 4\sqrt{3} - 3 = 0$

Self practice problem :

(11) Find the equation of s a line passing through P(–3, 5) and whose slope is –2.
Ans. 2x + y + 1 = 0

Slope-intercept form :

y = mx + c is the equation of a straight line whose slope is m & which makes an intercept c on the y–axis.

Example # 10 : Find the equation of a line with slope –3 and cutting off an intercept of 5 units on negative direction of y-axis.
Solution : Here m = –3 and c = – 5. So, the equation of the line is y = mx + c
i.e. y = –3x – 5 or 3x + y + 5 =0

Self practice problem :

(12) Find the equation of a straight line which cuts off an intercept of length 3 on y- axis and whose slope is – 3.
Ans. 3x + y – 3 = 0

Two point form :

$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1} (x - x_1)$ is the equation of a straight line which passes through the points (x_1, y_1) & (x_2, y_2).

Example # 11 : Find the equation of the line joining the points (3, 4) and (–2, 5)
Solution : Here the two points are $(x_1, y_1) = (3, 4)$ and $(x_2, y_2) = (–2, 5)$.
So, the equation of the line in two-point form is

$y - 4 = \frac{5-4}{-2-3} (x - 3) \Rightarrow x + 5y = 23$

Self practice problem :

(13) Find the equations of the sides of the triangle whose vertices are (–1, 8), (4, –2) and (–5, –3). Also find the equation of the median through (–1, 8)

Ans. $2x + y - 6 = 0$, $x - 9y - 22 = 0$, $11x - 4y + 43 = 0$, $21x + y + 13 = 0$

Determinant form : Equation of line passing through (x_1, y_1) and (x_2, y_2) is $\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0$

Example # 12 : Find the equation of line passing through (2, 4) & (– 1, 3).

Solution : $\begin{vmatrix} x & y & 1 \\ 2 & 4 & 1 \\ -1 & 3 & 1 \end{vmatrix} = 0 \quad \Rightarrow \quad x - 3y + 10 = 0$

Self practice problem :

(14) Find the equation of the passing through (– 2, 3) & (– 1, – 1).

Ans. $4x + y + 5 = 0$

Intercept form : $\frac{x}{a} + \frac{y}{b} = 1$ is the equation of a straight line which makes intercepts a & b on OX & OY respectively.

Example # 13 : Find the equation of the line which passes through the point (–3, 8) and the sum of its intercepts on the axes is 7.

Solution : Let the equation of the line be $\frac{x}{a} + \frac{y}{b} = 1$ ....(i)

This passes through (–3, 8), therefore $-\frac{3}{a} + \frac{8}{b} = 1$ ....(ii)

It is given that $a + b = 7 \Rightarrow b = 7 - a$.

Putting $b = 7 - a$ in (ii), we get $-\frac{3}{a} + \frac{8}{7-a} = 1$

$\Rightarrow a^2 + 4a - 21 = 0 \Rightarrow a = 3, -7$

For a = 3, b = 4 and for a = –7, b = 14

Putting the values of a and b in (i), we get the equations of the lines

$\frac{x}{3} + \frac{y}{4} = 1$ and $\frac{x}{-7} + \frac{y}{14} = 1$ or $4x + 3y = 12$ and $2x - y + 14 = 0$

Self practice problem :

(15) Find the equation of the line through (2, 3) so that the segment of the line intercepted between the axes is bisected at this point.

Ans. $3x + 2y = 12$.

Perpendicular/Normal form :

$x\cos\alpha + y\sin\alpha = p$ (where $p > 0$, $0 \le \alpha < 2\pi$) is the equation of the straight line where the length of the perpendicular from the origin O on the line is p and this perpendicular makes an angle α with positive x–axis.

Example # 14 : Find the equation of the line which is at a distance 3 from the origin and the perpendicular from the origin to the line makes an angle of 30º with the positive direction of the x-axis.

Solution : Here $p = 3$, $\alpha = 30º$

$\therefore$ Equation of the line in the normal form is

$x\cos 30º + y\sin 30º = 3$ or $x\frac{\sqrt{3}}{2} + \frac{y}{2} = 3$ or $\sqrt{3}x + y = 6$

Self practice problem :

(16) The length of the perpendicular from the origin to a line is 7 and the line makes an angle of 150º with the positive direction (clock-wise) of y-axis. Find the equation of the line.

Ans. $\sqrt{3}\,x - y + 14 = 0$

General Form : ax + by + c = 0 is the equation of a straight line in the general form

In this case, slope of line $= -\dfrac{a}{b}$

x - intercept $= -\dfrac{c}{a}$, y - intercept $= -\dfrac{c}{b}$

Example # 15 : Find slope, x-intercept & y-intercept of the line 2x – 3y + 5 = 0.

Solution : Here, a = 2, b = – 3, c = 5

$\therefore$ slope $= -\dfrac{a}{b} = \dfrac{2}{3}$

x-intercept $= -\dfrac{c}{a} = -\dfrac{5}{2}$

y-intercept $= \dfrac{5}{3}$

Self practice problem :

(17) Find the slope, x-intercept & y-intercept of the line 3x – 5y – 8 = 0.

Ans. (17) $\dfrac{3}{5}, \dfrac{8}{3}, -\dfrac{8}{5}$

Parametric form :

$P(r) = (x, y) = (x_1 + r\cos\theta,\ y_1 + r\sin\theta)$ or $\dfrac{x - x_1}{\cos\theta} = \dfrac{y - y_1}{\sin\theta} = r$ is the equation of the line in parametric form, where 'r' is the parameter whose absolute value is the distance of any point (x, y) on the line from the fixed point (x_1, y_1) on the line.

Remark : The above form is derived from point-slope form of line.

$y - y_1 = m(x - x_1)$ where $m = \tan\theta$ $\Rightarrow$ $y - y_1 = \dfrac{\sin\theta}{\cos\theta}(x - x_1)$

Example # 16 : Find the equation of the line through the point A(1, 4) and making an angle of 45º with the positive direction of x-axis. Also determine the length of intercept on it between A and the line x + y – 10 = 0

Solution : The equation of a line through A and making an angle of 45º with the x-axis is

$\dfrac{x-1}{\cos 45^\circ} = \dfrac{y-4}{\sin 45^\circ}$ or $\dfrac{x-1}{\frac{1}{\sqrt{2}}} = \dfrac{y-4}{\frac{1}{\sqrt{2}}}$ or $x - y + 3 = 0$

Suppose this line meets the line x + y – 10 = 0 at P such that AP = r. Then the co-ordinates of P are given by

$\dfrac{x-1}{\cos 45^\circ} = \dfrac{y-4}{\sin 45^\circ} = r$ $\Rightarrow$ $x = 1 + r\cos 45^\circ,\ y = 4 + r\sin 45^\circ$

$\Rightarrow$ $x = 1 + \dfrac{r}{\sqrt{2}}$, $y = 4 + \dfrac{r}{\sqrt{2}}$

Thus, the co-ordinates of P are $\left(1 + \dfrac{r}{\sqrt{2}},\ 4 + \dfrac{r}{\sqrt{2}}\right)$

Since P lies on x + y – 10 = 0, so $1 + \dfrac{r}{\sqrt{2}} + 4 + \dfrac{r}{\sqrt{2}} = 10$

$\Rightarrow \quad 5 + \sqrt{2}\,r = 10 \Rightarrow r = \dfrac{5}{\sqrt{2}} \Rightarrow$ length AP = $|r| = \dfrac{5}{\sqrt{2}}$

Thus, the length of the intercept = $\dfrac{5}{\sqrt{2}}$.

Self practice problem :

(18) A straight line is drawn through the point $A\left(\sqrt{3}, 2\right)$ making an angle of $\pi/6$ with positive direction of the x-axis. If it meets the straight line $\sqrt{3x} - 4y + 8 = 0$ in B, find the distance between A and B.

Ans. 6 units

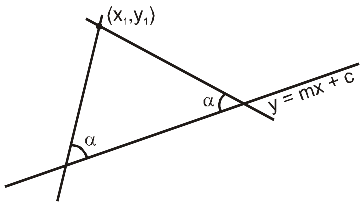
Angle between two straight lines in terms of their slopes:

If m_1 & m_2 are the slopes of two intersecting straight lines ($m_1 m_2 \neq -1$) & θ is the acute angle between them, then $\tan\theta = \left|\dfrac{m_1 - m_2}{1 + m_1 m_2}\right|$.

Notes : (i) Let m_1, m_2, m_3 are the slopes of three lines $L_1 = 0; L_2 = 0; L_3 = 0$ where $m_1 > m_2 > m_3$, then the tangent of interior angles of the Δ ABC formed by these lines are given by, $\tan A = \dfrac{m_1 - m_2}{1 + m_1 m_2}$; $\tan B = \dfrac{m_2 - m_3}{1 + m_2 m_3}$ & $\tan C = \dfrac{m_3 - m_1}{1 + m_3 m_1}$

(ii) The equation of lines passing through point (x_1, y_1) and making angle α with the line $y = mx + c$ are given by :

$(y - y_1) = \tan(\theta - \alpha)(x - x_1)$ & $(y - y_1) = \tan(\theta + \alpha)(x - x_1)$, where $\tan\theta = m$.



Example # 17 : The acute angle between two lines is $\pi/4$ and slope of one of them is $-1/3$. Find the slope of the other line.

Solution : If θ be the acute angle between the lines with slopes m_1 and m_2, then $\tan\theta = \left|\dfrac{m_1 - m_2}{1 + m_1 m_2}\right|$

Let $\theta = \dfrac{\pi}{4}$ and m_1 $-1/3$

$\therefore \quad \tan\dfrac{\pi}{4} = \left|\dfrac{-\frac{1}{3} - m_2}{1 - \frac{1}{3}m_2}\right| \Rightarrow 1 = \left|\dfrac{3m_2 + 1}{3 - m_2}\right| \Rightarrow \dfrac{3m_2 + 1}{3 - m_2} = 1 \text{ or } -1$

Now $\dfrac{3m_2 + 1}{3 - m_2} = 1 \Rightarrow m_2 = \dfrac{1}{2}$ and $\dfrac{3m_2 + 1}{3 - m_2} = -1 \Rightarrow m_2 = -2.$

$\therefore$ The slope of the other line is either 1/2 or –2

Example # 18 : Find the equation of the straight line which passes through the point (3,–2) and making angle 60º with the line $\sqrt{3}$ x + y =1.

Solution : Given line is $\sqrt{3}$ x + y =1..

$\Rightarrow$ Slope of (1) = $-\sqrt{3}$.

Let slope of the required line be m. Also between these lines is given to be 60º.

$$\Rightarrow \quad \tan 60^\circ = \left|\frac{m-(-\sqrt{3})}{1+m\ (-\sqrt{3})}\right| \quad \Rightarrow \quad \sqrt{3} = \left|\frac{m+\sqrt{3}}{1-\sqrt{3}m}\right| \quad \Rightarrow \quad \frac{m+\sqrt{3}}{1-\sqrt{3}m} = \pm\sqrt{3}$$

$$\frac{m+\sqrt{3}}{1-\sqrt{3}m} = \sqrt{3} \quad \Rightarrow \quad m + \sqrt{3} = \sqrt{3} - 3m \quad \Rightarrow \quad m = 0$$

the equation of the required line is y + 2 = 0

$$\frac{m+\sqrt{3}}{1-\sqrt{3}m} = -\sqrt{3} \quad \Rightarrow \quad m = \sqrt{3}$$

$\therefore$ The equation of the required line is y +2 = $\sqrt{3}$ (x–3) $\Rightarrow$ y $\sqrt{3}$ = x – 2 –3 $\sqrt{3}$

Self practice problem :

(19) A vertex of an equilateral triangle is (2, 3) and the equation of the opposite side is x + y = 2. Find the equation of the other sides of the triangle.

Ans. (19) (2 – $\sqrt{3}$)x – y + 2 $\sqrt{3}$ – 1 = 0 and (2 + $\sqrt{3}$) x – y – 2 $\sqrt{3}$ – 1 = 0.

Parallel Lines :

(i) When two straight lines are parallel their slopes are equal. Thus any line parallel to y = mx + c is of the type y = mx + d, where 'd' is a parameter.

(ii) Two lines ax + by + c = 0 and a′x + b′y + c′ = 0 are parallel if $\frac{a}{a'} = \frac{b}{b'} \neq \frac{c}{c'}$.

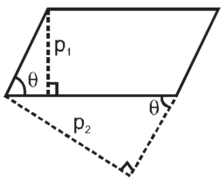
Thus any line parallel to ax + by + c = 0 is of the type ax + by + k = 0, where k is a parameter.

(iii) The distance between two parallel lines with equations ax + by + c_1 = 0 &

ax + by + c_2 = 0 is = $\left|\frac{c_1 - c_2}{\sqrt{a^2+b^2}}\right|$.

Note that coefficients of x & y in both the equations must be same.

(iv) The area of the parallelogram = $\frac{p_1 p_2}{\sin\theta}$, where p_1 & p_2 are distances between two pairs of opposite sides & θ is the angle between any two adjacent sides. Note that area of the parallelogram bounded by the lines y = m_1x + c_1, y = m_1x + c_2 and y = m_2x + d_1, y = m_2x + d_2 is given by $\left|\frac{(c_1-c_2)\ (d_1-d_2)}{m_1-m_2}\right|$.



Example # 19 : Find the equation of the straight line that has y-intercept 5 and is parallel to the straight line $3x - 7y = 8$.

Solution : Given line is $3x - 7y = 8$

$\therefore$ Slope of (1) is $\frac{3}{7}$

The required line is parallel to (1), so its slope is also 3/7, y-intercept of required line = 5

$\therefore$ By using $y = mx + c$ form, the equation of the required line is

$y = \frac{3}{7}x + 5$ or $3x - 7y + 35 = 0$

Example # 20 : Two sides of a square lie on the lines $5x - 12y + 6 = 0$ and $5x - 12y = 20$. What is its area ?

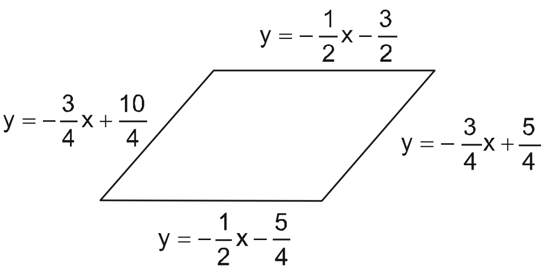
Solution : Clearly the length of the side of the square is equal to the distance between the parallel lines
$5x - 12y + 6 = 0$........(i) and $5x - 12y = 20$ ....(ii)
Now, Distance between the parallel lines

$= \frac{|6+20|}{\sqrt{5^2+(-12)^2}} = 2$

Thus, the length of the side of the square is 2 and hence its area = 4

Example # 21 : Find the area of the parallelogram whose sides are $x + 2y + 3 = 0$, $3x + 4y - 5 = 0$,
$2x + 4y + 5 = 0$ and $3x + 4y - 10 = 0$

Solution :



Here, $c_1 = -\frac{3}{2}$, $c_2 = -\frac{5}{4}$, $d_1 = \frac{10}{4}$, $d_2 = \frac{5}{4}$, $m_1 = -\frac{1}{2}$, $m_2 = -\frac{3}{4}$

$\therefore$ Area = $\left| \frac{\left(-\frac{3}{2}+\frac{5}{4}\right)\left(\frac{10}{4}-\frac{5}{4}\right)}{\left(-\frac{1}{2}+\frac{3}{4}\right)} \right| = \frac{5}{4}$ sq. units

Self practice problem :

(20) Find the area of parallelogram whose sides are given by $4x - 5y + 1 = 0$, $x - 3y - 6 = 0$,
$4x - 5y - 2 = 0$ and $2x - 6y + 5 = 0$

Ans. (20) $\frac{51}{14}$ sq. units

Perpendicular Lines:

(i) When two lines of slopes m_1 & m_2 are at right angles, the product of their slopes is -1, i.e. $m_1 m_2 = -1$. Thus any line perpendicular to $y = mx + c$ is of the form

$y = -\frac{1}{m}x + d$, where 'd' is any parameter.

(ii) Two lines $ax + by + c = 0$ and $a'x + b'y + c' = 0$ are perpendicular if $aa' + bb' = 0$. Thus any line perpendicular to $ax + by + c = 0$ is of the form $bx - ay + k = 0$, where 'k' is any parameter.

Example # 22 : Find the equation of the straight line that passes through the point (3, 4) and perpendicular to the line $3x + 2y + 5 = 0$

Solution : The equation of a line perpendicular to $3x + 2y + 5 = 0$ is

$2x - 3y + \lambda = 0$ ....(i)

This passes through the point (3, 4)

$\therefore$ $3 \times 2 - 3 \times 4 + \lambda = 0 \Rightarrow \lambda = 6$

Putting $\lambda = 6$ in (i), we get $2x - 3y + 6 = 0$, which is the required equation.

<u>Aliter</u> The slope of the given line is –3/2. Since the required line is perpendicular to the given line. So, the slope of the required line is 2/3. As it passes through (3, 4). So, its equation is $y - 4 = \frac{2}{3}(x - 3)$

or $2x - 3y + 6 = 0$

Self practice problem :

(21) The vertices of a triangle are A(10, 4), B (–4, 9) and C(–2, –1). Find the equation of its altitudes. Also find its orthocentre.

Ans. (21) $x - 5y + 10 = 0$, $12x + 5y + 3 = 0$, $14x - 5y + 23 = 0$, $\left(-1, \frac{9}{5}\right)$

Position of the point (x_1, y_1) relative of the line $ax + by + c = 0$:

If $ax_1 + by_1 + c$ is of the same sign as c, then the point (x_1, y_1) lie on the origin side of $ax + by + c = 0$. But if the sign of $ax_1 + by_1 + c$ is opposite to that of c, the point (x_1, y_1) will lie on the non–origin side of $ax + by + c = 0$.

In general two points (x_1, y_1) and (x_2, y_2) will lie on same side or opposite side of $ax + by + c = 0$ according as $ax_1 + by_1 + c$ and $ax_2 + by_2 + c$ are of same or opposite sign respectively.

Example # 23 : Show that (2, –1) and (–3, 3) lie on the opposite sides of the line $2x - 3y + 5 = 0$.

Solution : At (2, –1), the value of $2x - 3y + 5 = 4 + 3 + 5 = 12 > 0$.

At (–3, 3), the value of $2x - 3y + 5 = -6 - 9 + 5 = -10 < 0$

$\therefore$ The points (2, –1) and (–3, 3) are on the opposite sides of the given line.

Self practice problems :

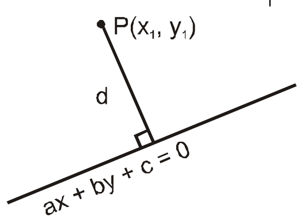
(22) Are the points (3, – 4) and (2, 6) on the same or opposite side of the line $3x - 4y = 8$?

(23) Which one of the points (1, 1), (–1, 2) and (2, 3) lies on the side of the line $4x + 3y - 5 = 0$ on which the origin lies?

Ans. (22) Opposite sides (23) (–1, 2)

Length of perpendicular from a point on a line :

The length of perpendicular from $P(x_1, y_1)$ on $ax + by + c = 0$ is $\left|\frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}}\right|$.



Example # 24 : Find the distance between the line 4x – 3y + 8 = 0 and the point (–2, 3)

Solution : The required distance = $\left|\dfrac{(-2)\times 4-3\times 3+8}{\sqrt{4^2+(-3)^2}}\right| = \dfrac{9}{5}$

Example # 25 : Find all points on x – y + 2 = 0 that lie at a unit distance from the line 12x – 5y + 9 = 0.

Solution : Note that the co-ordinates of an arbitrary point on x – y + 2 = 0 can be obtained by putting x = t (or y = t) and then obtaining y (or x) from the equation of the line, where t is a parameter.
Putting x = t in the equation x – y + 2 = 0 of the given line, we obtain y = 2 + t.
So, co-ordinates of an arbitrary point on the given line are P(t, 2 + t). Let P(t, 2 + t) be the required point. Then, distance of P from the line 12x – 5y + 9 = 0 is unity i.e.

$\Rightarrow \quad \left|\dfrac{12t-5(t+2)+9}{\sqrt{12^2+5^2}}\right| = 1 \quad \Rightarrow \quad |7t-1| = 13$

$\Rightarrow \quad 7t - 1 = \pm 13 \quad \Rightarrow \quad t = 2$ or $t = -12/7$

Hence, required points are (2,4) or (–12/7,2/7)

Self practice problem :

(24) Find the length of the altitudes from the vertices of the triangle with vertices : (–1, 1), (5, 2) and (3, –1).

Ans. (24) $\dfrac{16}{\sqrt{13}}, \dfrac{8}{\sqrt{5}}, \dfrac{16}{\sqrt{37}}$

Reflection of a point about a line :

(i) Foot of the perpendicular from a point (x_1, y_1) on the line ax + by + c = 0 is

$$\frac{x-x_1}{a} = \frac{y-y_1}{b} = -\left(\frac{ax_1+by_1+c}{a^2+b^2}\right)$$

(ii) The image of a point (x_1, y_1) about the line ax + by + c = 0 is

$$\frac{x-x_1}{a} = \frac{y-y_1}{b} = -2\left(\frac{ax_1+by_1+c}{a^2+b^2}\right)$$

Example # 26 : Find the foot of perpendicular of the line drawn from P (2, –3) on the line x – 2y + 5 = 0.

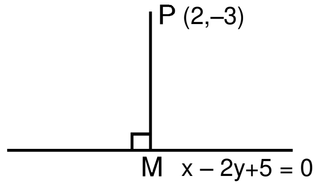
Solution : Slope of PM = – 2

$\therefore$ Equation of PM is

2x + y = 1 ....(i)

solving equation (i) with x – 2y + 5 = 0, we get

co-ordinates of M (–3/5, 11/5)



Aliter Here, $\dfrac{x-2}{1} = \dfrac{y+3}{-2} = -\dfrac{2+6+5}{1+4} \Rightarrow \dfrac{x-2}{1} = \dfrac{y+3}{-2} = -\dfrac{13}{5} \Rightarrow x = -3/5$, y = 11/5

Example # 27 : Find the image of the point P(–1, 2) in the line mirror 2x – 3y + 4 = 0.

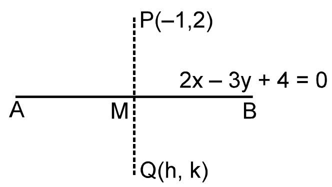
Solution : Let image of P is Q.

$\therefore$ PM = MQ & PQ $\perp$ AB

Let Q is (h, k)

$\therefore$ M is $\left(\dfrac{h-1}{2}, \dfrac{k+2}{2}\right)$

It lies on 2x – 3y + 4 = 0.



$\therefore \quad 2\left(\frac{h-1}{2}\right) - 3\left(\frac{k+2}{2}\right) + 4 = 0.$

or $\quad 2h - 3k = 0 \quad$ ....(i)

slope of PQ = $\frac{k-2}{h+1}$

PQ $\perp$ AB

$\therefore \quad \frac{k-2}{h+1} \times \frac{2}{3} = -1. \Rightarrow \quad 3h + 2k - 1 = 0. \quad$ ....(ii)

soving (i) & (ii), we get $h = \frac{3}{13}$, $k = \frac{2}{13}$

$\therefore$ Image of P(– 1, 2) is $Q\left(\frac{3}{13}, \frac{2}{13}\right)$

Aliter The image of P (– 1, 2) about the line

$2x - 3y + 4 = 0$ is $\frac{x+1}{2} = \frac{y-2}{-3} = -2\,\frac{[2(-1)-3(2)+4]}{2^2+(-3)^2}$

$\frac{x+1}{2} = \frac{y-2}{-3} = \frac{8}{13}$

$\Rightarrow \quad 13x + 13 = 16 \Rightarrow x = \frac{3}{13} \quad \& \quad 13y - 26 = -24 \quad \Rightarrow y = \frac{2}{13}$

$\therefore \quad$ image is $\left(\frac{3}{13}, \frac{2}{13}\right)$

Self practice problems :

(25) Find the foot of perpendicular of the line drawn from (– 2, – 3) on the line 3x – 2y – 1 = 0.

(26) Find the image of the point (1, 2) in y-axis.

Ans. (25) $\left(\frac{-23}{13}, \frac{-41}{13}\right)$ (26) (– 1, 2)

Centroid, Incentre & Excentre :

If A (x_1, y_1), B(x_2, y_2), C(x_3, y_3) are the vertices of triangle ABC, whose sides BC, CA, AB are of lengths a, b, c respectively, then the co-ordinates of the special points of triangle ABC are as follows :

Centroid $G \equiv \left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$

Incentre $I \equiv \left(\frac{ax_1+bx_2+cx_3}{a+b+c}, \frac{ay_1+by_2+cy_3}{a+b+c}\right)$, and

Excentre (to A) $I_1 \equiv \left(\frac{-ax_1+bx_2+cx_3}{-a+b+c}, \frac{-ay_1+by_2+cy_3}{-a+b+c}\right)$ and so on.

Notes : (i) Incentre divides the angle bisectors in the ratio, (b + c) : a; (c + a) : b & (a + b) : c.

(ii) Incentre and excentre are harmonic conjugate of each other w.r.t. the angle bisector on which they lie.

(iii) Orthocentre, Centroid & Circumcentre are always collinear & centroid divides the line joining orthocentre & circumcentre in the ratio 2 : 1.

(iv) In an isosceles triangle G, O, I & C lie on the same line and in an equilateral triangle, all these four points coincide.

(v) In a right angled triangle orthocentre is at right angled vertex and circumcentre is mid point of hypotenuse

(vi) In case of an obtuse angled triangle circumcentre and orthocentre both are out side the triangle.

Example # 28: Find the co-ordinates of (i) centroid (ii) in-centre of the triangle whose vertices are (0, 0), (5, 0) and (0, 12).

Solution : (i) We know that the co-ordinates of the centroid of a triangle whose angular points are (x_1, y_1), (x_2, y_2) (x_3, y_3) are $\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$

So the co-ordinates of the centroid of a triangle whose vertices are (0, 0), (5, 0) and (0, 12) are $\left(\frac{0+5+0}{3}, \frac{0+0+12}{3}\right)$ or $\left(\frac{5}{3}, 4\right)$.

(ii) Let A (0, 0), B (5, 0) and C(0, 12) be the vertices of triangle ABC.
Then c = AB = 5, b = CA =12
and a = BC = 13.

The co-ordinates of the in-centre are $\left(\frac{ax_1 + bx_2 + cx_3}{a+b+c}, \frac{ay_1 + by_2 + cy_3}{a+b+c}\right)$

or $\left(\frac{13\times0+12\times5+5\times0}{5+12+13}, \frac{13\times0+12\times0+5\times12}{5+12+13}\right)$ or (2, 2)

Self practice problems :

(27) Two vertices of a triangle are (3, –5) and (–7, 4). If the centroid is (2, –1), find the third vertex.

(28) Find the co-ordinates of the centre of the circle inscribed in a triangle whose vertices are (– 36, 7), (20, 7) and (0, – 8)

Ans. (27) (10, – 2) (28) (–1, 0)

Bisectors of the angles between two lines:

Equations of the bisectors of angles between the lines ax + by + c = 0 & $a'x + b'y + c' = 0$ $(ab' \neq a'b)$ are : $\frac{ax + by + c}{\sqrt{a^2 + b^2}} = \pm\frac{a'x + b'y + c'}{\sqrt{a'^2 + b'^2}}$

Note : Equation of straight lines passing through $P(x_1, y_1)$ & equally inclined with the lines $a_1x + b_1y + c_1 = 0$ & $a_2x + b_2y + c_2 = 0$ are those which are parallel to the bisectors between these two lines & passing through the point P.

Example # 29 : Find the equations of the bisectors of the angle between the straight lines 3x + y + 1 = 0 and x + 3y + 1 = 0.

Solution : The equations of the bisectors of the angles between 3x+y+1= 0 and x+3y+1=0 are $\frac{3x+y+1}{\sqrt{3^2+1^2}}$

$= \pm\frac{x+3y+1}{\sqrt{1^2+3^2}}$ or 3x + y + 1 = ± (x + 3y + 1)

Taking the positive sign, we get x = y as one bisector
Taking the negative sign, we get 2x + 2y + 1 = 0 as the other bisector.

Self practice problem :

(29) Find the equations of the bisectors of the angles between the following pairs of straight lines 3x + 4y + 13 = 0 and 12x – 5y + 32 = 0

Ans. (29) 21x – 77y – 9 = 0 and 99x + 27y + 329 = 0

Methods to discriminate between the acute angle bisector & the obtuse angle bisector:

(i) If θ be the angle between one of the lines & one of the bisectors, find $\tan\theta$.

If $|\tan\theta| < 1$, then $2\theta < 90°$ so that this bisector is the acute angle bisector.

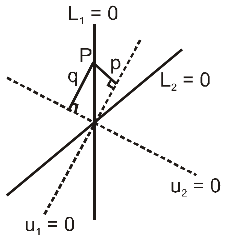
If $|\tan\theta| > 1$, then we get the bisector to be the obtuse angle bisector.

(ii) Let $L_1 = 0$ & $L_2 = 0$ are the given lines & $u_1 = 0$ and $u_2 = 0$ are the bisectors between $L_1 = 0$ & $L_2 = 0$. Take a point P on any one of the lines $L_1 = 0$ or $L_2 = 0$ and drop perpendicular on $u_1 = 0$ & $u_2 = 0$ as shown in figure. If,

$|p| < |q| \Rightarrow u_1$ is the acute angle bisector.

$|p| > |q| \Rightarrow u_1$ is the obtuse angle bisector.

$|p| = |q| \Rightarrow$ the lines L_1 & L_2 are perpendicular.



(iii) If $aa' + bb' < 0$, then the equation of the bisector of this acute angle is

$$\frac{ax + by + c}{\sqrt{a^2 + b^2}} = + \frac{a'x + b'y + c'}{\sqrt{a'^2 + b'^2}}$$

If, however, $aa' + bb' > 0$, the equation of the bisector of the obtuse angle is :

$$\frac{ax + by + c}{\sqrt{a^2 + b^2}} = + \frac{a'x + b'y + c'}{\sqrt{a'^2 + b'^2}}$$

Example # 30 : For the straight lines $2x - y + 1 = 0$ and $x - 2y - 2 = 0$, find the equation of the
(i) bisector of the obtuse angle between them;
(ii) bisector of the acute angle between them;

Solution : $2x - y + 1 = 0$ ....(1)

and $x - 2y - 2 = 0$ ....(2)

Here $a_1 = 2, a_2 = 1, b_1 = -1, b_2 = -2$

Now $a_1a_2 + b_1b_2 = 4 > 0$

$\therefore$ bisector of the obtuse angle between lines (1) and (2) will be

$$\frac{2x - y + 1}{\sqrt{2^2 + (-1)^2}} = \frac{x - 2y - 2}{\sqrt{1^2 + (-2)^2}}$$

or $x + y + 3 = 0$

and the equation of the bisector of the acute angle will be

$$\frac{2x - y + 1}{\sqrt{2^2 + (-1)^2}} = -\frac{x - 2y - 2}{\sqrt{1^2 + (-2)^2}}$$

or $3x - 3y = 1$

Self practice problem :

(30) Find the equations of the bisectors of the angles between the lines $x + y - 3 = 0$ and $7x - y + 5 = 0$ and state which of them bisects the acute angle between the lines.

Ans. (30) $x - 3y + 10 = 0$ (bisector of the obtuse angle);
$6x + 2y - 5 = 0$ (bisector of the acute angle)

Condition of Concurrency :

Three lines $a_1x + b_1y + c_1 = 0$, $a_2x + b_2y + c_2 = 0$ & $a_3x + b_3y + c_3 = 0$ are concurrent if $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0.$

Alternatively : If three constants A, B & C (not all zero) can be found such that
$A(a_1x + b_1y + c_1) + B(a_2x + b_2y + c_2) + C(a_3x + b_3y + c_3) \equiv 0$, then the three straight lines are concurrent.

Example # 31 : If the straight lines $x + 2y = 9$, $3x + 5y = 5$ and $ax - by = 1$ are concurrent. Then find the value of $35a + 22b$

Solution : Given lines are

$x + 2y = 9$ ....(1)

$3x + 5y = 5$ ....(2)

and $ax - by = 1$ ....(3)

Lines will be concurrent if $\Delta = 0$

$$\Delta = \begin{vmatrix} 1 & 2 & -9 \\ 3 & 5 & -5 \\ a & -b & -1 \end{vmatrix} = 0 \Rightarrow 35a + 22b = -1$$

Self practice problem :

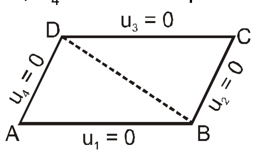
(31) Find the value of m so that the lines $4x - 3y + 2 = 0$, $3x + 4y - 4 = 0$ and $x + my + 6 = 0$ may be concurrent.

Ans. (31) –7

Family of Straight Lines :

The equation of a family of straight lines passing through the point of intersection of the lines,
$L_1 \equiv a_1x + b_1y + c_1 = 0$ & $L_2 \equiv a_2x + b_2y + c_2 = 0$ is given by $L_1 + k L_2 = 0$ i.e.
$(a_1x + b_1y + c_1) + k(a_2x + b_2y + c_2) = 0$, where k is an arbitrary real number.

Note : (i) If $u_1 = ax + by + c$, $u_2 = a'x + b'y + d$, $u_3 = ax + by + c'$, $u_4 = a'x + b'y + d'$ then $u_1=0$; $u_2= 0$; $u_3= 0$; $u_4= 0$ form a parallelogram. The diagonal BD can be given by $u_2u_3 - u_1u_4= 0$.



(ii) The diagonal AC is also given by $u_1 + \lambda u_4 = 0$ and $u_2 + \mu u_3 = 0$, if the two equations are identical for some real λ and μ.
[For getting the values of λ & μ compare the coefficients of x, y & the constant terms].

Example # 32 : If $3a + 2b + 5c = 0$ and the set of lines $ax + by + c = 0$ passes through a fixed point . Find co-ordinates of that point.

Solution : $3a + 2b + 5c = 0$ .... (i)

$ax + by + c = 0$ .... (ii)

Eliminating c, we get.

$$ax + by - \frac{1}{5}(3a + 2b) = 0 \Rightarrow a\left(x - \frac{3}{5}\right) + b\left(y - \frac{2}{5}\right) = 0 \Rightarrow \left(x - \frac{3}{5}\right) + \frac{b}{a}\left(y - \frac{2}{5}\right) = 0$$

It is of the form $L_1 + \lambda L_2 = 0$

Which passes through the point of intersection $\left(\frac{3}{5}, \frac{2}{5}\right)$ of $L_1= 0$ & $L_2= 0$ for all real values of a & b

Aliter : $3a + 2b + 5c = 0 \quad \Rightarrow \quad \frac{3}{5}a + \frac{2}{5}b + c = 0$

$\left(\frac{3}{5}, \frac{2}{5}\right)$ lies on the line $ax + by + c = 0$ Hence fixed point $\left(\frac{3}{5}, \frac{2}{5}\right)$

Example # 33 : Obtain the equations of the lines passing through the intersection of lines $3x + 7y = 17$ and $x + 2y = 5$ and is prependicular to the straight line $3x + 4y = 10$.

Solution : The equation of any line through the intersection of the given lines is
$(x + 2y - 5) + \lambda\,(3x + 7y - 17) = 0$
or $x\,(3\lambda + 1) + y\,(7\lambda + 2) - 17\lambda - 5 = 0$ ....(i)
This is prependicular to the line $3x + 4y = 10$
$\therefore \left(-\frac{3\lambda+1}{7\lambda+2}\right)\left(-\frac{3}{4}\right) = -1 \Rightarrow \lambda = -\frac{11}{37}$
Putting this value of λ in (i), the equation of required line $4x - 3y + 2 = 0$

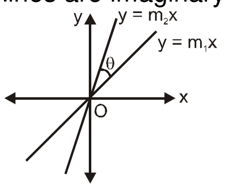
Self practice problem :

(32) Find the equation of the lines through the point of intersection of the lines $x - 3y + 1 = 0$ and $2x + 5y - 9 = 0$ and whose distance from the origin is $\sqrt{5}$

Ans. (32) $2x + y - 5 = 0$

A Pair of straight lines through origin:

(i) A homogeneous equation of degree two, "$ax^2 + 2hxy + by^2 = 0$" always represents a pair of straight lines passing through the origin if :
(a) $h^2 > ab \Rightarrow$ lines are real & distinct .
(b) $h^2 = ab \Rightarrow$ lines are coincident .
(c) $h^2 < ab \Rightarrow$ lines are imaginary with real point of intersection i.e. (0, 0)



This equation is obtained by multiplying the two equations of lines
$(m_1x - y)\,(m_2x - y) = 0$
$\Rightarrow m_1m_2x^2 - (m_1 + m_2)\,xy + y^2 = 0$

(ii) If $y = m_1x$ & $y = m_2x$ be the two equations represented by $ax^2 + 2hxy + by^2 = 0$, then;
$m_1 + m_2 = -\frac{2h}{b}$ & $m_1\,m_2 = \frac{a}{b}$.

(iii) If θ is the acute angle between the pair of straight lines represented by,
$ax^2 + 2hxy + by^2 = 0$, then $\tan\theta = \left|\frac{2\sqrt{h^2 - ab}}{a + b}\right|$.

(iv) The condition that these lines are :
(a) at right angles to each other is $a + b = 0$. i.e. co–efficient of x^2 +co–efficient of $y^2 = 0$.
(b) coincident is $h^2 = ab$.
(c) equally inclined to the axis of x is $h = 0$.i.e. coeff. of $xy = 0$.

Note that a homogeneous equation of degree n represents n straight lines passing through origin.

(v) The equation to the pair of straight lines bisecting the angles between the straight lines $ax^2 + 2hxy + by^2 = 0$ is $\frac{x^2 - y^2}{a - b} = \frac{xy}{h}$.

Example # 34 : Show that the equation $18x^2 - 9xy + y^2 = 0$ represents a pair of distinct straight lines, each passing through the origin. Find the separate equations of these lines.

Solution : The given equation is a homogeneous equation of second degree. So, it represents a pair of straight lines passing through the origin. Comparing the given equation with $ax^2 + 2hxy + by^2 = 0$, we obtain a = 18, b = 1 and 2h = – 9.

$\therefore$ $h^2 - ab = \frac{81}{4} - 18 = \frac{9}{4} > 0 \quad \Rightarrow \quad h^2 > ab$

Hence, the given equation represents a pair of distinct lines passing through the origin.

Now, $18x^2 - 9xy + y^2 = 0 \Rightarrow \left(\frac{y}{x}\right)^2 - 9\left(\frac{y}{x}\right) + 18 = 0 \Rightarrow \left(\frac{y}{x} - 6\right)\left(\frac{y}{x} - 3\right) = 0$

$\Rightarrow \quad - 6 = 0$ or $- 3 = 0 \Rightarrow y - 6x = 0$ or $y - 3x = 0$

So the given equation represents the straight lines y – 3x = 0 and y – 6x = 0.

Example # 35 : Find the equations to the pair of lines through the origin which are perpendicular to the lines represented by $6x^2 - xy - 12y^2 = 0$.

Solution : We have $6x^2 - xy - 12y^2 = 0$.

$\Rightarrow \quad 6x^2 - 9xy + 8xy - 12y^2 = 0 \quad \Rightarrow \quad (3x + 4y)(2x - 3y) = 0$

$\Rightarrow \quad 3x + 4y = 0$ & $2x - 3y = 0$

Thus the given equation represents the lines 3x + 4y = 0 and 2x – 3y = 0. The equations of the lines passing through the origin and perpendicular to the given lines are 4x – 3y = 0 & 3x + 2y = 0 there combined equations $12x^2 - xy - 6y^2 = 0$

Example # 36 : Find the angle between the pair of straight lines $x^2 - 3xy + 2y^2 = 0$

Solution : Given equation is $x^2 - 3xy + 2y^2 = 0$

Here a = coeff. of x^2 = 1, b = coeff. of y^2 = 2

and 2h = coeff. of xy = –3 $\quad \therefore \quad h = -\frac{3}{2}$

Now $\tan\theta = \left|\frac{2\sqrt{h^2 - ab}}{a+b}\right| = \left|\frac{2\sqrt{\frac{9}{4} - 2}}{1+2}\right| = \frac{1}{3}$

Where θ is the acute angle between the lines.

$\therefore$ acute angle between the lines is $\tan^{-1}\left(\frac{1}{3}\right)$ and obtuse angle between them is $\pi - \tan^{-1}\left(\frac{1}{3}\right)$

Example # 37 : Find the equation of the bisectors of the angle between the lines represented by $3x^2 - 5xy + 4y^2 = 0$

Solution : Given equation is $3x^2 - 5xy + 4y^2 = 0$ ....(1)

comparing it with the equation $ax^2 + 2hxy + by^2 = 0$ ....(2)

we have a = 3, 2h = –5; and b = 4

Now the equation of the bisectors of the angle between the pair of lines (1) is

$\frac{x^2 - y^2}{a - b} = \frac{xy}{h}$ or $\frac{x^2 - y^2}{3 - 4} = \frac{xy}{-\frac{5}{2}}$; or $\frac{x^2 - y^2}{-1} = \frac{2xy}{-5}$ or $5x^2 - 2xy - 5y^2 = 0$

Self practice problems :

(33) Find the area of the triangle formed by the lines $y^2 - 9xy + 18x^2 = 0$ and y = 9.

(34) If the pairs of straight lines $x^2 - 2pxy - y^2 = 0$ and $x^2 - 2qxy - y^2 = 0$ be such that each pair bisects the angle between the other pair, prove that pq = –1.

Ans. (33) $\frac{27}{4}$ sq. units

General equation of second degree representing a pair of Straight lines :

(i) $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represents a pair of straight lines if :

$$abc + 2fgh - af^2 - bg^2 - ch^2 = 0, \text{ i.e. if } \begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = 0.$$

Such an equation is obtained again by multiplying the two equation of lines $(a_1x + b_1y + c_1)(a_2x + b_2y + c_2) = 0$

(ii) The angle θ between the two lines representing by a general equation is the same as that between the two lines represented by its homogeneous part only.

Example # 38 : Prove that the equation $6x^2 + 13xy + 6y^2 + 8x + 7y + 2 = 0$ represents a pair of straight lines. Find the co-ordinates of their point of intersection.

Solution : Given equation is $6x^2 + 13xy + 6y^2 + 8x + 7y + 2 = 0$

Writing the equation (1) as a quadratic equation in x we have

$6x^2 + (13y + 8)x + 6y^2 + 7y + 2 = 0$

$$\therefore \quad x = \frac{-(13y+8) \pm \sqrt{(13y+8)^2 - 4.6(6y^2+7y+2)}}{12}$$

$$= \frac{-(13y+8) \pm \sqrt{169y^2 + 208y + 64 - 144y^2 - 168y - 48}}{12}$$

$$= \frac{-(13y+8) \pm \sqrt{25y^2 + 40y + 16}}{12} = \frac{-(13y+8) \pm (5y+4)}{12}$$

$\Rightarrow \quad 12x = -13y - 8 + 5y + 4$ or $12x = -13y - 8 - 5y - 4$

$\Rightarrow \quad 3x + 2y + 1 = 0$ or $2x + 3y + 2 = 0$

Hence equation (1) represents a pair of straight lines whose equation are

$3x + 2y + 1 = 0$ .....(1) and $2x + 3y + 2 = 0$ .....(2)

Solving these two equations, the required point of intersection is

Self practice problem :

(35) Find the combined equation of the straight lines passing through the point (1, 1) and parallel to the lines represented by the equation $x^2 - 5xy + 4y^2 + x + 2y - 2 = 0$ and find the angle between them.

Ans. (35) $x^2 - 5xy + 4y^2 + 3x - 3y = 0$, $\tan^{-1}\left(\frac{3}{5}\right)$

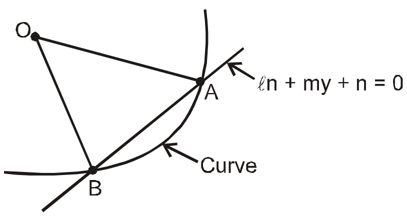
Homogenization :

This method is used to write the joint equation of two lines connecting origin to the points of intersection of a given line and a given second degree curve. The equation of a pair of straight lines joining origin to the points of intersection of the line $L \equiv \ell x + my + n = 0$ and a second degree curve

$S \equiv ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$

is $ax^2 + 2hxy + by^2 + 2gx\left(\frac{\ell x + my}{-n}\right) + 2fy\left(\frac{\ell x + my}{-n}\right) + c\left(\frac{\ell x + my}{-n}\right)^2 = 0.$

The equation is obtained by homogenizing the equation of curve with the help of equation of line.



Notes : (i) Here we have written 1 as $\frac{\ell x + my}{-n}$ and converted all terms of the curve to second degree expressions

(ii) Equation of any curve passing through the points of intersection of two curves $C_1 = 0$ and $C_2 = 0$ is given by $\lambda C_1 + \mu C_2 = 0$, where λ & μ are parameters.

Example # 39 : All chords of the curve $2x^2 - 3y^2 + 4x - 2y = 0$ which subtend a right angle at the origin passes through a fixed point. Find that point

Solution : Let $Ax + By = 1$ be a chord of the curve $2x^2 - 3y^2 + 4x - 2y = 0$

Equation of the given curve is $2x^2 - 3y^2 + 4x - 2y = 0$

and equation of the chord is $Ax + By = 1$ ....(i)

Making equation (1) homogeneous equation of the second degree in x any y with the help of (1), we have $2x^2 - 3y^2 + 4x\,(Ax + By) - 2y(Ax + By) = 0$

$x^2(2 + 4A) + y^2\,(-3 - 2B) + xy\,(4B - 2A) = 0$ ....(ii)

since line represented by (ii) are at right angle

$\Rightarrow$ coefficient of x^2+ coefficient of $y^2 = 0$

$\Rightarrow \quad 2 + 4A + (-3 - 2B) = 0$

$4A - 2B = 1$

chord $Ax + By = 1$

$x = 4 \quad y = -2$

fixed point $(4, -2)$

Self practice problems :

(36) Find the equation of the straight lines joining the origin to the points of intersection of the line $3x + 4y - 5 = 0$ and the curve $2x^2 + 3y^2 = 5$.

(37) Find the equation of the straight lines joining the origin to the points of intersection of the line $lx + my + n = 0$ and the curve $y^2 = 4ax$. Also, find the condition of their perpendicularity.

Ans. (36) $x^2 - y^2 - 24xy = 0$ (37) $4alx^2 + 4amxy + ny^2 = 0;\ 4al + n = 0$

Exercise-1

✍ **Marked questions are recommended for Revision.**

PART - I : SUBJECTIVE QUESTIONS

Section (A) : Distance formula, section formula, Area of Triangle & polygon, Collinearity, slope

A-1. (i) Prove that the points (2a, 4a), (2a, 6a) and $(2a + \sqrt{3}a, 5a)$ are the vertices of an equilateral triangle whose side is 2a.
(ii) Find the points which trisect the line segment joining the points (0, 0) and (9, 12).

A-2. (i) In what ratio does the point $\left(\frac{1}{2}, 6\right)$ divide the line segment joining the points (3, 5) and (–7,9)?
(ii) In which ratio P(2a – 2, 4a – 6)) divides Q (2a – 3, 3a – 7) and R(2a, 6a – 4).

A-3. (i) Find the value of λ such that points P(1, 2), Q(– 2,3) and R(λ + 1, λ) are not forming a triangle?
(ii) Find the ratio in which the line segment joining of the points (1, 2) and (– 2, 3) is divided by the line 3x + 4y = 7
(iii)✍ Find the harmonic conjugate of the point R (5, 1) with respect to points P (2, 10) and Q (6, – 2).

A-4. A and B are the points (3, 4) and (5, – 2) respectively. Find the co-ordinates of a point P such that PA = PB and the area of the triangle PAB = 10.

A-5. Find the area of the quadrilateral with vertices as the points given in each of the following :
(i) (0, 0), (4, 3), (6, 0), (0, 3) (ii) (0, 0), (a, 0), (a, b), (0, b)

Section (B) : Different forms of straight lines and Angle between lines

B-1. Reduce $x + \sqrt{3}\ y + 4 = 0$ to the :
(i) Slope intercepts form and find its slope and y-intercept.
(ii) Intercepts form and find its intercepts on the axes.
(iii) Normal form and find values of P and α .

B-2. Find number of straight line passing through (2, 4) and forming a triangle of 16 sq. cm with the coordinate axis.

B-3. Find the equation of the straight line that passes through the point A(– 5, – 4) and is such that the portion intercepted between the axes is divided by the point A in the ratio 1 : 2 (internally).

B-4. The co-ordinates of the mid-points of the sides of a triangle are (2, 1), (5, 3) and (3, 7). Find the length and equation of its sides.

B-5. Find the straight line cutting an intercept of one unit on negative x-axis and inclined at 45° (in anticlockwise direction) with positive direction of x-axis

B-6.✍ Through the point P(4, 1) a line is drawn to meet the line 3x – y = 0 at Q where $PQ = \frac{11}{2\sqrt{2}}$. Determine the equation of line.

B-7.✍ A line having slope '1' is drawn from a point A(–3, 0) cuts a curve $y = x^2 + x + 1$ at P & Q. Find ℓ(AP) ℓ(AQ).

B-8. Find the direction in which a straight line must be drawn through the point (1, 2), so that its point of intersection with the line x + y = 4 may be at a distance $\frac{\sqrt{6}}{3}$ from this point.

B-9. Through the point (3, 4) are drawn two straight lines each inclined at 45° to the straight line x – y = 2. Find their equations and also find the area of triangle bounded by the three lines.

B-10. Find the equation of a straight lines which passes through the point (2, 1) and makes an angle of $\pi/4$ with the straight line 2x + 3y + 4 = 0

B-11. From (1, 4) you travel $5\sqrt{2}$ units by making 135° angle with positive x-axis (anticlockwise) and then 4 units by making 120° angle with positive x-axis (clockwise) to reach Q. Find co-ordinates of point Q.

B-12. The ends of the hypotenuse of a right angled triangle are (6, 0) and (0, 6), then find the locus of third vertex of triangle.

B-13. A point moves in the x-y plane such that the sum of its distances from two mutually perpendicular lines is always equal to 3, then find the area enclosed by the locus of the point.

B-14. One side of a rectangle lies along the line 4x + 7y + 5 = 0. Two of its vertices are (–3, 1) and (1, 1). Then find the equations of other sides.

Section (C) : Position of point, linear inequation, perpendicular distance, image & foot, Area of Parallelogram

C-1. Plot the region
(i) $6x + 2y \geq 31$ (ii) $2x + 5y \leq 10$ (iii) $8x + 3y + 6 > 0$ (iv) $x > 2$

C-2. Find coordinates of the foot of perpendicular, image and equation of perpendicular drawn from the point (2, 3) to the line y = 3x – 4.

C-3. Starting at the origin, a beam of light hits a mirror (in the form of a line) at the point A(4, 8) and reflected line passes through the point B (8, 12). Compute the slope of the mirror.

C-4. Find the nearest point on the line 3x + 4y – 1 = 0 from the origin.

C-5. Find the position of the origin with respect to the triangle whose sides are x + 1 = 0, 3x – 4y – 5 = 0, 5x + 12y – 27 = 0.

C-6. Find the area of parallelogram whose two sides are y = x + 3, 2x – y + 1 = 0 and remaining two sides are passing through (0, 0).

C-7. Is there a real value of λ for which the image of the point $(\lambda, \lambda - 1)$ by the line mirror $3x + y = 6\lambda$ is the point $(\lambda^2 + 1, \lambda)$? If so find λ.

C-8. Find the equations of two straight lines which are parallel to x + 7y + 2 = 0 and at $\sqrt{2}$ distance away from it.

C-9. Prove that the area of the parallelogram contained by the lines 4y – 3x – a = 0, 3y – 4x + a = 0, 4y – 3x – 3a = 0 and 3y – 4x + 2a = 0 is $\frac{2}{7}a^2$.

Section (D) : Centroid, orthocentre, circumcentre, incentre, excentre, Locus

D-1. For triangle whose vertices are (0, 0), (5, 12) and (16, 12). Find coordinates of
(i) Centroid (ii) Circumcentre
(iii) Incentre (iv) Excentre opposite to vertex (5, 12)

D-2. Find the sum of coordinates of the orthocentre of the triangle whose sides are $x = 3$, $y = 4$ and $3x + 4y = 6$.

D-3. Find equations of altitudes and the co-ordinates of the othocentre of the triangle whose sides are $3x - 2y = 6$, $3x + 4y + 12 = 0$ and $3x - 8y + 12 = 0$.

D-4. A triangle has the lines $y = m_1x$ and $y = m_2x$ for two of its sides, where m_1, m_2 are the roots of the equation $x^2 + ax - 1 = 0$, then find the orthocentre of triangle.

D-5. Prove that the circumcentre, orthocentre, incentre & centroid of the triangle formed by the points A(–1, 11); B (–9, –8); C (15, –2) are collinear, without actually finding any of them.

D-6. Find locus of centroid of ΔAOB if line AB passes through (3, 2), A and B are on coordinate axes.

D-7. Find the locus of the centroid of a triangle whose vertices are (a cos t, a sin t), (b sin t, –b cos t) and (1, 0), where 't' is the parameter.

D-8. Show that equation of the locus of a point which moves so that difference of its distance from two given points (ae, 0) and (–ae, 0) is equal to 2a is $\frac{x^2}{a^2} - \frac{y^2}{a^2(e^2-1)} = 1$.

D-9. Find the locus of point of intersection of the lines $x \cos \alpha + y \sin \alpha = a$ and $x \sin \alpha - y \cos \alpha = b$, where α is a parameter.

D-10. The area of the triangle formed by the intersection of a line parallel to x-axis and passing through P(h, k) with the lines $y = x$ and $x + y = 2$ is $4h^2$. Find the locus of the point P.

Section (E) : Angle Bisector, condition of concurrency, family of straight lines

E-1. Find equations of acute and obtuse angle bisectors of the angle between the lines $4x + 3y - 7 = 0$ and $24x + 7y - 31 = 0$.

E-2. Find the equation of a straight line passing through the point (4, 5) and equally inclined to the lines $3x = 4y + 7$ and $5y = 12x + 6$.

E-3. The line $x + 3y - 2 = 0$ bisects the angle between a pair of straight lines of which one has equation $x - 7y + 5 = 0$,then find equation of other line

E-4. Find the value of λ such that lines $x + 2y = 3$, $3x - y = 1$ and $\lambda x + y = 2$ can not form a triangle.

E-5. Find values of λ for which line $y = x + 1$, $y = \lambda x + 2$ and $y = (\lambda^2 + \lambda - 1) x + 3$ are con-current.

E-6. Find the equation to the straight line passing through
(i) The point (3, 2) and the point of intersection of the lines $2x + 3y = 1$ and $3x - 4y = 6$.
(ii) The intersection of the lines $x + 2y + 3 = 0$ and $3x + 4y + 7 = 0$ and perpendicular to the straight line $y - x = 8$.

E-7. Find the locus of the circumcentre of a triangle whose two sides are along the co-ordinate axes and third side passes through the point of intersection of the lines $ax + by + c = 0$ and $\ell x + my + n = 0$.

Section (F) : Pair of straight lines, Homogenization

F-1. If the slope of one of the lines represented by $ax^2 + 2hxy + by^2 = 0$ be the n^{th} power of the other, then prove that $(ab^n)^{\frac{1}{n+1}} + (a^nb)^{\frac{1}{n+1}} + 2h = 0$

F-2. For what value of λ does the equation $12x^2 - 10xy + 2y^2 + 11x - 5y + \lambda = 0$ represent a pair of straight lines ? Find their equations, point of intersection, acute angle between them and pair of angle bisector.

F-3. (i) Find the integral values of 'h' for which $hx^2 - 5xy + 4hy^2 + x + 2y - 2 = 0$ represents two real straight lines.
(ii) If the pair of lines represented by equation $k(k - 3)\, x^2 + 16xy + (k + 1)y^2 = 0$ are perpendicular to each other, then find k.

F-4. Find the equation of the straight lines joining the origin to the points of intersection of the line $lx + my + n = 0$ and the curve $y^2 = 4ax$. Also, find the condition of their perpendicularity.

F-5. Find the condition that the diagonals of the parallelogram formed by the lines $ax + by + c = 0$; $ax + by + c' = 0$; $a'x + b'y + c = 0$, $a'x + b'y + c' = 0$ are at right angles. Also find the equation to the diagonals of the parallelogram.

PART - II : ONLY ONE OPTION CORRECT TYPE

Section (A): Distance formula, section formula, Area of Triangle & polygon, Collinearity, slope

A-1. Mid point of A(0, 0) and B(1024, 2048) is A_1. mid point of A_1 and B is A_2 and so on. Coordinates of A_{10} are.
(A) (1022, 2044) (B) (1025, 2050) (C) (1023, 2046) (D) (1, 2)

A-2. If the points (k, 2 – 2k), (1 – k, 2k) and (–k –4, 6 – 2k) be collinear, the number of possible values of k are
(A) 4 (B) 2 (C) 1 (D) 3

A-3. Given a ΔABC with unequal sides. P is the set of all points which is equidistant from B & C and Q is the set of all point which is equidistant from sides AB and AC. Then $n(P \cap Q)$ equals :
(A) 1 (B) 2 (C) 3 (D) Infinite

A-4. A line segment AB is divided internally and externally in the same ratio (> 1) at P and Q respectively and M is mid point of AB.
Statement-1: MP, MB, MQ are in G.P.
Statement-2 AP, AB and AQ are in HP.
(A) STATEMENT-1 is true, STATEMENT-2 is true and STATEMENT-2 is correct explanation for STATEMENT-1
(B) STATEMENT-1 is true, STATEMENT-2 is true and STATEMENT-2 is not correct explanation for STATEMENT-1
(C) STATEMENT-1 is true, STATEMENT-2 is false
(D) STATEMENT-1 is false, STATEMENT-2 is true
(E) Both STATEMENTS are false

A-5. Find the area of the triangle formed by the mid points of sides of the triangle whose vertices are (2, 1), (– 2, 3), (4, – 3)
(A) 1.5 sq. units (B) 3 sq. units (C) 6 sq. units (D) 12 sq. units

Section (B) : Different forms of straight lines and Angle between lines

B-1. A straight line through P (1, 2) is such that its intercept between the axes is bisected at P. Its equation is :
(A) $x + 2y = 5$ (B) $x - y + 1 = 0$ (C) $x + y - 3 = 0$ (D) $2x + y - 4 = 0$

B-2. The number of integral points (integral point means both the coordinates should be integer) exactly in the interior of the triangle with vertices (0, 0), (0, 21) and (21, 0), is
(A) 133 (B) 190 (C) 233 (D) 105

B-3. The line joining two points A (2, 0) and B (3, 1) is rotated about A in the anticlock wise direction through an angle of 15º. The equation of the line in the new position is :
(A) $x - \sqrt{3}\,y - 2 = 0$ (B) $x - 2y - 2 = 0$ (C) $\sqrt{3}\,x - y - 2\sqrt{3} = 0$ (D) $\sqrt{2}\,x - y - 2\sqrt{2} = 0$

B-4. In a ΔABC, side AB has the equation $2x + 3y = 29$ and the side AC has the equation $x + 2y = 16$. If the mid point of BC is (5, 6), then the equation of BC is
(A) $2x + y = 16$ (B) $x + y = 11$ (C) $2x - y = 4$ (D) $x + y = 10$

B-5. A square of side 'a' lies above the x-axis and has one vertex at the origin. The side passing through the origin makes an angle $\alpha\left(0 < \alpha < \frac{\pi}{4}\right)$ with the positive direction of x-axis. The equation of its diagonal not passing through the origin is :
(A) $y(\cos\alpha - \sin\alpha) - x(\sin\alpha - \cos\alpha) = a$ (B) $y(\cos\alpha + \sin\alpha) + x(\sin\alpha - \cos\alpha) = a$
(C) $y(\cos\alpha + \sin\alpha) + x(\sin\alpha + \cos\alpha) = a$ (D) $y(\cos\alpha + \sin\alpha) + x(\cos\alpha - \sin\alpha) = a$

B-6. Find equation of a straight line on which length of perpendicular from the origin is four units and the line makes an angle of 120º with the positive direction of x-axis.
(A) $\sqrt{3}\,x - y = 0$ (B) $\sqrt{3}\,x + y = 8$ (C) $x + \sqrt{3}\,y = 8$ (D) $x - \sqrt{3}\,y = 8$

B-7. The distance of the point (2, 3) from the line $2x - 3y + 9 = 0$ measured along a line $x - y + 1 = 0$ is :
(A) $5\sqrt{3}$ (B) $4\sqrt{2}$ (C) $3\sqrt{2}$ (D) $2\sqrt{2}$

B-8. If a point P(x, y) from where line drawn cuts coordinate axes at A and B (with A on x-axis and B on y-axis) satisfies $\alpha.\frac{x^2}{PB^2} + \beta\frac{y^2}{PA^2} = 1$, then $\alpha + \beta$ is
(A) 1 (B) 2 (C) 3 (D) 4

B-9. Two particles start from the point (2, – 1), one moving 2 units along the line $x + y = 1$ and the other 5 units along the line $x - 2y = 4$. If the particles move towards increasing y, then their new positions are
(A) $\left(2-\sqrt{2}, \sqrt{2}-1\right), \left(2\sqrt{5}+2, \sqrt{5}-1\right)$ (B) $\left(2\sqrt{5}+2, \sqrt{5}-1\right), \left(2\sqrt{2}, \sqrt{2}+1\right)$
(C) $\left(2+\sqrt{2}, \sqrt{2}+1\right), \left(2\sqrt{5}+2, \sqrt{5}+1\right)$ (D) none of these

B-10. Equation of a straight line passing through the origin and making with x – axis an acute angle twice the size of the angle made by the line y = (0.2) x with the x – axis, is :
(A) y = (0.4) x (B) y = (5/12) x (C) $6y - 5x = 0$ (D) $6y + 5x = 0$

B-11. The points A (1, 3) and C (5,1) are the oppositive vertices of rectangle. The equation of line passing through other two vertices and of gradient 2, is
(A) $2x+y-8 = 0$ (B) $2x-y-4 = 0$ (C) $2x-y+4 = 0$ (D) $2x+y = 0$

B-12. The point (–4,5) is the vertex of a square and one of its digonals is $7x-y+8 = 0$. The equation of the other diagonal is
(A) $7x-y+23 = 0$ (B) $7y + x = 30$ (C) $7y + x = 31$ (D) $x - 7y = 30$

B-13. Two straight lines $x + 2y = 2$ and $x + 2y = 6$ are given, then find the equation of the line parallel to given lines and divided distance between lines in the ratio 2 : 1 internally
(A) $3x + 6y + 8 = 0$ (B) $3x + 6y = 14$ (C) $3x + 6y + 14 = 0$ (D) $3x + 2y = 10$

Section (C) : Position of point, linear inequation, perpendicular distance, image & foot

C-1. The set of values of 'b' for which the origin and the point (1, 1) lie on the same side of the straight line, $a^2x + a\,by + 1 = 0\ \forall\ a \in R, b > 0$ are :
(A) $b \in (2, 4)$ (B) $b \in (0, 2)$ (C) $b \in [0, 2]$ (D) $(2, \infty)$

C-2. The point (a^2, a + 1) is a point in the angle between the lines 3x − y + 1 = 0 and x + 2y − 5 = 0 containing the origin, then
(A) $a \geq 1$ or $a \leq -3$ (B) $a \in (-3, 0) \cup (1/3, 1)$ (C) $a \in (0, 1)$ (D) $a \in (-\infty, 0)$

C-3. Find area of region represented by 3x + 4y > 12, 4x + 3y > 12 and x + y < 4
(A) $\frac{8}{7}$ (B) $\frac{4}{7}$ (C) $\frac{7}{8}$ (D) $\frac{8}{7}$

C-4. The image of the point A (1, 2) by the line mirror y = x is the point B and the image of B by the line mirror y = 0 is the point (α, β), then :
(A) $\alpha = 1, \beta = -2$ (B) $\alpha = 0, \beta = 0$ (C) $\alpha = 2, \beta = -1$ (D) $\alpha = 1, \beta = -1$

C-5. The equations of the perpendicular bisector of the sides AB and AC of a ΔABC are x – y + 5 = 0 and x + 2y = 0 respectively. If the point A is (1, –2), then the equation of the line BC is :
(A) 14x + 23y = 40 (B) 14x – 23y = 40 (C) 23x + 14y = 40 (D) 23x – 14y = 40

C-6. A light beam emanating from the point A(3, 10) reflects from the straight line 2x + y − 6 = 0 and then passes through the point B(4, 3). The equation of the reflected beam is x + 3y – λ = 0, then the value of λ is
(A) 11 (B) 12 (C) 13 (D) 14

Section (D) : Centroid, orthocentre, circumcentre, incentre, excentre, Locus

D-1. The orthocentre of the triangle ABC is 'B' and the circumcentre is 'S' (a, b). If A is the origin, then the co–ordinates of C are :
(A) (2a, 2b) (B) $\left(\frac{a}{2}, \frac{b}{2}\right)$ (C) $\left(\sqrt{a^2+b^2}, 0\right)$ (D) none

D-2. A triangle ABC with vertices A (− 1, 0),B (− 2, 3/4) & C (− 3, − 7/6) has its orthocentre H. Then the orthocentre of triangle BCH will be :
(A) (− 3, − 2) (B) (1, 3) (C) (− 1, 2) (D) none of these

D-3. Find locus of centroid of ΔABC, if B(1, 1), C(4, 2) and A lies on the line y = x + 3.
(A) 3x + 3y + 1 = 0 (B) x + y = 3 (C) 3x – 3y + 1 = 0 (D) x – y = 3

D-4. The locus of the mid-point of the distance between the axes of the variable line x cos α + y sin α = p, where p is constant, is
(A) $x^2 + y^2 = 4p^2$ (B) $\frac{1}{x^2} + \frac{1}{y^2} = \frac{4}{p^2}$ (C) $x^2 + y^2 = \frac{4}{p^2}$ (D) $\frac{1}{x^2} - \frac{1}{y^2} = \frac{2}{p^2}$

D-5. Find the locus of a point which moves so that sum of the squares of its distance from the axes is equal to 3.
(A) $x^2 + y^2 = 9$ (B) $x^2 + y^2 = 3$ (C) |x|+|y|=3 (D) $x^2 - y^2 = 3$

D-6. A variable straight line passes through a fixed point (a, b) intersecting the co–ordinates axes at A & B. If 'O' is the origin, then the locus of the centroid of the triangle OAB is :
(A) bx + ay – 3xy = 0 (B) bx + ay – 2xy = 0 (C) ax + by – 3xy = 0 (D) ax + by – 2xy = 0

D-7. Consider a triangle ABC, whose vertices are A(–2, 1), B(1, 3) and C(x, y). If C is a moving point such that area of ΔABC is constant, then locus of C is :
(A) Straight line (B) Circle (C) Ray (D) Parabola

D-8. If the equation of the locus of a point equidistant from the points (a_1, b_1) and (a_2, b_2) is $(a_1 - a_2)\,x + (b_1 - b_2)\,y + c = 0$, then the value of 'c' is :

(A) $\frac{1}{2}\,(a_2^2 + b_2^2 - a_1^2 - b_1^2)$ (B) $a_1^2 - a_2^2 + b_1^2 - b_2^2$

(C) $\frac{1}{2}\,(a_1^2 + a_2^2 + b_1^2 + b_2^2)$ (D) $\sqrt{a_1^2 + b_1^2 - a_2^2 - b_2^2}$

Section (E) : Angle Bisector, condition of concurrency, family of straight lines

E-1. The equation of bisectors of two lines L_1 & L_2 are $2x - 16y - 5 = 0$ and $64x + 8y + 35 = 0$. If the line L_1 passes through $(-11, 4)$, the equation of acute angle bisector of L_1 & L_2 is :

(A) $2x - 16y - 5 = 0$ (B) $64x + 8y + 35 = 0$ (C) $2x + 16y + 5 = 0$ (D) $2x + 16y - 5 = 0$

E-2. The equation of the internal bisector of $\angle BAC$ of ΔABC with vertices A(5, 2), B(2, 3) and C(6, 5) is

(A) $2x + y + 12 = 0$ (B) $x + 2y - 12 = 0$ (C) $2x + y - 12 = 0$ (D) $2x - y - 12 = 0$

E-3. The equation of the bisector of the angle between two lines $3x - 4y + 12 = 0$ and $12x - 5y + 7 = 0$ which contains the point $(-1, 4)$ is :

(A) $21x + 27y - 121 = 0$ (B) $21x - 27y + 121 = 0$

(C) $21x + 27y + 191 = 0$ (D) $\frac{-3x + 4y - 12}{5} = \frac{12x - 5y + 7}{13}$

E-4. The least positive value of t so that the lines $x = t + a$, $y + 16 = 0$ and $y = ax$ (where a is real variable) are concurrent is

(A) 2 (B) 4 (C) 16 (D) 8

E-5. Consider the family of lines $5x + 3y - 2 + \lambda_1 (3x - y - 4) = 0$ and $x - y + 1 + \lambda_2(2x - y - 2) = 0$. Equation of a straight line that belong to both families is -

(A) $25x - 62y + 86 = 0$ (B) $62x - 25y + 86 = 0$ (C) $25x - 62y = 86$ (D) $5x - 2y - 7 = 0$

E-6. The equation of a line of the system $2x + y + 4 + \lambda (x - 2y - 3) = 0$ which is at a distance $\sqrt{10}$ units from point A(2, –3) is

(A) $3x + y + 1 = 0$ (B) $3x - y + 1 = 0$ (C) $y - 3x + 1 = 0$ (D) $y + 3x - 2 = 0$

E-7. The lines $ax + by + c = 0$, where $3a + 2b + 4c = 0$, are concurrent at the point

(A) $\left(\frac{1}{2}, \frac{3}{4}\right)$ (B) (1, 3) (C) (3, 1) (D) $\left(\frac{3}{4}, \frac{1}{2}\right)$

E-8. Find the equation of a straight line which passes through the point of intersection of the straight lines $x + y - 5 = 0$ and $x - y + 3 = 0$ and perpendicular to the straight line intersecting x-axis at the point (–2, 0) and the y-axis at the point (0, –3),

(A) $2x + 3y + 10 = 0$ (B) $2x - 3y + 10 = 0$ (C) $2x - 5y + 10 = 0$ (D) $2x + 5y + 10 = 0$

E-9. The line parallel to the x-axis and passing through the intersection of the lines $ax + 2by + 3b = 0$ and $bx - 2ay - 3a = 0$, where $(a, b) \neq (0, 0)$ is

(A) Above the x-axis at a distance of 3/2 from it
(B) Above the x-axis at a distance of 2/3 from it
(C) Below the x-axis at a distance of 3/2 from it
(D) Below the x-axis at a distance of 2/3 from it

Section (F) : Pair of straight lines, Homogenization

F-1. If the slope of one line of the pair of lines represented by $ax^2 + 10xy + y^2 = 0$ is four times the slope of the other line, then a =
(A) 1 (B) 2 (C) 4 (D) 16

F-2. The combined equation of the bisectors of the angle between the lines represented by $(x^2 + y^2)\sqrt{3} = 4xy$ is
(A) $y^2 - x^2 = 0$ (B) $xy = 0$ (C) $x^2 + y^2 = 2xy$ (D) $\frac{x^2 - y^2}{\sqrt{3}} = \frac{xy}{2}$

F-3. The equation of second degree $x^2 + 2\sqrt{2}\,xy + 2y^2 + 4x + 4\sqrt{2}\,y + 1 = 0$ represents a pair of straight lines. The distance between them is
(A) 4 (B) $\frac{4}{\sqrt{3}}$ (C) 2 (D) $2\sqrt{3}$

F-4. The straight lines joining the origin to the points of intersection of the line $2x + y = 1$ and curve $3x^2 + 4xy - 4x + 1 = 0$ include an angle :
(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{4}$ (D) $\frac{\pi}{6}$

PART - III : MATCH THE COLUMN

1.

	Column –I	**Column – II**	
(A)	The points A(–4, –1), B (–2, –4), C (4, 0) and D(2, 3) are the vertices of	(p)	Square
(B)	The figure formed by the lines $2x + 5y + 4 = 0$, $5x + 2y + 7 = 0$, $2x + 5y + 3 = 0$ and $5x + 2y + 6 = 0$ is	(q)	Rectangle
(C)	A quadrilateral whose diagonal are perpendicular bisectors of each other must be	(r)	Rhombus
(D)	A quadrilateral whose diagonals are angle bisector of sides and bisect each other must be	(s)	Parallelogram

2. **Match The Followings :**

	Column –I	**Column – II**	
(A)	P lies on the line y = x and Q lies on y = 2x. The equation for the locus of the mid point of PQ, if |PQ| = 4, is $25x^2 - \lambda xy + 13y^2 = 4$, then λ equals	(p)	36
(B)	The line $(K + 1)^2 x + ky - 2K^2 - 2 = 0$ passes through a point (α, β) regardless of value K. Then $(\alpha - \beta)$ equals :	(q)	6
(C)	Let two lines be $C_1 : 3x - 4y + 1 = 0$ and $C_2 : 8x + 6y + 1 = 0$ suppose (α, β) is a point which is at unit distance from each of the given lines. then sum of all possible value of α is	(r)	–4/5
(D)	If distance between the pair of parallel lines $x^2 + 2xy + y^2 - 8ax - 8ay - 9a^2 = 0$ is $25\sqrt{2}$, then 'a/5' is equal to	(s)	– 1

Exercise-2

✍ **Marked questions are recommended for Revision.**

PART - I : ONLY ONE OPTION CORRECT TYPE

1. On the portion of the straight line x + 2y = 4 intercepted between the axes, a square is constructed on the side of the line away from the origin. Then the point of intersection of its diagonals has co−ordinates :

(A) (2, 3) (B) (3, 2) (C) (3, 3) (D) (2,2)

2. If the straight line ax + by + p = 0 and xcosα + ysinα = p enclosed an angle of $\frac{\pi}{4}$ and the line xsinα – ycosα = 0 meets them at the same point, then $a^2 + b^2$ is

(A) 4 (B) 3 (C) 2 (D) 1

3. A ΔABC is formed by the lines 2x – 3y – 6 = 0 , 3x – y + 3 = 0 and 3x + 4y – 12 = 0. If the points P(α, 0) and Q(0, β) always lie on or inside the ΔABC, then ;

(A) α ∈ [–1, 2] & β∈ [–2, 3] (B) α ∈ [–1, 3] & β∈ [–2, 4]
(C) α ∈ [–2, 4] & β∈ [–3, 4] (D) α ∈ [–1, 3] & β∈ [–2, 3]

4.✍ If $P\left(1+\frac{t}{\sqrt{2}}, 2+\frac{t}{\sqrt{2}}\right)$ be any point on a line, then the range of values of t for which the point P lies between the parallel lines x + 2y = 1 and 2x + 4y = 15 is

(A) $-\frac{4\sqrt{2}}{3} < t < \frac{5\sqrt{2}}{6}$ (B) $0 < t < \frac{5\sqrt{2}}{6}$ (C) $-\frac{4\sqrt{2}}{5} < t < 0$ (D) $-\frac{4\sqrt{2}}{3} < t < \frac{\sqrt{2}}{6}$

5. The point A(4, 1) undergoes following transformations successively :
(i) reflection about line y = x
(ii) translation through a distance of 3 units in the positive direction of x-axis
(iii) rotation through an angle 105° in anti-clockwise direction about origin O.
Then the final position of point A is

(A) $\left(\frac{1}{\sqrt{2}}, \frac{7}{\sqrt{2}}\right)$ (B) $\left(-2, 7\sqrt{2}\right)$ (C) $\left(-\frac{1}{\sqrt{2}}, \frac{7}{\sqrt{2}}\right)$ (D) $\left(-2\sqrt{6}, 2\sqrt{2}\right)$

6.✍ Given two points A ≡ (–2, 0) and B ≡ (0, 4), then find coordinate of a point P lying on the line 2x – 3y = 9 so that perimeter of Δ APB is least.

(A) $\left(\frac{42}{13}, -\frac{11}{3}\right)$ (B) $\left(\frac{84}{13}, -\frac{74}{13}\right)$ (C) $\left(\frac{21}{17}, -\frac{37}{17}\right)$ (D) (0, – 3)

7.✍ A ray of light is sent from the point (1, 4). Upon reaching the x-axis, the ray is reflected from the point (3, 0). This reflected ray is again reflected by the line x + y = 5 and intersect y-axis at P. Find the co-ordinate of P.

(A) $\left(\frac{1}{2}, 0\right)$ (B) $\left(0, \frac{-1}{2}\right)$ (C) $\left(0, \frac{1}{3}\right)$ (D) $\left(2, \frac{1}{-2}\right)$

8.✍ AB is a variable line sliding between the co-ordinate axes in such a way that A lies on X-axis and B lies on Y-axis. If P is a variable point on AB such that PA = b, PB = a and AB = a + b, then equation of locus of P is

(A) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ (B) $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ (C) $x^2 + y^2 = a^2 + b^2$ (D) $x^2 - y^2 = a^2 + b^2$

9. If the distance of any point (x, y) from the origin is defined as d(x, y) = max { |x|, |y|}, d(x, y) = a (where 'a' is non-zero constant), then the locus is
(A) A circle (B) Straight line (C) A square (D) A triangle

10. Two ends A & B of a straight line segment of constant length 'c' slide upon the fixed rectangular axes OX & OY respectively. If the rectangle OAPB is completed. Then find locus of the foot of the perpendicular drawn from P to AB.
(A) $x^{2/3} + y^{2/3} = c^{2/3}$ (B) $x^{2/3} + y^{2/3} = c^{1/3}$ (C) $x^{1/3} + y^{1/3} = c^{2/3}$ (D) $x^{1/3} + y^{1/3} = c^{1/3}$

11. Let the line $\frac{x}{a}+\frac{y}{b}=1$ cuts the x and y axes at A and B respectively. Now a line parallel to the given line cuts the coordinate axis at P and Q and points P and Q are joined to B and A respectively. The locus of intersection of the joining lines is
(A) $\frac{x}{a}-\frac{y}{b}=0$ (B) $\frac{x}{a}+\frac{y}{b}=0$ (C) $\frac{x}{b}-\frac{y}{a}=0$ (D) $\frac{x}{b}+\frac{y}{a}=0$

12. A variable line whose slope is –2 cuts the x and y axes respectively at points A and C. A rhombus ABCD is completed such that vertex B lies on the line y = x. Then the locus of vertex D is
(A) 2x + y = 1 (B) x – y = 0 (C) x + y = 0 (D) x + 2y = 0

13. ABCD is a square away from origin of side length ‘a’. Its side AB slides between x and y–axes in first quadrant with A on x-axis and B on y-axis. The locus of the foot of perpendicular dropped from the point E on the diagonal AC (where E is the midpoint of the side AD), is
(A) $(y - x)^2 + (x - 3y)^2 = a^2$ (B) $(y - x)^2 + (x - 3y)^2 = \frac{a^2}{2}$
(C) $(y - x)^2 + (x - 3y)^2 = \frac{a^2}{4}$ (D) None of these

14_. The locus of circumcentre of the triangle formed by vertices A((–pq – p – q), –(1 + p)(1 + q)), B(pq + p – q, (1 + p)(1 + q)), C(pq + q – p, (1 + p)(1 + q)) is
(A) y + x = 0 (B) y – x = 0 (C) $x^2 + y^2 = 1$ (D) xy = 1

15_. Let two sides of rectangle of area 20 units are along lines x – y = 0 and x + y = 2, then the locus of point of intersection of diagnals is
(A) $(x -1)^2 + (y -1)^2 = 10$ or $(y -1)^2 + (x -1)^2 = 10$
(B) $(x -1)^2 - (y -1)^2 = 10$ or $(y -1)^2 - (x -1)^2 = 10$
(C) $(x +1)^2 - (y +1)^2 = 10$ or $(y +1)^2 - (x +1)^2 = 10$
(D) $(x -1)^2 + (y -1)^2 = 10$ or $(y +1)^2 - (x +1)^2 = 10$

16. Area of the triangle formed by the line x + y = 3 and angle bisectors of the pair of straight lines $x^2 - y^2 + 2y = 1$ is
(A) 2 sq units (B) 4 sq. units (C) 6 sq. units (D) 8 sq. units

17. Equation of the line pair through the origin and perpendicular to the line pair $xy - 3y^2 + y - 2x + 10 = 0$ is :
(A) $xy - 3y^2 = 0$ (B) $xy + 3x^2 = 0$ (C) $xy + 3y^2 = 0$ (D) $x^2 - y^2 = 0$

18. Find the equation of the two straight lines which together with those given by the equation $6x^2 - xy - y^2 + x + 12y - 35 = 0$ will make a parallelogram whose diagonals intersect in the origin.
(A) $6x^2 - xy - y^2 - x - 12y - 35 = 0$ (B) $6x^2 - xy - y^2 - x - 12y + 35 = 0$
(C) $6x^2 - xy - y^2 - x + 12y - 35 = 0$ (D) $6x^2 - xy - y^2 + x - 12y - 35 = 0$

19. The curve passing through the points of intersection of $S_1 \equiv \frac{x^2}{a^2}+\frac{y^2}{b^2}-1 = 0$ and $S_2 \equiv x^2 + y^2 + 2gx + 2fy + c = 0$ represents a pair of straight lines which are
(A) equally inclined to the x - axis (B) perpendicular to each other
(C) parallel to each other (D) Not equally inclined to y-axis

PART - II : SINGLE AND DOUBLE VALUE INTEGER TYPE

1. If the points $\left(\frac{a^3}{a-1}, \frac{a^2-3}{a-1}\right)$, $\left(\frac{b^3}{b-1}, \frac{b^2-3}{b-1}\right)$ and $\left(\frac{c^3}{c-1}, \frac{c^2-3}{c-1}\right)$ are collinear for three distinct values a, b, c and $a \neq 1$, $b \neq 1$ and $c \neq 1$, then find the value of abc– (ab + bc + ac) + 3 (a + b + c).

2. Find number of integral values of λ if $(\lambda, \lambda + 1)$ is an interior points of ΔABC, where $A \equiv (0, 3)$, $B \equiv (-2, 0)$ and $C \equiv (6, 1)$.

3. Let ABC be a triangle such that the coordinates of the vertex A are (– 3, 1). Equation of the median through B is $2x + y - 3 = 0$ and equation of the angular bisector of C is $7x - 4y - 1 = 0$. Find the slope of line BC.

4. A(3, 4), B(0, 0) and C(3, 0) are vertices of ΔABC. If 'P' is the point inside the ΔABC, such that $d(P, BC) \leq \min. \{d(P, AB), d(P, AC)\}$. Then the maximum of d (P, BC) is.
(where d(P, BC) represent distance between P and BC).

5. Drawn from the origin are two mutually perpendicular straight lines forming an isosceles triangle together with the straight line $2x + y = 5$. Then find the area of the triangle.

6. On the straight line $y = x + 2$, a point (a, b) is such that the sum of the square of distances from the straight lines $3x - 4y + 8 = 0$ and $3x - y - 1 = 0$ is least, then find value of 11 (a + b).

7. Parallelogram ABCD is cut by (2n –1) number of parallel lines in which one is diagnal AC. Distance between any two nearest lines is same which is also equal to distance of B, D from respective nearest line anong these. Ratio of area of smallest triangle so formed to area of parallelogram is $\frac{1}{32}$. Find n.

8. A is a variable point on x-axis and B(0,b) is a fixied point. A equilateral triangle ABC is completed with vertex C away from origin. If the locus of the point C is $\alpha x + \beta y = b$, then
$\alpha^2 + \beta^2$ is

9. Two lines (L_1 and L_2) are drawn from point (α, α) making an angle 45^0 with the lines $L_3 \equiv x + y - f(\alpha) = 0$ and $L_4 \equiv x + y + f(\alpha) = 0$. L_1 intersects L_3 and L_4 at A and B and L_2 intersects L_3 and L_4 at C and D respectively ($|2\alpha| > |f(\alpha)|$). If the area of trapezium ABDC is independent of α. if $f(\alpha) = \lambda\alpha^q$, where λ is a constant, then |q| is

10. The portion of the line $ax + by - 1 = 0$, intercepted between the lines $ax + y + 1 = 0$ and $x + by = 0$ subtends a right angle at the origin and the condition in a and b is $\lambda a + b + b^2 = 0$, then find value of λ.

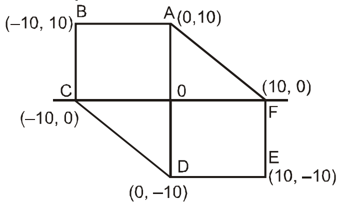
11. If the straight lines joining the origin and the points of intersection of the curve
$5x^2 + 12xy - 6y^2 + 4x - 2y + 3 = 0$ and $x + ky - 1 = 0$ are equally inclined to the x-axis, then find the value of | k |.

12. If the points of intersection of curves $C_1 = 4y^2 - \lambda x^2 - 2xy - 9x + 3$ and $C_2 = 2x^2 + 3y^2 - 4xy + 3x - 1$ subtends a right angle at origin, then find the the value of λ.

13. A parallelogram is formed by the lines $ax^2 + 2hxy + by^2 = 0$ and the lines through (p, q) parallel to them and the equation of the diagonal of the parallelogram which doesn't pass through origin is
$(\lambda x - p)(ap + hq) + (\mu y - q)(hp + bq) = 0$, then find the value of $\lambda^3 + \mu^3$.

14. The equation $9x^3 + 9x^2 y - 45x^2 = 4y^3 + 4xy^2 - 20y^2$ represents 3 straight lines, two of which passes through origin. Then find the area of the triangle formed by these lines

15. Let the integral points inside or on the boundary of region bounded by straight lines as shown in figure is equal to k, then $\sqrt{k-7}$ is equal to



PART - III : ONE OR MORE THAN ONE OPTIONS CORRECT TYPE

1. Point P(2, 3) lies on the line 4x + 3y = 17. Then find the co-ordinates of points farthest from the line which are at 5 units distance from the P.
(A) (6, 6) (B) (6, –6) (C) (2, 0) (D) (–2, 0)

2. Find the equation of the line passing through the point (2, 3) & making intercept of length 2 units between the lines y + 2x = 3 & y + 2x = 5.
(A) 3x – 4y = 18 (B) x = 2 (C) 3x + 4y = 18 (D) x + 2 = 0

3. In a triangle ABC, co-ordinates of A are (1, 2) and the equations to the medians through B and C are x + y = 5 and x = 4 respectively. Then the co-ordinates of B and C will be
(A) (– 2, 7), (4, 3) (B) (7, – 2), (4, 3) (C) (2, 7), (– 4, 3) (D) (2, – 7), (3, – 4)

4. A is a point on either of two rays $y + \sqrt{3}\,|x| = 2$ at a distance of $\frac{4}{\sqrt{3}}$ units from their point of intersection. The co–ordinates of the foot of perpendicular from A on the bisector of the angle between them is/are
(A) $\left(-\frac{2}{\sqrt{3}}, 2\right)$ (B) (0, 0) (C) $\left(\frac{2}{\sqrt{3}}, 2\right)$ (D) (0, 4)

5. If one side of a square is parallel to 3x – 4y = 0 & its area being 16 while centre being (1, 1), then find equation of sides of square.
(A) 3x – 4y + 11 = 0 (B) 3x – 4y – 9 = 0 (C) 4x + 3y + 3 = 0 (D) 4x + 3y – 17 = 0

6. Find the equations of the sides of a triangle having (4, –1) as a vertex, if the lines x – 1 = 0 and x – y – 1 = 0 are the equations of two internal bisectors of its angles.
(A) 2x – y + 3 = 0 (B) x + 2y – 6 = 0 (C) 2x + y – 7 = 0 (D) x – 2y – 6 = 0

7. A straight line L with negative slope passes through the point (8, 2) and cuts the positive coordinate axes at points P and Q, then the correct statement(s) among the following is/are (O is origin)
(A) The absolute minimum value of OP + OQ, where O is origin is $18\sqrt{2}$
(B) Minimum area of ΔOPQ is 32
(C) The absolute minimum value of OP + OQ, where O is origin is 18
(D) Area of ΔOPQ is minimum for slope $\left(\frac{-1}{4}\right)$.

8. The equation of the diagonals of a rectangle are y + 8x – 17 = 0 and y – 8x + 7 = 0. If the area of the rectangle is 8 sq. units, find the equation of the sides of the rectangle.
(A) y = 1 (B) y = 9 (C) x = 1 (D) x = 2.

9. Two adjacent sides of a rhombus are 2x + 3y = a – 5 and 3x + 2y = 4 – 2a and its diagonals intersect at the point (1, 2), then a can be –
(A) – 16 (B) 16 (C) $-\frac{10}{3}$ (D) $\frac{10}{3}$

10. A line $L_1 \equiv 3y - 2x - 6 = 0$ is rotated about its point of intersection with y–axis in clockwise direction to make it L_2 such that the area formed by L_1, L_2, x–axis and line x = 5 is $\frac{49}{3}$ sq units if its point of intersection with x = 5 lies below x–axis then points lying on the equation of L_2 are
(A) (3, – 1) (B) (4,2) (C) (1,1) (D) (3,3)

11. Let $D(x_4, y_4)$ be a point such that ABCD is a square & M & P are the midpoints of the sides BC & CD respectively, then
(A) Ratio of the areas of ΔAMP and the square is 3 : 8
(B) Ratio of the areas of ΔMCP & ΔAMD is 1 : 1
(C) Ratio of the areas of ΔABM & ΔADP is 1 : 1
(D) Ratio of the areas of the quandrilateral AMCP and the square is 1 : 3

12. The equations of perpendicular of the sides AB & AC of Δ ABC are $x - y - 4 = 0$ and $2x - y - 5 = 0$ respectively. If the vertex A is (– 2, 3) and circumcenter is $\left(\frac{3}{2}, \frac{5}{2}\right)$, then which of the following is true.
(A) equation of median of side AB is $x - y + 1 = 0$ (B) centroid of triangle ABC is (3, 1)
(C) vertex C is (4, 0) (D) Area of triangle ABC is 12.

13. Triangle ABC lies in the cartesian plane and has an area of 70 sq. units. The coordinates of B and C are (12, 19) and (23, 20) respecitvely and the coordinates of A are (p, q). The median to the side BC has slope – 5, then which can be corrected.
(A) p + q = 47 (B) p + q = 27 (C) p – q = 17 (D) p – q = 13

14. All the points lying on or inside the triangle formed by the points (1, 3), (5, 6) and (–1, 2) satisfy
(A) $3x + 2y \geq 0$ (B) $2x + y + 1 \geq 0$ (C) $2x + 3y - 12 \geq 0$ (D) $2x + 11 \geq 0$

15. $A \equiv (4, 2)$ and $B \equiv (2, 4)$ are two given points and a point P on the line $3x + 2y + 10 = 0$ is given then which of the following is/are true.
(A) (PA + PB) is minimum when $P\left(\frac{-14}{5}, \frac{-4}{5}\right)$ (B) (PA + PB) is maximum when $P\left(\frac{-14}{5}, \frac{-4}{5}\right)$
(C) |PA – PB | is maximum when P(– 22, 28) (D) (PA – PB) is minimum when P(– 22, 28).

16. A line passing through $P = (\sqrt{3}, 0)$ and making an angle of 60° with positive direction of x-axis cuts the parabola $y^2 = x + 2$ at A and B, then :
(A) PA + PB = $\frac{2}{3}$ (B) |PA – PB| = $\frac{2}{3}$
(C) (PA) (PB) = $\frac{4(2+\sqrt{3})}{3}$ (D) $\frac{1}{PA} + \frac{1}{PB} = \frac{2-\sqrt{3}}{2}$

17. Let $u \equiv ax + by + a\sqrt[3]{b} = 0$, $v \equiv bx - ay + b\sqrt[3]{b} = 0$, where $a, b \in R$ be two straight lines, then find the equations of the bisectors of the angles formed by $k_1u - k_2v = 0$ & $k_1u + k_2v = 0$ for non zero real k_1 & k_2 are :
(A) u = 0 (B) $k_2u + k_1v = 0$ (C) $k_2u - k_1v = 0$ (D) v = 0

18. The sides of a triangle are the straight line x + y = 1, 7y = x and $\sqrt{3}y + x = 0$. Then which of the following is an interior points of triangle ?
(A) circumcentre (B) centroid (C) incentre (D) orthocentre

19. The line 'ℓ_1' passing through the point (1, 1) and the 'ℓ_2' passes through the point (– 1, 1). If the difference of the slope of lines is 2. Find the locus of the point of intersection of the ℓ_1 and ℓ_2.
(A) $x^2 = y$ (B) $y = 2 - x^2$ (C) $y^2 = x$ (D) $x = 2 - y^2$

20. The two lines pairs $y^2 - 4y + 3 = 0$ and $x^2 + 4xy + 4y^2 - 5x - 10y + 4 = 0$ enclose a 4 sided convex polygon, then the correct statement among the following is/are
(A) Area of polygon is 6 (B) Length of its diagonals are $\sqrt{5}$ & $\sqrt{53}$
(C) Point of intersection of diagonals is $(-2, 2)$ (D) Polygon is parallelogram.

21. If the distance between the lines represented $9x^2 - 24xy + 16y^2 + k(6x - 8y) = 0$ is 4, then k may be
(A) 3 (B) 10 (C) –10 (D) 7

PART - IV : COMPREHENSION

Comprehenssion # 1 (Q. NO. 1 to 3)

Let ABC be an acute angled triangle and AD, BE and CF are its medians, where E and F are the points (3, 4) and (1, 2) respectively and centroid of Δ ABC is G(3, 2), then answer the following questions :

1. The equation of side AB is
(A) $2x + y = 4$ (B) $x + y - 3 = 0$ (C) $4x - 2y = 0$ (D) none of these

2. Co-ordinates of D are
(A) (7, –4) (B) (5, 0) (C) (7, 4) (D) (– 3, 0)

3. Height of altitude drawn from point A is (in units)
(A) $4\sqrt{2}$ (B) $3\sqrt{2}$ (C) $6\sqrt{2}$ (D) $2\sqrt{3}$

Comprehension # 2 (Q. No. 4 to 6)

Given two straight lines AB and AC whose equations are $3x + 4y = 5$ and $4x - 3y = 15$ respectively. Then the possible equation of line BC through (1, 2), such that ΔABC is isosceles, is $L_1 = 0$ or $L_2 = 0$, then answer the following questions

4. If $L_1 \equiv ax + by + c = 0$ & $L_2 \equiv dx + ey + f = 0$ where a, b, c, d, e, f $\in$ I, and a, b, d, f > 0 and HCF(a, b) = HCF(d, f) = 1, then c + f =
(A) 1 (B) 2 (C) 3 (D) 4

5. A straight line through P(2, c + f – 1), inclined at an angle of 60° with positive Y-axis in clockwise direction. The co-ordinates of one of the points on it at a distance (c + f) units from point P is (c, f obtained from previous question)
(A) $(2 + 2\sqrt{3}, 5)$ (B) $(3 + 2\sqrt{3}, 3)$ (C) $(2 + 3\sqrt{3}, 4)$ (D) $(2 + 3\sqrt{3}, 3)$

6. If (a, b) is the co-ordinates of the point obtained in previous question, then the equation of line which is at the distance |b – 2a – 1| units from origin and make equal intercept on co-ordinate axes in first quadrant, is
(A) $x + y + 4\sqrt{6} = 0$ (B) $x + y + 2\sqrt{6} = 0$ (C) $x + y - 4\sqrt{6} = 0$ (D) $x + y - 2\sqrt{6} = 0$

Comprehension # 3 (Q.No. 7 to 9)

If vertices of triangle are $P(p_1, p_2)$, $Q(q_1, q_2)$, $R(r_1, r_2)$, then area of $\Delta PQR = \frac{1}{2}\begin{vmatrix} p_1 & p_2 & 1 \\ q_1 & q_2 & 1 \\ r_1 & r_1 & 1 \end{vmatrix}$ and if P, Q, R are collinear, then $\begin{vmatrix} p_1 & p_2 & 1 \\ q_1 & q_2 & 1 \\ r_1 & r_1 & 1 \end{vmatrix} = 0$.

On the basis of above answer the following question.

7. If A(x_1, y_1), B(x_2, y_2), C(x_3, y_3) are the vertices of the triangle then find equation of median through A.

(A) $\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} - \begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0.$ (B) $\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} + \begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$

(C) $\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} + \begin{vmatrix} x & y & 1 \\ x_3 & y_3 & 1 \\ x_1 & y_1 & 1 \end{vmatrix} = 0$ (D) None of these

8. If A(x_1, y_1), B(x_2, y_2), C(x_3, y_3) are the vertices of the triangle then find equation of line through A and parallel to BC

(A) $\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} - \begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0.$ (B) $\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} + \begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$

(C) $\begin{vmatrix} x & y & 1 \\ x_2 & y_2 & 1 \\ x_1 & y_1 & 1 \end{vmatrix} + \begin{vmatrix} x & y & 1 \\ x_3 & y_3 & 1 \\ x_1 & y_1 & 1 \end{vmatrix} = 0$ (D) $\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = \begin{vmatrix} x & y & 1 \\ x_3 & y_3 & 1 \\ x_1 & y_1 & 1 \end{vmatrix}$

9. If A(x_1, y_1), B(x_2, y_2), C(x_3, y_3) are the vertices of the triangle then find the equation of internal angle bisector through A

(A) $b\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} - c\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$ (B) $c\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} + b\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$

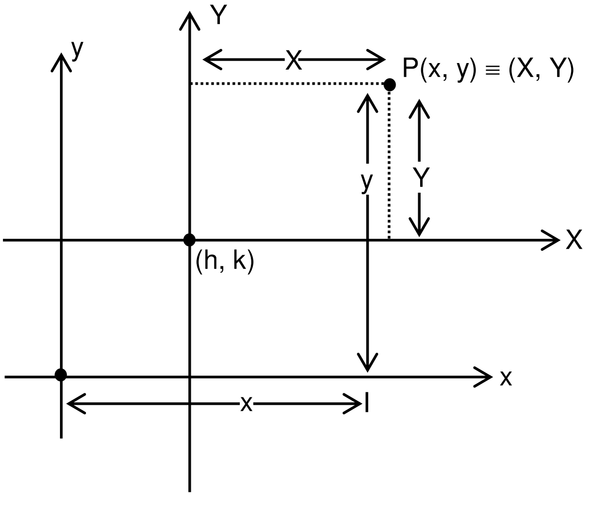
(C) $b\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} + c\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$ (D) $c\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} - b\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$

Comprehension # 4 (10 & 11)

Origin of coordinate system xy is shifted to (h, k) to make new coordinate system XY. X and Y are parallel to x and y. New co-ordinates of point P(x, y) are P(X, Y). x, y, X, Y are related as given below.

X = x – h

Y = y – K



10. Co-ordinates of (–7, 9) if origin is shifted to (2, 4) without changing direction of axes, are

(A) (–5,13) (B) (–7,4) (C) (–9,5) (D) (–9,13)

11. If co-ordinate axes are so translated such that ordinate of (4, 12) becomes zero while abscissa remains same. Then new coordinates of point (–8, –2) are

(A) (–8, 14) (B) (–8, 10) (C) (–8, –14) (D) (–8, –10)

Exercise-3

PART - I : JEE (ADVANCED) / IIT-JEE PROBLEMS (PREVIOUS YEARS)

*** Marked Questions may have more than one correct option.**

1. Let O(0, 0), P(3, 4), Q(6, 0) be the vertices of the triangle OPQ. The point R inside the triangle OPQ is such that the triangles OPR, PQR, OQR are of equal area. The co-ordinates of R are

[IIT-JEE - 2007, P-II, (3, – 1), 81]

(A) $\left(\frac{4}{3}, 3\right)$ (B) $\left(3, \frac{2}{3}\right)$ (C) $\left(3, \frac{4}{3}\right)$ (D) $\left(\frac{4}{3}, \frac{2}{3}\right)$

2. Lines L_1 : y – x = 0 and L_2 : 2x + y = 0 intersect the line L_3 : y + 2 = 0 at P and Q, respectively. The bisector of the acute angle between L_1 and L_2 intersects L_3 at R. **[IIT-JEE - 2007, P-II, (3, – 1), 81]**

STATEMENT – 1 : The ratio PR : RQ equals $2\sqrt{2} : \sqrt{5}$.

because

STATEMENT – 2 : In any triangle, bisector of an angle divides the triangle into two similar triangles.

(A) Statement - 1 is True, Statement - 2 is True; Statement - 2 **is** a correct explanation for Statement - 1

(B) Statement - 1 is True, Statement - 2 is True; Statement - 2 is **NOT** a correct explanation for Statement - 1

(C) Statement - 1 is True, Statement - 2 is False

(D) Statement - 1 is False, Statement - 2 is True

3. Consider three points **[IIT-JEE - 2008, P-II, (3, – 1), 81]**

P = $(-\sin(\beta - \alpha), -\cos\beta)$, Q = $(\cos(\beta - \alpha), \sin\beta)$ and R = $(\cos(\beta - \alpha + \theta), \sin(\beta - \theta))$, where

$0 < \alpha, \beta, \theta < \frac{\pi}{4}$. Then,

(A) P lies on the line segment RQ (B) Q lies on the line segment PR

(C) R lies on the line segment QP (D) P, Q, R are non-collinear

4.✍ The locus of the orthocentre of the triangle formed by the lines **[IIT-JEE - 2009, Paper-2, (3, –1), 80]**

$(1 + p)x - py + p(1 + p) = 0$, $(1 + q)x - qy + q(1 + q) = 0$ and $y = 0$, where $p \neq q$, is

(A) a hyperbola (B) a parabola (C) an ellipse (D) a straight line

5. A straight line **L** through the point (3, –2) is inclined at an angle 60º to the line $\sqrt{3}x + y = 1$. If **L** also intersects the x-axis, then the equation of **L** is **[IIT-JEE 2011, Paper-1, (3, –1), 80]**

(A) $y + \sqrt{3}x + 2 - 3\sqrt{3} = 0$ (B) $y - \sqrt{3}x + 2 + 3\sqrt{3} = 0$

(C) $\sqrt{3}y - x + 3 + 2\sqrt{3} = 0$ (D) $\sqrt{3}y + x - 3 + 2\sqrt{3} = 0$

6. For $a > b > c > 0$, the distance between (1, 1) and the point of intersection of the lines $ax + by + c = 0$ and $bx + ay + c = 0$ is less than $2\sqrt{2}$. Then **[JEE (Advanced) 2013, Paper-1, (2, 0)/60]**

(A) $a + b - c > 0$ (B) $a - b + c < 0$ (C) $a - b + c > 0$ (D) $a + b - c < 0$

7. For a point P in the plane, let $d_1(P)$ and $d_2(P)$ be the distance of the point P from the lines $x - y = 0$ and $x + y = 0$ respectively. The area of the region R consisting of all points P lying in the first quadrant of the plane and satisfying $2 \le d_1(P) + d_2(P) \le 4$, is **[JEE (Advanced) 2014, Paper-1, (3, 0)/60]**

PART - II : JEE (MAIN) / AIEEE PROBLEMS (PREVIOUS YEARS)

1. The lines $p(p^2 + 1)x - y + q = 0$ and $(p^2 + 1)^2 x + (p^2 + 1)y + 2q = 0$ are perpendicular to a common line for: **[AIEEE - 2009 (4, –1), 144]**

(1) exactly one value of p (2) exactly two values of p

(3) more than two values of p (4) no value of p

2. Three distinct points A, B and C are given in the 2-dimensional coordinate plane such that the ratio of the distance of any one of them from the point (1, 0) to the distance from the point (–1, 0) is equal to $\frac{1}{3}$.

Then the circumcentre of the triangle ABC is at the point : **[AIEEE - 2009 (4, –1), 144]**

(1) $\left(\frac{5}{4}, 0\right)$ (2) $\left(\frac{5}{2}, 0\right)$ (3) $\left(\frac{5}{3}, 0\right)$ (4) 0, 0

3. The line L given by $\frac{x}{5} + \frac{y}{b} = 1$ passes through the point (13, 32). The line K is parallel to L and has the equation $\frac{x}{c} + \frac{y}{3} = 1$. Then the distance between L and K is **[AIEEE - 2010 (8, –2), 144]**

(1) $\sqrt{17}$ (2) $\frac{17}{\sqrt{15}}$ (3) $\frac{23}{\sqrt{17}}$ (4) $\frac{23}{\sqrt{15}}$

4. The line $L_1 : y - x = 0$ and $L_2 : 2x + y = 0$ intersect the line $L_3 : y + 2 = 0$ at P and Q respectively. The bisector of the acute angle between L_1 and L_2 intersects L_3 at R. **[AIEEE - 2011, I(4, –1), 120]**

Statement-1 : The ratio PR : RQ equals $2\sqrt{2} : \sqrt{5}$

Statement-2 : In any triangle, bisector of an angle divides the triangle into two similar triangles.

(1) Statement-1 is true, Statement-2 is true ; Statement-2 is correct explanation for Statement-1

(2) Statement-1 is true, Statement-2 is true ; Statement-2 is **not** a correct explanation for Statement-1

(3) Statement-1 is true, Statement-2 is false

(4) Statement-1 is false, Statement-2 is true

5. The lines x + y = | a | and ax – y = 1 intersect each other in the first quadrant. Then the set of all possible values of a is the interval : **[AIEEE - 2011, II(4, –1), 120]**

(1) $(0, \infty)$ (2) $[1, \infty)$ (3) $(-1, \infty)$ (4) $(-1, 1]$

6. If A(2, –3) and B(–2, 1) are two vertices of a triangle and third vertex moves on the line 2x + 3y = 9 , then the locus of the centroid of the triangle is : **[AIEEE - 2011, II(4, –1), 120]**

(1) x – y = 1 (2) 2x + 3y = 1 (3) 2x + 3y = 3 (4) 2x – 3y = 1

7. If the line 2x + y = k passes through the point which divides the line segment joining the points (1, 1) and (2, 4) in the ratio 3 : 2, then k equals : **[AIEEE-2012, (4, –1)/120]**

(1) $\frac{29}{5}$ (2) 5 (3) 6 (4) $\frac{11}{5}$

8. A line is drawn through the point (1, 2) to meet the coordinate axes at P and Q such that it forms a triangle OPQ, where O is the origin. if the area of the triangle OPQ is least, then the slope of the line PQ is : **[AIEEE-2012, (4, –1)/120]**

(1) $-\frac{1}{4}$ (2) – 4 (3) – 2 (4) $-\frac{1}{2}$

9. A ray of light along $x + \sqrt{3}y = \sqrt{3}$ gets reflected upon reaching x-axis, the equation of the reflected ray is **[AIEEE - 2013, (4, –1),360]**

(1) $y = x + \sqrt{3}$ (2) $\sqrt{3}y = x - \sqrt{3}$ (3) $y = \sqrt{3}x - \sqrt{3}$ (4) $\sqrt{3}y = x - 1$

10. The x-coordinate of the incentre of the triangle that has the coordinates of mid points of its sides as (0, 1) (1, 1) and (1, 0) is : **[AIEEE - 2013, (4, –1),360]**

(1) $2 + \sqrt{2}$ (2) $2 - \sqrt{2}$ (3) $1 + \sqrt{2}$ (4) $1 - \sqrt{2}$

11. Let PS be the median of the triangle with vertices P(2, 2), Q (6, – 1), and R (7, 3). The equation of the line passing through (1, – 1) and parallel to PS is : **[JEE(Main) 2014, (4, – 1), 120]**

(1) 4x + 7y + 3 = 0 (2) 2x – 9y – 11 = 0 (3) 4x – 7y – 11 = 0 (4) 2x + 9y + 7 = 0

12. Let a, b, c and d be non-zero numbers. If the point of intersection of the lines 4ax + 2ay + c = 0 and 5bx + 2by + d = 0 lies in the fourth quadrant and is equidistant from the two axes then :

[JEE(Main) 2014, (4, – 1), 120]

(1) 3bc – 2ad = 0 (2) 3bc + 2ad = 0 (3) 2bc – 3ad = 0 (4) 2bc + 3ad = 0

13. The number of points, having both co-ordinates as integers, that lie in the interior of the triangle with vertices (0, 0), (0, 41) and (41, 0) is **[JEE(Main) 2015, (4, – 1), 120]**

(1) 901 (2) 861 (3) 820 (4) 780

14. Two sides of a rhombus are along the lines, $x - y + 1 = 0$ and $7x - y - 5 = 0$. if its diagonals intersect at $(-1, -2)$, then which one of the following is a vertex of this rhombus ? **[JEE(Main) 2016, (4, – 1), 120]**

(1) $(-3, -8)$ (2) $\left(\frac{1}{3}, -\frac{8}{3}\right)$ (3) $\left(-\frac{10}{3}, -\frac{7}{3}\right)$ (4) $(-3, -9)$

15. Let k be an integer such that the triangle with vertices $(k, -3k)$, $(5, k)$ and $(-k, 2)$ has area 28 sq. units. Then the orthocentre of this triangle is at the point : **[JEE(Main) 2017, (4, – 1), 120]**

(1) $\left(2, -\frac{1}{2}\right)$ (2) $\left(1, \frac{3}{4}\right)$ (3) $\left(1, -\frac{3}{4}\right)$ (4) $\left(2, \frac{1}{2}\right)$

16. A straight line through a fixed point (2,3) intersects the coordinate axes at distinct points P and Q. If O is the origin and the rectangle OPRQ is completed, then the locus of R is

[JEE(Main) 2018, (4, – 1), 120]

(1) $3x + 2y = xy$ (2) $3x + 2y = 6xy$ (3) $3x + 2y = 6$ (4) $2x + 3y = xy$

17. Consider the set all lines $px + qy + r = 0$ such that $3p + 2q + 4r = 0$. Which one of the following statements is true ? **[JEE(Main) 2019, Online (09-01-19),P-1 (4, – 1), 120]**

(1) The lines are not concurrent

(2) The lines are all parallel

(3) The lines are concurrent at the point $\left(\frac{3}{4}, \frac{1}{2}\right)$

(4) Each the line passes through the origin.

18.✍ Let the equations of two sides of a triangle be $3x - 2y + 6 = 0$ and $4x + 5y - 20 = 0$. If the orthocentre of this triangle is at (1, 1), then the equation of its third side is :

[JEE(Main) 2019, Online (09-01-19),P-2 (4, – 1), 120]

(1) $26x - 122y - 1675 = 0$ (2) $26x + 61y + 1675 = 0$

(3) $122y - 26x - 1675 = 0$ (4) $122y + 26x + 1675 = 0$

19.✍ Two vertices of a triangle are (0, 2) and (4, 3). If its orthocentre is at the origin, then its third vertex lies in which quadrant ?

(1) third (2) second (3) first (4) fourth

[JEE(Main) 2019, Online (10-01-19),P-2 (4, – 1), 120]

Answers

EXERCISE - 1

PART - I

Section (A) :

A-1. (ii) (3, 4) (6, 8) **A-2.** (i) 1:3 internally (ii) 1:2

A-3. (i) $\frac{3}{2}$ (ii) 4 : 1 internally (iii) (8, – 8). **A-4.** (7, 2) or (1, 0)

A-5. (i) 15 sq. units (ii) |ab| sq. units

Section (B) :

B-1. (i) $y = -\frac{1}{\sqrt{3}}x - \frac{4}{\sqrt{3}}$, slope $= -\frac{1}{\sqrt{3}}$, y-intercept $= -\frac{4}{\sqrt{3}}$

(ii) $\frac{x}{-4} + \frac{y}{-4/\sqrt{3}} = 1$, x-intercept = – 4, y-intercept $= -\frac{4}{\sqrt{3}}$

(iii) $x \cos 240^\circ + y \sin 240^\circ = 2$, P = 2, $\alpha = 240^\circ$

B-2. 3 **B-3.** 8x + 5y + 60 = 0, 2x + 5y + 30 = 0

B-4. Equations are 2x – 3y + 15 = 0, 2x + y – 5 = 0 , 6x – y – 27 = 0

Length of sides are $2\sqrt{13}$, $4\sqrt{5}$, $2\sqrt{37}$

B-5. x – y + 1 = 0 **B-6.** x + y = 5, x – 7y + 3 = 0 **B-7.** 14 **B-8.** $\pi/12$, $5\pi/12$

B-9. x = 3, y = 4, 9/2 sq. units **B-10.** x – 5y + 3 = 0, 5x + y – 11 = 0 **B-11.** $\left(-6, 9 - 2\sqrt{3}\right)$

B-12. $x^2 + y^2 - 6x - 6y = 0$. **B-13.** 18 **B-14.** 7x – 4y + 25 = 0, 7x – 4y – 3 = 0, 4x + 7y = 11

Section (C) :

C-2. Foot $\left(\frac{23}{10}, \frac{29}{10}\right)$, Image $\left(\frac{13}{5}, \frac{14}{5}\right)$, x + 3y – 11 = 0 **C-3.** $\frac{1+\sqrt{10}}{3}$ **C-4.** $\left(\frac{3}{25}, \frac{4}{25}\right)$

C-5. Inside **C-6.** 3 sq. unit **C-7.** 2 **C-8.** x + 7y + 12 = 0, x + 7y – 8 = 0

Section (D) :

D-1. (i) (7, 8) (ii) $\left(\frac{21}{2}, \frac{8}{3}\right)$ (iii) (7, 9) (iv) (27, –21) **D-2.** 7

D-3. $2x + 3y + 8 = 0$, $4x - 3y - 7 = 0$, $8x + 3y + 9 = 0$, orthocentre $\left(-\frac{1}{6}, -\frac{23}{9}\right)$. **D-4.** (0, 0)

D-6. $\frac{3}{x} + \frac{2}{y} = 3$ **D-7.** $(3x - 1)^2 + 9y^2 = a^2 + b^2$ **D-9.** $x^2 + y^2 = a^2 + b^2$

D-10. $y = 2x + 1$ or $y = -2x + 1$

Section (E) :

E-1. acute $2x + y - 3 = 0$, obtuse $x - 2y + 1 = 0$, origin lies in obtuse angle bisector.

E-2. $9x - 7y = 1$, $7x + 9y = 73$ **E-3.** $5x + 5y - 3 = 0$ **E-4.** $\frac{6}{5}, \frac{1}{2}, -3$

E-5. 0 **E-6.** (i) $43x - 29y = 71$ (ii) $x + y + 2 = 0$

E-7. $2xy(ma-b\ell) + x(an - \ell c) + y(mc - bn) = 0$.

Section (F) :

F-2. $\lambda = 2$, $3x - y + 2 = 0$, $4x - 2y + 1 = 0$, point of intersection $\left(-\frac{3}{2}, -\frac{5}{2}\right)$,

$\tan^{-1}\left(\frac{1}{7}\right)$, $2x^2 + 4xy - 2y^2 + 16x - 4y + 7 = 0$.

F-3. (i) h = 1 (ii) k = 1 **F-4.** $4alx^2 + 4amxy + ny^2 = 0$; $4al + n = 0$

F-5. $a^2 + b^2 = (a')^2 + (b')^2 \Rightarrow (a + a')x + (b + b')y + c + c' = 0 \Rightarrow (a - a')x + (b - b')y = 0$

PART - II

Section (A) :

A-1. (C) **A-2.** (B) **A-3.** (B) **A-4.** (A) **A-5.** (A)

Section (B) :

B-1. (D) **B-2.** (B) **B-3.** (C) **B-4.** (B) **B-5.** (D) **B-6.** (B) **B-7.** (B)

B-8. (B) **B-9.** (A) **B-10.** (B) **B-11.** (B) **B-12.** (C) **B-13.** (B)

Section (C) :

C-1. (B) **C-2.** (B) **C-3.** (A) **C-4.** (C) **C-5.** (A) **C-6.** (C)

Section (D) :

D-1. (A) **D-2.** (D) **D-3.** (C) **D-4.** (B) **D-5.** (B) **D-6.** (A) **D-7.** (A)

D-8. (A)

Section (E) :

E-1.	(A)	**E-2.**	(C)	**E-3.**	(A)	**E-4.**	(D)	**E-5.**	(D)	**E-6.**	(B)	**E-7.**	(D)
E-8.	(B)	**E-9.**	(C)										

Section (F) :

F-1.	(D)	**F-2.**	(A)	**F-3.**	(C)	**F-4.**	(A)

PART - III

1. (A)→(q, s); (B)→(r, s); (C)→(r, s); (D)→(r, s) **2.** (A) → (p); (B) → (q); (C) → (r); (D) → (s)

EXERCISE - 2

PART - I

1.	(C)	**2.**	(C)	**3.**	(D)	**4.**	(A)	**5.**	(D)	**6.**	(C)	**7.**	(B)
8.	(A)	**9.**	(C)	**10.**	(A)	**11.**	(A)	**12.**	(C)	**13.**	(C)	**14.**	(A)
15.	(B)	**16.**	(A)	**17.**	(B)	**18.**	(A)	**19.**	(A)				

PART - II

1.	0	**2.**	2	**3.**	18	**4.**	1	**5.**	5	**6.**	52	**7.**	4
8.	4	**9.**	1	**10.**	2	**11.**	1	**12.**	19	**13.**	16	**14.**	30
15.	18												

PART - III

1.	(AD)	**2.**	(BC)	**3.**	(B)	**4.**	(B)	**5.**	(ABCD)	**6.**	(ACD)	**7.**	(BCD)
8.	(ABCD)	**9.**	(AD)	**10.**	(AC)	**11.**	(AC)	**12.**	(C)	**13.**	(ABD)	**14.**	(ABD)
15.	(AC)	**16.**	(BC)	**17.**	(AD)	**18.**	(BC)	**19.**	(AB)	**20.**	(ABD)	**21.**	(BC)

PART - IV

1.	(A)	**2.**	(B)	**3.**	(C)	**4.**	(D)	**5.**	(A)	**6.**	(C)	**7.**	(B)
8.	(A)	**9.**	(C)	**10.**	(C)	**11.**	(C)						

EXERCISE - 3

PART - I

1.	(C)	**2.**	(C)	**3.**	(D)	**4.**	(D)	**5.**	(B)	**6.**	(A) or (C) or Bonus
7.	(6)										

PART - II

1.	(1)	**2.**	(1)	**3.**	(3)	**4.**	(3)	**5.**	(2)	**6.**	(2)	**7.**	(3)
8.	(3)	**9.**	(2)	**10.**	(2)	**11.**	(4)	**12.**	(1)	**13.**	(4)	**14.**	(2)
15.	(4)	**16.**	(1)	**17.**	(3)	**18.**	(1)	**19.**	(2)				

High Level Problems (HLP)

1. The vertices of a triangle OBC are O(0,0) B(–3,–1) and C(–1,–3). Find the equation of line parallel to BC and intersecting the sides OB and OC, whose perpendicular distance from the point (0,0) is $\frac{1}{2}$.

2. A variable line, drawn through the point of intersection of the straight lines $\frac{x}{a} + \frac{y}{b} = 1$ and $\frac{x}{b} + \frac{y}{a} = 1$, meets the coordinate axes in A & B. Show that the locus of the mid point of AB is the curve $2xy(a + b) = ab(x + y)$.

3. From the vertices A, B, C of a triangle ABC, perpendiculars AD, BE, CF are drawn to any straight line. Show that the perpendiculars from D, E, F to BC, CA, AB respectively are concurrent.

4. A triangle is formed by the lines whose equations are AB : $x + y - 5 = 0$, BC : $x + 7y - 7 = 0$ and CA : $7x + y + 14 = 0$. Find the bisector of the interior angle at B and the exterior angle at C. Determine the nature of the interior angle at A and find the equation of the bisector.

5. The coordinates of the feet of $\perp$ from the vertices of a Δ on the opposite sides are (20, 25), (8, 16) and (8, 9). Then find the coordinates of a vertices of the Δ

6. Let P is any point inside the triangle ABC of side lengths 6, 5, 5 units and p_1, p_2, p_3 be the lengths of perpendiculars drawn from P to the sides of triangle. Find the maximum value of $p_1.p_2.p_3$.

7. Let in ΔPAB, A is (0, 0), B is (a, 0) and P is variable such that $\angle$PBA is equal to three times $\angle$PAB the, find the locus of P.

8. Through a fixed point any straight line is drawn meeting two given parallel straight lines in P and Q, through P and Q straight lines are drawn in fixed directions, meeting in R. Prove that the locus of R is straight line.

9. Through the origin O a straight line is drawn to cut the lines $y = m_1 x + C_1$ and $y = m_2 x + C_2$ at Q and R. respectively. Find the locus of the point P on this variable line, such that OP is the geometric mean of OQ and OR.

10. The sides of a triangle are $L_r \equiv x\cos\alpha_r + y\sin\alpha_r - p_r = 0$ for r = 1, 2, 3.
Show that its orthocentre is given by
$L_1 \cos(\alpha_2 - \alpha_3) = L_2\cos(\alpha_3 - \alpha_1) = L_3 \cos(\alpha_1 - \alpha_2)$.

11. A line passes through a fixed point R intersecting a fixed line at P. A point Q on RP such that $\frac{RP}{RQ}$ is constant. Then show that locus of Q is a straight line.

12. A triangle ABC with a = 8, b = 6 and c = 10 slides on the coordinate axes with vertices A and B on the x–axis and the y–axis respectively. Find the locus of the vertex C.

13. The line $L_1 \equiv 4x + 3y - 12 = 0$ intersects the x and the y-axis at A and B respectively. A variable line perpendicular to L_1 intersects the x and the y-axis at P and Q respectively. Find the locus of the circumcentre of triangle ABQ.

14. Show that the orthocentre of Δ formed by the straight lines, $ax^2 + 2hxy + by^2 = 0$ and $\ell x + my = 1$ is a point (x', y') such that $\frac{x'}{\ell} = \frac{y'}{m} = \frac{a+b}{am^2 - 2h\ell m + b\ell^2}$.

15. Show that the lines joining the origin to the other two points of intersection of the curves $ax^2+2hxy+by^2+2gx = 0$ and $a'x^2 + 2h'xy + b'y^2 + 2g'x = 0$ will be at right angles to one another if $g(a' + b') = g'(a + b)$

16. The distance of a point (x_1, y_1) from each of two straight lines which passes through the origin of co–ordinates is δ, find the combined equation of these straight lines.

17. If the equation $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$, represents a pair of straight lines, prove that the third pair of straight lines (excluding $xy = 0$) passing through the points where these meet the axes is $ax^2 - 2hxy + by^2 + 2gx + 2fy + c + \frac{4fg}{c} \cdot xy = 0$.

18. A point moves so that the distance between the feet of the perpendiculars from it on the lines $ax^2 + 2h\,xy + by^2 = 0$ is a constant 2 d. Show that the equation to its locus is,
$(x^2 + y^2)(h^2 - ab) = d^2\{(a - b)^2 + 4h^2\}$.

19. Show that the pair of lines given by $a^2x^2 + 2h(a + b)xy + b^2y^2 = 0$ is equally inclined to the pair given by $ax^2 + 2hxy + by^2 = 0$.

20. All the chords of the curve $3x^2 - y^2 - 2x + 4y = 0$ which subtend a right angle at the origin are concurrent. Does this result also hold for the curve, $3x^2 + 3y^2 - 2x + 4y = 0$? If yes, what is the point of concurrence.

21. The straight lines $(A^2 - 3B^2)x^2 + 8AB\,xy + (B^2 - 3A^2)y^2 = 0$ form a Δ with the line $Ax + By + C = 0$, then prove that

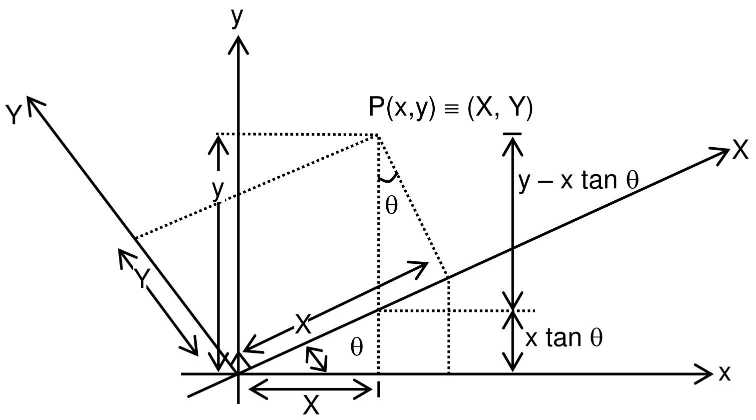
(i) Area of Δ is $\frac{C^2}{\sqrt{3}\left(A^2 + B^2\right)}$

(ii) Δ is equilateral

(iii) The orthocentre of Δ does not lie on one of its vertexs

Comprehension (Q. 22 & 23)

If coordinate system xy is being rotated through an angle θ in anti clock wise direction about the origin as shown in the diagram, Coordinates of P(x, y) has been change to P(X, Y) in new coordinate system XY, then x, y, X, Y are related as given below.



$X = x \sec\theta + (y - x\tan\theta)\sin\theta$ and $Y = (y - x\tan\theta)\cos\theta$

$= x\sec\theta + y\sin\theta - \dfrac{x\sin^2\theta}{\cos\theta}$ $= y\cos\theta - x\sin\theta$

$\boxed{X = x\cos\theta + y\sin\theta}$ $\boxed{Y = -x\sin\theta + y\cos\theta}$

22. If the axes are rotated through 60º in anticlockwise direction about origin. Find co-ordinats of point (2, 6) in new co-ordinate axes.

23. If axes are rotated through an acute angle in clockwise direction about origin so that equation $x^2 + 2xy + y^2 - 2x + 2y = 0$ becomes free from xy in its new position, then find equation in new position

24. Find the acute angle between two straight lines passing through the point M(−6, −8) and the points in which the line segment $2x + y + 10 = 0$ enclosed between the co−ordinate axes is divided in the ratio 1 : 2 : 2 in the direction from the point of its intersection with the x−axis to the point of intersection with the y−axis.

25_. Let A lies on $3x - 4y + 1 = 0$. B lies on $4x + 3y - 7 = 0$ and C is (–2, 5). If ABCD is rhombus, then find locus of D.

26_. Let D is point on line $\ell_1 : x + y - 2 = 0$ and S(3, 3) is fixed point. ℓ_2 is the line perpendicular to DS and passing through S. If M is another point on line ℓ_1 (other than D), then find locus of point of intersection of ℓ_2 and angle bisector of $\angle MDS$.

27_. A variable line cuts the line $2y = x - 2$ and $2y = -x + 2$ in points A and B respectively. If A lies in first quadrant, B lies in 4[th] quadrant and area of ΔAOB is 4, then find locus of

(i) mid point of AB (ii) centroid of ΔOAB

28. An equilateral triangle PQR is formed where P (1, 3) is fixed point and Q is moving point on line x = 3. Find the locus of R.

Answers

1. $x + y + \frac{1}{\sqrt{2}} = 0.$

4. $3x + 6y - 16 = 0$; $8x + 8y + 7 = 0$; $12x + 6y - 11 = 0$

5. (5, 10). (50, – 5), (15, 30)

6. $p_1p_2p_3 \le \frac{256}{75}$

7. $4x^3 - 4xy^2 - 3ax^2 + ay^2 = 0.$

9. $(y - m_1x)(y - m_2x) = c_1c_2$

12. vertex C lies on the line $\frac{y}{x} = \frac{3}{4}$ or $3x - 4y = 0$.

13. $6x - 8y + 7 = 0$

16. $(y_1^2 - \delta^2)x^2 - 2x_1y_1xy + (x_1^2 - \delta^2)y^2 = 0$

20. (1, – 2), yes हाँ (1/3, – 2/3)

22. $(1 + 3\sqrt{3}, -\sqrt{3} + 3)$

23. $x^2 + \sqrt{2}y = 0$

24. $\pi/4$

25. $25((x + 2)^2 + (y - 5)^2) = (3x - 4y + 1)^2$

26. $(x - 3)^2 + (y - 3)^2 = \left(\frac{x + y - 2}{\sqrt{2}}\right)^2$

27. (i) $(x - 1)^2 - 4y^2 = 9$ (ii) $(x - 2/3)^2 + 4y^2 = 4$

28. $(x - 2) = \pm\sqrt{3}(y - 3 \pm \sqrt{3})$