CONTENT

Probability

JEE (Advanced) Syllabus

Probability : Addition and multiplication rules of probability, conditional probability, independence of events, computation of probability of events using permutations and combinations.

JEE (Main) Syllabus

Probability : Probability of an event, addition and multiplication theorems of probability, Baye's theorem, probability distribution of a random variate, Bernoulli trials and Binomial distribution.

Probability

Lest men suspect your tale untrue Keep probability in view........Gay, John

There are various phenomena in nature, leading to an outcome, which cannot be predicted apriori e.g. in tossing of a coin, a head or a tail may result. Probability theory aims at measuring the uncertainties of such outcomes.

(I) Important terminology :

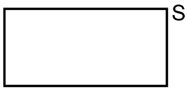
(i) Random experiment:

It is a process which results in an outcome which is one of the various possible outcomes that are known to us before hand e.g. throwing of a die is a random experiment as it leads to fall of one of the outcome from {1, 2, 3, 4, 5, 6}. Similarly taking a card from a pack of 52 cards is also a random experiment.

(ii) Sample space :

It is the set of all possible outcomes of a random experiment e.g. {H, T} is the sample space associated with tossing of a coin.

In set notation it can be interpreted as the universal set.



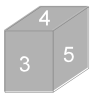
Example # 1 : Write the sample space of the experiment 'A coin is tossed and a die is thrown'.
Solution : The sample space S = {H1, H2, H3, H4, H5, H6, T1, T2, T3, T4, T5, T6}.

Example # 2 : Write the sample space of the experiment 'A coin is tossed, if it shows head a coin tossed again else a die is thrown.
Solution : The sample space S = {HH, HT, T1, T2, T3, T4, T5, T6}

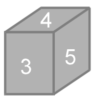
Example # 3 : Find the sample space associated with the experiment of rolling a pair of dice (plural of die) once. Also find the number of elements of the sample space.
Solution : Let one die be blue and the other be green. Suppose '1' appears on blue die and '2' appears on green die. We denote this outcome by an ordered pair (1, 2). Similarly, if '3' appears on blue die and '5' appears on green die, we denote this outcome by (3, 5) and so on. Thus, each outcome can be denoted by an ordered pair (x, y), where x is the number appeared on the first die (blue die) and y appeared on the second die (green die). Thus, the sample space is given by

S = {(x, y) x is the number on blue die and y is the number on green die}

We now list all the possible outcomes (figure)



	1	2	3	4	5	6
1	(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
2	(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
3	(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
4	(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
5	(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)
6	(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)



Figure

Number of elements (outcomes) of the above sample space is 6×6 i.e., 36

Self practice problems :

(1) A coin is tossed twice, if the second throw results in head, a die is thrown then write sample space of the experiment.

(2) An urn contains 3 red balls and 2 blue balls. Write sample space of the experiment 'Selection of a ball from the urn at random'.

Ans. (1) {HT, TT, HH1, HH2, HH3, HH4, HH5, HH6, TH1, TH2, TH3, TH4, TH5, TH6}.

(2) $\{R_1, R_2, R_3, B_1, B_2\}$. (Here the balls are distinguished from one and other by naming red balls as R_1, R_2 and R_3 and the blue balls as B_1 and B_2.)

(iii) Event :

It is subset of sample space. e.g. getting a head in tossing a coin or getting a prime number in throwing a die. In general if a sample space consists 'n' elements, then a maximum of 2^n events can be associated with it.

(iv) Complement of event :

The complement of an event 'A' with respect to a sample space S is the set of all elements of 'S' which are not in A. It is usually denoted by A', or A^C.

(v) Simple event :

If an event covers only one point of sample space, then it is called a simple event e.g. getting a head followed by a tail in throwing of a coin 2 times is a simple event.

(vi) Compound event :

When two or more than two events occur simultaneously, the event is said to be a compound event. Symbolically $A \cap B$ or AB represent the occurrence of both A & B simultaneously.

Note : "$A \cup B$" or A + B represent the occurrence of either A or B.

Example # 4 : Write down all the events of the experiment 'tossing of a coin'.

Solution : S = {H, T}

the events are ϕ, {H}, {T}, {H, T}

Example # 5 : A die is thrown. Let A be the event ' an odd number turns up' and B be the event 'a number divisible by 3 turns up'. Write the events (a) A or B (b) A and B

Solution : A = {1, 3, 5}, B = {3, 6}

$\therefore$ A or B = $A \cup B$ = {1, 3, 5, 6}

A and B = $A \cap B$ = {3}

Self practice problems :

(3) A coin is tossed and a die is thrown. Let A be the event 'H turns up on the coin and odd number turns up on the die' and B be the event ' T turns up on the coin and an even number turns up on the die'. Write the events (a) A or B (b) A and B.

(4) In tossing of two coins, let A = {HH, HT} and B = {HT, TT}. Then write the events (a) A or B (b) A and B.

Ans. (3) (a) {H1, H3, H5, T2, T4, T6} (b) ϕ

(4) (a) {HH, HT, TT} (b) {HT}

(vii) Equally likely events :

If events have same chance of occurrence, then they are said to be equally likely.

e.g

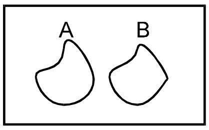
(i) In a single toss of a fair coin, the events {H} and {T} are equally likely.

(ii) In a single throw of an unbiased die the events {1}, {2}, {3} and {4}, are equally likely.

(iii) In tossing a biased coin the events {H} and {T} are not equally likely.

(viii) Mutually exclusive / disjoint / incompatible events :

Two events are said to be mutually exclusive if occurrence of one of them rejects the possibility of occurrence of the other i.e. both cannot occur simultaneously.
In the vein diagram the events A and B are mutually exclusive. Mathematically, we write $A \cap B = \phi$
Events $A_1, A_2, A_3, ... A_n$ are said to be mutually exclusive events iff
$A_i \cap A_j = \phi \ \forall \ i, j \in \{1, 2, ..., n\}$ where $i \neq j$



Note : If $A_i \cap A_j = \phi \ \forall \ i, j \in \{1, 2, ..., n\}$ where $i \neq j$, then $A_1 \cap A_2 \cap A_3 .... \cap A_n = \phi$ but converse need not to be true.

Example # 6 : In a single toss of a coin find whether the events {H}, {T} are mutually exclusive or not.
Solution : Since $\{H\} \cap \{T\} = \phi$,
$\therefore$ the events are mutually exclusive.

Example # 7 : In a single throw of a die, find whether the events {1, 2}, {2, 3} are mutually exclusive or not.
Solution : Since $\{1, 2\} \cap \{2, 3\} = \{2\} \neq \phi$ $\therefore$ the events are not mutually exclusive.

Self practice problems :

(5) In throwing of a die write whether the events 'Coming up of an odd number' and 'Coming up of an even number' are mutually exclusive or not.

(6) An experiment involves rolling a pair of dice and recording the numbers that come up. Describe the following events :
A : the sum is greater than 8.
B : 2 occurs on either die.
C : the sum is at least 7 and a multiple of 3.
Also, find $A \cap B$, $B \cap C$ and $A \cap C$.
Are (i) A and B mutually exclusive ?
(ii) B and C mutually exclusive ?
(iii) A and C mutually exclusive ?

Ans. (5) Yes
(6) A = {(3, 6), (4, 5), (5, 4), (6, 3), (4, 6), (5, 5), (6, 4), (5, 6), (6, 5), (6, 6)}
B = {(1, 2), (2, 2), (3, 2), (4, 2), (5, 2), (6, 2), (2, 1), (2, 3), (2, 4). (2, 5), (2, 6)}
C = {(3, 6), (6, 3), (5, 4), (4, 5), (6, 6)}
$A \cap B = \phi$, $B \cap C = \phi$, $A \cap C$ = {(3, 6), (6, 3), (5, 4), (4, 5), (6, 6)}
(i) Yes (ii) Yes (iii) No.

(ix) Exhaustive system of events :

If each outcome of an experiment is associated with at least one of the events $E_1, E_2, E_3, ...E_n$, then collectively the events are said to be exhaustive. Mathematically we write
$E_1 \cup E_2 \cup E_3.........E_n = S$. (Sample space)

Example # 8 : In throwing of a die, let A be the event 'even number turns up', B be the event 'an odd prime turns up' and C be the event 'a numbers less than 4 turns up'. Find whether the events A, B and C form an exhaustive system or not.
Solution : $A \equiv \{2, 4, 6\}$, $B \equiv \{3, 5\}$ and $C \equiv \{1, 2, 3\}$.
Clearly $A \cup B \cup C = \{1, 2, 3, 4, 5, 6\} = S$. Hence the system of events is exhaustive.

Example # 9 : Three coins are tossed. Describe
(i) two events A and B which are mutually exclusive
(ii) three events A, B and C which are mutually exclusive and exhaustive.
(iii) two events A and B which are not mutually exclusive.
(iv) two events A and B which are mutually exclusive but not exhaustive.
(v) three events A, B and C which are mutually exclusive but not exhaustive.

Ans. (i) A : “getting at least two heads” B : “getting at least two tails”
(ii) A : “getting at most one heads” B : “getting exactly two heads”
C : “getting exactly three heads”
(iii) A : “getting at most two tails” B : “getting exactly two heads”
(iv) A : “getting exactly one head” B : “getting exactly two heads”
(v) A : “ getting exactly one tail” B : “getting exactly two tails”
C : “getting exactly three tails”
[Note : There may be other cases also]

Self practice problems :

(7) In throwing of a die which of the following pair of events are mutually exclusive?
(a) the events ‘coming up of an odd number’ and ‘coming up of an even number’
(b) the events ‘coming up of an odd number’ and ‘coming up of a number ≥ 4’

(8) In throwing of a die which of the following system of events are exhaustive ?
(a) the events ‘an odd number turns up’, ‘a number ≤ 4 turns up’ and ‘the number 5 turns up’.
(b) the events ‘a number ≤ 4 turns up’, ‘a number > 4 turns up’.
(c) the events ‘an even number turns up’, ‘a number divisible by 3 turns up’, ‘number 1 or 2 turns up’ and ‘the number 6 turns up’.

Ans. (7) (a) (8) (b)

(II) Classical (a priori) definition of probability :

If an experiment results in a total of (m + n) outcomes which are equally likely and if ‘m’ outcomes are favorable to an event ‘A’ while ‘n’ are unfavorable, then the probability of occurrence of the event ‘A’, denoted by P(A), is defined by $\frac{m}{m+n} = \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$

i.e. $P(A) = \frac{m}{m+n}$.

Here we say that odds in favour of ‘A’ are m : n, while odds against ‘A’ are n : m.

Note that $P(\bar{A})$ or $P(A')$ or $P(A^c)$, i.e. probability of non-occurrence of A = $\frac{n}{m+n}$ = 1 – P(A)

In the above we shall denote the number of out comes favourable to the event A by n(A) and the total number of out comes in the sample space S by n(S).

$\therefore \quad P(A) = \frac{n(A)}{n(S)}$.

Example # 10 : In throwing of a fair die find the probability of the event ‘ a number ≤ 3 turns up’.

Solution : Sample space S = {1, 2, 3, 4, 5, 6} ; event A = {1, 2, 3}

$\therefore \quad n(A) = 3$ and $n(S) = 6$

$\therefore \quad P(A) = \frac{n(A)}{n(S)} = \frac{3}{6} = \frac{1}{2}$.

Example # 11 : In throwing of a fair die, find the probability of turning up of an odd number ≥ 4.

Solution : S = {1, 2, 3, 4, 5, 6}

Let E be the event ‘turning up of an odd number ≥ 4’

then E = {5}

$\therefore \quad P(E) = \frac{n(E)}{n(S)} = \frac{1}{6}$.

Example # 12 : In throwing a pair of fair dice, find the probability of getting a total of 10.

Solution : When a pair of dice is thrown the sample space consists

{(1, 1) (1, 2) .... (1, 6)
(2, 1,) (2, 2,)......... (2, 6)
....
....
(6, 1), (6, 2) .... (6, 6)}

Note that (1, 2) and (2, 1) are considered as separate points to make each outcome as equally likely.

To get a total of '10', favourable outcomes are, (4, 6) (5, 5) and (6, 4)

Hence required probability = $\frac{3}{36} = \frac{1}{12}$

Example # 13 : A five digit number is formed using the digits 0, 1, 2, 3, 4,5 without repetition. Find the probability that it is divisible by 4

Solution : Total 5 digit numbers formed

↓	↓	↓	↓	↓
5	5	4	3	2

Total ways =5×5×4×3×2 = 600

Now, A number is divisible by 4, if last two digits of the number is divisible by 4

Hence we can have

			0	4	→ first 3 places can be filled in 4×3×2 = 24 ways
			1,3 or 5	2	→ first 4 places can be filled in 3×3×2×3 = 54 ways
			2 or 4	0	→ first 4 places can be filled in 4×3×2×2 = 48 ways
			2	4	→ first 3 places can be filled in 3×3×2 = 18 ways

Total number of ways 144 ways

probability = $\frac{\text{favorable outcomes}}{\text{Total outcomes}} = \frac{144}{600} = \frac{6}{25}$ Ans.

Self practice problems :

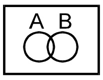
(9) A bag contains 4 white, 3 red and 2 blue balls. A ball is drawn at random. Find the probability of the event the ball drawn is blue or red

(10) In throwing a pair of fair dice find the probability of the events ' a total of of less than or equal to 9"'.

Ans. (9) 5/9 (10) 5/6.

(III) Addition theorem of probability :

If 'A' and 'B' are any two events associated with an experiment, then

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$ 

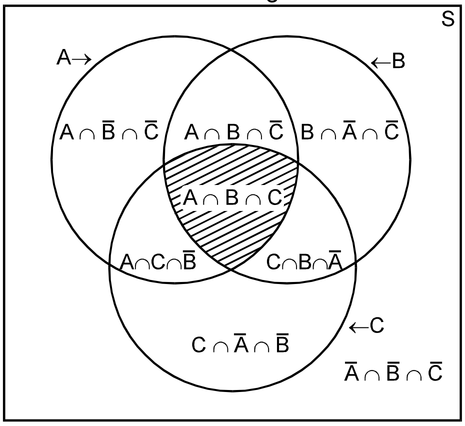
De Morgan's laws : If A & B are two subsets of a universal set U, then

(a) $(A \cup B)^c = A^c \cap B^c$
(b) $(A \cap B)^c = A^c \cup B^c$

Distributive laws :

(a) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
(b) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

For any three events A, B and C we have the figure



(i) P(A or B or C) = $P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$
(ii) P (at least two of A, B, C occur) = $P(B \cap C) + P(C \cap A) + P(A \cap B) - 2P(A \cap B \cap C)$
(iii) P(exactly two of A, B, C occur) = $P(B \cap C) + P(C \cap A) + P(A \cap B) - 3P(A \cap B \cap C)$
(iv) P(exactly one of A, B, C occur) =
$P(A) + P(B) + P(C) - 2P(B \cap C) - 2P(C \cap A) - 2P(A \cap B) + 3P(A \cap B \cap C)$

Example # 14 : A bag contains 4 white, 3red and 4 green balls. A ball is drawn at random. Find the probability of the event 'the ball drawn is white or green'.
Solution : Let A be the event 'the ball drawn is white' and B be the event 'the ball drawn is green'.
P(The ball drawn is white or green) = $P(A \cup B) = P(A) + P(B) - P(A \cap B) = \frac{8}{11}$

Example # 15 : In throwing of a die, let A be the event 'an odd number turns up', B be the event 'a number divisible by 3 turns up' and C be the event 'a number ≤ 4 turns up'. Then find the probability that atleast one of A, B, C occur.
Solution : Event A = {1, 3, 5}, event B = {3, 6} and event C = {1, 2, 3, 4}
∴ $A \cap B = \{3\}$, $B \cap C = \{3\}$, $A \cap C = \{1, 3\}$ and $A \cap B \cap C = \{3\}$.
Thus
P(A or B or C) = $P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$
$$= \frac{3}{6} + \frac{2}{6} + \frac{4}{6} - \frac{1}{6} - \frac{1}{6} - \frac{2}{6} + \frac{1}{6} = 1$$

Self practice problems :

(11) In throwing of a die, let A be the event 'an odd number turns up', B be the event 'a number divisible by 3 turns up' and C be the event 'a number ≤ 4 turns up'. Then find the probability that atleast two of A, B and C occur.

(12) In the problem number 11, find the probability that exactly one of A, B and C occurs.

Ans. (11) $\frac{1}{3}$ (12) $\frac{2}{3}$

(IV) Conditional probability

If A and B are two events, then $P(A/B) = \frac{P(A \cap B)}{P(B)}$.

Note that for mutually exclusive events P(A/B) = 0.

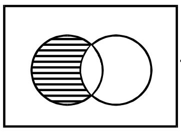
Example # 16 : If P(A/B) = 0.2 and P(B) = 0.5 and P(A) = 0.2. Find $P(A \cap \bar{B})$.
Solution : $P(A \cap \bar{B}) = P(A) - P(A \cap B)$
Also $P(A/B) = \frac{P(A \cap B)}{P(B)}$ ⇒ $P(A \cap B) = 0.1$
From given data,
$P(A \cap \bar{B}) = 0.1$

Example # 17 : If P(A) = 0.25, P(B) = 0.5 and $P(A \cap B) = 0.14$, find probability that neither 'A' nor 'B' occurs. Also find $P\left(A \cap \bar{B}\right)$

Solution : We have to find $P\left(\bar{A} \cap \bar{B}\right) = 1 - P(A \cup B)$ (by De-Morgan's law)

Also, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

putting data we get, $P\left(\bar{A} \cap \bar{B}\right) = 0.39$



The shaded region denotes the simultaneous occurrence of A and $\bar{B}$

Hence $P\left(A \cap \bar{B}\right) = P(A) - P(A \cap B) = 0.11$

Self practice problem :

(13) If $P(\bar{A}/\bar{B}) = 0.3$, $P(A \cup B) = 0.8$, then find $P(A \cap \bar{B})$?

Ans. $\frac{7}{15}$

(V) Independent and dependent events

If two events are such that occurence or non-occurence of one does not affect the chances of occurence or non-occurence of the other event, then the events are said to be independent. Mathematically : if $P(A \cap B) = P(A)\,P(B)$, then A and B are independent.

Note: (i) If A and B are independent, then
(a) A′ and B′ are independent,
(b) A and B′ are independent and
(c) A′ and B are independent.

(ii) If A and B are independent, then P(A / B) = P(A).

If events are not independent then they are said to be dependent.

Independency of three or more events

Three events A, B & C are independent if & only if all the following conditions hold :

$P(A \cap B) = P(A) \,.\, P(B)$; $P(B \cap C) = P(B) \,.\, P(C)$

$P(C \cap A) = P(C) \,.\, P(A)$; $P(A \cap B \cap C) = P(A) \,.\, P(B) \,.\, P(C)$

Example # 18 : A pair of fair coins is tossed yielding the equiprobable space S = {HH, HT, TH, TT}. Consider the events:

A = {head on first coin} = {HH, HT}, B = {head on second coin} = {HH, TH}
C = {head on exactly one coin} = {HT, TH}
Then check whether A, B, C are independent or not.

Solution : $P(A) = P(B) = P(C) = \frac{2}{4} = \frac{1}{2}$.

Also $P(A \cap B) = \frac{1}{4} = P(A)\,P(B)$, $P(A \cap C) = \frac{1}{4} = P(A)\,P(C)$, $P(B \cap C) = \frac{1}{4} = P(B)\,P(C)$

but $P(A \cap B \cap C) = 0 \neq P(A)\,P(B)\,P(C)$

$\therefore$ A, B & C are not independent

Example # 19 : In drawing two balls from a box containing 7 red and 4 white balls without replacement, which of the following pairs is independent ?
(a) Red on first draw and red on second draw
(b) Red on first draw and white on second draw

Solution : Let E be the event 'Red on first draw', F be the event 'Red on second draw' and G be the event 'white on second draw'.

$P(E) = \frac{7}{11}$, $P(F) = \frac{7}{11}$, $P(G) = \frac{4}{11}$

(a) $P(E \cap F) = \frac{^7P_2}{^{11}P_2} = \frac{21}{55}$

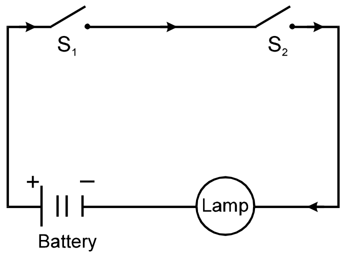
$P(E) . P(F) = \frac{7}{11} \times \frac{7}{11} = \frac{49}{121} \neq \frac{21}{55}$ $\Rightarrow$ E and F are not independent

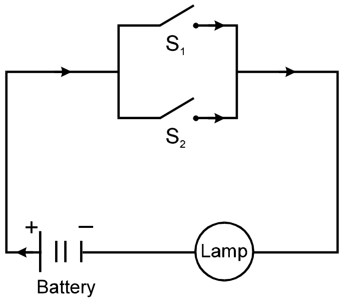
(b) $P(E) . P(G) = \frac{7}{11} \times \frac{4}{11} = \frac{28}{121}$

$P(E \cap G) = \frac{^7P_1 \times ^4P_1}{^{11}P_2} = \frac{14}{55}$

$\therefore$ $P(E) . P(G) \neq P(E \cap G)$ $\Rightarrow$ E and G are not independent

Example # 20 : If two switches S_1 and S_2 have respectively 90% and 80% chances of working. Find the probabilities that each of the following circuits will work.





Solution : Consider the following events :

A = Switch S_1 works,

B = Switch S_2 works,

We have,

$P(A) = \frac{90}{100} = \frac{9}{10}$ and $P(B) = \frac{80}{100} = \frac{8}{10}$

(i) The circuit will work if the current flows in the circuit. This is possible only when both the switches work together. Therefore, Required probability

$= P(A \cap B) = P(A)\,P(B)$ [$\because$ A and B are independent events]

$= \frac{9}{10} \times \frac{8}{10} = \frac{72}{100} = \frac{18}{25}$

(ii) The circuit will work if the current flows in the circuit. This is possible only when at least one of the two switches S_1, S_2 works. Therefore,

Required Probability

$= P(A \cup B) = 1 - P(\bar{A})\,P(\bar{B})$ [$\because$ A, Bare independent events]

$= 1 - \left(1 - \frac{9}{10}\right)\left(1 - \frac{8}{10}\right) = 1 - \frac{1}{10} \times \frac{2}{10} = \frac{49}{50}$

Example # 21 : A speaks truth in 60% of the cases and B in 90% of the cases. In what percentage of cases are they likely to contradict each other in stating the same fact?

Solution : Let E be the event that A speaks truth and F be the event that B speaks truth. Then E and F are independent events such that

$P(E) = \frac{60}{100} = \frac{3}{5}$ and $P(F) = \frac{90}{100} = \frac{9}{10}$

A and B will contradict each other in narrating the same fact in the following mutually exclusive ways:

(i) A speaks truth and B tells a lie i.e. $E \cap \bar{F}$

(ii) A tells a lie and B speaks truth lie i.e. $\bar{E} \cap F$

$\therefore$ P(A and B contradict each other)

$= P(I \text{ or } II) = (I \cup II) = P[(E \cap \bar{F}) \cup (\bar{E} \cap F)]$

$= P(E \cap \bar{F}) + P(\bar{E} \cap F)$ [$\because E \cap \bar{F}$ and $\bar{E} \cap F$ are mutually exclusive]

$= \quad P(E)\,P(\bar{F}) + P(\bar{E})\,P(F) \qquad [\because$ E and F are independent$]$

$= \quad \frac{3}{5} \times \left(1-\frac{9}{10}\right) + \left(1-\frac{3}{5}\right) \times \frac{9}{10} = \frac{3}{5} \times \frac{1}{10} + \frac{2}{5} \times \frac{9}{10} = \frac{21}{50}$

Example # 22 : A box contains 5 bulbs of which two are defective. Test is carried on bulbs one by one untill the two defective bulbs are found out. Find the probability that the process stops after
(i) Second test (ii) Third test

Solution : (i) Process will stop after second test. Only if the first and second bulb are both found to be defective

probability $= \frac{2}{5} \times \frac{1}{4} = \frac{1}{10}$ (Obviously the bulbs drawn are not kept back.)

(ii) Process will stop after third test when either

(a) DND $\rightarrow \frac{2}{5} \times \frac{3}{4} \times \frac{1}{3} = \frac{1}{10}$ Here 'D' stands for defective

or (b) NDD $\rightarrow \frac{3}{5} \times \frac{2}{4} \times \frac{1}{3} = \frac{1}{10}$ and 'N' is for not defective.

or (c) NNN $\frac{3}{5} \times \frac{2}{4} \times \frac{1}{3} = \frac{1}{10}$

hence required probability $= \frac{3}{10}$

Example # 23 : If E_1 and E_2 are two events such that $P(E_1) = \frac{1}{4}$; $P(E_2) = \frac{1}{2}$; $P\left(\frac{E_1}{E_2}\right) = \frac{1}{4}$, then choose the correct options.

(i) E_1 and E_2 are independent (ii) E_1 and E_2 are exhaustive

(iii) E_1 and E_2 are mutually exclusive (iv) E_1 & E_2 are dependent

Also find $P\left(\frac{\bar{E}_1}{E_2}\right)$ and $\left(\frac{E_2}{\bar{E}_1}\right)$

Solution : Since $P\left(\frac{E_1}{E_2}\right) = P(E_1) \Rightarrow$ E_1 and E_2 are independent of each other

Also since $P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1) \cdot P(E_2) \neq 1$

Hence events are not exhaustive. Independent events can't be mutually exclusive.

Hence only (i) is correct

Further since E_1 & E_2 are independent; E1 and $\bar{E}_2$ or $\bar{E}_1$, E_2 are $\bar{E}_1, \bar{E}_2$ are also independent.

Hence $P\left(\frac{\bar{E}_1}{E_2}\right) = P\left(\bar{E}_1\right) = \frac{3}{4}$ and $P\left(\frac{E_2}{\bar{E}_1}\right) = P(E_2) = \frac{1}{2}$

Example # 24 : If cards are drawn one by one from a well shuffled pack of 52 cards without replacement, until an ace appears, find the probability that the fourth card is the first ace to appear.

Solution : Probability of selecting 3 non-Ace and 1 Ace out of 52 cards is equal to $\frac{{}^{48}C_3 \times {}^4C_1}{{}^{52}C_4}$

Since we want 4th card to be first ace, we will also have to consider the arrangement, Now 4 cards in sample space can be arranged in 4! ways and, favorable they can be arranged in 3 ! ways as we want 4th position to be occupied by ace

Hence required probability $= \frac{{}^{48}C_3 \times {}^4C_1}{{}^{52}C_4} \times \frac{3\,!}{4\,!}$

Aliter :
'NNNA' is the arrangement then we desire in taking out cards, one by one
Hence required chance is $\frac{48}{52} \times \frac{47}{51} \times \frac{46}{50} \times \frac{4}{49}$

Self practice problems :

(14) An urn contains 7 red and 4 blue balls. Two balls are drawn at random with replacement. Find the probability of getting
(i) 2 red balls (ii) 2 blue balls (iii) one red and one blue ball

(15) Probabilities of solving a specific problem independently by A and B are $\frac{1}{2}$ and $\frac{1}{3}$ respectively. If both try to solve the problem independently, find the probability that
(i) the problem is solved (ii) exactly one of them solves the problem.

(16) In throwing a pair of dies find the probability of getting an odd number on the first die and a total of 7 on both the dies.

(17) In throwing of a pair of dies, find the probability of getting a doublet or a total of 4.

(18) A bag contains 8 marbles of which 3 are blue and 5 are red. One marble is drawn at random, its colour is noted and the marble is replaced in the bag. A marble is again drawn from the bag and its colour is noted. Find the probability that the marbles will be
(i) blue followed by red (ii) blue and red in any order (iii)of the same colour.

(19) A coin is tossed thrice. In which of the following cases are the events E and F independent ?
(i) E : "the first throw results in head".
F : "the last throw result in tail".
(ii) E : "the number of heads is two".
F : "the last throw result in head".
(iii) E : "the number of heads is odd ".
F : "the number of tails is odd".

Ans. (14) (i) $\frac{49}{121}$ (ii) $\frac{16}{121}$ (iii) $\frac{56}{121}$ (15) (i) $\frac{2}{3}$ (ii) $\frac{1}{2}$

(16) $\frac{1}{12}$ (17) $\frac{2}{9}$ (18) (i) $\frac{15}{64}$ (ii) $\frac{15}{32}$ (iii) $\frac{17}{32}$

(19) (i)

(VI) Total probability theorem

If an event A can occur with one of the n mutually exclusive and exhaustive events $B_1, B_2, \ldots, B_n$ and the probabilities $P(A/B_1), P(A/B_2) \ldots P(A/B_n)$ are known, then

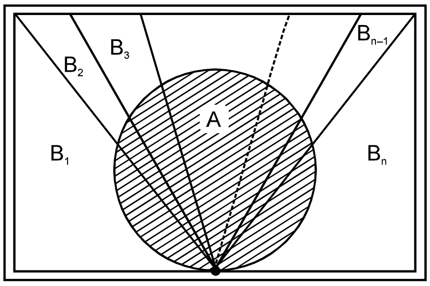
$$P(A) = \sum_{i=1}^{n} P(B_i) \; . \; P(A/B_i)$$

Proof :
The event A occurs with one of the n mutually exclusive and exhaustive events
$B_1, B_2, B_3, \ldots\ldots, B_n$
$A = (A \cap B_1) \cup (A \cap B_2) \cup (A \cap B_3) \cup \ldots\ldots \cup (A \cap B_n)$

$$P(A) = P(A \cap B_1) + P(A \cap B_2) + \ldots\ldots + P(A \cap B_n) = \sum_{i=1}^{n} P(A \cap B_i)$$

Now,
$P(A \cap B_i) = P(A) \cdot P(B_i/A) = P(B_i) \cdot P(A/B_i)$

$$\therefore \quad P(A) = \sum_{i=1}^{n} P(B_i) \; . \; P(A/B_i)$$



Example # 25 : Box - I contains 6 red and 3 white balls while box - II contains 4 red and 3 white balls. A fair die is thrown. If it turns up a multiple of 3, a ball is drawn from box - I else a ball is drawn from box - II. Find the probability that the ball drawn is white.

Solution : Let A be the event 'a multiple of 3 turns up on the die' and R be the event 'the ball drawn is white'

then P (ball drawn is white)

$$= P(A) \,.\, P(R / A) + P(\bar{A})\ P(R / (\bar{A})\ ) = \frac{2}{6} \times \frac{3}{9} + \left(1 - \frac{2}{6}\right)\frac{3}{7} = \frac{25}{63}$$

Example # 26 : Cards of an ordinary deck of playing cards are placed into two heaps. Heap - I consists of all the red cards and heap - II consists of all the black cards. A heap is chosen at random and a card is drawn, find the probability that the card drawn is a king.

Solution : Let I and II be the events that heap - I and heap - II are choosen respectively. Then

$P(\text{I}) = P(\text{II}) = \frac{1}{2}$

Let K be the event 'the card drawn is a king'

$\therefore$ $P(K / \text{I}) = \frac{2}{26}$ and $P(K / \text{II}) = \frac{2}{26}$

$\therefore$ $P(K) = P(\text{I})\, P(K / \text{I}) + P(\text{II})\, P(K / \text{II}) = \frac{1}{2} \times \frac{2}{26} + \frac{1}{2} \times \frac{2}{26} = \frac{1}{13}$.

Self practice problems :

(20) Box - I contains 3 red and 6 blue balls while box - II contains 5 red and 4 blue balls. A fair coin is tossed. If it turns up head, a ball is drawn from box - I, else a ball is drawn from box - II . Find the probability that the ball drawn is red.

(21) There are 5 brilliant students in class XI and 8 brilliant students in class XII. Each class has 50 students. The odds in favour of choosing the class XI are 2 : 3. If the class XI is not chosen then the class XII is chosen. Find the probability of selecting a brilliant student.

Ans. (20) $\frac{4}{9}$ (21) $\frac{17}{125}$.

(VII) Bayes' theorem :

If an event A can occur with one of the n mutually exclusive and exhaustive events $B_1, B_2, \ldots, B_n$ and the probabilities $P(A/B_1), P(A/B_2) \ldots P(A/B_n)$ are known, then $P(B_i / A) = \dfrac{P(B_i)\ .\ P(A/B_i)}{\sum\limits_{i=1}^{n} P(B_i)\ .\ P(A/B_i)}$

Proof :

The event A occurs with one of the n mutually exclusive and exhaustive events

$B_1, B_2, B_3, \ldots, B_n$

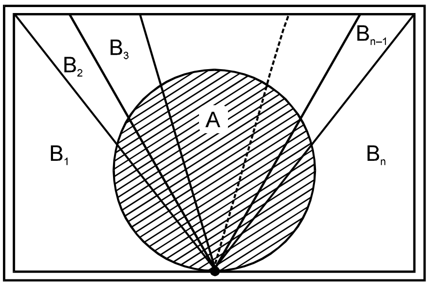
$A = (A \cap B_1) \cup (A \cap B_2) \cup (A \cap B_3) \cup \ldots \cup (A \cap B_n)$

$P(A) = P(A \cap B_1) + P(A \cap B_2) + \ldots + P(A \cap B_n) = \sum\limits_{i=1}^{n} P(A \cap B_i)$

Now, $P(A \cap B_i) = P(A) \, . \, P(B_i/A) = P(B_i) \, . \, P(A/B_i)$

$$P(B_i/A) = \frac{P(B_i) \; . \; P(A/B_i)}{P(A)} = \frac{P(B_i) \; . \; P(A/B_i)}{\sum_{i=1}^{n} P(A \cap B_i)}$$

$$P(B_i/A) = \frac{P(B_i) \; . \; P(A/B_i)}{\sum P(B_i) \; . \; P(A/B_i)}$$



Example # 27 : Pal's gardener is not dependable, the probability that he will forget to water the rose bush is $\frac{2}{3}$. The rose bush is in questionable condition any how, if watered the probability of its withering is $\frac{1}{2}$, if not watered, the probability of its withering is $\frac{3}{4}$. Pal went out of station and upon returning, he finds that the rose bush has withered, what is the probability that the gardener did not water the bush.

[Here result is known that the rose bush has withered, therefore. Bayes's theorem should be used]

Solution : Let A = the event that the rose bush has withered

Let A_1 = the event that the gardener did not water.

A_2 = the event that the gardener watered.

By Bayes's theorem required probability,

$$P(A_1/A) = \frac{P(A_1) \; . \; P(A/A_1)}{P(A_1) \; . \; P(A/A_1) + P(A_2) \; . \; P(A/A_2)} \qquad .....(i)$$

Given, $P(A_1) = \frac{2}{3}$ $\quad \therefore \quad$ $P(A_2) = \frac{1}{3}$

$P(A/A_1) = \frac{3}{4}$, $P(A/A_2) = \frac{1}{2}$ From (1), $P(A_1/A) = \dfrac{\frac{2}{3} \, . \, \frac{3}{4}}{\frac{2}{3} \, . \, \frac{3}{4} + \frac{1}{3} \, . \, \frac{1}{2}} = \frac{6}{6+2} = \frac{3}{4}$

Example # 28 : There are 5 brilliant students in class XI and 8 brilliant students in class XII. Each class has 50 students. The odds in favour of choosing the class XI are 2 : 3. If the class XI is not chosen then the class XII is chosen. A student is chosen and is found to be brilliant, find the probability that the chosen student is from class XI.

Solution : Let E and F be the events 'Class XI is chosen' and 'Class XII is chosen' respectively.

Then $P(E) = \frac{2}{5}$, $P(F) = \frac{3}{5}$

Let A be the event 'Student chosen is brilliant'. Then $P(A / E) = \frac{5}{50}$ and $P(A / F) = \frac{8}{50}$.

$\therefore$ $P(A) = P(E) \, . \, P(A / E) + P(F) \, . \, P(A / F) = \frac{2}{5} \, . \, \frac{5}{50} + \frac{3}{5} \, . \, \frac{8}{50} = \frac{34}{250}$.

$\therefore$ $P(E / A) = \dfrac{P(E) \; . \; P(A/E)}{P(E) \; . \; P(A/E) + P(F) \; . \; P(A/F)} = \frac{5}{17}$.

Example # 29 : A pack of cards is counted with face downwards. It is found that one card is missing. One card is drawn and is found to be red. Find the probability that the missing card is red.

Solution : Let A be the event of drawing a red card when one card is drawn out of 51 cards (excluding missing card.) Let A_1 be the event that the missing card is red and A_2 be the event that the missing card is black.

Now by Bayes's theorem, required probability,

$$P(A_1/A) = \frac{P(A_1)\ .\ (P(A/A_1)}{P(A_1)\ .\ P(A/A_1)+P(A_2)\ .\ P(A/A_2)} \qquad ....(i)$$

In a pack of 52 cards 26 are red and 26 are black.

Now $P(A_1)$ = probability that the missing card is red = $\frac{^{26}C_1}{^{52}C_1} = \frac{26}{52} = \frac{1}{2}$

$P(A_2)$ = probability that the missing card is black = $\frac{26}{52} = \frac{1}{2}$

$P(A/A_1)$ = probability of drawing a red card when the missing card is red.

$= \frac{25}{51}$

[$\because$Total number of cards left is 51 out of which 25 are red and 26 are black as the missing card is red]

Again $P(A/A_2)$ = Probability of drawing a red card when the missing card is black = $\frac{26}{51}$

Now from (i), required probability, $P(A_1/A) = \dfrac{\frac{1}{2}\ .\ \frac{25}{51}}{\frac{1}{2}\ .\ \frac{25}{51}+\frac{1}{2}\ .\ \frac{26}{51}} = \frac{25}{51}$

Example # 30 : A bag contains 6 white and an unknown number of black balls (≤ 3). Balls are drawn one by one with replacement from this bag twice and is found to be white on both occassion. Find the probability that the bag had exactly '3' Black balls.

Solution : A priori, we can think of the following possibilies

(i)	E_1	6W	,	0 B
(ii)	E_2	6W	,	1 B
(iii)	E_3	6W	,	2 B
(iv)	E_4	6W	,	3 B

Clearly $P(E_1) = P(E_2) = P(E_3) = P(E_4) = \frac{1}{4}$

Let 'A' be the event that two balls drawn one by one with replacement are both white therefore we have to find $P\left(\frac{E_4}{A}\right)$

By Baye's theorem $P\left(\frac{E_4}{A}\right) = \dfrac{P\left(\frac{A}{E_4}\right)\times P(E_4)}{P\left(\frac{A}{E_1}\right)\times P(E_1)+P\left(\frac{A}{E_2}\right).P(E_2)+P\left(\frac{A}{E_3}\right).P(E_3)+P\left(\frac{A}{E_4}\right).P(E_4)}$

$P\left(\frac{A}{E_4}\right) = \frac{6}{9}\times\frac{6}{9}$; $\quad P\left(\frac{A}{E_3}\right) = \frac{6}{8}\times\frac{6}{8}$; $\quad P\left(\frac{A}{E_2}\right) = \frac{6}{7}\times\frac{6}{7}$; $\quad P\left(\frac{A}{E_1}\right) = \frac{6}{6}\times\frac{6}{6}$;

Putting values $P\left(\frac{E_4}{A}\right) = \dfrac{\frac{1}{81}\times\frac{1}{4}}{\frac{1}{4}\left[\frac{1}{81}+\frac{1}{64}+\frac{1}{49}+\frac{1}{36}\right]}$

Self practice problems :

(22) Box-I contains 3 red and 2 blue balls while box-II contains 2 red and 3 blue balls. A fair coin is tossed. If it turns up head, a ball is drawn from box-I, else a ball is drawn from box-II. If the ball drawn is red, then find the probability that the ball is drawn from box-II.

(23) Cards of an ordinary deck of playing cards are placed into two heaps. Heap - I consists of all the red cards and heap - II consists of all the black cards. A heap is chosen at random and a card is drawn, if the card drawn is found to be a king, find the probability that the card drawn is from the heap - II.

Ans. (22) $\frac{2}{5}$ (23) $\frac{1}{2}$

(VIII) Binomial probability theorem :

If an experiment is such that the probability of success or failure does not change with trials, then the probability of getting exactly r success in n trials of such an experiment is ${}^nC_r\, p^r\, q^{n-r}$, where 'p' is the probability of a success and q is the probability of a failure in one particular experiment. Note that $p + q = 1$.

Example 31 : A pair of dice is thrown 7 times. Find the probability of getting a doublet thrice

Solution : In a single throw of a pair of dice probability of getting a doublet is $\frac{1}{6}$

considering it to be a success, $p = \frac{1}{6}$

$\therefore \quad q = 1 - \frac{1}{6} = \frac{5}{6}$

number of success r = 3

$\therefore \quad P(r = 3) = {}^7C_3\, p^3\, q^4 = 35 \left(\frac{1}{6}\right)^3 \left(\frac{5}{6}\right)^4$

Example # 32 : A pair of dice is thrown 4 times. If getting 'a total of 7' in a single throw is considered as a success then find the probability of getting 'a total of 7' thrice.

Solution : p = probability of getting 'a total of 7' $= \frac{6}{36} = \frac{1}{6}$

$\therefore \quad q = 1 - \frac{1}{6} = \frac{5}{6}$

r = 3, n = 4

$\therefore \quad P(r = 3) = {}^4C_3\, p^3\, q = 4 \left(\frac{1}{6}\right)^3 \left(\frac{5}{6}\right) = \frac{5}{324}$

Example # 33 : In an examination of 10 multiple choice questions (1 or more can be correct out of 3 options). A student decides to mark the answers at random. Find the probability that he gets exactly two questions correct.

Solution : A student can mark 7 different answers to a MCQ with 3 option i.e. ${}^3C_1 + {}^3C_2 + {}^3C_3 = 7$

Hence if he marks the answer at random, chance that his answer is correct $= \frac{1}{7}$ and being incorrecting $\frac{6}{7}$ Thus $\quad p = \frac{1}{7}, q = \frac{6}{7}$

$P\text{ (2 success)} = {}^{10}C_2 \times \left(\frac{1}{7}\right)^2 \times \left(\frac{6}{7}\right)^8$

Example # 34 : A family has three children. Event 'A' is that family has at most one boy, Event 'B' is that family has at least one boy and one girl, Event 'C' is that the family has at most one girl. Find whether events 'A' and 'B' are independent. Also find whether A, B, C are independent or not.

Solution : A family of three children can have

(i) All 3 boys (ii) 2 boys + 1 girl (iii) 1 boy + 2 girls (iv) 3 girls

(i) P (3 boys) = ${}^3C_0 \left(\frac{1}{2}\right)^3 = \frac{1}{8}$ (Since each child is equally likely to be a boy or a girl)

(ii) P (2 boys +1girl) = ${}^3C_1 \times \left(\frac{1}{2}\right)^2 \times \frac{1}{2} = \frac{3}{8}$

(Note that there are three cases BBG, BGB, GBB)

(iii) P (1 boy + 2 girls) = ${}^3C_2 \times \left(\frac{1}{2}\right)^1 \times \left(\frac{1}{2}\right)^2 = \frac{3}{8}$

(iv) P (3 girls) = $\frac{1}{8}$

Event 'A' is associated with (iii) & (iv). Hence P(A) = $\frac{1}{2}$

Event 'B' is associated with (ii) & (iii). Hence P(B) = $\frac{3}{4}$

Event 'C' is associated with (i) & (ii). Hence P(C) = $\frac{1}{2}$

$P(A \cap B) = P(iii) = \frac{3}{8} = P(A) . P(B)$. Hence A and B are independent of each other

$P(A \cap C) = 0 \neq P(A) . P(C)$. Hence A, B, C are not independent

Self practice problems :

(24) A box contains 4 red and 5 blue balls. Two balls are drawn successively without replacement. If getting 'a red ball on first draw and a blue ball on second draw' is considered a success, then find the probability of 2 successes in 3 performances.

(25) Probability that a bulb produced by a factory will fuse after an year of use is 0.2. Find the probability that out of 5 such bulbs exectly 2 bulb will fuse after an year of use.

Ans. (24) $\frac{325}{1944}$ (25) $\frac{640}{3125}$

(IX) Probability distribution :

A probability distribution spells out how a total probability of 1 is distributed over several values of a random variable (i.e. how .... possibilities)

(X) Expectation :

If there are n possibilities $A_1, A_2, ... A_n$ in an experiment having the probabilities $p_1, p_2, ...p_n$ respectively. If value $M_1, M_2, ..., M_n$ are associated with the respective possibility. Then the expected value of the experiment is given by $\sum_{i=1}^{n} p_i \;.\; M_i$

Note: (i) Mean of any probability distribution of a random variable is given by :

$$\mu = \frac{\Sigma\ p_i\ x_i}{\Sigma\ p_i} = \Sigma\, p_i\, x_i \qquad (\text{Since } \Sigma\, p_i = 1)$$

Its also known as expectation.

(ii) Variance of a random variable is given by, $\sigma^2 = \Sigma\,(x_i - \mu)^2 . p_i$

$\therefore \quad \sigma^2 = \Sigma\, p_i\, x_i^2 - \mu^2$ (Note that SD = $+\sqrt{\sigma^2}$)

(iii) The probability distribution for a binomial variate 'X' is given by :
$P(X = r) = {}^nC_r\, p^r\, q^{n-r}$ where $P(X = r)$ is the probability of r successes.
The recurrence formula $\frac{P(r+1)}{P(r)} = \frac{n-r}{r+1} \cdot \frac{p}{q}$, is very helpful for quickly computing $P(1)$. $P(2)$. $P(3)$ etc. if $P(0)$ is known.
Mean of Binomial Probability Distribution = np ; variance of Binomial Probability Distribution = npq.

(iv) If p represents a person's chance of success in any venture and 'M' the sum of money which he will receive in case of success, then his expectations or probable value = pM

Example # 35 : A random variable X has the following probability distribution :

X	0	1	2	3	4	5	6	7
P(X)	0	2k	3k	k	0	k^2+k	$2k^2$	$5k^2$

Determine
(i) k (ii) $P(X < 3)$ (iii) $P(X \geq 6)$ (iv) $P(0 < X \leq 3)$

Hints : Use $\sum P(X) = 1$ to determine k, $P(X < 3) = P(0) + P(1) + P(2)$, $P(X \geq 6) = P(6) + (P(7)$,
$P(0 < X \leq 3) = P(0) + P(1) + P(2) + P(3)$

Ans. (i) $\frac{1}{8}$ (ii) $\frac{5}{8}$ (iii) $\frac{7}{64}$ (iv) $\frac{3}{4}$

Example # 36 : A fair die is tossed. If 2, 3 or 5 occurs, the player wins that number of rupees, but if 1, 4, or 6 occurs, the player loses that number of rupees. First complete probability distribution table. Hence find expectation.

Solution :

A_i	2	3	5	1	4	6
M_i	2	3	5	–1	–4	–6
P_i	1/6	1/6	1/6	1/6	1/6	1/6

Then expected value E of the game payoffs for the player

$$= 2\left(\frac{1}{6}\right) + 3\left(\frac{1}{6}\right) + 5\left(\frac{1}{6}\right) - 1\left(\frac{1}{6}\right) - 4\left(\frac{1}{6}\right) - 6\left(\frac{1}{6}\right) = -\left(\frac{1}{6}\right)$$

Since E is negative therefore game is unfavorable to the player.

Example # 37 : There are 50 tickets in a raffle (Lottery). There is 1 prize each of Rs. 800/-, Rs. 300/- and Rs. 200/-. Remaining tickets are blank. Find the expected price of one such ticket.

Solution : Expectation = $\sum p_i M_i$

Probability of 1 ticket = $\frac{1}{50}$

expected price = $800 \times \frac{1}{50} + 300 \times \frac{1}{50} + 200 \times \frac{1}{50} + \frac{47}{50} \times 0 = 26$ Rs.

Example # 38 : A purse contains four coins each of which is either five rupees or two rupees coin. Find the expected value of a coin in that purse.

Solution : Various possibilities of coins in the purse can be

$$\left.\begin{array}{cc} 5\text{Rs.} & 2\text{Rs.} \\ 0 & 4 \\ 1 & 3 \\ 2 & 2 \\ 3 & 1 \\ 4 & 0 \end{array}\right\} \text{equally likely } 1/5$$

expected value per coin

$$= \frac{1}{5}[5\times 0+4\times 2] + \frac{1}{5}[5\times 1+3\times 2] + \frac{1}{5}[5\times 2+2\times 2] + \frac{1}{5}[5\times 3+2\times 1]\ \frac{1}{5}[5\times 4+2\times 0] = 14$$

Example # 39 : A pair of dice is thrown 7 times. If getting a doublet is considered as a success, then find the mean and variance of successes.

Solution : In a single throw of a pair of dice, probability of getting a doublet = $\frac{1}{6}$

considering it to be a success, p = $\frac{1}{6}$ $\quad\therefore\quad$ q = 1 – $\frac{1}{6}$ = $\frac{5}{6}$

mean = 7 × $\frac{1}{6}$ = $\frac{7}{6}$, $\quad$ variance = 7 × $\frac{1}{6}$ × $\frac{5}{6}$ = $\frac{35}{36}$

Example # 40 : A pair of dice is thrown 4 times. If getting a total of 7 in a single throw is considered as a success then find the mean and variance of successes.

Solution : p = probability of getting a total of 7 = $\frac{6}{36} = \frac{1}{6}$

$\therefore \quad q = 1 - \frac{1}{6} = \frac{5}{6}$

$\therefore \quad$ mean = np = 4 × $\frac{1}{6}$ = $\frac{2}{3}$

variance = npq = 4 × $\frac{1}{6}$ × $\frac{5}{6}$ = $\frac{5}{9}$

Example # 41 : Difference between mean and variance of a Binomial variate is '1' and difference between their squares is '11'. Find the probability of getting exactly three success

Solution : Mean = np & variance = npq

therefore, $\quad np - npq = 1$ ....(i)

$n^2p^2 - n^2p^2q^2 = 11$ ....(ii)

Also, we know that p + q = 1 ....(iii)

Divide equation (ii) by square of (i) and solve, we get, q = $\frac{5}{6}$, p = $\frac{1}{6}$ & n = 36

Hence probability of '3' success = ${}^{36}C_3 \times \left(\frac{1}{6}\right)^3 \times \left(\frac{5}{6}\right)^{33}$

Self practice problems :

(26) From a bag containing 2 one rupee and 3 two rupee coins a person is allowed to draw 2 coins simultaneously ; find the value of his expectation.

(27) A box contains 2 red and 3 blue balls. Two balls are drawn successively without replacement. If getting 'a red ball on first draw and a blue ball on second draw' is considered a success, then find the mean and variance of successes.

(28) Probability that a bulb produced by a factory will fuse after an year of use is 0.2. If fusing of a bulb is considered an failure, find the mean and variance of successes for a sample of 10 bulbs.

(29) A random variable X is specified by the following distribution law :

X	2	3	4
P(X = x)	0.3	0.4	0.3

Find the variance of this distribution.

Ans. (26) Rs. 3.20 $\quad$ (27) mean = 2.1, σ^2 = .63 $\quad$ (28) mean = 8 and variance = 1.6

(29) 0.6

Exercise-1

✍ **Marked questions are recommended for Revision.**

PART - I : SUBJECTIVE QUESTIONS

Section (A) : Problems based on Classical definition of Probability (PRCD)

A-1. Write the sample space of the following experiment

(i) 'Three coins are tossed'.

(ii) 'Selection of two children from a group of 3 boys and 2 girls without replacement'.

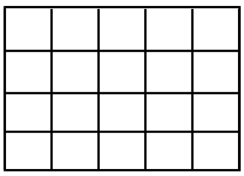
A-2. There are three events A, B, C, one of which must, and only one can, happen; the odds are 8 to 3 against A, 5 to 2 against B : find the odds against C.

A-3. If the letters of the word BANANA are arranged randomly, then find the probability that the word thus formed does not contain the pattern BAN.

A-4. Nine cards are labelled 0, 1, 2, 3, 4, 5, 6, 7, 8. Two cards are drawn at random and put on a table in a sucessive order, and then the resulting number is read say 07(seven), 42(fourty two) and so on. Find the probability that the number is even.

A-5. Three persons A, B and C speak at a function along with 5 other persons. If the persons speak at random, find the probability that A speaks before B and B speaks before C

A-6. (i) A rectangle is randomly selected from the grid of equally spaced squares as shown.



Find the probability that the rectangle is a square.

(ii) Three of the six vertices of a regular hexagon are chosen at random. Then the probability that the triangle with three vertices is equilateral is 'p' then 100p equals

A-7. In throwing a pair of dice, find whether the two events
(i) E_1 : 'coming up of an odd number on first dice' and E_2 : 'coming up of a total of 8'.
(ii) E_1 : 'coming up of 4 on first dice' and E_2 : 'coming up of 5 on the second dice'.
are mutually exclusive or not

A-8. In throwing of a pair of dice, find the probability of the event : total is 'not 8' and 'not 11'.

A-9.✍ Before a race the chance of three runners, A, B, C were estimated to be proportional to 5, 3, 2, but during the race A meets with an accident which reduces his chance to 1/3. What are the respective chance of B and C now?

A-10. Tickets are numbered from 1 to 100. One ticket is picked up at random. Then find the probability that the ticket picked up has a number which is divisible by 5 or 8.

A-11. Three cards are drawn at random from a pack of well shuffled 52 cards. Find the probability that
(i) all the three cards are of the same suit;
(ii) one is a king, the other is a queen and the third a jack.

A-12. In a throw of a pair of dice, then find the probability of 'A total of 8 but not 11'.

A-13. Six boys and six girls sit in a row at random. Find the probability that boys and girls sit alternately

A-14. Four persons draw 4 cards from an ordinary pack find the chance
(1) that a card is of each suit
(2) that no two cards are of equal value.

Section (B) : Problems based on venn diagram & set theory (PRVD)

B-1. Prove that
$P(A - B) = P(A) - P(A \cap B) = P(A \cup B) - P(B) = P(A \cap \bar{B}) = 1 - P(\bar{A} \cup B)$

B-2 If $P(A) = 0.7$ and $P(A \cap B) = 0.5$ find
(i) $P(A - \bar{B})$ (ii) $P(\bar{A} \cup B)$

B-3. If $P(A) = 0.4$, $P(B) = 0.48$ and $P(A \cap B) = 0.16$, then find the value of each of the following :
(i) $P(A \cup B)$ (ii) $P(A' \cap B)$
(iii) $P(A' \cap B')$ (iv) $P((A \cap B') \cup (A' \cap B))$

B-4. There are three clubs A,B,C in a town with 40, 50, 60 members respectively 10 people are members of all the three clubs, 70 members belong to only one club. A member is randomly selected. Find the probability that he has membership of exactly two clubs

Section (C) : Problems based on Conditional Probability/ Total probability & Bayes' theorem

C-1. (i) In a two child family, one child is a boy. What is the probability that the other child is a girl ?

(ii) If the older child is a boy, then probability that the second child is a girl is

C-2. A fair dice is thrown untill a score of less than 5 points is obtained. Find the probability of obtaining not less than 2 points on the last throw.

C-3. A card is drawn from a well shuffled ordinary deck of 52 playing cards. Find the probability that the card drawn is :
(i) A king or a queen (ii) A king or a spade

C-4. The odds against a certain event are 5 to 2, and the odds in favor of another event independent of the former are 6 to 5 : find the chance that one at least of the events will happen.

C-5. A, B, C in order draws a card from a pack of cards, replacing them after each draw, on condition that the first who draws a spade shall win a prize : find their respective chances.

C-6. 6 persons A,B,C,D,E,F are arranged in row. Find the conditional probability that C & D are separated given that A & B are together.

C-7. There are 5 brilliant students in class XI and 8 brilliant students in class XII. Each class has 50 students. The odds in favour of choosing the class XI are 2 : 3. One of the classes is chosen randomly and then a student is randomly selected. Find the probability of selecting a brilliant student.

C-8. Box – I contains 5 red and 2 blue balls while box – II contains 2 red and 6 blue balls. A fair coin is tossed. If it turns up head, a ball is drawn from box–I, else a ball is drawn from box–II. Find the probability of each of the following :
(i) A red ball is drawn
(ii) Ball drawn is from box–I if it is blue

C-9. Two cards are drawn successively from a well-shuffled ordinary deck of 52-playing cards without replacement and is noted that the second card is a king. Find the probability of the event 'first card is also a king'.

C-10. 12 cards, numbered 1 to 12, are placed in a box, mixed up throughly and then a card is drawn at random from the box. If it is known that the number on the drawn card is more than 3, find the probability that it is an even number.

C-11. In a building programme the event that all the materials will be delivered at the correct time is M, and the event that the building programme will be completed on time is F. Given that P(M) = 0.8 & $P(M \cap F) = 0.65$. If P(F) = 0.7, find the probability that the building programme will be completed on time if all the materials are not delivered at the correct time.

Section (D) : Problem based on Binomial Distribution / Expectation / mean & Variance

D-1. South African cricket captain lost the toss of a coin 13 times out of 14. Then find the chance of this happening.

D-2. In an examination of 10 multiple choice questions (1 or more can be correct out of 4 options). A student decides to mark the answers at random. Find the probability that he gets exactly two questions correct. (Assume he attempts all the questions)

D-3. Three cards are drawn successively with replacement from a well shuffled deck of 52 playing cards. If getting a card of spade is considered a success, find the probability distribution of the number of successes.

D-4. A had in his pocket a 100 Rupee and four 10 rupee notes; taking out two notes at random he promises to give them to B and C. What is the worth of C's expectation?

D-5. A box contains 2 red and 3 blue balls. Two balls are drawn successively with replacement. If getting 'a red ball on first draw and a blue ball on second draw' is considered a success, then write the probability distribution of successes. It is given that the above experiment is performed 3-times,

D-6. A coin is tossed 5-times. Find the mean and variance of the probability distribution of appearance of heads on the tosses.

PART - II : ONLY ONE OPTION CORRECT TYPE

Section (A) : Problems based on Classical definition of Probability (PRCD)

A-1. In drawing of a card from a well shuffled ordinary deck of playing cards the events 'card drawn is spade' and 'card drawn is an ace' are
(A) mutually exclusive (B) equally likely
(C) forming an exhaustive system (D) none of these

A-2. A 9 digit number using the digits 1, 2, 3, 4, 5, 6, 7, 8 & 9 is written randomly without repetition. The probability that the number will be divisible by 9 is:
(A) 1/9 (B) 1/2 (C) 1 (D) $9!/9^9$

A-3. Entries of a 2 × 2 determinant are chosen from the set {–1, 1}. The probability that determinant has zero value is
(A) $\frac{1}{4}$ (B) $\frac{1}{3}$ (C) $\frac{1}{2}$ (D) none of these

A-4. A dice is thrown a fixed number of times. If probability of getting even number 3 times is same as the probability of getting even number 4 times, then probability of getting even number exactly once is
(A) $\frac{1}{4}$ (B) $\frac{3}{128}$ (C) $\frac{5}{64}$ (D) $\frac{7}{128}$

A-5. A and B throw with two dice ; if A throws 9, then B's chance of throwing a higher number equals

(A) $\frac{1}{6}$ (B) $\frac{1}{3}$ (C) $\frac{1}{2}$ (D) $\frac{2}{3}$

A-6. If an integer q is chosen at random in the interval $-10 \le q \le 10$, then the probability that the roots of the equation $x^2 + qx + \frac{3q}{4} + 1 = 0$ are real is

(A) $\frac{16}{21}$ (B) $\frac{15}{21}$ (C) $\frac{14}{21}$ (D) $\frac{17}{21}$

A-7. The chance that a 13 card combination from a pack of 52 playing cards is dealt to a player in a game of bridge, in which 9 cards are of the same suit, is

(A) $\frac{4\ .\ ^{13}C_9\ .\ ^{39}C_4}{^{52}C_{13}}$ (B) $\frac{4!\ .\ ^{13}C_9\ .\ ^{39}C_4}{^{52}C_{13}}$ (C) $\frac{^{13}C_9\ .\ ^{39}C_4}{^{52}C_{13}}$ (D) $2.\frac{^{13}C_9\ .\ ^{39}C_4}{^{52}C_{13}}$

A-8. A bag contains 7 tickets marked with the numbers 0, 1, 2, 3, 4, 5, 6 respectively. A ticket is drawn & replaced. Then the chance that after 4 drawings the sum of the numbers drawn is 8 is:

(A) 165/2401 (B) 149/2401 (C) 3/49 (D) 1/49

A-9. A & B having equal skill, are playing a game of best of 5 points. After A has won two points & B has won one point, the probability that A will win the game is:

(A) 1/2 (B) 2/3 (C) 3/4 (D) 2/5

Section (B) : Problems based on venn diagram & set theory (PRVD)

B-1. If two subsets A and B of set S containing n elements are selected at random, then the probability that $A \cap B = \phi$ and $A \cup B = S$ is

(A) $\frac{1}{2}$ (B) $\frac{1}{2^n}$ (C) $\left(\frac{3}{4}\right)^4$ (D) $\frac{1}{3^n}$

B-2. If $P(A) = \frac{3}{5}$ and $P(B) = \frac{2}{3}$ then –

(i) The range of values of $P(A \cap B)$ is

(A) $\left[\frac{2}{5}, \frac{9}{10}\right]$ (B) $\left[\frac{2}{3}, 1\right]$ (C) $\left[0, \frac{1}{3}\right]$ (D) $\left[\frac{4}{15}, \frac{3}{5}\right]$

(ii) The range of values of $P(A \cup B)$ is

(A) $\left[\frac{2}{5}, \frac{9}{10}\right]$ (B) $\left[\frac{2}{3}, 1\right]$ (C) $\left[0, \frac{1}{3}\right]$ (D) $\left[\frac{4}{15}, \frac{3}{5}\right]$

(iii) The range of values of $P(A \cap B')$ is

(A) $\left[\frac{2}{5}, \frac{9}{10}\right]$ (B) $\left[\frac{2}{3}, 1\right]$ (C) $\left[0, \frac{1}{3}\right]$ (D) $\left[\frac{4}{15}, \frac{3}{5}\right]$

B-3 Let X = {1,2, ... , 10}, if set A and B are formed from elements of X the probability that $n(A \cap B) = 2$, is

(A) $\frac{^{10}C_2}{4^{10}}$ (B) $5.\left(\frac{3}{4}\right)^{10}$ (C) $\left(\frac{3}{4}\right)^{10}$ (D) $\frac{3^8}{4^{10}}$

B-4 If probability that exactly one of events A, B, C occurs, is 0.6 and probability that none of A,B,C occur is 0.2, then probability that atleast two of A,B,C occur is

(A) 0.6 (B) 0.4 (C) 0.8 (D) 0.2

Section (C) : Problems based on Conditional Probability/ Total probability & Bayes' theorem

C-1. The odds that a book will be favourably reviewed by three independent critics are 5 to 2, 4 to 3, and 3 to 4 respectively. Then the probability that of the three reviews a majority will be favourable.
(A) $\frac{163}{343}$ (B) $\frac{209}{343}$ (C) $\frac{209}{387}$ (D) $\frac{208}{387}$

C-2. In throwing a pair of dice, the events 'coming up of 6 on Ist dice' and 'a total of 7 on both the dice' are
(A) mutually exclusive (B) forming an exhaustive system
(C) independent (D) dependent

C-3. A dice is thrown twice and the sum of the numbers appearing is observed to be 8. The conditional probability that the number 5 has appeared at least once is
(A) 1/6 (B) 2/5 (C) 3/5 (D) 1/2

C-4. An instrument consists of two units. Each unit must function for the instrument to operate. The reliability of the first unit is 0.9 and that of the second unit is 0.8. The instrument is tested & fails. The probability that "only the first unit failed & the second unit is sound" is "
(A) $\frac{1}{7}$ (B) $\frac{2}{7}$ (C) $\frac{3}{7}$ (D) $\frac{4}{7}$

C-5. A pack of cards is counted with face downwards. It is found that one card is missing. One card is drawn and is found to be red. Then the probability that the missing card is red.
(A) $\frac{25}{51}$ (B) $\frac{26}{51}$ (C) $\frac{1}{2}$ (D) $\frac{25}{52}$

C-6. Pal's gardner is not dependable, the probability that he will forgot to water the rose bush is 2/3. The rose bush is in questionable condition. Any how if watered, the probability of its withering is 1/2 & if not watered then the probability of its withering is 3/4. Pal went out of station & after returning he finds that rose bush has withered. Then the probability that the gardner did not water the rose bush is.
(A) 3/4 (B) 2/5 (C) 1/4 (D) 1/2

C-7. A dice is weighted so that the probability of different faces to turn up is as given

Number	1	2	3	4	5	6
Probability	0.2	0.1	0.1	0.3	0.1	0.2

If $P(A/B) = p_1$ and $P(B/C) = p_2$ and $P(C/A) = p_3$ then the values of p_1, p_2, p_3 respectively are -
Take the events A, B & C as A = {1, 2, 3}, B = {2, 3, 5} and C = {2, 4, 6}
(A) $\frac{2}{3}, \frac{1}{3}, \frac{1}{4}$ (B) $\frac{1}{3}, \frac{1}{3}, \frac{1}{6}$ (C) $\frac{1}{4}, \frac{1}{3}, \frac{1}{6}$ (D) $\frac{2}{3}, \frac{1}{6}, \frac{1}{4}$

Section (D) : Problem based on Binomial Distribution / Expectation / mean & Variance

D-1. A bag contains 2 white & 4 black balls. A ball is drawn 5 times, each being replaced before another is drawn. The probability that atleast 4 of the balls drawn are white is:
(A) 4/81 (B) 10/243 (C) 11/243 (D) 8/243

D-2. In a series of 3 independent trials the probability of exactly 2 success is 12 times as large as the probability of 3 successes. The probability of a success in each trial is:
(A) 1/5 (B) 2/5 (C) 3/5 (D) 4/5

D-3. A coin is tossed n times, what is the chance that the head will present itself an odd number of times.
(A) $\frac{1}{2}$ (B) $\frac{2}{3}$ (C) $\frac{1}{4}$ (D) $\frac{2}{5}$

D-4. From a bag containing 2 one rupee and 3 two rupee coins a person is allowed to draw 2 coins randomly then the value of his expectation.
(A) Rs. 5.10 (B) Rs. 2.30 (C) Rs. 4.30 (D) Rs. 3.20

D-5. A & B throw with one dice for a stake of Rs. 99/- which is to be won by the player who first throws 4. If A has the first throw then their respective expectations of rupees are:
(A) 50 & 49 (B) 54 & 45 (C) 45 & 54 (D) 33 & 66

D-6. A fair coin is tossed 99 times. If X is the number of times heads occur, if P (X = r) is maximum then sum of possible values of r is
(A) 98 (B) 99 (C) 101 (D) 104

PART - III : MATCH THE COLUMN

1. **Column – I** **Column – II**

	Column – I		Column – II
(A)	If the probability that units digit in square of an even integer is 4 is p, then the value of 5p is	(p)	1
(B)	If A and B are independent events and $P(A \cap B) = \frac{1}{6}$, $P(A) = \frac{1}{3}$, then $6P\left(\frac{B}{\bar{A}}\right) =$	(q)	2
(C)	One mapping is selected at random from all mappings of the set S = {1, 2, 3, ..., n} into itself. If the probability that the mapping is one-one is $\frac{3}{32}$, then the value of n is	(r)	3
(D)	A boy has 20% chance of hitting at a target. Let p denote the probability of hitting the target for the first time at the n^{th} trial. If p satisfies the inequality $625p^2 - 175p + 12 < 0$, then value of n is	(s)	4

2. **Column – I** **Column – II**

	Column – I		Column – II
(A)	A pair of dice is thrown. If total of numbers turned up on both the dice is 8, then the probability that the number turn up on the second dice is 5' is	(p)	5/16
(B)	A box contains 4 white and 3 black balls. Two balls are drawn successively and is found that second ball is white, then the probability that Ist ball is also white is	(q)	1/3
(C)	A biased coin with probability p, 0 < p < 1 of heads is tossed until a head appears for the first time. If the probability that the number of tosses required is even is 2/5, then p equals	(r)	$\frac{1}{2}$
(D)	A coin whose faces are marked 3 and 5 is tossed 4 times : what is the probability that the sum of the numbers thrown being less, than 15?	(s)	$\frac{1}{5}$

Exercise-2

✍ **Marked questions are recommended for Revision.**

PART - I : ONLY ONE OPTION CORRECT TYPE

1.✍ A local post office is to send M telegrams which are distributed at random over N communication channels, (N > M). Each telegram is sent over any channel with equal probability. Chance that not more than one telegram will be sent over each channel is:

(A) $\frac{{}^{N}C_{M}\ .\ M\,!}{N^{M}}$ (B) $\frac{{}^{N}C_{M}\ .\ N\,!}{M^{N}}$ (C) $1-\frac{{}^{N}C_{M}\ .\ M\,!}{M^{N}}$ (D) $1-\frac{{}^{N}C_{M}\ .\ N\,!}{N^{M}}$

2. A cube painted red on all sides, is cut into 125 equal small cubes. A small cube when picked up is found to show red colour on one of its faces. Then the probability that two more faces also show red colour.

(A) $\frac{4}{49}$ (B) $\frac{4}{120}$ (C) $\frac{8}{49}$ (D) $\frac{3}{49}$

3.✍ A car is parked by an owner in a parking lot of 25 cars in a row, including his car not at either end. On his return he finds that exactly 15 placed are still occupied. The probability that both the neighboring places are empty is

(A) $\frac{91}{276}$ (B) $\frac{15}{184}$ (C) $\frac{15}{92}$ (D) $\frac{17}{92}$

4.✍ A has 3 tickets in a lottery containing 3 prizes and 9 blanks; B has 2 tickets in a lottery containing 2 prizes and 6 blanks. Compare their chances of success

(A) 952 / 715 (B) 950 / 952 (C) 952 / 710 (D) 425/952

5. A 2n digit number starts with 2 and all its digits are prime, then the probability that the sum of all 2 consecutive digits of the number is prime, is

(A) 4×2^{3n} (B) 4×2^{-3n} (C) 2^{3n} (D) 2^{2n}

6. A fair coin is tossed eight times, then find the probability that resulting sequence of heads and tails looks the same when viewed from the beginning or from the end.

(A) 1/8 (B) 1/16 (C) 1/4 (D) 1/2

7.✍ An urn contains 'm' green and 'n' red balls. K (< m, n) balls are drawn and laid aside, their colur being ignored. Then one more ball is drawn. Then the probability that it is green.

(A) $\frac{m}{m+n}$ (B) $\frac{n}{m+n}$ (C) $\frac{2n}{m+n}$ (D) $\frac{3n}{m+n}$

8.✍ In a regular decagon find the probability that the two diagonal chosen at random will intersect inside the polygon.

(A) $\frac{6}{17}$ (B) $\frac{12}{17}$ (C) $\frac{5}{17}$ (D) $\frac{3}{17}$

9.✍ An urn contains m white and n black balls. A ball is drawn at random and is put back into the urn along with k additional balls of the same colour as that of the ball drawn. A ball is again drawn at random. Then the probability that the ball drawn now is white.

(A) $\frac{m}{m+n}$ (B) $\frac{n}{m+n}$ (C) $\frac{2n}{m+n}$ (D) $\frac{2m}{m+n}$

10. There are two urns. There are m white & n black balls in the first urn and p white & q black balls in the second urn. One ball is taken from the first urn & placed into the second. The probability of drawing a white ball from the second urn is -

(A) $\frac{(p+1)n+pm}{(m+n)\,(p+q+1)}$ (B) $\frac{(p+1)m+pn}{(m+n)\,(p-q+1)}$

(C) $\frac{(p+1)m+2pn}{(m+n)\,(p+q+1)}$ (D) $\frac{(p+1)m+pn}{(m+n)\,(p+q+1)}$

11. The chance that the top card in the deck is a diamond given that the fourth card from the top is a eight in well shuffled deck.

(A) $\frac{1}{4}$ (B) $\frac{2}{3}$ (C) $\frac{1}{2}$ (D) $\frac{2}{5}$

12. A fair coin is tossed 9 times the probability that at least 5 consecutive heads occurs is

(A) $\frac{5}{64}$ (B) $\frac{3}{32}$ (C) ${}^9C_5\left(\frac{1}{2}\right)^9$ (D) $\frac{5}{2^9}$

13. A man has 10 coins and one of them is known to have two heads. He takes one at random and tosses it 5 times and it always falls head. Then the chance that it is the coins with two heads.

(A) $\frac{32}{41}$ (B) $\frac{32}{51}$ (C) $\frac{23}{32}$ (D) $\frac{19}{32}$

14. 2 hunters A & B shot at a bear simultaneously. The bear was shot dead with only one hole in its hide. Probability of A shooting the bear 0.8 & that of B shooting the bear is 0.4. The hide was sold for Rs. 280/-. If this sum of money is divided between A & B in a fair way, then find the share of A

(A) 130 (B) 240 (C) 200 (D) 190

15. A number is chosen at random from the numbers 10 to 99. A number whose product of digits is 12 will be called a good number. If he choose three numbers with replacement then the probability that he will choose a good number at least once is

(A) 0.872 (B) 0.127 (C) 0.562 (D) 0.461

PART - II : SINGLE AND DOUBLE VALUE INTEGER TYPE

1. Seven digits from the number 1, 2, 3, 4, 5, 6, 7, 8 & 9 are written in random order. The probability that this seven digit number is divisible by 9 is $\frac{p}{q}$ then (p + q) equals. (Where p & q are co-prime natural numbers)

2. Three numbers are chosen at random without replacement from {1, 2, 3,......, 10}. The probability that the minimum of the chosen numbers is 3 or their maximum is 7 is $\frac{p}{q}$ then (q – 2p) equals.

(Where p & q are co-prime natural numbers)

3. In a multiple choice question there are 4 alternative answers of which 1, 2, 3 or all may be correct. A candidate will get marks in the question only if he ticks all the correct answer. The candidate decides to tick answers at random. If he is allowed upto 5 chances to answer the question, If the probability that he will get the marks in the question is p then 3p equals

4. 3 firemen X, Y and Z shoot at a common target. The probabilities that X and Y can hit the target are 2/3 and 3/4 respectively. If the probability that exactly two bullets are found on the target is 11/24, then the probability of Z to hit the target is 'λ' then 6λ equals

5. A mapping is selected at random from all the mappings defined on the set A consisting of three distinct elements. Probability that the mapping selected is one to one is $\frac{p}{q}$ (where p and q are co-prime natural numbers) then p + q is

6. A card is drawn from a pack, the card is replaced & the pack shuffled. If this is done 6 times, the probability that the cards drawn are 2 hearts, 2 diamonds & 2 black cards is $\frac{p}{q}$ then total number of proper divisors of (pq). (Where p & q are co-prime natural numbers):

7. There is a three volume dictionary among 40 books arranged on a shelf in random order. Then the raciprocal of probability of these volumes standing in increasing order from left to right (the volumes are not necessarily kept side by side) is

8. There are 4 urns. The first urn contains 1 white & 1 black ball, the second urn contains 2 white & 3 black balls, the third urn contains 3 white & 5 black balls & the fourth urn contains 4 white & 7 black balls. The selection of each urn is not equally likely. The probability of selecting i^{th} urn is $\frac{i^2+1}{34}$ (i=1,2,3, 4). If we randomly select one of the urns & draw a ball, then the probability of ball being white is $\frac{p}{q}$ then sum of digits of p is. (Where p & q are co-prime natural numbers) :

9. In a Nigerian hotel, among the english speaking people 40% are English & 60% Americans. The English & American spellings are **"RIGOUR" & "RIGOR"** respectively. An English speaking person in the hotel writes this word. A letter from this word is chosen at random & found to be a vowel. If the probability that the writer is an Englishman is a/b, then (b – a) equals. (Where a & b are co-prime natural numbers)

10. Mr. Modi is a professional tea taster. When given a high grade tea, he will identify it with probability 0.9 correctly as high grade and will mistake it for a low grade tea with probability 0.1 . When given a low grade tea, he will identify it with probability 0.8 correctly as low grade tea and will mistake it for a high grade tea with probability 0.2. Suppose that Mr. Modi is given ten unlabelled cups of tea, three with high grade and seven with low grade tea. He randomly picks a cup, tries the tea and solemnly says "high grade tea". If the probability that the tea he tasted was low grade tea is express in form of a/b then (3a – b) equals. (Where a & b are co-prime natural numbers)

11. A gambler has one rupee in his pocket. He tosses an unbiased normal coin unless either he is ruined or unless the coin has been tossed for a maximum of five times. If for each head he wins a rupee and for each tail he looses a rupee, then if the probability that the gambler is ruined is $\frac{p}{q}$ (where p and q are co-prime natural numbers) then p + q is

12. 3 cards are given, one of them is red on both sides, one is blue on both sides & one is blue on one side and red on the other side. One of them is chosen randomly & put on the table. It shows red colour on the upper side. If probability of the other side of the card being red is $\frac{p}{q}$ (where p and q are co-prime natural numbers) then q – p is

13. In a purse there are 10 coins, all 5 paise except one which is a rupee. In another purse there are 10 coins all 5 paise. 9 coins are taken out from the former purse & put into the latter & then 9 coins are taken out from the latter & put into the former. Then the chance that the rupee is still in the first purse is p then 19p – 9 equals :

14. A Teacher wrote either of words “PARALLELOGRAM” or “PARALLELOPIPED” on board but due to malfunction of marker words is not properly written and only two consecutive letters "RA" are visible then the chance that the written word is “PARALLELOGRAM” is $\frac{p}{q}$ then (p + q) equals. (Where p & q are co-prime natural numbers) :

15. The numbers ‘a and b ’ are randomly selected from the set of natural numbers. Probability that the number $3^a + 7^b$ has a digit equal to 8 at the units place, is $\frac{p}{q}$ then p + q is : (Where p & q are co-prime natural numbers)

16. Suppose that of all used cars of a particular year, 30% have bad brakes. You are considering buying a used car of that year. You take the car to a mechanic to have the brakes checked. The chance that the mechanic will give you wrong report is 20%. Assuming that the car you take to the mechanic is selected "at random" from the population of cars of that year. The odds in favor of chance that the car's brakes are good given that the mechanic says its brakes are good is m : n then (m – 7n) equals. (Where m & n are co-prime natural numbers)

17. A bag contains (n + 1) coins. It is known that one of these coins has a head on both sides, whereas the other coins are normal. One of these coins is selected at random & tossed. If the probability that the toss results in head, is 7/12, then the value of n is.

18. In a certain factory machines A, B and C produce bolts of their production A, B and C produce 2%, 1% and 3% defective bolts respectively. Machine A produces 35% of the total output of bolts machine B produces 25% and machine C produces 40%. A bolts is chosen at random from the factory’s production and its found to be defective. The odds in favor that it was produced on machine C is m : n then (m –n) equals. (Where m & n are co-prime natural numbers)

19. In each of a set of games it is 2 to 1 in favor of the winner of the previous game. If the probabilty that the player who wins the first game shall win three at least of the next four is $\frac{p}{q}$ then pq equals :
(Where p & q are co-prime natural numbers)

20. A couple has one or two or three children with probability $\frac{1}{4}$, $\frac{1}{2}$ and $\frac{1}{4}$ respectively. Probability of a couple having exactly four grandchildren in such a type of society is $\frac{p}{q}$ then sum of digit of q equals.
(Where p & q are co-prime natural numbers)

PART - III : ONE OR MORE THAN ONE OPTIONS CORRECT TYPE

1. In throwing a dice let A be the event ‘coming up of an odd number’, B be the event ‘coming up of an even number’, C be the event ‘coming up of a number $\geq$ 4’ and D be the event ‘coming up of a number < 3’, then
(A) A and B are mutually exclusive and exhautive
(B) A and C are mutually exclusive and exhautive
(C) A, C and D form an exhautive system
(D) B, C and D form an exhautive system

2. If M & N are any two events, then which one of the following represents the probability of the occurence of exactly one of them ?
(A) $P(M) + P(N) - 2P(M \cap N)$
(B) $P(M) + P(N) - P(M \cap N)$
(C) $P(\bar{M}) + P(\bar{N}) - 2P(\bar{M} \cap \bar{N})$
(D) $P(M \cap \bar{N}) + P(\bar{M} \cap N)$

3. Let $0 < P(A) < 1$, $0 < P(B) < 1$ & $P(A \cup B) = P(A) + P(B) - P(A).\ P(B)$, then:
(A) $P(B/A) = P(B) - P(A)$
(B) $P(A^C \cup B^C) = P(A^C) + P(B^C)$
(C) $P((A \cup B)^C) = P(A^C).\ P(B^C)$
(D) $P(A/B) = P(A)$

4. A box contains 11 tickets numbered from 1 to 11. Two tickets are drawn similtaneously at random.
Let E_1 denotes the event that the sum of the numbers on the tickets drawn is even
and E_2 denotes the event that the sum of the numbers on the tickets drawn is odd
Which of the following hold good?
(A) $P(E_1/E_2) = P(E_2/E_1)$
(B) E_1 and E_2 are exhaustive
(C) $P(E_2) > P(E_1)$
(D) E_1 and E_2 are equally likely

5. The probabilities of events, $A \cap B$, A, B & $A \cup B$ are respectively in A.P. with second term equal to the common difference. Therefore A & B are :
(A) mutually exclusive
(B) independent
(C) such that one of them must occur
(D) such that one is twice as likely as the other

6. A bag contains four tickets marked with numbers 112, 121, 211, 222. One ticket is drawn at random from the bag. Let E_i(i = 1, 2, 3) denote the event that ith digit on the ticket is 2. Then
(A) E_1 and E_2 are independent
(B) E_2 and E_3 are independent
(C) E_3 and E_1 are independent
(D) E_1, E_2, E_3 are independent

7. In an experimental performance of a single throw of a pair of unbiased normal dice, three events E_1, E_2 & E_3 are defined as follows:
E_1 : getting a prime numbered face on each dice
E_2 : getting the same number on each dice
E_3 : getting a sum total of dots on two dice equal to 8. Then:
(A) the events E_1, E_2 & E_3 are mutually exclusive
(B) the events E_1, E_2 & E_3 are not pairwise mutually exclusive
(C) the events E_1, E_2 are independent
(D) $P(E_3 | E_1) = 2/9$.

8. The probability that a bulb produced by a factory will fuse after an year of use is 0.1. Then the probability that out of 4 such bulbs
(A) None of then bulb will fuse after an year of use is $\frac{9^4}{10^4}$
(B) More then three bulbs will fuse after an year of use is $\frac{1}{10^4}$
(C) Not more then three bulbs will fuse after an year of use is $\frac{9999}{10000}$
(D) All the bulbs will fuse after an year of use is $\frac{1}{10^4}$

9. If 4 whole numbers taken at random are multiplied together
(A) Probability that the last digit in the product is 1, 3, 7 or 9 is $\frac{16}{625}$
(B) Probability that the last digit in the product is '5' is $\frac{369}{10^4}$
(C) Probability that the last digit in the product is 0 is $\frac{3727}{10^4}$
(D) Probability that the last digit in the product is 0 is $\frac{2357}{10^4}$

10. The probability that 4th power of a positive integer ends in the digit λ is $P(\lambda)$

(A) $P(6) = \frac{4}{10}$ (B) $P(1) = \frac{4}{10}$ (C) $P(5) = \frac{1}{10}$ (D) $P(0) = \frac{1}{10}$

11. Mean and variance of a Binomial variate of 10 trials of the experiment are in the ratio of 3 : 2.

(A) The most probable number of happening of variable is 3

(B) Sum of the mean and variance is 10

(C) Probability of getting exactly 5 success is $\frac{8064}{3^{10}}$

(D) The most probable number of happening of variable is 5

12. A student appears for tests I, II & III. The student is successful if he passes either in tests I & II or tests I & III. The probabilities of the student passing in the tests I, II & III are p, q & 1/2 respectively. If the probability that the student is successful is 1/2, then:

(A) p = 1, q = 0 (B) p = 2/3, q = 1/2

(C) p = 3/5, q = 2/3 (D) there are infinitely many values of p & q.

13. A student has to match three historical events i.e. Dandi March, Quit India Movement and Mahatma Gandhi's assasination with the years 1948, 1930 and 1942 and each event happens in different years. The student has no knowledge of the correct answers and decides to match the events and years randomly. Let E_i : $(0 \le i \le 3)$ denote the event that the student gets exactly i correct answer, then

(A) $P(E_o) + P(E_3) = P(E_1)$ (B) $P(E_o) \,.\, P(E_1) = P(E_3)$

(C) $P(E_0 \cap E_1) = P(E_2)$ (D) $P(E_0) + P(E_1) + P(E_3) = 1$

14. For any two events A & B defined on a sample space,

(A) $P(A/B) \ge \frac{P(A) + P(B) - 1}{P(B)}$, $P(B) \ne 0$ is always true

(B) $P(A \cup \bar{B}) = P(A) - P(A \cap B)$

(C) $P(A \cup B) = 1 - P(A^c).\, P(B^c)$, if A & B are independent

(D) $P(A \cup B) = 1 - P(A^c).\, P(B^c)$, if A & B are disjoint

15. An unbiased coin is tossed n times. Let X denote the number of times head occurs. If P(X = 4), P (X = 5) and P(X = 6) are in AP, then the value of n can be

(A) 7 (B) 10 (C) 12 (D) 14

16. A random variable x takes values 0, 1, 2, 3, ..., with probability proportional to $(x + 1)\left(\frac{1}{5}\right)^x$, then

(A) $P(x = 0) = \frac{16}{25}$ (B) $P(x \le 1) = \frac{112}{125}$ (C) $P(x \ge 1) = \frac{9}{25}$ (D) $E(x) = \frac{25}{32}$

17. Let X be a set containing 'n' elements. If two subsets A and B of X are picked at random. The probability of A and B having same number of elements

(A) $\frac{{}^{2n}C_n}{2^{2n}}$ (B) $\frac{1}{{}^{2n}C_{cn}}$ (C) $\frac{1.3.5.....(2n-1)}{2^n.n!}$ (D) $\frac{3^n}{4^n}$

18. A square matrix of order 3 × 3 is formed using the elements of the set {–2016, 0, 2016}

(A) Probability of getting a matrix which is symmetric $\frac{1}{3^3}$ is

(B) Probability of getting a matrix which is skew symmetric $\frac{1}{3^6}$ is

(C) Probability of getting a matrix which has maximum trace is $\frac{1}{3^3}$

(D) Probability of getting a matrix which has minimum trace is $\frac{1}{3^3}$

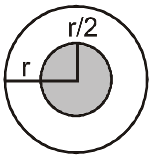
PART - IV : COMPREHENSION

Comprehension # 1

If sample space contains infinite number of points then with the help of geometry that is length, area, volume many problems of probibility can be solved

For example : A point is selected randomely inside the circle then the probability that it is nearer to centre then its circumeference

Probability = $\dfrac{\text{Favourable} \quad \text{Area}}{\text{Total} \quad \text{Area}} = \dfrac{\frac{\pi r^2}{4}}{\pi r^2} = \dfrac{1}{4}$



1. A sphere is circumscribed over a cube. Find the probability that a point lies inside the sphere, lies outside the cube.

(A) $1-\dfrac{2}{\pi\sqrt{3}}$ (B) $1-\dfrac{1}{\pi\sqrt{3}}$ (C) $1-\dfrac{1}{2\pi\sqrt{3}}$ (D) $1-\dfrac{2}{2\pi\sqrt{3}}$

2. A parallelogram is inscribed inside a circle of radius 10 cm. One side of parallelogram being 12 cms. Then the probability that a point inside the circle also lies inside the parallelogram.

(A) $\dfrac{48}{25\pi}$ (B) $\dfrac{24}{25\pi}$ (C) $\dfrac{42}{25\pi}$ (D) $\dfrac{1}{2}$

3. The sides of a rectangle are chosen at random, each less than 10 cm, all such lengths being equally likely. The chance that the diagonal of the rectangle is less than 10 cm is

(A) 1/10 (B) 1/20 (C) $\pi/4$ (D) $\pi/8$

Comprehension # 2

A JEE aspirant estimates that he will be successful with an 80 percent chance if he studies 10 hours per day, with a 60 percent chance if he studies 7 hours per day and with a 40 percent chance if he studies 4 hours per day. He further believes that he will study 10 hours, 7 hours and 4 hours per day with probabilities 0.1, 0.2 and 0.7 respectively.

4. The chance he will be successfull is

(A) 0.28 (B) 0.38 (C) 0.48 (D) 0.58

5. Given that he is successful the chance that he studied for 4 hours, is

(A) $\dfrac{6}{12}$ (B) $\dfrac{7}{12}$ (C) $\dfrac{8}{12}$ (D) $\dfrac{9}{12}$

6. Given that he does not achieve success, the chance that he studied for 4 hour, is

(A) $\dfrac{18}{26}$ (B) $\dfrac{19}{26}$ (C) $\dfrac{20}{26}$ (D) $\dfrac{21}{26}$

Comprehension # 3

A bag contain 6 Red and 4 White balls. 4 balls are drawn one by one without replacement and were found to be atleast 2 white.

7.✍ Then the probablity that next draw of a ball from this bag will give a white ball.

(A) $\dfrac{34}{115}$ (B) $\dfrac{19}{115}$ (C) $\dfrac{90}{115}$ (D) $\dfrac{24}{115}$

8.✍ If the next draw of a ball from this bag will give a white ball then the probablity that the drawn of four balls initially contain two white and two red balls.

(A) $\frac{8}{17}$ (B) $\frac{15}{17}$ (C) $\frac{13}{17}$ (D) $\frac{13}{34}$

Comprehension # 4

Eight digit number can be formed using all the digits 1,1,2,2,3,3,4,5.

9. A number is selected at random then the probability such that no two identical digits appear together

(A) $\frac{37}{84}$ (B) $\frac{43}{84}$ (C) $\frac{17}{84}$ (D) $\frac{23}{84}$

10. A number is selected at random then the probability that it has exactly two pair of identical digits occuring together

(A) $\frac{1}{7}$ (B) $\frac{2}{9}$ (C) $\frac{3}{5}$ (D) $\frac{4}{9}$

Exercise-3

✍ **Marked questions are recommended for Revision.**

PART - I : JEE (ADVANCED) / IIT-JEE PROBLEMS (PREVIOUS YEARS)

* **Marked Questions may have more than one correct option.**

Comprehension (Q.1 to 3)

A fair die is tossed repeatedly until a six is obtained. Let X denote the number of tosses required.

1. The probability that X = 3 equals **[IIT-JEE 2009, Paper-1, (4, –1), 80]**

(A) $\frac{25}{216}$ (B) $\frac{25}{36}$ (C) $\frac{5}{36}$ (D) $\frac{125}{216}$

2. The probability that $X \geq 3$ equals **[IIT-JEE 2009, Paper-1, (4, –1), 80]**

(A) $\frac{125}{216}$ (B) $\frac{25}{36}$ (C) $\frac{5}{36}$ (D) $\frac{25}{216}$

3. The conditional probability that $X \geq 6$ given $X > 3$ equals **[IIT-JEE 2009, Paper-1, (4, –1), 80]**

(A) $\frac{125}{216}$ (B) $\frac{25}{216}$ (C) $\frac{5}{36}$ (D) $\frac{25}{36}$

4.✍ Let ω be a complex cube root of unity with $\omega \neq 1$. A fair die is thrown three times. If r_1, r_2 and r_3 are the numbers obtained on the die, then the probability that $\omega^{r_1} + \omega^{r_2} + \omega^{r_3} = 0$ is **[IIT-JEE 2010, Paper-1, (3, –1), 84]**

(A) $\frac{1}{18}$ (B) $\frac{1}{9}$ (C) $\frac{2}{9}$ (D) $\frac{1}{36}$

5. A signal which can be green or red with probability $\frac{4}{5}$ and $\frac{1}{5}$ respectively, is received by station A and then transmitted to station B. The probability of each station receiving the signal correctly is $\frac{3}{4}$. If the signal received at station B is green, then the probability that the original signal was green is

[IIT-JEE 2010, Paper-2, (5, –2), 79]

(A) $\frac{3}{5}$ (B) $\frac{6}{7}$ (C) $\frac{20}{23}$ (D) $\frac{9}{20}$

Comprehension (Q.6 & 7)

Let U_1 and U_2 be two urns such that U_1 contains 3 white and 2 red balls, and U_2 contains only 1 white ball. A fair coin is tossed. If head appears then 1 ball is drawn at random from U_1 and put into U_2. However, if tail appears then 2 balls are drawn at random from U_1 and put into U_2. Now 1 ball is drawn at random from U_2. **[IIT-JEE 2011, Paper-1, (3, –1), 80]**

6. The probability of the drawn ball from U_2 being white is

(A) $\frac{13}{30}$ (B) $\frac{23}{30}$ (C) $\frac{19}{30}$ (D) $\frac{11}{30}$

7. Given that the drawn ball from U_2 is white, the probability that head appeared on the coin is

(A) $\frac{17}{23}$ (B) $\frac{11}{23}$ (C) $\frac{15}{23}$ (D) $\frac{12}{23}$

8.* Let E and F be two independent events. The probability that exactly one of them occurs is $\frac{11}{25}$ and the probability of none of them occurring is $\frac{2}{25}$. If P(T) denotes the probability of occurrence of the event T, then **[IIT-JEE 2011, Paper-2, (4, 0), 80]**

(A) $P(E) = \frac{4}{5}$, $P(F) = \frac{3}{5}$ (B) $P(E) = \frac{1}{5}$, $P(F) = \frac{2}{5}$

(C) $P(E) = \frac{2}{5}$, $P(F) = \frac{1}{5}$ (D) $P(E) = \frac{3}{5}$, $P(F) = \frac{4}{5}$

9*. A ship is fitted with three engines E_1, E_2 and E_3. The engines function independently of each other with respective probabilities $\frac{1}{2}, \frac{1}{4}$ and $\frac{1}{4}$. For the ship to be operational at least two of its engines must function. Let X denote the event that the ship is operational and let X_1, X_2 and X_3 denotes respectively the events that the engines E_1 E_2 and E_3 are functioning. Which of the following is (are) true ?

(A) $P\left[X_1^c \mid X\right] = \frac{3}{16}$

(B) P[Exactly two engines of the ship are functioning | X] $= \frac{7}{8}$

(C) $P[X \mid X_2] = \frac{5}{16}$

(D) $P[X \mid X_1] = \frac{7}{16}$ **[IIT-JEE 2012, Paper-1, (4, 0), 70]**

10. Four fair dice D_1, D_2, D_3 and D_4 each having six faces numbered 1,2,3,4,5 and 6 are rolled simultaneously. The probability that D_4 shows a number appearing on one of D_1, D_2 and D_3 is

[IIT-JEE 2012, Paper-2, (3, –1), 66]

(A) $\frac{91}{216}$ (B) $\frac{108}{216}$ (C) $\frac{125}{216}$ (D) $\frac{127}{216}$

11*. Let X and Y be two events such that $P(X \mid Y) = \frac{1}{2}$, $P(Y \mid X) = \frac{1}{3}$ and $P(X \cap Y) = \frac{1}{6}$. Which of the following is (are) correct ? **[IIT-JEE 2012, Paper-2, (4, 0), 66]**

(A) $P(X \cup Y) = \frac{2}{3}$ (B) X and Y are independent

(C) X and Y are not independent (D) $P(X^c \cap Y) = \frac{1}{3}$

12. Four persons independently solve a certain problem correctly with probabilities $\frac{1}{2}, \frac{3}{4}, \frac{1}{4}, \frac{1}{8}$. Then the probability that the problem is solved correctly by at least one of them is

[JEE (Advanced) 2013, Paper-1, (2, 0)/60]

(A) $\frac{235}{256}$ (B) $\frac{21}{256}$ (C) $\frac{3}{256}$ (D) $\frac{253}{256}$

13. Of the three independent events E_1, E_2 and E_3, the probability that only E_1 occurs is α, only E_2 occurs is β and only E_3 occurs is γ. Let the probability p that none of events E_1, E_2 or E_3 occurs satisfy the equations $(\alpha - 2\beta)\, p = \alpha\beta$ and $(\beta - 3\gamma)\, p = 2\beta\gamma$. All the given probabilities are assumed to lie in the interval (0, 1).

Then $\frac{\text{Probability of occurrence of } E_1}{\text{Probability of occurrence of } E_3} =$ **[JEE (Advanced) 2013, Paper-1, (4, – 1)/60]**

Comprehension (Q.14 & 15)

A box B_1 contains 1 white ball, 3 red balls and 2 black balls. Another box B_2 contains 2 white balls, 3 red balls and 4 black balls. A third box B_3 contains 3 white balls, 4 red balls and 5 black balls.

14. If 1 ball is drawn from each of the boxes B_1, B_2 and B_3, the probability that all 3 drawn balls are of the same colour is **[JEE (Advanced) 2013, Paper-2, (3, –1)/60]**

(A) $\frac{82}{648}$ (B) $\frac{90}{648}$ (C) $\frac{558}{648}$ (D) $\frac{566}{648}$

15. If 2 balls are drawn (without replacement) from a randomly selected box and one of the balls is white and the other ball is red, the probability that these 2 balls are drawn from box B_2 is

[JEE (Advanced) 2013, Paper-2, (3, –1)/60]

(A) $\frac{116}{181}$ (B) $\frac{126}{181}$ (C) $\frac{65}{181}$ (D) $\frac{55}{181}$

16. Three boys and two girls stand in a queue. The probability, that the number of boys ahead of every girl is at least one more than the number of girls ahead of her, is **[JEE (Advanced) 2014, Paper-2, (3, –1)/60]**

(A) $\frac{1}{2}$ (B) $\frac{1}{3}$ (C) $\frac{2}{3}$ (D) $\frac{3}{4}$

Comprehension (Q.17 & 18) **[JEE (Advanced) 2014, Paper-2, (3, –1)/60]**

Box 1 contains three cards bearing numbers 1, 2, 3; box 2 contains five cards bearing numbers 1, 2, 3, 4, 5; and box 3 contains seven cards bearing numbers 1, 2, 3, 4, 5, 6, 7. A card is drawn from each of the boxes. Let x_i be the number on the card drawn from the i^{th} box, i = 1, 2, 3.

17. The probability that $x_1 + x_2 + x_3$ is odd, is

(A) $\frac{29}{105}$ (B) $\frac{53}{105}$ (C) $\frac{57}{105}$ (D) $\frac{1}{2}$

18. The probability that x_1, x_2, x_3 are in an arithmetic progression, is

(A) $\frac{9}{105}$ (B) $\frac{10}{105}$ (C) $\frac{11}{105}$ (D) $\frac{7}{105}$

19. The minimum number of times a fair coin needs to be tossed, so that the probability of getting at least two heads is at least 0.96, is **[JEE (Advanced) 2015, P-1 (4, 0) /88]**

Comprehension (Q.20 & 21)

Let n_1 and n_2 be the number of red and black balls, respectively, in box I. Let n_3 and n_4 be the number of red and black balls, respectively, in box II. **[JEE (Advanced) 2015, P-2 (4, –2)/ 80]**

20*. One of the two boxes, box I and box II, was selected at random and a ball was drawn randomly out of this box. The ball was found to be red. If the probability that this red ball was drawn from box II is $\frac{1}{3}$, then the correct option(s) with the possible values of n_1, n_2, n_3 and n_4 is(are)

(A) $n_1 = 3, n_2 = 3, n_3 = 5, n_4 = 15$ (B) $n_1 = 3, n_2 = 6, n_3 = 10, n_4 = 50$
(C) $n_1 = 8, n_2 = 6, n_3 = 5, n_4 = 20$ (D) $n_1 = 6, n_2 = 12, n_3 = 5, n_4 = 20$

21*. A ball is drawn at random from box I and transferred to box II. If the probability of drawing a red ball from box I, after this transfer, is $\frac{1}{3}$, then the correct option(s) with the possible values of n_1 and n_2 is(are)

(A) $n_1 = 4$ and $n_2 = 6$ (B) $n_1 = 2$ and $n_2 = 3$
(C) $n_1 = 10$ and $n_2 = 20$ (D) $n_1 = 3$ and $n_2 = 6$

22. A computer producing factory has only two plants T_1 and T_2. Plant T_1 produces 20% and plant T_2 produces 80% of the total computers produced. 7% of computers produced in the factory turn out to be defective. It is known that
P(computer turns out to be defective given that it is produced in plant T_1)
= 10 P(computer turns out to be defective given that it is produced in Plant T_2),
where P(E) denotes the probability of an event E. A computer produced in the factory is randomly selected and it does not turn out to be defective. Then the probability that it is produced in plant T_2 is
[JEE (Advanced) 2016, Paper-1, (3, –1)/62]

(A) $\frac{36}{73}$ (B) $\frac{47}{79}$ (C) $\frac{78}{93}$ (D) $\frac{75}{83}$

Comprehension (Q.23 & 24)

Football teams T_1 and T_2 have to play two games against each other. It is assumed that the outcomes of the two games are independent. The probabilities of T_1 winning, drawing and losing a game against T_2 are $\frac{1}{2}$, $\frac{1}{6}$ and $\frac{1}{3}$, respectively. Each team gets 3 points for a win, 1 point for a draw and 0 point for a loss in a game. Let X and Y denote the total points scored by teams T_1 and T_2, respectively, after two games. **[JEE (Advanced) 2016, Paper-2, (3, –1)/62]**

23. P (X > Y) is

(A) $\frac{1}{4}$ (B) $\frac{5}{12}$ (C) $\frac{1}{2}$ (D) $\frac{7}{12}$

24. P (X = Y) is

(A) $\frac{11}{36}$ (B) $\frac{1}{3}$ (C) $\frac{13}{36}$ (D) $\frac{1}{2}$

25*. Let X and Y be two events such that $P(X) = \frac{1}{3}$, $P(X|Y) = \frac{1}{2}$ and $P(Y|X) = \frac{2}{5}$. Then

[JEE(Advanced) 2017, Paper-1,(4, –2)/61]

(A) $P(Y) = \frac{4}{15}$ (B) $P(X'|Y) = \frac{1}{2}$ (C) $P(X \cup Y) = \frac{2}{5}$ (D) $P(X \cap Y) = \frac{1}{5}$

26. Three randomly chosen nonnegative integers x, y and z are found to satisfy the equation x + y + z = 10. Then the probability that z is even, is **[JEE(Advanced) 2017, Paper-2,(3, –1)/61]**

(A) $\frac{1}{2}$ (B) $\frac{36}{55}$ (C) $\frac{6}{11}$ (D) $\frac{5}{11}$

Comprehension (Q.27 & 28)

There are five students S_1, S_2, S_3, S_4 and S_5 in a music class and for them there are five seats R_1, R_2, R_3, R_4 and R_5 arranged in a row, where initially the seat R_i is allotted to the student S_i, i = 1, 2, 3, 4, 5. But, on the examination day, the five students are randomly allotted the five seats.

(*There are two questions based on PARAGRAPH* "A"*, the question given below is one of them*)

[JEE(Advanced) 2018, Paper-1,(3, –1)/60]

27. The probability that, on the examination day, the student S_1 gets the previously allotted seat R_1, and **NONE** of the remaining students gets the seat previously allotted to him/her, is

(A) $\frac{3}{40}$ (B) $\frac{1}{8}$ (C) $\frac{7}{40}$ (D) $\frac{1}{5}$

28. For i = 1, 2, 3,4, let T_i denote the event that the students S_i and S_{i+1} do **NOT** sit adjacent to each other on the day of the examination. Then, the probability of the event $T_1 \cap T_2 \cap T_3 \cap T_4$ is

(A) $\frac{1}{15}$ (B) $\frac{1}{10}$ (C) $\frac{7}{60}$ (D) $\frac{1}{5}$

PART - II : JEE (MAIN) / AIEEE PROBLEMS (PREVIOUS YEARS)

1. In a binomial distribution $B\left(n,\ p = \frac{1}{4}\right)$, if the probability of at least one success is greater than or equal to $\frac{9}{10}$, then n is greater than : **[AIEEE 2009 (4, –1), 144]**

(1) $\frac{1}{\log_{10} 4 + \log_{10} 3}$ (2) $\frac{9}{\log_{10} 4 - \log_{10} 3}$ (3) $\frac{4}{\log_{10} 4 - \log_{10} 3}$ (4) $\frac{1}{\log_{10} 4 - \log_{10} 3}$

2. One ticket is selected at random from 50 tickets numbered 00, 01, 02, ..., 49. Then the probability that the sum of the digits on the selected ticket is 8, given that the product of these digits is zero, equal :

[AIEEE 2009 (4, –1), 144]

(1) $\frac{1}{7}$ (2) $\frac{5}{14}$ (3) $\frac{1}{50}$ (4) $\frac{1}{14}$

3. Four numbers are chosen at random (without replacement) from the set {1,2,3,.....,20}.
Statement -1 : The probability that the chosen numbers when arranged in some order will form an AP is $\frac{1}{85}$. **[AIEEE 2010 (8, –2), 144]**
Statement -2 : If the four chosen numbers form an AP, then the set of all possible values of common difference is {±1, ±2, ±3, ±4, ±5}
(1) Statement- is true, Statement-2 is true ; Statement -2 is not a correct explanation for Statement -1.
(2) Statement-1 is true, Statement-2 is false.
(3) Statement -1 is false, Statement -2 is true.
(4) Statement -1 is true, Statement -2 is true; Statement-2 is a correct explanation for Statement-1.

4. An urn contains nine balls of which three are red, four are blue and two are green. Three balls are drawn at random without replacement from the urn. The probability that the three balls have different colours is **[AIEEE 2010 (4, –1), 144]**
(1) $\frac{2}{7}$ (2) $\frac{1}{21}$ (3) $\frac{2}{23}$ (4) $\frac{1}{3}$

5. Consider 5 independent Bernoulli's trials each with probability of success p. If the probability of at least one failure is greater than or equal to $\frac{31}{32}$, then p lies in the interval : **[AIEEE 2011, I, (4, –1), 120]**
(1) $\left(\frac{1}{2}, \frac{3}{4}\right]$ (2) $\left(\frac{3}{4}, \frac{11}{12}\right]$ (3) $\left[0, \frac{1}{2}\right]$ (4) $\left(\frac{11}{12}, 1\right]$

6. If C and D are two events such that $C \subset D$ and $P(D) \neq 0$, then the correct statement among the following is : **[AIEEE 2011, I, (4, –1), 120]**
(1) $P(C|D) = P(C)$ (2) $P(C|D) \geq P(C)$ (3) $P(C|D) < P(C)$ (4) $P(C|D) = \frac{P(D)}{P(C)}$

7. Let A, B, C be pairwise independent events with $P(C) > 0$ and $P(A \cap B \cap C) = 0$. Then $P(A^c \cap B^c / C)$.
[AIEEE 2011, II, (4, –1), 120]
(1) $1 - P(B^c)$ (2) $P(A^c) + P(B^c)$ (3) $P(A^c) - P(B^c)$ (4) $P(A^c) - P(B)$

8. Three numbers are chosen at random without replacement from {1, 2, 3, ..., 8}. The probability that their minimum is 3, given that their maximum is 6, is : **[AIEEE-2012, (4, –1)/120]**
(1) $\frac{3}{8}$ (2) $\frac{1}{5}$ (3) $\frac{1}{4}$ (4) $\frac{2}{5}$

9. A multiple choice examination has 5 questions. Each question has three alternative answers of which exactly one is correct. The probability that a student will get 4 or more correct answers just by guessing is :
[AIEEE - 2013, (4, – 1) 120]
(1) $\frac{17}{3^5}$ (2) $\frac{13}{3^5}$ (3) $\frac{11}{3^5}$ (4) $\frac{10}{3^5}$

10. Let A and B be two event such that $P\left(\overline{A \cup B}\right) = \frac{1}{6}$, $P(A \cap B) = \frac{1}{4}$ and $P\left(\overline{A}\right) = \frac{1}{4}$, where $\overline{A}$ stands for the complement of the event A. Then the events A and B are : **[JEE(Main)2014,(4, – 1), 120]**
(1) independent but not equally likely (2) independent and equally likely
(3) mutually exclusive and independent (4) equally likely but not independent

11. If 12 identical balls are to be placed in 3 identical boxes, then the probability that one of the boxes contains exactly 3 balls is **[JEE(Main)2015, (4, – 1), 20]**
(1) $\frac{55}{3}\left(\frac{2}{3}\right)^{11}$ (2) $55\left(\frac{2}{3}\right)^{10}$ (3) $220\left(\frac{1}{3}\right)^{12}$ (4) $22\left(\frac{1}{3}\right)^{11}$

12. Let two fair six-faced dice A and B be thrown simultaneously. If E_1 is the event that die A shows up four, E_2 is the event that die B shows up two and E_3 is the event that the sum of numbers on both dice is odd, then whic h of the following statements is NOT True ? **[JEE(Main)2016,(4, – 1), 120]**

(1) E_2 and E_3 are independent (2) E_1 and E_3 are independent
(3) E_1, E_2 and E_3 are independent (4) E_1 and E_2 are independent

13. For three events A, B and C, P(Exactly one of A or B occurs) = P(Exactly one of B or C occurs) = P(Exactly one of C or A occurs) = $\frac{1}{4}$ and P (All the three events occur simultaneously) = $\frac{1}{16}$. Then the probability that at least one of the events occurs, is : **[JEE(Main)2017,(4, – 1), 120]**

(1) $\frac{7}{32}$ (2) $\frac{7}{16}$ (3) $\frac{7}{64}$ (4) $\frac{3}{16}$

14. It two different numbers are taken from the set {0,1,2,3,....., 10}; then the probability that their sum as well as absolute difference are both multiple of 4, is **[JEE(Main)2017,(4, – 1), 120]**

(1) $\frac{6}{55}$ (2) $\frac{12}{55}$ (3) $\frac{14}{45}$ (4) $\frac{7}{55}$

15. A box contains 15 green and 10 yellow balls. If 10 balls are randomly drawn, one-by-one, with replacement, then the variance of the number of green balls drawn is **[JEE(Main) 2017, (4, – 1), 120]**

(1) $\frac{12}{5}$ (2) 6 (3) 4 (4) $\frac{6}{25}$

16. A bag contains 4 red and 6 balck balls. A ball is drawn at random from the bag, its colour is observed and this ball along with two additional balls of the same colour are returned to the bag. If now a ball is drawn at random from the bag, then the probability that this drawn ball is red, is

[JEE(Main)2018,(4, – 1), 120]

(1) $\frac{1}{5}$ (2) $\frac{3}{4}$ (3) $\frac{3}{10}$ (4) $\frac{2}{5}$

17. Two cards are drawn successively with replacement from a well-shuffled deck of 52 cards. Let X denote the random variable of number of aces obtained in the two drawn cards. Then P(X = 1) + P(X = 2) equals : **[JEE(Main) 2019, Online (09-01-19),P-1 (4, – 1), 120]**

(1) 52/169 (2) 24/169 (3) 49/169 (4) 25/169

18. Let S = {1,2,…..,20}. A subset B of S is said to be "nice", if the sum of the elements of B is 203. Then the probability that a randomly chosen subset of S is " nice" is :

[JEE(Main) 2019, Online (11-01-19),P-2 (4, – 1), 120]

(1) $\frac{4}{2^{20}}$ (2) $\frac{5}{2^{20}}$ (3) $\frac{7}{2^{20}}$ (4) $\frac{6}{2^{20}}$

19. In a random experiment a fair die is rolled until two fours are obtained in succession the probability that the experiment will end in the fifth throw of the die is equal

[JEE(Main) 2019, Online (12-01-19),P-1 (4, – 1), 120]

(1) $\frac{200}{6^5}$ (2) $\frac{175}{6^5}$ (3) $\frac{150}{6^5}$ (4) $\frac{225}{6^5}$

20. In a game, a man wins Rs. 100 if he gets 5 or 6 on a throw of a fair die and loses Rs. 50 for getting any other number on the die. If he decides to throw the die either till he gets a five or a six or to a maximum of three throws, then his expected gain/loss (in rupees) is –

[JEE(Main) 2019, Online (12-01-19),P-2 (4, – 1), 120]

(1) 0 (2) $\frac{400}{9}$ loss (3) $\frac{400}{3}$ gain (4) $\frac{400}{3}$ loss

Answers

EXERCISE - 1

PART - I

Section (A) :

A-1. (i) {HHH, HHT, HTH, THH, HTT, THT, TTH, TTT}

(ii) $\{B_1 B_2,\ B_1 B_3,\ B_1 G_1,\ B_1 G_2,\ B_2 B_3,\ B_2 G_1,\ B_2 G_2,\ B_3 G_1,\ B_3 G_2,\ G_1 G_2\}$

A-2. 43 to 34 **A-3.** $\frac{4}{5}$ **A-4.** $\frac{5}{9}$ **A-5.** $\frac{1}{6}$ **A-6.** (i) $\frac{4}{15}$ (ii) 10

A-7. (i) No (ii) No **A-8.** $\frac{29}{36}$ **A-9.** $B = \frac{2}{5}$, $C = \frac{4}{15}$ **A-10.** $\frac{3}{10}$

A-11. (i) $\frac{22}{425}$ (ii) $\frac{16}{5525}$ **A-12.** $\frac{5}{36}$ **A-13.** $\frac{1}{462}$ **A-14.** (i) $\frac{2197}{20825}$ (ii) $\frac{{}^{13}C_4 \times 4^4}{{}^{52}C_4}$

Section (B) :

B-2 (i) 0.5 (ii) 0.8 **B-3.** (i) 0.72 (ii) 0.32 (iii) 0.28 (iv) 0.56 **B-4.** 5/21

Section (C) :

C-1. (i) $\frac{2}{3}$ (ii) $\frac{1}{2}$ **C-2.** $\frac{3}{4}$ **C-3.** (i) $\frac{2}{13}$ (ii) $\frac{4}{13}$ **C-4.** $\frac{52}{77}$

C-5. $\frac{16}{37}, \frac{12}{37}, \frac{9}{37}$ **C-6.** 3/5 **C-7.** $\frac{17}{125}$ **C-8.** (i) $\frac{27}{56}$ (ii) $\frac{8}{29}$ **C-9.** $\frac{1}{17}$

C-10. $\frac{5}{9}$ **C-11.** $\frac{1}{4}$

Section (D) :

D-1. $\frac{7}{2^{13}}$ **D-2.** ${}^{10}C_2 \cdot \frac{(14)^8}{15^{10}}$ **D-3.**

X	0	1	2	3
P(X)	$\frac{27}{64}$	$\frac{27}{64}$	$\frac{9}{64}$	$\frac{1}{64}$

D-4. 28 rupees

D-5.

x_i	0	1	2	3
p_i	$\left(\frac{19}{25}\right)^3$	$18 \times \frac{19^2}{25^3}$	$108 \times \frac{19}{25^3}$	$\frac{216}{25^3}$

D-6. mean = 2.5, variance = 1.25

PART - II

Section (A) :

A-1. (D) **A-2.** (C) **A-3.** (C) **A-4.** (D) **A-5.** (A) **A-6.** (D)

A-7. (A) **A-8.** (B) **A-9.** (C)

Section (B) :

B-1. (B) **B-2.** (i) (D) (ii) (B) (iii) (C) **B-3** (B) **B-4** (D)

Section (C) :

C-1. (B) **C-2.** (C) **C-3.** (B) **C-4.** (B) **C-5.** (A) **C-6.** (A)

C-7. (D)

Section (D) :

D-1. (C) **D-2.** (A) **D-3.** (A) **D-4.** (D) **D-5.** (B) **D-6.** (B)

PART - III

1. (A) $\rightarrow$ (q), (B) $\rightarrow$ (r), (C) $\rightarrow$ (s), (D) $\rightarrow$ (r)
2. (A) $\rightarrow$ (s), (B) $\rightarrow$ (r), (C) $\rightarrow$ (q), (D) $\rightarrow$ (p)

EXERCISE - 2

PART - I

1. (A) **2.** (A) **3.** (C) **4.** (A) **5.** (B) **6.** (B) **7.** (A)

8. (A) **9.** (A) **10.** (D) **11.** (A) **12.** (B) **13.** (A) **14.** (B)

15. (B)

PART - II

1. 10 **2.** 18 **3.** 1 **4.** 3 **5.** 11 **6.** 59 **7.** 6

8. 20 **9.** 6 **10.** 1 **11.** 27 **12.** 1 **13.** 1 **14.** 32

15. 19 **16.** 7 **17.** 5 **18.** 5 **19.** 36 **20.** 11

PART - III

1. (AC) **2.** (ACD) **3.** (CD) **4.** (ABC) **5.** (AD) **6.** (ABC) **7.** (BD)

8. (ABCD) **9.** (AB) **10.** (ABCD) **11.** (AC) **12.** (ABCD) **13.** (ABCD)

14. (AC) **15.** (AD) **16.** (ABC) **17.** (AC) **18.** (ABCD)

PART - IV

1. (A) **2.** (A) **3.** (C) **4.** (C) **5.** (B) **6.** (D) **7.** (A)

8. (B) **9.** (A) **10.** (A)

EXERCISE - 3

PART - I

1.	(A)	**2.**	(B)	**3.**	(D)	**4.**	(C)	**5.**	(C)	**6.**	(B)	**7.**	(D)
8.*	(AD)	**9*.**	(BD)	**10.**	(A)	**11*.**	(AB)	**12.**	(A)	**13.**	6	**14.**	(A)
15.	(D)	**16.**	(A)	**17.**	(B)	**18.**	(C)	**19.**	8	**20.**	(AB)	**21.**	(CD)
22.	(C)	**23.**	(B)	**24.**	(C)	**25.**	(AB)	**26.**	(C)	**27.**	(A)	**28.**	(C)

PART - II

1.	(4)	**2.**	(4)	**3.**	(2)	**4.**	(1)	**5.**	(3)	**6.**	(2)	**7.**	(4)
8.	(2)	**9.**	(3)	**10.**	(1)	**11.**	(1)	**12.**	(3)	**13.**	(2)	**14.**	(1)
15.	(1)	**16.**	(4)	**17.**	(4)	**18.**	**(2)**	**19.**	**(2)**	**20.**	**(1)**		

High Level Problems (HLP)

1. Urn A contains 6 red & 4 black balls and urn B contains 4 red & 6 black balls. One ball is drawn at random from urn A & placed in urn B. Then one ball is drawn at random from urn B & placed in urn A. If one ball is now drawn at random from urn A, then find the probability that it is red.

2. Let p be the probability that a man aged x years will die in a year time. Then find the probability that out of 'n' men A_1, A_2, A_3,......, A_n each aged 'x' years. A_1 will die & will be the first to die.

3. A Sudoku matrix is defined as a 9 × 9 arrary with entries from {1, 2, 3 . . . . . 9} and with the constraint that each row, each column and each of the nine 3 × 3 boxes that tile the array contains each digit from 1 to 9 exactly once. A Sudoku matrix is chosen at random (so that every Sudoku matrix has equal probability of being chosen). We know two of square in this matrix as shown. Then find the probability that the square marked by ? contains the digit 3.

4. 5 girls and 10 boys sit at random in a row having 15 chairs numbered as 1 to 15, then find the probability that end seats are occupied by the girls and between any two girls an odd number of boys sit.

5. Team A plays with 5 other teams exactly once. Assuming that for each match the probabilities of a win, draw and loss are equal then find the probability that A wins and losses equal no. of matches.

6. Suppose that S be the set of all the ordered 4-tuples (x, y, z, w) of the +ve integers, which are the solutions of x + y + z + w = 21. One such ordered tuple of solution is selected at random from S. Then find the probability that x > y.

7. In a betting game in an exhibition two dice P and Q are being used. Dice P has four red faces and two white faces where as dice Q has two red and four white faces. A fair coin is tossed once. If it shows head the game continues by throwing dice P. If it falls tail dice Q is thrown. If first n throws of the die all turns up red then find the probability that P is being used.

8. On a particular day, six persons pick six different books, one each from different counters at a public library. At the closing time, they arbitrarily put their books to the vacant counters. Then find the probability that exactly two books are at their previous places.

9. A dice has one 1, two 2's and three 3's on its faces. A player throws it till he gets three consecutive 1's. If p_n is the probability that no 3 consecutive 1's appear in n throws, then prove that

(i) $p_1 = p_2 = 1$ and $p_3 = \dfrac{215}{216}$

(ii) $p_n = \dfrac{5}{216}\left[p_{n-3} + 6p_{n-2} + 36p_{n-1}\right],\ n > 3$

10. n students filled their forms for a competitive exam. Probability that exactly r students will not appear in the exam is proportional to r. If probability that out of remaining n–r students exactly i students are selected is proportional to i. Prove that the probability of exactly two students finally getting selected is

$$\frac{8}{n(n+1)}\left[n\left(\frac{1}{2}-\frac{1}{n}\right)-\left(\frac{1}{3}+\frac{1}{4}+......+\frac{1}{n}\right)\right]$$

11. In an organization number of women are μ times that of men. If n things are to be distributed among them then the probability that the number of things received by men are odd is $\left(\dfrac{1}{2}-\left(\dfrac{1}{2}\right)^{n+1}\right)$ Evaluate μ.

12. The color of a person's eyes is determined by a single pair of genes. If they are both blue eyed genes, then the person will have blue eyes ; if they are both brown -eyed genes, then the person will have brown eyes; and if one of them is a blue-eyed gene and the other a brown-eyed gene, then the person will have brown eyes. (Because of the latter fact we say that the brown-eyed gene is dominant over the blue-eyed one.) A newborn child independently receives one eye gene from each of its parents and the gene it receives from a parent is equally likely to be either of the two eye genes of that parent. Suppose that Smith and both of his parents have brown eyes, but Smith's sister has blue eyes. Suppose that Smith's wife has blue eyes. Find

(i) What is the probability that both of Smith's parents has one blue-eyed gene and one brown eyed gene?

(ii) What is the probability that Smith's possesses a blue-eyed gene ?

(iii) What is the probability that Smith's first child will have blue eyes ?

(iv) If Smith's first child has brown eyes, what is the probability that both Smith's genes are brown-eyed genes?

(v) If Smith's first child has brown-eyes, what is the probability that Smith's next child will also have brown eyes ?

13. Each square of a 3×3 board is coloured either red or blue at random (each having probability 1/2). Then find the probability that there is no 2×2 red square.

14. A fair coin is tossed $(2m + 1)$ times, then find the probability of getting at least m consecutive heads.

15. In a single throw of three dice find the probability of the event 'a total of 8.

16. Suppose A & B shoot independently until each hits his target. They have probabilities $\frac{3}{5}, \frac{5}{7}$ of hitting the target at each shoot. Find the probability that B will require more shots than A.

17. A quadratic equation is chosen by selecting two real numbers as its roots such that the quadratic equation doesn't change by squaring the numbers. Now find probability that both roots are equal.

18. Five team of equal strength play against each other in a tournament and each match either ends in a win or loss for a team. Find the probability that no team win all its games or loss all its game.

19. If A and B has $(n + 1)$ and n fair coins respectively. Then find the probability that A gets more heads then B.

20. Let the probability p_n that a family has exactly n children be αp^n, where $n \geq 1$ and $p_0 = 1 - \alpha p(1 + p + p^2 + ...)$ $(0 < \alpha, p < 1)$. Giving birth to a boy and girl is equally likely. If $k \geq 1$, then find the probability that the family has exactly k boys.

21. A bear hides itself either behind bush A with probability $\frac{9}{25}$ or behind bush B with probability $\frac{16}{25}$. A hunter have 5 bullets each of which can be fired either at bush A or B. Hunter hits each target independtly with an accuracy of 1/4. How many bullets can be fired at bush A to hit the bear with max. probability.

22. Set A : {randomly choosen 100 years in which 76 are simple and 24 are leap years}
Set B : {randomly choosen 100 years in which 75 are simple and 25 are leap years}
An year is chosen from either set A or set B and is found to have 53 sundays. Probability that the chosen year was a leap year.

23. In ten trials of an experiment, if the probability of getting '4 successes' is maximum, then find the range of probability of success in each trial.

24. There are two lots of identical articles with different amounts of standard & defective articles. There are N articles in the first lot, n of which are defective & M articles in the second lot, m of which are defective. K articles are selected from the first lot & L articles from the second & a new lot results. Find the probability that an article selected at random from the new lot is defective.

25. Find the chance of throwing 10 exactly in one throw with 3 dice.

26. Two players of equal skill, A and B, are playing a set of games; they leave off playing when A gets 3 points and B gets 2 (in each game winner get one point). If the stake is Rs.1600, what share ought each to take?

27. A family has three children. Event 'A' is that family has at most one boy, Event 'B' is that family has at least one boy and one girl, Event 'C' is that the family has at most one girl. Find whether events 'A' and 'B' are independent. Also find whether A, B, C are independent or not.

28. A line segment of length a is divided in two parts at random by taking a point on it, find the probability that no part is greater than b, where 2b > a

29. Two ants are on the opposite corners of a grid of size 8 × 8 if they move then what is the probability that they will meet after each travelled eight steps (Assuming that they do not move in backward direction)

33. **Match :** A box contains n coins. Let $P(E_i)$ be the probability that exactly i out of n coins are biased. If $P(E_i)$ is directly proportional to $i(i+1)$; $1 \le i \le n$.

(A)	Proportionality constant k is equal to	(p)	$\frac{(3n+1)}{4n}$
(B)	If P be the probability that a coin selected at random is biased then P is	(q)	$\frac{3}{n(n+1)(n+2)}$
(C)	If a coin selected at random is found to be biased then the probability that it is the only biased coin in the box is	(r)	$\frac{3}{n+2}$
(D)	$P(E_n)$ is equal to	(s)	$\frac{24}{n(n+1)(n+2)(3n+1)}$

Answers

1. $\frac{32}{55}$ **2.** $\frac{1-(1-p)^n}{n}$ **3.** $\frac{2}{21}$ **4.** $\frac{20\times10!\times5!}{15!}$ **5.** $\frac{17}{81}$ **6.** $\frac{35}{76}$

7. $\frac{2^n}{2^n+1}$ **8.** 3/16 **11.** 3 **12.** (i) 1 (ii) 2/3 (iii) 1/3 (iv) 1/2 (v) 2/3

13. $\frac{417}{512}$ **14.** $\frac{(m+3)2^m-1}{2^{2m+1}}$ **15.** 7/72 **16.** $\frac{6}{31}$ **17.** $\frac{1}{2}$ **18.** $\frac{17}{32}$

19. ½ **20.** $\frac{2\alpha p^k}{(2-p)^{k+1}}$ **21.** 1, 2 **22.** $\frac{98}{249}$ **23.** $p \in \left[\frac{4}{11},\frac{5}{11}\right]$

24. $\frac{KnM + LmN}{MN(K+L)}$ **25.** 1/8 **26.** 500 Rs. & 1100 Rs.

27. A and B are independent but A,B,C are not independent **28.** $\frac{2b-a}{a}$ **29.** $\frac{^{16}C_8}{2^{16}}$

30. $\frac{1}{256}$ **31.** $\frac{1}{256}$ **32.** $\frac{9}{64}$ **33.** A–q ; B–p, C–s, d–r