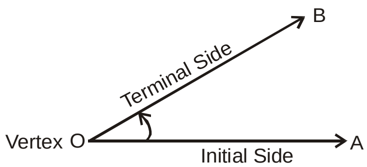
Trigonometry

When writing about transcendental issues, be transcendentally clear....... Descartes, Rene

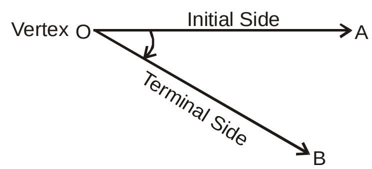
The word 'trigonometry' is derived from the Greek words 'trigon' and ' metron' and it means 'measuring the sides and angles of a triangle'.

Angle :

Angle is a measure of rotation of a given ray about its initial point. The original ray is called the initial side and the final position of the ray after rotation is called the terminal side of the angle. The point of rotation is called the vertex. If the direction of rotation is anticlockwise, the angle is said to be positive and if the direction of rotation is clockwise, then the angle is negative.



(i) Positive angle (anticlockwise measurement)



(ii) Negative angle (clockwise measurement)

Systems For Measurement of Angles :

An angle can be measured in the following systems.
One complete rotation is equal to 360 degree = 400 grade = 2 π radian

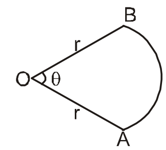
Relation between radian, degree and grade :

To / From	Sexagesimal System (British system)	Centesimal System (French system)	Circular System (Radian Measurement)
Sexagesimal System (British system)		1 degree = $\frac{400}{360}$ grade	1 degree (1°) = $\frac{\pi}{180}$ radian 1min(1′)= $\frac{1}{60}$ degree (1°=60′) 1 sec(1″) = $\frac{1}{60}$ min (1′ = 60″)
Centesimal System (French system)	1 grade = $\frac{360}{400}$ degree		
Circular System (Radian Measurement)	1 radian = $\frac{180}{\pi}$ degree 1 degree = 60 min (1°=60′) 1 min = 60 sec (1′ = 60″)	1 radian = $\frac{200}{\pi}$ grade 1 grade=100 min(1^g = 100') 1 min = 100 sec(1' =100")	

Note : # The minutes and seconds in the Sexagesimal system are different with the minutes and seconds respectively in the Centesimal System. Symbols in both systems are also different.

If no symbol is mentioned while showing measurement of angle, then it is considered to be measured in radians.

e.g. $\theta = 15$ implies 15 radian



Arc length AB = $\ell = r\theta$

Area of circular sector = $\frac{1}{2}r^2\theta$ sq. units

Trigonometric Ratios for Acute Angles :

Let a revolving ray OP starts from OA and revolves into the position OP, thus tracing out the angle AOP.

In the revolving ray take any point P and draw PM perpendicular to the initial ray OA.

In the right angle triangle MOP, OP is the hypotenuse, PM is the perpendicular, and OM is the base.

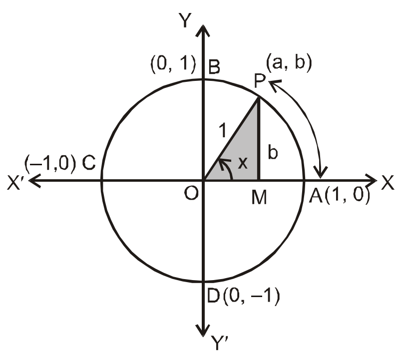
The trigonometrical ratios, or functions, of the angle AOP are defined as follows :

sin(∠AOP)	cos(∠AOP)	tan(∠AOP)	cot(∠AOP)	sec(∠AOP)	cosec(∠AOP)
$\frac{\text{Perp}}{\text{Hyp}} = \frac{MP}{OP}$	$\frac{\text{Base}}{\text{Hyp}} = \frac{OM}{OP}$	$\frac{\text{Perp}}{\text{Base}} = \frac{MP}{OM}$	$\frac{\text{Base}}{\text{Perp}} = \frac{OM}{MP}$	$\frac{\text{Hyp}}{\text{Base}} = \frac{OP}{OM}$	$\frac{\text{Hyp}}{\text{Prep}} = \frac{OP}{MP}$

It can be noted that the trigonometrical ratios are all real numbers.

Trigonometric ratios for angle $\theta \in$ R :

We will now extend the definition of trigonometric ratios to any angle in terms of radian measure and study them as trigonometric functions. (also called circular functions) Consider a unit circle (radius 1 unit) with centre at origin of the coordinate axes. Let at origin of the coordinate axes. Let P(a, b) be any point on the circle with angle AOP = x radian, i.e., length of arc AP = x We define cos x = a and sin x = b Since Δ OMP is a right triangle, we have $OM^2 + MP^2 = OP^2$ or $a^2 + b^2 = 1$ Thus, for every point on the unit circle, we have $a^2 + b^2 = 1$ or $\cos^2 x + \sin^2 x = 1$



Since one complete revolution subtends an angle of 2π radian at the centre of the circle, $\angle AOB = \frac{\pi}{2}$, $\angle AOC = \pi$ and $\angle AOD = \frac{3\pi}{2}$. All angles which are integral multiples of $\frac{\pi}{2}$ are called quadrantal angles. The coordinates of the points A, B, C and D are, respectively, (1, 0), (0, 1), (–1, 0) and (0, –1). Therefore, for quadrantal angles, we have

$\cos 0 = 1$ $\qquad$ $\sin 0 = 0$,

$\cos \frac{\pi}{2} = 0$ $\qquad$ $\sin \frac{\pi}{2} = 1$

$\cos \pi = -1$ $\qquad$ $\sin \pi = 0$

$\cos \frac{3\pi}{2} = 0$ $\qquad$ $\sin \frac{3\pi}{2} = -1$

$\cos 2\pi = 1$ $\qquad$ $\sin 2\pi = 0$

Now if we take one complete revolution from the position OP, we again come back to same position OP. Thus, we also observe that if x increases (or decreases) by any integral multiple of 2π, the values of sine and cosine functions do not change. Thus, $\sin (2n\pi + x) = \sin x$, $n \in Z$, $\cos (2n\pi + x) = \cos x$, $n \in Z$. Further, $\sin x = 0$, if $x = 0, \pm \pi, \pm 2\pi, \pm 3\pi$ ....., i.e., when x is an integral multiple of π and $\cos x = 0$, if $x = \pm \frac{\pi}{2}, \pm \frac{3\pi}{2}, \pm \frac{5\pi}{2}$, ...i.e., cos x vanishes when x is an odd multiple of $\frac{\pi}{2}$. Thus $\sin x = 0$ implies $x = n\pi$, where n is any integer $\cos x = 0$ implies $x = (2n + 1)\frac{\pi}{2}$, where n is any integer.

We now define other trigonometric functions in terms of sine and cosine functions :

$\operatorname{cosec} x = \frac{1}{\sin x}$, $x \neq n\pi$, where n is any integer.

$\sec x = \frac{1}{\cos x}$, $x \neq (2n + 1)\frac{\pi}{2}$, where n is any integer.

$\tan x = \frac{\sin x}{\cos x}$, $x \neq (2n + 1)\frac{\pi}{2}$, where n is any integer.

$\cot x = \frac{\cos x}{\sin x}$, $x \neq n\pi$, where n is any integer.

We have shown that for all real x, $\sin^2 x + \cos^2 x = 1$

It follows that $1 + \tan^2 x = \sec^2 x$ (Think !) $\{x \neq (2n + 1)\frac{\pi}{2}\ ;\ n \in Z\}$

$1 + \cot^2 x = \operatorname{cosec}^2 x$ (Think !) $\{x \neq n\pi\ ;\ n \in Z\}$

Sign of The Trigonometric Functions

(i) If θ is in the first quadrant then P(a, b) lies in the first quadrant. Therefore $a > 0$, $b > 0$ and hence the values of all the trigonometric functions are positive.

(ii) If θ is in the II quadrant then P(a, b) lies in the II quadrant. Therefore $a < 0$, $b > 0$ and hence the values sin, cosec are positive and the remaining are negative.

(iii) If θ is in the III quadrant then P(a, b) lies in the III quadrant. Therefore $a < 0$, $b < 0$ and hence the values of tan, cot are positive and the remaining are negative.

(iv) If θ is in the IV quadrant then P(a, b) lies in the IV quadrant. Therefore $a > 0$, $b < 0$ and hence the values of cos, sec are positive and the remaining are negative.

	$\sin\theta$	$\cos\theta$	$\tan\theta$	$\cot\theta$	$\sec\theta$	$\operatorname{cosec}\theta$
I^st Quadrant	+	+	+	+	+	+
II^nd Quadrant	+	–	–	–	–	+
III^rd Quadrant	–	–	+	+	–	–
IV^th Quadrant	–	+	–	–	+	–

Values of trigonometric functions of certain popular angles are shown in the following table :

	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
sin	$\sqrt{\frac{0}{4}} = 0$	$\sqrt{\frac{1}{4}} = \frac{1}{2}$	$\sqrt{\frac{2}{4}} = \frac{1}{\sqrt{2}}$	$\sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}$	$\sqrt{\frac{4}{4}} = 1$
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	N.D.

N.D. implies not defined

The values of cosec x, sec x and cot x are the reciprocal of the values of sin x, cosx and tan x, respectively.

Trigonometric Ratios of allied angles

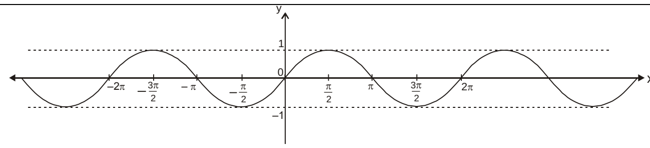
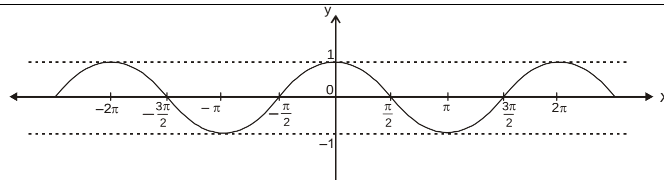
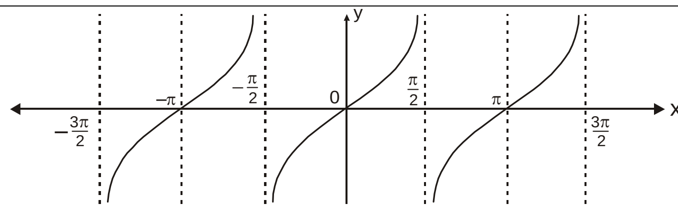
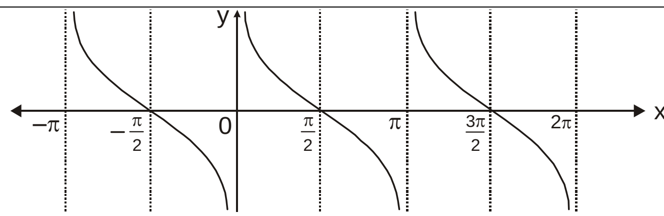
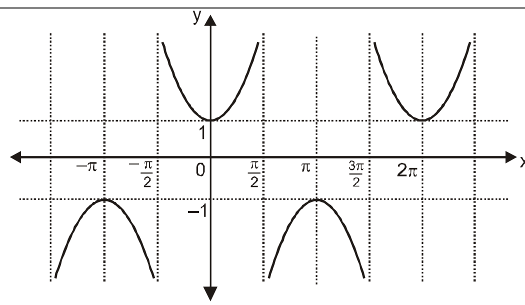
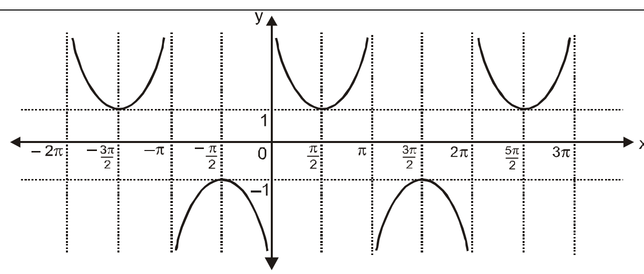
If θ is any angle, then $-\theta$, $\frac{\pi}{2} \pm \theta$, $\pi \pm \theta$, $\frac{3\pi}{2} \pm \theta$, $2\pi \pm \theta$ etc. are called allied angles.

	$-\theta$	$\frac{\pi}{2} - \theta$	$\frac{\pi}{2} + \theta$	$\pi - \theta$	$\pi + \theta$	$\frac{3\pi}{2} - \theta$	$\frac{3\pi}{2} + \theta$	$2\pi - \theta$	$2\pi + \theta$
sin	$-\sin\theta$	$\cos\theta$	$\sin\theta$	$\sin\theta$	$-\sin\theta$	$-\cos\theta$	$-\cos\theta$	$-\sin\theta$	$\sin\theta$
cos	$\cos\theta$	$\sin\theta$	$-\cos\theta$	$-\cos\theta$	$-\cos\theta$	$-\sin\theta$	$\sin\theta$	$\cos\theta$	$\cos\theta$
tan	$-\tan\theta$	$\cot\theta$	$-\tan\theta$	$-\tan\theta$	$\tan\theta$	$\cot\theta$	$-\cot\theta$	$-\tan\theta$	$\tan\theta$
cot	$-\cot\theta$	$\tan\theta$	$-\cot\theta$	$-\cot\theta$	$\cot\theta$	$\tan\theta$	$-\tan\theta$	$-\cot\theta$	$\cot\theta$
sec	$\sec\theta$	$\mathrm{cosec}\,\theta$	$-\sec\theta$	$-\sec\theta$	$-\sec\theta$	$-\mathrm{cosec}\,\theta$	$\mathrm{cosec}\,\theta$	$\sec\theta$	$\sec\theta$
cosec	$-\mathrm{cosec}\theta$	$\sec\theta$	$\mathrm{cosec}\theta$	$\mathrm{cosec}\theta$	$-\mathrm{cosec}\theta$	$-\sec\theta$	$-\sec\theta$	$-\mathrm{cosec}\theta$	$\mathrm{cosec}\theta$

Think, and fill up the blank blocks in following table.

	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{11\pi}{6}$	2π
sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1									
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0									
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	N.D.									

Trigonometric functions :

	Domain	Range	Graph
y = sinx	R	[–1, 1]	
y = cosx	R	[–1, 1]	
y = tanx	R– $\left\{(2n+1)\frac{\pi}{2},\ n \in I\right\}$	R	
y = cotx	R – {$n\pi,\ n \in I$}	R	
y = secx	R– $\left\{(2n+1)\frac{\pi}{2},\ n \in I\right\}$	(– ∞, – 1] ∪ [1, **∞)**	
y=cosecx	R – {$n\pi,\ n \in I$}	(– ∞, – 1] ∪ [1, **∞)**	

Trigonometric functions of sum or difference of two angles:

(a) $\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$
(b) $\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$
(c) $\sin^2 A - \sin^2 B = \cos^2 B - \cos^2 A = \sin(A+B) . \sin(A-B)$
(d) $\cos^2 A - \sin^2 B = \cos^2 B - \sin^2 A = \cos(A+B) . \cos(A-B)$
(e) $\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$ (f) $\cot(A \pm B) = \dfrac{\cot A \cot B \mp 1}{\cot B \pm \cot A}$
(g) $\sin(A+B+C) = \sin A \cos B \cos C + \sin B \cos A \cos C + \sin C \cos A \cos B - \sin A \sin B \sin C$
(h) $\cos(A+B+C) = \cos A \cos B \cos C - \cos A \sin B \sin C - \sin A \cos B \sin C - \sin A \sin B \cos C$
(i) $\tan(A+B+C) = \dfrac{\tan A + \tan B + \tan C - \tan A \tan B \tan C}{1 - \tan A \tan B - \tan B \tan C - \tan C \tan A}$.

(j) $\tan(\theta_1 + \theta_2 + \theta_3 + \dots\dots + \theta_n) = \dfrac{S_1 - S_3 + S_5 - \dots\dots}{1 - S_2 + S_4 - \dots\dots}$

where S_i denotes sum of product of tangent of angles taken i at a time

Example # 1 : Prove that
(i) $\sin(45^\circ + A)\cos(45^\circ - B) + \cos(45^\circ + A)\sin(45^\circ - B) = \cos(A - B)$
(ii) $\tan\left(\dfrac{\pi}{4} + \theta\right) \tan\left(\dfrac{3\pi}{4} + \theta\right) = -1$

Solution : (i) Clearly $\sin(45^\circ + A)\cos(45^\circ - B) + \cos(45^\circ + A)\sin(45^\circ - B)$
$= \sin(45^\circ + A + 45^\circ - B) = \sin(90^\circ + A - B) = \cos(A - B)$
(ii) $\tan\left(\dfrac{\pi}{4} + \theta\right) \times \tan\left(\dfrac{3\pi}{4} + \theta\right) = \dfrac{1 + \tan\theta}{1 - \tan\theta} \times \dfrac{-1 + \tan\theta}{1 + \tan\theta} = -1$

Self practice problems :

(1) If $\cos\alpha = \dfrac{2\sqrt{2}}{3}$, $\sin\beta = \dfrac{4}{5}$, then find $\cos(\alpha + \beta)$ (2) Find the value of $\cos 375^\circ$

(3) Prove that $1 + \tan A \tan \dfrac{A}{2} = \tan A \cot \dfrac{A}{2} - 1 = \sec A$

Answers : (1) $\dfrac{\pm 6\sqrt{2} \pm 4}{15}$ (2) $\dfrac{\sqrt{3} + 1}{2\sqrt{2}}$

Transformation formulae :

(i) $\sin(A+B) + \sin(A-B) = 2 \sin A \cos B$
(ii) $\sin(A+B) - \sin(A-B) = 2 \cos A \sin B$
(iii) $\cos(A+B) + \cos(A-B) = 2 \cos A \cos B$
(iv) $\cos(A-B) - \cos(A+B) = 2 \sin A \sin B$

(a) $\sin C + \sin D = 2 \sin \dfrac{C+D}{2} \cos \dfrac{C-D}{2}$
(b) $\sin C - \sin D = 2 \cos \dfrac{C+D}{2} \sin \dfrac{C-D}{2}$
(c) $\cos C + \cos D = 2 \cos \dfrac{C+D}{2} \cos \dfrac{C-D}{2}$
(d) $\cos C - \cos D = 2 \sin \dfrac{C+D}{2} \sin \dfrac{D-C}{2}$

Example # 2 : Prove that $\cos 7A + \cos 8A = 2\cos\left(\dfrac{15A}{2}\right)\cos\left(\dfrac{A}{2}\right)$

Solution : L.H.S. $\cos 7A + \cos 8A = 2\cos\left(\dfrac{15A}{2}\right)\cos\left(\dfrac{A}{2}\right)$
[$\because \cos C + \cos D = 2\cos\dfrac{C+D}{2}\cos\dfrac{C-D}{2}$]

Example # 3 : Find the value of $2\sin 3\theta \sin\theta - \cos 2\theta + \cos 4\theta$
Solution : $2\sin 3\theta \sin\theta - \cos 2\theta + \cos 4\theta = 2\sin 3\theta \sin\theta - 2\sin 3\theta \sin\theta = 0$

Example # 4 : Prove that

(i) $\dfrac{\sin 8\theta \cos\theta - \sin 6\theta \cos 3\theta}{\cos 2\theta \cos\theta - \sin 3\theta \sin 4\theta} = \tan 2\theta$

(ii) If A + B = 45º then prove that (1 + tanA) (1 + tanB) = 2

Solution : (i) $\dfrac{2\sin 8\theta \cos\theta - 2\sin 6\theta \cos 3\theta}{2\cos 2\theta \cos\theta - 2\sin 3\theta \sin 4\theta} = \dfrac{\sin 9\theta + \sin 7\theta - \sin 9\theta - \sin 3\theta}{\cos 3\theta + \cos\theta - \cos\theta + \cos 7\theta} = \dfrac{2\sin 2\theta \cos 5\theta}{2\cos 5\theta \cos 2\theta} = \tan 2\theta$

(ii) A + B = 45º

$\tan(A + B) = 1 \Rightarrow \dfrac{\tan A + \tan B}{1 - \tan A \tan B} = 1$

$\tan A + \tan B = 1 - \tan A \tan B \quad \Rightarrow \quad \tan A + \tan B + \tan A \tan B + 1 = 2$

$(1 + \tan A)(1 + \tan B) = 2$

Self practice problems

(4) Prove that

(i) $\cos 8x - \cos 5x = -2 \sin \dfrac{13x}{2} \sin \dfrac{3x}{2}$

(ii) $\dfrac{\cos A - \cos 3A}{\sin A - \sin 3A} = -\tan 2A$

(iii) $\dfrac{\sin 2A + \sin 4A + \sin 6A + \sin 8A}{\cos 2A + \cos 4A + \cos 6A + \cos 8A} = \tan 5A$

(iv) $\dfrac{\sin A + 2\sin 3A + \sin 5A}{\sin 3A + 2\sin 5A + \sin 7A} = \dfrac{\sin 3A}{\sin 5A}$

(v) $\dfrac{\sin A - \sin 5A + \sin 9A - \sin 13A}{\cos A - \cos 5A - \cos 9A + \cos 13A} = \cot 4A$

(5) Prove that $\sin \dfrac{\theta}{2} \sin \dfrac{7\theta}{2} + \sin \dfrac{3\theta}{2} \sin \dfrac{11\theta}{2} = \sin 2\theta \sin 5\theta$

(6) Prove that cos A sin (B – C) + cos B sin (C – A) + cos C sin (A – B) = 0

(7) Prove that $2 \cos \dfrac{\pi}{13} \cos \dfrac{9\pi}{13} + \cos \dfrac{3\pi}{13} + \cos \dfrac{5\pi}{13} = 0$

Multiple and sub-multiple angles :

(a) $\sin 2A = 2 \sin A \cos A$ Note : $\sin\theta = 2 \sin \dfrac{\theta}{2} \cos \dfrac{\theta}{2}$ etc.

(b) $\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$

Note : $2\cos^2 \dfrac{\theta}{2} = 1 + \cos\theta$, $2\sin^2 \dfrac{\theta}{2} = 1 - \cos\theta$.

(c) $\tan 2A = \dfrac{2 \tan A}{1 - \tan^2 A}$ Note : $\tan\theta = \dfrac{2 \tan \frac{\theta}{2}}{1 - \tan^2 \frac{\theta}{2}}$

(d) $\sin 2A = \dfrac{2 \tan A}{1 + \tan^2 A}$, $\cos 2A = \dfrac{1 - \tan^2 A}{1 + \tan^2 A}$

(e) $\sin 3A = 3 \sin A - 4 \sin^3 A$

(f) $\cos 3A = 4 \cos^3 A - 3 \cos A$

(g) $\tan 3A = \dfrac{3 \tan A - \tan^3 A}{1 - 3 \tan^2 A}$

Example # 5 : Prove that

(i) $\dfrac{\sin 2A}{1+\cos 2A}$ = tan A (ii) tan A + cot A = 2 cosec 2 A

(iii) $\dfrac{1-\cos A+\cos B-\cos(A+B)}{1+\cos A-\cos B-\cos(A+B)} = \tan\dfrac{A}{2}\cot\dfrac{B}{2}$

Solution : (i) L.H.S. $\dfrac{\sin 2A}{1+\cos 2A} = \dfrac{2\sin A\cos A}{2\cos^2 A}$ = tan A

(ii) L.H.S. tan A + cot A = $\dfrac{1+\tan^2 A}{\tan A} = 2\left(\dfrac{1+\tan^2 A}{2\tan A}\right) = \dfrac{2}{\sin 2A}$ = 2 cosec 2 A

(iii) L.H.S. $\dfrac{1-\cos A+\cos B-\cos(A+B)}{1+\cos A-\cos B-\cos(A+B)} = \dfrac{2\sin^2\dfrac{A}{2}+2\sin\dfrac{A}{2}\sin\left(\dfrac{A}{2}+B\right)}{2\cos^2\dfrac{A}{2}-2\cos\dfrac{A}{2}\cos\left(\dfrac{A}{2}+B\right)}$

$$= \tan\frac{A}{2}\left[\frac{\sin\frac{A}{2}+\sin\left(\frac{A}{2}+B\right)}{\cos\frac{A}{2}-\cos\left(\frac{A}{2}+B\right)}\right] = \tan\frac{A}{2}\left[\frac{2\sin\frac{A+B}{2}\cos\left(\frac{B}{2}\right)}{2\sin\frac{A+B}{2}\sin\left(\frac{B}{2}\right)}\right] = \tan\frac{A}{2}\cot\frac{B}{2}$$

Self practice problems

(8) Prove that $\dfrac{\sin 4\theta+\sin 2\theta}{1+\cos 4\theta+\cos 2\theta} = \dfrac{2\tan\theta}{1-\tan^2\theta}$

(9) Prove that $\sin\dfrac{\pi}{18}\sin\dfrac{3\pi}{18}\sin\dfrac{5\pi}{18}\sin\dfrac{7\pi}{18} = \dfrac{1}{16}$

(10) Prove that tan 3A tan 2A tan A = tan 3A – tan 2A – tan A

(11) Prove that $\tan\left(45^\circ+\dfrac{A}{2}\right)$ = sec A + tan A

Important trigonometric ratios of standard angles :

(a) $\sin n\pi = 0$; $\cos n\pi = (-1)^n$; $\tan n\pi = 0$, where $n \in I$

(b) sin 15° or $\sin\dfrac{\pi}{12} = \dfrac{\sqrt{3}-1}{2\sqrt{2}}$ = cos 75° or $\cos\dfrac{5\pi}{12}$;

cos 15° or $\cos\dfrac{\pi}{12} = \dfrac{\sqrt{3}+1}{2\sqrt{2}}$ = sin 75° or $\sin\dfrac{5\pi}{12}$;

$\tan 15^\circ = \dfrac{\sqrt{3}-1}{\sqrt{3}+1} = 2-\sqrt{3}$ = cot 75° ; $\tan 75^\circ = \dfrac{\sqrt{3}+1}{\sqrt{3}-1} = 2+\sqrt{3}$ = cot 15°

(c) $\sin\dfrac{\pi}{10}$ or sin 18° = $\dfrac{\sqrt{5}-1}{4}$ = cos 72°

cos 36° or $\cos\dfrac{\pi}{5} = \dfrac{\sqrt{5}+1}{4}$ = sin 54°

Conditional identities:

If $A + B + C = \pi$ then :

(i) $\sin 2A + \sin 2B + \sin 2C = 4 \sin A \sin B \sin C$

(ii) $\sin A + \sin B + \sin C = 4 \cos \frac{A}{2} \cos \frac{B}{2} \cos \frac{C}{2}$

(iii) $\cos 2A + \cos 2B + \cos 2C = -1 - 4 \cos A \cos B \cos C$

(iv) $\cos A + \cos B + \cos C = 1 + 4 \sin \frac{A}{2} \sin \frac{B}{2} \sin \frac{C}{2}$

(v) $\tan A + \tan B + \tan C = \tan A \tan B \tan C$

(vi) $\tan \frac{A}{2} \tan \frac{B}{2} + \tan \frac{B}{2} \tan \frac{C}{2} + \tan \frac{C}{2} \tan \frac{A}{2} = 1$

(vii) $\cot \frac{A}{2} + \cot \frac{B}{2} + \cot \frac{C}{2} = \cot \frac{A}{2} . \cot \frac{B}{2} . \cot \frac{C}{2}$

(viii) $\cot A \cot B + \cot B \cot C + \cot C \cot A = 1$

Example # 6 : If $A + B + C = 90°$, Prove that, $\tan A \tan B + \tan B \tan C + \tan C \tan A = 1$

Solution : $A + B = 90° - C$

$$\frac{\tan A + \tan B}{1 - \tan A \tan B} = \cot C$$

$\tan A \tan B + \tan B \tan C + \tan C \tan A = 1$

Example # 7 : If $x + y + z = xyz$, Prove that $\frac{2x}{1-x^2} + \frac{2y}{1-y^2} + \frac{2z}{1-z^2} = \frac{2x}{1-x^2} . \frac{2y}{1-y^2} . \frac{2z}{1-z^2}$.

Solution : Put $x = \tan A$, $y = \tan B$ and $z = \tan C$,
so that we have
$\tan A + \tan B + \tan C = \tan A \tan B \tan C \Rightarrow A + B + C = n\pi$, where $n \in I$
Hence L.H.S.

$$\therefore \quad \frac{2x}{1-x^2} + \frac{2y}{1-y^2} + \frac{2z}{1-z^2} = \frac{2\tan A}{1-\tan^2 A} + \frac{2\tan B}{1-\tan^2 B} + \frac{2\tan C}{1-\tan^2 C} .$$

$= \tan 2A + \tan 2B + \tan 2C$ $\quad [\because A + B + C = n\pi]$

$= \tan 2A \tan 2B \tan 2C = \frac{2x}{1-x^2} . \frac{2y}{1-y^2} . \frac{2z}{1-z^2}$

Self practice problem

(12) If $A + B + C = 180°$, prove that

(i) $\sin(B + 2C) + \sin(C + 2A) + \sin(A + 2B) = 4\sin \frac{B-C}{2} \sin \frac{C-A}{2} \sin \frac{A-B}{2}$

(ii) $\frac{\sin 2A + \sin 2B + \sin 2C}{\sin A + \sin B + \sin C} = 8 \sin \frac{A}{2} \sin \frac{B}{2} \sin \frac{C}{2}$.

(13) If $A + B + C = 2S$, prove that

(i) $\sin(S - A) \sin(S - B) + \sin S \sin (S - C) = \sin A \sin B$.

(ii) $\sin(S - A) + \sin (S - B) + \sin(S - C) - \sin S = 4\sin \frac{A}{2} \sin \frac{B}{2} \sin \frac{C}{2}$.

Sine and Cosine series:

(i) $\sin \alpha + \sin (\alpha + \beta) + \sin (\alpha + 2\beta) + ... + \sin \{\alpha + (n-1) \beta\} = \frac{\sin \frac{n\beta}{2}}{\sin \frac{\beta}{2}} \sin \left(\alpha + \frac{n-1}{2} \beta\right)$

(ii) $\cos \alpha + \cos (\alpha + \beta) + \cos (\alpha + 2\beta) + ... + \cos \{\alpha + (n-1) \beta\} = \frac{\sin \frac{n\beta}{2}}{\sin \frac{\beta}{2}} \cos \left(\alpha + \frac{n-1}{2} \beta\right)$

where : $\beta \neq 2m\pi$, $m \in I$

Example # 8 : (i) Prove that $\sin\theta + \sin3\theta + \sin5\theta + ... + \sin(2n-1)\theta = \dfrac{\sin^2 n\theta}{\sin\theta}$

(ii) Find the average of sin2°, sin4°, sin6° , ... , sin180°

(iii) Prove that $\cos\dfrac{\pi}{11} + \cos\dfrac{3\pi}{11} + \cos\dfrac{5\pi}{11} + \cos\dfrac{7\pi}{11} + \cos\dfrac{9\pi}{11} = \dfrac{1}{2}$

Solution : (i) $\sin\theta + \sin3\theta + \sin5\theta + ... + \sin(2n-1)\theta = \dfrac{\sin n\left(\frac{2\theta}{2}\right)\sin\left(\frac{\theta+(2n-1)\theta}{2}\right)}{\sin\left(\frac{2\theta}{2}\right)} = \dfrac{\sin^2 n\theta}{\sin\theta}$

(ii) $= \dfrac{\sin 2^\circ + \sin 4^\circ + ... + \sin 180^\circ}{90} = \dfrac{\sin 90^\circ(\sin 91^\circ)}{90\sin 1^\circ} = \dfrac{\cos 1^\circ}{90\sin 1^\circ} = \dfrac{\cot 1^\circ}{90}$

(iii) $\cos\dfrac{\pi}{11} + \cos\dfrac{3\pi}{11} + \cos\dfrac{5\pi}{11} + \cos\dfrac{7\pi}{11} + \cos\dfrac{9\pi}{11} = \dfrac{\cos\frac{10\pi}{22}\sin\frac{5\pi}{11}}{\sin\frac{\pi}{11}} = \dfrac{\sin\frac{10\pi}{11}}{2\sin\frac{\pi}{11}} = \dfrac{1}{2}$

Self practice problem

Find sum of the following series :

(14) $\cos\dfrac{\pi}{2n+1} + \cos\dfrac{3\pi}{2n+1} + \cos\dfrac{5\pi}{2n+1} + ...$ up to n terms.

(15) $\sin2\alpha + \sin3\alpha + \sin4\alpha + ... + \sin n\alpha$, where $(n + 2)\alpha = 2\pi$

Answers : (14) $-\dfrac{1}{2}$ (15) 0.

Product series of cosine angles

$$\cos\theta . \cos 2\theta . \cos2^2\theta . \cos2^3\theta ...... \cos2^{n-1}\theta = \frac{\sin 2^n\theta}{2^n \sin\theta}$$

Range of trigonometric expression:

$E = a \sin\theta + b \cos\theta$

$\Rightarrow \quad E = \sqrt{a^2+b^2}\left\{\dfrac{a}{\sqrt{a^2+b^2}}\sin\theta + \dfrac{b}{\sqrt{a^2+b^2}}\cos\theta\right\}$

Let $\dfrac{b}{\sqrt{a^2+b^2}} = \sin\alpha$ & $\dfrac{a}{\sqrt{a^2+b^2}} = \cos\alpha$

$\Rightarrow \quad E = \sqrt{a^2+b^2}\sin(\theta+\alpha)$, where $\tan\alpha = \dfrac{b}{a}$

Hence for any real value of θ,

$$-\sqrt{a^2+b^2} \le \mathbf{E} \le \sqrt{a^2+b^2}$$

Example # 9 : (i) If $\alpha + \beta = 90^\circ$ then find the maximum value of $\sin\alpha \sin\beta$

(ii) Find maximum and minimum value of $1 + 2\sin x + 3\cos^2 x$

Solution : (i) $\sin\alpha \sin(90^\circ - \alpha) = \sin\alpha\cos\alpha = \dfrac{1}{2} \times \sin2\alpha$

maximum value = $\dfrac{1}{2}$

(ii) $1 + 2\sin x + 3\cos^2 x = -3\sin^2 x + 2\sin x + 4 = -3\left(\sin^2 x - \frac{2\sin x}{3}\right) + 4 = -3\left(\sin x - \frac{1}{3}\right)^2 + \frac{13}{3}$

Now $0 \le \left(\sin x - \frac{1}{3}\right)^2 \le \frac{16}{9} \Rightarrow -\frac{16}{9} \le -3\left(\sin x - \frac{1}{3}\right)^2 \le 0$

$-1 \le -3\left(\sin x - \frac{1}{3}\right)^2 + \frac{13}{3} \le \frac{13}{3}$

Self practice problems

(16) Find maximum and minimum values of following

(i) $3 + (\sin x - 2)^2$

(ii) $9\cos^2 x + 48\sin x \cos x - 5\sin^2 x - 2$

(iii) $2\sin\left(\theta + \frac{\pi}{6}\right) + \sqrt{3}\cos\left(\theta - \frac{\pi}{6}\right)$

Answers : (i) max = 12, min = 4. (ii) max = 25, min = –25

(iii) max = $\sqrt{13}$, min = $-\sqrt{13}$

Trigonometric Equation :

An equation involving one or more trigonometric ratios of an unknown angle is called a trigonometric equation.

Solution of Trigonometric Equation :

A solution of trigonometric equation is the value of the unknown angle that satisfies the equation.

e.g. if $\sin\theta = \frac{1}{\sqrt{2}} \Rightarrow \theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{9\pi}{4}, \frac{11\pi}{4}, ...$

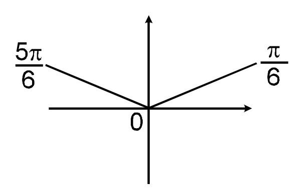
Thus, the trigonometric equation may have infinite number of solutions (because of their periodic nature) and can be classified as :

(i) Principal solution (ii) General solution.

Principal solutions :

The solutions of a trigonometric equation which lie in the interval $[0, 2\pi)$ are called Principal solutions.

e.g. Find the Principal solutions of the equation $\sin x = \frac{1}{2}$.



Solution **:**

$\because$ $\sin x = \frac{1}{2}$

$\because$ there exists two values

i.e. $\frac{\pi}{6}$ and $\frac{5\pi}{6}$ which lie in $[0, 2\pi)$ and whose sine is $\frac{1}{2}$

$\therefore$ Principal solutions of the equation $\sin x = \frac{1}{2}$ are $\frac{\pi}{6}, \frac{5\pi}{6}$

General Solution :

The expression involving an integer 'n' which gives all solutions of a trigonometric equation is called General solution. General solution of some standard trigonometric equations are given below.

General Solution of Some Standard Trigonometric Equations :

(i) If $\sin\theta = \sin\alpha \Rightarrow \theta = n\pi + (-1)^n\alpha$ where $\alpha \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, $n \in I$.

(ii) If $\cos\theta = \cos\alpha \Rightarrow \theta = 2n\pi \pm \alpha$ where $\alpha \in [0, \pi]$, $n \in I$.

(iii) If $\tan\theta = \tan\alpha \Rightarrow \theta = n\pi + \alpha$ where $\alpha \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, $n \in I$.

(iv) If $\sin^2\theta = \sin^2\alpha \Rightarrow \theta = n\pi \pm \alpha$, $n \in I$.

(v) If $\cos^2\theta = \cos^2\alpha \Rightarrow \theta = n\pi \pm \alpha$, $n \in I$.

(vi) If $\tan^2\theta = \tan^2\alpha \Rightarrow \theta = n\pi \pm \alpha$, $n \in I$. [**Note:** α is called the principal angle]

Some Important deductions :

(i) $\sin\theta = 0 \Rightarrow \theta = n\pi$, $n \in I$

(ii) $\sin\theta = 1 \Rightarrow \theta = (4n + 1)\frac{\pi}{2}$, $n \in I$

(iii) $\sin\theta = -1 \Rightarrow \theta = (4n - 1)\frac{\pi}{2}$, $n \in I$

(iv) $\cos\theta = 0 \Rightarrow \theta = (2n + 1)\frac{\pi}{2}$, $n \in I$

(v) $\cos\theta = 1 \Rightarrow \theta = 2n\pi, n \in I$

(vi) $\cos\theta = -1 \Rightarrow \theta = (2n + 1)\pi$, $n \in I$

(vii) $\tan\theta = 0 \Rightarrow \theta = n\pi$, $n \in I$

Example # 10: Solve $\cos\theta = \frac{1}{2}$

Solution : $\because \cos\theta = \frac{1}{2} \Rightarrow \cos\theta = \cos\frac{\pi}{3}$ $\therefore \theta = 2n\pi \pm \frac{\pi}{3}, n \in I$

Example # 11 : Solve : $\sec 2\theta = -\frac{2}{\sqrt{3}}$

Solution : $\because \sec 2\theta = -\frac{2}{\sqrt{3}} \Rightarrow \cos 2\theta = -\frac{\sqrt{3}}{2}$

$\Rightarrow \cos 2\theta = \cos\frac{5\pi}{6} \Rightarrow 2\theta = 2n\pi \pm \frac{5\pi}{6}$, $n \in I \Rightarrow \theta = n\pi \pm \frac{5\pi}{12}, n \in I$

Example # 12 : Solve $\tan\theta = \frac{3}{4}$

Solution : $\because \tan\theta = \frac{3}{4}$ ....(i)

Let $\frac{3}{4} = \tan\alpha \Rightarrow \tan\theta = \tan\alpha$

$\Rightarrow \theta = n\pi + \alpha$, where $\alpha = \tan^{-1}\left(\frac{3}{4}\right)$, $n \in I$

Self Practice Problems :

(17) Solve $\cot\theta = -1$ (18) Solve $\cos 4\theta = -\frac{\sqrt{3}}{2}$

Answers : (17) $\theta = n\pi - \frac{\pi}{4}$, $n \in I$ (18) $\frac{n\pi}{2} \pm \frac{\pi}{24}, n \in I$

Example # 13 : Solve $\tan^2\theta = 1$

Solution : $\because$ $\tan^2\theta = 1$ $\Rightarrow$ $\tan^2\theta = (1)^2$

$\Rightarrow$ $\tan^2\theta = \tan^2\frac{\pi}{4}$ $\Rightarrow$ $\theta = n\pi \pm \frac{\pi}{4}$, $n \in I$

Example # 14 : Solve $4\sec^2\theta = 5 + \tan^2\theta$

Solution : $\because$ $4\sec^2\theta = 5 + \tan^2\theta$ ....(i)

For equation (i) to be defined $\theta \neq (2n + 1)\frac{\pi}{2}$, $n \in I$

$\because$ equation (i) can be written as:

$4(1 + \tan^2\theta) = 5 + \tan^2\theta$

$3\tan^2\theta = 1$

$\tan^2\theta = \tan^2\pi/6$

$\theta = n\pi \pm \frac{\pi}{6}$, $n \in I$

Self Practice Problems :

(19) Solve $\frac{\tan 3x - \tan 2x}{1 + \tan 3x \ \tan 2x} = 1$

(20) Solve $2\cos^2x + \sin^2 2x = 2$

Answers : (19) no Solution (20) $n\pi$, $n \in I$ or $n\pi \pm \frac{\pi}{4}$, $n \in I$

Types of Trigonometric Equations :

Type -1

Trigonometric equations which can be solved by use of factorization.

Example # 15 : $\frac{\sin^3\frac{x}{2} - \cos^3\frac{x}{2}}{2 + \sin x} = \frac{\cos x}{3}$

Solution : $\frac{\sin^3\frac{x}{2} - \cos^3\frac{x}{2}}{2 + \sin x} = \frac{\cos x}{3} \Rightarrow \frac{\left(\sin\frac{x}{2} - \cos\frac{x}{2}\right)\left(\sin^2\frac{x}{2} + \cos^2\frac{x}{2} + \sin\frac{x}{2}\cos\frac{x}{2}\right)}{2 + \sin x} = \frac{\cos x}{3}$

$\frac{\left(\sin\frac{x}{2} - \cos\frac{x}{2}\right)(2 + \sin x)}{2(2 + \sin x)} = \frac{\cos x}{3} \Rightarrow 3\left(\sin\frac{x}{2} - \cos\frac{x}{2}\right) - 2\left(\cos^2\frac{x}{2} - \sin^2\frac{x}{2}\right) = 0$

$\left(\sin\frac{x}{2} - \cos\frac{x}{2}\right)\left(3 + 2\sin\frac{x}{2} + 2\cos\frac{x}{2}\right) = 0 \Rightarrow \sin\frac{x}{2} = \cos\frac{x}{2} \Rightarrow \tan\frac{x}{2} = 1$

$\frac{x}{2} = n\pi + \frac{\pi}{4}$, $n \in I$ $\Rightarrow$ $x = 2n\pi + \frac{\pi}{2}$, $n \in I$

Self Practice Problems :

(21) Solve $\cos^3x + \cos^2x - 4\cos^2\frac{x}{2} = 0$

(22) Solve $\tan^2\theta + 3\sec\theta + 3 = 0$

Answers : (21) $(2n + 1)\pi$, $n \in I$

(22) $2n\pi \pm \frac{2\pi}{3}$, $n \in I$ or $(2n + 1)\pi$, $n \in I$

Type - 2

Trigonometric equations which can be solved by reducing them in quadratic equations.

Example # 16 : Solve $\sin^2x - \frac{\cos x}{4} = \frac{1}{4}$

Solution : $\sin^2x - \frac{\cos x}{4} = \frac{1}{4}$

$4(1 - \cos^2x) - \cos x = 1$

$4\cos^2x + \cos x - 3 = 0$

$(\cos x + 1)(4\cos x - 3) = 0$

$\cos x = -1$, $\cos x = \frac{3}{4}$

$x = (2n+1)\pi$, $x = (2m\pi \pm \alpha)$ where $\alpha = \cos^{-1}\frac{3}{4}$, m, n $\in$ I

Self Practice Problems :

(23) Solve $4\sin^2\theta + 2\sin\theta\left(\sqrt{3}-1\right) - \sqrt{3} = 0$

(24) Solve $4\cos\theta - 3\sec\theta = \tan\theta$

Answers : (23) $n\pi + (-1)^n \frac{\pi}{6}$, n $\in$ I or $n\pi + (-1)^n\left(\frac{-\pi}{3}\right)$, n $\in$ I

(24) $n\pi + (-1)^n \alpha$ where $\alpha = \sin^{-1}\left(\frac{-1-\sqrt{17}}{8}\right)$, n $\in$ I

or $n\pi + (-1)^n \beta$ where $\beta = \sin^{-1}\left(\frac{-1+\sqrt{17}}{8}\right)$, n $\in$ I

Type - 3

Trigonometric equations which can be solved by transforming a sum or difference of trigonometric ratios into their product.

Example # 17 : Solve $\cos x + \cos 3x - 2\cos 2x = 0$

Solution : $\cos x + \cos 3x - 2\cos 2x = 0$

$2\cos 2x \cos x - 2\cos 2x = 0$

$2\cos 2x (\cos x - 1) = 0$

$\cos 2x = 0$, $\cos x = 1$

$x = (2n + 1), \frac{\pi}{2}$ $x = 2m\pi$, m, n $\in$ I

Self Practice Problems :

(25) Solve $\sin 7\theta = \sin 3\theta + \sin\theta$ (26) Solve $1 + \cos 3x = 2\cos 2x$

(27) Solve $8\cos x \cos 2x \cos 4x = \frac{\sin 6x}{\sin x}$

Answers : (25) $\frac{n\pi}{3}$, n $\in$ I or $\frac{n\pi}{2} \pm \frac{\pi}{12}$, n $\in$ I

(26) $n\pi \pm \frac{\pi}{6}$, n $\in$ I or $2n\pi$, n $\in$ I (27) $\frac{n\pi}{7} + \frac{\pi}{14}$, n $\in$ I

Type - 4

Trigonometric equations which can be solved by transforming a product of trigonometric ratios into their sum or difference.

Example # 18 : Solve $\sec 4\theta - \sec 2\theta = 2$

Solution : $\frac{1}{\cos 4\theta} - \frac{1}{\cos 2\theta} = 2$

$\cos 2\theta - \cos 4\theta = 2 \cos 4\theta \cos 2\theta$

$\cos 2\theta - \cos 4\theta = \cos 6\theta + \cos 2\theta$

$\cos 6\theta + \cos 4\theta = 0$

$2\cos 5\theta \cos\theta = 0$

$\cos 5\theta = 0$ or $\cos\theta = 0$

$5\theta = (2n + 1)\frac{\pi}{2}$ $\quad\quad \theta = (2m + 1)\frac{\pi}{2}$ m, n $\in$ I

Type - 5

Trigonometric Equations of the form a sinx + b cosx = c, where a, b, c $\in$ R, can be solved by dividing both sides of the equation by $\sqrt{a^2 + b^2}$.

Example # 19 : Solve $\sin x + 2\cos x = \sqrt{5}$

Solution : $\because$ $\sin x + 2\cos x = \sqrt{5}$ ....(i)

Here a = 1, b = 2.

$\therefore$ divide both sides of equation (i) by $\sqrt{5}$, we get

$\sin x \cdot \frac{1}{\sqrt{5}} + 2\cos x \cdot \frac{1}{\sqrt{5}} = 1 \quad \Rightarrow \quad \sin x.\sin\alpha + \cos x.\cos\alpha = 1 \quad \Rightarrow \cos(x - \alpha) = 1$

$\Rightarrow \quad x - \alpha = 2n\pi,\ n \in I \quad \Rightarrow \quad x = 2n\pi + \alpha,\ n \in I$

$\therefore$ Solution of given equation is $2n\pi + \alpha,\ n \in I$ where $\alpha = \tan^{-1}\left(\frac{1}{2}\right)$

Note : Trigonometric equation of the form a sinx + b cosx = c can also be solved by changing sinx and cosx into their corresponding tangent of half the angle.

Example # 20 : Solve $3\cos x + 4\sin x = 5$

Solution : $\because$ $3\cos x + 4\sin x = 5$ ....(i)

$\because$ $\cos x = \frac{1 - \tan^2\frac{x}{2}}{1 + \tan^2\frac{x}{2}}$ & $\sin x = \frac{2\tan\frac{x}{2}}{1 + \tan^2\frac{x}{2}}$

$\therefore$ equation (i) becomes

$\Rightarrow \quad 3\left(\frac{1 - \tan^2\frac{x}{2}}{1 + \tan^2\frac{x}{2}}\right) + 4\left(\frac{2\tan\frac{x}{2}}{1 + \tan^2\frac{x}{2}}\right) = 5$ ....(ii)

Let $\tan\frac{x}{2} = t$

$\therefore$ equation (ii) becomes $3\left(\frac{1 - t^2}{1 + t^2}\right) + 4\left(\frac{2t}{1 + t^2}\right) = 5$

$\Rightarrow \quad 4t^2 - 4t + 1 = 0 \quad \Rightarrow \quad (2t - 1)^2 = 0$

$\Rightarrow \quad t = \frac{1}{2} \quad \because \quad t = \tan\frac{x}{2}$

$\Rightarrow \quad \tan\frac{x}{2} = \frac{1}{2} \quad \Rightarrow \quad \tan\frac{x}{2} = \tan\alpha$, where $\tan\alpha = \frac{1}{2}$

$\Rightarrow \quad \frac{x}{2} = n\pi + \alpha \quad \Rightarrow \quad x = 2n\pi + 2\alpha$ where $\alpha = \tan^{-1}\left(\frac{1}{2}\right)$, n $\in$ I

Self Practice Problems :

(28) Solve $2\sqrt{2}\cos x + \sin x = 3$

(29) Solve $\sin x + \tan\dfrac{x}{2} = 0$

Answers : (28) $2n\pi + \alpha,\ n \in I$ where $\alpha = \tan^{-1}\left(\dfrac{1}{2\sqrt{2}}\right)$

(29) $x = 2n\pi,\ n \in I$

Type - 6

Trigonometric equations of the form P(sinx ± cosx, sinx cosx) = 0, where p(y, z) is a polynomial, can be solved by using the substitution sinx ± cosx = t.

Example # 21 : Solve $\sin 2x + 3\sin x = 1 + 3\cos x$

Solution : $\sin 2x + 3\sin x = 1 + 3\cos x$

$\sin 2x + 3(\sin x - \cos x) = 1$ .... (i)

Let $\sin x - \cos x = t$

$\Rightarrow \sin^2 x + \cos^2 x - 2\sin x.\cos x = t^2 \quad \Rightarrow \quad \sin 2x = 1 - t^2$

Now put $\sin x - \cos x = t$ and $\sin 2x = 1 - t^2$ in (i)

$1 - t^2 + 3t = 1$

$t^2 - 3t = 0$

$t = 0$ or $t = 3$ (not possible)

$\sin x - \cos x = 0$

$\tan x = 1 \quad \Rightarrow \quad x = n\pi + \dfrac{\pi}{4},\ n \in I$

Self Practice Problems:

(30) Solve $1 - \sin 2x + 2\sin x - 2\cos x = 0$

(31) Solve $2\cos x + 2\sin x + \sin 3x - \cos 3x = 0$

(32) Solve $(1 - \sin 2x)(\cos x - \sin x) = 1 - 2\sin^2 x$.

Answers : (30) $n\pi + \dfrac{\pi}{4},\ n \in I$

(31) $n\pi - \dfrac{\pi}{4}$ or $\dfrac{n\pi}{2} + (-1)^n\dfrac{\pi}{12},\ n \in I$

(32) $2n\pi + \dfrac{\pi}{2},\ n \in I$ or $2n\pi,\ n \in I$ or $n\pi + \dfrac{\pi}{4},\ n \in I$

Type - 7

Trigonometric equations which can be solved by the use of boundness of the trigonometric ratios sinx and cosx.

Example # 22 : Solve $\sin 2x + \cos 4x = 2$

Solution : $\sin 2x + \cos 4x = 2$

Now equation will be true if $\sin 2x = 1$ and $\cos 4x = 1$

$\Rightarrow 2x = (4n + 1)\dfrac{\pi}{2},\ n \in I$ and $4x = 2m\pi,\ m \in I$

$\Rightarrow x = (4n + 1)\dfrac{\pi}{4},\ n \in I$ and $x = \dfrac{m\pi}{2},\ m \in I \Rightarrow (4n + 1)\dfrac{\pi}{4} = \dfrac{m\pi}{2} \Rightarrow m = \dfrac{4n+1}{2}$

Which is not possible for $m, n \in I$

Self Practice Problems :

(33) Solve $\cos^{50}x - \sin^{50}x = 1$

(34) Solve $12\sin x + 5\cos x = 2y^2 - 8y + 21$ for x & y

Answers : (33) $n\pi,\ n \in I$ (34) $x = 2n\pi + \alpha$ where $\alpha = \cos^{-1}\left(\dfrac{5}{13}\right),\ n \in I\ \ y = 2$

IMPORTANT POINTS :

1. Many trigonometrical equations can be solved by different methods. The form of solution obtained in different methods may be different. From these different forms of solutions, the students should not think that the answer obtained by one method are wrong and those obtained by another method are correct. The solutions obtained by different methods may be shown to be equivalent by some supplementary transformations.

To test the equivalence of two solutions obtained from two methods, the simplest way is to put values of
n = ...–2, –1, 0, 1, 2, 3....... etc. and then to find the angles in $[0, 2\pi]$. If all the angles in both solutions are same, the solutions are equivalent.

2. While manipulating the trigonometrical equation, avoid the danger of losing roots. Generally, some roots are lost by cancelling a common factor from the two sides of an equation. For example, suppose we have the equation tanx = 2 sinx. Here by dividing both sides by sinx, we get cosx = $\frac{1}{2}$. This is not equivalent to the original equation. Here the roots obtained by sinx = 0, are lost. Thus in place of dividing an equation by a common factor, the students are advised to take this factor out as a common factor from all terms of the equation.

3. While equating one of the factors to zero, take care of the other factor that it should not become infinite. For example, if we have the equation sinx = 0, which can be written as cos x tan x = 0. Here we cannot put
cosx = 0, since for cos x = 0, tanx = sinx/ cosx is infinite.

4. Avoid squaring : When we square both sides of an equation, some extraneous roots appear. Hence it is necessary to check all the solutions found by substituting them in the given equation and omit the solutions not satisfying the given equation.
For example : Consider the equation,
$\sin\theta + \cos\theta = 1$ .....(1)
Squaring we get
$1 + \sin 2\theta = 1$ or $\sin 2\theta = 0$ .....(2)
i.e. $2\theta = n\pi$ or $\theta = n\pi/2$,
This gives $\theta = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, ...$
Verification shows that π and $\frac{3\pi}{2}$ do not satisfy the equation as $\sin\pi + \cos\pi = -1, \neq 1$
and $\sin\frac{3\pi}{2} + \cos\frac{3\pi}{2} = -1, \neq 1$.
The reason for this is simple.
The equation (2) is not equivalent to (1) and (2) contains two equations : $\sin\theta + \cos\theta = 1$ and $\sin\theta + \cos\theta = -1$. Therefore we get extra solutions.
Thus if squaring is must, verify each of the solution.

5. Some necessary restrictions :
If the equation involves tanx, secx, take cosx $\neq 0$. If cot x or cosec x appear, take sinx $\neq 0$.
If log appear in the equation, i.e. log $[f(\theta)]$ appear in the equation, use $f(\theta) > 0$ and base of log $> 0, \neq 1$.
Also note that $\sqrt{[f(\theta)]}$ is always positive, for example $\sqrt{\sin^2\theta} = |\sin\theta|$, not $\pm \sin\theta$.

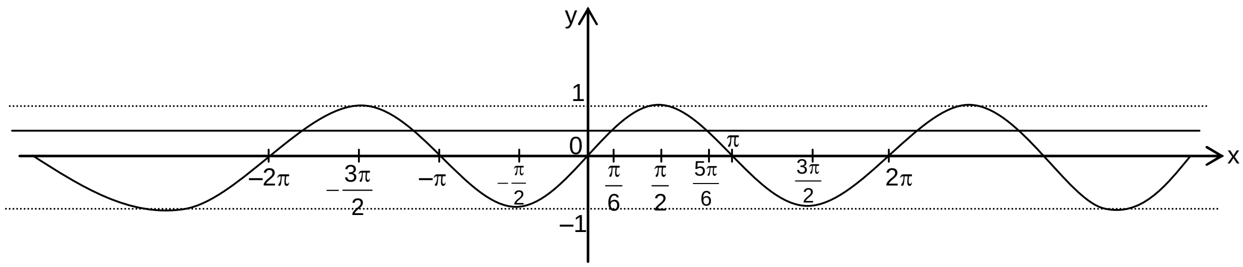
6. Verification : Student are advice to check whether all the roots obtained by them satisfy the equation and lie in the domain of the variable of the given equation.

Trigonometric Inequalities :

To solve a trigonometric inequality, transform it into many basic trigonometric inequalities. The transformation process proceeds exactly the same as in solving trigonometric equations. The common period of a trigonometric inequality is the least common multiple of all periods of the trigonometric functions presented in the inequality. **For example :** the trigonometric inequality
sinx + sin2x + cos x/2 < 1 has 4π as common period. Unless specified, the solution set of a trigonometric inequality must be solved, at least, within one whole common period.

Example : Find the solution set of inequality sinx > 1/2.

Solution : When sinx = 1/2, the two values of x between 0 and 2π are π/6 and 5π/6.



y = sinx

From, the graph of y = sinx, it is obvious that, between 0 and 2π, sinx > 1/2 $\Rightarrow$ π/6 < x < 5π/6.

Hence sinx >1/2 $\Rightarrow$ 2nπ + π/6 < x < 2nπ + 5π/6, n $\in$ I .

The required solution set is $\bigcup\limits_{n\in I}$ (2nπ + π/6, 2nπ + 5π/6)

Self practice problems

(35) Solve the following inequations

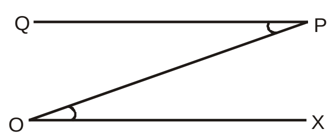
(i) (sinx – 2) (2sinx–1) < 0 (ii) sinx + $\sqrt{3}$ cosx $\geq$ 1

Ans. (i) $x \in \bigcup\limits_{n\in I}\left(\frac{\pi}{6}+2n\pi, \frac{5\pi}{6}+2n\pi\right)$ (ii) $x\in \bigcup\limits_{n\in I}\left[-\frac{\pi}{6}+2n\pi, 2n\pi+\frac{\pi}{2}\right]$

Heights and distances :

Angle of elevation and depression :

Let OX be a horizontal line and P be a point which is above point O. If an observer (eye of observer) is at point O and an object is lying at point P then $\angle$XOP is called angle of elevation as shown in figure. If an observer (eye of observer) is at point P and object is at point O then $\angle$QPO is called angle of depression.



Exercise-1

✍ **Marked questions are recommended for Revision.**

PART - I : SUBJECTIVE QUESTIONS

Section (A) : Measurement of Angles & Allied angles

A-1. Find the radian measures corresponding to the following degree measures

(i) 15° (ii) 240° (iii) 530°

A-2. Find the degree measures corresponding to the following radian measures

(i) $\frac{3\pi}{4}$ (ii) -4π (iii) $\frac{5\pi}{3}$ (iv) $\frac{7\pi}{6}$

A-3. Prove that :

(i) $2\sin^2\frac{\pi}{6} + \text{cosec}\,\frac{7\pi}{6}\cos^2\frac{\pi}{3} = 0$

(ii) $\cot^2\frac{\pi}{6} + \text{cosec}\,\frac{5\pi}{6} + 3\tan^2\frac{\pi}{6} = 6$

A-4. Find the value of :

(i) cos 210° (ii) sin 225° (iii) tan 330° (iv) cot (– 315°)

A-5. Prove that

(i) $\dfrac{\cos(\pi+\theta)\cos(-\theta)}{\sin(\pi-\theta)\cos\left(\frac{\pi}{2}+\theta\right)} = \cot^2\theta.$

(ii) $\cos\theta + \sin(270° + \theta) - \sin(270° - \theta) + \cos(180° + \theta) = 0.$

(iii) $\cos\left(\frac{3\pi}{2}+\theta\right)\cos(2\pi+\theta)\left[\cot\left(\frac{3\pi}{2}-\theta\right)+\cot(2\pi+\theta)\right] = 1.$

A-6. If $\tan\theta = -5/12$, θ is not in the second quadrant, then show that

$$\frac{\sin(360^0-\theta)+\tan(90^0+\theta)}{-\sec(270^0+\theta)+\text{cosec}(-\theta)} = \frac{181}{338}$$

Section (B) : Graphs and Basic Identites (sin(A±B), cos(A±B), tan(A±B))

B-1. Sketch the following graphs :

(i) $y = 3\sin 2x$ (ii) $y = 2\tan x$ (iii) $y = \cos\pi x$

B-2. Find number of solutions of equation $\sin x = -4x + 1$

B-3.✍ If $\tan\theta + \sec\theta = \frac{2}{3}$ then $\sec\theta$ is

B-4. Show that :

(i) $\sin 20° \cdot \cos 40° + \cos 20° \cdot \sin 40° = \sqrt{3}/2$

(ii) $\cos 100° \cdot \cos 40° + \sin 100° \cdot \sin 40° = 1/2$

B-5. Show that : $\cos 2\theta\cos\frac{\theta}{2} - \cos 3\theta\cos\frac{9\theta}{2} = \sin 5\theta\sin\frac{5\theta}{2}$.

B-6 If $A + B = 45^0$, prove that $(1 + \tan A)(1 + \tan B) = 2$ and hence deduce that $\tan 22\frac{1}{2}^0 = \sqrt{2} - 1$

B-7. Eliminate θ from the relations $a\sec\theta = 1 - b\tan\theta$, $a^2\sec^2\theta = 5 + b^2\tan^2\theta$

Section (C) : $\sin^2 A - \sin^2 B$, Multiple angles upto 3A, 2sinA cosB, sinC – sinD

C-1. Show that :
(i) $\sin^2 75^0 - \sin^2 15^0 = \sqrt{3}/2$ (ii) $\sin^2 45^0 - \sin^2 15^0 = \sqrt{3}/4$

C-2. Find the value of
(i) $4 \sin 18° \cos 36°$ (ii) $\cos^2 72° - \sin^2 54°$
(iii) $\cos^2 48° - \sin^2 12°$

C-3. If α and β are the solution of $a \cos\theta + b \sin\theta = c$, then show that $\cos(\alpha + \beta) = \dfrac{a^2 - b^2}{a^2 + b^2}$

C-4. Show that : $\sin^2\left(\dfrac{\pi}{8} + \dfrac{A}{2}\right) - \sin^2\left(\dfrac{\pi}{8} - \dfrac{A}{2}\right) = \left(\dfrac{1}{\sqrt{2}}\right)\sin A$

C-5. Show that : $\cos^2\alpha + \cos^2(\alpha + \beta) - 2\cos\alpha \cos\beta \cos(\alpha + \beta) = \sin^2\beta$.

C-6. Prove that
(i) $\dfrac{\sin^2 A - \sin^2 B}{\sin A \cos A - \sin B \cos B} = \tan(A + B)$
(ii) $\cot(A + 15°) - \tan(A - 15°) = \dfrac{4\cos 2A}{1 + 2\sin 2A}$

C-7. If $0 < \theta < \pi/4$, then show that $\sqrt{2 + \sqrt{2(1 + \cos 4\theta)}} = 2\cos\theta$

C-8. Prove that $\dfrac{\cos^3 A - \cos 3A}{\cos A} + \dfrac{\sin^3 A + \sin 3A}{\sin A} = 3$

C-9. Prove that
(i) $\left\{\dfrac{1 - \cot^2\left(\dfrac{\alpha - \pi}{4}\right)}{1 + \cot^2\left(\dfrac{\alpha - \pi}{4}\right)} + \cos\dfrac{\alpha}{2} \cot 4\alpha\right\}\sec\dfrac{9\alpha}{2} = \operatorname{cosec} 4\alpha.$

(ii) $\dfrac{1}{\tan 3\alpha - \tan\alpha} - \dfrac{1}{\cot 3\alpha - \cot\alpha} = \cot 2\alpha.$

(iii) $\dfrac{\sec 8A - 1}{\sec 4A - 1} = \dfrac{\tan 8A}{\tan 2A}$

(iv) $\dfrac{\cos A + \sin A}{\cos A - \sin A} - \dfrac{\cos A - \sin A}{\cos A + \sin A} = 2\tan 2A$

C-10. Prove that $\sin\theta = \dfrac{\sin 3\theta}{1 + 2\cos 2\theta}$ and hence deduce the value of sin 15°.

C-11. Prove that $4(\cos^3 20° + \cos^3 40°) = 3(\cos 20° + \cos 40°)$

C-12. Prove that :
(i) $\dfrac{\tan 3x}{\tan x} = \dfrac{2\cos 2x + 1}{2\cos 2x - 1}$ (ii) $\dfrac{2\sin x}{\sin 3x} + \dfrac{\tan x}{\tan 3x} = 1$

C-13. Prove that :
$\tan\theta \tan(60^0 + \theta) \tan(60^0 - \theta) = \tan 3\theta$ and hence deduce that $\tan 20^0 \tan 40^0 \tan 60^0 \tan 80^0 = 3$.

C-14. Prove that :

(i) (cosec θ – sin θ) (sec θ – cos θ) (tan θ + cot θ) = 1

(ii) $\dfrac{2\sin\theta \quad \tan\theta \quad (1-\tan\theta)+2\sin\theta \quad \sec^2\theta}{(1+\tan\theta)^2} = \dfrac{2\sin\theta}{(1+\tan\theta)}$

(iii) $\sqrt{\dfrac{1-\sin A}{1+\sin A}}$ = ± (sec A – tan A)

(iv) $\dfrac{\cos A \operatorname{cosec} A - \sin A \sec A}{\cos A + \sin A}$ = cosec A – sec A

(v) $\dfrac{1}{\sec\alpha - \tan\alpha} - \dfrac{1}{\cos\alpha} = \dfrac{1}{\cos\alpha} - \dfrac{1}{\sec\alpha + \tan\alpha}$

(vi) $\dfrac{\cos^3 A + \sin^3 A}{\cos A + \sin A} + \dfrac{\cos^3 A - \sin^3 A}{\cos A - \sin A} = 2$

Section (D) : Conditional Identities & Trigonometric Series

D-1. For all values of α, β, γ prove that,

$\cos\alpha + \cos\beta + \cos\gamma + \cos(\alpha+\beta+\gamma) = 4\cos\dfrac{\alpha+\beta}{2} \cdot \cos\dfrac{\beta+\gamma}{2} \cdot \cos\dfrac{\gamma+\alpha}{2}$.

D-2. If x + y + z = $\dfrac{\pi}{2}$ show that, sin 2x + sin 2y + sin 2z = 4cosx cosy cosz.

D-3. If x + y = π + z, then prove that $\sin^2 x + \sin^2 y - \sin^2 z = 2 \sin x \sin y \cos z$.

D-4. If A + B + C = 2S then prove that

$\cos(S - A) + \cos(S - B) + \cos(S - C) + \cos S = 4\cos\dfrac{A}{2}\cos\dfrac{B}{2}\cos\dfrac{C}{2}$

D-5. If A + B + C = 0° then prove that sin 2A + sin 2B + sin 2C = – 4 sin A sin B sin C

D-6. If ϕ is the exterior angle of a regular polygon of n sides and θ is any constant, then prove that sin θ + sin (θ + ϕ) + sin (θ +2ϕ) + ... up to n terms = 0

D-7. Prove that $\sin^2\theta + \sin^2 2\theta + \sin^2 3\theta + ... + \sin^2 n\theta = \dfrac{n}{2} - \dfrac{\sin n\theta \cos(n+1)\theta}{2\sin\theta}$

D-8. Prove that :

(i) $\cos\dfrac{2\pi}{7}\cos\dfrac{4\pi}{7}\cos\dfrac{6\pi}{7} = \dfrac{1}{8}$

(ii) $\cos\dfrac{\pi}{11}\cos\dfrac{2\pi}{11}\cos\dfrac{3\pi}{11}\cos\dfrac{4\pi}{11}\cos\dfrac{5\pi}{11} = \dfrac{1}{32}$

Section (E) : Range of Trigonometric Expressions

E-1. Find the extreme values of $\cos x \cos\left(\dfrac{2\pi}{3}+x\right)\cos\left(\dfrac{2\pi}{3}-x\right)$

E-2. Find the maximum and minimum values of following trigonometric functions

(i) $\cos 2x + \cos^2 x$ (ii) $\cos^2\left(\dfrac{\pi}{4}+x\right) + (\sin x - \cos x)^2$

E-3. Find the greatest and least value of y

(i) $y = 10\cos^2 x - 6\sin x \cos x + 2\sin^2 x$

(ii) $y = 3\cos\left(\theta + \dfrac{\pi}{3}\right) + 5\cos\theta + 3$

Section (F) : Trigonometric Equations

F-1. What are the most general values of θ which satisfy the equations:

(i) $\sin\theta = \frac{1}{\sqrt{2}}$ (ii) $\tan(x-1) = \sqrt{3}$

(iii) $\tan\theta = -1$ (iv) $\text{cosec}\theta = \frac{2}{\sqrt{3}}$.

(v) $2\cot^2\theta = \text{cosec}^2\theta$

F-2. Solve

(i) $\sin 9\theta = \sin\theta$ (ii) $\cot\theta + \tan\theta = 2\text{cosec}\theta$

(iii) $\sin 2\theta = \cos 3\theta$ (iv) $\cot\theta = \tan 8\theta$

(v) $\cot\theta - \tan\theta = 2$. (vi) $\text{cosec}\theta = \cot\theta + \sqrt{3}$

(vii) $\tan 2\theta \tan\theta = 1$

(viii) $\tan\theta + \tan 2\theta + \sqrt{3}\tan\theta\tan 2\theta = \sqrt{3}$.

F-3. Solve

(i) $\sin\theta + \sin 3\theta + \sin 5\theta = 0$.

(ii) $\cos\theta + \sin\theta = \cos 2\theta + \sin 2\theta$.

(iii) $\cos^2 x + \cos^2 2x + \cos^2 3x = 1$.

(iv) $\sin^2 n\theta - \sin^2(n-1)\theta = \sin^2\theta$, where n is constant and $n \neq 0, 1$

F-4. Solve

(i) $\tan^2\theta - (1+\sqrt{3})\tan\theta + \sqrt{3} = 0$

(ii) $4\cos\theta - 3\sec\theta = 2\tan\theta$

(iii) $\tan x \,.\, \tan\left(x+\frac{\pi}{3}\right) . \tan\left(x+\frac{2\pi}{3}\right) = \sqrt{3}$.

F-5. Solve

(i) $\sqrt{3}\sin\theta - \cos\theta = \sqrt{2}$ (ii) $5\sin\theta + 2\cos\theta = 5$

Section (G) : Trigonometric Inquations and Height & Distance

G-1. Solve $\tan^2 x \leq 1$

G-2. Solve $2\sin^2 x - \sin x - 1 > 0$

G-3. Solve $\sqrt{\sqrt{3}\cot\theta} < 1$

G-4. Two pillars of equal height stand on either side of a roadway which is 60 m wide. At a point in the roadway between the pillars, the angle of elevation of the top of pillars are 60° and 30°. Then find height of pillars -

G-5. If the angles of elevation of the top of a tower from two points distance a and b from the base and in the same straight line with it are complementary, then find the height of the tower :

G-6. From the top of a cliff 25 m high the angle of elevation of a tower is found to be equal to the angle of depression of the foot of the tower. Then find height of the tower -

PART - II : ONLY ONE OPTION CORRECT TYPE

Section (A) : Measurement of Angles & Allied angles

A-1. $\cos(540° - \theta) - \sin(630° - \theta)$ is equal to

(A) 0 (B) $2\cos\theta$ (C) $2\sin\theta$ (D) $\sin\theta - \cos\theta$

A-2. The value of tan 1° tan 2° tan 3° ... tan 89° is

(A) 1 (B) 0 (C) ∞ (D) $\frac{1}{2}$

A-3. If x = y cos $\frac{2\pi}{3}$ = z cos $\frac{4\pi}{3}$, then xy + yz + zx is equal to

(A) – 1 (B) 0 (C) 1 (D) 2

A-4. If 0° < x < 90° & cosx = $\frac{3}{\sqrt{10}}$, then the value of $\log_{10}\sin x + \log_{10}\cos x + \log_{10}\tan x$ is

(A) 0 (B) 1 (C) – 1 (D) 2

A-5. If $\tan\alpha + \cot\alpha = a$ then the value of $\tan^4\alpha + \cot^4\alpha$ =

(A) $a^4 + 4a^2 + 2$ (B) $a^4 - 4a^2 + 2$ (C) $a^4 - 4a^2 - 2$ (D) $a^4 - 2a^2 + 2$

Section (B) : Graphs and Basic Identites (sin(A±B), cos(A±B), tan(A±B))

B-1. **STATEMENT-1 :** sin 2 > sin 3

STATEMENT-2 : If x, y $\in \left(\frac{\pi}{2}, \pi\right)$, x < y, then sin x > sin y

(A) STATEMENT-1 is true, STATEMENT-2 is true and STATEMENT-2 is correct explanation for STATEMENT-1

(B) STATEMENT-1 is true, STATEMENT-2 is true and STATEMENT-2 is not correct explanation for STATEMENT-1

(C) STATEMENT-1 is true, STATEMENT-2 is false

(D) STATEMENT-1 is false, STATEMENT-2 is true

B-2. If cosecθ – cotθ = α, then cotθ is :

(A) $\frac{1}{2}\left(\frac{1}{\alpha}+\alpha\right)$ (B) $\frac{1}{2}\left(\frac{1}{\alpha}-\alpha\right)$ (C) $\left(\frac{1}{\alpha}+\alpha\right)$ (D) $\left(\frac{1}{\alpha}-\alpha\right)$

B-3. If a cosθ + b sinθ = 3 & a sinθ – b cosθ = 4 then $a^2 + b^2$ has the value =

(A) 25 (B) 14 (C) 7 (D) 10

B-4. $\dfrac{\tan\left(x-\frac{\pi}{2}\right).\cos\left(\frac{3\pi}{2}+x\right)-\sin^3\left(\frac{7\pi}{2}-x\right)}{\cos\left(x-\frac{\pi}{2}\right).\tan\left(\frac{3\pi}{2}+x\right)}$ when simplified reduces to:

(A) $\sin x\cos x$ (B) $-\sin^2 x$ (C) $-\sin x\cos x$ (D) $\sin^2 x$

B-5. The expression $3\left[\sin^4\left(\frac{3\pi}{2}-\alpha\right)+\sin^4(3\pi+\alpha)\right]-2\left[\sin^6\left(\frac{\pi}{2}+\alpha\right)+\sin^6(5\pi+\alpha)\right]$ is equal to

(A) 0 (B) 1 (C) 3 (D) sin 4α + sin 6α

B-6. The value of the expression $\left(1+\cos\frac{\pi}{10}\right)\left(1+\cos\frac{3\pi}{10}\right)\left(1+\cos\frac{7\pi}{10}\right)\left(1+\cos\frac{9\pi}{10}\right)$ is

(A) $\frac{1}{8}$ (B) $\frac{1}{16}$ (C) $\frac{1}{4}$ (D) 0

B-7. The value of $\dfrac{\sin 24^\circ\cos 6^\circ - \sin 6^\circ\sin 66^\circ}{\sin 21^\circ\cos 39^\circ - \cos 51^\circ\sin 69^\circ}$ is

(A) –1 (B) 1 (C) 2 (D) 0

B-8. If tan A and tan B are the roots of the quadratic equation $x^2 - ax + b = 0$, then the value of $\sin^2(A + B)$.

(A) $\frac{a^2}{a^2 + (1-b)^2}$ (B) $\frac{a^2}{a^2 + b^2}$ (C) $\frac{a^2}{(b + c)^2}$ (D) $\frac{a^2}{b^2 (1 - a)^2}$

B-9. If tan A – tan B = x and cot B – cot A = y, then cot (A – B) is equal to

(A) $\frac{1}{y} - \frac{1}{x}$ (B) $\frac{1}{x} - \frac{1}{y}$ (C) $\frac{1}{x} + \frac{1}{y}$ (D) $\frac{1}{x + y}$

B-10. If tan 25° = x, then $\frac{\tan 155° - \tan 115°}{1 + \tan 155° \ \tan 115°}$ is equal to

(A) $\frac{1-x^2}{2x}$ (B) $\frac{1+x^2}{2x}$ (C) $\frac{1+x^2}{1-x^2}$ (D) $\frac{1-x^2}{1+x^2}$

B-11. If A + B = 225°, then the value of $\left(\frac{\cot A}{1+\cot A}\right) . \left(\frac{\cot B}{1+\cot B}\right)$ is

(A) 2 (B) $\frac{1}{2}$ (C) 3 (D) $-\frac{1}{2}$

B-12. The value of tan 203° + tan 22° + tan 203° tan 22° is
(A) – 1 (B) 0 (C) 1 (D) 2

Section (C) : $\sin^2 A - \sin^2 B$, Multiple angles upto 3A, 2sinA cosB, sinC – sinD

C-1. The value of $\frac{1 - \tan^2 15°}{1 + \tan^2 15°}$ is

(A) 1 (B) $\sqrt{3}$ (C) $\frac{\sqrt{3}}{2}$ (D) 2

C-2. If A lies in the third quadrant and 3 tan A – 4 = 0, then 5 sin 2A + 3sinA + 4 cosA is equal to

(A) 0 (B) $-\frac{24}{5}$ (C) $\frac{24}{5}$ (D) $\frac{48}{5}$

C-3. If cos A = 3/4, then the value of $16\cos^2 (A/2) - 32 \sin (A/2) \sin (5A/2)$ is
(A) – 4 (B) – 3 (C) 3 (D) 4

C-4. If $\tan^2 \theta = 2 \tan^2 \phi + 1$, then the value of $\cos 2\theta + \sin^2 \phi$ is
(A) 1 (B) 2 (C) – 1 (D) Independent of ϕ

C-5. If $\alpha \in \left[\frac{\pi}{2}, \pi\right]$ then the value of $\sqrt{1 + \sin\alpha} - \sqrt{1 - \sin\alpha}$ is equal to:

(A) $2 \cos \frac{\alpha}{2}$ (B) $2 \sin \frac{\alpha}{2}$ (C) 2 (D) none of these

C-6. The value of $\frac{1}{\cos 290°} + \frac{1}{\sqrt{3} \sin 250°}$ is

(A) $\frac{2\sqrt{3}}{3}$ (B) $\frac{4\sqrt{3}}{3}$ (C) $\sqrt{3}$ (D) none

C-7. The value of tan 3A – tan 2A – tan A is equal to
(A) tan 3A tan 2A tan A (B) – tan 3A tan 2A tan A
(C) tan A tan 2A – tan 2A tan 3A – tan 3A tan A (D) none of these

C-8. $\dfrac{\cos 20^\circ + 8\sin 70^\circ \sin 50^\circ \sin 10^\circ}{\sin^2 80^\circ}$ is equal to:

(A) 1 (B) 2 (C) 3/4 (D) 0

C-9. The numerical value of sin 12° . sin48° . sin 54° is equal to

(A) $\dfrac{1}{2}$ (B) $\dfrac{1}{4}$ (C) $\dfrac{1}{16}$ (D) $\dfrac{1}{8}$

C-10. If A = tan 6° tan 42° and B = cot 66° cot 78°, then

(A) A = 2B (B) A = $\dfrac{1}{3}$ B (C) A = B (D) 3A = 2B

Section (D) : Conditional Identities & Trigonometric Series

D-1. In a triangle tan A + tan B + tan C = 6 and tan A tan B = 2, then the values of tan A, tan B and tan C are respectively

(A) 1, 2, 3 (B) 2, 3, 1 (C) 1, 2, 0 (D) none of these

D-2. $\tan\alpha + 2\tan 2\alpha + 4\tan 4\alpha + 8\cot 8\alpha =$

(A) $\tan\alpha$ (B) $\cot\alpha$ (C) $\cot 16\alpha$ (D) $16\cot\alpha$

D-3. The value of $\cos 0 + \cos\dfrac{\pi}{7} + \cos\dfrac{2\pi}{7} + \cos\dfrac{3\pi}{7} + \cos\dfrac{4\pi}{7} + \cos\dfrac{5\pi}{7} + \cos\dfrac{6\pi}{7}$ is

(A) 1/2 (B) −1/2 (C) 0 (D) 1

D-4. The value of $\cos\dfrac{\pi}{10}\cos\dfrac{2\pi}{10}\cos\dfrac{4\pi}{10}\cos\dfrac{8\pi}{10}\cos\dfrac{16\pi}{10}$ is :

(A) $\dfrac{\sqrt{10+2\sqrt{5}}}{64}$ (B) $-\dfrac{\cos(\pi/10)}{16}$ (C) $\dfrac{\cos(\pi/10)}{16}$ (D) $-\dfrac{\sqrt{10+2\sqrt{5}}}{16}$

D-5. The value of $\cos\dfrac{\pi}{19} + \cos\dfrac{3\pi}{19} + \cos\dfrac{5\pi}{19} + ... + \cos\dfrac{17\pi}{19}$ is equal to :

(A) 1/2 (B) 0 (C) 1 (D) 2

Section (E) : Range of Trigonometric Expressions

E-1. If $f(\theta) = \sin^4\theta + \cos^2\theta$, then range of $f(\theta)$ is

(A) $\left[\dfrac{1}{2}, 1\right]$ (B) $\left[\dfrac{1}{2}, \dfrac{3}{4}\right]$ (C) $\left[\dfrac{3}{4}, 1\right]$ (D) None of these

E-2. Range of function $f(x) = \cos^2 x + 4\sec^2 x$ is

(A) $[4, \infty)$ (B) $[0, \infty)$ (C) $[5, \infty)$ (D) $(0, \infty)$

E-3. The difference between maximum and minimum value of the expression $y = 1 + 2\sin x + 3\cos^2 x$ is

(A) $\dfrac{16}{3}$ (B) $\dfrac{13}{3}$ (C) 7 (D) 8

E-4. The maximum value of $12\sin\theta - 9\sin^2\theta$ is -

(A) 3 (B) 4 (C) 5 (D) None of these

E-5. The greatest and least value of $y = 10\cos^2 x - 6\sin x\cos x + 2\sin^2 x$ are respectively

(A) 11, 1 (B) 10, 2 (C) 12, – 4 (D) 11, – 1

Section (F) : Trigonometric Equations

F-1. The solution set of the equation $4\sin\theta.\cos\theta - 2\cos\theta - 2\sqrt{3}\ \sin\theta + \sqrt{3} = 0$ in the interval $(0, 2\pi)$ is

(A) $\left\{\frac{3\pi}{4}, \frac{7\pi}{4}\right\}$ (B) $\left\{\frac{\pi}{3}, \frac{5\pi}{3}\right\}$ (C) $\left\{\frac{3\pi}{4}, \pi, \frac{\pi}{3}, \frac{5\pi}{3}\right\}$ (D) $\left\{\frac{\pi}{6}, \frac{5\pi}{6}, \frac{11\pi}{6}\right\}$

F-2. All solutions of the equation $2\sin\theta + \tan\theta = 0$ are obtained by taking all integral values of m and n in:

(A) $2n\pi + \frac{2\pi}{3}$, $n \in I$

(B) $n\pi$ or $2m\pi \pm \frac{2\pi}{3}$ where $n, m \in I$

(C) $n\pi$ or $m\pi \pm \frac{\pi}{3}$ where $n, m \in I$

(D) $n\pi$ or $2m\pi \pm \frac{\pi}{3}$ where $n, m \in I$

F-3. Total number of solutions of equation $\sin x \cdot \tan 4x = \cos x$ belonging to $(-\pi, 2\pi)$ are :

(A) 4 (B) 7 (C) 8 (D) 15

F-4. If $x \in \left[0, \frac{\pi}{2}\right]$, the number of solutions of the equation $\sin 7x + \sin 4x + \sin x = 0$ is:

(A) 3 (B) 5 (C) 6 (D) 4

F-5. The general solution of equation $\sin x + \sin 5x = \sin 2x + \sin 4x$ is :

(A) $\frac{n\pi}{2}$; $n \in I$ (B) $\frac{n\pi}{5}$; $n \in I$ (C) $\frac{n\pi}{3}$; $n \in I$ (D) $\frac{2\ \ n\pi}{3}$; $n \in I$

F-6. The general solution of the equation $2\cos 2x = 3.2\cos^2 x - 4$ is

(A) $x = 2n\pi$, $n \in I$ (B) $x = n\pi$, $n \in I$ (C) $x = n\pi/4$, $n \in I$ (D) $x = n\pi/2$, $n \in I$

F-7. If $2\cos^2(\pi + x) + 3\sin(\pi + x)$ vanishes then the values of x lying in the interval from 0 to 2π are

(A) $x = \pi/6$ or $5\pi/6$ (B) $x = \pi/3$ or $5\pi/3$ (C) $x = \pi/4$ or $5\pi/4$ (D) $x = \pi/2$ or $5\pi/2$

F-8. $\frac{\cos 3\theta}{2\cos 2\theta - 1} = \frac{1}{2}$ if

(A) $\theta = n\pi + \frac{\pi}{3}$, $n \in I$ (B) $\theta = 2n\pi \pm \frac{\pi}{3}$, $n \in I$ (C) $\theta = 2n\pi \pm \frac{\pi}{6}$, $n \in I$ (D) $\theta = n\pi + \frac{\pi}{6}$, $n \in I$

F-9. If $\cos 2\theta + 3\cos\theta = 0$, then

(A) $\theta = 2n\pi \pm \alpha$ where $\alpha = \cos^{-1}\left(\frac{\sqrt{17}-3}{4}\right)$

(B) $\theta = 2n\pi \pm \alpha$ where $\alpha = \cos^{-1}\left(\frac{-\sqrt{17}-3}{4}\right)$

(C) $\theta = 2n\pi \pm \alpha$ where $\alpha = \cos^{-1}\left(\frac{\pm\sqrt{17}-3}{4}\right)$

(D) $\theta = 2n\pi \pm \alpha$ where $\alpha = \cos^{-1}\left(\frac{\sqrt{17}+3}{4}\right)$

F-10. If $\sin\theta + 7\cos\theta = 5$, then $\tan(\theta/2)$ is a root of the equation

(A) $x^2 - 6x + 1 = 0$ (B) $6x^2 - x - 1 = 0$ (C) $6x^2 + x + 1 = 0$ (D) $x^2 - x + 6 = 0$

F-11. The most general solution of $\tan\theta = -1$ and $\cos\theta = \frac{1}{\sqrt{2}}$ is :

(A) $n\pi + \frac{7\pi}{4}$, $n \in I$

(B) $n\pi + (-1)^n \frac{7\pi}{4}$, $n \in I$

(C) $2n\pi + \frac{7\pi}{4}$, $n \in I$

(D) $2n\pi + \frac{3\pi}{4}$, $n \in I$

F-12. A triangle ABC is such that $\sin(2A + B) = \frac{1}{2}$. If A, B, C are in A.P. then the angle A, B, C are respectively.

(A) $\frac{5\pi}{12}, \frac{\pi}{4}, \frac{\pi}{3}$ (B) $\frac{\pi}{4}, \frac{\pi}{3}, \frac{5\pi}{12}$ (C) $\frac{\pi}{3}, \frac{\pi}{4}, \frac{5\pi}{12}$ (D) $\frac{\pi}{3}, \frac{5\pi}{12}, \frac{\pi}{4}$

Section (G) : Trigonometric Inquations and Height & Distance

G-1. The complete solution of inequality $\sec^2 3x < 2$ is

(A) $x \in \left(\frac{n\pi}{3}-\frac{\pi}{12}, \frac{n\pi}{3}+\frac{\pi}{12}\right)$, $n \in I$

(B) $x \in \left(\frac{n\pi}{3}-\frac{\pi}{12}, \frac{n\pi}{3}+\frac{\pi}{6}\right)$, $n \in I$

(C) $x \in \left(n\pi-\frac{\pi}{12}, n\pi+\frac{\pi}{12}\right)$, $n \in I$

(D) $x \in \left(\frac{n\pi}{3}-\frac{\pi}{6}, \frac{n\pi}{3}+\frac{\pi}{6}\right)$, $n \in I$

G-2. The complete solution of inequality $2\cos^2 x - 7\cos x + 3 < 0$ is

(A) $n\pi - \frac{\pi}{3} < x < \frac{\pi}{3} + n\pi$

(B) $2n\pi - \frac{\pi}{6} < x < \frac{\pi}{6} + 2n\pi$

(C) $2n\pi - \frac{\pi}{3} < x < \frac{\pi}{3} + 2n\pi$

(D) $n\pi - \frac{\pi}{6} < x < \frac{\pi}{6} + n\pi$

G-3. The complete solution of inequality $\cos 2x \le \cos x$ is

(A) $x \in \left[2n\pi-\frac{\pi}{3},\ 2n\pi+\frac{\pi}{3}\right]$

(B) $x \in \left[2n\pi-\frac{2\pi}{3},\ 2n\pi+\frac{2\pi}{3}\right]$

(C) $x \in \left[2n\pi,\ 2n\pi+\frac{2\pi}{3}\right]$

(D) $x \in \left[2n\pi-\frac{2\pi}{3},\ 2n\pi\right]$

G-4. Which of the following set of values of x satisfy the inequation $\tan^2 x - (1 + \sqrt{3}) \tan x + \sqrt{3} < 0$

(A) $\left(\frac{(4n+1)\pi}{4}, \frac{(3n+1)\pi}{3}\right)$, $(n \in Z)$

(B) $\left(\frac{(2n+1)\pi}{4}, \frac{(2n+1)\pi}{3}\right)$, $(n \in Z)$

(C) $\left(\frac{(4n+1)\pi}{4}, \frac{(4n+1)\pi}{3}\right)$, $(n \in Z)$

(D) $x \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right]$

G-5. A tree 12 m high, is broken by the wind in such a way that its top touches the ground and makes an angle 60º with the ground. The height from the bottom of the tree from where it is broken by the wind is appoximately

(A) 5.57 m (B) 5.21 (C) 5.36 (D) 5.9

G-6. AB is a vertical pole and C is the middle point. The end A is on the level ground and P is any point on the level ground other than A. The portion CB subtends an angle β at P. If AP : AB = 2 : 1, then β is equal to-

(A) $\tan^{-1}\left(\frac{1}{9}\right)$ (B) $\tan^{-1}\left(\frac{4}{9}\right)$ (C) $\tan^{-1}\left(\frac{5}{9}\right)$ (D) $\tan^{-1}\left(\frac{2}{9}\right)$

G-7. A round ballon of radius r subtends an angle α at the eye of the observer, while the angle of elevation of its centre is β. The height of the centre of ballon is-

(A) $r \text{ cosec } \alpha \sin \frac{\beta}{2}$ (B) $r \sin \alpha \text{ cosec} \frac{\beta}{2}$ (C) $r \sin \frac{\alpha}{2} \text{ cosec } \beta$ (D) $r \text{ cosec } \frac{\alpha}{2} \sin \beta$

G-8. If the angle of elevation of a cloud from a point 200 m above a lake is 30º and the angle of depression of its reflection in the lake is 60º, then the height of the cloud above the lake, is

(A) 200 m (B) 500 m (C) 30 m (D) 400 m

G-9. A man on the top of a vertical tower observes a car moving at a uniform speed coming directly towards it. If it takes 12 minutes for the angle of depression to change from 30º to 45º, then the car will reach the tower in

(A) 17 minutes 23 seconds

(B) 16 minutes 23 seconds

(C) 16 minutes 18 seconds

(D) 18 minutes 22 seconds

PART - III : MATCH THE COLUMN

1. **Column - I** | **Column - II**

	Column - I		Column - II
(A)	$\tan 9° - \tan 27° - \tan 63° + \tan 81°$	(p)	1
(B)✍	$\text{cosec } 10° - \sqrt{3} \sec 10°$	(q)	2
(C)	$2\sqrt{2} \sin 10° \left[\dfrac{\sec 5°}{2} + \dfrac{\cos 40°}{\sin 5°} - 2 \sin 35°\right]$	(r)	3
(D)	$\sqrt{3}\,(\cot 70° + 4 \cos 70°)$	(s)	4

2.

	Column – I		Column – II
(A)	If for some real x, the equation $x + \dfrac{1}{x} = 2 \cos \theta$ holds, then $\cos \theta$ is equal to	(p)	2
(B)	If $\sin \theta + \text{cosec } \theta = 2$, then $\sin^{2008} \theta + \text{cosec}^{2008} \theta$ is equal to	(q)	1
(C)	Maximum value of $\sin^4\theta + \cos^4\theta$ is	(r)	0
(D)	Least value of $2 \sin^2\theta + 3 \cos^2\theta$ is	(s)	– 1

3.

	Column – I		Column – II
(A)	Number of solutions of $\sin^2\theta + 3 \cos \theta = 3$ in $[-\pi, \pi]$	(p)	0
(B)	Number of solutions of $\sin x . \tan 4x = \cos x$ in $(0, \pi)$	(q)	1
(C)	Number of solutions of equation $(1 - \tan \theta)(1 + \tan \theta) \sec^2\theta + 2^{\tan^2 \theta} = 0$ where $\theta \in \left(-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right)$	(r)	4
(D)	If $[\sin x] + [\sqrt{2} \cos x] = -3$, where $x \in [0, 2\pi]$ then $[\sin 2x]$ equals (Here [.] denotes G.I.F.)	(s)	5

Exercise-2

✍ **Marked questions are recommended for Revision.**

PART - I : ONLY ONE OPTION CORRECT TYPE

1. In a triangle ABC if $\tan A < 0$ then:

(A) $\tan B . \tan C > 1$ (B) $\tan B . \tan C < 1$ (C) $\tan B . \tan C = 1$ (D) Data insufficient

2. If $\sin \alpha = 1/2$ and $\cos \theta = 1/3$, then the values of $\alpha + \theta$ (if θ, α are both acute) will lie in the interval

(A) $\left[\dfrac{\pi}{3}, \dfrac{\pi}{2}\right]$ (B) $\left[\dfrac{\pi}{2}, \dfrac{2\pi}{3}\right]$ (C) $\left[\dfrac{2\pi}{3}, \dfrac{5\pi}{6}\right]$ (D) $\left[\dfrac{5\pi}{6}, \pi\right]$

3. If $\frac{\sin A}{\sin B} = \frac{\sqrt{3}}{2}$ and $\frac{\cos A}{\cos B} = \frac{\sqrt{5}}{2}$, $0 < A, B < \pi/2$, then tan A + tan B is equal to

(A) $\sqrt{3}/\sqrt{5}$ (B) $\sqrt{5}/\sqrt{3}$ (C) 1 (D) $(\sqrt{5}+\sqrt{3})/\sqrt{5}$

4. In a right angled triangle the hypotenuse is $2\sqrt{2}$ times the perpendicular drawn from the opposite vertex. Then the other acute angles of the triangle are

(A) $\frac{\pi}{3}$ & $\frac{\pi}{6}$ (B) $\frac{\pi}{8}$ & $\frac{3\pi}{8}$ (C) $\frac{\pi}{4}$ & $\frac{\pi}{4}$ (D) $\frac{\pi}{5}$ & $\frac{3\pi}{10}$

5. If 3 cos x + 2 cos 3x = cos y, 3 sin x + 2 sin 3x = sin y, then the value of cos 2x is

(A) – 1 (B) $\frac{1}{8}$ (C) $-\frac{1}{8}$ (D) $\frac{7}{8}$

6. If $\cos\alpha + \cos\beta = a$, $\sin\alpha + \sin\beta = b$ and $\alpha - \beta = 2\theta$, then $\frac{\cos 3\theta}{\cos\theta} =$

(A) $a^2 + b^2 - 2$ (B) $a^2 + b^2 - 3$ (C) $3 - a^2 - b^2$ (D) $(a^2 + b^2)/4$

7. If $\frac{3\pi}{4} < \alpha < \pi$, then $\sqrt{2\cot\alpha + \frac{1}{\sin^2\alpha}}$ is equal to

(A) $1 + \cot\alpha$ (B) $-1 - \cot\alpha$ (C) $1 - \cot\alpha$ (D) $-1 + \cot\alpha$

8. For $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$, $\frac{\sin\theta + \sin 2\theta}{1 + \cos\theta + \cos 2\theta}$ lies in the interval

(A) $(-\infty, \infty)$ (B) $(-2, 2)$ (C) $(0, \infty)$ (D) $(-1, 1)$

9. The number of all possible triplets (a_1, a_2, a_3) such that $a_1 + a_2 \cos 2x + a_3 \sin^2 x = 0$ for all x is

(A) 0 (B) 1 (C) 2 (D) infinite

10. If $A + B + C = \frac{3\pi}{2}$, then cos 2A + cos2B + cos2C is equal to

(A) 1 – 4cos A cosB cosC (B) 4 sin A sin B sin C

(C) 1 + 2 cos A cos B cos C (D) 1– 4 sin A sin B sin C

11. If $A + B + C = \pi$ & cosA = cosB. cosC then tanB. tanC has the value equal to:

(A) 1 (B) 1/2 (C) 2 (D) 3

12. The general solution of the equation $\tan^2\alpha + 2\sqrt{3}\tan\alpha = 1$ is given by:

(A) $\alpha = \frac{n\pi}{2}$, $n \in I$ (B) $\alpha = (2n+1)\frac{\pi}{2}$, $n \in I$

(C) $\alpha = (6n+1)\frac{\pi}{12}$, $n \in I$ (D) $\alpha = \frac{n\pi}{12}$, $n \in I$

13. The general solution of the equation $\tan x + \tan\left(x + \frac{\pi}{3}\right) + \tan\left(x + \frac{2\pi}{3}\right) = 3$ is

(A) $\frac{n\pi}{4} + \frac{\pi}{12}$, $n \in I$ (B) $\frac{n\pi}{3} + \frac{\pi}{6}$, $n \in I$ (C) $\frac{n\pi}{3} + \frac{\pi}{12}$, $n \in I$ (D) $\frac{n\pi}{3} + \frac{\pi}{4}$, $n \in I$

14. The complete solution set of the equation $1 + 2\ \text{cosec}\ x = -\dfrac{\sec^2 \frac{x}{2}}{2}$ is

(A) $2n\pi - \dfrac{\pi}{2}, n \in I$ (B) $n\pi - \dfrac{\pi}{2}, n \in I$ (C) $2n\pi + \dfrac{\pi}{2}, n \in I$ (D) $n\pi + \dfrac{\pi}{2}, n \in I$

15. The principal solution set of the equation $2 \cos x = \sqrt{2 + 2\sin 2x}$ is

(A) $\left\{\dfrac{\pi}{8}, \dfrac{13\pi}{8}\right\}$ (B) $\left\{\dfrac{\pi}{4}, \dfrac{13\pi}{8}\right\}$ (C) $\left\{\dfrac{\pi}{4}, \dfrac{13\pi}{10}\right\}$ (D) $\left\{\dfrac{\pi}{8}, \dfrac{13\pi}{10}\right\}$

16. The solution of $|\cos x| = \cos x - 2\sin x$ is

(A) $x = n\pi, n \in I$

(B) $x = n\pi + \dfrac{\pi}{4}, n \in I$

(C) $x = n\pi + (-1)^n \dfrac{\pi}{4}, n \in I$

(D) $x = (2n + 1)\pi + \dfrac{\pi}{4}, n \in I$

17. The solution of inequality $4^{\tan x} - 3.2^{\tan x} + 2 \le 0$ is

(A) $x \in \left[n\pi,\ n\pi + \dfrac{\pi}{4}\right]; n \in I$

(B) $x \in \left[n\pi,\ n\pi - \dfrac{\pi}{4}\right]; n \in I$

(C) $x \in \left[n\pi,\ n\pi + \dfrac{\pi}{6}\right]; n \in I$

(D) $x \in \left[n\pi,\ n\pi - \dfrac{\pi}{6}\right]; n \in I$

18. The solution of inequality $\sqrt{5 - 2\sin x} \ge 6 \sin x - 1$ is

(A) $[\pi (12n - 7)/6, \pi (12n + 7)/6]$ $(n \in Z)$

(B) $[\pi (12n - 7)/6, \pi (12n + 1)/6]$ $(n \in Z)$

(C) $[\pi (2n - 7)/6, \pi (2n + 1)/6]$ $(n \in Z)$

(D) $[\pi (12n - 7)/3, \pi (12n + 1)/3]$ $(n \in Z)$

PART - II : NUMERICAL VALUE QUESTIONS

INSTRUCTION :

❖ The answer to each question is **NUMERICAL VALUE** with two digit integer and decimal upto two digit.

❖ If the numerical value has more than two decimal places **truncate/round-off** the value to **TWO** decimal placed.

1. If $19 \sin\alpha = 29 \sin\beta$, then find the value of $\dfrac{\tan\dfrac{\alpha+\beta}{2}}{\tan\dfrac{\alpha-\beta}{2}}$.

2. If α, β $(\alpha - \beta \ne 2n\pi, n \in I)$ are different values of θ satisfying the equation $5 \cos \theta - 12 \sin \theta = 11$. then absolute value of $\sin (\alpha + \beta)$ is

3. If $x \in \left(\pi, \dfrac{3\pi}{2}\right)$ then $4 \cos^2 \left(\dfrac{\pi}{4} - \dfrac{x}{2}\right) + \sqrt{4\sin^4 x + \sin^2 2x}$ is always equal to

4. If three angles A, B, C are such that
$\cos A + \cos B + \cos C = 0$ and if
$\cos A \cos B \cos C = \lambda (\cos 3A + \cos 3B + \cos 3C)$ then value of λ is :

5. Find sum of square of all possible integral values of λ for which equation $4\cos x + 3 \sin x = 2\lambda + 1$ has a solution.

6. If $a \cos^3 \alpha + 3a \cos\alpha \sin^2 \alpha = m$ and $a \sin^3 \alpha + 3a \cos^2 \alpha \sin \alpha = n$. if $(m+n)^{2/3} + (m - n)^{2/3} = pa^q$, then find value of $p^3 + q^3$

7. If 2 cos x + sin x = 1, then find the sum of all possible values of 7 cos x + 6 sin x.

8. The number of roots of the equation $\cot x = \frac{\pi}{2} + x$ in $\left[-\pi, \frac{31\pi}{2}\right]$ is ,

9. If $2\tan^2 x - 5 \sec x - 1 = 0$ has 7 different roots in $\left[0, \frac{n\pi}{2}\right]$, $n \in N$, then find the greatest value of n.

10. Sum of all possible integral values of a for which the equation cos 2x + a sin x = 2a − 7 possesses a solution.

11. The number of solutions of the equation |sinx| = | cos3x| in [–2π, 2π] is

12. In any triangle ABC, which is not right angled $\sum$cosA .cosecB.cosecC is equal to

13. If A + B + C = π, then find value of tan B tan C + tan C tan A + tan A tan B – sec A sec B sec C.

14. If the arithmetic mean of the roots of the equation $4\cos^3 x - 4\cos^2 x - \cos(\pi + x) - 1 = 0$ in the interval [0, 315] is equal to kπ , then find the value of k

15. $\cos(\alpha - \beta) = 1$ and $\cos(\alpha + \beta) = \frac{1}{e}$, where $\alpha, \beta \in [-\pi, \pi]$. Then number of ordered pairs (α, β) which satisfy both the equations.

16. Sum of all possible value of θ between 0° and 360° which satisfy the equation
$\sec^2\theta \,.\text{cosec}^2\theta + 2\,\text{cosec}^2\theta = 8$

17. Find the number of all values of $\theta \in [0, 10.5]$ satisfying the equation
$\cos 6\theta + \cos 4\theta + \cos 2\theta + 1 = 0$.

18. In (0, 6π), find the number of solutions of the equation tanθ + tan 2θ + tan 3θ = tan θ.tan2θ.tan3θ

19. If $0 \le x \le 3\pi$, $0 \le y \le 3\pi$ and cos x . sin y = 1, then find the possible number of values of the ordered pair (x, y)

20. Find the number of values of θ satisfying the equation sin3θ = 4sinθ. sin2θ. sin4θ in $0 \le \theta \le 2\pi$

21. Sum of all possible values of x which satisfy equation $\cos 6x + \tan^2 x + \cos 6x \,.\, \tan^2 x = 1$ in the interval [0, 2π].

22. Consider $\tan\theta + \sin\phi = \frac{3}{2}$ & $\tan^2\theta + \cos^2\phi = \frac{7}{4}$, find maximum value of (θ + ϕ) if $\theta + \phi \in (0, 2\pi)$.

23. If range of values of n so that sinx(sinx + cosx) = n has at least one solution is [p,q] then values of $p^3 + q^3$ is

24. Find the number of values of x in (0, 2π) satisfying the equation cot x − 2 sin 2x = 1 .

25. Find the number of solutions of sinθ + 2sin2θ + 3sin3θ + 4sin4θ = 10 in(0, π).

26. Find the values of x satisfying the equation $2 \sin x = 3x^2 + 2x + 3$.

27. Find the number of solution of sinx cosx – 3 cosx + 4 sinx – 13 > 0 in [0,2π] .

PART - III : ONE OR MORE THAN ONE OPTIONS CORRECT TYPE

1. The value of $\dfrac{(\cos 11° + \sin 11°)}{(\cos 11° - \sin 11°)}$ is

(A) $-\tan 304°$ (B) $\tan 56°$ (C) $\cot 214°$ (D) $\cot 34°$

2. If $\sin t + \cos t = \dfrac{1}{5}$ then $\tan \dfrac{t}{2}$ can be

(A) -1 (B) $-\dfrac{1}{3}$ (C) 2 (D) $-\dfrac{1}{6}$

3. The value of $\dfrac{\sin x + \cos x}{\cos^3 x}$ =

(A) $1 + \tan x + \tan^2 x - \tan^3 x$
(B) $1 + \tan x + \tan^2 x + \tan^3 x$
(C) $1 - \tan x + \tan^2 x + \tan^3 x$
(D) $(1 + \tan x) \sec^2 x$

4. If (sec A + tan A) (sec B + tan B) (sec C + tan C) = (sec A – tan A) (sec B – tan B) (sec C – tan C) then each side can be

(A) 1 (B) –1 (C) 0 (D) none

5. Which of the following is correct ?

(A) $\sin 1° > \sin 1$ (B) $\sin 1° < \sin 1$ (C) $\cos 1° > \cos 1$ (D) $\cos 1° < \cos 1$

6. If $\sin x + \sin y = a$ & $\cos x + \cos y = b$, then which of the following may be true.

(A) $\sin (x + y) = \dfrac{2ab}{a^2 + b^2}$
(B) $\tan \dfrac{x-y}{2} = \sqrt{\dfrac{4 - a^2 - b^2}{a^2 + b^2}}$
(C) $\tan \dfrac{x-y}{2} = -\sqrt{\dfrac{4 - a^2 - b^2}{a^2 + b^2}}$
(D) $\cos (x + y) = \dfrac{2ab}{a^2 + b^2}$

7. If $\cos (A - B) = \dfrac{3}{5}$ and $\tan A \tan B = 2$, then which of the following is/are correct

(A) $\cos A \cos B = -\dfrac{1}{5}$
(B) $\sin A \sin B = \dfrac{2}{5}$
(C) $\cos (A + B) = -\dfrac{1}{5}$
(D) $\sin A \cos B = \dfrac{4}{5}$

8. If $P_n = \cos^n\theta + \sin^n\theta$ and $Q_n = \cos^n\theta - \sin^n\theta$, then which of the following is/are true.

(A) $P_n - P_{n-2} = -\sin^2\theta \cos^2\theta\, P_{n-4}$
(B) $Q_n - Q_{n-2} = -\sin^2\theta \cos^2\theta\, Q_{n-4}$
(C) $P_4 = 1 - 2 \sin^2\theta \cos^2\theta$
(D) $Q_4 = \cos^2\theta - \sin^2\theta$

9. $\tan^2\alpha + 2\tan\alpha . \tan 2\beta = \tan^2\beta + 2\tan\beta . \tan 2\alpha$, if

(A) $\tan^2\alpha + 2\tan\alpha . \tan 2\beta = 0$
(B) $\tan \alpha + \tan \beta = 0$
(C) $\tan^2\beta + 2\tan\beta . \tan 2\alpha = 1$
(D) $\tan \alpha = \tan \beta$

10. If the sides of a right angled triangle are $\{\cos 2\alpha + \cos 2\beta + 2\cos(\alpha + \beta)\}$ and $\{\sin 2\alpha + \sin 2\beta + 2\sin(\alpha + \beta)\}$, then the length of the hypotenuse is :

(A) $2[1+\cos(\alpha - \beta)]$ (B) $2[1 - \cos(\alpha + \beta)]$ (C) $4 \cos^2 \dfrac{\alpha - \beta}{2}$ (D) $4\sin^2 \dfrac{\alpha + \beta}{2}$

11. For $0 < \theta < \pi/2$, $\tan \theta + \tan 2\theta + \tan 3\theta = 0$ if

(A) $\tan \theta = 0$ (B) $\tan 2\theta = 0$ (C) $\tan 3\theta = 0$ (D) $\tan \theta \tan 2\theta = 2$

12. (a + 2) sin α + (2a – 1) cos α = (2a + 1) if tan α =

(A) $\frac{3}{4}$ (B) $\frac{4}{3}$ (C) $\frac{2a}{a^2+1}$ (D) $\frac{2a}{a^2-1}$

13. If tan x = $\frac{2b}{a-c}$, ($a \neq c$)

y = a cos²x + 2b sin x cos x + c sin²x

z = a sin²x – 2b sin x cos x + c cos²x, then

(A) y = z (B) y + z = a + c (C) y – z = a – c (D) y – z = $(a – c)^2 + 4b^2$

14. The value of $\left(\frac{\cos A+\cos B}{\sin A-\sin B}\right)^n + \left(\frac{\sin A+\sin B}{\cos A-\cos B}\right)^n$ is

(A) 2 $\tan^n \frac{A-B}{2}$ (B) 2 $\cot^n \frac{A-B}{2}$: n is even

(C) 0 : n is odd (D) 0 : n is even

15. The equation $\sin^6x + \cos^6x = a^2$ has real solution if

(A) $a \in (–1, 1)$ (B) $a \in \left(-1, -\frac{1}{2}\right)$ (C) $a \in \left(-\frac{1}{2}, \frac{1}{2}\right)$ (D) $a \in \left(\frac{1}{2}, 1\right)$

16. If sin(x – y) = cos(x + y) = 1/2 then the values of x & y lying between 0 and π are given by:

(A) x = $\pi/4$, y = $3\pi/4$ (B) x = $\pi/4$, y = $\pi/12$

(C) x = $5\pi/4$, y = $5\pi/12$ (D) x = $11\pi/12$, y = $3\pi/4$

17. If 2 $\sec^2 \alpha – \sec^4 \alpha – 2\,\text{cosec}^2 \alpha + \text{cosec}^4 \alpha$ = 15/4, then tan α can be

(A) $1/\sqrt{2}$ (B) 1/2 (C) $1/2\sqrt{2}$ (D) $–1/\sqrt{2}$

18. If 3 sin β = sin (2α + β), then tan (α + β) – 2 tan α is

(A) independent of α (B) independent of β

(C) dependent of both α and β (D) independent of α but dependent of β

19. If $\alpha + \beta + \gamma = 2\pi$, then

(A) $\tan\frac{\alpha}{2} + \tan\frac{\beta}{2} + \tan\frac{\gamma}{2} = \tan\frac{\alpha}{2}\tan\frac{\beta}{2}\tan\frac{\gamma}{2}$

(B) $\tan\frac{\alpha}{2}\tan\frac{\beta}{2} + \tan\frac{\beta}{2}\tan\frac{\gamma}{2} + \tan\frac{\gamma}{2}\tan\frac{\alpha}{2} = 1$

(C) $\tan\frac{\alpha}{2} + \tan\frac{\beta}{2} + \tan\frac{\gamma}{2} = -\tan\frac{\alpha}{2}\tan\frac{\beta}{2}\tan\frac{\gamma}{2}$

(D) $\tan\frac{\alpha}{4}\tan\frac{\beta}{4} + \tan\frac{\beta}{4}\tan\frac{\gamma}{4} + \tan\frac{\gamma}{4}\tan\frac{\alpha}{4} = 1$

20. If x + y = z, then $\cos^2 x + \cos^2 y + \cos^2 z$ – 2 cos x cos y cos z is equal to

(A) $\cos^2 z$ (B) $\sin^2 z$ (C) cos (x + y – z) (D) 1

21. If tanA + tan B + tan C = tan A. tan B. tan C, then

(A) A, B, C may be angles of a triangle (B) A + B + C is an integral multiple of π

(C) sum of any two of A, B, C is equal to third (D) none of these

22. Which of the following values of ' t ' may satisfy the condition 2 sin t = $\frac{1-2x+5x^2}{3x^2-2x-1}$, $t \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

(A) $\left[-\frac{\pi}{2}, -\frac{\pi}{10}\right]$ (B) $\left[0, \frac{\pi}{2}\right]$ (C) $\left[\frac{3\pi}{10}, \frac{\pi}{2}\right]$ (D) $\left[\frac{-\pi}{10}, \frac{3\pi}{10}\right]$

23. sinx, sin2x, sin3x are in A.P if

(A) $x = n\pi/2, n \in I$ (B) $x = n\pi, n \in I$ (C) $x = 2n\pi, n \in I$ (D) $x = (2n+1)\pi, n \in I$

24. $\sin x + \sin 2x + \sin 3x = 0$ if

(A) $\sin x = 1/2$ (B) $\sin 2x = 0$ (C) $\sin 3x = \sqrt{3}/2$ (D) $\cos x = -1/2$

25. $\cos 4x \cos 8x - \cos 5x \cos 9x = 0$ if

(A) $\cos 12x = \cos 14x$ (B) $\sin 13x = 0$ (C) $\sin x = 0$ (D) $\cos x = 0$

26. $\sin x - \cos^2 x - 1$ assumes the least value for the set of values of x given by:

(A) $x = n\pi + (-1)^{n+1}(\pi/6)$, $n \in I$ (B) $x = n\pi + (-1)^n (\pi/6)$, $n \in I$

(C) $x = n\pi + (-1)^n (\pi/3)$, $n \in I$ (D) $x = n\pi - (-1)^n (\pi/6)$, $n \in I$

27. Let $0 \le \theta \le \frac{\pi}{2}$ and $x = X\cos\theta + Y\sin\theta$, $y = X\sin\theta - Y\cos\theta$ such that $x^2 + 4xy + y^2 = aX^2 + bY^2$,

where a, b are constants then

(A) $a = -1, b = 3$ (B) $\theta = \pi/4$ (C) $a = 3, b = -1$ (D) $\theta = \frac{\pi}{3}$

28. If the equation $\sin(\pi x^2) - \sin(\pi x^2 + 2\pi x) = 0$ is solved for positive roots, then in the increasing sequence of positive root

(A) first term is $\frac{-1+\sqrt{7}}{2}$ (B) first term is $\frac{-1+\sqrt{3}}{2}$

(C) third term is 1 (D) third term is $\frac{-1+\sqrt{11}}{2}$

29. The general solution of the equation $\cos x \cdot \cos 6x = -1$, is :

(A) $x = (2n+1)\pi$, $n \in I$ (B) $x = 2n\pi$, $n \in I$

(C) $x = (2n-1)\pi$, $n \in I$ (D) none of these

30. Which of the following set of values of x satisfy the inequation $\sin 3x < \sin x$.

(A) $\left(\frac{(8n-1)\pi}{4}, 2n\pi\right)$, $n \in I$ (B) $\left(\frac{(8n-1)\pi}{4}, \frac{(8n+1)\pi}{4}\right)$, $n \in I$

(C) $\left(\frac{(8n+1)\pi}{4}, \frac{(8n+3)\pi}{4}\right)$, $n \in I$ (D) $\left((2n+1)\pi, \frac{(8n+5)\pi}{4}\right)$, $n \in I$

31. The equation $2\sin\frac{x}{2} \cdot \cos^2 x + \sin^2 x = 2\sin\frac{x}{2} \cdot \sin^2 x + \cos^2 x$ has a root for which

(A) $\sin 2x = 1$ (B) $\sin 2x = -1$ (C) $\cos x = \frac{1}{2}$ (D) $\cos 2x = -\frac{1}{2}$

32. $\cos 15x = \sin 5x$ if

(A) $x = -\frac{\pi}{20} + \frac{n\pi}{5}$, $n \in I$ (B) $x = \frac{\pi}{40} + \frac{n\pi}{10}$, $n \in I$

(C) $x = \frac{3\pi}{20} + \frac{n\pi}{5}$, $n \in I$ (D) $x = -\frac{3\pi}{40} + \frac{n\pi}{10}$, $n \in I$

33. $5\sin^2 x + \sqrt{3}\sin x \cos x + 6\cos^2 x = 5$ if

(A) $\tan x = -1/\sqrt{3}$ (B) $\sin x = 0$

(C) $x = n\pi + \pi/2$, $n \in I$ (D) $x = n\pi + \pi/6$, $n \in I$

34. $\sin^2x + 2 \sin x \cos x - 3\cos^2x = 0$ if
(A) $\tan x = 3$
(B) $\tan x = -1$
(C) $x = n\pi + \pi/4,\ n \in I$
(D) $x = n\pi + \tan^{-1}(-3),\ n \in I$

35. Solution set of inequality $\sin^3 x \cos x > \cos^3 x \sin x$, where $x \in (0, \pi)$, is
(A) $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$
(B) $\left(\frac{3\pi}{4}, \pi\right)$
(C) $\left(0, \frac{\pi}{4}\right)$
(D) $\left(\frac{\pi}{2}, \frac{3\pi}{4}\right)$

36. $4 \sin^4x + \cos^4x = 1$ if
(A) $x = n\pi$; $(n \in I)$
(B) $x = n\pi \pm \frac{1}{2}\cos^{-1}\left(\frac{1}{5}\right)$; $(n \in I)$
(C) $x = \frac{n\pi}{2}$; $(n \in I)$
(D) $x = -n\pi$; $(n \in I)$

37. $\sin x + \sin 2x + \sin 3x = \cos x + \cos 2x + \cos 3x$ if
(A) $\cos x = -\frac{1}{2}$
(B) $\sin 2x = \cos 2x$
(C) $x = \frac{n\pi}{2} + \frac{\pi}{8}$
(D) $x = 2n\pi \pm \frac{2\pi}{3}$, $(n \in I)$

PART - IV : COMPREHENSION

Comprehenssion # 1

Let p be the product of the sines of the angles of a triangle ABC and q is the product of the cosines of the angles.

1. In this triangle $\tan A + \tan B + \tan C$ is equal to
(A) $p + q$
(B) $p - q$
(C) $\frac{p}{q}$
(D) none of these

2. $\tan A \tan B + \tan B \tan C + \tan C \tan A$ is equal to
(A) $1 + q$
(B) $\frac{1+q}{q}$
(C) $1 + p$
(D) $\frac{1+p}{p}$

3. The value of $\tan^3 A + \tan^3 B + \tan^3 C$ is
(A) $\frac{p^3 - 3pq^2}{q^3}$
(B) $\frac{q^3}{p^3}$
(C) $\frac{p^3}{q^3}$
(D) $\frac{p^3 - 3pq}{q^3}$

Comprehension # 2

Let $a, b, c, d \in R$. Then the cubic equation of the type $ax^3 + bx^2 + cx + d = 0$ has either one root real or all three roots are real. But in case of trigonometric equations of the type $a \sin^3 x + b \sin^2 x + c \sin x + d = 0$ can possess several solutions depending upon the domain of x.

To solve an equation of the type $a \cos\theta + b \sin\theta = c$. The equation can be written as $\cos(\theta - \alpha) = c/\sqrt{(a^2+b^2)}$.

The solution is $\theta = 2n\pi + \alpha \pm \beta$, where $\tan \alpha = b/a$, $\cos \beta = c/\sqrt{(a^2+b^2)}$.

4. On the domain $[-\pi, \pi]$ the equation $4\sin^3 x + 2 \sin^2 x - 2\sin x - 1 = 0$ possess
(A) only one real root
(B) three real roots
(C) four real roots
(D) six real roots

5. In the interval $[-\pi/4, \pi/2]$, the equation, $\cos 4x + \frac{10\tan x}{1+\tan^2 x} = 3$ has
(A) no solution
(B) one solution
(C) two solutions
(D) three solutions

6. $|\tan x| = \tan x + \frac{1}{\cos x}$ $(0 \le x \le 2\pi)$ has
(A) no solution
(B) one solution
(C) two solutions
(D) three solutions

Comprehension # 3

To solve a trigonometric inequation of the type sin x $\geq$ a where $|a| \leq 1$, we take a hill of length 2π in the sine curve and write the solution within that hill. For the general solution, we add $2n\pi$. For instance, to solve $\sin x \geq -\frac{1}{2}$, we take the hill $\left[-\frac{\pi}{2}, \frac{3\pi}{2}\right]$ over which solution is $-\frac{\pi}{6} < x < \frac{7\pi}{6}$. The general solution is $2n\pi - \frac{\pi}{6} < x < 2n\pi + \frac{7\pi}{6}$, n is any integer. Again to solve an inequation of the type sin x $\leq$ a, where $|a| \leq 1$, we take a hollow of length 2π in the sine curve. (since on a hill, sinx $\leq$ a is satisfied over two intervals). Similarly cos x $\geq$ a or cosx $\leq$ a, $|a| \leq 1$ are solved.

7. Solution to the inequation $\sin^6 x + \cos^6 x < \frac{7}{16}$ must be

(A) $n\pi + \frac{\pi}{3} < x < n\pi + \frac{\pi}{2}$ (B) $2n\pi + \frac{\pi}{3} < x < 2n\pi + \frac{\pi}{2}$

(C) $\frac{n\pi}{2} + \frac{\pi}{6} < x < \frac{n\pi}{2} + \frac{\pi}{3}$ (D) none of these

8. Solution to inequality cos 2x + 5 cos x + 3 $\geq$ 0 over $[-\pi, \pi]$ is

(A) $[-\pi, \pi]$ (B) $\left[\frac{-5\pi}{6}, \frac{5\pi}{6}\right]$ (C) $[0, \pi]$ (D) $\left[\frac{-2\pi}{3}, \frac{2\pi}{3}\right]$

9. Over $[-\pi, \pi]$, the solution of $2\sin^2\left(x + \frac{\pi}{4}\right) + \sqrt{3}\cos 2x \geq 0$ is

(A) $[-\pi, \pi]$ (B) $\left[\frac{-5\pi}{6}, \frac{5\pi}{6}\right]$

(C) $[0, \pi]$ (D) $\left[-\pi, \frac{-7\pi}{12}\right] \cup \left[-\frac{\pi}{4}, \frac{5\pi}{12}\right] \cup \left[\frac{3\pi}{4}, \pi\right]$

Exercise-3

Marked questions are recommended for Revision.

*** Marked Questions may have more than one correct option.**

PART - I : JEE (ADVANCED) / IIT-JEE PROBLEMS (PREVIOUS YEARS)

1. The maximum value of the expression $\frac{1}{\sin^2\theta + 3\sin\theta\cos\theta + 5\cos^2\theta}$ is

[IIT-JEE-2010, Paper-1, (3, 0)/84]

2. The positive integer value of n > 3 satisfying the equation

$\frac{1}{\sin\left(\frac{\pi}{n}\right)} = \frac{1}{\sin\left(\frac{2\pi}{n}\right)} + \frac{1}{\sin\left(\frac{3\pi}{n}\right)}$ is **[IIT-JEE 2011, Paper-1, (4, 0), 80]**

3. The number of values of θ in the interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ such that $\theta \neq \frac{n\pi}{5}$ for n = 0, ±1, ± 2 and $\tan\theta = \cot 5\theta$ as well as $\sin 2\theta = \cos 4\theta$ is **[IIT-JEE-2010, Paper-1, (3, 0)/84]**

4. Let P = $\{\theta : \sin\theta - \cos\theta = \sqrt{2}\cos\theta\}$ and Q = $\{\theta : \sin\theta + \cos\theta = \sqrt{2}\sin\theta\}$ be two sets. Then

(A) $P \subset Q$ and $Q - P \neq \varnothing$ (B) $Q \not\subset P$

(C) $P \not\subset Q$ (D) P = Q **[IIT-JEE 2011, Paper-1, (3, –1), 80]**

5.* Let $\theta, \phi \in [0, 2\pi]$ be such that $2\cos\theta(1 - \sin\phi) = \sin^2\theta \left(\tan\frac{\theta}{2} + \cot\frac{\theta}{2}\right)\cos\phi - 1$, $\tan(2\pi - \theta) > 0$ and $-1 < \sin\theta < -\frac{\sqrt{3}}{2}$. Then ϕ cannot satisfy **[IIT-JEE 2012, Paper-1, (4, 0), 70]**

(A) $0 < \phi < \frac{\pi}{2}$ (B) $\frac{\pi}{2} < \phi < \frac{4\pi}{3}$ (C) $\frac{4\pi}{3} < \phi < \frac{3\pi}{2}$ (D) $\frac{3\pi}{2} < \phi < 2\pi$

6. For $x \in (0, \pi)$, the equation $\sin x + 2\sin 2x - \sin 3x = 3$ has
[JEE (Advanced) 2014, Paper-2, (3, –1)/60]

(A) infinitely many solutions (B) three solutions
(C) one solution (D) no solution

7. The number of distinct solutions of the equation $\frac{5}{4}\cos^2 2x + \cos^4 x + \sin^4 x + \cos^6 x + \sin^6 x = 2$ in the interval $[0, 2\pi]$ is **[JEE (Advanced) 2015, P-1 (4, 0) /88]**

8. The value of $\sum_{k=1}^{13} \frac{1}{\sin\left(\frac{\pi}{4} + \frac{(k-1)\pi}{6}\right)\sin\left(\frac{\pi}{4} + \frac{k\pi}{6}\right)}$ is equal to

[JEE (Advanced) 2016, Paper-2, (3, –1)/62]

(A) $3 - \sqrt{3}$ (B) $2\left(3 - \sqrt{3}\right)$ (C) $2\left(\sqrt{3} - 1\right)$ (D) $2\left(2 + \sqrt{3}\right)$

9. Let $S = \left\{x \in (-\pi, \pi) : x \neq 0, \pm\frac{\pi}{2}\right\}$. The sum of all distinct solutions of the equation $\sqrt{3}\sec x + \operatorname{cosec} x + 2(\tan x - \cot x) = 0$ in the set S is equal to
[JEE (Advanced) 2016, Paper-1, (3, –1)/62]

(A) $-\frac{7\pi}{9}$ (B) $-\frac{2\pi}{9}$ (C) 0 (D) $\frac{5\pi}{9}$

10. Let α and β be nonzero real numbers such that $2(\cos\beta - \cos\alpha) + \cos\alpha\cos\beta = 1$. Then which of the following is/are true? **[JEE(Advanced) 2017, Paper-2,(4, –2)/61]**

(A) $\sqrt{3}\tan\left(\frac{\alpha}{2}\right) - \tan\left(\frac{\beta}{2}\right) = 0$ (B) $\tan\left(\frac{\alpha}{2}\right) - \sqrt{3}\tan\left(\frac{\beta}{2}\right) = 0$

(C) $\tan\left(\frac{\alpha}{2}\right) + \sqrt{3}\tan\left(\frac{\beta}{2}\right) = 0$ (D) $\sqrt{3}\tan\left(\frac{\alpha}{2}\right) + \tan\left(\frac{\beta}{2}\right) = 0$

11. Let a, b, c be three non-zero real numbers such that the equation $\sqrt{3}a\cos x + 2b\sin x = c$, $x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, has two distinct real roots α and β with $\alpha + \beta = \frac{\pi}{3}$. Then, the value of $\frac{b}{a}$ is ______.
[JEE(Advanced) 2018, Paper-1,(3, 0),60]

12. For non-negative integer n, let $f(n) = \frac{\sum_{k=0}^{n}\sin\left(\frac{k+1}{n+2}\pi\right)\sin\left(\frac{k+2}{n+2}\pi\right)}{\sum_{k=0}^{n}\sin^2\left(\frac{k+1}{n+2}\pi\right)}$

Assuming $\cos^{-1}x$ takes values in $[0, \pi]$ which of the following options is/are correct?
[JEE(Advanced) 2019, Paper-2 ,(4, –1)/62]

(A) $f(4) = \frac{\sqrt{3}}{2}$ (B) If $\alpha = \tan(\cos^{-1} f(6))$, then $\alpha^2 + 2\alpha - 1 = 0$

(C) $\sin(7\cos^{-1} f(5)) = 0$ (D) $\lim_{n\to\infty} f(n) = \frac{1}{2}$

For Q. 13 & 14

Let $f(x) = \sin(\pi\cos x)$ and $g(x) = \cos(2\pi \sin x)$ be two functions defined for $x > 0$. Define the following sets whose elements are written in increasing order

$X = \{x : f(x) = 0\}$, $Y = \{x : f'(x) = 0\}$

$Z = \{x : g(x) = 0\}$, $W = \{x : g'(x) = 0\}$

List-I contains sets X,Y,Z and W List-II contains some information regarding these set.

[JEE(Advanced) 2019, Paper-2 ,(4, –1)/62]

List - I		**List - II**
(I)	X	(P) $\supseteq \left\{\frac{\pi}{2}, \frac{3\pi}{2}, 4\pi, 7\pi\right\}$
(II)	Y	(Q) an arithmetic progression
(III)	Z	(R) NOT an arithmetic progression
(IV)	W	(S) $\supseteq \left\{\frac{\pi}{6}, \frac{7\pi}{6}, \frac{13\pi}{6}\right\}$
		(T) $\supseteq \left\{\frac{\pi}{3}, \frac{2\pi}{3}, \pi\right\}$
		(U) $\supseteq \left\{\frac{\pi}{6}, \frac{3\pi}{4}\right\}$

13. Which of the following is the only correct combination ? **[JEE(Advanced) 2019, Paper-2 ,(4, –1)/62]**

(1) IV – (Q), (T) (2) III – (R), (U) (3) III – (P), (Q), (U) (4) IV – (P), (R), (S)

14. Which is the following is only CORRECT combination ? **[JEE(Advanced) 2019, Paper-2 ,(4, –1)/62]**

(1) I – (Q), (U) (2) I – (P), (R) (3) II – (Q), (T) (4) II – (R), (S)

PART - II : JEE (MAIN) / AIEEE PROBLEMS (PREVIOUS YEARS)

1. Let $\cos(\alpha + \beta) = \frac{4}{5}$ and let $\sin(\alpha - \beta) = \frac{5}{13}$, where $0 \le \alpha, \beta \le \frac{\pi}{4}$. Then $\tan 2\alpha$ =

[AIEEE 2010 (4, –1), 144]

(1) $\frac{56}{33}$ (2) $\frac{19}{12}$ (3) $\frac{20}{7}$ (4) $\frac{25}{16}$

2. If $A = \sin^2 x + \cos^4 x$, then for all real x : **[AIEEE 2011 (4, –1), 120]**

(1) $\frac{3}{4} \le A \le 1$ (2) $\frac{13}{16} \le A \le 1$ (3) $1 \le A \le 2$ (4) $\frac{3}{4} \le A \le \frac{13}{16}$

3. In a ΔPQR, if $3 \sin P + 4 \cos Q = 6$ and $4 \sin Q + 3 \cos P = 1$, then the angle R is equal to :

[AIEEE-2012, (4, –1)/120]

(1) $\frac{5\pi}{6}$ (2) $\frac{\pi}{6}$ (3) $\frac{\pi}{4}$ (4) $\frac{3\pi}{4}$

4. The expression $\frac{\tan A}{1-\cot A} + \frac{\cot A}{1-\tan A}$ can be written as : **[AIEEE - 2013, (4, –1),360]**

(1) sinA cosA + 1 (2) secA cosecA + 1 (3) tanA + cotA (4) secA + cosecA

5. Let $f_k(x) = \frac{1}{k}(\sin^k x + \cos^k x)$ where $x \in R$ and $k \ge 1$. Then $f_4(x) - f_6(x)$ equals

[JEE(Main)2014,(4, – 1), 120]

(1) $\frac{1}{4}$ (2) $\frac{1}{12}$ (3) $\frac{1}{6}$ (4) $\frac{1}{3}$

6. If the angles of elevation of the top of a tower from three collinear points A, B and C, on a line leading to the foot of the tower, are 30º, 45º and 60º respectively, then the ratio, AB : BC , is

[JEE(Main)2015,(4, – 1), 120]

(1) $\sqrt{3}:1$ (2) $\sqrt{3}:\sqrt{2}$ (3) $1:\sqrt{3}$ (4) 2 : 3

7. A man is walking towards a vertical pillar in a straight path, at a uniform speed. At a certain point A on the path, he observes that the angle of elevation of the top of the pillar is 30°, After walking for 10 minutes from A in the same direction, at a point B, he observes that the angle of elevation of the top of the pillar is 60°. Then the time taken (in minutes) by him, from B to reach the pillar, is :

[JEE(Main)2016,(4, – 1), 120]

(1) 10 (2) 20 (3) 5 (4) 6

8. If $0 \le x < 2\pi$, then the number of real values of x, which satisfy the equation $\cos x + \cos 2x + \cos 3x + \cos 4x = 0$, is **[JEE(Main)2016,(4, – 1), 120]**

(1) 5 (2) 7 (3) 9 (4) 3

9. If $5(\tan^2 x - \cos^2 x) = 2\cos 2x + 9$, then the value of cos4x is : **[JEE(Main)2017,(4, – 1), 120]**

(1) $\frac{-3}{5}$ (2) $\frac{1}{3}$ (3) $\frac{2}{9}$ (4) $-\frac{7}{9}$

10. Let a vertical tower AB have its end A on the level ground. Let C be the mid-point of AB and P be a point on the ground such that AP = 2AB. If $\angle BPC = \beta$, then $\tan\beta$ is equal to

[JEE(Main)2017,(4, – 1), 120]

(1) $\frac{6}{7}$ (2) $\frac{1}{4}$ (3) $\frac{2}{9}$ (4) $\frac{4}{9}$

11. If sum of all the solutions of the equation $8\cos x.\left(\cos\left(\frac{\pi}{6}+x\right).\cos\left(\frac{\pi}{6}-x\right)-\frac{1}{2}\right) = 1$ in $[0, \pi]$ is $k\pi$, then k is equal to : **[JEE(Main)2018,(4, – 1), 120]**

(1) $\frac{8}{9}$ (2) $\frac{20}{9}$ (3) $\frac{2}{3}$ (4) $\frac{13}{9}$

12. PQR is a triangular park with PQ = PR = 200 m. A T.V. tower stands at the mid-point of QR. If the angles of elevation of the top of the tower at P, Q and R are respectively 45º, 30º and 30º, then the height of the tower (in m) is : **[JEE(Main)2018,(4, – 1), 120]**

(1) $100\sqrt{3}$ (2) $50\sqrt{2}$ (3) 100 (4) 50

13. The sum of all values of $\theta \in \left(0,\frac{\pi}{2}\right)$ satisfying $\sin^2 2\theta + \cos^4 2\theta = \frac{3}{4}$ is :

[JEE(Main) 2019, Online (10-01-19),P-1 (4, – 1), 120]

(1) π (2) $\frac{\pi}{2}$ (3) $\frac{3\pi}{8}$ (4) $\frac{5\pi}{4}$

14. The value of $\cos\frac{\pi}{2^2}.\cos\frac{\pi}{2^3}.\ .....\ .\cos\frac{\pi}{2^{10}}.\sin\frac{\pi}{2^{10}}$ is :

[JEE(Main) 2019, Online (10-01-19),P-2 (4, – 1), 120]

(1) $\frac{1}{1024}$ (2) $\frac{1}{2}$ (3) $\frac{1}{512}$ (4) $\frac{1}{256}$

15. If $\sin^4\alpha + 4\cos^4\beta + 2 = 4\sqrt{2}\sin\alpha\cos\beta$; $\alpha,\beta \in [0, \pi]$, then $\cos(\alpha+\beta) - \cos(\alpha-\beta)$ is equal to

[JEE(Main) 2019, Online (12-01-19),P-2 (4, – 1), 120]

(1) $-\sqrt{2}$ (2) 0 (3) $\sqrt{2}$ (4) –1

16. The number of solutions of the equation $1 + \sin^4 x = \cos^2 3x$, $x \in \left[-\frac{5\pi}{2}, \frac{5\pi}{2}\right]$ is

[JEE(Main) 2019, Online (12-04-19),P-1 (4, – 1), 120]

(1) 4 (2) 5 (3) 7 (4) 3

17. The value of sin10°sin30°sin50°sin70° is : **[JEE(Main) 2019, Online (09-04-19),P-2 (4, – 1), 120]**

(1) $\frac{1}{18}$ (2) $\frac{1}{32}$ (3) $\frac{1}{16}$ (4) $\frac{1}{36}$

18. All the pairs (x, y) that satisfy the inequality $2^{\sqrt{\sin^2 x - 2\sin x + 5}} \cdot \frac{1}{4^{\sin^2 y}} \le 1$ also satisfy the equation :

[JEE(Main) 2019, Online (10-04-19),P-1 (4, – 1), 120]

(1) sin x = |siny| (2) 2|sin x| = 3 siny (3) sin x = 2 sin y (4) 2 sin x = 2 sin y

19. Two vertical poles of heights, 20 m and 80 m stand a part on a horizontal plane. The height (in meters) of the point of intersection of the lines joining the top of each pole to the foot of the other, from this horizontal plane is : **[JEE(Main) 2019, Online (08-04-19),P-2 (4, – 1), 120]**

(1) 18 (2) 16 (3) 15 (4) 12

20. Let α and β be two real roots of the $(k+1)\tan^2 x - \sqrt{2} \cdot \lambda \tan x = (1 - k)$, where $k(\neq -1)$ and λ are real numbers. If $\tan^2(\alpha + \beta) = 50$, then a value of λ is:

[JEE(Main) 2020, Online (07-01-20),P-1 (4, – 1), 120]

(1) $10\sqrt{2}$ (2) $5\sqrt{2}$ (3) 10 (4) 5

21. The value of $\cos^3\left(\frac{\pi}{8}\right) . \cos\left(\frac{3\pi}{8}\right) + \sin^3\left(\frac{\pi}{8}\right) . \sin\left(\frac{3\pi}{8}\right)$ is :

[JEE(Main) 2020, Online (09-01-20),P-1 (4, –1), 120]

(1) $\frac{1}{2\sqrt{2}}$ (2) $\frac{1}{\sqrt{2}}$ (3) $\frac{1}{4}$ (4) $\frac{1}{2}$

22. The number of distinct solutions of the equation, $\log_{1/2}|\sin x| = 2 - \log_{1/2}|\cos x|$ in the interval $[0, 2\pi]$, is

[JEE(Main) 2020, Online (09-01-20),P-1 (4, 0), 120]

Answers

EXERCISE - 1

PART - I

Section (A) :

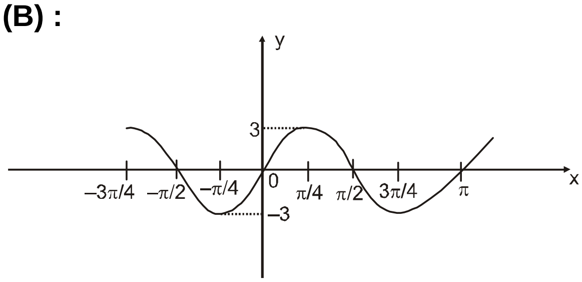
A-1. (i) $\frac{\pi}{12}$ (ii) $\frac{4\pi}{3}$ (iii) $\frac{53\pi}{18}$

A-2. (i) 135° (ii) – 720° (iii) 300° (iv) 210°

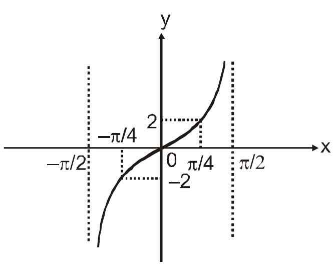
A-4. (i) $\left(\frac{-\sqrt{3}}{2}\right)$ (ii) $-\frac{1}{\sqrt{2}}$ (iii) $-\frac{1}{\sqrt{3}}$ (iv) 1

Section (B) :

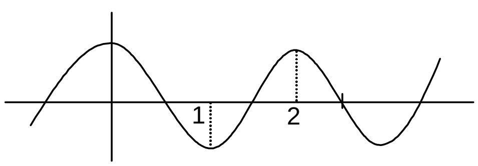
B-1. (i)



(ii)



(iii)



B-2. 1 **B-3.** $\frac{13}{12}$ **B-7.** $a^2b^2 + 4a^2 = 9b^2$

Section (C)

C-2. (i) 1 (ii) $-\sqrt{5}/4$ (iii) $\frac{\sqrt{5}+1}{8}$ **C-10.** $\frac{\sqrt{3}-1}{2\sqrt{2}}$

Section (E) :

E-1. $-\frac{1}{4}, \frac{1}{4}$ **E-2.** (i) 2, –1 (ii) 3, 0

E-3. (i) $y_{max} = 11; y_{min} = 1$ (ii) $y_{max} = 10; y_{min} = -4$

Section (F) :

F-1. (i) $n\pi + (-1)^n \frac{\pi}{4}, n \in I$ (ii) $n\pi + \frac{\pi}{3} + 1, n \in I$

(iii) $n\pi - \frac{\pi}{4}, n \in I$ (iv) $n\pi + (-1)^n \frac{\pi}{3}, n \in I$ (v) $n\pi \pm \frac{\pi}{4}, n \in I$

F-2. (i) $\frac{m\pi}{4}, m \in I$ or $\frac{(2m+1)\pi}{10}$, $m \in I$

(ii) $2n\pi \pm \frac{\pi}{3}, n \in I$

(iii) $\left(2n+\frac{1}{2}\right)\frac{\pi}{5}, n \in I$ or $2n\pi - \frac{\pi}{2}, n \in I$

(iv) $\left(n+\frac{1}{2}\right)\frac{\pi}{9}, n \in I$ (v) $\left(n+\frac{1}{4}\right)\frac{\pi}{2}, n \in I$

(vi) $2n\pi + \frac{2\pi}{3}, n \in I$ (vii) $n\pi \pm \frac{\pi}{6}$

(viii) $\left(n+\frac{1}{3}\right)\frac{\pi}{3}, n \in I$

F-3. (i) $\frac{n\pi}{3}, n \in I$ or $\left(n \pm \frac{1}{3}\right)\pi, n \in I$

(ii) $2n\pi, n \in I$ or $\frac{2n\pi}{3} + \frac{\pi}{6}, n \in I$

(iii) $x = (2n+1)\frac{\pi}{4}, n \in I$ or $x = (2n+1)\frac{\pi}{2}, n \in I$ or $x = n\pi \pm \frac{\pi}{6}, n \in I$

(iv) $m\pi, m \in I$ or $\frac{m\pi}{n-1}, m \in I$ or $\left(m+\frac{1}{2}\right)\frac{\pi}{n}, m \in I$

F-4. (i) $n\pi + \frac{\pi}{3}, n \in I$ or $n\pi + \frac{\pi}{4}, n \in I$

(ii) $n\pi + (-1)^n \frac{\pi}{10}, n \in I$ or $n\pi - (-1)^n \frac{3\pi}{10}, n \in I$

(iii) $x = \frac{n\pi}{3} - \frac{\pi}{9}, n \in I$

F-5. (i) $n\pi + \frac{\pi}{6} + (-1)^n \frac{\pi}{4}, n \in I$

(ii) $2n\pi + \frac{\pi}{2}, n \in I$ or $2n\pi + 2\alpha$ where $\alpha = \tan^{-1}\frac{3}{7}, n \in I$

Section (G) :

G-1.	$x \in \left[n\pi - \frac{\pi}{4}, n\pi + \frac{\pi}{4}\right] : n \in I$	**G-2.**	$\left(2n\pi + \frac{7\pi}{6},\ 2n\pi + \frac{11\pi}{6}\right)$
G-3.	$\theta \in (n\pi + \pi/3, n\pi + \pi/2]$	**G-4.**	$15\sqrt{3}$ m
G-5.	$\sqrt{ab}$	**G-6.**	50 m

PART - II

Section (A) :

A-1. (A) **A-2.** (A) **A-3.** (B) **A-4.** (C) **A-5.** (B)

Section (B) :

B-1. (A) **B-2.** (B) **B-3.** (A) **B-4.** (D) **B-5.** (B) **B-6.** (B) **B-7.** (A)

B-8. (A) **B-9.** (C) **B-10.** (A) **B-11.** (B) **B-12.** (C)

Section (C) :

C-1. (C) **C-2.** (A) **C-3.** (C) **C-4.** (D) **C-5.** (A) **C-6.** (B) **C-7.** (A)

C-8. (B) **C-9.** (D) **C-10.** (C)

Section (D) :

D-1. (A) **D-2.** (B) **D-3.** (D) **D-4.** (B) **D-5.** (A)

Section (E) :

E-1. (C) **E-2.** (C) **E-3.** (A) **E-4.** (B) **E-5.** (A)

Section (F) :

F-1. (D) **F-2.** (B) **F-3.** (D) **F-4.** (B) **F-5.** (C) **F-6.** (B) **F-7.** (A)

F-8. (B) **F-9.** (A) **F-10.** (B) **F-11.** (C) **F-12.** (B)

Section (G) :

G-1. (A) **G-2.** (C) **G-3.** (B) **G-4.** (A) **G-5.** (A) **G-6.** (D) **G-7.** (D)

G-8. (D) **G-9.** (B)

PART - III

1. (A) $\rightarrow$ (s), (B) $\rightarrow$ (s), (C) $\rightarrow$ (s), (D) $\rightarrow$ (r)

2. (A) $\rightarrow$ (q, s), (B) $\rightarrow$ (p), (C) $\rightarrow$ (q), (D) $\rightarrow$ (p)

3. (A) $\rightarrow$ (q), (B) $\rightarrow$ (s), (C) $\rightarrow$ (r), (D) $\rightarrow$ (p)

EXERCISE - 2

PART - I

1. (B) **2.** (B) **3.** (D) **4.** (B) **5.** (A) **6.** (B) **7.** (B)

8. (A) **9.** (D) **10.** (D) **11.** (C) **12.** (C) **13.** (C) **14.** (A)

15. (A) **16.** (D) **17.** (A) **18.** (B)

PART - II

1.	04.80	**2.**	00.71	**3.**	02.00	**4.**	00.08	**5.**	19.00	**6.**	08.29 or 08.30
7.	08.00	**8.**	17.00	**9.**	15.00	**10.**	20.00	**11.**	24.00	**12.**	02.00
13.	01.00	**14.**	50.00	**15.**	04.00	**16.**	25.13	**17.**	17.00	**18.**	17.00
19.	06.00	**20.**	15.00	**21.**	21.99	**22.**	04.45	**23.**	01.75	**24.**	06.00
25.	00.00	**26.**	00.00	**27.**	00.00						

PART - III

1.	(ABCD)	**2.**	(BC	**3.**	(BD)	**4.**	(AB)	**5.**	(BC)	**6.**	(ABC)
7.	(BC)	**8.**	(ABCD)	**9.**	(BCD)	**10.**	(AC)	**11.**	(CD)	**12.**	(BD)
13.	(BC)	**14.**	(BC)	**15.**	(BD)	**16.**	(BD)	**17.**	(AD)	**18.**	(AB)
19.	(AD)	**20.**	(CD)	**21.**	(AB)	**22.**	(AC)	**23.**	(ABCD)	**24.**	(BD)
25.	(ABC)	**26.**	(AD)	**27.**	(BC)	**28.**	(BC)	**29.**	(AC)	**30.**	(ACD)
31.	(ABCD)	**32.**	(ABCD)	**33.**	(AC)	**34.**	(CD)	**35.**	(AB)	**36.**	(ABD)
37.	(ABCD)										

PART - IV

1.	(C)	**2.**	(B)	**3.**	(D)	**4.**	(D)	**5.**	(C)	**6.**	(B)	**7.**	(C)
8.	(D)	**9.**	(D)										

EXERCISE - 3

PART - I

1.	2	**2.**	(n = 7)	**3.**	3	**4.**	(D)	**5.**	(ACD)	**6.**	(D)	**7.**	8
8.	(C)	**9.**	(C)	**10.**	Bonus	**11.**	(0.5)	**12.**	(ABC)	**13.**	(4)	**14.**	(3)

PART - II

1.	(1)	**2.**	(1)	**3.**	(2)	**4.**	(2)	**5.**	(2)	**6.**	(1)	**7.**	(3)
8.	(2)	**9.**	(4)	**10.**	(3)	**11.**	(4)	**12.**	(3)	**13.**	(2)	**14.**	(3)
15.	(1)	**16.**	(2)	**17.**	(3)	**18.**	(1)	**19.**	(2)	**20.**	(3)	**21.**	(1)
22.	8												

High Level Problems (HLP)

1. Prove that :

(i) $\sec^4 A\,(1 - \sin^4 A) - 2\tan^2 A = 1$ (ii) $\dfrac{\cot^2\theta\;(\sec\theta - 1)}{1+\sin\theta} = \sec^2\theta\;\dfrac{1-\sin\theta}{1+\sec\theta}$.

2. Simplify the expression $\sqrt{\sin^4 x + 4\cos^2 x} - \sqrt{\cos^4 x + 4\sin^2 x}$

3. Let a, b, c, d be numbers in the interval $[0, \pi]$ such that $\sin a + 7\sin b = 4(\sin c + 2\sin d)$, $\cos a + 7\cos b = 4(\cos c + 2\cos d)$. Prove that $2\cos(a - d) = 7\cos(b - c)$.

4. Prove that $(4\cos^2 9^\circ - 3)(4\cos^2 27^\circ - 3) = \tan 9^\circ$

5. If $\cos(\alpha + \beta) = \dfrac{4}{5}$; $\sin(\alpha - \beta) = \dfrac{5}{13}$ & α, β lie between 0 & $\dfrac{\pi}{4}$, then find the value of $\tan 2\alpha$.

6. If α & β are two distinct roots of the equation $a\tan\theta + b\sec\theta = c$, then prove that $\tan(\alpha + \beta) = \dfrac{2ac}{a^2 - c^2}$.

7. If $\tan\alpha = \dfrac{p}{q}$ where $\alpha = 6\beta$, α being an acute angle, prove that;

$\dfrac{1}{2}(p\,\text{cosec}\,2\beta - q\sec 2\beta) = \sqrt{p^2 + q^2}$.

8. If $\sin(\theta + \alpha) = a$ & $\sin(\theta + \beta) = b$ $(0 < \alpha, \beta, \theta < \pi/2)$ then find the value of $\cos 2(\alpha - \beta) - 4ab\cos(\alpha - \beta)$

9. Show that:

(i) $\cot 7\dfrac{1^\circ}{2}$ or $\tan 82\dfrac{1^\circ}{2} = \left(\sqrt{3} + \sqrt{2}\right)\left(\sqrt{2} + 1\right)$ or $\sqrt{2} + \sqrt{3} + \sqrt{4} + \sqrt{6}$

(ii) $\tan 142\dfrac{1^\circ}{2} = 2 + \sqrt{2} - \sqrt{3} - \sqrt{6}$.

10. If $\tan\beta = \dfrac{\tan\alpha + \tan\gamma}{1 + \tan\alpha \cdot \tan\gamma}$, prove that $\sin 2\beta = \dfrac{\sin 2\alpha + \sin 2\gamma}{1 + \sin 2\alpha \cdot \sin 2\gamma}$.

11. If α & β satisfy the equation $a\cos 2\theta + b\sin 2\theta = c$ then prove that: $\cos^2\alpha + \cos^2\beta = \dfrac{a^2 + ac + b^2}{a^2 + b^2}$.

12. Show that: $4\sin 27^\circ = (5 + \sqrt{5})^{1/2} - (3 - \sqrt{5})^{1/2}$

13. If $xy + yz + xz = 1$, then prove that

$$\frac{x}{1-x^2} + \frac{y}{1-y^2} + \frac{z}{1-z^2} = \frac{4xyz}{(1-x^2)(1-y^2)(1-z^2)}.$$

14. Let $a = \dfrac{\pi}{7}$

(a) Show that $\sin^2 3a - \sin^2 a = \sin 2a \sin 3a$
(b) Show that $\text{cosec}\,a = \text{cosec}\,2a + \text{cosec}\,4a$
(c) Evaluate $\cos a - \cos 2a + \cos 3a$
(d) Prove that $\cos a$ is a root of the equation $8x^3 - 4x^2 - 4x + 1 = 0$
(e) Evaluate $\tan a \tan 2a \tan 3a$
(f) Evaluate $\tan^2 a + \tan^2 2a + \tan^2 3a$
(g) Evaluate $\tan^2 a \tan^2 2a + \tan^2 2a \tan^2 3a + \tan^2 3a \tan^2 a$
(h) Evaluate $\cot^2 a + \cot^2 2a + \cot^2 3a$

15. In a ΔABC, prove that $\sin\frac{A}{2} + \sin\frac{B}{2} + \sin\frac{C}{2} = 1 + 4\sin\left(\frac{\pi - A}{4}\right)\sin\left(\frac{\pi - B}{4}\right)\sin\left(\frac{\pi - C}{4}\right)$

16. Evaluate

$\cos a \cos 2a \cos 3a..........\cos 999a$, where $a = \frac{2\pi}{1999}$

17. Prove that the average of the numbers
$2 \sin 2^\circ, 4 \sin 4^\circ, 6 \sin 6^\circ, ...180 \sin 180^\circ$ is $\cot 1^\circ$

18. Solve $\tan 2\theta = \tan\frac{2}{\theta}$.

19. Find the general values of x and y satisfying the equations
$5 \sin x \cos y = 1$, $4 \tan x = \tan y$

20. Solve $\frac{\sin^3\frac{x}{2} - \cos^3\frac{x}{2}}{2 + \sin x} = \frac{\cos x}{3}$.

21. Solve the system of equations :

$x + y = \frac{2\pi}{3}$, $\sin x + \sin y = \frac{3}{2}$ and $x, y \in \left[0, \frac{\pi}{2}\right]$

22. Solve the following system of simultaneous equations for x and y:
$4^{\sin x} + 3^{1/\cos y} = 11$
$5.16^{\sin x} - 2.3^{1/\cos y} = 2$

23. Solve $\cos\theta + \sin\theta = \cos 2\theta + \sin 2\theta$.

24. Solve $8 \sin x = \frac{\sqrt{3}}{\cos x} + \frac{1}{\sin x}$

25. Solve the equation $\sin^3 x \cos 3x + \cos^3 x \sin 3x + 0.375 = 0$.

26. Solve the equation $\frac{\sqrt{3}}{2}\sin x - \cos x = \cos^2 x$.

27. Solve the equation $\sin^4 x + \cos^4 x - 2\sin^2 x + \frac{3}{4}\sin^2 2x = 0$

28. Solve for *x*, the equation $\sqrt{13 - 18\tan x} = 6\tan x - 3$, where $-2\pi < x < 2\pi$.

29. Solve the equation $3 - 2\cos\theta - 4\sin\theta - \cos 2\theta + \sin 2\theta = 0$

30. Solve the equation $\sin^2 4x + \cos^2 x = 2 \sin 4x \cdot \cos^4 x$

31. Prove that : $\cos 5A = 16\cos^5 A - 20\cos^3 A + 5\cos A$

32. If $\cos\theta = \frac{1}{2}\left(a + \frac{1}{a}\right)$ and $\cos 3\theta = \frac{1}{2}\left(a^k + \frac{1}{a^k}\right)$ then number of natural numbers 'k' less than 50 is (given $a \in R$)

33. Consider the equation for $0 \le \theta \le 2\pi$; $\left(\sin 2\theta + \sqrt{3}\cos 2\theta\right)^2 - 5 = \cos\left(\frac{\pi}{6} - 2\theta\right)$. If greatest value of θ is $\frac{k\pi}{p}$ (k, p are coprime), then find the value of (k + p).

Answers

2. $\cos^2 x - \sin^2 x = \cos 2x$ **5.** $\frac{56}{33}$ **8.** $1 - 2a^2 - 2b^2$

14. (c) $\frac{1}{2}$ (e) $\sqrt{7}$ (f) 21 (g) 35 (h) 5

16. $\frac{1}{2^{999}}$ **18.** $\frac{n\pi}{4} \pm \sqrt{1+\frac{n^2\pi^2}{16}}$, $n \in I$

20. $x = (4n + 1)\frac{\pi}{2}$, $n \in I$ **21.** $x = \frac{\pi}{2}$, $y = \frac{\pi}{6}$ or $x = \frac{\pi}{6}$, $y = \frac{\pi}{2}$.

22. $x = n\pi + (-1)^n \frac{\pi}{6}$, $y = 2n\pi \pm \frac{\pi}{3}$ **23.** $2n\pi$, $n \in I$ or $\frac{2n\pi}{3} + \frac{\pi}{6}$, $n \in I$

24. $x = n\pi + \frac{\pi}{6}$, $n \in I$, $x = \frac{n\pi}{2} - \frac{\pi}{12}$, $n \in I$ **25.** $x = \frac{n\pi}{4} + (-1)^{n+1} \cdot \frac{\pi}{24}$; $n \in I$

26. $x = (2n + 1)\pi : n \in I$, $x = 2n\pi + \frac{\pi}{3}$, $n \in I$ **27.** $x = n\pi \pm \frac{1}{2}\cos^{-1}\left(2-\sqrt{5}\right)$, $n \in I$

28. $\alpha - 2\pi;\ \alpha - \pi,\ \alpha,\ \alpha + \pi$, where $\tan\alpha = \frac{2}{3}$ **29.** $\theta = (4n + 1)\pi/2$, $\theta = 2n\pi$, $n \in I$

30. $x = (2n + 1)\frac{\pi}{2}$, $n \in I$ **32.** 25 **33.** 31